1. INTRODUCTION

The most reliable method of determining the initial spin period of a pulsar is to have a historical association of the supernova explosion, combined with a measurement of the pulsar braking index: for the Crab, Lyne et al. (1993) estimated a birth period of $P_0 = 19$ ms, whereas a value of $P_0 = 63$ ms was determined for PSR B1509–58. The pulsar current technique was introduced by Phinney & Blandford (1981) and Vivekanand & Narayan (1981) but suffers from small-number statistics (Lorimer et al. 1993). Narayan (1987) and Emmering & Chevalier (1989) proposed a technique based on the pulsar luminosity function technique, injection, and the presence of pulsar wind nebulae (PWNe), so that injection at $P_0 = \sim \text{a few hundred milliseconds}$ would result in the absence of a PWN. Van der Swaluw & Wu (2001) have shown that the PWN radius is a function of the birth period of a pulsar, and that the ratio of this radius relative to the supernova remnant forward-shock radius provides a measure of the birth period as well.

In this Letter we show that high-energy $\gamma$-ray observations of PWNe in the GeV domain can also give us a measurement of the birth period of the associated PWNe. The reasoning behind this is simple: the relatively low energy leptons radiating GeV $\gamma$-rays via inverse Compton scattering on the cosmic microwave background (CMBR) and the Galactic radiation fields do not suffer from radiation losses as, e.g., X-ray synchrotron emitting electrons do. In this case the GeV flux should be proportional to the total energy in leptons ejected since birth, which in turn is a fraction $\eta$ of the total rotational kinetic energy $E_{\text{rot}} = I(\Omega_0^2 - \Omega^2)/2$ released since birth. Here $\Omega_0 = 2\pi/P_0$ and $\Omega$ are respectively the birth and current pulsar angular frequencies.

Normally such a technique would suffer from uncertainties in PWN parameters, but we will show that if we assume a particle injection spectral index of 2 above the radio domain, the expected GeV flux can be expressed as a simple analytic formula, in which case the uncertainty in $P_0$ scales linearly with the uncertainty in the distance, but less sensitively on other less known parameters such as $\eta$, the maximum particle energy $\gamma_{\text{max}}m_e^c$, and the intrinsic spectral break energy $\gamma_b m_e^c$ above the radio domain. The latter typically divides the injection spectrum into a $\sim \gamma^{-2}$ component above this break, but a much harder spectrum at $\gamma \ll \gamma_b$ to account for the flat spectra seen from radio PWNe. The requirement for such an intrinsic break in the Crab Nebula and other PWNe was discussed respectively by Atoyan & Aharonian (1996) and Chevalier (2004).

Whereas such an analytical expression helps us with population studies and to get approximate estimates for $P_0$, a GLAST LAT detection with a clear multiwavelength PWN association should not rely on this expression to obtain an accurate estimate of $P_0$; the multiwavelength synchro-Compton spectrum should give us more information about particle spectra, break energies, and conversion efficiencies through proper numerical modeling, although the principle remains the same as expressed below.

2. THE ELECTRON SPECTRUM CONTRIBUTING TO THE GLAST LAT SPECTRAL RANGE

We adopt the injection spectrum of Venter & de Jager (2006) for electrons at the pulsar wind shock,

$$Q(\gamma, t) = \begin{cases} \left(\frac{Q_0(t)}{\gamma_{\text{break}}(t)}\right)^{-\alpha} & \text{for } \gamma < \gamma_{\text{break}}(t) \ , \\ \left(\frac{Q_0(t)}{\gamma_{\text{break}}(t)}\right)^{-\alpha} & \text{for } \gamma_{\text{break}}(t) < \gamma < \gamma_{\text{max}} \ , \end{cases}$$

with $\gamma_{\text{break}}m_e^c$ the intrinsic break energy. For $\gamma_{\text{break}} \sim 10^{14}$ we retrieve the typical flat radio spectra of PWNe where $F_\gamma \propto \nu^{-\alpha}$ with $\alpha \sim 0$. The high-energy spectrum is steeper and can extend up to ultrahigh energies ($\gamma_{\text{max}}m_e^c \sim 10^{14}$ to $>10^{15}$ eV). The cor-

1 South African Department of Science and Technology and National Research Foundation Research Chair: Astrophysics and Space Science.
responding spectral index $p_2 \sim 2$ as a result of particle acceleration at or near the pulsar wind shock. This is supported by observations at X-ray and very high energy $\gamma$-ray energies: At the pulsar wind shock (radius $R_\bullet$) we observe X-ray and VHE $\gamma$-ray photon indices $\Gamma_\gamma = (p_2 + 1)/2 \sim 1.5$, whereas for $r \gg R_\bullet$, the electron spectrum cools due to synchrotron losses ($p_2$ increases by unity), resulting in a value of $\Gamma_\gamma = (p_2 + 2)/2 \sim 2$ as illustrated by de Jager & Djannati-Ataï (2008), who combined the results from several PWNe in the form of a plot of $\Gamma_\gamma$ versus $\Gamma_\gamma$. This reveals a cluster near $\Gamma_\gamma \sim 1.5$, evolving to $\Gamma_\gamma \sim 2$. In this Letter we will assume $p_2 = 2$, resulting in a logarithmic suppression of uncertainties in $\gamma_\gamma$ and $\gamma_{\rm max}$, which is important for a relatively accurate determination of the birth period. Note however that there are deviations: There are a few cases for which $p_2 \sim 3$, in which case a multiwavelength/numerical modeling approach is called for, as discussed in § 1.

In general we find a critical energy $\gamma_{\rm crit}m_e c^2$, such that radiation losses are unimportant for $\gamma < \gamma_{\rm crit}$ (the “uncooled” domain) and $R_\bullet < r < R_{\rm PWNe}$ (with $R_{\rm PWNe}$ the PWN radius), in which case the total observed spectrum is reflected by the injection indices $p_\gamma$ and $p_2$. For $\gamma \gg \gamma_{\rm crit}$ (the “cooled” domain), the corresponding electron spectral index steepens to $p_2 + 1$ due to radiative cooling, in which case total nebular photon spectra of the form $E_\gamma^{-2}$ are observed. An estimate of $\gamma_{\rm crit}$ can be obtained by setting the timescale for radiative losses (assuming isotropic lepton pitch angles) equal to the age $\tau_{\rm age}$ of the region and by rewriting this electron energy in terms of its inverse Compton scattered $\gamma$-ray energy, giving (assuming scattering on the CMBR)

$$E_\gamma(\text{rad}) = \frac{4}{3}(2.7\,kT)\gamma_{\text{rad}}^{\frac{1}{2}} \sim (0.50\,\text{TeV})\left(\frac{B}{10\,\mu G}\right)^{-1} \left(\frac{\tau_{\text{age}}}{10\,\text{k yr}}\right)^{-2}.$$  

(2)

For $B < 3\,\mu G$ this expression must be corrected for inverse Compton losses on the CMBR. The spectral breaks of the two mature H.E.S.S. sources HESS J1825−137 [$E_\gamma(\text{rad}) \sim 2\,\text{TeV}$, $\tau_{\text{age}} = 20\,\text{k yr}$; Aharonian et al. 2006b] and the cocoon of Vela X [$E_\gamma(\text{rad}) \sim 13\,\text{TeV}$, $\tau_{\text{age}} = 11\,\text{k yr}$; Aharonian et al. 2006a] can be explained with field strengths between 4 and 5 $\mu G$. Furthermore, the relatively faint X-ray synchrotron flux from these sources support such low field strengths (see also de Jager & Djannati-Ataï 2008). The newly detected PWN PSR J1718−3825 also shows a spectral break at 7 TeV (Aharonian et al. 2007), and the discovery of an X-ray PWNe around the associated pulsar PSR J1718−3825 implies a field strength $<5\,\mu G$ (Hinton et al. 2007). For a characteristic age of 90 kyr the predicted break energy is however in the multiple GeV range. If the birth period is not much less than the current 75 ms period, it would be possible to reproduce the break (Hinton et al. 2007), but for this we need to know the pulsar braking index.

Assuming evolved PWNe with radiation breaks not lower than 10 GeV and $B \sim 5\,\mu G$, GLAST LAT should be able to detect PWNe between the intrinsic break energy and $\sim 10$ GeV with a (“uncooled”) photon index near 1.5 if the age

$$\tau_{\text{age}} \gtrsim (280\,\text{k yr})\left(\frac{B}{5\,\mu G}\right)^{-2}.$$  

(3)

Having set the constraints for detecting the specified injection spectral indices, we can now proceed with the high-energy flux calculations: Following Sefako & de Jager (2003) and Venter & de Jager (2006) the energy equation can be written in terms of the present-day spin-down power $L(t)$ giving

$$m_e c^2 \int Q(\gamma_\gamma, t)\gamma_\gamma d\gamma = \eta L(t).$$  

(4)

From this expression the normalization constant $Q_\gamma(t)$ for $p_2 = 2$ follows,

$$Q_\gamma(t) = \frac{\eta L(t)}{m_e c^2 \gamma_{\text{crit}}^{p_2}}, \quad \text{with } a = \frac{1}{2} - p_1 + \ln \left(\frac{\gamma_{\text{max}}}{\gamma_{\gamma}}\right),$$  

(5)

and $\eta$ the conversion efficiency of spin-down power into energetic particles. Since we observe at electron energies well below $\gamma_{\gamma}$ (assuming escape losses are also unimportant), the total nebular particle spectrum at $r > R_\bullet$ can be derived from the transport equation to give

$$\frac{dN(\gamma_\gamma, \tau_{\text{age}})}{d\gamma} = \int_{0}^{\tau_{\text{age}}} Q(\gamma_\gamma, t) dt \approx \frac{\bar{\Delta} E_{\text{red}}(kT)}{a m_e c^2 \gamma_{\gamma}^{-2}}.$$  

(6)

The time integral $\int L(t) dt$ up to the present age is given by the integral in rotational kinetic energy $\Delta E_{\text{rot}} = h(\Omega_0^2 - \Omega^2)/2 = 2\pi^2 h(1/R_{\bullet}^3 - 1/P^2)$.  

3. THE EXPECTED $\gamma$-RAY FLUX AND THE SENSITIVITY OF LAT TO BIRTH PERIODS

The differential $\gamma$-ray spectrum resulting from the inverse Compton scattering of the CMBR (with $T = 2.7\,\text{K}$) is given by Blumenthal & Gould (1970). We then replace the electron spectral normalization constant with the above-mentioned rotational kinetic energy to give a total differential photon rate of

$$\frac{dN_{\gamma}}{dt dE_{\gamma}} = \frac{r_0^2}{\pi^3 h m_e c^4 a} \Delta E_{\text{rot}}(kT)^{\gamma/2} F(p_2) E_{\gamma}^{1.5}.$$  

(7)

Note the photon spectral index of 1.5 resulting from electrons radiating with a $p_2 = 2$ spectral index. The function $F(p_2) = 5.3$ for $p_2 = 2$. De Jager et al. (1995) introduced the necessity of including the contribution of far-infrared (FIR) photons from Galactic dust grains (at a temperature of $\sim 25\,\text{K}$) to the inverse Compton process for PWNe for which external Compton scattering is more important than the synchrotron-self-Compton process. In general, if $U_i$ (in units of eV cm$^{-3}$) is the energy density of an ambient target photon field with corresponding temperature $T_i$, the additional flux in the Thomson limit due to species $i$ would then be given by (in the energy range where $p_2 = 2$ contribute to the $\gamma$-ray flux; Blumenthal & Gould 1970)

$$\frac{dN_{\gamma}(T_i)}{dt dE_{\gamma}} \sim 6.5 T_i^{-1/2} U_i \frac{dN_{\gamma}(T = 2.7\,\text{K})}{dt dE_{\gamma}}.$$  

(8)

With this derivation we assumed that the energy density at temperature $T_i$ is a constant times that of a pure blackbody at the same temperature. The energy density of this FIR is $U \sim 1\,\text{eV cm}^{-3}$ within a galactocentric radius of $\sim 4\,\text{kpc}$, decreasing to $\sim 0.3\,\text{eV cm}^{-3}$ toward the solar neighborhood (Porter et al.
2006). From this expression for the $\gamma$-ray flux it is clear that high-temperature photon fields do not contribute, unless the corresponding photon energy density is relatively large. We will therefore only add the contributions for the CMBR and FIR dust fields.

By selecting $p_2 = 2$, the constant $a$ in $Q_\nu(t)$ can vary between 6 and 12 for $P_i \sim 1$, $\gamma_{\text{inj}}$, $mc^2 = 10^{15}$ eV (Vela-like) to $2 \times 10^{15}$ eV (Crab-like), and a break energy of $\gamma_{\text{m}}mc^2$ between $10^{12}$ and $3 \times 10^{12}$ eV to reproduce the observed break frequencies near $10^{13}$ and $10^{16}$ Hz, respectively, in the Crab Nebula. We will normalize $a$ to a value of 10 by setting $a = 10a_p$. In general, since the birth period scales as $a^{-1/2}$, uncertainties in the above-mentioned parameters are significantly suppressed since we take the square root of a logarithm.

We can now write the total high-energy $\gamma$-ray spectrum from a PWN in the GLAST LAT GeV domain (i.e., where $\gamma_b > \gamma_{\text{rad}}$), where the electron spectral index over the whole nebula remains $\sim 2$, so that the photon index is $\sim 1.5$. If the pulsar is also sufficiently spun down ($P_b \ll P$), we can neglect the $P$-term in $\Delta KE_{\gamma\gamma}$, so that

$$\frac{dN_\gamma}{dt \ dE_\gamma} = 4 \times 10^{-8} \frac{1 + 1.3 U_{25} I_{45} \delta E_{\gamma}^{-1.5}}{a_{10} P_{40} d_{10}^2} \left(\frac{d}{10 \ \text{kpc}}\right)^{-1} \ (9)$$

in units of $\text{cm}^{-2} \ \text{s}^{-1} \ \text{GeV}^{-1}$. Here $I_{45}$ is the moment of inertia normalized to $10^{45}$ g cm$^2$, whereas the birth period is normalized to 40 ms, $P_i = (40 \ \text{ms})P_{0}\nu$, following the finding of van der Swaluw & Wu (2001) that many pulsars in composite remnants were born with $P_b \sim 40$ ms. From the integral flux above 1 GeV we can calculate the LAT sensitivity to detect a given birth period: For a 50 hr, 5 $\sigma$ flux sensitivity of $2 \times 10^{-10}$ cm$^{-2} \ \text{s}^{-1}$ above 1 GeV, it is clear that LAT should be sensitive to birth periods in the range

$$P_b < (44 \ \text{ms})(1 + 1.3 U_{25})^{1/2} \left(\frac{\delta}{0.3} a_{10}^{-1/2} \left(\frac{d}{10 \ \text{kpc}}\right)^{-1} \cdot \ (10)$$

For example, for a source at a distance of 10 kpc, but within a galactocentric radius of 4 kpc where $U_{25} \sim 1$ eV cm$^{-3}$, GLAST should be able to detect birth periods less than 67 ms, whereas for a source at a distance of 2 kpc ($U_{25} \sim 0.3$ eV cm$^{-3}$) birth periods as long as 260 ms can be detected, assuming a realistic conversion efficiency of $\delta = 0.3$.

Assuming birth periods $P_b \leq \epsilon P$ (with $\epsilon < 1$), we can use this flux prediction (including the $P$-term) to determine the number of pulsars with expected GeV fluxes above the GLAST LAT sensitivity threshold. Using the ATNF pulsar catalog of Manchester et al. (2005), we select all pulsars with characteristic ages less than 280 kyr (according to the condition set by eq. [3]) and for each of these pulsars we calculate the integral flux above 1 GeV from equation (9) for a given period and selected value of $\epsilon$ (the current period was included for this calculation). Note that we only take the total number from this catalog and do not correct for selection effects such as dispersion measure constraints and off-beam pulsars. Adding such potentially unseen pulsars should increase the numbers presented below. The result from the existing catalog is a total of 50 detectable pulsars for $\epsilon = 0.5$, but only 30 for $\epsilon = 0.75$. Note, however, that the true birth periods are not expected to scale as a fixed fraction $\epsilon$ of the current period; the only purpose of these estimates is to show that a relatively large number of PWNe are expected to be visible even if $P_b$ differs by only 25% from $P$.

4. DISCUSSION AND CONCLUSION

In this Letter we derived the expected GeV $\gamma$-ray spectrum of PWN under the assumption that the particle spectrum in the uncooled domain ($\gamma < \gamma_{\text{rad}}$) is $p_2 \sim 2$, as observed from the injection spectra at most PWN shocks. When observing mature PWNe (i.e., those for which the age is significantly larger than the spin-down timescale), we arrive at a fresh new perspective with respect to the different multiwavelength domains:

For the GLAST LAT domain above 1 GeV, the observed $\gamma$-ray luminosity would scale as the change in rotational kinetic energy $(\Omega_{\text{rad}}^4 - \Omega^4)/2$, rather than the present-day spin-down power $\dot{\Omega}$. This is because the electrons contributing to the LAT energy domain survive since the earliest epoch when the spin period was close to the birth period. We therefore predict that the GeV luminosity and $\dot{\Omega}$ would be uncorrelated. There is, however, a caveat to this argument: The field strength during earlier epochs is expected to be larger, so that GeV-emitting electrons may have been burned off during an earlier epoch of high $B$. However, for the PWN of PSR B1509$-$58 the nebular field strength is between 8 $\mu$G (du Plessis et al. 1995; Gaensler et al. 2002) and 17 $\mu$G (Aharonian et al. 2005), given an age of $\sim 1700$ yr, whereas equation (2) requires a field strength of $B < 64 \mu$G after 1700 yr to force a radiation spectral break above 10 GeV. It is thus clear that this requirement is easily achieved for this example. The same can be shown for other relatively young VHE-emitting PWNe, such as Kes 75, G21.5$-$0.9, and G09.9+0.1, except for the Crab Nebula, for which $B \sim 2 \times 10^{-4}$ G after 950 yr. In general, the expansion history of the PWN, given the environment into which the associated supernova remnant forward-shock expands, is expected to determine the time evolution of PWN field strength. See also Reynolds & Chevalier (1984).

For the very high energy $\gamma$-ray (ground based) domain, we detect inverse Compton radiation from electrons around the radiation maximum $\gamma_{\text{rad}}$ as reviewed by de Jager & Djinjati-Atai (2008) for PWNe older than 10 kyr (see the discussion on Vela X and HESS J1825$-$137). This is possibly also the case for HESS J1718$-$385 (Aharonian et al. 2007; Hinton et al. 2007). Below this maximum the photon spectral index is near 1.5 due to relatively old electrons ejected since birth. This hard component should extend into the GLAST LAT range as discussed in this Letter. However, electrons with energies $\gamma \gg \gamma_{\text{rad}}$ have lifetimes shorter than the age of the pulsar, so that they are expected to reflect the current spin-down power. Thus, for the VHE domain some correlation may be expected, with the strength of the correlation depending on the magnitude of $\gamma_{\text{rad}}$.

In the X-ray domain we observe only synchrotron radiation from freshly ejected electrons (i.e., $\gamma \gg \gamma_{\text{rad}}$), in which case we expect a relatively tight correlation with the current spin-down power. In fact, Cheng et al. (2004) isolated the PWN X-ray contribution and found a correlation between X-rays and spin-down power of the form $L_X \propto (\dot{\Omega} \Omega)^{1/2}$.

Recently Huang & Wu (2003) derived birth periods for normal pulsars in the range $P_b \sim 0.6$–2.6 ms. For such birth periods this study would predict very bright PWNe at a distance of 10 kpc—even by EGRET standards, unless $\eta < 1$ during early epochs after birth. However, observations of the Crab Nebula do not support a low value of $\eta$ at birth.

Mature (Vela-like) PWNe appear to have field strengths be-
If we also require the spectral turnover for such PWNe due to radiative losses to be above 10 GeV, the corresponding age of the PWNe should be \( \lesssim 300 \) kyr as derived from equation (3). Assuming that mature PWNe evolve to such low field strengths in general, we infer that \( \sim 50 \) PWNe should be detectable for GLAST LAT if we assume a birth period \( P_0 \leq 0.5 \) kyr, whereas this number reduces to 30 for \( P_0 \leq 0.75 \) kyr. More may be visible if a significant amount of pulsars are off-beam, in which case we may not see any pulsed counterpart. Another implication of an evolved PWN with age near 300 kyr is that the current spin-down power and magnetic field strength are too low for a detectable synchrotron nebula, whereas the relatively low energy relic lepton component would still inverse Compton scatter external radiation fields into the GeV domain, which would lead to a population of “dark high energy \( \gamma \)-ray sources,” i.e., those without multiwavelength counterparts.

To avoid source confusion and claim sources with confidence it would be important to follow the LAT spectrum into the VHE \( \gamma \)-ray domain. The advantages for such an association would be the following: (1) we detect the spectral turnover associated with \( \gamma_{\text{rad}} \); (2) the positional agreement of the images between GeV and the higher resolution TeV energies makes the claim more significant; and (3) the size is expected to shrink for photon energies \( \gg 3kT_{\gamma_{\text{rad}}} \), with the image centroid converging toward the associated pulsar as observed from HESS J1825−137 (Aharonian et al. 2006b). Finally, to improve the accuracy in determining \( P_0 \), one should also add multiwavelength spectral information, if observable.

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REFERENCES

Aharonian, F. A., et al. (H.E.S.S. Collaboration). 2005, A&A, 435, L17
———. 2006a, A&A, 448, L43
———. 2006b, A&A, 460, 365
———. 2007, A&A, 472, 489
Atoyan, A. M., & Aharonian, F. A. 1996, MNRAS, 278, 525
Blumenthal, G. R., & Gould, R. J. 1970, Rev. Mod. Phys., 42, 237
Cheng, K. S., Taam, R. E., & Wang, W. 2004, ApJ, 617, 480
Chevalier, R. A. 2004, Adv. Space Res., 33, 456
de Jager, O. C., & Djannati-Ataï, A. 2008, in Neutron Stars and Pulsars: 40 Years after Discovery, ed. W. Becker (Berlin: Springer), in press (arXiv: 0803.0116)
de Jager, O. C., Harding, A. K., Baring, M., & Mastichiadis, A. 1995, in Proc. 24th Int. Cosmic Ray Conf. (Rome), 2, 528
du Plessis, I., de Jager, O. C., Buchner, S., Nel, H. I., North, A. R., Raubenheimer, B. C., & van der Walt, D. J. 1995, ApJ, 453, 746
Emmering, R. T., & Chevalier, R. A. 1989, ApJ, 345, 931
Gaensler, B. M., et al. 2002, ApJ, 569, 878
Hinton, J. A., Funk, S., Carrigan, S., Gallant, Y. A., de Jager, O. C., Kosack, K., Lemièrre, A., & Pühlhofer, G. 2007, A&A, 476, L25
Huang, Z.-K., & Wu, X.-J. 2003, Chinese J. Astron. Astrophys., 3, 166
Lorimer, D. R., Bailes, M., Dewey, R. J., & Harrison, P. A. 1993, MNRAS, 263, 403
Lyne, A. G., Pritchard, R. S., & Graham-Smith, F. 1993, MNRAS, 265, 1003
Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, AJ, 129, 1993
Narayan, R. 1987, ApJ, 319, 162
Phinney, E. S., & Blandford, R. D. 1981, MNRAS, 194, 137
Porter, T. A., Moskalenko, I. V., & Strong, A. W. 2006, ApJ, 648, L29
Reynolds, S. P., & Chevalier, R. A. 1993, MNRAS, 259, 726
Sefako, R. R., & de Jager, O. 2003, ApJ, 593, 1013
van der Swaluw, E., & Wu, Y. 2001, ApJ, 555, L49
Venter, C., & de Jager, O. C. 2006, in Proc. 363rd WE-Heraeus Seminar, Neutron Stars and Pulsars, ed. W. Becker & H. H. Huang (MPE Rep. 291; Garching: MPE), 40
Vivekanand, M., & Narayan, R. 1981, J. Astrophys. Astron., 2, 315