Ferromagnetic Luttinger Liquids

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We study weak itinerant ferromagnetism in one-dimensional Fermi systems using perturbation theory and bosonization. We find that longitudinal spin fluctuations propagate ballistically with velocity \( v_{\sigma} \ll v_F \), where \( v_F \) is the Fermi velocity. This leads to a large anomalous dimension in the spin-channel and strong algebraic singularities in the single-particle spectral function and in the transverse structure factor for momentum transfers \( q \approx 2\Delta/v_F \), where \( 2\Delta \) is the exchange splitting.

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Recently several authors presented conductance measurements in ultra low-disorder semiconductor quantum wires and suggested that an unusual feature in the range \( 0.5-0.7\times2e^2/h \) of conductance can be explained in terms of spontaneous ferromagnetism [1, 2, 3]. At first sight this interpretation seems to contradict the Lieb-Mattis theorem [4], which rules out magnetized ground states for electrons moving on a line, as well as for one-band lattice models in one dimension (1d) with nearest-neighbor hopping and interactions involving densities. However, there is no fundamental principle that forbids ferromagnetic ground states in quasi 1d systems with finite width or one-band lattice models in 1d with more general hoppings. Indeed, numerical studies [5] show that the ground state of the 1d Hubbard model with hopping between nearest and next-nearest neighbors can be ferromagnetic in a substantial range of densities and on-site interactions \( U \). Clearly, the precise form of the energy dispersion \( \epsilon_k \) plays an important role in stabilizing ferromagnetism [3, 4, 5]. In principle, it should therefore be possible to design metallic systems with ferromagnetic ground states by properly adjusting the hopping integrals between the relevant orbitals. A promising class of 1d materials where this might be achieved are certain types of organic polymers [6], whose molecular structure can be designed in a controlled manner in the laboratory. Motivated by these new developments, in this work we shall use a combination of perturbation theory and bosonization to derive some physical properties of itinerant ferromagnets in 1d.

Let us briefly consider this problem from a renormalization group (RG) point of view. The usual RG approach to 1d metals is based on the assumption that their long-wavelength and low-energy properties are determined by wavevectors \( k \) in the vicinity of the Fermi wavevectors \( \pm k_F \). Given a general energy dispersion \( \epsilon_k \), it therefore seems reasonable to expand for \( k \) close to \( k_F \)

\[
\epsilon_k = \epsilon_{k_F} + v_F (k - k_F) + \frac{(k - k_F)^2}{2m^*} + \frac{\lambda}{6} (k - k_F)^3 + \ldots ,
\]

and similarly for \( k \approx -k_F \). By power counting, the Fermi velocity \( v_F \) is a marginal coupling, while the inverse effective mass \( 1/m^* \) and the cubic parameter \( \lambda \) are irrelevant in the RG sense. In the field-theoretical formulation of the RG [10], these irrelevant couplings are simply ignored. However, as shown below, the cubic term in Eq. (1) is crucial to stabilize a ferromagnetic ground state in 1d, so that a proper RG treatment of itinerant ferromagnetism should include also the irrelevant couplings associated with band curvature effects. Therefore methods which cannot properly handle these couplings, such as the field-theoretical RG [11] or bosonization, lose much of their power. Nevertheless, as shown below, in certain regimes bosonization is still useful to obtain nonperturbative results for correlation functions.

We consider the following Hamiltonian describing interacting electrons on a 1d lattice with length \( L \),

\[
\hat{H} = \sum_{k\sigma} \epsilon_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \frac{1}{2L} \sum_{q,ij} f_{ij} \hat{\rho}_i (-q) \hat{\rho}_j (q) , \quad (2)
\]

where \( \hat{c}_{k\sigma}^\dagger \) and \( \hat{c}_{k\sigma} \) are creation and annihilation operators for electrons with momentum \( k \) and spin \( \sigma \). The labels \( i \) and \( j \) assume values in \( \{n,m\} \), where \( n \) corresponds to the charge density \( \hat{\rho}_n (q) = \sum_{k\sigma} \epsilon_{k\sigma} ^\dagger \hat{c}_{k+q\sigma} \), and \( m \) denotes the spin density \( \hat{\rho}_m (q) = \sum_{k\sigma} \sigma \epsilon_{k\sigma} ^\dagger \hat{c}_{k+q\sigma} \). To discuss spontaneous symmetry breaking we should start from a spin-rotationally invariant \( \hat{H} \), which constrains the bare \( f_{ij} \) to satisfy \( f_{nm} = f_{mn} = 0 \) and precludes any momentum-dependence of \( f_{nm} \). We also take \( f_{nm} \equiv f_{nm} \) to be momentum-independent [11].

As a first step, we study the ferromagnetic instability within Hartree-Fock theory. Adding and subtracting the counterterm \( \Delta\sigma (m) = f_{nn} + \sigma f_{mm} \), where \( n = \langle \hat{\rho}_n (0) \rangle / L \) is the density and \( m = \langle \hat{\rho}_m (0) \rangle / L \) is the spin density, we may write \( \hat{H} = \hat{H}_0 + \hat{H}_1 \), with

\[
\hat{H}_0 - \mu \hat{N} = \sum_{k\sigma} \xi_{k\sigma} \epsilon_{k\sigma} ^\dagger \hat{c}_{k\sigma} - \frac{L}{2} \left[ f_{nn} n^2 + f_{mm} m^2 \right] , \quad (3)
\]

and \( \hat{H}_1 = (2L)^{-1} \sum_q \sum_{i,j} f_{ij} \delta \hat{\rho}_i (-q) \delta \hat{\rho}_j (q) \). Here \( \xi_{k\sigma} = \epsilon_k - \mu + \Delta\sigma (m) \) is the Hartree-Fock energy, \( \delta \hat{\rho}_i (q) = \hat{\rho}_i (q) - \delta \hat{\rho}_i (\hat{\rho}_0 (0)) \), and \( \hat{N} = \hat{\rho}_n (0) \). In the ferromagnetic state the Fermi wavevectors \( k_{\sigma} \) and velocities \( v_{\sigma} \) are defined by...
\[ \epsilon_{k\sigma} - \mu + \Delta_{\sigma}(m) = 0 \] and \( v_\sigma = \partial \epsilon_k / \partial k |_{k_F} \), while in the normal state \( \epsilon_{k\sigma} - \mu + \Delta_{\sigma}(0) = 0 \) and \( v_\sigma = \partial \epsilon_k / \partial k |_{k_F} \).

Hence \( \epsilon_{k\sigma} - \epsilon_{k\sigma + f_\sigma \delta n} = \sigma \Delta \) where \( \Delta = -f_m m \) and \( \delta n = n(m) - n(0) \). For convenience we keep the chemical potential \( \mu \) constant, so that the density \( n \) is a function of \( m \). The two equations \( \epsilon_{k\sigma} - \epsilon_{k\sigma + f_\sigma \delta n} = \sigma \Delta \) fix the four quantities \( k_\uparrow, k_\downarrow, \delta n, \) and \( m \).

Throughout this work we shall assume \( m \ll n \) (weak ferromagnetism). The low-energy properties are then determined by wavevectors in the vicinity of the Fermi surface, as discussed in the classic work by Dzyaloshinskii and Kondratenko [12]. Hence we may expand \( \epsilon_k \) around \( \pm k_F \). To leading order, it is sufficient to truncate the expansion at the third order, see Eq. [1]. Keeping in mind that \( \pi m \ll k_F \) and defining \( q_m = \Delta / v_F \) we obtain

\[
k_F - k_F = \sigma q_m - \frac{\lambda_1 q_m^2}{2(1 + F_0)} - 2\sigma A q_m^3 + \ldots,
\]

where \( A = \frac{1}{12}(\lambda_2 - \frac{3\lambda_3}{1 + F_0}) \), with \( \lambda_1 = \frac{1}{m^* v_F} \), \( \lambda_2 = \lambda / v_F \), and \( F_0 = 2 f_n / \pi v_F \). The Fermi velocities are

\[
v_\sigma / v_F = 1 + \sigma \lambda_1 q_m + \frac{1}{2} \left( \lambda_2 - \frac{\lambda_2}{1 + F_0} \right) q_m^2 + \ldots.
\]

Substituting Eq. (1) into \( m = \pi^{-1}(k_F - k_\downarrow) \), it is easy to see that, besides the solution \( m = 0 \), there is a non-trivial solution \( m_0 = [2 l_0 - 1]/(2 l_0 A) \) \( 1/2 \), provided the radicand is positive. Here \( l_0 = 2 f_n / \pi v_F \) is the dimensionless Stoner parameter [3]. To see whether the solution \( m_0 \) is stable, consider the energy change \( \delta \Omega_0(m) = \Omega_0(m) - \Omega_0(0) \) due to a finite value of \( m \), where \( \Omega_0(m) = \langle \hat{H}_0 - \mu \hat{N} \rangle \). We obtain

\[
\delta \Omega_0(m) = \frac{L v_F}{4\pi} \left[ -l_0 (l_0 - 1) (m^2) + \frac{A}{4} l_0 \left[ (m)^3 \right] + \ldots \right].
\]

Obviously, a necessary condition for \( m_0 \) to represent a minimum of \( \Omega_0(m) \) is \( A \geq 0 \). In addition, the square root \( [2 l_0 - 1]/(2 l_0 A) \) is only real if either \( l_0 < 0 \) or \( l_0 > 1 \). For consistency, we should also require that \( \pi m_0 \ll k_F \) and that the band-structure is such that the higher order corrections in Eq. (3) are small. For some special form of \( \epsilon_k \) it should be possible to satisfy these conditions even for small negative \( l_0 \) provided \( k_F^2 A \gg \pi^2 |l_0|^{-3} \).

We shall not further consider this case, but focus instead on the regime close to the Stoner threshold, where \( l_0 \) is slightly larger than unity. The distance from the critical point is then measured by the small parameter \( \delta l_0 = 2(l_0 - 1)/l_0 \). Interestingly, the numerical results of Ref. [3] indeed show a critical \( l_0 \) of order unity for not too large densities, which suggests that even in 1d the Stoner criterion can be a reasonable estimate for the ferromagnetic instability.

For simplicity we now set \( f_n = -f_m = f_0 > 0 \), corresponding to a repulsive Hubbard on-site interaction [11]. Note that close to the phase transition \( l_0 = F_0 = 1 + \mathcal{O}(\delta l_0) \). Let us first consider the density-density \( (\chi_{nn}) \) and the longitudinal spin-spin \( (\chi_{mm}) \) correlation functions. Within the random-phase approximation (RPA) we obtain

\[
\chi^{\text{RPA}}_{nn}(q, i\omega) = \frac{[\lambda_{+}^0 + \chi_{+}^0 - 4f_0 \lambda_{+}^0 \chi_{+}^0(1+\delta l_0)]}{D},
\]

\[
\chi^{\text{RPA}}_{mm}(q, i\omega) = \frac{[\lambda_{+}^0 + \chi_{+}^0 + 4f_0 \lambda_{+}^0 \chi_{+}^0(1+\delta l_0)]}{D},
\]

where \( D(q, i\omega) = 1 - 4 f_0^2 \lambda_{+}^0 \chi_{+}^0 \).

\[
\chi_{\sigma\sigma}(q, i\omega) = \frac{1}{L} \sum_k \frac{f(\xi_{k+q/2,\sigma'}(\omega') - f(\xi_{k-q/2,\sigma} - i\omega)}{\xi_{k+q/2,\sigma'} - \xi_{k-q/2,\sigma} - i\omega}.
\]

Here \( f(E) \) is the Fermi function. For small \( q \) and \( \omega \) we may approximate

\[
\chi_{\sigma\sigma}^0(q, i\omega) \approx \frac{v_n}{\pi} \frac{q^2}{(v_F q)^2 + \omega^2}.
\]

For \( \omega > 0 \) the dynamic structure factors \( S_{ii}^{\text{RPA}}(q, \omega) = \pi^{-1} \text{Im} \chi_{ii}^{\text{RPA}}(q, \omega + i0) \) can then be written as

\[
\delta_i^{\text{RPA}}(q, \omega) = Z_\omega |q| \delta (\omega - v_n |q|),
\]

with \( Z_\omega = \frac{\pi v_F (1 + F_0)}{\pi v_F F_0} \), \( v_n = v_F (1 + F_0) \), and \( Z_m = \frac{\pi v_F F_0}{\pi v_F F_0} \). Note that \( S_{ii}^{\text{RPA}}(q, \omega) \) satisfy the sum rules [13] \( \lim_{q\to0} \int_0^\infty \frac{d\omega}{\pi} \frac{S_i^{\text{RPA}}(q, \omega)}{\omega} = \chi_i \), with the compressibility \( \chi_m = [\pi v_F (1 + F_0)]^{-1} \) and the spin susceptibility \( \chi_m = 2/(\pi v_F F_0) \). The latter is related to the Hartree-Fock energy [1] via \( \chi_m^{-1} = L^{-1} \partial^2 \Omega_0(m) / \partial m^2 |_{m_0} \). We conclude that longitudinal spin fluctuations in 1d can propagate ballistically, with velocity \( v_n \ll v_F \). In contrast, in 3d itinerant ferromagnets the longitudinal spin mode can decay into particle-hole pairs and is therefore strongly Landau-damped [5].

Next, let us calculate the transverse spin-spin correlation function \( \chi_{\perp\perp}(q, \omega) \) within the ladder approximation shown in Fig. [4], which yields

\[
\chi_{\perp\perp}^{\text{LAD}}(q, \omega) = \frac{1}{2\Delta} \left( \frac{\omega}{2\Delta} - B q^2 \right) + \ldots,
\]

with the nonuniversal constant [4] \( B = \frac{1}{12} (\lambda_2 - \lambda_3) \). Note that \( B \geq A > 0 \). Using \( \Delta = f_{m0} \) we obtain

\[
\chi_{\perp\perp}^{\text{LAD}}(q, i\omega) = \frac{\pi m_0}{\omega} \frac{\pi m_0/\omega - B q^2}{\omega - B q^2},
\]

where \( b = 2\Delta B \) is the spin wave stiffness. This implies a \( \delta \)-function peak in the dynamic structure factor, \( S_{\perp\perp}(q, \omega) = m_0 \delta (\omega - B q^2) \), which exhausts the sum rule.
The existence of well-defined transverse spin waves in the symmetry broken phase follows from general hydrodynamic arguments \[13\]. How-

\[
\int_0^\infty \frac{dw}{2\pi} S_{\uparrow\downarrow}(q, \omega) = m_0/bq^2.
\]

The existence of well-defined transverse spin waves in the symmetry broken phase follows from general hydrodynamic arguments \[13\]. However, in $1d$ it may well be that interactions lead to anomalous damping of spin waves and a breakdown of hydro-

\[
\chi_{\sigma}(q, \omega) \equiv \frac{\int d\epsilon \rho_{\sigma}(\epsilon) e^{i\epsilon q} e^{-i\omega t}}{\int d\epsilon \rho_{\sigma}(\epsilon) e^{i\epsilon q}}.
\]

Due to the linearized energy dispersion and the irrelevance of scattering processes with large momentum transfers, the single-particle Green function $G_\sigma(x, \tau)$ can be calculated exactly using bosonization in real space and imaginary time. For $\max(\{|x|, v_m|\tau|\}) \gg r_0$ we obtain

\[
G_\sigma(x, \tau) = \frac{1}{2\pi i} \left[ \frac{r_0^2}{x^2 + v_m^2 \tau^2} \right]^{\eta_m/2} \left[ \frac{r_0^2}{x^2 + v_m^2 \tau^2} \right]^{\eta_m/2} \times \sum_\alpha \frac{\epsilon^{iksx}}{[\alpha x + iv_m \tau]^{1/2}[\alpha x + iv_m \tau]^{1/2}}.
\]

Here $\eta = \frac{1}{4}(K_1 + K_{\perp}^{-1} - 2)$, with $K_{\perp} = [I + 1]^{-1/2}$ and $K_1 = [2(I - 1)]^{-1/2}$. Note that the anomalous dimension $\eta_m$ of the spin channel diverges for $I \to 1$. This singularity is also found directly from the universal Luttinger liquid relation \[17\] $\chi_i = 2K_i/\pi v_i$, together with the above RPA results for $\chi_m$ and $v_m$. We note that an analogous scenario has recently been found for the charge channel of the 1d $t$-$J$ model in the vicinity of the phase separation instability \[18\].

Finally, let us consider the transverse spin-spin correlation function $\chi_{\perp \perp}(x, \tau)$, which, due to the linearized energy dispersion, can also be calculated for large $x$ and $\tau$ by bosonization. Following Ref. \[19\] we obtain for $\max(\{|x|, v_m|\tau|\}) \gg r_0$

\[
\chi_{\perp \perp}(x, \tau) = \frac{-1}{(2\pi)^2} \left[ \frac{r_0^2}{x^2 + v_m^2 \tau^2} \right]^{2\eta_m} \sum_\alpha \frac{\epsilon^{ia(k_\perp - k_\perp)x}}{[\alpha x + iv_m \tau]^{1/2} [\alpha x + iv_m \tau]^{1/2}}.
\]

The leading diagrams taken into account in Eq. \[15\] are shown in Fig. 2: they contain the ladder diagrams of Fig. 1 as a subset, but include in addition self-energy corrections, screening bubbles, and complicated vertex corrections. It is important to realize that Eq. \[15\] can only be used to obtain the Fourier transform $\chi_{\perp \perp}(x, \tau) = \int dx\,d\tau \, e^{-i(\sigma - \omega)\tau} \chi_{\perp \perp}(x, \tau)$ for wavevectors close to $\pm(k_\perp - k_\perp)$, i.e. for $|q - (k_\perp - k_\perp)| < q_m$.
the spin wave regime $|q| \ll q_m$ the transverse spin-spin correlation function cannot be calculated using abelian bosonization with linearized energy dispersion, because (i) the ladder approximation suggests that the spin wave dispersion depends on the nonlinear terms of the energy dispersion, and (ii) the existence of spin waves follows from the spontaneous breaking of spin-rotational invariance, so that their dispersion cannot be obtained using a method which explicitly violates this symmetry. On the other hand, for $|q| \lesssim (k_f - k_i) \lesssim q_m$ the Fourier transform of Eq. (15) yields an accurate approximation for the transverse dynamic structure factor $S_{\perp}(q, \omega)$. For $\omega > 0$ we obtain

$$
S_{\perp}(q, \omega) = C_m (\omega - v_m |q| - k_f + k_i) \\
\times [\omega - v_m (|q| - k_f + k_i)]^{2q_m-1} \\
\times [\omega + v_m (|q| - k_f + k_i)]^{2q_m+1},
$$

with

$$
C_m = \left[4\pi v_m \Gamma(2q_m) \Gamma(2 + 2q_m) (r_0/v_m)^{4q_m} \right],
$$

which region $S_{\perp}(q, \omega)$ is finite represents the 1$d$ Stoner continuum. The complete picture of low-energy spin excitations is depicted in Fig. 3 and is qualitatively quite similar to its 3$d$ counterpart [6]. However, in 1$d$ there is no Landau damping and the structure factor shows anomalous scaling associated with broken spin-rotational symmetry of a Luttinger liquid phase.

For an outlook from a renormalization-group perspective, we note that while the ferromagnetic ground state is stabilized by nonlinear terms in the energy dispersion close to the Fermi points, the flow of the corresponding irrelevant couplings is not accessible within the usual field-theoretical RG [10]. However, using modern formulations of the RG [20] based on Wilson’s idea of eliminating degrees of freedom and rescaling, it should be possible to examine the subtle role played by irrelevant couplings in stabilizing a ferromagnetic ground state in 1$d$.

In conclusion, we presented the effective low-energy theory of weakly ferromagnetic Luttinger liquids. Many of their properties only depend on the effective Stoner parameter $I$, i.e., on the distance $\delta = 2(I - 1)/I \ll 1$ from the ferromagnetic instability. Neutron scattering experiments should be able to test our predictions for spin-spin correlation functions. Furthermore the propagating longitudinal mode with small velocity $v_m \propto \delta^{1/2}$ and large residue $Z_m \propto \delta^{-1/2}$ dominates some thermodynamic quantities, for example through the divergence of the uniform spin susceptibility, $\chi_m \propto Z_m/v_m \propto \delta^{-1}$. The discussed features of the weakly ferromagnetic regime should be accessible in specially designed organic polymers [3], for which the effective Stoner parameter $I$ can be controlled by adjusting the density via external gate voltages. Our predictions are also relevant to semiconductor quantum wires which are believed to show spontaneous ferromagnetism [12, 13].

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