The Ideal Mixing Departure in Vector Meson Physics

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Abstract

In this work we study the departure for the ideal \( \phi - \omega \) mixing angle in the frame of the Nambu-Jona-Lasinio model. We have shown that in that context, the flavour symmetry breaking is unable to produce the shifting in the mixing angle. We introduce a nonet symmetry breaking in the neutral vector sector to regulate the non-strange content of the \( \phi \) meson. The phenomenon is well reproduced by our proposal.

It is well known that it is not possible to use perturbative expansions of QCD to describe low energy hadronic phenomena. For that reason, a great deal of effective theories, preserving the symmetries of QCD, have been developed to account for the main properties of hadrons. One of the basic aspects common to both schemes, is the chiral invariance of the strong interactions in the massless limit. This symmetry is explicitly broken if quarks are massive.

Chiral effective models [1] have demonstrated to reproduce the low energy hadron phenomena. The effective four fermion Lagrangian, proposed by Nambu-Jona-Lasinio (NJL) [2], is an intermediate step between QCD and effective mesons theories. From NJL Lagrangian is it possible to obtain an effective meson Lagrangian after proper bosonization [3].

The NJL model is a profitable arena to study various phenomena related to symmetry breakdown in hadron physics. In particular, the \( \omega - \phi \) mixing was largely studied by theoretical and experimental points of view since the beginning of the sixties [4]. Measurements show that the \( \phi \) meson decays into \( \pi^+\pi^- \), violating isospin conservation and OZI rule [5,6]. In the present paper we will study the departure of the \( \omega - \phi \) ideal mixing angle in the NJL model, allowing a non strange content in the \( \phi \) meson. We will focus our attention in the vector meson sector where the phenomenology we are interested to describe takes place. It is important to note that, in the framework of most theoretical models, the \( \rho \) and \( \omega \) mesons are composed by \( u \) and \( d \) light quarks, whereas the quark content of \( \phi \) meson is purely strange, which do not coincide with experimental data [7]. Let us note that some authors predict the departure from ideal mixing angle within different approaches [8–10].

Our purpose is to explore the origin of the shifting in the \( \phi - \omega \) mixing angle in the frame of the NJL Lagrangian and its connection with chiral symmetry. The goal is to find the relation
of that effect with the explicit breakdown of the $SU(3)$ flavour symmetry to $SU(2)$ symmetry with the assumption $m_u = m_d \neq m_s$. The NJL model predicts the ideal mixing angle in the vector sector in the process of diagonalization of the neutral sector [11,12].

In order to investigate the source of the departure of the ideal mixing angle in the vector sector, we explore different scenarios. First of all we present a brief summary of our previous results [12] where we have studied the explicit chiral symmetry breaking in the NJL model when considering $m_u = m_d \neq m_s$. We revisit that scheme focusing on the $\phi - \omega$ mixing angle.

As a second step we studied how QCD vertex corrections modify the NJL coupling constant and its relation with the $\phi - \omega$ mixing angle. In the hadron scale we are dealing with, the non perturbative gluon propagator can be approximated by an universal constant leading to a local NJL Lagrangian [13]. The chiral symmetry breakdown has no further effect in the tree level approximation. However, the effective NJL coupling constant is affected if we consider the QCD vertex corrections when chiral and flavor symmetry breaking is considered. We model the contributions emerging as a consequence of explicit breakdown of the $SU(3)$ flavor symmetry to $SU(2)$ isospin symmetry, in four fermion interactions coming from vertex corrections at QCD level. As a consequence the symmetry breakdown becomes explicit in the coupling constant of NJL Lagrangian. We will see in our present work that, neither the explicit chiral symmetry breaking in the NJL Lagrangian when considering quark masses, nor the consideration of QCD vertex corrections, modify the ideal mixing angle in the vector sector.

Then, as a final step, with the objective of investigating the source of the departure of the ideal mixing angle for the vector sector, we modify the coupling constant in the NJL Lagrangian inspired by models for $\eta - \eta'$ physics [14]. We add a new parameter redefining the coupling constant in order to separate the singlet state from the octet.

Let us start with the NJL Lagrangian [2]

$$\mathcal{L} = \bar{q}(i \not{D} - \hat{m}_0)q + 2G_1 \left[ (\bar{q} \frac{1}{2} \gamma^a q)^2 + (\bar{q} i \gamma_5 \frac{1}{2} \lambda^a q)^2 \right] - 2G_2 \left[ (\bar{q} \gamma_\mu \frac{1}{2} \lambda^a q)^2 + (\bar{q} \gamma_\mu \gamma_5 \frac{1}{2} \lambda^a q)^2 \right], \quad (1)$$

where $q$ denotes the $N$-flavour quark spinor, $\lambda^a$, $a = 0, \ldots, N^2 - 1$ are the generators of the $U(N)$ flavor group (we normalize $\lambda^0 = \sqrt{2/N} \mathbf{1}$) and $\hat{m}_0$ stands for the current quark mass matrix. The coupling constants $G_1$ and $G_2$, as well as the quark masses, are introduced as free parameters of the model. In the absence of the mass term, the NJL Lagrangian shows at the quantum level the $SU(N)_A \otimes SU(N)_V \otimes U(1)_V$ symmetry characteristic of massless QCD.

It is possible to reduce the fermionic degrees of freedom to bosonic ones by bosonization technique. By means of the Stratonovich identity, the vector–vector coupling in (1) can be transformed as

$$-2G_2 \left( \bar{q} \gamma_\mu \frac{1}{2} \lambda^a q \right)^2 \rightarrow -\frac{1}{4G_2} \text{Tr} V_\mu^2 + i \bar{q} \gamma_\mu V_\mu q, \quad (2)$$

where

$$V_\mu \equiv -i \sum_{a=0}^{8} V_\mu^a \lambda^a / 2. \quad (3)$$
The spin 1 fields $V^a_\mu$ can be identified with the usual nonet of vector mesons as in [12], which transform in such a way to preserve the chiral symmetry of the original NJL Lagrangian (and therefore that of QCD). Notice that the first term in the right hand side of Eq. (2) is nothing but a mass term for the vector fields $V^a_\mu$, thus the vector–meson masses are governed by the coupling $G_2$ in the NJL Lagrangian. It can be seen that these masses are degenerate in the limit where the quark masses are degenerate. The quark fields can be integrated out, leading to an effective Lagrangian which only contains bosonic degrees of freedom. This procedure can be carried out by taking into account the generating functional and performing the calculation of the fermion determinant (a detailed analysis can be found in [3]). A similar procedure can be followed for the full NJL Lagrangian (1), leading to the interactions involving scalar, pseudoscalar and axial–vector bosons. In this way, the final effective Lagrangian is written only in terms of spin 0 and spin 1 colorless hadron fields.

In our previous work [12] we have studied the explicit chiral symmetry breaking in NJL model when $m_u = m_d \neq m_s$, obtaining the ideal mixing in the process of diagonalizing the neutral vector meson sector. We have performed the bosonization by carrying out an expansion of the fermion determinant, which gives rise to a set of one–loop Feynman diagrams [15]. The generating functional gives rise to effective kinetic terms for the spin-1 vector mesons via one–loop diagrams, giving the following contribution to the vector meson self-energy

$$iN_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \frac{k - p + m_1}{(k - p)^2 - m_1^2} \lambda^a \gamma_\mu \frac{k + m_2}{k^2 - m_2^2} \lambda^b \gamma_\nu,$$

where $m_1$ and $m_2$ are the constituent masses of the quarks entering the loop, $N_c$ is the number of colors, and the trace acts over the flavor and Dirac indices.

We will take only the leading order in the external momentum $p$, which means to evaluate the integral at $p = 0$ after extracting the relevant kinematical factors. In this case, this is equivalent to consider only the divergent piece of (4):

$$\Pi^{(V)}_{\mu \nu} = I_2(m_1, m_2) \left[ \frac{1}{3} \left( p_\mu p_\nu - p^2 g_{\mu \nu} \right) + \frac{1}{2} (m_2 - m_1)^2 \right],$$

where

$$I_2(m_i, m_j) \equiv -i \frac{N_c}{(2\pi)^4} \int d^4 k \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)}.$$

In order to regularize the divergence we use the proper–time regularization scheme with a cut–off $\Lambda$, which will be treated as a free parameter of the model. We obtain

$$I_2(m_i, m_j) = \frac{N_c}{16\pi^2} \int_0^1 dx \frac{1}{\Lambda} \left( 0, \frac{(m_i^2 - m_j^2)x + m_j^2}{\Lambda^2} \right).$$

From (5), the kinetic terms for the vector mesons in the effective Lagrangian are given by

$$\mathcal{L}_{kin}^{(V)} = -\frac{1}{4} \frac{2}{3} I_2(m_u, m_u) \left[ \rho_{\mu \nu} \rho^{\mu \nu} + 2 \rho_+^{\mu \nu} \rho_-^{\mu \nu} + \omega_{\mu \nu} \omega^{\mu \nu} + \alpha \phi_{\mu \nu} \phi^{\mu \nu} + 2 \beta \left( K^{+ \mu}_{\mu} K^{- \mu \nu} + K^{0 \mu}_{\mu} K^{\mu \nu} \right) \right],$$

where $\rho_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $A_\mu$ is the electromagnetic field.
where $V^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$, and

$$\alpha = \frac{I_2(m_s, m_u)}{I_2(m_u, m_u)}$$

$$\beta = \frac{I_2(m_s, m_u)}{I_2(m_u, m_u)}$$

(9)

parameterize the magnitude of the chiral symmetry breaking.

The kinetic Lagrangian in (8) has been expressed in terms of the vector fields, with the additional rotation

$$\omega_8 = \phi \cos \theta_0 + \omega \sin \theta_0$$

$$\omega_1 = -\phi \sin \theta_0 + \omega \cos \theta_0$$

(10)

which diagonalizes the neutral sector. It is easy to see that in this model the rotation is “ideal”, i.e., the spin 1 mass eigenstates $\rho$ and $\omega$ are composed by pure light $u$ and $d$ quarks, while the $\phi$ meson is a bound state $\bar{s}s$. The ideal rotation angle is given by the well known relation $\sin \theta_0 = 1/\sqrt{3}$.

The mass terms for the vector mesons are given by the $(V^\mu V^\mu)^2$ term in (2), plus a divergent one–loop contribution given by the second term in the square brackets in (5), which vanishes in the chiral limit. This leads to

$$L_{mass}^{(V)} = \frac{1}{8G_2} \left[ \rho_{\mu} \rho^\mu + 2\rho^+_{\mu} \rho^-_{\mu} + \omega_{\mu} \omega^\mu + \phi_{\mu} \phi^\mu + 2K^{*+}_{\mu} K^{*-\mu} + 2K^{*0}_{\mu} K^{*00}_{\mu} \right]$$

$$(m_s - m_u)^2 \beta I_2(m_u, m_u) (K^{*+}_{\mu} K^{*-\mu} + K^{*0}_{\mu} K^{*00}_{\mu}) + \alpha I_2(m_u, m_u) = \alpha Z^{-1}_\rho.$$ 

(11)

Notice that (as expected) the mass terms turn out to be diagonal in the $(\omega, \phi)$ basis.

We proceed now to the wave function renormalization required by the kinetic terms in (8). The vector meson fields can be properly redefined by $V^\mu \rightarrow Z^{1/2}_V V^\mu$, with

$$Z^{-1}_\rho = \frac{2}{3} I_2(m_u, m_u)$$

$$Z^{-1}_\omega = \frac{2}{3} I_2(m_u, m_u)$$

$$Z^{-1}_K = \frac{2}{3} \beta I_2(m_u, m_u) = \beta Z^{-1}_\rho$$

$$Z^{-1}_\phi = \frac{2}{3} \alpha I_2(m_u, m_u) = \alpha Z^{-1}_\rho.$$ 

(12)

Then from (11) one obtains

$$m^2_\rho = m^2_\omega = \frac{Z_\rho}{4G_2}, \quad m^2_K = \frac{m^2_\rho}{\beta} + \frac{3}{2} (m_s - m_u)^2, \quad m^2_\phi = \frac{m^2_\rho}{\alpha},$$

(13)

thus the $\phi$ meson mass can be written in terms of the $\rho$ mass and the chiral symmetry breaking parameter $\alpha$. In the case of the $K^*$, the corresponding mass relation includes both the parameter $\beta$ and a quark–mass dependent contribution that arises from the loop.

From these previous results, the explicit chiral symmetry breaking in the NJL model taking into account the explicit breakdown of the $SU(3)$ flavour symmetry to $SU(2)$ isospin symmetry, do not leads to a departure of the ideal mixing angle in the vector meson sector.
As we mentioned before, another mechanism should be responsible for such effect. Our aim is to perform a further analysis, focusing our attention to the four-fermion interaction in the NJL model. We will study how four-fermion interaction is modified when considering the flavour symmetry breaking and its consequences on the vector mixing angle. The effect of chiral symmetry breakdown, when considering quark mass terms, do not modify the coupling. However, one can expect that the explicit flavour symmetry breaking at QCD level should manifest in the quarks couplings in the NJL model through the vertex corrections.

In QCD, vertex corrections depend on quark propagators and consequently, on quarks masses, as shown in Fig. 1. If quarks masses are degenerated, the QCD vertex contributions are the same for all couplings. Nevertheless, if we consider the case of explicit breakdown of the \(SU(3)\) flavor symmetry to \(SU(2)\) isospin symmetry, the internal lines in Fig. 1 will have different propagators due to different quarks masses. As a consequence, the contributions coming from vertex corrections at QCD level should modify the four fermion interaction in the NJL Lagrangian. We modelled that effect introducing in the NJL Lagrangian as new chiral symmetry breaking parameter in the strange current.

Let us start writing the terms containing vector current-current interaction in (1). As we are interested in the departure of the \(\phi - \omega\) ideal mixing angle, we will only concentrate in the neutral vector current-current terms; the extension to the full Lagrangian is straightforward

\[
-2G_2 \left( \bar{q} \gamma_\mu \frac{\lambda_a}{2} q \right)^2 = -2G_2 \left[ \left( \bar{u} \, \bar{d} \, \bar{s} \right) \gamma_\mu \frac{\lambda_a}{2} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \right]^2 ;
\]

(14)
as we are considering three flavors (u, d, s), \(\lambda_a\) are the Gell-Mann matrices with the normalization \(\lambda_0 = \sqrt{2/N} \cdot 1\). Let us rewrite the diagonal terms, i.e. with \(\lambda_0\), \(\lambda_3\), \(\lambda_8\),

\[
J_0 = \frac{\sqrt{2}}{2\sqrt{3}} \bar{u} \gamma_\mu u + \frac{\sqrt{2}}{2\sqrt{3}} \bar{d} \gamma_\mu d + \frac{\sqrt{2}}{2\sqrt{3}} \bar{s} \gamma_\mu s
\]

\[
J_3 = \frac{1}{2} \bar{u} \gamma_\mu u - \frac{1}{2} \bar{d} \gamma_\mu d
\]

\[
J_8 = \frac{1}{2\sqrt{3}} \bar{u} \gamma_\mu u + \frac{1}{2\sqrt{3}} \bar{d} \gamma_\mu d - \frac{2}{2\sqrt{3}} \bar{s} \gamma_\mu s,
\]

(15)

here we have set \(J_a = \bar{q} \gamma_\mu (\lambda_a/2) q\) (for simplicity we have omitted the index \(\mu\)). Defining \(U = \bar{u} \gamma_\mu u\), \(D = \bar{d} \gamma_\mu d\) and \(S = \bar{s} \gamma_\mu s\), it is easy to see that when \(m_u = m_d = m_s\), the sum of the these three terms (squared) presents flavor symmetry \(SU(3)\) and it is proportional to \((U^2 + D^2 + S^2)\).

We will focus on the case when \(m_u = m_d \neq m_s\). We choose the \(S\) current-current interaction in the NJL Lagrangian in the following way

\[
S^2 \rightarrow (1 + \epsilon)^2 S^2
\]

(16)
or equivalently
\[ s \rightarrow (1 + \epsilon)s, \]  
\[ (U^2 + D^2 + S^2) \rightarrow (U^2 + D^2 + S^2) + \epsilon(2 + \epsilon)S^2. \]  

We rewrite the diagonal current–current interactions at \( \mathcal{O}(\epsilon^2) \)

\[
\begin{align*}
J_0 &= \frac{\sqrt{2}}{2\sqrt{3}}(U + D + S) \rightarrow J'_0 = \frac{\sqrt{2}}{2\sqrt{3}}(U + D + S + \epsilon S) = (1 + \frac{\epsilon}{3})J_0 - \frac{\sqrt{2}}{3}\epsilon J_8 \\
J_3 &= (U - D) \\
J_8 &= \frac{\sqrt{2}}{2\sqrt{3}}(U + D - 2S) \rightarrow J'_8 = \frac{\sqrt{2}}{2\sqrt{3}}(U + D - 2S - 2\epsilon S) = (1 + \frac{2\epsilon}{3})J_8 - \frac{\sqrt{2}}{3}\epsilon J_0,
\end{align*}
\]

where the primed currents are expressed in terms of a mixture of the non primed ones, regulated by \( \epsilon \) parameter.

The NJL Lagrangian vector terms can be expressed in terms of the primed currents, containing the \( \epsilon \) dependence, as follows

\[
\mathcal{L}^{(V)}_{\text{NJL}} = \bar{q} (i \slashed{\partial} - \hat{m}_0) q - 2G_2 \sum_{a=0}^{8} J_a^2.
\]

In this case, following the procedure presented in [12], the quark fields can be integrated out, leading to an effective Lagrangian which only contains bosonic degrees of freedom. We have performed the bosonization by carrying out an expansion of the fermion determinant, which gives rise to a set of 1-loop Feynman diagrams. Taking into account (2), the final Lagrangian can be written in terms of the bosonic fields

\[
\begin{align*}
\omega_1 &= \left(1 + \frac{\epsilon}{3}\right)\omega'_1 - \frac{\sqrt{2}}{3}\epsilon \omega'_8 \\
\omega_8 &= \left(1 + \frac{2\epsilon}{3}\right)\omega'_8 - \frac{\sqrt{2}}{3}\epsilon \omega'_1,
\end{align*}
\]

warranting that kinetic energy has the same expression as in (8), with no dependence in \( \epsilon \) parameter. Note that, from (3), identifying \( V_{\mu}^a \) as the usual nonet of vector mesons [12], \( V_0 \) and \( V_8 \) correspond to \( \omega_1 \) and \( \omega_8 \) respectively.

Then, keeping the first order in the \( \epsilon \) power expansion, the neutral vector mass terms are

\[
\mathcal{L}^{(V)}_{\text{mass}} = \frac{1}{8G_2} \left[(1 - \frac{2\epsilon}{3})\omega_1^2 + \rho^2 + (1 - \frac{4\epsilon}{3})\omega_8^2 + \frac{4\sqrt{2}}{3}\epsilon \omega_1\omega_8\right].
\]

Both kinetic and mass terms are non diagonal in the neutral sector. In order to diagonalize them simultaneously it is necessary to introduce the following rotation which includes two different angles.
\[ \omega_8 = \phi \cos \theta_1 + \omega \sin \theta_2 \]
\[ \omega_1 = -\phi \sin \theta_1 + \omega \cos \theta_2 . \]  

(23)

In the literature, some authors [8–10] obtain by different procedures two different mixing angles. Replacing (23) in (22) we obtain the following form

\[ -2 \left( 1 - \frac{2\epsilon}{3} \right) \sin \theta_1 \cos \theta_2 + 2 \left( 1 - \frac{4\epsilon}{3} \right) \sin \theta_2 \cos \theta_1 + \frac{4\sqrt{2}}{3} \epsilon (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = 0. \]

(24)

which is the mass terms diagonalization condition.

The above expression is again satisfied with the ideal mixing angle as well as the kinetic terms. As a consequence, the non strange content of \( \phi \) meson does not arise from the coupling constant modification when considering QCD vertex corrections with \( m_u = m_d \neq m_s \).

In order to estimate the magnitude of the \( \epsilon \), we express the vector meson masses in terms of that parameter. We proceed to the wave function renormalization required by kinetic terms of (8), redefining the meson fields as in (12). Considering the mass contributions coming from the divergent 1-loop contribution [12], the vector meson masses, expressed in terms of the chiral symmetry breaking parameters are

\[ m_\rho^2 = m_\omega^2 = \frac{Z_\rho}{4G_2}, \quad m_{K^*}^2 = \frac{m_\rho^2(1-\epsilon)}{\beta} + \frac{3}{2}(m_s - m_u)^2, \quad m_\phi^2 = \frac{m_\rho^2(1-2\epsilon)}{\alpha}. \]  

(25)

Taking into account the experimental value for the \( \phi \) mass [7], we can estimate the value for \( \epsilon \). In our calculation we have supposed that the chiral symmetry breaking parameters \( \alpha \) and \( \beta \), are not modified by considering the vertex corrections at QCD level (here we use the phenomenological values for these parameters obtained in [12]). In this way, we estimate the value for the \( \epsilon \) parameter

\[ \epsilon \simeq -0.03. \]  

(26)

Therefore, the coupling constant in the neutral sector (those terms in NJL Lagrangian with \( S \) current-current interactions) and in the charged vector sector are 0.94\( G_2 \) and 0.97\( G_2 \) respectively. However, those tiny but non-vanishing vertex corrections that modify meson masses, are not able to shift the ideal mixing angle.

Our results lead us to conclude that the mechanism responsible for the ideal mixing departure has no source neither in the explicit chiral symmetry breaking in the NJL Lagrangian [12] when considering quark masses throughout QCD vertex corrections. The inclusion of chiral symmetry breaking parameters \( \alpha \) and \( \beta \) in [12], as well as the parameter which takes into account the QCD vertex corrections \( \epsilon \), do not lead to any change in the mixing angle in the vector sector. That means that another mechanism will be responsible to allow a non vanishing \( u, d \) content of meson \( \phi \) to permit decays as \( \phi \rightarrow \pi^+\pi^- \) [7]. In our opinion, inspired by the phenomenology of pseudoscalar sector, one possible approach to solve that puzzle, is considering a new parameter. In the pseudoscalar sector, the presence of the \( U(1) \) anomaly breaks the
$U(3)$ symmetry down to $SU(3)$, leading to the mass splitting between the observed $\eta$ and $\eta'$ physical states. In the NJL model this effect is included throughout the 't Hooft interaction [14]. Another way to take into account the anomaly is introducing the $\eta - \eta'$ mixing angle as a parameter of the model. Inspired in this peculiar physics, we have tested the sensibility of mixing angle in the vector sector including a parameter $\delta$ to force a nonet symmetry breaking. We proceed as follows

$$-2G_2 J^2_0 \rightarrow -2G'_2 J^2_0$$

(27)

with $G'_2 \equiv \delta G_2$. This new parameter, after proper bosonization, can be absorbed in the quadratic term as follows

$$-2G'_2 J^2_0 \rightarrow \left( \frac{-1}{4G'_2} V^2_0 + V'_0 J'_0 \right).$$

(28)

We have kept the $\epsilon$ dependence in mass terms and we will see the consequences in our calculations. Then proceeding as before, let us rewrite the neutral mass terms including the new parameter $\delta$

$$\mathcal{L}^{(V)}_{\text{mass}} = \frac{1}{8G_2} \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \omega^2_1 + \rho^2 + \left( 1 - \frac{4\epsilon}{3} \right) \omega^2_8 + \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \omega_1 \omega_8 \right].$$

(29)

Note that the mixing in quadratic neutral sector term is regulated by the $\delta$ and $\epsilon$ parameters. Is easy to see that in the limit $\delta \rightarrow 1$ we reobtain (22). As before, we have performed the following rotation to diagonalize both kinetic and quadratic terms, considering two different angles

$$\omega_8 = \phi \cos \theta_1 + \omega \sin \theta_2$$
$$\omega_1 = -\phi \sin \theta_1 + \omega \cos \theta_2.$$  

(30)

We found that kinetic terms of (8) are diagonal when the following condition is satisfied

$$(1 + 2\alpha) \tan \theta_2 - (2 + \alpha) \tan \theta_1 + \sqrt{2}(1 - \alpha)(1 - \tan \theta_1 \tan \theta_2) = 0,$$

(31)

and mass terms are diagonal if

$$-\frac{2}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \sin \theta_1 \cos \theta_2 + 2 \left( 1 - \frac{4\epsilon}{3} \right) \sin \theta_2 \cos \theta_1 +$$
$$+ \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = 0.$$

(32)

It is straightforward to see that the ideal mixing angle does not satisfy the above conditions simultaneously. The above relations become trivial when $m_u = m_d = m_s$, then $\alpha = 1$ and $\epsilon = 0$ are excluded in the following calculations. If $\delta = 1$ is considered, we reobtain (24).
We proceed now to obtain the vector meson masses in terms of $\delta$ and the two mixing angles. Our intention is computing the magnitude of those parameters. Considering the wave function renormalization (12) as before, from (29) we obtain

\[ m_{\rho}^2 = \frac{Z_\rho}{4G_2} \quad m_{K^*}^2 = \frac{m_\rho^2 (1 - \epsilon)}{\beta} + \frac{3}{2} (m_s - m_u)^2 \]

\[ m_\omega^2 = m_\rho^2 \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \cos^2 \theta_2 + \left( 1 - \frac{4\epsilon}{3} \right) \sin^2 \theta_2 + \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \sin \theta_2 \cos \theta_2 \right] \]

\[ m_\phi^2 = \frac{m_\rho^2}{\alpha} \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \sin^2 \theta_1 + \left( 1 - \frac{4\epsilon}{3} \right) \cos^2 \theta_1 - \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \sin \theta_1 \cos \theta_1 \right] . \]  

(33)

Experimentally, the mixing angle is near $35^\circ$ [7] (-0,3$^\circ$ apart from the ideal mixing angle), then we choose

\[ \tan \theta_1 = \frac{1}{\sqrt{2}} + x \]  

(34)

as the shifting in the ideal mixing angle. Replacing (34) in condition (31), we obtained

\[ \tan \theta_2 = \frac{1}{\sqrt{2}} + \frac{x}{\alpha} + O(x^2). \]  

(35)

We have replaced both $\tan \theta_1$ and $\tan \theta_2$ in condition (32), together with the definition $d = (1/\delta) - 1$. Then, at $O(d^2)$ and neglecting terms containing $d\epsilon$, the expression for $x$ is the following

\[ x \simeq -\frac{\sqrt{2} d}{2 \left( 1 - \frac{1}{\alpha} \right)}. \]  

(36)

It is interesting to express the physical states $\phi_F$ and $\omega_F$ in terms of the “ideal ones” ($\phi_I$ and $\omega_I$) when considering a shifting in the ideal mixing angle

\[ \phi_F = \phi_I - \frac{1}{\alpha} x \omega_I \]

\[ \omega_F = \omega_I + x \phi_I. \]  

(37)

From the above expressions we can see that the non strange decays of the $\phi$ meson is controlled by $x$, i. e. the shifting in $\theta_1$ mixing angle. Then, from (34) and (36) together with the experimental value for the shifting in the ideal mixing angle, we determined the phenomenological value for the parameter $\delta$, that leads to the following relation between the coupling constant

\[ G'_2/G_2 \simeq 1.005. \]  

(38)

Though tiny, this difference in the coupling constant becomes crucial to model the departure from the ideal mixing angle.
Summing up, we have devoted our phenomenological analysis to study the departure from the ideal mixing angle in the vector meson sector in the frame of the NJL model, considering the flavour symmetry breaking when $m_u = m_d \neq m_s$. We have analyzed different mechanisms separately and together.

As a starting point we have revisit our previous results [12] focusing on the $\phi - \omega$ mixing angle. We show that the explicit chiral symmetry breaking when considering quark masses in NJL Lagrangian, does not lead to a non strange content of $\phi$ meson.

As a second step, to explore another possible source of the ideal mixing departure, we have include in our analysis the phenomenology associated with vertex corrections at QCD level which accounts of flavour symmetry breaking. As a consequence, the effective couplings in NJL Lagrangian containing strange quarks in the currents, are modified throughout the parameter $\epsilon$. After bosonization, both kinetic and mass terms are diagonalized again with the ideal mixing angle. Consequently, in this framework, the $\phi$ meson is still composed by $s\bar{s}$ quarks, then, another mechanism should be responsible for the ideal-mixing departure. Throughout the expressions for the vector meson masses, we have estimated the phenomenological value for the parameter $\epsilon$.

As a final step, inspired in the peculiar physics of the neutral pseudoscalar sector, we have forced a nonet symmetry breaking in the vector sector. For that purpose we have included the $\delta$ parameter to test the sensibility of the $\phi - \omega$ mixing angle. In this scheme, after proper bosonization, we have diagonalized simultaneously both kinetic and mass terms with two different non ideal angles, allowing a non strange content of $\phi$ meson. We have expressed de physical $\phi$ and $\omega$ mesons, in terms of the shifting in the ideal mixing. From the expressions (37) it can be seen that only one of the mixing angles is related with physical observables. We have obtained a phenomenological value for the parameter $\delta$, in terms of the shifting in the ideal mixing angle, $\alpha$ and $\epsilon$ parameters.

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FIG. 1. One–loop correction to quark-gluon vertex in QCD.
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