Fire spread and percolation in polydisperse compartment structures

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Abstract. In this paper, we employ a cellular automata and percolation analysis to model fire spread in polydisperse amorphous massively multi-compartmented structures (e.g. naval vessels, high-rise buildings, warehouses, or nuclear plants). Various shapes and sizes of compartments are considered. Each compartment is composed of \( n_c \) equal-size cells. It is found that increasing \( n_c \) increases fingering and lacunarities of fire patterns, and subsequently front roughness. However, this also increases the probability of fire propagation throughout the system as the percolation threshold presents a power-law decrease with \( n_c^{-1} \) for small values of \( n_c \). For large polydisperse compartments, the propagation/non propagation transition seems to be size-independent. A special emphasis is put on the dynamics of fire propagation. Further study is needed to evaluate network properties that should help in developing better strategies to reduce fire consequences.

1. Introduction

Existing probabilistic models, namely epidemic theory, random walk theory, Markov or cellular automata ([1]-[7], to name but a few) can successfully describe certain aspects of fire spread in buildings. Without going into details, there are some disadvantages when simulating fire propagation in amorphous massively multi-compartmented structures using these models [6]. From a theoretical point of view, the effect of stick inclusions (here, compartments) on the percolation threshold \( p_c \) was examined since the eighties [8]. It was found in particular that \( p_c \) is inversely proportional to the excluded volume in the limit of vanishing stick aspect ratio [9]. More recently, Mecke and Seyfried considered inclusions with non-random shapes and sizes: squares and sticks in two-dimensional networks, and cubes, plaquettes, and sticks in three-dimensional networks [10]. A power-law decrease of \( p_c \) with the size of the inclusions, \( \lambda \), was found: \( p_c \lambda^D = \text{const} \), the exponent being the Euclidean dimension of space. In order to correctly describe percolation and fire spread in polydisperse amorphous multi-compartmented structures, we propose in this study to generalize the model of spatially extended inclusions of Mecke and Seyfried to random shape and size inclusions.

The percolation dependence on polydispersity and spatial arrangement is determined numerically and analytically. Diffusion properties are also examined around the percolation threshold.

2. Model

Percolation threshold and diffusion properties are examined by using a cellular automata model on a square or cubic lattice of basic cells of edge length \( a \), with a density \( p \) of occupied cells. Active compartments (i.e. which contain combustible materials) are composed of one or more \( (n_c) \) occupied cells and are randomly oriented and distributed throughout the network. As shown in Figure 1, various types of compartments are considered: sticks, worms and squares in two dimensions, and worms in...
three dimensions. The spatial arrangement of worms may be nonrandom or random, and their size is either fixed or randomly and uniformly generated from 1 to $n_c$.

![Diagram of different compartment shapes](image)

**Figure 1.** Shapes of compartments used in this study.

Fire is ignited from a randomly selected compartment. After a certain period of time, $\tau$, which depends on the fire intensity and on the resistance of compartment walls, fire propagates to the nearest neighboring compartments. To avoid unnecessary complication, this time duration is taken equal to unity. For the same reason, the fire residence time is assumed to be greater than the propagation duration. This fire model can be extended to any growth pattern with invasion percolation [11]. As the percolation is a second order phase transition, its threshold corresponds to the maximum fluctuations of any physical quantity. Here $p_c$ is determined from the maximum fluctuations of fire propagation duration.

As an illustration, Figure 2 shows 2D fire patterns obtained for some percolating systems. As $n_c$ increases, fingering and lacunarity increase, which in turn increases the roughness of fire front structure.

3. Results

System sizes up to 1000 are considered. The statistical averaging process is carried out by generating 100 sample configurations, which ensures that fluctuations are small far from the percolation threshold. Let us first examine the effects of polydispersity and random shape of compartments on the percolation threshold $p_c$ for 2D systems. Percolation threshold behaviors for systems with stick and square inclusions are in qualitative agreement with previous results [10], [12]. For systems with stick and worm compartments, whatever the shape, the percolation threshold seems to be shifted to smaller densities as $n_c$ increases (Figure 3). Unlike sticks, saturation is observed as $n_c$ becomes of the order of 10 for worms. On the contrary, for square compartments $p_c$ seems to increase slightly for large values of $n_c$. 
Figure 2. 2D fire patterns after 120 time steps (\(\tau\)) of propagation for 100a\(\times\)100a square lattices with single cell compartments for \(p=0.6\) (top left), random worm compartments with \(n_c=5\) and \(p=0.55\) (top right), and \(n_c=10\) and \(p=0.52\) (bottom).

Systems with stick compartments seem to behave as power-law with \(n_c\) for large \(n_c\) with an exponent close to 0.5 as that found by Becklehimer and Pandey [13]. However, the fit is rather poor and is influenced by \(p_c\) for \(n_c=1\). A better fit is shown in the inset of Figure 3, indicating the power-law dependence on \(n_c\) of the percolation threshold deviation (\(\Delta p_c = p_c(1) - p_c(n_c)\)) for stick and nonrandom worm compartments for small values of \(n_c\) (a slight deviation from this power law behavior is observed for random worm compartments). The dependence of \(\Delta p_c\) on \(n_c\) may be analytically deduced from the following relationship

\[
\Delta p_c = p_c(1) - p_c(n_c) \sim (C)^{-1/\nu(1)} - (C)^{-1/\nu(n_c)} + C'(n_c - 1)^\alpha
\]

where \(C\) and \(C'\) are constants depending on the correlation length of the system, \(\nu(n_c)\) is the correlation length exponent of the system and \(\alpha\) depends on the correlation length and the effective dimension (\(d_{eff}\)) of the compartment as

\[
\alpha = 1/\nu(1)d_{eff}
\]
Equation (1) allows checking the universality of the percolation phase transition and estimating the average effective dimension of active compartments. A universal phase transition corresponds to a constant value of the correlation length exponent (ν(n) = ν(L)), which leads to the elimination of the two first terms of the right hand side of Eq. (1). The percolation threshold variation, Δp_c, has thus a power-law dependence on n_c^-1, with an exponent expected to be inversely proportional to the compartment effective dimension d_{eff}. Therefore the power-law behavior observed in Figure 3 for stick and non-random worm compartments (i.e. n_c constant) indicates a universal percolation phase transition in such 2D systems. The value of the exponent α for stick inclusions (d_{eff}=1) is 0.73±0.01 (Figure 3), which is compatible with the inverse of the 2D correlation length exponent 1/ν=3/4 (see Eq.(2)). The exponent value for non-random worm inclusions (α = 0.411 ± 0.003) yields an effective dimension of 1.83±0.02. A slight deviation from the power-law behavior is observed for systems with polydisperse random worm inclusions. For networks with square compartments, the percolation threshold remains almost constant for small n_c (Figure 3). The symmetry of such networks is similar to that of single cell compartment networks, which explains why, after renormalization, the usual percolation threshold holds [7].

Let us now examine the dependence of the percolation threshold for three-dimensional networks of thickness h. 100α×100α×h systems with random and non-random worm compartments are considered (Figure 1). The parameter n_c is arbitrarily set equal to 5, and a wide range of thickness is possible, from 1 to 20. The percolation threshold is plotted in Figure 4 as a function of the system thickness. Three regions may be distinguished: a region of small thicknesses, a crossover region and a saturation region (for h comparable with L). As the thickness is limited to 20 cells, the scaling behavior in saturation region (which starts above h/L=0.1) cannot be clearly observed. Predicted 3D percolation thresholds, p_c (3D), are approximately of 0.23 and 0.20 for non-random and random worm inclusions, respectively. For small thickness systems (h ≤ n_c a, with n_c = 5; see Figure 5a), p_c is power-law decreasing as
where the exponent values are $0.386 \pm 0.003$ and $0.366 \pm 0.05$ for non-random and random worm inclusions. The exponent value for non-random worm inclusions is compatible with $\frac{1}{\nu_2} - \frac{1}{\nu_3} = 0.386$, where $\nu_2$ and $\nu_3$ are 2D and 3D correlation length exponents. For random worm inclusions there is a slight difference that can be attributed to the non-universal percolation transition observed for 2D systems (shown in Figure 3).

\[ p_c(h) = p_c(2D)h^{-x} \]  

(5)

Now let us examine the dynamics of invasion. Time evolution of the number of burning cells is shown in Figure 5 for 2D and 3D random worm systems. At $p_c$, the number of affected cells is power-law increasing with time for both 2D and 3D systems, with an exponent of about 1.5. This behavior is caused by the contribution of single compartments to propagation at each time step through the critical links. Above $p_c$, this power-law regime also appears, but it is followed by a constant rate regime with an exponent of around 2, due to the collective contributions of active compartments. The diffusion process seems to be not influenced by long-range connections through large compartments for both $p=p_c$ and $p>>p_c$.

4. Conclusion

Fire spread process is numerically and analytically investigated for 2D and 3D structures with dispersive compartments composed of $n_c$ basic cells. Various types of compartments are considered, namely sticks, random and nonrandom worms and squares.

Some conclusions can be drawn.

- For 2D systems, power-law deviations from the usual percolation threshold are obtained for small values of $n_c$. The percolation threshold saturates for large values $n_c$ due to smearing effects. The slight deviation from the power-law behavior for random worm polydisperse systems corresponds to a non-universal phase transition.
Figure 5. Invasion dynamics in random worm systems: a) Burned area versus time for $n_c=1$ and 10, at $p_c$ and above $p_c$. b) Burned volume versus time for $h=8a$, at $p_c$, below and above $p_c$.

- For 3D systems, the percolation threshold is power-law decreasing from $p_c(2D)$ for small thicknesses, and saturates to $p_c(3D)$ for larger thicknesses. The power-law exponent seems to correspond to the difference between 3D and 2D inverse correlation length critical exponents, $1/\xi_3 - 1/\xi_2$. For the same value of $n_c$, the percolation threshold value for polydisperse worm systems is greater than that of stick systems.
- The propagation dynamics seems to be independent of $n_c$. The active zone is power-law increasing with time at $p_c$ due to the single connections of the active compartments. Above $p_c$, the collective contribution of active compartments may induce a second power-law regime.

Further study is required to explore the influence of the weighting effects induced by the variability of the ignition time $r$ on network properties (i.e. percolation threshold, super-nodes or highly-connected nodes, and critical channels along which the fire will propagate preferentially percolation) and propagation dynamics. Evaluating network properties should help in developing better strategies to reduce fire consequences. The present project is a preliminary phase of a long-term research, intended to provide designers and managers with a decision making tool allowing them to reduce the vulnerability of buildings and infrastructure.

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