The Determination of $\sin^2 \theta_W$ in Neutrino Scattering: no more anomaly

A. W. Thomas

ARC Centre of Excellence in Particle Physics at the Tera-scale (CoEPP) and CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

Abstract. We review the corrections to the NuTeV determination of $\sin^2 \theta_W$, concluding that it is no longer appropriate to present it as an “anomaly”. Indeed, when well understood corrections associated with charge symmetry violation and the iso-vector nuclear force are properly included, the measurement is completely consistent with the Standard Model.

Keywords: Standard Model test, neutrinos, charge symmetry violation, iso-vector EMC effect, strange quark asymmetry

INTRODUCTION

The famous expression of Paschos and Wolfenstein [1] relating the ratio of neutral-current and charge-changing neutrino interactions on isoscalar targets to the Weinberg angle takes the form:

$$R^- \equiv \frac{\rho_0^2 \left( \langle \sigma^{\nu N_0}_{\text{NC}} \rangle - \langle \sigma^{\nu N_0}_{\text{CC}} \rangle \right)}{\langle \sigma^{\nu N_0}_{\text{CC}} \rangle - \langle \sigma^{\nu N_0}_{\text{NC}} \rangle} = \frac{1}{2} - \sin^2 \theta_W. \quad (1)$$

In Eq.(1), $\langle \sigma^{\nu N_0}_{\text{NC}} \rangle$ and $\langle \sigma^{\nu N_0}_{\text{CC}} \rangle$ are respectively the neutral-current and charged-current inclusive, total cross sections for neutrinos on an isoscalar target (assuming charge symmetry is exact and that there is no strange quark asymmetry). The quantity $\rho_0 \equiv M_W / (M_Z \cos \theta_W)$ is one in the Standard Model.

The application of this relation in a test of the Standard Model took almost 30 years, with the NuTeV collaboration reporting the measurement of neutrino charged-current and neutral-current cross sections on iron [2]. Surprisingly, the result that they found, namely $\sin^2 \theta_W = 0.2277 \pm 0.0013 \, \text{(stat)} \pm 0.0009 \, \text{(syst)}$, differed from the expectation within the Standard Model by $3\sigma$. This has been widely treated as an indication of the need to go beyond the Standard Model.

Unfortunately, there are several corrections to the Paschos-Wolfenstein relation (PWR) that were omitted in the NuTeV analysis [4]. The first of these arises because in the Standard Model the $u$ and $d$ masses are not equal and so charge symmetry, a necessary condition for the PWR, is broken [5, 6]. Indeed, two independent estimates of the effect of charge symmetry violation in parton distributions, which had been published a decade before NuTeV [7, 8] and which reduce the discrepancy to $2\sigma$ or less, were ignored in the NuTeV analysis. The dubious response to that criticism has been
that “there were no reliable calculations”. We address that claim in the next section, showing that it is clearly incorrect.

Of course, the steel target used by NuTeV is not iso-scalar and standard corrections were made for the neutron excess. However, the second major correction to the NuTeV analysis involves a critical piece of physics which was not appreciated at the time. That is, the EMC effect produced by those extra neutrons on every nucleon in the nucleus and which is therefore not taken into account by subtracting their contribution to the nuclear structure function. Because the iso-vector nuclear force is repulsive between neutrons and $d$-quarks, the effect of this new nuclear correction is to shift momentum from all $u$ to all $d$ quarks in the nucleus \[10\]. Thus, as far as the determination of $\sin^2 \theta_W$ goes, it has the same sign and similar magnitude to the CSV correction already mentioned.

In this brief review we first outline the reasons why the original calculation of the size of CSV was considered to be very reliable by the authors. We then describe the recent direct measurement of this effect within the framework of lattice QCD, which agreed beautifully with the earlier bag model based calculations. Next we explain the role of the iso-vector EMC effect. Finally, we make some concluding remarks.

**MODEL INDEPENDENCE OF THE CHARGE SYMMETRY CORRECTION**

The first calculations of CSV in parton distributions were made independently by Sather and Rodionov \textit{et al.} \[7, 8\]. Both were based on a calculation of the parton distribution functions (PDFs) at a low momentum scale, appropriate to a valence-dominated quark model, and followed by QCD evolution to generate the CSV distributions at the $Q^2$ values appropriate for the NuTeV experiment. However, the two calculations were based on very different levels of approximation. The first \[7\] used several approximations to simplify the evaluation and which have the effect that the model distributions do not have the correct support. These approximations were not necessary in the work of Rodionov \textit{et al.} \[8\], in which energy-momentum conservation was ensured. Nevertheless, the two calculations agreed on the extent of CSV rather well, with the result a correction to the NuTeV result $\Delta R_{CSV} \sim -0.0015$. This reduces the reported effect from 3 to 2 standard deviations. After this was pointed out, NuTeV made their own estimate of the CSV parton distributions, using a rather different procedure \[9\]. They obtained a much smaller correction, $\Delta R_{CSV} \sim +0.0001$. On the basis of the large discrepancy between these two results it was suggested that the CSV correction might be strongly model dependent.

This question was investigated by Londergan and Thomas \[13\], in order to check just how model dependent the size of the CSV correction could be. We briefly recall their argument. The charge symmetry violating contribution to the Paschos-Wolfenstein ratio has the form

$$\Delta R_{CSV} = \left[ 3\Delta_u^2 + \Delta_d^2 + \frac{4\alpha_s}{9\pi} (g_L^2 - g_R^2) \right] \left[ \frac{\delta U_V - \delta D_V}{2(U_V + D_V)} \right]$$  \(2\)

where

$$\delta Q_V = \int_0^1 x \delta q_V(x) \, dx$$
\[ \delta d_V(x) = d_V^u(x) - u_V^d(x) ; \quad \delta u_V(x) = u_V^u(x) - d_V^d(x) . \] (3)

The denominator in the final term in Eq. (2) gives the total momentum carried by up and down valence quarks, while the numerator gives the charge symmetry violating momentum difference – for example, \( \delta U_V \), is the total momentum carried by up quarks in the proton minus the momentum of down quarks in the neutron. This ratio is completely independent of \( Q^2 \) and can be evaluated at any convenient value.

Using the analytic approximation to the charge symmetry violating valence parton distributions initially proposed by Sather [7], one can evaluate Eq.(2) at a low scale, \( Q_0^2 \), appropriate for a (valence dominated) quark or bag model [12, 14]. The advance over earlier work was to realize that for NuTeV one needs only the first moments of the CSV distribution functions and these can be obtained analytically. The result for the moment of the CSV down valence distribution, \( \delta D_V \), is

\[ \delta D_V = \int_0^1 x \left[ -\frac{\delta M}{M} \frac{d}{dx} (x d_V(x)) - \frac{\delta m}{M} \frac{d}{dx} d_V(x) \right] dx \]

\[ = \frac{\delta M}{M} \int_0^1 x d_V(x) dx + \frac{\delta m}{M} \int_0^1 d_V(x) dx = \frac{\delta M}{M} D_V + \frac{\delta m}{M} , \] (4)

while for the up quark CSV distribution it is

\[ \delta U_V = \frac{\delta M}{M} \left[ \int_0^1 x \left( -\frac{d}{dx} [x u_V(x)] + \frac{d}{dx} u_V(x) \right) dx \right] \]

\[ = \frac{\delta M}{M} \left( \int_0^1 x u_V(x) dx - \int_0^1 u_V(x) dx \right) = \frac{\delta M}{M} (U_V - 2) . \] (5)

(Here \( \delta M = 1.3 \text{ MeV} \) is the neutron-proton mass difference, and \( \delta m = m_d - m_u \sim 4 \text{ MeV} \) is the down-up quark mass difference.)

Equations (4) and (5) show that the CSV correction to the Paschos-Wolfenstein ratio depends only on the fraction of the nucleon momentum carried by up and down valence quarks. At no point do we have to calculate specific CSV distributions. At the bag model scale, \( Q_0^2 \approx 0.5 \text{ GeV}^2 \), the momentum fraction carried by down valence quarks, \( D_V \), is between 0.2 − 0.33, and the total momentum fraction carried by valence quarks is \( U_V + D_V \sim .80 \). From Eqs. (4) and (5) this gives \( \delta D_V \approx 0.0046, \delta U_V \approx -0.0020 \). Consequently, evaluated at the quark model scale, the CSV correction to the Paschos-Wolfenstein ratio is

\[ \Delta R_{CSV} \approx 0.5 \left[ 3\Delta_u^2 + \Delta_d^2 \right] \frac{\delta U_V - \delta D_V}{2(U_V + D_V)} \approx -0.0020 . \] (6)

Once the CSV correction has been calculated at some quark model scale, \( Q_0^2 \), the ratio appearing in Eq. (2) is independent of \( Q^2 \), because both the numerator and denominator involve the same moment of a non-singlet distribution.

Note that both Eqs. (4) and (5) are only weakly dependent on the choice of quark model scale – through the momentum fractions \( D_V \) and \( U_V \), which are slowly varying functions of \( Q_0^2 \) and, in any case, not the dominant terms in those equations. This, together with the \( Q^2 \)-independence of the Paschos-Wolfenstein ratio (Eq. (2)) under QCD
evolution, explains why the previous results, obtained by Londergan and Thomas with different models and at different values of $Q^2$ [3], were so similar. Finally, Londergan and Thomas also demonstrated that the acceptance function calculated by NuTeV does not introduce any significant model dependence to this result.

In spite of the power of this demonstration the effect of CSV is still omitted in the official NuTeV collaboration analysis of $\sin^2 \theta_W$.

**Lattice QCD**

Recent simulations of PDFs within lattice QCD have included not just the nucleon but all members of the baryon octet, over a range of values of the strange quark mass. Using SU(3) symmetry one can use these results to investigate CSV in a novel way. Indeed, by viewing the s-quark as effectively a heavier light quark, one can extract the moments of the CSV PDFs. This is illustrated in Fig. 1, where we show the linear dependence of the second moment of the CSV $u$ and $d$ distributions as a function of the $d - u$ mass difference. From the slopes of these lines Horsley *et al.* deduced the first moments of the $C$-positive CSV moments $\delta u = -0.0023 \pm 0.0006$ and $\delta d = 0.0020 \pm 0.0003$ [15],
in excellent agreement with the values $\delta u^- = -0.0014$ and $\delta d^- = 0.0015$ (at 4 GeV$^2$) found within the MIT bag model [7, 8].

In summary, the explicit lattice simulations of the extent of charge symmetry violation arising through the $u - d$ mass difference is in remarkable agreement with the earlier calculations based on the MIT bag model. Given the demonstration of the degree of model independence of the results for the moment relevant to the NuTeV experiment this should not be a surprise. It certainly provides a convincing counter to the suggestions that there have been no reliable calculations. This level of agreement between lattice QCD simulation and phenomenology can leave no doubt concerning the sign and magnitude of the CSV correction which must therefore be included in any serious analysis of the NuTeV data.

**HADRONs IN-MEDIUM**

A remarkably effective approach to the nuclear many body problem, based on the underlying quark structure of hadrons, begins with the realization that at some density (perhaps 3 to 5 times nuclear matter density) nuclear matter will make a transition to quark matter – a phase transition which may have dramatic effects on the observable properties of neutron stars. One therefore constructs a theory of the nuclear many-body system starting with a description of hadron structure at the quark level and considers the self-consistent modification of that structure in a nuclear medium. This is the approach taken within the QMC (quark-meson coupling) model [16, 17]. A remarkable advantage of this approach is that no new parameters are needed to calculate the effective density dependent forces [18] between any hadrons whose quark structure is known. Indeed, it has been possible to develop a remarkably successful derivation of realistic Skyrme forces [18, 19] for comparison with low energy nuclear phenomenology – while the fully relativistic underlying theory successfully predicts key features of hypernuclear physics and allows the study of the appearance of hyperons in dense matter [20].

The QMC model has the additional advantage that one can address not only those low energy properties such as binding energies and charge densities but it can also be used to calculate the nuclear modification of the form factors [21] and deep inelastic structure functions [22]. Extensions of the QMC approach based upon a covariant, confining version of the NJL model have produced a satisfactory description of the EMC effect in finite nuclei [23]. In the present context it has also produced the remarkable prediction that there will be a component of the EMC effect which is isovector in nature if one has a target with $N \neq Z$ [10]. Most importantly, because the EMC effect involves a change in the structure of the bound nucleon, that isovector EMC correction will persist even if one derives data for an effectively isoscalar nucleus by subtracting the contribution of the excess neutrons.

In terms of the Paschos-Wolfenstein relation and the NuTeV determination of $\sin^2 \theta_W$ this leads to a shift of momentum from all the $u$ quarks in the nucleus to all the $d$ quarks. This is the same sign as the effect of CSV and also reduces the NuTeV anomaly by around one standard deviation. Figure 2 shows the result of a recent reanalysis of the NuTeV anomaly [11] in which this correction was applied to the data along with a correction for genuine charge symmetry violation associated with the mass difference
between $u$ and $d$ quarks, as well as the effect of photon radiation. Clearly there remains no significant discrepancy between the corrected measurement and the Standard Model.

### CONCLUSION

We have briefly reviewed the latest state of play in the analysis of what used to be known as the NuTeV anomaly. It should be clear that well understood corrections associated with charge symmetry violation and the iso-vector EMC effect unambiguously remove any significant deviation from the Standard Model. The one major remaining uncertainty concerns the possible asymmetry between the strange and anti-strange PDFs [24]. Even though the model independent behaviour of these distributions under chiral symmetry suggests that there must be some asymmetry [25], it is experimentally very poorly determined. Working toward a precise measurement of $s(x) - \bar{s}(x)$ should be a very high experimental priority for a number of reasons, including its relevance to the NuTeV experiment. However, on grounds that are admittedly somewhat model dependent, we find it extremely unlikely that this asymmetry could be large enough to make a large contribution. In particular, there is no known perturbative QCD mechanism to produce a large asymmetry. In terms of non-perturbative mechanisms, chiral effects are controlled by factors like $m_K/m_N$ and $m_A/m_N$ and hence any change in sign in $s(x) - \bar{s}(x)$ is
unlikely to occur at values of Bjorken $x$ much below 0.1. With such a constraint the NuTeV analysis of their own di-muon data yields a tiny value for the $s-\bar{s}$ asymmetry [5].

Of course, while the theoretical results presented here are compelling, as always it will be important to carry out careful experimental investigations of CSV of the $u$ and $d$ PDFs as well as the iso-vector EMC effect. Further lattice simulations of the moments of $s-\bar{S}$, complemented by dedicated experimental studies are also vital.

ACKNOWLEDGMENTS

This work was supported by the University of Adelaide and by the Australian Research Council through the award of an Australian Laureate Fellowship.

REFERENCES

1. E. A. Paschos and L. Wolfenstein, Phys. Rev. D 7, 91 (1973).
2. G. P. Zeller et al., Phys. Rev. Lett. 88, 091802 (2002).
3. J. T. Londergan and A. W. Thomas, Phys. Lett. B 558, 132 (2003) [arXiv:hep-ph/0301147].
4. S. Davidson, S. Forte, P. Gambino, N. Rius and A. Strumia, JHEP 0202, 037 (2002) [arXiv:hep-ph/0112302].
5. J. T. Londergan, J. C. Peng, A. W. Thomas, Rev. Mod. Phys. 82, 2009-2052 (2010). [arXiv:0907.2352 [hep-ph]].
6. J. T. Londergan, A. W. Thomas, Prog. Part. Nucl. Phys. 41, 49-124 (1998). [hep-ph/9806510].
7. E. Sather, Phys. Lett. B 274, 433 (1992).
8. E. N. Rodionov, A. W. Thomas and J. T. Londergan, Mod. Phys. Lett. A 9, 1799 (1994).
9. G.P. Zeller et al., Phys. Rev. D65, 111103 (2002).
10. I. C. Cloet, W. Bentz, A. W. Thomas, Phys. Rev. Lett. 102, 252301 (2009). [arXiv:0901.3559 [nucl-th]].
11. W. Bentz, I. C. Cloet, J. T. Londergan, A. W. Thomas, Phys. Lett. B693, 462-466 (2010). [arXiv:0908.3198 [nucl-th]].
12. A. I. Signal and A. W. Thomas, Phys. Rev. D 40 (1989) 2832.
13. J. T. Londergan and A. W. Thomas, Phys. Rev. D 67, 111901 (2003) [arXiv:hep-ph/0303155].
14. A. W. Schreiber, A. I. Signal and A. W. Thomas, Phys. Rev. D 44, 2653 (1991).
15. R. Horsley et al., Phys. Rev. D83, 051501 (2011). [arXiv:1012.0215 [hep-lat]].
16. P. A. M. Guichon, K. Saito, E. N. Rodionov et al., Nucl. Phys. A601, 349-379 (1996). [nucl-th/9509034].
17. K. Saito, K. Tsushima, A. W. Thomas, Prog. Part. Nucl. Phys. 58 (2007) 1-167. [hep-ph/0506314].
18. P. A. M. Guichon et al., Nucl. Phys. A772, 1-19 (2006). [nucl-th/0603044].
19. P. A. M. Guichon, A. W. Thomas, Phys. Rev. Lett. 93, 132502 (2004). [nucl-th/0402064].
20. P. A. M. Guichon, A. W. Thomas, K. Tsushima, Nucl. Phys. A814, 66-73 (2008). [arXiv:0712.1925 [nucl-th]].
21. D. -H. Lu, K. Tsushima, A. W. Thomas, A. G. Williams, K. Saito, Phys. Rev. C60, 068201 (1999). [nucl-th/9807074]; D. -H. Lu, A. W. Thomas, A. G. Williams, Phys. Rev. C57, 2628-2637 (1998). [nucl-th/9706019].
22. K. Saito, A. Michels, A. W. Thomas, Phys. Rev. C46, 2149-2152 (1992).
23. I. C. Cloet, W. Bentz, A. W. Thomas, Phys. Lett. B642, 210-217 (2006). [nucl-th/0605061].
24. A. I. Signal, A. W. Thomas, Phys. Lett. B191, 205 (1987).
25. A. W. Thomas, W. Melnitchouk, F. M. Steffens, Phys. Rev. Lett. 85, 2892-2894 (2000). [hep-ph/0005043].