Forward-backward multiplicity distribution with the Chou-Yang model for \( pp \) collisions at \( \sqrt{s} = 0.9, 7 \) and \( 8 \) TeV from the CMS experiment

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Abstract

A Chou-Yang type multiplicity distribution comprising a total multiplicity component and a binomial asymmetry component is used to describe charged hadron multiplicity data at \( \sqrt{s} = 0.9, 7 \) and \( 8 \) TeV from the CMS experiment at CERN. The data was obtained and processed from the CERN Open Data Portal. For the total multiplicity component, it was found that a convex sum of a Negative Binomial Distribution (NBD) and a Furry-Yule Distribution (FYD) is able to describe the shoulder-like structure characteristic of KNO scaling violation well. The mean cluster size produced from collisions was also found to increase with collision energy. A prediction is given for \( pp \) collisions at \( \sqrt{s} = 14 \) TeV.

1 Introduction

The Chou-Yang model was proposed by T.T. Chou and C.N. Yang in 1984 \[1\] to describe the forward-backward multiplicity distribution of high energy collisions. In their paper, they defined the variables

\[
n = n_F + n_B, \quad z = n_F - n_B, \]

(1)

(2)

to describe an event with total multiplicity \( n \) and forward-backward (FB) asymmetry \( z \) respectively. \( n_F \) and \( n_B \) are the number of particles produced in the forward \((\eta > 0)\) and backward \((\eta < 0)\) regions. They found that the \( p\bar{p} \) collision data at \( \sqrt{s} = 540 \) GeV from the UA5 collaboration seemed to obey

\[
\langle z^2 \rangle \text{ at fixed } n = 2n,
\]

(3)

which led them to postulate that the asymmetry parameter \( z \) is governed by a binomial distribution. Hence, they proposed that the \( P(n, z) \) distribution be composed of two components – one to describe the production of \( n \) charged particles, and another to describe \( z \) (i.e. how they are distributed in the FB direction):

\[
P(n, z) = \Psi \left( \frac{n}{\langle n \rangle} \right) C^{n/2}_{n_F/2} B \left( \frac{n}{2} \right)^{-1},
\]

(4)

where \( \Psi \left( \frac{n}{\langle n \rangle} \right) \) is the KNO scaling function describing \( n \), \( C^{n/2}_{n_F/2} = C_{(n+z)/4} \) is a combinatorial factor describing the binomial distribution of \( z \), and \( B \) is factor that normalises the latter,

\[
B \left( \frac{n}{2} \right) = \sum_{n_F} C_{n_F/2}^{n/2}
\]

(5)

such that the overall probability distribution \( P(n, z) \) is normalised.

1.1 Proposed modification to the Chou-Yang Model: total multiplicity \( n \)

Presently, it is well-established that the multiplicity distributions of \( pp \) collisions no longer obey KNO scaling \[2\] at UA5 energies and beyond. This led to several studies that replaced the KNO scaling function in \( P(n, z) \) with other analytic distributions. Lim et al. \[3, 4\] suggested replacing the KNO
scaling function with the Negative Binomial Distribution (NBD, equation 6), and Lai et al. [5] and Phang et al. [6] developed the approach further by replacing the NBD with the Generalised Multiplicity Distribution (GMD, equation 7):

\[
P_{\text{NBD}}(n; \pi, k) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \left[ \frac{\pi}{\pi+k} \right]^n \left[ \frac{k}{\pi+k} \right]^k,
\]

\[
P_{\text{GMD}}(n; \pi, k, k') = \frac{\Gamma(n+k)}{\Gamma(n-k'+1)\Gamma(k+k')} \left[ \frac{\pi - k'}{\pi} \right]^{n-k'} \left[ \frac{k+k'}{\pi+k} \right]^{k+k'},
\]

where \( \Gamma \) is the Gamma function, which generalises the arguments of the combinatorial prefactors of the NBD and GMD from positive integers to positive real numbers. Thus, we now have

\[
P(n, z) = P(n)C_{n/F}^{n/2} \left[ B \left( \frac{n}{2} \right) \right]^{-1},
\]

where \( P(n) \) is either \( P_{\text{NBD}}(n) \) or \( P_{\text{GMD}}(n) \).

The GMD is defined for values of \( n \) such that \( n \geq k' \). This mathematical requirement might severely constraint the overall ability of \( P(n, z) \) to describe the data, since it would cause several bins in \( P(n_F, n_B) \) to be empty. Hence, we propose using a \( P(n) \) constructed from a convex sum of the two limiting forms of the GMD (which becomes the NBD when \( k' = 0 \), and the FYD when \( k = 0 \)), so that all values of \( n \) can be described:

\[
P(n) = \alpha P_{\text{NBD}}(n; \pi_{\text{NBD}}, k) + (1 - \alpha) P_{\text{FYD}}(n; \pi_{\text{FYD}}, k'),
\]

where \( \alpha \in [0,1] \).

### 1.2 Mean cluster size \( r \)

Lim et al. [3, 4] considered the prospect that the factor ‘2’ in equation 5 could take on other numbers. They replaced it with \( r \), which results in a reinterpretation of the original binomial model: the original \( n \) particles are first grouped into \( r \) clusters, each of which then gets distributed in \( z \) according to the modified Chou-Yang scheme. This has come to be known as the “cluster model”; thus, in Chou and Yang’s original paper [1], we have \( r = 2 \) clusters for the special case of \( p\bar{p} \) collisions at \( \sqrt{s} = 540 \text{ GeV} \) from UA5.

To this end, the overall Chou-Yang model has been modified to become

\[
P(n, z) = P(n)C_{n/r}^{n/r} \left[ B \left( \frac{n}{r} \right) \right]^{-1},
\]

where the mean cluster size \( r \) is to be determined from experimental data. Equation 8 also becomes modified to be

\[
\langle z^2 \rangle \text{ at fixed } n = rn.
\]

### 2 Investigation at LHC energies

We wish to make two measurements. The first is to investigate how our proposed NBD-FYD distribution (equation 9) is able to describe the collision data, and if it is able to offer some physical insight into the multiparticle production process.

The second measurement concerns the asymmetry part of the generalised version of the Chou-Yang model,

\[
P(z; r) = C_{n/r}^{n/r} \left[ B \left( \frac{n}{r} \right) \right]^{-1}.
\]

Specifically, we wish to find out how \( r \) varies with collision energy in the CMS Run 1 data. This will be obtained by extracting the 2-dimensional probability distribution \( P(n_F, n_B) \) from data, plotting \( \langle z^2 \rangle \) vs. \( n \), and performing linear regression to obtain \( r \) in equation 11.
Table 1: Best-fit parameters for NBD-FYD model

| √s   | 900 GeV     | 7 TeV       | 8 TeV       |
|------|-------------|-------------|-------------|
| α    | 0.541 ± 0.001 | 0.814 ± 0.001 | 0.846 ± 0.002 |
| k    | 32.18 ± 0.04  | 29.56 ± 0.04  | 43.2 ± 0.1   |
| k'   | 4.9999904 ± 0.0000008 | 3.99963 ± 0.0002 | 2.81 ± 0.03  |

Note: The large number of decimal figures for k' for √s = 900 GeV and 7 TeV are the optimiser’s attempts to generate as large a real number below 5 and 4 respectively, so as not to sacrifice one data point from the FYD component (which is defined only for n ≥ k').

3 About the data

This analysis is performed on Run 1 data from the CMS collaboration processed from the CMS Open Data Portal, covering centre-of-mass energies √s = 0.9, 7 and 8 TeV. The analysis method follows largely that of CMS [7], which analysed minimum-bias (MinBias), non-single diffractive (NSD) multiplicity distributions.

NSD events were selected by requiring that at least one forward hadron (HF) calorimeter tower on each side of the detector have at least 3 GeV of energy deposited in the event. The primary vertex was chosen as the vertex with the highest number of associated tracks, which must also be within 15 cm of the reconstructed beamspot in the beam axis and be of good reconstruction quality (ndof > 4).

Good quality tracks were selected by requiring them to carry the highPurity label. Furthermore, we select for tracks with <10% relative error on the transverse momentum (p_T) measurement (σ_p_T/p_T < 0.1) to reject low-quality and badly reconstructed tracks. Secondaries were removed by requiring a small impact parameter with respect to the selected primary vertex. Also, tracks were required to have p_T > 500 MeV/c, which will be extrapolated to zero via unfolding.

Finally, unfolding was performed using an iterative “Bayesian unfolding method”, which is more accurately known as “D’Agostini iteration with early stopping” and described in [8]. This infers the original charged hadron multiplicity distribution (MinBias NSD) from the charged track multiplicity distribution measured.

Tables 3 and 4 in the Appendix summarise the datasets used.

4 Results and discussion

4.1 Multiplicity component

Table 1 summarises the best-fit parameters found for the NBD-FYD model in describing the data, and Figure 1 shows the collision data, the fitted NBD-FYD model and its constituent NBD and FYD components. In optimising the parameters, only data points within 1 ≤ n ≤ 120 were used; n = 0 was excluded from the fitting due to the high values of P(0) which are not described by the model, while data points at n > 120 suffered from low statistics and were not smooth, which would interfere adversely with the optimisation process. The χ^2/d.o.f. values are much greater than unity due to the small magnitudes of the errors (due to underestimating the systematic errors). However, it can be ascertained visually that the model describes the data very well. Most importantly, the model is able to describe the shoulder-like structure (from KNO scaling violation) of the data.

It is interesting to note the manner in which the NBD-FYD model brings about the shoulder-like structure. The NBD component (green dots in Figure 1) provides the general overall downward-sloping profile, and the FYD component (blue dots) provides the protruding “head”, which also defines the “shoulder”. As √s increases, the FYD component becomes narrower to manifest a more pronounced change in gradient (i.e. a more pronounced shoulder-like structure). This narrowing is brought about by an increase in α and a decrease in k'. It is also worth noting that πFYD remains relatively constant at all values of √s.
Figure 1: Fits of the NBD-FYD model to data for $1 \leq n \leq 120$, with their NBD and FYD components also shown.
Table 2: Measured mean cluster size $r$ for hadronic collisions, compiled with previous studies at other energies

| Collaboration | $\sqrt{s}$ (GeV) | $r$ | Source |
|--------------|------------------|-----|--------|
| ISR (pp)     | 24               | 1.14 ± 0.04 | |
|              | 31               | 1.33 ± 0.05 | |
|              | 45               | 1.29 ± 0.04 | [3] |
|              | 53               | 1.49 ± 0.04 | |
|              | 63               | 1.55 ± 0.05 | |
| UA5 (p$\bar{p}$) | 200            | 1.88 ± 0.07 | [3] |
|              | 546              | 2.23 ± 0.07 | |
|              | 900              | 2.28 ± 0.07 | |
| E735 (p$\bar{p}$) | 300            | 2.15 ± 0.23 | |
|              | 546              | 2.78 ± 0.26 | [11] |
|              | 1000             | 2.81 ± 0.30 | |
|              | 1800             | 2.62 ± 0.12 | |
| CMS (pp)     | 900              | 2.3 ± 0.2  | |
|              | 7000             | 3.07 ± 0.03 | (New measurement) |
|              | 8000             | 3.28 ± 0.03 | |

**Interpretation of NBD-FYD model**

The above findings raise an issue of interpretation that is potentially incompatible with the GMD. Chan and Chew’s analysis [9] showed that as collision energies increase towards the TeV scale, the GMD will approach the FYD (via decreasing $k$ and increasing $k'$), indicating significant contribution from gluon branching. The NBD-FYD model proposed here seems to suggest the opposite, with a diminishing FYD component. While the NBD-FYD model describes the data excellently, it seems to be incompatible with the GMD’s notion of quark and gluon branching from the NBD and FYD components respectively.

It is useful to reconsider the origins of the NBD and FYD. As Hwa pointed out [10], these are particular solutions to the basic evolution equation

$$\frac{\partial P_n(t)}{\partial t} = a_{n+1}P_{n+1} + c_{n-1}P_{n-1} - (a_n + c_n)P_n,$$

which has been adapted to describe the evolution of the multiplicity distribution $P_n(t)$, with $t$ as the QCD evolution parameter. The coefficient $c_n$ is proportional to “creation” processes, which describes birth, emission, fragmentation or branching in the context of multiparticle production. $a_n$ is proportional to “annihilation” processes, which describes death, absorption, recombination or decay [10].

In this framework, the NBD is the solution to the evolution equation describing birth, death and immigration processes, having coefficients of the form $a_n = \alpha n$, $c_n = \beta n + \gamma$. The FYD is the solution describing only birth processes, having coefficients of the form $a_n = 0$, $c_n = \beta n$.

When considered with the results in Table 1, the NBD-FYD model paints a physical picture of an increasingly complex mode of multiparticle production. At lower energies, multiparticle production mainly involves pure.birth processes, as described by the FYD; at higher energies, additional death and immigration processes come into play, as described by the NBD.

### 4.2 FB component

Figure 2 shows the unfolded (and unnormalised) 2D multiplicity distribution plots of the data. From this, $\langle z^2 \rangle$ vs. $n$ can be computed, and the results are shown in Figure 3. The values of $r$ obtained from the data are summarised in Table 2 and compiled alongside results from previous studies. Figure 4 shows how $r$ varies with collision energy $\sqrt{s}$ for hadronic collisions. The increase in $r$ seems to follow the relation

$$r = \alpha \log \sqrt{s} + \beta,$$

with $\alpha = 0.34 \pm 0.03$ and $\beta = 0.2 \pm 0.2$.

\[1\] The maximum values of $n_F$ and $n_B$ were restricted to 107 due to the memory limitations (in computer RAM) of the ROOT system. This will affect the computation of high values of total multiplicity $n$ due to truncation effects.
Figure 2: 2D multiplicity distribution $P(n_F, n_B)$ for $\sqrt{s} = 900$ GeV, 7 TeV and 8 TeV, with the colour legend in log scale.
Figure 3: $\langle z^2 \rangle$ vs. $n$ for $\sqrt{s} = 900$ GeV, 7 TeV and 8 TeV. The slope of the best-fit line is equal to the mean cluster size $r$. Data points have been included for $n \leq 120$ to avoid truncation effects from the finiteness of $P(n_F,n_B)$.

Figure 4: Variation of mean cluster size $r$ as a function of $\log \sqrt{s}$, for hadronic collisions with $\sqrt{s}$ ranging from 24 GeV to 8 TeV.
5 Conclusion

We presented a model that describes the multiplicity distribution as a convex sum of a Furry-Yule Distribution (FYD) and a Negative Binomial Distribution (NBD). It was found to describe data well. The FYD component diminishes with increasing energy while the NBD component becomes more prominent, the latter suggesting an increase in complexity of multiparticle production with death and immigration processes starting to emerge.

An increase in mean cluster size $r$ has also been observed in hadronic collisions from $\sqrt{s} = 24$ GeV to 8 TeV. The increase in $r$ seems to follow the relation $r = \alpha \log \sqrt{s} + \beta$, with $\alpha = 0.35 \pm 0.03$ and $\beta = 0.1 \pm 0.2$. Based on this, we predict that we would have $r = 3.4 \pm 0.5$ at $\sqrt{s} = 14$ TeV, and we look forward to analysing 14 TeV data when it becomes available.

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A Datasets used

Table 3: Summary of CMS collider datasets used from CERN Open Data Portal

| $\sqrt{s}$ (TeV) | Dataset | Ref. |
|------------------|---------|------|
| 0.9              | /MinimumBias/Commissioning10-07-JunReReco_900GeV/RECO | [12] |
| 7                | /MinimumBias/Run2010A-Apr21ReReco-v1/AOD | [13] |
| 8                | /MinimumBias/Run2012B-22Jan2013-v1/AOD | [14] |

Table 4: Summary of Monte Carlo datasets used from CERN Open Data Portal

| $\sqrt{s}$ (TeV) | Dataset | Ref. |
|------------------|---------|------|
| 0.9              | /MinBias_TuneZ2_900GeV_pythia6_cfl_py_GEN_SIM_START311_V2_Dec11_v2 | [15] |
| 7                | /MinBias_TuneZ2star_7TeV_pythia6/Summer12-LowPU2010_DR42-PU_S0_START42_V17B-v1/AODSIM | [16] |
| 8                | /MinBias_TuneZ2star_8TeV-pythia6/Summer12_DR53X-PU_S10_START53_V7A-v1/AODSIM | [17] |

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