LOW-MOMENTUM NUCLEON-NUCLEON POTENTIAL
AND HARTREE-FOCK CALCULATIONS

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A low-momentum nucleon-nucleon (NN) potential $V_{\text{low}-k}$ is derived from modern realistic NN potentials by integrating out the high-momentum modes. The smooth $V_{\text{low}-k}$ may be used as input for nuclear structure calculations instead of the usual Brueckner $G$ matrix. Such an approach eliminates the energy dependence one finds in the $G$-matrix approach, allowing this interaction to be used directly in Hartree-Fock calculations. Bulk properties of $^{16}\text{O}$ have been calculated starting from different NN potentials. Our results, obtained including up to second order contributions in the Goldstone expansion, are in good agreement with experiment.

1. Introduction

A central problem in nuclear theory has long been the calculation of nuclear structure properties starting from realistic nucleon-nucleon (NN) potentials. There are various models for these potentials, such as the Bonn$^1$ and CD-Bonn,$^2$ Paris,$^3$ Nijmegen,$^4$ and the new Idaho potential.$^5$ They all describe the observed deuteron and $NN$ scattering data very accurately. However, owing to their strong repulsive core, none of them can be used directly in nuclear structure. To overcome this difficulty, the Brueckner $G$ matrix has traditionally been the starting point, but, as is well known, its energy dependence is an undesirable feature, in particular when dealing with Hartree-Fock calculations.

In this paper, we make use of a different approach to renormalize the short-range repulsion of realistic NN potentials, which is motivated by the recent applications of effective field theory (EFT) and renormalization group (RG) to low-energy nuclear systems.$^6,7,8,9,10$
A fundamental theme of the RG-EFT approach is that the details of the short-distance dynamics are irrelevant for physics in the infrared region. One can therefore have infinite theories that differ substantially at short distance, but still give the same low-energy physics.\textsuperscript{6,10} In nuclear physics, various meson models for $V_{NN}$, sharing the same one-pion tail, give the same phase shifts and deuteron binding energy even if they differ significantly in the treatment of the shorter distance pieces.

Motivated by these considerations, we derive a low-momentum $NN$ potential $V_{\text{low}-k}^{\text{11,12}}$ by integrating out the high-momentum components of $V_{NN}$ in the sense of the RG,\textsuperscript{6,10} and use this smooth potential in nuclear structure.

In this paper, we calculate the bulk properties of $^{16}\text{O}$ in the framework of the Hartree-Fock theory, using $V_{\text{low}-k}$’s derived from different $NN$ potentials. The paper is organized as follows. In section 2 we describe our method for carrying out the high-momentum integration. Section 3 is devoted to the comparison of our results with experiment. A summary of our conclusions is given in the last section.

2. Outline of Calculations

The first step in our approach is to integrate out the high-momentum components of $V_{NN}$. According to the general definition of a renormalization group transformation, the decimation must be such that low-energy observables calculated in the full theory are preserved exactly by the effective theory. Once the relevant low-energy modes are identified, all remaining modes or states have to be integrated out.

For the nucleon-nucleon problem in vacuum, we require that the deuteron binding energy, the low-energy phase shifts, and the low-momentum half-on-shell $T$ matrix calculated from $V_{NN}$ must be reproduced by $V_{\text{low}-k}$.

The full-space nuclear Schrödinger equation may be written as

$$H\Psi_{\mu} = E_{\mu}\Psi_{\mu}; \quad H = H_0 + V_{NN},$$

(1)

where $H_0$ is the unperturbed Hamiltonian, in this case the kinetic energy. The above equation can be reduced to a model-space one of the form

$$PH_{\text{eff}}P\Psi_{\mu} = E_{\mu}P\Psi_{\mu}; \quad H_{\text{eff}} = H_0 + V_{\text{eff}},$$

(2)

where $P$ denotes the model-space, which is defined by momentum $k \leq k_{\text{cut}} = \Lambda$, $k$ being the relative momentum and $k_{\text{cut}}$ a cut-off momentum.
The half-on-shell $T$ matrix of $V_{NN}$ is defined as

$$T(k', k, k^2) = V_{NN}(k', k) + \int_0^\infty q^2 dq V_{NN}(k', q) \frac{1}{k^2 - q^2 + i0^+} T(q, k, k^2),$$

(3)

and the corresponding model-space $T$ matrix given by $V_{low-k}$ is

$$T_{low-k}(p', p, p^2) = V_{low-k}(p', p) + \int_0^\Lambda q^2 dq V_{low-k}(p', q) \frac{1}{p^2 - q^2 + i0^+} T_{low-k}(q, p, p^2).$$

(4)

Note that for $T_{low-k}$ the intermediate states are integrated up to $\Lambda$.

It is required that, for $p$ and $p'$ both belonging to $P (p, p' \leq \Lambda)$, $T(p', p, p^2) = T_{low-k}(p', p, p^2)$. In Refs. 11,12 it has been shown that the above requirements are satisfied when $V_{low-k}$ is given by the folded-diagram series

$$V_{low-k} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} + ...,$$

(5)

where $\hat{Q}$ is an irreducible vertex function, in the sense that its intermediate states must be outside the model space $P$. The integral sign represents a generalized folding operation, and $\hat{Q}'$ is obtained from $\hat{Q}$ by removing terms of first order in the interaction $V_{NN}$.

The above $V_{low-k}$ can be calculated by means of iterative techniques. In Ref. 14 some iteration methods have been proposed which are suitable for non-degenerate model spaces. The method that we use here, which we refer to as Andreozzi-Lee-Suzuki (ALS) method, consists in constructing solutions of the Lee-Suzuki type. The ALS method converges to the lowest (in energy) $d$ states of $H$, $d$ being the dimension of the $P$ space. We have found this method to be very convenient for our calculations.

Since the $V_{low-k}$ obtained by this technique is non-hermitian, we have hermitized it by using the simple and numerically convenient procedure suggested in Ref. 14.

We have verified that the deuteron binding energy and the phase shifts up to the cut-off momentum $\Lambda$ are preserved by $V_{low-k}$. By way of illustration, we compare in Fig. 1 some $^1S_0$ phase shifts calculated using a cut-off momentum $\Lambda = 2 \text{ fm}^{-1}$ (squares), with those obtained using the full Idaho potential (continuous line). It can be seen that the phase shifts from the full $V_{NN}$ are well reproduced by those obtained from $V_{low-k}$ up to the value of $E_{lab}$ corresponding to the cut-off momentum.
This evidences the physical equivalence of $V_{low-k}$ and $V_{NN}$ in the sense of RG.

An important issue is what value one should use for $\Lambda$. Guided by general EFT arguments, the minimum value for $\Lambda$ must be large enough so that $V_{low-k}$ explicitly contains the necessary degrees of freedom for the physical system under investigation. For nuclear structure calculations $\Lambda$ turns out to be around 2 fm$^{-1}$. This is consistent with the following considerations. Most $NN$ potentials are constructed to fit empirical phase shifts up to $E_{lab} \approx 350$ MeV. Since $E_{lab} \leq 2\hbar^2\Lambda^2/M$, $M$ being the nucleon mass, and one requires $V_{low-k}$ to reproduce the empirical phase shifts, a choice of $\Lambda$ in the vicinity of 2 fm$^{-1}$ seems to be appropriate. In our calculations we have used $\Lambda = 2.1$ fm$^{-1}$.

3. Realistic Hartree-Fock Calculations

As stated before, $V_{low-k}$ is energy independent and well-behaved, so that one can use it directly to compute the bulk properties of $^{16}$O in the framework of the self-consistent Hartree-Fock (HF) theory. The HF equations are solved using a basis of harmonic-oscillator wave-functions. The single-particle states $|i\rangle$ are expanded in a finite series of oscillator wave-functions.
\(|\alpha\rangle\)

\(|i\rangle = \sum_\alpha C^i_\alpha |\alpha\rangle .

(6)

The expansion coefficients \(C^i_\alpha\) and the oscillator parameter \(\hbar \omega\) are determined solving self-consistently the HF equations

\[
\sum_\beta \left( \langle \alpha | T | \beta \rangle + \sum_{j_h} \langle \alpha, j_h | V_{\text{low} - k} | \beta, j_h \rangle \right) C^i_\beta = \epsilon_i C^i_\alpha .
\]

(7)

Since we assume that \(^{16}\text{O}\) is a spherical nucleus, the \(|i\rangle\)'s have good orbital and total angular momentum. The Coulomb correction is included exactly by taking the total two-body interaction as \(V_{\text{low} - k}\) plus the Coulomb potential.

Once Eqs.(11) have been solved, we use the HF wave-functions to calculate both the binding energy and r.m.s. mass radius. We also evaluate the second-order contribution to the above quantities within the framework of the Goldstone linked cluster perturbation expansion\(^{15}\). In Fig. 2 we report the results obtained using all the \(NN\) potentials mentioned in the Introduction. It is worth noting that the differences between the various \(NN\) potentials are not very relevant, and that second-order corrections improve remarkably the agreement with experiment. The latter point confirms the need to go beyond a mean-field description of ground-state properties of finite nuclei\(^{18,19,20,21}\).
4. Summary and Conclusions

In the present work, we have used a new technique to derive a low-momentum $NN$ potential $V_{\text{low-}k}$ from realistic potentials by integrating out the high-momentum modes. With this smooth $V_{\text{low-}k}$ we have calculated the bulk properties of $^{16}$O in the framework of the Goldstone expansion using a self-consistent Hartree-Fock basis. The quality of the results, obtained using several modern realistic $NN$ potentials, is quite good. We conclude that this new method, that it has also been successfully employed in shell-model calculations\textsuperscript{12,22}, may contribute to a better understanding of the role of realistic $NN$ interactions in nuclear structure.

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