Fast Parametric Learning with Activation Memorization

Jack W Rae 1 2 Chris Dyer 1 Peter Dayan 3 Timothy P Lillicrap 1 2

Abstract

Neural networks trained with backpropagation often struggle to identify classes that have been observed a small number of times. In applications where most class labels are rare, such as language modelling, this can become a performance bottleneck. One potential remedy is to augment the network with a fast-learning non-parametric model which stores recent activations and class labels into an external memory. We explore a simplified architecture where we treat a subset of the model parameters as fast memory stores. This can help retain information over longer time intervals than a traditional memory, and does not require additional space or compute. In the case of image classification, we display faster binding of novel classes on an Omniglot image curriculum task. We also show improved performance for word-based language models on news reports (GigaWord), books (Project Gutenberg) and Wikipedia articles (WikiText-103) — the latter achieving a state-of-the-art perplexity of 29.2.

1. Introduction

Neural networks can be trained to classify discrete outputs by appending a softmax output layer. This is a linear map projecting the d-dimensional hidden output of the network to m outputs, where m is the number of distinct classes. A softmax operator (Bridle, 1990) is then applied to produce a probability distribution over classes. The parameters in this softmax layer are typically optimized with the network’s parameters by gradient descent.

We can think of the weights in the softmax layer \( \theta \in \mathbb{R}^{m \times d} \) as a set of m vectors \( \theta[i]; \ i = 1, \ldots, m \) that each correspond to a given class. When trained with a supervised loss, such as cross-entropy, each step of gradient descent pulls the parameter \( \theta[y] \), corresponding to the class label \( y \), towards having a greater inner product with the network output \( h \), and pushes all other parameters \( \theta[j], \ j \neq y \) towards having a smaller inner product with \( h \).

One shortcoming of neural network classifiers trained with backpropagation is that they require many input examples for a given class in order to predict it with reasonable accuracy. That is, many positive class examples and optimization steps are required to pull \( \theta[i] \) towards a point in space where class \( i \) can then be recognized. While the learner will have many opportunities to organize \( \theta[i] \) parameters associated with frequent classes, infrequent class parameters will be poorly estimated. In domains where new classes are frequently introduced, or large-scale classification problems where some classes are very infrequently observed, this estimation problem is potentially quite serious.

One approach to speed up learning, which has received revived interest, is meta-learning. Here, meta-learning refers to algorithms which learn to produce or manipulate learning algorithms (Thrun, 1998; Hochreiter et al., 2001), and it operates by learning over a distribution of tasks or datasets. A meta-learner applies knowledge from the global distribution of tasks to produce or optimize algorithms which specialize to a given task instance. Meta-learning of neural networks has seen promising results for applications such as parameter optimization (Andrychowicz et al., 2016; Finn et al., 2017) and classification (Santoro et al., 2016; Vinyals et al., 2016; Zhou et al., 2018). For classification, the networks are augmented with a differentiable external memory, and are trained with many rounds of data — with class labels permuted between episodes.

Meta-learning can be very powerful for few-shot learning in cases where there is a set of similar prior data to meta-learn over, however it may not be practical for standalone datasets. For example, if one wants to model the grammar of computer code, it is unclear that a meta-learning system trained over natural language will be useful. Also memory-based meta-learning requires backpropagating from the read time to the original write time, which is not well suited to applications where writes and reads are separated by long time steps of conditional computation. In the case of modelling language, for example, infrequent words will not occur for large time intervals — rendering memory-based...
Fast Parametric Learning with Activation Memorization

![Diagram](image)

**Figure 1.** Mixture model of parametric and non-parametric classifiers connected to a recurrent language model. The non-parametric model (right hand side) stores a history of past activations and associated labels as key, value pairs. The parametric model (left hand side) contains learnable parameters \( \theta \) for each class in the output vocabulary \( V \). We can view both components as key, value memories — one slow-moving, optimized with gradient descent, and one rapidly updating but ephemeral.

meta-learning challenging.

The task of statistical language modelling itself is interesting to investigate issues of binding new or infrequent classes, because most classes (words) are infrequent (Zipf, 1935) and new classes naturally emerge over time. Recent approaches to improve neural language models have involved augmenting the network with a non-parametric cache, which stores past hidden activations \( h_{1:n}, \ldots, h_{t-1} \) and corresponding labels, \( y_{1:n}, \ldots, y_{t-1} \) (Vinyals et al., 2015; Merity et al., 2016; Grave et al., 2016b; Kawakami et al., 2017; Grave et al., 2017). Attention over this cache provides better modelling of infrequent words that occur in a recent context, including previously unknown words (Gulcehre et al., 2016). However there is a diminishing return to increasing the cache size (Grave et al., 2016b), and once rare words fall outside the recent context the boost in predictive performance expires.

Motivated from these memory systems, we explore a very simple optimization procedure where the network accumulates activations \( h_t \) directly into the softmax layer weights \( \theta[y_t] \) when a class \( y_t \) has been seen a small number of times, and uses gradient descent otherwise. Accumulating or smoothing network activations into the weights actually corresponds to the well-known Hebbian learning update rule \( W[i,j] \leftarrow \frac{1}{n} \sum_{t=1}^{n} x_t^i x_t^j \) (Hebb, 1949) in the special case of classification on the output layer, where \( W, x_t^i, x_t^j \) correspond to \( \theta, h_t, y_t \) respectively. We see that mixing the two rules provides better initial representations and can also preserve these representations for much longer time spans. This is because memorized activations for one class are not competing for space with activations from other (more frequent, say) classes — unlike a conventional external memory. In this sense, the parameters become an instance of a quickly updated compressed memory, we explore this idea in Section 3.2

We demonstrate this model adapts quickly to novel classes in a simple image classification task using handwritten character from Omniglot (Lake et al., 2015). We then show it improves overall test perplexity for two medium-scale language modelling corpora, WikiText103 (wikipedia articles) (Merity et al., 2016) and Project Gutenberg (books) alongside a large-scale corpus GigaWord v5 (news articles) (Parker et al., 2011). By splitting accuracy over word frequency buckets, we see improved perplexity for less frequent words.

2. Background

2.1. Memory

There has been recent interest in models which store past hidden activations through time \( h_1, h_2, \ldots, h_{t-1} \) into a memory matrix and query the contents with a differentiable attention mechanism. This has been applied to machine translation (Bahdanau et al., 2014), program induction (Graves et al., 2014; 2016), and question answering (Sukhbaatar et al., 2015). Memory-augmented neural networks have also been successfully applied to language modelling (Vinyals et al., 2015; Kawakami et al., 2017; Merity et al., 2016; Grave et al., 2016b; 2017) to facilitate the learning of unknown words, capture the tendency for globally rare words to be repeated in close proximity, and to quickly adapt the network to contextually relevant prior text (Sprechmann et al., 2018).

There are many variants of how to read from memory and mix this information with the network computations. One approach is to retrieve hidden activations and mix these with network activations in latent space (Gulcehre et al., 2016). Another approach is a classic mixture model, as shown in Figure 1; the output probability distribution can be obtained by interpolating the probabilities \( p_p, p_{np} \) from the parametric model and memory respectively.

For intuition we briefly explain a particular architecture, the Neural Cache (Grave et al., 2016b), whose operation is related to our model. The cache is a store of the last \( n \) hidden activations along with their corresponding target output (next word) from a trained parametric language model, such as the Long Short Term Memory (LSTM) (Hochreiter & Schmidhuber, 1997). The conditional probability of a word \( w \) occurring is proportional to the sum over kernalized inner

---

1Project Gutenberg. (n.d.). Retrieved January 2, 2018, from www.gutenberg.org
product similarities between the current hidden state \( h_t \) and past hidden states when word \( w \) occurred.

\[
p_c(w \mid h_t) \propto \sum_{i=1}^{t-1} e^{h_t^T h_i} \mathbb{I}\{y_i = w\} \tag{1}
\]

Where \( \mathbb{I}\{p\} = 1 \) if \( p \) is true, 0 otherwise. This is then interpolated with the parametric language model using a fixed hyper-parameter, swept over during validation. Although the cache is of fixed size \( n \), it can be defined to be very large with sparse attention and efficient data-structures (Rae et al., 2016; Kaiser et al., 2017; Grave et al., 2017).

2.2. Language modelling

We can model a sequence of text as the product of conditional word probabilities,

\[
p(w_1, w_2, \ldots, w_t) = \prod_{i=1}^{t} p(w_i \mid w_1, w_2, \ldots, w_{i-1})
\]

which are estimated separately. Traditional \( n \)-gram models take frequency-based estimates of these conditional probabilities with truncated contexts \( p_n = p(w_i \mid w_{i-n}, \ldots, w_{i-1}) \) and smooth between them to estimate the full conditional probability, \( p(w_i \mid w_1, \ldots, w_{t-1}) = \sum_{j=1}^{n} \lambda_j p_j \). A popular approach is Kneser-Ney smoothing (Kneser & Ney, 1995). More recently, neural language models such as LSTMs and convolutional neural networks directly model the conditional probabilities through sequence-to-sequence training and achieve state-of-the-art performance in many established benchmarks (Collobert & Weston, 2008; Sundermeyer et al., 2012; Kalchbrenner et al., 2014; Jozefowicz et al., 2016; Dauphin et al., 2016; Melis et al., 2017).

3. Model

We propose Hebbian Softmax, a modification of the traditional softmax layer with an updated learning rule. Hebbian Softmax contains the same linear map from the hidden state to the output vocabulary, but learns by smoothing hidden activations into the weight parameters for novel classes whilst concurrently applying gradient descent. This is to facilitate faster binding of novel classes, and improve learning of infrequent classes. We note this corresponds to a learning rule that transitions from Hebbian learning to gradient descent, and we will show that the combination of the two learning rules works better than either one in isolation.

Many of the features of Hebbian Softmax are motivated from memory systems, and the theory of complementary learning systems in the brain (McClelland et al., 1995). During training, the weights corresponding to a given class will initially correspond to a compressed episodic memory store — with new activations memorized and older activations eventually forgotten.

The parameters of the softmax layer are treated both as regular slow-adapting network parameters through which gradients flow to the rest of the network, and fast-adapting memory slots which are updated sparsely without altering the rest of the network. In comparison to an external memory, the advantage of Hebbian Softmax is that it is simple to implement and requires almost no additional space or computation.

We will describe the learning rule in detail, and contrast the conditional probabilities from Hebbian Softmax to those generated by a non-parametric cache. We also generalize the memorization procedure in Section 3.3 as an instance of a secondary fast-learning overfitting procedure with respect to a euclidean objective, and explore several promising variant objective functions.

3.1. Update Rule

Given the weights of a linear projection \( \theta \in \mathbb{R}^{d \times m} \) in the final softmax layer of a network, we calculate the gradient descent update with respect to a cross-entropy loss,

\[
\hat{\theta}_{t+0.5}[i] \leftarrow \begin{cases} \theta_t[i] - \alpha (p_t - 1) h_t & i = y_t \\ \theta_t[i] - \alpha p_t h_t & i \neq y_t \end{cases} \tag{2}
\]

where \( p_t = \frac{e^{h_t^T \theta_t}}{\sum_{j=1}^{n} e^{h_t^T \theta_j}} \) is the probability output from the softmax, and \( \alpha \) is the learning rate. In practice the gradient descent update \( \hat{\theta}_{t+0.5} \) can be calculated with adaptive optimizers, such as RMSProp (Tieleman & Hinton, 2012). This is interpolated with the previous layer’s hidden state to the output vocabulary, but learns by smooth-

---

2 The memory is compressed because multiple activations corresponding to the same class are smoothed into one vector, instead of being stored separately.
Fast Parametric Learning with Activation Memorization

activation $h_t$ for the active class $y_t$,

$$
θ_{t+1}[i] \leftarrow \begin{cases} 
λ_t h_t + (1 - λ_t) \hat{θ}_{t+0.5}[i] & i = y_t \\
\hat{θ}_{t+0.5}[i] & i \neq y_t,
\end{cases}
$$

(3)

as illustrated in Figure 2. When $λ_t = 1$ this corresponds to the rule $θ_{t+1} \leftarrow h_t$, $\|{y_t}\|$ where $\|{y_t}\| \in {0, 1}^m$ is a one-hot target vector. In this case Hebbian update rule, $W_{ij} \leftarrow x_i x_j$ for $x_i = h_t$ the hidden output and $x_j = \|{y_t}\|$ the target. Naturally when $λ = 0$ this is gradient descent, and so we see Hebbian Softmax is mixture of the two learning rules. All remaining parameters in the model are optimized with gradient descent as usual.

When mixing the two learning rules, we would like to benefit from fast initial learning of classes that have not been seen many times, along with stable consolidation of frequently seen classes. As such we do not want $λ_t$ to be constant, but instead something that is eventually annealed to zero. We add an additional counter array $c \in \mathbb{Z}^m$ which counts class occurrences, and propose an annealing function of

$$λ_t = \max(1 / c[y_t], γ) \cdot \mathbb{I}[c[y_t] < T]$$

(4)

where $γ$, $T$ are tuning parameters. $T$ is the number of class occurrences before switching completely to gradient descent and $γ$ is the minimum activation mixing parameter. Although heuristic, we found this worked well in practice vs. a constant $λ$ or pure annealing $λ_t = 1/c[y_t]$. If training from scratch, we suggest setting $γ = 1/N_{min}$ and $T = N_{min} \times$ (#epochs until convergence) where $N_{min}$ is the minimum number of occurrences of any class in a training epoch. This is to ensure we smoothen over many class examples in a given epoch, and the memorization of activations continues until the representation of $h_t$ stabilizes. We describe the full algorithm in Algorithm 1, including details for training with minibatches.

The final layer trains with a two-speed dynamic. For some training steps the full network will be optimized slowly via gradient descent as usual (when frequently-encountered classes are observed), and for other time steps a sparse subset of parameters will rapidly change. The remaining network parameters are optimized with gradient descent.

It is worth noting that simply increasing the learning rate of the softmax layer, or running multiple steps of optimization on rare class inputs, would not achieve the same effect. The value $θ[y_t]$ would indeed be pulled towards a large inner product with $h_t$, however neighbouring parameters $θ[i]$: $i \neq y_t$ would be pushed towards a large negative inner product with $h_t$ and this could lead to catastrophic forgetting of previously consolidated classes. Instead we allow gradient descent to slowly push neighbouring parameters away, and thus disambiguate similar classes in a gradual fashion.

**Algorithm 1** Hebbian Softmax batched update

--- At iteration $0$

$γ \leftarrow$ min. discount (hyper-parameter)

$T \leftarrow$ smoothing limit (hyper-parameter)

$M \leftarrow$ num. classes

$B \leftarrow$ batch size

$c_0[i] \leftarrow 0$; $i = 1, \ldots, M$

--- At iteration $t$

$w_{1:B} \leftarrow$ softmax inputs

$p_{t,1:B} \leftarrow$ softmax outputs

$y_{t,1:B} \leftarrow$ target labels

$θ_{t+0.5} \leftarrow \text{SGD}(θ_t, h_{t,1:B}, p_{t,1:B}, y_{1:B})$

for $i = 1, \ldots, M$

$n_{t,i} \leftarrow \sum_{j=1}^B \mathbb{I}[y_{t,j} = i]$

if $n_{t,i} > 0$

$λ_{t,i} \leftarrow \max(1/c_i[1], γ) \mathbb{I}[c_i[1] < T]$

$\hat{h}_{t,i} \leftarrow \frac{1}{n_{t,i}} \sum_{j=1}^B h_{t,j}[y_{t,j} = i]$

$θ_{t+1} \leftarrow θ_{t+0.5} + (1 - λ_{t,i}) \hat{θ}_{t+0.5}[i]$

else

$θ_{t+1} \leftarrow θ_{t+0.5}[i]$

end if

$c_{t+1}[i] \leftarrow c_t[i] + n_{t,i}$

end for

3.2. Relation to cache models

We can consider the weights constructed from the above optimization procedure as a compressed memory storing historic activations. We contrast the output probabilities of Hebbian Softmax with those produced from a non-parametric cache model.

Recall the conditional probability of a class, $w$, given a cache of previous activations (1). If we set $I_w(j)$ to be the time step of $j$-th most recent occurrence of $w$, then we can re-write the cache probability,

$$p_c(w | h_t) \propto \sum_{i=t-n}^{t-1} e^{g_t h_t \cdot 1[y_i = w]}$$

$$= \sum_{j=1}^{N_w} e^{g(j) h_t \cdot I_{iw}(j)}$$

(5)

where $g(j) = -∞$ if $j < t-n$ and $1$ otherwise, is a weighting function which places uniform weight to the attention over classes in the past $n$ time steps. However if we wish to characterize infrequent classes, we may want a weighting scheme with a larger time horizon that has a smooth decay.

If we modified the cache to have infinite memory capacity and used a geometric weighting scheme to decay the contribution of the $j$-th most recent activation corresponding to the given class, e.g. $g(j) = λ(1 - λ)^{j-1}$, then the resulting
conditional probability is,

\[ \hat{p}_c(w \mid h_t) \propto \sum_{j=1}^{N_w} e^{\lambda(1-\lambda)^{t-1} \lambda h_t^T h_{w(j)}} \tag{6} \]

where \( N_w \) is the total number of occurrences of class \( w \). Let us now consider the conditional probability from Hebbian Softmax for class \( w \), where \( h_t \) has been observed less than \( T \) times. If \( \theta_t \) has not received large gradients from the occurrence of nearby neighboring classes, and we fix \( \lambda_t = \lambda \) over time, then (3) gives

\[ \theta_t \approx \sum_{j=1}^{N_w} \lambda (1-\lambda)^{t-1} h_{w(j)}, \]

plugging this into our softmax conditional probability,

\[ p_\theta(w \mid h_t) \propto e^{h_t^T \theta_w} \approx e^{h_t^T \sum_{j=1}^{N_w} \lambda (1-\lambda)^{t-1} h_{w(j)}} = \prod_{j=1}^{N_w} e^{\lambda(1-\lambda)^{t-1} h_t^T h_{w(j)}}. \]

we see the parametric Hebbian Softmax actually becomes a proxy for the conditional probability output by the non-parametric infinite cache model \( \hat{p}_c \). Past activations now have a geometric contribution to the probability, versus the cache’s arithmetic reduction (6). This form is useful because we can compute \( p_{sm} \) much more efficiently than \( \hat{p}_c \) and it does not require storing the entire history of past activations.

### 3.3. Alternate Objective Functions

We briefly discuss a generalization of the Hebbian Softmax for class \( w \), where \( h_t \) has been observed less than \( T \) times. If \( \theta_t \) has not received large gradients from the occurrence of nearby neighboring classes, and we fix \( \lambda_t = \lambda \) over time, then (3) gives

\[ \theta_t \approx \sum_{j=1}^{N_w} \lambda (1-\lambda)^{t-1} h_{w(j)}, \]

plugging this into our softmax conditional probability,

\[ p_\theta(w \mid h_t) \propto e^{h_t^T \theta_w} \approx e^{h_t^T \sum_{j=1}^{N_w} \lambda (1-\lambda)^{t-1} h_{w(j)}} = \prod_{j=1}^{N_w} e^{\lambda(1-\lambda)^{t-1} h_t^T h_{w(j)}}. \]

we see the parametric Hebbian Softmax actually becomes a proxy for the conditional probability output by the non-parametric infinite cache model \( \hat{p}_c \). Past activations now have a geometric contribution to the probability, versus the cache’s arithmetic reduction (6). This form is useful because we can compute \( p_{sm} \) much more efficiently than \( \hat{p}_c \) and it does not require storing the entire history of past activations.

### 4. Results

#### 4.1. Image Curriculum

We apply Hebbian Softmax to the problem of image classification. We create a simple curriculum task using Omniglot data (Lake et al., 2015), where a subset of classes (30) are initially provided, and 5 new classes are added when test performance exceeds a threshold (60%). Although this is a toy setup, it allows us to investigate the basic properties of fast class binding without other confounding factors, found in real-world problems.

Omniglot contains handwritten characters from 50 alphabets, totalling 1,623 unique character classes. There are 20 examples per class. We partition the first 5 examples per class to a test set, and assign the rest for training.

We use the same architectural setup as Matching Networks (Vinyals et al., 2016) where the images are re-sized to \( 28 \times 28 \) and a 4 layer convolutional neural network is used. Each layer has 64 filters, \( 3 \times 3 \) convolutions, batch normalization, ReLU activations, and \( 2 \times 2 \) max pooling. Each channel maps the input to a scalar, so the resulting hidden size is 64. All weight parameter in the softmax are initialized with Glorot initialization (Glorot & Bengio, 2010). Models were trained with 20% dropout on the final layer and a small amount of data augmentation was applied to training examples (rotation \( \in [-30, 30] \), translation) to avoid overfitting. Otherwise the models quickly plateau on a low level.
For the Hebbian Softmax update, we store the pristine hidden activation pre-dropout. Unlike many one-shot Omniglot papers, we do not train in a meta-learning setup — namely, labels are not shuffled between episodes.

We trained the convnet classifier with RMSProp and swept over learning rates $\alpha \in [1e^{-4}, 4e^{-2}]$ to find the fastest-learning baseline softmax model. A value of $\alpha = 8e^{-3}$ was the largest learning rate to provide stable learning (see Figure 8 in Appendix B). We then compared the regular softmax layer with Hebbian Softmax, both placed on the deep convnet.

If we inspect the number of steps spent on each level averaged over 10 seeds, we see in Figure 3 that the model is noticeably more data efficient after 80 total classes. Although it is still far from one-shot, there is a $1 - 2X$ data efficiency gain on average. In Appendix B, Figure 9 we show the progression of the curriculum in terms of the number of classes shown versus training steps. Hebbian Softmax progresses through the curriculum at identical speed to the softmax until around 1,000 steps of training, from then on it begins to bind new classes and complete the each level faster.

4.2. Language Modelling

We would like to evaluate Hebbian Softmax in the context of a large-scale classification task, where some classes are infrequently observed. Word-level language modelling is an ideal fit because it satisfies both criteria, and there are established performance benchmarks. Some large-scale language modelling corpora require the use of efficient softmax approximations, such as the adaptive softmax (Grave et al., 2016a) or hierarchical softmax (Goodman, 2001) due to the very large vocabulary size. To reduce confounding factors, we restrict ourselves to applications where the full softmax can be used. We investigate two medium-sized corpora, WikiText-103 which contains just over 100M tokens derived from Wikipedia articles (Merity et al., 2016), and Gutenberg which contains a subset of open-access texts from Project Gutenberg listed in Appendix A.2. The idea is that Wikipedia articles should cover factual information, where the style of writing is somewhat consistent and named entities may appear across many articles; whereas books should be more self-contained (unique named entities) and stylistically different. We also consider a very large corpus, GigaWord v5, which is a collection of articles from eight press associations exceeding a decade’s worth of global news.

4.2.1. Model details

For WikiText-103 we swept over LSTM hidden sizes \{1024, 2048, 4096\}, no. LSTM layers \{1, 2\}, embedding dropout \{0, 0.1, 0.2, 0.3\}, use of layer norm (Ba et al., 2016b) \{True, False\}, and whether to share the input/output embedding parameters \{True, False\} totalling 96 parameters. A single-layer LSTM with 2048 hidden units with tied embedding parameters and an input dropout rate of 0.3 was selected, and we used this same model configuration for the other language corpora. We trained the models on 8 P100 Nvidia GPUs by splitting the batch size into 8 sub-batches, sending them to each GPU and summing the resulting gradients. The total batch size used was 512 and a sequence length of 100 was chosen. We did not pass the state of the LSTM between sequences during training, however the state is passed during evaluation.

4.2.2. WikiText-103

The WikiText-103 corpus contains 267,735 unique words and each word occurs at least three times in the training set. We take the best LSTM parameter configura-
We also investigate the model evaluated dynamically on (b) Memory-based Parameter Adaptation (MbPA) (Sprechvocabulary of 242 token. We test out some of the alternate inner objective functions We evaluate Hebbian Softmax on a large-scale language modelling corpus. GigaWord is interesting because it is a vast collection of news articles, and there is a natural temporal order. We pre-process the dataset (Appendix A.5), select all articles from 2000–2009 for the training set, and test on all articles from 2010. The total number of training tokens is 4.0B and the total number of test tokens is 260M. The total unique tokens (after pre-processing) for the training set reaches 6M, however for parity with the other experiments we choose a vocabulary size of 250K. We use the same LSTM hyper-parameters and Hebbian Softmax hyper-parameters, and train the model for 6B steps, after which the models plateau in evaluation performance. We observe a 9.8-point drop in perplexity, from 53.5 to 43.7, illustrated in Table 4.2.2.

4.4. Alternate Objective Functions

We test out some of the alternate inner objective functions described in Section 3.3. These are described in (7), and the
inner objective functions include Euclidean, Inner Product, SVM, (sparse) Softmax. These could be applied to any of the described experiments, we chose the WikiText-103 language modelling task because it is more comparable to prior work.

Although more expressive objective functions appear promising, in practice we see (Figure 7 in Appendix A.4) that validation performance is roughly equivalent between all inner objective functions. This suggests the network activation $h_t$ naturally do not land too close to other class parameters, and the norm of activations is not too large or small, in comparison to the model parameters $\theta$. The latter may be due to the use of layer normalization from the LSTM.

5. Related Work

Few-shot classification has been investigated in a meta-learning setup with a mixture model of a parametric neural network and a non-parametric memory (Santoro et al., 2016; Vinyals et al., 2016). Here, a subset of classes are used with permuted labels per episode, activations are stored to memory, and gradients are passed through the memory. This allows the network to shape its activations to be conducive to accurate retrieval and classification. Here, we do not meta-learn the activations stored into network parameters and instead rely on their representation being rich enough from regular parametric training. We do this to avoid back-propagating through time to the point of writing to memory, as the parameters may contain memories stored millions of time steps ago in the case of rare words.

In natural language processing memory-augmented models have been shown to improve the modelling of unknown words and adaptation to new domains (Grave et al., 2016b; Merity et al., 2016; Kawakami et al., 2017). However in these works the memory is typically small and models the recent past. During evaluation the test activations and corresponding labels are stored in memory, and the model is evaluated dynamically — adapting to the test data on the fly. Whilst dynamic evaluation provides insights into domain transfer, it is limited in applicability as the model may not receive ground-truth labels when launched into production.

More recent work has investigated methods of memorizing and searching over the training set to enhance performance (Kaiser et al., 2017; Grave et al., 2017; Gu et al., 2017). These approaches typically require complex engineering to efficiently index this memory store. Part of the benefit of Hebbian Softmax is implementation simplicity.

Prior literature on the softmax operator for language modelling computational efficiency (Chen et al., 2015; Grave et al., 2016a) or tricks such as smoothing across many softmax layers (Yang et al., 2017). However these do not focus on increasing the data-efficiency or faster learning of infrequent classes.

Other architectures have been considered for fast learning, such as the ‘fast weights’ auto-associative memory (Ba et al., 2016a). This focuses on fast adaptation to recent information that persists over a short window of time. The LEABRA architecture (O’Reilly, 1996a) contains a mixture of contrastive Hebbian learning (GENEREC) (O’Reilly, 1996b) and gradient descent for fast and slow learning, however this cognitively-inspired model has not been shown to scale to large-scale classification problems.

6. Discussion

This paper explores one way in which we can achieve fast parametric learning in neural networks, and preserve this knowledge over time. We show that activation memorization is useful for vision in the binding of newly introduced classes, beating a well tuned adaptive learning rate optimizer, RMSProp.

For language we show improvement in the modelling of text with an extensive vocabulary. In the latter we show the model beats a very strong LSTM benchmark on three stylistically different corpora, and achieves state of the art on WikiText-103. This is achieved with effectively no additional compute or memory resources. Breaking down perplexity over word frequency bucket, we see that less frequent words are better modelled, as hypothesized. We suggest that Hebbian Softmax could be applied to any classification domain with infrequent classes, or non-stationary data. It may also be useful in quickly adapting a pre-trained classifier to a new task / set of classes — however this is beyond the scope of our initial investigation.

It would also be interesting to explore activation memorization deeper within the network, and thus in more general scenarios to classification. In this case, there is no direct feedback from a ground-truth class label and the update rule would not necessarily be an instance of Hebbian learning. A natural first step would be to generalize the ideas to large-scale softmax operators that are internal to the network — such as attention over a large memory.

Acknowledgements

The authors would like to thank Gabor Melis, Greg Wayne, Oriol Vinyals, Pablo Sprechmann, Siddhant Jayakumar, Charles Blundell, Koray Kavukcuoglu, Shakir Mohamed, Adam Santoro, Phil Blunsom, Felix Hill, Angeliki Lazaridou, James Martens, Ozlem Aslan, Guillaume Desjardins, and Chloe Hillier for their comments and assistance during the course of this project. Peter Dayan is currently on a leave of absence at Uber Technologies; Uber was not involved in this study.
References

Andrychowicz, Marcin, Denil, Misha, Gomez, Sergio, Hoffman, Matthew W, Pfau, David, Schaul, Tom, and de Freitas, Nando. Learning to learn by gradient descent by gradient descent. In Advances in Neural Information Processing Systems, pp. 3981–3989, 2016.

Ba, Jimmy, Hinton, Geoffrey E, Mnih, Volodymyr, Leibo, Joel Z, and Ionescu, Catalin. Using fast weights to attend to the recent past. In Advances in Neural Information Processing Systems, pp. 4331–4339, 2016a.

Ba, Jimmy Lei, Kiros, Jamie Ryan, and Hinton, Geoffrey E. Layer normalization. arXiv preprint arXiv:1607.06450, 2016b.

Bahdanau, Dzmitry, Cho, Kyunghyun, and Bengio, Yoshua. Neural machine translation by jointly learning to align and translate. arXiv preprint arXiv:1409.0473, 2014.

Bai, Shaojie, Kolter, J. Zico, and Koltun, Vladlen. Convolutional sequence modeling revisited, 2018. URL https://openreview.net/forum?id=rk8wKK-R-.

Bridle, John S. Training stochastic model recognition algorithms as networks can lead to maximum mutual information estimation of parameters. In Advances in neural information processing systems, pp. 211–217, 1990.

Chen, Welin, Grangier, David, and Auli, Michael. Strategies for training large vocabulary neural language models. arXiv preprint arXiv:1512.04906, 2015.

Collobert, Ronan and Weston, Jason. A unified architecture for natural language processing: Deep neural networks with multitask learning. In Proceedings of the 25th international conference on Machine learning, pp. 160–167. ACM, 2008.

Dauphin, Yann N, Fan, Angela, Auli, Michael, and Grangier, David. Language modeling with gated convolutional networks. arXiv preprint arXiv:1612.08083, 2016.

Finn, Chelsea, Abbeel, Pieter, and Levine, Sergey. Model-agnostic meta-learning for fast adaptation of deep networks. arXiv preprint arXiv:1703.03400, 2017.

Glorot, Xavier and Bengio, Yoshua. Understanding the difficulty of training deep feedforward neural networks. In Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics, pp. 249–256, 2010.

Goodman, Joshua. Classes for fast maximum entropy training. In Acoustics, Speech, and Signal Processing, 2001. Proceedings.(ICASSP’01). 2001 IEEE International Conference on, volume 1, pp. 561–564. IEEE, 2001.

Grave, Edouard, Joulin, Armand, Cissé, Moustapha, Grangier, David, and Jégou, Hervé. Efficient softmax approximation for gpus. arXiv preprint arXiv:1609.04309, 2016a.

Grave, Edouard, Joulin, Armand, and Usunier, Nicolas. Improving neural language models with a continuous cache. arXiv preprint arXiv:1612.04426, 2016b.

Grave, Edouard, Cisse, Moustapha M, and Joulin, Armand. Unbounded cache model for online language modeling with open vocabulary. In Advances in Neural Information Processing Systems, pp. 6044–6054, 2017.

Graves, Alex, Wayne, Greg, and Danihelka, Ivo. Neural turing machines. arXiv preprint arXiv:1410.5401, 2014.

Graves, Alex, Wayne, Greg, Reynolds, Malcolm, Harley, Tim, Danihelka, Ivo, Grabska-Barwińska, Agnieszka, Colmenarejo, Sergio Gómez, Grefenstette, Edward, Ramalho, Tiago, Agapiou, John, et al. Hybrid computing using a neural network with dynamic external memory. Nature, 538(7626):471, 2016.

Gu, Jiatao, Wang, Yong, Cho, Kyunghyun, and Li, Victor OK. Search engine guided non-parametric neural machine translation. arXiv preprint arXiv:1705.07267, 2017.

Gulcehre, Caglar, Ahn, Sungjin, Nallapati, Ramesh, Zhou, Bowen, and Bengio, Yoshua. Pointing the unknown words. arXiv preprint arXiv:1603.08148, 2016.

Hebb, Donald O. The organization of behavior: A neurophysiological approach, 1949.

Hochreiter, Sepp and Schmidhuber, Jürgen. Long short-term memory. Neural computation, 9(8):1735–1780, 1997.

Hochreiter, Sepp, Younger, A Steven, and Conwell, Peter R. Learning to learn using gradient descent. In International Conference on Artificial Neural Networks, pp. 87–94. Springer, 2001.

Jozefowicz, Rafal, Vinyals, Oriol, Schuster, Mike, Shazeer, Noam, and Wu, Yonghui. Exploring the limits of language modeling. arXiv preprint arXiv:1602.02410, 2016.

Kaiser, Lukasz, Nachum, Ofir, Roy, Aurko, and Bengio, Samy. Learning to remember rare events. International Conference on Learning Representations, 2017.

Kalchbrenner, Nal, Grefenstette, Edward, and Blunsom, Phil. A convolutional neural network for modelling sentences. arXiv preprint arXiv:1404.2188, 2014.

Kawakami, Kazuya, Dyer, Chris, and Blunsom, Phil. Learning to create and reuse words in open-vocabulary neural language modeling. arXiv preprint arXiv:1704.06986, 2017.
Kneser, Reinhard and Ney, Hermann. Improved backing-off for m-gram language modeling. In *Acoustics, Speech, and Signal Processing, 1995. ICASSP-95.*, 1995 International Conference on, volume 1, pp. 181–184. IEEE, 1995.

Lake, Brenden M, Salakhutdinov, Ruslan, and Tenenbaum, Joshua B. Human-level concept learning through probabilistic program induction. *Science*, 350(6266):1332–1338, 2015.

McClelland, James L, McNaughton, Bruce L, and O’reilly, Randall C. Why there are complementary learning systems in the hippocampus and neocortex: insights from the successes and failures of connectionist models of learning and memory. *Psychological review*, 102(3):419, 1995.

Melis, Gábor, Dyer, Chris, and Blunsom, Phil. On the state of the art of evaluation in neural language models. *arXiv preprint arXiv:1707.05589*, 2017.

Merity, Stephen, Xiong, Caiming, Bradbury, James, and Socher, Richard. Pointer sentinel mixture models. *arXiv preprint arXiv:1609.07843*, 2016.

O’Reilly, Randall C. Biologically plausible error-driven learning using local activation differences: The generalized recirculation algorithm. *Neural computation*, 8(5):895–938, 1996b.

O’Reilly, Randall C. *The Leabra model of neural interactions and learning in the neocortex*. PhD thesis, Carnegie Mellon University, Pittsburgh, PA, USA, 1996a.

Parker, Robert, Graff, David, Kong, Junbo, Chen, Ke, and Maeda, Kazuaki. English gigaword fifth edition ldc2011t07. dvd. *Philadelphia: Linguistic Data Consortium*, 2011.

Rae, Jack, Hunt, Jonathan J, Danihelka, Ivo, Harley, Timothy, Senior, Andrew W, Wayne, Gregory, Graves, Alex, and Lillicrap, Tim. Scaling memory-augmented neural networks with sparse reads and writes. In *Advances in Neural Information Processing Systems*, pp. 3621–3629, 2016.

Santoro, Adam, Bartunov, Sergey, Botvinick, Matthew, Wierstra, Daan, and Lillicrap, Timothy. One-shot learning with memory-augmented neural networks. *arXiv preprint arXiv:1605.06065*, 2016.

Sprechmann, Pablo, Jayakumar, Siddhart, Rae, W. Jack, Pritzel, Alexander, Puigdomenech, Adria Badia, Uría, Benigno, Vinyals, Oriol, Hassabis, Demis, Pascanu, Razvan, and Blundell, Charles. Memory-based parameter adaptation. *International Conference on Learning Representations*, 2018.
Appendix

A. Language Modelling

A.1. WikiText-103

A.1.1. Dynamic Evaluation Parameters

For the Neural Cache, we swept over the hyper-parameters:

- Softmax inverse temperature: \( \theta_{cache} \in \{0.1, 0.2, 0.3\} \)
- Cache output interpolation: \( \lambda_{cache} \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35\} \)
- Cache size \( n_{cache} \in \{1000, 5000, 8000, 9000, 10000\} \)

and chose \( \theta_{cache} = 0.3 \), \( \lambda_{cache} = 0.1 \), \( n_{cache} = 10000 \) by sweeping over the validation set.

For the mixture of Neural Cache and MbPA we swept over the same cache parameters, alongside:

- MbPA output interpolation: \( \lambda_{mbpa} \in \{0.02, 0.04, 0.06, 0.08, 0.10\} \)
- Number of neighbours retrieved from memory: \( K \in \{512, 1024\} \)
- Number of MbPA steps: \( n_{mbpa} \in \{1, 2\} \)

and selected \( \lambda_{mbpa} = 0.04 \), \( \lambda_{cache} = 0.1 \), \( \theta_{cache} = 0.3 \), \( K = 1024 \), \( n_{mbpa} = 1 \), \( n_{cache} = 10000 \). We also selected the MbPA learning rate \( \alpha_{lr} = 0.3 \), and the L2-regularization \( \beta_{mbpa} = 0.5 \) on the MbPA-modified parameters. The memory size for MbPA was chosen to be equal to the cache size.

A.2. Gutenberg

A.2.1. Splits

We downloaded a subset of books (listed below) from Project Gutenberg on January 2, 2018 from a mirror site (https://www.gutenberg.org/MIRRORS.ALL). We selected 2042 English-language books under the /1 subdirectory. Each book has a unique id, we shuffled the books and split out a reasonably sized train, validation and test set. The book ids are listed below for these splits.

Test (13 books, 526,646 tokens):

11959, 12211, 10912, 11015, 12585, 10827, 10268, 11670, 126, 1064, 11774, 12505, 11931

Validation (12 books, 609,545 tokens):

11399, 10003, 1202, 12213, 1177, 1285, 10516, 11635, 12315, 11804, 11249, 11163

Train (2017 books, 175,181,505 tokens):

10000, 10064, 1019, 10538, 10482, 10598, 10675, 10787, 10864, 10929, 10, 11112, 11190, 11257, 11363, 11449, 11526, 11612, 11715, 11825, 11920, 11987, 1204, 12118, 1218, 12249, 12236, 12405, 12478, 12582, 12691, 12806, 12895, 10001, 10065, 101, 1035, 10483, 10595, 10676, 10788, 10865, 10930, 11007, 11113, 11191, 11258, 11364, 11451, 11527, 11613, 11716, 11826, 11921, 11988, 12050, 1211, 12185, 12252, 12327, 12406, 1247, 12583, 12692, 12807, 12896, 10002, 10066, 10201, 10363, 10489, 1059, 1067, 10789, 10867, 10931, 11008, 11114, 11192, 11259, 11365, 11452, 11528, 11634, 1171, 11827, 11922, 11989, 12051, 12121, 12186, 12253, 12284, 12409, 12486, 12584, 1269, 12808, 12897, 10067, 10202, 10365, 1048, 105, 10644, 1078, 10868, 10932, 11009, 11115, 11193, 11260, 11366, 11454, 1152, 11615, 1172, 11828, 11923, 1198, 12052, 12122, 12187, 12254, 12329, 1240, 1248, 12697, 12809, 12898, 10004, 10068, 1020, 10366, 10490, 10600, 10687, 10790, 10869, 10933, 11010, 11119, 11194, 11263, 11367, 11455, 11530, 11623, 11734, 11829, 11924, 11990, 12054, 12128, 12288, 12526, 1232, 12412, 12490, 12589, 12699, 1280, 12899, 10005, 10069, 10210, 10367, 10491, 10601, 1068, 10791, 10870, 10934, 11012, 11210, 11195, 11264, 11368, 11456, 11531, 11624, 11738, 1182, 11926, 11991, 12055, 12124, 12189, 12257, 12330, 12413, 12491, 1258, 1269, 12810, 128, 10006, 1006, 10211, 10368, 10493, 10602, 10690, 10792, 10871, 10935, 11013, 11121, 11196, 11265, 11369, 11459, 11533, 11625, 11736, 11830, 11929, 11992, 12056, 12125, 1218, 12259, 12333, 12414, 12498, 12590, 12811, 12900, 10007, 10070, 10212, 10369, 1049, 10603,
Fast Parametric Learning with Activation Memorization
Fast Parametric Learning with Activation Memorization
Figure 5. Validation learning curves for the Gutenberg corpus. All word classes have been observed after around 4B training tokens and we observe the performance of Hebbian Softmax return to that of the vanilla LSTM thereafter, as all parameters are optimized by gradient descent.

A.3. GigaWord

Figure 6. Test perplexity on GigaWord v5 corpus. Each model is trained on all articles from 200 − 2009 and tested on 2010. Because the test set is very large, a random subsample of articles are used per evaluation cycle. For this reason, the measurements are more noisy.

A.4. Alternate Objective Functions
Fast Parametric Learning with Activation Memorization

Figure 7. Validation learning curves for WikiText-103 comparing different overfitting objectives. Surprisingly there is not a significant improvement in performance by choosing inner objectives which relate to the overall training objective, e.g. Softmax, vs $L_2$.

A.5. Data pre-processing

For Project Gutenberg and GigaWord v5 we used a very simple python script to pre-process and tokenize the data using NLTK. We post the Gutenberg script here for ease of reproduction. The GigaWord v5 script excludes the Project Gutenberg-specific selection of start / end markers to extract the text. The NLTK library \(^3\) is used to split out sentence and word tokens, the resulting text contains lower-case text with one sentence per line.

```python
# Copyright 2018 Google LLC.
# # Licensed under the Apache License, Version 2.0 (the "License");
# # you may not use this file except in compliance with the License.
# # You may obtain a copy of the License at
# # https://www.apache.org/licenses/LICENSE-2.0
# # Unless required by applicable law or agreed to in writing, software
# distributed under the License is distributed on an "AS IS" BASIS,
# WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.
# See the License for the specific language governing permissions and
# limitations under the License.

def process_text(text):
    import nltk
    start_text = "START OF THIS PROJECT GUTENBERG EBOOK"
    start = text.find(start_text) + len(start_text)
    end = text.find("END OF THIS PROJECT GUTENBERG EBOOK")
    text = text[start:end]
    text = text.decode("utf-8", "ignore")
    text = text.replace("\r", " ")
    text = text.replace("\n", " ")
    final_text_list = []
    sent_text_tokens = nltk.sent_tokenize(text)
    for sentence in sent_text_tokens:
        final_text_list.extend(nltk.word_tokenize(sentence) + ["\n"])
    return " ".join(final_text_list).lower().encode("utf-8")
```

\(^3\)https://www.nltk.org/
B. Omniglot

Figure 8. Omniglot curriculum performance versus learning rate for a regular softmax architecture. Values of $1e^{-3}$ to $8e^{-3}$ are similarly fast to learn and are stable. Stability breaks down for larger values.

Figure 9. Omniglot curriculum task. Starting from 30 classes, 5 new classes are added when total test error exceeds 60%. Each line shows a 2-σ confidence band obtained from 10 independent seed runs.