Electron hole instability as a primordial step towards sustained intermittent turbulence in linearly subcritical plasmas

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Abstract
Electron and ion holes are highly stable nonlinear structures met omnipresently in driven collisionless hot plasmas. A mechanism destabilizing small perturbations into holes is essential for an often witnessed but less understood subcritically driven intermittent plasma turbulence. In this paper we show how a tiny, eddy-like, non-topological electron seed fluctuation can trigger an unstable evolution deep in the linearly damped region, a process being controlled by the trapping nonlinearity and hence being beyond the realm of the Landau scenario. After a (transient) transition phase modes of the privileged spectrum of cnoidal electron and ion holes are excited which in the present case consist of a solitary electron hole (SEH), two counter-propagating ‘Langmuir’ modes (plasma oscillation), and an ion acoustic mode. A quantitative explanation involves a nonlinear dispersion relation with a forbidden regime and the negative energy character of the SEH, properties being inherent in Schamel’s model of undamped Vlasov–Poisson structures identified here as lowest order trapped particle equilibria. An important role in the final adaption of nonlinear plasma eigenmodes is played by a deterministic response of trapped electrons which facilitates transfer of energy from electron thermal energy to an ion acoustic nonuniformity, accelerating the SEH and positioning it into the right place assigned by the theory.

1. Introduction
Subcritically driven turbulence of plasma state remains a less understood process, often presenting its strong signatures in nature [1, 2], experiments [3–5] and in simulations [6–11] of collisionless hot plasmas. Underlying this are instabilities of nonlinear collective eigenmodes of nonthermal distributions rather than those of the normal linear eigenmodes of a thermalized distribution f₀, recoverable by selecting the corresponding poles of dispersion function to perform the Landau integral, yielding f₀(ω) = ∂f₀/∂v as a unique driver for the microinstabilities. Explanation of this stronger nonlinear basis of the turbulence threshold is explored both by stochastic [9, 12] as well as deterministic approaches [13, 14], prescribing the growth largely linked to species’ ∂f. With these criteria often defied by the evolution, no basis is known for quantitatively exploring drivers of rather complex uncontrollable subcritical evolution [15] of coherent phase-space perturbations constituting fundamental nonlinear collective structures in hot nonthermal collisionless plasma [16], inevitably unstable if they possessed a forbidden regime or violated the negative energy state condition [17] in certain regimes. By first recovery of these two characteristic attributes of electron phase-space holes in our simulations, we have quantitatively applied, to the observed evolution, a formulation implementing a stochastic scale cut-off to approach fundamental smallest nonlinear unit of phase-space perturbations [18]. We have thus characterized the subcritically unstable response in terms of parameters that allow generalization to ensembles, or large scale nonthermal phase-space equilibria.

We present results of three cases of high-resolution Vlasov simulations initialized with small phase-space perturbations capable of developing into unstable hole structures. Present investigation shows that how tiny
electron hole (EH) structures have forbidden parameter regimes in which they adapt by variation of several parameters, essentially traversing a manifold of potential, and associated distribution function, solutions \( \{ \phi(x - x_0 \varepsilon t), f(x - x_0 \varepsilon t, \nu) \} \) in phase-space or position–velocity \((x-\nu)\) space such that their stability boundary is rather complex in function space, an unstable subcritical evolution that we term as multifaceted \([13, 19]\). To provide an analogy, an evolving non-multifaceted (orthogonal) linear eigenmode has freedom only of either growing or damping in its amplitude, because of analytic invariance, e.g., of frequency \( \omega \), wave-vector \( k \) or propagation velocity \( \nu_p \), essentially enforced by the choice of linear approximation. The smallest amplitude modes in these terms are therefore essentially non-orthogonal and coherent nonlinear structures. Here we term such structures which satisfy a nonlinear dispersion relation (NDR) as nonlinear eigenmodes.

The second part of observations shows that the EHs can also be destabilized by parametric coupling to conventional collective modes of collisionless plasmas. In all cases the phase velocity \( \nu_p \) of the finally settled solitary EH (SEH) exceeds the electron drift and is hence located at the right wing of \( f_{eq} \), which has a negative slope that, according to standard wave theory, would imply disappearance by Landau damping \([20]\). We hence have observed a nonlinear evolution beyond the generally accepted Landau scenario for the plasma turbulence.

In dealing with subcritical turbulence in compressible phase-space flows rather than in 2D-incompressible shear flow (as by Pringle et al \([21]\) and Barkley et al \([22]\)), the main analogy we note between the two is that a localized shearing flow is introduced in both the systems. In present treatment, however, it is in the phase-space flows of non-topological kind and done through the fluctuation rather than through the background. The phase-flow analog of the linear regime of incompressible flow system \([21]\) is the regime of commonly accepted standard linear Vlasov wave theory \([20]\). The acceleration of EH structures is thus shown to result specifically from an unstable evolution of such phase-space perturbations. The acceleration of a stable but faster EH due to the momentum exchange mechanism of electron and ion through scattering is also reported in some recent works \([14, 23]\).

The present paper is organized as follows: in section 2 we discuss the initial perturbations for three different cases used in the simulations for EH generation, normalization factors and corresponding simulation results. In sections 3 and 4 the analytic model for a privileged SEH and a NDR are presented which are applied to and compared with the simulation results. Section 4.1 describes the existence of unstable forbidden region for a EH solution on the basis of NDR and explains the mechanism of acceleration of SEH after emerging from the forbidden region due to parametric coupling of conventional plasma modes with the nonlinear SEH mode which makes the solution unstable. Negative energy character of stable EH solution is discussed in section 4.3 which is consistent with our simulation results and explains the absence of SEH with a valid very low velocity \((B > 0)\) in case 3. Time evolution of the trapped species dispersion parameter \( B \) is discussed in section 5. Three different regions are identified and energy exchange mechanism in the unstable regions are also discussed. Finally, summary and conclusions are discussed in section 6.

2. Development of trapped electron eigenmodes

For the present exact mass ratio simulations \((\delta = m_e/m_i = 1/1836)\) we have used a well localized initial perturbation in the electron distribution function of the following analytic form,

\[
f_e(x, \nu) = -\epsilon \, \text{sech} \left( \frac{\nu - \nu_i}{L_1} \right) \text{sech}^2 \left[ k(x - x_i) \right],
\]

where \( \epsilon \) is the amplitude of the perturbation, \( L_1 \) is the width of the perturbation in the velocity dimension and \( k^{-1} \) is its spatial width. We use the Debye length \( \lambda_D \), inverse electron plasma frequency \( \omega_p^{-1} \) and electron thermal velocity \( \nu_{th} = \sqrt{T_e/m_e} \) as normalizations for length, time and electron velocities, respectively. The background electron and ion velocity distributions are Maxwellian with a finite electron drift \( \nu_D \),

\[
f_{eq}(\nu) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{(\nu - \nu_D)^2}{2} \right],
\]

\[
f_{th}(u) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{u^2}{2} \right],
\]

where \( u = \sqrt{\theta/\delta} \nu \) and \( \nu_D = 0.01 \) is chosen below the critical linear threshold \( \nu_D^* = 0.053 \) \([24]\) for \( \theta = T_e/T_i = 10 \) used by us.

Here we solved the nonlinear 1D Vlasov equation using flux balance technique \([25, 26]\) for both electron and ion in \( x-\nu \) space with \((16384 \times 32768)\) mesh grid. We first present the evolution of the total electron distribution \( f_e = f_{eq} + f_1 \) in three cases, 1, 2, and 3, where \( \nu_1 = 0.05, 0.01, \) and 0.004, respectively, i.e., the perturbation located beyond the maximum of \( f_{eq}(\nu) \) far in the decreasing tail in case 1, just at its maximum in case 2 and in the rising slope in case 3. Moreover, in case 3 \( f_1 \) is situated between the two peaks of the unperturbed
distributions of electron and ion. It is additionally initiated from the center, \( x = 15 \), of the simulation box of length \( L = 30 \). Figure 1 presents the initial electron distribution for three different cases. The phase-space widths of the perturbation is chosen as \( L = 0.01 \) along \( \nu \) and \( k^{-1} = 10 \) along \( x \) in expression (1) with the perturbation strength \( \epsilon = 0.06 \).

Despite the fact that we are well in the linearly Landau damped region we expect the nonlinear excitation of an EH mode, as suggested by our previous publication [15]. This EH is indeed recovered (figure 2) in case 1 (apart from a completely decoupled undamped electron plasma oscillation in all the three cases) where a much faster saturation of ion expulsion (potential decay) is achieved resulting in an immediate set up of a coherently propagating structure. For both the second and third cases the slower perturbation, however, the phase-space structure presented in left column of figures 3 and 4 are seen accelerating to a higher velocity after a noticeable change in its topology in the phase-space. For all these cases, the removal of electrons (\( f_1 \ll f_0 \)) from a small velocity interval translates in an electron density dip (potential hump) at \( x_1 \), instantly introducing a phase-space separatrix about \( (x_1, \nu_1) \). A slowly varying separatrix corresponds to an adiabatic invariant, with a response time (time for it to modify) longer than that of untrapped ions (\( \nu_{\text{adiabatic}} \gg \omega_{pi}^{-1} \)). While ions can be expelled faster to restore quasineutrality, an inward flux of them is also expected, driven by deficiency of thermal electrons at \( x_1 \) that must allow ions to easily bunch at \( x_1 \) [27]. Clearly, in a stably propagating solitary electron hole structure, these two fluxes must balance and a co-moving ion density hump must exist, as seen in figure 2(j). However, an unstable, subcritically evolving and accelerating perturbation recovered in figures 3 and 4, in clear contrast to figure 2, is subject of this paper.

Note that in all the three cases the \( u_1 \)'s are sufficiently large \((6.78, 1.36 \) and \( 0.54v_{\text{thi}})\), to neglect ion trapping in first approximation. However, since the ion sound speed \( c_i = 3.16v_{\text{thi}} \) in case 1 the perturbation is moving supersonically, in cases 2 and 3 we have subsonic propagation. This implies that the ion mobility can be largely neglected in case 1 but plays an important role in cases 2 and 3. Consequently, the time scales of the evolution are rather distinct in all the cases, being determined essentially by \( \omega_{pi}^{-1} \) for case 1 where the EH has settled in about \( 10 \omega_{pi}^{-1} \), but by \( \omega_{pi}^{-1} \) for cases 2 and 3, where the settling occurs in about \( 3.6 \omega_{pi}^{-1} \approx 156 \omega_{pe}^{-1} \) and \( 14 \omega_{pi}^{-1} \approx 598 \omega_{pe}^{-1} \), respectively.

In the simulation \( f_1 \) represents tiny granular patches of shear flow in phase-space representing individual particle configurations (fine texture) on the Debye length scale. This is in accordance with, e.g., the approach of Pines and Bohm [28] which describe a (dense) electron gas by a sufficiently smeared out Vlasov–Plasma equilibrium superimposed on a fluctuating background caused by its individual particle structure. While in most of the previous publications the starting point for wave excitation is always sought in linear Vlasov theory and its nonlinear extensions, with our \( f_1 \) we not only catch at least initially a deeper level of plasma description rather distinct in all the cases, being determined essentially by \( \omega_{pi}^{-1} \) for case 1 where the EH has settled in about \( 10 \omega_{pi}^{-1} \), but by \( \omega_{pi}^{-1} \) for cases 2 and 3, where the settling occurs in about \( 3.6 \omega_{pi}^{-1} \approx 156 \omega_{pe}^{-1} \) and \( 14 \omega_{pi}^{-1} \approx 598 \omega_{pe}^{-1} \), respectively.

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circumstances linear theory, being still the major theory for the bulk of plasma physics according to which no structures should survive in the subcritical regime, is violated. The present simulations therefore present the formation of EH structures in the subcritical regime and also indicates a gap of existence for the EH solutions which is now addressed for the first time well within the analytic model for equilibrium solutions of the Vlasov–Poisson system presented by Schamel where the trapped particle effects are retained in the distributions.

### 3. Standard SEH formulation and its stability

With extendability of the fundamental model of trapped species distribution $f_{\alpha}$ to more deterministic forms (discussed further below), the distributions $f_{\alpha \epsilon}$ are written by Schamel as function of total energy $\epsilon_{\alpha \epsilon}$ hence satisfying the Vlasov equation (see [18] and references therein). Using them in Poisson’s equation one can derive the NDR (see equation (24) of [29]),

\[
k_{\alpha}^2 = \frac{1}{2} Z'_{\epsilon}(\nu_D / \sqrt{\epsilon}) - \frac{\theta}{2} Z'_{\epsilon}(u_0 / \sqrt{\epsilon}) \\
= \frac{16}{15} \left[ \frac{3}{2} b(\alpha, u_0) \theta^{1/2} + b(\beta, \nu_D) \right] \psi^{1/2},
\]

where $Z_{\epsilon}(x)$ is the real part of the plasma dispersion function, $\nu_D := \nu_D - u_0$ and $\nu_D$ describes a given constant drift between electron and ion existing already in unperturbed state. The quantity $b$ is function of trapping parameters $\beta$ (or $\alpha$) of electrons (or ions), and has the form

![Figure 2. Evolution of the electron phase-space perturbation (subplots (a), (c), (e), (g), (i) and (k)) and the density perturbations (subplots (b), (d), (f), (h), (j) and (l)) initially introduced at $(x_1, v_1) = (15, 0.05)$. The dashed line in surface plots marks the initial seed velocity. In the density evolution plot the solid red line represents electron density and solid blue line represents ion density.](image)
such that $b(\alpha, u_0) = 0$ for no trapping effects of ions. The NDR (4) determines the phase velocity of structures ($v_0$ or $u_0$) in terms of $v_D$, $k_0$, $\theta$, $\psi$, $\alpha$ and $\beta$.

In generality, we meet a two parametric solution (described by the parameters $k_0^2$ and $B = \frac{16}{\pi} b(\beta, v_D) (\sqrt{\psi})$), which is termed cnoidal EH (CEH) because it can be expressed by Jacobian elliptic functions such as $cn(x)$ or $sn(x)$. It incorporates as special cases the familiar SEH, when $k_0^2 = 0$ and $B > 0$ [30, 31], the harmonic wave, when $B = 0$, as well as the special solitary potential dip, when $k_0^2 = -\frac{4\theta^2}{\varphi^2} > 0$ demanding $B < 0$. The compliance with the NDR (4) ensures the net charge flux balance resulting in time independent solutions in the frame of structures. The structure stability (time independence) is therefore subject to its parameters satisfying equation (4).

4. Saturated holes as valid SEH solutions

We now validate holes settled in equilibrium states as described by above analytic model. Since our code is periodic the lowest available wavenumber is $k_0 = \frac{2\pi}{L} = 0.21, (k_0^2 = 0.04)$, to approximate SEH with. We moreover recognize that both $v_D$ and $v_0$, and hence $\bar{v}_D$, are small quantities such that $-\frac{1}{2} Z_i'(\bar{v}_D / \sqrt{2}) \approx 1$ to a good approximation, while noticing that $Z_i'(x)$ is an even function. Under these special conditions our NDR simplifies and becomes, in case of negligible ion trapping:

$b(\beta, \bar{v}_D) = \frac{1}{\sqrt{\pi}} (1 - \beta - \bar{v}_D^2) \exp(-\bar{v}_D^2/2)$
An inspection of the $-\frac{1}{2}Z'(x)$ shows (see figure 1 of [18]) that $D$ is negative, corresponding to $1.307 < u_0$, provided that $0 < B < 1.04$. Taking the ideal SEH solution, $\phi(x) = \psi \sech\left(\frac{x}{\Delta}\right)$ with $\Delta = \frac{15}{\sqrt{\pi}}$, this amounts to $\Delta > 3.92$. Since the spatial width of our perturbation is essentially maintained during the evolution we can take the initial width and approximate $\Delta$ by $\Delta \approx \frac{4}{3} = 10$ such that $B$ becomes $B \approx 0.16$. On the other hand, $B$ is given in the present situation by $B = \frac{1641 - \beta \sqrt{\psi}}{15 \sqrt{\pi}}$, which gives, for $\psi \approx 10^{-4}$, a value of the electron trapping parameter $\beta \approx -25.6$. Analytically, we hence get a depression of the electron distribution in the resonant or trapping region, as observed. The corresponding phase velocity is for this case with $D \approx -0.09$ is found to be $u_0 \approx 3.7$ or $v_0 \approx 0.027$, i.e. in the observed range.

Figure 4. Evolution of the electron phase-space perturbation (subplots (a), (c), (e), (g), (i) and (k)) and density perturbations (subplots (b), (d), (f), (h), (j) and (l)) initially introduced at $(x_1, v_1) = (15, 0.004)$. The dashed line in surface plots marks the initial seed velocity. In the density evolution plot the solid red line represents electron density and solid blue line represents ion density.

$$-\frac{1}{2}Z'(u_0/\sqrt{2}) = \frac{1}{\theta} [B - (1 + k_0^2)] \equiv \frac{B - 1.04}{10} = D. \quad (5)$$
4.1. The acceleration of SEH
The function $-\frac{1}{2}Z'(x)$ has a minimum of $-0.285$ at $x = 1.5$, which corresponds in terms of $u_0$ in (5) to $u_0 = 2.12$. This yields, by use of (5), $B = -1.81$ which is outside the admissible range of $B$, $0 < B < 1.04$. There is hence a gap in $u_0$ in which no equilibrium (quasi) SEH can exist. The lowest value of $B$ for which a solution is $B = 0^+$ corresponding to $D = -0.104$ or $u_{0f} = 1.48$ ($x_1 = 1.05$) and $u_{0f} = 3.61$ ($x_\beta = 2.55$), hence a gap bounded by these slow and fast velocities, $1.48 < u_{0f} < 3.61$. This explains why a slow perturbation in case 2 ($v_1 = 0.01 \equiv u_1 = 1.36$) and case 3 ($v_1 = 0.004 \equiv u_1 = 0.54$), which despite acquiring an adiabatic character, cannot settle below $u_0 = 3.61$. The result presented in case 3 with much slower perturbation $v_1 = 0.004$ additionally showed that the acceleration continues despite the condition $f'_h f''_h < 0$ [13] was violated when EH velocity crossed $v_2$. It remains to be shown as to why the hole must accelerate, instead of decaying by phase mixing or decelerating. Quantitatively supported by the energy balance presented further below, the mechanism underlying this acceleration is well explained by the simulated phase-space evolution of the hole, illustrated more clearly in the schematic figure 5. While the net charge flux is balanced (zero) for the fast moving structures (left), in a slow moving structure (right) the inbound ion flux limited by finite $T_e$ is too weak to balance the outbound ion flux generated by a longer exposure to hole electric field, $\Delta t \sim 4\pi/v_0 k$. With finite trapped electron population, this insufficient ion influx in a slow moving hole is supplemented by the deterministic response of trapped electrons which create an effective flux by beginning to update their phase-space orbits. The spatial distribution of trapped electrons keeps modifying until the saturation, effectively increasing $|\beta|$, and hence increasing the hole velocity [31]. Note that interpreting $\beta^{-1}$ as trapped electron temperature (i.e. $f_{ej}$, a maximum entropy state), lets the EH represent an structure of infinitesimal scale below which no internal phase-space structures are considered. For treating a deterministic (Vlasov) prescription of internally structured finite amplitude EH, this opens possibility of generalizing Schamel approach by defining a multitude of $f_{ej}$, $j = e, i$, (in mutual equilibrium, e.g., in phase locked states [32, 33]) with an associated set of $\beta$ and $\alpha$.

4.2. Parametric phase of EH instability
Additionally seen in our results is a further acceleration continuing beyond $t = 92.3$ (g), (h) in figure 3 where $v_0 = 0.028$ or $u_0 = 3.8$) when $B$ changes sign to become positive. The further increase in $v_0 (u_0)$ at later times is due to an increase of $D$ (decrease of $|D|$) or increase of $B = \frac{16\gamma_1 - \beta \sqrt{\gamma}}{15 \sqrt{\gamma^2}}$. The latter can have two sources, an increase of $\psi$ and an increase of $(1 - \beta) = (1 + |\beta|)$, corresponding to a deeper (or sharper, with large $k$) depression in the phase-space vortex center. This additional acceleration essentially corresponds to a net imbalance of ion flux across the separatrix of a valid hole ($B > 0$) due to difference in ion density at two ends of the hole, or an ambient ion density gradient, that must cause further trapped electron response, and hence the acceleration. The model [29] can hence explain both, a gap in $u_0$ and a further acceleration along the fast dispersion branch.

4.3. Negative energy character of settled hole
As derived in [17, 34, 35] the total energy density $w$ of a SEH carrying plasma is changed by

$$ \Delta w = \frac{\psi}{2} \left[ 1 + \frac{1}{2} Z'(\frac{u_0}{\sqrt{2}}) (1 - u_0^2) \right] $$

with respect to the unperturbed, homogeneous state. This expression is negative when it holds: $2.12 < u_0$ (see figure 1 of [17]), and is satisfied for all of our settled SEHs. In figure 7 we additionally compare the evolution of $B$ for the case 3 with that in cases 1 and 2 where in case 3 with $v_1 = 0.004$, we have shown that a SEH which is initially placed in an allowed low velocity range ($B > 0$), follows the route of a further release of free energy, moves over the forbidden range and finally settles only in the high energy tail of $f_0$, which are energetically preferred.

Figure 5. Schematic of (left) valid fast SEH with $v_0 \geq 0.028$ (right) unstable slow SEH in the forbidden regime $v_0 < 0.028$. 
5. Time evolution of trapped species dispersion $B$

In figure 6(a) we have presented the time variation of $B$, calculated using equation (5) and rest of the quantities available from the simulation data. This can be noted that for the case 1 (represented by the dashed line) $B$ is uniform and positive at all times as required for the valid SEH eigenmode. For the case 2, however, the value of $B$ (solid line) has negative value in a finite interval (region II) indicating no valid SEH eigenmodes, explaining the unshielded phase of the SEH during this initial interval in case 2. The initial $B > 0$ phase ($t < 10$ or region I) in this case still has an inappropriate SEH eigenmode that violates the negative energy state condition $v_0 > 0.016$ as described by (6) [17]. The value of $B$ can still be seen changing once the unshielded phase (region II) is over and $B > 0$ is achieved which is attributed to interaction of valid SEH with the background ion acoustic structure created during the unshielded phase. This is evident from the saturation in the $B$ variation that exactly corresponds to the time of exit of the SEH from the region of a positive ion density gradient ($t = 153 \omega_{pe}^{-1}$ in figures 3 and 6(a)).

We now show that the energy to accelerate ions and growth of an ion acoustic perturbation is derived from the thermal energy of electrons, establishing the unstable hole evolution as a fundamental mechanism for the plasma destabilization driven by a source of free energy. The resulting ion acoustic structure in turn interacts with the SEH and accelerates it further in $B > 0$ regime. This exchange is mediated by the deterministically modifying trapped electron phase-space orbits that allow them spend longer time away-from/close-to center in unstable $B < 0$ regime, to supplement the incoming ion flux (pushed by excess thermal electrons) that would balance the outgoing ion flux in a valid plasma eigenmode. The higher $|\beta|$ values correspond to larger dispelled density of trapped electrons and, in turn, to higher hole velocity, explaining the hole acceleration for $t < \tau$, that continues in region III due to coupling with ion density nonuniformity. This conversion of electron thermal energy to ion kinetic energy however need not be 100% as a fraction of variation in the total thermal energy $\delta KE_{tot} = \delta KE_e + \delta KE_i$ balances that in the sum $\delta PE$ of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{(a) Time evolution of parameter $B$ and (subplot) velocity $v_0$ of the SEH. (b) Time variation of change from initial value of kinetic energy of ions (black line), and electrons (blue, averaged over fast electron oscillations). The subplot shows variation of potential (gray and black, total and averaged, respectively) and total kinetic energy (magenta). (c) and (d) $\delta f_i$ in the ion phase-space at indicated times and black solid line is contour of $f_i$ representing the electron hole.}
\end{figure}
electrostatic energies of the SEH and the developing ion compression wave structures (plotted for case 2 in the subplot of figure 6(b) as magenta line and black line, respectively). We have also presented the entire process in the ion phase-space by plotting $\delta f_i = f_i - f_{ig}$ at two time points in figures 6(c) and (d). The contour of SEH separatrix is superimposed at both the times on the contours of $\delta f_i$ where an SEH with $B < 0$ ($t < \tau$) can be seen coinciding large $\partial f_i / \partial x$, while a valid SEH, with trapped electrons coinciding the ion density hump is seen for $B > 0$ ($t > \tau$). Figure 7 presents the time evolution of $B$ (left hand side of the nonlinear dispersion relation (NDR) Eq. ~4) for all the three cases. The subplot presents the time evolution of the velocity of the electron hole for all the cases.

6. Summary and conclusions

To summarize, we have indicated the presence of a new forbidden regime of nonlinear EH structures at smaller velocities in linearly subcritical collisionless plasmas. We have presented for the first time that a hole structure already in the transient, adiabatic state, follows the cnoidal hole theory represented by a NDR and satisfies an energy expression in which the trapping nonlinearity plays the major role. Generally speaking, the nonlinear stability is still an unsettled issue. In the most recent work of Schamel [36] it is shown that harmonic EHs, satisfying a thumb-like NDR, are unconditionally marginally stable with respect to linear perturbations in a current-driven plasma. This together with their negative energy density character favors the occurrence of sustained intermittent plasma turbulence beyond the realm of linear Vlasov scenario. To be in a thermal state, a plasma must hence be sufficiently quiet and composed of a broad harmonic spectrum of incoherent fluctuations to omit structure formation. A corresponding (i.e. mathematically consistent) stability analysis for more general CEHs, inclusively that of solitary holes, is not yet available.

Here the evolution is shown to be a manifestation of an already predicted [15] multifaceted nonlinear instability. Importantly, the independence of the nonlinear evolution from the $f'$ and the role of trapped particles that facilitate conversion of thermal energy to coherent modes imply an evolution beyond the realm of linear Landau scenario. Finally we mention that our analysis rests on the availability of a NDR, which is provided by the use of Schamel’s kinetically upgraded pseudo-potential method. A BGK-analysis, was given by Bernstein, Greene and Kruskal in 1957, could not be applied because of the lack of a NDR of the latter, which is a consequence of the strong slope singularity of the derived $f_{ig}$ within the BGK method [37]. By establishing negative energy SEHs the plasma gains free energy and resides in a metastable, structural thermodynamic state. In the long term run, when dissipative processes are no longer negligible, this enables the plasma to heat electrons and approach the thermodynamic equilibrium state faster than without this intermediate structural state. The latter property is suggested by the existence of separatrices around which collisionality is appreciably enhanced.
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