Banerjee, Soumyarup; Chakraborty, Kalyan; Hoque, Azizul
An analogue of Wilton’s formula and values of Dedekind zeta functions. (English)
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Authors’ abstract: J. R. Wilton obtained an expression for the product of two Riemann zeta functions. This expression played a crucial role to find the approximate functional equation for the product of two Riemann zeta functions in the critical region. We find analogous expressions for the product of two Dedekind zeta functions and then use these expressions to find some expressions for Dedekind zeta values attached to arbitrary real as well as quadratic number fields at any positive integer.

Reviewer’s remarks: Papers about any kind of zeta-functions together with their functional equations, happen to be always very detailed. The paper under review produces developments from the past (see the literature list) up to our days, in that the authors extend know work with their own inputs. We refrain in giving details. The interested reader and teachers at universities can use the contents of the paper in student seminars or so. The contents of the paper are very well worked out. One also should take knowledge of the contents of the sources mentioned in the literature list.

Reviewer: Robert W. van der Waall (Huizen)

MSC:
11R42 Zeta functions and L-functions of number fields
11B68 Bernoulli and Euler numbers and polynomials
11M06 \(\zeta(s)\) and \(L(s, \chi)\)
11A07 Congruences; primitive roots; residue systems
11E45 Analytic theory (Epstein zeta functions; relations with automorphic forms and functions)
11F20 Dedekind eta function, Dedekind sums

Keywords:
Wilton’s formula; Dedekind zeta function; special values of Dedekind zeta function; Riesz sums; Nakajima dissection

Full Text: DOI arXiv

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