The b-quark and Symmetries of the Strong Interaction

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abstract

Applications of HQET and NRQCD to fragmentation are briefly reviewed. The special role of the b-quark in applications of heavy quark symmetry is discussed. Predictions of HQET for semileptonic B decays to excited charmed mesons are considered.

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1 HQET

The heavy quark effective theory (HQET) is a limit of the theory of the strong interactions appropriate for hadrons containing a single heavy quark $Q$. In such hadrons the light degrees of freedom typically have momentum of order $\Lambda_{QCD}$. Interactions of the heavy quark with the light degrees of freedom cause changes in its four-velocity $v$ of order $\Delta v \sim \Lambda_{QCD}/m_Q$. Consequently for these hadrons it is a reasonable approximation to take the limit of QCD where $m_Q \to \infty$ with the heavy quark’s four-velocity fixed.

The part of the QCD Lagrange density involving the heavy quark field is

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q.$$  \hfill (1)

The QCD heavy quark field is related to its HQET counterpart by

$$Q = e^{-im_{Q}v \cdot x} \left[ 1 + \frac{i\not{D}}{2m_Q} + \ldots \right] Q_v,$$  \hfill (2)

where

$$\not{v}Q_v = Q_v.$$  \hfill (3)

Putting Eq. (2) into the QCD Lagrange density and using eq. (3) yields

$$\mathcal{L} = \mathcal{L}_{HQET} + \delta_1 \mathcal{L} + \ldots,$$  \hfill (4)

where the HQET Lagrange density is [1]

$$\mathcal{L}_{HQET} = \bar{Q}_v iv \cdot DQ_v.$$  \hfill (5)

If there are several heavy flavors a sum over different flavors of heavy quarks is understood. This Lagrange density is independent of the heavy quark mass and spin and has the spin-flavor symmetry [2] of HQET. $\delta_1 \mathcal{L}$ contains corrections to the $m_Q \to \infty$ limit suppressed by a single power of the heavy quark mass. Explicitly [3]

$$\delta_1 \mathcal{L} = \frac{1}{2m_Q} [O_{kin,v}^{(Q)} + O_{mag,v}^{(Q)}],$$  \hfill (6)

where the kinetic energy term is

$$O_{kin,v}^{(Q)} = \bar{Q}_v(iD_\perp)^2 Q_v.$$  \hfill (7)
Here, $D^\mu_\perp = D^\mu - v^\mu (v \cdot D)$ are the components of the covariant derivative perpendicular to the four-velocity. The chromomagnetic energy term is

$$O^{(Q)}_{\text{mag},v} = \bar{Q}_v g \frac{g}{2} \sigma_{\alpha\beta} G^{\alpha\beta A} T^A Q_v.$$  \hspace{1cm} (8)$$

Note that the part of $\delta_1 \mathcal{L}$ involving $O^{(Q)}_{\text{kin},v}$ breaks the flavor symmetry but not the spin symmetry. $O^{(Q)}_{\text{mag},v}$ breaks both symmetries.

In the limit $m_Q \to \infty$ the angular momentum of the light degrees of freedom,

$$\vec{S}_\ell = \vec{J} - \vec{S}_Q,$$

is conserved \[4\]. Therefore, in this limit, hadrons occur in doublets with total angular momentum

$$j_\pm = s_\ell \pm 1/2.$$  

Here $\vec{J}^2 = j(j+1)$ and $\vec{S}_\ell^2 = s_\ell (s_\ell + 1)$. In the case of mesons with $Q\bar{q}$ flavor quantum numbers, the ground state doublet has spin-parity of the light degrees of freedom $s_\ell^{\pi_\ell} = \frac{3}{2}^-$. For $Q = c$ this doublet contains the $D$ and $D^*$ mesons with spin 0 and 1 respectively and for $Q = b$ they are the $B$ and $B^*$ mesons. An excited doublet of mesons with $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ has also been observed. In the $Q = c$ case this doublet contains the $D_1(2420)$ and $D_2^*(2460)$ with spin 1 and spin 2 respectively. The analogous $Q = b$ mesons are called $B_1$ and $B_2^*$.  

2 NRQCD

For quarkonia (i.e., $Q\bar{Q}$ hadrons) physical properties are usually predicted using an expansion in $v/c$ where $v$ is the magnitude of the heavy quarks’ relative velocity and $c$ is the speed of light \[3\]. So the appropriate limit of QCD to take in this case is the $c \to \infty$ limit \[1\]. In eq. (1) the speed of light was set to unity. Making the factors of $c$ explicit it becomes

$$\mathcal{L} = c\bar{Q}(i\not{D} - m_Q c)Q,$$

where

$$\partial_0 = \frac{1}{c} \frac{\partial}{\partial t},$$  

and the covariant derivative

$$D_\mu = \partial_\mu + \frac{ig}{c} A^A_\mu T^A.$$  

\hspace{1cm} (12)$$
Note that the strong coupling $g$ has the same units as $\sqrt{c}$. The full QCD heavy quark field $Q$ is related to its NRQCD counterpart by

$$Q = e^{-i m_Q c^2 t} \left[ 1 + \frac{i \mathcal{P}_ \perp}{2m_Q c} + \ldots \right] \begin{pmatrix} \psi \\ 0 \end{pmatrix}, \quad (13)$$

where $\psi$ is a two component Pauli spinor and $D_ \perp = (0, \mathbf{D}_ \perp)$. Putting eq. (13) into eq. (10) gives

$$\mathcal{L} = \mathcal{L}_{NRQCD} + \ldots, \quad (14)$$

where

$$\mathcal{L}_{NRQCD} = \bar{\psi} \left( i \left( \frac{\partial}{\partial t} + i g A_0 A^A \right) + \frac{\vec{\nabla}^2}{2m_Q} \right) \psi. \quad (15)$$

The $c \to \infty$ limit of QCD is called non-relativistic quantum chromodynamics (NRQCD). Since the kinetic energy appears as a leading term in NRQCD this theory does not have a heavy quark flavor symmetry; however, it still has a heavy quark spin symmetry. The gluon field $A_0$ in eq. (15) is not a propagating field. It gives rise to a Coulomb potential between the heavy quarks. All the interactions of the propagating transverse gluons with the heavy quarks are suppressed by powers of $1/c$. The leading interaction of the propagating transverse gluons with the heavy quarks is also invariant under heavy quark spin symmetry.

# 3 Special Role of the Bottom Quark

The $c, b$ and $t$ quarks can be considered heavy. Unfortunately the top is so heavy that it decays before forming a hadron. Heavy quark symmetry is not a useful concept for the $t$-quark. The charm quark mass is not large enough for one to be confident that predictions based on heavy quark symmetry will work well. For charmonium $v^2/c^2 \sim 1/3$ and $\Lambda_{QCD}/m_c \sim 1/7$. However, for the b-quark, corrections to predictions based on heavy quark symmetry should be small. This “special role” of the b-quark is illustrated nicely by comparing with experiment the predictions of heavy quark symmetry for fragmentation.

Heavy quark symmetry implies that the probability $P_{h_Q \to h_s}^{(H)}$ for heavy quark $Q$ with spin along the fragmentation axis (i.e., helicity) $h_Q$ to fragment to a hadron $H$ with spin of the light degrees $s_\ell$, total spin $s$ and helicity $h_s$ is

$$P_{h_Q \to h_s}^{(H)} = P_{Q \to s_\ell | q} \langle \langle s_Q, h_Q; s_\ell, h_\ell | s, h_s \rangle \rangle^2. \quad (16)$$

In eq. (16) $P_{Q \to s_\ell}$ is the probability for the heavy quark to fragment into the doublet with spin of the light degrees of freedom $s_\ell$. $p_{h_\ell}$ is the probability for the helicity of the light...
degrees of freedom to be \( h_\ell = h_s - h_Q \), given that the heavy quark fragments to this doublet. Parity invariance of the strong interactions implies that

\[
p_{h_\ell} = p_{-h_\ell},
\]

(17)

and the definition of a probability implies that

\[
\sum_{h_\ell} p_{h_\ell} = 1.
\]

(18)

The constraints in eqs. (18) and (17) imply that there are \( s_\ell - 1/2 \) independent probabilities \( p_{h_\ell} \).

For the \( c\bar{q} \) ground state meson doublet \( p_{1/2} = p_{-1/2} = 1/2 \) and the relative fragmentation probabilities are

\[
P^{(D)}_{1/2 \rightarrow 0} : P^{(D^*)}_{1/2 \rightarrow 1} : P^{(D^*)}_{1/2 \rightarrow -1} = \frac{1}{4} : \frac{1}{2} : \frac{1}{4} : 0
\]

(19)

For the excited \( s_\ell^\pi = \frac{3}{2}^+ \) doublet the relative fragmentation probabilities can be expressed using eq. (16) in terms of \( w_{3/2} \). This parameter is defined by \( p_{3/2} = p_{-3/2} = (1/2) w_{3/2} \) and \( p_{1/2} = p_{-1/2} = (1/2)(1 - w_{3/2}) \).

In the charm system only part of eq. (19) is in agreement with experiment. While the experimental value for the relative probability to fragment to longitudinal and transverse \( D^* \) helicities agrees with eq. (19), the experimental values for the probabilities to fragment to \( D \) and \( D^* \) are approximately equal \[7\] instead of in the ratio 1:3 that eq. (19) predicts. This discrepancy is probably due to the \( D^*-D \) mass difference which suppresses fragmentation to the \( D^* \). Recent LEP data shows that predictions for fragmentation based on heavy quark symmetry work better in the b-quark case \[8\]. The experimental value for the probabilities to fragment to the \( B \) and \( B^* \) are in the ratio 1:3 .

Experimental information on \( D^{**} \) production provides the bound, \( w_{3/2} < 0.24 \[7\]. It would be very interesting to have an experimental determination of the Falk-Peskin fragmentation parameter \( w_{3/2} \).

Heavy quark spin symmetry also makes predictions for the alignment of quarkonia produced by gluon fragmentation. At leading order \( v/c \) the gluon fragments to \( Q\bar{Q} \) in a color singlet configuration. Two hard gluons occur in the final state to conserve color and charge conjugation , giving a fragmentation probability to \( ^3S_1 \) quarkonia of order \( (\alpha_s(m_Q)/\pi)^3(v/c)^3 \). However, a term higher order in \( v/c \) is much more important because it is lower order in
\( \alpha_s(m_Q)/\pi \). The gluon can fragment to the \( Q\bar{Q} \) pair in a color octet with two soft propagating NRQCD gluons in the final state (each with typical momentum of order \( m_Qv/v/c \) in the quarkonium rest frame). This color octet process \([4]\) gives a contribution to the \( ^3S_1 \) fragmentation probability of order \( (\alpha_s(m_Q)/\pi)(v/c)^7 \). The fragmenting gluon has large energy (compared with \( m_Q \)) and is almost real. Real gluons are transversely aligned. Because the leading interactions of the NRQCD propagating gluons preserve spin symmetry the final state \( ^3S_1 \) quarkonium is also transversely aligned \([10]\). (There are \( \alpha_s(m_Q) \) and \( v/c \) corrections \([11]\) that reduce this alignment.) It may be possible to test this prediction in the \( Q=c \) case from large \( p_\perp \) data on \( J/\psi \) and \( \psi' \) production at the Tevatron \([12]\).

4 \( B \rightarrow D_1(2420)e\bar{\nu}_e \) and \( B \rightarrow D^*_2(2460)e\bar{\nu}_e \) Decay

Semileptonic B decays have been extensively studied. The semileptonic decays \( B \rightarrow De\bar{\nu}_e \) and \( B \rightarrow D^*e\bar{\nu}_e \) have branching ratios of \((1.8 \pm 0.4)\% \) and \((4.6 \pm 0.3)\% \) respectively \([13]\). They amount to about 60% of the semileptonic decays. The differential decay rates are determined by matrix elements of the \( b \rightarrow c \) weak axial-vector and vector currents. These matrix elements are usually written in terms of Lorentz scalar form factors and the differential decay rates are expressed in terms of them. For comparisons with the predictions of HQET it is convenient to write the form factors in terms of \( w = v \cdot v' \). In the limit \( m_Q \rightarrow \infty \) heavy quark spin symmetry implies that all six form factors can be written in terms of a single function of \( w \) \([2]\). Furthermore, heavy quark flavor symmetry implies that this function is normalized to unity \([2, 14]\) at zero recoil, \( w = 1 \). The success of these predictions \([15]\) indicates that in this case treating the charm quark mass as large is a reasonable approximation. At order \( 1/m_{c,b} \) several new functions occur but the normalization of the zero recoil matrix elements is preserved.

In the \( m_Q \rightarrow \infty \) limit zero recoil matrix elements of the weak axial vector and vector currents from the B-meson to any excited charmed meson vanish because of heavy quark spin symmetry. Since most of the phase space for such decays is near zero recoil (e.g., for B decay to the \( s^\pi_+ = \frac{3}{2}^+ \) mesons \( D_1(2420) \) and \( D^*_2(2460), 1 < w < 1.3 \)) the \( \Lambda_{QCD}/m_{c,b} \) corrections are very important.

The decay \( B \rightarrow D_1e\bar{\nu}_e \) has been observed. CLEO and ALEPH, respectively, find the branching ratios \([16]\) \( Br(B \rightarrow D_1e\bar{\nu}_e) = (0.49 \pm 0.14)\% \) and \((0.74 \pm 0.16)\% \). For \( Br(B \rightarrow D^*_2e\bar{\nu}_e) \) there are only upper limits.
The form factors that parametrize the $B \to D_1$ and $B \to D^*_2$ matrix elements of the weak currents $V^\mu = \bar{c}\gamma^\mu b$ and $A^\mu = \bar{c}\gamma^\mu\gamma^5 b$ are defined by

\[
\begin{align*}
\langle D_1(v', \varepsilon) | V^\mu | B(v) \rangle / \sqrt{m_{D_1} m_B} &= f_{V_1} \varepsilon^\mu + (f_{V_2} v^\mu + f_{V_3} v'^\mu)(\varepsilon^\ast \cdot v), \\
\langle D_1(v', \varepsilon) | A^\mu | B(v) \rangle / \sqrt{m_{D_1} m_B} &= i f_A \varepsilon^{\mu\alpha\beta\gamma} \varepsilon^\ast_{\alpha\beta} v_\gamma^\prime, \\
\langle D^*_2(v', \varepsilon) | A^\mu | B(v) \rangle / \sqrt{m_{D^*_2} m_B} &= k_{A_1} \varepsilon^{\mu\alpha} v_\alpha + (k_{A_2} v^\mu + k_{A_3} v'^\mu)\varepsilon^\ast_{\alpha\beta} v^\sigma v^\beta, \\
\langle D^*_2(v', \varepsilon) | V^\mu | B(v) \rangle / \sqrt{m_{D^*_2} m_B} &= i k_V \varepsilon^{\mu\alpha\beta\gamma} \varepsilon^\ast_{\alpha\sigma} v^\sigma v^\prime_\gamma.
\end{align*}
\]

(20)

The form factors $f_i$ and $k_i$ are functions of $w$. In the $m_{c,b} \to \infty$ limit they can be written in terms of a single function $\tau(w)$ [17],

\[
\begin{align*}
\sqrt{6} f_A &= -(w+1)\tau, \quad k_V = -\tau, \\
\sqrt{6} f_{V_1} &= (1-w^2)\tau, \quad k_{A_1} = -(1+w)\tau, \\
\sqrt{6} f_{V_2} &= -3\tau, \quad k_{A_2} = 0, \\
\sqrt{6} f_{V_3} &= (w-2)\tau, \quad k_{A_3} = \tau.
\end{align*}
\]

(21)

Only the form factor $f_{V_1}$ contributes at zero recoil. Surprisingly one can predict its value [18]

\[
\sqrt{6} f_{V_1}(1) = -\frac{4(\bar{\Lambda}' - \bar{\Lambda})\tau(1)}{m_c},
\]

(22)

in terms of the $m_{c,b} \to \infty$ Isgur–Wise function $\tau$ and the difference between the mass of the light degrees of freedom in the excited $s_\pi^\ast = \frac{3}{2}^+$ doublet $\bar{\Lambda}'$ and the mass of the light degrees of freedom in the ground state doublet $\bar{\Lambda}$. Experimentally the difference $\bar{\Lambda}' - \bar{\Lambda} \simeq 0.39$ GeV. (It can be expressed in terms of measured hadron masses.) A detailed discussion of the $1/m_{c,b}$ corrections to these decays can be found in Refs. [18]. They enhance the rate to $B \to D_1 e\bar{\nu}_e$ (compared with the $m_{c,b} \to \infty$ limit) and lead to the expectation that its branching ratio is greater than that for $B \to D^*_2 e\bar{\nu}_e$. This may explain why semileptonic decays to the $D^*_2$ have not been observed.

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