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Gauge dependence of the on-shell renormalized mixing matrices

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It was recently pointed out that the on-shell renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the method by Denner and Sack causes a gauge parameter dependence of the amplitudes. We analyze the gauge dependence of the on-shell renormalization of the mixing matrices both for fermions and scalars in general cases, at the one-loop level. We then show that this gauge dependence can be avoided by fixing the counterterms for the mixing matrices in terms of the off-diagonal wave function corrections for fermions and scalars after a rearrangement, in a similar manner to the pinch technique for gauge bosons. We finally present explicit calculation of the gauge dependence for two cases: the CKM matrix in the standard model, and left-right mixing of scalar quarks in the minimal supersymmetric standard model.

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I. INTRODUCTION

Particles in the same representation under unbroken symmetries can mix with each other. The neutral gauge bosons, quarks, and massive neutrinos in the standard model (SM) are well-known examples. New particles in extensions of the standard model also show the mixings. For example, in the minimal supersymmetric (SUSY) standard model (MSSM) [1], a very promising extension, superpartners of most SM particles show the mixing [1,2]. The mixing of particles is expressed in terms of the mixing matrix, which represents the relations between the gauge eigenstates and the mass eigenstates of the particles. The mixing matrices always appear at the couplings of these particles in the mass eigenbasis.

Because of the fact that mass eigenstates at the tree-level mix with each other by radiative corrections, the mixing matrices have to be renormalized [3,4] to obtain ultraviolet (UV) finite amplitudes. Denner and Sack have proposed [4] a simple scheme to renormalize the mixing matrix of Dirac fermions at the one-loop level, which is usually called the on-shell renormalization scheme. They have required the counterterm for the renormalized mixing matrix to completely absorb the antihermitian part of the wave function correction $\delta Z_{\alpha\beta}$ for the external on-shell fields. This definition works very well for the subtraction of the ultraviolet divergence and dependence on the renormalization scale. The renormalization procedure is universal for any processes with the particles as external states. It also absorbs the $O(1/(m_i^2-m_j^2))$ terms which are singular for the case $m_i=m_j$. The on-shell scheme was also applied to the mixing of other fields, such as Majorana fermions [5] and complex scalar particles [6].

However, it has recently been pointed out [7–9] that in the on-shell scheme of Ref. [4] the counterterms for the Cabibbo-Kobayashi-Maskawa (CKM) matrix [10] is dependent on the gauge fixing parameter and that, as a consequence, the amplitudes of charged current interactions of quarks are also gauge dependent in this scheme. This fact motivated these authors to introduce other ways for the UV finite renormalization of the CKM matrix [7,9]. However, their method cannot be directly applied to mixings of other particles.

In this paper we study the gauge parameter dependence of the on-shell renormalized mixing matrices in general cases. We demonstrate that this gauge dependence is a general feature for the on-shell mixing matrices. Nevertheless, at the one-loop level the on-shell mixing matrices by Ref. [4] can be modified to be gauge independent by the following procedure. First, we split the gauge-dependent parts of the wave function corrections in the similar way to the “pinch technique” [11–13]. They are then rearranged into the corresponding vertex corrections in the amplitudes and cancelled. Next, we give the counterterm for the on-shell renormalized mixing matrices in terms of the remaining, gauge-independent part of $\delta Z_{\alpha\beta}$. The subtraction of the UV divergence and of the $O(1/(m_i^2-m_j^2))$ singularity is not affected by this modification. This method can be applied in a similar manner for mixings of both fermions and of scalars.

This paper is organized as follows. In Sec. II we review the one-loop on-shell renormalization of the mixing matrices for scalars and fermions in general case. In Sec. III their gauge dependences are analyzed by using the Nielsen identities [14–16] for self-energies of scalars and fermions. We then show that the gauge dependences of the off-diagonal wave function corrections and, in consequence, of the on-shell mixing matrices can be split by the rearrangement of the loop corrections. Sections IV and V present two explicit calculations of the gauge dependence of mixing matrices: CKM matrix of quarks in the SM and left-right mixing of scalar quarks (squarks) in the MSSM. Section VI gives our conclusion.

II. ON-SHELL RENORMALIZATION OF MIXING MATRICES

Let $\psi_\alpha$ (with index $\alpha$) be fields in gauge eigenstates, either real or complex scalars, or chiral components of Dirac or Majorana fermions. The fields in common representation under unbroken symmetries may mix with each other to form mass eigenstates. The relation between gauge eigenstates $\psi_\alpha$ and tree-level mass eigenstates $f_i$ with masses $m_i$ is expressed by an unitary matrix $U$ as
\[ f_i = U_{ia} \psi_a, \quad \psi_a = U_{ia}^* f_i. \]  

(1)

The mixing matrix \( U \) is determined such that the tree-level mass matrix for \( f_i \) is diagonal. The couplings of \( f_i \) are always multiplied by \( U \). For example, an amplitude \( \mathcal{M}_i \) with one incoming external \( f_i \) is expressed as

\[ \mathcal{M}_i = \sum_a \mathcal{M}_{ia} U_{ia}^*. \]  

(2)

where \( \mathcal{M}_a \) has no \( U \) dependence. \( U \) is therefore very important parameter for \( f_i \). Note that, when \( f \) are fermions, the mixing matrices \( U^L \) and \( U^R \) for chiral components \( f_L \) and \( f_R \), respectively, are generally different from each other.

By radiative corrections, the wave functions of \( f_i \) should be renormalized. The on-shell renormalized fields \( f_i \) are related to the unrenormalized \( f_i \) by, at the one-loop level

\[ f_i^{(0)} = (\delta_{ij} + \frac{1}{2} \delta Z_{ij}) f_j. \]  

(3)

The off-diagonal parts of \( \delta Z_{ij} (i \neq j) \) represent the mixing between \( f_i \) and \( f_j \). For the relation (1) is modified as

\[ \psi_a = U_{ia}^{(0)*} f_i^{(0)} = U_{ia}^{(0)*} (\delta_{ij} + \frac{1}{2} \delta Z_{ij}) f_j. \]  

(4)

the wave function correction to the amplitude (2) is expressed as the replacement of \( U \) by

\[ U_{ia}^{(0)*} \rightarrow U_{ia}^{(0)*} (\delta_{ij} + \frac{1}{2} \delta Z_{ij}). \]  

(5)

This correction is universal in any processes involving on-shell external \( f_i \).

The explicit from of \( \delta Z_{ij} \) for \( i \neq j \) is given in terms of the off-diagonal, flavor-mixing parts of the self energy\(^1\) of the fields \( f \). For scalars with unrenormalized, dimensionally regularized self energy \( \Pi_{ij}(p^2) \), we have

\[ \frac{1}{2} \delta Z_{ij} = \frac{1}{m_i^2 - m_j^2} \Pi_{ij}(m_i^2). \]  

(6)

For Dirac fermions with self-energy

\[ \Sigma_{ij}(p^2) = \Sigma_{Lij}(p^2) \gamma^\mu P_L + \Sigma_{Rij}(p^2) \gamma^\mu P_R + \Sigma_{DLij}(p^2) P_L + \Sigma_{DRij}(p^2) P_R, \]  

(7)

the corrections to chiral components of the wave functions \((f_{IL}, f_{IR})\) are \([17]\)

\[ \frac{1}{2} \delta Z_{ij}^L = \frac{1}{m_i^2 - m_j^2} \left[ (m_j^2 + m_i^2) \Sigma_{Lij}(m_j^2) + m_i m_j (\Sigma_{Rij}(m_j^2) + \Sigma_{DLij}(m_j^2)) \right] + m_i m_j (\Sigma_{DLij}(m_j^2) + \Sigma_{DRij}(m_j^2)), \]  

\[ \frac{1}{2} \delta Z_{ij}^R = \frac{1}{m_i^2 - m_j^2} \left[ m_i m_j (\Sigma_{Lij}(m_j^2) + \Sigma_{DRij}(m_j^2)) \right] + m_i m_j (\Sigma_{DLij}(m_j^2) + \Sigma_{DRij}(m_j^2)), \]  

(9)

respectively. Both Eqs. (6) and (9) have the factor \( 1/(m_i^2 - m_j^2) \) which is unique for the off-diagonal wave function corrections. These \( \delta Z_{ij} \) are UV divergent and depend on the gauge fixing parameters \( \xi \) for the massive gauge bosons. Note also that \( \delta Z_{ij} \) superficially diverge when the masses \((m_i, m_j)\) of \( f_i \) and \( f_j \), respectively, become close to each other. For the case of Majorana fermions \([5]\), the self-energy (7) obeys additional conditions

\[ \Sigma_{Lij}(p^2) = \Sigma_{Rij}(p^2), \quad \Sigma_{DLij}(p^2) = \Sigma_{DLij}(p^2), \quad \Sigma_{DRij}(p^2) = \Sigma_{DRij}(p^2). \]  

(10)

The condition for the wave function corrections, \( \delta Z_{ij}^L = \delta Z_{ij}^R \), which is necessary for keeping the Majorana condition \( U^L = U^R \) after renormalization, then follows from Eqs. (9), (10). All subsequent discussions in this and the next sections remain unchanged by the conditions (10).

For the cancellation of the UV divergence of off-diagonal \( \delta Z_{ij} \) in Eq. (5), the mixing matrix \( U \) has to be renormalized \([3,4]\). Assume that the renormalized \( U \) is related to the bare \( U^{(0)} \) by

\[ U_{ia}^{(0)} = (\delta_{ij} + \delta u_{ij}) U_{ja}. \]  

(11)

Since both \( U^{(0)} \) and \( U \) are unitary, the counterterm \( \delta u \) should be antihermite. The correction factor (5) is then rewritten as

\[ U_{ja}^{(0)*} (\delta_{ij} + \frac{1}{2} \delta Z_{ij}) = U_{ja}^{(0)*} (\delta_{ij} + \frac{1}{2} \delta Z_{ij} - \delta u_{ij}). \]  

(12)

The UV divergent part of \( \delta u \) is determined \([4]\) such as to cancel that of the antihermitian part of \( \delta Z \). For fermions, also the UV divergence of the diagonal \( CP \)-violating part

\[ \frac{i}{2} \text{Im}(\delta Z_{ii}^L) = - \frac{i}{2} \text{Im}(\delta Z_{ii}^R) = \frac{i}{2m_i} \text{Im}[\Sigma_{DLij}(m_i^2)], \]  

(13)

in the convention\(^2\) which is valid both for Dirac and Majorana fermions, has to be cancelled by \( \delta u_{ii} \). The earlier UV divergence of \( \delta u \) is consistent with the running of the mass

\[^1\text{We assume that the absorptive part of the self energy is negligible. For its correct inclusion one has to treat } f \text{'s as unstable intermediate states.}\]

\[^2\text{For Dirac fermions, one may make the shift } (\delta Z_{ii}^L, \delta Z_{ii}^R) \rightarrow (\delta Z_{ii}^L + i \theta_i, \delta Z_{ii}^R + i \theta_i) \text{ by an arbitrary imaginary number } i \theta_i. \text{ This is equivalent to the phase rotation } (f_{IL}, f_{IR}) \rightarrow (e^{i \theta_i} f_{IL}, e^{i \theta_i} f_{IR}) \text{ in Eq. (3). This freedom is killed by the Majorana condition. See Ref. [5] for details.}\]
matrix of $f$ in the gauge eigenbasis [18,19,5]. The renormal-
ized mixing matrix $U$ is then given by specifying the finite part of $\delta u$.

The modified minimal subtraction ($\overline{MS}$) scheme is sim-
plest and proven to give gauge-independent renormalized pa-
rameters and different parts of the amplitude is often quite
delicate and complicated. In addition, the
parameters
special and proven to give gauge-independent renormalized pa-

Equation

\begin{equation}
\frac{1}{4} (\delta Z_{ij} - \delta Z_{ji}^\dagger). \tag{14}
\end{equation}

This is usually called the on-shell renormalization of the mixing matrix. Equation (5) is then rewritten as

\begin{equation}
U_{fa}^{(0)*} = \left(\begin{array}{c}
\delta u_{fi} + \frac{1}{4} (\delta Z_{fi} + \delta Z_{fi}^\dagger) \end{array}\right) \cdot (U_{fa}^{(0)*})_{ji} \cdot \left(\begin{array}{c}
\delta u_{ji} + \frac{1}{4} (\delta Z_{ji} + \delta Z_{ji}^\dagger) \end{array}\right). \tag{15}
\end{equation}

One important feature of Eq. (15) is that all $O(1/(m_f^2 - m_i^2))$ singularities in $\delta Z_{fi}$ are absorbed into the renormal-
ized $U^{(0)*}$. Also, $U^{(0)*}$ is independent of the MS renormaliza-
tion scale. These properties are equally valid for both fermi-
ons and scalars.

The mixing of quarks in different generations needs special-
care for there is no unique “gauge eigenbasis” for them. Instead, one can discuss only the difference between the mixing of left-handed up-type quarks and that of down-type quarks, namely the CKM matrix $V_{ij} = (U_{fa}^{(0)*})_{ji}$. The counterterm for the on-shell CKM matrix is then given by [4,7]

\begin{equation}
\delta V_{ij} = \delta u_{ik}^{d*} V_{kj} + \delta u_{jk}^{d*} V_{ik}, \tag{16}
\end{equation}

where $\delta u^{d*}$ are given by Eq. (14).

III. GAUGE DEPENDENCE OF WAVE FUNCTION CORRECTIONS AND ON-SHELL MIXING MATRICES

Since the proposal in Ref. [4], however, the dependence of the on-shell mixing matrix on the gauge fixing parameters $\xi$ has not been examined for a long time. Recent studies [7–9] showed that the on-shell renormalization of the CKM matrix introduces gauge dependence into one-loop amplitudes for the $W^+ \rightarrow u_i \bar{d}_j$ decays through the counterterm $\delta V_{ij}$. They proposed alternative definitions for quark mixing matrix which are independent of the renormalization scale. References [7,8] used a modified process-independent definition for the CKM matrix. As shown in this section, their definition strongly relies on the gauge representation of quarks. Reference [9] fixed the renormalized CKM matrix by using the amplitudes of the decays $W^+ \rightarrow u_i \bar{d}_j$ (or $t \rightarrow W^+ d_j$). To keep the renormalized CKM matrix unitary, four processes have to be selected out of nine possible ones. As a result, the forms of the corrected amplitudes become very asymmetric with respect to generation indices $(i,j)$. Thus, both methods cannot be directly applied for the renormaliza-
tion of other mixing matrices. In this section we show an-
other way to avoid the problem of gauge dependence of the

on-shell scheme of Ref. [4].

We first investigate the gauge parameter dependences of the wave function correction $\delta Z$ and of the counterterm $\delta u$ for the on-shell mixing matrix in general cases. We use the fact that, in the $R_\xi$ gauge, the dependence of the one-particle irreducible Green functions on the gauge parameters $\xi$ is controlled by the Nielsen identities [14,15], a kind of the Slavnov-Taylor identities which follow from the extended Becchi-Rouet-Stora (BRS) symmetry [15] of the theory. The identity for the gauge parameter dependence of the inverse propagator $\Gamma_{ij}(p)$ for the transition $f_j \rightarrow f_i$ takes the following form [16]:

\begin{equation}
\frac{\partial}{\partial \xi} \Gamma_{ij}(p) = \partial \Gamma_{ij}(p) / \partial \xi \notag
\end{equation}

Here $\Gamma_{ij}(p)$ is the vertex function with $f_i, \chi$, the “BRS variation” of the gauge parameter $\xi$ [15,16], and $K_j$, the source associated with the BRS variation of $f_j$. $\Gamma_{ij}(p)$ is its conjugate. Since the identity (17) is determined by the form of the gauge-fixing terms [16], it holds for general gauge theories in the $R_\xi$ gauge fixing. In Eq. (17) $\xi$s are assumed to be physical fields with gauge-independent masses, not the would-be Nambu-Goldstone (NG) bosons, Fadeev-Popov ghosts, or longitudinal modes of gauge bosons. Under this condition $\Gamma_{ij}(p)$ has no tree level con-
tribution. It is also required that the renormalization does not introduce additional gauge dependence [16]. Especially, the
shift of the vacuum expectation values (VEVs) of Higgs bosons by tadpole graphs should be cancelled in a gauge-

independent way.

The gauge dependence of the one-loop two-point func-
tions $\Sigma_{ij}(p)$ of fermions is, in the tree-level mass basis, de-


\begin{equation}
\partial \Sigma_{ij}(p) = \Lambda_{ij}(p) (\not p - m_j) + (\not p - m_i) \tilde{\Lambda}_{ij}(p), \tag{18}
\end{equation}

where $\Lambda(p)$ and $\tilde{\Lambda}(p)$ are some one-loop Dirac spinors. Af-
ter the decomposition

\begin{equation}
\Lambda_{ij}(p) = \Lambda_{ij}(p^2) \not p L + \Lambda_{Rij}(p^2) \not p R + \Lambda_{DLij}(p^2) P_L + \Lambda_{DLij}(p^2) P_R, \tag{19}
\end{equation}

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and similar one for $\bar{\Lambda}$, the $\xi$ dependence of the components of $\Sigma$ in Eq. (7) is [16]

$$\partial_\xi \Sigma_{Lij} = -m_j \Lambda^\times_{Lij} - m_i \bar{\Lambda}_{Lij} + \Lambda^\times_{DRij} + \bar{\Lambda}^\times_{DLij},$$

$$\partial_\xi \Sigma_{Rij} = -m_j \Lambda^\times_{Rij} - m_i \bar{\Lambda}_{Rij} + \Lambda^\times_{DLij} + \bar{\Lambda}^\times_{DRij},$$

$$\partial_\xi \Sigma_{DLij} = -p^2 \Lambda^\times_{Rij} + p^2 \bar{\Lambda}_{Rij} - m_i \Lambda^\times_{DLij} - m_i \bar{\Lambda}^\times_{DLij},$$

$$\partial_\xi \Sigma_{DRij} = p^2 \Lambda^\times_{Lij} + p^2 \bar{\Lambda}_{Lij} - m_j \Lambda^\times_{DRij} - m_j \bar{\Lambda}^\times_{DRij}.$$  

(20)

The relations

$$\bar{\Lambda}^\times_{Lij} = \Lambda^\times_{Lji}, \quad \bar{\Lambda}^\times_{Rij} = \Lambda^\times_{Rji}, \quad \bar{\Lambda}^\times_{DLij} = \Lambda^\times_{DLji}, \quad \bar{\Lambda}^\times_{DRij} = \Lambda^\times_{DRji},$$

(21)

follow from the hermiticity of the effective action.

By substituting them into Eq. (9), we obtain [7] for $i \neq j$

$$\frac{1}{2} \partial_\xi (\partial_\xi Z_{ij}) = -m_j \bar{\Lambda}_{Rij}(m_j^2) - \bar{\Lambda}^\times_{DLij}(m_j^2),$$

(22)

and similar result for $\partial_\xi Z^\times_{ij}$. As a result, the original definition of the on-shell renormalized fermion mixing matrices in Eq. (14) has gauge parameter dependence. Explicit calculation shows that the gauge dependence of the counterterm $\partial_\xi u^\times_{ij}$ is equal to

$$\frac{1}{2} \partial_\xi (\partial_\xi Z_{ij} - \partial_\xi Z^\times_{ij}) = \frac{1}{2} \left[ -m_j \bar{\Lambda}_{Rij}(m_j^2) - \bar{\Lambda}^\times_{DLij}(m_j^2) + m_i \bar{\Lambda}^\times_{Rij}(m_i^2) + \Lambda^\times_{DLji}(m_i^2) \right],$$

(23)

which does not vanish in general. This is also the case for $\partial_\xi u^\times_{ij}$ and $\partial_\xi u_{ji}$.

A remarkable fact in Eq. (22) is that the factor $1/(m_i^2 - m_j^2)$, which characterizes the off-diagonal $\partial_\xi Z_{ij}$, is cancelled for the gauge dependence. This is expected from the gauge independence of the total amplitudes [21] with gauge-dependent renormalization of the couplings. Since the gauge dependence of Eq. (22) has to be cancelled by that from other parts of the amplitudes which do not have the factor $1/(m_i^2 - m_j^2)$, the factor cannot remain in Eq. (22). Similar cancellation occurs in the gauge dependence of the diagonal part $\partial_\xi u^\times_{ii} = -\partial_\xi u_{ii}$, which is equal to

$$\frac{i}{2} \partial_\xi (\text{Im } \partial_\xi Z^\times_{ii}) = \frac{i}{2} \text{Im} \left[ -m_i \bar{\Lambda}_{Rii}(m_i^2) + m_i \bar{\Lambda}^\times_{Lii}(m_i^2) + \bar{\Lambda}^\times_{DRii}(m_i^2) - \bar{\Lambda}^\times_{DLii}(m_i^2) \right].$$

(24)

The factor $1/m_j$ in Eq. (13), which characterizes $\text{Im}(\partial_\xi a^\times)$, is cancelled in Eq. (24). Another important point is that Eqs. (23, 24) are UV finite.

The mixing matrices of the scalars can be analyzed in the similar way. The one-loop two-point function $\Pi_{ij}(p^2)$ for scalars in the tree-level mass basis obeys the relation [16]

$$\partial_\xi \Pi_{ij}(p^2) = \Lambda_{ij}(p^2)(p^2 - m_i^2) + \Lambda_{ij}^\times(p^2),$$

(25)

from the Nielsen identity. We assume that there are no mixings with unphysical modes. By substitution we obtain for $i \neq j$

$$\frac{1}{2} \partial_\xi (\partial_\xi Z_{ij}) = -\Lambda_{ij}^\times(m_j^2).$$

(26)

The gauge dependence of the counterterm (14) for the on-shell mixing matrix for scalars is therefore

$$\partial_\xi (\partial_\xi u_{ij}) = -\frac{1}{2} \left[ \Lambda_{ij}^\times(m_j^2) - \Lambda_{ij}(m_j^2) \right],$$

(27)

which is UV finite but does not cancel in general. However, the factor $1/(m_i^2 - m_j^2)$ is again cancelled in Eq. (27).

According to the earlier observation, we can define the gauge-independent one-loop on-shell mixing matrices for fermions and scalars as follows. First, we split gauge-dependent parts without the factor $1/(m_i^2 - m_j^2)$ from $\partial_\xi Z_{ij}$ and regard them as parts of the corrections to the attached vertex. They are eventually cancelled by the gauge dependence of the vertex and other corrections. Second, we give the counterterms for mixing matrices in terms of the remaining, gauge-independent part of $\partial_\xi Z_{ij}$. This procedure gives the one-loop corrected amplitudes which are expressed in terms of the on-shell mixing matrices and manifest gauge independence. Of course, the choice of the gauge-independent parts of $\partial_\xi Z_{ij}$ has arbitrariness. For example, we can regard the results in the $R \xi$ gauge with a given $\xi$ as the gauge-independent parts.

Here we propose a method to specify the gauge-invariant parts of $\partial_\xi Z_{ij}$, inspired by the pinch technique [11–13] to define gauge-independent form factors for gauge bosons. We consider a general process with the external on-shell particle $f_j$ which is either a fermion or a scalar, with incoming momentum $p$. One source of the gauge dependence of $\partial_\xi Z_{ij}$ is the graph of Fig. 1(a). As pointed out in Refs. [11,12], the longitudinal part of the propagator of the (massive) gauge boson $A$ triggers the Ward identity at the vertex $\mu$ as

$$k = -(\not{p} - m_i) + (\not{k} + \not{p} - M) + (M - m_i),$$

(28)

for fermions, or

$$k^\mu(k + 2p)_\mu = -(p^2 - m_i^2) + (k^\mu + (p^\mu)^2 - M^2) + (M^2 - m_i^2),$$

(29)

for scalars, respectively. The first two terms of Eqs. (28, 29) cancel the propagators of $f_j$ and of the intermediate particle $F$ with a mass $M$, respectively, and yield the contributions of Figs. 1(b, c) (pinching). The last terms of Eqs. (28, 29) are the effect of the spontaneous breaking of the gauge symmetry for $A$ and are proportional to the couplings to the associated NG boson. The part of Fig. 1(a) where the last terms are picked up at the vertex $\mu$ is further decomposed into three parts by the Ward identity (28, 29) at $v$. The part which cancels $f_j$ propagator vanishes in on-shell amplitudes, while that which cancels $F$ propagator is included in the type of Fig. 1(c). The remaining part where the last terms of the Ward identity are picked up at both vertices does not fit into Figs. 1(b, c). To satisfy the Nielsen identities (18, 25), this part has to be combined with the contribution from Fig. 1(d)

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FIG. 1. The gauge-dependent contributions to δZij for a general process with external on-shell fj, which is either a fermion or a scalar, from the loops of massive gauge boson A and intermediate particle F. Graphs (b, c) are the “pinch terms” stemming from (a). Graph (d) is a contribution of the NG boson φA associated with A. Graph (e) represents the shift of the VEV of Higgs bosons h by the loops of A, φA, and Faddeev-Popov ghosts. Inclusion of (e) is necessary for gauge-independent renormalization of the Higgs VEVs.

by the NG boson φA to yield a gauge-independent sum. This result should be thus equal to the contribution of Fig. 1(d) in the ξ = 1 gauge.

The contribution of Fig. 1(b) is manifestly consistent with the Nielsen identity. In contrast, the remaining gauge-dependent part, Fig. 1(c), cannot satisfy the identity by itself because of its p independence. This part has to be cancelled by the contributions from the Higgs VEV shift [Fig. 1(e)] by the loops of unphysical modes for A and, in the case of scalars, by the “seagull” contributions with four-point couplings fμνAfμνAμ [the same topology as Fig. 1(c)] and fμν fhφA. Again, the result should be gauge independent and therefore equal to the one in the ξ = 1 gauge. We have verified that, for the cases discussed in Secs. IV and V, the earlier cancellation of the gauge dependence really occurs and that the contribution of Fig. 1(b) is equal to the difference from the result in the ξ = 1 gauge.

It is then natural to identify the contribution of Fig. 1(b) to δZij, as the gauge-dependent pinch term, in analogy to Ref. [12], and to regard this as a part of the vertex corrections. Then, in this manner, we may regard the on-shell mixing matrices in the ξ = 1 gauge as the gauge-independent ones. The cancellation of the UV divergence, renormalization scale dependence, and the O(1/(m2i − m2j)) singularity is not affected by this modification of the original definition of the on-shell mixing matrices. Note that the agreement of the ξ = 1 and the pinch technique results has been observed for the QCD correction to the off-shell quark propagator [13]. Note also that we have not considered, for scalars, the possible trigger of the Ward identity (29) at the vertex μ by the momentum (k + 2p)μ at the vertex ν in Fig. 1(a), which was done for the couplings of the gauge and NG bosons [12] to satisfy the Ward identities among correlated vertices.

We finally comment on other definitions for the UV finite and process-independent mixing matrices for fermions. As the first example, Ref. [22] proposed a definition of the UV finite and momentum-dependent effective mixing matrices [Ũ(p2), ŨR(p2)] for fermions. The counterterms for Ũ are given by, instead of Eq. (9),

$$\delta \bar{u}_{ij}^T(p^2) = \frac{1}{m_i^2 - m_j^2} \left[ \frac{1}{2} (m_i^2 + m_j^2) \Sigma_{Lij}(p^2) + m_i \Sigma_{Rij}(p^2) + m_i \Sigma_{DLij}(p^2)^2 + m_i \Sigma_{DRIj}(p^2)^2 \right]$$

and similar form for δũR(p2). Similar to the on-shell U by Ref. [4], Ũ(p2) absorbs the O(1/(m2i − m2j)) singularity when the couplings of fi are expressed in terms of Ũij(p2) = m2i.

Unfortunately, this definition also shows gauge dependence. From Eq. (20) we obtain

$$\delta \bar{u}_{ij}^T(GGM) = \frac{1}{m_i^2 - m_j^2} \left[ \frac{1}{2} (m_i^2 + m_j^2) \Sigma_{Lij}(0) + m_i \Sigma_{DLij}(0) \right]$$

This definition gives the renormalized CKM matrix which is gauge-independent and UV finite. However, its validity relies on the fact that quark couplings to W± are purely left-handed. Moreover, Eq. (32) does not absorb the O(1/(m2i − m2j)) singularity. Thus, this definition has to be greatly modified for the renormalization of other mixing matrices.

IV. CKM MATRIX: EXAMPLE FOR FERMION MIXING

In this and the next sections we show the explicit form of the gauge dependence of the on-shell mixing matrices, both for fermions and for scalars. In this section we discuss the on-shell CKM matrix, following previous studies [7,8].

The off-diagonal parts of the one-loop self energies Σij(p) of the quarks receive gauge-dependent contribution from the W± loops [3,4]. The ξW dependent part of Σij(p) for up-type quarks $u_i = (u, c, t)$, namely the difference from
the result in the $\xi_W = 1$ gauge, takes the following form:

$$\Sigma_{ij}^u(p)|_{\xi_W = (1 - \xi_W)} \frac{g_2^2}{32\pi^2} \sum_k V_{ik} V^*_{jk} \left( \hat{p} - m_u \right) \times \beta_{Wd_i}^{(1)}(p^2) \hat{p} P_R(\hat{p} - m_u) - (\hat{p} - m_u)$$

$$\times P_L \left[ m_d^2 \beta_{Wd_i}^{(0)}(p^2) - m_u \beta_{Wd_i}^{(1)}(p^2) \hat{p} + \frac{1}{2} \alpha_w \right]$$

$$\times P_R(\hat{p} - m_u) \right) \right\}. \tag{33}$$

Here we define

$$\frac{i}{16\pi^2} \alpha_i = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_i^2)(q^2 - \xi m_i^2)}, \tag{34}$$

$$\frac{i}{16\pi^2} \beta_{ij}^{(0)}(p^2)$$

$$= \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_i^2)(q^2 - m_j^2)(q^2 - \xi m_i^2)(q^2 - \xi m_j^2)(q + p)^2 - m_j^2}, \tag{35}$$

$$\frac{i}{16\pi^2} \beta_{ij}^{(1)}(p^2) \beta_{ij}^{(1)}(p^2) \mu$$

$$= \int \frac{d^4q}{(2\pi)^4} \frac{(q + p)_{\mu}}{(q^2 - m_i^2)(q^2 - \xi m_i^2)(q^2 - \xi m_j^2)(q^2 - \xi m_j^2)(q + p)^2 - m_j^2}, \tag{36}$$

where $n = 4 - 2\epsilon$. $\Sigma_{ij}^d(p)|_{\xi_W = \xi_W}$, for down-type quarks $d_i = (d, s, b)$ is obtained by replacing $(u, u, d, k, V_{ik} V^*_{jk})$ in Eq. (33) by $(d, d, u, u, k, V^*_{ik} V_{jk})$. Equation (33) is equivalent to the results in Refs. [8,16], except that Eq. (33) includes the gauge-dependent part of the HiggsVEV shift in $\Sigma_{ij}^d$, by tadpoles with $W^\pm$ and associated unphysical modes. This corresponds to defining renormalized Higgs VEV as the minimum of the tree-level potential [23,24,16], which is gauge-independent in the MS scheme. By the addition of the Higgs VEV shift, Eq. (33) manifestly satisfies the Nielsen identity (18). Instead, one may also add the counterterms for pole masses of quarks to the diagonal elements to satisfy Eq. (18). This difference does not affect the present discussion.

The counterterm for the on-shell CKM matrix in the original definition [4], without separating Eq. (33), has gauge dependence as

$$\delta V_{ij} = X_{ik} V_{kj} + V_{il} X_{lj}^d. \tag{37}$$

$X_{ik}^d$ (i $\neq k$) is obtained from Eq. (33) as

$$X_{ik}^n = (1 - \xi_W) \frac{g_2^2}{64\pi^2} V_{ik} V^*_{jl} \left[ m_u^2 \beta_{Wd_i}^{(0)}(m_u^2) + m_u^2 \beta_{Wd_i}^{(1)}(m_u^2) \right] + m_d^2 \beta_{Wd_i}^{(0)}(m_u^2) - m_d^2 \beta_{Wd_i}^{(1)}(m_u^2) \right]. \tag{38}$$

$X_{ij}^d$ has a similar form. Equation (37) causes gauge-dependent amplitudes for the $W\mu, d$ interactions [7–9]. Numerically, Eq. (37) is greatly suppressed, partly by the Glashow-Iliopoulos-Maiani (GIM) mechanism [25], and completely negligible in practice [4]. The relative corrections are largest to $(V_{cb}, V_{ub}, V_{td}, V_{td})$, but are at most $O(10^{-6})$. Nevertheless, this is not satisfactory for theoretical point of view. The study in previous section shows, however, that one can give the counterterm $\delta V$ in terms of $\xi = 1$ parts of $\Sigma_{ij}^d$ and $\Sigma_{ij}^d$. The original calculation in Ref. [4] is thus interpreted as a gauge-independent one after the rearrangement.

V. LEFT-RIGHT MIXING OF SQUARKS: EXAMPLE FOR SCALAR MIXING

We next consider the renormalization of the left-right mixing of squarks in the MSSM, for an example for the mixing of scalar particles. For simplicity, we treat the mixing of two eigenstates of the top squarks, ignoring $CP$ violation and mixing with different generations.

The gauge eigenstates ($\tilde{q}_L, \tilde{q}_R$) of squarks, which are the superpartners of a quark $q$, mix with each other by spontaneous breaking of SU(2)$\times$U(1) gauge symmetry [1,2]. Their mass eigenstates $\tilde{q}_i$ ($i = 1, 2$) are related to the gauge eigenstates $\tilde{q}_a$ ($a = L, R$) by $\tilde{q}_i = R^a_{i\alpha} \tilde{q}_a$ with the left-right mixing matrix

$$R_{i\alpha} = \begin{pmatrix} \cos \theta_\alpha & \sin \theta_\alpha \\ -\sin \theta_\alpha & \cos \theta_\alpha \end{pmatrix}. \tag{39}$$

The renormalization of the squark sector is often performed by specifying the poles masses of $\tilde{q}_1, \tilde{q}_2$ and the mixing angle $\theta_\alpha$, as in Refs. [26–32,6,33,34]. Following the result in Sec. II, the counterterm $\delta \theta_\alpha$ is given by [6,34]

$$\delta \theta_\alpha = \delta r_{1\alpha} = \frac{1}{2} \frac{\Pi_{10}^2(m_{\tilde{q}_1}^2 + \Pi_{10}^2(m_{\tilde{q}_2}^2))}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \right\}. \tag{40}$$

with $\Pi_{10}^2(p^2)$ the off-diagonal self-energy of squarks in the tree-level mass basis. Although many other on-shell definitions [26–32] have been used in the studies of the SUSY QCD corrections, they are either unable to be applied for other loop corrections, or too specific for the squark processes considered there.

We consider the on-shell mixing matrix for top squarks $\tilde{T}_1$. The gauge-dependent part of the unrenormalized two-point function $\Pi_{10}^2(q^2)$, namely the difference from the results in the $\xi = \xi_W = 1$ gauge [35], takes the following form:
\[
\Pi_{ij}^\ell(p^2)|_\xi = \frac{\hat{s}_W^2}{16\pi^2} (1 - \xi_Z) \sum_k (R_k^i R_k^j T_{3\ell} - \delta_{k3} \delta_{ij} s_W^2 Q_(R_k^i R_k^j T_{3\ell} - \delta_{k3} \delta_{ij} s_W^2 Q_i)
\]
\[
\times \left[ -\frac{1}{2} (2p^2 - m_{t_i}^2 - m_{t_j}^2) \alpha_Z + \left((p^2 - m_{t_i}^2)(p^2 - m_{t_j}^2) + (p^2 - m_{t_j}^2) \right) \right. \\
\times (m_{t_i}^2 - m_{t_j}^2) + (m_{t_j}^2 - m_{t_i}^2)(p^2 - m_{t_j}^2) \beta_{Zi}^{(0)}(p^2) \] \\
\left. + \frac{\hat{g}_2^2}{32\pi^2} (1 - \xi_W) R_k^i R_k^j \right] \\
\times \left[ -\frac{1}{2} (2p^2 - m_{t_i}^2 - m_{t_j}^2) \alpha_W + \sum_k (R_k^i R_k^j T_{3\ell} - \delta_{k3} s_W^2 Q_i) \right. \\
\times (m_{t_i}^2 - m_{t_j}^2) + (m_{t_j}^2 - m_{t_i}^2)(p^2 - m_{t_j}^2) \beta_{Wb}^{(0)}(p^2) \] \\
+ (p^2 - m_{t_i}^2)(m_{t_j}^2 - m_{b_i}^2) + (m_{t_i}^2 - m_{b_i}^2)(p^2 - m_{t_j}^2) \beta_{Wb}^{(0)}(p^2) \right) .
\]

Here \( T_{3\ell} = 1/2, Q_\ell = 2/3 \), and \( s_W^2 = \sin^2 \theta_W \). As before, Eq. (41) includes the gauge-dependent shifts of the two Higgs VEVs for the on-shell renormalization. In contrast to the SM case, they also contribute to the gauge-dependent parts. The result (41) satisfies the Nielsen identity (25).

The magnitude of the gauge dependence of the on-shell \( \delta \theta_i \) is very sensitive to squark parameters. For a parameter choice \( (M_0, M_1, M_2) = (350,300,400) \) GeV, \( \tan \beta = 4 \), \( (\mu, A_1, A_0) = (-400,300,0) \) GeV, and \( 0 < \xi < 10 \), \( \xi_w \) and \( \xi_Z \) depend on \( \delta \theta_i \); they may be as large as 0.008, and 0.003, respectively. Although too small for realistic phenomenology, they are much larger than the \( \xi_w \) dependence of the on-shell CKM matrix. This is partly due to the absence of the GIM cancellation, following from that \( \tilde{t}_L \) and \( \tilde{t}_R \) have different gauge representations. As is already shown, these gauge dependence of \( \delta \theta_i \) can be avoided by removing the contribution of Eq. (41) from off-diagonal wave function corrections \( \delta Z_{12} \) for top squarks, cancelling it by other gauge dependences of the amplitude, and then giving \( \delta \theta_i \) by the remaining part of \( \delta Z_{12} \).

VI. CONCLUSION

In this paper, we investigated the gauge parameter dependence of the on-shell renormalized mixing matrices for scalars and fermions at the one-loop level. It has been shown recently that the on-shell renormalization of the CKM matrix in the definition by Ref. [4] is gauge dependent. By using the Nielsen identities for self-energies, we demonstrated that this gauge dependence exists for the on-shell mixing matrices in general cases. We also showed that this gauge dependence can be avoided by the following procedure: split the gauge-dependent parts from the off-diagonal wave function corrections in the manner similar to the pinch technique, and then give the counterterm for the mixing matrix in terms of the remaining, gauge-independent parts. The subtraction of the UV divergence and \( O(1/(m^2_{t_i} - m^2_{t_j})) \) singularity is not affected by this modification. The on-shell scheme in Ref. [4] in the \( \xi = 1 \) gauge can then be regarded as gauge-independent one. Finally, we presented explicit calculation of the gauge dependence of the mixing matrices in two cases, CKM matrix and left-right mixing of squarks, and verified the result from the Nielsen identities.

We did not treat the mixings of the gauge bosons and of the Higgs bosons. In principle, our method would also be applicable for these mixings. When applied to the mixing of the gauge bosons \( y \) and \( Z \), the square of the renormalized mixing angle \( \sin^2 \theta_W \) (OS) agrees with the effective angle \( s_W^2(m_Z^2) \) defined in Ref. [36], at the one-loop level. But the inclusion of the absorptive part of the Z boson propagator is necessary for realistic studies. The correction to the mixing of the MSSM Higgs bosons in diagrammatic calculation [37–39] is a very interesting subject. However, due to the mixing of physical Higgs bosons with unphysical modes, a separate consideration is necessary. We expect to study the case of the MSSM Higgs bosons in the future.

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