Oscillatory vortex interaction in a gapless fermion superfluid with spin population imbalance

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Using effective field theory approach we study a homogeneous superfluid state with a single (gapless) Fermi surface, recently suggested as a possible phase for an ultracold Fermi gas with spin-population imbalance. We find an unconventional form of the interaction between vortices.

The presence of gapless fermions gives rise to an additional, predominantly attractive, potential oscillating in space, analogous to the RKKY magnetic interaction in metals. Our study then leads to an interesting question as to the nature of the vortex lattice in the presence of the competition between the usual repulsive logarithmic Coulomb and the fermion-induced attractive oscillatory interactions.

Recently a beautiful array of vortices has been observed in the MIT experiment of ultracold Fermi gases, providing a definitive evidence for superfluidity in this class of systems. [1] When fermionic excitations are either fully gapped or located at a few nodal points such as in a d-wave superconductor, the physics of vortices belongs to the universality class of the XY model where the phase of the superfluid order parameter plays the dominant role. In this case, the vortex sector of the XY model is described by the logarithmic Coulomb interaction between the vortices.

The development of imbalanced (spin-polarized) cold Fermi gases is set to alter this standard picture in a fundamental fashion, with a number of exotic superfluid states recently proposed or revisited [2, 3, 4]. One of those states commonly found in various theoretical approaches [2, 3, 4] is a homogeneous gapless fermion superfluid with a single Fermi surface (FS) on the molecular side of the Feshbach resonance (the BEC regime). We will call this phase BP1 (breached-pairing state with single Fermi surface) on the molecular side of the Feshbach resonance (the BEC regime).

In the present work, we study the effect of gapless fermionic excitations on the vortices in the BP1 phase by explicitly incorporating these excitations in the low-energy effective field theory. The resulting interaction between vortices is no longer of the pure Coulomb form, but contains an additional fermion-induced contribution that is attractive (between the vortices with the same vorticity) and oscillates on a length scale set by the spin polarization, analogous to the Ruderman-Kittel-Kasuya-Yosida (RKKY) magnetic interaction in metals.

The Bogoliubov-quasiparticle energy spectrum in a system of two fermion gases with an equal mass and unequal chemical potentials is given by [2] (ℏ ≡ 1)

\[ E_k^± = \sqrt{\left(\frac{k^2}{2m} - \mu\right)^2 + \Delta^2 \pm \delta} \tag{1} \]

where \( \mu = (\mu_\uparrow + \mu_\downarrow)/2 \) is the average chemical potential and \( \delta = \mu_\downarrow - \mu_\uparrow \) the chemical potential mismatch.

Since our treatment concerns the superfluid phase realized on the BEC side of the Feshbach resonance, \( \mu < 0 \) in what follows. For definiteness, we assume that \( \delta > 0 \).

If \( \delta \) is sufficiently large (i.e., if \( \delta/2 > \sqrt{\mu^2 + \Delta^2} \)), the lower branch \( E_k^\downarrow \) of the above dispersion is gapless, with a single effective FS. For brevity, we hereafter denote it as \( \varepsilon_k \). When linearized in the vicinity of the effective FS, this gapless dispersion adopts the generic form \( \varepsilon_k = v_b(|k| - k_b) \), parameterized by the “Fermi velocity” \( v_b \) and the radius \( k_b = [6\pi^2 n_b]^{1/3} \) of the “breached-pairing Fermi ball” in momentum space, where \( n_b = n_\downarrow - n_\uparrow \) (here \( n_\downarrow > n_\uparrow \), as follows from \( \delta > 0 \)).

In the present context, the superfluid phase field represents the phase of the complex Cooper pair amplitude \( (\psi_\uparrow \psi_\downarrow) = (|\psi_\uparrow \psi_\downarrow| |e^{i\theta}|) \) and can be separated into a (singlet) vortex part and a smoothly fluctuating field \( \phi \) via \( \nabla \theta(x, \tau) = -a + \nabla \phi(x, \tau) \), where \( a \) is the vortex gauge field. Let us consider a static configuration of vortex lines of unit winding number, all parallel to the z-axis, with a “vortex charge” density distribution \( \rho(x) = 2\pi \sum_o \delta^{(2)}(x - x_o) \). The gauge field \( a \) is related to \( \rho \) by the standard equation \( \nabla \times a = -\nabla \rho(x) e_z \), where \( \kappa_0 = \pi\hbar/m \) (\( \hbar \) restored for clarity) is the circulation quantum. In momentum space this reads \( q \times a_q = ik_0 \hat{\rho}(q) e_z \), where \( q \) is a two-dimensional wave-vector \( (q \cdot e_z = 0) \) of the phase fluctuations and \( \hat{\rho}(q) \) the Fourier transform of \( \rho(x) \). For convenience, we adopt the Coulomb gauge \( \nabla \cdot a = 0 \) in which the vector field \( a_q \) is purely transverse \( (q \cdot a_q = 0) \).

We start from a low-energy effective Lagrangian for
the gapless branch of the Bogoliubov quasiparticles (described by the field $\chi(x)$) and the superfluid phase field. This Lagrangian obeys two global $U(1)$ symmetries, one of which corresponds to the total atom number conservation ($U_c(1)$), and the other one to the conservation of spin population imbalance ($U_s(1)$). Son and Stephanov [11] first constructed an effective Lagrangian of this kind for the case of small gapless fermion density (corresponding to small spin population imbalance) to obtain a phase diagram. Here, we extend their approach to an arbitrary spin population imbalance. In the imaginary-time path-integral formalism, our effective Lagrangian reads

$$\mathcal{L} = \chi^* [\partial_t + \varepsilon(-i\nabla)]\chi + c_1 (\partial_x \theta)^2 + c_2 (\nabla \theta)^2 + c_3 \chi \left[ i \partial_t \phi + \frac{1}{2m_p} (\nabla \theta)^2 \right] + \nabla \phi \cdot \mathbf{j} + \cdots , \quad (2)$$

where $\cdots$ stands for higher-order derivative terms of the $\theta$ field; $\mathbf{j} = (\chi^* \nabla \chi - \nabla \chi^* \chi)/(2m_p)$, where $m_p = 2m$ is the mass of the Cooper pair; $\varepsilon(-i\nabla)$ is the gapless fermion dispersion, written in the coordinate representation. The phenomenological parameters $c_1$, $c_2$, and $c_3$ are not constrained by the $U(1)$ symmetries, but can be determined from the Galilean-invariance of the fermion-independent part of the above Lagrangian : $c_1 = \partial_n / \partial \mu$ (where $n = n_\uparrow + n_\downarrow$), while $c_2$ and $c_3$ are related to the superfluid density in a manner to be described shortly. Under Galilean boost with velocity $\mathbf{u}$ we have the following transformation properties: $\partial_x \to \partial_{x'} = \partial_x - i(\mathbf{u} \cdot \nabla), \nabla \to \nabla' = \nabla - \chi(x, \tau) \to \chi'(x', \tau') = \chi(x, \tau), \theta(x, \tau) \to \theta'(x', \tau') = \theta(x, \tau) - m_p \mathbf{u} \cdot \mathbf{X} - \frac{1}{2} m_p \mathbf{u}^2 \tau$. An alternative effective theory, formulated in terms of majority-species original fermions rather than Bogoliubov quasiparticles, is given in Ref. [12].

Because the Lagrangian has to be invariant under the $U_c(1)$ particle number symmetry $\theta \to \theta + \alpha$, it contains the coordinate and time derivatives of $\theta$, but not $\theta$ itself. Recast in terms of $\phi$ and $a$, it adopts the form

$$\mathcal{L} = \chi^* [\partial_t + \varepsilon(-i\nabla)]\chi + c_1 (\partial_x \phi)^2 + c_2 (\nabla \phi - a)^2 + c_3 \chi \left[ i \partial_t \phi + \frac{\nabla \phi - a}{2m_p} \right] + (\nabla \phi - a) \cdot \mathbf{j} . \quad (3)$$

We derive our effective phase-only action $S[\theta] \equiv S[\phi, a]$ by integrating out the fermionic degrees of freedom: $e^{-S[\theta]} = \int D(\chi^*, \chi) e^{-S[\chi, \theta]} \left[ S[\chi, \theta] = \int_0^\infty d\tau \int d\mathbf{x} \mathcal{L} \right]$. The Euclidean action corresponding to the last Lagrangian ($\beta \equiv (k_B T)^{-1}$). In order to accomplish this, we first note that the field $\chi$ enters the Lagrangian through a quadratic form $\chi^* K \chi = \chi^* (-\partial_\tau - \varepsilon(-i\nabla))^2 - \chi$, where $\phi_0 = [-\partial_\tau - \varepsilon(-i\nabla)]^{-1}$ is the noninteracting fermion propagator, and $X = X^{(1)} + X^{(2)}$ where $X^{(1)} = c_3 \partial_t \phi + \frac{\nabla \phi - a}{2m_p} \nabla$ and $X^{(2)} = \frac{c_3}{2m_p} (\nabla \phi - a)^2$ are respectively of the first and second order in fields $\phi$ and $a$. Integrating out the $\chi$ fields gives rise to the fermion contribution to the action $S[\phi, a]$:

$$S_F[\phi, a] = -\text{tr} \ln K = \text{const.} + \sum_{n=1}^\infty \frac{1}{n} \text{tr}[(\gamma_0 X)^n] . \quad (4)$$

We expand $\gamma_0$ to first order in $X^{(2)}$ (tree level) and to second order in $X^{(1)}$ (one-loop order). The effective phase-only action $S[\phi, a]$ is then obtained by gathering $S_F[\phi, a]$ and the fermion-independent terms of the original action. In momentum-frequency space $(q \equiv (\mathbf{q}, i\omega_n))$

$$S[\phi, a] = \sum_q \left\{ \left( c_2 + \frac{n_b}{2m_p} c_3 \right) q^2 + \frac{1}{2m_p^2} R_{ij}(q) q_i q_j + \left( c_1 - \frac{\Pi(q)}{2} \right) \omega_n^2 \delta_{ij} \right\} \phi_i \phi_j - \sum_q \left\{ c_2 + \frac{n_b}{2m_p} c_3 + \frac{P(q)}{2m_p^2} \right\} a_i \cdot a_{-q} . \quad (5)$$

where the first and second terms correspond to the propagating Goldstone mode and the topological vortex part of the broken $U_c(1)$ symmetry, respectively. [Summation over repeated indices in the last equation is implicit.] Here $\Pi(q)$ is the fermion density polarization bubble, while

$$R_{ij}(q) = \frac{1}{\beta V} \sum_k G_0(k)G_0(k + q)(k_i + \frac{q_i}{2})(k_j + \frac{q_j}{2}) \quad (6)$$

$$P(q) = \frac{1}{\beta V} \sum_k G_0(k)G_0(k + q) \frac{k^2}{2} \quad (7)$$

represent the longitudinal and transverse current-current correlation functions, respectively, with $k_\perp$ being the transverse component of the three-dimensional vector $k$ with respect to $q$.

It is important to point out that there is no RPA-type correction from the interaction vertex $\mathbf{j} \cdot \nabla \phi$ to the transverse current-current correlation function; this is manifest in our choice of the Coulomb gauge for the topological gauge field $a$.

The magnitude of wave-vector $q$ of the phase fluctuations has an upper cutoff of order $k_\Delta = (2m \Delta)^{1/2}$, a momentum scale corresponding to the pairing gap. This choice of momentum cutoff is a consequence of the BP1 phase being realized in the strong-coupling regime on the BEC side of the Feshbach resonance, where $\Delta$ is related to the binding energy of a Feshbach molecule. It is also important to emphasize that typical $|q|$ is not necessarily small with respect to $k_\Delta$; the latter is controlled by the spin imbalance and can be arbitrarily small.

It is straightforward to show that $R_{ij}(q) = R(q) \delta_{ij}$. Consequently, the phase-only action in Eq. (5) in the

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The text is a continuation of the previous discussion on the effective Lagrangian and the effective phase-only action. It explains how the Lagrangian is derived and how it relates to the phase-only action, providing equations and explanations for each step. The text also discusses the implications of the choice of the Coulomb gauge for the topological gauge field and the significance of the momentum cutoff. The final part of the text notes the relationship between the phase-only action and the correlation functions, and how the RPA-type correction is treated.
zero-temperature static limit reduces to
\[
S[\phi, a] = \sum_q \left( c_2 + \frac{n_b}{2m_p} c_3 + \frac{R_{q=0}^0}{2m_p^2} \right) q^2 \phi_q \phi_{-q} + \sum_q \left( c_2 + \frac{n_b}{2m_p} c_3 + \frac{P_{q=0}^0}{2m_p^2} \right) a_q \cdot a_{-q} ,
\] (8)
where \(P_{q=0}^0\) and \(R_{q=0}^0\) are the zero-temperature static limits of \(P(q)\) and \(R(q)\), respectively.

The superfluid mass density \(\rho_s = n_s m_p\), which plays the role of rigidity in the present problem ("spin-wave" stiffness in the XY-model terminology), can be identified from the long-wavelength (\(q \to 0\)) limit through the relation
\[
\frac{\rho_s}{2} = c_2 + \frac{n_b}{2m_p} c_3 + \frac{P_{q=0}^0}{2m_p^2} .
\] (9)
This equation constraints the coefficients \(c_2\) and \(c_3\), which reduces the phase-only action in Eq. (8) to
\[
S[\phi, a] = \sum_q \left( \frac{n_s}{2m_p} q^2 \phi_q \phi_{-q} \right) + \sum_q \left( \frac{n_s}{2m_p} + \frac{P_{q=0}^0}{2m_p^2} \right) a_q \cdot a_{-q} ,
\] (10)
which is free of the phenomenological parameters of the original theory.

Integrating out the phase field \(\phi\) in the action (10), taking account of the fact that \(P_{q=0}^0 = P_{q=0}\), leads to the following effective action for vortices
\[
S_{\text{eff}}[a] = \sum_q \left( \frac{n_s}{2m_p} + \frac{P_{q=0}^0 - P_{q=0}^0}{2m_p^2} \right) a_q \cdot a_{-q} .
\] (11)
(Since the vortex gauge field belongs to the classical sector of the theory, the derived effective action contains only the \(\omega l = 0\) part). The last result, combined with the identity \(\bar{\rho}(q)\bar{\rho}(-q) = (q^2/\sqrt{2})(a_q \cdot a_{-q})\), yields the effective vortex interaction potential:
\[
V_{\text{eff}}(q) = k_b^2 \left( \frac{n_s}{2m_p} q^2 + \frac{1}{2m_p^2} \frac{P_{q=0}^0 - P_{q=0}^0}{q^2} \right) .
\] (12)
Thus besides the conventional long-range component proportional to \(1/q^2\), we have an additional component
\[
V_{\text{ind}}(q) = \frac{k_b^2}{2m_p} \frac{P_{q=0}^0 - P_{q=0}^0}{q^2} \] (13)
induced by the gapless fermions.

Response function \(P_{q=0}^0\) has to be calculated numerically. Yet, prior to numerical computations, \(P_{q=0}^0\) can be reduced to a two-dimensional principal-value integral
\[
P_{q=0}^0 = \frac{k_b^3}{(2\pi)^2} \int_0^1 |k|^4 dk \int_0^{\pi} \frac{1 - \cos^2 \theta}{\varepsilon_k - \varepsilon_{k+q}} \sin \theta d\theta ,
\] (14)
where momenta \(k\) and \(q\) are expressed in units of \(k_b\). For small \(q\) (\(|q| < 0.1k_\Delta\)), numerical evaluation becomes troublesome because of the strongly singular character of the integrand in Eq. (14). However, in the limiting case \(|q| \to 0\), by replacing dispersion \(\varepsilon_k\) with its linearized form \((\varepsilon_k \to v_b(|k| - k_b))\) we can derive the analytical result
\[
P_{q=0}^0 \to -\frac{m k_b^3}{6\pi^2} \sqrt{\left( \frac{k^2}{2m} - \mu \right)^2 + \Delta^2} \quad (|q| \to 0) .
\] (15)
Some typical results of numerical evaluation of the response function \(P_{q=0}^0\) for \(|q| \geq 0.1k_\Delta\) are displayed in Fig. 1 (where \(2k_b < k_\Delta\)). The salient characteristic of these results is a knee-like feature at \(|q| = 2k_b\), which reflects the existence of an effective FS of diameter \(2k_b\). It bears analogy to the \(2k_F\)-feature of the paramagnetic spin susceptibility in 3D, responsible for the RKKY indirect-exchange interaction in metals, albeit the \(2k_b\)-feature found here comes from the current-current correlator so that it is not directly related to the RKKY interaction. The values of \(P_{q=0}^0\) obtained analytically in \(|q| \to 0\) limit differ just slightly from numerical values at \(|q| = 0.1k_\Delta\), indicating that \(P_{q=0}^0\) can be approximated as a constant in this numerically-inaccessible region \(0 < |q| < 0.1k_\Delta\).

FIG. 1: Transverse current response function \(P_{q=0}^0\) as a function of dimensionless momentum, for \(m = 1.0\) and \(\Delta/|\mu| = 2.0\). Values of \(k_b\) are given in units of \(k_\Delta\).

By exploiting the rotational invariance of the induced potential in momentum space, in real space we obtain
\[
V_{\text{ind}}(r) = \frac{1}{2\pi} \int_0^{k_b} |q| |V_{\text{ind}}(|q|)| J_0(|q|r)d|q| ,
\] (16)
where \(J_0(x)\) is the zeroth-order Bessel function of the first kind. Our numerical calculations for different values of relevant parameters \((k_b, \Delta)\) show that the induced potential has RKKY-like oscillating behavior, with attractive character \((dV_{\text{ind}}/dr > 0)\) at short distances. To
calculate the effective vortex interaction potential, given by the sum of \( V_{\text{ind}} \) and the repulsive logarithmic part \( V_0(r) = -\left( k_\Delta^2 \rho_s/(2m_f) \right) \ln(k_\Delta r) \), we have computed the superfluid density by adapting the method of Ref. [13] to the special case of a single gapless FS.

In order to elucidate the realm of validity of our effective theory and make contact with experiments, it is useful to estimate the physical healing length \( \xi = (8\pi n_s a_m)^{-1/2} \) (\( a_m \) being the molecular scattering length), with the inverse of the momentum scale \( k_\Delta \). It is known that in the strong-coupling BEC regime of a superfluid Fermi gas with equal populations of two hyperfine spin components the molecular scattering length is given by \( a_m = 0.6 a_f \) (\( a_f \) being the scattering length between fermionic atoms). [14] For a polarized Fermi gas, as shown by Sheehy and Radzihovsky, \( a_m \) decreases monotonously as a function of the polarization and vanishes at the boundary of first-order phase transition to the phase separated state. Therefore, right at the boundary to phase separation and in the immediate vicinity of it the coherence length becomes much greater than \( k_\Delta^{-1} \), thus making the quantitative implications of our theory not directly applicable in this special case. Taking \( a_m(P = 0) \) in place of \( a_m \), together with typical values of \( n_s/n \) and \( \Delta/\epsilon_F \) \( (\epsilon_F = k_F^2/(2m) \), where \( k_F = (3\pi^2 n)^{1/3} \) is the momentum scale set by the total fermion density) in the BEC regime, we estimate that \( \xi \) is of the same order as \( k_\Delta^{-1} \) when \( |\kappa| \sim 1-100 \). By taking into account the fact that \( \xi/k_\Delta^{-1} \propto (a_m/a_f)^{-1/2} \), we can infer that the above estimate is just slightly modified as a result of \( a_m \) decreasing in the vicinity of the phase separation line.

As our calculations demonstrate, the effective vortex-vortex interaction shows three characteristic types of behavior, i.e. three polarization-dependent regimes. The critical polarizations corresponding to the boundaries between these different regimes are not universal but depend on the actual location in the part of the phase diagram pertaining to the BP1 phase.

In the regime of relatively low polarization, the total potential is dominated by the conventional repulsive logarithmic part; the effective vortex interaction is repulsive \( (dV_{\text{eff}}/dr < 0) \) at all distances. The resulting vortex phase is accordingly expected to be conventional, with triangular vortex arrangement. An example is shown in Fig. 2a.

In the other extreme - the regime of high polarization, the induced potential plays a dominant role at short and intermediate distances. This renders the total potential attractive at short distances, with pronounced oscillating features resembling the RKKY interaction, as illustrated in Fig. 2b.

The attractive nature of two-body interaction already at short distances suggests an instability of the vortex lattice. However, whether this instability really occurs is still an open question for the following reasons. The physics at distances shorter than the healing length \( \xi \) (to be discussed in the next section) is not captured by our effective theory; also, the multi-vortex interactions, not considered here but certainly allowed as higher orders in the effective vortex action, may support unusual vortex phases.

Apart from these two extreme regimes, in a narrow window of parameters the total potential is repulsive at short distances \( (r \approx (2 - \xi)^k \) and becomes attractive at intermediate ones (Fig. 3).

Due to the finite range of the RKKY-like induced potential, the truly long-distance dependence of the effective potential (for all polarizations) is governed by the infinite-range repulsive logarithmic interaction.

![FIG. 2: Effective vortex interaction potential in real space (in units of \( \Delta \)), for \( n = 1.0 \) and \( \Delta/|\mu| = 2.0 \). Values of \( k_\Delta \) in both plots are expressed in units of \( k_\Delta \): (a) \( k_\Delta = 0.623; 0.724; 0.846 \) correspond to polarizations \( P = 0.155; 0.227; 0.314 \), and (b) \( k_\Delta = 1.500; 1.352; 1.205 \) correspond to polarizations \( P = 0.763; 0.702; 0.626 \), respectively.](image)
FIG. 3: Effective vortex interaction potential in real space (in units of $\Delta$), for $m = 1.0$ and $\Delta/|\mu| = 1.0$. $k_0 = 1.022$ (in units of $\kappa_\Delta$) corresponds to $P = 0.504$.

therefore, opens up an intriguing question about the vortex lattice structure in the spin-imbalanced gapless fermion superfluid state.

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