The New Minimal Standard Model

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We construct the New Minimal Standard Model that incorporates the new discoveries of physics beyond the Minimal Standard Model (MSM): Dark Energy, non-baryonic Dark Matter, neutrino masses, as well as baryon asymmetry and cosmic inflation, adopting the principle of minimal particle content and the most general renormalizable Lagrangian. We base the model purely on empirical facts rather than aesthetics. We need only six new degrees of freedom beyond the MSM. It is free from excessive flavor-changing effects, CP violation, too-rapid proton decay, problems with electroweak precision data, and unwanted cosmological relics. Any model of physics beyond the MSM should be measured against the phenomenological success of this model.

The last several years have brought us revolutionary new insights into fundamental physics: the discovery of Dark Energy, neutrino masses and bi-large mixings, a solid case for non-baryonic Dark Matter, and mounting evidence for cosmic inflation. It is now clear that the age-tested Minimal Standard Model (MSM) is incomplete and needs to be expanded.

There exist many possible directions to go beyond the MSM: supersymmetry, extra dimensions, extra gauge symmetries (e.g., grand unification), etc. They are motivated to solve aesthetic and theoretical problems of the MSM, but not necessarily to address empirical problems. It is embarrassing that all currently proposed frameworks have some phenomenological problems, e.g., excessive flavor-changing effects, CP violation, too-rapid proton decay, disagreement with electroweak precision data, and unwanted cosmological relics.

In this letter, we advocate a different and conservative approach to physics beyond the MSM. We include the minimal number of new degrees of freedom to accommodate convincing (e.g., > 5σ) evidence for physics beyond the MSM. We do not pay attention to aesthetic problems, such as fine-tuning, the hierarchy problem, etc. We stick to the principle of minimality seriously to write down the Lagrangian that explains everything we know. We call such a model the New Minimal Standard Model (NMSM). In fact, the MSM itself had been constructed in this spirit, and it is a useful exercise to follow through with the same logic at the advent of the major discoveries we have witnessed. Of course, we require it to be a consistent Lorentz-invariant renormalizable four-dimensional quantum field theory, the way the MSM was constructed.

We should not forget that the MSM is a tremendous success and string theory because of the cosmological overabundance of cosmology for various reasons. There is now compelling evidence for a non-baryonic matter component [1].

• Dark Matter has been suggested as a necessary ingredient of cosmology for various reasons. There is now compelling evidence for a non-baryonic matter component [1].
• Dark Energy is needed based on the concordance of data from cosmic microwave anisotropy [1], galaxy clusters (see, e.g., [2]), and high-redshift Type-Ia supernovae [3, 4].
• Atmospheric [5] and solar neutrino oscillations [6] have been established, with additional support from reactor anti-neutrinos [7], demonstrating neutrino masses and mixings.
• The cosmic baryon asymmetry \( \eta = n_B/s = 9.2 \pm 0.4 \times 10^{-10} \).
which cannot be explained in the MSM, has been known for many decades.

• The nearly scale-invariant, adiabatic, and Gaussian density fluctuations (see, e.g., [3]) point to cosmic inflation. This has not been proven, but we find the evidence compelling.

There are many other hints for physics beyond the MSM at a few sigma levels which we do not try to incorporate.

We now apply our principle of minimal particle content to address each of the issues. First, we discuss Dark Matter. It is clear that the MSM does not have a candidate degree of freedom. The minimal way to add a new degree of freedom in a quantum field theory is a real Klein–Gordon (KG) field. To make it stable, we must assign it a symmetry; the only such possibility for a real KG field is a $Z_2$ parity. Therefore, we introduce a singlet field $S$ completely neutral under the gauge group and odd under a $Z_2$ parity. Then its most general renormalizable Lagrangian is

$$\mathcal{L}_S = \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} m_S^2 S^2 - \frac{k}{2} |H|^2 S^2 - \frac{h}{4!} S^4. \quad (2)$$

It is encouraging that this model indeed had been proposed to explain the cosmological Dark Matter in the past [3] [17] [18]. Remarkably, this model can explain the correct abundance, the lack of its detection so far, and the lack of observation at high-energy accelerators. We will show later that the model is still viable. This is clearly the minimal model of Dark Matter.

The next issue is Dark Energy. Because we do not concern ourselves with aesthetic issues such as naturalness and fine-tuning in constructing the NMSM, we simply postulate a cosmological constant of the observed size, approximately

$$\mathcal{L}_\Lambda = (2.3 \times 10^{-3} \text{ eV})^4. \quad (3)$$

This is a relevant operator in the Lagrangian, consistent with all known symmetries. Hence, it cannot be left out in a most general Lagrangian. Its renormalized value at the Hubble scale needs to be the one given above.

The third issue is the neutrino masses and bi-large mixings. We have strong evidence for two mass-squared splittings, one from atmospheric neutrinos $\Delta m^2 \approx 2.5 \times 10^{-3} \text{ eV}^2$, and the other from solar neutrinos (and reactor anti-neutrinos) $\Delta m^2 \approx 7 \times 10^{-5} \text{ eV}^2$. Because the Planck-scale operator $(LH)(\bar{LH})/M_{Pl}$ gives only $m_{\nu} \lesssim 10^{-5} \text{ eV}$, too small to explain the data, we need new degrees of freedom to generate neutrino masses. There is no evidence that all three neutrinos are massive, and one of them may be massless. We hence need only two right-handed neutrinos $N_\alpha (\alpha = 1, 2)$, or four new degrees of freedom, to write down the mass terms. We still have to make a choice whether the mass terms are of Dirac or Majorana type. Based on the minimality alone, either of them is perfectly valid. In the case of Dirac neutrinos, we need to impose a global lepton number symmetry, while for Majorana neutrinos, we write down all possible renormalizable terms. The next minimal way of generating Majorana neutrino masses requires a triplet scalar exchange [12] with six new degrees of freedom. Therefore, adding two right-handed neutrinos is the minimal choice.

Next, we have to explain the baryon asymmetry of the universe. We might have insisted that the baryon asymmetry was the initial condition of the universe. However, this is not possible because we will accept the inflationary paradigm. We will come back to this point later. Therefore, the asymmetry needs to be explained. In fact, having accepted two right-handed neutrinos, we can let them produce the baryon asymmetry via leptogenesis [13] [14] [15]. This is possible only for Majorana neutrinos with seesaw mechanism without additional degrees of freedom, unlike leptogenesis with Dirac neutrinos [16]. Therefore, we do not have a choice: the neutrinos are Majorana, and the decays of right-handed neutrinos in the early universe, coupled with the electroweak anomaly, is responsible for creating the baryon asymmetry. The NMSM Lagrangian, hence, must also include

$$\mathcal{L}_N = \bar{N}_\alpha i \gamma^\mu N_\alpha \left( \frac{M_\alpha^2}{2} N_\alpha N_\alpha + h_{\alpha i}^\alpha N_\alpha L_i \tilde{H} + c.c. \right). \quad (4)$$

Because the left-handed neutrino Majorana mass matrix has rank two, there is one massless state. The other two neutrino masses can be determined from the solar and atmospheric neutrino data, and there is only one Majorana phase. In the basis where the charged-lepton and right-handed-neutrino mass matrices are real and diagonal, there are eleven real parameters in Eq. (4), after rephasing of three lepton doublets. Since there are only seven real parameters for light neutrinos, two masses, three mixing angles, one Dirac and one Majorana phase, we have enough parameters to accommodate the current data. In order to produce the observed baryon asymmetry via leptogenesis, the lighter right-handed neutrino should be heavier than $10^{10} \text{ GeV}$ to have enough CP asymmetry [15] [17].

Finally, nearly scale-invariant, adiabatic, and Gaussian density fluctuations need to be generated in order to explain the observed structure, velocity field, and cosmic microwave background anisotropy. We adopt inflation for this purpose. We do not see any candidate scalar field to drive inflation in the MSM nor among the new particles introduced above. Therefore, we have to introduce at least another degree of freedom. The minimal new particle content is again a real KG field, and its most general renormalizable Lagrangian is

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\mu}{3!} \phi^3 - \frac{\kappa}{4!} \phi^4. \quad (5)$$

Here, the possible linear term has been absorbed by a shift. This potential can drive inflation, e.g., if the field starts with a trans-Planckian amplitude; this is nothing but the chaotic inflation model [18]. Current data prefer the quadratic term to drive inflation [19] [20] with $m \approx 1.8 \times 10^{13} \text{ GeV}$ [21], while $\mu \lesssim 10^{16} \text{ GeV}$ and $\kappa \lesssim 10^{-14}$ [22].

The only possible renormalizable couplings of the inflaton to other fields in the NMSM allowed by symmetries are

$$V_{RH} = \mu_1 |H|^2 + \mu_2 \phi^2 + \kappa_H \phi^2 |H|^2 + \kappa_S \phi^2 S^2 + (g_{\alpha}^{\alpha} \phi N_\alpha N_\beta + c.c.). \quad (6)$$
Reheating after inflation can take place by couplings $\mu_1, \mu_2$, or $y^H_{\alpha\beta}$. For thermal leptogenesis to take place, the reheating temperature must be higher than the mass of the lighter right-handed neutrino, say $10^{10}$ GeV, requiring either $\mu_{1,2} \gtrsim 10^9$ GeV or $y^H_{\alpha\beta} \gtrsim 10^{-4}$; they do not spoil the flatness of the inflaton potential if $\kappa_{H,S} \lesssim 10^{-6}$. Moreover, $y^H_{\alpha\beta}$ lets the inflaton decay directly to the right-handed neutrinos, whose subsequent decay can produce the asymmetry \cite{22,23}, allowing for even smaller couplings. This is a non-trivial cross check that the inflation and the leptogenesis are consistent within our model.

Let us come back to the question if the baryogenesis is necessary. Even if we accept the inflationary paradigm, one may still hope that a large initial baryon asymmetry before the inflation may be retained to account for the observed value. We can exclude this possibility on purely empirical grounds. Even if we set aside the desire to explain the horizon and flatness puzzles, which are after all aesthetic issues which we disregard in this letter, we have just accepted inflation as the source of nearly scale-invariant density fluctuations to account for the cosmic microwave background anisotropies, large scale structures, and eventually galaxy formation. Therefore we need the $e$-folding of the inflation to be larger than the logarithm of the ratio of the cosmological scale to the galactic scale, conservatively $N \gtrsim \ln(10\text{Gpc}/10\text{pc}) = 14$. On the other hand, the large initial baryon asymmetry before the inflation can only be in the form of a Fermi-degenerate gas. Its energy density $\rho_B \simeq \mu_F^4$, where $\mu_F$ is the Fermi momentum, behaves as radiation. In order for the inflation to start, the energy density of the Fermi-degenerate gas must be less than that of the inflaton $\rho_\phi$. Assuming that they were approximately the same, the energy density of the baryon gas is suppressed by $\mu_F^4/\rho_\phi \simeq e^{-4N}$ at the end of the inflation. Reheating will further dilute the baryon asymmetry and hence we conservatively assume that the reheating was instantaneous. Then the maximum baryon asymmetry one can obtain is $\eta \simeq \mu_F^4/\rho_\phi^{3/4} \simeq e^{-3N} \lesssim 10^{-18}$, insufficient to explain the observed asymmetry of $\eta \simeq 10^{-10}$. Therefore, baryon asymmetry cannot be explained by the initial condition based on purely empirical arguments once inflation is accepted as the source of the density fluctuations.

It is remarkable that the MSM Lagrangian Eq. \ref{LM}, supplemented by the most general renormalizable Lagrangian in Eqs. \ref{Ls}--\ref{Lphi}, for two right-handed neutrinos $N_\alpha$, one $Z_2$ odd real scalar $S$, and another real scalar $\varphi$, \begin{equation}
\mathcal{L}_{\text{NMSM}} = \mathcal{L}_{\text{MSM}} + \mathcal{L}_S + \mathcal{L}_\lambda + \mathcal{L}_N + \mathcal{L}_\varphi - V_{\text{RH}}, \tag{7}
\end{equation}
explains everything we currently know about our universe.

This model is supposed to describe all known physics including classical gravity. Note that quantum gravity effects have not empirically been observed and hence are beyond the scope of the model, but we expect them to be there. Thus we assume there is no new physics beyond the NMSM up to the Planck scale. All higher dimension operators from the cut-off scale are suppressed by the Planck scale. Hence it is free from excessive flavor-changing effects, CP violation, too-rapid proton decay, and problems with electroweak precision data.

Now we come to another non-trivial consistency check of the model, that is the addition of the scalar $S$ does not conflict with empirical requirements. For the MSM to be valid up to the Planck scale, various authors have studied constraints from the instability and triviality of the Higgs potential (see, e.g., \cite{24}). We do the same for the NMSM. At one-loop level, the gauge coupling constants and the top Yukawa coupling $y$ run the same way as in the MSM. The couplings in the scalar sector run as \begin{align}
(4\pi)^2 \frac{d\lambda}{dt} &= 12\lambda^2 + 12\lambda y^2 - 12y^4 - 3\lambda(g^2 + 3g^2) \\
&+ \frac{3}{4} [2g^4 + (g^2 + y^2)^2] + k^2, \tag{8}
(4\pi)^2 \frac{dk}{dt} &= k \left[ 4k + 6\lambda + h + 6y^2 - \frac{3}{2} (g^2 + g^2) \right], \tag{9}
(4\pi)^2 \frac{dh}{dt} &= 3h^2 + 12k^2, \tag{10}
\end{align}
with $t = \log \mu$. We require that none of the couplings be driven negative below the Planck scale (stability bound) and stay below 10 (triviality bound). The region of $(m_S, k(m_Z))$ is shown in Fig. \ref{fig1} for three values of $h(m_Z) = 0, 1, 1.2$. The region disappears when $h(m_Z) \gtrsim 1.3$. The Higgs boson is predicted to be light, at most 180 GeV, while heavier than 130 GeV. This range is in complete accordance with the precision electroweak fits $m_h \lesssim 200$ GeV \cite{25}, while beyond the LEP-II reach \cite{25} and is not probed experimentally yet.

The Dark Matter annihilation cross section is proportional to $k^2$ and depends on $m_S$ and $m_h$ \cite{10}. We have improved the abundance calculation using HDECAY \cite{30} and included the $s$-channel Higgs exchange diagram in $SS \rightarrow hh$, absent in \cite{10} even though it is not qualitatively important. Preferred values of $(k(m_Z), m_h)$ are shown for $\Omega h^2 = (\Omega_m - \Omega_b)h^2 = 0.11$ as curves in Fig. \ref{fig1} for various $m_S$. Note that $m_S = 75$ GeV allows for annihilation through Higgs pole and has a special behavior. To be consistent with the triviality and stability bounds, we find $m_S \simeq 5.5$ GeV–1.8 TeV.

Now we have demonstrated that all new elements we have added to the MSM do not cause any tensions among themselves nor with the empirical constraints. The new scalar we added at the TeV-scale is consistent with the electroweak data even after we imposed the triviality and stability bounds, while it can give the required cosmological density without conflicting the direct search limits. It does not induce any flavor-changing effects or new CP violation that typically haunt models with new degrees of freedom at the TeV scale. The inflation model we adopted can successfully reheate to a high-enough temperature to account for leptogenesis for parameters consistent with neutrino oscillation data, while the required coupling for the reheating does not spoil the required flatness of the inflaton potential. We also pointed out that inflation, even with a conservative requirement on the $e$-folding based on purely empirical grounds, actually requires baryogenesis.
Are there new observable consequences of the NMSM? The Higgs boson may decay invisibly $h \to SS$ [11]. It will be subject to search at the LHC via $W$-boson fusion, or more promisingly at a Linear Collider. If the singlet is heavier than $m_h/2$, the search at collider experiments becomes exceedingly difficult. One possibility is the $W$-boson fusion processes $gg \to q\bar{q}SS + g$ or $ggSS + \gamma$, where forward jets are tagged, large missing $p_T$ is seen, together with additional isolated photon or jet. It may not cover the entire range up to 1.8 TeV. The scattering of $S$ on nuclei is dominated by the Higgs boson exchange, as worked out in [13,11]. The prediction for $m_h = 150$ GeV is shown in Fig. 2; it is clear that the model is consistent with the current limit from CDMS-II [25]. It cannot explain, however, the controversial data from DAMA [26]. Because the Higgs boson is light thanks to the triviality bound, the scattering cross section is promising for the underground Dark Matter searches for $m_S \lesssim m_h/2$.

The spectrum index of the $\varphi^2$ chaotic inflation model is predicted to be 0.96. This may be confirmed in improved cosmic-microwave background anisotropy data, with more years of WMAP and Planck. The tensor-to-scalar ratio is $0.16 \pm 0.03$, again within the reach of near future observations. For other inflationary scenarios, predictions vary. The equation of state of Dark Energy is predicted to be exactly $w = -1$.

Neutrinos are Majorana fermions and hence we expect neutrinoless double beta decay at some level. Because one of the neutrino masses exactly vanishes (ignoring tiny Planck suppressed effects), the signal in the near-future experiments is possible only for the inverted hierarchy [51].

Here we list a few future observations that could rule the NMSM incomplete. Obviously, discovering any particles at the electroweak scale other than $h$ and $S$ at a collider will require an extension of the model. A Higgs mass inconsistent with the bounds in Fig. 1 will also be a smoking gun for additional physics. Confirmation of the DAMA signal would require a different Dark Matter candidate. Signals of some rare decays, such as $\mu \to e\gamma$, would require extra flavor-changing effects. Observation of new sources of CP violation beyond the CKM and MNS phases is another avenue, e.g., an electron electric dipole moment or a discrepancy in $\sin 2\beta$ between $B \to \phi K_S$ and $\psi K_S$ modes. As for the neutrino sector, a confirmation of the LSND results by the Mini-BooNE experiment would require new degrees of freedom beyond the NMSM. Positive signal for neutrino mass at KATRIN would require masses for all three neutrinos. A future observation by a satellite experiment, such as Planck, of $\Omega_{DM}h^2 = 0.11$ that represents a tuning with an accuracy of $\sim 1\%$.

It needs to be mentioned that the NMSM does require an extreme degree of fine-tuning. The cosmological constant represents a tuning with an accuracy of $10^{-120}$. The hierarchy between the electroweak and the Planck scales should also be fine-tuned at the level of $10^{-32}$. Fermion mass hierarchies and mixings are not explained. The QCD vacuum angle is simply chosen to be $\theta < 10^{-10}$. The $Z_2$ symmetry on the singlet is imposed by hand. The parameters in the inflation potential are chosen to be small. Nonetheless, the model is empirically successful in describing everything we know about fundamental physics, and needs to be taken seriously. Any new physics beyond the NMSM that may address the aesthetic issues mentioned here should not spoil the success of the NMSM.

Neutrinoless double beta decay could rule out the one-field inflationary scenario of the NMSM. A positive signal for neutrino mass at KATRIN would require masses for all three neutrinos. A future observation by a satellite experiment, such as Planck, of $\Omega_{DM}h^2 = 0.11$ that represents a tuning with an accuracy of $\sim 1\%$.

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terest, since both collider and direct Dark Matter searches are challenging. It would require a detailed Monte Carlo study of the annihilation products in the Sun. A lighter $S$ can be seen in the invisible decay of the Higgs boson at a Linear Collider, while its mass measurement would require an off-shell Higgs process which needs to be investigated. Other possibilities for the one-field inflationary scenario may warrant further study.

In summary, we have presented the New Minimal Standard Model of particle physics and cosmology that incorporates Dark Matter, Dark Energy, neutrino masses and mixings, baryon asymmetry, and nearly scale-invariant Gaussian density fluctuations, based on the principle of minimal particle content and the most general renormalizable Lagrangian. Remarkably, it requires only six new degrees of freedom. Any model of physics beyond the Minimal Standard Model should be judged against the empirical success of this model.

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[32] It may well be possible to achieve successful inflation also with small field amplitudes (small-field models), but many existing models require more than one degree of freedom; we do not pursue this interesting possibility further in this letter.