Beyond the quark model of hadrons from lattice QCD

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Abstract. Lattice QCD can give direct information on OZI-violating contributions to mesons. Here we explore the contributions that split flavour singlet and non-singlet meson masses. I discuss in detail the spectrum and decays for scalar mesons (i.e. including glueball effects). I also review the status of hybrid mesons and their decays.

PACS. 12.38.Gc Lattice QCD calculations – 12.39.Mk Glueball and non-standard multi-quark/gluon states

1 Introduction

Lattice QCD is a first-principles approach to solving QCD non-perturbatively. It enables theorists to explore the consequences of QCD for quarks of different mass and so complements experimental studies. It is especially valuable for validating phenomenological models. Direct comparison with experiment is hampered by two obstructions: (i) the continuum limit (lattice spacing $a \rightarrow 0$) should be taken but this is not computationally feasible in many cases and (ii) the $u$ and $d$ quark masses are too light to be successfully explored on a lattice so that one has to rely on chiral perturbation theory and extrapolate from heavier quark masses. Nevertheless, lattice QCD enables exploration of the hadron spectrum and matrix elements and can be used to quantify the success (and failure) of quark models. One area where the quark model is deficient is in flavour singlet mesons. Here the gluonic degrees of freedom are likely to make a significant contribution.

2 Flavour singlet mesons

In lattice QCD, one studies mesons by creating a quark anti-quark pair at time 0 with the quantum numbers of that meson and then annihilating the quark anti-quark pair at time $t$. The quark sources and sinks are then combined using the fully non-perturbative quark propagators determined on the lattice. For flavour singlet mesons, there will be two contributions: disconnected $D(t)$ and connected $C(t)$ as illustrated in fig. 1.

At large $t$ where ground state contributions dominate these measured correlations satisfy $C(t) = ce^{-m_1 t}$ and $C(t) + D(t) = de^{-m_0 t}$ where $m_0$ is the flavour singlet mass and $m_1$ the flavour non-singlet mass. Then by a study of $D/C$ which is given by $(d/c) \exp((m_1 - m_0) t) - 1$ one can explore the mass splitting between flavour singlet and non-singlet which is a measure of the OZI violating gluonic contribution arising from the disconnected diagram.

From a lattice study of this for all mesonic quantum numbers the only significant disconnected contributions were found for scalar and for pseudoscalar mesons. This is not unexpected: both from the phenomenology of the observed meson masses and from the expectation that the lightest glueball has scalar quantum numbers and that topological charge fluctuations have pseudoscalar quantum numbers.

For a review of the situation concerning pseudoscalar mesons see ref [2]. We now discuss the glueballs and their impact on the meson spectrum.

3 Glueballs and scalar mesons

Glueballs are defined to be hadronic states made primarily from gluons. The full non-perturbative gluonic interaction is included in quenched QCD. In the quenched approximation, there is no mixing between such glueballs and quark-antiquark mesons. A study of the glueball spectrum in quenched QCD is thus of great theoretical value.
significant reduction in the lightest scalar mass, as shown in

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glueball spectrum as dynamical quarks are included \cite{7}. 

a study with

est, we expect the largest effects here and one can explore

$J^P C = 0^{++}$ meson (glueball) from quenched data ($N_f = 0$) \cite{1,2,3} in units of $r_0$ 

where $r_0 \approx 0.5 \text{ fm}$. The straight line shows a fit describing the 

approach to the continuum limit as $a \rightarrow 0$. Results \cite{3,4,5,6} for 

the lightest scalar meson with $N_f = 2$ flavours of sea quarks are also shown.

This has been studied extensively \cite{2,11} and the con-

sensus is that the lightest glueball has scalar quantum 

numbers with the tensor ($J^{PC} = 2^{++}$) and pseudoscalar 

glueballs ($J^{PC} = 0^{-+}$) next in mass. The quenched re-

sults have been explored to very small lattice spacings 

and there is convincing evidence that the continuum limit 

values have been extracted - see fig. 1. Since the quenched 

approximation does not reproduce experiment, different 

ways to set the scale will differ - by $\pm 10\%$. Using a con-

ventional scale assignment ($r_0 = 0.5 \text{ fm}$) gives masses of 

around 1.6 GeV for the scalar glueball and 2.2 GeV for 

the tensor.

One signal of great interest would be a glueball with 

$J^{PC}$ not allowed for $q\bar{q}$ - a spin-exotic glueball or odd-

ball - since it would not mix with $q\bar{q}$ states. These states 

are found \cite{2,3,11} to be high lying: considerably above 

$2m(0^{++})$. Thus they are likely to be in a region very dif-

ficult to access unambiguously by experiment.

Within the quenched approximation, the glueball states 

are unmixed with $q\bar{q}$, $q\bar{q}q\bar{q}$, etc. Furthermore, the $q\bar{q}$ states 

have degenerate flavour singlet and non-singlet states in 

the quenched approximation. This gives rise to anomalies 

in the quenched approximation: for example the scalar 

meson propagation can have the wrong sign \cite{2} because 

the $\eta\sigma$ intermediate state is mistreated. Once quark loops 

are allowed in the vacuum then these anomalies are re-

moved. Indeed, for the favour-singlet states of any given 

$J^{PC}$, there will be mixing between the $s\bar{s}$ state, the $u\bar{u}+d\bar{d}$ 

state and the glueball. Since the scalar glueball is light-

est, we expect the largest effects here and one can explore 

this by measuring directly the scalar mass eigenstates in 

a study with $N_f = 2$ flavours of sea-quark.

Most studies have shown no significant change of the 

glueball spectrum as dynamical quarks are included \cite{2}. 

However the larger lattice spacing result \cite{8} shows a sig-

nificant reduction in the lightest scalar mass, as shown in 

fig. 1. Before concluding that this implies a lower scalar 

mass in the continuum limit, one needs to check whether 

an enhanced order $a^2$ correction might be present. Studies 

using the same approach at a finer lattice spacing \cite{3,4,5} 

do suggest that this large order $a^2$ effect is significant, but 

studies even nearer to the continuum or with improved 

actions are needed to resolve this fully.

Let us now discuss the mixing of the scalar glueball and 

scalar mesons. In quenched QCD there is the problem de-

scribed above concerning scalar mesons. For heavy enough 

quarks this is unimportant and one can measure the mixing 

strength on a quenched lattice even though no mixing 

actually occurs. On a rather coarse lattice ($a^{-1} \approx 1.2 

\text{ GeV}$), two groups have attempted this \cite{13,8}. Their re-

sults expressed as the mixing for two degenerate quarks 

of mass around the strange quark mass are similar, namely 

$E \approx 0.36 \text{ GeV}$ \cite{3} and 0.44 GeV \cite{8}. Opinions differ \cite{13,8} as to whether this large mixing which would shift the 

glueball mass down by 20\% will persist to the continuum 

limit.

From dynamical fermion studies with $N_f = 2$, one 

can determine the flavour singlet and non-singlet mass 

spectrum. No glueball, as such, can be defined. What we 

find \cite{8,10} is that the lightest flavour-singlet scalar meson 

is lighter than the lightest flavour non-singlet. This is in 

qualitative agreement with the mixing scenario described 

above.

As well as this mixing of the glueball with $q\bar{q}$ states, 

there will be mixing with $qq\bar{q}\bar{q}$ states which will be re-

sponsible for the hadronic decays. A first attempt to study 

this \cite{14} in quenched QCD yields an estimated width for 

decay to two pseudoscalar mesons from the scalar glueball 

of order 100 MeV. A more realistic study would involve 

taking account of mixing using gluonic, $q\bar{q}$ and $qq\bar{q}\bar{q}$ oper-

ators in full QCD.

4 Hybrid mesons

A hybrid meson has the gluonic degrees of freedom which 

are excited non-trivially. The most significant consequence 

of this, experimentally, will be mesons with $J^{PC}$ values 

not allowed in the quark model for a $q\bar{q}$ system (such as 

$1^{-+}$).

On the lattice one can easily deal with relatively light 

quarks (down to the strange quark mass) or extremely 

heavy quarks (treated as static or using NRQCD). I first 

discuss hybrid mesons with static heavy quarks where the 

description can be thought of as an excited colour string. 

The lattice results \cite{3,4,5,6} can then be presented as po-

tential energy versus quark-antiquark separation $R$. The 

ground state will correspond to the usual interquark po-

tential whereas excited states will have non-trivial repre-

sentations of the symmetries, e.g. the lightest excited state 

($\Pi_u$) has colour flux which is a difference of paths from 

quark to antiquark of the form $\bigtriangledown - \bigtriangleup$.

From the potential corresponding to these excited glu-

onics states, one can determine the spectrum of hybrid 

quarkonia using the Schrödinger equation in the Born-

Oppenheimer approximation. The $\Pi_u$ symmetry state will
produce a degenerate set of eight hybrid mesons of which those with $J^{PC}=1^{-+}$, $0^{++}$ and $2^{--}$ are spin-exotic and hence will not mix with $QQ$ states. They thus form a very attractive goal for experimental searches for hybrid mesons.

Within the quenched approximation, the lattice evidence for $b\bar{b}$ quarks points to a lightest hybrid spin exotic meson $H$ with $J^{PC}=1^{-+}$ at an energy given by $\frac{(m_H - m_{2S})r_0}{2} = 1.9 \pm 0.1$. This has been checked with $N_f = 2$ studies \[18\] which find similar mass ratios $(m_H - m_{2S})/(m_{1S} - m_{2S})$ but a larger splitting in terms of $r_0$, namely $\frac{(m_H - m_{2S})r_0}{2} = 2.4 \pm 0.2$. Using the experimental mass of the $\Upsilon(2S)$, these results imply that the lightest spin exotic hybrid is in the mass range 10.7 to 10.9 GeV. Above this energy there will be many more hybrid states, many of which will be spin exotic.

Within this static quark framework, one can explore the decay mechanisms. One special feature is that the symmetries of the quark and colour fields about the static quark-antiquark state. They thus form a degenerate set of eight hybrid mesons of which those with $J^{PC}=1^{-+}$, $0^{++}$ and $2^{--}$ are spin-exotic and hence will not mix with $QQ$ states. They thus form a very attractive goal for experimental searches for hybrid mesons.

Several lattice groups \[18, 20, 21, 22\] have studied hybrid spectra for light quarks and find the lightest spin-exotic hybrid to have $J^{PC}=1^{-+}$ and mass (for s-quarks) of around 2GeV. The corresponding light-quark $(u, d)$ state would be around 120 MeV lighter. The light quark results have also been extrapolated to charm quarks and masses near 4.4GeV are found for the corresponding state. A recent study has confirmed this value but with a very long extrapolation. These mass estimates can be compared to naive estimates $\frac{1}{6}$ of the spin-exotic charm state mass of 4.0 GeV from the static quark approach which will have an uncontrolled systematic error.

It is not easy to reconcile these lattice results with experimental indications \[18\] for resonances at 1.4 GeV and 1.6 GeV, especially the lower mass value. Mixing with $qqq\bar{q}$ states such as $\pi\pi$ is not included for realistic quark masses in the lattice calculations. This can be interpreted, dependent on one’s viewpoint, as either that the lattice calculations are incomplete or as an indication that the experimental states may have an important meson-meson component in them.

An attractive prospect to study hybrid mesons is from $p\bar{p}$ annihilation. Spin-exotic hybrids, which provide the best signal, will only be produced in association with other hadrons. The decay channel to $\chi_{c} + \pi + \pi$ with the two $\pi$ mesons in an $S$-wave is a promising detection channel. Lattice studies give first estimates of the masses and decay widths but more work needs to be done to constrain the systematic errors on these estimates.

### References

1. UKQCD Collaboration, C. McNeile, C. Michael and K.J. Sharkey, Phys. Rev. D65, (2002) 014508.
2. UKQCD Collaboration; C McNeile and C Michael, Phys. Lett. B491, (2000) 123.
3. P. De Forcrand et al., Phys. Lett. B152, (1985) 107.
4. C. Michael and M. Teper, Nucl. Phys. B314 (1989) 347.
5. UKQCD collaboration, G. Bali et al., Phys. Lett. B309 (1993) 378.
6. H. Chen et al., Nucl. Phys. B (Proc. Suppl.) 34 (1994) 357; A. Vaccarino and D. Weingarten, Phys. Rev. D60 (1999) 114501.
7. SESAM and TcL Collaboration, G. Bali et al., Nucl. Phys. B (Proc. Suppl.) 63 (1998) 209; Phys. Rev. D62 (2000) 054503.
8. C. McNeile and C. Michael, Phys. Rev. D63, (2001) 114503.
9. UKQCD Collaboration, A. Hart and M. Teper, Phys. Rev. D 65, (2002) 034502.
10. UKQCD collaboration; A. Hart, C. McNeile and C. Michael, (in preparation).
11. C. Morningstar and M. Peardon, Phys. Rev. D56 (1997) 4043; ibid., D60 (1999) 034509.
12. W. Bardeen et al., Phys. Rev. D65 (2002) 014509.
13. W. Lee and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 63,194 (1998); hep-lat/9805028.
14. J. Sexton, A. Vaccarino and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 279; Phys. Rev. Lett. 75 (1995) 4563.
15. L.A. Griffiths, C. Michael and P.E.L. Rakow, Phys. Lett. B129 (1983) 351.
16. S. Perantonis and C. Michael, Nucl. Phys. B347 (1990) 854.
17. K. Juge, J. Kuti and C. Morningstar, Phys. Rev. Lett. 82 (4400) 1999; Nucl. Phys. B (Proc. Suppl.) 83 (2000) 304.
18. CP-PACS Collaboration, T. Manke et al., Phys. Rev. Lett. 82 (1999) 4396; Phys. Rev. D64 (2001) 097505.
19. UKQCD Collaboration, P. Lacock, C. Michael, P. Boyle and P. Rowland, Phys. Rev. D54 (1996) 6997; Phys. Lett. B401 (1997) 308.
20. C. Bernard et al., Phys. Rev. D56 (1997) 7039; Nucl. Phys. B (Proc. Suppl.) 73 (1999) 264.
21. P. Lacock and K. Schilling, Nucl. Phys. B (Proc. Suppl.) 73 (1999) 261.
22. Z.H. Mei and X.Q. Luo, hep-lat/0206012.
23. D. Thompson et al., Phys. Rev. Lett. 79 (1997) 1630; S. U. Chung et al., Phys. Rev. D60 (1999) 092001; D. Adams et al., Phys. Rev. Lett. 81 (1998) 5760; E.I.Ivanov etal., Phys. Rev. Lett. 86 (2001) 3977.
24. C. Michael, Proc. Heavy Flavours 8, Southampton, (ed. P. Daunecz and C. Sachrajda), JHEP, PRHEP-hf8/001, 1-10 (2000); hep-ph/9911219.
25. UKQCD Collaboration, C. McNeile, C. Michael and P. Pennanen, Phys. Rev. D65, (2002) 094505.