Einstein-Infeld-Hoffman method and soliton dynamics in a parity noninvariant system

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Abstract

We consider slow motion of a pointlike topological defect (vortex) in the nonlinear Schrödinger equation minimally coupled to Chern-Simons gauge field and subject to external uniform magnetic field. It turns out that a formal expansion of fields in powers of defect velocity yields only the trivial static solution. To obtain a nontrivial solution one has to treat velocities and accelerations as being of the same order. We assume that acceleration is a linear form of velocity. The field equations linearized in velocity uniquely determine the linear relation. It turns out that the only nontrivial solution is the cyclotron motion of the vortex together with the whole condensate. This solution is a perturbative approximation to the center of mass motion known from the theory of magnetic translations.

1. Introduction

In this paper we are going to discuss the reliability of the Einstein-Infeld-Hoffman (EIH) method [1], which was developed in the theory of general relativity for analysis of the motion of point sources of gravitational field, to dynamics of point-like topological defects in parity noninvariant systems. The essence of the EIH method, which remains relevant in the present context, can be outlined as follows. Let us assume that there is a characteristic velocity \( v \) in the model. In a relativistic model \( v \) is the velocity of light while in a nonrelativistic model \( v \) can be say a sound velocity. To consider time evolution which is slow as compared to the characteristic velocity, one can develop a perturbative expansion with \( 1/v \) as an expansion parameter. The fields are expanded in powers of \( 1/v \) and the time \( t \) is replaced by a rescaled time \( \tau \), \( t = v\tau \). The time derivatives are rescaled as \( \partial_t = \frac{1}{v} \partial_{\tau} \). The \( n \)-th order time derivative of a given quantity is formally \( n \) orders of magnitude smaller than the quantity itself. In particular acceleration is always negligible as compared to velocity. However in a parity noninvariant system like a system in an external uniform magnetic field one can imagine a soliton performing a cyclotron kind of motion. In the cyclotron motion acceleration is always a linear form of velocity so that this kind of motion can not be described within the formal expansion in powers of \( 1/v \).

The standard approach to the dynamics of point particles interacting with fields is to take in the first step arbitrary particle trajectories and find from field equations the fields produced by the assumed currents. In this first step the parameter \( 1/v \) can be employed to calculate the fields an expansions in powers of particle velocities. The second step is to substitute the calculated fields into particles equations of motion. In this way one obtains purely mechanical equations which can be solved to obtain particle trajectories. Such a method was applied to derive the Darwin Lagrangian in electrodynamics [2] or in the context of general relativity [1] to mention only the most important cases.

It turns out that in the case of topological defects such an off-shell calculation is not possible. One can try to find solutions of field equations for a given soliton trajectory but it turns out that a regular solution exists but only for a very special trajectory

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which is thus the solution of the soliton dynamics problem. This property has been observed first in the case of relativistic Chern-Simons vortices and independently for relativistic membranes. The origin of the problem in the case of solitons can be traced back to the fact that, unlike for point particles interacting with fields, there is no a priori equation of motion for the topological defects. Unlike point particles solitons are composed of the same fields as the fields which mediate interactions between them. Their dynamics is implicit in the field equations. Our calculation in Section 3 provides another example. Its advantage is that there is an exact solution at hand to be compared with the perturbative result.

2 Model and boost in an external magnetic field

Let us consider the model, which is a field-theoretical description of the quantum Hall effect of polarized electrons. The electrons are described in terms of bosonic fields fermionized by coupling to the auxiliary Chern-Simons fields. Let us consider the model which is a field-theoretical description of the quantum Hall effect of polarized electrons. The electrons are described in terms of bosonic fields fermionized by coupling to the auxiliary Chern-Simons fields.

\[ L = \frac{\kappa}{2} \varepsilon^{\alpha\beta\gamma} a_\gamma \partial_\alpha a_\beta + \frac{i}{2} \{ \psi^*(D_0\psi) - \psi(D_0\psi)^* \} - \frac{1}{2m} (D_k\psi)^*(D_k\psi) - \frac{\lambda}{2} (\rho_0 - \psi^*\psi)^2. \]

\( \rho_0, \) which is a condensate density, is related to the external magnetic field by \( \rho_0 = -\kappa B/e. \) In what follows we assume \( \kappa \) to be negative. The covariant derivative couples the scalar field to both the external electromagnetic field and the Chern-Simons field. The Greek indices run over space-time indices 0, 1, 2 while the Latin indices denote planar coordinates 1, 2. Our convention is \( \varepsilon^{012} = +1 \) and we assume the signature \((+,-,-).\)

In the formulation of the ground state of the theory in the external magnetic field B,

\[ A_0(t, x) = 0, \]
\[ A_k(t, x) = \frac{1}{2} B \varepsilon_{kl} x^l, \]

is the uniform condensate \( \Psi^*\Psi = \rho_0 \) with the external magnetic field being screened by the Chern-Simons field, \( a_k = -A_k, \) \( a_0 = 0. \) The theory is a Galilean invariant system in an external magnetic field. As such it has the following symmetry. If the set of fields \( \psi(t, \vec{x}), a_\mu(t, \vec{x}) \) is a solution of the model, then the following boosted fields are also solutions of the model:

\[ \tilde{\psi}(t, \vec{x}) = \psi(t - \vec{R}(t)), \]
\[ \tilde{a}_0(t, \vec{x}) = a_0[t, \vec{x} - \vec{R}(t)] - \vec{R}^k a_k[t, \vec{x} - \vec{R}(t)], \]
\[ \tilde{a}_k(t, \vec{x}) = a_k[t, \vec{x} - \vec{R}(t)], \]
\[ \chi_B = -\frac{1}{2} m (\vec{R}^k \dot{\vec{R}}^k) + \frac{e}{m} \int_{t_0}^t d\tau \dot{\vec{R}}^k(\tau) A_k[\vec{x} - \vec{R}(\tau)] . \]

provided that the trajectory \( \vec{R}(t) \) satisfies the equation of motion

\[ m \ddot{\vec{R}}^k = -eB \varepsilon^{kl} \dot{\vec{R}}^l. \]

The last equation is simply the equation of motion of a planar electron in uniform magnetic field. Its solution is the cyclotron motion with the cyclotron frequency \( \omega_c = eB/m. \) If an unboosted solution is say the uniform condensate, then the solution after the boost is a condensate performing a cyclotron motion. If the unboosted solution contained a vortex then after the boost the vortex would move together with the condensate.

3 Slow vortex motion in perturbative approximation

In this section we are going to describe a perturbative scheme which can be applied in investigations of slow motions of topological defects in parity noninvariant systems. The perturbative method is quite general, we think it may find applications in some other models. The method has been already described in our earlier article devoted to the dynamics of relativistic self-dual Chern-Simons vortices. However, the calculation in is relatively complicated and what is more there is no exact solution at hand to be compared with the perturbative result. The magnetic boost provides us with a nice example of exact solution, which confirms the perturbative result and the perturbative method as such. The perturbative calculation shows also uniqueness of the magnetic boost.

Before we describe the perturbative calculation let us make a small rearrangement in the model. Namely, let us replace \( a_{\mu} + A_{\mu} = a_{\mu}. \) This replacement and the use of the definition \( \rho_0 = -\frac{\partial \chi}{\partial B} \) leads to the equivalent version of the model:

\[ L = \frac{\kappa}{2} \varepsilon^{\alpha\beta\gamma} a_\gamma \partial_\alpha a_\beta - e \rho_0 a_0 + \frac{i}{2} \{ \psi^*(D_0\psi) - \psi(D_0\psi)^* \} - \frac{1}{2m} (D_k\psi)^*(D_k\psi) - \frac{\lambda}{2} (\rho_0 - \psi^*\psi)^2, \]
where we have already neglected the hats. The covariant derivative is simplified to \(D_\mu = \partial_\mu - i e a_\mu\). The advantage of this formulation is that there is just the Chern-Simons gauge field to be handled with. The external magnetic field is replaced by the uniform background charge density now.

We concentrate on the model \(\mathcal{F}\) from now on. The field equations of the model are

\[
i\partial_t \Psi + e a_0 \Psi + \frac{1}{2m} D_k D_k \Psi + \lambda (\rho_0 - \Psi^* \Psi) \Psi = 0 ,
\]

\[
\kappa \varepsilon_{kli} \partial_0 a_l + e (\rho - \rho_0) = 0 ,
\]

\[
\kappa \varepsilon^{k\alpha\beta} \partial_\alpha a_\beta + e J^k = 0 ,
\]

where \(\rho = \Psi^* \Psi\) is the particle density and the current is

\[
J^k = i \frac{1}{2m} \{ \Psi (D_k \Psi)^* - \Psi^* (D_k \Psi) \} = \frac{\rho}{m} (\partial_k \chi - e a_k) .
\]

\(\chi\) is the phase of the scalar field, \(\Psi = \sqrt{\rho} \exp i \chi\). The model admits the uniform condensate solution \(\Psi = \sqrt{\rho_0}, a_\mu = -A_\mu\). Because there is a nonvanishing condensate, the model also admits topological vortex solutions. The Ansatz for a vortex solution with the winding number minus one can be taken as

\[
\Psi^{(0)} (\vec{x}) = [\rho(r)]^{1/2} e^{-i \theta} ,
\]

\[
a_\theta^{(0)} (\vec{x}) = A(r) ,
\]

\[
a_0^{(0)} (\vec{x}) = A_0(r) ,
\]

\[
a_\nu^{(0)} (\vec{x}) = 0 .
\]

The function \(\rho(r)\) interpolates between \(\rho(0) = 0\) and \(\rho(\infty) = \rho_0\). The gauge potential vanishes at the origin, \(A(0) = 0\), and tends to a pure gauge at infinity, \(A(r) \approx -\frac{1}{r}\). In the Bogomol’nyi limit, \(\lambda = \frac{\pi^2}{\rho_0 m}\), the equations fulfilled by the profile functions in Eq.\((8)\) can be derived \(\mathcal{F}\) from

\[
\nabla^2 \ln \rho = - \frac{2e^2}{\kappa} (\rho - \rho_0) ,
\]

\[
A(r) = \frac{1}{er} + \frac{\rho'}{2e\rho} ,
\]

\[
A_0(r) = \frac{e}{2m\kappa} (\rho - \rho_0) .
\]

The primes denote derivatives with respect to \(r\), \(\rho'' = \frac{d^2}{dr^2}\). Once the solution of the first equation in \(\mathcal{F}\) is known the functions \(A(r)\) and \(A_0(r)\) can be expressed through \(\rho(r)\).

The aim of the perturbative calculation is to find an approximate trajectory of a vortex in the limit of slow motion. The perturbative method consists of two main ingredients. The first of them is rather classic. As we are interested in slowly moving vortices, we expand all the quantities in powers of vortex velocity \(\vec{R}(t)\). The zero order approximation to a moving vortex solution would be just

\[
\Psi(t, \vec{x}) \equiv \Psi^{(0)} [\vec{x} - \vec{R}(t)] ,
\]

\[
a_\mu(t, \vec{x}) = a_\mu^{(0)} [\vec{x} - \vec{R}(t)] .
\]

It is a solution to field equations but only when we neglect all the terms in field equations linear (or higher order) in vortex velocities. An exact solution \(\Psi(t, \vec{x}), a_\mu(t, \vec{x})\), if exists, differs from the fields in Eqs.\((10)\),

\[
\Psi(t, \vec{x}) = \Psi(t, \vec{x}) + \psi(t, \vec{x}) ,
\]

\[
a_\mu(t, \vec{x}) = a_\mu(t, \vec{x}) + u_\mu(t, \vec{x}) .
\]

The deviations \(\psi, u_\mu\) are at least of first order in vortex velocity. The field equations \(\mathcal{F}\), when linearized both in the vortex velocity and in the deviations \(\psi, u_\mu\), become a set of inhomogenous differential equations

\[
i\partial_t \psi + e a_0 \psi + e \Psi u_0 + \frac{1}{2m} [D_k D_k \psi - 2ie(D_k \Psi) u_k - ie\Psi (D_k u_k)] + \lambda \rho_0 \psi - 2\lambda \Psi^* \Psi \psi - \lambda \Psi^2 \psi^* = -i \partial_t \Psi ,
\]

\[
\kappa \varepsilon_{kli} \partial_0 u_l + e (\vec{\Psi}^* \vec{\psi} + \vec{\Psi} \vec{\psi}^*) = 0 ,
\]

\[
\kappa \varepsilon^{k\alpha\beta} \partial_\alpha u_\beta - e \vec{\Psi}^* \vec{\Psi} u_k + e (\partial_k \chi - e a_k)(\vec{\Psi}^* \vec{\psi} + \vec{\Psi} \vec{\psi}^*) = \kappa \varepsilon^{kli} \partial_l a_i ,
\]

\(\square\)

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where \( \hat{D}_k = \partial_k - ie\bar{a}_k \). The deviations \( \psi, u_\mu \) can be expanded in powers of vortex velocity components \( \dot{R}^k(t) \). The leading terms in the expansion read

\[
\psi(t, \bar{x}) = \dot{R}^k(t)\psi^{(k)}[\bar{x} - \bar{R}(t)] , \\
u_\mu(t, \bar{x}) = \dot{R}^k(t)\nu^{(k)}[\bar{x} - \bar{R}(t)] .
\]

Before we substitute the Ansatz (13) to the linearized equations (12), we have to introduce the second and key ingredient of the perturbative method. Namely we allow the vortex acceleration to be manifestly linear in velocity

\[
\dot{R}^k(t) = \omega^{kl}\dot{R}^l(t) + O(\dot{R}^k\dot{R}^k) .
\]

The matrix \( \omega \) is time-independent and velocity-independent. In other words we are going to consider accelerations as being of the same order in magnitude as velocities.

The assumption that the acceleration and the velocity are of the same order contradicts customary claims in the literature. The usual reasoning, which can be traced back to the Einstein-Infeld-Hoffman method in general relativity [4], is as follows. We assume there is a characteristic velocity in the model, say \( v \). To consider slow time-evolution, we develop a perturbative expansion in the parameter \( 1/v \), which we consider to be small. The perturbative expansion has two ingredients. First of all the fields are expanded around boosted static solutions in the powers of \( 1/v \). Second, the real time \( t \) is replaced by a rescaled time \( \tau = v t \). The time derivatives are then rescaled as \( \partial_t = \frac{1}{v}\partial_\tau \). For a given quantity \( Q \), its \( n \)-th time derivative is formally \( n \) orders smaller than the quantity itself, \( \partial^n_t Q = v^{-n}\partial^n_\tau Q \). In particular in this perturbative scheme the acceleration is always regarded to be negligible as compared to velocity or in other words the matrix \( \omega^{kl} \) in Eq.(14) is implicitly assumed to be zero. Thus the commonly accepted perturbative scheme rules out any solution like a motion along circular orbit.

We still do not have a satisfactory perturbative scheme to describe eventual vortex motion with radiation in which the back-reaction could be self-consistently taken into account. Thus in this paper we will restrict to nonradiative trajectories. When there is no radiation the energy of the vortex must be conserved. For a single vortex in an uniform condensate, translational invariance implies that its acceleration must be perpendicular to velocity

\[
\omega^{kl} = \omega_{c}^{kl} ,
\]

where \( \omega \) is a constant. We do not know the value of the constant. It has to be fixed by a solvability condition.

Let us then try to solve the Eqs.(12). The examination of sources on the RHS’s of Eqs.(12)

\[
-i\partial_t \bar{\Psi} = \frac{i\rho'}{2}\dot{R}^1 \cos \theta + \dot{R}^2 \sin \theta + \frac{\rho'^2}{r}[-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta] , \\
\partial_\tau \bar{a}_1 = [\dot{R}^1 \cos \theta + \dot{R}^2 \sin \theta]A' \sin \theta + [-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta]\frac{A}{r} \cos \theta , \\
\partial_\tau \bar{a}_2 = -[\dot{R}^1 \cos \theta + \dot{R}^2 \sin \theta]A' \cos \theta - [-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta]\frac{A}{r} \sin \theta
\]

shows that without loss of generality we can adopt the following Ansatz for the first order field deviations (we assume the gauge in which the deviation of the scalar field’s phase is zero)

\[
\dot{R}^k f^{(k)}(\bar{x}) = \rho^{l/2}(r)s(r)[-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta] , \\
\dot{R}^k u_0^{(k)}(\bar{x}) = \omega\rho)[-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta] , \\
\dot{R}^k u_1^{(k)}(\bar{x}) = \omega\rho s[-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta] , \\
\dot{R}^k u_2^{(k)}(\bar{x}) = \omega\rho c[-\dot{R}^1 \sin \theta + \dot{R}^2 \cos \theta] .
\]

The substitution of the above Ansatz to Eqs.(12) yields

\[
s'' + \frac{s'}{r} - \frac{s}{r^2} + \frac{\rho'}{\rho}(s' - c) + 2a - 4\rho s = \frac{2}{r} , \\
b' + \frac{b}{r} - \frac{c}{r} + \frac{\rho'}{\rho}(b - \frac{s}{r}) + 2\omega s = -\frac{\rho'}{\rho} , \\
c' + \frac{c}{r} - \frac{b}{r} - 2\rho s = 0 , \\
a' - \omega b - \rho s - \rho c = \frac{A}{r} , \\
a - \omega c - \rho b = A'.
\]
We use from now on the rescaled units in which all the constants \( m, (-\kappa), \rho_0 \) and \( e \) are set equal to 1. We have also restricted to the Bogomol’nyi limit, which in the rescaled units corresponds to \( \lambda = 1 \). It is by no means necessary but it makes the formulas more compact. In particular the second equation in the set (9), which holds only in the Bogomol’nyi limit, can serve for many simplifications.

The second equation in the set (18) is not independent, it can be derived from the last three equations. The last equation in the set (18) can be used to express \( a(r) \) by other functions

\[
a = \omega rc + r \rho b + A' r .
\]

Once \( a(r) \) is expressed like in Eq.(19), it can be eliminated from the first, third and fourth equation in the set (18). Finally we are left with only three independent equations

\[
\begin{align*}
b' + \frac{b}{r} - \frac{c}{r} + 2\omega s + \frac{\rho'}{\rho} [b - \frac{s}{r} + 1] &= 0, \\
c' + \frac{c}{r} - \frac{b}{r} - 2\rho s &= 0, \\
s'' + \frac{s'}{r} - \frac{s}{r^2} + \frac{\rho'}{\rho} (s' - c) - 4\rho s + 2r \rho b + 2\omega rc &= 2r (\frac{1}{r^2} - A').
\end{align*}
\]

The regular asymptotes close to \( r = 0 \) are

\[
\begin{align*}
s(r) &\approx (1 + \alpha)r + \ldots, \\
b(r) &\approx \alpha + \ldots, \\
c(r) &\approx \alpha + \ldots,
\end{align*}
\]

where the higher order terms in \( r \) were neglected. There is only one free parameter \( \alpha \) in the asymptote.

To find out the asymptote at infinity we go back to Eqs.(18). At infinity \( \rho \approx 1 \), up to terms decaying exponentially. The asymptotic form of Eqs.(18) is

\[
\begin{align*}
s'' + \frac{s'}{r} - \frac{s}{r^2} + 2a - 4s &= \frac{2}{r}, \\
b' + \frac{b}{r} - \frac{c}{r} + 2\omega s &= 0, \\
c' + \frac{c}{r} - \frac{b}{r} - 2s &= 0, \\
a' + \frac{1}{r^2} &= \omega b + c, \\
a - \frac{1}{r^2} &= b + \omega c.
\end{align*}
\]

This set of asymptotic equations implies the following

\[
\begin{align*}
s'' + \frac{s'}{r} - \frac{s}{r^2} - 4s + 2a &= \frac{2}{r}, \\
a'' + \frac{a'}{r} - \frac{a}{r^2} - 2(1 - \omega^2)s &= 0.
\end{align*}
\]

Once \( s(r) \) and \( a(r) \) are known, one can find \( b(r) \) and \( c(r) \) solving with respect to them the last two equations in the set (22). The solution is unique provided that \( \omega^2 \neq 1 \).

One can diagonalize Eqs.(23). The eigenvalues turn out to be

\[
\lambda_{1,2} = -2(1 - \omega^2) .
\]

If \( \omega^2 > 1 \), one of the eigenvalues is positive and the other is negative. The asymptote must contain a linear combination of two Bessel functions and of two modified Bessel functions. Thus there is one divergent mode and two long-range unnormalisable modes to be removed. But we have only two adjustable constants, \( \alpha \) in the asymptote at the origin (21) and \( \omega \). Thus there is no normalisable solution for \( \omega^2 > 1 \).

If \( \omega^2 < 1 \), the general asymptote is a combination of two exponentially divergent modes and two normalisable modes. Thus the two constants \( \omega, \alpha \) might happen to be sufficient to remove the two divergent modes. Numerical analysis shows that it does not happen for \(-1 < \omega < 1\), however.
Thus we must restrict our attention to the case $\omega^2 = 1$. For $\omega = +1$ the asymptotic equations (22) are solved by

$$
a(r) \approx \frac{1}{r} + 2\varepsilon r,
$$

$$
s(r) \approx \beta_1 \frac{e^{2r}}{\sqrt{2r}} + \beta_2 \frac{e^{-2r}}{\sqrt{2r}} + \varepsilon r,
$$

$$
b(r) \approx -c(r) \approx \gamma r^{-2} + \frac{\varepsilon}{2} r^2.
$$

(25)

The two constants $\beta_1$ and $\varepsilon$ must be tuned to zero with just one parameter $\alpha$ in the asymptote close to the origin. There is no regular solution for $\omega = +1$.

For $\omega = -1$ the asymptote is

$$
a(r) \approx \frac{1}{r} + \frac{2\sigma}{r},
$$

$$
s(r) \approx \beta_1 \frac{e^{2r}}{\sqrt{2r}} + \beta_2 \frac{e^{-2r}}{\sqrt{2r}} + \frac{\sigma}{r},
$$

$$
b(r) = -c(r) \approx \delta.
$$

(26)

For nonzero $\delta$ the gauge fields’ deviations become $\tilde{R}^{k}_{ij} = \delta \tilde{R}_{ij}$, compare with Eq. (17). Once the constant $\beta_1$ is tuned to zero with the help of the free parameter $\alpha$, the asymptote of the electric current becomes $J^k = \delta e^2 \rho_0 \tilde{R}^k$. Thus at any given moment of time the vortex velocity is parallel to the condensate current. What is more, for $\omega = -1$ one can give an explicit unique solution to Eqs. (24), namely $b(r) = c(r) = -1$ and $s(r) = 0$. According to Eq. (19)

$$
a(r) = r[1 - \rho(r) + A'(r)] = -A(r).
$$

(27)

The last equality holds thanks to the Gauss law. Thus the constant $\delta$ is fixed as $\delta = -1$. The vortex moves together with the negatively charged condensate.

Note that $\omega = -1$ so the condesate moves along a cyclotron orbit, $\tilde{R}^k = -\varepsilon^{kl} \tilde{R}^l$. The solution is a perturbative approximation to the exact deformed Galilean boost. Such a solution could not be obtained in a perturbative calculation if accelerations were neglected as compared to velocities. At the same time, the analysis shows that the deformed Galilean boost is the only solution which can be obtained by a perturbative expansion in velocities. In principle this does not exclude possibility of a dissipative noncyclotron motion.

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