Stable central structures in topologically nontrivial Anti-de Sitter spacetimes

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Abstract

We investigate stable central structures in multiply-connected, anti de Sitter spacetimes with spherical, planar and hyperbolic geometries. We obtain an exact solution for the pressure in terms of the radius when the density is constant. We find that, apart from the usual simply-connected spherically symmetric star with a well-behaved metric at $r = 0$, the only solutions with non-singular pressure and density have a wormhole topology. However these wormhole solutions must be composed of matter which violates the weak energy condition. Admitting this type of matter, we obtain a structure which is maintained via a balance between its cohesive tension and its repulsive negative matter density. If the tension is insufficiently large, this structure can collapse to a black hole of negative mass.
1 Introduction

Multiply-connected spacetimes are attracting an increasing amount of attention among gravitational physicists and cosmologists. Although the idea that our universe could be topologically non-trivial has been around for quite some time [1], the possibility of setting observational constraints on this topology by performing a careful search for particular correlations in the cosmic microwave background is a recent development [2]. Further interest has been spurred by the realization that domain walls in the early universe can give rise to pair-production of black holes with event horizons whose topology is non-trivial [4]. Such objects have been referred to as Topological Black Holes, or TBHs.

It is the formation of these objects that we are concerned with in this paper. In an earlier paper, we demonstrated that a dust cloud in a multiply-connected, anti de Sitter spacetime could collapse to a TBH in a process that is analogous to the usual Oppenheimer-Snyder collapse [3]. The solutions obtained matched a static exterior spacetime to a dynamic collapsing cloud. The question of whether corresponding static solutions exist was left unresolved.

In this paper, we investigate the existence of stable perfect fluid solutions in multiply-connected, anti de Sitter spacetime. The spatial sections of such spacetimes have the topology $R \times H_g$ where $H_g$ is a two-dimensional compact space of genus $g$. Such a space may be described by a metric which is either flat, spherically or hyperbolically symmetric. The spherically symmetric case is simply connected and has genus $g = 0$. The flat and hyperbolic cases are made compact via appropriate identifications in those two dimensions. Perfect fluid solutions in such spacetimes are centrally located, separated from the exterior spacetime at a constant radius. In the $g = 0$ case they correspond to a ball of fluid surrounded by a cosmological vacuum spacetime; in the $g \neq 0$ case the analogous objects may be referred to as topological stars.

We find that the only stable solutions for $g > 0$ necessitate the development of a wormhole inside the star. A constant positive matter density throughout the star necessarily implies an infinite pressure somewhere in its interior, excluding such objects as stable solutions.

The situation is considerably different if we consider matter which violates the weak energy condition. It has been shown that negative concentrations of stress-energy can collapse to black holes of negative mass provided the
cosmological constant is sufficiently large in magnitude \[5\]. We find that it is possible to construct topological (wormhole) stars with constant negative matter density and have finite negative pressure throughout. The negative pressure is a tension which acts to hold together the negative density, which is gravitationally self-repulsive.

Section 2 discusses the three spacetimes of interest, as well as their topologies. The spherically symmetric case has much in common with earlier studies, but we dropped the demand that the metric be well-behaved at the origin. Section 3 introduces the interior metrics and their Einstein equations. In section 4, the structure of the solutions for a variable matter density is found using perturbative techniques. Section 5 produces a solution for the pressure within wormholes of constant density as a function of radius. It is shown in section 6 that this must necessarily become infinite somewhere within a wormhole with constant positive density. The question of wormholes with negative matter density is addressed in section 7.

2 Fluid Topology

The universe described by the static solution presented here is an asymptotically anti de Sitter spacetime with a nontrivial topology. The exterior metric in this universe, adapted from \[6\]

\[
\begin{align*}
\left(\frac{\Lambda}{3G}\hat{R}^2 + b - \frac{2M}{R}\right) dT^2 + \frac{d\hat{R}^2}{-\frac{4}{3}R^2 + b - \frac{2M}{R}} + R^2 (d\hat{\theta}^2 + s(b, \hat{\theta})^2 d\hat{\phi}^2) \\
\end{align*}
\]

where \(T\) and \(R\) are the time and radial coordinates. \(M\) represents the mass of the star and \(\Lambda\) is the cosmological constant (where \(\Lambda > 0\) corresponds to the de Sitter case). \(\hat{\phi}\) assumes values between 0 and 2\(\pi\).

When \(b = +1\), the universe takes on the familiar spherically symmetric form, and the \((\hat{\theta}, \hat{\phi})\) sector has constant positive curvature. The cosmological constant may assume either sign, but we will only examine behavior resulting from negative values in the models explored here. When \(b = 0\), the space is flat, with \(\Lambda\) less than zero. In order to produce the central star to be
examined, a space with two sides identified is considered. The outer edge of
the star will be an identified flat plane of constant ‘radius,’ $R$. Identification
requires the edges of this plane to be geodesics, which are in this case straight
lines. The sum of the angles must be $2\pi$, yielding a parallelogram, or in the
simplest case, a square or rectangle with opposite sides identified, creating
a toroidal topology, as in figure 1A. The exterior universe will maintain this
topology with two compact and one infinite spatial dimensions.

When $b = -1$, the $(\hat{\theta}, \hat{\phi})$ sector is a space with constant negative cur-
vature, also known as a hyperbolic plane, a discussion of which is available
in Balasz and Voros [7]. Geodesics are intersections between the hyperbolic
plane and planes through the origin. A compact surface is formed from the
hyperbolic plane by identifying opposite sides of a suitable polygon whose
edges are geodesics. The polygon must have a minimum of eight sides, and
the number of sides must be a multiple of four to avoid conical singularities.
An identified polygon with $4g$ sides is of genus $g$. The genus determines the
topology of the compact space. The surface genus $g = 2$ has two ‘holes’, and
so is a double-holed doughnut or a pacifier. The surface with $g = 3$ has three
‘holes’ and has a pretzel shape, and so on, as shown in figure 1. This type of
identification of hyperbolic surfaces is described in more detail in [3].

The fluid cloud in these identified spacetimes is located in a central po-
sition, analogous to the central sphere in a spherically symmetric spacetime.
The boundary between the fluid and the exterior universe is an identified
flat or hyperbolic plane. The universe outside the fluid maintains the same
topology of the cloud boundary itself. The fluid forms a pacifier within a
pacifier with the holes lined up, or the banana cream in our doughnut, if
you will. The situation is analogous for higher genus topologies. Beyond the
cloud, the radial coordinate may range to infinity.

3 Calculations of the Einstein Equations

The standard metric of an arbitrary, static spacetime with spherical, toroidal
or hyperbolic symmetry was used in conjunction with the Einstein equations
to generate interior solutions. The metric is given by:

$$ds^2 = -F(r, b)dt^2 + H(r, b)dr^2 + r^2(d\theta^2 + s(b, \theta)^2d\phi^2),$$

(2)
Figure 1: Examples of higher genus two-surfaces. The dotted lines represent the identifications of edges. A. The torus, $g = 1$, is a representation of a square or rectangle in flat space identified in two dimensions. B. The pseudosphere, $g = 2$, is the simplest appropriate hyperbolic two-surface, an octagon, after identification. C. This $g = 3$ surface is a slightly more complicated possibility.
with \( s(b, \theta) \) as given in (3) above. Here, \( t \) is the time coordinate, \( r \) is the radial coordinate, and \( \theta \) and \( \phi \) are coordinates on a two-surface of constant positive, zero or negative curvature, where \( \phi \) has a range of 0 to \( 2\pi \). The matching of the metrics is analogous to that in [3], and is carried out under the conditions that the metrics and the extrinsic curvatures match smoothly across the boundary.

The cosmological constant will be absorbed into definitions of the density and pressure, so that
\[
\rho = \rho_m - \frac{|\Lambda|}{8\pi G}, \quad P = P_m + \frac{|\Lambda|}{8\pi G},
\]
where \( \rho_m \) and \( P_m \) are the density and pressure due to matter respectively. Since the \( \Lambda < 0 \) case is of interest here, the absolute value of \( \Lambda \) is used for the remainder of this paper. Einstein equations are therefore simply
\[
G_{\mu\nu} = -8\pi GT_{\mu\nu} \tag{4}
\]
\[
T_{\mu\nu} = P g_{\mu\nu} + (\rho + P)u_\mu u_\nu \tag{5}
\]
with the fluid 4-velocity given by \( u^\mu = -\sqrt{F(dt)^\mu} \).

This means that the components of the Einstein tensor are,
\[
G_{00} = -F\left(\frac{H'}{H^2r} - \frac{1}{r^2} (1/H - b)\right) = -8\pi \rho F \tag{6}
\]
\[
G_{11} = -H\left(\frac{F'}{FHr} + \frac{1}{r^2} (1/H - b)\right) = -8\pi P H \tag{7}
\]
\[
G_{22} = -\frac{r}{4F^2H^2} \left(2F'FH - 2H'F^2 + 2rF''FH - rF'^2H - rF'H'F\right) \\
= -8\pi Pr^2 = \frac{G_{33}}{\sinh^2 \theta} \tag{8}
\]
where the primed variables refer to the derivative, \( d/dr \).

4 Variable density solutions

Equation (3) produces the following solution for \( H(r) \):
\[
H = \frac{|\Lambda|}{3G} r^3 + br + c - \int_{r_m}^r 8\pi \rho_m r^2 \tag{9}
\]
where $c$ is a constant of undetermined sign, and $r_m$ is the minimum value of the radius. In order to examine behavior of the metric, expand about the minimum radius with the infinitesimal, $r = r_m + \epsilon$, and use a Taylor series approximation for the integral. The spatial metric then becomes

\[
 ds^2 = \frac{(r_m + \epsilon)d\epsilon^2}{A + B\epsilon + C\epsilon^2 + D\epsilon^3 + \ldots} + (r_m + \epsilon)^2 d\Omega^2. \tag{10}
\]

where $d\Omega^2$ is the appropriate angular spatial section. The behavior at small $r$ will depend on the parameters, $A$, $B$, $C$ and $D$, as well as the value of $r_m$. The parameters are given by

\[
 A = \frac{|\Lambda|}{3G} r_m^3 + b r_m + c, \quad C = \frac{|\Lambda|}{G} r_m - 8\pi \rho_m r_m - 4\pi \rho'_m r_m^2, \tag{11}
\]
\[
 B = \frac{|\Lambda|}{G} r_m^2 + b - 8\pi \rho_m r_m^2, \quad D = \frac{|\Lambda|}{3G} - \frac{8}{3} \pi \rho_m - \frac{16}{3} \pi \rho'_m r_m - \frac{4}{3} \pi \rho''_m r_m^2.
\]

Small $r$ behavior is tabulated in Table 1. A finite throat refers to the situation in which the radial coordinate reaches a minimal value at some finite distance from the outer edge of the fluid, and then expands again into another universe. In the genus $g = 0$ case a series of spheres of smaller and smaller proper radii are encountered, as an observer travels into the star. Eventually a sphere of minimum size is encountered, beyond which the spheres begin to grow once more. In the genus $g > 0$ case, the situation is the same, except that the spheres are replaced with pseudospheres (or planes) which are identified under the action of a discrete isometry group, as described in section 2. Hence the spacetime within the fluid has a wormhole structure, and may be matched at each end of the wormhole (where $r = R$) to an exterior spacetime whose metric is given by (1).

In the infinite trumpet, the wormhole is infinitely long; i.e. the center of the star is an infinite proper distance from its surface. The infinite hourglass is a wormhole of infinite proper length connecting two spaces where the radius may grow to infinity, i.e. two universes.

From table 1, it is readily apparent that the interesting cases, the finite throats, are those for which the behavior of the metric at small $r$ is independent of $\rho'$. We will next examine the constant density case in more detail to see if the solutions are indeed stable.
| $r_m = 0$ | $r_m \neq 0$ |
|---|---|
| cusp | finite throat |
| no solution | finite throat |
| infinite hourglass | infinite trumpet |

| Ricci scalar | $2b(\frac{2}{3}c\ell)^{-4/3}$ | $\frac{b}{r_m^2}$ |
|---|---|---|
| $A \neq 0$ | $A = 0, B \neq 0$ | $A = B = 0, C \neq 0$ |
| no solution | no solution | infinite hourglass |

Table 1: A summary of small $r$ behavior for stars with variable $\rho_m$. When $r_m$ is non-zero, the Ricci scalar becomes $2b/r_m^2$, while in the cusp case, it is inversely proportional to the proper length, $\ell$, to the four-thirds power. For $A = 0$ and $B \neq 0$ (second column) when $r_m = 0$ a spherically symmetric star in an anti de Sitter spacetime may form, with the Ricci scalar vanishing at the center. For the infinite hourglass (fourth column) only $b = 0$ is allowed. In every case, the relevant nonzero constant $A, B, C, or D$ must be positive, restricting the possible values of $\rho_m$ and $|\Lambda|$.

## 5 Constant Density Stars and the Buchdahl Identity

Equation (6) leads to a solution for the function $H$, if the density is taken as constant,

$$H = \left(\beta r^2 + b + \frac{\alpha}{r}\right)^{-1}$$

in which $\alpha$ an arbitrary constant and

$$\beta = -\frac{8}{3}\pi \rho = -\frac{8}{3}\pi \rho_m + \frac{|\Lambda|}{3G}. \quad (13)$$

Here, $\rho$ is the net density. For now, we assume the matter density is positive or zero. $H$ must always be greater than zero, to preserve the signature of the metric.

If we let $F = e^{2\Phi}$, then equation (7) will be

$$\frac{d\Phi}{dr} = \frac{8\pi Pr^3 - \beta r^3 - \alpha}{2r(\beta r^3 + br + \alpha)} \quad (14)$$

The final Einstein tensor equation, (8), along with equation (12) leads to a
solution for the change in pressure as a function of $r$:

$$
\frac{dP}{dr} = -(P + \rho) \frac{d\Phi}{dr} = -\frac{(8\pi P - 3\beta)(8\pi Pr^3 - \beta r^3 - \alpha)}{16\pi r(\beta r^3 + br + \alpha)} \quad (15)
$$

In order to solve this equation, note that it can be put in the form

$$
\left[ \frac{16\pi (\beta r^3 + b + \alpha)}{r} \frac{dP}{8\pi P - 3\beta} \right] dr + \left[ \frac{1}{r^2} \left( \frac{8\pi P r^3}{8\pi P - 3\beta} - \beta r^3 - \alpha \right) \right] dr = 0. \quad (16)
$$

and the integrating factor

$$
\mu = \left( \frac{r}{\beta r^3 + br + \alpha} \right)^{3/2} \frac{-1}{8\pi P - 3\beta}. \quad (17)
$$

applied. By integrating from the outer edge of the star, where the pressure is $|\Lambda|/8\pi G$ to an arbitrary radius within the star, we find that

$$
P = \frac{\rho_m}{\sqrt{(\beta r^2 + b + \alpha/r)} - 4\pi \rho_m \sqrt{(\beta r^2 + b + \alpha/R)} \int_r^R \frac{rdr}{(\beta r^2 + b + \alpha/r)^{3/2}}} + \frac{3\beta}{8\pi}. \quad (18)
$$

The condition that $H$ be real for all radii, forces the pressure to be everywhere real.

The Buchdahl identity is normally found by demanding the central pressure be finite in a simply connected spacetime, where $g = 0$. Furthermore $\alpha$ is assumed to vanish, allowing the metric to be well-behaved at $r = 0$. In this case it is straightforward to explicitly carry out the integral in (18) to obtain

$$
P = \rho \left[ K \sqrt{1 - \frac{8}{3}\pi \rho R^2} - \sqrt{1 - \frac{8}{3}\pi \rho r^2} \right],
K = \frac{P(R) + \rho}{3P(R) + \rho} = \frac{4\pi \rho_m}{4\pi \rho_m + |\Lambda|/G}. \quad (19)
$$

The pressure should be positive definite, meaning

$$
\frac{1}{3} \leq K \sqrt{1 - \frac{8}{3}\pi \rho R^2} < 1 \quad (20)
$$
The right hand inequality constrains $K$ most strongly at the edge of the cloud, and the left at the center. This second constraint may be translated as a limit on the mass in terms of the radius,

$$M = \int_0^R 4\pi \rho r^2 \, dr > \frac{9K^2 - 1}{18K^2} R$$

(21)

which reduces to the familiar $M > 4R/9$ limit when $\Lambda = 0$. By demanding that $\frac{dP_m}{d\rho_m} \geq 0$, that $M(0) = 0$ and that the pressure was non-negative and bounded everywhere, Hiscock was able to obtain the stronger constraint

$$M/R \leq \frac{2}{9} \left[ 1 - \frac{3|\Lambda|}{4G} R^2 + \left( 1 + \frac{3|\Lambda|}{4G} R^2 \right)^{1/2} \right]$$

(22)

for the $b = +1$ case. [9]

We will not demand $\alpha = 0$. The $b = +1$ case has already been examined for nonzero $\alpha$, in which stable stars with $g = 0$ may form, by Hiscock. Consider now the higher-genus cases. When $g = 0$, $b = 0$ and the parameter $\beta$ is forced to be positive. When $g \geq 2$, $b = -1$, implying $\beta > 1/R^2$. The equation for the pressure becomes, for any genus,

$$P = \rho \left( \frac{\sqrt{\beta r^2 + b} - K \sqrt{\beta R^2 + b}}{3K \sqrt{\beta R^2 + b} - \sqrt{\beta r^2 + b}} \right),$$

(23)

provided $\alpha = 0$. The analogous Buchdahl identity, found by demanding that the pressure is positive definite, is then

$$1/3 < K \sqrt{\frac{\beta R^2 + b}{\beta r^2 + b}} \leq 1$$

(24)

for all $r$.

When $R = r$, the left hand equality demands that $|\Lambda|/G < 8\pi \rho_m$ but maintaining a metric with the correct signature throughout requires $|\Lambda|/G > 8\pi \rho_m$. This contradiction rules out the possibility of a genus $g > 0$ stable star with $\alpha = 0$ and positive pressure everywhere.

6 Constant density solutions

When $\rho$ is constant, the parameters $A$, $B$, $C$ and $D$ from section 4 may still be nonzero. The behavior of the solution will depend on the character of the
lower cutoff of the positive region of $\beta r^3 + br + \alpha$ being examined. If that function is cut off by a double or triple root, an infinite throat will result. If the region in which the function if positive is ended by the $y$-axis, a cusp will be formed. If, however, the function has a non-degenerate, positive root as the lower bound to its positive region, the star will have a finite throat, with a minimum radius given by that root.

None of these wormholes will have a well behaved pressure for a positive matter density. Note that we can rewrite (18) as

$$P_m = \frac{\rho_m}{\sqrt{\frac{(\beta r^2 + b + \alpha/r)}{(\beta R^2 + b + \alpha/R)}}} \int_r^R \frac{\tilde{r}d\tilde{r}}{\left(\beta \tilde{r}^2 + b + \alpha/\tilde{r}\right)^{3/2}} - \rho_m. \quad (25)$$

The first term in the denominator varies from a value of unity at the outer radius to zero at the throat. The integral in the second term is a well-behaved positive function, so the second term will be zero at the outer radius and some positive number at the throat. To see this, approximate the behavior of the term near a single root, $r_0$.

$$\sqrt{\beta r^2 + b + \alpha/r} \int_r^R \frac{rdr}{(\beta r^2 + b + \alpha/r)^{3/2}} \approx \frac{r_0^2\sqrt{r - r_0}}{2r_0^2} + \frac{1}{2} \int_r^R \frac{dr}{(r - r_0)^{3/2}} \approx \frac{qr_0^2 + sr_0 + t}{2r_0^2}. \quad (26)$$

It is therefore unavoidable that the two terms in the denominator will become equal at some value of $r$, at which point the pressure will be infinite. This type of wormhole can never be stable. Sample plots displaying this behavior were obtained by numerical integration, and are shown in figure 2.

7 Negative matter density wormholes

Wormhole solutions have long been known to require the existence of exotic matter (matter which violates the weak energy condition) [10], so it is not surprising that this case also requires such. A study of wormhole solutions in topologically trivial spacetimes with nonzero cosmological constant also led to this conclusion, although the actual conditions on the exotic matter are
Figure 2: The pressure, as found in (25), is evaluated for the special case where the largest root of \( H(r, b) \) is a single root, \( r_m = 1 \). The equation used is
\[
H(r, b) = -\frac{1}{2} (\frac{8}{3} \pi \rho_m - |\Lambda|) r_m^2 (x - 1) [x^2 + x + 1 - b r_m^2 (\frac{8}{3} \pi \rho_m - |\Lambda|)^{-1}] .
\]
All the curves have \( |\Lambda| = 1 \), \( \rho_m = .5 \), \( R = 2 \) and \( r_m = 1 \). The dashed curve has \( b = -1 \), the solid curve has \( b = 0 \), and the dotted curve has \( b = +1 \)

modified \[11\]. The necessity of exotic matter is an unpleasant but not prohibitive situation, the Casimir effect being perhaps the best known example of a manifestation of the violation of the energy conditions.

The most likely situation in which topological black holes have physical relevance is in the early universe \[6\] is also one in which quantum fluctuations may produce (temporarily at least) regions in which the weak energy condition is violated. It is therefore natural to consider in more detail topological “stars” in which the energy conditions are violated. Indeed, a study of a dust cloud of negative energy density indicated that exotic matter may behave in a counter-intuitive manner, collapsing to form black holes \[4\].

The simplest case which requires \( \rho_m \) to be negative is that for which the parameter \( \alpha \) vanishes, as referred to earlier. The behavior of the pressure in this situation is representative of the more complicated \( \alpha \neq 0 \) cases. Here the matter density is forced to be negative in order that the metric be real. The pressure takes on the simple form
\[
P_m = -|\rho_m| \left( \frac{\sqrt{\beta r^2 + b} - \sqrt{\beta R^2 + b}}{|\rho_m| - |\Lambda|/(4\pi G)} \sqrt{\beta R^2 + b} - \sqrt{\beta r^2 + b} \right) . \tag{27}
\]
In this case the pressure is well-behaved. It vanishes at the star’s edge and decreases to a finite central value at \( r = r_m \). The pressure here is always negative, so in fact it is a tension. Generalizing the constraint from ref.\[9\] to
Figure 3: The matter pressure as a function of matter density for $b = 0, +1, -1$, represented by the solid, dotted and dashed lines. All plots use $|\Lambda|/G = 1$, $r = 1.001$, $R = 2$ and $r_m = 1$ in the equation shown for figure 2. The cutoff of the allowed region is at $\rho_m = -1.06, -1.45, \text{ and } -0.69$ for $b = 0, +1$ and $-1$ respectively.

include negative matter pressure and density yields

\[
\frac{dP_m}{d\rho_m} \geq 0 \geq \frac{dP_m}{d|\rho_m|}. \tag{28}
\]

This will guarantee that the matter pressure $P_m$ must become more negative as the matter density becomes more negative, as is a physically reasonable. This constraint may also be interpreted as a limit on the matter density, since when it becomes too negative the inequality will no longer hold throughout the star. This constraint is most easily evaluated numerically.

The behavior of the matter pressure when $\alpha$ is non-zero is qualitatively similar. The function is well-behaved, zero at the outer edge and finite at the throat. The same constraint (28) as before is employed to produce physically reasonable results. The strongest constraint arises at a radius near the throat radius, as was done in figure 3.

8 Conclusions

Although the collapse of a pressureless dust cloud of positive energy to a topological black hole proceeds in a manner somewhat analogous to the usual spherical case (with genus $g = 0$) [3], the formation of stable central
structures in topologically non-trivial anti de Sitter spacetimes differs considerably from the topologically trivial case. Indeed geometric requirements are in conflict with energy positivity requirements, implying that there are no stable central structures formed from a perfect fluid respecting the energy conditions and whose exterior metric is given by (1).

The only “stable” solutions are those in which the matter density within the star is negative, with a magnitude smaller than a critical value determined by (28). In the case the topological star consists of a fluid of gravitationally repulsive negative energy, held together by a sufficiently large tension (i.e. negative pressure). Both pressure and density are finite everywhere throughout the star. This is the reverse of a normal star, whose gravitationally self-attractive density is prevented from collapsing by its pressure. Should the pressure of the negative-mass star decrease below a certain threshold during its evolution, it will either explode due to gravitational self-repulsion or collapse to a black hole of negative mass. The former situation will occur if the magnitude of the density is sufficiently large relative to $|\Lambda|/G$. Otherwise the evolution of the star should proceed along the lines described in ref. [5], ultimately reaching a black hole of negative mass as its final state.

The most likely physical situation in which any of these scenarios is relevant is in the early universe. Topological black holes can be formed via pair-production in the presence of domain walls [6] or from the collapse of a
dust cloud (of either positive or negative density) in a topologically suitable setting \[3, 5\]. However, if it is possible to produce exotic matter in such settings, the results of this paper indicate that stable (wormhole-type) central structures can form.

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