Wilson Loops in $3d$ QFT from D-branes in $\text{AdS}_4 \times \text{CP}^3$

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Abstract: We study the Wilson loops in the three dimensional QFT from the D-branes in the $\text{AdS}_4 \times \text{CP}^3$ geometry. We find out explicit D-brane configurations in the bulk which correspond to both straight and circular Wilson lines extended to the boundary of $\text{AdS}_4$. We analyze critically the role of boundary contributions to the D2-branes with various topology and fundamental string actions.

Keywords: D-branes, AdS-CFT Correspondence
1. Introduction and Summary

Recently, Aharony, Bergman, Jafferis and Maldacena (ABJM) [1] have proposed a new class of gauge-string duality between \( \mathcal{N} = 6 \) Chern-Simons theory and type IIA string theory on \( \text{AdS}_4 \times \text{CP}^3 \). More precisely, ABJM theory has been conjectured to be dual to M-theory on \( \text{AdS}_4 \times S^7/\mathbb{Z}_k \) with \( N \) units of four-form flux which for \( 1 << N << k^4 \) can be compactified to type IIA theory on \( \text{AdS}_4 \times \text{CP}^3 \), where \( k \) is the level of Chern-Simon theory with gauge group \( SU(N) \). The ABJM theory is weakly coupled for \( \lambda << 1 \), where \( \lambda = N/k \) is the 't Hooft coupling.

In proving \( \text{AdS}_5/\text{CFT}_4 \) duality, the integrability of both the string and the gauge theory side has played a key role. The semiclassical string states in the bulk side has been used to look for suitable gauge theory operators in the dual side, in establishing the correspondence. This important observation makes one to believe that perhaps a similar structure of integrability can be employed in the recently proposed \( \text{AdS}_4/\text{CFT}_3 \) to understand it better. Indeed in \([2, 3, 4]\) there has been attempts along this and it seems quite interesting to study the semiclassical rotating strings in particular sectors of the theory (see for example \([5]-[16]\)). For example, the giant magnon solution and spike string solutions has been studied and they would certainly correspond to the trace operators in the three dimensional CFT\(^1\).

Further in the gauge theory, Wilson loop operators are non-local gauge invariant operators in gauge theory in which the theory can be formulated. In the absence of matter, the Wilson loops in Chern-Simons theory compute topological objects as knot invariants.

\(^1\)for more related work see \([17]-[37]\)
and are somewhat less interesting than in four dimensions, where they can be used as an order parameter for confinement. For theories coupled to matter, on the other hand, we expect to find a similar structure to the four-dimensional case of $N = 4$ SYM, where the definition of these operators involves the scalar fields in a non-trivial way. In fact, one defines a Wilson loop as the trace in an arbitrary representation $R$ of the gauge group $G$ of the holonomy matrix associated with parallel transport along a closed curve $C$ in spacetime. Since the beginning of the proposed AdS/CFT correspondence \cite{39}, it is known that Wilson loops in $N = 4$ SYM theory can be calculated in dual description using macroscopic strings \cite{10, 11}. This prescription is based on a picture of the fundamental string ending on the boundary of AdS$_5$ along the path $C$ specified by the Wilson loop operator. The description of this Wilson loop in terms of a fundamental string is a well established part of the AdS/CFT dictionary. In a recent interesting paper \cite{42}, a class of new open string solutions in AdS$_5$ were found which end at the boundary on various Wilson lines. It was found that these configurations arising out of the solutions to the equations of motion corresponding to fundamental strings, they describe Wilson loops in the fundamental representations.

Motivated by the recent development of AdS$_4$/CFT$_3$ duality, we would like to find out these Wilson line solutions in the AdS bulk, which will correspond to non-local gauge invariant operators in the CFT side. However, we will look at various D-branes in the AdS side and argue for the Wilson loop solutions. Some time back it was argued in very interesting paper \cite{43}, for type IIB theory, that Wilson loops have a gravitational dual description in terms of D5-branes or alternatively in terms of D3-branes in AdS$_5 \times S^5$ background \cite{2}. More precisely, in \cite{57} it was argued that a Wilson loop with matter in the rank $l$ symmetric representation is better described as a D3-brane embedded in AdS$_5$ with $l$ units of electric flux. Further, it was argued in \cite{2, 59, 60} that a Wilson loop with matter in the rank $l$ antisymmetric representation is better described by a D5-brane whose world-volume is a minimal surface in the AdS part of the geometry times an $S^4$ inside the $S^5$ and that has a support from $l$-units of world-volume electric flux.

Hence, it seems that perhaps there are D-brane representations in type IIA theory which correspond to Wilson lines in the dual 3d CFT. In fact various particle like branes has been found out in \cite{11} wrapping various cycles in CP$^3$. There are D0-branes, D2-brane wrapped $CP^1 \subset CP^3$, D4-branes wrapped on $CP^2 \subset CP^3$ and D6-brane wrapped on $CP^3$ and so on. We will however, find out solutions for the D-branes which correspond to Wilson lines in the 3d boundary theory. We study the D2-brane completely in the Euclidean AdS$^4$ in analogy with the recent paper \cite{57}. Moreover, by adding a surface term, in accordance with \cite{57} we find that this solution has similar form as D3-brane configuration in AdS$_5$ and it corresponds to line operator in $N = 4$ SYM theory. We also argue that the induced geometry on the world-volume of D2-brane is AdS$_2 \times S^1$. Moreover we also show that the action evaluated on the classical configuration vanishes and hence we can expect that the vacuum expectation value of dual line operator in 3d theory is equal to one. Then our results can be interpreted as a support for an existence of line operators in 3d theory. It

\footnote{For closely related works, see \cite{44, 13, 10, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58}.}
would be really interesting to find their explicit form.

As the next step we propose another D2-brane configurations with topology $\text{AdS}_2 \times S^1$, corresponding to the straight and circular Wilson lines. These after adding the boundary terms to cancel the divergence of the action while evaluated on a straight surface have shown to behave like Wilson lines that end on the boundary of AdS.

The rest of the paper is organized as follows. In next section (2) we review basic facts about ABJM theory. Then in section (3) we review description of Wilson loops using fundamental string in dual geometry. Then in section (4) we study the D2-brane inside the Euclidean $\text{AdS}_4$. We show that this configuration corresponds to the dual surface operator of dimension one. Then we study the D2-brane configuration which has $\text{AdS}_2 \times S^1$ topology. We find the D2-branes which correspond to straight and circular Wilson lines solutions. We study the boundary terms in both the cases. We conclude in section (5).

2. Basic Facts About ABJM Theory

The purpose of this section is to outline the basic facts about the ABJM theory. It is a 3d superconformal Chern-Simons-matter theory with explicit $N = 6$ supersymmetry and it can be interpreted as a theory describing $N$ coincident M2-branes at the singularity of the orbifold $C^4/Z_k$. This theory has following basic properties:

- Gauge and global symmetries:

  \[
  \begin{align*}
  \text{gauge symmetry} : & \quad U(N) \times \overline{U(N)}, \\
  \text{global symmetry} : & \quad SU(4).
  \end{align*}
  \]

  (2.1)

  We also denote trace over $U(N)$ and $\overline{U(N)}$ as $\text{Tr}$ and $\overline{\text{Tr}}$ respectively.

- The on-shell fields are gauge fields, together with complexified Hermitian scalars and Majorana spinors ($A = 1, 2, 3, 4$):

  \[
  \begin{align*}
  A_\mu : & \quad \text{Adj}(U(N)), \quad \overline{A}_\mu : \quad \text{Adj}(\overline{U(N)}), \quad \mu, \nu = 0, 1, 2, \\
  Y^A : & \quad (X^1 + iX^5, X^2 + iX^6, X^3 - iX^7, X^4 - iX^8) : \quad (N, \overline{N}; 4), \\
  Y^A_\dagger : & \quad (X^1 - iX^5, X^2 - iX^6, X^3 + iX^7, X^4 + iX^8) : \quad (\overline{N}, N; 4), \\
  \Psi_A : & \quad (\psi^2 + i\chi^2, -\psi^1 - i\chi^1, \psi_4 - i\chi_4, -\psi_3 + i\chi_3) : \quad (N, \overline{N}; 4), \\
  \Psi^A_\dagger : & \quad (\psi_2 - i\chi_2, -\psi_1 + i\chi_1, \psi^4 + i\chi^4, -\psi^3 - i\chi^3) : \quad (\overline{N}, N; 4).
  \end{align*}
  \]

  (2.2)

- The action of ABJM theory takes the form ($\kappa = \frac{k}{2\pi}$)

  \[
  S = \kappa \int d^3x \left[ \epsilon^{\mu \nu \lambda} \text{Tr} \left( \frac{1}{2} A_\mu \partial_\nu A_\lambda + \frac{i}{3} A_\mu A_\nu A_\lambda \right) - \epsilon^{\mu \nu \lambda} \overline{\text{Tr}} \left( \frac{1}{2} \overline{A}_\mu \partial_\nu \overline{A}_\lambda + \frac{i}{3} \overline{A}_\mu \overline{A}_\nu \overline{A}_\lambda \right) + \frac{1}{2} \overline{\text{Tr}} \left( - D_\mu Y^A_\dagger D^\mu Y^A + i \Psi^A_\dagger \gamma^\mu D_\mu \Psi_A \right) + \frac{1}{2} \text{Tr} \left( - D_\mu Y^A (D^\mu Y)^A_\dagger + i \overline{\Psi}_A \gamma^\mu D_\mu \overline{\Psi}^A \right) - V_F - V_B \right],
  \]

  (2.3)
where the covariant derivatives are defined as
\[ D_\mu Y^A = \partial_\mu Y^A + iA_\mu Y^A - iY^A \mathcal{A}_\mu, \quad (D_\mu Y)^A = \partial_\mu Y^A + i\mathcal{A}_\mu Y^A - iY^A A_\mu \] (2.4)
and similarly for fermions \( \Psi^A, \Psi^A \). Finally, the potential terms are
\[ V_F = i \text{Tr} \left[ Y^A \Psi^B \Psi_B - 2Y^A \Psi^B \Psi_B + \epsilon^{ABC} Y^A \Psi_C \Psi_B \right] - \]
\[ - i \text{Tr} \left[ Y^A \Psi_B \Psi^B - 2Y^A \Psi_B \Psi_B + \epsilon^{ABC} Y^A \Psi^B \Psi^C \right] \] (2.5)
and
\[ V_B = - \frac{1}{3} \text{Tr} \left[ Y^A Y^B Y^C Y^D - 6Y^A Y^B Y^C Y^D + 4Y^A Y^B Y^C Y^D \right] \] (2.6)

3. Fundamental string as Wilson line

In this section we review the description of Wilson loop using fundamental string in the bulk of \( AdS_4 \). We are interested in the configuration when string sigma model is embedded in \( AdS_4 \) only. Since \( AdS_4 \) can be trivially embedded in \( AdS_5 \) it is clear that many results that were derived for Wilson loops in \( N = 4 \) SYM are valid also for Wilson loops in \( N = 6 \) CS.

We begin with Nambu-Gotto action
\[ S_F = \tau_F \int d\tau d\sigma \sqrt{\det a_{\mu\nu}}, \] (3.1)
where
\[ a_{\mu\nu} = \partial_\mu X^m \partial_\nu X^n g_{mn}, \quad \mu, \nu = \tau, \sigma. \] (3.2)
and where \( \tau_F = \frac{1}{2\pi} \) in our units. In order to describe infinite Wilson line we consider an ansatz
\[ X^0 = \tau, \quad Y = \sigma, \quad X^1 \equiv X = X(\sigma) \] (3.3)
while remaining fields are constant. For this ansatz we have
\[ a_{\tau\tau} = \frac{\hat{R}^2}{\sigma^2}, \quad a_{\sigma\sigma} = g_{yy} \partial_\sigma Y \partial_\sigma Y + g_{xx} \partial_\sigma X \partial_\sigma X = \frac{\hat{R}^2}{\sigma^2} (1 + X'^2) \] (3.4)
and hence the action (3.1) takes the form
\[ S_F = \tau_F \hat{R}^2 \int d\tau d\sigma \frac{1}{\sigma^2} \sqrt{1 + X'^2}. \] (3.5)

Further, the equation of motion for \( X \) takes the form
\[ \partial_\sigma \left[ \frac{X'}{\sigma^2 \sqrt{1 + X'^2}} \right] = 0 \] (3.6)
that has clearly solution as \(X' = 0 \Rightarrow X = \text{const.}\) Then this ansatz above describes infinite long Wilson line. For this line the action takes the form

\[
S_F = \tau_F \tilde{R}^2 \int_{-T/2}^{T/2} d\tau \int d\sigma \frac{1}{\sigma^2} \sqrt{1 + X'^2} = \tau_F \tilde{R}^2 T \int_0^\infty d\sigma \frac{1}{\sigma^2} = \tau_F T \tilde{R}^2 \frac{1}{\epsilon},
\]

(3.7)

where in order to derive finite result we imposed condition that Wilson line is extended in time interval \((-T/2,T/2)\).

Let us add boundary term to the action. Recall that we have firstly evaluate this term for arbitrary \(Y\) and \(X\), then include the ansatz given above and finally evaluate it on the surface \(Y = \sigma = \epsilon\)

\[
\delta S_F = - \int_{-T/2}^{T/2} d\tau \frac{\delta \mathcal{L}}{\delta \sigma} Y = - \tau_F T g_{yy} \partial_\sigma Y \frac{\sqrt{g_{00}(\partial_\sigma X^0)^2}}{\sqrt{g_{yy}(\partial_\sigma Y)^2 + g_{xx}(\partial_\sigma X)^2}} = - \tau_F T \tilde{R}^2 \frac{1}{\epsilon}
\]

(3.8)

and we obtain familiar result that

\[
S_F + \delta S_F = 0
\]

(3.9)

that shows that the expectation value of corresponding Wilson line is \(< W > = 1\) so that this Wilson line preserves some fraction of supersymmetry. We discuss the number of unbroken supersymmetries below.

It is important property of superconformal theories that straight Wilson loop can be conformally transformed to space-like circular Wilson loop that has following string theory description. To begin with let us consider the metric in the form

\[
ds^2 = \frac{R^2}{y^2} (dy^2 + dr^2 + r^2 d\phi^2 + d(x^0)^2).
\]

(3.10)

Then we consider an ansatz

\[
Y = \sigma, \quad X^0 = \text{const}, \quad \tau = \phi, R = R(\sigma).
\]

(3.11)

Then

\[
a_{\sigma\sigma} = \frac{R^2}{y^2} ((\partial_\sigma Y)^2 + (\partial_\sigma R)^2), \quad a_{\tau\tau} = \frac{R^2 R^2}{Y^2}
\]

(3.12)

and hence the action (3.1) takes the form

\[
S_F = \tau_F \tilde{R}^2 \int d\tau d\sigma \frac{R}{Y^2} \sqrt{(\partial_\sigma Y)^2 + (\partial_\sigma R)^2}
\]

(3.13)

From this action we easily determine the equation of motion for \(R\)

\[
\frac{1}{Y^2} \sqrt{1 + (\partial_\sigma R)^2} - \partial_\sigma \left[ \frac{R}{Y^2} \frac{\partial_\sigma R}{\sqrt{1 + (\partial_\sigma R)^2}} \right] = 0.
\]

(3.14)
Then it can be easily shown that (3.14) can be solved with the ansatz
\[ R^2 = K^2 - \sigma^2, \]
for constant \( K \). Let us now evaluate the action for the solution (3.15)
\[ S_F = \tau_F \tilde{R}^2 \int_0^{2\pi} d\tau \int_k d\sigma \frac{R}{\sigma^2} \sqrt{1 + (\partial_{\sigma} R)^2} = -2\pi \tau_F \tilde{R}^2 K \left( \frac{1}{K} - \frac{1}{\epsilon} \right). \]
(3.16)

Further, we add to the action the boundary term in the form
\[ \delta S_F = - \int_0^{2\pi} d\tau \frac{\delta L}{\delta Y} Y = -2\pi \tau_F \tilde{R}^2 \frac{R}{Y^2} \frac{\partial_{\sigma} Y Y}{\sqrt{(\partial_{\sigma} Y)^2 + (\partial_{\sigma} R)^2}} = -2\pi \tau_F \frac{K}{\epsilon}, \]
(3.17)

Then we obtain that the whole action is finite and equal to
\[ S_F + \delta S_F = -2\pi \tau_F \tilde{R}^2 = -\pi \sqrt{2\lambda}. \]
(3.18)
using the fact that \( \tau_F = \frac{1}{2\pi} \) and \( \tilde{R}^2 = \pi \sqrt{2\lambda} \).

Let us now briefly review the explanation why the expectation values of the straight and circular Wilson lines are different [67], at least in case of \( CFT_4/AdS_5 \) correspondence. As was argued there the origin in the difference is in the application of the conformal transformation that maps line to the circle. In fact, in order to map the line to the circle we have to add the point at infinity to the line. Then there is a small difference in the calculation of the perturbative theory when we have to add a total derivative to the propagator. Then it was shown in [67] that the perturbative calculation on SYM side agrees with the string theory description of this Wilson line.

Let us now briefly discuss the space-time symmetries that are preserved by these classical string solutions. It can be shown that each string configuration wraps an appropriate \( AdS_2 \) submanifold in \( AdS_4 \) so that it preserves \( SL(2, R) \times SO(2) \) symmetry of isometry \( SO(2, 3) \) of \( AdS_4 \). On the other hand the string is localized at \( CP^3 \) so that it breaks the original isometry \( U(4) \) of \( CP^3 \) to \( U(1) \times U(3) \). Further, it can be shown that both these string configurations preserve 12 supercharges from 24 supercharges of original background.

With analogy with \( AdS_5/CFT_4 \) correspondence we suggest that this string configuration is dual straight and circular Wilson line in dual theory. However we leave the analysis of the properties of these objects for further works.

4. D-branes in Euclidean \( AdS_4 \times CP^3 \)

We start by writing down the metric for \( AdS_4 \times CP^3 \), which in a particular parametrization reads
\[
\begin{align*}
 ds^2 &= \tilde{R}^2 (ds^2_{AdS_4} + 4 ds^2_{CP^3}) \\
 ds^2_{AdS_4} &= -\cosh^2 \rho \ dt^2 + d\rho^2 + \sinh^2 \rho \ (d\theta^2 + \sin^2 \theta d\phi^2) \\
 ds^2_{CP^3} &= \end{align*}
\]
\[
\begin{split}
 ds^2_{CP^4} &= d\xi^2 + \cos^2 \xi \sin^2 \xi \left( d\psi + \frac{1}{2} \cos \theta_1 d\phi_1 - \frac{1}{2} \cos \theta_2 d\phi_2 \right)^2 \\
 & \quad + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) ,
\end{split}
\]

(4.1)

where
\[
0 \leq \xi \leq \frac{\pi}{2} , \quad 0 \leq \phi_i \leq 2\pi , \quad 0 \leq \theta_i \leq \pi , \quad ,i = 1, 2 ,
\]

(4.2)

and where
\[
\tilde{R}^2 = \frac{R^3}{4k} , \quad e^{2\Phi} = \frac{R^3}{k^3} .
\]

(4.3)

The 't Hooft coupling constant is \( \lambda \equiv N/k \) where \( k \) is the level of the 3-dimensional \( N=6 \) ABJM model. The relation between the parameters of the string background and of the field theory are (for \( \alpha' = 1 \))
\[
\tilde{R}^2 = \pi \sqrt{\frac{2N}{k}} = \pi \sqrt{2}\lambda .
\]

(4.4)

At the same time the two-form field strength is given by
\[
F^{(2)} = k( - \cos \xi \sin \xi d\xi \wedge (2d\psi + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2) -
\]
\[
- \frac{k}{2} (\cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\phi_1 + \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\phi_2)
\]

(4.5)

and the four-form field
\[
F^{(4)} = \frac{3R^3}{8} d\Omega_{AdS_4} ,
\]

(4.6)

where \( d\Omega_{AdS_4} \) is unit volume form of the \( AdS_4 \). There exist freedom in determination of the three-form \( C^{(3)} \) and we choose following one
\[
C^{(3)} = \frac{R^3}{8} \frac{1}{y^3} dx^0 \wedge dx^1 \wedge dx^2 .
\]

(4.7)

The dynamics of Dp-brane in general background is governed by following Dirac-Born-Infeld type of action including the Wess-Zumino term:
\[
S = S_{DBI} + S_{WZ} ,
\]

\[
S_{DBI} = -\tau_p \int d^{p+1}\xi e^{-\Phi} \sqrt{\text{det} A} ,
\]

\[
A_{\alpha\beta} = \partial_\alpha x^M \partial_\beta x^N G_{MN} + (2\pi \alpha') \mathcal{F}_{\alpha\beta} ,
\]

\[
\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - (2\pi \alpha')^{-1} B_{MN} \partial_\alpha x^M \partial_\beta x^N ,
\]

\[
S_{WZ} = \tau_p \int e^{(2\pi \alpha')\mathcal{F}} \wedge C ,
\]

(4.8)

where \( \tau_p \) is Dp-brane tension, \( \xi^\alpha , \alpha = 0, 1, \ldots, p \) are the \( (p+1) \) world-volume coordinates and where \( A_\alpha \) is gauge field living on the world-volume of Dp-brane. Note also that \( C \) in the last line in (4.8) means collection of Ramond-Ramond fields.
4.1 D2-brane

Our goal is to find D-brane description of Wilson lines in dual 3d theory. In order to do this we will consider Euclidean version of $AdS_4$ and also write it in the following form

$$ds^2_{AdS_4} = \frac{\tilde{R}^2}{y^2} (dy^2 + (dx^\mu)^2) = \frac{\tilde{R}^2}{y^2} [(dx^0)^2 + dy^2 + dr^2 + r^2 d\alpha^2], \quad \mu = 0, 1, 2, \quad (4.9)$$

where the boundary of $AdS_4$ is at $y = 0$. Let us consider following D2-brane configuration

$$x^0 = \xi^0, \quad r = \xi^1, \quad \alpha = \xi^2, \quad y = y(r),$$

$$\xi = \text{const}, \quad \theta_i = \text{const}, \quad \phi_2 = \text{const}, \quad \phi_1 = \phi_1(\alpha), \quad (4.10)$$

where $\xi, \theta_1, \theta_2, \phi_1, \phi_2$ are the coordinates of $CP^3$. This configuration should correspond to a topological two dimensional operator in the dual CFT on the boundary of the $AdS_4$.

Further, for the ansatz (4.10) the matrix $A$ takes the form

$$A_{00} = \frac{\tilde{R}^2}{y^2}, \quad A_{rr} = \frac{\tilde{R}^2}{y^2}(1 + y'^2),$$

$$A_{\alpha\alpha} = \frac{\tilde{R}^2}{y^2} \left( 1 + y^2 \left( \cos^2 \xi \sin^2 \xi \cos^2 \theta_1 + \cos^2 \xi \sin^2 \theta_1 \right) \dot{\phi}_1^2 \right), \quad (4.11)$$

where $y' = \frac{dy}{dr}, \dot{\phi}_1 = \frac{d\phi_1}{d\alpha}$. Then it is easy to see that the D2-brane action takes the form

$$S_{D2} = \tau_2 \int d^3 \zeta e^{-\Phi_0} \sqrt{\det A} - \tau_2 \int C^{(3)} =$$

$$= \tau_2 \frac{R^3}{8} \int dx^0 dr d\alpha \left( \frac{1}{y^3} \sqrt{(1 + y'^2) \left( r^2 + y^2 \left( \cos^2 \xi \sin^2 \xi \cos^2 \theta_1 + \cos^2 \xi \sin^2 \theta_1 \right) \dot{\phi}_1^2 \right) - \frac{r}{y^3}} \right).$$

(4.12)

In order to simplify the analysis we note that the equation of motion for $\theta_1$ for non-zero $\xi$ has two solutions $\theta_1 = 0$ and $\theta_1 = \pi$ and we choose $\theta_1 = 0$. Then the equation of motion for $\xi$ has solutions for $\xi = 0, \frac{\pi}{2}, \frac{\pi}{2}$, or $\xi = \frac{\pi}{4}$, and in order to find non-trivial configuration we choose $\xi = \frac{\pi}{4}$. Then the action (4.12) simplifies considerably

$$S_{D2} = \tau_2 \frac{R^3}{8} \int dx^0 dr d\alpha \left( \frac{1}{y^3} \sqrt{(1 + y'^2) \left( r^2 + \frac{y^2}{4} \dot{\phi}_1^2 \right) - \frac{r}{y^3}} \right).$$

(4.13)

Then the equation of motion for $y$ takes the form

$$- \frac{3}{y^4} \sqrt{(1 + y'^2) \left( r^2 + \frac{y^2}{4} \dot{\phi}_1^2 \right)} - \frac{d}{dr} \left[ \frac{y' \sqrt{r^2 + \frac{y^2}{4} \dot{\phi}_1^2}}{y^3 \sqrt{1 + y'^2}} \right] +$$

$$+ \frac{1}{4y^2} \sqrt{y^2 \dot{\phi}_1^2 + \frac{3r}{y^4}} = 0$$

(4.14)
while the equation of motion for $\phi$ is equal to
\[ \frac{d}{d\alpha} \left[ \frac{\sqrt{1 + y'^2} \phi_1}{y\sqrt{r^2 + \frac{y'^2}{4} \phi_1^2}} \right] = 0 \quad (4.15) \]
and it can be easily shown that the equations (4.14) and (4.15) can be solved with the ansatz
\[ y = \kappa r \, , \quad \phi_1 = 2\alpha \quad (4.16) \]
Similar half-BPS configuration was previously studied in [64] in the case of AdS$_5 \times$ S$^5$ background. The induced metric on the world-volume of D2-brane is equal to
\[ ds^2 = g_{\alpha \beta} \partial_{\alpha} \partial_{\beta} X^N = \frac{\tilde{R}^2}{\kappa^2} (1 + \kappa^2)(d\xi^2)^2 + \frac{\tilde{R}^2}{\kappa^2(\xi^1)^2} [(d\xi^0)^2 + (1 + \kappa^2)(d\xi^1)^2] \quad (4.17) \]
that clearly shows that this D2-brane configuration has a topology $AdS_2 \times S^1$. Let us now evaluate the action on this ansatz
\[ S = \tau_2 \frac{R^3}{8\kappa} 2\pi T \int_\epsilon^\infty \frac{dr}{r^2} = \frac{R^3}{8\kappa} 2\pi T \frac{1}{\epsilon} , \quad (4.18) \]
where $T$ is regularized interval in $x^0$ direction and where we have introduced a regulator $\epsilon << 1$. It is however important to stress that since we consider D2-brane that has a finite extend we should take the boundary terms into account. We will discuss the boundary contributions in more details bellow. Here we only stress that, following [57], that we should add to the action the boundary contribution in the form
\[ \delta S = - \left. \frac{\delta L}{\delta y'} \right|_{y = \kappa \epsilon} = -\tau_2 \frac{R^3}{8} 2\pi T \left. \frac{y' \sqrt{r^2 + \frac{y'^2}{4} \phi_1^2}}{y^3 \sqrt{1 + \frac{y'^2}{4} \phi_1^2}} \right|_{y = \kappa \epsilon} = -\tau_2 \frac{R^3}{8} 2\pi T \frac{1}{\kappa \epsilon} . \quad (4.19) \]
Consequently we obtain the action after adding the boundary contribution
\[ S = S + \delta S = 0 . \quad (4.20) \]
We see that the action vanishes and hence the vacuum expectation value of the dual line operator corresponding to the D2-brane explained above, is equal to one. This is a strong indication that the dual line operator is stable and preserves some fractions of supersymmetry. It would be really interesting to study these operators from the point of view of 3d QFT, following [60, 61].
4.2 D2-brane with topology $\text{AdS}_2 \times S^1$

In this section we present another example of D2-brane solution that has a topology $\text{AdS}_2 \times S^1$. Once again we start with the metric on $\text{AdS}_4$ as

$$ds^2_{\text{AdS}_4} = \frac{\tilde{R}^2}{Y^2} [dY^2 + (dX^\mu)^2], \quad \mu = 0, 1, 2.$$  \hfill (4.21)

This D2-brane can be described by the following parametrization:

$$X^0 = \xi^0, \quad Y = \xi^1 \equiv \sigma, \quad X^1 \equiv X(\sigma), \quad \xi^2 = \phi_1,$$

$$\theta = \theta_1 = \theta_2 = \psi = \phi_2 = \text{const}, \quad F_{01} = -F_{10} = F.$$  \hfill (4.22)

The DBI part of the D2-brane action is written as

$$S_{\text{DBI}} = \tau_2 e^{-\Phi_0} \int d^3 \zeta \sqrt{\text{det} A}.$$  \hfill (4.23)

Further, there is a WZ term in the following form

$$S_{\text{WZ}} = -i\tau_2 \int F \wedge C^{(1)} = -\frac{i\tau_2 k}{2} \int d^3 \zeta (2\pi\alpha') F_{01} \cos^2 \xi \cos \theta_1,$$

where the non-zero one form R-R field is

$$C^{(1)}_{\phi_1} = \frac{k}{2} \cos^2 \xi \cos \theta_1.$$  \hfill (4.24)

The fact that the WZ term in Euclidean signature contains factor $i$ implies that we have to consider imaginary electric flux so that we can write

$$A_{00} = \tilde{R}^2 \frac{1}{\sigma^2}, \quad A_{11} = \tilde{R}^2 \left(1 + X'Y^2\right), \quad A_{01} = -A_{10} = i(2\pi\alpha')F,$$

$$A_{22} = \tilde{R}^2 \left(\cos^2 \xi \sin^2 \xi \cos^2 \theta_1 + \cos^2 \xi \sin^2 \theta_1\right),$$  \hfill (4.26)

where $X' = \partial X/\partial \sigma$. Hence we obtain

$$S_{\text{DBI}} = \tau_2 \int d^3 \zeta \tilde{R}^2 \sqrt{A_{22}} \sqrt{\frac{1}{\sigma^4} (1 + X'^2) - \frac{(2\pi\alpha')^2}{R^4} F^2}.$$  \hfill (4.27)

Let us now solve the equations of motion coming from the above DBI action including the WZ term. First of all the variation of the action with respect to $\theta_1$ implies

$$\tau_2 e^{-\Phi_0} \frac{\tilde{R}^3 \cos^4 \xi \sin \theta_1 \cos \theta_1}{\sin^2 \xi \cos^2 \theta_1 + \cos^2 \xi \sin^2 \theta_1} \sqrt{\frac{1}{\sigma^4} (1 + X'^2) - \frac{(2\pi\alpha')^2}{R^4} F^2}$$

$$- \frac{\tau_2 k}{2} (2\pi\alpha') F \cos^2 \xi \sin \theta_1 = 0.$$  \hfill (4.28)

When $\cos \xi \neq 0$, the equation above is solved with $\theta_1 = 0, \pi$ and we choose for our convenience $\theta_1 = 0$. Further the equation of motion for $\xi$ implies

$$\tau_2 e^{-\Phi_0} (\cos^2 \xi - \sin^2 \xi) \tilde{R}^2 \sqrt{\frac{1}{\sigma^4} (1 + X'^2) - \frac{(2\pi\alpha')^2}{R^4} F^2} - \tau_2 k (2\pi\alpha') F \cos \xi \sin \xi = 0.$$  \hfill (4.29)
Now let us consider the equations of motion for $A_0, A_1$ that again imply the existence of a conserved electric flux $\Pi$:

\[
e^{-\Phi_0} \frac{\tilde{R}^2}{R^4} \sqrt{A_{22}} \frac{(2\pi\alpha')^2 F}{\sqrt{\frac{1}{\sigma^4}(1 + X'^2) - \frac{(2\pi\alpha')^2 F^2}{R^4}}} - \frac{2\pi\alpha' k}{2} \cos^2 \xi = \Pi \tag{4.30}
\]

Simplifying the above equations one gets:

\[
\sqrt{\frac{1}{\sigma^4}(1 + X'^2) - \frac{(2\pi\alpha')^2 F^2}{R^4}} = \sqrt{1 + X'^2} \sigma^2 \sqrt{\frac{e^{2\Phi_0}}{R^2 \sin^2 \xi} \left( \frac{\Pi}{2\pi\alpha' \cos \xi} + \frac{k}{2} \cos \xi \right)^2}
\]

Inserting the above result into the equation of motion for $\xi$ we obtain

\[
\frac{\Pi}{2\pi\alpha'} = -\frac{k}{4}. \tag{4.32}
\]

Note that this equation is obeyed for any $\xi$. Notice also that in deriving the above, we have used the fact that

\[
e^{2\Phi_0} \frac{4}{R^2} = \frac{4}{k^2}. \tag{4.33}
\]

Finally we consider equation of motion for $X$ that takes the following simple form

\[
\partial_\phi \left[ X' / \sigma^4 \sqrt{\ldots} \right] = 0. \tag{4.34}
\]

It is clear that this equation has natural solution $X' = 0 \Rightarrow X = \text{const.}$. Then the metric induced on the world-volume of D2-brane takes the form

\[
ds^2_{\text{in}} = g_{\alpha\beta} \partial_\alpha X^m \partial_\beta X^n d\xi^\alpha d\xi^\beta = \frac{\tilde{R}^2}{\sigma^2} ((d\xi^0)^2 + d\sigma^2) + \tilde{R}^2 \sin^2 \xi \cos^2 \xi (d\xi_2)^2
\]

that clearly has a form $\text{AdS}_2 \times S^1$.

Let us again compute the current $J^{01}$ which takes the form

\[
J^{01} = -\tau_2 \frac{1}{2\pi\alpha'} \left( \Pi + \frac{2\pi\alpha' k}{2} \cos^2 \xi \right) + \frac{\tau_2 k}{2} \cos^2 \xi = -\frac{\tau_2 \Pi}{(2\pi\alpha')} = \frac{\tau_2 k}{4}. \tag{4.35}
\]

Note that it is again proportional to the number of fundamental strings $k$.

Let us now evaluate the action on the solution given above

\[
S = \tau_2 e^{-\Phi_0} \tilde{R}^3 \int_0^{2\pi} d\phi \int_{-T/2}^{T/2} d\xi^0 \int_0^\infty d\sigma \sin \xi \cos \xi \sqrt{\frac{1}{\sigma^4}(1 + X'^2) - \frac{(2\pi\alpha' F)^2}{R^4}} + \\
+ \frac{\tau_2 k}{2} \int_0^{2\pi} \int_{-T/2}^{T/2} dt \int_0^\infty d\sigma (2\pi\alpha') F \cos^2 \xi
\]
\[ \frac{2\pi T \tau_2 e^{\Phi_0} \tilde{R}}{\sin \xi \cos \xi} \left( \frac{k^2}{4} \sin^2 \xi + k \frac{\Pi}{4\alpha'} + \frac{k^2}{4} \cos^2 \xi \right) \cos^2 \xi \times \right. \\
\times \int_\epsilon^\infty d\sigma \frac{\sqrt{1 + \epsilon^2 \sigma^2}}{\sqrt{1 + \frac{e^{2\Phi_0}}{R^2 \sin^2 \xi} \left( \frac{\Pi}{2\pi\alpha'} + \frac{k}{2} \cos \xi \right)^2}} \\
= \frac{2\pi T \tau_2 e^{\Phi_0} \tilde{R} k^2}{4 \cos^2 \xi \frac{1}{\epsilon}}. \]  

Where deriving above we have used the following identities
\[ \frac{e^{2\Phi_0}}{R^2} = \frac{4}{k^2}, \quad 1 + \frac{e^{2\Phi_0}}{R^2 \sin^2 \xi} \left( \frac{\Pi}{2\pi\alpha'} + \frac{k}{2} \cos \xi \right)^2 = \frac{1}{4 \sin^2 \xi \cos^2 \xi} \]  

Notice that this action derived above is proportional to the world volume electric flux \( \Pi \).

### 4.3 Boundary terms

The D2-brane solution that we found corresponds to D2-brane that extends all the way to the boundary of AdS\(_4\) and ends there along a one-dimensional curve. Since this D2-brane action has finite extent we have to discuss the possibility of adding boundary terms to the action. These boundary terms should not change the equations of motion and hence the solution will still be the same, but the value of the action when evaluated at this solution will in general depend on the boundary terms.

As it is well known that when we calculate the Wilson loop using string surfaces the bulk action is divergent but this divergence can be canceled by boundary term as we will review in the Appendix. Explicitly, the string that corresponds to Wilson loop has to satisfy three Dirichlet boundary conditions on the three directions parallel to the boundary of AdS\(_4\), and the seven Neumann ones that combine the radial coordinate of AdS\(_4\) and coordinates along CP\(_3\) and hence it is the appropriate action assuming that we have Dirichlet boundary conditions. So we have to add appropriate boundary terms that change the boundary conditions.

In case of the fundamental string we have the coordinate \( Y \) with Neumann boundary condition (before imposing the static gauge). Then it is natural to define \( p_Y \) as the momentum conjugate to it
\[ p_Y = \frac{\delta S}{\delta \partial_\sigma Y} , \]
where \( n \) is normal derivative to the boundary. The new action including the term that changes the boundary conditions is
\[ \tilde{S} = S - Y_0 \int d\tau p_Y \]  

where the integral is over the boundary at a cutoff \( Y = Y_0 \). In fact, the variation of the original action is
\[ \delta S = \int d^2\sigma \left[ \frac{\delta L}{\delta Y} - \partial_\alpha \frac{\delta L}{\delta \partial_\alpha Y} \right] \delta Y + \int d\tau \frac{\delta L}{\delta \partial_\sigma Y} \delta Y = \int d\tau \frac{\delta L}{\delta \partial_\sigma Y} \delta Y , \]  

\[ (4.40) \]
where we used the fact that the field $Y$ obeys the equation of motion in the bulk. The boundary term above shows (since it is proportional to $\delta Y$) that on-shell action is functional of $Y$. Then including the boundary term we obtain

$$\delta \tilde{S} = \int d\tau [p_Y \delta Y - \delta p_Y Y - p_Y \delta Y] = -\int d\tau \delta p_Y Y_0$$ (4.41)

and hence the new action is functional of $p_Y$ as it should be.

Taking a lesson from the review of the boundary term for the fundamental string mentioned above we now return to D2-brane action. In case of D2-brane action there is another subtlety. The DBI action is a functional of the gauge field, but the Wilson loop observable should depend on the variable $\Pi$. Then in order to find correct form of the action we have to add to it following boundary term

$$\delta S = -\int dt d\phi \left[ \frac{\delta L}{\delta \sigma} Y Y + \frac{\delta L}{\delta \sigma} A_0 \right] = -2\pi T \left( \frac{\delta L}{\delta \sigma} Y + 2\pi T \tau_2 \int_{\epsilon}^{\infty} d\sigma \Pi F_{0\sigma} = -\tau_2 e^{\Phi_0} 2\pi T \tilde{R} \frac{k^2}{4} \cos^2 \xi \frac{1}{\epsilon}, \right)$$ (4.42)

where $\frac{\delta L}{\delta \sigma} A_0 = -\Pi = \text{const}$. Now if we collect all these terms together we obtain

$$S + \delta S = 0 \quad \text{(4.43)}$$

We find the straight Wilson line has vanishing action after adding the boundary term which is the same result as in case of $AdS_5/CFT_4$ correspondence.

### 4.4 Circular Wilson line

In order to find this solution we consider following form of $AdS_4$ metric

$$ds^2_{AdS_4} = \frac{1}{y^2} (dy^2 + dr^2 + r^2 d\phi^2 + d(x^0)^2)$$ (4.44)

and consider the Wilson line that is sitting at $x^0 = 0$. Our goal is to find Wilson line that for $y = 0$ takes the form of circle with $r^2 = R^2$. In order to find such a configuration we consider an ansatz

$$\sigma^0 = \phi, \quad y = \sigma^1 = \sigma, \quad r = r(\sigma), \quad F_{01} = -F_{10} = iF, \quad \phi^2 = \xi^2, \quad \theta_1 = \theta_2 = \psi = \phi_1 = \text{const}.$$ (4.45)

Then we easily obtain

$$A_{00} = \tilde{R}^2 \frac{r^2}{y^2}, \quad A_{11} = \tilde{R}^2 \frac{1}{y^2} (1 + r^2),$$
$$A_{01} = i(2\pi \alpha')F, \quad A_{10} = -i(2\pi \alpha')F,$$
$$A_{22} = \tilde{R}^2 (\cos^2 \xi \sin^2 \xi \cos^2 \theta_1 + \cos^2 \xi \sin^2 \theta_1),$$ (4.46)
where $r' = \partial_\sigma r$. Then we obtain
\[ \det A = \tilde{R}^4 A_{22} \left( \frac{r^2}{y^2} (1 + r'^2) - \frac{(2\pi \alpha')^2 F^2}{\tilde{R}^4} \right) \] (4.47)
so that DBI part of the action takes the form
\[ S = \tau_2 e^{-\Phi_0} \tilde{R}^2 \int d^2\sigma dy \sqrt{A_{22}} \frac{r^2}{y^2} (1 + r'^2) - \frac{(2\pi \alpha')^2 F^2}{\tilde{R}^4} \] (4.48)
Further, there is a coupling to $C^{(1)}$ field in the form
\[ S_{WZ} = -i \tau_2 \int (2\pi \alpha') F \wedge C = -i \frac{\tau_2 (2\pi \alpha') k}{2} \int d^3\zeta F_{01} \cos^2 \xi \cos^2 \theta_1 . \] (4.49)
As in the case of straight Wilson line we start with the equation of motion for $\theta_1$ and we found that it is solved for $\theta_1 = 0$, or $\theta_1 = \pi$ and we choose for our convenience $\theta_1 = 0$. Further the equation of motion for $\xi$ implies
\[ \tau_2 e^{-\Phi_0} (\cos^2 \xi - \sin^2 \xi) \tilde{R}^3 \sqrt{\frac{r^2}{y^2} (1 + r'^2) - \frac{(2\pi \alpha')^2 F^2}{\tilde{R}^4}} - \tau_2 k (2\pi \alpha') F \cos \xi \sin \xi = 0 . \] (4.50)
Further, the equations of motion for $A_0, A_1$ imply a conserved electric flux $\Pi$
\[ e^{-\Phi_0} \tilde{R}^2 \sqrt{A_{22}} \frac{(2\pi \alpha')^2 F}{\sqrt{\frac{r^2}{y^2} (1 + r'^2) - \frac{(2\pi \alpha')^2 F^2}{\tilde{R}^4}}} = \frac{2\pi \alpha' k}{2} \cos^2 \xi = \Pi \] (4.51)
that allows us to find
\[ \frac{2\pi \alpha' F}{\tilde{R}^2} = \frac{e^{2\Phi_0}}{R \sin \xi} \left( \frac{\Pi}{2 \pi \alpha' \cos \xi} + \frac{k}{2} \cos \xi \right) - \frac{r \sqrt{1 + r'^2}}{y^2} \left( \frac{1}{\sqrt{1 + \frac{e^{2\Phi_0}}{R^2 \sin^2 \xi} \left( \frac{\Pi}{2 \pi \alpha' \cos \xi} + \frac{k}{2} \cos \xi \right)^2}} \right) ; \] (4.52)
Inserting the above results into the equation of motion for $\xi$ we again obtain
\[ \frac{\Pi}{2 \pi \alpha'} = - \frac{k}{4} . \] (4.53)
Note that in deriving the above, we have used the fact that
\[ \frac{e^{2\Phi_0}}{\tilde{R}^2} = \frac{4}{k^2} . \]
Finally we determine the equation of motion for $r$
\[ \frac{1}{y^2} \sqrt{1 + r'^2} - \frac{d}{d\sigma} \left[ \frac{rr'}{y^2 \sqrt{1 + r'^2}} \right] = 0 \] (4.54)
Now we will argue that the ansatz $r^2 = R^2 - y^2$ solves the equation above. Indeed, using the fact that $r' = -\frac{y}{r}$ we one can check that the above equation is identically zero. Let us now evaluate the action for the D2-brane configuration on this solution

$$
S = \tau_2 e^{-\phi_0} \int d\phi d^2 y \tilde{R}^2 \sin \xi \cos \xi \sqrt{\frac{r^2}{y^2}(1 + r'^2)} + \left(\frac{2\pi \alpha'}{R^4}\right) F^2 + 2\pi \alpha' k \tilde{R}^2 \int d\phi d^2 y \left(1 - \frac{1}{R} + \frac{1}{\epsilon}\right).
$$

Further, we have the first boundary contribution at $\sigma = \epsilon$. We again proceed as in previous section for the boundary contributions. Namely, we evaluate the contribution to the action for general $y$, then insert the ansatz $y = \sigma$ and finally evaluated the action at $y = \epsilon$

$$
\delta S_Y = -4\pi^2 \tau_2 e^{\phi_0} \tilde{R}^2 \frac{k^2 R^2 - \epsilon^2}{\epsilon R}.
$$

In the same way the boundary contribution from the gauge fields takes the form

$$
\delta S_A = \tau_2 \int d\phi d^2 y \int_\epsilon^R d\sigma \Pi F_{\tau\sigma} = 4\pi^2 \tau_2 \tilde{R} e^{\phi_0} \frac{k^2}{16} (1 - 2 \cos^2 \xi) \int_\epsilon^R \frac{d\sigma}{\sigma^2} \frac{2\tilde{R}}{1} \int_\epsilon^R \frac{d\sigma}{\sigma^2}
$$

Collecting all these terms together we obtain that the divergent terms cancel as in the case of straight Wilson line. On the other hand we find finite contribution to the action in the form

$$
S + \delta S_Y + \delta S_A = -4\pi^2 \tau_2 e^{\phi_0} \tilde{R}^2 \frac{k^2}{8} = \frac{k}{4} S_{FS},
$$

where we used the convention that $\tau_2 = \frac{1}{4\pi^2}$ and where $S_{FS}$ is the fundamental string action evaluated on circular Wilson line. Borrowing the interpretation of Wilson lines in $AdS_5/CFT$ correspondence using D3-branes we can argue that our solution describes Wilson line in symmetric $\frac{k}{4}$ representation where $k$ is level of CS action. It would be certainly very interesting to study the problem whether there exists D-brane description of Wilson loops in arbitrary representations. We hope to return to these problems in future.

5. Conclusions

We have studied in this paper various D-brane configurations corresponding to Wilson loops and lines in the boundary of $AdS_4$. We have taken D2-brane as an example and have
shown various configurations of this in the Euclidean AdS$_4 \times$ CP$^3$ correspond to straight and circular Wilson line solutions. We analyzed the D2-brane which has a topology of AdS$_2 \times$ S$^1$ and corresponds to straight Wilson line gives vanishing action after adding the boundary term. We have also studied the circular Wilson line solution and show that the action gives a non-zero contribution after adding the boundary term. One can study the D4-brane which correspond to both straight and circular Wilson line in the boundary AdS$_4$, however we mention that the action will take a similar form like that of D2-brane and hence the analysis is very similar to the one performed in the present paper.

Note added: While we were finalizing to submit our paper, we came across [12] and [13] which have some overlap with our present work.

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