Hidden Vehicles Positioning via Asynchronous V2V Transmission: A Multi-Path-Geometry Approach

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Abstract—Accurate vehicular sensing is a basic and important operation in autonomous driving. Unfortunately, the existing techniques have their own limitations. For instance, the communication-based approach (e.g., transmission of GPS information) has high latency and low reliability while the reflection-based approach (e.g., RADAR) is incapable of detecting hidden vehicles (HVs) without line-of-sight. This is arguably the reason behind some recent fatal accidents involving autonomous vehicles. To address this issue, this paper presents a novel HV-sensing technology that exploits multi-path transmission from a HV to a sensing vehicle (SV). The powerful technology enables the SV to detect multiple HV-state parameters including position, orientation of driving direction, and size. Its implementation does not even require transmitter-receiver synchronization like the conventional microwave positioning techniques. Our design approach leverages estimated information on multi-path (namely their AoA, AoD, and ToA) and their geometric relations. As a result, a complex system of equations or optimization problems, where the desired HV-state parameters are variables, can be formulated for different channel-noise conditions. The development of intelligent solution methods ranging from least-square estimator to disk/box minimization yields a set of practical HV-sensing techniques. We study their feasibility conditions in terms of the required number of paths. Furthermore, practical solutions, including sequential path combining and random directional beamforming, are proposed to enable HV-sensing given insufficient paths. Last, realistic simulation of driving in both highway and rural scenarios demonstrates the effectiveness of the proposed techniques.

I. INTRODUCTION

Autonomous driving (auto-driving) aims at reducing car accidents, traffic congestion, and greenhouse gas emissions by automating the transportation process [2]. The potential impact of the cross-disciplinary technology has attracted heavy R&D investments not only by leading car manufacturers (e.g., Tesla) but also Internet companies (e.g., Google). One primary operation of auto-driving is vehicular positioning [3]–[6], namely positioning nearby vehicles and tracking other parameters such as sizes and trajectories. The information then serves as inputs for computing and control tasks such as navigation and accident avoidance. There exist diversified approaches for vehicular positioning but they all have their own drawbacks. One conventional approach is to exchange the absolute location information among a group of nearby vehicles by using vehicle-to-vehicle (V2V) transmission [7]. Its main drawbacks include high latency due to the data exchanging process and low reliability arising from inaccurate Global Positioning System (GPS) information in e.g., dense urban areas or tunnels [2]. Another approach is to deploy sensors ranging from Light Detection and Ranging (LiDAR) to Radio Detection And Ranging (RADAR). As illustrated in Fig. 1, they are capable of sensing the line-of-sight (LoS) vehicles, but cannot “see through” a large solid object (e.g., a truck) to detect hidden vehicles (HVs). A comprehensive discussion of existing approaches is given in the sequel. Motivated by their drawbacks, this paper presents a novel technology for accurately sensing a HV including detecting its position, orientation of driving direction, and size by exploiting the multi-path geometry of V2V transmission.

A. Wireless Transmission and Vehicular Positioning

Wireless transmission underpins different approaches for vehicular positioning. They are discussed as follows and their relative strengths and limitations are summarized in Table I.

1) Communication-Based Vehicular Positioning: Nearby vehicles can position each other by vehicle-to-everything (V2X) communication to exchange position information obtained using the GPS [8], [9]. The state-of-art protocol for connecting vehicles is the low-latency dedicated short-range communication (DSRC) [10], a variant of the IEEE 802.11 Wi-Fi random-access protocol. The effectiveness of the protocol has been examined in the scenario of two-lane rural highway [11]. Recently, the 3rd Generation Partner Project (3GPP) has initiated a standardization process to realize cellular-based V2X (C-V2X) communication supporting higher data rate and larger coverage than DSRC [12]. To boost data rate, the technology aims at implementation in the millimetre-wave (mmWave) spectrum where abundant of bandwidth is available [13].

Figure 1: Hidden vehicle scenario with multi-path NLoS channels.
training and alignment to cope with fast fading in channels between high-speed vehicles [14]. To reduce this overhead, a fast beam-alignment scheme is proposed in [15] that leverages matching theory and swarm intelligence to efficiently pair vehicles. An alternative technique for accelerating beam alignment is to leverage information generated by either onboard RADAR [16] or GPS [17] to deduce useful channel information. It is also possible to use location specific information stored in the cloud [18] to help beam training, alignment, and tracking.

The main challenge faced by the communication-based approach is that their reliability and latency may not meet the requirements of mission-critical scenarios such as accident avoidance at high speeds. Consider the reliability issue. The GPS information exchanged between vehicles can be highly inaccurate due to the blockage of GPS signals from satellites in urban areas or tunnels. Instead of using GPS satellites, an alternative solution, mobile positioning, using multiple access points (APs) in a cellular network requires close-to-perfect synchronization for accurate positioning, which is difficult to be guaranteed in practice [19]. Next, consider the latency issue. The implementation of V2V communication in a dense vehicular network incurs high communication overhead due to many practical factors including complex protocols, packet loss and retransmission, limited link life time, etc. Presently, efficient resource management and multi-access in vehicular networks overlaying V2V and V2X communications still face many open challenges [20]. Due to the above issues, the application of the communication-based approach is limited to pedestrian positioning but not yet suitable for a latency-sensitive task like vehicular positioning.

2) Reflection-Based Vehicular Positioning: Vehicles capable of auto-driving are typically equipped with RADAR and LIDAR among other sensing devices. The two sensing technologies both adopt the reflection-based approach, namely that the sensors detect the reflections from vehicles and objects in the environment but using different mediums, microwaves and laser light, respectively.

LIDAR steers ultra-sharp laser beams to scan the surrounding environment and generates a dynamic high-resolution three-dimensional (3D) map for navigation [21]. The main drawback of LIDAR is its ineffectiveness under hostile weather conditions due to the difficulty of light in penetrating fog, snow, and rain [2]. In addition, LIDAR is currently too expensive to be practical and the huge amount of generated data is challenging to be processed within the ultra-low latency required for safe driving.

RADAR can locate a target object as well as estimate its velocity via sending a designed waveform [e.g., frequency modulated continuous waveform (FMCW)] and analyzing its reflection by the object [22]. Particularly, the metal surfaces of vehicles are capable of reflecting microwaves with negligible absorption. For the reason, RADAR is popularly used for long-range sensing. Recent breakthroughs in mmWave RADAR make it feasible to deploy large-scale but highly impact arrays for sharp beamforming to achieve a high positioning accuracy [23]. Compared with LIDAR, RADAR can provide longer sensing ranges (up to hundreds-of-meter) and retain the effectiveness under hostile weather conditions or in an environment with poor lighting. One disadvantage of RADAR is that it may incorrectly recognize a harmless small metal object as a much larger object due to scattering, leading to false alarms [24].

In the context of auto-driving, perhaps the most critical limitation of LIDAR, RADAR and other reflection-based technologies is that they can detect only vehicles with LoS since neither microwaves nor laser light can penetrate a large solid object such as a truck or a building. However, detecting HVs with non-LoS (NLoS) is crucial for collision avoidance in complex scenarios such as overtaking and cross junctions as illustrated in Fig. 1.

3) Vehicular Positioning by Synchronous Transmission: There exist signal processing techniques for positioning via synchronous transmission where a receiver attempts to estimate the position of a transmitter. To be specific, a receiver estimates the position of the transmitter from the prior knowledge of the transmitted waveform and the received signal (see e.g., [25], [26]). The effectiveness of this approach hinges on the key assumption of perfect synchronization between the transmitter and receiver. The reason is that the distance of each propagation path can be directly converted from its propagation delay (for pulse transmission) [27] or frequency variation (for FMCW transmission) [28], which can be easily measured given the said synchronization. On the other hand, spatial parameters of individual paths, including the angles-of-arrival (AoA) and angles-of-departure (AoD), can be estimated if the transmitter and receiver are equipped with antenna arrays to perform spatial filtering. Combining the distances and spatial parameters of multi-paths and exploiting their geometric relations enable a receiver to estimate the position of a transmitter despite the lack of LoS [25], [26].

The assumption of perfect transmitter-receiver synchronization is reasonable for collocated or static transmitter and receiver (e.g., in the case of bistatic RADAR). However, in the auto-driving scenario, the transmitter and receiver are a HV and a sensing vehicle (SV), respectively, and their synchronization is impractical especially at high speeds. This renders the approach based on synchronous transmission unsuitable for vehicular positioning. In view of the prior work, sensing a HV remains largely an open problem and tackling it is the
theme of this work.

B. Main Contributions

In this work, we aim at tackling the open challenges faced by existing vehicular positioning techniques as summarized below.

1) **No LoS and lack of synchronization**: None of the existing techniques (see Table I) can be effective for sensing a HV that has neither LoS nor synchronization (with the SV).

2) **Efficient detection of HV size and orientation**: Except for LIDAR, other existing vehicular positioning approaches are incapable of detecting the size and orientation of a HV, which are important safety information required in auto-driving. However, LIDAR is an inefficient technique for mission critical services due to its high cost and processing delay. This calls for development of an economical and low-latency solution.

3) **Insufficient multi-path**: Similar to the approach based on synchronous transmission, the current technology positions a HV by exploiting multi-path propagation. Such approaches may not be feasible in the scenario with sparse scattering and hence insufficient paths. Tackling the challenge is important for making the technologies robust.

To tackle the above challenges, we propose a novel technology, called HV-sensing, to enable a SV to simultaneously sense the position, orientation of driving direction, and size of a HV without requiring SV-HV synchronization. The SV leverages the information of multi-path signals [including AoA, AoD and time-of-arrival (ToA)] as well the derived geometry relations between the paths so as to construct tractable systems of equations or optimization problems, where the HV position, orientation of driving direction, and size are unknown variables to be found. A set of HV-sensing techniques are designed for operation in different practical settings ranging from low to high signal-to-noise ratios (SNRs), single-cluster to multi-cluster HV arrays, and small to large waveform sets. The differences between the proposed HV-sensing technology and conventional approaches are summarized in Table I.

The main contributions of this work are summarized as follows.

1) **Sensing HV position and orientation**: Consider the case that the HV array contains a cluster of collocated antennas. The goal is to simultaneously estimate HV’s position and orientation even the absence of SV-HV synchronization. HV transmits orthogonal waveforms over different antennas to enable the SV to estimate the multi-path information (AoA, AoD, and ToA). Then given the information and when noise is negligible, a complex system of equations is constructed and solved in sequential steps at SV to obtain the desired HV-state parameters. On the other hand, when noise is present, the sensing problem is suitably formulated as least-square (LS) estimation and also solved in a sequential procedure. To make the HV sensing feasible, the required number of paths is at least 4 for 2D propagation or at least 3 for 3D propagation.

2) **Sensing HV position, orientation, and size**: Consider the case that the HV array contains multiple clusters of antennas that are distributed over the vehicular body. In this case, the goal is to further estimate the HV size along with its position and orientation. Two specific schemes are presented. The first assumes the transmission of multiple orthogonal waveform sets so that the SV can group the paths according to their originating HV antenna clusters. Then the scheme can build on the preceding design in 1) to estimate the HV size via efficiently positioning the HV antenna clusters, which also yields the HV position and orientation. The second assumes the transmission of an identical waveform set such that the said path-grouping is infeasible. Then alternative size detection techniques are proposed based on efficient disk or box minimization under the constraint that the disk or box encloses the HV array. The required numbers of paths for the first sensing scheme is found to be 6 and that for the second scheme is 4. Nevertheless, when both of two schemes are feasible, the former outperforms the latter as multiple orthogonal waveform sets help improve sensing accuracy.

3) **Coping with insufficient multi-path**: To make the proposed HV-sensing techniques more reliable, we further propose practical solutions for increasing the number of available paths in the case where they are insufficient for meeting the feasibility requirements of the above HV-sensing techniques. The first solution is to combine paths exploited in multiple time instants and the second is to apply random directional beamforming for uncovering hidden paths invisible in the case of isotropic HV transmission. The solutions are complementary and can be jointly implemented to maximize the number of significant paths for enhancing the sensing accuracy.

4) **Realistic simulation**: The proposed HV-sensing techniques are evaluated using practical simulation models of highway and rural scenarios and found to be effective.

The remainder of the paper is organized as follows. Section II introduces the system model and problem formulation for HV-sensing. Sections III and IV present the HV-sensing techniques for the cases of single-cluster and multi-cluster HV arrays, respectively. The solutions for the practical issue of insufficient multi-path for HV-sensing are developed in Section V. Simulation results are presented in Section VI followed by concluding remarks in Section VII.

II. System Model

We consider a two-vehicle system where a SV attempts to detect the (relative) position, size, and orientation of a HV blocked by obstacles such as trucks or buildings as illustrated in Fig. 1. An antenna cluster refers to a set of collocated antennas where the half-wavelength antenna spacing is negligible compared with vehicle sizes and propagation distances. An array can comprises a single or multiple antenna clusters, referred to as a single-cluster and a multi-cluster array, respectively. The deployment of a single-cluster array at the HV enables the SV to detect the HV’s position and orientation. On the other hand, a multi-cluster HV array can further make it possible for the SV to estimate the HV size. For the purpose of exposition, in the case of multi-cluster HV array, we consider 4 clusters located at the vertices of a rectangle representing the vehicle. The principle of HV-sensing design in the sequel is based on the efficient detection of the clusters’ positions and thus can be straightforwardly extended to other
clusters’ topologies with irregular clusters’ distributions. For HV-sensing in both scenarios, the SV requires only a single-cluster array. Signal propagation is assumed to be contained in the 2D plane and the results are subsequently extended to the 3D propagation. The channel model, V2V transmission, and sensing problem are described in the following sub-sections.

### A. Multi-Path NLoS Channel

The NLoS channel between SV and HV contains multi-paths reflected by a set of scatterers. Consider the characteristics of V2V channel [30]. We make the following assumption.

**Assumption 1 (Single-Bound Scattering).** The single-bound scattering is used to model the V2V propagations that the NLoS signals are assumed to have only once reflection due to scatterers.

Assumption 1 is widely used in the literature of localization via NLoS paths [25], [26], [31] and geometry-based V2V channel [32]. Various methods have been proposed to detect and extract the single-bound scattering paths from among the multiple-bound scatterings paths. A two-step proximity detection scheme is used in [26] to detect and discard such multiple-bound scattterings paths. The single-bound scattering paths can also be extracted by jointly using ToA and received signal strength information [33], [34] due to the fact that paths from multiple-bound scatterings usually have small signal power and larger ToA due to the severe attenuation and longer propagation distance. Based on Assumption 1, a 2D Cartesian coordinate system is considered as illustrated in Fig. 2 where the SV array is located at the origin and the X-axis is aligned with the orientation of SV. Further consider a typical antenna cluster at the HV. Each NLoS signal path from the HV antenna cluster to the SV array can be characterized by the following five parameters: the AoA at the SV denoted by \( \theta \); the AoD at the HV denoted by \( \varphi \); the orientation of the HV denoted by \( \omega \); and the propagation distance denoted by \( d \) which is divided into the propagation distance after reflection, denoted by \( \nu \), and the remaining distance \( (d - \nu) \). The AoD and AoA are defined as azimuth angles relative to orientations of HV and SV, respectively. Fig. 2 graphically shows the definitions of the above parameters.

### B. Hidden Vehicle Transmission

To enable sensing at the SV, the HV transmits a set of waveforms defined as follows. Each antenna cluster at HV has \( M_i \) antennas with at least half-wavelength spacing between adjacent antennas. Consider a typical antenna cluster. A set of orthogonal waveforms are transmitted over different antennas. Let \( s_m(t) \) be the finite-duration baseband waveform in \( t \in [0, T_s] \) assigned to the \( m \)-th HV antenna with the bandwidth \( B_s \). Then the waveform orthogonality is specified by \( \int_0^{T_s} s_{m_1}(t)s_{m_2}^\ast(t)dt = \delta(m_1 - m_2) \) with the delta function \( \delta(x) = 1 \) if \( x = 0 \) and 0 otherwise. The transmitted waveform vector for the \( k \)-th HV antenna cluster is \( s_{c(k)}(t) = [s_{1(k)}(t), \ldots, s_{M_k}(t)]^T \). In the case of multi-cluster HV array, the waveform sets for different clusters are either identical or orthogonal with each other. The use of orthogonal waveform sets allows SV to group the detected paths according to their originating antenna clusters as elaborated in the sequel, and hence this case is referred to as decoupled clusters. Then the other case is called coupled clusters. With the prior knowledge of transmitted waveforms, the SV with \( M_f \) antennas can scan the received signal due to the HV transmission to resolve multi-path as discussed in the next sub-section.

The expression of the received signal is obtained as follows. Consider a typical HV antenna cluster. Based on the far-field propagation model, the cluster response vector is a function of the AoA \( \varphi \) defined as

\[
a(\varphi) = [\exp(j2\pi f c_1(\varphi)), \ldots, \exp(j2\pi f c_{M_f}(\varphi))]^T,
\]

where \( f_c \) denotes the carrier frequency and \( \alpha_{m}(\varphi) \) is the difference in propagation time to the corresponding scatterer between the \( m \)-th HV antenna and the first HV antenna in the same cluster, i.e., \( \alpha_{1}(\varphi) = 0 \). The response vector of the SV array is written in terms of AoA \( \theta \) as

\[
b(\theta) = [\exp(j2\pi f c_1\beta_1(\theta)), \ldots, \exp(j2\pi f c_{M_f}\beta_{M_f}(\theta))]^T,
\]

where \( \beta_m(\theta) \) refers to the difference of propagation time from the scatterer to the \( m \)-th SV antenna than the first SV antenna. We assume that SV has prior knowledge of both the HV-and-SV array configurations and thereby the response functions \( a(\varphi) \) and \( b(\theta) \). This is feasible by standardizing the vehicular arrays’ topology. In addition, the Doppler effect is ignored based on the assumption that the Doppler frequency shift is much smaller than the waveform bandwidth and thus does not affect waveform orthogonality.\(^1\)

Let \( k \) denote the index of the HV antenna cluster and \( p(k) \) denote the number of received paths originating from the \( k \)-th HV antenna cluster. The total number of paths arriving at SV is \( P = \sum_{k=1}^{K} p(k) \). Represent the received signal vector at SV as \( r(t) = [r_1(t), \ldots, r_{M_f}(t)]^T \) that can be written in terms of s(t), a(\( \varphi \)) and b(\( \theta \)) as

\[
r(t) = \sum_{k=1}^{K} \sum_{p=1}^{p(k)} \gamma_p \cdot b_{p}(\theta_p) a^\ast(\varphi_p) s(t - \lambda_p^k) + n(t),
\]

\(^1\)The doppler frequency \( f_D = \frac{v}{c} \) with the relative velocity of HV to SV \( v \) and the light speed \( c = 3 \times 10^8 \) (m/sec) is ignored because \( f_D \ll f_c \) and the resultant Doppler phase shifts in \( a(\varphi) \) and \( b(\theta) \) are almost zero within the waveform duration.
where \( \gamma_p^{(k)} \) and \( \lambda_p^{(k)} \) denote the complex channel coefficient and ToA of path \( p \) originating from the \( k \)-th HV antenna cluster, respectively, and \( n(t) \) represents channel noise. With no synchronization between the HV and SV, the SV has no information of HV’s transmission timing. Therefore, it is important to note that \( \lambda_p^{(k)} \) differs from the corresponding propagation delay, denoted by \( \tau_p^{(k)} \), with \( \tau_p^{(k)} = \frac{d_p^{(k)}}{c} \) and \( d_p^{(k)} \) being the propagation distance. Given an unknown clock-synchronization gap between the HV and SV denoted as \( \Gamma \), \( \tau_p^{(k)} = \lambda_p^{(k)} - \Gamma \).

**Remark 1** (Multi-Access in a Vehicular Network). Building the above transmission scheme, multi-access techniques can be designed to allow the implementation of the proposed HV-sensing technology in a vehicular network. For instance, an AP periodically broadcasts the lists of available orthogonal waveform sets, each of which includes different \( M_1 \) orthogonal waveforms. Then by sensing the ambient signal, each HV can select and transmit one available waveform set unused by nearby vehicles with random timing. Then a SV can decouple the signals from different HVs provided 1) they transmit different waveform sets or 2) their waveforms do not overlap in time upon arrival at the SV which is likely due to bursty transmissions.

### C. Estimations of AoA, AoD, and ToA

The sensing techniques in the sequel assume that the SV has the knowledge of AoA, AoD, and ToA of each receive NLoS signal path, say path \( p \), denoted by \( \{\theta_p, \varphi_p, \lambda_p\} \) where \( p \in \mathcal{P} = \{1, 2, \ldots, P\} \). The knowledge can be acquired by applying classical parametric estimation techniques briefly described as follows.

1. **Sampling**: The received analog signal \( r(t) \) and the waveform vector \( s(t) \) are sampled at the Nyquist rate \( B_s \) and converted to the digital signal vectors \( r[n] \) and \( s[n] \), respectively.

2. **Matched Filter**: The sequence of \( r[n] \) is matched-filtered by \( s[n] \). The resultant \( M_r \times M_t \) coefficient matrix \( Y[z] \) is given by \( Y[z] = \sum_n r[n]s[n]^H[n-z] \). The sequence of ToAs \( \{\lambda_p\} \) can be estimated by detecting peak points of the matrix norm \( \|Y[z]\| \), denoted by \( \{z_p\} \), which can be converted into time by multiplexing with the time resolution \( \frac{1}{B_s} \). One peak point can contain multiple signal paths if the signals arrive within the same sampling interval.

3. **Multi-Path Estimation**: Given \( \{Y[z_p]\} \), AoAs and AoDs are jointly estimated using a 2D-multiple signal classification (MUSIC) algorithm, which is the most widely used subspace-based detection method [35]. The estimated AoA \( \{\theta_p\} \) and AoD \( \{\varphi_p\} \) are paired with the corresponding estimated ToA \( \{\lambda_p\} \), which jointly characterize path \( p \).

### D. Hidden Vehicle Sensing Problem

The SV attempts to sense the HV’s position, size, and orientation, which can be obtained by using parameters of AoA \( \theta \), AoD \( \varphi \), orientation \( \omega \), distances \( d \) and \( \nu \), and location of multi-cluster HV array. Noting the first two parameters are obtained based on the estimations in Section II-C and the goal is to estimate the remaining parameters.

### III. Sensing Hidden Vehicle with a Single-Cluster Array

In this section, we consider the scenario that a single-cluster array is deployed at HV. Then this section focuses on designing the sensing techniques for SV to detect 1) the HV position (i.e., position of the single-cluster array), specified by the coordinate \( p = (x, y) \), and 2) the HV orientation, specified by \( \omega \) (see Fig. 2). Based on the multi-path-geometry, \( p \) is described as

\[
\begin{align*}
    x &= \nu_p \cos(\theta_p) - (d_p - \nu_p) \cos(\varphi_p + \omega), \\
    y &= \nu_p \sin(\theta_p) - (d_p - \nu_p) \sin(\varphi_p + \omega),
\end{align*}
\]

\( \forall p \in \mathcal{P} \). (3)

The prior knowledge that SV has for sensing is the parameters of \( P \) NLoS paths estimated as described in Section II-C. Each path, say path \( p \), is determined by the parametric set \( \{\theta_p, \varphi_p, \lambda_p\} \) and orientation \( \omega \) as [3] shows. Then given the equations in [3], the **sensing problem** for the current scenario reduces to

\[
\bigcup_{p \in \mathcal{P}} \{\theta_p, \varphi_p, \lambda_p\} \Rightarrow \{p, \omega\}. \tag{4}
\]

### A. Sensing Feasibility Condition

In this subsection, it is shown that for the HV-sensing to be feasible, there should exist at least four NLoS paths. To this end, by using [3] and multi-path-geometry, we can obtain the following system of equations:

\[
\begin{align*}
    x_p &= \nu_1 \cos(\theta_1) - (d_1 - \nu_1) \cos(\varphi_1 + \omega), \\
    y_p &= \nu_1 \sin(\theta_1) - (d_1 - \nu_1) \sin(\varphi_1 + \omega),
\end{align*}
\]

where \( (x_p, y_p) \) denotes the coordinate characterized via path \( p \). The number of equations in [3] is \( 2(P - 1) \), and the above system of equations has a unique solution when the dimensions of unknown variables are less than \( 2(P - 1) \). Since the AoAs \( \{\theta_p\} \) and AoDs \( \{\varphi_p\} \) are known, the number of unknowns is \( (2P + 1) \) including the propagation distances \( \{d_p\} \), \( \{\nu_p\} \), and orientation \( \omega \). To further reduce the number of unknowns, we use the propagation time difference between signal paths also known as TDoAs, denoted by \( \{\rho_p\} \), which can be obtained from the difference of ToAs as \( \rho_p = \lambda_p - \lambda_1 \) where \( \rho_1 = 0 \). The propagation distance of signal path \( p \), say \( d_p \), is then expressed in terms of \( d_1 \) and \( \rho_p \) as

\[
d_p = c(\lambda_p - \Gamma) = c(\lambda_1 - \Gamma) + c(\lambda_p - \lambda_1) = d_1 + c\rho_p, \tag{5}
\]

for \( p \in \{2, \ldots, P\} \). Substituting the above \( (P - 1) \) equations into [3] eliminates the unknowns \( \{d_2, \ldots, d_P\} \) and hence reduces the number of unknowns from \( (2P + 1) \) to \( (P + 2) \). As a result, [3] has a unique solution when \( 2(P - 1) \geq P + 2 \).

**Proposition 1** (Sensing Feasibility Condition). To sense the position and orientation of a HV equipped with a single-cluster array, at least four NLoS signal paths are required: \( P \geq 4 \).

**Remark 2** (Asynchronization and TDoA). Recall that one sensing challenge is asynchronization between HV and SV represented by \( \Gamma \), which is a latent variable we cannot observe explicitly. Considering TDoA helps solve the problem by...
avoiding the need of considering $\Gamma$ by exploiting the fact that all NLoS paths experience the same synchronization gap.

### B. Hidden Vehicle Sensing without Noise

Consider the case of a high receive signal-to-noise ratio (SNR) where noise can be neglected, i.e., the estimations of AoA/AoD/ToA $\{\theta_p, \varphi_p, \lambda_p\}$ are perfect. Then the HV-sensing problem in (4) is translated to solve the system of equations in (6). One challenge is that the unknown orientation $\omega$ introduces nonlinear relations, namely $\cos(\varphi_p + \omega)$ and $\sin(\varphi_p + \omega)$, in the equations. To overcome the difficulty, we adopt the following two-step approach: 1) estimate the correct orientation $\omega^*$ via its discriminant introduced in the sequel; 2) given $\omega^*$, the equation becomes linear and thus can be solved via LS estimator, giving the position $p^*$. To this end, the equations in (6) can be arranged in a matrix form as

$$A(\omega)z = B(\omega),$$

(E2)

where $z = (v, d_1)^T \in \mathbb{R}^{(P+1) \times 1}$ and $v = \{\nu_1, \cdots, \nu_P\}$. For matrix $A(\omega)$, we have

$$A(\omega) = \begin{bmatrix} A^{(\cos)}(\omega), A^{(\sin)}(\omega) \end{bmatrix}^T \in \mathbb{R}^{2(p-1) \times (p+1)},$$

(6)

where $A^{(\cos)}(\omega)$ is

$$A^{(\cos)}(\omega) = \begin{bmatrix} a_1^{(\cos)} & -a_2^{(\cos)} & \cdots & 0 & a_{1,2}^{(\cos)} \\ a_1^{(\cos)} & 0 & -a_3^{(\cos)} & \cdots & 0 & a_{1,3}^{(\cos)} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ a_1^{(\cos)} & 0 & 0 & \cdots & -a_p^{(\cos)} & a_{1,p}^{(\cos)} \end{bmatrix},$$

with $a_1^{(\cos)} = \cos(\theta_p) + \cos(\varphi_p + \omega)$ and $a_1^{(\cos)} = \cos(\varphi_p + \omega) - \cos(\varphi_1 + \omega)$, and $A^{(\sin)}(\omega)$ is obtained by replacing all cos operations in (7) with sin operations. Next,

$$B(\omega) = \begin{bmatrix} B^{(\cos)}(\omega), B^{(\sin)}(\omega) \end{bmatrix}^T \in \mathbb{R}^{2(p-1) \times 1},$$

(8)

where $B^{(\cos)}(\omega) = \begin{pmatrix} c_p \cos(\varphi_1 + \omega) & c_p \cos(\varphi_2 + \omega) & \cdots & c_p \cos(\varphi_p + \omega) \end{pmatrix}^T$, and $B^{(\sin)}(\omega)$ is obtained by replacing all cos in $B^{(\cos)}(\omega)$ with sin.

1) Computing $\omega^*$: Note that (E2) becomes an over-determined linear system of equations if $P \geq 4$ (see Proposition 2), providing the following discriminant of orientation $\omega$. Since the equations in (6) are based on the geometry of multi-path propagation and HV orientation as illustrated in Fig. 2, there exists a unique solution for the equations. Then we can obtain from (6) the following result, which is useful for computing $\omega^*$.

**Proposition 2 (Discriminant of Orientation).** With $P \geq 4$, the unique $\omega^*$ exists when $B(\omega^*)$ is orthogonal to the null column space of $A(\omega^*)$ denoted by $\ker(A(\omega^*)^T) \subseteq \mathbb{R}^{2(p-1) \times (p-3)}$:

$$\ker(A(\omega^*)^T) = \ker(B(\omega^*)) = 0.$$

(9)

Given this discriminant, a simple 1D search can be performed over $[0, 2\pi]$ to find $\omega^*$.

2) Computing $p^*$: Given $\omega^*$, (E2) can be solved by

$$z^* = [A(\omega^*)^T A(\omega^*)]^{-1} A(\omega^*)^T B(\omega^*).$$

(10)

Then the HV position $p^*$ can be computed by substituting (9) and (10) into (5) and (E1).

### C. Hidden Vehicle Sensing with Noise

In the presence of significant channel noise, the estimated AoAs/AoDs/ToAs contain errors. Consequently, HV-sensing is based on the noisy versions of matrix $A(\omega)$ and $B(\omega)$, denoted by $\tilde{A}(\omega)$ and $\tilde{B}(\omega)$, which do not satisfy the equations in (E2) and (9). To overcome the difficulty, we develop a sensing technique by converting the equations into minimization problems whose solutions are robust against noise.

1) Computing $\omega^*$: Based on (9), we formulate the following problem to find the orientation $\omega$:

$$\omega^* = \arg \min_{\omega} \|\tilde{A}(\omega)^* \tilde{A}(\omega)^T \tilde{B}(\omega)\|^2.$$

(11)

Solving the problem relies on a 1D search over $[0, 2\pi]$.

2) Computing $p^*$: Next, given $\omega^*$, the optimal $z^*$ can be derived by using the LS estimator that minimizes the squared Euclidean distance as

$$z^* = \arg \min_{z} \|\tilde{A}(\omega)^* z - \tilde{B}(\omega)^*\|^2 = [\tilde{A}(\omega)^* \tilde{A}(\omega)^T]^{-1} \tilde{A}(\omega)^* \tilde{B}(\omega)^*,$$

(12)

which has the same structure as (10). Last, the origins of all paths $\{(x_p, y_p)\}$ can be computed using the parameters $\{z^*, \omega^*\}$ as illustrated in (E1). Averaging these origins gives the estimate of the HV position $p^* = (x^*, y^*)$ with $x^* = \frac{1}{p} \sum_{p=1}^{P} x_p$ and $y^* = \frac{1}{p} \sum_{p=1}^{P} y_p$.

### D. Extension to 3D Propagation

Consider the scenario that propagation paths lie in the 3D Euclidean space instead of the 2D plane previously assumed. As shown in Fig. 3, the main differences from the 2D scenario are that the elevation angles are added to the AoAs, AoDs, and HV orientation. Specially, the AoA includes two angles: $\theta$ (azimuth) and $\psi$ (elevation) and AoD consists $\varphi$ (azimuth) and $\psi$ (elevation). The estimations of AoAs and AoDs in the 3D model can be jointly estimated via various approaches, e.g., MUSIC algorithm for 3D signal detection (see e.g., [36]). The HV orientation also includes two unknowns: $\omega$ (azimuth) and $\psi$ (elevation). The coordinates of HV, denoted by $p = (x, y, z)^T$, are given as

$$\begin{align*}
&x = \nu_p \sin(\varphi_p) \cos(\theta_p) - (d_p - \nu_p) \sin(\psi_p + \varphi_p) \\
y = \nu_p \sin(\varphi_p) \sin(\theta_p) - (d_p - \nu_p) \cos(\psi_p + \varphi_p) \\
z = \nu_p \cos(\varphi_p) - (d_p - \nu_p) \cos(\psi_p + \varphi_p),
\end{align*}$$

(13)

where $\forall p \in P$.

Similar to (E1), the following system of equations is constructed for 3D propagation:

$$\begin{align*}
&\nu_p \sin(\varphi_p) \cos(\theta_p) - (d_p - \nu_p) \sin(\psi_p + \varphi_p) = \nu_1 \sin(\varphi_1) \cos(\theta_1) - (d_1 - \nu_1) \sin(\psi_1 + \varphi_1) \\
&\nu_p \sin(\varphi_p) \sin(\theta_p) - (d_p - \nu_p) \cos(\psi_p + \varphi_p) = \nu_1 \sin(\varphi_1) \sin(\theta_1) - (d_1 - \nu_1) \cos(\psi_1 + \varphi_1) \\
&\nu_p \cos(\varphi_p) - (d_p - \nu_p) \cos(\psi_p + \varphi_p) = \nu_1 \cos(\varphi_1) - (d_1 - \nu_1) \cos(\psi_1 + \varphi_1),
\end{align*}$$

(E3)
where $\forall p \in P$.

It is shown that the number of equations and the number of unknown variables are $3(P - 1)$ and $(P + 3)$, respectively. For the HV-sensing problem to be solvable, we require $3(P - 1) \geq P + 3$, which leads to the following proposition.

**Proposition 3** (Sensing Feasibility Condition for 3D). Consider the 3D propagation model. To sense the position and orientation of HV provisioned with a single-cluster array, at least three NLoS signal paths are required, i.e., $P \geq 3$.

Compared with 2D propagation, the minimal number of required signal paths is reduced because extra information can be extracted from one additional dimension (i.e., elevation angles information of AoAs, AoDs) of each signal path. A similar methodology described in Sections [II-B] and [III-C] can be easily modified for 3D propagation by applying a 2D search based discriminant to find $\omega$ and $\vartheta$ over $[0, 2\pi]$ and $[0, \pi]$, respectively.

IV. SENSING HIDDEN VEHICLE WITH A MULTI-CLUSTER ARRAY

The preceding section targets the scenario that the HV is provisioned with a single-cluster array, allowing the SV to sense the HV position and orientation. In this section, we consider the scenario where a multi-cluster array is deployed at HV so that SV can sense HV’s array size (approximating the HV size) in addition to its position and orientation. Sensing techniques are designed separately for two cases, namely decoupled and coupled HV antenna clusters, in the following sub-sections.

A. Case 1: Decoupled HV Antenna Clusters

Consider the case of decoupled HV antenna clusters via transmission of orthogonal waveform sets over different clusters. As a result, the SV is capable of grouping detected paths according to their originating clusters. This simplifies the HV-sensing in the sequel by building on the techniques in the preceding section.

Recall that four HV antenna clusters are located at the vertices of a rectangle with length $L$ and width $W$ that represents the HV shape (see Fig. 4). The vertex locations are represented as $\{p^{(k)} = (x^{(k)}, y^{(k)})\}^{K}_{k=1}$. Different orthogonal waveform set is assigned to each cluster, allowing SV with prior knowledge on the waveform sets to differentiate the signals transmitted by different clusters. The more challenging case where all clusters are assigned an identical waveform set is studied in the next section. Let each path be ordered based on HV array index such that $P = \{P^{(1)}, P^{(2)}, P^{(3)}, P^{(4)}\}$, where $P^{(k)}$ represents the set of received signals from the $k$-th HV antenna cluster. Note that the vertices determine the HV size and their centroid that gives the HV position. Therefore, the sensing problem can be represented as

$$\bigcup_{k=1}^{4} \bigcup_{p \in P^{(k)}} \{\theta_{p}, \varphi_{p}, \lambda_{p}\} \Rightarrow \{\{p^{(k)}\}^{4}_{k=1, \omega}\}. \tag{14}$$

A naive sensing approach is to exploit the orthogonality of multiple waveform sets to decompose the sensing problem into separate positioning of HV antenna clusters using the technique designed in the preceding section. In the following subsection, we propose a more efficient sensing technique exploiting the prior knowledge of the HV clusters’ rectangular topology.

1) Sensing Feasibility Condition: Without loss of generality, assume that the received signal from the first HV antenna cluster, indexed by the set $P^{(1)}$, is not empty and $1 \in P^{(1)}$. Based on the rectangular configuration of $\{p^{(k)}\}^{4}_{k=1}$ (see Fig. 4), a system of equations is formed:

$$\begin{align*}
\nu_{p} \cos(\theta_{p}) - (d_{p} - \nu_{p}) \cos(\varphi_{p} + \omega) + \eta_{p}(\omega, L, W) \\
= \nu_{1} \cos(\theta_{1}) - (d_{1} - \nu_{1}) \cos(\varphi_{1} + \omega), \\
\nu_{p} \sin(\theta_{p}) - (d_{p} - \nu_{p}) \sin(\varphi_{p} + \omega) + \zeta_{p}(\omega, L, W) \\
= \nu_{1} \sin(\theta_{1}) - (d_{1} - \nu_{1}) \sin(\varphi_{1} + \omega),
\end{align*}$$

(14)

where $\eta_{p}(\omega, L, W) = 0, L \cdot \cos(\omega), L \cdot \cos(\omega) - W \cdot \sin(\omega)$, and $-W \cdot \sin(\omega)$ when $p \in P^{(1)}, p \in P^{(2)}, p \in P^{(3)}$, and $p \in P^{(4)}$, respectively. $\zeta_{p}(\omega, L, W)$ is obtained via replacing all cos and sin in $\eta_{p}(\omega, L, W)$ with sin and $-\cos$, respectively. The number of signal paths is given as $P = |P| = \sum_{k=1}^{4} |P^{(k)}|$. Compared with [E4], the number of equations in [E4] is the same as $2(P - 1)$ while the number of unknowns increases from $(P + 2)$ to $(P + 4)$ since $L$ and $W$ are also unknown. Consequently, [E4] has a unique solution when $2(P - 1) \geq P + 4$. 

![Figure 3: 3D propagation model.](image)

![Figure 4: Rectangular configuration of a 4-cluster HV array and the corresponding multi-path propagations.](image)
Proposition 4 (Sensing Feasibility Condition). Consider the scenario that the HV is provisioned with a 4-cluster array and orthogonal waveform sets are transmitted from different clusters. To sense the position, size, and orientation of the HV, at least six paths are required: \( P \geq 6 \).

Remark 3 (Advantage of Array-Topology Knowledge). The separate positioning of individual HV antenna clusters requires at least 16 NLoS paths (see Proposition 1). On the other hand, the proposed sensing technique reduces the number of required paths to only 6 by exploiting the prior knowledge of the rectangular configuration of antenna clusters.

2) Hidden Vehicle Sensing: Consider the case where channel noise is negligible. The system of equations in (E4) can be rewritten in a compact matrix form:

\[
\hat{A}(\omega)\hat{z} = B(\omega),\]

where \( \hat{z} = (v, d_1, L, W)\) \( \in \mathbb{R}^{(P+3)\times 1} \) with \( v \) following the index ordering of \( P \), and \( B(\omega) \) is given in (E3). The matrix \( \hat{A}(\omega) \) can be decomposed as

\[
\hat{A}(\omega) = \begin{bmatrix} A(\omega) & L(\omega) & W(\omega) \end{bmatrix} \in \mathbb{R}^{2(P-1)\times (P+3)},
\]

where \( A(\omega) \) follows (4). Moreover, \( L(\omega) \in \mathbb{R}^{2(P-1)\times 1} \) is given as \( [L(\cos)(\omega), L(\sin)(\omega)]^T \) with

\[
L(\cos)(\omega) = \begin{bmatrix} 0, \cdots, 0, -\cos(\omega), \cdots, -\cos(\omega), 0, \cdots, 0 \end{bmatrix}^T, \\
L(\sin)(\omega) = \begin{bmatrix} |P(1)| - 1, |P(2)| + |P(3)|, |P(4)| \end{bmatrix}^T,
\]

where \( |P(k)| \) counts the number of elements in \( P(k) \) and \( L(\sin)(\omega) \) is obtained by replacing all \( \cos(\omega) \) in \( L(\cos)(\omega) \) with \( \sin(\omega) \). \( W(\omega) \) can be written as \( [W(\sin)(\omega), W(\cos)(\omega)]^T \) where

\[
W(\sin)(\omega) = \begin{bmatrix} 0, \cdots, 0, \sin(\omega), \cdots, \sin(\omega) \end{bmatrix}^T, \\
W(\cos)(\omega) = \begin{bmatrix} |P(1)| + |P(2)| - 1, |P(3)| + |P(4)| \end{bmatrix}^T,
\]

and \( W(\cos)(\omega) \) is obtained by replacing all \( \sin \) in \( W(\sin)(\omega) \) with \( -\cos \).

1) Computing \( \omega^* \): Noting that (E5) is over-determined when \( P \geq 6 \), the resultant discriminant of the orientation \( \omega \) is similar to that in Proposition 2 and given as follows.

Proposition 5 (Discriminant of Orientation). With \( P \geq 6 \), the unique \( \omega^* \) exists when \( B(\omega^*) \) is orthogonal to the null column space of \( \hat{A}(\omega^*) \) denoted by \( \text{null}(\hat{A}(\omega^*))^T \in \mathbb{R}^{2(P-1)\times (P+1)} \):

\[
\text{null}(\hat{A}(\omega^*))^T \hat{B}(\omega^*) = 0.
\]

Given this discriminant, a simple 1D search can be performed over the range \([0, 2\pi]\) to find \( \omega^* \).

2) Computing \( \{p(k)\}_{k=1}^4 \): Given the \( \omega^* \), (E5) can be solved by

\[
\hat{z}^* = \left[ \hat{A}(\omega^*)^T \hat{A}(\omega^*) \right]^{-1} \hat{A}(\omega^*)^T \hat{B}(\omega^*).
\]

The positions of HV antenna clusters, say \( \{p(k)\}_{k=1}^4 \), can be computed by substituting (18) and (19) into (5) and (E4). Extending the above sensing technique to the case with channel noise is straightforward by modifying (18) to a minimization problem as in Section III-C.

B. Case 2: Coupled HV Antenna Clusters

It is desired to reduce the number of orthogonal waveform sets used by a HV so as to facilitate multi-access by dense HVS. Thus, in this section, we consider the resource-limited case of coupled HV antenna clusters where a identical waveform set is shared and transmitted by all HV antenna clusters. The design of HV-sensing is more challenging since the SV is incapable of grouping the signal paths according to their originating HV antenna clusters. For tractability, the objectives of HV-sensing for this scenario is redefined as: 1) positioning of the centroid of HV multi-cluster array denoted by \( p_0 = (x_0, y_0) \); 2) sensing the HV size by estimating the maximum distance between HV antenna clusters and \( p_0 \) denoted by \( R = \max_k |p(k) - p_0| \); 3) estimating the HV orientation \( \omega \). It follows that the sensing problem can be formulated as

\[
\bigcup_{p \in P} \{\theta_p, \varphi_p, \lambda_p\} \Rightarrow \{p_0, R, \omega\}. \tag{20}
\]

To solve the problem, we adopt the following two-step approach:

Step 1: By assuming that all signals received at SV originate from the same transmitting location, it is treated as the HV array centroid and estimated together with the orientation \( \omega \) using the technique in Section III.

Step 2: Given \( \omega \) and \( p_0 \), the size parameter \( R \) can be estimated by solving optimization problems based on bounding the HV array by either a disk or a box.

The techniques for Step 2 are designed in following subsections.

1) HV Size Sensing by Disk Minimization: Note that the HV array is outer bounded by a disk. Then the problem of estimating the HV size parameter \( R \) at SV can be translated into the optimization problem of minimizing the bounding-disk radius. As shown in Fig. 5(a), we define a sensing disk \( C(p_0, R) \) centered at the estimated centroid \( p_0 \) with the radius \( r \):

\[
C(p_0, R) = \left\{ (x, y) \mid (x-x_0)^2 + (y-y_0)^2 \leq r^2 \right\}. \tag{21}
\]

A constraint is applied that all HV antennas, or equivalently the origins of all signal paths received at the SV, should lie within the disk. Then estimating the HV size \( R \) can be translated into the following problem of disk minimization:

\[
R = \min_{d_1, \nu_p} r \;
\text{s.t.} \;
(x_p - x_0)^2 + (y_p - y_0)^2 \leq r^2, \\
0 < \nu_p < d_1 + c \rho_p, \\
(x_p, y_p) \text{ satisfies } \{3\} \text{ with } d_p = d_1 + c \rho_p, \forall p \in P, \tag{E6}
\]

where the first constraint is as mentioned above and the second represents the distance after the reflection \( \nu_p \) cannot exceed the total propagation distance \( d_p \) represented in terms of \( d_1 \) and TDoA \( \rho_p \) as \( d_p = d_1 + c \rho_p \) with \( \{\rho_p\} \) being the TDoAs [see (5)]. One can observe that Problem (E6) is a problem of second-order cone programming (SOCP). Thus, it is a convex optimization problem and can be efficiently solved numerically e.g., using the efficient MatLab toolbox such as CVX.
Analyzing the problem structure can shed light on the number of required paths for HV-sensing in the current scenario. The existence and uniqueness of the optimal solution \( r^* \) for Problem \( \text{E6} \) can be explained intuitively by considering the feasible range of the optimization variable \( d_1 \). Let \( \mathcal{S}_p(r) \) represent the feasible range of \( d_1 \) given the disk radius \( r \) by considering only path \( p \):

\[
\mathcal{S}_p(r) = \{ d_1 | \text{constraints in } \text{E6} \}. \tag{22}
\]

Then the feasible range of \( d_1 \), denoted by \( \mathcal{S}(r) \), can be written as \( \mathcal{S}(r) = \bigcap_{p \in P} \mathcal{S}_p(r) \). It is straightforward to show the following monotonicity of \( \mathcal{S}(r) : \mathcal{S}(r_1) \subseteq \mathcal{S}(r_2) \) if \( r_1 \leq r_2 \) with \( \mathcal{S}(0) = \emptyset \). Based on the monotonicity, there always exists an optimal and unique solution \( r^* \) for Problem \( \text{E6} \) such that \( \mathcal{S}(r) \neq \emptyset \) if \( r \geq r^* \) or otherwise \( \mathcal{S}(r) = \emptyset \). In other words,

\[
r^* = \inf \{ r > 0 | \mathcal{S}(r) \neq \emptyset \} = \sup \{ r > 0 | \mathcal{S}(r) = \emptyset \}. \tag{23}
\]

The value \( r^* \) corresponds to the critical case where there exist two feasible range sets \( \mathcal{S}_p(r^*) \) and \( \mathcal{S}_p'(r^*) \) each contact only other at their boundaries such that \( \mathcal{S}(r) \) contains a single feasible point \( d_1^* \) that corresponds to \( r^* \). This leads to the following proposition.

**Proposition 6** (HV Size Sensing by Disk Minimization). Given the solution \( r^* \) for Problem \( \text{E6} \), there always exist at least two paths, say \( p_1 \) and \( p_2 \), whose originating positions lie on the boundary of the minimized disk \( \mathcal{C}(p_0, r^*) \):

\[
(x_{p_1} - x_0)^2 + (y_{p_1} - y_0)^2 = (x_{p_2} - x_0)^2 + (y_{p_2} - y_0)^2 = (r^*)^2. \tag{24}
\]

Instead of the earlier intuitive argument, Proposition 6 can be proved rigorously using the Karush-Kuhn-Tucker (KKT) conditions as shown in Appendix A.

**Remark 4** (Feasible Condition of HV Sensing by Disk Minimization). Though only two paths are required to determine the optimal disk radius \( R = r^* \) based on Proposition 6, at least four paths are required for estimating the required centroid \( p_0 \) (see Proposition 3).

**Remark 5** (Extension to 3D Propagation). The extension to 3D propagation model in Section III-D is straightforward by using a sphere instead of a disk [see Fig. 5(b)]. The resultant sphere minimization problem has the same form as Problem \( \text{E6} \) except that the first constraint modified as \( (x_{p} - x_0)^2 + (y_{p} - y_0)^2 + (z_{p} - z_0)^2 \leq r^2, \quad \forall p \in P \), where the centroid \( p_0 = (x_0, y_0, z_0) \) is estimated using the technique in Section III-D. Again, the problem can be optimally solved since it still follows SOCP structure.

2) HV Size Sensing by Box Minimization: In the preceding subsection, the HV size is estimated by bounding the HV array by a disk and then minimizing it. In this sub-section, the disk is replaced by a box and the HV size estimation is translated into the problem of box minimization. Compared with disk minimization, the current technique improves the estimation accuracy since a vehicle typically has a rectangular shape. Let \( L \) and \( W \) be the length and width of the rectangular where the HV antenna clusters are placed at its vertices (see Fig. 5(c)). Then the problem of HV size sensing is to estimate both \( L \) and \( W \). Recall that the HV array centroid \( p_0 \) and orientation \( \omega \) are estimated in Step 1 of the proposed sensing approach as mentioned. Given \( p_0 \) and \( \omega \), we define a sensing box for bounding the HV array, denoted as \( B(p_0, \omega, \ell, w) \), as an \( \omega \)-rotated rectangle centered at \( p_0 = (x_0, y_0) \) and having the length \( \ell \) and width \( w \):

\[
B(p_0, \omega, \ell, w) = \left\{ (x, y) \Big| -\frac{1}{2} (\ell, w)^T \preceq R(\omega) \left[ x - x_0, y - y_0 \right]^T \leq \frac{1}{2} (\ell, w)^T \right\}, \tag{25}
\]

where \( R(\omega) \) is the counterclockwise rotation matrix with the rotation angle \( \omega \) given as

\[
R(\omega) = \begin{bmatrix}
\cos(\omega) & \sin(\omega) \\
-\sin(\omega) & \cos(\omega)
\end{bmatrix}, \tag{26}
\]

and \( \preceq \) represents an element-wise inequality. Like disk minimization in the previous subsection, finding the correct \( L \) and \( W \) is transformed into the following box minimization problem:

\[
\{ L, W \} = \arg \min_{d_1, (v_{p}), \ell, w} (\ell^2 + w^2) \quad \text{s.t.} \quad -\frac{1}{2} (\ell, w)^T \preceq R(\omega) \left[ x_{p} - x_0, y_{p} - y_0 \right]^T \preceq \frac{1}{2} (\ell, w)^T, \quad 0 \leq v_{p} < d_1 + c_{r_{p}}, \quad \forall p \in P, \tag{E7}
\]

where the first constraint represents that all origins of signal paths \( \{ (x_{p}, y_{p}) \} \) should be inside \( B(p_0, \omega, \ell, w) \) defined in (25) and the second one is the same as in \( \text{E6} \). Problem \( \text{E7} \) can be solved by quadratic programming (QP), which is a convex optimization problem and can be efficiently solved using a software toolbox such as MatLab CVX. A result similar to
that in Proposition 6 can be obtained for HV size sensing by box minimization as shown below.

**Proposition 7 (HV Size Sensing by Box Minimization).** Given the solution \( \{ \ell^*, w^* \} \) for Problem E7 there always exist at least two paths, say \( p_1 \) and \( p_2 \), whose originating positions lie on two different vertices of the minimized box:

\[
\mathbf{R}(\omega) \begin{bmatrix} x_{p_1} - x_{p_2} \\ y_{p_1} - y_{p_2} \end{bmatrix} = \begin{bmatrix} \ell^* \\ w^* \end{bmatrix} \text{ or } \begin{bmatrix} -\ell^* \\ w^* \end{bmatrix} \text{ or } \begin{bmatrix} \ell^* \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ w^* \end{bmatrix} .
\]

(27)

**Proof:** See Appendix B.

**Remark 6 (Feasible Condition of HV-Sensing by Box Minimization).** A similar remark as Remark 2 for disk minimization also applies to the current technique. Specifically, though two paths are required to determine the optimal box length \( L = \ell^* \) and width \( W = w^* \) based on Proposition 7 at least four paths are required for estimating the required HV centroid \( p_0 \) and orientation \( \omega \) (see Proposition 1).

**Remark 7 (Sensing Box Minimization for Decoupled Antenna Clusters).** The technique of HV size sensing by box minimization developed for the case of coupled HV antenna clusters can be also modified for use in the case of decoupled clusters. Roughly speaking, the modified technique involves separation minimization of four boxes corresponding to the positioning of four antenna clusters. As the modification is straightforward, the details are omitted for brevity. The resultant advantage with respect to the original sensing technique proposed in Section IV-A is to reduce the minimum number of required paths from 6 (see Proposition 1) to 4.

**Remark 8 (Extension to 3D Propagation).** Similar to Remark 8 for disk minimization, the technique of HV size sensing by box minimization originally designed for 2D propagation can be extended to 3D propagation model by using a cuboid instead of a box, yielding the problem of *cuboid minimization* as illustrated in Fig. 5(d). Compared with E7, the objective function of the cuboid minimization is \( \ell^2 + w^2 + h^2 \) where the new variable \( h \) is added to represent the height of the cuboid. In addition, the first constraint in E7 is modified as

\[
-\frac{1}{2} \begin{bmatrix} \ell \\ w \\ h \end{bmatrix} \leq \mathbf{R}_{3D}(\omega, \varrho) \begin{bmatrix} x_p - x_0 \\ y_p - y_0 \\ z_p - z_0 \end{bmatrix} \leq \frac{1}{2} \begin{bmatrix} \ell \\ w \\ h \end{bmatrix} , \quad \forall p \in \mathcal{P}
\]

(28)

where \( \mathbf{R}_{3D}(\omega, \varrho) \) is the 3D counterclockwise rotation matrix with the rotation angles \( \omega \) and \( \varrho \) as

\[
\mathbf{R}_{3D}(\omega, \varrho) = \begin{bmatrix} \cos(\omega) & -\sin(\omega) \cos(\varrho) & \sin(\omega) \sin(\varrho) \\ \sin(\omega) \cos(\varrho) & \cos(\omega) & -\cos(\omega) \sin(\varrho) \\ 0 & \sin(\varrho) & \cos(\varrho) \end{bmatrix},
\]

(29)

and the centroid \( p_0 = (x_0, y_0, z_0) \) can be obtained by the technique in Section II-D. The cuboid minimization is still QP and the solution approach is similar to that for the 2D counterpart.

**V. COPING WITH INSUFFICIENT MULTI-PATH**

The HV-sensing techniques designed in the preceding sections require at least four propagation paths to be effective. In practice, it is possible that the number of received paths may be insufficient, i.e., \( P < 4 \), due to either sparse scatterers or that most paths are severely attenuated by multiple reflections. To address this practical issue, two solutions are proposed in the following sub-sections, called sequential path combining and random directional beamforming. For simplicity, we focus on the case of single-cluster HV array while the extension to the case of multi-cluster array is straightforward.

**A. Sequential Path Combining**

As shown in Fig. 6 the technique of sequential path combining implemented at the SV merges paths from repeated transmissions of HV till a sufficient number of paths is identified for the purpose of subsequent HV-sensing. Let \( Q \) denote the number of HV’s \( Q \) repetitive transmissions with a constant interval denoted by \( \Delta \). The interval is chosen to be much larger than the delay-spread of each transmission, enabling SV to differentiate the arrival paths according to their transmission time instants. Let \( t_q \) and \( P_q \) denote the time instant of the \( q \)-th transmission and the corresponding set of detected paths, respectively. Assume that the relative orientation of driving direction and velocity of HV with respect to SV, namely \( \omega \) and \( v \), remain constant within the entire duration of \( Q \) intervals \( Q \Delta \). Then the following system of equations are formed:

\[
\begin{cases}
\nu_p \cos(\theta_1) - (d_p - \nu_p) \cos(\varphi_p + \omega) + v(q - 1) \cos(\omega) \\
\nu_p \sin(\theta_1) - (d_p - \nu_p) \sin(\varphi_p + \omega) + v(q - 1) \sin(\omega)
\end{cases}
\]

where \( p \in \mathcal{P}_q \) and \( q = 1, 2, \ldots, Q \). They can be solved following a similar procedure as in Section IV-A. Let \( P_{1:q} \) be the total number of paths identified due to the \( q \) transmissions, i.e., \( P_{1:q} = |\mathcal{P}_1| + |\mathcal{P}_2| + \cdots + |\mathcal{P}_q| \). Noting that the number of equations above is \( 2P_{1:q} - 1 \) and the number of unknowns are \( (P_{1:q} + 3) \) including \( \{\nu_p\}, d_p, \omega \) and \( v \). As a result, the condition for the SV collecting sufficient paths for HV sensing is \( 2P_{1:q} - 1 \leq (P_{1:q} + 3) \) or equivalently \( P_{1:q} \geq 5 \). So path combining over multiple sequential transmissions overcomes the practical limitation of insufficient paths.

**B. Random Directional Beamforming**

To further enhance the effectiveness of sequential path combining, a directional beam can be randomly steered at HV over sequential transmissions. Its purpose is to reveal some paths that are otherwise hidden to SV due to severe attenuation. The beam width can be set as ranging from \( 90^\circ \) to \( 30^\circ \) with gain ranging from \( 3 \) dB to \( 10 \) dB [37], which helps reach faraway scatterers by focusing the transmission power in their directions and thereby mitigating path loss [38]. Note that a single trial of randomly steered beam may not find enough paths. Thus, it is important to combine the technique with sequential path combining designed in the preceding
sub-section for the former to be effective. Their integrated operation is illustrated in Fig. 6 and its effectiveness is verified by simulation in the sequel.

VI. SIMULATION RESULTS

In this section, the performance of the proposed HV-sensing techniques are evaluated by realistic simulation. We adopt the geometry-based stochastic channel model proposed in [30] for modelling the practical scatterers distribution and V2V propagation channels, which has been validated by real measurement data. The two typical driving scenarios, highway and rural, are considered by following the settings in [30]. As illustrated in Fig. 7(a), in the highway scenario, the scatterers are vehicles randomly distributed on the same highway as the SV and HV. On the other hand, in the rural scenario illustrated in Fig. 7(b), the scatterers include random vehicles on the highway as well as relatively denser stationary objects off the highway (e.g., road signs and buildings). Note that the scatterers in the highway scenario are relatively sparse compared with those in rural scenario. We set the carrier frequency \( f_c = 5.9 \) GHz, the transmission bandwidth \( B_s = 100 \) MHz, \( M_s = M_t = 20 \), and the per-cluster transmission power is \( 23 \) dBm.  We consider the case of 4-cluster HV array with the antenna clusters following the rectangular configuration. The size of HV is \( L \times W = 3 \times 6 \) m² and distance between SV and HV is \( 50 \) m unless stated otherwise.

A. Hidden Vehicular Positioning

The metric for measuring positioning accuracy is defined as the average Euclidean squared distance of estimated positions of HV antenna clusters to their true locations: \( \frac{1}{4} \sum_{k=1}^{4} ||p_s^{(k)} - p^{(k)}||^2 \), named average positioning error. Note that the metric also indirectly measures the accuracy of estimated HV size and orientation that are determined by the clusters positions.

Fig. 8 shows the performance of the proposed HV-sensing techniques in the 2D propagation model. The curves of average positioning error versus varying number of NLoS paths and SV-HV distance are plotted in Fig. 8(a) and 8(b), respectively. For the case of decoupled HV antenna clusters using different orthogonal waveform sets, the LS estimator given in

Figure 6: Proposed solutions for coping with insufficient number of propagation paths.

Figure 7: Two typical driving scenarios considered in simulation.

Figure 8: The performance of the proposed HV-sensing techniques for varying number of paths and SV-HV distance.
the positioning accuracy for the 3D case is worse and the error gap between the highway and rural scenarios becomes larger. Due to relatively low scatterer height compared with the HV-SV distance, most elevation angles are around $\frac{\pi}{4}$, and the resultant positioning accuracy tends to be sensitive to noisy angle detection. In addition, angle detection for 3D propagation is less accurate on the highway due to the larger propagation loss.

C. Hidden Vehicular Size Sensing

In the preceding results, we assume decoupled HV antenna clusters by using multiple orthogonal waveform sets for transmission. Next, we consider the case of coupled clusters and evaluate the performance of the HV size sensing techniques developed in Section IV-B. To this end, we define the average sizing error as the difference between the diagonal length of the real rectangular configuration of HV array and the estimated HV sizes. Fig. 10 shows the performance of HV sizing via disk minimization and box minimization by plotting the average sizing error versus HV-SV distance. Compared with the exact size of HV (dash-dot line, $\sqrt{3^2 + 6^2} \approx 6.7$ m), both the sizes of sensing disk and sensing box are increasing with HV-SV distance due to the same reason discussed for Fig. 8b). Box minimization provides better sizing performance by exploiting the HV array configuration that is neglected by disk minimization.
in the case of isotropic transmission, at least the feasible condition of positioning is satisfied. For example, a sharper beam is used (Q increases and approaches to one as Fig. 11(a), it is observed that the sensing success probability time instants should be larger than sensing is feasible when the aggregate number of paths over case of random beamforming with sequential path combining, sensing success probability is defined as the probability that transmission and serves as the benchmark. The metric of 2 time instants. We consider the random beamforming with the beam-forming and combining, 2 time instants. Beamforming+combining, 4 time instants. Beamforming+combining, 8 time instants. Figure 11: Coping with insufficient number of multi-path by sequential path combining and random directional beamforming.

D. Coping with Insufficient Multi-Path

We evaluate the performance of the integrated solutions of sequential path combining and random directional beamforming both proposed in Section V in the case where scatterers are sparse and there are insufficient paths for HV sensing. The HV is equipped with a single-cluster array and transmits Q repetitive signals with the transmission interval δ = 0.2 (sec). We consider the random beamforming with the beam-width 2π/Q and Q time instants sequential combining with Q = {1, 2, 4, 8}. Note that Q = 1 corresponds to isotropic transmission and serves as the benchmark. The metric of sensing success probability is defined as the probability that the feasible condition of positioning is satisfied. For example, in the case of isotropic transmission, at least 4 paths should be detected at each time instant. On the other hand, in case of random beamforming with sequential path combining, sensing is feasible when the aggregate number of paths over Q time instants should be larger than 5 (see Section V-A). In Fig. 11(a), it is observed that the sensing success probability increases and approaches to one as Q increases. Moreover, Fig. 11(b) shows that the positioning accuracy is improved as sharper beam is used (Q increases) since the angle detections become more accurate.

VII. CONCLUDING REMARKS

This work presents the technologies for sensing a HV, namely its position, orientation, and size by relying on V2V transmission and exploiting multi-path-geometry. Different techniques have been designed to support tradeoffs between the sensing accuracy and varying practical requirements in terms of bandwidth, signalling complexity, and array configuration. We have also addressed practical issue of insufficient multi-path for HV-sensing via developing effective techniques exploiting path randomness in time domain. This work opens a new area of HV-sensing and points to many promising research topics on advanced HV-sensing such as velocity detection by estimating Doppler frequencies and simultaneous sensing of multiple HVs.

APPENDIX

A. Proof of Proposition 6

Using the KKT conditions, the optimal solution of Problem E6 should satisfy:

\[
\sum_{p \in P} \gamma_p \left[ 2 (x_p - x_0) \frac{\partial x_p}{\partial d_1} + 2 (y_p - y_0) \frac{\partial y_p}{\partial d_1} \right] = 0, \quad (30) \\
\gamma_p \left[ 2 (x_p - x_0) \frac{\partial x_p}{\partial \nu_p} + 2 (y_p - y_0) \frac{\partial y_p}{\partial \nu_p} \right] = 0, \quad \forall p \in P, \quad (31) \\
\gamma_p \left( (x_p - x_0)^2 + (y_p - y_0)^2 - \bar{r}^2 \right) = 0, \quad \forall p \in P, \quad (32)
\]

where \{ \gamma_p \} represents the Lagrangian multipliers of the first constraint in Problem E6. At least two \gamma_p should be strictly positive to satisfy (30) and (31). From (32), it is obvious to lead (24).

B. Proof of Proposition 7

Using KKT conditions, the optimal solution of Problem E7 should satisfy:

\[
\sum_{p \in P} \left[ \tilde{\gamma}_p \left( \cos(\omega) \frac{\partial x_p}{\partial d_1} + \sin(\omega) \frac{\partial x_p}{\partial d_1} \right) \right. \\
\left. + \tilde{\mu}_p \left( -\sin(\omega) \frac{\partial x_p}{\partial d_1} + \cos(\omega) \frac{\partial x_p}{\partial d_1} \right) \right] = 0, \quad (33) \\
\left[ \bar{\gamma}_p \left( \cos(\omega) \frac{\partial x_p}{\partial \nu_p} + \sin(\omega) \frac{\partial x_p}{\partial \nu_p} \right) \right. \\
\left. + \bar{\mu}_p \left( -\sin(\omega) \frac{\partial x_p}{\partial \nu_p} + \cos(\omega) \frac{\partial x_p}{\partial \nu_p} \right) \right] = 0, \quad (34)
\]

where \tilde{\gamma}_p = \gamma_p^{(+) - \gamma_p^{(-)}}, \tilde{\mu}_p = \mu_p^{(+) - \mu_p^{(-)}} with Lagrangian multipliers of the first constraint represented by \gamma_p^{(+)}, \gamma_p^{(-)}, \mu_p^{(+)}, \mu_p^{(-)}. The above Lagrangian multipliers are positive only when the corresponding equalities are satisfied. In other words, either \gamma_p^{(+)}, \mu_p^{(+)}, \gamma_p^{(-)}, \mu_p^{(-)} should be zero. Some observations are made. First, to satisfy (33) and (34) simultaneously, at least two origins should be located in the boundary. Next, it is shown in (34) such that if \tilde{\gamma}_p \neq 0 then
its counterpart multiplier \( \hat{\mu}_p \neq 0 \), implying that the origin located in the boundary should be on the vertex. Last, the origin located at the vertex is equivalent to \( \{27\} \).

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