The Composition of Functions
Preliminaries and Objectives

Preliminaries

• Functions
• Function Notation

Objectives

• Define the composition of functions
Functions and Notation

\[ f(x) = x^2 \]

\[ f(t) = t^2 \]

\[ f(-3) = 9 \]

\[ f\left(g(x)\right) = \left[g(x)\right]^2 \]
Composition of Functions

\[ f(x) = x^2 \quad g(x) = x - 3 \]

\[ f(g(x)) = (x - 3)^2 \]
Composition of Functions

\[ f(x) = x^2 \quad \quad \quad g(\cdot) = \cdot - 3 \]

\[ f(g(x)) = (x - 3)^2 \]

\[ g(f(x)) = x^2 - 3 \]
Example 2

\[ f(x) = \sqrt{x} \quad g(x) = 3x \]

Find \( f(g(x)) \) and \( g(f(x)) \)

\[ f(g(x)) = \sqrt{3x} \]
\[ g(f(x)) = 3\sqrt{x} \]
Example 3

\[
\begin{align*}
  f(x) &= \sqrt{x} \\
  g(x) &= x - 6 \\
  h(x) &= 3x
\end{align*}
\]

Find \( f(g(h(x))) \)

\[
\begin{align*}
  h(x) &= 3x \\
  g(h(x)) &= g(3x) = 3x - 6 \\
  f(g(h(x))) &= f(3x - 6) \\
  &= \sqrt{3x - 6}
\end{align*}
\]
Recap

To find $f(g(x))$, use the output of $g(x)$ as the input to $f(x)$. 
