Cooling a magnetic nanoisland by spin-polarized currents

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(Dated: January 23, 2014)

We investigate cooling of a vibrational mode of a magnetic quantum dot by a spin-polarized tunneling charge current, exploiting the interaction between the magnetization and the vibration. The spin-polarized charge current polarizes the magnetic nanoisland, lowering its energy. Inevitable Ohmic energy losses due to the charge current flow will heat up the vibration. A small but finite coupling between the vibration and the local magnetic moment then permits an energy exchange, resulting in a lower energy, i.e., cooling, of the vibrational mode. We determine parameter regimes for the cooling of the vibration below 50\% of its initial value. Lowest final phonon temperature is observed for weak electron-phonon-coupling but similar magnetization-phonon coupling strength. The cooling rate, thereby, increases at first with the magnetization-phonon coupling and then saturates.

The present and ongoing miniaturization of electronic devices will sooner or later have to face the problem of how to efficiently remove inevitable heating from the devices. Up to now, essentially all electronic devices operate on the basis of dumping heat via passive sinks, i.e., the substrate is placed on top of the electronic device. Active or dynamical nano-cooling has received little attention, although passive thermal transport is inefficient at the nanoscale. At the same time, most present nanoelectronic or -magnetic devices function at low temperatures only. Efficient applications will thus demand active or dynamic nanorefrigerators. Active nano-cooling devices would also facilitate many new experiments which are not conceivable today due to the spacious equipment required for cooling.

Various forms of nanorefrigerators in which heat is carried by an electronic charge current have been proposed\textsuperscript{[1]}. We, in contrast, propose to use the spin degree of freedom instead of the charge for cooling, similar to the macroscopic magnetocaloric demagnetization cooling\textsuperscript{[2]}. Typically, cooling requires to open and close heat links which is realized at the macroscale by mechanically moving parts or using coolants. This becomes impractical at the nanoscale. Instead, we propose to use spin-polarized currents to polarize a magnetic nanoisland, thereby lowering its energy in a magnetic field. Subsequent energy exchange between the magnetic and vibrational degrees of freedom then lowers the energy of the latter. Electric losses give rise to Ohmic heating during the polarization process. Thus, a net cooling is reached when the polarization of the magnetic moment of the device is more efficient, i.e., faster than the Ohmic heating due to losses.

In detail, we investigate the nonequilibrium quantum dynamics of a magnetic quantum dot with a single electronic level, a local magnetic moment and a single vibrational mode as sketched in Fig. 1. The magnetic quantum dot is weakly coupled to ferromagnetic spin-polarized electronic leads via tunnelling. Charge and spin currents through quantum dots and their effects on the single electronic level with an additional magnetic moment or an additional vibrational mode have been studied extensively\textsuperscript{[3,4]}, but the full model including both has been unexplored up to present. For weak tunneling contacts, the sequential tunnelling processes dominate and a description in terms of classical or quantum master equations is adequate\textsuperscript{[10,11]}. Spin-polarized currents can polarize the dot’s magnetic moment even against an externally applied magnetic field and thus can lower its energy. At the same time the vibration gets excited to higher energies, which can be viewed as heating due to Ohmic losses. The final goal is to cool the vibrational degree of freedom which is eventually achieved by the exchange of energy between the local magnetic moment and the vibration due to a small but finite coupling. Such a reasonably strong coupling has recently been observed for a magnetic molecule covalently bound to a carbon nanotube suspended between two leads\textsuperscript{[12]}. Finally, we determine parameter regimes for cooling of the vibration below its initial temperature. Lowest final phonon temperature is observed for weak electron-phonon coupling with similar magnetization-phonon coupling strength $\xi$. The cooling rate increases with $\xi$ at first and then saturates.

\textbf{Model} We have set-up a minimal model in order to demonstrate the possible cooling of the phonon mode in
the sample, see Fig.1 for a sketch. The quantum dot is given as a single electronic level with energy $\epsilon_0$. For small dots, the local charging energy exceeds all other energies of the model, and double occupancy of the dot is forbidden. Particularly, the Hamiltonian takes the form $H_t = \epsilon_0 \left( a_\uparrow a_\uparrow + a_\downarrow a_\downarrow \right) + \sum_{k,\alpha} B_{\downarrow} \xi_{k,\alpha}^{\dagger} \xi_{k,\alpha}$, with electron annihilation and creation operators $a_\sigma$, $a_\sigma^\dagger$, respectively, the g-factor $g$, and the Bohr magneton $\mu_B$. The spin projection quantum number is $\sigma$ and $s_z$ the z-component of the electron spin $s = \frac{\hbar}{2} \sum_\sigma a_\sigma^\dagger a_\sigma$. A small external field $B$ splits the spin states along the quantization axis. The localized magnetic moment is here taken as parallel, anti-parallel or perpendicular to the source. In order to have an overall quantization axis, the tunneling Hamiltonian depends explicitly on spin rotation matrices as $H_t = \sum_{k,\alpha} = \frac{\hbar}{2} B_{\downarrow} \lambda (s \cdot J)$ where the electronic spin and the magnetization are coupled by spin-magnetization interaction of strength $g$. Within the most simple form of the interaction the coupling constant is chosen to be isotropic. In order to study the dynamical heating, a single phonon mode of frequency $\omega$ is coupled to the quantum dot. Using bosonic annihilation and creation operators $b$ and $b^\dagger$, phonon degrees of freedom are appearing as $H_{ph} = \hbar \omega b b^\dagger + \lambda \left( b + b^\dagger \right) \sum_\sigma a_\sigma b a_\sigma$. Ganzhorn et al. [12] have realized a set-up with a small magnetic molecule covalently bound to a carbon nanotube suspended between two leads. They observe a sizeable coupling between the molecular magnetic moment and the vibration. Thus, we include a corresponding coupling Hamiltonian

$$H_{J-\text{ph}} = \frac{\zeta}{\hbar} \left( b + b^\dagger \right) \left( J_+ + J_- \right),$$

where $J_\pm = \left( J_x \pm i J_y \right)/2$ are spin-$1/2$ ladder operators inducing transitions to higher/lower spin states of the magnetic impurity when at the same time the vibrational mode undergoes a transition to a lower/higher angular momentum state. The quantum dot is tunnel coupled to ferromagnetic leads on the left and right side. In general, the magnetization directions of the left and right lead are nonlinear. They are modelled as non-interacting electron reservoirs $H_{\text{leads}} = \sum_k \left( \epsilon_{k,\alpha} - \mu_\alpha \right) \left( c_{k,\alpha}^{\dagger} c_{k,\alpha} + c_{k,\alpha}^{\dagger} c_{k,\alpha} \right)$, where $c_{k,\alpha}$ ($c_{k,\alpha}^{\dagger}$) are annihilation (creation) operator for an electron with the wave number $k$ and the majority/minority spin in the lead $\alpha = L, R$ and $\mu_{L/R} = \pm eV/2$ is the chemical potential of the leads shifted by the applied bias voltage $V$. Due to the ferromagnetic nature of the leads, different spin species have different density of states at the Fermi energy. It is useful to define the polarization $p_\alpha$ of lead $\alpha$ by the relative difference in the density of states $\nu_{\alpha,\pm}$ for majority / minority spins at the Fermi energy according to $p_\alpha = \left( \nu_{\alpha, +} - \nu_{\alpha, -} \right)/\left( \nu_{\alpha, +} + \nu_{\alpha, -} \right)$. In the following, we set $p_L = p_R = p$. All energies are measured with respect to the Fermi energy at zero polarization. Spin dynamical effects, which in turn influence the population dynamics of the phonons are influenced by the relative angle between the magnetization directions of the leads due to the appearance of the exchange field on the dot [3]. The source lead polarization is chosen to be antiparallel to the dot quantization axis, whereas we consider three setups for the drain: parallel, anti-parallel or perpendicular to the source. In order to have an overall quantization axis, the tunneling Hamiltonian depends explicitly on spin rotation matrices as $H_t = \sum_{k,\alpha} = \frac{\hbar}{2} B_{\downarrow} \lambda (s \cdot J)$ with $A = (a_\uparrow, a_\downarrow) \alpha, C_{k,\alpha} = (c_{k,\alpha}^+, c_{k,\alpha}^-)$ and $\Lambda^{(L)} = \Lambda^{(R,\text{para})} = \mathbb{1}$, $\Lambda^{(R,\text{anti})} = \sigma_x$ and $\Lambda^{(R,\text{perp})} = (1 - i \sigma_y)/\sqrt{2}$ for the three setups. The hybridization with the dot state in the wide-band limit is given by $\Gamma_{\alpha} = 2\pi |t_{\alpha}|^2 (\nu_{\alpha,+} + \nu_{\alpha,-})$.

**Method** Spin polarization and phonon excitations are dynamical processes. Since the lead electrons are assumed noninteracting, they are integrated out. The time evolution of the reduced density matrix covering the degrees of freedom for electrons on the dot, the impurity and phonons obeys the kinetic equation

$$\partial_t \rho_{\chi_2}(t) = -i (\epsilon_{\chi_1} - \epsilon_{\chi_2}) \rho_{\chi_2}(t) - \int_{t_0}^{t} dt' \sum_{\chi_1, \chi_2} M_{\chi_1, \chi_2}^{\chi_2, \chi_1}(t, t') \rho_{\chi_1}(t'),$$

which covers (i) all appearing quantum coherences of the system as well as (ii) nonequilibrium effects due to the leads. In the absence of the leads, $M = 0$, the time evolution is governed by the eigenenergies $\epsilon_{\chi_i}$ of the system only. In practice, we diagonalize the Hamiltonian of dimension $6n$ numerically, where $n$ is the number of phonon states taken into account. The maximum number $n$ necessary to obtain converged results depends mainly on the electron-phonon coupling $\lambda$, the temperature $T$ and the applied bias voltage $eV$. For all results present in this work a maximum number of phonon states $n = 6$ is sufficient. In the presence of the FM leads but for small tunneling couplings, i.e., $\Gamma_{\alpha} \ll (B, q, \lambda)$, the tensor $M$ is expanded to lowest orders in $\Gamma_{\alpha}$. Taking into account the finite bias voltage between source and drain, this gives irreducible self energies on the Keldysh contour [10][11]. The leads are held at temperature $T$ at all times.

In the Markovian limit, we can solve the kinetic equation for the model either in the stationary limit or obtain the full time evolution of the observables of interest. Occupation probabilities for the subspaces of the dot, impurity and phonon are calculated by further tracing over the remaining respective degrees of freedom. This allows us to monitor the spin polarization of the dot and the heating (population of higher vibrational states) as a function of the system parameters in nonequilibrium. The calculation of the electronic tunneling current is straightforward, see Ref. [3] for the details.
is, however, obtained for the antiparallel alignment (\(\downarrow \uparrow\)). Here, the minimal steady state current and the largest magnetic polarization of the dot against an applied magnetic field and, thus, lowers the magnetic energy of the moment. Second, the spin blockade suppresses the charge current. This significantly improves the cooling efficiency since each transmitted electron can contribute to the polarization and thus to the cooling process. Additionally, the heating effect due to Ohmic losses is directly connected to the charge current and therefore suppressed as well.

We are focussed on weak to intermediate electron-phonon coupling where phonon blockade is absent. When neglecting the magnetization-phonon coupling, i.e., for \(\xi = 0\), we observe for all set-ups an increasing phonon energy \(\langle H_{\text{ph}} \rangle / \epsilon_0\) with a stronger heating for larger electron-phonon coupling. For finite magnetization-phonon coupling, energy exchange with the polarized / cooled local moment takes place and a net cooling of the phonon mode is possible. Fig. 2 (d) shows the phonon energy for three different electron-phonon couplings \(\lambda\) with finite magnetization-phonon coupling in the optimal \(\downarrow \uparrow\)-set-up. Here, cooling is observed in all cases. However, again with increasing electron-phonon coupling the cooling diminishes since heating of the phonon by the current increases. In total, these results illustrates the proof of principle of the idea of cooling a magnetic nanodevice by a spin-polarized current.

In order to quantify the cooling, we define an effective phonon temperature \(T_{\text{ph}}\) assuming a Boltzmann distribution in the steady state as

\[
T_{\text{ph}} = \frac{\langle H_{\text{ph}} \rangle_{\text{stat}}}{k_B}.
\]

Fig. 3 shows the ratio of the effective phonon temperature and the initial temperature \(T_{\text{init}}\) as a function of electron-phonon \(\lambda\) and magnetization-phonon coupling \(\xi\). Cooling is achieved in the full parameter regime depicted. As expected, with increasing electron-phonon coupling \(\lambda\), the effective phonon temperature increases. Surprisingly, given a finite electron-phonon coupling, we observe

![FIG. 2. (a) Polarization \(\langle J_z \rangle / \hbar\) of the localized magnetic moment along the z-direction, (b) charge current \(\langle I \rangle / \Gamma\), (c) phonon energy \(\langle H_{\text{ph}} \rangle / \epsilon_0\) as a function of time for the three different lead setups and (d) for different electron-phonon coupling strengths \(\lambda / \epsilon_0\).](image)

![FIG. 3. Effective phonon temperature \(T_{\text{ph}}/T_{\text{init}}\) as a function of electron-phonon \(\lambda\) and magnetization-phonon coupling \(\xi\). The remaining parameters are as in Fig. 2](image)
FIG. 4. Effective phonon temperature $T_{\text{ph}}$ in the stationary limit versus magnetic field $g\mu_B B$ and spin-magnetization coupling $q$. The dashed lines indicate resonances between spin and phonon states for the uncoupled system $(\xi, \lambda \to 0)$. The used parameters are $\xi = 0.12\epsilon_0$, $\lambda = 0.1\epsilon_0$, with the remaining parameters as in Fig. 2.

A nonmonotonic behaviour regarding the magnetization-phonon coupling. At first, the effective phonon temperature decreases with magnetization-phonon coupling. However, a minimum is reached and for further increasing magnetization-phonon coupling the effective phonon temperature increases again. In Fig. 4 the effective phonon temperature is shown as a function of the magnetic field $g\mu_B B$ and the spin-magnetization coupling $q$. For higher magnetic field values the cooling effect is increased as expected since the energy gain due to spin polarization is directly proportional to the magnetic field. Additional fine structures can be observed and traced back to resonances between spin flips and phonon transitions which increase or decrease the effective phonon temperature depending on the direction of the transition. The dashed lines in Fig. 4 indicate such resonances for the case of vanishing magnetization-phonon and electron-phonon coupling $(\xi, \lambda \to 0)$.

An effective temperature as in Eq. 3 can be defined at all times. We find an exponential behaviour towards the steady state for the effective temperature. Thus, we can determine the effective cooling rate $\Gamma_{\text{cool}}$ by a fit to our numerical results, $T_{\text{ph}}(t) \simeq T_{\text{ph}}(\infty) + e^{-\Gamma_{\text{cool}}(T_{\text{init}} - T_{\text{ph}}(\infty))}$. Fig. 5 (a) depicts the cooling rate versus magnetization-phonon coupling for two values of electron-phonon coupling, i.e., $\lambda = 0.1\epsilon_0$ and $\lambda = 0.2\epsilon_0$. We observe initially a strong increase of the cooling rate with increasing magnetization-phonon coupling. For $\xi \gtrsim 0.1$, however, the cooling rate saturates.

The initial and finite effective phonon temperature as a function of the lead temperature $T$ are shown in Fig. 5 (b). For the whole depicted temperature range the initial phonon temperature $T_{\text{init}}$ exceeds the steady state value $T_{\text{ph}}$, thus maintaining the cooling effect for a range of lead temperatures of at least one order of magnitude.

We also observe that our definition of the initial phonon temperature is not directly proportional to the lead temperature. This behaviour originates from the preparation of our initial state which includes all couplings and therefore leads to a nonlinear behaviour of the phonon temperature with respect to the lead temperature.

**Conclusion** In this Letter, we have established a simple model to illustrate the principle of cooling a magnetic nanoisland by a spin-polarized charge current. It is based on the polarization of the island magnetization by the flowing polarized electron spins and a subsequent removal of thermal energy from the phonon, i.e., a net cooling arises. Interestingly, Ohmic heating can be overcome by a magnetization-phonon coupling, which leads to a significant lowering of the energy stored in the vibration. We also find that the cooling rate saturates as a function of the magnetization-phonon coupling, implying that not very strong couplings are required to observe the proposed effect. We are confident that this mechanism could be realized in magnetic molecular nanojunctions by present day technology.

We acknowledge financial support by the DFG via the Schwerpunktprogramm Spin Caloric Transport (SPP 1538). P.N. thanks the BSH Bosch und Siemens Hausgeräte GmbH for interesting and useful insights into cooling technology.
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