On Primordial Density Perturbation and Decaying Speed of Sound

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The decaying speed of sound can lead to the emergence of the primordial density perturbation in any expanding phase, even if the expansion is decelerated. Recently, some proposals have been given to implement this mechanism, in which it was found that the primordial spectrum of scalar perturbation can be scale invariant. In this note, we will give more insights for the details of this seeding mechanism.

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In principle, it seems impossible that there is a causal origin of primordial perturbations in a decelerated expansion phase. However, if the perturbation is induced by the matter content with a decaying speed $c_s$ of sound, the conclusion will be altered. In this case, the nearly scale invariant primordial perturbations can be surely obtained \cite{1, 2}, in which the parameter $z \sim a/c_s$ in the perturbation equation, where $a$ is the scale factor. In a simple case, the enough e-folding number for the primordial perturbation can be hardly acquired, however, this embarrassment can be avoided in a slightly complex example \cite{2}. Recently, the same mechanism was studied in Ref. \cite{3, 4}, in which $z \sim a/c_s$ in the perturbation equation. In Ref. \cite{3, 5}, the relation of this seeding mechanism with the varying speed of light theory \cite{1, 2} was argued. In Ref. \cite{6}, there is a discussion on the connection of the varying speed of sound with deformed special relativity. It seems that this mechanism can be interesting, since it not only provides an alternative to special relativity. It seems that this mechanism can be implemented in a possible high energy theory. How- ever, with this in the mind, it will be required, and also significant to recheck the details of this mechanism, and its compares with the observations \cite{7}.

We begin with the motion equation of curvature perturbation induced by the matter content with the speed of sound $c_s$, which is

$$ u''_k + \left( c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0, \quad (1) $$

see Refs. \cite{8, 10}, where $u_k$ is related to the curvature perturbation $\zeta$ by $u_k \equiv z \zeta_k$ and the prime denotes the derivative with respect to the conformal time $\eta$, and $z = \sqrt{\rho/(\rho + p)}$, where $\rho$ is the energy density and $h$ is the Hubble parameter. For simplicity, we will take $a \sim t^n$ and $c_s \sim t^p$ for discussion \cite{2}, where both $n$ and $p$ are constants, which are $a \sim \eta^{3/2}$ and $c_s \sim \eta^{1/2}$ after being turned to the conformal time. In this case, $\sqrt{\rho + p} \sim h$, thus we have $z \sim a/c_s$.

The general solutions of $\zeta$ given by Eq. (1) are the combination of $\zeta_k \simeq c_1$ and $\zeta_k \simeq c_2 \int \frac{d\eta}{z}$, where $c_1$ denotes the constant mode and $c_2$ denotes the mode changed with time. Both are only $c_s, k$-dependent functions. $c_2$ is actually a decreasing mode for inflation, which is also the reason that we generally do not consider it in calculating the inflationary perturbation. When the constant mode $c_1(c_s, k)$ is dominated, the spectrum of curvature perturbation is given by

$$ P_\zeta^{1/2} \simeq \frac{k^{3/2}}{\sqrt{2\pi}} |c_1(\omega)| \simeq f(n) \frac{h}{2\pi \sqrt{c_s}} (c_s k \eta)^{3/2-v}, \quad (2) $$

where $f(n) \sim \frac{m_p}{\sqrt{c_s m_s^2}}$ is only the function of $n$ and $m_p^2 = 1$ has been set. Eq. (1) is a deformed Bessel equation, not like in usual inflation model, since here $c_s$ is rapidly changed with the time. Thus in principle $v$ is determined not only by $z''/z$, but also by the evolution of $c_s^2$ before $k^2$, see Ref. \cite{2} for some relevant details. Here $v = \frac{[2p + 1 - 3n]/2[p + 1 - n]}{1 - n}$ can be obtained by the calculations. The nearly scale invariance of spectrum requires $v \simeq 3/2$, thus we have $p \simeq -2$ for all $n$. This means that during the generation of primordial perturbation the change of $c_s$ corresponds to $c_s \sim (1/t)^2 \sim h^2 \sim \rho$ \cite{3}. This result indicates that in order to have a nearly scale invariant spectrum $c_s/\rho$ must be constant, which recently

\footnote{This can be implemented in such a model of field theory. We take the Langrangian of $\varphi$ field as

$$ L = p_\varphi^2 \sqrt{X} + \frac{\mu_\varphi^2}{\sqrt{X}} - m_\varphi^2 \varphi^2, \quad (3) $$

where $X = \frac{\partial \varphi^2}{2}$. For $X \gg 1$, the term $\frac{\mu_\varphi^2}{\sqrt{X}}$ is negligible. In this case, this corresponds to the cuscuton model proposed in Refs. \cite{13, 14}. There is a scale solution for this Lagrangian, which is $h^2 \sim \rho \sim \varphi^2 \sim 1/t^2$. Thus $\varphi \sim 1/t$ can be obtained, which implies $X \sim 1 \sim 1/t^4$. Thus we have $\rho \sim \sqrt{X}$. While the speed of sound is given by

$$ c_s^2 = \frac{L_{\varphi \varphi}}{L_{\varphi} - 2XL_{\varphi X}X} \sim X. \quad (4) $$

Thus $\rho \sim c_s$ is naturally presented.}

\footnote{For the thermal fluctuation, e.g. in Refs. \cite{11, 12}, in order to assure the emergence of primordial perturbation, the decaying speed of sound has been also applied \cite{13, 14}.}
has been pointed out in Ref. [3], and actually also was implicitly showed in an earlier Ref. [4] in which one can obtain \(c_s^2 \sim \rho^2\) from its Eq.(12) for \(\alpha = 6\). From Eq.(2), one can find this ratio determines the amplitude of primordial perturbation. For any \(n\) not approaching 0 or 1, we need to have \(P \sim \rho / c_s \sim 10^{-10}\), which is given by the observations [17]. In addition, also note that when \(p = 0\), \(v \approx 3/2\) means that \(n\) must approach infinite, thus the usual results for inflation are recovered. In this case we have \(f(n) \approx 1 / \sqrt{2} a\) since here \(\epsilon = 1 / n\), thus Eq.(3) will exactly equals to that of k-inflation [9, 10].

The \(c_1\) mode is used to calculate the resulting spectrum is reasonable only when the \(c_2\) mode is the decreasing mode. Thus it is required to examine whether here or in what case the \(c_2\) mode is decreased, which was not done in previous references. The integral \(\int \frac{d\eta}{\eta'}\) may be straightly calculated, since \(z \sim a / c_s\) is the function of \(\eta\). The result indicates that in order to make the \(c_2\) mode decreased with the time, we need to impose the condition \(2p < 3n - 1\). Thus for a given \(n\), if the constant mode is dominated, there must be an upper bound for \(p\). In order to have the scale invariant spectrum we need \(p \approx -2\), which means that for \(0 \lesssim n < 1\) the condition \(2p < 3n - 1\) is always satisfied here. Thus the validity of Eq.(2) can be assured.

The emergence of primordial perturbation can be explained as follows. In the regime \(c_s k \eta \gg 1\), we have \(a / k \ll c_s / h\). Thus though \(a / k \gg 1 / h\), i.e. the physical wave length of perturbation mode is larger than the horizon, it is actually smaller than the sound horizon \(c_s / h\) since the speed \(c_s\) of sound is large. Thus a causal relation can be established on superhorizon scale. When \(c_s\) is decreased the corresponding mode will leave the sound horizon and can be able to be responsible for the seed in observable universe. In this case the horizon \(1 / h\) in the conventional consideration should be replaced by the sound horizon \(c_s / h\). The scale invariant spectrum requires \(c_s \sim h^2\). Thus this replacement can be equally written as \(1 / h \rightarrow c_s / h\), since \(c_s / h \sim h\). We generally have \(a \sim \left(\frac{1}{n}\right)^{\frac{n+1}{2n}} \sim \left(\frac{c_s}{ah}\right)^{\frac{1}{1+n}}\). Thus Eq.(3) in Ref. [2] is replaced as

\[
\ln\left(\frac{c_s}{ah}\right) = -\frac{1}{n} + 1 \ln a, \quad (5)
\]

which is plotted in Fig.1 as the black dashed lines. Thus in principle the rapid decaying of \(c_s\) is included for a region of decelerated expansion, see the blue region below the \(\ln a\) axis, corresponds to map this region to a dual superinflation region, see the same blue one beyond the \(\ln a\) axis, by a dual replacement \(1 / h \rightarrow c_s / h\). The results in Eq.(3) in Ref. [2] and Eq.(5) are just a reflection of this duality, which has been plotted in Fig.1. This duality is not illustrated in Ref [2]. For example, for an expansion dominated by radiation, we have \(n = 0.5\) and thus \(\ln (1 / ah) = \ln a\), see the black thick solid line in Fig.1. When \(c_s \sim h^2\) is introduced, with Eq.(5), we have \(\ln (c_s / ah) = -3 \ln a\), see the black dashed line in Fig.1.

The efolding number can be defined as the ratio of the physical wavelength corresponding to the present observable scale to that at the end time of the generating phase of perturbations, which in general is actually not the eefolding number of scale factor, but is that of primordial perturbation. The efolding number is

\[
N = \ln\left(\frac{c_s}{c_s(\epsilon)} \cdot \frac{k_{\epsilon}}{k}\right) = (n + 1) \ln\left(\frac{h}{h_{\epsilon}}\right), \quad (6)
\]

where the subscript ‘\(\epsilon\)’ denotes the end time of the generating phase of perturbations, and \(h \sim 1 / a^{\frac{\epsilon}{2}}\) has been used. Here \(k = ah\) and \(c_s \sim h^2\), thus during this seeding mechanism \(N\) is actually equal to the change of \(\ln (h / a)\), which is consistent with Eq.(5) and also the black dashed line in Fig.1. The prefactor of Eq.(6) is distinguished from that of Eq.(12) in Ref. [2]. The reason of difference is that there we straightforwardly introduce a scalar field with separating time derivative and space derivative, which leads to that \(z \sim a\) defined in Eq.1 is independent of \(c_s\), while here \(z \sim a / c_s\). In Ref. [2], it was showed that in a simple case the corresponding model can hardly have enough eefolding number. We will see, however, that this difficulty dose not exist here. The key point is here to obtain the scale invariance of spectrum \(c_s\) has to decrease more rapidly than that in Ref. [2], which thus will lead
FIG. 2: The figure of the log($\frac{m_p}{T_{e}}$) with respect to $n$ showing how enough efolding number is obtained. The solid line is for the case that the energy scale $\sim m_p$ at the beginning time of phase generating the primordial perturbations and the dashed line is that of the energy scale $\sim m_p/10^3$. The region above the corresponding line is that with enough efolding number.

In Eq. (6), the resulting $\mathcal{N}$ depends on the ratio $h$ to $h_e$, which must be large enough to match the requirement of observable cosmology. $\mathcal{N}$ required is generally determined by the evolution of standard cosmology after reheating. Here the “reheating” means that the matter content generating the primordial perturbation, e.g. that in footnote 3, decays into radiation. In principle the lower $T_e$ is, where $T_e$ is the reheating scale, the smaller the efolding number required is. For an idealistic case, in which after the generating phase of perturbations ends the universe will rapidly be linked to an usual evolution of standard cosmology, $\mathcal{N} \simeq 68.5 + 0.5 \ln(h_e/m_p)$ can be obtained [18], which actually approximately equals to $\mathcal{N} \simeq 36 + \ln(\frac{T_e}{T_{ev}})$ given by Ref. [17]. We can substituting it into Eq. (6) to cancel $\mathcal{N}$, and obtain a relation between $h_e$ and $n$. We plot Fig.2 in which for various $n$ in the region $0 \lesssim n < 1$, the log($\frac{m_p}{T_{e}}$) required for enough efolding number is given, where $m_p$ is the Planck scale. We can see from Fig.2 that when taking the initial energy scale as the Planck scale or grand united scale, enough efolding number can be easily obtained, since in principle $T_e$ may low to the nucleosynthesis scale in which log($\frac{m_p}{T_{e}}$) $\simeq 22$ while in above two cases log($\frac{m_p}{T_{e}}$) $\lesssim 13$. For example, for $n = 0.5$, we have $T_e \sim 10^{12}$ Gerv for $m_p$ and $T_e \sim 10^{10}$ Tev for grand united scale, respectively.

In conclusion, it is rechecked that the primordial density perturbation can be generated in a decelerated expanding phase with decaying $c_s$. We give more insights for this seeding mechanism, and show that, for this mechanism, the enough efolding number of primordial perturbation can be obtained. However, it should be mentioned that this mechanism only serves the generation of primordial perturbation, it can not solve all problems of standard cosmology, as has been explained in inflation scenario. Thus unless there are some other mechanisms to give the solutions of above problems, it seems inevitable that we still needs a following period of inflation. However, in the latter case, the seeding of perturbation and the stage of inflation may be actually decoupled, which might help to relax the bounds for inflation model itself leaded by the observations. In some sense, this work may be interesting for coming endeavor of embedding such a seeding mechanism into a possible fundamental theory.

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