Research on Modeling of Equipment Damaging Caused by Environment Factor with the Distribution of Weibull

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Abstract. Working environment is one important factor to equipment failure. In order to quantify the environment influence to equipment failure, this paper qualitatively analyzed some environment factors’ effect to equipment failure and drew the graphs about the relationship between equipment failure and environment factors. Based on bathtub curve principle, progressively increasing curve and Weibull distribution principle which are basic theories in reliability engineering, this paper built a mathematical model about environment influence to equipment failure and put forward a calculating method to the unknown parameters in the model.

1. Introduction
Every kind of equipment has its environment conditions fit to work. These conditions include temperature, humidity, pressure and dust content etc. When these conditions are beyond the regular level, equipment's damage will increase obviously. In some conditions equipment damage caused by environment factors is very serious. In fact, the problem that equipment must to fit for environment conditions perplex people for a long period of time. There are many examples that equipment failed to run due to environment factors. A lot of special investigation carried out by USA DoD in 1960s has revealed that the equipment damage caused by environment factors is over 50%, which is even more serious than damage caused by fight. Some data indicates that plane’s fault will increase obviously working over 15 hours in a dust atmosphere. In the Gulf war the multinational force headed by the United States’ planes often work wrong because of the dust environment. These failures rarely happen in usually time [1] [2]. Above all, to quantity the influence of environment to equipment failure is a significant issue.

2. Analysis of environment factors according to bathtub curve

2.1. Conditions and Parameters Setting
(1) Only consider the factor of temperature, other environment factors are considered to be ideal.
(2) $P$: Equipment failure rate.
(3) $T$: Temperature.
(4) $T_{\text{min}}$: Minimum temperature.
(5) $T_{\text{max}}$: Maximum temperature.
(6) There are temperature points ‘$T_0$, $T_1$, $T_2$, $T_3$’. These temperature points divide the whole temperature area to 5 continuously districts ‘$T_{\text{min}} < T_0 < T_1 < T_2 < T_3 < T_{\text{max}}$’. 

(7) $T_0$: The lowest temperature to equipment failure.
For $T < T_0$
Then $P = 1$

(8) $T_1$: The highest temperature to equipment failure.
For $T > T_1$
Then $P = 1$.

(9) The suitable temperature for equipment working is the following temperature range. $T \in [T_1, T_2]$

(10) Equipment works in the certain temperature for a period of time.

2.2. Equipment Failure Analysis According to the Temperature Factor

(1) For $T \in [T_{\text{min}}, T_0]$, then $P = 1$. For $T \in [T_0, T_1]$, equipment failure rate will decrease with the temperature increasing.

(3) For $T \in [T_1, T_2]$. The equipment works in a suitable condition and has a low failure rate.

(4) For $T \in [T_2, T_3]$. The equipment failure rate is increasing with the temperature increasing.

(5) For $T \in [T_3, T_{\text{max}}]$. Then $P = 1$.

According to analysis above, one simple graph as figure 1 can be got.

![Figure 1. Equipment failure rate according to temperature](image)

2.3. Preliminary conclusions

Figure 1 is a typical bathtub curve. From the curve we can draw conclusions that under the premise that other conditions is ideal, in a certain temperature district such as ‘$[T_1, T_2]$’ equipment can work stably and the equipment failure rate is low. When temperature deviate the district to left or right such as ‘$[T_0, T_1]$’ or ‘$[T_2, T_3]$’, the equipment failure rate will increase too.

3. Analysis of environment factors according to progressively increasing or decreasing curve

3.1. Conditions and Assumptions
The dust evenly distributes in atmosphere and the unit of dust content is g/m3. The rain and snow evenly distributes in atmosphere and the unit of rain or snow content is g/m3. There is the same
pressure in the same area. The pressure is expressed as Pa. When one environment factor is analyzed, other factors are supposed to be ideal which can be ignored. Equipment runs for a period of time in certain conditions.

3.2. Qualitative analysis

(1) The influence of dust content

When the dust content is lower, the equipment failure rate will be lower. As the dust content increasing, the equipment failure rate will increase. If the dust content increased to one certain degree, the equipment would not work. From above, a progressive increasing curve can be got as figure 2. The trend of equipment failure rate changing with dust content changing is that the rate increasing slowly at beginning, then increasing faster progressively until the equipment breakdown.

![Figure 2. Equipment failure rate according to dust content](image)

(2) The influence of rain and snow content

The influence of rain and snow content to equipment failure rate is alike as dust content’s influence. As the content of rain and snow in atmosphere is increasing, the equipment failure rate is increasing. When the content is lower, the influence is less. When the content is higher, the influence is larger. When the content is to one certain degree, the equipment will breakdown. From the changing trend figure 3 can be got.

![Figure 3. Equipment failure rate according to rain and snow content](image)

(3) The influence of pressure

Equipment can stably work in certain pressure range. In plateau area as pressure declining, equipment failure rate will increase progressively. If the pressure decline to certain degree, equipment would not work. From the trend figure 4 can be got.

![Figure 4. Equipment failure rate according to pressure](image)
3.3. Preliminary conclusion
The intuitive analysis and figure 1-4 above shows the influence of dust content, rain and snow content to equipment failure rate is according to progressively increasing curve, and the influence of pressure to equipment failure rate is according to progressively decreasing curve.

4. Analysis of environment factors according to progressively increasing or decreasing curve

4.1. Analysis about the applicability of Weibull distribution
(1) Basic graphs of Weibull distribution
In reliability theory there are graphs of failure rate function, failure density function, and failure probability function [3] [4]. And there are 3 parameters $\beta$, $\eta$ and $\gamma$ in Weibull distribution function. The shape of the functions depend on the parameter ‘$\beta$’. In order to draw the graph conveniently, in this paper the value of $\eta$ and $\gamma$ is set as following.

$\eta = 1$, $\gamma = 0$, then the value of $\beta$ is given respectively as $\beta = 1$ $\beta > 1$ or $\beta < 1$. Set the value of the parameters, give a serious of values to the environment factor, use the software tool ‘Matlab’, figures 5-7 can be got.

(2) Applicability analysis of Weibull distribution
Conclusions can be drawn according to Figure 5-7.

Figure 5. Failure density function

Figure 6. Failure probability function

Figure 7. Failure rate function

(2) Applicability analysis of Weibull distribution
Conclusions can be drawn according to Figure 5-7.
When $\beta < 1$, the failure rate is decreasing progressively. This is according to the early failure time of bathtub curve. When $\beta = 1$, the failure rate is certain. This is according to the occasional failure time of bathtub curve. (3) When $\beta > 1$, the failure rate is increasing progressively. This is according to the tired failure time of bathtub curve. From above, Weibull distribution can fit curves of failure rate increasing, failure rate decreasing and failure rate stable through change the value of $\beta$. So failure rate according to the regular of increasing progressively or decreasing progressively or stably can be forecast through Weibull distribution [5].

4.2. Mathematical model based on Weibull distribution

(1) Assumptions

Equipment damage caused by environment factor meets bathtub curve. Environment factor is independent. In traditional failure graphs, the data of the horizontal axis is time. When replace time with one environment factor such as temperature, pressure or dust content, one distribution about equipment failure caused by the environment factor can be got. Simultaneously in the model of Weibull distribution function the variable parameter of time is replaced by the environment factor data. So based on Weibull distribution theory one model to quantify the influence of equipment failure caused by environment factor can be got.

(2) Equipment failure function

Because the Weibull distribution model with 3 parameters have better adaptability and easy to solve [6], this paper set up Weibull distribution model with 3 parameters. Equation 1 is the function of equipment failure density.

$$f(x) = \frac{\beta}{\eta} \left(\frac{x - \gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x - \gamma}{\eta}\right)\right]$$

In function 1 $\beta$ is a shape parameter that can determine the shape of the figure, $\gamma$ is a place parameter that can determine the place of the figure, $\eta$ is a scale parameter that can determine the scale of the figure [7].

(3) Method to parameter estimation

Use maximum likelihood estimation method to calculate the parameters in function 1.

Setting $\theta = (\beta, \eta, \gamma)$. Then set up likelihood function.

$$L(\theta; x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(\beta, \eta, \gamma; x_i)$$

$$= \prod_{i=1}^{n} \frac{\beta}{\eta} \left(\frac{x_i - \gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x_i - \gamma}{\eta}\right)\right]$$

Find logarithm of the function as following.

$$\ln L(\beta, \eta, \gamma; x_i) = n \ln \frac{\beta}{\eta} + (\beta - 1) \sum_{i=1}^{n} \ln \left(\frac{x_i - \gamma}{\eta}\right) - \sum_{i=1}^{n} \left(\frac{x_i - \gamma}{\eta}\right)^{\beta}$$

$$= n \ln \beta - n \beta \ln \eta + (\beta - 1) \sum_{i=1}^{n} (x_i - \gamma) - \sum_{i=1}^{n} \left(\frac{x_i - \gamma}{\eta}\right)^{\beta}$$
Equation group 4 as following can be got through Seeking partial derivative of function 3.

$$
\frac{\partial \ln L(\beta, \eta, \gamma; x_i)}{\partial \beta} = 0
$$

$$
\frac{\partial \ln L(\beta, \eta, \gamma; x_i)}{\partial \eta} = 0
$$

$$
\frac{\partial \ln L(\beta, \eta, \gamma; x_i)}{\partial \gamma} = 0
$$

Equation group 5 as following can be got form function 3 and equation group 4.

$$
\sum_i \left[ \frac{1}{\beta} - \ln \eta + \ln \left( x_i - \gamma \right) - \left( \frac{x_i - \gamma}{\eta} \right)^\beta \ln \left( \frac{x_i - \gamma}{\eta} \right) \right] = 0
$$

$$
\sum_i \left[ \frac{\beta \left( \frac{x_i - \gamma}{\eta} \right)^\beta}{\eta} - \frac{\beta}{\eta} \right] = 0
$$

$$
\sum_i \left[ \frac{\beta \left( \frac{x_i - \gamma}{\eta} \right)^{\beta-1}}{x_i - \gamma} - \frac{\beta-1}{\gamma} \right] = 0
$$

In equation group 5, the parameter can be solved by computer procedure. And the estimated value can be expressed as $\hat{\theta} = (\hat{\beta}, \hat{\eta}, \hat{\gamma})$.

4.3. Discussion on the estimated parameters

(1) Discussion on $\beta$

When $\hat{\beta} < 1$, the influence to equipment damage is decreasing progressively. For example, when equipment works in a lower temperature than normal condition, the failure rate will increase with temperature decreasing. When $\hat{\beta} = 1$, the influence of environment factor to equipment failure is stable and lower. For example, equipment works in the normal condition. When $\hat{\beta} > 1$, the influence of environment factor to equipment failure is increasing progressively. For example, equipment is working in the condition of dust, snow, rain or high temperature.

(2) Discussion on $\eta$

In Weibull distribution as scale parameter, the parameter $\eta$ decides the changing velocity of the graph. In the model of environment factor influence equipment failure, the estimated parameter $\hat{\eta}$ decides the sensitivity about equipment failure to environment factor.

(3) Discussion on $\gamma$

As place parameter, $\hat{\gamma}$ decides the parameter index about the equipment breakdown.

5. Conclusion

Start from the qualitative analysis on the environment influence to equipment failure, in terms of the principle that Weibull distribution can be used to forecast typical distribution failure rate, one model to quantify the influence that equipment failure caused by environment factors is put forward. In the model time dimension in Weibull distribution is replaced by environment factor. The model is fit for the situation that environment factors’ influence to equipment failure according with the rue of increasing progressively curve, decreasing progressively curve and bathtub curve.
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