Split Supersymmetry from Anomalous $U(1)$

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Abstract

We present a scenario wherein the anomalous $U(1)$ $D$–term of string origin triggers supersymmetry breaking and generates naturally a Split Supersymmetry spectrum. When the gaugino and the Higgsino masses (which are of the same order of magnitude) are set at the TeV scale, we find the scalar masses to be in the range $(10^6 - 10^8)$ GeV. The $U(1)$ $D$–term provides a small expansion parameter which we use to explain the mass and mixing hierarchies of quarks and leptons. Explicit models utilizing exact results of $N = 1$ suersymmetric gauge theories consistent with anomaly constraints, fermion mass hierarchy, and supersymmetry breaking are presented.

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1 Introduction

It is widely believed that supersymmetry may be relevant to Nature. There are four major observations which may justify this belief: (i) Supersymmetry (SUSY) can stabilize scales associated with spontaneous symmetry breaking. (ii) Unification of gauge couplings works well in the minimal SUSY extension of the Standard Model (SM). (iii) SUSY provides a natural candidate for cold dark matter. (iv) Supersymmetry is a necessary ingredient of superstring theory, which may eventually lead to a consistent quantum theory of gravity. Among these, reasoning (i), when applied to stabilize the electroweak scale, would suggest that all superpartners of the SM particles must have masses below or around a TeV. This is indeed what was assumed in almost all applications of supersymmetry to particle physics in the past twenty five years. The second and third observations above would only require that a subset of superpartners be lighter than a TeV, while the last one allows SUSY to be broken anywhere below the Planck scale, \( M_{Pl} = 2.4 \times 10^{18} \) GeV. This is because, among the superpartners, if the split members of a unifying group (\( SU(5) \), \( SO(10) \), etc), namely the gauginos and the Higgsinos, are lighter than a TeV, while the complete multiplets (the scalar partners of SM fermions) are much heavier, unification of gauge couplings would work just as well. The lightest of these SUSY particles would still be a natural candidate for cold dark matter.

A scenario dubbed as “Split Supersymmetry”, in which the spin 1/2 superparticles, namely, the gauginos and the Higgsinos, have masses of order TeV while the spin zero superparticles (squarks and sleptons) are much heavier, has recently been advocated \([1]\). This scenario gives up the conventionally employed naturalness criterion, since the light SM Higgs boson is realized only by fine–tuning. Such a finely tuned scenario, it is argued, may not be as improbable as originally thought \([1]\). This is because in any theory with broken SUSY one has to cope with another, even more severe, fine-tuning, in the value of the cosmological constant. A cosmic selection rule, an anthropic principle \([2]\), may be active in this case. If so, a similar argument may also explain why the SM Higgs boson is light \([3]\). Supersymmetry plays no role in solving the hierarchy problem here. Recent realization of a string landscape \([4]\), which suggests the existence of a multitude of string vacua, may justify this approach. Probabilistically, the chances of finding a vacuum with a light SM Higgs (along with a small cosmological constant) may not be infinitesimal, given the existence of a large number of string vacua \([5]\).

Split Supersymmetry has a manifest advantage over TeV scale supersymmetry: Un-
acceptably large flavor changing neutral current (FCNC) processes \cite{6}, fermion electric dipole moments, and $d = 5$ proton decay rate, which generically plague TeV scale SUSY are automatically absent in Split Supersymmetry. Various aspects of this scenario have been analyzed by a number of authors \cite{7,8}.

In this paper we take the Split Supersymmetry scenario from a theoretical point of view. Perhaps the most important question in this context is a natural realization of the split spectrum. Although it may be argued that $R$–symmetries would protect masses of the spin 1/2 SUSY fermions and not of the squarks and sleptons, in any specific scenario for SUSY breaking there is very little freedom in choosing the relative magnitudes of the two masses. We will focus on SUSY breaking triggered by the anomalous $U(1)$ $D$–term of string origin coupled to a SUSY QCD sector \cite{9}. Each sector treated separately would preserve supersymmetry, but their cross coupling breaks it. We make extensive use of exact results known for $N = 1$ SUSY QCD \cite{10}. In this scenario, the squarks and sleptons receive SUSY breaking masses at the leading order from the anomalous $U(1)$ $D$–term, while the gauginos acquire masses only at higher order. The Higgsino mass also arises at higher order and is similar in magnitude to the gaugino mass. Thus, a naturally split spectrum is realized. The anomalous $U(1)$ $D$–term also provides a small expansion parameter which we use to explain the mass and mixing hierarchies of quarks and leptons. We present complete models which are consistent with anomaly cancelation, and which lead to naturally split SUSY spectrum.\footnote{A somewhat similar analysis has recently been carried out in Ref. \cite{11}, our approach is different in that we present complete models without assuming a hidden sector and address the fermion masses and mixing hierarchy problems. Our spectrum is also quite different, especially as regards the gravitino mass.}

We note that with flavor–dependent charges, the anomalous $U(1)$ $D$–term contributions to the squark and slepton masses generically lead to large FCNC processes with sub–TeV scalars \cite{12}, this problem is absent in the Split Supersymmetry scenario.

2 Supersymmetry breaking by anomalous $U(1)$ and gaugino condensation

In this section we review supersymmetry breaking induced by the $D$–term of anomalous $U(1)$ symmetry \cite{9,13} coupled to the strong dynamics of $N = 1$ SUSY gauge theory \cite{10}. Each sector separately preserves supersymmetry, so an expansion parameter (the...
cross coupling) is available. Exact results of supersymmetric gauge theories can then be applied. Here we focus on the global supersymmetric limit, in Sec. 2.1 we extend the analysis to supergravity. In addition to the SM fields, these models contain an $SU(N_c)$ gauge sector with $N_f$ flavors. The “quark” ($Q$) and “antiquark” ($\tilde{Q}$) fields of the $SU(N_c)$ sector are also charged under the $U(1)_A$. $U(1)_A$ is broken by a SM singlet field $S$ carrying $U(1)_A$ charge of $-1$. The Standard Model fields carry flavor–dependent $U(1)_A$ charges so that the hierarchy in fermion masses and mixings is naturally explained. A small expansion parameter $\epsilon \sim 0.2$ is provided by the ratio $\epsilon = \langle S \rangle / M_{Pl}$ by the induced Fayet–Iliopoulos $D$–term for the $U(1)$. To see this, we recall that the apparent anomalies in $U(1)_A$ are canceled by the Green–Schwarz (GS) mechanism [14]. Heterotic superstring theory when compactified to four dimensions contains the Lagrangian terms $L \supset \phi(x) \sum_i k_i F_i^2 + i\eta(x) \sum_i k_i F_i \tilde{F}_i$, where $k_i$ are the Kac–Moody levels, $\phi(x)$ is the dilaton field and $\eta(x)$ is its axionic partner. The GS mechanism makes use of the transformation $\eta(x) \rightarrow \eta(x) - \theta(x) \delta_{GS}$, and the gauge variation for the $U(1)_A$ gauge field, $V_\mu \rightarrow V_\mu + \partial_\mu \theta(x)$. The anomalies are canceled if the following conditions are satisfied:

$$\frac{A_i}{k_i} = \frac{A_N}{k_N} = \frac{A_A}{3k_A} = \frac{A_{\text{gravity}}}{24} = \delta_{GS},$$

(1)

where $A_i (i = 1, 2, 3)$, $A_N$, $A_A$ and $A_{\text{gravity}}$ are the anomaly coefficients for $SM^2 \times U(1)_A$, $SU(N_c)^2 \times U(1)_A$, $U(1)_A^3$ and gravity$^2 \times U(1)_A$. Here $A_{\text{gravity}}$ is the gravitational anomaly, given by the sum of the anomalous charges of all fields in the theory. All other anomalies must vanish. These conditions put severe restrictions on the choice of $U(1)_A$ charges.

String loop effects induce a nonzero Fayet–Iliopoulos $D$–term for the $U(1)_A$ given by [15, 16]

$$\xi = \frac{g_{st}^2 M_{Pl}^2}{192 \pi^2} A_{\text{gravity}},$$

(2)

where $g_{st}$ is the string coupling at the unification scale $M_{Pl}$, related to the SM gauge couplings at that scale as

$$k_i g_i^2 = 2 g_{st}^2.$$  

(3)

The scalar potential receives a contribution from the $D$-term given by

$$V_D = \frac{D_A^2}{2} = \frac{g_{st}^2}{2} \left(\xi - |S|^2 + q_Q |Q_i|^2 + q_Q |\tilde{Q}_i|^2 + \sum_i q_i |\phi_i|^2\right)^2.$$  

(4)

Here $S$ is the flavon field with charge $-1$, $Q_i$ and $\tilde{Q}_i$ are the “quark” and “antiquark” fields belonging to the fundamental and antifundamental representations of an $SU(N_c)$
gauge group with $U(1)$ charges $q_Q$ and $q_{\bar{Q}}$. $\phi_i$ in Eq. (4) stand for all the other fields, and includes the SM sector.

In our models, all fields except $S$, will have positive $U(1)_A$ charges, so $\xi$ will turn out to be positive. The potential of Eq. (4) will minimize to preserve supersymmetry by giving the negatively charged $S$ field a vacuum expectation value (VEV), which would break the $U(1)_A$ symmetry. To zeroth order in SUSY breaking parameters, $\langle S \rangle = S_0$, where

$$S_0 \equiv \sqrt{\xi} = \sqrt{\frac{g_{\text{st}}^2 A_{\text{gravity}}}{192\pi^2}} M_{\text{Pl}} \equiv \epsilon M_{\text{Pl}}. \quad (5)$$

Here $\epsilon \sim 0.2$ will provides a small expansion parameter to explain the hierarchy of quark and lepton masses and mixings.

As for the $N = 1$ SUSY QCD sector, we consider the gauge group $SU(N_c)$ with $N_f$ flavors of quarks and antiquarks, and apply the well–known exact results of Ref. [10]. For concreteness we choose $N_f < N_c$. These results have been applied to TeV scale SUSY breaking by Binetruy and Dudas in Ref. [9] in the presence of anomalous $U(1)$ symmetry. These models actually lead to a Split Supersymmetry spectrum, as we will show. We also generalize the results of Ref. [9] to include supergravity corrections (in Sec. 2.2). In Sec. 3, we apply these results to explicit and complete models.

The effective superpotential we consider has two pieces:

$$W_{\text{eff}} = W_{\text{tree}} + W_{\text{dynamical}}, \quad (6)$$

where $W_{\text{tree}}$ is the tree–level superpotential, while $W_{\text{dynamical}}$ is induced dynamically by nonperturbative effects. Since the $Q$ and the $\bar{Q}$ fields are charged under $U(1)_A$, a bare mass term connecting them is not allowed. A mass term will arise through the coupling

$$W_{\text{tree}} = \frac{\text{Tr} \left( \lambda_{Q\bar{Q}} S^n \right)}{M_*^{n-1}} \quad (7)$$

when $\langle S \rangle = S_0$ is inserted. Here the trace is taken over the $N_f$ flavor indices of the $Q_i$ and $\bar{Q}_i$ fields. $M_*$ is a mass scale at which this term is induced. The most natural value of $M_*$ is $M_{\text{Pl}}$, which is what we will use for our numerical analysis, but we allow $M_*$ to be different from $M_{\text{Pl}}$ for generality. We have used the definition

$$n = q_Q + q_{\bar{Q}} \quad (8)$$

for the sum of the $U(1)$ charges of $Q$ and $\bar{Q}$. As we will see later the choice $n = 1$, which would correspond to a renormalizable superpotential will be phenomenologically
unacceptable. From the results of Ref. [10], the dynamically generated superpotential is known to be (for $N_f < N_c$)

$$W_{\text{dynamical}} = (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{\det(Q\tilde{Q})} \right)^{1/(N_c-N_f)}.$$  

(9)

Here $\Lambda$ is the dynamically induced scale below which the $SU(N_c)$ sector becomes strongly interacting:

$$\Lambda \sim M_{Pl} e^{-\frac{\alpha_{N_c}(N_c-N_f)}{2}},$$

(10)

where $\alpha_{N_c}$ is the $SU(N_c)$ gauge coupling constant at $M_{Pl}$. For $N_f = N_c - 1$, the gauge symmetry is completely broken, and Eq. (9) is induced by instantons. For $N_f < N_c - 1$, the gauge symmetry is reduced to $SU(N_c - N_f)$ and the gaugino condensate of this symmetry induces Eq. (9).

Below the scale $\Lambda$ the effective theory can be described in terms of $N_f \times N_f$ mesons $Z_i^j$:

$$Z_i^j = Q_j \tilde{Q}_i \text{ with } (i, j = 1, \ldots, N_f).$$

(11)

Neglecting small supersymmetry breaking effects, we can describe the theory below $\Lambda$ along the $D$–flat directions $Q_i = \tilde{Q}_i$ in terms of the $Z$ fields. We can make the following replacements in the $D$–term and the superpotential: $q_Q |Q_i|^2 + q_{\tilde{Q}} |\tilde{Q}_i|^2 \to n \text{Tr} \left( Z^\dagger Z \right)^{1/2}$ and $Q_j \tilde{Q}_i \to Z_i^j$. We use the notation

$$m = \lambda \frac{S^n}{M^{n-1}},$$

(12)

with $m$ identified as the mass matrix of the $Z$ field (upto small supersymmetry breaking effects). Then the $F$–term for the $Z$ fields, defined as $(F_Z)_j^i = 2 \left[ \left( Z^\dagger Z \right)^{1/2} \right]^{ik} \partial W/\partial Z_k^j$, is found to be

$$(F_Z)_j^i = 2 \left[ \left( Z^\dagger Z \right)^{1/2} \left( m - \left( \frac{\Lambda^{3N_c-N_f}}{\det(Z)} \right)^{1/(N_c-N_f)} \left( \frac{1}{Z} \right) \right) \right]^{\dagger j}_i.$$ 

(13)

This theory preserves supersymmetry, as $F_Z = 0$ can be realized with $\langle Z \rangle \neq 0$ and given by

$$\left( Z_0 \right)_j^i \equiv \left( \det(m) \Lambda^{3N_c-N_f} \right)^{1/N_c} \left( \frac{1}{m} \right)_j^i.$$ 

(14)
Note that this result holds only in the presence of a nonvanishing VEV $\langle S \rangle$, so that $m$ is nonzero.

So far we treated the $U(1)_A$ $D$–term and the ensuing superpotential for the $Z$ fields separately. The two sectors are however coupled through $W_{\text{tree}}$ of Eq. (7). Owing to this coupling, supersymmetry is actually broken. This is evident by examining the $F$–term of the $S$ field,

$$F_S = n \frac{Tr(mZ_0)}{S_0} \neq 0.$$  \hfill (15)

Similarly $F_Z$ is also nonzero. The VEVs of $S$ and $Z$ fields will shift from the supersymmetry preserving values of Eqs. (5) and (14) when the full potential is minimized jointly. To find the soft SUSY breaking parameters we need to calculate these corrections.

The scalar potential of the model in the global limit is given by

$$V = |F_S|^2 + \frac{1}{2} \text{Tr}(F_Z(Z^\dagger Z)^{-1/2}F_Z^\dagger) + \frac{1}{2} D_A^2.$$  \hfill (16)

We expand the fields around the SUSY preserving minima:

$$S = S_0 + \delta S \quad Z_j = (Z_0 + \delta Z)_j$$  \hfill (17)

with $\delta S / S_0 \ll 1$, $\delta Z / Z_0 \ll 1$. For simplicity we assume the coupling matrix $\lambda$ to be an identity matrix, $\lambda^i_j = \delta^i_j$, in which case $Z^i_j = Z \delta^i_j$ can be chosen. The VEV $\langle Z \rangle = Z_0$ arising from Eq. (14) in this case becomes

$$Z_0 = \frac{\Lambda^3}{m} \left( \frac{m}{\Lambda} \right)^{N_f/N_c}.$$  \hfill (18)

We make an expansion in the supersymmetry breaking parameter $\Delta$ defined as

$$\Delta = \frac{Z_0/S_0^2}{mS_0^2} \left( \frac{m}{\Lambda} \right)^{N_f/N_c} \ll 1.$$  \hfill (19)

From the minimization of the scalar potential with respect to these shifted fields, we find

$$\langle S^\dagger S \rangle = S_0^2 \left[ 1 + \Delta (nN_f) - \Delta^2 \left( \frac{n^2N_f^2}{2N_c^2g_A^2} \right) \left\{ g_A^2 n (N_c - N_f) (2N_c - N_f) ight. ight.$$  

$$- 2N_c (N_c - nN_f) \frac{m^2}{S_0^2} \left. \right] + O(\Delta^3),$$  \hfill (20)

$$\langle Z \rangle = Z_0 \left[ 1 - \Delta \left( \frac{n^2N_f (N_c - N_f) (2N_c - N_f)}{2N_c^2} \right) + O(\Delta^2) \right].$$

This agrees with the results of Ref. [9], except that there are two apparent typos in Eq. (2.22) of that paper.
Now the $F$ and the $D$–terms are given by

$$\langle F_S \rangle = mS_0 \Delta (nN_f) \left( 1 + \Delta \frac{nN_f}{2} \left( n - 1 + \frac{nN_f (N_c - N_f) (2N_c - N_f)}{N_c^2} \right) \right),$$

$$\langle F_Z \rangle = mZ_0 \Delta \left( n^2 N_f \right) \left( \frac{N_f}{N_c} - 1 \right),$$

$$\langle D_A \rangle = m^2 \Delta^2 (nN_f)^2 \left( \frac{nN_f}{N_c} - 1 \right) / g_A. \tag{21}$$

Consequently, the scalar soft masses induced from the $D$–term of anomalous $U(1)$ are

$$m^2_{f_i} = q_f m^2_0, \tag{22}$$

where

$$m^2_0 = m^2 \Delta^2 (nN_f)^2 \left( \frac{nN_f}{N_c} - 1 \right). \tag{23}$$

There is a simple interpretation of these results in terms of the gaugino condensate (for $N_f < N_c - 1$), which is given by \[17\]

$$\langle \lambda^\alpha \lambda^\alpha \rangle = e^{2i\pi k/(N_c - N_f)} \Lambda^3 \left( \frac{m}{\Lambda} \right)^{N_f/N_c}, \quad k = 1 - (N_c - N_f). \tag{24}$$

The soft scalar masses are simply proportional to the gaugino condensate. We will make use of these results in Sec. 3. Note that had we chosen $n = 1$ these results would have led to negative squared masses for scalars. Note also that the $D$–term contributions are proportional to the $U(1)_A$ charges, so they are zero for particles with zero charge.

### 2.1 Gravity corrections to the soft parameters

In this section we work out the supergravity corrections to the soft parameters found in the global SUSY limit in the previous section. Our reasons for this extension are two–fold. First, we wish to show explicitly that supergravity corrections do not destabilize the minimum of the potential that we found in the global limit. Second, the main contribution to the masses of scalars with zero $U(1)$ charge will arise from supergravity corrections. In our explicit models, we do have particles with zero charge.

It is conventional in supergravity to add a constant term to the superpotential in order to fine–tune the cosmological constant to zero:

$$W = W_{\text{global}} + \beta. \tag{25}$$
We separate the constant into two parts, $\beta = \beta_0 + \beta_1$, such that $\beta_0$ cancels the leading part of the superpotential in which case $\langle W \rangle = \beta_1$. The $F$-term contribution to the scalar potential in supergravity is given by

$$V_F = M_{Pl}^4 e^G \left( G_i \left( G^{-1} \right)_j^i G^j - 3 \right), \quad (26)$$

where

$$G^i \equiv \partial G / \partial \phi_i^* , \; G_i \equiv \partial G / \partial \phi^i , \; G^i_j \equiv \partial^2 G / \partial \phi_i^* \partial \phi^j . \quad (27)$$

We will assume for illustration the minimal form of the Kähler potential. In our model it is given by

$$G = \frac{|S|^2}{M_{Pl}^2} + 2 \frac{\text{Tr}(Z\dagger Z)^{1/2}}{M_{Pl}^2} + \sum_i \frac{|\phi|^2}{M_{Pl}^2} + \ln \left( \frac{|W|^2}{M_{Pl}^2} \right) . \quad (28)$$

Then the scalar potential is given by

$$V = V_F + V_D , \quad (29)$$

with

$$V_F = e^{(|S|^2/2 + \text{Tr}(Z\dagger Z)^{1/2} + \sum_i |\phi|^2)/M_{Pl}^2} \left( \left| F_S + S^* \frac{W}{M_{Pl}^2} \right|^2 + \ldots \right) \quad (30)$$

$$+ \frac{1}{2} \text{Tr} \left[ (F_Z + Z \frac{W}{M_{Pl}^2}) (Z\dagger Z)^{-1/2} (F_Z + Z\dagger \frac{W}{M_{Pl}^2}) \right] \quad (31)$$

$$+ \sum_i \left| F_{\phi_i} + \phi_i^* \frac{W}{M_{Pl}^2} \right|^2 - 3 \frac{|W|^2}{M_{Pl}^2} \quad (32)$$

and

$$V_D = \frac{g^2}{2} \left( G_i (T_a)_j^i \phi^j \right)^2 M_{Pl}^4 . \quad (33)$$

In our case for $G^i = \phi^i / M_{Pl}^2 + \partial W / \partial \phi_i^* / W$, so $M_{Pl}^2 G_i (T_a)_j^i \phi^j = \phi_i^* (T_a)_j^i \phi^j$, which is identical to the $D$ term of global supersymmetry (Note that the term $\partial W / \partial \phi_i^* (T_a)_j^i \phi^j$ vanishes due to the gauge invariance of $W$).

Including these supergravity corrections, by minimizing the potential we find

$$\langle S\dagger S \rangle = \langle S\dagger S \rangle_{global} + 2 \Delta^2 S_0^2 \left[ - \frac{n^2 N_f^2}{4g_A^2 N_c^2 S_0} \left\{ n g_A^2 (N_c - N_f) \right\} + 2 N_c (n N_f + N_c) \frac{m^2}{S_0} \right]$$

$$- \frac{\tilde{\beta}_1 n N_f}{4g_A^2 N_c^2} \left( g_A^2 (N_c - N_f)^2 (2 N_c + n (N_c - N_f)) + 2 N_c (n N_f - 4 N_c) \frac{m^2}{S_0} \right) , \quad (34)$$

$$\langle Z \rangle = \langle Z \rangle_{global} - \Delta (Z_0 e^2) \frac{N_c - N_f}{2 N_c^2} \left[ n^2 N_f (N_c - N_f) + \tilde{\beta}_1 \{ 2 N_c + n (N_c - N_f) \} \right] ,$$
where the subscript “global” denotes the contributions found in global SUSY case in Eq. (20). Here we have introduced a dimensionless parameter $\tilde{\beta}_1$ defined through the relation

$$\beta_1 = \left( \tilde{\beta}_1 m S_0^2 \right) \Delta. \quad (35)$$

From the condition that the vacuum energy is zero at the minimum for the vanishing of the cosmological constant, $\tilde{\beta}_1$ is found to be

$$\tilde{\beta}_1 \simeq \pm \frac{nN_f}{\sqrt{3} \epsilon} \left( 1 \pm \frac{\epsilon}{\sqrt{3}} + \frac{2}{3} \epsilon^2 \right). \quad (36)$$

Eq. (35) ensures that the cosmological constant remains zero to the scale of strong dynamics. With these corrections the soft scalar masses from the $D$–term are now given by

$$m_f^2 = (m_f^2)_{\text{global}} + q_f m_0^2 \frac{\epsilon^2}{nN_f - N_c} \left[ N_c + n N_f + \tilde{\beta}_1 \left( 1 - 4 N_c / (n N_f) \right) \right]. \quad (37)$$

Note that the shifts in the masses are small, suppressed by a factor of $\epsilon \simeq 0.2$.

The gravitino mass is determined to be

$$m_{3/2} \simeq m \frac{\tilde{\beta}_1 \Delta S_0^2}{M_{Pl}^2} \simeq \frac{nN_f \Lambda^3}{\sqrt{3} S_0} \left( \frac{m}{\Lambda} \right)^{N_f / N_c}. \quad (38)$$

In addition to the $D$–term corrections, all scalar fields receive a contribution to their soft masses from the term

$$\left| \phi_i^* \frac{W}{M_{Pl}^2} \right|^2 = m_{3/2}^2 |\phi_i|^2. \quad (39)$$

For particles neutral under the anomalous $U(1)_A$ these are the leading source for soft masses. With the assumed minimal Kähler potential, note that these soft masses are equal to the gravitino mass.

So far we assumed the minimal form of the Kähler potential for illustration. There is no justification for this assumption. In fact, within Split Supersymmetry, since there are no excessive FCNC processes, an arbitrary form for the Kähler potential is permissible phenomenologically. The effects of such a nonminimal $G$ can be understood in terms of higher dimensional operators suppressed by the Planck scale. Scalar fields can acquire soft SUSY breaking masses through the terms

$$\mathcal{L} \supset \int (\phi_i^* \phi^i) \frac{|S|^2}{M_{Pl}^2} d^4 \theta. \quad (40)$$

The resulting masses are $m_{3/2}^2 = c_i m_{3/2}^2$, with $c_i$ being order one (flavor–dependent) coefficients. We will allow for such corrections.


3 Explicit models

In this section we consider a class of models based on flavor–dependent anomalous $U(1)$ symmetry and apply the results of the previous section. These models were developed to address the pattern of fermion masses and mixings \[18, 19\]. As noted earlier, the anomalous $U(1)$ $D$–term provides a small expansion parameter $\epsilon = \langle S \rangle / M_{Pl} \sim 0.2$, which can be used to explain the mass hierarchy. We assign charge $q_i$ to fermion $f_i$ and charge $q_c^j$ to fermion $f_c^j$, such that the mass term $f_i f_c^j H$ will arise through a higher dimensional operators with the factor $(S/M_{Pl})^{q_i+q_c^j}$ and thus suppressed by a factor $\epsilon^{q_i+q_c^j}$. By choosing the charges appropriately the observed mass and mixing hierarchy can be explained, even with all Yukawa coefficients being of order one.

With sub–TeV supersymmetry this approach to fermion mass and mixing hierarchy cannot be combined with supersymmetry breaking triggered by anomalous $U(1)$, since the $D$–terms will split the masses of scalars leading to unacceptable FCNC. Within Split Supersymmetry, however, these two approaches can be combined, which is what we analyze now.

The superpotential of the class of models under discussion has the following form:

$$ W = \sum_f y_f^i f_i^c H f_j (S/M_{Pl})^{n_{ij}} + \frac{M_{Rij}}{2} \nu^c_i \nu^c_j (S/M_{Pl})^{n_{ij}} + \mu H_u H_d + \text{Tr} (\lambda Z) S^n + (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{\text{det} (Z)} \right)^{1/(N_c-N_f)} + W_A (S, X_k). $$ (41)

Here $X_k$ are the SM singlet fields necessary for the cancelation of gravitaitonal anomaly. We will focus on the sub-class of such models studied in Ref. \[19\]. The mass matrices for the various sectors in Ref. \[19\] are given (in an obvious notation) by:

$$ M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^{8-2\alpha} & \epsilon^{6-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^{6-\alpha} & \epsilon^4 & \epsilon^2 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^{4-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^{4-\alpha} & \epsilon^3 & \epsilon^2 \\ \epsilon^4 & \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, $$

$$ M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^{3} & \epsilon \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \quad M_{\nu D} \sim \langle H_u \rangle \epsilon^p \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, $$

$$ M_{\nu c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \Rightarrow M_{\nu}^{\text{light}} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2p} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. $$ (42)
Anomalous flavor charges

| Field               | Anomalous flavor charges |
|---------------------|--------------------------|
| $10_1, 10_2, 10_3$  | 4 $-\alpha$, 2, 0       |
| $\overline{5}_1, \overline{5}_2, \overline{5}_3$ | 1 $+p$, $p$, $p$        |
| $\nu_1^c, \nu_2^c, \nu_3^c$ | 1, 0, 0                |
| $H_u, H_d, S, Q, \tilde{Q}$ | 0, 0, $-1$, $n/2$      |

Table 1: The flavor $U(1)_A$ charge assignment for the MSSM fields, the $SU(N_c)$ fields $Q$ and $\tilde{Q}$ and the flavon field $S$ in the normalization where $q_S = -1$.

Although not unique, these mass matrices would lead to small quark mixings and large neutrino mixings. Note that the neutrino masses are hierarchical in this scheme.

The charge assignment which leads to these mass matrices is given in Table 1. Here we use $SU(5)$ notation for the fields in the first column for simplicity, although we do not explicitly assume $SU(5)$ unification. There are two parameters, $p$ and $\alpha$, which can take a set of discrete values. The parameter $p$ takes values $p = 2$ (1, 0) corresponding to low (medium, high) value of $\tan \beta$ (the ratio of the two Higgs VEVs). Actually, in Split Supersymmetry, since $\tan \beta \sim 1$ is also permitted, $p = 3$ is also allowed. $\alpha$ appears in the mass of the up–quark, both $\alpha = 0$ and $\alpha = 1$ give reasonable spectrum. We also consider the case where the charge of $\overline{5}_1$ is $p$ (rather than $1 + p$) in Table 1. This case would have mass matrices which are very similar to those in Eq. (42). The main difference in this case is that all elements of $M_{\nu}^{light}$ will be of the same order, which would lead to larger $U_{e3}$. This scenario has been widely studied [20], sometimes under the name of neutrino mass anarchy [21]. The charge assignment of Table 1, as well as its above–mentioned variant, explain naturally the mass and mixing hierarchy of quarks and leptons, including small quark mixings and large neutrino mixings.

The Green–Schwarz anomaly cancelation conditions for these models are given by

$$\frac{A_1}{k_1} = \frac{A_i}{k_i} = \frac{A_{N_c}}{k_N} = \frac{n N_f}{2 k_N} = \frac{19 - 3\alpha + 3p}{2k_i} \text{ or } \frac{18 - 3\alpha + 3p}{2k_i}$$

with $A_i$ being the $(SM)^2 \times U(1)_A$ anomalies for $i = 2 - 3$. Their equality is automatically satisfied, due to the $SU(5)$ compatibility of charges, provided that the Kac–Moody levels $k_i$ for the SM gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ are chosen to be, for example, $5/3$, 1 and 1 respectively. For $A_{gravity}$, one needs to introduce extra heavy matter $X_k$ (with charge +1) which decouple at or near the Planck scale (see Ref. [19] for a detailed discussion). In Eq. (43) the first $p$–dependent factor applies to the charge assignment of Table
1, while the second one corresponds to the variant with $\bar{5}_1$ carrying charge \( p \). For every choice of charge we can compute the expansion parameter \( \epsilon \) from \( \epsilon = \sqrt{g^2 \text{gravity}/(192\pi^2)} \). We find for \( \alpha = 0 \) and for the charges of Table 1, \( \epsilon = 0.174 (0.187, 0.199) \) for \( p = 0 (1, 2) \). The results are very similar for other choices.

Eq. (43) allows for only a finite set of choices for \( n, N_c \) and \( N_f \). First of all, all these must be integers. Secondly, the mass parameter \( m \) of the meson fields of \( SU(N_c) \) must be of order \( \Lambda \) or smaller, otherwise these mesons will decouple from the low energy theory, affecting its dynamics. Thirdly, the dynamical scale \( \Lambda \) is determined for any choice of charges, due to the string unification condition, Eq. (3). (We will confine to Kac–Moody level 1 for the \( SU(N_c) \) as well as the SM sectors.) This should lead to an acceptable SUSY breaking spectrum. Consistent with these demands, we find four promising cases. (i) \( n = 5, N_f = 5, p = 2, \alpha = 0 \); (ii) \( n = 6, N_f = 4, p = 2, \alpha = 0 \); (iii) \( n = 7, N_f = 3, p = 1, \alpha = 1 \); and (iv) \( n = 6, N_f = 3, p = 1, \alpha = 1 \). Here (i) has \( \bar{5}_1 \) charge equal to \( p + 1 \), while the other three cases has it to be equal to \( p \). We will see that the choices \( N_c = 6 \) or \( 7 \) yield reasonable spectrum.

3.1 The spectrum of the model

Now we turn to the spectrum of the model. We set the gaugino masses at the TeV scale. (The Higgsinos will turn out to have masses of the same order.) We then seek possible values of the scale \( \Lambda \) and the mass parameter \( m_0 \) (the scalar mass) that would induce the TeV scale gaugino masses. The spectrum will turn out to be that of Split Supersymmetry. The main reason for this is that the leading SUSY breaking term, the \( U(1)_A \) \( D \)-term, generates squark and slepton masses, but not gaugino and Higgsino masses.

Supersymmetry breaking trilinear \( A \) terms are induced in the model by the same superpotential \( W \) (Eq. (41)) that generates quark and lepton masses, once the \( S \) field acquires a nonzero \( F \) component:

\[
\mathcal{L} \supset y_{ij}^f \int d^2 \theta f_i f_j^c H \left( \frac{S}{M_{Pl}} \right)^{n_{ij}^f} = Y_{ij}^f \left( q_i^f + q_j^c \right) \tilde{f}_i \tilde{f}_j^c H \frac{F_S}{S}. \tag{44}
\]

Here \( Y_{ij}^f \simeq y_{ij}^f e^{n_{ij}^f} \) are the effective MSSM Yukawa couplings, with \( n_{ij} = q_i + q_j^c \), the sum of the anomalous charge of the SM fermions \( f_i \) and \( f_j^c \). Substituting results from the previous section, Eqs. (20) and (21), we find

\[
A_{ij}^f = Y_{ij}^f \left( q_i^f + q_j^c \right) n N_f \frac{\Lambda^3}{S_0^2} \left( \frac{m}{\Lambda} \right)^{N_f/N_c}. \tag{45}
\]
These $A$–terms are induced at the scale $\Lambda$. The messengers of supersymmetry breaking are the meson fields of the $SU(N_c)$ sector, which have masses of order $\Lambda$. In the momentum range $m_0 \leq \mu \leq \Lambda$, the spectrum is that of the MSSM and there is renormalization group running of all SUSY breaking parameters as per the MSSM beta functions. This implies that once the $A$–terms are induced, they will generate nonzero gaugino masses through two–loop MSSM interactions. These are estimated from the two–loop MSSM beta functions to be\footnote{The one–loop finite corrections arising from diagrams involving the top–quark and the stop–squark are negligible since $A_t = 0$ and $\mu \sim \text{TeV} \ll m_t$.}

\[
M^i_\tilde{g}(m_0) \simeq -\frac{g^2}{(16\pi^2)^2} \left( C^b_i Y_b^2 + C^\tau_i Y_\tau^2 \right) \frac{m_0}{\sqrt{nN_f/N_c - 1}} \ln \left( \Lambda^2/m_0^2 \right),
\]

where $C^b_i = (14/5, 6, 4)$ and $C^\tau_i = (18/5, 6, 0)$ for $i = 1 - 3$. $Y_b$ and $Y_\tau$ are the MSSM Yukawa couplings of the $b$–quark and the $\tau$–lepton. From the requirement that $M^i_\tilde{g} \sim 1$ TeV we can estimate $\Lambda$ and $m_0$, which will enable us to obtain the full spectrum of the model. Assuming that $m \sim \Lambda$, for the Bino mass we obtain (for $p = 2$, or $\tan \beta \sim 5$):

\[
M_{\tilde{B}}(m_0) \sim -10^{-5} m_0.
\]

The mass of the Wino is somewhat larger than this, and that of the gluino is somewhat smaller (compare the coefficients $C^b_i$ and $C^\tau_i$), all at the scale $m_0$. There is significant running of these masses below $m_0$ down to the TeV scale. This running is the largest for the gluino \footnote{The one–loop finite corrections arising from diagrams involving the top–quark and the stop–squark are negligible since $A_t = 0$ and $\mu \sim \text{TeV} \ll m_t$.} which increases its mass, while it is the smallest for the Bino, which decreases its mass. Consequently, at the TeV scale, we have the normal mass hierarchy $M_{\text{Bino}} \leq M_{\text{Wino}} \leq M_{\text{gluino}}$.

In addition to the SM gauge interactions, the gauginos receive masses from the anomaly mediated contributions \footnote{The one–loop finite corrections arising from diagrams involving the top–quark and the stop–squark are negligible since $A_t = 0$ and $\mu \sim \text{TeV} \ll m_t$.}. These contributions may be suppressed in specific setups such as in 5 dimensional supergravity \footnote{The one–loop finite corrections arising from diagrams involving the top–quark and the stop–squark are negligible since $A_t = 0$ and $\mu \sim \text{TeV} \ll m_t$.}. We will allow for both a suppressed and an unsuppressed anomaly mediated contributions to gaugino masses. These contributions are given by

\[
M_{\text{gaugino}} = \frac{\beta(g)}{g} F_\phi
\]

where $F_\phi$ is the $F$–component of the compensator superfield. With our setup as described in the previous section, $F_\phi$ is equal to the gravitino mass, so the Wino mass, for eg., will be about $3 \times 10^{-3}$ of the gravitino mass, or about $10^{-3} m_0$. If we set the Wino mass at 1 TeV, $m_0$ will be of order $10^6$ GeV in such a scenario.
As we stated in the previous section, only a limited choice of \( n \) and \( N_f \) are allowed from the mixed anomaly cancelation conditions. We have considered four cases with \( nN_f = 25, 24, 21, \) or 18. Our results for the spectrum are listed in Table 2. In each case we studied different values of \( N_c \) and \( N_f \). \( N_c = 6, 7 \) give the correct dynamical scale \( \Lambda \) which leads to TeV scale gauginos. The scalar masses are found to be of order \( 10^6 \) GeV in the case of unsuppressed anomaly mediated contribution (cases 1 and 3), and of order \( 10^8 \) GeV for the suppressed case (all the other cases). Clearly this is a Split Supersymmetry spectrum. In the computation of Table 2 we assumed \( g_N^2/(4\pi) = 1/28 \) at \( M_{Pl} = 2.4 \times 10^{18} \) GeV. The mass \( m \) for the meson fields is computed in terms of an effective coupling \( \hat{\lambda} \equiv \lambda \left( \frac{M_{Pl}}{M_*} \right)^{n-1} \). We expect \( \hat{\lambda} \) to be of order one from naturalness, if \( M_* \) is the same as \( M_{Pl} \). We list the mass \( m \) in terms of \( \hat{\lambda} \) in the third column in Table 2. Note that the scalar masses from anomalous \( U(1) \) \( D \)-term are proportional to the \( U(1) \) charges, and therefore vanish for \( H_u, H_d \) and 10 fields. These fields will however acquire masses from supergravity corrections.

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The \( U(1)_A \) symmetry does not forbid a bare \( \mu \) term in the superpotential. However, it can be banished by a discrete \( Z_4 \) \( R \)-symmetry \([23]\). Under this \( Z_4 \), all the SM fermion superfields (scalar components) have charge +1, the gauginos have charge +1, the \( Z \) field has charge +2 and the SM Higgses and the \( S \) fields have charge zero. This symmetry has no anomaly, as a consequence of discrete Green–Schwarz anomaly cancelation. The \( G^2 \times Z_4 \) anomaly coefficients are \( A_3 = 3, A_2 = 2 - 1 = 1 \) and \( A_{N_c} = N_c \). The GS condition for discrete \( Z_4 \) anomaly cancelation is that the differences \( A_i - A_j \) should be an integral

| \((p, \alpha, n, N_f, N_c)\) | \(\Lambda \) (GeV) | \(m\) (GeV) /\(\hat{\lambda}\) | \(m_0\) | \(M_{Pl}(m_0)\) (GeV) | \(\mu\) (GeV) /\(\hat{\lambda}\) |
|----------------|-----------------|-----------------|--------------|----------------------|----------------------|
| \((2, 0, 5, 5, 6)\) | \(3 \times 10^{12}\) | \(8 \times 10^{14}\) | \(6 \times 10^5\hat{\lambda}^{5/6}\) | \(5\hat{\lambda}^{5/6}\) | \(600/\hat{\lambda}^{1/6}\) |
| \((2, 0, 5, 5, 7)\) | \(4 \times 10^{13}\) | \(8 \times 10^{14}\) | \(9 \times 10^7\hat{\lambda}^{5/7}\) | \(600\hat{\lambda}^{5/7}\) | \(9 \times 10^4/\hat{\lambda}^{2/7}\) |
| \((2, 0, 6, 4, 6)\) | \(8 \times 10^{12}\) | \(1 \times 10^{14}\) | \(7 \times 10^5\hat{\lambda}^{2/3}\) | \(5\hat{\lambda}^{2/3}\) | \(700/\hat{\lambda}^{1/3}\) |
| \((2, 0, 6, 4, 7)\) | \(8 \times 10^{12}\) | \(1 \times 10^{14}\) | \(1 \times 10^8\hat{\lambda}^{4/7}\) | \(640\hat{\lambda}^{4/7}\) | \(10^5/\hat{\lambda}^{3/7}\) |
| \((1, 0, 7, 3, 6)\) | \(2 \times 10^{13}\) | \(3 \times 10^{13}\) | \(1 \times 10^6\hat{\lambda}^{1/2}\) | \(100\hat{\lambda}^{1/2}\) | \(1600/\hat{\lambda}^{1/2}\) |
| \((1, 0, 7, 3, 7)\) | \(1 \times 10^{14}\) | \(3 \times 10^{13}\) | \(2 \times 10^8\hat{\lambda}^{3/7}\) | \(10^4\hat{\lambda}^{3/7}\) | \(2 \times 10^5/\hat{\lambda}^{4/7}\) |
| \((1, 1, 6, 3, 6)\) | \(2 \times 10^{13}\) | \(1 \times 10^{14}\) | \(2 \times 10^6\hat{\lambda}^{1/2}\) | \(200\hat{\lambda}^{1/2}\) | \(3000/\hat{\lambda}^{1/2}\) |
multiple of 2, which is automatic when \( N_c \) is odd.

One can write the following effective Lagrangian for the \( \mu \) term that is consistent with the \( Z_4 \) \( R \) symmetry:

\[
L \supset \int d^2 \theta H_u H_d \frac{\text{Tr}(\lambda_{\mu} Z) S^n}{M_{Pl}^{n+1}} = \lambda_{\mu} N_f \frac{(Z S^n)}{M_{Pl}^{n+1}} H_u H_d.
\]  

(49)

This leads to

\[
\mu = \lambda_{\mu} \epsilon^n N_f \Lambda^3 \left( \frac{m}{\Lambda} \right)^{N_f/N_c}.
\]  

(50)

The numerical results for \( \mu \)–term are given in the last column of Table 2 using this relation.

The SUSY breaking bilinear Higgs coupling, the \( B\mu \) term, arises from the Lagrangian

\[
L \supset \int d^4 \theta H_u H_d \frac{\lambda_{B_1} |S|^2 + \lambda_{B_2} \text{Tr} (Z^\dagger Z)^{1/2}}{M_{Pl}^2} = \frac{\lambda_{B_1} |F_S|^2 + \lambda_{B_2} N_f |F_Z|^2/|Z_0|}{M_{Pl}^2} H_u H_d,
\]  

(51)

leading to

\[
B\mu = m_0^2 \left( \frac{N_c}{n N_f - N_c} \right)^2 \left( \lambda_{B_1} \epsilon^2 + \lambda_{B_2} N_f \frac{\Lambda^3}{m M_{Pl}^2} \left( \frac{m}{\Lambda} \right)^{N_f/N_c} \right).
\]  

(52)

The second term in Eq. (52) is small compared to the first. From this we see that the 2 \( \times \) 2 Higgs boson mass matrix has its off–diagonal entry of the same order as its diagonal entries. Recall that the diagonal entries are of order \( m_{3/2}^2 \), since the \( U(1)_A \) charges of \( H_u \) and \( H_d \) are zero. Fine–tuning can then be done consistently so that one of the Higgs doublets remain light, with mass of order \( 10^{2} \) GeV.

Even when the \( Z_4 \) \( R \) symmetry is not respected by gravitational corrections, the induced \( \mu \) term and gaugino masses are of order TeV. There can be a new contribution to the \( \mu \) term in this case, arising from

\[
L \supset \int d^4 \theta H_u H_d \frac{(Z S^n)^*}{M_{Pl}^{n+2}}.
\]  

(53)

This \( \mu \) term is however smaller than that from Eq. (49). Similarly, gaugino masses can arise from

\[
L \supset \int d^4 \theta W_\alpha W_\alpha \frac{Z S^n}{M_{Pl}^{n+2}}
\]  

(54)

which is also smaller than the SM induced corrections.

For the scalars neutral under \( U(1)_A \) (\( H_u \), \( H_d \) and \( 10_3 \)), the \( D \)–term contribution to the soft masses vanish. We should take account of the subleading supergravity corrections.
then. Since these corrections are suppressed by a factor of $\epsilon^2$ in the mass–squared, we should worry about potentially large negative corrections proportional to the other soft masses arising from SM interactions through the RGE in the momentum range $m_0 \leq \mu \leq \Lambda$. We have examined this in detail and found consistency of the models.

For the masses of zero charge fields we write

$$m^2_{\tilde{\phi}_i} = c_i m^2_{\tilde{\phi}_i}/2 + \delta \left( m^2_{\tilde{\phi}_i} \right)$$  \hspace{1cm} (55)

with $\delta \left( m^2_{\tilde{\phi}_i} \right)$ denoting the MSSM RGE corrections. The most prominent one–loop radiative corrections are

$$\delta \left( m^2_{\tilde{f}_3} \right)^{1\text{-loop}} \simeq - (Y_b)^2 \frac{m^2_0}{16\pi^2} \frac{p}{(nN_f/N_c - 1)} \frac{nN_f}{N_c} \ln \left( \frac{\Lambda^2}{m^2_{\tilde{f}}/2} \right),$$  \hspace{1cm} (56)

$$\delta \left( m^2_{H_d} \right)^{1\text{-loop}} \simeq - \left\{ 3(Y_b)^2 + (Y_\tau)^2 \right\} \frac{m^2_0}{16\pi^2} \frac{p}{(nN_f/N_c - 1)} \frac{nN_f}{N_c} \ln \left( \frac{\Lambda^2}{m^2_{\tilde{f}}/2} \right)$$

where $\tilde{f}_3 = (\tilde{Q}_3, \tilde{\tau}_i)$. Similar corrections for $H_u$ and $\tilde{u}_i$ scalar components are small. Since $p = 2$, we have low $\tan \beta \sim 5$, so these corrections are not large, although not negligible. For example, for the down–type Higgs bosons we have

$$\delta \left( m^2_{H_d} \right)^{1\text{-loop}} \sim -2 \times 10^{-3} m^2_0.$$  \hspace{1cm} (57)

If the supergravity corrections to the mass–squared of $H_d$ is larger than $3 \times 10^{-2} m_0$, it will remain positive down to the scale $m_0$.

There is an important two–loop correction to the scalar masses arising from the gauge sector:

$$\delta \left( m^2_{\tilde{\phi}} \right)^{2\text{-loop}} \simeq - g^4 \frac{m^2_0}{(16\pi^2)^2} (nN_f) K_\phi \ln \left( \frac{\Lambda^2}{m^2_{\tilde{f}}} \right)$$  \hspace{1cm} (58)

where $K_\phi = (63/15, 16/5, 6/5$ and $9/5)$ for $\tilde{\phi} = (\tilde{Q}, \tilde{u}^c, \tilde{e}^c$ and $H_u$). This correction is estimated to be

$$\left( m^2_{\tilde{f}} \right)^{2\text{-loop}} \sim - 10^{-2} m^2_0.$$  \hspace{1cm} (59)

We see that these corrections are, although close to the gravitino contribution, at a safe level. We conclude that Split Supersymmetry is realized consistently in these models.
4 Conclusion

In this paper we have proposed concrete models for supersymmetry breaking making use of the anomalous $U(1)$ $D$–term of string origin. The anomalous $U(1)$ sector is coupled to the strong dynamics of an $N = 1$ SUSY gauge theory where exact results are known. The complete models we have presented also address the mass and mixing hierarchy of quarks and leptons. We have generalized the analysis of Ref. [9] to include supergravity corrections, which turns out to be important for certain fields in these models which carry zero $U(1)$ charge. Table 2 summarizes our results on the spectrum of these models. This spectrum is that of Split Supersymmetry. The gaugino and the Higgsino masses are of the same order, when these are set at the TeV scale, the squarks and sleptons have masses in the range $(10^6 - 10^8)$ GeV. This provides an explicit realization of part of the parameter space of split supersymmetry [11].

The experimental and cosmological implications of Split Supersymmetry have been widely studied [7, 8, 6, 11]. We conclude by summarizing the salient features that apply to our framework. (i) Gauge coupling unification works well, in fact somewhat better than in the MSSM. When embedded into $SU(5)$ symmetry, proton decay via dimension six operators will result, with an estimated lifetime for $p \rightarrow e^+ \pi^0$ of order $(10^{35} - 10^{36})$ yrs. There is no observable $d = 5$ proton decay in these models. (ii) The lightest neutralino, which is charge and color neutral, is a natural and consistent dark matter candidate. (iii) The gluino lifetime is estimated to be of order $10^{-7}$ seconds or shorter in these models. There is no cosmological difficulty with such a mass. (iv) The gravitino mass is of order $10^7$ GeV, thus there is no cosmological gravitino abundance problem. (v) The low energy theory is the SM plus the neutralinos and the charginos of supersymmetry. All other particles acquire masses either near the Planck scale or through strong dynamics at a scale $\Lambda \sim 10^{14}$ GeV.

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