Bad-metal relaxation dynamics in a Fermi lattice gas

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Electrical current in conventional metals is carried by electrons that retain their individual character. Bad metals, such as the normal state of some high-temperature superconductors, violate this scenario, and the complete picture for their behavior remains unresolved. Here, we report phenomena consistent with bad-metal behaviour in an optical-lattice Hubbard model by measuring the transport lifetime for a mass current excited by stimulated Raman transitions. We demonstrate incompatibility with weak-scattering theory and key characteristics of bad metals: anomalous resistivity scaling consistent with $T$-linear behavior, the onset of incoherent transport, and the approach to the Mott-Ioffe-Regel limit. Our work demonstrates a direct method for determining the transport lifetime, which is critical to theory but difficult to measure in materials, and exposes minimal ingredients for bad-metal behavior.

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Landau’s Fermi liquid theory successfully describes the behavior of interacting fermionic particles for a wide range of materials, such as electrons in simple metals and liquid helium-3. In some metals, Fermi liquid theory fails when strong correlations or fluctuations are present. Also known as bad, or strange, metals, these states present anomalous properties such as resistance that does not follow the Fermi liquid prediction $T^2$, sometimes scaling as $T$ instead or exhibiting more complex phenomena. The resistivity of bad metals also does not saturate as temperature is increased into the regime where the Mott–Ioffe–Regel (MIR) limit is violated and the apparent mean-free-path is shorter than the interatomic spacing. This lack of saturation implies that quasiparticles are absent (e.g., refs. 7–9, for an overview), as does the lack of particle-like excitations in photoemission spectroscopy.

Understanding the origin of bad-metal behavior is a key problem in condensed matter physics, which may be important to resolving questions related to high-temperature superconductivity and Mott quantum criticality. This problem has been studied using varied theoretical frameworks, including antiferromagnetic space-conformal field theory (AdS-CFT) holographic duality, the Sachdev–Ye–Kitaev model[14], high-temperature series expansions[15], and dynamical mean-field theory (DMFT) (e.g., refs. 16,17). DMFT, in particular, has shown $T$-linear resistivity that can exceed the MIR limit and a regime below the MIR where saturation implies that quasiparticles are absent (e.g., refs. 7–9). Despite this extensive activity, there has been relatively little progress in understanding the source of the failure of Fermi liquid theory in these states.

Ultracold fermionic atoms trapped in optical lattices, which realize the Fermi–Hubbard model[23–25], provide a well-controlled platform free of phonons and impurities with well controlled and understood microscopic parameters to study bad metal phenomenology[13]. In ultracold gas experiments with fermionic atoms, photoemission spectroscopy has been used to probe the spectral function in the Bose–Einstein condensate (BEC)–Bardeen–Cooper–Schrieffer (BCS) crossover for a trapped gas, and a failure of Fermi liquid theory was discovered[26]. Transport measurements such as diffusion in a 2D lattice gas[27], shear viscosity in a unitary Fermi gas[28], and spin diffusion[29] have also explored the effect of strong interactions on various relaxation processes. In this paper, we describe a method for measuring the decay rate of a mass current and inferring the analog of electrical resistivity for a two-component fermionic gas composed of $^{40}$K atoms trapped in a cubic optical lattice. A net current consisting of a flow of spin-polarized atoms shifted in quasimomentum is created using stimulated Raman transitions (Fig. 1). By fully resolving the decay dynamics of the current, we deduce the transport lifetime induced by collisions with atoms in the other spin state. The analog of resistivity is inferred from the transport lifetime and the atomic density.

**Results**

**Mass current generation.** We prepare the gas in a metallic state by slowly superimposing the optical lattice after cooling in an optical dipole trap to temperatures $T = 0.2–1.2 T_F$, where $E_B = k_B T_F$ is the Fermi energy (see Methods). The temperature of the gas is sufficiently low for the atoms to realize a single-band Hubbard model described by the Hamiltonian

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left( \hat{c}_i^\dagger \hat{c}_j \right) + U \sum_i n_i \left( n_i - 1 \right) + \sum_{i, \sigma} m \omega_i r_i^2 n_i / 2,$$

where $i$ indexes the lattice sites, $\langle \rangle$ indicates a sum over neighboring sites, $\sigma = \uparrow, \downarrow$ indexes spin, $\omega_i$ is the geometric mean of the dipole trap frequencies, $r_i$ is the distance from site $i$ to the trap center, $\hat{c}_i^\dagger (\hat{c}_i)$ creates (annihilates) an atom from site $i$, $n_i = \hat{c}_i^\dagger \hat{c}_i$ is the number operator, $t$ is the Hubbard tunneling energy, and $U$ is the on-site Hubbard interaction energy. Two hyperfine states $| F = 9/2, m_F = 9/2 \rangle (\langle \uparrow \rangle)$ and $| F = 9/2, m_F = 7/2 \rangle (\langle \downarrow \rangle)$ play the role of the electron spin. The metallic regime is achieved by tuning the number of atoms $N$ so that $E_B = 6t$ (corresponding to 0.5 particles of each spin per site in the center of the lattice at $T = 0$) and the lattice potential depth $s$ to sample $U/t \approx 2.3–9.0$. To create a well-characterized initial state, the gas is spin polarized by removing the $| \downarrow \rangle$ atoms before turning on the lattice. We use exact eigenstates[30,31] and measurements of $N$ and $T$ to estimate an effective chemical potential $\mu$ and temperature $T$ of the initial metallic lattice gas (see Supplementary Note 1).

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**Fig. 1** Schematic diagram of method used to generate current via Raman transition. **a** A spin-polarized $| \uparrow \rangle$ (blue) gas is prepared in a metallic state in the ground band (with dispersion $\varepsilon$) of the lattice. A pair of Raman beams (red arrows) with frequencies $\omega_0$, $\omega_2$ and wavevectors $\mathbf{k}_1$, $\mathbf{k}_2$ are used to quickly transfer atoms from the $| \uparrow \rangle$ to the $| \downarrow \rangle$ (red) state via the $4S_{1/2} \rightarrow 4P_{1/2}$ electronic transition. The frequency difference $\delta \omega = \omega_1 - \omega_2$ is tuned to be resonant, and the Raman pulse equally samples all quasimomenta $q$ in the Brillouin zone, which ranges along one direction from $-q_0$ to $q_0 = \pi a/d$, where $d \approx 390$ nm is the lattice spacing. The Raman transition introduces a momentum shift $\Delta q$ for the atoms in the $| \downarrow \rangle$ state, resulting in a net current of $| \downarrow \rangle$ atoms. **b** The quasimomentum distribution in 3D Brillouin zone is shown before (left) and after (right) the Raman pulse. The initial quasimomentum shift has approximate magnitude of 0.5 $q_0$ and is aligned with the direction of $q_x$ and $\delta \mathbf{k}$, which is along the $[-1, -1, -1]$ direction. The imaging procedure projects the quasimomentum distributions along the $[1, -\sqrt{2}, 1]$ direction, so that the Brillouin zone has a hexagonal shape in the imaging plane, which has axes $q_x$ and $q_y$.
A current consisting of approximately 30% of the atoms transferred to the $|\downarrow\rangle$ state and shifted in quasimomentum is created using a pulse of two laser beams focused onto the gas (Fig. 1). Based on semi-classical, non-interacting thermodynamics, we estimate that the Raman excitation increases the total energy of the gas by less than 10%. The quasimomentum profiles of the atoms in the $|\uparrow\rangle$ and $|\downarrow\rangle$ states are separately imaged after evolution time $t_{\text{hold}}$ in the lattice using band mapping\textsuperscript{31} and spin-resolved time-of-flight imaging (see Methods). Sample images for $s = 4E_F$, corresponding to $U/t = 2.3$, and $T/T_F = 0.23$ before turning on the lattice ($k_B T = 1.3t$) are shown in Fig. 2a. The quasimomentum distribution of the $|\uparrow\rangle$ gas is unaffected by the Raman pulse, while the $|\downarrow\rangle$ gas is displaced along the wavevector difference $\delta \mathbf{k} = \mathbf{k}_\downarrow - \mathbf{k}_\uparrow$ between the Raman beams. The $|\downarrow\rangle$ atoms, therefore, form a net current proportional to their average quasimomentum $q_\downarrow$ (Fig. 2b), which is determined by fitting the images to a Gaussian function (see Methods).

**Transport lifetime.** The decay of the current caused by momentum-changing collisions between atoms in $|\downarrow\rangle$ and $|\uparrow\rangle$ states is apparent for different evolution times following the Raman pulse (Fig. 2b). The characteristic decay time, which is the transport lifetime $\tau_t$, in our experiments is a few milliseconds. Effects besides interactions play a minor role in the excitation dynamics. The relaxation is too fast for the trap oscillations to play a significant part in the dynamics or for the motion of the $|\uparrow\rangle$ atoms to be affected for most of the temperatures and interaction strengths we sample. We have also checked that dephasing of atomic trajectories with different initial quasimomenta and trap anharmonicity do not significantly contribute to the relaxation via classical dynamics simulations and measurements employing spin-polarized gases (see Methods).

The Boltzmann formalism provides an intuitive picture to relate the transport lifetime with microscopic scattering processes and resistivity\textsuperscript{1}. In solids, $\tau_t$ is usually inferred from resistivity, but here we measure it directly. The decay rate of $q_\downarrow$ is the inverse of the transport lifetime $\tau_t$, averaged over the density profile (see Methods). To determine $\tau_t$, data such as those shown in Fig. 2b are therefore fit to a model based on the Boltzmann equation (see Methods), which is similar to an approach that has been used to determine collision cross sections\textsuperscript{32} and observe Pauli blocking\textsuperscript{33,34} in weakly interacting Fermi gases.

Measurements of $\tau_t$ for different temperatures at fixed $U/t = 2.3$ (corresponding to $s = 4E_F$) are shown in Fig. 3. We compare these data to the thermal-limit scaling prediction for scattering between trapped quasiparticles and a Fermi’s golden rule (FGR) calculation that accounts for collisions between atoms in quasimomentum states (see Supplementary Note 2). In the Maxwell-Boltzmann limit ($T > T_F$), the scattering time between quasiparticles scales as $1/\langle\nu\rangle n_{\text{dwa}} \propto T^{3/2}$, where $\langle\nu\rangle$ and $n_{\text{dwa}}$ are the thermally averaged speed and density-weighted (thermally averaged) density (see Methods). The more sophisticated FGR calculation, which has no free parameters, agrees with this behavior at high temperature. A FGR approach has been used to accurately calculate relaxation times for trapped gases in the weakly interacting regime\textsuperscript{33}, but may be expected to fail for the strong interactions ($U \geq 2t$) sampled by our measurements. Our FGR calculation has no free parameters, fully accounts for the trap and quantum statistics, and averages over a thermodynamic distribution of quasimomenta based on the inferred $T$ and $\mu$. Based on general principles, this approach predicts that $\tau_t$ decreases for stronger interactions, since the rate of scattering events increases, and that $\tau_t$ increases at higher temperatures, because the density is reduced. Furthermore, while Pauli blocking limits the phase space for scattering and causes the equilibrium collision rate to vanish at zero temperature\textsuperscript{33}, $\tau_t$ for our measurement remains finite at $T = 0$ because the Raman excitation creates unfilled quantum states.

**Fig. 2** Measurement of transport lifetime. **a** Quasimomentum distribution for the $|\uparrow\rangle$ and $|\downarrow\rangle$ components for different evolution times after the Raman pulse for $T/T_F = 0.23$ and $U/t = 2.3$ ($s = 4E_F$). The dashed lines mark $q = 0$, and the hexagons are the first Brillouin zone (BZ) projected onto the imaging plane. The color bar shows the measured optical depth (OD). Fits to these quasimomentum profiles are used to determine the average momentum $q_\downarrow$ of the $|\downarrow\rangle$ component (see Methods). **b** The insets shows how momentum-changing collisions between atoms in different spin states relax the initial current (left) so that the net quasimomentum vanishes at long times (right). The momentum $q_\downarrow$ is fit (solid line) to a solution of the Boltzmann equation to determine the transport lifetime. Each point is the average of 5–10 measurements, and the error bars show the standard error of the mean.

**Fig. 3** Transport lifetime at varied temperature and fixed interaction strength $U/t = 2.3$. A logarithmic scale is used to display the data. The lifetime $\tau_t$ is shown in units of the tunneling time (left axis) and in ms (right axis). The slope from a linear fit to the data (not shown) has a negative slope at greater than a 99.5% confidence level. The measurements are compared with a FGR weak-scattering calculation (black solid line) and a scaling prediction (red dashed line, which was fixed to match the FGR result at the highest temperature point) in the Maxwell-Boltzmann limit. For the FGR calculation, $N$ is fixed to the average value from the data. The vertical error bars show the uncertainty in the fit used to determine $\tau_t$ and the horizontal error bars display the uncertainty in $T$ from time-of-flight thermometry.
The temperature dependence of $\tau_f$ shown in Fig. 3 shows a trend strikingly opposite to that predicted by scattering theory: at greater than a 99.5% confidence level, the transport lifetime decreases for higher temperatures. While this behavior is standard in solid metals, it is surprising for trapped gas in this temperature regime—a quantity proportional to the mean time between collisions such as $\tau_f$ is expected to increase at higher temperatures because the density decreases as the gas expands into a larger volume of trap. For these data, we vary the temperature of the gas before turning on the lattice from $T/T_F = 0.2$ to $1.2$, which leads to $k_B T/\hbar \approx 1-8$. The upper end of this regime cannot be explored in solid metals, where $T_F = (1-15) \times 10^4$ K, which is well above the melting temperature. The measured transport lifetime agrees with weak scattering theory within 30% at the lowest temperatures. As the temperature is increased, $\tau_f$ decreases by approximately a factor of two, while the weak scattering calculation predicts that $\tau_f$ increases by a factor of 6, leading to a disagreement of over 60 standard errors at the highest temperature. This discrepancy cannot be explained by an error in density—we have verified that the density of the gas decreases across this range and is consistent with thermodynamic calculations via in-situ imaging (see Supplementary Note 1).

**T-linear resistivity.** A higher-than-expected increase in scattering with temperature is characteristic of bad metals. The onset of another key signature of bad metals—incoherent transport—is also apparent in Fig. 3. Transport in a metal becomes incoherent\(^{35}\) when the lifetime of states with well-defined momentum is comparable to the characteristic single-particle timescale, which is the tunneling time $\hbar/\tau$. This regime is approached at high temperatures (Fig. 3) and at high interaction strengths (see Methods). The commensurability of timescales signals the breakdown of a central assumption underlying Fermi liquid theory and the failure of the quasiparticle picture. The inverse dependence of $\tau_f$ on $T$ evident in Fig. 3 cannot be explained by any known quasiparticle theory, and therefore suggests that quasiparticles are absent.

To expose other bad-metal behaviors and compare with DMFT predictions, we infer a dimensionless resistivity $\rho$ from the measured $\tau_f$ and the relationship between the Kubo and Landauer (solid line) are shown in Fig. 4. The inset in Fig. 4b shows the ratio $\tau_{MIR}/\tau_n$ (solid line) fit to DMFT scaling law $\rho^2$ with reduced $\chi^2 = 0.99$. The temperature scaling, fit (solid line) to DMFT scaling law $\rho$ with reduced $\chi^2 = 1.76$. The vertical error bars indicate the uncertainty in the fit used to determine $\tau_n$ and the uncertainty in $\rho_{MIR}$ from the measurements of $\rho$ and $T$. Orthogonal distance regression is used for fitting to accommodate the horizontal error bars in b. The plots along the right side of the figure show the local spectral function $\rho(\omega)$ using a constant vertical scale, and the energy $\omega$ relative to the Fermi energy $E_F$ in units of the half bandwidth $W$. $\rho(\omega)$ is calculated at $U/t = 2.3$ for b. The inset in b shows the ratio between the dimensionless resistivity and the MIR limit.

**Approaching the MIR limit.** The onset of another characteristic of bad metals is also evident in Fig. 4: continual growth toward the MIR limit as temperature is increased. The MIR limit defines the regime in which semiclassical transport theory is valid and current-carrying particles are a legitimate concept\(^{4,6}\). In solids, the MIR limit is $l = d^{1/2}$, where $l$ is the mean-free path and $d$ is the atomic lattice spacing. This condition must be modified to $l = n^{-1/3}$ for optical lattices, which are free from impurities and phonons, and the only scattering is between particles with separation $n^{-1/3}$. Using this definition, we show the MIR-limited
resistivity $\rho_{\text{MIR}}$ in Fig. 4 determined from $\tau_{\text{MIR}} = (n_1^{1/3})/(\langle v \rangle)$. The resistivity steadily escalates toward $\rho_{\text{MIR}}$ with interaction strength (Fig. 4a). The behavior in Fig. 4b is more subtle, since $\rho_{\text{MIR}}$ is a strong function of temperature. The ratio $\rho/\rho_{\text{MIR}}$ shown in the inset reveals that $\rho$ continuously approaches the MIR limit as the temperature of the gas is increased. While we measure a steady increase of resistivity with temperature and interaction strength, a violation of the MIR limit is not evident in our data. We cannot sample higher temperatures and interaction strengths, where a MIR violation may occur, because the mass current (which vanishes in the $T/U/t \to \infty$ limits) becomes too small to resolve.

Discussion

One way to understand the effect of the strong interactions on the system is through the change in the local spectral function $A(\omega)$ (see Supplementary Note 3). This quantity captures changes in the density of states caused by interactions and temperature. At low temperature, $A(\omega)$ consists of a single band centered at the Fermi energy, which is broadened by interactions from the non-interacting bandwidth $2W$ (Fig. 4a). As the temperature is raised, spectral weight is redistributed to peaks centered at approximately $\pm U/2$ (Fig. 4b). This reduction in the spectral weight near the Fermi surface gives rise to a resistivity that depends on the changing nature of the quasiparticles in addition to their scattering rate, providing insight into the qualitative failure of the weak scattering calculation.

To our knowledge, our measurement of scaling consistent with $T$-linear samples the highest temperatures relative to the Fermi temperature, and, along with the concurrent work reported in ref. 39, is evidence for this behavior in an ultracold-gas Hubbard model. Because this system has precisely known microscopic parameters and is well isolated from the environment, our measurements provide direct evidence—consistent with the predictions from DMFT and other techniques (e.g., refs. 15,40)—that the minimal ingredients of strongly interacting lattice fermions contained in the Fermi-Hubbard Hamiltonian are sufficient to cause some characteristic bad-metal dynamics. In the future, rf spectroscopy measurements in this system may reveal information about $A(\omega)$ directly26. Furthermore, additional effects present in solids can be added in a controllable fashion. The influence of disorder can be investigated via, for example, applying optical speckle11, and the impact of phonons could be explored using mixtures of different species12.

Methods

Lattice gas and mass current preparation. Ultracold gases composed of $^{40}$K atoms are cooled to temperatures below $T_F$ in a crossed-beam $1084\text{nm}$ optical dipole trap using standard techniques. A 3 G static magnetic field is applied. The final trap depth during evaporative cooling is adjusted to control the temperature and atom number. After cooling, the optical trap depth is slowly increased during evaporative cooling is adjusted to control the temperature and atom number. After cooling, the optical trap depth is slowly increased after the Raman pulses are delivered. The timing of the Raman pulses, pulse power, and $\Delta \omega$ are controlled using acousto-optic modulators. The Raman transition is a superposition of the $|\uparrow\rangle$ and $|\downarrow\rangle$ states, and the relative amplitude in each state depends on the initial quasimomentum. Subsequently, the coherence of the superposition decays after the pulse. The decoherence timescale measured using a Ramsey pulse sequence is approximately 0.08 ms, and therefore the Raman transition can be treated as an instantaneous process for the relaxation measurements.

Transport lifetime measurement. After holding atoms in the lattice for variable times following the Raman transition, we ramp down the lattice potential in $0.1\text{ ms}$ and release the gas from the trap. A magnetic field gradient is applied along the $y$ direction during time-of-flight expansion to spatially separate atoms in the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. The images of each spin component are separately fit to a Gaussian distribution to obtain the center-of-mass (COM) position $y_1$ ($\langle v \rangle$ for the $|\uparrow\rangle$ ($|\downarrow\rangle$) atoms. This COM position is translated into a net quasimomentum shift as $\eta_0 \propto m F \tau y_1 \tau$ where $\tau = t_{\text{exp}}$ is the expansion time, and $\tau y_1 \tau$ is the COM position without a Raman pulse. The $|\downarrow\rangle$ atoms remain nearly at rest for all of the data—the maximum $\eta_0$ is an order of magnitude smaller than $q_j$ after the Raman pulse. The net current is therefore proportional to $q_j$.

We fit the time evolution of $q_j$ to a solution of the Boltzmann equation.32

$$ q_j(t) = \frac{\eta_0}{2\sqrt{\pi} \vert \phi(t) \vert} \left[ 1 + \sqrt{\phi(t)} e^{\frac{\phi(t)}{\sqrt{4\pi}} - \left(1 + \sqrt{\phi(t)} e^{\frac{\phi(t)}{\sqrt{4\pi}}}ight)} \right] $$

Here, $\phi(t) = 1 - 4t^2 F^2$, and $\tau_\phi, \eta_0$, and an offset are free parameters in the fit. Nonlinear effects are small—the data fit well to this model of linear dissipation with an adjusted $R^2 = 0.7-1.0$ for all the data used in this work and the fit residuals display no systematic effects (Supplementary Note 5). An offset is necessary to account for drifts in the center of the equilibrium gas caused by changes in the dipole trap and strain magnetic field gradients. We find that this offset corresponds to less than a pixel in the images for all the data used in this work.

Interaction dependence. Independent measurements of how $\tau_j$ depends on interaction strength are used to infer $\rho$ for Fig. 4a. For these measurements, shown in Fig. 5, $U/t = 2.3-9$ is tuned at fixed $T/t = 0.25$ by changing the lattice potential depth from $s = 4-7 E_R$. As is the case at high temperature, the incoherent regime is achieved high $U$. The data are compared with the FGR weak scattering calculation (solid line). With $E_F$ fixed to approximately $6\tau$, the weak scattering calculation predicts $\tau_j \propto U/\mu^2$. The measured $\tau_j$ normalized to the tunneling time $\hbar/t$ agrees within 10% with the weak scattering prediction at the lowest interaction strength. While $\tau_j$ follows the same trend as weak scattering theory, it does not decrease with increasing interaction strength as rapidly as the weak scattering prediction. Across the range we sample, the weak scattering theory predicts that $\tau_j$ decreases by a factor of 10 (see inset), while the measured value changes only by a factor of 2, leading to a disagreement of five standard errors at $s = 7 E_R$. This discrepancy may be explained by the change in $A(\omega)$ shown in Fig. 4. The FGR calculation assumes that $A(\omega)$ is a delta-function, while DMFT predicts a very broad peak at high $U/t$, which implies a higher scattering rate.

Dephasing time for a spin-polarized gas. We have assumed that the atoms have a free-particle dispersion in this approach in order to develop a simple, closed form solution for the dipole force from the Raman beams is too small to affect our analysis.

We fit the time evolution of $q_j$ to a solution of the Boltzmann equation.32
fitting function for the data. Because of the tight-binding dispersion, the gas in the lattice is not harmonic, and hence Kohn’s theorem does not hold. Therefore, center-of-mass motion such as we excite will decay as individual atomic trajectories dephase. We experimentally probe and theoretically model this dephasing time to determine \( \tau \).

A summary of the measured and simulated dephasing times is shown in Fig. 6c for different temperatures in an \( s = 4 \) lattice. The data shown in Fig. 5 will behave similarly to the lowest temperature point. For comparison, the corresponding measured transport lifetimes from Fig. 3b are displayed. As expected, the dephasing time is shorter at higher temperatures since a wider range of quasimomenta are present. The agreement between simulated and measured dephasing times for spin-polarized gases indicates that the simulation accurately describes the dephasing dynamics. The simulated dephasing times have much smaller uncertainties than the measurements and therefore are a useful benchmark for estimating the impact of dephasing on our measurements. The simulated dephasing time is at least four times longer than the measured transport lifetime. At high temperature, where the deviation from weak scattering theory is largest, the simulated dephasing time is six times longer than the measured transport lifetime. We conclude that dephasing has a minor impact on our measurements.

**Analog of resistivity.** A complication is that measurements on solids and the DMFT prediction we compare with involve a spatially uniform system, while our measurements are averaged over the inhomogeneous atomic density profile. We must, therefore, correct our measured \( \tau \) for this average. We use an approach that assumes a null hypothesis: weak binary s-wave scattering for a trapped gas. In that limit, the collision rate per atom is \( n_\text{indep}(\mu, \nu) \), where \( \langle \nu \rangle \) is the mean relative speed between colliding partners and \( \sigma \) is the collision cross-section, and \( \nu \) will be independent of temperature (ignoring the effects of Pauli blocking, which are minimal for our measurements). Our construction for resistivity will not correctly account for density or energy-dependent scattering processes, which is precisely the type of phenomena we wish to expose by measuring changes in \( \rho \) with temperature.

For any process that generates resistivity through independent two-body scattering, without losing generality, we can write \[ 1/\rho = (\langle n_{\nu} \rangle / M_{\text{lat}}) \rho(\mu, \nu) \]

where \( \rho(\mu, \nu) \) is the density of scatterers. The resistivity is proportional to \( M^2 \tau_{\nu} \), where \( M \) is the effective mass. The challenge in determining resistivity for strongly correlated systems is evaluating \( \rho(\mu, \nu) \).

In our case, \( n_{\nu} = n_{\text{indep}}(\nu) \), which varies across the gas. Our measured transport lifetime is a weighted average \( M \int d\nu n_{\text{indep}}(\nu) \rho(\nu) / M_{\text{lat}} \) over the \( \rho(\nu) \) density profile, which links our measurements with M. Here, \( n_{\text{indep}} \) is the density-weighted density, and \( N_{\nu} \) is the number of atoms in the \( \nu \) state. To compare with DMFT simulations, which involves a uniform system with fixed electron density, we, therefore, divide the measured \( \tau_{\nu} \) by \( 1/n_{\text{indep}} \). In addition, since \( m^\text{eff} \), we absorb a factor of \( t \) into a dimensionless \( 1/(\hbar t) \), and define the dimensionless resistivity \( \rho = (\langle n_{\nu} \rangle / M_{\text{lat}})^{-1} \).

Our analysis also assumes that \( \langle \nu \rangle \) is independent of temperature. We also assume that the thermally averaged speed of the \( \nu \) component is also fixed with respect to changes in temperature for our determination of \( \rho_{\text{data}} \).

Variation in the thermally averaged speeds of the particles is suppressed because \( E_\ell = 6t \) and there is a maximum allowed speed in this single-band system. For our experimental parameters, we have used non-interacting thermodynamics to determine that \( \langle \nu \rangle \) is fixed to within 4% and the thermally averaged speed for the \( \nu \) component is fixed to within 1%.

Calculations that allow for variation away from half-filling (unlike the code we employ) predict a density-dependent resistivity. This effect, combined with the inhomogeneous density profile and change in density with temperature, would lead to non-\( T \)-linear behavior for our measurements. To estimate the magnitude of this effect, we use the density dependence \( \rho \propto \ln(1 - n) \) predicted in ref. 15 for the \( U \to \infty \) and high-temperature limit. This dependence averaged across the \( \langle \nu \rangle \) density profile would induce a 10% change in the trap-averaged resistivity at the lowest temperature sampled in Figs. 3 and 4b and a 1% change at the highest temperature. We conclude that this is a minor effect and the \( T \)-linear scaling predicted by the TRIQS code should apply to the trap-averaged \( \rho \).

**Data availability**

The data and computer code that support the findings of this study are available from the corresponding author upon reasonable request.

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**References**

1. Mahan, G. D. *Many-Particle Physics*. (Springer, US, 2000).
2. Schofield, A. J. Non-Fermi liquids. *Contemp. Phys.* 40, 95–115 (1999).
3. Bruin, J. A. N., Sakai, H., Perry, R. S. & Mackenzie, A. P. Similarity of scattering rates in metals showing T-linear resistivity. *Science* 339, 804–807 (2013).
4. Emery, V. J. & Kivelson, S. A. Superconductivity in bad metals. *Phys. Rev. Lett.* 74, 3253–3256 (1995).

5. Gunnarsson, O., Calandra, M. & Han, J. E. Colloquium: Saturation of electrical resistivity. *Rev. Mod. Phys.* 75, 1085–1099 (2003).

6. Ioffe, A. F. & Regel, A. R. Non-crystalline, amorphous and liquid electronic semiconductors. *Prog. Semicond.* 4, 237–291 (1960).

7. Hussey, N. E., Takenaka, K. & Takagi, H. Universality of the Mott–Ioffe–Regel line in metals. *Philos. Mag.* 84, 2847–2864 (2004).

8. Johnson, C. V. & Steinberg, P. What black holes teach about strongly coupled particles. *Phys. Today* 63, 29–33 (2010).

9. Liu, H. From black holes to strange metals. *Phys. Today* 65, 68–69 (2012).

10. Damascelli, A., Hussain, Z. & Shen, Z. X. Angle-resolved photoemission studies of the cuprate superconductors. *Rev. Mod. Phys.* 75, 473–543 (2003).

11. Lee, P. A., Nagosa, N. & Wózniak, X. G. Doping a Mott insulator: Physics of high-temperature superconductivity. *Rev. Mod. Phys.* 78, 17–85 (2006).

12. Vučičević, J., Tanasković, D., Rozenberg, M. J. & Dobrosavljević, V. Bad-metal behavior reveals Mott quantum criticality in doped Hubbard models. *Phys. Rev. Lett.* 114, 246402 (2015).

13. Hartnoll, S. A. & Karch, A. Scaling theory of the cuprate strange metals. *Phys. Rev. B* 91, 155126 (2015).

14. Parcollet, O. & Georges, A. Non-fermi-liquid regime of a doped Mott insulator. *Phys. Rev. B* 59, 5341–5360 (1999).

15. Perepelitsky, E. et al. Transport and optical conductivity in the hubbard model: A high-temperature expansion perspective. *Phys. Rev. B* 94, 235115 (2016).

16. Georges, A., Kotliar, G., Krauth, W. & Rozenberg, M. J. Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions. *Rev. Mod. Phys.* 68, 13–125 (1996).

17. Pakhira, N. & McKenzie, R. H. Absence of a quantum limit to charge diffusion in bad metals. *Phys. Rev. B* 91, 075124 (2015).

18. Deng, X. et al. How bad metals turn good: Spectroscopic signatures of resilient quasiparticles. *Phys. Rev. Lett.* 110, 086401 (2013).

19. Jaramillo, R., Ha, S. D., Silevitch, D. M. & Ramanathan, S. Origins of bad metal conductivity and the insulator-metal transition in the rare-earth nickelates. *Nat. Phys.* 10, 304–307 (2014).

20. Wermann, Y. & Berg, E. Mott–Ioffe–Regel limit and resistivity crossover in a tractable electron-phonon model. *Phys. Rev. B* 93, 075109 (2016).

21. Wermann, Y., Kivelson, S. A. & Berg, E. Quantum chaos in an electron-phonon bad metal. Preprint at https://arxiv.org/abs/1705.07895 (2017).

22. Wermann, Y., Kivelson, S. A. & Berg, E. Non-quasiparticle transport and resistivity saturation: a view from the large-n limit. *npj Quantum Mater.* 2, 7 (2017).

23. Jaksch, D., Bruder, C., Cirac, J. I., Gardiner, C. W. & Zoller, P. Cold bosonic atoms in optical lattices. *Phys. Rev. Lett.* 81, 3108–3111 (1998).

24. Esslinger, T. Fermi-Hubbard physics with atoms in an optical lattice. *Annu. Rev. Condens. Mater. Phys.* 1, 129–152 (2010).

25. Lewenstein, M. et al. Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond. *Adv. Phys.* 56, 243–379 (2007).

26. Sagi, Y., Drake, T. E., Paudel, R., Chapurin, R. & Jin, D. S. Breakdown of the Fermi liquid description for strongly interacting fermions. *Phys. Rev. Lett.* 114, 075301 (2015).

27. Schneider, U. et al. Fermionic transport and out-of-equilibrium dynamics in a homogeneous Hubbard model with ultracold atoms. *Nat. Phys.* 8, 213–218 (2012).

28. Cao, C. et al. Universal quantum viscosity in a unitary Fermi gas. *Science* 331, 58–61 (2011).

29. Sommer, A., Ku, M., Roati, G. & Zwierlein, M. W. Universal spin transport in a strongly interacting Fermi gas. *Nature* 472, 201–204 (2011).

30. Rey, A. M., Pupillo, G., Clark, C. W. & Williams, C. J. Ultracold atoms confined in an optical lattice plus parabolic potential: A closed-form approach. *Phys. Rev. A.* 72, 033616 (2005).

31. McCoy, D., White, M. & DeMarco, B. Lattice thermodynamics for ultracold atoms. *Phys. Rev. A.* 79, 063605 (2009).

32. DeMarco, B., Bohn, J. L., Burke, J. P., Holland, M. & Jin, D. S. Measurement of p-wave threshold law using evaporatively cooled fermionic atoms. *Phys. Rev. Lett.* 82, 4208–4211 (1999).

33. DeMarco, B., Papp, S. B. & Jin, D. S. Pauli blocking of collisions in a quantum degenerate fermion gas. *Phys. Rev. Lett.* 86, 5409–5412 (2001).

34. Gensermer, S. D. & Jin, D. S. Transition from collisionless to hydrodynamic behavior in an ultracold Fermi gas. *Phys. Rev. Lett.* 87, 173201 (2001).

35. Hartnoll, S. A. Theory of universal incoherent metallic transport. *Nat. Phys.* 11, 54–61 (2015).

36. Thouless, D. J. Relation between the Kubo-Greenwood formula and the Boltzmann equation for electrical conductivity. *Philos. Mag.* 32, 877–879 (1975).

37. De Filippis, G., Cataldi, V., Candel, A., de, Mishchenko, A. S. & Nagosa, N. Alternative representation of the Kubo formula for the optical conductivity: A shortcut to transport properties. *Phys. Rev. B* 90, 014310 (2014).

38. Parcollet, O. et al. TRIQS: A toolbox for research on interacting quantum systems. *Comput. Phys. Commun.* 196, 398–415 (2015).

39. Brown, P. T. et al. Bad metallic transport in a cold atom Fermi-Hubbard system. *Science* 363, 379–382 (2018).

40. Pállsson, G. & Kotliar, G. Thermoelcetric response near the density driven Mott transition. *Phys. Rev. Lett.* 80, 4775–4778 (1998).

41. Kondov, S. S., McGehee, W. R., Xu, W. & DeMarco, B. Disorder-induced localization in a strongly correlated atomic Hubbard gas. *Phys. Rev. Lett.* 114, 083002 (2015).

42. Günter, K., Stöferle, T., Moritz, H., Köhl, M. & Esslinger, T. Bose-Fermi mixtures in a three-dimensional optical lattice. *Phys. Rev. Lett.* 96, 180402 (2006).

43. Reif, F. *Fundamentals of Statistical and Thermal Physics*. (McGraw-Hill, 1965).

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