The Nature of Dark Energy from deep Cluster Abundance

P. Solevi, R. Mainini & S.A. Bonometto

ABSTRACT

We show that using the redshift dependence of the deep cluster abundance to detect the nature of Dark Energy is a serious challenge. We compare the expected differences between flat ΛCDM models, with different Ω_{mo}, with the difference between ΛCDM and dynamical DE models. In the former case, cluster abundances in comoving volume and geometrical factors act in the same direction, yielding a significant difference between the expected angular densities. On the contrary, when we keep a constant Ω_{mo} and change the DE nature, abundances in comoving volume and geometrical factors act in the opposite direction, so that the expected differences in angular densities reduce to small factors.

Subject headings: cosmological parameters

1. Introduction

High redshift supernovae, data on the cosmic microwave background (CMB), as well as on the large-scale galactic distribution (Riess et al. 1988, Perlmutter et al. 1988, Tegmark et al. 2001, De Bernardis et al. 2000, Hanany et al. 2000, Halverson et al. 2001, Spergel et al. 2003, Percival et al. 2002, Efstathiou et al. 2002) indicate that ∼ 70% of the world contents are due to a smooth component with negative pressure, such that the ratio \( w \equiv p/\rho < \sim -0.8 \). The nature of this component, dubbed dark energy (DE), is still open for debate. Candidates range from false vacuum, yielding a positive cosmological constant \( \Lambda \), to a self-interacting scalar field \( \phi \) (Ratra & Peebles 1988; Wetterich 1988) to even more exotic physics of extra dimensions (e.g., Dvali & Turner 2003).

While the observed value of \( \Lambda \) implies a dramatic fine-tuning of vacuum, at the latest phase transition, ΛCDM models provide an excellent fit to data. Alternative viable DE models must, first of all, rival their success. Therefore, their phaenomenology must be and is hardly distinguishable from ΛCDM. This calls for tests able to discriminate between different DE natures.
In recent work, the evolution of the cluster mass function has been shown to have a significant dependence on DE nature. This has been shown first on the basis of a Press & Schechter (PS) formulation (Mainini, Macciò & Bonometto 2003), then using n–body simulations (Klypin et al. 2003; see also Linder & Jenkins 2003), which confirmed PS findings.

There are a number of other cosmological measures which can contribute to discriminate between different DE natures. This is important also on the light of the point that we wish to make in this paper. In fact, while the number density of clusters \( n(M, z) \), in a comoving volume, has a significant \( z \) dependence, the measurable signal is much less significant. On the light of observational uncertainties, it seems unlikely that the nature of DE can be easily discriminated along this pattern.

In a sense, this contradicts the expectations for a number of deep sky surveys (see, e.g., Davis, Gerke & Newman 2004) based on the ancient intuition of Hubble (1926), that the number–redshift relation can be used to determine the geometry of the world. Discriminating the DE nature is much harder than fixing the matter density parameter \( \Omega_m \) in a flat \( \Lambda \)CDM model.

It is not so because of a reduced impact of DE nature on geometry or on \( n(M, z) \), but because their effects on geometry and \( n(M, z) \) tend to erase each other.

2. Geometrical effects

Let us consider a family of objects whose (cumulative) mass function is \( n(M, z) \). Independently of the actual \( z \) dependence, the angular number density of objects belonging to such family, with redshift between \( z \) and \( z + \Delta z \), in a spatially flat geometry, reads

\[
N(M, z, \Delta z) = \int_z^{z+\Delta z} dz' D(z') r^2(z') n(M, z')
\]  

(1)

with \( D(z) = dr/dz \). For the models we are considering, it is useful to show that

\[
D(z) = \frac{c}{H_o} \sqrt{\frac{\Omega_m(z)}{\Omega_{mo}(1 + z)^3}}
\]  

(2)

and, therefore,

\[
r(z) = \frac{c}{H_o} \int_0^z dz' \sqrt{\frac{\Omega_m(z')}{\Omega_{mo}(1 + z')^3}}.
\]  

(3)
In fact, for flat models, on the past light cone, \( a dr = -c dt \) and, therefore, by dividing the two sides by \( dz = -da/a^2 \), we obtain \( a dr/dz = a^2 c dt/da \), so that:

\[
D(z) = \frac{dr}{dz} = \frac{c}{H(z)} \quad \text{(here, as usual, } H = \frac{\dot{a}}{a}).
\]  

(4)

In turn

\[
H^2(z) = \frac{8\pi G}{3} \rho_m(z) = \frac{8\pi G}{3} \rho_{mo}(1+z)^3 =
\]

\[
= H_o^2 \frac{\Omega_{mo}(1+z)^3}{\Omega_m(z)},
\]

(5)

so that eqs. (2) and (3) follow.

These equations lead to handable expressions when DE has a state equation \( w = p/\rho \) with constant \( w \). It is then easy to see that

\[
D(z) = (c/H_o)(1 + z)^{-3/2}[\Omega_{mo} + (1 - \Omega_{mo})(1 + z)^3w]^{-1/2},
\]

(6)

while \( r(z) \) can be worked out by integrating from 0 to \( z \). We can also easily differentiate both with respect of \( \Omega_{mo} \) and with respect to \( w \), finding that

\[
\frac{\partial D^2}{\partial \Omega_{mo}} = -\left(\frac{c}{H_o}\right)^2 \frac{1 - (1 + z)^{3w}}{(1 + z)^3[\Omega_{mo} + (1 - \Omega_{mo})(1 + z)^3w]^2}
\]

(7)

and

\[
\frac{\partial D^2}{\partial w} = -\left(\frac{c}{H_o}\right)^2 \frac{(1 - \Omega_{mo})3 \ln(1 + z)(1 + z)^{3w}}{(1 + z)^3[\Omega_{mo} + (1 - \Omega_{mo})(1 + z)^3w]^2},
\]

(8)

so that we expect that the geometrical factors increase when \( \Omega_{DEo} = 1 - \Omega_{mo} \) increases, whenever \( w < 0 \), and decrease when \( w \) increases. Notice that eq. (8) is not a functional derivative, as the expression (6) holds just for constant \( w \).

An extension to dynamical DE can be however performed by using the interpolating expressions yielding \( \Omega_m(z) \), for RP (Ratra & Peebles, 1988, 1995) and SUGRA (Brax & Martin 1999, 2001, Brax, Martin & Riazuelo 2000) models, provided by Mainini et al (2004). In these cases DE is due to a scalar field, self–interacting through a potential whose expression depends on an energy scale \( \Lambda \) (see Appendix A for details on the dynamical DE models considered here). Using the interpolating expressions or, equivalently, direct numerical integration, we obtain the results shown in Figure 1. Here the \( z \) dependence of the geometrical factor \( D(z)r^2(z) \) is shown for three \( \Lambda \)CDM models (\( \Lambda \)CDM 06, \( \Lambda \)CDM 07, \( \Lambda \)CDM 08, with \( \Omega_{mo} = 0.4, 0.3 \) and 0.2 respectively), as is obtainable from eq. (6). In the same Figure we show the \( z \) dependence of geometrical factors also for SUGRA and RP models, with \( \Omega_{mo} = 0.3 \) and with the \( \Lambda \) parameter fixed at \( 10^3 \)GeV. \( H_o \) is 70 km/s/Mpc in all models.
In the absence of number density evolution, Fig. 1 would also show the dependence of the angular number density on $z$.

The $\Lambda$CDM models considered are characterized by quite different values of $\Omega_{mo}$. Among them, only 0.7 approaches available data, which are coherent with an $\Omega_{mo}$ interval not wider than 0.02–0.03. In spite of that, the discrepancy between $\Lambda$CDM 07 and $\Lambda$CDM 06 only marginally exceeds the discrepancy between SUGRA and $\Lambda$CDM 07 and is smaller than the difference between RP and $\Lambda$CDM 07. Notice, in particular, that the RP model can be considered as discrepant from data as $\Lambda$CDM 06 or $\Lambda$CDM 08.

Altogether, Fig. 1 shows how strongly the nature of DE affects geometrical factors and that, in the presence of a non–evolving population, an insight into the DE equation of state can be provided by the $z$ dependence of their angular number density.

3. Cluster number evolution

A fair insight into the evolution of the number of clusters with the redshift $z$, can be obtained by using a PS expression. Sheth & Tormen (1999, 2002) as well as Jenkins et al (2001) provided expressions more closely fitting n–body simulations (which are already reasonably approached by PS results). The latter expressions are more complex and include more parameters, while their use is unessential for the present aims.

The expected (differential) cluster number density $n(M)$, at a given time, is then given by the expression

$$f(\nu)\nu d\log \nu = \frac{M}{\rho_m} n(M) M d\log M . \tag{9}$$

Here $\rho_m$ is the matter density, $\nu = \delta_c/\sigma_M$ is the so–called bias factor, $M$ is the mass scale considered. $\sigma_M$ is the r.m.s. density fluctuation on the scale $M$ and $\delta_c$ is the amplitude that, in the linear theory, fluctuations should have in order that, assuming spherical evolution, full recollapse is attained exactly at the time considered (in a standard CDM model this value is $\sim$1.68; the difference, in other model, ranges around a few percent). As usual, we took a Gaussian $f(\nu)$ distribution.

Together with eq. (9), we must take into account the virialization condition, which yields significantly different density contrasts $\Delta_c$ in different DE models. Further details can be found in Mainini et al (2003).

In Figure 2 we show the cumulative cluster number density, $n(> M, z)$, obtained by integrating $n(M)$, for the same models of Fig. 1, for $M = 10^{14} M_\odot h^{-1}$. All models are normalized to the same cluster number today and the redshift dependence of $n(> M, z)$
is clearly understandable, on qualitative bases: When ΛCDM models are considered, the evolution is faster as we approach standard CDM. Hence, ΛCDM 08 (ΛCDM 06), being the most distant (nearest) model to standard CDM, has the slowest (fastest) evolution. ΛCDM 07 just stands in between.

When we compare dynamical DE models to ΛCDM 07, we know that they also yield a slower evolution, i.e. structures form earlier in models with \( w > -1 \).

Accordingly, apart of numerical details, the behaviors shown in Fig. 2 reasonably fit our expectations.

4. Conclusions

Let us now put together the results shown by Figs. 1 & 2 and evaluate the angular number densities of clusters in redshift intervals \( z, z + \Delta z \) with \( \Delta z = 0.1 \). If ΛCDM models are considered, the geometrical factor \( D(z) r^2(z) \) and the evolutionary factor \( n(> M, z) \) vary in the same directions, when \( \Omega_{mo} \) is modified.

On the contrary, for dynamical DE models, the two terms vary in the opposite directions and, therefore, model differences tend to erase.

Accordingly, the results shown in Figure 3 are essentially expected. It is then easy to argue that testing the differences between cluster numbers is really challenging, when different DE models are compared. On the contrary, different \( \Omega_{mo}'s \) cause comparatively huge shifts. Data on cluster masses are obtainable either from temperatures \( T \) or luminosities \( L \) (see, e.g., Pierpaoli et al 2004). Finding a fair cluster mass is perhaps the main reason for uncertainty, but also the cluster redshift determination is subject to errors. In Figure 4 we compare the expected mass functions, for the ΛCDM 07 and the two dynamical DE models considered, at two mass scales \( (10^{14}M_\odot h^{-1} \) and \( 4.2 \cdot 10^{14}M_\odot h^{-1}) \), showing also the uncertainty in the halo mass function caused by a 5% indetermination in the mass: \( \delta N = \left[ Mn(M)/N(> M, z, \Delta z) \right] \delta M/M, \) still for \( \Delta z = 0.1 \). Of course, \( \delta M/M \sim 0.05 \) is not easily reachable, on the basis of the present capacity to reconstruct cluster dynamics. The plots also assume that a complete cluster sample in a solid angle of 1 steradian is available.

Using deep cluster distributions, to discriminate DE models, is therefore hard. A comparison between the two panels of Figure 4 however shows that differences become wider as we go to smaller masses. Accordingly, on group mass scales, a further enhancement can be expected. An even stronger difference can be expected, when angular number densities of galaxies are being compared. In particular, comparing the two panels shows that, when
lowering the mass scale considered, differences in trends (vs. redshift) are enhanced. On galaxy mass scales, therefore, the very peaks of distributions could fall at different redshifts. Predictions for galaxy scales cannot be formulated only using a PS recipe, but should include an accurate consideration of how large halos split into galactic objects. This will be the topic of future research.

Before concluding let us however outline that, here above, the dependence on $z$ of cluster features has been assumed not to depend on DE nature. As a matter of fact, when we consider different state equations for DE, we change the $z$–$t$ relation and the same redshift corresponds to a different time. This is true also when $\Omega_{mo}$ is changed, of course. However, while, in the latter case, this change of the $z$–$t$ relation has a counterpart in a change of halo angular densities, there is no equivalent counterpart in the former case. Comparing the $z$–$t$ relation, obtained from a study of cluster morphologies or galactic evolution, with number density predictions can be therefore a way to discriminate between different DE natures.

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### A. Dynamical DE models

Dynamical DE is to be ascribed to a scalar field, $\phi$, self–interacting through an effective potential $V(\phi)$, whose dynamics is set by the Lagrangian density:

$$\mathcal{L}_{DE} = -\frac{1}{2} \sqrt{-g} \left( \partial^\mu \phi \partial_\mu \phi + V(\phi) \right).$$  \hfill (A1)

Here $g$ is the determinant of the metric tensor $g_{\mu\nu} = a^2(\tau) dx_\mu dx_\nu$ ($\tau$ is the conformal time). In this work we need to consider just a spatially homogeneous $\phi$ ($\partial_i \phi \ll \dot{\phi}$; $i = 1, 2, 3$; dots denote differentiation with respect to $\tau$); the equation of motion then reads:

$$\ddot{\phi} + 2\frac{a}{a} \dot{\phi} + 2a^2 \frac{dV}{d\phi} = 0. \hfill (A2)$$

Energy density and pressure, obtained from the energy–momentum tensor $T_{\mu\nu}$, are:

$$\rho = -T^0_0 = \frac{\dot{\phi}^2}{2a} + V(\phi), \quad p = \frac{1}{3} T^i_i = \frac{\dot{\phi}^2}{2a} - V(\phi), \hfill (A3)$$

so that the state parameter

$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2/2a - V(\phi)}{\dot{\phi}^2/2a + V(\phi)}. \hfill (A4)$$
changes with time and is negative as soon as the potential term \( V(\phi) \) takes large enough values.

The evolution of dynamical DE depends on details of the effective potential \( V(\phi) \). Here we referred to models proposed by Ratra & Peebles (RP: 1988, 1995), yielding a rather slow evolution of \( w \), and Brax & Martin (SUGRA: 1999, 2001, see also Brax, Martin & Riazuelo 2000) yielding a much faster evolving \( w \). Altogether, RP and SUGRA potentials cover a large spectrum of evolving \( w \). They read

\[
V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \quad \text{RP},
\]

\[
V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \exp(4\pi G \phi^2) \quad \text{SUGRA}.
\]

These potentials allow tracker solutions, yielding the same low–\( z \) behavior, almost independently of initial conditions. In eqs. (A5) and (A6), \( \Lambda \) is an energy scale, currently set in the range \( 10^2 \text{–} 10^{10} \) GeV, relevant for the physics of fundamental interactions. The potentials depend also on the exponent \( \alpha \). Fixing \( \Lambda \) and \( \alpha \), the DE density parameter \( \Omega_{DE} \) is determined. Here we rather use \( \Lambda \) and \( \Omega_{DE} \) as independent parameters. In particular, numerical results are given for \( \Lambda = 10^3 \) GeV.

The RP model with such \( \Lambda \) value is in slight disagreement with low-\( l \) multipoles of the CMB anisotropy spectrum data. Agreement may be recovered with smaller \( \Lambda \)'s, which however loose significance in particle physics. The SUGRA model considered here, on the contrary, is in fair agreement with all available data.

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Fig. 1.— Redshift dependence of geometrical factors for ΛCDM and dynamical DE models.
Fig. 2.— Cluster number, in comoving volumes, in $\Lambda$CDM and dynamical DE models. In this and in the following plots $\Delta z = 0.1$. 
Fig. 3.— Cluster number, on the celestial sphere, in ΛCDM and dynamical DE models.
Fig. 4.— Cluster number, on the celestial sphere, in ΛCDM and dynamical DE models with \( \Omega_m = 0.7 \). Effects due to an uncertainty of 5% in the mass determination are also shown. In the range \( 1 < z < 2 \) and for \( M = 10^{14}h^{-1}M_\odot \), discrimination may be possible after suitable improvements of data and theory.