Neutrino Magnetic Moment Model and Solar Neutrino Problem

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Abstract

We examine the time variation problem of solar neutrinos in the spin-flavour precession mechanism taking into account the $\nu_e-\nu_\mu$ mixing. The models with the small and large mixing angle are studied. It shows that the models which realize $m_{\nu_e\nu_e} \sim m_{\nu_\mu\nu_\mu} \ll m_{\nu_e\nu_\mu}$ and $m_{\nu_\mu\nu_\mu} \sim m_{\nu_\mu\nu_\mu} \gg m_{\nu_e\nu_\mu}$ seem to have preferable time variation features. It is very interesting that the former type of models can give the large magnetic moment to neutrinos and also suppress the radiative mass correction naturally.

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1. Introduction

The solar neutrino problem is one of the challenging problems in the recent neutrino physics\[1\]. So called solar neutrino problem has two aspects. One of them is the depletion of the solar neutrino flux in comparison with that predicted from the standard solar model (SSM)\[2\]. This has been pointed out by the radio chemical $^{37}$Cl experiments of Davis and his colaborators\[3\]. Recently it was confirmed by the water Čerenkov detector in Kamioka\[4\]. Another aspect is about the existence of the time variation of the solar neutrino flux anti-correlated with the sunspot activity. The $^{37}$Cl experiment shows this anti-correlation\[3\]. On the other hand the data of KamiokandeII experiment suggest that it is constant in time\[4\]. At a first sight there seems to be a discrepancy if we consider these results seriously.

For the former problem there is a very simple solution called MSW\[3\]\[5\], which uses the resonance enhancement of the neutrino oscillation in the matter. But this mechanism cannot explain any kind of time variation. To explain the time variation problem, we need other mechanisms. If neutrino has considerably large magnetic moment, neutrinos precess into the sterile or other flavor neutrinos in the magnetic field in the sun as suggested in some papers\[7\]\[8\]\[9\]\[10\]. This mechanism seems to be able to explain both aspects of the problem except for the discrepancy about the time variation between two detectors. However, it needs rather large extensions of the standard model to make the neutrino magnetic moment large. There are many trials toward the model construction\[11\]\[12\]. Simple minded extensions bring also the large mass correction and are ruled out from the experimental neutrino mass bound.

In the previous work\[13\] we showed the possibility that in the spin-flavor precession mechanism there are some parameter regions which can explain the difference in the time variation of solar neutrino between two experiments. In that explanation, essential points are that two experiments use the different processes for the detection of neutrinos and also different energy neutrinos have different precession features in the magnetic field\[13\]\[14\].

The $^{37}$Cl detector is sensitive only to $\nu_e$ neutrinos above 0.86MeV. The average energy of $^8$B neutrinos is $\sim$10MeV. The total average energy of $^7$Be, $^{13}$N, $^{15}$O and pep neutrinos is $\sim$1MeV. Therefore all of them are included in this region. On the other hand the threshold of KamiokandeII detector is 7.5MeV and sensitive only to $^8$B neutrinos but it can react to $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$ also. These differences can produce the apparent discrepancy
between two experimental results.

In this paper we extend this analysis into the case where $\nu_e-\nu_\mu$ mixing exists and study the relation between the typical neutrino magnetic moment model and its time variation feature of solar neutrino flux. We suggest that if we take such a view point, we may have a possibility to obtain the information on the particle physics models which produce the large neutrino magnetic moment on the basis of the solar neutrino experiments.

2. The mixing angle

As is well-known, to give neutrinos the large magnetic moment of order $10^{-11}\mu_B$ which is an experimental upper bound, we must extend the standard model somehow. However, if we can make the neutrino magnetic moment large by a certain radiative correction, the same graph eliminated the photon line which contributes to the magnetic moment brings in general the large mass correction to neutrinos. To forbid such a mass correction, it is usual to introduce a Lagrangian symmetry $\mathcal{G}$ which may break down to its subgroup $\mathcal{H}$ \cite{11} \cite{12}. In the most models $\mathcal{H}$ contains $L_e - L_\mu$ symmetry or its discrete subgroup.

If $\mathcal{H}$ is exactly conserved, only off-diagonal mass appears and there is no mass difference between the neutrino flavors and then the solar neutrino problem cannot be explained in the resonant spin-flavor precession \cite{4} \cite{10}. As the result we must break $\mathcal{H}$ softly to explain it. In this situation taking account of the radiative mass correction, there appears inevitably the extremely large $\nu_e-\nu_\mu$ mixing.

The mass matrices in the mass eigenstates $(\nu_1, \nu_2)$ and weak eigenstates $(\nu_e, \nu_\mu)$ are related as

$$
\begin{pmatrix}
  m_1 & 0 \\
  0 & m_2
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
  m_{\nu_e\nu_e} & m_{\nu_e\nu_\mu} \\
  m_{\nu_e\nu_\mu} & m_{\nu_\mu\nu_\mu}
\end{pmatrix}
\begin{pmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{pmatrix}.
$$

(1)

From this we get

$$
\Delta m^2 \equiv m_2^2 - m_1^2 = (m_{\nu_\mu\nu_\mu} + m_{\nu_e\nu_e}) \sqrt{(m_{\nu_e\nu_\mu} - m_{\nu_e\nu_e})^2 + 4m_{\nu_e\nu_\mu}^2},
$$

(2)

and

$$
\tan 2\theta = \frac{2m_{\nu_e\nu_\mu}}{m_{\nu_\mu\nu_\mu} - m_{\nu_e\nu_e}}.
$$

(3)

In the model which has the restrictive symmetry, $m_{\nu_e\nu_e}$, $m_{\nu_\mu\nu_\mu}$ and $m_{\nu_e\nu_\mu}$ can be considerably constrained. As an example, we take the model proposed by Babu et al. \cite{12}.
Their model has custodial $SU(2)$ as $H$ which is softly broken. Because of this symmetry, they get $m_{\nu_e\nu_\mu} > m_{\nu_e\nu_\tau} \sim 10^{-7}$ eV and $m_{\nu_e\nu_\mu} \sim 1$ eV. This realizes $\Delta m^2 \lesssim O(10^{-7})$ eV$^2$ which is needed to explain the solar neutrino deficit due to the resonant spin-flavor precession. But the above value gives $\tan 2\theta \sim 10^7$. This feature seems to be general in the models which have the softly broken $L_e - L_\mu$ symmetry or its discrete subgroup as the subgroup of $H$. It is a non-trivial problem what time variation behavior the models with non-zero $\sin^2 \theta$, in particular, $\sin^2 \theta \sim 1$ show even if $\Delta m^2 \lesssim O(10^{-7})$ eV$^2$. If we relate this to the above mentioned time variation problem and analyze them, we may find some informations about the features of the various models of large transition magnetic moment. This is the subject in the following arguments.

3. Time evolution of the solar neutrino

We consider the solar neutrino problem in the framework of the spin-flavor precession mechanism with non-zero $\nu_e - \nu_\mu$ mixing angle $\theta$. As discussed in the previous part, we should note that this is the general situation in the model with large transition magnetic moment. Here we restrict our study to two flavors case.

The time evolution equation of the neutrinos in the magnetic field $B$ is expressed in the weak interaction basis as

$$ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} a_e & \frac{\Delta m^2}{4E} \sin 2\theta & 0 & B\mu \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{2E} \cos 2\theta + a_\mu & -B\mu & 0 \\ 0 & -B\mu & -a_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ B\mu & 0 & \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{2E} \cos 2\theta - a_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix}. $$

(4)

$a_e$ and $a_\mu$ are the coherent scattering effect of neutrinos with matter and expressed as

$$ a_e = \sqrt{2} G_F (n_e - \frac{1}{2} n_n), \quad a_\mu = \frac{1}{\sqrt{2}} G_F (-n_n). $$

(5)

$G_F$ is Fermi coupling constant. $n_e$ and $n_n$ are the electron and neutron number densities in the sun, respectively. $n_e$ is known to be approximately represented \(^2\) as

$$ n_e = 2.4 \times 10^{26} \exp(-r/0.09 R_\odot) \text{ cm}^{-3}. $$

(6)

Since we consider the neutrino relativistic, we take $r = t$. In our discussion we assume that $B$ is constant throughout the sun and $B\mu \sim 10^{-16+y}$ eV. This corresponds approximately

\(^2\)In the following study we use $n_n \simeq \frac{1}{5} n_e$ in the sun.
to $\mu \sim 10^{-11}\mu_B$ and $B \sim 10^{4+\gamma}G$. $y$ represents the change of the magnetic field and we take $0 < y < 3$.

We are interesting in the time variation feature of the solar neutrino flux, particularly, and then for a while we focus our attention on $(\nu_e, \bar{\nu}_\mu)$ sector of eq.(4),

$$\frac{d}{dt} \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} a_e & B\mu \\ B\mu & \frac{\Delta m^2}{2E} \cos 2\theta - a_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix}. \quad (7)$$

There are two possibilities for the large $\nu_e \rightarrow \bar{\nu}_\mu$ transition. One is the resonant spin-flavor precession in the sun and the other case has no resonance in the sun but has the large precession because of $B\mu > \frac{\Delta m^2}{2E} \cos 2\theta - a_\mu - a_e$.

At first we consider the former possibility. As suggested in refs.[9][10], this precession $\nu_e \rightarrow \bar{\nu}_\mu$ has the resonance structure. The resonance occurs at $\frac{\Delta m^2}{2E} \cos 2\theta = a_e + a_\mu$. If we require this resonance point to exist in the convection zone ($0.7R_\odot \leq r \leq R_\odot$), $\frac{\Delta m^2}{2E} \cos 2\theta$ should satisfy

$$4.0 \times 10^{-16}eV \leq \frac{\Delta m^2}{2E} \cos 2\theta \leq 1.1 \times 10^{-14}eV. \quad (8)$$

This condition is necessary to be satisfied to explain the sufficient time variation of solar neutrinos anti-correlated with sunspot numbers. To see the qualitative feature of $\nu_e \rightarrow \bar{\nu}_\mu$ transition we solve eq.(7) around the resonance point following Landau and Zener[15].

The transition amplitude is expressed as

$$P_x = \exp[-2\pi\gamma(1 - \tan^2\beta_{N=0})] \quad (9)$$

where

$$\gamma = \frac{(B\mu)^2}{\Delta m^2 \cos 2\theta \left| \frac{1}{N} \frac{dN}{dt} \right|_r}, \quad N = a_e + a_\mu. \quad (10)$$

$\beta$ is the angle which is used to diagonalize the matrix in the right-hand side of eq.(7) and

$$\tan 2\beta = \frac{2B\mu}{\frac{\Delta m^2}{2E} \cos 2\theta - N}. \quad (11)$$

Here we introduced the correction $1 - \tan^2\beta_{N=0}$ which is due to the specific electron density function we used here[9]. After substitution of eqs.(10) and (11) into eq.(9), we get

$$P_x = \exp[-\pi \frac{(2B\mu)^2}{\left(\frac{\Delta m^2}{2E} \cos 2\theta + \sqrt{\left(\frac{\Delta m^2}{2E} \cos 2\theta\right)^2 + (2B\mu)^2} \left| \frac{1}{N} \frac{dN}{dt} \right|_r}\right)]. \quad (12)$$
For the electron number density (6) we have $\left| \frac{1}{N} \frac{dN}{dt} \right|_{\nu_e} \sim 3.2 \times 10^{-15}$ eV. Following the procedure of Parke[16], we can express the probability $P_s$ such that $\nu_e$ remains $\nu_e$ on the surface of the sun as follows

$$P_s = \frac{1}{2} + \left( \frac{1}{2} - P_x \right) \cos 2\beta_1 \cos 2\beta_2. \quad (13)$$

$\beta_1$ and $\beta_2$ are the values of $\beta$ at the core and the surface of the sun. The value of $\beta$ can generally change from $\beta_1 \sim \frac{\pi}{2}$ to $\beta_2(< \frac{\pi}{2})$ when the neutrino travels from the core to the surface of the sun through the resonance point. Here it should be noted that $P_s$ has the neutrino energy dependence as shown in eq.(12).

Next we consider the case in which no resonance is in the sun. To see the qualitative feature we consider that $a_e$ and $a_\mu$ are constants for a while. If we put $A = \frac{\Delta m^2}{2E} \cos 2\theta - N$ and solve eq.(7), we get

$$P_s = 1 - \frac{(2B\mu)^2}{A^2 + (2B\mu)^2} \sin^2\left( \frac{(A^2 + (2B\mu)^2)^{\frac{1}{2}}}{2} R \right). \quad (14)$$

For an effective $\nu_e \rightarrow \nu_\mu$ transition $B\mu > |A|$ should be satisfied and then the neutrino energy dependence in $P_s$ becomes very small. Here $R$ corresponds to the distance where such a condition is satisfied in the sun.

In both cases we need to take account of the vacuum oscillation on the way from the sun to the earth due to the non-zero mixing angle $\theta$. The $\nu_e$ preserving probability on the earth is expressed as

$$P_e = \tilde{P}_s + \frac{1}{2} [(1 - 2\tilde{P}_s) \sin 2\theta - \sqrt{\bar{P}_s(1 - \bar{P}_s)} \sin 4\theta], \quad (15)$$

if the averaging procedure of oscillation parts is justified $(\frac{\Delta m^2}{4E} D > \pi)$. $D$ is the distance from the surface of the sun to the earth ($\simeq 1.5 \times 10^8$ km). $\tilde{P}_s$ stands for the value in which the MSW transition effect is also considered.

4. Time variation features

We proceed to the study of time variation features in the various models. Before it, it is useful to note that there exists the interesting situation where there is no time variation of $P_s$ associated with the change of magnetic field even if the neutrino has the large magnetic moment. In eq.(13) this is related to the adiabaticity($P_x = 0$) of the spin-flavor precession[13][14]. If $\cos 2\beta_2 \sim 0$ is realized in this adiabatic situation, eq.(13)
shows $P_s \cong \frac{1}{2}$. On the other hand in eq.(14) it occurs as the averaging effect when $B\mu R > \pi (B\mu > |A|)$ and $P_s \cong \frac{1}{2}$. These mean that the neutrino large magnetic moment does not necessarily induce the time variation of the solar neutrino flux according to the change of the magnetic field of the sun. Thus the determination of the parameter range where the time variation becomes large is the crucial point to explain the solar neutrino problem in the spin-flavor precession framework.

In order to study the relation between the neutrino magnetic moment models and the solar neutrino time variation, it is very instructive to consider the two extreme cases, that is, $\sin 2\theta \sim 0$ and $\sim 1$.

(i) $\sin 2\theta \sim 0$ case.

In this case the main transition of $\nu_e$ occurs almost through $\nu_e \rightarrow \bar{\nu}_\mu$. Equations(11) and (12) become

$$\tan 2\beta \sim \frac{2B\mu}{\frac{\Delta m^2}{2E} - N},$$

$$P_x \sim \exp\left[-\pi \frac{(2B\mu)^2}{(\frac{\Delta m^2}{2E} + \sqrt{(\frac{\Delta m^2}{2E})^2 + (2B\mu)^2})\frac{1}{N}\frac{dN}{dt}}\right].$$

Because of $\sin 2\theta \sim 0$, there is no vacuum oscillation and $P_e$ in eq.(15) is equivalent to the value on the surface of the sun $P_e \cong P_s$. The resonance condition (8) is now $4.0 \times 10^{-16}\text{eV} < \frac{\Delta m^2}{2E} < 1.1 \times 10^{-14}\text{eV}$. As a typical $\Delta m^2$ value, let’s take $\Delta m^2 = 2 \times 10^{-8}\text{eV}$. Then $\frac{\Delta m^2}{2E} = 10^{-14}\text{eV (E = 1MeV)}$ and $\frac{\Delta m^2}{2E} = 10^{-15}\text{eV (E = 10MeV)}$. As easily checked by eqs.(16) and (17), these values show that for 1MeV neutrinos $P_x$ is nonadiabatic and can have rather large $B\mu$ dependence in a certain $B\mu$ range but for 10MeV neutrinos $P_x$ is nearly adiabatic and depends on $B\mu$ weakly in comparison with 1MeV neutrinos (Fig.1). Moreover the very small $\cos 2\beta_2$ is realized for 10MeV neutrinos but it is very large for 1MeV neutrinos (Fig.2). As a result, in this case we can find parameter regions where the low energy neutrinos generally show rather larger time variation than the high energy neutrinos (Fig.3). Numerical studies have been done in $\sin 2\theta = 0$ case and very interesting results have been obtained for the time variation problem as predicted from above qualitative arguments. In ref. $\text{[13]}$ $\sin 2\theta = 0.1$ case has also been studied. There it is shown that there is no essential difference from $\sin 2\theta = 0$. This case corresponds to the model with $m_{\nu_e\nu_e}, m_{\nu_\mu\nu_\mu} \gg m_{\nu_e\nu_\mu}$ ($\Delta m^2 \sim m_{\nu_e\nu_e}^2 - m_{\nu_\mu\nu_\mu}^2$). However, within our knowledge this type of model with both of the large transition magnetic moment and the natural suppression of the radiative mass correction has not been proposed by now.
(ii) \( \sin 2\theta \cong 1 \) case.

In this case, from eq.(4) the neutrino flux can be expected to be sufficiently reduced \( (P_s < \frac{1}{2}) \) due to both the large mixing and the large magnetic moment effect as in pseudo Dirac case\[17\]. And also it seems to be possible to exhibit the time variation unless \( B_\mu \) becomes too large and the adiabaticity or the averaging effect appears. Moreover, in the almost maximal mixing case there is an interesting feature. Kamiokande detector can interact with all of \( \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu \) in contrast with \( ^{37}\text{Cl} \) detector which reacts only to \( \nu_e \). These cross sections for \( 10\text{MeV} \) neutrinos are\[18\]

\[
\sigma(\bar{\nu}_e e) = 0.42\sigma(\nu_e e), \quad \sigma(\nu_\mu e) = 0.17\sigma(\nu_e e), \quad \sigma(\bar{\nu}_\mu e) = 0.14\sigma(\bar{\nu}_e e).
\]

As pointed out in ref.\[17\], in the case of almost maximal mixing, independently of the initial values on the surface of the sun, the vacuum oscillation from the sun to the earth causes the probabilities \( a, a, \frac{1-2a}{2}, \frac{1-2a}{2} \) for \( \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu \), respectively. Thus the detection ratio of two detectors are

\[
P_{\text{Cl}} = a, \quad P_{K-II} = 0.61a + 0.28.
\]

This means that \( 0.2 < P_{\text{Cl}} < 0.56 \) when \( 0.4 < P_{K-II} < 0.6 \). We can expect this amount of depletion and variation through the cooperation of the almost maximal mixing and \( B_\mu \) effect as the pseudo Dirac case in ref.\[14\].

Now we examine the relation between the models with \( \sin 2\theta \cong 1 \) and time variation features in more detail. At first we consider the case in which the resonance exists in the sun. In this case the time variation can be expected through the resonance effect. But the neutrino energy dependence of \( P_s \) in eq.(12) disappears in contrast with the (i) case. The difference of the time variation between two experiment must be explained only by the difference of the processes used in the detection. We assume \( \cos 2\theta \sim 10^{-\alpha} \) \( (\alpha > 1) \). From the resonance condition (8), \( 8.0 \times 10^{-9+\alpha}\text{eV}^2 < \Delta m^2 < 2.2 \times 10^{-7+\alpha}\text{eV}^2 \) for \( 10\text{MeV} \) neutrinos. Following eqs.(2) and (3), we get \( \frac{2m_{\nu_\mu \nu_\mu}}{m_{\nu_\mu \nu_\mu} - m_{\nu_e \nu_e}} \sim 10^{\alpha} \) and \( m_{\nu_\mu \nu_\mu}^2 - m_{\nu_e \nu_e}^2 \sim 10^{-8} \) in the case of \( \Delta m^2 \sim 10^{-8+\alpha} \). We must impose electron neutrino mass bound on \( m_{\nu_e \nu_\mu} \lesssim 10\text{eV} \). This results in \( m_{\nu_\mu \nu_\mu} - m_{\nu_e \nu_e} \sim 10^{1-\alpha} \) and \( m_{\nu_\mu \nu_\mu} + m_{\nu_e \nu_e} \gtrsim 10^{-7+\alpha} \). To satisfy these simultaneously, \( \alpha > 4 \) is required. If \( \alpha \) become large, the extreme fine tuning between \( m_{\nu_\mu \nu_\mu} \) and \( m_{\nu_e \nu_e} \) is necessary. From this fact, \( m_{\nu_\mu \nu_\mu} \sim m_{\nu_e \nu_e} \sim 10^{-4}\text{eV} \) and \( m_{\nu_e \nu_\mu} \sim 1\text{eV} \) is considered.

\[\text{For instance this can be easily seen also from eq.(15).}\]

\[\text{As pointed out in refs.}\ [18] [19], \text{the fraction of } \bar{\nu}_e \text{ may be constrained from the data on the background of the KamiokandeII experiment. Although it may be crucial factor to select the neutrino large magnetic moment model, in our following analysis we do not take account of this constraint.}\]
as a natural possibility. However, as predicted from the pseudo Dirac case\[17\], when \(\Delta m^2 \gtrsim 10^{-7}\) eV the transition \(\nu_e \to \nu_\mu\) due to the maximal mixing effect becomes adiabatic and as a result we will not get the sufficient suppression of the flux. This case seems not to be so attractive.

Next we consider the case in which no resonance exists in the sun. From eq.\((8)\) this is realized when \(\frac{\Delta m^2}{2E} \cos 2\theta \lesssim 10^{-16}\). No resonance is in the convection zone in the sun but \(B_\mu > N\) is satisfied in the convection zone. If \(B_\mu R < \pi\) is satisfied, the change of magnetic field will be able to induce the sufficient \(\nu_e\) flux change. In fact as we found numerically in ref.\[13\], there is almost no dependence on \(\frac{\Delta m^2}{2E} \cos 2\theta\) in the form of \(P_s\) derived from eq.\((7)\) when \(\frac{\Delta m^2}{2E} \cos 2\theta < 10^{-16}\) eV. Thus we can apply that result in this case because of \(|A| \sim N\). Following it, the sufficient time variation (from 1 to 0.3) of \(P_s\) can be expected associated with the order two change of the magnetic field \((10^{-16}\text{ eV} \lesssim B_\mu \lesssim 10^{-14}\text{ eV})\). This seems to be very preferable feature. We can use this change of magnetic field fully to explain the time variation of solar neutrino because the flux depletion is the combined effect of the mixing and \(B_\mu\) in this case. As an example, in Fig.4 we give the numerical results in \(\sin 2\theta = 1\) case. We can easily see that our above predictions are realized in it. In the resonant solution case where \(\sin 2\theta = 0\), the necessary change of the magnetic field is only factor 2 or 3 and much smaller than the expected value.\[13\] This is because \(\nu_e \to \bar{\nu}_\mu\) transition is the only origin of the depletion of the neutrino flux. The neutrino energy independence in \(P_s\) does not seem to be matter for the explanation of the difference between two experiments because of eq.\((18)\). We should note that the model of ref.\[12\] and other models with similar kind of symmetries correspond to this interesting category of models. This model is realized naturally by \(m_{\nu_\mu\nu_\mu} \gg m_{\nu_e\nu_e} \sim m_{\nu_\mu\nu_\mu}\) without fine tunings.

Here we comment a bit on the maximal mixing case \((\sin 2\theta = 1)\). It can be realized in two ways, that is, \(m_{\nu_\mu\nu_\mu} = m_{\nu_e\nu_e} \neq 0(\Delta m^2 \neq 0)\) and \(m_{\nu_\mu\nu_\mu} = m_{\nu_e\nu_e} = 0(\Delta m^2 = 0)\). The former case corresponds to the softly broken \(\mathcal{H}\) type model and has the similar time variation property as \(\sin 2\theta \cong 1\). On the other hand the latter case which corresponds to the exact \(\mathcal{H}\) symmetry has no \(\nu_e \to \nu_\mu\) transition. It is similar to \(\sin 2\theta \sim 0\) case except for having no resonance.\[5\] No resonance means that \(P_s\) has no neutrino energy dependence.

\[5\] It should be noted that there might be a possibility to explain the solar neutrino problem even if \(\Delta m^2 = 0\).
Thus the ratio of the time variation between $^{37}\text{Cl}$ and KamiokandeII experiments will become smaller than that of $\sin 2\theta \sim 0$ case. The softly broken $\mathcal{H}$ is favorable for the explanation of the time variation problem also in the maximal mixing case.

5. Conclusions

From the study of extreme cases we found that the different $\theta$ models show the considerably different time variation of the solar neutrino flux due to the different reasons. And also our study suggests that only models with rather restricted $\sin 2\theta$ value are natural as the large neutrino magnetic moment models and also can produce the sufficient time variation of solar neutrino flux for a certain parameter range.

The typical models which give the large transition magnetic moment have the symmetry $\mathcal{H}$ which contains $L_e - L_\mu$ symmetry or its discrete subgroup. These symmetries are essential in these models to suppress the large mass correction and the rare process like $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and so on. On the other hand these symmetries lead to $\sin 2\theta \cong 1$ in general. Fortunately, from the above analysis we conclude that the softly broken $\mathcal{H}$ type model can induce the certain flux depletion and also can cause the different time variation results between two experiments associated with the change of magnetic field. The models with $\sin 2\theta \sim 0$ have the same property but in such models the natural suppression of the radiative mass correction is not known.

In summary, we investigated the time variation problem of the solar neutrino flux in the framework of the spin-flavor precession in the magnetic field. In the models which give neutrinos the large transition magnetic moment there exists the non-zero $\nu_e-\nu_\mu$ mixing angle $\theta$ generally. Taking account of this mixing we analyzed the solar neutrino experimental results of $^{37}\text{Cl}$ and KamiokandeII. As a result we found that models with $\sin 2\theta \cong 1$ for the large neutrino transition magnetic moment proposed by now seem to have a very interesting feature to induce the time variation of solar neutrino flux. This suggests that the models which realize $m_{\nu_e\nu_e} \sim m_{\nu_e\nu_\mu} \ll m_{\nu_\mu\nu_\mu}$ ($\sin 2\theta \cong 1$) are promising to explain the time variation of the solar neutrino flux. The $\sin 2\theta \sim 0$ model which is realized by $m_{\nu_e\nu_e} \sim m_{\nu_\mu\nu_\mu}$ ($\sin 2\theta \sim 0$) is also promising. However, it is very challenging problem how to construct a small $\sin 2\theta$ model which realize simultaneously the large transition magnetic moment and the natural suppression of the large mass correction. More detailed numerical investigation will be needed for more quantitative arguments.
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