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$$\hat{s}(M_Z)=0.1153\pm0.0017(\text{exp})\pm0.0023(\text{th})$$

$$5p^0|_0 = 0.5132\pm0.0115(\text{exp})\pm0.0381(\text{th})$$

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Hadronization effects in event shape moments

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Abstract

We study the moments of hadronic event shapes in $e^+e^-$ annihilation within the context of next-to-next-to-leading order (NNLO) perturbative QCD predictions combined with non-perturbative power corrections in the dispersive model. This model is extended to match upon the NNLO perturbative prediction. The resulting theoretical expression has been compared to experimental data from JADE and OPAL, and a new value for $\alpha_s(M_Z)$ has been determined, as well as of the average coupling $\alpha_0$ in the non-perturbative region below $\mu_I = 2$ GeV within the dispersive model:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017\text{(exp)} \pm 0.0023\text{(th)},$$

$$\alpha_0 = 0.5132 \pm 0.0115\text{(exp)} \pm 0.0381\text{(th)}.$$  

The precision of the $\alpha_s(M_Z)$ value has been improved in comparison to the previously available next-to-leading order analysis. We observe that the resulting power corrections are considerably larger than those estimated from hadronization models in multi-purpose event generator programs.
1 Introduction

Event shape variables measure geometrical properties of hadronic final states at high energy particle collisions. They have been studied extensively at $e^+e^-$ collider experiments, which provided a wealth of data at a variety of centre-of-mass energies. Exploiting this large energy range, one can attempt to disentangle perturbative and non-perturbative contributions (which scale differently with increasing energy) to event shape observables.

Apart from distributions of these observables, one can also study mean values and higher moments. The $n$th moment of an event shape observable $y$ is defined by

$$\langle y^n \rangle = \frac{1}{\sigma_{\text{had}}} \int_0^{y_{\text{max}}} y^n d\sigma \, dy ,$$

(1)

where $y_{\text{max}}$ is the kinematically allowed upper limit of the observable. Moments were measured for a variety of different event shape variables in the past. The most common observables $y$ of three-jet type are: thrust $T$ [1] (where moments of $y = (1 - T)$ are taken), the heavy jet mass $\rho = M_H^2/s$ [2], the $C$-parameter [3], the wide and total jet broadenings $B_W$ and $B_T$ [4], and the three-to-two-jet transition parameter in the Durham algorithm $Y_3$ [5]. Definitions for all observables are given in, for example, Ref. [6]. Moments with $n \geq 1$ have been measured by several experiments, most extensively by JADE [7,8] and OPAL [9], but also by DELPHI [10] and L3 [11]. A combined analysis of JADE and OPAL results has been performed in Ref. [12].

As the calculation of moments involves an integration over the full phase space, they offer a way of comparing to data which is complementary to the use of distributions, where in general cuts on certain kinematic regions are applied. Furthermore, the two extreme kinematic limits – two-jet-like events and multi-jet-like events – enter with different weights in each moment: the higher the order $n$ of the moment, the more it becomes sensitive to the multi-jet region. Therefore it is particularly interesting to study the NNLO corrections to higher moments of event shapes, as these corrections should offer a better description of the multi-jet region due to the inclusion of additional radiation at parton level.

Moments are particularly attractive in view of studying non-perturbative hadronization corrections to event shapes. In event shape distributions, one typically corrects for hadronization effects by using generic Monte Carlo event simulation programs. A recent study, carried out in the context of a precision determination of the strong coupling constant from event shape distributions [13], revealed large discrepancies between the standard event simulation programs used at LEP [14, 15] on one hand and more modern generators [16], which incorporate recent theoretical advances, on the other hand. In the event shape distributions, it is very difficult to disentangle hadronization corrections empirically, since they typically result in a distortion of the distribution, which can not be unfolded in a straightforward manner.

In event shape moments, one expects the hadronization corrections to be additive, such that they can be divided into a perturbative and a non-perturbative contribution,

$$\langle y^n \rangle = \langle y^n \rangle_{\text{pt}} + \langle y^n \rangle_{\text{np}} ,$$

(2)

where the non-perturbative contribution accounts for hadronization effects. Based upon the calculation of next-to-next-to-leading order (NNLO) QCD corrections to the event shape distributions, which became available recently [6, 17–21], the perturbative contribution to event shape moments is now known to NNLO [22, 23]. The non-perturbative part is suppressed by powers of $\lambda_p/Q^p$ ($p \geq 1$), where $Q \equiv \sqrt{s}$ is the centre of mass energy and $\lambda_1$ is of the order of $\Lambda_{\text{QCD}}$. The functional form of $\lambda_p$ has been discussed quite extensively in the literature, but as this parameter is closely linked to non-perturbative effects, it cannot be fully derived from first principles.

In this work, we use the dispersive model derived in Ref. [24–27] to compute hadronization corrections to event shape moments. This model provides analytical predictions for the power corrections,
and introduces only a single new parameter $\alpha_0$, which can be interpreted as the average strong coupling in the non-perturbative region. This model has been used extensively in combination with NLO QCD perturbative calculations to study event shape moments [9, 28–30]. To combine the dispersive model with the perturbative prediction at NNLO QCD, we extended its analytical expressions to compensate for all scale-dependent terms at this order. By comparing the newly derived expressions with experimental data on event shape moments, we perform a combined determination of the perturbative strong coupling constant $\alpha_s$ and the non-perturbative parameter $\alpha_0$. Compared to previous results at NLO, we observe that inclusion of NNLO effects results in a considerably improved consistency in the parameters determined from different shape variables, and in a substantial reduction of the error on $\alpha_s$.

In Section 2, we outline the structure of perturbative and non-perturbative contributions to event shape moments. The predictions of the dispersive model to power corrections are extended to NNLO in Section 3, and used to extract $\alpha_s$ and $\alpha_0$ from experimental data in Section 4. In Section 5 the results obtained within the dispersive model are compared to those from multi-purpose event generator programs.

2 Power corrections to event shape moments

Non-perturbative power corrections can be related to infrared renormalons in the perturbative QCD expansion for the event shape variable [24, 25, 31–36]. The analysis of infrared renormalon ambiguities suggests power corrections of the form $\lambda_p/Q_p$, but cannot make unique predictions for $\lambda_p$: it is only the sum of perturbative and non-perturbative contributions in (2) that becomes well-defined [37]. Different ways to regularise the IR renormalon singularities have been worked out in the literature [38–43].

One approach is to introduce an IR cutoff $\mu_I$ and to replace the strong coupling constant below the scale $\mu_I$ by an effective coupling such that the integral of the coupling below $\mu_I$ has a finite value [24–27]

$$\frac{1}{\mu_I} \int_0^{\mu_I} dQ \alpha_{\text{eff}}(Q^2) = \alpha_0(\mu_I).$$

This dispersive model for the strong coupling leads to a shift in the distributions

$$\frac{d\sigma}{dy}(y) = \frac{d\sigma_{\text{pt}}}{dy}(y - a_y P),$$

where the numerical factor $a_y$ depends on the event shape and is listed in Table 1, while $P$ is believed to be universal (universality breaking terms arise from hadron mass effects [44] in the moments of $\rho$, an estimate on these effects can be obtained from general-purpose event generator programs, e.g. from PYTHIA [14]) and scales with the CMS energy like $\mu_I/Q$.

By inserting (4) into the definition of the moments, one obtains:

$$\langle y^n \rangle = \int_0^{y_{\text{max}}} dy y^n \frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dy}(y)$$

$$= \int_{-a_y P}^{y_{\text{max}} + a_y P} dy (y + a_y P)^n \frac{1}{\sigma_{\text{had}}} \frac{d\sigma_{\text{pt}}}{dy}(y)$$

$$\approx \int_0^{y_{\text{max}}} dy (y + a_y P)^n \frac{1}{\sigma_{\text{had}}} \frac{d\sigma_{\text{pt}}}{dy}(y).$$

| event shape observable | $1 - T$ | $C$ | $Y_3$ | $\rho$ | $B_T$ | $B_W$ |
|------------------------|--------|-----|-------|-------|-------|-------|
| $a_y$                  | 2      | 3$\pi$ | 0     | 1     | 1     | 1/2   |

Table 1: The $a_y$ coefficients of the non-perturbative event shape moment prediction
discarding the integration over the kinematically forbidden values of \( y \). This leads to the non-perturbative predictions for the moments of \( y \):

\[
\begin{align*}
\langle y^1 \rangle &= \langle y^1 \rangle_{pt} + A_y P, \\
\langle y^2 \rangle &= \langle y^2 \rangle_{pt} + 2 \langle y^1 \rangle_{pt} (a_y P) + (a_y P)^2, \\
\langle y^3 \rangle &= \langle y^3 \rangle_{pt} + 3 \langle y^2 \rangle_{pt} (a_y P) + 3 \langle y^1 \rangle_{pt} (a_y P)^2 + (a_y P)^3, \\
\langle y^4 \rangle &= \langle y^4 \rangle_{pt} + 4 \langle y^3 \rangle_{pt} (a_y P) + 6 \langle y^2 \rangle_{pt} (a_y P)^2 + 4 \langle y^1 \rangle_{pt} (a_y P)^3 + (a_y P)^4, \\
\langle y^5 \rangle &= \langle y^5 \rangle_{pt} + 5 \langle y^4 \rangle_{pt} (a_y P) + 10 \langle y^3 \rangle_{pt} (a_y P)^2 + 10 \langle y^2 \rangle_{pt} (a_y P)^3 + 5 \langle y^1 \rangle_{pt} (a_y P)^4 + (a_y P)^5 \quad (8)
\end{align*}
\]

The perturbative contribution to \( \langle y^n \rangle \) is given up to NNLO in terms of the dimensionless coefficients \( \bar{A}_{y,n}, \bar{B}_{y,n} \) and \( \bar{C}_{y,n} \) as:

\[
\langle y^n \rangle_{pt}(s, \mu^2) = \left( \frac{\alpha_s(\mu)}{2\pi} \right) \bar{A}_{y,n} + \left( \frac{\alpha_s(\mu)}{2\pi} \right)^2 \left( \bar{B}_{y,n} + \bar{A}_{y,n} \beta_0 \log \frac{\mu^2}{s} \right) \\
+ \left( \frac{\alpha_s(\mu)}{2\pi} \right)^3 \left( \bar{C}_{y,n} + 2 \bar{B}_{y,n} \beta_0 \log \frac{\mu^2}{s} + \bar{A}_{y,n} \left( \beta_0^2 \log \frac{\mu^2}{s} + \beta_1 \log \frac{\mu^2}{s} \right) \right) \nonumber \right) + \mathcal{O}(\alpha_s^4).
\]

In here, \( s \) denotes the centre-of-mass energy squared and \( \mu \) is the QCD renormalisation scale. The NLO expression is obtained by suppressing all terms at order \( \alpha_s^3 \). The first two coefficients of the QCD \( \beta \)-function are

\[
\beta_0 = \frac{11C_A - 4T_R N_F}{6}, \\
\beta_1 = \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{6}, \quad (10)
\]

with \( C_A = N, \ C_F = (N^2 - 1)/(2N), \ T_R = 1/2 \) for \( N = 3 \) colours and \( N_F \) quark flavours.

The perturbative coefficients in (9) are independent on the centre-of-mass energy. They are obtained by integrating parton-level distributions, which were calculated recently to NNLO accuracy [6, 17, 19]. These parton-level calculations are based on a numerical integration of the relevant three-parton, four-parton and five-parton matrix elements, which are combined into a parton-level event generator [18, 20, 21] after subtraction of infrared singular configurations using the antenna subtraction method [45]. These NNLO event shape distributions were used subsequently for improved extractions of the strong coupling constant [13, 46–49], matched on all-order resummation of logarithmically enhanced corrections [48, 50], and used for power correction studies on the thrust distribution [47].

The coefficients entering the event shape moments are computed at a renormalisation scale fixed to the centre-of-mass energy, and are therefore just dimensionless numbers for each observable and each value of \( n \). For the first five moments of the six event shape variables considered here, they were computed up to NNLO in \([22, 23]\).

## 3 Dispersive model extended to NNLO

Up to now, the dispersive model for power corrections to event shapes was used in connection with NLO calculations of the perturbative part. In this context, one obtains the following, \( 1/Q \)-dependent power correction [26]:

\[
P = \frac{4C_F}{\pi^2} \cdot \mathcal{M} \cdot \left\{ a_0 - \left[ \alpha_s(\mu_R) + \frac{\beta_0}{\pi} \alpha_s^2(\mu_R) \left( \ln \frac{\mu_R}{\mu} + 1 + \frac{K}{2\beta_0} \right) + \mathcal{O}(\alpha_s^3) \right] \right\} \times \frac{\mu_I}{Q}. \quad (11)
\]
with the Milan factor $\mathcal{M} = 1.49 \pm 20\%$, which is known at two loops. Its uncertainty [51] accounts for currently unknown corrections beyond this loop order. The term in square brackets amounts to the renormalon subtraction in the power corrections, expanded to NLO.

The prediction of the dispersive model can be extended to match onto the NNLO perturbative prediction, and first steps in this direction were taken already in [47] for power corrections to the thrust distribution.

The perturbative ingredients to the dispersive model are the running of the coupling constant and the relation between the $\overline{\text{MS}}$-coupling and the effective coupling, whose definition [52] absorbs universal correction terms from the cusp anomalous dimension.

In the present context, we use the evolution of the coupling constant to two loops

$$
\mu^2 \frac{d\alpha_s(\mu)}{d\mu^2} = -\alpha_s(\mu) \left[ \beta_0 \left( \frac{\alpha_s(\mu)}{2\pi} \right) + \beta_1 \left( \frac{\alpha_s(\mu)}{2\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right].
$$

(12)

Moreover, the relation between $\overline{\text{MS}}$-coupling and effective coupling reads

$$
\alpha_s^{\text{eff}} = \alpha_s \left[ 1 + K \frac{\alpha_s}{2\pi} + L \left( \frac{\alpha_s}{2\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right]
$$

(13)

$$
K = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} N_F,
$$

(14)

$$
L = C_A^2 \left( \frac{245}{24} - \frac{67 \pi^2}{9} \right) + \frac{11}{6} \zeta_3 + \frac{11}{5} \left( \frac{\pi^2}{6} \right)^2 + C_F N_F \left( - \frac{55}{24} + 2 \zeta_3 \right) + C_A N_F \left( - \frac{209}{108} + \frac{10 \pi^2}{9} + \frac{7}{3} \zeta_3 \right) + N_F^2 \left( - \frac{1}{27} \right)
$$

(15)

The coefficient $L$ is obtained from the three-loop cusp anomalous dimension [53, 54], which can be extracted from the three-loop corrections to the partonic splitting functions [55] or to the quark and gluon form factors [56].

The derivation of a generic power correction starts from considering a dimensionless quantity

$$
F = \int_0^Q d\mu f(\mu)
$$

(16)

with

$$
f(\mu) \propto a_F \alpha_s(\mu) \frac{\mu^p}{Q^{p+1}}
$$

(17)

assuming $F$ to be dimensionless. The value of $p$ determines the scaling behaviour of the power correction, with $p = 0$ for the leading power correction to event shape variables.

The dispersive model assumes that in the non-perturbative range of (16) the perturbative strong coupling $\alpha_s(\mu)$ is replaced by an effective coupling that remains finite for all $\mu$ values. One defines then the value of the integral over this region by

$$
\int_0^{\mu_I} d\mu \alpha_{s,\text{IR}}(\mu) \frac{\mu^p}{Q^{p+1}} \equiv \frac{\mu_I^{p+1}}{Q^{p+1}(p+1)} \alpha_p(\mu_I)
$$

(18)

introducing an infrared matching scale $\mu_I, \Lambda_{\text{QCD}} \ll \mu_I \ll Q$ and $\alpha_p$ as a non-perturbative parameter. One has then to subtract the perturbative part of (16) in the range from 0 to $\mu_I$ from the whole integral, that is, the value of (18) with $\alpha_s$ replaced by $\alpha_{s,\text{IR}}$.

This perturbative contribution to (16) thus acquires a dependence on the renormalisation scale $\mu_R$ used in the strong coupling constant. By requiring $F$ to be scale-independent, one can then infer logarithmic
terms in the non-perturbative contribution to (16). Applied to the event-shape power correction $P$ (with $p = 0$), this results in

$$P = \frac{4C_F}{\pi^2} \mathcal{M} \left\{ \alpha_0 - \left[ \alpha_s(\mu_R) + \frac{\beta_0}{\pi} \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{2\beta_0} \right) \alpha_s^2(\mu_R) + \right. \right.
\left. \left. 2\beta_1 \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{L}{2\beta_1} \right) + 8\beta_0^2 \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{2\beta_0} \right) \right. \right.
\left. \left. + 4\beta_0^2 \ln \left( \frac{\mu_R}{\mu_I} \right) \left( \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{\beta_0} \right) \alpha_s^3(\mu_R) \right) \right\} \times \frac{\mu_I}{Q}. \tag{19}$$

Together with (9) this gives the full expression for the event shape observable moments, including perturbative and non-perturbative contributions.

For $B_T$ and $B_W$ there is a further correction to (19). It arises from the kinematical mismatch between parton direction and thrust direction used to define the hemispheres used in the broadening variables. Retaining (8), this modification can be accounted for by a modification to the power correction. In [27], this modification was computed to NLO for the first moment as

$$P_{(B_W)} = P \left( \frac{\pi}{8C_F \hat{\alpha}_s \left( 1 + \frac{K \hat{\alpha}_s}{2\pi} \right)} + \frac{3}{4} \frac{\beta_0}{6C_F} + \eta_0 \right), \tag{20}$$

$$P_{(B_T)} = P \left( \frac{\pi}{4C_F \hat{\alpha}_s \left( 1 + \frac{K \hat{\alpha}_s}{2\pi} \right)} + \frac{3}{4} \frac{\beta_0}{3C_F} + \eta_0 \right). \tag{21}$$

with $\hat{\alpha}_s(Q) = \alpha_s(e^{-\frac{Q}{4}})$ and $\eta_0 = -0.6137$. Corrections to higher moments have not been derived up to now, and we assume that they can be approximated by using the above modifications to the power correction in all moments. The full NNLO expression for these has not been calculated either. The potentially dominant NNLO terms can however be approximated by including the effective coupling to this order, resulting in

$$P_{(B_W)} = P \left( \frac{\pi}{8C_F \hat{\alpha}_s \left( 1 + \frac{K \hat{\alpha}_s + L \hat{\alpha}_s^2}{2\pi} \right)} + \frac{3}{4} \frac{\beta_0}{6C_F} + \eta_0 \right), \tag{22}$$

$$P_{(B_T)} = P \left( \frac{\pi}{4C_F \hat{\alpha}_s \left( 1 + \frac{K \hat{\alpha}_s + L \hat{\alpha}_s^2}{2\pi} \right)} + \frac{3}{4} \frac{\beta_0}{3C_F} + \eta_0 \right). \tag{23}$$

However, further NNLO corrections to this expression will reside in the coefficient $\eta_0$. Therefore, we will treat $B_W$ and $B_T$ separately from the other variables in the numerical studies in the following section.

## 4 Analysis of JADE and OPAL data

The theoretical expressions for event shapes derived in the previous section contain two parameters: the strong coupling constant $\alpha_s(M_Z)$ and the non-perturbative coupling parameter $\alpha_0$. Using experimental
data on event shape moments, it is possible to fit these parameters. The data from the JADE and OPAL experiments [8] consists of 18 points at centre-of-mass energies between 14.0 and 206.6 GeV for the first five moments of $T$, $C$, $Y_3$, $M_H$, $B_W$ and $B_T$, and have been taken from [29]. For each moment the NLO as well as the NNLO prediction was fitted with $\alpha_s(M_Z)$ and $\alpha_0$ as fit parameters, except for the moments of $Y_3$, which have no power correction and thus are independent of $\alpha_0$. For the heavy jet mass, we use only the even moments $\langle M_H^2 \rangle$ and $\langle M_H^4 \rangle$, since the theoretical prediction is in terms of $\rho = M_H^2/s$.

4.1 Fits

The fits were done using the program ROOT [57] and its $\chi^2$ fit method. The errors used for the fit were the total errors, composed of the experimental statistic and systematic errors, added in quadrature. Based on these, ROOT returned errors on the fit which are displayed in Tables 4-15 in the appendix together with the fit results. For $T$ and $C$ the NNLO values of $\alpha_s(M_Z)$ and $\alpha_0$ seem to be more stable throughout the moments, as at NNLO they increase less towards higher moments than at NLO. For $Y_3$ and $\rho$, where the values decrease at higher moments, this is not the case. These moments show $\alpha_s(M_Z)$ results which are significantly lower at NNLO than at NLO. For $B_W$ the $\alpha_s(M_Z)$ values at NNLO are much lower than the ones of the other observables, and do not change much from NLO to NNLO. For $B_T$ the $\alpha_s(M_Z)$ values at NNLO are lower than at NLO. Both are exceptionally stable throughout the different moments. The $\alpha_0$ values of all moments are higher at NNLO than at NLO.

4.2 Theoretical systematic errors

There are different parameters in the theoretical prediction which may influence the results displayed above, namely the matching scale $\mu_I$, the renormalisation scale $\mu_R$ and the Milan factor $\mathcal{M}$. In order to estimate the resulting theoretical uncertainty on $\alpha_s(M_Z)$ and $\alpha_0$, the fits were repeated, $\mu_I$, $\mu_R$ and $\mathcal{M}$ being separately varied by a certain amount.

For this purpose the scaling factor $x_\mu = \frac{\mu_R}{\mu_I}$ was introduced. The uncertainty on the corresponding parameter was then taken to be the difference between the nominal and the new value returned by ROOT. In order to get a total systematic error, the greater values of the up and down uncertainties were determined and quadratically added. As $\alpha_0$ depends directly on $\mu_I$ no error was determined for this variation. For $Y_3$ there is only an error on $\alpha_s(M_Z)$ coming from the $x_\mu$ variation, since the theoretical description of this observable does not contain a contribution from the leading power correction, and is thus independent on $\mu_I$ and $\mathcal{M}$. At NLO the fit to the moment $\langle C^3 \rangle$ suffers from a numerical instability by scaling up $\mathcal{M}$ by 20%. The numbers reported in Table 6 refer to an up variation of 19%.

The NLO error on $\alpha_s(M_Z)$ agrees well with the values of [8]. At NNLO, it is reduced by more than half throughout all event shape observables except $B_W$, confirming a good description by the NNLO prediction. Unfortunately, this is not the case for the error on $\alpha_0$. It does not change much from NLO to NNLO, even increasing a little in the first moments due to the higher $x_\mu$ uncertainty at NNLO and decreasing slightly at the higher moments, with exception, again, of $B_W$. Analysing the different sources of the systematical errors, we observe that the error on $\alpha_s(M_Z)$ is clearly dominated by the $x_\mu$ variation, while the largest contribution to the error on $\alpha_0$ comes from the uncertainty on the Milan factor $\mathcal{M}$. Since

| nominal value | up variation | down variation |
|---------------|--------------|----------------|
| $\mu_I$ (GeV) | 2            | 3              |
| $x_\mu$      | 1            | 2              |
| $\mathcal{M}$| 1.49         | 1.788 (+20%)   |
|              |              | 1.192 (-20%)   |

Table 2: Table of the $\mu_I$, $x_\mu$ and $\mathcal{M}$ variations
this uncertainty has not been improved in the current study, it is understandable that the systematic error on $\alpha_0$ remains unchanged. This finding clearly motivates the need for a three-loop calculation of the Milan factor. However, it is very important to note that the uncertainty on the Milan factor has little impact on the extraction of $\alpha_s(M_Z)$, thereby demonstrating the systematic decoupling of perturbative and non-perturbative effects in the dispersive model.

For the higher moments ($n \geq 2$) of the jet broadenings $B_W$ and $B_T$, the kinematical modifications to the power correction are not known at present. We have approximated them in the above fits by the corrections to the first moments, given to NLO and NNLO in the previous section. If we do not apply these correction to the higher moments, the mutual consistency of the parameter extractions from different moments of $B_T$ deteriorates considerably, while only minor improvements in consistency are observed on $B_W$.

Including empirical hadron mass corrections [44,58] from PYTHIA affects in particular the parameter extraction from $\rho$, resulting in values of $\alpha_s(M_Z)$ and $\alpha_0$ from $\rho$ much lower than from the other variables.
Since these corrections may interplay with other non-perturbative parameters in PYTHIA, we do not include them in our default fits or error estimates.

By taking the weighted means over the corresponding values from all moments of all observables one gets combined values for $\alpha_s(M_Z)$ and $\alpha_0$. The weights are given by the inverse of the total error squared and are normalized such that the sum over all weights is equal to one. For the errors one has to take care of the correlation between the errors of the single measurements. The correlation matrix for $\alpha_s(M_Z)$ and $\alpha_0$ is in first approximation equal to the correlation matrix for the event shape moments, since the variable transformation is linear in first approximation. The correlation matrix for the event shape moments is given in [29]. We first combine the measurements from different moments of the same observable. Figures 1 and 2 compare the combined NNLO results on the $\alpha_s(M_Z)$ and $\alpha_0$ measurements. Owing to the large correlation between individual moments of the same observable, the combined errors are only marginally smaller than the errors obtained from single measurements. The combined results and their errors are summarised in Table 3. From this Table, we clearly observe that the the theoretical error on the extraction of $\alpha_s(M_Z)$ from $\rho$, $Y_3$ and $B_W$ is considerably smaller than from $\tau$, $C$ and $B_T$. It was observed previously in [22] that the moments of the former three shape variables receive moderate
| Observable | $\alpha_s(M_Z)$ | Experimental Error | Theoretical Error | Total Error |
|------------|----------------|--------------------|------------------|-------------|
| $\tau$     | 0.1208         | 0.0018             | 0.0045           | 0.0048      |
| C          | 0.1181         | 0.0013             | 0.0046           | 0.0048      |
| $\rho$     | 0.1131         | 0.0024             | 0.0019           | 0.0031      |
| $Y_3$      | 0.1139         | 0.0016             | 0.0015           | 0.0022      |
| $B_T$      | 0.1161         | 0.0014             | 0.0036           | 0.0038      |
| $B_W$      | 0.1062         | 0.0021             | 0.0018           | 0.0027      |
| Total      | 0.1131         | 0.0017             | 0.0022           | 0.0028      |
| Total w/o $B_T,B_W$ | 0.1153 | 0.0017             | 0.0023           | 0.0028      |

| Observable | $\alpha_0$ | Experimental Error | Theoretical Error | Total Error |
|------------|------------|--------------------|------------------|-------------|
| $\tau$     | 0.5444     | 0.0184             | 0.0388           | 0.0430      |
| C          | 0.4841     | 0.0066             | 0.0347           | 0.0353      |
| $\rho$     | 0.6380     | 0.0270             | 0.0824           | 0.0867      |
| $Y_3$      | -          | -                  | -                | -           |
| $B_T$      | 0.4924     | 0.0102             | 0.0449           | 0.0460      |
| $B_W$      | 0.3362     | 0.0125             | 0.0338           | 0.0360      |
| Total      | 0.4604     | 0.0108             | 0.0359           | 0.0375      |
| Total w/o $B_T,B_W$ | 0.5132 | 0.0115             | 0.0381           | 0.0398      |

Table 3: Table of the $\alpha_s(M_Z)$ and $\alpha_0$ results for the individual moments and the global weighted average.

NNLO corrections for all $n$, while the NNLO corrections for the latter three are large already for $n = 1$ and increase with $n$. Consequently, the theoretical description of the moments of $\rho$, $Y_3$ and $B_W$ displays a higher perturbative stability, which is reflected in the theoretical uncertainty on $\alpha_S(M_Z)$ derived from them.

In a second step, we combine the $\alpha_s(M_Z)$ and $\alpha_0$ measurements obtained from different event shape variables. Taking the weighted mean over all values, but excluding the values for the moments of $B_W$ and $B_T$ where the theoretical description is incomplete, we obtain at NNLO:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017(\text{exp}) \pm 0.0023(\text{th}),$$

$$\alpha_0 = 0.5132 \pm 0.0115(\text{exp}) \pm 0.0381(\text{th}),$$

where the errors have been derived taking into account the correlation between the moments of different event shapes. Including the values for $B_W$ and $B_T$ modifies this result to:

$$\alpha_s^{B}(M_Z) = 0.1131 \pm 0.0017(\text{exp}) \pm 0.0022(\text{th}),$$

$$\alpha_0^{B} = 0.4604 \pm 0.0108(\text{exp}) \pm 0.0359(\text{th}).$$

These latter values are however quoted only to illustrate the impact of including the broadenings. The default fit result is (24), where only observables with a consistent theoretical description are included.

To illustrate the improvement due to the inclusion of the NNLO corrections, we also quote the corresponding NLO results. Based on $\tau$, $C$, $\rho$ and $Y_3$, we obtain:

$$\alpha_s^{\text{NLO}}(M_Z) = 0.1200 \pm 0.0021(\text{exp}) \pm 0.0062(\text{th}),$$

$$\alpha_0^{\text{NLO}} = 0.4957 \pm 0.0118(\text{exp}) \pm 0.0393(\text{th}),$$

while inclusion of $B_W$ and $B_T$ modifies this to

$$\alpha_s^{\text{NLO,B}}(M_Z) = 0.1147 \pm 0.0020(\text{exp}) \pm 0.0046(\text{th}),$$

$$\alpha_0^{\text{NLO,B}} = 0.4019 \pm 0.0130(\text{exp}) \pm 0.0296(\text{th}),$$
We compare the NLO and NNLO combinations in Figure 3. It can be seen very clearly that the measurements obtained from the different variables are consistent with each other within errors. The average of $\alpha_s(M_Z)$ is dominated by the measurements based on $\rho$ and $Y_3$, which have the smallest theoretical uncertainties. From NLO to NNLO, the error on $\alpha_s(M_Z)$ is reduced by a factor two, and the result shifts towards the lower end of the NLO error band, as was already the case in the individual measurements. No improvement and no shift in the central value between NLO and NNLO is seen on $\alpha_0$. 

Figure 3: Error band plot of the final results. The points for $\alpha_s(M_Z)$ are $C, T, Y_3, M_H$ and for $\alpha_0 C, T, M_H$. 
Figure 4: Comparison of fits with hadronization corrections from PYTHIA and power corrections from the dispersive model.

5 Comparison with PYTHIA hadronization corrections

The primary motivation for studying power corrections to moments of event shapes in the dispersive model comes from the observation that the commonly used method to derive hadronization corrections from multi-purpose event generator programs may be unreliable [13].

To quantify the difference of both approaches to hadronization corrections, we compare them on the example of the moments of $1 - T$. For this comparison, we extracted the PYTHIA [14] hadronization corrections to these moments from the ratio of PYTHIA hadron level and parton level results. Using
these corrections in combination with the NNLO perturbative expressions for the event shape moments, we repeated the fit of $\alpha_S(M_Z)$ on the different moments of $1 - T$. The results are displayed and compared with the fits in the dispersive model in Figure 4. We observe that both approaches yield a reasonable description of the experimental data, but that the resulting values of $\alpha_S(M_Z)$ are considerably larger when applying hadronization corrections extracted from PYTHIA. Given that the perturbative contribution increases monotonously with $\alpha_S(M_Z)$, this indicates that the hadronization corrections in PYTHIA are considerably smaller (and perhaps underestimated) than those obtained in the dispersive model.

This observation is quantified on the moments of $(1 - T)$ at $\sqrt{s} = M_Z$, displayed in Figure 5. Depending on the moment number, we observe that the PYTHIA hadronization corrections are between two and four times smaller than those obtained from the dispersive model. It can also be seen that the PYTHIA-based predictions are systematically below the experimental data, which perhaps indicates that, despite the decent agreement on the full range of energies, Figure 4, PYTHIA fails in the precise description of the energy dependence of the hadronization corrections.

Our comparison suggests strongly that hadronization corrections extracted from PYTHIA (or from other comparable multi-purpose event generator programs [15]) are lower than power corrections obtained from analytical hadronization models. As a consequence, using PYTHIA hadronization corrections to analyse data in view of precision extractions of $\alpha_S(M_Z)$ may result in anomalously large values, since the missing numerical magnitude of the power corrections must be compensated by a larger perturbative contribution.

6 Conclusions

In this paper, we studied the perturbative and non-perturbative contributions to the moments of event shapes in $e^+e^-$ annihilation. In view of the recently calculated NNLO perturbative contributions [22, 23] to the event shape moments, we extended the dispersive model for non-perturbative power corrections [24–26] to include all logarithmic corrections to this order. The normalisation of the power correction (the
Milan factor [26]) is however still restricted to NLO accuracy, and specific corrections [27] to the jet broadenings $B_W$ and $B_T$ are also only included to NLO.

We used this newly obtained theoretical description of the event shape moments to reanalyse data from JADE and OPAL in view of a determination of the strong coupling constant $\alpha_s(M_Z)$ and of the non-perturbative parameter $\alpha_0$. We observed that inclusion of the NNLO corrections results in a considerably better consistency among the values extracted from different moments of the same variable, and an improved consistency among the different variables. Averaging over the different moments and different shapes (excluding $B_W$ and $B_T$, where the theoretical description is incomplete, and taking proper account of the uncertainty due to missing terms in the Milan factor), we obtain the following combined values:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017(\text{fit}) \pm 0.0023(\text{th}),$$
$$\alpha_0 = 0.5132 \pm 0.0115(\text{fit}) \pm 0.0381(\text{th}),$$

Compared to previous NLO results, the theoretical error on $\alpha_s(M_Z)$ (which is dominated by the scale variation, improved at NNLO) is reduced by a factor of two, while the error on $\alpha_0$ (which is dominated by the uncertainty on the Milan factor) remains unchanged. We observed that the sources of uncertainty on $\alpha_s(M_Z)$ and $\alpha_0$ largely decouple. An improvement on $\alpha_0$ will only be achievable once the three-loop corrections to the Milan factor become available.

It is noteworthy that application of the dispersive model to hadronization corrections results in a considerably lower value of $\alpha_s(M_Z)$ from event shapes than previous studies based on Monte Carlo hadronization models [13], and in better agreement with measurements from other observables [59]. A direct comparison hints to an underestimation of hadronization effects in the Monte Carlo models. This feature has been observed previously also on the thrust distribution [47]. Revisiting the hadronization models in multi-purpose Monte Carlo programs appears to be mandatory for meaningful precision QCD studies at colliders.

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**A  Tables of results**

In this appendix, we collect the extractions of $\alpha_s(M_Z)$ and $\alpha_0$ at NLO and NNLO from individual moments of the six event shape variables: $\tau$, $C$, $\rho$, $Y_3$, $B_T$, $B_W$. 
| NLO       | $\langle \tau \rangle$ | $\langle \tau^2 \rangle$ | $\langle \tau^3 \rangle$ | $\langle \tau^4 \rangle$ | $\langle \tau^5 \rangle$ |
|----------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\chi^2$/dof | 1.0043 | 1.0565 | 0.8399 | 0.6459 | 0.4740 |
| $\alpha_s(M_Z)$ | 0.1242 | 0.1344 | 0.1416 | 0.1479 | 0.1534 |
| Experimental Error | 0.0018 | 0.0023 | 0.0028 | 0.0033 | 0.0038 |
| $x_\mu$ variation: $x_\mu = 0.5$ | -0.0054 | -0.0083 | -0.0101 | -0.0115 | -0.0129 |
| $x_\mu = 2.0$ | 0.0066 | 0.0102 | 0.0123 | 0.0143 | 0.0162 |
| $\mu_I$ variation: $\mu_I = 1.0$ Gev | 0.0025 | 0.0035 | 0.0038 | 0.0045 | 0.0054 |
| $\mu_I = 3.0$ Gev | -0.0019 | -0.0025 | -0.0027 | -0.0031 | -0.0037 |
| $\mathcal{M}$ variation: $\mathcal{M} - 20\%$ | 0.0012 | 0.0016 | 0.0017 | 0.0020 | 0.0024 |
| $\mathcal{M} + 20\%$ | -0.0011 | -0.0014 | -0.0016 | -0.0018 | -0.0021 |
| Theoretical Error | 0.0072 | 0.0109 | 0.0130 | 0.0151 | 0.0172 |
| $\alpha_0$ | 0.4782 | 0.5147 | 0.5359 | 0.5521 | 0.5744 |
| Experimental Error | 0.0151 | 0.0152 | 0.0189 | 0.0222 | 0.0243 |
| $x_\mu$ variation: $x_\mu = 0.5$ | 0.0017 | 0.0004 | -0.0038 | -0.0065 | -0.0081 |
| $x_\mu = 2.0$ | -0.0000 | -0.0001 | 0.0030 | 0.0051 | 0.0064 |
| $\mathcal{M}$ variation: $\mathcal{M} - 20\%$ | 0.0432 | 0.0423 | 0.0405 | 0.0377 | 0.0375 |
| $\mathcal{M} + 20\%$ | -0.0306 | -0.0307 | -0.0298 | -0.0284 | -0.0290 |
| Theoretical Error | 0.0433 | 0.0423 | 0.0406 | 0.0382 | 0.0384 |

Table 4: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $\tau$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.

| NNLO      | $\langle \tau \rangle$ | $\langle \tau^2 \rangle$ | $\langle \tau^3 \rangle$ | $\langle \tau^4 \rangle$ | $\langle \tau^5 \rangle$ |
|----------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\chi^2$/dof | 0.9889 | 0.9411 | 0.7284 | 0.5526 | 0.3997 |
| $\alpha_s(M_Z)$ | 0.1166 | 0.1202 | 0.1233 | 0.1267 | 0.1294 |
| Experimental Error | 0.0015 | 0.0018 | 0.0021 | 0.0024 | 0.0027 |
| $x_\mu$ variation: $x_\mu = 0.5$ | -0.0020 | -0.0034 | -0.0042 | -0.0048 | -0.0054 |
| $x_\mu = 2.0$ | 0.0025 | 0.0042 | 0.0051 | 0.0058 | 0.0065 |
| $\mu_I$ variation: $\mu_I = 1.0$ Gev | 0.0017 | 0.0017 | 0.0017 | 0.0019 | 0.0022 |
| $\mu_I = 3.0$ Gev | -0.0011 | -0.0011 | -0.0011 | -0.0013 | -0.0014 |
| $\mathcal{M}$ variation: $\mathcal{M} - 20\%$ | 0.0009 | 0.0010 | 0.0009 | 0.0011 | 0.0012 |
| $\mathcal{M} + 20\%$ | -0.0009 | -0.0009 | -0.0009 | -0.0010 | -0.0011 |
| Theoretical Error | 0.0032 | 0.0046 | 0.0054 | 0.0062 | 0.0070 |
| $\alpha_0$ | 0.5165 | 0.5408 | 0.5452 | 0.5512 | 0.5641 |
| Experimental Error | 0.0135 | 0.0152 | 0.0194 | 0.0223 | 0.0246 |
| $x_\mu$ variation: $x_\mu = 0.5$ | 0.0140 | 0.0075 | 0.0016 | -0.0008 | -0.0023 |
| $x_\mu = 2.0$ | -0.0078 | -0.0045 | -0.0001 | 0.0019 | 0.0031 |
| $\mathcal{M}$ variation: $\mathcal{M} - 20\%$ | 0.0415 | 0.0430 | 0.0396 | 0.0357 | 0.0347 |
| $\mathcal{M} + 20\%$ | -0.0298 | -0.0308 | -0.0286 | -0.0264 | -0.0261 |
| Theoretical Error | 0.0438 | 0.0436 | 0.0397 | 0.0358 | 0.0348 |

Table 5: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $\tau$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.
| NLO       | \langle C \rangle | \langle C^2 \rangle | \langle C^3 \rangle | \langle C^4 \rangle | \langle C^5 \rangle |
|-----------|------------------|------------------|------------------|------------------|------------------|
| \chi^2/dof | 1.1849           | 1.5245           | 1.5651           | 1.5446           | 1.4094           |
| \alpha_s(M_Z) | 0.1230           | 0.1308           | 0.1347           | 0.1374           | 0.1407           |
| Experimental Error | 0.0013           | 0.0016           | 0.0020           | 0.0023           | 0.0026           |
| \mu variation: | \mu = 0.5       | -0.0052          | -0.0079          | -0.0091          | -0.0100          | -0.0108          |
| x_\mu = 2.0 | 0.0063           | 0.0096           | 0.0111           | 0.0122           | 0.0134           |
| \mu_I variation: | \mu_I = 1.0 Gev | 0.0029           | 0.0045           | 0.0051           | 0.0057           | 0.0064           |
| \mu_I = 3.0 Gev | -0.0022          | -0.0031          | -0.0034          | -0.0038          | -0.0041          |
| M variation: | M - 20%         | 0.0013           | 0.0020           | 0.0022           | 0.0025           | 0.0028           |
| M + 20% | -0.0012          | -0.0018          | -0.0019          | -0.0022          | -0.0024          |
| Theoretical Error | 0.0071           | 0.0107           | 0.0124           | 0.0137           | 0.0151           |
| \alpha_0 | 0.4267           | 0.4632           | 0.4789           | 0.4839           | 0.4857           |
| Experimental Error | 0.0082           | 0.0064           | 0.0067           | 0.0069           | 0.0070           |
| \mu variation: | \mu = 0.5       | -0.0020          | -0.0033          | -0.0039          | -0.0043          | -0.0054          |
| x_\mu = 2.0 | -0.0029          | -0.0021          | 0.0007           | 0.0027           | 0.0042           |
| M variation: | M - 20%         | 0.0324           | 0.0359           | 0.0377           | 0.0376           | 0.0366           |
| M + 20% | -0.0236          | -0.0266          | -0.0268          | -0.0283          | -0.0278          |
| Theoretical Error | 0.0328           | 0.0360           | 0.0377           | 0.0377           | 0.0370           |

Table 6: Results for \alpha_s(Q) and \alpha_0 as obtained from fits to C moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.

| NNLO      | \langle C \rangle | \langle C^2 \rangle | \langle C^3 \rangle | \langle C^4 \rangle | \langle C^5 \rangle |
|-----------|------------------|------------------|------------------|------------------|------------------|
| \chi^2/dof | 1.1574           | 1.2418           | 1.2353           | 1.1735           | 1.0126           |
| \alpha_s(M_Z) | 0.1161           | 0.1180           | 0.1193           | 0.1202           | 0.1216           |
| Experimental Error | 0.0011           | 0.0013           | 0.0016           | 0.0017           | 0.0019           |
| \mu variation: | \mu = 0.5       | -0.0020          | -0.0033          | -0.0039          | -0.0043          | -0.0046          |
| x_\mu = 2.0 | 0.0025           | 0.0040           | 0.0047           | 0.0051           | 0.0056           |
| \mu_I variation: | \mu_I = 1.0 Gev | 0.0019           | 0.0022           | 0.0023           | 0.0025           | 0.0028           |
| \mu_I = 3.0 Gev | -0.0013          | -0.0014          | -0.0015          | -0.0016          | -0.0017          |
| M variation: | M - 20%         | 0.0011           | 0.0012           | 0.0013           | 0.0014           | 0.0015           |
| M + 20% | -0.0010          | -0.0011          | -0.0012          | -0.0013          | -0.0014          |
| Theoretical Error | 0.0033           | 0.0047           | 0.0054           | 0.0059           | 0.0064           |
| \alpha_0 | 0.4689           | 0.4897           | 0.4919           | 0.4877           | 0.4828           |
| Experimental Error | 0.0071           | 0.0063           | 0.0067           | 0.0069           | 0.0070           |
| \mu variation: | \mu = 0.5       | 0.0166           | 0.0095           | 0.0053           | 0.0027           | 0.0010           |
| x_\mu = 2.0 | -0.0105          | -0.0066          | -0.0033          | -0.0013          | 0.0001           |
| M variation: | M - 20%         | 0.0316           | 0.0360           | 0.0359           | 0.0346           | 0.0326           |
| M + 20% | -0.0234          | -0.0264          | -0.0265          | -0.0258          | -0.0246          |
| Theoretical Error | 0.0357           | 0.0372           | 0.0363           | 0.0347           | 0.0326           |

Table 7: Results for \alpha_s(Q) and \alpha_0 as obtained from fits to C moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.
### Table 8: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $\rho$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.

| NLO          | $\langle \rho \rangle$ | $\langle \rho^2 \rangle$ |
|--------------|------------------------|--------------------------|
| $\chi^2$/dof | 0.6587                 | 0.7547                   |
| $\alpha_s(M_Z)$ | 0.1164                 | 0.1152                   |
| Experimental Error | 0.0023                 | 0.0033                   |
| $x_\mu$ variation: $x_\mu = 0.5$ | -0.0028                | -0.0038                  |
| $x_\mu = 2.0$ | 0.0039                 | 0.0049                   |
| $\mu_I$ variation: $\mu_I = 1.0$ Gev | 0.0014                 | 0.0014                   |
| $\mu_I = 3.0$ Gev | -0.0011                | -0.0011                  |
| $M$ variation: $M - 20\%$ | 0.0006                 | 0.0006                   |
| $M + 20\%$ | -0.0006                | -0.0006                  |
| Theoretical Error | 0.0042                 | 0.0051                   |
| $\alpha_0$ | 0.5914                 | 0.5657                   |
| Experimental Error | 0.0268                 | 0.0361                   |
| $x_\mu$ variation: $x_\mu = 0.5$ | 0.0115                 | 0.0092                   |
| $x_\mu = 2.0$ | -0.0042                | -0.0047                  |
| $M$ variation: $M - 20\%$ | 0.0795                 | 0.0748                   |
| $M + 20\%$ | -0.0539                | -0.0508                  |
| Theoretical Error | 0.0803                 | 0.0753                   |

### Table 9: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $\rho$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.

| NNLO          | $\langle \rho \rangle$ | $\langle \rho^2 \rangle$ |
|--------------|------------------------|--------------------------|
| $\chi^2$/dof | 0.6750                 | 0.7607                   |
| $\alpha_s(M_Z)$ | 0.1142                 | 0.1113                   |
| Experimental Error | 0.0021                 | 0.0030                   |
| $x_\mu$ variation: $x_\mu = 0.5$ | -0.0009                | -0.0012                  |
| $x_\mu = 2.0$ | 0.0013                 | 0.0017                   |
| $\mu_I$ variation: $\mu_I = 1.0$ Gev | 0.0012                 | 0.0010                   |
| $\mu_I = 3.0$ Gev | -0.0008                | -0.0007                  |
| $M$ variation: $M - 20\%$ | 0.0007                 | 0.0006                   |
| $M + 20\%$ | -0.0006                | -0.0006                  |
| Theoretical Error | 0.0018                 | 0.0020                   |
| $\alpha_0$ | 0.6565                 | 0.6208                   |
| Experimental Error | 0.0224                 | 0.0316                   |
| $x_\mu$ variation: $x_\mu = 0.5$ | 0.0312                 | 0.0233                   |
| $x_\mu = 2.0$ | -0.0184                | -0.0143                  |
| $M$ variation: $M - 20\%$ | 0.0799                 | 0.0759                   |
| $M + 20\%$ | -0.0547                | -0.0518                  |
| Theoretical Error | 0.0858                 | 0.0794                   |
### Table 10: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $Y_3$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.

|         | $\langle Y_3^3 \rangle$ | $\langle Y_3^4 \rangle$ | $\langle Y_3^5 \rangle$ |
|---------|-------------------------|-------------------------|-------------------------|
| $\chi^2$/dof | 0.8616                  | 0.6386                  | 0.7771                  |
| $\alpha_s(M_Z)$ | 0.1183                  | 0.1172                  | 0.1165                  |
| Experimental Error | 0.0011                  | 0.0016                  | 0.0020                  |
| $x_{\mu}$ variation: $x_{\mu} = 0.5$ | -0.0040                  | -0.0042                  | -0.0040                  |
| $x_{\mu} = 2.0$ | 0.0053                  | 0.0054                  | 0.0052                  |
| Theoretical Error | 0.0047                  | 0.0048                  | 0.0046                  |

### Table 11: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $Y_3$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.

|         | $\langle Y_3^3 \rangle$ | $\langle Y_3^4 \rangle$ | $\langle Y_3^5 \rangle$ |
|---------|-------------------------|-------------------------|-------------------------|
| $\chi^2$/dof | 0.8577                  | 0.6581                  | 0.7948                  |
| $\alpha_s(M_Z)$ | 0.1156                  | 0.1136                  | 0.1136                  |
| Experimental Error | 0.0010                  | 0.0015                  | 0.0019                  |
| $x_{\mu}$ variation: $x_{\mu} = 0.5$ | -0.0005                  | -0.0008                  | -0.0006                  |
| $x_{\mu} = 2.0$ | 0.0015                  | 0.0017                  | 0.0015                  |
| Theoretical Error | 0.0010                  | 0.0013                  | 0.0011                  |

### Table 12: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $B_T$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.

|         | $\langle B_T \rangle$ | $\langle B_T^2 \rangle$ | $\langle B_T^3 \rangle$ | $\langle B_T^4 \rangle$ | $\langle B_T^5 \rangle$ |
|---------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\chi^2$/dof | 1.5775                  | 1.6741                  | 1.5926                  | 1.4005                  | 1.1996                  |
| $\alpha_s(M_Z)$ | 0.1199                  | 0.1276                  | 0.1308                  | 0.1327                  | 0.1347                  |
| Experimental Error | 0.0012                  | 0.0018                  | 0.0023                  | 0.0027                  | 0.0031                  |
| $x_{\mu}$ variation: $x_{\mu} = 0.5$ | -0.0037                  | -0.0078                  | -0.0093                  | -0.0101                 | -0.0108                 |
| $x_{\mu} = 2.0$ | 0.0049                  | 0.0094                  | 0.0112                  | 0.0123                  | 0.0133                  |
| $\mu_I$ variation: $\mu_I = 1.0$ Gev | 0.0021                  | 0.0032                  | 0.0036                  | 0.0038                  | 0.0041                  |
| $\mu_I = 3.0$ Gev | -0.0016                 | -0.0024                 | -0.0026                 | -0.0028                 | -0.0030                 |
| $M$ variation: $M - 20\%$ | 0.0010                  | 0.0015                  | 0.0016                  | 0.0017                  | 0.0018                  |
| $M + 20\%$ | -0.0009                 | -0.0014                 | -0.0015                 | -0.0016                 | -0.0017                 |
| Theoretical Error | 0.0054                  | 0.0101                  | 0.0119                  | 0.0130                  | 0.0140                  |
| $\alpha_0$ | 0.4252                  | 0.4897                  | 0.5180                  | 0.5193                  | 0.5088                  |
| Experimental Error | 0.0130                  | 0.0105                  | 0.0112                  | 0.0129                  | 0.0146                  |
| $x_{\mu}$ variation: $x_{\mu} = 0.5$ | 0.0154                  | 0.0083                  | 0.0005                  | -0.0050                 | -0.0093                 |
| $x_{\mu} = 2.0$ | -0.0106                 | -0.0074                 | -0.0015                 | 0.0031                  | 0.0070                  |
| $M$ variation: $M - 20\%$ | 0.0333                  | 0.0443                  | 0.0500                  | 0.0493                  | 0.0452                  |
| $M + 20\%$ | -0.0238                 | -0.0318                 | -0.0358                 | -0.0354                 | -0.0329                 |
| Theoretical Error | 0.0367                  | 0.0451                  | 0.0501                  | 0.0496                  | 0.0462                  |
| NNLO          | \langle B_T \rangle | \langle B_T^2 \rangle | \langle B_T^3 \rangle | \langle B_T^4 \rangle | \langle B_T^5 \rangle |
|--------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \chi^2/dof   | 1.6191              | 1.4765              | 1.3723              | 1.2059              | 1.0363              |
| \alpha_s(M_Z) | 0.1164              | 0.1158              | 0.1158              | 0.1158              | 0.1162              |
| Experimental Error | 0.0011              | 0.0014              | 0.0017              | 0.0019              | 0.0022              |
| x_\mu variation: x_\mu = 0.5 | -0.0012             | -0.0027             | -0.0033             | -0.0035             | -0.0037             |
| \mu_I variation: \mu_I = 1.0 Gev | 0.0017              | 0.0017              | 0.0018              | 0.0018              | 0.0019              |
| \mathcal{M} variation: \mathcal{M} - 20% | 0.0010              | 0.0010              | 0.0010              | 0.0010              | 0.0011              |
| Theoretical Error | 0.0025              | 0.0040              | 0.0046              | 0.0049              | 0.0052              |
| \alpha_0      | 0.4844              | 0.5053              | 0.5059              | 0.4938              | 0.4772              |
| Experimental Error | 0.0104              | 0.0094              | 0.0098              | 0.0108              | 0.0117              |
| x_\mu variation: x_\mu = 0.5 | 0.0491              | 0.0272              | 0.0190              | 0.0142              | 0.0109              |
| \mu_I variation: \mu_I = 1.0 Gev | -0.0295             | -0.0186             | -0.0129             | -0.0093             | -0.0066             |
| \mathcal{M} variation: \mathcal{M} - 20% | 0.0325              | 0.0419              | 0.0436              | 0.0415              | 0.0374              |
| Theoretical Error | 0.0589              | 0.0500              | 0.0476              | 0.0438              | 0.0390              |

Table 13: Results for \alpha_s(Q) and \alpha_0 as obtained from fits to B_T moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.

| NLO          | \langle B_W \rangle | \langle B_W^2 \rangle | \langle B_W^3 \rangle | \langle B_W^4 \rangle | \langle B_W^5 \rangle |
|--------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \chi^2/dof   | 1.5082              | 1.2870              | 1.1182              | 0.8965              | 0.6999              |
| \alpha_s(M_Z) | 0.1128              | 0.1077              | 0.1049              | 0.1023              | 0.1010              |
| Experimental Error | 0.0015              | 0.0020              | 0.0027              | 0.0033              | 0.0039              |
| x_\mu variation: x_\mu = 0.5 | 0.0007              | -0.0028             | -0.0026             | -0.0022             | -0.0019             |
| \mu_I variation: \mu_I = 1.0 Gev | 0.0018              | 0.0015              | 0.0016              | 0.0016              | 0.0016              |
| \mathcal{M} variation: \mathcal{M} - 20% | 0.0008              | 0.0007              | 0.0007              | 0.0007              | 0.0007              |
| Theoretical Error | 0.0021              | 0.0040              | 0.0039              | 0.0035              | 0.0032              |
| \alpha_0      | 0.3960              | 0.3552              | 0.3090              | 0.2550              | 0.2025              |
| Experimental Error | 0.0106              | 0.0132              | 0.0154              | 0.0180              | 0.0203              |
| x_\mu variation: x_\mu = 0.5 | 0.0870              | 0.0256              | 0.0137              | 0.0107              | 0.0089              |
| \mu_I variation: \mu_I = 1.0 Gev | -0.0401             | -0.0166             | -0.0097             | -0.0074             | -0.0059             |
| \mathcal{M} variation: \mathcal{M} - 20% | 0.0357              | 0.0308              | 0.0222              | 0.0112              | -0.0005             |
| Theoretical Error | 0.0941              | 0.0400              | 0.0261              | 0.0155              | 0.0089              |

Table 14: Results for \alpha_s(Q) and \alpha_0 as obtained from fits to B_W moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NLO predictions.
Table 15: Results for $\alpha_s(Q)$ and $\alpha_0$ as obtained from fits to $B_W$ moments measured by JADE and OPAL for centre-of-mass energies between 14.0 and 206.6 GeV using theoretical NNLO predictions.
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