Abstract

We have investigated the possible existence of a $^7\text{H}$ resonant state, considered as a five-body system consisting of a $^3\text{H}$ core with four valence neutrons. To this aim, an effective n-$^3\text{H}$ potential is constructed in order to reproduce the low energy elastic neutron scattering on $^3\text{H}$ phase shifts and the $^5\text{H}$ resonant ground state in terms of $^3\text{H}$-n-n system. The variational Gaussian Expansion Method is used to solve the 5-body Schrödinger equation, while the resonant state parameters were estimated by means of the stabilization method. We have not found any sign of a narrow low energy resonance in the vicinity of $^3\text{H}$+4n threshold. However, we have identified a very broad structure at $E_R \approx 9$ MeV above this threshold, which corresponds to the $^7\text{H}$ $J^\pi=1/2^+$ ground state. In the vicinity of this state, we have also identified a broad structure corresponding to the ground state of $^6\text{H}$ isotope with quantum numbers $J^\pi=2^-$. 

Keywords: $^4\text{H}$, $^5\text{H}$, $^6\text{H}$ and $^7\text{H}$, Gaussian Expansion Method, Stabilization method, Few-Nucleon problem, ab initio calculations

1. Introduction

Aside from the well-known stable isotopes - deuteron $^2\text{H}$ (d) and tritium $^3\text{H}$ (t) - the H isotopic chain extends beyond these two familiar nuclei with well-defined resonant states in $^4\text{H}$ – clearly seen in the low energy n-t cross section– and $^5\text{H}$ system $[1, 2, 3, 4]$. The possibility for the simplest nucleus, a single proton, to form an isotopic state as exotic as $^7\text{H}$ constitutes an exciting challenge both from experimental as well as from theoretical points of view.

The first indication of a resonant $^7\text{H}$ state came in 2003 from RIKEN $[5]$ study of $p(^8\text{He,}2p)^7\text{H}$ reaction. These authors conclude the existence of a $^7\text{H}$ resonance near the $t+4n$ threshold, although they were not able to determine the resonance parameters.

This was immediately followed by a series of experiments in GANIL using a $^8\text{He}$ beam of $\approx 15$ A MeV. The first one (E465S) $[6, 7]$ was based on the transfer reaction $d(^8\text{He},^3\text{He})^7\text{H}$. They measured a clear structure in the missing mass spectrum, which was attributed to the presence of a resonant state with $E_R=1.56 \pm 0.27$ above the $t+4n$ threshold and $\Gamma=1.74\pm 0.72$ MeV.

In the next GANIL experiment (E406S) $[8, 9]$, the $^{12}\text{C}(^8\text{He,}^{13}\text{N})^7\text{H}$ transfer reaction was studied by employing a $^8\text{He}$ beam of 15.4 A MeV and a $^{12}\text{C}$ gas target. Regardless of the small statistical evidence
(7 events), these authors suggested the existence of a very narrow resonant $^7\text{H}$ state at $E_R=0.57\pm0.42-0.21$ MeV and an estimated width $\Gamma=0.09+0.94-0.06$ MeV. The same result has been recently published in [10].

GANIL results were not fully affirmed in a 2010 RIKEN experiment [11]. Nevertheless, a well-pronounced peak was observed and associated with a $^7\text{H}$ state, resulting in a peculiar structure at $\sim2$ MeV above the t+4n threshold.

A recent experiment [12] at the Flerov Laboratory of Nuclear Reactions (JINR, Dubna) employed an $^8\text{He}$ beam of 26 A MeV to investigate the $^2\text{H}(^8\text{He},^3\text{He})^7\text{H}$ reaction and suggested the existence of a $^7\text{H}$ ground state at $E=2.0(5)$ MeV as well as the first excited state at $E^*=6.5(5)$ MeV with $\Gamma=2.0(5)$ MeV.

We should also mention the unsuccessful attempt of Ref. [13] to observe a long-living quasi stable $^7\text{H}$ nucleus in the reaction $^2\text{H}(^8\text{He},^7\text{H})^3\text{He}$. The authors of a former work estimated the $^7\text{H}$ lifetime to be less than 1 ns and a lower limit of 50-100 keV for the $^7\text{H}$ energy above the t + 4n breakup threshold.

From the theoretical side, little is known despite the growing interest in exotic nuclear systems. One needs to clarify the tendency – for or against the stability – of the H isotopic chain when moving from p+3n to p+4n and to p+6n. Our previous ab-initio calculation on $^4\text{H}$ and $^5\text{H}$ [14] shows a gain of stability (i.e. proximity to the E-real axis) in the computed S-matrix poles when moving from $^4\text{H}$ to $^5\text{H}$ ground state (see Fig 4 of this reference). If the results of the GANIL second experiment [8, 9] would be confirmed, they will indicate a further increase when moving to p+6n.

However, only a few attempts have been made in this direction. In Ref. [15] the energy of $^7\text{H}$ was roughly estimated to $E(^7\text{H})\sim1.4$ MeV (above t+4n) by using a purely S-wave Volkov potential [16] and an exponential extrapolation of energies calculated up to the Hyperspherical quantum number $K=K_{\text{min}}+6$.

Using a variant of the Volkov potential, one could also estimate the energy of $^7\text{H}$ in [5]. Since the experimental $^8\text{He}$ binding energy is 31.4 MeV, we can expect the binding energy of seven-body $^7\text{H}$ to be -5.4 MeV, which is about 3 MeV above the t+4n threshold.

A more realistic estimation was realized in [17] based on the same Volkov interaction. In order to solve the 7-body problem, these authors combined the Antisymmetrized Molecular Dynamics (AMD) approach with generator coordinate and stochastic variational methods. They found a state at $E(^7\text{H})\sim7$ MeV above the t+4n threshold. All the calculations were performed within the bound state approximation, failing to estimate the width of a such high energy state. Some years later, these authors [18] realized a more advanced AMD calculation, which suggested the presence of a state at $E(^7\text{H}) \sim 4$ MeV above the t+4n threshold, again without giving its decay width.

Finally, it is worth noticing the recent work [19] in which $^7\text{H}$ was considered as a state formed from a rigid $^3\text{H}$ core and four valence neutrons. The unbound $^4-^7\text{H}$ isotopes have been computed by using No-core Gamow Shell Model (NCGSM) techniques with different phenomenological NN interactions. These authors concluded that $^7\text{H}$ should be a very narrow resonant state, even the sharpest neutron resonance observed in the light nuclei chart. The value of $E_R$, the real part of the complex energy ground state, varied from 2-3 MeV depending on the NN model, and in all cases $\Gamma \approx 0.1$ MeV. This is a quite surprising result, since the predicted resonance is found to be well above the t + 4n threshold, above the resonant $^4\text{H}+3n$ and $^5\text{H}+2n$ decay channels and, nevertheless, its width is predicted to be much smaller than those of $^4\text{H}$ and $^5\text{H}$ resonant states.

Recently, an experimental group at RIBF (NP1512-SAMURAI134) have performed a study aiming to observe the $^7\text{H}$ resonant states. The analysis of the collected data is still in progress. Considering this situation, it is timely to study the structure of $^7\text{H}$.

Contrary to our previous ab-initio works on 3n [20], 4n [21, 22, 23, 24], $^4\text{H}$ and $^5\text{H}$ [14] we are not able
to consider the $^7$H system as a genuine 7-N problem. Our present work is thus an attempt to compute the $^7$H ground state properties in the 5-body cluster approximation (t-n-n-n-n), which anyway should be the dominant decay channel for a low energy resonant state. To this aim, we have first constructed a n-t local interaction that we have adjusted in order to reproduce the n-t phase shifts. We have also adjusted a simple t-n-n three-body force to reproduce the ground state properties of $^5$H, now described as a t-n-n system. It should be noted, however, that the experimentally determined energy and decay widths of $^5$H represent only an effective response to a particular reaction mechanism leading to the short-living formation and instantaneous decay of a $^5$H state. Its value is thus strongly dependent of the particular reaction mechanism. Therefore, the t-n-n three-body force used in our model was adjusted in order to reproduce the $^5$H resonance parameters computed in our previous ab initio calculation [14].

The paper is organized as follows. Section 2 contains the formal aspects of the problem. These are essentially the t-n potential (2.1), the Gaussian Expansion Method (GEM) used to solve the five-body Schrödinger equation for the t-n-n-n-n system (2.2) and the Stabilization Method (SM) used to estimate the complex energies of resonant states (2.3). Our numerical results for $^6$H and $^7$H resonant state are given in Section 3 and Section 4 contains some concluding remarks.

2. Formalism

An ab initio 7-body calculations of a resonant state is beyond the existing technical capabilities. Since the ground state of $^7$H decays essentially into the five-body $^3$H+4n channel, we have decided to describe its ground state as a 5-body system, based on a solid tritium core interacting with four valence nucleons.

At the current stage, our 5-body code based on Gaussian expansion method is not able to handle tensor interactions, and therefore our t-n interaction model had to omit such terms. On the one hand, it is well known that the tensor force is crucial in order to describe quantitatively the few-nucleon data. On the other hand, light nuclei fulfill quite well the criteria of Effective Field Theory [25, 26, 27, 28], according to which the main features he weakly bound nuclei rely on very few parameters of the NN interaction. Our previous studies of the lightest hydrogen isotopes [14] clearly confirm this point. Indeed, by considering the phenomenological MT I-III NN potential, which is limited to S-wave, we were able to qualitatively describe the n-t scattering phase shifts and the position of $^5$H resonant state. We have therefore decided to build our n-t effective interaction by adjusting its parameters to reproduce the t-n phase shifts calculated by solving the ab-initio four-nucleon scattering problem with MT I-III.

2.1. The n-t potential

For numerical convenience, we have constructed a n-t potential in terms of Gaussian regulators

$$V_{nt}^{SL}(r) = \delta_{L0} \phi_0 > \lambda_\infty < \phi_0 | + \sum_{i=1}^{3} \left[ V_i^c + (-)^L V_i^P + \frac{S^2}{2} V_i^S + (-)^L \frac{S^2}{2} V_i^{SP} \right] e^{-a_r r^2}$$

(1)

where $V_i^c$, $V_i^P$, $V_i^S$ and $V_i^{SP}$ are adjustable parameters, and

$$< \tilde{r} | \phi_0 > \equiv \phi_0(r) = e^{-a_0 r^2}$$

is an infinitely repulsive S-wave contribution accounting for the Pauli forbidden state. In principle, $\lambda_\infty$ should be infinite, however we found that value $\lambda_\infty=10^5$ MeV is sufficient to ensure an accuracy of 100 keV for the calculated $^7$H binding energies. Potential (1) is spin and angular momentum dependent but
it has neither spin-orbit nor tensor force. In spirit, it is rather similar to the MT I-III interaction used in our previous studies of $^4$H and $^5$H systems [14], and will in particular provide degenerate $J^p = 0, 1, 2^-$ triplet P-wave resonant states states in $^4$H (see Table I from [14]).

\[
\begin{array}{ccc}
  \alpha_i & \text{fm}^{-2} & 1 & 0.04458 & 0.271 & 0.1700 \\
  V^C_i & \text{MeV} & 2.4975 & 96.245 & -57.19 \\
  V^P_i & \text{MeV} & 2.4975 & -96.245 & 57.19 \\
  V^S_i & \text{MeV} & -1.00725 & -13.755 & 0 \\
  V^{SP}_i & \text{MeV} & -1.00725 & 13.755 & 0 \\
\end{array}
\]

Table 1: Parameters of the $V_{nt}$ effective potential (1).

The corresponding parameters, adjusted to reproduce the experimental n-t phase shifts, are provided in Table I with $\alpha_0 = 0.1979$ 068 fm$^{-2}$. The results for the n-t phase shifts obtained with this potential (and with $\hbar^2/2\mu_{nt} = 27.647$ MeV fm$^{-2}$) are plotted in Figure 1.

As expected, this potential does not support any bound state, however it has two broad P-wave resonant states: one in spin-singlet (S=0) and one in spin-triplet (S=1) channels. The corresponding complex energies of these resonant states are given in central column of Table 2. They are in qualitative agreement with the values (right column) obtained by solving the four-nucleon problem with the MT I-III potential and the Jost function/boundary condition method to determine the S-matrix pole positions.

If the agreement between the n-t two-body model and four-body calculations is not perfect, is essentially due to the fact that the n-t phase shifts displayed in Figure 1 are not perfectly reproduced by our two-body potential (1). Indeed we had to make some compromises when trying to fit simultaneously all the partial wave using the simple operator form (1). Moreover, when performing this fit we kept in

![Figure 1: The S- and P-wave n-t phase shifts (solid lines for spin singlet states and dashed lines for triplet ones) computed with the two-body potential (1) are compared with the results obtained by solving four-nucleon problem with MT I-III potential (circles and squares).](image)
mind to simultaneously reproduce within our two-body model the resonant state of $^5$H and an effort was made to minimize the effect of t-n-n three-body force. On the other hand, the four-nucleon calculations by realistic models predict a $J^\pi = 2^-$ resonant state at slightly higher energy ($E\approx1.2-2i$) than MT I-III, what justifies the slight discrepancies with our 2-body model prediction.

| $V_{nt}$ [1] | pnn [14] |
|------------|----------|
| $L = 1^-, S = 0$ | 1.28-2.61 i 0.88(5)-2.20(5) i |
| $L = 1^-, S = 1$ | 1.33-1.84 i 1.08(3)-2.03(3) i |

Table 2: Positions of $^4$H resonant states (central column) calculated with the effective 2-body n-t potential [1] and (right column) by solving the 4N problem with the MT I-III interaction. The parameters of [1] were adjusted to the n-t scattering phase shifts displayed in Fig. [1].

When describing the $^5$H and $^7$H systems, the n-t interaction has been completed with a neutron-neutron potential. We have taken the Minnesota model [29] which consists in a superposition of two Gaussians with different ranges and provides the neutron-neutron low energy parameters $a_{nn}=-16.90$ fm and $r_{nn}=2.88$ fm.

As it was found in previous studies [30, 31], we were not able to reproduce the experimental $^5$H ground state ($J^\pi=1/2^+$) resonance parameters when using the local $V_{nt}$ and $V_{nn}$ interactions, although we were explicitly trying to favour the appearance of $^5$H resonant state when adjusting parameters of $V_{nt}$. We have thus adjusted the position of the $^5$H ground state by adding a t-n-n 3-body force, having the same form as in Refs. [30, 31]:

$$V_{tnn}(\rho) = -V_0 e^{-\frac{\rho^2}{3}} \tag{2}$$

where

$$\rho^2 = \sum_{i<j} \frac{m_im_j}{mM} (\vec{r}_j - \vec{r}_i)^2 \quad M = \sum_i m_i \tag{3}$$

and $m$ and arbitrary mass. Setting $m = m_n$ one has

$$\rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M} r_{nt}^2 + \frac{m_t}{M} r_{nt}^2 \quad M = 2m_n + m_t$$

Figure 2: $^5$H as a t-n-n three-body system.
in which $r_{nn}, r_{nt}, r_{nt}$ denote the corresponding relative distances, as shown in figure (2)  

2.2. Solution of the 3- and 5-body problems with Gaussian expansion method

![Diagram](image)

Figure 3: Faddeev components and corresponding Jacobi coordinates of the t-n-n system.

We aim to solve the Schrödinger equation for $^5\text{H}$ and $^7\text{H}$ within the framework of t-n-n and t-n-n-n-n 3- and 5-body cluster models respectively, by means of the Gaussian Expansion Method (GEM) [32][33]. It consists in first splitting the total wave function $\Psi$ in terms of 3 (and 9) components, which correspond to the schematic decomposition of the 3- (and 5-body) systems in different clusters. These components are depicted in Figures 3 and 4 together with the corresponding set of Jacobi coordinates on which they are naturally expressed.

The total Hamiltonian and the Schrödinger equation are given by

$$H = K + \sum_{a,b} V_{ab} + \sum_{abc} V_{abc}$$

(4)

$$\left( H - E \right) \Psi^{JM}_{TT_z}(5,7\text{H}) = 0$$

(5)

where $K$ is the kinetic-energy operator, $V_{ab}$ is the interaction between the constituent particle $a$ and $b$ and $V_{abc}$ the three-body force.

The total wavefunctions of $^5\text{H}$ and $^7\text{H}$ are expressed as sum of amplitudes corresponding to the rearrangement channels shown in Figs 3 and 4. In the LS coupling scheme they read, respectively:

$$\Psi^{JM}_{TT_z}(5\text{H}) = \sum_{c=1}^{3} \sum_{n,N} \sum_{l,L} \sum_{S,\Sigma, I} C^{(c)}_{nlNLS\Sigma I}$$

$$\times \mathcal{A}_N \left[ \Phi(t)[\chi_2(t)[\chi_2(t)(n_1)\chi_2(t)(n_2)]_\Sigma \times [\phi_{nL}(r_c)\psi^{(c)}_{NL}(R_c)]_I \right]_{JM} \left[ \eta_2(t)[\eta_2(t)(n_1)\eta_2(t)(n_2)]_I \right]_{T,T_z}$$

(6)

$$\Psi^{JM}_{TT_z}(7\text{H}) = \sum_{c=1}^{9} \sum_{n,N} \sum_{l,L,\lambda, \alpha, \beta} \sum_{S,\Sigma, I, K} C^{(c)}_{nlNL\lambda\alpha\beta S\Sigma IK} \mathcal{A}_N$$

1Notice that in terms of Jacobi coordinates

$$\tilde{x}_3 = \sqrt{\frac{1}{m_1 + m_2}} \left( r_2 - r_1 \right)$$

$$\tilde{y}_3 = \sqrt{\frac{2m_3}{m_1 + m_2 M}} \left( r_3 - \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \right)$$

on has

$$\rho_j^2 = x_j^2 + y_j^2 = 2\rho^2$$
The Gaussian range parameters are chosen according to geometrical progressions:

\begin{align*}
 r_n &= r_1 a^{n-1} \quad (n = 1 - n_{\text{max}}), \\
 R_N &= R_1 A^{N-1} \quad (N = 1 - N_{\text{max}}), \\
 \rho_\nu &= \rho_1 \tilde{a}^{\nu-1} \quad (\nu = 1 - \nu_{\text{max}}), \\
 s_\beta &= s_1 \tilde{A}^{\kappa-1} \quad (\kappa = 1 - \kappa_{\text{max}}).
\end{align*}

Figure 4: Different topologies for the 5-body components and corresponding Jacobi coordinates of the t-n-n-n-n system.
The eigenenergy $E$ in Eq. 5 and the coefficients $C^{(c)}$ are determined by inserting respectively the expansions (6) and (7) in (5) and applying the Rayleigh-Ritz variational method. This results into a generalized eigenvalue equation $AC = \lambda BC$ of typical dimension $d \sim 60000$. It is worth noticing that the different, not orthogonal, components of the total wave function represented in Fig. 4 are all crucial in order to reach a converged solution of the Schrödinger equation with limited numerical resources.

2.3. The stabilization method

To access the complex energy eigenvalues of a resonant state, we have used the so-called stabilization (or real scaling) method [34, 35]. It consists in computing the real eigenvalues of the Hamiltonian matrix representation and studying their (approximate) ”stability” when varying a scaling parameter (generically denoted $\alpha$) of the basis state, for instance, the Gaussian range parameters $\nu_n \rightarrow \alpha \nu_n$.

![Stabilization Graph](image)

Figure 5: Example of stabilization graph as a function of the stabilization parameter $\alpha$

The method is based in the observation, illustrated in Figure 5, that when varying $\alpha$, the eigenvalue corresponding to the real part of a resonant state ($E_r$) has an avoided crossing with the real eigenvalues corresponding to the continuum ($E_c$). The analysis of this level repulsion near the crossing point, defined by the intersection of the asymptotes to both curves, provides an estimation of the resonance parameters ($E_R, \Gamma$). In Figure 5, the level crossing between a resonant state ($E_r$) and a continuous state ($E_c$) moving down, is produced at $\alpha = \alpha_c$ and defines the resonant energy $E_R$ (dashed horizontal line). This feature is a signature of a resonance state of the Hamiltonian with real energy $E_R$ and a width given by

$$\Gamma = 2\Delta E \frac{\sqrt{|S_r||S_c|}}{|S_r - S_c|}$$

where $\Delta E$ (in red) is the energy difference between the two curves at the crossing, and $S_r$ and $S_c$ are respectively the slopes of the resonant and continuous levels asymptotes at the crossing.

The stabilization method is closely connected with the Complex Scaling Method (CSM) [36] but lacks of a sound mathematical ground, providing only a rough estimation of the resonance width. This method has been recently applied, in conjunction with the Gaussian Expansion Method, to determine the resonance positions of a doubly heavy tetraquark system [37].
3. Results

Our initial effort was to establish properties of the $^5$H ground state within our model, i.e., by considering it as a t-n-n three-body system interacting via the t-n potential of eq. (1) and Minnesota nn pairwise interaction [29] in conjunction with the hyperradial 3-body force eq. (2). The solution was obtained, as in Ref. [31], by combining GEM and CSM, which directly provides the position of the S-matrix pole in the complex energy plate corresponding to the resonant state.

The complex energy values $E=E_R + i E_I$ of the $^5$H ground state as a function of the three-body force strength $V_0$, defined in eq (2), are displayed in Figure 6 in the $(E_R, E_I)$ plane for several values of the range parameter $b_3$. The numbers along the curves denote the $V_0$ values in MeV. The ab-initio value for this state computed in [14] is indicated by a filled blue square (■) and the experimental data from [2] by a filled green up-pointing triangle (△) with corresponding error bars.

As one can see, the best result is obtained with $b_3=8$ fm and $V_0=2.5$ MeV for which one gets $E(\text{^5H})=1.9 - 1.2$ i MeV in full agreement with our previous ab-initio theoretical result, obtained for the realistic NN interaction models [14]. This value is quite close to, but different from, the experimental one [2] as the later one reflects the footprint of the $^5$H resonant state in a complex nuclear reaction, which strongly depends on the reaction mechanism but also on the experimental protocol, while its relation with the S-matrix singularity is only indirect.

Notice that reproducing the phenomenology and the ab-initio results requires a relatively large value of the range parameter $b_3$ compared in comparison with Refs. [30, 31]. This is related to the choice of our n-t potential, with the particular functional form of eq. (1). The presence of strongly repulsive S-wave channels, manifested by the $\phi_o$ term with parameter $a_0 \approx 0.20$ fm$^{-2}$, implies the exclusion of a large spatial region to project out the symmetry forbidden states in $^4$H. Any attractive contribution,
aimed to account for the spin-dependent effects in the n-t S-waves, as well as to describe the energy dependence of the n-t phase shifts, will require an even larger interaction range to have a significant effect. Furthermore, any attempt to reproduce the $^5\text{H}$ resonance position also favored large interaction ranges. This is an obvious consequence of simulating the Pauli repulsion between the valence neutrons and those of the triton core. As the n-n interaction is resonant in $^1S_0$ state, dineutron correlations take place at rather large distances, and this generates also a rather large exclusion region.

Having fixed the parameters of the t-n and t-n-n interactions, we have estimated the complex energy of $^7\text{H}$ ground state by using the stabilization method, briefly described in section (2.3). An illustrative example of results provided by this approach in the case of $^7\text{H}$, is displayed in Figure 7. It reveals the stabilization graph $E(\alpha)$ – that is the lower values of the $^7\text{H}$ Hamiltonian spectra as a function of the stabilization parameter $\alpha$ – corresponding to the 3-body parameters $b_3=8$ fm and $V_0=3$ MeV.

![Figure 7: $^7\text{H}$ stabilization graph: lower eigenvalues of the $^7\text{H}$ hamiltonian as a function of the stabilization parameter $\alpha$ corresponding to the three-body force parameters $b_3=8$ fm and $V_0=3$ MeV in (2).](image)

In the last figure a level repulsion manifests at $\alpha \approx 1.1$ in the energy range $E \approx 7$-10 MeV with $\Delta E \approx 3$ MeV, indicated by a vertical double arrow. This is a signature of a resonant state at $E_R \approx 8.8$ MeV (denoted by an horizontal blue line) and an estimated width $\Gamma = 2E_I \approx 3.1$ MeV given by eq. (10).

Following this approach, we have computed the $^7\text{H}$ ground state energy as a function of the t-n-n three-body force strength parameter $V_0$. The range of $V_{\text{tun}}$ was fixed to $b_3=8$ fm and we have varied its strength $V_0$ from $V_0 = 7$ MeV until it reaches the value $V_0 = 2.5$, closely reproducing position of the $^5\text{H}$ resonant state established in our previous ab-initio calculations [14].

The final results are summarized in Figure 8. The complex energies of $^7\text{H}$ are indicated by black dots joined by black dashed line. For comparison we have also included in the same figure (blue dots joined by blue dashed line) the results of $^5\text{H}$, already described in Figure 6.

For $V_0=7$ MeV, the $^7\text{H}$ appears to be bound with respect to the t+4n threshold by $E \approx -0.5$ MeV. When decreasing $V_0$ this bound state turns into a resonance. When the strength of t-n-n force is further reduced to its realistic value $V_0=2.5$ – reproducing position of the $^5\text{H}$ – the $^7\text{H}$ resonance is found at $E_R \approx 9.5$ MeV with an estimated width $\Gamma = 2E_I \approx 3.5$ MeV. If we increase the value $V_0 = 3$ MeV, producing $^5\text{H}$ ground state at the lowest point compatible with the experiment, the $^7\text{H}$ resonance is found at $E_R = 8.5$ MeV and $\Gamma = 3$ MeV. These finding conflicts with a presence of a narrow $^7\text{H}$ resonant state in the vicinity of t+4n threshold. We still find a signature of a broad state at relatively
high energy sufficient to break triton core in single particles. This high energy domain is beyond the range of validity of our model, which supposes existence of a solid tritium core. One expects that the widths of $^7\text{H}$ state should become even larger in a more realistic model, where tritium core is allowed to break.

As one can see our results are in no way compatible with the existence of a narrow $^7\text{H}$ resonant state as being claimed in some experimental works, mainly [6, 7, 8, 11, 12]. Although all these experiments suffer from a very poor statistics, there is a huge gap between these two views. The only possible link between our results and the experimental claims is to identify our values with what the authors of Ref. [12] claim to be the first excited state of $^7\text{H}$ with $E^*=6.5(5)$ and $\Gamma=2.0(5)$.

In relation with the previous theoretical works, our conclusions are in the same line than those of Ref. [17] who obtain $E(7\text{H}) \sim 7$ MeV. Our findings are however in a sharp contrast with the recent work [19]. These authors provide an energy of the ground $^7\text{H}$ which varies from 2-3 MeV, depending on the NN interaction considered, and in all cases a very narrow width $\Gamma \approx 0.1$ MeV (see Fig 4 of this reference, with a factor 10 enhancement in the $^7\text{H}$ widths).

It is worth emphasizing that our differences with the results of [19] come already from the very input of both approaches. Indeed, in a former work [38], the same authors using the same method found very narrow $^4\text{H}$ resonant states (see Fig 1), placing them at $E_R \approx 1.7 - 0.9$ i MeV , in agreement with some R-matrix analysis [41]. However, it is well established that experimental R-matrix results and the S-matrix pole positions may differ significantly for broad resonant states, which is in particular true for $^4\text{H}$ case as stated in [40, 30, 14]. As a consequence of this original choice, [19] obtained also an extremely narrow $^5\text{H}$ resonance (see Fig 4), which is not supported neither by theory nor by the experiment.

In our opinion, there seems to exist an intrinsic limitation in the GSM used in [19], which makes this
method to systematically underestimate the widths of resonant states. It is due to the fact that GSM employs only single particle basis, which are not well adapted to describe collective cluster channels and thus limits the decay possibilities of a resonant states. In the same way, if we would have restricted the variation of the basis parameters in the stabilization method described in Section 2.3 for instance by just limiting to few selected components of Figure 4, we would have obtained a significant reduction of the estimated $^7\text{H}$ width.

For the sake of completeness, we have also computed the resonant ground state of $^6\text{H}$, considered as a t-n-n-n four-body system. By using the same n, t-n, and t-n-n interactions than those used to obtain the $^5\text{H}$ and $^7\text{H}$, in particular $V_0=2.5$ and $b_3=8$ fm, we estimate for $^6\text{H}$ ground state the resonance parameters $E_R=10$ MeV and $\Gamma=4$ MeV. Both, energy and width, are slightly larger than those of $^7\text{H}$, though compatible.

In summary, our theoretical analysis of the heavier well identified H-isotopes $^4\text{H}$, $^5\text{H}$, $^6\text{H}$ and $^7\text{H}$ can be described in Figure 9. As it is customary in the experimental tables, it represents the resonance energies $E_R$ of their ground states relative to the corresponding t+xn threshold, with an overimposed square denoting the total width $\Gamma$ (all units are in MeV).

The results of $^6\text{H}$ ($E_R=10$ MeV, $\Gamma=4$ MeV) and $^7\text{H}$ ($E_R=9.5$ MeV, $\Gamma=3.5$ MeV) are those obtained in the present work, within the t-n-n-n-n cluster approximation and $V_{tnn}$ parameters $b_3=8$ fm and $V_0=2.5$ MeV. They are compatible with the experimental value $E_{gs}=7.3\pm1$ MeV, $\Gamma_{gs}=5.8\pm2$ MeV obtained by $^{11}\text{B}(\pi^-,p)^{6}\text{H}$ reaction. They indicate that the big jump in $E_R$ already manifests in $^6\text{H}$.

The values for $^4\text{H}$ and $^5\text{H}$ correspond respectively to $2^-$ ($E_R=1.2$ MeV, $\Gamma=4$ MeV) and $1/2^+$ ($E_R=1.85$ MeV, $\Gamma=2.4$ MeV), both supposed to be the experimental ground states, as they are provided by the ab initio calculations and N3LO interaction in our previous work [14] (see Table I and II). For $^5\text{H}$, the energy fully coincides with the value of the t-n-n cluster approximation displayed in Figs 6 (blue squared corresponding to $b_3=8$ fm and $V_0=2.5$ MeV). For $^4\text{H}$, it is slightly different from the $J^p=0^-,1^-,2^-$ degenerate value of the two-body ground state ($E_R=1.3$ MeV, $\Gamma=3.8$ MeV) provided by the n-t potential [1].

The tendency to stability observed between $^4\text{H}$ and $^5\text{H}$, pointed out in our previous work [14], is totally reversed by the heavier isotopes. Both, $^6\text{H}$ and $^7\text{H}$ ground states are at relatively high energies ($E_R\sim9$ MeV) and are broad ($\Gamma\sim4$ MeV) resonant states, which suggest the end point of the H isotopic chain rather than a stability region opened by the neutron richer isotopes.

In view of the strong contradiction with previous experimental and theoretical works, it would be highly desirable that new high statistic experiments and independent calculations could bring some light into this disarming situation.
4. Conclusion

We have obtained the complex energy position of the $^7\text{H}$ resonant ground state considered as a 5-body $t+n+n+n+n$ system. To this aim, we first constructed a n-t local potential whose parameters were adjusted to reproduce the low energy neutron scattering on triton phase shifts. An additional 3-body force was needed to reproduce the $^5\text{H}$ ground state in the $t+n+n$ approximation.

The solution of the 5-body Schrödinger equation has been obtained by means of the variational Gaussian expansion approach and the resonance parameters have been estimated by using the stabilization (or real scaling) method.

Our estimated parameters of $^7\text{H}$ ground state are $E_R=9.5$ MeV and $\Gamma=3.5$ MeV, that is quite a broad state when compared with the lighter H isotopes ($^4\text{H}$ and $^5\text{H}$). Our findings are in sharp contrast with some experimental values $[5, 8, 11, 12]$ as well as with the theoretical results of $[19]$. By using the same interaction parameters than for $^7\text{H}$, a similar calculation was performed to describe the ground state of the $^6\text{H}$ isotope, considered as a $^3\text{H}+3n$ four-body system. The ground state was identified to have $J=2^-$ and is situated at $E_R \approx 10$ MeV and $\Gamma \approx 4$ MeV.

Even within the limits of our calculation, we must exclude the presence of a narrow resonant state in $^7\text{H}$. The comparison with $^4\text{H}$ and $^5\text{H}$ makes unlikely that the H isotopic chain could be continued, at least an experimentally observed state, beyond $^7\text{H}$.

Acknowledgements

We are sincerely grateful to V. Lapoux and F. M. Marqués for enlightening discussions concerning the experimental results. We were granted access to the HPC resources of TGCC/IDRIS under the
allocation A0110506006 by GENCI (Grand Equipement National de Calcul Intensif). This work was supported by french IN2P3 for a theory project "Neutron-rich light unstable nuclei", by the japanese Grant-in-Aid for Scientific Research on Innovative Areas (No.18H05407) and by the Pioneering research project 'RIKEN Evolution of Matter in the Universe Program'. It was completed during the program Living Near Unitarity at Kavli Institute for Theoretical Physics, University of Santa Barbara (California). We thank the organizers and the staff members of this Institute for their kind invitation and financial support, from the National Science Foundation Grant No. NSF PHY-1748958.

References

[1] P.G. Young et al., Phys. Rev. 173 (1968) 949.
[2] A.A. Korsheninnikov, et al., Phys. Rev. Lett. 87 (2001) 092501.
[3] M.S. Golovkov et al, Phys. Lett. B 566 (2003) 70, Phys. Rev. Lett. 93 (2004) 262501, Phys. Rev. C 72 (2005) 064612.
[4] A.H. Wuosmaa et al Phys. Rev. C 95 (2017) 014310.
[5] A. A. Korsheninnikov et al., Phys. Rev. Lett. 90, 082501 (2003)
[6] S. Fortier et al., AIP Conference Proceedings 912, 3 (2007); doi: 10.1063/1.2746575
[7] D. Baumel et al., International Symposium on Physics of Unstable Nuclei (ISPUN07), Jul 2007, Hoi An, Vietnam. pp.18-25. ?in2p3-00292987?
[8] M. Caamaño et al., Phys. Rev. Lett. 99 (2007) 062502.
[9] M. Caamaño et al., Phys. Rev. C 78 (2008) 044001
[10] M. Caamaño et al., Phys. Lett B29 (2022) 137067
[11] E. Yu. Nikolskii, A. A. Korsheninnikov, H. Otsu, H. Suzuki, et al., Phys. Rev C 81, 064606 (2010)
[12] A.A. Bezbakh et al., Phys. Rev. Lett. 124 (2020) 2, 022502
[13] M.S. Golovkov, L.V. Grigorenko, A.S. Fomichev, Yu.Ts. Oganessian, Yu.I. Orlov et al., Phys. Lett. B 588 (2004) 163-171
[14] R. Lazauskas, E. Hiyama, J. Carbonell, Phys. Lett B791 (2019) 325-341
[15] N.K. Timofeyuk, Phys. Rev. C 65 (2002) 064306
[16] A.B. Volkov, Nucl. Phys. 74, 33 (1965).
[17] S. Aoyama and N. Itagaki, Nucl. Phys. A738, 362 (2004)
[18] S. Aoyama and N. Itagaki, Phys. Rev. C 80, 021304(R) (2009)
[19] H.H. Li, J.G. Li, N. Michel and W. Zuo, Pys. Rev C 104, L061306 (2021)
[20] R. Lazauskas and J. Carbonell, Phys. Rev. C 71 (2005) 044004.
[21] R. Lazauskas and J. Carbonell, Phys. Rev. C 72 (2005) 034003.
[22] E. Hiyama, R. Lazauskas, J. Carbonell, N. Kamimura, Phys. Rev. C 93 (2016) 044004.
[23] J. Carbonell, R. Lazauskas, E. Hiyama, M. Kamimura, Few-Body Syst. 58, 2 (2017) 58.
[24] R. Lazauskas, E. Hiyama, J. Carbonell, Prog. Theor. Exp. Phys. 7 (2017) 073D03.
[25] U. van Kolck, Nucl. Phys. A 645 (1999) 273, arXiv:nucl-th/9808007.
[26] E. Braaten and H. W. Hammer, Phys. Rept. 428, 259 (2006), arXiv:cond-mat/0410417.
[27] P. Naidon and S. Endo, Rept. Prog. Phys. 80, 056001 (2017), arXiv:1610.09805.
[28] M. Gattobigio, A. Kievsky, M. Viviani, Phys. Rev. C 100 (2019) 034004.
[29] D.R. Thompson, M. Lemere and Y.C. Tang, Nucl. Phys. A 286, 1 (1977) 53-66.
[30] R. de Diego, E. Garrido, D.V. Fedorov, A.S. Jensen, Nucl. Phys. A 786 (2007) 71.
[31] E. Hiyama, S. Ohnishi, M. Kamimura, Y. Yamamoto, Nucl. Phys. A 908 (2013) 29.
[32] M. Kamimura, Phys. Rev. A 38, 621 (1988).
[33] E. Hiyama, M. Kamimura, Front. Phys. 13(6), 132106 (2018) and references therein.
[34] H. S. Taylor, Adv. Chern. Phys. 18, 91 (1970).
[35] Jack Simons, J. Chem. Phys. 75, 2465 (1981); https://doi.org/10.1063/1.442271.
[36] N. Moiseyev, Physics Reports, 302 (1998) 212.
[37] Qi Meng, Masayasu Harada, Emiko Hiyama, Atsushi Hosaka, Makoto Oka, Phys. Lett B824 (2022) 136800.
[38] J.G. Li, N. Michel, W. Zuo and F.R. Xu, Pys. Rev C 104, 024319 (2021).
[39] Y. B. Gurov, M. N. Behr, D. V. Aleshkin, B. A. Chernyshev, S. V. Lapushkin, P. V. Morokhov, V. A. Pechkurov, N. O. Poroshin, V. G. Sandukovsky and M. V. TelKushev, Eur. Phys. J. A 24, 231 (2005).
[40] D.R. Tilley, H.R. Weller, G.M. Hale, Nucl. Phys. A 541 (1992) 1.
[41] Y. B. Gurov, M. N. Behr, D. V. Aleshkin, B. A. Chernyshev, S. V. Lapushkin, P. V. Morokhov, V. A. Pechkurov, N. O. Poroshin, V. G. Sandukovsky and M. V. TelKushev, Eur. Phys. J. A24, 231 (2005).