Natural Inflation from Near Alignment in Heterotic String Theory

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Abstract

We propose a model for large field inflation in heterotic string theory. The construction applies the near alignment mechanism of Kim, Nilles, and Peloso. By including gaugino condensates and world-sheet instanton non-perturbative effects, we obtain a large effective axion decay constant.
1 Introduction

Cosmological inflation \[1\] uses a period of exponential expansion immediately after the Big Bang to account for why the Universe appears homogeneous, isotropic, and flat on large scales today. Scalar field models of inflation moreover require that the inflationary potential \( V(\varphi) \) be extremely flat \[2,3\]. This potential is characterized by the slow-roll parameters:

\[
\varepsilon \approx \frac{1}{2} M_{\text{Pl}}^2 \left( \frac{V'(\varphi)}{V(\varphi)} \right)^2 \ll 1, \quad |\eta| \approx M_{\text{Pl}}^2 \left| \frac{V''(\varphi)}{V(\varphi)} \right| \ll 1, \quad (1.1)
\]

where \( M_{\text{Pl}} \) is the reduced Planck mass. Current astrophysical data are beginning to illuminate the structure of the potential. The Planck satellite’s measurement of the spectral index \[4\] establishes that \( \varepsilon \) or \( \eta \) are of order \( 10^{-2} \) and favours power law potentials with leading exponent less than two. Intriguing preliminary results on B-modes from the BICEP2 collaboration \[5\] published earlier this year suggest that the energy scale for inflation is set by the tensor-to-scalar ratio \( r \) as follows:

\[
V \approx \left( \frac{r}{0.2} \right) \times (2.2 \times 10^{16} \text{ GeV})^4. \quad (1.2)
\]

However, dust within the galaxy contaminates the BICEP2 measurements at the same level as the signal \[6\] and therefore the observations in \[5\] should be regarded as a provisional hypothesis that awaits refinement through improved analysis.

Since the Lyth bound \[7,8\] indicates that the motion \( \Delta \varphi \) in field space is

\[
\Delta \varphi \approx \sqrt{\frac{r}{0.01}} M_{\text{Pl}}, \quad (1.3)
\]

the still inconclusive BICEP2 claim that \( r \approx 0.2 \) advocates a trans-Planckian fluctuation in the inflaton field \( \varphi \) over the course of inflation, while at the same time retaining the form of the potential implied by \( (1.1) \). Since \( M_{\text{Pl}} \) is the ultraviolet cutoff, trans-Planckian motion in field space raises questions about the validity of the effective field theory description of the physics. From the Wilsonian perspective, the flat potential ought to be destabilized by higher dimensional contributions to the effective action for the scalar. Indeed, we may write the effective action in terms of a Lagrangian density

\[
\mathcal{L} = -\frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi - \left[ V_0 + \frac{1}{2} m^2 \varphi^2 + \lambda \varphi^4 + \sum_{k=1}^{\infty} c_k \left( \frac{\varphi^{4+k}}{M_{\text{Pl}}^k} \right) \right], \quad (1.4)
\]

where the \( c_k \) are generically order one coefficients, and the terms in the sum become problematic for \( \Delta \varphi \gtrsim M_{\text{Pl}} \).

One scenario that yields a flat potential is so-called natural inflation \[9\], in which an axion serves as the inflaton. The existence of a shift symmetry under which the potential is invariant ensures its flatness. This symmetry persists (at least approximately) non-perturbatively
at the quantum level. Because the setup enforces symmetries on the ultraviolet completion, realizing the model within the architecture of a fundamental theory of quantum gravity such as string theory is remarkably challenging \cite{10,11}. Large field models such as N-flation \cite{12} or axion monodromy \cite{13,16} are supported in this scenario.

Kim, Nilles, and Peloso (KNP) \cite{17} propose a framework for large field models in which two axionic fields are balanced.\footnote{The idea of multi-field assisted inflation was earlier studied in \cite{18}.} The alignment mechanism advanced by KNP generates a large effective axion scale even though the axion decay constants are themselves sub-Planckian. Recently, \cite{19} constructed a realization of this alignment mechanism in type IIB superstring theory. Inspired by \cite{19}, we develop in this article the ideas of KNP \cite{17} (further explained in \cite{20}) in the context of the $E_8 \times E_8$ heterotic string. However, although the ten dimensional critical string theories are related by dualities, the construction in heterotic theory is notably different from \cite{19}. Similar recent attempts to embed natural inflation in type IIB string theory or string motivated supergravity can be found in \cite{21,24}.

While most investigations of cosmology in string theory are couched in the language of the type II superstring, the heterotic string has considerable advantages of its own. In particular, modeling cosmological inflation is only a part of a larger story. A resolution to the vacuum selection problem must also justify the existence of the Standard Model at low energies. Since \cite{25}, the heterotic string has offered a promising avenue toward this goal.\footnote{Of course, there is not a uniform opinion on how to arrive at the Standard Model from a theory of strings and D-branes, and numerous alternatives to heterotic compactification have been considered. See, for example, \cite{26}.} Several realistic constructions of a theory of particle physics with three generations of chiral matter that fall in representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$ with the appropriate field interactions stem from Calabi–Yau compactifications of the heterotic string \cite{27,32}. Moreover, within the heterotic setup, one can stabilize geometric moduli for the compactification \cite{33,35}. Engineering a stable, aligned, natural inflation scenario within heterotic string theory is therefore a step toward synthesizing model building in cosmology with model building in particle physics. Since the two endeavours are today mostly disparate enterprises, facilitating their marriage serves as a principal motivation for our work.

The organization of the paper is as follows. In Section 2, we briefly review natural inflation and the decay constant alignment mechanism. In Section 3, we construct a model in which a pair of axion fields correspond to Kähler moduli. The scalar potential within this construction has a large effective decay constant as a consequence of near alignment. In Section 4, we provide a numerical example with order one numbers to illustrate the naturalness of the construction. In Section 5, we discuss the results and indicate future work. The Appendix derives the form of the scalar potential.
2 Natural Inflation and the Alignment Mechanism

One way to guarantee a small value for the $\eta$ parameter is when the inflaton field is protected by a global symmetry. If inflation is driven by an axion $\tau$, then this is easy to achieve as the axion has a continuous shift symmetry, $\tau \rightarrow \tau + 2\pi f$, where $f$ is the axion decay constant. However, non-perturbative corrections break this continuous symmetry to a discrete shift symmetry. An example of a potential with such a shift symmetry is

$$V(\tau) = \Lambda^4 \left[1 \pm \cos \left(\frac{\tau}{f}\right)\right]. \quad (2.1)$$

For large values of $f$, this potential supports inflation. The higher order terms are suppressed by $f$, and this ensures a small value for the slow-roll parameter $\eta$.

In principle, it is difficult to generate such potentials with decay constants $f$ larger than the Planck scale from string theory. Typically, instanton corrections spoil the flatness of the potential \cite{10,11}. The KNP proposal \cite{17} is useful in this respect. It shows that if there is more than one axion with multiple non-perturbative terms in the potential generated for different linear combinations of these axions, then a large effective decay constant can be generated from one of the linear combinations. The other linear combinations have much smaller effective decay constants that produces high curvatures of the potential in those directions. These other linear combinations are frozen to some value by the potential.

For example, suppose, for two axions $\tau_1$ and $\tau_2$, we have a potential that has the following form:

$$V = \Lambda_A^4 \left[1 - \cos \left(\frac{\tau_1}{f_{A_1}} + \frac{\tau_2}{f_{A_2}}\right)\right] + \Lambda_B^4 \left[1 - \cos \left(\frac{\tau_1}{f_{B_1}} + \frac{\tau_2}{f_{B_2}}\right)\right]. \quad (2.2)$$

It might happen that the decay constants in this potential are **aligned**, i.e., they satisfy the following condition:

$$\frac{f_{A_1}}{f_{A_2}} = \frac{f_{B_1}}{f_{B_2}}. \quad (2.3)$$

If so, the same linear combination of axions appears in both of the cosine functions, and an orthogonal direction is the flat direction. It may happen that this alignment is slightly broken in a natural way, and then we get a nearly flat direction that can have a large effective decay constant. This supplies an inflaton field. Crucially, the parameters $f$ are sub-Planckian, and therefore the effective field theory description of the system is valid.

3 The Model

We start with $E_8 \times E_8$ heterotic string theory compactified on a Calabi–Yau threefold $X$. We shall assume that compactification on $X$ gives rise to an effective $S, T^1, T^2, Z^a$ model. We
denote by \( T^i \) the complex moduli fields coming from the Kähler structure deformations while \( Z^a \) denotes the moduli fields from the complex structure deformations. Here, we assume that \( X \) is chosen in such a way that there are two Kähler moduli fields but we shall not need to specify the number of fields coming from the complex structure deformations. We also have the universal axiodilaton field which we denote by \( S \). It is important to note that, instead of adopting the standard embedding, we shall loosely adopt the setting outlined in [33–35], in which moduli fixing is achieved by making a judicious choice of the gauge bundle and non-perturbative superpotentials coming from gaugino condensates and string instantons.

As mentioned previously, our construction shares some similarities with the type IIB construction advanced by [19]. Thus, we require the interplay between the classical superpotential and non-perturbative contributions. More specifically we shall need the classical superpotential to depend entirely on \( Z^a \) moduli. We also require two sources of non-perturbative potentials for the Kähler moduli, which we achieve by turning on gaugino condensations as well as string instantons. We shall assume that using the methods outlined in [34] and related works that we can fix all the complex structure moduli fields as well as the real parts of \( T^1 \) and \( T^2 \). We as well assume that we can fix \( S \) to a value that keeps us in the perturbative regime by imposing the F-term equation involving non-perturbative gaugino condensations [36]. Although we shall not make very precise assumptions for the values of \( Z^a \), we shall pick explicit values for the real parts of \( T^1 \), \( T^2 \) and \( S \) in our construction. We then tune the D-term proposed in [34] and generate an effective potential of the form (2.2) for the remaining two axion fields \( \tau_1 \) and \( \tau_2 \) coming from the two Kähler structure moduli fields.

### Superpotential and Kähler Potential

As we shall be using the axions from the Kähler sector for decay constant misalignment, it will be necessary for the perturbative superpotential \( W_p \) to be independent of the Kähler sector moduli. Apart from the fact that \( W_p \) has to be small but non-zero, our construction is independent of the details how that is achieved. In [34], an F-term classical superpotential was used for \( Z^a \) to stabilize to a supersymmetric Minkowski ground state. At this ground state their \( W_p \) is zero. However, upon closer inspection it appears that this may not be the most general solution (for example, the bundle moduli are all set to zero) and we assume that it is possible to modify their proposal in a way such that the stabilized values of \( Z^a \) yield a small but non-zero value for \( W_p \). Thus we choose the perturbative superpotential to be exclusively a function of the complex structure moduli:

\[
W_p = W_p(Z^a) .
\]  

\(^3\)Alternatively one can envision a scenario where fractional Wilson lines are used to generate a small \( W_p \) as in [37]. We are grateful to L. McAllister for bringing this work to our attention.
For definiteness, we take a Kähler potential:

\[ K = -\log (S + \bar{S}) - \log 8K - \log 8\bar{K}, \]  

(3.2)

where \( S \) is the axiodilaton, and \( K \) and \( \bar{K} \) are the Kähler potentials in the complex structure and the Kähler structure sectors, respectively. For this particular form of the superpotential we assume that the contribution from the complex structure moduli is such that \( e^{-\ln 8K} \sim 1 \), and we therefore neglect these effects in the potential that we calculate below.

In the large volume limit the quantum corrections to \( \bar{K} \) are small, and it takes the form

\[ \bar{K} = \frac{1}{8}d_{ijk}(T^i + T^j)(T^j + T^k)(T^k + T^k), \]  

(3.3)

where \( d_{ijk} \) are triple intersection numbers of the Calabi–Yau manifold \( X \).

We decompose the two Kähler structure moduli fields in terms of their component fields as

\[
\begin{align*}
T^1 &= t_1 + i\tau_1, \\
T^2 &= t_2 + i\tau_2. 
\end{align*}
\]

(3.4)

For illustrative purposes, we consider the case that all the intersection numbers except for \( d_{111} = d_{222} = 1 \) are zero. This is not a necessary assumption, but it will simplify our computations significantly and demonstrate the idea within a simple setup.

**Non-perturbative Terms**

To generate the desired potential for \( \tau^i \) we shall need to turn on non-perturbative terms for \( T^i \) coming from gaugino condensates of the gauge line bundles as well worldsheets instantons. By choosing our gauge bundle to be of the form \( V = U \oplus I \mathcal{L}_I \) where \( \mathcal{L}_I \) denotes \( U(1) \) line bundles, we generates non-perturbative gaugino condensate terms \[34,36\] in the superpotential of the form

\[ W_{\text{gaugino}}^{\text{np}} = A' e^{-\alpha(S - \beta_i T^i)}, \]  

(3.5)

where \( A' \) is a constant and \( \beta_i \) and \( \alpha \) are defined below. We can use this superpotential and \( W_p \) to fix the value of the moduli \( s \) to some value \( s^* \) by imposing the \( F \)-term equation for \( S \). We then absorb the \( s^* \) dependence into the constant prefactor. The constant \( \alpha \) is related to the one-loop beta-function, and \( \beta_i \) are given by

\[ \beta_i = \int_X (\text{ch}_2(V) - \frac{1}{2}\text{ch}_2(TX)) \wedge \omega_i, \]  

(3.6)

where \( \text{ch}_2 \) is the second Chern class of the bundle indicated in its argument and \( TX \) is the tangent bundle of \( X \). The \( \omega_i \) supply a basis of the second cohomology class. We shall also
need non-perturbative contributions from another sector, namely the worldsheet instanton sector. These contributions are of the form

$$W_{\text{worldsheet}}^{\text{np}} = Be^{-n_i T^i}.$$  \hfill (3.7)

where $B$ and $n^i$ are numbers. Here, we choose an integral basis $\{T^i\}$ for the Kähler moduli space. As a result $n_i$ are integers which count the number of times string worldsheets wrap holomorphically around two-cycles on the CY.

**Kinetic Terms**

The kinetic terms for the $\tau$ axions are:

$$\mathcal{L}_{\text{kinetic}} = -K_\beta \partial_\mu \tau_i \partial^\mu \tau_j^\beta ,$$ \hfill (3.8)

where $K_\beta$ is computed from (3.2) in the standard way and is given by

$$K_\beta = \frac{3}{4(t_1^3 + t_2^3)^2} \begin{pmatrix} t_1^4 & 2t_1t_2^3 & 3t_1^2t_2^2 \\ 2t_1t_2^3 & t_2^4 & 2t_2t_1^3 \end{pmatrix}.$$ \hfill (3.9)

We see that there is kinetic mixing between the two axions, and so we shall need to diagonalize and canonically normalize the kinetic terms of the fields.

**Scalar Potential**

As usual the F-term contribution to the scalar potential is given as follows:

$$V_F = e^K \left( K^\beta D_\beta W \overline{D_\beta \overline{W}} - 3|W|^2 \right) ,$$ \hfill (3.10)

where $D_\beta W = W_{;i} + K_{;i} W$ and $K^\beta$ is the inverse of the Kähler metric on the Kähler structure moduli space. Given the form of the superpotential and the fact that the Kähler sector satisfies the no-scale condition and assuming $W_p$ is larger compared to superpotential terms (3.5) and (3.7) coming from the non-perturbative sector, a computation similar to the one in [19] yields the following scalar potential:

$$V_F = e^K \left\{ K^\beta W_p K_{;i} \partial_\beta \overline{W}_{\text{np}} + \text{c.c.} \right\} ,$$ \hfill (3.11)

where $W_{\text{np}}$ is the sum of the non-perturbative superpotentials. If we decompose the axiodilaton in terms of real scalar fields:

$$S = s + i\sigma$$ \hfill (3.12)

and assume that $s$ is fixed to $s^*$, the prefactor in the potential (3.11) becomes

$$e^K = \frac{1}{2s^*(t_1^3 + t_2^3)^2} ,$$ \hfill (3.13)
where we have used the the explicit expression for $\tilde{K}$ is given by

$$\tilde{K} = \frac{1}{8}(t_1^3 + t_2^3).$$

With these ingredients it is straightforward to compute the scalar potential. However, in order to implement the decay constant misalignment mechanism we need to add to this scalar potential a supersymmetry breaking uplifting term $V_{up}$ whose precise form is computed in Appendix A. We shall assume the value of $s^*$ is chosen so that our gauge coupling constants remain in the perturbative regime, and that $\sigma$ is stabilized at $\frac{2n\pi}{\alpha}$, where $n$ is an integer.

Then our non-perturbative superpotential takes the form

$$W_{np} = Ae^{\alpha\beta T^i} + Be^{-n T^i},$$

with

$$A = A'e^{-\alpha s^*}.$$ (3.15)

The full scalar potential for the construction is then given by

$$V = V_F + V_{up}$$

$$= \Lambda_A^4[1 - \cos \{\alpha(\beta_1\tau_1 + \beta_2\tau_2)\}] + \Lambda_B^4[1 - \cos (n_1\tau_1 + n_2\tau_2)],$$

where the two $\Lambda$s are:

$$\Lambda_A^4 = \frac{2A|W_p|\alpha(\beta_1 t_1 + \beta_2 t_2)e^{\alpha(\beta_1 t_1 + \beta_2 t_2)}}{s^*(t_1^3 + t_2^3)^2}, \quad \Lambda_B^4 = -\frac{2B|W_p|(n_1 t_1 + n_2 t_2)e^{-(n_1 t_1 + n_2 t_2)}}{s^*(t_1^3 + t_2^3)^2}.$$ (3.18)

The uplifting term $V_{up}$ in the above equation is given by (see Appendix A)

$$V_{up} = 2e^K(M' + N'),$$

where we have defined

$$M' = 2e^{\alpha t^i}A\alpha t_i \beta_1 W_p,$$

$$N' = -2e^{-n t^i}Bt_i n_i W_p.$$ (3.20)

The source of the uplifting term will be explained below. For (3.17), in terms of the canonically normalized fields, the determinant of the Hessian of the potential at the minimum is given by

$$\det \partial_i \partial_j V|_{\tau=0} = \frac{8\alpha^2(n_2\beta_1 - n_1\beta_2)^2M'N'}{9s^2 t_1 t_2(t_1^3 + t_2^3)^2}.$$ (3.21)
The Uplifting Term

The uplifting term $V_{up}$ in the type IIB setting generically comes from anti-D-branes that break the supersymmetry. In a heterotic setting, since there are no D-branes it is customary to invoke the D-terms associated with the anomalous $U(1)$ factors [38] that are generic to heterotic compactifications to do the same job. Fortunately in the setting of [34] there are anomalous $U(1)$ line bundles that supply us with such D-terms. According to [34], the D-term associated with the $I$-th line bundle is

$$D_I = \frac{c_1^I(\mathcal{L}_I)d_{ijk}t^it^jt^k}{d_{ijk}t^it^jt^k} = \frac{8 (c_1^I(\mathcal{L}_I)t_1^2 + c_1^I(\mathcal{L}_I)t_2^2)}{(t_1^2 + t_2^2)},$$

(3.22)

where $c_1^I(\mathcal{L}_I)$ is related to the first Chern class $c_1(\mathcal{L}_I)$ of the $I$-th holomorphic line bundle in the following way:

$$c_1(\mathcal{L}_I) = c_1^I(\mathcal{L}_I)\omega_i.$$  

(3.23)

The $\omega_i$ are the basis for the second cohomology group of the Calabi–Yau manifold $X$. The D-terms contributes the following term to the potential

$$V_D = \frac{1}{2} \sum_I D_ID_I,$$

(3.24)

which must be tuned to give the uplifting term (3.19). It should be noted that whether this proposed $D$-term is able to act as $V_{up}$ is a model dependent statement and should be examined on a case by case basis.

Large Effective Decay Constant from Near Alignment

We can now compare our final potential (3.17) to (2.2). It is clear then that in our model we can identify $1/(\alpha\beta)$ or $1/n_i$ as the fundamental decay constants. In this subsection we discuss how we can choose the parameters of the potential (3.17) to give rise to a tiny decay constant misalignment, along the same lines as was done in [19] even though the physical origin of our parameters are very different from theirs.

Given that we have

$$\Lambda_A^4 = 2e^K\mathcal{M}' \quad \text{and} \quad \Lambda_B^4 = 2e^K\mathcal{N}',$$

(3.25)

we can choose

$$n_1 = n_2 = \alpha\beta_1 \equiv n, \quad \alpha\beta_2 = \frac{\alpha\beta_1}{1 + \delta}, \quad \Lambda_B^4 = \Lambda_A^4, \quad \text{and} \quad \Lambda_A^4 = \delta^p\Lambda_A^4,$$

(3.26)

where $0 < \delta \ll 1$ and $p > 0$. The mass eigenvalues corresponding to this choice are:

$$m_1^2 = \delta^{2+p} \frac{\Lambda_A^4n^2}{2} \left[ 1 + \mathcal{O}(\delta) \right], \quad m_2^2 = 2\Lambda_A^4n^2 \left[ 1 + \mathcal{O}(\delta) \right].$$

(3.27)
In terms of the mass eigenstates the potential can be rewritten as

\[ V = \Lambda^4 \left[ 1 - \cos \left( \sqrt{2} n \psi_2 + \frac{\delta \psi_2}{\sqrt{2}} \right) \right] + \delta \Lambda^4 \left[ 1 - \cos \left( -\sqrt{2} n \psi_2 + \frac{\delta n \psi_1}{\sqrt{2}} \right) \right]. \]  

(3.28)

The first term of this potential serves to fix \( \psi_2 \) to zero while the second term gives rise to and effective decay constant for \( \psi_1 \) whose value is given by

\[ f_{\text{eff}} \approx \frac{1}{\delta n}. \]  

(3.29)

In the next section we choose some plausible numbers into these formulae and generate a plot for the scalar potential that generates a large effective decay constant.

4 Example

In this section, we will generate an example for the near alignment mechanism. This should be considered a toy model with a discussion about how this could be implemented in a realistic Calabi–Yau manifold postponed to a future work. For the following values

\[ \alpha = \frac{1}{14}, \quad \beta_1 = 37, \quad \beta_2 = 18, \quad n_1 = 4, \quad n_2 = 2, \quad W_p = 0.1, \]

\[ A' = 0.001, \quad B = -3, \quad \tau_1 = 10, \quad \tau_2 = -18, \quad s^* = 25, \]  

(4.1)
we get the following mass eigenvalues:

\[ m_1 = 7.71 \times 10^{-4} \, M_{\text{Pl}}, \quad m_2 = 6.13 \times 10^{-7} \, M_{\text{Pl}}. \tag{4.2} \]

We have been careful to choose numbers that are close to order one. The potential in terms of the mass eigenstates can be written as:

\[
\frac{V}{M_{\text{Pl}}^4} = 8.04 \times 10^{-11} - (5.04 \times 10^{-12}) \cos \left[ \frac{57.38 \psi_1}{M_{\text{Pl}}} + \frac{0.27 \psi_2}{M_{\text{Pl}}} \right] \\
- (7.53 \times 10^{-11}) \cos \left[ \frac{87.61 \psi_1}{M_{\text{Pl}}} - \frac{0.011 \psi_2}{M_{\text{Pl}}} \right]. \tag{4.3} \]

The hierarchy between the decay constants allows us to integrate out the \( \psi_1 \) field and we are then left with \( \psi_2 \). Note that we started with decay constants that are sub-Planckian

\[
\frac{1}{\alpha \beta_1} = 0.38 M_{\text{Pl}}, \quad \frac{1}{\alpha \beta_2} = 0.77 M_{\text{Pl}}, \quad \frac{1}{n_1} = 0.25 M_{\text{Pl}}, \quad \frac{1}{n_2} = 0.50 M_{\text{Pl}}, \tag{4.4} \]

but the effective decay constant is roughly \( f_{\text{eff}} \approx 14 M_{\text{Pl}} \). Moreover, for these choices of the constants the non-perturbative superpotential is much smaller than the perturbative part.

\section{Discussion}

In this paper, we have proposed an elementary mechanism by which large field inflation can be realized in heterotic string theory. Our construction is inspired by that of \cite{19}. Like them, we make use of the scheme of natural inflation \cite{9} and the decay constant alignment mechanism proposed by \cite{17}. We do not address the problem of moduli stabilization explicitly in this paper, but propose that this issue can be addressed using the techniques developed in \cite{33,35}. While most string cosmology work in recent years has been performed in a type II setting, the heterotic setting described in our paper seems particularly simple and natural. Similar work has been recently done in \cite{39}.

The main ingredients of our construction are as follows: a classical superpotential that only depends on the complex structure moduli and non-perturbative superpotentials coming from gaugino condensation and string instantons for the the axions arising from the Kähler structure moduli.

These axions appear with two different linear combinations in the scalar potential that results from the interplay of these superpotential terms. When we tune the model so that these two linear combinations are slightly different, we can break the alignment and can generate a large value for the effective decay constant. In our setting we achieve uplifting of the vacuum using D-terms proposed in \cite{34}. We then demonstrate the effectiveness of our construction with a numerical example by choosing plausible values of the tuneable parameters that contribute to the model.
In this way, we are able to generate a large enough effective decay constant that realizes cosmological inflation. For the simple numerical example that we have presented, the effective decay constant is larger than $10M_{Pl}$ and $\Lambda$ is greater than the Hubble constant. If the BICEP2 observation of the tensor-to-scalar ratio $r$ turns out to be correct, then these numbers seem consistent with the inflationary models supported with that data [5].

Our analysis has been at tree level in $g_s$. An important issue for this type of construction is whether the $g_s$ threshold corrections will spoil the flatness of the potential. In [36], the authors argued that this threshold effect is actually subdominant with respect to the $\alpha'$ ones for large volume, so it may be reasonable to neglect their effect. As a future project, we believe that it would be an interesting to look for explicit examples of Calabi–Yau manifolds which would realize our construction in more concrete settings.

**Note:** As this work was in the final stages of completion, the preprint [40] appeared on arXiv. Their construction is completely different from our construction. They did not employ two axions with sub-Planckian decay constants and near alignment as KNP [17].

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A The Scalar Potential

In a supersymmetric theory the holomorphic superpotential $W$ gives rise to the scalar potential via

$$V_F = e^K (K^{IJ} D_I W D_J \overline{W} - 3|W|^2) .$$  \hspace{1cm} (A.1)

As explained in the main text, the superpotential for our model has the form:

$$W = W_p + W_{np} ,$$  \hspace{1cm} (A.2)
where we assume that the classical superpotential $W_p$ only depends on the complex structure moduli whose values have been fixed to yield a small $W_p$. Furthermore, we assume that the axiodilaton $S$ has been fixed by imposing its $F$-term equation. Thus, the expression for the potential in terms of the superfields now only contains derivatives with respect to the Kähler moduli. Furthermore, by using the identity $K^i K_j = 3$, we arrive at

$$e^{-K} V_F = K^i W_p K_j \partial_j \overline{W}_{np} + \text{c.c.},$$

(A.3)

where we have made the additional assumption that $W_{np}$ is smaller than $W_p$ so that we can ignore terms which are quadratic in $W_{np}$. A key observation here is that, as pointed out in (3.1), $W_p$ is function of only the complex structure moduli $Z^a$, whereas, after having fixed $S$, the non-perturbative superpotential (3.15) is exclusively a function of the Kähler structure moduli fields.

From the expression of the non-perturbative superpotential we have:

$$\partial_j \overline{W}_{np} = A \alpha \beta_j e^{\alpha \beta_k T^k} - B n_j e^{-n_k T^k}.$$  

(A.4)

Noting that

$$K^i K_i = -2 t_j$$  

(A.5)

as well as

$$M = \alpha \beta_i T^i,$$  

$$N = -n_i T^i,$$

(A.6)

we have

$$e^{-K} V_F = 2 t_i \left(-e^M A \alpha \beta_i + e^N B n_i\right) W_p + \text{c.c.}$$  

(A.7)

$$\equiv e^M \mathcal{M} + e^N \mathcal{N} + \text{c.c.}$$  

(A.8)

Next, using the decomposition (3.4), we isolate the part of the potential that is a function of $\tau_1$ and $\tau_2$:

$$e^{-K} V_F = -e^{-i \alpha (\beta_1 \tau_1 + \beta_2 \tau_2)} \mathcal{M}' - e^{i (n_1 \tau_1 + n_2 \tau_2)} \mathcal{N}' + \text{c.c.},$$  

(A.9)

where we have defined

$$\mathcal{M}' = -e^{\alpha \beta_i t^i} \mathcal{M},$$  

$$\mathcal{N}' = -e^{-n_i t^i} \mathcal{N}.$$  

(A.10)

We now assume that the functions $\mathcal{M}'$ and $\mathcal{N}'$ can be chosen to be real so that the potential can now be expressed as

$$V = e^K \left\{2 \mathcal{M}' \left[1 - \cos \alpha (\beta_1 \tau_1 + \beta_2 \tau_2)\right] + 2 \mathcal{N}' \left[1 - \cos (n_1 \tau_1 + n_2 \tau_2)\right]\right\},$$  

(A.11)

where we have set:

$$V_{np} = 2 e^K (\mathcal{M}' + \mathcal{N}') .$$  

(A.12)
References

[1] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys.Rev. D23 (1981) 347–356.

[2] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys.Lett. B108 (1982) 389–393.

[3] A. Albrecht and P. J. Steinhardt, *Cosmology for grand unified theories with radiatively induced symmetry breaking*, Physical Review Letters 48 (1982), no. 17, 1220.

[4] Planck Collaboration, P. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, Astron.Astrophys. (2014) [1303.5076].

[5] BICEP2 Collaboration, P. Ade et al., *Detection of B-Mode Polarization at Degree Angular Scales by BICEP2*, Phys.Rev.Lett. 112 (2014) 241101 [1403.3985].

[6] Planck Collaboration, R. Adam et al., *Planck intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes*, [1409.5738].

[7] D. H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, Phys.Rev.Lett. 78 (1997) 1861–1863 [hep-ph/9606387].

[8] D. Baumann and D. Green, *A Field Range Bound for General Single-Field Inflation*, JCAP 1205 (2012) 017 [1111.3040].

[9] K. Freese, J. A. Frieman and A. V. Olinto, *Natural inflation with pseudo Nambu-Goldstone bosons*, Phys. Rev. Lett. 65 (Dec, 1990) 3233–3236.

[10] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, *On the possibility of large axion decay constants*, JCAP 0306 (2003) 001 [hep-th/0303252].

[11] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The String landscape, black holes and gravity as the weakest force*, JHEP 0706 (2007) 060 [hep-th/0601001].

[12] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, *N-flation*, JCAP 0808 (2008) 003 [hep-th/0507205].

[13] E. Silverstein and A. Westphal, *Monodromy in the CMB: Gravity Waves and String Inflation*, Phys.Rev. D78 (2008) 106003 [0803.3085].

[14] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, Phys.Rev. D82 (2010) 046003 [0808.0706].
[15] N. Kaloper and L. Sorbo, *A Natural Framework for Chaotic Inflation*, Phys.Rev.Lett. **102** (2009) 121301 [0811.1989].

[16] N. Kaloper, A. Lawrence and L. Sorbo, *An Ignoble Approach to Large Field Inflation*, JCAP **1103** (2011) 023 [1101.0026].

[17] J. E. Kim, H. P. Nilles and M. Peloso, *Completing natural inflation*, JCAP **0501** (2005) 005 [hep-ph/0409138].

[18] A. R. Liddle, A. Mazumdar and F. E. Schunck, *Assisted inflation*, Phys.Rev. **D58** (1998) 061301 [astro-ph/9804177].

[19] C. Long, L. McAllister and P. McGuirk, *Aligned Natural Inflation in String Theory*, [1404.7852].

[20] R. Kappl, S. Krippendorf and H. P. Nilles, *Aligned Natural Inflation: Monodromies of two Axions*, Phys.Lett. **B737** (2014) 124–128 [1404.7127].

[21] M. Czerny, T. Higaki and F. Takahashi, *Multi-Natural Inflation in Supergravity and BICEP2*, [1403.5883].

[22] X. Gao, T. Li and P. Shukla, *Combining Universal and Odd RR Axions for Aligned Natural Inflation*, JCAP **1410** (2014), no. 10, 048 [1406.0341].

[23] Z. Kenton and S. Thomas, *D-brane Potentials in the Warped Resolved Conifold and Natural Inflation*, [1409.1221].

[24] A. Ashoorioon, H. Firouzjahi and M. Sheikh-Jabbari, *M-flation: Inflation From Matrix Valued Scalar Fields*, JCAP **0906** (2009) 018 [0903.1481].

[25] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, *Vacuum Configurations for Superstrings*, Nucl.Phys. **B258** (1985) 46–74.

[26] L. E. Ibanez and A. M. Uranga, *String theory and particle physics: An introduction to string phenomenology,*.

[27] V. Braun, Y.-H. He, B. A. Ovrut and T. Pantev, *A Heterotic standard model*, Phys.Lett. **B618** (2005) 252–258 [hep-th/0501070].

[28] V. Braun, Y.-H. He, B. A. Ovrut and T. Pantev, *The Exact MSSM spectrum from string theory*, JHEP **0605** (2006) 043 [hep-th/0512177].

[29] V. Bouchard and R. Donagi, *An SU(5) heterotic standard model*, Phys.Lett. **B633** (2006) 783–791 [hep-th/0512149].
[30] R. Blumenhagen, S. Moster and T. Weigand, *Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds*, Nucl.Phys. **B751** (2006) 186–221 [hep-th/0603015].

[31] L. B. Anderson, J. Gray, A. Lukas and E. Palti, *Heterotic Line Bundle Standard Models*, JHEP **1206** (2012) 113 [1202.1757].

[32] L. B. Anderson, A. Constantin, J. Gray, A. Lukas and E. Palti, *A Comprehensive Scan for Heterotic SU(5) GUT models*, JHEP **1401** (2014) 047 [1307.4787].

[33] L. B. Anderson, J. Gray, A. Lukas and B. Ovrut, *Stabilizing the Complex Structure in Heterotic Calabi-Yau Vacua*, JHEP **1102** (2011) 088 [1010.0255].

[34] L. B. Anderson, J. Gray, A. Lukas and B. Ovrut, *Stabilizing All Geometric Moduli in Heterotic Calabi-Yau Vacua*, Phys.Rev. **D83** (2011) 106011 [1102.0011].

[35] L. B. Anderson, J. Gray and B. A. Ovrut, *Transitions in the Web of Heterotic Vacua*, Fortsch.Phys. **59** (2011) 327–371 [1012.3179].

[36] M. Cicoli, S. de Alwis and A. Westphal, *Heterotic Moduli Stabilisation*, JHEP **1310** (2013) 199 [1304.1809].

[37] S. Gukov, S. Kachru, X. Liu and L. McAllister, *Heterotic moduli stabilization with fractional Chern-Simons invariants*, Phys.Rev. **D69** (2004) 086008 [hep-th/0310159].

[38] C. Burgess, R. Kallosh and F. Quevedo, *De Sitter string vacua from supersymmetric D terms*, JHEP **0310** (2003) 056 [hep-th/0309187].

[39] I. Ben-Dayan, F. G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, [1404.7773].

[40] H. Abe, T. Kobayashi and H. Otsuka, *Towards natural inflation from weakly coupled heterotic string theory*, [1409.8436].