Problem of oscillations presence at $CP$ violation in the system of $K^0$ mesons

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Abstract

In this work there are considered two approaches to the description of $K^0, \bar{K}^0$ meson transitions into $K_S(K^0)$ mesons at $CP$ violation in weak interactions. The first approach uses the standard theory of oscillations and the second approach supposes that $(K_S, K_L)$ states which arise at $CP$ violation are normalized but not orthogonal state functions then there arise interferences between these states but not oscillations. It is necessary to remark that the available experimental data are in good agreement with the second approach. So we came to the conclusion that oscillations do not arise at $CP$ violation in weak interactions in the system of $K^0$ mesons. Only interference between two - $K_S, K_L$ states takes place there.

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1 Introduction

Oscillations of $K^0$ mesons (i.e., $K^0 \leftrightarrow \bar{K}^0$) were theoretically [1] and experimentally [2] investigated in the 50-s and 60-s. Recently there has been achieved an understanding that these processes go as a double-stadium process [3, 4, 5, 6]. A detailed study of meson mixing and oscillations is very important since the theory of neutrino oscillations is built by analogy with the theory of meson oscillations.

Previously it was supposed that $P$ parity is a well number, however, after theoretical [7] and experimental [8] works it has become clear that in weak interactions $P$ parity is violated. Then in work [9] there was an advanced supposition that in weak interactions $CP$ parity is conserved but not $P$ parity. Work [10] has reported that in $K_L$ decays with a probability of about 0.2% there is two $\pi$ decay mode that is a detection of $CP$ parity violation.

A phenomenological analysis of $K^0$ meson processes was done in work [11] (see also [12]). There non unitary transformation and non orthogonal states where used at obtaining $K_S, K_L$ states. It was supposed that these states arise at $CP$ violation. In work [13] there was considered the same process in the framework of the standard scheme (theory) of $K^0$ meson oscillations.

This work is continuation of pervious work [13]. In this work we will consider elements of the theory of $K^0$ meson oscillations at strangeness ($S$) and $CP$ violations then the case
of $CP$ violation at absence of oscillations. At the same time we will fulfill the comparative analysis of the obtained results at $CP$ violation in the above two approaches and also we will fulfill comparison these results with the available experimental data.

2 $K^o_1, K^o_2$ meson vacuum oscillations at indirect violation of $CP$ invariance with taking into account width decays

The process of $K^o_1, K^o_2$ meson vacuum oscillations at indirect violation of $CP$ invariance with taking into account width decays in detail was considered in work [13]. Therefore we are considering main elements of these oscillations.

It is clear that we have to take into account $CP$ phase $\delta$. We can do it by using the parametrization of Kobayashi-Maskawa matrix [15] proposed by L. Maiani [16]. The expressions for $U, U^{-1}$ will then have the following form:

$$U = \begin{pmatrix} \cos\beta & -\sin\beta e^{-i\delta} \\ \sin\beta e^{i\delta} & \cos\beta \end{pmatrix}, \quad U^{-1} = \begin{pmatrix} \cos\beta & \sin\beta e^{-i\delta} \\ -\sin\beta e^{i\delta} & \cos\beta \end{pmatrix}.$$  (1)

Then at $CP$ violation $K^o_1, K^o_2$ mesons have to transform into superposition states of $K_S$ and $K_L$ mesons

$$K_S = \cos\beta K^o_1 - \sin\beta K^o_2 e^{-i\delta},$$
$$K_L = \sin\beta e^{i\delta} K^o_1 + \cos\beta K^o_2,$$  (2)

and at inverse transformation we get:

$$K^o_1 = \cos\beta K_S + \sin\beta e^{-i\delta} K_L,$$
$$K^o_2 = -\sin\beta e^{i\delta} K_S + \cos\beta K_L.$$  (3)

In work [13] it was shown that

$$m_2 - m_1 \simeq m_L - m_S.$$  (4)

If we take into account that $K_S, K_L$ decay and have the decay widths $\Gamma_S, \Gamma_L$, then $K_S, K_L$ mesons with masses $m_S$ and $m_L$ evolve in dependence on time according to the following formula:

$$K_S(t) = e^{-iE_S t - \frac{\Gamma_S}{2} t} K_S(0), \quad K_L(t) = e^{-iE_L t - \frac{\Gamma_L}{2} t} K_L(0),$$  (5)

where

$$E_k^2 = (p^2 + m_k^2), \quad k = S, L.$$  

If these mesons are moving without interactions, then

$$K^o_1(t) = \cos\beta e^{-iE_S t - \frac{\Gamma_S}{2} t} K_S(0) + \sin\beta e^{-iE_L t - \frac{\Gamma_L}{2} t} K_L(0),$$
$$K^o_2(t) = -\sin\beta e^{i\delta} e^{-iE_S t - \frac{\Gamma_S}{2} t} K_S(0) + \cos\beta e^{-iE_L t - \frac{\Gamma_L}{2} t} K_L(0).$$  (6)
Then using expressions (6) and (3) the probability that meson $K_1^\circ$ produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of $K_2^\circ$ meson is given the following expression

$$P(K_2^\circ \rightarrow K_1^\circ, t) = \frac{1}{4} \cos^2 \beta \sin^2 2\beta [e^{-\Gamma s t} + e^{-\Gamma L t} - 2e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)]$$

(7),

If suppose that $\cos^2 \beta \simeq 1$ and $\sin^2 \beta \simeq \varepsilon$ then

$$P(K_2^\circ \rightarrow K_1^\circ, t) \simeq \varepsilon [e^{-\Gamma s t} + e^{-\Gamma L t} - 2e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)],$$

(8)

and $P(K_2^\circ \rightarrow K_1^\circ, t) = P(K_1^\circ \rightarrow K_2^\circ, t)$.

Then the probability that meson $K_1^\circ$ produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of $K_1^\circ$ meson is given by the following expressions:

$$P(K_1^\circ \rightarrow K_1^\circ) = [\cos^4 \beta e^{-\Gamma s t} + \sin^4 \beta e^{-\Gamma L t} +$$

$$+2\sin^2 \beta \cos^2 \beta e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)],$$

(9)

further

$$P(K_1^\circ \rightarrow K_1^\circ) \simeq [e^{-\Gamma s t} + \varepsilon^2 e^{-\Gamma L t} + 2\varepsilon e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)],$$

(10)

and probability $P(K_2^\circ \rightarrow K_2^\circ)$ is

$$P(K_2^\circ \rightarrow K_2^\circ) = [\sin^4 \beta e^{-\Gamma s t} + \cos^4 \beta e^{-\Gamma L t} +$$

$$+2\sin^2 \beta \cos^2 \beta e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)],$$

(11)

further

$$P(K_2^\circ \rightarrow K_2^\circ) \simeq [\varepsilon^2 e^{-\Gamma s t} + e^{-\Gamma L t} + 2\varepsilon e^{-\frac{(\Gamma s + \Gamma L)t}{2}} \cos((E_L - E_S)t)].$$

(11')

In the all above expressions we have to add factor $\frac{1}{2}$ since it arises from the primary $K^\circ, \bar{K}^\circ$ mesons ($K^\circ = (K_1^\circ + K_2^\circ)/\sqrt{2}, \bar{K}^\circ = (K_1^\circ - K_2^\circ)/\sqrt{2}$).

When matrix transformation is unitary then $CP$ phase in the expressions for transition probabilities is absent. In expression (1) matrix $U$ is unitary, i. e., $UU^{-1} = 1$. In principle we can use the non unitary matrix, i. e., to use matrix $U$ and for back transformation to use matrix $U^T$ instead of $U^{-1}$ ($detU = detU^T = 1$), then

$$U = \begin{pmatrix} \cos \beta & -\sin \beta e^{-i\delta} \\ \sin \beta e^{i\delta} & \cos \beta \end{pmatrix}, U^T = \begin{pmatrix} \cos \beta & \sin \beta e^{i\delta} \\ -\sin \beta e^{-i\delta} & \cos \beta \end{pmatrix}.$$

(12)

Now instead of expr. (2) and (3) we get

$$K_S = \cos \beta K_1^\circ - \sin \beta K_2^\circ e^{i\delta};$$

$$K_L = \sin \beta e^{-i\delta} K_1^\circ + \cos \beta K_2^\circ,$$

(13)
\[ K_1^* = \cos \beta K_S + \sin \beta e^{-i\delta} K_L, \]
\[ K_2^* = -\sin \beta e^{i\delta} K_S + \cos \beta K_L. \] (14)

Now if mesons are moving without interactions, then
\[ K_1^*(t) = \cos \beta e^{-iE_{St} \frac{t}{2}} K_S(0) + \sin \beta e^{-iE_{Lt} \frac{t}{2}} K_L(0), \]
\[ K_2^*(t) = -\sin \beta e^{i\delta} e^{-iE_{St} \frac{t}{2}} K_S(0) + \cos \beta e^{-iE_{Lt} \frac{t}{2}} K_L(0). \] (15)

Then using expressions (15) and (13) for the probability that meson \( K_1^* \) produced at moment \( t = 0 \) will be at moment \( t \neq 0 \) in the state of \( K_2^* \) meson we get the following expression:
\[
P(K_1^* \rightarrow K_1^*) = \left[ \cos^4 \beta e^{-\Gamma_{St}t} + \sin^4 \beta e^{-\Gamma_{Lt}t} + 2\sin^2 \beta \cos^2 \beta e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right],
\] (16)

or \( \sin^2 \beta = \epsilon \) then
\[
P(K_1^* \rightarrow K_1^*) \approx \left[ \epsilon^2 e^{-\Gamma_{St}t} + e^{-\Gamma_{Lt}t} + 2\epsilon e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right],
\] (17)

and the probability of \( P(K_2^* \rightarrow K_2^*) \) transition is
\[
P(K_2^* \rightarrow K_2^*) = \left[ \sin^4 \beta e^{-\Gamma_{St}t} + \cos^4 \beta e^{-\Gamma_{Lt}t} + 2\sin^2 \beta \cos^2 \beta e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right]
\] (18)

or
\[
P(K_2^* \rightarrow K_2^*) \approx \left[ \epsilon^2 e^{-\Gamma_{St}t} + e^{-\Gamma_{Lt}t} + 2\epsilon e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right].
\] (19)

Then the probability that meson \( K_1^* \) produced at moment \( t = 0 \) will be at moment \( t \neq 0 \) in the state of \( K_2^* \) meson is given by the following expression:
\[
P(K_2^* \rightarrow K_1^*, t) = \frac{1}{4} \sin^2 2\beta \left[ e^{-\Gamma_{St}t} + e^{-\Gamma_{Lt}t} - 2e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right] \approx \epsilon \left[ e^{-\Gamma_{St}t} + e^{-\Gamma_{Lt}t} - 2e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos((E_L - E_S)t + 2\delta) \right],
\] (20)

and \( P(K_2^* \rightarrow K_1^*, t) = P(K_1^* \rightarrow K_2^*, t) \) (the above expression has taken into account that \( \cos^2 \beta \simeq 1, \sin^2 \beta \simeq \epsilon \)).

The length of oscillations in this case is
\[
R_{LS} \simeq \frac{\gamma}{2\Delta} \equiv \frac{2\pi \hbar c \gamma}{2\Delta}.
\] (21)

where \( \Delta = m_L - m_S \) and \( \gamma \) is usual relativistic factor. The expressions (12) \div (20) were obtained using the standard technique of oscillations and then they are analogous to the expression obtained in [11, 12] at violation of orthogonality of \( K_S, K_L \) states.
The graphics of transition probabilities $K_1^o \to K_1^o$ (expr. (10) – $P(K^o, K_1^o \to K_1^o, t) \simeq e^{-t} + (0.0023)^2 e^{-t/580} + 2 \cdot 0.0023(\cos(0.477t - 0.752)) e^{-t(581/1160)}$) and $K_2^o \to K_1^o$ (expr. (8) – $P(K^o, K_2^o \to K_1^o, t) \simeq e^{-t} + (0.0023)^2 e^{-t/580} - 2 \cdot 0.0023(\cos(0.477t - 0.752)) e^{-t(581/1160)}$) in dependence on $t_S = t/\tau_S$ ($\tau_S$ is $K_S$ life time) are given in Figure 1 (where $\varepsilon = 0.0023$ [14]). And the summary graphic of expressions (8) and (10) (line) normalized to the experimental data from [14] together with experimental data from [14] (open circles) is given in Figure 2 (for primary $K^o$ mesons). From this figure we see that total transition probability to $K_1^o$ obtained in the framework of oscillations theory are placed very far from experimental data from [14]. Then we can come to the conclusion that at $CP$ violation in weak interactions oscillations do not arise. In reality at drawing Figures 1, 2 it was taken into account that there is phase $\delta = 44^o$ (i.e. we used expressions (17) and (20)).

Figure 1. Line 1 is $K_2^o \to K_1^o$ transition probability (expression (8)) and line 2 is $K_1^o \to K_1^o$ transition probability (expression (10)) at presence of oscillations at $CP$ violation in weak interactions ($\varepsilon = 0.00223$) in dependence on $t_S$ for $t_S = t/\tau_S = 1 \div 20$. 
Figure 2. Summary transition probabilities $(K_1^o \rightarrow K_1^o) + (K_2^o \rightarrow K_2^o)$ (line) when oscillations take place (exprs. (8) + (10)) normalized to experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data (open circles) from [14] for $t_S = 1 \div 20$.

Now we can consider the case when $\varepsilon' = \varepsilon^2 = 4.97 \cdot 10^{-6}$ then

\[
P(K^o, \bar{K}^o, K_1^o \rightarrow K_1^o, t) = \exp(-t) + 0.00000497(\exp(-t) +
\exp(-t/580) \pm 2(\cos(0.477t - 0.752))\exp(-0.500862t)).\] (22)

In Figure 3 there is presented line obtained by using the above expression which normalized to the experimental data from [14] at $t_S = 1.22$ and experimental data from [14] for $P(\bar{K}^o, K_1^o \rightarrow K_1^o, t \equiv t_S)$. We see that in this case, the interference term which is present in the experimental data, is absent. Then we can make the conclusion that oscillations in this case also do not occur.
Figure 3. Summary transition probabilities \((K_1^o \rightarrow K_1^o) + (K_2^o \rightarrow K_1^o)\) (line) when oscillations take place (expres. (8)+(10)) normalized to experimental data from [14] at \(t_S = 1.22 (\varepsilon = 4.97 \cdot 10^{-6})\) and experimental data (open circles) from [14] for \(t_S = 1 \div 20\).

Now come to the consideration of the case when oscillations between \(K_1^o, K_2^o\) meson states do not arise at \(CP\) violation.

### 3 The case when at \(CP\) violation oscillations between \(K_1^o, K_2^o\) meson states do not arise

Above we considered the case when at \(CP\) violation there can arise oscillations. Now we are considering the case when superposition states arise but there are no oscillations. It arises when there cannot be realized the condition for realization of \(K\) meson oscillations. Here an analogue with Cabibbo [17] mixing matrix takes place with one exclusion, namely, since masses of \(\pi\) and \(K\) mesons differ very much then the interference between these states in contrast to \(K_S, K_L\) meson states cannot arise (by the way in full analogy with Cabibbo case we could use below the old \(K_1^o, K_2^o\) meson states instead of using the new \(K_S, K_L\) states).

We know that the parameter of \(CP\) violation is very small. Then new states \(K'_1 = \cos \beta K_S + \sin \beta K_L\) and \(K'_2 = -\sin \beta K_S + \cos \beta K_L\) are equivalent to \(K_1^o, K_2^o\) states \((\cos^2 \beta + \sin^2 \beta = 1)\). Where \(K_S, K_L\) states are states which arise at small violation of \(CP\) parity. They are not orthogonal but normalized quantum mechanic function of states \((K_S(0) = 1, K_L(0) = 1, |K_1^o(0)|^2 + |K_2^o(0)|^2 = |K_S(0)|^2 + |K_L(0)|^2\). Then

\[
|K_1^o|^2 \equiv |K'_1|^2 = |\cos \beta K_S + \sin \beta K_L|^2, \\
|K_2^o|^2 \equiv |K'_2|^2 = | - \sin \beta K_S + \cos \beta K_L|^2, \\
|K'_1 K'_2| \simeq 0.
\]
As we see in this case instead of oscillations we get interferences between $K_S$ and $K_L$ states. It is present interest to rewrite the above expressions with taking into account time dependence. Then taking into account that the standard expressions for $K_S(t)$ and $K_L(t)$ have the following form:

\[ K_S(t) = \exp(-iE_{St} - \frac{1}{2}\Gamma_{St}) \quad K_L(t) = \exp(-iE_{Lt} - \frac{1}{2}\Gamma_{Lt}), \tag{24} \]

and putting expressions (24) in (23) for a primary $K^o$ meson we get

\[ |K'_1(t)|^2 = \cos^2\beta \exp(-\Gamma_{St}) + \sin^2\beta \exp(-\Gamma_{Lt}) + 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t, \]

\[ |K'_2|^2 = \sin^2\beta \exp(-\Gamma_{St}) + \cos^2\beta \exp(-\Gamma_{Lt}) - 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t, \tag{25} \]

\[ |K'_1K'_2| \simeq 0. \]

Since $K^o = \frac{1}{\sqrt{2}}(K^o_1 + K^o_2)$ then for the case of a $K^o$ meson expressions (25) in normalized form get the following form:

\[ |K'_1(t)|^2 = \frac{1}{2}[\cos^2\beta \exp(-\Gamma_{St}) + \sin^2\beta \exp(-\Gamma_{Lt}) + 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t], \]

\[ |K'_2|^2 = \frac{1}{2}[\sin^2\beta \exp(-\Gamma_{St}) + \cos^2\beta \exp(-\Gamma_{Lt}) - 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t], \tag{26} \]

\[ |K'_1K'_2| \simeq 0. \]

For the case of a $\bar{K}^o$ meson we have

\[ |K'_1|^2 = |\cos\beta K_S - \sin\beta K_L|^2, \]

\[ |K'_2|^2 = |\sin\beta K_S + \cos\beta K_L|^2, \tag{27} \]

\[ |K'_1K'_2| \simeq 0. \]

Then using expression (24) for normalized case we get

\[ |K'_1(t)|^2 = \frac{1}{2}[\cos^2\beta \exp(-\Gamma_{St}) + \sin^2\beta \exp(-\Gamma_{Lt}) - 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t], \]

\[ |K'_2|^2 = \frac{1}{2}[\sin^2\beta \exp(-\Gamma_{St}) + \cos^2\beta \exp(-\Gamma_{Lt}) + 2\sin\beta\cos\beta \exp(\frac{1}{2}(\Gamma S + \Gamma L)t) \cos(E_L - E_S)t], \tag{28} \]

\[ 8 \]
\[ |K_1'K_2'| \approx 0. \]

So, we have obtained the above expressions without the renormalization of states by hand and without using non-unitary matrix for transformation in contrast to work [11].

It is present an interest the case when in expressions (23) will be present a supplementary \( CP \) phase. If this phase appears in the unitary form as it is in [15] in the form of [16]

\[
U = \begin{pmatrix} \cos \beta & -\sin \beta e^{-i\delta} \\ -\sin \beta e^{i\delta} & \cos \beta \end{pmatrix},
\]

then in the case of \( K^o \) meson instead of expressions (25) in the case of \( K^o \) meson we obtain:

\[
|K_1'(t)|^2 = \frac{1}{2} [\cos^2 \beta \exp(-\Gamma_{st}) + \sin^2 \beta \exp(-\Gamma_{lt}) + 2\sin \beta \cos \beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_l) t\right) \cos((E_L - E_S) + \delta t)],
\]

\[
|K_1'(t)|^2 = \frac{1}{2} [\cos^2 \beta \exp(-\Gamma_{st}) + \sin^2 \beta \exp(-\Gamma_{lt}) - 2\sin \beta \cos \beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_l) t\right) \cos((E_L - E_S) - \delta t)],
\]

and in the case of \( \bar{K}^o \) meson instead of expressions (26) we obtain:

\[
|K_1'(t)|^2 = \frac{1}{2} [\cos^2 \beta \exp(-\Gamma_{st}) + \sin^2 \beta \exp(-\Gamma_{lt}) - 2\sin \beta \cos \beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_l) t\right) \cos((E_L - E_S) + \delta t)],
\]

\[
|K_1'(t)|^2 = \frac{1}{2} [\cos^2 \beta \exp(-\Gamma_{st}) + \sin^2 \beta \exp(-\Gamma_{lt}) + 2\sin \beta \cos \beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_l) t\right) \cos((E_L - E_S) - \delta t)],
\]

where using the existing experimental data [14] we can write that the value for \( \sin \beta \) is about \( \sin \beta = \varepsilon \cong 2.23 \cdot 10^{-3} \).

In Figure 4 are given graphic of functions (31) - \( P(K^o \to K_1, t) \approx e^{-t} + (0.00223)^2 e^{-t/580} + 2 \cdot 0.00223(\cos(0.477t - 0.752)) e^{-t/(581/1160)} \) normalized to the experimental data from [14] at \( t_s = 1.22 \) together with experimental data from [14] for \( t_S = 1 \div 20 \) (\( t_S = t/\tau_S \), \( \tau_S \) is \( K_S \) meson life time).

In Figure 5 are given graphic of functions (33) - \( P(\bar{K}^o \to K_1, t) \approx e^{-t} + (0.00223)^2 e^{-t/580} - 2 \cdot 0.00223(\cos(0.477t - 0.752)) e^{-t/(581/1160)} \) normalized to the experimental data from [14] at \( t_s = 1.22 \) together with experimental data from [14] for \( t_S = 1 \div 20 \) (\( t_S = t/\tau_S \), \( \tau_S \) is \( K_S \) meson life time).

We see that the curves from expressions (31) and (33) are in quite satisfactory agreement with the experimental data obtained in [14] at \( \varepsilon \cong 2.23 \cdot 10^{-3} \).
Figure 4. Transition probabilities of primary $K^o$ mesons into $K_S$ ($P(K^o, K^o_1 \to K_S, t)$, expr. (31)) normalized to the experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data from [14] for $t_S = 1 \div 20$.

Figure 5. Transition probabilities of primary $K^o$ mesons into $K_S$ ($P(\bar{K}^o, K^o_1 \to K_S, t)$, expr. (33)) normalized to the experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data from [14] for $t_S = 1 \div 20$.

By the way, the signs of the additional $CP$ phase in our approach are different for $K'_1$ and $K'_2$ mesons in contrast to [11] where was used non-unitary matrix transformation.
in the case of $CP$ violation. Then there arises a question: what mechanism works at $CP$ violation? If it is possible to determine this sign in experiment for a $K'_2$ meson then we can obtain the answer to this question. If we use non-unitary matrix instead of unitary matrix (29)

$$U = \begin{pmatrix} \cos \beta & \sin \beta e^{-i \delta} \\ -\sin \beta e^{i \delta} & \cos \beta \end{pmatrix},$$

then for $K^o$ and $\bar{K}^o$ transition probabilities we obtain the same expressions that are in [11].

So, as it has been was stressed above expressions for transition probabilities (31), (33) are in good agreement with the experimental data from [14]. Then from expressions (31), (33) and Figure 3, 4 we can come to conclusion that at $CP$ violation in weak interactions the standard theory of oscillations is not realized. There takes place only interference between two - $K^o$, $K^o$ meson states.

At $CP$ violation in weak interactions the mixing states of $K_S$, $K_L$ mesons arise with very small angle mixing. These states are not orthogonal states. I.e., there takes place an analogy with Cabibbo matrix mixing [17] at $\pi$, $K$ meson mixings with one distinction: there arise interference between these states since the masses of these states are very close. Then we can in principle not introduce new $K_S$, $K_L$ states and use the old $K'_1$, $K'_2$ meson states as it was done in the case of $\pi$, $K$ mesons (or for $d$, $s$ quarks).

4 Conclusion

In this work have been considered two approaches for description of $K^o$, $\bar{K}^o$ meson transitions into $K'_1$ mesons at $CP$ violation in weak interactions. The first approach uses standard theory of oscillations and the second approach supposes that ($K_S$, $K_L$) states which arise at $CP$ violation are normalized but not but not orthogonal state functions, then there arise interferences between these states but not oscillations between them.

In the case of presence of oscillations the probability of $K^o$, $\bar{K}^o$ meson transition into $K'_1$ mesons is proportional to $\sin^2 \beta = \varepsilon = 2.23 \cdot 10^{-3}$ and at long distances oscillations occur. In the second case there arises an interference term between $K_S$ and $K_L$ meson states. This term is proportional to $\sin \beta = 2.23 \cdot 10^{-3}$ and it disappears at big distances. And at big distances there is present a term which is proportional to $\sin^2 \beta = \varepsilon^2$. As it was stressed above the available experimental data [14] are in good agreement with the second approach. So, we have come to the conclusion that at $CP$ violation in weak interaction in the system of $K^o$ mesons oscillations do not arise. There takes place only interference between two - $K_S$, $K_L$ meson states.
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