Local and Quasilocal Conserved Quantities in Integrable Systems

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We outline a procedure for counting and identifying a complete set of local and quasilocal conserved operators in integrable lattice systems. The method yields a systematic generation of all independent, conserved quasilocal operators related to time-average of local operators with a support on up to M consecutive sites. As an example we study the anisotropic Heisenberg spin-1/2 chain and show that the number of independent conserved operators grows linearly with M. Besides the known local operators there exist novel quasilocal conserved quantities in all the parity sectors. The existence of quasilocal conserved operators is shown also for the isotropic Heisenberg model. Implications for the anomalous relaxation of quenched systems are discussed as well.

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Introduction.— One expects that the long-time properties of generic systems are consistent with the Gibbs ensemble [14]. It means that the steady states are determined by very few conserved quantities (CQ), e.g., the total energy, the total spin and the particle number. However, in the integrable models there exist a macroscopic number of local CQ which can explain some anomalous transport properties [3]. Various studies have recently suggested that steady states of integrable systems [6–9] are fully specified by these local CQ. This conjecture is suggested that steady states of integrable systems [6–9] are fully specified by these local CQ. This conjecture is well-verified in the thermodynamic limit L → ∞. Hence we use an appropriate Hilbert-Schmidt inner product to select the relevant independent QLCQ. For arbitrary model parameters we find clear evidence for the existence of QLCQ in the even spin-flip parity sector, while odd parity QLCQ are found only in the easy-plane regime. On the other hand, the total number of CQ and QLCQ appears to scale linearly with M being close to the number of CQ in a noninteracting system, 2(M − 1), and far below the maximum number of mutually commuting (diagonal) operators ∝ 2M. Such a finding can have important consequences for relevance of extended GGE.

The method.— For concreteness we focus on a paradigmatic example of integrable quantum systems, the AHM

\[ H = J \sum_{j=1}^{L} \left\{ \frac{1}{2} (S_{j}^{+} S_{j+1}^{-} + S_{j}^{-} S_{j+1}^{+}) + \Delta S_{j}^{z} S_{j+1}^{z} \right\} , \]

where \( S_{j}^{\pm,\pm} \) are spin-1/2 operators and \( \Delta \) is the anisotropy. To avoid as much as possible degenerate states we use in the computations twisted b.c. \( S_{j+L}^{z} = S_{j}^{z}, S_{j+L}^{\pm} = e^{\pm i\phi} S_{j}^{\pm} \) introducing flux \( \phi \neq 0 \).

Within the space of all translationally invariant (TI) traceless observables, named as \( A_L \), we introduce the following (Hilbert-Schmidt) inner product

\[ (A|B) = \frac{1}{L} \frac{1}{2L} \text{tr} A_j^\dagger B_j , \]

which is equivalent to the infinite temperature \( (\beta \to 0) \) correlation \( (A|B) = \langle A_j^\dagger B_j \rangle_{\beta=0}/L \). The normalisation is chosen such that extensive local TI observables, like \( Q_m \), have finite norm \( \|A\|^2 = \langle A|A \rangle \) in the thermodynamic...
limit (TL) $L \to \infty$. We consider finiteness of TL of the norm as a general definition of quasi-locality of an observable $A$. We define a subspace $A^m_L$ of local TI observables with support of size $m$. In particular, we can define the basis of $A^m_L$ ($m$-local basis) to be composed of operators

$$O_{\Delta} = \sum_j \sigma_j^{s_j} \sigma_{j+1}^{s_{j+1}} \cdots \sigma_m^{s_m-1},$$  \hspace{1cm} (3)

where $\sigma_j^\pm = 2S_j^x, \sigma_j^z = \sqrt{2}S_j^y, \sigma_j^0 = 1_4$, $s_j \in \{+,-,0\}$ while $s_1, s_m \in \{+,-,z\}$. Note that for given $m$ there are $N_m = 3 \times 4^{m-2} \times 3$ different $O_{\Delta}$ and $\dim A^m_L = N_m$ for $m \leq L/2$. Definitions 23 imply that operators are orthonormal, i.e. $(O_{\Delta} \mid O_{\Delta'} = \delta_{\Delta, \Delta'}$.

Let us define the time-averaged operator of $\hat{A} \in A^m_L$

$$\bar{\hat{A}} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt e^{iHt} \hat{A} e^{-iHt} = \sum_{n,n'} \langle n \mid \hat{A} \mid n' \rangle \langle n' \rangle,$$  \hspace{1cm} (4)

which by construction gives a conserved operator $[\hat{H}, \bar{\hat{A}}] = 0$. In principle, $\bar{\hat{A}}$ can be reexpressed, using exact diagonalization (ED) $H\mid n \rangle = E_n \mid n \rangle$ via Eq. 4, in terms of operators 3. On this basis one could decide whether the operator $\bar{\hat{A}}$ is local ($\in B^M_L \equiv \bigoplus_{1 \leq m \leq L} A^m_L$ for some $L$-independent $M \geq m$) or quasi-local (with convergent sum of operators $\in A^m_L$ with increasing $m$) or generic nonlocal. Such a direct approach is, however, tedious and less transparent and in following we use a different protocol.

Picking $M > 0$, we calculate the complete set of $O_{\Delta}$ of $D_M = \sum_{m=1}^M N_m = 4^M - 1$ traceless operators spanning $B^M_L$. To answer how many of $O_{\Delta}$ are local or might be quasilocal, and are as well independent we evaluate Hermitian positive-definite $D_M \times D_M$ matrix

$$K_{\Delta, \Delta'} = (O_{\Delta} \mid O_{\Delta'}) = (O_{\Delta} \mid O_{\Delta'}).$$  \hspace{1cm} (5)

which can be considered as the generalized stiffness matrix (at $\beta \to 0$) in analogy with the (spin) current $J_s$ stiffness $D = \beta (J_s^\beta J_s)$. Orthonormal eigenvectors $u_{\Delta}$ of matrix $K$ corresponding to eigenvalues $\lambda_i > 0$ generate linearly independent conserved operators $Q^\perp_i$

$$Q^\perp_i = \sum_{\Delta} u_{\Delta} O_{\Delta} = \sum_{\Delta} v_{\Delta} O_{\Delta} + Q^\perp_i,$$  \hspace{1cm} (6)

where the operator $Q^\perp_i$ has support on more than $M$ sites, hence $\langle Q^\perp_i \mid Q^\perp_i \rangle = 0$. Calculating the inner product of $Q^\perp_i$ with $O_{\Delta'}$ and utilizing Eq. 4 one finds that $v_{\Delta} = \lambda_i u_{\Delta}$. Substituting this result back into Eq. 4 and calculating the (squared) norm of operators on both hands one finds that

$$\lambda_i = \lambda^2_i + \|Q^\perp_i\|^2.$$  \hspace{1cm} (7)

Local CQ with support on up to $M$ sites ($\|Q^\perp_i\| = 0$) has $\lambda_i = 1$ strictly independent of $L$. Contrary to this, $\lambda_i|_{L \to \infty} > 0$ corresponding to QLCQ ($\|Q^\perp\| > 0$) always smaller than 1 gradually approaching unity with growing $M$. The objective of our study is to establish how the number of CQ and QLCQ depends on $M$.

**Symmetries.** The matrix $K$ can be decomposed in terms of the symmetries of the system. Within the AHM 11, one CQ is the magnetisation $S^z_{\text{tot}} = \sum S^z_j$ implying that one may consider $S^z_{\text{tot}}$ preserving subspace $[S^z_{\text{tot}} \mid O_{\Delta} = 0$, i.e., subset of $O_{\Delta}$ with the constraint that the number of $\pm = \text{equals the number of } s = -$ in the sequence $\Delta$, reducing the dimensions $D_M$. More interesting is the spin-flip $\mathbb{Z}^2$ symmetry, generated by the parity operator $P = \Pi_j (S^+_j + S^-_j)$. $m$-local operator spaces $A^m_L$ can then be decomposed into even($E$)/odd($O$), with the bases generated by sets $\{O_{\Delta} \pm PO_{\Delta} \mid P\}$. Remarkably, all known local conserved operators $O_{\Delta}$ are even, while it has been shown recently [34–36] that odd QLCQ exist for $|\Delta| < 1$ which determine the properties of the spin current. Furthermore, AHM model is time-reversal invariant implying that the time-averaging matrix $K$ is real symmetric and its eigenoperators $\sum_{\Delta} v_{\Delta} O_{\Delta}$ can be classified as real (R) or imaginary (I), being separated spanned separately by the bases $O_{\Delta} + O_{\Delta}^\perp$, $i(O_{\Delta} - O_{\Delta}^\perp)$. Hence one ends up with four orthogonal sectors: $ER, EI, OR, OI$.

**Numerical procedure and results.** In order to reduce the computational effort we restrict our calculations to the Hilbert subspace with $S^z_{\text{tot}} = 0$. This requires a straightforward modification of the scalar product in Eq. (2) where the number of states $2^L$ should be replaced by $(L/2)^L$. Although the set $\{O_{\Delta}\}$ is not orthonormal within the chosen subspace, the overlap matrix $N_{\Delta, \Delta'} = (O_{\Delta} \mid O_{\Delta'})$ remains real, symmetric and positive-definite. Since it can be diagonalized by an orthogonal matrix $V$ one can introduce an orthonormal set of operators $O_{\mu} = \theta^{-1/2}_{\mu} \sum \nu V_{\mu, \nu} O_{\nu}$ where $\theta_{\mu}$ are eigenvalues of $N$. The numerical calculations have been performed via full ED of systems with sizes $L = 10, 20$ and the boundary condition twisted by $\phi = 10^{-4}$. For given $M$ and $L$ we start the procedure generating first all operators $O_{\Delta}$ then the orthonormal ones $O_{\mu}$ and finally the time-averaged $\bar{O}_{\mu}$. At the end the matrix $K_{\mu, \nu} = (\bar{O}_{\mu} \mid \bar{O}_{\nu})$ is evaluated and diagonalized, leading to the eigenvalues $\lambda_i \in [0,1]$. The number of non-vanishing eigenvalues is the quantity that we are looking for: the number of local or quasilocal mutually orthogonal conserved operators.

Let us first consider a more generic case as a test of the method. Adding to the AHM, Eq. 1 a next nearest neighbor interaction term $H' = \Delta_2 \sum_j S^+_{j+1} S^-_j$ the model becomes nonintegrable. In this case we expect the existence of a single CQ (besides $S^z_{\text{tot}}$) is the full Hamiltonian, i.e. $Q_2 = H + H'$. In Fig. 1 we show the results for eigenvalues $\lambda_i$ vs. $1/L$ for $L = 10, \ldots, 18$ for both $E/O$ sectors, choosing parameters $\Delta = 0.5, \Delta_2 = 0.5$. This implies the expectation that $\lambda_i = 1$ being independent of $L$ for $M \geq 3$, as confirmed in Fig. 1. All other $\lambda_l$
In both sectors vanish with increasing \( L \), predominantly exponentially, \( \lambda_1 \propto \exp(-\zeta L) \). There are exceptions decaying as \( \lambda \propto 1/L \) which might be related to powers of local operators, e.g., \((Q_2)^2\) which are however nonlocal quantities [35].

On the other hand, for the integrable AHM one finds several CQ and QLCQ as shown in the three topmost rows of Fig.1 as well as in Fig.2. The latter figure demonstrates in more detail how the spectra of the matrix \( K \) depend on \( M \) and \( L \). Results in Fig.1 show that the strictly local CQ exist only in the even sector. These are exactly the well known CQ described in Refs. [12, 20]. All the other (novel) CQ are quasi-local. In the easy-axis, \( \Delta > 1 \), or isotropic, \( \Delta = 1 \), cases QLCQ exist only in the even parity sector. We have confirmed the latter observation carrying out a finite–size scaling of tr\( K \) for the anisotropy \( \Delta \geq 1 \) (not shown). We have found a very clear vanishing of tr\( K \) for \( L \to \infty \), in the odd sector at any fixed \( M \), what excludes existence of QLCQ in these sectors. On the contrary, in the easy-plane regime, \(|\Delta| < 1 \), QLCQ exist in all parity and time-reversal sectors.

While the initial operators \( O_\mu \) as well as the orthonormal ones \( O_\mu \) have support on not more than \( M \) sites, the subsequent time–averaging may extend their support beyond this value. Therefore, before counting the number of QLCQ one should exclude the possibility that \( O_\mu \) can be expanded solely in terms of higher local CQ having support on more than \( M \) sites. In order to estimate this contribution we have calculated the first nontrivial eigenvalue of \( \tilde{K}_{\mu,\nu} = \sum_{m=1}^\infty \sum_{j} (O_\mu|Q_m)(Q_m|O_\nu)/\langle Q_m|Q_m \rangle \) (for computation we truncate at \( m = 18 \)), which would agree with \( K_{\mu,\nu} \) under the assumption that the orthogonal local operators \( Q_m \) are a complete set, i.e. that \( A = A \equiv \sum_m \langle Q_m|A|Q_m \rangle \). These leading eigenvalues of \( \tilde{K}_{\mu,\nu} \), which are shown in Fig.2 as open circles, are always well below eigenvalues of unprojected \( K_{\mu,\nu} \) clearly indicating that the set of known \( Q_m \) is incomplete also in the even sector. The central question is how the number of CQ and QLCQ grows with the size of the support \( M \) and, in particular, whether this growth is linear as in the case of noninteracting particles, where one finds \( 2M - 2 \) local operators, \( Q_m^{\mu} = \sum_k 2\cos(mk)n_k = \sum_j \sigma^{\mu}_j \cdot (c^\dagger_{j+m}c^\dagger_j + \text{H.c.}) \), \( Q_m^{\mu} = \sum_k 2\sin(mk)n_k = i\sum_j \sigma^\mu_j \cdot (c^\dagger_{j+m}c_j - \text{H.c.}) \), \( m = 1, \ldots, M - 1 \), re-written into the spin language via \( c^\dagger_{j+m}c_j = (1/2)(\sigma^\dagger_j \cdot \sigma^\dagger_{j+m} + \sigma^\dagger_{j+m} \cdot \sigma^\dagger_j) \).

Since the eigenvalues \( \lambda \leq 1 \), the total number of CQ and QLCQ is obviously bounded from below by tr\( K \). Moreover, as all CQ with support on up to \( M \) sites correspond to \( \lambda = 1 \), tr\( K \) should gradually approach the number of QC and QLCQ for large enough \( M \). In Fig.3 we show tr\( K \) obtained after the \( 1/L \) size scaling. For comparison we show also the number of nonzero \( \lambda \) counted directly after carrying out a finite–size scaling of individual leading eigenvalues. The number of directly counted CQ and QLCQ as well as tr\( K \) increase linearly with \( M \). We find it particularly surprising and suggestive that the in easy–plane regime tr\( K \) is very close to \( 2(M - 1) \), i.e. to the number of CQ in the systems of noninteracting particles. However, contrary to the latter systems tr\( K \) in AHM is not equally distributed among the parity sectors. Half of the total tr\( K \) comes from the known (even) QC, approximately one quarter originates from even QLCQ and one quarter from odd QLCQ. In the isotropic and Ising regimes (\( \Delta \geq 1 \)) QLCQ in the odd sector disappear, while the total number of CQ and QLCQ in the even sector is evidently larger than \( (M - 1) \) in noninteracting case. Our central observation concerns the main difference between interacting and noninteracting integrable systems. We have found that this difference does not consists in the number of QC (which is extensive in both cases) but rather in their locality and symmetry.

Identifying the novel conserved quantities.— It is evident that besides known \( Q_m, m = 2, 3 \ldots \) which are all in the even sector we did not find any other strictly local CQ. As expected we also confirm the existence of the QLCQ in the odd sector which has been analytically constructed [34, 37] and used in the extended GGE previously [10]. The simplest novel QLCQ which appears within the even sector in the isotropic case, \( \Delta = 1 \), starts with a \((m = 3)\)–local term \( Q' = \alpha \sum_j S_j \cdot S_{j+1} + \text{h.o.t.} \), where the amplitudes of all higher order terms are smaller than \( \alpha \). In other regimes, \( \Delta \neq 1 \), \( Q' \) starts already with the \((m = 2)\)–local term.

Steady state after a linear quench.— Let us assume that at time \( t = 0 \) the Hamiltonian is quenched from \( H - X_a \) to the integrable \( H_a \), where \( X_a \in \mathcal{A}_L \) is considered as a perturbation. The system is initially in the Gibbs state \( \rho_0 = \exp[-\beta(H - X_a)]/Z, \) where \( Z = \text{Tr}\exp[-\beta(H - X_a)] \), and it relaxes towards a steady state \( \overline{\rho}_a \). Since \( X_a \) commutes with \( H \) one easily finds the following linear expansion

\[
\exp[-\beta(H - X_a)] \simeq \exp[-\beta H](1 + \beta \overline{X}_a).
\]

Calculating the trace over the eigenstates of \( H \) one obtains an analogous approximation for the partition function \( Z = Z_T(1 + \beta(X_a)) \). Here \( Z_T = \text{Tr}\exp(-\beta H) \) and the average \( \langle \ldots \rangle \) is defined for the thermal state \( \rho_T = \exp(-\beta H)/Z_T \). At the end we obtain the linear approximation for the steady state:

\[
\overline{\rho}_a = \rho_T[1 + \beta(\overline{X}_a - \langle X_a \rangle)]
\]

Next, we consider some other extensive observable \( X_b \in \mathcal{A}_L \) and a related intensive one \( X_b/L \). We study whether the (possibly) nonthermal \( \overline{\rho}_a \) and the thermal \( \rho_T \) states can be distinguished by the measurement of \( X_b/L \). We calculate

\[
\beta K_{ab} = \text{Tr}\left[ (\overline{\rho}_a - \rho_T) \frac{X_b}{L} \right] = \frac{\beta}{L}(\overline{X}_a \overline{X}_b) - (\overline{X}_a \langle X_b \rangle).
\]
FIG. 1. (Color online) The size (1/L) scaling of leading eigenvalues $\lambda_l$ of matrix $K$ with $M = 6$, corresponding to symmetric (red circles) or antisymmetric (blue crosses) eigenoperators with respect to time reversal. Left/right column shows even/odd parity sectors, while rows indicate different regimes of integrable (upper three rows) and non-integrable (lower row) model with parameters indicated in the panel. Dashed lines indicate 1/L extrapolation to TL which in some cases provide clear indication of existence of QLCQ $\lambda_l|L\to\infty > 0$, beyond the local eigenoperators with $\lambda_l = 1$.

In the high–temperature regime ($\beta \to 0$) and for traceless operators $X$ the above correlation matrix $K$ coincides with the matrix of inner products in Eq. (5).

Conclusions.— We have presented a systematic procedure which allows to establish the existence of local and quasi local CQ in 1D many-body models with short range interactions. In spite of limitations of our results to finite sizes $L \leq 20$, explicitly performed only within the (un-polarised) subspace $S^z_{\text{tot}} = 0$, they allow for some firm conclusions for AHM. The method confirms besides the known strictly local CQ, being all in the even sector, also even and odd QLCQ. In the metallic regime $\Delta < 1$ the odd QLCQ are consistent with the finite spin stiffness $D > 0$ and analytical construction [34–37]. Our results clearly establish the convergence of at least one entirely novel QLCQ, $Q'$, existing in the whole $\Delta > 0$ range. This particular QLCQ alone can already explain at least part of deviations from GGE observed in quenched spin systems at $\Delta \gg 1$ [17, 32]. There are clear indications for the existence of further QLCQ which emerge with increasing support size $M$ but for $M \leq 6$ are not yet sufficiently converged to determine their explicit form. It is nevertheless plausible that in quenched or driven systems the major role in thermalisation will be related to CQ and QLCQ with smaller supports $M$ as identified in our study. Here the most important conclusion is that the number of local and quasilocal CQ appears to scale linearly with $M$, for $|\Delta| < 1$, being approximately $2(M-1)$ similar to a model of noninteracting fermions. Our results allow for a meaningful extension of GGE incorporating the full set of QLCQ.

FIG. 2. (Color online) Dependence of leading eigenvalues $\lambda_l$ of $K$ in ER sector for isotropic HM $\Delta = 1$. Different panels indicate decreasing support sizes $M = 6, 5, 4, 3$, while decreasing sizes of points and colors indicate the system size $L = 20$ (orange), 18 (red), 16 (green), 14 (cyan), 12 (blue), 10 (brown). Extrapolated $L \to \infty$ values are indicated with crosses if in the range of the plot. Open circles are explained in the text.

FIG. 3. (Color online) Left: The extrapolated 1/L $\to 0$ value of trK versus support size M for different regimes of AHM: $\Delta = 1/2$ (blue squares), where contributions for even/odd parity sector are shown separately with crosses/disks, $\Delta = 1$ (red circles), $\Delta = 3/2$ (diamonds). Right: similar plot for the number of non-vanishing extrapolated eigenvalues $\lambda_l$. Long/short dashed lines indicate number of known local CQ for interacting $(M-1)/$non-interacting $(2M-2)$ cases.
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