Structure Scalars for Charged Cylindrically Symmetric Relativistic Fluids

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Abstract

We investigate some structure scalars developed through Riemann tensor for self-gravitating cylindrically symmetric charged dissipative anisotropic fluid. We show that these scalars are directly related to the fundamental properties of the fluid. We formulate dynamical-transport equation as well as the mass function by including charge which are then expressed in terms of structure scalars. The effects of electric charge are investigated in the structure and evolution of compact objects. Finally, we show that all possible solutions of the field equations can be written in terms of these scalars.

Keywords: Relativistic dissipative fluids; Electromagnetic field; Cylindrically symmetric system.

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1 Introduction

It is believed that anisotropy plays a vital role for understanding the gravitation of those objects which have higher densities than neutron stars. Many phenomena such as the solid core, phase transition, mixture of two fluids, slow

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rotation and pion condensation can generate anisotropy in the star models [1]. Lemaitre [2] was the first who gave the idea about the tangential and radial pressures. Since the pioneering work of Bowers and Liang [3], there has been extensive literature devoted to the study of general anisotropic relativistic configuration both analytically and numerically [4].

Recent literature indicates interesting consequences of the inclusion of an electromagnetic field to discuss gravitational collapse. Bekenstein [5] extended the Oppenheimer-Volkoff equations of hydrostatic equilibrium [6] from the neutral to the charged case. Herrera et al. [7] provided a set of equations for the physical interpretation of models for collapsing charged spheres. Nath et al. [8] discussed charged gravitational collapse and concluded that electromagnetic field increases the formation of naked singularity. Sharif and his collaborators [9] investigated the effects of electromagnetic field on different aspects of spherically/cylindically and plane symmetric gravitational collapse.

The orthogonal splitting of the Riemann tensor was first considered by Bell [10]. Herrera and his collaborators [11] followed this idea to develop a relationship between structure scalars and the fluid properties. Also, they [12] analyzed the structure scalars for charged dissipative spherical fluids. Structure scalars, \(X_T\), \(X_{TF}\), \(Y_T\), \(Y_{TF}\) have important properties such as \(X_T\) is the energy density of the fluid, \(X_{TF}\) controls inhomogeneity in the fluid, \(Y_{TF}\) describes the effect of the local anisotropy of pressure as well as density inhomogeneity of the Tolman mass and \(Y_T\) turns out to be proportional to the Tolman mass density for systems in equilibrium or quasi-equilibrium. This ultimately provides a relationship between structure scalars and energy density inhomogeneity.

In a recent paper, Herrera et al. [13] discussed cylindrically symmetric relativistic dissipative fluids based on structure scalars. Here we take the effect of electromagnetic field to study the structure scalars with the same configuration. The paper is organized as follows. In the next section, we describe the Einstein-Maxwell field equations, kinematics and the Weyl tensor. Section 3 is devoted for the structure scalars and conservation laws. In section 4, we formulate the dynamical equations in terms of mass function and couple with transport equation. We discuss the possible static charged cylindrically symmetric solutions for the anisotropic fluid in section 5. In the last section, we summarize the results.
2 Charged Anisotropic Dissipative Fluid Cylinders

Here we review the basic general equations and some definitions. The general cylindrically symmetric spacetime is given as

\[ ds^2 = -A^2(t, r)(dt^2 - dr^2) + B^2(t, r)dz^2 + C^2(t, r)d\phi^2, \]  

(1)

where \(-\infty \leq t \leq \infty, \ 0 \leq r, \ -\infty < z < \infty, \ 0 \leq \phi \leq 2\pi\). We assume that the collapsing cylinder is filled with anisotropic and dissipative fluid for which the energy-momentum tensor is

\[ T_{\alpha\beta}^{(m)} = (\mu + P_r)V_\alpha V_\beta + P_r g_{\alpha\beta} + q_\alpha V_\beta + q_\beta V_\alpha + \Pi_{\alpha\beta}, \]  

(2)

where \(\Pi_{\alpha\beta} = (P_z - P_r)S_\alpha S_\beta + (P_\phi - P_r)K_\alpha K_\beta, \ q_\alpha = qL_\alpha, \ \mu\) is the energy density, \(P_r, \ P_z, \ P_\phi\) are the pressure in the radial, \(z\) and \(\phi\) directions, respectively, \(q_\alpha\) is the radial heat flux and \(V_\alpha\) is the four velocity. Also, \(S_\alpha, \ K_\alpha\) and \(L_\alpha\) are the unit four-vectors with

\[ V_\alpha V_\alpha = -1, \quad L_\alpha L_\alpha = S_\alpha S_\alpha = K_\alpha K_\alpha = 1, \]
\[ V_\alpha S_\alpha = V_\alpha K_\alpha = S_\alpha K_\alpha = 0. \]

For comoving coordinate system, we have

\[ V_\alpha = -A\delta_\alpha^0, \quad L_\alpha = A\delta_\alpha^1, \quad S_\alpha = B\delta_\alpha^2, \quad K_\alpha = C\delta_\alpha^3. \]

The energy-momentum tensor for electromagnetic field is

\[ T_{\alpha\beta}^{(em)} = \frac{1}{4\pi} \left( F_{\alpha\gamma} F_{\beta\gamma} - \frac{1}{4} F_{\gamma\delta} F_{\alpha\beta} g_{\gamma\delta} \right), \]

(3)

where \(F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}\) is the Maxwell field tensor and \(\phi_\alpha\) represents the four potential. The Maxwell field equations are

\[ F_{\alpha\beta} = \mu_0 J_\alpha, \quad F_{[\alpha\beta;\gamma]} = 0, \]

(4)

where \(\mu_0 = 4\pi\) is the magnetic permeability and \(J_\alpha\) is the four current. In comoving coordinate system, we can assume that the charge per unit length of the system is at rest, so the magnetic field will be zero. Thus one can choose the four potential and four current as

\[ \phi_\alpha = \phi_\alpha^0, \quad J_\alpha = \zeta V_\alpha, \]
φ is the scalar potential and ζ is the charge density, both are functions of \( t \) and \( r \). The only non-zero component of the Maxwell field tensor is

\[ F_{01} = -F_{01} = -\phi', \]

where prime is the differentiation with respect to \( r \). Using these values, the Maxwell field equations become

\[
\phi'' + \phi' \left( \frac{B'}{B} + \frac{C'}{C} - \frac{2A'}{A} \right) = 4\pi \zeta A^3, \tag{5}
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{A^2} \frac{\partial \phi}{\partial r} \right) + \left( \frac{1}{A^2} \frac{\partial \phi}{\partial r} \right) \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{2\dot{A}}{A} \right) = 0, \tag{6}
\]

where dot means differentiation with respect to \( t \). Integration of Eq. (5) yields

\[ \phi' = \frac{2sA^2}{BC}, \]

where

\[ s(r) = 2\pi \int_0^r \zeta ABC dr, \tag{7} \]

is the total amount of charge per unit length of the cylinder found through the conservation equation, \( J_{\mu}^{\nu} = 0 \). Also, \( \phi' \) identically satisfies Eq. (6). The Einstein-Maxwell field equations yield

\[
\kappa A^2 \left( \mu + \frac{s^2}{2\pi B^2 C^2} \right) = \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \frac{\dot{C}}{C} - \frac{B''}{B} - \frac{C''}{C}, \tag{8}
\]

\[
-kqA^2 = -\frac{B'}{B} - \frac{\dot{C}'}{C} + \frac{\dot{A}}{A} \left( \frac{B'}{B} + \frac{C'}{C} \right) + \frac{A'}{A} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right), \tag{9}
\]

\[
\kappa A^2 \left( P_r - \frac{s^2}{2\pi B^2 C^2} \right) = -\frac{\dot{B}}{B} - \frac{\dot{C}}{C} + \frac{\dot{A}}{A} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) - \frac{\dot{B}}{B} \frac{\dot{C}}{C} + \frac{A'}{A} \left( \frac{B'}{B} + \frac{C'}{C} \right) + \frac{B' C'}{B C}, \tag{10}
\]
κB² \left( P_z + \frac{s^2}{2\pi B^2 C^2} \right) = \left( \frac{B}{A} \right)^2 \left[ \frac{\ddot{A}}{A} - \frac{\dot{C}}{C} + \left( \frac{\dot{A}}{A} \right)^2 + \frac{A''}{A} + \frac{C''}{C} \right] - \left( \frac{A'}{A} \right)^2,

(11)

κC² \left( P_φ + \frac{s^2}{2\pi B^2 C^2} \right) = \left( \frac{C}{A} \right)^2 \left[ \frac{\ddot{A}}{A} - \frac{\dot{B}}{B} + \left( \frac{\dot{A}}{A} \right)^2 + \frac{A''}{A} + \frac{B''}{B} \right] - \left( \frac{A'}{A} \right)^2.

(12)

There are four kinematical variables for the description of fluid, i.e., expansion, acceleration, shear and rotation. Since we are using irrotational fluid, so the rotation will be zero. Rest are defined as

\[ \Theta = V_α^α, \quad a_α = V_α^β V_β^α, \quad σ_{αβ} = V_{(α;β)} + a_{(α} V_{β)} - \frac{1}{3} Θ h_{αβ}, \]

where \( h_{αβ} = g_{αβ} + V_α V_β \). Using Eq. (11), these quantities turn out to be

\[ Θ = \frac{1}{A} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right), \quad a_1 = \frac{A'}{A}, \quad a_2 = a^α a_α = \left( \frac{A'}{A^2} \right)^2, \quad a_α = a L_α. \]

The shear tensor can also be expressed as

\[ σ_{αβ} = σ_s \left( S_α S_β - \frac{1}{3} h_{αβ} \right) + σ_k \left( K_α K_β - \frac{1}{3} h_{αβ} \right), \]

\[ σ^{αβ} σ_{αβ} = \frac{2}{3} (σ_s^2 - σ_s σ_k + σ_k^2), \]

(13)

where

\[ σ_s = \frac{1}{A} \left( \frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right), \quad σ_k = \frac{1}{A} \left( \frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right). \]

The Weyl tensor is defined as

\[ C_ρ_{αβμ} = R_ρ_{αβμ} - \frac{1}{2} R_ρ_βμ_α + \frac{1}{2} R_{αβδ_ρ} - \frac{1}{2} R_{αμ} δ_ρ^β + \frac{1}{2} R_μ g_{αβ} + \frac{1}{6} (δ_ρ^ρ g_{αμ} - g_{αβ} δ_μ^ρ). \]

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which may be decomposed in its electric and magnetic parts as
\[ E_{\alpha\beta} = C_{\alpha\nu\beta} V^{\nu} V^{\delta}, \quad H_{\alpha\beta} = \frac{1}{2} \eta_{\alpha\nu\rho} C_{\beta}^{\rho\delta} V^{\nu} V^{\delta}, \]
respectively, where \( \eta_{\alpha\nu\rho} \) is the Levi-Civita tensor. These can also be written as
\[ E_{\alpha\beta} = E_{s}(S_{\alpha} S_{\beta} - \frac{1}{3} h_{\alpha\beta}) + E_{k}(K_{\alpha} K_{\beta} - \frac{1}{3} h_{\alpha\beta}), \quad H_{\alpha\beta} = H(S_{\alpha} K_{\beta} + S_{\beta} K_{\alpha}), \]
where
\[ E_{s} = \frac{1}{A^{2}B^{2}} C_{0202} - \frac{1}{A^{2}} C_{0101}, \quad E_{k} = \frac{1}{A^{2}C^{2}} C_{0303} - \frac{1}{A^{2}} C_{0101}, \quad H = -\frac{C_{0313}}{A^{2}C^{2}}. \]

The components of the Weyl tensor \( C_{0202}, C_{0101}, C_{0303}, C_{0313} \) are given in a recent paper [13].

3 Structure Scalars

In this section, we formulate structure scalars for the charged fluid from the orthogonal splitting of the Riemann tensor [13]. For this purpose, the following tensors are defined
\[ Y_{\alpha\beta} = R_{\alpha\nu\beta\delta} V^{\nu} V^{\delta}, \quad X_{\alpha\beta} = * R^{*}_{\alpha\gamma\beta\delta} V^{\gamma} V^{\delta} = \frac{1}{2} \eta_{\alpha\nu\rho} R^{*}_{\beta}^{\rho\delta} V^{\nu} V^{\delta}, \]
where \( R^{*}_{\alpha\beta\gamma\delta} = \frac{1}{2} \eta_{\nu\rho\gamma \delta} R_{\alpha \beta}^{\nu \rho} \). These can be expressed in trace and trace free parts as
\[ Y_{\alpha\beta} = \frac{1}{3} Y_{T} h_{\alpha\beta} + Y_{s}(S_{\alpha} S_{\beta} - \frac{1}{3} h_{\alpha\beta}) + Y_{k}(K_{\alpha} K_{\beta} - \frac{1}{3} h_{\alpha\beta}), \]
\[ X_{\alpha\beta} = \frac{1}{3} X_{T} h_{\alpha\beta} + X_{s}(S_{\alpha} S_{\beta} - \frac{1}{3} h_{\alpha\beta}) + X_{k}(K_{\alpha} K_{\beta} - \frac{1}{3} h_{\alpha\beta}). \]

Using the field equation (8)-(12) with (16), we obtain \( Y_{\alpha\beta} \) and \( X_{\alpha\beta} \) in terms of physical variables
\[ Y_{T} = \frac{\kappa}{2}(\mu + P_{z} + P_{\phi} + P_{r}) + \frac{4s^{2}}{B^{2}C^{2}}, \quad X_{T} = \kappa \mu + \frac{4s^{2}}{B^{2}C^{2}}, \]
\[ Y_{s} = E_{s} - \frac{\kappa}{2}(P_{z} - P_{r}) - \frac{4s^{2}}{B^{2}C^{2}}, \quad Y_{k} = E_{k} - \frac{\kappa}{2}(P_{\phi} - P_{r}) - \frac{4s^{2}}{B^{2}C^{2}}, \]
\[ X_{s} = -E_{s} - \frac{\kappa}{2}(P_{z} - P_{r}) - \frac{4s^{2}}{B^{2}C^{2}}, \quad X_{k} = -E_{k} - \frac{\kappa}{2}(P_{\phi} - P_{r}) - \frac{4s^{2}}{B^{2}C^{2}}. \]
The conservation law, $T^\alpha_\beta = 0$, yields

$$\mu^* + \Theta(\mu + P_r) + q^\alpha a_\alpha + \sigma_{\alpha\beta} \Pi^{\alpha\beta} + \frac{1}{3} \Theta \Pi^\alpha_\alpha = 0,$$

$$h^{\alpha\beta}(P_{r;\beta} + \Pi^{\mu}_{\beta;\mu} + q^*_\beta) + (\mu + P_r) a^\alpha + \frac{4}{3} \Theta q^\alpha + \sigma^\alpha_\mu q^\mu - \frac{ss'}{\pi AB^2C^2} = 0. \quad (20)$$

The last equation can be written in an alternative form as

$$P^*_r + q^* - \frac{1}{A} \left[ (P_z - P_r) \frac{B'}{B} + (P_\phi - P_r) \frac{C'}{C} \right] + (\mu + P_r) a$$

$$- \frac{1}{3} (\sigma_s + \sigma_k - 4\Theta) q - \frac{ss'}{\pi AB^2C^2} = 0, \quad (21)$$

where $f^* = f_\alpha V^\alpha$, $f^\dagger = f_\alpha L^\alpha$. There are two important differential equations relating the Weyl tensor to different physical variables. Herrera [13] found these relations for cylindrically symmetric spacetime which are generalized to charge distribution as

$$-(Y_s + Y_k - X_s - X_k)^\dagger + 3(Y_s - X_s) \frac{B'}{AB} - 3(Y_k - X_k) \frac{C'}{AC}$$

$$- 6H(\sigma_s - \sigma_k) = \kappa(2\mu + P_r + P_z + P_\phi)^\dagger + 3\kappa(\mu + P_r)a$$

$$+ 2\kappa q(\Theta - \sigma_s - \sigma_k) + 3\kappa q^* + \frac{3\kappa s}{\pi AB^2C^2} \left( s' - s \frac{B'}{B} - s \frac{C'}{C} \right), \quad (22)$$

$$(2Y_s - Y_k - 2X_s + X_k)^\dagger + 3(Y_s - X_s) \frac{B'}{AB} + 3a(Y_s - Y_k - X_s)$$

$$+ X_k) + 6H(\Theta - \sigma_k) + 6H^* = -\kappa(\mu - P_r - P_z + 2P_\phi)^\dagger$$

$$- 3\kappa (P_\phi - P_r) \frac{C'}{AC} + \kappa q(\Theta - \sigma_s + 2\sigma_k) + \frac{3\kappa s}{\pi AB^2C^2} \left( s \frac{B'}{B} - s' \right). \quad (23)$$

### 4 Mass Function and Dynamical-Transport Equation

Here, we develop equations that govern the dynamics of non-adiabatic cylindrically symmetric collapsing process. For this purpose, we define the velocity $U = \frac{\dot{C}}{\dot{A}} = C^*$. Using Eq.(10), we get

$$U^* = a \frac{C'}{A} - \kappa P_r C - \frac{C}{A^2} \left( \frac{\dot{B}}{B} - \frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} - \frac{B'C'}{BC} - \frac{A'B'}{AB} \right) + \frac{\kappa s^2}{2\pi B^2C}. \quad (24)$$
which turns out to be

\[ U^* = a \frac{C'}{A} - \kappa P_r C + \frac{C}{B^2} \left( \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} \right) + \frac{\kappa s^2}{2\pi B^2 C}. \]

Solving it for \( a \) and substituting into Eq. (21), we obtain

\[ (\mu + P_r) U^* = -(\mu + P_r) \left[ \kappa P_r C - \frac{C}{B^2} \left( \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} \right) \right] - C' \left[ \frac{P^i}{A} \right] - \frac{C'}{A} \left( \frac{B}{AB} \right) \left( P_z - P_r \right) - (P_\phi - P_r) \left( \frac{C''}{AC} \right) + \frac{C''}{A} \left[ -q^* + \frac{1}{3}(\sigma) \right] + \frac{ss'C'}{\pi B^2 C^2} + \frac{2s'}{2\pi B^2 C} + \frac{1}{3}(\mu + P_r) \frac{\kappa s^2}{2\pi B^2 C}. \]  

This equation yields the effect of different forces on the collapsing process and have the "Newtonian" form as

\[ \text{Force} = \text{Mass Density} \times \text{Acceleration}. \]

The term on the left is the density (the inertial or passive gravitational mass density) multiplied by the proper time derivative of the velocity \( U \). The terms on the right represent the force which is the contribution of four forces: the gravitational force (the pressure gradient plus the anisotropic contribution), the contribution from the dissipation and the contribution from the charge term.

Now we define a mass function \( m \) similar to the spherically symmetric case. For this purpose, we write the term \( \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} \) in the form of structure scalars as

\[ \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} = \frac{B^2}{3} (Y_T - X_T + X_s + 2Y_s + X_k - Y_k). \]

Using Eqs. (18) and (19), it follow that

\[ \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} = \frac{\kappa B^2}{3} \left( -\mu + 2P_r - P_z + \frac{P_\phi}{2} \right) + \frac{B^2}{3} (E_s - 2E_k) - \frac{4s^2}{C}. \]  

Inserting this value in the first square brackets on the right side of Eq. (24), we obtain

\[ \kappa P_r C - \frac{C}{B^2} \left( \frac{R_{0202}}{A^2} - \frac{R_{2323}}{C^2} \right) = \frac{\kappa P_r C}{2} + \frac{\kappa}{6} (\mu - P_\phi - P_r + 2P_z) C \]

\[ - \frac{C}{3} (E_s - 2E_k) + \frac{4s^2}{B^2 C}. \]
To define a mass function, we have followed the procedure of [7, 13]. We have compared Eq.(26) with the corresponding equation in the spherically symmetric case [7]. We have also assumed that the pressure effect remains the same as in spherically symmetric case which provides the definition of mass function as a possible extension of the Misner-Sharp mass function to the cylindrically symmetric case given by

$$m = \frac{C^3\kappa}{6}(\mu - P_\phi - P_r + 2P_z) - \frac{C^3}{3}(E_s - 2E_k) + \frac{4s^2C}{B^2}. \quad (27)$$

Making use of Eq.(19), this can be written as

$$\frac{3m}{C^3} = \frac{\kappa}{2}(\mu + P_\phi - 2P_r + P_z) - (Y_s - 2Y_k) + \frac{16s^2}{B^2C^2}. \quad (28)$$

or

$$\frac{3m}{C^3} = \frac{\kappa}{2}(\mu - 3P_\phi + 3P_z) - (2X_k - X_s) + \frac{8s^2}{B^2C^2}. \quad (29)$$

The mass function in terms of electric charge can be found by using Eqs.(19) and (21)-(23) as follows

$$(3Y_s - 3Y_k + X_s + X_k)\dagger = \kappa\mu\dagger + \kappa q(\sigma_k - \Theta - 2\sigma_s) - \frac{3B'}{AB}(Y_s + X_s)$$

$$+ \frac{3C'}{AC}(Y_k - X_k) - 6H(\Theta - \sigma_s) - 6H^* - 3a(Y_s - Y_k - X_s + X_k)$$

$$+ \frac{3\kappa s}{\pi A B^2 C^2} \left( s' - \frac{sC'}{C} \right). \quad (30)$$

Adding Eqs.(28) and (29), it follows that

$$\frac{6m}{C^3} = (\kappa\mu + 3Y_k - 3Y_s - X_k - X_s) + \frac{16s^2}{B^2C^2}.$$  

Applying the operator $\dagger$ ($f\dagger = f,\alpha L^\alpha$) on both sides of the above equation and then substituting in Eq.(30), after some manipulation, we obtain

$$\left( \frac{6m}{C^3} \right)\dagger = 3(Y_s + X_s)\frac{B'}{AB} + 6H(\Theta - \sigma_s) - 3(Y_k - X_k)$$

$$\times \frac{C'}{AC} + \kappa q(2\sigma_s - \sigma_k + \Theta) + 6H^* + 3a(Y_s - Y_k)$$

$$- X_s + X_k) - \frac{\kappa s}{\pi A B^2 C^2} \left( s' - \frac{sC'}{C} - 4sB' \right).$$
Integration leads to

\[ m = \frac{C^3}{6} \int A \left[ 3(Y_s + X_s) \frac{B'}{AB} + 6H(\Theta - \sigma_s) - 3(Y_k - X_k) \frac{C'}{AC} \right. \]

\[ + \kappa q(2\sigma_s - \sigma_k + \Theta) + 6H^* + 3a(Y_s - Y_k - X_s + X_k) - \frac{\kappa s}{C^2} \]

\[ \times \frac{1}{\pi AB^2} \left( s' - s \frac{C'}{C} - \frac{4sB'}{B} \right) \right] dr + \frac{C^3 \gamma(t)}{6}, \quad (31) \]

where \( \gamma \) is an arbitrary integration function of \( t \). This mass function shows its dependence on different factors, in particular, on electric charge. Inserting the values of \( X_s, Y_s, X_k, Y_k \) from Eq. (19), it can be expressed in terms of physical variables and the Weyl tensor

\[ m = \frac{C^3}{2} \int A \left[ \kappa(P_r - P_z) \frac{B'}{AB} - 2E_k \frac{C'}{AC} + 2H(\Theta - \sigma_s) + \frac{\kappa q}{3} \right. \]

\[ \times (2\sigma_s - \sigma_k + \Theta) + 2a(E_s - E_k) + 2H^* \right] dr - \frac{C^3}{6} \int \frac{\kappa s}{\pi B^2 C^2} \]

\[ \times \left( s' - s \frac{C'}{C} - \frac{4sB'}{B} \right) dr + \frac{C^3 \gamma(t)}{6}. \quad (32) \]

This gives the contribution of anisotropy, Weyl tensor, electric charge and dissipation.

The transport equation for dissipative fluids is given by [14]

\[ \tau h^{\alpha \beta} V^\gamma q_{\beta \gamma} + q^\alpha = -K h^{\alpha \beta}(T_{\beta} + Ta_{\beta}) - \frac{1}{2} KT^2 \left( \frac{\tau V^\beta}{KT^2} \right)_{,\beta} q^\alpha. \quad (33) \]

Here \( K, T \) and \( \tau \) indicate thermal conductivity, temperature and relaxation time, respectively. The only one independent component is

\[ \tau q^* + q = -K(T^* + Ta) - \frac{1}{2} KT^2 q \left( \frac{\tau}{KT^2} \right)^* - \frac{1}{2} q\tau T. \quad (34) \]

Substituting Eqs. (26), (27) and (34) in (24), it follows that

\[ (\mu + P_r) \left[ 1 - \frac{KT}{\tau(\mu + P_r)} \right] U^* = -(\mu + P_r) \left( \frac{\kappa P_r C^3}{2} + m \right) \frac{1}{C^2} \]

\[ \times \left[ 1 - \frac{KT}{\tau(\mu + P_r)} \right] + \frac{C'}{A} \left[ -P_r^* + (P_z - P_r) \frac{B'}{AB} + (P_\phi - P_r) \frac{C'}{AC} \right] \]
This shows that gravitational attraction and electric charge on any fluid element will decrease by the same factor as the inertial mass density.

We see that the charge increases the gravitational mass only if

\[ s' > s\kappa \, C(\mu + P_r) \left( -1 + \frac{K\tau}{\tau(\mu + P_r)} \right), \tag{36} \]

otherwise, it will decrease. This increase of gravitational mass causes rapid collapse. Bekenstein \[5\] noticed this strange effect for the Oppenheimer-Volkoff equations of hydrostatic equilibrium. We see that the charge does not enter into the term \( 1 - \frac{K\tau}{\tau(\mu + P_r)} \) but it affects the gravitational mass and shows how thermal effects reduce the effective inertial mass. We observe that as \( \frac{K\tau}{\tau(\mu + P_r)} \to 1 \), the inertial mass density of the fluid element tends to zero \[15\]. This shows that there is no inertial force and matter would experience a gravitational attraction which causes the collapse. For \( 0 < \frac{K\tau}{\tau(\mu + P_r)} < 1 \), the inertial mass density goes on decreasing while \( \frac{K\tau}{\tau(\mu + P_r)} > 1 \) indicates the increase of inertial mass density. By the equivalence principle, there should occur increase or decrease of mass. Thus one can easily distinguish between expanding and collapsing mechanism during the dynamics of dissipative process.

Assume that the collapsing cylinder evolves in such a way that the value of \( \frac{K\tau}{\tau(\mu + P_r)} \) increases and approaches to 1 for some region. During this process, the gravitational force term decreases and leads to a change of the sign of the right hand side of Eq.(35). This would happen for small values of the effective inertial mass density and implies a strong bouncing of that part of the cylinder \[16\]. This phenomenon causes the loss of energy from the system and hence the collapsing cylinder with non-adiabatic source leads to the emission of the gravitational radiations. Notice that the term \( 1 - \frac{K\tau}{\tau(\mu + P_r)} \) (in the inertial mass, gravitational force and electric charge) is related to the left of Eq.(33).
5 Static Charged Anisotropic Cylinders

Here we discuss all possible solutions of the field equations of the static charged anisotropic cylinders. The corresponding field equations are

\[
\kappa A^2 \left( \mu + \frac{s^2}{2\pi B^2 C^2} \right) = -\frac{B''}{B} - \frac{C''}{C} + \frac{A'}{A} \left( \frac{B'}{B} + \frac{C'}{C} \right) - \frac{B' C'}{B C},
\]

\[
\kappa A^2 \left( P_r - \frac{s^2}{2\pi B^2 C^2} \right) = \frac{A'}{A} \left( \frac{B'}{B} + \frac{C'}{C} \right) + \frac{B' C'}{B C},
\]

\[
\kappa A^2 \left( P_z + \frac{s^2}{2\pi B^2 C^2} \right) = \frac{A''}{A} + \frac{C''}{C} - \left( \frac{A'}{A} \right)^2,
\]

\[
\kappa A^2 \left( P_\phi + \frac{s^2}{2\pi B^2 C^2} \right) = \frac{A''}{A} + \frac{B''}{B} - \left( \frac{A'}{A} \right)^2.
\]

These equations, in terms of auxiliary variables, \( \omega = \frac{A'}{A}, \ \xi = \frac{B'}{B}, \ \eta = \frac{C'}{C}, \) can be written as

\[
\kappa A^2 \left( \mu + \frac{s^2}{2\pi B^2 C^2} \right) = -\xi' - \xi^2 - \eta' - \eta^2 + \omega \xi + \omega \eta - \xi \eta, \quad (37)
\]

\[
\kappa A^2 \left( P_r - \frac{s^2}{2\pi B^2 C^2} \right) = \omega \xi + \omega \eta + \xi \eta, \quad (38)
\]

\[
\kappa A^2 \left( P_z + \frac{s^2}{2\pi B^2 C^2} \right) = \omega' + \eta' + \eta^2, \quad (39)
\]

\[
\kappa A^2 \left( P_\phi + \frac{s^2}{2\pi B^2 C^2} \right) = \omega' + \xi' + \xi^2. \quad (40)
\]

Adding all these equations, we have

\[
\omega' + \omega \xi + \omega \eta = Y_T A^2. \quad (41)
\]

From Eqs. (38)-(40), it follows that

\[
\omega' + \xi' + \xi^2 - \omega \xi - \omega \eta - \xi \eta = \kappa (P_\phi - P_r) A^2 + \frac{8s^2 A^2}{B^2 C^2}, \quad (42)
\]

\[
\omega' + \eta' + \eta^2 - \omega \xi - \omega \eta - \xi \eta = \kappa (P_z - P_r) A^2 + \frac{8s^2 A^2}{B^2 C^2}, \quad (43)
\]

\[
\xi' + \xi^2 - \eta' + \eta^2 = \kappa (P_\phi - P_z). \quad (44)
\]
In terms of auxiliary variables, the scalars $E_s$ and $E_k$ become

$$E_s = \frac{1}{2A^2} \left[ -\omega' + \eta' + \eta^2 + \omega\xi - \omega\eta - \xi\eta \right],$$

$$E_k = \frac{1}{2A^2} \left[ -\omega' + \xi' + \xi^2 - \omega\xi + \omega\eta - \xi\eta \right].$$

(45)

Equations (19), (42)-(45) yield

$$Y_s A^2 = -\omega' + \omega\xi, \quad Y_k A^2 = -\omega' + \omega\eta.$$  

(46)

Integration of the first equation yields

$$A = \alpha e^{\int B \left( \int \frac{Y_s A^2}{B} dr \right) dr},$$

where $\alpha$ is an integration constant, which means $B = B(A)$ or $\xi = \xi(\omega)$ for any $Y_s$. When we integrate second of the above equation, we obtain

$$A = \gamma e^{\int C \left( \int \frac{Y_k A^2}{C} dr \right) dr},$$

where $\gamma$ is another integration constant implying $C = C(A)$ or $\eta = \eta(\omega)$ for any $Y_k$. Consequently we can express any $(\omega, \xi, \eta)$ in terms of each other. This leads to the similar result as that of paper [13] with the effect of charge. Thus any static anisotropic solution is determined by a triplet of scalars $(Y_k, Y_s, X_k)$ or $(Y_k, Y_s, X_s)$ as in the charged free case. Similarly, we can discuss the case of isotropic cylinders.

6 Conclusion

This paper investigates the effects of electromagnetic field on structure scalars of the cylindrically symmetric anisotropic dissipative fluid. The electric charge increases the inhomogeneity produced by local anisotropy. Following Herrera [13], we have formulated dynamical as well as transport equations and also mass function for the charged cylindrical system. It turns out that the coupled dynamical-transport equation has an extra factor due to charge. If we take $\frac{KT}{\tau(\mu + P_r)} = 1$, the inertial mass and gravitational force vanish while the gravitational mass reduces for $s' < s k C(\mu + P_r) \left( -1 + \frac{KT}{\tau(\mu + P_r)} \right)$. Further, we have discussed the static case in electromagnetic field which shows that any solution of the field equations can be expressed in terms of the scalar functions like charge free case. It is worth mentioning here that all our results reduce to charge free case [13] for $s = 0$. 

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