THREE-DIMENSIONAL HYDRODYNAMIC CORE-COLLAPSE SUPERNOVA SIMULATIONS FOR AN 11.2 $M_\odot$ STAR WITH SPECTRAL NEUTRINO TRANSPORT

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ABSTRACT

We present numerical results on three-dimensional (3D) hydrodynamic core-collapse simulations of an 11.2 $M_\odot$ star. By comparing one-dimensional (1D) and two-dimensional (2D) results with those of 3D, we study how the increasing spacial multi-dimensionality affects the postbounce supernova dynamics. The calculations were performed with an energy-dependent treatment of the neutrino transport that is solved by the isotropic diffusion source approximation scheme. In agreement with previous study, our 1D model does not produce explosions for the 11.2 $M_\odot$ star, while the neutrino-driven revival of the stalled bounce shock is obtained in both the 2D and 3D models. The standing accretion-shock instability (SASI) is observed in the 3D models, in which the dominant mode of the SASI is bipolar ($\ell = 2$) with its saturation amplitudes being slightly smaller than 2D. By performing a tracer-particle analysis, we show that the maximum residency time of material in the gain region becomes longer in 3D than in 2D due to non-axisymmetric flow motions, which is one of advantageous aspects of 3D models to obtain neutrino-driven explosions. Our results show that convecitive matter motions below the gain radius become much more violent in 3D than in 2D, making the neutrino luminosity larger for 3D. Nevertheless, the emitted neutrino energies are made smaller due to the enhanced cooling. Our results indicate whether these advantages for driving 3D explosions could or could not overwhelm the disadvantages is sensitive to the employed numerical resolutions. An encouraging finding is that the shock expansion tends to become more energetic for models with finer resolutions. To draw a robust conclusion, 3D simulations with much higher numerical resolutions and with more advanced treatment of neutrino transport and of gravity are needed, which could be practicable by utilizing forthcoming Petaflops-class supercomputers.

Key words: hydrodynamics – neutrinos – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Core-collapse supernovae have long drawn the attention of astrophysicists because they have many aspects that play important roles in astrophysics. They are the mothers of neutron stars and black holes, play an important role for the acceleration of cosmic rays, influence galactic dynamics that trigger further star formation, and are gigantic emitters of neutrinos and gravitational waves. They are also major sites for nucleosynthesis, so, naturally, any attempt to address human origins may need to begin with an understanding of core-collapse supernovae.

Ever since the first numerical simulation (Colgate & White 1966), the neutrino-heating mechanism, in which a stalled bounce shock is revived by a neutrino energy deposition to trigger explosions (Wilson 1985; Bethe & Wilson 1985; Bethe 1990), has been the working hypothesis of supernova theorists for ~45 years. However, one important lesson we have learned from Ramp & Janka (2000), Liebendörfer et al. (2001), Thompson et al. (2003), and Sumiyoshi et al. (2005), who implemented the best input physics and numerics to date, is that the mechanism fails to blow up canonical massive stars in spherical symmetric (one-dimensional, 1D) simulations. Pushed by mounting supernova observations of the blast morphology (e.g., Wang et al. 2001; Maeda et al. 2008; Tanaka et al. 2009 and references therein), it is now almost certain that breaking the spherical symmetry is the key to solving the supernova problem. So far a number of multi-dimensional (multi-D) hydrodynamic simulations have shown that hydrodynamic motions associated with convective overturn (e.g., Herant et al. 1994; Burrows et al. 1995; Janka & Müller 1996; Fryer et al. 2002; Fryer 2004) and the standing accretion-shock instability (SASI; e.g., Blondin et al. 2003; Scheck et al. 2004, 2006; Ohnishi et al. 2006, 2007; Foglizzo et al. 2006; Iwakami et al. 2008, 2009; Murphy & Burrows 2008; Fernández & Thompson 2009a, 2009b, and references therein) can help the onset of the neutrino-driven explosion.

In fact, the neutrino-driven explosions have been obtained in the following state-of-the-art two-dimensional (2D) simulations (e.g., Table 1 in Kotake (2011)). Using the MuDBaTH code, which includes one of the best available neutrino transfer approximations, Buras et al. (2006) first reported explosions for non-rotating low-mass (11.2 $M_\odot$) progenitor of Woosley et al. (2002), and then for a 15 $M_\odot$ progenitor of Woosley & Weaver (1995) with a moderately rapid rotation imposed (Marek & Janka 2009). By implementing a multi-group flux-limited diffusion algorithm to the CHIMERA code (e.g., Brenn et al. 2010), Yakunin et al. (2010) obtained explosions for non-rotating progenitors of Woosley et al. (2002) in the mass range of 12 $M_\odot$ and 25 $M_\odot$. More recently, Suwa et al. (2010) pointed out that a stronger explosion is obtained for a rapidly rotating 13 $M_\odot$ progenitor of Nomoto & Hashimoto (1988) compared to the corresponding non-rotating model, in which the isotropic diffusion source approximation (IDSA) for the spectral neutrino transport (Liebendörfer et al. 2009) is implemented in the ZEUS code.

This success, however, has lead to new questions. First of all, the explosion energies obtained in these 2D simulations are typically underpowered by one or two orders of magnitude...
to explain the canonical supernova kinetic energy ($\sim 10^{51}$ erg). Moreover, the softer nuclear equation of state (EOS), such as the Lattimer & Swesty (1991) (LS) EOS with an incompressibility at nuclear densities, $K$, of 180 MeV, is employed in those simulations. In addition to striking evidence that favors a stiffer EOS based on the nuclear experimental data ($K = 240 \pm 20$ MeV; Shlomo et al. 2006), the soft EOS may not account for the recently observed massive neutron star of $\sim 2 M_\odot$ (Demorest et al. 2010).\footnote{The maximum mass for the LS180 EOS is about 1.8 $M_\odot$ (e.g., O’Connor & Ott 2011; Knici & Kotake 2008).} Using a stiffer EOS, the explosion energy may be even lower as inferred by Marek & Janka (2009) who did not obtain the neutrino-driven explosion for their model with $K = 263$ MeV.\footnote{On the other hand, they obtained 2D explosions for Shen EOS ($K = 281$ MeV, H.-T. Janka, private communication).} What is then missing? The neutrino-driven mechanism would be assisted by other candidate mechanisms such as the acoustic mechanism (e.g., Burrows et al. 2006) or the magnetohydrodynamic mechanism (e.g., Kotake et al. 2004; Takiwaki et al. 2004, 2009; Burrows et al. 2007a; Guilet et al. 2011; Obergaulinger & Janka 2011; Takiwaki & Kotake 2011; see also Kotake et al. 2006 for collective references therein). We may find the answer by taking into account new ingredients, such as exotic physics in the core of the protoneutron star (PNS; e.g., Takahara & Sato 1988; Sagert et al. 2009), viscous heating by the magnetorotational instability (Thompson et al. 2005; Masada et al. 2012), or energy dissipation via Alfvén waves (Suzuki et al. 2008).

But before seeking alternative scenarios, it may be of primary importance to investigate how the explosion criteria extensively studied so far in 2D simulations could or could not be changed in three-dimensional (3D) simulations. Nordhaus et al. (2010) are the first to argue that the critical neutrino luminosity for producing neutrino-driven explosions becomes smaller in 3D than in 2D. They employed the CASTRO code with an adaptive mesh refinement technique, by which unprecedentedly high-resolution 3D calculations were made possible. Since it is generally computationally expensive to solve the neutrino transport in 3D, they employed a light-bulb scheme (e.g., Janka & Müller 1996) to trigger explosions, in which the heating and cooling by neutrinos are treated by a parametric manner. Since the light-bulb scheme can capture fundamental properties of neutrino-driven explosions (albeit on the qualitative grounds), it is one of the most prevailing approximations adopted in recent 3D models (e.g., Iwakami et al. 2008, 2009; Wongwathanarat et al. 2010). A number of important findings have been reported recently in these simulations, including a potential role of non-axisymmetric SASI flows in generating spins (Wongwathanarat et al. 2010; Rantsiou et al. 2011; see also Blondin & Mezzacappa 2007; Fernández 2010) and magnetic fields (Endeve et al. 2010) of pulsars, the stochastic nature of gravitational-wave (e.g., Kotake et al. 2009b, 2011; Müller et al. 2012), and neutrino emission (e.g., Duan & Kneller 2009).

To go beyond the light-bulb scheme, in this study we explore possible 3D effects in the supernova mechanism by performing 3D, multigroup, and radiation-hydrodynamic core-collapse simulations. For the multigroup transport, the IDSA scheme is implemented, which can be done in a rather straightforward manner by extending our 2D modules (Suwa et al. 2010; Suwa et al. 2011) to 3D. This can be made possible because we apply the so-called ray-by-ray approach (e.g., Buras et al. 2006) in which the neutrino transport is solved along a given radial direction assuming that the hydrodynamic medium for the direction is spherically symmetric. From a technical point of view, it is worth mentioning that the ray-by-ray treatment is highly efficient in parallelization\footnote{Along each radial ray.} on present supercomputers, most of which employ the message-passing-interface (MPI) routines. We focus here on the evolution of an 11.2 $M_\odot$ star of Woosley et al. (2002). We first choose such a lighter progenitor star, not only because we follow a pattern in the 2D literature (e.g., Buras et al. 2006; Burrows et al. 2006), but also because the neutrino-driven shock revival for the progenitor was reported to occur rather earlier after the bounce in the 2D models by Buras et al. (2006). We anticipate that the cost of 3D simulations would not be too expensive for the progenitor. By comparing with our 1D and 2D results, we study how the increasing multi-dimensionality could affect the postbounce supernova dynamics.

The paper begins with descriptions of the initial models and the numerical methods (Section 2). The main results are presented in Section 3. We summarize our results and discuss their implications in Section 4.
developed in the Lorene code (Grandclément & Novak 2009). For the calculations presented here, the monopole approximation is employed. The computational grid is comprised of 300 logarithmically spaced, radial zones that cover from the center up to 5000 km and 64 polar (θ) and 32 azimuthal (φ) uniform mesh points for our 3D model, which are used to cover the whole solid angle. To vary numerical resolutions, we run one more 3D model that has one-half of the mesh numbers in the φ direction (nφ = 16), while fixing the mesh numbers in other directions. In both 2D and 3D models we take the same mesh numbers in the polar direction (nθ = 64) so that we can see how the dynamics change due to the additional degree of freedom in the φ direction. For the spectral transport, we use 20 logarithmically spaced energy bins reaching from 3 to 300 MeV. For our non-rotating progenitor, the spectral transport is employed.

In the following (Section 3.1), we first outline hydrodynamic features in our 3D model. Then in Sections 3.2 and 3.3, we discuss how 3D effects impact the explosion dynamics by comparing with the 1D and 2D results.

3.1. 3D Dynamics from Core-collapse through Postbounce Turbulence Until Explosion

Figure 1 shows three snapshots, which are helpful to characterize hydrodynamic features in the 3D model. The top panel is for t = 15 ms after the bounce, showing that the bounce shock stalls (indicated by inward arrows in the top right panel) at a radius of 150 km. Note that the colors of the velocity arrows change from yellow to red as the absolute values become larger. By looking carefully at the top right panel, matter flows in supersonically (indicated by red arrows) in the standing shock (the central transparent sphere), and then advects subsonically (indicated by yellow arrows) to the PNS (or the unshocked core, the central blue region in the top left panel). As seen, the entropy (left panel) and density (right panel) configurations are essentially spherical at this epoch.8

The middle panels show an epoch (t = 65 ms) when the neutrino-driven convection is already active. From the right panel, turbulent motions can be seen (arrows in random directions) inside the standing shock, which is indicated by the boundary between red and yellow arrows. The entropy behind the standing shock becomes high by the neutrino heating (red regions in the left panel). The size of the neutrino-heated hot bubble becomes larger in a non-axisymmetric way later on, which is indicated by smaller structures encompassed by the stalled shock (i.e., inside the central green sphere in the left panel).

The bottom panels (t = 125 ms) show the epoch when the revived shock is expanding aspherically, which is indicated by the outgoing yellow arrows in the right panel. The asphericity of the expanding shocks could be more clearly visible by the sidewall panels. From the entropy distribution (left panel), the expanding shock is shown to touch a radius of ∼500 km (the projected back bottom panel). Inside the expanding shock (enclosed by the green membrane in the left panel), the bumpy structures of the hot bubbles are shown. In contrast to these smaller asphericities, the deformation of the shock surface is mild, which is a consequence of the SASI as will be discussed in Section 3.2.

Figure 2 shows the net neutrino heating rate (left panels) and the ratio of the residency timescale to the neutrino-heating timescale (right panels) for the two snapshots in Figure 1 (at t = 65 ms (top panels) and t = 125 ms (bottom panels)). Here, the residency timescale and the neutrino heating timescale9 are locally defined as

\[
t_{\text{res}}(r, \theta, \phi) = \begin{cases} \frac{r - r_{\text{gain}}(\theta, \phi)}{v_r} & \text{for } v_r < 0, \\ \frac{r_{\text{shock}}(\theta, \phi) - r}{v_r} & \text{for } v_r > 0, \end{cases}
\]

\[
t_{\text{heat}}(r, \theta, \phi) = \frac{[1 + v_r]}{Q_{\nu, \text{total}}},
\]

where \(r_{\text{gain}}\) is the gain radius that depends on θ and φ, \(r_{\text{shock}}\) is the shock radius, and \(v_r\) is the radial velocity. We take the above criteria in order to estimate the residency timescale for material with positive radial velocities (\(v_r > 0\)) behind the shock.10

The heating timescale can be rather straightforwardly defined by dividing the local binding energy \(e_{\text{bind}} = (1/2)v^2 + e - \rho\Phi\) [erg cm\(^{-3}\)] in the gain layer by the net neutrino-heating rate \(Q_{\nu, \text{total}}\) [erg cm\(^{-3}\) s\(^{-1}\)].

At t = 65 ms after bounce (top panels), the gain region is clearly formed (red region in the left panel), where the ratio of the two timescales exceeds unity (yellow region in the right panel). At t = 125 ms after bounce (bottom panels), the ratio reaches about 2 (red region in the bottom right panel) behind the shock (compare with the bottom right panel in Figure 1), which presents evidence that the shock revival is driven by the neutrino-heating mechanism. Recently, Pejcha & Thompson (2012) proposed an alternative definition of the onset time of the explosion, which is the so-called antersonic condition. From Figure 3, it can be seen that the criteria that the ratio of sound speed and escape velocity squared ∼0.2 is also satisfied in our 3D model when the neutrino-driven explosion sets in.

Figure 4 shows distributions of pressure perturbation (top) and vorticity (bottom) at t = 60 ms after the bounce. Here the pressure perturbation is estimated by \(\Delta p/p\), with \(\Delta p\) representing the deviation from the average pressure \((\langle p \rangle)\) at a given position. Here we define the angle average of variable A as

\[
\langle A \rangle = \frac{\int d\Omega A}{4\pi}.
\]

The positive and negative deviations are colored by red and blue, respectively (e.g., \(\langle \Delta p/p \rangle\) (red) or \(-\Delta p/p\) (blue)). The left and right panels are for an equatorial (\(\theta = \pi/2\) and \(\phi = 0\)) and a polar observer (\(\theta = 0\)), respectively. In each plot, the circle that screens between the colored region and the white

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8 It is approximately 5 ms after we remap the 1D data to the 3D grids.

9 Originally, the ratio of the advection to the neutrino-heating timescale was known as a useful quantity to diagnose the success \((\tau_{\text{adv}}/\tau_{\text{heat}} \sim 1)\). i.e., the neutrino-heating timescale is shorter than the advection timescale of material in the gain region or failure \((\tau_{\text{adv}}/\tau_{\text{heat}} \lesssim 1)\) of the neutrino-driven explosion (e.g., Burrows & Goshy 1993; Janka 2001; Thompson et al. 2005).

10 We take the word of “residency” timescale from Murphy & Burrows (2008), which we feel is more appropriate than the canonical “advective” timescale especially when we need to estimate the timescale regarding the postshock material with positive radial velocities.
Figure 1. Three-dimensional plots of entropy per baryon (left panels) and logarithmic density (right panels in unit of g cm$^{-3}$) for three snapshots (top: $t = 15$ ms, middle: $t = 65$ ms, and bottom: $t = 125$ ms measured after bounce ($t = 0$)) of our 3D model. In the right panels, velocities are indicated by arrows. The contours on the cross sections in the $x = 0$ (back right), $y = 0$ (back bottom), and $z = 0$ (back left) planes are, respectively, projected on the sidewalls of the graphs. For each snapshot, the linear scale is indicated along the axis in unit of km.
The Astrophysical Journal, 749:98 (17pp), 2012 April 20
Takiwaki, Kotake, & Suwa

Figure 2. Same as Figure 1 but for the net neutrino heating rate (left panels, logarithmic in unit of erg cm$^{-3}$ s$^{-1}$) and $\tau_{\text{adv}}/\tau_{\text{heat}}$ (right panels, see the text for details), which is the ratio of the advection to the neutrino heating timescale. The gain region (colored by red in the top left panel) is shown to be formed at around $t = 65$ ms after bounce, which coincides with the epoch approximately when the neutrino-driven shock revival initiates in our 3D model. The condition of $\tau_{\text{adv}}/\tau_{\text{heat}} \gtrsim 1$ is satisfied behind the aspherical shock, the low-mode deformation of which is characterized by the SASI (bottom right panel).

Figure 5 shows the spacetime diagrams of entropy dispersion ($\sigma_s$) for the 3D model. Note that the dispersion of quantity $A$...
with respect to angular variation is defined by

$$\sigma_A = \sqrt{\int d\Omega (A - \langle A \rangle)^2/(4\pi)},$$

where $\langle A \rangle$ represents the angle average (Equation (8)). It is rather uncertain where the entropy production actually takes place in the supernova core in the context of the advective-acoustic cycle (e.g., Sato et al. 2009). The primary position is the surface of the PNS, where the advecting material receives faster deceleration by the walls of the PNS due to the localized gravitational potential (e.g., Blondin et al. 2003). In addition, the infalling material could also receive faster deceleration just outside the gain radius, where the neutrino heating becomes maximum. Our 3D results show that both of the candidates are relevant indeed. As seen in Figure 5, the position where the entropy production takes place roughly coincides with the gain radius (the dotted gray line) before 125 ms after the bounce (indicated by the upward arrow). Later on, the position is shown to transit to the surface of the PNS surface (the dotted black line).

Until now, we have focused on the postbounce dynamics only for our 3D model. In the next sections, we discuss in more detail how they differ from the 1D and 2D results.

### 3.2. Blast Morphology and Explosion Dynamics

Figure 6 shows the blast morphology for our 3D (left panel), 2D (middle), and 1D (right) models, respectively. In the 2D model (middle panel), the morphology is symmetric around the coordinate symmetry axis. In contrast, non-axisymmetric structures are clearly shown in the 3D model (left panel). The direction of explosion is rather closely aligned with the polar axis in the 3D model. Owing to the use of the spherical coordinates, we cannot omit the possibility that the polar axis still gives a special direction in our 3D simulations. However, we suspect that the alignment might be just an accident, because the axis-free 3D explosions were obtained in a number of parametric 3D explosion models by using the same hydro-code (e.g., Iwakami et al. 2008, 2009; Kotake et al. 2009b, 2011). To clearly witness the stochastic natures concerning the explosion direction, we may need to investigate a number of 3D models by changing the initial perturbations and numerical resolutions systematically, which we think are an important extension of this study (T. Takiwaki et al., in preparation).

The left panel of Figure 7 shows mass-shell trajectories for the 3D (red lines) and 1D model (green line), respectively. At around 300 ms after bounce, the average shock radius for the 3D model exceeds 1000 km in radius. On the other hand, an explosion is not obtained for the 1D model, which is in agreement with Buras et al. (2006). The right panel of Figure 7 shows a comparison of the average shock radius with the postbounce time. In the 2D model, the shock expands rather continuously after bounce. This trend is qualitatively consistent with the 2D result by Buras et al. (2006; see their Figure 15 for model s112 128 f); however, the average shock of our 2D model expands much faster than theirs. We suspect that all of the neglected effects in this work, including general relativistic effects, inelastic neutrino-electron scattering, and cooling by heavy-lepton neutrinos, could give a more optimistic condition to produce explosions. Apparently these ingredients should be appropriately implemented, which we hope to be feasible in the next-generation 3D simulations.

Comparing the shock evolution between our 2D (green line in the right panel of Figure 7) and 3D model (red line), the shock is shown to expand much faster in 2D. The pink line labeled as “3D low” is for the low-resolution 3D model, in which the mesh numbers are taken to be half of the standard model (see Section 2). Compared with our standard 3D model (red line), the shock expansion becomes less energetic for the low-resolution model (later than $\sim$150 ms). The above results indicate that explosions are easiest to obtain in 2D, followed in order by 3D, and 3D (low). At first sight, this may seem contradictory to the finding by Nordhaus et al. (2010) that explosions could be more easily obtained in 3D than 2D. In the following section, we discuss the reason this discrepancy more in detail.

### 3.3. Comparison between 2D and 3D

In this section, we illuminate the key differences between our 2D and 3D models. We highlight the SASI (Section 3.3.1) and convective activities (Section 3.3.2), the residency (Section 3.3.3) and neutrino-heating timescales (Section 3.3.4), respectively.

#### 3.3.1. SASI Activities in 2D and 3D

To compare the SASI activities in 2D and 3D, we first perform the mode analysis of the shock wave. The deformation of the shock surface can be expanded as a linear combination of the spherical harmonics components $Y_{lm}(\theta, \phi)$:

$$R_S(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi),$$

where $Y_{lm}$ is expressed by the associated Legendre polynomial $P_{lm}$ and a constant $K_{lm}$ given as

$$K_{lm} = \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}}.$$
The Astrophysical Journal, 749:98 (17pp), 2012 April 20

Takiwaki, Kotake, & Suwa

Figure 4. Top panels show distributions of the pressure perturbation ($\Delta p/\langle p \rangle$, see the text for definition). Left and right panels correspond to a partial cutaway in the $YZ$ and $XY$ planes, respectively. Bottom panels show the vorticity distributions ($\mathbf{v} \times \mathbf{n}$, with $\mathbf{n}$ being $\phi$ or $\theta$ in the left and right panels, respectively). The circle that screens between the regions colored by blue and red and the white region outside corresponds to the surface of the stalled shock. Note that the positive and negative values are colored by red and blue, respectively (e.g., $+|\Delta p/\langle p \rangle|$ (red) or $-|\Delta p/\langle p \rangle|$ (blue) for the pressure perturbation). The linear scale and the time of these snapshots of the 3D model ($t = 60$ ms after bounce) are indicated in the top right and bottom right edge of each plot.

(A color version of this figure is available in the online journal.)

Here the expansion coefficients read,

$$c_{lm} = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta R_S(\theta, \phi) Y_{lm}^*(\theta, \phi),$$

where the superscript * denotes complex conjugation.

Figure 8 shows the time evolution of the expansion coefficients (Equation (10)) for the 3D (left panel) and 2D model (right panel), respectively. As can be seen, the amplitude of each mode grows exponentially until $\sim 100$–$150$ ms after bounce, which corresponds to the linear SASI phase.

The top panels show that the mode of $(\ell, m) = (2,0)$ (green line) is dominant when the SASI enters to the saturation phase, which is common both in our 2D and 3D models. The epoch when the SASI shifts from the linear to nonlinear phase is much delayed for 3D ($t \gtrsim 150$ ms) than 2D ($t \gtrsim 80$ ms), which was also seen in the parametric 3D models by Iwakami et al. (2008, e.g., their Figure 9). These transition timescales are also consistent with Buras et al. (2006) who employed the same progenitor model as ours in their 2D simulations in which detailed neutrino transport was solved (see their Figure 22). It is also worth mentioning that the timescale seems rather...
insensitive to the employed progenitor. In fact, Figure 5 in Marek & Janka (2009) shows the transition timescale to be around 150 ms for a 15 $M_{\odot}$ progenitor model. In the bottom panels of Figure 8, the saturation levels of the even modes of $(\ell, |m|) = (4,0), (4,2), (4,4)$ in 3D are shown to become much larger than those in 2D (pink line), while the odd mode of $(\ell, m) = (3,0)$ is much the same.

Based on the pioneering work by Houck & Chevalier (1992), the linear growth rate of the SASI in the core-collapse case was presented by Scheck et al. (2008). They pointed out that the cycle efficiency ($Q$) which represents how many times the average radius expands compared to the original position per a unit oscillation frequency ($\omega_{\text{osc}}$) of the SASI is an important quantity to characterize the linear growth rate. From Figure 8, $Q$ and $\omega_{\text{osc}}^{-1}$ in our simulation are approximately estimated to be 2 and 25 ms, respectively. Note that these values are in agreement with the ones obtained in 2D simulations by Scheck et al. (2008, e.g., their Figure 17). From the two quantities, the linear growth rate can be straightforwardly estimated as $\exp(\ln(Q) t \omega_{\text{osc}})$, which is shown in the top panels of Figure 8 as black-dotted lines. As can be seen, the growth rates observed both in our 3D (top left panel in Figure 8) and 2D simulations (top right) are close to the linear growth rate, which seems to be a rather generic trend for the low-modes ($\ell = 1, 2$) of the SASI. Note here that the normalized amplitude of the shock (the
value in the vertical axis of Figure 8) shortly after the bounce is actually as small as $10^{-6}$ to $10^{-5}$. Deduced from the linear growth rate above, the amplification due to the SASI in the linear phase is at most ~20 within 100 ms after the bounce. On the other hand, the amplitudes do increase by more than about $10^{-1}/10^{-3} \approx 10^4$ for the epoch. Thus, the SASI is not the only agent for the shock deformation. In fact, the amplitudes are observed to sharply increase from $10^{-3}$ to $\sim 5 \times 10^{-3}$ within 10 ms after the bounce, which is predominantly driven by the Rayleigh–Taylor instability behind the shock. To summarize, our results suggest that the rapid deformation after the bounce is triggered by the Rayleigh–Taylor instability and the subsequent deformation with much milder growth rates is predominantly determined by the SASI.

Concerning the saturation levels of the dominant mode of $(\ell, m) = (2,0)$ between 2D and 3D, it is slightly larger for 2D than 3D (top panels, green line). The dominance of this bipolar mode can also be seen in the blast morphology (Figure 6). The second-order mode of $(\ell, m) = (1,0)$ is shown to be much smaller for 3D than 2D (red lines in the top panels of Figure 8). This is qualitatively consistent with Nordhaus et al. (2010) who did not observe the dominance of the $(\ell, m) = (1,0)$ mode in their 3D models. Note that this agreement might be just by chance. Iwakami et al. (2008) observed the dominance of the $(\ell, m) = (1,0)$ mode in the saturation phase (see their Figure 12). From 3D results reported so far (Iwakami et al. 2008, 2009; Wongwathanarat et al. 2010; Fernández 2010), it seems almost certain that the low modes ($\ell = 1, 2$) are dominant in the 3D SASI-aided neutrino-driven explosions. However, it might be rather uncertain which of the two modes ($\ell = 1$ or 2) becomes dominant. This may reflect the fact that the explosion dynamics in 3D proceeds totally stochastically.

### 3.3.2. Convective Activities in 2D and 3D

To discuss convective activities, we compute the Brunt–Väisälä ($B-V$) frequency which is defined as (e.g., Buras et al. 2006)

$$\omega_{B-V} = \text{sgn}(C_L) \sqrt{\frac{C_L}{\rho} \frac{d\Phi}{dr}}, \quad (11)$$

with $d\Phi/dr$ being the local gravitational acceleration. $C_L$ is the Ledoux-criterion, which is given by

$$C_L = - \left( \frac{\partial \rho}{\partial P} \right)_{\rho,Y_l} \left[ \left( \frac{\partial P}{\partial s} \right)_{\rho,Y_l} \left( \frac{d s}{d r} \right) + \left( \frac{\partial P}{\partial Y_l} \right)_{\rho,Y_l} \left( \frac{d Y_l}{d r} \right) \right], \quad (12)$$

with $Y_l$ being the lepton fraction. It predicts instability in static layers if $C_L > 0$. The $B-V$ frequency denotes the linear growth rate of fluctuations if it is positive (instability). If it is negative (stable), it denotes the negative of the oscillation frequency of stable modes.

The left panel of Figure 9 shows the profile of the $B-V$ frequency for our 3D model at 10 ms after bounce. The negative entropy gradient (bottom left panel) between the gain radius ($\sim 100$ km in radius) and the stalled shock ($\sim 160$ km) drives convective instability. The region in the vicinity of the PNS
Figure 9. Profile of the Brunt–Väisälä (B–V) frequency (left top panel) and entropy (left bottom pane) at 10 ms after the bounce for our 3D model. The right panel shows the B–V frequency contributed only from the lepton gradient. Note that the angle-averaged quantities are plotted here. The horizontal green line in the top panels is shown just for the reference of $\omega_{B-V} = 0$.

Figure 10. Evolution of convective activities in our 3D (left) and 2D (right) models, respectively. The color scale is for anisotropic velocity (see the text for more details), in which higher anisotropy (colored by yellow to red) is related to active convective overturns. The dotted gray line represents the gain radius. The bottom panels are just zoom up of the top panels focusing on the central region.

By this definition, higher anisotropy comes from greater deviation in the radial motions ($v_r - \langle v_r \rangle$) or larger non-radial ($v_\theta$, $v_\phi$) motions.

From the top left panel of Figure 10, the initial formation of convectively unstable regions is shown to be around 15 ms after the bounce (seen as a sudden formation of the non-zero $v_{aniso}$). Subsequently, the convectively unstable regions are shown to advect to the center. At around 20–30 km in radius, the anisotropic velocities are strongly suppressed (seen as a change from yellow to blue region at ~30 ms after bounce) due to the stabilizing positive entropy gradient (see the left panel of Figure 9). As a result, the convective overturns are shown to persistently stay in the regions above the PNS (~20–30 km in radius) and below ~50 km in radius. This is seen as a
Figure 11. Streamlines of selected tracer particles advecting through the shock wave to the PNS for the 3D model. As in Figure 4, the left and right panels are for the equatorial and polar observer, respectively. Each panel shows several surfaces of constant entropy marking the position of the shock wave (green outside) and the PNS (indicated by the central sphere). The linear scale is given in the right bottom edge of each panel.

Having referred to the SASI and convective activities in 2D and 3D, we are now ready to perform an analysis of the residency and neutrino-heating timescales. We discuss the residency timescale in the next section.

### 3.3.3. Residency Timescales in 2D and 3D

Figure 11 depicts the streamlines of tracer particles advecting from the outer boundary of the computational domain through the shock wave down to the PNS. The number of the tracer particles that we actually injected is \( \sim 10^6 \); however, only the trajectories of selected particles are shown in Figure 11. As seen from the left panel, the tracer particles first go down to the shock wave, which is shown by the radial straight lines. Later on, as indicated by the tangled streamlines, they experience turbulence before falling to the PNS. The low-mode (here, \( t = 2 \)) oscillations of the accretion shock due to SASI activities (as discussed in Section 3.3.1, e.g., right panel in Figure 11) make the residency timescales much longer for multi-D models than 1D. If the right panel were for 2D models, the streamlines would be seen as a superposition of circles with different diameters. In contrast, the non-axisymmetric matter motions can be clearly seen, which is a genuine 3D feature.

The left panel of Figure 12 compares the number of the tracer particles with their individual residency timescales between the 2D and 3D models (for the same snapshot in Figure 11). As seen, the maximum residency time is longer for 3D \( (t_{\text{res}} \sim 92 \text{ ms}) \) than 2D \( (\sim 80 \text{ ms}) \), which is most likely to be the outcome of the non-axisymmetric matter motions in the gain region. As is well known, the longer residency time is good for producing neutrino-driven explosions because of the long exposure to the neutrino heating in the gain region.

The right panel of Figure 12 shows the comparison of the advection timescale conventionally employed in the literature (e.g., Equation (4) in Marek & Janka (2009), that is \( (R_s - R_g)/|\langle v_r \rangle| \) with \( R_s, R_g, \) and \( \langle v_r \rangle \) representing the angle average shock radius, gain radius, and postshock radial velocity, respectively. Against our anticipation, the averaged advection timescale is not always longer for 3D. Before \( \sim 70 \text{ ms} \) after the bounce, our 3D model (red line) has generally longer advection timescales. However, the advection timescale later on can be longer for 2D.

At around \( \sim 70 \text{ ms} \) after the bounce, the revived shock wave has already reached a radius of \( \sim 400 \text{ km} \) for 2D and \( \sim 320 \text{ km} \) for 3D, respectively (see the right panel of Figure 7). In such a shock expansion phase, the definition of the “advection timescale” would become rather vague. For example, the advection timescale is longer for 2D than 3D at \( t = 100 \text{ ms} \) (right panel of Figure 12); however, this simply reflects larger shock radii for 2D than 3D (e.g., right panel of Figure 7). Also in the above residency-time analysis, the longer residency time for 2D

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12 In some sense, artificially this makes the advection timescale longer by the larger distances between \( R_s \) and \( R_g \); however, this does not evidently mean that the 2D model can gain much more efficient neutrino-heating.
can be seen around $t_{\text{res}} = 70–80$ ms in the left panel of Figure 12 (seen as a dominance of the green line over the red line). If the onset time of explosion could be much delayed after the bounce (such as $\sim 600$ ms as in Marek & Janka 2009), the advection (or residency)-timescale analysis between 2D and 3D could have been made clearer in the long-lasting bubbling phase. In order to see the 3D effects more clearly, we plan to employ a more massive progenitor (such as $15 M_\odot$) as a follow-up of this study.

3.3.4. Neutrino-heating Timescales in 2D and 3D

Now we finally compare the neutrino-heating timescales between our 2D and 3D models. From the left panel of Figure 13, the heating timescale is shown to be longer for 3D (red line). As seen from the right panel, this is because the total net rate of the heating rate is generally smaller for our 3D model. To understand this feature, we analyze the neutrino luminosities ($L_\nu$) and mean energies ($\langle \epsilon_\nu \rangle$), since the neutrino heating rate can be symbolically expressed as $Q_+^\nu \propto L_\nu \langle \epsilon_\nu^2 \rangle$ (e.g., Equation (23) in Janka 2001)).

From the left panel of Figure 14, regardless of electron or anti-electron type the neutrino luminosities are shown to be generally larger for 3D than 2D. On the other hand, the mean neutrino energies are lower for 3D (right panel). Although the higher neutrino luminosity is advantageous for producing neutrino-driven explosions, the lower neutrino energies predominantly make the heating rate smaller, thus leading to the longer heating timescale in our 3D model.

The higher neutrino luminosity in 3D is due to the stronger convective activities as discussed in Section 3.3.2. The left panel of Figure 15 compares the velocity dispersion (top panel) and the average radial velocity (bottom panel) between 2D and 3D. Figure 15 is at 50 ms after the bounce, when the convection and SASI are actively in operation.

The top left panel in Figure 15 shows that convective motions are much more vigorous for 3D in a radius of 30–50 km (see also the yellow stripe in Figure 10). The top right panel shows that the neutrino cooling rate (top) as well as its dispersion there ($\sigma_{Q_+}^\nu$, bottom panel) is larger for 3D than 2D. For 3D models computed in this work, these features are generally maintained before the revived shock expands further out (typically $\sim 100$ ms after bounce). The bottom panel of Figure 15 shows that the entropy above 30 km is generally smaller for 3D (red line) than 2D. This is as a consequence of the weaker neutrino heating in 3D than 2D. For the time snapshot in Figure 15 (at 50 ms after bounce), the position of the electron (energy-averaged) neutrinosphere is about 75 km. So the convection deep below the neutrinosphere (30–50 km in radius) is the agent to affect the neutrino luminosity and the mean neutrino energy. This trend is akin to the one observed in the 2D simulations by Buras et al. (2006).

In Figure 16, we proceed to perform a more detailed analysis on matter mixing behind the shock and its impact on the emergent neutrino luminosity. Top panels show radial velocities for our 3D (left) and 2D (right) models within a radius of 100 km, in which downflows and upflows are colored by blue and red,

![Figure 12](image1.png)

**Figure 12.** Comparison between our 2D and 3D models at 100 ms after the bounce, showing the number of tracer-particles traveling in the gain region as a function of their individual residency time (left panel, see the text for details), and the average advection timescale as a function of the postbounce time (right panel). (A color version of this figure is available in the online journal.)

![Figure 13](image2.png)

**Figure 13.** Time evolutions of neutrino-heating timescale (left) and total net rate of neutrino heating (right) in our 2D (green line) and 3D models (red line). (A color version of this figure is available in the online journal.)
respectively. The central white regions correspond to the PNS, which is convectively stable (hence, with small radial velocities) due to the positive entropy gradient (e.g., Section 3.3.2 and Figure 10). In the vicinity of the PNS, downflows and upflows are visible near the equator and pole, respectively, in the 3D model (e.g., back left and back right panels in Figure 16 (top left)). The bottom left panel is same as the top left panel but for the normalized angle variation of $\delta Y_{\bar{\nu}_e}$. Here we define the normalized angle variation of quantity $A$ as

$$\delta A = (A - \langle A \rangle) / \sigma_A. \quad (14)$$

We focus on anti-electron neutrino ($\bar{\nu}_e$), because the luminosity of $\bar{\nu}_e$ dominates that of $\nu_e$ during the simulation time (e.g., left panel in Figure 14). Comparing the top left to the bottom left panel in Figure 16, it can be seen that the positive sign of $\delta Y_{\bar{\nu}_e}$ (red region in the bottom left panel) tends to have a correlation with the downflows (blue region in the top left panel), which is opposite for the upflows. This is because material with larger $Y_{\bar{\nu}_e}$ in the outer layers is mixed down to the vicinity of the PNS that possesses smaller $Y_{\bar{\nu}_e}$ due to convective overturns. This can be a possible explanation of the correlation between the gain (loss) in $Y_{\bar{\nu}_e}$ and the upflows (downflows) to the PNS. Note that this relation is also visible in our 2D model (right panels).

Figure 14. Same as Figure 13 but for luminosities (left) and mean energies (right) of radiated electron ($\nu_e$) or anti-electron ($\bar{\nu}_e$) type neutrinos. (A color version of this figure is available in the online journal.)

Figure 15. Important quantities for analyzing the properties of neutrino emission in multi-D models (see the text for more details). The top left panel compares the velocity dispersion (top) and the average radial velocity (bottom) between the 2D and 3D models. The top right panel shows the neutrino cooling rate (top) and its dispersion (bottom). The bottom panel compares the profiles of maximum and minimum entropies, which is indicated by $\langle s \rangle \pm \sigma_s$, respectively. These panels are for 50 ms after bounce. (A color version of this figure is available in the online journal.)
Figure 16. Analysis of flow patterns and matter mixing for our 3D (right) and 2D (left) models, respectively. Top panels show radial velocities, in which blue and red regions correspond to downflows and upflows that are distinguished from their local radial velocities (negative or positive). Similar to Figure 1, the contours on the cross sections in the $x = 0$ (back right), $y = 0$ (back left), and $z = 0$ (back bottom) planes are, respectively, projected on the sidewalls of the graphs to visualize 3D structures. Bottom panels show the relative angle variation of $Y_e$ ($\delta Y_e$, see the text for definition). In the regions with downflows (blue in the top panels), the sign of $\delta Y_e$ tends to be positive (colored by red in the bottom panels). All the panels are at 50 ms after bounce (same as Figure 15).

Figure 17 depicts angular variations of the flow patterns in Figure 16. The Mollweide projection (or $4\pi$-map) of various quantities is taken at a radius of 50 km. From the top left panel ($\delta v_r$), downflows are shown to flow from the pole (colored by blue), while upflows are rather uniformly distributed in the equator (seen like a horizontal red belt). From the bottom left panel, the color pattern of blue and red reverses with that of the top left panel. As already mentioned, this reflects the correlation between downflows (upflows) and gain (loss) in $Y_e$. Reflecting the gain or loss, the neutrino cooling rate ($\delta Q_{\nu e}$) has a positive correlation with $Y_e$ (compare the top right and the bottom left panel). The variation in the neutrino luminosity that is measured at the outermost boundary of the computation domain (bottom right panel) has a rough positive correlation with the neutrino cooling rate (top right panel), which may agree with one’s intuition.

Finally, we show the cross correlation that we take between the time evolution of the mass accretion rate to the PNS and the neutrino luminosity (Figure 18). As seen, a positive correlation is commonly seen during the simulation time. This plot may carry a
message that it is important to go beyond the light-bulb scheme, in which the input neutrino luminosity is usually kept constant with time (e.g., Iwakami et al. 2008, 2009; Nordhaus et al. 2010). To take into account the feedback between the mass accretion and the neutrino luminosity, the spectral IDSA scheme, which is beyond the gray transport scheme (e.g., Fryer et al. 2002; Fryer 2004), sounds quite efficient in the first-generation 3D simulations.

As suggested in the right panel of Figure 7, 3D explosions are more easily obtained for models with finer numerical resolutions. Our results would indicate whether the advantages for driving explosions mentioned above could or could not overwhelm the disadvantages should be tested by the next generation 3D simulations with much higher numerical resolutions. Needless to say, the 3D results (not to mention 2D results) should depend on the sophistication of the employed neutrino transport scheme. Regarding the gravity, we should first go over the monopole approximation. This may not be an easy task from a technical point of view because we need to implement a multi-grid approach to obtain a high scalability in the MPI computing. To go beyond the Newtonian gravity is also a challenging task (Müller et al. 2010). Our 3D results are only the very first step toward a more realistic 3D supernova modeling.

Figure 18. Time evolution of cross-correlation coefficient between the mass accretion rate ($\dot{M}$) to the PNS and the neutrino luminosity for our 3D model. The coefficient between two variables of $A$ and $B$ is defined by $c_{A,B} = \langle J \delta \dot{M} \delta A \rangle / \sqrt{\langle J \delta \dot{M}^2 \rangle \langle \delta A^2 \rangle}$. (A color version of this figure is available in the online journal.)

Figure 17. The 4π-maps of various quantities in Figure 16 visualized by the Mollweide projection at a radius 50 km. The top left, top right, and bottom left panels show the normalized angular variation of the radial velocity ($\delta v_r$), neutrino cooling rate ($\delta Q$), and $\bar{\nu}_e$ fraction ($\delta Y_{\bar{\nu}_e}$), respectively. The bottom right panel is the normalized angular variation of the (anti-electron) neutrino luminosity ($\delta L_{\nu_{\bar{e}}}$), which is measured at the outer boundary of the computational domain. (A color version of this figure is available in the online journal.)

4. SUMMARY AND DISCUSSION

We have presented numerical results on 3D hydrodynamic core-collapse simulations of an 11.2 $M_\odot$ star. By comparing our 1D and 2D results, we have studied how the increasing spatial multi-dimensionality affects the postbounce supernova dynamics. The calculations were performed with an energy-dependent treatment of the neutrino transport based on the IDSA scheme. In agreement with a previous study, our 1D model does not produce explosions for the 11.2 $M_\odot$ star, while the neutrino-driven revival of the stalled bounce shock is obtained both in 2D and 3D models. We showed that the SASI does develop in the 3D models; however, their saturation amplitudes are generally smaller than 2D. By performing a tracer-particle analysis, we showed that the maximum residency time of material in the gain region is shown to be longer for 3D than 2D due to non-axisymmetric flow motions, which is one of advantageous aspects in 3D to obtain neutrino-driven explosions. Our results showed that convective matter motions below the gain radius become much more violent in 3D than 2D, making the neutrino luminosity larger for 3D. Nevertheless the emitted neutrino energies are made smaller due to the enhanced cooling. Our results, which indicated whether these advantages for driving 3D explosions could or could not overwhelm the disadvantages, are sensitive to the employed numerical resolutions. An encouraging finding was 

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13 To say something important on this matter, we apparently need to study the effects of numerical resolutions in a more systematic manner. However, the result we reported here is the best what we can do for now, and it took more than four CPU months by keeping the currently available supercomputing facilities at our hand running.
that the shock expansion tends to become more energetic for models with finer resolutions. To draw a robust conclusion, 3D simulations with much higher numerical resolutions and also with more advanced treatment of neutrino transport as well as of gravity are needed.

Finally, we refer to the approximations adopted in this paper. As already mentioned, the omission of heavy lepton neutrinos, the inelastic neutrino scattering, and the ray-by-ray approach should be improved. The former two should act to suppress the explosion. The ray-by-ray approach may lead to the overestimation of the directional dependence of the neutrino anisotropies (see discussions in Marek et al. 2009). Although it would be very computationally expensive, the full-angle transport will give us the correct answer (e.g., Ott et al. 2008; Brandt et al. 2011). Our numerical grid in the azimuthal direction is only 32 to cover 360°. Such a low resolution could lead to a large numerical viscosity. The numerical viscosity is expected to be large especially in the vicinity of the standing accretion shock, which may affect the growth of the SASI. It could also affect the growth of the turbulence in the postshock convectively active regions, which is very important to determine the success or failure of the neutrino-driven mechanism. To clearly see these effects of numerical viscosity, we need to conduct a convergence test in which a numerical gridding is changed in a parametric way (e.g., Hanke et al. 2011).

A number of exciting issues remain to be studied in our 3D results, such as gravitational-wave signatures (e.g., Kotake et al. 2009a, 2011; Müller et al. 2012), neutrino emission and its detectability (e.g., Kistler et al. 2011), and the possibility of 3D SASI flows generating pulsar kicks and spins (Wongwathanarat et al. 2010). The dependence of progenitors (e.g., Buras et al. 2006; Burrows et al. 2007b) and equations of state (e.g., Marek & Janka 2009a) are important to be clarified in 3D computations. We are going to study these items one by one in the near future.

As of 2011 July, the K supercomputer in Kobe city of Japan is ranked as the top on the “TOP 500 list of World’s Supercomputers.”14 Early next year, we will be able to start using the facility for our 3D supernova simulations. Keeping with our efforts to overcome the caveats mentioned above, we plan to improve the numerical resolutions as much as possible in the forthcoming run, by which we hopefully gain a new insight into the long-veiled explosion mechanism.

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