Thermal distortion minimization by geometry optimization for water-cooled white beam mirror or multilayer optics

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Abstract. The temperature distribution and thermal deformation of optics subject to heat load largely depend on the system geometry. It is possible to optimize this geometry and therefore to minimize thermal distortion of both white beam mirrors and multilayer-based optics. The thermal slope error in an optimized water-cooled white beam mirror can be reduced by two to three orders of magnitude compared to a non-optimized mirror.

1. Introduction
Extremely high power X-ray beams generated by insertion devices at third generation light sources are a challenge for the design of the first optical beamline components, since the high heat load can induce thermal distortion of the optics and therefore degrade beam characteristics (beam size and divergence, photon flux). To minimize the thermal distortion, liquid nitrogen cooling has been widely used for silicon crystal monochromators with great success at many synchrotrons [1-5]. This approach can also be applied to white beam mirrors with silicon substrates, but implementation of such cryogenic cooling is much more expensive than water cooling and presents additional complexity. Consequently, at the ESRF, water cooling is the technique of choice for white beam mirrors. The typical optics dimensions and beam footprint sizes for multilayer-based systems are situated between those of mirrors and crystals. For instance at a photon energy of 14 keV, the Bragg angle for a silicon Si(111) crystal monochromator is 8.12° or 142 mrad, the grazing angle of a silicon mirror is about 2 mrad, while the incident angle for a W/B₄C multilayer (200 periods of 3.6 nm) is around 12 mrad. For the same beam size of 1.5 mm, the beam footprint sizes are 11 mm for the monochromator, 122 mm for the multilayer optics, and 750 mm for the mirror. The incident angle of the multilayer could be smaller and the footprint longer. It could be possible to extend the use of water cooling systems to this kind of optics. In this paper, we will make some fundamental and finite element analyses to show the reasons and logic of the evolution of the cooling schemes, the performance of these cooling schemes, leading to the mirror geometry optimization to minimize thermal distortion.

2. Fundamental analysis
2.1. Beam theory
White beam mirrors for hard X-ray applications at the ESRF are typically 500 mm long, 60 mm wide with thicknesses in the range of 40 to 80 mm. The length L is much larger than the width b and the
thickness t (see Figure 1). The mirror can be considered as a mechanical beam. The thermal deformation of the mirror for any temperature distribution can be calculated by following equation [6]:

\[
\frac{d^2 w}{dx^2} = -\frac{M_{Ty}}{EI_y} \quad \text{with} \quad M_{Ty} = \int_a^b aET(x, y, z)zdA \quad \text{and} \quad I_y = \int_a^b z^2dA = \frac{bt^3}{12}
\]

(1)

where \( w \) is the displacement in the z direction (normal to the mirror surface), \( M_{Ty} \) the thermal bending moment and \( I_y \) the moment of inertia about the y axis, \( \alpha \) and \( E \) are respectively the thermal expansion coefficient and Young’s modulus of the mirror substrate material. The angular deformation in the meridional direction, the thermal slope error \( \theta \), is the derivative of the displacement \( w \) over \( x \), and can be calculated from the integral of equation (1) as follows

\[
\theta(x) = \frac{1}{I} \int_0^{b/2} \left[ \int_{-t/2}^{t/2} \int_{z=-b/2}^{z=b/2} aET(x, y, z)zdA \right] dx = -\frac{\alpha}{T} \int_0^{b/2} \left[ \int_{-t/2}^{t/2} \int_{z=-b/2}^{z=b/2} T(x, y, z)dy \right] dz dx
\]

(2)

If the temperature distribution satisfies the following condition, it can be easily shown from equation (2) that the thermal slope error is zero at any position \( x \):

\[
\int_{-b/2}^{b/2} T(x, y, z)dy = f(x)
\]

(3)

Equation (3) means that the temperature averaged over the width is independent of the position \( z \) (depth), i.e. there is no gradient in depth of this average temperature. This implies that thermal deformation of the mirror is essentially caused by the temperature gradient along the mirror depth. In the case of constant material properties (thermal conductivity, thermal expansion coefficient and Young’s modulus), any variation in the convection cooling coefficient induces primarily a temperature offset for mirrors fabricated from materials with high thermal conductivity (e.g. silicon, copper, SiC,...), and therefore increasing the cooling coefficient is not effective to minimize the thermal slope error. That’s why a rectangular block heated uniformly remains perfect rectangular. Minimizing the temperature gradient in the depth of the mirror is the key to decreasing the thermal slope error.

2.2. Analysis of Different cooling schemes

The majority of white beam mirrors at 3\textsuperscript{rd} generation light sources are water cooled. The cooling schemes can be classified as direct cooling and indirect cooling. The latter are most common and have evolved from bottom cooling to side cooling, then to top side cooling as illustrated in the cross-sectional views of Figure 2. Here the orange zones represent cooling blocks. An ideal case of top face cooling is also shown in Figure 2. In this scheme, the cooled surfaces are two stripes of half width of the beam footprint, aligned along the edge of the mirror surface. In the following paragraphs we will compare the 4 cooling schemes shown in Figure 2. We consider a silicon mirror 500 mm long, 60 mm wide and thick, illuminated by a beam (HxV=4x2 mm\textsuperscript{2}) with a vertical grazing angle of 4 mrad. The footprint is then 500 mm long and 4 mm wide on the centre of the top surface. The mirror is fully illuminated in the meridional direction. The total absorbed power is 800 W, uniformly distributed over the footprint. The mirror is indirectly cooled by water with respectively the 4 schemes shown in Figure 2. We will focus on the pure thermal deformation by assuming that the crystal is free of mechanical constraints due to the cooling blocks. Constant material properties of the silicon at room temperature are used for the comparison of these 4 cooling schemes. It is possible to solve the heat...
transfer equations and to find analytical solutions of the temperature distributions, as well as the thermal deformations. But finite element modelling (FEM) is straightforward and offers convenient graphic display. The temperature distribution in the cross section of the mirror is also shown in Figure 2. The temperature gradient in the depth (vertical direction) is slightly decreased from bottom cooling to full side cooling, but remains high. In the case of top side and top face cooling, the temperature gradient is essentially from centre to edges in horizontal direction and in the upper part of the mirror. The temperature gradient in vertical direction (depth) is significantly reduced. The temperature gradient in the depth (vertical direction) is constant and independent of depth; since there is no gradient in depth of the averaged temperature over the width, the equation (3) is satisfied, and therefore thermal slope error in the meridional direction is zero. The thermal slope errors along the centre axis on the top surface in the meridional direction shown in Figure 3 confirm our analysis on the temperature distribution. Neglecting end effects, the mirrors acquire a convex cylindrical form. The peak-to-peak thermal slope errors in the centre part [-200 200] mm are, respectively, 188, 78, 4.9, 0.03 μrad for the bottom, full side, top side and top face cooling. The top face cooling significantly reduces the thermal slope error compared to the bottom and full side cooling. The top face cooling is a case in which the equation (3) is satisfied.

Figure 2. Cross section of water-cooled mirror, 4 cooling schemes: (a) bottom, (b) full side, (c) top side, (d) top face. Orange blocks represent cooling blocks. Beam footprint on the centre of top surface.

Figure 3. Thermal slope error along the centre axis on the top surface of the mirror for the 4 cooling schemes shown in Figure 2. (b) is the zoom of (a) around +/- 5μrad.

3. Thermal distortion minimization by geometry optimization

In practice, the top face cooling is not used for multiple reasons: the small cooling surface area on the edge leads to high temperature and it is practically complicated to introduce heat exchangers close to the active mirror surface. Introducing a cut area in the mirror substrate just below the top side cooled area can modify the temperature distribution. This cut area (characterised by notch width w) can be optimised to reach minimum thermal distortion. To illustrate this approach, consider a horizontal deflection mirror in silicon 800 mm long, 80 mm wide and thick, illuminated by a beam of 0.84 mm in
Figure 4. Mirror cross section shape and thermal distortion displacement along the centre of the footprint: without cut area, with optimized cut area, with smaller and larger cut area.

vertical and 2.48 mm in horizontal direction with a horizontal grazing angle of 3.1 mrad. The footprint is then 800 mm long and 0.84 mm wide on the centre of the optical surface. Based on a typical ESRF undulator source the total absorbed power is 834 W, with Gaussian distribution in the both horizontal and vertical directions. The mirror is top side cooled over a height of 20 mm by contact with a water cooling block. The effective cooling coefficient on the substrate is assumed to be 0.005 W/mm²/°C. To illustrate the cross section optimization, we have plotted the thermal distortion induced displacement normal to the mirror surface along the centre of the footprint in meridional direction for the 4 cross sections: without cut area, with optimized cut area, with smaller and larger cut area in Figure 4. The color maps in the cross section represent the temperature distribution. Results show that the deformed mirror shape under heat load is convex for the cross section without cut area, or with small cut area, and can be concave for the cross section with large cut area, and therefore can be flat with optimized cut area. The RMS slope error with optimized shape is 0.018 μrad, compared to 2.5 μrad without cut area (although the induced slope errors are associated with an approximately cylindrical deformation of the mirror figure). The optimization results (cut area) depend on the mirror dimensions, cooled surface area, power distribution especially in case of Gaussian parameter σ much smaller than beam size. The end effects, more visible on the slope error, can be eliminated by secondary slits downstream of the mirror. These end effects are limited in the region about 50 mm from the extremities. The geometry optimisation method presented here can also be used for multilayer optics in certain applications, for instance, when the beam incident angle is less than 15 mrad or even smaller. This technique is routinely used to minimize the thermal distortion of water-cooled white beam mirrors and most multilayer optics for ESRF upgrade beamlines.

4. Conclusion
White beam mirror and, in certain cases, multilayer optics can be water cooled along the top side of the substrate. It is possible to limit the thermal slope error to sub-μrad by optimising the optics geometry (dimension and cross section), fully illuminating the optics length and using secondary slits downstream of the optics. The thermal slope error in an optimized water-cooled white beam mirror can be reduced by two to three orders of magnitude compared to a non-optimized mirror.

5. References
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