Validity of Falling Ball Tester for Measuring Dynamic Elastic Modulus of Soil Subgrade Modulus of Highway

Yingqi Xue¹, Luwan Wang¹, Xiaokun Han¹ and Mengjie Wang¹

¹Research Institute of Highway Ministry of Transport, Beijing, 100088, China

*Corresponding author’s e-mail: 18804627386@163.com

Abstract: The deformation characteristics of geotechnical materials are very important for improving the safety of highways. To determine the reliability and detection accuracy of the falling ball tester, the paper uses standard method and falling ball tester to measure the dynamic elastic modulus of standard components, PPR. The theoretical result is obtained by measuring the dynamic elastic modulus of the one-dimensional member with the single-side reflection method. To calculate the Poisson’s ratio, the transmission method is used to measure the P wave velocity of PPR that is 3 dimensional. In the falling ball tests, two excitation hammers with a diameter of 0.05m and 0.06m are used to measure the dynamic elastic modulus of PPR. By comparing the theoretical results with the values obtained by D50 and D60, it is found that more accurate results can be obtained by D60.

1. Introduction
With the continuous development of the social economy, the construction scale and mileage of highways and high-speed railways are gradually increasing. It is particularly important to control the quality of subgrade projects to ensure the safety and comfort of high-speed railway. The filling materials commonly used in soil subgrade have many characteristics such as naturalness, diversity and complexity. The engineering quality is always affected by the uneven settlement occurring after the roller compacted. It is of great significance to improve the durability of the subgrade by the effective detection for the deformation characteristics of the filling materials and increasing the overall uniformity of the subgrade [1][2].

The falling ball detection technology was jointly developed by Wu Jiaye and researchers in Japan [3]. Since the development and maturity of the technology was mainly determined by the promotion and application in practice, the gradual promotion of the technology mainly benefited from the engineering practical application. Since 1999, falling ball tester and related technologies have been applied in more than 300 practical projects (railway, highways, embankments, etc.) at home and abroad, especially in Japan.

It is an urgent need for accurate, direct, quick and convenient technology to detect the mechanical properties of soil subgrade at the construction site. This paper is planned to test the correctness and feasibility of the falling ball tester scheme.

2. Testing Principle
The theory of falling ball tester to detect the deformation characteristics of geotechnical materials are derived from the elastic contact theory proposed by Hertz in 1881 [4][5]. When studying the elastic deformation of glass lens under the action of contact force, Hertz deduced the calculation formula of contact problem by mathematical elastic mechanics method. The assumptions [6] used in the theory are
as follows:

1) The material is uniform, isotropic and completely elastic;
2) The frictional force on the contact surface is negligible, and only the vertical pressure is applied;
3) The contact surface is elliptical.

According to the theory, when a sphere A with known rigidity collides with an object B with unknown rigidity, the greater the rigidity of the object B will lead to the shorter the contact time \( T_c \) during collision. The collision theory is shown in Figure 1.

![Figure 1. Schematic diagram of Hertz collision theory](image)

According to the contact time obtained by the falling ball tester, the deformation modulus of the test component can be obtained by the following equation [7].

\[
E = \frac{(1-\mu_2^2) m_1 E_1}{0.0719 E_1 \sqrt{R_1 v_0 T_c} - m_1 (1-\mu_1^2)}
\]

where \( E \) is the deformation modulus (unknown) of the testing component obtained by the tester; \( m_1 \) represent the mess (known) of the falling ball; \( E_1 \) is the known deformation modulus of the falling ball and \( R_1 \) is its radius. \( \mu_1 \) is the unknown passion ratio of the testing component and \( \mu_2 \) is the known passion ratio of the falling ball. \( T_c \) is the contact time obtained by the testing instrument. The contact velocity \( v_0 \), \( v_0 = \sqrt{2gH} \). And H is the height of the falling ball.

3. Testing setup
The dynamic elastic modulus of PPR obtained by falling ball tester is compared with the actual dynamic elastic modulus, and the relevant reference on the falling ball tester is provided by analyzing the difference between the both results.

To achieve the mentioned results, the test process is divided into 3 parts. The first is to calculate the dynamic elastic modulus of PPR material. Next is to measuring the dynamic elastic modulus of PPR material by using falling ball tester. At last, the comparison between the dynamic elastic modulus obtained by both methods is conducted out and the conclusion is summarized:

Firstly, the dynamic elastic modulus of PPR material can be obtained by calculating according to the relationship between the dynamic elastic modulus \( E_d \) of the one-dimensional member and the elastic wave (P wave) velocity \( V_{P1} \);

Secondly, the Poisson's ratio of PPR material is necessary to test the dynamic elastic modulus by falling ball tester. The Poisson's ratio can be calculated according to the relationship between the P-wave velocity, dynamic elastic modulus and Poisson's ratio when the test structure is 3-dimensional.

4. Testing process and analysis
At present, the falling ball tester is going to be adopted to test the dynamic elastic modulus of the material when its Poisson's ratio is known. To achieve the ultimate test results, the dynamic elastic modulus of
PPR is also needed to measure. Therefore, the two main parameters that is measured in this test are the dynamic elastic modulus $E_d$ and Poisson's ratio $\mu$ of PPR material.

If the material is a one-dimensional homogeneous elastomer, the relationship between the dynamic elastic modulus $E_d$ and the elastic wave (P wave) velocity $V_{p1}$ is expressed:

$$V_{p1} = \sqrt{\frac{E_d}{\rho}}$$  \hspace{1cm} (2)

where $V_{p1}$ is one-dimensional longitudinal wave (P wave) velocity of the material; $E_d$ represents the dynamic elastic modulus of the material; $\rho$ is the density of the material. When the density of PPR and the one-dimensional elastic wave velocity are known, the dynamic elastic modulus value of PPR can be calculated.

4.1 Calculation of PPR material density

Firstly, the density of PPR material should be calculated. Two sets of cylinder test blocks with PPR material are selected. The details are presented in Table 1.

| No. | Radius (m) | Height (m) | Mess (kg) |
|-----|------------|------------|-----------|
| PPR-3 | 0.05       | 0.103      | 0.80      |
| PPR-4 | 0.025      | 0.198      | 0.41      |

As shown in Table 1, it is not difficult to calculate the density $\rho$ of the two sets of PPR test blocks:

$$\rho_{PPR-3} = \frac{m}{V} = \frac{m}{\pi r^2 h} = \frac{0.8}{3.14 \times 0.05^2 \times 0.103} = 989.4 \text{ (kg/m}^3\text{)}$$

Similarly, we can get:

$$\rho_{PPR-4} = 1055.1 \text{ (kg/m}^3\text{)}$$

Here, the density of the PPR material calculated by simple averaging is:

$$\rho_{PPR} = (\rho_{PPR-3} + \rho_{PPR-4})/2 = (989.4 + 1055.1)/2 = 1022.3 \text{ (kg/m}^3\text{)}$$

Therefore, in the next calculation, we will use 1022.3kg/m$^3$ for the density $\rho$ of the PPR material.

4.2 One-dimensional wave velocity and dynamic elastic modulus test of PPR

The one-dimensional elastic wave velocity of the PPR that is a one-dimensional member. The test object was a one-dimensional member with a diameter of 0.05m and a height of 1.078m, named PPR-2, as shown in Figure 2.

![Figure 2. Standard components of PPR](image-url)
both collocations are shown in Figure 3.

Figure 3. Original oscillogram (top: D17+ST-S31SC; bottom: D30+ST-S31SC)

Spectral analysis was performed on the data obtained by the collocations of D17+ST-S31SC and D30+ST-S31SC respectively. And the density and related specifications of model PPR-2 were known at the same time. The one-dimensional wave velocity has been calculated, the detailed analysis of the dynamic elastic modulus of PPR-2 model is as depicted in Figure 4 and Figure 5.

Figure 4. Spectrum and wave velocity analysis of D17+ST-S31S
Figure 5. Spectrum and wave velocity analysis of D30+ST-S31S

By summing up Figure 4 and Figure 5, the one-dimensional velocity and dynamic elastic modulus of PPR-2 are shown in Table 2.

| No. | Collocation | Analysis Mode | One-dimensional velocity (km/S) | Dynamic elastic modulus (MPa) |
|-----|-------------|---------------|---------------------------------|-------------------------------|
| 1   | D17+ST-S31S | MEM/FFT       | 1.848                           | 3490                          |
| 2   | D30+ST-S31S | MEM/FFT       | 1.852                           | 3508                          |
| Mean| /           | /             | 1.850                           | 3499                          |

4.3 Calculation of Poisson's ratio of PPR materials

If the test object is 3-dimensional, the relationship between the 3-dimensional P-wave velocity, the dynamic elastic modulus and Poisson’s ratio is as follows:

$$V_{P3} = \sqrt{\frac{E_d \rho}{\rho} \frac{(1-\mu)}{(1+\mu)(1-2\mu)}}$$ (3)

where $V_{P3}$ can be tested by the double-sided transmission method, and the material density $\rho$ and the dynamic elastic modulus $E_d$ have been calculated. Therefore, the Poisson’s ratio of the PPR material can be calculated by the above equation.

When the PPR material is tested by the double-sided transmission method, D10 and D17 are used for excitation. The test results are presented in Figure 6 and Figure 7.
According to the test results of the three-dimensional P-wave velocity of D10 and D17, the three-dimensional P-wave velocity $V_{p3}$ of PPR material is 2.015 km/s. The relevant parameters of PPR material is summarized in Table 3.

Table 3. Parameters of PPR material

| Parameter | $V_{p3}$ (km/s) | $E_d$ (GPa) | $\rho$ (kg/m$^3$) |
|-----------|-----------------|------------|------------------|
| Value     | 2.015           | 3.499      | 1022.3           |

Therefore, according to the relevant parameters in Table 3 and equation (2), the Poisson's ratio of the PPR material can be calculated:

$$\mu = 0.243$$

4.4 Falling ball test elastic modulus

In the falling ball tests, the D50 and D60 hammers are used to replace the ball of the falling ball tester in the testing. The parameters of the two excitation hammers are shown in Table 4.
| No. | Diameter (m) | Mess (kg) |
|-----|--------------|-----------|
| D50 | 0.05         | 0.49      |
| D60 | 0.06         | 0.86      |

During the falling ball tests, the falling heights of the two spheres are 5cm, 10cm and 20cm respectively. The test results are shown in Table 5.

| No. | Falling height (m) | Mean    | Standard deviation | Equal value | Coefficient of variation |
|-----|--------------------|---------|--------------------|-------------|-------------------------|
| D50 | 0.05               | 2921.02 | 340.12             | 2888.34     | 0.116                   |
|     | 0.1                | 3156.30 | 460.31             | 3083.99     | 0.146                   |
|     | 0.2                | 2789.15 | 566.75             | 2695.98     | 0.203                   |
| D60 | 0.05               | 3423.04 | 248.9              | 3406.67     | 0.073                   |
|     | 0.1                | 3685.99 | 172.34             | 3678.28     | 0.047                   |
|     | 0.2                | 3635.19 | 370.75             | 3602.61     | 0.102                   |

Figure 8. Comparison between the theoretical and Measurement results

It can be seen in Table 5 and Figure 8, the dynamic elastic modulus of the PPR material measured with the D60 ball is more accurate than that obtained by D50 ball. Next, the nylon material can be used to verify again.

5. Conclusion
In the present study, authors use standard method and falling ball tester to measure the dynamic elastic modulus of standard components of PPR. The dynamic elastic modulus of PPR obtained by falling ball tester is compared with the actual dynamic elastic modulus and theoretical value. To achieve the measured results obtained by falling ball tester, the single-side reflection method is adopted to measure the one-dimensional velocity and dynamic elastic modulus of PPR-2. Then the transmission method is
used to achieve the P wave velocity which is necessary to calculate the Poisson’s ratio of PPR. At last, both balls with diameter of 0.05m and 0.06m are used to measure the dynamic elastic modulus of PPR. Though the comparison between the results obtained by D50 and D60, it is found that the results obtained by D60 are more accurate than the other ball. So in the next tests, it is recommended to adopt the ball with a diameter of 0.06m. In future work, another material, Nylon, is going to be used to validate the reliability of falling ball tester.

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