Mathematical model of the optical radiation propagation in
the microsatellite moving in near-earth orbit

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Abstract. The paper considers the problem of the laser radiation propagation from the Earth surface to a Luneburg lens moving at a speed $V$ along a geosynchronous orbit, and the registration by a ground-based observer of radiation reflected by a lens. Preliminary calculations show that the beams reflected by the moving lens deviate at different angles, which leads to the mixing of the beams on the Earth's surface in the region of the radiation recorded. That, in turn, affects the appearance of the interference pattern and the process of recording the radiation reflected by the lens. Therefore, the purpose of this work is to calculate the formed optical response in the reception area of the radiation, taking into account the optical effects of the moving media: the Fizeau effect, the Doppler effect, violation of the Snell's law, the kinematic effect of the beam points displacement on the optical surface caused by the final velocity of radiation in the lens.

1. Introduction

One of the ways to solve the problem of increasing the accuracy of global satellite positioning systems is “Laser GLONASS” \cite{1} — a set of technical methods and tools, which in general is the laser location of reference low-orbit satellites with a set of reflectors.

The BLITS satellite \cite{2}, which is a spherical Luneburg lens moving in a geosynchronous near-earth orbit, was used as a passive laser satellite. Such a satellite can be used as a laboratory to study the effects of moving-media optics.

For a theoretical description of the processes of electromagnetic radiation propagation in a moving lens and the formation of its optical response, it is necessary to make a mathematical model that includes:

1) Kinematic relations describing the beam propagation and non-uniform movement of optical surfaces \cite{3};
2) The coordinate solution of the dispersion optical equation of moving media to calculate the beam trajectories taking into account the violation of the Snell's law \cite{4};
3) Expressions for the polarization plane rotation during the optical radiation propagation in a moving medium \cite{5};
4) The system of equations which takes into account amplitude changes and the polarization plane at the interface sections boundaries \cite{6};
5) An expression which takes into account the trajectory curvature in the case of non-uniform medium motion \cite{7};
6) The equations of the multipath interference response [8].

The developed model includes a sufficient mathematical apparatus for solving modern problems in computational optics, such as calculating the multipath interference of a diffraction interferometer [9] or modeling the interaction of electromagnetic radiation with a moving dielectric [10].

Using the mathematical model (1–6), an algorithm can be compiled for calculating the parameters of radiation propagating in a moving heterogeneous medium and the intensity distribution in the area of the signal reception.

2. The algorithm for calculating the optical response

Consider the stages of solving the problem:

1) As the initial data, we have a set of \( N \) beams with given wave vectors \( \vec{k}_i \) and amplitudes \( A_i \), a set of \( M \) surfaces determined analytically (curves or surfaces of second order), with given refractive indices \( n \).

2) Solving the system of kinematic relations describing the propagation of a beam and non-uniform movement of optical surfaces [3], to obtain the coordinates of the beam incidence \((x_{ij}, y_{ij})\) \((j=1..M)\) on optical surfaces \( M \), the travel time \( t_{ij} \) and optical paths \( r_{ij} \) between the surfaces.

3) Obtaining new wave vectors \( \vec{k}_{ij} \) at the media interface using the coordinate solution of the dispersion optical equation of moving media [4].

4) Setting a uniform grid on the registration surface, with given amplitudes and phases in the cells \( A_{res}(x)=0 \) and \( \Phi_{res}(x)=0 \).

5) When a beam hits a cell, we recalculate the its amplitude and phase \( A_{res}, \Phi_{res} \) according to the following formulas:

\[
A_{res}(x) = \sqrt{A^2_{res}(x) + A^2_{i} + 2A_{res}A_{i} \cos(\Phi_{res} - \Phi_{i})}
\]

\[
\Phi_{res} = \Phi_{res} + \Phi_{i}
\]

where the incident beam phase \( \Phi_{i} \) is calculated as follows:

\[
\Phi_{i} = \sum_{j=1}^{M} \omega_{ij}t_{ij} - \omega_{iM}t_{iM} - \sum_{j=1}^{M} \vec{k}_{ij}r_{ij},
\]

where \( \Delta t \) is the difference between the passage time of all optical path sections by the beam and the passage time of the buffer beam.

6) After calculating all the beams, you can get the intensity distribution in the registration plane:

\[
I(x) = c \frac{(A_{res}(x) \cos(\Phi_{res}))^2}{8\pi}
\]

3. Results of numerical experiments for the “BLITS-M” satellite

The calculation was carried out in the central plane of the beam incidence on the Luneburg lens. We introduce additional parameters:

- effective deflection angle \( \theta \) - the angle between the central axis of the lens \( (OO_1) \) and the segment connecting its center and the point of the reflected beam incidence on the recording surface \( (OA) \) (Fig. 1)
- the shift of coordinates of beam incidence points in the reception area \( dl(AB) \), the cause of which is the non-parallelism of the beam reflected by the lens (Fig. 1)
- \( dx \) partitioning interval - uniform grid step on the registration plane for calculating interference for beams falling in one coordinate interval (Fig. 1)
Figure 1. Geometry of the beam propagation in the lens.

Figure 1 shows the beam path in the Luneburg lens, where \( S \) is the signal registration surface, \( O \) is the lens center, \( (x_i, y_i) \) is the point of beam incidents on optical surfaces, \( R_{1,2} \) is the radius of the outer meniscus and the central ball, \( \vec{V} \) is the speed of the lens, \( OO_i \) is the central axis of the lens, \( A \) is the point the reflected beam incidents on the registration surface, \( B \) is the projection of a point \( (x_f, y_f) \) on the registration surface, \( \theta \) is the angle of the beam incidence, \( \theta_e \) is the effective angle of deflection, \( dl \) is the shift of the coordinate of the beam incidence, \( dx \) is the split interval.

| Parameters                      | “BLITS-M” |
|---------------------------------|-----------|
| The radius of the central sphere \( R_1 \), mm | 63.9      |
| External meniscus radius \( R_2 \), mm          | 110.4     |
| Central sphere material        | TF105     |
| External meniscus material     | K108      |
| Orbit \( h \), km              | 1500      |
| Linear velocity in orbit \( V \), m/s        | 7100      |

As the results of a numerical experiment, we consider radiation patterns reflected by the BLITS-M satellite. The design parameters of the satellite are given in Table 1.
Figure 2. The type of radiation pattern on the universal measuring complex, lens speed $V=0 \text{ m/s}$.

The Fig. 2 shows the experimental dependence of the intensity $I$ on the angle of deviation $\theta$ obtained in the universal measuring complex in the JSC NPK “SPP”. Based on the type of experimental radiation pattern, it can be concluded that the distribution of the beam amplitudes of incident radiation is Gaussian.

The numerical experiment was conducted with the following parameters: the number of beams $N = 10^5$, the wavelength $\lambda = 532 \text{ nm}$, the intensity of the central beam $dl = 5 \times 10^{-7} \text{ W/m}^2$, the dependence of the amplitude on the coordinate of the beam incident on the lens has a Gaussian distribution with standard deviation $\sigma = 8 \times 10^{-4}$, the angles of incidence $\vartheta = 0..3^\circ$ on the left and on the right (Fig. 1), phase summation interval for the pattern is $d\theta = 0.5$ angular seconds.

The calculation was carried out in the near/far zone modes (10 m/1500 km, respectively), taking into account without taking into account the speed (satellite linear speed is 7100 m/s).

Figure 3. The radiation pattern of the reflected radiation in the near zone, without taking into account the satellite speed. The registration surface of the signal $S$ is located at a distance of 10 m from the lens center.

When calculating in the near zone without speed, the dependence of intensity $I$ on the effective deflection angle $\theta$ mainly determines the coordinate of the beam exit from the lens $x_7$, since $dl \parallel x_7$.
(Fig. 1). Note that the magnitude of the intensity of the side peaks is determined by the standard deviation and is not fundamental in the calculations.

![Figure 4](image1.png)

**Figure 4.** The radiation pattern of reflected radiation in the near zone with taking into account the speed. The registration surface of the signal $S$ is located at a distance of $10$ m from the lens center, the satellite speed is $V = 7100$ m/s.

In the case of the calculation in the near zone with speed, it is necessary to notice significant changes in the refraction angles of the beams in the lens, therefore in this case for off-center beams $dl \approx x_\gamma$ (Fig. 1), which is why the signal is blurring in the receiving area and the central peak of the intensity is divided (4·10^{-2} W/m²) for two side peaks (1.8·10^{-2}, ±11 angular seconds).

![Figure 5](image2.png)

**Figure 5.** The radiation pattern of the reflected radiation in the far zone without taking into account the speed. The registration surface of the signal $S$ is located at a distance of 1500 km from the lens center.
In the case of calculations in the far zone without speed, the following situation is observed: the exit coordinate of the beam \( x \) does not contribute, since the recording surface is at a relatively large distance, due to the lack of speed we have an almost parallel reflected beam, because of which the deviation of the fall coordinate \( dl \) is small (Fig. 1), this explains the smaller range of the effective deflection angle \( \theta \) in which the radiation is recorded, and due to the narrowing of the diagram, the intensity increases, the maximum of which in the diagram above reaches \( 7.5 \times 10^{-1} \) W/m².

**Figure 6.** The radiation pattern of the reflected radiation in the far zone with taking into account the speed. The registration surface of the signal \( S \) is located at a distance of 1500 km from the lens center, the satellite speed is \( V = 7100 \) m/s.

If the speed in the far zone is taken into account, a noticeable broadening of the detection region is observed. It is associated with an increase in the deviation angles of off-center beams, which significantly reduces the intensity in the region of small angles \( \theta \) (\( 1.6 \times 10^{-3} \) W/m² at \( \theta = 0 \)).

4. **Conclusion**

Calculations in the near and far zones have a fundamental difference, since different beams fall on the same coordinate interval on the registration plane in these cases (due to the dependence of \( dl \) (Fig. 1) on the distance between the lens and the surface \( S \)) regardless of the satellite speed. The speed also affects the redistribution of beams on the surface, which leads to significant changes in the intensity distribution, in particular, a significant drop in the signal-to-noise ratio in the signal receiving area (\( \theta = 0 \)), which may affect the reliability of the satellite’s laser locating method, therefore it is necessary to solve the inverse problem of correcting geometrical characteristics of the lens for focusing the reflected signal taking into account the optical effects of moving media.

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