AstroSat Observations of GRO J2058+42 during the 2019 Outburst

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Received 2019 July 18; revised 2020 May 23; accepted 2020 May 27; published 2020 July 3

Abstract

We present results from AstroSat observations of the recent outburst of GRO J2058+42, an X-ray pulsar in a Be-binary system. The source was observed by the LAXPC and SXT instruments on AstroSat on 2019 April 10 during the declining phase of its latest giant outburst. Light curves showed a strong pulsation of the pulsar with a period of 194.2201 ± 0.0016 s and a spin-up rate of (1.65 ± 0.06) × 10⁻¹¹ Hz s⁻¹. Intermittent flaring was detected in light curves between the 3 and 80 keV energy bands, with an increase in intensity of up to 1.8 times its average intensity. Pulse profiles obtained between the 3 and 80 keV energy bands of the pulsar showed strong dependence on energy. During AstroSat observations, a broad peak was consistently observed in the power density spectrum of the source with a peak oscillation frequency of 0.090 Hz along with its higher harmonics, which may be due to quasi-periodic oscillations, a commonly observed phenomenon in transient X-ray pulsars during their outburst. AstroSat observations also detected cyclotron absorption features in its spectrum corresponding to (9.7–14.4) keV, (19.3–23.8) keV, and (37.8–43.1) keV. The pulse-phase-resolved spectroscopy of the source showed a phase-dependent variation in its energy and the relative strength of these features. The spectrum was well fitted with an absorbed blackbody, Fermi–Dirac cutoff model and alternatively with an absorbed CompTT model. Both of these models were combined with an Fe line and three Gaussian absorption lines to account for the observed cyclotron resonance scattering features in the spectrum.

Unified Astronomy Thesaurus concepts: Binary pulsars (153); Neutron stars (1108); Be stars (142); X-ray sources (1822); High mass x-ray binary stars (733)

1. Introduction

Many Be-binary systems were observed during their outbursts, which offered interesting results (Bildsten et al. 1997; Reig 2011). Bright X-ray outbursts are observed in Be binaries, most likely during the periastron passage of its neutron star through the circumstellar disk of its companion. Depending on the amount of matter released from its companion and the geometry of the binary system, rare as well as regular outbursts are observed during its binary orbit (Okazaki & Negueruela 2001; Okazaki et al. 2002; Okazaki 2016). The pulse characteristics of some of these, such as EXO 2030+375 (Parmar et al. 1989), Cepheus X-4 (Mukerjee et al. 2000), and XTE J1946+274 (Paul et al. 2001), were studied in detail during their outburst activities. Studies on pulse characteristics offer information on the pulsar geometry and the mechanism underlying its emitted pulse profile. The shape of the emitted pulse depends on modes of accretion inflows, source luminosity, geometry of accretion column, and the configuration of its magnetic field with respect to an observer’s line of sight (Parmar et al. 1989). Therefore, such studies offer understanding of pulsars in binary system and disk–magnetosphere interaction during the process of mass accretion, which affects its emitted radiation. Quasi-periodic oscillations (QPOs) have been detected from many Be binaries, such as A0535+26 (Finger et al. 1996; Finger 1998), EXO 2030+375 (Angelini et al. 1989), 4U 0115+63 (Soong & Swank 1989; Heindl et al. 1999; Dougair et al. 2013), and V0032+53 (Qu et al. 2005). Studies of QPOs offer rich information about the accretion torque onto the neutron star, thermodynamic properties of the inner accretion disk, and electromagnetics of the disk–magnetosphere interaction of the neutron star. Details on sources with observed QPOs, their frequencies, and other features along with pulsar spin frequencies, etc. have been given in tabular form by Devis et al. (2011), Ghosh (1998), and Mukerjee et al. (2001). Some of these transient Be-binary pulsars such as A0535+26 (50 mHz, 9.7 mHz), 1A 1118–61 (92 mHz, 2.5 mHz), XTE J1858+034 (110 mHz, 4.53 mHz), EXO 2030+375 (200 mHz, 24 mHz), SWIFT J1626.6–5156 (1000 mHz, 65 mHz), XTE J0111.2–7317 (1270 mHz, 32 mHz), as well as a persistent Be binary, X Per (54 mHz, 1.2 mHz), and an OB-type binary, 4U 1907+09 (69 mHz, 2.27 mHz), showed higher QPO frequency compared to their respective spin frequency as mentioned in order inside parentheses (Devis et al. 2011). These cover an interestingly wide range of QPO frequencies between 50 and 1270 mHz for these pulsars. Studies of cyclotron absorption features, if present in the spectrum, enable us to determine the strength of the surface magnetic field of the neutron star and offer insight into the line-producing region and the structure of the accretion column and its geometry (Staubert et al. 2019). Cyclotron absorption features thus have provided an important diagnostic probe for detailed studies of neutron star binaries since their discovery in the spectrum of Her X-1 (Truemper et al. 1978). There are several reports on the detection of cyclotron absorption features in the spectrum of many Be binaries, starting at a lower energy, from ~10 keV (Jun et al. 2012; DeCesar et al. 2013), to a higher energy, at ~100 keV (La Barbera et al. 2001). A detailed compilation of such sources and studies is given in Staubert et al. (2019) and Maitra (2016). It has been observed in detailed studies that some sources show a wide variation in their cyclotron line energy with respect to their pulse phase and source luminosity, and time, such as Vela X-1, Cen X-3, and Her X-1 (Staubert et al. 2019). These interesting properties help in understanding the nature of these sources and also offer insight into their underlying physical properties governing such changes.
The high-mass X-ray binary GRO J2058+42 is a transient X-ray pulsar that was first discovered by BATSE on board the Compton Gamma-Ray Observatory (CGRO) during its giant outburst in 1995 September–October. The outburst lasted for about 46 days, peaking at 300 mCrab intensity (Wilson et al. 1998). The spin period of its neutron star decreased from 198 s to 196 s during its 46 day outburst (Wilson et al. 1998). This outburst was subsequently followed by five more bursts of lower intensity of about 15 mCrab and of shorter duration of 15 days observed at an interval of about 110 days. Additional shorter outbursts with peak intensity of about 8 mCrab were detected by BATSE, halfway between the first four outbursts of 15–20 mCrab (20–50 keV) intensity. In early 1998, two outbursts of lower intensity were observed with PCA and HEXTE on board RXTE (Wilson et al. 2005). There were, however, no reports of any giant outburst from the source. Thus, GRO J2058+42 has shown very rare giant outburst activity—only twice so far—since its discovery. The first was detected by BATSE (Wilson et al. 1998, 2005) and the more recent one was detected by Swift-BAT (Barthelmy et al. 2019; Kennea et al. 2019) and Fermi-GBM (Malacaria et al. 2019) in 2019 March. GRO J2058+42 was earlier suggested to be a high-mass X-ray binary due to its observed properties. It was subsequently confirmed to have a Be-star companion through optical observations (Reig et al. 2005). Very limited details are known so far about this source, due to its rare intense outbursts.

AstroSat observed the source on 2019 April 10 during the declining phase of the second giant outburst, for studies of some of the above described properties of a typical Be-binary pulsar. In this work, we present results from our studies on the spectral and timing properties of GRO J2058+42 using data from AstroSat observations. Incidentally, there have been no reports on the detection of any cyclotron absorption features from the earlier outbursts of GRO J2058+42. We, therefore, wish to check for the possible detection of cyclotron absorption features in the spectrum, in particular, and the presence of QPOs. After this manuscript was submitted, Molkov et al. (2019) reported the detection of cyclotron absorption features using NuSTAR observations of the source during the same outburst. The rest of the paper is organized as follows: Section 2 describes the observations and data analysis including the software tools used, Section 3 describes the results, and its implications are discussed in Section 4. Finally, Section 5 gives the conclusions from this study.

2. Observations and Data Analysis

GRO J2058+42 had a rare and long outburst during March–May, 2019 as its latest event, as seen in the Swift-BAT intensity curve shown in Figure 1. The light curve was downloaded from the Swift-BAT light curves archive, and relevant details about these curves are described by Krimm et al. (2013). As there was a data gap close to the peak of the outburst, the peak of the outburst was approximately determined by fitting a cubic B-spline with 32 uniformly distributed knots to the data while data points with large errors were dropped. It enabled us to determine the peak intensity and its corresponding time, which was found to be at around MJD 58574.8 with peak intensity reaching about 256 mCrab (Swift-BAT 0.0564 cts cm$^{-2}$ s$^{-1}$) in the 15–50 keV energy band. When we compared this giant outburst with the earlier outburst, it turns out that the main flare of this giant outburst lasted for about a similar duration of 46 days (Wilson et al. 1998) and was followed by a relatively weaker and a much narrower secondary flare as seen in Figure 1. There is also a small burst about 110 days before the main burst as seen in the Swift-BAT light curve (Figure 1). CGRO/BATSE observed the earlier giant outburst with a peak intensity of about 300 mCrab (Wilson et al. 1998, 2005), comparable to that of its latest outburst which was also followed by a secondary flare observed after about 52 days.

AstroSat observed the source on 2019 April 10 during the declining phase of the outburst from MJD 58583.11 to MJD 58584.67, as marked in Figure 1. It was about 8 days after the peak of the outburst, when the source intensity had decreased to about 170 mCrab, which is about 66% of its peak intensity as obtained above. LAXPC was used as the primary instrument for this observation, with a total effective exposure of 57 ks. Details of the LAXPC instruments on board AstroSat and its operation modes are described in detail by Antia et al. (2017). During the observation, only one LAXPC detector, i.e., LAXPC20, was working properly. LAXPCs were operated in its event-analysis mode. LAXPC20 was operating at a very low gain, and it barely detected pulsation in the source. LAXPC20 detected an average source count rate of 225.6 ± 0.1 cts s$^{-1}$ in the 6–70 keV energy band. The SXT was operated in its photon-counting mode during the whole observation, with an effective exposure of about 21 ks, and detected an average count rate of 3.78 ± 0.01 cts s$^{-1}$ in the 0.8–7 keV energy band. The SXT instrument along with its operational modes are described in detail by Singh et al. (2016). Thus, useful data from the AstroSat SXT and LAXPC20 instruments were used for timing and spectroscopic analyses, covering a total energy band of 0.8–80 keV. The AstroSat observation was made under a Target of Opportunity (TOO) proposal with the observation

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1 https://swift.gsfc.nasa.gov/results/transients/index.html
ID of 20190410_T03_098T01_9000002836. AstroSat data from the proposal are available from the AstroSat Data Archive.  
LAXPC data were analyzed using the LAXPC pipeline software, version 3.0. The LAXPC software can be downloaded from the LAXPC site. The software takes level 1 data files as input and also utilizes calibration and background files to generate level 2 data products, like the light curve and spectrum. The background files are available with the software, which also gives a suitable recommendation for selection of a background and associated response files that may be used for data product generation and analysis. The background observation during 2019 April 19 was used to estimate the contribution from the source. Events were extracted from all main anodes from all the layers of LAXPC20 for this analysis. The pipeline software was executed with default values. However, appropriate energy channels were selected to derive light curves for a particular energy band. The pipeline software also corrects for any shift in the gain based on calibration data and generates a corrected spectrum and background-subtracted light curves of the source within its set energy band.

The SXT data analysis pipeline software, version AS1SXTLevel2-1.4b, was used for a set of default parameters in this analysis. The spectral response file and auxiliary response files of the SXT including background files are offered along with data analysis tools and can be downloaded from the SXT payload operation center site. Updated CALDB files are directly linked to the pipeline processing software. All photon events were extracted by including a circular area covering a region of interest of 6° radius with respect to the source center. All grades between 0 and 12 were considered for selection of photon events. The pipeline analysis software takes level 1 data as its input along with other relevant files for the analysis of events in steps to finally produce clean and calibrated event lists. The final data products, such as the spectrum, light curve, and image, are produced by applying various default filters and appropriate screening of the data. From calibrated clean events, one can produce spectra and light curves in different energy bands covering 0.8–7.0 keV. The “Xselect” tool version 2.4.c was used to screen the data. The solar system barycentric corrections were applied to the time series for both SXT and LAXPC data to correct for arrival time delays of events prior to its detailed timing analysis, using the tool ‘as1bary,’ developed by the AstroSat science support cell.

Heasoft version 6.14 was used for analysis of this data. The standard software XRONOS 5.22 was used for timing analysis, lcurve for the generation of combined light curves, efsearch for determination of spin period of the pulsar, powspec for computing the power density spectrum, etc. The standard spectral analysis tool XSPEC version 12.8.1 was used for fitting spectral data with a combination of appropriate models as described below. Results obtained from these analysis are presented in the next section.

3. Results

3.1. X-Ray Light Curve and Folded Pulse Profiles

The extracted light curve of GRO J2058+42 showed regular and strong pulsations along with variation in its intensity. A light-curve segment of LAXPC20 with 10 s binning is shown in the left panel of Figure 2. The source intensity variation between the 3 and 80 keV energy bands for the entire observation duration of LAXPC20 is shown in the right panel of Figure 2, obtained by binning the data with its established spin period. A straight line over the binned light curve shows its average intensity. Intensity variations were clearly seen for the entire duration of AstroSat observation. The observed source count rate varied from 230 cts s\(^{-1}\) to about 520 cts s\(^{-1}\) with an average count rate of 294 cts s\(^{-1}\). Intermittent flaring and dips were also seen in the light curve, which are typical intrinsic properties of Be binaries. During the flares, intensity increased by up to 1.8 times the average intensity.

The pulsar spin period was derived after applying corrections to pulse arrival time delays with respect to the solar system barycenter. The average pulsar period was initially derived with epoch folding and verified using the Lomb-Scargle periodogram, on full AstroSat observation data. The spin period of the pulsar was found to be 194.180 ± 0.001 s with 1σ confidence limit, during AstroSat observation. The total duration of the observation time was then divided into five intervals to estimate spin-up rate of the pulsar during AstroSat observation. The pulsar spin period was determined for each data interval separately along with its estimated error. These periods were plotted with respect to the midvalue of the respective intervals in MJD. An average spin-up rate of (1.71 ± 0.14) × 10\(^{-11}\) Hz s\(^{-1}\) was derived by fitting a straight line to these five data points, along with its error estimated with 90% confidence limit. Then, starting with these initially measured values, the spin period at the beginning of observation at \(t = t_0\) and its time derivative were accurately determined by correcting the phase using

\[
\phi(t) = \phi_0 + \nu_0(t - t_0) + \nu (t - t_0)^2 \frac{1}{2},
\]

where \(\phi\) is the phase (in the range 0–1 over the period), \(\phi_0\) is the phase at initial time \(t_0\), \(\nu_0\) is the spin frequency at initial time, and \(\nu\) is its time derivative. The fit was performed by fitting a periodic signal with 20 harmonics of the basic period using Equation (2),

\[
c(t) = c_0 + \sum_{k=1}^{N} (a_k \sin(2\pi k\phi) + b_k \cos(2\pi k\phi)),
\]

where \(c(t)\) is the observed count rate, \(N\) is the number of harmonics included in the fit, and \(c_0, a_k,\) and \(b_k\) are the coefficients fitted, apart from \(\nu_0\) and \(\nu\). The light curve was obtained with a time bin of 1 s. The best-fit values for \(\nu_0\) and \(\nu\) were determined by varying both parameters to minimize the \(\chi^2\) deviation of the light curve from the model (Equation (2)). Various values of \(N\) were tried, and \(N = 20\) was found to be adequate to account for all pulsation signals. In principle, it is possible to use the F-test to decide the required value of \(N\), which gave a value of \(N = 14\) with a probability threshold of 0.05, but some components after \(k = 15\) were also significant. Beyond \(N = 20\), the next few components were not found to be
significant. That is why this value was adopted. To find the errors in the fitted values of ν and ν, we performed a Monte Carlo simulation by perturbing c(t) and repeating the process for 4000 different realizations of noise to find the distribution of parameter values. Using this distribution, we found the 90% confidence limits for the parameters. The same program was used to generate the pulse profiles as well as the GTI intervals for the different phase intervals using the fitted values of parameters. The GTI values were then used to calculate the spectra for different phase intervals for phase-resolved spectroscopy. The same program was also used to generate the light curve after subtracting the pulse profile as defined by Equation (2) to filter out the contribution of coherent pulsation in the resulting power density spectrum.

The fitted value of the period at t = t₀ corresponding to MJD 58583.1068148 was found to be 194.2201 ± 0.0016 s and ν = (1.65 ± 0.06) × 10⁻¹¹ Hz s⁻¹, where the error bars denote the 90% confidence limits. The orbit of the binary system is not known, and the orbital motion can also contribute to the period variation during the observation. However, the Fermi-GBM observations during this outburst suggest that the period variation was restricted to the duration of the giant outburst, and the change in ν during the outburst showed a good correlation with intensity, with a correlation coefficient of about 0.93. This implies that the variation in period is largely due to accretion during the outburst.

Light curves corresponding to different energy bands were extracted, and the respective pulse profiles were derived by folding the corresponding light curves in 64 bins with the derived spin period of its neutron star at the beginning of the epoch and its derivative. Two cycles of all such pulse profiles are shown in Figure 3 for clarity. Folded pulse profiles showed variation with respect to different energy bands. The average pulse profile covering the full 3–80 keV energy band is also shown at the bottom panel of Figure 3 for reference and relative comparison. We noticed that pulse profiles at lower energies showed pronounced and multiple pulses, which gradually merged into a double pulse and then into a single asymmetric broad pulse at higher energies. The rms pulse fraction measurements and source intensity with respect to energy is shown in Figure 4. The rms pulse fraction showed energy-dependent variation, showing an increase from about 20% to 30% between 3 and 20 keV while at higher energies it is nearly constant. Thus, the pulse profiles showed remarkable variability in shape and its pulse fraction with respect to the considered energy bands between 3 and 80 keV.

Pulse profiles derived from recent AstroSat observations can be compared with earlier RXTE observations at a much lower source intensity of about 10 mCrab. There is a general similarity in shape but structures are seen much more clearly than those reported from the 1996 November 28 RXTE observation between 2 and 60 keV during its earlier lower intensity outburst (Wilson et al. 1998). For the source intensity of about 17 mCrab, the rms pulse fractions were measured at different energy bands between 2 and 20 keV from the RXTE PCA observations of 1998 February 4. These were estimated to be 11.7% ± 2.6% (2–4 keV), 19.1% ± 4.1% (4–9 keV), and 27.2% ± 5.9% (9–20 keV). This can be compared with the recent observation by AstroSat LAXPC20, which gives 15.6% ± 3.3% (3–4 keV), 22.0% ± 4.7% (4–9 keV), and 28.6% ± 6.1% (9–20 keV), respectively. The RXTE/PCA and AstroSat/LAXPC20 measurements show that these are comparable within respective error limits, although source intensity was an order of magnitude higher during the AstroSat observation. Thus, the pulsar does not show a drastic change in the shape of pulse profiles and pulse fractions with a change in source luminosity in this range.

3.2. Power Density Spectrum and Detection of a QPO Feature

The source light curve along with its background between the 3 and 30 keV energy bands with a bin time of 1.0 s was used to derive the power density spectrum. The XRONOS tool “powspec” was used for this purpose. The normalization parameter value of −2 was selected so that power density spectra were normalized such that their integral gave the squared rms fractional variability. Therefore, the power was expressed in the units of (rms)² Hz⁻¹, and the expected white noise level was subtracted to obtain the rms fractional variability of the time series. For such a normalized power density spectrum where a QPO profile was modeled by a Lorentzian, the strength of a QPO signal was described by its fractional rms amplitude, which is proportional to its integrated power that contributes to its overall power density spectrum and often expressed in percent (Wang et al. 2014). The

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*https://gamma-ray.msfc.nasa.gov/gbm/science/pulsars/lightcurves/groj2058.html*
integrated power can be computed from the area under the Lorentzian profile. The amplitude of the Lorentzian multiplied by its FWHM and $\pi/2$ would determine the area under the profile in units of $\text{rms}^2$, hence its square root would give the rms amplitude of the QPO signal. The power density spectrum was therefore derived considering the above normalization and a total of 1024 bins per interval, and a total of 73 intervals in a frame were considered for the purpose of obtaining the power density spectrum. However, out of a total of 73 intervals, 16 intervals with less than 50% data were rejected for default window selection. The data gaps were padded with zeros as its default option. A geometric rebinning of 1.04 was applied to generate a power density spectrum to improve on statistical fluctuation at higher frequencies.

The average power density spectrum derived as above showed a spin frequency along with its several harmonics. It also showed a broad peak around 0.09 Hz and its two harmonics, which were identified as a QPO. The QPO signal was found to be better in the 3–30 keV energy band compared to a higher energy band, hence power spectra are presented for this energy band. To confirm the presence of a QPO, we removed the contribution of a coherent pulsation signal by subtracting the fitted pulse profile with 20 harmonics (Equation (2)) from the time series data as described in Section 3.1. The power density spectrum was then derived from this time series. The power density spectrum showed a clear detection of a QPO feature at $0.090 \pm 0.003$ Hz along with its two harmonics. As the pulsation signal was removed from the time series data, the power density spectrum did not show any coherent pulsations seen earlier. However, the continuum defined by the power-law index were found to be consistent within the 90% error limit, when a coherent

**Figure 3.** Folded pulse profiles derived from LAXPC20 at different energy bands covering 3–80 keV. The vertical lines in the lowest panel mark the four divisions of the pulse period used in phase-resolved studies.
A pulsation signal was present (power-law index $\approx -0.89 \pm 0.09$) and when the pulsation signal was removed (power-law index $\approx -0.78 \pm 0.07$). The observed QPO and its two higher harmonics defined by a Lorentzian function were also found to be consistent within their parameter uncertainty. The power density spectrum was modeled using a power law in combination with three Lorentzian functions to model the QPO and its two higher harmonics. The model parameter values and the QPO frequency and its corresponding two harmonics as derived from the fitted model are listed in Table 1. Because many intervals of data were averaged to generate the power density spectrum, the $\chi^2$ test for the fitted model therefore does not introduce significant biases as it is well known that in this case, the power density spectrum has a distribution close to normal (Barret & Vaughan 2012). The model was fitted and parameter values were established from the power density spectrum after its y-axis was normalized and expressed in units of $(\text{rms})^2 \text{Hz}^{-1}$ with frequency along the x-axis. However, the power density spectrum is shown in Figure 5 with the y-axis multiplied by a corresponding frequency to show the prominence of the QPO and its two harmonics. The Lorentzian function $L(f)$ used in the model is defined as

$$L(f) = \frac{\text{LN}}{1 + \left(\frac{f - \text{LC}}{\text{LW}}\right)^2},$$

where the parameters LN, LC, and LW represent its peak amplitude, centroid frequency, and its FWHM, respectively.

An alternate model, a broken power-law along with three Lorentzian functions, was also fitted to the same power density

![Figure 4. Count rate and rms pulse fraction derived from LAXPC20 at different energy bands covering 3–80 keV.](image)

Table 1

| Parameter | Value | Amplitude (rms%) | F-test FAP | Detection Significance ($\sigma$) |
|-----------|-------|------------------|------------|----------------------------------|
| Power Index | $-0.78 \pm 0.08$ | | | |
| Normalization (K) (at 1 Hz) | $0.010 \pm 0.001$ | | | |
| QPO $f_0$ Centroid LC (Hz) | $0.090 \pm 0.003$ | $12.1 \pm 1.0$ | $2.7 \times 10^{-11}$ | 6.7 |
| Sigma LW (Hz) | $0.011 \pm 0.012$ | | | |
| Normalization LN | $0.116 \pm 0.008$ | | | |
| QPO-$f_2$ Centroid LC (Hz) | $0.183 \pm 0.004$ | $5.2 \pm 0.9$ | $1.4 \times 10^{-3}$ | 5.3 |
| Sigma LW (Hz) | $0.048 \pm 0.015$ | | | |
| Normalization LN | $0.036 \pm 0.006$ | | | |
| QPO-$f_3$ Centroid LC (Hz) | $0.280 \pm 0.017$ | $4.2 \pm 1.5$ | $2.2 \times 10^{-4}$ | 3.7 |
| Sigma LW (Hz) | $0.11 \pm 0.07$ | | | |
| Normalization LN | $0.010 \pm 0.003$ | | | |

$\chi^2_{\text{red}}$ (dof) | 1.08 (62) |

Note. Errors with 90% confidence range for each parameter. FAP: false-alarm probability.
3.3. Spectrum and Detection of Cyclotron Absorption Features

RXTE derived the first X-ray spectrum of the source during the earlier outburst in 1996 covering the energy band between 2.7 and 50 keV. The spectrum was modeled adequately well with an absorbed thermal bremsstrahlung (\texttt{Phabs*brems}). However, there was no report on any detection of cyclotron absorption features from the RXTE observations. Therefore, to check for the presence of a cyclotron absorption feature in the spectrum, we took the ratio of counts in the spectrum of the source to that in the Crab spectrum which was observed by LAXPC20 on AstroSat; the results for the phase-resolved spectra are shown in Figure 6. The overall pulse phase was divided into four intervals covering structures of the folded pulse profile as shown by vertical lines on the 3–80 keV pulse profile in Figure 3. The phase intervals considered are 0.00–0.18 (phase 1), 0.18–0.43 (phase 2), 0.43–0.70 (phase 3), and 0.70–1.00 (phase 4). The ratio of spectral counts plotted against energy with respect to Crab was fitted with a polynomial of degree 3 along with a combination of Gaussians to define corresponding features in the ratio curve. The first phase shows two features around 10 keV and 20 keV and an insignificant feature around 38 keV. The feature around 38 keV is more prominent during other phases as well as in the phase-averaged case, while the two lower features are not significant during the last two phases. This is borne out by the results of fitting the spectrum. The variation in the properties of the features with phase effectively rules out instrumental effects or uncertainties in instrumental response as these effects would be independent of phase.

We then derived the X-ray spectrum covering a total energy band from 0.8–70 keV with AstroSat, combining SXT and LAXPC20 spectral data. The combined spectrum was fitted with two different models. The first (Model 1) is an absorbed Fermi–Dirac cutoff model, FD\textsc{cut} (Tanaka 1986) combined with a blackbody (\texttt{bbody}), a Gaussian for an iron emission line, and three Gaussian absorption features (\texttt{gabs}) which were introduced to account for the presence of cyclotron absorption features in the spectrum to improve the spectral fit. We also tried an alternative model (Model 2), an absorbed Comp\textsc{TT} model (Titarchuk 1994) combined with an iron emission line and three absorption features as described above for comparison and measurement of the centroid energy of the cyclotron absorption features. The spectral parameters derived from these two models are tabulated in Table 2 for the phase-averaged spectrum, while Table 3 shows the results of fitting the second model to phase-resolved spectra. Spectra along with the fitted models are shown for phase-averaged and phase-resolved spectra in Figures 7 and 8, respectively. Figure 9 shows phase-dependent variations in spectral residues, indicating the relative intensities of three absorption features individually when the optical depth of the corresponding line energy is made zero. In addition, the residue of the fitted model is also presented when all three \texttt{gabs} components, associated with cyclotron resonance scattering features, were removed. This depicts phase-dependent variations of the relative intensities of cyclotron absorption features for four different phases. The difference in $\chi^2$ values without the three \texttt{gabs} components were established for all four phases and given in Table 3. The difference in $\chi^2$ values clearly shows the overall significance of the presence of cyclotron absorption features in the spectrum. A systematic error of 2.5% was added to account for uncertainties in the response of the instruments. The relative difference in the normalization between instruments was accounted for by introducing a constant multiplicative factor for the two instruments and by fixing it for LAXPC20 to 1.0 during the fit. Thus, the normalization factor of SXT was found to be 0.31 when a circular field of view of diameter 40$''$ was selected. This is due to a fixed offset between the pointing axes of the two instruments. The region of interest of SXT, however, was restricted to 12$''$ in diameter to accumulate events mainly from the region where source photons dominate, to allow better constraints in the lower energy region of the overall spectrum. With this restricted region of interest, the normalization was found to be 0.18 for SXT with respect to that of LAXPC20, which is frozen at unity. Both spectral models offered a spectrum for comparison. The reduced $\chi^2$ value of 1.12 was found for 60 degrees of freedom (dof), compared with the power-law model along with three Lorentzian functions which yield the reduced $\chi^2$ of 1.08 for 62 dof. Thus, the power-law model resulted in a better fit, compared to the broken-power-law model. A break frequency of $0.31 \pm 0.01$ Hz was obtained from the broken-power-law model when fitted to the power density spectrum. The break frequency is much higher than the observed QPO frequency of 0.09 Hz, and hence, it cannot mimic the QPO.

Thus, rms amplitudes determined from the average power density spectrum, by fitting a power law and three Lorentzian functions to the QPO and its two harmonics, are also given in Table 1. The statistical significance of the detected QPOs was established using an F-test. This was done by determining the difference in $\chi^2$ with and without the inclusion of a Lorentzian function, used for modeling the detected QPO in the power density spectrum. A similar approach was followed for its other two harmonics, individually. The false-alarm probability (FAP) and detection significance of the QPO expressed in terms of $\sigma$ for an equivalent normal distribution are computed and listed in Table 1 for the QPO and its two harmonics. It turns out that the QPO frequency and the two harmonics are significant. It is observed that apart from the first harmonic, the other two peaks are rather broad.
reasonably good fit to the spectral data, and measurements of the respective energies of cyclotron absorption features are found to be consistent within their error limits. Considering the observed flux of $4.3 \times 10^{-9}$ ergs cm$^{-2}$ s$^{-1}$ in the 3–80 keV energy band and using the source distance of 9 kpc (Reig et al. 2005), the X-ray luminosity is determined to be $4.2 \times 10^{37}$ ergs s$^{-1}$ during AstroSat observations.

To estimate the statistical significance of the cyclotron absorption features that were detected, we initially tried the first two approaches described by Bhalerao et al. (2015) and then cross-verified these using the Monte Carlo technique as a reverification for a few cases, as described below.

1. The first approach was to use the F-test, which uses the reduction in $\chi^2$ value when the three parameters defining the cyclotron absorption feature using gabs (multiplicative model) were included. Based on the improvement in $\chi^2$ value by adding the feature, one could estimate its significance and FAP. However, this is to specifically mention that the F-test used here is additive and has certain limitations for multiplicative models such as the gabs model (Protassov et al. 2002; Orlandini et al. 2012). This test was applied for an initial assessment and for rough estimates of the significance of the three individual additional components, defined by the gabs model. In all cases, the significance of each feature was calculated independently, and we have calculated the probability of the signal being false and converted that to the significance in terms of the equivalent value of $\sigma$ in the normal distribution. The estimated significance from this F-test for the absorption features in the spectrum is tabulated for the phase-averaged spectrum and for four different phases in Tables 2 and 3, respectively.

2. In the second approach, we kept the centroid energy fixed for a cyclotron absorption feature and stepped through a grid of values of line depth and width to study the change in $\chi^2$ as a function of these two parameters using the XSPEC steppar command. The change in the minimum $\chi^2$ value corresponding to zero line depth was noted. Thus, using the minimum difference in $\chi^2$ required to get zero line depth, we could calculate the FAP and the significance of the line. The results are shown in Table 2 for the phase-averaged spectrum and in Table 3 for four different pulse phases of the pulsar. These estimates of significance of cyclotron absorption features are consistent with the estimates using option (1).

3. We also assessed the significance of cyclotron absorption features independently using Monte Carlo simulation. This technique has been used to establish the significance of cyclotron absorption features in the spectrum for many
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Table 2
Spectral Parameters for the Phase-averaged Spectrum

| Parameter                  | Units       | Model 1 | Model 2    |
|----------------------------|-------------|---------|------------|
|                            | (FDCUT)     | (CompTT)|            |
| Phabs(NH)                  | 10^{22} cm^{-2} | 0.869±0.045 | 0.650±0.046 |
| bbbody (E1)                | keV         | 0.83±0.04 | ...        |
| bbbody (B1)                | keV         | 0.83±0.04 | ...        |
| power-law (π)              | ...         | 1.04±0.03 | ...        |
| E_fold                     | keV         | 34.6±1.2 | ...        |
| E_cut                      | keV         | 10.95±0.27 | 0.45       |
| power-law (N1)             | keV pa^{-1} cm^{-2} | 0.10±0.01 | ...        |
|                            | at 1 keV    | ...     | 0 (fixed)  |
| CompTT                     | ...         | ...     | 0.02 (fixed) |
| CompTT (T0)                | keV         | 5.1±0.02 | ...        |
| CompTT (K1)                | keV         | 8.2±0.10 | ...        |
| CompTT (tau)               | ...         | 5.21±0.12 | ...        |
| CompTT                     | ...         | ...     | 0.2 (fixed) |
| Gauss (Line E)             | keV         | 6.5 (fixed) | 6.5 (fixed) |
| Gauss (Sigma)              | keV         | 0.24 (fixed) | 0.24 (fixed) |
| Gauss (Norm)               | ph cm^{-2} s^{-1} | 1.8×10^{-4} | 2.3×10^{-4} |
| gabs (Line E1)             | keV         | 10.95±0.24 | 10.81±0.48 |
| gabs (Sigma)               | keV         | 3.74±0.72 | 3.20±0.39 |
| gabs (Strength)            | ...         | 1.13±0.29 | 1.56±0.28 |
| gabs (Line E2)             | keV         | 21.7±0.84 | 20.84±0.54 |
| gabs (Sigma)               | keV         | 3.61±1.28 | 3.69±0.40 |
| gabs (Strength)            | ...         | 0.90±0.44 | 1.42±0.27 |
| gabs (Line E3)             | keV         | 38.73±0.81 | 38.38±1.14 |
| gabs (Sigma)               | keV         | 2.64±1.31 | 2.56±0.12 |
| gabs (Strength)            | ...         | 1.02±0.55 | 0.96±0.21 |
| Flux (3–80 keV)            | erg cm^{-2} s^{-1} | 4.33×10^{-9} | 4.33×10^{-9} |

χ^2                      | ...         | 879.79 | 877.36 |
χ^2 (dof)                 | ...         | 1.264(696) | 1.257(698) |

Note. Errors with 90% confidence range for each parameter. False-alarm probability: FAP.

Our results for phase 1 are in agreement with Molkov et al. (2019), whereby the first two features around 10 keV and 20 keV were detected with high significance and no other higher-energy features were detected. The mean energy, width, and depth of the features estimated by us is also consistent with those estimated by Molkov et al. (2019). Additionally, we detected an absorption feature around 38–40 keV for the rest of the three phases with high significance. However, we did not detect the presence of any cyclotron absorption feature around 30 keV as reported by Molkov et al. (2019). This was verified using phase 1 spectrum, where a third gabs model was introduced with the values of its energy and width frozen as reported by Molkov et al. (2019) and its optical depth allowed to vary. The additional gabs model fitted to the spectral data did not detect any significant optical depth corresponding to the energy ~30 keV. The individual spectrum, Figures 4 and 5 of

introduced to model the observed cyclotron absorption features. Only LAXPC20 data were considered to improve on the speed of simulation and convergence to the fit. We fixed the hydrogen column density (N_H = 0.81 ×10^{22} cm^{-2}) as derived from the fit to the combined SXT and LAXPC20 spectral data. We then used the XSPEC script “simfptest” to simulate 1000 spectra. The script does the fit with and without a cyclotron absorption features for each of the simulated data. From these simulations, one can estimate the change in χ^2 between a model with and without the cyclotron absorption feature and find the distribution of Δχ^2 values. These simulations were done to reconfirm the significance results for Phase 2 spectrum, corresponding to its ~22 keV and ~40 keV cyclotron absorption features independently. The change in the chi-squared (Δχ^2) distribution is plotted. Because the cyclotron absorption feature is defined by three free parameters, we expect the simulated Δχ^2 distribution to follow a χ^2 distribution with three dof. The observed distribution was found to be consistent with the χ^2 distribution as can be seen in Figure 10. In these two cases, the maximum values of Δχ^2 obtained in the simulation was 14 (22 keV line) and 10 (40 keV line), which are much lower than the observed values of 25.5 (22 keV line) and 36.7 (40 keV line) in the LAXPC20 data. By extrapolating the distribution, we estimate that a very large number of simulations would be required to get the observed value of Δχ^2 in one of them by chance. This could be 10^{−10} simulations required for Δχ^2 ≈ 26 and 10^{−9} for Δχ^2 ≈ 37. Therefore, it is not feasible to perform these many simulations to achieve desired values. The significance of the individual features for LAXPC20 data for phase 2 corresponding to these values is 4.38σ (FAP = 1.2 × 10^{−5}) for the 22 keV line and 5.4σ (FAP = 5.32 × 10^{−6}) for the 40 keV line. These simulations therefore confirmed the presence of these features with high significance in the LAXPC20 spectrum, which has cyclotron absorption features from the source detected in its sensitive energy band. It thus also confirms that the significance established from the first two methods as shown in Table 3 is consistent with the results obtained by the Monte Carlo simulation technique shown here as a test case. We tried a few more cases, and in all of these, the significance is consistent with those obtained using options (1) and (2).

The three phases with high signiﬁcance are introduced with the values of its energy and width frozen as derived from the ﬁt to the combined SXT and LAXPC20 spectral data. We then used the XSPEC script “simfptest” to simulate 1000 spectra. The script does the fit with and without a cyclotron absorption features for each of the simulated data. From these simulations, one can estimate the change in χ^2 between a model with and without the cyclotron absorption feature and find the distribution of Δχ^2 values. These simulations were done to reconfirm the significance results for Phase 2 spectrum, corresponding to its ~22 keV and ~40 keV cyclotron absorption features independently. The change in the chi-squared (Δχ^2) distribution is plotted. Because the cyclotron absorption feature is defined by three free parameters, we expect the simulated Δχ^2 distribution to follow a χ^2 distribution with three dof. The observed distribution was found to be consistent with the χ^2 distribution as can be seen in Figure 10. In these two cases, the maximum values of Δχ^2 obtained in the simulation was 14 (22 keV line) and 10 (40 keV line), which are much lower than the observed values of 25.5 (22 keV line) and 36.7 (40 keV line) in the LAXPC20 data. By extrapolating the distribution, we estimate that a very large number of simulations would be required to get the observed value of Δχ^2 in one of them by chance. This could be 10^{−10} simulations required for Δχ^2 ≈ 26 and 10^{−9} for Δχ^2 ≈ 37. Therefore, it is not feasible to perform these many simulations to achieve desired values. The significance of the individual features for LAXPC20 data for phase 2 corresponding to these values is 4.38σ (FAP = 1.2 × 10^{−5}) for the 22 keV line and 5.4σ (FAP = 5.32 × 10^{−6}) for the 40 keV line. These simulations therefore confirmed the presence of these features with high significance in the LAXPC20 spectrum, which has cyclotron absorption features from the source detected in its sensitive energy band. It thus also confirms that the significance established from the first two methods as shown in Table 3 is consistent with the results obtained by the Monte Carlo simulation technique shown here as a test case. We tried a few more cases, and in all of these, the significance is consistent with those obtained using options (1) and (2).
Table 3
Spectral Parameters for Phase-resolved Spectra

| Parameter                  | Units        | Phase 1              | Phase 2              | Phase 3              | Phase 4              |
|----------------------------|--------------|----------------------|----------------------|----------------------|----------------------|
| Phabs(nH)                  | $10^{22}$ cm$^{-2}$ | $0.83^{+0.11}_{-0.11}$ | $0.81^{+0.10}_{-0.12}$ | $0.65^{+0.10}_{-0.08}$ | $0.64^{+0.09}_{-0.08}$ |
| CompTT(Redshift)           | ...          | 0 (fixed)            | 0 (fixed)            | 0 (fixed)            | 0 (fixed)            |
| CompTT(T0)                 | keV          | $0.41^{+0.05}_{-0.11}$ | $0.42^{+0.06}_{-0.11}$ | $0.53^{+0.05}_{-0.04}$ | $0.47^{+0.04}_{-0.04}$ |
| CompTT(TK)                 | keV          | $10.03^{+0.22}_{-0.16}$ | $8.56^{+0.10}_{-0.14}$ | $8.45^{+0.15}_{-0.26}$ | $8.36^{+0.15}_{-0.18}$ |
| CompTT(tau)                | ...          | $4.60^{+0.16}_{-0.15}$ | $6.17^{+0.15}_{-0.16}$ | $5.91^{+0.15}_{-0.11}$ | $4.99^{+0.23}_{-0.23}$ |
| CompTT(approx)             | ...          | 0.2 (fixed)          | 0.2 (fixed)          | 0.2 (fixed)          | 0.2 (fixed)          |
| CompTT(Norm)               | ...          | $0.037^{+0.003}_{-0.003}$ | $0.055^{+0.006}_{-0.003}$ | $0.047^{+0.001}_{-0.003}$ | $0.042^{+0.002}_{-0.007}$ |
| Gauss(Line E)              | keV          | 6.5 (fixed)          | 6.5(fixed)           | 6.5 (fixed)          | 6.5 (fixed)          |
| Gauss(Sigma)               | keV          | 0.24 (fixed)         | 0.24 (fixed)         | 0.24 (fixed)         | 0.24 (fixed)         |
| Gauss(Norm)                | ph cm$^{-2}$ s$^{-1}$ | $2.10 \times 10^{-4}$ | $2.20 \times 10^{-4}$ | $2.88 \times 10^{-3}$ | $2.88 \times 10^{-3}$ |
| $\chi^2$                  | ...          | 663.48               | 528.95               | 526.29               | 625.61               |
| $\chi^2_{red}$ (dof)      |              | 1.14 (548)           | 0.96 (546)           | 0.96 (545)           | 1.14 (545)           |
| $\chi^2_{red}$ (with 3 gabs removed) | ...          | 224.0                | 68.76                | 66.54                | 62.79                |
| FAP                        |              | $6.08 \times 10^{-30}$ | $5.42 \times 10^{-11}$ | $1.24 \times 10^{-10}$ | $3.67 \times 10^{-8}$ |
| significance ($\sigma$)    |              | 11.37                | 6.55                 | 6.43                 | 5.51                 |

Significance test of cyclotron line using the F-test:

| E1 FAP                     |              | $9.67 \times 10^{-13}$ | $4.13 \times 10^{-4}$ | $9.29 \times 10^{-2}$ | $9.83 \times 10^{-3}$ |
| significance ($\sigma$)    |              | 7.14                 | 3.53                 | 1.68                 | 3.58                 |
| E2 FAP                     |              | $2.22 \times 10^{-7}$ | $1.58 \times 10^{-6}$ | $2.51 \times 10^{-3}$ | $1.02 \times 10^{-1}$ |
| significance ($\sigma$)    |              | 5.18                 | 4.80                 | 3.02                 | 1.64                 |
| E3 FAP                     |              | $9.9 \times 10^{-1}$ | $7.26 \times 10^{-13}$ | $1.56 \times 10^{-9}$ | $6.11 \times 10^{-9}$ |
| significance ($\sigma$)    |              | 0.01                 | 7.17                 | 6.04                 | 5.81                 |

Significance test of the cyclotron line using a nonzero line depth:

| E1 FAP                     |              | $4.05 \times 10^{-17}$ | $1.34 \times 10^{-4}$ | $5.13 \times 10^{-2}$ | $3.81 \times 10^{-3}$ |
| significance ($\sigma$)    |              | 8.41                 | 3.82                 | 1.95                 | 2.89                 |
| E2 FAP                     |              | $1.76 \times 10^{-9}$ | $3.89 \times 10^{-7}$ | $1.24 \times 10^{-3}$ | $2.27 \times 10^{-2}$ |
| significance ($\sigma$)    |              | 5.73                 | 4.96                 | 3.23                 | 2.21                 |
| E3 FAP                     |              | $9.26 \times 10^{-1}$ | $1.38 \times 10^{-13}$ | $5.70 \times 10^{-14}$ | $3.01 \times 10^{-11}$ |
| significance ($\sigma$)    |              | 0.09                 | 7.40                 | 7.08                 | 6.65                 |

Note. Errors with 90% confidence range for each parameter. False-alarm probability: FAP.

Molkov et al. (2019), does not appear to show a significant feature around 30 keV. The feature seen around 38–40 keV in the LAXPC spectrum could possibly be either the third harmonic (if the primary absorption feature is around 12 keV) or the fourth harmonic (if the primary feature is around 10 keV). In the latter case, we may have missed the third harmonic as it could possibly be very weak and hence not detectable. Even higher harmonics could be present, but due to lower sensitivity at high energies, it is difficult to say anything definitively about them. We thus detected a cyclotron absorption feature around 38–40 keV in the other three phases, which is contrary to the results of Molkov et al. (2019). This detection was possible due to the relatively higher effective area of LAXPC20 (Antia et al. 2017) by more than an order of magnitude, around 38–40 keV, compared to the FM modules of NuSTAR (Brejnholt et al. 2012; Harison et al. 2013). This is evidently from their Figures 4 and 5 (Molkov et al. 2019), where errors are relatively large around 38 keV, and hence it is likely that the 38 keV feature was not detected by NuSTAR. We detected the 38 keV absorption feature with high significance as reported in Table 2, even in the phase-averaged spectrum. We also noticed that the spectral ratio with respect to the Crab spectrum (Figure 6) clearly showed a depression around 38–40 keV, confirming the presence of this absorption feature in the spectrum and the variation of its shape with phase. This rules out the possibility of occurrence of the absorption feature due to the inaccuracy of the instrument response or the modeled background.
4. Discussion

4.1. Power Spectrum and QPO

Many accretion-powered pulsars show millihertz QPOs. X-ray pulsars with high-mass companions such as X Per (Takeshima 1998), 4U 1907+09 (Zand et al. 1998; Mukerjee et al. 2001), XTE J1858+034 (Paul & Rao 1998), A0535+26 (Finger et al. 1996), V0332+53 (Takeshima et al. 1994), and X0115+63 (Soong & Swank 1989) showed QPOs with their respective peak oscillation frequencies in the range 50–110 mHz. QPOs in X-ray pulsars offer an important diagnostic tool to probe accretion flows in these binaries, conditions in the inner region of its accretion disk, properties of accretion torques exerted on its neutron stars, and disk–magnetosphere coupling and their interaction (Ghosh 1996, 1998).

Now let us consider two main models and their applicability to the high-mass X-ray binary GRO J2058+42. These are the Keplerian frequency and beat frequency model (van der Klis 2000). The Keplerian frequency model, where QPOs are produced due to inhomogeneities at the inner edge of the Keplerian disk that modulate the X-ray flux at Keplerian frequency, is expressed as $\nu_{\text{QPO}} = \nu_k$ (van der Klis et al. 1987). In the case of the beat frequency model, the accretion flow onto the neutron star is modulated at the beat frequency between the Keplerian frequency at the inner edge of the accretion disk and the neutron star spin frequency, $\nu_{\text{QPO}} = \nu_k - \nu_s$ (Alpar & Shaham 1985). In the case of GRO J2058+42, the detected QPO frequency $\nu_{\text{QPO}} = 9.0 \times 10^{-2}$ Hz is much higher than the spin frequency of its neutron star, 5.15 $\times 10^{-3}$ Hz. It is, therefore, not possible to differentiate between a beat frequency and a Keplerian frequency model in the case of GRO J2058+42.

We can obtain the radius of the inner edge of the accretion disk $r_{\text{QPO}}$ using the expression for Keplerian orbital motion,

$$r_{\text{QPO}} = \left( \frac{GM}{4\pi^2} \right)^{1/3} \nu_k^{-2/3} \approx 8.0 \times 10^8 \text{ cm}, \quad (4)$$

where $M$ is $1.4M_\odot$ for a neutron star, $G$ is the gravitational constant, and $\nu_k$ its Keplerian frequency.

Alternatively, using the mass-accretion rate and strength of the magnetic field of the neutron star one, can also derive the radius of the inner edge of the Keplerian disk, which is equivalent to the magnetospheric radius $r_m$ of the neutron star as expressed by Equation 6.18 of Frank et al. (2002, p. 158):

$$r_m \approx 5.2 \mu_{30}^{4/7} M_{16}^{-2/7} m_s^{-1/7} \times 10^8 \text{ cm} = 2.4 \times 10^8 \text{ cm}, \quad (5)$$

where $m_s = M_c/M_\odot = 1.4$, $\mu_{30}$ is the magnetic moment of the neutron star expressed in units of $10^{30}$ G cm$^3$, and $M_{16}$ is its mass-accretion rate expressed in units of $10^{16}$ g s$^{-1}$. These are derived from observed values of the neutron star magnetic field and source luminosity, respectively, as detected by AstroSat observations.

The corotation radius $r_{\text{cor}}$ of an X-ray pulsar can be defined where the spin angular velocity of neutron star is equal to the Keplerian angular velocity of matter. It can be derived by equating the Keplerian velocity to the corotating Keplerian velocity,

$$r_{\text{cor}} = 1.7 \times 10^8 P^{2/3} \left( \frac{M}{1.4M_\odot} \right)^{1/3} \text{ cm}, \quad (6)$$

where $P$ is the spin period of the neutron star. Using the estimated pulse period from AstroSat observations and assuming a neutron star mass of $M = 1.4M_\odot$, one can obtain the corotation radius $r_{\text{cor}}$ for GRO J2058+42 as $5.7 \times 10^8$ cm (Equation (6)). It is therefore evident that the disk radius $r_{\text{QPO}}$ is about an order of magnitude smaller than the corotation radius $r_{\text{cor}}$. It suggests, therefore, that formation of such a transient disk is possible between the magnetosphere and corotation radius of the neutron star. The estimated value of the radius of the inner accretion disk derived from Keplerian orbital motion is $8.0 \times 10^8$ cm (Equation (4)), and using accretion torque theory, it is determined to be $2.4 \times 10^8$ cm (Equation (5)). These values are comparable considering the level of approximations involved, and uncertainties in mass, radius, and distance of the source as well as in the geometry of the magnetic field. It favors, however, the formation of a transient accretion disk around the neutron star, which could possibly explain the cause of the observed oscillations at 0.090 Hz in the X-ray flux to be either beat frequency or Keplerian frequency modulations as discussed above. Such properties are quite recurrent in many cases of Be-binary pulsars as mentioned above and as also observed in this case for the first time by the
LAXPC on board AstroSat. The neutron star magnetosphere radius established for some cases (Devasia et al. 2011; Nespoli & Reig 2011) using the measured strength of the magnetic field, source distance, and luminosity, assuming the canonical mass and radius of a typical neutron star (Equation (5)), was found to be very close to the inner radius of the accretion disk, as determined from the observed QPO frequency (Equation (4)) for some sources. Hence, for such cases, it was possible to establish consistency with the spectroscopically measured value of their magnetic field strength. The same appears to be true for GRO J2058+42 with the uncertainties involved in measured parameters and standard assumptions.

The formation of a transient disk may supply the necessary spin-up torque to the neutron star, if the disk rotates in the same direction as the pulsar spin. The expected torque ($N_{\text{char}}$) onto the neutron star due to the transfer of angular momentum of the accreted mass from such a transient disk can be calculated using the expression below, assuming the transfer of angular momentum of all accreted mass from such a disk with radius $r_{\text{QPO}}$ to its neutron star (Zand et al. 1998; Mukerjee et al. 2001):

$$N_{\text{char}} = \eta \dot{M} \left( GM_{\text{QPO}} \right)^{3/2},$$

where $\dot{M}$ is the mass-accretion rate and $\eta$ is the duty cycle for the applied torque.

If we assume that all of the potential energy of the accreted mass liberated during the outburst is transformed into radiation, then the luminosity can be expressed as

$$L_{37} = 1.33 \dot{M}_{17} \left(M/M_\odot \right) R_{6}^{-1} \text{ erg s}^{-1},$$

where $L_{37}$ is the luminosity in units of $10^{37}$ erg s$^{-1}$ and $R_{6}$ is the radius in units of $10^6$ cm. The mass-accretion rate can be calculated from the observed luminosity, which is derived from measurements of the flux from the AstroSat spectral model (Table 2) and the known distance to the star. The estimated value of $\dot{M}$ during the AstroSat observation was $2.3 \times 10^{17}$ g s$^{-1}$. Using these parameters, the value of $N_{\text{char}}$ is found to be $\eta \left(9.0 \times 10^{34}\right)$ g cm$^{-2}$ s$^{-2}$.

The observed torque $N_0$ of the pulsar can be expressed in terms of the moment of inertia, $I = 10^{45}$ g cm$^2$ (for a typical neutron star, having mass of $1.4 M_{\odot}$ and radius of 10 km) and $\dot{\nu}$ the observed rate of change of frequency of the neutron star,

$$N_0 = 2\pi I \dot{\nu}. \quad (9)$$

Now, if we consider the value of $\eta \approx 1.0$, representing the fact that the transient accretion disk was present and torque was applied for almost the whole duration of observation, then, by equating the above two torques (Equations (7) and (9)), one can estimate the average spin-up rate of the pulsar to be $\dot{\nu} = 1.43 \times 10^{-11}$ Hz s$^{-1}$ or $\dot{P} = -5.39 \times 10^{-7}$ s s$^{-1}$. This is comparable to the observed value of $\dot{\nu}$ during the AstroSat observation.
Considering the value of $\dot{\nu} = 1.65 \times 10^{-11} \text{ Hz s}^{-1}$ obtained during the AstroSat observation and assuming that the spin-up rate is proportional to the luminosity during the recent outburst, we could integrate $\dot{\nu}$ over the entire outburst to calculate the net change in spin frequency or period. Taking into account the observed period of 194.22 s during the beginning of the AstroSat observation, we could calculate that the period decreased from about 195.7 s before outburst ($MJD = 58,550$) to 193.4 s after this latest long outburst of 2019 ($MJD = 58,624$). This change in period is comparable to the measured period change from 195.6 to 193.5 by Fermi-GBM during this latest outburst. This is also comparable to the period change during the earlier outburst observed by BATSE in 1995. Thus, the formation and presence of such a transient accretion disk around a neutron star during the source outburst could cause a significant change in the pulsar period over the short duration of 46 days.

### 4.2. Pulse-phase-averaged Spectrum

The X-ray spectrum of the source was first derived by RXTE during the earlier outburst of 1996. The RXTE PCA and HEXTE data covering 2.7–25 keV and 11–50 keV, respectively, were used (Wilson et al. 2005). The spectral data were fitted with an absorbed thermal bremsstrahlung model ($\text{Phabs} \times \text{bremss}$) adequately well, particularly at higher intensities during its outburst. The source flux dropped sharply above 20 keV in most cases; therefore, other models such as a power law with a high-energy cutoff were not used with RXTE data. It was found that the absorption term $N_H$ was nearly constant, with its best-fit values in the range $(4.6–5.4) \times 10^{22} \text{ cm}^{-2}$. The temperature, $kT$, was found to increase with the source intensity, with best-fit values varying from $10.3 \pm 0.5 \text{ keV}$ at $2.9 \times 10^{-11} \text{ ergs cm}^{-2} \text{ s}^{-1}$ to $22.2 \pm 0.4 \text{ keV}$ at $2.6 \times 10^{-10} \text{ ergs cm}^{-2} \text{ s}^{-1}$ (Wilson et al. 2005). There was, however, no report on any cyclotron absorption feature from RXTE observations. This was very likely missed due to observations at a much lower intensity of the source during low-intensity outbursts, which are an order of magnitude lower than the latest outburst.

AstroSat data enabled us to derive the X-ray spectrum in the 0.8–70 keV energy band as shown in Figure 7. The spectrum was fitted reasonably well using two models. The first model was defined as an absorbed Fermi–Dirac cutoff model along with a blackbody, an Fe emission line, and three Gaussian absorption lines introduced to model cyclotron scattering features and its two higher harmonics as observed in the spectrum. The CompTT model was used as the second model in combination with an Fe emission line and three Gaussian absorptions lines as defined in the first model. The CompTT model is generally used for neutron-star-based low-mass X-ray binaries such as Z-type and atoll sources with a relatively lower
magnetic field (<10^{12} G) of the neutron star (Ferlinelli et al. 2008). However, the model could also successfully define the spectrum of some of the pulsars in Be binaries, for example, Cep X-4 (Jaiswal & Naik 2015), 4U 1907+09 (Varun et al. 2019), and GRO J2058+42 (Molkov et al. 2019). The parameters derived from the two models are tabulated in Table 2. The first model estimated a blackbody temperature of 0.83 ± 0.04 keV and detected the presence of a cyclotron resonance scattering feature and its harmonics. The Comptonization model, on the other hand, enabled us to determine the input photon Wien temperature of 0.52 ± 0.02 keV, the plasma temperature of 8.22 ± 0.10 keV, and the plasma optical depth of 5.21 ± 0.12 for the phase-averaged spectra. The Wien temperature, per the CompTT model, originates far from the neutron star surface and closer to the inner accretion disk i.e., at the outer transition layer; hence, the Wien temperature is always found to be relatively lower than the neutron star blackbody temperature as it originates closer to the inner transition layer, i.e., near the surface of the neutron star (Ferlinelli et al. 2008). For the CompTT model, bulk Comptonization occurs in the innermost part of the transition layer region, while thermal Comptonization is dominant in the outer transition layer and presumably within some extended region located above the accretion disk. Similar deviations were also observed in the case of Cep X-4 fitted with the CompTT model and a blackbody combined with the FDCUT model (Table 1 of Jaiswal & Naik 2015). However, the centroid energy of the cyclotron absorption features and its detected harmonics are found to be consistent within errors for the two models (Table 2).

Some of the accretion-powered X-ray pulsars showed additional features in emission between the 10 and 20 keV energy bands and more rarely in absorption between 8 and 10 keV in their respective residuals when fitted with a variety of continuum models (Coburn et al. 2002). Coburn et al. (2002) argued that such features may be caused by inadequacies of the continuum model rather than cyclotron resonance features. For example, it was observed that a single emission line at around 14 keV can fit two features around 10 and 20 keV for Vela X-1 (Kreykenbohm et al. 2002), Her X-1 (Coburn 2001), and Cep X-4 (McBride et al. 2007), and an absorption line between 8 and 10 keV can fit the features for 4U 1907+09, 4U 1538–52, and 4U 0352+309 as in Figure 6 of Coburn et al. (2002). In the case of GRO J2058+42, an introduction of a single Gaussian emission line in this range of energies could not appropriately fit the spectrum. The results described in Section 3.3 using ratios of spectral counts with respect to the Crab spectrum derived for four different phases of the pulsar (Figure 6) clearly indicate the presence of prominent depressions around 10 keV and 20 keV for phase 1 in particular, and its presence in other phases as well. Therefore, it confirms the presence of such absorption features in the spectral data associated with a physical origin and not due to any discrepancies of the continuum model as discussed above. Additionally, it also excludes the possibility of any uncertainty in the response matrix of the detector as the response matrix was not used to deconvolve the spectrum to calculate these ratios. The relative significance of these absorption features that was subsequently estimated after modeling the data and results is shown in Table 2 for the phase-averaged case and in Table 3 for the four different pulse phases. These observations strongly favor the presence and detection of these absorption features in their respective pulsar spectra.

The Be-binary pulsar 4U 0115+63 showed interesting features when its measured pulsed fraction was plotted against energy starting from 5 keV onwards. It showed a gradual increase in pulsed fraction with energy along with a sharp localized decrease around 22 keV. This localized decrease was attributed to cyclotron resonant scattering at the second Landau level. No such localized decrease was observed for its higher harmonics, likely due to low signal-to-noise ratio. Such decrease was also not observed corresponding to its fundamental line, despite its high count rate, as it could be due to competing effects such as photon spawning and cyclotron emission (Ferrigno et al. 2009). The pulse fractions measured at different energies between 3 and 80 keV from AstroSat observations of GRO J2058+42 (Figure 4) did not show any localized decrease in its pulse fraction corresponding to the fundamental (∼10 keV) or even for its observed higher harmonics (∼20 keV, ∼38 keV). These could possibly be due to similar reasons mentioned above for 4U 0115+63 for the fundamental and higher harmonics, or the resultant decrease in pulse fraction is small and it could not be detected within the error limits. This is very similar to another Be binary, V0332 +53, where no such localized change in the pulse fraction was observed corresponding to its cyclotron line energies, despite
their prominent optical depths (Tsygankov 2010). The pulse fraction for GRO J2058+42, however, showed a gradual increase with energy and then became nearly constant at higher energies above 20 keV as seen in Figure 4.

4.3. Pulse-phase-resolved Spectra

Cyclotron resonant scattering features are produced by the interaction of photons with electrons quantized in the Landau states formed in the accretion column in the presence of a strong magnetic field of a neutron star in accreting X-ray binaries (Schwarz et al. 2017). It can also be produced by the reflection of X-rays from the atmosphere of the neutron star (Poutanen et al. 2013). These interactions result in absorption features in their spectrum at a particular energy and produce the complex shape of the absorption features due to complex scattering cross sections. The line energies and their shapes change depending on the environment of the line-forming region typically located close to the neutron star, strength of the magnetic field of the neutron star, nature of the accretion column, source luminosity, and pulse phase of accreting X-ray pulsars (Nishimura 2003, 2005, 2013).

The pulse-phase-resolved spectra corresponding to four different phases of the accreting pulsar GRO J2058+42 are presented in Figure 8. AstroSat detected a cyclotron absorption feature and its two harmonics in its pulse-phase-resolved spectra, which were identified in the phased-averaged spectrum. Spectral parameters derived from the fitted CompTT model for the four different pulse phases are given in Table 3. The relative strengths and shape of these absorption features change with pulse phase as shown in Figure 9 for clarity. The line energies are found to vary within the ranges of (9.7–14.4) keV, (19.3–23.8) keV, and (37.3–43.1) keV, respectively, for the observed cyclotron scattering features and its higher harmonics. Their respective detection significance along with the FAP with respect to pulse phases are also given in Table 3. We noticed a higher value of the centroid energy of the fundamental cyclotron line energy at 13.17 for phase 2, but it was found to be consistent with that for phase 3 and phase 4 within its associated larger uncertainty. Overall, the pulse-phase-resolved spectra of GRO J2058+42 showed consistency with respect to cyclotron line energies and the continuum, except for phase 1 (Table 3).

The phase-averaged spectrum (Figure 7), fitted using the CompTT model for GRO J2058+52, determined the cyclotron fundamental line energy along with the harmonics from the AstroSat observation. Interestingly, the observed ratios between the fundamental and the two harmonics are found to cover a range of 1.93 ± 0.06 and 3.6 ± 0.12 respectively with a 1σ error limit. This indicates a nonharmonic ratio, observed for higher harmonics of the cyclotron resonance scattering feature with respect to its fundamental line energy. However, looking at the results from the pulse-phase-resolved spectra in Table 3, it is clear that the position of the spectral features as well as their strengths showed variation with phase, but when compared with the phase-averaged spectrum it may give misleading results. For example, the mean position of the first feature is different in phase 1 compared to other phases and its strength is highest in phase 1. Thus, the phase-averaged spectrum is likely to be biased toward the lower value in phase 1. On the other hand, the third feature around 38 keV is not seen in phase 1, while it is significant in other phases and the phase-averaged spectrum will reflect this value. If we consider the ratios between the energies of the different features in phase-resolved spectra (Table 3), the ratios for the first two features are 1.69 ± 0.16, 1.82 ± 0.14, and 1.84 ± 0.11, corresponding to phase 2, phase 3, and phase 4, respectively. Similarly, the ratios between the third and the first features, ignoring phase 1 where the third feature is not significant, are 3.04 ± 0.17, 3.10 ± 0.23, and 3.47 ± 0.20 for the three phases, respectively. Within the error bars, these ratios are consistent with those expected for harmonics. Hence, there is no evidence for nonharmonic ratios in GRO J2058+42. However, nonharmonic ratios in cyclotron absorption features were observed for some of the pulsars, for example, V0332+53 (Kreykenbohm et al. 2005; Pottschmidt et al. 2005; Nakajima 2010), Her X-1 (Enoto et al. 2008), and Cep X-4 (Vybornov et al. 2017). For some cases, marginal deviation in harmonicity could be explained by employing relativistic approximation of photon–electron scattering (Meszaros 1992), but for large deviations, it could possibly be explained by the influence of a nondipole magnetic field on the line-forming region when the field strength increases with height (Nishimura 2005, 2013).

The variation in cyclotron features with pulse phase could be due to the superposition of contributions from a number of lines formed at different heights of a line-forming region in a cylindrical accretion column geometry, influenced by a large gradient in its magnetic field. This could result in the observed variation of the centroid energy and shape depending on the visibility of the accretion column with respect to the line of sight that depends on the system’s orientation during the spin of the neutron star. This could occur either in the accretion column or in the neutron star atmosphere for an anisotropic injection of energy with the emission peak exiting in a particular direction (Nishimura 2015). In such cases, cyclotron scattering features could be observed only for the interval depending on its pulse phase as seen for pulse phase 1 for GRO J2058+42. There could be other possibilities where the accretion column itself could partially be eclipsed by the neutron star for certain pulse phases, such that only a portion of the accretion column could be visible to the observer (Mushtukov 2018). This may cause a dispersion of magnetic field strength across the visible portion of the accretion column and may cause a resultant change in the line energy and its shape with pulse phase. This suggests that the observed phase-dependent changes in the cyclotron line energy and its shape for GRO J2058+42 during the AstroSat observation could likely be due to changes in the geometry of the line-producing region with respect to the line of sight during its spin, for a stable luminosity of the source.

We find from Table 3 that the photoelectric absorption and input soft photon temperature CompTT(T0) for all four phases were almost constant within the 90% error limit. The plasma temperature CompTT(T) for phase 1 was, however, found to be relatively higher by ~1.5 keV compared to the average of the remaining three phases at 8.5 keV. The plasma optical depth for phase 2 was found to be higher at 6.17 compared to its average value of 4.8 for the other three phases. These measurements indicated that for phase 1, the difference in plasma temperature and possibly a local change in the configuration of its magnetic field could be responsible for the observed change in its continuum and line parameters as compared to the other three phases. The observed ratio of the spectral counts with respect to Crab for phase 1 also clearly
showed that there are significant changes in the spectrum relative to other phases (Figure 6). Therefore, the evident differences in its continuum and line parameters indicate that for this narrow phase 1, emission probably comes mainly from a different region and hence emission components are different with respect to the rest of the three phases. For example, the radiation could originate from one column instead of another, from its visible higher column height as opposed to near the stellar surface. The stellar surface area associated with a relatively hotter plasma accumulation may also be suggestive of a scenario where a magnetically more intense area produces a hotter region. This hotter region contributes to the substantial variation of the spectral shape due to the interaction of accelerated high-energy particles, which could upscatter soft photons, giving rise to a Comptonization spectrum with a change in its spectral shape. The recent work of Nishimura (2019) could explain more favorably the observed change in spectral continuum as well as the decrease in the centroid energy of the cyclotron lines in general and in particular that of phase 1. The velocity of bulk motion of infalling plasma in the cyclotron line-forming regions plays a dominant role. Line-forming regions are located near the walls of the cylindrical accretion column whereas spectral continua are formed above and around the accumulated mound close to the neutron star surface. Therefore, in the relative location of these two regions where continuum and cyclotron lines are formed with respect to a line of sight of the observer, one could see the observed change in the spectral continuum as well as the decrease in centroid energy of the cyclotron line during certain pulse phases of the pulsar depending on optimal conditions, such as velocity of the bulk motion of the infalling matter, effect of gravitational bending of the emitted radiation, and suitable variation of the local magnetic field (Nishimura 2019). The fundamental centroid energy of the cyclotron absorption feature showed an overall variation of ~30% with the pulse phase measured for GRO J2058+42. There are several other sources, namely Cen X-3 (Burderi et al. 2000; Suchy et al. 2008), Vela X-1 (Kreykenbohm et al. 2002; La Barbera et al. 2003), 4U 0115+63 (Heindl et al. 2004), GX 301–2 (Kreykenbohm et al. 2004; Suchy et al. 2012), and Her X-1 (Voges et al. 1982; Soong et al. 1990; Klochkov et al. 2008), which showed variations in their fundamental energy of about 10%–30% over the pulse phase. The cyclotron absorption features of GRO J2058+42 clearly showed variation in the cyclotron line energy and shape with the variation in its pulse phase like in other pulsars.

4.4. Determination of the Strength of the Pulsar Magnetic Field

The strength of the surface magnetic field $B$ of a neutron star can be obtained from

$$E_c \simeq 11.6 n \left( \frac{1}{1 + z} \right) \left( \frac{B}{10^{15} G} \right) \text{keV}. \quad (10)$$

We assume that the observed cyclotron absorption feature at $E_c$ is associated with its fundamental line ($n = 1$), and considering a gravitational redshift of $z = 0.3$ at the surface of a typical neutron star having a mass of $1.4 M_\odot$ and a radius of 10 km. From the measurements of the centroid energy at $10.81^{+0.48}_{-0.49}$ keV, for the phase-averaged case, we could thus establish the strength of the magnetic field of the pulsar as $(1.21^{+0.05}_{-0.06}) \times 10^{12}$ G. Considering the variation in energy of the first cyclotron line with the pulse phase, we get the range of $(9.7–14.4) \text{ keV}$ after including the error bars. This would translate to a variation in the magnetic field strength of $(1.1–1.6) \times 10^{12}$ G over the pulse phase of the pulsar. The strength of the magnetic field derived for GRO J2058+42 from its spectrum is comparable to the magnitude of those measured for other X-ray pulsars, such as KS 1947+300 (12.2 keV; Fürst et al. 2014), Swift J1626.3–5156 (10 keV; DeCesar et al. 2013), and 4U 0115+63 (12 keV; Jun et al. 2012; Ferrigno et al. 2009), where the respective energy values of their fundamental cyclotron absorption features are shown within parentheses.

5. Conclusion

AstroSat observed the Be-binary pulsar GRO J2058+42 on 2019 April 10, during its latest long outburst between 2019 March and May, using both LAXPC and SXT instruments for a total exposure of 57 ks. The source intensity during this observation declined to 170 mCrab, about 66% of its peak intensity of 256 mCrab. AstroSat observations showed a clear detection of strong pulsation from the source. The spin period of the pulsar was determined to be 194.2201 ± 0.0016 s, and its average spin-up rate was $\dot{\nu} = (1.65 ± 0.06) \times 10^{-11}$ Hz s$^{-1}$, corresponding to MJD 58583.10868148. Pulse profiles derived at different energy bands covering 3−80 keV showed pronounced and multiple pulse structures, which varied in shape and pulse fraction with energy. The rms pulse fraction varied from about 20% to 30% between 3 and 20 keV, beyond which it was approximately constant. Pulse profiles derived from recent AstroSat observations were found to be similar in shape to those reported from RXTE observations, which is at a relatively lower source intensity by an order of magnitude. Thus, the source did not show a drastic change in its pulse shape and pulse fraction within the observed variation of the source luminosity.

A broad QPO feature corresponding to a frequency of 0.090 Hz was detected for the first time from AstroSat observations. This provided evidence for the formation of a transient accretion disk around the neutron star. The QPO, therefore, offered us a tool to probe the inner region of the accretion disk and to quantify the transfer of angular momentum through such an accretion disk. This enabled us to estimate the torque applied to the neutron star by mass transfer from a transient accretion disk. This resulted in the prediction and determination of the observed change in spin period of the pulsar by 2.3 s during the recent 46 day outburst. Similar change in spin period was observed by BATSE during the earlier outburst in 1995 and also by the Fermi-GBM during this latest outburst. This, therefore, favors a scenario where the pulsar could be effectively spun up by such a magnitude, in a short span of 46 days.

The centroid energies of the cyclotron line and its harmonics were found to be consistent within errors for the two models used for spectral fits. The line energies are found to vary within the range of $(9.7–14.4) \text{ keV}$, $(19.3–23.8) \text{ keV}$, and $(37.8–43.1) \text{ keV}$, respectively, for the three observed absorption features. The detection of the cyclotron line led to the determination of the strength of the strong magnetic field of the neutron star. Therefore, using AstroSat observations, we could establish the strength of the magnetic field of the pulsar as $(1.1–1.6) \times 10^{12}$ G. Further observations are required to study...
the spectral variation with source luminosity and the variation of the magnetic field strength during its long outburst duration and with time in order to probe the nature of its accretion column and the geometry where cyclotron lines are produced.

We gratefully acknowledge all the support received from the Indian Space Research Organization (ISRO) for the successful realization of the AstroSat mission from the initial phase of instrument building, tests, and qualifications, to software developments and mission operations. We also acknowledge the support received from the LAXPC Payload Operation Center (POC), TIFR, Mumbai, for the release of verified data, calibration data products, and pipeline processing tools. This work has also utilized the data from the Soft X-ray Telescope (SXT) and hence thankfully acknowledge SXT POC at TIFR for releasing the data through the ISDC data archive center and providing the necessary software tools. We acknowledge software engineers Sanket Kotak, Ashutosh Bajpai, and Harshal Pawar for their vital services in software development activities and timely completion of the full SXT pipeline processing chain along with relevant documentation. We also acknowledge the support of the Department of Atomic Energy, Government of India, under project No. 12-R&D-TFR-5.02-0200. This research also made use of data obtained from Swift-BAT and RXTE through the High Energy Astrophysics Science Archive Research Center Online Services, provided by the NASA/Goddard Space Flight Center; we acknowledge their vital support. We also thank the Fermi-GBM team of NASA/MSFC for sharing monitoring data of the pulsar period measurements of this source during its outburst. We also acknowledge the generous support of NASA’s HEASARC for offering all of the useful software and tools for analysis of astronomical data. We thank the anonymous referee for critical comments, which have improved the manuscript significantly.

Facility: AstroSat.
van der Klis, M., Stella, L., White, N., Jansen, F., & Parmar, A. N. 1987, ApJ, 316, 411
Varun, Pradhan, P., Maitra, C., et al. 2019, ApJ, 880, 61
Voges, W., Pietsch, W., Reppin, C., et al. 1982, ApJ, 263, 803
Vybornov, V., Klochkov, D., Gornostaev, M., et al. 2017, A&A, 601, A126
Wang, D. H., Chen, L., Zhang, C. M., Lei, Y. J., & Qu, J. L. 2014, AN, 335, 168
Wilson, C. A., Finger, M. H., Harmon, B. A., Chakrabarty, D., & Strohmayer, T. 1998, ApJ, 499, 820
Wilson, C. A., Weisskopf, M. C., Finger, M. H., et al. 2005, ApJ, 622, 1024