Supplementary Material: RANS Equations

The kinematic eddy viscosity in SST k-ω model is defined as:

\[ \nu_T = \frac{a_1 k}{\max (a_1 \omega, SF_2)} \]  \hspace{1cm} (1)

The transport equation for turbulent kinetic energy is defined as:

\[ \frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P_k - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma_k \nu_T) \frac{\partial k}{\partial x_j} \right] \]  \hspace{1cm} (2)

The transport equations for specific turbulent dissipation rate are given as:

\[ \frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma_\omega \nu_T) \frac{\partial \omega}{\partial x_j} \right] \]

\[ + 2 (1 - F_1) \sigma_{\omega} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \] \hspace{1cm} (3)

where, the model coefficients and relations can be defined as given below:

\[ F_2 = \tanh \left( \max \left( \frac{2 \sqrt{K}}{\beta^* \omega y}, \frac{500 \nu}{y^2 \omega} \right) \right)^2 \]  \hspace{1cm} (5)

\[ P_k = \min \left( \tau_{ij} \frac{\partial U_i}{\partial x_j}, 10 \beta^* k \omega \right) \]  \hspace{1cm} (6)

\[ F_1 = \tanh \left\{ \min \left[ \max \left( \frac{\sqrt{K}}{\beta^* \omega y}, \frac{500 \nu}{y^2 \omega} \right), \frac{4 \sigma_{\omega} k}{CD_{k\omega} y^2} \right] \right\} \]  \hspace{1cm} (7)

\[ CD_{k\omega} = \max \left( 2 \rho \sigma_{\omega} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10} \right) \]  \hspace{1cm} (8)

\[ \varphi = \varphi_1 F_1 + \varphi_2 (1 - F_1) \]  \hspace{1cm} (9)

\[ \alpha_1 = \frac{5}{9}, \alpha_2 = 0.44 \]  \hspace{1cm} (10)

\[ \beta_1 = \frac{3}{40}, \beta_2 = 0.0828 \]  \hspace{1cm} (11)

\[ \beta^* = \frac{9}{100} \]  \hspace{1cm} (12)

\[ \sigma_{k1} = 0.85, \sigma_{k2} = 1 \]  \hspace{1cm} (13)

\[ \sigma_{\omega1} = 0.5, \sigma_{\omega2} = 0.856 \]  \hspace{1cm} (14)

where \( \nu \) and \( \nu_T \) are defined as the kinematic viscosity and the turbulent viscosity.