Noncommutative Sprott systems, their jerk dynamics, and their chaos synchronization by active control

Marcin Daszkiewicz

Institute of Theoretical Physics, University of Wroclaw, s.q. Max Born 9, Wroclaw 50-206, Poland
E-mail: marcin@ift.uni.wroc.pl

Abstract. In this article we remaind the 19 noncommutative Sprott models provided in paper [11] and further, we synchronize all of them by active control scheme. Particularly, we establish the proper so-called active controllers with use of the Lyapunov stabilization theory [30].

1. Introduction
In the last few decades there appeared a lot of papers dealing with so-called chaotic models, i.e., with such models whose dynamics is described by strongly sensitive with respect initial conditions, nonlinear differential equations. The most popular of them are: Lorenz system [1], Roessler system [2], Rayleigh-Benard system [3], Henon-Heiles system [4], jerk equation [5], Duffing equation [6], Lotka-Volter system [7], Liu system [8], Chen system [9] and Sprott system [10] respectively. However, the especially interesting among the above models seem to be the last ones, i.e., the Sprott systems. In fact, since they have been provided by systematic examination of three-dimensional ODEs with quadratic nonlinearities, they admit the most simple kind of chaotic dynamics listed in Table 1. Moreover, there has been introduced in article [11] the noncommutative counterpart of Sprott models defined on the quantum space of the form

\[ [t, \hat{x}_i] = 0, \quad [\hat{x}_i, \hat{x}_j] = i f_{ij}(t), \]

with arbitrary time-dependent real functions \( f_{ij}(t) \) satisfying \( f_{ij}(t) = -f_{ji}(t) \). Particularly, it has been demonstrated that such constructed systems are described by nonlinear and complex differential equations given in Table 4. Besides, the performed studies indicate that at least in the case of canonical \((f_{ij}(t) = \theta_{ij} = \text{const.})\) [12]-[15] space-time noncommutativity, the deformed Sprott models remain strongly sensitive with respect the change of initial data; i.e., it has been shown that they are in fact completely chaotic.

It is well known that one of the most important problem just of the chaos theory concerns so-called chaos synchronization phenomena. Since Pecora and Caroll [16] introduced a method to synchronize two identical chaotic systems, the chaos synchronization has received increasing attention due to great potential applications in many scientific discipline. Generally, there are known several methods of chaos synchronization, such as: OGY method [17], active control
method [18], [19], adaptive control method [20], [21], backstepping method [22], [23], sampled-data feedback synchronization method [24], time-delay feedback method [25] and sliding mode control method [26], [27]. All of them have been applied to the synchronization of many identical as well as different chaotic models, such as, for example, Sprott, Lorenz and Roessler systems respectively (see [28] and [29]).

In this article we remained the 19 noncommutative Sprott models provided in paper [11] and further, we synchronize all of them by active control scheme. Particularly, we establish the proper so-called active controllers with use of the Lyapunov stabilization theory [30].

The paper is organized as follows. In second Section we recall the main result of articles [10], [31], i.e., we provide Table 1 as well as Tables 2, 3 including chaotic flows of all 19 Sprott models and the corresponding jerk dynamics respectively. In Section 3 we review their noncommutative counterparts defined and analyzed in paper [11]. Section 4 concerns the basic concepts of active synchronization method, while in Section 5 we provide in Table 7 the active controllers, which synchronize all identical noncommutative Sprott models. The conclusions and final remarks are discussed in the last Section.

2. Sprott systems

In this Section we recall the main result of paper [10], in which there has been performed a systematic examination of general three-dimensional ODEs with quadratic nonlinearities. Particularly, it has been uncovered 19 distinct most simple examples of chaotic flows (so-called Sprott systems) listed in Table 1.

| type | 1st equation | 2nd equation | 3rd equation |
|------|--------------|--------------|--------------|
| A    | $\dot{x}_1 = x_2$ | $\dot{x}_2 = -x_1 + x_2 x_3$ | $\dot{x}_3 = 1 - x_3^2$ |
| B    | $\dot{x}_1 = x_2 x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = 1 - x_1 x_2$ |
| C    | $\dot{x}_1 = x_2 x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = 1 - x_2^2$ |
| D    | $\dot{x}_1 = -x_2$ | $\dot{x}_2 = x_1 + x_3$ | $\dot{x}_3 = x_1 x_3 + 3 x_2^2$ |
| E    | $\dot{x}_1 = x_2 x_3$ | $\dot{x}_2 = x_1^2 - x_2$ | $\dot{x}_3 = 1 - 4 x_1$ |
| F    | $\dot{x}_1 = x_2 + x_3$ | $\dot{x}_2 = -x_1 + 0.5 x_2$ | $\dot{x}_3 = x_2^2 - x_3$ |
| G    | $\dot{x}_1 = 0.4 x_1 + x_3$ | $\dot{x}_2 = x_1 x_3 - x_2$ | $\dot{x}_3 = -x_1 + x_2$ |
| H    | $\dot{x}_1 = -x_2 + x_3^2$ | $\dot{x}_2 = x_1 + 0.5 x_2$ | $\dot{x}_3 = x_1 - x_3$ |
| I    | $\dot{x}_1 = -0.2 x_2$ | $\dot{x}_2 = x_1 + x_3$ | $\dot{x}_3 = x_1 + x_3^2 - x_3$ |
| J    | $\dot{x}_1 = 2 x_3$ | $\dot{x}_2 = -2 x_2 + x_3$ | $\dot{x}_3 = -x_1 + x_2 + x_3^2$ |
| K    | $\dot{x}_1 = x_1 x_2 - x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = x_1 + 0.3 x_3$ |
| L    | $\dot{x}_1 = x_2 + 3.9 x_3$ | $\dot{x}_2 = 0.9 x_1^2 - x_2$ | $\dot{x}_3 = 1 - x_1$ |
| M    | $\dot{x}_1 = -x_3$ | $\dot{x}_2 = -x_1^2 - x_2$ | $\dot{x}_3 = 1.7 + 1.7 x_1 + x_2$ |
| N    | $\dot{x}_1 = -2 x_2$ | $\dot{x}_2 = x_1 + x_3^2$ | $\dot{x}_3 = 1 + x_2 - 2 x_1$ |
| O    | $\dot{x}_1 = x_2$ | $\dot{x}_2 = x_1 - x_3$ | $\dot{x}_3 = x_1 + x_1 x_3 + 2.7 x_2$ |
| P    | $\dot{x}_1 = 2.7 x_2 + x_3$ | $\dot{x}_2 = -x_1 + x_3^2$ | $\dot{x}_3 = x_1 + x_2$ |
| Q    | $\dot{x}_1 = -x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = 3.1 x_1 + x_2^2 + 0.5 x_3$ |
| R    | $\dot{x}_1 = 0.9 - x_2$ | $\dot{x}_2 = 0.4 + x_3$ | $\dot{x}_3 = x_1 x_2 - x_3$ |
| S    | $\dot{x}_1 = -x_1 - 4 x_2$ | $\dot{x}_2 = x_1 + x_3^2$ | $\dot{x}_3 = 1 + x_1$ |

Besides, the corresponding jerk dynamics were introduced in paper [31] by using of the proper nonlinear transformation rules of variables $x_1$, $x_2$ and $x_3$; they look as follows:
### Table 2. The jerk dynamics.

| type | jerk dynamics |
|------|---------------|
| A    |               |
| B    |               |
| C    |               |
| D    | $x_1^{(3)} = x_1 \ddot{x}_1 - \dot{x}_1 - 3\dot{x}_1^2 + x_1^2$ |
| E    |               |
| F    | $x_1^{(3)} = -0.5\ddot{x}_1 - 0.5\dot{x}_1 - 0.5x_1^2 + 2\dot{x}_1 x_1 - x_1$ |
| G    | $x_1^{(3)} = -0.6\ddot{x}_1 - 0.6\dot{x}_1 - 0.4x_1^2 + \dot{x}_1 x_1 - x_1$ |
| H    | $x_1^{(3)} = -0.5\ddot{x}_3 - 0.5\dot{x}_3 - 0.5x_3^2 + 2\dot{x}_3 x_3 - x_3$ |
| I    | $x_1^{(3)} = -\ddot{x}_1 - 0.2\dot{x}_1 - 5\dot{x}_1^2 - 0.4x_1$ |
| J    | $x_1^{(3)} = -2\ddot{x}_2 - \dot{x}_2 - 4x_2 + 2\ddot{x}_2 x_2$ |
| K    | $x_1^{(3)} = -0.7\ddot{x}_2 - 0.7\dot{x}_2 - x_2 + x_2 \ddot{x}_2 + 1.7x_2 x_2 - 0.3x_2^2 + \dot{x}_2^2$ |
| L    | $x_1^{(3)} = -\ddot{x}_1 - 3.9\dot{x}_1 - 3.9x_1 + 1.8x_1 \ddot{x}_1 + 3.9$ |
| M    | $x_1^{(3)} = -\ddot{x}_1 - 1.7\dot{x}_1 - 1.7x_1 + x_1^2 - 1.7$ |
| N    | $x_1^{(3)} = -2\ddot{x}_3 - 2\dot{x}_3 - 4x_3 + 2\ddot{x}_3 x_3 + 2$ |
| O    | $x_1^{(3)} = x_1 \ddot{x}_1 - 1.7\dot{x}_1 - x_1 - x_1^2$ |
| P    | $x_1^{(3)} = 2x_2 \ddot{x}_2 - 1.7\dot{x}_2 + 2\dot{x}_2^2 - x_2 - x_2^2$ |
| Q    | $x_1^{(3)} = -0.5\ddot{x}_2 - 2.6\dot{x}_2 - 3.1x_2 - x_2^2$ |
| R    | $x_1^{(3)} = -\ddot{x}_1 - 0.9x_1 + \dot{x}_1 x_1 - 0.4$ |
| S    | $x_1^{(3)} = -\ddot{x}_3 - 4\dot{x}_3 - 4x_3^2 + 4$ |

### Table 3. The jerk dynamics - remaining trajectories.

| type | first direction | second direction |
|------|----------------|-----------------|
| A    |               |                 |
| B    |               |                 |
| C    |               |                 |
| D    | $x_2 = -\ddot{x}_1$ | $x_3 = -\ddot{x}_1 - x_1$ |
| E    |               |                 |
| F    | $x_2 = 0.6(\ddot{x}_1 + \dot{x}_1 + x_1 - x_1^2)$ | $x_3 = 0.6(-\ddot{x}_1 + 0.5\dot{x}_1 - x_1 + x_1^2)$ |
| G    | $x_2 = \ddot{x}_1 - 0.4\dot{x}_1 + x_1$ | $x_3 = \ddot{x}_1 - 0.4x_1$ |
| H    | $x_1 = \ddot{x}_3 + x_3$ | $x_2 = -\ddot{x}_3 - \dot{x}_3 + x_3^2$ |
| I    | $x_2 = -5\ddot{x}_1$ | $x_3 = -5\ddot{x}_1 - x_1$ |
| J    | $x_1 = -\ddot{x}_2 - 2\dot{x}_2 + x_2 + x_2^2$ | $x_3 = 2\ddot{x}_2 + \dot{x}_2$ |
| K    | $x_1 = \ddot{x}_2 + x_2$ | $x_3 = -\ddot{x}_2 - \dot{x}_2 + x_2 \ddot{x}_2 + x_2^2$ |
| L    | $x_2 = -\ddot{x}_1 - 3.9x_1 + 0.9x_1^2 + 3.9$ | $x_3 = 0.2(\ddot{x}_1 + \dot{x}_1 + 3.9x_1 + -0.9x_1^2 - 3.9)$ |

3
M \quad x_2 = -\dot{x}_1 - 1.7x_1 - 1.7 \quad x_3 = -\dot{x}_1 \\
N \quad x_1 = \dot{x}_3 + 2\dot{x}_3 - x_3^2 \quad x_2 = \dot{x}_3 + 2x_3 - 1 \\
O \quad x_2 = \dot{x}_1 \quad x_3 = -\dot{x}_1 + x_1 \\
P \quad x_1 = -\dot{x}_2 + x_2^2 \quad x_3 = -\dot{x}_2 + 2x_2\dot{x}_2 - 2.7x_2 \\
Q \quad x_1 = \dot{x}_2 + x_2 \quad x_3 = -\dot{x}_2 - \dot{x}_2 \\
R \quad x_2 = -\dot{x}_1 + 0.9 \quad x_3 = -\dot{x}_1 - 0.4 \\
S \quad x_1 = \dot{x}_3 - 1 \quad x_2 = 0.25(-\dot{x}_3 - \dot{x}_3 + 1)

3. Noncommutative Sprott systems

Let us now turn to NC Sprott models defined on the quantum space-time (1). In the first step of our construction we remind (following article [11]), that the commutation relations (1) can be (formally) realized in the framework of Quantum Group Theory [32], with use so-called twist procedure [33]. Then, the quantum space (1) is represented by Hopf module equipped with the following $\star$-product for two arbitrary classical functions $f(x)$ and $g(x)$ [34]

$$f(x) \star g(y) = f(x) \exp \left( \frac{i}{2} f_{ij}(t) \partial_x^i \otimes \partial_y^j \right) g(y) .$$ (2)

Particularly, for $f(x) = x$ and $g(y) = y$ we have

$$x_i \star x_j = x_i \exp \left( \frac{i}{2} f_{kl}(t) \partial_x^k \otimes \partial_x^l \right) x_j = x_i x_j + i \frac{f_{ij}(t)}{2} ,$$ (3)

$$x_j \star x_i = x_j \exp \left( \frac{i}{2} f_{kl}(t) \partial_x^k \otimes \partial_x^l \right) x_i = x_j x_i + i \frac{f_{ji}(t)}{2} ,$$ (4)

and, consequently

$$[ \dot{x}_i, \dot{x}_j ] = [ x_i, x_j ]_\star = x_i \star x_j - x_j \star x_i = if_{ij}(t) .$$ (5)

It should be noted however, that the commutation relations (1) has been explicitly constructed only for particular form of function $f_{ij}(t)$ in papers [34], [35] and [36].

Next, we define the deformed Sprott systems by the following replacement

$$x_i x_j \rightarrow x_i \star x_j = x_i x_j + i \frac{f_{ij}(t)}{2} ; \; i \neq j ,$$ (6)

provided in all dynamical equations from Table 1. In such a way, we get:

| type | 1st equation | 2nd equation | 3rd equation |
|------|-------------|-------------|-------------|
| A    | $\dot{x}_1 = x_2$ | $\dot{x}_2 = -x_1 + x_2 \star x_3$ | $\dot{x}_3 = 1 - x_2^2$ |
| B    | $\dot{x}_1 = x_2 \star x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = 1 - x_1 \star x_2$ |
| C    | $\dot{x}_1 = x_2 \star x_3$ | $\dot{x}_2 = x_1 - x_2$ | $\dot{x}_3 = 1 - x_1^2$ |
| D    | $\dot{x}_1 = -x_2$ | $\dot{x}_2 = x_1 + x_3$ | $\dot{x}_3 = x_1 \star x_3 + 3x_2^2$ |
| E    | $\dot{x}_1 = x_2 \star x_3$ | $\dot{x}_2 = x_1^2 - x_2$ | $\dot{x}_3 = 1 - 4x_1$ |
Further, there have been also found in [11] the corresponding (noncommutative) jerk dynamics listed in the following two Tables:

| type | jerk dynamics |
|------|---------------|
| A    | $\dot{x}_1 = x_2 + x_3$ $\dot{x}_2 = -x_1 + 0.5x_2$ $\dot{x}_3 = x_1^2 - x_3$ |
| B    | $\dot{x}_1 = 0.4x_1 + x_3$ $\dot{x}_2 = x_1 \cdot x_3 - x_2$ $\dot{x}_3 = -x_1 + x_2$ |
| C    | $\dot{x}_1 = -x_2 + x_3^2$ $\dot{x}_2 = x_1 + 0.5x_2$ $\dot{x}_3 = x_1 - x_3$ |
| D    | $\dot{x}_1 = -0.2x_2$ $\dot{x}_2 = x_1 + x_3$ $\dot{x}_3 = x_1 + x_3^2 - x_3$ |
| E    | $\dot{x}_1 = 2x_3$ $\dot{x}_2 = -2x_2 + x_3$ $\dot{x}_3 = -x_1 + x_2 + x_3^2$ |
| F    | $\dot{x}_1 = x_1 \cdot x_2 - x_3$ $\dot{x}_2 = x_1 - x_2$ $\dot{x}_3 = x_1 + 0.3x_3$ |
| G    | $\dot{x}_1 = x_2 + 3.9x_3$ $\dot{x}_2 = 0.9x_3^2 - x_2$ $\dot{x}_3 = 1 - x_1$ |
| H    | $\dot{x}_1 = -3x_3$ $\dot{x}_2 = -x_1^2 - x_2$ $\dot{x}_3 = 1.7 + 1.7x_1 + x_2$ |
| I    | $\dot{x}_1 = -2x_2$ $\dot{x}_2 = x_1 + x_3^2$ $\dot{x}_3 = 1 + x_2 - 2x_1$ |
| J    | $\dot{x}_1 = 2x_2 + x_3$ $\dot{x}_2 = x_1 + x_3^2$ $\dot{x}_3 = 1 + x_1 \cdot x_3 + 2.7x_2$ |
| K    | $\dot{x}_1 = -3x_3$ $\dot{x}_2 = x_1 - x_2$ $\dot{x}_3 = 3.1x_1 + x_3^2 + 0.5x_3$ |
| L    | $\dot{x}_1 = 0.9 - x_2$ $\dot{x}_2 = 0.4 + x_3$ $\dot{x}_3 = x_1 \cdot x_2 - x_3$ |
| M    | $\dot{x}_1 = -x_1 - 4x_2$ $\dot{x}_2 = x_1 + x_3^3$ $\dot{x}_3 = 1 + x_1$ |

Table 5. The noncommutative jerk dynamics.
chaotic models. A

| type | first direction | second direction |
|------|----------------|-----------------|
| A    | x₂ = −x₁      | x₃ = −x₁ − x₁   |
| B    | x₂ = −x₁      | x₃ = −x₁ − x₁   |
| C    | x₂ = −x₁      | x₃ = −x₁ − x₁   |
| D    | x₂ = −x₁      | x₃ = −x₁ − x₁   |
| E    | x₂ = −x₁      | x₃ = −x₁ − x₁   |
| F    | x₂ = 0.6x₁    | x₃ = 0.6x₁      |
| G    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| H    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| I    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| J    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| K    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| L    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| M    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| N    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| O    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| P    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| Q    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| R    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |
| S    | x₂ = x₁ + x₂  | x₃ = x₁ + x₂    |

Of course, for functions \( f_{ij}(t) \) approaching zero the above models become classical.

4. Chaos synchronization by active control - general prescription

In this Section we remained the general scheme of chaos synchronization of two systems by so-called active control procedure \[18\], \[19\]. Let us start with the following master model

\[
\dot{x} = Ax + F(x) ,
\]

where \( x = [x₁, x₂, \ldots, xₙ] \) is the state of the system, \( A \) denotes the \( n \times n \) matrix of the system parameters and \( F(x) \) plays the role of the nonlinear part of the differential equation (7). The slave model dynamics is described by

\[
\dot{y} = By + G(y) + u ,
\]

with \( y = [y₁, y₂, \ldots, yₙ] \) being the state of the system, \( B \) denoting the \( n \)-dimensional quadratic matrix of the system, \( G(y) \) playing the role of nonlinearity of the equation (8) and \( u = [u₁, u₂, \ldots, uₙ] \) being the active controller of the slave model. Besides, it should be mentioned that for matrices \( A = B \) and functions \( F = G \) the states \( x \) and \( y \) describe two identical chaotic systems. In the case \( A \neq B \) or \( F \neq G \) they correspond to the two different chaotic models.

Let us now provide the following synchronization error vector

\[
e = y - x ,
\]

\( \text{(9)} \)
which in accordance with (7) and (8) obeys
\[ \dot{e} = By - Ax + G(y) - F(x) + u \, . \tag{10} \]

In active control method we try to find such a controller \( u \), which synchronizes the state of the master system (7) with the state of the slave system (8) for any initial condition \( x_0 = x(0) \) and \( y_0 = y(0) \). In other words, we design a controller \( u \) in such a way that for system (10) we have
\[ \lim_{t \to \infty} ||e(t)|| = 0 \, , \tag{11} \]
for all initial conditions \( e_0 = e(0) \). In order to establish the synchronization (10) we use the Lyapunov stabilization theory [30]. It means, that if we take as a candidate Lyapunov function of the form
\[ V(e) = e^T Pe \, , \tag{12} \]
with \( P \) being a positive \( n \times n \) matrix, then we wish to find the active controller \( u \) so that
\[ \dot{V}(e) = -e^T Qe \, , \tag{13} \]
where \( Q \) is a positive definite \( n \times n \) matrix as well. Then the systems (7) and (8) remain synchronized.

5. Chaos synchronization of identical noncommutative Sprott systems
The described in pervious Section algorithm can be applied to the case of all deformed Sprott systems. The obtained results are summarized in Table 7, i.e., there are listed controllers \( u_1 \), \( u_2 \) and \( u_3 \) for which the proper identical noncommutative Sprott systems become synchronized for arbitrary initial conditions \( x_1(0), x_2(0) \) and \( x_3(0) \) as well as \( y_1(0), y_2(0) \) and \( y_3(0) \).

| type | 1st controller | 2nd controller | 3rd controller |
|------|----------------|----------------|----------------|
| A    | \(-e_1 + e_2\) | \(e_1 - e_2 - y_2y_3 + x_2x_3\) | \(e_2(y_2 + x_2) - e_3\) |
| B    | \(x_2x_3 - y_2y_3 - e_1\) | \(-e_1\) | \(y_1y_2 - x_1x_2 - e_3\) |
| C    | \(x_2x_3 - y_2y_3 - e_1\) | \(-e_1\) | \((x_1 + y_1)e_1 - e_3\) |
| D    | \(e_2 - e_1\) | \(-e_1 + e_2 + e_3\) | \(x_1x_3 - 3(y_2 + x_2)\) |
| E    | \(x_1x_3 - y_1y_3 - e_1\) | \(-e_1 + x_1\) | \(4e_1 - e_2\) |
| F    | \(-e_1 + e_2 + e_3\) | \(e_1 - 1.5e_2\) | \(-e_1\) |
| G    | \(-1.4e_1 + e_3\) | \(x_1x_3 - y_1y_3\) | \(e_1 - (e_2 + e_3)\) |
| H    | \(e_2 - y_3 + x_3\) | \(-e_1 + 1.5e_2\) | \(-e_1\) |
| I    | 0.2e_2 - e_1 | \(-e_1 + e_2 + e_3\) | \(-e_1 + (y_2 + x_2)\) |
| J    | \(-e_1 + 2e_3\) | \(e_2 - e_3\) | \(e_1 - (1 + y_2 + x_3)\) |

Table 7. The active controllers for noncommutative Sprott systems.
6. Final remarks

In this article we remind the 19 noncommutative Sprott models provided in paper [11] and further, we synchronize all of them by active control scheme. Particularly, we find the corresponding so-called active controllers listed in Table 7.

It should be noted that the presented investigations can be extended in various ways. For example, one may perform synchronization of noncommutative Sprott models with use of another mentioned in Introduction methods. Besides, it seems senseable to consider the Lie-algebraically deformed counterpart of Sprott systems defined on the quantum space-time with two spatial directions commuting to space. The works in these directions already started and are in progress.

References

[1] Lorenz E N 1963 Deterministic Nonperiodic Flow J. Atmos. Sci. 20 130
[2] Roessler O E 1976 An equation for continuous chaos Phys. Lett. A 57 397
[3] Getling A V 1998 Rayleigh-Benard Convection: Structures and Dynamics (Singapore: World Scientific)
[4] Henon M and Heiles C 1964 The applicability of the third integral of motion: some numerical experiments A.J. 69 73
[5] Sprott J C 1997 Some simple chaotic jerk functions Am. J. Phys. 65 537
[6] Duffing G 1918 Erzwungene Schwingungen bei Veranderlicher Eigenfrequenz (Braunschweig: F. Vieweg u. Sohn)
[7] Volterra V 1926 Variations and fluctuations of the number of individuals in animal species living together. In Animal Ecology (New York: McGraw-Hill)
[8] Liu C, Liu T, Liu L and Liu K 2004 A new chaotic attractor Chaos, Solitons and Fractals 22 1031
[9] Chen G and Ueta T 1999 Yet another chaotic attractor Journal of Bifurcation and Chaos 9 1465
[10] Sprott J C 1994 Some simple chaotic flows Phys. Rev. E 50 647
[11] Daszkiewicz M 2018 Noncommutative Sprott systems and their jerk dynamics Mod. Phys. Lett. A 33 1850100
[12] Oeckl R 1999 Classification of differential calculi on $U_q(b_+)$, classical limits, and duality J. Math. Phys. 40 3588
[13] Douglas M R and Nekrasov N A 2001 Noncommutative field theory Rev. Mod. Phys. 73 977
[14] Szabo R 2003 Quantum field theory on noncommutative spaces Phys. Rep. 378 207
[15] Chaichian M, Kulish P P, Nohsijima K and Tureanu A 2004 On a Lorentz-invariant interpretation of noncommutative space-time and its implications on noncommutative QFT Phys. Lett. B 604 98
[16] Pecora L M and Carroll T L 1990 Synchronization in chaotic systems Phys. Rev. Lett 64 821
[17] Ott E, Grebogi C and Yorke J A 1990 Controlling chaos Phys. Rev. Lett 64 1196
[18] Ho M C and Hung Y C 2002 Synchronization of two different chaotic systems by using generalized active control Phys. Lett. A 301 424
[19] Chen H K 2005 Global Chaos Synchronization of New Chaotic Systems via Nonlinear Control Chaos, Solitons and Fractals 23 1245
[20] Liao T L and Tsai S H 2000 Adaptive synchronization of chaotic systems and its application to secure communication Chaos, Solitons and Fractals 11 1387
[21] Sundarapandian V 2011 Adaptive control and synchronization of Lius four-wing chaotic system with cubic nonlinearity Int. Journ. of Computer Science, Engineering and Applications 1 108
[22] Yu Y G and Zhang S C 2006 Adaptive backstepping synchronization of uncertain chaotic systems Chaos, Solitons and Fractals 27 1369
[23] Wu X and Lu J 2003 Parameter identification and backstepping control of uncertain Lu system Chaos, Solitons and Fractals 18 721
[24] Yang T and Chua L O 1999 Generalized synchronization of chaos via linear transformations Int. J. Bifur. Chaos 9 215
[25] Park J H and Kwon O M 2003 A Novel Criterion for Delayed Feedback Control of Time-delay Chaotic Systems Chaos, Solitons and Fractals 17 709
[26] Feki M 2009 Sliding mode control and synchronization of chaotic systems with parametric uncertainties Chaos, Solitons and Fractals 41 1390
[27] Sundarapandian V 2011 Global Chaos Synchronization of the Pehlivan Systems by Sliding Mode Control Int. Journ. of Comp. Science and Eng. 3 2163
[28] Xu D 2010 Chaos synchronization between two different Sprott systems Adv. Theor. Appl. Mech. 3 195
[29] Bai E and Longren K E 1997 Synchronization of two Lorenz systems using active control Chaos, Solitons and Fractals 8 51
[30] Lyapunov A M 1966 Stability of Motion (New-York/London: Academic Press)
[31] Eichhorn R, Linz S J and Hanggi P 1998 Transformations of nonlinear dynamical systems to jerky motion and its application to minimal chaotic flows Phys. Rev. E 58 7151
[32] Majid S 2000 Foundations of quantum group theory (Cambridge: Cambridge University Press)
[33] Reshetikhin N Yu 1990 Multiparameter quantum groups and twisted quasitriangular Hopf algebras Lett. Math. Phys. 20 331
[34] Oeckl R 2000 Untwisting Noncommutative $R^d$ and the Equivalence of Quantum Field Theories Nucl. Phys. B 581 559
[35] Daszkiewicz M 2009 Twist deformations of Newton-Hooke Hopf algebras Mod. Phys. Lett. A 24 1325
[36] Daszkiewicz M 2008 Canonical and Lie-algebraic twist deformations of Galilei algebra Mod. Phys. Lett. A 23 505