Sparkplug ignition of mono- and bi-dispersed aluminum-air suspensions

K M Moiseeva¹, A Yu Krainov¹, D A Krainov¹,²
¹Tomsk State University, Tomsk 634050, Russia
²Tomsk Polytechnic University, Tomsk 634050, Russia
E-mail: Moiseeva_KM@t-sk.ru

Abstract. In the present paper we numerically investigate the problem of spark ignition of monodispersed and bi-dispersed aluminum suspension in the air. The aim of the research is to determine the critical ignition conditions for an aluminum suspension depending on the size and mass concentration of the particles and the fraction distribution. The mathematical formulation of the problem is determined by a system of equations written in a cylindrical coordinate system, consisting of the gas continuity equation, the momentum and energy conservation equations for the gas and particles, oxygen and particles mass balance equations, the particle number density equation and the gas state equation. To solve the problem numerically we use the algorithms of arbitrary discontinuity decay and disintegration of a discontinuity in a medium devoid of its own pressure. We conduct parametric study on the minimum spark energy which is required to ignite the mixture and provide the subsequent steady propagation of the flame in the aluminum-air suspension.

1. Introduction
The important issues of aluminum powder combustion as an additive in fuels are the burning rate of aluminum particles in the oxidant gas stream and the critical ignition conditions. A rather detailed review of existing experiments on the combustion of an aluminum-gas suspension is given in [1]. The study provides data on the experimentally determined burning rates of the aluminum powder suspension which are collected in accordance with the size, mass concentration of the aluminum particles and the experimental conditions.

In recent studies related to the numerical simulation of the aluminum suspension combustion, great attention is paid to the detailed kinetics of oxidation and to the combustion of a single aluminum particle. Another approach to simulate the combustion of an aluminum suspension is to use the empirical law of aluminum particles combustion, for example, from [2]. Applying of the combustion law from [2] requires setting of the chemical reaction constant rate, which can be determined by comparing the calculation results with the experiment. The use of the empirical combustion law makes it possible to concentrate the investigation on the influence of the aluminum particle parameters on the burning rate of the aluminum suspension and on the critical conditions of the ignition which provides the stable burning of the suspension. In the present paper we numerically investigate the problem of spark ignition of an aluminum-air suspension using the empirical law of aluminum particle combustion from [2] and the basis of physico-mathematical statements from [3, 4]. The aim of the study is to determine the critical conditions of the ignition for monodisperse and bi-dispersed aluminum suspensions depending on the
size and mass concentration of the particles and on the ratio between the fractions in bi-dispersed aluminum suspension.

2. Mathematical model
The mathematical model is based on the dual-velocity two-phase model of the reacting gas-dispersion medium [5] and includes the system of equations written in a cylindrical coordinates: the gas conservation equation, conservation of momentum and energy for gas and particles, oxygen and particle mass balance equations, the particle number density equation and the gas state equation. The right-hand parts in the equations which are responsible for the chemical interaction between gas and particles are determined through the radius of aluminum in the reacting particle. The summands correspond to inertial and thermal interaction are determined through the radius of the particle itself, consisting of aluminum and oxide layer. The radius of the particle and the radius of aluminum in the particle during the burnup are determined according to [4]. The assumptions relating to the characteristics of spark ignition were consistent with [3]. The physico-mathematical formulation of the problem under the made assumptions has the following form:

\[
\frac{\partial \rho_g}{\partial t} + \frac{\partial \rho_g u_g}{\partial r} = -r \sum_{i=1,2} G_i, \quad (1)
\]

\[
\frac{\partial r (\rho_g u_g)}{\partial t} + \frac{\partial r (\rho_g u_g^2 + p)}{\partial r} = p - r \sum_{i=1,2} [\tau_{\nu,i} - G_i u_g], \quad (2)
\]

\[
\frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i u_i}{\partial r} = r \sum_{i=1,2} \left[ \frac{G_i}{G_{G}} \left( c_i, T_g + u_g^2/2 \right) - u_{k,i} \tau_{\nu,i} r_{\nu,i} + \alpha_{k,i} n_{k,i} S_{k,i} \right] \left( T_{k,i} - T_g \right), \quad (3)
\]

\[
\frac{\partial r \rho_{02}}{\partial t} + \frac{\partial r \rho_{02} u_g}{\partial r} = \frac{\partial}{\partial r} \left( rD_g \rho_g (T_g) \frac{\partial \alpha_{02}}{\partial r} \right) - r \sum_{i=1,2} G_i, \quad (4)
\]

\[
\frac{\partial r \rho_{k,i}}{\partial t} + \frac{\partial r \rho_{k,i} u_{k,i}}{\partial r} = r G_i, \quad i = 1, 2, \quad (5)
\]

\[
\frac{\partial r (\rho_{k,i} u_{k,i})}{\partial t} + \frac{\partial r (\rho_{k,i} u_{k,i}^2)}{\partial r} = r \tau_{\nu,i} + r G_i u_g, \quad i = 1, 2, \quad (6)
\]

\[
\frac{\partial \rho_{k,i}}{\partial t} + \frac{\partial \rho_{k,i} u_{k,i}}{\partial r} = -r \alpha_{k,i} S_{k,i} n_{k,i} \left( T_{k,i} - T_g \right) + \frac{r G_i}{\alpha_i} - r G_i \left( c_i, T_g + u_g^2/2 \right) + r \tau_{\nu,i} u_{k,i}, \quad i = 1, 2, \quad (7)
\]

\[
\frac{\partial r n_{k,i}}{\partial t} + \frac{\partial r n_{k,i} u_{k,i}}{\partial r} = 0, \quad i = 1, 2, \quad (8)
\]

\[
p = \rho_g R_g T_g. \quad (9)
\]

The initial and the boundary conditions for the equation system (1)–(9):
The following notations are used in (1)–(13): \( t \) is time, \( r \) is radius coordinate, \( \rho_g \), \( \rho_{O_2} \) are the gas and oxygen partial density, \( \rho_k \) is the aluminum particle mass per unit volume, \( p \) is the pressure, \( \lambda \) is the thermal conductivity coefficient, \( \lambda = \lambda_m (T/T_a)^{2/7} \), \( D \) is the diffusion coefficient, \( D_g (T_g) = \lambda_g (T_g)/(c_p \rho_g) \), \( Q \) is the heat of the reaction on the surface, \( T \) is temperature, \( \eta \) is the absolute viscosity coefficient of the gas. \( \alpha_k = Nu_k \lambda_g/(2r_t) \) is the gas-particles heat exchange coefficient, \( \epsilon_g = p/(\rho_g (\gamma - 1)) \) is the gas internal energy, \( \gamma = c_p/c_v \) is the adiabatic exponent, \( \epsilon_k = c_k T_k \) is the particle internal energy, \( G \) is the rate of particle mass changing during the combustion, \( S_n = 4 \pi r_t^2 \) is the particle surface area, \( S_m = \pi r_t^2 \) is the particle cross-sectional area, \( \tau_n = n_k F_n \) is the friction force, \( F_n = C_r S_m \rho_g (u_g - u_t)\|u_g - u_t\|/2 \) is the interaction force of a single particle with the gas, \( C_r = 24(1 + 0.15 \operatorname{Re}^{0.682})/ \operatorname{Re} \) is the friction coefficient, \( \operatorname{Re} \) is the Reynolds number, \( Nu_k = 2 + (Nu^2_g + Nu^2_r)^{1/2} \) is the Nusselt number, where \( Nu_k = 0.664 \operatorname{Re}^{0.8} \), \( Nu_r = 0.037 \operatorname{Re}^{0.8} \) [4], \( \alpha_k \) is the oxygen stoichiometric coefficient in reaction with aluminum particles. Indexes: \( b \) is the initial values of the parameters, \( k \) is the particle parameters, \( g \) is the gas parameters, \( Al \) is the aluminum parameters inside the oxide layer, \( i \) is the number of the fraction (for monodispersed suspension \( i \equiv 1 \), for the bispersed 1 is for the large particles, 2 small particles). The current radius of the unburnt aluminum \( r_{Al} \) and the radius of the whole particle \( r_k \) are defined as in [4]:

\[
T_g(r,t) = T_0 + \frac{Q_g}{4\pi r^2} \exp \left( -\frac{r^2}{4\lambda_g t} \right),
T_{k,i}(r,t) = T_0, \quad \rho_{O_2}(r,t) = \rho_{O_2,b},
\rho_{k,i}(r,t) = \rho_{k,b}, u_g(r,t) = u_{k,i}(r,t) = u_{k,b}, \quad \rho_g(r,t) = \rho_{g,b}, \quad n_{k,i}(r,t) = n_{k,b},
\]

\[
\frac{\partial \rho(0,t)}{\partial r} = \frac{\partial \rho_{O_2}(0,t)}{\partial r} = \frac{\partial T_g(0,t)}{\partial r} = 0, \quad u_g(0,t) = 0,
\]

\[
\frac{\partial n_{k,i}(0,t)}{\partial r} = \frac{\partial T_{k,i}(0,t)}{\partial r} = 0, \quad u_{k,i}(0,t) = 0,
\]

\[
\frac{\partial \rho_{O_2}(\infty,t)}{\partial r} = \frac{\partial T_{k}(\infty,t)}{\partial r} = 0.
\]

The current radius of the unburnt aluminum \( r_{Al} \) and the radius of the whole particle \( r_k \) are defined as in [4]:

\[
r_{Al,i,j} = \left[ \left( \frac{\mu_{Al} + 3/2 \mu_{O}}{\mu_{Al}} \right) r_{Al,i,j}^3 - \frac{\rho_{k,i}}{(4/3) \pi n_{k,i} \rho_{k,i}^6} \right]^{1/3},
\]

\[
r_{k,i} = \sqrt{\frac{3 \rho_{k,i}}{4\pi \rho_{k,i}^6}}.
\]
The rate of the particle mass change accounting the diffusional resistance of the reaction has the following form:

\[ G_i = \alpha_i n_i \rho_i S_{d_i,j} \frac{k(a_{o2, r_{d,i}}) \beta_{k,i}}{\left(k(a_{o2, r_{d,i}}) + \beta_{k,i}\right)}, \]  

where \( k(a_{o2, r_{d}}) = \frac{k_0 a_{o2}}{\sqrt{r_{d}}} \) is the combustion law of a single particle in the oxidant-rich environment, \( S_{d,i} = 4\pi r_{d,i}^2 \) is the area of the unburnt aluminum in the particle, \( k_0 \) is the constant of the chemical reaction rate, \( \beta_m = \lambda_g (T) \nu D / \left(c_s \rho_k \right) \) is the particle mass-transfer coefficient.

3. Results and discussion

The system of equation was solved numerically using the methods [6, 7]. The solution methodology and the computational mesh were corresponded to [3]. The initial radius of the aluminum particles and the initial mass concentration of the powder per unit volume were varied in the calculations. The minimum spark energy required for ignition and further propagation of the combustion front in the suspension was determined from the numerical investigation.

A series of calculations were performed to select the rate constant of the chemical reaction \( k_0 \) in the combustion law (16) before conducting the parametric investigation on the critical conditions for the ignition of aluminum suspension. The rate constant was chosen to provide the normal burning rate of the monodisperse aluminum powder corresponded to the experimental data [1]. According to the data [1], the dependence of the normal burning rate on the mass concentration of aluminum powder for particles with an average diameter \( d_{av} = 4 - 6 \mu m \) has the form shown in figure 1. A suspension with particle diameter of 4 \( \mu m \) was investigated in our study. The results of the calculations for the constant rate \( k_0 = 1.11 \times 10^{-6} \text{m}^{1.5}/\text{s} \) are shown in figure 2. There is fairly good agreement between obtained results and the results from the monograph [1] for the mass concentration of aluminum powder 0.1 – 0.3 kg/m\(^3\) (figures 1, 2). The further calculations were performed for the selected value of the constant \( k_0 \).

Figure 3 shows the dependence of the minimum spark ignition energy of monodisperse aluminum suspension with particle mass concentration of 0.3 kg/m\(^3\) on the initial particle size. The dependence of the minimum spark ignition energy for a suspension with the particle radius of 0.5 \( \mu m \) on the aluminum mass concentration of the mixture is presented in figure 4. An increase in the particle size leads to an increase in the minimum energy which is required to ignite the mixture and for subsequent propagation of the combustion front in the aluminum-air suspension. For a particle radius less than 0.5 \( \mu m \) the minimum energy of the spark ignition converges to the same value.
We have conducted the same investigation on the minimum spark-ignition energy of a bi-disperse aluminum-air suspension. The results of the calculation are shown in figure 5. The figure shows the dependence of the minimum spark ignition energy of the bi-disperse suspension on the percentage of large particles in it. It was assumed that the radius of the second fraction was 0.5 µm, the radius of the first fraction was 1 µm. The calculation was performed for the total mass concentrations of the particles $m_{dust} = \sum_{i=1,2} \rho_{kb,i} = 0.2 \text{ kg} / \text{ m}^3$ (dashed line) and $m_{dust} = \sum_{i=1,2} \rho_{kb,i} = 0.3 \text{ kg} / \text{ m}^3$ (solid line). The coordinate $x=0$ corresponds to a monodisperse aluminum-air suspension with particle radiuses 0.5 µm, $x=1$ corresponds to a monodisperse suspension with particle radiuses 1 µm.
According to figure 5 with an increase in the proportion of large particles in the mixture there is an increase in the minimum energy of spark ignition. An increase in the total mass concentration of aluminum powder in the mixture leads to a shortening of the minimum energy range depending on the percentage of the large particles in the mixture.

![Figure 5](image)

**Figure 5.** The dependence of the minimum spark ignition energy of the bi-dispersed suspension on the percentage of large particles.

4. Conclusions

We have numerically investigated the effect of the aluminum particle size on the minimum spark ignition energy of the aluminum-air suspension. The obtained result has shown that an increase in the aluminum mass concentration in the mixture leads to a decrease in the minimum energy of the spark ignition. For an aluminum-air suspension with particle radiuses less than 0.5 µm the minimum energy of the spark ignition converges to the same value with increasing mass concentration of aluminum. For a bi-dispersed suspension we have obtained the dependence of the minimum spark ignition energy on the percentage of large particles in it.

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