Consequences of $\mu - \tau$ Reflection Symmetry at DUNE

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Abstract

We consider minimal type - I seesaw framework to realize $\mu - \tau$ reflection symmetry in the low energy neutrino mass matrix, $M_\nu$. Considering DUNE experiment, we scrutinize its potential to measure the precision of 2-3 mixing angle, $\theta_{23}$ and the Dirac CP-phase, $\delta$ for the given symmetry. Later, we examine the precision of these two parameters considering NuFit-3.2 data as one of the concerned true point. To study the low energy phenomenology, we further discuss various breaking patterns of such an exact symmetry. Moreover, for each breaking scenario we perform the capability test of DUNE for the determination of $\theta_{23}$ and to establish the phenomenon of CP violation considering true benchmark point arising from the breaking of $\mu - \tau$ reflection symmetry. We also make remarks on the potential of DUNE to rule out maximal CP-violation or CP-conservation hypothesis at a certain confidence level for different scenarios.

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I. INTRODUCTION

During last a few years, there have been remarkable progress in the field of neutrino physics which guided us to understand some intriguing aspects of neutrinos in a comprehensive manner. It is now well established phenomenon from different experimental results that neutrinos possess non-zero mass and their different flavors are mixed [1]. However, the dynamical origin associated with neutrinos mass generation as well as mixing patterns are still unknown. There have been numerous theoretical attempts to understand the nature of tiny neutrino masses, among which seesaw mechanism is considered as highly appreciated one [2–6]. The simplest way to generate neutrino masses are to add at least two $SU(2)$ singlet right-handed neutrino fields (i.e. $N_{\mu R}, N_{\tau R}$) in the Standard Model (SM). The relevant SM gauge invariant Lagrangian containing the neutrino Yukawa matrix and the Majorana neutrino mass matrix can be written as

$$-\mathcal{L} \supset \bar{L}_\alpha L Y_\nu N_R \tilde{H} + \frac{1}{2} N_R^C M_R N_R + \text{h.c.},$$

where $L_\alpha = (\nu_\alpha, \alpha)^T_L$ is the left-handed lepton doublet, $Y_\nu$ denotes the neutrino Yukawa matrix, $\tilde{H} = i\sigma_2 H^*$ with $H$ being the Higgs doublet in the SM. Also, $M_R$ is the Majorana neutrino mass matrix and $C$ denotes the charge-conjugation operator. After spontaneous symmetry breaking, one obtains the Dirac neutrino mass term as $\bar{\nu}_L^\alpha M_D N_R + \text{h.c.}$, where $M_D = vY_\nu$ is the Dirac neutrino mass matrix with vacuum expectation value (vev), $v = \langle H \rangle \approx 174$ GeV [1]. Employing seesaw mechanism, one gets light neutrino mass matrix in type - I seesaw formalism as, $M_\nu \approx -M_D M_R^{-1} M_D^T$ and diagonalization of such $M_\nu$ leads to three active neutrino masses $m_i$ (for $i = 1, 2, 3$).

Furthermore, flavor symmetry based approaches get numerous attention to explain the observed neutrino mixing patterns as discussed in Refs. [7–11] and the references therein. Among number of such approaches, $\mu-\tau$ reflection symmetry attracts a lot of attention in recent times which was originally discussed in Ref. [12] (see Ref. [13] for a latest review). This symmetry predicts: the maximal atmospheric mixing angle $\theta_{23}$, i.e., $\theta_{23} = 45^\circ$ along with the maximal value of Dirac CP phase $\delta$, i.e. $\delta = \pm 90^\circ$; and trivial values for the two Majorana phases with non-zero $\theta_{13}$. Indeed, in recent times there are many attempts toward $\mu-\tau$ reflection symmetry as outlined in Refs. [14–30].

In this work, we embed $\mu-\tau$ reflection symmetry in minimal seesaw formalism such that one can address both neutrino masses and mixing patterns (see Ref. [31] for recent review).
Later, we study its consequences considering next-generation super beam Deep Underground Neutrino Experiment (DUNE). This statistically high potential experiment will improve the precision of the atmospheric mixing angle, $\theta_{23}$ and play a key role to probe the leptonic CP-violating phase, $\delta$ [32]. Because of this, DUNE can test various flavor symmetry models and helps us to understand some inherent physics associate with it.

At the given framework along with maximal $\delta$ and $\theta_{23}$, we also find remaining oscillation parameters both analytically as well as numerically. Considering this as true benchmark point, we depict the allowed area in ($\delta - \sin^2 \theta_{23}$) plane for DUNE at various confidence levels which serves our intention to inspect precision of these two less known parameters. This also show the potential of DUNE to know how well it can measure $\delta$ and $\theta_{23}$. Moreover, latest results of global-fit of neutrino oscillation data from NuFit-3.2 collaboration [33, 34] favors higher octant of $\theta_{23}$ along with non-maximal $\delta$\(^1\). Also, results of on-going neutrino oscillation experiments (e.g., T2K [35] and NO\(\nu\)A [36]) are in well agreement with the predictions of the concerned symmetry but, still show large uncertainties in their measurement of $\delta$ and $\theta_{23}$. Therefore, it is tenacious to accept the exact nature of $\mu - \tau$ reflection symmetry. In that respect, it is worthwhile to study various broken scenarios of such a symmetry.

To proceed with phenomenological study, we first perform our analysis considering global best-fit values as our benchmark point [33, 34]. Afterwards, we consider breaking of $\mu - \tau$ reflection symmetry by introducing explicit breaking parameter in the high energy neutrino mass matrices $M_D$, $M_R$, respectively. For each scenario, we find the set of neutrino oscillation parameters and perform the capability test of DUNE in ($\delta - \sin^2 \theta_{23}$) plane. Considering different cases, we analyze the potential of DUNE to rule out the possibility of maximal CP-violation (CPV) as well as CP-conservation hypothesis at a given confidence level. Some recent studies considering different flavor models in the context of long baseline experiments have been performed in [30, 37–45].

We organize rest of the paper as follows. In Section II, we present a general set-up of the $\mu - \tau$ reflection symmetry and perform our analysis in the given scenario for DUNE. We also present our numerical details in this section. We proceed to discuss our results considering NuFit-3.2 data in Section III. Furthermore, in subsequent subsections of Section III, we discuss the breaking of $\mu - \tau$ reflection symmetry by introducing explicit breaking parameter

\(^1\) Note that $\theta_{23} < 45^\circ$ is called as lower octant (LO) whereas $\theta_{23} > 45^\circ$ is called as higher octant (HO).
in $M_D$ and $M_R$, respectively and their implications in the context of DUNE. Finally, we summarize our noteworthy results in Section IV.

II. PHENOMENOLOGY AT $\mu - \tau$ REFLECTION SYMMETRY

The $\mu - \tau$ reflection symmetry at the low energy neutrino mass matrix, $M_\nu$ was first proposed in Ref. [12] which leads us to following four predictions:

\[
M_{ee} = M_{ee}^*, \quad M_{\mu\tau} = M_{\mu\tau}^*, \quad M_{e\mu} = M_{e\tau}^*, \quad M_{\mu\mu} = M_{\tau\tau}^* ,
\]

where $M_{\alpha\beta}$, with $\alpha, \beta = e, \mu, \tau$ are the elements of $M_\nu$. We consider minimal type - I seesaw mechanism to realize $\mu - \tau$ reflection symmetry at $M_\nu$. To achieve such symmetry, we extend the SM fields content by adding two right-handed neutrino fields which are singlet under the SM gauge group. Without loss of generality, we consider the following texture of $M_D$ to realize $\mu - \tau$ reflection symmetry,

\[
M_D = \begin{pmatrix} a & a^* \\ b & c \\ c^* & b^* \end{pmatrix} = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & be^{-i\phi_b} \end{pmatrix}.
\]

Also, we adopt diagonal $M_R$ of the form $M_R = \text{diag}(M_1, M_1)$ with degenerate heavy Majorana neutrino masses. Further, considering type-I seesaw mechanism, we obtain the effective neutrino mass matrix for the light neutrinos as

\[
-M_\nu = M_D M_R^{-1} M_D^T, = \frac{1}{M_1} \begin{pmatrix} 2a^2 \cos 2\phi_a & abe^{i(\phi_a+\phi_b)} + ace^{-i(\phi_a-\phi_c)} & abe^{-i(\phi_a+\phi_b)} + ace^{i(\phi_a-\phi_c)} \\ -b^2 e^{2i\phi_b} + c^2 e^{2i\phi_c} & 2bc \cos(\phi_b - \phi_c) \\ -b^2 e^{-2i\phi_b} + c^2 e^{-2i\phi_c} \end{pmatrix}.
\]

We notice that the elements of $M_\nu$ as given by Eq. (4) satisfy all the conditions of Eq. (2) and hence leads to $\mu - \tau$ reflection symmetry. In the standard PDG [1] parameterization,

\footnote{It is possible to find the considered mass textures using a suitable flavor group along with preferred $\mathbb{Z}_n$ cyclic group. As our intention is to study the impact of these textures rather their theoretical origin, hence we do not perform this study here.}

\footnote{Note that non-degenerate Majorana neutrino mass matrix does not satisfy all the conditions mentioned in Eq. (2) and thus does not lead to the concerned symmetry which we discuss in section III.}
the unitary mixing matrix which diagonalizes neutrino mass matrix, $M_\nu$, can be written as,

$$V = P_l U P_\nu,$$

$$= P_l \left( \begin{array}{ccc}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
    s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{array} \right) P_\nu,$$

(5)

where $c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$ (for $i < j = 1, 2, 3$). Here, $P_l$ contains three unphysical phases of the form, $P_l = diag(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$ which can be absorbed by the rephasing of charged lepton fields whereas $P_\nu = diag(e^{i\rho}, e^{i\sigma}, 1)$ contains two Majorana phases.

With the above form of $M_\nu$ as given by Eq. (4), one can find that there exist 6 predictions for the leptonic mixing angles and phases which are

$$\phi_e = 90, \quad \phi_\mu = -\phi_\tau = \phi, \quad \theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ, \quad \rho, \quad \sigma = 0^\circ \text{ or } 90^\circ. \quad (6)$$

Note that under $\mu - \tau$ reflection symmetry the value of $\theta_{13}, \theta_{12}$ remain unspecified. We find their analytical form in terms of model parameters as

$$\theta_{13} = \pm \tan^{-1} \left[ \frac{b^2 \sin 2\varphi_b + c^2 \sin 2\varphi_c}{a(b \sin \varphi_{ab} + c \sin \varphi_{ac})} \right],$$

$$\theta_{12} = \begin{cases} 
    \frac{1}{2} \tan^{-1} \left[ \frac{2\sqrt{2}a \cos 2\theta_{13}(b \sin \varphi_{ab} + c \sin \varphi_{ac})}{c_{13}[(b^2 \cos 2\varphi_b + c^2 \cos 2\varphi_c - 2bc \cos \varphi_{bc}) \cos 2\theta_{13} - (b^2 \cos 2\varphi_b + c^2 \cos 2\varphi_c + 2bc \cos \varphi_{bc})s_{13}^2 + 2a^2 \cos 2\phi_a s_{13}^2]} \right] ; & \text{for NMO} \\
    \frac{1}{2} \tan^{-1} \left[ \frac{2\sqrt{2}a(b \sin \varphi_{ab} + c \sin \varphi_{ac})s_{13}^2}{c_{13}[(b^2 \cos 2\varphi_b + c^2 \cos 2\varphi_c)(1 + s_{13}^2) + 2c_{13}^2bc \cos \varphi_{bc}]} \right] ; & \text{for IMO}
\end{cases}$$

(7)

where $\varphi_{b,c} = (\phi - \phi_{b,c}), \varphi_{ab,c} = (\phi - \phi_a - \phi_{b,c}), \varphi_{bc} = -(\phi_b - \phi_c)$.

Similarly, one can calculate masses of light neutrinos by diagonalizing $M_\nu$ of Eq.(4) as

$$V^\dagger M_\nu V^* = diag(m_1, m_2, m_3). \quad (8)$$

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4 For a detailed discussion on the adopted phase conventions see appendix of Ref. [28].

5 Note that mass pattern of the form $m_3 > m_2 > m_1$ is known as normal mass ordering (NMO) whereas $m_3 < m_1 \approx m_2$ pattern is known as inverted mass ordering (IMO).
where $m_i$’s $(i = 1, 2, 3)$ are the active neutrino masses. Further, the masses can be expressed for NMO as

$$m_1 = 0,$$

$$\tilde{m}_2 = \frac{2\sqrt{2}a(b \sin \varphi_{ab} + c \sin \varphi_{ac})}{c_{13} \sin 2\theta_{12} M_1},$$

$$m_3 = \frac{1}{M_1} \left[ 4bc \cos \varphi_{bc} + 2a^2 \cos 2\phi_a + \frac{2\sqrt{2}a(b \sin \varphi_{ab} + c \sin \varphi_{ac})}{c_{13} \sin 2\theta_{12}} \right], \quad (9)$$

whereas, expressions for IMO can be written as

$$m_1 = \frac{1}{M_1} \left[ 2bc \cos \varphi_{bc} - a^2 \cos 2\phi_a - \frac{2\sqrt{2}a(b \sin \varphi_{ab} + c \sin \varphi_{ac})}{c_{13} \sin 2\theta_{12}} \right],$$

$$\tilde{m}_2 = \frac{1}{M_1} \left[ -2bc \cos \varphi_{bc} - a^2 \cos 2\phi_a + \frac{2\sqrt{2}a(b \sin \varphi_{ab} + c \sin \varphi_{ac})}{c_{13} \sin 2\theta_{12}} \right],$$

$$m_3 = 0. \quad (10)$$

Here, $\tilde{m}_2 = m_2 e^{2i\sigma}$ and $\sigma$ can take value either $0^\circ$ or $90^\circ$. Also note that as the minimal seesaw formalism always predicts massless lightest neutrino, one has the freedom of eliminating one of the Majorana phases. Thus, in this study we do not consider phase, $\rho$.

To proceed further and to investigate low energy phenomenology, we first give here simulation and experimental details that are considered in this work. The principle strategy of our numerical analysis is to scan all the high energy variables of $Y_{\nu}$ and $M_R$ as free variables and later constrain the allowed space of high energy variables to find neutrino oscillation parameters which are compatible with the latest NuFit-3.2 data \cite{33, 34} at low energies. We vary different parameters as,

$$|a|, |b|, |c| \in [0, 1] \, v, \quad \phi_{a,b,c} \in [0, 360^\circ), \quad M_1 \in [10^{12}, 10^{15}] \, \text{GeV}. \quad (11)$$

We use the nested sampling package \texttt{Multinest} \cite{46–48} to guide the parameter scan with the built $\chi^2$ function considering latest NuFit-3.2 data \cite{33, 34}. The analytical expression of the Gaussian-$\chi^2_{\text{min}}$ function that we use in our numerical simulation is defined as,

$$\chi^2_{\text{min}} = \min \sum_i \frac{[\xi_{\text{True}} - \xi_{\text{Test}}]^2}{\sigma [\xi_{\text{True}}]^2}, \quad (12)$$

where $\xi = \{\theta_{12}, \theta_{13}, \theta_{23}, \Delta m^2_{21}, |\Delta m^2_{31}|\}$, represents the set of neutrino oscillation parameters. Here, $\xi_{\text{True}}$ represent the current best-fit values of the latest NuFit-3.2 data \cite{33, 34} and
\(\xi_i^{\text{Test}}\) correspond to the predicted values for a given set of parameters in theory. We also symmetrize standard deviation, \(\sigma [\xi_i^{\text{True}}]\) considering 1\(\sigma\) errors as given by Ref. [33, 34].

We consider here DUNE, which is a proposed next generation superbeam experiment at Fermilab, USA [32, 49] designing to detect neutrinos. This experiment will utilize existing NuMI (Neutrinos at the Main Injector) beamline design at Fermilab as a neutrino source. The far detector of DUNE will be placed at Sanford Underground Research Facility (SURF) in Lead, South Dakota, at a distance of 1300 km (800 mile) from neutrino source. DUNE collaboration has planned to use LArTPC (liquid argon time-projection chamber) detector. For the numerical simulation of the DUNE data, we use the GLoBES package [50, 51] along with the required auxiliary files presented in Ref. [49]. We perform our simulation considering 40 kton fiducial mass far detector. We also consider the flux corresponding to 1.07 MW beam power which gives \(1.47 \times 10^{21}\) protons on target (POT) per year due to 80 GeV proton beam energy. In addition, we adopt signal and background normalization uncertainties for appearance as well as disappearance channel as presented in DUNE CDR [49]. Further, we distribute the total exposure of DUNE (i.e., 300 kton-MW-years) in two scenarios ; (i) in first scenario, we perform our analysis only with neutrino mode considering 7 years of neutrino run, i.e., DUNE\([7\nu + 0\bar{\nu}]\), and (ii) in second scenario, we consider 3.5 years each of neutrino and antineutrino mode i.e., DUNE\([3.5\nu + 3.5\bar{\nu}]\). We also add 5\% prior on \(\sin^2 2\theta_{13}\) in our analysis.

The main steps to carry out our numerical analysis are to calculate set of neutrino oscillation parameters corresponding to minimum \(\chi^2(=\chi^2_{\text{min}})\), as defined by Eq.(12), using Multinest in this model. Later, considering this set of parameters as true benchmark value, we generate DUNE results using GLoBES and present the allowed parameter space in test \((\delta - \sin^2 \theta_{23})\)-plane. We utilize GLoBES inbuilt \(\chi^2\)-function for the data analysis. In this study, we marginalize all the oscillation parameters over their 3\(\sigma\) range as given by Table II. In addition, we marginalize \(\delta\) in the range \(\delta \in [0^\circ, 360^\circ]\) for each scenario unless otherwise stated.

In Fig.1, we present our results in the framework of \(\mu - \tau\) reflection symmetry. We calculate the numerical values for the set of neutrino oscillation parameters in the given scenario corresponding to \(\chi^2_{\text{min}}\) as given in Table I. Considering these true set of parameters, we find the allowed area in \((\delta - \sin^2 \theta_{23})\)-plane in case of DUNE which we have depicted in Fig.1. The green, pink and blue colored contours represent 1\(\sigma\), 3\(\sigma\) and 5\(\sigma\) allowed parameter.
| Parameters | NMO ($\chi^2_{\text{min}} = 0.10$) | IMO ($\chi^2_{\text{min}} = 0.82$) |
|-----------|-------------------------------|-------------------------------|
| $\Delta m^2_{21} [10^{-5}\text{eV}^2]$ | 7.401 | 7.50 |
| $|\Delta m^2_{31}| [10^{-3}\text{eV}^2]$ | 2.498 | 2.465 |
| $\sin^2 \theta_{12}$ | 0.304 | 0.303 |
| $\sin^2 \theta_{23}$ | 0.50 | 0.50 |
| $\sin^2 \theta_{13}$ | 0.02217 | 0.02218 |
| $\delta$ [deg] | 90 | 270 |

TABLE I: Set of neutrino oscillation parameters at $\chi^2_{\text{min}} = 0.10$ ($\chi^2_{\text{min}} = 0.82$) for NMO (IMO) in the $\mu - \tau$ reflection symmetry scenario.

space, respectively whereas red-star point represents the true value of $(\delta, \sin^2 \theta_{23})$. Further, top and bottom row show our results for DUNE$[7\nu + 0\overline{\nu}]$, and DUNE$[3.5\nu + 3.5\overline{\nu}]$, respectively. Also, vertical black-dashed lines represent maximal CPV corresponding to $\delta = 90^\circ$ and $270^\circ$, respectively. Similarly, blue-dotted line signifies CP-conserving value, $\delta = 180^\circ$ whereas horizontal black-dashed line represents $\sin^2 \theta_{23} = 0.5$. Note that we consider similar color details throughout this work.

Considering maximal value of $(\delta, \sin^2 \theta_{23})$ as true benchmark point, we notice from the first row of Fig.1 that 7 years of neutrino run of DUNE can rule out CP-conservation hypothesis at 1\(\sigma\) C.L. for both the mass ordering (i.e., NMO, IMO) as shown by green contour. This observation remains true even at 3\(\sigma\) C.L. for both the mass ordering as presented in pink contour. To justify this point, we notice from upper panel that pink contour does not intersect with vertical blue-dotted line which provides clear evidence of the ruling out of CP-conservation hypothesis at the same confidence level. Besides this, we notice from the 5\(\sigma\) contour (see blue contour) that DUNE can not exclude CP-conservation hypothesis for both the mass orderings. In addition, we also notice that the precision of CP-phase, $\delta$ is marginally better in case of IMO compare to NMO whereas $\sin^2 \theta_{23}$ shows almost similar precision for both cases at 5\(\sigma\) C.L. From second row of Fig.1, we notice that DUNE can rule out CP-conservation hypothesis even at 5\(\sigma\) C.L. for IMO (see right panel) whereas in case NMO, it can almost exclude the same except for some regions around $(\delta = 0^\circ/360^\circ, \sin^2 \theta_{23} = 0.5)$. Finally, we notice from top row that DUNE can rule out one half-plane of $\delta$ at 3\(\sigma\) C.L. whereas at 5\(\sigma\) C.L. it can exclude almost the same for both the

\footnote{Note that authors of Ref. [52] have performed a detailed analysis on the sensitivity of these less known parameters considering various combinations of $(\nu + \overline{\nu})$ for DUNE.}
FIG. 1: Allowed parameter space of DUNE in $(\delta - \sin^2 \theta_{23})$-plane in $\mu - \tau$ reflection symmetry scenario. Here green, pink and blue color represent $1\sigma, 3\sigma$ and $5\sigma$ allowed contours and ‘red-*’ signifies true value of $(\delta, \sin^2 \theta_{23})$. Also left (right) column represents normal (inverted) mass ordering and top (bottom) row shows our results for DUNE[$7\nu + 0\bar{\nu}$] ( DUNE[$3.5\nu + 3.5\bar{\nu}$] ).

mass orderings. In case of NMO (for true $\delta = 90^\circ$ ), we observe that DUNE can rule out $\delta$ in the range, $\delta \in [180^\circ, 360^\circ]$ whereas for IMO (for true $\delta = 270^\circ$ ), it can rule out $\delta$ in the range, $\delta \in [0^\circ, 180^\circ]$ at $3\sigma$ C.L. Similarly, from bottom row we notice that the same conclusion remains true even at $5\sigma$ C.L. except a small regions for NMO.

Having discussed our results in the $\mu - \tau$ reflection symmetry scenario considering DUNE, in the following section we proceed to perform our analysis by utilizing current oscillations data. Later, we also examine different symmetry breaking scenarios where we will discuss the impact of breaking parameter on the poorly measured parameters, $\delta$ and $\sin^2 \theta_{23}$. 
III. PHENOMENOLOGY BEYOND $\mu - \tau$ REFLECTION SYMMETRY

In this section, we discuss our results beyond $\mu - \tau$ reflection symmetry considering DUNE. As the current best-fit value of neutrino oscillation data prefers non-maximal value of $\delta$, $\sin^2 \theta_{23}$, we start the discussion considering this as our true benchmark point. Furthermore, in subsequent subsections we perform our study considering different breaking scenarios of $\mu - \tau$ reflection symmetry and its impact in the context of DUNE.

A. Analysis of global best-fit data

In Table II, we give the latest results of global-fit of neutrino oscillation data as obtained by NuFit-3.2 [33] collaboration. We notice from the table that the best-fit points of latest analysis favor higher octant for the 2-3 mixing angle, $\theta_{23}$ and non-maximal value for the Dirac CP phase, $\delta$ for both the mass orderings.

| Oscillation Parameters | NMO Best-fit | IMO Best-fit | Any Ordering 3σ |
|------------------------|-------------|-------------|------------------|
| $\Delta m^2_{21} [10^{-5} \text{eV}^2]$ | 7.40 | 7.40 | 6.80 $\rightarrow$ 8.02 |
| $|\Delta m^2_{31} [10^{-3} \text{eV}^2]|$ | 2.494 | 2.465 | 2.399 $\rightarrow$ 2.593 (NMO) 2.395 $\rightarrow$ 2.536 (IMO) |
| $\sin^2 \theta_{12}$ | 0.307 | 0.307 | 0.272 $\rightarrow$ 0.346 |
| $\sin^2 \theta_{23}$ | 0.538 | 0.554 | 0.418 $\rightarrow$ 0.613 |
| $\sin^2 \theta_{13}$ | 0.02206 | 0.02227 | 0.019 $\rightarrow$ 0.0243 |
| $\delta$ [deg] | 234 | 278 | 144 $\rightarrow$ 374 |

TABLE II: The best-fit values and $3\sigma$ range of neutrino oscillation parameters [33]

In Fig. 2, we present our results in ($\delta - \sin^2 \theta_{23}$)-plane for DUNE considering best-fit values of NuFit-3.2 data as our true benchmark point. Here red-star represents true value of ($\delta, \sin^2 \theta_{23}$) i.e., (234°, 0.538) and (278°, 0.554) corresponding to NMO and IMO, respectively. From first plot of top row, we notice that DUNE can exclude the possibility of having maximal CP-violation as well as CP-conservation hypothesis at 1σ C.L. as shown by green contour for NMO. On the other hand, it can not exclude either of these hypotheses at 3σ C.L. as can be seen from the pink contour which intersects with $\delta = 180°$ vertical blue-dotted line and $\delta = 270°$ vertical black-dashed line. Investigating bottom row for normal mass ordering, we notice from first plot that DUNE can exclude maximal CP-violation at 1σ C.L. similar as only neutrino mode of DUNE. Apart from this it can exclude CP-conservation hypothesis
at $3\sigma$ C.L. but not at higher confidence levels.

In case of IMO as shown in the right column, we notice that DUNE can not exclude the phenomenon of maximal CP-violation even at $1\sigma$ C.L. as depicted by green contour. But, it can exclude CP-conservation hypothesis approximately at $5\sigma$ C.L. as the blue contour marginally touches $\delta = 180^\circ$. On the other hand, it can reject CP-conservation hypothesis even at $5\sigma$ C.L. as described by blue contour of last plot for inverted mass ordering with 3.5 years each of neutrino and antineutrino run of DUNE. Furthermore, normal mass ordering of DUNE[3.5$\nu$+3.5$\bar{\nu}$] can marginally exclude lower octant (LO) of $\theta_{23}$ (i.e., when $\sin^2 \theta_{23} \leq 0.5$) at $1\sigma$ C.L. as depicted by green contour. Moreover, in case of IMO, we notice that it can rule out LO of $\theta_{23}$ clearly at $1\sigma$ C.L. but not at higher confidence levels.

FIG. 2: Allowed parameter space of DUNE in ($\delta - \sin^2 \theta_{23}$)-plane considering latest NuFit-3.2 data [33]. Remaining details are same as Fig. 1.
B. Breaking of $\mu - \tau$ reflection symmetry through $M_D$

We discuss here three different scenarios to break $\mu - \tau$ reflection symmetry by introducing explicit breaking parameter in the Dirac neutrino mass matrix, $M_D$. Further, for each case we perform precision study to determine $\delta, \sin^2 \theta_{23}$ considering DUNE. We study them as follows.

- Broken Scenario-1 (BS1): After assigning breaking parameter in the (12) position of $M_D$, the new Dirac neutrino mass matrix, $\hat{M}_D$ can be written as

\[
\hat{M}_D = \begin{pmatrix}
  a e^{i\phi_a} & a(1 + \epsilon)e^{-i\phi_a} \\
  b e^{i\phi_b} & c e^{i\phi_c} \\
  c e^{-i\phi_c} & b e^{-i\phi_b}
\end{pmatrix}.
\]  

(13)

The above texture of $\hat{M}_D$ leads to low energy neutrino mass matrix $\hat{M}_\nu$ of the form,

\[
\hat{M}_\nu \simeq M_\nu - \epsilon \frac{a e^{-i\phi_a}}{M_1} \begin{pmatrix}
  2a e^{-i\phi_a} & c e^{i\phi_c} & b e^{-i\phi_b} \\
  c e^{i\phi_c} & 0 & 0 \\
  b e^{-i\phi_b} & 0 & 0
\end{pmatrix} + O(\epsilon^2).
\]  

(14)

Now to find masses and mixing angles in presence of breaking term $\epsilon$, we diagonalize $\hat{M}_\nu$ with $\hat{V}$. Note that $\hat{V}$ has similar form as $V$ in the absence of $\epsilon$ as described by Eq. (5). In Table III, we give the expressions of modified masses and mixing angles for both the mass orderings. Note that for simplicity, we only consider the leading order corrections in terms of $\epsilon, \theta_{13}$ and $\xi_1 = m_2/m_3(\xi_2 = \Delta m^2_{21}/m^2_2)$ for NMO (IMO).

Afterwards, we proceed to find the set of neutrino oscillation parameters numerically in this scenario. We also emphasize here that the numerical analysis throughout this work are based on exact formula not on any leading order approximations. The numerical best-fit values at $\chi^2_{min}$ for both the mass orderings are tabulated in Table IV. Considering these set of values as the true benchmark point, we present allowed area in test $(\delta - \sin^2 \theta_{23})$-plane for DUNE in Fig. 3.

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7 We vary the breaking term $\epsilon$ in the range, [-1,1] along with other high energy parameters, as mentioned in Eq. (11).

8 Note that one can also perform various correlations study considering neutrino oscillation parameters in different broken scenarios. Recently, authors in Ref. [28] have performed different correlation study.
From first plot of Fig. 3, we notice that in BS1 scenario, DUNE can exclude the theory of maximal CPV at $3\sigma$ C.L. (see pink contour) for NMO even only with neutrino run. On the other hand at $5\sigma$ C.L., this case is unable to exclude both the concerned hypotheses in neutrino mode. With the combined equal neutrino and antineutrino mode analysis of DUNE, we observe that it can exclude the possibility of maximal CPV hypothesis at $3\sigma$ C.L. whereas CP-precision becomes poorer at $5\sigma$ C.L. as shown in first plot of second row. We also notice from both the plots of first column that as the best-fit value of $\delta$ is marginally away from CP conserving value (i.e. $\delta = 360^\circ$), this scenario can not exclude CP-conservation hypothesis even at $1\sigma$ C.L. In case of IMO, considering the best-fit values as given by third column of Table IV as the benchmark point, we notice that DUNE can exclude the phenomenon of CP-conservation at $3\sigma$.
FIG. 3: Allowed parameter space of DUNE in $(\delta - \sin^2 \theta_{23})$-plane in BS1 scenario. Here green, pink and blue color represent $1\sigma$, $3\sigma$ and $5\sigma$ allowed contours and ‘red-*’ signifies true value of $(\delta, \sin^2 \theta_{23})$.

C.L. but not at $5\sigma$ C.L. which is depicted in first plot of second column by pink contour. From second plot of right column, we observe that DUNE can reject CP-conservation hypothesis even at $5\sigma$ C.L. Further, both the cases of IMO can not reject the value corresponding to maximal CPV even at $1\sigma$ C.L. We also notice that precision of $\delta$ improves significantly when one chooses IMO over NMO and it gets even better with the combined mode of DUNE run as shown in the last plot. Finally, here we point out that DUNE can exclude $\delta$ in the range, $\delta \in [180^\circ, 360^\circ]$ at $3\sigma$ C.L. for IMO (see first plot of right column) whereas same conclusion remains permissible even at $5\sigma$ C.L. with combined $(\nu + \bar{\nu})$ analysis of DUNE (see second plot of right column).

- Broken Scenario-2 (BS2): In this scenario, we introduce breaking term, $\epsilon$, in the (22)
position of $M_D$ and this modifies $M_D$ (which we renamed $\hat{M}_D$) as

$$
\hat{M}_D = \begin{pmatrix}
ae^{i\phi_u} & ae^{-i\phi_u} \\
be^{i\phi_b} & c(1 + \epsilon)e^{i\phi_c} \\
be^{-i\phi_b} & \end{pmatrix}.
$$

(15)

Using the form of $\hat{M}_D$ as given by Eq. 15, we find modified $\hat{M}_\nu$ as,

$$
\hat{M}_\nu \simeq M_\nu - \epsilon \frac{cc^{\phi_c}}{M_1} \begin{pmatrix}
0 & ae^{-i\phi_u} & 0 \\
ae^{-i\phi_u} & 2cc^{\phi_c} & be^{-i\phi_b} \\
0 & be^{-i\phi_b} & 0
\end{pmatrix} + \mathcal{O}(\epsilon^2).
$$

(16)

To find modified masses and mixing angles in the given scenario, we follow the similar steps as discussed in subsection III B. In the following Table V, we give their expressions for both the mass orderings. The subleading order term in $\epsilon$ shows the corrections in active neutrino masses and mixing angles for this broken pattern.

| Parameters($S2$) | NMO | IMO |
|------------------|-----|-----|
| $\hat{m}_1 \simeq$ | 0 | $m_1 + \epsilon \frac{c}{M_1} \sqrt{2a} s_{12}^2 \sin \varphi_{ac}$ |
| $\hat{m}_2 \simeq$ | $m_2 + \epsilon \frac{c}{M_1} [-\sqrt{2a}s_{12}c_{12} \sin \varphi_{ac}^\mu + c_{12}(c \cos (2(\phi_c - \mu) - b \cos \varphi_{bc})]$ | $m_2 + \epsilon \frac{c}{M_1} [-\sqrt{2a}s_{12}c_{12} \sin \varphi_{ac}^\mu + c_{12}(c \cos (2(\phi_c - \mu) - b \cos \varphi_{bc})]$ |
| $\hat{m}_3 \simeq$ | $m_3 - \epsilon \frac{c}{M_1}[b \cos \varphi_{bc} + c \cos (2(\phi_c - \mu))]$ | 0 |
| $\hat{\theta}_{13} \simeq$ | $\theta_{13} - \epsilon \frac{ac}{\sqrt{2m_3 M_1}} \cos \varphi_{ac}^\mu$ | $\theta_{13} + \epsilon \frac{ac}{\sqrt{2m_2 M_1}} \cos \varphi_{ac}^\mu$ |
| $\hat{\theta}_{12} \simeq$ | $\theta_{12} - \epsilon \frac{c}{2m_2 M_1 \xi_1} c(\cos (2(\phi_c - \mu) - b \cos \phi_{bc})$ | $\theta_{12} - \epsilon \frac{c}{m_2 M_1 \xi_2} c(\cos (2(\phi_c - \mu) - b \cos \phi_{bc})$ |
| $\hat{\theta}_{23} \simeq$ | $45^\circ - \epsilon \frac{c^2}{m_3 M_1} \cos 2(\phi_c - \phi_\mu)$ | $45^\circ - \epsilon \frac{c^2}{m_3 M_1} \cos 2(\phi_c - \phi_\mu)$ |

TABLE V: Modified masses and mixing angles in $\text{BS2}$ scenario. Notation adopted here are same as Eq. 7 and Table III.

After finding analytical expressions, we now evaluate the numerical set of neutrino oscillation parameters in the broken scenario $\text{BS2}$. We calculate the best-fit values corresponding to $\chi^2_{min}$ numerically and present them in Table VI.

Moreover, considering the given set of values as our true benchmark point, we show allowed parameter space of DUNE in $(\delta - \sin^2 \theta_{23})$-plane for NMO as well as IMO in
| Parameters                  | NMO ($\chi^2_{\text{min}} = 1.01$) | IMO ($\chi^2_{\text{min}} = 4.58$) |
|-----------------------------|------------------------------------|------------------------------------|
| $\Delta m^2_{21}[10^{-5}\text{eV}^2]$ | 7.428                              | 7.56                               |
| $|\Delta m^2_{31}|[10^{-3}\text{eV}^2]$  | 2.499                              | 2.450                              |
| $\sin^2 \theta_{12}$       | 0.305                              | 0.301                              |
| $\sin^2 \theta_{23}$       | 0.49                               | 0.51                               |
| $\sin^2 \theta_{13}$       | 0.0218                             | 0.0229                             |
| $\delta \ [\text{deg}]$     | 89                                 | 125                                |

**TABLE VI**: Set of neutrino oscillation parameters corresponding to $\chi^2_{\text{min}} = 1.01 (= 4.58)$ for NMO (IMO) in BS2 scenario.

**FIG. 4**: Allowed parameter space of DUNE in $(\delta - \sin^2 \theta_{23})$-plane in BS2 scenario. Here green, pink and blue colors represent $1\sigma$, $3\sigma$ and $5\sigma$ allowed contours and red-* signifies true value of $(\delta, \sin^2 \theta_{23})$.

Fig. 4. In case of NMO, as described in left panel, we notice that as the concerned scenario predicts, $\delta = 89^\circ$ and $\sin^2 \theta_{23} = 0.49$ neutrino mode of DUNE can not rule out maximal CPV hypothesis even at $1\sigma$ C.L. This observation prolongs further even
with the inclusion of antineutrino run with neutrino as depicted by second plot of first panel. Whereas neutrino mode of it can exclude CP-conservation hypothesis at $3\sigma$. In addition, combined $(\nu + \bar{\nu})$ run can reject the same scenario approximately at $5\sigma$ C.L. as shown by blue contour. We also notice by comparing both the plots of left column that DUNE can reject $\delta$ in the range, $\delta \in [180^\circ, 360^\circ]$ at $3\sigma$, $5\sigma$ C.L. considering only neutrino and combined $(\nu + \bar{\nu})$ mode run of DUNE, respectively. From right panel (which is for IMO), we find that both cases can rule out maximal CPV as well as CP-conservation hypothesis only at $1\sigma$ C.L. Besides this, we notice that only neutrino mode data of DUNE can exclude CP-conservation hypothesis at $3\sigma$ C.L. but not at $5\sigma$ C.L. Whereas combined effect of $(\nu + \bar{\nu})$ can reject the same hypothesis at $5\sigma$ C.L. as shown in bottom right panel by blue contour. At the end, we notice from right panel that CP precision improves significantly with the combined effect of neutrino and antineutrino run for DUNE and it can successfully exclude $\delta$ in the range, $\delta \in [180^\circ, 360^\circ]$ at $5\sigma$ C.L. In addition, comparing both the columns we find here that NMO shows better CP precision over IMO.

- **Broken Scenario-3 (BS3):** Here, we assign breaking parameter in the (32) position of $M_D$ and we write the new Dirac neutrino mass matrix, $\tilde{M}_D$ as

\[
\tilde{M}_D = \begin{pmatrix}
ae^{i\phi_a} & ae^{-i\phi_a} \\
b e^{i\phi_b} & ce^{i\phi_c} \\
ce^{-i\phi_c} b(1 + \epsilon) e^{-i\phi_b}
\end{pmatrix},
\]

(17)

$\tilde{M}_D$ given by Eq. (17), leads us to the following $\tilde{M}_\nu$ through type - I seesaw formalism,

\[
\tilde{M}_\nu \simeq M_\nu - \frac{\beta e^{-i\phi_b}}{M_1} \begin{pmatrix}
0 & 0 & ae^{-i\phi_a} \\
0 & 0 & ce^{i\phi_c} \\
b e^{-i\phi_a} & ce^{i\phi_c} & 2be^{-2i\phi_b}
\end{pmatrix} + \mathcal{O}(\epsilon^2).
\]

(18)

Now we diagonalize $\tilde{M}_\nu$ as given by Eq. (18) to find corrections in masses and mixing angles. Here also we perform similar study as discussed in subsection IIIB. In Table VII, we give analytical expressions for masses and mixing angles considering both the mass orderings where $\mathcal{O}(\epsilon)$ term shows the correction in active neutrino masses and mixing angles for the concerned scenario.
\[ \hat{m}_1 \simeq 0 \]

\[ \hat{m}_2 \simeq m_2 + \frac{b}{M_1} \left[ \sqrt{2} s_{12} c_{12} \sin \phi_{ab} \right. \\
+ \left. c_{12}^2 (b \cos (\phi_b - \phi_\mu) - c \cos \phi_{bc}) \right] \]

\[ \hat{m}_3 \simeq m_3 - \frac{b}{M_1} \left[ b \cos (\phi_b - \phi_\mu) + c \cos \phi_{bc} \right] \]

\[ \hat{\theta}_{13} \simeq \theta_{13} - \frac{ab}{\sqrt{2} m_3 M_1} \cos \phi_{ab} \]

\[ \hat{\theta}_{12} \simeq \theta_{12} + \frac{b}{2 m_3 M_1 \xi_1} \left[ (c \cos \phi_{bc} - b \cos 2(\phi_b - \phi_\mu)) \sin 2\theta_{12} \\
+ \sqrt{2} a \cos 2\theta_{12} \sin \phi_{ab} \right] \]

\[ \hat{\theta}_{23} \simeq 45^\circ + \frac{b^2}{m_3 M_1} \cos 2(\phi_b - \phi_\mu) \]

**TABLE VII:** Modified masses and mixing angles in BS3 scenario. Notation adopted here are same as Eq. 7 and Table III.

| Parameters (S3) | NMO | IMO |
|----------------|-----|-----|
| \( \hat{m}_1 \) | 0   |     |
| \( \hat{m}_2 \) | \( m_2 + \frac{b}{M_1} \left[ \sqrt{2} s_{12} c_{12} \sin \phi_{ab} \right. \\
+ \left. c_{12}^2 (b \cos (\phi_b - \phi_\mu) - c \cos \phi_{bc}) \right] \) | \( m_2 + \frac{b}{M_1} \left[ \sqrt{2} s_{12} c_{12} \sin \phi_{ab} \right. \\
+ \left. c_{12}^2 (b \cos (\phi_b - \phi_\mu) - c \cos \phi_{bc}) \right] \) |
| \( \hat{m}_3 \) | \( m_3 - \frac{b}{M_1} \left[ b \cos (\phi_b - \phi_\mu) + c \cos \phi_{bc} \right] \) | 0 |
| \( \hat{\theta}_{13} \) | \( \theta_{13} - \frac{ab}{\sqrt{2} m_3 M_1} \cos \phi_{ab} \) | \( \theta_{13} + \frac{ab}{\sqrt{2} m_3 M_1} \cos \phi_{ab} \) |
| \( \hat{\theta}_{12} \) | \( \theta_{12} + \frac{b}{2 m_3 M_1 \xi_1} \left[ (c \cos \phi_{bc} - b \cos 2(\phi_b - \phi_\mu)) \sin 2\theta_{12} \\
+ \sqrt{2} a \cos 2\theta_{12} \sin \phi_{ab} \right] \) | \( \theta_{12} + \frac{b}{m_2 M_1 \xi_2} \left[ (c \cos \phi_{bc} - b \cos 2(\phi_b - \phi_\mu)) \sin 2\theta_{12} \\
+ \sqrt{2} a \cos 2\theta_{12} \sin \phi_{ab} \right] \) |
| \( \hat{\theta}_{23} \) | \( 45^\circ + \frac{b^2}{m_3 M_1} \cos 2(\phi_b - \phi_\mu) \) | \( 45^\circ + \frac{b^2}{m_2 M_1} \cos 2(\phi_b - \phi_\mu) \) |

**TABLE VIII:** Set of neutrino oscillation parameters corresponding to \( \chi^2_{min} = 0.62 \) (= 5.39) for NMO (IMO) in BS3 scenario.

| Parameters | NMO \( (\chi^2_{min} = 0.62) \) | IMO \( (\chi^2_{min} = 5.39) \) |
|------------|-----------------|-----------------|
| \( \Delta m^2_{21} [10^{-5} eV^2] \) | 7.49 | 7.28 |
| \| \( \Delta m^2_{31} [10^{-3} eV^2] \) | 2.493 | 2.428 |
| \( \sin^2 \theta_{12} \) | 0.311 | 0.316 |
| \( \sin^2 \theta_{23} \) | 0.56 | 0.51 |
| \( \sin^2 \theta_{13} \) | 0.0219 | 0.0229 |
| \( \delta \ [deg] \) | 252 | 140 |

Having discussed analytical results, we proceed to find the set of neutrino oscillation parameters in the broken scenario BS3. We calculate the best-fit values corresponding to \( \chi^2_{min} \) numerically and present them in Table VIII. Using this set of true benchmark point, we examine allowed parameter space of DUNE in \((\delta - \sin^2 \theta_{23})\)-plane for both the mass orderings as shown in Fig. 5 (see figure caption for the adopted color convention and other minutes details).

We observe from first plot of left panel that DUNE with only neutrino mode data is not able to exclude the phenomenon of maximal CPV even at 1\( \sigma \) C.L. (see green contour for NMO). Whereas it can exclude CP-conservation hypothesis at 3\( \sigma \) C.L. (see pink contour for NMO) but not at 5\( \sigma \) C.L. as blue contour intersect with blue-dotted vertical
FIG. 5: Allowed parameter space of DUNE in \((\delta - \sin^2 \theta_{23})\)-plane in BS3 scenario. Here green, pink and blue colors represent 1\(\sigma\), 3\(\sigma\) and 5\(\sigma\) allowed contours and red-* signifies true value of \((\delta, \sin^2 \theta_{23})\).

We find that similar conclusion remains permissible for the combined effect of \((\nu + \bar{\nu})\) run of DUNE as shown in first plot of second row. In case of IMO with 7-years neutrino run, we find that DUNE can reject both the concerned hypotheses at 1\(\sigma\) C.L. whereas at higher confidence levels it fails to rule out any of these hypothesis as depicted in the first plot of second panel. Investigating right hand side plot of second row, we notice that at 1\(\sigma\) C.L. it shows similar behaviour as neutrino mode whereas at 3\(\sigma\) C.L. it is able to rule out CP-conservation hypothesis but not maximal CPV as shown by pink contour. Finally, we observe a noteworthy outcome in this scenario compare to former two breaking patterns that this scenario can exclude lower octant of \(\theta_{23}\) at 1\(\sigma\) C.L. for NMO even with 7-years of neutrino mode data of DUNE.
C. Breaking of $\mu - \tau$ reflection symmetry through $M_R$

We discuss here the breaking of $\mu - \tau$ reflection symmetry by introducing explicit breaking parameter in the Majorana neutrino mass matrix, $M_R$. We discuss the scenario as below.

- Broken Scenario-4 (BS4): After assigning breaking parameter in the (22) position of $M_R$, the modified Majorana neutrino mass matrix, $\hat{M}_R$ becomes,

$$\hat{M}_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_1(1 + \epsilon) \end{pmatrix}. \quad (19)$$

Note here that in this scenario $\hat{M}_R$ becomes non-degenerate. After integrating out heavy right-handed neutrino fields, the low energy neutrino mass matrix in the type-I seesaw formalism can be written as

$$\hat{M}_\nu \simeq M_\nu - \frac{\epsilon}{M_1} \begin{pmatrix} a^2 e^{-2i\phi_a} & ace^{-i(\phi_a - \phi_c)} & abe^{-i(\phi_a + \phi_b)} \\ -b^2 e^{-2i\phi_b} & bce^{-i(\phi_b - \phi_c)} \end{pmatrix} + O(\epsilon^2). \quad (20)$$

| Parameters          | NMO ($\chi^2_{min} = 0.53$) | IMO ($\chi^2_{min} = 3.91$) |
|---------------------|-----------------------------|-----------------------------|
| $\Delta m^2_{31}$ [10^{-5}eV^2] | 7.31                        | 7.38                        |
| $|\Delta m^2_{31}|[10^{-3}eV^2]$ | 2.497                       | 2.456                       |
| $\sin^2 \theta_{12}$ | 0.302                       | 0.303                       |
| $\sin^2 \theta_{23}$ | 0.53                        | 0.50                        |
| $\sin^2 \theta_{13}$ | 0.02179                     | 0.02228                     |
| $\delta$ [deg]     | 280                         | 33                          |

TABLE IX: Set of neutrino oscillation parameters corresponding to $\chi^2_{min} = 0.53$ (= 3.91) for NMO (IMO) in BS4 scenario.

In this framework, we notice from Eq. (20) that as all the entries of $O(\epsilon)$ term are non-zero, it is highly non-trivial to perform analytical study and to find expressions for modified neutrino masses and mixing angles. Therefore, we proceed to employ only numerical study unlike previous subsections where both analytical as well as numerical study were performed. The set of neutrino oscillation parameters at $\chi^2_{min}$ for possible mass ordering are tabulated in Table IX. We notice from table that best-fit values corresponding to $\chi^2_{min}$ deviates from maximal ($\delta, \theta_{23}$) for NMO whereas for IMO the given mass textures still favor maximal $\theta_{23}$ but not maximal $\delta$. 

20
After finding set of best-fit values at $\chi^2_{\text{min}}$, we proceed to analyze its impact on DUNE. Performing similar kinds of analysis as illustrated in the former broken scenarios, here also we show the allowed parameter space of DUNE considering two poorly determined parameters, viz, $\delta$ and $\sin^2 \theta_{23}$. We show our results in Fig. 6 considering test $(\delta - \sin^2 \theta_{23})$-plane. Now from both the plots of first column, we notice that as the given mass textures have chosen the value of Dirac CP-phase, $\delta$ slightly away from its maximal value at $\chi^2_{\text{min}}$, DUNE fails to rule out the phenomenon of maximal CPV even at 1$\sigma$ C.L. In fact, it can rule out CP-conservation hypothesis at 3$\sigma$ C.L. even with only neutrino run as shown in first plot of top row by pink contour. We see similar conclusion from the second plot of first column. We also notice here that DUNE with 3.5 years of each neutrino and antineutrino mode data can approximately exclude $\delta$
in the range, \( \delta \in [0^\circ, 180^\circ] \) at 5\( \sigma \) C.L. for the normal mass ordering. In case of IMO as depicted in the right column, we find that DUNE can exclude both the concerned phenomena, viz, maximal CPV and CP-conservation at 1\( \sigma \) C.L. but not at higher confidence levels. Also, none of the cases are able to rule out lower octant of \( \sin^2 \theta_{23} \) even at 1\( \sigma \) C.L. In addition, we find here that NMO shows better CP-precision over IMO.

We add a remark here that as the Majorana neutrino mass matrix is always symmetric, addition of non-zero off-diagonal entry still respect \( \mu - \tau \) flavor symmetry and predicts maximal \( \delta, \sin^2 \theta_{23} \). Hence, here we do not include this as an additional scenario.

We now summarize our results in Table X for the different scenarios which are depicted in Figs. 1-6. We show the possibility of ruling out maximal CP-violation (mCPV) or CP-

| Scenarios | mCPV (CPC) |
|-----------|------------|
|           | 1\( \sigma \) | 3\( \sigma \) | 5\( \sigma \) |
|           | (7\( \nu \) + 0\( \nu \)) | (3.5\( \nu \) + 3.5\( \nu \)) | (7\( \nu \) + 0\( \nu \)) | (3.5\( \nu \) + 3.5\( \nu \)) |
| NMO       | \( \times(\sqrt{\checkmark}) \) | \( \times(\checkmark) \) | \( \times(\checkmark) \) | \( \times(\checkmark) \) |
| GF        | \( \sqrt{\checkmark} \) | \( \sqrt{\checkmark} \) | \( \checkmark \) | \( \checkmark \) |
| BS1       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS2       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS3       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS4       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |

| Scenarios | mCPV (CPC) |
|-----------|------------|
|           | 1\( \sigma \) | 3\( \sigma \) | 5\( \sigma \) |
|           | (7\( \nu \) + 0\( \nu \)) | (3.5\( \nu \) + 3.5\( \nu \)) | (7\( \nu \) + 0\( \nu \)) | (3.5\( \nu \) + 3.5\( \nu \)) |
| IMO       | \( \times(\sqrt{\checkmark}) \) | \( \times(\checkmark) \) | \( \times(\checkmark) \) | \( \times(\checkmark) \) |
| GF        | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS1       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS2       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS3       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |
| BS4       | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) | \( \checkmark \) |

TABLE X: The possibility of ruling out maximal CP-violation (mCPV) or CP-conservation (CPC) hypothesis for both the mass orderings at different C.L. in case of DUNE. We denote the concerned hypothesis (i.e. mCPV/CPC) by ‘\( \sqrt{\checkmark} \)’ (‘\( \checkmark \)’) mark when DUNE is able (unable) to rule out the given scenario. Also parenthesis in bracket shows our result for CPC hypothesis. Note that here ‘\( \mu - \tau \)’ refers to symmetry scenario and abbreviation ‘GF’ stands for scenario corresponding to global-fit data.
TABLE XI: Precision table of $\delta, \sin^2 \theta_{23}$ for all the considered scenarios of $(\delta, \sin^2 \theta_{23})$ in case of DUNE[$7\nu + 0\bar{\nu}$] and DUNE[$3.5\nu + 3.5\bar{\nu}$] at $3\sigma$ C.L.

| Scenarios | NMO (in %) | IMO (in %) |
|-----------|------------|------------|
|           | $P(\delta)$ | $P(\sin^2 \theta_{23})$ | $P(\delta)$ | $P(\sin^2 \theta_{23})$ |
| $\mu - \tau$ | (7$\nu + 0\bar{\nu}$) | (3.5$\nu + 3.5\bar{\nu}$) | (7$\nu + 0\bar{\nu}$) | (3.5$\nu + 3.5\bar{\nu}$) |
| GF        | 32.5       | 31.9       | 8.7       | 9.3       |
| BS1       | 36.1       | 25.0       | 9.2       | 8.9       |
| BS2       | 31.4       | 31.9       | 9.3       | 9.8       |
| BS3       | 34.2       | 32.7       | 11.5      | 11.3      |
| BS4       | 41.1       | 38.6       | 9.0       | 9.3       |
|           | 37.5       | 32.2       | 8.7       | 9.1       |
|           | 35.0       | 31.6       | 11.6      | 10.5      |
|           | 38.8       | 30.0       | 8.9       | 9.4       |
|           | 37.5       | 35.9       | 8.9       | 8.5       |
|           | 40.8       | 36.9       | 8.9       | 8.8       |
|           | 37.5       | 33.8       | 8.9       | 9.4       |

conservation (CPC) hypothesis by ticks-mark (✓) considering DUNE. Whereas if DUNE fails to rule out a concerned hypothesis, we mark this with cross-mark (✗). Note that the parenthesis in bracket shows our results for CPC hypothesis (see table caption for details).

Finally, we calculate the precisions of the two poorly measured parameter $\delta$ and $\sin^2 \theta_{23}$. The precision ($P$) can be defined as

$$P(\delta) = \frac{\delta_{\text{max}} - \delta_{\text{min}}}{360^\circ} \times 100\% ,$$

$$P(\sin^2 \theta_{23}) = \frac{(\sin^2 \theta_{23})_{\text{max}} - (\sin^2 \theta_{23})_{\text{min}}}{(\sin^2 \theta_{23})_{\text{max}} + (\sin^2 \theta_{23})_{\text{min}}} \times 100\% .$$

Here, max (min) refers to the maximum (minimum) value of the concerned parameter in a given contour. Also, we present the precision table considering $3\sigma$ confidence level for all the cases that we have considered here around their true values.

### IV. CONCLUSION

In this paper we present an elaborate discussion on the capability of DUNE experiment to test the consequences of $\mu - \tau$ reflection symmetry considering two different modes namely, (i) 7-years of neutrino run and (ii) 3.5-years each of neutrino and antineutrino run. In addition, to realize $\mu - \tau$ reflection symmetry in the low energy neutrino mass matrix under minimal type - I seesaw formalism, we add two heavy right-handed neutrino fields in the SM. This symmetry predicts maximal atmospheric mixing angle (i.e., $\theta_{23} = 45^\circ$) and Dirac CP phase (i.e., $\delta = \pm 90^\circ$) along with trivial Majorana phases in the leptonic sector. In
this framework, we also find remaining oscillation parameters both analytically as well as numerically. Later, considering numerical best-fit values of neutrino oscillation parameters as our true benchmark point, we find the allowed area in $(\delta - \sin^2 \theta_{23})$ plane for DUNE. Further, as the latest global best-fit data prefer non-maximal $\delta$ as well as $\theta_{23}$, we perform our study considering global best-fit values as one of our true benchmark point in the context of DUNE. Subsequently, we extend our study to break $\mu - \tau$ reflection symmetry by introducing explicit breaking term in the high energy Dirac and Majorana neutrino mass matrices, respectively. Given the breaking scenario, we calculate the set of neutrino oscillation parameters and considering this set as true benchmark point we find the allowed area in test $(\delta - \sin^2 \theta_{23})$ plane for DUNE. It is noteworthy to make a note here that allowed parameter space in test $(\delta - \sin^2 \theta_{23})$ plane also gives an idea about the precision of these two poorly determined parameters for DUNE. Later, we examine the potential of DUNE to rule out maximal CP-violation (CPV) or CP-conservation hypothesis in each broken scenario.

We summarize DUNE’s capability to test concerned hypotheses for all considered cases in Table X. Given the framework of $\mu - \tau$ reflection symmetry, we notice that DUNE can rule out CP-conservation hypothesis at $3\sigma$ confidence level even with only neutrino mode run for both the mass orderings, respectively. Whereas, DUNE$[3.5\nu + 3.5\overline{\nu}]$ mode can reject the same at $5\sigma$ only in case of IMO. Further, considering global best-fit values as one of our concerned case, we find that both the considered modes of DUNE can exclude both hypothesis at $1\sigma$ C.L. only for NMO. Whereas it can exclude CP-conservation hypothesis at $5\sigma$ C.L. for IMO with $(3.5\nu + 3.5\overline{\nu})$ mode of DUNE but not in case of NMO. Later, by inspecting broken scenario BS1, we notice that DUNE can exclude the phenomenon of maximal CPV at $3\sigma$ C.L but not the phenomenon of CP-conservation even at $1\sigma$ C.L. for NMO. Subsequently for IMO, we find that it can rule out CP-conservation hypothesis even at $5\sigma$ C.L. with DUNE$[3.5\nu + 3.5\overline{\nu}]$ but not maximal CPV hypothesis. Moving to BS2 scenario, we observe that both the specifications of DUNE can exclude CP-conservation hypothesis at $3\sigma$ C.L. for NMO as well as IMO. Besides this, it can rule out theory of CP-conservation even at $5\sigma$ C.L. only for inverted mass ordering. Examining both BS3, BS4 scenarios, we come to conclusion that DUNE can exclude either of maximal CP-violation or CP-conservation hypothesis at $1\sigma$ C.L. for IMO. Whereas, both of the scenarios can rule out CP-conservation hypothesis at $3\sigma$ C.L. only for NMO. In case of IMO, BS3 can rule out CP-conservation hypothesis at $3\sigma$ C.L. whereas BS4 can exclude maximal CPV hypothesis
at 3σ C.L. considering DUNE[3.5ν + 3.5ν̄]. In addition, by inspecting all the scenarios for both the mass orderings, we notice that none of the scenarios of NMO can exclude any of the concerned hypothesis at 5σ C.L. Whereas, except BS3, BS4, remaining scenarios of IMO can exclude CP-conservation hypothesis with DUNE[3.5ν + 3.5ν̄] at the same confidence level.

Afterwards, we also examine the precision of both the less known parameters, δ, θ_{23} and as a case study we present our results at 3σ confidence level in Table XI. By scrutinizing all the possibilities, we notice that case BS4 gives worst precision on the Dirac CP-phase, δ of 41.1% in case of DUNE[7ν + 0ν̄] for NMO. Whereas BS1 comes with best precision of 25.0% among all concerned cases considering DUNE[3.5ν + 3.5ν̄] for NMO. Similarly, for 2-3 mixing angle, θ_{23}, we find that global best-fit value with DUNE[7ν + 0ν̄] mode gives worst precision of 11.6% for IMO. Whereas BS2 for IMO gives best precision of 8.5% for DUNE[3.5ν + 3.5ν̄]. Also, by investigating all scenarios, we notice that scenario BS3 is able to exclude the lower octant of θ_{23} at 1σ C.L. for NMO and analysis of global best-fit value shows similar conclusion in context of IMO. Note that results discussed here can be used to test DUNE’s potential for the discrimination of different scenarios.

Finally, we conclude this work with a remark that with the available data in the neutrino oscillation sector, μ - τ reflection symmetry possesses as one of the finest theoretically favored approach to study some intriguing aspects of neutrinos. On the other hand forthcoming experiment, like DUNE with its high statistics and ability to measure (δ, θ_{23}) with high precisions serves as an impeccable experiment to test numerous predictions of different models.

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