Arbitrariness in the gravitational Chern-Simons-like term induced radiatively

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Lorentz violation through a radiatively induced Chern-Simons-like term in a fermionic theory embedded in linearized quantum gravity with a Lorentz- and CPT-violating axial-vector term in the fermionic sector proportional to a constant field $b_\mu$ has been recently studied. In a similar fashion as for the extended-QED model of Carroll-Field-Jackiw, we explicitly show that neither gauge invariance nor the more stringent momentum routing invariance condition on underlying Feynman diagrams fix the arbitrariness inherent to such induced term at one loop order. We present the calculation in a nonperturbative expansion in $b_\mu$ and within a framework which besides operating in the physical dimension (and thus avoiding $\gamma_5$ matrix Clifford algebra ambiguities), judiciously parametrizes regularization dependent arbitrary parameters usually fixed by symmetries.

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I. INTRODUCTION

In the Standard Model of particle physics, Lorentz and CPT are regarded as fundamental symmetries. However, since the early 90's possible violations of such symmetries have been studied $^{[1]-[30]}$. The first model, introduced by Sean M. Carroll et al $^{[6]}$, considered the theoretical and phenomenological consequences of adding to QED a Chern-Simons-like term proportional to a constant four-vector. They found out that such model predicts vacuum birefringence. However, the stringent bounds to this value obtained by astrophysical data tell us how small this kind of deviations from Lorentz and CPT symmetries actually are $^{[4, 5]}$. Such small effects would come from spontaneous symmetry breaking of Lorentz symmetry in a more complete theory such as string theory $^{[1]}$.

One interesting aspect which has been vastly investigated is whether this CS-like term can be radiatively produced. One example of this mechanism occurs in extended QED with a Lorentz violating axial term, in which the CS-like term appears when we consider radiative corrections to the photon propagator. However, different results for the CS-like coefficient have been found (see, for example, $^{[3, 8-14]}$), i.e. the coefficient of the induced term, coming from the cancellations of divergences, is regularization dependent. Using Pauli-Villars method, for instance, this coefficient is found to be zero $^{[3, 12]}$, while the result using dimensional regularization depends on $\gamma_5$ matrix algebra dimensional continuation (see $^{[31, 32]}$ and references therein).

Following the idea of an induced CS-like term in extended QED, it has also been discussed if a gravitational CS-like term can be radiatively induced in a fermionic theory in curved space. Phenomenologically, the existence of such term would imply that gravitational waves possess two degrees of polarization instead of four $^{[15]}$. The coefficient of such induced term turns out to depend on details involving the regularization of intermediate divergencies $^{[16, 17]}$.

In this work, we compute the 1-loop correction to the graviton propagator in the weak field approximation, using a more general approach called Implicit Regularization. Since it does not specify any particular regularization technique, allowing the reproduction of other results by choosing the method at the end of the calculation, it permits us to identify the sources of the ambiguities. We find that the induced gravitational CS-like term depends on a set of surface terms which, coming from a difference of divergent integrals, are arbitrary.

Following $^{[12]}$, arbitrary parameters that appear in finite radiative corrections must be fixed either by phenomenology or symmetries of the underlying model. By demanding gauge invariance of the action, which enforces transversality of the graviton self-energy, we find that all but one surface term can be fixed. Momentum routing invariance (MRI) of the diagram which has been shown to lead to vector gauge invariance $^{[34]}$ is a stronger symmetry condition which could be imposed. We show that it does lead to an additional constraint which however does not fix the arbitrariness. This is the same result obtained in the case of extended QED in flat space. In such case requiring transversality of the final amplitude, as well as MRI, does not determine the coefficient of the
Carroll-Field-Jackiw term.

The paper is organized as follows: in section II, we carry out a review of the calculation of the induced CS-like term in flat space extended QED with arbitrary loop momentum routing which serves the purpose of both pedagogical presentation and comparison with the model we study in section III, namely fermions in linearized quantum gravity with a Lorentz violating extension. We compute the 1-loop correction to the graviton propagator in detail in a regularization independent fashion to study the resulting CS-like term in this case. In section IV, we conclude and leave details of the integrals that appear in this work to appendix V.

II. REVISITING THE CS-LIKE INDUCED TERM IN EXTENDED QED

In order to motivate our line of reasoning, we revisit the induction of the Chern-Simons-like term (also called Carroll-Field-Jackiw term) in extended QED. For simplicity, we show here the massless model whose action reads $S_{QED} \propto \int d^4x \bar{\psi} (i \partial - \mathcal{A} - \bar{\gamma}_5 \mathcal{A}) \psi$. This calculation is well known to be ambiguous and many different methods have been applied, furnishing various results. Here, we take as an example the calculation of [13], in which the Implicit Regularization scheme has been used. If the fermion is non-massive, its propagator can be decomposed as [13]

$$\frac{i}{k - \not{p} \gamma_5} = \frac{i}{k - \not{p}} P_L + \frac{i}{k + \not{p}} P_R,$$

where we are using the chiral projectors

$$P_{R,L} = \frac{1 \pm \gamma_5}{2}.$$

Here, we carry out the calculation with an arbitrary loop routing. The full one-loop photon self-energy is given by

$$\Pi^{\mu \nu} = \frac{1}{2} \{ \Pi_{+}^{\mu \nu} + \Pi_{-}^{\mu \nu} + \Pi_{+5}^{\mu \nu} + \Pi_{5}^{\mu \nu} \},$$

with

$$\Pi_{+}^{\mu \nu}(p, \alpha p + b) = \int_k \frac{\gamma^{\nu}(k + \alpha \not{p} + \not{b}) \gamma^{\mu} [k + (\alpha + 1) \not{p} + \not{b}]}{(k + \alpha p + b)^2 [k + (\alpha + 1) p + b]^2}$$

and

$$\Pi_{5}^{\mu \nu}(p, \alpha p + b) = \pm \int_k \frac{\gamma^{\nu}(k + \alpha \not{p} + \not{b}) \gamma^{\mu} [k + (\alpha + 1) \not{p} + \not{b}]}{(k + \alpha p + b)^2 [k + (\alpha + 1) p + b]^2}$$

where $k \equiv \int \frac{d^4k}{(2\pi)^4}$ and the superscript $\Lambda$ is used to indicate that some four dimensional regularization has been applied (say a cutoff) just to justify algebraic operations at the level of the integrands. For a particular momentum routing in the loop, a variable $\alpha$ is fixed.

The induction of the CS-term comes from the $\Pi_{5}^{\mu \nu}$ parts, so that we have

$$\Pi_{5}^{\mu \nu} = \frac{1}{2} [\Pi_{+5}^{\mu \nu}(p, \alpha p + b) + \Pi_{5}^{\mu \nu}(p, \alpha p - b)]$$

$$= \frac{1}{2} [\Pi_{+5}^{\mu \nu}(p, b_1) - \Pi_{+5}^{\mu \nu}(p, b_2)],$$

with $b_1 = \alpha p + b$ and $b_2 = \alpha p - b$. So, let us calculate $\Pi_{+5}^{\mu \nu}(p, b)$, which, after Dirac algebra, can be written as

$$\Pi_{+5}^{\mu \nu}(p, b) = 4i p_\beta \epsilon^{\alpha \mu \nu \beta}$$

with

$$I_4 I_\alpha = \int_k \frac{1}{(k + \not{b})^2 (k + \not{b} + \not{b})^2}.$$

The results of these integrals by means of Implicit Regularization are given by

$$I = I_{log}(\lambda^2) - \frac{i}{16\pi^2} \ln \left( - \frac{p^2}{\lambda^2} \right) - 2$$

and

$$I_4 I_\alpha = \frac{(p + 2b)_\alpha}{2} \left( I_{log}(\lambda^2) - \frac{i}{16\pi^2} \ln \left( - \frac{p^2}{\lambda^2} \right) - v_0 \right),$$

where

$$v_0 \eta^{\mu \nu} = T_0^{\mu \nu} \equiv \int \frac{\partial k^\mu}{k_{\mu}} \left( k^\mu - m^2 \right)$$

$$= \eta^{\mu \nu} I_{log}(m^2) - 4I^{\mu \nu}(m^2),$$

$$I_{log}^{\cdot \cdot \cdot n}(\lambda^2) \equiv \int_k \frac{k^\mu \cdot \cdot \cdot k^{2n}}{(k^2 - \lambda^2)^{2+n}},$$

and $\lambda^2$ is a dimensionful parameter which plays the role of renormalization scale and cancels out through the calculations in the case of finite radiative corrections as it should. Substituting these results in equation (6), we get

$$\Pi_{+5}^{\mu \nu}(p, b) = 4iv_0 b_\alpha p_\beta \epsilon^{\alpha \mu \nu \beta}.$$ 

So, we obtain

$$\Pi_{5}^{\mu \nu} = \frac{1}{2} [4iv_0 (\alpha p + b)_\alpha p_\beta \epsilon^{\alpha \mu \nu \beta} - 4iv_0 (\alpha p - b)_\alpha p_\beta \epsilon^{\alpha \mu \nu \beta}]$$

$$= 4iv_0 b_\alpha p_\beta \epsilon^{\alpha \mu \nu \beta}.$$ 

The induced coefficient of the Carroll-Field-Jackiw term will then be given by

$$\Delta c_\mu = 2iv_0 b_\mu.$$
We see that the coefficient of the CS-type generated term is proportional to the surface term $v_0$. It is important to note that, in this nonperturbative calculation in the Lorentz-violating vector $b_\mu$, there is no dependence on the loop routing parameter $\alpha$. This fact has already been observed in $^8$. This situation will be confronted with the induced gravitational CS-like term, as discussed in the next section.

III. ARBITRARINESS IN THE INDUCED CS GRAVITY TERM

We consider a massless fermionic theory in a gravitational background with a CPT-violating term,

$$ S = \int d^4x \left( \frac{i}{2} e^\mu \bar{\psi} \gamma^\mu \psi D_\mu \psi - e^\mu_\alpha b_\mu \bar{\psi} \gamma^\alpha \gamma_5 \psi \right), \quad (16) $$

where $e^\mu_\alpha$ is the tetrad, $e = \det e^\mu_\alpha$ and $b_\mu$ is a constant four-vector.

In equation (16), in order to couple fermions with the gravitational field, we need to define the covariant derivative,

$$ D_\mu \psi = \partial_\mu \psi + \frac{1}{2} \omega_{\mu ab} \sigma^{ab} \psi, \quad (17) $$

where $\omega_{\mu ab}$ is the spin connection, which depends on the tetrad, and $\sigma^{ab} = \frac{1}{4} [\gamma^a, \gamma^b]$.

In the weak field approximation, we use the following expansions for the metric and the tetrad:

$$ g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}, \quad (18) $$

and

$$ e_{\mu a} = \eta_{\mu a} + \frac{1}{2} \kappa h_{\mu a}. \quad (19) $$

Therefore, the action (16) can be reexpressed as

$$ S = \int d^4x \left\{ \frac{1}{2} i \bar{\psi} \gamma^\mu \psi - \frac{1}{2} i \kappa \left[ h \bar{\psi} \gamma^\mu \psi - \frac{1}{2} h^{\mu \alpha} \bar{\psi} \gamma^\alpha \psi \right] + \frac{1}{4} \bar{\psi} \partial_\nu h \bar{\psi} \gamma^\nu \psi - \frac{1}{4} \bar{\psi} \partial_\nu h \gamma^\nu \psi \right\} + O(\kappa^2). \quad (20) $$

The Lorentz indices between brackets stand for symmetrization of the tensor, i.e. $A_{\{\alpha_1 \cdots \alpha_n\}} = A^{\alpha_1 \cdots \alpha_n} + \sum$ over permutations of indices $\alpha_1 \cdots \alpha_n$. Feynman rules can be readily derived from equation (21) as

A. Fermion propagator

$$ = S(p) = \frac{i}{\not{p} - \not{\kappa} 5}, \quad (21) $$

B. Graviton-fermion vertices

$$ V^{\alpha \beta}(k_2, k_3) = \frac{i\kappa}{8} \left[ 2\eta^{\alpha \beta}(k_2 + k_3) - \gamma^\alpha (k_2 + k_3) \beta - \gamma^\beta (k_2 + k_3) \alpha \right] \quad (22) $$

$$ = V^{\alpha \beta \mu \nu}(k_1, k_2, k_4) = i\kappa^2 \left[ \frac{5}{16} (k_1 + k_2) \left( \frac{2}{5} \eta^{\alpha \beta} \eta^{\mu \nu} + \frac{1}{4} \gamma^{\{\alpha \beta\}} \{\mu, \nu\} + \frac{1}{4} \gamma^{\{\mu, \nu\}} \{\alpha, \beta\} \right) ight] $$

$$ + \frac{1}{32} \gamma^{\{\alpha \beta\}} (\mu (3k_4 + 4k_2 - 3k_1)^\nu - \frac{1}{16} \gamma^{\{\alpha \beta\}} (2k_4 + k_1)^\beta ) \eta^{\mu \nu} $$

$$ + \frac{1}{16} \gamma^{\{\mu, \nu\}} (\alpha (3k_4 + 4k_1 - 3k_2)^\beta - \frac{1}{16} \gamma^{\{\mu, \nu\}} (2k_4 + k_2)^\beta ) \eta^{\alpha \beta} $$

$$ - \frac{1}{16} \gamma^{\{\alpha \beta\}} (\eta^{\mu \nu} \eta^{\alpha \beta} - \eta^{\mu \alpha} \eta^{\nu \beta} \right] \quad (23) $$

In order to obtain the induced Chern-Simons-like term, we have to compute the 1-loop correction for the graviton propagator which is linear in $b_\mu$. Figure shows the two diagrams that contribute.
Figure 1. 1-loop corrections for the graviton propagator. The double waved line and the solid line stand for the graviton and the fermion, respectively.

Their amplitudes read

\[ \Pi_{(a)}^{\mu\nu\alpha\beta}(p) = i \int \frac{d^4k}{(2\pi)^4} Tr [V^{\mu\nu}(k+p,k)S(k+p) \times V^{\alpha\beta}(k, k+p)S(k)] \]  \hspace{1cm} (24)

and

\[ \Pi_{(b)}^{\mu\nu\alpha\beta}(p) = i \int \frac{d^4k}{(2\pi)^4} Tr [S(k)V^{\alpha\beta\mu\nu}(p, p, k)]. \] \hspace{1cm} (25)

We write the following expansion for the fermion propagator

\[ \frac{i}{\slashed{k} - i\gamma_5} = \sum_{n=0}^{\infty} i \left\{ -i\gamma_5 \frac{i}{\slashed{k} - i\gamma_5} \right\} = \sum_{n=0}^{\infty} S_n(k). \] \hspace{1cm} (26)

Since the CS-like term we are interested in is linear in \( b_\mu \), we can write

\[ \Pi_{(a)CS}^{\mu\nu\alpha\beta}(p) = i \int \frac{d^4k}{(2\pi)^4} Tr [V^{\mu\nu}(k+p, k)S_0(k+p) \times V^{\alpha\beta}(k, k+p)S_0(k)] \]

\[ + i \int \frac{d^4k}{(2\pi)^4} Tr [V^{\alpha\beta}(k, k+p)S_0(k) \times V^{\mu\nu}(k+p, k)S_0(k+p) \times \beta\gamma_5 S_0(k+p)] \] \hspace{1cm} (27)

and

\[ \Pi_{(b)CS}^{\mu\nu\alpha\beta}(p) = i \int \frac{d^4k}{(2\pi)^4} Tr [S_0(k)\beta\gamma_5 S_0(k) V^{\alpha\beta\mu\nu}(p, p, k)]. \] \hspace{1cm} (28)

These amplitudes are symmetric under the exchange \( \mu \leftrightarrow \nu \) and \( \alpha \leftrightarrow \beta \) as they should. The amplitude \( \Pi_{(b)CS}^{\mu\nu\alpha\beta}(p) \) is null after the trace operation. The amplitude \( \Pi_{(a)CS}^{\mu\nu\alpha\beta}(p) \) is superficially cubically divergent.

We apply the Implicit Regularization framework\footnote{\textsuperscript{32}} to treat these amplitudes. In this scheme, we assume the existence of an implicit regulator (\( \Lambda \)) in order to use the following identity to separate UV divergent basic integrals from the finite part:

\[ \int \frac{1}{(k + p)^2 - m^2} = \int \frac{1}{k^2 - m^2} - \int \frac{1}{k^2 - m^2} \left[ \frac{(p^2 + 2p \cdot k)}{(k^2 - m^2)(k + p)^2 - m^2} \right], \] \hspace{1cm} (29)

The amplitude is infrared safe, but the above expression without mass will break the original integral in two infrared divergent parts. In order to deal with this problem, we have also added a fictitious mass \( m \) to regularize the infrared divergence. The limit \( m \rightarrow 0 \) is taken in the end. In this process a renormalization scale \( \lambda \neq 0 \) is introduced and readily cancels out throughout the calculations. The result of the integrals obtained after taking the trace can be found in the appendix. Besides a finite part in the UV limit, we get basic divergent integrals which are defined as

\[ I_{log}^{\mu_1 \cdots \mu_{2n}}(m^2) = \int k^{\mu_1} \cdots k^{\mu_{2n}} \] \hspace{1cm} (30)

and

\[ I_{quad}^{\mu_1 \cdots \mu_{2n}}(m^2) = \int k^{\mu_1} \cdots k^{\mu_{2n}} \] \hspace{1cm} (31)

The basic divergences with Lorentz indices can be judiciously combined as differences between integrals with the same superficial degree of divergence, according to the equations below. They automatically define surface terms which are shown to be related to MRI in Feynman diagrams\footnote{\textsuperscript{34}}:

\[ T_{2w}^{\mu\nu} = \eta^{\mu\nu} I_{2w}(m^2) - 2(2 - w) I_{2w}^{\mu\nu}(m^2) \equiv v_{2w} \eta^{\mu\nu}, \] \hspace{1cm} (32)

\[ \Xi_{2w}^{\mu\nu\alpha\beta} = \eta^{(\mu\nu, \alpha\beta)} I_{2w}(m^2) - 4(3 - w)(2 - w) I_{2w}^{\mu\nu\alpha\beta}(m^2) \equiv \xi_{2w} \eta^{(\mu\nu, \alpha\beta)}, \] \hspace{1cm} (33)

\[ \Sigma_{2w}^{\mu\nu\alpha\beta\gamma\delta} = \eta^{(\mu\nu, \alpha\beta, \gamma\delta)} I_{2w}(m^2) - 8(4 - w)(3 - w)(2 - w) I_{2w}^{\mu\nu\alpha\beta\gamma\delta}(m^2) \equiv \sigma_{2w} \eta^{(\mu\nu, \alpha\beta, \gamma\delta)}. \] \hspace{1cm} (34)

In the expressions above, \( 2w \) is the degree of divergence of the integrals and for the sake of brevity, we substitute the subscripts log and quad by 0 and 2, respectively. Surface terms can be conveniently written as integrals of total derivatives, namely

\[ v_{2w} \eta^{\mu\nu} = \int \frac{\partial}{\partial k^\mu} \frac{k^\nu}{(k^2 - m^2)^{2-w}}, \] \hspace{1cm} (35)
(ξ_{2w} - v_{2w})\eta^{[\mu\nu, \eta^{\alpha\beta}} = \int_k \frac{\partial}{\partial k^\nu} \left( \frac{2(2 - w)}{k^2 - m^2} \right)^{3-w} k^\alpha k^\beta, \quad (36)

and

(σ_{2w} - ξ_{2w})\eta^{(\mu\nu, \eta^{\alpha\beta})} = \int_k \frac{\partial}{\partial k^\nu} \left( \frac{4(3 - w)(2 - w)}{k^2 - m^2} \right) k^\alpha k^\beta k^\gamma k^\delta \quad (37)

We see that equations (32) - (34) are undetermined because they are differences between divergent quantities. Each regularization scheme gives a different value for these terms. However, as physics should not depend on the schemes applied, we leave these terms to be arbitrary until the end of the calculation fixing them by symmetry constraints or phenomenology when it applies.

Thus, the result of the regularized amplitude Π_{(a)CS}^{\mu\nuαβ}(\rho) is given by

Π_{(a)CS}^{\mu\nuαβ}(\rho) = \frac{-i}{8} \kappa^2 \left[ \left( \frac{i}{48\pi^2} - 64\sigma_0 - 4\nu_0 + 4\xi_0 \right) p^\alpha p^\beta - \left( \frac{i}{48\pi^2} + 32\xi_0 \right) \eta^{\alpha\beta} p^2 \right] + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta, \mu \leftrightarrow \nu).

(38)

Obviously this result contains arbitrariness expressed by surface terms. To fix them we demand gauge invariance expressed by the transversality of the final amplitude in order to verify whether it is a sufficient condition to give an unambiguous value for the CS-like term. Explicitly, we have

p_α Π_{(a)CS}^{\mu\nuαβ}(\rho) = \frac{-i}{8} \kappa^2 (4\xi_0 - 4\nu_0 - 96\sigma_0) × \left( \epsilon^{\lambda\rho\beta\mu} p^\nu + \epsilon^{\rho\beta\nu} p^\mu \right) p^2 b_\lambda p_\rho = 0 \quad (39)

In order to satisfy equation (39), we must have ξ_0 = v_0 = σ_0 = 0 or ξ_0 - v_0 = 24\sigma_0. The former condition determines the CS-like term and the latter does not. If we replace this expression in equation (38) the result is

Π_{(a)CS}^{\mu\nuαβ}(\rho) = \frac{-i}{24} \kappa^2 \epsilon^{\lambda\rho\beta\mu} b_\lambda p_\rho \left( \frac{i}{16\pi^2} + 96\sigma_0 \right) \left( p^\alpha p^\beta - \eta^{\alpha\beta} p^2 \right) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta, \mu \leftrightarrow \nu). \quad (40)

We see that transversality is not sufficient to fix all surface terms leaving us an arbitrary result. Depending on the choice of the arbitrary term σ_0, we can either recover other results found in the literature \[16\]-\[18\] or even get zero.

Since transversality does not fix all the arbitrary terms, we may wonder if there is another constraint we could enforce in order to fix the remaining surface term. One possibility is demanding Momentum Routing Invariance (MRI) of the Feynman diagrams, a condition that was shown to be closely related to the surface terms defined before \[34\]. Therefore, we now calculate the diagrams depicted in figure 1 with an arbitrary momentum routing. As before, we need only to consider the first diagram, which is represented in figure 2.

![Figure 2. 1-loop correction to the graviton propagator with an arbitrary loop momentum routing.](image)

After computing the amplitude, MRI is enforced by the condition

Π_{(a)}^{\mu\nuαβ}(\rho, l) = Π_{(a)}^{\mu\nuαβ}(\rho, l') = 0, \quad (41)

which must be respected by any set of values of l, l’. As shown in \[34\], the general expression for the equation above contains only surface terms, which must be related in a precise manner in order to comply with MRI. Obviously, setting all surface terms to zero is a possible solution. However, there may be other less restrictive relations as well. In particular, one may wonder if demanding MRI is equivalent to demand transversality of the amplitude in the sense that the same conditions for the surface terms must be respected. Assuming l = cp, where c is an arbitrary constant, we obtain:

Π_{(a)CS}^{\mu\nuαβ}(\rho, l) = -\frac{3}{4} (c^2 - c) i \kappa^2 \epsilon^{\mu\nu} (v_0 - ξ_0 + 24\sigma_0) × \left( p^\alpha p^\beta - \eta^{\alpha\beta} p^2 \right) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta, \mu \leftrightarrow \nu) + Π_{(a)CS}^{\mu\nuαβ}(\rho, l = 0) \quad (42)

As can be easily seen, requiring MRI gives us two equations, ξ_0 - v_0 = 24\sigma_0 and 0 = 24\sigma_0. The first equation is the one we obtained before by imposing transversality. Since we have tree parameters to determine and only two equations, it is not possible to fix the remaining arbitrary term.

The four terms of equation (40) assure the symmetry of the amplitude under the change μ ↔ ν and α ↔ β. The consequent CS-like effective action is

$$L_{CS} = \left( \frac{1}{96\pi^2} - 16i\sigma_0 \right) \kappa^2 b^{\lambda\mu} \epsilon_{\alpha\mu\lambda\rho} \partial^\rho (\partial^2 h^\alpha - \partial_\alpha \partial_\lambda h^{\lambda\alpha})$$

(43)

If we set σ_0 = 0, this result agrees with the one of reference [10] where dimensional regularization was employed. Such behavior should be expected since the surface terms are zero if explicitly evaluated by this technique.
One comment is in order. In the case of the extended QED in flat space-time, the transversality of the vacuum polarization tensor is trivially respected by the Carroll-Field Jackiw (CFJ) term, because of the presence of only one antisymmetric Lévi-Civitá tensor contracted with the external momentum. In that case, this symmetry was not an alternative to try to determine the remaining surface term. The use of an arbitrary loop momentum routing was also innocuous, since the parameter $\alpha$ also disappeared due to a product between symmetric and antisymmetric tensors, leaving the result undetermined. The case of the Lorentz-violating model in a gravitational background is different, since the satisfaction of these symmetries is not trivial. It was necessary to enforce relations among three parameters so as to satisfy them. An interesting fact to be noted is that the conditions for momentum routing invariance are sufficient to guarantee the transversality of the part of the amplitude which is linear in the vector $b_\mu$. On the other hand, the satisfaction of transversality is not enough to enforce MRI of the amplitude.

IV. CONCLUDING REMARKS

In this work we study the induction of a CS-like term by radiative corrections for a massless fermionic theory embedded in a curved spacetime with an axial-vector CPT and Lorentz violating term in the fermionic sector. We adopt the framework of Implicit Regularization which clearly parametrizes regularization dependent terms. We imposed gauge invariance through transversality of the polarization tensor and momentum routing invariance of the amplitude to fix the coefficient of the induced Lorentz-violating term. However, after enforcing these symmetries, the relations to be satisfied by the surface terms are not sufficient to determine the coefficient of the induced CS gravity term, leaving a free parameter.

This result should be compared with the one of the induction of a CFJ term in the extended QED in flat space. In that case, the satisfaction of transversality and momentum routing invariance of the amplitude is trivial due to products involving symmetric and antisymmetric tensors. This is not the case here, since it was necessary to enforce relations among three parameters so as to satisfy these symmetries. An interesting fact to be noted is that the conditions for momentum routing invariance are sufficient to guarantee the transversality of the part of the amplitude which is linear in the vector $b_\mu$. On the other hand, the satisfaction of transversality is not enough to enforce MRI of the amplitude. In other words, we verified in this case that momentum routing invariance is a more stringent condition than gauge symmetry.

V. APPENDIX

The result of the regularized integrals, after taking the trace, are:

\[
\int \frac{k^2}{k^4(k+p)^2} = I_{\log}(\lambda^2) + 2\beta - \beta \ln(-\frac{p^2}{\lambda^2}),
\]

(44)

\[
\int \frac{k^2k^\alpha}{k^4(k+p)^2} = \frac{1}{2} p^\alpha \left[ -I_{\log}(\lambda^2) + v_0 - 2\beta + \beta \ln(-\frac{p^2}{\lambda^2}) \right],
\]

(45)

\[
\int \frac{k^\alpha k^\beta}{k^4(k+p)^2} = \frac{1}{4} \eta^{\alpha\beta} \left[ I_{\log}(\lambda^2) - v_0 + 2\beta - \beta \ln(-\frac{p^2}{\lambda^2}) \right] + \frac{1}{2} \frac{p^\alpha p^\beta}{p^2},
\]

(46)

\[
\int \frac{k^\kappa k^\kappa k^\beta}{k^4(k+p)^2} = \frac{1}{12} p^{(\mu\eta^{\alpha\beta})} \left[ -I_{\log}(\lambda^2) + \xi_0 + \beta \ln(-\frac{p^2}{\lambda^2}) - \frac{5}{3} \right] - \frac{1}{3} \frac{\lambda^2 p^\mu p^\nu p^\rho}{p^2},
\]

(47)

\[
\int \frac{k^\mu k^\nu k^\kappa k^\beta}{k^4(k+p)^2} = -\frac{1}{4} \eta^{\mu\nu\kappa\beta} p^2 [I_{\log}(\lambda^2) - v_0] + \frac{1}{6} (p^2 \eta^{\alpha\beta} + 2 p^\alpha p^\beta) [I_{\log}(\lambda^2) - \xi_0] + \frac{1}{2} \frac{\lambda^2 p^\mu p^\nu}{p^2},
\]

(48)

\[
\int \frac{k^\mu k^\nu k^\kappa k^\beta}{k^4(k+p)^2} = -\frac{1}{24} \eta^{(\mu\nu\eta^{\alpha\beta})} p^2 [I_{\log}(\lambda^2) - \xi_0] + \frac{1}{48} (p^2 \eta^{(\alpha\beta\eta^{\mu\nu})} + p^{(\alpha\beta\eta^{\mu\nu})}) [I_{\log}(\lambda^2) - \xi_0 - 24\xi_0] + \frac{1}{2} \frac{\lambda^2 p^\mu p^\nu}{p^2},
\]

(49)
\[ \int \frac{k^2 k_{\mu} k_{\nu} k_{\beta}}{(k^2 + k_{\beta})^2} = \frac{1}{6} i \epsilon^{\mu \nu \alpha \beta} \frac{1}{2} \left[ I_{\text{log}}(\lambda^2) - \xi_0 \right] - \frac{1}{8} \left( p^2 p_{\mu} \epsilon^{\alpha \beta} + 2 p^{\alpha} p^{\beta} p^{\mu} \right) \left[ I_{\text{log}}(\lambda^2) - \xi_0 - 24 \sigma_0 \right] - \frac{1}{4} \bar{b} p^2 \epsilon^{\alpha \beta} \left[ \frac{1}{2} \ln \left( -\frac{p^2}{\lambda^2} \right) - \frac{4}{9} \right] + \bar{b} p^2 p^\mu \left[ \frac{1}{4} \ln \left( -\frac{p^2}{\lambda^2} \right) - \frac{7}{12} \right] , \tag{50} \]

where \( \lambda \) is mass scale and \( \bar{b} \equiv \frac{i}{(4\pi)^2} \).

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