Outrunning planning by network management in Industry 4.0 concept

V V Provotorov¹, L B Raijhelgauz², A A Fedotov³, S N Makarova² and O Ja Kravets³

¹ Voronezh state university, 1, Universitetskaya sq., 394006, Voronezh, Russian Federation
² Orel state university, 95, Komsomol'skaya str., Orel, 302026, Russian Federation
³ Voronezh State Technical University, 14 Moscow ave., Voronezh, 394026, Russian Federation

E-mail: wwprov@mail.ru

Abstract. You need a digital twin to solve the problem of efficient planning in Industry 4.0. It is based on the use of scientifically proved algorithms and complicated mathematical models. The current trend for the consolidation of engineering enterprises in network structures is caused by the competition in this segment of production. Those business participants, who apply the latest solutions to the support planning based on artificial intelligence systems will get the advantage. A spectral approach is presented in the problem of finding approximations of the eigenvalues of the Sturm-Leewill problem with an unknown operator. The approach is based on the spectral features of the elliptic operator of the initial-boundary objective and the method for solving the inverse spectral objective of reconstructing the Sturm-Leewill operator by two sequences of eigenvalues matching two sets of boundary conditions. The method, presented in this article can be successfully used in the objectives of optimal controlling. The obtained results are fundamental in solving the objectives of optimal controlling by evolutionary systems in networks and the analysis of network models.

1. Introduction

The paper presents a new approach in determining the approximations of the cellip operator's own values on some data of the solution of the operator equation at the internal point of the spatial coordinate change [1]. At the same time, the operator is considered unknown, i.e. its coefficients are unknown. This made it possible to use the classical results of the recovery of multidimensional systems [2-5].
2. The main formalisms

Let the function \( T(x,t) \in C^{1,0}(\Omega_x) \cap C^{2,1}(\Omega_x) \), \( \Omega_x = [0, \ell] \times [0, \infty) \) satisfy the differential equation

\[
\frac{a(x) \frac{\partial T(x,t)}{\partial t}}{=} \frac{\partial}{\partial x} \left( b(x) \frac{\partial T(x,t)}{\partial x} \right), \quad x \in (0, \ell), \quad t > 0,
\]

(1)

the initial and boundary conditions:

\[
T(x,0) = 0, \quad x \in [0, \ell]
\]

(2)

\[
\frac{\partial T(0^+,t)}{\partial x} = \alpha \left( T(0^+,t)-T_0(t) \right), \quad \frac{\partial T(\ell^-,t)}{\partial x} = -\beta \left( T(\ell^-,t)-T_\ell(t) \right), \quad t > 0.
\]

(3)

To simplify the presentation, the initial condition (2) is uniform \([6-8]\). Boundary conditions (3) are determined by functions \( T_0(t), T_\ell(t), t > 0 \) and parameters \( \alpha, \beta \). In the future, we assume that \( a(x), b(x) \) are positive on \([0, \ell]\) and \( a(x) \in C[0, \ell], b(x) \in C^1[0, \ell] \); continuous on \([0, \infty)\) function \( T_0(t), T_\ell(t) \) are such that there exist constant \( C_1 > 0, C_2 \) and there are ratios

\[
|T_0(t)| \leq c_1 e^{c_2 t}, \quad |T_\ell(t)| \leq c_1 e^{c_2 t}, \quad c_1 > 0, \quad c_2 - \text{const.}
\]

(4)

The initial-boundary value problem formed by the differential system (1)–(3), has the only solution \([9,10]\). The initial-boundary value problem (1)–(3) transformed according to Laplace in the half-plane \( \mathbb{R}_+ = \{ p : \text{Re} \, p \geq \gamma_0, \nu > -\gamma_0 \} \) takes

\[
\frac{d}{dx} \left( b(x) \frac{dT^*(x,p)}{dx} \right) = p a(x) T^*(x,p), \quad x \in (0, \ell),
\]

(5)

\[
\frac{dT^*(0,p)}{dx} = \alpha \left( T^*(0,p)-T_0^*(p) \right),
\]

\[
\frac{dT^*(\ell,p)}{dx} = -\beta \left( T^*(\ell,p)-T_\ell^*(p) \right).
\]

(6)

where \( \gamma_0 \) is the smallest eigenvalue of the operator's, generated by the differential expression

\[
\frac{d}{dx} \left( b(x) \frac{du(x)}{dx} \right) [11-14] \text{ conditions (4) guarantees the application of the Laplace conversion in the present case: } c_2 < \gamma_0 \). Here, the subscript \( (\ast) \) designate the Laplace conversion of function \( T(x,t) \).

Let \( h \) it be some positive number (task approximation step (5), (6)). The approximation of the task (5), (6) present as a system

\[
b(x) T^*(x+h,p)-[b(x)+b(x-h)]T^*(x,p)+b(x-h)T^*(x,p) = 0,
\]

(7)

\[
T^*(h,p)-T^*(0,p) = \alpha h \left( T^*(0,p)-T_0^*(p) \right),
\]

(8)

Let \( N \) is the number of partition of segment \([0, \ell]\) and \( h \) is the step of partition. Let the fixed point \( x_0 \in (0, \ell) \) coincide with one of the nodes of partition \( (x_0 \) is the point where information about the initial-boundary value problem is known (1)–(3)). Let's mark through
\[ x^0 = 0, \quad x^i = i h, \quad i = 1, 2, \ldots, N-1, \quad x^N = \ell, \]
\[ u_i = T^*(x^i, p), \quad u_i = u_i(p), \quad a_i = a(x^i), \quad b_i = b(x^i), \quad i = 0, 1, 2, \ldots, N, \]
then out (7), (8) follows that
\[ b_i u_{i+1} - [b_i + b_{i+1} + p h^2 a_i] u_i + b_{i+1} u_{i+2} = 0, \quad i = 1, 2, \ldots, N-1, \quad (9) \]
\[ u_i - u_0 = \alpha h (u_0 - T^*_0(p)), \quad u_N - u_{N-1} = -\beta h (u_N - T^*_\ell(p)). \quad (10) \]

Value at point \( x_0 \in (0, \ell) \) of function \( f^*(x, p) \) a given continuous function \( f(x, t) \) mark through
\[ u_0 = f^*(p), \quad p \in \mathbb{R}_+, \quad (11) \]
in here \( 0 < x_0 < N - 1 \) it takes a fixed value that corresponds to the value \( x_0 \). The resulting system can be written in the form of a matrix equation [14-17]
\[ (-B + pA)U(p) = \alpha T^*_0(p)B_0 + \beta T^*_\ell(p)B_1, \]
\[ U = \text{col}(u_1, u_2, \ldots, u_N), \quad (12) \]

where
\[ B_0 = \frac{1}{h^2} \text{col} \left( \frac{h b_0}{b_0 + \alpha h}, 0, \ldots, 0 \right), \quad B_1 = \frac{1}{h^2} \text{col} \left( 0, \ldots, 0, \frac{h b_{N-1}}{b_n + \beta h} \right) \]

are \((N-1)\)-dimensional vector columns; \( A \) is \((N-1)\times(N-1)\)-diagonal matrix with diagonal elements \( a_i \); \( B \) is \((N-1)\times(N-1)\)-three-diagonal matrix with diagonal elements:
if \( i = 1 \) then \( d_{i1}^1 = b_1 + b_1 - \frac{b_0^2}{b_0 + \alpha h}, \quad d_{12}^1 = -b_1 \),
if \( i = 2, N-2 \) then \( d_{ii}^i = b_{i-1}, \quad d_{i1}^i = b_{i-1} + b_i, \quad d_{i+1}^i = -b_i \),
if \( i = N-1 \) then \( d_{N-1, N-2}^{N-1} = -b_{N-2}, \quad d_{N-1, N-1}^{N-1} = b_{N-1} + b_N - \frac{b_{N-1}b_N}{b_N + \beta h} \).

Remark. The characteristic numbers of the equation (12) coincide with the characteristic numbers of the matrix \(-A^{-1}B\) [18-23], which are approximations of the eigenvalue of (5), (6).

Thus, the task of finding of the eigenvalue of the different analogue of the Sturm-Leewill task (5), (6) is to determine the characteristic numbers of the matrix \(-A^{-1}B\) centers.

3. The main result
Take place the following fundamental statements.

Theorem 1. The eigenvalues \( \lambda_i \) and eigenvectors \( U_i = \text{col}(u_1^i, u_2^i, \ldots, u_N^i) \), \( i = 1, N-1 \), of matrix \(-A^{-1}B\) satisfy ratios
\[ h^2 \sum_{n=1}^{N-1} \frac{1}{(\lambda_n + \mu)^m} u_n^m(p) \sum_{s=1}^{N-1} u_s^n(p) a_{rs}(p) = \frac{(-1)^m r_{ij}^{(m)}(p)}{m!}, \quad (13) \]
\[ m = 2, 3, \ldots, \quad 1 \leq i, j \leq N - 1, \]
where \( r_{ij}^{(m)}(p) \) is \( m \)-order derivative of the element \( r_{ij}(p) \) of matrix \((B + pA)^{-1}\), \( p \in \mathbb{R}_+ \).
The system (13) allows you to reduce the task of determining the eigenvalues $\gamma_n$ to the task of finding the real solutions of the polynomial with a degree $N - 1$.

Denote $\tau_i = \frac{1}{\gamma_i + p}$, $p \in \mathbb{R}$, $i = 1, N - 1$.

**Theorem 2.** Let $T_\tau (t) = 0$ and let the system

$$\sum_{n=1}^{N-1} \alpha_{n+m} c_m = -d_m \quad (m = 0, 1, 2, \ldots), \quad (14)$$

where $d_m = \frac{(-1)^m}{(m + 2)!} \left( \frac{f^* (p)}{T_\tau^* (p)} \right)^{(m+2)}$, has a solution $\{c_1, c_2, \ldots, c_{N-1}\}$.

Then $\gamma_i = -\frac{1}{\tau_i} - p$, $i = 1, N - 1$, for arbitrary fixed $p \in \mathbb{R}$, where $\tau_i$ are the real solutions of the polynomial $\psi (\tau) = \sum_{n=1}^{N-1} c_n \tau^m \quad (c_0 = 1)$.

4. Conclusions

The approach allows us to write the system (9), (10) (which means approximation of the initial-edge task (1) - (3)) in the form of a matrix equation, for which it is necessary to determine the characteristic numbers. Characteristic numbers coincide with the characteristic numbers of the known matrix. These numbers are approximations of eigenvalues of the spectral problem (5), (6), which are determined by system solutions (13) and (14). The method presented in the work of determining the approximations of eigenvalues can be successfully used both in the tasks of optimal control [13-15] and in the tasks of stabilization [22-25] and stability of solutions partial differential equations [21, 22]. It should also be noted that presented in the works [23-25] the approaches and methods of analysis of dynamic systems presented in the works allow partial use of the results of this work. This applies primarily to the widespread use of operational calculus techniques in the reverse tasks of spectral analysis. Note that the results complement (and in some ways develop) the idea of restoring one-dimensional operators when analyzing the recovery of multidimensional systems [26-28].

References

[1] Provotorov V V, Sergeev S M and Part A A 2019 Solvability of hyperbolic systems with distributed parameters on the graph in the weak formulation *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes* 14(1) 107–17
[2] Yanenko M 2016 Cost-Based Brand Management *International Business Management* 10(26) 5991-5
[3] Sergeev S, Kirillova T and Krasyuk I 2019 Modelling of sustainable development of megacities under limited resources *TPACEE-2018 E3S Web of Conferences* 91 05007
[4] Alexandrova I and Zhabko A 2018 A new LKF approach to stability analysis of linear systems with uncertain delays *Automatica* 91 173-8
[5] Krasnov S, Zotova E, Sergeev S, Krasnov A and Draganov M 2019 Stochastic algorithms in multimodal 3PL segment for the digital environment *IOP Conference Series Materials Science and Engineering* 618(1) 012069
[6] Kravets O J, Barkalov S A, Butyrina N A, Sekerin V D and Gorokhova A E 2018 Processes of multidimensional classification of scoring objects with heterogeneous features based on the neural networks modeling *International Journal of Pure and Applied Mathematics* 119(7a) 875-9
[7] Kamachkin A M and Yevstafyeva V V 2000 Oscillations in a relay control system at an external disturbance Control Applications of Optimization 2000 Proceedings of the 11th IFAC Workshop 2 459-62
[8] Krasnov S, Sergeev S, Titov A and Zotova Y 2019 Modelling of digital communication surfaces for products and services promotion IOP Conference Series Materials Science and Engineering 497(1) 012032
[9] Pilipenko O V, Provotorova E N, Sergeev S M and Rodionov O V 2019 Automation engineering of adaptive industrial warehouse J. Phys.: Conf. Ser. 1399 044045
[10] Krasnov S, Sergeev S, Zotova E and Grashchenko N 2019 Algorithm of optimal management for the efficient use of energy resources E3S Web of Conferences 110 02052
[11] Aleksandrov A and Platonov A 2009 On stability and dissipativity of some classes of complex systems Automation and Remote Control 70(8) 1265-80
[12] Borisoglebskaya L N, Provotorov V V, Sergeev S M and Kosinov E S 2019 Mathematical aspects of optimal control transference processes in spatial networks IOP Conf. Ser.: Mater. Sci. Eng. 537 042025
[13] Provotorov V V 2008 Eigenfunctions of the Sturm-Liouville problem on a star graph Mathematics 199(10) 1523-45
[14] Krasyuk I A, Bakharev V V, Kozlova N A and Mirzoeva D D 2017 Staffing in the sphere of trade: the main issues and prospects of solution Proceedings of 2017 IEEE 6th Forum Strategic Partnership of Universities and Enterprises of Hi-Tech Branches (Science. Education. Innovations) SPUE 2017 6 48-50
[15] Borisoglebskaya L N, Provotorova E N, Sergeev S M and Khudyakov A P 2019 Automated storage and retrieval system for Industry 4.0 concept IOP Conf. Ser.: Mater. Sci. Eng. 537 032036
[16] Artemov M A, Baranovskii E S, Zhabko A P and Provotorov V V 2019 On a 3D model of non-isothermal flows in a pipeline network Journal of Physics. Conference Series 1203 012094
[17] Provotorov V V, Ryazhskikh V I and Gnilitskaya Yu A 2017 Unique weak solvability of nonlinear initial boundary value problem with distributed parameters in the netlike domain // Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes 13(3) 264-77
[18] Kapustina I V, Kirillova T V, Ilyina O V, Razzhivin O A and Smelov P A 2017 Features of Economic Costs of Trading Enterprise: Theory and Practice International Journal of Applied Business and Economic Research 15(11) 1-10
[19] Provotorov V V and Provotorova E N 2017 Optimal control of the linearized Navier-Stokes system in a netlike domain Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes 13(4) 428-41
[20] Borisoglebskaya L N, Provotorova E N and Sergeev S M 2019 Promotion based on digital interaction algorithm IOP Conf. Ser.: Mater. Sci. Eng. 537 042032
[21] Aleksandrov A, Aleksandrova E and Zhabko A 2016 Asymptotic stability conditions of solutions for nonlinear multiconnected time-delay systems Circuits, Systems and Signal Processing 35(10) 3531-54
[22] Karelin V V 2010 Penalty functions in the control problem of an observation process Vestnik of Saint Petersburg University. Applied mathematics. Computer science. Control processes 4 109-14
[23] Kamachkin A M, Potapov D K and Yevstafyeva V V 2016 Existence of solutions for second-order differential equations with discontinuous right-hand side Electronic Journal of Differential Equations 124 1-9
[24] Zhabko A P, Nurtazina K B and Provotorov V V 2019 About one approach to solving the inverse problem for parabolic equation Vestnik of Saint Petersburg University. Applied mathematics. Computer science. Control processes 15(3) 322-35
[25] Avdonin S, Murzabekova G and Nurtazina K 2017 Source Identification for the Differential
Equation with Memory New Trends in Analysis and Interdisciplinary Applications. Trends in Mathematics Dang P, Ku M, Qian T and Rodino L (Birkhäuser, Cham) 111-20

[26] Sergeev S and Kirillova T 2019 Information support for trade with the use of a conversion funnel IOP Conference Series: Materials Science and Engineering 666 012064

[27] Aleksandrov A and Zhabko A 2003 On stability of solutions to one class of nonlinear difference systems Siberian Mathematical Journal 44(6) 951-8

[28] Borisoglebskaya L N, Provotorova E N and Sergeev S M 2019 Commercial software engineering under the digital economy concept J. Phys.: Conf. Ser. 1399 033029