Lie symmetry analysis, conservation laws and analytical solutions for chiral nonlinear Schrödinger equation in (2 + 1)-dimensions

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Abstract. In this work, we consider the chiral nonlinear Schrödinger equation in (2 + 1)-dimensions, which describes the envelope of amplitude in many physical media. We employ the Lie symmetry analysis method to study the vector field and the optimal system of the equation. The similarity reductions are analyzed by considering the optimal system. Furthermore, we find the power series solution of the equation with convergence analysis. Based on a new conservation law, we construct the conservation laws of the equation by using the resulting symmetries.

Keywords: the chiral nonlinear Schrödinger equation in (2+1)-dimensions, Lie symmetry analysis, symmetry reductions, the power series solutions, conservation laws.

1 Introduction

The nonlinear Schrödinger equation, which plays a very important role in nonlinear evolution equations (NLEEs), has been fully applied in many phenomena, such as fluid dynamics, nonlinear fiber, molecular biology, quantum mechanics, deep water modeling, etc. [9, 10, 30]. Up to now, finding for the exact solution of NLEEs still plays a very important role in the study dynamics of nonlinear phenomena. In the last few decades, the exact solutions of NLEEs have been extensively studied. The main methods used are Darboux transformation, the inverse scattering method, Hirota bilinear method, Lie

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symmetry group method [1, 4, 11, 19, 23, 26]. Among them, the Lie symmetry group method can simultaneously obtain the symmetry, exact solutions and conservation laws of NLEEs through some effective calculations [3, 7, 12, 16, 29, 38–45, 49].

In the past few decades, the conservation laws has played an increasingly important role in the research of NLEEs. At the same time, various methods for solving conservation law of the NLEE are also produced, such as Noether’s theorem, characteristic method, variational approach, conservation theorem [5, 8, 15, 24, 25, 48, 50], etc. The famous Noether’s theorem establishes the connection between symmetries of NLEEs and conservation laws. But the disadvantage of the Noether’s theorem is that it is not suitable for solving NLEE without Lagrangian. In order to solve this NLEE without Lagrangian, Ibragimov entered a new method for solving the conservation law in 2007. This new method relies on the notion of Lie symmetry generators, the adjoint equation and formal Lagrangians of NLEEs. Therefore, this new conservation law will also play an important role in solving the conservation laws of NLEEs.

In this work, we mainly study the chiral nonlinear Schrödinger (NLS) equation in (2 + 1)-dimensions

\[ i q_t + a(q_{xx} + q_{yy}) + i [b_1(q q_x^* - q^* q_x) + b_2(q q_y^* - q^* q_y)] q = 0, \tag{1} \]

where \( q = q(x, y, t) \), \( a \) means the coefficient of dispersion term, and \( b_1, b_2 \) are the coefficients of nonlinear coupling terms. In [6], the bright and dark soliton solution of the chiral NLS equation in (2 + 1)-dimensions (1) is obtained using the constant coefficient method. In [14], the singular periodic solution of the chiral NLS equation in (2 + 1)-dimensions (1) is obtained by using the trial solution method. As we all know, the Lie symmetry and conservation laws of equation (1) have not been studied. Therefore, in this work, we will mainly study the Lie symmetry and conservation laws of equation (1).

The rest of the paper is structured as follows. In Section 2, we first transform equation (1) into a form of equations, and then vector field and optimal system are constructed by using the Lie symmetry analysis method. In Section 3, the symmetry reductions of equation (1) is obtained by using the optimal system. In Section 4, we obtain the power series solution of the system by using the power series method. In Section 5, the conservation law of the equation is obtained by the new conservation law. In Section 6, we give some summaries and discussions.

2 Lie symmetries analysis

In this section, Lie symmetry analysis will be performed on the chiral NLS equation in (2 + 1)-dimensions (1). Firstly, we consider the complex-valued function \( q(x, y, t) \) in the following form:

\[ q(x, y, t) = u(x, y, t) + i v(x, y, t), \tag{2} \]

where \( u(x, y, t) \) and \( v(x, y, t) \) are real-valued functions, and \( q^* \) represents the conjugate of \( q \). Substituting equation (2) into equation (1) and equating the real and imaginary parts,
we can obtain
\[ u_t + a(v_{xx} + v_{yy}) + 2b_1(uvv_x - v^2u_x) + 2b_2(uvv_y - v^2u_y) = 0, \]
\[ -v_t + a(u_{xx} + u_{yy}) + 2b_1(u^2v_x - uvu_x) + 2b_2(u^2v_y - uvu_y) = 0. \]

(3)

To construct the point symmetry of equation (1), we first introduce a Lie group with a one-parameter Lie transformation group

\[ x \to x + \varepsilon \xi^1(x, y, t, u, v) + O(\varepsilon^2), \]
\[ y \to y + \varepsilon \xi^2(x, y, t, u, v) + O(\varepsilon^2), \]
\[ t \to t + \varepsilon \xi^3(x, y, t, u, v) + O(\varepsilon^2), \]
\[ u \to u + \varepsilon \eta^1(x, y, t, u, v) + O(\varepsilon^2), \]
\[ v \to v + \varepsilon \eta^2(x, y, t, u, v) + O(\varepsilon^2), \]

where \( \varepsilon \ll 1 \) means a group parameter, and \( \xi^1, \xi^2, \xi^3, \eta^1 \) and \( \eta^2 \) are the infinitesimal generators. The vector field corresponding to the above group of transformation as

\[ V = \xi^1(x, y, t, u, v) \frac{\partial}{\partial x} + \xi^2(x, y, t, u, v) \frac{\partial}{\partial y} + \xi^3(x, y, t, u, v) \frac{\partial}{\partial t} + \eta^1(x, y, t, u, v) \frac{\partial}{\partial u} + \eta^2(x, y, t, u, v) \frac{\partial}{\partial v}, \]

(4)

where \( \xi^1(x, y, t, u, v), \xi^2(x, y, t, u, v), \xi^3(x, y, t, u, v), \eta^1(x, y, t, u, v) \) and \( \eta^2(x, y, t, u, v) \) are functions of coefficient to be determined. For system (3), pr\(^2\) will be the second prolongation, then its invariance condition is

\[ \text{pr}^2 V(\Delta_1)|_{\Delta_1=0} = 0, \quad \text{pr}^2 V(\Delta_2)|_{\Delta_2=0} = 0, \]

(5)

where

\[ \Delta_1 = u_t + a(v_{xx} + v_{yy}) + 2b_1(uvv_x - v^2u_x) + 2b_2(uvv_y - v^2u_y), \]
\[ \Delta_2 = -v_t + a(u_{xx} + u_{yy}) + 2b_1(u^2v_x - uvu_x) + 2b_2(u^2v_y - uvu_y). \]

On the basis of Lie’s theory, the second prolongation of equation (4) can be written as

\[ \text{pr}^2 V = \eta^1 \frac{\partial}{\partial u} + \eta^2 \frac{\partial}{\partial v} + \eta^1_t \frac{\partial}{\partial u_t} + \eta^1_x \frac{\partial}{\partial u_x} + \eta^1_y \frac{\partial}{\partial u_y} + \eta^2_x \frac{\partial}{\partial v_x} + \eta^2_y \frac{\partial}{\partial v_y} + \eta^2_{xx} \frac{\partial}{\partial v_{xx}} + \eta^2_{yy} \frac{\partial}{\partial v_{yy}}, \]
\[ \text{pr}^2 V = \eta^1 \frac{\partial}{\partial u} + \eta^2 \frac{\partial}{\partial v} + \eta^2_t \frac{\partial}{\partial v_t} + \eta^2_x \frac{\partial}{\partial v_x} + \eta^2_y \frac{\partial}{\partial v_y} + \eta^2_{xx} \frac{\partial}{\partial v_{xx}} + \eta^2_{yy} \frac{\partial}{\partial v_{yy}}, \]

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Theorem 1. The Lie algebra of infinitesimal symmetry of equation (1) are spanned by the following six linear independent operators:

\[
\begin{align*}
V_1 &= \frac{\partial}{\partial t}, \\
V_2 &= \frac{\partial}{\partial x}, \\
V_3 &= \frac{\partial}{\partial y}, \\
V_4 &= v \frac{\partial}{\partial u} - u \frac{\partial}{\partial v}, \\
V_5 &= \frac{1}{2} x \frac{\partial}{\partial x} + \frac{1}{2} y \frac{\partial}{\partial y} + \frac{t}{4} \frac{\partial}{\partial t} - \frac{1}{4} u \frac{\partial}{\partial u} - \frac{1}{4} v \frac{\partial}{\partial v}, \\
V_6 &= t \frac{\partial}{\partial x} - \frac{b_1 t}{b_2} \frac{\partial}{\partial y} + \frac{v(b_1 y - b_2 x)}{2ab_2} \frac{\partial}{\partial u} - \frac{u(b_1 y - b_2 x)}{2ab_2} \frac{\partial}{\partial v}.
\end{align*}
\]
Based on the commutator operator \( [V_k, V_j] = V_k V_j - V_j V_k \), we can get the commutator table of system (3) (see Table 1).

Based on the commutator relations in Table 1, we want to get the adjoint representations of the vector fields by using the following Lie series:

\[
\text{Ad}(\exp(\epsilon V_k)) V_j = V_j - \epsilon [V_k, V_j] + \frac{1}{2} \epsilon [V_k, [V_k, V_j]] - \cdots.
\]

Then we have

\[
\begin{array}{ccccccc}
\text{Lie} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\
V_1 & 0 & 0 & 0 & 0 & 0 & V_1 - \frac{b_1}{b_2} V_3 \\
V_2 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} V_2 - \frac{1}{2a} V_4 \\
V_3 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} V_3 + \frac{b_3}{2ab_2} V_4 \\
V_4 & -V_1 & 0 & 0 & 0 & 0 & 0 \\
V_5 & \frac{b_1}{b_2} V_3 - V_2 & 0 & 0 & 0 & 0 & \frac{1}{2} V_6 \\
V_6 & \frac{b_1}{b_2} V_3 - V_2 & 0 & 0 & 0 & 0 & \frac{1}{2} V_6 \\
\end{array}
\]

Table 1. Lie bracket of system (3).
Therefore, we can get the optimal system of (3) according to the adjoint representation of the vector field (8), then have get $V_1$, $V_2$, $V_3$, $V_1 + hV_2$, $V_1 + hV_3$, $V_2 + hV_3$, $V_5 - yV_3/2 - tV_1$, $V_5 - xV_2/2 - tV_1$, where $h$ is arbitrary constant.

3 Symmetry reductions

In the previous section, we mainly obtained the vector field and the optimal system of equation (1). Therefore, in this section, we will mainly do the symmetry reduction of these optimal systems.

3.1 The generator $V_1$

For the generator $V_1$, we can get
\[ u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau), \tag{9} \]
where $\xi = x$ and $\tau = y$. Inserting (9) into (3), we can get system (3) of ordinary differential equations (ODEs) in which $F$ and $G$ satisfy
\[
aG'' - 2b_1G^2F' + 2b_2FGG' = 0, \\
aF'' - 2b_1FGF' + 2b_2F^2G' = 0. \tag{10} \]

3.2 The generator $V_2$

For the generator $V_2$, we can obtain
\[ u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau), \tag{11} \]
where $\xi = y$ and $\tau = t$. Inserting (11) into (3), we can get system (3) of ODEs in which $F$ and $G$ satisfy
\[
-2b_2G^2F' = 0, \quad -G' + aF'' - 2b_2FGF' = 0, \tag{12} \]
where $F' = dF/d\xi$, $F'' = d^2F/d\xi^2$ and $G' = dG/d\tau$. Solving system (12), we can get $F(\xi) = c_1, G(\tau) = c_2$ or $F(\xi) = c_1\xi + c_2, G(\tau) = 0$. Therefore, we get the solution of equation (1) as
\[ q(x, y, t) = c_1 + ic_2 \quad \text{or} \quad q(x, y, t) = c_1y + c_2, \]
where $c_1$ and $c_2$ are arbitrary functions.

3.3 The generator $V_3$

For the generator $V_3$, we can obtain
\[ u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau), \tag{13} \]
where $\xi = x$ and $\tau = t$. Inserting (13) into (3), we can get system (3) of ODEs in which $F$ and $G$ satisfy
\[
-2b_1G^2F' = 0, \quad -G' + aF'' - 2b_1FGF' = 0, \tag{14} \]
where \( F' = \frac{dF}{d\xi}, \quad F'' = \frac{d^2F}{d\xi^2} \) and \( G' = \frac{dG}{d\tau} \). Solving system (14), we can get \( F(\xi) = c_1, \quad G(\tau) = c_2 \) or \( F(\xi) = c_1 \xi + c_2, \quad G(\tau) = 0 \). Therefore, we get the solution of equation (1) as

\[
q(x, y, t) = c_1 + ic_2 \quad \text{or} \quad q(x, y, t) = c_1 y + c_2,
\]

where \( c_1 \) and \( c_2 \) are arbitrary functions.

### 3.4 The generator \( V_1 + hV_2 \)

For the generator \( V_1 + hV_2 \), we can get

\[
u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau),
\]

where \( \xi = y \) and \( \tau = t - x/h \). Inserting (15) into (3), we can get system (3) of ODEs in which \( F \) and \( G \) satisfy

\[
\frac{a}{h^2} G'' - \frac{2b_1}{h} FGG' - 2b_2 G^2 F' = 0,
\]

\[-G' + aF'' - \frac{2b_1}{h} F^2 G' - 2b_2 FGF' = 0.
\]

### 3.5 The generator \( V_1 + hV_3 \)

For the generator \( V_1 + hV_3 \), we can get

\[
u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau),
\]

where \( \xi = x \) and \( \tau = t - y/h \). Inserting (16) into (3), we can get system (3) of ODEs in which \( F \) and \( G \) satisfy

\[
\frac{a}{h^2} G'' - 2b_1 G^2 F' - \frac{2b_2}{h} FGG' = 0,
\]

\[-G' + ah^2 F'' + aF'' + 2b_1 FGF' - \frac{2b_2}{h} F^2 G' = 0.
\]

### 3.6 The generator \( V_2 + hV_3 \)

For the generator \( V_2 + hV_3 \), we can get

\[
u(x, y, t) = F(\xi), \quad v(x, y, t) = G(\tau),
\]

where \( \xi = -hx + y \) and \( \tau = t \). Inserting (17) into (3), we can get system (3) of ODEs in which \( F \) and \( G \) satisfy

\[
2b_1 h G^2 F' - 2b_2 G^2 F' = 0,
\]

\[-G' + ah^2 F'' + aF'' + 2b_1 h FGF' - 2b_2 FGF' = 0.
\]
where $F' = dF/\partial \xi$, $F'' = d^2F/\partial \xi^2$ and $G' = dG/\partial \tau$. Solving system (18), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 0$ or $F(\xi) = c_1$, $G(\tau) = c_2$. Therefore, we get the solution of equation (1) as

$$q(x, y, t) = c_1(hx + y) + c_2 \quad \text{or} \quad q(x, y, t) = c_1 + ic_2,$$

where $c_1$ and $c_2$ are arbitrary functions.

### 3.7 The generator $V_5 - yV_3/2 - tV_1$

For the generator $V_5 - (1/2)yV_3 - tV_1$, we can obtain

$$u(x, y, t) = \frac{F(\xi)}{x^{1/2}}, \quad v(x, y, t) = \frac{G(\tau)}{x^{1/2}}, \quad (19)$$

where $\xi = y$ and $\tau = t$. Inserting (19) into (3), we can get system (3) of ODEs in which $F$ and $G$ satisfy

$$\frac{3}{4}ax^{-1} - 2b_2GF' = 0, \quad (20)$$

$$-G' + \frac{3}{4}ax^{-2}F + aF'' - 2b_2x^{-1}FF' = 0,$$

where $F' = dF/\partial \xi$, $F'' = d^2F/\partial \xi^2$ and $G' = dG/\partial \tau$. Solving system (20), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 3a/(8b_2c_1x)$. Therefore, we get the solution of equation (1) as

$$q(x, y, t) = \frac{c_1y + c_2}{x^{1/2}} + \frac{3ia}{8b_2c_1x^{3/2}}, \quad (21)$$

where $c_1$ and $c_2$ are arbitrary functions.

### 3.8 The generator $V_5 - xV_2/2 - tV_1$

For the generator $V_5 - (1/2)xV_2 - tV_1$, we can obtain

$$u(x, y, t) = \frac{F(\xi)}{y^{1/2}}, \quad v(x, y, t) = \frac{G(\tau)}{y^{1/2}}, \quad (22)$$

where $\xi = x$ and $\tau = t$. Inserting (22) into (3), we can get system (3) of ODEs in which $F$ and $G$ satisfy

$$\frac{3}{4}ay^{-1} - 2b_1GF' = 0, \quad (20)$$

$$-G' + \frac{3}{4}ay^{-2}F + aF'' - 2b_1y^{-1}FGF' = 0,$$

where $F' = dF/\partial \xi$, $F'' = d^2F/\partial \xi^2$ and $G' = dG/\partial \tau$. Solving system (20), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 3a/(8b_1c_1y)$. Therefore, we get the solution of equation (1) as

$$q(x, y, t) = \frac{c_1y + c_2}{y^{1/2}} + \frac{3ia}{8b_1c_1y^{3/2}},$$

where $c_1$ and $c_2$ are arbitrary functions.
4 The power series solutions

Based on the symbolic calculation methods [13, 17, 18, 22, 27, 28, 31–37, 46, 47], we study the analytical solution of ODE by through the power series method. When we get the analytical solution of ODE, we can easily obtain the power series solutions of the original partial differential equation.

According to (10), we can get

\[ aG'' - 2b_1 G^2 F' + 2b_2 FGG' = 0, \]
\[ aF'' - 2b_1 FGF' + 2b_2 F^2 G' = 0. \]  

(23)

Below we will use the following hypothetical form to calculate the solution of (23)

\[ F(\xi) = \sum_{n=0}^{\infty} P_n \xi^n, \quad G(\tau) = \sum_{n=0}^{\infty} Q_n \tau^n, \]  

(24)

where the coefficients \( P_n \) and \( Q_n \) (\( n = 0, 1, \ldots \)) are all constants.

Inserting (24) into (23) yields

\[
\begin{align*}
a & \sum_{n=0}^{\infty} (n+1)(n+2)Q_{n+2} \tau^n - 2b_1 \left( \sum_{n=0}^{\infty} Q_n \tau^n \right)^2 \sum_{n=0}^{\infty} (n+1)P_{n+1} \xi^n \\
& + 2b_2 \sum_{n=0}^{\infty} Q_n \tau^n \sum_{n=0}^{\infty} P_n \xi^n \sum_{n=0}^{\infty} (n+1)Q_{n+1} \tau^n = 0, \\
\end{align*}
\]

\[
\begin{align*}
a & \sum_{n=0}^{\infty} (n+1)(n+2)P_{n+2} \xi^n - 2b_1 \sum_{n=0}^{\infty} Q_n \tau^n \sum_{n=0}^{\infty} P_n \xi^n \sum_{n=0}^{\infty} (n+1)P_{n+1} \xi^n \\
& + 2b_2 \left( \sum_{n=0}^{\infty} P_n \xi^n \right)^2 \sum_{n=0}^{\infty} (n+1)P_{n+1} \tau^n = 0.
\end{align*}
\]

When \( n = 0 \), we compare coefficients of \( \xi \) to get

\[
Q_2 = \frac{1}{a} \left( b_1 Q_0^2 P_1 - b_2 Q_0 P_0 Q_1 \right), \quad P_2 = \frac{1}{a} \left( b_1 Q_0 P_0^2 P_1 - b_2 P_0^2 Q_1 \right).
\]  

(25)

Generally, when \( n \geq 1 \), we can obtain

\[
Q_{n+2} = \frac{M_Q}{a(n+1)(n+2)}, \quad P_{n+2} = \frac{M_P}{a(n+1)(n+2)}
\]  

(26)

with

\[
M_Q = 2b_1 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1)Q_j Q_{k-j} P_{n-k+1} \\
- 2b_2 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1)Q_j P_{k-j} Q_{n-k+1},
\]

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From the above derivation we can see that all the coefficients $a, b$ of (24) can be represented by

$$M_P = 2b_1 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1)Q_j P_{k-j} P_{n-k+1}$$

$$- 2b_2 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1)P_j P_{k-j} Q_{n-k+1}.$$

Then we can get the following results:

when $n = 1$,

$$Q_3 = \frac{1}{3a} \left( 2b_1 Q_0^2 P_2 + b_1 Q_0 Q_1 P_1 - 2b_2 Q_0 P_0 P_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0 \right),$$

$$P_3 = \frac{1}{3a} \left( 2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 + b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1 \right),$$

when $n = 2$,

$$Q_4 = \frac{1}{6a} \left( 3b_1 Q_0^2 P_3 + 4b_1 Q_0 Q_1 P_2 + 2b_1 Q_0 P_2 P_1 + b_1 Q_1^2 P_1 - 3b_2 Q_0 Q_3 P_0 - 2b_2 Q_0 P_1 Q_2 - 3b_2 Q_1 P_0 Q_2 - b_2 Q_0 Q_2 P_1 - b_2 Q_1^2 P_1 \right),$$

$$P_4 = \frac{1}{6a} \left( 3b_1 Q_0 P_0 P_3 + 3b_1 Q_0 P_1 P_2 + 2b_1 Q_1 P_1 P_2 + b_1 Q_1 P_2^2 + b_1 Q_0 P_2 P_1 - 3b_2 P_0^2 Q_3 - 4b_2 P_0 P_1 Q_2 - 2b_2 P_0 P_2 Q_1 - b_2 P_1^2 Q_1 \right).$$

From the above derivation we can see that all the coefficients $(P_1, Q_1)$ in the power series solution of (24) can be represented by $a, b_1, b_2, Q_0, Q_1, P_0, P_1$, where $a, b_1, b_2, Q_0, Q_1, P_0, P_1$ are arbitrary constants. Besides, on the basis of [2, 21], we can also prove the convergence of the coefficients determined by (25)–(26). Thus, we obtain that a power series solution (24) is the power series solution of (23). Then a power series solution of (24) can be rewritten into

$$F(\xi) = P_0 + P_1 \xi + \frac{1}{a} (b_1 Q_0 P_0 P_1 - b_2 P_0^2 Q_1) \xi^2$$

$$+ \frac{M_P}{a(n+1)(n+2)} \xi^{n+2} + \frac{1}{3a} \left( 2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 \right)$$

$$+ b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1 \xi^3 + \cdots,$$

$$G(\tau) = Q_0 + Q_1 \tau + \frac{1}{a} (b_1 Q_0^2 P_1 - b_2 Q_0 P_0 Q_1) \tau^2$$

$$+ \frac{M_Q}{a(n+1)(n+2)} \tau^{n+2} + \frac{1}{3a} \left( 2b_1 Q_0^2 P_2 + 2b_1 Q_0 Q_1 P_1 \right)$$

$$- 2b_2 Q_0 P_0 Q_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0) \tau^3 + \cdots.$$
Then the power series solution of equation (1) is

\[
q(x, y, t) = \left[ P_0 + P_1 x + \frac{1}{a} \left( b_1 Q_0 P_0 P_1 - b_2 P_0^2 Q_1 \right) x^2 
+ \frac{M_P}{a(n + 1)(n + 2)} x^{n+2} + \frac{1}{3a} \left( 2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 
+ b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1 \right) x^3 + \ldots \right] 
+ i \left[ Q_0 + Q_1 y + \frac{1}{a} \left( b_1 Q_2 P_1 - b_2 Q_0 P_0 Q_1 \right) y^2 
+ \frac{M_Q}{a(n + 1)(n + 2)} y^{n+2} + \frac{1}{3a} \left( 2b_1 Q_0^2 P_2 + 2b_1 Q_0 Q_1 P_1 
- 2b_2 Q_0 P_0 Q_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0 \right) y^3 + \ldots \right],
\]

where \( a, b_1, b_2, Q_0, Q_1, P_0, P_1 \) are arbitrary constants, and other coefficients determined by (25)–(26).

Based on the previous detailed derivation, we can obtain the following theorem.

**Theorem 2.** The chiral NLS equation in \((2 + 1)\)-dimensions (1) has the following power series solution:

\[
q(x, y, t) = \sum_{n=0}^{\infty} P_n x^n + i \sum_{n=0}^{\infty} Q_n y^n, \tag{27}
\]

where \( a, b_1, b_2, Q_0, Q_1, P_0, P_1 \) are arbitrary constants, and other coefficients determined by (25)–(26).

Next, by choosing the appropriate parameters, we draw the graph of the power series solution and thus illustrate its properties (see Figs. 1, 2).

![Figure 1](http://www.journals.vu.lt/nonlinear-analysis)
5 Conservation laws

In this section, if we want to derived the conservation law of equation (1), it is necessary to first find the conservation law of system (3). Therefore, we will use Lie point symmetry (8) to construct the conservation law of system (3).

A vector $C = (C^t, C^x, C^y)$ is called a conserved vector for equation (1) if it satisfy the following conservation equations:

$$D_t (C^t) + D_x (C^x) + D_y (C^y) = 0.$$  

In [20], Ibragimov proposes a new conservation theorem, that is, constructing a conservation law without a Lagrangian quantity in a differential equation. Then on the basis of [20], the Lagrangian of system (3) can be written as follows:

$$L = \phi(x, y, t)\left[-v_t + a(u_{xx} + u_{yy}) + 2b_1(u^2 v_x - uvu_x) + 2b_2(u^2 v_y - uvu_y)\right]$$

$$+ \psi(x, y, t)\left[u_t + a(v_{xx} + v_{yy}) + 2b_1(uvv_x - v^2 u_x) + 2b_2(uvv_y - v^2 u_y)\right].$$  

where $\phi(x, y, t)$ and $\psi(x, y, t)$ are two new dependent variables. The adjoint equations of system (3) can be written as following form:

$$F^* = \frac{\delta L}{\delta u} = 0, \quad G^* = \frac{\delta L}{\delta v} = 0$$

with

$$\frac{\delta L}{\delta u} = \frac{\partial L}{\partial u} - D_t \frac{\partial L}{\partial u_t} - D_x \frac{\partial L}{\partial u_x} - D_y \frac{\partial L}{\partial u_y} + D_x^2 \frac{\partial L}{\partial u_{xx}} + D_y^2 \frac{\partial L}{\partial u_{yy}},$$

$$\frac{\delta L}{\delta v} = \frac{\partial L}{\partial v} - D_t \frac{\partial L}{\partial v_t} - D_x \frac{\partial L}{\partial v_x} - D_y \frac{\partial L}{\partial v_y} + D_x^2 \frac{\partial L}{\partial v_{xx}} + D_y^2 \frac{\partial L}{\partial v_{yy}}.$$
Combining with (28) and adjoint equations (29), we can obtain
\[ F^* = 6b_1v^2\phi + 6b_2uv\phi_y + 6b_1vv_x\psi + 6b_2vu_y\psi + 2b_1uv\phi_x + 2b_1v^2\psi_x \\
+ 2b_2uv\phi_y + 2b_2v^2\psi_y - \psi_t + a\phi_{xx} + a\phi_{yy}, \]
\[ G^* = -6b_1uv_x\phi - 6b_2uv\phi_y - 6b_1vu_x\psi - 6b_2vu_y\psi - 2b_1uv\psi_x - 2b_1u^2\phi_x \\
- 2b_2uv\psi_y - 2b_2u^2\phi_y + \phi_t + a\psi_{xx} + a\psi_{yy}. \] (30)

In the above system (30), if we substitute \( v \) instead of \( \phi \) and \( u \) instead of \(-\psi\), we can get system (3). In [20], we know that the conservation vector \( C = (C^1, C^2, C^3, \ldots) \) has the following form:
\[ C^n = \xi^n L + W^\alpha \left[ \frac{\partial L}{\partial u_{ij}^\alpha} - D_j \left( \frac{\partial L}{\partial u_{ij}^\alpha} \right) + D_j D_k \left( \frac{\partial L}{\partial u_{ijk}^\alpha} \right) - \cdots \right] \\
+ D_j(W^\alpha) \left[ \frac{\partial L}{\partial u_{ij}^\alpha} - D_k \left( \frac{\partial L}{\partial u_{ijk}^\alpha} \right) + \cdots \right] + D_j D_k(W^\alpha) \left[ \frac{\partial L}{\partial u_{ijk}^\alpha} - \cdots \right], \]
where \( W^\alpha = \eta^\alpha - \xi^j u_j^\alpha (\alpha = 1, 2, \ldots, m) \) are shown in [20].

Using the above formula, we can further write about the conservation vector of (28) as
\[ C^t = \xi^t L + W^u \frac{\partial L}{\partial u_t} + W^v \frac{\partial L}{\partial v_t}, \]
\[ C^x = \xi^x L + W^u \left( \frac{\partial L}{\partial u_x} - D_x \frac{\partial L}{\partial u_{xx}} \right) + D_x(W^u) \frac{\partial L}{\partial u_{xx}} \\
+ W^v \left( \frac{\partial L}{\partial v_x} - D_x \frac{\partial L}{\partial v_{xx}} \right) + D_x(W^v) \frac{\partial L}{\partial v_{xx}}, \] (31)
\[ C^y = \xi^y L + W^u \left( \frac{\partial L}{\partial u_y} - D_y \frac{\partial L}{\partial u_{yy}} \right) + D_y(W^u) \frac{\partial L}{\partial u_{yy}} \\
+ W^v \left( \frac{\partial L}{\partial v_y} - D_y \frac{\partial L}{\partial v_{yy}} \right) + D_y(W^v) \frac{\partial L}{\partial v_{yy}}. \]

In which \( W^u \) and \( W^v \) are the Lie characteristic functions.

In order to obtain the conservation vector of system (3), we can use the symmetry generators \( V_1, V_2, V_3, V_4, V_5 \) and \( V_6 \) as an example to illustrate.

**Case 1.** For the generator \( V_1 = \partial/\partial t \), we can get the following Lie characteristic functions:
\[ W^u = -u_t, \quad W^v = -v_t. \] (32)

Inserting (32) into (31), we can get the following conserved vector:
\[ C^t_1 = auu_{xx} + avu_{yy} - auu_{xx} - auu_{yy}, \]
\[ C^x_1 = auv_x - auv_{tx} - au_x + au_t, \]
\[ C^y_1 = auv_y - avu_{ty} - au_y v_t + au_{ty}. \] (33)
After calculation, we can find the following equation:

\[ D_t(C^l_1) + D_x(C^l_1) + D_y(C^y_1) = 0. \]

Thus, we know that (33) is a conservation law of system (3). Inserting \( u = (q + q^*)/2 \) and \( v = (q - q^*)/(2i) \) into (33), we can obtain conservation laws of equation (1) as

\[
T^t_1 = \frac{a}{2i}(qq^*_{xx} + q^*_{yy} - q^*_{xx} - q^*_{yy}), \\
T^x_1 = \frac{a}{2i}(q_{t}q^*_t + q^*_t q_{tx} - q_t q^*_x - q^*_t q_x), \\
T^y_1 = \frac{a}{2i}(q_{y}q^*_t + q^*_y q_{ty} - q_{t}q^*_y - q^*_t q^*_y).
\]

**Case 2.** For the generator \( V_2 = \partial/\partial x \), we can get the following Lie characteristic functions:

\[
W^u = -u_x, \quad W^v = -v_x.
\] (34)

Inserting (34) into (31), we can get the following conserved vector:

\[
C^t_2 = uu_x + vv_x, \quad C^x_2 = -vv_t - uu_t + avu_{yy} - awv_{yy}, \\
C^y_2 = au_x v_y - avu_{xy} - au_y v_x + auv_{xy}.
\] (35)

After calculation, we can find the following equation:

\[ D_t(C^l_2) + D_x(C^x_2) + D_y(C^y_2) = 0. \]

Thus, we know that (35) is a conservation law of system (3). Inserting \( u = (q + q^*)/2 \) and \( v = (q - q^*)/(2i) \) into (35), we can obtain conservation laws of equation (1) as

\[
T^t_2 = \frac{1}{2}(qq^*_{xx} + q^*_{xy}), \quad T^x_2 = -\frac{1}{2}(qq_{t}q^*_t + q^*_t q_{tx}) + \frac{a}{2i}(qq^*_{yy} - q^*_{yy}), \\
T^y_2 = \frac{a}{2i}(q_{y}q^*_x + q^*_{xy} - q_{x}q^*_y - q^*_x q_{xy}).
\]

**Case 3.** For the generator \( V_3 = \partial/\partial y \), we can get the following Lie characteristic functions:

\[
W^u = -u_y, \quad W^v = -v_y.
\] (36)

Inserting (36) into (31), we can get the following conserved vector:

\[
C^t_3 = uu_y + vv_y, \quad C^x_3 = au_y v_x - avu_{xy} - au_x v_y + auv_{xy}, \\
C^y_3 = -vv_t - uu_t + avu_{xx} - auv_{xx}.
\] (37)

After calculation, we can find the following equation:

\[ D_t(C^l_3) + D_x(C^x_3) + D_y(C^y_3) = 0. \]
Thus, we know that (37) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2i)$ into (37), we can obtain conservation laws of equation (1) as

$$T_t^3 = \frac{1}{2} (qq_y^* + q^* q_y), \quad T_x^3 = \frac{a}{2i} (q_x q_y^* + q^* q_{xy} - q_y q_x^* - q_{xy}^*),$$

$$T_y^3 = -\frac{1}{2} (qq_t^* + q^* q_t) + \frac{a}{2i} (qq_{xx}^* - q^* q_{xx}).$$

**Case 4.** For the generator $V_4 = v\partial/\partial u - u\partial/\partial v$, we can get the following Lie characteristic functions:

$$W^u = v, \quad W^v = -u. \quad (38)$$

Inserting (38) into (31), we can get the conserved vector

$$C_t^4 = 0, \quad C_x^4 = 0, \quad C_y^4 = 0. \quad (39)$$

After calculation, we can find the following equation:

$$D_t (C_t^4) + D_x (C_x^4) + D_y (C_y^4) = 0.$$

Thus, we know that (39) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2i)$ into (39), we can obtain conservation laws of equation (1) as

$$T_t^4 = 0, \quad T_x^4 = 0, \quad T_y^4 = 0.$$

**Case 5.** For the generator $V_5 = (1/2)x\partial/\partial x + (1/2)y\partial/\partial y + t\partial/\partial t - (1/4)u\partial/\partial u - (1/4)v\partial/\partial v$, we can get the following Lie characteristic functions:

$$W^u = -\frac{1}{2} xu_x - \frac{1}{2} yu_y - tu_t - \frac{1}{4} u, \quad W^v = -\frac{1}{2} xv_x - \frac{1}{2} yv_y - tv_t - \frac{1}{4} v. \quad (40)$$

Inserting (40) into (31), we can get the following conserved vector:

$$C_t^5 = atvu_{xx} + atvu_{yy} - atvu_{xx} - atvu_{yy} + \frac{1}{4} u^2 + \frac{1}{2} xu_x + \frac{1}{2} yu_y + \frac{1}{4} v^2 + \frac{1}{2} xv_x + \frac{1}{2} yv_y,$$

$$C_x^5 = -\frac{1}{2} xv_t - \frac{1}{2} xu_t + \frac{1}{2} axvu_{yy} - \frac{1}{2} axuv_{yy} + \frac{1}{2} ayu_y v_y + atvu_x u_t - avu_x - \frac{1}{2} ayvu_{xx} - atvu_{xt} - \frac{1}{2} ayu_x v_y$$

$$- atu_x v_t + auv_x + \frac{1}{2} ayuv_{xy} + atuv_{xt}, \quad (41)$$

$$C_y^5 = -\frac{1}{2} yv_v - \frac{1}{2} yu_t + \frac{1}{2} ayvu_{xx} - \frac{1}{2} ayuv_{xx} + \frac{1}{2} axv_y u_x + atvu_y u_t - avu_y - \frac{1}{2} axvu_{xy} - atvu_{ty} - \frac{1}{2} axu_y v_x$$

$$- atu_y v_t + auv_y + \frac{1}{2} axuv_{xy} + atuv_{ty},$$
After calculation, we can find the following equation:

\[ D_t(C_5^t) + D_x(C_5^x) + D_y(C_5^y) = 0. \]

Thus, we know that (41) is a conservation law of system (3). Inserting \( u = (q + q^*)/2 \) and \( v = (q - q^*)/(2i) \) into (41), we can obtain conservation laws of equation (1) as

\[ T_5^t = \frac{at}{2i}(qq_{xx}^* - q^* q_{xx} + qq_{yy}^* - q^* q_{yy}) \]
\[ + \frac{1}{4}[x(qq_x^* + q^* q_x) + y(qq_y^* + q^* q_y) + qq^*], \]
\[ T_5^x = \frac{ax}{4i}(qq_{yy}^* - q^* q_{yy}) + \frac{ay}{4i}(q_x q_y^* - q_y q_x^*) + \frac{a}{2i}(q^* q_x - q_x^*), \]
\[ T_5^y = \frac{ay}{4i}(qq_{xx}^* - q^* q_{xx}) + \frac{ax}{4i}(q_y q_x^* - q_x q_y^*) + \frac{a}{2i}(q^* q_y - q_y^*). \]

**Case 6.** For the generator \( t \partial / \partial x - (b_1 t/b_2) \partial / \partial y + (v(b_1 y - b_2 x)/(2ab_2)) \partial / \partial u - ((u(b_1 y - b_2 x))/(2ab_2)) \partial / \partial v, \) we can get the following Lie characteristic functions:

\[ W^u = -tu_x + \frac{b_1 t}{b_2} u_y + \frac{v(b_1 y - b_2 x)}{2ab_2}, \]
\[ W^v = -tv_x + \frac{b_1 t}{b_2} v_y - \frac{u(b_1 y - b_2 x)}{2ab_2}. \]

Inserting (42) into (31), we can get the following conserved vector:

\[ C_6^t = t(uu_x + vv_x) - \frac{b_1 t}{b_2} (uu_y + vv_y), \]
\[ C_6^x = -t(vv_t + uu_t) + at(vv_{yy} - uu_{yy}) - \frac{1}{2}(u^2 + v^2) \]
\[ + \frac{ab_1 t}{b_2} (u_x v_y - v_x u_y + u_xy v - uv_{xy}), \]
\[ C_6^y = \frac{b_1 t}{b_2} (vv_t + uu_t) + \frac{ab_1 t}{b_2} (uv_{xx} - u_{xx} v) + \frac{b_1}{2b_2} (u^2 + v^2) \]
\[ + at(u_x v_y - v_x u_y + v_{xy} u - u_{xy}). \]

After calculation, we can find the following equation:

\[ D_t(C_6^t) + D_x(C_6^x) + D_y(C_6^y) = 0. \]
Thus, we know that (43) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2i)$ into (43), we can obtain conservation laws of equation (1) as

$$T_6^t = \frac{t}{2}(qq_x^* + q^* q_x) - \frac{b_1 t}{2b_2}(q q_y^* + q^* q_y),$$

$$T_6^x = -\frac{t}{2}(qq_t^* + q^* q_t) + \frac{at}{2i}(qq_{yy}^* - q^* q_{yy}) - \frac{1}{2}qq^* + \frac{ab_1 t}{2ib_2}(q_y q_x^* - q_x q_y^* - q^* q_{xy} + qq_{xy}^*),$$

$$T_6^y = \frac{b_1 t}{2b_2}(qq_t^* + q^* q_t) + \frac{ab_1 t}{2ib_2}(qq_{yy}^* - q^* q_{yy}) + \frac{b_1}{2b_2}qq^* + \frac{at}{2i}(q_y q_x^* - q_x q_y^* + q^* q_{xy} - qq_{xy}^*).$$

### 6 Conclusions and discussions

As we mentioned above, the bright and dark soliton solutions of the chiral NLS equation (1) have been obtained using the constant coefficient method in [6]. The singular periodic solution of equation (1) has been obtained by using the trial solution method in [14]. Compared with previous literatures [6, 14], we have obtained some new results, such as vector field, optimal system, similarity reduction solutions, power series solutions with convergence analysis, and conservation laws of equation (1). Firstly, we have transformed the complex model (1) to the real system (3) by using the transformation $q(x, y, t) = u(x, y, t) + iv(x, y, t)$. Then, through the Lie symmetry analysis method, we have constructed the optimal systems and symmetry reductions of system (3). In addition, we have also obtained the power series solution of equation (1) by the power series method. In Figs. 1, 2, when $n = 4, 5$, we have obtained a perspective view of the real part of the power series solution and the wave propagation pattern of the wave along the $x$-axis by selecting the appropriate parameter values. Subsequently, we have obtained the conservation law related to the lie symmetry of equation (1) by using the new conservation law method introduced by Ibragimov in [20]. The new results presented in this work can be used to describe soliton dynamics in nuclear physics and other optical experiments. Therefore, it is hoped that all the research results in this work can be used to enrich the dynamic behavior of nonlinear Schrödinger-type equations in engineering and mathematical physics.

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