Modified GR and Helium Nucleosynthesis

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Abstract

We show that a previously proposed cosmological model based on general relativity with non vanishing divergence for the energy-momentum tensor is consistent with the observed values for the nucleosynthesis of helium for some values of the arbitrary parameter $\alpha$ presented in this model. Further more values of $\alpha$ can be accommodated if we adopt the Randall-Sundrum single brane model.

Motivated by the desire to seek a solution to the entropy problem within standard classical cosmology we have previously proposed a simple modification of GR; namely relaxation of the condition of the vanishing of the divergence of the energy-momentum tensor [1]

$$T_{\mu\nu};\mu = 0 \quad (1)$$

This can be motivated by noting that the assumptions that lead to (1) are all questionable, including the principle of equivalence [2]. Relaxation of (1) does not upset the success of general relativity either in or outside cosmology. It does, therefore, appear that the covariant conservation has not been specifically tested by observation. The construction of a theory in which (1) does not necessarily hold would thus provide an opportunity for testing this condition. In fact the relaxation of this condition, introduces a single arbitrary constant, $\alpha$, where $0 < \alpha < 1$. The field equations in this model become [1]:

$$R_{\mu\nu} - \frac{1}{2} \gamma g_{\mu\nu} = -KT_{\mu\nu} \quad (2)$$

Where:

$$\gamma = \frac{2 - \alpha}{3 - 2\alpha}, \quad K = \frac{8\pi G}{\alpha} \quad (3)$$

Imposing the Bianchi identity one obtains:

$$T^{\mu\nu};\mu = -\frac{1}{2}(1 - \alpha)T_{\mu\nu} \quad (4)$$

Standard GR has $\alpha = 1$.

The vacuum field equations remain the same as in standard GR. Consequently the crucial tests of GR and the important analytic features (such as the existence of singularities and black holes) are maintained. The Field equations (2) are also the same as the standard field equations for systems with traceless energy-momentum tensors, except that $G$ is replaced by
This shift in the effective value of $G$ to $G/\alpha$ in the early universe is expected to change the prediction of the primordial nucleosynthesis. In this paper we use a simple, approximate and semi analytical method to check the consistency of this model with the observed values for the nucleosynthesis of helium. We show that at least for some values of $\alpha$ the model is consistent with the observed values. This problem was investigated in ref. [3] using a different approach.

The modified filed equations, in the radiation dominated era in this model become:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{8\pi G}{3\alpha} \rho_r$$  \hspace{1cm} (5)

This represent an increase in density as related to standard model.

The primordial production of $^4$He is controlled by a competition between the weak interaction rates and the expansion rate of the universe. As long as the weak interaction rates are faster than the expansion rate, the neutron-to-proton $\left(\frac{n}{p}\right)$ ratio tracks its equilibrium value. Eventually as the universe expands and cools, the expansion rate comes to dominate and $\frac{n}{p}$ essentially freezes out at the so called freeze out temperature. However the nucleosynthesis chain which begins with the formation of deuterium through the process $p + n \rightarrow D + Y$ is delayed past the point where the temperature has fallen below the deuterium binding energy $E_B$ since there are many photons in the exponential tail of the photon energy distribution with energies $E > E_B$ despite the fact that the average photon energy are less than $E_B$. (see for example [4]).

The increase in density is expected to modify the expansion of the universe during nucleosynthesis by modifying both the freeze out temperature and the time available for the decay of neutron. These two factors will lead to an increase in the nucleosynthesis of Helium.

Equation (5) can be written as:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho_r + \frac{8\pi G}{3} \frac{1}{\alpha} \rho_r$$  \hspace{1cm} (6)

The perturbation in density presented in Equation. (5) can be taken as a modification of the gravitational constant $\delta G$, where

$$\delta G = \frac{1 - \alpha}{\alpha} G$$  \hspace{1cm} (7)

Use of the conservation equation for radiation with equation (6) leads to:

$$t_m = \frac{t_s}{\alpha^2}$$

Where $t_m$ is the modified time from freeze out to start of nucleosynthesis and $t_s$ is the time calculated from Standard Model without the extra energy.

The calculation of primordial element abundances is a highly nonlinear problem with many coupled nuclear reactions, and requires a numerical analysis. There are many different numerical codes for doing this calculation starting with Wagoner [5]. They mainly differ in the different
factors they include in their calculations and particularly the different values they use for the neutron half life \cite{6,7}. Here we only present a simplified and approximate method. In this approximation, the primordial helium abundance $Y_p$ is given by:

$$Y_p = \left( \frac{2N_n}{N_n + N_p} \right) F \exp[-\lambda(t_m - t_F)]$$

(8)

$$\cong \left( \frac{2N_n}{N_n + N_p} \right) F \exp(-\lambda t_m)$$

(9)

Where $F$ represent freeze out and we ignored $t_F$ because it is of two order less than $t_m$.

Equation (9) can be written as

$$Y = \left( \frac{2x}{1 + x} \right) F \exp(-\frac{\lambda t_s}{\sqrt{\alpha}}) \text{ where } x = \frac{N_n}{N_p}$$

(10)

And thus

$$\delta Y = \frac{2\delta x}{(1 + x)^2} \exp(-\frac{\lambda t_s}{\sqrt{\alpha}})$$

(11)

$$x = \frac{N_n}{N_p} = \exp(\frac{-1.5}{T_{f10}^3})$$

where $T_{f10}$ is the freeze out temperature \( T_n = \frac{T}{10^3 K} \) and is determined by equating the Hubble constant $H$ to the weak reaction rate $\eta$ for $n \leftrightarrow p$ conversions.

Now,

$$H \propto G^{1/2} T_{f10}^2 \text{ and } \eta \propto T_{f10}^5$$

So that: $T_{f10}^3 = G^{1/2}$

$$\therefore \delta Y = \frac{(x \ln x)e^{-\frac{\lambda t_s}{\sqrt{\alpha}}}}{3(1 + x)^2} \frac{\delta G}{G}$$

(12)

which from equation (4) becomes:

$$\delta Y = \frac{(x \ln x)e^{-\frac{\lambda t_s}{\sqrt{\alpha}}}}{3(1 + x)^2} \left( \frac{1 - \alpha}{\alpha} \right)$$

(13)

Putting $x = 0.14$ (From $Y = 0.246$ \cite{6}), $\lambda = 78 \times 10^{-5}$, $t_s \cong 120s$, and if we take a value for $\alpha = 0.9$, equation (13) gives:

$$\delta Y = 0.0071$$

which is quite within the margin of difference between measured and observed values. This value, as expected will increase, for smaller values of $\alpha$. 


Discussion:

Big bang nucleosynthesis (BBN) is one of the most sensitive available probes of physics beyond the standard model. The $^4$He abundance in particular, has often been used as a sensitive probe of new physics [8–12]. This is due to the fact that nearly all available neutrons at the time of BBN end up in $^4$He and the neutron-to-proton ratio is very sensitive to the competition between the weak interaction rate and the expansion rate. Observations of light element abundances have improved dramatically over the past few years. The recent all-sky, high-precision measurements of microwave background anisotropies by WMAP has opened the possibility for new precision analysis of BBN. Using BBN prediction gives a powerful constraint over the various cosmological parameters. The possibility of the physical ‘constants’ taking different values at different times in the universe history has recently received much attention with the apparent observation that fine structure constant had a different value in the distant past [13]. Variation of the gravitational constant, that was originally started by Dirac [14] in his so called ”Large Number” theory is now constrained by the observations of the light element abundances [15-18].

Copi et al [16] uses the recent measurements of the primordial deuterium abundances ($D/H$) in conjunction with the WMAP determination of $\eta$ to set a Limit for $G/G_0$, where $G$ is the value for the gravitational constant during nucleosynthesis and $G_0$ its value at present. They found that $G/G_0$ is constrained in the range 1.21 to 0.85 at the 68.3% confidence level and between 1.43 and 0.71 at the 95% confidence level Assuming a simple power dependence $G \sim t^{-x}$, $x$ was constrained to the range $-0.004 < x < 0.005$ at the 68.3% confidence level, $-0.009 < x < 0.01$ at the 95% confidence level. In our model taking $\alpha = 0.9$ gives $G/G_0 = 1.11$, which is within the range obtained by Copi et al .Further Cyburt et al [17] using $^4$He abundance set a more restrictive limit for $\Delta G/G_0$ of 13%.

Finally we mention that if one adopted the recent ideas of brane model, where the universe is supposed to be embedded in a higher dimensional bulk, particularly the Randal Sunundrun type II model [19], one could accommodate more smaller values for $\alpha$. The predicted expansion rate in the early universe can be significantly modified, particularly with the presence of a dark radiation term [20,21], which may take negative values and can contribute as high as 27% of the background photon energy density [22].

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