An axially symmetric spacetime with causality violation

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Abstract

We present an axially symmetric spacetime which contains closed timelike curves, and hence violates the causality condition. The metric belongs to type III in the Petrov classification scheme with vanishing expansion, shear and twist. The matter-energy represents a pure radiation field with a negative cosmological constant. The spacetime is asymptotically anti-de Sitter space in the radial direction.

1. Introduction

Time-travel is amongst the weirdest predictions of general relativity (GR). There are numerous solutions of the field equations of GR that allow for the formation of closed timelike curves (CTCs); trajectories in spacetime that allows a material object to return to a point in its past. The Gödel solution [1], describing a rotating universe, was the first one to be studied that contained CTCs. Van Stockum’s solution [2], found years before Gödel’s, was later shown to contain CTCs [3]. Some other prominent examples include the Kerr and Kerr-Newmann black holes solution where CTCs form inside the event horizon [4, 5], the NUT-Taub metric [6], Gott’s solution [7] of two infinitely long cosmic strings and the Alcubierre warp-drive [8] solution. All the above mentioned time machine spacetimes belong to the class of eternal time machines, where CTCs pre-exist. ‘True’ time machine spacetimes are those where CTCs form at some particular instant of time. The Ori time machine spacetime [9], which is locally isometric to plane wave spacetimes, is a prime example in this category. Some other examples would be the spacetimes discussed in [10–17]. It should be noted here that a workable model of a time machine must be a spacetime where CTCs appear at a definite instant of time.

Most of the time machine spacetimes discussed in the literature violate one or more of the energy conditions. The various energy conditions of the general theory with their interpretations are [18]

(i) Weak Energy Condition (WEC): Local mass-energy density must not be negative.
(ii) Strong energy condition (SEC): Gravitationally active matter and fields should tend to focus rather than defocus a collimated beam of light.
(iii) Dominant energy condition (DEC): Energy must not flow faster than light.
(iv) Null energy condition (NEC): Stress-energy experienced by a light ray must not be negative.

The WEC is violated in the spacetimes discussed in [8, 19–22], while the SEC is violated in [23–25]. It should be mentioned here that there are quite a number of causality violating solutions where there are no violations of the standard energy conditions of GR. A handful of such solutions would be the spacetimes discussed in [9, 13, 17, 26, 27]. The point of discussion here is that the violation of energy conditions is not a prerequisite for obtaining CTCs [9, 28]. The spacetime discussed in this work satisfies the energy conditions. The matter-energy of the spacetime represents a pure radiation field with a negative cosmological constant.

It is easy to fathom that time travel in the form of CTCs may lead to paradoxical circumstances [29]. The infamous grandfather paradox [30], which essentially posits that a time traveller could travel to the past and kill...
her grandfather which ultimately leads to the impossibility of her own birth, has always been a bitter pill to swallow for physicists.

A prominent theoretical result concerning CTCs is the Novikov’s self-consistency principle [31]. It states that the only events that can occur along a CTC are those which are globally self-consistent, i.e., a globally consistent solution of the local physical laws must exist. Very recently, the authors in [32] have shown how the Novikov’s principle is realized in quantum mechanics. Furthermore, they have demonstrated how the grandfather paradox can be resolved when we invoke quantum mechanics into the classical gravitation picture.

A spacetime with a negative cosmological constant is termed anti-de Sitter (AdS)-like. The AdS spacetime has huge theoretical implications, particularly after the advent of the AdS/CFT correspondence [33] in superstring theory. The AdS/CFT correspondence provides a connection between a quantum theory of gravity on an asymptotically AdS spacetime and a lower-dimensional conformal field theory (CFT) on the boundary of the spacetime.

In this work, we present an axially symmetric, Petrov type III solution of the field equations, which acts as a time machine spacetime. A family of Petrov type III solutions with a non-vanishing cosmological constant and aligned pure radiation field has been obtained in [34]. A few number of causality violating spacetimes with a pure radiation field as matter-energy content, with or without a cosmological constant or any fluid content, has been studied in [14, 17, 27, 35–37]. In [17] and [37], the studied spacetimes are of type II while the spacetimes in [14, 27, 35, 36] belong to type N in the Petrov classification scheme. The matter-energy content of the presented solution represents a pure radiation field in an asymptotically AdS spacetime. Here CTCs evolve from causally well-behaved initial conditions.

2. Analysis of the spacetime

The axially symmetric metric in \((r, \phi, z, t)\) coordinates is given by

\[
ds^2 = \frac{dr^2}{\alpha^2 + r^2} + r^2 dz^2 + \left(-2 r^2 dt + \frac{\beta z dr}{r^2} - t r^2 d\phi\right) d\phi
\]

(1)

Here, the constants \(\alpha\) and \(\beta\) are non-zero real numbers with \(\beta > 0\). The coordinates are labeled \(x^1 = r\), \(x^2 = \phi\), \(x^3 = z\) and \(x^4 = t\). The ranges of the coordinates are \(0 \leq r < \infty\), \(-\infty < z < \infty\), \(-\infty < t < \infty\) and \(\phi\) is a periodic coordinate \(\phi \sim \phi + \phi_0\) with \(\phi_0 > 0\). It is to be noted that the spacetime (1) has a coordinate singularity at \(r = 0\).

The non-zero components of the Ricci tensor for the spacetime (1) are:

\[
R_{rr} = -\frac{3}{r^2}, \quad R_{r\phi} = -\frac{3 \alpha^2 \beta z}{2 r^2}, \quad R_{\phi\phi} = \alpha^2 \left(24 r^6 t + \beta^2\right), \quad R_{\phi\phi} = 3 \alpha^2 r^2 = -R_{zz}
\]

(2)

The non-zero Einstein tensor components of the spacetime (1) are:

\[
G_{\mu\nu} = 3\alpha^2, \quad G_{\phi\phi} = -\frac{\alpha^2 \beta^2}{8 r^6}
\]

(3)

The determinant of the corresponding metric tensor \(g_{\mu\nu}\) is given by

\[
det g = -\frac{r^4}{\alpha^2}
\]

(4)

We have calculated some of the curvature invariants of the above metric (1), and find that they are constant everywhere:

\[
R_{\mu\nu} R^{\mu\nu} = 36 \alpha^4
\]

(5)

\[
R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 24 \alpha^4
\]

(6)

\[
R_{\mu\nu\rho\sigma;\lambda} R^{\mu\nu\rho\sigma;\lambda} = 0
\]

(7)

Thus the presented spacetime belongs to the class of solutions with constant curvature invariants (CCI spacetimes).

We proceed to show that the spacetime represented by (1) is axially symmetric. Consider the Killing vector \(\eta = \partial_\phi\) having the normal form
\( \eta^\mu = (0, 1, 0, 0) \) \hspace{1cm} (8)

Its covector is

\[
\eta_\mu = \left( \frac{\beta z}{2 r^2}, -t, 0, -r^2 \right)
\]

\hspace{1cm} (9)

The vector (8) satisfies the Killing equation \( \eta_{\mu \rho} + \eta_{\rho \mu} = 0 \). A cyclically symmetric spacetime admits a Killing vector with spacelike, closed orbits. Axial symmetry means that the spacetime contains a non-empty axis of symmetry. This is ensured if the norm of \( \eta^\mu \) vanishes on the symmetry axis, i.e., at \( r = 0 [38–43] \). In our case we find that the norm is

\[
\eta_\mu \eta^\mu = -r^2 t
\]

\hspace{1cm} (10)

Closed orbits of the above are spacelike for \( t < 0 \) and the norm vanishes on the symmetry axis. For \( t > 0 \) the norm of \( \eta_\mu \) changes sign, which indicates the formation of closed timelike curves. We will elaborate on this topic later.

Another Killing vector, which happens to be a null vector, allowed by the metric (1) is

\[
\xi^\mu = (0, 0, 0, e^{-\phi/2})
\]

\hspace{1cm} (11)

Furthermore, another Killing vector is the linear combination of \( \eta^\mu \) and \( \xi^\mu \)

\[
\xi^\mu = (0, 1, 0, e^{-\phi/2})
\]

\hspace{1cm} (12)

The field equations of gravitation are given by

\[
G^{\mu \nu} + \Lambda g^{\mu \nu} = T^{\mu \nu}, \quad \mu, \nu = 1, 2, 3, 4
\]

\hspace{1cm} (13)

where the various symbols have their usual meanings. The energy-momentum tensor of pure radiation field is given by [18]

\[
T^{\mu \nu} = \rho \xi^\mu \xi^\nu
\]

\hspace{1cm} (14)

where \( \xi^\mu \) is a null vector defined by

\[
\xi^\mu = (0, 0, 0, 1)
\]

\hspace{1cm} (15)

Using equations (14), (13) can be stated as

\[
R^{\mu \nu} = \Lambda g^{\mu \nu} + \rho \xi^\mu \xi^\nu
\]

\hspace{1cm} (16)

The non-zero component of the energy-momentum tensor is given by

\[
T^t_t = -\rho r^2
\]

\hspace{1cm} (17)

From (3), (13) and (16), we get

\[
\Lambda = -3 \alpha^2,
\]

\hspace{1cm} (18)

and

\[
\rho = \frac{\alpha^2 \beta^2}{8 r^8}
\]

\hspace{1cm} (19)

Here \( \rho \) is always positive and decreases with increase in \( r \), and vanishes as \( r \to \infty \), indicating that the presented spacetime (1) is asymptotically AdS radially. The matter-energy of the pure radiation field always satisfies the energy conditions [17].

The field equations for the metric (1) are solved by an energy-momentum tensor with components

\[
T^t_t = T^t_t = T^t_t = p = 3 \alpha^2
\]

\hspace{1cm} (20)

\[
T^t_t = -\alpha^2 \beta^2
\]

\hspace{1cm} (21)

\[
T^t_t = -\rho t = 3 \alpha^2
\]

\hspace{1cm} (22)

where \( p \) is the isotropic pressure and \( \rho \) is the energy density.

From (22), it is clear that \( \rho = \Lambda = -3 \alpha^2 \). This indicates a negative energy density, which apparently violates the weak energy condition (WEC). Now, this runs contrary to our assertion that the spacetime (1) satisfies all of the energy conditions. Actually the cosmological constant \( \Lambda \) could be viewed in two equivalent ways: (a) as a fixed energy density throughout the Universe, or (b) as a curvature of spacetime that was just a natural aspect of the Universe. We adhere to point (b) here, i.e., \( \Lambda \) is treated as part of the geometry.
3. Classification of the spacetime and its physical interpretation

In order to classify the spacetime metric (1), it is convenient to construct a null tetrad \((\mathbf{k}, \mathbf{l}, \mathbf{m}, \mathbf{n})\). These null tetrad vectors are mutually orthogonal except that \(k_\mu l_\nu = -1\) and \(m_\mu n_\nu = 1\). Explicitly these vectors are:

\[
k_\mu = (0, 1, 0, 0)
\]

\[
l_\mu = \left(\frac{-\beta z}{2r^2}, \frac{r^2}{2}, 0, r^2\right)
\]

\[
m_\mu = \frac{1}{\sqrt{2}}\left(\frac{1}{\alpha r}, 0, i r, 0\right)
\]

\[
\bar{m}_\mu = \frac{1}{\sqrt{2}}\left(\frac{-1}{\alpha r}, 0, -i r, 0\right)
\]

The set of null tetrad vectors defined above is such that the metric tensor for the line element (1) can be expressed as

\[
g_{\mu \nu} = -k_\mu l_\nu - l_\mu k_\nu + m_\mu \bar{m}_\nu + \bar{m}_\mu m_\nu
\]

Using the above set of null tetrad vectors we calculate the five Weyl scalars. Of these, the only non-zero scalars are:

\[
\Psi_3 = \frac{i \alpha^2 \beta}{4 \sqrt{2} r^2}
\]

and

\[
\Psi_4 = -\frac{\alpha \beta (i + 2r z \alpha)}{8 r^2}
\]

Thus the spacetime (1) belongs to type III in the Petrov classification scheme. Type III regions are associated with a kind of longitudinal gravitational radiation [18]. The vector \(k_\mu\) is not only null, but also satisfies the geodesic condition \(k_\mu k^\nu = 0\). A congruence is called geodesic null congruence if the tangent vector field at each point of the congruence is parallely propagated along it and that the congruence is affinely parametrized.

In GR optical scalars refer to a set of three scalar functions \(\Theta\), \(\omega^2\) and \(\sigma\) describing the propagation of a geodesic null congruence. For a vacuum field (with or without \(\Lambda\)), containing gravitational radiation, it is appropriate to apply such an analysis (of the gravitational field) to null rays, particularly if they are geodesic and aligned with a repeated principal null direction of the spacetime [44]. The optical scalars of the presented spacetime (1) are as given below:

\[
\Theta = \frac{1}{2} k_\mu k^\mu = 0
\]

\[
\omega^2 = \frac{1}{2} (k_\mu k^\nu - k^\mu k_\nu) = 0
\]

\[
\sigma = \frac{1}{2} (k_\mu k^\nu + k^\mu k_\nu) - \Theta^2 = 0
\]

Hence, the spacetime admits an expansion-free, twist-free and shear-free null geodesic congruence.

Furthermore, the following relations hold true for the spacetime metric (1):

\[
R_{\mu \nu} k^\mu = \Lambda k_\nu, \quad \Lambda = -3 \alpha^2
\]

\[
R_{\mu \nu} k^\rho k^\sigma = 0 = R_{\mu \nu} k^\mu k^\nu
\]

Note that if we allow for \(\beta = 0\) in (1), we obtain an axially symmetric, locally anti-de Sitter (AdS) spacetime which acts as a time machine spacetime. Such a solution has been obtained in [15].

4. Spacetime generating CTCs

An intriguing property of the presented spacetime (1) is that it generates CTCs that appear after a definite instant of time, thus exhibiting time machine-like behaviour, and hence violating the causality condition. Consider closed orbits of constant \(r = r_0 > 0\), \(z = z_0\) and \(t = t_0\) given by the 1D line element

\[
ds^2 = g_{\phi \phi} d\phi^2 = -t r^2 d\phi^2
\]

These orbits are null curves for \(t = t_0 = 0\), spacelike throughout \(t = t_0 < 0\), but become timelike when \(t = t_0 > 0\), which indicates that CTCs are formed at an instant when \(t = t_0 > 0\).
One can ensure that the CTCs evolve from an initially spacelike \( t = \text{constant} \) hypersurface. This can be ascertained by calculating the norm of \( \nabla_t t \), or equivalently by noting the sign of the inverse metric tensor component \( g^{tt} \). For the metric (1), we have

\[
g^{tt} = \frac{4 t}{r^2} + \frac{\alpha^2 \beta^2}{4} \frac{z^2}{r^6}
\]  

(36)

A hypersurface \( t = \text{constant} \) is spacelike when \( g^{tt} < 0 \) for \( t < 0 \), null at \( g^{tt} = 0 \) when \( t = 0 \) and timelike when \( g^{tt} > 0 \) for \( t > 0 \). To conform with the above, we choose constant \( z \)-planes, \( z = z_0 \), where \( z_0 \) equals zero. Now from (36),

\[
g^{tt} = \frac{4 t}{r^2}
\]  

(37)

The spacelike \( t = \text{constant} < 0 \) hypersurface can be chosen as the initial hypersurface over which initial data is specified. There is a Cauchy horizon at \( t = t_0 = 0 \), called the Chronology horizon, which separates the causal past and future in the past directed and a future directed manner. In other words, the Chronology horizon separates the spacetime into a chronal region where there are no CTCs and a nonchronal region where CTCs are formed [45]. Hence the spacetime evolves from a partial Cauchy surface (i.e., Cauchy spacelike hypersurface) in a causally well-behaved manner up to a moment, i.e., a null hypersurface \( t = t_0 = 0 \), and the formation of CTCs takes place from causally well-behaved initial conditions.

The metric (1) can be regarded as a 4D extension of the 2D Misner space [46]. The metric for the Misner space in 2D is given by

\[
d^2 s_{\text{Mis}} = -2 \, dT \, d\psi - T \, d\psi^2
\]  

(38)

where \( -\infty < T < \infty \) and \( \psi \) is a periodic coordinate. The curves \( T = T_0 \), where \( T_0 \) is a constant, are closed since \( \psi \) is periodic. The curves \( T < 0 \) are spacelike, \( T > 0 \) are timelike while the null curves \( T = 0 \) form the chronology horizon, indicating the formation of CTCs at \( T = 0 \).

For constant \( r \) and \( z \), our metric (1) reduces to the form

\[
d^2 s = r^2 (-2 \, dt \, d\phi - t \, d\phi^2) = \Omega (-2 \, dt \, d\phi - t \, d\phi^2),
\]  

(39)

a conformal Misner space, with \( \Omega = r^2 \) as the conformal factor. Hence, the formation of CTCs in our spacetime is analogous to that of the Misner space in 2D.

5. Conclusion and outlook

Realization of a time-machine is a huge academic challenge in gravitation physics. Closed timelike curves (CTCs) are worldlines of massive objects, such that one traveling in such a trajectory returns back to a spacetime point in its past. This results in time travel paradoxes [29]. To do away with the paradoxes, Hawking proposed the Chronology Protection Conjecture [47], which states that the laws of physics will always forbid the occurrence of CTCs in a solution of GR. However, a general proof of this conjecture is yet to be obtained. In fact, the authors in [48] have shown that time travel via CTCs is possible without the associated paradoxes.

Our primary motivation in this paper is to highlight causality violation in a solution of GR. It is shown here that the axially symmetric solution (1) behaves as a time machine spacetime where CTCs evolve from causally well-behaved initial conditions. The spacetime is a 4D extension of the 2D Misner space, having identical causality violating properties. The matter-energy content of the spacetime is pure radiation well-behaved initial conditions. The spacetime has vanishing expansion, shear and twist and it belongs to type III in the Petrov classification scheme.

It is shown here that the presented solution has constant curvature invariants. However, the spacetime contains a parallelly propagated (p.p.) curvature singularity [49] at \( r = 0 \). The divergence of the energy density \( \rho \) of the pure radiation field (19) is a signal for the existence of a p.p. curvature singularity. Let \( k_\mu \) be a tangent vector (23) of a null geodesic \( \gamma \) and let \( \{ k_\mu, p_\mu, n_\mu, m_\mu \} \) be basis vectors which define a parallelly propagated pseudo-orthonormal frame along \( \gamma \) satisfying

\[
k_{\mu}, \, k^\mu = p_{\mu}, \, p^\mu = 0, \quad k_{\mu}, \, p^\mu = -1, \quad n_{\mu}, \, n^\mu = m_{\mu}, \, m^\mu = 1
\]  

(40)

\[
k_{\mu}, \, n^\mu = k_{\mu}, \, n^\mu = p_{\mu}, \, n^\mu = m_{\mu}, \, \bar{n}^\mu = 0,
\]  

(41)

\[
k_{\mu}, \, k^\nu = k_{\mu}, \, p^\nu = n_{\mu}, \, k^\nu = m_{\mu}, \, k^\nu = 0
\]  

(42)

Equation (42) is the condition for \( \{ k_{\mu}, \, p_{\mu}, \, n_{\mu}, \, m_{\mu} \} \) being parallelly propagated along \( \gamma \). Note that \( l_{\mu}, \, m_{\mu}, \, m_{\mu} \) described by (24)–(26) are not parallelly propagated along \( \gamma \).
For the presented spacetime (1), \( R_{\rho \sigma} p^\rho p^\sigma \) diverges at \( r = 0 \) because

\[
R_{\rho \sigma} p^\rho p^\sigma = (\rho \quad \zeta_\mu \quad \zeta_\nu) \quad p^\mu \quad p^\nu = r^4 \quad \rho \quad k_\mu \quad k_\nu \quad p^\mu \quad p^\nu = \frac{\alpha^2 \beta^2}{8 \ r^4},
\]

where we have used the fact that \( \zeta^\mu \) given by (13) satisfies \( \zeta^\mu = -r^2 \ k^\mu \). Furthermore, since the relation \( R_{\rho \sigma} p^\rho p^\sigma = 2 \ R_{\mu \nu \rho \sigma} p^\rho p^\sigma \ p^\mu p^\nu \) holds [49], one of the pseudo-orthonormal components of the Riemann tensor blows up at \( r = 0 \) and hence it is a p.p. curvature singularity.

Regarding the presence of a p.p. curvature singularity we just noted above, and keeping in mind the presence of CTCs in the presented solution (1), a small discussion regarding causality violation and naked curvature singularities warrants itself. We note that the presence of a naked curvature singularity in a solution of the field equations may break down the causality condition, paving the way for the formation of CTCs [14]. Clarke and de Felice [50] analyzed the possibilities that a naked curvature singularity may give rise to a cosmic time machine. Mallary and Khanna [28] have examined CTCs and ‘effective’ superluminal travel in a spacetime containing naked line singularities.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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