Reduction of Torque Ripple for Interior Permanent Magnet Motor with Fractional-Slot Concentrated-Windings

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Abstract. Fractional slot concentrated winding motors have advantages of compact structure, high efficiency and high power density, and are widely used in various fields. Due to the large torque fluctuation caused by improper design, it is necessary to pay attention to the torque ripple of the motor. This paper analyses the torque ripple of an 18-slot 12-pole motor, and provides a new method to reduce cogging torque and torque ripple torque through optimizing the harmonic magnetic field generated by permanent magnets. Finally, the validity of the method is verified by the FEM method.

1. Introduction

Interior permanent magnet synchronous machine(IPMSM) has been widely used in electric vehicles because of high power density, high efficiency and wide speed range. Meanwhile, fractional slot concentrated winding PMSM offers significant advantages such as short end winding, high slot filling factor and low cogging torque [1]. For hub motors, the IPMSM with fractional slot concentrated winding is the preferred solution. However, cogging torque and torque ripple will cause vibration and noise of permanent magnet hub motors, which will seriously affect the control accuracy of the system and the running quality of the motor.

In the field of motor design at this stage, many methods to reduce torque ripple have been proposed and applied to the design. such as the skewing of rotor magnets or stator slots, PM shifting, different slot width pairing etc. have been proposed and investigated [2-5]. The use of these techniques can reduce the amplitude of torque ripple, but these methods will inevitably reduce the average torque or increase the production cost. Rotor slant pole or stator skew is another effective method to reduce torque ripple and cogging torque components, and is widely used. However, for PMSM, when the rotor is skewed, only segmented rotor skew can be used. Therefore, rotor skew increases the production cost of the machine. In addition, the stator skew will also increase the cost, it also affects the length of the stator winding and the gap filling factor, and therefore increases the loss of the machine winding.

This paper analyzes the torque ripple of the 18-slot 12-pole IPMSM with concentrated winding based on the torque calculation model, and provides a novel method to reduce cogging torque and torque ripple torque via reducing flux harmonics generated by permanent magnet. Ignoring the effects of stator slotting, multi-objective genetic algorithm is used to optimize the rotor geometric parameters, reduce the air gap flux density harmonics, and reduce the cogging torque and torque ripple of PMSM.
2. Analysis model and methodology

2.1. Prototype model

The investigation is carried out on an IPM hub motor, which has stringent requirements on torque ripples. The peak-peak (p-p) value of cogging torque must be lower than 3Nm, and the on-load torque ripple p-p value must be lower than 3% of the rated torque average value. The magnets are shaped as illustrated in Figure 1 and the parameter of the prototype motor as shown in Table 1.

![Figure 1. The IPM and the V-shape permanent magnet rotor used as an analysis model](image)

2.2. Analytical Model of Torque

Assuming a permanent magnet synchronous motor is a linear system, and the magnetic permeability of ferromagnetic materials is infinite. The electromagnetic torque calculation model based on energy method is expressed as follows (1):

\[ T = T_{em} + T_{cog} = N_p (\Psi_d i_q - \Psi_q i_d) + T_{cog} \]

\[ = N_p \left[ (\Psi_{fd} i_q - \Psi_{fq} i_d) + (L_d - L_q) i_d i_q \right] + T_{cog} \]

where \( N_p \) is the number of rotor pole pairs, \( i_d \) and \( i_q \) are the current component on the d-axis and q-axis, \( \Psi_d \) and \( \Psi_q \) are the component of the flux generated by the stator on the d-axis and q-axis, \( \Psi_{fd} \) and \( \Psi_{fq} \) are the component of the magnetic flux generated by the permanent magnet on the d-axis and q-axis, \( L_d \) and \( L_q \) are the inductance component on the d-axis and q-axis. From the electromagnetic torque expression of (1), if the stator current is assumed to be an ideal sine wave, the ripple torque is caused by the harmonics component of inductance and permanent magnet flux linkage.

Cogging torque can be calculated by analytical model based on magnetic energy variation as (2)[6].

\[ T_{cog} = \frac{\partial W}{\partial \theta_m} = - \frac{\pi z L_{af} (R_o^2 - R_i^2)}{4 \mu_0} \sum_{n=1}^{\infty} n \left[ G_{an} B_{sn} \sin(n \pi \theta_m) \right] \]

where \( L_{af} \) is the axial length of armature, \( R_o \) the outer radius of rotor, \( R_i \) the inner radius of armature, \( z \) the number of slots, \( \theta_m \) the rotor position, \( G_{an} \) the \( n^{th} \) Fourier expansion coefficients of air gap relative permeability, \( B_{sn} \) the \( n^{th} \) Fourier expansion coefficients of air gap magnetic density. From (2), if the components of \( B_{sn} \) can be made be zero, the torque can be eliminated. Therefore, according to the harmonic order corresponding to the cogging torque, the analysis of the magnetic field harmonics causing the fluctuation and suppressing it can achieve the purpose of reducing the cogging torque.

| Table 1. Specifications of the IPMSM |
|--------------------------------------|
| Parameter               | Unit | Specification |
|----------------------------|------|---------------|
| Stator Diameter           | mm   | 308           |
| Axial Length              | mm   | 60            |
| Rated Output              | kW   | 30            |
| Power                     | rpm  | 2750/6000     |
| Rated/Max Speed           | rpm  | 2750/6000     |
| Pole/Slot Number          | --   | 12/18         |
2.3. Torque ripple and cogging torque analysis

Figure 2 shows the cogging torque and torque ripple FEM and FFT results for a 12-pole 18-slot prototype motor. The peak-to-peak cogging torque fluctuations are as high as 10Nm, and the peak-to-peak torque fluctuations under rated current are as high as 20Nm. It can be found that the torque fluctuations are mainly 6th and 12th in Figure 2(b).

![Figure 2. Cogging torque and Torque ripple of Base model](image)

The spatial harmonics of the air-gap magnetic induction intensity generated by the permanent magnets will generate flux-chain time harmonics of the same frequency in the windings. The third harmonic of the flux chain has no effect on the three-phase motor connected to the star, so they need not be considered. The 5th and 7th harmonics of the magnetic flux are converted into the 6th harmonics of the d-axis and q-axis fluxes, which results in a 6th order torque ripple, which is derived as follows:

\[
\begin{align*}
\psi_a &= \psi_1 \sin(\omega t + \theta_1) + \psi_5 \sin(-5\omega t + \theta_5) \\
&\quad + \psi_7 \sin(7\omega t + \theta_7) + \cdots \\
\psi_b &= \psi_3 \sin(\omega t + \theta_1 - \frac{2\pi}{3}) + \psi_5 \sin(-5\omega t + \theta_5 - \frac{2\pi}{3}) \\
&\quad + \psi_7 \sin(7\omega t + \theta_7 - \frac{2\pi}{3}) + \cdots \\
\psi_c &= \psi_5 \sin(\omega t + \theta_1 + \frac{2\pi}{3}) + \psi_7 \sin(-5\omega t + \theta_5 + \frac{2\pi}{3}) \\
&\quad + \psi_7 \sin(7\omega t + \theta_7 + \frac{2\pi}{3}) + \cdots 
\end{align*}
\]  

(3)

where \(\psi_1, \psi_5, \psi_7\) are the amplitudes of the fundamental wave, the 5th harmonic, and the 7th harmonic; \(\theta_1, \theta_5, \theta_7\) are the initial phase angles of the fundamental wave, 5th harmonic, and 7th harmonic flux. The PMSM three-phase winding flux linkages A, B, and C are transformed using the Park transformation with constant amplitude constraints to the fundamental wave dq synchronous rotation coordinate axis system. The following results are obtained:

\[
\begin{align*}
\psi_d &= \psi_{d1} + \psi_{d5}\cos(-6\omega t + \theta_5) + \psi_{d7}\cos(6\omega t + \theta_7) + \cdots \\
\psi_q &= \psi_{q1} + \psi_{q5}\sin(-6\omega t + \theta_5) + \psi_{q7}\sin(6\omega t + \theta_7) + \cdots 
\end{align*}
\]  

(4)

where, \(\psi_{d1}, \psi_{q1}\) are the components of the d-axis and q-axis of the fundamental wave flux linkage in the coordinate system of the fundamental wave dq synchronous rotation coordinate axis system. The 5th and 7th harmonic components in the three-phase stationary coordinate system appear as the 6th harmonic in the fundamental wave dq synchronous rotation coordinate system.
In the same way, the 12th torque ripple originates from the 11th and 13th harmonics of the flux linkage, so the shape and size of the rotor and permanent magnets need to be optimized to reduce the content of the permanent magnet flux linkages in the air gap.

From the torque formula (1), we can know that to reduce the torque ripple of the permanent magnet synchronous motor, the motor can be optimized in terms of the design of the motor body, and the characteristics of the flux linkage and cogging torque in the motor can be improved. Rotor shape optimization can improve the strength waveform of air-gap magnetic steel, reduce the torque ripple of the motor, and is suitable for different rotor structures. This method is also used in this paper to suppress the torque ripple of the IPMSM.

3. Proposed design method to decrease cogging torque and torque ripple

This paper adopts the method of optimizing both the rotor magnetic bridge and the permanent magnet to reduce the air gap magnetic induction intensity harmonics generated by the permanent magnet of the IPMSM. Since the air gap magnetic density generated by the permanent magnet is very sensitive to the size parameters of the rotor and the saturation degree of the magnetic bridge, it is necessary to consider and optimize the parameters uniformly. We know that the rotor structure parameters that mainly affect torque fluctuations include V shape angle, pole arc coefficient and magnetic bridge length, etc. Therefore, this paper uses genetic algorithms to optimize these rotor parameters. The optimized parameters are marked in the finite element numerical calculation model shown in Figure3. It should be noted that in addition to the parameters marked in Figure3, the optimized parameters include the eccentricity $H_p$ of the rotor outer circle. To simplify the model, the stator slotting effect is ignored, therefore the stator core is simplified to a slotless solid structure in the finite element calculation model, the optimization function is shown:

$$\begin{align}
\text{Objective:} & \quad \left( B_5^2 + B_7^2 + B_{11}^2 + B_{13}^2 \right) / B_1^2 \rightarrow \text{minimum} \\
\text{Constrains:} & \quad B_i \geq 0.9
\end{align}$$

where $B_i$ is the $i$th harmonic component amplitude of air gap magnetic density.

After parameterized calculation, the optimized results of the rotor parameters are shown in Table2. To verify the results of the optimization and analyze the effect of the optimization method, the optimized in-wheel motor and the optimized front rotor shape motor are analyzed and compared, as shown in Figure4. The rated operating point of the modify model is at the saturation point of the silicon steel sheet material, which proves that the silicon steel sheet material has a better utilization rate, as shown in Figure5.

By optimizing the magnetic circuit of the rotor, the flux harmonics generated by the permanent magnets have been optimized. The 5th, 7th, 11th and 13th harmonics of air gap density have been reduced...
as shown in Figure 6. Meanwhile, as shown in Figure 7 and Figure 8, the no-load back-EMF waveform is improved, and the 5th, 7th, and 11th harmonics are significantly reduced, which proves the effectiveness of the optimization method.

The Optimization method also effectively improves the torque performance of the IPMSM, as shown in Figure 9 and Figure 10. The 6th and 12th harmonic of cogging torque and torque ripple at the rated current is significantly reduced. From Table 3, the peak-to-peak cogging torque is reduced by 8.26Nm; the peak-to-peak torque ripple at rated current is reduced by 7.06Nm; the ripple-rate between base model and modify model has been reduced 21% to 15%. However, the average torque in rated current is reduced by 8.46Nm because of the reduction of reluctance torque.

Table 2. Optimization results of the rotor core of the 12-pole/18-slot in-wheel IPM motor

| Size/Parameters Value | Size/Parameters Value |
|------------------------|------------------------|
| Aeg (degree) 83        | Lt (mm) 1.5            |
| Hm (mm) 5             | Rib (mm) 1.5           |
| Wm (mm) 22            | Rib (mm) 2.1           |
| Inset (mm) 0.75       | Hp (mm) 32             |

Table 3. Comparison between the base model and the modified model

| Item               | Base Model | Modified Model |
|--------------------|------------|----------------|
| Cogging Torque     | 9.96Nm     | 1.70Nm         |
| Rated Load Torque  | 97.64Nm    | 89.18Nm        |
| Ripple             | 20.46Nm    | 13.40Nm        |
Figure 10. Comparison of torque ripple

From Figure 11, the reluctance torque was extracted using the frozen permeability method, and it can be found that the average value of the reluctance torque decreased by 15Nm. The outer circle of the optimized model rotor is eccentric, and the q-axis air gap of the motor increases, which causes the q-axis inductance to decrease. According to Formula (1), the difference between $L_q$ and $L_{q0}$ becomes smaller, so the reluctance torque is reduced.

4. Conclusion
This paper analyzes the cogging torque and torque ripple of 12-pole and 18-slot IPMSM, and provides a new optimization method to reduce cogging torque and torque ripple based on reducing the harmonics of permanent magnet flux. The rotor structure of the motor was optimized by the optimization method. The cogging torque and the 6th and 12th harmonics of the total torque fluctuation were effectively reduced. But in this method, the ripple of reluctance torque has not been effectively improved, and it is more suitable for the optimization of permanent magnet torque fluctuation.

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