A joint theoretical–experimental study focusing on the description of the ferromagnetic resonance response of thin films in the presence of periodic perturbations introduced on the upper film surface is presented. From the viewpoint of theory, these perturbations may exist in the form of any kind of one- or two-dimensional rectangular defect arrays patterned onto one surface of the magnetic film. Indeed, the defects may be pits or bumps, or ion-implanted regions with a lower saturation magnetization. The complete set of response functions, given by the components of the frequency and wave-vector dependent dynamic magnetic susceptibility tensor of the film exposed to microwave excitation, are provided and are used to explain the experimental data. This allows us to obtain the response of the system due to microwave absorption, from which the zero wave-vector spin-wave modes in the field-frequency spectra, including their intensity, are calculated. Explicit calculations for periodic defects featuring the shape of stripes, dots and rectangles are given.
in detail, as well as experimental results for stripe-like defects prepared either by
topographical depressions or by ion implantation of thin magnetic films. The
excellent agreement of the theoretical and experimental results manifests the
validity of the presented model.

1. Introduction

Magnetic nanostructures are currently produced by means of many advanced techniques,
e.g. electron beam lithography (EBL), deep-ultraviolet photo lithography, focused ion beam
patterning or ion beam erosion. With these and other techniques, various types of nanostructures
can be fabricated, including ultrathin films, multilayer structures, nanowires, nanotubes and dot
and antidot lattices, among several other patterned structures [1]. Their static and dynamic
magnetic properties depend strongly on their size and topology and can be further tailored
by incorporating geometrical perturbations on the nanometer scale or by combining different
materials and processing capabilities. A prominent step in this regard has been the patterning
of thin magnetic films to form one-dimensional (1D) or two-dimensional (2D) magnonic
crystals (MCs)—a new class of periodically modulated magnetic materials that, due to the
unique properties of spin-waves, should offer relevant new functionalities that are currently
unavailable in photonic or electronic devices [2–14]. For example, the spin-wave spectrum
in MCs can be manipulated through a periodic magnetic field, whose origin may be in the
stray fields created by the magnetization at the edges of the periodic structure. In fact the
spin-wave spectra in MCs show tunable band gaps, analogous to photons in a photonic crystal
and electrons in a crystal [2–4]. In such MCs, usually the entire height of the magnetic layer
is structured in contrast to perturbed or modulated samples. Typical MCs that are currently
being investigated include (i) hybrid structures composed by alternating materials [5–9],
(ii) dipolarly coupled nanowire arrays [10–12], (iii) nanostripes with periodically varying
width [13], (iv) antidot lattices [4, 11, 14–16] and (v) thin films with 1D and 2D arrays of
periodic perturbations [17, 18]. Periodic defect structures do not necessarily need to be of
geometrical type, like magnetic films on a grooved [19, 20] or ripple substrate [21], but can
consist of a variation of the magnetic parameters, i.e. a periodic modulation of the saturation
magnetization \( M_s \), as created by ion implantation [17].

It is well established that a uniformly magnetized ferromagnetic thin film exposed to
microwaves will absorb part of the microwave power at certain frequencies and dc fields.
This physical process takes place since the applied magnetic fields induce a precessional
motion on the magnetization. Since the magnetization’s (or spin’s) precession is intrinsically
(or extrinsically) damped, the driving microwaves provide the energy to compensate for
the dissipation due to the damping processes. In this regard, the resonant absorption of
microwave energy can be realized when the natural frequency of precession is equal to
the microwave frequency. Usually, ferromagnetic resonance (FMR) measurements probe the
uniform precession mode, corresponding to a spin-wave with wave vector \( \mathbf{k} = 0 \). Nowadays,
FMR is a powerful technique from which, through measurement of the absorption spectra,
useful information can be inferred, comprising magnetic anisotropies, interlayer coupling,
g-factor, spin-torque and damping mechanisms, for example [22–27].
The spin damping mechanisms are usually associated with the width of the FMR absorption line, i.e. the linewidth. The most studied case corresponds to a flat ferromagnetic thin film, where, in the case that solely phenomenological Gilbert damping is present in the sample, the Landau–Lifshitz–Gilbert equation gives a linewidth directly proportional to the microwave frequency [23–27]. However, it has been well-known for decades that FMR measurements in yttrium iron garnet (YIG) spheres gave linewidths much larger than expected from a pure intrinsic damping mechanism [28]. After polishing the YIG surface, the measured linewidth was decreased considerably, reducing the extrinsic contributions that arose from defects on the surface. In 1961, Sparks et al [29] presented a theory to explain extrinsic relaxation mechanisms, where two-magnon scattering (TMS) between the uniform FMR mode and degenerate magnons was activated by defects in the sample. Within the physical picture of TMS, defects can be viewed as scattering centers where the uniform mode relaxes to degenerate magnon states [24, 30–38].

In this paper, we present a generalization of the theory of spin-waves in 1D arrays of stripe defects presented in [18], where the striking peaks observed recently by Barsukov et al [17] in the frequency variation of the linewidth of periodically modulated permalloy films, were explained, providing insights into their origin. According to TMS-processes, the theory presented here can be utilized to consider both kinds of defect structures, either the geometrical one (with $M_s' = 0$), or defects created by ion implantation (with $M_s' < M_s$), where $M_s$ is the saturation magnetization of the ferromagnetic material and $M_s'$ is the saturation magnetization within the defects. We extend these ideas to arbitrary defect shapes as well as 2D arrays covering a wide range of periodically patterned magnetic thin films (see figure 1). We then use the theory to obtain an insight into the origin of the splitting of FMR modes and their connection to the magnonic band-gaps. As examples, we discuss the cases where the perturbations are in the form of dots, squares and stripes, corresponding to zones on the surface with a reduced saturation magnetization ($M_s'$), different from the value for the unperturbed film ($M_s$). The theory presented here is then used to explain recent FMR data in thin films with stripe-like defects, obtaining a good agreement between theory and experiment. Two kinds of patterned samples were fabricated—ion-implanted and topographic (etched) films—where for both cases EBL was used to mask the defect areas. The outline of the paper is as follows. In section 2, the theory is presented where general and specific formulas are given for the defect arrays (technical details are left in the appendix). In section 3, our main theoretical findings are shown and discussed and in section 4, we compare the theory with experimental results. Final remarks are summarized in section 5.

2. Theoretical description

In this section, we present a theoretical description of the influence that a rectangular array of periodic defects patterned onto a thin ferromagnetic film has on the FMR response. We start with a general discussion that considers defects with arbitrary shape. Then, we derive analytical expressions to the particular cases of circular and rectangular shaped defects, and we show that our picture can also be applied to a film patterned with a 1D array of stripe defects, as reported recently [17, 18]. It is worth mentioning here that our calculations also provide information about the FMR intensities of the spin-wave modes and not only about the selection criteria for the modes.
Figure 1. Schematic representation of a 2D rectangular array of defects on the upper surface of the film. In (a), an overview is shown, while in (b), the top view shows the magnetization orientation, which lies in the plane of the film and makes an angle $\psi$ with the $z$ direction. The coordinate system ($x$, $y$, $z$) is oriented according to the defect array, while ($X$, $Y$, $Z$) is given by the equilibrium magnetization. The periodicities in the $x$ and $z$ direction are given by $a_x$ and $a_z$, respectively, while the widths of the defects are denoted as $w_x$ and $w_z$. In (c), the cross section of the film of thickness $d$ and magnetization $M_s$ is illustrated. Here, the defects correspond to the perturbed zones with magnetization $M'_s$ and depth $h$.

2.1. General model for arbitrary defect shape

We consider the case where the surface of a magnetic thin film is periodically perturbed, as shown in figure 1. If a magnetic field is applied in the film plane, the static magnetization will be oriented along that field and will induce a periodic distribution of magnetic charges at the lateral surfaces (or interfaces) of each defect. The dipolar energy associated with the system in figure 1 can be evaluated from

$$E_d = -\frac{1}{2} \int \mathbf{M} \cdot \mathbf{h}_d \, d^3r,$$

where the magnetization can be written as $\mathbf{M} = \mathbf{M}^{(0)} + \mathbf{M}^{(1)} + \mathbf{M}^{(2)}$ and generates the dipolar fields $\mathbf{h}_d(\mathbf{M}^{(0)})$, $\mathbf{h}_d(\mathbf{M}^{(1)})$ and $\mathbf{h}_d(\mathbf{M}^{(2)})$, respectively. Here, the upper index indicates the order in the spin deviation or dynamic magnetization. Furthermore, we have to separate the magnetization into two parts. These are the magnetization of the perfect film and the magnetization inside the defects, namely $\mathbf{M}^{(i)} = \mathbf{M}^{(i)}_{\text{film}} + \mathbf{M}^{(i)}_{\text{def}}$. Notice that $\mathbf{M}^{(i)}_{\text{film}}$ is zero outside the film of thickness $d$, while $\mathbf{M}^{(i)}_{\text{def}}$ is zero outside the defects (zones with $M'_s$ in figure 1). Then, considering only terms of quadratic order in the dynamic magnetization, we can write the total
dipolar energy as

\[ E_d = E_d^{\text{film}} + V, \tag{2} \]

where the first term is associated with the perfect film, while the second term is related to the defects, and corresponds to the TMS matrix [30]

\[ V = -\int_{v_{\text{film}}} \left[ M_{\text{film}}^{(2)} \cdot h_d^{(0)} + M_{\text{film}}^{(1)} \cdot h_d^{(1)} \right] d^3r - \int_{v_{\text{def}}} \left[ M_{\text{def}}^{(2)} \cdot h_d^{(0)} + \frac{1}{2} M_{\text{def}}^{(1)} \cdot h_d^{(1)} \right] d^3r. \tag{3} \]

Here we have used the reciprocity theorem and the notation \( h_d^{(0,1)} = h_d(M_{\text{def}}^{(0,1)}) \). The first integral in equation (3) corresponds to the interaction between the film and the defects and is proportional to \((h/d)^2\), while the second integral stems from the interaction between the defects and is proportional to \((h/d)^2\). Therefore, the latter can be set aside if the condition \( h \ll d \) is fulfilled. In any case, we have to obtain the dipolar fields created by the static \( (h_d^{(0)}) \) and dynamic \( (h_d^{(1)}) \) magnetization of the defects.

The theory we present focuses on systems as shown in figure 1. It is based on a previous study [18] for 1D defect structures with ridges on the upper surface of the film that can be considered as zones without magnetic matter, and hence \( M' = 0 \). This picture is now extended to include 2D defect arrays with \( M' = 0 \), as well as the cases with \( M' \neq 0 \), corresponding to defects with magnetic matter, but with different properties. This can include, for instance, defects with a different magnetic material, and defects created by ion-implantation (see the experiments in section 4), where the saturation magnetization is modified depending on the details of the ion-implantation procedure [17]. Then, within the defects, the saturation magnetization should be a function of the normal coordinate: \( M' = M'_x(y) \). As the character of these samples is quite complex, it is very difficult to know the precise dependence of \( M'_x(y) \) with the normal coordinate, since it depends on the penetration length of the ions and on the concentration beneath the surface [17]. As an approximation, we model the ion implanted zones as having a depth \( h \) and an average magnetization \( M' \), which is in general lower than the saturation magnetization of the perfect film. When the magnetization has a component perpendicular to the interface around a given defect, there will be a divergence in the magnetization and magnetic charges will appear creating a periodic dipolar field. In order to determine the dipolar field \( h_d^{(0)} \) created by the zero order magnetization of the defects, we expand the components of this static magnetization

\[ M'_z = M'_z \cos \psi \sum_{pq} \tilde{C}_{pq}(y) \exp \left[i g^{pq} \cdot \mathbf{r}\right], \]
\[ M'_x = M'_x \sin \psi \sum_{pq} \tilde{C}_{pq}(y) \exp \left[i g^{pq} \cdot \mathbf{r}\right]. \tag{4} \]

The reciprocal lattice vector of the array of defects is denoted by \( g^{pq} = g^{pq}_x \hat{x} + g^{pq}_z \hat{z} \), with \( g^{pq}_x = 2\pi p/a_x \) and \( g^{pq}_z = 2\pi q/a_z \), where \( a_x \) and \( a_z \) are the lattice constants of the 2D array, while \( \hat{x} \) and \( \hat{z} \) represent unit vectors. Here, \( p \) and \( q \) are integer numbers, \( y \) is the coordinate normal to the plane, where \( y = 0 \) corresponds to the lower surface of the film, while \( \mathbf{r} \) is a vector in the plane of the film defined by the coordinates \( x \) and \( z \), as shown in figure 1. As the magnetization is uniformly distributed within \( d - h < y < d \) (see figure 1(c)), the coefficients \( \tilde{C}_{pq}(y) \) satisfy

\[ \tilde{C}_{pq}(y) = C_{pq} \Theta(y - d + h) \Theta(d - y), \tag{5} \]
where $\Theta(y)$ is the Heaviside step function and $C_{pq}$ is a Fourier coefficient that depends on the specific shape of the defects, as we shall see in section 2.2. Overall, the coefficient $\tilde{C}_{pq}(y)$ is such that the magnetization components (4) are well defined within the defects. On the other hand, for the rest of the film (with saturation magnetization $M_s$) we have

$$M_z = M_s \cos \psi \left( 1 - \sum_{pq} \tilde{C}_{pq}(y) \exp \left[ i g_{pq} \cdot r \right] \right),$$

$$M_x = M_s \sin \psi \left( 1 - \sum_{pq} \tilde{C}_{pq}(y) \exp \left[ i g_{pq} \cdot r \right] \right).$$

Note that the first terms $M_z \cos \psi$ and $M_x \sin \psi$ represent the equilibrium magnetization of a perfect film of thickness $d$ in such a way that including the negative second terms, we finally obtain the magnetization components for the film with $M_s$, excluding the defect zones. As we are interested in the evaluation of the TMS matrix, that is the second term in equation (2) that stems from the defects, the first terms in equations (6) do not enter in the following discussions, since they only contribute to the Hamiltonian of the perfect film $E^\text{film}_d$. Thus, the presence of the defects at the surface of the film can be written as a superposition of the two above magnetizations (equations (4) and (6))

$$M_z'' = M_z' \cos \psi \sum_{pq} \tilde{C}_{pq}(y) \exp \left[ i g_{pq} \cdot r \right],$$

$$M_x'' = M_x' \sin \psi \sum_{pq} \tilde{C}_{pq}(y) \exp \left[ i g_{pq} \cdot r \right],$$

where $M_z'' = M_z' - M_s$ is the difference of the defects magnetization and that of the perturbed film. To find the dynamic dipolar field $h_d^{(1)}$, we can write the dynamic components of the defect’s magnetization as

$$m_{X,Y}''(r) = \frac{1}{\sqrt{L^2 d}} \sum_{k,p,q} m_{X,Y}''(k) \tilde{C}_{pq}(y) e^{i(k+g_{pq}) \cdot r},$$

where $L^2$ is the area of the film and $m_{X,Y}''(k) = m_{X,Y}'(k) - m_{X,Y}(k)$. Also, as we are neglecting the variation of the magnetization along the coordinate normal to the film, and in the case of $M_z' \neq 0$, we can assume that $\mathbf{M}(r) / M_z = \mathbf{M}'(r) / M_z'$, and then, $m_{X,Y}(r) / M_z = m_{X,Y}'(r) / M_z'$.

In the general case we have periodicity in the $x$ and $z$ directions, but for stripe-like defects just one direction is necessary to describe such periodicity. In the magnetic charge distribution, $\tilde{C}_{pq}(y)$ is a coefficient associated with the Fourier series expansion and can be calculated once the shape of the periodic defects is specified, as we discuss in section 2.2 for rectangular, circular and stripe-like defects. With the distribution of magnetic surface charges localized at the edges of the defects, as described by equations (5) and (7), it is possible to evaluate the magnetostatic potential associated with such charges (see appendix for details). Then, we calculate the dipole field and the corresponding dipole energy, which constitutes the mechanism that activates TMS [30, 36]. Using the notation introduced by Arias and Mills [30, 36], the corresponding Hamiltonian of this perturbation reads

$$V = \frac{1}{2} \sum_{k,k'} \left[ V_{XX}(k', k)m_x^*(k')m_x(k) + V_{YY}(k', k)m_y^*(k')m_y(k) \right],$$

where $V_{XX}$ and $V_{YY}$ are the potentials arising from the interaction of $m_x$ and $m_x$ or $m_y$ and $m_y$, respectively.
where \( \mathbf{k} \) is the in-plane wave vector of the spin-waves and \( m_{X,Y}(\mathbf{k}) \) are the Fourier components of the dynamic magnetization perpendicular to the equilibrium direction \( Z \). Here we have used the fact that, for our system, the elements of the TMS matrix are related by \( V_{XY}(\mathbf{k}, \mathbf{k}') = V_{YX}(\mathbf{k}', \mathbf{k}) = 0 \). Thus, for a 2D array of periodic defects of any sort, the elements of the perturbation matrix can be generally represented by

\[
V_{XX}(\mathbf{k}', \mathbf{k}) = \frac{h'}{d} \sum_{pq} \left[ \Gamma_{pq}^{(0)} + \Gamma_{pq}^{(1)} \right] C_{pq} \delta_{k', k + \mathbf{g}_{pq}},
\]

and

\[
V_{YY}(\mathbf{k}', \mathbf{k}) = \frac{h'}{d} \sum_{pq} \Gamma_{pq}^{(0)} C_{pq} \delta_{k', k + \mathbf{g}_{pq}},
\]

where we have defined an effective depth \( h' \) as

\[
h' = h \left( 1 - \frac{M'}{M_s} \right).
\]

In the case of geometrical defects (\( M_s' = 0 \)), the effective depth \( h' \) coincides with \( h \). We have also defined the functions

\[
\Gamma_{pq}^{(0)} = 2\pi \left( 1 - \frac{e^{-|\mathbf{g}_{pq}|d}}{|\mathbf{g}_{pq}|^2} \right) \left( g^p_x \cos \psi + g^p_y \sin \psi \right)^2,
\]

and

\[
\Gamma_{pq}^{(1)} = 4\pi \left( \frac{e^{-|\mathbf{g}_{pq}|d} - 1}{|\mathbf{k} + \mathbf{g}_{pq}|^2} \right) \left[ (k_x + g^p_x) \cos \psi - (k_y + g^p_y) \sin \psi \right]^2.
\]

Here, \( \Gamma_{pq}^{(0)} \) and \( \Gamma_{pq}^{(1)} \) are associated to the dipolar fields \( \mathbf{h}_d^{(0)} \) and \( \mathbf{h}_d^{(1)} \), respectively.

On the other hand, following the theoretical framework given in [18], one can show that, with the perturbation treated to the lowest order, the response function in the presence of a 2D array of periodic perturbations can be written in the same form as the 1D array

\[
S_{XX}(\mathbf{k}; \Omega) \approx \frac{S_{XX}^{(0)}(\mathbf{k}; \Omega)}{1 - \sum_{\alpha\beta} S_{\alpha\beta}^{(0)}(\mathbf{k}; \Omega) \Sigma_{\beta\alpha}(\mathbf{k}; \Omega)},
\]

wherein \( S_{\alpha\beta}^{(0)}(\mathbf{k}; \Omega) \) are the response functions for the perfect film given by equation (2) in [18], while the elements of the proper self-energy \( \Sigma_{\alpha\beta} \) are

\[
\Sigma_{\alpha\beta}(\mathbf{k}, \Omega) = \sum_{k''} \left[ V_{\alpha\alpha}(\mathbf{k}'', \mathbf{k}) S_{\alpha\beta}^{(0)}(\mathbf{k}''; \Omega) \right] V_{\beta\beta}(\mathbf{k}, \mathbf{k}''),
\]

where \( \alpha, \beta = X, Y \). Note that the set of response functions \( S_{\alpha\beta}(\mathbf{k}; \Omega) \) are the frequency and wave vector dependent dynamic susceptibilities, which relate the dynamic magnetization with the microwave driving field. It is worth mentioning that the response function (15) can be used to understand Brillouin light scattering (BLS) experiments and to study the dispersion relations with the related magnonic band gaps. These issues will be published elsewhere, as in this paper we focus on the investigation of the \( \mathbf{k} = 0 \) limit, appropriate to FMR measurements. In this case, \( S_{XX}(\mathbf{k} = 0; \Omega) \equiv \tilde{S}_{XX}(\Omega) \) and the dynamic susceptibility reduces to

\[
\tilde{S}_{XX}(\Omega) = \frac{\gamma M_s \left[ \gamma H_Y(0) - i\Omega \right]}{\Omega^2 \gamma^2 - \Omega^2 - F_R - i \left[ \Lambda_0 + F_i \right]},
\]
Here the functions $F_R$ and $F_I$ are displayed in equations (20) and (21) and depend on the microwave frequency and the reciprocal lattice vectors $g^{pq}$. Also, $\gamma$ is the absolute value of the gyromagnetic ratio, $\alpha_G$ the dimensionless Gilbert damping parameter, $\Omega_0 \equiv \Omega(0)$ the FMR angular frequency with spin-wave dispersion $\Omega(k) = \gamma \sqrt{H_X(k)H_Y(k)}$, and $\Lambda_0 \equiv \Lambda(0)$, where $\Lambda(k) = \alpha_G \gamma^2 [H_X(k) + H_Y(k)]$. Note that we use later the frequency $f(k)$ instead of the angular frequency $\Omega(k) = 2\pi f(k)$, where $f_0 \equiv f(0)$ is the FMR frequency. The stiffness fields $H_{X,Y}(k)$ depend on the specific directions of the applied field, magnetization, wave vector and relevant anisotropies in the sample. In the case that the field is applied in the plane of a thin film without in-plane anisotropy, these fields are given by Landeros et al [36], Lindner et al [37] and Barsukov et al [38]

$$H_X(k) = H_X(0) + 4\pi M_s \left(1 - \frac{1 - e^{-k d}}{k d}\right) \sin^2 \phi_k + Dk^2$$

and

$$H_Y(k) = H_Y(0) - 4\pi M_s \left(1 - \frac{1 - e^{-k d}}{k d}\right) + Dk^2.$$  (19)

Here, $H_X(0) = H_0$ and $H_Y(0) = H_0 + 4\pi M_s - H_s$, with $H_0$ being the strength of the external field applied in the $Z$ direction and $H_s$ being the magnitude of an out-of-plane uniaxial anisotropy field with easy axis along the normal of the sample. Note that a film with in-plane anisotropy can be considered by a negative $H_s$. The angle between the magnetization and the wave vector is denoted by $\phi_k$, and $D = 2A/M_s$ is the exchange stiffness constant [36–38].

The contribution to the dynamic susceptibility (17) of the array of defects is contained in the function $F$, whose real and imaginary parts are

$$F_R = \left(\frac{h'}{d}\right)^2 \sum_{pq} C_{pq}^2 \frac{\gamma^2 M_s^2 \Omega_{pq} \Lambda (g^{pq}) + \Omega^2 (g^{pq}) - \Omega^2 \left[\Omega^2 (g^{pq}) - \Omega^2 \right]^2 + \Lambda^2 (g^{pq})}{\Omega^2 (g^{pq}) - \Omega^2}$$

and

$$F_I = \left(\frac{h'}{d}\right)^2 \sum_{pq} C_{pq}^2 \frac{\gamma^2 M_s^2 \Omega_{pq} \Lambda (g^{pq}) - \Omega^2 (g^{pq}) - \Omega^2 \left[\Omega^2 (g^{pq}) - \Omega^2 \right]^2 + \Lambda^2 (g^{pq})}{\Omega^2 (g^{pq}) - \Omega^2},$$

(21)

where the functions $\gamma^R_{pq}$ and $\gamma^I_{pq}$ are given by

$$\gamma_{pq}^R = \gamma^2 M_s^2 \left[2\Omega^2 \Gamma_{pq}^0 (\Gamma_{pq}^0 + \Gamma_{pq}^1) + \gamma^2 H_X^2 (0) \left(\Gamma_{pq}^0 \right)^2 + \gamma^2 H_Y^2 (0) \left(\Gamma_{pq}^0 + \Gamma_{pq}^1 \right)^2 \right]$$

and

$$\gamma_{pq}^I = 2\alpha_G \gamma^3 M_s^2 \Omega \left[H_Y (0) \left(\Gamma_{pq}^0 + \Gamma_{pq}^1 \right)^2 + H_X (0) \left(\Gamma_{pq}^0 \right)^2\right],$$

(22)

(23)

where $\Gamma_{pq}^1$ must be evaluated at $k = 0$. Note that $F_I$ and $F_R$ have an appreciable effect on the susceptibility whenever the microwave frequency $\Omega$ is degenerate with $\Omega(g^{pq})$.

The FMR dynamic susceptibility given in equation (17) has exactly the same structure as equation (22) of [18], which was obtained for a 1D array of stripe defects. Therefore, we have proven here that equation (17) can be used to describe the dynamic response of thin films with
1D or 2D arrays of patterned defects. From equation (17) we can obtain the real and imaginary parts of the dynamic susceptibility, which can be written as

\[
\bar{S}^R_{XX} = \gamma M_s \frac{\gamma H_y(0) [\Omega_0^2 - \Omega^2 - F_R] + \alpha \Omega \Lambda_0 + F_I}{[\Omega_0^2 - \Omega^2 - F_R]^2 + [\Lambda_0 + F_I]^2}
\]

(24)

and

\[
\bar{S}^I_{XX} = \gamma M_s \frac{\gamma H_y(0) [\Lambda_0 + F_I] - \alpha \Omega [\Omega_0^2 - \Omega^2 - F_R]}{[\Omega_0^2 - \Omega^2 - F_R]^2 + [\Lambda_0 + F_I]^2}.
\]

(25)

From the denominator we can clearly see that the resonance condition is given by

\[
\Omega_0^2 - \Omega^2 - F_R = 0.
\]

(26)

As \(F_R\) is frequency-dependent, we will find more than one absorption peak, which constitutes the appearance of additional spin-wave branches, as we will see later. This behavior, i.e. the existence of additional branches in the FMR spectra, has been discussed by some authors in periodically modulated films [15, 16, 18, 19].

So far we have not mentioned anything about the shape of the perturbations. We rather have derived all previous expressions considering only a periodic potential produced by the defects. Note also that the particular shape of the defect enters in the response functions (equations (17), (24) and (25)) through \(F_I\) and \(F_R\) (equations (20) and (21)), which contain the functions \(\Gamma^{(0,1)}_{pq}\) given by equations (13) and (14). Here, the only term that depends on the shape of the defects is the coefficient \(C_{pq}\), so that our theory is universal for any 1D or 2D periodic structure when their influence has a perturbative character, so that linear response theory is applicable. Furthermore, the general formulation presented here can be applied to thin films made of other ferromagnetic materials with other anisotropies and with the applied field in any direction, just by introducing the appropriate stiffness fields \(H_{X,Y}(k)\).

On the other hand, one should note that the theory allows us to determine the influence of the defects when the perturbations are either pits or bumps. For the case of bumps with magnetization \(M'_s\) above a flat film of thickness \(d\), a simple analysis shows that redefining \(h'\) as \(h' = -h(M'_s/M_s)\) allows us to represent such a case. Note that the minus sign means that the quantity \(h\) is referred to as a ‘height’ above the film instead of a depression into the film.

2.2. Examples of specific shapes

In this section we are going to derive analytical expressions if the defects acquire the shape of stripes, rectangles and circular dots. Thus, according to the previous discussions, we need to calculate \(C_{pq}\) (see equation (5)) for each specific defect shape.

2.2.1. Stripes. For a 1D periodic array of perturbations with the shape of stripes, we can model the distribution of the magnetization in the plane, writing the components as

\[
M'_z(z) = M'_s \cos \psi \sum_{q=-\infty}^{\infty} \xi_q(z),
\]

\[
M'_x(z) = M'_s \sin \psi \sum_{q=-\infty}^{\infty} \xi_q(z),
\]
where the function $\zeta_{st}^q(z)$ is given by [18]

$$\zeta_{st}^q(z) = \Theta \left( z - \left[ qa_z - \frac{w_z}{2} \right] \right) \Theta \left( \left[ qa_z + \frac{w_z}{2} \right] - z \right).$$

(27)

Thus, the coefficient $C_{st}^{pq}$ is readily computed as

$$C_{st}^{pq} = \frac{1}{a_xa_z} \int_{\frac{a_x}{2}}^{\frac{a_z}{2}} \Theta \left( \frac{w_z}{2} + z \right) \Theta \left( \frac{w_z}{2} - z \right) e^{-ig_zz} dx \, dz$$

and the result is

$$C_{st}^{q} = \frac{2}{g_qa_z} \sin \left( g_q \frac{w_z}{2} \right).$$

(28)

(29)

This coefficient depends only on the index $q$ in the sum, which is a consequence of the 1D nature of the stripe defects.

2.2.2. Rectangles. In the case of rectangles and dots, the procedure is similar but the algebra is a bit more tedious. The defects with a rectangular shape can be structured by replacing $\zeta_{st}^q(z)$ by $\zeta_{re}^{pq}(x, z)$, where

$$\zeta_{re}^{pq}(x, z) = \Theta \left( x - \left[ pa_x - \frac{w_x}{2} \right] \right) \Theta \left( \left[ pa_x + \frac{w_x}{2} \right] - x \right) \Theta \left( z - \left[ qa_z - \frac{w_z}{2} \right] \right) \Theta \left( \left[ qa_z + \frac{w_z}{2} \right] - z \right)$$

and after some algebraic manipulations we find the following coefficient for rectangular defects:

$$C_{re}^{pq} = \frac{2}{g_q^x a_x} \sin \left( g_q^x \frac{w_x}{2} \right) \frac{2}{g_q^z a_z} \sin \left( g_q^z \frac{w_z}{2} \right).$$

(30)

Note that in the particular (1D) case $w_x = w$, we obtain

$$\frac{2}{g_q^x a_x} \sin \left( g_q^x \frac{w_x}{2} \right) = \delta_{p,0}$$

which is non-zero only for $p = 0$. Then, in this limit, we end up with the coefficient for stripes (29).

2.2.3. Circular dots. For the case of circular dots, we have $w_x = w_z = w$ and then

$$\zeta_{do}^{pq}(x, z) = \Theta \left( z - \left[ qa_z - \sqrt{\left( \frac{w}{2} \right)^2 - (x - pa_x)^2} \right] \right) \Theta \left( \left[ qa_z + \sqrt{\left( \frac{w}{2} \right)^2 - (x - pa_x)^2} \right] - z \right)$$

and as in the previous case, we find

$$C_{do}^{pq} = \frac{\pi w}{a_xa_z |g^{pq}|} J_1 \left( \frac{w}{2} |g^{pq}| \right).$$

(31)

where $J_1(x)$ is the Bessel function of the first kind.

The theory presented in this section allows us to describe and explain experimental results in thin films with 1D and 2D arrays of periodic perturbations in the case that the defects are considered as small perturbations. In this sense, the systems of interest here are slightly different from the extensively studied MCs, which have been modeled so far by micromagnetic simulations [39–41] and described theoretically by the plane wave method [5, 6, 11].
have been used to explain BLS experiments [42] or spin-wave transmission data in MCs, where usually the periodic structures are antidot lattices (where the holes extend over the full film thickness) [11], bicomponent MCs [7, 8] or nanowire arrays [11, 13]. It is important to mention that any periodic perturbation on a film can be understood with our theory, as in the case of thin films with ion eroded ripple surfaces [21, 43].

3. Results and discussion

With the model at hand, we can now discuss the FMR response of the thin films with patterned periodic perturbations for several geometries. We use typical parameters for permalloy (Py = Ni$_{81}$Fe$_{19}$): $4\pi M_s = 10000$ G ($M_s = 796$ kA m$^{-1}$) for the saturation magnetization, $D = 2.45 \times 10^9$ G nm$^2$ ($D = 24.5$ T nm$^2$) for the stiffness constant; the damping constant is taken as $\alpha_G = 0.007$ and the gyromagnetic ratio is $\gamma/2\pi = 2.95$ GHz kG$^{-1}$ ($\gamma = 185.354$ GHz T$^{-1}$). The thickness of the film is $d = 30$ nm, while $h'$ is varied between 0 and 3 nm.

It is well known that the microwave absorption in an FMR experiment is proportional to the imaginary part of the dynamic susceptibility (equation (25)), which is shown color-coded in figure 2 as a function of the frequency and external dc field. The dark color represents a low response and the brighter color (white) a higher response, corresponding to an absorption peak. In this figure, we can see the influence of the periodic perturbations when they have the shape of stripes (figures 2(a)–(c)), squares (figures 2(d)–(f)) and circular dots (figures 2(g)–(i)). The field is oriented in the plane along the $z$-axis ($\psi = 0$), whereas in the insets of figures 2(b), (e) and (h), the angle $\psi$ is swept under the assumption of vanishing dragging of the static magnetization. As a reference, in the left column we show the responses of defect-free films, corresponding to a depth $h = 0$. One can notice that, as a general rule, a splitting of the main FMR mode is observed and additional branches appear in the spectrum, which already has been reported [12, 14–16, 18, 19]. This splitting is enhanced by the defects’ effective depth $h'$, since the magnetic charges that reside at the lateral surfaces of the defects naturally increase with $h'$, thus increasing the periodic dipole field that splits the FMR mode. Hence, for the geometries considered in this paper, it is clear that the stripes (the upper panels in figure 2) show the strongest mode splitting in the FMR response. When the field is applied parallel to the stripes (not shown), we do not see any splitting [17, 18], in contrast to the case of 2D defects. Also, one can note that there are field ranges (usually at low fields) where the lower resonance branch dominates in intensity, while the second one dominates at intermediate fields, the third one at higher fields, and so on. Of course, in the limit of high fields and high frequencies, the periodic perturbations do not have a relevant influence anymore (the functions $F_R$ and $F_I$ become small), and the strongest mode observed is very close to the uniform Kittel mode.

The insets in the central column of figure 2 show the microwave absorption as a function of the external field and angle $\psi$ for a fixed frequency equal to 9.4 GHz. In the case of stripe defects, the angular dependence shows that the splitting of spin-wave modes disappears close to 18$^\circ$. This can be easily understood in the framework of the so-called critical angle $\phi_c$, a special value of the angle $\phi$, such that for $\phi_c > \phi_c$, there are no degenerate magnon states, and hence no TMS [30]. In other words, the function $F$ will be relevant whenever the microwave frequency $\Omega$ is close to $\Omega (g^q)$, which for the case of stripe defects occurs if $\phi_{k_g} < \phi_c$. Here, the reciprocal vector points along the $z$ direction and the angle $\phi_{k_g}$ is just the angle $\psi$ (neglecting the dragging of the magnetization). Therefore, when $\psi$ is above the critical angle, the frequency $\Omega$ cannot be equal to $\Omega (g^q)$, and the influence of the defects on the FMR-response is negligible.
Figure 2. Imaginary part of the dynamic susceptibility (color-coded) as a function of the microwave frequency \( f \) and external field \( H_0 \), calculated for three specific defect depths: \( h' = 0, 1.5 \) and \( 3 \) nm, for a field applied in the plane, along the \( z \) direction (\( \psi = 0 \)). The defect shapes are (a)–(c) stripes, (d)–(f) squares and (g)–(i) dots. For \( h' > 0 \) additional branches appear in the resonant response with the strongest splitting for the film with stripe defects. In (b) and (c), the splitting appears at several fields: around \( H_0 = 36, 70 \) and \( 150 \) mT. The insets in the central column (a), (e), (h) show the angular dependence of the FMR response at a fixed frequency of \( f = 9.4 \) GHz (dotted line).

The insets in (c), (f), (i) show the FMR response at \( f = 9.4 \) GHz as a function of \( H_0 \) and the ratio between the defect width and periodicity \( w_z/a_z \). We use \( w_z = 170 \) nm in (a)–(i) and a Py film thickness \( d = 30 \) nm. Parameters for Py are given in the text.

In our case, the critical angle for a field around 92 mT is equal to 16.9°, which fully explains the angular dependence in the inset of figure 2(b). For the 2D arrays of defects (squares and dots), the situation is more complex because the reciprocal lattice vectors have \( x \) and \( z \) components, and it is difficult to know the actual direction of the reciprocal vector \( \mathbf{g}_{pq} \), so the critical angle concept is not useful in those cases, since \( \phi_{k=qg_{pq}} \) is not well defined.

The dependence on \( w_z/a_z \) (insets in figures 2(c), (f) and (i)) is associated with the coefficients \( C_{pq} \), since the defect widths \( w_z \) and \( w_x \) enter only in this quantity. In the inset of figure 2(c), we observe several modes, each one associated with a given branch in figure 2(c), where we use the ratio \( w_z/a_z = 0.582 \), and thus four branches are observed. Note that in the case \( w_z/a_z = 1/2 \), just three branches are expected, as in the case \( w_z/a_z = 1/3 \) and \( 2/3 \). This
The upper panels show $\Omega_0^2 - \Omega^2$ (black solid), $F_I$ (blue dash-dotted) and $F_R$ (red dashed) for stripe structures as a function of the frequency for three values of the external field: $H_0 = 20$, 50 and 80 mT, respectively. The intersections between the solid and dashed lines indicate the resonance frequencies (see equation (25)), where $\Omega_0^2 - \Omega^2 - F_R = 0$. In (a) and (b) we can see three such points, which are labeled by A–C. The lower panels (d)–(f) show the corresponding imaginary part of the susceptibility given by equation (25). In the case of a perfect film ($h' = 0$), the resonant frequency is given by $\Omega_0^2 - \Omega^2 = 0$, which is shown by the solid green line, showing a single mode, while for $h' = 1.5$ nm we clearly see the appearance of additional spin-wave modes.

Figure 3. The upper panels show $\Omega_0^2 - \Omega^2$ (black solid), $F_I$ (blue dash-dotted) and $F_R$ (red dashed) for stripe structures as a function of the frequency for three values of the external field: $H_0 = 20$, 50 and 80 mT, respectively. The intersections between the solid and dashed lines indicate the resonance frequencies (see equation (25)), where $\Omega_0^2 - \Omega^2 - F_R = 0$. In (a) and (b) we can see three such points, which are labeled by A–C. The lower panels (d)–(f) show the corresponding imaginary part of the susceptibility given by equation (25). In the case of a perfect film ($h' = 0$), the resonant frequency is given by $\Omega_0^2 - \Omega^2 = 0$, which is shown by the solid green line, showing a single mode, while for $h' = 1.5$ nm we clearly see the appearance of additional spin-wave modes.

issue will be discussed later and stems from the pre-factor $\sin(q w_z/2) = \sin(q \pi w_z/a_z)$, which vanishes if $q w_z/a_z$ is an integer number, as happens for $w_z/a_z = 1/2$ and $q = 2, 4, 6, \ldots$ and also for $w_z/a_z = 1/3$ or $2/3$ and $q$ being a multiple of 3. Thus, the value of $w_z/a_z$ alters the strength of the splitting and it can also suppress some branches. In the case of dots (see the inset of figure 2(i)) the coefficient $C_{pq}$ is different to the square case, and therefore the behavior of the response function as a function of $w_z$ is different as well.

A deeper understanding of the results presented in figure 2 can be obtained by analyzing the dynamic susceptibility (equation (17)) and FMR spectra at fixed fields, as illustrated in figure 3, where we show in (a–c) the imaginary and real parts of the function $F$ (blue dashed-dotted and red dashed curves), i.e. equations (21) and (20), together with the quantity $\Omega_0^2 - \Omega^2$ (solid black curve) as a function of the frequency $f$. Clearly, the resonant condition for a perfect film ($F = 0$) is given by $\Omega_0^2 - \Omega^2 = 0$, which can be seen in figures 3(a)–(c) when the solid line crosses the abscissa, corresponding to the single resonance peak in $S_{XX}$ (the solid lines in figures 3(d)–(f)). Nevertheless, in the presence of defects it is possible to see additional peaks in the microwave absorption, resembling a splitting of the FMR mode, as is shown by the black dotted lines in figures 3(d)–(f). In figures 3(a)–(c), the intersection of the solid and dashed lines identify the resonant points (labeled A–C) where—depending on the field strength—we can see differences in the intensity of the absorption peaks in the panels below. For example, in figure 3(d), the low frequency peak dominates in intensity, while for stronger fields (figure 3(e) or (f)), the higher frequency peak dominates. An important conclusion that we extract from the theory presented here is that some modes are very low in intensity and just some modes seem to be relevant.
This behavior can be better understood by realizing that the resonance frequencies satisfy the condition (26), which includes the perturbation term $F_R$. For the resonance frequencies, the amplitude of the imaginary part of the response function at the peaks is given by

$$\tilde{S}_\perp^{i} = \gamma M_s \frac{\gamma H_f (0)}{\Lambda_0 + F_1}. \tag{32}$$

Then, at resonance, the intensity of the peak depends mainly on the imaginary part of the perturbation $F_1$ (blue dashed-dotted line), which in this case is always positive, meaning that the peaks in an implanted sample with stripe defects are always lower in intensity than the susceptibility peak in the perfect film. For the three labeled points in figure 3(a), we observe the following: (i) at point A, the function $F_1$ is close to zero and, as $F_1$ is in the denominator of equation (32), it does not influence the amplitude of $\tilde{S}_\perp^{i}$. We observe a strong peak in the perfect film (solid green line). (ii) At point C, the function $F_1$ is also small, but is greater than at point A. So, a smaller peak in $\tilde{S}_\perp^{i}$ is observed close to 5 GHz. (iii) In contrast to points A and C, point B does not produce a strong resonance peak in $\tilde{S}_\perp^{i}$ since this point is quite close to a maximum in $F_1$. As explained above, a large $F_1$ reduces $\tilde{S}_\perp^{i}$ due to the denominator of (32). Therefore a notable decrease in the response function is given in this case. The above arguments can be applied to understand figures 3(e) and (f) as well. Looking at figure 3(c) calculated for $H_0 = 80 \text{ mT}$, it is easy to note that the effect of the perturbation is small and the resonant frequency is closer to the resonant frequency for the perfect film. Besides that, the small peak visible in figure 3(f) around 8 GHz is due to the still relatively small difference between $\Omega_0^2 - \Omega^2$ (solid line) and $F_R$ (dashed line) in figure 3(c).

Despite the fact that the theoretical picture presented here just considers the FMR response, a deeper analysis provides us with key information about the existence of forbidden zones or magnonic band gaps in the dispersion relation of periodic systems. The strong peaks of the imaginary part of the defect function (21) ($F_1$, blue dashed-dotted curves in figure 3) always occur when the microwave frequency becomes $f = f(g^q)$, where $f(k) = \Omega(k)/(2\pi)$. In this case, the function $F_1$ is a maximum (and $F_R \approx 0$), and then the response function is considerably reduced, since it is inversely proportional to $F_1$. As a consequence, around this frequency the film does not respond to the excitation, which means that there are no spin-waves propagating at this special combination of field and frequency. Figure 4 shows the calculated FMR spectra of a film with stripe defects, where we also include $f(g^{q=1})$ (solid line), $f(g^{q=2})$ (dashed line) and $f(g^{q=3})$ (dot-dashed line). Then, if we superimpose the FMR mode for the perfect film (dotted curve in figure 4), we can see that the splitting appears at $f_0 = f(g^q)$, that is exactly at their crossing point. This is shown in figure 4, where we suggest that the splitting of the Kittel (uniform FMR) mode into different spin-wave modes is closely related to the formation of magnonic band gaps.

Within the physical picture of TMS [30–38], the following is happening: at the special points given by $f_0 = f(g^q)$, the FMR mode is degenerate with higher wave vector modes matching the periodicity of the defect array [18]. These special points are responsible for the splitting of the branches, and therefore allow us to explain the formation of new branches as well as the appearance of the magnonic band gaps. A complete understanding of the stepwise magnonic band gap formation detected by BLS will be published elsewhere. Note that this aspect provides also the answer why one can see more than one branch in the $f(H_0)$ plots of figure 2, regardless of the specific array. Moreover, it suggests that the band gap width depends on the strength of the perturbation—in our case the defect’s effective depth $h'$. 

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Our main conclusion is that for a given field, as the magnons cannot access the forbidden regions, only magnons with surrounding frequencies may respond to the external excitation and therefore this behavior induces a splitting of the main uniform mode of a perfect film.

4. Comparison between theory and experiment

In this section we compare the experimental results with the theoretical model for samples featuring stripe-like defects induced either by ion implantation (section 4.1) or the topographic structuring (section 4.2) of thin Py films. The first one is a sample of 30 nm thickness with implanted stripe defects, whereas the second one is a 20 nm thin film featuring topographic stripe defects on its surface.

4.1. Implanted samples ($M'_s \neq 0$)

The implanted stripe samples have been prepared by a combination of EBL and ion implantation, as described in detail in [17]. First, a $1 \times 1$ mm$^2$ square window was patterned by EBL into a polymethylmethacrylate (PMMA) resist on a Si(001) substrate. Then 30 nm permalloy capped by 1.5 nm Cr were grown by molecular beam epitaxy with subsequent lift-off. In the next step, a stripe pattern was written by EBL into the 100 nm thick PMMA resist and developed. In the last step, Cr$^+$ ions were implanted with a fluence of $5 \times 10^{15}$ ions cm$^{-2}$. The ion energy of 5 keV was selected such that the Cr$^+$ ions just penetrate the first 5–7 nm of the uncovered stripes of the Py/Cr sample or otherwise get absorbed within the PMMA stripes, respectively. The implanted Cr$^+$ ions cause an intermixing of the Cr–Py interface and
therefore a local reduction of the saturation magnetization [44, 45]. Hence, these implanted stripes are periodic perturbations of the magnetization without modulating the morphology (surface). Clearly, in theory we use an effective depth \( h' \) as is shown in equation (12), since the implanted zone has a finite magnetization \( M'_s \). The final stripe width of \( w = 170(20) \) nm and periodicity \( a_z = 292(40) \) nm were determined by scanning electron microscopy (not shown).

The FMR measurements were performed on a broadband FMR setup based on an Agilent E8364B vector network analyzer. The samples were placed face-down on the center of a 80 \( \mu \)m wide coplanar waveguide. We recorded the complex microwave transmission parameter \( S_{21} \), which is proportional to the imaginary part of the dynamic susceptibility component \( S_{XX} \), as the FMR signal sweeping the external dc magnetic field at constant microwave frequency. Figure 5(a) shows the absolute value \( |S_{21}| \) color coded as function of frequency and field \( H_0 \). One can clearly see four bright resonance branches. Figure 5(b) shows the corresponding calculation of the resonances using the Py parameters as mentioned in section 3 and the same \( w \) and \( a_z \) as the sample. Setting the effective defect depth to \( h' = 1.5 \) nm, we get an almost perfect match between theory and experiment. The red circles in both diagrams denote the split points of the resonance branches, which fall exactly onto each other. There is a slight deviation for small fields below 15 mT, which is most probably due to the difference of uniform magnetization assumed in the model and non-uniform distribution in the real sample.

4.2. Topographic stripe defect sample \((M'_s = 0)\)

The second sample considered here consists of a thin Py film exhibiting topographical stripe defects on its surface. This directly corresponds to the case of a 1D array with a defect shape of stripes as described in section 2.2.1 and vanishing defect magnetization \((M'_s = 0)\). For preparation, a polycrystalline 20 nm thick Py film is grown on a SiO/Si(100)-substrate by molecular beam epitaxy and is covered by a stripe-patterned PMMA mask using EBL. Subsequently, additional Py is evaporated on the patterned mask. After the lift-off process, a thickness modulated thin film remains, which can be viewed as geometrical stripe defects on
top of a flat thin film. The height, width and periodicity of the defects were determined to be $h = 5 \pm 1$, $w = 101.5 \pm 10$ and $a_0 = 308 \pm 5$ nm respectively from cross-sectional transmission electron microscopy analysis.

Figure 6 shows the comparison between theory and experiment for this sample. In figure 6(a), a measured map of the absorbed microwave power as a function of the external field and frequency is shown. Here, the splitting of the FMR mode is also apparent as for the implanted sample, and is very well reproduced by the theory (figure 6(c)). Figures 6(b) and (d) show the in-plane angular dependence of the absorbed microwave power from experiment and theory, respectively. As was mentioned in section 3, the splitting vanishes at a given angle, in this case near to $20^\circ$. This can be explained by analyzing the angle $\phi_g$ in the dispersion relation of the perfect film. Whenever the angle $\phi_g$ exceeds a given critical value $\phi_c$, degenerate magnon states are not likely to appear since the TMS process is not operative [30, 36]. In our case this angle is given by $\psi$, so that for an applied field of 96 mT the critical angle is about $17.2^\circ$ and therefore above this value the splitting disappears, as can be seen in figures 6(b) and (d) (see vertical dashed lines). Then, when the field is tipped out of the perpendicular orientation to the stripes ($\psi \neq 0$), the angle $\phi_g$ increases as well, and the mode splitting is no longer observed if
\( \psi > \phi_c \). At this point the well-known Kittel behavior is recovered. We also note that the modes labeled by 1, 2 and 3, have in general different intensities that depend on the frequency and the angle.

5. Summary

We have obtained analytical expressions for the magnetic resonance response in ferromagnetic films with periodical perturbations (defects) at the surface. The theory for periodically perturbed films presented here, which allows us to calculate the response for different 1D and 2D shapes of the defects, was tested for the case of a 1D (stripe-like) defect structure prepared either by ion implantation (implanted sample) or by molecular beam epitaxy (topographical sample), for which we found that the calculations very well reproduce the experimental results gained from FMR measurements. The results manifest an interesting behavior of splitting resonance branches, similar to the ones observed in conventional MCs. Our analytical theory provides a physical interpretation of the appearance of these new branches based on the concept of forbidden zones, whose existence has been proven both theoretically and experimentally for MCs. We thus believe that the limit of small perturbations is a very promising one to also study the physics within MCs. We also demonstrate that FMR as an experimental tool is useful in observing key aspects of MCs.

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Appendix

The magnetostatic potential \( \phi = \phi(\mathbf{r}) \) can be calculated from

\[
\phi(\mathbf{r}) = \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}',
\]

where the magnetic charge density associated with the periodic array of defects can be extracted from equation (7), that is

\[
\rho(\mathbf{r}) = -iM_s'' \sum_{pq} \tilde{C}_{pq}(y) \left( g_q^z \cos \psi + g_p^x \sin \psi \right) e^{ig^{pq} \mathbf{r}}
\]

and therefore, the potential reads

\[
\phi(\mathbf{r}) = -iM_s'' \sum_{pq} \left( g_q^z \cos \psi + g_p^x \sin \psi \right) \int \tilde{C}_{pq}(y') \frac{e^{ig^{pq} \mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'.
\]

Clearly in the limit of \( d \gg h \), \( y' \) may be replaced by \( d \) and therefore, after integration in the \( y' \) coordinate, we have

\[
\phi(\mathbf{r}) = iM_s h \sum_{pq} C_{pq} \left( g_q^z \cos \psi + g_p^x \sin \psi \right) \int \frac{dx' dz' e^{ig^{pq} \mathbf{r}'}}{\sqrt{(x-x')^2 + (y-d)^2 + (z-z')^2}}.
\]
Now, we replace $x' - x \rightarrow x''$ and $z' - z \rightarrow z''$, with $(r'')^2 = (x'')^2 + (z'')^2$ and thus

$$
\phi(r) = i M_s h' \sum_{pq} C_{pq} \left( g^q_z \cos \psi + g^p_x \sin \psi \right) e^{i g^{pq} \cdot r} \int \frac{d^2 r'' e^{i g^{pq} \cdot r''}}{\sqrt{(r'')^2 + (y - d)^2}}. \quad (A.5)
$$

Then, in polar coordinates the potential becomes

$$
\phi(r) = i M_s h' \sum_{pq} C_{pq} \left( g^q_z \cos \psi + g^p_x \sin \psi \right) e^{i g^{pq} \cdot r} \int r'' d\theta' e^{i |g^{pq}| \theta' \cos \theta} \frac{r'' d\theta}{\sqrt{(r'')^2 + (y - d)^2}}, \quad (A.6)
$$

where the following integral

$$
I = \int \frac{e^{i |g^{pq}| \theta' \cos \theta}}{\sqrt{(r'')^2 + (y - d)^2}} r'' d\theta
$$
gives

$$
I = \int \frac{2 \pi J_0(|g^{pq}| r'')}{\sqrt{(r'')^2 + (y - d)^2}} r'' = \frac{2 \pi e^{-|g^{pq}| |d - y|}}{|g^{pq}|}, \quad (A.7)
$$

wherewith we obtain

$$
\phi(r) = 2 \pi i M_s h' \sum_{pq} C_{pq} \left( g^q_z \cos \psi + g^p_x \sin \psi \right) e^{-|g^{pq}| |d - y|} |g^{pq}| e^{i g^{pq} \cdot r}. \quad (A.8)
$$

Now, the components of the static dipole field \( h^{(0)}_d \) (averaged over the film thickness) can be obtained using the potential (A.8), i.e. \( h^{(0)}_z = -\partial_z \phi(x, y, z) \) and \( h^{(0)}_x = -\partial_x \phi(x, y, z) \), whereupon

$$
h^{(0)}_z = 2 \pi M_s h' \sum_{pq} C_{pq} \left( g^q_z \cos \psi + g^p_x \sin \psi \right) g^q_z \frac{1 - e^{-|g^{pq}| d}}{|g^{pq}|^2 d} e^{i g^{pq} \cdot r}, \quad (A.9)
$$

and

$$
h^{(0)}_x = 2 \pi M_s h' \sum_{pq} C_{pq} \left( g^q_z \cos \psi + g^p_x \sin \psi \right) g^p_x \frac{1 - e^{-|g^{pq}| d}}{|g^{pq}|^2 d} e^{i g^{pq} \cdot r}. \quad (A.10)
$$

According to the geometry shown in figure 1, only the component \( h^{(0)}_z = h^{(0)}_x \sin \psi + h^{(0)}_z \cos \psi \), which is parallel to the magnetization, is relevant in the evaluation of the dipolar energy. The magnetization component of the perfect film in such a direction is obviously reduced by the spin-waves, and can be expressed as

$$
M_Z(r) = M_s \left[ m^Z_X(r) + m^Z_Y(r) \right]. \quad (A.11)
$$

Then, the first term of equation (3) can be written as \([18]\)

$$
V = \frac{1}{2M_s} \int d^3 r h_Z \left[ m^2_X(r) + m^2_Y(r) \right] \quad (A.12)
$$

while the dynamic components of the magnetization are expanded in a Fourier series, i.e. \([18, 30, 36]\)

$$
m_{X,Y}(r) = \frac{1}{\sqrt{L^2 d}} \sum_k m_{X,Y}(k) e^{i k \cdot r} \quad (A.13)
$$
and
\[ m_{X,Y}^* (r) = \frac{1}{\sqrt{L^2d}} \sum_k m_{X,Y}^* (k) e^{-ik \cdot r}, \] (A.14)

where \( L^2 \) is the area of the film and the vectors \( r \) and \( k \) lie in the film plane. Now, using the following identity
\[ \int dx \ dz \ e^{ig_{pq} \cdot r} e^{i(k-k') \cdot r} = L^2 \delta_{k,k'+g_{pq}}, \] (A.15)

we obtain an expression for the function \( \Gamma^{(0)}_{pq} \) given in equation (13), which, as was shown previously, is associated with the static dipolar field \( h^{(0)}_d \). Finally, using the dynamic components given in equation (8) and under the same procedure, we can determine the function \( \Gamma^{(1)}_{pq} \) given in equation (14).

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