Two-dimensional topological superconductivity with antiferromagnetic insulators

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Two-dimensional topological superconductivity has attracted great interest due to the emergence of Majorana modes bound to vortices and propagating along edges. However, due to its rare appearance in natural compounds, experimental realizations rely on a delicate artificial engineering involving materials with helical states, magnetic fields and conventional superconductors. Here we introduce an alternative path using a class of three-dimensional antiferromagnet to engineer a two-dimensional topological superconductor. Our proposal exploits the appearance of solitonic states at the interface between a topologically trivial antiferromagnet and a conventional superconductor, which realize a topological superconducting phase when their spectrum is gapped by intrinsic spin-orbit coupling. We show that these interfacial states do not require fine-tuning, but are protected by asymptotic boundary conditions.

Topological matter represents one of the most intriguing frameworks to realize unconventional physics, due to d − 1-dimensional excitations originating from topological properties of the d-dimensional systems. [1–3] This allows to realize electronic spectra in solid state platforms which resemble those found for some elementary particles in high energy physics and exhibit often an even larger variety. Topologically non-trivial band structures give rise to chiral modes in Chern insulators, [4] helical modes in quantum spin Hall insulators [5] and Majorana modes in topological superconductors. [6–8] In particular, Majorana zero-energy modes in one-dimensional (1D) topological superconductors have fostered intense efforts both in their detection [9–11] and manipulation. [12–15] motivated by their potential for topological quantum computing. [16–17] Yet, one of the biggest challenges is that nature lacks materials with 1D topological superconductivity, and thus, the only hope for its experimental realization relies on an artificial engineering in nano-structures. [18–25]

Two-dimensional (2D) topological superconductors share the exciting phenomena of their 1D counterparts, while providing additional flexibility. On the one hand, Majorana bound states can be found in vortex cores, [26–28] which display properties of interest for topological superconductivity, and thus, the only hope for its experimental realization relies on an artificial engineering in interacting Majorana fermions. [30–33] However, natural 2D topological superconductors are rather elusive, [34] rendering artificial engineering of 2D topological superconductors an important milestone, very much like their 1D counterparts. This further motivated extensions of the original mechanisms for 1D topological superconductivity to two dimensions, based on topological insulators, [8] Yu-Shiba-Rusinov lattices [25] or 2D electron gases with Rashba spin-orbit coupling. [36]

In this letter we introduce an alternative route to create topological superconductivity, exploiting an interface between two bulk ordered phases. Our proposal consists of a heterostructure formed by an insulating bulk antiferromagnet and a conventional bulk superconductor (Fig. 1). Individually, both systems have an excitation gap, both in the bulk as well as at the surface. However, for a special class of antiferromagnetic insulators, as we will discuss below, protected gapless Andreev bound states emerge at the interface between the two 3D systems. These states are mathematically similar to the Jackiw-Rebbi soliton, [36] so that interfacial zero modes exist independently on how the respective magnitudes and spatial profiles between the two electronic orders are. Furthermore, once intrinsic spin-orbit coupling is introduced, the interface states open a gap, giving rise to a topological superconducting state (Fig. 1b). Therefore, this mechanism shows that antiferromagnetic insulators, commonly overlooked, are potential candidates to engineer topological superconductors.

The key ingredient for our proposal is the existence of Dirac lines [37–45], lines of points in the Brillouin zone where the low energy model is a Dirac equation, in the non-magnetic state of the antiferromagnet. There is no specific requirement for the superconductor, apart from having a conventional s-wave Cooper pairing. For the sake of concreteness, we start by introducing a minimal model that exemplifies such a phenomenology. For this purpose, we take an antiferromagnetic diamond lattice with lattice constant a, which can be viewed as a three dimensional analog of the antiferromagnetic honeycomb lattice. [46] Such a structure would be the minimal model for an antiferromagnetic spinel XYZ4, with the magnetic ions sitting in the X sites. [47–51] In order to describe the antiferromagnet-superconductor heterostructure, we propose a Hamiltonian consisting of electron hopping Hkin, antiferromagnetic ordering HAF, superconducting s-wave pairing HSC, and spin-orbit coupling HSOC: [52]

\[ H = H_{\text{kin}} + H_{\text{AF}} + H_{\text{SC}} + H_{\text{SOC}} \]

\[ H_{\text{kin}} = \sum_{(ij)} t_{ij} c_{i,s}^{\dagger} c_{j,s} - \sum_{i,s} \mu(z_i) c_{i,s}^{\dagger} c_{i,s} \]

\[ H_{\text{AF}} = \sum_{i,j,s,s'} m(z_i) \tau_z^{i,s} \sigma_z^{j,s'} c_{i,s}^{\dagger} c_{j,s'} \]

\[ H_{\text{SC}} = \sum_i (\Delta(z_i) [c_{i,\uparrow} c_{i,\downarrow}^{\dagger} + c_{i,\downarrow} c_{i,\uparrow}^{\dagger}]) \]

\[ H_{\text{SOC}} = \sum_{(ij)} i \Delta \sigma^z \cdot (\vec{r}_i \times \vec{r}_j) c_{i,s}^{\dagger} c_{j,s'} \]

(1)
The parameters are chosen so that the Hamiltonian describes an insulating antiferromagnet for \( z < 0 \), with magnetization perpendicular to the interface, and a conventional superconductor for \( z > 0 \). In this way, the electronic spectra of the previous Hamiltonian has an antiferromagnetic gap for \( z = -\infty \) and a superconducting gap for \( z = +\infty \). We may take \( m(z) = m_0(1 - \tanh(z/W))/2 \) the antiferromagnetic order parameter, \( \Delta(z) = \Delta_0(1 + \tanh(z/W))/2 \) the superconducting order parameter, \( \mu(z) = \mu_0(1 + \text{sign}(z))/2 \) the chemical potential fixing half-filling on the antiferromagnetic side. The parameter \( W \) controls the smoothness of the change between the two orders, which in the limit \( W \to 0 \) becomes sharp. Spin-orbit coupling enters as a next-nearest neighbor hopping \([52]\) between sites \( i \) and \( j \), and \( \vec{r}_{ij} (\vec{r}_{ij}) \) is the vector between nearest neighbors \( i \) (\( j \)) and \( l \). We denote \( \vec{r} \) and \( \vec{\sigma} \) the Pauli matrices for the sublattice (A and B) and the spin, respectively. The heterostructure within the \( fcc \) lattice is chosen so that the interface (perpendicular to the \( z \)-axis) consists only of sites belonging to one of the two sublattices, i.e., a zigzag-like interface. Using the standard \( fcc \) lattice vectors, \( \vec{a}_1, \vec{a}_2, \vec{a}_3 \) we can also define the interface plane by two of them, say \( \vec{a}_1 \) and \( \vec{a}_2 \) such that the \( z \)-axis is parallel to \( \vec{a}_1 \times \vec{a}_2 \).

The first interesting finding is that, in the absence of spin-orbit coupling (\( \Lambda = 0 \)), the spectrum of the combined structure develops gapless quasiparticle excitations at the interface [Fig. 1(c)]. These gapless Andreev modes are protected against different choices of the interface profile for the antiferromagnetic order, the superconducting order and the chemical potential. Due to their robustness and structure shown below, we refer to these protected Andreev modes as solitonic states. Switching on spin-orbit coupling (\( \Lambda \neq 0 \)) leads to a fully gapped spectrum for the solitonic states [Fig. 1(d)]. The second remarkable observation is the appearance of the topological Chern invariant \( \mathcal{C} = 2 \) for the gapped system, indicating the presence of two propagating Majorana modes at the edges of the interface [Fig. 1(e)]. This chiral state relies on the broken time reversal symmetry due to the antiferromagnetic order.

The emergence of this topological insulating state by combining two topologically trivial insulating systems is the main finding of our manuscript. This topological superconducting state is robust upon changing parameters [Fig. 1(f)], raising two questions. First, why the interface between the two topologically trivial gapped materials shows robust zero energy modes? Second, why including a small spin-orbit coupling gives rise to a topological superconducting state?

We first address the origin of the gapless interface states, starting with the Bloch Hamiltonian for the prismatic diamond lattice \( \hat{H}_{\text{kin}} = \sum_{\vec{k},s} f(\vec{k}) c_{\vec{A},\vec{k},s}^\dagger c_{\vec{B},\vec{k},s} + c.c. \), where \( f(\vec{k}) = t[1 + e^{i\vec{k} \cdot \vec{a}_1} + e^{i\vec{k} \cdot \vec{a}_2}] + t' e^{i\vec{k} \cdot \vec{a}_3} \), with \( \vec{a}_1, \vec{a}_2, \vec{a}_3 \) the lattice vectors of the \( fcc \) lattice, and \( t'/t \) corresponds to the cubic symmetry. The spectrum possesses lines in \( k \)-space where the valence and conduction band touch. The projected two-dimensional Brillouin zone (BZ) perpendicular to the \( z \)-axis is hexagonal with the \( \Gamma \) point (line) in the center and the \( K \) and \( K' \) points (lines) at the boundary. Depending on the ratio \( t'/t \), one Dirac line forms around the \( \Gamma \) point or two disconnected Dirac lines form around \( K \) and \( K' \) points [Figs. 2(a,b)]. [52] Focusing on such a Dirac line, we can formulate an effective low-energy Hamiltonian \( \hat{H}_D = \sum_{\vec{k},s} (p_z - ip_x) c_{\vec{A},\vec{k},s}^\dagger c_{\vec{B},\vec{k},s} + c.c. \). We use that the momentum \( p \) is tied to the reference frame of the line,
such that $p_\phi$ is tangential to the line, $p_r$ perpendicular to the line and the $z$-axis and $p_z$ perpendicular to the two other components, slightly tilted with respect to the $z$-axis. This low-energy model allows us to study the interface between the superconductor and antiferromagnet analytically. Using the spatially dependent order parameters $\Delta(z)$ and $m(z)$ as introduced above, the effective Hamiltonian takes the form

$$H = \sum_{i,j,s} \left[ \tau_z^{i,j} p_z + \tau_y^{i,j} p_r \right] c_{i,k_{||}}^\dagger s c_{j,k_{||}+} + m(z) \tau_z^{i,k} \sigma_z^{s,s} c_{i,k_{||}+} c_{j,k_{||}+} + \sum_i \Delta(z)c_{i,k_{||}+}c_{i,k_{||}+} + c.c. \right]$$

where $\vec{k}_{||}$ is the conserved Bloch momentum parallel to the interface, $i,j$ sum runs over the two sites $A,B$ and $\mu = 0$.

The Hamiltonian defines a system which is inhomogeneous along the $z$-direction, where for $z \to -\infty$ the Hamiltonian is purely antiferromagnetic and for $z \to \infty$ purely superconducting. Remarkably, for $p_r = 0$ and a profile fulfilling these asymptotic conditions, two solitonic zero-energy Andreev modes exist localized at the interface (Fig. 2), with the following ansatz

$$\Psi_{\alpha,\vec{k}_{||}}(z) = g(z) \left[ c_{A,\vec{k}_{||}+}^\dagger - c_{A,\vec{k}_{||}+} + (-1)^\alpha c_{B,\vec{k}_{||}+}^\dagger + c.c. \right]$$

where $g(z) = C \exp\left[ \int_0^z [m(z') - \Delta(z')]dz' \right]$, $C$ as the normalization constant and $\alpha = 1, 2$ as branch index. Note that although these states are pinned to zero energy, they are not Majorana modes. Furthermore, such states will also exist in the more generic case $|\Delta(z \to \infty)| > |m(z \to \infty)|$ and $|m(z \to -\infty)| > |\Delta(z \to -\infty)|$. Away from $p_r = 0$, the two solitonic wave functions have a finite energy dispersion in the direction of the radial momentum $p_r$ (Fig. 2b), yielding the effective Hamiltonian $H = -\sum_{\alpha} (-1)^\alpha \alpha_{\alpha,B} \Psi_{\alpha,\vec{k}_{||}}^\dagger \Phi_{\alpha,\vec{k}_{||}}$. The existence of these states for each point of the Dirac line implies that the zero mode surface of the heterostructure reflects the original Dirac lines of the antiferromagnet. Thus, any change of the Dirac line structure would be reflected in these zero-energy interface modes, as shown in Figs. 2ab.

In a next step, we introduce the intrinsic spin-orbit coupling, equivalent to a momentum and sublattice dependent exchange field. In the vicinity of the Dirac lines it takes the effective form $H_{\text{SOC}} \propto \alpha \sigma_z^{s,s'} \left[ -\sin(\phi) \sigma_y^{s,s'} + \cos(\phi) \sigma_y^{s,s'} \right] c_{i,k_{||}+}^\dagger s c_{i,k_{||}+} s'$, where $\phi$ denotes the position on
vanishing Chern number, if
of a chiral gap function. In this way, the superconductor, since the spin-orbit coupling
λ Hamiltonian has the structure of a chiral p-wave super-
ical potential and the profile width
ear in the spin-orbit coupling and depends on the chem-
zero modes were located [Figs.3(cd)]. The gap is lin-
gap in the spectra and finite Berry curvature where the
gap closing at this transition point. This specific
transition point depends on the chemical potential µ as
shown in Fig. 4(c), depicting a phase diagram with two
topological phase transitions, from a gapless supercon-
ductor to the topological sector \( C = 2 \) and then \( C = -1 \).
Our calculations demonstrate that the topological phases
are robust, and their existence does not depend on de-
tails of the electronic structure of the superconductor,
but is determined by the topology of the Dirac lines of the
magnetic side. Since the symmetric case \( t'/t = 1 \)
belongs to the sector \( C = 2 \), the sector \( C = -1 \) could
be reached through uniaxial strain perpendicular to the
interface, increasing \( t'/t \).

A last important issue, especially for future experimen-
tal realizations, is whether topological phases are sensi-
tive to the quality of the interface. To test this we now
consider a saw-shaped interface, i.e. a tilted interface ori-
entation yielding a periodicity \((3, 1) \times (3, 1)\) of the original
unit cell. We observe that even for this “imperfect” het-erostructure the interface develops a topological phase
with \( C = -1 \) for \( t' = t \) [Fig. 4(d)]. The inter-valley scat-
ering induced by the interface supercell shifts the sys-
tem to the sector \( C = -1 \). Similar results are obtained
for other interface orientation, with the exception of the
armchair interface where the two sublattice sites appear
in equal number at the interface. This result demon-
strates that the topological phase can be ascribed to the
robustness of the parent solitonic states and generically
requires an imbalance between the two sublattice sites.

Using a minimal model, we have shown how to engi-
neer topological superconductivity connecting an insulating
antiferromagnet with a conventional superconductor.
While we use a single-orbital model, multi-orbital exten-
sions of Eq. 1 could, for example, capture the physics of
antiferromagnetic spinels, such as CoAl2O4, that realizes
an insulating antiferromagnetic diamond lattice,18 but
it is unclear so far whether this material generates in the
paramagnetic state the necessary Dirac lines. Finally, it
is important to notice that the phenomenology presented
here is not restricted to antiferromagnets on diamond lat-
tices, but will emerge in generic systems displaying this
kind of Dirac lines, which would enlarge the range of

the Dirac line as shown in Fig. 2(a,b). We note that for
the different situations of the Dirac lines the SOC takes
a vortex-like profile, but with opposite vorticities around
\( \Gamma \) and \( K \), \( K' \). By projecting the spin-orbit coupling term
onto the solitonic basis, we arrive to the following low-
energy Hamiltonian,

\[
H(p_r, \phi) = \sum_{1, k_j} \begin{pmatrix} \Psi_{1, k_j}^\dagger & \Psi_{2, k_j}^\dagger \end{pmatrix} \begin{pmatrix} v_r p_r & -i \lambda e^{i \phi} \\ i \lambda e^{-i \phi} & -v_r p_r \end{pmatrix} \begin{pmatrix} \Psi_{1, k_j} \\ \Psi_{2, k_j} \end{pmatrix}
\]

(4)

where \( \lambda(\Lambda, \mu, W) = \pm |\langle \Psi_1 | H_{SOC} | \Psi_2 \rangle| \propto \Lambda \) generates a
gap in the spectra and finite Berry curvature where the
zero modes were located [Figs.3(cd)]. The gap is lin-
ear in the spin-orbit coupling and depends on the chem-
ical potential and the profile width \( W \) [Fig. 4(a)]. This
Hamiltonian has the structure of a chiral p-wave super-
conductor, since the spin-orbit coupling \( \lambda \) takes the form
of a chiral gap function. In this way, the superconduct-
ing phase in the interface acquires chirality with a non-
vanishing Chern number, if \( \lambda \neq 0 \).

The Lifshitz transition found in the paramagnetic

phase of the antiferromagnetic side by varying the hopping ratio \( t'/t \) [Figs.2(a,b)] has a final consequence for the
gapped interface modes: this Lifshitz transition in-
troduces a topological transition for the superconduct-
ing phase of the heterostructure. For \( t' < t \) each of the
two Dirac lines contributes through a single phase wind-
ing adding together to a Chern number \( C = 2 \) [Fig.3(c)],
while for \( t' > t \) there is only a single Dirac line winding in
opposite orientation around \( \Gamma \) leading to \( C = -1 \) for the
interface superconductor [Fig.3(d)]. Due to corrections
to the low energy model, the topological phase transition
found by exactly solving the model does not coincide per-
fectly with the bulk Lifshitz transition, but happens at
\( t' \) slightly higher than \( t \), as visible in Fig. 4(b),
which shows a gap closing at this transition point. This specific
transition point depends on the chemical potential \( \mu \) as
shown in Fig. 4(c), depicting a phase diagram with two
topological phase transitions, from a gapless supercon-
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paramagnetic state the necessary Dirac lines. Finally, it
is important to notice that the phenomenology presented
here is not restricted to antiferromagnets on diamond lat-
tices, but will emerge in generic systems displaying this
kind of Dirac lines, which would enlarge the range of
potential candidate materials.\cite{37, 43} Such kind of antiferromagnets would constitute an invaluable building block to engineer two-dimensional topological superconductors without fine tuning requirements, robust against imperfections and changes of materials, providing a new platform to study Majorana physics.

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**APPENDIX**

**Structure of the diamond lattice and the heterostructure**

Here we briefly discuss the spatial structure of the tight binding scheme employed to model a Dirac line material. We use a single orbital tight binding model, whose sites are located in a diamond lattice. This model can be understood as a three-dimensional extension of a honeycomb lattice. The unit cell has two sites, labeled A and B (Fig. 5a). The bulk structure can be understood as two interpenetrating fcc lattices as shown in Fig. 5b. The heterostructures considered in our manuscript are obtained by growing the diamond lattice along the (1,1,1) direction of the fcc lattice shown in Fig. 5b. A sketch of such heterostructure is shown in Fig. 5c, where the purple line mark the interface plane between the antiferromagnetic and superconducting parts. The heterostructure can be understood as stacked buckled honeycomb lattices. Finally, Fig. 5d shows a top view of the interface, showing a triangular lattice whose Brillouin zone will be hexagonal.

**Spectra of isolated AF and SC**

We first briefly discuss the electronic spectra for isolated bulk superconducting and antiferromagnetic states separately. In the absence of spin-orbit coupling, both electronic spectra show a gap, as shown in Fig. 6 (a,b), whose magnitude is controlled by $m$ for the antiferromagnet and $\Delta$ for the superconductor. Including spin-orbit coupling slightly modifies the spectra but maintains the gap [Fig. 6 (c,d)]. This highlights that spin-orbit coupling does not create a strong change in the electronic structure for the bulk system.

**Spin texture induced by spin-orbit coupling**

We now discuss in more in detail the effect of spin-orbit coupling, that enters in the Hamiltonian as a momentum and sublattice dependent exchange coupling $\vec{m}(\vec{k})$. In Fig. 7 we show the spin-texture induced by the spin-orbit coupling, projected onto the Bloch momentum parallel to the interface $\vec{k}_I$. We observe opposite sign for the spin texture on the two sublattices, naturally connected by the combination of time reversal and inversion symmetry operation. The exchange field $\vec{m}(\vec{k})$ induces a vortex structure of the spin textures around the $K$ and $K'$ point as well as around the $\Gamma$ point. This is responsible for the chirality and the structure of the gapped topological interface states. The reciprocal spin texture vanishes in the $M$ point as required by time-reversal symmetry, allowing for a gap closing in the context of the Lifshitz and the topological phase transition.

This in-plane spin texture round the $K$, $K'$ and $\Gamma$ points can be captured by a reduced effective Hamiltonian of the form $\hat{H}_{SOC} \propto \tau_z^{\sigma_{x,x'}} [- \sin(\phi) \sigma_{x,x'}^x + \cos(\phi) \sigma_{y,y'}^y] c_i^{\dagger} \tau_z^{\sigma_{x,x'}} c_i^{\dagger} \tau_z^{\sigma_{x,x'}}$. The winding is visible through the phase factor of the off-diagonal matrix elements parametrized by the polar angles $\phi^K$ and $\phi^\Gamma$ for a circle around the corresponding centers. Ignoring corrections due to irrelevant details of the band structure, the
spin-orbit coupling term can be written as

\[ H_{SO}^{K} = \lambda \sigma^{i}_{x}[-\sin(\phi^{K})\sigma^{s,s'}_{x} + \cos(\phi^{K})\sigma^{s,s'}_{y}]c_{i,\vec{k},s}^{\dagger}c_{i,\vec{k},s'} \]

in the vicinity of the \( K (K') \) point and

\[ H_{SO}^{S} = -\lambda \sigma^{i}_{x}[-\sin(\phi^{S})\sigma^{s,s'}_{x} + \cos(\phi^{S})\sigma^{s,s'}_{y}]c_{i,\vec{k},s}^{\dagger}c_{i,\vec{k},s'} \]

around \( \Gamma \). Note that the sign change in the effective exchange field for the two cases, that is visible in the corresponding vorticity in Fig. 7.

**Interface zero-energy states**

We now discuss the analysis of the interface states based on an effective one-dimensional Dirac equation with a spatially dependent antiferromagnetic ordering and onsite s-wave pairing, that would arise for the different \( \vec{k}_{||} \) contained in the Dirac line. This model allows for a decoupling into two separate sectors, one for spin-up electrons and spin-down holes and another for spin-down electrons and spin-up holes, with operators \( \varphi_1 \) and \( \varphi_2 \), respectively,

\[ \mathcal{H}_{1} = \varphi_{1}^\dagger h_{1} \varphi_{1} \quad \mathcal{H}_{2} = \varphi_{2}^\dagger h_{2} \varphi_{2} \]

where

\[ \varphi_{1}^\dagger = \left( c_{A,\vec{k},\uparrow}^{\dagger}, c_{B,\vec{k},\uparrow}^{\dagger}, c_{A,\downarrow}^{\dagger}, c_{B,\downarrow}^{\dagger} \right) \]

\[ \varphi_{2}^\dagger = \left( c_{A,\vec{k},\uparrow}^{\dagger}, c_{B,\vec{k},\uparrow}^{\dagger}, -c_{A,\downarrow}^{\dagger}, -c_{B,\downarrow}^{\dagger} \right) \]

\[ h_{1} = \begin{pmatrix} m & p_{z} & \Delta & 0 \\ p_{z} & -m & 0 & \Delta \\ \Delta & 0 & m & -p_{z} \\ 0 & \Delta & -p_{z} & m \end{pmatrix} \]

\[ h_{2} = \begin{pmatrix} -m & p_{z} & \Delta & 0 \\ p_{z} & m & 0 & \Delta \\ \Delta & 0 & -m & -p_{z} \\ 0 & \Delta & -p_{z} & m \end{pmatrix} \]

at \( p_{z} = 0 \). The superconducting and antiferromagnetic ordering are general functions of \( z \), so that \( m = m(z) \) and \( \Delta = \Delta(z) \). We will impose the asymptotic conditions \( m(\infty) = m_{0} \), \( m(\infty) = 0 \), \( \Delta(\infty) = 0 \) and \( \Delta(\infty) = \Delta_{0} \), with \( \Delta_{0} \) and \( m_{0} \) positive real numbers. Taking \( p_{z} = -i\partial_{z} \), the following two spinors \( u_{1}, u_{2} \)

\[ u_{1}(z) = \frac{1}{2} \begin{pmatrix} -i \\ 1 \\ -1 \\ i \end{pmatrix} e^{i \int_{0}^{z}(m(z')-\Delta(z'))dz'} \]

\[ u_{2}(z) = \frac{1}{2} \begin{pmatrix} 1 \\ i \\ 1 \\ -i \end{pmatrix} e^{i \int_{0}^{z}(m(z')-\Delta(z'))dz'} \]

are eigenvectors fulfilling \( h_{1}u_{1} = 0 \) and \( h_{2}u_{2} = 0 \) as expected for a zero-energy state. It is important to note that such zero-energy solutions exist for any profile \( m(z) \) and \( \Delta(z) \) provided the asymptotic conditions are fulfilled, and can be understood as solitonic solutions between a Dirac antiferromagnet and a Dirac superconductor. In terms of field operators, the zero-energy solutions take the simple product form

\[ \Psi_{1}(z) = g(z)[c_{A,\vec{k},\uparrow}^{\dagger} - ic_{B,\vec{k},\uparrow}^{\dagger}, -c_{A,\downarrow}^{\dagger}, -c_{B,\downarrow}^{\dagger}] \]

\[ \Psi_{2}(z) = g(z)[c_{A,\vec{k},\uparrow}^{\dagger} + ic_{B,\vec{k},\uparrow}^{\dagger}, c_{A,\downarrow}^{\dagger}, c_{B,\downarrow}^{\dagger}] \]

with \( g(z) = Ce^{\int_{0}^{z}[m(z')-\Delta(z')]dz'} \) and \( C \) a normalization constant as stated in the main text. We finally note that a zero energy solution generically exists provided \( \Delta(\infty) > m(\infty) \) and \( \Delta(\infty) < m(\infty) \). Therefore, our
FIG. 7. Expectation value of the spin in reciprocal space for the spin-orbit coupling, for a finite slab grown along $\vec{a}_3$. Arrows denote the spin components projected in the plane defined by $\vec{a}_1$ and $\vec{a}_2$, whereas the color denotes the perpendicular component. The expectation values are projected onto the A sublattice (a,c,e) and B sublattice (b,d,f). It is clearly observed that spin texture changes between valleys as required by time reversal symmetry, and between sub-lattices as required by inversion symmetry. Panels (e,f) show a minimal sketch of the vortex-like exchange field around $K, K'$.

Gap opening in interfacial states

Next we consider possible terms in the Hamiltonian that could open a gap in the interfacial edge states. For that, we will consider several one-body perturbations, and we will project them onto the solitonic subspace. We take a basis that accounts for the two interfacial states as they will not be independent anymore,

\begin{equation}
\varphi = \begin{pmatrix}
c_A,\vec{k}_{i1},\uparrow \\
c_B,\vec{k}_{i1},\uparrow \\
c_A,\vec{k}_{i1},\downarrow \\
c_B,\vec{k}_{i1},\downarrow \\
c_A,\vec{k}_{i1},\uparrow \\
c_B,\vec{k}_{i1},\downarrow \\
-c_A,\vec{k}_{i1},\uparrow \\
-c_B,\vec{k}_{i1},\downarrow \\
\end{pmatrix}
\end{equation}

In this basis, the two interface states are now represented as $\Psi_\alpha^\dagger = \varphi^\dagger \psi_\alpha$ with

\begin{equation}
\psi_1 = \frac{1}{2} \begin{pmatrix}
1 \\
-i \\
-1 \\
-i \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix},
\psi_2 = \frac{1}{2} \begin{pmatrix}
0 \\
0 \\
0 \\
i \\
1 \\
1 \\
-i \\
\end{pmatrix}
\end{equation}

such that the projection operator $P$ in the manifold $(\Psi_1^\dagger, \Psi_2^\dagger) = \Psi_\alpha^\dagger P$ takes the form

\begin{equation}
P = \frac{1}{2} \begin{pmatrix}
1 & 0 \\
-i & 0 \\
-1 & 0 \\
-i & 0 \\
0 & 1 \\
0 & i \\
0 & 0 \\
0 & -i \\
\end{pmatrix}
\end{equation}

Given a certain perturbation of the form $V = \varphi^\dagger V \varphi$ in the original basis, its representation in the solitonic basis, $W = \Psi^\dagger W \Psi$, is obtained as $W = P^\dagger V P$. With the previous representation, projecting the different operators simply consists of multiplying the relevant matrix elements. The first perturbation that we will consider is a sublattice independent exchange field, which takes the form

\begin{equation}
\mathcal{U}(\vec{k}_i) = \sum_{i=A,B} f(\vec{k}_i) c_{i,\vec{k}_i,\uparrow}^\dagger c_{i,\vec{k}_i,\downarrow}^\dagger + c.c
\end{equation}
with \( f(\vec{k}) \) a generic function. Projecting this in the matrix representation yields immediately zero due to the sublattice structure of \( \Psi \). Hence no sublattice independent local exchange can break the degeneracy of the solitonic states, at least to first order. In particular, this implies that an external in-plane magnetic field and a sublattice-independent Rashba spin-orbit coupling will not open up a gap.

The next perturbation that we will consider is a sublattice dependent exchange field, even in momentum,

\[
\mathcal{V}(\vec{k}) = g(\vec{k})[c^\dagger_{A,\vec{k}+\uparrow}c_{A,\vec{k}+\downarrow} - c^\dagger_{B,\vec{k}+\uparrow}c_{B,\vec{k}+\downarrow}] + \text{c.c} \tag{19}
\]

with \( g(\vec{k}) = g(-\vec{k}) \). Once more the projection on the solitonic states is zero, but now the reason lies in the relative phases between electron and hole sectors. This perturbation would arise from rotating the axis of the staggered moment of the antiferromagnet. The degeneracy results from the spin rotational symmetry of the energy spectra.

Finally, we consider a sublattice dependent exchange field that is odd in momentum,

\[
\mathcal{W}(\vec{k}) = h(\vec{k})[c^\dagger_{A,\vec{k}+\uparrow}c_{A,\vec{k}+\downarrow} - c^\dagger_{B,\vec{k}+\uparrow}c_{B,\vec{k}+\downarrow}] + \text{c.c} \tag{20}
\]

with \( h(\vec{k}) = -h(-\vec{k}) \). In this case the projection does not vanish as it fits to the relative phase structure of electrons and holes. Such an odd-momentum exchange term arises from spin-orbit coupling, which includes the sign change between the sublattices due to inversion symmetry. To summarize, a perturbation \( \mathcal{W}(\vec{k}) \) opening a gap in the solitonic states must fulfill the following conditions

\[
\hat{\Theta} : \mathcal{W}(\vec{k}) \rightarrow -\mathcal{W}(-\vec{k})
\]

\[
\hat{\Theta}\hat{P} : \mathcal{W}(\vec{k}) \rightarrow \mathcal{W}(\vec{k})
\]

where \( \hat{\Theta} \) and \( \hat{P} \) are time reversal and inversion symmetry operators.

### Calculation of the Chern number

We start with the effective Hamiltonian for the low-energy Andreev modes around the Dirac lines \( \mathcal{H}(p_x, \phi) = \Psi^\dagger \mathcal{H} \Psi \) with

\[
\mathcal{H} = \begin{pmatrix} v_r p_r & -i\lambda e^{i\phi} \\ i\lambda e^{-i\phi} & -v_r p_r \end{pmatrix}
\]

where \( p_r \in (-\infty, \infty) \) and \( \phi \in (0, 2\pi) \). By performing a change of variable \( \cos \theta = \frac{v_r p_r}{\sqrt{v_r^2 + v_r^2}} \), the Hamiltonian can be rewritten as

\[
\tilde{\mathcal{H}} = f(\theta) \begin{pmatrix} \cos \theta & -i \sin \theta e^{i\phi} \\ i \sin \theta e^{-i\phi} & -\cos \theta \end{pmatrix}
\]

with \( f(\theta) > 0 \) and \( \theta \in (0, \pi) \). This Hamiltonian describes a skyrmion in reciprocal space with the variables \( \theta \) and \( \phi \). The Berry curvature associated with the low-energy state of this Hamiltonian corresponds to a magnetic monopole in reciprocal space. Thus, the calculation of the Chern number simply yields the charge of the monopole \( \pm 1 \). This leads to a Chern number \( \mathcal{C}_{K,K'} = 1 \) for the \( K \) and \( K' \) point which add up to \( \mathcal{C} = 2 \). For the \( \Gamma \)-point the skyrmion is reversed leading to the Chern number \( \mathcal{C} = -1 \). The topological phase transition connects the phase with \( \mathcal{C} = 2 \) and \( \mathcal{C} = -1 \).

### Spatial dependence of the gap

In this section we show how the topological gap evolves as one moves in the heterostructure from superconducting to the antiferromagnetic part in the \( z \) direction. In the topological state, the gap remains open as one goes
The uniaxial strain FIG. 9. (a) Evolution of the gap of the heterostructure with circle in (a). It is observed that the interface modes remain heterostructure at the gap closing point marked with a purple circle at the M point. Panel (b) shows the band structure of the as well as the gap just at the M point (blue). It is observed 

\[ \Lambda = 0 \]

Correction of the antiferromagnet the gap converges to away from the interface in the z-direction. In the direction of the antiferromagnet the gap converges to \( m_0 \), whereas in the direction of the superconductor it converges to \( \Delta_0 \). To rationalize this, it is illustrative to compute the density of states (DOS) the heterostructure as shown in Fig. 8c, where show two different situations: a gapless state which arises for zero spin-orbit coupling \( \Lambda = 0 \) (Figs. 8bc), and a gapped situation that arises when taking \( \Lambda \neq 0 \) (Figs. 8ce). The computation of the density of states can be performed by means of the Green function of the heterostructure as 

\[ \text{DOS}(z, \omega) \propto \text{Im}(\int_{BZ} G(\omega, \vec{k}, z) d^2 \vec{k}) \]

with \( G(\omega, \vec{k}, z) \) the Green function of the heterostructure, \( \omega \) the energy, \( \vec{k} \) the in-plane Bloch momenta and \( z \) the vertical coordinate in the heterostructure.

Both in the absence (\( \Lambda = 0 \)) and presence (\( \Lambda \neq 0 \)) of spin-orbit coupling, it is observed that in the antiferromagnetic region \( z < 0 \), the local gap converges to the antiferromagnetic gap \( m_0 \), whereas in the superconducting region \( z > 0 \) it converges to the superconducting gap \( \Delta_0 \) (Figs. 8ce). In the absence of spin-orbit coupling (Figs. 8bc), the density of states at the interface \( z = 0 \) becomes gapless, signaling the existence of the gapless modes in that region. In comparison, for \( \Lambda \neq 0 \) (Figs. 8ce) the density of states remains gapped at the interface. Therefore, in the case of a topological gap (Figs. 8bc), the system remains fully gapped for every point of the heterostructure.

### Strain driven topological phase transition

Here we briefly discuss details on the gap closing for the topological phase transition with strain between \( C = 2 \) and \( C = -1 \). The gap closing occurs at the M points, where the spin-orbit coupling vanishes for symmetry reasons. To illustrate this, we show in Fig. 9, the evolution of the gap with uniaxial strain, both in the M point and in the full Brillouin zone. In this way, when the interfacial zero modes are located at the M point, that happens at the critical strain marker with a purple circle in Fig. 9, the system remains gapless even in the presence of spin-orbit coupling, as shown in Fig. 9b. For strains in which there is a finite gap, the gap is generically not located at the M point but at some arbitrary point in the Brillouin zone, close to the location of the Dirac lines.

### Possible candidate materials

The main limitation of our model is that we cannot unequivocally assess if a specific complex material would be suitable for our proposal. An analogous analysis to the one presented in our manuscript could be performed by combining density functional theory and Wannierization, which allows obtaining a multiorbital Hamiltonian from first principles. In that way, it would be possible to assess from first principles if a certain material would be suitable for the mechanism presented.

As mentioned in the manuscript, a possible candidate for our proposal is CoAl\(_2\)O\(_4\), that is experimentally known to realize an antiferromagnetic diamond lattice, giving rise to a multiorbital version of the tight binding model of our manuscript. Interestingly, spinels compounds have been proposed to show Dirac and Weyl like crossings, in particular CaO\(_2\)O\(_4\), SrO\(_2\)O\(_4\) and HgCr\(_2\)Se\(_4\). Assessing whether if CoAl\(_2\)O\(_4\) has Dirac lines in the paramagnetic state requires first principles density functional theory calculations, which is beyond the scope of our study. Nevertheless, given that similar compounds are known to show Dirac-like physics, it is likely that CoAl\(_2\)O\(_4\) could realize the required electronic structure for our proposal.

Assuming that CoAl\(_2\)O\(_4\) hosts the necessary gapped Dirac lines, it is still necessary to assess the value of the topological gap, controlled by spin-orbit coupling of the antiferromagnetic and superconductor. In the following we will take as the antiferromagnet CoAl\(_2\)O\(_4\) and as superconductor the spinel compound LiTi\(_2\)O\(_4\), that has a superconducting gap \( \Delta_0 = 1.9 \) meV. In a low energy Hamiltonian, the effective spin-orbit coupling can be reduced by an order of magnitude with respect to the atomic value due to its interplay with crystal field effects. Taking into account that the atomic spin-orbit coupling in 3d transition metals is on the order of 20 meV, we would have \( \Lambda = 2 \) meV for the effective low energy Hamiltonian. Therefore, according to the previous discussion for a CoAl\(_2\)O\(_4\)/LiTi\(_2\)O\(_4\) heterostructure, we may expect a topological gap on the order of 0.4 meV, which can be observed experimentally and is on the same order of magnitude of state-of-the-art
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