Controllable chiral Majorana modes by noncollinear magnetic configuration in a topological Josephson junction

M Y Li¹, Q Y Yu¹, J G Hu¹, T M Liu¹, Y C Tao¹,⁴ and J Wang¹,⁴
¹ Department of Physics, Nanjing Normal University, Nanjing 210023, People’s Republic of China
² Department of Physics, Yangzhou University, Yangzhou 225009, People’s Republic of China
³ Department of Physics, Southeast University, Nanjing 210096, People’s Republic of China
⁴ These authors contributed equally to this work and should be considered co-first authors.

E-mail: yctao88@163.com and jwang@seu.edu.cn

Keywords: chiral Majorana modes, noncollinear magnetic configuration, topological Josephson junction

Abstract
We theoretically present controllable chiral Majorana modes (CMMs) by noncollinear magnetic configuration in a Josephson junction on a topological insulator, where two ferromagnetic insulators (FIs) are sandwiched in between two superconductors (SCs). It is shown that an additional phase shift is induced by the different chiralities of the CMMs at the two FI/SC interfaces. Its magnitude is determined by the misorientation angle θ, which can result in the 0 – π state transition and φ₀ supercurrent. Particularly, due to the selective equal spin Andreev reflection, the coexistence of fully spin-polarized spin-triplet and -singlet correlations is exhibited. However, for the two magnetizations only along the y-axis, there exist no additional phase shift and the 100% supercurrent magnetoresistance can be exhibited. The results can be employed to not only identify the topological SCs but also design a perfect topological supercurrent spin valve device.

1. Introduction
The Josephson effect contributes to identifying the pairing symmetry of superconductors (SCs), therefore it has been a fundamental and central topic in the field of superconductivity. Especially, in recent years, the anomalous Josephson effect has been triggering much attention. Compared with the current–phase relation (CPR) of conventional Josephson junctions, its CPR is written as \( I(\phi) = I_c \sin(\phi + \phi_0) \) with an extra phase shift \( \phi_0 \) [1–4]. The anomalous Josephson effect has emerged in the Josephson junctions based on such materials as black phosphorus and tilted Weyl semimetals [5–9]. The \( \phi_0 \) state can be leveraged to determine the Majorana fermions, type of spin–orbit coupling, order of spin–orbit coupling (linear or cubic), and pairing symmetries [10–12].

On the other hand, Fu and Kane studied the superconductivity on the surface of a topological insulator (TI) induced by the proximity effect [13, 14]. They predicted the appearance of the chiral Majorana state as an Andreev bound state (ABS) at the interface between the ferromagnetic insulator (FI) and conventional SC. The state is called the chiral Majorana mode (CMM) [15], with a dispersion along the interface and a confinement along the direction perpendicular to the interface [15, 16]. The SC/FI/SC junction formed on a three-dimensional (3D) TI has been extensively studied in references [15–17]. It was found that the CMM arising in the junction is very sensitively manipulated by the direction of the magnetization \( \mathbf{m} \) in the FI region [15]. A phase shift generated in the CPR of the junction being neither 0 nor \( \pi \), can be continuously controlled by the component of \( \mathbf{m} \) perpendicular to the interface. It was also shown that the 4π periodic ABS can still be present at high electron densities due to tunneling via Majorana states, and finite angle incidence usually gives rise to opening of a gap at zero energy and resultantly a 2π periodic ABS [17]. However, the chiralities of the CMMs at the two interfaces of FI/SC are thoroughly the same. A TI-based Josephson junction in which a proximity-induced in-plane helical magnetization is sandwiched in between...
two SCs, was proposed by Zyuzin et al. [18]. They found that $0 - \pi$ state transitions are induced by the magnetization strength, junction thickness, and parameters inherent to the helical modulation and surface states. However, in their junction, there exist no CMMs at the two interfaces thanks to the absence of component $m_z$ of $\mathbf{m}$ along the $z$-axis. Therefore, only considering two FIs can produce the two CMMs with different chiralities. Although Bobkova et al. [19] subsequently put forward a Josephson junction with two weakly coupled uniformly magnetized superconducting surfaces based on a 3D TI, no CMMs emerge similarly on account of no existence of $m_z$.

Particularly, theory recently has predicted that Majorana fermions induce selective equal spin Andreev reflections (SESARs) [20], specifically, the incoming electrons and counterpropagating Andreev-reflected holes have the same spin polarization direction. The spin polarization direction of the electrons of this Andreev-reflected channel is selected by the Majorana fermions, which was probed by experiments [21] but is expected to be further probed and confirmed experimentally from different methods and setups. Topological superconductivity may be induced on surfaces of such 3D TIs as Bi$_2$Se$_3$, Bi$_2$Te$_3$, and Sb$_2$Te$_3$ via the proximity effect [22–24], and the localized Majorana zero mode at the vortex core has been studied by using scanning tunneling microscopy or spectroscopy. Fu and Kane proposed the existence of the localized Majorana zero mode at the interface of a TI and an $s$-wave SC [13]. This indicates that the CMM at the SC/FI on a TI also gives rise to the SESAR. It is therefore highly desirable to find whether in the TI-based Josephson hybrid structures with two FIs sandwiched in between two SCs, the noncollinear magnetic configurations [25] can bring about a plenty of peculiar features due to the SESARs, which has not been studied heretofore. And thus, the features may be experimental signatures for not only identifying the Majorana fermions in topological SCs but also distinguishing the topological SC from the trivial one. As expected, such features have been exhibited in the following parts.

In this work, therefore, we study the supercurrent properties of SC/FI/TI/FI/SC Josephson junction on a 3D TI. It is found that an additional phase shift is caused by the different chiralities of the CMMs at the two FI/SC interfaces, which can be tuned by the misorientation angle $\theta$. As a result, not only the $0 - \pi$ state transition but also $\phi_0$ supercurrent is induced by $\theta$. Most interestingly, thanks to the SESAR, the fully spin-polarized spin-triplet correlation emerges with the fully spin-polarized spin-singlet one. Furthermore, as the two magnetizations are magnetized along the $y$-axis, the additional phase shift is not exhibited any longer, and a considerably large supercurrent magnetoresistance (even up to 100%) is displayed.

2. Topological superconducting spin-valve

We consider an SC/F$_1$/TI/F$_2$/SC Josephson junction based on a 3D TI with F$_1$ and F$_2$ being FIs, which is stacked along the $x$-axis. Because of the proximity effect, the superconductivity and ferromagnetism are induced on the surface of the 3D TI. The length of the central region is $L$ and the lengths of the F$_1$ and F$_2$ regions are assumed identical, expressed as $d$. The corresponding magnetic exchange fields on the surface of the 3D TI are written as $\mathbf{m}_s = (m_{1x}, m_{1y}, m_{1z})$ and $\mathbf{m}_f = (m_{2x}, m_{2y}, m_{2z})$, which can be respectively chosen as $(0, 0, m)$ and $(m \sin \theta, 0, m \cos \theta)$ for the out-of-plane situation of figure 1. Experimentally, F$_1$ and F$_2$ can be FIs with very strong easy-axis anisotropy and soft magnet effect, respectively, which can be all controlled by a weak external magnetic field, so do the magnetic configurations.

The excited states of the electron and hole can be given by the following Bogoliubov–de Gennes equation [15, 16]

$$
\begin{pmatrix}
\hat{H}(k) + \hat{M} \\
- \hat{\Delta}
\end{pmatrix}
\begin{pmatrix}
\hat{\Delta}^* \\
- \hat{H}^*(\mathbf{k}) - \hat{M}^*
\end{pmatrix} \psi = E \psi,
$$

(1)

where $\psi = (\psi_{c1}, \psi_{c2}, \psi_{h1}, \psi_{h2})^T$ is the four-spinor wave function, $\hat{H}(k) = \hbar \nu_F (\hat{\sigma}_x k_x + \hat{\sigma}_y k_y) - \mu_i$ with $\hat{\sigma}$ Pauli matrix, $\nu_F$ Fermi velocity, $\mu_i$ chemical potential for the three regions (the subscript $i = $ SC, TI, and FI) being controlled by the gate voltage, $\hat{M} = \mathbf{m}_j \cdot \hat{\sigma}$ with the subscript $j = 1, 2$, and $E$ is the energy of the

![Figure 1. Schematic of the system, SC/F$_1$/TI/F$_2$/SC Josephson junction on a 3D TI with the black arrows indicating magnetization orientations, where the F$_1$ region is magnetized along the $z$-axis and F$_2$ region magnetized in the $x$–$z$ plane.](image-url)
quasiparticle relative to the chemical potential \( \mu_c \). The pair symmetries of the left and right SCs are all s-wave, whose pair potentials are written as \( \Delta = i \delta_E [\Delta e^{i \phi_L} (x - d) + \Delta e^{i \phi_R} (x - L)] \) with \( \theta(x) \) the step function and \( \phi_L, \phi_R \) the microscopic phase of the left (right) SC. As usual, \( \phi_L \) is assumed to be zero, thus the microscopic phase difference \( \phi = \phi_R - \phi_L = \phi_R \). The temperature dependence of the superconducting pair potentials is given by \( \Delta \equiv \Delta(T) = \Delta_0 \tanh \left( 1.74 \sqrt{T_c/T - 1} \right) \).

In light of the above-mentioned assumptions and quasi-classical approximation, we can obtain the wave functions for all the regions by solving equation (1). In the left superconducting region,

\[
\psi_{SC}^L(x) = r_e \begin{pmatrix} \frac{v}{-ve^{i\theta}} \\ e^{-i\theta} \end{pmatrix} e^{-i\delta x} + r_i \begin{pmatrix} 1 \\ \frac{ve^{i\theta}}{-ve^{i\theta}} \end{pmatrix} e^{i\delta x},
\]

for \( x < -d \), where \( v = \left( E + \sqrt{E^2 - \Delta^2} \right)/\Delta \), \( k = \mu_c \cos \alpha'/h(\nu_F) \) standing for the wave vector of the \( x \) direction, and \( \alpha' \) represents the propagating angle. In the \( F_1 \) region,

\[
\psi_{F_1}(x) = f_1 \begin{pmatrix} \eta_+ \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{(\kappa_+ - i\eta_\mp/m_F) x} + f_2 \begin{pmatrix} -\eta_-^{-1} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-(\kappa_+ + i\eta_\mp/m_F) x} + f_3 \begin{pmatrix} 0 \\ 0 \\ \eta_- \\ 1 \end{pmatrix} e^{-(\kappa_- - i\eta_\mp/m_F) x} + f_4 \begin{pmatrix} 0 \\ 0 \\ -\eta_-^{-1} \\ 1 \end{pmatrix} e^{(\kappa_- + i\eta_\mp/m_F) x},
\]

for \( -d < x < 0 \), where \( \kappa_\pm = \sqrt{m_F^2 + (\nu_F k_y \pm m_F)^2 - \mu_F^2} \)/\( (\nu_F) \) and \( \eta_\pm = i \left[ \sqrt{m_F^2 + (\nu_F k_y \pm m_F)^2 - \mu_F^2} - (\nu_F k_y \pm m_F) \right]/(\mu_F - m_F) \). In the central region,

\[
\psi_{TI}(x) = a_1 \begin{pmatrix} 1 \\ \frac{1}{e^{\alpha}} \\ 0 \\ 0 \end{pmatrix} e^{i\varphi x} + a_2 \begin{pmatrix} 1 \\ -e^{-i\alpha} \\ 0 \\ 0 \end{pmatrix} e^{-i\varphi x} + a_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ -e^{-i\alpha} \end{pmatrix} e^{i\varphi x} + a_4 \begin{pmatrix} 0 \\ 0 \\ 1 \\ e^{i\alpha} \end{pmatrix} e^{-i\varphi x},
\]

for \( 0 < x < L \), where \( q = \mu_{TI} \cos \alpha'/h(\nu_F) \) standing for the \( x \)-component of wave vector for the electron and hole in \( E \ll \mu, \alpha \) represents the incident angle, \( k_y = \mu_{TI} \sin \alpha'/h(\nu_F) \) the \( y \) component of the wave vector, and \( \alpha' = \arcsin (\mu_{TI} \sin \alpha'/\mu_c) \) due to the conservation of momentum \( k_y \) from the translational invariance along the \( y \)-axis. In the \( F_2 \) region,

\[
\psi_{F_2}(x) = g_1 \begin{pmatrix} \lambda_+ \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{(\rho_+ - i\eta_\mp/m_{FI}) x} + g_2 \begin{pmatrix} -\lambda_-^{-1} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-(\rho_+ + i\eta_\mp/m_{FI}) x} + g_3 \begin{pmatrix} 0 \\ 0 \\ \lambda_- \\ 1 \end{pmatrix} e^{-(\rho_- - i\eta_\mp/m_{FI}) x} + g_4 \begin{pmatrix} 0 \\ 0 \\ -\lambda_-^{-1} \\ 1 \end{pmatrix} e^{(\rho_- + i\eta_\mp/m_{FI}) x},
\]

for \( L < x < L + d \), where \( \rho_\pm = \sqrt{m_{FI}^2 + (\nu_F k_y \pm m_{FI})^2 - \mu_{FI}^2} \)/\( (\nu_F) \),

\[
\lambda_\pm = i \left[ \sqrt{m_{FI}^2 + (\nu_F k_y \pm m_{FI})^2 - \mu_{FI}^2} - (\nu_F k_y \pm m_{FI}) \right]/(\mu_{FI} - m_{FI}), \quad \text{and } \mu_{FI} \text{ is taken to be zero in the following parts as in reference [15].}
\]

In the right superconducting region,

\[
\psi_{SC}^R(x) = t_e \begin{pmatrix} ve^{i\theta} \\ ve^{i\theta} e^{i\varphi} \\ -e^{-i\alpha} \\ 1 \end{pmatrix} e^{i\delta x} + t_i \begin{pmatrix} 1 \\ -e^{-i\alpha} \\ ve^{-i\alpha} e^{-i\varphi} \\ ve^{-i\alpha} \end{pmatrix} e^{-i\delta x},
\]

for \( x > L + d \).
for \( x > L + d \). The coefficients \( r_i, t_h, t_e \), and \( t_b \) are, respectively, the amplitudes of normal reflection as an electron-like quasiparticle (ELQ), the AR as a hole-like quasiparticle (HLQ), transmission to the right SC as an ELQ, and transmission to the right SC as a HLQ. The coefficients \( f_i, a_i, \) and \( g_i \) with \( i = 1 \text{–} 4 \) stand for the amplitudes of electrons and holes propagating in the two magnetic and central regions, respectively. All the above-mentioned coefficients will be determined by the following matching boundary conditions

\[
\begin{align*}
\psi_{SC}^{L}|_{x=-d} &= \psi_{F1}^{L}|_{x=-d}, \\
\psi_{F1}|_{x=0} &= \psi_{TI}|_{x=0}, \\
\psi_{TI}|_{x=L} &= \psi_{F2}|_{x=L}, \\
\psi_{F2}|_{x=L+d} &= \psi_{SC}^{R}|_{x=L+d}.
\end{align*}
\]

After substituting equations (2)–(6) into equation (7), we can attain a system of homogeneous linear equations about the scattering coefficients, i.e. \( Ax = 0 \), where \( A \) is a 16 × 16 matrix and \( x \) is a column vector including the 16 coefficients. The Andreev bound energy \( E(\phi) \) versus \( \phi \) with \( i = (1, 2) \) can be acquired from \( \det(A) = 0 \). Under the situation of the short junction, only the contribution from the AR to the Josephson current is considered, which is given by [26]

\[
I(k_y, \phi) = \frac{2e}{\hbar} \sum_{k_y \neq 0} \frac{\partial E_i}{\partial \phi} \frac{1}{e^{E_i/k_B T} + 1}.
\]

Due to the symmetry of the electron–hole, equation (8) reduces to

\[
I(k_y, \phi) = \frac{2e}{\hbar} \sum_{k_y \neq 0} \frac{\partial E_i}{\partial \phi} \tanh \frac{E_i}{2k_B T},
\]

thus we can obtain the total Josephson current,

\[
I(\phi) = \int dk_y I(k_y, \phi).
\]

In addition, the ground state phase of the Josephson junction is determined by the free energy being minimum,

\[
F(k_y, \phi) = -\frac{1}{\beta} \ln \left[ \prod_{i} \left( 1 + e^{-\beta E_i(\phi)} \right) \right],
\]

with \( \beta = 1/k_B T \).

3. Controllable CMMs by the misorientation angle

We first consider the out-of-plane situation shown in figure 1 with the \( F_1 \) region magnetized along the \( z \)-axis and \( F_2 \) region magnetized in the \( x-z \) plane, where \( \theta \) is the angle between the two magnetizations. Figures 2(a)–(c) illustrate the Andreev bound surface energies as a function of the microscopic phase difference \( \phi \) at different incident angles \( \alpha \) for three magnetic configurations (\( \theta = 0, \pi, \) and \( \pi/2 \)), respectively. For the ferromagnetic (F) configuration (\( \theta = 0 \)), the features are found to be the same as those of the SC/FI/SC junctions on 3D TIs with the FI region magnetized only along the \( z \)-axis [15–17]. More specifically, for the perpendicular incidence (\( \alpha = 0 \)), a 4\( \pi \) periodic ABS with a gapless dispersion is presented, whereas a gap is always caused by a nonzero angle of incidence (\( \alpha \neq 0 \)), which indicates a 2\( \pi \) periodic ABS. The larger the incident angle \( \alpha \) is, the larger the opened gap becomes. However, for the noncollinear magnetic configuration, for instance, a typical angle \( \theta = \pi/2 \), although the above-presented features remain, all the curves shift along \( \phi \)-axis by \( \pi/2 \). More interestingly, for the antiferromagnetic (AF) configurations (\( \theta = \pi \)), the phase shift along \( \phi \)-axis is \( -\pi \). The new physics could be given next, which is easily understood by considering the formation of an ABS in the hybrid structure accompanied by the presence of the CMMs.

We first discuss the presence of the CMMs in the present hybrid structure briefly. As mentioned above, the presence of CMM is due to the magnetism \( m_r \) along the \( z \)-axis, which only leads to opening of the gap but not the spin splitting. At the left SC/FI interface, there is always the CMM because the \( F_1 \) region is magnetized along the \( z \)-axis. However, the \( F_2 \) region is magnetized in the \( x-z \) plane with the angle \( \theta \) varying from 0 to \( \pi \), which means that chirality of the CMM at the right FI/SC interface can turn opposite and the CMM disappears at \( \theta = \pi/2 \).

Let us observe the phase shift along a classical trajectory for a right-going electron at \( x = +0 \). The electron moves right and gets a phase \( e^{i\theta} \) at the interface \( x = L + 0 \). However, the electron runs through the
F₂ region and arrives at the interface x = L + d − 0, there is no additional phase since the F₂ region is a topological FI and the wave function is evanescent. The electron collides with the SC and is Andreev-reflected as a hole with an additional phase [27], \( \phi_{\text{hl}} = -\arctan \left( \frac{\sqrt{\Delta_1^2 - E^2}}{E} \right) - \phi_0 \). The Andreev-reflected hole runs left and comes back to the central region x = L − 0. Then the hole moves left through the central region by acquiring a phase \( e^{-i\phi_0} \). The hole continues to run left in the F₁ region to the interface x = −d + 0 with no additional phase. The hole is Andreev-reflected as an electron with an additional phase \( \phi_{\text{he}} = -\arctan \left( \frac{\sqrt{\Delta_1^2 - E^2}}{E} \right) + \phi_L \), and then the electron moves through the F₁ region again and back to the starting position x = +0, finishing a cycle.

The energy level \( E \) of the ABS satisfies the quantum condition, i.e. the phase shift along a closed path of the classical trajectory is a multiple of \( 2\pi \) [27–30]

\[
-\arctan \left( \frac{\sqrt{\Delta_1^2 - E^2}}{E} \right) - \arctan \left( \frac{\sqrt{\Delta_1^2 - E^2}}{E} \right) \pm \phi + 2\theta_1(E) = 2n\pi, \quad (12)
\]

where \( n \) is an integer, \( \theta_1(E) \) indicates the phase shift during the round-trip path in the central region and \( \phi = \phi_0 - \phi_0 \). Since \( \theta_1(E) = qL - qL = 0 \) in \( E \ll \mu_L \), \( \phi \) is not dependent on the regions sandwiched in between the two superconducting regions, as shown in figure 2(a), which is also the same for the AF configurations. Therefore, the phase shift \( -\pi \) along \( \phi \)-axis for the curve of \( E \) vs \( \phi \) in figure 2(b) for the AF configurations is obviously connected with the superconducting regions, which stems from that the chiralities of the two CMMs formed at the left SC/FI interface and right FI/SC interface are opposite. For the noncollinear magnetic configuration (\( \theta = \pi/2 \)), the situation is complex. Specifically, not only the chiralities of the two CMMs but also the \( x \)-component of the exchange magnetic field in F₂ region leads to a phase. For the former, the phase is \( -\pi/2 \), and for the latter, is just \( 2\pi \) in the context of \( d = 0.04\pi \xi_0 \) with \( \xi_0 = 2/\pi \Delta_0 \), giving rise to a net phase shift \( 3\pi/2 \) or \( -\pi/2 \). The induced phase by the latter stems from the fact that the term \( \kappa_+ + \frac{i\mu_R}{m^*} \) at \( \mu_R = 0 \) in equation (5) becomes a complex number in the presence of \( m_0 \), which indicates that the wave function at the Fermi energy is damped oscillatory with \( L \).

The ABS can be experimentally investigated by virtue of the CPR \( I(\phi) \), which just in turn confirms the presence of the CMMs and their contributions to supercurrent. According to equation (10), we calculate \( I(\phi) \) by using the Andreev bound energies \( E(\phi) \) of figures 2(a)–(c), which are shown in figure 3(a) for the three magnetic configurations. The sinusoidal shape with periodicity \( 2\pi \) is no more exhibited for each magnetic configuration. Despite a \( 4\pi \) periodicity in the ABS for the zero incidence angle, the \( 2\pi \) periodic
character for the other channels dominates the angle-averaged supercurrent, leading to the current with the periodicity $2\pi$ in measurements \[17\]. The position of the maximum (critical) Josephson current $I_c$ for the F configuration is strongly away from $\phi = \pi/2$ and close to $\pi$. The behavior is similar to that of the SC-short normal metal-SC junctions with fully transparent interface \[31\] or unconventional SC-short junctions with the low transmission in the presence of zero energy surface ABSs \[32–34\]. In light of the result of the ABS (see figure \ref{fig:2}(a)), this behavior can be attributed to the CMMs appearing in the superconducting gap. The minimum free energy is at $\phi = 0$ as shown in figure \ref{fig:2}(d), and thus the corresponding junction lies in 0 state, where the chiralities of two CMMs at the left SC/F1 and right F2/SC interfaces are the same and thus lead no phase shift. On the other hand, for the AF configuration, the maximum current $I_c$ is also strongly away from $\phi = \pi/2$ but close to 0, particularly, compared with the F configuration, the direction of the supercurrent turns reversal, which can be explained by the ABS (see figure \ref{fig:2}(b)). It is found from figure \ref{fig:2}(d) that the minimum free energy is at $\phi = \pi$, so the corresponding junction is in $\pi$ state, where the chiralities of two CMMs at the SC/F1 and F2/SC interfaces are opposite, giving rise to the $\pi$ phase shift. More interestingly, for the noncollinear magnetic configuration ($\theta = \pi/2$), the Josephson current displays an anomalous effect, $I(\phi) \neq 0$ at $\phi = 0$, with an extra phase shift $\phi_0$, being neither 0 nor $\pi$, where for the present value of $d$, $\phi_0 = -\pi/2$. The corresponding maximum current $I_c$ is found to be slightly away from $\phi = \pi/2$ due to the shift of the ABS along $\phi$-axis by $-\pi/2$, and the minimum free energy is not located at 0 ($\pi$), giving rise to the $\phi_0$ supercurrent. Under this situation, there exists the CMM only at the SC/F1 interface but not at the F2/SC interface, inducing the $\pi/2$ phase shift. The extra shift $\phi_0$ is manipulated by not only the magnetic region length $d$ but also the misorientation angle $\theta$. Although the $0-\pi$ state transition can also emerge in similar structures without TIs \[35\], the mechanism of the $0-\pi$ state transition induced by the misorientation angle $\theta$ is obviously different, particularly, there exists no $\phi_0$ supercurrent in the latter.

In order to get a deep insight into $\phi_0$ of the ground state, $\phi_0$ is shown as a function of $\theta$ in figure \ref{fig:3}(b). We find that with the enhancement of $\theta$, $\phi_0$ slowly increases from 0, then linearly mounts in the middle region of $\theta$, finally rises slowly again up to $\pi$. Obviously, $\phi_0 < \theta$ for $\theta < \pi/2$, while $\phi_0 > \theta$ for $\theta > \pi/2$. Particularly, at approaching the F (AF) configurations, $\phi_0$ keeps unchanged and is 0 ($\pi$). All these indicate the strong influence of ferromagnetic configuration on the CMMs.

Now we observe the influence of the chemical potentials on the CPR. Figures \ref{fig:4}(a) and (b) display the CPRs at fixed $\mu_{SC}$ for different $\mu_{FI}$, respectively, from which and figure \ref{fig:3}(a), we find that with the increase of $\mu_{FI}$, the feature is not varied except for the magnitude, specifically, the supercurrent $I(\phi)$ generally first
Figure 4. The variations of the Josephson current $I$ with $\phi$ at different $\theta$. For the fixed $\mu_{SC} = 150\Delta_0$, (a) $\mu_{FI} = 50\Delta_0$ and (b) $\mu_{FI} = 150\Delta_0$, and for the fixed $\mu_{FI} = 0$, (c) $\mu_{SC} = 50\Delta_0$ and (d) $\mu_{SC} = 100\Delta_0$. Here, the other parameters are the same as in figure 2.

decreases and then increases. However, from figures 3(a) and 4(c) and (d) at fixed $\mu_{FI}$ for different $\mu_{SC}$, respectively, we notice that with the enhancement of $\mu_{SC}$, although the feature is also unchanged, the supercurrent $I(\phi)$ generally first increases and then decreases. This is much different from the situation for the different $\mu_{FI}$.

4. The supercurrent switch effect controlled by the magnetic exchange field’s $y$-component in the F configuration

In what follows, we study the in-plane situation, in which the two identical FIs are assumed to be only magnetized along the $y$-axis or its negative direction for simplicity. Similarly, there exists the F (AF) configuration with parallel (antiparallel) magnetizations. An energy gap cannot be opened by $m_y$, which only shifts the Fermi energy. Particularly, not only the CMMs at the two SC/FI interfaces but also the additional phase in the $F_{1(2)}$ region is not induced. The latter can be understood by that in the term complex number $\kappa_+ + i\frac{m_{1(2)y}}{m_y}$ of equations (3) and (5), $m_{1(2)y}$ lies in the real part, while $m_{1(2)x}$ is zero in the imaginary one.

Figure 5 is corresponding to the F configuration. In figure 5(a), we plot the Andreev bound energy $E$ as a function of $\phi$ at the exchange field magnitude $m = 50\pi\Delta_0$ and different $\alpha$. For $\alpha = 0$, the feature of the ABS energy level is the same as that for the out-plane situation, which is natural. However, for $\alpha \neq 0$, some peculiar features are exhibited. Specifically, although the gap of the ABS energy level is opened, the amplitude varies non-monotonically with increasing $\alpha$. The maximum of the gap lies at $\phi = \pi (3\pi)$, while the minimum locates at $\phi = 0 (2\pi)$, indicating that the Andreev bound energy level reverses. The shifts of the two ABS levels are different with increasing $\alpha$, for big $\alpha$, one almost keeps immoveable, and the other has a large shift. Particularly, the whole ABS levels generally shift downward. Compared with the results of figures 2(a)–(c), this is much different and induces the direction of the supercurrent at $\alpha > 0$ opposite to the one at $\alpha = 0$ in light of equation (10), thus leading to the decrease of total Josephson current. The minimum of the corresponding free energy is located at $\phi = 0$ as shown in figure 5(b), implying the Josephson junction is in the 0 state.

In figure 5(c), the Josephson currents $I(\phi)$ at different $m$ are displayed, which shows that with the enhancement of $m$, the Josephson current can be remarkably suppressed, being consistent with the situation of figure 5(a). Particularly, when $m \geq 68\pi\Delta_0$, the ABS energy is thoroughly suppressed, resulting in the disappearance of the total Josephson current. This implies adjusting $m$ can produce the switch effect of supercurrent.
Figure 5. For the F configuration, the variations of (a) the Andreev bound energy $E$ at $m = 50\pi \Delta_0$ and different $\alpha$, (b) free energy $F$, and (c) Josephson current $I$ at different $m$ with $\phi$. Here, the parameters are the same as in figure 2 except for $d = 0.03\pi \xi_0$ and $L = 0.02\pi \xi_0$.

5. Supercurrent spin valve effect

Figure 6(a) illustrates the variations of Josephson current $I(\phi)$ in the F and AF configurations at $m = 40\pi \Delta_0$. It is shown that the maximum current $I_c$ is greatly influenced by magnetic configuration, and the influence in the F configuration is much larger than that in the AF one. In order to further exhibit the effect of magnetic configurations on the Josephson current, we plot $I_c$ versus $m$ for the F and AF configurations at $d = 0.03\pi \xi_0$ and $0.045\pi \xi_0$ in figure 6(b). We find that the amplitudes of the decrease with $m$ for $I_c$ in different magnetic configurations are different. The amplitude in the F configuration is a lot larger than that in the AF one, particularly, $I_c$ is found to decrease gradually up to zero with the enhancement of $m$, which is consistent with the situation in figure 5(c). Furthermore, with the increase of $d$, the magnetic exchange field $m$ for $I_c$ being zero in the F configuration decreases remarkably, being in favor of the experimental manipulation. Here, we define the supercurrent magnetoresistance

$$\text{SMR} = [(I_c^{AF} - I_c^{F})/I_c^{AF}] \times 100\%,$$

where $I_c^{F(AF)}$ is the Josephson maximum current in the F (AF) configuration. Because $I_c^{F(AF)}$ can be experimentally measured, so it is with the SMR. In figure 6(c), the SMR versus $m$ at different $d$ is presented, which shows that with increasing $m$, the SMR gradually increases up to 100%. Particularly, with the enhancement of $d$, the magnetic exchange field $m$ for the SMR attaining to 100% decreases apparently. Due to the transformation from the F into AF configuration being easily by the external magnetic field, our results provide a perfect supercurrent spin valve.
Figure 6. The same magnetizations as in figure 5. In the F and AF configurations, the variations of (a) Josephson current $I$ with the phase difference $\phi$ at $m = 40 \pi \Delta_0$ and $d = 0.03 \pi \xi_0$ and (b) the critical Josephson current $I_c$ with $m$ at $d = 0.03 \pi \xi_0$ (0.045 $\pi \xi_0$ in the inset). (c) The dependence of the SMR corresponding to (b) on $m$. Here, the other parameters are taken the same as in figure 5.

6. Fully spin-polarized spin-triplet and -singlet correlations

We now turn our attention to the spin structure. In the present structure, the most important point is the presence of CMM due to the magnetism $m_z$ along the $z$-axis, which only leads to opening of the gap but not the spin splitting. For the incidence of an ELQ or a HLQ with a certain spin, the corresponding Andreev-reflected quasiparticle has the fixed spin (the same as or opposite to the incident spin) as known to us all. The former corresponds to the spin-triplet AR, while the latter corresponds to the spin-singlet one. In the present hybrid structure, no spin-splitting of the ABS is induced by ferromagnetism (see figure 2(a)), indicating the spin degeneracy, particularly, only the incidence with a kind of spin can take place. In the presence of the CMMs, the spin of corresponding Andreev-reflected quasiparticle is the same as that of incident quasiparticle due to the SESAR, which indicates the fully spin-polarized spin-triplet AR. For the nonlinear magnetic configurations, the transformation of the spin quantum axis in fact takes place, as in reference [36]. Therefore, in the noncollinear configuration, the fully spin-polarized spin-triplet and -singlet correlations are mixed, specifically, for the right-going incident quasiparticle in the F$_1$ region, there exists not only the corresponding spin-triplet Andreev-reflected quasiparticle but also the spin-singlet one. The F and AF configurations are corresponding to fully spin-polarized spin-triplet and -singlet correlations, respectively. This is a peculiar feature of the present structure, which is characteristic of different chiralities of the CMMs. However, under the conditions without $m_z$ but with only $m_y$, the features are different, specifically, the fully spin-polarized AR, and consequently fully spin-polarized spin-triplet and -singlet correlations, no longer exist.

Next, we present the contributions to the CPR from the spin-triplet and -singlet correlations at different misorientation angles $\theta$, as shown in figures 7(a) and (b), respectively, which can be obtained by the transformation of the spin quantum axis [36]. It is found that at $\theta = 0$, there is only the full spin-polarized spin-triplet correlation on account of the SESAR. With increasing $\theta$ from 0, the spin-singlet one begins to appear, accompanied by its increasing fraction, while the fraction of the spin-triplet correlation gradually falls down. At $\theta = \pi/2$, the two fractions are equal, i.e. 50%. With the further increase of $\theta$, the fraction of the spin-triplet correlation gradually decreases to zero at $\theta = \pi$, while the situation for the spin-singlet correlation is just contrary. The spin-singlet correlation with 100% fraction is also fully spin-polarized due to the SESAR.

7. Experimental feasibility

Finally, we briefly discuss the experimental feasibility. In light of actual materials, the EuO or EuS can be chosen for the FI [37, 38], and the monocrystalline Bi$_2$Se$_3$ and Bi$_2$Te$_3$ reported experimentally, or HgTe quantum wells can be chosen for the 3D TI [39–42]. The bulk energy gaps of the TIs are smaller and dependent on the concrete materials, such as about 300 meV, 100 meV, and 22 meV for the
above-mentioned three ones, respectively. For the $s$-wave SC, we can utilize the experimentally reported Al or Ti material, whose critical temperature is $T_c \simeq 1$ K. By virtue of the proximity between the SC and TI, the superconductivity on the surface of the TI can be experimentally induced. Due to their lattice mismatch, the induced superconducting pair potential $\Delta_0$ could be decreased drastically, which is estimated at $\Delta_0 \simeq 0.1$ meV [16]. Therefore, in the calculation, the taken chemical potentials in all regions are all smaller than the bulk energy gap, implying the demand that Fermi energy of the TI must lie in the bulk energy gap can be satisfied. In order to fit the actual experimental parameters, we evaluate $v_F \simeq 5 \times 10^5$ m s$^{-1}$ [39], from which the superconducting coherence length is attained as $\xi_0 \approx 3 \times 10^3$ nm, much larger than the corresponding $d = L \approx 360$ nm that could be experimentally realized. At the fixed temperature $T = 0.1T_c$, the thermal excitation energy $k_B T \simeq 0.057\Delta_0$, which is comparable with $5.7$ $\mu$eV and a lot smaller than the bulk energy gap. To sum up, the proposed setup could be experimentally realized.

8. Conclusions

In summary, we have proposed one setup for Josephson effect based on a 3D TI, which reveals controllable CMMs by the misorientation angle $\theta$ of the magnetizations of two FIs sandwiched in between the two SCs. An additional phase shift is induced by the different chiralities of the CMMs at the two FI/SC interfaces. The phase manipulated by the misorientation angle $\theta$ can be demonstrated by the ABS energies. It is also shown that the setup has three resultant features: (1) the misorientation angle $\theta$ leads to not only the $0 - \pi$ state transition but also $\phi_0$ supercurrent, (2) owing to the SESAR, the fully spin-polarized spin-triplet and -singlet correlations blend, and (3) for the two magnetizations only along the $y$-axis, there exist no additional phase shift and a large supercurrent magnetoresistance (even up to 100%) is exhibited by tuning the exchange field strength $m$ and FI length $d$. The setup can be therefore used to not only probe and discern the topological SCs but also design a perfect topological supercurrent spin valve device.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant Nos. 11874221, 11805103, 12104232, 12174051 and 12074332, and Postgraduate Research & Practice Innovation Program of Jiangsu Province No. 1812000024441.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

M Y Li https://orcid.org/0000-0001-6351-3098
J G Hu https://orcid.org/0000-0003-3829-5177
Y C Tao https://orcid.org/0000-0001-5539-3815
J Wang https://orcid.org/0000-0002-6278-0677
References

[1] Szombati D B, Nadj-Perge S, Car D, Plissard S R, Bakkers E P A M and Kouwenhoven L P 2016 Nat. Phys. 12 568
[2] Mazanik A A et al 2020 Phys. Rev. Appl. 14 014003
[3] Goldobin E, Koelle D and Kleiner R 2015 Phys. Rev. B 91 214511
[4] Dolcini F, Houzet M and Meyer J S 2015 Phys. Rev. B 92 035428
[5] Alidoust M and Hamzehpour H 2017 Phys. Rev. B 96 165422
[6] Alidoust M, Willatzen M and Jauho A P 2018 Phys. Rev. B 98 085414
[7] Shukrinov Y M, Rahmonov I R and Sengupta K 2019 Phys. Rev. B 99 224513
[8] Alidoust M 2018 Phys. Rev. B 98 245418
[9] Huang C S, Wei Y J, Tao Y C and Wang J 2021 Phys. Rev. B 103 035418
[10] Alidoust M and Halterman K 2020 Phys. Rev. B 101 035120
[11] Alidoust M 2020 Phys. Rev. B 101 155123
[12] Alidoust M, Shen C and Zutić I 2021 Phys. Rev. B 103 060503
[13] Fu L and Kane C L 2008 Phys. Rev. Lett. 100 096407
[14] Fu L and Kane C L 2009 Phys. Rev. Lett. 102 216403
[15] Tanaka Y, Yokoyama T and Nagaosa N 2009 Phys. Rev. Lett. 103 107002
[16] Linder J et al 2010 Phys. Rev. B 81 184525
[17] Snelder M et al 2013 Phys. Rev. B 87 104507
[18] Zyuzin A, Alidoust M and Loss D 2016 Phys. Rev. B 93 214502
[19] Bobkova I V et al 2016 Phys. Rev. B 94 134506
[20] He J J et al 2014 Phys. Rev. Lett. 112 037001
[21] Sun H H et al 2016 Phys. Rev. Lett. 116 257003
[22] Wang M X et al 2012 Science 336 52
[23] Xu J P et al 2014 Phys. Rev. Lett. 112 217001
[24] Xu J P et al 2015 Phys. Rev. Lett. 114 017001
[25] Wei Y J, Liu T M, Huang C S, Tao Y C and Qi F H 2021 Phys. Rev. Res. 3 033131
[26] Annunziata G et al 2011 Phys. Rev. B 83 144520
[27] Kashiwaya S and Tanaka Y 2000 Rep. Prog. Phys. 63 1641
[28] Furusaki A and Tsukada M 1991 Solid State Commun. 78 299
[29] Furusaki A and Tsukada M 1991 Phys. Rev. B 43 10164
[30] Kashiwaya S et al 1995 J. Appl. Phys. Japan 34 4555
[31] Toki T et al 2016 Phys. Rev. B 100 104518
[32] Kuklik O I and Omelianchuk A N 1978 Fiz. Nizk. Temp. 4 296–311
[33] Tanaka Y and Kashiwaya S 1996 Phys. Rev. B 53 R11957
[34] Tanaka Y and Kashiwaya S 1997 Phys. Rev. B 56 892
[35] Bergeret S, Volkov A F and Efetov K B 2005 Rev. Mod. Phys. 77 1321
[36] Tao Z et al 2018 J. Phys.: Condens. Matter. 30 225302
[37] Todorov P, Tkaczyk J E and Kumar A 1986 Phys. Rev. Lett. 56 1746
[38] Haugen H, Huertas D and Brataas A 2008 Phys. Rev. B 77 115406
[39] Zhang H et al 2009 Nat. Phys. 5 438
[40] Xia Y et al 2009 Nat. Phys. 5 398
[41] Chen Y L et al 2010 Science 329 659
[42] Brune C et al 2011 Phys. Rev. Lett. 106 126803