Short-range spin correlations and induced local spin-singlet amplitude in the Hubbard model

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In this paper, from the microscopic Hubbard Hamiltonian we extract the local spin-singlet amplitude due to short-range spin correlations, and quantify its strength near half-filling. As a first application of the present approach, we study a problem of the energy dispersion and its d-wave modulation in the insulating cuprates, Sr$_2$CuO$_2$Cl$_2$ and Ca$_2$CuO$_2$Cl$_2$. Without any adjustable parameters, most puzzling issues are naturally and quantitatively explained within the present approach.

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Recent discovery of a normal state pseudogap in underdoped copper oxides has attracted considerable attention both from experimentalists and theoretical physicists for many years. For these materials, the low frequency spectral weight begins to be strongly suppressed below some characteristic temperature $T^*$ higher than $T_c$. This anomalous behavior has been observed through various experimental probes such as angle resolved photoemission spectroscopy (ARPES), specific heat, NMR, and optical conductivity. One of the most puzzling questions in pseudogap phenomena is why $T^*$ has a completely different doping dependence from $T_c$, in spite of possibly their close relation.

There are many theoretical scenarios to attempt to understand the pseudogap phenomena. These include the spinon pair formation without the Bose-Einstein condensation of holons in the slave boson (or fermion) mean-field theory, strong superconducting (SC) phase fluctuations, strong magnetic fluctuations near the antiferromagnetic (AF) instability, and so on. At present there is no consensus on the origin of the pseudogap and its relationship with the SC long-range order. Among several scenarios, the slave boson (or fermion) mean-field theory may shed some insight into the problem. This is because the pseudogap is closely related to a spin gap, the predicted phase diagram is, at least, qualitatively consistent with experiments, and furthermore it starts from the microscopic model ($t-J$ Hamiltonian) as opposed to other phenomenological models. On the other hand, the recombination of a spinon and a holon into a physical electron is nontrivial, and also the constraint of no-double-occupancy at each site is difficult to impose at a microscopic level. In deriving a local spin-singlet amplitude which may be responsible for the pseudogap behavior, in this paper we use the Hubbard Hamiltonian instead of the $t-J$ Hamiltonian. Local spin-singlet amplitude is induced directly from short-range spin correlations in the normal state and its strength is quantified near half-filling. In this paper short-range spin correlation means that when site $i$ is occupied by an electron with up-spin (or down-spin), then the nearest sites predominantly by electrons with the opposite spin.

We start by defining the one-band Hubbard model proposed by Anderson as the simplest model which might capture the correct low energy physics of copper oxides. The Hubbard model is described by the Hamiltonian where $c_{i,\sigma}$ destroys an electron at site $i$ with spin $\sigma$ on a two-dimensional square lattice

$$H = -t \sum_{\langle i,j \rangle,\sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i c_{i,\uparrow}^\dagger c_{i,\uparrow} c_{i,\downarrow}^\dagger c_{i,\downarrow}. \quad (1)$$

t is a hopping parameter between nearest neighbors $\langle i,j \rangle$ and $U$ denotes local Coulomb repulsion. It is believed that the realistic strength of the Coulomb repulsion lies in between the weak and strong coupling regimes, namely, $U \sim W - 2W$ where $W$ is the bandwidth of $8t$ in two dimensions.

As a first step to the microscopic understanding of the pseudogap, it is important to answer a more fundamental question: How can spin-singlet tendency appear directly from the Coulomb repulsion, presumably without the exchange of bosonic degrees of freedom such as spin fluctuations? As a simple example to illustrate this point qualitatively, let us consider the $U \gg t$ limit at half-filling where only an up-spin or down-spin electron is allowed at each site. The typical spin-singlet structure (with $d$-wave form factor) in which we are interested in this paper is

$$\Delta_g(i) = \frac{1}{2} \sum_i g(\delta)(c_{i+\delta,\uparrow}^\dagger c_{i,\downarrow} - c_{i+\delta,\downarrow}^\dagger c_{i,\uparrow}),$$

where $g(\delta)$ is an appropriate structure factor. For a $d$-wave symmetry, for instance,

$$g(\delta) = \begin{cases} 
1/2 & \text{if } \delta = (\pm 1, 0), \\
-1/2 & \text{if } \delta = (0, \pm 1), \\
0 & \text{otherwise}. 
\end{cases} \quad (2)$$

Then the local spin-singlet amplitude for strongly correlated electrons is

$$\langle \Delta_g^\dagger(i) \Delta_g(i) \rangle = (1 \times 1 + 0 \times 0)/2 = 1/2$$
is increased over its noninteracting value (\(\langle |\Delta_g(0)|^2 \rangle_0 = (1/2 \times 1/2 + 1/2 \times 1/2)/2 = 1/4\) for both spins).

To make this argument more quantitative, we introduce a spin-singlet correlation function \[\chi_g(i, \tau) = \langle T_\tau \Delta_g(i, \tau) \Delta_g^\dagger(0, 0) \rangle, \tag{3}\]
where \(T_\tau\) is the imaginary time ordering operator. In this paper we consider only the local spin-singlet amplitude \(\langle |\Delta_g(0)|^2 \rangle\), which may be obtained by \(\chi_g(i \to 0, \tau \to 0^-)\). Now we examine whether there is an increase in \(\langle |\Delta_g(0)|^2 \rangle\) for strongly correlated electrons with respect to \(\langle |\Delta_g(0)|^2 \rangle_0\) for the noninteracting electrons, which is similar in spirit to a renormalization group (RG) approach \[\[\[\].\] As already illustrated in the previous paragraph, the increase in \(\langle |\Delta_g(0)|^2 \rangle\) for strongly correlated electrons crucially depends on the short-range spin correlations. Although there exists no controlled way of obtaining the wave function in general, certain local or short-range static quantities such as double occupancy (or equivalently local spin amplitude) or the nearest neighbor correlations are reasonably well captured by the mean-field state with AF order. Before going further, it is important to establish the validity of the current approximation by explicitly comparing the above quantities calculated in the mean-field state (with AF order) with available quantum Monte Carlo (QMC) results.

The double occupancy \(D = \langle n_{i,\uparrow} n_{i,\downarrow} \rangle\) plays an important role in gauging the degree of strong correlations in the Hubbard-type model. For instance, \(D \to 0\) at half-filling for \(U \to \infty\), while \(D_0\) evaluated in the noninteracting state is \((n/2)^2 = 0.25\). In Fig. 1(a) our calculations (solid curve) are compared with (virtually exact) QMC results (open circles) by White et al. \[\[\] for \(n = 1, N = 4 \times 4\), and \(T = t/16\). For purely interaction induced effect, \(D_0\) is subtracted from \(D\). Above \(U \simeq 3t\) the agreement with QMC data is increasingly better and for \(U \geq 8t\) the two results are almost indistinguishable. For the nearest neighbor correlations between \(i\) and \(j\), we calculate \(t_{eff}/t = \langle \phi_{\alpha_0}^\dagger(\vec{k}) | f(E_{-(\vec{k})}) + f(E_{+(\vec{k})}) \rangle - \text{sign}[\phi_{\alpha_0}(\vec{k} + \vec{Q})/\phi_{\alpha_0}(\vec{k})]\]
\[
\times \frac{\Delta_{sdw}}{2UN} \sum_{\vec{k}} \phi_{\alpha_0}^2(\vec{k}) \Delta_{sdw} [f(E_{-(\vec{k})}) - f(E_{+(\vec{k})})], \tag{4}\]
where
\[
\lambda(\vec{k}) = \sqrt{((\epsilon(\vec{k}) - \epsilon(\vec{k} + \vec{Q}))/2 + \Delta_{sdw}^2},
E_{\pm}(\vec{k}) = (\epsilon(\vec{k}) + \epsilon(\vec{k} + \vec{Q}))/2 \pm \lambda(\vec{k}). \tag{5}\]
The spin density wave (SDW) approximation will be used below only to capture the reasonable local and short-range correlations between electrons.

We should also point out some limitations in this approach. The physical reason that the nearest neighbor correlation is overestimated (up to by 10%) compared with QMC data is as follows. Suppose site \(i\) is occupied by an electron with up-spin in the strong coupling limit. Then the nearest sites are occupied by down-spin electrons as majority and also by up-spin electrons as minority. As the distance from site \(i\) increases, the effective polarization of electron spins decreases in magnitude. In the mean-field state with AF order, however, the effective polarization is the same for all the distances from site \(i\). This means that physical quantities evaluated in the mean-field state become progressively overestimated for increasingly distant correlations. Another limitation is that the mean-field critical doping \(x_c\) for AF mean-field order vanishes increases with increasing \(U\). This is inconsistent with more advanced treatment of the Hubbard model \[\[\] where with increasing \(U\), \(x_c\) decreases after reaching its maximum around at \(x \approx 0.21\). Hence our approximation based on the Hubbard model is valid near half-filling (probably up to \(x \approx 0.1 - 0.15\)). For the \(t - J\) Hamiltonian the current approximation will be valid even far away from half-filling, because this strong coupling feature is already taken into account there.

Then for \(d\)-wave type symmetries, the local spin-singlet amplitude \(\langle |\Delta_g(0)|^2 \rangle\) evaluated in the mean-state with AF order becomes
\[
\frac{n}{4N} \sum_{\vec{k}} \phi_{\alpha_0}^2(\vec{k}) | f(E_{-(\vec{k})}) + f(E_{+(\vec{k})}) \rangle - \text{sign}[\phi_{\alpha_0}(\vec{k} + \vec{Q})/\phi_{\alpha_0}(\vec{k})]\]
\[
\times \frac{\Delta_{sdw}}{2UN} \sum_{\vec{k}} \phi_{\alpha_0}^2(\vec{k}) \Delta_{sdw} [f(E_{-(\vec{k})}) - f(E_{+(\vec{k})})], \tag{4}\]
where
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\lambda(\vec{k}) = \sqrt{((\epsilon(\vec{k}) - \epsilon(\vec{k} + \vec{Q}))/2 + \Delta_{sdw}^2},
E_{\pm}(\vec{k}) = (\epsilon(\vec{k}) + \epsilon(\vec{k} + \vec{Q}))/2 \pm \lambda(\vec{k}). \tag{5}\]
the strategy is to find \( V \) amplitude gives the same interaction induced local spin-singlet amplitude is in good agreement with QMC data for intermediate to strong coupling the mean-field approximation for the repulsive Hubbard model which was discussed in approximations, respectively. The SDW approximation and attractive Hubbard models in the SDW and BCS which has been obtained through study on the repulsive numerical results. Then one may use general knowledge above Hamiltonian. Unfortunately there exist no such approximation with available numerical data for the attractive Hubbard model. However, the numerical results of nearest neighbor correlation is spilt into two (relatively broad) peaks instead of long as the energy dispersion is concerned, it is basically different from that point of view. Because the induced local AF correlations are introduced. Furthermore a d-wave-like modulation in the insulating cuprates, Sr\(_2\)CuO\(_2\)Cl\(_2\) and Ca\(_2\)CuO\(_2\)Cl\(_2\). Recent ARPES experiments for an insulating cuprate Sr\(_2\)CuO\(_2\)Cl\(_2\) [21] clearly show that the near isotropy and the overall band dispersion along \((\pi/2, \pi/2) - (0, 0)\) and \((\pi/2, \pi/2) - (0, 0)\) cannot be explained by considering only AF order or its fluctuations, unless some adjustable fitting parameters such as \(t'\) and \(t''\) are introduced. Furthermore a d-wave-like modulation of the insulating gap in Ca\(_2\)CuO\(_2\)Cl\(_2\) is totally mysterious from that point of view. Because the induced local spin-singlet and spin amplitudes increase with decreasing doping, half-filling and at low temperatures, both short-range spin-singlet and AF fluctuations are strong and coexist, leading to a resonating valence bond (RVB) type state [8]. Our general experience [22] tells that as long as the energy dispersion is concerned, it is basically identical in a strongly fluctuating state (or in the pseudogap state) and in a long-range ordered state. The only difference is that in the former state the gap is filled with some spectral weight (pseudogap) and the spectral function is split into two (relatively broad) peaks instead of two delta functions. Thus at half-filling and at low temperatures, a hole strongly interacts with both short-range correlations between electrons, as before in the SDW approximation.

In the BCS approximation for the above Hamiltonian, \( \langle |\Delta_d(0)\rangle^2 \) is given as

\[
\frac{n}{4N} \sum_k \phi_d^2(\vec{k}) \left[ \frac{1}{2} - \frac{\varepsilon(\vec{k})}{2E(\vec{k})} \tanh(E(\vec{k})/2T) \right] + \left[ \frac{1}{N} \sum_k \phi_d(\vec{k}) \Delta_d(\vec{k}) \tanh(E(\vec{k})/2T) \right]^2,
\]

where \( E(\vec{k}) = \sqrt{\varepsilon^2(\vec{k}) + \Delta_d^2(\vec{k})} \). As before, the d-wave gap \( \Delta_d(\vec{k}) = \Delta\phi_d(\vec{k}) \) and the chemical potential \( \mu \) are self-consistently determined through the gap and number equations for given \( V_{ind}, T \) and \( n \). In Fig. 2(b) \( V_{ind} \) is plotted as a function of doping \( x = 1 - n \) for \( T = 0 \) and \( U = 1.5W = 12t \) near half-filling. The BCS approximation \( (|V_{ind}| \geq 3t) \) is expected to be accurate for \( x \leq 0.1 \) and to be less accurate beyond it. The strength of the induced local spin-singlet amplitude rapidly decreases with doping, just as the induced AF correlations do. Beyond \( x \sim 0.1 \sim 0.15 \), \( |V_{ind}| \) will decrease even faster when the strong coupling effect is correctly included, as was mentioned in Fig. 2(a). For \( U > t \) at \( n = 1 \), \( V_{ind} \) saturates to be \(-7.64t \) and \( \langle |\Delta_d(0)\rangle^2 \rightarrow 0.25 \), consistent with our previous result \( \langle |\Delta_d(0)\rangle = 0.5 \) based on qualitative argument. \( V_{ind} \) and its doping dependence have some interesting consequences which may answer some of the puzzling issues in the high temperature superconductors.

As a first application of the present approach, we attack a problem of the energy dispersion and its d-wave modulation in the insulating cuprates, Sr\(_2\)CuO\(_2\)Cl\(_2\) and Ca\(_2\)CuO\(_2\)Cl\(_2\). Recent ARPES experiments for an insulating cuprate Sr\(_2\)CuO\(_2\)Cl\(_2\) [21] clearly show that the near isotropy and the overall band dispersion along \((\pi/2, \pi/2) - (0, 0)\) and \((\pi/2, \pi/2) - (0, 0)\) cannot be explained by considering only AF order or its fluctuations, unless some adjustable fitting parameters such as \(t'\) and \(t''\) are introduced. Furthermore a d-wave-like modulation of the insulating gap in Ca\(_2\)CuO\(_2\)Cl\(_2\) [21] is totally mysterious from that point of view. Because the induced local spin-singlet and spin amplitudes increase with decreasing doping, half-filling and at low temperatures, both short-range spin-singlet and AF fluctuations are strong and coexist, leading to a resonating valence bond (RVB) type state [8]. Our general experience [22] tells that as long as the energy dispersion is concerned, it is basically identical in a strongly fluctuating state (or in the pseudogap state) and in a long-range ordered state. The only difference is that in the former state the gap is filled with some spectral weight (pseudogap) and the spectral function is split into two (relatively broad) peaks instead of two delta functions. Thus at half-filling and at low temperatures, a hole strongly interacts with both short-range
spin-singlet and AF fluctuations, yielding an energy dispersion similar to that in the coexistence state of the SC and AF long-range order. 22 The energy dispersion is then given by

$$\sqrt{\varepsilon^2(\vec{k}) + \Delta_{sdw}^2 + \Delta^2(\cos k_x - \cos k_y)^2}. \quad (8)$$

From the overall bandwidth ($\approx 320$ meV) experimentally measured from $(\pi/2, \pi/2)$ to $(0,0)$ along which $\phi_d(\vec{k})$ vanishes, the Coulomb repulsion $U$ (and $\Delta_{sdw} \approx U/2$) is determined to be 11.4$t$ for $t = 250$ meV. For $U = 11.4t$, $n = 1$ and $T = 0$, the effective strength $V_{ind}$ of the induced spin-singlet amplitude is completely determined to give $V_{ind} = -6.09t$ and $\Delta = 2.07t$. Figure 3 clearly demonstrates that the energy dispersion and its near isotropy along $(\pi/2, \pi/2) - (\pi,0)$ and $(\pi/2, \pi/2) - (0,0)$ are in excellent agreement with experiments without any adjustable parameters. Our energy dispersion along $(\pi/2, \pi/2) - (\pi,0)$ is proportional to $|\cos k_x - \cos k_y|^2$ (solid curve in the inset) at low energies as opposed to $|\cos k_x - \cos k_y|$ (dashed curve) predicted from the flux-phase in the $t-J$ model. 24 Recent experiments show a significant deviation of the energy dispersion from the cusp-like form along $(\pi/2, \pi/2) - (0,0)$. A similar result to ours was recently obtained in the context of SO(5) symmetry. 22 The state with a spin gap (or pseudogap) of $(\Delta \phi_d)^2/U \approx J\phi_d^2$ and also with a Mott-Hubbard gap of order $U$ at half-filling is expected to continuously evolve into a state with a relatively smaller spin gap (or pseudogap) but without a charge gap away from half-filling.

In this paper we have considered only the local spin-singlet amplitude induced by short-range spin correlations and its consequences. The long-range $d$-wave superconductivity is beyond the scope of the present approach. How local spin-singlets acquire local SC phases and eventually establish their long-range phase coherence is a challenging problem to the theory of high temperature superconductivity. Quantitative analytical study of the Hubbard model in the physically relevant regime and of the interplay between AF and $d$-wave pairing correlations beyond a mean-field level is not available at present, mainly due to the absence of a small parameter. Certainly it is a subject for future study. It is also desirable to perform numerical simulations for a Hamiltonian with $d$-wave symmetry such as Eq. 6 if it is possible. Then the mean-field BCS approximation used here to obtain reasonable local and short-range static correlations between electrons will be tested for its range of validity.

In summary, from the microscopic Hubbard Hamiltonian, the local spin-singlet amplitude due to short-range spin correlations was explicitly extracted and its strength was quantified near half-filling. As a first application of the present approach, a problem of the energy dispersion and its $d$-wave modulation in the insulating cuprates, $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ and $\text{Ca}_2\text{CuO}_2\text{Cl}_2$ was studied. Without any adjustable parameters, most puzzling issues on the energy dispersion were naturally and quantitatively explained within the present approach.

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[1] T. Timusk and B. Statt, Rep. Prog. Phys. 62, 61 (1999).
[2] H. Ding, T. Yokoya, J. C. Campuzano, T. Takahashi, M. Randeria, M. R. Norman, T. Mochiku, K. Kadowaki, and J. Giapintzakis, Nature 382, 51 (1996).
[3] A. G. Loeser, Z. -X. Shen, D. S. Dessau, D. S. Marshall, C. H. Park, P. Fournier, A. Kapitulnik, Science 273, 325 (1996).
[4] J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang, Phys. Rev. Lett. 71, 1740 (1993).
[5] Ch. Renner, B. Revaz, J. -Y. Genoud, K. Kadowaki, and ÖFischer, Phys. Rev. Lett. 80, 149 (1998).
[6] M. Takigawa, P. C. Hammel, R. H. Heffner, and Z. Fisk, Phys. Rev. B 43, 247 (1991).
[7] C. C. Homes and T. Timusk, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. Lett. 71, 1645 (1993).
[8] P. W. Anderson, Science 235, 1196 (1987).
[9] G. Baskaran, Z. Zou, and P. W. Anderson, Solid State Commun. 69, 973 (1987); P. W. Anderson, G. Baskaran, Z. Zou, and T. Hsu, Phys. Rev. Lett. 58, 2790 (1987); A. E. Ruckenstein, P. J. Hirshfield, and J. Appel, Phys. Rev. B 36, 857 (1987); G. Kotliar and J. Liu, Phys. Rev. B 38 5142 (1988); Y. Suzumura, Y. Hasegawa, and H. Fukuyama, J. Phys. Soc. Jpn. 57, 401 (1988).
[10] V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995).
[11] J. Schmalian, D. Pines, and B. Stojkovic, Phys. Rev. Lett. 80, 3839 (1998).
[12] Spin-singlet tendency is not necessarily associated with superconductivity. As will be demonstrated shortly, local spin-singlet can arise entirely from short-range spin correlations without any explicit attractive interaction.
[13] N. E. Bickers, D. J. Scalapino, and S. R. White, Phys. Rev. Lett. 62, 961 (1989).
[14] The spin-singlet correlation function (with $d$-wave symmetry) is more general than the $d$-wave pair correlation function in the sense that the spin-singlet correlations can be also induced by short-range spin correlations even in the absence of pairing interactions.
[15] H. J. Schulz, Europhys. Lett. 4, 609 (1987); I. Dzyaloshinskii, Sov. Phys. JETP, 66, 848 (1987); P. Ledemer, G. Montambaux, and D. Poilblanc, J. Phys. (Paris) 48, 1613 (1987); J. Gonzalez, F. Guinea, and M. A. H.
For local $s$-wave and extended $s$-wave symmetries, there are additional contributions to the above equation. But the second term is still the major factor determining whether $\langle |\Delta_g(0)|^2 \rangle$ or not.

For $n=1$, $N = 4 \times 4$, and $T = t/16$. For purely interaction induced effect, $D_0$ and unity are subtracted from $D$ and $t_{eff}/t$, respectively.

FIG. 1. (a) Double occupancy $D = \langle n_{i\uparrow}n_{i\downarrow} \rangle$ and (b) nearest neighbor correlation $t_{eff}/t = \langle c_{i\sigma}c_{j\sigma}^\dagger \rangle_U / \langle c_{i\sigma}c_{j\sigma}^\dagger \rangle_U = 0$ calculated in the mean-field state with AF order (solid curve) and in the QMC simulations by White et al. [16] (open circles) for $n=1$, $N = 4 \times 4$, and $T = t/16$.

FIG. 2. (a) Local spin-singlet amplitude $\langle |\Delta_d|^2 \rangle$ calculated in the mean-field state with AF order for $U = 12t$, $T = 0$. $\langle |\Delta_d|^2 \rangle_0$ is subtracted for purely interaction induced effect. (b) Effective strength $V_{ind}$ of the local spin-singlet amplitude induced by short-range spin correlations for $U = 12t$, $T = 0$. 

[16] S. R. White, D. J. Scalapino, R. L. Sugar, E. Y. Loh, J. E. Gubernatis, and R. T. Scalettar, Phys. Rev. B 40, 506 (1989).

[17] G. Kotliar and A. E. Ruckenstein, Phys. Rev. Lett. 57, 1362 (1986); R. Meyer and P. Entel, Phys. Rev. B 47, 1099 (1993); J. K. Freericks and M. Jarrell, cond-mat/9407101 (unpublished).

[18] B. O. Wells, Z.-X. Shen, A. Matsuura, D. M. King, M. A. Kastner, M. Greven, and R. J. Birgeneau, Phys. Rev. Lett. 74, 964 (1995).

[19] F. Ronning, C. Kim, D. L. Feng, D. S. Marshall, A. G. Loeser, L. L. Miller, J. N. Eckstein, I. Bozovic, Z.-X. Shen, Science, 282, 2067 (1998).

[20] B. Kyung, S. Allen, A.-M. S. Tremblay, cond-mat/0010001 and to appear in Phys. Rev. B.

[21] B. Kyung, Phys. Rev. B, 62, 9083 (2000).
FIG. 3. Calculated energy dispersion for an insulating cuprate $\text{Ca}_2\text{CuO}_2\text{Cl}_2$. The dispersion is measured from $(\pi/2, \pi/2)$ point with $\Delta_{sdw} = U/2$ and $\Delta = 2.07t$. The solid and dashed curves in the inset denote the dispersions calculated from our approach, and from the flux phase in the $t-J$ model, respectively.