COINCIDENCE OF THE UNIVERSE DESCRIPTION
STEMMING FROM D-BRANES THEORY AND ENU MODEL

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Abstract. The contribution provides a comparison of consequences stemming from D-brane theories and Expansive Nondecelerative Universe model, and calls attention to coincidence of the results arising from the mentioned approaches to a description of the Universe. It follows from the comparison that the effects of quantum gravitation should appear at the energy near to 2 TeV.

D-brane theories of the Universe [1] are founded on the principle declaring that there are more than 3 space dimensions, and that the introduction of any further extra dimension limits its range. For two extra dimensions the related range is close to 1 mm, when dealing with seven extra dimensions, the range drops to $10^{-14}$ m. It follows from the mathematical treatment of the issue by the D-brane theoretical approaches that some changes in the Newtonian gravitational law should occur on a scale below 1 mm. It is supposed that in the region $10^{-19}$ m which corresponds to the energy of about 1 TeV, the effects of quantum gravitation should appear.

Due to the Vaidya metric application [2], the model of Expansive Nondecelerative Universe (ENU) [2-5] enables to localize and quantify the energy of the gravitational field. For weak field conditions it adopts the form

$$\varepsilon_g = -\frac{R_c^4}{8\pi G} = -\frac{3m_c^2}{4\pi a r^2}$$

(1)

where $\varepsilon_g$ is the density of gravitational field energy generated by a body with a mass $m$ at a distance $r$, $R$ is the scalar curvature and $a$ is the gauge factor being at present

$$a \cong 1.3 \times 10^{26} \text{ m}$$

(2)

It is worth mentioning that the magnitude of $\varepsilon_g$ given by (1) is closed to the $\theta_0^i$ component of the Einstein energy-momentum pseudotensor describing the
density of gravitational energy also for strong field conditions [6]. In the ENU the critical density of gravitational energy is expressed as

\[ \varepsilon_{\text{crit}} = \frac{3c^4}{8\pi G a^2} \]  

(3)

For the instances when

\[ |\varepsilon_g| = \varepsilon_{\text{crit}} \]  

(4)

it follows that the effective gravitational range \( R_{ef} \) of a body having the gravitational radius \( R_g \) is related to the gauge factor as

\[ R_{ef} = (R_g a)^{1/2} \]  

(5)

It follows of the above that at present there should exist a particle with the Compton wavelength \( \lambda_x \) being equal to its effective gravitational range. It represents the lightest particle able to exert gravitational influence on its surrounding. The mass of the particle is expressed by

\[ m_x = \left( \frac{\hbar^2}{G a} \right)^{1/3} \]  

(6)

At the time being

\[ m_x \approx 10^{-28} \text{kg} \]  

(7)

and

\[ \lambda_x \approx 3.235 \times 10^{-15} \text{m} \]  

(8)

Based on the assumption that the ENU model is compatible with the superstring theory M [7], in which a number of space dimensions is 10, i.e. a number of extra dimensions is 7, we postulate that just relation (8) expresses their magnitude. In the case when

\[ r \leq \lambda_x \]  

(9)

in 10-dimensional space it must hold

\[ E_g \approx r^{-8} \]  

(10)

At these conditions the gravitational energy attracting two particles with a mass \( m \) may be expressed as
\[ E_g = \frac{G.m^2.\lambda^2_o}{r^8} \quad (11) \]

and \( r \) must be equal to the Compton wavelength of given particles,

\[ r = \frac{\hbar}{m.c} \quad (12) \]

In the limiting case, the energy \( E_g \) will be identical to the rest energy \( E_o \) of the particles, i.e.

\[ E_o = \frac{\hbar.c}{r} \quad (13) \]

It follows from the identity of (11) and (13) that

\[ r^9 = l^2_P.\lambda^7_x \quad (14) \]

where the Planck length is

\[ l_P = \left( \frac{G.\hbar}{c^3} \right)^{1/2} \approx 1.614 \times 10^{-35} \text{m} \quad (15) \]

It follows from (8), (14) and (15) that

\[ r \approx 9.66 \times 10^{-20} \text{m} \quad (16) \]

which corresponds to the energy value

\[ E \approx 2.0 \text{TeV} \quad (17) \]

We suppose that just at this energy, the effects of quantum gravitation will manifest themselves at present.

A significant argument of our considerations lies in time evolution of a number of extradimensions. At the initial stage of the Universe expansion, all dimensions, including extradimensions, had to be of Planck size which is in full agreement with relations (8), (14), (8), (14), (13), and (17) since they describe the phenomena dependent on the cosmological time.
Conclusions

1. The present contribution documents compatibility of predictions stemming from the ENU model and D-branes theory, particularly that related to the energy (2 TeV) at which the effects of quantum gravitation appears.

2. Inevitability of time evolution of the size of extra dimensions and its rationalization is manifested.

References

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