Derivative-Coupling Models and the Nuclear-Matter Equation of State

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Abstract

The equation of state of saturated nuclear matter is derived using two different derivative-coupling Lagrangians. We show that both descriptions are equivalent and can be obtained from the $\sigma-\omega$ model through an appropriate rescaling of the coupling constants. We introduce generalized forms of this rescaling to study the correlations amongst observables in infinite nuclear matter, in particular, the compressibility and the effective nucleon mass.

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1 Introduction

The equation of state (EOS) of dense matter is known, experimentally, at just one density value \( \rho_0 = 0.15/\text{fm}^3 \) (symmetric nuclear matter) for a binding energy per particle \( E/A(\rho_0) = -16 \text{ MeV} \). The compression modulus is more loosely determined ranging from as low as \( \kappa = 100 \text{ MeV} \) \cite{1} to the highest estimate \( \kappa = 344 \text{ MeV} \) \cite{2}. The last piece of experimental information is the effective mass which, from the energy dependence of the proton-nucleus optical potential, can be constrained to lie in the range \( M^*/M = 0.6-0.9 \).

Theoretically, the EOS is obtained using as starting point either non-relativistic interactions of the Skyrme type \cite{3}, or relativistic mean-field models (RMF) of baryons and mesons \cite{4, 5, 6, 7}. As an example of the non-relativistic calculations, the Skm*-interaction \cite{8}, used successfully to reproduce ground-state nuclear properties, gives \( M^*/M = 0.78 \) and a compressibility \( \kappa = 217 \text{ MeV} \). As for the RMF calculations, the original linear \( \sigma - \omega \) model \cite{4} gives for the same observables, a smaller effective mass \( M^*/M = 0.556 \) and a larger compressibility \( \kappa = 540 \text{ MeV} \).

The relativistic treatment of the EOS becomes necessary at extreme conditions of density and temperature such as those found in heavy-ion reactions at energies above 400 MeV/nucleon and type-II supernovas. In what follows we will concentrate on it.

The linear \( \sigma - \omega \) (Walecka) model satisfactorily explains many properties of nuclear matter and finite nuclei with two free parameters. The resulting compressibility at saturation density, however, exceeds the experimental bound. A way out of this difficulty is to introduce non-linear scalar self-couplings \cite{9, 10}. The resulting non-linear \( \sigma - \omega \) model reproduces well ground state nuclear properties and is renormalizable (though it shows some instabilities at high densities for low values of the compression modulus, \( \kappa < 200 \text{ MeV} \)). The number of free parameters in this case is four.

One alternative approach which renders a satisfactory compression modulus without increasing the number of parameters is that advanced in Refs. \cite{11, 12, 13}. It employs the same degrees of freedom and the same number of independent couplings present in the Walecka
model. The difference is that it introduces a non-renormalizable derivative coupling to the baryon field later adjusted to reproduce the experimental conditions at saturation. The results for the compression modulus and effective mass compare well with the Skyrme-type calculations. The spin-orbit splitting in finite-nuclei, however, turns out to be smaller than required by the data. We shall refer to this description as Model I.

A qualitative different model, though similar in spirit, obtains if instead of modifying the covariant derivative only the baryonic kinetic energy term is redefined. As in Model I, a suitable rescaling of the baryonic field leads to a Lagrangian describing baryons with effective mass $M^*$. Such a model was originally suggested as a possibility by Zimanyi and Moskowski in Ref. [13] (appendix) and implemented for a particular case by Delfino et al. [14]. The experimental nuclear matter saturation density and binding energy per nucleon are used to fit the two free parameters; the compressibility comes out rather small but within the bounds (156 MeV) and the spin-orbit splitting doubles the value obtained in Model I (though still short of the data). To this model we shall refer hereafter as Model II.

The aim of this paper is to provide a unified discussion of models I and II. We will show that both are to be understood as generalized Walecka models where the meson couplings ($g_\sigma$ and $g_\omega$) become effective coupling constants. In Model I only the scalar coupling is affected; in Model II both scalar and vector couplings are modified. The discussion will lead us naturally to consider a generalization of the models above effected by the introduction of a third parameter $\alpha$ which rescales the effective coupling constants. We will explore the results for $M^*$, $\kappa$, and the mean-field vector (V) and scalar (S) potentials, within both models for different values of the parameter $\alpha$.

Our basic motivation is to sum up in a phenomenological meson-baryon model, with as few parameters as possible, the ability to reproduce the four nuclear-matter observables and to give a good description of ground-state properties of finite nuclei without the presence of instabilities at high densities. It is not a substitute to the more fundamental relativistic Brueckner-Hartree-Fock (RBHF) method [15, 16, 17] but a simplifying alternative to explore the nuclear matter
It is important to stress that the same idea of density-dependent coupling constants underlies two recent approaches. The first one, known as relativistic density-dependent Hartree-Fock [18], describes finite nuclei and nuclear matter saturation properties using coupling constants that are fitted, at each density value, to the RBHF self-energy terms. The good agreement obtained for the ground state properties of spherical nuclei lends support to a description involving these density-dependent coupling constants. In the second approach [19], a similar density dependence of the parameters is achieved through projection of the meson-nucleon vertices into positive-energy space weighted with a parameter adjusted to reproduce saturation properties. Here also, the generalization of the model through the inclusion of adjustable parameters at the vertices permits to study the density dependence in an analytical manner. Both approaches have natural partners in the relativistic density-dependent Hartree-Fock of Li and Zhuo [20] and in the Gmuca’s model [21]. Our treatment of models I and II belongs to this category of descriptions involving density-dependent coupling constants.

The outline of the paper is as follows: in the next section we present the two models, establish their form and relationship and give expressions for the generalized EOS and compressibility. In section 3 we present our results and discuss the correlation between the calculated nuclear matter observables. Finally, in the last section we present our conclusions.

2 The Models

We begin by introducing the non-linear Lagrangian density of Model I,

\[ \mathcal{L}_{NL} = \bar{\psi} \{ \mathcal{D}(g_\omega - m^*(\sigma) M) \} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega^\mu \omega^\mu + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^2 \sigma^2 \right), \]  

(1)

where \( \mathcal{D}(g_\omega) = \gamma^\mu D_\mu (g_\omega) = \gamma^\mu (i \partial_\mu - g_\omega \omega_\mu) \) is the usual covariant derivative and the degrees of freedom are the baryon field \( \psi \), the scalar-meson field \( \sigma \), and the vector-meson field \( \omega^\mu \). The real function \( m^*(\sigma) \) is unknown except for the fact that it must go to unity as the density
goes to zero and has to vanish at large densities where the effective mass approaches zero asymptotically. Indeed, the Dirac equation obtained from the Lagrangian density (1) gives

\[ m^*(\sigma) = \frac{M^*}{M}, \]  

(2)

where \( M \) and \( M^* \) are the bare and effective baryonic mass, respectively.

We show next that performing a transformation on the spinor field \( L_{NL} \) can be derived from a Lagrangian density with derivative scalar coupling (DSC). The proposed Lagrangian density \[ \text{13} \] which generates \( L_{NL} \) is given by

\[ L = \bar{\psi} \left\{ [m^*(\sigma)]^{-1} \mathcal{D}(g_\omega) - M \right\} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2_\omega \omega_\mu \omega^\mu + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^2_\sigma \sigma^2 \right). \]  

(3)

Introducing the rescaled baryonic field \( \psi \rightarrow [m^*(\sigma)]^{1/2} \psi \), we obtain from Eq.(3) the rescaled Lagrangian density

\[ L_R = L_{NL} + \Im, \]  

(4)

where \( \Im \) is an imaginary contribution given by

\[ \Im = \frac{i}{2} (\bar{\psi} \gamma_\mu \psi) \partial^\mu \ln(m^*(\sigma)). \]  

(5)

This term does not carry any physical content and can be transformed away by substituting the baryonic kinetic energy term in Eq.(1) for the symmetric derivative \( \frac{i}{2} \{ \bar{\psi} \gamma_\mu \partial^\mu \psi - (\partial^\mu \bar{\psi}) \gamma_\mu \psi \}. \) The imaginary contribution \( \Im \) cancels after the field scaling. Once performed the field rescaling is equivalent to the replacement,

\[ \left\{ [m^*(\sigma)]^{-1} \mathcal{D}(g_\omega) - M \right\} \rightarrow \{ \mathcal{D}(g_\omega) - m^*(\sigma) M \}. \]  

(6)

The Lagrangian densities given by Eqs.(1) and (3) are, therefore, completely equivalent; they describe the same physics whether we deal with infinite nuclear matter or finite nuclei. The equations of motion obtained from Eqs.(1) and (3) give rise to the same hadronic dynamics. The modified kinetic energy of Eq.(3) is not arbitrary; it describes the motion of a baryon of mass \( M^* \) instead of the bare mass \( M \). Equation (1) just carries this information to the scalar-baryon coupling fields.
The Walecka model can be obtained as a particular case of either Lagrangian density by making the choice $m^*(\sigma) = (1 - g_\sigma \sigma / M)$. More generally, recent nonlinear models like those presented in Refs. [13, 22, 23, 24] (which are variations of the DSC model) may be interpreted as modified Walecka models where,

$$L_{NL} \equiv L_{Walecka}(g_\sigma \rightarrow g_\sigma^*),$$

and $g_\sigma^*$ (hereafter we will interpret * as referring to effective coupling constants in the medium) is now a function of $\sigma$ related to $m^*(\sigma)$ by

$$m^*(\sigma) = 1 - g_\sigma^* \sigma / M.$$

By itself, this establishes a class of models since $m^*(\sigma)$ is still a generic real function. In the usual Zimanyi-Moszkowski (ZM) model [13], for example,

$$m_{ZM}^*(\sigma) = (1 + g_\sigma \sigma / M)^{-1},$$

and $g_\sigma^* = g_\sigma m_{ZM}^* (\sigma)$. Likewise, the identification of $g_\omega^*$ and $m^*(\sigma)$ for the other models [22, 23, 24] can be easily done.

We introduce next a modified version of Model I that we refer to as Model II. We keep the generalized factor $m^*(\sigma)$ but, following the suggestion given in Ref. [13], we restrict the $m^*(\sigma)$ dependence in $L$ to the fermionic kinetic energy term. To this end we define a modified covariant derivative

$$\partial_m (g_\omega) \equiv (\partial_m^* (g_\omega) - 1)i \partial_m - g_\omega \partial_m \psi$$

and write the Lagrangian in the form

$$L = \bar{\psi} \{\partial_m^* (g_\omega) - M\} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_m^\mu \omega_m^\mu + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right).$$

A rescaling similar to the previous one, $\psi \rightarrow [m^*(\sigma)]^{1/2} \psi$, transforms

$$\{\partial_m^* (g_\omega) - M\} \rightarrow \{\partial_m^* (g_\omega^*) - m^*(\sigma) M\},$$

thus generating a new class of nonlinear models. The connection with the Walecka model is an extension of the previous one and proceeds through the change,

$$L_{NL} \equiv L_{Walecka}(g_\sigma \rightarrow g_\sigma^*; g_\omega \rightarrow g_\omega^*).$$
Here $g^*_\sigma$ is connected with $m^*(\sigma)$ by Eq. (8) and
\[
\frac{g^*_\sigma}{g_\omega} = m^*(\sigma). \tag{13}
\]
Equations (7) and (12) help to understand the different types of nonlinear couplings showing that they are, in fact, effective Walecka models. An important point to emphasize is that given the Lagrangian of Eq. (10), the scaling of the effective vector coupling constant in the medium is fixed by (13).

We now explore the possibility of extending the same scaling to the effective scalar coupling constant in the medium. Thus, we look for a function $m^*(\sigma)$ such that the following constraint is satisfied:
\[
\frac{g^*_\sigma}{g_\sigma} = \frac{g^*_\omega}{g_\omega} = m^*. \tag{14}
\]
It turns out that the only function that fulfills this requirement is $m^*(\sigma) = m^*_{ZM}(\sigma)$. We call the Model II Lagrangian with $m^*(\sigma) = m^*_{ZM}(\sigma)$ the modified ZM model (MZM) and is the first hadronic model which exhibits this property. It couples the scalar $\sigma$ to the vector $\omega$ field and gives results for the nuclear matter observables substantially different from the usual ZM model. A comparison follows.

We took for the usual ZM model, as given by Eq. (8), the coupling constants used in Ref. [13]. For the MZM model, Eq. (12), we fit the parameters to the saturation density and the experimental binding energy per nucleon to obtain, $C_\sigma^2 \equiv g^2_\sigma M^2/m^2_\sigma = 443.3$ and $C_\omega^2 \equiv g^2_\omega M^2/m^2_\omega = 305.5$ [14]. In the table below we show, for both models, the numbers obtained for the compressibility $\kappa$, the scalar potential $S$, the vector potential $V$ (in MeV) and the baryonic effective mass $m^*$:

| Model | $m^*$ | $\kappa$ | $S$ | $V$ |
|-------|-------|----------|-----|-----|
| ZM    | 0.85  | 225      | -141| 82  |
| MZM   | 0.72  | 156      | -267| 204 |
Compared to ZM, the MZM model gives, simultaneously, a smaller $m^*$ and a smaller $\kappa$. This is unlike what is observed when comparing ZM with the Walecka model. This difference in predictions is explained by the nonlinear scalar-vector coupling contained in the MZM Lagrangian. Furthermore, it is known that the ZM model gives poor results for the spin-orbit splitting (which is strongly dependent on the quantity $V - S$) when used in finite nuclei calculations [24]. In the MZM model, this quantity more than doubles that of ZM, suggesting that MZM may improve upon the former in this particular direction.

It is also interesting to remark that at low energies the slope of the real optical potential (given by $1 - m^*$) provides information regarding the expected value of $m^*$. The experimental value, in the limit of infinite mass number and zero radius, gives $m^*$ around 0.6 [23, 25]. This result does not lend support to the ZM model, but tend to favour the Walecka and MZM models instead [14]. Finally, $\kappa$ is smaller in the MZM model than the “empirical” prediction $\kappa = 210 \pm 30$ MeV. Despite this, the advantage of MZM here is that, unlike the nonlinear $\sigma - \omega$ model [26], it does not present anomalies in the EOS for any value of $\kappa$.

We proceed, now, to generalize the ZM and the MZM models. We recall that both originate from Eq.(8) for the particular choice $m^*(\sigma) = m^*_{ZM}(\sigma)$. As a consequence of this choice the scalar effective coupling constant scales as $(g^*_\sigma/g_{\sigma}) = m^*$. However, different choices of $m^*(\sigma)$ can be made meeting the requirement of Eq.(8).

### 2.1 Model I

From Eq.(7) we generate a family of models by choosing a scaling given by

$$\text{Model I : } g^*_\sigma/g_{\sigma} = m^* \alpha \quad g^*_\omega/g_{\omega} = 1$$

The equation of state for such a family of models will be shown in the next section in a general expression along with that of Model II. For Model I we consider two particular cases: the Walecka model ($\alpha = 0$) and ZM model ($\alpha = 1$).
2.2 Model II

Defined by Eq.(12) and the scaling

\[ \frac{g_\sigma^*}{g_\sigma} = m^\alpha \quad \frac{g_\omega^*}{g_\omega} = m^* \]  

(16)

(the MZM model is obtained for \( \alpha = 1 \)). For a given value of \( \alpha \), the function \( m^*(\sigma) \) is defined by Eq.(16) together with (8) as in the Model I case.

2.3 The Equations of State for the Models

When the meson fields in the Lagrangian of both models are substituted by their mean values we arrive at the mean field approximation (MFA). In Model I the scalar-meson field \( \sigma \) is a function of the scalar density (\( \rho_s \)) exclusively. In Model II, \( \sigma \) depends on \( \rho_s \) but also on the baryonic density \( \rho_b \). For rotationally and translationally invariant symmetric nuclear matter, the MFA equation for the scalar fields reads

\[ \sigma = \frac{g_\sigma}{m_\sigma^2 M} \frac{m^\alpha+1}{(1-\alpha)m^* + \alpha} \left[ M \rho_s + \beta \left( \frac{g_\omega}{m_\omega} \right)^2 m^* \rho_b^2 \right] , \]  

(17)

with \( \beta = 0 \) or 1 for Model I or Model II, respectively. Equation (17) shows clearly how Model II and its generalization mix the scalar and vector fields. The scalar and baryonic densities are related through,

\[ \frac{\rho_s}{\rho_b} = - \left( \frac{C_\omega^2}{C_\sigma^2} \right) \left( \frac{1}{m^{2\alpha+1}} \right) \left( \frac{S}{V} \right) - \beta \left( \frac{V}{M m^*} \right) , \]  

(18)

where

\[ S = -g_\sigma^* \sigma = -M(1 - m^*) , \]  

(19)

and

\[ V = (C_\omega^2/M^2)m^{2\beta}/\rho_b . \]  

(20)

This density ratio weighs the content of each model. Model II has an additional term making its contribution to the \( \sigma \)-field, and consequently to the effective mass, larger than that of Model I.
The expressions for the energy density and pressure at a given temperature \( T \) can be found, as usual, by the MFA average of the energy-momentum tensor. They read,

\[
\mathcal{E} = \frac{C^2_2}{2M^2} m^2 \rho^2_b + \frac{M^4}{2C^2_\alpha} \left( \frac{1 - m^*}{m^{*\alpha}} \right)^2 + \frac{\gamma}{(2\pi)^3} \int d^3k E^* (k) (n_k + \bar{n}_k) , \tag{21}
\]

and

\[
p = \frac{C^2_2}{2M^2} m^{*2\beta} \rho^2_b - \frac{M^4}{2C^2_\sigma} \left( \frac{1 - m^*}{m^{*\alpha}} \right)^2 + \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int d^3k \frac{k^2}{E^* (k)} (n_k + \bar{n}_k) , \tag{22}
\]

where

\[
\rho_b = \frac{\gamma}{(2\pi)^3} \int d^3k (n_k - \bar{n}_k) . \tag{23}
\]

Here \( \gamma \) is the degeneracy factor (\( \gamma = 4 \) for nuclear matter; \( \gamma = 2 \) for neutron matter), \( \bar{n}_k \) and \( n_k \) stand for the Fermi-Dirac distribution for antibaryons and baryons with exponent \( (E^* \pm \nu)/T \), respectively. The energy \( E^* (k) \) is given by

\[
E^* (k) = (k^2 + M^{*2})^{1/2} , \tag{24}
\]

while the effective chemical potential which preserves the number of baryons and antibaryons in the ensemble is defined by \( \nu = \mu - V \), with \( \mu \) the thermodynamical chemical potential. According the Hugenholtz-van Hove theorem [27], the Fermi energy must be equal to the energy per baryon at saturation density. Therefore, the following relation has to be satisfied,

\[
\frac{\mathcal{E}}{\rho_o} = V + E^* (\rho_o) . \tag{25}
\]

We end the section by presenting an analytical general expression for the compressibility valid in both models,

\[
K = 9V + 3 \frac{k_F^2}{(k_F^2 + (M + S)^2)^{1/2}} + 9 \left( \frac{\partial S}{\partial \rho} \left( \frac{1}{(k_F^2 + (M + S)^2)^{1/2}} + 2\beta \frac{V}{(M + S)} \right) \right) , \tag{26}
\]

where \( k_F \) is the Fermi momentum and all quantities are calculated at nuclear matter saturation density \( \rho_o \).
3 Results and Discussions

We have conducted calculations with these generalized models at $T = 0$, for different values of $\alpha$, requiring $E_b = -15.75$ MeV at $\rho_c = 0.15$ fm$^{-3}$. We begin this section by discussing Model I. The question we ask ourselves is whether, by varying $\alpha$, we can obtain a simultaneous fit for $\kappa$, $M^*$ and $V - S$. The answer to this question is negative. This can be better appreciated by looking at Fig. 1 where these quantities are plotted as a function of $\alpha$. No region in the plot gives a simultaneous “reasonable” agreement for the discussed quantities. By “reasonable” we mean $m^*$ around 0.6 (which, we shall see later, fixes the values of $V - S$ at approximately 680 MeV) and a compressibility $\kappa = 210 \pm 30$ MeV which is the value of derived from the energy of the breathing mode of doubly magic nuclei [28]. This conclusion agrees with that of Greiner and Reinhard [22] using a generalized “ansatz” applied to the same model. A word about the nuclear compressibility; using Fermi-liquid Landau theory, Brown [29] has argued strongly in favour of a lower value of $\kappa$. Since the subject is still under debate we use the value of Ref.[28] as an approximate upper limit.

The results for Model II are shown in Fig. 2. The compressibility shows a weak dependence on $\alpha$ in the region of $\alpha > 1$, and reaches a minimum close to $\alpha = 1$ which corresponds to the MZM case. Particularly interesting are the results for the region $\alpha < 1$ where $m^*$ and the difference $V - S$ show an improvement over the results of the MZM model given in the previous section. For example, for $\alpha = 0.9$ Model II gives $M^* = 558.3$ MeV, $V - S = 687.6$ MeV and a compressibility $\kappa = 166.2$ MeV. It is also interesting to note that for $\alpha = 0.88$ one gets $M^* = 507.0$ MeV, and $V - S = 785.1$ MeV which are very close to the values obtained in the usual linear Walecka model but, this time, with a softer compressibility, $\kappa = 181.3$ MeV.

Figure 2 shows also that for $\alpha < 0.79$ nuclear matter does not saturate. This includes the $\alpha=0$ case, which corresponds to the Walecka model if $m^*(\sigma)$ is as in Eq.(8). In addition, Figs. 1 and 2 indicate that $M^*$ increases with $\alpha$ thus decreasing the vector potential due to (23). We have allowed for higher values of $\alpha$ in the calculations to reach the curious situation where $V$
vanishes, and model I and II become degenerate. Nuclear matter saturation is achieved with just the scalar field. This situation occurs for $\alpha \approx 12.8$, which is the maximum value that this parameter can have (beyond it $V$ becomes negative), and for $C_0^2 = 315.36$. The results for this special case are: $M^* = 885.8\text{ MeV}$, $V - S = -S = 52.4\text{ MeV}$ and $K = 65.8\text{ MeV}$.

With some insight into the two models, we discuss here if in any $M^*$ could give model-independent properties of nuclear matter. As we argued before the effective mass is a manifestation of the dynamical relativistic content of any particular model. On the other hand, if two models have the same $M^*$, they have the same density ratio as given by Eq.(18). This quantity is shown in Fig. 3 as a function of $\alpha$. We notice that Model II, unlike Model I, can acquire a strong relativistic character. Going back to Figs. 1 and 2 we notice that the same $M^*$ can be obtained in both models for different values of $\alpha$. Consequently, they will give the same relativistic ratio. If the situation is such that two different models give the same $M^*$, the question is whether other observables can be determined likewise. For instance, for a given $M^*$, the potentials $S$ and $V$ are fixed by Eqs.(19) and (25), respectively. Therefore, the quantity $V - S$ and $M^*$ are directly correlated and carry the same physical information for each particular model. As for the correlation between $\kappa$ and $M^*$, the situation is different having a more model-dependent character.

In order to learn about the compressibility $\kappa$, we have calculated its individual contributions according to Eq.(26). To the first three terms of (26) we will refer as $\kappa_1$, $\kappa_2$ and $\kappa_3$, respectively. An additional fourth term ($\kappa_4$) is present for Model II ($\beta = 1$). These quantities are plotted in Figs. 4 and 5. $\kappa_1$ and $\kappa_2$ are completely determined by $M^*$ since they depend directly on $S$ and $V$; $\kappa_3$ and $\kappa_4$, however, show an extra dependence on the slope of $M^*(\rho)$ which is negative and carries information about the scalar field. For Model I, Fig. 1 shows that as $M^*$ increases the compressibility $\kappa$ decreases. This explains why, in the Walecka model where $M^*$ is small, $\kappa$ is so high. In the usual ZM model the opposite is true. In Model II the changes in $\kappa_3$ and $\kappa_4$ are such that despite the increase in $M^*$ the total compressibility remains almost constant.

Finally, in Fig.6 we show the effective mass $m^*$ —solution of Eq.(8)— for different values
of $\alpha$.

4 Conclusions

In summary, we have shown the equivalence between two descriptions of derivative scalar coupling models which we now interpret as effective Walecka models. In them the effective coupling constants depend on the density and are completely determined by the effective nucleon mass $m^*$. The first model has only one effective coupling $g^*_\sigma$ related to $m^*$ through $m^* = 1 - g^*_\sigma \sigma / M$. The second includes also an effective vector coupling $g^*_\omega$ which is always given by $g^*_\omega = m^* g_\omega$.

We have also shown that for a particular choice of $m^* = m^*_{\sigma \omega} (\sigma)$ both effective meson coupling constants scale with $m^*$. We have generalized this scaling by introducing an extra parameter $\alpha$ and concluded that, in a theory with three free parameters, Model I can not provide good results for the level splitting in finite nuclei and at the same time adjust the experimental $\kappa$ and $m^*$. We showed, however, that it is possible to choose the parameter $\alpha$ so as to give values of $m^*, \kappa$ and $V - S$ similar to those obtained from the nonlinear $\sigma - \omega$ model. We conclude that changing only the scalar coupling (Model I) definitely improves $\kappa$ but with no hope of reproducing the spin-orbit splitting for finite nuclei, thus supporting the conclusions of Ref.\cite{22}. This is a direct consequence of the large values $M^*$ that Model I gives when $\kappa$ goes in the right direction as $\alpha$ changes. Model II, instead, through the inclusion of a mixed coupling between the scalar and the vector fields may be the way to expect improvement in the calculation of the observables.

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Figure 1: The nucleon effective mass ($M^*$), the difference between the vector and scalar potentials ($V - S$) and the compressibility $\kappa$ as function of $\alpha$ for Model I.

Figure 2: The nucleon effective mass ($M^*$), the difference between the vector and scalar potentials ($V - S$) and the compressibility $\kappa$ as function of $\alpha$ for Model II.

Figure 3: The relativistic ratio between scalar and baryonic densities ($\rho_s/\rho_b$) as a function of $\alpha$ for both models.

Figure 4: The components $\kappa_1$, $\kappa_2$ and $\kappa_3$ of the compressibility $\kappa$ as a function of $\alpha$ for Model I.

Figure 5: The components $\kappa_1$, $\kappa_2$, $\kappa_3$ and $\kappa_4$ of the compressibility $\kappa$ as a function of $\alpha$ for Model II.

Figure 6: The effective nucleon mass as a function of the scalar sigma field ($u = g_\sigma \sigma/M$) for different values of $\alpha$. 

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Model 1

$K, M^*, V-S$ (GeV)
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Fig. 3 - A. Delfino et al.
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Fig. 5 - A. Delfino et al.
Fig. 6 - A. Delfino et al.

\[ m^*(u) = g_0 \sigma / M \]

![Graph showing the relationship between \( m^*(u) \) and \( u = g_0 \sigma / M \) for different values of \( \alpha \).]