Analysis of error sources on orbital atomic clocks’ stability
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As a rule, stability calculation of atomic clock requires observations with equivalent sampling interval. Apart from atomic clocks in laboratory, orbital atomic clock stability calculations are impacted of raw data sampling intervals, non-continuous time series, non-data segment, frequency drift, and other factors. So, the calculated stability results are not so exact. In this article, the impacts of kinds of error sources on Allan and Hadamard variances are analyzed using global positioning system satellite precise clock offset data. And the laws of variety are summarized.

Keywords: atomic clocks; Allan variance; Hadamard variance; frequency drift

1. Introduction

Atomic clocks are a core component of satellite navigation systems. Satellite navigation, positioning, and timing cannot be achieved without atomic clocks (1). Satellite navigation systems as compared with laboratory or general ground engineering applications require a longer life, higher reliability, and higher accuracy from onboard atomic clocks. Analyzing the influences from various factors on atomic clocks has important significance for objectively understanding their capability and applicable for onboard atomic clock offset determination and forecasting (2).

Atomic clock frequency stability analysis describes the random fluctuations of output frequency generated by noise influences. The deterministic part of an atomic clock time series can be expressed as quadratic polynomial, while random variation can usually be described by seven independent energy spectrum noises (3). Stability analysis can be processed using more than a dozen methods including standard variance, Allan variance series, Hadamard variance series, and so on. These methods are comprehensively described in many literatures, and will not be described in this paper (2–6). In comparison with atomic clocks in ground laboratories, the stability calculation of atomic clocks onboard is also influenced by specific factors such as the satellite visibility in addition to the influences from the seven kinds of noise. Since data collection is limited by the original data sampling interval, noncontinuous time series, no data segment, frequency drift, and other conditions (7), therefore, the main purpose of this study is analyzing the influences of specific factors on the stability of onboard atomic clocks. These results will provide a reference for the practical application.

2. Influence analysis of stability

2.1. Data adopted

In the process of stability analysis, three months (92 days) global positioning system (GPS) satellites’ precise clock offset data from 6 July 2008 to 5 October 2008 were downloaded from the International Global navigation satellite system Service (IGS) website. The data sampling interval was 30 s. To facilitate calculation and analysis, data were processed based on these clock offset data files. The clock offset time series at a sampling interval of 2, 5, and 10 min were extracted from these data. By using precise ephemeris within the same time period, visibility is calculated and precision clock offset data within the arc which can be observed by IGS Beijing tracking station were also extracted (8, 9). At a data sampling interval of 30 s, the minimum elevation angle was set to 5° and 10°, respectively. The IGS site coordinates of the Beijing tracking station are (−2,160,025.221, 4,383,318.161, and 4,085,370.342 m). GPS satellites used for calculation are pseudo-random number (PRN) 7 and PRN 16, satellites with the fewest missing data points.

2.2. The impact of the original data sampling interval

For analyzing the impact of raw data sampling interval on atomic clock stability results, the stabilities with smoothing time of 1000, 10,000 s, and one day of PRN 7 and PRN 16 satellites with sampling interval of 30 s,
2, 5, and 10 min are compared in this paper. The results are shown in Table 1. During stability calculation, the pretreatment work, including atomic clock data conversion from the time domain to frequency domain, $5\sigma$ gross error exclusion, and frequency drift exclusion were carried out (6). Unless specifically noted, these data preprocessing methods are used throughout this paper.

The “Dif” column in Table 1 gives the percentage change compared between the 30 s sampling interval results and other sampling interval results. As shown in Table 1, during data conversion from the time to frequency domain, the high frequency noise which frequency domain data contained is relatively weakened by the averaging effect. So, most of the results are improved with an increased sampling interval. Using the calculated results of PRN 7 and PRN 16 satellites as an example, the increasement of the PRN 7 satellite results is relatively low. The maximum increase extent of stabilities with smoothing time of 1000, 10,000 s, and one day can reach 4.48, 5.63, and 9.52%, respectively, while the raw data sampling interval increased from 30 s to 10 min. The change is higher for PRN 16 satellite data. The maximum increase can reach 6.06, 3.59, and 48.78%. Thus, the impact of the raw data sampling interval on the Allan variance and Hadamard variance is considerable.

Because of the large differences on the impact of sampling interval between the two satellites, the main noise types and noise coefficients are analyzed and shown in Table 2.

The random variation parts of atomic clocks are usually described by seven independent energy spectrum noises. These noises are white noise phase modulation (WH PM), flicker noise phase modulation (FL PM), white noise frequency modulation (WH FM), flicker noise frequency modulation (FL FM), random walk noise frequency modulation (RW FM), flicker walk noise frequency modulation (FW FM), and random run noise frequency modulation (RR FM). The order of noise frequency is decreasing one by one (10, 11). As shown in Table 2, the PRN 16 satellite is mainly influenced by high-frequency WH PM and FL PM noise. The noise coefficient has a three order decrease when compared with the PRN 7 satellite. Therefore, the influence of the sampling interval is larger.

### 2.3. The impact of noncontinuous time series

For navigation constellations with nonglobal tracking stations, onboard atomic clock data usually appear to be a noncontinuous sampling phenomenon because of the limit of visible arc of satellite tracking stations. The interrupted time interval of each data break off is not nearly the same. A noncontinuous time series not only effects stability calculation, but also reduces the maximum available smoothing factor. In this section, PRN 7 satellite’s onboard atomic clock data of IGS Beijing

| Stability        | Smoothing time(s) | Sampling interval | Dif (%) | Dif (%) | Dif (%) | Dif (%) | Dif (%) |
|------------------|------------------|-------------------|---------|---------|---------|---------|---------|
| **PRN 7**        |                  |                   |         |         |         |         |         |
| Allan variance   | 1000 s           | 21.0              | 20.5    | −2.44   | 20.1    | −4.48   | 20.1    | −4.48   |
|                  | 10,000 s         | 3.94              | 3.94    | 0.00    | 3.83    | −2.87   | 3.75    | −5.07   |
|                  | One day          | 1.84              | 1.80    | −2.22   | 1.81    | −1.66   | 1.84    | 0.00    |
| Hadamard variance| 1000 s           | 21.0              | 21.0    | 0.00    | 21.0    | 0.00    | 21.0    | 0.00    |
|                  | 10,000 s         | 3.94              | 3.94    | 0.00    | 3.82    | −3.14   | 3.73    | −5.63   |
|                  | One day          | 1.84              | 1.74    | −5.74   | 1.68    | −9.52   | 1.76    | −4.54   |
| **PRN 16**       |                  |                   |         |         |         |         |         |
| Allan variance   | 1000 s           | 21.0              | 20.0    | −5.00   | 20.2    | −3.96   | 19.8    | −6.06   |
|                  | 10,000 s         | 6.40              | 6.22    | −2.89   | 6.30    | −1.58   | 6.30    | −1.58   |
|                  | One day          | 1.04              | 0.699   | −48.78  | 0.732   | −42.07  | 0.766   | −35.77  |
| Hadamard variance| 1000 s           | 21.7              | 20.5    | −5.85   | 21.0    | −3.33   | 20.5    | −5.85   |
|                  | 10,000 s         | 6.63              | 6.52    | −1.69   | 6.40    | −3.59   | 6.57    | −0.91   |
|                  | One day          | 1.04              | 0.702   | −48.14  | 0.705   | −47.51  | 0.755   | −37.74  |

| Satellite | WH PM | FL PM | WH FM | FL FM | RW FM | FW FM | RR FM |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| 7         | 2.11e−10 |       | 1.87e−11 |       | 2.02e−12 |       | 1.90e−17 |       |       |       |       |       |
| 16        | 1.94e−10 |       | 1.94e−11 |       | 5.16e−15 |       |       |       |       |       |       |
station within its visible arcs are used to analyze the impact of noncontinuous time series on stability calculation. The minimum elevation angle restrictions of the visible arc were set to 5° and 10°. The calculated Allan variance results are shown in Table 3.

When the minimum elevation angles were restricted to 5° and 10°, only 32.8 and 26.4% of the PRN 7 satellite is visible from the IGS Beijing station. The “Dif” column in Table 3 gives the percentage change compared between the results calculated using all data and the results calculated using only the data within the visible arc. As shown in Table 3, noncontinuous atomic clock data has little effect on short-term stability within 8 min. But has greater impact on the stability with smoothing time between 10³ and 10⁵ s. The maximum drop can reach an order of magnitude. The drop of stability with smoothing time of 1000 s is about 35% and the drop of stability with smoothing time of 10,000 s is about 84%. So we should avoid using noncontinuous time series.

### 2.4. The impact of non-data segment

In order to analyze the impact of the non-data segment on stability calculation, the central part of PRN 7 and PRN 16 satellites’ full data was deleted for the purpose of this paper. The nondata segments were then formatted. The deleted segments were one day and five days. The results are shown in Table 4. The “Dif” column in Table 4 gives the percentage change compared between the calculated results using all data and the results calculated using data with the non-data segment.

As shown in Table 4, the difference can reach 13.58% with five days’ data missing for the PRN 7 satellite. The PRN 16 satellite, however, was less affected by

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**Table 3. The impact of noncontinuous time series on stability analysis.**

| Smoothing time (s) | All data | 5° limit | Dif (%) | 10° limit | Dif (%) |
|-------------------|----------|----------|---------|-----------|---------|
| 3.0000e+01        | 2.14e−12 | 2.07e−12 | −3.38   | 2.06e−12  | −3.88   |
| 6.0000e+01        | 1.89e−12 | 1.82e−12 | −3.85   | 1.81e−12  | −4.41   |
| 1.2000e+02        | 1.29e−12 | 1.25e−12 | −3.20   | 1.24e−12  | −4.03   |
| 2.4000e+02        | 7.61e−13 | 7.71e−13 | 1.30    | 7.66e−13  | 0.65    |
| 4.8000e+02        | 4.47e−13 | 4.62e−13 | 3.25    | 4.64e−13  | 3.66    |
| 9.6000e+02        | 2.24e−13 | 2.87e−13 | 21.95   | 2.81e−13  | 20.28   |
| 1.9200e+03        | 1.19e−13 | 2.41e−13 | 50.62   | 2.12e−13  | 43.86   |
| 3.8400e+03        | 6.92e−14 | 2.55e−13 | 72.86   | 1.78e−13  | 61.12   |
| 7.6800e+03        | 4.77e−14 | 1.80e−13 | 73.50   | 1.98e−13  | 75.90   |
| 1.5360e+04        | 3.21e−14 | 1.67e−13 | 80.78   | 2.85e−13  | 88.73   |
| 3.0720e+04        | 2.51e−14 | 9.14e−14 | 72.54   | 1.16e−13  | 78.36   |
| 6.1440e+04        | 1.84e−14 | 3.14e−14 | 41.40   | 3.77e−14  | 51.19   |
| 1.2288e+05        | 1.52e−14 | 2.19e−14 | 30.59   | 2.60e−14  | 41.53   |
| 2.4576e+05        | 1.22e−14 | 1.55e−14 | 21.29   | 1.70e−14  | 28.23   |
| 4.9152e+05        | 1.33e−14 | 1.34e−14 | 0.75    | 1.42e−14  | 6.33    |
| 9.8304e+05        | 1.87e−14 | 1.66e−14 | −12.65  | 1.70e−14  | −10.00  |
| 1.0000e+03        | 2.10e−13 | 2.92e−13 | 28.08   | 2.97e−13  | 29.29   |
| 1.0000e+04        | 3.94e−14 | 2.44e−13 | 83.85   | 2.31e−13  | 82.94   |
| 8.6400e+04        | 1.84e−14 | 2.27e−14 | 18.94   | 2.86e−14  | 35.66   |

**Table 4. The impact of the non-data segment on stability analysis.**

| No data length | Allan variance | Hadamard variance |
|----------------|----------------|-------------------|
|                | 1000 s | 10,000 s | One day | 1000 s | 10,000 s | One day |
| PRN 7 No missing data | 2.10e−13 | 3.94e−14 | 1.84e−14 | 2.10e−13 | 3.94e−14 | 1.84e−14 |
| PRN 7 1 day | Stability | 2.10e−13 | 3.97e−14 | 1.87e−14 | 2.26e−13 | 4.00e−14 | 1.84e−14 |
| PRN 7 5 days | Stability | 2.06e−13 | 3.92e−14 | 1.71e−14 | 2.16e−13 | 3.97e−14 | 1.62e−14 |
| PRN 16 No missing data | 2.10e−13 | 6.40e−14 | 1.04e−14 | 2.17e−13 | 6.63e−14 | 1.04e−14 |
| PRN 16 1 day | Stability | 2.09e−13 | 6.43e−14 | 1.04e−14 | 2.17e−13 | 6.63e−14 | 1.05e−14 |
| PRN 16 5 days | Stability | 2.10e−13 | 6.25e−14 | 1.01e−14 | 2.16e−13 | 6.47e−14 | 1.03e−14 |

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the non-data segment. Mainly because the frequency drift rate for the PRN 7 satellite is $2.55 \times 10^{-16}/s^2$. The frequency drift rate for the PRN 16 satellite is $2.48 \times 10^{-17}/s^2$. As shown in Figures 1 and 2, the drop in the frequency drift rate can reach one order of magnitude. Therefore, the non-data segment has greater impact on atomic clock stability calculation with a large frequency drift.

2.5. The impact of frequency drift

The formula of Allan variance $\sigma_y^2(\tau)$ and Hadamard variance $H\sigma_y^2(\tau)$ are expressed as (5, 10):

$$\sigma_y^2(\tau) = \frac{1}{2(N - 2m)\tau^2} \sum_{i=1}^{N-2m} [\Delta_x^2(i)]^2$$  \hspace{1cm} (1)

$$\Delta_x^2(i) = x_i + 2x_{i+1} - 2x_{i+2} + x_{i+3}$$

$$H\sigma_y^2(\tau) = \frac{1}{6\tau^2(N - 3m)} \sum_{i=1}^{N-3m} [\Delta_x^2(i)]^2$$  \hspace{1cm} (2)

where $\tau = mt_0$ is smoothing time, $m$ is the smoothing factor, $x_i$ is $i$th value of the atomic clock time series. The atomic clock time series were expressed as the variation parts and random variation parts, i.e.:

$$x_t = x_0 + y_0 t + \frac{1}{2}D\tau^2 + e(t)$$  \hspace{1cm} (3)

where $x_0$ is the time deviation, $y_0$ is the frequency deviation, $D$ is the frequency drift, and $e(t)$ is random variation part. Putting formula (3) into formulas (1) and (2), we can derive that frequency drift has no effect on the two variance when $x_0$ and $y_0$ are constant. But the influence on variance will no longer be zero when frequency difference changes over time. As for the frequency drift term $D$:

$$\Delta_x^2(t) = D\tau^2 \hspace{1cm} \Delta_y^2(t) = 0$$  \hspace{1cm} (4)

As shown in formula (4), the Hadamard variance is a triple sampling variance, while the influence of frequency drift on the Hadamard variance is zero. But the impact on the estimated Allan variance is no longer zero. The magnitude of impact is (12):

$$\delta\sigma_y^2(\tau) = \frac{D^2\tau^2}{2}$$  \hspace{1cm} (5)

In this paper, the simulated frequency domain data were used to inflect the impact of frequency drift. Sample interval ($\tau_0$) of simulated data is set to 300s. The frequency deviation is set to 1.1e-10. Frequency drift is 1.7e-17/\tau_0. The WH PM coefficient is 2.0e-10. The FL PM coefficient is 2.0e-10. The number of simulated data points was 1.0e5. Allan and Hadamard variance results were calculated using data with frequency deviation “not eliminated” and “eliminated” are shown in Figures 3 and 4. “ADEV” represents the Allan variance and “HDEV” represents the Hadamard variance.
As can be seen from Figure 3, the Allan variance calculated using data with frequency drift “not eliminated” deviates from the real stability when smoothing time greater than the 200 s. The difference increases with time. The Allan variance calculated using data with frequency drift “eliminated” matched well with the Hadamard variance.

3. Conclusion

(1) Compared with laboratory and ground engineering applications, onboard atomic clock stability and other performance evaluations are limited by measurement noise, visible arcs, and other conditions with unique characteristics.

(2) Increasing raw data sampling intervals, the high frequency noise which frequency domain data contains are weakened by the averaging effect. Stability was improved. The impacts on Allan variance and the Hadamard variance are considerable.

(3) Noncontinuous atomic clock data have little effect on short-term stability. But, it has greater impact on the stability with smoothing time between $10^3$ and $10^5$ s. The maximum drop can reach an order of magnitude. So, we should avoid using noncontinuous time series.

(4) The non-data segment has greater impact on atomic clock stability calculations with large frequency drift. When there is a non-data segment, use a zero-padding operation on the raw data in order to improve accuracy.

(5) Not eliminating frequency drift in the original data will lead to long-term Allan variance stability deviation, but it has no effect on the Hadamard variance.

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