Quantifying productivity of a gravity dispenser

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Abstract. In processing lines for the manufacturing of concentrated feed for farm animals, there is often an issue of operations coordination of machines according to their performance. Particularly, when delivering feed components to the mixer, it is essential to consider the performance of the dispenser distributing each of them. The productivity of the dispenser has to be no higher than the productivity of the mixer. In given article we have provided an example of calculating the theoretical performance of a gravity dispenser in a truncated cone-shaped hopper with dampers in the lower part. Calculations let us conclude that the main element affecting the productivity of the dispenser is the radius of the hopper outlet, since it is included in the formula in the second degree.

1 Introduction

The gravity dispenser of the production line for concentrated feed for farm animals is constructed in the form of a truncated cone-shaped hopper with flaps in the lower part [1-4]. We will theoretically identify its performance [5-7]. Consider the flow movement of bulky material from the hopper in the similarity of a liquid, we assume the movement is steady.

2 Materials and methods

The movement of material is carried into the atmosphere by gravity (G). The studied flow of bulky material is bounded by the walls of the hopper and two flat sections (figure 1) [8,9].

The pressure distribution in the highlighted sections is hydrostatic, the sections are normal to the z axis of the flow and are at an elementary distance (dz) from each other.

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Fig. 1. Acting forces diagram for the selected piece of material.

In section 1-1 acts the pressure force $P_1 \cdot \omega_1$. It is directed into the interior of the isolated feed mass. In section 2-2 acts $P_2 \cdot \omega_2$ force, (where $P_1, P_2$ - pressure at the center of section masses; $\omega_1, \omega_2$ – area of these sections). On the elementary site $dS$ of the walls of the hopper are impacted by pressure forces on the side surface of the feed flow volume under consideration with the sides of its bounding walls $p_n dS$, directed properly to this surface, and the motion resistance force (frictional force) $\tau ds$, (where $p_n$ – local pressure on the walls. The movement is due to gravity force $G$ (weight of the feed volume by height $dz$ bounded by sections 1-1 and 2-2). It is directed straight down and is equal to $G = \rho g dW$ (where $dW$ – elementary volume of the selected mass of feed flow to the hopper). Let's sum up the projections of all the acting forces on the axis $OZ$ and $OX$.

\[
\sum Z = 0 \quad p_n \cdot ds \cdot \sin \alpha + \tau \cdot ds \cdot \cos \alpha + P_1 \omega_1 - P_2 \omega_2 - \rho \cdot g \cdot dW = 0 \quad (1)
\]

\[
\sum X = 0 \quad p_n \cdot ds \cdot \cos \alpha - \tau \cdot ds \cdot \sin \alpha - q ds = 0 \quad (2)
\]

As far as is known, the frictional force is connected with the reaction of the wall by the dependence:

\[
\tau \cdot ds = F_m = f \cdot p_n \cdot ds \quad (3)
\]

where $f$ – friction index of feed against the hopper wall.

Using equation (1) and taking into account expression (3), we get

\[
p_n \cdot ds \cdot (\sin \alpha + f \cdot \cos \alpha) + P_1 \omega_1 - P_2 \omega_2 - \rho \cdot g \cdot dW = 0 \quad (4)
\]

Using equation (2) and taking into account expression (3), we get:
Jointly solving equations (4) and (5), we get:

\[
q \cdot \frac{(\sin \alpha + f \cdot \cos \alpha)}{\cos \alpha - f \cdot \sin \alpha} + P_1 \omega_1 - P_2 \omega_2 - \rho \cdot g \cdot dW = 0
\]  

(6)

Take that the feed particles move like a liquid at the same speed, equal to the average flow rate. Practically, the gravity dispensers are usually made in the form of a truncated pyramid, prism, or cone. The most favorable from the uniformity of pressure distribution and minimal arching is the shape of the hopper in the form of a truncated cone. The cross-sections of such a hopper in height are not constant, but depend on the coordinate \( z \). To solve the same equation (1), it is required to express changes in the cross-section area from the coordinate \( z \).

A truncated cone is made by rotating a rectangular trapezium around the side adjacent to the axis \( z \). The base of the truncated cone is formed by rotating the bases of the trapezium, and the side surface is formed by rotating the side. In this case the ordinate \( x \) will be:

\[
x = \frac{R-r}{h} \cdot z + r
\]

or in differential form

\[
dx = \frac{R-r}{h} \cdot dz
\]

(7)

where \( R, r \) – radii of the larger and smaller bases, respectively,

\( h \) – the height of the cone,

\( l \) – cone generator.

As far as is known, the area \( S \) of the side surface of a truncated cone and the volume \( W \) are defined by the formulas:

\[
S = \pi \cdot (R + r) \cdot l, \quad W = \frac{1}{3} \pi \cdot \left( R^2 + Rr + r^2 \right)
\]

(8)

Differentiating expressions (8) by coordinate, we get:

\[
dS = \pi (R + r) \cdot dl,
\]

\[
dW = \frac{1}{3} \pi \cdot \left( R^2 + Rr + r^2 \right) dz,
\]

(9)

Take that the radius of the hopper in section 1-1 is equal to \( R_1 \), and the area respectively \( \omega_1 = \pi R_1^2 \). In section 2-2, the radius will be (see figure) \( R_2 = R_1 + dx = R_1 + tg \alpha \cdot dz \).

Respectively the sectional area 2-2 will be:

\[
\omega_2 = \pi (R_1 + tg \alpha \cdot dz)^2 = \pi (R_1^2 + 2R_1tg \alpha \cdot dz + (tg \alpha \cdot dz)^2)
\]

(10)

The last summand in parentheses in equation (10) tends to zero, as an infinitely small quantity squared, so the sectional area in the first approximation will be:

\[
\omega_2 = \pi (R_1^2 + 2R_1tg \alpha \cdot dz)
\]

(11)

Take that the section 1-1 is under pressure \( P+dP \), and pressure \( P \) is in the section 2-2, then the expression (6), considering (8) and (9) and after simplification, will be the following:
\[
q \cdot \frac{(\sin \alpha + f \cdot \cos \alpha)}{\cos \alpha - f \cdot \sin \alpha} \cdot \pi \cdot (R + r) \cdot dl + \pi \cdot R^2 \cdot dP - 2\pi \cdot P \cdot R \cdot \tan \alpha \cdot dz - \frac{\rho \cdot g}{3} \cdot \pi \cdot (R^3 + R \cdot r + r^2) \cdot dz = 0 \quad (12)
\]

The figure shows that 
\[
dl = \frac{dz}{\cos \alpha}, \quad R_1 = \frac{R - r}{h}, \quad z + r = z \cdot \tan \alpha + r
\]

After that, considering the above expressions, expression (12) will be the following:
\[
q \cdot \frac{(\sin \alpha + f \cdot \cos \alpha)}{\cos \alpha - f \cdot \sin \alpha} \cdot (R + r) + (z \cdot \tan \alpha + r)^2 \cdot \frac{dP}{dz} - 2 \cdot P \cdot (z \cdot \tan \alpha + r) \cdot \tan \alpha - \frac{\rho \cdot g}{3} \cdot (R^3 + R \cdot r + r^2) = 0 \quad (13)
\]

Given that the outflow of feed material from the hopper is owing to gravity at atmospheric pressure, to simplify the solution, we take that the pressure of the brace (side pressure) \(q = q_c = \text{const} \).

Then we insymbolize the notation:
\[
q \cdot \frac{(\sin \alpha + f \cdot \cos \alpha)}{\cos \alpha - f \cdot \sin \alpha} \cdot (R + r) - \frac{\rho \cdot g}{3} \cdot (R^3 + R \cdot r + r^2) = A,
\]

\(\tan \alpha = a\), then, according to the accepted notation, equation (13) will be the following:
\[
\frac{dP}{dz} \cdot (az + r)^2 - 2a \cdot (az + r) \cdot P + A = 0
\]

Or
\[
2a \cdot (az + r) \cdot P - \frac{dP}{dz} \cdot (az + r)^2 = A
\]

We will solve the linear differential equation (14) using the Bernoulli method [10,11]. Ultimately, we get an expression for determining the pressure in the hopper when the bulky material flows out of it:
\[
P = \left( \frac{q (\sin \alpha + f \cdot \cos \alpha) (R + r)}{3 (\cos \alpha + f \cdot \sin \alpha)} - \frac{\rho \cdot g (R^2 + R \cdot r + r^2)}{9 \tan \alpha} \right) \cdot \left( \frac{(h \cdot \tan \alpha + R)^3 - (z \cdot \tan \alpha + r)^3}{(z \cdot \tan \alpha + r) \cdot (h \cdot \tan \alpha + r)} \right) \quad (15)
\]

We have derived an expression for determining the pressure of feed material in any section of a cone-shaped hopper.

In view of the fact that the operating hoppers of mixers are small in size, the knowledge of the pressure distribution over height is rather of academic interest in this event. For practical purposes, it is interesting to know the capacity of the hopper as a gravity dispenser for proportional supply of the feed mixer.

Expression (15) consists of the lateral pressure or expansion pressure occurring in the feed stream from the action of axial forces and the reaction of the hopper walls. This pressure is usually expressed as a fraction of the axial pressure generated in the hopper, i.e.
\[
q = \varepsilon \cdot P
\]

where \(\varepsilon\) – the coefficient of side thrust.

The side pressure coefficient of the bulky material may be defined by the formula [12]:

\[
\varepsilon = \frac{q}{\rho \cdot g \cdot \tan \alpha}
\]
where $\varphi$ – internal angle of treatment of the material.

The value of the side pressure coefficient lies according to various researchers in the range of $\varepsilon = 0.3...0.5$.

Considering the relation (16), the expression (15) will be the following:

$$P(1 - \left(\frac{\varepsilon(\sin \alpha + f \cos \alpha)(R + r)}{3(\cos \alpha - f \sin \alpha)\sin \alpha}\right)(\frac{(htg \alpha + r)^3 - (ztg \alpha + r)^3}{(ztg \alpha + r)(htg \alpha + r)^3}) =$$

$$- \left(\frac{\rho g(R^2 + Rr + r^2)}{9tg \alpha}\right)(\frac{(htg \alpha + r)^3 - (ztg \alpha + r)^3}{(ztg \alpha + r)(htg \alpha + r)^3})$$

(17)

Solving the expression (17) relative to the feed axial pressure in the hopper, we get:

$$P = \left(\frac{\rho g(R^2 + Rr + r^2)}{9tg \alpha}\right)(\frac{(htg \alpha + r)^3 - (ztg \alpha + r)^3}{(ztg \alpha + r)(htg \alpha + r)^3})$$

$$\left(\frac{\varepsilon(\sin \alpha + f \cos \alpha)(R + r)}{3(\cos \alpha - f \sin \alpha)\sin \alpha}\right)(\frac{(htg \alpha + r)^3 - (ztg \alpha + r)^3}{(ztg \alpha + r)(htg \alpha + r)^3}) - 1$$

(18)

Under the influence of pressure, the numerical values of which are defined from the expression (18), the feed stream enters the loading hole of the spiral-screwed mixer.

The flow capacity of the hopper as a gravity dispenser for uniform feeding of the feed material point over a certain period of time $\Delta t$ and is equal to the momentum of the force acting on this point for the same time

$$m(\vartheta - \vartheta_0) = F_p \cdot \Delta t$$

(19)

where $F_p$ – acting force, H.

The resulting pressure in the hopper is the effect of the acting forces such as gravity force, drag forces, and wall reactions. Thus, due to the resulting pressure, the feed stream moves out of the hopper. Considering the above, we take that $F_p = P \omega$; then the expression (19) will be the following

$$P \omega = \frac{m(\vartheta - \vartheta_0)}{\Delta t} \quad \text{or} \quad P \omega = Q(\vartheta - \vartheta_0)$$

(20)
where $Q$—flow capacity of the hopper (output), kg / s.

$\vartheta, \vartheta_0$—final and initial feed speed in the hopper, m/s.

Given the movement of the feed over a period of time from the horizontal section of the hopper at a height of $z= h$ to the section $z=0$, then the feed speed on the height $z=h$ will be zero, and at the outlet of the hopper ($z=0$) - $\vartheta$. At the same time, the sign should be reversed, since the direction of the $OZ$ coordinate axis and the direction of the feed flow are opposite.

Jointly solving expressions (18) and (20), we get:

$$\vartheta^2 = \frac{g \left( R^2 + Rr + r^2 \right)}{9 t \alpha} \cdot \left( \frac{\left( h t \alpha + r \right)^3 - r^3}{r \left( h t \alpha + r \right)^3} \right) \frac{\sin \left( \alpha + f \cos \alpha \right) (R + r)}{\frac{3}{3} \left( \cos \alpha - f \sin \alpha \right) \sin \alpha} \left( \frac{\left( h t \alpha + r \right)^3 - r^3}{r \left( h t \alpha + r \right)^3} \right) - 1$$

Obtained expression enables you to specify the speed value at the outlet of the hopper.

Let's adjust the expression (21). Given that $t \alpha = \frac{R - r}{h}$

$$\vartheta^2 = \frac{gh \left( R^2 + Rr + r^2 \right)}{9(R - r)} \cdot \left( \frac{R^3 - r^3}{rR^3} \right) \frac{\sin \left( \alpha + f \cos \alpha \right) (R + r)}{3 \left( \cos \alpha - f \sin \alpha \right) \sin \alpha} \left( \frac{R^3 - r^3}{rR^3} \right) - 1$$

Then we use formulas of abridged multiplication for the difference of cubes, we get

$$\vartheta = \frac{R^2 + Rr + r^2}{3} \sqrt{\frac{gh}{rR^3} \left( \frac{\sin \left( \alpha + f \cos \alpha \right) (R + r)}{\frac{3}{3} \left( \cos \alpha - f \sin \alpha \right) \sin \alpha} \left( \frac{R^3 - r^3}{rR^3} \right) - 1 \right)}$$

This expression enables you to specify the speed value at the outlet of the hopper. Based on this expression, the feed flow rate depends on the geometric parameters of the hopper and the physical and mechanical features of the feed material.

### 3 Results and discussion

We design a graphical interpretation of expression (23) and at the same time also construct a graph of the dependence used in practice (Toricelli's formula) for the hydraulic flow of the material $\vartheta = k \sqrt{2gh}$, where $k$ — an empirical coefficient called the outflow coefficient.

The outflow coefficient is reliant on the properties of the bulky material. Highest value $\lambda=0.55…0.65$ is used for light bulky materials. Grit materials with irregular pieces in the dry state and a pulverized powdery state have $\lambda = 0.2…0.25$ [13]. Figure 2 illustrates the dependencies without considering the material outflow coefficient.
Fig. 2. Graphical dependence of the speed change ($\mathcal{G}$) of the feed from its height in the hopper ($h$).

Seen from the graphical dependencies presented in figure 2, the pattern of velocity changes is almost identical. The difference in them may be offset by a coefficient of 1.18. This gives reason to believe that in calculations for the hydraulic flow of material, it is feasible to determine the feed rate at the outlet of the hopper also using the well-known Torricelli’s formula. If the height is $h=1m$, the flow rate will be stabilized. For this reason, in the operating hopper of the gravity dispenser, the height of the feed component layer of one meter should be supported for stable loading of the spiral-screwed mixer.

Flow capacity (outlet) of a gravity hopper dispenser may be specified by the following expression.

$$Q = \mathcal{G} \omega = \mathcal{G} \pi r^2 = \frac{(R^2 + Rr + r^2)}{3} \sqrt{\frac{gh}{rR^3} \left( \frac{\varepsilon \sin \alpha + f \cos \alpha}{3 \cos \alpha - f \sin \alpha} \sin \alpha \left( \frac{R^3 - r^3}{rR^3} - 1 \right) \right)} \pi r^2$$ (24)

4 Conclusions

The given formula (24) provides for determining the productivity of the gravity dispenser depending on the design characteristics of the hopper and the physical and mechanical qualities of the material. Analysis of the formula makes it possible to deduce that the fundamental factor affecting the productivity of the dispenser is the radius of the hopper outlet, since it is included in the formula in the second degree.

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