Effects of negative energy components in two-body deuteron photodisintegration

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(Dated: March 30, 2022)

Abstract

Several observables in two-body deuteron photodisintegration are investigated in the framework of the Bethe-Salpeter formalism. Apart from keeping throughout Lorentz covariance, a special attention is paid to inclusion of both the positive energy and negative energy partial-wave components of the deuteron state. Using the Bethe-Salpeter equation for deuteron in the ladder approximation with one-boson exchanges as a driving force, the contribution of the negative energy states is studied for the unpolarized differential cross as well as the linear photon and tensor target asymmetries. These states are found to have an impact on the observables and, thus, should be taken into account in a complete theoretical development of the reaction in the intermediate energy regime.

PACS numbers: 25.10+s; 25.20.Dc; 21.45+v; 11.10.St
I. INTRODUCTION

An application of the Bethe-Salpeter equation (BSE) for spinors particles and Mandelstam’s theory to the analysis of two-body deuteron photodisintegration has been elaborated in the previous works \[1, 2\], where the contributions of relativistic effects to the differential cross section have been estimated by using the Bethe-Salpeter (BS) formalism as well as the equal-time approximation to the BSE. The analysis shows some deficiency of without taking into account the full Dirac structure of the two-nucleon bound state. It is well known that while dealing with the BSE the problem of many coupled states arises since there are four types of solutions of the Dirac equation for a given momentum, two positive energy states and two negative energy states. In our case this deficiency is ascribed to the making use of the BSE in the context of the relativistic separable interaction kernel with leaving out various negative energy states \[3\]. The present paper is to shed a light on the effect of the inclusion of negative energy components of the deuteron state on the observables of deuteron photodisintegration. Their driving force is the one-boson exchange (OBE) kernel, in which the pion-nucleon coupling is described with the help of axial-vector (A) theory.

The relativistic covariant BSE with a superposition of \(\pi, \omega, \rho, \eta,\) and \(\delta\) exchanges originally has been applied to a description of low energy NN scattering (see Ref. \[4\] and references therein). A good agreement with the experimental data is shown to be achieved for partial waves with \(J > 0\) considering that the coupling for pion-nucleon vertex to be an A type, i. e. a weak \(N\bar{N}\pi\) coupling. Then the same model has been implemented to a research on the deuteron electromagnetic (EM) form factors in the relativistic framework \[5\]. These investigations have inspired different authors to carry out studies of the EM properties of two nucleon system \[6\] and development of an effective theory of strongly interacting particles at momentum transfers of a few GeV/c \[7\]. Moreover, a deuteron resulting from this model has been obtained from the solution of the BSE in the ladder approximation with basic mesons and by adding one more meson, the \(\sigma\) \[8\]. Based on this numerical solution, extended calculations of the relativistic corrections to the deuteron static properties, such as the magnetic and quadrupole moments of deuteron, have been made \[9\].

The work introduced by this paper is an realization of this relativistic framework for studying two-body deuteron photodisintegration. As energy and momentum transfers involved in the process are held to be below one GeV, the appropriate effective degrees of freedom in this energy range are the mesons and nucleons. Working in the plane-wave one-body approximation, we give an accurate and thorough treatment of the differential cross section, the photon beam as well as three tensor asymmetries. The effects of various relativistic contributions to these observables are examined using the BSE with two rather different kernels: the OBE and separable ones. Our ultimate objective is to establish a role played by the negative energy states of the relativistic wave function of deuteron.

The next section briefly refers to the approach used. Definitions of the polarization observables, the linear photon asymmetry \(\Sigma^l\) and tensor-target asymmetries \(T_{2M} (M = 0, 1, 2)\) are given in terms of reaction amplitudes. Since the chief source of the relativistic effects comes from the relativistic wave function of deuteron, we discuss decomposition of the full amplitude into partial waves. The deuteron properties in the framework of the BS formalism are also specified. Sec. \[10\] shows the general structure of the reaction amplitudes in the plane-wave case with the one-body EM current contribution. In particular, the inclusion of both the positive and negative energy states of the deuteron vertex function as well as boost operator is considered. In Sec. \[11\] we give numerical results for the observables with
an emphasis on the leading contribution coming from the dominant negative-energy triplet $P$-component for the OBE kernel in A theory. Due to some core reasons, our numerical work is restricted to a case when one keeps the relative energy variable of the deuteron vertex function equal to zero. A comparison with the previous calculations, performed for the separable kernel, is also carried out and differences are discussed. Sec. V concludes the paper with a number of authors’ remarks on the subject.

II. REVIEW OF THE MODEL

We briefly describe the basics of the relativistic model presented in Refs. [1, 2].

A. Definition of the polarization observables

The description of all the possible polarization observables in the reaction $\gamma + D \rightarrow P + N$ with polarized photons and oriented deuterons has been given in Ref. [10]. The treatment uses a standard coordinate system with $z$-axis, chosen as quantization one, in the direction of the incoming photon c. m. three momentum $\mathbf{q}$ ($|\mathbf{q}| \equiv \omega$) and $x$ axis in the direction of maximal linear polarization of the photon. Spin indices of the initial state are specified by the photon polarization $\lambda = \pm 1$ and the deuteron spin projection $m_d = 0, \pm 1$ with respect to the quantization axis. The final state is given by a free neutron-proton ($np$) with the relative three momentum $\mathbf{p}$ and by the total spin $S = 0, 1$ and its $z$-projection $m_s$. Concerning only angular dependence, the reduced reaction $t$ matrix elements are expressed in terms of the reaction $T$ matrix for two-body photodisintegration of deuteron in the c.m. frame as follows

$$ t_{Sm_s, \lambda m_d}(\Theta_p) = e^{-i(\lambda + m_d)\Phi_p}T_{Sm_s, \lambda m_d}(\Phi_p, \Theta_p), $$

where $\Theta_p$ and $\Phi_p$ are the spherical angles of the $np$ pair relative three momentum with respect to the frame of reference.

Before the form of $T_{Sm_s, \lambda m_d}$ is specified in the framework of the BS theory and approximation is discussed, we write the most general expressions for the polarization observables in question as expressed of products of the reaction amplitudes. First, it is the unpolarized differential cross section

$$ \frac{d\sigma_0}{d\Omega_p} = \frac{\alpha}{16\pi s} \frac{|\mathbf{p}|}{\omega} F(\Theta_p), \quad F(\Theta_p) = \frac{1}{3} \sum_{Sm_s m_d} |t_{Sm_s, \lambda = 1 m_d}(\Theta_p)|^2, \quad (2) $$

where $\alpha = e^2/(4\pi)$ is fine structure constant and $s$ is the square of the total energy of the $np$ pair. Second, the photon asymmetry for linearly polarized photons and the tensor target asymmetries

$$ -\Sigma^l F(\Theta_p) = \frac{1}{3} \sum_{Sm_s m_d} t^*_{Sm_s, 1 m_d} t_{Sm_s, -1 m_d}, \quad (3) $$

$$ T_{2M} F(\Theta_p) = \frac{\sqrt{5}}{3} \sum_{Sm_s m_d C_{1M+m_d}} C_{1m_2 M} \text{Re} \{ t^*_{Sm_s, 1 m_d} t_{Sm_s, 1 M + m_d} \} (2 - \delta_{M0}), \quad (M \geq 0). \quad (4) $$
In the Eqs. (2)-(3) it is assumed that the observables also depend on the photon lab. energy $E_\gamma$, for which defines
\[ \omega = \frac{M_d}{\sqrt{s}} E_\gamma \quad \text{with} \quad s = M_d(M_d + 2E_\gamma), \]
where $M_d$ is the deuteron rest mass.

The reaction $T$ matrix is expressed in terms of the matrix elements of the EM current between the final and initial 2N states. The procedure for the calculation of the matrix elements in the BS theory is based on Mandelstam’s theory and the reduction formalism. It also preserves the consistence between the amplitudes and current operators from the very outset, see the example in Refs. [11, 12]. In the deuteron breakup one deals with the matrix elements of the EM current between two-body bound state (deuteron) and asymptotically free $np$ scattering state
\[ T_{Sm, \lambda m} = \frac{i}{4\pi^3} \int d^4k d^4u \chi_{Sm_a}(u; \hat{p} \mathbb{P}) \epsilon_\lambda \cdot J(u, k; q, \mathbb{K}) \chi_{m_d}(k; \mathbb{K}), \]
where $J$ is the irreducible EM vertex, $\chi_{m_d}(k; \mathbb{K})$ is the BS amplitudes for deuteron with the total momentum $\mathbb{K} = (E_d, -\omega)$ with $E_d = \sqrt{M_d^2 + \omega^2}$ and $\chi_{Sm_a}(k; \hat{p} \mathbb{P})$ denotes the conjugate BS amplitude of $np$ pair with $\hat{p} = (0, p)$ and $\mathbb{P} = (\sqrt{s}, 0)$ being the on-mass-shell relative and the total four momenta.

The EM vertex operator $J$ is split up into two pieces
\[ J = J^{[1]} + J^{[2]}, \]
where $J^{[1]}$ is a free part (a nucleon couples individually to the radiation field without interacting with another) and $J^{[2]}$ is a two-body EM vertex operator. Owing to the facts that the deuteron is an isoscalar $I = 0$ target and EM interaction does not conserve the total isospin $I$, the two-body vertex correction contributes to the Eq. (3). The gauge independence of the reaction amplitude $q \cdot T = 0$ is necessarily guaranteed by the one- and two-body Ward-Takahashi identities, which the EM vertexes $J^{[1,2]}$ have to satisfy. The sufficient condition of the gauge independence demands both amplitudes to be solutions of the BSE with the same irreducible interaction kernel $\mathcal{V}$. In the ladder approximation the corresponding equations read
\[ \chi_{m_d}(k; \mathbb{K}) = \frac{i}{4\pi^3} \int d^4u S^{(1)}(\frac{\mathbb{K}}{2} + u) S^{(2)}(\frac{\mathbb{K}}{2} - u) \mathcal{V}(k, u) \chi_{m_d}(u; \mathbb{K}), \]
\[ \chi_{Sm_a}(k; \hat{p} \mathbb{P}) = \chi_{Sm_a}^{(0)}(k; \hat{p} \mathbb{P}) + \frac{i}{4\pi^3} \int d^4u S^{(1)}(\frac{\mathbb{K}}{2} + u) S^{(2)}(\frac{\mathbb{K}}{2} - u) \mathcal{V}(k, u) \chi_{Sm_a}(u; \hat{p} \mathbb{P}), \]
where $S^{(l)}$ is the free nucleon propagator ($l = 1, 2$) and $\chi_{Sm_a}^{(0)}(k; \hat{p} \mathbb{P})$ stands for the amplitude for the motion of free particles.

**B. The bound state vertex function**

Now we outline ideas concerning the partial-wave decomposition of the relativistic amplitude of deuteron. One needs the vertex function in order to calculate the EM current
matrix elements. Its form can be readily determined in a general moving frame, after it is obtained in the deuteron rest frame.

The vertex function is related to the BS amplitude through the relation

\[
\Gamma_{md}(k; \mathcal{K}) = \left[ S^{(1)} \left( \frac{\mathcal{K}}{2} + k \right) S^{(2)} \left( \frac{\mathcal{K}}{2} - k \right) \right]^{-1} \chi_{md}(k; \mathcal{K}). \tag{10}
\]

There are two ways to describe the partial-wave decomposition of the vertex function in the Eq. (10): the direct product and matrix ones. The latter allows to nicely absorb the angular-dependence factors into the specification of the partial-wave states entirely. Relevant explanations can be found in Ref. [13]. In both cases the general forms of the eigenstates of the BSE for a given total angular momentum \(J\) are classified according to the spatial parity and "exchange" quantum number that embodies the Pauli principle for two identical relativistic particles [14]. For the coupled physical channels \(3S_1-3D_1\) the relativistic wave function consists of eight states. In the momentum space, the deuteron vertex function is given by

\[
\Gamma_{md}(k_0, \mathbf{k}; \mathcal{K}_{(0)}) = \sum_{\alpha=1}^{8} g(k_0, |\mathbf{k}|, \alpha) \Gamma_{md}(-\mathbf{k}, \alpha) \zeta_{0}^{\alpha}, \tag{11}
\]

where \(\mathcal{K}_{(0)} = (M_d, 0)\), \(\Gamma_{md}(-\mathbf{k}, \alpha)\) are the normalized spin-angular momentum eigenfunctions involving the Dirac \(u\) and \(v\) spinors, \(\zeta_{0}^{\alpha}\) denotes the normalized eigenstate of the total isospin \(I\) and \(I_3\). Index labeled as \(\alpha = 2^{S+1}L_{\gamma=1}^{\rho_{l} \rho_{s}}\) runs over all symmetrical under interchanged of particle 2N states with the total orbital momentum \(L=0–2\), the spin \(S = 0, 1\) and the energy spins of both particles \(\rho_{s} = \pm 1\):

\[
1: 3S_1^{++}, \quad 2: 3D_1^{++}, \quad 3: 3S_1^{--}, \quad 4: 3D_1^{--}, \quad 5: 3P_1^{++}, \quad 6: 3P_1^{--}, \quad 7: 1P_1^{++}, \quad 8: 1P_1^{--}. \tag{12}
\]

The first two states have the nonrelativistic counterparts. The rest six states, corresponding to the negative energy one-particle states, are the relativistic components of the deuteron vertex function.

The partial-wave decomposition of the BSE (13) over states in Eq. (12) yields the set of the coupled two-fold integral equations in the relative energy \(k_0\) and modulo of the relative three-momentum \(|\mathbf{k}|\)

\[
g(k_0, |\mathbf{k}|, \alpha) = \frac{i}{4\pi^3} \int dq_0 dq |\mathbf{q}| \sum_{\beta, \gamma} \mathcal{V}(k_0, |\mathbf{k}|, \alpha; q_0, |\mathbf{q}|, \beta) G_0(q_0, |\mathbf{q}|, \beta, \gamma) g(q_0, |\mathbf{q}|, \gamma), \tag{13}
\]

where \(\mathcal{V}(k_0, |\mathbf{k}|, \alpha; q_0, |\mathbf{q}|, \beta)\) and \(G_0(q_0, |\mathbf{q}|, \beta, \gamma)\) are the partial-wave projections of the interaction kernel \(\mathcal{V}\) and the two-particle spinor propagator \(G_0 = S^{(1)}S^{(2)}\) onto the basis two-nucleon states (12). The coupling between positive and negative energy states occurs directly through the interaction kernel matrix and indirectly through the propagator matrix. The term \(G\) has a simple form independent of angle and spin variables, as it depends only on the \(\rho\)-spin indices. The corresponding structure of the matrix \(\mathcal{V}(k_0, |\mathbf{k}|, \beta; q_0, |\mathbf{q}|, \gamma)\) for pseudoscalar and scalar exchanges can be found in Ref. [14] and for axial-vector, vector and tensor exchanges in Ref. [13]. As for the numerical part, there have been three solutions of the BSE (13) up to now. Two of them are appropriate for the interaction kernel.
which is a superposition of the one-boson exchanges \[4, 5\]. The third one is done for the multi-rank separable interaction kernel, which is a relativistic covariant generalization of the nonrelativistic separable Graz-II potential for the NN system in the coupled \(3S_1\) and \(3D_1\) states \[3\].

In this paper we also present the research on the question how strongly the observables of deuteron break-up are sensitive to the different inputs of the deuteron vertex functions. Fig. 1 shows the positive energy components \(g(k_0, |k|, \alpha) (\alpha = 1, 2)\) for two classes of the interaction plotted against the momentum \(|k|\) at \(k_0 = 0\). It is seen that the behavior in certain regions essentially depends on the type of interaction model used for a description of the NN system. The structures are similar at low three-momentum \(|k| \leq 250\) MeV. A ‘tail’ at high momenta of the \(3D_1^+\)-component for the separable interaction is much harder than those for the OBE one\[21\]. The \(3S_1^+\)-component for the OBE interaction has a more pronounced dip with respect to that of the separable interactions in the region \(0.5 \leq |k| \leq 1\) GeV. These differences have an perceptible effect upon the observables.

In further discussion it is worth introducing states, which symmetrical and antisymmetrical with respect to the relative energy \(k_0\). These states are labelled by numbers \(n = 2S+1L^\rho\) with \(\rho = (+), (-), (e)\) and \((o)\) being the projection of the total energy spin of the 2N system. In this case the notation of the components of the vertex function is as follows,

\[
1 : 3S_1^+, \quad 2 : 3D_1^+, \quad 3 : 3S_1^-, \quad 4 : 3D_1^-, \quad 5 : 1P_{1e}, \quad 6 : 3P_{1o}, \quad 7 : 1P_{1o}, \quad 8 : 3P_{1e}. \tag{14}
\]

The first six states are even in the relative energy, while the last two change signs as \(k_0 \rightarrow -k_0\), i. e. they are referred to as odd. The normalization condition for the vertex function is the matrix element of the charge operator between the deuteron state at the zero momentum transfer

\[
\frac{1}{2\pi^2 M_d} \sum_{n=1}^{8} \int_{-\infty}^{+\infty} dk_4 \int_0^{\infty} d|k| |k|^2 \omega_\rho(E_k) \left[ \phi(k_4, |k|, n) \right]^2 = 1, \tag{15}
\]

where \(k_4 = ik_0\), \(\omega_\rho = \frac{1}{2}(\rho_1 + \rho_2)E_k - \frac{M_d}{2}\) and \(E_k = \sqrt{m^2 + |k|^2}\). The radial components of the vertex function \(g(n)\) are related to those \(\phi(n)\) of the BS amplitude as follows

\[
g(1, 2) = G_0(+)^{-1} \phi(1, 2) \tag{16}
\]

for the \(S\)- and \(D\)-positive energy states,

\[
g(3, 4) = G_0(-)^{-1} \phi(3, 4) \tag{17}
\]

for the \(S\)- and \(D\)-negative energy states,

\[
g(5, 7) = G_0(e,e)\phi(5, 7) - G_0(e,o)\phi(6, 8) \tag{18}
\]

\[
G_0(e,e)^2 + G_0(e,o)^2
\]

for the \(P\)-states even in the relative energy,

\[
g(6, 8) = G_0(e,o)\phi(5, 7) + G_0(e,e)\phi(6, 8) \tag{19}
\]

\[
G_0(e,e)^2 + G_0(e,o)^2
\]
for the $P$-states odd in the relative energy. In the Eqs. (14)-(19) the partial wave projections of the two-nucleon propagator are labelled as a matrix in $\rho$-subspace

$$
G_0(\pm, \pm) = \frac{1}{(\frac{M^4}{2} - E_k)^2 + k_4^2}, \quad G_0(-, -) = \frac{1}{(\frac{M^4}{2} + E_k)^2 + k_4^2},
$$

$$
G_0(e, e) = \frac{1}{2} [G_0(\pm, \pm) + G_0(-, -)], \quad G_0(e, o) = \frac{1}{2} [G_0(\pm, \pm) - G_0(-, -)].
$$

(20)

The normalization condition (15) defines the $n$-state probability. According to Ref. [5] each can be understood as a measure of the effective charge of the state. The corresponding numerical values are listed in Table I. The upper limit of integration in Eq. (15) is limited to 3 GeV both for the relative energy and momentum. One can see that the probabilities for the relativistic components of the vertex function are negative, and they will be further referred to as "pseudoprobabilities". As a consequence, inclusion of the negative energy states reinforce the contribution to the charge of the positive energy ones. Their resultant contribution is greater than one. The smallness pseudoprobabilities of the negative energy states is explained by the use of the axial-vector coupling for the $\pi N$-vertex. The dominant relativistic component is the triplet $3P_0^0$-wave with $P_0 = -0.08\%$, and contribution of the $3S_1^-$ and $3D_1^-$-states are negligible. In the case of the $\pi N$ interaction due to pseudoscalar coupling, values of the pseudoprobabilities of the negative-energy partial states are expected to be characteristically different. In that case the qualitative analysis shows that $3S_1^-$-state may have the largest pseudoprobability, while all other negative-energy states may have been dismissed [6].

In Table I we also quote the numerical values for the quadrupole $Q_d$ and the magnetic $\mu_d$ moments (without meson current contributions) as well as the asymptotic $D/S$-state ratio $\rho_{D/S}$. The moments $Q_d$ and $\mu_d$ are given with taking into account of the relativistic corrections. The experimental values are cited according to the ones given in Table XVII of Ref. [17]. In case of the OBE interactions the contribution of the negative energy components is considered. It is purely of the relativistic nature and it is negative in sign, thus reinforcing the discrepancy with the empirical value. As the Graz-II interaction favors the $D$-state probability of 5\%, we have focused our attention on the separable vertex function calculated for this case. The asymptotic $D/S$-state ratio has been calculated for each vertex function explicitly. In momentum space, the ratio of the $3D_1^+$ over the $3S_1^+$ component of the vertex function with both particles on the mass-shell is defined by the formula [3]

$$
\rho_{D/S} = \frac{g(k_4, |k|, 2)}{g(k_4, |k|, 1)} \bigg|_{k_4=0, E_k = \frac{M_d}{2}}.
$$

(22)

Extrapolation of the radial vertex functions to the unphysical value of $|k|^2 = \frac{M_d^2}{4} - m^2$ is achieved in two ways: directly and by expansion of the functions in the Taylor series up to terms of the third order.

It is interesting to compare the pseudoprobabilities of the relativistic components of the deuteron vertex function of Refs. [5] and [9]. Both functions are calculated using the BSE in the ladder approximation with the same superposition of the meson exchanges, except that in Ref. [9] one more $\sigma$ meson is added. Difference in the pseudoprobabilities of the spin singlet and triplet $P$-components reaches one order of magnitude. Compare the total strength of these states $P_\perp = -2.5 \times 10^{-2}$ of Ref. [5], which is mostly due to the spin-singlet
even and odd $P$-states, with $P = -1.1 \times 10^{-1}$ of Ref. [3]. For the latter value the major contribution is due to the spin-triplet even $P$-states. The source of such difference may reside in the structure of the interaction matrix $V$ in Eq. (13). For the axial-vector, vector and tensor exchanges (opposite to the scalar one) there is a direct coupling between the partial-wave states that are even in relative energy and states that are odd. Consequently, effects as turning off the $\frac{2q^\mu}{\mu_\pi^2}$ term in the vector propagators or a tiny change of the $\pi N$-coupling in $\Lambda$ theory may influence on a quite sensitive changes in strengths of the negative energy channel.

III. THE REACTION AMPLITUDE

Having determined the vertex function in the rest frame of the deuteron, we can calculate the reaction amplitudes (6) and, as a result, obtain the differential cross section (2), photon and the target asymmetries (3) and (4).

In the plane-wave case with the one-body EM vertex one finds that (the detail derivation is given in Ref. [1])

$$T_{S_{m_s} \lambda m_d} = \sum_{l=1,2} \bar{\chi}_{S_{m_s}}(p) \zeta^+_s \Lambda(\mathcal{L}) \Gamma^{(l)}_{\lambda}(q^2 = 0) S^{(l)} \left( \frac{\mathbb{K}(0)}{2} - (1)^l k_l \right) \Gamma_{m_d}(k_l; \mathbb{K}(0))$$

$$- \sum_{l=1,2} (-1)^l \bar{\chi}_{S_{m_s}}(-p) \zeta^+_v \Lambda(\mathcal{L}) \Gamma^{(l)}_{\lambda}(q^2 = 0) S^{(l)} \left( \frac{\mathbb{K}(0)}{2} + (1)^l k_l \right) \Gamma_{m_d}(-k_l; \mathbb{K}(0)),$$

where the conjugate of the 2N continuum amplitude $\bar{\chi}^{(0)} = \gamma_0^{(1)} \gamma_0^{(2)} \chi^{(0)}$ is expressed in terms of the two free Dirac positive energy spinors

$$\chi_{S_{m_s}}(p) = \sum_{\lambda_1, \lambda_2} C^{S_{m_s}}_{\lambda_1 \lambda_2} u_{\lambda_1}(p) u_{\lambda_2}(-p)$$

and the two combinations of the isospin singlet and triplet functions $\zeta_s = \zeta^0_0 + \zeta^1_0$, $\zeta_v = \zeta^0_0 - \zeta^1_0$.

The operator $\Lambda(\mathcal{L}) = \Lambda^{(1)}(\mathcal{L}) \Lambda^{(2)}(\mathcal{L})$ represents the Lorentz transformation $\mathcal{L}$ on the Dirac subspace with

$$\Lambda^{(l)}(\mathcal{L}) = \left( \frac{E_d + M_d}{2M_d} \right)^{1/2} \left[ 1 + \frac{\gamma_0 \gamma \cdot q}{E_d + M_d} \right]^{(l)}$$

The matrix $\mathcal{L}$, defined as $\mathbb{K} = \mathcal{L} \mathbb{K}(0)$, boosts the initial BS amplitude from the rest frame to the c. m. frame, in which the deuteron moves with a velocity $\omega/M_d$.

In deuteron break-up we deal with the half off-mass-shell photon-nucleon vertex $\Gamma^{(l)}_{\lambda}$, since the knocked-out nucleon is taken as the physical one. Moreover, $\Gamma^{(l)}_{\lambda}$ takes on the on-shell form at the real photon point $q^2 = 0$ as a consequence of gauge invariance [18]

$$\Gamma(q^2 = 0) = \epsilon^\mu_{\lambda} \gamma_\mu \frac{1 + \tau_3}{2} + \frac{i}{2m} \sigma_{\mu\nu} \epsilon^\mu_{\lambda} q^\nu \kappa_s + \kappa_v \tau_3,$$

where $\kappa_s = \kappa_p + \kappa_n$ and $\kappa_v = \kappa_p - \kappa_n$ with the anomalous part of the proton (neutron) magnetic moments in units of the nuclear magneton $1/(2m)$ denoted as $\kappa_{p(n)}$.  

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As the result of the boosting of the bound state wave function along the negative \( z \) axis, the relative four-momentum \( k_l \) in Eq. (23) is “contracted”. One can find

\[
\begin{align*}
  k_{l0} &= \frac{\omega}{M_d} p_z + (-1)^l \frac{\sqrt{s}}{2M_d} \omega, \\
  k_{lx} &= p_x, \quad k_{ly} = p_y, \\
  k_{lz} &= \frac{\sqrt{s} - \omega}{M_d} p_z + (-1)^l \frac{\sqrt{s}}{2M_d} \omega.
\end{align*}
\]  

(27)

We proceed with introducing the matrix representation for each partial wave of the deuteron vertex function (12) and for the amplitude of \( np \) pair (24). The reaction amplitudes (23) related to \( \Delta I = 0 \) transitions interfere with those having \( \Delta I = 1 \). Evaluating the isospin part of matrix elements, the amplitudes \( T_{S_{m_d},\lambda m_d}^a \) for the isoscalar \( a = 1 \) and isovector \( a = 2 \) transitions are cast into traces of \( \gamma \)-matrix terms. Due to the symmetry consideration, the first two terms in Eq. (23) are identical to the latter two. Further evaluation of amplitudes for the given set of the polarization indices \( \lambda, m_d \) and \( m_s \) has been analytically carried out with the help of the formulae manipulating language REDUCE [19]. The resulting expressions have the form

\[
\begin{align*}
  t_{S_{m_d},\lambda m_d}^a &= \sum_{l=1,2} \sum_{\alpha=1}^8 \left[ \tilde{\Gamma}_{S_{m_d},\lambda m_d}^a (\omega, k_l; l, \alpha) S_{\rho_i}^{(l)} (k_{l0}, |k_l|) g(k_{l0}, |k_l|; \alpha) \\
  &\quad + \tilde{\Gamma}_{S_{m_d},\lambda m_d}^a (\omega, -k_l; l, \alpha) S_{\rho_i}^{(l)} (-k_{l0}, |k_l|) g(-k_{l0}, |k_l|; \alpha) \right],
\end{align*}
\]  

(28)

where quantities \( S_{\rho_i}^{(l)} \) are the positive (\( \rho_i = +1 \)) and negative (\( \rho_i = -1 \)) scalar parts of the free one-nucleon propagator

\[
S_{\rho_i}^{(l)} (k_{l0}, |k_l|) = \frac{1}{E_{k_l} - \rho_i \left( \frac{M_d^2}{2} - (-1)^l k_{l0} \right)}.
\]

(29)

and factors \( \tilde{\Gamma}_{S_{m_d},\lambda m_d}^a (\omega, k_l; l, \alpha) \) absorb the spin-angular part of the reaction amplitude. They also depend on the Lorentz boost parameter \( \sqrt{E^2/s^{1/4}} \).

A reduce code is set up for the explicit analytical evaluation of the spin-angular factors with free polarization indices in square brackets of the formula (28) for each isospin \( a \), particle \( l \) and state \( \alpha \) numbers.

IV. RESULTS

It finally remains to numerically calculate the Eq. (28). After that one can proceed with studying the effects on the differential cross section and polarization observables due to various relativistic contributions. Beforehand it is worthwhile to note that the BSE for the OBE interaction as previously described is solved in the energy-momentum space using the Wick rotation in the relative energy variable. That implies that the deuteron vertex function is obtained along the imaginary relative energy axis \( k_4 \). As seen from Eq. (28) in calculating the reaction amplitude, we are to obtain the vertex function at some real \( k_{l0} \) axis. But it is rather hard to do, since numerical procedure is very unstable. For not high \( k_{l0} \) values by expanding the deuteron vertex function in a Taylor series around \( k_4 = 0 \), we can...
principally find the function at given value of the relative energy variable. In numerical work we may use the analytical parameterization of the BS amplitude for the deuteron presented in Ref. [16]. Unfortunately, our examination shows that the derivatives of eight components $g(k_4, |k|; \alpha)$ of the vertex function with respect to the relative energy can not be calculated with a sufficient controlled accuracy. In Ref. [1] it was shown that a very good approximation to the exact result is the zeroth order approximation for the BS amplitude (BS-ZO). It comes to keeping the $k_0$-dependence of the one-particle propagator in Eq. (28), but keeping the relative energy $k_0l$ in the radial part $g$ of the vertex function equal to zero. That means that both nucleons are forced to stay equally off the mass-shell. The Lorentz boost on the angular momentum part of the vertex function is included as well. This approximation implies that the retardation due to dependence of the radial part of the vertex function on the $k_0$ is far less important than the boost on the one-particle propagator due to recoil. It is also justified by conclusions drawn from the studies of the elastic electron-deuteron scattering [3]. When $k_0 = 0$, the odd in the relative energy $^1P_1^o$ and $^3P_1^e$ components of the vertex function vanish. We may hope that this is a minor drawback, since the net pseudoprobability of these states $P_{odd}$ is roughly one order of magnitude less than that of the two even $P$-states (compared with the value of $P_{even}$ in Table I). As for the separable interaction we can only do exact calculations of the deuteron vertex function in the $^3S_1^+^1D_1^+$ channel at a given value of the relative energy variable. The negative-energy states are completely left behind in this case.

In Figs. 2 the results for the angular distributions of the differential cross section $d\sigma_\gamma/d\Omega_\gamma$ and the linear photon asymmetry $\Sigma$ at three different values of lab photon energies are presented. The BS-ZO calculation in the OBE model is given by the solid curve. The dotted line shows the effect of shutting off all the negative energy states of the OBE vertex function of the deuteron. That leaves only two positive energy $L = 0$ and $L = 2$ states. The dot-dashed curve should be compared with the dotted one. The former depicts the calculation in the separable model with the inclusion of all the retardation factors and relativistic effects apart from those generated by the negative energy partial waves. As can be seen from Fig. 2 the global structure of the observable is defined by the positive energy components of the deuteron vertex function. The negative energy components increase markedly the cross section but leave its global structure untouched. Comparing the dotted and dot-dashed line, we observe the systematic deficiency of the calculations with the separable positive energy states of the vertex function with respect to the OBE one. Explanations can be found observing Fig. [3]. As follows form the Eq. (3) the differential cross section is proportional to square of the modulus of the partial-wave components of the vertex function multiplied by the over-all kinematic factor. At $E_\gamma = 50$ MeV the absolute value of the three relative momentum of the $l$th bound nucleon $|k_l|$, see the Eq. (27), lies in the range $200$–$265$ MeV. At this momentum range the separable and OBE vertex function are almost identical and, thus, both functions give the same result. The tiny rise of the differential cross section in the case of the OBE interaction is due to the smallness of the negative energy components of the deuteron vertex function. Results start to be closely interaction kernel dependent as the photon energy rises. At $E_\gamma = 500$ MeV the absolute value of $|k_l|$ is within the interval $450$–$950$ MeV. As seen in Fig. 2 the $^3S_1^+^1D_1^+$-partial wave in case of the OBE interactions has a more pronounced minimum than that of the Graz-II interaction kernel. We can conclude that the cross section is rather sensitive to the momentum behavior of the partial-wave components of the vertex function. As for the negative energy components the $^1P_1^e$- and $^3P_1^o$-states have a persistent superiority over the positive-energy states at high
three momentum. As the contribution of the latter is diminished, the former increases the cross section. Effects produced $^3S_1^-$- and $^3D_1^-$-states are completely negligible. Shutting off the negative energy states has a minor effects, particularly, on the photon asymmetry in a wide photon energy range. Though the reaction amplitude is subject to a change when the relativistic components of the vertex function are included, we are to suppose that they cancel each other in the photon asymmetry.

Next, we discuss three observables associated with tensor polarization of the deuteron. Fig. 3 shows the angular dependence of the tensor target asymmetries. The solid curve is the calculation with the six partial wave components of the OBE vertex function of deuteron. Comparison with the dot curve shows what differences arise when the negative energy states are invoked to describe the physical process. Inclusion of these states at a moderate photon energy effects to a change of the $T_{2M}$ at the forward and backward proton c.m. angles $\Theta_p$. At higher energies these are visible in the whole $\Theta_p$ range. As in the case of the differential cross section, the negative energy states leave the global structure inalterable. Again, the dot-dashed line should be compared with the dotted one. For the photon energies about 500 GeV two curves demonstrate sensitivity bearing by the observables on the model of the NN interaction. The dissimilarity between the OBE and separable models clearly shows up in the observable $T_{22}$. For $M = 2$ in the Eq. (11) there are not interference terms between amplitudes $t_{s_{m_{s}}m_{d}}$ with different $m_d$. The Clebsch-Gordan coefficient in the Eq. (11) for the observable $T_{22}$ selects reaction amplitudes $t_{s_{m_{s}}\lambda=1m_{d}}$ with $m_d = -1$. In contrast with that the $T_{20}$ is the sum of products of reaction amplitudes with all allowed values of $m_d$.

V. CONCLUDING REMARKS

In the previous sections we presented the results of the relativistic covariant calculation of the polarization observables in two-body deuteron photodisintegration. Our aim has been to study contribution of the relativistic components of the deuteron amplitude, which does not have counterparts in the nonrelativistic physics. These negative energy components comply with requirements of covariance, since the full Dirac structure should be taken into account in the partial-wave analysis of the full BS amplitude of the NN system.

In this paper we have studied the influence of the relativistic effects on the photon beam and three tensor target asymmetries as well as the differential cross section within the framework of the BS formalism. The significance of the negative energy components is examined using the BS equation for the OBE interaction kernel, which couples two positive and six negative energy states of the deuteron vertex function. Moreover, the importance in particularities of the behavior of the positive energy components is tested as well. We compare the results obtained for the observables in the OBE model with those of the multirank separable model of the NN interaction; in the separable model the negative energy states are switched off.

Unfortunately, our results do not account properly for the role of the relative energy variable $k_0$ of the deuteron vertex function. A fundamental obstacle here is posed via the numerical treating of the BSE. After the partial-wave decomposition of the equation, the numerical procedure in solving the eigenvalue problem is based on the Wick rotation. The components of the vertex function are computed along the imaginary $k_4 = ik_0$ axis. This technique prevents a direct application and practical use of the vertex function in some physical process involving deuteron. Considering reliable possibilities of making approximate calculations, we employ the zeroth order approximation for the radial part of the vertex
function. It allows one to take into account six out of eight partial wave states as well as other very important relativistic effects.

In studying the cross section and asymmetries, we establish the following results. Adding the negative energy states of the OBE vertex function to the positive energy, we find lead to a sizeable increasing (up to 10 %) of the differential cross section in a wide range of the proton c. m. angles. This tendency becomes more pronounced at higher photon energies. The asymmetries are less influenced by these states and, thus, show minor changes. However, modifications become noticeable in the tensor asymmetries at the deuteron break-up in the forward and backward directions. On the other hand, the results indicate strong dependence on the behavior of the $^3S_1^+ - ^3D_1^+$-partial waves as functions of the relative three momentum of nucleons.

Numerical results of this paper has been obtained in the plane-wave approximation with one-body EM current operator. Since the one-body current is not conserved, the choice of a gauge for the radiation field does become important. We have used the transverse gauge, that in combination with the one-body current leads to a too small value for the matrix elements. The two-body nucleonic current as well as exchange currents should be called into play to restore the gauge independence of the reaction amplitude and the Siegert limit (see, for example, Refs. [11, 20]). Therefore, an inclusion of the two-body EM currents into the relativistic analysis is of the prime importance. Corresponding numerical work will be done in future and results are going to be reported in a forthcoming paper.

Acknowledgments

We would like to thank A.A. Goy and S.Eh. Schirmovsky for helpful conversations. We also wish to thank S.M. Dorkin and L.P. Kaptari for consultations in conducting the numerical part of this work. We are grateful to O.V. Chelomina for the assistance rendered in preparation of the paper. This research was supported in part by Grant No. 015.02.01.022, "Universities of Russia".

[1] K.Yu. Kazakov and S.Eh. Shirmovsky, Phys. Rev. C 63, 014002 (2000).
[2] K.Yu. Kazakov and S.Eh. Sus’kov, Sov. J. Nucl. Phys. 64, 1 (2001).
[3] G. Rupp and J.A. Tjon, Phys. Rev. C 41, 472 (1990).
[4] J. Fleischer and J.A. Tjon, Phys. Rev. D 21, 87 (1980).
[5] M.J. Zuilhof and J.A. Tjon, Phys. Rev. C 22, 2369 (1980).
[6] N. Honzawa and S. Ishida, Phys. Rev. C 45, 47 (1992).
[7] F. Gross, J.W. Van Orden and K. Holinde, Phys. Rev. C 45, 2094 (1992).
[8] A.Yu. Umnikov, L.P. Kaptari, K.Yu. Kazakov, and F.C. Khanna, Phys. Lett. B 334, 309 (1994).
[9] L.P. Kaptari, A.Yu. Umnikov, S.G. Bondarenko, K.Yu. Kazakov, F.C. Khanna, and B. Kämpfer, Phys. Rev. C 54, 986 (1996).
[10] H. Arenhövel, Few-Body Syst. 4, 55 (1988).
[11] W. Bentz, Nucl. Phys. A 446, 678 (1985).
[12] A.Yu. Korchin and A.V. Shebeko, Sov. J. Nucl. Phys. 54, 214 (1991).
[13] S.G. Bondarenko, V.V. Burov, M. Beyer and S.M. Dorkin, Few-Body Syst. 26, 185 (1999).
[14] J.J. Kubis, Phys. Rev. D 6, 547 (1972).
[15] J. Fleischer and J.A. Tjon, Phys. Rev. D 15, 2537 (1977).
[16] A.Yu. Umnikov, Z. Phys. A 357, 333 (1997).
[17] R. Machleidt, Phys. Rev. C 63, 024001 (2001).
[18] S.I. Nagorny and A.E.L. Dieperink, Eur. Phys. J. A 5, 417 (1999).
[19] A.C. Hearn, REDUCE User’s Manual, Version 3.6 (RAND Pub. CP78 Rev. 7195, 1995).
[20] S. Ying, E.M. Henley and G.A. Miller, Phys. Rev. C 38, 1584 (1988).
[21] The factor $i^2$ is absorbed in the definition of the $^3D_1^+$ partial-wave component.
FIG. 1: The momentum dependence of the radial components of the deuteron vertex function ($^1S_1^+ - ^1S_1^+$ channel) is shown for the relative energy variable $p_4 = 0$. Solid and dash curves depict the behavior of the vertex functions for the one-boson exchange and relativistic separable interactions, respectively.

FIG. 2: The differential cross section and the linear photon asymmetry in the plane wave one-body approximation at different lab. photon energies $E_\gamma$. Curves: solid line, the OBE interaction vertex function computed in the BS-ZO approximation (6 partial-wave states are included); dotted line, the OBE interactions with taking into account only two positive-energy partial-wave states of the deuteron vertex function; dot-dashed line, calculation with the deuteron vertex function in case of the separable interactions.
FIG. 3: The tensor target asymmetries. Notations are the same as in Fig. 2.

Tables

TABLE I: Deuteron properties in the framework of the BS formalism, compared with experiment.

|                      | OBE   | Graz-II | Empirical          | Reference(s) |
|----------------------|-------|---------|--------------------|---------------|
| Binding energy $\varepsilon_d$ (MeV) | 2.2250 | 2.2250  | 2.224575(9)        | [3, 9, 17]    |
| Asymptotic D/S ratio $\rho_{D/S}$       | 0.02497 | 0.02691 | 0.0256(4)          | [3, 16, 17]   |
| Quadrupole moment $Q_d$ (fm$^2$)        | 0.2678$^a$ | 0.2774$^a$ | 0.2859(3)          | [3, 16, 17]   |
| Magnetic moment $\mu_d$ ($\frac{e}{2m}$) | 0.8561$^a$ | 0.8512$^a$ | 0.857506(1)        | [3, 9, 17]    |
| D-state probability $P_D$ (%)            | 5.10   | 5.0     | —                  | [3, 16]       |
| Pseudoproductility $P_-$ (%)             | -0.0050 | —       | —                  | [16]          |
| Pseudoproductility $P_{even}$ (%)        | -0.0920 | —       | —                  | [16]          |
| Pseudoproductility $P_{odd}$ (%)         | -0.0230 | —       | —                  | [16]          |

$^a$Without meson current contributions and with relativistic corrections.