Shear Viscosity in a Gluon Gas

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The elliptic flow measurements at the Relativistic Heavy Ion Collider (RHIC) indicate that the new matter created is a nearly perfect quark gluon plasma fluid [1]. Quarks and gluons should be strongly coupled [2]. The reason for it is still open. Attempts to understand the phenomena by using perturbative QCD (PQCD), which include elastic $gg \rightarrow gg$ interactions, failed since $gg \rightarrow gg$ interactions cannot even drive the system toward thermalization [3]. Also, the shear viscosity to entropy ratio for elastic processes [4, 5] is much larger due to interactions among strongly coupled systems, which suggest strongly coupled systems, are due to the gluon bremsstrahlung incorporated.

In this Letter we first derive a useful formula for the shear viscosity coefficient to the recently introduced transport rate is derived within relativistic kinetic theory. We calculate the shear viscosity over entropy ratio $\eta/s$ for a gluon gas, which involves elastic $gg \rightarrow gg$ perturbative QCD (PQCD) scatterings as well as inelastic $gg \rightarrow ggg$ PQCD bremsstrahlung. For $\alpha_s = 0.3$ we find $\eta/s = 0.13$ and for $\alpha_s = 0.6$, $\eta/s = 0.076$. The small $\eta/s$ values, which suggest strongly coupled systems, are due to the gluon bremsstrahlung incorporated.

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where the neglected terms are of second order in spatial gradients when assuming that $f_1$ in (13) only contains terms of first order in spatial gradients. Following $\partial_\mu T^{\mu\nu} = 0$, Eq. (10) is rewritten in the rest frame

\[ v^\mu \partial_\mu f \simeq f v^\mu \partial_\mu \ln \lambda + f_0 (1 \pm f_0) \frac{\partial \beta}{\partial \epsilon} \]

\[ \times [\delta_0 (\ln \lambda) E \epsilon + 3 \delta_0 (\ln \lambda) P^\mu T^{\mu\nu}] \]

\[ + f_0 (1 \pm f_0) \beta E v^\nu v^\nu \]

\[ \times \left[ \frac{1}{2} (\partial_i u^j + \partial_j u^i) - \frac{1}{3} \delta_{ij} \vec{\nabla} \cdot \vec{u} \right], \quad (11) \]

where $i, j = 1, 2, 3$ and the equation of state $\epsilon = 3 P$ is used for massless particles. For $\lambda = 1$ Eq. (11) is identical with the term derived by Eckard et al. in [4].

Integrating Eq. (11) over momentum using (11) gives

\[ \int \frac{d^3 p}{(2\pi)^3} I = \int \frac{d^3 p}{(2\pi)^3} v^\mu \partial_\mu f \simeq \frac{1}{4} n \partial_\mu (\ln \lambda), \quad (12) \]

where

\[ n = \int \frac{d^3 p}{(2\pi)^3} f \quad (13) \]

is the particle density. In the derivation for (12) we assumed that there is no particle flow in the rest frame following Eckard’s definition of $u^\mu$ [11]. Integrating (11) by weight $v_z^2$ gives

\[ \int \frac{d^3 p}{(2\pi)^3} v_z^2 I = \int \frac{d^3 p}{(2\pi)^3} v_z^2 v^\mu \partial_\mu f \]

\[ \approx \frac{2}{15} n \left( 3 \partial_z u_z - \vec{\nabla} \cdot \vec{u} \right) + \left( \frac{1}{4} - \langle v_z^2 \rangle \right) n \partial_\mu (\ln \lambda), \quad (14) \]

where $\langle \cdot \rangle$ denotes the average over particles. We then obtain the relation

\[ 3 \partial_z u_z - \vec{\nabla} \cdot \vec{u} \simeq \frac{15}{2} \left( \frac{1}{3} - \langle v_z^2 \rangle \right) \left( \sum R^{tr} + \frac{3}{4} n \partial_\mu (\ln \lambda) \right), \quad (15) \]

where

\[ \sum R^{tr} = \int \frac{d^3 p}{(2\pi)^3} v_z^2 I - \langle v_z^2 \rangle \int \frac{d^3 p}{(2\pi)^3} I \]

\[ n \left( \frac{1}{4} - \langle v_z^2 \rangle \right), \quad (16) \]

is the total transport collision rate, which was introduced in (10) as the characteristic quantity describing momentum isotropization. In kinetic equilibrium, Eq. (15) becomes an exact equation.

Inserting (15) into (7) we obtain

\[ \eta \simeq \frac{1}{5} n \left( \frac{E}{\epsilon} - \langle v_z^2 \rangle \right) \sum R^{tr} + \frac{1}{4} n \partial_\mu (\ln \lambda). \quad (17) \]

This expression constitutes our major formula calculating the shear viscosity coefficient $\eta$ and gives a direct correspondence of $\eta$ to the transport rate $R^{tr}$: $\eta$ is inversely proportional to the sum of the total transport collision rate and the chemical equilibration rate, and is roughly proportional to the energy density. If the chemical equilibration is governed by $2 \leftrightarrow 3$ processes,

\[ \frac{3}{4} n \partial_\mu (\ln \lambda) = 3 \int \frac{d^3 p}{(2\pi)^3} I = \frac{3}{2} R_{23} - R_{32}, \quad (18) \]

which might even become negative for systems with over-saturation. However, if the system is not far away from chemical equilibrium, the total transport collision rate is most dominant and, thus, determines the shear viscosity.

If $f$ or $f_1$ in (13) is known and $f_1 \neq 0$, $\eta$ can be in principle calculated using (7). A way to get $f_1$ is to solve the linearized Boltzmann equation

\[ f v^\mu \partial_\mu \ln \lambda - f_0 (1 \pm f_0) v^\mu \partial_\mu (\beta u_\nu p^\nu) = I (f_1) \quad (19) \]

as a variational problem for $f_1$ [11]. Often used is a simple ansatz [1, 12] for $f_1$ such as the function (21) below. One notices that $\eta$ obtained from (7) using $f$ from (19) is identical with that from (17). Another more complicated method to get $f$ is to solve the Boltzmann equation (11) numerically performing extensive transport simulations, which is, in principle, more reliable than the first method except for numerical uncertainties. Since the spatial gradients are needed in (7) and their extractions from transport simulations are difficult, it is at present more convenient to use (17) to calculate $\eta$. Such calculations will be presented in a forthcoming publication.

In this Letter we consider a static particle system, which is initially disturbed from equilibrium and is relaxing again to thermal equilibrium. For a Boltzmann gas in chemical equilibrium $f$ is assumed to have the form [13]

\[ f = e^{\frac{-\beta \sqrt{E^2 + \chi}}{2}} \]

(20)

in the rest frame. For small $\chi$

\[ f \approx e^{\frac{-\beta E}{2}} \left( 1 - \frac{\chi}{2} \beta \frac{p^2}{E} \right). \]

(21)

Comparing (21) with (13) $f_1 = -\frac{1}{2} \beta \frac{p^2}{E}$, which satisfies Eq. (19) with appropriate $\chi$. Using (21) in (17) we calculate $\eta$ in thermal equilibrium as the limit when letting $\chi \to 0$. In this limit Eq. (17) becomes an exact equation identical to Eq. (7). We obtain

\[ \eta = \frac{4}{15} \epsilon l^{tr}, \]

(22)

where $l^{tr}$ is the inverse of the total transport collision rate and is called the mean transport path.

We now apply Eq. (22) to calculate the shear viscosity in a gluon gas. For the sake of simplicity we assume gluons to be Boltzmann particles. The boson enhancement is neglected, which is a good approximation for gluons
at high temperatures. Interactions among gluons include elastic $gg \rightarrow gg$ PQCD scatterings and inelastic $gg \rightarrow ggg$ PQCD bremsstrahlung processes, which are screened by the Debye mass

$$m_D^2 = \frac{\pi d_G \alpha_s}{(2\pi)^3} \int \frac{\beta G}{N_c N_f} \frac{d \beta}{\beta} = \frac{1}{2\pi} d_G N_c \alpha_s \frac{1}{\beta^2},$$

where $d_G = 16$ denotes the gluon degeneracy factor and $N_c = 3$. The Landau-Pomeranchuk-Migdal suppression of bremsstrahlung is taken into account as a lower cutoff in the momentum of the radiated gluon $[14]$. The matrix elements of the transitions can be found in [13, 10]. We obtain $\sum R^{tr}$ in thermal equilibrium as the term in 0th order of $\chi$ in Eq. (16) when using Eq. (21) for the integrals. Because the collision term is additive, $\sum R^{tr} = R^{tr} = R^{tr}_{gg \rightarrow gg} + R^{tr}_{gg \rightarrow ggg} + R^{tr}_{gg \rightarrow ggg}$, where $R^{tr}_{gg \rightarrow gg} = \frac{2}{3} R^{tr}_{gg \rightarrow ggg}$ due to detailed balance $[10]$.

The shear viscosity to entropy ratio at equilibrium is

$$\eta/s = \left(5\beta \sum R^{tr} \right)^{-1} = \left(5\beta R^{tr}_{gg \rightarrow gg} + \frac{25}{3} \beta R^{tr}_{gg \rightarrow ggg} \right)^{-1},$$

where $s = \frac{4}{3}\beta \epsilon$ is used. Because the transport collision rates scale with the temperature, $\beta R^{tr}_{gg \rightarrow gg}$, $\beta R^{tr}_{gg \rightarrow ggg}$ and, thus, $\eta/s$ depend only on the coupling constant $\alpha_s$.

The upper panel of Fig. 1 shows the shear viscosity to entropy ratio for $gg \rightarrow gg$ and $gg \rightarrow ggg$ processes, respectively. $\eta/s$ for $gg \rightarrow ggg$ processes is roughly a factor of 7 smaller than that for the elastic collisions, which implies that compared to the elastic scatterings the PQCD inspired bremsstrahlung is the leading process in relaxing the system to equilibrium. For $\alpha_s = 0.3$, which might be appropriate at RHIC energy, $\eta/s = 1.03$ for $gg \rightarrow gg$ only and $\eta/s = 0.13$ when including $gg \rightarrow ggg$ processes. To match the lower bound of $\eta/s = 1/4\pi$ from the AdS/CFT conjecture $[6]$ $\alpha_s = 0.6$ has to be chosen. The $\eta/s$ ratios in the two cases correspond to the mean transport path $t^{tr} = 1/\sum R^{tr} = 0.32$ fm and $t^{tr} = 0.2$ fm at $T = 1/\beta = 400$ MeV for $\alpha_s = 0.3$ and $\alpha_s = 0.6$, respectively. From the collision rates shown in the lower panel of Fig. 1 with $n = d_G T^3/\pi^2$ we obtain $\langle v_{rel} \sigma_{gg \rightarrow gg} \rangle = $ $R_{gg \rightarrow gg}/n = 0.82$ mb and $\langle v_{rel} \sigma_{gg \rightarrow ggg} \rangle = $ $R_{gg \rightarrow ggg}/n = 0.57$ mb for $\alpha_s = 0.3$, and $\langle v_{rel} \sigma_{gg \rightarrow ggg} \rangle = 1.27$ mb and $\langle v_{rel} \sigma_{gg \rightarrow ggg} \rangle = 0.73$ mb for $\alpha_s = 0.6$ at $T = 400$ MeV. Hence, perturbative interactions can drive gluons to behave like a strongly coupled system with a small $\eta/s$ ratio at RHIC.

From Fig. 1, we also obtain that $R^{tr}_{gg \rightarrow gg}/R_{gg \rightarrow gg} = 0.36(0.46)$ and $R^{tr}_{gg \rightarrow ggg}/R_{gg \rightarrow ggg} = 2.1(2.7)$ for $\alpha_s = 0.3(0.6)$, respectively. The wide difference in the behavior of the $R^{tr}/R$ ratio for the $gg \rightarrow gg$ and $gg \rightarrow ggg$ processes is essential for the different contributions to $\eta/s$. Because $R^{tr}$ contains an indirect relationship with the distribution of the collision angle $\theta$, we decompose the transport collision rate to

$$R^{tr}_{gg \rightarrow gg} = A_i n \langle v_{rel} \sigma_i \rangle, \quad i = gg \rightarrow gg, \; gg \rightarrow ggg,$$

with $\sigma_i = \int d\sigma \sin^2 \theta$ defined as the transport cross section $[13]$ and $A_i$ being a multiplication factor. Figure 2 shows $\beta n \langle v_{rel} \sigma_i \rangle$ and $A_i$ as function of $\alpha_s$. For

![FIG. 1: Upper panel: Shear viscosity to entropy ratio for $gg \rightarrow gg$ and $gg \rightarrow ggg$ processes. $\eta/s$ for $gg \rightarrow gg$ is divided by a factor of 7. Lower panel: Collision rate to temperature ratio for $gg \rightarrow gg$ and $gg \rightarrow ggg$ processes.](image1)

![FIG. 2: Scaled transport cross section (upper panel) and transport collision rate to scaled transport cross section ratio (lower panel) for $gg \rightarrow gg$ and $gg \rightarrow ggg$ processes.](image2)
\( \alpha_s = 0.3(0.6) \langle \nu_{\text{rel}} \sigma_{gg-gg}/\nu_{\text{rel}} \sigma_{gg-gg} \rangle = 0.3(0.35) \) and \( \langle \nu_{\text{rel}} \sigma_{gg-ggg}/\nu_{\text{rel}} \sigma_{gg-ggg} \rangle = 0.71(0.78) \), which indicate that for the chosen \( \alpha_s \) values \( gg \to gg \) favors small-angle scattering and \( gg \to ggg \) favors large-angle bremsstrahlung.

The factors \( A_i \) in Eq. (25) have weak dependences on \( \alpha_s \) and are around the isotropic distribution values. (For isotropic angular distribution \( A_{gg-gg} = 9/8 \) and \( A_{gg-ggg} = 27/16 \).) For \( \alpha_s = 0.3(0.6) \) we obtain \( A_{gg-ggg}/A_{gg-gg} = 2.5(2.7) \), which are significantly larger than 1.

Parametrically, \( \beta R_{gg-gg}^{\text{tr}} \) is fitted by (0.68 + 2.8\( \alpha_s \))\( \alpha_s^2 (\ln \alpha_s)^2 \) from \( \alpha_s = 0.001 \) up to 0.3, whereas \( \beta R_{gg-gg} \approx (3 - 36\alpha_s/\pi)\alpha_s \) for \( \alpha_s < 0.1 \). Thus, \( R_{gg-gg}^{\text{tr}}/R_{gg-gg} \sim O(\alpha_s) \) for \( \alpha_s < 0.1 \), which indicates again small-angle \( gg \to gg \) scatterings, because for those scatterings \( R_{gg-gg}^{\text{tr}}/R_{gg-gg} \sim \theta^2 \) with \( \theta^2 \sim 4q^2/sm \approx 4m_3^2/s_m \sim \alpha_s \), where \( q \) is the momentum transfer.

Including the Landau-Pomeranchuk-Migdal effect for the PQCD inspired bremsstrahlung within the Bethe-Heitler regime \( \beta R_{gg-gg} \sim \alpha_s^2 (\ln \alpha_s)^2 \). For small-angle bremsstrahlung \( \beta R_{gg-gg} \) should be of the order \( \alpha_s^3 (\ln \alpha_s)^2 \), which is clearly not the case for \( \alpha_s > 0.1 \) as seen from Fig. 2 as within this \( \alpha_s \) range the transport cross sections for \( gg \to gg \) and \( gg \to ggg \) have almost the same \( \alpha_s \) dependence, namely \( O(\alpha_s^3 (\ln \alpha_s)^2) \). Thus, \( R_{gg-ggg}^{\text{tr}}/R_{gg-gg} \sim O(1) \) for \( \alpha_s > 0.01 \), which shows that in the chosen \( \alpha_s \) interval the bremsstrahlung favors isotropic angular distribution. For smaller \( \alpha_s \), however, the collision angle tends to be distributed forward. This can be observed in Fig. 2 where \( \beta \eta_n \langle \nu_{\text{rel}} \sigma_{gg-gg}^{\text{tr}} \rangle \) becomes steeper than \( \beta \eta_n \langle \nu_{\text{rel}} \sigma_{gg-gg} \rangle \) at \( \alpha_s < 0.01 \). This implies that the “collinear” bremsstrahlung where \( R_{gg-ggg}^{\text{tr}}/R_{gg-gg} \sim O(\alpha_s) \) will occur at extreme small \( \alpha_s \) and only then will have smaller contribution to the transport coefficients than the \( gg \to gg \) processes.

Finally, the ratio of the collisional width to the mean gluon energy, \( \Gamma/\langle E \rangle = (R_{gg-gg} + R_{gg-ggg} + R_{gg-ggg})/3/\langle E \rangle \), can be calculated from the lower panel of Fig. 1 where \( R_{gg-gg} = 1.5 R_{gg-ggg} \) due to detailed balance. For \( \alpha_s = 0.3(0.6) \) we obtain \( \Gamma/\langle E \rangle = 0.5(0.69) \). These ratios are smaller than, but, close to 1, which indicates that PQCD with \( \alpha_s = 0.3-0.6 \) is at the edge of its applicability. For larger values of \( \alpha_s \) one obtains \( \Gamma > \langle E \rangle \) and the PQCD calculations with on shell kinematics are no longer applicable, as for such a strong coupling regime the Heisenberg uncertainty principle has to be taken care of by a full quantum transport treatment.

The higher order processes such as \( ggg \to ggg \) and \( gg \leftrightarrow gggg \) will certainly modify the total transition rate. However, their contributions are suppressed by higher order of \( \alpha_s \). Because the full diagrammatic many-body theory for higher order collisions becomes rather complex, at present, the incorporation of higher order multiparticle interactions is considered in a phenomenological manner.

In summary, the shear viscosity is derived within relativistic kinetic theory, which is proportional to energy density and inversely proportional to the total transport collision rate. We calculated the shear viscosity to entropy ratio \( \eta/s \) for a gluon gas, and found \( \eta/s = 0.13(0.076) \) for \( \alpha_s = 0.3(0.6) \). Perturbative QCD interactions can drive the gluon matter to a strongly coupled system with an \( \eta/s \) ratio as small as the lower bound from the AdS/CFT conjecture. The PQCD inspired gluon bremsstrahlung is responsible for small \( \eta/s \) ratios, and, thus, can explain that the quark gluon plasma created at RHIC behaves like a nearly perfect fluid.

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