Beyond the Standard Model Effective Field Theory: The Singlet Extended Standard Model

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S. Adhikari, I.M. Lewis, M. Sullivan, Physical Review D (2021) 075027

Phenomenology Symposium 2021
May 26, 2021
Approaches for Beyond the Standard Model Physics

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- Alternative approaches: Simplified models and effective field theories.
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Alternative approaches: Simplified models and effective field theories.

Simplified Models:

- Assume only one or two particles accessible at the LHC, the rest are too heavy.
- Choose particles that are ubiquitous in more complete beyond the Standard Model models.

Effective Field Theory:

Assume only Standard Model particles accessible at the LHC.

Write a power expansion in inverse powers of a heavy new physics scale $\Lambda$:

$$L = L_{SM} + \sum_{k} c_{1,k} \Lambda^{-1} + \sum_{k} c_{2,k} \Lambda^{-2} + \cdots$$

The operators consist of Standard Model fields and are invariant under Standard Model symmetries.

Any new high scale physics will induce these operators: the Standard Model Effective Field Theory is inevitable.

Note: you can also classify according to topology.
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  N. Craig, P. Draper, KC Kong, Y. Ng, D. Whiteson, arXiv:1610.09392; 
  J.H. Kim, KC. Kong, B. Nachman, D. Whiteson JHEP 04 (2020) 030
Simplified Models

- Assume only one or two particles accessible at the LHC, the rest are too heavy.

\[
\begin{align*}
\text{Other New Physics} & \gtrsim \Lambda \\
\text{LHC Energy} & \sim 13 \text{ TeV} \\
\text{Handful new states} & \\
\text{SM} & \lesssim v
\end{align*}
\]

- As with Standard Model Effective Field Theory, the new physics beyond the LHC reach will inevitably manifest itself as an EFT:

\[
L = L_{\text{ren}} + \sum_k \frac{c_{1,k}}{\Lambda} O_{1,k} + \sum_k \frac{c_{2,k}}{\Lambda^2} O_{2,k} + \cdots
\]

Now \( L_{\text{ren}} \) is the renormalizable theory, and the operators \( O_{n,k} \) consist of the fields of and are invariant under the symmetries of the simplified model.

- The goal: use EFT methods to test the assumptions of the simplified models:
  - Can the effects of heavy new physics be ignored?
First, review the renormalizable model.

Add a real gauge singlet, scalar singlet $S$ to SM:

$$V(\Phi, S) = V_\Phi(\Phi) + V_{\Phi S}(\Phi, S) + V_S(S)$$

Higgs potential:

$$V_\Phi(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Scalar singlet potential:

$$V_S(S) = b_1 S + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4$$

Mixing terms:

$$V_{\Phi S}(\Phi, S) = \frac{a_1}{2} \Phi^\dagger \Phi S + \frac{a_2}{2} \Phi^\dagger \Phi S^2$$

After electroweak symmetry breaking, have two mass eigenstates:

- $h_1$ with mass $m_1 = 125$ GeV.
- $h_2$ with mass $m_2 > m_1$.
- SM Higgs and singlet scalar mix with a mixing angle $\theta$. 
Relevant Feynman Diagrams

- **Couplings to fermions:**

\[
\begin{align*}
  h_1 & \quad f \quad -i \cos \theta \frac{m_f}{v} \\
  h_2 & \quad f \quad -i \sin \theta \frac{m_f}{v}
\end{align*}
\]

- **Couplings to gauge bosons:**

\[
\begin{align*}
  h_1 & \quad V \quad i \cos \theta \frac{2m_V^2}{v} g_{\mu
u} \\
  h_2 & \quad V \quad i \sin \theta \frac{2m_V^2}{v} g_{\mu
u}
\end{align*}
\]

- All SM-like Higgs rates suppressed $\cos^2 \theta$ relative to SM predictions.

- Since $h_2$ couplings to fermions and gauge bosons proportional to SM coupling, it is produced through same mechanisms as SM Higgs boson. Again, search predictions are relatively straight forward.
Interpretation of Fits

- Bound parameters by a $\chi^2$ fit to Higgs signal strengths:

$$\mu_i^f = \frac{\sigma_i(pp \rightarrow h_1)}{\sigma_{i,SM}(pp \rightarrow h_1)} \frac{BR(h_1 \rightarrow f)}{BR_{SM}(h_1 \rightarrow f)}$$

- Without effective operators:

$$\sigma(pp \rightarrow h_1 + X) = \cos^2 \theta \sigma_{SM}(pp \rightarrow h_1 + X) \quad BR(h_1 \rightarrow XX) = BR_{SM}(h_1 \rightarrow XX)$$

$\theta$ is the mixing angle between Scalar singlet and SM Higgs.

- Then signal strengths to all initial and final states are the same:

$$\mu_i^f = \cos^2 \theta$$

- Hence, we have a simple interpretation at 95% CL:

$$|\sin \theta| < 0.24$$

- No longer true with effective operators. Different production and decay channels have different dependencies on the EFT, changing the interpretation of fits considerably [Dawson, Lewis PRD95 (2017) 015004]
Adding EFT operators

- What if we add non-renormalizable interactions to dimension-5?
  - Perturb the model and see how stable our conclusions are.

\[
L = g_s^2 \frac{f_{GG}}{16 \pi^2 \Lambda} S G^{\mu \nu, a} G^a_{\mu \nu} + g'^2 \frac{c_{BB}}{16 \pi^2 \Lambda} S B^{\mu \nu} B_{\mu \nu} + \frac{g^2}{16 \pi^2 \Lambda} c_{WW} \frac{16 \pi^2 \Lambda}{S W^{\mu \nu, a} W^a_{\mu \nu}}.
\]

\[
- \left( \frac{\sqrt{2} m_t}{\nu} \frac{f_f}{\Lambda} S \bar{Q}_{3L} \tilde{\Phi}_{tR} + \sum_{f=\tau, \mu, b} \frac{\sqrt{2} m_f}{\nu} \frac{f_f}{\Lambda} S F_L \Phi f_R + \text{h.c} \right)
\]

\[
- \left( \frac{a_3}{2 \Lambda} \Phi^\dagger \Phi S^3 + \frac{a_4}{2 \Lambda} (\Phi^\dagger \Phi)^2 S + \frac{b_5}{5 \Lambda} S^5 \right)
\]

- After scalar mixing, these operators introduce new interactions between the gauge bosons and the Higgs.

- See also Baur, Butter, Gonzalez-Fraile, Plehn, Rauch PRD95 (2017) 055011 with dimension-6 terms, or Dawson, Lewis PRD95 (2017) 015004 for my previous work.
2-D Fits to $f_{GG}$, other parameters profiled over. 2-D Fits to $f_t$, other parameters profiled over.

- Combination of all Higgs measurements from ATLAS and CMS.
- Renormalizable model corresponds to Wilson coefficients set to zero.
- Non-zero Wilson coefficients change bounds on scalar mixing angle.
Combination of all Higgs measurements from ATLAS and CMS.
- Black and red: EFT
- Blue: Renormalizable model
- Clearly the EFT changes in interpretation of measurements and searches.
  - High scale new physics can have large impact on the simplified model.
- There are also direct searches for heavy scalar resonances that must be accounted for.
What is usually done:

Accept point if $\sigma \leq \sigma_{\text{obs}}$, Reject point if $\sigma > \sigma_{\text{obs}}$

where $\sigma_{\text{obs}}$ is the observed 95% CL upper limit.

However, statistically, when combining many measurements you can have 2-sigma fluctuations, and this prescription does not allow for it.
We proposed a new way of incorporating bounds from Brazilian bands:
First, work in Gaussian limit and assume no large upper fluctuations.
Assuming all data is SM-like, then every search is a SM measurement.
Each measurement has a 95% uncertainty band.
The upper limit of these error bands is the 95% limit on how large an additional signal can be on top of the signal.

Make a series of assumptions:
Assume data in good agreement with the Standard Model.
Assume Gaussianity.
We ignored interference between signal and background.

Hence, the $\chi^2$ for direct searches:

$$\chi^2 = \begin{cases} \frac{(\sigma_{\text{sig}} - \sigma_{\text{obs}} + \sigma_{\text{exp}})^2}{(\sigma_{\text{exp}}/1.96)^2} & \text{if } \sigma_{\text{obs}} \geq \sigma_{\text{exp}} \\ \frac{\sigma_{\text{sig}}^2}{(\sigma_{\text{obs}}/1.96)^2} & \text{if } \sigma_{\text{obs}} < \sigma_{\text{exp}} \end{cases}$$

$\sigma_{\text{sig}}$: signal cross section, $\sigma_{\text{obs}}$: observed upper limit, $\sigma_{\text{exp}}$: expected upper limit.
Can check for one measurement and one degree of freedom the 95% CL gives $\sigma_{\text{sig}} < \sigma_{\text{obs}}$, consistent with usual approach.
Scalar searches: combined CMS and ATLAS measurements of all relevant final states.

Renormalizable model corresponds to Wilson coefficients set to zero

Non-zero Wilson coefficients open up new regions of allowed mixing angle
Higgs fits only.

- **Black and red**: EFT
- **Blue**: Renormalizable model

Clearly the EFT changes in interpretation of measurements and searches.
Conclusions

- The LHC has completed two very successful runs and the data analysis is under way.
- Still may expect to see new physics. Two interesting ways forward:
  - Simplified models: only a few new particles at LHC energies.
  - Effective field theory: only Standard Model and LHC energies.

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- Scalar Singlet EFT: Investigated the stability of the interpretation of Higgs precision data in the singlet extended model. Interpretation of Higgs physics is considerably changed, even with multi-TeV new physics.
- Direct searches of scalar singlets depend on the couplings differently than precision Higgs data. Could have complementary information.

- Another example of BSM EFT: top partner can couple to top and gluons/photons via a chromomagnetic and magnetic dipole operator: Introduces new decay channels can open: $T \rightarrow t\gamma$ and $T \rightarrow tg$.

Kim, Lewis, JHEP 05 (2018) 095; Alhazmi, Kim, Kong, Lewis JHEP 01 (2019) 139.

- New production modes: $gg \rightarrow Tt$. Kim, Lewis, JHEP 05 (2018) 095; also see Xing Wang's talk yesterday.
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- The EFTs can drastically change the phenomenology of the simplified models.

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Thank You
First, review regular EFT counting with amplitude to dimension-6:
- Amplitude has terms up to $\Lambda^{-2}$.
- Amplitude squared includes terms that go as $\Lambda^{-4}$:

$$|A|^2 \sim |g_{SM} + \frac{c_{dim-6}}{\Lambda^2}|^2 \sim g_{SM}^2 + g_{SM} \times \frac{c_{dim-6}}{\Lambda^2} + \frac{c_{dim-6}^2}{\Lambda^4}$$

- $g_{SM}$ is a generic Standard Model coupling.
Comment on Counting

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  - $g_{SM}$ is a generic Standard Model coupling.
  - Same order as dimension-8 contributions:
    \[ |\mathcal{A}|^2 \sim |g_{SM} + \frac{c_{\text{dim}-6}}{\Lambda^2} + \frac{c_{\text{dim}-8}}{\Lambda^4}|^2 \]
    \[ \sim g_{SM}^2 + g_{SM} \times \frac{c_{\text{dim}-6}}{\Lambda^2} + \frac{c_{\text{dim}-6}^2}{\Lambda^4} + g_{SM} \times \frac{c_{\text{dim}-8}}{\Lambda^4} + O(\Lambda^{-6}) \]
  - Validity of keeping dimension-6 squared without dimension-8:
    - Strongly interacting theory: $c \gg g_{SM}$ so that $c_{\text{dim}-6}^2 \gg c_{\text{dim}-8} \times g_{SM}$.
    - Or the UV completion suppresses the dimension-8 terms.
Beyond the SM EFT Counting

Consider production or decay of an $h_1$. Then to dimension-6 there are three contributions:

- Renormalizable amplitude proportional to SM amplitude: $A_{\text{ren}} \sim \cos \theta A_{SM}$.
- Dimension-5 amplitude from the new scalar: $A_{5,S}$
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- Dimension-6 amplitude from SMEFT: $A_{6,SM}$.
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Contributions from new scalar suppressed by $\sin \theta$.
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Full amplitude:

$$A_{h_1} \sim \cos \theta A_{SM} + \cos \theta \frac{A_{6,SM}}{\Lambda^2} + \sin \theta \left( \frac{A_{5,S}}{\Lambda} + \frac{A_{6,S}}{\Lambda^2} \right) + O(\Lambda^{-3})$$

Amplitude squared:

$$|A_{h_1}|^2 \sim \cos^2 \theta |A_{SM}|^2 + \sin \theta \cos \theta \frac{A_{SM}A_{5,S}}{\Lambda}$$

$$+ \frac{1}{\Lambda^2} \left( \sin^2 \theta |A_{5,S}|^2 + \sin \theta \cos \theta A_{SM}A_{6,S} + \cos^2 \theta A_{SM}A_{6,SM} \right) + O(\Lambda^{-3})$$
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Large mixing angle limit: \( \sin \theta \to \pm 1 \) and \( \cos \theta \to 0 \)

- The amplitude becomes:

\[ |A_{h_1}|^2 \to \frac{|A_{5,S}|^2}{\Lambda^2} + O(\Lambda^{-3}) \]

- The square of the dimension five term is dominant, dimension-6 terms can be safely neglected.
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- Very different from SMEFT, where dimension-6 squared is same order as dimension-8.
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- In the intermediate and small mixing angle limit, the interference can not be ignored and \( |A_{5,S}|^2 \) will not necessarily dominate over dimension-6.

- Counting depends on the angle.
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- Very different from SMEFT, where dimension-6 squared is same order as dimension-8.
- In the intermediate and small mixing angle limit, the interference can not be ignored and $|A_{5,S}|^2$ will not necessarily dominate over dimension-6.
- Counting depends on the angle.
- For $h_2$ production $\sin \theta \leftrightarrow \cos \theta$ up to signs. The $|A_{5,S}|^2$ dominates at small angles.
Direct Contributions to Important Branching Ratios

Nonzero effective $W$ coupling.

Nonzero effective hypercharge coupling.

Adhikari, Lewis, Sullivan, arXiv:2003.10449
Indirect Contributions to Important Branching Ratios

Nonzero effective b-quark coupling.

Nonzero effective gluon coupling.

Adhikari, Lewis, Sullivan, arXiv:2003.10449
With this logic, we can construct a $\chi^2$:

$$\chi^2 = \frac{(\sigma_{SM+sig} - \hat{\sigma}_{SM+sig})^2}{(\sigma_{exp}/1.96)^2}$$

- $\sigma_{SM+sig}$ is the predicted SM+signal cross section
- $\hat{\sigma}_{SM+sig}$ is the measured rate in a signal search
- $\sigma_{exp}$ is the expected 95% CL upper limit.
Ignoring interference, we can approximate the SM+signal cross section as the addition of the SM and signal cross sections

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$$\sigma_{SM+sig} = \sigma_{SM} + \sigma_{sig}$$

For the measurement, we assume that it is mostly SM like and that any deviation is reflected in the deviation between the expected and observed 95% CL:

$$\hat{\sigma}_{SM+sig} = \sigma_{SM} + \sigma_{obs} - \sigma_{exp},$$

where $\sigma_{obs}$ is the observed 95% CL.
Detour: Combining Higgs Fits with Direct Search Limits

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- The \( \chi^2 \) then becomes:

\[ \chi^2 = \frac{(\sigma_{SM+sig} - \hat{\sigma}_{SM+sig})^2}{(\sigma_{exp}/1.96)^2} = \frac{(\sigma_{sig} - \sigma_{obs} + \sigma_{exp})^2}{(\sigma_{exp}/1.96)^2} \]
Problem: If observed limit is smaller than expected limit, best fit value for signal cross section is negative:

\[ \chi^2 = \frac{(\sigma_{\text{sig}} - \sigma_{\text{obs}} + \sigma_{\text{exp}})^2}{(\sigma_{\text{exp}}/1.96)^2} \]

In this case, we interpret the best fit for the signal cross section to be zero, and the uncertainty to be the observed upper limit:

\[ \chi^2 = \frac{(\sigma_{\text{obs}} - \sigma_{\text{exp}} + \sigma_{\text{obs}})^2}{(\sigma_{\text{obs}}/1.96)^2} \]

Hence, the \( \chi^2 \) for direct searches:

\[
\chi^2 = \begin{cases} 
(\sigma_{\text{sig}} - \sigma_{\text{obs}} + \sigma_{\text{exp}})^2 & \text{if } \sigma_{\text{obs}} \geq \sigma_{\text{exp}} \\
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