Suppression and enhancement of decoherence in an atomic Josephson junction

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Abstract

We investigate the role of interatomic interactions when a Bose gas, in a double-well potential with a finite tunneling probability (a ‘Bose–Josephson junction’), is exposed to external noise. We examine the rate of decoherence of a system initially in its ground state with equal probability amplitudes in both sites. The noise may induce two kinds of effects: firstly, random shifts in the relative phase or number difference between the two wells and secondly, loss of atoms from the trap. The effects of induced phase fluctuations are mitigated by atom–atom interactions and tunneling, such that the dephasing rate may be suppressed by half its single-atom value. Random fluctuations may also be induced in the population difference between the wells, in which case atom–atom interactions considerably enhance the decoherence rate. A similar scenario is predicted for the case of atom loss, even if the loss rates from the two sites are equal. We find that if the initial state is number-squeezed due to interactions, then the loss process induces population fluctuations that reduce the coherence across the junction. We examine the parameters relevant for these effects in a typical atom chip device, using a simple model of the trapping potential, experimental data, and the theory of magnetic field fluctuations near metallic conductors. These results provide a framework for mapping the dynamical range of barriers engineered for specific applications and set the stage for more complex atom circuits (‘atomtronics’).

1. Introduction

Circuits for neutral atoms (coined ‘atomtronics’), e.g., [1–4] are becoming more feasible due to tremendous advances in the control and manipulation of ultracold atoms using magnetic and optical fields. The idea of atomtronics is inspired by the analogy between ultracold atoms confined in optical or magnetic potentials and solid-state systems based on electrons in various forms of conductors, semiconductors or superconductors. For example, ultracold atoms in optical lattices exhibit a Mott insulator to superfluid transition, or display spin–orbit coupling as in solid-state systems [5, 6]. Another example is a Bose–Einstein condensate (BEC) of neutral atoms in a double-well potential, which is analogous to a Josephson junction of coupled superconductors [7–11]. On the other hand, the quantum properties of ultracold atoms as coherent matter waves enable systems that are equivalent to optical circuits, which are based on waveguides and beam-splitters for interferometric precision measurements in fundamental science and technological applications [12, 13].

A promising platform for accurately manipulating matter waves in a way that would enable integrated circuits for neutral atoms is an atom chip [14–17]. Such a device facilitates precise control over magnetic or optical potentials on the micrometer scale. This length scale, which is on the order of typical atomic de-Broglie wavelengths in a BEC, permits control of important dynamical parameters, such as the tunneling rate through a
potential barrier. For a network consisting of static magnetic fields, such control over the dynamics requires loading the atoms into potentials just a few micrometers from the surface of the chip [14, 18, 19]. The ability to load such potentials, while maintaining spatial coherence, was recently shown to be possible [20]. This achievement is facilitated by the weak coupling of neutral atoms to the environment [14, 21, 22]. Yet, in view of the fact that spatial coherence is one of the most vulnerable properties of quantum systems made of massive particles, it is quite surprising that a BEC of thousands of atoms preserves spatial coherence for a relatively long time in the very close proximity of a few micrometers from a conducting surface at room temperature.

Here we examine the interplay between coupling to external noise and the internal parameters—tunneling rate and atom–atom interactions—of a BEC in a double-well potential (a ‘Bose–Josephson junction’). Such a system, consisting of a potential barrier between two potential wells, provides one of the fundamental building blocks of atomic circuits and comprises one of the basic models for studying a simple system of many interacting particles occupying only two modes. The present study unravels some general many-body effects, and at the same time enables insights into limits for the practical use of circuits of a trapped BEC near an atom chip surface.

Macroscopic one-particle coherence is the hallmark of Bose–Einstein condensation. The Penrose–Onsager criterion states that as the condensate forms, one of the eigenvalues of the reduced one-particle density matrix becomes dominant, resulting in a pure state in which all atoms occupy the same quantum ‘orbital’. Whenever two condensates expand and overlap with each other, their detection yields an interference pattern. However, such patterns exhibit deterministic, repeatable, fringe positions only if the two parts of the atomic cloud have a fixed phase relation, namely that they constitute a single condensate [23, 24]. Once such a system is prepared however, its one-particle coherence can be lost via entanglement with an external environment (‘decoherence’) [21, 25–27] or by internal entanglement between condensate atoms due to interactions (‘phase diffusion’) [28, 29]. While each of these processes has been extensively studied in the literature in a general context or in the specific context of ultracold atoms, this work will focus on the interplay between these two decoherence mechanisms.

The interplay between non-Hamiltonian decoherence and the Hamiltonian dynamics of interacting particles is often rich and intricate. The combined effect is rarely additive and depends strongly on the details of the coupling mechanisms. For example, decoherence may be used to protect one-particle coherence by suppressing interaction-induced squeezing in a quantum-Zeno-like effect [30, 31]. It may also induce stochastic resonances which enhance the system’s response to external driving [32].

Reversing roles, one may ask how interactions affect the dephasing or dissipation of a BEC due to its coupling to the environment. In this work we consider a BEC in a double well using a two-mode Bose–Hubbard approximation, and investigate how the loss of one-particle coherence due to the external noise is affected by interparticle interactions. We find that many-body dynamics can either enhance or suppress decoherence, depending on the nature of the external noise. We consider several sources of such noise and their effects: thermal (Johnson) noise, inherent in the metallic structures used for atom chips, has a short correlation length near the chip surface and can give rise to phase differences between the two wells via shifts in the Zeeman energy levels. Technical noise, typically originating in the power supplies which drive the atom chip currents, has a long correlation length but can cause asymmetric deformation of the double-well potential that lead to a similar dephasing effect. Both types of noise give rise to atom loss due to spin-flip transitions. In the presence of atom–atom interactions this loss process leads to fluctuations in the number difference between the wells and consequently to dephasing.

In light of these fundamental effects, this work also attempts to construct a framework for combining our theoretical model with practical experimental parameters for realistic magnetic potentials and magnetic noise on an atom chip. We therefore present some experimental results for atom loss at distances of a few micrometers from an atom chip, which we analyze in the context of key parameters that are relevant for maintaining coherence in practical atomic circuits using similar platforms.

This paper is structured as follows: in section 2, we describe the basic constituents of the system we are about to study. In section 3 we review the theoretical model and fundamental properties of a BEC in a double well and in section 4 we derive its coupling to magnetic noise. Section 5 then combines these effects to present the main results of this work: how decoherence in an atomic Josephson junction can be suppressed or enhanced by atom–atom interactions. The range of validity and accessible range of parameters of our model are discussed in section 6. This discussion is supplemented by experimental measurements of magnetic noise in atomic traps (section 7). Finally, our discussion in section 8 includes examples of practical and fundamental implications of the predicted effects.

2. Description of the system

We consider a BEC of atoms with mass $m$ in a double-well potential. The potential is modeled by a cylindrically symmetric harmonic transverse part $V_\perp = \frac{1}{2}m\omega_\perp^2[y^2 + (z - z_0)^2]$ centered at a distance $z_0$ from the surface of
an atom chip, and a longitudinal part \( V_{\parallel} \) representing a barrier of height \( V_0 \) between two wells with minima at \( x = \pm d/2 \). This potential, which is symmetric under \( x \to -x \) reflection, may be modeled as:

\[
V(x) = \begin{cases} 
  V_0 \cos^2(\pi x/d) & |x| \leq d/2, \\
  \frac{1}{2}m\omega^2_s(|x| - d/2)^2 & |x| > d/2, 
\end{cases}
\]

(1)

where the longitudinal frequency \( \omega_s \), characterizing the curvature of the potential far from the barrier, is typically smaller than the transverse frequency \( \omega_w \). Such a system is usually referred to as a Josephson junction; it exhibits Josephson oscillations with frequency \( \omega_J \) when the number of atoms in the two wells deviates slightly from equilibrium.

Some of the most important properties of the condensate in the potential can be derived from the assumption that a 'macroscopic' number of atoms occupy a single mode whose wave function \( \phi_0(\mathbf{r}) \) satisfies the Gross–Pitaevskii (GP) equation, whose static form is

\[
-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + gN|\phi_0(\mathbf{r})|^2 - \mu \phi_0(\mathbf{r}) = 0, 
\]

(2)

where \( \mu \) is the chemical potential, which is the energy of a single atom in the effective (mean-field) potential \( V_{\text{eff}} = V(\mathbf{r}) + gN|\phi_0(\mathbf{r})|^2 \). Here \( N \) is the total number of atoms and \( g = 4\pi\hbar^2a_s/m \) is the collisional interaction strength, with \( a_s \) being the s-wave scattering length.

As long as the barrier height is not too high and the temperature is low enough, the atoms predominantly occupy the condensate mode \( \phi_0(\mathbf{r}) \) and the system is coherent, namely, the phase between the two sites is well defined. However, when the barrier height grows and tunneling is suppressed, more atoms occupy other spatial modes and the one-particle coherence drops. In order to understand this effect, we use a set of spatial modes defined by higher-energy solutions of the GP equation

\[
-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + gN|\phi_j(\mathbf{r})|^2 - \mu_j \phi_j(\mathbf{r}) = E_j \phi_j(\mathbf{r}),
\]

(3)

where \( \phi_j(\mathbf{r}) \) is the condensate mode with \( E_0 = 0 \) and the other modes \( j > 0 \) represent excited single-atom states in the mean-field potential \([33] \). These modes form a complete set which may serve as a basis for any calculation. Furthermore, this specific choice is useful because it can describe the ground state of the system for all barrier heights.

When the barrier is low (or does not exist) the condensate approximation holds for the ground state. However, the nature of the ground state changes when the barrier becomes higher than the longitudinal ground-state energy

\[
\mu \equiv \int d^3r \, \phi_0(\mathbf{r}) \hat{H}_{\text{eff}}^0 \phi_0(\mathbf{r});
\]

\[
\hat{H}_{\text{eff}}^0 \equiv -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + gN|\phi_0(x, y = 0, z = z_0)|^2.
\]

(4)

As demonstrated in figure 1, when \( \mu \lesssim V_0 \) the energy \( E_1 \) of the (anti-symmetric) first excited mode \( \phi_1 \) becomes very small compared to the other excited modes and the configuration space can be described by the two modes \( \phi_0 \) and \( \phi_1 \), or alternatively by their superpositions \( \phi_L \) and \( \phi_R \) localized predominantly in the left- and right-hand wells, respectively. In this regime, collisional interactions play a major role in determining the ground-state configuration beyond the spatial effects accounted for by the mean-field potential, as will be described in section 3.

In this work we focus on the interplay between the intrinsic parameters of the system and the coupling to the environment. The latter may appear in different shapes and forms. Basically, the strongest coupling is between the magnetic moment of the atom and magnetic field noise originating from current fluctuations in the atom chip device that creates the trapping potentials from current-carrying wires. We distinguish between macroscopic current fluctuations generated by external drivers (‘technical noise’) and Johnson noise, i.e. microscopic fluctuations of thermal origin in the metallic layers of the chip itself, which are typically at room temperature or higher. Johnson noise has a short correlation length \([21] \) and can therefore cause direct loss of coherence over short length scales. When the magnetic field fluctuates perpendicular to the quantization axis (along which the atomic spin is typically aligned), it may induce transitions between Zeeman sub-levels and cause atoms to leave the trap, as discussed in more detail in section 4. This loss mechanism may also cause dephasing, as we discuss in section 5.2. Although technical noise has a long correlation length, it may also contribute to dephasing through the latter mechanism. However, as we discuss in section 4, technical noise may also lead to direct dephasing if the corresponding current fluctuations induce an asymmetric deformation in the trapping potential as a result of summing magnetic field vectors from nearby microwires and more distant sources.
3. The two-mode model

Our framework for the analysis of decoherence of a Bose gas in a double well is the two-site Bose–Hubbard model, which is based on the assumption that all atoms occupy one of two spatial modes, as described in the previous section. Before we analyze decoherence in the system we review this model and its main predictions relevant to this work. The validity of the model for typical scenarios presented in this paper is discussed further in section 6.

3.1. Interaction and tunneling Hamiltonian

Consider $N$ bosonic atoms in a double well such that two spatial modes may be occupied, $\phi_L(r)$ in the left well and $\phi_R(r)$ in the right well (see section 2 above). The dynamics is governed by the two-site Bose–Hubbard Hamiltonian \[ H = \frac{\epsilon}{2} (\hat{n}_L - \hat{n}_R) - \frac{J}{2} (\hat{a}_L^{\dagger} \hat{a}_R + \hat{a}_R^{\dagger} \hat{a}_L) + \frac{U}{2} \sum_{j=L,R} \hat{n}_j (\hat{n}_j - 1), \] where $\epsilon$ is the energy imbalance (per particle) between the two wells, $J$ is the tunneling matrix element and $U$ is the on-site interaction energy per atom pair. Here $\hat{a}_j$ and $\hat{n}_j = \hat{a}_j^{\dagger} \hat{a}_j$ are the bosonic annihilation operators of atoms in the two modes $\phi_j$ and $\hat{n}_j = \hat{a}_j^{\dagger} \hat{a}_j$ are the corresponding number operators.

Since the Hamiltonian (equation (3)) conserves the total number of particles in the two wells, we can write it in terms of the pseudo-spin operators $\hat{S}_z = \frac{1}{2} (\hat{a}_L^{\dagger} \hat{a}_R + \hat{a}_R^{\dagger} \hat{a}_L)$, $\hat{S}_+ = -\frac{1}{2} (\hat{a}_L^{\dagger} \hat{a}_R - \hat{a}_R^{\dagger} \hat{a}_L)$ and $\hat{S}_- = \frac{1}{2} (\hat{n}_L - \hat{n}_R) \equiv \hat{n}$, whereupon the Hamiltonian takes the form \[ \hat{H} = \epsilon \hat{S}_z + U \hat{S}_z^2. \] (6)

Note that the total spin $\hat{S}_z^2 + \hat{S}_+^2 + \hat{S}_-^2 = s(s + 1)$ is fixed by the total atom number $2s = N = \hat{n}_L + \hat{n}_R$, as it commutes with the Hamiltonian in equations (5) and (6). The Hilbert space of the two-mode model is spanned by the $\hat{S}_z$ eigenstates $|s, n\rangle$, corresponding to the number states basis $|n_L, n_R\rangle$ with $n_{L,R} = N/2 \pm n$. The total spin $s = n = \pm s$.

3.2. Equivalence to the top and pendulum Hamiltonians

The spin Hamiltonian in equation (6) is equivalent to a quantized top, whose spherical phase space is described by the conjugate non-canonical coordinates $(\tilde{\theta}, \tilde{\varphi})$ defined through

\[ \tilde{\theta} = \sin^{-1} \left( \frac{\hat{S}_z}{\sqrt{s(s+1)}} \right), \quad \tilde{\varphi} = \tan^{-1} \left( \frac{\hat{S}_+}{\hat{S}_-} \right). \]
\[
\dot{\hat{S}}_3 = \frac{N}{2} \cos \hat{\theta},
\]
\[
\dot{\hat{S}}_1 = \frac{N}{2} \sin \hat{\theta} \cos \hat{\varphi},
\]
in analogy to the definitions on the Bloch sphere. Equation (6) is thus transformed into the top Hamiltonian \[36, 37,\]
\[
\hat{H}(\hat{\theta}, \hat{\varphi}) = \frac{N}{2} \left[ \epsilon \cos \hat{\theta} - J \sin \hat{\theta} \cos \hat{\varphi} + \frac{NU}{2} \cos^2 \hat{\theta} \right],
\]
If the population imbalance between the two wells \(\frac{1}{2}(\hat{n}_L - \hat{n}_R) = \hat{S}_3\) remains small during the dynamics, such that \(\cos \hat{\theta} \ll 1\) and \(\sin \hat{\theta} \sim 1\), then the spherical phase space reduces to the ‘equatorial region’, where it may be described by the cylindrical coordinates \(\hat{n} = \hat{S}_1\) and its canonically conjugate angle \(\hat{\varphi}\), satisfying \([\hat{n}, \hat{\varphi}] = 1\).

The Hamiltonian (equation (9)) is then approximated by the Josephson pendulum Hamiltonian [26]
\[
\hat{H}_J(\hat{n}, \hat{\varphi}) = U (\hat{n} - n_e)^2 - \frac{1}{2} JN \cos \hat{\varphi},
\]
where \(n_e = -\epsilon / 2U\).

In the classical limit with a large number of atoms, the physics described by the Hamiltonian in equation (10) is equivalent to that of a Josephson junction of two coupled superconductors and to a classical pendulum, as demonstrated experimentally [9]. Depending on the parameters, the dynamics of the system is either characterized by small oscillations of the population \(\langle \hat{n} \rangle\) and the phase \(\langle \hat{\varphi} \rangle\) (‘Josephson oscillations’) or by self-trapping of a relatively high population imbalance accompanied by a monotonic growth of the phase between the two wells (‘DC Josephson effect’).

3.3. Classical phase space and interaction regimes

The classical states of the double-well system, described by the full Hamiltonian (equations (6) or (9)), are SU(2) spin coherent states \(|\theta, \varphi\rangle [38]\). Dynamics restricted to such states satisfies the mean-field approximation of the GP equation, where all the atoms are in a single mode—a superposition of \(|\phi_L\rangle\) and \(|\phi_R\rangle\)—and \(O(1/N)\) fluctuations are neglected. The operators in equations (6), (9), and (10) are hence replaced by corresponding real numbers. Thus, the GP dynamics may be viewed as the classical limit of the quantum many-body Hamiltonian, with an effective Planck constant \(\hbar \to 2/N\). This regime is quite different to single-particle tunneling dynamics widely studied in the spin-boson problem [26].

The qualitative features of the classical phase-space structure change drastically with the interaction strength [39]. It proves advantageous to make use of dimensionless characteristic parameters which determine both stationary and dynamic properties; these are
\[
u \equiv NU / J \quad \text{and} \quad \epsilon \equiv \epsilon / J.
\]
If \(|\epsilon| < \epsilon_c \equiv (\nu^{2/3} - 1)^{1/2}\), then for a strong enough interaction \((\nu > 1)\) two regions appear in the spherical \((\hat{\theta}, \hat{\varphi})\) phase space, divided by a separatrix, as shown in figure 2. Motion within the region close to the equator (small population imbalance) is dominated by linear (tunneling) dynamics characterized by Josephson oscillations of population between the wells. Conversely, motion between the separatrix and the poles is dominated by nonlinear (interaction) dynamics of self-trapping characterized by a monotonic phase change with a non-vanishing time-averaged population imbalance.

The relative size of these phase-space regions can be estimated from the dimensionless interaction parameter \(u\) (equation (11)). In the case of zero energy imbalance \((\epsilon = 0)\), on which we will focus in this work, three interaction regimes may be identified [8]:

Rabi regime : \(u < 1\),
Josephson regime : \(1 < u < N^2\),
Fock regime : \(u > N^2\).

While the Josephson regime is characterized by the appearance of the two dynamical regions, in the Rabi regime the entire phase space is dominated by nearly linear population oscillations between the wells. At the opposite extreme, in the Fock regime, almost the entire phase space is dominated by the nonlinear self-trapping dynamics. The region dominated by nearly linear dynamics has an area less than \(1/N\) and cannot accommodate quantum states. In the absence of tunneling between the wells, spatial coherence cannot be sustained even if external noise does not exist. We will therefore focus on the phase-space region with oscillatory dynamics which exists in the Rabi and Josephson regimes.
3.4. Semiclassical dynamics

Beyond strictly classical evolution, much of the full quantum dynamics is captured by a semiclassical approach. This amounts to classical Liouville propagation of an ensemble of points corresponding to the initial Wigner distribution in phase space \(39\). Thus, while classical mean-field theory assumes the propagation of coherent states (minimal Gaussians centered at \(q_j(\theta, \varphi)\) with fixed uncertainties), the semiclassical method permits the deformation of the phase-space distribution, accounting for its squeezing, folding, and spreading. The semiclassical evolution fails to accurately describe the fully quantum evolution only in the case where the Wigner function may become negative \(39\).

3.5. Ground state and excitation basis

As an alternative point of view that complements the semiclassical approach we also use a fully quantum treatment of the two-mode Bose-Hubbard model, which takes a simple analytic form when the energy is small and only the lowest energy eigenstates of the Hamiltonian are populated. We assume again a balanced potential \(\equiv \int_0^\infty\) and derive the structure of the ground state and the low-energy excitations.

In the Fock regime, the ground state and low-lying excitations are number states. On the other hand, throughout the Rabi and Josephson regimes, the ground state and low-lying excitations are close to coherent states and are characterized by a small phase uncertainty \(\approx 1\). We therefore make the approximation \(\approx -\cos 1/2\), which converts the Josephson Hamiltonian (equation (10)) to an oscillator in the canonical variables \(\hat{n} \equiv \hat{S}_3\) and \(\hat{\varphi}\),

\[
\hat{H}_1 \approx \frac{1}{2M} (\xi \hat{n})^2 + \frac{M}{2} \omega_1^2 (\hat{\varphi}/\xi)^2,
\]

where \(\xi\) is the squeezing factor, given by

\[
\xi = (1 + \nu)^{1/4},
\]

\(\omega_1\) is the Josephson frequency

\[
\omega_1 = \sqrt{J (\nu + N U)} = \xi J,
\]

and \(M \equiv N/2\omega_1\). The ground state of this Hamiltonian is Gaussian in \(\xi \hat{n}\) and \(\xi \hat{\varphi}\), and the excitation energies in this approximation are integer multiples of \(\hbar \omega_1\).

More explicitly, we may define a bosonic operator

\[
\hat{b} \equiv \frac{\sqrt{N}}{2 \xi} \hat{\varphi} + \frac{i \xi}{\sqrt{N}} \hat{n},
\]
such that from $[\hat{\varphi}, \hat{n}] = i$ we obtain $[\hat{b}, \hat{b}^\dagger] = 1$. The Josephson Hamiltonian can then be written as

$$\hat{H}_1 \approx \omega_j (\hat{b}^\dagger \hat{b} + \frac{1}{2}),$$

and the pseudo-spin operators of equation (6) are identified as

$$\hat{S}_1 = \frac{N}{2} \sin \hat{\varphi} \cos \hat{\varphi},$$

$$\approx \frac{N}{2} - \frac{1}{4} [\xi^2 (\hat{b} + \hat{b}^\dagger)^2 - \xi^{-2} (\hat{b} - \hat{b}^\dagger)^2 - 2],$$

(20)

$$\hat{S}_2 = \frac{N}{2} \sin \hat{\varphi} \sin \hat{\varphi} \approx \frac{1}{2} \sqrt{N} \xi (\hat{b} + \hat{b}^\dagger),$$

(21)

$$\hat{S}_3 = \hat{n} \approx \frac{\sqrt{N}}{2 \xi} (\hat{b} - \hat{b}^\dagger).$$

(22)

Here we have used a second-order expansion in $\hat{n}/N$ and $\hat{\varphi}$ such that $\sum_{j=0}^{\infty} \hat{S}_j^2 = \frac{N}{2} \left( \frac{N}{2} + 1 \right)$.

### 3.6. Coherence

We are interested in the one-body spatial coherence, which determines the ability to observe macroscopic Josephson oscillations when an imbalance between the two wells is induced. In the context of ultracold atoms in a double well, coherence may be defined as the visibility of a fringe pattern formed by averaging many events in which the atoms are released from the double-well trap. If we neglect experimental imperfections related to the release process or imaging, then the coherence is the relative magnitude of the interference term of the momentum distribution of the atoms in the trap [40]. If the two spatial modes $\phi_L$ and $\phi_R$ are confined primarily to the left and right sites, respectively, of the potential, then the coherence is given by

$$g_{LR}^{(1)}(\tilde{n}) = \frac{|\langle \tilde{n}_L \rangle \langle \tilde{n}_R \rangle|}{\sqrt{\tilde{n}_L \tilde{n}_R}} = \frac{|\langle \hat{S}_1 + i \hat{S}_2 \rangle|}{[N^2/4 - (\hat{S}_1)^2]^{1/2}},$$

(23)

where $\tilde{n}_j \equiv \langle \tilde{n}_j \rangle$ is the average number of atoms in the two modes.

For a symmetric double well ($\epsilon = 0$) with equal populations of the two modes ($\langle \hat{S}_j \rangle = 0$), the definition of equation (23) coincides with the normalized length $S/s = 2S/N$ of the Bloch vector $S \equiv (\langle \hat{S}_1 \rangle, \langle \hat{S}_2 \rangle, \langle \hat{S}_3 \rangle)$. Let us note that $s$ is the maximal allowed Bloch vector length (e.g., after preparation of a coherent state), whereas $S$ is the actual Bloch vector length, i.e., after time evolution under dephasing or if the prepared state was squeezed.

For the ground state preparations with $\langle \varphi \rangle = 0$ considered below, symmetry implies that the Bloch vector remains aligned along the $S_1$ axis, so that $g_{LR}^{(1)} = S/s = \langle \hat{S}_1 \rangle / s$ throughout the time evolution.

Semiclassically, for any Gaussian phase-space distribution, the one-particle coherence is related to the variances $\Delta_{s_1}, \Delta_{s_0}$ of the distribution along its principal axes $s_1, s_0$ as [41]

$$S/s = \exp \left[-\frac{1}{2} (\Delta^2 - 2/N) \right],$$

(24)

where $\Delta^2 = \Delta_{s_0}^2 + \Delta_{s_1}^2$. A coherent state with isotropic $\Delta_{s_0}^2 = \Delta_{s_1}^2 = 1/N$ thus has $S = s$, i.e., $g_{LR}^{(1)} = 1$.

Otherwise, for a squeezed Gaussian distribution with $\Delta_{s_0}^2 = \xi^2/N$, $\Delta_{s_1}^2 = \xi^{-2}/N$, the coherence drops to $g_{LR}^{(1)} = \exp \left[-(\xi^2 + \xi^{-2})/2N \right] < 1$. For example, the ground state of the Josephson Hamiltonian is described by a Gaussian phase-space distribution which is squeezed along the principal axes with the squeezing factor $\xi$ in equation (16), corresponding to a coherent state ($\xi = 1$) only in the non-interacting case ($\mu = 1$). It is evident that in order to see significant reduction in the ground state coherence, $\xi^2 = \sqrt{\mu} + 1$ should be comparable to $N$. Thus, the coherence of the ground state drops only in the transition from the Josephson to the Fock regimes ($\mu > N^2$) [8]. This crossover in the finite size system is the simplest version of the superfluid to Mott insulator transition [42].

The coherence drops in the Josephson regime when the phase-space distribution spreads out. This may result from a decoherence process due to external noise, that will be described in section 5, or due to the thermal population of excited states. If the widths of the distribution increase by a factor $D$ so that $\Delta_{s_0}^2 = D \xi^2/N$, $\Delta_{s_1}^2 = D \xi^{-2}/N$, equation (24) gives a coherence $g_{LR}^{(1)} = \exp \left[-(D \xi^2 + D \xi^{-2} - 2)/2N \right]$. For a thermal state $D = 2n_T + 1$, with $n_T \equiv \langle \hat{b}^\dagger \hat{b} \rangle_T \approx \exp(h\omega_0/k_B T) - 1]^{-1}$ being the average occupation number of the excited levels.
4. Dephasing and loss due to magnetic noise

The interaction of the magnetic field fluctuations with an atom is given by the Zeeman Hamiltonian

$$\hat{V}_Z (r, t) = -\hat{\mu} \cdot B(r, t),$$

(25)

where $\hat{\mu}$ is the magnetic moment of the atom. If the magnetic field is not too strong then the atom stays in a specific hyperfine state $|F_i, m_i\rangle$ and its magnetic moment is proportional to the total angular momentum $\mathbf{F}$ (orbital, electronic and nuclear spins) through $\hat{\mu} = \mu_F \mathbf{F} = g_F \mu_B \mathbf{F}, \mu_F$ being the Landé factor and $\mu_B$ the Bohr magneton. For an ensemble of atoms with translational degrees of freedom, we adopt the language of second quantization and write the interaction Hamiltonian as

$$\hat{V}_Z (t) = -\mu_F \sum_{q=0, \pm 1} \sum_m \int d^3r \hat{F}^{m'}_{q}(r) B_q (r, t),$$

(26)

where the three operators

$$\sum_{m} \hat{F}^{m'}_{q} \equiv \sum_{m} \psi^{+}_{m+q} \langle r \rangle F^{m+q}_{q} \psi_{m}(r)$$

(27)

are the components of the magnetization density. Here, $q = 0, \pm 1$ labels the components of the magnetic field and the angular momentum operator. $B_q$ is the field component parallel to the local quantization axis (i.e. the direction of static trapping field), whereas $B_{\pm 1}$ represents transverse magnetic field components. The matrix elements $F^{m'}_q (-F \leq m, m' \leq F)$ arise from the angular momentum operators $\hat{F}_m$ and $\hat{F}_m \equiv \hat{F}_1 \mp i \hat{F}_2$ and are non-zero for $m' = m \pm q$. The operators $\hat{\psi}_{m}(r)$ and $\hat{\psi}^{+}_{m}(r)$ are field operators for atoms in the Zeeman sublevel $|m\rangle$, satisfying bosonic commutation relations $[\hat{\psi}_{m}(r), \hat{\psi}^{+}_{m'}(r')] = \delta_{m,m'} \delta (r - r')$.

In magnetic traps the Zeeman Hamiltonian (equation (26)), with $B$ as the magnetic trapping field, determines the trapping potential (e.g., equation (1)) for atoms in a Zeeman sublevel whose magnetic moment is aligned parallel to the local magnetic field. We will now examine the effect of magnetic field fluctuations, such that $B$ in equation (26) will represent changes in the magnetic field relative to a static trapping field. These are responsible either for fluctuations of the trapping potential (for parallel magnetic field components ($q = 0$) or slowly varying fields in any direction), or for transitions between different Zeeman states (transverse components ($q = \pm 1$) at the transition frequency for transitions with positive or negative angular momentum changes).

We assume that the trapped atoms are initially in the state $|F_i, m = F\rangle$ and predominantly occupy the two lowest energy modes $\phi_L$ and $\phi_R$ of the double-well potential. The field operator for the trapped level can in general be expanded as

$$\hat{\psi}_{m}(r) = \phi_L(r) \hat{a}_L + \phi_R(r) \hat{a}_R + \sum_{k} \phi_k(r) \hat{a}_k,$$

where $\hat{a}_L$, $\hat{a}_R$ are the bosonic annihilation operators for the left and right modes, while $\phi_L(r)$ and $\hat{a}_L$ represent higher energy modes and their annihilation operators, respectively. The atomic magnetization component $q \equiv 0$ of equation (27) is then given by

$$\hat{F}^{F}_{0}(r) = F \left\{ \sum_{i=\text{L,R}} \phi^{\dagger}_{i}(r) \phi_{i}(r) \hat{a}^\dagger_{i} \hat{a}_{i} \right\} + \left\{ \sum_{j=\text{L,R}} \sum_{k} \phi^{\dagger}_{k}(r) \phi_{k}(r) \hat{a}^\dagger_{k} \hat{a}_{k} + \text{h.c.} \right\},$$

(28)

In the first line, terms with $i = j$ correspond to fluctuating energy shifts that potentially dephase coherent superpositions of left and right sites, whereas terms with $i \neq j$ describe transitions between the right and left modes. The second line describes transitions between the two lowest modes and higher ones, corresponding to spatial excitations of the atoms in each well. Here we assume that the magnetic field is fairly homogeneous over each single site, such that the orthogonality between the spatial modes implies that these terms vanish in the integral of equation (26). We therefore assume that in any dynamics driven by the $B_0$ field the total number of atoms in the two modes $\phi_L$ and $\phi_R$ is conserved so that the two-mode Bose–Hubbard model described in section 3 is valid.

Magnetic field fluctuations perpendicular to the trapping field drive spin-flip transitions between the trapped level $m = F$ and other levels, which may be untrapped or more weakly trapped ($F > 1$). Here we will assume for simplicity that the level $m = F - 1$ is untrapped so that each atom transferred to this level immediately disappears from the trap. Using the matrix element $F^{F-1,F} = \sqrt{F}$ in equation (27) and expanding the field operator for the $m = F - 1$ level as $\hat{\psi}_{F-1}(r) = \sum_{k} \zeta_{k}(r) \hat{c}_{k}$, where $\zeta_{k}(r)$ are the untrapped spatial modes and $\hat{c}_{k}$ the corresponding annihilation operators, the magnetization density coupled to $B_{-1}$ becomes
\[ \hat{\mathcal{F}}_{l-1}^{F} = \sqrt{F} \sum_{k} \zeta_k^{(l)}(r) \tilde{c}_k^{+} \sum_{j=L,R} \phi_j(r) \hat{a}_j. \]  

(29)

Note that slowly varying magnetic fields perpendicular to the local trapping field would not cause transitions to untrapped states because the atomic spin direction would adiabatically follow the local direction of the magnetic field vector, such that the net effect of these slow fluctuations would be a change of the potential in a manner similar to magnetic fluctuations parallel to the trapping field.

We will now examine the master equation for the dynamics of the two processes: number-conserving processes dominated by energy/phase fluctuations, and loss processes.

### 4.1. Dephasing

The number-conserving evolution follows from the \( q = 0 \) term in the Hamiltonian (equation (26)), with \( \mathcal{F}_{l=0}^{m=F} \) given in equation (28), where only the low-lying modes \( \phi_L \) and \( \phi_R \) are kept, as explained above. One way to calculate the evolution would be to express the perturbation in terms of the eigenstates of the system Hamiltonian (section 3), and then distinguish between terms of the perturbation responsible for transitions among eigenstates ('relaxation') and terms responsible for phase fluctuations between eigenstates ('dephasing'), as was done in some other studies (see [26] for a review). In order to investigate the dynamic interplay between the effects of noise and atom–atom interactions, here we choose to keep the basis of left/right states and solve master equations in Lindblad form for the density matrix of the system.

The master equation for the density matrix \( \rho \) that follows from the number-conserving term proportional to \( \hat{\mathcal{F}}_{l}^{F} \) in the Hamiltonian has the form \( \dot{\rho} = -i [\hat{H}, \rho] + \mathcal{L}_N \rho \), where \( \mathcal{L}_N \) is the number-conserving operator

\[ \mathcal{L}_N \rho = -\sum_{jkl} \frac{\gamma_{ijkl}^{N}}{2} \left[ \hat{a}_i^{+} \hat{a}_j \hat{a}_k \hat{a}_l \rho + \rho \hat{a}_j^{+} \hat{a}_k \hat{a}_l \hat{a}_i - 2 \hat{a}_k \hat{a}_l \hat{a}_j \hat{a}_i \rho \right] \]  

(30)

and involves transition rates given by

\[ \gamma_{ijkl}^{N} = \frac{\mu F^2}{\hbar^2} \int d^3r \int d^3r' \times \phi_i^*(r) \phi_j(r) \phi_k^*(r') \phi_l(r') \mathcal{B}_{ijkl}(r, r', \omega_{ijkl}), \]

where \( \mathcal{B}_{ijkl}(r, r', \omega) \) is the two-point correlation spectrum of the \( q = 0 \) component of the magnetic field fluctuations, which is given for two general components \( q, q' \) by [43]

\[ \mathcal{B}_{q,q'}(r, r', \omega) = \int d\tau e^{i\omega \tau} \langle \mathcal{B}_q(r, t) \mathcal{B}_{q'}(r', t + \tau) \rangle. \]  

(32)

The spectrum in equation (31) is evaluated at the frequencies \( \omega_{ijkl} \equiv \omega_i + \omega_k - \omega_j - \omega_l \).

To proceed, we assume that the magnetic field fluctuations have a flat spectrum over the frequency scale of the lowest excitations in the trap (the Josephson frequency), and again use the assumption that their spatial variation is weak across the trap region. Then the orthogonality of the modes \( \phi_j \) implies that only the terms with \( i = j \) and \( k = l \) in equation (31) survive and we get \( \omega_{ijkl} = 0 \). Note that even if \( \mathcal{B}_{ijkl}(r, r', 0) \) varies over the trap region, the spatial modes are usually well localized in the two wells and overlap only in the region of the barrier, where their amplitude is small. We may therefore assume that \( \gamma_{ijkl}^{N} = \delta_{ij} \delta_{kl} \gamma_{i,j}^{N} \), such that the dissipative term in the master equation can be approximated as

\[ \mathcal{L}_N \rho \approx \frac{-\gamma_{ij}^{N}}{2} \sum_{k=L,R} \alpha_{jk} [\hat{n}_j \hat{n}_k \rho + \rho \hat{n}_j \hat{n}_k - 2 \hat{n}_j \rho \hat{n}_k]. \]  

(33)

Here \( \gamma_{ij}^{N} \) and \( \alpha_{jk} \) are defined as

\[ \gamma_{ij}^{N} = \frac{\mu F^2}{\hbar^2} \mathcal{B}_{00}(r, r, 0) \]  

(34)

\[ \alpha_{jk} = \int d^3r \int d^3r' \mathcal{A}(r, r') |\phi_j(r)|^2 |\phi_k(r')|^2, \]  

(35)

where \( r \) represents a typical location in which \( \mathcal{B}_{00} \) may be maximal, and the dimensionless function \( \mathcal{A}(r, r') \) represents the spatial shape of the correlation function relative to its value at \( r \).

In the limit where the magnetic field fluctuations are smooth over the trap region, the normalization \( \int d^3r |\phi_j(r)|^2 = 1 \) implies that \( \alpha_{jk} \) is constant. In this case the dissipative term in equation (33) becomes

\[ \mathcal{L}_N \rho = -\frac{\gamma_{ij}^{N}}{2} [\hat{N}^2 \rho + \rho \hat{N}^2 - 2 \hat{N} \rho \hat{N}], \]

where \( \hat{N} \equiv \hat{n}_L + \hat{n}_R \) is the total number of atoms in the two wells. This term does not affect the relative number of atoms between the wells, and it completely vanishes in the case where the number of atoms is fixed or if the density matrix is diagonal in the total number-\( N \) basis. If one adopts,
however, a symmetry-breaking approach to BEC [44], where the density matrix involves superpositions between different number states, these would decay under this interaction. This scenario thus provides a typical example of a dynamically emerging ‘superselection rule’ stating that superpositions of different atomic numbers are forbidden.

Since we may assume that the total number of particles is constant, we can add a constant term to $\alpha_{jk}$ without affecting the master equation. Let us then replace $\alpha_{jk} \rightarrow \tilde{\alpha}_{jk} \equiv \alpha_{jk} - \alpha_{jkr}$. Since the correlation function $A(r, r')$ is symmetric under the transformation $r \leftrightarrow r'$ (see equation (32)), it follows that $\alpha_{jkr} = \alpha_{rjk}$ and therefore $\tilde{\alpha}_{jk}$ vanishes for $j \neq k$. The form of the dissipative term then becomes

$$\mathcal{L}_N \rho = -\sum_{j=l,R}^{2N} \tilde{\alpha}_{jk} [\hat{n}_j^2 \rho + \rho \hat{n}_j^2 - 2\hat{n}_j \rho \hat{n}_j],$$

$$= -\gamma_c (S_i^2 \rho + \rho S_i^2 - 2S_i \rho S_i),$$

(36)

where

$$\gamma_c = \frac{2N}{\gamma} (\alpha_{LL} + \alpha_{RR} - 2\alpha_{jkr})$$

(37)

and we have used $\tilde{n}_{l,R} = N/2 \pm \hat{S}_j$ with the total number $N$ being conserved. As expected, magnetic fluctuations along the quantization axis are responsible for phase fluctuations between the two wells. In a stochastic (Langevin) picture, this process may be viewed as random rotations about the $S_j$-axis of the Bloch sphere, and the dissipative term (equation (36)) generates the corresponding phase diffusion in the density operator.

Let us now consider two typical types of noise. The first is Johnson noise from thermal current fluctuations near a conducting surface. Let us focus for simplicity on a double-well potential whose sites are equally close to the surface. The correlation function $A(r, r')$ then depends only on the distance $|r - r'|$ and decays to zero on a length scale $\lambda_c$ comparable to the distance of the trap from the surface [21]. Consider for example the simple model $A(r, r') \propto \exp(-|r - r'|/\lambda_c)$. When the correlation length $\lambda_c$ is smaller than the distance $d$ between the two sites, the off-diagonal terms in the matrix $\alpha_{jk}$ (equation (35)) decay like $\alpha_{jkr} \sim e^{-d/\lambda_c}$, while the diagonal terms scale like $\alpha_{ij} \sim \mathcal{V}_c / \mathcal{V}_0$, the fraction of the volume occupied by the mode $\phi_l$ lying within a radius $\lambda_c$ around the trap center. For very short correlation lengths, $\mathcal{V}_c \sim \lambda_c^3$. It follows that when the correlation length is smaller than the distance $d$, the off-diagonal terms $\alpha_{jkr}$ become exponentially small while the diagonal terms are relatively large, so that a dephasing process takes place. This conclusion also applies when the correlation function $A(r, r')$ is not exponential, but Lorentzian, as computed in [21].

The second example is noise with a long correlation length that changes the effective magnetic potential, as happens typically with so-called ‘technical noise’. Assume that one (or more) of the parameters in the potential of equation (1), such as $\omega_d$, $\mathcal{V}_0$ or $d$, is fluctuating, or that another fluctuating term such as $\delta \mathcal{V}(t) = f(t) x$ is added to the potential. If any of these variables (denoted by a generic name $\nu$) fluctuates as $\delta \mathcal{V}(t)$, where $\langle \delta \mathcal{V} \rangle = 0$ and $\int dt \delta (\delta \mathcal{V}(t) \nu(t + \tau)) = \eta > 0$, then the magnetic field correlation function has the form

$$B_{\nu}(r, r', 0) \propto (\delta \mathcal{V}(r) / \partial \nu)(\delta \mathcal{V}(r') / \partial \nu) \eta \delta (r - r')$$

so that $A(r, r')$ in equation (35) can be factorized into a product of a function of $r$ and the same function at $r'$. This implies that $\alpha_{ij}$ can be also factorized as $\alpha_{ijk} = \beta_i \beta_j$, where $\beta_j \propto \int d^3 r (\delta \mathcal{V}(r) / \partial \nu)(\phi_j(r))^2$. We then get a dephasing rate $\gamma_c \propto (\beta_L - \beta_R)^2$. It follows that for noise of technical origin, dephasing is expected whenever the potential changes due to the magnetic fluctuations are asymmetric. For example, if the magnetic field fluctuations create a linear slope, $\delta \mathcal{V}(r) = f(t) x$, then with $f(t) f(t') = \eta \delta (t - t')$ we obtain $\gamma_c = \frac{1}{2}(d/h)^2 \eta$, where $d$ is the distance between the centers of the two wells. This is indeed what we expect for phase fluctuations with a white noise spectrum $\delta \nu(t) \sim f(t) d / h$ between the two sites.

If the left and right modes of the double-well potential are not well separated, then the variation of the field fluctuations may lead to the appearance of cross terms $\hat{S}_i \hat{S}_k (i \neq j$ and $k \neq l)$ in the dissipative term of equation (30), which correspond to transitions between the wells rather than energy differences. These are driven by spatial gradients of the magnetic field (see [45, 46] for a more detailed discussion). Such cross terms can be interpreted as fluctuations of the tunneling rate, appearing as $\hat{S}_i$ and $\hat{S}_j$ terms in the Hamiltonian (6). On the Bloch sphere, they correspond to random rotations around an axis in the ‘equatorial plane’ which cause changes in the populations in the wells. For completeness, we include such terms in the following discussion. Although they are expected to be small, they may be amplified by atomic collisions, as we shall see below.
4.2. Loss

The magnetic fields transverse to the quantization axis lead to a loss interaction

\[ \hat{V}_{\text{loss}} = \sum_k g_{kk}^i \hat{a}_k^i \hat{a}_k + g_{kk}^r \hat{a}_k \hat{a}_k + \text{h.c.}, \]  

(38)

where

\[ g_{ij}^j(t) = -\mu_F \sqrt{F} \int d^3r B_{-1}(r, t) \zeta_i^j(r) \phi_j(r), \]  

(39)

and \( B_{-1} \) is a complex-valued circular component that lowers the angular momentum by one unit \( (B_{-1} = (B_x + iB_y)/\sqrt{2} \) if the quantization axis is along \( z \)).

The relevant loss term in the master equation for the atoms in the two wells is then given by

\[ L_{\text{loss}} \rho = -\sum_{ij=L,R} \frac{\gamma_{ij}^{\text{loss}}}{2} [\hat{a}_i^+ \hat{a}_j + \rho \hat{a}_i^+ \hat{a}_j - 2\hat{a}_i \rho \hat{a}_j^+], \]  

(40)

where the loss rates are given by

\[ \gamma_{ij}^{\text{loss}} = \frac{1}{\hbar^2} \sum_k \int d\tau e^{i\omega_k \tau} (g_{kk}^i(t) g_{jk}^r(t + \tau)) \]

\[ = \frac{\mu_F^2}{\hbar^2} \sum_k \int d\tau' \int d\tau' \zeta_i^k(r) \phi_j^*(r) \phi_j(r') \zeta_k^r(r') B_{-1}(r, \tau', \omega_k), \]

(41)

\[ B_{-1}(r, \tau', \omega) = \int d\tau e^{i\omega \tau} \langle B_{-1}(r, t) B_{-1}(r', t + \tau) \rangle, \]

(42)

where \( B_{-1}(r, \tau', \omega) \) is the spectral density of transverse magnetic field fluctuations, \( B_{+1} = B_{+1}^* \), and \( \omega_k \) contains the Zeeman and kinetic energies of the non-trapped level.

If we assume that the magnetic spectrum is flat over the range of energies \( \omega_k \), then we use the completeness relation of the non-trapped states \( \sum_k \zeta_k^r(r) \zeta_k^r(r') = \delta(r - r') \) to obtain

\[ \gamma_{ij}^{\text{loss}} = \frac{\mu_F^2}{\hbar^2} \int d\tau \phi_j^*(r) \phi_j(r) B_{-1}(r, r, \omega_k), \]

(43)

where \( \omega_k \) is the average transition energy to the untrapped Zeeman level. If the magnetic noise spectrum depends weakly on the position \( r \) over the trap region, the orthogonality relations between the modes \( \phi_L \) and \( \phi_R \) imply that the off-diagonal loss rates vanish and we are left with

\[ \gamma_{ij}^{\text{loss}} = \delta_{ij} \frac{\mu_F^2}{\hbar^2} B_{-1}(r, r, \omega_L), \]

(44)

where \( r \) represents the region occupied by the atoms.

Compared to the dephasing rate \( \gamma_p \) (equation (37)) of the previous section, loss is harder to suppress: it occurs whenever the spectrum of the magnetic fluctuations has significant components at the transition frequency \( \omega_L \) and does not depend on the correlation length of the field. However, for Johnson noise at trap-surface distances of the same order as the distance between the wells, the two rates happen to be similar since the magnetic noise spectrum is typically quite flat in frequency and not strongly anisotropic [43]. Let us also note that several suggestions exist on how to suppress the overall Johnson noise (e.g., [47]), and in addition, how to suppress specific components of the field, such as those contributing to \( \gamma_p \) [48]. For experimental measurements of loss rates, see section 7.

5. Combined dynamics

Here we investigate the effect of noise on the coherence of a BEC in the presence of tunneling and interaction dynamics. We describe the main features of the dynamics semiclassically and make both analytical and numerical quantitative estimations of the decoherence rate. We consider two types of noise: number-conserving noise, which does not change the total number of particles in the trap, and noise-inducing loss of particles from the trap, which is most common in atom chip traps as we show in section 7. The dynamics is initialized in the ground state of the balanced two-site Bose–Hubbard Hamiltonian equation (6) that we compute numerically for a typical atom number \( N = 50 \rightarrow 200 \).

In this work we use a simple white-noise model, which is appropriate for magnetic fields whose sources have a temperature much larger than the ultracold atoms [25, 43]. This corresponds to the classical (high-temperature) limit of an Ohmic bath in the language of past work on dissipative processes in Josephson junctions based on the spin–boson model ([25, 26] and references therein). Note that due to the high bath
temperature, the atomic system does not reach equilibrium within the time of measurement, and eventually loses coherence.

5.1. Number conserving noise (no loss)

For visualizing the combined effects of the nonlinear Hamiltonian dynamics of section 3 and the magnetic noise, a semiclassical picture involving the distribution over the spherical phase space (Bloch sphere) is illuminating. For example, the master equation generated by the dissipative term of equation (36) can be represented by adding a stochastic term $f(t)\hat{S}_j$ to the Hamiltonian, which can also be written as

$$f(t)\hat{S}_j = \frac{f(t)}{2} (\hat{n}_L - \hat{n}_R).$$ \hspace{1cm} (45)

This term can be viewed as a fluctuating energy bias driving phase fluctuations. Here $f(t)$ is an erratic driving amplitude that we take as a Markovian random process with zero average and short correlation time [41],

$$\langle f(t)f(t') \rangle = 2\gamma_3 \delta(t - t').$$ \hspace{1cm} (46)

On the Bloch sphere, such a driving term corresponds to a random sequence of rotations around the ‘north-south’ (or $S_1$-) axis. The phase-space distribution in figure 2 then diffuses in the $\varphi$-direction so that the relative phase between the left and right wells gets randomized. The parameter $\gamma_3$ is the corresponding phase diffusion rate.

Similarly, we also consider random rotations around other axes of the Bloch sphere, which transfer population between the two wells. For example

$$f(t)\hat{S}_2 = -\frac{f(t)}{2} (\hat{a}_R^+ \hat{a}_R - \hat{a}_L^+ \hat{a}_L)$$ \hspace{1cm} (47)

corresponds to inter-site hopping with a random amplitude and may also be generated by fluctuating inhomogeneous magnetic fields (section 4). On the Bloch sphere of figure 2, we then have, near $\varphi = 0$, random kicks that spread the phase-space distribution along the ‘north-south’ (or $\theta$) direction. They change the population difference which is proportional to $\cos \theta \approx \pi/2 \approx \theta$.

As long as the dynamics is constrained to the ground state and lowest energy excitations of the Bose–Hubbard Hamiltonian, the approximations leading to the Josephson Hamiltonian (equation (10)) are valid. In this case the operators $\hat{S}_1$ and $\hat{S}_2$ may be replaced by the canonical operators $\hat{n}$ and $\hat{\varphi}$, respectively. Noise coupled to $\hat{n}$ is responsible for random changes in the conjugate variable $\varphi$ (phase fluctuations), while noise coupled to $\hat{\varphi}$ is responsible for random changes in the conjugate variable $n$ (population fluctuations). The effect of $\hat{S}_1$ in this phase-space region, when coupled to random noise, amounts to random fluctuations of the tunneling rate $J$ in the Hamiltonian (equation (5)).

The actual dynamics must also consider the other parts of the Bose–Hubbard Hamiltonian (equation (5)). Consider first the interaction-free case $U = 0$ with the squeezing parameter $\xi = 1$. Classical trajectories in the vicinity of the ground state are perfect circles on the Bloch sphere centered on the $S_1$-axis. The Rabi–Josephson rotation is faster than the diffusion rate, both types of noise above produce isotropic 2D diffusive spreading of the phase-space distribution, with a diffusion rate $\gamma_3$ ($\alpha = 2, 3$). Thus, at large times $t$, one obtains a circular Gaussian phase-space distribution whose radial variance grows linearly as $\Delta^2 = 2\gamma_3 t$. Substitution into equation (24) gives the expected exponential decay $S/s \propto \exp[-\gamma_3 t]$. This regime flattens out when the state has spread over the entire Bloch sphere, $\Delta \sim \pi$.

Now, when interactions are present, it is evident that the two types of noise will give different results. The Josephson trajectories are squeezed, with the width in the $\theta$-direction being a factor $1/\xi^2$ smaller than in the $\varphi$-direction (figure 2). The combined action of phase fluctuations ($\alpha = 3$) and Josephson rotation gives an elliptical distribution whose variances are $\Delta_\varphi^2 = 2\gamma_3 t$ and $\Delta_\theta^2 = 2\gamma_3 t/\xi^4$ (inner blue ellipse in figure 2). In contrast, for noise driving population fluctuations ($\alpha = 2$), the distribution at time $t$ has variances $\Delta_\varphi^2 = 2\xi^4\gamma_3 t$ and $\Delta_\theta^2 = 2\gamma_3 t$ (outer blue ellipse in figure 2). Therefore, using equation (24) the one-particle coherence decays as

$$S/s \propto \exp(-\Gamma_{3,2} t),$$ \hspace{1cm} (48)

with effective decay rates

$$\Gamma_{3,2} = \gamma_3 (1 + \xi^4).$$ \hspace{1cm} (49)

Thus, for noise driving phase (number) fluctuations the decoherence rate is suppressed (enhanced) in the presence of interactions with respect to the interaction-free case. The best suppression factor one can expect in the case of phase fluctuations is a factor of two, i.e. $\Gamma_3 = \gamma_3/2$. This limiting case can be understood in the
The following way: in figure 2 the interactions can restrict the vertical spreading (θ-direction) but not the horizontal spreading. This evolution then leads to a distribution with a relatively small θ. It should be noted that our model does not take into account processes that relax the system towards its ground state and actually create phase coherence. This kind of relaxation may occur in models that take into account coupling of the system to low-temperature reservoirs, as mentioned at the beginning of this section, but are not included in our white-noise model.

The predictions of equations (48) and (49) are borne out by an exact numerical analysis shown in figure 3. We have solved the master equation with the dissipative term having the form of equation (36) with \( N = 50 \) particles in a symmetric double-well potential. A pronounced feature of the evolution is the oscillatory decay of the coherence. In the derivation of equation (49), it was assumed that the Josephson oscillations spread the semiclassical ensemble throughout the pertinent classical orbit. However, it may happen that diffusion due to the noise leads to an ensemble with an oscillating width. This is particularly noticeable at short times and if the noise is switched on faster than the Josephson period. The combination of anisotropic diffusion and motion along squeezed elliptical classical trajectories leads to a ‘breathing’ ensemble and a coherence that oscillates with half the Josephson period about the decaying coherence value of equation (48).

A more general quantitative analysis confirms the predictions of the semiclassical description and the features shown in figure 3. We use the master equation with a dissipative term of the structure of equation (36) to...
derive rate equations for the relevant bilinear observables of the Josephson junction (equation (18)): the occupation \( \hat{b} \dagger \hat{b} \) and the ‘anomalous occupation’ \( \hat{b} \hat{b} \). Let us begin with noise-driving population fluctuations, where the \( \hat{S}_3 \) operator in equation (36) is replaced by \( \hat{S}_2 \) (compare equations (45) and (47)).

We obtain the dynamical equations

\[
\frac{d}{dt} \langle \hat{b} \dagger \hat{b} \rangle = \gamma_2 \frac{N \xi^2}{4},
\]

\[\frac{d}{dt} \langle \hat{b}^2 \rangle = -2i\omega_1 \langle \hat{b}^2 \rangle - \gamma_2 \frac{N \xi^2}{4},\]

where \( \gamma_2 \) is the diffusion rate related to population fluctuations. This has the simple solution

\[
\langle \hat{b} \dagger \hat{b} \rangle_t = \langle \hat{b} \dagger \hat{b} \rangle_0 + \gamma_2 \frac{N \xi^2}{4} t,
\]

\[
\langle \hat{b}^2 \rangle_t = \langle \hat{b}^2 \rangle_0 e^{-2i\omega_1 t} + i \frac{\gamma_2 N \xi^2}{8\omega_1} (1 - e^{-2i\omega_1 t}).
\]

Substitution into equation (20) gives the coherence evolution for an initial state where \( \langle \hat{b}^2 \rangle_0 = 0 \) (this applies to the ground state and thermal equilibrium)

\[
\langle \hat{S}_3 \rangle_t = \langle \hat{S}_3 \rangle_0 - \frac{\gamma_2 \xi^2}{4} \left[ (\xi^2 + \xi^{-2})t - \frac{\sin 2\omega_1 t}{2\omega_1} (\xi^2 - \xi^{-2}) \right],
\]

implying that

\[
\Gamma_2(t) \equiv -\frac{\partial}{\partial t} \langle \hat{S}_3 \rangle_t = \frac{\gamma_2 \xi^2}{2} [\xi^{-2} \cos^2 \omega_1 t + \xi^2 \sin^2 \omega_1 t].
\]

Thus, the decay rate oscillates between \( \gamma_2 \) and \( \gamma_2 \xi^4 \) with the average rate given by \( \gamma_2 (1 + \xi^4)/2 \), as in equation (49).

Similarly we obtain for phase fluctuations (random rotations around the \( \hat{S}_3 \)-axis)

\[
\Gamma_3(t) = \frac{1}{2} [\xi^2 \cos^2 \omega_1 t + \xi^{-2} \sin^2 \omega_1 t],
\]

implying oscillations of the decay rate between \( \gamma_2 \) and \( \gamma_2 \xi^4 \) with an average rate of \( \gamma_2 (1 + \xi^4)/2 \).

As we showed in section 4.1, magnetic noise in a double-well trap leads mainly to phase fluctuations represented by an erratic field coupling to \( \hat{S}_3 \). However, in the spirit of the general discussion above and for completeness of the discussion, we have also calculated the evolution of the coherence for \( \hat{S}_3 \) and \( \hat{S}_1 \) as driving forces: random rotations around the \( \hat{S}_3 \)-axis that generate population fluctuations, and around the \( \hat{S}_1 \)-axis that generate tunneling-rate fluctuations. In figure 3(a) we present the evolution of coherence for given parameters (see caption) and different rotation axes.

The decay rates \( \Gamma_3(t) \equiv -\partial/\partial t \log \langle \hat{S}_3 \rangle_t \) for the noise rotating around \( \hat{S}_3 \) are presented in figure 3(b) \( \gamma_3 = \gamma \) for \( \alpha = 1, 2, 3 \). The decoherence rate for rotations around \( \hat{S}_3 \) (cyan curve) oscillates between \( \Gamma_1 = 0 \) and \( \Gamma_1 = \gamma \) and then starts to increase gradually. The decoherence rate \( \Gamma_3 \) (phase noise, blue curve) oscillates between \( \gamma \) and \( \gamma / \xi^4 \) and then stabilizes with the average value \( \Gamma_3 \approx \gamma (1 + \xi^4)/2 \) of equation (49). The fastest decay of coherence is found for number noise (red curve, \( \alpha = 2 \)) because random rotations around this axis displace the squeezed ensemble significantly from its equilibrium position. The rate first oscillates between \( \gamma \) and \( \gamma / \xi^4 \) and then stabilizes with the average value \( \Gamma_2 = \gamma (1 + \xi^4)/2 \). It gradually decreases to smaller values when the coherence is already small.

The decay of the decoherence rate oscillations may be interpreted within the semiclassical approach as following from the dispersion of Josephson periods for classical trajectories with different perimeters. In the linearized excitation approach, this corresponds to unequally spaced energy levels due to the deviation from the harmonic approximation. Note that similar oscillations have been observed in [49] for an oscillator and a certain scenario of spatial decoherence.

The decay of the coherence in the case of tunneling rate fluctuations (random rotations around \( \hat{S}_3 \)), which leave the system invariant in the absence of interactions (circular distribution in figure 2), can also be explained by the squeezing. The rotation of an elliptical ground state changes it and excites higher energy states (or classical trajectories). After some time these excitations become similar to number excitations and the decay rate increases.

5.2. Decoherence induced by particle loss

Loss of atoms from traps due to noise-induced transitions to untrapped internal atomic states is a very common process in atom chip traps. The nature of this process was discussed above in section 4.2. Here we examine the
possibility that a loss process leads not only to the reduction of the total number of atoms in the trap, but also to decoherence of the remaining BEC. Particle loss was previously shown to prevent revival of coherence in the presence of interaction-induced phase diffusion [50] and to affect the rate of decoherence in the presence of other decoherence effects [51, 52]. However, here we will show how decoherence emerges from the interplay between particle loss and interactions even when decoherence would not occur in the absence of each of these factors alone.

Let us begin by noting that while a loss process does not usually heat up a BEC by transitions into higher-energy trap levels, it does increase the uncertainty in the number of remaining atoms. If we imagine two BECs in two separate traps which initially have a fixed number of atoms, then independent loss from both traps will lead to uncertainty in the relative numbers in the two traps. In the presence of interactions this leads to an uncertainty of the chemical potential and thereby the rate of phase evolution in the two traps. On the other hand, due to the number-phase complementarity, number uncertainty is essential for the establishment of coherence. It should be then asked: does this process of independent loss from the two traps, which is described by equation (40), indeed lead to decoherence as shown above for noise inducing population fluctuations?

Consider a trapped Bose gas of $N$ atoms in a double well. Atom loss from the left (right) well is described by the application of the annihilation operator $\hat{a}_\ell$ ($\hat{a}_r$) to the system state. We now denote $\hat{a}_- \equiv \hat{a}_\ell$ and $\hat{a}_+ \equiv \hat{a}_r$, as subtracting an atom from the left (right) well reduces (increases) the population imbalance $\hat{n} \equiv \hat{S}_r$. In the resulting state with $N-1$ atoms, the expectation value of a pseudo-spin operator is

$$\langle \hat{S}_r \rangle_{N-1}^+ = \frac{\langle \hat{a}_+^\dagger \hat{S}_r \hat{a}_+ \rangle_N}{\langle \hat{a}_+ \hat{a}_+ \rangle_N}. \quad (57)$$

This can be worked out as

$$\langle \hat{S}_r \rangle_{N-1}^+ \approx \frac{N-1}{N} \langle \hat{S}_r \rangle_N \mp \frac{\langle \hat{S}_r \hat{S}_r \rangle_N + \langle \hat{S}_r \hat{S}_r \rangle_N}{N}, \quad (58)$$

$$\langle \hat{S}_r \rangle_{N-1}^+ \approx \langle \hat{S}_r \rangle_N \pm \frac{1}{2} \left(1 - \frac{4\langle \hat{n} \rangle_N}{N} \right). \quad (59)$$

where we have dropped terms of order $\langle \hat{n} \rangle / N$, assuming nearly equilibrated populations in the wells. Equation (59) implies that if the state of the $N$-atom system has a Poissonian population distribution with $\langle \hat{n} \rangle_N = N/4$, as in a coherent state (the ground-state of a non-interacting system), then the second term in the right-hand side vanishes and the loss of one atom does not change the population imbalance. Conversely, if the $N$-atom state has a sub-Poissonian population distribution, as in a number-squeezed state, then subtraction of a particle from one well changes the population imbalance in favor of the other well. For a number-squeezed state this implies a change of the population imbalance $n_\ell - n_r = 2 \langle \hat{S}_r \rangle$ by $\pm (1 - 1/\xi^2)$ for each atom that is lost from either of the wells. It then follows that the loss process from an interacting system is equivalent to random kicks along the population ($\theta$) direction, which was discussed in section 5.1 above. By analogy to equation (55) the rate of decoherence would then be

$$\Gamma_{\text{dec}} \sim \frac{\gamma_{\text{loss}}}{2N} \left[c(t) + s(t) \xi^4 \right], \quad (60)$$

where $c(t)$ and $s(t)$ are functions that we expect to behave like $\cos^2 \omega_1 t$ and $\sin^2 \omega_1 t$ for short times and then stabilize with their average value $1/2$.

In figure 4 we present a numerical solution for the time evolution of the coherence due to a loss process described by a dissipative term as in equation (40), which is driven by the annihilation operators $\hat{a}_\ell$ and $\hat{a}_r$ from both wells. For the parameters considered in figure 4, the decoherence rate is much smaller than the loss rate, but it grows with the strength of the interaction $u$. The results of the numerical simulation show that the initial rate of decoherence is close to zero ($c(0) = 0$ in equation (60)), rather than to the value $\gamma_{\text{loss}} (1 - 1/\xi^2)/2N$. However, the value of $s(t)$ is close to the expected value $\sin^2 \omega_1 t$.

For relatively strong interactions, the decoherence rate may become larger than the loss rate itself. This occurs when $\xi^2 / 2N \sim u / 2N > 1$, and may even happen in the Josephson regime (see equation (13)).

In this work we have considered only the case where the initial state of the system is the ground state, whose coherence is relatively high compared to, for example, the equilibrium states at non-zero temperature. In the cases considered above, direct phase changes due to loss are negligible (as $\langle \hat{S}_r \hat{S}_r \rangle \approx \langle \hat{S}_r \hat{S}_r \rangle \approx 0$ in equation (58)), while subtracting a particle from one well always increases the population imbalance in favor of the other well (equation (59)). However, states with higher temperature and reduced coherence may have super-Poissonian population fluctuations ($\langle \hat{n}^2 \rangle > N/4$). The subtraction of an atom from one well would then change the population imbalance in favor of the same well. In such cases we may also expect scenarios where the losses lead to the reduction of the population uncertainty and the increase of coherence. These kinds of processes will not be discussed further here.
6. Realizability and validity

The two-site Bose–Hubbard model, which was used in the previous sections, is fully valid (namely, applicable over the whole phase space shown in figure 2) only if the atom–atom interaction energy is much smaller than the energy of excited modes in each well, such that higher-energy spatial modes are not excited by the interaction. For a single-well frequency \( \omega \) this criterion implies \( n \omega \ll \hbar \omega \), where \( n \) is the peak atomic density. This requirement is equivalent to the healing length \( l_c = \hbar / \sqrt{mgn} \) being much longer than the trap width \( L \sim \sqrt{\hbar / mn \omega} \). However, dynamics similar to that predicted by the two-mode model, such as Josephson oscillations and phase oscillations of self-trapped populations, appear in mean-field (GP) calculations even if the interactions are much stronger, as long as the atomic density changes during the evolution are small. Since the GP approximation represents the classical limit of the two-site model, this suggests that the model is valid for such dynamics, in which the populations in the two sites do not deviate much from their equilibrium values and the spatial density can be derived from the two stationary modes \( f_L \) and \( f_R \). Here we are interested in slow dynamics during which the population does not vary considerably, since the main change is in the one-particle coherence. With the aid of the mean-field approach described in section 2, we therefore require that the dynamics involves only low-lying excitations of the two-mode system, whose energy is lower than the energy splitting of higher spatial modes \( E_j (j \geq 2) \) (see equation (3)), such that only the pair \( f_0 \) and \( f_1 \) is populated.

In addition, the Hamiltonian (equation (5)) neglects interaction terms involving different modes, such as a term proportional to \( \hat{n}_L \hat{n}_R \) [33], which may be significant if the two main modes \( f_L \) and \( f_R \) are not well separated in space. We therefore define a simple validity criterion: that the barrier height \( V_0 \) is larger than the
longitudinal energy $\mu_N$ of the condensate mode $\phi_N$. With this condition we can show that the Josephson frequency $\omega_J$, which defines the excitation energy in the two-mode system, is much smaller than the higher-mode excitation energy. This ensures that, as long as low-energy states of the two-mode system are involved, higher spatial modes are not excited and the system may still be described by the two modes.

In figures 5 and 6 we show the parameters of the Bose–Hubbard and Josephson Hamiltonians in their validity range, as calculated for $^{87}$Rb atoms in a potential having the form of equation (1) with $d = 5 \mu m$ and $\omega_x = 2\pi \times 200$ Hz, as in figure 1. The strength of the interaction is determined both by the total number of atoms $N$ and by the transverse frequency $\omega_x/2\pi$, which is equal to 500 Hz in figure 5 and 100 Hz in figure 6. In all cases the Josephson frequency is not larger than 30 Hz, which is smaller than the excitation energy of higher modes, and thus ensures the validity of the model. However, for the strongly interacting BEC in figure 5 the squeezing factor $\xi$ may become larger than $N$, implying that for such high barrier heights and low tunneling rates the system is in the Fock regime, where the coherence of the ground state drops to zero. In this work we are interested in the Josephson regime of equation (13), where $\xi^2 < N$. When $\xi^2 > 2\sqrt{N}$, loss-induced decoherence is faster than the loss rate, as happens for most of the curves in (d).

Two differences between figure 5 and figure 6 are most noticeable. First, the chemical potential of the BEC in tight confinement is much larger than that of the BEC in weaker confinement, implying that the barrier height for which the two-mode model is valid is much lower for the latter. Second, the amount of squeezing is correspondingly much larger for the tightly confined BEC in figure 5. It follows that for a given rate $\gamma_{\text{loss}}$ of atoms from the trap, we would expect a high decoherence rate for the tightly confined BEC, since $\gamma_{\text{dec}}/\gamma_{\text{loss}} \propto \xi^4/4N > 1$ for most of the parameter range in figure 5. Conversely, we should expect very low decoherence rates relative to the loss rate in the case of weak confinement as in figure 6.

In the next section we present some experimental results that give realistic loss rates near an atom chip. This will enable us to discuss expected decoherence rates for atomic Josephson junctions in similar scenarios.

7. Measurements of atom loss near an atom chip

Spatially coherent interferometric signals from double-well potentials on an atom chip have been observed so far only for distances greater than $50 \mu m$ from the atom chip surface [10, 11, 53–55]. Most of these experiments were based on double-well potentials with radio-frequency-dressed potentials and the decoherence was mainly due to phase diffusion caused by atom–atom interactions when the two parts of the BEC were completely...
separated. Although some properties of a partially split BEC, such as number squeezing, were measured in a purely magnetic double-well potential very close to an atom chip [56], coherent interference signals were not reported. Realizations of spin coherence have been much more successful [22, 57, 58], giving some indications about magnetic noise near the chip. Here we present new measurements of atom loss and an analysis of its possible sources, together with comparisons to previous experimental and theoretical studies. Since atom loss and dephasing under these conditions have a common source, i.e., fluctuations of the magnetic field (see section 4), the results reported here may assist in making predictions related to the spatial decoherence of matter waves for experimental parameters typical of atom chips. In addition, particle loss by itself may be a dominant source of decoherence, as we have shown in section 5.2, so the measurements of loss rates reported here are directly relevant for future experiments where spatial coherence is required.

We investigate the nature of magnetic fluctuations near the surface by measuring the lifetime of atomic clouds as a function of the distance \( z_0 \) from an atom chip. Thermal clouds initially containing about \( 10^4 \) atoms of \( ^{87}\text{Rb} \) in the \( |F = 2, m = 2 \rangle \) state are prepared at a temperature of about 2 \( \mu \text{K} \). We control \( z_0 \) using a current of 4–25 mA in an 8 \( \mu \text{m} \)-wide gold wire on the atom chip. The magnetic field at the trap minimum and the longitudinal trap frequency are maintained at about 1 G and 110 Hz respectively, using external bias coils and currents through a pair of U-shaped wires on the second level of our atom chip, 300 \( \mu \text{m} \) further from the atoms [59]. The transverse trap frequency changes with the wire current but the corresponding radius of the cloud is always <20\% of the distance to the chip (at the level of 1\% of the peak atomic density), thereby inhibiting atom loss to the chip surface.

Distances \( z_0 \gtrsim 9 \mu \text{m} \) are measured by reflection imaging; smaller distances are obtained by calibration with the wire current. Lifetimes are obtained by fitting atom-loss measurements as a function of trap holding time. Our data, adjusted for the lifetime due to vacuum of about 30 s, are shown in figure 7. The lifetime of a BEC was also measured for \( z_0 = 5 \mu \text{m} \).

For comparison with theory, let us for the moment ignore cascading effects amongst Zeeman sub-levels \( |F, m \rangle \) that can re-populate the initial state, and the geometry of the current-carrying wire, i.e., we assume a thin gold layer of infinite width. Assuming that the measured lifetimes \( \tau_{\text{meas}} \) are due to Johnson and technical noise, we can write [61, 62]:

\[
\tau_{\text{meas}}^{-1} = \tau_{\text{Johnson}}^{-1} + \tau_{\text{tech}}^{-1}, \quad \text{with} \quad \frac{1}{\tau_{\text{Johnson}}} = \frac{3}{8} \frac{\hbar}{\omega} + 1 \left( \frac{\epsilon}{\omega} \right)^3 \frac{2 \hbar}{\delta^2 z_0^3} = \frac{c_1}{z_0^2} \\
\frac{1}{\tau_{\text{tech}}} = \left( \frac{\mu_0 B \delta F}{2\pi \hbar} \right)^2 \frac{\omega}{z_0^2} = \frac{c_2}{z_0^2}
\]

Figure 6. Same as figure 5 with transverse frequency \( \omega_L = 2\pi \times 100 \text{ Hz} \). Here \( \xi^2 < 2\sqrt{N} \) for all curves, implying weak loss-induced decoherence over the parameter range.
and where $\bar{n}_h$ and $\tau_0$ are the mean thermal occupation number and the free-space lifetime, respectively, at the spin-flip frequency $\omega$. In this simple case we can re-write equation (62) extensively, since $\tau_0$ in [61] is proportional to $(e/\omega)^2$ and $n_h = [e^{\hbar \omega / k_\text{B} T} - 1]^{-1} \approx k_\text{B} T / \hbar \omega$ for typical values of $\omega / 2 \pi \approx 1 \text{ MHz}$ and $T \approx 300 \text{ K}$. Additionally, given the nearly linear dependence for the resistivity of gold near room temperature, $\rho(T) \approx \rho_T$, the skin depth can be written as $\delta = [2 \delta T / (\mu_0 \rho)]^{1/2} \approx 76 \mu\text{m}$. These substitutions render the Johnson lifetime independent of $\omega$ and $T$, noted as case (iii) in [47] and demonstrated in figure 1 there. We exploit measures that $\rho_T = 0.5 \mu\text{m}$, so $h \ll z_0 < \delta$ for all our measurements, and the Johnson lifetime is expected to scale with distance as $z_0^2$ [61]. The lifetime due to technical noise also scales as $z_0^2$ and involves the current noise spectrum $I$, $\mu_B$ is the Bohr magneton and $g_F$ is the atomic Landé factor for the hyperfine level $F$.

The experimental data of figure 7 are fitted on a logarithmic scale and conform very well to the quadratic scaling $\tau_{\text{meas}} \propto z_0^2$. The fit yields $c_1 + c_2 \approx 65 \mu\text{m}^2 \text{s}^{-1}$. Estimating the coefficient for Johnson noise from equation (62) as $c_1 \approx 8.5 \mu\text{m}^2 \text{s}^{-1}$, we find in turn that $c_2 \approx 56 \mu\text{m}^2 \text{s}^{-1}$. For $z_0 = 5 \mu\text{m}$, these values correspond to lifetimes for Johnson and technical noise of 3 s and 0.4 s respectively. The $S$-introducing cascading transitions, i.e., $|F, m\rangle = |2, 2\rangle \leftrightarrow |2, 1\rangle \leftrightarrow |2, 0\rangle$, and geometrical effects [47], doubles the estimated lifetime to 6 s for the Johnson noise component in these atom chip measurements (green curve in figure 7, see caption for details).

We conclude that in our experiment, technical noise is the dominant cause of atom loss, by at least a factor of 6. We note that both the geometric and cascading factors lengthen the calculated lifetime due to Johnson noise, reinforcing the conclusion that technical noise dominates our losses. This may also exploit the longer lifetimes measured in [60], noting especially that those longer lifetimes were measured near a zero-current metallic surface, so the corresponding technical noise should be much lower. From equation (63), our value of $c_2$ corresponds to a current noise spectral density of $I = 0.9 \text{ nA/}\sqrt{\text{Hz}}$. This is much larger than shot noise, by at least one order of magnitude, strengthening the hypothesis of a technical origin for the current noise.

The measurements reported in this section imply that at a distance of 5 $\mu\text{m}$ from the chip Johnson noise gives rise to weak loss and therefore it is expected to give rise to weak decoherence. However, decoherence on the time scale of less than a second may still be expected from technical noise, either by direct phase fluctuations if the current configuration is not symmetric with respect to the double-well axis, or by the loss process itself if the atom–atom interactions are strong. We discuss the practical implications of these results in the discussion section below.

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6 We use $\rho = 7.7 \times 10^{-11} \Omega \text{ m}^{-1}$ for gold, based on NIST recommended resistivity values at 350 K, intermediate between substrate temperatures commonly cited in the literature. See [63]. The value of $\rho$ is $\rho(T) / T$ is constant to within $\pm 4\%$ in the range 200–400 K.

7 Reference [61] gives $\tau_0 = 3 \times 10^{25} \text{s}$ at $\omega / 2\pi = 400 \text{ kHz}$, whereupon the frequency- and temperature-independent Johnson lifetime is $\tau_{\text{Johnson}} = \sqrt{2} \tau_0 / h \times (7.62 \times 10^{14} \text{s})$, with all quantities in SI units.
8. Discussion

In this work we have investigated the effect of noise on the decoherence of a BEC in a double-well potential with non-zero tunneling. It is convenient to characterize the many-body state by pseudo-spin operators with \( \hat{S}_x \) representing the population difference between the wells, and the relative phase \( \phi \) being encoded in the operator \( \hat{S}_z + i\hat{S}_y \propto \exp(i\phi) \) (equation (23)). Specifically, we have discussed three dephasing mechanisms: (a) direct dephasing from inhomogeneous magnetic field fluctuations at low frequency (technical or Johnson noise). This can be represented by random rotations about the \( S_z \)-axis of the Bloch sphere, where the phase-space distribution in figure 2 diffuses in the \( \phi \)-direction so that the relative phase between the left and right wells gets randomized, and (b) dephasing due to induced relative number fluctuations, which are transformed into relative phase fluctuations by atomic tunneling (Josephson oscillations). On the Bloch sphere, these correspond to random rotations around the \( S_y \)-axis and lead to spreading of the phase-space distribution along the \( \theta \) direction, which is translated into spreading along the \( \phi \)-direction by the tunneling dynamics. These fluctuations may originate from an overlap between the spatial modes of atoms in the two sites and fluctuating magnetic fields with a short correlation length. While figure 3 shows that decoherence due to random rotations about the \( S_y \)-axis may be considerably enhanced by atom–atom interactions, the physical source of these number fluctuations is weak and its contribution may be assumed to remain small even when enhanced. The third dephasing mechanism (c) is due to atom losses induced by both Johnson and technical noise. This decoherence process involves induced relative number fluctuations initiated by a non-Poissonian initial number distribution (number squeezing). This process leads to interaction-enhanced dephasing as in mechanism (b) above. Since particle loss is very common in atom traps, this process may become dominant for strong atom–atom interactions.

The two decoherence mechanisms that we expect to be dominant ((a) and (c)) are affected by atom–atom interactions in opposite ways. The decoherence rate due to direct dephasing (a) is suppressed due to atom–atom interactions by a factor up to \( 2 \). Interactions play a quite different role in decoherence induced by particle loss. While particle loss induced by noise has no effect on the coherence of the remaining atoms if the initial state is a coherent state (squeezing factor \( \xi = 1 \)), it may produce considerable decoherence if the initial state is a squeezed one, as typical for interacting atoms. Such enhanced decoherence would appear for any kind of noise which tends to change (randomize) the relative number of particles in the two wells. We note that our results apply only to the case where the initial state is the squeezed many-body ground state of the double well, while different types of evolution may be expected for other initial states, such as a thermal state with a super-Poissonian population distribution or in two-mode systems that do not include tunneling, such as a binary mixture of condensates [52].

As far as number-conserving processes are concerned, our results are well established, as long as the two-mode model is valid, since their main characteristics are derived independently by three different approaches: semiclassical phase-space methods, analytical calculations using an approximate mapping to a Josephson oscillator, and exact numerical calculations. However, we note two limitations of our model. First, it does not take into account possible effects of heating (or cooling) of the Bose gas due to transitions to (or from) higher-energy spatial modes in the trap. Such transitions may be driven by components of the external noise which have correlation lengths smaller than the single trap width, or by external magnetic fields which cause deformations of the potential with higher-order spatial dependence. Such processes may affect the dynamics of decoherence in the double-well trap and are beyond the scope of this paper. Second, our model is valid for a BEC with a macroscopic number of atoms, while dephasing in a system of a few atoms, which may be relevant to atomic circuits, would have to be treated separately.

Finally, the results of this work allow an estimation of the accessible range of parameters for an atomic Josephson junction permitting operation over a reasonable duration of time without significant decoherence. As we have shown in section 7, typical values of the Johnson noise at a distance of \( 5 \mu m \) from the surface cause losses at a rate of less than \( 0.5 \text{ s}^{-1} \). At this distance, the rate of dephasing due to Johnson noise is expected to be of the same order as the loss rate, and we therefore expect that such dephasing will enable coherent operation for a time scale of a few seconds. This time scale could even be doubled in the presence of squeezing due to interactions, as predicted by our theory in section 5.1. The main source of noise in our experiments is found to be technical noise, which is not expected to directly cause dephasing due to its long correlation length, as long as the electric currents that form the magnetic trap are fully symmetric with respect to the double-well axis (see section 4).

Technical noise, however, induces loss, and decoherence due to loss is expected to be significant if the BEC is strongly interacting due to tight transverse confinement as in figure 5. In this case we expect that the squeezing factor is so large that the coherence time is shorter than the trapping lifetime. However, in the case of weak confinement, as in figure 6, the squeezing factor is relatively small (\( \xi^2 \ll \sqrt{N} \)) and therefore the decoherence rate due to loss is smaller than the loss rate \( \Gamma_{\text{dec}}/\Gamma_{\text{loss}} \sim \xi^4/4N < 1 \) (see equation (60)).

To conclude, the above discussion of typical loss and decoherence rates, together with the results of section 6 for a potential with two wells split by \( 5 \mu m \) which may be formed and controlled at a distance of a few...
micrometers from the chip, allows us to estimate the range of accessible Josephson junction operational parameters near the chip. At this distance, for which accurately controllable tunneling barriers may be formed, figure 6 provides an estimate of parameters for which the total spatial decoherence rate is equal or smaller than the loss rate: tunneling rates of about 0.1–10 Hz, or Josephson oscillation frequencies $\omega_j/2\pi$ of about 2–25 Hz, depending on the barrier height, degree of transverse confinement, and number of atoms. This provides a wide dynamic range in the operation of a tunneling barrier for atomtronics, while not excluding parameters beyond this range that could be accessed, for example, with lower atom numbers or smaller distances between the wells.

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