An Improved Shrunken Primal-Dual Subgradient (i-SPDS) algorithm for Optimization in Large-Scale Networked Environment

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Abstract. Large-scale optimization in networked environment is challenged by scalability. To address this challenge, this paper sets its aim on developing algorithms for optimization problems with strongly coupled objective function and global constraints. As a radical improvement of the state-of-the-art algorithms, this paper develops an improved shrunken-primal-dual subgradient (i-SPDS) algorithm that establishes fast convergence and eliminates optimality gap. The developed algorithm is rather generic in that it can be readily implemented in any applications that involve large-scale networked environment. Simulations are conducted to demonstrate the efficacy and efficiency of the proposed i-SPDS.

Keywords: Large-scale optimization; Distributed energy resources; Decentralized optimization.

1. Introduction
Large-scale optimization commonly exists in practical engineering problems, such as optimal power flow (OPF) problems over the distribution networks and traffic flow optimization problems. Take the power system for example, a popular research area is controlling distributed energy resources (DER) for the provision of grid reliability services, which is normally casted into a large-scale OPF problem. In controlling DERs for grid services, most early works adopt the centralized control scheme [1]–[9], where a control center is in place to gather the information of and compute real/reactive power control signals for all participating DERs. Though centralized schemes are easy to design and realize from the algorithmic perspective, a major issue that prevents them from being implemented in practice is scalability. The scalability issue originates from the fact that a vast number of controllable units are involved in the control framework and near-real-time operation is normally required. With the increase of the number of controllable units, the amount of computation and computational complexity of centralized optimization increase sharply. This will ultimately lead to delays in the computation of control signals and the computational power at the control center may not be sufficient. In the future digitalized Internet of Things (IoT) era, millions of controllable units (agents) need to be coordinated to cooperate in order to achieve system-level objectives. This motivates the development of scalable optimizations frameworks, specifically the decentralized and distributed optimization algorithms therein.

Investigations on decentralized and distributed optimization algorithms for various problem formulations largely exist in the literature, e.g., [10]–[17], to name a few. Unfortunately, these algorithms, despite their merits, either require special problem formulations, e.g., decision variables must be separable in the objective function [16], or for special use cases, e.g., consensus problems [17].
Therefore, their applicability in generic problem formulation or problems that cannot be formulated into those convenient forms are rather limited. Specifically, in generic optimization problems, decision variables of different agents not necessarily reach consensus; the objective function which represents system-level goals normally strongly couples decision variables of all agents; and global constraints which maintain the network safety and security normally strongly couples decision variables of all agents. In addition, co-existence of local and global objectives as well as local and global constraints makes decentralized algorithms design even more challenging.

Three lines of research in the optimization community have attempted to solve this problem. Koshal et al. [18] developed a regularized primal-dual subgradient (RPDS) algorithm by adding regularization terms to Lagrangian primal and dual variables. This ground-breaking work later inspired many adoptions and follow-up improvements, e.g., [19]–[22]. The major merit of RPDS is in its guaranteed convergence. Despite the guaranteed convergency, RPDS inevitably introduces relative convergence errors to the optimal solution due to the usage of regularization, and the errors would be enlarged as the problem dimension increases. Second, a range of research builds the decentralized algorithms relying on the alternating direction method of multipliers (ADMM) by reformulating the strongly coupled problem into consensus problems and creating local copies of global variables [23]–[27]. These approaches, though, carry over the convergence attribute of ADMM, require frequent information exchange between agents and two-layer communication networks. These characteristics severely complicate the communication and pose computing burdens to an additional layer of controllers. In addition, the distributed algorithm based on ADMM will always encounter a large number of iterations. The third line of research is built on the shrunken primal-dual subgradient (SPDS) algorithm [28]–[31]. SPDS-based approaches iterate between agents and a central coordinator, which solves strongly coupled convex optimization problems, and it is no need to communicate between agents. Comparing with RPDS and ADMM, SPDS has been validated effectively in reducing iteration times, alleviating communication loads, and establishing monotone convergence property. Comparing with RPDS and ADMM, SPDS has been proved to be effective in reducing the number of iterations, alleviating communication loads, and establishing monotone convergence property. Despite the merits, we recently found that existing SPDS-based approaches still encounter slight optimality gap due to the introduced shrinkage-expansion procedure.

The aforementioned observations motivate us to develop a new decentralized algorithm that has guaranteed convergence, needs only a small number of iterations, and is free of optimality gap. The major contribution of this paper is that it investigates convergence-error-free decentralized algorithms for optimization problems with strongly coupled constraints and objective function for the first time. The improved SPDS (i-SPDS) algorithm solves a type of optimization problems composed of inseparable objective functions, strong coupling network opportunity constraints and local opportunity constraints, and has good versatility. The algorithm fundamentally eliminates convergence error, establishes monotonic and fast convergence, requires low computational load, making it appreciate for real- or near real-time implementation.

2. Problem Formulation and Main Results

2.1. Problem Formulation

In this paper, we consider a generic convex optimization problem in the form of

$$\min_{x_i} F(x) = f(x_1, x_2, \ldots, x_n, y) + \sum_{i=1}^{n} f_i(x_i)$$

s.t. $g(x_1, x_2, \ldots, x_n, y) \leq 0$

$x_i \in X_i, \forall i = 1, 2, \ldots, n$ (1)

where $n$ denotes the total number of agents in the network, $x_i \in \mathbb{R}^m$ is the decision variable of agent $i$ with the dimension $m$, $y \in \mathbb{R}^m$ is the system uncontrollable term, mapping $f(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^1$ is the global objective function, mapping $f_i(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^1$ is the local objective function, mapping $g(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^p$ denotes the global constraint function, and $X_i$ denotes the local constraint set.
In the most generic form of (1), the global objective function \( f(\cdot) \) strongly couples all decision variables \( x_1, \ldots, x_n \) in the way that they are not separable. For example, in controlling DERs, we can define \( f(\cdot) = \sum_{i=1}^{n} x_i + y - z \) for tracking a desired trajectory \( z \) and define \( f(\cdot) = \| \sum_{i=1}^{n} a_i x_i + by \|_2^2 \) for power loss minimization; in controlling traffic flows, we can define \( F(\cdot) = x^T A^T A x \) to represent the cost of congestion, where \( x = \text{col}(x_1, \ldots, x_n) \) denotes the collection of all decision variables and \( A \) is the network connectivity matrix. The global constraint function \( g(\cdot) \) also strongly couples the decision variables. For example, we can define \( g(\cdot) = \sum_{i=1}^{n} A_i x_i + b y \) based on linearized power flow equations to represent nodal voltage magnitude violations in the distribution network caused by DER generation; we can define \( g(\cdot) = \sum_{i=1}^{n} A_{il} x_i \leq C_l \) to denote traffic flow constraint where \( l \) is the traffic network branch indicator. This type of problem universally exists in a range of practical application problems. However, the strongly coupling attribute of the generic problem formulation (1) makes the decentralized optimization algorithm design extremely challenging.

2.2. Preliminaries

In this subsection, we briefly present the state-of-the-art decentralized algorithm [18], i.e., RSPDS, and state its limitations. The RPDS developed in [18] suggests to iteratively update the dual and primal variables by following

\[
\begin{align*}
\lambda^{i+1} &= \Pi_D (\lambda^i + \beta \nu \lambda^i) \\
\lambda^{i+1} &= \Pi_D (\lambda^i + \beta \nu \lambda^i)
\end{align*}
\]

where \( \nu > 0 \), \( \epsilon > 0 \), and \( \lambda \) is the Lagrange multiplier vector associated with the global constraint \( g(x) \leq 0 \). The terms \( \frac{\nu}{2} \| x \|_2^2 \) and \( -\frac{\epsilon}{2} \| \lambda \|_2^2 \) are the introduced regularizations. The RPDS developed in [18] suggests to iteratively update the primal and dual variables by following

\[
\begin{align*}
x^{i+1} &= \Pi_x \left( x^i - \alpha \nabla_x L(x^i, \lambda^i) \right) \\
\lambda^{i+1} &= \Pi_D \left( \lambda^i + \beta \nu \lambda^i \right)
\end{align*}
\]

where \( \ell \) denotes the iteration number, \( \Pi (\cdot) \) is the Euclidean projection operator, \( D \) denotes the constraint set for the dual variable \( \lambda \), and \( \alpha \) and \( \beta \) are updating step sizes. Owing to the regularization terms in \( L_{\nu,\epsilon}(x, \lambda) \), RPDS has guaranteed convergence given appropriately chosen \( \alpha \) and \( \beta \). However, due to those regularization terms, an inevitable convergence error (optimality gap) exists. In addition, RPDS may suffer from a huge number of iterations. For example, in [18], around \( 10^5 \) iterations are needed for a 9-link-5-agent network. In the digitalized era, a large number of iterations will not only increase the communication burden, but also pose potential cyber-security risks as private information is potentially transmitted. In this paper, we aim to develop a new algorithm that has guaranteed convergence and optimality, while within a limited number of iterations.

2.3. Algorithm design

The decentralized algorithm proposed in this paper is based on the most recent research on SPDS [28]. The SPDS has significantly improved RPDS in that it attenuates the optimality gap introduced by the regularization terms by creating an extra shrinking-expanding procedure. SPDS is designed based on the unregularized locally unconstrained Lagrangian

\[ L(x, \lambda) = F(x) + \lambda^T g(x), \]

and it suggests to iteratively update the dual and primal variables by following

\[
\begin{align*}
x^{i+1} &= \Pi_x \left( \frac{1}{\tau_x} \Pi_x \left( \tau_x x^{i+1} - \alpha \nabla_x L(x^{i+1}, \lambda^{i+1}) \right) \right) \\
\lambda^{i+1} &= \Pi_D \left( \frac{1}{\tau_\lambda} \Pi_D \left( \tau_\lambda \lambda^{i+1} + \beta \nu \lambda^{i+1} \right) \right)
\end{align*}
\]

where \( \tau_x \) and \( \tau_\lambda \) are updating step sizes. The terms \( \frac{1}{2} \| x \|_2^2 \) and \( -\frac{\epsilon}{2} \| \lambda \|_2^2 \) are the introduced regularizations. The RPDS developed in [18] suggests to iteratively update the dual and primal variables by following

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\begin{align*}
x^{i+1} &= \Pi_x \left( x^i - \alpha \nabla_x L(x^i, \lambda^i) \right) \\
\lambda^{i+1} &= \Pi_D \left( \lambda^i + \beta \nu \lambda^i \right)
\end{align*}
\]

These terms ensure guaranteed convergence given appropriately chosen \( \alpha \) and \( \beta \). However, due to those regularization terms, an inevitable convergence error (optimality gap) exists. In addition, RPDS may suffer from a huge number of iterations. For example, in [18], around \( 10^5 \) iterations are needed for a 9-link-5-agent network. In the digitalized era, a large number of iterations will not only increase the communication burden, but also pose potential cyber-security risks as private information is potentially transmitted. In this paper, we aim to develop a new algorithm that has guaranteed convergence and optimality, while within a limited number of iterations.
where \(0 < \tau_x < 1\) and \(0 < \tau_\lambda < 1\) are the shrinking parameters for the dual and primal variables, respectively. The convergence analysis of SPDS can be found in [28]. SPDS has been widely applied in electric vehicle (EV) charging control and DER control for distribution grid services. Recently, we have found that SPDS still encounters slight convergence errors due to the shrinkage and expansion, though the optimality gap is shrunken comparing with RPDS. Therefore, in this paper, we aim to improve SPDS so as to fundamentally eliminate the optimality gap.

An interesting observation of the optimality gap in SPDS is that it is linearly correlated with the degree of shrinkage, i.e., the values of \(\tau_x\) and \(\tau_\lambda\). An intuitive solution is to choose \(\tau_x = 1\) and \(\tau_\lambda = 1\). This is, however, not a viable choice as the convergence of SPDS depends on the fact that \(\tau_x\) and \(\tau_\lambda\) are strictly less than 1. An appealing convergence property of SPDS is that its convergence is monotone. That being said, the convergence error defined as \(\varepsilon^{(\ell+1)} = \| \text{col}(x^{(\ell+1)}, \lambda^{(\ell+1)}) - \text{col}(x^{(\ell)}, \lambda^{(\ell)}) \|_2\) is monotonically decreasing. Inspired by this, we propose to expand the shrinking parameters asymptotically to 1 in (5). This leads to the improved SPDS (i-SPDS) updates, represented as

\[
x^{(\ell+1)} = \Pi_x \left[ \frac{1}{\varepsilon^{(\ell)}} \Pi_x \left( \tau_x^{(\ell)} x^{(\ell)} - \alpha \nabla_x L (x^{(\ell)}, \lambda^{(\ell)}) \right) \right]
\]

\[
\lambda^{(\ell+1)} = \Pi_\lambda \left[ \frac{1}{\varepsilon^{(\ell)}} \Pi_\lambda \left( \tau_\lambda \lambda^{(\ell)} + \beta \nabla_\lambda L (x^{(\ell)}, \lambda^{(\ell)}) \right) \right]
\]

Herein, the shrinking parameters \(\tau_x\) and \(\tau_\lambda\) will also update by following

\[
\tau_x^{(\ell)} = \tau_x^{(\ell-1)} + \frac{\varepsilon^{(\ell)}}{\varepsilon^{(\ell-1)}} \left( 1 - \tau_x^{(\ell-1)} \right)
\]

\[
\tau_\lambda^{(\ell)} = \tau_\lambda^{(\ell-1)} + \frac{\varepsilon^{(\ell)}}{\varepsilon^{(\ell-1)}} \left( 1 - \tau_\lambda^{(\ell-1)} \right)
\]

By leveraging the monotonicity of SPDS, Eqn. (7) guarantees that both shrinking parameters are asymptotically converging to 1. This leads the primal variables to asymptotically converge to the global optimum, i.e., the optimality gap is eliminated. The proposed i-SPDS algorithm is summarized in Algorithm 1.

3. Numerical Example
In this section, we will use a traffic flow optimization problem adopted in to show the efficacy and efficiency of the proposed i-SPDS, as well as to contrast with the state-of-the-art RPDS. Note that i-SPDS in this paper can be generically implemented in any networked optimization problem, especially the OPF problem in DER control.
Algorithm 1 Improved SPDS (i-SPDS) Algorithm

1: Iteration number $\ell = 0$; Agents initialize $x_i^{(0)}$; Central coordinator initializes $\lambda^{(0)}$; Iteration tolerance $\tau_x$; Initial shrinking parameters $0 < \lambda_x^{(0)} < 1$; Initial error $\varepsilon = 10^9$; Maximum iteration $\ell_{\text{max}}$.
2: procedure
3: while $\varepsilon > \tau_x$ and $\ell \leq \ell_{\text{max}}$ do
4: Each agent transmits local $x_i^{(\ell)}$ to the central coordinator.
5: Central coordinator computes $\nabla_x \mathcal{L}(x^{(\ell)}, \lambda^{(\ell)})$, $\lambda^{(\ell)}$, and $\varepsilon^{(\ell)}$, then broadcasts them to all agents.
6: All agents perform (7a) to update the shrinking parameters to $\tau_x^{(\ell)}$.
7: All agents perform (6a) to update local decision variables to $x_i^{(\ell+1)}$.
8: The central coordinator performs (7b) to update the shrinking parameter to $\tau_\lambda^{(\ell+1)}$ and performs (6b) to update the dual variable to $\lambda_i^{(\ell+1)}$.
9: $\varepsilon^{(\ell+1)} = \| \text{col}(x^{(\ell+1)}, \lambda^{(\ell+1)}) - \text{col}(x^{(\ell)}, \lambda^{(\ell)}) \|_2$.
10: $\ell = \ell + 1$.
11: end while
12: end procedure

Herein, we consider a network comprising $n$ agents sharing a set of links in the set $\mathcal{L}$. The connectivity of this network is shown in Fig. 1.

![Figure 1. A network with 5 agents and 9 links.][18]

Each Agent $i$ has a local cost function $f_i(x_i) : \mathbb{R}^1 \mapsto \mathbb{R}$ of its traffic (or flow) rate $x_i \in \mathbb{R}^1$ given by

$$f_i(x_i) = -k_i \log(1 + x_i).$$

Each agent selects an origin-destination pair of nodes on this network and faces congestion based on the links traversed along the prescribed path connecting the selected origin-destination nodes. In this case, the congestion cost is considered in the form of

$$c(x) = \sum_{i=1}^{n} \sum_{l \in \mathcal{L}} x_{il} \sum_{j=1}^{n} x_{ij},$$

where $x_{ij}$ is the flow of agent $j$ on link $l$. Consequently, the total cost of the network is given by
\[
F(\mathbf{x}) = \sum_{i=1}^{n} f_i(x_i) + c(\mathbf{x})
= \sum_{i=1}^{n} -k_i \log(1 + x_i) + \sum_{i \in \mathcal{L}} \sum_{j=1}^{n} x_{ij}.
\]

Following the similar lines as in [18], we let \( \mathbf{A} \) denote the adjacency matrix that specifies the set of links traversed by the traffic generated by the agents. Specifically, \( A_{ij} = 1 \) if the traffic of agent \( i \) goes through link \( l \) and 0 otherwise. Therefore, a compact form of (9) can be represented as
\[
c(\mathbf{x}) = \mathbf{x}^{T} \mathbf{A}^{T} \mathbf{A} \mathbf{x},
\]
where \( \mathbf{x} = \text{col}(x_1, x_2, \ldots, x_n) \). The agent traffic (or flow) rates are coupled through the constraint of the form \( \sum_{i=1}^{N} A_{il} x_i \leq C_l \) for all \( l \in \mathcal{L} \), where \( C_l \) is the maximum aggregate traffic through link \( l \). The constraint can be compactly written as \( \mathbf{Ax} \leq \mathbf{C} \), where \( \mathbf{C} \) is the link capacity vector and is given by \( \text{col}(1,1,1,1,1,1,1,1,1) \). Thus, the illustrative problem can be concisely represented as
\[
\min_{x_1, \ldots, x_n} F(\mathbf{x}) \quad \text{s.t.} \quad \mathbf{g}(\mathbf{x}) = \mathbf{Ax} - \mathbf{C} \leq \mathbf{0}
\]
where \( \mathcal{X}_i \) is any convex local constraint set for \( x_i \). Herein, we chose \( \mathcal{X}_i := \{ x_i : 0 \leq x_i \leq 1 \} \). In setting up the simulation, as shown in Fig. 1, we chose Agent 1 to traverse links L2, L3, and L6 with \( k_1 = 10 \); Agent 2 to traverse links L2, L5, and L9 with \( k_2 = 0 \); Agent 3 to traverse links L1, L5, and L9 with \( k_3 = 10 \); Agent 4 to traverse links L6, L4, and L9 with \( k_4 = 10 \); and Agent 5 to traverse links L8 and L9 with \( k_5 = 10 \).

To apply i-SPDS to the problem in (12), we chose \( \alpha = 0.02, \beta = 0.5, \tau_\mathbf{x}^{(0)} = 0.98, \) and \( \tau_\lambda^{(0)} = 0.97 \). The error tolerance is set to \( 10^{-4} \). The updates of five decision variables can be shown in Fig. 2.

**Figure 2.** Convergence of \( x_1, x_2, x_3, x_4, \) and \( x_5 \) in 87 iterations.

The global optimal solution of (12) is at \( \mathbf{x}^* = \text{col}(0.8211,0,0.3595,0.1789,0.4617) \). As shown in Fig. 2, after only 87 iterations, the decision variables converged to the global optimum. In contrast, the best-case scenario in [18] for the same problem establishes convergence in about \( 1.11 \times 10^5 \) iterations. This verifies the superior fast convergence of i-SPDS. Convergence of the dual variable \( \lambda \) is shown in Fig. 3.
Figure 3. Convergence of the dual variable $\lambda$ in 87 iterations.

The zero optimality gap of i-SPDS depends on the monotonicity of the convergence of SPDS. Though the shrinking parameters are improved in i-SPDS to update iteratively, this monotonicity still holds. The convergence error of the illustrative example is shown in Fig. 4.

Figure 4. Convergence error of i-SPDS.

It can be readily seen that the convergence error reaches 0 at the 87th iteration, and the convergence is monotone.

Remark: Some potential applications include controlling the reactive and real power of solar PV inverters to minimize the distribution network power loss and controlling the charging process of EVs to provide valley filling services.

4. Conclusions

In this paper, a new decentralized algorithm is proposed, i.e., i-SPDS, for optimization algorithms with strongly coupled objective function and global constraints. Comparing with RPDS and SPDS, i-SPDS radically improved the state of the art by establishing fast convergence and eliminating optimality gap. The superiority of the developed i-SPDS is also represented by its monotone convergence. Simulations on a 9-link-5-agent traffic flow optimization problem demonstrated the convergence and optimality of i-SPDS. The developed algorithm is rather generic in that it can be readily implemented in any applications that involve large-scale networked environment.
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References
[1] M. E. Baran and I. M. El-Markabi, “A multiagent-based dispatching scheme for distributed generators for voltage support on distribution feeders,” IEEE Transactions on Power Systems, vol. 22, no. 1, pp. 52–59, 2007.
[2] K. Turitsyn, P. S’ulec, S. Backhaus, and M. Chertkov, “Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration,” in Proceedings of the IEEE PES General Meeting, Providence, RI, USA, July 25-29 2010, pp. 1–6.
[3] K. Clement-Nyns, E. Haesen, and J. Driesen, “The impact of charging plug-in hybrid electric vehicles on a residential distribution grid,” IEEE Transactions on Power Systems, vol. 25, no. 1, pp. 371–380, 2010.
[4] P. Richardson, D. Flynn, and A. Kaene, “Optimal charging of electric vehicles in low-voltage distribution systems,” IEEE Transactions on Power Systems, vol. 27, no. 1, pp. 268–279, 2012.
[5] Sharma, C. Canizares, and K. Bhattacharya, “Smart charging of PEVs penetrating into residential distribution systems,” IEEE Transactions on Smart Grid, vol. 5, no. 3, pp. 1196–1209, 2014.
[6] J. Hu, S. You, M. Lind, and J. Østergaard, “Coordinated charging of electric vehicles for congestion prevention in the distribution grid,” IEEE Transactions on Smart Grid, vol. 5, no. 2, pp. 703–711, 2014.
[7] X. Luo and K. Chan, “Real-time scheduling of electric vehicles charging in low-voltage residential distribution systems to minimise power losses and improve voltage profile,” IET Generation, Transmission & Distribution, vol. 8, no. 3, pp. 516–529, 2013.
[8] J. Quirós-Tortós, L. Ochoa, S. Alnaser, and T. Butler, “Control of EV charging points for thermal and voltage management of LV network,” IEEE Transactions on Power Systems, vol. 31, no. 4, pp. 3028–3039, 2016.
[9] S. Bansal, M. Zeilinger, and C. Tomlin, “Plug-and-play model predictive control for electric vehicle charging and voltage control in smart grids,” in Proceedings of the IEEE Conference on Decision and Control, Los Angeles, CA, USA, December 15-17 2014, pp. 5894–5900.
[10] D. Jia and B. H. Krogh, “Distributed model predictive control,” in Proceedings of American Control Conference, Arlington, VA, USA, Jun. 25-27 2001, pp. 2767–2772.
[11] M. Liu, Y. Shi, and X. Liu, “Distributed mpc of aggregated heterogeneous thermostatically controlled loads in smart grid,” IEEE Transactions on Industrial Electronics, vol. 63, no. 2, pp. 1120–1129, 2016.
[12] J. Rivera, P. Wolfrum, S. Hirche, C. Goebel, and H.-A. Jacobsen, “Alternating direction method of multipliers for decentralized electric vehicle charging control,” in Proceedings of the IEEE Conference on Decision and Control, Florence, Italy, Dec. 10-13 2013, pp. 6960–6965.
[13] C. Chen, M. Li, X. Liu, and Y. Ye, “Extended ADMM and BCD for nonseparable convex minimization models with quadratic coupling terms: Convergence analysis and insights,” arXiv preprint arXiv:1508.00193v3 [math.OC], pp. 1–21, 2016.
[14] X. Gao and S. Zhang, “First-order algorithms for convex optimization with nonseparate objective and coupled constraints,” arXiv preprint arXiv:1605.05969v1 [math.OC], pp. 1–39, 2016.
[15] Y. Cui, X. Liu, D. Sun, and K.-C. Toh, “On the convergence properties of a majorized ADMM for linearly constrained convex optimization problems with coupled objective functions,” arXiv preprint arXiv:1502.00098v1 [math.OC], pp. 1–26, 2015.
[16] N. Li and J. Marden, “Decoupling coupled constraints through utility design,” IEEE Transactions on Automatic Control, vol. 59, no. 8, pp. 2289–2294, 2014.
[17] T.-H. Chang, A. Nedić, and A. Scaglione, “Distributed constrained optimization by consensus-based primal-dual perturbation method,” IEEE Transactions on Automatic Control, vol. 59, no. 6, pp. 1524–1538, 2014.
[18] J. Koshal, A. Nedić, and U. Shanbhag, “Multiuser optimization: Distributed algorithms and error analysis,” SIAM Journal on Optimization, vol. 21, no. 3, pp. 1046–1081, 2011.
[19] E. Dall’Anese and A. Simonetto, “Optimal power flow pursuit,” IEEE Transactions on Smart Grid, vol. 9, no. 2, pp. 942–952, 2018.

[20] X. Zhou, E. Dall’Anese, L. Chen, and A. Simonetto, “An incentive-based online optimization framework for distribution grids,” IEEE Transactions on Automatic Control, vol. 63, no. 7, pp. 2019–2031, 2018.

[21] T. Chen, A. Mokhtari, X. Wang, A. Ribeiro, and G. B. Giannakis, “Stochastic averaging for constrained optimization with application to online resource allocation,” IEEE Transactions on Signal Processing, vol. 65, no. 12, pp. 3078–3093, 2017.

[22] J. Koshal, A. Nedić, and U. V. Shanbhag, “Regularized iterative stochastic approximation methods for stochastic variational inequality problems,” IEEE Transactions on Automatic Control, vol. 58, no. 3, pp. 594–609, 2013.

[23] L. Zhang, V. Kekatos, and G. B. Giannakis, “Scalable electric vehicle charging protocols,” IEEE Transactions on Power Systems, vol. 32, no. 2, pp. 1451–1462, 2017.

[24] Hassan, Y. Dvorkin, D. Deka, and M. Chertkov, “Chance-constrained ADMM approach for decentralized control of distributed energy resources,” in Proceedings of the Power Systems Computation Conference, Dublin, Ireland, June 11-15 2018, pp. 1–7.

[25] A. Robbins and A. D. Domínguez-García, “Optimal reactive power dispatch for voltage regulation in unbalanced distribution systems,” IEEE Transactions on Power Systems, vol. 31, no. 4, pp. 2903–2913, 2016.

[26] P. Šulc, S. Backhaus, and M. Chertkov, “Optimal distributed control of reactive power via the alternating direction method of multipliers,” IEEE Transactions on Energy Conversion, vol. 29, no. 4, pp. 968–979, 2014.

[27] E. Dall’Anese, S. V. Dhople, B. B. Johnson, and G. B. Giannakis, “Decentralized optimal dispatch of photovoltaic inverters in residential distribution systems,” IEEE Transactions on Energy Conversion, vol. 29, no. 4, pp. 957–967, 2014.

[28] M. Liu, P. K. Phanivong, Y. Shi, and D. S. Callaway, “Decentralized charging control of electric vehicles in residential distribution networks,” IEEE Transactions on Control Systems Technology, vol. 27, no. 1, pp. 266–281, 2019.

[29] M. Liu, P. K. Phanivong, and D. S. Callaway, “Electric vehicle charging control in residential distribution network: A decentralized event-driven realization,” in Proceedings of the IEEE Conference on Decision and Control, Melbourne, VIC, Australia, December 12-15 2017, pp. 214–219.

[30] M. Liu, P. K. Phanivong, and D. S. “Customer-and network-aware decentralized ev charging control,” in Proceedings of the Power Systems Computation Conference, Dublin, Ireland, June 11-15 2018, pp. 1–7.

[31] M. Liu, “Chance-constrained SPDS-based decentralized control of distributed energy resources,” in Proceedings of the IEEE Conference on Decision and Control, Nice, France, December 11-13 2019, pp. 3272–3278.