Obtaining the sensitivity of a calibrator for interferometric gravitational wave

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Abstract: Interferometric gravitational wave detectors (IGWD) are a very complex detector, the need to lock the detector in a dark fringe condition besides the vibrations that affect the mirrors, creates the necessity of using active suspension systems. These active systems make the system reach the desired sensitivity but make the calibration of such detectors much more difficult. To solve this problem a calibrator is proposed, a resonant mass gravitational wave detector could be used to detect the same signal in a narrower band and use the measured amplitude to calibrate the IGWD, as resonant mass gravitational wave detectors are easily calibrated. This work aims to obtain the expected sensitivity of such a calibrator by using lumped modelling in such mechanical detectors. The calibrator is modelled as a spring mass system and the sensitivity curve is presented calculated in by a matlab program. The curve shows that using state of art parameters for the detector the final sensitivity is close to the quantum limit and can be used to calibrate the IGWDs.

1. Introduction

Soon after Albert Einstein developed his theory of General Relativity, there was many developments, one of such developments was the prediction of gravitational waves, as space time can be deformed and dictate how matter should move, if this medium is perturbed, such perturbations should propagated with the speed equal to the speed of light. So after one hundred years experiments to measure such speed are being proposed [1,2].

The authors are part of a Brazilian research group called GRAVITON devoted to the study of gravity. In the search to understand, GRAVITON group members make many contributions to the field of
gravitational waves; such efforts can be seen from reference [3] to [26] covering all sorts of aspects in gravitational wave detection.

This expertise gave the author capability to develop a way to calibrate the big detectors that are claimed to have achieved the first detection of these waves. Section 2 will explain how such detectors work and why it is difficult to make its calibration as a whole, section 3 will show how to calculate the sensitivity, section 4 will present the results and section 5 is the conclusion.

2. The Laser Interferometric Gravitational Wave Detectors

Fig. 1 shows the experimental setup: it works as an interferometer, there is a laser source that emits a laser that is divided in two in a beam splitter then the laser enters in two optical cavities called arms, the laser is storage in both arms then they returns to the beam splitter and one fourth of the original laser power goes to the photodetector. The experiment is set in a dark fringe interference at the photodetector. When a gravitational wave arrives perpendicular to the apparatus it changes the length of the arms changing the interference pattern. The problem of the experiment arises when the vibration is taken into account. Vibration of any kind (seismic, thermal, etc) also changes the interference condition of the experiment. To avoid that, clever suspension systems were designed including active systems. The problem is that the active system could respond to the gravitational wave system as well making the calibration of the system quite difficult.

Nevertheless the experiments are taking place and many transient signals were detected as can be seen in Fig. 2. The figure shows the signal in both domains: time and frequency, vertical for the frequency and horizontal for the time. In the figure can be observed that the highest intensity of the signal happens around the frequency of 125 Hz.
These were not the only gravitational wave detectors. There are the resonant mass gravitational wave detectors, they worked for many decades. They consist of a mass that, when the gravitational wave passes through it, vibrates if the system is in the same frequency of the gravitational wave (because of that the name resonant) [10]. There were many detectors working in coincidence around the world, an example of one of these detectors can be seen in Fig. 3. This detector was called Allegro and operated for more than two decades in the Louisiana State University.

No gravitational wave was detected with this kind of detector probably because the operational frequency was in a different range, they operated close to 1 kHz frequency, and as can be seen in Fig. 2 a good detection window will be close to 100 Hz. The frequency of 1 kHz came for historical reasons, the idea was to use a bar as a detector, and a bar that vibrates at 1 kHz is impractical to be constructed. Nevertheless, this kind of detectors are easily calibrated.

It has been proposed a new kind of resonant mass antenna to build a gravitational wave detector and use the signal detected by such a detector to calibrate the laser interferometric gravitational wave detectors. This resonant mass is called the antenna of the detector.
There are two main options for the construction of such antenna: Sapphire with the properties: specific mass of 3.98 g.cm\(^{-3}\), sound velocity of 9.4 km.s\(^{-1}\), mechanical quality factor (Q\(_M\)) of 3\(\times\)10\(^9\), Young modulus of 431.80 GPa, shear modulus of 170 GPa, Poisson ratio of 0.27, tensile yield strength of 15.5 GPa and bulk modulus of 312.9 GPa; and Niobium, with the properties: specific mass of 8.57 g.cm\(^{-3}\), sound velocity of 3.48,4 km.s\(^{-1}\), elasticity modulus of 98.6 GPa, ultimate tensile strength of 172 MPa, yield strength of 103.0 MPa, QM of 2 \(\times\) 10\(^8\), Poisson ratio of 0.38 and hardness (nano identification) of 2.5GPa Q\(_M\) = 2 \(\times\) 10\(^8\).

Sapphire presents the best properties to build such an antenna but is impractical to get a very massive detector using this material because of manufacturing limitations, then the Niobium is the next possible choice.

The proposed geometry for the new antenna can be seen in Fig. 4. This figure shows a normal mode simulation of the vibration mode of interest. It's a color figure with colors showing the displacement from the central position of the mass; the redder the higher the displacement and the more blue the smaller it is.

**Figure 4:** Chosen geometry for the mechanical antenna. Here the figure shows the antenna on its highest displacement as it vibrates. The authors.

### 3. Calculation of the sensitivity

The following sections are the result of [4]. The calculations are made considering the system as a spring mass model. To make possible measuring the displacement of the side masses of the system, one of those will be connected to a system of smaller resonators that mechanically amplifies the vibration. It will be a set of two resonators whose masses will diminish with the same proportion from the mass of the initial mass. This will lead us to three coupled equations of motion that can be solved numerically in the frequency domain, in the same way that was done in [4].

To the calculation was incorporated thermal noise, back action noise (from the microwave source to be used to measure the displacement of the last mass), phase noise (from the microwave source) and the electronic noise from the amplifier.

In order to calculate the sensitivity, the follow characteristics were implemented:

- \(M_0 = 10,000\) kg;
- \(M_1 = 10\) kg;
- \(M_2 = 0.01\) Kg;
- \(Q_{Mb} = 250 \times 10^6\); (mechanical quality factor of the side bars);
4. Result

Fig. 5 shows the displacement sensitivity of such a detector, the unit is strain divided by the square root of Hertz because the sensitivity should be squared before making the integration and obtain the total sensitivity which is equal to $1.31 \times 10^{-21}$. This result is adimensional because it represents the proportion of same length that will deform in the presence of a gravitational wave, if the length is one kilometer the displacement will be of the order of one thousandth of an atomic nucleus. The quantum limit for this measurement is equal to $1.29 \times 10^{-21}$. It means the experiment will be measuring on phonon of the vibration of the mechanical antenna.

5. Conclusion

The calculation for the strain sensitivity shows that the measurement of a gravitational wave is possible with such a detector, as the first gravitational wave ever detected had a strain of $10^{-18}$. Nevertheless it was a rare event, but other events were detected and the strain sensitivity of this new detector reaches the strain sensitivity of the laser interferometric gravitational wave detectors, meaning that the wave detected by then will be detected by this calibrator. Then the signal will be used to calibrate these detectors.

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References

[1] Frajuca C et al 2020 Journal of Physics: Conference Series 1730 012025
[2] Fabricio Junior C A et al 2020 Journal of Physics: Conference Series 1730 012082
[3] Aguiar O D et al. 2006 Journal Class. Quantum Grav. 23, 239
[4] Frajuca C et al. 2004 Class. Quantum Grav. 21 1107
[5] Velloso W F, Aguiar OD and Magalhaes NS Proc. First International Workshop for an Omnidirectional Gravitational Radiation Observatory 1997 Singapore:World Scientific
[6] Magalhaes N S et al. 1997 Astrophysical Journal 475, 462
[7] Magalhaes N S et al. 1995 MNRAS 274, 670
[8] Magalhaes N S et al 1997 Gen. Relat. Grav. 29 1511
[9] Aguiar O D et. al. 2005 Class. Quantum Grav. 22, 209
[10] Frajuca et al 2002 Class. Quantum Grav. 19 1961
[11] Ribeiro K L et al. 2004 Class. Quantum Grav. 21, 1225
[12] Aguiar O D et al. 2012 Journal of Physics: Conference Series 363, 012003
[13] Frajuca C, Bortoli F S, Magalhaes N S 2005 Brazilian Journal of Physics 35 1201
[14] Frajuca C, Bortoli F S, Magalhaes N S 2006 Journal of Physics: Conference Series 32 319
[15] Aguiar O D et al. 2004 Class. Quantum Grav. 21 459
[16] Bortoli F S et al 2010 Journal of Physics: Conference Series 228 012011.
[17] Andrade L A et al. 2004 Class. Quantum Grav. 21, 1215
[18] Aguiar O D et al 2002 Brazilian Journal of Physics 32 866
[19] Bortoli F S et al. 2019 Brazilian Journal of Physics 49 133
[20] Frajuca C, Bortoli F S, Magalhaes N S, Horiguti A M 2008 Journal of Physics: Conference Series 122 012029
[21] Magalhaes N S, Miranda T A, Frajuca C 2012 Astrophysical Journal 755, 54[26] Frajuca C, Bortoli F S 2006 Journal of Physics: Conference Series 32 315
[22] Bortoli F S et al. 2016 Brazilian Journal of Physic 46 308
[23] Frajuca C et al. 2018 Journal of the Brazilian Society of Mechanical Sciences and Engineering 40 319
[24] Bortoli F S et al 2020 Brazilian Journal of Physic 50 541
[25] Aguiar O D et al 2002 The Ninth Marcel Grossmann Meeting: On Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories 1891
[26] Frajuca C, Bortoli F S 2019 Journal of Physics:Conference Series 1391 012029