Natural top-bottom mass hierarchy in composite Higgs models

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We consider composite two-Higgs doublet models based on gauge-Yukawa theories with strongly interacting fermions generating the top-bottom mass hierarchy. The model features a single “universal” Higgs-Yukawa coupling, $g$, which is identified with the top quark $g \equiv g_t \sim \mathcal{O}(1)$. The top-bottom mass hierarchy arises by soft breaking of a $Z_2$ symmetry by a condensate of strongly interacting fermions. A mass splitting between vectorlike masses of the confined technifermions controls this top-bottom mass hierarchy. This mechanism can be present in a variety of models based on vacuum misalignment. For concreteness, we demonstrate it in a composite two-Higgs scheme.

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I. INTRODUCTION

The relative hierarchy of the top mass to the other standard model (SM) fermion masses has long been of interest [1–10]. The mass of the top quark is of the same order as the electroweak symmetry breaking (EWSB) scale $v_{EW} = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, while in some “leading order” the other fermion masses are small. In the SM, the observed mass spectrum of the fermions is achieved by inputting a hierarchy of the Higgs-Yukawa (HY) coupling constants, and while technically natural, no dynamical rationale for the hierarchy is offered.

In this paper we explore a mechanism for generating the top-bottom mass hierarchy with a universal HY coupling constant $g \equiv g_t = g_b \sim \mathcal{O}(1)$. Our background interest is multi-Higgs models, composites due to something like a new strong force or gravity, which is flavor blind and generates a large spectrum with universal HY couplings. The resulting fermion mass hierarchies and Cabibbo-Kobayashi-Maskawa (CKM) matrix are then relegated to the problem of the multi-Higgs masses and mixings [9,10]. We explore a realization of this in a composite two-Higgs doublet models (2HDM) with soft breaking of an associated $Z_2$, and based on a new strong dynamics. Here the Higgs bosons arise as pseudo-Nambu-Goldstone bosons (pNGB) from a spontaneously broken global symmetry. The genealogy of Higgses-as-pNGBs includes “composite Higgs” (CH) [11], “partially composite Higgs” (pCH) [12–14], “little Higgs” [15,16], “holographic extra dimensions” [17,18], “twin Higgs” [19] and “elementary Goldstone Higgs models” [20].

We start with two composite Higgs doublets $H_t$ and $H_b$ that are, respectively, even and odd under a discrete $Z_2$ symmetry. The Higgs doublet $H_t$ is identified with the observed SM doublet, where its neutral, electroweak (EW) vacuum expectation value (VEV). A small “tadpole” VEV can be induced in $H_b^0$ by introducing a $H_t - H_b$ mixing term that softly breaks $Z_2$ symmetry.

By demanding the left- and right-handed top and bottom quarks transform as $Q_{L,3} \equiv (t_L, b_L) \rightarrow Q_{L,3}, t_R \rightarrow t_R$ and $b_R \rightarrow -b_R$ under the $Z_2$ symmetry, the $H_b^0$ only couples to the top quark while $H_b^0$ only to the bottom quark. The HY couplings are generated by effective operators involving the quarks and the composite Higgs doublets with one universal HY coupling constant, $g \sim \mathcal{O}(1)$. The mass hierarchy $m_t/m_b \approx 40$ is thus generated naturally without fine-tuning.

For concreteness, we consider the minimal composite 2HDM fulfilling the above requirements, the SU(6)/Sp(6) model of [21]. In this model, the top-bottom mass hierarchy can be controlled by a mass splitting in the explicit vectorlike masses of the confining fermions. This mass splitting also leads to a heavy isodoublet $H_b^0 \sim \mathcal{O}(1)$ TeV, which is possibly observable at LHC or its upgrades. The model dynamically generates the top-bottom mass hierarchy and simultaneously alleviates the hierarchy problem of EWSB.

Finally, we will briefly consider ideas to future studies such as: (i) making an UV complete theory for these
models that explains the origin of the $Z_2$ symmetry breaking term and the effective Higgs-Yukawa terms, and (ii) extending this kind of models such that they also include the mass hierarchy of the other SM fermions.

II. COMPOSITE TWO-HIGGS DOUBLET MODELS WITH $Z_2$ SYMMETRY

We presently focus on CH models with misalignment based on an underlying gauge description of strongly interacting fermions (technifermions). Different chiral symmetry breaking patterns in these CH models are discussed in Refs. [22,23], and we note the following minimal cosets with a Higgs candidate and custodial symmetry: SU(4)/Sp(4) [24], SU(5)/SO(5) [25], SU(6)/Sp(6) [21], SU(6)/SO(6) [26], and SU(4) × SU(4)/SU(4) [27]. Two composite Higgs doublets and a $Z_2$ symmetry are present in the three latter cases [21,26,28], where the coset SU(6)/Sp(6) generates the minimal number of pNGBs that simultaneously fulfills our requirements. With an unbroken $Z_2$ symmetry, this kind of model may also provide (a)symmetric dark matter candidates [29]. Note that our proposal is rather general because the above requirements can also be fulfilled in other realizations that do not have a simple gauge-fermion underlying description, e.g., the models in Refs. [30,31], and they can also be fulfilled in fundamental realizations.

We remind the reader that the basic idea of natural composite Higgs models, based on vacuum misalignment [32,33], is that the SM Higgs doublet will be identified as composite pNGBs. These appear in the cosets G/H of global symmetry groups. The model consists of $N_f$ new Weyl technifermions which form a representation of a new strongly interacting “technicolor” gauge group $G_{TC}$. The choice of either real, pseudoreal or a complex representation, determines the breaking pattern of a global symmetry G to H. The model is constructed such that the EW gauge symmetry SU(2)$_L$ × U(1)$_Y$ is contained in H for a certain alignment, known as the EW unbroken alignment limit.

This particular alignment, however, is not stable because there exists an explicit breaking of G in the form of gauge interactions, top couplings to the strong sector, and explicit masses for the technifermions, etc. The EW gauge interactions, the SM fermion couplings to the strong sector and the explicit masses of the technifermions contribute to the effective Higgs potential. These terms are responsible for generating a VEV of the Higgs and corresponds to a misalignment of the vacuum. We describe this by an angle, sin $\theta = v_{EW}/(2\sqrt{2} f)$ [11], where $v_{EW} = 246$ GeV and $f$ is the decay constant of the pNGBs depending on the confinement of the underlying strong dynamics. From electroweak precision measurements [34,35] this angle can be fixed typically to sin $\theta \lesssim 0.2$, which also fixes $2\sqrt{2} f \gtrsim 1.2$ TeV, however, it may also be allowed for smaller scales [36,37].

If we assume that the VEV hierarchy of the Higgs doublets only arises from vacuum misalignment with two separate angles, $\sin \theta_1 = v_{EW}/(2\sqrt{2} f)$ and $\sin \theta_2 = v_b/(2\sqrt{2} f)$, then we require a very fine-tuned angle $\theta_b \ll \theta_1 \lesssim 0.2$. We assume therefore the vacuum is only misaligned along the SM Higgs direction while a smaller tadpole VEV of the second doublet is provided by breaking the $Z_2$ symmetry. The coset structure can be schematically represented by a $N_f \times N_f$ matrix,

$$
\begin{pmatrix}
G_0/H_0 & Z_2\text{-odd} \\
Z_2\text{-odd} & pNGBs
\end{pmatrix}.
$$

where $G_0/H_0$ is one of the two minimal cosets SU(4)/Sp(4) or SU(5)/SO(5), with one composite Higgs doublet. The $Z_2$ symmetry can be understood in terms of the underlying technifermions $\psi_i$, $i = 1, \ldots N_f$, that condense: $\psi_{5,\ldots N_f}$ are $Z_2$-odd while the technifermions that participate to the minimal coset are $Z_2$-even. Among the $Z_2$-odd pNGBs must be contained the $Z_2$-odd Higgs doublet $H_b$.

III. A CONCRETE COMPOSITE 2HDM

In the following, we focus on the concrete SU(6)/Sp(6) model [21] as a template for this mechanism. We assume four Weyl fermions are arranged in SU(2)$_L$ doublets, $\Psi_1 \equiv (\psi_1, \psi_2)^T$ and $\Psi_2 \equiv (\psi_5, \psi_6)^T$, and two in SU(2)$_L$ singlets, $\psi_{3,4}$, with hypercharges $\mp 1/2$. We have listed in Table I the representations of the gauge groups and parity of the fermions of this model.

By assuming the Weyl fermions are in fundamental representation of the new strongly interacting gauge group $G_{TC} = SU(2)^{\text{TC}}$ or Sp(N)$^{\text{TC}}$, which is the pseudoreal representation, we can construct an SU(6) flavor multiplet by arranging the six Weyl fermions into an SU(6) vector $\Psi \equiv (\psi^1, \psi^2, \psi^3, \psi^4, \psi^5, \psi^6)^T$. This results in the chiral symmetry breaking SU(6) → Sp(6) when the

TABLE I. The technifermions and the quarks in the SU(6)/Sp(6) template model labeled with their representations of $G_{TC} \times SU(3)_c \times SU(2)_L \times U(1)_Y$ and parity under the $Z_2$ symmetry.

| $G_{TC}$ | SU(3)$_C$ | SU(2)$_L$ | U(1)$_Y$ | Z$_2$ |
|----------|----------|----------|----------|-------|
| $\psi_1 \equiv (\psi_1, \psi_2)^T$ | $\Box$ | $1$ | $\Box$ | $0$ | $+1$ |
| $\psi_3$ | $\Box$ | $1$ | $1$ | $-1/2$ | $+1$ |
| $\psi_4$ | $\Box$ | $1$ | $1$ | $+1/2$ | $+1$ |
| $\Psi_2 \equiv (\psi_5, \psi_6)^T$ | $\Box$ | $1$ | $\Box$ | $0$ | $-1$ |
| $Q_{L,3} \equiv (t_L, b_L)^T$ | $1$ | $\Box$ | $\Box$ | $+1/6$ | $+1$ |
| $t_R$ | $1$ | $\Box$ | $1$ | $+2/3$ | $+1$ |
| $b_R$ | $1$ | $\Box$ | $1$ | $-1/3$ | $-1$ |
The fermions confine. The fermions develop a nontrivial and antisymmetric vacuum condensate [24]
\[
\langle \psi_{a,a}^\dagger \psi_{b,b} \rangle e^{i\theta_0} c_{ab} \sim \Phi_{CH}^{ij},
\]
where \( \alpha, \beta \) are spinor indices, \( a, b \) are technicolor (TC) indices, and \( I, J \) are flavor indices. We will suppress the contractions of these indices for simplicity. The CH vacuum of the model, giving rise to the EW VEV of \( H_U^0 \) by misalignment, can be written as [24]
\[
\begin{pmatrix}
0 & 0 \\
0 & \sigma_2 c_0 \\
-\sigma_2 s_0 & 0 \\
0 & -\sigma_2 c_0
\end{pmatrix}
\]
where from now on we use the definitions \( s_x \equiv \sin x, c_x \equiv \cos x \) and \( t_x \equiv \tan x \).

The chiral symmetry breaking \( SU(6) \rightarrow Sp(6) \) results in 14 pNGBs, \( \pi_a \) with \( a = 1, \ldots, 14 \), and thus 14 SU(6) broken generators, \( X_a \). The Goldstone bosons around the EW vacuum are parametrized as \( \Sigma = \text{exp}[i\pi_a X_a/f] \Phi_{CH} \) with the decay constant \( f \) of them. This model preserves a \( Z_2 \) symmetry generated by the SU(6) matrix, which is
\[
P = \text{Diag}(1,1,1,1,-1,-1),
\]
where the \( Z_2 \)-odd fields of the model are
\[
H^0, (H_U^0)^*, \Delta^0, \Delta^\pm = \phi^0, \eta^0, \eta'', \mathrm{b}_R.
\]

We have listed in Table II the quantum numbers and parity of the pNGBs divided into the various groupings in Eq. (1) for the EW unbroken \( (\theta = 0) \) and the TC vacuum \( (\theta = \pi/2) \). The neutral components of \( H_I, b \) in unitary gauge are as follows:
\[
H_U^0 = \frac{h}{\sqrt{2}}, \quad H_U^0 = \frac{h_b + i\chi_b}{\sqrt{2}}.
\]

| \begin{tabular}{ll}
  \text{pNGBs} & \text{EW vacuum} \\ 
  \text{TC vacuum} & \text{TC vacuum} \\ 
  \text{}\( \theta = 0 \) & \text{}\( \theta = \pi/2 \) \\
  \text{G}_3/H_0 & \text{H}_I = (2,1/2) \_ \\
  \text{Z}_2-odd pNGBs & \text{Z}_2-odd pNGBs \\
  \text{Z}_2-even pNGBs & \text{Z}_2-even pNGBs \\
  \text{G}_3/H_0 & \text{H}_I = (2,1/2) \_ \\
  \text{Z}_2-odd pNGBs & \text{Z}_2-odd pNGBs \\
  \text{Z}_2-even pNGBs & \text{Z}_2-even pNGBs
\end{tabular} |

In terms of the six-Weyl spinors, \( \Psi \), under the global flavor group \( SU(6) \) the underlying fermionic Lagrangian can be written as
\[
\mathcal{L} = \Psi^\dagger i\gamma^\mu D_\mu \Psi + \delta \mathcal{L} + \delta \mathcal{L}_m,
\]
where the covariant derivatives involve the techniquarks and the SU(2)_L and U(1)_Y gauge generators. The terms \( \delta \mathcal{L} \) are additional interactions including the four-fermion operators responsible for Yukawa couplings of the top and bottom and the \( Z_2 \) symmetry breaking term given in Eqs. (12) and (15). Finally, we have collected the Lagrangian terms with vectorlike masses \( m_{1,2,3} \) for \( \Psi \) in \( \delta \mathcal{L}_m \) and will consider
\[
\delta \mathcal{L}_m = \frac{1}{2} \Psi^T M_\Psi \Psi + \mathcal{H} \mathrm{c}.,
\]
where \( M_\Psi = \text{Diag}(im_1 \sigma_2, -im_2 \sigma_2, im_3 \sigma_2) \).

Below the condensation scale (\( \Lambda_{TC} \sim 4\pi f \)), Eq. (7) yields the effective Lagrangian:
\[
L_{\text{eff}} = L_{\text{kin}} - V_{\text{eff}}.
\]
The gauge-kinetic terms in \( L_{\text{kin}} \) besides providing the kinetic terms for the pNGBs, induce the masses of the EW gauge bosons and their couplings with the pNGBs (including the SM Higgs),
\[
m_W^2 = 2g_W^2 f^2 s_0^2, \quad m_Z^2 = m_{h_0}^2/c_{h_0}^2,
\]
where \( v_{EW} = 2\sqrt{2} f s_0 = 246 \mathrm{GeV} \), \( g_W \) is the weak gauge coupling, and \( g_W \) is the Weinberg angle. The vacuum misalignment angle \( \theta \) parameterizes the corrections to the Higgs couplings to the EW gauge bosons and is constrained by LHC data [38]. This would require a small \( \theta (s_\theta \lesssim 0.3) \), but even smaller according to the electroweak precision measurements [34,35] (\( s_\theta \lesssim 0.2 \)). Misalignment of \( \theta \) to a small value is controlled by the contributions from the EW gauge interactions, the SM fermion couplings to the strong sector and the vectorlike masses of the technifermions. These terms contribute all to the effective potential in Eq. (9),
\[
V_{\text{eff}} = V_{\text{gauge}} + V_{\text{top}} + V_{\text{bottom}} + V_{m} + \cdots.
\]

We further require operators that generate the top and bottom Yukawa couplings and the fermion loop contributions to the potential in Eq. (11). These operators could be analogous to the four-fermion interactions in [39],
\[
\frac{\gamma_f}{\Lambda_f} (Q_L^a r_{LR}^a) (\psi^T P_{I}^a \psi) + \frac{\gamma_b}{\Lambda_b} (Q_L^b r_{LR}^b) (\psi^T P_{I}^b \psi),
\]
where it is assumed \( y_f \equiv y_f = y_b \) and there is one scale \( \Lambda_f \equiv \Lambda_f = \Lambda_b \) from an underlying mechanism which we for now leave unspecified. The spurions, \( P_{I}^a \), project out
the EW components such that $\Psi^T P_t^{\mu} \Psi$ transform as the two Higgs doublets. When the technifermions condense, these terms generate the top contribution to the effective potential in Eq. (11) and the following operators:

$$g f [(Q_L t_R)^{\mu}_a \text{Tr}[P^a_t \Sigma] + (Q_L b_R)^{\mu}_a \text{Tr}[P^a_t \Sigma]]$$ (13)

with $\vartheta \equiv 4 \pi N A (\Lambda_{TC}/\Lambda_f)^2 y_f$ [40] where $A$ is an integration constant arising from the condensation and $N$ is the number of technicolors. So far, the model only generates the top quark mass, while the bottom quark is massless,

$$m_t = \frac{g f s_{\theta}}{2 \sqrt{2}}, \quad m_b = 0.$$ (14)

We generate a tadpole VEV for $h_b$ by adding an operator that weakly mixes $H_1$ and $H_2$, and thus softly breaking the $Z_2$ symmetry in Eq. (4). We introduce a four-fermion interaction that generates the $Z_2$ breaking terms,

$$\frac{g_Z}{\Lambda_Z} (\Psi^T S_1 \Psi)(\Psi^T S_5 \Psi) - \frac{1}{2} \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & -\sigma_2 & 0 \\ 0 & 0 & 0 \end{array} \right) \beta,$$

where $S_3$ and $S_{15}$ are unbroken generators of the unbroken global group $Sp(6)$, which are

$$S_3 = \frac{1}{2} \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & -\sigma_2 & 0 \\ 0 & 0 & 0 \end{array} \right), \quad S_{15} = \frac{1}{2} \left( \begin{array}{ccc} 0 & 0 & \sigma_2 \\ 0 & 0 & 0 \\ -\sigma_2 & 0 & 0 \end{array} \right).$$

After the condensation, we obtain the following $Z_2$ breaking terms in Eq. (11):

$$V_Z = \frac{g_Z f^3}{8 s_{\theta}/2 \beta} \left[ S_{15} \Sigma S_{15} \Sigma \right]$$

$$= \frac{g_Z f^3}{8 s_{\theta}/2 \beta} \left[ \frac{g_Z f^2}{2 \sqrt{2}} (c_{\theta}/2 + 3 c_{3 \theta/2}) \beta + \cdots \right]$$ (16)

with $g_Z (4 \pi)^4 N^2 A^2 (\Lambda_{TC}/\Lambda_Z)^2 \beta$ [40]. The above operator provides a mixing term between $h$ and $h_b$, and therefore a tadpole VEV for $h_b$. By minimizing the total Higgs potential in Eqs. (11) and (16), we obtain the tadpole VEV of $h_b$, which is determined by

$$v_b = \frac{g_Z f^3 (c_{\theta}/2 + 3 c_{3 \theta/2})}{16 \sqrt{2} \pi Z m_{23} + 2 \sqrt{2} f (C_g g_0^3 (3 + t_{\eta_0}) \beta - 2 C_f g^2 c_{\theta})},$$

where $m_{ij} \equiv m_i + m_j$ with $i, j = 1, 2, 3$. The loop factors $C_g, C_f$ and the constant $Z$ occurring in the gauge, SM fermion loop corrections and effective vectorlike mass terms in the effective potential [Eq. (11)], respectively, are nonperturbative $O(1)$ coefficients. These coefficients can be suggested by lattice simulations of the underlying strong dynamics.

Hence, both the top and bottom quarks obtain masses,

$$m_t = \frac{g f s_{\theta}}{2 \sqrt{2}}, \quad m_b = \frac{g v_{EW}}{2 \sqrt{2}}, \quad c_{\theta/2}.$$ (17)

Notice that the tadpole VEV $v_b$ can be reduced by increasing the vectorlike mass $m_i$ for the second technifermion doublet $\Psi_2$, and the bottom mass can thus be tuned down to its observed value.

### IV. Numerical Results of the Concrete Composite 2HDM

We turn to a numerical analysis of the mass spectrum of the concrete composite 2HDM that generates the top-bottom mass hierarchy. For simplicity, we assume the vectorlike masses in the minimal coset are equal, $m_1 = m_2$, and the unknown constant $Z = 1$ [expected to be $O(1)$]. The expressions for the EW VEV [Eq. (10)], the SM Higgs mass, and the top and bottom mass [Eq. (17)] can be fixed to their observed values [41], and $m_{12}$ can be eliminated by the vacuum misalignment condition for $\theta$ from the minimization of the effective potential in Eq. (11). Here, we have collected these expressions:

$$v_{EW} = 2 f s_{\theta} \approx 246 \text{ GeV}, \quad m_h \approx 125 \text{ GeV},$$

$$m_t = \frac{g f^2}{2 \sqrt{2}} \approx 173 \text{ GeV}, \quad m_b = \frac{g v_{EW}}{2 \sqrt{2}} c_{\theta/2} \approx 4.2 \text{ GeV},$$

$$m_{12} = \frac{f (2 C_f g^2 - C_g g_0^3 (3 + t_{\eta_0}) c_{\theta})}{8 \pi Z}.$$ (16)

where $m_h$ is the mass of the physical SM Higgs, $\tilde{h}$, which is the mass eigenstate of a mass matrix $M_0^2$ in the basis $(h, h_b, \Delta^0)$. The SM Higgs $\tilde{h}$ consists mostly of the $Z_2$-even field $h$ and less of the $Z_2$-odd field $h_b$ while the $Z_2$-even field $h_b$ also mixes with the neutral field of the triplet, $\Delta^0$. Further simplicity, we also fix the vacuum misalignment angle to $s_{\theta} = 0.1$ ($\Lambda_{TC} \sim 4 \pi f = 10.9$ TeV) such that we only have the coupling of the $Z_2$ breaking terms, $g_Z$, as free parameter.

In Fig. 1, the mass spectrum of the neutral fields (upper panel) and the vectorlike masses of the technifermions (lower panel) are shown for varying $g_Z$ for fixed $m_i = 125$ GeV. The ratio of the nonperturbative coefficients for the fermion loops and the gauge loops is $C_f/C_g = 2, \ldots, 4$ for the interval $g_Z = 0.1, \ldots, 10$ and therefore they can be $O(1)$.

For $g_Z = 1$, the mass splitting between the vectorlike masses is $m_1 = m_2 = 314$ GeV and $m_3 = 1326$ GeV. This mass splitting generates the observed masses of the top and bottom quark. In this case, the masses of all the composite fields are above 1 TeV (except of the SM Higgs), while the mass of $\tilde{h}$ is 1.7 TeV. In Refs. [9,10], it is emphasized that the LHC may already have the capability of ruling out an $\tilde{h}$ with $g \sim 1$ of mass $\sim 1$ TeV with current integrated...
luminosities, ~200 fb^{-1}, set by the constraints of the $h_b$ production with same couplings like in our theory. Our theory is therefore testable and should provide motivation to go further and deeper into the energy frontier with LHC upgrades with a 100 TeV $pp$ and/or high energy lepton colliders.

Finally, we estimate the couplings $y_f$ and $g_Z$ for the four-fermion operators in Eqs. (12) and (15). For $A \sim 1$ and $N = 2$, we find that

$$y_f \sim 0.1 \left( \frac{\Lambda_f}{\Lambda_{TC}} \right)^2, \quad g_Z \sim 10^{-5} \left( \frac{\Lambda_Z}{\Lambda_{TC}} \right)^2 g_Z.$$  

Assuming $y_f \sim 1$, we obtain the scale $\Lambda_f \sim 40$ TeV for $s_\theta = 0.1$. Then for $\tilde{g}_Z = 1$ and $\Lambda_Z = \Lambda_f \sim 40$ TeV, we estimate $g_Z \sim 10^{-4}$. The smallness of this coupling is technically natural according to 't Hooft's naturalness principle [42] because the $Z_2$ symmetry of the condensate is recovered when we take the limit $g_Z \to 0$. Note, however, small technically natural couplings such as $g_Z$ arise dynamically from power-law suppressions of large initial couplings such as $\tilde{g}_Z$.

### V. Conclusions and Future Work

We have relegated the detailed study of the origin of the four-fermion operators in Eqs. (12) and (15) to future work. Probably these operators can be derived from a fundamental partial compositeness theory like in Refs. [43,44] with global symmetry group SU$(6) \times \text{Sp}(12)_S$. In contrast to Ref. [44], there will be two complex color- and TC-charged scalars, a $Z_2$-even $S_i$ and -odd $\tilde{S}_i$ that are in the fundamental representation of a global symmetry $\text{Sp}(12)_S$ while the technifermions transform under SU$(6)_C$. We can potentially introduce new renormalizable terms with one coupling constant involving the technifermions, techniscalars and the SM fermion. These terms will dynamically generate similar Higgs-Yukawa terms like in Eq. (13). By introducing a weak $Z_2$-breaking term that mixes the fundamental fields $S_i$ and $\tilde{S}_i$ will potentially generate effective operators similar to them generated by the four-fermion operator in Eq. (15).

The concrete SU$(6)/\text{Sp}(6)$ model can be extended to describe the masses of the other SM fermions by adding a SU$(2)_L$ technifermion doublet for each SM fermion. Such a CH model will contain a SU$(2)_L$ technifermion doublet $\Psi_i$ for each SM fermion with $i = b, c, s, \ldots$, one SU$(2)_L$ doublet $\Psi_t$ for the top quark and two SU$(2)_L$ technifermion singlets. For each $\Psi_i$, there will exist a $Z_2^{(i)}$ symmetry of the condensate. Assuming these $Z_2^{(i)}$ symmetries are softly broken by four-fermion operators with one small coupling $g_Z$, we create a mass hierarchy between the SM fermion masses controlled by the hierarchy of the vectorlike masses of the technifermion doublets, while the CKM mixing of the SM quarks may be generated by providing vectorlike masses across the technifermion doublets, $\Psi_i$ and $\Psi_t$. Such a model remains nonconformal up to eight technifermions when the strongly interacting gauge group of the model is SU$(2)_TC$ [45]. If we want a model with more technifermions to remain asymptotically free or even nonconformal then we may need an underlying $\text{Sp}(N)_TC$ gauge description of the technifermions [46]. Finally, the $Z_2^{(i)}$-odd doublets $H_{s, u, d, \ldots}$, consist of technifermions with vectorlike masses much larger than the condensation scale, $\Lambda_{TC}$, which are no longer pNGBs coming from the chiral symmetry breaking, but they are technihadrons described by an effective theory with a heavy-technifermion symmetry containing unbroken $Z_2^{(i)}$ symmetries. This flavor symmetry is similar to the heavy-quark symmetry in QCD for the heavy SM quarks [47]. While for the $Z_2^{(i)}$-odd doublets $H_{t, b, c, \ldots}$, that consist of technifermions with small vectorlike masses compared to $\Lambda_{TC}$, the chiral symmetry is an approximate flavor symmetry.

In conclusion, we have presented a novel mechanism for generation of the top-bottom mass hierarchy with one natural Higgs-Yukawa coupling, $g \equiv 2$. This top-bottom mass hierarchy is provided by breaking a $Z_2$ symmetry of the condensate of new confining technifermions and in
principle controlled by the vectorlike masses of these technifermions. This mechanism can also be present in a wide variety of other models based on vacuum misalignment.

We have considered an SU(6)/Sp(6) CH template model where we showed this model can naturally explain the top-bottom mass hierarchy without fine-tuning of Yukawa coupling constants. Finally, we have briefly discussed the possibility of an underlying theory generating the four-fermion operators in this template model, and furthermore the possibility to extend it to describe the hierarchy of the other SM fermion masses.

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