Correlation of normal neutrino mass ordering with upper octant of \( \theta_{23} \) and third quadrant of \( \delta \) via RGE-induced \( \mu-\tau \) symmetry breaking

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Abstract: The recent global analysis of three-flavor neutrino oscillation data indicates that the normal neutrino mass ordering is favored over the inverted one at the 3\( \sigma \) level, and the best-fit values of the largest neutrino mixing angle \( \theta_{23} \) and the Dirac CP-violating phase \( \delta \) are located in the higher octant and third quadrant, respectively. We show that all these important issues can be naturally explained by the \( \mu-\tau \) reflection symmetry breaking of massive neutrinos from a superhigh energy scale down to the electroweak scale owing to the one-loop renormalization-group equations (RGEs) in the minimal supersymmetric standard model (MSSM). The complete parameter space is explored for the first time in both the Majorana and Dirac cases, by allowing the smallest neutrino mass \( m_1 \) and the MSSM parameter \( \tan\beta \) to vary within their reasonable regions.

Keywords: renormalization group equations, \( \mu-\tau \) reflection symmetry, lepton flavor mixing

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1 Introduction

The striking phenomena of solar, atmospheric, reactor and accelerator neutrino oscillations have all been observed in the past twenty years [1], demonstrating that the standard model (SM) of particle physics is by no means complete, and it must be extended to explain both the origin of tiny but finite neutrino masses and of large lepton flavor mixing effects. Qualitatively, the smallness of neutrino masses might be attributed to the existence of some heavy degrees of freedom at a superhigh energy scale: a popular idea known as the seesaw mechanism [2]. The largeness of neutrino mixing angles and CP-violating phases might originate from an underlying flavor symmetry [3, 4], which should also manifest itself at a superhigh energy scale. Thus, a combination of the seesaw and flavor symmetry conjectures is the most likely phenomenological approach to understanding what is behind the observed spectrum of neutrino masses and the observed pattern of lepton flavor mixing. In this case, the renormalization-group equations (RGEs) are imperative to bridge the gap between the (theoretically suggestive) superhigh and (experimentally measurable) low energy scales [5, 6].

Because the oscillation experiments are insensitive to the Dirac or Majorana nature of massive neutrinos, one may describe the link between the three known neutrinos (\( \nu_e, \nu_\mu, \) and \( \nu_\tau \)) and their mass eigenstates (\( \nu_1, \nu_2, \) and \( \nu_3 \)) in terms of a \( 3 \times 3 \) unitary matrix, namely the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [7]

\[
V = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23}c_{13} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\
s_{13}s_{23} - c_{13}s_{12}c_{23}e^{i\delta} & -c_{13}s_{23} - s_{13}s_{12}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix},
\]

(1)

with \( c_{ij} \equiv \cos\theta_{ij} \) and \( s_{ij} \equiv \sin\theta_{ij} \) (for \( ij = 12, 13, 23 \)). A global analysis of currently available data on neutrino oscillations indicates that the normal neutrino mass ordering \( (m_1 < m_2 < m_3) \) is favored over the inverted one \( (m_3 < m_1 < m_2) \) at the 3\( \sigma \) level\(^4\). Furthermore, the best-fit value of the largest neutrino mixing angle \( \theta_{23} \)

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\(^{4}\) We admit that the inverted neutrino mass ordering is currently still allowed at the 2\( \sigma \) confidence level, but here we give preference to the normal ordering.

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RGE-induced quadrant can naturally be correlated and explained via \( \Lambda \) sive neutrinos; that is, via the one-loop RGE evolution normal neutrino mass ordering, the slightly higher octant of model building. In this study, we show that the investigate their implications independently of any de- global-fit results of neutrino oscillations seriously, and to particle physics vant experimental data often points to the truth in par- strength of leptonic CP violation.

Although the same topic has been discussed previ- ously, the present study differs from previous works at least in the following aspects:

- Based on more reliable experimental data, espe- cially the 3\( \sigma \) indication of the normal neutrino mass ordering, our work is the first to numerically ex- plore the almost complete parameter space in both the Majorana and Dirac cases, by allowing the smallest neutrino mass \( m_1 \) and the MSSM parameter \( \tan \beta \) to vary within their reasonable regions. In comparison, previous work such as Ref. [6] has only estimated the RGE correction to \( \theta_{23} \) by assuming that \( \theta_{13} = 0 \). This is an approximation that has now been excluded. Such work also did not con- sider all four distinct cases of the Majorana phases in the \( \mu-\tau \) reflection symmetry limit, nor the RGE correction to \( \delta \).

- While all previous studies have simply assumed some special values of \( m_1 \) and \( \tan \beta \) to “illustrate” a possible correlation of the neutrino mass order- ing with the octant of \( \theta_{23} \) and the quadrant of \( \delta \) based on the RGE-induced \( \mu-\tau \) reflection symme- try breaking, our present work has made remark- able progress in “figuring out” which part of the parameter space is favored by current neutrino oscil- lation data, and which part is unfavored or ruled out. The outcome of our in-depth analysis is there- fore more up-to-date, suggestive, and useful for model building.

- Our statistical analysis shows that currently the best-fit points of \( \theta_{23} \) and \( \delta \) [8, 9] can be explained by the \( \mu-\tau \) reflection symmetry breaking induced by the RGE running from \( \Lambda_{\mu} \) down to \( \Lambda_{EW} \), but this simple flavor symmetry itself is in slight dis- agreement (at the 1\( \sigma \) confidence level) with the low-energy data. Although the best-fit points may shift when new data becomes available, the analy- sis method itself will remain useful.

- The upcoming precision measurement of \( \theta_{23} \) and an experimental determination of \( \delta \) will test the considered scenario and help to locate the correct regions in parameter space for the smallest neutrino mass \( m_1 \), the MSSM parameter \( \tan \beta \), and even the Majorana phases. No previous approaches were able to achieve this.

The rest of the paper is organized as follows. In Section 2 we present the main analytical results of the \( \mu-\tau \) reflection symmetry breaking caused by the RGE run- ning effects. In Section 3 the parameter space of \( \tan \beta \) and \( m_1 \) is extensively explored and constrained with the help of current experimental data. Section 4 presents a summary and some concluding remarks.

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1) As in most literature, in this case the so-called \( \mu-\tau \) reflection symmetry actually means the \( \nu_{\mu}\nu_{\tau} \) reflection symmetry in the neutrino sector. One may refer to the discussions above Eq. (2) or Eq. (9) in Section 2, and to Refs. [10, 11] for the building of specific models that realize this interesting discrete flavor symmetry without involving charged leptons.

2) One successful example of this kind was the global-fit “prediction” for an unsuppressed value of \( \theta_{13} \), made in 2008 [18], which proved to be essentially true after the direct measurement of \( \theta_{13} \) was reported in 2012 [19]. This work has recently been recognized by the prestigious Bruno Pontecorvo Prize.
2 RGE-induced $\mu$-$\tau$ reflection symmetry breaking

2.1 The Majorana case

Let us assume that the tiny masses of the three known neutrinos originate from a viable seesaw mechanism at a superhigh energy scale $\Lambda_{\mu\tau} \sim 10^{14}$ GeV. Without loss of generality, we choose the basis in which the mass eigenstates of the three charged leptons are identical with their flavor eigenstates. In this case, only the neutrino sector is responsible for lepton flavor mixing and CP violation. If the effective Majorana neutrino mass term is invariant under the charge-conjugation transformations $\nu_{\alpha L} \leftrightarrow \nu_{\alpha R}^{c}, \nu_{\mu L} \leftrightarrow \nu_{\mu R}^{c}$ and $\nu_{\tau L} \leftrightarrow \nu_{\tau R}^{c}$, then the corresponding mass matrix must take the form

$$M_\nu \equiv \begin{pmatrix} 
\langle m \rangle_{ee} & \langle m \rangle_{en} & \langle m \rangle_{\tau e} \\
\langle m \rangle_{en} & \langle m \rangle_{en} & \langle m \rangle_{\tau e} \\
\langle m \rangle_{\tau e} & \langle m \rangle_{\tau e} & \langle m \rangle_{\tau e} 
\end{pmatrix}$$

(2)

$$I_0 = \exp \left[ \frac{1}{16\pi^2} \int_{0}^{\ln(\Lambda_{\mu\tau}/\Lambda_{EW})} y_\mu^2(\chi) \, d\chi \right],$$

$$I_\alpha = \exp \left[ -\frac{1}{16\pi^2} \int_{0}^{\ln(\Lambda_{\mu\tau}/\Lambda_{EW})} y_{\alpha}^2(\chi) \, d\chi \right].$$

Here, $\chi = \ln(\mu/\Lambda_{\mu\tau})$, where $\mu$ is an arbitrary renormalization scale between $\Lambda_{EW}$ and $\Lambda_{\mu\tau}, g_1$ and $g_2$ denote the gauge couplings, and $y_\mu$ and $y_\alpha$ (for $\alpha = e, \mu, \tau$) denote the Yukawa coupling eigenvalues of the top quark and charged leptons, respectively. The smallness of $y_\mu$ and $y_\alpha$ assures that $I_\alpha \approx I_\mu \approx 1$ holds to an excellent degree of accuracy. Note that

$$\Delta_\alpha \equiv 1 - I_\alpha \approx \frac{1}{16\pi^2} \int_{0}^{\ln(\Lambda_{\mu\tau}/\Lambda_{EW})} y_\alpha^2(\chi) \, d\chi$$

(5)

is also a fairly small quantity, but may affect the running behaviors of the flavor mixing parameters in an appreciable manner [20]. To illustrate, Fig. 1 presents the two-dimensional maps of $\Delta_\alpha$ (left panel) and $I_0$ (right panel) versus the variables $\Lambda_{\mu\tau}$ and $\tan\beta$. One can observe that $I_0$ does not change significantly with different settings of $\Lambda_{\mu\tau}$ and $\tan\beta$. In comparison, $\Delta_\alpha$ can change from 0.001 to 0.05. Note that shifting the energy scale is equivalent to altering $\tan\beta$, and the outputs at $\Lambda_{\mu\tau}=10^{14}$ GeV and $\Lambda_{\mu\tau}=10^{16}$ GeV are reasonably similar in magnitude. If we shift the energy scale from $\Lambda_{\mu\tau}=10^{14}$ GeV to $\Lambda_{\mu\tau}=10^{16}$ GeV, then $\Delta_\alpha$ will lie in the range (0.001,0.05) instead of (0.001,0.05). In the following numerical calculations, we shall fix $\Lambda_{\mu\tau} \sim 10^{14}$ GeV as the $\mu$-$\tau$ flavor symmetry scale.

One may diagonalize the neutrino mass matrix at $\Lambda_{EW}$, and then obtain the mass eigenvalues $(m_1, m_2, m_3)$, flavor mixing angles ($\theta_{13}, \theta_{12}, \theta_{23}$), and CP-violating phases ($\delta, \rho, \sigma$). Here, we define $\Delta\theta_{ij} \equiv \theta_{ij}(\Lambda_{EW}) - \theta_{ij}(\Lambda_{\mu\tau})$ for $ij = 12, 13, 23$, $\Delta\delta \equiv \delta(\Lambda_{EW}) - \delta(\Lambda_{\mu\tau})$, $\Delta\rho \equiv \rho(\Lambda_{EW}) - \rho(\Lambda_{\mu\tau})$, and $\Delta\sigma \equiv \sigma(\Lambda_{EW}) - \sigma(\Lambda_{\mu\tau})$ to measure the strengths of the RGE-induced corrections to the parameters of $U$. As a good approximation, the three neutrino masses at $\Lambda_{EW}$ are found to be

$$m_1(\Lambda_{EW}) \approx I_0^1 \left[ 1 - \Delta\theta_{23}(1-c^2_{13}c^2_{12}) \right] m_1(\Lambda_{\mu\tau}),$$

$$m_2(\Lambda_{EW}) \approx I_0^2 \left[ 1 - \Delta\theta_{13}(1-c^2_{13}c^2_{12}) \right] m_2(\Lambda_{\mu\tau}),$$

$$m_3(\Lambda_{EW}) \approx I_0^3 \left[ 1 - \Delta\theta_{12}(c^2_{13}) \right] m_3(\Lambda_{\mu\tau}),$$

(6)

where $\theta_{12}$ and $\theta_{13}$ take their values at $\Lambda_{EW}$. Unless otherwise specified, the nine physical flavor parameters appearing in the subsequent text and equations are all those at $\Lambda_{EW}$. In a reasonable analytical approximation, we can also arrive at

$$\Delta\theta_{12} \approx \Delta\theta_{12} \approx \frac{\Delta}{2} c_{13}s_{12} \left( \frac{c^2_{13}}{\Delta\theta_{13}} - \frac{s^2_{13}c^2_{12}}{\Delta\theta_{12}} \right) \left[ c_{13}^2 - n_{\eta_{12}} \right].$$

1) Note that our analytical results are not exactly the same as those obtained in Ref. [15], where a different phase convention for the PMNS matrix has been adopted.
\[ \Delta \theta_{13} \approx \frac{\Delta \tau}{2} \left( c_{12}^2 s_{13} \left( s_{23}^2 + c_{23}^2 \right) \right) , \]
\[ \Delta \theta_{23} \approx \frac{\Delta \tau}{2} \left( s_{12}^2 c_{23} - s_{12} c_{23} \right) \]

for the deviations of the three flavor mixing angles between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\mu\tau} \), and

\[ \Delta \delta \approx \frac{\Delta \tau}{2} \left[ c_{12}^2 s_{13} \left( s_{23}^2 - c_{23}^2 \right) \right] \]
\[ \Delta \rho \approx \frac{\Delta \tau}{s_{12}} \left[ c_{12}^2 s_{13} \left( s_{23}^2 - c_{23}^2 \right) \right] \]
\[ \Delta \sigma \approx \frac{\Delta \tau}{2 c_{12}} \left[ c_{12}^2 s_{13} \left( s_{23}^2 - c_{23}^2 \right) \right] \]

for the deviations of the three CP-violating phases between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\mu\tau} \), where \( \eta_\rho \equiv \cos 2\rho = \pm 1 \) and \( \eta_\sigma \equiv \cos 2\sigma \). The possible options for \( \rho \) and \( \sigma \) in their \( \mu-\tau \) symmetry limit at \( \Lambda_{\mu\tau} \), and the ratios \( \xi_{ij} \equiv (m_j - m_i)/(m_i + m_j) \) are defined with \( m_i \) and \( m_j \) at \( \Lambda_{\text{EW}} \) (for \( i, j = 1, 2, 3 \)). In obtaining Eqs. (6)–(8) the \( \mu-\tau \) reflection symmetry conditions \( \theta_{23}(\Lambda_{\mu\tau}) = 45^\circ \) and \( \delta(\Lambda_{\mu\tau}) = 270^\circ \) have also been applied.

2.2 The Dirac Case

Although the Majorana nature of massive neutrinos is well motivated from a theoretical viewpoint, it is also interesting to consider the possibility of a pure Dirac mass term for the three known neutrinos, and combine this with a certain flavor symmetry that can be realized at a superhigh energy scale \( \Lambda_{\mu\tau} \). In this case, the \( \mu-\tau \) reflection symmetry means that the Dirac neutrino mass term is invariant under the charge-conjugation transformations \( \nu_{\mu L} \leftrightarrow (\nu_{\mu L})^c \), \( \nu_{\tau L} \leftrightarrow (\nu_{\tau L})^c \), and \( \nu_{\mu L} \leftrightarrow (\nu_{\mu L})^c \) for the left-handed neutrino fields and \( N_{\mu R} \leftrightarrow (N_{\mu R})^c \), \( N_{\tau R} \leftrightarrow (N_{\tau R})^c \) for the right-handed neutrino fields. The resultant Dirac neutrino mass matrix is

\[ M_\nu \equiv \begin{pmatrix} \langle m\rangle_{ee} & \langle m\rangle_{e\mu} & \langle m\rangle_{e\tau} \\ \langle m\rangle_{\mu e} & \langle m\rangle_{\mu\mu} & \langle m\rangle_{\mu\tau} \\ \langle m\rangle_{\tau e} & \langle m\rangle_{\tau\mu} & \langle m\rangle_{\tau\tau} \end{pmatrix} , \]

and is constrained by \( \langle m\rangle_{ee} = \langle m\rangle_{e\mu} = \langle m\rangle_{e\tau} \), \( \langle m\rangle_{\mu e} = \langle m\rangle_{\mu\mu} = \langle m\rangle_{\mu\tau} \), \( \langle m\rangle_{\tau e} = \langle m\rangle_{\tau\mu} = \langle m\rangle_{\tau\tau} \) [17]. One can observe that in general \( M_\nu \) is neither symmetric nor Hermitian, even though it possesses the \( \mu-\tau \) reflection symmetry. However, one may diagonalize the Hermitian combination \( M_\nu M_\nu^T \) by means of the unitary transformation \( U' M_\nu U' = \text{Diag}(m_1^2, m_2^2, m_3^2) \), where \( U' = P_V \), with \( P_V = \text{Diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \) being an unphysical phase matrix and \( V \) taking the form given in Eq. (1). Then, the \( \mu-\tau \) reflection symmetry constraints on \( M_\nu \) naturally lead us to the constraints on the PMNS matrix \( U' \); namely, \( \theta_{23} = 45^\circ \), \( \delta = 90^\circ \) or \( 270^\circ \), and \( 2\phi_1 - \phi_2 - \phi_3 = 180^\circ \) at \( \Lambda_{\mu\tau} \). Given the global-fit preference for \( \sin \delta < 0 \) [8], we focus only on the possibility that \( \delta(\Lambda_{\mu\tau}) = 270^\circ \).

In the MSSM, the evolution of the Dirac neutrino mass matrix \( M_\nu \) from \( \Lambda_{\mu\tau} \) down to \( \Lambda_{\text{EW}} \) via the one-loop RGE can be described as [17]

\[ M_\nu(\Lambda_{\text{EW}}) = I_0 \left[ T_i M_\nu(\Lambda_{\mu\tau}) \right] , \]

where the definitions of \( I_0 \) and \( T_i \) are the same as those in Eqs. (3) and (4), respectively. Diagonalizing \( M_\nu(\Lambda_{\text{EW}}) \) will yield the seven physical flavor parameters at \( \Lambda_{\text{EW}} \). Utilizing the same notations as in the Majorana case, we...
summarize our approximate analytical results as follows:

\[ m_1(\Lambda_{\text{EW}}) \approx f_0 \left[ 1 - \frac{1}{2} \Delta \nu, (1 - c_{12}^2 c_{13}^2) \right] m_1(\Lambda_{\nu}), \]

\[ m_2(\Lambda_{\text{EW}}) \approx f_0 \left[ 1 - \frac{1}{2} \Delta \nu, (1 - s_{12}^2 c_{13}^2) \right] m_2(\Lambda_{\nu}), \]

\[ m_3(\Lambda_{\text{EW}}) \approx f_0 \left[ 1 - \frac{1}{2} \Delta \nu, c_{13}^2 \right] m_3(\Lambda_{\nu}) \quad (11) \]

for the masses of the three neutrinos,

\[ \Delta \theta_{12} \simeq \frac{\Delta}{4} s_{12} c_{12} \left[ s_{13}^2 (\zeta_{12} + \zeta_{21}^2) - s_{13}^2 (\zeta_{12} + \zeta_{21}^2 - \zeta_{32} - \zeta_{31}^2) \right], \]

\[ \Delta \theta_{13} \simeq \frac{\Delta}{4} s_{13} c_{13} \left[ s_{13}^2 (\zeta_{13} + \zeta_{31}^2) + c_{12}^2 (\zeta_{13} + \zeta_{32}^2) \right], \]

\[ \Delta \theta_{23} \simeq \frac{\Delta}{4} [ c_{12} (\zeta_{23}^2 + \zeta_{13}^2) + s_{12} (\zeta_{23}^2 + \zeta_{13}^2) ] \quad (12) \]

for the deviations of the three flavor mixing angles between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\nu} \), and

\[ \Delta \delta \simeq \frac{\Delta}{4} \left[ c_{12} \left( s_{12}^2 - s_{12}^2 c_{13}^2 \right) (\zeta_{13} + \zeta_{32}^2) \right] \]

\[ - \frac{s_{12} \left( c_{12}^2 - s_{12}^2 c_{13}^2 \right)}{c_{12} s_{13}^2} (\zeta_{32} + \zeta_{31}^2) \]

\[ - \frac{s_{13} \left( c_{12}^2 - s_{12}^2 c_{13}^2 \right)}{c_{12} s_{13}^2} (\zeta_{23} + \zeta_{31}^2) \] \quad (13)

for the deviation of \( \delta \) between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\nu} \). In obtaining Eqs. (11)–(13), we have taken into account the \( \mu-\tau \) reflection symmetry conditions \( \theta_{23}(\Lambda_{\nu}) = 45^\circ \) and \( \delta(\Lambda_{\nu}) = 270^\circ \).

The analytical approximations presented in Eqs. (7) and (8) for Majorana neutrinos or in Eqs. (12) and (13) for Dirac neutrinos are instructive, and helpful for understanding the RGE corrections to relevant flavor mixing and CP-violating parameters. However, the accuracy will be rather poor if the neutrino masses are strongly degenerate. In the subsequent section, we shall numerically evaluate the effects of \( \mu-\tau \) symmetry breaking, and explore the allowed parameter space to fit current experimental data.

## 3 Numerical exploration of the parameter space

In the framework of the MSSM, we numerically run the RGEs from \( \Lambda_{\nu} \sim 10^{14} \) GeV down to \( \Lambda_{\text{EW}} \sim 10^2 \) GeV by taking into account the initial conditions \( \theta_{23} = 45^\circ \) and \( \delta = 270^\circ \), as well as the initial values of \( \rho \) and \( \sigma \) in the following four different cases: Case A \( \rho = \sigma = 0^\circ \), Case B \( \rho = 90^\circ \), Case C \( \rho = 0^\circ \) and \( \sigma = 90^\circ \), and Case D \( \rho = 90^\circ \) and \( \sigma = 0^\circ \). For any given values of the MSSM parameter \( \tan \beta \) and the smallest neutrino mass \( m_1 \) at \( \Lambda_{\text{EW}} \), the other relevant neutrino oscillation parameters, such as \( \{ \sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m^2_{\text{sol}}, \Delta m^2_{\text{atm}} \} \) at \( \Lambda_{\nu} \), are scanned over suitably wide ranges with the help of the MultiNest program [21]. Here, we have adopted the notations \( \Delta m^2_{\text{sol}} \equiv m^2_2 - m^2_1 \) and \( \Delta m^2_{\text{atm}} \equiv m^2_3 - (m^2_1 + m^2_2)/2 \), as defined in Ref. [8] \( 1 \).

For each scan, the neutrino flavor parameters at \( \Lambda_{\text{EW}} \) are obtained, and these are immediately compared with their global-fit values by minimizing

\[ \chi^2 \equiv \sum_i \frac{(\xi_i - \bar{\xi}_i)^2}{\sigma_i^2}, \quad (14) \]

where the \( \xi_i \) terms denote the parameters at \( \Lambda_{\text{EW}} \) produced from the RGE evolution, the \( \bar{\xi}_i \) represent the best-fit values of the global analysis, and the \( \sigma_i \) are the corresponding symmetrized 1\( \sigma \) errors (i.e., \( \sigma_i = (\sigma_i^+ + \sigma_i^-)/2 \)).

In our numerical calculations, we concentrated on the normal neutrino mass ordering \( 2 \) and adopt the best-fit values and 1\( \sigma \)-level deviations of the six neutrino oscillation parameters \( 8 \):

\[ \sin^2 \theta_{12} = 3.04^{+0.14}_{-0.13} \times 10^{-1}, \]

\[ \sin^2 \theta_{13} = 2.14^{+0.09}_{-0.09} \times 10^{-2}, \]

\[ \sin^2 \theta_{23} = 5.11^{+0.10}_{-0.07} \times 10^{-1}, \]

\[ \delta = 1.32^{+0.23}_{-0.18} \pi, \]

\[ \Delta m^2_{\text{sol}} = 7.34^{+0.17}_{-0.14} \times 10^{-5} \text{ eV}^2, \]

\[ \Delta m^2_{\text{atm}} = 2.455^{+0.030}_{-0.032} \times 10^{-3} \text{ eV}^2. \]

In this case, the smallest neutrino mass \( m_1 \) is allowed to take values in the range [0, 0.1] eV, and the MSSM parameter \( \tan \beta \) may vary from 10 to 50 based on a reasonable phenomenological argument \( 3 \). It should be pointed out that our numerical results are independent of the analytical approximations obtained in the previous section, but the latter will be helpful for understanding some salient features of the former.

### 3.1 The Majorana case

The strategy of our numerical analysis is rather straightforward. Let us first examine how significantly \( \theta_{23} \) and \( \delta \) at \( \Lambda_{\text{EW}} \) can deviate from their initial values at \( \Lambda_{\nu} \), incorporating the recent global-fit results. To this end, we only need to take into account the

1) In the normal neutrino mass ordering case, one may therefore express \( m_2 \) and \( m_3 \) in terms of \( m_1 \) as \( m_2 = \sqrt{m_1^2 + \Delta m^2_{\text{sol}}} \) and \( m_3 = \sqrt{m_1^2 + 0.5 \Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}} \).

2) This is not only our phenomenological preference but also a recent 3\( \sigma \) global-fit indication that the true neutrino mass spectrum should exhibit a normal ordering \( m_1 < m_2 < m_3 \).

3) Note that \( \tan \beta > 50 \) is unfavored, because the heavy-quark Yukawa couplings would fall into the non-perturbative region, while for \( \tan \beta < 10 \) the RGE-induced corrections to the relevant neutrino parameters are negligibly small, and thus less interesting for our purpose.
global-fit information concerning the parameter set \( \xi = \{ \sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m_{sol}^2, \Delta m_{atm}^2 \} \). One can observe that the RGE running effects always push \( \theta_{23} \) to the higher octant, and in most cases lead \( \delta \) to the third quadrant. This is the right direction as indicated by the best-fit values of these two quantities [8]. After this preliminary diagnosis is reached, the experimental information on \( \theta_{23} \) and \( \delta \) will be included in order to evaluate the statistical compatibility between the RGE-triggered \( \mu-\tau \) reflection symmetry breaking and the global-fit values of \( \theta_{23} \) and \( \delta \).

For each given value of \( m_1 \) or \( \tan \beta \) at \( \Lambda_{EW} \), we obtain the associated \( \chi_2^2 \) from Eq. (14), which is minimized over the chosen parameter set \( \xi \). In Fig. 2, we illustrate \( \chi_2^2 \) with respect to the neutrino mass \( m_1 \) for Case C, (i.e., \( \rho = 0^\circ \) and \( \sigma = 90^\circ \) at \( \Lambda_{\mu\tau} \)), where \( \tan \beta = 50 \) has been typically input.

For each given value of \( m_1 \) or \( \tan \beta \) at \( \Lambda_{EW} \), we obtain the associated \( \chi_2^2 \) from Eq. (14), which is minimized over the chosen parameter set \( \xi \). In Fig. 2, we illustrate \( \chi_2^2 \) with respect to the neutrino mass \( m_1 \) for Case C, (i.e., \( \rho = 0^\circ \) and \( \sigma = 90^\circ \) at \( \Lambda_{\mu\tau} \)), where \( \tan \beta = 50 \) has been typically input.

The RGE correction to \( \theta_{23} \) illustrated in Fig. 3 can be effectively understood with the help of the analytical approximations obtained in Eq. (7) if the neutrino masses are not too degenerate and the RGE evolution is not too strong. Eq. (7) tells us that the sign of \( \Delta \theta_{23} \) is positive, because \( \Delta \), and \( \zeta_{31} \approx \zeta_{32} \) are all positive for the normal neutrino.
mass ordering. The factor $\Delta_\tau$ is essentially proportional to $\tan^2 \beta$, because $y_\tau^2 \propto (1 + \tan^2 \beta) \approx \tan^2 \beta$ for $\tan \beta \gtrsim 10$, and therefore $\Delta \theta_{23}$ always increases with $\tan^2 \beta$. On the other hand, the dependence of $\theta_{23}$ on the neutrino mass $m_1$ is different for the four options of $\rho$ and $\sigma$ at $\Lambda_{\mu\tau}$. For example, $\zeta_{31}^{\|}$ is proportional to $m_1$ for $\rho(\Lambda_{\mu\tau}) = 0^\circ$, but is proportional to $1/m_1$ when $\rho(\Lambda_{\mu\tau}) = 90^\circ$. In the region of small $m_1$ and $\tan \beta$, the radiative correction to $\theta_{23}$ is proportional to $m_1$ for Cases A, C, and D, but is inversely proportional to $m_1$ in Case B with $\eta_\rho = \eta_\sigma = -1$.

In Fig. 4, we plot the allowed region of $\delta$ at $\Lambda_{\text{EW}}$. Note that for each point in the $m_1$-$\tan \beta$ plane, $\delta$ and $\theta_{23}$ are determined concurrently. Some remarks are in order.

- The RGE-induced corrections to $\delta$ in Cases A and B are very weak, being only approximately $0.5^\circ$. Even though the higher octant of $\theta_{23}$ (including its best-fit value) can easily be reached in Case A, it is impossible to approach the best-fit value of $\delta$ (i.e., $\delta \approx 238^\circ$). However, the best-fit value of $\delta$ can be reached in both Cases C and D. Case D warrants highlighting, where the best-fit point $(\theta_{23}, \delta) \approx (48^\circ, 238^\circ)$ is reachable from the same settings of $m_1$ and $\tan \beta$.

- Similar to the case of $\theta_{23}$, the radiative correction to $\delta$ is also proportional to $\tan^2 \beta$ regardless of its sign. However, the dependence of $\delta$ on $m_1$ in Eq. (8) is not as straightforward as that of $\theta_{23}$ in Eq. (7). There are two terms in $\Delta \delta$, with one enhanced by $1/\sin \theta_{13}$ and the other suppressed by $\sin \theta_{13}$, but the latter can become dominant in some cases. In Case A, the first term $\propto 1/\sin \theta_{13}$ is positive and dominant when the neutrino mass $m_1$ is relatively small, while the second term $\propto \sin \theta_{13}$ is negative, and will gradually dominate when the value of $m_1$.

Fig. 3. (color online) The allowed region of $\theta_{23}$ at $\Lambda_{\text{EW}}$ owing to the RGE-induced $\mu$-$\tau$ reflection symmetry breaking, where the dashed curves show the contours for some typical values of $\theta_{23}$, and the blue one is compatible with the best-fit result of $\theta_{23}$ obtained in Ref. [8].
increases. These analytical features can explain the numerical evolution behavior of \( \delta \) for Case A shown in Fig. 4. In Case B with \( \eta_\rho = \eta_\sigma = -1 \), both terms of \( \Delta \delta \) are negative and inversely proportional to \( m_1 \). Note that the first term of \( \Delta \delta \) in either Case A or B is suppressed, owing to the cancellation between \( \zeta_{32}^{\nu_\tau} \) and \( \zeta_{31}^{\nu_\tau} \), and this largely explains the smallness of \( \Delta \delta \). In Case C (or Case D), the first term of \( \Delta \delta \) is negative (or positive) and initially dominant, but the second term containing \( \zeta_{21}^{\nu_\mu} = \zeta_{21}^{\nu_\tau} \) will eventually dominate for relatively large values of \( m_1 \). Hence, the RGE-induced corrections to \( \delta \) in these two cases can be considerably enhanced by \( 1/\Delta m^2_{\text{sol}} \).

To see the correlation between \( \theta_{23} \) and \( \delta \) at \( \Lambda_{\text{EW}} \), let us marginalize \( m_1 \) and \( \tan \beta \) over the reasonable ranges \( m_1 \in [0, 0.1] \) eV and \( \tan \beta \in [10, 50] \). Our numerical outputs are summarized in Fig. 5, where the recent global-fit results [8] are plotted as black lines with the \( 1\sigma \) (dashed), \( 2\sigma \) (solid), and \( 3\sigma \) (dotted) contours. The corresponding best-fit point of each analysis is marked as a black star. We note that the \( \mu_\tau \) reflection symmetry point \( (\theta_{23}, \delta) = (45^\circ, 270^\circ) \) at \( \Lambda_{\text{EW}} \), which is marked by the red circled cross in the plot, lies on the dashed contour. This means that \( \theta_{23}(\Lambda_{\text{EW}}) = 45^\circ \) and \( \delta(\Lambda_{\text{EW}}) = 270^\circ \) are statistically unfavored at the \( 1\sigma \) level [8]. The \( \theta_{23} \)–\( \delta \) correlation at low energies, which arises from RGE-induced \( \mu_\tau \) symmetry breaking, is described by the pink or green region. In the pink region, the best-fit values of \( \{\sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m^2_{\text{sol}}, \Delta m^2_{\text{atm}}\} \) can be simultaneously reached (i.e., \( \chi^2_{\text{min}} = 0 \)). If the value of \( \chi^2_{\text{min}} \) is relaxed to 11.83 (i.e., the \( 3\sigma \) confidence level for two degrees of freedom), then the wider green region for \( \theta_{23} \) and \( \delta \) will be allowed. In the two upper panels of Fig. 5, which correspond to Cases A and B, the allowed range of \( \delta \) is very narrow. This feature is compatible with the two upper panels of Fig. 4, where \( \delta \) varies less than \( 1^\circ \). In these
Fig. 5. (color online) The correlation between $\theta_{23}$ and $\delta$ at $\Lambda_{EW}$ compared with the recent global-fit results (abbreviated as “CLMP”) [8], where $m_1$ and $\tan \beta$ have been marginalized over $[0, 0.1]$ eV and $[10, 50]$, respectively, and the red circled cross denotes the point $(\theta_{23}, \delta) = (45^\circ, 270^\circ)$. The pink region is allowed for $\theta_{23}$ and $\delta$ when $\{\sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m_{sol}^2, \Delta m_{atm}^2\}$ at $\Lambda_{EW}$ take their best-fit values, and the green region is allowed when these four observables deviate from their best-fit values by a $3\sigma$ level (i.e., $\chi^2 = 11.83$ for two degrees of freedom).

To numerically verify the compatibility between our $\mu$-$\tau$ symmetry breaking scenario and the current experimental data, or how well the RGE-triggered $\mu$-$\tau$ reflection symmetry breaking effect can fit the complete set of data at $\Lambda_{EW}$, now we include the global-fit information on $\theta_{23}$ and $\delta$. Namely, we define $\tilde{\chi}^2 = \chi^2 + \chi^2_{\theta_{23}} + \chi^2_{\delta}$, where $\chi^2$ is formed with the parameter set $\{\sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m_{sol}^2, \Delta m_{atm}^2\}$ as before, and $\chi^2_{\theta_{23}}$ and $\chi^2_{\delta}$ are the contributions from $\theta_{23}$ and $\delta$, respectively. In this case, the minimum of $\tilde{\chi}^2$ can be calculated for each of the four cases of $\rho$ and $\sigma$ at $\Lambda_{\mu\tau}$ by marginalizing the relevant quantities over $\tan \beta \in [10, 50]$, and the result is plotted in Fig. 6 as a function of $m_1$. For the special point $(\theta_{23}, \delta) = (45^\circ, 270^\circ)$ at $\Lambda_{EW}$, the corresponding $\tilde{\chi}^2_{\min}$, value is 2.08. Fig. 6 demonstrates the very effective reduction of $\tilde{\chi}^2_{\min}$ by incorporating the RGE running effect in the framework of the MSSM. Among the four cases under discussion, the red curve for Case D with $\rho = 90^\circ$ and $\sigma = 0^\circ$ at $\Lambda_{\mu\tau}$ is most favorable. Even given the Planck limit on the sum of the neutrino masses $\sum \equiv m_1 + m_2 + m_3 < 0.23$ eV at the 95% confidence level [22], it is still possible to reduce the value of $\tilde{\chi}^2_{\min}$ to almost 0.05.

two cases, the green region almost overlaps with the pink region. In the two lower panels of Fig. 5, corresponding to Cases C and D, the RGE-induced corrections are significant. Note that there is a separate shaded region around $\theta_{23} \simeq 50^\circ$ in Fig. 5(C), and this is associated with the small upper-right corner of the parameter space in Fig. 3(C) or Fig. 4(C). There is a similar separate shaded region in Case D, but this is outside the chosen ranges of $\theta_{23}$ and $\delta$ in plotting Fig. 5(D), and its confidence level is significantly weaker, being outside the 3$\sigma$ region of the global analysis.
3.2 The Dirac case

Because there is only a single CP-violating phase in the Dirac case, it is considerably easier to perform a numerical analysis of the parameter space, which is constrained by both the RGE-induced $\mu$-$\tau$ reflection symmetry breaking effect and the recent global-fit results of neutrino oscillation data. Fig. 7 illustrates the allowed regions of $\theta_{23}$ and $\delta$ at $\Lambda_{\text{EW}}$, and their intimate correlation is illustrated in Fig. 8. Here, the evolution behaviors of these two parameters can be understood in a more straightforward manner than in the Majorana case, simply because of the absence of the two Majorana phases. In the leading-order approximation, the analytical expressions of $\Delta\theta_{23}$ and $\Delta\delta$ in Eqs. (12) and (13) simplify to

$$\Delta\theta_{23} \simeq \frac{\Delta\tau}{2} \frac{m_2^2 + m_3^2}{\Delta m_{\text{atm}}^2},$$

$$\Delta\delta \simeq -\frac{\Delta\tau}{2} \frac{s_{13}}{c_{12} s_{12}} \frac{m_1^2 + m_2^2}{\Delta m_{\text{sol}}^2}.$$  

(17)

It becomes obvious that $(\theta_{23}, \delta)$ may have larger deviations from $(45^\circ, 270^\circ)$ for bigger values of $m_1$ and $\tan\beta$. In particular, $\theta_{23}$ and $\delta$ are always located in the upper octant and third quadrant, respectively. The gray regions in Fig. 7 exist for the same reason as in the Majorana case, as discussed above. Similar to Case D in the Majorana scenario, the RGE-induced corrections can take $(\theta_{23}, \delta)$ very close to their best-fit point, and the corresponding $\chi^2_{\text{min}}$ is represented in Fig. 6 by the gray dashed line.

![Fig. 6](image1)

Fig. 6. (color online) The minimal $\chi^2$ by marginalizing over $\tan\beta \in [10, 50]$. The colored curves denote the four Majorana cases, and the dashed gray curve denotes the Dirac case. The vertical dashed line is derived from the cosmological limit on the sum of the neutrino masses [22], and the horizontal line represents $\chi^2_{\text{min}}$ for the point $(\theta_{23}, \delta) = (45^\circ, 270^\circ)$ at $\Lambda_{\text{EW}}$.

![Fig. 7](image2)

Fig. 7. (color online) The allowed regions of $\theta_{23}$ (left panel) and $\delta$ (right panel) in the Dirac case at $\Lambda_{\text{EW}}$ owing to the RGE-induced $\mu$-$\tau$ reflection symmetry breaking, where the dashed curves represent the contours for some typical values of $\theta_{23}$ and $\delta$, and the blue one is compatible with the best-fit result of $\theta_{23}$ or $\delta$ obtained in Ref. [8].

![Fig. 8](image3)

Fig. 8. (color online) The correlation of the broken values of $(\theta_{23}, \delta)$ for the Dirac case. The notations are the same as for Fig. 5.
4 Concluding remarks

In neutrino physics, it is usually necessary (and popular) to introduce some heavy degrees of freedom and certain flavor symmetries at a superhigh energy scale in order to explain the tiny masses of the three known neutrinos and the striking pattern of lepton flavor mixing observed at low energies. In this case, it is also necessary to employ the RGEs as a powerful tool to bridge the gap between these two considerably different energy scales. Such RGE-induced quantum corrections may naturally break the given flavor symmetry, thus leading to some phenomenologically interesting consequences, including a possible correlation between the neutrino mass ordering and flavor mixing parameters.

In this work, we have considered the intriguing \(\mu-\tau\) reflection symmetry and its RGE-induced breaking as an instructive playground to realize the aforementioned idea, especially in view of the fact that the recent global analysis of neutrino oscillation data has indicated a remarkable preference for the normal neutrino mass ordering at the 3\(\sigma\) level, together with a slightly higher octant of \(\theta_{23}\) and the possible location of \(\delta\) in the third quadrant. We have shown that all these important issues can be naturally correlated and explained by the RGE-triggered \(\mu-\tau\) reflection symmetry breaking from a superhigh energy scale \(\Lambda_{\mu-\tau} \sim 10^{14}\) GeV down to the electroweak scale \(\Lambda_{\text{EW}}\) \(\sim 10^2\) GeV in the MSSM. Unlike previous attempts along these lines, our study represents the first numerical exploration of the complete parameter space in both the Majorana and Dirac cases, by allowing the smallest neutrino mass \(m_1\) and the MSSM parameter \(\tan\beta\) to vary within their respective reasonable regions \([0, 1]\) eV and \([10, 50]\). We believe that direct measurements of the neutrino mass ordering and precision measurements of \(\theta_{23}\) and \(\delta\) in the near future will test the simple but suggestive scenario under consideration.

Of course, some of our main observations are subject to the MSSM itself, and the current best-fit values of \(\theta_{23}\) and \(\delta\). The reason why we have chosen the MSSM instead of the SM is three-fold: (a) in the SM it is extremely difficult to generate an appreciable value of \(\Delta\theta_{23}\) via the RGE-induced \(\mu-\tau\) symmetry breaking effect, no matter which neutrino mass ordering is considered; (b) in the SM the running direction of \(\theta_{23}\) from \(\Lambda_{\mu-\tau}\) down to \(\Lambda_{\text{EW}}\) seems to be “wrong” if one takes today’s best-fit result \(\theta_{23} > 45^\circ\) seriously in the normal mass ordering case; and (c) the SM itself may suffer from the vacuum-stability problem, as the energy scale is above \(10^{10}\) GeV \([23]\). When the two-Higgs doublet models (2HDMs) are considered \([24]\), one may perform a similar analysis to reveal the correlation between the neutrino mass ordering, the octant of \(\theta_{23}\), and the quadrant of \(\delta\) via the RGE-induced breaking effects. The deviations of \(\theta_{23}\) and \(\delta\) from their values in the \(\mu-\tau\) reflection symmetry limit can be quite different from those in the MSSM case, depending largely on which 2HDM scenario is taken into account. Ref. [17] has provided an explicit example of this kind in the type-II 2HDM scenario.

On the other hand, we admit that the best-fit values of \(\theta_{23}\) and \(\delta\) will unavoidably “fluctuate” in the coming years, when more accurate experimental data are accumulated and incorporated into the global analysis framework. That is why we have numerically explored the complete parameter space to illustrate the tolerable ranges of \(m_1\) and \(\tan\beta\) that allow us to correlate the normal neutrino mass ordering with the higher octant of \(\theta_{23}\) and the third quadrant of \(\delta\). If the inverted neutrino mass ordering, the lower octant of \(\theta_{23}\), and (or) another quadrant of \(\delta\) turned out to be favored by future precision measurements, it would be straightforward to consider a different correlation scenario for them, either within or beyond the MSSM. In the same spirit, one may study other interesting flavor symmetries and their RGE-induced breaking, in order to effectively link model building at high-energy scales to neutrino oscillation experiments at low energies.

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