Numerical research method of an impact device model

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Abstract. The article describes the numerical and analytical research methods of an impact device model. The hammer with the tool is considered as rods of uniform cross section, connected by elastic and dissipative elements. The model also takes into account the elastic and viscous resistance of the working medium. The process of interaction in the system of hammer – tool – working medium is described by wave equations connected to initial and boundary conditions. A finite-difference method is used to solve initial-boundary value problems. A mixed difference scheme with weight coefficient is applied. The choice of the optimal parameters of the difference scheme is carried out using model problems whose solutions are found by the Fourier method. The difference problem is solved by the succession-sweep method, which is adapted to two connected equation systems. There are oscillations of low and high frequencies in the solutions of initial-boundary value problems. Coarse and fine grids are used to register these oscillations. The algorithm of numerical and analytical methods is realized in the Mathcad system.

1. Introduction

The rod model researches of various impact devices were conducted in studies [1-7]. The Fourier and d'Alembert methods were used in this case. These methods can get a solution if only there is a spring linkage. The presence of more complex linkage, for example, dissipative or spring with discontinuous characteristics, complicates significantly the application of analytical methods. A numerical method is given for studying impact device model of rod type with the hammer interaction with the tool and implement of the tool into the working medium. The mathematical model is represented by a wave equations system in partial derivatives. Hard and dissipative links are modelled by boundary conditions. Model problems are given to ensure the results reliability. The analytical solutions are found by the Fourier method [8-10]. The solutions obtained for initial-boundary value problems are compared with the results of applying a mixed difference scheme for certain limiting values of the main parameters. The difference scheme parameters are chosen from the condition of the minimum relative error compared with the Fourier method for the linear problem.

2. Materials and methods

2.1. Mathematical model

The diagram of the rod impact device and the design model are shown in figure 1. The hydraulic system provides a pulse effect on the hammer (1) and tool (2).
Here $\rho_i$ – the density of the material, $E_i$ – elasticity modulus, $S_i$ – the cross sectional area $i$-rod, $i=1,2$. Suppose $U(t,x)$ – the cross-section displacement of the rod $x$ (1) from the equilibrium; $V(t,y)$ – the cross-section displacement of the rod $y$ (2) from the equilibrium, $t$ – the time. Let us write the oscillation equations for rods:

$$\frac{\partial^2 U(t,x)}{\partial t^2} = a_i^2 \frac{\partial^2 U(t,x)}{\partial x^2}, \quad 0 \leq x \leq L_1,$$

$$\frac{\partial^2 V(t,y)}{\partial t^2} = a_i^2 \frac{\partial^2 V(t,y)}{\partial y^2}, \quad 0 \leq y \leq L_2, \quad t \in [0,T], \quad i=1,2.$$  

The initial conditions are that the first rod, the hammer moves at a speed $W_0$, and the second, the tool is stationary or moves at a lower speed $W_1$ (until the moment of rods contact). After the contact, their joint motion occurs:

$$U(0,x) = 0, \quad \frac{\partial U}{\partial t}(0,x) = W_0,$$

$$V(0,y) = 0, \quad \frac{\partial V}{\partial t}(0,y) = 0.$$  

The boundary condition for the left end of the first rod has the form

$$S_1 E_1 \frac{\partial U}{\partial x}(t,0) = 0.$$  

The condition (5) means that the left end of the first rod is free. At the right end of the first rod, we take into account the resistance of the spring and the dissipative element:

$$S_1 E_1 \frac{\partial U}{\partial x}(t,L_1) = K_0 (V(t,0) - U(t,L_1)) + b_0 \left( \frac{\partial V}{\partial t}(t,0) - \frac{\partial U}{\partial t}(t,L_1) \right).$$  

For the left end of the second rod, let us write the same conditions:

$$S_2 E_2 \frac{\partial V}{\partial x}(t,0) = S_1 E_1 \frac{\partial U}{\partial x}(t,L_1).$$  

Conditions on the right end of the second rod model the interaction process of the tool with the working medium:
\[
S_2 E_2 \frac{\partial V}{\partial t}(t, L_2) = -c_1 V(t, L_2) - b_1 \frac{\partial V}{\partial t}(t, L_2) .
\] (8)

2.2. Difference problem
Let us introduce a grid in each of the coordinate systems \( Ox \) and \( Oy \) with parameters:
\[
h_1 = \frac{L_1}{N_1} , \quad x_0 = 0 , \quad i = 1, 2, ..., N_1 , \quad x_i = x_{i-1} + h_1 ; \quad h_2 = \frac{L_2}{N_2} , \quad y_0 = 0 , \quad j = 1, 2, ..., N_2
\]
\[
y_j = y_{j-1} + h_2 ; \quad \tau = \frac{T}{M} , \quad n = 1, 2, ..., M , \quad t_0 = 0 , \quad t_n = t_{n-1} + \tau.
\]

We write the difference equations (a mixed two-layer difference scheme was chosen for the approximation of differential equations [11]):
\[
\frac{U_{i+1}^{n+1} - 2U_i^n + U_i^{n-1}}{\tau^2} = \sigma_i \left( \frac{U_{i+1}^{n+1} - U_{i+1}^{n-1}}{h_1^2} + (1 - \sigma_i) \frac{U_{i-1}^{n+1} - U_{i-1}^{n-1}}{h_1^2} \right), \quad i = 0, 1, ..., N_1 - 1 ,
\]
\[
\frac{V_{j+1}^{n+1} - 2V_j^n + V_j^{n-1}}{\tau^2} = \sigma_j \left( \frac{V_{j+1}^{n+1} - V_{j+1}^{n-1}}{h_2^2} + (1 - \sigma_j) \frac{V_{j-1}^{n+1} - V_{j-1}^{n-1}}{h_2^2} \right), \quad j = 0, 1, ..., N_2 - 1.
\]

The differential equations of oscillations are approximated with the second order in \( \tau \), \( h_1 \), and \( h_2 \). A two-layer scheme with weight coefficients was chosen, which ensures the stability and efficiency of the method under conditions of modeling high-frequency oscillations [11]. The weight coefficients \( \sigma, \sigma \), and \( \sigma, \), as well as the parameters \( \tau, h_1 \), and \( h_2 \), are selected according to analysis results of the model problems solutions by the Fourier method. They are selected to ensure the circuit stability and acceptable accuracy. A coarse grid is used while registering low frequencies; high frequencies require a reduction in time by an order of magnitude, so it requires the use of a fine grid.

The approximation of the initial conditions is taken in the form:
\[
U_i^0 = 0 , \quad V_j^0 = 0 ,
\]
\[
(U_i^0 - U_i^{-1}) = W_0 , \quad i = 1, 2, ..., N_1 - 1 , \quad (V_j^0 - V_j^{-1}) = 0 , \quad j = 1, 2, ..., N_2 - 1.
\]

Approximation of boundary conditions:
\[
S_1 E_1 \left( U_i^{n+1} - U_0^{n+1} \right) h_1^{-1} = 0 ,
\]
\[
S_1 E_1 \left( U_i^{n+1} - U_{N_1}^{n+1} \right) h_1^{-1} = K_0 \left( V_0^{n+1} - V_0^n \right) + b_1 \left( V_0^{n+1} - V_0^n \right) - (U_i^{n+1} - U_i^n) \right) h_1^{-1} ,
\]
\[
S_2 E_2 \left( V_i^{n+1} - V_0^{n+1} \right) h_2^{-1} = S_1 E_1 \left( U_i^{n+1} - U_i^{n+1} \right) h_1^{-1} ,
\]
\[
S_2 E_2 \left( V_i^{n+1} - V_{N_2}^{n+1} \right) h_2^{-1} = -c_1 \left( V_i^{n+1} - V_i^n \right) - b_1 \left( V_i^{n+1} - V_i^n \right) \right) h_2^{-1} , \quad n = 0, 1, 2, ..., M - 1.
\]

The initial and boundary conditions are approximated with the first order in \( \tau, h_1 \), and \( h_2 \). Increasing the approximation order leads to the algorithm complication for solving a difference problem. In this work, the problem of increasing the approximation order of initial and boundary conditions was not considered.

2.3. The algorithm for solving a difference problem
Systems of equations (9) and (10) will be reduced to the standard form for the use of the sweep method:
Let us introduce the following coefficient notation for the system (9):

\[ \begin{align*}
A_i &= -\alpha_i^2 \tau_i^2 h_i^{-2}, \\
b_i &= -1 - 2\alpha_i^2 \tau_i^2 h_i^{-2}, \\
c_i &= -\alpha_i^2 \tau_i^2 h_i^{-2}, \\
D_i &= -2U_i^n + U_i^{n-1} - (1-\sigma)\alpha_i^2 \tau_i^2 \left( U_{i+1}^n - 2U_i^n + U_{i-1}^n \right) h_i^{-2}.
\end{align*} \]  

For the system (10):

\[ \begin{align*}
a_j &= -\sigma_j \alpha_j^2 \tau_j^2 h_j^{-2}, \\
b_j &= -1 - 2\sigma_j \alpha_j^2 \tau_j^2 h_j^{-2}, \\
c_j &= -\sigma_j \alpha_j^2 \tau_j^2 h_j^{-2}, \\
d_j &= -2V_j^n + V_j^{n-1} - (1-\sigma_j)\alpha_j^2 \tau_j^2 \left( V_{j+1}^n - 2V_j^n + V_{j-1}^n \right) h_j^{-2}.
\end{align*} \]

It is difficult to apply the sweep method in a standard form for each of the two systems of algebraic equations connected by boundary conditions. The adaptation of this method for this problem is that a sequential sweep is performed for two systems of equations taking into account the approximation of the boundary conditions on each time layer.

Let us consider the sweep method for both systems and find the connection between the unknown parameters. We seek the solution of system (9) in the form

\[ U_{i-1}^{n+1} = a_{i-1} U_i^{n+1} + B_{i-1}, \quad i = 1, 2, \ldots, N_1. \]  

Then, the formulas for determining the sweep method coefficients are written in the form [11]:

\[ \alpha_i = A_i \left( B_i - C_i a_{i-1} \right)^{-1}, \quad \beta_i = (C_i a_{i-1} + D_i) \left( B_i - C_i a_{i-1} \right)^{-1}. \]  

To calculate these coefficients, it is necessary to find \( \alpha_0 \) and \( \beta_0 \).

The condition (13) and the recurrent formula (18) with \( i = l \) imply the system of equations

\[ U_0^{n+1} = U_1^{n+1}, \quad U_0^{n+1} = \alpha_0 U_1^{n+1} + \beta_0. \]

From this system, we obtain \( \alpha_0 = 1, \beta_0 = 0 \). We calculate \( \alpha_i \) and \( \beta_i \), for \( i = 1, 2, \ldots, N_1 - 1 \) using formulas (19).

It is necessary to calculate \( U_{N_1}^{n+1} \) taking into account conditions (14) and (15) to implement the reverse motion of sweep method. We seek the system solution (10) in the form

\[ V_j^{n+1} = \delta_j V_{j+1}^{n+1} + \gamma_j, \quad j = 0, 1, 2, \ldots, N_2 - 1. \]  

Let us write the system of four equations: (14), (15), (18) with \( i = N_j \) and (20) with \( j = 0 \):

\[ \begin{align*}
S_1 E_1 \left( U_{N_1}^{n+1} - U_{N_1-1}^{n+1} \right) h_1^{-1} &= S_2 E_2 \left( V_1^{n+1} - V_0^{n+1} \right) h_2^{-1}, \\
S_1 E_1 \left( U_{N_1}^{n+1} - U_{N_1-1}^{n+1} \right) h_1^{-1} &= K_0 \left( V_0^n - U_0^n \right) + b_0 \left( V_0^{n+1} - V_0^n - \left( U_{N_1}^{n+1} - U_{N_1}^n \right) \right) h_2^{-1}, \\
U_{N_1-1}^{n+1} &= \alpha_{N_1-1} U_{N_1-1}^{n+1} + \beta_{N_1-1}, \\
V_0^{n+1} &= \delta_0 V_{0}^{n+1} + \gamma_0.
\end{align*} \]  

Using the elimination method, we find the system solution (21), expressed in terms of the unknown reverse-sweep coefficients for system (20) \( \delta_0 \) and \( \gamma_0 \).
\[
U_{N_i}^{n+1} = \frac{b_2 q(1-\delta_0) + \gamma_0 b_0 p + \beta_{N_i-1} q(\varphi(1-\delta_0) - \delta_0 b_0)}{\alpha_{N_i-1} q(\delta_0 b_0 - \varphi(1-\delta_0)) + (b_0 p(1-\delta_0) - q \delta_0 b_0 + \nu p(1-\delta_0))},
\]
(22)

\[
U_{N_i}^{n+1} = \alpha_{N_i-1} U_{N_i} + \beta_{N_i-1},
\]

\[
V_1^{n+1} = -q \left( \frac{1}{p} + \frac{\tau}{b_0} \right) U_{N_i-1}^{n+1} + \left( 1 + \frac{q}{p} + \frac{\nu}{b_0} \right) U_{N_i}^{n+1} - \frac{b_2 \tau}{b_0},
\]
(23)

\[
V_0^{n+1} = V_1^{n+1} - \frac{q}{p} U_{N_i}^{n+1} + \frac{q}{p} U_{N_i-1}^{n+1},
\]

where \( p = S_2 E_2 h_2^{-1}, \quad q = S_1 E_1 h_1^{-1} \), \( b_2 = K_0 \left( V_0^n - U_{N_i}^n \right) + b_0 \left( U_{N_i}^n - V_0^n \right) \).

To determine the coefficients \( \delta_0 \) and \( \gamma_0 \), it is necessary to take into account the boundary condition (16) on the right end of the second rod. Let us write these conditions together with the putative solution of sweep method of equitation system (10):

\[
S_2 E_2 \left( V_2^{n+1} - V_{N_2-1}^{n+1} \right) h_2^{-1} = -c_1 V_1^{n+1} - b_1 \left( V_{N_2}^{n+1} - V_{N_2-1}^{n+1} \right) \tau^{-1},
\]
(24)

\[
V_{N_2-1}^{n+1} = \delta_{N_2-1} V_{N_2}^{n+1} + \gamma_{N_2-1}.
\]

Let us write the system (24) in the form

\[
\begin{align*}
V_{N_2-1}^{n+1} &= \frac{E_2 S_2}{E_2 S_2} + c_1 h_2 + b_1 h_2 \tau^{-1} \frac{V_{N_2}^{n+1}}{E_2 S_2} - \frac{b_1 h_2}{\tau E_2 S_2} V_{N_2}^{n} - \frac{b_1 h_2}{\tau E_2 S_2} V_{N_2}^{n}, \\
V_{N_2-1}^{n+1} &= \delta_{N_2-1} V_{N_2}^{n+1} + \gamma_{N_2-1}. 
\end{align*}
\]
(25)

By this means,

\[
\delta_{N_2-1} = 1 + \frac{c_1 h_2}{E_2 S_2} + \frac{b_1 h_2}{\tau E_2 S_2}, \quad \gamma_{N_2-1} = -\frac{b_1 h_2}{\tau E_2 S_2} V_{N_2}^{n}. 
\]

From the second equation system (10), the following formulas are valid for the coefficients:

\[
\delta_j = a_j \left( b_j - c_j \delta_{j-1} \right)^{-1}, \quad \gamma_j = \left( c_j \gamma_{j-1} + d_j \left( b_j - c_j \delta_{j-1} \right) \right)^{-1}.
\]
(26)

To calculate the remaining formula coefficients (26), let us rewrite it in the form

\[
\delta_{j-1} = (b_j \delta_j - a_j) \left( c_j \delta_{j-1} \right)^{-1}, \quad \gamma_{j-1} = \left( c_j \left( b_j - c_j \delta_{j-1} \right) - d_j \right) \delta_{j-1}^{-1}, \quad j = N_2 - 1, N_2 - 2, \ldots, 2, 1.
\]

Then, we calculate the displacement values of the cross sections of the first rod by the formula

\[
U_{j-1}^{n+1} = \alpha_{j-1} U_{j}^{n+1} + \beta_{j-1}, \quad j = N_1, N_1 - 1, \ldots, 2, 1.
\]

The displacement values of the cross sections of the second rod are calculated in the reverse order by the formula

\[
V_{j+1}^{n+1} = \frac{1}{\delta_j} V_{j}^{n+1} - \frac{\gamma_j}{\delta_j}, \quad j = 1, 2, \ldots, N_2 - 1.
\]

Thus, the calculations at each time layer are made according to the following pattern.

1. The sweep coefficients of the first system are calculated in the direction to the left center (center – the contact end of the rod);
2. The sweep coefficients of the second system are calculated in the direction to the right center;
3. To determine the central values, the system of equations is solved (approximation of the boundary conditions);
4. The displacement values from the center to the left for the first system and from the center to the right for the second system are calculated.

3. Results and discussion

3.1. Rod oscillations with an impulse load

In the first model problem, an imitation of the first rod motion is performed in the presence of only elastic resistance at its right end. This sets a large cross-sectional area and the second rod length. The impulse $P$ acts on the left end of the rod (1) and is modeled by the initial velocity distribution at length $\varepsilon=(0.05 - 0.2)L_1$ [8, 12]. The Fourier solution with an impulse load on the left end of the rod for such a scheme is given in [8, 10, 12]. In this case, the oscillations of the first rod are obtained and they are similar to the rod oscillations with elastic resistance at the end. The oscillations of the second rod are practically absent. Low-frequency ($a$) and high-frequency ($b$) oscillations of the right end of the rod (1) are given in figure 2.

![Figure 2](image)

The reduction of the second rod cross-sectional area and the presence of dissipative resistance leads to the second rod motion (figures 3, 4). There is a decrease of the rod oscillation amplitude in time. The intensity of an amplitude decrease of low-frequency oscillations increases with equal cross-section area.

![Figure 3](image)
Figure 4. High frequency oscillations of the second rod $S_2/S_1 = 0.3$, $L_1 = 0.96 \, m$, $P = 1005 \, Ns$, $K_0 = 600000 \, N/m$, $c_1 = 600000 \, N/m$, $b_0 = 2000 \, Ns/m$, $b_1 = 4000 \, Ns/m$.

1 – oscillations of the left end, 2 – oscillations of the right end; (a) $L_2 = 1 \, m$, (b) $L_2 = 10 \, m$.

3.2. Fourier method

Let us consider the problem of the impacting of two rods through an of a high rigidity element, and the solution of this problem by the Fourier method. The design scheme is given in figure 5.

Figure 5. Rod impact through a rigid element.

A special feature of the design scheme in comparison with the work [1] is that an elastic element (spring) is fixed on the right end of the second rod.

The equation of joint rod motion has the form

$$ \frac{\partial^2 U(t,x)}{\partial t^2} = a^2 \frac{\partial^2 U(t,x)}{\partial x^2}, \quad 0 \leq x \leq L_1 + L_2, \quad t \in [0, T]. \quad (27) $$

The first boundary condition is written in the form

$$ SE \frac{\partial U}{\partial x}(t,0) = 0, \quad (28) $$

Here $S_1 = S_2 = S$, $a_1 = a_3 = a$, $E_1 = E_2 = E$.

The condition (28) means that there is no voltage at the left end. The resistance of the elastic element (spring) is taken into consideration on the right end of the first end:

$$ SE \frac{\partial U}{\partial x}(t,L_1) = K_0(U(t,L_1 + H) - U(t,L_1)). \quad (29) $$

For the left end of the second rod, let us write the same conditions
\[ SE \frac{\partial U}{\partial x}(t, L_1 + H) = K_0 (U(t, L_1 + H) - U(t, L_1)). \]  

(30)

Conditions on the right end of the second rod:

\[ SE \frac{\partial U}{\partial x}(t, L_1 + L_2) = -c U(t, L_1 + L_2). \]  

(31)

We find the solution to problem (27) - (31) by the Fourier method. In [1], this method is used to solve the problem when the second rod is free.

The solution of the differential equation (27) can be equated \[ U(t, x) = T(t) \cdot X(x). \]

Separation of variables leads to two ordinary differential equations.

\[ \frac{\partial^2 T}{\partial t^2} + \lambda^2 a^2 T = 0, \quad \frac{\partial^2 X}{\partial x^2} + \lambda^2 X = 0. \]

(32)

For each equation, under condition \( \lambda \neq 0 \), the general solution will have the form

\[ T(t) = A_1 \cos \lambda a t + B_1 \sin \lambda a t, \quad X(x) = A_2 \cos \lambda x + B_2 \sin \lambda x. \]

The constant coefficients \( A_1, B_1, A_2, B_2 \) are determined from the initial and boundary conditions. Thus, we arrive at a solution

\[ U(t, x) = A \sin \lambda a t \cdot \cos \lambda x. \]

Let us use the second boundary condition (31) and obtain the equation

\[ ES \lambda e^{-1} = \text{ctg} \lambda (L_1 + L_2). \]  

(33)

Let us perform the permutation \( \mu = \lambda (L_1 + L_2) \), and then we get the equation

\[ ES \lambda (L_1 + L_2)^{-1} \mu = \text{ctg} \mu. \]

The method for finding the roots of equation (33) in the Mathcad system is described in detail in works [10, 12] (applying the built-in function of \text{Root} (...)).

Suppose \( \mu_1, \mu_2, ..., \mu_n \) be the roots of equation (33), then the eigenvalues of the problem are equal to

\[ \lambda_n = \mu_n (L_1 + L_2)^{-1}. \]

By this means, we obtain a set of particular solutions of the wave equation

\[ U_n(t, x) = A_n \sin \lambda_n a t \cdot \cos \lambda_n x. \]

The general solution can be performed as the sum of particular solutions.

\[ U(t, x) = \sum_{n=1}^{\infty} A_n \sin a \lambda_n t \cdot \cos \lambda_n x. \]

To determine the constants \( A_n \), the condition is used

\[ \frac{\partial U}{\partial t}(0, x) = F(x) = \begin{cases} W_0, & x \in [0, L_1] \\ W_1, & x \in [L_1, L_1 + L_2] \end{cases} \]

After transformations we obtain

\[ A_n = \frac{4 \cdot (L_1 + L_2)}{a (2 \mu_n + \sin 2 \mu_n)} \cdot \mu_n \cdot \left( W_0 - W_1 \right) \sin \frac{\mu_n L_1}{L_1 + L_2} + W_1 \sin \mu_n. \]
Comparison of solutions obtained by the difference method with high rigidity of the intermediate element and the Fourier method is shown in figure 6. The practical coincidence of the graphs indicates the correct choice of the difference scheme parameters.

Figure 6 shows the high-frequency oscillations of the rods (a) and the voltage distribution along the rod length at a specified interval (b), obtained by the Fourier method. The time points are given in the table 1.

| №  | 1    | 2    | 3    | 4    | 5    |
|----|------|------|------|------|------|
| \( t, s \) | 0.00001 | 0.00002 | 0.0001 | 0.0004 | 0.002 |

The functional scheme of the algorithm implementation in the Mathcad system is shown in figure 7.
4. Conclusions

1. Initial boundary value problems are formulated. They model the interaction process between the elements of an impact device (hammer, tool) with the working medium under impulse loads. A hammer and a tool are considered as rods of constant cross section. The interaction of the hammer with the tool is modelled by elastic and dissipative linkages. The resistance of the working medium is represented by elastic and dissipative components. An approximate solution of initial-boundary value problems is found by the finite difference method.

2. Model problems are formulated. They are used to compare the solutions obtained by the Fourier method and the difference method. The comparison is performed with the limiting values of the parameters and the presence of only elastic linkage. This comparison allows to select the difference scheme parameters, providing an acceptable efficiency and accuracy of the method. For registration of low-frequency oscillations, a coarse grid is used, for high-frequency oscillations – a fine grid.

3. A difference scheme with weighting coefficients and a double successive sweep method is effective for solving an initial-boundary value problem describing the interaction of a tool and a hammer in the presence of constraints. The algorithm of comparing solutions obtained by the difference method and the Fourier method is realized in the Mathcad system.

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