1. INTRODUCTION

Supermassive black hole binaries formed from galactic mergers can coalesce if angular momentum loss – initially to stars and gas and subsequently via gravitational radiation (Begelman, Blandford & Rees 1980) – is fast enough to conclude the process within at most a Hubble time, or more stringently before a subsequent merger creates a 3-body system that is unstable to a black hole ejection (Valtonen et al. 1994). Due to the formation of a loss cone, stars on their own are likely to fail to meet this requirement for binary separations of the order of $r \sim 1$ pc and binary masses $M \gtrsim 10^5 M_\odot$ (Milosavljevic & Merritt 2001; Yu 2002; Berczik, Merritt & Spurzem 2005), parameters which today are characteristic of massive ($\sigma \sim 200$ km s$^{-1}$) galaxies (Gebhardt et al. 2000; Ferrarese & Merritt 2000). For this subset of galaxies, angular momentum loss to gas provides a plausible mechanism that can yield a prompt merger (Begelman, Blandford & Rees 1980; Gould & Rix 2003; Armitage & Natarajan 2002; Escala et al. 2004; 2005). At lower masses, stellar dynamic processes are in principle sufficient to effect mergers, but gas may still be involved if disks are present and remove angular momentum efficiently on a time scale significantly shorter than stellar interactions.

Gas disks that catalyze binary mergers may generate observable electromagnetic counterparts to the primary gravitational wave signature detectable with the Laser Interferometer Space Antenna (LISA). Induced accretion or outflows from small accretion disks surrounding the individual holes might yield precursors to the gravitational wave event (Armitage & Natarajan 2003), while the inflow of circumbinary gas left stranded as the binary contracts under gravitational radiation losses provides a robust prediction of an afterglow (Milosavljevic & Phinney 2005). On a longer time scale, impulsive changes to the spin of the hole following merger (Hughes & Blandford 2003) are likely to change the direction of any jets launched from the immediate vicinity of the black hole (Merritt & Ekers 2002). Such counterparts are of interest, in part, because their detection in coincidence with LISA events would overcome LISA’s limited capability to spatially localize signals, and pin down the galaxy within which a merger had occurred. Unambiguous identification of merger sites significantly enhances the potential cosmological utility of merger events (Holz & Hughes 2003; Koels et al 2005), and substantially improves the ability of LISA to trace the growth of black holes as a function of galactic environment and morphology.

In this paper, we study the evolution of binary eccentricity in the event that angular momentum loss to gas dominates the inspiral immediately prior to the onset of rapid shrinkage via gravitational radiation. In §2 we show that the gravitational interaction between the binary and a surrounding gas disk is likely to excite black hole binaries to eccentricities $e > 0.1$, a result anticipated from both analytic studies (Artymowicz 1992; Goldreich & Sari 2003), simulations of pre-main sequence binary stars (Artymowicz et al. 1991), and simulations of massive disk-embedded planets (Papaloizou, Nelson & Masset 2001). In §3 we argue that although gravitational radiation will damp these eccentricities prior to coalescence, the transition between disk-driven and gravitational wave-driven inspiral can occur at small enough radii that a small but significant eccentricity survives when the separation is only tens of gravitational radii. Detection of this eccentricity in the gravitational wave signal would provide strong circumstantial evidence for the role of gas in driving mergers, even in the absence of other electromagnetic counterparts.
We refer to these as the ‘hot’, ‘warm’ and ‘cold’ disk models of the black holes (Artymowicz & Lubow 1994). We exclude the region close to radial scaling, which mainly extends from eccentricity truncation occurs at around 2a, where a is the binary semi-major axis (Artymowicz & Lubow 1994). The binary, and the inner edge of the circumbinary disk, shrink as angular momentum is transferred from the orbit to the angular momentum of the black holes into the gas. This phase is qualitatively similar to the Type II migration (i.e. involving a gap) of massive planets, with the major difference being that a close black hole binary interacts with a relatively smaller mass of disk gas than a typical massive planet. The mismatch between the disk mass and the binary mass slows the rate of both gas inflow and orbital decay.

The first phase ends when angular momentum loss to gravitational radiation – which on its own drives decay of the semi-major axis a at a rate \( \dot{a} \propto a^3 \) – first exceeds that due to circumbinary disk torques. The binary then enters a second phase in which gravitational wave losses control the rate of inspiral. Initially, the circumbinary gas, although now dynamically unimportant, flows inward fast enough to remain in contact with the tidal barrier created by the binary. Any initial eccentricity excited during the first phase will start to damp. Eventually, however, the decay of the binary separation becomes too fast, and the binary detaches from the circumbinary gas, which through coalescence remains approximately frozen in the configuration it had at the moment of detachment. We adopt it purely for numerical simplicity. The magnitude of \( \nu \) is set by reference to the Shakura & Sunyaev (1973) viscosity prescription,

\[
\nu = \frac{\alpha c_s^2}{\Omega}
\]

(2)

We choose \( \nu \) such that the equivalent \( \alpha = 0.1 \) at \( r = 2a \), where a is the initial binary separation. Consistent with the choice of a constant viscosity, we set the disk surface density \( \Sigma(r) \) to be uniform in the initial state, so that the initial accretion rate \( M(r) \propto \nu \Sigma \) is constant with radius. The disk models with varying \( h/r \) are set up with the same initial surface density.

The dynamics of the binary, including forces from the circumbinary gas, are integrated using a simple second-order scheme that is adequate for the relatively short duration (\( t \sim 10^2 \) orbits) of our simulations. Over these time scales, realistic surface densities of the circumbinary disk are small enough that they would lead to negligible changes in the binary semi-major axis a and eccentricity e. For example, a steady-state Shakura-Sunyaev disk (Shakura & Sunyaev 1973) around a black hole of mass \( 10^6 M_\odot \), accreting at the Eddington limit with \( \alpha = 0.1 \), has a surface density at \( 10^3 r_s \) of the order of \( 10^3 \) g cm\(^{-2} \). This means that the mass in the region of the disk that interacts with the binary is only a very small fraction of the binary mass, and evolution is slow. In order to see significant evolution within of the order of 10\(^3\) orbits, we therefore need to adopt a higher scaling for the surface density. However, this does not affect the rate of eccentricity growth (or damping), measured as \( d e /dt \), because the perturbing accelerations that alter \( a \) and e are linear in the disk mass (this is evident explicitly in Gauss’ equations, given in simplified form as equations (1)-(3) of Artymowicz et al. 1991). Choosing a higher surface density scaling therefore accelerates the time scale for changes in both \( a \) and e by the same factor. Large disk masses do lead to some wandering of the binary as the eccentricity grows – we suppress this and maintain the binary center of mass at \( r = 0 \) throughout the runs.

The initial conditions are set up as a uniform disk in Keplerian rotation (including the correction due to the pressure gradient) outside \( r = 2a \), where a is the initial binary separation. Inside \( 2a \), we exponentially truncate the surface density. For the hot disk run, we evolve the disk for 40 binary orbits before turning on (smoothly over an additional 5 orbits) the backreaction from the disk in the binary integration. For the cooler disks, we start with the initial conditions for the hot disk and linearly reduce the sound speed over the first 25 orbits of the calculation until it reaches its final, fixed value. These procedures are intended to minimize dynamical time scale transients created as the initially uniform disk adjusts to the non-axisymmetric forcing from the binary. The inner edge of the circumbinary disk, for example, is able to adjust its position in response to the forcing on such a short time scale.

At the relatively small radii we are most interested in (\( r \lesssim 10^3 r_s \), where \( r_s \) is the Schwarzschild radius), angular momentum transport within the circumbinary disk probably arises from turbulence driven by the Magnetorotational Instability (MRI) (Balbus & Hawley 1991), which cannot be simulated in two dimensions. Instead we model angular momentum transport within the disk using a constant kinematic viscosity \( \nu \), which operates on the azimuthal component of the momentum equation only (Papaloizou & Stanley 1986). There is no reason to think that this, or any other simple viscosity prescription, reproduces accurately the multi-dimensional effects of angular momentum transport from MHD turbulence (Ogilvie 2003). We adopt it purely for numerical simplicity. The magnitude of \( \nu \) is set by reference to the Shakura & Sunyaev (1973) viscosity prescription,

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Considering longer time scales, however, the initial disk surface density profile— which corresponds to a steady-state solution around a single central mass—is not in equilibrium. Over a viscous time scale, gas whose inflow is impeded by tidal forces from the binary would accumulate near the inner edge of the circumbinary disk, increasing the surface density. Over the duration of our simulations, this effect is small but noticeable. It is not present for the hottest disk, which is able to overflow the gap toward the individual binary components most easily.

2.2. Results

Figure 1 shows a map of the surface density at $t = 40$ orbits for the hot disk, which surrounds a binary with mass ratio $q = M_2/M_1 = 1/3$, initial semi-major axis $a = 1.02$, and initial eccentricity $e = 0.02$. The computational domain extended from $r_{in} = 1.15$ to $r_{out} = 20$. Torques from the binary truncate the inner edge of the circumbinary disk at around $2a$, although there is significant overflow across the gap (Artymowicz & Lubow 1996) and through the inner boundary of the simulated disk. The cooler disk runs are qualitatively similar, although the thickness of the spiral arms and the extent of mass overflow both decrease as the disk becomes thinner.

Figure 2 shows the evolution of $a$ and $e$ for binaries embedded within the cold, warm, and hot disks. Once the back-reaction forces from the disk are turned on after the initial equilibration period, the eccentricity grows and the orbit decays in all three cases. As expected, the rate of orbital decay is fastest for the hottest, most viscous disk model, while eccentricity grows most rapidly for the coldest disk. This is consistent with the analytic expectation that eccentricity growth is driven by resonant interaction with gas at relatively large distance from the binary. For all three of our disk models, we find that the inner edge of the disk at the end of the equilibration period remains close to $r \approx 2a$. The dominant ac-

![Fig. 1.— Surface density at $t = 40$ orbits, just prior to starting active integration of the binary, from the hottest disk with $h/r = 0.2$. The image is $8a$ on a side, where $a$ is the initial binary separation. For this run the mass ratio $q = M_2/M_1 = 1/3$, and the initial eccentricity was $e = 0.02$.](image1)

![Fig. 2.— Evolution of the binary eccentricity (upper panel) and semi-major axis (lower panel) for simulations of binaries embedded within disks of varying geometric thickness. The eccentricity has been averaged over the orbital period to filter out small short time scale variations. The rate of eccentricity growth increases with decreasing disk thickness, while the rate of decay of the separation is reduced. Note that the normalization of the disk surface density has been chosen (arbitrarily) to yield measurable evolution over the ~100 orbit time span of the simulations.](image2)
to the equilibrium disk structure that occur over a viscous time scale \cite{Syer1993}. Viscous time scale equilibrium, which is not achieved in our simulations, is discussed more thoroughly in §3. The rate of change of eccentricity is expected to scale as \cite{Goldreich1980},

\[
\frac{d\varepsilon}{dt} \propto q^2 \dot{q} \Omega \varepsilon. \tag{5}
\]

Comparing these expressions we expect eccentricity growth to be most rapid for nearly equal mass ratio binaries in geometrically thin and/or relatively inviscid disks.

### 2.3. Comparison with previous work

Our simulations are similar in spirit to those of \cite{Armitage1991} and \cite{Gunther2004}. Using smooth particle hydrodynamics simulations, \cite{Armitage1991} derived an eccentricity growth rate \( \dot{\varepsilon}/\dot{a} \approx -4.4 \) for a binary with a mass ratio of \( q = 3/7 \) and \( e = 0.1 \), embedded within a disk in which \( h/r \approx 0.03 \) and \( \alpha \approx 0.1 \). This disk is modestly colder than our ‘cold’ case. Their growth rate, shown as an arrow in Figure 3, is approximately a factor of two greater than obtained in our ‘cold’ simulation. Our smaller growth rate is consistent with that of such a seed eccentricity is speculative, although there are enough possibilities that we think it unlikely that a binary could evade the instability by remaining perfectly circular. First, the endpoint of the prior stage of stellar dynamical hardening might leave the binary with non-zero eccentricity, although the results of \cite{Berczik2005} suggest that any such eccentricity would be small. Second, the binary could acquire eccentricity while embedded within the disk, for example via a Kozai resonance with a misaligned gas or stellar disk at larger radius in the nucleus \cite{Blaes2002}. Finally, non-axisymmetry in the disk itself – due to self-gravity at large radii \cite{Goodman2003}, eccentric instabilities in individual circumstellar disks \cite{Lubow1991}, or instabilities at the edge of the circumbinary disk \cite{Goldreich2003} – appears to be almost unavoidable. Such non-axisymmetry in the disk is readily exchanged into the binary \cite{Papaloizou2001}, and, if it is large enough, could serve as the initial seed for subsequent growth.

### 3. ECCENTRICITY OF BINARIES CLOSE TO COALESCENCE

Any eccentricity attained by the binary during the phase of gas-driven orbital decay will be damped during the final stage of gravitational wave-driven inspiral. The extent of the damping depends upon the separation of the binary (in gravitational radii) at the time when decoupling from the gas occurs. To gain a rough idea of the magnitude of the damping, we initially consider a toy model in which the binary decouples instantaneously from the disk at a radius \( a_{\text{crit}} \), which for now we regard as a free parameter. For \( a < a_{\text{crit}} \), the binary orbit evolves solely under the influence of gravitational radiation losses. For a binary with energy \( E \) and angular momentum \( J \), at such a stage \cite{Padmanabhan2001},

\[
\frac{dE}{dt} = \frac{32G^2 M_1^2 M_2^2 (M_1 + M_2)}{5e^2 a^5} \times \frac{1}{(1-e^2)^{\frac{5}{2}}} \times \left[ 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right], \tag{6}
\]

\[
\frac{dJ}{dt} = \frac{32G^2 M_1^2 M_2^2 \sqrt{M_1 + M_2}}{5e^2 a^5} \times \frac{1}{(1-e^2)^{\frac{5}{2}}} \times \left[ 1 + \frac{7}{4} e^2 \right]. \tag{7}
\]

These equations are readily integrated to yield the residual eccentricity at a fiducial time (typically either one week or one year) prior to coalescence, as a function of \( a_{\text{crit}}, q \), and the assumed initial eccentricity generated during the earlier gas-dominated phase. The formulae apply in the
weak field limit, which is appropriate provided that we do not consider times extremely close to the instant of coalescence (note that at one week before coalescence, the separation is still $a \approx 40\text{GM}/c^2$). In the strong field regime, $e$ tends to increase in anticipation of the final ‘plunge’ (Cutler, Kennefick & Poisson 1994).

Figure 4 shows the predicted binary eccentricity at one week and one year before coalescence for different $q$, as a function of the decoupling radius $a_{\text{crit}}$. An initial eccentricity $e \approx 0.25$ was assumed, which seems reasonable given our simulations and previous work (Artymowicz et al. 1991; Artymowicz 1992; Papaloizou, Nelson & Masset 2001). Binaries observed through coalescence with LISA are predicted to have a mass ratio distribution that peaks around $q \approx 0.1$. If these binaries in fact have a gas-driven inspiral phase, decoupling within $10^3$ gravitational radii typically yields a final eccentricity of $e \approx 0.01$. At fixed $a_{\text{crit}}$, binaries with more extreme mass ratios yield smaller final values of the eccentricity, reflecting the fact that at a given time prior to coalescence the separation of $q \ll 1$ systems is smaller than for the $q = 1$ case.

To go further, we need to estimate how $a_{\text{crit}}$ depends upon the binary and disk parameters. We begin by noting that, in the absence of a binary, gas in an accretion disk flows inward at a rate given by the viscous inflow speed,

$$a_{\text{visc}} = \frac{3}{2} \left( \frac{H}{r} \right)^2 \alpha \nu K. \quad (8)$$

The separation of a circular binary shrinking under the influence of gravitational radiation decays at a rate,

$$a_{\text{GW}} = \frac{64G^2M_1M_2(1+M_2)}{5c^5a^3}. \quad (9)$$

Equating these expressions determines a \textit{detachment} radius, defined via $a_{\text{visc}}(a_{\text{detach}}) = a_{\text{GW}}(a_{\text{detach}})$. The detachment radius is the smallest radius for which the inflowing gas can ‘keep up’ with the shrinkage of the binary. Interior to this radius, the binary orbit continues to contract within a growing hole while the circumbinary gas remains essentially frozen (Armitage & Natarajan 2002; Milosavljevic & Phinney 2005). The detachment radius is typically quite small. For example, Milosavljevic & Phinney (2005) estimate that the shrinking binary leaves a hole of size $\approx 120\text{GM}/c^2$, which lies well within the innermost, radiation pressure dominated disk.

For dynamical purposes we are more interested in the decoupling radius $a_{\text{crit}} - \epsilon$ within which the rate of angular momentum loss to gravitational waves exceeds that due to interaction with the circumbinary disk. To calculate this we need to set $a_{\text{GW}}$ equal to the disk-driven inspiral velocity $a_{\text{inspiral}}$. The inspiral velocity equals the viscous inflow speed in the limit where the secondary black hole is of low enough mass that it can be treated as a test particle within the disk. If the unperturbed disk surface density at the orbital radius $r$ of the secondary is $\Sigma_0$, test particle behavior will occur for $M_2 \lesssim \pi r^2 \Sigma_0$. For more massive secondaries, the large reservoir of binary angular momentum reduces the inspiral velocity to a fraction of the viscous inflow speed. The binary then acts as a slowly moving dam which impedes the free inflow of gas.

For black hole binaries the massive secondary limit is the relevant one for small enough separations. To estimate the inspiral velocity in this regime, we apply the analysis of Sver & Clarke (1995). For a disk in which the unperturbed surface density at radius $r$ is $\Sigma_0$, Sver & Clarke (1995) define a measure of disk dominance,

$$B \equiv 4\pi r^2 \Sigma_0/M_2,$$

where $M_2$ is the secondary mass. Then if the unperturbed disk solution can be locally written as,

$$\Sigma_0 \propto M_2^\beta r^\gamma,$$

with $\beta > 0$, the inspiral velocity is,

$$a_{\text{inspiral}} = a_{\text{visc}} \left( B > 1 \right)$$

$$a_{\text{inspiral}} = a_{\text{visc}} B^{\beta/(1+\gamma)} \left( B \leq 1 \right). \quad (12)$$

The correction factor in the last equation accounts for the slowdown in orbital decay when the secondary is massive compared to the local disk. The slowdown is usually less than the full factor of $B$, because gas accumulates just outside the slowly moving tidal barrier created by the binary. This pile-up increases the torque relative to that obtained from an unperturbed disk solution (Pringle 1991).

The above equations allow us to calculate the decoupling radius for a binary given an estimate for $\alpha$ and a model for the accretion disk structure. Here, we adopt the simplest Shakura-Sunyaev (1973) model. We normalize the accretion rate to the Eddington limiting value,

$$\dot{m} \equiv \frac{M}{M_{\text{Edd}}} \quad (13)$$

$$M_{\text{Edd}} = 2.2 \times 10^{8} \left( \frac{M}{M_\odot} \right) M_\odot \text{yr}^{-1}, \quad (14)$$

and measure the mass $m \equiv M/M_\odot$ in Solar units. Then in the inner, radiation pressure-dominated disk, away from the inner
boundaries (Padmanabhan 2001b).

\[ \frac{h}{r_s} = 7.46 \dot{m}_s, \]

\[ \Sigma = 0.42 \alpha^{-1} m_1^{-1} (r/r_s)^{3/2} \text{ g cm}^{-2}, \]

whereas at larger radii, where gas pressure dominates and the opacity is due to electron scattering,

\[ \frac{h}{r_s} = 1.6 \times 10^{-2} \alpha^{-1/10} m_1^{1/5} m_2^{-1/10} (r/r_s)^{21/20}, \]

\[ \Sigma = 9.7 \times 10^4 \alpha^{-4/5} m_1^{3/5} m_2^{-1/5} (r/r_s)^{-3/5} \text{ g cm}^{-2}. \]

Here \( r_s = 2GM/c^2 \) is the Schwarzschild radius, and the transition between the two regimes occurs at,

\[ \frac{r_1}{r_s} = 360 \alpha^{2/21} m_1^{16/21} m_2^{2/21}. \]

These relations allow us to calculate \( \dot{a}_{\text{inspiral}} \) in both the radiation pressure and gas pressure dominated regions of the disk. In the gas pressure dominated middle disk we can also directly estimate \( \dot{a}_{\text{inspiral}} \). In this region \( \beta \) in equation (11) is 0.6, so the slowdown in the inspiral velocity when the secondary is massive scales as \( B_0.375 \). For representative disk and binary parameters \( (M_1 = 9 \times 10^5 M_\odot, M_2 = 10^5 M_\odot, \alpha = 0.1, \dot{m} = 1) \) we find that the disk drives inspiral at a rate reduced from the viscous rate by about a factor of 50. We cannot use the same formula to calculate the slowdown in the radiation pressure dominated inner disk, since in this region \( \beta < 0 \). For an estimate, we simply continue to use the slowdown factor derived for the middle disk in this region. We note that the structure of accretion disks in the innermost radiation-dominated regime remains subject to large uncertainties (Turner 2004, Turner et al. 2005), so any statements about this region are at best semi-quantitative.

The solid curves in Figure 5 show our best estimates for the estimated eccentricity at one week and one year prior to coalescence. We take \( \alpha = 0.01, \dot{m} = 1 \), and consider binaries of varying \( q \) but fixed total mass \( M_1 + M_2 = 10^5 M_\odot \). Calculations suggest that LISA should detect a significant number of mergers in which the black holes have masses in this range (Sesana et al. 2005), whereas events involving more massive holes \( (M \sim 10^7 M_\odot) \) will be rare. As previously, we assume that \( e = 0.25 \) at the epoch when gravitational radiation becomes the dominant angular momentum loss mechanism and the binary decouples from the gas. With these assumptions, we find that for roughly equal mass binaries \( e \approx 7 \times 10^{-3} \) at one week before coalescence, while a year earlier \( e \approx 0.02 \). Higher terminal eccentricities are predicted for more extreme mass ratios, first because gravitational radiation is relatively less efficient, and second because the slowdown in disk driven inflow at small radii is smaller for a less massive secondary (equation 12). The eccentricity at one year before coalescence could exceed \( e \approx 0.1 \) for \( q \lesssim 0.02 \). We also note that there is a substantial separation between the decoupling radius – when the gas ceases to be dynamically important for the evolution of the binary – and the detachment radius when the gas and the binary become physically separated.

With these parameters the derived decoupling radius \( a_{\text{crit}} \) moves from modestly interior to \( r_s \) (for \( q = 1 \)), to well inside the radiation pressure dominated disk (for \( q \ll 1 \)). The uncertainties in calculating \( \dot{a}_{\text{inspiral}} \) in this region therefore have a direct impact on the derived values of \( a_{\text{crit}} \), and on the final estimated eccentricity. Cognizant of this, we also plot in

**Figure 5**—The predicted binary eccentricity at 1 year (upper solid curve) and 1 week (lower solid curve) prior to coalescence as a function of mass ratio \( q \). We assume that \( \dot{m} = 1, \alpha = 0.01 \), set the initial (gas-driven) eccentricity to \( e = 0.25 \), and the total system mass \( M_1 + M_2 = 10^5 M_\odot \). With these parameters, the transition between disk-driven and gravitational wave-driven orbital decay occurs close to the transition radius in the disk between radiation and gas pressure domination (for \( q = 1 \)), and moves to smaller radii for \( q \approx 1 \). Since the migration rate in the radiation pressure dominated disk is uncertain, we also plot a lower limit to the final eccentricity (dashed curves), obtained by assuming that gravitational radiation always dominates in the radiation pressure dominated disk. The dashed curves are evaluated at 1 year and 1 week prior to merger.

**4. DISCUSSION AND SUMMARY**

In this paper, we have argued that observation of the gravitational waves from merging black hole binaries could provide circumstantial evidence of the uncertain astrophysical processes that (probably) allow supermassive black hole binaries to coalesce. Specifically, we suggest that a small but non-zero eccentricity would signal that angular momentum loss to gas was the dominant decay process immediately preceding the final gravitational wave-driven inspiral. If, alternatively, interaction with stars precedes the final inspiral, the final eccentricity is likely to be very small. Most studies suggest that orbital decay via stellar dynamical processes is accompanied by circularization (Polnarev & Rees 1992, Makino 1997, Quinlan & Hernquist 1997), although Aarseth (2003) has reported eccentricity growth in some recent N-body simulations.

In the case of gas-driven mergers, eccentricity growth appears to be a robust prediction of simulations
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Artymowicz et al. [1991], Papaloizou, Nelson & Masset [2001] provided that (a) the mass ratio is not very extreme (q ∼ 0.1 appears to be safely in the ‘growth’ regime), and (b) the surrounding gas is not so hot and dense as to completely swamp the binary and thereby fill in the gap whose existence is necessary for eccentricity growth (Escala et al. 2004). One caveat is that existing simulations do not treat the angular momentum transport within the disk from first principles, and it is possible that this omission might alter the conditions needed for eccentricity growth. Direct simulations of binaries embedded within MHD turbulent disks are computationally possible (although still difficult) only in the gas pressure dominated regime (Nelson & Papaloizou 2003). Winters, Balbus & Hawley [2003], and would be valuable. There is even more uncertainty as to how the binary interacts with a radiation pressure dominated disk.

The presence of significant eccentricity for black hole binaries close to merger has additional consequences that are potentially observable. As noted by Artymowicz & Lubow (1996) both in general, and for the specific case of the quasar OJ 287, accretion from the circumbinary disk on to the individual disks around the holes is strongly time-variable if the binary is eccentric. The time scales can be interesting even when the binary is still far from coalescence. For example, a circumbinary disk surrounding a binary with $M_1 + M_2 = 10^8 M_\odot$ and $q = 1/4$ would remain gas pressure dominated until the separation was less than $\approx 860 r_s$, at which point we would predict the disk torque to dominate and the eccentricity to be significant. The inspiral time for these parameters is of the order of $10^5$ yr, but the orbital time scale is still small – approximately 1 month. This is also the time scale on which the individual disks might be expected to vary. If gas is responsible for merging a large fraction of supermassive black hole binaries, such systems, because they are much further away from merger, must be much more numerous than galaxies that will host LISA events. Although still rare, they might be identified via their characteristic X-ray variability.

The eccentricity is one of the intrinsic parameters of merging black holes that can be measured via detection of the gravitational wave emission, potentially to high accuracy. As an illustration, for a stellar mass black hole merging with a $10^8 M_\odot$ supermassive black hole from an orbit with $e = 0.1$, LISA may achieve an accuracy of $\Delta e \sim 10^{-4}$ for an event with signal-to-noise ratio of 30 (Barack & Cutler 2004). Miller et al. [2005] discuss how LISA measurements of the eccentricity of compact objects merging with supermassive black holes can be used to distinguish between different capture mechanisms. Similar measurements for mergers of two supermassive holes may allow the role of gas in the evolution of binary black holes to be studied as a function of the masses of the holes involved.

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