How can one find nonlocality in Bohmian mechanics?

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Abstract

We perform the probabilistic analysis of the pilot wave formalism. From the probabilistic point of view it is not so natural to follow D. Bohm and to consider nonlocal interactions between parts of (e.g.) a two-particle system. It is more natural to consider dependence of corresponding preparation procedures and the propagation of the initial correlations between preparation procedures. In the pilot wave formalism it is more natural to speak about correlations of the initial pilot waves which propagate with time.

1. Introduction

Traditionally the pilot-wave quantum formalism, Bohmian mechanics (see, e.g., [1]-[3]), is considered as a nonlocal model, see, e.g., D. Bohm and B. Hiley [2], p. 58: “In our interpretation of quantum theory, we see that the interaction of parts is determined by something that cannot be described solely in terms of there parts and their preassigned

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interrelationships.” 1

In this note we analyse the pilot wave model and Bohm’s arguments about nonlocality from the probabilistic viewpoint. Our analysis demonstrated that Bohm’s nonlocal interpretation of many-particles systems is not so natural. Instead of a nonlocal interaction, it would be more natural to consider dependence produced by the preparation procedure for the initial state and propagation of this dependence in time (with modifications owed to interactions of subsystems).

2. Nonlocality or dependence?

Let us consider two particle systems in which particles do not interact. This model can be described by the Hamiltonian:

\[ \hat{H} = \hat{H}_1 + \hat{H}_2 = \left( \frac{-\hbar^2}{2m_1} \Delta_1 + V_1 \right) + \left( \frac{-\hbar^2}{2m_2} \Delta_2 + V_2 \right), \]

where \( \Delta_j \) are Laplacians and \( V_j = V_j(x_j), j = 1, 2, \) are potentials.

The important thing is that the operators \( \hat{H}_1 \) and \( \hat{H}_2 \) commute, \( [\hat{H}_1, \hat{H}_2] = 0 \). Thus:

\[ e^{-\alpha \hat{H}_1} e^{-\alpha \hat{H}_2} = e^{-\alpha \hat{H}} \]

Let us now consider the corresponding Schrödinger equation:

\[ i\hbar \frac{\partial \psi}{\partial t}(t, x_1, x_2) = \hat{H} \psi(t, x_1, x_2) \]

\[ \psi(0, x_1, x_2) = \psi_0(x_1, x_2). \]

Suppose now that at the initial instant of time particles composing a two particle system were prepared independently2

\[ \psi_0(x_1, x_2) = \varphi_1(x_1) \varphi_2(x_2). \]

This condition implies that

\[ \mathbf{P}_{\psi_0}(x_1, x_2) = \mathbf{P}_{\varphi_1}(x_1) \mathbf{P}_{\varphi_2}(x_2). \]

1It is interesting to notice that J. Bell was very excited by the Bohmian nonlocality. This was one of the main sources of the Bell’s inequality. I also understand well that people belonging to the Bohmian community have various views to Bohmian nonlocality and these views are in the process of permanent changing, see, e.g. [4]. Here I present just the common view to the Bohmian mechanics as a nonlocal theory.

2Well there always can exist some correlations in preceding histories of systems, but we assume that it is possible to neglect all those correlations.
Thus the probability to find parts of a two-particle system having the state \( \psi \) in points \( x_1 \) and \( x_2 \), respectively, is factorized into probabilities for the position observations on particles in the states \( \varphi_1 \) and \( \varphi_2 \), respectively.

We remark that the factorization condition (2) for a wave function also implies the factorization condition in the momentum representation (and vice versa):

\[
\psi_0(p_1, p_2) = \varphi_1(p_1) \varphi_2(p_2),
\]

where

\[
\psi_0(p_1, p_2) = \int e^{ip_1x_1 + ip_2x_2} \psi_0(x_1, x_2) dx_1 dx_2.
\]

Thus the condition (2) also implies the factorization of the probability for the momentum measurements:

\[
P_{\psi_0}(p_1, p_2) = P_{\varphi_1}(p_1) P_{\varphi_2}(p_2)
\]

Hence under the condition (2) preparations are independent both with respect to the position and momentum.

I did not perform detailed analysis, but it seems that conditions (3) and, (5) together imply the condition (2) (and hence (4)). Thus by considering the factorization (2) of the wave function we, in fact, consider preparations which are independent with respect to both fundamental variables, the position and the momentum.

By (1) we have

\[
\psi(t, x_1, x_2) = e^{-\frac{i\mathcal{H}t}{\hbar}} \psi_0(x_1, x_2) = \varphi_1(t, x_1) \varphi_2(t, x_2),
\]

where \( \varphi_1(t) \) and \( \varphi_2(t) \) are solutions of the Schrödinger equations with Hamiltonians \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) and the initial conditions \( \varphi_1 \) and \( \varphi_2 \).

Thus independence (both in the position and momentum representations) is preserved in the process of evolution. We see that if there were no correlations between preparations of statistical ensembles of particles composing two-particle systems then (in the absence of interactions between particles, i.e., \( \hat{V} = \hat{V}_1 + \hat{V}_2, [\hat{V}_1, \hat{V}_2] = 0 \)) there is no any trace of the Bohmian nonlocality.

Even if we use the individual interpretation of a wave function and associate a wave function with a single (e.g. two particle) system as a pilot wave, then in the light of the presented probabilistic analysis it is more natural to consider correlation between initial pilot waves (which propagate according
to the Schrödinger equation) and not a nonlocal interaction in the process of evolution. Not particles are “guided in a correlated way”, cf. [2], p. 57, but initially correlated waves propagate according to the Schrödinger equation.

Of course, if the initial state cannot be factorized, i.e., the preparation of the initial ensemble of two-particle systems could not be split into two independent preparation procedures (both with respect to the position and the momentum variables), then the initial dependence will propagate with time.

If we use the language of the Bohmian mechanics we can say if initially pilot waves corresponding to parts of two-particle systems were correlated their correlation would not disappear immediately and it will propagate in time (by the law given by the Schrödinger equation).

Conclusion. The pilot wave formalism need not be interpreted in the Bohmian way\(^3\) – as a formalism where “interactions can therefore be described as nonlocal.”

It is natural to interpret this formalism by considering propagation of correlations induced by state preparations.

I would like to thank B. Hiley and D. Dürr for a discussion on nonlocality of Bohmian mechanics. Especially I am thankful to them for the explanation that (despite a rather common opinion) indistinguishability of particles cannot be considered as a fundamental feature of quantum multi-particle systems which induces ”nonclassical probabilistic behaviour”.

References
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\(^3\)As I know from some private conversations, L. De Broglie personally did not support the nonlocal interpretation of the pilot wave approach (unfortunately I do not have a precise reference).