A supermassive scalar star at the Galactic Center?

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ABSTRACT

We explore whether supermassive non-baryonic stars (in particular boson, mini-boson and non-topological soliton stars) might be at the center of some galaxies, with special attention to the Milky Way. We analyze, from a dynamical point of view, what current observational data show, concluding that they are compatible with a single supermassive object without requiring it to be a black hole. Particularly, we show that scalar stars fit very well into these dynamical requirements. The parameters of different models of scalar stars necessary to reproduce the inferred central mass are derived, and the possible existence of boson particles with the adequate range of masses is commented. Accretion to boson stars is also analyzed, and a comparison with another non-baryonic candidate, a massive neutrino ball, which is also claimed as an alternative to the central black hole, is given. Both models are capable to explain the nature of the object in Sgr A* without invoking the presence of a singularity. One difficult issue is why the accreted materials will not finally produce, in a sufficiently long time, a black hole. We provide an answer based on stellar disruption in the case of boson stars, and comment several suggestions for its possible solution in neutrino ball scenarios. Finally, we discuss the prospects for the observational detection of these supermassive scalar objects, using the new generation of X-ray and radio interferometry satellites.

Subject headings: galaxies: nuclei — stars: boson stars — black holes

1. Introduction

During the last years, the possible existence of a single large mass in the Galactic Center has been favored as the upper bound on its size tightens, and stability criteria rule out complex clusters. Although it is commonly believed that this central mass is a supermassive black hole, it is not yet established, as we discuss below, on a firm observational basis.

The aim of this paper is to present an alternative model for the supermassive dark object in the center of our Galaxy, formed by self-gravitating non-baryonic matter composed by bosons. This kind of objects, so-called boson stars, are well known to physicists, but up to now, observational astrophysical consequences have hardly been explored. The main characteristics of this model are:

1. It is highly relativistic, with a size comparable to (but slightly larger than) the Schwarzschild radius of a black hole of equal mass.
2. It has neither an event horizon nor a singularity, and after a physical radius is reached, the mass distribution exponentially decreases.

3. The particles that form the object interact between each other only gravitationally, in such a way that there is no solid surface to which falling particles can collide.

It is the purpose of this work to show that these features are able to produce a Galactic Center model which can be confronted with all known observational constraints, and to point the way in which such a center could be differentiated from a usual supermassive black hole.

1.1. Why scalar fields?

Interesting models for dark matter use weakly interacting bosons, and primordial nucleosynthesis show that most of the mass in the universe should be non-baryonic if $\Omega \sim 1$. Most models of inflation make use of scalar fields. Scalar-tensor gravitation is the most interesting alternative to general relativity. Recent results from supernovae, which in principle were thought of to favor a cosmological constant (Perlmutter et al. 1999), can as well be supported by a variety of models, some of them with scalar fields too (Célerier 1999). Particle physicist expect to detect the scalar Higgs particle in the next generation of accelerators. Scalar dilatons appear in low energy unified theories, where the tensor field $g_{\mu\nu}$ of gravity is accompanied by one or several scalar fields, and in string effective super-gravity. The axion is a scalar with a long history as a dark matter candidate, and Goldstone bosons have also already inferred masses. Symmetry arguments, which once led to the concept of neutron stars, may force to ask whether there could be stellar structures made up of bosons instead of fermions.

In recent works, Schunck and Liddle (1997), Schunck and Torres (2000), and Capozziello, Lambiase and Torres (2000) analyzed some of the observational properties of boson stars, and found them notoriously similar to black holes. In particular, the gravitational redshift of the radiation emitted within a boson star potential, and the rotational curves of accreted particles, were studied to assess possible boson star detection. The interest in observational properties of boson stars also led to investigate them as a possible lens in a gravitational lensing configuration (Dąbrowski and Schunck 2000). Recent studies are putting forward the gravitational lensing phenomenon in strong field regimes (Virkhadra et al. 1998; Virkhadra and Ellis 2000; Torres et al. 1998a,b). This would be the case for boson stars, which are genuine relativistic objects.

This work is organized as follows. Sec. II is a brief summary of the main observational results concerning Sgr A*, and the main hypotheses in order to explain it are outlined there. Sec. III analyzes what dynamical observational data show, and what kind of models can support them. Sec. IV gives the basic ingredients to theoretically construct scalar stars, shows mass and radius estimates, and study effective potentials and orbits of particles. In Sec. V we study the center of the Galaxy, show which are the stable scalar star models able to fit such a huge mass, and comment on the possible existence of boson particles with the required features. We also provide there an assessment of the disruption processes in boson star scenarios. Discussion and conclusions are given in Sections VI and VII.

2. The Galactic Center

2.1. Main observational facts

The Galactic Center is a very active region toward the Sagittarius constellation where, at least, six very energetic radio sources are present (Sgr A, A*, B1, B2, C, and D). Furthermore, there are several supernova remnants, filaments, and very reach star clusters. Several observational campaigns (Genzel et al. 1994) have identified the exact center with the supermassive compact dark object in Sgr A*, an extremely loud radio source. The mass and the size of the object has been established to be $(2.61 \pm 0.76) \times 10^6 M_\odot$ concentrated within a radius of 0.016 pc (about 30 lds)(Ghez 1998; Genzel et al. 1996).

More precisely, Ghez et al. (1998) have made extremely accurate velocity measurements in the central square arcsecs. From this bulk of data, it is possible to state that a supermassive compact dark object is present at the Galactic Center. It is revealed by the motion of stars moving within projected distances around 0.01 pc from the radio source Sgr A*, at projected velocities in excess of
1000 km s$^{-1}$. In other words, a high increase in the velocity dispersion of the stars toward the dynamical center is revealed. Furthermore, a large and coherent counter–rotation, especially of the early–type stars, is shown, supporting their origin in a well–defined epoch of star formation. Observations of stellar winds nearby Sgr A$^*$ give a mass accretion rate of $dM/dt = 6 \times 10^{-6} M_\odot$ yr$^{-1}$ (Genzel et al. 1996). Hence, the dark mass must have a density $\sim 10^6 M_\odot$ pc$^{-3}$ or greater, and a mass–to–luminosity ratio of at least 100$M_\odot/L_\odot$. The bottom line is that the central dark mass seems to be a single object, and that it is statistically very significant ($\sim 6 - 8\sigma$).

2.2. Domain of the black hole?

Such a large density contrast excludes that the dark mass could be a cluster of almost $2 \times 10^8$ neutron stars or white dwarfs. Detailed calculations of evaporation and collision mechanisms give maximal lifetimes of the order of $10^8$ years, much shorter than the estimated age of the Galaxy (Sanders 1992; Maoz 1995).

As a first conclusion, several authors state that in the Galactic Center there is either a single supermassive black hole or a very compact cluster of stellar size black holes (Genzel et al. 1996). We shall come back to this paradigm through the rest of the paper (particularly in Section III).

Dynamical evidence for central dark objects has been published for 17 galaxies, like M87 (Ford et al. 1994; Macchetto et al. 1997), or NGC4258 (Greenhill et al. 1995), but proofs that they really are black holes requires measurement of relativistic velocities near the Schwarzschild radius, $r_s \approx 2M_\odot/(10^8 M_\odot)$ AU (Kormendy and Richstone 1995), or seeing the properties of the accretion disk. Due to the above mentioned mass accretion rate, if Sgr A$^*$ is a supermassive black hole, its luminosity should be more than $10^{40}$ erg s$^{-1}$, provided the radiative efficiency is about 10%. On the contrary, observations give a bolometric luminosity less than $10^{37}$ erg s$^{-1}$, already taking into account the luminosity extinction due to interstellar gas and dust. This discrepancy is the so–called “blackness problem” which has led to the notion of a “black hole on starvation”. Standard dynamics and thermodynamics of the spherical accretion onto a black hole must be modified in order to obtain successful models (Falcke and Heinrich 1994). Recent observations, we recall, probe the gravitational potential at a radius larger than $4 \times 10^4$ Schwarzschild radii of a black hole of mass $2.6 \times 10^8 M_\odot$ (Ghez 1998).

2.3. Domain of the massive neutrino?

An alternative model for the supermassive compact object in the center of our Galaxy has been recently proposed by Tsiklauri and Viollier (1998). The main ingredient of the proposal is that the dark matter at the center of the galaxy is non–baryonic (but in any case fermions, e.g. massive neutrinos or gravitinos), interacting gravitationally to form supermassive balls in which the degeneracy pressure balances their self–gravity. Such neutrino balls could have formed in early epochs, during a first–order gravitational phase transition and their dynamics could be reconciled, with some adjustments, to the Standard Model of Cosmology.

Several experiments are today running to search for neutrino oscillations. LSND (White 1998) found evidence for oscillations in the $\nu_e - \nu_\mu$ channel for pion decay at rest and in flight. On the contrary KARMEN (Zeitnitz 1998) seems to be in contradiction with LSND evidence. CHORUS and NOMAD at CERN are just finishing the phase of 1994–1995 data analysis. It is very likely that exact predictions for and $\nu_\mu - \nu_\tau$ oscillations will be available in next years. Thanks to the fact that it is possible to give correct values for the masses to the quarks up, charm, and top, it is possible to infer reasonable values of mass for $\nu_e$, $\nu_\mu$, and $\nu_\tau$. In order to explain the characteristics of Sgr A$^*$, we need fermions whose masses range between 10 and 25 keV which, cosmologically, fall into the category of warm dark matter. It is interesting to note that a good estimated value for the mass of $\tau$–neutrino is

$$m_{\nu_\tau} = m_{\nu_\mu} \left( \frac{m_\mu}{m_e} \right)^2 \approx 14.4 \text{ keV}. \quad (1)$$

Choosing fermions like neutrinos or gravitinos in this mass range allows for the formation of supermassive degenerate objects from $10^6 M_\odot$ to $10^9 M_\odot$.

The theory of heavy neutrino condensates, bound by gravity, can be easily sketched considering a Thomas–Fermi model for fermions (Viollier et al. 1992). We can set the Fermi energy equal
to the gravitational potential which binds the system, and see that the number density is a function of the gravitational potential. Such a gravitational potential will obey a Poisson equation where neutrinos (and anti-neutrinos) are the source term. This equation is valid everywhere except at the origin. By a little algebra, the Poisson equation reduces itself to the radial Lané–Emden differential equation with polytropic index \( n = 3/2 \), which is equivalent to the Thomas–Fermi differential equation of atomic physics, except for a minus sign that is due to the gravitational attraction of the neutrinos. The Newtonian potential, which is the solution of such an equation, is \( \Phi(r) \sim r^{-4} \).

Considering a standard accretion disk, if Sgr A* is a neutrino star with radius \( R = 30.3 \) lds \(( \sim 10^3 \) Schwarzschild radii), mass \( M_{GC} = 2.6 \times 10^6 M_{\odot} \), and luminosity \( L_{GC} \sim 10^{37} \text{erg sec}^{-1} \), it should consist of neutrinos with masses \( m \geq 12.0 \) keV for \( g_\nu = 4 \), or \( m \geq 14.3 \) keV for \( g_\nu = 2 \) (Bilić 1998; Viollier et al. 1992). It is specially appealing that with the same mass for neutrinos, several galactic centers can be modeled. For instance, similar results hold also for the dark object \( (M \sim 3 \times 10^9 M_{\odot}) \) inside the center of M87. Due to the Thomas–Fermi theory, the model fails at the origin: we have to consider the effect of the surrounding baryonic matter which, in some sense, has to stabilize the neutrino condensate. In fact, the solution \( \Phi(r) \sim r^{-4} \) is clearly unbounded from below.

The model proceeds assuming a thermodynamical phase where a constant neutrino number density can be taken into consideration. This is quite natural for a Fermi gas at temperature \( T = 0 \). The Poisson equation can be recast in a Lané–Emden form with polytropic index \( n = 0 \). The solution of such an equation is \( \Phi(r) \sim r^2 \), which is clearly bounded (Capozziello and Iovane 1999). For \( T \neq 0 \), we get a solution of the form \( \Phi(r) \sim r^{-4} \). Matching these two results it is possible to confine the neutrino ball. On the other hand, a similar result is recovered using the Newton Theorem for a spherically symmetric distribution of matter of radius \( R \) (Binney and Tremaine 1987). In that case, the potential goes quadratic inside the sphere while it goes as \( \Phi(r) \sim r^{-1} \) matching on the boundary. In our case, the situation is similar assuming the matching with a steeper potential.

If such a neutrino condensate exists in the center of Galaxy, it could act as a spherical thick lens (a magnifying glass) for the stars behind it, so that their apparent velocities will be larger than in reality. In other words, depending on the line of sight, it should be possible to correct the projected velocities by a gravitational lensing contribution, so trying to explain the bimodal distribution (early and late type stars) actually observed (Ghez 1998; Genzel et al. 1996). Since the astrophysical features of the object in Sgr A* are quite well known, accurate observations by lensing could contribute to the exact determination of particle constituents. A detailed model and comparison with the data was presented by some of us (Capozziello and Iovane 1999).

3. Observational status

3.1. Brief review on dynamical data and what are they implying

An important information on the central objects of galaxies (particularly in active galactic nuclei) is the short timescale of variability. This has the significance of putting an upper bound –known as causality constraint– on the size of the emitting region: If a system of size \( L \) suddenly increases its emissivity at all points, the temporal width with which we receive it is \( L/c \) and thus, a source can not fluctuate in a way that involves its entire volume in timescales shorter than this (unless \( c \) is not a limiting velocity). This finally yields a corresponding maximum lengthscale, typically between \( 10^{-4} \) to 10 pc (Krolik 1999). Autocorrelation in the emission argue against the existence of a cluster of objects (unless only one member dominates the emissivity), and the small region in which the cluster should be allows two body encounters to be very common and produces the lost of the stability of complex clusters. These facts can be used to conclude that a single massive object must be in the center of most galaxies. What observations show in this case is that the size of the emitting regions are very small. This does not directly implicate black holes as such.

The way in which we expect to detect the influence of a very massive object is through its gravity. The “sphere of influence” of a large mass is defined by the distance at which its potential significantly affects the orbital motions of stars and gas, and is
given by

$$R_\ast = \frac{GM}{\sigma_z^2} \sim 4M_7\sigma_{\ast,100}^2pc,$$  \hspace{1cm} (2)$$

where the central mass is normalized to $10^7M_\odot$ and $\sigma_{\ast,100}$ is a rms orbital speed in units of 100 km s$^{-1}$. Thus, even very large masses have a small sphere of influence. Within this sphere, the expected response to a large nuclear mass can be divided in two groups. Firstly, the response of interstellar gas can hardly be anything different from an isotropic motion in the center of mass frame. Random speeds are often thought to be much smaller than this overall speed, which far from the sphere of influence is just given by $(GM/r)^{1/2}$. If the energy produced by random motions is radiated away, the gas behaves as a whole and tend to flatten itself, conserving the angular momentum.

Observations of the rotational velocity of gas as a function of the radius can thus provide a measure of the total mass at the center. Then, what observation searches is the existence of a Keplerian potential signature in the flattened gas velocity distribution. Examples of this are the radio galaxies M87 (Ford et al. 1994), NGC 4258 (Miyoshi et al. 1995), and many others. Then, any massive object producing a velocity distribution with a Keplerian sphere of influence. Within this sphere, the expected response to a large nuclear mass can be divided in two groups. Firstly, the response of interstellar gas can hardly be anything different from an isotropic motion in the center of mass frame. Random speeds are often thought to be much smaller than this overall speed, which far from the sphere of influence is just given by $(GM/r)^{1/2}$. If the energy produced by random motions is radiated away, the gas behaves as a whole and tend to flatten itself, conserving the angular momentum.

Secondly, we consider the response of the stars. In this case, peculiar velocities are larger or comparable to the bulk streaming and it is harder to actually differentiate the stellar response to a nuclear mass from the observable properties of a pure stellar potential. However, stellar motions, contrary the to that of gas, is not affected by other forces (as those produced by magnetic fields), and are better tracers of mass distributions. What observations show in this case are the stellar density and the velocity distribution. Use of the collisionless Boltzmann equation allows one to get Jean’s relation (Binney and Tremaine 1987; Krolik 1999):

$$\frac{GM(r)}{r} = V_{rot}^2 - \sigma_r^2 \left[ \frac{d\ln n(r)}{d\ln r} + \frac{d\ln \sigma^2}{d\ln r} + 2 - \frac{\sigma_i^2}{\sigma^2_r}\right]$$

$$\hspace{1cm} (3)$$

here, $n(r)$ is the spherically symmetric distribution of stars, $V_{rot}$ is the mean rotational speed, $\sigma_r, \sigma_\theta, \sigma_\phi$ are velocity dispersions and $\sigma_i^2 = \sigma_\theta^2 + \sigma_\phi^2$. Measurements of $V$ and random velocities determine that $M(r)$ decline inwards until a critical radius, where $M$ becomes constant. What really happens, however, if that even if this region is not scrutinized, the mass to light ratio becomes sufficiently large to suggest the presence of a large dark mass. A combination of this with other measurements strongly suggest that this dark mass is a single object.

To summarize, usual measurements point to inform us that there exist a single supermassive object in the center of some galaxies but do not definitively state its nature. As Kormendy and Richstone (1995) have stated, the black hole scenario has become our paradigm. But suggestions that the dark objects are black holes are based only on indirect astrophysical arguments, and surprises are possible on the way to the center.

### 3.2. The Galaxy

We follow Genzel et al. (1996) and parameterize the stellar density distribution as,

$$n(r) = \frac{\Sigma_0}{R_0} \frac{1}{1 + (R/R_0)^\alpha}. \hspace{1cm} (4)$$

Note that $R_0$ is related to the core radius through $R_{core} = b(\alpha)R_0$. Genzel et al. found that the best fit parameters for the observed stellar cluster are a central density of $4 \times 10^6 M_\odot$pc$^{-3}$, a core radius of 0.38 pc, and a value $\alpha = 1.8$ ($b(\alpha) = 2.19$). With this distribution, they found that a dark mass of about $2.5 \times 10^6 M_\odot$ was needed to fit the observational data.

The cluster distribution, and the cluster plus a black hole constant mass is shown in Fig. 1. Black boxes represent Genzel et al. 1996 (Table 10) and Eckart and Genzel 1997 (Fig. 5) data. A dashed line stands for the stellar cluster contribution, while a dot-dashed line represent an enclosed point-like black hole mass. The solid line in the right half of the figure stands for the mass distribution both for a black hole and a boson star plus the stellar cluster. The mass dependence we are plotting for the boson star is obtained in Section IV, and represents the mass distribution of a mini-boson star ($\Lambda = 0$, $m[GeV] = 2.81 \times 10^{-26}$) with dimensionless central density $\sigma(0)$ equal to 0.1. Other boson star configurations, with appropriate choice of the boson field mass $m$, yield to the same results. Note the break of more than three orders of magnitude in the x-axis, this is caused because the boson star distribution, further out of the equivalent Schwarzschild radius, behaves as a
black hole. It begins to differ from the black hole case at radius more than three orders of magnitude less than the innermost data point, that is why the x-axis has to have a break. From the mass distribution, a boson star in the center of the galaxy is virtually indistinguishable from a black hole.

Tsiklauri and Viollier (1998) have shown that the same observational data can also be fitted using an extended neutrino ball. In that case, differences begin to be noticed just around the innermost data point. It is then hard to determine whether the central object is a black hole, a neutrino ball, or a boson star based only on dynamical data now at hand, and other problems concerning the accretion disk have to considered (see below). Even harder is the situation for deciding – using only this kind of data – if the supermassive object is a boson star instead of a black hole: as a boson star is a relativistic object, the decay of the enclosed mass curve happens close the center. This, however, will provide an equivalent picture than a black hole for disruption and accretion processes; we shall comment on it in the next sections.

To apply Eq. (3) to the observational data we have to convert intrinsic velocity dispersions \((\sigma_r(r))\) and volume densities \((n(r))\) to projected ones (Binney and Tremaine 1987), these are the ones we observe. We shall also consider that \(\delta = 1 - \sigma_0^2/\sigma_\phi^2\), the anisotropy parameter, is equal to 0, and we are assuming implicitly that \(\sigma_\theta = \sigma_\phi\). We take into account the following Abel integrals,

\[
\Sigma(p) = 2\int_p^\infty \frac{n(r)r}{\sqrt{r^2 - p^2}}dr, \tag{5}
\]

\[
\Sigma(p)\sigma_r(p) = 2\int_p^\infty \frac{n(r)\sigma_r(r)^2r}{\sqrt{r^2 - p^2}}dr. \tag{6}
\]

\(\Sigma(p)\) denotes surface density, \(\sigma_r(p)\) is the projected velocity dispersion, and \(p\) is the projected distance. We adopt Genzel et al.’s (1996) and Tsiklauri and Viollier’s (1998) parameterization for \(\sigma_r(r)\),

\[
\sigma_r(r) = \sigma(\infty)^2 + \sigma(2')^2 \left( \frac{R}{2''} \right)^{-2\beta}. \tag{7}
\]

What one usually does is to numerically integrate Eqs. (5,6) and fit the observational data \((\sigma_r(p) vs. p)\). To do so one also has to assume a density dependence for the cluster (as in Eq. (4)), and the dark mass. With a point-like dark mass of \(2.5 \times 10^6 M_\odot\), the parameters of the fit result to be \(\sigma(\infty) = 55\ \text{km s}^{-1}\), \(\sigma(2'') = 350\ \text{km s}^{-1}\), and \(\beta = 0.95\).

If one now changes the central black hole for a neutrino ball, or a boson star, one has to consider the particular density dependence for these objects. In this way, \(n(r) = n_{\text{stellar cluster}}(r) + n_{\text{dark mass}}(r)\). We obtained \(n_{\text{boson star}}(r)\) as \(M(r)/(4/3\pi r^3)\), where \(M(r)\) is the fitted mass dependence of the boson star as explained in the next section. It is now useful to consider that the fitting of \(\sigma_r(p)\) is made taking into account observational data points in regions where the boson star generated space-time is practically indistinguishable from a black hole. Then, we may expect that the actual parameters, \(\sigma(\infty)\), \(\sigma(2'')\), and \(\beta\), will be very close to those obtained for the black hole. For our purposes, it is enough to take the same \(\sigma_r(r)\) as in the black hole case, and compute \(\sigma_r(\text{boson star})(p)\) using Eqs. (5,6), with the adequate total \(n(r)\).

In Fig. 2 we show the observational data of Eckart and Genzel (1997) (filled black boxes) and Genzel et al. (1996) (hollow circles) superimposed with the curve for \(\sigma_r(p)\) that we obtain with a mini-boson star \((A = 0, \sigma(0) = 0.1, m[GeV] = 2.81 \times 10^{-26}\) in the galactic center. Other boson star configurations, with appropriate choice of the boson field mass \(m\), yield to the same results. For this configuration, we used, as will be explained in Section IV, a mass distribution given by a Boltzmann-like equation with \(A_2 = 2.5 \times 10^6 M_\odot, A_1 = -0.237 \times 10^6 M_\odot, R_0 = 1.19210^{-6}\) pc, and \(\Delta R = 4.16310^{-6}\) pc. In the range plotted, and in which data is available, the differences between boson and black hole theoretical curves is undetectable. They only begin to deviate from each other at \(p \sim 10^{-4}\) pc, well beyond the last observational data point. Even the deviation in such a region is as slight as 1 km s\(^{-1}\), and it only becomes more pronounced when \(p\) values are closer to the center. However, we should recall that it has no sense to go to such extreme values of \(p\): the stars will be disrupted by tidal forces (see the discussion in Section VI) in those regions.
4. Boson stars

4.1. Basic concepts and configurations

Let us study the Lagrangian density of a massive complex self-gravitating scalar field (taking \( h = c = 1 \)),

\[
\mathcal{L} = \frac{1}{2} \sqrt{|g|} \left[ \frac{m^2}{8\pi} R + \partial_{\mu} \psi^* \partial^\mu \psi - U(|\psi|^2) \right],
\]

where \( R \) is the scalar of curvature, \(|g|\) the modulus of the determinant of the metric \( g_{\mu\nu} \), and \( \psi \) is a complex scalar field with potential \( U \). Using this Lagrangian as the matter sector of the theory, we get the standard field equations,

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{8\pi}{m^2} T_{\mu\nu}(\psi),
\]

\[
\Box \psi + \frac{dU}{d|\psi|^2} \psi = 0,
\]

where the stress energy tensor is given by,

\[
T_{\mu\nu} = (\partial_{\mu} \psi^*)(\partial_{\nu} \psi) + \frac{1}{2} g_{\mu\nu} \left[ g^{\alpha\beta}(\partial_{\alpha} \psi^*)(\partial_{\beta} \psi) - U(|\psi|^2) \right],
\]

and

\[
\Box = \partial_{\mu} \left[ \sqrt{|g|} g^{\mu\nu} \partial_{\nu} \right] / \sqrt{|g|}
\]

is the covariant d’Alembertian. Because of the fact that the potential is a function of the square of the modulus of the field, we obtain a global \( U(1) \) symmetry. This symmetry, as we shall later discuss, is related with the conserved number of particles. The particular form of the potential is what makes the difference between mini-boson, boson, and soliton stars. Conventionally, when the potential is given by

\[
U = m^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4,
\]

where \( m \) is the scalar mass and \( \lambda \) a dimensionless constant measuring the self-interaction strength, mini-boson stars are those spherically symmetric equilibrium configurations with \( \lambda = 0 \). Boson stars, on the contrary, have a non-null value of \( \lambda \). The previous potential with \( \lambda \neq 0 \) was introduced by Colpi et al. (1986), who numerically found that the masses and radius of the configurations were deeply enlarged in comparison to the mini-boson case.

Soliton (also called non-topological soliton) stars are different in the sense that, apart from the requirement that the Lagrangian must be invariant under a global \( U(1) \) transformation, it is required that—in the absence of gravity—the theory must have non-topological solutions; i.e. solutions with a finite mass, confined to a finite region of space, and non-dispersive. An example of this kind of potentials is the one introduced by Lee and his coworkers (Friedberg et al. 1987),

\[
U = m^2 |\psi|^2 \left( 1 - \frac{|\psi|^2}{\Phi_0^2} \right)^2,
\]

where \( \Phi_0 \) is a constant. In general, boson stars accomplish the requirement of invariance under a \( U(1) \) global transformation but not the solitonic second requirement. To fulfill it, it is necessary that the potential contains attractive terms. This is why the coefficient of \( (\psi^*)^2 \) of the Lee form has a negative sign. Finally, when \(|\psi| \to \infty \), \( U \) must be positive, which leads, minimally, to a sixth order function of \( \psi \) for the self-interaction. It is usually assumed, because of the range of masses and radius for soliton stars in equilibrium, that they are huge and heavy objects, although this finally depends on the choice of the different parameters.

We shall now briefly explain how these configurations can be obtained (Kaup 1968; Ruffini and Bonazzola 1969; Lee and Pang 1992; Liddle and Madsen 1992; Mielke and Schunck 1998). We adopt a spherically symmetric line element

\[
ds^2 = e^{\nu(r)} dt^2 - e^{\mu(r)} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

with a scalar field time-dependence ansatz consistent with this metric:

\[
\psi(r, t) = \sigma(r) e^{-i\omega t},
\]

where \( \omega \) is the (eigen-)frequency. This form of the field ensures us to be working in the configurations of minimal energy (Friedberg et al. 1987).

The non-vanishing components of the energy-momentum tensor are

\[
T_{0}^{0} = \rho = \frac{1}{2} [\omega^2 \sigma^2(r) e^{-\nu} + \sigma'^2(r) e^{-\mu} + U],
\]

\[
T_{1}^{1} = p_r = \frac{1}{2} [\omega^2 \sigma^2(r) e^{-\nu} + \sigma'^2(r) e^{-\mu} - U],
\]

\[
T_{2}^{2} = T_{3}^{3} = p_\perp = -\frac{1}{2} [\omega^2 \sigma^2(r) e^{-\nu} - \sigma'^2(r) e^{-\mu} - U].
\]
where \( ' = \frac{d}{dr} \). One interesting characteristic of this system is that the pressure is anisotropic; thus, there are two equations of state \( p_r = \rho - U \) and \( p_\perp = \rho - U - \sigma^2(r)e^{-\mu} \). The non-vanishing independent components of the Einstein equation are

\[
n' + \mu' = \frac{8\pi}{m_{Pl}^2}(\rho + p_r)re^{\mu}, \quad (20)
\]

\[
\mu' = \frac{8\pi}{m_{Pl}^2} \rho e^\mu - \frac{1}{r}(e^\mu - 1). \quad (21)
\]

Finally, the scalar field equation is

\[
\sigma'' + \left( \frac{\nu' - \mu'}{2} + \frac{2}{r} \right) \sigma' + e^{\mu - \nu} \omega^2 \sigma - e^{\mu} \frac{dU}{d\sigma^2} = 0.
\]

To do numerical computations and order of magnitude estimates, it is useful to have a new set of dimensionless variables. We adopt here

\[
x = mr,
\]

for the radial distance, we redefine the radial part of the boson field as

\[
\sigma = \sqrt{4\pi} \sigma/m_{Pl}, \quad (24)
\]

and introduce

\[
\Lambda = \lambda m_{Pl}^2 / 4\pi m^2, \quad \Omega = \frac{\omega}{m}. \quad (25)
\]

In order to obtain solutions which are regular at the origin, we must impose the following boundary conditions \( \sigma'(0) = 0 \) and \( \mu(0) = 0 \). These solutions have two fundamental parameters: the self-interaction and the central density (represented by the value of the scalar field at the center of the star). The mass of the scalar field fixes the scale of the problem. Boundary conditions representing asymptotic flatness must be applied upon the metric potentials, these determine –what is actually accomplished via a numerical shooting method—the initial value of \( \nu = \nu(0) \). Then, having defined the value of the self interaction, or alternatively, the form of the soliton potential, the equilibrium configurations are parameterized by the central value of the boson field. As this central value increases, so does the mass and radius of the the star. This happens until a maximum value is reached in which the star looses its stability and disperses away (the binding energy being positive). Up to this value of \( \sigma_0 \), catastrophe theory can be used to show that these equilibrium configurations are stable (Kusmartsev et al. 1991). As an example of boson star configurations, we show in Fig. 3, the mass and number of particles (see below) for a \( \Lambda = 10 \) Colpi et al.'s potential, and in Fig. 4, the stability analysis.

When \( \Lambda \gg 1 \), as in some of the cases we explore in the next sections, we must follow an alternative dimensionalization (Colpi et al. 1986). For large \( \Lambda \), we shift to the following set of variables,

\[
\sigma_* = \sigma \Lambda^{1/2},
\]

\[
x_* = x \Lambda^{-1/2},
\]

\[
M_* = M \Lambda^{-1/2}.
\]

Here, \( M_* \) is defined by

\[
e^{\nu} = \left( 1 - 2 \frac{M_*}{x_*} \right)^{-1}, \quad (29)
\]

which corresponds to the Schwarzschild mass (see below). Ignoring terms \( \mathcal{O}(\Lambda^{-1}) \), the scalar wave equation is solved algebraically to yield,

\[
\sigma_* = (\Omega^2 e^{-\nu} - 1)^{1/2},
\]

and up to the same accuracy, the field equations are

\[
\frac{dM_*}{dx_*} = \frac{1}{4} x_*^2 (3\Omega^2 e^{-\nu} + 1)(\Omega^2 e^{-\nu} - 1), \quad (31)
\]

\[
\frac{d\nu}{dx_*} e^{-\mu} - \frac{1}{x_*} \left( 1 - e^{-\mu} \right) = \frac{1}{2} (\Omega^2 e^{-\nu} - 1)^2. \quad (32)
\]

The system now depends only on one free parameter \( \Omega^2 e^{-\nu(0)} \). Numerical solutions show that the maximum mass corresponding to a stable star is given by \( M_{\text{max}} \sim 0.22\Lambda^{1/2}m_{Pl}^2/m \).

4.2. Masses estimates

The invariance of the Lagrangian density under a global phase transformation \( \psi \rightarrow \psi e^{-i\vartheta} \) of the complex scalar field gives (via the Noether’s theorem) a locally conserved current \( \partial_\mu j^\mu = 0 \), and a conserved charge (number of particles). We need to study the number of particles because it is essential to determine whether the configurations are stable or not. A necessary requirement towards the stability of the configurations is a negative binding energy \( (BE = M - mN) \), i.e. the
star must be energetically more favorable than a group of unbound particles of equal mass. From the Noether theorem, the current \( j^\mu \) is given by

\[
    j^\mu = \frac{i}{2} \sqrt{|g|} g^{\mu\nu} \left[ \psi^* \partial_\nu \psi - \psi \partial_\nu \psi^* \right] .
\]

and the number of particles is

\[
    N := \int j^0 d^3x .
\]

For the total gravitational mass of localized solutions, we may use Tolman’s expression (Tolman 1934), or equivalently, the Schwarzschild mass:

\[
    M = \int (2T_0^0 - T_\mu^\mu) \sqrt{|g|} d^3x .
\]

In Fig. 5, we show the mass of a boson star with \( \Lambda = 0 \) as a function of \( x \). We have fitted this curve with a Boltzmann-like function

\[
    M_{\text{fit}}(x) = A_2 + \frac{A_1 - A_2}{1 + e^{(x-x_0)/\Delta x}},
\]

which is reliable except in regions very near the center, where \( M_{\text{fit}}(x) \) becomes slightly negative. The \( \chi^2 \)-parameter of the fitting is around \( 10^{-5} \), and values for \( A_1, A_2, \) and \( \Delta x \) are given in the figure. It is this formula what we have used in Fig. 2 to analytically get the \( \sigma_r(p) \) dependence of the boson star (in the range we use the approximation, the actual mass and the fitting differ negligibly). Note also what the fitting is physically telling us: it represents a black hole of mass \( A_1 \) plus an inner exponentially decreasing correction. This is the first time that such a Boltzmann-like fitting is done, and we think it could be usefully applied in other analytical computations. Models with different \( \Lambda \) and \( \sigma \) can be equally well fitted.

Since boson stars are prevented from gravitational collapse by the Heisenberg uncertainty principle, we may make some straightforward mass estimates (Mielke and Schunck 1998): For a boson to be confined within the star of radius \( R_0 \), the Compton wavelength has to satisfy \( \lambda_\psi = (2\pi \hbar/mc) \leq 2R_0 \). In addition, the star radius must be of the order of the last stable Kepler orbit \( 3R_S \) around a black hole of Schwarzschild radius \( R_S := 2GM \). In the case of a mini-boson star of effective radius \( R_0 \) close to its Schwarzschild radius one obtains the estimate

\[
    M_{\text{crit}} \cong (2/\pi)m_{\text{Pl}}^2/m \geq 0.633 M_{\text{Pl}}^2/m .
\]

The exact value in the second expression was found only numerically.

For a mass of \( m = 30 \text{ GeV} \), one can estimate the total mass of this mini–boson star to be \( M \simeq 10^{10} \) kg and its radius \( R_0 \simeq 10^{-17} \) m, amounting a density \( 10^{48} \) times that of a neutron star. In the case of a boson star \( (\lambda \neq 0) \), since \( |\psi| \simeq m_{\text{Pl}}/\sqrt{8\pi} \) inside the boson star (Colpi et al. 1986), the energy density is

\[
    \rho \simeq m^2 m^2_{\text{Pl}} (1 + \Lambda/8) .
\]

Equivalently, we may think that this corresponds to a star formed from non–interacting bosons with re-scaled mass \( m \rightarrow m/\sqrt{1 + \Lambda/8} \) and consequently, the maximal mass scales with the coupling constant \( \Lambda \) as,

\[
    M_{\text{crit}} \simeq \frac{2}{\pi} \sqrt{1 + \Lambda/8} \frac{m^2_{\text{Pl}}}{m} .
\]

There is a large range of values of mass and radius that can be covered by a boson star, for different values of \( \Lambda \) and \( \sigma_0 \). For instance, if \( m \) is of the order of the proton mass and \( \lambda \simeq 1 \), this is in the range of the Chandrasekhar limiting mass \( M_{\text{Ch}} := m^3_{\text{Pl}}/m^2 \simeq 1.5M_\odot \).

Larger than these estimates is the range of masses that a non-topological soliton star produces, this is because the power law dependence on the Planck mass is even higher: \( \sim 10^{-2}(m^4_{\text{Pl}}/m^2 \Phi^2_0) \). These configurations are static and stable with respect to radial perturbations. It is by no means true, however, that these are the only stable equilibrium configurations that one can form with scalar fields. Many extensions of this formalism can be found. It is even possible to see that boson stars are a useful setting where to study gravitation theory in itself (Torres 1997; Torres et al. 1998c,d; Whinnett and Torres 1999). Most importantly for our hypothesis is that rotating stable relativistic boson stars can also be found with masses and radii comparable in magnitude to their static counterparts (Schunck and Mielke 1996; Yoshida and Eriguchi 1997). In astrophysical settings it is usual to expect some induced rotation of stellar objects, and it is important that these rotation may not destabilize the structure. Other interesting generalization is that of electrically charged boson stars introduced by Jetzer and van der Bij (1989). Although it is usually assumed that selective accretion will
quickly discharge any astrophysical object, some recent results by Punsly (1998) suggest this may not always be the case.

From these simple considerations, we can set a difference between boson and fermion condensations (e.g. neutrino condensation): in the second case we can have an extended object (in the case of Sgr A*, its size can be of about \( \sim 30 \)d) and it can be quite diluted. In the case of a boson star, the object is “strongly” relativistic, extremely compact, and its size is comparable with its Schwarzschild radius. We shall discuss the consequences of this point widely in the next section.

### 4.3. Effective potentials

The motion of test particles can be obtained from the Euler-Lagrange equations taking into account the conserved canonical momenta (Shapiro and Teukolsky 1983). In a general spherically symmetric potential, the invariant magnitude squared of the four velocity \( u^2 = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \), which is 1 for particles with non-zero rest mass and 0 for massless particles, yields to

\[
\dot{r}^2 = \frac{1}{g_{rr}} \left[ \frac{E_\infty^2}{g_{tt}} - u^2 - \frac{l^2}{g_{\phi\phi}} \right],
\]

where \( l \) and \( E_\infty \) are constants of motion given by \(-g_{\phi\phi} \sin^2 \theta \dot{\phi} \) and \( g_{tt} \dot{t} \) (angular momentum and energy at infinity, both per unit mass) respectively, and a dot stands for derivation with respect to an affine parameter. This equation can be transformed to

\[
\frac{1}{2} \dot{r}^2 + V_{\text{eff}} = \frac{1}{2} E_\infty^2 e^{-\mu - \nu},
\]

where \( V_{\text{eff}} \) is an effective potential. This name comes from the fact that in the Schwarzschild solution \( e^{-\mu - \nu} \equiv 1 \) and thus, the previous equation can be understood as a classical trajectory of a particle of energy \( E_\infty^2/2 \) moving in a central potential. This is not so in a more general spherically symmetric case, like these non-baryonic stars. In the boson star case, for instance, typical metric potentials are shown in Fig. 6; note that for them \( e^{-\mu - \nu} \neq 1 \). We can see, however, that \( e^{-\mu - \nu} < e^{-\mu(0) - \nu(0)} = C \), where \( C \) is a constant, and then, usual classical trajectories can be looked at, in the sense that we may construct an equation of the form

\[
\dot{r}^2/2 + V_{\text{eff}} < 1/2 E_\infty^2 e^{-\mu(0) - \nu(0)},
\]

and it will be always satisfied.

The effective potential that massive or massless particles would feel must be very different from the black hole case. In the case of a massless particle, the effective potential is given by

\[
V_{\text{eff}} = e^{-\mu} \frac{l^2}{2r^2},
\]

while for a massive test particle it is

\[
V_{\text{eff}} = \frac{1}{2} e^{-\mu} \left( 1 + \frac{l^2}{r^2} \right).
\]

We show both, boson (for the case of \( \Lambda = 0 \)) and black hole potentials in Figs. 7 and 8. The mass of the central object is fixed to be the same in both cases (see captions) and the curves represent a fourth order Runge-Kutta numerical integration of the equations we previously derived with \( \Lambda = 0 \) and \( \sigma(0) = 0.1 \). In the case of massive test particles we use \( l^2/M^2 = 0, 12 \) and 15; thus explicitly showing the change in the behavior of \( V_{\text{eff}} \) for the black hole case. As \( l \) increases, this shape changes from a monotonous rising curve to one that has a maximum and a minimum before reaching its asymptotic limit. These extrema disappear for \( l^2/M^2 < 12 \). In the case of boson stars, however, if \( l \neq 0 \) we have a divergence in the center at \( r = 0 \) and only one extremum -a minimum-, which occurs at rising values of \( r/M \) as \( l \) grows. The curve \( V_{\text{eff}} \) for \( l = 0 \) -radially moving objects- is not divergent, and we can see that the particles may reach the center of the star with a non-null velocity, given by \( 1/2 \dot{r}^2 = 1/2 E_\infty^2 e^{-\mu(0) - \nu(0)} - V_{\text{eff}}(0) > 0 \), and will then traverse the star unaffected.

For massless particles, differences are also notorious: a black hole produces a negative divergence and a boson star a positive one. Radial motion of massless particles is insensitive to \( V_{\text{eff}} \), being this equal to zero, as in the Schwarzschild case. In both cases, for massive and massless particles, outside the boson star the potential mimics the Schwarzschild one.

### 4.4. Particle orbits

In the case of a black hole, we can use the Newtonian analogy. Orbits can be of three types: if the energy is bigger than the effective potential at all points, particles are captured. If the energy is such that the energy equals the effective potential just once, particles describe an unbound orbit, and
the point of equality is known as turning point. If there are two such points, orbits are bound around the black hole. Orbits in which the energy equals the potential in a minimum of the latter are circular and stable ($\dot{r} = 0, V_{\text{eff}} \leq 0$).

We can not, because of the different relationship between the metric potentials we commented above, do the same analysis using $V_{\text{eff}}$ for the boson star. We can, however, note that in most cases the total equation for the derivative of $r$ will be modified in a trivial way: If we look at the cases where the effective potential has a divergence at the center (and because the metric coefficients do not diverge), there is no other possibility for the particles more than found a turning point. Orbits can then be bound or unbound depending on the energy, but we can always find a place where $\dot{r} = 0$ and then it has to reverse its sign. In the particular cases in which the equality happens at the minimum of the potential we have again stable circular orbits. Only in the case where $l = 0$ particles can traverse the scalar star unaffected. For $m \neq 0$ there is still the possibility of finding a turning point, if the energy is low enough. However, if the particle is freely falling from infinity, with $E_\infty = 1$, all energy is purely rest mass, it will radially traverse the star, as would do a photon.

We conclude that all orbits are not of the capture type. They can be circular, or unbound, and they all have at least one turning point. This helps to explain why a non-baryonic object will not develop a singularity while still being a relativistic object (comparable effective potentials, equivalently, comparable relativisticity coefficient $GM(r)/r$).

5. Galactic parameters and mass

One interesting fact, which seems to be not referred before, is that for all these scalar stars, their radius is always related with the mass in the same way: $R \gtrsim Mm_{\Pi}^{-2}$. This is indeed the statement -contrary to what it is usually assumed- that not all interesting astrophysical ranges of mass and radius can be modeled with scalar fields. In the scalar star models, from the given central mass, the radius we obtain for the star is comparable to that of the horizon, $R \sim m_{\Pi}^{-2} \times 2.61 \times 10^6 M_\odot \sim 3.9 \times 10^{11}$ cm.

The question now is for which values of the parameters we can obtain a scalar object of such a huge mass. For the case of mini-boson star we need an extremely light boson:

$$m[\text{GeV}] = 1.33 \times 10^{-25} \frac{M(\infty)}{M_{\text{BH}}}.$$  \hspace{1cm} (44)

Given a central density $\sigma(0)$, $M(\infty)$ stands for the dimensionless value of the boson star mass as seen by an observer at infinity. $M_{\text{BH}}$ is the value of the black hole mass (in millions of solar masses), obtained by fitting observational data. Then, we are requiring that the total mass of the boson star equals that of the black hole. For instance, in Fig. 1, we have taken Eckart and Genzel’s (1997) and Genzel et al.’s (1996) data, and fit them with a mini-boson star with $\sigma(0) = 0.1$, which yields to a boson mass given by $m[\text{GeV}] = 2.81 \times 10^{-26}$; the total mass of the star (without the cluster contribution) is $2.5 \times 10^6 M_\odot$.

In the case of boson stars, and using the critical mass dependence, $\propto \sqrt{\lambda m_{\Pi}^3/m^2}$, the requirement of a 2.5 million mass star yields to the following constraint,

$$m[\text{GeV}] = 7.9 \times 10^{-4} \left(\frac{\lambda}{4\pi}\right)^{1/4}.$$  \hspace{1cm} (45)

It is possible to fulfill the previous relationship, for instance, with a more heavy boson of about 1 MeV and $\lambda \sim 1$. A plot of this relation -for some values of $\lambda$- is shown in Fig. 9. Note that in this case, the value of the dimensionless parameter $\Lambda$ is huge, and special numerical procedures, as explained above, must be used to obtain solutions. The characteristics of these solutions have proven to be totally similar to those with $\Lambda = 0$ which were used in Fig. 1, just the adimensionalization differs.

Finally, in the case of a non-topological soliton we obtain the following constraint,

$$m[\text{GeV}] = \frac{7.6 \times 10^{12}}{\Phi_0[\text{GeV}]^2}.$$  \hspace{1cm} (46)

For the usually assumed case, in which the order parameter $\Phi_0$ is of equal value than the boson mass, we need very heavy bosons of unit mass $m = 1.2 \times 10^4 \text{GeV}$. Other possible pairs are shown in Fig. 10.
5.1. Boson candidates?

Based only on the constraints imposed by the mass–radius relationship valid for the scalar stars analyzed, we may conclude that:

1. if the boson mass is comparable to the expected Higgs mass (hundreds of GeV), then the Center of Galaxy could be a non-topological soliton star;
2. an intermediate mass boson could produce a super-heavy object in the form of a boson star;
3. for a mini-boson star to be used as central objects for galaxies it is needed the existence of an ultra-light boson.

These conclusions should be considered as order of magnitude estimations. Several reasons force us to make this warning. Firstly, we are just considering static and uncharged stars, this is just a model (the simplest), but more complicated ones can modify the actual constraints. Secondly, we do not know the exact form of the self-interaction, or in the case of non-topological stars, the value of $\Phi_0$. For instance, consider the Higgs mass. In the electro-weak theory a Higgs boson doublet ($\Phi^+, \Phi^0$) and its anti-doublet ($\Phi^-, \Phi^0$) are necessary ingredients in order to generate masses for the $W^\pm$ and $Z^0$ gauge vector bosons. Calculations of two-loop electro-weak effects have lead to an indirect determination of the Higgs mass (Gambino 1998). For a top quark mass of $M_t = 173.8 \pm 5$ GeV, the Higgs mass is $m_h = 104^{+93}_{-45}$ GeV. However, experimental constraints are weak. Fermilab's tevatron (Han and Zhang 1999) has a mass range of $135 < m_h < 186$ GeV, and together with LHC at Cern, they could decide if these Higgs particles (in the given range) exist in nature. Interestingly is to note that as a free particle, the Higgs boson is unstable with respect to the decays $h \to W^+ + W^-$ and $h \to Z^0 + Z^0$. However, as remarked by Mielke and Schunck (2000), in a compact object, and in full analogy with neutron stars —where there is equilibrium of $\beta$ and inverse $\beta$ decay— these decay channels are expected to be in equilibrium with the inverse process $Z^0 + Z^0 \to h$.

We should also mention the possible dilatons appearing in low energy unified theories, where the tensor field $g_{\mu\nu}$ of gravity is accompanied by one or several scalar fields. In string effective super-gravity (Ferrar et al. 1994), for instance, the mass of the dilaton can be related to the super-symmetry breaking scale $m_{\text{susy}}$ by $m_{\varphi} \simeq 10^{-3}(m_{\text{susy}}/\text{TeV})^2$ eV. Finally, a scalar with a long history as a dark matter candidate is the axion, which has an expected light mass $m_\sigma = 7.4 \times (10^7\text{GeV}/f_\sigma)$ eV $> 10^{-11}$ eV with decay constant $f_\sigma$ close to the inverse Planck time. Goldstone bosons have also inferred mass in the range of eV and less, $m_\eta < 0.06 - 0.3$ eV (Umeda et al. 1998).

If boson stars really exist, they could be the remnants of first-order gravitational phase transitions and their mass should be ruled by the epoch when bosons decoupled from the cosmological background. The Higgs particle, besides its leading role in inflationary theories, should be the best and natural candidate as constituent of a boson condensation if the phase transition occurred in early epochs. A boson condensation should be considered as a sort of topological defect relic. In this case, as we have seen, Sgr A* could be a soliton star. If soft phase-transitions took place during cosmological evolution (e.g. soft inflationary events), the leading particles could have been intermediate mass bosons and so our supermassive objects should be genuine boson stars. If the phase transitions are very recent, the ultra-light bosons could belong to the Goldstone sector giving rise to mini-boson stars.

We shall not discuss here any further the formation processes of boson stars. The reader is referred to the recent paper by Mielke and Schunck (2000) and references therein. It is apparent that for every possible boson mass in the particle spectrum there is a boson star model able to fit the galactic center constraints, at least in order of magnitude.

6. Accretion and luminosity

6.1. Relativistic rotational velocities

For the static spherically symmetric metric considered here circular orbit geodesics obey,

$$v_\varphi^2 = \frac{r
u^\nu e_{\nu} - e_{\nu} e_{\mu} + \frac{8\pi}{m_{\text{Pl}}^2} \rho r^2 e_{\mu} e_{\nu}}{2}$$

$$\simeq \frac{M(r)}{r} + \frac{8\pi}{m_{\text{Pl}}^2} \rho r^2 e_{\mu} e_{\nu}$$

(47)
These curves increase up to a maximum followed by a Keplerian decrease. Liddle and Schunck (1997) found that the possible rotation velocities circulating within the gravitational boson star potential are quite remarkable: their maximum reaches more than one-third of the velocity of light. Schunck and Torres (2000) proved that these high velocities are quite independent of the particular form of the self-interaction and are usually found in general models of boson stars. For instance, for \( \Lambda = 0,300 \) of the Colpi et al. 's (1986) choice, \( U_{\text{cosh}} = 2a\beta m^{2}[\cosh(\beta\sqrt{\rho}) - 1] \), and \( U_{\text{exp}} = 2a\beta m^{2}[\exp(\beta^2\rho) - 1] \), the maximal velocities are: 122990 km/s at \( x = 20.1 \) for \( \Lambda = 300 \), 102073 km/s at \( x = 4.1 \) for \( \Lambda = 0 \), 104685 km/s at \( x = 4.2 \) for \( U_{\text{exp}} \), and 102459 km/s at \( x = 5.9 \) for \( U_{\text{cosh}} \) (Schunck and Torres 2000).

With such high velocities, the matter possesses an impressive kinetic energy, of about 6% of the rest mass; i.e. to obtain the required luminosity we would need that about \( 10^{-8} \) to \( 10^{-7} \) solar masses per year be transformed into radiation. Note that the required matter-radiation transfer is at least two orders of magnitude smaller than the accretion rate towards Sgr A*.

The maximum rotational speed is attained well outside the physical radius of the star, as can be seen by computing the dimensionless \( x \) value for the star radius (e.g. it happens between \( x \approx 5 \) and 15 for \( \Lambda \) going from 0 to 300). It is interesting to note also that the dependence of the maximum velocity on \( \Lambda \) is not very critical, and the same process can be operative with mini-boson stars. The rotational velocity is dependent on the central density, increasing with a higher value of \( \sigma_0 \).

To obtain large rotational velocities, it is needed that the central density of the star be highly relativistic, for Newtonian solutions velocities are low and quite constant over a larger interval. This is consistent with the density constraint of the dark object in Sgr A*.

6.2. The black hole danger

How can one justify that the accretion onto the central object—a neutrino ball or a boson star—will not create a black hole in its center anyway. Interstellar gas and stars, while spiraling down towards the center of the object, will begin to collide with each other, and may glue together at the center, what could be the seed for a very massive black hole. Even a black hole of small mass can spiral inwards, and if it remains at the center, that black hole itself could be the seed. On the other hand, stellar formation of massive stars would yield, after evolution, to a black hole. Then, we need to consider whether there is a mechanism that prevents the formation of a very massive baryonic object -leading inevitably to a black hole- in the center of the galaxy. Below we provide an answer to this question.

The key aspect to consider here is disruption. A star interacting with a massive object can not be treated as a point mass when it is close enough to the object such that it becomes vulnerable to tidal forces. Such effects become important when the pericentre \( r_{\text{min}} \) is comparable to the tidal radius (Rees 1988),

\[
r_t = 5 \times 10^{12} M_6^{1/3} \left( \frac{R_*}{R_\odot} \right) \left( \frac{M_*}{M_\odot} \right)^{-1/3} \text{ cm. (48)}
\]

Here, \( R_*/\odot \) stands for the radius of the star and the sun respectively, while \( M_*/\odot \) for the masses. \( M_6 \) is the mass of the central object in millions of solar masses. \( r_t \) is the distance from the center object at which \( M/r^3 \) equals the mean internal energy of the passing star. Alternatively, we may compute the Roche criterion, a comparison between the smoothest out mass of the disruptor and the internal mean energy of the disruptee (Krolik 1999).

This catastrophe radius is

\[
R_c \sim R_g 2 \left( \frac{M}{10^8 M_\odot} \right)^{-2/3} \left( \frac{\rho_*}{\rho_\odot} \right)^{-1/3}, \quad (49)
\]

where \( R_c \) is given in units of the event horizon, \( R_g \), of the black hole, and the falling star has density \( \rho_* \). Only for black holes with masses smaller than \( 10^8 M_\odot \) and for certain low density stars -red giants- we can expect that the disruption happens outside the event horizon. This is the reason why supplying the material at lower densities, with the black hole gravity dominating the situation, can generate more power. Rees has also given an estimate of how frequently a star enters this zone. When star velocities are isotropic, the frequency with which a solar-like star pass within a distance \( r_{\text{min}} \) is

\[
\sim 10^{-4} M_6^{4/3} \left( \frac{N_*}{10^6 \text{pc}^{-3}} \right) \left( \frac{\sigma}{100 \text{km s}^{-1}} \right) \left( \frac{r_{\text{min}}}{r_t} \right) \text{ yr}^{-1}, \quad (50)
\]
where \( N_\ast \) is the star density and \( \sigma \) the velocity distribution. Disruptions are rare events that happen once in about 10000 years.

6.2.1. Scalar stars

Because of the similar metric potentials, far from the center of the non-baryonic star, the accretion mechanism will be the same than that operative in the Schwarzschild case. If a boson star is in the center of the galaxy, the characteristics of the tidal radius and the timescale of disruption occurrence will be similar to those of a black hole of equal mass. Stars falling inwards will all be disrupted after they approach a minimum radius. Contrary to black holes of big masses, which can swallows stars as a whole and disrupt them behind their event horizons, boson stars will disrupt all stars—most stars outside and a few inside the Schwarzschild radius of a black hole of equal mass—at everyone sight.

In the case of black holes, the most recent simulations (Ayal et al. 2000) show that up to 75% of the mass that once formed the disrupted star become unbound. For boson stars, we argue that once the star is disrupted to test particles (with masses absolutely negligible to that of the central object), and because there is no capture orbits, all particles follow unbound trajectories. In this sense, all material is diverted from the center and the formation of a black hole is avoided.

It could be worth of interest to perform numerical simulations changing the central object from a black hole to a boson star, to see the aftermath and the fate of the debris in an actual boson-star-generated disruption. This, however, could well not be an easy task: In black hole simulations, the minimum radius is maintained still far from the center (\( \sim 10 \) Schwarzschild radius), where Newtonian or Post-Newtonian approximations are valid. To actually see the difference between a black hole and a boson star it could be necessary to attain inner values of radii, where the behavior is completely relativistic.

One case could merit further attention: the possible spiraling of a black hole of stellar size. This case may complicate the situation since a black hole can not be disrupted. However, differences in masses are so large that it will behave as a test particle for the boson central potential, and will be also diverted from the center. Moreover, being of stellar size, it will be appreciably influenced by other intruding stars, also making it to left an static position at the boson star center.

Finally, it is worth of interest to study other possible observational consequences of boson star disruption. The ejecta now is 100% of the star mass, and may possible produce some long term effects in the surrounding medium. In black hole scenarios, the bound debris will create a flare as it accretes. Does the same happen here? While we defer this kind of analysis to another work, we mention that if boson stars exists, disruption, what was once thought of as an inevitable concomitant of black holes, could now happen in non-baryonic environments.

6.2.2. Neutrino balls

If the center of the galaxy is a neutrino ball, one also has to obtain a mechanism that prevents the formation of a very massive baryonic object. However, we have to expect very crucial differences with a boson star case, caused by the fact that a neutrino ball is an extended object and that the gravitational potential is shallower. The first thing to note is that \( r_t \) is well within the neutrino ball, and then, stars will traverse the exterior parts of the ball without being disrupted. In doing so, however, the central mass that they see at the center will be less than the total mass of the ball, and at a distance \( r = r_t \) the mass enclosed is negligible. Disruption can not proceed and other mechanism have to devised. We note then that the observation of disruption processes in the center of a galaxy is then indicative that a neutrino ball is not there.

When the mass enclosed by the neutrino ball is small enough (say, \( \mathcal{O}(10^3)M_\odot \)), the accretion disk will be unstable. This happens about 0.1 – 1 light years from the center. There, stars which could actually form at a rate of 1 per hundred thousand years (the actual number will depend on the mass of the star) will be probably kicked off by intruding stars (Viollier 2000). The absence of the disruption mechanism makes this problem worse, since given an enough amount of time, it is hard to think in compelling reasons by which gas and stars are expelled from the center.

In a published paper, Tsiklauri and Viollier...
(1999) suggested that matter arriving at the center would be diverted in the form of non-radiating jets generated by pressure of the inner accretion disk. However, it is not clear that gravity attractive forces of the spiraling objects will be smaller than gas pressure exerted by the disk. Moreover, even when the baryonic mass acquired by the central object during the entire age of the universe is at least two orders of magnitude smaller than the mass of the central neutrino ball, one should be worried if this yields to a black hole of that mass, since it may be the seed for a further collapse, or appreciably influence the dynamics. The mechanism considered in the previous paragraph seems more adequate than this for solving this question, if that is finally possible. To us, it is yet an unclear issue in the neutrino ball scenario.

7. Discussion: how to differentiate among these models?

One of the easiest things one may think of is to follow the trajectory of a particular star. This has been done by Munyaneza et al. (Munyaneza et al. 2000) in the case of a neutrino ball. The trajectory of S1, a fast moving star near Sgr A*, offers the possibility of distinguishing between a black hole and a neutrino condensate, since Newtonian orbits deviate from each other by several degrees in a period of some years. However, as soon as the central object is not so extended, as in the boson star case, this technique is useless (in every case in which the pericentre is far than the tidal radius), and other forms of detecting their possible presence have to be devised.

It has been already noted that X-ray astronomy can probe regions very close to the Schwarzschild radius. It is only observations in the X-ray band that can study the inner accretion disk as close to the center as an event horizon would be. Recent results from the Japanese-US ASCA mission have revealed a broadened iron line feature that comes from so close to the event horizon that a gravitationally redshift is observed. This is 10000 times closer into the black hole than what can be pictured by HST. In particular, Iwasawa et al. (1996) claimed that ASCA observations to Seyfert 1 galaxy MCG-6-30-15 got data from 1.5 gravitational radii, and conclude that the peculiar line profile suggests that the line-emitting region is very close to a central spinning (Kerr) black hole where enormous gravitational effects operate. By the way, this is stating that a neutrino ball can not be the center of that galaxy. However, as was already noted (Schunck and Liddle 1997), a boson star could well be a possible alternative, and X-ray could be used to map out in detail the form of the potential well. The NASA Constellation-X (Constellation-X web page 2000) mission, to be launched in 2008, is optimized to study the iron K line feature discovered by ASCA and, if they are there, will determine the black hole mass and spin for a large number of systems. Still, Constellation-X will provide an indirect measure of the properties of the region within a few event horizon radii. A definite answer in this sense will probably be given by NASA-planned MAXIM mission (Maxim web page 2000), a μ-arcsec X-ray imaging mission, that would be able to take direct X-ray pictures of regions of the size of a black hole event horizon. Both of these space mission will have the ability to give us proofs of black hole existence, or to provide evidence for more strange objects, like boson stars.

Very recently, Falcke et al. (2000) have noted that gravitational lensing observations of very large baseline interferometry (VLBI) could give the signature to discriminate among these models. Falcke et al. assumed that the overall specific intensity observed at infinity is an integration of the emissivity (taken as independent of the frequency or falling as $r^{-2}$) times the path length along geodesics. Defining the apparent boundary of a black hole as the curve on the sky plane which divides a region where geodesics intersects the horizon from a region whose geodesics miss the horizon, they noted that photons on geodesics located within the apparent boundary that can still escape to the observer will experience strong gravitational redshift and a shorter total path length, leading to a smaller integrated emissivity. On the contrary, photons just outside the apparent boundary could orbit the black hole near the circular photon radius several times, adding to the observed intensity. This is what produces a marked deficit of the observed intensity inside the apparent boundary, which they refer to as the “shadow” of the black hole. The apparent boundary of the black hole is a circle of radius $27R_g$ in the Schwarzschild case, which is much larger.
than the event horizon due to strong bending of light by the black hole. This size is enough to consider the imaging of it as a feasible experiment for the next generation of mm and sub-mm VLBI. While the observation of this shadow would confirm the presence of a single relativistic object, a non-detection would be a major problem for the current black hole paradigm.

Concerning the ideas put forward in this work, Falcke et al.’s shadow concept is appealing. In the case of a boson star, we might expect some diminishing of the intensity right in the center, this would be provided by the effect on relativistic orbits, however, this will not be as pronounced as if a black hole is present: for that case, many photons are really gone through the horizon and this deficit also shows up in the middle. If a boson star is there, some photons will traverse it radially, and the center region will not be as dark as in the black hole case. A careful analysis of Falcke et al.’s shadow behavior replacing the central black hole with a boson star model would be necessary to get any further detail, and eventually an observable prediction.

We also mention that the project ARISE (Advanced Radio Interferometry between Space and Earth) is going to use the technique of Space VLBI to increase our understanding of black holes and their environments. The mission, to be launched in 2008, will be based on a 25-meter inflatable space radio telescope working between 8 and 86 GHz (Ulvestad 1999). It will study gravitational lenses at resolutions of tens of μarcsecs, yielding information on the possible existence (and signatures) of compact objects with masses between $10^3 M_\odot$ and $10^6 M_\odot$.

Another possible technique for detecting boson stars from other relativistic objects could be gravitational wave measurements (Ryan 1997). If a particle with stellar mass is observed to spiral into a spinning object with a much larger mass and a radius comparable to its Schwarzschild length, from the emitted gravitational waves, one could in principle obtain the lowest multipole moments. The black hole no-hair theorem says that all moments are determined by its lowest two, the mass and angular momentum (assuming the charge equal to zero). Should this not be so, the central object would not be a black hole, and as far as we know, the only remaining viable candidate would be a boson star. In ten years time, perhaps, a combination of gravitational wave measurements, better determination of stellar motions, and mm and sub-mm VLBI techniques could give us a definite picture of the single object at the center of our Milky Way.

From a theoretical point of view, developments on gravitational lensing theory in very strong field regimes will be of extreme importance since, for objects like Sgr $A^*$, the standard weak field theory does not hold and new effects have to be expected (Virbhadra and Ellis 2000).

8. Conclusions

We have shown that boson stars provide the basic necessary ingredients to fit dynamical data and observed luminosity of the center of the Galaxy. They constitute viable alternative candidates for the central supermassive object, producing a theoretical curve for the projected stellar velocity dispersion consistent with Keplerian motion, relativistic rotational velocities, and having an extremely small size.

Other singularity-free models were as well considered, as non-baryonic fermion stars (e.g. neutrino, or gravitino condensations). In this case, the object is sustained by its Fermi energy, while in the boson star case, it is the Heisenberg’s Uncertainty Principle which prevents the system from collapsing to a singularity. Due to this fact, boson stars are genuine relativistic objects where a strong gravitational field regime holds.

This difference in the relativistic status of both objects is not trivial. While fermion neutrino balls are extended objects, boson stars mimic a black hole. Disruption processes can not happen in a fermion condensation, and it has the unpleasant consequence of not providing with a straightforward mechanism by which stars could be diverted from the center, and through which finally avoid the formation of a massive black hole inside the condensate.

The formation of boson stars, neutrino balls, and black holes can all be competitive processes. Then, it might well be that even if we discover that a black hole is in the center of the galaxy, others galaxies could harbor non-baryonic centers. In the case of boson stars, only after the discovery of the boson mass spectrum we shall be in position to de-
termine a priori which galaxies could be modeled by such a center. Observations of galactic centers could then suggest the existence of boson scalars much before than their discovery in particle physicists labs.

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A. Appendix

For reader’s convenience, we quote here the dimensional conversion for the radius and the mass of a boson star. Using the value of 1 GeV in cm$^{-1}$, and taking into account the dimensionless parameter $x = mr$, we get

$$r[\text{pc}] = \frac{x}{m[\text{GeV}]} 6.38 \times 10^{-33}. \quad (A1)$$

For the mass, recalling that $M = M(x)m_{Pl}^2/m$, we get

$$M[10^6M_\odot] = \frac{M(x)}{m[\text{GeV}]}1.33 \times 10^{-25}. \quad (A2)$$

In the case where $\Lambda \gg 1$, both right hand sides of the previous formulae get multiplied by $\Lambda^{1/2}$. 
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Fig. 1.— Enclosed mass in the center of the galaxy together with observational data points. See discussion in the main text.

Fig. 2.— Projected velocity dispersions: observational data and fit using a boson star model. See discussion in the main text.

Fig. 3.— Mass and number of particles, in dimensionless units, for a $\Lambda = 10$ boson star (Colpi et al. (1986)) potential.

Fig. 4.— Stability analysis for $\Lambda = 10$ configurations. Catastrophe theory ensures us that the first branch, which includes the (0,0) point, is the only stable one.

Fig. 5.— Boson star mass as a function of $x$, for a $\Lambda = 0$, $\sigma(0) = 0.1$ model. We show a Boltzmann-like fitting (dotted curve, but almost everywhere on the solid curve) and its residue (solid lower line).

Fig. 6.— Boson star metric potentials $g_{rr} = e^\mu$ (dashed) and $g_{tt} = e^\nu$ (solid). Boson star parameters are $\Lambda = 0$, $\sigma(0) = 0.1$.

Fig. 7.— Boson (solid) and Schwarzschild (dashed) effective potentials for massless particles, $b = l^2/E_\infty^2$. The mass for both, the black hole and the boson star, was taken as $M = 0.62089m_\odot^2/m$. The maximum in the black hole case happens, independently of $l$, for $r/M = 3$.

Fig. 8.— Boson (solid) and Schwarzschild (dashed) effective potentials in the case of massive particles. The mass of the central object is as in the previous figure. The three curves correspond to $l^2/M^2 = 0, 12$ and 15 (from bottom to top).

Fig. 9.— Constraint in the boson star fundamental parameters which gives rise to an object of two million solar masses within approximately ten solar radius, consistent with the mass of the central object in our galaxy.

Fig. 10.— Constraint in the non-topological soliton fundamental parameters which gives rise to an object consistent with the mass of the central object in our galaxy. It is specially marked the usual case in which $\Phi = m$. 

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$\Lambda=0$, $\sigma(0)=0.1$

Model: Boltzmann

$\chi^2 = 0.00002$

$A_1 = -0.05021\pm0.00044$

$A_2 = 0.53161\pm0.00009$

$x_0 = 5.24932\pm0.00414$

$\Delta x = 1.83296\pm0.00293$
