On finite $N = 1, 2$ BRST transformations: Jacobians and Standard Model with gauge-invariant Gribov horizon

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Abstract

We review the concept and properties of finite field-dependent BRST and BRST-antiBRST transformations introduced in our recent study [1, 2, 3, 4, 5, 6, 7] for gauge theories. Exact rules for calculating the Jacobian of a corresponding change of variables in the partition function are presented. Infrared peculiarities under $R_\xi$-gauges in the Yang–Mills theory and Standard Model are examined in a gauge-invariant way with an appropriate horizon functional and unaffected local $N = 1, 2$ BRST symmetries.

1 Introduction

BRST transformations [8,9] for gauge theories in Lagrangian formalism with a field-dependent (FD) Grassmann parameter $\mu$ were considered for the first time within the BV method [10] in the infinitesimal case, so as to prove the invariance of the partition function $Z_\Psi$: $Z_\Psi = Z_{\Psi+\delta\Psi}$ with respect to small variations of the gauge (in terms of the gauge fermion $\Psi$) under the choice $\mu = -\frac{i}{\hbar}\delta\Psi$. FD BRST transformations in the case of a finite functional parameter were introduced in Yang–Mills (YM) theories, within the family of generalized $R_\xi$-gauges [11], as a sequence of infinitesimal FD BRST transformations. Developed initially as a special $N = 1$ SUSY transformation, and being a change of the field variables $\phi^A \rightarrow \phi'^A = \phi^A + \delta_\mu \phi^A$ in the integrand of $Z_\Psi = \int d\phi \exp\left\{\frac{i}{\hbar}S_\Psi(\phi)\right\}$ with a quantum action $S_\Psi(\phi)$, BRST transformations were extended by means of antiBRST transformations [12,13] in YM theories to the case of $N = 2$ BRST (BRST-antiBRST) transformations (in YM [14] and general gauge theories [15]), parametrized by an Sp(2) doublet of Grassmann parameters, $\mu_a, a = 1, 2$.

The study of [16], which suggested to analyze so-called soft BRST symmetry breaking in YM theories, with account taken of the Gribov problem [17] in the infrared region of field configurations, discussed in the Zwanziger recipe [18] by adding a BRST-non-invariant horizon functional $H$ to the quantum action, attracted the attention of A.A.R. (whose e-mail communication of 11.03.2011 initiated the joint study [19] together with P.M. Lavrov and O. Lechtenfeld). Among the results of [19] in the field-antifield formalism related to [16], an equation was derived in softly broken BRST symmetry (SB BRST) for a bosonic term

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$M(\phi, \phi^*)$ added to the quantum action $S_\Psi(\phi, \phi^*)$ of a general gauge theory. The validity of this equation, involving the generating functional of Green’s functions $Z_{\Psi,M}(J, \phi^*)$ depending on sources $J_A$, preserves the gauge-independence of the respective vacuum functional $Z_{\Psi,M}(0)$ and the effective action, depending on external antifields, $\Gamma_M = \Gamma_M(\phi, \phi^*)$, and evaluated

on the extremals,

$$
\left[ M_A(\frac{\hbar}{i} \overleftarrow{\partial}(J), \phi^*) \left( \overrightarrow{\partial}^{\ast} A - \frac{i}{\hbar} M^{\ast} A(\frac{\hbar}{i} \overleftarrow{\partial}(J), \phi^*) \right) \right] \delta \Psi \left( \frac{\hbar}{i} \overleftarrow{\partial}(J), \phi^* \right) \right]
Z_{\Psi,M}(J, \phi^*) = 0 \quad (1)

\implies Z_{\Psi,M}(0) = Z_{\Psi + \delta \Psi,M + \delta M(0)} \quad (2)

\text{with } Z_{\Psi,M}(J, \phi^*) = \int d\phi \exp \left\{ \frac{\hbar}{i} \left( S_\Psi(\phi, \phi^*) + M(\phi, \phi^*) + J_A \phi^A \right) \right\} \quad (3)

\text{where it is assumed that } [M_A, M^{\ast} A] \equiv [M \overleftarrow{\partial}_A, \overrightarrow{\partial}^{\ast} A M], \text{ and } \phi^A = \frac{\hbar}{i} \overleftarrow{\partial}(J) \ln Z \text{ are average fields. [11] has the same form when the horizon functional } H(A) \text{ for YM fields } A^m(x) \text{ is used as } M(\phi, \phi^*) \text{. In terms of the vacuum expectation value, in the presence of external sources } J_A \text{ and with a given gauge } \Psi, \text{ relation [11] acquires the form}

$$
\langle \delta M + M^{\ast} s_{\hbar} \delta \Psi(\phi) \rangle = \langle \delta M - M^{\ast} s_{\mu} \mu(\delta \Psi) \rangle = 0, \text{ for } s_{\hbar} = \overleftarrow{\partial}_A \overrightarrow{\partial}^{\ast} A S_\Psi : \delta_\mu A^A \equiv \phi^A s_{\mu} \Psi, \quad (4)

\text{with } s \text{ being the generator of BRST transformations. In fact, the horizon functional in the family of } R_\xi \text{-gauges for small } \xi \text{ was derived explicitly in [19] by Eq. (5.20) therein. By using FD BRST transformations with a small odd-valued parameter, this was established in [1]. A.A.R. drew the attention of his coauthors (communication of 05.12.2011 to P.M. Lavrov) in [19] to the study of [20] which attempted to use FD BRST transformations [11] for relating}

\text{the vacuum functionals of YM and GZ (Gribov–Zwanziger) theories in the same gauge. Explicit calculations of the functional Jacobian for a change of variables induced by FD BRST transformations with a small odd-valued parameter, this was established in [1].}

\text{The present article reviews the study of finite } N = 1, 2 \text{ BRST transformations (including the case of FD parameters) and the way they influence the properties of the quantum action and path integral in conventional quantization. We suggest a quantum action for the YM theory and the Standard Model with an } N = 1, 2 \text{ BRST-invariant horizon functional in terms of gauge-invariant transverse fields } (A^h)_{\mu}^\circ(x) \text{, with the initial BRST symmetry under } R_\xi \text{-gauges, in a way different from the recipe of [22]. We use the DeWitt condensed notation and the conventions of [1, 2], e.g., } \epsilon(F), \overleftarrow{\partial} A, \overrightarrow{\partial}^A A \text{ and } \overrightarrow{\partial}(J) \text{ are used to denote the respective value of the Grassmann parity of a quantity } F \text{ and derivatives with respect to (anti)field variables } \phi^A, \phi^A \text{, and sources } J_A \text{. The raising and lowering of Sp(2) indices, } (s_{a}^{\ast}, s_{b}^{\ast}) = (\epsilon^{ab}s_{a}^{\ast}, \epsilon_{ab}s_{b}^{\ast}), \text{ are carried out by the antisymmetric tensor } \epsilon^{ab}, \epsilon^{ac} = \delta_{cb}, \epsilon^{12} = 1.

2 \text{ Finite FD BRST transformations for the integrand in [3] at } J = M = 0

$$
\delta_\mu \phi^A = \phi^A s_{\mu}, \quad s_{\mu} = \overleftarrow{\partial}_A S_{\Psi}^A \text{ with } (s_{\mu})^2 = \overleftarrow{\partial}_A (S_{\Psi}^{A} \overleftarrow{\partial}_B) S_{\Psi}^{B} \neq 0, \quad (5)

\text{with a finite Grassmann parameter } \mu(\phi, \phi^*) \text{, depending on external antifields } \phi^A, \epsilon(\phi^A) + 1 = \epsilon(\phi^A) = \epsilon_A, \text{ and internal fields } \phi^A \text{, were introduced in [1] and made it possible to solve

\footnote{The variables } \phi^A \text{ contain the classical fields } A^i, \quad i = 1, \ldots, n, \text{ with gauge transformations } \delta A^i = R^i_{\alpha}(A) \xi^\alpha, \quad \alpha = 1, \ldots, m < n, \text{ as well as the ghost, antighost, and Nakanishi–Lautrup fields } C^\alpha, \overline{C}^\alpha, B^\alpha, \text{, as well as the antighost, and Nakanishi–Lautrup fields } C^\alpha, \overline{C}^\alpha, B^\alpha,$
the problem of SB BRST symmetry in general gauge theories. The master equation for $S_\varphi$, $\Delta \exp \{ \frac{i}{\hbar} S_\varphi \} = 0$ with $\Delta = (-1)^{\varphi} \partial_A \overleftarrow{J}^A$, reflects the absence of nilpotency for the generator $\overleftarrow{s}$, which reduces at $\varphi^* = 0$ to the usual generator $\overleftarrow{s}$ of BRST transformations.

Construction of finite $N = 2$ BRST Lagrangian transformations solving the same problem within a suitable quantization scheme (starting from YM theories) was problematic in view of BRST-antiBRST-non-invariance for the gauge-fixed quantum action $S_F$ in a form more than linear in $\mu_a$, $S_F(g_\mu(\mu_a) \phi) = S_F(\phi) + O(\mu_1 \mu_2)$, with the gauge condition encoded by a gauge boson $F(\phi)$. This problem was solved by finite $N = 2$ BRST transformations in an Abelian supergroup form, $\{ g(\mu_a) \}$, using an appropriate set of variables $\Gamma^p$, according to \[ G(\Gamma(\mu_a)) = G(\Gamma) \text{ and } G(s_a^0) = 0 \implies g(\mu_a) = 1 + s_a^0 \mu_a + \frac{1}{4} s_a^2 \mu^2 = \exp \{ s_a^0 \mu_a \}, \] where $G(\Gamma)$ is an arbitrary regular functional; $\mu^2 \equiv \mu_a \mu_a$, $s_a^2 \equiv s_a^0 s_a^2$, and $s_a^0$ are the generators of BRST-antiBRST transformations in the space of $\Gamma^p$.

In YM theories, the construction of finite $N = 2$ BRST transformations \[ \text{uses an explicit form of generators } s_a^0 \text{ (satisfying} \{ s_a^0, s_b^0 \} = 0 \text{ in the space of fields} \phi^A = (F^I, C_\alpha, \bar{C}_\alpha, B^\alpha) \text{ arranged in} \mathrm{Sp}(2)\text{-symmetric tensors,} (\bar{A}^m, C^m, B^m), \text{ as follows}\] \[ S_F(\phi) = S_0(A) - \frac{1}{2} F_\xi s_a^2, \quad S_0(A) = -\frac{1}{4} \int d^Dx \ G_{\mu \nu} G^{\mu \nu}, \quad G_{\mu \nu} = \partial_\mu A_\nu + f_{mnl} A_\mu^m A_\nu^n (7), \]
\[ F_\xi(\phi) = \frac{1}{2} \int d^Dx \left( - A^m_{\mu} A^\mu_{m} + \xi_{ab} C^{ma} C^{mb} \right) \leftrightarrow R_\xi \text{ - gauges,} (8) \]
\[ \Delta A^m = D^m_{\mu n} C^{n a} \mu_a - \frac{1}{2} \left( D^m_{\mu n} B^n + \frac{1}{6} f_{mnl} C^{da} D_\mu^{ab} C^{hb} \xi_{ba} \right) \mu^2, (9) \]
\[ \Delta B^m = -\frac{1}{2} \left( f_{mnl} B^{l} C^{ma} + \frac{1}{6} f_{mnl} f^{irs} C^{sb} C^{ra} C^{mc} \xi_{ab} \right) \mu_a, \]
\[ \Delta C^{ma} = \left( \varepsilon^{ab} B^m - \frac{1}{2} f_{mnl} C^{da} C^{mb} \right) \mu_b - \frac{1}{2} \left( f_{mnl} B^{l} C^{ma} + \frac{1}{6} f_{mnl} f^{irs} C^{sb} C^{ra} C^{mc} \xi_{ab} \right) \mu^2, (11) \]
for $\eta_{\mu \nu} = \text{diag} (-, +, \ldots, +)$ and the totally antisymmetric $su(N)$ structure constants $f_{mnl}$, $l, m, n = 1, \ldots, \hat{N}^2 - 1$. In general gauge theories, such as reducible theories or theories with an open gauge algebra, the corresponding space of triplectic variables $\Gamma^p_{\nu} = (\phi^A, \phi^{\dagger}_A, \bar{\phi}_A, \pi^{\dagger}_A, \lambda^A)$ in the Sp(2)-covariant Lagrangian quantization scheme \[ \text{contains, in addition to} \phi^A, \text{ also 3 sets of antifields} \phi^{\dagger}_A, \bar{\phi}_A, \varepsilon(\phi^{\dagger}_A, \bar{\phi}_A) = (\epsilon_A + 1, \epsilon_A), \text{ as sources to} \text{ BRST, antiBRST and mixed BRST-antiBRST transformations, and 3 sets of Lagrangian multipliers} \pi^{\dagger}_A, \lambda^A, \varepsilon(\pi^{\dagger}_A, \lambda^A) = (\epsilon_A + 1, \epsilon_A), \text{ introducing the gauge. The corresponding generating functional of Green’s functions,} Z_F(J) = \int \mathcal{Z}^{(F)}(J), \text{ with the bosonic functional}\]
\[ W(\phi, \phi^{\dagger}_a, \bar{\phi}) \text{ being related to the gauge-invariant classical action} S_0(A) \text{ as} W(\phi, 0, 0) = S_0(A), \]
\[ Z_F(J) = \int d\Gamma \exp \left\{ \frac{i}{\hbar} \left[ W + \phi^{\dagger}_a \pi^a + \bar{\phi} \lambda - \frac{1}{2} F^2 + J(\phi) \right] \right\}, \overleftarrow{U}^a = \overleftarrow{\partial}_A \pi^a + \varepsilon^{ab} \overleftarrow{\partial}^{(\pi)}_{Ab} \lambda^A, (12) \]
is invariant at $J = 0$ with respect to finite $N = 2$ BRST transformations (for constant $\mu_a$)

\[ \epsilon(\phi^A, \phi^{\dagger}_A, \bar{\phi}, \lambda^A) = (\epsilon_i, \epsilon_a, \epsilon_a + 1, \epsilon_a + 1), \text{ along with additional towers of fields, depending on the (ir)reducibility of the theory.} \]

\[ \text{The transformations} \Gamma^p \rightarrow \Gamma^p g(\mu_a), \text{ however, cannot be presented in terms of an exp-like relation for an Sp}(2)\text{ doublet of functional parameters} \mu_a(\Gamma), \text{ due to} \mu_a s_b \neq 0. \]

\[ N = 2 \text{ and } N = 1 \text{ BRST-invariant actions of YM theories coincide only in Landau gauge,} \xi = 0. \]
in the space of $\Gamma_{tr}^p$, which are obtained from \cite{1} with a functional $G_{tr} = G(\Gamma_{tr}^p)$:

$$\Gamma_{tr}^p \rightarrow \Gamma_{tr}^{p'} = \Gamma_{tr}^p \left(1 + \frac{\kappa e}{\kappa} \mu_a + \frac{1}{4} \frac{\kappa}{\kappa} \mu^2\right) \equiv \Gamma_{tr}^p g(\mu_a) \implies \mathcal{I}_{tr}^{(F)}(0) = \mathcal{I}_{tr}^{(F)}(0), \quad (13)$$

where $\frac{\kappa e}{\kappa} = (\overrightarrow{\partial}_A, \overrightarrow{\partial}^{(s)}_{(\phi^s)}, \overrightarrow{\partial}^A_{(\phi^s)}, \overrightarrow{\partial}^{A}_{(\phi)}) \left(\pi^{Aa}, W_A(-1)^{\epsilon_A}, \varepsilon^{ab}\phi_{Ab}(-1)^{\epsilon_{A+1}}, \varepsilon^{ab}\lambda^A\right)^T, \{\frac{\kappa e}{\kappa}, \frac{\kappa}{\kappa}\} \neq 0,$

provided that $\left(\Delta^a + (\mu/\hbar)\varepsilon^{ab}\phi_{Ab} \overrightarrow{g}^A\right) \exp \left\{\frac{\kappa}{\hbar} W\right\} = 0$, for $\Delta^a = (-1)^{\epsilon_A} \frac{\kappa}{\kappa} \overrightarrow{g}_A \overrightarrow{g}^{*A}$. \quad (14)

3. Jacobians of FD $N = 1, 2$ BRST transformations

The Jacobian induced by a change of variables $\phi^A \rightarrow \phi'^A = \phi^A(1 + \frac{\kappa e}{\kappa} \mu)$ is given by \cite{1}

$$\text{Sdet} \left|\phi'^A \overrightarrow{g}^A_B\right| = \exp \left\{\text{Str} \left(\delta^A_B + (S^A_\Psi \mu) \overrightarrow{g}^A_B\right)\right\} = \exp \left\{\text{Str} \sum_{n=1}^{(\kappa e)^{-1}} \left(1 + \frac{\kappa e}{\kappa}\right)^n\right\}$$

$$= \left(1 + \frac{\kappa e}{\kappa}\right)^{-1} \left(1 + \frac{\kappa e}{\kappa} \mu\right)^{-1} \left\{1 + (\Delta S_\Psi) \mu\right\} = J_{\mu(\phi)} \quad (15)$$

and reduces, in a rank-1 theory with a closed gauge algebra, $[\Delta S_\Psi, \frac{\kappa e}{\kappa}] = [0, 0]$, where $\frac{\kappa e}{\kappa} = \frac{\kappa}{\kappa}$, to the form $\text{Sdet} \left|\Phi'^A \overrightarrow{g}^A_B\right| = \left(1 + \frac{\kappa e}{\kappa}\right)^{-1}$, which is the same as in YM theories \cite{21}.

The Jacobian (15) allows one to solve the problem of SB BRST symmetry in general gauge theories \cite{1} and was examined in detail \cite{3} for an equivalent representation of $Z_{\Psi,M}(J, \phi^*)$ with BRST transformations $\Gamma^p \rightarrow \Gamma^{p'} = \Gamma^p(1 + \frac{\kappa e}{\kappa} \mu)$, for $\mu(\Gamma)$ and $\Gamma^p \overrightarrow{g} = (\phi^A, \phi*'_{s}, \lambda^A) \overrightarrow{g} = (\lambda^A, S^{*A} \overrightarrow{g}_A, 0)$, in an extended space $\Gamma^p$ of fields $\phi^A$, internal antifields $\phi^{'s}_{s}$, and Lagrangian multipliers $\lambda^A$ for Abelian hypergauge conditions, $G_A(\phi, \phi^*) = \phi^* - \Psi(\phi) \overrightarrow{g}_A$, with the result

$$Z_{\Psi,M}(J, \phi^*) = \int d\Gamma \exp \left\{\frac{\kappa}{\kappa} S(\phi, \phi^*) + G_A(\phi, \phi^{\prime s} + \phi^*) \lambda^A + M(\phi, \phi^*) + J\phi\right\}, \quad (16)$$

$$J_{\mu(\Gamma)}^{BV} = \text{Sdet} \left|\Gamma^{p'} \overrightarrow{g}^A_B\right| = \left(1 + \frac{\kappa e}{\kappa}\right)^{-1} \left(1 + \frac{\kappa e}{\kappa} \mu\right)^{-1} \left\{1 + (\Delta S_\Psi) \mu\right\}. \quad (17)$$

The Jacobian (17) coincides with (15), except for the $U$-exact term $\overrightarrow{U} = \frac{\kappa}{\kappa}|_{\phi, \lambda}$, $\overrightarrow{U}^2 = 0$, with the hypergauge $G_A = (\phi^*_{s}\phi^A + \Psi) \overrightarrow{U}$, which is $\Delta$-exact, thus making the Jacobian $J_{\mu(\phi)}$ unique.

We suggest a so-called soft nilpotency condition, $\bar{\mu}(\frac{\kappa e}{\kappa})^2 = 0$, for the parameter $\bar{\mu}(\bar{\phi})$ which transforms the set of $G_w = \{g(\mu(\phi)) : \mu(\frac{\kappa e}{\kappa})^2 = 0\}$ into a group with the Jacobian

$$J_{\bar{\mu}(\phi)} = J_{\mu(\phi)}|_{G_w} = \left(1 + \frac{\kappa e}{\kappa}\right)^{-1} \left\{1 + (\Delta S_\Psi) \mu\right\}, \quad (18)$$

being more general than in a rank-1 theory and formally identical with $J_{\mu(\phi, \lambda)}^{BV}$; see (17).

For $N = 2$ BRST transformations in YM theories, the technique of calculating the Jacobian was first examined in the case of functionally-dependent parameters $\mu_a = \Lambda(\phi)^{\frac{\kappa}{\kappa}} a$.
with an even-valued functional $\Lambda$ in [2]. The result is given by, $\phi^A \equiv \phi^A g(\Lambda(\phi)^\frac{\Lambda}{s_a})$,

$$J_{\Lambda(\phi)\pi_a} = \text{Sdet} \left| \frac{\partial A}{\partial B} \right| = \exp \left\{ \text{Str} \left( \Delta^A B + M^A \right) \right\} \text{, for } M_B^A = P_B^A + Q_B^A + R_B^A (19)$$

$$= \phi^A \delta^a_s (\mu_a \frac{\Lambda}{s_a}) \partial_B + \mu_a \left[ \phi^A \delta^a_s (\partial_B) \right] + \left[ (\phi^A \delta^a_s ) \partial_B \right] (1)^{n+1} \frac{1}{2} \mu^2 \delta^a_s (\partial_B),$$

$$\text{Str}(P + Q + R)^n = \text{Str}(P + Q)^n + C_n^1 \text{Str} P^{n-1} R, \quad \text{for } C_n^k = n!/(k!/(n-k)!), \quad (20)$$

$$\text{Str}(P + Q)^n = \left\{ \begin{array}{ll} \text{Str} P^n + n \text{Str} P^{n-1} Q + C_n^2 \text{Str} P^{n-2} Q & n = 2, 3, \\
\text{Str} P^n + n \sum_{k=0}^{n-2} \text{Str} P^n - k Q^n + K_n \text{Str} P^{n-3} Q P Q & n > 3 \end{array} \right. \quad (21)$$

$$\implies J_{\Lambda(\phi)\pi_a} = \exp \left( \sum_{n=1} (-1)^{n-1} n \text{Str}(P_B^n) \right) = (1 - \frac{1}{2} \Lambda^a_s )^{-2}, \quad (22)$$

where $K_n = \left[ \frac{n+1}{2} - 2 \right] C_n^1 + ((n+1) \mod 2) C_n^1 [2]$, with $[x]$ being the integer part of $x \in \mathbb{R}$. For functionally-independent FD parameters, $\mu_a(\phi) \neq \Lambda_{\pi_a}$, the above algorithm (19)-(22) involves a generalization of (21), examined separately for odd- and even-valued $n$, which leads to (6)

$$J_{\mu_a} = \exp \left\{ \text{tr} \left( \sum_{n=1} (-1)^{n-1} n \text{Str}(P_B^n) \right) \right\} = \exp \left\{ - \text{tr} \ln(e + m) \right\}, \quad m^a_b = \mu_b^{\pi_a}, \quad (23)$$

where $(e)^a_b$ and $\text{tr}$ denote $\delta^a_b$ and trace over $\text{Sp}(2)$ indices. The Jacobian (23) is generally not BRST-antiBRST-exact; however, it is identical at $\mu_a = \Lambda_{\pi_a}$ with $J_{\Lambda_{\pi_a}}$, due to $\text{tr} m^a_b = -\Lambda^a_s$.

In general gauge theories (12)-(14), the calculation of Jacobians induced by FD $N = 2$ BRST transformations was first carried out in [3, 5] with functionally-dependent parameters $\mu_a = \Lambda(\phi, \pi, \lambda) \hat{U}_a$, the restricted generators $\hat{U}^a = \gamma^{s=2} \phi, \pi, \lambda$, satisfying the algebra \{ \hat{U}_a, \hat{U}_b \} = 0, and then in [6] with arbitrary parameters $\mu_a(\Gamma_{tr})$, including functionally-independent $\mu_a(\phi, \pi, \lambda)$. The result is given by

$$J_{\Lambda_{\hat{U}_a}} = \text{Sdet} \left| \left[ \Gamma_{tr}, g(\Lambda_{\hat{U}_a}) \right] \right| = \exp \left[ - (\Delta^a W) \mu_a - \frac{1}{4} (\Delta^a W) \gamma^{s=2} \mu^2 \right] \left( 1 - \frac{1}{2} \Lambda^a_s \right)^{-2}, \quad (24)$$

$$J_{\mu_a(\phi, \pi, \lambda)} = \exp \left\{ - (\Delta^a W) \mu_a - \frac{1}{4} (\Delta^a W) \gamma^{s=2} \mu^2 - \text{tr} \ln(e + m) \right\}, \quad (25)$$

$$J_{\mu_a(\Gamma_{tr})} = \exp \left\{ - \text{tr} \ln(e + m) \right\} g(\mu_a(\Gamma_{tr})) \exp \left\{ - (\Delta^a W) \mu_a - \frac{1}{4} (\Delta^a W) \gamma^{s=2} \mu^2 \right\}. \quad (26)$$

The group-like element $g(\mu_a(\Gamma_{tr}))$ in (26) draws a difference between the Jacobians $J_{\mu_a(\phi, \pi, \lambda)}$ and $J_{\mu_a(\Gamma_{tr})}$, because $\gamma^{s=2}$ are not reduced to the nilpotent $\hat{U}_a$ as they act on $\Gamma^a_{tr}$. In generalized Hamiltonian formalism, the Jacobians of corresponding FD BRST-antiBRST transformations were calculated from first principles by the rules (19), (23) in [1, 6].

4 On soft nilpotency and gauge-independent Standard Model with GZ horizon

For FD parameters, finite BRST transformations allow one to obtain a new form of the Ward identity and to establish the gauge-independence of the path integral under a finite change of the gauge, $\Psi \rightarrow \Psi + \Psi'$, provided that the SB BRST symmetry term $M = M_{\Psi}$ transforms to $M_{\Psi + \Psi'} = M_{\Psi}(1 + \gamma^{s=2} \mu(\Psi'))$, with $\mu(\Psi')$ being a solution of a so-called compensation equation,

$$Z_{\Psi, M_{\Psi}}(0, \phi^*) = Z_{\Psi + \Psi', M_{\Psi + \Psi'}}(0, \phi^*) \Rightarrow \Psi' = \Psi + \lambda \mu = \frac{b}{i} \left[ \sum_{n=1} \frac{(-1)^{n-1}}{n} (\mu^{\gamma^{s=2}})^{n-1} \right] \mu, \quad (27)$$
for representations of the path integral (16). The Ward identity, depending on the FD parameter \( \mu(\Psi') = -1/g(y)\Psi' \), for \( g(z) = 1 - \exp\{z\}/z \), \( z = (i/\hbar)\Psi'^s, \) and the gauge-dependence problem are described by the respective expressions [3]

\[
\left\{ 1 + \frac{i}{\hbar} [J_A \phi^A + M] \frac{\hbar}{s} \mu(\Psi') \right\} (1 + \mu(\Psi') \frac{\hbar}{s})^{-1} \mid_{\Psi,M,\lambda} = 1 \quad \text{and} \quad \left\{ (J_A \phi^A + M) \frac{\hbar}{s} \right\}_{\Psi,M,\lambda} = 0,
\]

as one makes averaging with respect to \( Z_{\Psi,M}\phi(J,\phi^*) \). The above equations are equivalent to those of [1] in the representation (3) if we restrict ourselves by the set \( G_w \) of \( N = 1 \) BRST transformations with soft nilpotency imposed on \( \bar{\mu}(\phi) \). Indeed, from (18) and (2.40) in [1], we find the compensation equation

\[
\hbar \ln \left( 1 + \frac{\hbar}{s} \right) = \left( \exp \left\{ - \left[ \Delta, \Psi' \right] \right\} - 1 \right) S_{\psi},
\]

whose resolvability implies that its right-hand side should be

\[
\frac{\hbar}{s} - \text{closed} : \left\{ \left( \exp \left\{ - \left[ \Delta, \Psi' \right] \right\} - 1 \right) S_{\psi} \right\} = 0.
\]

For an infinitesimal change \( \Psi' \), this amounts to a soft nilpotency condition: \( \Psi' \left( \frac{\hbar}{s} \right)^2 = 0 \). For admissible changes of the gauge \( \Psi' \) satisfying this condition, the solution to (29) for an unknown \( \bar{\mu} \), with accuracy up to a total derivative (\( F \delta_A \)), has the form

\[
\bar{\mu}(\Psi'|\phi) = \frac{\Psi'}{\Psi' \frac{\hbar}{s} e} \left\{ \exp \left\{ \frac{i}{\hbar} \left( \exp \left\{ - \left[ \Delta, \Psi' \right] \right\} - 1 \right) S_{\psi} \right\} - 1 \right\}.
\]

This allows one to obtain a new form of Ward identity depending on \( \bar{\mu}(\Psi') \) and to specify gauge dependence, with similar results developed in [1].

\( N = 2 \) FD BRST transformations solve the same problem under a finite change of the gauge, \( F \rightarrow F + F' \), provided that the SB BRST-antiBRST symmetry term \( M_F \) transforms to \( M_{F+F'} = M_F (1 + \frac{\hbar}{s} a \mu_a(F) + \frac{1}{2} \frac{\hbar}{s} \mu_2(F')) \), with \( \mu_a(F', \phi, \pi, \lambda) = \Lambda \tilde{U}_a \) being a solution to the corresponding compensation equation based on (12):

\[
Z_F(0) = Z_{F+F'}(0) \Rightarrow F'(\phi, \pi, \lambda|\mu_a) = 4\hbar \left[ \sum_{n=1}^{(N-1)/2} \frac{(-1)^{n-1}}{2^n n!} \left( \Lambda \tilde{U}_a \right)^{n-1} \Lambda \right].
\]

As a result, the Ward identity with the FD parameters \( \mu_a(F') = \frac{i}{2\hbar} g(z) F' \tilde{U}_a, \Lambda(\Gamma|F') = \frac{i}{2\hbar} g(z) F' \) for \( z \equiv (i/4\hbar) F' \tilde{U}_a^2 \), acquires the form [5]

\[
\left\{ 1 + \frac{i}{\hbar} J_A \phi^A \left[ \tilde{U}_a \mu_a(\Lambda) + \frac{1}{4} \tilde{U}_a^2 \mu_2(\Lambda) \right] - \frac{1}{4} \left( \frac{i}{\hbar} \right)^2 J_A \phi^A \tilde{U}_a J_B(\phi^B) \tilde{U}_a \mu_2(\Lambda) \right\}
\]

\[
\times \left( 1 - \frac{1}{2} \Lambda \tilde{U}_a^2 \right)^{-2} \right\}_{F,J} = 1,
\]

and allows one to solve the gauge-dependence problem [5] with a source-dependent average expectation value with respect to \( Z_F(J) \), corresponding to a gauge-fixing \( F(\phi) \).

Using the \( N = 1,2 \) SB BRST symmetry term \( M(\phi) \) as the horizon functional \( H(A) \) [18],

\[
H(A) = \gamma^2 \int d^D x (d^D y f^{mnk} A^m_n(x) (K^{-1})^m_l(x,y) f^{ljk} A^l_j(y) + D(N^2-1))
\]

with the inverse Faddeev–Popov operator \( (K^{-1})^{mn}(x,z) \) in Landau gauge, the Gribov mass \( \gamma \), and gauge-independent \( Z_{H,\Psi}, Z_{H,F} \) in [27], [32], we find in a new gauge

\[
H_{\Psi}(\phi) = H(A) \left\{ 1 + \frac{\hbar}{s} \mu(\Psi') \right\} \quad \text{or} \quad H_{F'}(\phi) = H(A) \left\{ 1 + \frac{\hbar}{s} a \mu_a(F') + \frac{1}{4} \frac{\hbar}{s} \mu_2(F') \right\}.
\]
At the same time, we can suggest a new $N = 1$ or $N = 2$ BRST-invariant and gauge-independent extension of YM theory, by using a gauge-invariant horizon $H(A^h) = H(A)|_{A \rightarrow A^h}$ in terms of gauge- and BRST-invariant transverse fields $A^h_\mu = (A^h)^n T^n$, with $su(N)$ generators $T^m$ and a coupling constant $g$; see [23]:

$$A_\mu = A^h_\mu + A^L_\mu, \quad A^h_\mu = (\eta_{\mu\nu} - \frac{\partial_\nu}{\partial x^\nu}) \left( A^\nu - ig \left[ \frac{\partial A^\nu}{\partial x^\nu}, A^\nu - \frac{1}{2} \partial^\nu \frac{\partial A}{\partial x^\nu} \right] \right) + O(A^3): \quad A^h_\mu \frac{s}{\bar{s}} = 0,$$

$$H(A) = H(A^h) + \gamma^2 \int d^D x \, d^D y \, R^m(A, \partial A, \partial x^m; x, y) \partial^\mu A^m(y), \quad H(A^h) \frac{s}{\bar{s}} = 0,$$

with a non-local function $R^m(x, y)$ in [22]. The structure of the second term in $H(A)$ allows one to add it to the gauge term $B^m(\partial^\mu A^m_\mu)$ in the Faddeev–Popov action $S_0 + \Psi \frac{s}{\bar{s}}$ (or the $N = 2$ BRST action $S_F$), in such a way that the change of variables in $Z_{H, \Psi}$ is a shift, $B^m \rightarrow B^m + \gamma^2 R^m$, with the unity Jacobian completely eliminating the dependence on the SB BRST symmetry term in $Z_{H, \Psi}$. Therefore, the action

$$\hat{S}_{GZ}(\phi) = S_0 + \int d^D x \overline{\phi} \gamma \frac{s}{\bar{s}} + H(A^h), \quad \text{for } \phi \frac{s}{\bar{s}} = (D^\mu C^m - \frac{1}{2} f^{mnl} C^l C^n, B^m, 0)$$

provides the gauge-independence in the YM theory and Standard Model [6] under $\xi$-gauages, with the same Faddeev–Popov operator $(K)^{mn}(x, y)$ and unaffected $N = 1$ (with $\Psi \frac{s}{\bar{s}}$ replaced by $-\frac{1}{2} F^2 \xi^2$), in [38] for $N = 2$) BRST symmetry, for which one may expect the unitarity of the theory within the Faddeev–Popov quantization rules [24]. The same results concerning the problems of unitarity and gauge-independence may be achieved within the local formulation of Gribov–Zwanziger theory [18] when the horizon functional is localized by means of a quartet of auxiliary fields $\phi_{aux} = (\varphi^m_\mu, \varphi^m_\mu, \omega^m_\mu, \bar{\omega}^m_\mu)$, having opposite Grassmann parities, $\epsilon(\varphi, \bar{\varphi}) = \epsilon(\omega, \bar{\omega}) + 1 = 0$, and being antisymmetric in $m, n$:

$$\hat{S}_{GZ}(\phi, \phi_{aux}) = S_0(A) + \int d^D x \overline{\phi} \gamma \frac{s}{\bar{s}} + S_\gamma(A^h, \phi_{aux}),$$

$$S_\gamma = \int d^D x \left( \varphi^m_\mu K^{ml}(A^h) \varphi^{ln}_\mu - \bar{\omega}^m_\mu K^{ml}(A^h) \omega^{ln}_\mu + \gamma^2 D(\hat{N}^2 - 1) \right).$$

The part $S_\gamma$ additional to the Faddeev–Popov action is explicitly $N = 1$ BRST invariant, because of a trivial (vanishing) definition of $N = 1$ ($N = 2$) BRST transformations for the auxiliary fields: $\phi_{aux} \frac{s}{\bar{s}} = 0$ ($\phi_{aux} \frac{s}{\bar{s}} = 0$).

Notice in conclusion that $N = 1, 2$ FD BRST transformations make it possible to study their explicit influence on the Standard Model, reducible theories (such as the Friedman–Townsend model), the concept of average effective action [1, 2, 3, 5, 6], and also allow one to extend themselves to the case of $N = m$ BRST transformations for arbitrary $m > 2$, along the lines of [25].

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4These results were obtained in June 2016, but were not included in the SFP’16 Proceedings due to the limitations for a paper volume.
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