Protected Area Biodiversity Conservation: Population Dynamics

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Abstract. One of the methods for protecting the environment and conserving species biodiversity is the creation of protected areas, the task of which is to conserve rare species of animals and plants. The dynamics of populations can be studied quite effectively with the mathematical models. Modeling allows to study all possible scenarios of interaction and development of populations, to evaluate the influence of the environment on the spatial-temporal dynamics of biological communities. The paper considers a non-linear model of a protected population, the feature of which is the presence of a bilocal range and the exchange of species between its parts. It is a system of nonlinear parabolic partial differential equations that describes the dependence of migration flows on the uneven distribution of species in an area with a generalized resource. A complex of programs was coded in Python programming language for numerical study of diffusion models and visualization of the results obtained. Numerical simulation results of cross-diffusion effects and influence of generalized resource are analyzed in detail. The obtained results of research can be used for studying the processes in the protected population, for assessing the size of the protected areas being created, determining the conditions of population degeneration and recommendations for their stable existence.

1. Introduction

Industrial exploitation of natural resources leads to a change the size of the areas of various populations. For example, carrying out the gas pipeline or cutting of a glade for a power line leads to division of an area into parts. The borders of protected areas can also be considering as ecological barriers. Mathematical models of the populations distribution by area are built on the base of partial differential equations system [1, 2, 3, 4, 5, 6]. The migration of individuals is considered as diffusion. The diffusion equations are approximate by difference operators [7, 8, 9]. The introduction of nonlinear and cross-diffusion parameters allows one to describe migration with an uneven distribution of individuals. Numerical methods are used to solve non-linear problems of the space-time population dynamics. This allows for the most accurate and quick to simulate the situation using the constructed models [10, 11]. In [12], the area is regarded as a homogeneous area. In this article, we consider a generalization of this mathematical model. The parameter of a generalized resource is introduced into the system of parabolic equations. This allows us to take into account the heterogeneity of the habitat. The introduction of a favorable zone (with abundant food resources) in the protected part of the area affects the dynamics of migration throughout the area.
2. Mathematical model

Let the linear area of the population be divided under the influence of certain conditions into two parts. In a mathematical model, such an area can be represented as a straight-line segment. The model describing population development in a bilocal area defined by generalized resource has the form of a system of non-linear parabolic equations:

\[
\begin{align*}
\frac{\partial u}{\partial t} & = \frac{\partial}{\partial x} \left( \epsilon_1(u, v) \frac{\partial u}{\partial x} \right) - p_1 \frac{\partial}{\partial x} \left( u \frac{\partial v}{\partial x} \right) - q_1 \frac{\partial}{\partial x} \left( u \frac{\partial f_1}{\partial x} \right) + f_1(u, v), \\
\frac{\partial v}{\partial t} & = \frac{\partial}{\partial x} \left( \epsilon_2(u, v) \frac{\partial v}{\partial x} \right) - p_2 \frac{\partial}{\partial x} \left( v \frac{\partial u}{\partial x} \right) - q_2 \frac{\partial}{\partial x} \left( v \frac{\partial f_2}{\partial x} \right) + f_2(u, v).
\end{align*}
\]  

(1)

Here \( u(x, t), v(x, t) \) population densities, divided into two non-secured and secured, respectively. Migrational flows are defined as in work [12, 13]. The diffusion and cross-diffusion coefficients \( \epsilon_1(u, v), \epsilon_2(u, v), p_1, p_2 \) are define directed migration. A feature of this one-dimensional system is the presence of a generalized resource function \( r(x) \). The effect of the resource influence is given by the values \( q_1 \) and \( q_2 \). The maximums of the resource function curve define favorable zones in the segment. Natural growth functions of population have the form \( f_1 \) and \( f_2 \):

\[
\begin{align*}
f_1(u, v) & = m_1 u + d_1(v - u) - c_1 u^2 - a_1 u - b_1, \\
f_2(u, v) & = m_2 v + d_2(u - v) - c_2 v^2.
\end{align*}
\]

(2)

The functions \( f_1 \) and \( f_2 \) are the right-parts of the point model considered in [14, 15]. Interactions of a population parts without migration are described by the coefficients of the system (2). Fertility \( (m_1, m_2) \), internal competition \( (c_1, c_2) \) and exchange between parts \( (d_1, d_2) \), the influence of human activity \( (a_1, b_1) \) are taken into account in this equations. The signs \( m_1, m_2 \) indicates the growth or decrease of the population parts. Other coefficients are non-negative.

Stable stationary states in a system of differential equations (in point models) were studied in [14, 15]. The obtained values of the parameters from the point model allow us to describe stable scenarios in the distributed model. The values of the parameters at which a bifurcation occurs are revealed.

Diffusion coefficients \( \epsilon_1(u, v) \) and \( \epsilon_2(u, v) \) selected as nonlinear functions:

\[
\epsilon_i(u, v) = \alpha_{i1} + \alpha_{i2} uv, \quad i = 1, 2.
\]

Here \( \alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22} \) are presented as matrixes of second derivatives with non-negative elements.

On the boundaries of segments \( \Omega = [0, l] \) conditions of Dirichlet and Neumann were established. Initial values for the system (1)-(2) are given as:

\[
u(x, 0) = u^0(x), \quad v(x, 0) = v^0(x).
\]

Normal distributions functions were established as initial distributions of the population parts densities. Other functions may also serve as initial values.

Numerical solution of nonlinear differential equations for given boundary conditions can be constructed using variational or grid methods. Various methods for approximating a continuous grid function and discretization of differential operators are used, with a subsequent reduction of the solution of the boundary value problem for differential equations to the solution of a system of nonlinear equations. Numerical solve of one-dimensional problem and graphic visualization of the results has been realized in Python.

3. Results

The system (1)-(2) depicts dynamics of the population parts in non-homogeneous bilocal area. The
density distributions $u(x, t)$ and $v(x, t)$ are formed at:

$$\alpha_{11} = \alpha_{21} = 0.03, \quad \alpha_{12} = \alpha_{22} = 0.01, \quad q_1 = q_2 = 0.09,$$

$$m_1 = 0.75, m_2 = 1.1, d_1 = 0.7, d_2 = 0.5, c_1 = 0.25, c_2 = 0.65, a_1 = 0.15, b_1 = 0.35$$  \(3\)

The all bilocal area in question can possess a few zones abundant with resources. Also, may be one strip resource zone which encompasses only one or both parts of the habitat.

Figure 1 show distributions of densities $u(x, t)$ and $v(x, t)$, when the abundant resource zone mostly located in secured territories or vice versa. The calculations indicated, that in the case of different initial values $u_0(x)$ and $v_0(x)$ (fig. 1a) and parameters of system (1)-(2), the final distributions of population parts reach extremum in the abundant resource zone. Profiles of final distributions are become similar to the profile of resource function $r(x)$ (fig. 1b).

**Figure 1.** Density distributions $u$ and $v$ under one prosperous zones with one extremum, $p_1 = p_2 = 0$.

Further, under conditions of parameters (3) and absence of a flow on the boundaries of line segment, resource function $r(x)$ - defines two prosperous zones, and its influence on a migrations of a secured population is investigated (fig. 2a). Figure 2b shows that the final distributions of population parts concentrated at right part of abundant resource zone.

**Figure 2.** Density distributions $u$ and $c$ under two prosperous zones, $p_1 = p_2 = 0$.

Figure 3 shows a case of influence cross-diffusion and resource function defining two abundant zone on distributions of population parts. Final distributions reach steady state at about $t = 20$. With under $p_1 > 0$, $p_2 > 0$, the population tends to completely fill the abundant zones, no matter where they are in the habitat at initial time.
Figure 3. Influence of cross-diffusion parameters \((p_1 > 0, p_2 > 0)\) for density distributions \(u\) and \(v\) under two prosperous zones.

Figure 4 shows the final distributions under opposite signs of cross-diffusion parameters \(p_1 > 0, p_2 < 0\) \((p_1 < 0, p_2 > 0)\). The positive value of the cross-diffusion parameter means the directed migration of one part of the population towards the highest concentration of the other part, and vice versa, a negative sign of the cross-diffusion parameter of one part of the population means a directed migration to the smallest concentration of the other part. The resource function \(r(x)\) significantly affects to the final distribution of population parts.

Figure 5 shows initial and final distributions \(u(x, t)\) and \(v(x, t)\) at values \(p_1 = -0.02, p_2 = -0.03, q_1 = 0.09, q_2 = 0.05\). The resource function sets two abundant zones. In the dynamics, it turns out that population parts first occupy adjacent abundance zone. At about \(t = 20\) more distinctive extrema come to light in final distributions. The negative values of cross-diffusion parameters describe the process of population individuals scattering from each other. That is, in places where one part accumulates, a low concentration of another part is noticed. At the same time, the resource function continues to act on the dynamics of the population distribution.

The positive impact of the generalized resource on the survival of the population at strong negative anthropogenic influence is observed. In the case of population degeneration (it is revealed at bifurcation parameter value \(b_1 = 0.68\) and higher) the introduction of the resource function helps to preserve the non-zero density of the population distribution (fig. 6a). Figure 6b shows of zero density \(u(x, t)\) without resource influence at \(b_1 = 0.68\).
Figure 5. Influence of cross-diffusion parameters \((p_1 < 0, p_2 < 0)\) for density distributions \(u\) and \(v\) under two prosperous zones.

Figure 6. Influence of resource function to the final density distributions at presence of bifurcation in ODE systems, \(b_1 = 0.68\).

4. Conclusion
The protected and unprotected subpopulations distributions by area are strong depends on the resource distribution over territory. It also depends by boundary conditions in the mathematical formulation of the problem. In the case of bifurcations, the degeneration of an unguarded subpopulation slows down due to the resource presence. For a constant coefficient of cross-diffusion, a study was conducted. To describe the effects of herd formation and competition, it is necessary to introduce functional dependency into the description of directed migration.

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