\( \sigma(e^+e^- \rightarrow \text{hadrons}) \) AT LOW ENERGY:

EXPERIMENTAL STATUS AND PROSPECTS
FOR THE FUTURE.

ITS INFLUENCE ON \( \alpha_{\text{QED}}(M_Z^2) \) AND \((g - 2)_\mu^*\)

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Abstract

In this talk I will review the recent experimental status of \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) at \( \sqrt{s} < 10 \text{GeV} \) and the prospects for the future. The influence on \( \alpha_{\text{QED}}(M_Z^2) \) and \((g - 2)_\mu \) is also discussed.

1 Why \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) at low energy is still interesting?

An undeniable trend of the high energy physics community is the exploration of high energy ranges by constructing more and more powerful machines and detectors. However, beside that, there is still a considerable effort on precise physics at low energies, which uses \( e^+e^- \) annihilation in the region below 10 GeV: DAΦNE and VEPP-2M at \( \sqrt{s} < 1.4 \text{GeV} \); BEPC at \( 2 < \sqrt{s} < 5 \text{GeV} \) and CSR, KEKB, and PEP-II colliders at \( \sqrt{s} = 10 \text{GeV} \). Though the main motivation for \( \phi \) and \( B \) factories concerns the CP violation studies, R-measurement at low energy \( (R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}) \) is renewing its interest due to the precision reached to test the Standard Model at LEP and SLC and also to the new experimental result of \( (g - 2)_\mu \) at Brookhaven. The experimental accuracy reached so far asks for a precise determination of the theoretical estimation of both \( \alpha_{\text{QED}}(M_Z^2) \) and \((g - 2)_\mu \), whose main error comes from the non-perturbative computation of the hadronic contributions, which can be computed by using \( e^+e^- \) data at low energy. A precise measurement of R in this region is therefore mandatory, and is also one of the main reason for new projects: VEPP2000 \( (\sqrt{s} < 2 \text{GeV}) \), PEP-N \( (1.4 < \sqrt{s} < 2.5 \text{GeV}) \), BEPCII \( (2 < \sqrt{s} < 5 \text{GeV}) \), CLEO-C \( (3 < \sqrt{s} < 5 \text{GeV}) \).

R has been measured by many laboratories in the last 20 years, as shown in Tab. 1.

Fig. 1(b) shows an up-date compilation of these data done by Burkhart and Pietrzyk. The main improvements come in the region below 5 GeV, in particular between 2 – 5 GeV where the BESII coll. has reduced the error to \( \sim 7\% \) (before was \( \sim 15\% \)), and in the region below 1 GeV, where the CMD2 coll. has measured the pion form factor with

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a systematical error of 1.4% (see Fig. 1(a)). Both these new results have significant impact on the updated calculation of $\alpha_{QED}(M_{Z}^{2})$ and $(g - 2)_{\mu}$. While the data between 2-5 GeV are now closer to perturbative QCD, the error in the 1-2 GeV region is still 15%: a reduction of this error to few percent will be very important both for $\alpha_{QED}(M_{Z}^{2})$ and $(g - 2)_{\mu}$ calculations.

Table 1: Overview of R measurements.

| Place         | Ring      | Detector          | $\sqrt{s}(GeV)$ | pts | Year     |
|---------------|-----------|-------------------|-----------------|-----|----------|
| Novosibirsk   | VEPP-2M   | CMD2, SND         | 0.28-1.4        | 128 | '97-'99  |
|               | VEPP-2    | OLYA, ND, CMD     | 0.28-1.4        | -   | '78-'87  |
| Frascati      |           | $\gamma\gamma2$, MEA, BOSON, BCF | 1.42-3.09  | 31  | '78-'82  |
| Orsay         | DCI       | M3N,DM1,DM2       | 1.35-2.13       | 33  | 1978     |
| Beijing       | BEPC      | BESII             | 2-5             | 85  | 1998-99  |
| SLAC          | Spear     | MARKI             | 2.8-7.8         | 78  | 1982     |
| Hamburg       | DORIS     | DASP              | 3.1-5.2         | 64  | 1979     |
|               | PLUTO     | 3.6-4.8,9.46      | 27              | 1977|
|               | C.BALL    | 5.0-7.4           | 11              | 1990|
|               | LENA      | 7.4-9.4           | 95              | 1982|
| Novosibirsk   | VEPP-4    | MD-1              | 7.23-10.34      | 30  | 1991     |

2 Effective $\alpha_{QED}$ and precision test of the Standard Model

The precision reached for the measurements performed at LEP and SLC allows a stringent test of the Standard Model and to predict the Higgs mass. As discussed many times in this conference, the QED coupling constant at $\sqrt{s} = M_{Z}$, $\alpha_{QED}(M_{Z}^{2})$, is now the limiting factor for the fit of the SM. The uncertainty of $\alpha_{QED}(M_{Z}^{2})$ arises from the low energy contribution of the five quarks, $\Delta\alpha^{(5)}_{had}(M_{Z}^{2})$, which cannot be reliably calculated using perturbative QCD:

$$\alpha(M_{Z}^{2}) = \frac{\alpha(0)}{1 - \Delta\alpha_{l}(M_{Z}^{2}) - \Delta\alpha^{(5)}_{had}(M_{Z}^{2}) - \Delta\alpha_{top}(M_{Z}^{2})}$$

The leptonic contribution is computed to the third order, while the top contribution depends on the mass of the top quark, which is a parameter of the fit.

The hadronic contribution $\Delta\alpha^{(5)}_{had}(M_{Z}^{2})$ can be however evaluated by using $e^{+}e^{-}$ data, via a dispersion integral:

$$\Delta\alpha^{(5)}_{had}(M_{Z}^{2}) = -\frac{\alpha M_{Z}^{2}}{3\pi} Re \int_{4m_{t}^{2}}^{\infty} ds \frac{R(s)}{s(s - M_{Z}^{2} - i\epsilon)} =$$

$$\int_{E_{cut}^{2}}^{\infty} ds \frac{R_{data}(s)}{s(s - M_{Z}^{2} - i\epsilon)} + Re \int_{E_{cut}^{2}}^{\infty} ds \frac{R_{pQCD}(s)}{s(s - M_{Z}^{2} - i\epsilon)}$$

The above integral has been intentionally split into two parts to emphasize the role of the energy cut above which perturbative QCD (pQCD) is used: theoretical computation
of $\Delta \alpha^{(5)}_{\text{had}}(M_Z^2)$ depends not only on the experimental precision on $R_{\text{data}}(s)$, but also on the choice of the energy cut, leading to different predictions [2].

### 3 Hadronic contribution to the anomalous magnetic moment of the muon

In February 2001, Farley and colleagues [3], reported a new experimental value of the anomalous magnetic moment of the muon $a_\mu = (g - 2)/2 = (11659202 \pm 14 \pm 6) \times 10^{-10}$ using a positive muon beam, which is $2.6\sigma$ away from what is expected from SM [4]. Also in this case the main contribution to the theoretical error is given by the low energy hadronic contribution to the vacuum polarization, which again can be computed using experimental $e^+e^-$ data:

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left( \int_{4m_e^2}^{E_{\text{cut}}^2} ds \frac{R_{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{E_{\text{cut}}^2}^{\infty} ds \frac{R_{\text{PQCD}}(s) \hat{K}(s)}{s^2} \right)$$

The kernel $\hat{K}(s)$ is a smooth bounded function; the $1/s^2$ dependence in the above integral enhances low energy contributions, i.e. mainly $\sqrt{s} < 1 \text{ GeV}$ (the $\rho$ contributes for 62% of $a_\mu^{\text{had}}$). Recent evaluations have been computed using different approaches; a conservative data based approach using new data from BESII and CMD-2 found $a_\mu^{\text{had}} = (698.75 \pm 11.11) \times 10^{-10}$ [4] with an error still dominated by the $\sqrt{s} < 1.4 \text{ GeV}$ region.
4 Comments on the recent experimental results

Recent results from VEPP-2M

Many hadronic channel were measured at VEPP-2M by CMD2 and SND collaborations in the region 0.4-1.4 GeV, as shown Tab. 2. As said before, the main contribution for \( a_\mu \) comes from the region below 1.4 GeV, in particular from the \( e^+e^- \rightarrow \pi^+\pi^- \) channel; the systematical error is 1.4%, and it’s expected to go down to 0.6% in the near future. In order to achieve such a precision the systematics were carefully checked, for example the error coming from the energy beam is reduced by the resonance depolarisation technique. The main contribution to the systematical error comes now from the theoretical uncertainty to the radiative corrections: keeping the error below 1% is a challenging task.

Recent results from BESII at BEPC

The BESII collaboration has recently published a new measurement of \( R \) in the region 2-5 GeV, based on 85 points taken between February and June 99, with an average precision of 6.6%, a factor 2 better of the previous results. \( R \) was determined inclusively, from the number of observed hadronic events, \( N^\text{obs}_{\text{had}} \):

\[
R = \frac{N^\text{obs}_{\text{had}} - N_{\text{bckg}} - \sum N_{ll} - N_{\gamma\gamma}}{\sigma^0_{\mu\mu} \cdot L \cdot \epsilon_{\text{had}} \cdot (1 + \delta)}
\]

where \( N_{\text{bckg}} \) is the number of beam-associated background events; \( \sum N_{ll} \) and \( N_{\gamma\gamma} \) are respectively the background coming from misidentified events in one and two photons processes; \( L \) is the integrated luminosity; \( \delta \) is the radiative correction and \( \epsilon_{\text{had}} \) is the overall detector efficiency. In order to keep the error to \( \sim 7\% \) a big effort was done on: (a) Monte Carlo simulation to better understand detector efficiency; (b) estimation of \( N_{\text{bckg}} \) by means of separated beam and single beam operation; (c) radiative correction by comparing different schemes.
5 Conclusion: what we expect from the future?

We will now conclude by showing what we expect in the next years on hadronic cross section measurements at low energy:

0.4-1.4 GeV region

- **DAΦNE - LNF-Frascati (KLOE):**
  - Measurement of $|F_π|^2$ at $\sqrt{s} < 1\ GeV$ via radiative return [3];
  - upgrade for energy scan (2004?).

- **VEPP2M -Novosibirsk (CMD2, SND):**
  - Measurement of $|F_π|^2$ with 0.6% of systematic error and refined results on other channels;
  - new collider proposed VEPP-2000 (2003?) with $\sqrt{s}$ up to 2 GeV and expected luminosity of $10^{31} - 10^{32}\ cm^{-2}\ sec^{-1}$;

1.4 - 2.5 GeV region

**PEP-N:** new asymmetric $e^+e^-$ collider proposed at SLAC (2005?)

([http://www.slac.stanford.edu/grp/rd/epac/LOI/](http://www.slac.stanford.edu/grp/rd/epac/LOI/): expected $\delta_R/R \sim 2.5\%$;

2 - 5 GeV:

- **BEPC and BES upgraded (BEPCII and BESIII):** expected $\delta_R/R \leq 3\%$;

- **CLEO-C:** Modify CESR for high $L$ in 3-5 GeV region (2003?)

([http://www.lns.cornell.edu/public/CLEO/CLEO-C/index.html](http://www.lns.cornell.edu/public/CLEO/CLEO-C/index.html))

below 10 GeV:

Use ISR at B-factories to scan the region below Υ(4s) [7]

All the future results (within the existing or with the new detectors) will contribute to determine an new exciting era for the hadronic cross section measurement.

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