Three Point Functions of Chiral Primary Operators 
in $d = 3, \mathcal{N} = 8$ and $d = 6, \mathcal{N} = (2, 0)$ SCFT 
at Large $N$

Fiorenzo Bastianelli and Roberto Zucchini

Dipartimento di Fisica, Università degli Studi di Bologna

via Irnerio 46, I-40126 Bologna, Italy

and

I. N. F. N., Sezione di Bologna

Abstract

We use the AdS/CFT correspondence to calculate three point functions of chiral primary operators at large $N$ in $d = 3, \mathcal{N} = 8$ and $d = 6, \mathcal{N} = (2, 0)$ superconformal field theories. These theories are related to the infrared fixed points of world-volume descriptions of $N$ coincident M2 and M5 branes, respectively.

The computation can be generalized by employing a gravitational action in arbitrary dimensions $D$, coupled to a $(p + 1)$-form and appropriately compactified on $AdS_{(D-p-2)} \times S^{(p+2)}$. We note a surprising coincidence: this generalized model reproduces for $D = 10$, $p = 3$ the three point functions of $d = 4, \mathcal{N} = 4$ SYM chiral primary operators at large $N$. 
The AdS/CFT correspondence [1][2][3] has been quite useful in learning various properties of strongly coupled quantum fields theories at large $N$ (for reviews see [4][5].) In particular, it has been employed in [6] to compute two and three point functions at large $N$ for chiral primary operators (CPO) of $d = 4$, $N = 4$ super Yang-Mills (SYM) theory by making use of type IIB supergravity compactified on $AdS_5 \times S^5$.

This interesting result has been generalized in [7] to the case of the $d = 6$, $N = (2,0)$ superconformal field theory (SCFT) related to the infrared fixed point of the world-volume description of $N$ coincident M5 branes. This is a remarkable theory which seems to require a generalization of non-abelian gauge theories, where the gauge potential should be though of a two-form with self-dual field strength. A lagrangian formulation of this theory is not known, yet, but there exists a DLCQ Matrix description as quantum mechanics on the moduli space of instantons [8]. The AdS/CFT conjecture has provided new clues on this somewhat unaccessible theory.

Both theories described above have maximal supersymmetry, corresponding to the existence of 16 real supercharges (for a review see [3] and references therein). In this letter we complete the program of computing two and three point functions of chiral primary operators at large $N$ for the remaining known maximally supersymmetric conformal field theory: the $d = 3$, $\mathcal{N} = 8$ SCFT. This theory can be realized as the infrared fixed point of $N$ coincident M2 branes. It is a quite mysterious theory without an explicitly known lagrangian realization, though it can be seen to describe the strongly coupled infrared fixed point of $d = 3$, $\mathcal{N} = 8$ SYM. A kind of DLCQ Matrix description is also unknown, and the AdS/CFT conjecture offers a unique chance to learn more about its properties. With this aim in mind, we set out to compute the three point functions of chiral primary operators.

For this purpose the AdS/CFT correspondence instruct us to: i) identify the Kaluza-Klein tower of scalar excitations with lowest mass arising from 11D supergravity on $AdS_4 \times S^7$, ii) compute the quadratic and cubic part of their action. This will produce the correlation functions of interest by using the AdS/CFT relations. The conformal dimensions of the chiral primary operators have already been identified by using Maldacena’s conjecture in [10].

We start from the bosonic part of the 11D supergravity action which is given by

$$S = \frac{1}{2\kappa^2} \int d^{11}x \left[ \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{48} F^2 \right) + \frac{1}{3\sqrt{2}} A \wedge F \wedge F \right]$$

(1)

where $F = dA_3$ is the field strength of the 3-form $A_{mnp}$. One can check that on the $AdS_4 \times S^7$ background the Chern-Simon term is never excited by the quadratic and cubic
fluctuations of the scalar field of interest, which we denote by $s$. Thus, for our purposes, the Chern-Simon term can be dropped from the beginning. At this stage we also notice that a more convenient dual formulation with a 6-form $A_6$, instead of the original 3-form $A_3$, can be used. As shown in [11], this has the advantage that the scalar field $s$ is suitably described off-shell by a linear combination of two scalars (namely, the deformation of the trace of the metric on the sphere and a scalar deformation of the 6-form potential on the sphere; in the original 3-form formulation this second scalar excitation would have been described off-shell by a vector field).

The proposed simplified model captures all of the physical information we need to extract from 11D supergravity. It is also evident that one can generalize it by considering a gravitational action in arbitrary dimensions $D$ with metric $g_{mn}$ coupled to a $(p+1)$-form $A_{m_1\ldots m_{p+1}}$

$$S = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2(p+2)!} F^2 \right],$$

(2)

where $F = dA_{p+1}$ is the $p + 2$ form describing the field strength. For $D = 11$, $p = 5$ this gives the dual formulation described above, and constitutes the main object of our analysis. For $D = 11$, $p = 2$ it gives the usual description of 11D supergravity with fermions and Chern-Simon term set to zero.

The equation of motion obtained from eq. (2) are

$$R_{mn} = \frac{1}{(p+1)!} F_{m\ldots F_{n\ldots}} - \frac{(p+1)}{(p+2)!(D-2)} g_{mn} F^2,$$

$$\nabla^n F_{nm_1\ldots m_{p+1}} = 0.$$  

(3)

They are easily seen to admit the $AdS_{(D-p-2)} \times S^{(p+2)}$ solution induced by a Freund-Rubin type of ansatz for the $(p+2)$-rank antisymmetric tensor field $F$

$$F_{\alpha_1\ldots\alpha_{p+2}} = e \epsilon_{\alpha_1\ldots\alpha_{p+2}}, \quad \text{other } F = 0,$$

$$AdS_{(D-p-2)}: \quad R_{\mu\nu\lambda\rho} = -a_1 (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}), \quad a_1 = \frac{(p+1)}{(D-2)(D-p-3)} \bar{e}^2,$$

$$S^{(p+2)}: \quad R_{\alpha\beta\gamma\delta} = a_2 (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}), \quad a_2 = \frac{(D-p-3)}{(p+1)(D-2)} \bar{e}^2,$$

(4)

where $\epsilon_{\alpha_1\ldots\alpha_{p+2}}$ denotes the standard volume form on the sphere and $e$ is an arbitrary mass scale parameterizing the compactification. Note the splitting of $D$-dimensional coordinates $x^m = (z^\mu, y^\alpha) \sim (AdS_{(D-p-2)}, S^{(p+2)})$. Also, it will be convenient to use the notation $\bar{e}^2 = a_2$, since $\bar{e}$ gives directly the inverse radius of the sphere. The $(p+1)$-form $A_{p+1}$
couples naturally to "electric" $p$-branes, though the solution just described is generated by "magnetic" $(D - p - 4)$-brane sources. In fact, the $F_{p+2}$ field strength has non-trivial flux on the sphere $S^{(p+2)}$ surrounding the $(D - p - 4)$-dimensional source.

Bulk fields of interest are fluctuations about the $AdS_{(D−p−2)} × S^{(p+2)}$ background. For our needs, we define them as follows (now we denote field variables as $g_{mn}, A_{m_1⋯m_{p+1}}, F_{m_1⋯m_{p+2}}$ and their background values as $\bar{g}_{mn}, \bar{A}_{m_1⋯m_{p+1}}, \bar{F}_{m_1⋯m_{p+2}}$):

$$h_{mn} = g_{mn} - \bar{g}_{mn}, \quad h_2 = \bar{g}^{\alpha\beta}h_{\alpha\beta}, \quad H_{\mu\nu} = h_{\mu\nu} + \frac{1}{(D - p - 4)}\bar{g}_{\mu\nu}h_2,$$

$$\delta F_{m_1⋯m_{p+2}} = F_{m_1⋯m_{p+2}} - \bar{F}_{m_1⋯m_{p+2}}, \quad \delta F_{m_1m_2⋯m_{p+2}} = (p + 2)\bar{\nabla}[m_1a_{m_2⋯m_{p+2}}], \quad (5)$$

$$ea_{\alpha_1⋯\alpha_{p+1}} = -\epsilon_\beta e_{\alpha_1⋯\alpha_{p+1}}\bar{\nabla}^\beta b.$$ 

In the following we will drop bars on background fields, since no confusion can arise. Last equation defines the scalar field $b$. It is the unique physical fluctuation contained in $a_{\alpha_1⋯\alpha_{p+1}}$, since we have chosen to fix reparametrization and $(p + 1)$-form gauge invariance on the sphere by imposing the gauge choices $\nabla^\alpha h_{(\alpha\beta)} = \nabla^\alpha h_{\alpha\mu} = \nabla^\alpha a_{\alpha m_2⋯m_{p+1}} = 0$, where $h_{(\alpha\beta)}$ denotes the traceless part of $h_{\alpha\beta}$. We use the notation $\Box_1 \equiv \nabla_\mu \nabla^\mu$ and $\Box_2 \equiv \nabla_\alpha \nabla^\alpha$ to denote the d’alembertian on $AdS$ and the laplacian on $S$, respectively (subscripts 1 and 2 refer to the first and second factor of the $AdS \times S$ manifold).

Using this notation we find from eqs. (3) the following linearized coupled equations for the $(h_2, b)$ scalars

$$\left(\Box_1 + \Box_2 - \frac{2(p + 1)(D - p - 3)}{(D - 2)}e^2\right)h_2 - \frac{4(p + 2)(D - p - 3)}{(D - 2)}\Box_2 b = 0,$$

$$\left(\Box_1 + \Box_2\right)b + \frac{(p + 1)}{(p + 2)}e^2h_2 = 0. \quad (6)$$

These equations can be easily diagonalized. Introducing a complete orthonormal set of scalar spherical harmonics $Y^I$ satisfying $\Box_2 Y^I = -e^2 k(k + p + 1)Y^I$, where $k$ is a non-negative integer depending on the index $I$, and the harmonic expansions

$$h_2(x) = \sum_I h_2^I(z)Y^I(y), \quad b(x) = \sum_I b^I(z)Y^I(y), \quad (7)$$

we find that the linear combinations

$$s^I = \frac{1}{(2k + p + 1)}\left(\frac{1}{2(p + 2)(D - p - 3)}h_2^I + \frac{(k + p + 1)}{(p + 1)(D - 2)}b^I\right),$$

$$t^I = \frac{1}{(2k + p + 1)}\left(\frac{1}{2(p + 2)(D - p - 3)}h_2^I - \frac{k}{(p + 1)(D - 2)}b^I\right) \quad (8)$$
obey the equations of motion

\[ \Box_1 s^I = m_{sI}^2 s^I, \]
\[ \Box_1 t^I = m_{tI}^2 t^I, \]

with masses given by

\[ m_{sI}^2 = \bar{e}^2 k(k - p - 1), \]
\[ m_{tI}^2 = \bar{e}^2 (k + p + 1)(k + 2p + 2). \]

We have chosen a suitable normalization of the diagonal fields such that the inverse relations are simple:

\[ h_2^I = 2(p + 2)(D - p - 3)(ks^I + (k + p + 1)t^I), \]
\[ b^I = (p + 1)(D - 2)(s^I - t^I). \]

As a check, we notice that for this particular set of scalar fields we have reproduced the masses worked out in refs. [12] for the AdS$_4 \times S^7$ compactification, and in ref. [13] for the AdS$_7 \times S^4$ one. The fields $s^I$ describe the Kaluza-Klein tower of AdS scalars with lowest mass. To linear order they correspond to the chiral primary operators in SCFT, and they will be the focus of our analysis throughout the rest of this paper. The scalars $t^I$, on the other hand, correspond to descendents of these chiral primary operators, and for the present purposes they can be set to zero.

Having obtained some insights into the diagonal fields $s^I$, we now derive their quadratic off-shell action starting from eq. (2). Proceeding as described in refs. [11][14], one can decouple the $(h_2, b)$ sector from $H_{\mu\nu}$ by parameterizing the latter as

\[ H_{\mu\nu} = \phi_{\mu\nu} + \nabla_\mu \nabla_\nu \zeta + \frac{1}{(D - p - 2)} g_{\mu\nu} \eta. \]

Requiring a decoupling at the quadratic level between $(h_2, b)$ and $\phi_{\mu\nu}$ fixes $\zeta$ and $\eta$ appropriately. At higher orders the decoupling can be dealt with field redefinitions. Thus, one can immediately proceed to identify the quadratic $s^I$ off-shell action together with their cubic self-couplings. A laborious calculation produces the following action (with $d = D - p - 3$)

\[ S = \frac{n}{2\kappa^2} \int_{AdS} d^{d+1}z \sqrt{-g_1} \left[ \frac{1}{2} \sum_I A_1 s^I (\Box_1 - m_{I}^2) s^I + \sum_{l_1l_2l_3} \frac{1}{3} G_{l_1l_2l_3} s^{l_1} s^{l_2} s^{l_3} \right]. \]
where
\[ n = (D - 2)(p + 1)(D - p - 3)^2 \]
\[ A_I = \frac{k(k-1)(2k+p+1)}{(D-p-3)k + (p+1)} z_I \]
\[ m_I^2 = e^2 k(k-p-1) \]

and cubic couplings given by
\[ G_{I_1I_2I_3} = \frac{e^2(D-p-3)^2}{2(p+1)} \left( \prod_{i=1}^{3} \frac{\alpha_i}{(D-p-3)k_i + p+1} \right) (4\alpha^2 - (p+1)^2) \]
\[ \left[ (D-2)(2\alpha)^2 - (p+1)(D-2p-4)2\alpha - 2(p+1)^2 + 2\Theta(k_1k_2 + k_2k_3 + k_3k_1) \right] \]
\[ \times \left( (D-2)2\alpha - (D-2p-4)(p+1) \right) - 4\Theta[1-p(D-p-4)]k_1k_2k_3 a_{I_1I_2I_3} \langle C^{I_1}C^{I_2}C^{I_3} \rangle \]
with
\[ \Theta = (p+1)(D-p-3) - 2(D-2). \]

The modes with \( k = 0,1 \) decouple from the action and correspond to some global gauge degree of freedom. In obtaining these results we have used various definitions for the spherical harmonics, similar to those employed in [6]. Namely, we describe the \( n \)-sphere of radius \( \rho = e^{-1} \) by \( S^n \equiv \{ \vec{y}^2 = \rho^2 \mid \vec{y} \in \mathbb{R}^{n+1} \} \), and use scalar spherical harmonics defined by \( Y^I = C^I_{i_1...i_k} x^{i_1}...x^{i_k} \), where the coordinates \( x^i = \vec{y}^i \) live on the unit sphere and the tensors \( C^I_{i_1...i_k} \) form an orthonormal basis for completely symmetric traceless tensors. Orthonormality reads as \( C^I_{i_1...i_k} C^J_{i_1...i_k} = \delta^I_J \), while \( \langle C^{I_1}C^{I_2}C^{I_3} \rangle \) denotes the unique \( SO(n+1) \) scalar contraction of three tensors \( C^I_{i_1...i_k} \). We need to compute
\[ \int_{S^n} d^n y \sqrt{g_2} Y^I Y^J = z_I \delta^{IJ}, \]
\[ \int_{S^n} d^n y \sqrt{g_2} Y^{I_1} Y^{I_2} Y^{I_3} = a_{I_1I_2I_3} \langle C^{I_1}C^{I_2}C^{I_3} \rangle, \]
and obtain
\[ z_I = \omega_n \frac{(n-1)!k!}{(2k+n-1)!!} \]
\[ a_{I_1I_2I_3} = \omega_n \frac{(n-1)!}{(2\alpha+n-1)!!} \frac{k_1!k_2!k_3!}{\alpha_1!\alpha_2!\alpha_3!} \]
where \( \alpha_1 = \frac{1}{2}(k_2 + k_3 - k_1) \), and so on in a cyclically symmetric fashion, \( \alpha = \alpha_1 + \alpha_2 + \alpha_3 \) and \( \omega_n \) the volume of the sphere

\[
\omega_n = \int_{S^n} d^n y \sqrt{g_2} = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})} \left( \frac{1}{e} \right)^n .
\]  

(19)

Returning to the values of the cubic couplings \( G_{I_1I_2I_3} \) given in eq. (15), we note that they simplify for the \( AdS_{4,7,5} \times S^{7,4,5} \) cases, since then the coefficient \( \Theta = 0 \). In particular, defining \( G_{I_1I_2I_3} = \bar{G}_{I_1I_2I_3} a_{I_1I_2I_3} \langle C^{I_1} C^{I_2} C^{I_3} \rangle \), we obtain

\[
\begin{align*}
AdS_4 \times S^7 : & & \bar{G}_{I_1I_2I_3} = 24\bar{\epsilon}^2 \left( \prod_{i=1}^3 \frac{\alpha_i}{k_i + 2} \right) (\alpha + 2)(\alpha^2 - 1)(\alpha^2 - 9) \\
AdS_7 \times S^4 : & & \bar{G}_{I_1I_2I_3} = 3\bar{\epsilon}^2 \left( \prod_{i=1}^3 \frac{\alpha_i}{2k_i + 1} \right) (2\alpha - 2)(4\alpha^2 - 1)(4\alpha^2 - 9) \\
AdS_5 \times S^5 : & & \bar{G}_{I_1I_2I_3} = 16\bar{\epsilon}^2 \left( \prod_{i=1}^3 \frac{\alpha_i}{k_i + 1} \right) \alpha(\alpha^2 - 1)(\alpha^2 - 4).
\end{align*}
\]  

We are now ready to compute two and three point functions in the SCFTs using the \( AdS/CFT \) correspondence. The general formulas derived in [15] and adapted to the action (13) give (with \( d = D - p - 3 \) and \( AdS \) radius set to 1)

\[
\langle O^I(x)O^J(y) \rangle = \frac{n A_I}{2\kappa^2} \frac{2\Delta - d}{\pi^d} \frac{\Gamma(\Delta)}{\Gamma(\Delta - \frac{d}{2})} \frac{(w^I)^2\delta^{IJ}}{|x-y|^{2\Delta}}
\]  

(21)

and

\[
\langle O^{I_1}(x)O^{I_2}(y)O^{I_3}(z) \rangle = \frac{R_{I_1I_2I_3}}{|x-y|^{\Delta_1+\Delta_2-\Delta_3}|y-z|^{\Delta_2+\Delta_3-\Delta_1}|z-x|^{\Delta_3+\Delta_1-\Delta_2}},
\]  

(22)

where

\[
R_{I_1I_2I_3} = \left( \frac{n}{2\kappa^2} \right)^{\frac{1}{2}} \frac{1}{\pi^{\frac{d}{2}}} \frac{\Gamma(\frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3))\Gamma(\frac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1))\Gamma(\frac{1}{2}(\Delta_3 + \Delta_1 - \Delta_2))}{\Gamma(\Delta_1 - \frac{d}{2})\Gamma(\Delta_2 - \frac{d}{2})\Gamma(\Delta_3 - \frac{d}{2})} \frac{\Gamma(\frac{1}{2}(\Delta_1 + \Delta_2 + \Delta_3 - d))G_{I_1I_2I_3} w^{I_1}w^{I_2}w^{I_3}}{\delta^{IJ}}
\]  

(23)

The factors \( w^I \) parameterize unknown proportionality constants which relate the fields \( s^I \) to the sources of the operators \( O^I \), as in [3]. These factors can presumably be fixed by carefully studying absorption processes on the branes [16]. However, for the present purposes we follow ref. [3] and fix them to normalize the two point functions as

\[
\langle O^I(x)O^J(y) \rangle = \frac{\delta^{IJ}}{|x-y|^{2\Delta}}.
\]  

(24)
We first study the case of $AdS_4 \times S^7$, which corresponds to SCFT$_3$ at large $N$ and constitutes our main interest. To match the $AdS_4 \times S^7$ solution to the near horizon geometry of $N$ M2 branes we tune the gravitational coupling $\frac{1}{2\pi^2} = \frac{N^2}{2^{16/3}\pi^2}$. Also, to employ the formulas given above, we need to fix the $AdS$ radius to one by setting the mass parameter $\bar{e} = \frac{1}{2}$. The masses of the scalars $s^I$ fix the conformal dimensions of the operators $O^I$ to be $\Delta_I = \frac{k}{2}$. We obtain

$$R_{I_1I_2I_3} = \frac{\pi}{N^2} \frac{2^{-\alpha - \frac{1}{2}}}{\Gamma(\frac{\alpha}{2} + 1)} \left( \prod_{i=1}^{3} \frac{\Gamma(\alpha_i + 2)}{\Gamma(2\alpha_i + 1)} \right) \langle O^{I_1} O^{I_2} O^{I_3} \rangle. \quad (25)$$

This is the $AdS$/CFT prediction for the normalized CPO three point functions in SCFT$_3$ at large $N$.

Our general formulation can also produce the CPO three point functions in SCFT$_6$. In fact, the Chern-Simon terms doesn’t give any contribution in this case as well, and it can be neglected from the start. Thus, we turn immediately to the $AdS_7 \times S^4$ case and set the $AdS_7$ radius to one by fixing $\frac{1}{2\pi^2} = \frac{4N^3}{\pi^2}$ and $\bar{e} = 2$. The masses of the scalars $s^I$ fix the conformal dimensions of the operators $O^I$ to be $\Delta_I = 2k$ so that

$$R_{I_1I_2I_3} = \frac{2^{2\alpha - 2}}{(\pi N)^2} \frac{\Gamma(\alpha)}{\prod_{i=1}^{3} \frac{\Gamma(\alpha_i + 1)}{\sqrt{\Gamma(2\alpha_i - 1)}}} \langle O^{I_1} O^{I_2} O^{I_3} \rangle. \quad (26)$$

This essentially confirms the results already obtained in [7] (apart from a factor $\pi^{3/2}2^{\frac{\alpha + 1}{2}}$ present there for which we cannot find agreement). The three point couplings for the $s^I$ scalars at level $k = 2$ (and the corresponding three point functions) could in principle be extracted also from ref. [17], where the complete consistent Kaluza-Klein reduction of 11D supergravity on $AdS_7 \times S^4$ has been worked out. However, it is not straightforward to extract the cubic couplings from the lagrangian given there, since one should first perform some gauge-fixing to identify the physical scalars $s^I$.

Finally, we consider our model on $AdS_5 \times S^5$. Using the $AdS$/CFT correspondence we can obtain correlation functions for the operators $O^I$ corresponding to the scalars $s^I$. The result will not be obviously related to $d = 4$, $\mathcal{N} = 4$ SYM theory, since in the latter case the sources for the dual gravitational model must be self-dual 3-branes, i.e. branes with both electric and magnetic charges. Nevertheless, to satisfy our curiosity we compute the correlation functions. We set the $AdS$ radius to 1 by fixing $\bar{e} = 1$. We also set the gravitational coupling to $\frac{1}{2\pi^2} = \frac{N^2}{4\pi^2}$, as would have been appropriate to the near horizon
geometry of $N$ D3 branes. This time the $s^I$ masses fix the conformal dimensions of the operators to be $\Delta_I = k$ and we get

$$R_{I_1I_2I_3} = \frac{1}{N} \sqrt{k_1 k_2 k_3} \langle C^{I_1} C^{I_2} C^{I_3} \rangle.$$  \hspace{1cm} (27)

Surprisingly, this reproduces the $d = 4, \mathcal{N} = 4$ SYM results at large $N$ obtained in [8]. We have no immediate explanation for this coincidence.

To conclude, we have used the $AdS$/CFT conjecture to predict the normalized three point correlation functions of the chiral primary operators in SCFT$_3$ and SCFT$_6$. The first result is new, while the latter essentially confirms the findings of ref. [7]. Adapting our calculation to the $AdS_5 \times S^5$ case, we have noticed that our model reproduces the normalized three point functions at large $N$ for the $d = 4, \mathcal{N} = 4$ SYM theory.

We plan to present more details of our computation in a future publication, possibly producing additional correlation functions involving scalar operators related to the $t^I$ fields.
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