Theoretical modeling and experimental research on the depth of radial material removal for flexible grinding

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Abstract
As a difficult-to-cut metallic material with excellent characteristics, titanium alloy has been widely used in the field of aero-engines. In this study, a series of grinding experiments of titanium alloy was carried out with the abrasive belt flap wheel. The effects of spindle speed \( n \), feed speed \( V_w \), workpiece curvature radius \( R_2 \), and radial theoretical grinding depth \( a_e \) on the material removal depth are investigated. Meanwhile, a novel approach to determine the depth of radial material removal was established based on the Hertz elastic contact theory, Preston equation, and the principle of equivalent material removal volume. The results obtained show that the radial material removal depth increases with the increase of the radial theoretical grinding depth and decreases with the increase of the workpiece curvature radius and the spindle speed, and the feed speed has almost no effect on it. Finally, the measured value is compared with the calculated value, and the absolute error is mostly less than 10\%, which verifies the reliability and accuracy of the prediction model.

Keywords · Titanium alloy · Flexible grinding · Depth of radial material removal · Prediction model · Experiment analysis

1 Introduction
The blade is an important part of aero-engine; its surface integrity and profile accuracy will directly affect the performance of the aero-engine. The blade with poor surface quality is prone to fatigue failure, deformation, or fracture in an environment of high temperature, high pressure, and high speed, which leads to performance degradation, short service life, and even damage of aero-engine [1]. Now, multi-axis CNC milling machining methods are widely applied to blades of aero-engine in domestic and overseas, but there will be obvious residual milling knife marks on the surface of blades. Therefore, it is necessary to use grinding technology to remove residual milling knife marks, which improves the surface quality of blades [2]. At present, the grinding technology of blades is still mainly manual grinding. However, manual grinding has disadvantages such as low efficiency, poor consistency of machining accuracy, and high labor intensity. So, it is very important to develop new grinding technology in the field of aero-engine blade machining.

Zhang et al. [3] proposed a collision-free planning algorithm for the robot’s ground path, and the amount of collision detection of this algorithm is 3.86\% less than that of a complete collision layer. Ding et al. [4] based on the surface normal vector to adjust the speed of electric spindle in real time to ensure the uniformity of surface material removal rate at different normal vectors. Tian et al. [5] established a polishing pressure control model based on the relationship between removal rate and polishing pressure and realized the constant pressure polishing on the curved surface. Xiao and Huang [6] established a model of life cycle material removal that can effectively predict the material removal of titanium alloy materials during the belt machining process. Zou et al. [7] proposed an adaptive floating method to reduce the accuracy error of robotic abrasive belt grinding. This method solved the problem of machining accuracy under multiple weak stiffness systems. The above researches contribute greatly to
improve the machining accuracy of aero-engine blades, but the robot has low rigidity and large machining accuracy errors, which is not suitable for machining high-precision parts. In addition, the abrasive belt is easy to interfere with the blade due to the large size of the grinding head and is not suitable for machining the blisk.

Hung et al. [8] conducted grinding experiments on Ti-6Al-4V with a CBN wheel at different grinding parameters. The results showed that the feed rate, radial grinding depth, and cooling conditions had significant effects on the surface roughness, and wet grinding can achieve better surface quality. Shi et al. [9] developed a new type of grinding wheel to improve the thermal conductivity during the grinding of titanium alloys. Dai et al. [10] analyzed the grinding performance from the perspective of undeformed chip thickness. A novel approach of single grain was proposed at m/s and nanoscale depth of cut, and the scratching speeds were three to six orders magnitude higher than those used in nanoscratching [11]. This new approach opened a new pathway to investigate the fundamental mechanisms of abrasive machining. Force, stress, depth of cut, and damaged plastic size were calculated on both onsets of grinding and brittle-to-ductile transition [12]. A novel model on the maximum undeformed chip thickness was built, and series formulas were deduced. After grinding with silicon [13], cadmium zinc telluride, and mercury cadmium telluride [14, 15], the proposed model and deduced formulas were verified, and the results between theoretical and experimental results were in good agreement with each other. With the guidance of theories and models, novel diamond wheels and approaches were developed [16–18]. These works contribute greatly to grinding, as well as abrasive machining. However, the rigid grinding wheel cannot fit the surface of the blade well, and it is easy to cause “overgrinding” and “undergrinding,” resulting in poor surface quality and contour accuracy.

As a new type of grinding method, the abrasive belt flap wheel grinding has the characteristics of simple structure, low material removal rate, and small volume. It can be used for machining narrow inlet and outlet channels of blisk on five-axis CNC machine tools [19]. In addition, the abrasive belt flap wheel has good ductility when rotating and can be closely attached to the surface of the blade to ensure the consistency of the blade profile. Huai et al. [20, 21] determined the influence degree of various processing parameters. A novel predictive model on the surface roughness ratio was built and determined the adaptive flexible polishing path of the blade. After grinding with TC4, the proposed model and polishing path were verified. Zhang et al. [22] established a roughness prediction model based on experimental results and analyzed the interactive influence of process parameters on surface roughness using response surface methodology. The above works contributed greatly to the grinding processing parameters of the abrasive belt flap wheel, but they are all studied with the grinding force as the starting point. The noise generated by the machine tool in the grinding will affect the collection of the grinding force signal, resulting in greater results error. Therefore, this paper studies the actual radial grinding depth based on the material removal rate. First, the material removal rate model is built based on Hertz elastic contact theory [23–25] and Preston equation [26]. Then, according to the principle of equivalent material removal volume, a new model on the depth of radial material removal is built. Finally, the experiments for both plane and curved surfaces are implemented on a vertical machining center. The results indicate that the prediction model can better predict the depth of radial material removal.

2 Depth of radial material removal model

2.1 Grinding mechanism of the abrasive belt flap wheel

The abrasive belt flap wheel is a flexible abrasive tool that bonds the abrasive belt flap to the substrate. It has the characteristics of abrasive belt grinding and flexible grinding. Meanwhile, as the speed increases, the radius of the abrasive belt flap wheel increases under the action of the centripetal force, as shown in Fig. 1. In the grinding process, the grinding force is produced by the contact between the abrasive belt flap wheel and the surface of the workpiece, and the size of the grinding force directly affects the depth of radial material removal [27]. After the specification of the abrasive belt flap wheel is determined, the size of the grinding force can be controlled by the deformation of the abrasive belt flap wheel. The material removal of the abrasive belt flap wheel is very small so the radial theoretical grinding depth $a_e$ can be approximated to the maximum elastic deformation $\delta_{max}$ of the abrasive belt flap wheel that $\delta_{max} = a_e$.

The surface of the aero-engine blade is a complex curved surface with great curvature variation, while the abrasive belt flap wheel with significant flexibility, which can fit the blade surface well, thereby increasing the contact area and improving the grinding efficiency. Therefore, the contact of the abrasive belt flap wheel and the blade can be approximated by the elastic contact between the elastic cylinder and the rigid cylinder. Based on the Hertz elastic contact theory, the relationship between normal grinding force and contact width is shown in Eq. (1).

$$
F_n = \frac{\pi E_{wb} b a^2}{4 R_e}
$$

$$
E_{wb} = \left(\frac{1-v_w^2}{E_w} + \frac{1-v_b^2}{E_b}\right)^{-1}
$$

$$
R_e = \frac{1}{R_1} + \frac{1}{R_2}
$$

where $b$ is the contact length, $a$ is the half-contact width, $v_w$ is Poisson’s ratio of the workpiece, $E_w$ is the elastic modulus of the
workpiece, $v_b$ is Poisson’s ratio of abrasive belt flap wheel, $E_b$ is the elastic modulus of abrasive belt flap wheel, $R_1$ is the radius of abrasive belt flap wheel, and $R_2$ is the curvature radius of the workpiece at the contact point.

2.2 Average grinding pressure

The material removal model with the abrasive belt flap wheel is established based on the Preston equation, as shown in Eq. (2). Equations (2) and (3) reveal that the material removal is mainly affected by contact pressure and relative motion velocity. The contact pressure is mainly generated by grinding force, and the radial theoretical grinding depth is the main factor of grinding force. Therefore, in the abrasive belt flap wheel grinding, maintaining a constant radial theoretical grinding depth is important for the stability of the material removal rate.

$$\frac{dh}{dt} = KPV$$  \hspace{1cm} (2)

$$P = \frac{F_n}{A}$$  \hspace{1cm} (3)

where $h$ is the thickness of material removal, $K$ is the Preston coefficient, determined by the material properties of abrasive belt flap wheel and the material properties of the workpiece, $V$ is the relative motion velocity of abrasive belt flap wheel and workpiece, and $A$ is the contact area.

During the machining process, the material removal rate of the single contact zone between the abrasive belt flap wheel and the workpiece exhibits a parabolic distribution. For the convenience of research, the average material removal rate of a single contact zone is taken as the material removal rate of the abrasive belt flap wheel, as shown in Eq. (4).

$$H_m = KP_mV_m$$  \hspace{1cm} (4)

where $P_m$ is the average grinding pressure in the contact zone and $V_m$ is the average relative motion velocity.

Figure 2a shows the contact between the abrasive belt flap wheel and the convex surface of the workpiece. Set the point $M_1$ coordinate is $(x_{m1}, y_{m1})$, it can be seen from Fig. 2a that the point $O_1$ coordinate is $(0, R_1 - \delta_{\text{max}})$; the point $O_2$ coordinate is $(0, -R_2)$, from the point $M_1$ on both the surface of the abrasive belt flap wheel and the surface of the workpiece, as shown in Eq. (5).

$$\begin{cases} x_{m1}^2 + (y_{m1} - R_1 + \delta_{\text{max}})^2 = R_1^2 \\ x_{m1}^2 + (y_{m1} + R_2)^2 = R_2^2 \end{cases}$$  \hspace{1cm} (5)

where $\delta_{\text{max}}$ is the maximum deformation of the abrasive belt flap wheel.

It can be seen from Fig. 2 that point $M_1$ is on the left side of the coordinate system, and the value of $x_{m1}$ can be obtained from Eqs. (3) and (5).
Fig. 2 Contact pressure between abrasive belt flap wheel and workpiece

\[
x_{m1} = -\left( \frac{2R_1 \delta_{\text{max}} - \delta_{\text{max}}^2}{2\delta_{\text{max}} - 2R_1 - 2R_2} \right)^2 - \frac{2R_1 R_2 \delta_{\text{max}} - R_2 \delta_{\text{max}}^2}{\delta_{\text{max}} - 2R_1 - 2R_2} \]

(6)

Contact half-width of the convex surface is:

\[
a = |x_m| = \left[ \frac{2R_1 \delta_{\text{max}} - \delta_{\text{max}}^2}{2\delta_{\text{max}} - 2R_1 - 2R_2} \right]^2 - \frac{2R_1 R_2 \delta_{\text{max}} - R_2 \delta_{\text{max}}^2}{\delta_{\text{max}} - 2R_1 - 2R_2} \]

(7)

According to Eqs. (1), (3), and (7), \( P_{m1} \) can be written as:

\[
P_{m1} = \frac{\pi E_{wb}}{8R_e} \left[ \frac{2R_1 \delta_{\text{max}} - \delta_{\text{max}}^2}{2\delta_{\text{max}} - 2R_1 - 2R_2} \right]^2 - \frac{2R_1 R_2 \delta_{\text{max}} - R_2 \delta_{\text{max}}^2}{\delta_{\text{max}} - 2R_1 - 2R_2} \]

(8)

Figure 2b shows the contact between the abrasive belt flap wheel and the concave surface of the workpiece. Similar to the solution of convex surface contact, the average pressure of the contact between the abrasive belt flap wheel and the concave surface can be expressed as:

\[
P_{m2} = \frac{\pi E_{wb}}{8R_e} \left[ \frac{2R_1 \delta_{\text{max}} - \delta_{\text{max}}^2}{2\delta_{\text{max}} - 2R_1 + 2R_2} \right]^2 + \frac{2R_1 R_2 \delta_{\text{max}} - R_2 \delta_{\text{max}}^2}{\delta_{\text{max}} - R_1 + R_2} \]

(9)

Figure 3 shows the variation law of pressure in the contact zone and the deformation of the abrasive belt flap wheel. Within the deformation range of the abrasive belt flap wheel, the contact pressure increases with the increase of the deformation, and the contact area increases with the increase of the deformation; this slows down the growth trend. This result is consistent with the actual situation.

2.3 Relative grinding speed

The abrasive belt flap wheel has large flexibility, and the radius of idle rotation is needed to reduce the experimental error before grinding. In the grinding process, the abrasive belt flap wheel will elastically deform under the action of the grinding force, thereby generating a certain effect on the linear velocity of each point at the contact zone, as shown in Eq. (10).

\[
V_e = 2\pi n (R_1 - \delta)
\]

(10)

where \( V_e \) is the linear velocity of the contact point of the abrasive belt flap wheel, \( n \) is the speed of the abrasive belt flap wheel, and \( \delta \) is the deformation of the abrasive belt flap wheel, respectively, \( 0 \leq \delta \leq a_e \). The grinding feed rate \( V_w \) is much less than the linear velocity of the abrasive belt flap wheel so the linear velocity of the contact point of the abrasive belt flap wheel can be approximated as the relative motion velocity.

When the abrasive belt flap wheel contacts the surface of the workpiece, the projection of the contact area on the XOZ plane is rectangular, as shown in Fig. 4a.

Figure 4b and c respectively show the analysis of grinding speed when the abrasive belt flap wheel contacts the surface of the convex and concave. It is assumed that point A is any point on the contact zone; the velocity equations of point A on the surface of convex and concave can be obtained according to the geometric relationship in Fig. Figure 4b and c, as shown in Eqs. (11) and (12).

\[
V_1 = 2\pi n \left[ R_1^2 + (R_1 + R_2 - \delta_{\text{max}})^2 \right] \left\{ 2(R_1 + R_2 - \delta_{\text{max}}) \sqrt{R_2^2 - x^2} \right\}^{1/2}
\]

(11)
Therefore, the average relative motion velocity of convex surface contact and concave surface contact are respectively shown in Eqs. (13) and (14).

\[
V_1 = 2\pi n \left[ R_2^2 + (R_2 + \delta_{\text{max}} - R_1)^2 \right] - 2(R_2 + \delta_{\text{max}} - R_1) \sqrt{R_2^2 - x^2} \right) \frac{1}{2} \right.
\]

\[
V_{1m} = \frac{\pi n}{a} \int_{-a}^{a} \left[ R_2^2 + (R_1 + R_2 - \delta_{\text{max}})^2 \right] - 2(R_1 + R_2 - \delta_{\text{max}}) \sqrt{R_2^2 - x^2} \right) \frac{1}{2} dx
\]

Material removal rate can be calculated from Eqs. (4), (8), and (13) as:

\[
H_{mi} = \frac{K\pi^2 nE_w b}{8R_e} \int_{-a}^{a} \left[ R_2^2 + (R_1 + R_2 - \delta_{\text{max}})^2 \right] - 2(R_1 + R_2 - \delta_{\text{max}}) \sqrt{R_2^2 - x^2} \right) \frac{1}{2} dx
\]

Fig. 4 Relative motion velocity between abrasive belt flap wheel and workpiece
The total volume of material removed can be written as:

\[ H_v = H_m b t dL \]  

where \( H_v \) is the material removal volume, \( t \) is the contact time, and \( dL \) is the unit distance of the abrasive belt flap wheel moving on the workpiece.

According to the principle of equal material removal volume of the abrasive belt flap wheel during processing, it is combined with the traditional material removal volume formula, \( H_v = a_p b V_t \). The relationship between the depth of radial material removal and the material removal rate is obtained, as shown in Eq. (18).

\[ a_p = \frac{H_m}{V} dL \]  

where \( a_p \) is the depth of radial material removal.

### 3 Verification experiments

#### 3.1 Experimental platform

The grinding material removal rate experiments are carried out without cooling liquid on the three-axis vertical machining center VMP-40A, which has a maximum spindle speed of 10,000 rpm. The workpiece materials are the block of TC4 and cylinders of TC4 with different radii; the clamping methods are shown in Figs. 5a and b. The grinding parameters of the experiments are shown in Table 1. To reduce the error of the elastic deformation caused by the speed of the abrasive belt flap wheel in the experiment, it is necessary to measure the relationship between the radius of the abrasive belt flap wheel and the speed before the experiment, as shown in Fig. 6.

#### 3.2 Experimental methods

The orthogonal experimental method was used in this study, and 29 grinding experiments were conducted in total, including 4 groups of Preston coefficient solving experiments, 9 groups of TC4 block experiments, and 16 groups of cylindrical surface experiments. The curvature radius of the workpiece, the spindle speed, the feeding speed, the depth of radial theoretical grinding, and other parameters were used to analyze the influence on the material removal depth of titanium alloy during the abrasive belt flap wheel grinding process. Because the material removal rate of the abrasive belt flap wheel was very small, it is difficult to measure with traditional measurement methods, and thus, the depth of material removal is calculated by measuring the contour value of each workpiece before and after the experiment by the WENZEL coordinate measuring machine. The coefficient \( K \) of the Preston equation in the theoretical model is solved through the experiment. The experimental parameters are shown in Table 2. The average Preston equation coefficient is \( K = 6.633 \times 10^{-5} \).

#### 3.3 Analysis of results

To test the depth of material removal equation’s statistical significance for planes and cylinders in the abrasive belt flap wheel grinding, the F-testing method was used to test the prediction model. The sum of squared deviations \( S_s \) is the sum of squares of the difference between the actual measured value and the total average value. For statistical tests, the sum of squared deviations needs to be decomposed, which can be divided into two parts: the sum of regression squares \( S_r \) and the sum of residual squares \( S_e \).
\[ S_s = S_1 + S_2 \]  \hspace{1cm} (19)

\[ S_1 = \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2 \]  \hspace{1cm} (20)

\[ S_2 = \sum_{i=1}^{n} \left( \bar{x}_i - \bar{x} \right)^2 \]  \hspace{1cm} (21)

where \( x_i \) is the measured depth of material removal of each group, \( \bar{x} \) is the average depth of material removal of all groups measured, and \( \bar{x}_i \) is the predicted values of each group of experimental parameters.

By adopting the F-testing method, the following is obtained:

\[ F = \frac{S_1 (n-q-1)}{S_2 q} \sim F(q, n-q-1) \]  \hspace{1cm} (22)

where \( n \) is the number of experimental groups and \( q \) is the number of variables. According to Eq. (13), the F-testing results of plane grinding and cylindrical grinding were \( F_p = 7.233 \) and \( F_c = 13.654 \), when the test significance level is \( \alpha = 0.05 \), \( F_{0.05} (3, 5) = 6.59 \) and \( F_{0.05} (4, 11) = 3.36 \) in the look-up table, which are smaller than the calculated value. Thus, the equation is significant.

To reduce the measurement error of the plane and cylindrical, five points are randomly selected on the surface of the workpiece for measurement, and the average value is
calculated, as shown in Tables 3 and 4. As shown in Fig. 7a and b, most of the absolute prediction errors are within 10%. The average absolute prediction errors of planes and cylinders are 6.54% and 7.42%, and the maximum absolute prediction errors are 16.29% and 10.58%. The depth of radial material removal model had high reliability for both plane workpieces and cylindrical workpieces were proved.

Figure 8a and b are the main effect analysis diagrams of plane grinding and cylindrical grinding.

Radial theoretical grinding depth is the greatest influence on plane grinding and cylindrical grinding. The depth of the groove is mainly related to the grinding force has been proposed by Wang et al. [28], while the radial theoretical grinding depth mainly affects the grinding force. The curvature radius of the workpiece has a greater influence on the depth of radial material removal for cylindrical grinding, and the depth of radial material removal decreases with the increase of the curvature radius of the workpiece.

### Table 2 Preston coefficient K solution experiment

| No. | n (r/min) | \( V_w \) (mm/min) | \( a_e \) (mm) | \( a_p \) (μm) | K       |
|-----|-----------|---------------------|---------------|--------------|----------|
| 1   | 6000      | 100                 | 0.8           | 0.298        | 6.57×10⁻⁵ |
| 2   | 6000      | 100                 | 1.0           | 0.338        | 6.65×10⁻⁵ |
| 3   | 8000      | 100                 | 0.8           | 0.301        | 6.74×10⁻⁵ |
| 4   | 8000      | 100                 | 1.0           | 0.329        | 6.57×10⁻⁵ |

### Table 3 TC4 plane experimental combinations and results

| No. | n (r/min) | \( V_w \) (mm/min) | \( a_e \) (mm) | Measure \( a_p \) (μm) | Predictive \( a_p \) (μm) | Error (%) |
|-----|-----------|---------------------|---------------|------------------------|--------------------------|----------|
| 1   | 6000      | 100                 | 0.6           | 0.237                  | 0.26                     | -9.7     |
| 2   | 6000      | 150                 | 0.8           | 0.281                  | 0.301                    | -7.12    |
| 3   | 6000      | 200                 | 1.0           | 0.352                  | 0.337                    | 4.26     |
| 4   | 7500      | 100                 | 0.8           | 0.305                  | 0.298                    | 2.3      |
| 5   | 7500      | 150                 | 1.0           | 0.324                  | 0.334                    | -3.09    |
| 6   | 7500      | 200                 | 0.6           | 0.221                  | 0.257                    | -16.29   |
| 7   | 9000      | 100                 | 1.0           | 0.306                  | 0.33                     | -7.84    |
| 8   | 9000      | 150                 | 0.6           | 0.245                  | 0.254                    | -3.67    |
| 9   | 9000      | 200                 | 0.8           | 0.281                  | 0.294                    | -4.63    |

### Table 4 TC4 cylinder experimental combinations and results

| No. | \( R_2 \) (mm) | n (r/min) | \( V_w \) (mm/min) | \( a_e \) (mm) | Measure \( a_p \) (μm) | Predictive \( a_p \) (μm) | Error (%) |
|-----|---------------|-----------|---------------------|---------------|------------------------|--------------------------|----------|
| 1   | 10            | 6000      | 100                 | 0.6           | 0.346                  | 0.379                    | -9.54    |
| 2   | 10            | 8000      | 200                 | 0.8           | 0.468                  | 0.437                    | 6.62     |
| 3   | 12.5          | 6000      | 100                 | 0.6           | 0.325                  | 0.359                    | -10.46   |
| 4   | 12.5          | 8000      | 200                 | 0.8           | 0.428                  | 0.413                    | 3.5      |
| 5   | 15            | 6000      | 200                 | 0.6           | 0.312                  | 0.345                    | -10.58   |
| 6   | 15            | 8000      | 100                 | 0.8           | 0.408                  | 0.397                    | 2.7      |
| 7   | 20            | 6000      | 200                 | 0.6           | 0.299                  | 0.326                    | -9.03    |
| 8   | 20            | 8000      | 100                 | 0.8           | 0.395                  | 0.375                    | 5.06     |
| 9   | 25            | 8000      | 100                 | 0.6           | 0.292                  | 0.311                    | -6.51    |
| 10  | 25            | 6000      | 200                 | 0.8           | 0.381                  | 0.365                    | 4.2      |
| 11  | 35            | 8000      | 100                 | 0.6           | 0.274                  | 0.296                    | -8.03    |
| 12  | 35            | 6000      | 200                 | 0.8           | 0.380                  | 0.348                    | 8.42     |
| 13  | 40            | 8000      | 200                 | 0.6           | 0.273                  | 0.292                    | -6.96    |
| 14  | 40            | 6000      | 100                 | 0.8           | 0.376                  | 0.342                    | 9.04     |
| 15  | 45            | 8000      | 200                 | 0.6           | 0.265                  | 0.288                    | -8.68    |
| 16  | 45            | 6000      | 100                 | 0.8           | 0.373                  | 0.338                    | 9.38     |
The half-width of the contact between the abrasive belt flap wheel and the workpiece increases with the increase of the curvature radius of the workpiece and the greater the tangential grinding force required for material removal. The increase in spindle speed leads to the decrease of radial material removal depth. This result is contrary to the statement by Tian et al. [29] regarding the relationship between grinding speed and material removal. Because of the abrasive belt flap wheel with great flexibility, as the speed increases, the grinding force in contact with the workpiece decreases. Drazumeric et al. [30] found that grinding cylinders and non-cylinders have a great influence on the maximum chip thickness. Therefore, the difference in the grinding geometric profile causes the depth of radial material removal to increase slightly during the cylindrical grinding. The feed speed has little effect on the depth of radial material removal because the feed speed is much lower than the linear velocity in the actual grinding process.

4 Conclusions

(1) Within a certain speed range, the radius of the abrasive belt flap wheel will increase linearly with the increase of speed. The pressure in the contact zone shows a parabolic distribution with the deformation of the abrasive belt flap wheel.

(2) The radial theoretical grinding depth has a great influence on the stability of material removal rate in the grinding of the abrasive belt flap wheels.

(3) Based on the Preston equation and Hertz elastic theory, the relationship between the radial theoretical grinding depth and the material removal rate is derived. And, according to the principle of conservation of material removal, the depth of the radial material removal model is obtained.

(4) Through the experiment of the amount of grinding material removed by planes and cylinders, the reliability and applicability of the depth of the radial material removal model are verified.
(5) Analyzed the influence of speed, feed rate, theoretical radial grinding depth, and workpiece curvature radius on the depth of radial material removal, which provided a reference for the titanium alloy blisk grinding process and precise control of contour accuracy.

**Author contributions** Gang Zheng put forward the method and thought of establishing the theoretical model.

Keyan Chen conducted experiments and data collection, and wrote the paper.

Xiaojian Zhang participated in the modeling and experiment together, and provided the experimental conditions.

Xu Zhang revised the paper.

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**Declarations**

**Ethics approval** Not applicable.

**Competing interests** The authors declare no competing interests.

**References**

1. Huang H, Gong Z, Chen X, Zhou L (2002) Robotic grinding and polishing for turbine-vane overhual. J Mater Process Technol 127:140–145. https://doi.org/10.1016/S0924-0136(02)00114-0

2. Bigerelle M, Gautier A, Hagege B, Favergeon J, Bounichane B (2009) Roughness characteristic length scales of belt finished surface. J Mater Process Technol 209(20):6103–6116. https://doi.org/10.1016/j.jmatprotec.2009.04.013

3. Zhang T, Su J (2018) Collision-free planning algorithm of motion path for the robot belt grinding system. Int J Adv Robot Syst 15(4):172988141879377. https://doi.org/10.1186/s12206-018-0481-2

4. Ding Y, Min X, Fu W, Liang ZL (2018) Research and application on force control of industrial robot polishing concave curved surfaces. Proc Inst Mech Eng B J Eng Manuf. 233(6):1674–1686. https://doi.org/10.1177/0954405418802309

5. Tian F, Li Z, Lv C, Liu G (2016) Polishing pressure investigations of robot automatic polishing on curved surfaces. Int J Adv Manuf Technol 87(1-4):639–646. https://doi.org/10.1007/s00170-016-8527-2

6. Xiao G, Huang Y (2017) Experimental research and modelling of life-cycle material removal in belt finishing for titanium alloy. J Manuf Process 30:255–267. https://doi.org/10.1016/j.jmapro.2017.09.030

7. Zhou L, Liu X, Ren X, Huang Y (2020) Investigation of robotic abrasive belt grinding methods used for precision machining of aluminum blades. Int J Adv Manuf Technol. https://doi.org/10.1007/s00170-020-05632-z

8. Hung P, Dung H, Trung N (2020) The study on surface grinding process of Ti-6Al-4V alloy with resinoid CBN grinding wheel. Int J Mod Phys B 34:2040135. https://doi.org/10.1142/s0217979220401359

9. Shi Y, Chen L, Xin H, Yu T, Sun Z (2020) Investigation on the grinding properties of high thermal conductivity vitrified bond CBN grinding wheel for titanium alloy. Int J Adv Manuf Technol 107:1539–1549. https://doi.org/10.1007/s00170-020-05134-y

10. Dai C, Ding W, Zhu J, Xu J, Yu HW (2017) Grinding temperature and power consumption in high speed grinding of Inconel 718 nickel-based superalloy with a vitrified CBN wheel. Precis Eng 52:192–200. https://doi.org/10.1016/j.precisioneng.2017.12.005

11. Zhang Z, Wang B, Kang R, Zhang B, Guo D (2015) Changes in surface layer of silicon wafers from diamond scratching. CIRP Ann Manuf Technol 64(1):349–352. https://doi.org/10.1016/j.cirp.2015.04.005

12. Zhang Z, Guo D, Wang B, Kang R, Zhang B (2015) A novel approach of high speed scratching on silicon wafers at nanoscale depths of cut. Sci Rep-UK 5:16395. https://doi.org/10.1038/srep16395

13. Zhang Z, Huo Y, Guo D (2013) A model for nanogrinding based on direct evidence of ground chips of silicon wafers. Sci China Technol Sci 56(9):2099–2108. https://doi.org/10.1007/s11431-013-5286-2

14. Zhang Z, Huo F, Zhang X, Guo D (2012) Fabrication and size prediction of crystalline nanoparticles of silicon induced by nanogrinding with ultrafine diamond grits. Ser Mater 67(7–8):657–660. https://doi.org/10.1016/j.scriptamat.2012.07.016

15. Zhang Z, Song Y, Xu C, Guo D (2012) A novel model for undeformed nanometer chips of soft-brittle HgCdTe films induced by ultrafine diamond grits. Ser Mater 67(2):197–200. https://doi.org/10.1016/j.scriptamat.2012.04.017

16. Zhang Z, Huang S, Wang S, Wang B, Bai Q, Zhang B, Kang R, Guo D (2017) A novel approach of high-performance grinding using developed diamond wheels. Int J Adv Manuf Technol 91(9-12):3315–3326. https://doi.org/10.1007/s00170-017-04037-3

17. Zhang Z, Du Y, Wang B, Wang Z, Kang R, Guo D (2017) Nanoscale Wear Layers on Silicon Wafers Induced by Mechanical Chemical Grinding. Tribol Lett 65:132. https://doi.org/10.1007/s12249-017-0911-z

18. Zhang Z, Cui J, Wang B, Wang Z, Kang R, Guo D (2017) A novel approach of mechanical chemical grinding. J Alloys Compd 726:514–524. https://doi.org/10.1016/j.jallcom.2017.08.024

19. Zhang J, Shi Y, Lin X, Li Z (2017) Five-axis abrasive belt flap wheel polishing method for leading and trailing edges of aerograde blade. Int J Adv Manuf Technol 93(9-12):3383–3393. https://doi.org/10.1007/s00170-017-1017-7

20. Huai W, Shi Y, Tang H, Lin X (2016) Prediction of surface roughness ratio of polishing blade of abrasive cloth wheel and optimization of processing parameters. Int J Adv Manuf Technol 90(1-4):699–708. https://doi.org/10.1007/s00170-016-9397-3

21. Huai W, Shi Y, Tang H, Lin X (2019) An adaptive flexible polishing path programming method of the blisk blade using elastic grinding tools. J Mech Sci Technol 33(7):3487–3495. https://doi.org/10.1007/s12206-019-0643-0

22. Zhang J, Shi Y, Lin X, Li Z (2017) Parameter optimization of five-axis polishing using abrasive belt flap wheel for blisk blade. J Mech Sci Technol 31(10):4805–4812. https://doi.org/10.1007/s12206-017-0928-0

23. Johnson KL (1985) Contact Mechanics. Cambridge University Press, New York

24. Chang WR, Elision I, Bogy DB (1987) An elastic-plastic model for the contact of rough surfaces. J Tribol 109(2):257. https://doi.org/10.1115/1.3261348

25. Dražušerić R, Badger J, Reineinen R, Krajnik P (2020) On geometry and kinematics of abrasive processes: the theory of aggressive-ness. Int J Mach Tools Manuf 154:103567. https://doi.org/10.1016/j.ijmachtools.2020.103567

26. Preston FW (1927) The theory and design of plate glass polishing machines. J Soc Glas Technol 11(1):214–256
27. Rogelio L, Steven Y, Wu X, Xia P, David G (2007) Grinding force and power modeling based on chip thickness analysis. Int J Adv Manuf Technol 33(5-6):449–459. https://doi.org/10.1007/s00170-006-0473-y

28. Wang G, Zhou X, Meng G, Yang X (2017) Modeling surface roughness for polishing process based on abrasive cutting and probability theory. Mach Sci Technol 22(1):86–98. https://doi.org/10.1080/10910344.2017.1336629

29. Tian L, Fu Y, Xu J, Li H, Ding W (2015) The influence of speed on material removal mechanism in high speed grinding with single grit. Int J Mach Tools Manuf 89:192–201. https://doi.org/10.1016/j.ijmachtools.2014.11.010

30. Drazumeric R, Badger J, Krajnik P (2014) Geometric, kinematical and thermal analyses of non-round cylindrical grinding. J Mater Process Technol 214(4):818–827. https://doi.org/10.1016/j.jmatprotec.2013.12.007

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