A COMPLETE ASYMPTOTICALLY SAFE EMBEDDING OF THE STANDARD MODEL

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Abstract. We present and discuss the “Tetrad Model”, a large colour/flavour embedding of the Standard model which has an interacting ultraviolet fixed point. It is shown that its extended-Pati-Salam symmetry is broken radiatively via the Coleman-Weinberg mechanism, while the remaining electroweak symmetry is broken when mass-squared terms run negative. In the IR the theory yields just the Standard Model, augmented by the fact that the Higgs fields carry the same generation indices as the matter fields. It is also shown that the Higgs mass-squareds develop a hierarchical structure in the IR, from a UV theory that is asymptotically flavour symmetric, opening up an interesting direction for explaining the emergence of the observed flavour structure.

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1. Introduction and Overview of Embedding

A recent series of papers [1,2] suggested a framework for embedding the Standard Model (SM) in an asymptotically safe ultra-violet (UV) completion [3]. (For some earlier discussions of asymptotic safety applications see [4,21]. For recent reviews see [22,23].) The framework is partially perturbative based on the weak ultra-violet (UV) fixed points of [12,16,17] (hereafter LS). These are the UV counterparts of the better known Caswell-Banks-Zaks infra-red (IR) fixed points, in a large colour and flavour Veneziano limit. By suitable adjustment of the numbers of colours and flavours, a UV fixed point can be achieved that is arbitrarily weakly coupled. Coupled with the “large flavour” fixed points of [24–29] operating for the electroweak gauge couplings, one finds an asymptotically safe extension of the Pati-Salam (PS) theory, that has a UV fixed point with gauge group \( SU(N_C) \times SU(2)_L \times SU(2)_R \), and a natural breaking down to the SM gauge group in the IR driven partially by radiative symmetry breaking. The main observation of [2] was that the two kinds of fixed points (Veneziano and large \( N_f \)) do not interfere with each other.

Despite this attractive framework for embedding the Standard Model, the theories presented in [1,2] did not provide a mechanism for fully removing the extraneous degrees of freedom in the IR to leave purely the SM. In particular in this simplest realisation, there remain in the low energy theory a large multiplicity of electroweak \( SU(2) \) doublets, that are unmatched and hence massless.

In this paper we provide a complete phenomenological framework, by an enhancement that yields a theory flowing from an asymptotically safe fixed point in the UV to precisely the SM, augmented only by additional Higgses. In particular there are no other light superfluous states remaining in the theory. The additional Higgses are furnished with the same generation numbers as the matter fields, so their VEVs may therefore ultimately be able to explain flavour hierarchies (although we do not attempt this in the present paper).

Moreover symmetry breaking can be entirely radiative. It can happen in two ways (or by a combination of them). One possibility is the traditional radiative symmetry breaking mechanism of Coleman and Weinberg [30,31]. This can be shown to occur analytically driven by a single quartic coupling running negative and generating a minimum according to the pattern discussed in [31]. This can be responsible for the bulk of the breaking of the extended PS gauge group. At the same time the PS breaking generates a positive mass-squared for the Higgs at the high scale due to a portal-like coupling between the electroweak Higgs and the PS Higgs. This can run negative in the IR due to large Yukawa couplings from its initially positive boundary value at the PS-scale. The alternative possibility is that radiative symmetry breaking is instead dominated by the dimensionful couplings (i.e. mass-squared terms) and the PS breaking minimum is generated radiatively when they run negative due to the large Yukawa couplings that have to be present in the PS gauge-Yukawa theories. This was the mechanism discussed in [1], which is essentially the asymptotically safe version of the radiative symmetry breaking in the supersymmetric Standard Model [33]. Thus one appears to have the freedom to turn on as much or as little of the classically dimensionful operators as desired in the symmetry breaking.

In order to present our model we will also advocate in this paper the use of “quiver” diagrams. Such diagrams can greatly alleviate the generic problem that the UV of asymptotically safe models is complicated because they necessarily have to include extra degrees of freedom, and as a consequence the structure is often hard to appreciate (or present), even though it may in reality be relatively simple. Although (depending on the model in question) it may only be possible to represent part of the gauge groups in quiver diagrams (if some of the states do not easily fall into bi-fundamentals), their use can greatly ease the construction of phenomenologies within an asymptotically safe framework.

2. The “Tetrad” Model (TM)

2.1. Structure in the UV. We begin by recapping the LS fixed point of [16], whose field content is shown in Table 1, where mid-alphabet latin indices \( i, j, k \ldots \) are used to label flavour, while early-alphabet latin indices \( a, b, c \ldots \) label colour. The particle content is represented as in a conventional quiver diagram, in Figure 1. As usual, the circular nodes represent the \( SU(N_C) \) gauge factor, which is crucial in establishing the LS fixed point. The square nodes represent the flavour groups, \( SU(N_F)_L \otimes SU(N_F)_R \), which will become partially gauged in order to accommodate the electroweak gauge factors of the SM. In [1,2] the LS model was augmented by coloured scalars in order to break the gauge group down to the SM. However as mentioned in the Introduction, there remain in such models light doublets which are charged under the electroweak SM gauge groups.

\[\begin{array}{c|c}
\text{SU}(N_C) & \text{SU}(2)_L \times \text{SU}(2)_R \\
\hline
\text{SU}(2)_L & \text{SU}(2)_R \\
\end{array}\]
Table 1. Fields in the arbitrarily weakly coupled asymptotic safe fixed point of [16].

|          | $SU(N_C)$ | $SU(N_F)_L$ | $SU(N_F)_R$ | spin |
|----------|------------|-------------|-------------|------|
| $Q_{as}$ | □          | □           | □           | 1/2  |
| $Q^{pa}$ | □          | □           | □           | 1/2  |
| $H_i$    | 1          | □           | □           | 0    |

Figure 1. Quiver diagram of the fixed point theory of [16]. Solid lines represent fermions, dashed lines represent bosons.

Let us now proceed directly to the phenomenologically viable augmented model that we will propose in this paper. As we shall see the model leaves no light states, other than those appearing directly in the SM, beyond an enhanced Higgs sector (with the Higgs fields carrying the same generation indices as the matter fields). In this section we will lay out the spectrum and pattern of VEVs that need to be achieved in order to realise the Standard Model in the IR, and then in the following section we consider the dynamics that achieves them. The augmented model is shown in Table ?? and its corresponding quiver diagram in Figure2. It contains four elements, hence we refer to it as the Tetrad Model (TM). As in [2] it is an extension of the PS model to a larger unified group. Note that the PS gauge unification to $SU(2)_R$ is adopted to take advantage of the $SU(2)$ large-flavour fixed points, introduced in [24,25]. We will use a Weyl notation and display the left and right fermions explicitly. We use the following nomenclature for the spectrum: Fermions will be denoted with $Q$ and $q$’s, while scalars will be denoted with $\tilde{S}$ and $H$. The flavour indices $i = 1...N_F$ have three generations of components gauged under electroweak $SU(2)_L$ and $SU(2)_R$. However we have to gauge the right-handed component of the electroweak gauge group in the correct way to yield the SM spectrum. Indeed the “squarks” $\tilde{S}$ have their own $SU(N_S)$ flavour symmetry, and the first two flavours also have to be charged under $SU(2)_R$ in order to give the correct PS breaking. The simplest solution is then to identify $SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{\text{diag}}$. This leads to hypercharge $Y = \left(2T^{(3)}_R + B - L\right)$ and charge $Q_{\text{em}} = \frac{1}{2} \left(2T^{(3)}_R + 2T^{(3)}_L + B - L\right)$, where $T^{(3)}_{L/R} = \text{diag}(\frac{1}{2}, -\frac{1}{2})$ and $B - L$ is the diag$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, 0, 0, ..., 0)$ generator of $SU(N_C)$. As we shall see, for the LS gauge-Yukawa fixed point to be weakly coupled we require $N_F \approx \frac{N_C}{4}$.

The necessity of the additional fermionic fields $q, \tilde{q}$ can be deduced from the requirement that they are able to remove the unwanted light fermionic degrees of freedom while maintaining the chiral symmetry.

\footnote{The $\tilde{S}$ scalars were referred to as $\tilde{Q}$ in [2], but in the present context this would cause confusion.}
Table 2. Fields in the asymptotically safe “Tetrad” Model, where \( N_S = N_C - 2 \) and \( N_F \approx \frac{4}{3} N_C \). The top 2\( n_q \) = 6 components of flavour \( SU(N_F) \) correspond to \( SU(2) \) multiplets, where \( n_q \) is the generation number. The gauging for the usual Pati-Salam \( SU(2)_R \) group is identified as \( SU(2)_R = [SU(2)_L \otimes SU(2)_L]_{\text{diag}} \).

| Field | \( SU(N_C) \) | \( SU(N_F)_L \otimes SU(N_F)_L \) | \( SU(N_F)_R \otimes SU(N_F)_R \) | \( SU(N_F) = SU(N_F - 4)_R \otimes SU(2)_R \) | Spin |
|-------|----------------|---------------------------------|---------------------------------|---------------------------------|------|
| \( Q_{ai} \) | \( \Box \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box \) | 1/2 |
| \( \tilde{Q}^{\alpha} \) | \( \Box \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box \) | 1/2 |
| \( H^1_i \) | \( 1 \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box \) | 0 |
| \( S_{a,L=1-N_F} \) | \( \Box \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box = \Box_{N_C-4} \otimes \Box \) | 0 |
| \( \tilde{q}_i^a \) | \( 1 \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box = \Box_{N_C-4} \otimes \Box \) | 1/2 |
| \( q_i^a \) | \( 1 \) | \( \Box \otimes \Box \otimes \Box \) | \( \Box \otimes \Box \) | \( \Box = \Box_{N_C-4} \otimes \Box \) | 1/2 |

The allowed couplings one can consider for the generation of the UV-fixed point are

\[
\mathcal{L}_{\text{UVFP}} \supset \mathcal{L}_{\text{KR}} + \frac{y}{\sqrt{2}} \text{Tr} \left[ (QH) \cdot \tilde{Q} \right] + \frac{\tilde{y}}{\sqrt{2}} \text{Tr} \left[ qH^1 \tilde{q} \right] - \frac{\tilde{Y}}{\sqrt{2}} \text{Tr} \left[ (\tilde{S} \cdot \tilde{Q}) \tilde{q} \right] - \frac{Y}{\sqrt{2}} \text{Tr} \left[ (\tilde{Q} \cdot \tilde{S}^\dagger) q \right] - u_1 \text{Tr} \left[ H^1 H \right]^2 - u_2 \text{Tr} \left[ H^1 H H^1 H \right] - v_1 \text{Tr} \left[ H^1 H \right] \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right] - w_1 \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right]^2 - w_2 \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \cdot \tilde{S} \cdot \tilde{S} \right],
\]

(1)

where the trace is over the flavour indices and the dot refers to colour contraction. As we shall see the \( Y \) and \( \tilde{Y} \) Yukawa couplings are responsible for giving masses to the unwanted degrees of freedom in the IR once \( \tilde{S} \) gets a VEV. They are written above somewhat schematically as clearly they cannot couple all the flavour components in the same way due to the \( SU(2)_R \) gauge invariance. They will be treated explicitly below.

As in (2) we will not consider the flavour breaking coupling (schematically)

\[
\mathcal{L}_{\text{SU(N_F)}} = -v_2 \text{Tr} \left[ H^1 H \tilde{S}^\dagger \cdot \tilde{S} \right].
\]

(2)

This coupling can be fixed to be precisely zero, where it will remain along the flow. (It can of course be forbidden on grounds of preservation of flavour symmetry which we will associate with the classically relevant operators only.) As we shall see the flavour conserving portal coupling \( v_1 \) can generate a mass-squared for the electroweak Higgses, and we keep it in the analysis.

We can in addition include the aforementioned dimensionful “soft-terms”. Unlike the classically dimensionless couplings these will be allowed to explicitly violate the flavour symmetry. They can be written most generally in the form

\[
\mathcal{L}_{\text{soft}} = -m_{h_0}^2 \text{Tr} \left[ H^1 H \right] - \sum_{a=1}^{N^2 -1} \Delta_a^2 \text{Tr} \left[ H T^a \right] \text{Tr} \left[ H^1 T^a \right],
\]

(3)

where \( T^a \) are the generators of the \( SU(N_F)_{\text{diag}} \) flavour group. Being classically relevant, the soft terms cannot disrupt the UV fixed point, but can serve to generate symmetry breaking themselves, and also remove any Goldstone modes associated with the spontaneously broken global flavour symmetries.

2.2. Structure in the IR – emergence of the SM from the TM. Next let us confirm that the SM emerges in the IR from the Tetrad Model. It is useful for this purpose to explicitly write the particle content in terms of SM quantum numbers in order to discuss the couplings, and determine the required values for
Figure 2. The “Tetrad” quiver that gives the Standard Model in the IR. Note that it is not possible to illustrate the gauging of the electroweak symmetries on such a diagram. On the right, the gauging is on the SU(2)_R = [SU(2)_c ⊗ SU(2)_S]_diag factor, with the top 2n_q indices of SU(N_F)_L,R flavour transforming as doublets under SU(n_q) ⊗ SU(2)_L,R.

N_F, N_S: the explicit representations are (c.f. the usual PS model in for example [34])

\[
Q = \begin{pmatrix}
q_1 & \ell_1 & \cdots & \psi_\pm & \cdots & \psi_{-1}
\end{pmatrix},
\]

\[
N_F ; \quad \tilde{Q} = \begin{pmatrix}
\psi^c & \nu^c & \cdots & \chi^c & \cdots
\end{pmatrix},
\]

\[
\begin{array}{c}
N_S = N_C - 2
\end{array}
\]

\[
q = \begin{pmatrix}
\psi_0 & \psi_1 & \psi_\pm & \cdots & \psi_{-1}
\end{pmatrix},
\]

\[
N_F ; \quad \tilde{q} = \begin{pmatrix}
\tilde{\psi}_0 & \tilde{\psi}_1 & \tilde{\psi}_\pm & \cdots & \tilde{\psi}_{-1}
\end{pmatrix},
\]

\[
(4)
\]

\[
(5)
\]
Given that the gauge symmetry can be broken as required, one can focus on the excess states that need to form during the flow, so that they become dominant in the IR.

The universal part of such operators flows to relatively smaller absolute values, "exposing" flavour hierarchies. We therefore need a re-analysis of the UV fixed point behaviour of the theory so we instead adopt the philosophy that at this stage the gauge symmetry is broken to the Standard Model as

\[ \langle \tilde{S} \rangle = \begin{pmatrix} \bar{S}_{PS} \\ \Phi_0 \end{pmatrix} = \left( \begin{array}{cc}
\left( \begin{array}{c}
\bar{\Phi}_c \\
\bar{\Phi}_c
\end{array} \right) & \left( \begin{array}{c}
\phi^0_c \\
\phi^0_c
\end{array} \\
\phi^0_c \\
\phi^0_c
\end{array} \right)
\right) \right) \]

\[ N_S = N_C - 2 \quad \text{, (6)} \]

where \( H_0 \) is an \((N_F - 6) \times (N_F - 6)\) scalar which is uncharged under the SM gauge groups, and the suffices denote \( Q_{e.m.} \). The assignment of the remaining fields is obvious.

First note that the top 2\( n_g \) (where \( n_g = 3 \) is the number of generations, but it is often useful to leave it generic) entries of flavour are charged under the SU(2) gauge groups. Therefore, given the couplings and matter content, there are \( n_g \) generations of SM Higgs doublets in the top 2\( n_g \times 2 n_g \) components of \( H \).

Assuming that \( n_g = 3 \), this corresponds to 18 separate Higgs SU(2)\(_L\) doublets. Clearly one ultimately requires these to be lifted in a hierarchical way so that there is one dominant lighter Higgs which gets a VEV, which will be a mixture of the 18 original ones. In contrast with \( \tilde{S} \) we will assume that the scalars \( \tilde{S} \) are gauged only under colour except for the first two flavours which are charged under the gauged SU(2)\(_R\). (The latter choice is flexible.)

We repeat that we are assuming flavour degeneracy in all the couplings of \( \tilde{S} \). One could instead for example take the \( y \) Yukawa couplings to break SU(\( N_F \))\(_L\) \times SU(\( N_F \))\(_R\) symmetry, but this would require a re-analysis of the UV fixed point behaviour of the theory so we instead adopt the philosophy of [2]. As there is a pair of Higgs multiplets for each generation, this is indeed an attractive possibility for introducing SM-flavour structure. Moreover as shown in [2] and expanded upon below, the flavour universal part of such operators flows to relatively smaller absolute values, "exposing" flavour hierarchies during the flow, so that they become dominant in the IR.

There are two elements to the gauge symmetry breaking. First there are VEVs for \( \tilde{S} \). We must choose \( N_S = N_C - 2 \), so that they can be rearranged by suitable colour and SU(\( N_S \)) flavour rotations into the form

\[ \langle \tilde{S} \rangle = \tilde{V} \left( \begin{array}{c}
0 \\
0 \\
0 \\
0 \\
0 \\
\cdots \\
1
\end{array} \right) \]

\[ N_S = N_C - 2 \quad \text{, (8)} \]

with the VEV \( \phi_0 \) in [6] being of the form \( \langle \phi_0 \rangle = 0 \cdots 0 \), where \( \tilde{V} \) is a constant. The SU(2)\(_R\) orientation simply determines the direction corresponding to the massless right-handed "sneutrino", so one may always choose a basis in which the \( \tilde{\nu} \) and \( N_C - 4 \) of the \( \phi_0 \)’s on the diagonal get a VEV. (Obviously the case \( N_C = 4 \) is the standard, non-asymptotically free, Pati-Salam model.)

At this stage the gauge symmetry is broken to the Standard Model as

\[ SU(N_C) \times SU(2)_L \times SU(2)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y \].

(9)

Given that the gauge symmetry can be broken as required, one can focus on the excess states that need to be made massive in order to end up with the Standard Model in the IR. In particular there are of course
(by design) very many $SU(2)_L$ and $SU(2)_R$ doublets that should be removed at low scales. The second component of symmetry breaking that accomplishes this is that the block $H_0$ of the Higgs multiplets also acquire VEV along the diagonal,

$$ (H_0) = V_0 I_{N_F - 6}. $$  \hspace{1cm} (10)

Thanks to the $y$ coupling, this gives the $N_F - 6$ generations of complete non-doublet $SU(N_C)$ multiplets masses $\frac{\sqrt{2} y}{\sqrt{2} V}$, leaving untouched $n_g(N_C - 4)$ of the $SU(2)_L$ doublets in the $Q_L$, and $SU(2)_R$ doublets in the $Q_R$. Indeed in these remaining $n_g$ generations of $SU(N_C)$-coloured multiplets, only the first $SU(4)$ components are to be identified as matter fields, as in (11). The remaining states get masses $\frac{\sqrt{2} \tilde{y}}{\sqrt{V}}$ and $\frac{\sqrt{2} \tilde{V}}{\sqrt{V}}$ from the $\tilde{Y}$ and $\tilde{V}$ couplings respectively to which we now return, writing them with explicit indices:

$$ \mathcal{L}_{UVFP} \supset \frac{\tilde{Y}}{\sqrt{2}} Q \tilde{S} \tilde{q} - \frac{Y}{\sqrt{2}} \tilde{S}^j \tilde{q}^j, $$

$$ = \frac{\tilde{Y}}{\sqrt{2}} \left( k_{\alpha} \tilde{S}^j \tilde{q}^k_{\alpha} \right) - \frac{Y}{\sqrt{2}} \left( \tilde{Q}_{k\alpha} \tilde{S}^j \tilde{q}^k_{\alpha} \right), $$

$$ \equiv \frac{\tilde{Y}}{\sqrt{2}} \left( \psi_+ - \frac{1}{2} \gamma_k \phi_{0\alpha} \left( \frac{\psi_+}{\psi_-} \frac{\tilde{y}}{\sqrt{2}} \psi_ \right) \right) - \frac{Y}{\sqrt{2}} \left( \psi_+ - \frac{1}{2} \gamma_k \phi_{0\alpha} \left( \frac{\psi_+}{\psi_-} \right) \right), $$

$$ = \frac{\tilde{Y}}{\sqrt{2}} \left( \psi_+ - \frac{1}{2} \gamma_k \phi_{0\alpha} \left( \frac{\psi_+}{\psi_-} \right) \right) - \frac{Y}{\sqrt{2}} \left( \psi_+ - \frac{1}{2} \gamma_k \phi_{0\alpha} \left( \frac{\psi_+}{\psi_-} \right) \right), $$  \hspace{1cm} (11)

where $\alpha = 1 \ldots N_C$ are colour indices, $\alpha = 5 \ldots N_C$ are the $N_C - 4$ colour indices beyond the PS degrees of freedom, $j = 1 \ldots N_C - 4$ are the $N_C - 4$ flavour indices of $\tilde{S}$ that are not charged under $SU(2)_R$, the indices $\alpha = 1, 2$ are the $SU(2)_L$ indices, and $k = 1 \ldots n_g$ are generation indices. Note that as promised chiral symmetry dictates the choice $N_S = N_C - 4$, because $N_S$ flavour is locked to $SU(N_C)$ colour by the VEV of $\tilde{S}$.

It is easy to check that with this choice of colours and flavours, and these VEVs, the remaining content in the IR is that of the SM with Higgses carrying $SU(n_g)$ generation indices for the left and right handed fields.

3. Flow from the UV fixed points and symmetry breaking

3.1. The Tetrad Model contains the Coleman Weinberg mechanism. Next let us turn to the dynamics, first illustrating the appearance of traditional radiative symmetry breaking. As there are many couplings involved, it is useful to break down the evolution under RG flow into self-contained units. Indeed the crucial aspect of the flow from the UV fixed point is that it is actually controlled by two fixed points of the gauge and Yukawa couplings, which form a closed system by themselves.

It will be convenient to define rescaled couplings as follows:

$$ \alpha_g = \frac{N_C \tilde{g}^2}{(4\pi)^2}; \quad \alpha_y = \frac{N_C \tilde{y}^2}{(4\pi)^2}; \quad \alpha_\tilde{g} = \frac{N_C \tilde{g}^2}{(4\pi)^2}; \quad \alpha_\tilde{y} = \frac{N_C \tilde{y}^2}{(4\pi)^2}; \quad \alpha_\tilde{Y} = \frac{N_C \tilde{Y}^2}{(4\pi)^2}. $$

$$ \alpha_{u_1} = \frac{N_F u_1}{(4\pi)^2}; \quad \alpha_{u_2} = \frac{N_F u_2}{(4\pi)^2}; \quad \alpha_{v_1} = \frac{N_F v_1}{(4\pi)^2}; \quad \alpha_{v_2} = \frac{N_F v_2}{(4\pi)^2}; \quad \alpha_{w_1} = \frac{N_F w_1}{(4\pi)^2}; \quad \alpha_{w_2} = \frac{N_F w_2}{(4\pi)^2}. $$  \hspace{1cm} (12)

To determine their fixed points, we require their RG equations to order $\alpha^3 \equiv \alpha^2$ in $\beta_y$ and order $\alpha^2 \equiv \alpha$ in $\beta_{\tilde{y}, \tilde{Y}}$: defining $\epsilon = -11/2 + x_F + x_q/4 = x_F - 1/4$ and $\tilde{\Upsilon} = \sqrt{\alpha_g \alpha_\tilde{g} \alpha_Y \alpha_\tilde{Y}}$, and taking $n_g = 3$, the beta functions are found to be

$$ \beta_y = \alpha_y \left( \frac{4}{3} \epsilon + (26/3) x_F - 20 \right) \alpha_g - \alpha_\tilde{y} \alpha_Y - x_F \alpha_Y - x_F \alpha_\tilde{Y}, $$

$$ \beta_\tilde{y} = \alpha_y \left( 1 + x_F \alpha_Y + \alpha_\tilde{g} + \alpha_Y + 6 \alpha_g \right), $$

$$ \beta_\tilde{Y} = x_F \tilde{\Upsilon} + \alpha_\tilde{Y} \left( 2 (1 + x_F) \alpha_Y + (1/2) \alpha_\tilde{Y} + (1/2) \alpha_y + (1/2) \alpha_\tilde{g} - 3 \alpha_y \right), $$

$$ \beta_\tilde{\Upsilon} = 2 x_F \tilde{\Upsilon} + \alpha_\tilde{Y} \left( 2 (1 + x_F) \alpha_Y + (1/2) \alpha_\tilde{Y} + (1/2) \alpha_y + (1/2) \alpha_\tilde{g} + 2 \alpha_Y - 3 \alpha_y \right). $$  \hspace{1cm} (13)
The collection of UV fixed points for the gauge and Yukawa couplings: schematically the flow is from A → B → C,D → E. Fixed points C,D,E are pseudo-fixed points in the sense that the quartic scalar couplings do not have a fixed point there. The only true non-trivial fixed point is the LS fixed point of B.

| Label | $\alpha_y^*$ | $\alpha_Y/\alpha_y$ | $\alpha_g/\alpha_y$ | $\alpha_Y/\alpha_g$ | $\alpha_{\tilde{Y}}/\alpha_g$ |
|-------|--------------|---------------------|---------------------|---------------------|---------------------|
| A     | 0            | 0                   | 0                   | 0                   | 0                   |
| B     | 0.0823       | 0                   | 0                   | 0                   | 0                   |
| C     | 0.0831       | 0                   | 0                   | 0                   | 0                   |
| D     | 0.0831       | 0                   | 0                   | 0                   | 0                   |
| E     | 0.0831       | 0                   | 0                   | 0                   | 0                   |

Since it is positive, the equation for $\beta_y = 0$ can only be consistently met with $\alpha_y = Y = 0$. Moreover if any of the other couplings are non-zero, it flows to zero in the IR, so along the RG trajectory from any eligible fixed point it must remain zero. There are also by inspection no positive solutions with $\alpha_y = 0$. In addition the last two equations allow a fixed point if $\alpha_Y = \alpha_{\tilde{Y}}$ or one or both couplings vanish. Hence one finds the possible flows shown in Table 3 (where $\alpha_y^*$ is the fixed point value of the gauge coupling, taking $x_F \rightarrow 21/4$).

Note that we do not require all the couplings to be non-zero in order to have a non-trivial UV fixed point, but we definitely need to reproduce the gauge-Yukawa behaviour which was observed in [10], while at the same time having negative beta functions for the couplings that are required to be non-zero in the IR, for phenomenological reasons. Therefore we can reject the Gaussian fixed point A. The second of these options, fixed point B, was the LS fixed point that was utilized in [2], and leads to

$$B: \frac{\beta_Y}{\alpha_Y} = \frac{\beta_{\tilde{Y}}}{\alpha_{\tilde{Y}}} \approx \frac{3}{1 + x_F} \alpha_y \rightarrow -\frac{12}{25} \alpha_y < 0 \ ,$$

so that both the $Y$ and $\tilde{Y}$ couplings flow away from fixed point B in the IR. Hence this fixed point is an interesting Gaussian option for the $Y$ and $\tilde{Y}$ couplings. In order to assess the other possible fixed points, note that

$$\frac{\beta_Y - \beta_{\tilde{Y}}}{\alpha_Y - \alpha_{\tilde{Y}}} = 2(1 + x_F)(\alpha_Y + \alpha_{\tilde{Y}}) > 0 \ .$$

Hence $\alpha_Y - \alpha_{\tilde{Y}}$ shrinks in the IR, so if the flow begins in the UV at C or D, it will be attracted to fixed point E. We conclude that from the perspective of the couplings $g, y, \tilde{y}, Y, \tilde{Y}$, any of B,C,D,E are suitable for an asymptotically safe fixed point but, as it flows to the IR, the system is attracted to the trajectory emerging from fixed point E, driven by the Yukawa couplings $Y, \tilde{Y}$. A numerical evolution showing this cross-over for the Yukawa gauge couplings is shown in Figure 3.

Next we turn to the scalar couplings. Their beta functions are given by

$$\beta_{u_1} = 4\alpha_{u_1} \left[ 8\alpha_{u_1} + 8\alpha_{u_2} + (\alpha_y + \alpha_{\tilde{y}}) \right] + 32\alpha_{v_1}^2 x_F^2 + 6\alpha_{u_2}^2 \ ,$$

$$\beta_{u_2} = 2\alpha_{u_2} \left[ 4\alpha_{u_2} + (\alpha_y + \alpha_{\tilde{y}}) - \frac{1}{2} x_F (\alpha_y^2 + \alpha_{\tilde{y}}^2) \right] \ ,$$

$$\beta_{w_1} = 4\alpha_{w_1} \left[ 8\alpha_{w_1} + 24\alpha_{w_2} + 2x_F (\alpha_Y + \alpha_{\tilde{Y}}) - 3\alpha_y \right] + 32\alpha_{v_1}^2 x_F^2 + 48\alpha_{w_2}^2 + \frac{3}{8} \alpha_y^2 \ ,$$

$$\beta_{w_2} = 2\alpha_{w_2} \left[ 12\alpha_{w_2} + 2x_F (\alpha_Y + \alpha_{\tilde{Y}}) - 3\alpha_y \right] - \frac{1}{2} x_F (\alpha_y^2 + \alpha_{\tilde{y}}^2) + \frac{3}{16} \alpha_y^2 \ ,$$

$$\beta_{v_1} = 2\alpha_{v_1} \left[ 16\alpha_{v_1} + 8\alpha_{u_2} + 16\alpha_{w_1} + 24\alpha_{w_2} + (\alpha_y + \alpha_{\tilde{y}}) + 2x_F (\alpha_Y + \alpha_{\tilde{Y}}) - 3\alpha_y \right] - \frac{1}{2} (\alpha_y + \alpha_{\tilde{y}})(\alpha_Y + \alpha_{\tilde{Y}}) - Y \ .$$

Analysis of these RGEs shows that there is only a real solution for a fixed point when $\alpha_Y = \alpha_{\tilde{Y}} = 0$, corresponding to fixed point B, namely the original LS fixed point studied in [2]. Along the trajectory
Figure 3. The flow in gauge and Yukawa coupling-space from the true fixed point B on to the trajectory emanating from the pseudo-fixed E, as specified in Table 3. During the flow the system crosses over from the (red-dashed) trajectory emanating from B, onto the blue-dotted trajectory emanating from E, inducing non-zero $\alpha_Y, \bar{\alpha}_Y$. This evolution (which ignores the accompanying flow of the quartic couplings) is idealised: the system radiatively develops a minimum before reaching trajectory E.

From B, the couplings assume the following values (with actually two stable branches for $w_1$):

$$\alpha_{u_1} = \frac{-6\sqrt{22} + 3\sqrt{19} + 6\sqrt{22}}{100}\alpha_g,$$
$$\alpha_{u_2} = \frac{3}{25}\left(\sqrt{22} - 1\right)\alpha_g,$$
$$\alpha_{w_1} = \frac{3 \pm \sqrt{3\left(4\sqrt{2} - 5\right)}}{16\sqrt{2}}\alpha_g,$$
$$\alpha_{w_2} = \frac{1}{16}\left(2 - \sqrt{2}\right)\alpha_g,$$
$$\alpha_{v_1} = 0.$$  (17)

It is important for later reference that, as discussed in [16, 17], the negative value of $\alpha_{u_1}$ at the minimum does not induce instability in $H$ because it is off-set by the much larger positive value of $\alpha_{u_2}$.

Once the Yukawa flow in Figure 3 begins, the scalar couplings also begin to flow: indeed Figure 3 is somewhat idealised in the sense that the quartic couplings now rapidly induce radiative symmetry breaking. In order to show this analytically, one may use the relations between the Yukawas and gauge couplings corresponding to trajectory E which yields an effective set of beta functions:

$$\beta_{u_1} = 32\alpha_{u_1}^2 + 6\alpha_{u_2}^2 + 6\alpha_{u_1}\alpha_{u_2} + \frac{1056}{277}\alpha_{u_1}\alpha_g,$$
$$\beta_{u_2} = 8\alpha_{u_2}^2 + \frac{528}{277}\alpha_{u_2}\alpha_g - \frac{182952}{76729}\alpha_{u_1}\alpha_g,$$
$$\beta_{w_1} = 32\alpha_{w_1}^2 + 48\alpha_{w_2}^2 + 96\alpha_{w_1}\alpha_{w_2} - \frac{2820}{277}\alpha_{w_1}\alpha_g + \frac{3}{8}\alpha_{g}^2,$$
$$\beta_{w_2} = 24\alpha_{w_2}^2 - \frac{1410}{277}\alpha_{w_2}\alpha_g + \frac{227163}{1227664}\alpha_{g}^2,$$
$$\beta_{v_1} = 16\alpha_{v_1}\left(2\alpha_{u_1} + \alpha_{u_2} + 2\alpha_{u_1} + 3\alpha_{w_1} - \frac{441}{2216}\alpha_g\right) - \frac{1584}{76729}\alpha_{g}^2.$$  (18)

These show that, once the system is kicked onto the E trajectory, the quartic couplings $w_2$ and $w_2$ flow to “quasi-fixed points”, that is trajectories that are determined entirely by the slowly varying value of $\alpha_g$. Indeed $\alpha_g$ is parametrically slowly flowing compared to the quartics (because its beta function is order $\epsilon^2$), so we may approximate it as constant, with the quartic couplings starting close to the boundary values in (17). Solving for $\alpha_{u_2}$ and $\alpha_{w_2}$ we see that they can both asymptote (as tanh functions) to
Figure 4. The flow for the quartic couplings once the theory leaves trajectory B (at $t = 0$). On the left, $\alpha_{u_2}$ in red/dashed and $\alpha_{w_2}$ in blue/solid both flow to new fixed points (beginning from positive values at $t = 0$). The $\alpha_{w_2}$ quasi-fixed point is much larger, which among other contributions induces $\alpha_{w_1}$ in blue/solid (on the right) to run negative and form a minimum radiatively for $\hat{S}$. Meanwhile $\alpha_{u_1}$ in red/dashed on the right is only moderately changed, not enough to destabilise $H$, while $\alpha_{v_1}$ in red/dotted runs slightly positive, inducing a small positive mass-squared for the Higgs at the PS breaking scale. This example has $\epsilon = 0.01$.

While the first quasi-fixed value for $u_2$ is very close to its starting point in (17), the quasi-fixed value for $w_2$ is much larger. (We should repeat that the analysis is approximate because the Yukawa couplings have not yet reached their new trajectory; in a full numerical evolution the running of the quartics will be delayed because the Yukawas have not yet reached trajectory E, but it is expected to be qualitatively the same.) This evolution is shown numerically in the left panel of Figure 4. In the right panel we show the effect on the remaining couplings $u_1$, $w_1$, $v_1$. Because $w_2$ appears only in the RGE for $w_1$, the two couplings $u_1$ and $v_1$ are changed only very slightly, with $v_1$ becoming slightly positive. Importantly no instability can be induced radiatively for $H$ at this stage, because (as was the case above on trajectory B) the negative contribution of $u_1$ to the potential is still off-set by the positive contribution of $u_2$. On the other hand $w_2$ runs more positive in the IR, and as can be seen from (18), this is a positive contribution to $\beta_{w_1}$, adding to several other positive contribution to $\beta_{w_1}$. The nett result is that $w_1$ runs negative (regardless of $w_2$ in fact) and inevitably at some point overcomes the positive approximately constant contribution to the potential from the $w_2$ term itself, forming a radiative minimum as in Figure 5 (as per [31]).

Thus (extended) PS breaking is induced radiatively in the TM, and at this scale a small positive mass-squared is generated via the $v_1$ “portal” coupling. It is natural for the latter to then be driven negative itself below the PS breaking scale, due to the coupling of $H$ to the $N_F - n_y N_S \approx \frac{2}{3} N_C$ pairs of $Q, \tilde{Q}$ fields that remain light because (by chiral symmetry) they cannot receive a mass from the $Y, \tilde{Y}$ couplings. This flow would be similar to that of the other mass-squared operators above the scale of PS breaking which we discuss in the following subsection: a more complete analysis of the running of the “portal” Higgs mass-squared below the PS scale will be undertaken in a later phenomenological study.

3.2. The behaviour of the relevant couplings – emergent flavour hierarchies from flavour symmetric fixed points. We now turn to the behaviour of classically relevant operators. These are allowed in the model as they are unable to disrupt the UV fixed point. (In any asymptotically safe theory such classically
relevant operators are simply part of the collection of non-predictive parameters in the theory.) They renormalise multiplicatively and can themselves initiate radiative symmetry breaking, as described in \cite{1}. As in the minimal supersymmetric SM, one can begin with a set of entirely positive mass-squareds in the UV and have them run negative due to the large Yukawa couplings in the model.

In this subsection we shall perform a more complete analysis of the flow of these operators to show how one should incorporate their flavour dependence. In particular we are interested in the possible generation of flavour/generation hierarchies in $H$, which in any viable model will be required to satisfy phenomenological constraints.

Rather than write the explicit flavour dependence as in (3), we wish to consider smaller flavour structures that are closed under RG. To see how to do this, as a warm-up consider the completely $SU(N_F)$ symmetric terms in \cite{2}, which were mass-squareds of the form

$$M_{H_{ij}H_{kl}}^2 = m_0^2 \delta_{ij} \delta_{lk} + 2 \Delta^2 \sum_a T_{ji}^a T_{kl}^a = m_0^2 \delta_{ij} \delta_{kl} + \Delta^2 \left( \delta_{ij} \delta_{kl} - \frac{1}{N_F} \delta_{ji} \delta_{kl} \right).$$

Defining real and imaginary parts, $H_{ij} = \frac{1}{\sqrt{2}}(h_{ij} + ip_{ij})$, and $h_a + ip_a = \sqrt{2} T_{ij}^a (h_{ij} + ip_{ij})$, the corresponding operators can be written as

$$m_0^2 H^\dagger H = \frac{m_0^2}{2} \text{Tr} (h^2 + p^2),$$

$$\sum_a \frac{\Delta^2}{2} (h_a^2 + p_a^2) = \sum_a 2 \Delta^2 \text{Tr}(T_a H) \text{Tr}(T_a H^\dagger) = \frac{\Delta^2}{2} \left[ \text{Tr} (h^2 + p^2) - \frac{(\text{Tr} h)^2 + (\text{Tr} p)^2}{N_F} \right].$$

It is now clear that one can proceed to break flavour in a way that commutes with the RG equations, by arranging the flavour breaking in $SU(n)$ subgroups, where the $SU(n)$ generators are in the $n \times n$ upper-left $n \times n$ block of the parent $SU(N_F)$, where $1 < n \leq N_F$. This gives degenerate masses for the generators of each nested $SU(n)$ flavour subgroup, where we envisage an explicit breaking

$$SU(N_F) \supset SU(N_F - 1) \ldots \supset SU(n) \ldots$$

So without loss of generality we can express the new Cartan generators introduced for each $SU(n)$ as

$$T_{ij}^{(n^2-1)} = \frac{1}{\sqrt{2n(n-1)}} \begin{pmatrix} 1 & \cdots & 1 \\ \cdots & 1 & 1 - n \\ 0 & \cdots & \end{pmatrix},$$

\footnote{Note that $h_a$ and $p_a$ are not simply related to $h_{ij}$ and $p_{ij}$. That is, while $p_a$ is the coefficient of the anti-hermitian parts of $H$, $p_{ij}$ is the coefficient of the imaginary parts of $H$.}
with the non-Cartan generators being defined accordingly in the obvious way. Defining the trace over the SU(n) block of the generators as

$$\text{Tr}_n(O_{ij}) = \sum_{i=1}^{n} O_{ii},$$  \hspace{1cm} (23)

the flavour breaking generalisation of \((21)\) becomes

$$V^{(2)} = m_0^2 \frac{N_F}{2} \text{Tr}_{N_F} \left( h^2 + p^2 \right) + \sum_{n=1}^{N_F-1} m_n^2 \frac{N_F}{2} \left[ \left( \text{Tr}_n h \right)^2 + \left( \text{Tr}_n p \right)^2 \right]$$

$$+ \sum_{n=2}^{N_F} \frac{\Delta_n^2}{N_F} \left[ \text{Tr}_n \left( h^2 + p^2 \right) - \frac{\left( \text{Tr}_n h \right)^2 + \left( \text{Tr}_n p \right)^2}{n} \right].$$  \hspace{1cm} (24)

These operators form a system closed under RG, and we may now determine their coefficients in \(16\pi^2 \partial_t V\), relevant for solving the Callan-Symanzik equation: these are shown in Table 4. One can now solve the RG equations along trajectory B for these parameters to see how they evolve before their flow is cut off by the radiative symmetry breaking (regardless of how it arises): as for any relevant parameter the flow will be expressed in terms of a set of RG-invariants. In this case, defining \(f_y = \alpha_y / \alpha_g \approx 0.46, f_{u_1} = \alpha_{u_1} / \alpha_g \approx -0.30, f_{u_2} = \alpha_{u_2} / \alpha_g \approx 0.44\), and

$$f = 2f_y + 4f_{u_1} \left( 1 + \frac{1}{N_F^2} \right) + 8f_{u_2} \approx 3.22,$$

$$f_\Delta = 2f_y + \frac{4}{N_F^2} f_{u_1} \approx 0.92,$$

$$f_n = 8f_{u_2} \frac{n}{N_F} (1 - \delta_{nN_F})$$  \hspace{1cm} (25)

the RG-invariants are found to be

$$\tilde{m}_n^2 = \tilde{m}^2(0) \left( \tilde{\Omega}(0) \right)^{-f},$$  \hspace{1cm} (26)

$$\sigma_{n^2} = \left[ m_n^2(0) + (n^2 - 1) \Delta_n^2(0) \right] \left( \tilde{\Omega}(0) \right)^{-f_{\Delta} + f_n},$$  \hspace{1cm} (27)

$$\rho_{n^2} = \left[ \Delta_n^2(0) - m_n^2(0) \right] \left( \tilde{\Omega}(0) \right)^{-f_{\Delta}},$$  \hspace{1cm} (28)

where

$$\tilde{\Omega}(t) = \left( \frac{\alpha_y^2}{\alpha_g} - 1 \right)^{-3/4t}.$$  \hspace{1cm} (29)

In terms of these we find the following solutions for the operators in \((24)\):

$$m_0^2 = \left( \tilde{\Omega}(t) \Omega(0) \right)^f \tilde{m}_n^2 - \frac{1}{N_F} \sum_{n=1}^{N_F} \frac{\sigma_{n^2}}{1 + 2f_{\Delta}(1 - n/N_F)} \tilde{\Omega} / \Delta_n + f_n,$$

$$\Delta_n^2 = \frac{1}{n^2} \left( \rho_{n^2} \tilde{\Omega} / \Delta_n + \sigma_{n^2} \tilde{\Omega} / \Delta_n + f_n \right),$$

$$m_n^2 = \frac{1}{n^2} \left( \rho_{n^2}^2 (1 - n^2) \tilde{\Omega} / \Delta_n + \rho_{n^2}^2 \tilde{\Omega} / \Delta_n + f_n \right).$$  \hspace{1cm} (30)
This is the desired form, since it assumes nothing about the “starting values”, which are simply values chosen at an arbitrary point in renormalization time, and it properly encapsulates all the non-predictive parameters in the theory. The entire flow is determined by these parameters and \( \tilde{\Omega}(t) \), which just determines where one is in renormalization time.

The interesting feature of these solutions is that \( \tilde{\Omega} \to 0 \) in the IR. Simple flavour hierarchies can therefore be generated much like the mechanism for radiative symmetry breaking in [11]. That is the exponent \( f \) is much larger than \( f_n \) or \( f_\Delta \). Therefore \( m_3^2 \) runs to zero in the IR much more quickly than \( \Delta^2_n \) or \( m_3^2 \). Meanwhile in the deep IR one can see from these solutions and the corresponding operators in Table 2 that hierarchies are naturally driven into the trace components in the potential which becomes dominant:

\[
V \longrightarrow \sum_{n>1} \Delta^2_n \left[ \text{Tr}_n \left( h^2 + p^2 \right) - n \left( (\text{Tr}_n h)^2 + (\text{Tr}_n p)^2 \right) \right].
\]

This supports the intriguing possibility that flavour hierarchies originate within the VEVs of the Higgs sector, which would themselves become correspondingly hierarchical.

4. Conclusions

In this paper we have presented a model, the Tetrad Model (TM), which is asymptotically safe, and which descends directly to the Standard Model via radiative symmetry breaking. In terms of a convenient “quiver-like” interpretation, the model contains 4-units, with matter and electroweak Higgs fields falling into an extended Pati-Salam GUT structure, based on the gauge group \( SU(N_C) \times SU(2)_L \times SU(2)_R \), and a fourth unit that provides the PS breaking. (The electroweak gauging of a subgroup of the flavour symmetry can not be shown on the quiver, but nevertheless the language is useful for understanding the overall structure of the gauge-Yukawa UV fixed point.) At low energies the model is able to yield the Standard Model enhanced only in the Higgs fields, which carry the same generation indices as the matter fields.

Remarkably radiative symmetry breaking (i.e. the Coleman-Weinberg mechanism) operates in the model with no further adjustment. The (extended) PS Higgs naturally develops a VEV radiatively while the electroweak Higgs gains a positive “boundary value” mass-squared at the PS scale, due to a portal coupling that runs from zero at the UV fixed point. This mass-squared can itself then be driven negative in the IR. It was also found that it is natural to generate hierarchies among the electroweak Higgs VEVs due to the enhancement of mass-squared hierarchies as the theory runs to the IR.

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