VARIATION OF ACOUSTIC CUTOFF PERIOD WITH HEIGHT IN THE SOLAR ATMOSPHERE: THEORY VERSUS OBSERVATIONS

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Abstract

Recently Wiśniewska et al. demonstrated observationally how the acoustic cutoff frequency varies with height in the solar atmosphere including the upper photosphere and the lower and middle chromosphere, and showed that the observational results cannot be accounted for by the existing theoretical formulas for the acoustic cutoff. In order to reproduce the observed variation of the cutoff with atmospheric height, numerical simulations of impulsively generated acoustic waves in the solar atmosphere are performed, and the spectral analysis of temporal wave profiles is used to compute numerically changes of the acoustic cutoff with height. Comparison of the numerical results with the observational data shows good agreement, which clearly indicates that the obtained results may be used to determine the structure of the background solar atmosphere.

Key words: hydrodynamics – methods: numerical – Sun: atmosphere – waves

1. INTRODUCTION

Propagation of acoustic waves in the solar atmosphere has been the subject of many analytical and numerical studies over several decades. The main goal of these studies has been to understand the transfer of wave energy from the solar convection zone, where the waves are generated, to the solar atmosphere, where they may dissipate their energy and heat the background atmosphere. The concept of acoustic cutoff frequency has played an important role in these studies, because it is this cutoff frequency that uniquely determines the propagation conditions for acoustic waves in the solar atmosphere.

The acoustic cutoff (period) frequency was originally introduced by Lamb (1909, 1910), who first considered an isothermal atmosphere and showed that the resulting cutoff frequency is global (the same over the entire atmosphere) and is defined as the ratio of the sound speed to twice the scale height of pressure or density, these heights being the same. Lamb (1910, 1932) extended his studies of acoustic waves to an atmosphere with uniform temperature gradients and demonstrated how to define the acoustic cutoff in such a non-uniform medium. Lamb’s work was followed by many others; specific applications related to solar physics were carried out by Moore & Spiegel (1964), Souffrin (1966), Summers (1976), Campos (1986), Fleck & Schnitz (1993), and more recently by Musielak et al. (2006), Fawzy & Musielak (2012), and Routh & Musielak (2014). Different expressions for the cutoff were derived analytically and used in different studies of acoustic waves in the solar atmosphere. However, a recent work by Wiśniewska et al. (2016) clearly demonstrated that these analytically obtained formulas failed to account properly for the observed variation of the cutoff with height in the solar atmosphere reported by these authors.

There are also a number of numerical studies of the propagation of acoustic waves in the solar atmosphere (e.g., Ulmschneider 1971; Ulmschneider et al. 1978; Carlsson & Stein 1997; Cuntz et al. 1998; Fawzy et al. 2002) in which the authors tried to determine the role played by acoustic waves of different frequencies in atmospheric heating. Typically, propagating acoustic waves were considered, which means that the authors did not have to dwell upon the concept of the acoustic cutoff frequency. Variations of the acoustic cutoff frequency with height in the solar atmosphere have just recently been reported by Wiśniewska et al. (2016), who performed observations using the Helioseismological Large Regions Interferometric Device operating at the Vacuum Tower Telescope located on Tenerife. The paper shows clear observational evidence for the existence of the cutoff in the solar atmosphere and its variation with atmospheric height. In previous work, Jiménez (2006) and Jiménez et al. (2011) presented variations of the cutoff with the solar cycle.

The main goal of this paper is to perform numerical simulations of impulsively generated acoustic waves in the solar atmosphere, and use the spectral analysis of temporal wave profiles to calculate numerically variations of the acoustic cutoff frequency with height. The obtained numerical results are compared with the observational data. With good agreement between the theory and data, it is concluded that the results of this paper may become a basis for using the waves to determine the structure of the background solar atmosphere.

This paper is organized as follows: our model of the solar atmosphere and numerical results are presented in Sections 2 and 3, respectively; our conclusions are given in Section 4.

2. MODEL OF THE SOLAR ATMOSPHERE

Our one-dimensional model of the solar atmosphere contains a gravitationally stratified and magnetic field-free plasma, which is described by the Euler equations with adiabatic index γ = 1.4, gravity g = (0, –g, 0) with its solar value g = 274 m s−2, and a mean particle mass m specified by a mean molecular weight 1.24. Our assumption of one-dimensionality can be justified because we consider acoustic waves propagating over the atmospheric height of 1 Mm. This height is comparable with the average size of a solar granule, which is taken to be a source of the waves.

As we aim to study a quiet solar region, we assume that initially, at time t = 0 s, low layers of the solar atmosphere are...
free of magnetic field and they are in static equilibrium (with velocity \( V = 0 \)) in which the equilibrium mass density and gas pressure are specified by a realistic, semi-empirical model of the plasma temperature \( T(y) \) developed by Avrett & Loeser (2008).

The atmospheric equilibrium described above is perturbed by a Gaussian pulse in the vertical, \( y \)-component of the velocity given by

\[
V_y(y, t = 0) = A_v \exp \left( -\frac{(y - y_0)^2}{w_y^2} \right),
\]

where \( y \) is the vertical coordinate, \( A_v \) is the amplitude of the pulse, \( y_0 \) is its initial position, and \( w_y = 50 \) km denotes its width along the vertical direction. This initial pulse corresponds to a packet of waves with its Gaussian spectrum characterized a wavenumber \( k \). Since locally a different \( k \) corresponds to a different cyclic frequency \( \omega \), we actually have a packet of waves with different \( \omega \). Once this packet propagates through the solar atmosphere, the atmosphere filters those wave frequencies that correspond to propagating acoustic waves; waves that become evanescent do not appear at greater atmospheric heights. It is this very characteristic behavior of the waves that is considered here to determine variations of the acoustic cutoff with height, and compare the numerically obtained wave periods to the observational data reported by Wiśniewska et al. (2016).

3. NUMERICAL RESULTS

We solve the equations of hydrodynamics numerically by using the PLUTO code, in which we adopted the HLLD Riemann solver and minmod flux-limiter (Mignone et al. 2012). Numerical simulations are performed in the model of the solar atmosphere described in Section 2. The simulation region is set as \(-0.5 < y < 40 \) Mm. At the bottom and top boundaries we set all plasma quantities to their equilibrium values. The region \(-0.5 < y < 6.68 \) Mm is covered by 1536 uniform grid points, while the top level is represented by 512 numerical cells that grow in size with height. Such a stretched grid works as a sponge, absorbing the incoming signal, and it results in negligibly small wave reflection from the top boundary. This very long domain and the boundary type are not relevant for this simulation, because although the waves reach the upper boundary within the time range of interest they are strongly diffused in the top region, and therefore they do not affect the wave behavior below the transition region. As a result of the initial pulse given by Equation (1), acoustic waves are generated (Figure 1, top panels) and they propagate in the model of the solar atmosphere.

As shown first by Lamb (1909), the presence of gravity leads to the appearance of the acoustic cutoff period, \( P_{ac} = 4\pi \Lambda / c_a \), where \( \Lambda \sim T \) is the pressure scale height, which becomes responsible for the propagation of the waves if their period \( P \) is smaller than \( P_{ac} \), or their evanescence if \( P \) becomes comparable to or larger than \( P_{ac} \). It must be noted that in the realistic solar atmosphere considered here, \( P_{ac} \) is a local quantity (e.g., Musielak et al. 2006; Routh & Musielak 2014) that varies significantly with height. Lamb (1909, 1932) also showed that an initial pulse results in a wavefront that propagates away from the launching region. The wavefront is followed by an oscillating wake, which oscillates at the wave period \( P_{ac} \) and whose amplitude declines in time.

We analyze the time signal of \( V_y(y, t) \) that is collected at two altitudes: \( y = 0.4 \) and \( 0.525 \) Mm (Figure 1). The leading wavefront and oscillating wake are clearly seen in the time signatures (top panels). These time signatures are analyzed spectrally to obtain power spectra (Figure 1, bottom panels) that allow us to determine the dominant wave period \( P \) for each detection point. Note that for \( y = 0.4 \) Mm the maximum of \( P \approx 220 \) s is followed by a smaller local maximum at \( P \approx 170 \) s. For \( y = 0.525 \) Mm; the second local maximum
has already become the dominant wave period, while the former maximum at \( P \approx 220 \, \text{s} \) is now a local maximum. This simply means that at \( y = 0.4 \, \text{Mm} \) most of the wave energy is associated with waves of longer period, but just above this, mainly at \( y = 0.525 \, \text{Mm} \), waves of shorter period become dominant.

Figure 2 illustrates the numerically evaluated dominant wave period, \( P \), which is plotted versus altitude \( y \); the observational data of Wiśniewska et al. (2016) are represented by diamonds, and the acoustic cutoff wave period, \( P_{ac} \), as a dashed-dotted line. The numerical data correspond to the following cases: (a) \( A_v = 0.1 \, \text{km s}^{-1} \) and \( y_0 = -150 \, \text{km} \) (asterisks), \( A_v = 0.25 \, \text{km s}^{-1} \) and \( y_0 = -150 \, \text{km} \) (squares), \( A_v = 0.1 \, \text{km s}^{-1} \) and \( y_0 = -250 \, \text{km} \) (circles), and the observational data of Wiśniewska et al. (2016) (diamonds). The dashed-dotted line represents the acoustic cutoff wave period, \( P_{ac} \).

4. CONCLUSIONS

In this paper, we simulated numerically the behavior of acoustic waves in low layers of the solar atmosphere that are free of magnetic field and invariant along horizontal directions. Our main goal was to reconcile theory with the most recent observations performed by Wiśniewska et al. (2016), who demonstrated how the acoustic cutoff varies with height in the solar atmosphere. In our approach, the waves are excited by a single initial pulse in the vertical component of velocity with amplitude \( A_v = 0.1 \, \text{km s}^{-1} \) or \( A_v = 0.25 \, \text{km s}^{-1} \), and the pulse leads to a spectrum of acoustic waves of different periods that propagate throughout the background solar atmosphere. During this propagation, the spectrum is filtered by the atmosphere, and we used its non-propagating part representing standing acoustic waves to determine the resulting acoustic cutoff period, which varies with height. The numerically obtained decreasing trend of the dominant wave period generally matches the observational data of Wiśniewska et al. (2016). The agreement clearly indicates that the obtained numerical results may be used as a basis to determine the structure of the background solar atmosphere.

Finally, we want to point out that all presented results were obtained with fixed \( \gamma = 1.4 \) and that the OPAL equation of state would lead to the adiabatic index in the solar atmosphere varying between 1.1 and 1.66 within the first 1.5 Mm above the solar surface. This variation may affect the acoustic cutoff period by 20% or less as compared to the case of constant \( \gamma \), and will probably lead to a change of similar magnitude in the wave behavior in our numerical simulations.

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REFERENCES

Avrett, E. H., & Loeser, R. 2008, ApJS, 175, 229
Campos, L. M. B. C. 1986, RevMP, 58, 117
Carlsson, M., & Stein, R. F. 1997, ApJ, 481, 500
Cuntz, M., Ulmschneider, P., & Musielak, Z. E. 1998, ApJL, 493, L117
Fawzy, D. E., & Musielak, Z. E. 2012, MNRAS, 421, 159
Fawzy, D. E., Rammacher, W., Ulmschneider, P., Musielak, Z. E., & Stepieni, K. 2002, A&A, 386, 971
Fleck, B., & Schmitz, F. 1993, A&A, 273, 487
Jiménez, A. 2006, ApJ, 646, 1398
Jiménez, A., García, R. A., & Pallé, P. L. 2011, ApJ, 743, 99
Lamb, H. 1909, Proc. London Math. Soc., 7, 122
Lamb, H. 1910, RSPSA, 34, 551
Lamb, H. 1932, Hydrodynamics (New York: Dover)
Mignone, A., Zanni, C., Tzeferacos, P., et al. 2012, ApJS, 198, 31
Moore, D. W., & Spiegel, E. A. 1964, ApJ, 139, 48
Musielak, Z. E., Musielak, D. E., & Mobashi, H. 2006, PhRvE, 73, 036612
Routh, S., & Musielak, Z. E. 2014, AN, 335, 1043
Souffrin, P. 1966, AnAp, 39, 55
Summers, D. 1976, QJMM, 29, 117
Ulmschneider, P. 1971, A&A, 14, 275
Ulmschneider, P., Schnitz, E., Kalkofen, W., & Bohn, H. U. 1978, A&A, 70, 487
Wiśniewska, A., Musielak, Z. E., Staiger, J., & Roth, M. 2016, ApJL, 819, L23