Conflict-Based Search for Connected Multi-Agent Path Finding

Arthur Queffelec\textsuperscript{1} Ocan Sankur\textsuperscript{2}
François Schwarzentruber\textsuperscript{1}
\textsuperscript{1}Univ Rennes, CNRS, IRISA
\textsuperscript{2}Univ Rennes, CNRS, INRIA, IRISA
firstname.lastname@inria.fr

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Abstract

We study a variant of the multi-agent path finding problem (MAPF) in which agents are required to remain connected to each other and to a designated base. This problem has applications in search and rescue missions where the entire execution must be monitored by a human operator. We re-visit the conflict-based search algorithm known for MAPF, and define a variant where conflicts arise from disconnections rather than collisions. We study optimizations, and give experimental results in which we compare our algorithms with the literature.

1 Introduction

In information-gathering missions, a group of robots are used to retrieve data at particular locations of an area (e.g. farm, building, etc). An application is search & rescue missions which are often assisted by human operators, and can be realized by unmanned aerial vehicles or other types of robots. In some of these applications the robots must continuously remain connected, for instance, in order to ensure a real-time video stream and to allow human operators to make quick decisions [ABB17].

In this paper, we consider the problem of computing paths for a set of agents in which they remain connected at all steps. We call this problem the Connected Multi-Agent Path Finding problem (CMAPF). CMAPF was initially introduced and studied in [HS12]. CMAPF does not consider collisions. As advocated by Hollinger et al., a discretization can take into account the geometry of the agents such that collisions can be avoided by an on-board collision avoidance system. Also, with a small amount of agents, collisions can be ignored by letting agents operate at different altitudes (e.g. in drone applications). Thus, we suppose that several agents are allowed to share the same position at a given time as done in [HS12, TBR\textsuperscript{+}18].

Hollinger and Singh showed the NP-hardness of the problem given a bound on the length of the execution, and provided an online algorithm. Tateo et al., in [TBR\textsuperscript{+}18],
showed the PSPACE-completeness of the general decision problem and gave two sub-optimal sampling-based algorithms and an optimal DFS-based algorithm. In [CQSS19], Charrier et al. show the problem is in LOGSPACE when restricted to so-called sight-moveable graphs, but the bounded version of the problem remains NP-hard.

One may solve our problem offline with A* [HNR68]. However, the group of agents is seen as one, and the algorithm is then exponential in the number of agents and the number of their available moves. That is an approach similar to the DFS-based algorithm of [TBR+18], which we compare to our algorithm.

In our work, we propose to take the approach of Conflict-Based Search (CBS) algorithm introduced in [SSFS12a]. CBS solves another problem called the Multi-Agent Path Finding (MAPF) problem, that focuses on collision-free paths. CBS starts by computing optimal paths for each agent separately; when a collision between two agents is detected at a location, the algorithm constrains one of the agents away from this location. CBS is, intuitively, exponential in the number of conflicts. In general, CBS has significantly better performance in practice than A*.

Our contribution is a complete and optimal algorithm called Connectivity-Conflict-Based Search (CCBS). CCBS relies on ideas similar to CBS (including the bypass optimization [BFSS15]), but manipulates connectivity constraints instead of collision constraints.

CMAPF is more challenging than MAPF. In MAPF, a collision conflict concerns only a pair of agents. That is why CBS solves directly a collision conflict in one step. In our case, a connectivity conflict concerns an arbitrary subset of agents. As solving a connectivity conflict is more demanding, we opt for strategies that do not solve directly a connectivity constraint, but guide the search towards a connected configuration.

More precisely we consider three intuitive strategies when facing a disconnected configuration:

- **NEG**: constrain one of the agent $a$ to not be in its current location;
- **SELF**: constrain the disconnected agent $a$ to be connected to some other agent or to the base;
- **OTHER**: constrain another agent $b$ to be connected to the disconnected agent $a$.

As in CBS, strategy NEG creates negative constraints. As in done in [LHS+19], strategies SELF and OTHER create positive constraints.

In our experiments, we consider the (optimal) variant CCBS$_N$ of CCBS that applies only NEG, and the (incomplete) variant CCBS$_{SO}$ that applies only SELF and OTHER. We compared our algorithm to A* with operator decomposition. Surprisingly, our experiments show that:

- CCBS clearly outperforms CCBS$_N$, which outperforms A*;
- CCBS$_{SO}$, although incomplete, has the similar behavior as CCBS and outputs optimal plans in almost all cases.

Finally, we discuss an optimization (called partial and selected splitting) that saves 9% of the memory consumption on average.
Outline We give the definition of the CMAPF problem, in Section 2. Then, in Section 3, we recall the CBS algorithm. We present our algorithm CCBS, in Section 4. In Section 5, we discuss completeness and optimality. Finally, in Section 6, we show our experimental results, and finish with discussions, in Section 7.

2 Connected Multi-Agent Path Finding

In this section, we formalize CMAPF [HS12, TBR+18, CQSS19]. The input is a topological graph specifying how agents can move (via movement edges) and how they can communicate (via communication edges) with each other.

Definition 1 (Topological Graph). A topological graph is a tuple $G = (V, E_m, E_c)$, with $V$ a finite set of vertices containing a distinguished element $B$ called the base, $E_m \subseteq V \times V$ a set of undirected movement edges and $E_c \subseteq V \times V$ a set of undirected communication edges.

In this work, we restrict to graphs in which all vertices contain a movement self-loop. This means that agents can always idle at any vertex. Figure 1 gives an example of a topological graph with 7 vertices.

Definition 2 (Execution). An execution exec of length $\ell$ with $k$ agents in a topological graph $G$ is a collection of $k$ paths of length $\ell$, one for each agent.

As we are interested in the makespan (that is the maximum of the lengths), in Definition 2, we suppose w.l.o.g. that paths are of the same lengths; if not, simply consider that agents can idle at their destinations. Agents are numbered from 1 to $k$.

Given an execution exec, exec$_a$ is the path of agent $a$. We denote by exec$_a[t]$, the vertex occupied by agent $a$ at the $t$-th step. We denote by exec[$t$], the positions of all the agents at time $t$, i.e. exec[$t$] is the vector $(exec_1[t], \ldots, exec_k[t]) \in V^k$. Such a vector $c \in V^k$ is called a configuration.

Definition 3. A configuration $c \in V^k$ is connected if the vertices $B, c_1, \ldots, c_k$ form a connected subgraph w.r.t communication edges ($E_c$). Otherwise, we say that $c$ is disconnected. An execution exec of length $\ell$ is said to be connected if exec[$t$] is connected, for all $1 \leq t \leq \ell$.

Definition 3 captures agents that are connected to the base via multi-hop (that is, an agent $a$ is connected to the base if there is a sequence of agents connecting $a$ to the base). An example of an execution using multi-hop connection is depicted in Example 1.

We define the following optimization problem called connected multi-agent path finding problem (CMAPF), in which we require the group of agents to be connected via communication edges with the base during the entire execution. We minimize the makespan of the execution.

Definition 4 (CMAPF). Given a topological graph $G = (V, E_m, E_c)$, number $k$ of agents, two configurations $s, g \in V^k$, find a connected execution exec of minimum length $\ell$ with $k$ agents in $G$ such that exec[1] = $s$ and exec[\ell] = $g$. 


Figure 1: An example of a topological graph where plain edges are movement edges, and dotted ones are communication edges.

Example 1. Consider the topological graph of Figure 1. Consider the instance of CMAPF with $k = 2$ agents with starting configuration $s = (v_1, v_4)$ and goal configuration $g = (v_3, v_6)$. The execution $\{(v_1, v_2, v_3), (v_4, v_5, v_6)\}$ is not connected. Indeed, at the second step the configuration is $(v_2, v_5)$: the first agent is disconnected, that is the set of vertices $\{v_2, v_5, B\}$ do not form a connected graph with the communication relation. However, the execution $\{(v_1, v_2, v_3, v_5), (v_4, v_4, v_5, v_6)\}$ is connected: the sets $\{v_1, v_4, B\}$, $\{v_2, v_4, B\}$, $\{v_3, v_5, B\}$ and $\{v_3, v_6, B\}$ all form connected graphs with the communication relation. Actually, that execution is an optimal solution of this CMAPF instance.

3 Conflict-Based Search

We recall the conflict-based search (CBS) algorithm [SSFS12a] that solves the MAPF problem under collision constraints but without connectivity. CBS is composed of two levels: the high-level builds a constraint tree while the low-level finds optimal single-agent paths.

The constraint tree $CT$ is composed of nodes $n$ with the following attributes:

- $n$.constraints - A finite set of constraints;
- $n$.exec - An execution as defined in Definition 2;
- $n$.cost - The current cost of the execution (i.e. its length).

Initially, the root node contains no constraints, and its execution consists of a set of shortest paths computed independently for each agent. The cost is the maximum of the lengths of these paths. Here, we extend all paths to have the same length by letting agents idle at their goal vertices.

A conflict of an execution is a time point where a pair of agents are in collision. If a constraint tree node without conflicts is created, then the algorithm returns the execution. Otherwise, it chooses a conflict; say agents $a, a'$ collide at time $t$. The algorithm creates two successor nodes obtained by adding the negative constraints requiring, respectively, that agent $a$ must not be at $\text{exec}_a[t]$ at time $t$, and agent $a'$ must not be at $\text{exec}_{a'}[t]$ at time $t$. Since all solutions must satisfy one of these constraints, if the optimal solution is compatible with the current node, it must be compatible with one of these successors.
For each successor node, one updates the shortest path for the agent with the new constraint. This computation consists in the lower level of CBS, and is typically done using time-space A* [Sil05].

**Bypassing Conflict** The bypassing conflict optimization [BFSS15] works as follows. If the execution $n'.\text{exec}$ in a successor $n'$ of $n$ has the same cost but has fewer conflicts than the execution $n.\text{exec}$ of $n$, then one replaces $n.\text{exec}$ by $n'.\text{exec}$ and deletes all children of $n$.

## 4 Connectivity-Conflict-Based Search

In this section, we describe our algorithm called Connectivity-Conflict-Based Search (CCBS) for the CMAPF problem. Our algorithm is adapted from conflict-based search.

### 4.1 Positive and Negative Constraints

Connectivity constraints are more demanding than collision constraints found in CBS. Connectivity is a property involving all agents, rather than being a local property involving only a pair of agents.

That is why, we do not try to solve a connectivity constraint in one single step. Instead, we help the search to reach connected configurations. It seems natural that solving disconnection requires to enforce some agent to be at a connected location. In addition to negative constraints, we also use positive ones that require an agent to be at a position, typically connected to another agent.

Thus, our constraints are of the form $\langle a, v, t, \beta \rangle$, where $a$ is the constrained agent, $v$ the vertex on which it is constrained, $t$ the time-step at which the constraint applies and $\beta \in \{\top, \bot\}$ is a Boolean value that specifies whether the constraint is positive or negative. The constraint $\langle a, v, t, \top \rangle$ (resp. $\langle a, v, t, \bot \rangle$) means that the agent $a$ should be (resp. should not be) at vertex $v$ at time $t$.

Positive constraints were recently used in CBS [LHS+19] as an optimization to create disjoint splits of a node. We here apply them in a different context, but we use their low-level algorithm to compute constrained shortest paths.

### 4.2 The Low-Level: Constrained Shortest Paths

We use the algorithm described in [LHS+19] to compute the constrained shortest paths for individual agents.

Given a set of positive and negative constraints, a start and a goal vertex, we use the positive constraints as timely ordered *landmarks*. We compute the path from the start location to the first landmark, then from the first landmark to the second and so on up to the goal. This is done iteratively with the original low-level of CBS using the negative constraints [SSFS12a]. No time bound is put on the path to the goal, while one is used on landmarks, which correspond to the time given in the positive constraint. Remark that if a landmark cannot be reached in the given time then there is no path satisfying the constraints.
4.3 The High-Level: The Conflict Tree

In the original CBS, a conflict involves two agents, thus CBS creates two successor nodes to solve it. In our case, a conflict in an execution is a disconnected configuration; thus, conflicts involve all agents. So we create a larger number of successors reflecting the many ways to solve the conflict.

Let us explain the strategies used to generate constraints to handle connectivity. First, as in CBS, we add negative constraints but for all agents. More precisely, for a given disconnected configuration \( c \) at time \( t \), we create a successor node for each agent \( a \), with the negative constraint that forbids agent \( a \) to be at \( c_a \) at time \( t \).

This strategy, called \( \text{NEG} \), makes the search complete but, as we will see, it is inefficient alone to solve CCBS. Indeed, when CBS encounters a collision between two agents, in MAPF, the usage of a negative constraint is enough to solve this collision. However, the use of a negative constraints, in CMAPF, does not guarantee that the disconnection is solved immediately. The idea is rather to guide the search towards connected configurations by letting agents stay together.

To obtain an efficient algorithm, we will add positive constraints, according to two strategies called \( \text{SELF} \) and \( \text{OTHER} \). For a given conflict, we consider a disconnected agent \( a \). The strategy \( \text{SELF} \) constrains agent \( a \) to be at a position connected to some other agent or to the base. The strategy \( \text{OTHER} \) forces an arbitrary agent to a position where it becomes connected to agent \( a \). The \( \text{SELF} \) and \( \text{OTHER} \) strategies can be seen as shortcuts to negative constraints.

Algorithm 1 High-Level of CCBS

Require: A topological graph \( G = (V, E_m, E_c) \), an initial configuration \( s \) and a goal configuration \( g \) (considered as global variables)

1: \text{INSERTROOT}
2: \text{while OPEN is not empty do}
3: \( n := \text{best node from OPEN} \)
4: \text{if } n \text{ has no conflict then}
5: \text{return } n.\text{exec}
6: \text{CHILDREN := empty list}
7: \( (t, a) := \text{a time-step and a disconnected agent in a conflict in } n.\text{exec} \)
8: \text{SELF}(n, t, a)
9: \text{OTHER}(n, t, a)
10: \text{NEG}(n, t)
11: \text{if BYPASS was raised then}
12: \text{discard CHILDREN}
13: \text{else}
14: \text{insert all nodes in CHILDREN into OPEN}

The overall algorithm is given in Algorithm 1, which shows how the constraint tree is created. It maintains a priority queue called \( \text{OPEN} \) which stores the set of leaf nodes that have not been expanded yet. It runs as long as a solution has not been found and this queue is non-empty. At each iteration, a best node is picked (Line 3). If that node has no conflict, it means that a solution is found (Line 5). If the execution of the node
contains conflicts, then a conflict is chosen (arbitrarily) and child nodes are created with the strategies (from Line 8 to Line 10). The last part of the algorithm shows the bypass optimization.

The INSERTROOT procedure in Algorithm 2 shows the initialization step where the root of the constraint tree is created. Here, CSP refers to the Constrained Shortest Path computation of the low-level described in Subsection 4.2. This call returns a shortest path satisfying the given set of constraints between the starting node \( s_a \) and the target node \( g_a \) of agent \( a \). The procedures SELF, OTHER and NEG describe the creation of conflicts of respective types. Furthermore, each one of these procedures call CREATECHILD, which is responsible for creating child nodes and detecting the BYPASS condition (for which the bypass optimization is applied).

**Algorithm 2 Sub-procedures**

```plaintext
1: procedure INSERTROOT
2: root := new node
3: root.constraints := ∅
4: for all agents \( a \) do
5:   root.exec\(_a\) := CSP(\( s_a, g_a, ∅ \))
6: root.cost := max\{ |root.exec\(_a| \) where \( a \) is an agent \}
7: insert root to OPEN

7: procedure CREATECHILD(node \( n, \langle a, v, t, β \rangle \))
8: \( n′ \) := new node
9: \( n′.constraints \) := \( n \).constraints \( ∪ \{ \langle a, v, t, β \rangle \} \)
10: for all agents \( b \) do \( n′.exec\(_b\) := n.exec\(_b\) \)
11: \( n′.exec\(_a\) := CSP(\( s_a, g_a, n′.constraints \))
12: \( n′.cost \) := max\{ |\( n′.exec\(_a| \) where \( a \) is an agent \}
13: if \( n′.cost \neq n.cost \) and \( n′ \) has less conflicts than \( n \) then
14:   n.exec\(_a\) := \( n′.exec\(_a\)
15: raise BYPASS
16: insert \( n′ \) to CHILDREN

17: procedure SELF(node \( n, \langle a, v, t, β \rangle \))
18: for all agents \( b \) different from \( a \) do
19:   for all vertices \( v′ \) s.t. \( (n.\text{exec}_{b}[t], v′) \) \( E_c \) do
20:     if \( v′ \neq \text{exec}_{b}[t] \) then
21:       CREATECHILD(n, \( \langle a, v', t, T \rangle \))
22: for all vertices \( v'' \) s.t. \( (B, v'') \) \( E_c \) do
23:   if \( v'' \neq \text{exec}_{a}[t] \) then
24:     CREATECHILD(n, \( \langle a, v'', t, T \rangle \))

25: procedure OTHER(node \( n, \langle a, v, t, β \rangle \))
26: for all agents \( b \) different from \( a \) do
27:   for all vertices \( v' \) s.t. \( (n.\text{exec}_{a}[t], v') \) \( E_c \) do
28:     if \( v' \neq \text{exec}_{a}[t] \) then
29:       CREATECHILD(n, \( \langle b, v', t, T \rangle \))

30: procedure NEG(node \( n, \langle a, v, t, β \rangle \))
31: for all agents \( a \) do
32:   CREATECHILD(n, \( \langle a, n.\text{exec}_{a}[t], t, ⊥ \rangle \))
```
While the strategy \( \text{NEG} \) creates \( k \) child nodes, the number of child nodes created by \( \text{SELF} \) and \( \text{OTHER} \) is rather quadratic, in \( O(k \times |V|) \) in each case, where \( k \) is the number of agents and \( |V| \) is the number of vertices. Thus, the constraint tree might grow too quickly. We will show that despite this high branching factor, we do obtain satisfactory results in our benchmarks. See also discussion in Section 6.

5 Discussion on Completeness and Optimality

In this subsection, we consider the variant of CCBS called CCBS\(_N\) in which only strategy \( \text{NEG} \) is applied, but not \( \text{SELF} \) and \( \text{OTHER} \) (we remove lines 8 and 9 in Algorithm 1. We also consider the variant CCBS\(_{SO}\) in which only strategies \( \text{SELF} \) and \( \text{OTHER} \) are applied, but not strategy \( \text{NEG} \) (line 10 is deleted). As long as we apply the strategy \( \text{NEG} \), we obtain a complete and optimal algorithm. This can be proved similarly as in [SSFS12a]:

Theorem 1. Both CCBS\(_N\) and CCBS are complete and optimal.

5.1 Incompleteness of CCBS\(_{SO}\)

However, strategies \( \text{SELF} \) and \( \text{OTHER} \) alone lead to an incomplete algorithm. We will mainly study the variant CCBS\(_{SO}\) obtained by omitting the strategy \( \text{NEG} \). We observe that the remaining positive constraints guide the search very quickly towards a solution in our experiments.

We distinguish a class of topological graphs, called sight-moveable graphs. In fact, in a typical radius discretization, if the agents restrict their communication to links that do not cross obstacles then the obtained topological graph is sight-moveable. While being a strong restriction, such communication may guarantee a more reliable connection. Furthermore, the agents can still exploit links crossing obstacles while not being required during the execution. The formal definition is given below.

Our original motivation was to develop an efficient algorithm for this class. In fact, the theoretical complexity of deciding the existence of a connected plan was shown to be in LOGSPACE for sight-moveable graphs [CQSS19], while it is PSPACE-complete for general graphs [TBR + 18]. We will first introduce this class and then formally study the properties of CCBS\(_{SO}\).

Sight-Moveable Graphs

We recall the class of sight-moveable topological graphs introduced in [CQSS19]. Whenever an agent can communicate with another node, then it can also move to that node while maintaining the communication. A sight-moveable graph can be obtained on a discretized graph by restricting communication in line of sight.

Formally, a sight-moveable topological graph has an undirected movement relation such that for all vertices \( v, v' \in V \), for all \( v E_c v' \), there is a sequence of vertices \( \pi = \langle \pi_0, \pi_1, \ldots, \pi_\ell \rangle \) of size \( \ell \) such that \( \pi_0 = v \), \( \pi_\ell = v' \), \( \pi_i E_{m} \pi_{i+1} \) and \( \pi_i E_c v' \), for all \( 0 \leq i < \ell \).
Figure 2: A sight-moveable topological graph where plain edges represent movement edges, and the dotted ones represent communication edges.

**Example 2.** Figure 2 shows a sight-moveable graph. For instance, $B \cdots q_5$ and, indeed, there is a path $BE_mq_4E_mq_5$ with $B \cdots q_4$. In contrast, the graph in Figure 1 is not sight-moveable, since $v_4 \cdots v_2$ but it is not possible to go from $v_4$ to $v_2$ while maintaining the communication with $v_2$.

**Theorem 2.** CCBS$_{so}$ is not complete.

**Proof.** Consider the sight-moveable topological graph of Figure 2. Consider the instance with two agents: $s = (q_4, q_3)$ and $g = (B, B)$.

This instance has a solution, for instance, the connected execution from configuration $s$ to $g$ made up of the following paths: $(q_4, q_5, q_6, B, B)$ for agent $a_1$, and $(q_3, q_2, q_1, B)$ for agent $a_2$.

However, let us show that CCBS$_{so}$ does not find a solution. The first shortest paths generated by the algorithm are, respectively, $(q_4, B)$ and $(q_3, q_2, q_1, B)$. Thus agent $a_2$ is disconnected at the second step. The constraints added by the SELF strategies are not satisfiable. In fact, agent $a_2$ can only move to $q_2$ and $q_7$ at step 2, and none of them are connected to $B$ (where agent 1 is at step 2.) Furthermore, there is only one constraint of type OTHER added by the algorithm, and it consists at placing agent $a_1$ at $q_6$ at step 2, so as to connect it to agent 2 at $q_2$. However, this is not satisfiable either since agent $a_1$ cannot move there in one step. So the search is stuck and no solution is returned.

Interestingly, even if CCBS$_{so}$ is not complete in theory, our experiments promote the use of SELF and OTHER alone: CCBS$_{so}$ is slightly faster while outputting optimal plans in almost all cases.

### 5.2 Completeness of CCBS$_5$ when all agents start at the base

On the bright side, we show that the algorithm is complete on sight-moveable graphs if all agents start at the base $B$. This is an interesting case since in some applications, agents (say, drones) are all launched from the base. Thus, on a typical mission, reaching a given configuration from the base would be the initial task. The following lemma shows that only the SELF constraints are required to ensure completeness. The variant CCBS$_{so}$ is complete a fortiori.
Figure 3: Example of an ordering of nodes in $V' = \{g_1, \ldots, g_6, B\}$.

**Theorem 3.** CCBS$_s$ is complete on sight-moveable graphs when all agents start at the base.

**Proof.** Consider a sight-moveable topological graph $G$, and a connected configuration $g = (g_1, \ldots, g_k)$. We are going to construct a particular execution from $B^k$ to $g$, in the same way as in Prop. 19 in [CQSS19]. Let us order the nodes $B, g_1, \ldots, g_k$ into $g_i_0, g_i_1, g_i_2, \ldots, g_i_k$ with $g_i_0 = B$, such that for all $1 \leq j \leq k$, $g_i_j$ is connected to some $g_i_k$ with $0 \leq k < j$. This order induces a tree, and can be obtained by breath-first search in a multi-spanning tree of $V'$ at root $B$; see Figure 3 for an example. Let us fix such a spanning tree $T_g$.

By the sight-moveable property, for any pair $g_i, g_j$ connected in this tree, there is a path $\rho_{g_i, g_j}$ along which an agent can move from $g_i$ to $g_j$ while staying connected with $g_i$ along the way. Informally, the executions we define have the following property. Any agent who arrives at their destination stays there. Furthermore, for any agent $a$, if $a'$ denotes the agent such that $g_i_a'$ is the parent of $g_i_a$ in $T_g$, then, agent $a$ strictly shares the same position as $a'$ until $a'$ reaches their target. Agents move in groups following paths $\rho_{g_i_1, g_i_2}$ to move from one target configuration to another. For instance, in Fig 3, the executions for agents $i_4, i_5, i_6$ consist in them moving all together to $g_i_4$ (via $\rho_{g_i_0, g_i_4}$) then $i_5, i_6$ moving together to $g_i_5$ (via $\rho_{g_i_4, g_i_5}$), and $i_6$ to moving alone to $g_i_6$ (via $\rho_{g_i_5, g_i_6}$). The sight-moveable (SM) property ensures that the agents are always connected along this execution. We are going to define a set of executions based on this particular execution.

Let $\rho_i$ denote the path $\rho_{g_{\alpha_0}, g_{\alpha_1}} \rho_{g_{\alpha_1}, g_{\alpha_2}} \cdots \rho_{g_{\alpha_{k-1}}, g_{\alpha_k}}$ with $\alpha_0 = 0, \alpha_k = i$, and $g_{\alpha_k}$ is the path from the root to node $g_i$ in $T_g$. Intuitively, this is the path that is taken by agent $i$.

Let exec denote the execution constructed above. Observe that all configurations of exec are connected by the SM property. In fact, all nodes visited along $\rho_{g_{\alpha_0}, g_{\alpha_k}}$ are directly connected to the base since $\alpha_0 = 0$ and $g_0$ is the base. By construction, an agent moves along the path $\rho_{g_{\alpha_0}, g_{\alpha_k}}$ only when the agent whose target is $g_{\alpha_k}$ has reached this node and remains there. By the SM property, all nodes of this path are connected with $g_{\alpha_k}$. This argument, applied inductively, shows that all reached configurations are connected (see [CQSS19]).

We are going to show that in the CT created by CCBS$_s$, there is a branch $b$ such that along all its nodes $n$, either there is a solution or the following invariant holds:

- exec satisfies $n.$constraints,
- for all agents $a$, $\forall t \geq |\text{exec}_a|, n.\text{exec}_a[t] = g_a$.

That is, exec is compatible with current constraints, moreover, once an agent reaches their goal node, they remain there.
The invariant holds at the initial node since there are no constraints, and since the algorithm produces a shortest path for each agent separately, which are shorter or equal in length than the paths prescribed by exec.

Assume that the property holds at some node $n$. If there is no conflict, then the algorithm has found a solution. Otherwise, consider any conflict picked by the algorithm, say, $(t, a)$ (i.e. agent $a$ is disconnected at step $t$). We argue that $\text{SELF}(n, t, a)$ creates a child node with constraint $\langle a, v, t, \top \rangle$ where $v$ is the $t$-th vertex of $\rho_a$. For each $1 \leq k \leq k$, let $g_{\alpha_0}, g_{\alpha_1}, \ldots, g_{\alpha_k}$ be the path from the root to $\alpha_k$ in $T_g$. If $t \leq |\rho_{g_{\alpha_0}, g_{\alpha_1}}|$, then $v$ is connected to the base, so the above constraint will be added. Notice that exec is compatible with the newly added constraint; furthermore, when the algorithm recomputes a shortest path for agent $a$, it must find one whose length is not more than that of $\text{exec}_a$ since the latter is a candidate path satisfying the constraints. The invariant thus holds in the child node.

Otherwise, let $j$ denote the largest index such that

$$|\rho_{g_{\alpha_0}, g_{\alpha_1}} \rho_{g_{\alpha_1}, g_{\alpha_2}} \cdots \rho_{g_{\alpha_{j-1}}, g_{\alpha_j}}| \leq t.$$ 

By the invariant, agent $\alpha_j$ is at vertex $g_{\alpha_j}$ at time $t$. But $g_{\alpha_j}$ is connected to $v$ since $v$ is on the path $\rho_{g_{\alpha_j}, g_{\alpha_{j+1}}}$. Thus, some child node of $n$ will be created with the above constraint. The execution exec satisfies the newly added constraint by definition. Furthermore, exec$_a$ is a path from the base to $g_a$ so the newly computed shortest path for $a$ must be of length at most $|\text{exec}_a|$, which shows the second part of the invariant. 

6 Experimental Results

In this section, we evaluate optimal algorithms CCBS, CCBS$_N$, and CCBS$_{SO}$ that is incomplete.

6.1 Benchmarks

The experiments were carried out on 4 different benchmark maps depicted in Figure 4. Both Coast and Maze come from the Moving AI Lab benchmarks\(^1\). Coast is extracted from Dragon Age 2, in the same spirit as in [Stu12]. Both Offices and Open maps were used for experimental analysis in [HS12] and [TBR+18]. Offices is a map of the SDR offices from the Radish data set [HR03]. Open is a map of the McKenna MOUT site.

We discretized the maps as follows. The movement edges follow an 8-way grid (i.e. the agents can move in the 8 directions). Concerning communications, we adopted two practical settings, the distance-based and the line-of-sight-based (LOS) ones. Both are standard; see e.g. [ABB17]. We thus obtain $4 \times 2$ topological graphs.

- In the distance-based communication, an agent communicates with another one up to a certain maximal distance, called the range (e.g. as in Wi-Fi); the range is displayed below the maps in Figure 4;

\(^1\)https://movingai.com/benchmarks/mapf/index.html (w_woundedcoast.map and maze-32-32-2.map)
Coast:

- Nodes: 2174
- Movement Edges: 8260
- Communication Edges: 181070
- Distance: 96565

Maze:

- Nodes: 666
- Movement Edges: 2318
- Communication Edges: 8736
- Distance: 6422

Offices:

- Nodes: 1494
- Movement Edges: 2419
- Communication Edges: 29962
- Distance: 25660

Open:

- Nodes: 2205
- Movement Edges: 4107
- Communication Edges: 39310
- Distance: 39299

Figure 4: Benchmarks. The four maps used to obtain topological graphs. Obstacles are in black. For each map, we generate two topological graphs that share the same nodes and movement edges, but the communication edges correspond either to a distance-based communication (the range we used are depicted below the maps) or a LOS-based communication.

- In the LOS-based communication, an agent communicates with agents that are in line of sight, (do not pass through obstacles).

Note that both communication models are used in applications. LOS is particularly interesting to obtain a conservative model without false negatives, that is, assumed communication links are likely to exist in the real-world. In contrast, in the distance-based communication model, some obstacles can prevent communication between two locations or lower its quality. See the discussion in [ABB17].

6.1.1 Methodology

The algorithms were implemented single-threaded in Java. The experiments were done sequentially and the time of initialization of the algorithms (e.g. parsing of the graph, etc) was not counted in its execution time. These experiments were done on an Intel Xeon W-2104 CPU at 3.20GHz with 16 GB of memory.

We compare the three main algorithms (CCBS$_{3D}$, CCBS, CCBS$_S$) and A* algorithm [HNR68] with the operator decomposition (OD) optimization [Sta10], which consists in moving a single agent per step. We run the algorithms with 2 to 10 agents and then from 10 to 50 agents (10, 15, . . . , 45, 50), on the 8 topological graphs obtained from the 4 maps listed above with either a distance-based or LOS-based communication on 100 instances with a time out of 30 seconds to solve 10 instances.

The success rate of an algorithm on these benchmarks is defined as the percentage of instances for which it found an execution in the allocated time. These executions are optimal for CCBS and CCBS$_S$ but not necessarily optimal for CCBS$_{3D}$.

In Figure 5a, we report the success rates of CCBS$_S$, CCBS, CCBS$_{3D}$ and the A* algorithm on the 4 maps with a distance-based communication. In Figure 5b, we report the same experiments with the LOS-based communication.
6.2 Results

6.2.1 A∗ Algorithm

Our optimal algorithms CCBS and CCBS\textsubscript{N} outperform A∗ by an order of magnitude on all maps. We believe that the main reason for the inefficiency of A∗ is the branching factor which is at most 9 for moving a single agent (8 directions and idle), but $9^n$ for a joint move of $n$ agents. Notice that the performance of A∗ was slightly better in Maze maps with LOS communication which has a smaller branching factor. Another reason could be that the makespan objective prevents A∗ from distinguishing better executions. In fact, a long path taken by a single agent can shadow improvements in the paths of other agents. The A∗ algorithm is our implementation of the DFS-based algorithm of [TBR+18].

6.2.2 CCBS v.s. CCBS\textsubscript{N}

One can observe, first, that CCBS outperforms CCBS\textsubscript{N} in all benchmarks. Indeed, the addition of the SELF and OTHER strategies allows CCBS to gain 16% of success rate in average. Both algorithms have a similar behavior on some maps as the Maze and Coast with distance-based communication. However, on the map Open, CCBS performs better.

A second observation is the difference in success rates depending on the type of communication. In our experiments, the algorithms performed generally better when communication was LOS-based rather than distance-based. In particular, on the Maze map, almost no execution were generated past 20 agents with the distance-based communication, while with the LOS-based communication, CCBS is still above 40% of success with 50 agents.

6.2.3 CCBS v.s. CCBS\textsubscript{SO}

The sub-optimal CCBS\textsubscript{SO} is 4% better in total average of success rate than CCBS. Interestingly, over 13600 executions, CCBS and CCBS\textsubscript{SO} compute the same result except for 4 cases. For our benchmarks, the non-optimal algorithm CCBS\textsubscript{SO} found optimal executions in 99.97% cases. This indicates that despite its incompleteness (Theorem 2), CCBS\textsubscript{SO} seems suitable in practice.

6.2.4 Size of the constraint tree

The strategies SELF, OTHER used in CCBS lead to a larger branching factor compared to CCBS\textsubscript{N}. In fact, rather than selecting a disconnected agent and forbidding its current location, these strategies select a disconnected agent and a candidate vertex. Figure 6 shows a comparison of the number of nodes of the constraint trees generated by both algorithms. Despite the large branching factor, CCBS often quickly finds a solution which means that the depth of the conflict tree is small; while CCBS\textsubscript{N} generates significantly larger trees.
Figure 5: Success Rate on the benchmark Coast, Maze, Offices and Open with the two definitions for communication edges. CCBS generally outperforms CCBS\textsubscript{N}. CCBS\textsubscript{SO} generally slightly outperforms CCBS. Our algorithms outperform A*. 

(a) Distance-based communication.

(b) LOS-based communication.
6.3 Partial and Selective Splitting

Now, we discuss an optimization used to lower the memory usage of the algorithm CCBS.

In a variant of the A* algorithm from [YMI00] and its enhanced version [FGS + 12], the authors introduce an optimization which consists in partially generating the children of nodes based on their costs. When a node \( n \) is chosen for splitting, one can partially split it (that is, generate only some of its child nodes) and put the node back in OPEN so that it is split again later (to generate the rest of its child nodes).

In our setting, if the child node \( \text{NEG} \) generated for agent \( a \) has an execution strictly longer than that of the parent node, then we know that the executions of agent \( a \) at all \( \text{SELF} \) and \( \text{OTHER} \) nodes that constrain agent \( a \) are longer as well. We thus suggest the following optimization. Given a conflict at time \( t \), if the path of an agent \( a \) with the new constraint \( \langle a, \text{exec}_a[t], t, \perp \rangle \) is longer than its previous path then we partially split the node by omitting the generation of all child nodes in which agent \( a \) is constrained. The current node is put back in OPEN by incrementing its cost.

This is particularly useful in our case given the large amount of \( \text{SELF} \) and \( \text{OTHER} \) constraints created. In our experiments, this partial and selective splitting optimization reduced the number of created nodes by 9% in average over all benchmarks with slightly better success rate (the average success rate is 59% with optimization and 57% without).

7 Conclusion

We presented the optimal algorithm Connected-Conflict-based Search (CCBS) (Algorithm 1) for solving the Connected Multi-Agent Path Finding problem (Definition 4). We then investigated the impact of the strategies \( \text{SELF} \) and \( \text{OTHER} \). Omitting these yields CCBS\(_N\) that uses only \( \text{NEG} \), which is still optimal but has worse performance, although it does outperform A*. Using \( \text{SELF} \) and \( \text{OTHER} \) but not \( \text{NEG} \) yields CCBS\(_{SO}\) that is incomplete (Theorem 2) but produced optimal results in most cases in our ex-
periments, and had a better success rate than CCBS.

Note that SELF and OTHER strategies have tendency to increase the branching factor. Fortunately, these constraints also significantly improved the execution time in practice. In contrast, in some work, e.g. the bypass optimization [BFSS15] for CBS or the operator decomposition for A* [Sta10], the authors aim at reducing the branching factor to improve performance. This is why, while our specific strategies adding positive constraints might not be fitted for the original MAPF problem, finding new types of constraints might improve the performance both for CBS and CCBS. As shown in Figure 6, strategies SELF and OTHER reduce the number of generated nodes.

Interestingly, our Algorithm 1 can be easily tuned to handle both collision and communication conflicts. For this, Line 7 needs to be modified to detect the two different types of conflicts. If the conflict is a collision, apply the original strategy of CBS and jump to Line 11; otherwise, pursue from Line 8. Our algorithm can also be adapted for other definitions of execution costs, such as the sum of the path lengths. The experimental study of those extensions is left for future work.

Several optimizations for CBS are worth to be extended for CCBS: i) grouping agents as meta-agents (MA-CBS [SSFS12b]), ii) prioritizing conflicts (ICBS [BFS15]), iii) and adding an heuristic to the search (CBS-h [FLB18]).
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