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1. Introduction

When one tries to go further into the discussions and concepts related to Power Quality, one comes across basic questions about the voltage and current measurements. Such issues do not emerge only because of the evolution of sensors and digital techniques, but mainly because of the need to better understand the phenomena related with three-phase circuits under asymmetrical and/or distorted waveform conditions.

These issues are fundamental, both for establishing disturbance indicators as well as for power components formulation under non-sinusoidal and/or asymmetrical waveforms. This can be verified by the various conferences that have been dedicated to this topic and the growing number of articles published about this subject (Depenbrock, 1993; Akagi et al, 1993; Ferrero, 1998; Emanuel, 2004; Czarneck, 2008; IEEE Std 1459, 2010; Tenti et al., 2010; Marafão et al., 2010).

Several discussions have shown that the choice of the voltage reference point can influence the definitions and calculation of different power terms and power factor (Emmanuel, 2003; Willems & Ghijselen, 2003; Willems, 2004; Willems et al., 2005). Consequently, it may influence applications such as revenue metering, power conditioning and power systems design. Taking into account two of the most relevant approaches (Depenbrock, 1993; IEEE Std 1459, 2010), regarding to, e.g., the power factor calculation, it can be seen that quantitative differences are practically irrelevant under normal operating conditions, as discussed and demonstrated in (Moreira et al., 2006). However, under severe voltage and current deterioration, particularly in case of power circuits with a return conductor, the differences may result significant.

Nevertheless, the matter of voltage referential is much more extensive than the definitions or calculations of power terms and it can have a direct effect on many other power system’s applications, such as: power quality instrumentation and analysis, protection, power conditioning, etc.

Thus, this chapter deals with the selection of the very basic voltage referential and its influence of the quantification of some power quality indicators, as well as, in terms the apparent power definition.

The analysis of some power quality indices will illustrate how the selection of the voltages referential may influence the evaluation of, e.g., the total harmonic distortion, unbalance
factors and voltage sags and swells, especially in case of three-phase four-wire circuits. Such
case deserves special attention, both, from instrumentation and regulation points of view.
Finally, based on the classical Blakesley’s Theorem, a possible methodology will be
presented in order to allow the association of the most common voltage measurement
approaches, in such a way that the power quality (PQ) and power components definitions
would not be improperly influenced.

2. Choosing the voltage referential in three phase power systems

It is not possible to discuss the choice of a circuit voltage referential, without first recalling
Blondel’s classic definition (Blondel, 1893), which demonstrates that in a polyphase system
with “m” wires between source and load, only “m-1” wattmeters were needed to measure
the total power transferred from source to load. In this case, one of the wires should be
taken as the referential, be it either a phase or a return (neutral) conductor (Fig. 1).

![Diagram of voltage referential in three phase power systems]

Fig. 1. Illustration of the measuring method according to Blondel

This hypothesis was extended to various other power system applications and it is also
currently used, as can be seen, for example, in (IEEE Std 1459, 2010). However, other
proposals have also been discussed, such as the utilization of a referential external to the
power circuit (Depenbrock, 1993; Willems & Ghijselen, 2003; Blondel, 1893; Marafão, 2004).

2.1 External voltage referential

In this case, all wires, including the neutral (return), should be measured to a common
point outside the circuit (floating), as shown in Fig. 2. This common point was designated by
Depenbrock as a virtual reference or a virtual star point (*). In the same way as Blondel’s work,
the author originally dealt with the problem of choosing the voltage referential from the
point of view of power transfer.

In practice, this method requires that an external point (*) be used as the voltage referential.
This point can be obtained connecting “m” equal resistances (or sensor’s impedances)
among each wire on which the voltage should be measured. Voltage drops over these
resistors correspond to the voltages that characterize the electromagnetic forces involved.
Depenbrock has demonstrated that such measured voltages always sum up to zero,
according to Kirchhoff’s Voltage Law (Depenbrock, 1998).

Therefore this method is applicable to any number of wires, independently of the type of
connection (Y-n, Y ou Δ). It must be emphasized that measured voltages in relation to the
virtual point can be interpreted as virtual phase voltages, although they do not necessarily
equal the voltages over each branch of a load connected in Y-n, Y or Δ, especially when they
are unbalanced. Thus, the use of voltages in relation to the virtual point needs to be treated in a special way so as to arrive at phase or line quantities, as will be shown further on.

![Fig. 2. Voltages measurement considering a virtual star point (*)](image)

### 2.2 Internal voltage referential

Based on Blondel’s proposals, recent discussions and recommendations made by Standard 1459 (IEEE Std 1459, 2010) suggest that voltage should be measured in relation to one of the system’s wires, resulting in phase to phase voltages (line voltage) or phase to neutral voltages, according to the topology of the system used. In this approach, the number of voltage sensors is smaller than in the case of measurements in relation to a virtual point. Fig. 3 shows a measuring proposal considering one of the system’s conductor as the reference.

![Fig. 3. Voltage measurement considering an internal referential](image)

Note that, in case of 4 wire the phase voltages and currents may not sum zero. Where \( v^0 \) and \( i^0 \) are the zero sequence voltage and current components.

### 3. Considerations on three phase power system without return conductor

In this circuit topology, the lack of a return conductor allows either the selection of a virtual reference point (Fig. 2a) or a phase conductor reference (Fig. 3a). Apart from the fact that there is no zero-sequence current circulation, in the three-phase three-wire connection...
(system without a return wire), the zero-sequence voltage is also eliminated from the quantities measured between the phases. This is a direct consequence of Kirchhoff’s laws. Thus, considering three-wire systems and taking into account different applications, both measuring methods can have advantages and disadvantages. For example:

- With regard to low voltage applications one can conclude that the measurement of line quantities (Fig. 3a) results in the reduction of costs associated to voltage transducers;
- Assuming a common external point (Fig. 2a), the measurements need to be manipulated (adjusted) to obtain line voltages;
- However if we take into account high and medium voltage applications, measurements based on the scheme shown in Fig. 3a may not be the most adequate. Usually at these levels of voltage two methods are employed: the first requires the use of Voltage Transformers (VTs), which have a high cost, since they handle high line voltages. The second strategy, which is cheaper, is to employ capacitive dividers, which, in general, use the physical grounding of the electric system as a measuring reference. The problem is that this type of grounding is the natural circulation path for transient currents, leakage currents, atmospheric discharges, etc. resulting in a system with low protection levels for the measuring equipment;
- Therefore, when considering the previous case (high and medium voltage), the use of a virtual reference point may be a good strategy, since it would guarantee that the equipment is not subjected to disturbances associated to the grounding system. However, this connection with a floating reference point could cause safety problems to the instrument operator, since during transients the voltage of the common point could fluctuate and reach high values in relation to the real earth (operator).

### 4. Considerations on three phase power system with return conductor

The presence of the return conductor allows the existence of zero-sequence fundamental or harmonic components (homopolars: $v^0$ and $i^0$), and in this case, it is extremely important that these components are taken into account during the power quality analyses or even in the calculation of related power terms.

According to Fig. 3b, the reference in the return wire allows the detection of zero-sequence voltage ($v^0$) by adding up the phase voltages. According to Fig. 2b, the detection of possible homopolar components would be done directly through the fourth transducer to the virtual point ($v_{n*}$), which represents a common floating point, of which the absolute potential is irrelevant, since only voltage differences are imposed on the three-phase system. In the same way as for three-wire systems, there are some points that should to be discussed in case of four-wire systems:

- Considering the costs associated to transducers, it is clear that the topology suggested in Fig. 3b would be more adequate because of the reduction of one voltage sensor.
- On the other hand, many references propose the measurement of phase voltage (a,b,c,) and also of the neutral (n). The problem in this case is that it is not always clear which is the voltage reference and which is the information contained in such neutral voltage measurements. Usually phase voltages are considered in relation to the neutral wire and neutral voltage is measured in relation to earth or a common floating point (*). This cannot provide the same results. In order to attend the Kirchhoff’s Law, the sum of the measured voltages must be zero, which can only happen when voltages are measured in relation to the same potential.
• Comparing the equations related to Figs. 2b and 3b, we would still ask: what is the relationship between \( v_n \) e \( 3v^0 \), since the voltages measured in relation to the virtual point are different from those measured in relation to the neutral conductor? Therefore, taking into account these two topologies, it is essential the discussion about the impact of the voltage’s referential on the assessment of homopolar components (zero-sequence), as well as on the RMS value calculation or during short-duration voltage variations. As will be shown, the measured voltages in relation to an external point has its homopolar components (fundamental or harmonic) attenuated by a factor of \( 1/m \) (\( m \) = number of wires), which has direct impact on the several power quality indicators.

5. Apparent power definitions using different voltage referential

To analyze the influence of the voltage referential for apparent power and power factor calculations, two different apparent power proposals have been considered: the FBD Theory and the IEEE Std 1459. The following sections bring a briefly overview of such proposals.

5.1 Fryze-Buchholz-Depenbrock power theory (FBD-Theory)

The FBD-Theory collects the contribution of three authors (Fryze, 1932; Buchholz, 1950, Depenbrock, 1993) and it was proposed by Prof. Depenbrock (Depenbrock, 1962, 1979), who extended the Fryze’s concepts of active and non active power and current terms to polyphase systems. At the same time, Depenbrock exploited some of the definitions of apparent power and collective quantities which were originally elaborated by Buchholz. The FBD-Theory can be applied in any multiphase power circuit, which can be represented by an uniform circuit on which none of the conductors is treated as an especial conductor. In this uniform circuit, the voltages in the m-terminals are referred to a virtual star point “*”. The single requirement is that Kirchhoff’s laws must be valid for the voltages and currents at the terminals (Depenbrock, 1998).

Considering the three-phase four-wire systems (Fig. 2b), the collective instantaneous voltage and current have been defined as:

\[
\begin{align*}
   v_{\Sigma}(t) &= \sqrt{v_a^2 + v_b^2 + v_c^2 + v_n^2}, \\
   i_{\Sigma}(t) &= \sqrt{i_a^2 + i_b^2 + i_c^2 + i_n^2}
\end{align*}
\]

Thus leading straight to the collective RMS voltage and current

\[
\begin{align*}
   V_{\Sigma} &= \sqrt{V_a^2 + V_b^2 + V_c^2 + V_n^2}, \\
   I_{\Sigma} &= \sqrt{I_a^2 + I_b^2 + I_c^2 + I_n^2}
\end{align*}
\]

Differently from conventional definitions of apparent power, the Collective Apparent Power has been defined as:

\[
S_{\Sigma} = V_{\Sigma}I_{\Sigma} = \sqrt{V_a^2 + V_b^2 + V_c^2 + V_n^2} \sqrt{I_a^2 + I_b^2 + I_c^2 + I_n^2}
\]
Considering the existing asymmetries in real three-phase systems and the high current level which can circulate through the return conductor (when it is available), this definition also takes into account the losses in this path, which is not common in many other definitions of apparent power. According to various authors, this definition is the most rigorously presented up to that time, since it takes into account all the power phenomena which take place in relation to currents and voltages in the electric system (losses, energy transfer, oscillations, etc.).

The (collective) active power was given by:

$$ P_\Sigma = \frac{1}{T} \int_0^T (v_a i_a + v_b i_b + v_c i_c + v_n i_n) dt \quad (4) $$

For three-wire systems (Fig. 2a) $I_n = 0$ and $V_n = 0$ the expressions (3) and (4) become:

$$ S_\Sigma = V_\Sigma I_\Sigma = \sqrt{V_a^2 + V_b^2 + V_c^2} \sqrt{I_a^2 + I_b^2 + I_c^2} \quad (5) $$

and

$$ P_\Sigma = \frac{1}{T} \int_0^T (v_a i_a + v_b i_b + v_c i_c) dt \quad (6) $$

The collective active power has the same meaning and becomes identical to the conventional active (average) power ($P$), for both three- or four-wire systems, as indicated in (4) and (6). Finally the collective power factor has been defined as:

$$ \lambda_\Sigma = \frac{P_\Sigma}{S_\Sigma} \quad (7) $$

And it represents the overall behavior (or efficiency) of the polyphase power circuit.

### 5.2 IEEE Standard 1459

One of the main contributions of STD 1459 is the recommendation of the use of "equivalent" voltage and current for three-phase three- and four-wire systems (Emanuel, 2004; IEEE Std 1459, 2010). These values are based on a model of a balanced equivalent electric system, which should have exactly the same losses and/or use of power as the real unbalanced system (Emanuel, 2004; IEEE Std 1459, 2010).

Considering a three-phase four-wire system, the STD 1459 recommends using the values of the equivalent or effective voltage and current as:

$$ V_e = \sqrt{\frac{3(V_{a_n}^2 + V_{b_n}^2 + V_{c_n}^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{18}} \quad (8) $$

$$ I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} $$

The voltage and current equivalent variables were initially defined by Buchholz and Goodhue (Emmanuel, 1998) in a similar formula and as an alternative way by Depenbrock.
(2). Note that the effective current depends on all line and return currents and the effective voltage represents an equivalent phase voltage, which is based on all phase-to-neutral and line voltages. Thus, the Effective Apparent Power has been defined as:

$$S_e = 3V_e I_e = 3\sqrt{\frac{3(V_{an}^2 + V_{bn}^2 + V_{cn}^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{18}} \sqrt{\frac{l_a^2 + l_b^2 + l_c^2}{3}}$$  \((9)\)

This effective apparent power represents the maximum active power which can be transmitted through the three-phase system, for a balanced three-phase load, supplied by an effective voltage \(V_e\), keeping the losses constant in the line. And the active power is:

$$P = \frac{1}{T} \int_0^T (v_{an}i_a + v_{bn}i_b + v_{cn}i_c)dt$$  \((10)\)

For three-wire systems \(I_n = 0\). Then, considering only the line voltages the STD 1459 suggests using the following equation for the effective apparent power:

$$S_e = 3V_e I_e = 3\sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} \sqrt{\frac{l_a^2 + l_b^2 + l_c^2}{3}}$$  \((11)\)

and

$$P = \frac{1}{T} \int_0^T (v_{ab}i_a + v_{cb}i_c)dt$$  \((12)\)

Consequently, the Effective Power Factor has been defined as:

$$PF_e = \frac{P}{S_e}$$  \((13)\)

Equation (13) represents the relationship between the real power to a maximum power which could be transmitted whilst keeping constant the power losses in the line. In the same way as in (7), the effective power factor indicates the efficiency of the overall polyphase power circuit.

### 5.3 Comparison between the FBD and IEEE STD 1459 power concepts

Accordingly to the previous equations and based on the Blondel theorem (Blondel, 1893), it is possible to conclude that the active power definitions from FBD or STD do not depend on the voltage referential, which could be arbitrary at this point. It means that:

$$P_\Sigma = \frac{1}{T} \int_0^T (v_{a+}i_a + v_{b+}i_b + v_{c+}i_c + v_{n+}i_n)dt = \frac{1}{T} \int_0^T (v_{an}i_a + v_{bn}i_b + v_{cn}i_c)dt = P$$  \((14)\)

Considering the analyses of the collective and effective currents and voltages by means of symmetrical components, the following relations could be extracted from (Willems et al. 2005):
\[ I_\Sigma = \sqrt{3} I_e = \sqrt{(I^+)^2 + (I^-)^2 + 4(I^0)^2} \]  

(15)

where the positive sequence, negative sequence and zero-sequence components are indicated by the subscripts +, - and 0, respectively.

Moreover, in case of unbalanced three phase sinusoidal situation, the collective RMS values of the voltage (FBD) can also be expressed by means of the sequence components, such as:

\[ V_\Sigma = \sqrt{(V^+)^2 + (V^-)^2 + \frac{1}{4}(V^0)^2} \]  

(16)

Now, assuming the equivalent voltage from the STD:

\[ V_e = \sqrt{(V^+)^2 + (V^-)^2 + \frac{1}{2}(V^0)^2} \]  

(17)

It is possible to observe that the equivalent and collective currents match for both proposals, except for the factor \( \sqrt{3} \), which indicates the difference between the single and three-phase equivalent models of the STD and FBD, respectively. However, from (16) and (17) one can notice that the equivalent voltages differ for these two proposals.

Consequently, the choice of the voltage referential affects the zero-sequence components calculation and therefore, it affects the effective and collective voltages definitions, as well as the apparent power and power factor calculations in both analyzed proposals.

Next sections will illustrate the influence of the voltage referential in terms of several power quality indicators.

6. The influence of the voltage referential on power quality analyses

In this section, several simulations will be presented and discussed considering three-phase three- and four-wire systems. The main goal is to focus on the effect of different voltage referentials (return conductor or virtual star point) on the analyses of some Power Quality (PQ) Indicators. The resulting voltage measurements and PQ indicators using both voltage referentials will be also compared to the voltages at the load terminals. The main disturbances considered in the analysis are: harmonic distortions, voltage unbalances and voltage sag.

The analyses of such disturbance can be exploited in terms of the following indicators:

- RMS value:

\[ V = \sqrt{\frac{1}{T} \int_0^T v^2(t) \, dt} \]  

(18)

- Total Harmonic Distortion (THDv):

\[ THD_v = \sqrt{\frac{\sum_{k=2}^{\infty} V_k^2}{V_1^2}} \]  

(19)
• Voltage Unbalance Factors:

\[ K^- = \frac{V^-}{V^+} \]

\[ K^0 = \frac{V^0}{V^+} \] (20)

In the first case, events in the voltage source are generated to quantify the impact of the voltage reference on the occurrence of voltage sags. In the second case, distortions are generated in the voltage source by injecting odd harmonics up to the fifth order with amplitudes of 50% of the fundamental. In the third case, imbalances are imposed through the voltage source, generating negative- and zero-sequence components.

6.1 Three phase power system without return conductor

Considering the line quantities estimation (load in delta configuration) and assuming the voltage measurements referred to a virtual point, an adaptation of the algorithm is necessary since these voltages are virtual phase voltages, and the line voltages can be expressed as:

\[ v_{a*} - v_{b*} = v_{ab} \]

\[ v_{b*} - v_{c*} = v_{bc} \] (21)

\[ v_{c*} - v_{a*} = v_{ca} \]

and the RMS values are:

\[ V_{ab} = \frac{1}{T} \int_{0}^{T} (v_{a*} - v_{b*})dt \]

\[ V_{bc} = \frac{1}{T} \int_{0}^{T} (v_{b*} - v_{c*})dt \] (22)

\[ V_{ca} = \frac{1}{T} \int_{0}^{T} (v_{c*} - v_{a*})dt \]

In the case of an under-voltage event, Fig. 4 shows that both measuring methods adequately represent the impact effectively experienced by load (superimposed curves), either in terms of their magnitude or duration of the voltage sag. On the other hand, Fig. 5 shows that both measuring topologies being discussed are equivalent with regard to the measuring of harmonic components, thus representing their impact on the loads (superimposed spectra).

To assess the performance of both methodologies with regard to the unbalance factors, the three-phase source was defined with amplitude and phase angle as indicated in Table 1.
b) Reference at phase \( b \) 

Fig. 4. Evolution RMS values during voltage sag between phases \( b \) and \( c \) from 220V to 100V (4 cycles).

b) Reference at phase \( b \) 

Fig. 5. Spectral analysis with each measuring topology (3 wires)

| Source Voltage | Test 1 | Test 2 |
|---------------|--------|--------|
| \( v_a \) | 179.61 V | 179.61 V |
| \( v_b \) | 159.81 V | 159.81 V |
| \( v_c \) | 208.59 V | 208.59 V |

| Test 3 | Test 4 |
|--------|--------|
| \( v_a \) | 197.57 V | 197.57 V |
| \( v_b \) | 171.34 V | 171.34 V |
| \( v_c \) | 171.34 V | 171.34 V |

Table 1. Voltages and phase angles programmed at the power source

In this case the negative-sequence unbalance factor (K) is identical for both measuring methodologies (vide table 2), which also coincides with the theoretical value and the
measurements at the load terminals. As it was expected, the zero-sequence unbalance factor \(K_0\) is nil due to the lack of a return conductor.

| Test | Theoretical Value | Reference at phase b | Reference at the virtual Point | Measurement at load terminals |
|------|-------------------|----------------------|-------------------------------|-------------------------------|
|      | \(K^-(\%)\) | \(K^0(\%)\) | \(K^-(\%)\) | \(K^0(\%)\) | \(K^-(\%)\) | \(K^0(\%)\) | \(K^-(\%)\) | \(K^0(\%)\) | \(K^-(\%)\) | \(K^0(\%)\) |
| 1    | 15.92 | 0.00 | 15.92 | 0.00 | 15.92 | 0.00 | 15.92 | 0.00 |
| 2    | 19.49 | 0.00 | 19.49 | 0.00 | 19.49 | 0.00 | 19.49 | 0.00 |
| 3    | 10.00 | 0.00 | 10.00 | 0.00 | 10.00 | 0.00 | 10.00 | 0.00 |
| 4    | 0.00  | 0.00 | 0.00  | 0.00 | 0.00  | 0.00 | 0.00  | 0.00 |

Table 2. Unbalance factor calculated according to each measurement method

6.2 Three phase power system with return conductor

Fig. 6a shows that the voltage measurement using the return wire as the reference, correctly detects the presence of odd harmonics, with 50% amplitudes. Therefore, it is in this scenery that the real impact on the load Fig. 6 is being quantified. Note that when the virtual point is used as voltage reference (Fig. 6b) the harmonics multiples of 3 are not correctly detected. These homopolar components are attenuated by a factor of \(\frac{1}{4}\) in relation to the expected voltage spectrum on the load. The other harmonic components do not suffer attenuation, because they either are of positive- or negative-sequence.

![Fig. 6. Spectrum analysis with each measuring topology (4 wires)](https://www.intechopen.com)
Fig. 7. Evolution of RMS values during a voltage sag between phases b and c from 127V to 50V (4 cycles)

Fig. 7a shows when the reference is set in the return conductor the event is correctly detected and quantified (amplitude and duration) in all phases, thus representing the exact impact on the load. However, with the use of a virtual point as the voltage reference the event is detected, but it does not show how it is generated or how it could affect the load (Fig. 7b). Thus, this measuring method affects the assessment of the impact during voltage sag.

According to Table 3 both the voltage reference on the return wire and on the virtual point detected equal imbalances for the negative component (K-). However, the zero-sequence indicator (K0), calculated by means of the virtual reference point voltages is different from expected. It is attenuated by a factor of ¼ (1/m).

| Test | Theoretical Value | Reference at the neutral conductor | Reference at the virtual point | Measurement at load terminals |
|------|------------------|------------------------------------|--------------------------------|-----------------------------|
|      | K - (%) | K0 (%) | K - (%) | K0 (%) | K - (%) | K0 (%) | K - (%) | K0 (%) |
| 1    | 15.92 | 0.00   | 15.92  | 0.00   | 15.92  | 0.00   | 15.92  | 0.00   |
| 2    | 19.49 | 8.02   | 19.49  | 8.02   | 19.49  | 2.01   | 19.49  | 8.02   |
| 3    | 10.00 | 0.00   | 10.00  | 0.00   | 10.00  | 0.00   | 10.00  | 0.00   |
| 4    | 0.00  | 10.00  | 0.00   | 10.00  | 0.00   | 2.50   | 0.00   | 10.00  |

Table 3. Unbalance factor calculated according to each type of measurement

7. Attenuation and recovery of the zero-sequence component

From the previous results it can be concluded that in case of three-phase four wire circuits and in the presence of zero-sequence components (fundamental or harmonic), there is a clear difference between the two voltage referential methods. Therefore, it is important to provide a careful analysis of the two methodologies and the differences found between them.

Consider a set of three-phase and periodic voltage sources \(v_a\), \(v_b\) e \(v_c\), connected as in Fig. 8. In terms of symmetric components these voltages can be expressed as:
\[ v_a = v_a^+ + v_a^- + v^0 \]
\[ v_b = v_b^+ + v_b^- + v^0 \]  
(23)
\[ v_c = v_c^+ + v_c^- + v^0 \]

Fig. 8. Three-phase four-wire system

Considering the measurements with voltage reference at the neutral, Fig. 9 shows a circuit on which the neutral is utilized as the voltage reference, where \( R \) is the resistance of the voltage meter.

Assuming that the value of \( R \) is much greater than the values of the load impedances, we can take into account only the links formed by the voltage sources and the measuring instruments, and substitute the voltages of the sources by their respective sequence components. Thus, the circuit of Figure 9 can be represented as in Fig. 10.
Since $v_{an}$, $v_{bn}$ e $v_{cn}$ are the voltage drops over each instrument's resistances, it follows that:

\[
\begin{align*}
\nu_{an} &= \nu_{a}^+ + \nu_{a}^- + \nu^0 \\
\nu_{bn} &= \nu_{b}^+ + \nu_{b}^- + \nu^0 \\
\nu_{cn} &= \nu_{c}^+ + \nu_{c}^- + \nu^0
\end{align*}
\]

(24)

In this way, it can be seen that the measured voltages in relation to the neutral correspond to the imposed voltages by the source, containing all sequence components (positive, negative and zero), as it has been shown earlier in the sag, harmonics and unbalance tests.

On the other hand, Fig. 11 shows a circuit on which the virtual point is used as the voltage referential. As in the circuit of Fig. 10, we can represent the circuit shown in Fig. 11 through its sequence components (Fig. 12).
Fig. 12. Equivalent circuit for measuring to the virtual point

As it is known, the negative- and zero-sequence components are indicators of abnormal conditions (imbalances and/or harmonics) of an electric circuit. If we consider that the negative-sequence components "see" practically the same circuit as the positive-sequence components, the return (neutral) wire therefore is not necessary, as opposed to the zero-sequence current that only occurs in the presence of a return wire.

In this way, if we consider the superposition theorem, we can decompose the circuit in Fig. 12 into a circuit containing positive- and negative- components (Fig. 13) and another circuit containing only zero-sequence components (Fig.14).

Fig. 13. Decomposition: positive- and negative-sequence circuit by superposition theorem
Fig. 14. Decomposition: zero-sequence circuit by superposition theorem

From the circuit in Fig. 13 we have the following:

\[ \nu_a^\pm = \nu_a^+ + \nu_a^- \]
\[ \nu_b^\pm = \nu_b^+ + \nu_b^- \]  \hspace{1cm} (25)
\[ \nu_c^\pm = \nu_c^+ + \nu_c^- \]

According to the superposition theorem (Fig. 13 and 14), the measured voltages to a virtual point can be written as:

\[ \nu_a* = \nu_a^* + \nu_a^0 \]
\[ \nu_b* = \nu_b^* + \nu_b^0 \]  \hspace{1cm} (26)
\[ \nu_c* = \nu_c^* + \nu_c^0 \]

On the other hand, Fig. 14 can also be represented by the circuit shown in Fig. 15, based on Blakesley transform (Blakesley, 1894).

Thus, for the circuit in Fig. 15 we can apply the voltage divider rule:

\[ \nu_a^0 = \nu_b^0 = \nu_c^0 = \frac{R_{eq}}{R_T} \nu^0 = \frac{1}{4} \nu^0 \]  \hspace{1cm} (27)
\[ \nu_n* = \frac{R}{R_T} \nu^0 = -\nu_n = -\frac{3}{4} \nu^0 \]
Equation (27) indicates the zero-sequence components of phase and neutral voltages regarding to the virtual point. In this way, the total voltages (measured to the virtual point), taking into account positive-, negative- and zero-sequence components, can be obtained by substituting (27) in (26):

\[ v_{a*} = v_{a}^+ + v_{a}^- + \frac{1}{4}v^0 \]
\[ v_{b*} = v_{b}^+ + v_{b}^- + \frac{1}{4}v^0 \]  \hspace{1cm} (28)
\[ v_{c*} = v_{c}^+ + v_{c}^- + \frac{1}{4}v^0 \]

Note that the zero sequence component is attenuated by a factor of \( \frac{1}{4} \) of its real value, which means that for applications where its quantification is necessary, the measured value must be corrected. This can be done by adding \( \frac{3}{4}v^0 \) on both sides of the equation (29):

\[ v_{a*} + \frac{3}{4}v^0 = v_{a}^+ + v_{a}^- + v^0 \]
\[ v_{b*} + \frac{3}{4}v^0 = v_{b}^+ + v_{b}^- + v^0 \]  \hspace{1cm} (29)
\[ v_{c*} + \frac{3}{4}v^0 = v_{c}^+ + v_{c}^- + v^0 \]

From (31) we have:

\[ \frac{3}{4}v^0 = -v_n* \]  \hspace{1cm} (30)
Equation (30) finally provides the relationship between the measured voltage between the neutral and the virtual point and the zero-sequence component (homopolar). This equation allows us, to compare both measuring methodologies, as well as to provide algorithms for the measuring and monitoring equipments, which are correct, independently of the type of connection chosen by the end user.

Due to the differences in the apparent power, as indicated by (16) and (17), inclusion of equation (30) may be necessary in order to avoid miscalculation of the power terms and possible costumer’s penalization.

8. Conclusions

It has been shown that in case of three-phase three-wire systems (without a return wire), both voltage references (neutral or virtual point) provide identical measurements due to the lack of homopolar components (zero-sequence), which are filtered by the topology of the system itself. However for return-wire systems, there is a need to take certain aspects into consideration, as for example, the attenuation of homopolar components (zero-sequence) if measuring the voltages to a virtual star point.

In this way, to measure voltage in modern installations with the presence of distortions and imbalances, the choice of a reference point must be made very carefully and its implications must be taken into account in applications such as pricing, measurement, power quality monitoring, compensation, protection, etc.

Despite of the demonstration of how to recover the homopolar components, attenuated by the virtual point measurements, the connection referenced to the neutral continues to be the best option, especially for low voltage applications, due to the fact that it needs less one measuring channel. However, considering applications in high-voltage systems (3 wires), the use of an external virtual point may be an interesting option, from the point of view of the protection of the measuring equipments.

Finally, it is worth pointing out that the proposed methodology to associate two methods for measuring voltages, by using Blakesley Theorem, can also be used in order to find a convergence point between the different power theories.

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