Powerful Outflows and Feedback from Active Galactic Nuclei

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Abstract  Active Galactic Nuclei (AGN) represent the growth phases of the supermassive black holes in the center of almost every galaxy. Powerful, highly ionized winds, with velocities $\sim 0.1 - 0.2c$ are a common feature in X-ray spectra of luminous AGN, offering a plausible physical origin for the well known connections between the hole and properties of its host. Observability constraints suggest that the winds must be episodic, and detectable only for a few percent of their lifetimes. The most powerful wind feedback, establishing the $M - \sigma$ relation, is probably not directly observable at all. The $M - \sigma$ relation signals a global change in the nature of AGN feedback. At black hole masses below $M - \sigma$ feedback is confined to the immediate vicinity of the hole. At the $M - \sigma$ mass it becomes much more energetic and widespread, and can drive away much of the bulge gas as a fast molecular outflow.

Keywords  Supermassive black holes, accretion, $M - \sigma$ relation, X-ray winds, molecular outflows, quenching of star formation

1 INTRODUCTION

1.1 SMBH Scaling relations

Astronomers now generally agree that the center of almost every galaxy but the smallest contains a supermassive black hole (SMBH). In recent years it has become clear that the mass $M$ of the hole correlates strongly with physical properties of the host galaxy. In particular the hole mass $M$ appears always to be a fairly constant fraction of the stellar bulge mass $M_b$, i.e.

$$M \sim 10^{-3} M_b$$

(H"aring & Rix, 2004). Even more remarkably, observations give a tight relation of the form

$$M \simeq 3 \times 10^8 M_\odot \sigma_{200}^{\alpha}$$

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between the SMBH mass and the velocity dispersion $\sigma = 200\sigma_{200}$ km s$^{-1}$ of the host galaxy’s central bulge, with $\alpha \simeq 4.4 \pm 0.3$ (Ferrarese & Merritt 2000, Gebhardt et al. 2000; see Kormendy & Ho 2013 for a recent review). For many practical cases the relation is more conveniently written as

$$M \simeq 2 \times 10^7 M_\odot \sigma_{100}^\alpha$$

with now $\sigma = 100\sigma_{100}$ km s$^{-1}$.

Since observationally determining the SMBH mass generally involves resolving its sphere of influence, of radius

$$R_{\text{inf}} \simeq \frac{GM}{\sigma^2} \simeq \frac{8}{\sigma_{200}^2} M_8 \text{ pc} \simeq 3 \frac{M_7}{\sigma_{100}^2} \text{ pc},$$

with $M = 10^8 M_8 M_\odot = 10^7 M_7 M_\odot$. (2) may represent a maximum SMBH mass for a given velocity dispersion $\sigma$ (Batcheldor 2010).

### 1.2 Binding Energies

At first sight the relations (1,2) appear surprising. For (4) shows that the black hole’s gravity has a completely negligible effect on its host galaxy, which in most ways must be quite unaware of its existence. But we know (Soltan 1982) that the hole grew largely as a result of luminous accretion of gas. This released energy

$$E_{\text{BH}} \simeq \eta M c^2 \sim 2 \times 10^{61} M_8 \text{ erg},$$

where $\eta \simeq 0.1$ is the accretion efficiency, far larger than the binding energy

$$E_{\text{bulge}} \sim M_8 \sigma_{200}^2 \sim 8 \times 10^{58} M_8 \sigma_{200}^2 \text{ erg}$$

of a host bulge of stellar mass $M_8 \sim 10^8 M_\odot$.

The vast difference in these two numbers suggests that the host must notice the presence of the hole through its energy output, even though it is utterly insignificant in all other ways. We can already see how the black hole mass might correlate with galaxy properties – the hole grows by accreting gas, but in doing this communicates some of its huge binding energy $E_{\text{BH}}$ back to the gas reservoir, and so potentially limits its own growth. This suggests that the most relevant quantity to compare with $E_{\text{BH}}$ is not $E_{\text{bulge}}$, but instead the gravitational binding energy of the bulge gas alone, i.e.

$$E_{\text{gas}} = f_g E_{\text{bulge}}$$

where $f_g < 1$ is the gas fraction. In the following we take this as $f_g \sim 0.16$, the cosmological mean value, giving a typical relation

$$E_{\text{BH}} \sim 2000 E_{\text{gas}}$$

for a black hole close to the $M - \sigma$ relation (the rhs has an implicit factor $\sim \sigma_{200}^4 / M_8$).

This picture requires the black hole to communicate some of its accretion energy to its host. But this process cannot be very efficient, as otherwise the hole could disrupt the host entirely, or at the very least remove a large fraction of its gas. In this sense, the galaxy bulge leads a precarious existence. For much of its life it can ignore the threat that the SMBH poses, but we will see that in the end this is always decisive if accretion continues.
1.3 Communicating the Energy: Feedback

There are two main ways that the SMBH binding energy can potentially interact with its surroundings. By far the larger is direct radiation: after all, this is how all the accretion energy is initially released. But we know from observation that most light escapes relatively freely from active galactic nuclei (AGN). This suggests that radiation is in general not the main way the SMBH affects its host, and we will discuss in detail why this is so in Section 7.4. The second form of coupling SMBH binding energy to a host bulge is mechanical. The huge SMBH accretion luminosity drives powerful gas flows into the host, making collisions and communication inevitable. One form of flow often mentioned is jets – highly collimated flows driven from the immediate vicinity of the SMBH (see Fabian 2012 for a review). To turn these into a way of affecting most of the bulge requires a way of making the interaction relatively isotropic, perhaps with changes of the jet direction over time. Here we will mainly consider another form of mechanical communication which automatically has this property already. This is the observed presence in many AGN of near–isotropic winds carrying large momentum fluxes.

1.4 Powerful ionized winds

Early X-ray observations of AGN yielded soft X-ray spectra frequently showing the imprint of absorption from ionized gas, the ‘warm absorber’; hereafter WA (Halpern 1984, Reynolds & Fabian 1995). More recent observations have found at least 50% of radio-quiet AGN showing WAs in their soft X-ray (~0.3-2 keV) spectra. The limited spectral resolution of the Einstein Observatory and ASCA observations prevented important parameters of the WAs, in particular the outflow velocity and mass rate, to be determined with useful precision. The higher resolution and high throughput afforded by contemporary X-ray observatories, Chandra, XMM-Newton and Suzaku has transformed that situation over the past decade, with the WA being shown, typically, to be dominated by K-shell ions of the lighter metals (C, N, O, etc) and Fe–L, with outflow velocities of several hundred km s\(^{-1}\) (Blustin et al. 2005, McKernan et al. 2007).

A more dramatic discovery made possible with the new observing capabilities was the detection of blue-shifted X-ray absorption lines in the iron K band, indicating the presence of highly ionized outflows with velocities \(v \sim 0.1 - 0.25c\) (Chartas et al. 2002; Pounds et al. 2003; Reeves et al. 2003). In addition to adding an important dimension to AGN accretion studies, the mechanical power of such winds, which for a radial flow depends on \(v^3\), was quickly recognized to have a wider potential importance in galaxy feedback.

Additional detections of high velocity AGN winds were delayed by the low absorption cross section of such highly ionized gas, combined with strongly blue-shifted lines in low-redshift objects coinciding with falling telescope sensitivity above \(~7\) keV. However, further extended observations, particularly with XMM-Newton, found evidence in 5 additional AGN for outflow velocities of \(\sim 0.1 - 0.2c\) (Cappi et al. 2006). Some doubts remained as to how common high velocity outflows were, as the majority of detections were of a single absorption line (with consequent uncertainty of identification), and had moderate statistical significance, raising concerns of ‘publication bias’ (Vaughan and Uttley 2008). In addition, only
for PG1211+143 had a wide angle outflow been directly measured, confirming a high mass-rate and mechanical energy in that case (Pounds & Reeves 2007, 2009)

These residual doubts were finally removed following a blind search of extended AGN observations in the XMM-Newton archive (Tombesi et al. 2010), finding compelling evidence in 13 (of 42) radio quiet objects for blue-shifted iron K absorption lines, with implied outflow velocities of $\sim 0.03 - 0.3c$. A later search of the Suzaku data archive yielded a further group of strong detections, with a median outflow velocity again $\sim 0.1c$ (Gofford et al. 2013). In addition to confirming that high velocity, highly ionized AGN winds are common, the yield from these archival searches shows the flows must typically have a large covering factor, and therefore be likely to involve substantial mass and energy fluxes.

The observed distributions of velocity, ionization parameter and column density are compatible with Eddington winds launched from close to the black hole, where the optical depth $\tau_{es} \sim 1$, and carrying the local escape velocity (King & Pounds 2003). However, as the mean luminosity in most low-redshift AGN is on average sub-Eddington, such winds are likely to be intermittent, a view supported by repeated observations and by the range of observed column densities.

For the best-quantified high-velocity outflow (the luminous Seyfert PG1211+143), in which a wide-angle flow was directly measured (Pounds & Reeves 2007, 2009), the wind appeared to have more energy than needed to unbind the likely gas mass of the observed stellar bulge. This suggested that the energy coupling of wind to bulge gas must be inefficient, as seen in the discussion following equation 5. Evidence that the fast wind in NGC 4051 is shocked at a distance of $\sim 0.1$ pc from the black hole offers an explanation of why such powerful winds do not disrupt the bulge gas: strong Compton cooling by the AGN radiation field removes most of the wind energy before it can be communicated.

2 THE OBSERVATIONAL EVIDENCE FOR ULTRA FAST OUTFLOWS

As noted above, the requirement of X-ray observations with high sensitivity and good spectral resolution over a wide energy band delayed the discovery of powerful, highly ionized winds from non-BAL AGN until the launch of Chandra and XMM-Newton. A decade after the first reports (Pounds et al. 2003, Reeves et al. 2003), high-velocity ($v \sim 0.1c$), highly-ionized winds are now established to be common in low redshift AGN.

2.1 The fast outflow in PG1211+143

Exploring the nature of the 'soft excess' in a sample of luminous Palomar Green AGN was a primary target in the Guaranteed Time programme awarded to Martin Turner, Project Scientist for the EPIC Camera on XMM-Newton (Turner et al. 2001). At that time the X-ray spectrum in AGN above $\sim 1$ keV was expected to be a rather featureless power law apart from a fluorescent emission line at $\sim 6.4$ keV from near-neutral Fe. One source, PG1211+143, showed a surprisingly 'noisy' X-ray spectrum which one of us (KP) volunteered to explore.

PG1211+143, at a redshift of 0.0809 (Marziani et al. 1996), is one of the brightest AGN at soft X-ray energies. It was classified (Kaspi et al. 2000) as a
Narrow Line Seyfert 1 galaxy (FWHM Hβ ≈ 1800 km s⁻¹), with black hole mass $\sim 4 \times 10^7 M_\odot$ and bolometric luminosity $4 \times 10^{45}$ erg s⁻¹, indicating a mean accretion rate close to Eddington.

Analysis of the unusual spectral structure in the 2001 XMM-Newton observation of PG1211+143 showed it to be dominated by blue-shifted absorption lines of highly ionized metals, providing the first evidence for a high velocity ionized outflow in a non-BAL AGN, with the initial identification of a deep blue-shifted Fe Lyman-α absorption line indicating an outflow velocity of $\sim 0.09c$ (Pounds et al. 2003). That observation, closely followed by the detection of a still higher outflow velocity from the luminous QSO PDS 456 (Reeves et al. 2003), attracted wide attention, potentially involving the ejection of a significant fraction of the bolometric luminosity, and perhaps characteristic of AGN accreting near the Eddington rate (King & Pounds 2003).

 Appropriately for such an unexpected discovery, the validity of the high velocity in PG1211+143 was not unchallenged. The near-coincidence of the observed absorption line blueshift and the redshift of the host galaxy was a concern, notwithstanding the uncomfortably high column density of heavy metals implied by a local origin. Then, in a detailed modelling of the soft X-ray RGS data, Kaspi & Behar (2006) found only a much lower velocity. Any doubts relating to the absorption being local were removed, however, by a revised velocity of $0.13 - 0.15c$ based on the inclusion of additional absorption lines from intermediate-mass ions (Pounds & Page 2006), and when repeated observations of PG1211+143 demonstrated that the strong Fe K absorption line was variable over several years (Reeves et al. 2008).

Here we use the 2001 XMM-Newton observation of PG1211+143 with the pn camera (Struder et al. 2001) to illustrate the two methods used then – and since – to parameterise the ionized outflow. Figure 1 shows the ratio of EPIC pn data to a simple power law continuum, with a deep absorption line seen near 7 keV and additional spectral structure at $\sim 1 - 4$ keV.

Fitting a negative Gaussian to the deep $\sim 7$ keV absorption line (figure 2, top panel), finds an observed line energy of $7.06 \pm 0.02$ keV, or $7.63 \pm 0.02$ keV at the AGN redshift of 0.0809. The line is clearly resolved, with $1\sigma$ width $\sim 100 \pm 30$ eV. Assuming identification with the Fe XXV resonance (6.70 keV rest energy), the blueshifted line corresponds to an AGN outflow velocity $v \sim 0.122 \pm 0.005c$. The most likely a priori alternative identification, with the Fe XXVI Lyman–α line (6.97 keV rest energy), conservatively adopted in the initial analysis (Pounds et al. 2003), yields a lower outflow velocity $v \sim 0.095 \pm 0.005c$.

An alternative procedure, which also provides additional parameters of the gas flow, requires full spectral modelling, as in Pounds & Page (2006), and more widely in recent outflow studies (Section 2.3). For the 2001 XMM-Newton pn spectrum of PG1211+143, modelling the absorption from 1–10 keV with a photoionized gas derived from the XSTAR code of Kallman et al. (1996) gives an excellent fit, for a column density $N_H \sim 3.2 \times 10^{23}$ cm⁻², ionization parameter $log\xi = 2.7 \pm 0.1$ erg cm s⁻¹, and an outflow velocity (in the AGN rest frame) $v \sim 0.149 \pm 0.003c$. The model profile (figure 2, lower panel) shows significant inner shell absorption components to the low energy wing of the 1s-2p resonance line which explains why simply identifying the absorption near 7 keV with the 6.7 keV rest energy of Fe XXV gives too low a velocity.

Although individually weaker than the Fe absorption, the combination of resonance lines of He– and H–like Mg, Si, S and Ar in the broadband spectral fit
is evidently driving the spectral fit. That conclusion is confirmed with Gaussian fits to corresponding absorption features in figure 1, which find a weighted observed blueshift of $0.055 \pm 0.05$ and outflow velocity (at the AGN redshift) of $v \sim 0.14 \pm 0.01c$, a value consistent with that found from spectral modelling, but significantly higher than from simply identifying the $\sim 7$ keV absorption line with the resonance 1s-2p transitions of either FeXXV or FeXXVI.

An interesting by-product of the XSTAR modelling in the above case is that the observed broadening of the $\sim 7$ keV absorption line does not require high turbulence (we used grid 25 with $v_{\text{turb}}$ of 200 km s$^{-1}$) or an accelerating/decelerating flow. Instead, intrinsically narrow absorption components remain consistent with a radial outflow, coasting post-launch.

2.2 Mass rate and mechanical energy in the PG1211+143 outflow

Although the detection of high-speed winds in a substantial fraction of bright AGN suggests most such flows have a large covering factor, PG1211+143 is one of very few where a wide angle flow has been demonstrated directly.

Using stacked data from 4 XMM-Newton observations between 2001 and 2007, Pounds & Reeves (2007, 2009) examined the relative strength of ionized emission and absorption spectra modelled by XSTAR to estimate the covering factor and collimation of the outflowing ionized gas. The summed pn data of PG1211+143 also shows a well defined P Cygni profile in the Fe K band (TBTF 3), the classical
Fig. 2 (top) A Gaussian fit to the $\sim 7$ keV absorption feature finds a line energy of $7.06 \pm 0.02$ keV with (1$\sigma$) width $100 \pm 30$ eV. Identification with the Fe XXV 1s-2p resonance line (6.70 keV rest energy) gives an outflow velocity $v \sim 0.12 \pm 0.01\,c$. (lower)Alternative modelling with a photoionised gas over the wider 1–10 keV spectral band yields a good fit with a relatively high column density $N_H \sim 3.2 \pm 0.7 \times 10^{23}$ cm$^{-2}$, moderate ionisation parameter $\log \xi = 2.7 \pm 0.1$ erg cm s$^{-1}$, and outflow velocity of $v \sim 0.15 \pm 0.01\,c$. The Fe XXV absorption line profile is seen to include lower energy components due to the addition of one or more L-shell electrons, showing why the simple Gaussian fit gives too low a velocity.
signature of an outflow, with emission and absorption components of comparable equivalent width. Both methods indicated a covering factor $b(= \Omega/2\pi)$ of $0.75 \pm 0.25$. Analysis of a Suzaku observation of PG1211+143 gives a similar result (Reeves et al. 200TBTF), with an intrinsic emission component of $\sim 6.5$ keV and width of $\sigma \sim 250$ eV, corresponding to a flow cone of half angle $\sim 50$ deg, assuming velocity broadening in a radial flow.

The outflow mass rate and mechanical energy can then be estimated, since for a uniform radial outflow of velocity $v$ the mass rate is:

$$\dot{M}_{\text{out}} \approx 4\pi bn r^2 m_p v,$$

where $n$ is the gas density at a radial distance $r$, and $m_p$ is the proton mass.

The observed values for PG1211+143 find a mass loss rate of $\dot{M}_{\text{out}} \sim 7 \times 10^{25}$ gm s$^{-1}$ ($\sim 2.5 M_\odot$ yr$^{-1}$), and mechanical energy $\sim 4.5 \times 10^{44}$ erg s$^{-1}$ (Pounds & Reeves 2009).

The mass loss rate is comparable to the Eddington accretion rate $\dot{M}_{\text{Edd}} = 1.3 M_\odot$ yr$^{-1}$ for a supermassive black hole of mass $\sim 4 \times 10^7 M_\odot$ accreting at an efficiency of 10%, while the outflow mechanical energy is only $\sim 6\%$ of the Eddington luminosity, close to that predicted by continuum driving (equation 5 in Section 3 below). As noted elsewhere that energy flow rate would be more than sufficient to unbind the gas of the host galaxy bulge if all its energy were efficiently communicated.
2.3 High speed winds are common

The evidence for high velocity winds as an important property of AGN remained dependent on the prototype case of PG1211+143 for several years, with fast outflows in two BAL AGN (Chartas et al. 2002) and in the most luminous low redshift QSO PDS 456 (Reeves et al., 2003 O’Brien et al. 2005) seen as rare objects. That began to change with the detection of a highly significant outflow of velocity $\sim 0.1c$ in the Seyfert 1 galaxy IC4329A (Markowitz et al. 2006), and several outflow detections in the range $\sim 0.14 - 0.2c$ in multiple observations of Mrk 509 (Dadina et al. 2005). A review in 2006 (Cappi et al. 2006) listed 7 non-BAL objects with outflows of $v \sim 0.1c$ and several with red-shifted absorption lines.

A major step forward came with the results of an XMM-Newton archival search of bright AGN by Tombesi et al. (2010), finding strong statistical evidence in 15 of 42 radio-quiet objects of blue-shifted iron K absorption lines, identification with FeXXV or XXVI resonance absorption lines implying ultrafast outflow (UFO) velocities up to $\sim 0.3c$, and clustering near $v \sim 0.1c$. A later analysis based on broad-band modelling with XSTAR photoionized grids (Tombesi et al. 2011) led to several revised velocities and confirmed that the outflows were typically highly ionized, with log $\xi \sim 3 - 6$ erg cm s$^{-1}$, with column densities in the range $N_H \sim 10^{22} - 10^{24}$ cm$^{-2}$. A similar search of the Suzaku data archive (Gofford et al. 2013) yielded a further group of UFO detections, finding significant absorption in the Fe K band in 20 (of 51) AGN with velocities up to $\sim 0.3c$ and a flatter distribution than the XMM-Newton sample, but a median value again $v \sim 0.1c$.

Figure 4 brings together the results from the spectral modelling analyses of the XMM-Newton and Suzaku surveys. We follow Tombesi et al. (2011) in defining UFOs as having outflow velocities greater than $10^4$ km s$^{-1}$, to discriminate against WAs or post-shock flows (Sections 2.4 and 4). The velocity plot shows a peak at $\sim 0.1c$, with a tail extending to $\sim 0.3c$. In terms of the continuum-driving Black Hole Winds model (King & Pounds 2003) the higher velocities would imply a higher value of the accretion efficiency $\eta$, with the future potential for such observations to provide a measure of black hole spin. Equation (22) also suggests the low velocity tail in both the Tombesi et al. and Gofford et al. distributions could relate to primary outflows formed at a higher accretion ratio (but see Section 3.1).

Figure 4 also shows the distribution of ionization parameter and absorption column density from the surveys of Tombesi et al. (2011) and Gofford et al. (2013). The high ionization parameter, peaking near log $\xi \sim 4$, explains why the detection of UFOs has been almost exclusively from X-ray observations in the Fe K band, leaving open the possibility that fully ionized outflows (also consistent with continuum driving) will become detectable when the AGN luminosity (and hence ionization) falls. In assessing observational data it is important to note that for a radial outflow the observed column density is a line-of-sight integration over the flow time, dominated by the higher density at small radii, while the ionization parameter is governed by the current AGN luminosity. The column density, which generally lies below $N_H \sim 10^{24}$ cm$^{-2}$, can vary rapidly and turns out to be a powerful diagnostic of the flow history and dynamics. We return to the observability of UFOs in Section 3.3.
2.4 Evidence for a shocked flow

The mechanical energy in a fast wind, such as that in PG1211+143, was noted in Section 2.2 to be incompatible with the continued growth of the black hole and stellar bulge of the host galaxy, unless the flow is short-lived or the coupling of wind energy to bulge gas is highly inefficient. A recent XMM-Newton observation of the narrow-line Seyfert galaxy NGC 4051 has provided the first evidence of a fast ionized wind being shocked, with subsequent strong cooling leading to most of the initial flow energy being lost before it can be communicated to the bulge gas. We outline a possible scenario for that event below.

NGC 4051 was found in the XMM-Newton archival search to have a high velocity wind during an observation in 2002 when the source was in an unusually low state, the initial identification with Fe XXVI Lyman-α in Tombesi et al. (2010) indicating a velocity of ∼0.15c. In a full spectral fit (Tombesi et al. 2011) identification with Fe XXV was preferred, with an increased velocity ∼0.20c. Significantly, in a 2001 observation of NGC 4051, when the X-ray flux was much higher, a strong outflow was detected at ∼6000 km s⁻¹, but no ultra-fast wind was seen.
It seems that the detection of a UFO in NGC 4051 is unusually dependent on the source flux, with evidence for a high velocity wind \((v \sim 0.13c)\) again found only during periods when the ionizing continuum was low during a further XMM-Newton observation in 2009 (Pounds & Vaughan 2012). An additional factor may be the low redshift \((z=0.00234)\) of NGC 4051, which makes a high velocity wind more difficult to detect with current observing facilities.

The 600 ks XMM-Newton observation of the Seyfert 1 galaxy NGC 4051 in 2009, extending over 6 weeks and 15 spacecraft orbits, broke new ground by finding an unusually rich absorption spectrum with multiple outflow velocities, in both RGS (den Herder et al. 2001) and EPIC spectra, up to \(\sim 9000 \text{ km s}^{-1}\) (Pounds & Vaughan 2011a). Inter-orbit variability is seen in both absorption and emission lines, with strong recombination continua (RRC) and velocity-broadened resonance lines providing constraints on the dynamics and geometry of the putative post-shock flow (Pounds and Vaughan 2011b, 2012).

2.5 A self-consistent model for the shocked wind in NGC 4051

More complete modelling of both RGS and EPIC pn absorption spectra of NGC 4051 found a highly significant correlation of outflow velocity and ionization state (figure 5), as expected from mass conservation in a post–shock flow (King 2010, Pounds & King 2013). The additional analysis also found a range of column densities to be required by the individual XSTAR absorption components, suggesting

\[ \text{Fig. 5 The outflow velocity and ionization parameters for 6 XSTAR photoionised absorbers used to fit the RGS and EPIC spectra of NGC 4051, together with a high point representative of the pre-shock wind, show the linear correlation expected for a mass-conserved cooling flow (see Pounds and King 2013).} \]
Fig. 6 Fe K profiles from observations of NGC 4051 several days apart show an increased level of ionization coinciding with a hard X-ray flare (data from Pounds and Vaughan 2012). The ratio of resonance absorption lines of Fe XXV and Fe XXVI is a sensitive measure of the ionization state of the absorbing gas.

an inhomogeneous shocked flow, perhaps with lower ionization gas clumps or filaments embedded in a more extended, lower density and more highly ionized flow.

Theoretical considerations suggested a key factor in determining the structure of the post-shock flow was likely to be the cooling time, as discussed in more detail in Section 4. In particular, the fate of a fast wind depends on the distance it travels before colliding with the ISM or slower-moving ejecta, with Compton cooling dominating for a shock occurring sufficiently close to the AGN continuum source.

Importantly, flux-linked variations in the ratio of FeXXV to Fe XXVI absorption in the 2009 XMM-Newton observation (figure 6) provided a measure of the Compton cooling time, the mean flow speed then determining the shell thickness of the hotter, more highly ionized flow component. The detection of strong recombination continua (RRC) in the soft X-ray spectra furthermore suggested an increasing density in the decelerating post-shock flow, with two-body cooling becoming increasingly important.

To pursue that idea we note that at the (adiabatic) shock the free–free (thermal bremsstrahlung) and Compton cooling times are

\[
t_{ff} \simeq 3 \times 10^{11} \frac{T^{1/2}}{N} \text{ s} = 20 \frac{R_{16}^2}{M_7 \dot{m}} \text{ yr}
\]

and

\[
t_C = 10^{-4} \frac{R_{16}^2}{M_8} \text{ yr}
\]

respectively (see King et al. 2011: here \(T, N\) are the postshock temperature and number density, \(R_{16}\) is the shock radius in units of \(10^{16}\) cm, \(M_7\) is the black hole mass in units of \(10^7 M_\odot\), and \(\dot{m} \sim 1\) is the Eddington factor of the mass outflow rate).

After the adiabatic shock, the gas cools rapidly via inverse Compton cooling, while its density rises as \(N \propto T^{-1}\) (pressure is almost constant in an isothermal shock), and

\[
t_{ff} \propto \frac{T^{1/2}}{N} \propto T^{3/2},
\]

(12)
which means that the free–free cooling time decreases sharply while the Compton
time does not change. Eventually free–free (and other atomic two–body processes)
become faster than Compton when \( T \) has decreased sufficiently below the original
shock temperature \( T_s \sim 1.6 \times 10^{10} \) K. From (10, 11) above this requires
\[
\frac{T^3}{T_s^3/2} < 5 \times 10^{-5} \tag{13}
\]
or
\[
T < 2 \times 10^7 \text{ K}. \tag{14}
\]

The temperature of ionization species forming around a few keV is therefore
likely to be determined by atomic cooling processes rather than Compton cool-
ing. The strong recombination continua in NGC 4051 (Pounds & Vaughan 2011b,
Pounds & King 2013) are direct evidence for that additional cooling, with the RRC
flux yielding an emission measure for the related flow component. In particular,
the onset of strong two-body cooling results in the lower-ionization, lower-velocity
gas being confined in a relatively narrow region in the later stages of the post-
shock flow. The structure and scale of both high and low ionization flow regions
can be deduced from the observations and modelling parameters.

For the highly ionized post-shock flow, the iron Ly–\( \alpha \) to He–\( \alpha \) ratio will be
governed by the ionizing continuum and recombination time. Significant varia-
tions in this ratio are found on inter-orbit timescales (Pounds & Vaughan 2012),
with an example shown in figure 6. For a mean temperature of \( \sim 1 \) keV, and
recombination coefficient of \( 4.6 \times 10^{-12} \) cm\(^3\) s\(^{-1}\) (Verner & Ferland 1996),
the observed recombination timescale of \( \sim 2 \times 10^5 \) s corresponds to an average
particle density of \( \sim 4 \times 10^6 \) cm\(^{-3}\). Comparison with a relevant absorption column
\( N_H \sim 4 \times 10^{22} \) cm\(^{-2}\) from the XSTAR modelling indicates a column length scale of
\( \sim 10^{16} \) cm. Assuming a mean velocity of the highly ionized post-shock flow of 6000
km s\(^{-1}\), the observed absorption length corresponds to a flow time \( \sim 1.7 \times 10^7 \) s
(0.6 yr). Equation (11) finds a comparable cooling time for NGC 4051 at a shock
radius \( R \sim 10^{17} \) cm.

For the low-ionization flow component, decay of strong RRC of NVII, CVI
and CV (Pounds & Vaughan 2011b, Pounds & King 2013), occurs over \( \sim 2 – 6 \) days.
With an electron temperature from the mean RRC profile of \( \sim 5 \) eV, and
recombination coefficient for CVI of \( \sim 10^{-11} \) cm\(^3\) s\(^{-1}\) (Verner & Ferland 1996),
the observed RRC decay timescale corresponds to a (minimum) electron density
of \( \sim 2 \times 10^6 \) cm\(^{-3}\). A column density of \( 1.5 \times 10^{21} \) cm\(^{-2}\) from modelling absorption
in the main low-ionization flow component then corresponds to an absorbing path
length of \( 7 \times 10^{14} \) cm.

The RRC emission flux provides a consistency check on the above scaling.
Assuming solar abundances, and 30 percent of recombinations direct to the ground
state, a CVI RRC flux of \( \sim 10^{-5} \) photons cm\(^{-2}\) s\(^{-1}\) corresponds to an emission
measure of \( \sim 2 \times 10^{62} \) cm\(^{-3}\), assuming a Tully–Fisher distance to NGC 4051 of
15.2 Mpc. With a mean particle density of \( \sim 2 \times 10^6 \) cm\(^{-3}\) the emission volume
\( (4\pi R^2 \Delta R) \) is \( \sim 5 \times 10^{19} \) cm\(^3\). Assuming a spherical shell geometry of the flow,
with fractional solid angle \( b \), shell thickness \( \Delta R \sim 7 \times 10^{14} \) cm, and shell radius
\( R \sim 10^{17} \) cm, the measured RRC flux is reproduced for \( b \sim 0.5 \).

Although this excellent agreement may be fortuitous given the approximate
nature and averaging of several observed and modelled parameters, the mutual
Fig. 7 Schematic view of the shock pattern resulting from the impact of a black hole wind (blue) on the interstellar gas (red) of the host galaxy. The accreting supermassive black hole drives a fast wind (velocity $v \sim \eta c/\dot{m} \sim 0.1c$), whose ionization state makes it observable in X–ray absorption lines. It collides with the ambient gas in the host galaxy and is slowed in a strong shock. The inverse Compton effect from the quasar’s radiation field rapidly cools the shocked gas, removing its thermal energy and strongly compressing and slowing it over a very narrow radial extent. In the most compressed gas, two–body cooling becomes important, and the flow rapidly cools and slows over an even narrower region. In NGC 4051 this region is detected in the soft X–ray spectrum, where absorption (and emission) are dominated by the lighter metals. The cooled gas exerts the preshock ram pressure on the galaxy’s interstellar gas and sweeps it up into a dense shell (‘snowplow’). The shell’s motion then drives a milder outward shock into the ambient interstellar medium. This shock ultimately stalls unless the SMBH mass has reached the value $M_\sigma$ satisfying the $M \sim \sigma$ relation (from Pounds and King 2013)

Given that only blueshifted RRC emission is seen, $b \sim 0.5$ is consistent with a wide-angle flow, visible only on the near side of the accretion disc.

Figure 7 illustrates the main features of the overall NGC 4051 outflow, a fast primary wind being shocked at a radial distance of order 0.1pc, within the zone of influence of an SMBH of $1.7 \times 10^9 M_\odot$. The initially hot gas then cools in the strong radiation field of the AGN, with a Compton cooling length determining the absorption columns of Fe and the other heavy metal ions. Two–body recombination provides additional cooling as the density rises downstream, eventually becoming dominant. Absorption (and emission) in the soft X–ray band is located primarily in this thinner outer layer of the post–shock flow.
It is interesting to note that similar shocking of fast outflows provides a natural link between UFOs and the equally common ‘warm absorbers’ in AGN (Tombesi et al. 2013). While the onset of strong two–body cooling, resulting in the intermediate column densities being small, might explain why evidence for intermediate-flow velocities has awaited an unusually long observation of a low mass AGN, the accumulated ‘debris’ of shocked wind and ISM could be a major component of the ‘warm absorber’. See Section 7.4.1 for a discussion.

2.6 Variability of UFOs

While it is likely that powerful winds blow continuously in AGN in rapid growth phases, it is important to note that the existing observations of UFOs are restricted to bright, low-redshift AGN, \( z \leq 0.1 \), where the X-ray fluxes are sufficient to yield high quality spectra. Repeated observation of several bright AGN frequently show changes in the equivalent width in the primary Fe K absorption line.

Variability in the strength of blueshifted Fe-K absorption over several years in PG1211+143 was first noted in a comparison of the initial XMM-Newton and Chandra observations (Reeves et al. 2008), and confirmed by repeated XMM-Newton observations (Pounds and Reeves 2009). Multiple observations of the luminous Seyfert 1 galaxy Mrk 509 (Cappi et al. 2009) found variations in both intensity and blueshift of Fe K absorption lines. The archival searches provide the most comprehensive variability data, with repeated observations of several AGN demonstrating that variability of absorption line equivalent width (EW) over several years is common. More rapid variability in EW, over a few months, is reported in the XMM-Newton archive for Mrk 509, Mrk 79 and Mrk 841, with both velocity and EW change in \( \leq 2 \) days for Mrk766.

In addition, the ‘hit rate’ of UFOs for multiply-observed AGN in the archival XMM-Newton data search (Tombesi et al. 2010) was relatively low, being 1 of 6 observations for NGC4151, MCG-6-30-15 (0/5), Mrk509 (3/5), NGC 4051(1/2), Mrk79 (1/3), Mrk205 (1/3), and Mrk290 (1/4). Overall, though 101 suitably extended observations yielded 36 narrow absorption line detections in the Fe K band, only 22 were observed at \( > 7 \) keV. While the UFO ‘hit rate’ of \( \sim 22\% \) is a lower limit set by the sensitivity of available exposures it seems clear that the fast outflows currently being detected in low-redshift AGN are far from continuous.

3 BLACK HOLE WINDS

3.1 The Eddington Accretion Ratio in AGN

We have seen from Section 2 that a large fraction of observed AGN show ultrafast outflows. Unless we view every AGN from a very particular angle (so implying a much larger total population) this must mean that these winds have large solid angles \( 4\pi b \) with \( b \sim 1 \), i.e. they are quasispherical. We recall that UFOs are observed to have total scalar momenta \( \sim L_{\text{Edd}}/c \), where \( L_{\text{Edd}} \) is the Eddington luminosity of the SMBH. We can argue that on quite general grounds, SMBH mass growth is likely to occur at accretion rates close to the value \( \dot{M}_{\text{Edd}} = L_{\text{Edd}}/\eta c^2 \) which would produce this luminosity. As we noted earlier, the Soltan (1982) relation shows
that the largest SMBH gained most of their mass by luminous accretion, i.e. during AGN phases. But the fraction of AGN among all galaxies is small, strongly suggesting that when SMBH grow, they are likely to do so as fast as possible. The maximum possible rate of accretion from a galaxy bulge with velocity dispersion $\sigma$ is the dynamical value

$$\dot{M}_{\text{dyn}} \simeq f_g \sigma^3 \frac{G}{R}, \quad (15)$$

where $f_g$ is the gas fraction. This rate applies when gas which was previously in gravitational equilibrium is disturbed and falls freely, since one can estimate that the gas mass was roughly $M_g \sim \sigma^2 f_g R/G$. Once this is destabilized it must fall inwards on a dynamical timescale $t_{\text{dyn}} \sim R/\sigma$. This gives the result (15), since $\dot{M}_{\text{dyn}} \sim M_g/t_{\text{dyn}}$.

Numerically we have

$$\dot{M}_{\text{dyn}} \simeq 280 \sigma_{200}^3 M_\odot \text{yr}^{-1} \quad (16)$$

where we have taken $f_g = 0.16$, the cosmological baryon fraction of all matter. We have

$$\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{\eta c^2} = \frac{4\pi GM}{\eta c} \quad (17)$$

where $L_{\text{Edd}}$ is the Eddington luminosity and $\kappa$ is the electron scattering opacity. With $\eta = 0.1$ and black hole masses $M$ close to the observed $M - \sigma$ relation (2) we find

$$\dot{M}_{\text{Edd}} \simeq 4.4 \sigma_{200}^4 M_\odot \text{yr}^{-1} \quad (18)$$

and an Eddington accretion ratio

$$\dot{m} < \frac{\dot{M}_{\text{dyn}}}{\dot{M}_{\text{Edd}}} \simeq \frac{64}{\sigma_{200}} \simeq 54 \frac{1}{M_\odot^{1/4}} \quad (19)$$

Thus even dynamical infall cannot produce extremely super–Eddington accretion rates on to supermassive black holes. But the rate (15) is already a generous overestimate, since it assumes that the infalling gas instantly loses all its angular momentum. Keeping even a tiny fraction of this instead forces the gas to orbit the black hole and form an accretion disc, which slows things down drastically. Gas spirals inwards through a disc on the viscous timescale

$$t_{\text{visc}} = \frac{1}{\alpha} \left( \frac{R}{H} \right)^2 \left( \frac{R^3}{GM} \right)^{1/2} \quad (20)$$

where $\alpha \sim 0.1$ is the Shakura–Sunyaev viscosity parameter, while the disc aspect ratio $H/R$ is almost constant with radius, and typically close to $10^{-3}$ for an AGN accretion disc (e.g. Collin–Souffrin & Dumont 1990). Then $t_{\text{visc}}$ approaches a Hubble time even for disc radii of only 1 pc. Gas must evidently be rather accurately channelled towards the SMBH in order to accrete at all, constituting a major problem for theories of how AGN are fuelled.

Given all this, we see that while it is possible for AGN accretion rates to reach Eddington ratios $\dot{m} \sim 1$, significantly larger ones are unlikely unless the SMBH mass is far below the $M - \sigma$ value appropriate to the galaxy bulge it inhabits. In other words, only relatively modest values $\dot{m} \sim 1$ of the Eddington ratio are likely in SMBH growth episodes.
Indirect evidence supporting this view comes from stellar–mass compact binary systems. The dynamical rate is relatively much larger here, as the equivalent of \( \dot{M} \approx v_{\text{orb}}^3 \) is \( \dot{M} \approx \frac{v_{\text{orb}}^3}{G} \sim \frac{M^2}{P} \), with \( v_{\text{orb}} \) the orbital velocity of a companion star in a binary of period \( P \). This implies rates approaching a solar mass per few hours in many cases, if dynamical accretion ever occurs. These systems have highly super–Eddington apparent luminosities, probably as the result of geometric collimation (cf King et al. 2001). But significantly there are no obvious AGN analogues of the ultra-luminous X-ray sources (ULXs), suggesting that Eddington ratios \( \dot{m} \gg 1 \) are very unusual or absent in AGN.

3.2 Eddington Winds

Given this, we can crudely model the UFOs discussed in Section 2 as quasispherical winds from SMBH accreting at modest Eddington ratios \( \dot{m} = \dot{M}/M_{\text{Edd}} \sim 1 \). Winds like this have electron scattering optical depth \( \tau \sim 1 \), measured inwards from infinity to a distance of order the Schwarzschild radius (cf eq. (27) below). So on average every photon emitted by the AGN scatters about once before escaping to infinity. Since electron scattering is front–back symmetric, each photon on average gives up all its momentum to the wind, and so the total (scalar) wind momentum should be of order the photon momentum, or

\[
\dot{M}_w v \simeq \frac{L_{\text{Edd}}}{c},
\]

where \( v \) is the wind’s terminal velocity. The winds of hot stars obey relations like this. For accretion from a disc, as here, the classic paper of Shakura & Sunyaev (1973) finds a similar result at super–Eddington mass inflow rates: the excess accretion is expelled from the disc in a quasispherical wind.

Equation (17) now directly gives the wind terminal velocity as

\[
v \simeq \frac{\eta \dot{m} c}{1} \sim 0.1c.
\]

From eq. (22) we get the instantaneous wind mechanical luminosity as

\[
L_{\text{BH wind}} = \frac{\dot{M}_w v^2}{2} \simeq \frac{L_{\text{Edd}} v}{2} \sim \frac{\eta}{2} L_{\text{Edd}} \simeq 0.05 L_{\text{Edd}}.
\]

This relation turns out to be highly significant (see Sections 5.3, 7.3).

Ohsuga & Mineshige (2011) show in detail that winds with these properties (their Models A and B) are a natural outcome of mildly super–Eddington accretion. In particular their Model A and B winds are predicted (cf their Figure 3) to have mechanical luminosities \( \sim 0.1 L_{\text{Edd}} \), in rough agreement with equation (23).

Compared with the original disparity \( E_{\text{BH}} = \eta \dot{m} c^2 \sim 2000 E_{\text{gas}} \) between black hole and bulge gas binding energies outlined in the Introduction, we now have a relation

\[
E_{\text{BH wind}} \simeq \frac{\eta \dot{m}^2}{2} Mc^2 \sim 100 E_{\text{gas}}
\]

between the available black hole wind mechanical energy and the bulge binding energy. Although the mismatch is less severe, it still strongly suggests that the bulge gas would be massively disrupted if it experienced the full mechanical luminosity.
emitted by the black hole for a significant time. So the coupling of mechanical energy to the host ISM cannot be efficient all the time (see the discussion in Section 2). We show how this works in Section 4.

3.3 Observability

As we have seen, observations frequently give the hydrogen column density $N_H$ through a UFO wind from the X–ray absorption spectrum. We can show that this quantity determines whether a given UFO wind is observable or not. Using (21) in the mass conservation equation

$$\dot{M}_w = 4\pi b r^2 v \rho(r),$$

where $\rho(r)$ is the mass density, we find the equivalent hydrogen column density of the wind as

$$N_H \simeq \int_{R_{in}}^{\infty} \rho m_p d\rho = \int_{R_{in}}^{\infty} \frac{\dot{M}_w}{4\pi b r^2 v} d\rho \simeq \frac{L_{Edd}}{4\pi b m_p R_{in} c^2 r^2},$$

where $R_{in}$ is its inner radius, $m_p$ is the proton mass, and we have used (21) at the last step. From the definition of $L_{Edd}$ we find the wind electron scattering optical depth

$$\tau = N_H \sigma_T \simeq \frac{GM}{b v^2 R_{in}}$$

with $\sigma_T \simeq \kappa m_p$ the Thomson cross-section. This shows self-consistently that the scattering optical depth $\tau$ of a continuous wind is $\sim 1$ (cf King & Pounds 2003, equation 4) at the launch radius $R_{launch} \sim GM/bv^2 = (c^2/2bv^2)R_s \simeq 50R_s$.

The measured values of $N_H$ (Tombesi et al. 2011, Gofford et al. 2013, Figure 4) are always smaller than the value $N_H \simeq 1/\sigma_T \simeq 10^{24}$ cm$^{-2}$ for a continuous wind, and actually lie in the range $N_{22} \sim 0.3 - 30$, where $N_{22} = N_H/10^{22}$ cm$^{-2}$. It is perhaps not surprising that observations do not show any UFO systems with $N_H > 10^{24}$ cm$^{-2}$. These AGN would be obscured at all photon energies by electron scattering, and perhaps difficult to see. Although such systems might be common, we probably cannot detect them. To have a good chance of seeing a UFO system we need a smaller $N_H$, so from (27) the inner surface $R_{in}$ of the wind must be larger than $R_{launch}$. This is only possible if all observed UFOs are episodic, i.e. we see them some time after the wind from the SMBH has switched off. In this sense UFOs are more like a series of sporadically–launched quasispherical shells than a continuous outflow. The $N_H$ value of each shell is dominated by the gas near its inner edge (cf (26), so we probably at most detect only the inner edge of the most recently–launched shell. We can quantify this by setting $R_{in} = vt_{off}$, where $t_{off}$ is the time since the launching of the most recent wind episode ended. Using (27) gives

$$t_{off} = \frac{GM}{b v^2 N_H \sigma_T} \simeq \frac{3M_7}{b_0 c_0 v_{0.1} N_{22}} \text{ months},$$

where $v_{0.1} = v/0.1c$.

As seen in Figure 4, all UFOs have $N_{22} \sim 0.3 - 30$, and most of the SMBH masses are $\sim 10^7 M_\odot$. Evidently the launches of most observed UFO winds halted weeks or months before the observation. At first sight this is surprising. The
strength of the characteristic blueshifted absorption features defining UFOs is closely related to $N_H$. These features would be still stronger if there were UFOs with $N_{22} > 100$, but none are seen. We note from (28) that observing a UFO like this would require us to catch it within days of launch. Given the relatively sparse coverage of X-ray observations of AGN this is unlikely. So the apparent upper limit to the observed $N_H$ may simply reflect a lack of observational coverage, and implies that most UFOs are short-lived.

The lower limit to $N_H$ in the Tombesi et al. sample is also interesting. Once $N_H$ is smaller than some critical value, any blueshifted absorption lines must become too weak to detect. The strongest are the resonance lines of hydrogen– and helium–like iron, which have absorption cross–sections $\sigma_{Fe} \simeq 10^{-18} \text{cm}^2$. Given the abundance by number of iron as $Z_{Fe} = 4 \times 10^{-5}$ times that of hydrogen, the condition that one of these lines should have significant optical depth translates to $Z_{Fe}N_H\sigma_{Fe} > 1$ or $N_{22} > 2.5$. This is similar to the lowest observed values. From (28) this means that current observations cannot detect UFO winds launched more than a few months in the past, because the blueshifted iron lines will be too weak. Even these observed UFOs should gradually decrease their $N_H$ and become unobservable if followed for a few years. We see in Section 5 that the UFO wind typically travels $\sim 10M_7$ pc or more before colliding with the host galaxy’s interstellar gas, which takes $t_{\text{coll}} / v \sim 300M_7v_{-1}^{-1}$ yr. Finally, a UFO may be unobservable simply because it is too strongly ionized, so that no significant $N_H$ can be detected.

All this means that the state of the AGN seen in a UFO detection does not necessarily give a good idea of the conditions required to launch it. In particular, the AGN may be observed at a sub–Eddington luminosity, even though one might expect luminosities $\sim L_{\text{Edd}}$ to be needed for launching the UFO. This may be the reason why AGN showing other signs of super–Eddington phenomena (e.g. narrow–line Seyfert 2 galaxies) are nevertheless seen to have sub–Eddington luminosities most of the time (e.g. NGC 4051; Denney et al. 2009): the rather short wind episodes are launched in very brief phases in which accretion is slightly super–Eddington, whereas the long–term average rate of mass gain may be significantly sub–Eddington.

In summary, it is likely that current UFO coverage is remarkably sparse. We cannot see a continuous wind at all. We can only see an episodic wind shell shortly after launch, and then only for a tiny fraction $t_{\text{off}} / t_{\text{coll}} \sim 10^{-3}v_1^{-1}N_{-22}^{-1}$ of its $\sim 300 – 3000$ yr journey to collision with the host ISM. So it seems that the vast majority of UFO wind episodes remain undetected: more AGN must produce them than we observe, and the known UFO sources may have far more episodes than we detect.

All this has important consequences for how we interpret observations in discussing feedback. The most serious is that the most powerful form of feedback – from AGN at the Eddington limit producing continuous winds – is probably not directly observable at all.

3.4 The Wind Ionization State, and BAL QSOs

The ionization parameter

$$\xi = \frac{L_i}{NHR^2}$$ (29)
essentially fixes the ionization state of a black hole wind, and so determines
which spectral lines are observed. Here $L_i = l_i L_{\text{Edd}}$ is the ionizing luminosity, with
$l_i < 1$ a dimensionless parameter specified by the quasar spectrum, and $N = \rho/\mu m_p$
is the number density of the UFO gas. We use (22, 25) to get
\[
\xi = 3 \times 10^4 \eta_{0.1} l_2 \dot{m}^{-2} = 3 \times 10^3 \eta_{0.1} l_2
\]  
where $l_2 = l_i/10^{-2}$, and $\eta_{0.1} = \eta/0.1$.

This relation shows how the wind momentum and mass rates determine its
ionization parameter and so its line spectrum as well as its speed $v$. Given a quasar
spectrum $L_\nu$, the ionization state has to arrange that the threshold photon energy
defining $L_i$, and the corresponding ionization parameter $\xi$, together satisfy (30). This
shows that the excitation must be high: a low threshold photon energy (say in the infrared)
would imply a large value of $l_2$, but then (30) gives a high value of $\xi$
and so predicts the presence of very highly ionized species, physically incompatible
with such low excitation.

For suitably chosen continuum spectra (30) has a range of solutions. A given
spectrum might in principle allow more than one solution, the applicable one being
specified by initial conditions. For a typical quasar spectrum, an obvious self–
consistent solution of (30) is $l_2 \simeq 1$, $\dot{m} \simeq 1$, $\xi \simeq 3 \times 10^4$. Here the quasar radiates
the Eddington luminosity. We can also consider situations where the quasar’s
luminosity has decreased after an Eddington episode but the wind is still flowing,
with $\dot{m} \simeq 1$. Then the ionizing luminosity $10^{-2} l_2 L_{\text{Edd}}$ in (30) is smaller, implying
a lower $\xi$. As an example, an AGN of luminosity $0.3 L_{\text{Edd}}$ would have $\xi \sim 10^4$.
This gives a photon energy threshold appropriate to FeXXV and Fe XXVI (i.e.
$h\nu_{\text{threshold}} \sim 9$ keV). We conclude that Eddington winds from AGN are likely to
have velocities $\sim 0.1c$, and show the presence of helium– or hydrogen-like iron in
accord with the absorption reported in Section 2. Zubovas & King (2013) show
that this probably holds even for AGN which are significantly sub–Eddington.

We can see from (22) that a larger Eddington factor $\dot{m}$ is likely to produce
a slower wind. From comparison with ULXs (see Section 3.1) we also expect
the AGN radiation to be beamed away from a large fraction of the UFO, which should
therefore be less ionized, and as a result more easily detectable than the small
fraction receiving the beamed radiation. These properties – slower, less ionized winds – characterize BAL QSO outflows, perhaps suggesting that systems with
larger $\dot{m} > 1$ appear as BAL QSOs. Zubovas & King (2013) tentatively confirm
this idea.

4 THE WIND SHOCK

4.1 Momentum– and Energy–Driven Flows

So far we have only studied the black hole wind. But we know that this wind
must have a significant effect on the host galaxy when it impacts directly on its
interstellar medium (ISM). In this Section we model the wind and host ISM as
roughly spherically symmetric, and consider the effects of deviations from this
simple picture later.

The pattern of the wind–ISM interaction (Figure 7) is qualitatively identical to
that of a stellar wind hitting the interstellar medium around it (see e.g. Dyson &
Fig. 8 Schematic picture of momentum–driven (top) and energy–driven (bottom) outflows. In both cases a fast wind (velocity $\sim 0.1c$) impacts the interstellar gas of the host galaxy, producing an inner reverse shock slowing the wind, and an outer forward shock accelerating the swept–up gas. In the momentum–driven case (top), corresponding to the UFOs discussed in Section 2, the shocks are very narrow and rapidly cool to become effectively isothermal. Only the ram pressure is communicated to the outflow, leading to very low kinetic energy $\sim (\sigma/c)L_{Edd}$. In an energy–driven outflow (bottom), the shocked regions are much wider and do not cool. They expand adiabatically, communicating most of the kinetic energy of the wind to the outflow (in simple cases divided in a ratio of about 1:2 between the shocked wind and the swept–up gas). The outflow radial momentum flux is therefore greater than that of the wind. Momentum–driven conditions hold for shocks confined to within $\sim 1$ kpc of the AGN, and establish the $M - \sigma$ relation (King, 2003; King, 2005). Once the supermassive black hole mass attains the critical $M - \sigma$ value, the shocks move further from the AGN and the outflow becomes energy–driven. This produces the observed large–scale molecular outflows which probably sweep the galaxy clear of gas. (From Zubovas & King, 2012a).

Williams 1997). The black hole wind (shown in blue) is abruptly slowed in an inner (reverse) shock where the temperature approaches $\sim 10^{11}$ K if ions and electrons reach equipartition (but see the discussion below). The shocked wind gas acts like a piston, sweeping up the host ISM (shown in red) at a contact discontinuity moving ahead of it. Because this swept–up gas moves supersonically into the ambient ISM, it drives an outer (forward) shock into it (see Figs. 7 and 8 [top]).

The dominant interaction here is the reverse shock slowing the black hole wind, and injecting energy into the host ISM. The nature of this shock differs sharply depending on whether or not some form of cooling (typically radiation) removes significant energy from the hot shocked gas on a timescale shorter than its flow time. If the cooling is strong in this sense (‘momentum–driven flow’), most of the
Preshock kinetic energy is lost (usually to radiation). The very rapid cooling means that the shocked wind gas is highly compressed, making the postshock region geometrically narrow (see the upper part of Figure 8). This kind of narrow, strongly cooling region is often idealised as a discontinuity, known as an ‘isothermal shock’ (cf Dyson & Williams, 1997). As momentum must be conserved, the postshock gas transmits just its ram pressure $p_{\text{wind}} \sim \sigma/c$ to the host ISM. We will see that this amounts to transfer of only a fraction $\sim \sigma/c \sim 10^{-3}$ of the mechanical luminosity $E_{\text{BH wind}} \approx 0.05L_{\text{Edd}}$ (cf equation 24) to the ISM. In other words, in the momentum-driven limit, only energy

$$E_{\text{mom}} \sim \frac{\sigma}{c} E_{\text{BH wind}} \sim \frac{\sigma}{c} \frac{M c^2}{2} \sim 5 \times 10^{-5} M c^2 \sim 0.1 E_{\text{gas}}$$

(31)

is injected into the bulge ISM, i.e. about 10% of the bulge gas binding energy $f_{200} M_8 \sigma^2$ for black holes close to the $M - \sigma$ relation (there is now an implicit factor $\sigma_{200}/M_8$ on the rhs). Thus momentum-driven flows do not threaten the bulge’s integrity. Indeed we will see that they never interact with most of it, so there is no danger that the black hole will drive away the gas and suppress accretion. A momentum-driven regime is a stable environment for black hole mass growth.

In the opposite limit where cooling is negligible, the postshock gas retains all the mechanical luminosity

$$E_{\text{wind}} \approx 0.05L_{\text{Edd}} \approx 100 E_{\text{gas}}$$

(32)

(cf equation 24) thermalized in the shock, and instead expands adiabatically into the ISM. The postshock gas is now geometrically extended (see the lower part of Figure 8), unlike the momentum-driven (‘isothermal’) case. This ‘energy-driven flow’ is much more violent than momentum-driven flow. The estimate (32) is for a black-hole mass near the $M - \sigma$ relation: a hole with mass a factor of 100 below this would already unbind the bulge in doubling its mass. Unless the shock interaction is markedly aspherical, a black hole in an energy-driven environment is unlikely to reach observed SMBH masses.

Given these starkly different outcomes we must decide under what conditions we have momentum- or energy-driven outflows. Simple estimates immediately show that ordinary atomic two-body processes have no significant effect in cooling the wind shock. But the wind shock is exposed to the radiation field of an Eddington-accreting supermassive black hole. This has a characteristic temperature of no more than $\sim 10^7$ K, far lower than the wind’s immediate post-shock temperature of $\sim 10^{10} - 10^{11}$ K. Electrons in the postshock gas lose energy to these photons through the inverse Compton effect (cf Ciotti & Ostriker, 1997), at a rate dependent on the radiation density. For wind shocks close to the SMBH, the accretion radiation field is intense enough that this effect cools the postshock wind gas in less than the momentum-driven flow time $\sim R/\sigma$ (see below), and we are self-consistently in the momentum-driven regime, provided that the postshock gas is in equipartition, i.e. that electron and ion temperatures remain effectively equal. (We consider this further in Section 4.2 below.)

For shocks at larger radii $R$ the radiation energy density decreases as $R^{-2}$, increasing the cooling time as $R^2$. The flow time increases only as $R$, so for $R$ greater than the critical cooling radius

$$R_C \sim 500 M_8^{1/2} \sigma_{200} \text{ pc}$$

(33)
AGN Outflows and Feedback

(King, 2003, 2005; King et al 2011, Zubovas & King 2012b) the cooling time is longer than the flow time, and the flow must be energy–driven. So we have the general result that momentum–driven flows are confined to a small region $R < R_C$, while energy–driven flows must be large–scale. This is just as one would expect, given that a momentum–driven flow allows stable black hole mass growth, while an energy–driven one is likely to expel most of the bulge gas.

It is plausible then that the observed UFO winds can lead to momentum–driving through strongly cooled shocks close to the SMBH. In this picture all but a small fraction of the mechanical luminosity of the black hole wind is eventually radiated away as an inverse Compton continuum with characteristic photon energy $\sim 1$ keV. Pounds & Vaughan (2011) report a possible detection of this spectral component in the Seyfert 1 galaxy NGC 4051. As required for consistency, the luminosity of this component is comparable to the expected mechanical luminosity of the wind in that system.

Cooling shocks are called ‘isothermal’ because the gas temperature rapidly returns to something like its preshock value. Momentum conservation requires that the gas is also strongly slowed and compressed as it cools. So the postshock velocity of the X–ray emitting gas should correlate with its temperature (or roughly, ionization) while Compton cooling is dominant. Once this has compressed the gas sufficiently, two–body processes such as free–free and bound–free emission must begin to dominate, since they go as the square of the density, and their cooling times decrease with temperature also (Pounds & King 2013). Section 2.5 above shows that there is direct observational evidence for both of these effects in NGC 4051. So this object (uniquely) shows three signatures of a cooling shock: an inverse Compton continuum, an ionization–velocity correlation, and the appearance of two–body processes in the spectrum.

4.2 Shock Cooling

Cooling (or the lack of it) has a defining effect on the physics of the interaction between the black hole wind and the host ISM, so we must check the simple picture above. In particular the inverse Compton effect acts only on electrons, but the energy of the postshock gas is initially almost all in its ions. We assumed above that the electron and ion temperatures quickly come into equipartition after the shock, allowing the inverse Compton effect to drain energy from the ions.

This assumption can be questioned. Faucher–Giguère & Quataert (2012) show that if the only process coupling electrons and ions is Coulomb collisions, there is a significant parameter space where equipartition does not occur, although they do not rule out substantial momentum–conserving phases. An important consideration here is that many processes other than direct Coulomb collisions may rapidly equilibrate electron and ion temperatures. Faucher–Giguère & Quataert (2012) attempt to put limits on the incidence of such collisionless coupling by appealing to observations of the solar wind, but this is an area of considerable physical uncertainty.

Another way of using observations to decide if shock cooling is effective is to look for the inverse Compton spectral component directly revealing the cooling. Bourne et al. (2013) argue that the apparent lack of such a component in most AGN spectra rules out cooling shocks. But we recall from Section 3.3 that the coverage
of UFOs is extremely sparse. Actually observing a collision and so the inverse Compton emission is inevitably a very rare event. It appears that observationally ruling out Compton shock cooling is so far inconclusive.

5 THE $M - \sigma$ RELATION

5.1 Reaching $M - \sigma$: the Momentum–Driven Phase

We are now equipped to discuss the impact of a UFO on the host interstellar gas. We already noted that the wind impact implies a pair of shocks each side of the contact discontinuity between the wind and the host ISM. Initially the wind shock is close the hole, and we assume that inverse Compton cooling from the AGN radiation field cools it rapidly and puts the flow in the momentum–driven regime. The region of gas between the wind shock and the contact discontinuity, where it impacts and sweeps up the host ISM, is very narrow (cf the upper panel of Figure 8). The outer shock accelerating the ISM is also strongly cooled, so that the ‘snowplow’ region of swept–up ISM is narrow as well. So we can treat the whole region between the inner and outer shocks as a single narrow, outward–moving gas shell, whose mass grows as it sweeps up the host ISM (see Fig. 9).

As a simple model of a bulge, we assume that the matter of the host galaxy is distributed with an isothermal profile of velocity dispersion $\sigma$, with mass density

$$\rho(r) = \frac{f_g \sigma^2}{2\pi Gr^2}, \quad (34)$$

so that the mass within radius $R$ is

$$M(R) = \frac{2\sigma^2 R}{G}, \quad (35)$$

A distribution like this is expected if the bulge results from mergers. For a roughly constant gas fraction $f_g$, the mass of the narrow swept–up gas shell at radius $R$ is

$$M_g(R) = \frac{2f_g \sigma^2 R}{G}, \quad (36)$$

so that the shell has the equation of motion

$$\frac{d}{dt}[M_g(R)\dot{R}] + \frac{GM_g(R)[M + M(R)]}{R^2} = \frac{L_{Edd}}{c}, \quad (37)$$

where $M$ is the SMBH mass. From (35, 36) and the definition of $L_{Edd}$ (equation 17) this simplifies to

$$\frac{d}{dt}(R\dot{R}) + \frac{GM}{R} = -2\sigma^2 \left(1 - \frac{M}{M_\sigma}\right), \quad (38)$$

where

$$M_\sigma = \frac{f_g \kappa}{\pi G^2 \sigma^4}. \quad (39)$$

Multiplying through by $R\dot{R}$ and integrating once gives the first integral

$$R^2 \dot{R}^2 = -2GM R - 2\sigma^2 \left(1 - \frac{M}{M_\sigma}\right) R^2 + \text{constant} \quad (40)$$
For large $R$ we have
\[ \dot{R}^2 \approx -2\sigma^2 \left[ 1 - \frac{M}{M_\sigma} \right] \] (41)
which has no solution for $M < M_\sigma$. Physically this says that if the SMBH mass is below $M_\sigma$ the swept–up shell of interstellar gas cannot reach large radius because the Eddington thrust of the black hole wind is too small lift its weight against the galaxy bulge potential. The SMBH cannot remove the gas from its surroundings, and goes on accreting. Any gas shell it drives outwards eventually becomes too massive, and so tends to fall back and probably fragment. This is likely to stimulate star formation in the shell remnants.

The precise value $M_\sigma$ depends on the average gas fraction $f_g$. For a protogalaxy forming at high redshift we expect $f_g = \Omega_{\text{baryon}}/\Omega_{\text{matter}} \simeq 0.16$ (Spergel et al., 2003). Galaxies forming at later times may have larger $f_g$ if they have gained a lot of gas, or smaller $f_g$ if they have been largely swept clear of gas, or have turned a lot of their gas into stars. With the gas fraction $f_g$ fixed at the cosmological value $f_c = 0.16$, the expression
\[ M_\sigma = \frac{f_g \kappa}{\pi G} \sigma^4 \simeq 3.2 \times 10^8 M_\odot \sigma_{200}^4 \] (42)
is remarkably close to the observed relation (2), even though it contains no free parameter. We shall see in Section 5.5 why observations tend to give an exponent of $\sigma$ slightly larger than the value 4 derived here. This agreement strongly suggests that SMBH growth stops at this point, although we must do some more work to show this (see Sections 5.2, 5.3 and 5.5 below).

The derivation here took the simplest possible description of a galaxy spheroid as an isothermal sphere (cf equation 35). We should ask if things change significantly if the galaxy bulge is more complicated than this. If the potential is spherically symmetric but the cumulative mass $M(R)$ is not simply linear in $R$, we still get a first integral of the equation of motion simply by multiplying through by $M(R)\dot{R}$, giving the giving the condition for a swept–up momentum–driven shell to reach large radii. Relations very like (42) emerge in each case, so we expect qualitatively similar behavior. McQuillen & McLaughlin (2012) show this explicitly for three widely–used density distributions (Hernquist 1990; Navarro et al. 1996, 1997; and Dehnen & McLaughlin 2005): the results are in practice scarcely distinguishable from (42).

Whatever the bulge geometry, the black hole always communicates its presence only through the ram pressure of its wind, so we are always dealing with strongly radial forces in the solid angles exposed to this wind (this is not true of gas pressure, as we shall see in Section 5.3). It is likely that the orientation of the accretion disc with respect to the host galaxy changes with each new episode of accretion (so–called chaotic accretion, King & Pringle 2006, King et al. 2008), tending to isotropize the long–term effect of momentum feedback. Together with the sudden huge increase in the spatial scale as the critical black hole mass is reached (see the next Section) this may explain why the simple spherically symmetric prediction (42) seems to give a surprisingly accurate estimate of the critical mass.
5.2 What Happens When $M = M_\sigma$?

The result (42) is so close to observations that it strongly suggests that feedback somehow cuts off the growth of the black hole at a mass very close to this value. Some feeding may continue from gas in the immediate vicinity of the hole which is too dense to be affected by the ram pressure of the black hole wind. A thin accretion disc has this property for example, but cannot have a gas mass larger than $\sim (H/R)M \ll M$ without fragmenting and forming stars.

But we still have to explain precisely how the gas is expelled. For example, one might worry that although momentum–driving can push the ISM away and inhibit central accretion, some kind of infall and SMBH accretion might restart shortly after momentum–driving is switched off, perhaps leading to alternating stages of quiescence and growth, eventually to masses far above $M_\sigma$. Observations show that black hole accretion occurs preferentially in gas–rich galaxies (cf Vito et al. 2014), so it seems that the black hole must largely clear the galaxy bulge of gas to terminate its growth. We will see later (Section 7) that if no other process than momentum–driving operated, this requirement would indeed lead to black hole masses significantly larger than $M_\sigma$, in conflict with observation.

This last point means that the way black hole growth influences the host galaxy must change radically when $M = M_\sigma$. It is straightforward to see why it should. We saw above that for $M < M_\sigma$ the Eddington thrust cannot push the wind shocks to large $R$. As a result the wind shock remains efficiently Compton cooled, enforcing momentum–driving. It follows that the SMBH cannot stifle its own growth if $M < M_\sigma$. But all this changes once the SMBH exceeds the critical mass (42).

Now even for a very small increment ($O(R_{\text{inf}}/R_C) \sim 10^{-2}$) of $M$ above $M_\sigma$, a momentum–driven shell can reach the critical radius $R_C$. Crucially, this means that the wind shocks are no longer efficiently cooled: they become energy–driven. The shocked wind gas can now use all of its energy to push the interstellar gas as it expands into the host bulge. This motion becomes explosive and rapidly reaches kiloparsec lengthscales, comparable with the size of the bulge itself, rather the much smaller (parsec) scales of the momentum–driven phase.

So the real significance of the $M - \sigma$ relation is that it marks the point where outflows undergo a global transition from momentum– to energy–driving.

5.3 Clearing Out a Galaxy: the Energy–Driven Phase

We know from (44) that an energy–driven outflow has more than enough energy to remove the interstellar gas entirely, and so presumably suppress further SMBH growth. Here we examine how this works in detail.

Once $M > M_\sigma$ the outflow geometry changes completely (see Figure 8). The shocked wind region is no longer narrow (as in the upper panel of Figure 8), but large and expanding because of its strong thermal pressure (lower panel of Figure 8). The shocked wind’s thermal expansion pushes its shock inwards where it must hover at the cooling radius $R_C$ (Zubovas et al. 2013). If it tries to move within $R_C$, momentum driving instantly pushes it out again (remember $M > M_\sigma$).

The shocked wind rapidly evens out its internal pressure as it expands at its sound speed $\sim 0.03c$, so we take this pressure $P$ as uniform over this region (but changing with time). The contact discontinuity at the outer edge of the shocked
wind sweeps up the surrounding shocked ISM as before, but now has the equation of motion
\[
\frac{d}{dt}\left[M_g(R)\dot{R}\right] + \frac{GM_g(R)M(R)}{R^2} = 4\pi R^2 P,
\]
where the pressure \(P\) is much larger than the ram pressure \(\rho v^2\) appearing in (37).

In the second term on the lhs we have neglected the contribution \(GM_g M/R^2\) of the black hole gravity, as \(R \gg R_C \gg R_{\text{inf}}\). To make the problem determinate we need the energy equation. (This did not appear explicitly in the momentum–driven case because it was equivalent to the defining condition that all the wind energy not associated with the ram pressure was rapidly lost to radiation.) Here the energy equation constrains the pressure \(P\) by specifying the rate that energy is fed into the shocked gas, minus the rate of \(PdV\) working on the ambient gas and against gravity:
\[
\frac{d}{dt}\left[4\pi R^3 \frac{3}{2} P\right] = \frac{\eta^2 L_{\text{Edd}}}{2} - P \frac{d}{dt}\left[4\pi R^3 \frac{3}{2}\right] - 4f_g \frac{\sigma^4}{G}.
\]

We take a specific heat ratio \(\gamma = 5/3\), use (23) for the energy input from the outflow and (35) to simplify the gravity term \(GM(R)M(R)/R^2\). Now we use (43) to eliminate \(P\) from (44), and replace the gravity terms as before using the isothermal expression for \(M(R)\). We take the AGN luminosity as \(L_{\text{Edd}}\) to allow for small deviations from the Eddington value. This gives
\[
\frac{\eta^2 L_{\text{Edd}}}{2} = \dot{R} \frac{d}{dt}\left[M(R)\dot{R}\right] + 8f_g \frac{\sigma^4}{G} \dot{R} + \frac{d}{dt}\left[\frac{R}{2} \dot{R} \frac{d}{dt}\left[M(R)\dot{R}\right] + 2f_g \frac{\sigma^4}{G} R\right]
\]
and so
\[
\frac{\eta^2 L_{\text{Edd}}}{2} = 2f_g \frac{\sigma^2}{G} \left\{\frac{1}{2} R^2 \dot{R} + 3R \dot{R} R + \frac{3}{2} \dot{R} \right\} + 10f_g \frac{\sigma^4}{G} \dot{R}.
\]

This describes the motion of the interface (‘contact discontinuity’ in the lower panel of Figure 8) between wind and interstellar gas in the energy–driven case, replacing equation (57) in the momentum–driven case.

The energy–driven regime applies as soon as the SMBH mass reaches \(M_\sigma\), and we will see that the host ISM is now quickly removed. We assume \(M = M_\sigma\) in \(L_{\text{Edd}}\), and see that (46) has a solution \(R = v_e t\) with
\[
2\eta c = 3\frac{v_e^2}{\sigma^2} + 10v_e
\]
The assumption \(v_e <\) \(\sigma\) leads to a contradiction (\(v_e \approx 0.01\sigma\)), so
\[
v_e \approx \left[\frac{2\eta \sigma^2 c}{3}\right]^{1/3} \approx 925\Omega_1^{1/3} \sigma_200^{2/3} \text{ km s}^{-1}
\]
This solution is an attractor. Figure 9 shows that all solutions quickly converge to it, regardless of initial conditions. Physically, its meaning is that if shock cooling is ineffective, the extra gas pressure accelerates the previously momentum–driven gas shell to this new higher velocity. Figure 9 also confirms that if the driving by the AGN switches off when the contact discontinuity is at radius \(R_0\), it decelerates as predicted by the analytic solution of (47) with \(L_{\text{Edd}} = 0\) found by King et al. (2011):
\[
\dot{R}^2 = 3\left(v_e^2 + \frac{10}{3}\sigma^2\right)\left(\frac{1}{x^2} - \frac{2}{3x^3}\right) - \frac{10}{3}\sigma^2
\]
where \( x = R/R_0 \geq 1 \). Noting that \( v_e \) depends only weakly (as \( v_e \sim l^{1/3} \)) on the luminosity, these results show that fluctuations – or even the intermittent disappearance – of the AGN luminosity have almost no effect on the outflow once it has started, because the flow still has a large reservoir of thermal energy available for driving. In particular an outflow can persist long after the central AGN has turned off, and the real agency driving an observed outflow may have been an AGN even if this is currently observed to be weak or entirely absent.

The solutions (48) or (49) describe the motion of the contact discontinuity where the shocked wind encounters swept–up interstellar gas (see Figures 8 and 9). This interface is strongly Rayleigh–Taylor (RT) unstable, because the shocked wind gas is highly expanded and has much lower mass density than the swept–up interstellar gas outside it, so that we have a light fluid underneath a heavy one. The RT instability leads to strong overturning motions even on small scales, and so is difficult to handle numerically. Deductions concerning the mean velocity and energy of the outflow, and its average spatial scale \( R(t) \), are likely to be believable, and agree closely with observations (see below) but we should be very cautious about results depending strongly on the detailed nature of the interface between the shocked wind and the swept–up interstellar medium. The RT instability is probably the reason that the high–speed (\( \sim 1000 \text{ km s}^{-1} \)) outflows with prodigious mass rates we predict here are generally seen with much of the outflowing gas in molecular form. Apparently the interstellar gas entering the forward shock is efficiently cooled by two–body radiation processes. A preliminary analysis (Zubovas & King 2014) suggests that the interstellar gas overtaken by the forward shock is likely to have a multiphase structure. Most of it cools all the way from the shock temperature \( \sim 10^7 \) K back to low temperatures, ending in largely molecular form, even though it is entrained in an outflow with the \( \sim 1000 \text{ km s}^{-1} \) velocity of the forward shock. But cooling is affected by the topology of the gas flow and the total area of interfaces between different gas phases. A full numerical calculation of this is currently impossible, so for the time being we can only make comparison with simple estimates, as here.

The mass outflow rate is fixed by how fast the outer shock overtakes the ISM and entrains new interstellar gas ahead of the contact discontinuity. The ISM ahead of the shock is at rest, so this runs into it at a speed giving a velocity jump by a factor \( (\gamma + 1)/(\gamma - 1) \) in the shock frame (where \( \gamma \) is the specific heat ratio: see e.g. Dyson & Williams 1997 for a derivation). This fixes its velocity as

\[
v_{\text{out}} = \frac{\gamma + 1}{2} \dot{R} \simeq 1230 \sigma_{200}^{-2/3} \left( \frac{f_e}{f_g} \right)^{1/3} \text{ km s}^{-1}
\]

(50)

(where we have used \( \gamma = 5/3 \) in the last form, and \( f_e \) is the cosmological value of \( f_g \)). This implies a shock temperature of order \( 10^7 \) K for the forward (ISM) shock (as opposed to \( \sim 10^{10–11} \) K for the wind shock). Since the outer shock and the contact discontinuity were very close together as energy–driven flow took over from momentum–driven flow (see Figure 8) this means that the outer shock is at

\[
R_{\text{out}}(t) = \frac{\gamma + 1}{2} R(t) = \frac{\gamma + 1}{2} v_e t.
\]

(51)

This gives the mass outflow rate as

\[
\dot{M}_{\text{out}} = \frac{dM(R_{\text{out}})}{dt} = \frac{(\gamma + 1) f_g \sigma^2}{G} \dot{R}.
\]

(52)
Fig. 9 Evolution of an energy–driven shock pattern for the case $\sigma = 200 \text{ km s}^{-1}, f_g = 10^{-2}$ computed numerically from the full equation (46). Top: radius vs time, middle: velocity vs time, bottom: velocity vs radius. The curves refer to different initial conditions: black solid $- R_0 = 10 \text{ pc}, v_0 = 400 \text{ km s}^{-1}$; blue dashed $- R_0 = 100 \text{ pc}, v_0 = 1000 \text{ km s}^{-1}$; red dot–dashed $- R_0 = 50 \text{ pc}, v_0 = 200 \text{ km s}^{-1}$. All these solutions converge to the attractor (48). The vertical dashed line marks the time $t = 10^6 \text{ yr}$ when (for this case) the quasar driving is switched off. All solutions then follow the analytic solution (49). A case where the quasar remains on for a Salpeter times $\sim 4 \times 10^7 \text{ yr}$ would sweep the galaxy clear of gas. (From King et al., 2011)
For comparison the mass rate of the black hole wind, assuming \( M = M_\sigma \), is

\[
\dot{M}_w \equiv \dot{m}_w \dot{M}_{\text{Edd}} = \frac{4f_e \dot{m} \sigma^4}{\eta c G}, \tag{53}
\]

This is much smaller than the outflow rate \( \dot{M}_{\text{out}} \) it drives, so we define a mass-loading factor as the ratio of the mass flow rate in the shocked ISM to that in the wind:

\[
f_L \equiv \frac{\dot{M}_{\text{out}}}{\dot{M}_w} = \frac{\eta (\gamma + 1)}{4} \frac{f_g \dot{R} c}{f_e \sigma^2}.
\]

Then we have

\[
\dot{M}_{\text{out}} = f_L \dot{M}_w = \frac{\eta (\gamma + 1)}{4} \frac{f_g \dot{R} c}{f_e \sigma^2} \dot{M}_{\text{Edd}}. \tag{54}
\]

If the AGN radiates at a luminosity \( \sim L_{\text{Edd}} \), we have \( \dot{R} = v_e \), and (48) gives

\[
f_L = \left( \frac{2yc}{3\sigma} \right)^{4/3} \left( \frac{f}{f_e} \right)^{2/3} \frac{\dot{P}_w}{\dot{P}_{\text{out}}} \simeq 460\sigma_{200}^{-2/3} \dot{m}^{1/3} \tag{56}
\]

and

\[
\dot{M}_{\text{out}} \simeq 460\sigma_{200}^{10/3} \dot{m}^{1/3} M_\odot \text{ yr}^{-1} \tag{57}
\]

for typical parameters, \( f_g = f_e \) and \( \gamma = 5/3 \). The total gas mass in the bulge is roughly \( M_g \sim 10^9 f_g M_\sigma \) (from equation 1). Clearly if the outflow persists for a time \( t_{\text{clear}} \sim M_g/\dot{M}_{\text{out}} \sim 1 \times 10^7 \sigma_{200}^{-2/3} \text{ yr} \), it will sweep away a large fraction of the galaxy’s gas. The precise outflow duration needed for this depends on both the type and the environment of the galaxy, in practice leading to three parallel but slightly offset \( M - \sigma \) relations (see Section 5.5 below).

Equations (50, 57) give

\[
\frac{1}{2} \dot{M}_w v^2 \simeq \frac{1}{2} \dot{M}_{\text{out}} v_{\text{out}}^2. \tag{58}
\]

So most of the wind kinetic energy ultimately goes into the mechanical energy of the outflow, as we would expect for energy driving. The continuity relations across the contact discontinuity show that if the quasar is still active, the shocked wind retains \( 1/3 \) of the total incident wind kinetic energy \( \dot{M}_w v^2/2 \), giving \( 2/3 \) to the swept-up gas represented by \( \dot{M}_{\text{out}} \).

Equation (58) means that the swept-up gas must have a scalar momentum rate greater than the Eddington value \( L_{\text{Edd}}/c \), since we can rewrite it as

\[
\frac{\dot{P}_w^2}{2M_w} \geq \frac{\dot{P}_{\text{out}}^2}{2\dot{M}_{\text{out}}}, \tag{59}
\]

where the \( \dot{P} \)'s are the momentum fluxes. With \( \dot{P}_w = L_{\text{Edd}}/c \), we have

\[
\dot{P}_{\text{out}} = \dot{P}_w \left( \frac{\dot{M}_{\text{out}}}{\dot{M}_w} \right)^{1/2} = \frac{L_{\text{Edd}}}{c} f_e^{1/2} \simeq 20 \frac{L_{\text{Edd}}}{c} \sigma_{200}^{-1/3} \dot{m}^{1/6}. \tag{60}
\]

Observations of molecular outflows consistently show \( \dot{M}_{\text{out}} v_{\text{out}} > L_{\text{Edd}}/c \), and in particular Cicone et al. (2014) find that momentum rates \( 20L/c \) are common. This is an inevitable consequence of mass-loading \( (f_L > 1) \). These high momentum rates are important, as they are probably the way that the galaxy resists the accretion
that cosmological simulations suggest still continues at large scales (Costa et al. 2014).

Recent infrared observations show abundant evidence for molecular outflows with speeds and mass rates similar to (50) and (57). Feruglio et al. (2010), Rupke & Veilleux (2011) and Sturm et al. (2011) find large-scale (kpc) flows with $v_{\text{out}} \sim 1000 \text{ km s}^{-1}$ and $M_{\text{out}} \sim 1000 \text{ M}_\odot \text{ yr}^{-1}$ in the nearby quasar Mrk 231. Other galaxies show similar phenomena (cf Lonsdale et al. 2003, Tacconi et al. 2002, Veilleux et al. 2009 Riffel & Storchi–Bergmann (2011a, b) and Sturm et al. 2011: see Tables 1 and 2 of Zubovas & King 2012a for a detailed comparison with the theoretical predictions). In each case it appears that AGN feedback is the driving agency. There is general agreement for Mrk231 for example that the mass outflow rate $\dot{M}_{\text{out}}$ and the kinetic energy rate $\dot{E}_{\text{out}} = \dot{M}_{\text{out}} v_{\text{out}}^2 / 2$ are too large to be driven by star formation, but comparable with values predicted for AGN feedback.

It appears that energy–driven outflows from SMBH which have just reached their $M - \sigma$ masses should be able to sweep galaxy spheroids clear of gas. A robust observational test of this is the expected mechanical luminosity (cf equation (23))

$$L_{\text{mech}} \sim \frac{\eta}{2} L_{\text{Edd}} \simeq 0.05 L,$$

where $L = L_{\text{Edd}}$ is the observed AGN luminosity. This is investigated by Ciccone et al. (2014). As their Figure 12 shows, observation does largely confirm the relation (61). If the AGN are close to their Eddington luminosities (so that $L \propto M \propto \sigma^4$ and $I \simeq 1$), the clearout rate $\propto \sigma^{10/3}$ (equation 57) should scale linearly with the driving luminosity $L$. Figure 9 of Ciccone et al. shows evidence for this correlation, with normalization close to that predicted.

5.4 Effects of a Galaxy Disc: Stimulated Star Formation and Outflow Morphology

We have so far discussed galaxy spheroids in isolation. This is in line with the observational evidence (see Kormendy & Ho 2013 for a review) that the SMBH scaling relations apply only to this component of a galaxy, and are essentially unaffected by the presence of a galaxy disc. In particular we suggest that the critical $M - \sigma$ black hole mass is set by small-scale momentum–driven outflows interacting with only a very small central part of the bulge. But the energy–driven outflows we considered in the last subsection are global: they expand to far greater scales, and unless the galaxy is an elliptical must inevitably encounter its disc as they expand. In a gas–rich galaxy the gas in the innermost disc at radius $R_0$ must be close to self–gravitating. Assuming that the potential is roughly isothermal, it is straightforward to show that this implies a gas density $\rho_d \sim 2\sigma^2 / R_0$, i.e. greater than the bulge gas density by the factor $\sim 1/f_g \sim 10$. We see from equation (46) that higher gas densities mean lower spherical outflow velocities, as they meet greater resistance. So when an initially spherical outflow encounters a high–density gas disc it flows around it, over its plane upper and lower faces. But the pressure in the outflow is at least initially far higher than in the disc: we can read off the pressure at the contact discontinuity from equation (49) as

$$P_{\text{CD}} = \frac{f_g \sigma^2 (2\sigma^2 + v_e^2)}{\pi G R^2} \simeq \frac{f_g \sigma^2 v_e^2}{\pi G R^2},$$

(62)
and estimate the pressure at the forward shock into the ISM as

\[ P_{fs} = \frac{4}{3} \rho(R) v_e^2 = \frac{2 f_0 \sigma^2 v_e^2}{3 \pi G R^2} \approx \frac{2}{3} P_{CD}. \]  

(63)

By contrast the mid–plane pressure in a disc close to self–gravitating is

\[ P_{\text{disc}} \sim \rho c_s^2 \sim \rho \sigma^2 \sim 2 \sigma^4 G R_d^2 \]  

(64)

where we have assumed the sound speed \( c_s \sim \sigma \) and the self–gravity condition \( G \rho \sim \Omega^2 \) with \( \Omega = \sqrt{2 \sigma / R_d} \) the Kepler frequency at disc radius \( R_d \). Thus when the outflow shock arrives at \( R = R_d \) its pressure is a factor \( (v_e/\sigma)^2 \sim 25 \) larger than the disc’s, and this remains true until the outflow shock has travelled out to radii \( R > R_d v_e/\sigma \sim 5 R_d \). Any such compression must trigger a burst of star formation in the disc (cf Thompson et al. 2005, Appendix B), and here it rises to values

\[ \dot{\Sigma}_* \sim 2000 \epsilon_* \sigma_{200}^{10/3} \sigma_{R_d}^{-2} \]  

(65)

(Zubovas et al. 2013), where \( \epsilon_* = 10^{-3} \epsilon_{-3} \) is the efficiency of massive stars in converting mass into radiation, and we have substituted for \( v_e \) using (48). Zubovas et al. (2013) show that this leads to a starburst of total luminosity

\[ L_* \sim 5 \times 10^{47} L_{46}^{-1/6} \]  

(66)

where \( L_{46} \) is the AGN luminosity in units of \( 10^{46} \) erg s\(^{-1}\). Such systems would appear as ULIRGs.

This suggests that in a galaxy with both a bulge and a disc, the clearout phase leaves the galaxy bulge without gas, but may be accompanied by a starburst in the disc. Recent observations of dusty QSOs appear to show this, with the black hole mass already on the \( M - M_b \) relation (1), and so fully grown (Bongiorno et al. 2014). In an elliptical on the other hand, clearout must leave the galaxy ‘red and dead’.

Since a galaxy disc is a major obstacle to an outflow, it follows that it may be able to divert a quasi–spherical outflow into a bipolar shape. This is particularly true in cases where the SMBH mass grows only a little, in a minor accretion event. Zubovas et al. (2011) suggest that the gamma–ray emitting bubbles disposed symmetrically about the plane of the Milky Way (Su et al. 2010) may be the remnants of a relatively recent and rather weak event like this.

5.5 The Three \( M - \sigma \) relations

So far in this Section we have seen that the arrival of the black hole mass at the \( M - \sigma \) relation means that its feedback makes a radical change from momentum–driving to energy-driving. The energy–driven phase which clears the gas from a galaxy bulge is short and violent. But it is clear that the black hole must inject a non–negligible amount of energy to eject the gas, and this requires accretion energy, i.e. some black hole mass growth. Evidently if the mass increment \( \Delta M \) needed for this is \( \gg M_* \) we will have failed to explain the \( M - \sigma \) relation.

The mass increment \( \Delta M \) is influenced by two factors. First, it must require significantly less SMBH growth to remove the gas from a spiral galaxy with a
Fig. 10 The four (in reality three, as cluster spirals are rare) $M - \sigma$ relations (solid lines) and their combined effect on observational fits (dashed line). All solid lines have slopes $M \propto \sigma^4$ and the dashed line has $M \propto \sigma^6$. The grey area is the approximate locus of data points in Figure 3 of McConnell et al. (2011). (From Zubovas & King, 2012b)

relatively small bulge, than for example an elliptical, where the much larger bulge mass means that energy-driving by the central SMBH wind must continue for longer in order to expel the remaining gas. Zubovas & King (2012b) find that energy-driving, and therefore SMBH mass growth above $M_\sigma$, must continue only for about 4 Myr (about 0.1 Salpeter times) in a typical spiral, but for about 2 Salpeter times in an elliptical. So the final SMBH mass in a spiral is close to $M_\sigma$ but in an elliptical it can reach

$$M_{\text{final}} \sim e^2 M_\sigma \sim 7.5 M_\sigma$$ (67)

The second factor affecting $M, M_b$ is the galaxy environment. Cluster ellipticals can gain gas as they orbit through the intracluster gas. Some Brightest Cluster Galaxies (BCGs), which are near the centre of the cluster potential, are known to contain unusually massive SMBH (McConnell et al. 2011). Taking account of the extra black hole mass growth required to remove the bulge, and the mass a galaxy may gain from its surroundings, implies three parallel but slightly offset $M - \sigma$ relations for spirals, field and cluster ellipticals (see Fig 10). In principle there is also a relation for cluster spirals, but these are rare. We see from the Figure that the spread in offsets means that an observed sample drawn from galaxies of all three types would tend to produce a slope slightly bigger than the individual ones for each type, perhaps accounting for the slight discrepancy between the observed
Fig. 11 The four (in reality three, as cluster spirals are rare) $M - \sigma$ relations (solid lines) and their combined effect on observational fits (dashed line). All solid lines have slopes $M \propto \sigma^4$ and the dashed line has $M \propto \sigma^6$. The grey area is the approximate locus of data points in Figure 3 of McConnell et al. (2011). (From Zubovas & King, 2012b)

overall slope $\alpha = 4.4 \pm 0.3$ and the theoretical value of 4. All three types of galaxies obey a similar $M - M_b$ relation within the errors, as growth of the SMBH above $M_\sigma$ goes together with higher $M_b$.

6 THE SMBH – BULGE MASS RELATION

6.1 Feedback and the $M – M_b$ relation

In the Introduction we noted the observed proportionality between $M$ and $M_b$ as well as the $M - \sigma$ relation. So far we have concentrated almost entirely on the second of these relations, and suggested that it arises because the black hole feedback itself directly limits the mass reservoir available for black-hole growth. Quite independently of details, almost every discussion of this relation adopts this view (see Section 7 below).

But the character of the $M – M_b$ relation must be very different. Since we are assuming that feedback ensures that the black hole mass $M$ is set by $\sigma$ we cannot argue that $M$ is independently set by $M_b$. But reversing the argument to suggest that the black hole mass $M$ sets $M_b$ is also implausible, since $M_b$ is in the form of stars.
So there can be no directly causal connection between the black hole mass $M$ and the stellar bulge mass $M_b$. (Indeed one view – see Section 6.3 below – asserts that the connection is purely statistical.) Instead, their relation must arise because whatever determines $M_b$ makes it proportional to $\sigma^4$. Empirically, we already know that this is approximately true, at least for elliptical galaxies, the largest spheroids of all, because these are observed to obey the Faber–Jackson (1976) relation

$$L_* \sim 2 \times 10^{10} L_\odot \sigma_{200}^4.$$  \hfill (68)

Here $L_*$ is the total stellar luminosity and mass of an elliptical, so for mass–to–light ratios $\sim 5$ we immediately get the stellar mass as

$$M_* \sim 1 \times 10^{11} M_\odot \sigma_{200}^4 \sim 10^3 M.$$  \hfill (69)

There is now general agreement that this relation, like the $M – \sigma$ relation, may result from feedback inhibiting and ultimately suppressing the process that produces it. The difference is that here the feedback is from stars, and what ultimately has to be suppressed is star formation. Several papers make this point, starting with Murray et al. (2005). Power et al. (2011) show that this approach gives a bulge stellar mass

$$M_b \sim 0.14 f_b t_H \sigma^4,$$  \hfill (70)

where $\epsilon* \simeq 2 \times 10^{-3}$ measures the total luminous energy yield from a main–sequence star in terms of its rest–mass energy $M_* c^2$, and $t_H$ is the Hubble time. Comparing with (69) we get

$$M \simeq M_* \sim \frac{1.8 \kappa \epsilon_* c}{\pi G t_H} M_b \sim 10^{-3} M_b,$$  \hfill (71)

which is similar to observational estimates (cf equation 1). Both the $M – \sigma$ and the $M – M_b$ relations hold for elliptical galaxies, so equation (70) automatically reproduces the Faber–Jackson (1976) relation for typical mass–to–light ratios. In this view, the similarity of the SMBH and stellar (Faber–Jackson) $M, M_b \propto \sigma^4$ relations (2, 69) follows directly because both result from momentum feedback, and the ratio $M/M_B \sim 10^{-3}$ reflects the relative efficiencies of the black hole and stellar versions.

6.2 The $M – \sigma$ relation for Nucleated Galaxies

A similar argument (McLaughlin et al. 2006) shows that for nucleated galaxies (i.e. those whose central regions are dominated by nuclear star clusters, with no detectable sign of the presence of a supermassive black hole) there should be an offset $M – \sigma$ relation between the mass of the cluster and the velocity dispersion, i.e.

$$M_c \simeq 20 M_* \simeq 6 \times 10^6 \sigma_{200}^4 M_\odot.$$  \hfill (72)

Typically these galaxies are small, with $\sigma < 120$ km s$^{-1}$. The factor $\sim 20$ offset in cluster mass for a given $\sigma$ arises because momentum–driving by an ensemble of cluster stars is about 20 times less efficient per unit mass than from a black hole accreting at the Eddington rate.
6.3 Mergers and the $M - M_b$ relation

Jahnke & Macciò (2011) offer a radically different interpretation of the $M - M_b$ relation. Building on earlier work of Peng (2007) they assume that black hole and bulge masses are built up by repeated mergers of smaller galaxies with uncorrelated $M$ and $M_b$. They follow this evolution using dark matter halo merger trees, and as a result of the central limit theorem find that $M$ is roughly proportional to $M_b$, with the scatter decreasing for larger masses, where there have been more mergers. They conclude that the SMBH - bulge scaling relation may have an explanation that is largely or even entirely non–causal.

But it is hard to accept that there is no more physics in the SMBH scaling relations than this. First, the actual ratio $M/M_b$ is left undetermined by this procedure. Second, to get from the $M - M_b$ relation to $M - \sigma$ requires one to assume something like the $M_b \propto \sigma^4$ relation (69) implied by Faber–Jackson, so physics presumably must enter here too (cf the subsection above). Finally, it would seem a remarkable coincidence that the outcome of this indirect process by chance produces an $M - \sigma$ relation exactly equivalent to requiring that the SMBH Eddington thrust should just balance the weight of the bulge gas.

7 MOMENTUM, ENERGY OR RADIATION?

The study of AGN outflows and their effects on the host galaxy has two main aims. A viable picture must explain both the scaling relations, and simultaneously the fact that galaxy spheroids appear ultimately to be largely swept clear of gas by high–speed molecular outflows which have significantly greater scalar momenta $\dot{M} v$ than the radiative value $L/c$ of the central AGN (the clearout problem). The discussion given above offers plausible physical grounds that the shock interaction characterising the black hole wind feedback changes from momentum–driven, acting on small spatial scales near the black hole, to energy–driven, instead acting globally on the whole galaxy bulge and producing a high–energy clearout of its gas. The $M - \sigma$ relation marks the point where this transition occurs in a given galaxy. We will argue below (Section 8) that observations support this picture of local–global transition in several ways, but before accepting this conclusion we should consider other possibilities.

First, the switch from momentum to energy–driving depends on the details of gas cooling. It is sometimes argued (e.g. Silk & Nusser 2010) that strong cooling of the ambient interstellar medium enforces momentum–driving by the central SMBH throughout. In fact cooling the ambient gas is not relevant to the question of energy or momentum driving: as we have seen, it is the cooling of the shocked wind gas which decides this. But as we emphasised in Section 4, at least some of the physics of the suggested momentum–energy switch is still beyond a full numerical treatment with realistic parameters. It is sensible then to check our treatment above by considering the momentum–driven and energy–driven cases in isolation, and then the effect of direct radiation pressure.
7.1 Wind Momentum Driving

We first simply assume that a black hole wind acts on its surroundings by pure momentum–driving alone, at all radii. For generality we let the pre–shock wind have speed \( v_w \) and take its mechanical luminosity \( M_w v_w^2/2 \) as a fixed fraction \( a \) of \( L_{\text{Edd}} \), i.e. we do not explicitly assume that the wind has the Eddington momentum, as seems to hold for UFOs. Then the momentum feedback first becomes important at a critical black hole mass \( M_{\text{crit}} \) roughly given by equating the wind thrust \( M_w v_w = 2a L_{\text{Edd}}/v \) to the weight

\[
W = \frac{GM(R)M_g(R)}{R^2} = \frac{4f_g\sigma^4}{G}
\]

(73)

of the overlying gas in an isothermal potential (cf equation 37). With \( L_{\text{Edd}} = 4GM_{\text{crit}}c/\kappa \) we get

\[
M_{\text{crit}} = \frac{v_w}{2ac} M_g.
\]

(74)

By definition \( a = \dot{M}_w v_w^2/2L_{\text{Edd}} \) and \( L_{\text{Edd}} = \eta \dot{M}_{\text{Edd}} c^2 \), so

\[
M_{\text{crit}} = \frac{v_w}{2ac} M_g = \frac{\eta c \dot{M}_{\text{Edd}}}{v_w M_w} M_g,
\]

(75)

(cf Fabian 1999). We see that for general wind parameters the critical mass differs from \( M_s \). We find \( M_{\text{crit}} = M_s \) only if \( v_w = \eta c \dot{M}_{\text{Edd}}/M_w \), which is equation (22). This immediately implies that the wind momentum is Eddington, i.e. \( M_w v_w = \eta \dot{M}_{\text{Edd}} c = L_{\text{Edd}}/c \). In other words, assuming pure momentum driving gives the critical mass as \( M_s \) if and only if the driving wind has the Eddington momentum, i.e. has the properties observed for UFOs.

But pure momentum driving is unable to drive off the bulge gas without a significant increase of the black hole mass above \( M_{\text{crit}} \). Several authors have reached this conclusion (cf Silk & Nusser 2010, McQuillin & McLaughlin 2012). Moreover, if galaxy bulges accrete at the rates suggested by cosmological simulations it seems unlikely that any hypothetical momentum–driven outflows would have enough thrust to prevent infall and so could not turn off star formation definitively (cf Costa et al. 2014). We conclude that pure momentum–driving, even given the lack of a likely shock cooling process, probably does not give a realistic picture of the interaction between SMBH and their hosts.

7.2 Wind Energy Driving

The direct opposite case from that considered in the last subsection is pure energy–driving by winds, where radiative cooling is assumed negligible throughout. This was often the implicit assumption in early treatments (e.g. Silk & Rees 1998, Haehnelt et al. 1998). The equation of motion for this case is \( \dot{E}_{\text{dot}} = \dot{M}_w c^2 \). This shows that gas is always driven out at constant speed for any SMBH mass, however small: setting \( R = v_c t \) and using the definition of \( L_{\text{Edd}} \) (cf equation 17) gives the speed \( v_c \) as

\[
v_c^3 = \frac{\pi G^2 c \eta M}{3f_g \kappa \sigma^2}.
\]

(76)
This expresses the fact that the adiabatically expanding shocked gas pushes the interstellar gas away as a hot atmosphere for any SMBH mass. One can easily find the corresponding mass outflow rate by setting $\dot{M}_{\text{out}} v_e^2/2 \sim L_w$, since we know that the outflow mechanical luminosity is a significant fraction of the driving wind mechanical luminosity $L_w = \eta L_{\text{Edd}}/2$.

To define a critical SMBH mass for energy–driven outflow one has to impose a further condition. This is usually taken as $v_e \sim \sigma$, defining some kind of escape velocity. But it is not obvious that this is appropriate: the outflow is driven by pressure, so the ballistic escape velocity is not relevant. Even if the AGN switches off, the residual gas pressure still drives outflow for a long time (cf Fig. 9). If we nevertheless impose this condition we find a critical mass

$$M_{\text{energy}} = \frac{3f_\phi \kappa}{\pi G^2 \eta c} \sigma^5 = \frac{3\sigma}{\eta_c} M_\sigma = 0.02 M_\sigma = 6 \times 10^6 M_\odot \sigma_{200}^5$$  \hspace{1cm} (77)

which is a factor $3\sigma/\eta_c \sim 1/50$ too small in comparison with observations.

Silk & Rees (1998) considered the growth of a protogalaxy (i.e. gas with $f_g \sim 1$) around a supermassive seed black hole which formed earlier, but their argument applies to the coevolution of the SMBH and host also, provided we take $f_g \sim 0.1$. They assume the wind sweeps mass into a shell with speed $v_s$, and implicitly neglect pressure work, and the fact that energy is shared between the shocked wind and the swept–up outflow. This would imply a relation

$$L_w = \frac{2\pi r^2}{4\pi r^2} f_g \rho(r)v_s^3,$$

as each new shell of mass $4\pi r^2 \rho(r)v_s^2$ now simply acquires kinetic energy $v_s^2/2$ as it is swept up. Using the isothermal relation (34) and requiring $v_s \sim \sigma$ gives

$$M_{\text{SR}} \sim \frac{f_g \kappa}{4\pi G^2 f_w c} \sigma^5 \sim 5 \times 10^4 \left( \frac{f_g}{0.16} \right) M_\odot \sigma_{200}^5$$  \hspace{1cm} (79)

where $f_w = L_w/L_{\text{Edd}}$. The neglect of pressure work overestimates the wind–driving efficiency, so this mass is even smaller than (77). It is clear that wind energy–driving of this type does not correctly reproduce the $M - \sigma$ relation, giving a critical mass too low by factors 50 – 100.

A more promising approach has recently been outlined by Nayakshin (2014), Zubovas & Nayakshin (2014) and Bourne et al. (2014), who consider the effects of strong inhomogeneity of the bulge gas. They assume first that inverse Compton shock cooling may not be effective because of two–temperature effects (but see Section 4.2 above). Second, they suggest that sufficiently dense clouds of interstellar gas would feel a net outward force $\sim \rho v^2$ per unit area when overtaken by a free–streaming UFO wind of preshock density $\rho$ and speed $v$, thus mimicking a momentum–driven case. The density of these clouds is a factor $1/f_g \sim 6$ below the star–formation threshold. If most of the ISM gas mass is in the form of such clouds, SMBH feeding must stop when the outward force overcomes gravity, which gives a relation like (39) up to some numerical factor. This idea throws up several gas–dynamical problems. First, a cogent treatment must explain the origin and survival of clouds at densities close to but just below the star formation threshold, which must contain most of the interstellar gas. The clouds must be completely immersed in the wind, so the net outward force on them is a surface drag, which is dimensionally also $\sim \rho v^2$ per unit area. Estimating this surface drag requires
knowledge of how the cloud–wind interfaces evolve on very small scales. Since these are formidable theoretical tasks, we should ask for observational tests. The main difference from the quasispherical momentum–driven case is that instead of being radiated away, most of the energy of the UFO wind now continuously drives the tenuous intercloud part of the ISM out of the galaxy at high speed. If this tenuous gas is a fraction $f$ of the total gas content, equations (50, 57) show that for SMBH masses not too far from $M_\sigma$ this outflow should have speed $v_{\text{out}} \sim 1230 f^{-1/3}$ km s$^{-1}$ and mass–loss rate $M_{\text{out}} \sim 4000 f t$ M$_\odot$ yr$^{-1}$, and so be potentially observable for many AGN spheroids. From the work of Section 5.4 one might also expect a continuously elevated star formation rate in the central parts of their galaxy discs also, which is not in general observed.

### 7.3 Cosmological Simulations

Cosmological simulations often produce an empirical $M – \sigma$ relation as part of much larger structure formation calculations. Limits on numerical resolution inevitably require a much more broad–brush approach then adopted here. The effect of the SMBH on its surroundings is usually modelled by distributing energy over the gas of the numerical ‘galaxy’ at a certain rate. This injected mechanical luminosity is then iterated until the right relation appears. This empirical approach (e.g. di Matteo et al. 2005) seems always to require a mechanical luminosity $0.05 L_{\text{Edd}}$ to produce the observed $M – \sigma$ relation. This is precisely what we expect (cf equation 23) for a black hole wind with the Eddington momentum $M_{\text{out}} v = L_{\text{Edd}} / c$.

But the success of this procedure is puzzling. If the ambient gas absorbed the full numerically injected mechanical luminosity $0.05 L_{\text{Edd}}$ the resulting outflows would give the energy–driven (32) or Silk–Rees mass (79) above, which are too small compared with observations. The fact that cosmological simulations instead actually iterate roughly to the observed $M – \sigma$ value (42) suggests that they somehow arrange that the injected energy only couples to the gas at the very inefficient rate which occurs in momentum driving, or possibly that the numerical gas distribution is highly inhomogenous. The real physics determining this in both cases operates at lengthscales far below the resolution of any conceivable cosmological simulation, so the inefficiency must be implicit in some of the ‘sub–grid’ physics which all such simulations have to assume (cf Costa et al. 2014, Appendix B).

### 7.4 Radiation Driving

#### 7.4.1 Electron scattering opacity

We remarked in the Introduction that in principle direct radiation pressure is the strongest perturbation that a black hole makes on its surroundings, but its effects are more limited than this suggests. As we already suggested, the reason is that in many situations radiation decouples from matter before it has transferred significant energy or momentum. This is particularly likely for radiation emitted by an AGN in the center of a galaxy bulge. The gas density (cf equation 54) is sharply peaked towards the center, and so is its tendency to absorb or scatter the
radiation from the accreting black hole. The electron scattering optical depth from gas outside a radius $R$ for example is
\[
\tau(R) = \int_R^\infty \kappa \rho(r) dr = \frac{\kappa f g \sigma^2}{2\pi G R},
\] (80)
which is mostly concentrated near the inner radius $R$. This means that gas initially close to the AGN is probably swept into a thin shell by its radiation, and so at radius $R$ has optical depth
\[
\tau_{sh}(R) \approx \frac{\kappa f g \sigma^2}{\pi G R},
\] (81)
very similar to the undisturbed gas outside $R$ (cf equation 80). Gas distributed in this way has large optical depth near the black hole when its inner edge $R$ is small (i.e. less than the value $R_{tr}$ specified in equation 82 below). Then the accumulating accretion luminosity $L$ of the AGN is initially largely trapped and isotropized by electron scattering, producing a blackbody radiation field whose pressure grows as the central AGN radiates. This growing pressure pushes against the weight $W = 4 f g \sigma^4 / G$ (equation 73) of the swept–up gas shell at radius $R$. This is exactly like the material energy–driving we discussed in Section 5.3, except here the photon ‘gas’ has $\gamma = 4/3$ rather than $\gamma = 5/3$ there. Clearly the effectiveness of this radiation driving is eventually limited because the shell’s optical depth falls off like $1/R$ as it expands. The force exerted by the radiation drops as it begins to leak out of the cavity, until for some value $\tau_{\text{tot}}(R) \sim 1$ it cannot drive the shell further.

This shows that the sweeping up of gas by radiation pressure must stop at a ‘transparency radius’
\[
R_{tr} \sim \frac{\kappa f g \sigma^2}{\pi G} \approx 50 \left( \frac{f g}{0.16} \right) \sigma^2_{200} \text{ pc},
\] (82)
where (up to a logarithmic factor) the optical depth $\tau_{\text{tot}}$ is of order 1, so that the radiation just escapes, acting as a safety valve for the otherwise growing radiation pressure. This process is discussed in in detail by King & Pounds (2014), who suggest that the stalled gas at $R_{tr}$ may be the origin of the ‘warm absorber’ phenomenon (cf Tombesi et al. 2013). The radius $R_{tr}$ is so small that very little accretion energy is needed to blow interstellar gas to establish this structure, and to adjust it as the galaxy grows and changes $\sigma$.

### 7.4.2 Dust

At larger radii much of the cold diffuse matter in the galaxy bulge may be in the form of dust. The absorption coefficient of dust depends strongly on wavelength and is far higher than electron scattering in the ultraviolet, but decreases sharply in the infrared (e.g. Draine & Lee, 1984). The energy of an ultraviolet photon absorbed by a dust grain may be re–emitted almost isotropically as many infrared photons, which then escape freely. The net effect is that dusty gas feels only the initial momentum of the incident UV photon, while most of the incident energy escapes. Then a spherical shell around an AGN would experience a radial force $\approx L/c$, where $L$ is the ultraviolet luminosity, as long as it remained optically thick to this kind of dust absorption. This is dynamically similar to wind–powered
flows in the momentum–driven limit, and this type of radiation–powered flow is often also called ‘momentum–driven’, even though the physical mechanism is very different.

An important distinction between the wind and radiation–powered cases is that ambient gas in the path of a wind cannot avoid feeling its effects, whereas this is not true for radiation, as the gas may be optically thin. Galaxies are generally optically thin to photons in various wavelength ranges, and a radiation–driven shell may stall at finite radius because its optical depth \( \tau \) becomes so small that the radiation force decouples, as we saw in the electron–scattering case. Ishibashi & Fabian (2012, 2013, 2014) appeal to this property to suggest that star formation in massive galaxies proceeds from inside to outside as radiation–momentum driven shells of dusty gas are driven out and then stall at the dust transparency radius \( R_{\text{dust}} \approx (\kappa_d/\kappa) R_{\text{tr}} \). For large dust opacities \( \kappa_d \sim 1000 \kappa \) this can give \( R_{\text{dust}} \sim 50 \text{ kpc} \). In contrast galaxies are probably never ‘optically thin’ to winds, and the density of a black hole wind is always diluted as \( 1/R^2 \), so it inevitably shocks against a swept–up shell of interstellar gas at large \( R \).

The mathematical similarity (cf eq 37) between wind–powered and radiation–powered momentum driving allows an empirical estimate of an \( M – \sigma \) relation for the latter if we assume that observed AGN define the relation, and that their observed luminosities correspond to \( L/L_{\text{Edd}} \sim 0.1 – 1 \). This gives \( M_{\text{crit}} = (L_{\text{Edd}}/L)M_\sigma \sim 1 – 10M_\sigma \) (Murray et al. 2005). Optical depth effects might narrow this range closer to the observed one (Debuhr et al. 2012). This suggests that radiation driving might be compatible with the \( M – \sigma \) relation, but a momentum–driven outflow like this can never simultaneously reproduce the high–velocity molecular outflows characterising the clearout phase. In particular its momentum is \( L/c < L_{\text{Edd}}/c \), considerably smaller than the observed \( \sim 20L_{\text{Edd}}/c \) of such flows (see Section 5.3). In other words, we have the usual difficulty that momentum–driving can accommodate the \( M – \sigma \) relation, but not simultaneously solve the clearout problem.

One way of possibly overcoming this (e.g. Faucher–Giguère et al. 2012; Faucher–Giguère & Quataert, 2012) is to assume (cf Roth et al. 2012) that instead of degrading incident high–energy photons to lower–energy ones that escape freely, the effect of dust absorption is to retain much of the incident radiant energy. Then if the dust is distributed spherically and is in a steady state the radiation force on it is \( \tau L/c \), where \( \tau \) is the radial optical depth of the dust (cf Roth et al. 2012). This form of radiation driving of optically thick dust can in principle produce outflows whose scalar momenta are boosted above that of the driving luminosity \( L/c \) by a factor \( \sim \tau \) because photons may be reabsorbed several times. For \( \tau \gg 1 \) the radiation field is effectively trapped and presumably approaches a blackbody form (cf the discussion of the electron scattering case above), limiting the boost.

Evidently for radiation driving of dust to solve simultaneously both the SMBH scaling and clearout problems requires a sharp transition in the properties of the dust opacity at the critical \( M – \sigma \) mass. This must change the outflows from effectively momentum–driven (incident photons are absorbed but their energy escapes as softer photons) to energy–driven (incident photons trapped) at this point, in a switch analogous to the turnoff of Compton cooling in the wind–driven case. There have so far been no suggestions of how this might happen, but the physics of dust opacity is sufficiently complex that this is perhaps not surprising.
8 SMALL vs LARGE SCALE FEEDBACK

We have shown that UFO winds are very common in AGN, despite quite restrictive conditions on their observability. They provide an obvious way for the central supermassive black hole to communicate its presence to its host. This in turn suggests ways of understanding both the SMBH – galaxy scaling relations, and the need to expel gas from the galaxy spheroid to terminate star formation. It is clear that this AGN feedback must operate at times on small scales, and at others on large scales. Our discussion of feedback points to a natural association between momentum–driving and small scales, and between energy–driving and large scales. Small–scale phenomena naturally explained by wind momentum–driving include

1. Super–solar elemental abundances in AGN spectra. Wind momentum–driving automatically sweeps up and compresses the same gas many times before the black hole mass reaches \( M_\sigma \). Generations of massive stars forming out of the same swept–up gas can repeatedly enrich the gas close to the SMBH with nuclear–processed material before the \( M_\sigma \) mass is reached, and momentum–driving changes to energy–driving.

2. Dark matter cusp removal. The same repeated sweeping–up of a gas mass comparable to the SMBH mass, followed by fallback, has a strong tendency to weaken dark matter cusps. Because the baryonic mass involved is much larger, this is a more powerful version of the mechanism invoked by Pontzen & Governato (2012) (see also Garrison–Kimmel et al. 2013), who considered supernovae near the SMBH.

3. Quiescence of AGN hosts. Most AGN hosts do not show dramatically elevated star formation in the central regions of their galaxy discs, or so far much evidence for high–speed (\( \sim 1000 \) km s\(^{-1}\)) and massive (\( \sim \) few \( 100 M_\odot \) yr\(^{-1}\)) outflows on large scales. This is compatible with wind driving by momentum but not energy.

Large–scale phenomena suggesting the action of energy–driving include

4. Metals in the circumgalactic medium. These must be made in galaxies and only later expelled to make the CGM. This suggests that expulsion through energy–driving acts only after stellar evolution has had time to enrich a significant fraction of the galaxy bulge gas.

5. Mechanical luminosities of galaxy–scale molecular outflows. These are observed to be close to 5% of their central AGN luminosities \( L_c \), just as expected for energy–driving, with momenta close to the predicted \( 20L/c \).

6. Suppression of cosmological infall. Energy–driven outflows at large radii probably prevent galaxies accreting indefinitely (Costa et al. 2014).

This list seems to favor a combination of momentum and energy driving, with some kind of switch between them. The suppression of inverse Compton shock cooling at the point when the black hole mass reaches \( M_\sigma \) appears promising, but requires further work on how observable the cooling is, and the possibility of two–fluid effects. Radiation driving on dust could produce similar behavior, although the physics controlling the required switch between momentum and energy driving is so far unexplained.

It is worth stressing that even a detailed understanding of the dual role of AGN feedback in establishing both the SMBH – host scaling relations and the quenching of star formation would solve only half of the problem. For a full picture of how black holes and galaxies influence each other we need to know what physical
mechanism can produce a supply of gas with so little angular momentum that much of it can accrete on to the central supermassive black hole within a few Salpeter times (see Section 3.1, and equation [20]). We saw in Section 1.2 that the hole’s gravity is far too weak to influence the galaxy on the mass scale needed for this. Only feedback can do this, perhaps suggesting that SMBH feedback may ultimately cause SMBH feeding (cf Dehnen & King 2013).

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