Metal-Insulator Transitions in Interacting Disordered Systems

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Metal-insulator transitions that occur in the limit of zero temperature in a variety of electronically disordered solids as a function of composition, dopant concentration, magnetic field, stress, or some other tuning parameter, have been the focus of study for many decades and continue to be a central problem in condensed matter physics. A full overview of so broad a field would require considerably more space than I have been allotted. I will thus limit my presentation to two specific areas that have been of particular interest to me. In the first part I will give a brief overview of the recent history and current status of the so-called “critical exponent puzzle” in three-dimensional doped semiconductors and amorphous metal-semiconductor mixtures, including some interesting recent developments. The second part will be a brief summary of new and exciting findings in dilute 2D systems, such as silicon MOSFETs and GaAs/AlGaAs heterostructures, where the resistivity exhibits a metallic temperature dependence in some ranges of electron (hole) densities, raising the possibility of an unexpected metal-insulator transition in two dimensions. An equally intriguing property of these strongly interacting, low density 2D systems is their enormous magnetoresistance: for magnetic fields applied parallel to the electron plane, the resistivity increases dramatically by several orders of magnitude in response to relatively modest fields on the order of a few Tesla, saturating to a constant value at higher fields.

I. THE CRITICAL EXPONENT PUZZLE.

Based on the Ioffe-Regel criterion, which asserts that the mean free path of an electron in a metallic system cannot be shorter than its wavelength, Mott proposed in 1972 that there exists a minimum value of the conductivity, \( \sigma_{\text{min}} = C e^2 / h a \), below which a system cannot be in the metallic phase; here \( e \) is the electron charge, \( h \) is Planck’s constant, \( a \) is the average distance between electrons at the critical concentration \( n_c \), and \( C \) was estimated by Mott to be a number between 0.15 and 0.3. The conductivity is thus expected to drop discontinuously to zero at a Mott transition driven by interactions. Some years later, the scaling theory of Abrahams, Anderson, Licciardello and Ramakrishnan for disordered systems of non-interacting electrons implied that the transition is instead a continuous one, so that arbitrarily small values of conductivity are possible in the metallic phase. Theory soon followed which showed that the electrons (or holes) generally localize even more strongly when weak interactions are included. Experiments in amorphous metal-semiconductor mixtures such as AuGe and NbSi, as well as in some doped semiconductors, most notably the elegant stress-tuning measurements by Paalanen et al. in Si:P, provided strong evidence that the transition is indeed continuous.

Following these early seminal contributions, however, the field has been plagued for nearly two decades by controversy and conflicting results. On the one hand, a consensus emerged rather quickly regarding the behavior of the amorphous metal-semiconductor mixtures: tunneling experiments documented the appearance of a square-root singularity in the density of states as the transition is approached from the metallic side, consistent with theoretical predictions for interacting electrons: the critical exponent \( \mu \) which characterizes the (continuous) approach of the conductivity to the transition, \( \sigma \approx (n - n_c)^\mu \), was found to be close to 1 in essentially all the amorphous systems examined. In contrast, conflicting results have been reported in doped semiconductors, and these materials continue to be the subject of debate and uncertainty as discussed in more detail below. It should be noted, however, that a number of investigators, most actively A. Moebius, continue to question the generally accepted view; based on a scaling analysis of the conductivity of amorphous CrSe, this school maintains that the transition is discontinuous with a minimum metallic conductivity, as originally postulated by Mott.

Electron interactions are known to play a role in the case of doped semiconductors. This was demonstrated by tunneling experiments in Si:B which yielded results very similar to those found for GeAu and NbSi: a square-root singularity develops as the transition is approached from the metallic side, and a “Coulomb gap” appears in the insulator as the dopant concentration is further decreased. However, a great deal of disagreement persists concerning the critical behavior of the conductivity, with various laboratories reporting different results on different, and sometimes even the same, semiconductor systems.
Typical curves are shown in Fig. 1, where the conductivity of Si:B is plotted as a function of temperature $T$ in frame (a), and as a function of $T^{1/2}$ in frame (b). The conductivity is given by

$$\sigma(T) = \sigma(0) + \Delta \sigma_{\text{int}} + \Delta \sigma_{\text{loc}} = \sigma(0) + mT^{1/2} + BT^{p/2}$$

The second term on the right-hand side is due to electron-electron interactions and the last term is the correction to the zero-temperature conductivity due to localization. The exponent $p$ reflects the temperature dependence of the scattering rate, $\tau^{-1} \propto T^p$ of the dominant phase-breaking mechanism responsible for delocalization, such as electron-phonon scattering or spin-orbit scattering. The last term is assumed small at very low temperatures, and the conductivity is generally plotted as a function of $T^{1/2}$, as in Fig. 1 (b). There is little theoretical justification for using this expression within the critical range, where it is applied “by default” in a region where the behavior of the conductivity is not known. We note that the slope $m$ that determines the conductivity at low temperatures changes sign from negative to positive as the metal-insulator transition is approached. The significance of this change of sign has been a matter of debate, and lies at the heart of the controversy regarding the critical behavior.

The critical conductivity exponent $\mu$ is generally determined by the following procedure. For each sample with a given concentration (or for each value of stress, magnetic field, or other tuning parameter), a single, zero-temperature extrapolated value of the conductivity is deduced from data obtained at finite temperatures. The zero-temperature
extrapolations are then plotted as a function of concentration (or stress, or field) to obtain the critical behavior, as indicated in the inset to Fig. 1 (a). This experimental protocol has yielded conflicting values for $\mu$. Important examples are shown in the next two figures.

**FIG. 2.** Data of Paalanen et al. (ref. 6) for the conductivity as a function of temperature of uniaxially stressed Si:P very near the metal-insulator transition; the inset shows zero-temperature extrapolations plotted as a function of stress, yielding an exponent $\mu = 1/2$.

Figure 2 shows early classic measurements of the conductivity of uniaxially-stressed Si:P taken down to unusually low temperatures by the Bell group [6]; fitted to the square-root temperature dependence of Eq. (1), the zero-temperature extrapolations yielded a critical exponent $\mu = 1/2$ (see the inset). If the correlation length exponent $\nu$ is equal to the conductivity exponent $\mu$, as expected within Wegner scaling [9], this violates a lower bound $\nu > 2/3$ in three dimensions calculated by Chayes et al. [10]. A possible solution to the exponent puzzle was subsequently proposed by the Karlsruhe group of H. v. Lohneysen [11] based on measurements in unstressed Si:P shown in Fig. 3. Stupp et al. suggested that only those samples for which the low temperature slopes of the conductivity are positive are in the critical region and should be used to deduce the critical behavior. As shown in Fig. 3 (b), the Karlsruhe experiments yielded a much larger exponent, $\mu \approx 1.3$, based on a restricted range of dopant concentrations very near a critical concentration that is assumed to be substantially smaller than the generally accepted value. The critical exponent near 1/2 obtained by the Bell group was attributed by these authors to the improper inclusion of samples outside the critical region. The Bell group strongly disputed this claim, and attributed the large Karlsruhe exponent to inhomogeneities that cause “rounding” near the transition [12,13]. Indeed, the unknown breadth of the critical region had been a source of some concern, and the Karlsruhe ansatz offered a relatively simple and attractive solution to a vexing problem. At the same time, however, carefully executed investigations by Itoh et al. [14] of the conductivity of neutron transmutation doped Ge:Ga on both sides of the metal-insulator transition provided strong...
FIG. 3. Data of Stupp et al. (ref. 11) for the conductivity as a function of temperature of a series Si:P samples with closely spaced dopant concentrations very near the metal-insulator transition. (b) The conductivity extrapolated to zero-temperature as a function of dopant concentration. Stupp et al. (1993) claim that restricting the analysis to samples whose resistivities have positive slopes at low temperatures yields a true critical conductivity exponent \( \mu \approx 1.3 \) (solid line), while including points further from the transition (claimed to be outside the critical region) results in an apparent exponent \( \mu \approx 1/2 \) (dashed line).

evidence for the smaller exponent around 1/2. To complicate matters further, Castner [15] contended that some of the positive-slope curves that were classified as metallic actually obey Mott variable-range hopping, placing these on the insulating side of the transition. The few who were directly involved in the issue divided rather sharply into opposing camps, while the rest of the community began to lose interest in a problem that was making little apparent headway.

It has been difficult to obtain reliable determinations of the critical behavior for a number of reasons. One important issue is whether the distribution of dopant atoms is statistically random. This problem can be minimized by doping through neutron transmutation, as has been done in Ge:Ga [4]. Another difficulty is that the approach to the transition is most often controlled experimentally by varying the dopant concentration, \( n \), near its critical value, \( n_c \), a method that entails the use of a discrete set of carefully characterized samples. This makes it difficult to do systematic, controlled studies on closely spaced samples very near the transition within the critical regime. This problem has been circumvented in a few studies where individual samples have been driven through the transition using a different tuning parameter such as uniaxial stress or magnetic field. The central problem, however, is that zero-temperature conductivities deduced from extrapolations from finite temperature measurements are uncertain and unreliable, particularly in the absence of any theory known to be valid in the critical region.
FIG. 4. For different values of the tuning parameter, $S$, the normalized conductivity, $\sigma/\sigma_c$, of uniaxially stressed samples of Si:B is shown on a log-log scale as a function of the scaling variable $[(\Delta S)/S_c]/T^{1/\nu}$, with $\nu = 3.2$. Here $\Delta S = (S - S_c)$, where $S_c$ is the critical stress; the critical temperature dependence at the transition is $S_c \propto T^{1/2}$. The inset shows the unscaled conductivity as a function of temperature on a log-log scale for different values of stress.
A full scaling analysis which uses data obtained at all temperatures obviates the need for extrapolations to zero $T$. Attempts to apply finite-temperature scaling to the conductivity of crystalline doped semiconductors had largely failed until quite recently (except in the presence of an externally applied magnetic field). Application by Belitz and Kirkpatrick [17] of finite temperature scaling to data for Si:P gave satisfactory results only over a severely restricted range of temperature, and yielded critical conductivity exponents $\mu = 0.29$ for the Bell data, and $\mu = 1$ for the Karlsruhe data [13]. No data were available from either group for the insulating side of the transition; as shown below, the availability of data on both sides of the transition imposes important constraints.

Potentially important progress on this question was recently achieved in experiments on stress-tuned Si:B [18]. As shown in Fig. 4, full scaling of the conductivity with temperature and stress of the form

$$\sigma(n, T) = \sigma(n_c, T)F(\Delta n/T^{1/2})$$

was demonstrated on both sides of the transition. Here $\sigma(n_c) \propto T^{\nu/z}$, $n_c$ is the critical concentration, $\Delta n = (n - n_c)$, $\nu$ is the critical exponent that characterizes the divergence of the length scale, and $z$ and $\mu$ are the dynamical exponent and critical conductivity exponent, respectively. On a log-log scale, the inset to Fig. 4 shows the (unscaled) conductivity for various values of stress. The critical curve is denoted by the straight corresponding to a power law; the temperature dependence at the critical point is found to be $\sigma \propto T^{1/2}$ in stressed Si:B. It is worth emphasizing again that the power of this method lies with the fact that all the data obtained at all temperatures are used in the scaled curves of Fig. 4, rather than a single zero-temperature extrapolation for each stress deduced from a full curve of the conductivity as a function of temperature. This imposes more stringent constraints, particularly when data are available on both sides of the transition, and yields critical exponents that are far more reliable and robust.

It is puzzling that the experiment on stress-tuned Si:B yielded a critical conductivity exponent $\mu = 1.6$ considerably larger than any previous determination. Instead of answering old questions, this raises new ones. For example, does
the use of stress as a tuning parameter yield the same physics as varying the concentration, as had always been assumed? Although it has not resolved the controversy regarding the value of $\mu$, the Bogdanovich et al. experiment demonstrated that finite temperature-scaling can be applied. This has triggered a number of new attempts to use the same method in other cases. Among these are reports from Karlsruhe \[19\] of scaling for stressed Si:P (yielding $\mu = 1$), and by Itoh’s group in Ge:Ga \[20\].

I will close this section by showing some surprising results of a reanalysis of old data taken at City College on unstressed samples of Si:B. Encouraged by the full scaling form that was successively applied to stressed Si:B, an equivalent analysis was attempted for data obtained earlier by Peihua Dai where dopant concentration rather than stress was used to tune through the transition. As mentioned earlier, in the case of stressed Si:B the critical curve exhibits a power-law dependence on temperature, $\sigma \propto T^{1/2}$; more generally, the critical temperature dependence of the conductivity has been reported in various different semiconductor systems as either $T^{1/2}$ or $T^{1/3}$. Although the data can be manipulated to yield scaled curves for either the metallic or the insulating branches, neither of these choices for the critical T-dependence yields scaling on both sides of the transition for unstressed Si:B. Surprisingly, the full scaling that was obtained for the stressed samples appears not to hold for the unstressed case. On the other hand, as shown in Fig. 5, all the data obtained for insulating samples down to 0.75$n_c$ (which is clearly outside the critical region) collapse onto a single curve if one chooses to plot $\sigma$ itself rather than $\sigma/T^x$ with $x = \mu/z\nu = 1/2$ or 1/3. The conductivity for concentrations ranging from 0.75$n_c$ to $n_c = 4.38 \times 10^{18}$ cm$^{-3}$ collapses onto a single curve; although three positive-slope samples (the lowest three curves of Fig. 1) are included that have generally been assumed to be on the metallic side of the transition, further careful work is required to determine whether the collapse holds or whether it breaks down very near the transition. The conductivity of unstressed Si:B on the insulating side of the transition is thus given by $\rho = \rho_0 F(T^*(n)/T)$ with $T^* \propto (n_c - n)^{\nu}$, $F(0) = 1$, and a prefactor $\rho_0$ that is independent of temperature $T$ and dopant concentration $n$; this implies a resistivity $\rho = \rho_0$ that is independent of temperature at the critical point $n = n_c$. Deep in the insulating phase the conductivity obeys exponentially activated Efros-Shklovskii variable-range hopping, $\rho = \rho_0 \exp[-(T^*/T)^{1/2}]$.

It is remarkable that, despite considerable effort over a period of decades, a complete and satisfactory understanding of the behavior of doped semiconductors near the metal-insulator transition has not yet emerged; this remains one of the most interesting and important open questions in condensed matter physics.

II. NOVEL PHENOMENA IN DILUTE 2D SYSTEMS: NEW PHYSICS OR OLD?

While we continue our efforts to understand the metal-insulator transition in three dimensions, new developments in disordered, dilute two-dimensional systems have opened an entirely new area of exciting physics, launched by the availability of silicon MOSFETs with unusually high mobilities fabricated in the (former) Soviet Union. These samples allowed measurements at substantially lower densities than had been accessible earlier, a regime where interaction energies are very large compared with the Fermi energy. The experimental findings have called into question our availability of silicon MOSFETs with unusually high mobilities fabricated in the (former) Soviet Union. These samples.

The resistivity of a high-mobility silicon metal-oxide-semiconductor field-effect transistor (MOSFET) is shown in Fig. 6 (a) as a function of density at different temperatures, and in Fig. 6 (b) as a function of temperature at different densities. The crossing point in frame (a) indicates a critical density $n_c$ below which the behavior is insulating, and above which the resistivity decreases with decreasing temperature, behavior that is normally associated with a metal (see frame (b)). The curves shown in Fig. 1 (b) as a function of temperature $T$ can be collapsed onto two branches by applying a single scaling parameter $T_\alpha$, a feature generally associated with quantum phase transitions. These claims first met with considerable skepticism, but were soon confirmed in MOSFETs obtained from different sources, and similar behavior was subsequently found for other two-dimensional systems (p-GaAs, n-GaAs, p-SiGe, etc.).

As is true for many other interesting open questions in condensed matter physics, both interactions and disorder play a role, and their relative importance is unclear: (i) the transition from insulating to metallic temperature-dependence occurs at very low electron (hole) densities ($\approx 10^{11}$ cm$^{-2}$ or lower), where interaction energies are much larger than kinetic energies; (ii) the resistivity is on the order of $\hbar/e^2$, suggesting that disorder plays a role.
FIG. 6. (a) Resistivity as a function of electron density for the two-dimensional system of electrons in a high-mobility silicon MOSFET; different curves correspond to different temperatures. (b) Resistivity as a function of temperature; here different curves are for different electron densities.

FIG. 7. For different electron densities, the resistivity of a silicon MOSFET at 0.3 K as a function of magnetic field applied parallel to the plane of the 2D electron system. The top three curves are insulating in zero field while the lower curves are conducting.
A second important feature of these dilute 2D systems is their dramatic response to external magnetic fields applied parallel to the plane of the electrons [23]. As shown in Fig. 7, the resistivity increases by several orders of magnitude with increasing field and saturates to a constant plateau value above a density-dependent magnetic field on the order of 2 or 3 Tesla. The magnetoresistance is larger at lower temperatures and in higher mobility samples. It should be noted that the curves of Fig. 7 span densities that have both metallic and insulating temperature dependence in zero field. The dramatic field-dependence thus appears to be a general feature of these dilute two-dimensional electron systems that is distinct from the temperature-dependence.

A lively debate has ensued concerning the significance of these findings: whether they represent fundamentally new physics or whether they can be explained by an extension of physics that is already understood. A view held by many is that these features signal a true zero-temperature quantum phase transition to a novel ground state at T=0 (such as a “perfect” metal, a superconductor, a spin liquid, a Wigner glass, etc.) [24]. Others argue that the anomalous metallic behavior can be explained within a single-particle description in terms of a temperature-dependent Drude conductivity, and that ”metallic” behavior is observed in a restricted range of temperatures so that localization prevails in the limit of zero temperature. Suggestions include scattering at charged traps, temperature-dependent screening, interband scattering, and thermal smearing of a percolation threshold [21].

The high mobilities that are now attainable in MOSFETs and heterostructures have opened a new area of investigation in low density 2D electron (hole) systems where interactions are very strong. Regardless of how many-body effects will be incorporated into a full description in this regime, the behavior of these materials and the physics that is emerging are new and fascinating. My closing remark as an experimentalist is that much more experimental information is needed before serious progress can be made.

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