The lithium problem: new insight in the big bang nucleosynthesis (BBN) beyond the standard model

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Abstract. The production of the light elements in the framework of the standard Big Bang nucleosynthesis model (SBBN) matches the observed abundances except in case of $^7\mathrm{Li}$, where observations lie a factor 2.4-4.3 below SBBN+WMAP (Wilkinson Microwave Anisotropy Probe) predictions. This so-called “Lithium problem” needs to be resolved beyond the SBBN. In this contribution we focus on the effect of degenerate neutrinos and the addition of dark component, including dark energy density and dark entropy. We find that the effect of the degeneracy parameter is significant if chemical potentials of neutrino families are different. Concerning the dark component, the major effect comes from adding dark entropy.

1. Introduction
The standard Big Bang nucleosynthesis model (SBBN) represents one cornerstone of the Big Bang theory. It leads to the synthesis of light elements ($^2\mathrm{D}$, $^4\mathrm{He}$, $^6\mathrm{Li}$, $^7\mathrm{Li}$ and $^7\mathrm{Be}$) during the first three minutes after Big Bang. The SBBN is a parameter-free theory, since it depends only on the baryon to photon ratio $\eta$ which is meanwhile well determined by the WMAP. It is found that $\eta = (6.23 \pm 0.17) \times 10^{-10}$ [1, 2]. The observed abundances of $^4\mathrm{He}$, $^7\mathrm{He}$ and $^2\mathrm{D}$ are in good agreement with SBBN+WMAP predictions but not in case of $^7\mathrm{Li}$. The observation of $^7\mathrm{Li}$ in old Halo stars [3], yields $^7\mathrm{Li}/H = (1.23^{+0.08}_{-0.03} \times 10^{-10})$ where the SBBN prediction $^7\mathrm{Li}/H = (4.681 \pm 0.335) \times 10^{-10}$.

Some dispersion is found for deuterium: according to [5] $^2\mathrm{D}/H = 2.8^{+0.8}_{-0.6} \times 10^{-5}$, while recent observations by [6] shows $^2\mathrm{D}/H = (2.53 \pm 0.04) \times 10^{-5}$. The element $^4\mathrm{He}$ is strongly constrained primordial value of $X(\ ^4\mathrm{He}) = 0.228 \pm 0.005$ as obtained from seven metal-poor galaxies [7]. Other investigation by [8] indicate a range $0.232 \leq X(\ ^4\mathrm{He}) \leq 0.258$. Not only the difference is between observations and predictions but observing $^7\mathrm{Li}$ in very metal poor stars does not confirm the known plateau [9]. This constitutes presently “the Lithium problem”. The overproduction of $^7\mathrm{Li}$ seems not to be resolved by the SBBN, for example on astrophysical or nuclear physics grounds, a non-standard scenario of the SBBN seems to be needed to resolve the $^7\mathrm{Li}$ problem. In particular, modifying neutrinos properties (neutrinos degeneracy, oscillations, non-vanishing mass…) and adding the effect of a dark component [10-12].

2. Non-standard BBN
2.1. Non-vanishing neutrinos chemical potential
According to [13], neutrinos degeneracy affects elements production in two ways:
(i) It increases the energy density leading to speed up the expansion rate of the universe as appears in the Friedmann equation. The Hubble parameter is then given by:
\[
\left( \frac{2}{3} \right)^2 = H^2 = \frac{8\pi G}{3} \rho_{\text{rad}} = kT^4,
\]
where \( g \)'s are the relativistic degrees of freedom, \( a \) is the scale parameter and \( \rho \) is the total energy density. The energy density of fermions and anti-fermions is given by [14]:

\[
\rho_{f} + \rho_{\bar{f}} = \frac{7\pi^2}{120} gT^4 \left[ 1 + \frac{30\beta^2}{7\pi^2} + \frac{15\beta^4}{7\pi^4} \right],
\]
where \( \beta = \frac{\mu_f}{T} \) is the degeneracy parameter. In the SBBN, \( \beta = 0 \) (non-degenerate neutrinos). Clearly including \( \beta \) increases the energy density (or \( k \)) and this is the only way in which \( \mu_f \) and \( \nu_e \) modify the SBBN. As a consequence of a larger expansion rate, freeze-out temperature will be higher leading to higher neutrons mass fraction. This will result in increasing \( \text{D} \) and \( ^4\text{He} \) but \( ^7\text{Li} \) will decrease as shown in table 2.

(ii) A second effect concerns the electron neutrinos, which are involved in the weak interactions rates:

\[
n + \nu_e \leftrightarrow p + e^-, \quad n + e^+ \leftrightarrow p + \bar{\nu}_e, \quad n \leftrightarrow p + e^+ + \bar{\nu}_e
\]
A new equilibrium ratio is obtained (see [14]):

\[
\frac{n}{p} = \exp[-(\beta_{\nu_e} + \frac{Q}{T})]
\]
where \( Q = m_n - m_p = 1.29 \text{MeV} \), \( T \) in MeV, and \( \beta_{\nu_e} \) is the electron neutrino degeneracy parameter.

The rate of the first reaction can be written as [14]:

\[
\lambda_{n\nu} \approx 1.63 \left( \frac{T_{\nu}}{Q} \right)^3 \left( \frac{T_{\nu}}{Q} + 0.25 \right)^2 \text{s}^{-1}
\]

The other rates are related to it. Taking \( -0.5 < \beta_{\nu_e} < 0.5 \), equation (5) becomes:

\[
\lambda_{n\nu} \approx 1.63 \times \exp(\beta_{\nu_e}) \left( \frac{T_{\nu}}{Q} \right)^3 \left( \frac{T_{\nu}}{Q} + 0.25 \right)^2 \text{s}^{-1}
\]
It is seen that the rate is enhanced exponentially when \( \beta_{\nu_e} > 0 \), thus it becomes faster and more neutrons are converted into protons, so that the freeze-out temperature is delayed (see table 1). As table 1 shows the effect on \( ^7\text{Li}/\text{H} \) is significant. Using equation (6) we calculate the mass fraction of neutrons at freeze-out:

\[
X_n^{*} = \int_{0}^{\infty} e^{-5.42k^{-3/2}e^{\beta_{\nu_e}} \int_{0}^{y} (x+0.25)^2 \left( 1 + e^{-\beta_{\nu_e} \exp(-\frac{1}{2})} \right) dx} \frac{dy}{2y^2(1+\cosh(\beta_{\nu_e}+1/y))}
\]

And to get the freeze-out temperature \( T^* \) we use the modified

\[
X_n^{eq} = \frac{1}{1 + \exp(\beta_{\nu_e}) \exp(\frac{Q}{T^*})}
\]

The integral in equation (7) does not allow an explicit dependence of \( X_n^{*} \) on \( \beta_{\nu_e}'s \). For this reason we took different values of \(-0.5 < \beta_{\nu_e} < 0.5 \) and \( 0 < \beta_{\nu_e}, \beta_{\nu_e} < 2.5 \) (and or \( 3.53 < k < 8.6 \) and we evaluated this integral in the above ranges, where some results are given in tables 1 and 2.

**Table 1.** Effect of adding \( (\beta_{\nu_e}) \) on freeze-out temperature and light elements (numbers in brackets are power of ten in all tables and figures).

| \( \beta_{\nu_e} \) | \( X_n^{*} \) | \( T^* \) | \( X( ^4\text{He}) \) | \( \text{D}/\text{H} \) | \( ^7\text{Li}/\text{H} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| -0.2            | 0.1987          | 0.8110          | 2.987(-1)       | 2.832(-5)       | 5.224(-10)      |
| 0 or (SBBN)     | 0.1571          | 0.7695          | 2.464(-1)       | 2.509(-5)       | 4.681(-10)      |
| 0.2             | 0.1217          | 0.7279          | 2.030(-1)       | 2.291(-5)       | 4.073(-10)      |

**Table 2.** Effect of adding \( (\beta_{\nu_\mu} = \beta_{\nu_e}) \) on freeze-out temperature and light elements.

| \( \beta_{\nu_\mu} = \beta_{\nu_e} \) | \( K \) | \( X_n^{*} \) | \( T^* \) | \( X( ^4\text{He}) \) | \( \text{D}/\text{H} \) | \( ^7\text{Li}/\text{H} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 2.5             | 7.6511          | 0.1864          | 0.8773          | 3.133(-1)       | 4.882(-5)       | 2.710(-10)      |
| 1.0             | 4.0690          | 0.1624          | 0.7881          | 2.592(-1)       | 2.826(-5)       | 4.317(-10)      |
| SBBN            | 3.53            | 0.1571          | 0.7695          | 2.464(-1)       | 2.509(-5)       | 4.681(-10)      |
Table 3. Effect of adding $\beta_{\nu_e} = 0.28$ and $\beta_{\nu_{\mu}} = \beta_{\nu_{\tau}} = 2$ together on light elements.

| X($^4$He) | D/H   | $^3$He/H | $^7$Li/H |
|-----------|-------|----------|----------|
| 2.219(−1)| 3.361(−5)| 1.127(−5)| 2.814(−10) |

Adding $\beta_{\nu_e}$ or $\beta_{\nu_{\mu}}, \beta_{\nu_{\tau}}$ independently will have effect on $^7$Li but it will violate observational constraints on D and $^4$He. Therefore, adding $\beta_{\nu_e} = 0.28$ along with $\beta_{\nu_{\mu}} = \beta_{\nu_{\tau}} = 2$ together will lead to $^7$Li depletion without violating observational constraints (see table 3).

2.2. Adding dark energy density

It is known that dark matter and dark energy play a decisive role in the evolution of the universe. Due to the fact that the identity of dark matter is not well known a unified adiabatically expanded fluid is adopted so that the dark energy and dark matter can be considered as two different aspects of the same component. To illustrate this model, temperature-dependent dark energy density is added to radiation density as follows [10,11]:

$$\rho_{\nu}(\chi) = \frac{n_p}{T_{T_0}^3},$$

(9)

where $T_0 = 1.0$ Mev =1.16 $\times$ $10^{10}$ K, and $n_p$ is a constant characterizing the power law. In case of $n_p = 4$, the dark component mimics a radiation density. The case $n_p = 3$ describes a matter density behavior, while $n_p = 6$ describes a scalar field. With these assumptions the Friedmann equation becomes:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3} (\rho_{\text{rad}} + \rho_D),$$

(10)

where $\rho_{\text{rad}}(T)$ is as in equation (1). Including $\rho_D(T)$, two parameters will be introduced: $n_p = 3(w_D + 1)$, describing the behavior of the dark fluid ($w_D = \frac{p_D}{\rho_D}$ and P is the pressure of the dark component) and $k_p = \frac{\rho_D(T_0)}{\rho_{\text{rad}}(T_0)}$, which is the ratio of effective dark fluid density over the total radiation density at BBN time (taken to be $T_0 = 1$ Mev).

Then equation (9) becomes:

$$\rho_D(T) = k_p \rho_{\text{rad}}(T_0) \left(\frac{T}{T_0}\right)^{n_p},$$

(11)

Knowing that the universe was radiation dominated during the time of BBN requires the constraints $n_p \geq 4.0$ and $K_p < 1.0$ [10]. Adding dark density alone will not be sufficient to resolve the Lithium problem. The point is that the dark energy density being temperature dependent will increase the expansion rate leading higher $^4$He abundance and to higher production of $^7$Li (see table 4). This is contrary to the case when adding $\beta_{\nu_{\mu}}$ and $\beta_{\nu_{\tau}}$ as described above where the abundance of $^7$Li decreases because of higher abundance of Deuterium. These two different outcomes have different effects on the expansion rate and the freeze-out temperature and this illustrate how sensitive the Lithium production to these assumptions.

Table 4. Effect of adding $\rho_D(T)$ with $k_p = 0.5$ and $n_p = 6$.

| X($^4$He) | D/H       | $^3$He/H | $^7$Li/H |
|-----------|-----------|----------|----------|
| 2.920(−1)| 2.816(−5)| 1.016(−5)| 5.082(−10) |

2.3. Adding dark entropy

In analogy to the dark energy component, a dark entropy component can be added as well. This is described by [4] and [12] as:

$$s_D(T) = k_s s_{\text{rad}}(T_0) \left(\frac{T}{T_0}\right)^{n_s},$$

(12)

where $s_{\text{rad}}(T) = h_{\text{eff}}(T) \frac{2\pi^2}{45} T^3$, and $s_{\text{rad}}(T)$ is the radiation entropy density at a given time $T$.
and \( h_{\text{eff}}(T) \) are the effective degrees of freedom characterizing the contribution of particles to the entropy density. Then, the total entropy becomes:

\[
s_{\text{tot}}(T) = s_{\text{rad}}(T) + s_{D}(T)
\]

The direct effect of adding dark entropy is altering the time-temperature relation and clearly the light elements abundances. In this case the energy conservation equation will read:

\[
\frac{ds_{\text{tot}}}{dt} = -3Hs_{\text{tot}} \quad \text{or} \quad \frac{d}{dt}(\rho_{\text{tot}}a^3) + P_{\text{tot}} \frac{d}{dt}(a^3) + T \frac{d}{dt}(s_D a^3) = 0,
\]

where the third term corresponds to the dark entropy. The effect of adding dark entropy is increasing the temperature of the universe, altering the freeze-out of neutrons and the time when it takes place. Deuterium bottleneck (the highest D abundance where \( \frac{D}{H} \sim 10^{-7} \) at \( T \approx 0.07 \text{ Mev} \)) is shifted which leads to higher final D abundance. In addition, \(^7\text{Li} \) abundance decreases to match the observational range which is shown in table 5 and figure 1 without violating observational constraints.

**Table 5.** Effect of adding \( s_{D}(T) \) with \( k_s = 5.5 \times 10^{-4} \) and \( n_s = 5 \).

| X\(^{4}\text{He} \) | D/H | \(^3\text{He}/\text{H} \) | \(^7\text{Li}/\text{H} \) |
|-----------------|-----|----------------|-----------------|
| 2.306(-1)       | 3.489(-5) | 1.154(-5) | 2.781(-10) |

**Figure 1.** \(^7\text{Li} \) as function of temperature in SBBN and when adding dark entropy \((k_s = 0.00075, n_s = 5)\). Notice that \(^7\text{Li} \) decreases to match the range of observations.

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