Modular Invariance and Nonrenormalizable Interactions

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ABSTRACT

We examine the modular properties of nonrenormalizable superpotential terms in string theory and show that the requirement of modular invariance necessitates the nonvanishing of certain Nth order nonrenormalizable terms. In a class of models (free fermionic formulation) we explicitly verify that the nontrivial structure imposed by the modular invariance is indeed present. Alternatively, we argue that after proper field redefinition, nonrenormalizable terms can be recast as to display their invariance under the modular group. We also discuss the phenomenological implications of the above observations.

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1. Introduction

It is one thing to believe that string theory is the Theory of Everything and another to prove it. The plethora and multiple latitude of problems that one encounters is well-known: selection of a specific classical vacuum, stability of the selected vacuum under non-perturbative string effects, supersymmetry breaking, connection with the experimental reality of low-energy physics \( E \ll M_{Pl} \), etc. While nobody can claim a panacea for all these problems, remarkable progress has been made on several fronts. For instance, (semi) realistic vacua have been identified [1] with no obvious or presently identifiable stability problems, and different realistic scenarios have been proposed for ways to break supersymmetry in string theory and to communicate this seed of supersymmetry breaking to the low-energy spectrum [2,3].

Some time ago we started to explore ways of connecting the aloof string theory at the Planck scale with the low-energy physics world [4,5,6]. We found that one of the most convenient frameworks to address such issues is the so-called free fermionic formulation of superstrings in four dimensions [7]. Within this formulation we elucidated the methods to be used in calculating superpotential terms at the cubic level [4,8] as well as at arbitrarily high orders [8]. The importance of this program should be clear, since it is by the inclusion of nonrenormalizable terms (which arise by integrating out massive string modes) that we expect to obtain a realistic fermion mass spectrum, quark mixing, proton stability, etc [9]. This program has been decisive in the identification of realistic models [5].

String theory is more subtle than regular field theory. It contains symmetries beyond the usual gauge symmetries that impose tight constraints on the allowed effective action. Indeed, it has been found that string duality or target space modular invariance [10,11,12] imposes considerable restrictions on the four-dimensional effective action [13]. In this paper we determine the modular invariant properties of the cubic and higher-order superpotential in models built within the free fermionic formulation. We then use these facts to obtain powerful new results about the \( T \)-dependence of the calculated superpotential couplings. These results may have far-reaching phenomenological consequences.

The task of constructing a string theory effective action is marred by a plethora of subtle and not so subtle problems. If one were to begin with the standard supergravity action obtained from string [14](either by dimensional reduction of by the sigma model approach),

\[
\mathcal{G} = K(T, \bar{T}, m, m^\dagger) + \ln W(m, T) + \ln \bar{W}(m^\dagger, \bar{T}),
\]  

(1.1)
where $T$ are the moduli fields, $m$ generically denotes the matter fields, and $K$ is the Kähler function given by

$$K(T, \bar{T}, m, m^\dagger) = -3 \ln(T + \bar{T}) + f(T, \bar{T})mm^\dagger,$$

(1.2)

it would naively seem that the effective action for the moduli fields obtained through $G$ is invariant under the modular transformations of the $T$ fields, since these are part of the Kähler transformations, that is

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc \in \mathbb{Z},$$

(1.3a)

$$K \rightarrow -3 \ln(T + \bar{T}) + 3 \ln(icT + d) + 3 \ln(-ic\bar{T} + d),$$

(1.3b)

$$W \rightarrow (icT + d)^{-3}W.$$  

(1.3c)

The above transformation properties are contained in $T \rightarrow g(T)$, where $g$ is an arbitrary function, and $K \rightarrow K + F(T) + \mathcal{F}(\mathcal{T})$. However, as it can be easily shown, the transformations in Eqs. (1.3) have an anomaly at the one-loop level [15]. For a consistent treatment of the modular transformation, this anomaly must be incorporated into the effective action. Furthermore, the inclusion of nonperturbative effects of a strongly interacting gauge theory requires additional modifications of the effective action [3]. In fact, the focus of the efforts involving modular invariance and effective string theory actions has been directed towards these modifications [16]. However, a tacit assumption made in the above procedure is to presume that the superpotential $W$ is indeed such that it transforms covariantly as in Eq. (1.3c), which may or may not be the case.

In a supergravity theory [17], one may “choose” the superpotential such that it is already endowed with covariant properties under modular (or Kähler) transformations. However, in string theory one does not have that luxury. For a large class of models, the superpotential can be explicitly and unambiguously calculated [8]. Hence for the superstring effective action to be invariant under modular transformations, the explicit calculation of the superpotential must exhibit the covariant properties which are by no means obvious. Additionally, as it has been noted before, to go from a ‘generic’ string theory model to a specific phenomenologically interesting one, it is necessary to explore the superpotential $W$ in much greater depth [8][13][13]. Important physical implications of the model such as the scale of gauge symmetry breaking and the fermion mass matrices, can only be extracted after the superpotential is obtained in great detail. Again, the modular invariance properties of $W$ are a crucial ingredient in the analysis.
2. General remarks

As a starting point, let us consider the superpotential at the trilinear level only

\[ W_3 = c_{ijk} m_i m_j m_k. \]  

(2.1)

It is then clear that a trivial choice of modular weights \((-1)\) for each matter field \(m_i\), combined with the cubic nature of the superpotential leads to a modular covariant expression

\[ m_i \rightarrow (icT + d)^{-1} m_i, \]  

(2.2a)

\[ W_3 \rightarrow (icT + d)^{-3} W_3. \]  

(2.2b)

Hence, we see that the strong constraints that modular transformations might lead to cannot be uncovered until one considers higher-order (nonrenormalizable) terms in the superpotential.

Over the past few years there has been considerable progress in evaluating nonrenormalizable terms in a large class of string theory models. Notable among these are (2,2) symmetric orbifolds [18,19], Calabi-Yau manifolds [20], asymmetric orbifolds [18], and free fermionic formulations [8]. In all except free fermionic models, nonrenormalizable terms cannot be unambiguously calculated due to nonperturbative instanton corrections [9]. For the free fermionic case there exists a powerful machinery that allows one to evaluate explicitly nonrenormalizable terms up to very high orders [8]. In addition, the results thusly obtained enjoy strong nonrenormalization theorems [21], making them especially suited to study the interplay of modular invariance and \(W\).

Succinctly put, the method employed to evaluate a typical nonrenormalizable term relies on the calculation of the S-matrix elements between different fields of interest. Starting with a vertex operator (which generates properly normalized one-particle states), the string sigma model Lagrangian can be used to calculate the correlators among different fields which are then used to extract the possible nonrenormalizable terms that may be present. The key point to note here is that these vertex operators generate states which are already normalized, whereas the states in the supergravity action carry a nontrivial \(T\) dependence [22,11]. This is an important distinction whose true significance will soon become clear. To avoid confusion we will denote the matter fields in the string (supergravity) basis with (un)primed fields.
For theories possessing (2,2) worldsheet supersymmetry the identification of the moduli fields is reasonably straightforward \[23\]. However, for theories with only (2,0) supersymmetry (e.g., free fermionic formulations and asymmetric orbifolds) no simple, full-proof method exists for this purpose. In what follows we will consider a scalar massless field with zero potential as a possible candidate for a modulus field. The modulus field \( \Phi \) is generated by a vertex operator
\[
V_{\Phi(-1)}(z, \bar{z}) = e^{-c \, G_L(z) G_R(\bar{z})} e^{i k \cdot x},
\]
where \( G_L(z) \) and \( G_R(\bar{z}) \) are conformal fields of dimension \((1/2, 0)\) and \((0, 1)\) respectively, and \( c \) is the ghost field. For the \( \Phi \) field to be a modulus field we must also have
\[
\langle (V_{\Phi})^n \rangle = 0,
\]
in the zero momentum limit, i.e., \( \Phi \) has no potential. An effective Lagrangian for the \( \Phi \) field may have a form
\[
\mathcal{L} = \partial \Phi \partial \Phi^\dagger + A \Phi \Phi^\dagger \partial \Phi \partial \Phi^\dagger + \cdots,
\]
(\text{where the existence of the second term can be explicitly checked by evaluating the correlator } \langle V_{\Phi} V_{\Phi} V_{\Phi} V_{\Phi} V_{\Phi}^\dagger \rangle). \text{ From our experience with no-scale supergravity theories we know that the above Lagrangian is actually symmetric under a noncompact symmetry group } SU(1,1)/U(1) \[17\]. (To realize this symmetry a field redefinition of \( \Phi \) is necessary \[22,11\].) The modular symmetry of interest is actually a subgroup of the above mentioned continuous symmetry. In string theories, as opposed to no-scale supergravity theories, this continuous symmetry is broken to the discrete subgroup of modular transformations in Eq. \[13\] by the higher order terms in the superpotential and higher derivative terms in the effective action.

The Kähler potential reflecting the invariance of the theory under the noncompact continuous symmetry is
\[
K = -3 \ln(T + \overline{T}).
\]
The transformation or the (non-holomorphic) field redefinition necessary to show the equivalence between the effective Lagrangian of Eq. \[2.5\] and the one obtained through \[2.6\] is \[22,24\]
\[
\Phi = \frac{1 - T}{1 + T}.
\]
The inclusion of the matter fields is straightforward. If, for a moment, we just concentrate on the Kähler function $K$, in the string basis one has

$$K = \Phi \Phi^\dagger + A(\Phi \Phi^\dagger)^2 + m' m'^\dagger + B(m' m'^\dagger)(\Phi \Phi^\dagger) + \cdots,$$  \hspace{1cm} (2.8)

where $m'$ is a matter field (in the string basis). On the other hand, in the supergravity basis $K$ describes a sigma model on the coset space $SU(1, N)/SU(N) \times U(1)$ with Kähler potential given by \[25\]

$$K = -3 \ln(T + \overline{T} - \sum_{i=1}^{N} m_i m_i^\dagger).$$ \hspace{1cm} (2.9)

It is clear that in order for $K$ to be invariant (up to a Kähler transformation) under the $PSL(2, \mathbb{Z})$ duality transformation, the fields $m_i$ must transform with modular weight $-1$,

$$m_i \rightarrow e^{i\lambda(a,b,c,d)} \frac{icT+d}{icT} m_i,$$ \hspace{1cm} (2.10)

where $\lambda$ is a phase factor (called the multiplier system). Note that the matter field $m'$ in the string basis must also undergo a field redefinition for it to have the simplified transformation properties given in Eq. (2.10) \[22,11\] (the exact transformation \[24\] is not critical for the present discussion).

Now we turn our attention to the main topic of this paper, the modular transformation properties of the superpotential $W$. The point that the detailed knowledge of $W$ and its transformation properties are critically important in any serious analysis does not need to be belabored. The dichotomy of the situation is that the detailed information about $W$ can only be obtained in the string basis, whereas the symmetry properties of $W$ under modular transformations are manifest only in the supergravity basis.

The invariance of the effective Lagrangian and consequently of $G$ under modular transformations implies that the superpotential $W$ must transform as

$$W \rightarrow \frac{1}{(icT+d)^3} W,$$ \hspace{1cm} (2.11)

\text{i.e., with modular weight }-3.\text{ For a trilinear term in the superpotential (if all the matter fields in the theory have modular weight }-1)\text{ the modular covariance is automatic. It is clear that no higher order term in the superpotential can be constructed which has modular weight }-3.\text{ However, if an explicit calculation (albeit in the string basis) requires}
a quartic and/or higher order nonrenormalizable term to be present in the superpotential, consistency of the theory will require that the quartic term $W_4$ be of the form

$$W_4 = m_1 m_2 m_3 m_4 \eta^2(T) \mathcal{H}_4(T),$$

(2.12)

where $\eta(T)$ is the Dedekind function of the first kind and has transformation property

$$\eta(T) \to e^{i\pi/12}(icT + d)^{1/2}\eta(T) \quad \text{as} \quad T \to \frac{aT - ib}{icT + d},$$

(2.13)

and $\mathcal{H}_4(T)$ is an arbitrary modular invariant function. That is, the superpotential at the Nth order must be accompanied by appropriate number of $\eta^2(T)$ powers to ensure that the overall invariance of $W$ is maintained,

$$W = W_3 + \eta^2(T) \mathcal{H}_4(T) W_4 + \eta^4(T) \mathcal{H}_5(T) W_5 + \cdots,$$

(2.14)

where $W_N$ is the Nth order nonrenormalizable term and the $\mathcal{H}_N(T)$ are arbitrary modular invariant functions. As shown below, generalization of the above construction to include twisted fields (matter fields with modular weight $\neq -1$) and to include more than one type of modulus field is straightforward.

When examined from the the vantage point of the string basis, the above construction implies a remarkable structure. In terms of explicit correlator calculations, the above statement entails that a nonzero value of the quartic (or higher order) term in the superpotential necessitates the existence of an infinite string of nonvanishing correlators, i.e.,

$$\langle m'_1 m'_2 m'_3 m'_4 \rangle \neq 0 \Rightarrow \langle m'_1 m'_2 m'_3 m'_4 \Phi^n \rangle \neq 0, \quad \text{for all } n.$$

(2.15)

That is, a quartic (or higher order) term calculation in the string basis will necessarily receive corrections due to the moduli fields. Note that since the trilinear term is modular covariant by itself, no such correction is necessary. Due to the arbitrariness of the modular function that may appear in the superpotential in the supergravity basis (i.e., the $\mathcal{H}_N(T)$ in Eq. (2.14)) and due to the (non-holomorphic) field redefinition involved, it is not always possible to compare specific numerical values obtained in these two different bases. However, a great deal of information about the structure of the superpotential can still be obtained as we will show shortly.

In Ref. [27], an explicit calculation of the superpotential for a class of models showed that the trilinear terms did not receive higher order moduli-dependent corrections, i.e., they were stable. Here we see that it is the modular invariance of the theory which is responsible for the observed stability. Furthermore, this same symmetry of the theory also dictates that the quartic and higher order terms must receive corrections if the modulus field is away from its canonical value (Eq. (2.7): $T \neq 1 \Rightarrow \Phi \neq 0$).
3. The free fermionic case

The salient features of the free fermionic formulation of the heterotic string (in four dimensions) \[7\] include the existence of 18 real two-dimensional left-moving fermions $\chi^k, y^k, w^k \ (k = 1, \ldots, 6)$ transforming in the adjoint representation of $SU(2)^6$. A model can be further divided into ‘complex fermions’ or ‘real fermions’ model depending on whether or not all two-dimensional fermions can be paired up to give bosonic fields. Bosonization of the fields

$$\frac{1}{\sqrt{2}}(\chi^k + i\chi^{k+1}) = e^{iS_{k,k+1}}, \quad k = 1, 3, 5$$  \ (3.1)

plays a special role since

$$i\partial_z(S_{12} + S_{34} + S_{56}) = J(z),$$  \ (3.2)

where $J(z)$ is the conserved $U(1)$ current of the N=2 worldsheet supersymmetry algebra \[8\]. A vertex operator for the scalar component of a chiral superfield in the canonical ghost picture is given by \[8\]

$$V_{-1}^b = e^{-c} e^{i\alpha S_{12}} e^{i\beta S_{34}} e^{i\gamma S_{56}} G e^{i\frac{k \cdot K}{2}} e^{i\frac{k \cdot X}{2}},$$  \ (3.3)

where $G$ is a conformal field of dimension $(h(G), 1)$, with $h(G) = (1 - \alpha^2 - \beta^2 - \gamma^2)/2$, $\alpha, \beta, \gamma \in \{0, \pm\frac{1}{2}, \pm1\}$, $c$ is the ghost field, and $\alpha + \beta + \gamma = 1$.

This class of models can be shown to have modular group of at least $PSL(2, \mathbb{Z})^3$ and three associated moduli fields which will be denoted by $T_I, I = 1, 2, 3$. The modular weight of each matter field is related to its charge under $\partial_z S_{k,k+1}, \ k = 1, 3, 5$. A consistent choice is \[24\]

$$m_i \to m_i \prod_I (i\epsilon_I T_I + d_I)^{-w_i^I},$$  \ (3.4)

where $w_i^I$ is the charge of the vertex operator of the scalar component of the $m_i$ superfield under the current $\partial_z S_{2I-1,2I}$ (i.e., the $\alpha, \beta, \gamma$ in Eq. (3.3)). This choice automatically assures that all the trilinear couplings obtained using the S-matrix approach are modular invariant since the conservation of $U_J(1)$ charge requires that the superpotential carries charges $(-1, -1, -1)$ under $(S_{12}, S_{34}, S_{56})$. \[7\] That is, Eq. (2.11) is generalized to

$$W \to \prod_I (i\epsilon_I T_I + d_I)^{-1} W.$$  \ (3.5)

\[1\] As $U_J(1)$ and hence $(S_{12}, S_{34}, S_{56})$ are $R$-type charges, $\theta$ carries a nontrivial charge under these transformations. Thus, for $\int d^2 \theta W$ to be invariant, $W$ must carry a nonzero charge.
In terms of supergravity fields the Kähler function is analogously generalized to \[22,11,24\]

\[
K = \sum_I -\ln(T_I + \bar{T}_I) + \sum_i \prod_I (T_I + \bar{T}_I)^{-w} m_i m_i^\dagger + \cdots. \tag{3.6}
\]

As far as the trilinear terms are concerned, no term consistent with the gauge symmetry and other symmetries present in the theory is prohibited or constrained in any way due to modular symmetry. This follows from the conservation of \(U_J(1)\) charged mentioned above. By the same token it is also clear that no quartic or higher order term will be trivially covariant under the modular symmetry. Since each superfield has modular weight \(w = -\sum_I w_I = -1\), Nth order nonrenormalizable terms will have weight \(-N\). The overall modular weight of the superpotential must add up to \(-3\) (or \((-1, -1, -1)\) under each \(PSL(2, \mathbb{Z})\) separately). Hence, if the theory is expected to accommodate modular invariance, then either the offending Nth order term is zero or it must be accompanied by a coupling which would depend on the moduli fields \(T_i\) and would cancel the excess modular charge of \(-N + 3\).

From explicit S-matrix calculations we know that some of these nonrenormalizable terms are definitely nonzero, we conclude then that their coupling must be such as to screen out the modular weight. The simplest case of one modulus field was discussed above (see Eq. (2.15)). For the case of more than one modulus field the previous statement becomes more precipitous. Knowing the \(w_I\) charges of the fields it is straightforward to determine the “deficit” modular weight of a particular term. The “deficit” will dictate the form and argument of the modular function necessary to cast the term under consideration into modular covariant form. In the string basis this implies the existence of correlators of the type

\[
\langle m'_1 m'_2 \cdots m'_N \Phi_i^{p} \Phi_j^{k} \rangle \neq 0 \quad \forall p, k, \tag{3.7}
\]

where \(i, j\) depend on the modular “deficit” charge.

We now show how the above general remarks apply to a specific string model built in the free fermionic formulation, namely the flipped \(SU(5)\) string model \[4\]. The fields \(\Phi_1\) and \(\Phi_2\) (in the string basis) correspond to the moduli fields \(T_1\) and \(T_2\) (in the supergravity basis). We defer until later the discussion on the third modulus field \(T_3\) (which appears to be related to the fields \(\Phi_4\) and \(\Phi_5\)). The whole set of quartic superpotential couplings and their respective (nonvanishing) coefficients is known in this model \[5\]. For instance, the quartic term \(cF_1 \bar{f}_1 \bar{h}_{45} \phi_1\) has modular weight \((-1, -2, -1)\) and therefore its coefficient must contain \(\eta^2(T_2) \mathcal{H}'(T_2)\) to make it modular covariant. This implies that in the string
basis the correlators $\langle F_1 \bar{F}_1 \bar{h}_{45} \phi_1 \phi_2^n \rangle$, \(\forall n\) should all be generically nonvanishing, a fact that has been verified explicitly for small \(n\). The actual quartic coefficient will then be \(c(T_2) = \alpha \eta^2(T_2) / \eta^2(1) \eta'(1)\). We have confirmed that all quartic terms present in the superpotential do indeed receive higher order corrections due to the moduli fields as anticipated by the ‘modular deficit’ argument. A similar analysis for the quintic and higher order terms also bears out the premise that all nonrenormalizable terms are accompanied by a proper modular function such as to make the superpotential modularly covariant.

The flipped SU(5) model discussed here differs from a typical model obtained in the free fermionic formulation in one important respect, namely the presence of an anomalous \(U_A(1)\) in the gauge group. The model appears to only have \(PSL(2, \mathbb{Z})^2\) symmetry instead of \(PSL(2, \mathbb{Z})^3\) expected in free fermionic models of this class since there is no \(T_3\) field that can be readily identified as a modulus field. We attribute the possible non-existence of the “modulus” field \(T_3\) (related to \(\Phi_4\) and \(\Phi_5\) in the string basis) to the ill understood anomalous \(U_A(1)\) phenomenon. Although it is quite conceivable that since the theory does have additional flat directions, the modular symmetry group may be \(g' \times PSL(2, \mathbb{Z})^2\). However, we will not address this possibility here. The cancellation of the \(U_A(1)\) anomaly requires a ‘discrete’ shift in the string vacuum to a different nearby vacuum \([28]\). This mechanism, which albeit cancels the anomaly and restores the supersymmetry, spontaneously breaks the modular symmetry from \(PSL(2, \mathbb{Z})^2\) to either \(PSL(2, \mathbb{Z})\) or a smaller subgroup thereof depending on the choice of vacuum expectation values.

The phenomenological implications of the structure of the nonrenormalizable terms dictated by modular invariance are self evident. For example, while looking for quartic or higher order terms, one need not consider terms of the type \(\langle m_1 m_2 m_3 \Phi \rangle\) since they would automatically be zero. \(N\)th order \((N \geq 4)\) terms of the type \(\langle m_1 \cdots m_N \Phi^p \rangle\) are nonzero only if \(\langle m_1 \cdots m_N \rangle \neq 0\). The strength of this observation lies in the fact that the terms of the type \(\langle m_1 \cdots m_N \Phi^p \rangle\) are typically not forbidden by any other symmetry of the theory. In Ref. \([29]\) this structure of the nonrenormalizable terms was explicitly used to determine the specific value of the gaugino condensate. One expects that such information about the structure of the theory will prove invaluable in any detailed phenomenological inquiry.

4. Conclusions

In conclusion, we have argued that the requirement of modular invariance imposes a strong constraint on the \(N\)th order nonrenormalizable terms that may be nonvanishing.
We have verified explicitly that for free fermionic models, the structure of the nonrenormalizable terms implied by modular invariance is indeed present. The most useful phenomenological advantage of the above result is that it allows one to examine the physical implications of a model under a small shift of the moduli fields. We further argued that the effective action obtained in the string basis (after a suitable field redefinition) can be recast to manifestly display its invariance under modular transformations.

The complete effective action essentially contains three separate pieces. The gauge and ‘kinetic energy’ (Kähler function) part, nonperturbative effects of strong gauge interactions, and the superpotential. The modular properties of the gauge and kinetic energy part were examined in [30,26,15,31,24] in the guise of ‘threshold corrections’, whereas the modular properties of the nonperturbative gauge interaction terms have been known for some time. Here we complete the picture by explicitly showing how the nonrenormalizable part of the superpotential transforms under the modular group. As expected, all three pieces alluded to above orchestrate themselves to embody modular invariance in the effective theory. Further study of modular invariance, especially in the context of its possible phenomenological implications will indubitably prove to be very useful.

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