Tunable single-photon diode by chiral quantum physics

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We investigate the single photon scattering by an emitter chirally coupled to a one-dimensional waveguide. The single-photon transport property is essentially different from the symmetrical coupling case. The single photons propagating towards the emitter in opposite directions show different transmission behaviors, which is a manifestation of the single-photon diode. In the ideal chiral coupling case, the transmission probability of the single photon transport in one direction is zero by critical coupling, while in the opposite direction it is unity. The diode works well only when the single-photon frequency meets certain conditions. For a two-level emitter, the diode works well when the single photon is nearly resonant to the emitter. For a Λ-type three-level emitter, when the single-photon frequency is greatly altered, we can adjust the parameters of the external laser to ensure the diode works well. The latter provides a manner to realize a single-photon switch, in which the single-photon transmission probability can reach zero or unity although the emitter’s decay is considered.

I. INTRODUCTION

Optical diode, which allows unidirectional propagation of light, requires the ability to break Lorentz reciprocity. Nonreciprocity in light propagation has been extensively studied by various physical mechanisms. Single photons are considered as the ideal carrier of quantum information. The single-photon optical diode with low losses is an indispensable element for future quantum networks. Recently, the single-photon diode has been successfully achieved, such as in [24–29]. These diodes work well at a given frequency. If the input frequency is greatly altered, the devices should be programmed and actively reconfigured. Therefore, the largely tunable single-photon diode still needs to be explored.

For this purpose, we propose a scheme to realize a largely tunable single-photon diode. Our nonreciprocal system is realized by chiral quantum optics. In chiral quantum optics, the light propagating towards opposite directions could be coupled to the emitter with different strengths. The chiral coupling is underpinned by the spin-momentum locking of the transversely confined light and the polarization dependent dipole transitions of the emitter. In our scheme, the photon is largely confined in a one-dimensional(1D) waveguide, which is chirally coupled to an emitter. The photon scattering in 1D waveguide symmetrically coupled to emitters has been extensively studied. In the quantum network, the waveguide can act as a channel, and the emitter as a node. In the chirally coupling case, the single-photon shows essentially different transport properties compared to the symmetrical coupling cases. We theoretically study the single-photon scattering in the 1D waveguide chirally coupled to a two-level emitter and a Λ-type three-level emitter, respectively. When the decays from the emitter’s excitation to the other channels except the waveguide are neglected, the nonreciprocity in single-photon propagation can not be achieved. The single-photon reflection probabilities can not reach unity due to the chiral coupling. When the decays are considered, the transmission probabilities of the single photons transporting towards the opposite directions are not equal. Under certain conditions, the transmission probability for one of the directions is zero due to the critical coupling, while the transmission probability for the other direction reaches a near unity value. In the ideal chiral coupling case, the emitter is decoupled to the single photon transporting in one of the directions. The single photon transporting in one direction completely decays to the other channels except waveguide, while the single photon transporting in the other direction will be completely transmitted due to the decoupling. For the scheme composed by a 1D waveguide chirally coupled to a Λ-type three-level emitter, an external laser is employed to drive the emitter. It is significant that the single-photon diode works well at different frequencies by programming and actively adjusting the laser parameters.

Our scheme also shows certain advantages of the single-photon switch. The control of single-photon transport in 1D waveguide has been extensively investigated, such as. It is known that when the emitter’s decay is neglected, the single-photon transmission prob-

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ability can be zero or unity. However, when the emitter’s decay is considered, this perfect outcome cannot be realized. Having considered the emitter’s decay, the transmission probability of the single photon, which transports in the 1D waveguide chirally coupled to a A-type three-level emitter, can be zero by critical coupling or be unity by EIT (Electromagnetically Induced Transparency).

II. MODEL AND SINGLE-PHOTON SCATTERING

![Diagram of a tunable single-photon diode with A-type three-level emitter driven by an external laser and chirally coupled to a 1D waveguide.](image)

We consider a A-type three-level emitter chirally coupled to a 1D waveguide. The emitter’s states are denoted by |b⟩, |a⟩ and |c⟩, with the corresponding level frequencies ωₘ (m = a, b, c), respectively. The ground-state energy is for reference so that ω₀ is taken to be zero. The right-moving and left-moving photons in the waveguide are coupled to the level transition |b⟩ ←→ |a⟩ with strengths gₐ and gₐ, respectively. The coupling strengths gₐ and gₐ are not equal, distinguishing from the symmetrical coupling case. For simplicity and without generality, we assume Γₐ ≥ Γₐ in this paper. In the ideal chirally coupling case, the photon is perfectly circularly polarized at the position of the emitter and hence the polarization is orthogonal for opposite propagation. Consequently, the level transition |b⟩ ←→ |a⟩ can be solely coupled to one propagation direction photon. To measure the chiral coupling character, we bring in the parameter C = Γₐ + Γₐ. Obviously, C = 0 when Γₐ = Γₐ, 0 < C ≤ 1 when Γₐ ≠ Γₐ, and C = 1 in the ideal chiral coupling case.

The Hamiltonian of the waveguide coupled to the level transition is

\[
H = -i \int dx a_R^†(x)a_R(x) + i \int dx a_L^†(x)a_L(x) + (\omega_a - i\frac{\gamma_a}{2}) \sigma^{aa} + (\omega_c + \omega_L) \sigma^{cc} + \Omega \sigma^{ac} + h.c.,
\]

(1)

with σ^{mn} = |m⟩⟨n| (m, n = a, b, c) being the raising, lowering and energy level population operators of the emitter. The operators a_R^†(x) and a_L^†(x) create a right-moving and left-moving photons in the waveguide at the site x, respectively. The parameter γ_a accounts for the loss from the emitter’s excitation to the other channels except the waveguide, such as the spontaneous emission to the free space. We have taken h = 1, and the photonic group velocity v_g = 1. The first line of the Hamiltonian (1) denotes the free part of the waveguide photon. The second line is the emitter’s energy including the intrinsic dissipation, which is represented by adding the imaginary part −i\frac{\gamma_a}{2} to the corresponding level energy in the quantum jump picture. Here we assume that the states |b⟩ and |c⟩ are long-live states, and |a⟩ is the excited state. The third line represents the coupling of the waveguide photon to the emitter. In this paper, initially, the emitter is in the state |b⟩ and a single photon is injected into the waveguide from left or right side. The frequency of photon transporting in the waveguide is far away from the cutoff frequency of the waveguide so that the photonic dispersion relation is approximately linearized.

The system scattering eigenstate has the form

\[
|\Psi⟩ = \int dx [α_R(x)a_R^†(x) + α_L(x)a_L^†(x)]
+ β_aσ^{ab} + β_cσ^{ac}] |φ⟩,
\]

with α_R(x), α_L(x), β_a and β_c being probability amplitudes. The state |φ⟩ denotes that the emitter is in its ground state |b⟩ and the number of the photon transporting in the waveguide is zero. The probability amplitudes can be obtained from the eigenfunction H |Ψ⟩ = E |Ψ⟩, with eigenvalue E = v_g |k|.

When the input photon is injected from the left side of the waveguide, the spatial dependence of the amplitudes α_R(x) and α_L(x) are taken as α_R(x) = |θ(−x) + t_Rθ(x)|e^{ikx} and α_L(x) = r_Rθ(−x)e^{-ikx}, respectively. The function θ(x) is the Heaviside step function. The parameters t_R and r_R represent the single-photon transmission and reflection probability amplitudes, respectively. The subscript R in t_R and r_R denotes that the input photon is right moving. The expressions of t_R and r_R are

\[
t_R = \frac{\Delta_k (δ_k - i\frac{\gamma_a}{2} + i\frac{Γ_R + Γ_L}{2} + Ω^2)}{\Delta_k (δ_k - i\frac{\gamma_a}{2} - i\frac{Γ_R + Γ_L}{2} + Ω^2)}
+ \frac{Ω^2}{\Delta_k (δ_k - i\frac{\gamma_a}{2} + i\frac{Γ_R + Γ_L}{2} + Ω^2)}
+ \frac{Ω^2}{\Delta_k (δ_k - i\frac{\gamma_a}{2} - i\frac{Γ_R + Γ_L}{2} + Ω^2)}
\]

(3)

where \(\Delta_k = \sqrt{R^2 + L^2 - 2RL}\).
where $\delta_k = \omega_{ab} - v_g |k|$ and $\Delta_k = \Delta - \delta_k$ are detunings, with $\Delta = \omega_{ac} - \omega_L$. The parameters $\Gamma_R = \frac{\delta_k}{v_g}$ and $\Gamma_L = \frac{\delta_k}{v_g}$ account for the spontaneous emissions from the emitter's excitation into the waveguide right-moving and left-moving channels, respectively.

Similarly, when the input photon is injected from the right side, the single-photon transmission amplitude $t_L$ and reflection amplitude $r_L$ are

$$t_L = \frac{\Delta_k (\delta_k - i \frac{\gamma_a}{2} + i \frac{\Gamma_R - \Gamma_L}{2}) + \Omega^2}{\Delta_k (\delta_k - i \frac{\gamma_a}{2} - i \frac{\Gamma_R + \Gamma_L}{2}) + \Omega^2},$$

$$r_L = r_R. \quad (4)$$

If the decay rates from the emitter's excitation to the other channels are neglected, i.e., $\gamma_a = 0$, the single-photon transmission probabilities $T_R = |t_R|^2$ and $T_L = |t_L|^2$ are equal. The single-photon diode can not be achieved although the emitter are chirally coupled to the 1D waveguide. However, when the decay rate $\gamma_a$ is not negligible, it is interesting that $T_R$ and $T_L$ are different from each other due to the chiral coupling. The reflection probabilities $R_R = |r_R|^2$ and $R_L = |r_L|^2$ are equal in any case. For simplicity, we label $R = R_R = R_L$. When $\Gamma_R = \Gamma_L$, our results agree with the outcomes derived in the symmetrical coupling case [63].

### III. SINGLE-PHOTON DIODE

If the external laser is shut off, the emitter's level $|c\rangle$ never participates in the dynamic process. Consequently, our scheme is a 1D waveguide coupled to a two-level emitter. In this case, the single photon transport properties are

$$t_R = \frac{\delta_k - i \frac{\gamma_a}{2} + i \frac{\Gamma_R - \Gamma_L}{2}}{\delta_k - i \frac{\gamma_a}{2} - i \frac{\Gamma_R + \Gamma_L}{2}},$$

$$t_L = \frac{\delta_k - i \frac{\gamma_a}{2} + i \frac{-\Gamma_R - \Gamma_L}{2}}{\delta_k - i \frac{\gamma_a}{2} - i \frac{-\Gamma_R + \Gamma_L}{2}},$$

$$r_R = r_L = \frac{i \sqrt{\Gamma_R \Gamma_L}}{\delta_k - i \frac{\gamma_a}{2} - i \frac{\Gamma_R + \Gamma_L}{2}}. \quad (5)$$

In the symmetrical coupling case, i.e., $\Gamma_R = \Gamma_L$, the single-photon transport in a waveguide coupled to a two-level emitter has been extensively studied. It is known that when the emitter's decay to other modes except waveguide mode is neglected, the single photon moving towards the emitter will be fully reflected by interference in the resonance case [59]. In the chiral coupling case, the input single photon can not be fully reflected in any case. From the expression (5), the maximum value of $R$ is obtained as $1 - C^2$ in the resonance case. In Fig. 2(a), we plot the single photon reflection probabilities against the detuning $\delta_k$ for a two-level emitter coupled to a 1D waveguide. The spectra are shaped like the Lorentzian line. As the value of $C$ decreases, the maximum value of $R$ increases. This can also be understood from the fact that the input right-moving (left-moving) photon is converted into left-moving (right-moving) photon by the emitter-waveguide interaction. The reflection probability $R$ is essentially the conversion efficiency. From the investigations in [54, 70], the conversion efficiency, which relates to the difference between the coupling strengths $g_R$ and $g_L$, can reach unity only in the symmetrical coupling case.

For the two-level emitter, the critical coupling condition can not be satisfied for any nonzero value of $\gamma_a$ in the symmetrical coupling case. In the chiral coupling case, when $\delta_k = 0$ and $\gamma_a = \Gamma_R - \Gamma_L$, we obtain $T_R = 0$, $T_L = (\frac{\Gamma_R - \Gamma_L}{2})^2$, and $R_R = R_L = \frac{\Gamma_R}{\Gamma_R + \Gamma_L}$. The resonant single photon injected into the left port of the waveguide can not be received from the right port due to the critical coupling. However, the single photon injected into the right port will be received from the left port with a near unity probability when $C \rightarrow 1$. In the ideal chiral coupling case, the photon injected from the left side will completely decay out of the waveguide, i.e., $R = 0$. The photon injected from the right hand will be completely transmitted because it is decoupled to the emitter. The difference between the transmission probabilities corresponding to opposite transport directions is $\Delta T = |T_R - T_L| = \frac{\gamma_a (\Gamma_R - \Gamma_L)}{\delta_k + (\frac{\gamma_a}{2} - \frac{\Gamma_R + \Gamma_L}{2})^2}$. We can see that $\Delta T$ reaches its maximum value when $\delta_k = 0$ and
The single-photon frequency should satisfy the relation \( \omega_a = \Gamma_R - \Gamma_L \). Fig. 2(b) shows the transmission probabilities \( T_R, T_L \) and \( \Delta T \) against \( \delta_k \) for a two-level emitter coupled to a 1D waveguide when \( \Gamma_L/\Gamma_R = 0.1 \) and \( \gamma_a/\Gamma_R = 0.9 \). When the external laser is turned on, our scheme is a 1D waveguide chirally coupled to a Λ-type three-level emitter. In this case, the single-photon transmission and reflection probabilities have been obtained in Eqs. (3) and (4). When \( \Delta k = 0 \) the single photon transporting towards either directions will be fully transmitted no matter what the values of \( C \) and \( \gamma_a \) are due to the interference, which corresponds to EIT. We plot the single-photon reflection probabilities against the detuning \( \delta_k \) when \( \gamma_a = 0 \) in Fig. 2(c). The spectra split due to the interaction between the emitter and the laser. Similar to the two-level emitter, the symmetrical coupling reduces the maximum value of \( R \). When \( \gamma_a = \Gamma_R - \Gamma_L \) and \( \delta_k = \Delta k \sqrt{\frac{\gamma_a}{\Gamma_R - \Gamma_L}} \), we obtain \( T_R = 0 \) and \( T_L = (\Gamma_L/\Gamma_R)^2 \). In this case, \( \Delta T = \frac{\gamma_a (\Gamma_R - \Gamma_L)}{\gamma_a (\Gamma_R - \Gamma_L) + \Gamma_R (\Gamma_R + \Gamma_L)} \) reaches its maximum value. In Fig. 2(d) we plot the single-photon transmission probabilities against \( \delta_k \) for a Λ-type three-level emitter. The outcomes provide a manner to realize the single-photon switch. By adjusting the laser frequency, we can ensure the single photon is fully transmitted by EIT. Similarly, by adjusting the frequency and Rabi frequency of the laser, we can ensure the single photon can not be transmitted by critical coupling. It is interesting that the emitter’s decay is considered for these operations. The maximum values of \( \Delta T \) and \( T_L \) are equal to the corresponding values of the two-level emitter. However, the Λ-type three-level emitter provides a control to various input frequencies.

To ensure the diode works well, the frequency of the single photon can not be arbitrary. For a two-level emitter, the single-photon should be nearly resonant to the emitter, i.e., \( \delta_k \simeq 0 \). For a Λ-type three-level emitter, the single-photon frequency should satisfy the relation \( \delta_k = \frac{\Delta k \sqrt{\frac{\gamma_a}{\Gamma_R - \Gamma_L}}}{\Gamma_R (\Gamma_R + \Gamma_L)} \). The latter shows the advantage that it is largely tunable. For various values of the single-photon frequencies, we can adjust the frequency and Rabi frequency of the laser to satisfy this condition. Although this condition can not be satisfied for any arbitrary value of the single-photon frequency, it can be satisfied in a large range of values. This feasible range is enough to obtain a single-photon diode which is feasible for various single-photon frequencies. We plot the probabilities \( T_R \) and \( \Delta T \) against the laser parameters in Fig. 3. It shows that the single-photon diode works well at various values of the laser frequencies and Rabi frequencies.

The single-photon transport scattering by a Λ-type three-level emitter can be understood by the dressed-state representation. Our scheme is considered as the waveguide chirally coupled to a two-level emitter with states \(|\pm\rangle, |\rangle, |-\rangle\). The transitions between the ground state and dressed states are driven by the photons in the waveguide. The states \(|\alpha\rangle \) and \(|\alpha\rangle \) are the eigenstates of the Hamiltonian \( H_0 = \omega_a \sigma_{aa} + \omega_c \sigma_{cc} + \Omega (\sigma_{ac} + \sigma_{ca}) \), with frequencies \( \omega_a - \Delta \pm \frac{\Delta k \sqrt{\frac{\gamma_a}{\Gamma_R - \Gamma_L}}}{\Gamma_R (\Gamma_R + \Gamma_L)} \). The condition \( \delta_k = \frac{\Delta k \sqrt{\frac{\gamma_a}{\Gamma_R - \Gamma_L}}}{\Gamma_R (\Gamma_R + \Gamma_L)} \) implies that the single photon resonantly drives one of the transitions \(|\pm\rangle \leftrightarrow |\rangle\).

For two- or three-level emitters coupled to a waveguide, the decay match condition \( \gamma_a = |\Gamma_R - \Gamma_L| \) plays an important role. The rate \( \gamma_a \) mainly relates to the environment surrounding the emitter. The decay rates \( \Gamma_R \) and \( \Gamma_L \) relate to the position of the emitter relative to the waveguide. Here, the state \(|\sigma\rangle \) is considered long-lived. When \( |\sigma\rangle \) is an excited state, the dissipation can be incorporated by introducing an extra nonhermitian term \(-i \frac{\gamma_a}{2} \sigma_{cc} \) into the Hamiltonian. In addition, the term \( \Delta k \) in the results \( 3 \) and \( 4 \) should be replaced by \( \Delta k + i \frac{\omega_c}{\omega_a} \). The tunable single-photon diode can also be achieved in this case. We will not cover it again.

**IV. CONCLUSIONS**

We propose a scheme to investigate the tunable single-photon diode. This diode is composed by an emitter chirally coupled to a 1D waveguide. We study the single-photon scattering by a two-level and Λ-type three-level emitter. The single-photon diode is underpinned by the Λ-type emitter. The single-photon diode can work well at various single-photon frequencies by adjusting the external laser parameters. By tuning the laser parameters, the single-photon transmission probability can be tuned to zero or unity. Different from the few-photon diode, the single-photon diode property is not affected by the nonlinear effect in the Λ-type three-level emitter. The Λ-type emitter in the tunable single-photon diode can also be replaced by a three-level emitter in cascade configu-
ration.

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