Chasing CHOOZ\textsuperscript{1}

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We relate the MNS and CKM mixing matrices using ideas from grand unification. We catalog models in terms of the family symmetries of the down quark mass matrices, and emphasize the role of the Cabibbo angle in the lepton mixing matrix. We find a large class of models with an observable CHOOZ angle \( \sim \lambda/\sqrt{2} \).

1 Introduction

Since 1998 when SuperKamiokande announced incontrovertible evidence of neutrino oscillations, a series of experiments have unravelled most of the fundamental neutrino parameters. Two important pieces of information still await determination: the absolute value of the masses, together with their hierarchy, and the third mixing angle, for which there only exists a limit set by the CHOOZ experiment. From the theoretical side, neutrino masses are hardly surprising in the context of the grand unified\textsuperscript{[1] [2] [3] [4]} generalizations of the Standard Model. While some of us predicted one large mixing angle in the MNS lepton mixing matrix, the recent determination of two large mixing angles comes as a surprise, and poses unforeseen theoretical challenges. In the following we offer several remarks as to the theoretical implications of the present data set and note that with a modest infusion of grand unification ideas, it is possible to relate the CKM and MNS matrices. In a wide class of models, the CHOOZ angle is even predicted in terms of the Cabibbo angle.

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2 The Data

Five years of neutrino experiments\textsuperscript{5, 6, 7, 8, 9, 10} can be succinctly summarized\textsuperscript{11}: the MNS lepton mixing matrix

\[ U_{\text{MNS}} = \begin{pmatrix}
\cos \phi & \sin \phi & \epsilon \\
-\cos \theta \sin \phi & \cos \theta & \cos \phi \sin \theta \\
\sin \theta \sin \phi & -\sin \theta \cos \phi & \cos \theta 
\end{pmatrix}, \]

has two large angles, one of which may be maximal. The size of the third is limited by the CHOOZ\textsuperscript{8} reactor experiment

\[ \sin^2 2\theta_\odot > 0.85; \quad 0.30 < \tan^2 \phi_\odot < 0.65; \quad |\epsilon|^2 < 0.005. \]

It is theoretically intriguing to produce a $3 \times 3$ matrix with such angles, and it may be a powerful hint towards the resolution of the riddle of flavor. Oscillation experiments determine only mass differences

\[ \Delta m^2_\odot = |m_{\nu_2}^2 - m_{\nu_1}^2| \sim 7 \times 10^{-5} \text{ eV}^2 \]
\[ \Delta m^2_\odot = |m_{\nu_3}^2 - m_{\nu_1}^2| \sim 3 \times 10^{-3} \text{ eV}^2, \]

but WMAP\textsuperscript{12} is the only experiment to set a limit on the absolute value of their masses

\[ \sum m_{\nu_i} < .71 \text{ eV} \]

These allow for three three possible mass patterns\textsuperscript{13}:

- Hierarchical with $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$
- Inverted with $m_{\nu_3} < m_{\nu_2} \simeq m_{\nu_1}$
- Hyperfine with $m_{\nu_1} \simeq m_{\nu_2} \simeq m_{\nu_3}$

In view of the strong theoretical reasons for linking quarks and leptons, the difference of their mixings is quite striking. In the context of the seesaw\textsuperscript{14} mechanism, these have possibly fundamental implications as to the structure of the electroweak singlet masses of the right-handed neutrinos. In the following we explore the causes of these differences, and offer some theoretical prediction for the size of the CHOOZ angle.

3 CKM and MNS Mixings

The $\Delta I_w = 1/2$ Higgs doublet breaking of electroweak symmetry yields the quark Yukawa Matrices
\[ M^{(2/3)} = U_{2/3} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} V^{\dagger}_{2/3} \]

\[ M^{(-1/3)} = U_{-1/3} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V^{\dagger}_{-1/3} \]

from which the observable quark mixing matrix is deduced.

\[ U_{CKM} = U_{2/3}^{\dagger} U_{-1/3} \]

Experimentally, \( U_{CKM} \) is nearly equal to the unit matrix, up to small powers of \( \lambda \), the Cabibbo angle, allowing for its Wolfenstein expansion which uses the unit matrix as a starting point. The lesson here is that family mixing is roughly the same for charge \( 2/3 \) and \( -1/3 \) quarks. The charged leptons Yukawa Matrix

\[ M^{(-1)} = U_{-1} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} V^{\dagger}_{-1} \]

has hierarchical mass eigenvalues, as in the quark sector. To explain neutrino masses, it is simplest to add one right-handed neutrino per family. This implies a neutral leptons Yukawa Matrix

\[ M^{(0)}_{\text{Dirac}} = U_0 D_0 V_0^{\dagger} = U_0 \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_0^{\dagger} \]

and a lepton mixing matrix that is similar to the CKM matrix

\[ U_{\text{MNS}} = U_{-1} U_0 \]

Can such a matrix be so different from its sister matrix in the quark sector? Most right-thinking theorists think that the right-handed neutrino masses

\[ M^{(0)}_{\text{Majorana}} \sim \Delta L = 0 \]

radically change the landscape: unlike those of quarks and charged leptons, the right-handed neutrinos masses are unconstrained by electroweak quantum numbers. They only have lepton number, and since there are very good limits on lepton number conservation, one expects these masses to be very large. This leads to the seesaw mechanism, according to which the neutrino mass matrix is

\[ M^{(0)}_{\text{Seesaw}} = M^{(0)}_{\text{Dirac}} \frac{1}{M^{(0)}_{\text{Majorana}}} M^{(0)\dagger}_{\text{Dirac}}. \]

We can rewrite it as
\[ M_{\text{Seesaw}}^{(0)} = U_0 \mathcal{C} U_0^\dagger, \]

where we have introduced the Central Matrix

\[ \mathcal{C} = D_0 V_0^\dagger \frac{1}{M_{\text{Majorana}}^{(0)}} V_0^\dagger D_0. \]

It is diagonalized by a unitary matrix \( \mathcal{F} \)

\[ \mathcal{C} = \mathcal{F} D_\nu \mathcal{F}^t, \]

while the physical neutrino masses are contained in the diagonal \( D_\nu \). This alters the observable MNS lepton mixing matrix to

\[ U_{\text{MNS}} = U_{\nu}^\dagger U_0 \mathcal{F}. \]

The seesaw adds to the MNS matrix the extra unitary matrix \( \mathcal{F} \). This suggests we put models of neutrino masses in three classes:

- Models of type 0 for which \( \mathcal{F} \) contains no large angle,
- Models of type I for which only one large angle is in \( \mathcal{F} \),
- Models of type II, for which both large angles reside in \( \mathcal{F} \).

Type I models seem more generic to us than type II, since it is natural to have matrices with one, three or no large mixing angles.

### 4 A Pinch of Grand Unification

If we want to relate the quark and lepton observables, we need to add some theoretical prejudice. Such is not hard to find, as it is made obvious both by anomaly cancellation and the quantum number structures: grand unification.

While there is a great deal of uncertainty as to the nature of the grand unified group and the mechanism by which its large symmetries are broken, there is no doubt that such a group is to be found (cosmologically speaking) in our distant past. It is therefore important to study the patterns implied.

Grand Unification relates \( \Delta I_w = 1/2 \) quark and lepton Yukawa matrices. In the simplest case, \( SU(5) \), we have

\[ M^{(-1/3)} \sim M^{(−1)} t, \]

where \( t \) is the transpose, while in \( SO(10) \) which naturally includes right-handed neutrinos,

\[ M^{(2/3)} \sim M^{(0)}_{\text{Dirac}}. \]

Assuming only these two simple patterns, we infer
\[ \mathcal{M}^{(-1/3)} = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^2 & a & b \end{pmatrix}, \]

and

\[ \mathcal{M}^{(-1/3)} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \]

\[ \mathcal{M}^{(2/3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_t \end{pmatrix}, \]

\[ \mathcal{M}^{(-1/3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a & b \end{pmatrix}, \]

and
\[ U_{\text{CKM}} = U_{2/3}^1 U_{-1/3} = 1 \]

Then \( SU(5) \) suggest a non-symmetric charge \(-1\) matrix as well,

\[ M^{(-1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & b \end{pmatrix}, \]

so that

\[ U_{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}, \quad \tan \theta = \frac{a}{b}, \]

contains one unsuppressed mixing angle.

\[ U_{\text{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} F. \]

It is natural from this point of view\cite{17, 18} to expect unsuppressed atmospheric neutrino mixing, although this approach does not fix the value of the angle.

The relation between the CKM and MNS matrices suggest that the Cabibbo angle plays an important role and should not be neglected. In particular it could very well be that the CHOOZ angle is solely a Cabibbo effect. In type I Models, this hypothesis suggests that

\[ U_{\text{MNS}} = \begin{pmatrix} 1 & \lambda^{\alpha} & \lambda^{\beta} \\ \lambda^{\alpha} & \cos \theta & \sin \theta \\ \lambda^{\beta} & -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi & \lambda^{\gamma} \\ -\sin \phi & \cos \phi & \lambda^{\delta} \\ \lambda^{\gamma} & \lambda^{\delta} & 1 \end{pmatrix}, \]

where the exponents are completely unknown. As a result the CHOOZ angle is given by \( \lambda \) to an unknown power with an unknown prefactor, and our hypothesis is not very useful.

However, in type II models, where the \( \Delta I = 1/2 \) Yukawa matrices are family symmetric, we find a more definitive prediction

\[ U_{\text{MNS}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi & \lambda^{\gamma} \\ -\cos \theta & \sin \phi & \cos \phi & \cos \theta \\ \sin \theta & \sin \phi & -\sin \theta & \cos \phi & \cos \theta \end{pmatrix}, \]

so that plausibly

\[ \text{CHOOZ angle} \sim \lambda \sin \theta \sim \frac{\lambda}{\sqrt{2}}, \]

assuming \( \gamma < 2 \). This conclusion is particularly exciting as it can be tested in the foreseeable future; if correct it will also open the way to observable CP violation in neutrino physics!
We end this section with two additional remarks: one is that it will be desirable to fix a Wolfenstein parametrization for the MNS matrix. The second is that there is another class of models where the charge $-1/3$ Yukawa matrix is not family-symmetric. These are Family Cloning models where each family starts with its own gauge group. A tri-chiral order parameter naturally explains

$$SU(3)_1 \times SU(3)_2 \times SU(3)_3 \rightarrow SU(3)_{1+2+3},$$

but falls short in unifying the three weak isospins into one

$$SU(2)_1 \times SU(2)_2 \times SU(2)_3 \rightarrow SU(2)_{1+2} \times SU(2)_3,$$

resulting in a Standard Model with lopsided gauge symmetry

$$SU(2)_{1+2} \times SU(2)_3 \times SU(3)_{1+2+3},$$

yielding asymmetric Yukawa matrices and one large angle in $U_{-1}$.

### 5 Large Angles and $F$

Finally we would like to discuss the theoretical implications of large angles in $F$. $\Lambda(3 \times 3)$ matrix generically contains one or three large angles, but not two. From this point of view type I models are more desirable.

As in our earlier work, consider a simplified case with two families. Recall that

$$C(C \equiv D_0 V_0^\dagger \frac{1}{M^{(0)}_{\text{Majorana}}} V_0^* D_0),$$

and that the $\Delta I_w = 1/2$ Neutral Dirac Mass is hierarchical:

$$D_0 = m \begin{pmatrix} a \lambda^\alpha & 0 \\ 0 & 1 \end{pmatrix}.$$

If we call $M_1$ and $M_2$ the right-handed Majorana masses, simple algebra yields the central matrix

$$C = \begin{pmatrix} \left(\frac{c^2}{M_1} + \frac{s^2}{M_2}\right) a^2 \lambda^{2\alpha} & \left(\frac{c s}{M_1} - \frac{c s}{M_2}\right) a \lambda^\alpha \\ \left(\frac{c s}{M_1} - \frac{c s}{M_2}\right) a \lambda^\alpha & \left(\frac{s^2}{M_1} + \frac{c^2}{M_2}\right) a \lambda^\alpha \end{pmatrix},$$

where $s,c$ are the sine and cosine of a mixing angle.

It is diagonalized by a large mixing angle in one of two cases:

- $C_{11} \sim C_{22} \sim C_{12}$. This implies
  
  $$s \sim b \lambda^\alpha, \quad c \sim 1,$$

  and
\[
\frac{M_1}{M_2} \sim \lambda^{2\beta}.
\]

Hence the Majorana masses must be hierarchical: there is a correlated hierarchy between the \( \Delta I_w = 1/2 \) and \( \Delta I_w = 0 \) sectors. Then

\[
\mathcal{C} = \lambda^{2\alpha} \frac{m^2}{M_1} \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}, \quad \beta > \alpha
\]

\[\bullet \mathcal{C}_{11}, \mathcal{C}_{22} \prec \mathcal{C}_{12}. \] This is the level crossing case. We obtain

\[
\lambda^\alpha \prec s \preceq \lambda^{\alpha - \beta}
\]

so that

\[
\mathcal{C} = \frac{\lambda^\alpha m^2}{\sqrt{-M_1 M_2}} \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix},
\]

naturally leading to maximal mixing. Could the right-handed neutrinos of two families be Dirac partners?

## 6 Conclusions

The neutrino data set presents new theoretical challenges and hopefully hints towards the resolution of the family puzzle. It points to family mixing at very high scales and a hierarchy among the right-handed neutrinos, in accordance with the grand-unified paradigm. This is a welcome feature in terms of modern theories of leptogenesis.

We close with some suggestions to model builders. Go directly to the Planck scale (or just below). There you will find branes awaiting you with Weyl fermions we can right-handed neutrinos. Their interactions and masses are key to understanding the family riddles and symmetries, and even better laboratory neutrino physics opens a window to their masses. Only then should you worry about the Standard Model; and for that you can always use \( SO(10) \) or any such model\([21, 22]\).

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