The onset of the non-linear regime in unified dark matter models

P.P. Avelino,¹,2 L.M.G. Beça,¹ J.P.M. de Carvalho,³,4 C.J.A.P. Martins,⁵,6 and E.J. Copeland⁷

¹Centro de Física do Porto e Departamento de Física da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007, Porto, Portugal
²Astronomy Centre, University of Sussex, Brighton BN1 9QJ, United Kingdom
³Centro de Astrofísica da Universidade do Porto, R. das Estrelas s/n, 4150-762 Porto, Portugal
⁴Departamento de Matemática Aplicada da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007, Porto, Portugal
⁵Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
⁶Institut d’Astrophysique de Paris, 98 bis Boulevard Arago, 75014 Paris, France
⁷Centre for Theoretical Physics, University of Sussex, Brighton BN1 9QJ, United Kingdom

(Dated: 24 June 2003)

We discuss the onset of the non-linear regime in the context of unified dark matter models involving a generalised Chaplygin gas. We show that the transition from dark matter-like to dark energy-like behaviour will never be smooth. In some regions of space the transition will never take place while in others it may happen sooner or later than naively expected. As a result the linear theory used in previous studies may break down late in the matter dominated era even on large cosmological scales. We study the importance of this effect showing that its magnitude depends on the exact form of the equation of state in the low density regime. We expect that our results will be relevant for other unified dark matter scenarios particularly those where the quartessence candidate is a perfect fluid.

I. INTRODUCTION

There is growing evidence that the Universe we live in is (nearly) flat and presently undergoing an accelerating phase.¹² A ‘dark’ energy component is thought to be responsible for this acceleration and either a ‘standard’ cosmological constant, or a quintessence scalar field, or a k-essence field are usually put forth to account for it. However, all current explanations necessarily face some level of fine-tuning, which motivates a search for further alternatives (see for example §¹¹). The Chaplygin gas provides one such interesting alternative. It bears the exotic equation of state

\[ p = -\frac{C}{\rho^\alpha}, \]

(1)

where \( \rho \) is the density, \( C \) is a positive constant and \( 0 \leq \alpha \leq 1 \). In a homogeneous universe its main property (cosmologically speaking) is that of mimicking cold dark matter at early times but progressively evolving into a cosmological constant later on. If \( \alpha = 0 \), the Chaplygin gas model is in fact identical to a familiar ΛCDM scenario where \( \Omega_m^* = 1 - \Omega \) is the equivalent matter density (with \( \Omega = C/\rho_0^\alpha \) where \( \rho_0 \) is the Chaplygin gas density at the present time).

This particular behaviour indicates that dark energy and dark matter could just be different manifestations of a unique perfect fluid known as ‘quartessence’. In this regard, the Chaplygin gas¹¹ (or simple generalisations thereof)³ has attracted considerable attention as a quartessence prototype since a connection between string theory and the original Chaplygin gas has been proposed (see for example §¹ and references therein).

The simple picture described above has been taken for granted in all the work on this subject so far, namely when confronting models with cosmological observations.¹⁵,¹⁶,¹⁷,¹⁸,¹⁹,²⁰,²¹,²²,²³,²⁴,²⁵,²⁶,²⁷ There is, however, a caveat to it which may have important implications. As we previously stated, the Chaplygin gas starts out behaving as ΛCDM, with linear perturbations growing in the usual way as long as this regime holds. In linear theory, once the background evolution starts switching to a Λ-like behaviour, perturbations in the Chaplygin gas will become heavily damped while the growth of baryonic perturbations is slowed considerably. This would all be fine, in the sense that assuming linear theory to hold, regions of parameter space have been shown to exist where structure formation proceeds in a way that is in agreement with observations. However, there is no a priori guarantee that non-linear effects will not significantly alter these conclusions.

Specifically, in this letter we show that in a class of unified dark matter scenarios this transition from dark matter-like to dark energy-like behaviour will never be smooth. The reason is simple: at some point, density perturbations on a given scale reach the non-linear regime and consequently a large fraction of the mass is then expected to be incorporated in collapsed objects whose evolution is effectively decoupled from the background. In these regions the Chaplygin gas never evolves into the dark energy-like stages (which only happens at sufficiently low densities). On the other hand, in low density
regions the absolute value of the (negative) pressure may be much larger than the average value thus having the opposite effect on the dynamics of the universe (favouring an earlier accelerating phase).

It is the importance of this effect that we set out to quantify in this letter. While the analysis will be done for the case of the Chaplygin gas, our results will be relevant for any similar scenarios where the quartessence candidate is a perfect fluid. We note that concerns about the relevance of the non-linear evolutionary phase in a different context (condensation and metamorphism cosmologies) have also been made in ref. [28].

II. THE CRITICAL SCALES

It is straightforward to show that in a homogeneous and isotropic universe the Chaplygin gas energy density evolves as

\[ \rho = \rho_0 \left[ \frac{\alpha}{C/\rho_0} (1 + z)^{3(1 + \alpha)} \right]^{1/1 + \alpha}, \]

(2)

where \( \alpha = C/\rho_0 \) and \( \rho_0 \) is the present density. In Fig. 1 we plot the evolution of the sound speed squared \( c_s^2 = \alpha C/\rho^{1+\alpha} = -\alpha p/\rho \) as a function of the redshift \( z \) taking \( \alpha = 0.71 \) for three different values of \( \alpha \) (0.2, 0.5, and 1). At early times the sound speed squared is

\[ c_s^2 \propto (1 + z)^{-3(1 + \alpha)}, \]

(3)

which determines how the inclination of the curves in Fig. 1 depends on \( \alpha \). We clearly see that deep in the matter era the equation of state of the Chaplygin gas closely resembles that of CDM (with \( |p/\rho| \ll 1 \)).

Using a first order perturbative analysis, the evolution in Fourier space, of density perturbations in the Chaplygin gas, can be described by the following equation (see [23] for details):

\[ \delta''_k + \left[ 2 + \xi - 3\left(2w - c_s^2\right)\right]\delta'_k = \left[ \frac{3}{2}(1 - 6c_s^2 + 8w - 3w^2) - \left(\frac{kc_s}{aH}\right)^2 \right] \delta_k. \]

(4)

which we rewrite as \( \delta''_k + A\delta'_k + B\delta_k = 0 \). Here \( a \) is the scale factor, \( H \) is the Hubble parameter, \( \dot{\xi} = d/d\ln a \), \( \xi \equiv H'/H \) and \( w \equiv p/\rho \). This second order differential equation can easily be transformed into a non-autonomous dynamical system of the form \( \dot{x}' = A x \)

where \( x \equiv (\delta, \dot{\delta}) \) and \( A \) is the matrix of the coefficients, which are functions of both the wave number \( k \) and the redshift \( z \).

We shall look for critical scales associated with the Chaplygin gas by calculating the eigenvalues \( \lambda \) of \( A \) for fixed instants of time. These are the roots of the Cayley polynomial \( \Lambda^2 + A\Lambda + B = 0 \). Since the sign of the real part of the eigenvalue determines whether or not a mode grows or decays, the condition \( \text{Re}(\lambda(k_x, a)) = 0 \) sets the critical scale for that \( a \). Fig. 2 contains the real part of the two eigenvalues for several times. A bifurcation pattern emerges. At early times the Chaplygin gas acts as CDM and so all relevant scales are gravitationally unstable. As time goes by, this critical scale starts to diminish until there are no gravitational unstable scales.

We note that in this linear analysis both \( c_s \) and \( w \) are evaluated to zeroth order. We will show that this is no longer a good approximation when the fluctuations in the Chaplygin gas component become large.

III. NON-LINEAR EFFECTS

Having gained some intuition for the relevant length and time scales in the problem, we now proceed with
parameters are an equivalent matter density Ω\_m = 1 - Ω\_A + Ω\_b = 0.29, a baryon density Ω\_b = 0.047, an equivalent cosmological constant density Ω\_A = 1 - Ω\_m\_NL = 0.71, a Hubble parameter h = 0.72, a normalisation σ\_8 = 0.9, and perturbation spectral index n\_s = 0.99.

We start by determining how the linear density perturbations of the Chaplygin gas and baryon components evolve with redshift (the details of such an analysis are described in Sect. II of our earlier work [30]). We emphasise that the use of linear theory early in the matter era is actually a good approximation on large enough scales. The effects of the breakdown of linear theory will only be important (on large cosmological scales) at late times when a smooth transition from a dark matter to a dark energy dominated universe would naively be expected to take place.

Hence, we use linear theory in order to compute the value of the dispersion of the density fluctuations in the baryon and Chaplygin gas components, σ\(R, a\), as a function of R and a. This is plotted in Fig. 3 for the particular case of the original Chaplygin gas with α = 1. We see that since the Chaplygin gas behaves as matter at early stages (see Fig. 1), embedded perturbations will grow proportionally to the scale factor, a, and in tune with those in the baryonic component. Therefore, both fluids evolve in the same way earlier on and have approximately the same value of σ on all relevant scales. During this stage we have σ ∝ a = (1 + z)^{-1}. Later on, the pressure of the Chaplygin gas will have increased dramatically preventing further collapse of the Chaplygin gas component. However, the baryon fluctuations can still keep growing (at a slower pace). We also see in Fig. 3 that the Chaplygin gas component becomes non-linear on small scales early in the matter era. It is clear that when this happens a significant fraction of the Chaplygin gas will have collapsed and decoupled from the background so that a transition from a dark matter-like to a dark energy-like stage (which necessarily requires lower densities) never happens in those regions. Using the Press-Schechter framework [29] we can show that for σ = 1 the fraction of the equivalent mass that is already incorporated in collapsed objects is already close to 0.1.

In the non-linear regime an initial gaussian density field is better described by a lognormal one-point probability distribution function, \(\mathcal{P}(\delta)\), given by (see for example [31] and references therein)

\[
\mathcal{P}(\delta) = \frac{(1 + \delta)^{-1}}{\sqrt{2\pi \ln(1 + \sigma_{nl}^2)}} \exp \left( -\frac{\ln^2 (1 + \delta) \sqrt{1 + \sigma_{nl}^2}}{2 \ln(1 + \sigma_{nl}^2)} \right),
\]

where \(\sigma_{nl}^2 = \exp(\sigma^2) - 1\) and σ is computed using linear theory. We use (5) in order to calculate the ratio between the average value of p and its zeroth order background value \(\langle p \rangle/p_b\), as a function of the dispersion, σ, of the linear density fluctuations in the Chaplygin gas component for various values of α (1, 0.8, 0.6, 0.4, 0.2 and 0 from top to bottom). For α > 0 we clearly see that \(\langle p \rangle/p_b\) rapidly diverges from unity if σ is large enough.
diverges from its zeroth order background value as soon as $\sigma$ becomes large enough. This means that the average values of both $w$ and $c_s^2$ will also move away from their zeroth order value (except in the $\alpha = 0$ case) causing the breakdown of linear theory. The magnitude of this effect becomes more pronounced at late times when the negative pressure starts to become dynamically important on all scales.

We further note that in the $\alpha \neq 0$ case and for very small densities the sound speed, $c_s \propto \rho^{(1-\alpha)/2}$, will become greater than the speed of light, $c = 1$. Hence, we may expect that in the context of realistic models there will be a cut-off to this power law behaviour. One equation of state that resembles that of eqn. (1) at high densities while switching off to a cosmological constant like equation of state at low densities is

$$p = -\frac{\Delta^{1+\beta} \rho}{(\rho + \Delta)^{1+\beta}}, \quad (7)$$

where $\Delta \geq 0$ is a low density cut-off and $0 \leq \beta \leq 1$. Note that in this case the absolute values of both $c_s^2$ and $w$ are always smaller than unity. Another interesting case is

$$p = -\frac{C}{(\rho + \Delta)^{\beta}}, \quad (8)$$

for which $c_s^2 \leq 1$ (assuming that $C/\Delta^{\beta+1} \leq 1$) but the absolute value of $w$ can grow arbitrarily large. As we have shown the importance of the non-linear effects discussed in this letter for cosmological predictions will strongly depend on the exact form of the equation of state in the low density regime.

IV. CONCLUSIONS

We have shown that previous work on the Chaplygin gas has a caveat which may have crucial implications for the predicted observational consequences of the model. The linear theory used in previous treatments breaks down at late times even on large cosmological scales except in the $\alpha = 0$ case. This means that non-linear effects should be taken into account when confronting the model with cosmological observations.

Although the present analysis has been done for the particular case of a Chaplygin gas we expect our results to be relevant for other unified dark matter scenarios particularly those where the quintessence candidate is a perfect fluid.

Acknowledgments

C.M. is funded by FCT (Portugal), under grant FMRH/BPD/1600/2000. Additional support came from FCT under contract CERN/POCTI/49507/2002.

[1] S. Perlmutter et al., Astrophys. J. 517, 565 (1999), astro-ph/9812133.
[2] A. G. Riess et al., Astrophys. J. 560, 49 (2001), astro-ph/0104455.
[3] J. L. Tonry et al. (2003), astro-ph/0305008.
[4] D. N. Spergel et al. (2003), astro-ph/0302209.
[5] S. M. Carroll, Living Rel. Rev. 4, 1 (2001), astro-ph/0004075.
[6] L. Wang, R. R. Caldwell, J. P. Ostriker, and P. J. Steinhardt, Astrophys. J. 530, 17 (2000), astro-ph/9901388.
[7] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, Phys. Rev. D63, 103510 (2001), astro-ph/0006373.
[8] M. Bucher and D. N. Spergel, Phys. Rev. D60, 043505 (1999), astro-ph/9812022.
[9] P. P. Avelino and C. J. A. P. Martins, Astrophys. J. 565, 661 (2002), astro-ph/0106274.
[10] J. S. Alcaniz, D. Jain, and A. Dev, Phys. Rev. D67, 063509 (2003), astro-ph/0209395.
[11] J. S. Alcaniz, D. Jain, and A. Dev, Phys. Rev. D67, 043514 (2003), astro-ph/0210476.
[12] V. Gorini, K. A., and U. Moschella, Phys. Rev. D67, 063509 (2003), astro-ph/0209395.
[13] J. C. Fabris, S. V. B. Goncalves, and P. E. De Souza, Gen. Rel. Grav. 34, 53 (2002), gr-qc/0103083.
[14] J. C. Fabris, S. V. B. Goncalves, and P. E. De Souza, Gen. Rel. Grav. 34, 211 (2002), astro-ph/0203441.
[15] M. Makler, S. Quinet de Oliveira, and I. Waga, Phys. Lett. B555, 1 (2003), astro-ph/0209486.
[16] P. P. Avelino, L. M. G. Beca, J. P. M. de Carvalho, C. J. A. P. Martins, and P. Pinto, Phys. Rev. D67, 023511 (2003), astro-ph/0208528.
[17] A. Dev, J. S. Alcaniz, and D. Jain, Phys. Rev. D67, 023515 (2003), astro-ph/0209379.
[18] V. Gorini, K. A., and U. Moschella, Phys. Rev. D67, 063509 (2003), astro-ph/0209395.
[19] J. S. Alcaniz, D. Jain, and A. Dev, Phys. Rev. D67, 043514 (2003), astro-ph/0210476.
[20] D. Carturan and F. Finelli (2002), astro-ph/0211626.
[21] H. Sandvik, M. Tegmark, M. Zaldarriaga, and I. Waga (2002), astro-ph/0212114.
[22] R. Bean and O. Dore (2003), astro-ph/0301308.
[23] M. C. Bento, O. Bertolami, and A. A. Sen (2003), astro-ph/0303538.
[24] L. M. G. Beca, P. P. Avelino, J. P. M. Carvalho, and C. J. A. P. Martins, Phys. Rev. D67, 101301 (2003), astro-ph/0303564.
[25] L. Amendola, F. Finelli, C. Burigana, and D. Carturan (2003), astro-ph/0304325.
[26] B. A. Bassett, M. Kunz, D. Parkinson, and C. Ungarelli (2002), astro-ph/0211303.
[27] W. H. Press and P. Schechter, Astrophys. J. 187, 425
[30] V. Sahni and P. Coles, Phys. Rep. 262, 1 (1995), astro-ph/9505005.