Non-steady state model of global temperature change: Can we keep temperature from rising more than on two degrees?

Alexei V. Karnaukhov* and Elena V. Karnaukhova
Institute of Cell Biophysics, Russian Academy of Sciences, Puschino, Russian Federation

Elena P. Popova
Astronomy Research Center, Bernardo O’Higgins University, Santiago, Chile

Mikhail S. Blinnikov
Geography and Planning Department,
St. Cloud State University, St. Cloud, MN United States

Konstantin A. Shestibratov
M. A. Shemyakin and Yu. A. Ovchinnikov Institute of Bioorganic Chemistry,
Russian Academy of Sciences, Moscow, Russian Federation

Sergei I. Blinnikov
Institute for Theoretical and Experimental Physics,
National Research Center ”Kurchatov Institute”, Moscow, Russian Federation

Vladimir N. Reshetov
Institute for Laser and Plasma Technologies,
Moscow Engineering Physics Institute, Moscow, Russian Federation

Sergei F. Lyuksyutov*
Physics Department, University of Akron, Akron OH, United States

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Abstract

We propose a non-steady state model of the global temperature change. The model describes Earth’s surface temperature dynamics under main climate forcing. The equations were derived from basic physical relationships and detailed assessment of the numeric parameters used in the model. It shows an accurate fit with observed changes in the surface mean annual temperature (MAT) for the past 116 years. Using our model, we analyze the future global temperature change under scenarios of drastic reductions of CO$_2$. The presence of non-linear feed-backs in the model indicates on the possibility of exceeding two degrees threshold even under the carbon dioxide drastic reduction scenario. We discuss the risks associated with such warming and evaluate possible benefits of developing CO$_2$-absorbing deciduous tree plantations in the boreal zone of Northern Hemisphere.

Keywords: Climate sensitivity; non-steady state climate model; global temperature; albedo

* sfl@uakron.edu; alexeikarnaukhov@yandex.ru
I. INTRODUCTION

The need to plan specific mitigation measures for the expected global rise in land surface temperatures requires basic understanding of the climate change science for its presentation in a simple format, accessible to decision making institutions and governments worldwide. The standard approach is to use the results of 3D general circulation models (GCMs) using an ensemble of expected temperature changes under, for example, doubling of CO2, or expected change in the concentrations of all major greenhouse gases (GG). The IPCC report [1] has not changed the expected global Earth surface temperature rise of $\Delta T_{2xCO_2} = (1.5 \div 4.5)$ K.

This prediction can be written in a logarithmic form as:

$$\Delta T(t) = \sigma_{IPCC} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right)$$

where $\Delta T(t)$ – average global Earth surface temperature relative to the pre-industrial levels, $\Delta T(t) \equiv T(t) - T(t_0)$;

$\rho_{CO_2}(t_0)$ – pre-industrial CO2 concentration;

$\rho_{CO_2}(t)$ – observed level of CO2 concentration at a given moment of time $t$;

$\sigma_{IPCC} \equiv \frac{\Delta T_{2xCO_2}}{\ln 2} = (2.16 \div 6.5)$ K – climate sensitivity parameter estimated from models.

A “canonical” form of the equation (1) does not take into account some important facts known about the Earth climate system. A major weakness of GCMs is their inability to adequately quantify certain feed-backs, although the models are constantly improving. For example, Meraner et al. [2] note that the latest generation of climate models consistently exhibits an increasing equilibrium climate sensitivity (ECS) in warmer climates due to a strengthening of the water-vapor feedback with increasing surface temperatures. The increasing ECS is replicated in their work as a one-dimensional radiative-convective equilibrium model, which further shows that the enhanced water-vapor feedback follows from the rising of the tropopause in a warming climate. This feedback challenges the notion of linear radiative response of the Earth climate. We propose a non-steady state model. Colman and McAvaney [3] found that, as climate warms, climate sensitivity weakens (although not monotonically); albedo feedback weakens; water vapor feedback strengthens; and lapse rate feedback increases negatively. The latter change essentially offsets the water vapor increase. Ingram [4] proposed a new approach to quantifying the non-cloud long wave (LW) water vapor feed-backs that avoids the common problem of the feedback breakdown into lapse
rate change and direct water vapor radiative feedback near ground. We follow these authors to design a simple, yet robust, model of the Earth temperature change that accounts CO$_2$, thermal inertia of climate system, water vapor, and albedo effects. We also use our earlier results [5, 6], in which we found precise solutions to the differential greenhouse effect in the optically dense atmosphere if the surface input is negligibly small.

Our approach differs from many other in the usage of a renormalization calculation of climate model parameters that allows to explicitly include water vapor feedback [7] that is “typically neglected” [8]. Not only this increases the value of MAT, but does also lengthen the relaxation time required for the temperature curve leveling off. Our results show the substantially non-steady state response of the Earth’s climate to the current forcing that can be clearly observed.

In this work, we consider a simple zero-dimensional non-steady state model (see Eq.(2) below in Sec.II A) that can better describe the general sensitivity of the Earth’s temperature to the change of CO$_2$ concentrations using global feedbacks [8]. Unlike the spatially explicit GCMs, this conceptual model’s strength is its applicability as a working tool to a wide range of climate policy decision making institutions and climate change mitigation planning.

II. MODEL AND ITS RESULTS

A. Main parameters of the model

In terms of major parameters, our model can be simply introduced as the following differential equation:

$$\frac{d}{dt} \Delta T(t) = \frac{1}{\tau} \cdot \left( \sigma_{CO_2} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right) + \sigma_a \cdot \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1 - a} \right) - \Delta T(t) \right),$$  \hspace{1cm} (2)

where new parameters relative to Eq. (1) describe:

$\Delta a(t) = a(t) - a_0(t)$ – change in the spherical albedo of the Earth ($a \simeq a(t_0) \simeq a(t)$, $\Delta a(t) \ll a$);

$\Delta S(t) = S(t) - S_0(t)$ – change in the solar constant (power of Solar radiation falling on top of the Earth’s atmosphere $S \simeq S(t_0) \simeq S(t)$, $\Delta S(t) \ll S$);

$\sigma_{CO_2}$ – climate sensitivity to the change in CO$_2$ concentration;

$\sigma_a$ – climate sensitivity to the change in the albedo;
τ – relaxation time constant of the Earth’s climate system.

This model is described by only three key global climate parameters: $\sigma_{CO_2}, \sigma_a$, and $\tau$, which are generally considered most significant in determining temperature change [1, 9].

The starting point of the main equations of the non-stationary model of global climate is the condition of overall conservation of energy within the Earth system:

$$\frac{dT(t)}{dt} = \frac{1}{C} \cdot (W_{in}^S(t) - W_{out}^T(t)), \quad (3)$$

where $C$ is surface heat capacity, $W_{in}^S(t)$ – power of incoming short-wave solar radiation, $W_{out}^T(t)$ – power of outgoing infrared radiation.

The equation (3) can be used both for global estimate of the total heat capacity and power, and also for deriving average values per unit of area. The most common application is the latter approach. In this case, the power of incoming radiation $W_{in}^S(t)$ and outgoing radiation $W_{out}^T(t)$ can be represented as:

$$W_{in}^S(t) = (1 - a(t))S(t); \quad (4a)$$
$$W_{out}^T(t) = \sigma T_{eff}^4(t), \quad (4b)$$

where $S(t)$ – solar constant; $a(t)$ – spherical albedo; $\sigma$ – Boltzmann constant; $T_{eff}(t)$ – effective Earth temperature. The latter $T_{eff}(t)$ differs from the average global surface temperature $T(t)$ by the amount of greenhouse effect $\Delta T_G(t)$ by definition:

$$T_{eff}(t) = T(t) - \Delta T_G(t). \quad (5)$$

The amount of greenhouse effect can in turn be expressed through the concentration of greenhouse gases (GG), especially CO$_2$ and water vapor H$_2$O:

$$\Delta T_G(t) = \Delta T_G(t_0) + \sigma'_{CO_2} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right) + \sigma'_{H_2O} \cdot \ln \left( \frac{\rho_{H_2O}(t)}{\rho_{H_2O}(t_0)} \right) + \ldots, \quad (6)$$

where the corresponding values at the moment of time $t_0$ are considered as pre-industrial $\Delta T_G(t_0) \approx 35^\circ C$, $\rho_{CO_2}(t_0) \approx 280$ ppm [10]. The parametrization of the greenhouse gases effect as in (6) not only fits the traditional approach of (1), but also is a result of theoretical assessment given by both the radiation-convection models for small values of the differential greenhouse effect, $|\Delta T_G(t) - \Delta T_G(t_0)| < \Delta T_G(t_0)$, and also the radiation-adiabatic model, describing a strong greenhouse effect in the optically dense atmospheres [5, 6, 11, 12]. We will also note that the parameter $\sigma'_{CO_2}$ in (6) is a basic value of the temperature sensitivity climate constant, which can be very different from the renormalized $\sigma_{CO_2}$, used in the equation (2).
B. Derivation of the non-steady state model main equations of global climate considering an impact of the positive water vapor feedback

In order to move from the condition of energy balance to the model of (2) we must, first of all, set the starting moment of time $t_0$ (pre-industrial period), when the incoming and outgoing radiation were balanced, $W_{\text{in}}^S(t_0) - W_{\text{out}}^T(t_0)$, and we can then formulate the energy balance equation (3) as:

$$\frac{dT(t)}{dt} = \frac{1}{C} \cdot \left( (W_{\text{in}}^S(t) - W_{\text{in}}^S(t_0)) - (W_{\text{out}}^T(t) - W_{\text{out}}^T(t_0)) \right), \quad (7)$$

Using expressions (4a) and (4b) for $W_{\text{in}}^S(t_0)$ and $W_{\text{out}}^T(t_0)$ and considering that the changes of these corresponding parameters are small, we may present (7) as:

$$\frac{dT(t)}{dt} = \frac{1}{C} \cdot \left( ((1 - a(t))S(t) - (1 - a(t))S(t_0)) - (\sigma T_{\text{eff}}^4(t) - \sigma T_{\text{eff}}^4(t_0)) \right)$$

$$= \frac{1}{C} \cdot \left( ((1 - a)S) \cdot \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1 - a} \right) - 4 \sigma T_{\text{eff}}^3 \cdot \Delta T_{\text{eff}}(t) \right), \quad (8)$$

where we introduce new values of $T_{\text{eff}}$ and $\Delta T_{\text{eff}}(t)$: $T_{\text{eff}} \simeq T_{\text{eff}}(t_0) \simeq T_{\text{eff}}(t)$, $\Delta T_{\text{eff}}(t) = T_{\text{eff}}(t) - T_{\text{eff}}(t_0) \ll T_{\text{eff}}$, analogous to the earlier ones for albedo and the solar constant used in (2).

Using (4a) and (4b) and introducing $W_0 \simeq W_{\text{in}}^S(t_0) \simeq W_{\text{out}}^T(t_0)$, equation (8) can be expressed as:

$$\frac{dT(t)}{dt} = \frac{1}{C} \cdot \left( W_0 \cdot (\frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1 - a}) - 4 \frac{W_0}{T_{\text{eff}}} \cdot \Delta T_{\text{eff}}(t) \right). \quad (9)$$

It should be noted that $\Delta T_{\text{eff}}(t)$ in (8) and having (5) in mind may be expressed through the magnitude of greenhouse effect $\Delta T_G(t)$, which in turn depends on the concentration of the main greenhouse gases (6) (in this paper we only consider the two main greenhouse gases – CO$_2$ and water vapor):

$$\Delta T_{\text{eff}}(t) = T_{\text{eff}}(t) - T_{\text{eff}}(t_0) = (T(t) - T_{\text{eff}}(t)) - (T(t_0) - T_{\text{eff}}(t_0)) =$$

$$= T(t) - T(t_0) - \sigma'_{\text{CO}_2} \cdot \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) - \sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right). \quad (10)$$

With the help of this equation for $\Delta T_{\text{eff}}(t)$, and placing the multiplier $4W_0/T_{\text{eff}}$ outside the parentheses and keeping in mind that $dT(t)/dt = d\Delta T(t)/dt$, the equation (9) can be presented as:

$$\frac{d\Delta T(t)}{dt} = \frac{4W_0}{CT_{\text{eff}}} \left( \frac{T_{\text{eff}}}{4} \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1 - a} \right) - \Delta T(t) + \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + \sigma'_{\text{H}_2\text{O}} \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right) \right). \quad (11)$$
Using now:

\[ \tau' \equiv \frac{CT_{\text{eff}}}{4W_0}; \quad (12a) \]

\[ \sigma'_a \equiv \frac{T_{\text{eff}}}{4}, \quad (12b) \]

and changing the order in the right-hand side of the equation (11), we obtain the resulting equation for \( \Delta T(t) \) that differs from the basic equation (2) only by having the parameter that expresses the water vapor feedback:

\[
\frac{d}{dt} \Delta T(t) = \frac{1}{\tau'} \left( \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + \sigma'_{\text{H}_2\text{O}} \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right) + \sigma'_a \cdot \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1 - a} \right) - \Delta T(t) \right). \quad (13) \]

We note that unlike other dynamic parameters in (13), water vapor concentration in the atmosphere \( \rho_{\text{H}_2\text{O}} \) strongly depends on the mean air temperature \( \text{MAT} \) and can be excluded from (13). In order to do that, we must first express saturated water vapor pressure \( \rho_{\text{H}_2\text{O}}^{\text{s}} \), as a function of temperature, using the Boltzmann distribution:

\[
\rho_{\text{H}_2\text{O}}^{\text{s}}(T) = \rho_{\text{H}_2\text{O}}^{L} \cdot \exp \left( -\frac{\Delta E}{R \cdot T} \right), \quad (14) \]

where \( \rho_{\text{H}_2\text{O}}^{L} \) is the density of liquid water, \( \Delta E \) – molar energy of the phase change liquid to gas, \( R \) – universal gas constant. Second, we need to use a linear approximation to describe the dependence of the mean relative humidity \( \delta(T(t)) \) on the change in MAT:

\[
\delta(T(t)) = \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}^{\text{s}}(T(t))} \simeq \delta_0 - \delta_1 \cdot \Delta T. \quad (15) \]

Indeed, using (15) the parameter that is dependent on \( \rho_{\text{H}_2\text{O}}(t) \) is expressed as:

\[
\sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right) = \sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{(\delta_0 - \delta_1 \cdot \Delta T(t)) \cdot \rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0) + \Delta T(t))}{\delta_0 \rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0))} \right) =
\]

\[
= \sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{(\delta_0 - \delta_1 \cdot \Delta T(t)) \cdot \rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0) + \Delta T(t))}{\delta_0 \rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0))} \right) =
\]

\[
= \sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{\delta_0 - \delta_1 \cdot \Delta T(t)}{\delta_0} \right) + \ln \left( \frac{\rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0) + \Delta T(t))}{\rho_{\text{H}_2\text{O}}^{\text{s}}(T(t_0))} \right). \quad (16) \]

We can now transform (16) by using linear approximation: \( \ln(1 + k \cdot x) \simeq k \cdot x \), and substitution of (14) for \( \rho_{\text{H}_2\text{O}}^{\text{s}}(T) \) into the equation (16):

\[
\sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right) \simeq \sigma'_{\text{H}_2\text{O}} \cdot \left( \ln \left( \frac{\rho_{\text{H}_2\text{O}}^{L} \cdot \exp \left( -\frac{\Delta E}{R \cdot (T(t_0) + \Delta T(t))} \right)}{\rho_{\text{H}_2\text{O}}^{L} \cdot \exp \left( -\frac{\Delta E}{R \cdot T(t_0)} \right)} \right) - \frac{\delta_1}{\delta_0} \cdot \Delta T(t) \right) =
\]

\[
= \sigma'_{\text{H}_2\text{O}} \cdot \left( \ln \left( \frac{\Delta E}{R \cdot (T(t_0) + \Delta T(t))} - \frac{\Delta E}{R \cdot T(t_0)} \right) \right) - \frac{\delta_1}{\delta_0} \cdot \Delta T(t). \quad (17) \]
With $\ln(\exp(x)) = x$ and $\frac{1}{1+x} \simeq 1-x$, we finally arrive at the relatively simple expression:

\[
\sigma'_{\text{H}_2\text{O}} \cdot \ln \left( \frac{\rho_{\text{H}_2\text{O}}(t)}{\rho_{\text{H}_2\text{O}}(t_0)} \right) \simeq \sigma'_{\text{H}_2\text{O}} \cdot \left( \frac{\Delta E}{R \cdot T(t_0)} \left( 1 - \left( 1 - \frac{\Delta T(t)}{T(t_0)} \right) \right) - \frac{\delta_1}{\delta_0} \cdot \Delta T(t) \right)
\]

\[
= \sigma'_{\text{H}_2\text{O}} \cdot \left( \frac{\Delta E}{R \cdot T^2(t_0)} - \frac{\delta_1}{\delta_0} \right) \cdot \Delta T(t). \quad (18)
\]

Using (18), our model of (13), becomes:

\[
\frac{d}{dt} \Delta T(t) = \frac{1}{\tau'} \left( \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + \sigma'_{\text{H}_2\text{O}} \left( \frac{\Delta E}{R \cdot T^2(t_0)} - \frac{\delta_1}{\delta_0} \right) \Delta T(t) \right)
\]

\[
+ \sigma'_a \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1-a} \right) - \Delta T(t). \quad (19)
\]

Introducing further:

\[
k \equiv \frac{1}{1-\xi}, \quad \text{and} \quad \xi \equiv \sigma'_{\text{H}_2\text{O}} \left( \frac{\Delta E}{R \cdot T^2(t_0)} - \frac{\delta_1}{\delta_0} \right), \quad (20)
\]

the equation (19) may become even simpler:

\[
\frac{d}{dt} \Delta T(t) = \frac{1}{\tau'} \left( \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + \sigma'_a \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1-a} \right) - (1-\xi) \Delta T(t) \right)
\]

\[
= \frac{1}{\tau'} \left( \frac{1}{1-\xi} \cdot \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + \frac{1}{1-\xi} \cdot \sigma'_a \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1-a} \right) - \Delta T(t) \right)
\]

\[
= \frac{1}{k\tau'} \left( k \cdot \sigma'_{\text{CO}_2} \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) + k \cdot \sigma'_a \left( \frac{\Delta S(t)}{S} - \frac{\Delta a(t)}{1-a} \right) - \Delta T(t) \right), \quad (21)
\]

which is basically similar to our initial model equation (2) considering:

\[
\sigma_{\text{CO}_2} \equiv k\sigma'_{\text{CO}_2}, \quad (22a)
\]

\[
\sigma_a \equiv k\sigma'_a, \quad (22b)
\]

\[
\tau \equiv k\tau'. \quad (22c)
\]

Our model thus expressed highlights the significance of the water vapor feedback in making climate system highly sensitive to temperature increases in a non-linear way. Besides enhancing the role of greenhouse effect of CO$_2$ and other GG (22a), the feedback also amplifies other climate forcings, such as changes in albedo $\Delta a(t)$ and the incoming solar insolation $\Delta S(t)$, which results in renormalization of the constant of climate sensitivity (22b). Therefore, $k$ plays a role of the amplification coefficient ($\xi$ – coefficient of positive feedback).

A major consequence of the model discussed above is the sensitivity of relaxation time $\tau$, which is basically a measure of the thermal inertia of the climate system (22c).
C. Estimation of numeric values of the key model parameters

We now have three key parameters in the model (2) that must be estimated numerically: $\sigma_{CO_2}$, $\sigma_a$, $\tau$, because they determine MAT dynamic $\Delta T(t)$, under changing main climate forcings ($\rho_{CO_2}(t)$, $\Delta a(t)$, $\Delta S(t)$).

The main challenge in estimating these values is the estimate of the coefficient $k$. This is not the result of complex calculations for $\xi$ (20), rather the outcome of the mathematical formula that gives a major error in $k$ with even slight error in $\xi$ at values $\xi \approx 1$. Moreover, if $\xi \geq 1$ climate system described in the equation (21) loses stability and generates potentially unlimited runaway values for temperature.

The fact of reasonably stable climate system of Earth for the past 4 billion years despite high variability of climate parameters speaks against such values. In other words, based on the Earth’s track record, neither $\xi$, nor $k$ can be too big, or else the Earth temperature regime as we know it would have ended long time ago. Below we describe a way to estimate the current values for the three key parameters of the model ($\sigma_{CO_2}, \sigma_a, \tau$) based on the data from IPCC and NASA.

First of all, we need to estimate the original, non-renormalized climate sensitivities $\sigma'_{CO_2}, \sigma'_a$, and time constant $\tau'$.

D. CO$_2$-dependent climate sensitivity constant $\sigma'_{CO_2}$ (original value)

Changing CO$_2$ concentration $\rho_{CO_2}(t_0) \rightarrow \rho_{CO_2}(t)$ under constant MAT of the Earth surface $\Delta T(t) = 0$, albedo $\Delta a(t) = 0$ and other factors set as in (5) and (6) will result in a change in the effective temperature $\Delta T_{eff}$:

$$\Delta T_{eff}(t) = -\sigma'_{CO_2} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right). \quad (23)$$

This, in turn, will alter the intensity of the outgoing thermal radiation $\Delta W_{out}^T$, which according to (4b) and (23):

$$\Delta W_{out}^T \approx \frac{\partial W_{out}^T}{\partial T_{eff}} \Delta T_{eff} = -4\sigma_{eff}^3 \sigma'_{CO_2} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right) = -\frac{4W_{out}^T}{T_{eff}} \sigma'_{CO_2} \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right). \quad (24)$$

From (24) it follows that the CO$_2$-dependent climate sensitivity constant will be:

$$\sigma'_{CO_2} = \frac{\Delta W_{RF} \cdot T_{eff}}{4W_{out}^T \cdot \ln \left( \frac{\rho_{CO_2}(t)}{\rho_{CO_2}(t_0)} \right)}, \quad (25)$$
where $\Delta W^{RF}$ is the radiation forcing. In our case, $\Delta W^{RF} = -W^{T}_{\text{out}}$, because the reduction in the intensity of outgoing thermal radiation, from the energy balance perspective, is equivalent to an increase in incoming solar radiation forcing. To estimate $\sigma'_{\text{CO}_2}$ (Table I) we used the published values of the parameters in (25) from IPCC [1, 13] ($T_{\text{eff}} \simeq 254$ K is also published by NASA [14]).

From Table I, the estimate of the non-renormalized CO$_2$-dependent climate sensitivity is:

$$\sigma'_{\text{CO}_2} = (1.31 \pm 0.28) \text{ K}$$

(26)

E. Albedo-dependent climate sensitivity $\sigma'_a$ (non-renormalized value)

This parameter $\sigma'_a$ can be directly estimated from (12b) and the value of $T_{\text{eff}}$ from Table I:

$$\sigma'_a = \frac{T_{\text{eff}}}{4} = 63.5 \text{ K}.$$  

(27)

| $t$ | $\rho_{\text{CO}_2}(t_0)$ | $\rho_{\text{CO}_2}(t)$ | $T_{\text{eff}}$ | $W^{T}_{\text{out}}$ | $\Delta W^{RF}$ | $\sigma'_{\text{CO}_2}$ |
|-----|------------------|-----------------|----------------|-----------------|----------------|----------------|
| (year) | (ppmv) | (ppmv) | (K) | (W/m$^2$) | (W/m$^2$) | (K) |
| 1994 | 280$^a$ | 358$^a$ | 254$^b$ | 235$^c$ | 1.5$^d$ | 1.64 |
| 2011 (min) | 278$^e$ | 391$^f$ | 254$^b$ | 239$^g$ | 1.33$^h$ | 1.03 |
| 2011 | 278$^e$ | 391$^f$ | 254$^b$ | 239$^g$ | 1.68$^h$ | 1.31 |
| 2011 (max) | 278$^e$ | 391$^f$ | 254$^b$ | 239$^g$ | 2.05$^h$ | 1.59 |

$^a$ IPCC 1995 [13] (pp. 15, 78)

$^b$ NASA [14]

$^c$ IPCC 1995 [13] (p. 57)

$^d$ IPCC 1995 [13] (pp. 17, 24, 117)

$^e$ IPCC 2013 [1] (p. 467)

$^f$ IPCC 2013 [1] (p. 11)

$^g$ IPCC 2013 [1] (p. 181)

$^h$ IPCC 2013 [1] (p. 14)
F. Relaxation time constant of the Earth climate system $\tau'$ (non-renormalized value)

This parameter $\tau'$ is estimated based on (12a) and the estimates from the last IPCC report [1] (Table II). Besides that, we need to analyze the thermal heat storage capacity of the Earth surface $C$ because in (12a) the values are normalized per unit of surface area:

$$C \simeq \frac{\Delta Q_{40}}{\Delta T_{40}} \cdot \frac{1}{S_E} = \frac{1}{\Delta T_{40}} \cdot \frac{\Delta Q_{40}}{4\pi R^2_{mE}},$$

(28)

where $\Delta Q_{40}$ - change in thermal heat storage capacity and $\Delta T_{40}$ - change of the average MAT for 40 years (from 1971 to 2010); $S_E = 4\pi R^2_{mE} = 5.10 \cdot 10^{14}$ m² - surface area of the Earth; $R_{mE} = 6.371 \cdot 10^6$ m - average Earth radius [14].

Thus, the estimate of the time constant (relaxation time) $\tau'$ based on the Fifth report of the IPCC [1] will be:

$$\tau' = \frac{CT_{\text{eff}}}{4W_0} \cdot \frac{1}{n_\tau} \simeq 9.44 \text{ [5.78 to 18.1] years.}$$

(29)

G. Renormalization coefficient $k$

Direct estimate of coefficient $k$ (20) is not straightforward. The suggested method includes using empirical observations of the key climate parameters. For example, an important fact

| $\Delta Q_{40}$ | $\Delta Q_{40}/A_E$ | $\Delta T_{40}$ | $C$ | $T_{\text{eff}}$ | $W_0 \simeq W_{\text{out}}^T$ | $\tau'$ |
|-----------------|-------------------|----------------|-----|----------------|-----------------|--------|
| (J)             | (J/m²)            | (K)            | (J/m²K) | (K) | (W/m²) | (year) |
| min $\tau'$     | $19.6 \cdot 10^{22}$a | $3.84 \cdot 10^8$ | $0.56^b$ | $6.86 \cdot 10^8$ | $254^c$ d         | $239^e$ | $5.78$ |
| $\tau'$         | $27.4 \cdot 10^{22}$a | $5.37 \cdot 10^8$ | $0.48^b$ | $11.2 \cdot 10^8$ | $254^c$ d         | $239^e$ | $9.44$ |
| max $\tau'$     | $35.1 \cdot 10^{22}$a | $6.88 \cdot 10^8$ | $0.32^b$ | $21.5 \cdot 10^8$ | $254^c$ d         | $239^e$ | $18.1$ |

Note: in geophysical literature it is common to use years for time constant. Our formula (12a) gives time values in seconds. Conversion coefficient: $n_\tau = 3.16 \cdot 10^7$ sec/yr.

a IPCC 2013 [1] (p. 39)
b IPCC 2013 [1] (p. 37)
c NASA [14]
d IPCC 1995 [13] (p. 57)
e IPCC 2013 [1] (p. 181)
is the empirical curve of the growth of CO\textsubscript{2} (Fig. 1).

Fig. 1A shows that our model’s curve fits the growth of the observed CO\textsubscript{2} values rather well. The red curve is based on:

\[ \rho_{\text{CO}_2}(t) \simeq \rho_{\text{CO}_2}(t_0) + \Delta \rho_{\text{CO}_2}(t_1) \cdot \exp\left(\frac{t - t_1}{\tau_{\text{CO}_2}}\right), \quad (30) \]

where: \( \rho_{\text{CO}_2}(t_0) \simeq 280 \text{ ppm} - \text{pre-industrial concentration of CO}_2; \Delta \rho_{\text{CO}_2}(t_1) \simeq 86.34 \text{ ppm} - \text{empirically observed addition of CO}_2 \text{ by the year} 2000 (t_1 = 2000 \text{ yr}); \tau_{\text{CO}_2} \simeq 46.8 \text{ yr} - \text{time constant of CO}_2 \text{ concentration growth in the post-industrial period. Using (30), the expression } \frac{d}{dt} \Delta T(t) \text{ can be integrated over time for the growth of MAT due to the increase in CO}_2. \text{ Indeed, if } \Delta T(t) \text{ is:}

\[ \Delta T(t) = \Delta T(t_1) \cdot \exp\left(\frac{t - t_1}{\tau_{\text{CO}_2}}\right), \quad (31) \]

we can substitute it and also the expression for \( \rho_{\text{CO}_2}(t) \) (30) into (21):

\[ \frac{d}{dt} \Delta T(t) = \frac{\Delta T(t_1)}{\tau_{\text{CO}_2}} \cdot \exp\left(\frac{t - t_1}{\tau_{\text{CO}_2}}\right) \simeq \frac{1}{k \tau'} \left(k \cdot \sigma'_{\text{CO}_2} \frac{\Delta \rho_{\text{CO}_2}(t_1)}{\rho_{\text{CO}_2}(t_0)} - \Delta T(t_1)\right) \cdot \exp\left(\frac{t - t_1}{\tau_{\text{CO}_2}}\right), \quad (32) \]

and this will establish the connection between \( \Delta T(t_1) \) and \( \rho_{\text{CO}_2}(t_1) \):

\[ \Delta T(t_1) \simeq \frac{\tau_{\text{CO}_2} \sigma'_{\text{CO}_2}}{\tau' + \tau_{\text{CO}_2}/k} \cdot \ln \left(\frac{\rho_{\text{CO}_2}(t_0) + \Delta \rho_{\text{CO}_2}(t_1)}{\rho_{\text{CO}_2}(t_0)}\right). \quad (33) \]

We note that the expression (33) is true for any values of \( t_1 \) and can be considered as a universal relationship between \( \Delta T(t_1) \) and \( \Delta \rho_{\text{CO}_2}(t_1) \) analogous to (1). Comparing (33) and (1) allows us to write down the expression (34) below, which gives us an estimate for \( k \) (Table III):

\[ \sigma_{\text{IPCC}} = \frac{\tau_{\text{CO}_2} \sigma'_{\text{CO}_2}}{\tau' + \tau_{\text{CO}_2}/k} \quad \Rightarrow \quad k = \frac{\tau_{\text{CO}_2} \sigma_{\text{IPCC}}}{(\tau_{\text{CO}_2} \sigma'_{\text{CO}_2} - \tau' \sigma_{\text{IPCC}})} \simeq 10. \quad (34) \]

H. Resulting renormalized values of the three key parameters of the non-steady state model of global climate temperature change

To sum up the previous section, the following values are estimated for the three key parameters of the non-stationary model of global temperature change (2):

\[ \sigma_{\text{CO}_2} = k \cdot \sigma'_{\text{CO}_2} \approx 13.1 \text{ K} \quad (35a) \]
\[ \sigma_a = k \cdot \sigma'_a \approx 635 \text{ K} \]  

(35b)

\[ \tau = k \cdot \tau' \approx 100 \text{ yrs} \]  

(35c)

To improve our estimates for \( \sigma_{\text{IPCC}} \), we can use the instrumental temperature observations (Fig. 1C) for \( \Delta T(\text{MAT}) \). Using regression, we estimate \( \sigma_{\text{IPCC}} = 2.71 \text{ K} \), which allows a more exact value for \( \tilde{k} \) (Table III). We note that such more exact estimation of \( \sigma_{\text{IPCC}} \) can only be done within our approach, because expression (1) is only a particular case of the more general non-stationary model of global temperature change (2) within the period of rapid, quasi-exponential growth of both GHG concentrations and MAT (red curves on Fig. 1 B and C). In the traditional stationary approach the same model (1) required different estimates for \( \sigma_{\text{IPCC}} \).

The obtained more exact values of \( \tilde{k} \) allow to calculate a few scenarios of the future global temperature change based on the non-stationary model (2), for example under radical reduction of anthropogenic GHG emissions by half in 2050 and 90\% by 2100. Fig. 1A shows the rate of increase of \( \text{CO}_2 \) under such scenario. This would allow to stabilize \( \text{CO}_2 \) at about 500 ppm (Fig. 1B, grey curve), if assuming constant albedo and solar forcing \( \Delta S(t) \equiv \Delta a(t) \equiv 0 \), the main equation (2) is:

\[
\frac{d}{dt} \Delta T(t) = \frac{1}{\tau} \cdot \left( \sigma_{\text{CO}_2} \cdot \ln \left( \frac{\rho_{\text{CO}_2}(t)}{\rho_{\text{CO}_2}(t_0)} \right) - \Delta T(t) \right) .
\]  

(36)

Depending on the estimate of \( \tilde{k} \) (Table III), using formulae 22a-c we can obtain different temperature growth estimates (Fig. 1C, grey curves 1, 2, and 3).

| TABLE III. Key parameters needed to estimate coefficient \( k \) and \( \tilde{k} \). |
|-----------------------------------------------|
| \( \tau_{\text{CO}_2} \) | \( \tau' \) | \( \sigma'_{\text{CO}_2} \) | \( \sigma_{\text{IPCC}} \) | \( k \) |
| (yr) | (yr) | (K) | (K) |       |
| max \( k \) | 46.8 | 18.1 | 1.03 | 6.5 | \( \infty \) |
| \( k \) | 46.8 | 9.44 | 1.31 | 4.33 | 9.9 |
| min \( k \) | 46.8 | 5.78 | 1.58 | 2.16 | 1.64 |
| max \( \tilde{k} \) | 46.8 | 16.8 | 1.31 | 2.71 | 8 |
| \( \tilde{k} \) | 46.8 | 9.44 | 1.31 | 2.71 | 3.55 |
| min \( \tilde{k} \) | 46.8 | 5.78 | 1.38 | 2.71 | 2.6 |

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The main conclusion from our model is that the growth of global MAT is most likely going to increase 2 K, and under some scenarios, rather substantially (Curve 3 on Fig. 1C). This growth will continue due to inertia in the model even under flat or reducing emissions of CO$_2$ in the future due to non-linearity of the temperature response in our model.

FIG. 1. A. Annual increase of CO$_2$ concentration under drastic anthropogenic GHG reduction scenario (50% by 2050 and 90% by 2100 relative to 2010).

B. CO$_2$ concentration change under uncontrolled (red curve) or limited emissions (grey curve). Grey dots are MAT instrumental records from NOAA ESRL.

C. Instrumental values of MAT [15, 16] and future temperature curves under various modeled scenarios using our model (Model (36) parameters: $\sigma_{CO_2} = \tilde{k} \cdot \sigma'_{CO_2}$; $\tau = \tilde{k} \cdot \tau'$; $\sigma'_{CO_2}$; $\tau'$; $\tilde{k}$ from Table III).
III. DISCUSSION

A. Why MAT > 2 K can be significant?

An increase in MAT > 2 K by 2100 is an alarming prospect [1]. The danger is not only the resulting increases in sea levels or biodiversity crisis. At stake is the very survival of humanity, especially considering the extreme rapidity of change. Non-linear feedbacks in climate systems and probable existence of yet unknown critical thresholds [17–19] makes any direct predictions problematic. Our model highlights the importance of constraining the available fluxes of carbon in limestone carbonates (over 3·10^7 Gt), ocean pool CO_2 (4·10^4 Gt), methane hydrates (0.5 to 2.5·10^3 Gt), permafrost organics (<0.5 Gt), peat (<0.5 Gt), soil organics (<0.5 Gt) and wood biomass (about 0.6 Gt). One of the understudied feedbacks is the influence of temperature rise on trees worldwide, both temperate and tropical and, in turn, the shifting albedo values on climate as a result of tree species replacement [20]. For boreal forests of North America and Eurasia one mitigation measure may be preventative planting of fast growth species of broadleaf forest plantations (e.g., GMO poplar or aspen) in lieu of conifers [21].

B. Fast growth broad-leaf tree plantations and their possible role in climate mitigation

Such plantations were initially proposed as a way of dealing with increasing fire threats in the warming climate. However, they can also be an efficient way of rapid carbon sequestration. For example, if their total area reaches \( A_f = 5 \text{ million km}^2 \), which is about 1% of the Earth land surface, a realistic value for the boreal forest zone of the Earth, we may assess the total amount of sequestered carbon annually using (37) below.

Let us denote by \( P \) the production of lumber by these plantations. It was shown that \( P \) is approximately 20 m^3 ha\(^{-1}\) year\(^{-1}\) or 2·10^4 m^3 km\(^{-2}\) year\(^{-1}\) (\( P \) (1 km\(^2\)=100 ha) [22–24].

Considering that one m^3 of wood contains approximately 200 kg of carbon \( \chi_C \approx 200 \text{kg/m}^3 \), about 2 Gt/year of carbon could be thus sequestered we denote this quantity as(\( M_C \)): 
Two Gt of carbon equals about 20% of modern anthropogenic carbon emissions. Another benefit is the change in albedo of the boreal zone, with the estimates of broad-leaf native albedo values averaging 0.15, while coniferous forests average about 0.1 [25] (a difference of almost 35%). Matthies and Valsta [26] provided estimates of the difference between coniferous and deciduous temperate forest albedo ranging from 0.02 to 0.07.

Thus, by replacing coniferous forests with deciduous leaf tree plantations the albedo of the surface increases by about 50%. Considering that the spherical albedo of the Earth is approximately $a = 0.3$, and the terrestrial land surface contributes 25% of all outgoing short-wave radiation, we can roughly estimate the change in albedo due to plantations as

$$\Delta a \approx a \cdot 25\% \cdot 1\% \cdot 50\% = 3.75 \cdot 10^{-4}.$$  \hspace{1cm} (38)

Applying our parameters from Table III to the non-stationary model (28) we may estimate the impact of albedo change on the asymptotic value of MAT as

$$\Delta T = \tilde{k} \sigma_a' \left( \frac{\Delta a(t)}{1 - a} \right) \approx 2.6 \left( 3.55, \ 8 \right) \cdot 63.5 \text{K} \cdot \left( \frac{3.75 \cdot 10^{-4}}{1 - 0.3} \right) = 0.09 \left( 0.12, \ 0.27 \right) \text{K}. \hspace{1cm} (39)$$

The main conclusion from this consideration is that the change in albedo due to fast-growing deciduous tree plantations may trigger a constraining effect on the global temperature rising from 10% ($\tilde{k} = 2.6$) to 30% ($\tilde{k} = 8$) less as compared to the current rate of temperature increase with respect to the pre-industrial level. These would-be fast-growing tree plantations could potentially mitigate climate change by providing bio-fuels, reducing fire risks, and decreasing overall Earth surface albedo in the vast boreal zone.

**IV. SUMMARY**

In this paper we propose a non-steady state model of global temperature change as a baseline for a zero-dimensional climate model. Main equations of the model were derived considering an impact of water vapor feedback. The albedo dependent climate sensitivity

$$M_C = \chi PS_f \simeq 200 \frac{\text{kg}}{\text{m}^3} \cdot 2 \cdot 10^3 \frac{\text{m}^3}{\text{km}^2 \cdot \text{yr}} \cdot 5 \cdot 10^6 \text{km}^2 = 2 \cdot 10^{12} \frac{\text{kg}}{\text{yr}} = 2 \text{Gt/yr}. \hspace{1cm} (37)$$
and the relaxation time constant of Earth’s climate system were introduced and estimated on the basis of the derived equations.

It is highly likely that our approach may improve existing 3D General Circulation models. Our model asserts ongoing increase of the mean average temperature even after the concentration of (CO$_2$) presumably stopped or even began to decrease. This increases the probability of the global temperature exceeding 2 K limit even under aggressive constraining of greenhouse gases. Projects, such as planting rapidly growing deciduous trees in the boreal zone of Northern Hemisphere may prove beneficial to offset such temperature rising because of the albedo feed-backs. We provide an estimate of temperature increase slowdown because of the potential albedo variations. Our model is conceptually simple and thus could provide solutions for Climate community, international organizations, governments and beyond.

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- We used publicly available datasets from the IPCC 5th Assessment http://ipcc.ch/publications_and_data/publications_and_data_other.shtml and NASA http://data.giss.nasa.gov/

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