Privacy of Existence of Secrets: 
Introducing Steganographic DCOPs 
and Revisiting DCOP Frameworks

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Abstract
Here we identify a type of privacy concern in Distributed Constraint Optimization (DCOPs) not previously addressed in literature, despite its importance and impact on the application field: the privacy of existence of secrets. Science only starts where metrics and assumptions are clearly defined. The area of Distributed Constraint Optimization has emerged at the intersection of the multi-agent system community and constraint programming. For the multi-agent community, the constraint optimization problems are an elegant way to express many of the problems occurring in trading and distributed robotics. For the theoretical constraint programming community, the DCOPs are a natural extension of their main object of study, the constraint satisfaction problem. As such, the understanding of the DCOP framework has been refined with the needs of the two communities, but sometimes without spelling the new assumptions formally and therefore making it difficult to compare techniques. Here we give a direction to the efforts for structuring concepts in this area.

Introduction
We identify the “privacy of existence of secrets” as a not yet formally explored issue in the area of the Distributed Constraint Optimization problems (DCOPs). In order to scientifically draw conclusions about a concept or procedure, it is essential to formally define the involved concepts, metrics, and assumptions. Consequently, we also revisit some common concepts to suggest a direction for the possible structuring of DCOP frameworks.

The distributed constraint optimization has emerged at the confluence of research in Multi-Agent Systems and research in Constraint Programming. The multi-agent community has found in the DCOP framework an elegant way to specify requirements existing in the area of cooperating robot teams, negotiation, and trading. For the Constraint Programming community, the DCOP is a natural extension of a framework that is at the center of their field, the constraint satisfaction problem (CSP). The reasons and the expectations from the new DCOP framework are slightly different for the researchers in the involved areas, differences that for various reasons are sometimes not spelled formally inside the DCOP definitions, making comparison and reference difficult and error-prone for new researchers in this booming area.

The Constraint Programming community has developed an impressive body of knowledge and set of techniques focused on improving the efficiency of problems represented as constraint satisfaction problems (CSP) and for various extensions of such problems including the egalitarian Fuzzy CSPs (FCSPs), Probabilistic CSPs (PCSPs), dynamic CSPs (DCSPs), the utilitarian Weighted CSPs (WCSPs), and the general Valued CSPs (VCSPs) [Schiex et al. 1995] and Semiring based CSPs [Bistarelli et al. 1999]. The Constraint Programming community is careful in formally defining each extension to CSPs and clearly stating each framework by specifying formally and exactly the inputs and unambiguously defining outputs and evaluation metrics. This allows for work in the Constraint Programming field to be largely unambiguously comparable, and technology transfer between extensions to be possible without need of repeating work. However, most research in that area concerns efficiency while privacy and secrecy of constraints are generally not part of the CSP community criteria. An early set of extensions of the CSP family to the multi-agent world addressed parallelism [Kasif 1990], and continued to be focused solely on efficiency, as constraint were considered shared between agents.

The seminal work in [Yokoo et al. 1992] brought the concept of a distribution of CSPs where privacy was mentioned as a motivation, besides natural distributions of the constraints and control of variables to agents, as a reason to not attempt sharing constraints except on a need basis during a distributed backtracking effort. While this is also seen as an extension of the CSP family, the efficiency was no longer a sufficient criteria and direct comparison with traditional centralized constraints was seen as irrelevant, despite a lack of formal specification of the impediments to additional constraint sharing in terms of either technical cost or quantified privacy constraints. This lack of specification gave birth to a flurry of works exploiting various amounts of constraint information without quantifica-
tion and with efficiency gains whose trade-off was not always clear (Silaghi, Sam-Haroud, and Faltings 2000a) in terms of the privacy or in terms of the technical cost of constraint data centralization.

In a concern to make the field coherent and easily maintain scientific soundness, the constraint community recommended at the CP2000 distributed constraint satisfaction workshop panel to name the new family of extensions by appending distributed in front of each CSP extension, yielding:

• distributed constraint satisfaction (DisCSP)
• distributed fuzzy constraint satisfaction (DisFCSP)
• distributed probabilistic constraint satisfaction (DisPCSP)
• distributed dynamic constraint satisfaction (DisDCSP)
• distributed weighted constraint satisfaction (DisWCSP)
• distributed valued constraint satisfaction (DisVCSP)
• distributed semiring-based constraint satisfaction (DisSCSP)

Works had started to confusingly use the acronym DCSP for distributed CSPs, conflicting with the acronym for dynamic CSPs. It was recommended for the new frameworks to use the prefix "Dis" rather than "D" as a way to differentiate from dynamic CSPs which were intensively studied at the time. It was suggested that the latter be referred as DyCSP to further reduce confusion. Later, additional types of problem emerged, highlighting the wisdom of the suggestion, namely:

• dynamic distributed constraint satisfaction (DyDisCSP)
• dynamic distributed dynamic constraint satisfaction (DyDisDCSP)

Where besides a dynamism in constraint emergence there also exist a dynamism in agent involvement (i.e., churning) or in constraint redistribution (Silaghi et al. 2001; Silaghi and Faltings 2002b).

However, while privacy was already a stated motivation for distributed CSPs, (and by extension assumed present in all these frameworks), it remained unspecified formally, perpetuating the difficulty of correct comparison and evaluation of proposed techniques.

Meanwhile, the multi-agent community adopted the new direction and developed techniques coining the name “distributed constraint optimization” (DCOP) for a framework of distributed weighted constraints. For the early DCOPs definitions, privacy was also not formally stated, and the names DCOP and DisWCSP coexisted for a while until the community largely adopted the DCOP acronym. Extensions in this community are built on the DCOP concept, such as in the case of Leximin DCOPs (Matsui et al. 2013).

Further research noticed the negative effects from lack of formal definitions and quantification of privacy criteria in the distributed CSP family. Attempts to fix the problem either redefined the framework (Silaghi et al. 2005) without changing its name/acronym (in an effort to redeem or adopt earlier work that stressed privacy), or alternatively proposed new names and acronyms. New proposed names/acronyms appeared in the case of distributed Privacy CSPs (DisPrivCSPs) or distributed privacy DCOPs (DisPrivCOPs) (Doshi et al. 2008) and Utilitarian CSPs (UCSPs) or Utilitarian DCOPs (UD-COP) (Savaux et al. 2016; Savaux et al. 2017). Work has also addressed the classification of the nature of privacy concerns, as in (Léauté and Faltings 2013).

In this work, privacy is defined as a value:

**Definition 1** Privacy is the utility that agents draw from conserving the secrecy of their personal information.

In the Background section, we review the original and main formal definitions of distributed constraint satisfaction (DisCSP) and distributed constraint optimization problems (DCOPs/DisWCSPs), as well as the main informal privacy classifications in literature. We also introduce the concept of steganography, which is more intensively studied in the area of Cryptology. In the Steganographic DCOP section we then present motivation and define formally the Steganographic DCOPs. Further in the Frameworks Disambiguation section we propose a direction for the structure of the nomenclature and definitions for various concepts in the DCOP areas. We conclude with a summary of the contributions.

**Background**

We start by reviewing the main constraint optimization frameworks as emerging from constraint satisfaction. Later we review the various classifications of privacy found in literature, and the definitions we used for privacy and steganography.

**Formal Distributed Constraint Optimization Frameworks**

Several distributed CSPs are based on the CSP framework:

**Definition 2 (CSP (Yokoo et al. 1998))** A CSP consists of n variables $x_1, x_2, ..., x_n$, whose values are taken from finite, discrete domains $D_1, D_2, ..., D_n$ respectively, and a set of constraints on their values. A constraint is defined by a predicate. That is, the constraint $p_k(x_{k1}, ..., x_{kj})$ is a predicate that is defined on the Cartesian product $D_{k1} \times \ldots \times D_{kj}$. This predicate is true if the value assignment of these variables satisfies this constraint. Solving a CSP is equivalent to finding an assignment of values to all variables such that all constraints are satisfied.
The first distributed CSP definition from Yokoo et al. 1992 [Yokoo et al. 1998] is:

Definition 3 (DCSP (Yokoo et al. 1998)) There exist \( m \) agents: 1, 2, ..., \( m \). Each variable \( x_j \) belongs to one agent \( i \) (this relation is represented as \( \text{belongs}(x_j, i) \)). Constraints are also distributed among agents. The fact that an agent \( i \) knows a constraint predicate \( p_k \) is represented as \( \text{known}(p_k, i) \).

A Distributed CSP is solved iff the following conditions are satisfied:
\[ \forall i, \forall x_j \text{ where } \text{belongs}(x_j, i), \text{ the value of } x_j \text{ is assigned to some value } d_j, \text{ and } \forall i, \forall p_k \text{ where } \text{known}(p_k, i), p_k \text{ is true under the assignment } x_j = d_j. \]

As observed, even as Yokoo et al. 1998 has a section dedicated to privacy motivations, the formal definition does not specify a way to quantify privacy requirements and costs. Further, the semantic of variable ownership specified with the \( \text{belongs} \) predicate is not further formally defined except as implied by the fact that in the proposed algorithms, new assignments for a variable are only proposed by the agents to which the variable \( \text{belongs} \).

While this definition assumes that the number of agents equals the number of variables, with each agent keeping secret the domain of exactly one variable, other versions studied the case of unequal numbers of variables and agents with no secret domains but with secret constraints (Silaghi, Sam-Haroud, and Faltings 2000b).

Definition 4 (DisCSP (Silaghi and Faltings 2005)) Distributed constraint satisfaction problems (DisCSPs) is defined by:
- a set of \( n \) variables \( X = \{x_1, \ldots, x_n\} \),
- a set of \( n \) domains, \( D = \{d_1, \ldots, d_n\} \), for the variables,
- a set of \( t \) constraints, \( C = \{c_i = (x_1, x_2, \ldots, c_i)\} \), each of which is a subset of the set of variables linked with a relation, and
- a set of \( t \) relations, \( R = \{r_1, \ldots, r_t\} \). \( r_i \) gives the allowed value combinations for the corresponding constraint \( c_i \).
- a set of \( m \) independent but communicating agents \( A = \{A_0, \ldots, A_m\} \)
- an ownership mapping \( M: X \cup C \rightarrow P(A) \) that assigns each variable or constraint to the subset of agents that own it. \( P(A) \) is a common notation for the set of subsets of \( A \).

A solution to a CSP is an assignment of values from the corresponding domains to each variable such that for all constraints, the combination of assigned values is allowed by the corresponding relation.

The Distributed Constraint Optimization Framework was introduced in Modi et al. 2002.

Definition 5 (DCOP (Modi et al. 2002)) A Distributed Constraint Optimization Problem (DCOP) consists of \( n \) variables \( V = \{x_1, x_2, \ldots, x_n\} \), each assigned to an agent, where the values of the variables are taken from finite, discrete domains \( D_1, D_2, \ldots, D_n \), respectively. Only the agent who is assigned a variable has control of its value and knowledge of its domain.

The goal is to choose values for variables such that an objective function is minimized or maximized. The objective function described is addition over costs, but can be any associative, commutative, monotonic aggregation operator defined over a totally ordered set of valuations, with minimum and maximum element (described by Schiex, Fargier and Verfaillie as Valued CSPs (Schiex et al. 1993)).

The definition in Modi et al. 2002 makes a formal link to the Weighted CSP instance of the Valued CSP framework while claiming generality in terms of application to remaining CSP extensions.

However, while further stressing the control of values, it limits input secrecy to domains and does not further quantify effects of privacy loss or penalties for discussions by agents about values that they do not “control”.

Despite the framework not quantifying privacy, measurements of privacy loss based on qualitative comparisons were made in Silaghi and Faltings 2002a, and based on using assumptions from information theory, outside the standard DCOP definitions, in Freuder, Minca, and Wallace 2001, Franzin et al. 2002, Wallace and Silaghi 2004, Maheswaran et al. 2006, and in Greenstadt, Pearce, and Tambe 2006, Faltings, Léauté, and Petcu 2008.

An effort to redefine DisCSPs to formalize privacy requirements is in Silaghi et al. 2005.

Definition 6 (DisCSP (Silaghi et al. 2005)) A Cryptographic Distributed CSP (DisCSP) is defined by six sets (\( A, X, D, C, I, O \)) and an algebraic structure \( F \). \( A = \{A_1, \ldots, A_m\} \) is a set of agents. \( X, D, \) and the solution are defined like for CSPs.
\[ I = \{I_1, \ldots, I_n\} \text{ is a set of secret inputs. } I_i \text{ is a tuple of } o_i \text{ secret inputs (defined on } F) \text{ from the agent } A_i. \text{ Each input } I_i \text{ belongs to } F^{o_i}. \]

Like for CSPs, \( C \) is a set of constraints. There may exist a public constraint in \( C, \phi_0 \), defined by a predicate \( \phi_0(\varepsilon) \) on tuples of assignments \( \varepsilon \), known to everybody. However, each constraint \( \phi_i, i > 0, \) in \( C \) is defined as a set of known predicates \( \phi_i(\varepsilon, I) \) over the secret inputs \( I \), and the tuples \( \varepsilon \) of assignments to all the variables in a set of variables \( X_i, X_i \subseteq X \).
\[ O = \{o_1, \ldots, o_n\} \text{ is the set of outputs to the different agents. } \text{Let } m \text{ be the number of variables. } o_i : D_1 \times \ldots \times D_m \rightarrow F^{o_i} \text{ is a function receiving as parameter a solution and returning } \omega_i \text{ secret outputs (from } F) \text{ that will be revealed only to the agent } A_i. \]

This definition uses the same name and acronym as previously encountered definitions of distributed CSPs, but formally specifies that solutions may only be revealed to specific agents (introducing a concept now referred in literature as privacy of deci-
sion (Léauté and Faltings 2013). The formulation requires that there should not be any kind of privacy loss, being mainly appropriate for cryptographic techniques.

The extension of such privacy formalization to DCOPs is reported in (Silaghi 2005a):

Definition 7 (DisWCSP (Silaghi 2005a)) A distributed constraint satisfaction problem (DisWCSP) is defined by six sets (A, X, D, C, I, O), an arithmetic structure F, and a set of acceptable solution qualities B, that can be often represented as an interval [B1, B2]. A = {A1, ..., An} is a set of agents. X = {x1, ..., xm} is a set of variables and D = {D1, ..., Dm} is a set of finite domains such that xi can take values only from Di = {v1i, ..., vdi}.

An assignment is a pair (xi, vki) meaning that the variable xi is assigned the value vki.

A tuple is an ordered set. I = {I1, ..., In} is a set of secret inputs. Ii is a tuple of αi secret inputs (defined on a set Fi) from the agent Ai. Each input Ii belongs to Fiαi.

C = {ϕ0, ..., ϕn} is a set of constraints. A constraint ϕi weights the legality of each combination of assignments to the variables of an ordered subset Xi of the variables in X, X i ⊆ X. ϕ0 is a public constraint defined by a function ϕ0(ε) on tuples of assignments ε, known to everybody. Each constraint ϕi, i > 0, in C is defined as a known function ϕi(ε, I) over the secret inputs I, and the tuples ε of assignments to all the variables in a set of variables Xi, X i ⊆ X.

The projection of a tuple ε of assignments over a tuple of variables X i is denoted ε[Xi]. A solution is:

ε∗ = argminε∈D1×...×Dn i=1 c ϕi(ε[Xi]),

if ∑i=1c ϕi(ε∗[Xi]) ∈ [B1...B2].

O = {o1, ..., on} is a set of outputs to the different agents. o1: I × D1 × ... × Dm → Fω1 is a function receiving as parameter the inputs and a solution, and returning ω1 secret outputs (from F) that will be revealed only to the agent Ai. The problem is to generate O.

The above DCOP framework was designed to be addressed with cryptographic protocols and allows for chaining multiple DCOPs for solving complex problems such as Vickrey Auctions (Silaghi 2005a). It does not support non-cryptographic privacy aware solvers as it does not model costs for partial privacy leaks.

Frameworks for DCOP With Privacy for Non-cryptographic Solvers The quantification of privacy in distributed CSPs is introduced in (Savvaux et al. 2010), to enable the evaluation of non-cryptographic solvers:

Definition 8 (UDCSP) A UDisCSP is formally defined as a tuple ⟨A, V, D, C, U, R⟩ where:

• A = {A1, ..., An} is a vector of n agents
• V = {x1, ..., xn} is a vector of n variables. Each agent Ai controls the variable xi.
• D = {D1, ..., Dn} where Di is the domain for the variable xi, known only to Ai, and a subset of {1, ..., d}.
• C = {C1, ..., Cm} is a set of interagent constraints.
• U = {u1,1, ..., un,d} is a matrix of privacy costs where ui,j is the cost of agent Ai for revealing whether j ∈ Di.
• R = {r1, ..., rn} is a vector of rewards, where ri is the reward agent Ai receives if an agreement is found.

An agreement as a set of assignments for all the variables with values from their domain, such that all the constraints are satisfied.

The state of agent Ai includes the subset of Di that it has revealed, as well as the achievement of an agreement.

The problem is to define a set of communication actions and a policy for each agent such that their utility is maximized.

Another framework that allows for privacy loss but quantifies it was proposed in (Doshi et al. 2008). It is defined formalizing the concept of revelation:

Definition 9 (Revelation) Given a set of Boolean (propositional) secrets S and a set of agents A, a possible revelation R(A, S) is a function R(A, S): A × S → [0, 1] which maps each peer agent and a secret to the probability learned by that agent about the secret.

Definition 10 (DPCOP (Doshi et al. 2008)) A (minimization) Distributed Private Constraint Optimization Problem (DPCOP) is defined by a tuple (A, X, C, P, U). A is a set of agents {A1, ..., Ak}. X is a set of variables {x1, ..., xn}, and D is a set of domains {D1, ..., Dn} such that each variable xi may take values only from the domain Di. The variables are subject to a set C of sets of weighted constraints {C0, C1, ..., CK}, where Ci = {ϕ1i, ..., ϕki} holds the secret weighted constraints of agent Ai, and C0 holds the public constraints. Each weighted constraint is defined as a function ϕi: Xi → R+, where Xi ⊆ X. The value of such a function in an input point is called constraint entry, and each Ci can be seen as a set C of such constraint entries. P is a set of privacy loss cost functions {P1, ..., PK}, one for each agent. Pi defines the cost inflicted to Ai by each revelation r of its secrets, i.e., Pi(r): R(A, Ci) → R+. U is a set {U1, ..., UK}. Ui is the reward received by Ai if a solution is found (used for deciding to abandon the search).

A solution is an agreement between agents in A on a tuple ε∗ of assignments of values to variables that minimizes the total cost:

ε∗ = argminε ∑ i (∑ j ϕj(ε) + Pi j(ε))

where ∏ i j(ε) is the revelation R(A, Ci) during the process leading to the agreement on the assignments ε.
This definition quantifies privacy and drops the concept of control by agents on variables, which we describe as unclear in previous definitions. A variant related to UDCSP is described and evaluated in (Savaux et al. 2017).

Privacy Classifications
There are several efforts to classify privacy concepts in distributed constraint optimization (Leauté and Faltings 2013).

• **Domain Privacy As Existence of Values** The explanation in (Yokoo et al. 1998) concerning the first distributed CSP definition can be interpreted as describing a privacy concerning the very existence of values in domains. This goes to the point where the constraints of other agents cannot be defined extensionally due to the impossibility to enumerate these domains (e.g., impossibility to enumerate all meeting places that an agent controlling the corresponding variable may think about). This kind of domain privacy was highlighted in (Brito and Meseguer 2003).

• **Domain Privacy As Unary Constraint** The first types of privacy mentioned in (Yokoo et al. 1998) are domain and constraint privacy. Domain privacy is frequently understood as a unary constraint on the values of a variable (Yokoo et al. 1998). This interpretation is assumed in many subsequent works, where agents’ constraints are assumed to be extensionally defined using mutual knowledge of the values in domains of each others’ variables (Silaghi, Sam-Haroud, and Faltings 2000b; Brito and Meseguer 2003).

• **Constraint Privacy** Constraint privacy is linked to the knows predicate in the original distributed CSP definition (Yokoo et al. 1998). Later work propose to drop the variable controlling assumptions in exchange for focusing on constraint privacy (Silaghi, Sam-Haroud, and Faltings 2000a; Silaghi, Sam-Haroud, and Faltings 2000b). Intermediary frameworks allowing for some constraint privacy by distribution in order to maintain the notion of control of variables have also been proposed by splitting binary constraints into private halves, but it is unclear if the under-defined control of variables offers a tradeoff for the reduced constraint privacy.

• **Privacy of Existence of Solution** The fact that the problem in not solvable may in itself be a secret and impact on revelation of information. This privacy was addressed first in (Silaghi 2005b), based on stochastically discarding solutions even in problems found solvable by systematic solvers, to hide solution existence.

• **Privacy of Decision** The decision, as to what value is assigned to an individual variable in the final solution, only needs to be revealed to a predefined set of agents, and this was introduced in (Silaghi 2004b; Silaghi et al. 2005).

• **Privacy of Assignments** The individual values being tried during search by an agent can be secret and certainly reveal secrets about unary constraint of the proposer. This privacy is arguably at least partially achieved by the original ABT algorithm in (Yokoo 1992) where assignments are only announced to relevant agents. A twist where values are only revealed by relation was sometimes also claimed to increase this privacy (Silaghi, Sam-Haroud, and Faltings 2000b; Brito and Meseguer 2003). It is unclear whether cryptographic algorithms offer this privacy or it is simply irrelevant for them, as no proposal is made in them (Silaghi 2004a; Silaghi and Mitra 2004).

• **Privacy of Algorithms** The privacy of algorithm is typically defined as a privacy with respect to what reasoning processes are performed by an agent, other than the constraints on possible messages enforced by the commonly agreed solver protocol. Such privacy of algorithms is described in (Silaghi and Faltings 2002a). In solutions based on cryptographic protocols the equivalent concept for enforcing privacy of algorithm is introduced in (Silaghi and Friedrich 2005; Silaghi 2005a), allowing for the definition of a stronger level of privacy: requested privacy.

**Definition 11 (requested privacy)** Given secret inputs σ, the prior knowledge Γ of t colluders and a multi-party computation process Π with answer α that can be decomposed in a desired data α* and an algorithmic dependent unrequested data α̅, we say that an algorithm A achieves requested t-privacy if the probability distribution of the secrets that an attacker controlling any at most t participants can learn is conditionally independent on Π. A and α given requested data α* and prior knowledge Γ.

\[ P(\sigma \mid \alpha, \Gamma, \Pi, A) = P(\sigma \mid \alpha^*, \Gamma) \]

This concept of requested privacy is the strongest known relevant privacy concept and can be relaxed for computational purposes to:

**Definition 12 (non-uniform requested privacy)** Given secret inputs σ, the prior knowledge Γ of t colluders and a multi-party computation process Π with answer α that can be decomposed in a desired data α* and an algorithmic dependent unrequested data α̅, we say that an algorithm A achieves non-uniform requested t-privacy if for any secret σ ∈ Σ that is not deterministically revealed given requested data α and prior knowledge Γ, it is also not deterministically revealed given Π, A, and α̅, to any attacker controlling any at most t participants.
\( \forall \sigma \in \sigma P(\sigma | \alpha^*, \Gamma) < 1 \Rightarrow P(\sigma | \alpha, \Gamma, \Pi, A) < 1 \)

- **Privacy of Constraint Topology**
  It was shown in (Silaghi 2004a, Silaghi, Faltings, and Petcu 2006) that erasing topological information by combining all constraints into a big \( n \)-ary function maximizes privacy. Further, the protection of topological information can be a secret in itself, with application to protecting trade secrets, as in (Silaghi et al. 2001, Silaghi and Faltings 2002c).

- **Agent Identity Privacy** Hiding the real identity of the agents participating in a DCOP is a concern discussed in (Léauté and Faltings 2013). This refers to the general concept of anonymity, where software agents cannot be linked to human owners.

**Steganographic DCOPs**

Besides the previous types of privacy, we raise the next one:

- **Privacy of Existence of Secrets** In an observation we introduce here, note that for many of the studied DCOP frameworks, the agents that participate in the computation may need to suggest or admit that there exist secrets involved in the computation, without which the additional costs of the setup and execution would not be warranted.

But admitting that secrets may be involved is sometimes a significant privacy loss in itself, for example when agents want to claim openness for political reasons. This concept is introduced in this paper, with the Steganographic DCOP.

Steganography is the area of cryptography concerned with hiding the existence of secret information. While the oldest example of steganography dates from Histiaeus’ 499BC message tattooed under the hair of a slave’s head, as described by Herodotus, the best known modern techniques range from invisible ink to hiding of data in poetry and images.

Steganography is more appropriate then cryptography for the case of secrets with social impact, as is often the case with distributed constraint optimization (DCOP). With social impacts, an agent may lose status simply by stating that it has secrets, and trying to safeguard them. Politicians (and not only them) try to claim openness and lack of secrets.

In our experience, the need to protect existence of secrets proved to be a key impediment to the large scale adoption of cryptographic DCOP solvers, since many potential users find it difficult to publicly admit a need, or to call for secret problem solving.

In this work we identify the steganographic potential of traditional DCOP solvers where the technical distribution of the problem can be claimed as the unique cause of avoiding decentralization and straightforward full revelation, allowing to hide privacy needs.

The basic framework recommended for steganographic DCOPs, rephrasing variable ownership requirements as unary constraint, is:

**Definition 13 (StegDomDCOP)** A Steganographic Domain Distributed Constraint Optimization Problem (StegDomDCOP) consists of \( n \) agents \( A = \{a_1, ..., a_n\} \) and \( n \) variables \( V = \{x_1, x_2, ..., x_n\} \). Each agent \( a_i \) has a unary constraint on \( x_i \) and its domain \( D_i \). Agents know weighed constraints on subsets of these variables, specifying costs and rewards induced by assignments to concerned variables. Each agent can also identify, for each cost it associates with a total assignment, a secret reward for not revealing it.

The goal is to choose values for variables such that an objective function is minimized or maximized, while each agent’s participation is rational, performing only acts with positive expected sum of costs and rewards. The objective function described is addition over costs, but can be any associative, commutative, monotonic aggregation operator defined over a totally ordered set of valuations, with minimum and maximum elements.

The requirement of having the number of variables equal the number of agents can be dropped, together with the requirement that each agent has a unary constraint on a distinct variable, obtaining an equivalent framework that maps easier to certain problems, like to the problems in (Silaghi, Sam-Haroud, and Faltings 2000b):

**Definition 14 (StegCosDCOP)** A Steganographic Cost Distributed Constraint Optimization Problem (StegCosDCOP) consists of \( m \) agents \( A = \{a_1, ..., a_m\} \) and \( n \) variables \( V = \{x_1, x_2, ..., x_n\} \). Agents know weighed constraints on subsets of these variables, specifying costs and rewards induced by assignments to concerned variables. Each agent can also identify, for each cost it associates with a total assignment, a secret reward for not revealing it.

The goal is to choose values for variables such that an objective function is minimized or maximized, while each agent’s participation is rational, performing only acts with positive expected sum of costs and rewards. The objective function described is addition over costs, but can be any associative, commutative, monotonic aggregation operator defined over a totally ordered set of valuations, with minimum and maximum elements.

The fact that the last two frameworks are equivalent in expression power is guaranteed by the theory of primal dual CSP conversions (Bacchus and Van Beek 1998). However, modeling strategies and solvers may perform significantly better with one rather than the other approach, since the transformation between representations can be laborious.

The StegDCOP framework has the advantage that its agent problems can be masqueraded as instances of any common DCOP framework, and can be used in asynchronous search protocols to protect...
privacy without making other agents suspicious, as in [Silaghi and Faltings 2002a].

The framework enables the support of privacy of constraints and privacy of domains in a similar way to UD-COP and DCSPs.

Frameworks Disambiguation

Distributed Constraint Optimization Frameworks can be classified along the following independent dimensions:

- $X_1$ semiring/value system: Boolean, Fuzzy, Weighted, Probabilistic
- $X_2$ structure of the problem: Static, Dynamic_Local (D_L), Dynamic_Topology (D_T)
- $X_3$ whether domains are known only by some agents, by everyone with variable ownership (enabling extensional constraint representation), or with distribution of constraints. Combinations are possible. The corresponding distribution reasons are named: Domains, Variables, Costs, Domains_Costs (D_C), ...
- $X_4$ privacy of decision specification, for opening all assignments in the adopted solution to everyone, or only to specified agents: Open, Closed
- $X_5$ privacy management, where participants claim no privacy and do not secretly quantify privacy concerns, where participants claim privacy and secretly quantify its loss, where participants claim no privacy but secretly quantify privacy concerns, and where secrecy is admitted and enforced with cryptography: Public, Quantified, Steganographic, Cryptographic
- $X_6$ optimization function with utilitarian summation of constraints, egalitarian Leximin, or egalitarian Theil index. Utilitarian, Leximin, Theil

A nomenclature can be designed to easily differentiate between the different types of frameworks, or to enable studies of framework relations and hierarchy.

To maximize compatibility with past practice and the logic of the framework design, the proposed name structure is:

$$X_6X_5X_4DisX_3X_2X_1COP$$

(1)

where each of the names can be in camel case on any unambiguous prefix of the above properties, to enable disambiguation and easy extensions. The new nomenclature (1) is long and shorter versions can be obtained by selecting as defaults: $X_6=$Utilitarian, $X_5=Public$, $X_4=Open$, $X_3=Domain$, $X_2=Static$, $X_1=Weighted$. Each default parameter can be skipped if all other parameters it separates from the base string “Dis” are at their default value. Further, the Boolean-CSP can also be historically named as CSP.

With these conventions, for example:

- DCOP. The common DCOP in Definition [Modi et al. 2002] becomes Utilitarian-Public-Open-Dis-Domains-Static-Weighted-COP or shorter UPODisDSWCOP. With the use of defaults, the acronym becomes DisCOP.
- UD-COP. UD-COPs [Savaux et al. 2017] with full camel case specifiers becomes Utilitarian-Quantified-Open-Dis-Domains-Static-Weighted-COP or shorter, USQODisDSWCOP. With the use of defaults, the acronym becomes QODisCOP.
- The new StegDomDCOP framework in Definition [13] becomes Utilitarian-Steganographic-Open-Dis-Domains-Static-Weighted-COP. With the use of defaults, the acronym becomes SODisCOP.
- The new StegCosDCOP framework in Definition [14] becomes Utilitarian-Steganographic-Cryptographic-Closed-Dis-Costs-Static-Weighted-COP or UCCDisCSWCOP. With the use of defaults, the acronym becomes CDDisCCOP.
- DCSP. With this proposal, the original DCSP in Definition [Silaghi, Sam-haroud, and Faltings 2001] denoted Utilitarian-Public-Open-Dis-Domains-Static-Boolean-COP or UPODisDSBCOP. With the use of defaults, the acronym becomes DisCSP, as that acronym was used in several previous works [Silaghi, Sam-haroud, and Faltings 2001].
- DisCSP With the use of defaults, the DisCSPs with private constraints from Definition [Silaghi and Faltings 2005] is denoted Utilitarian-Public-Open-Dis-Domains-Static-Boolean-COP or UPODisDSBCOP and would be denoted shortly DisD_CCSP.

We note that the term “Open” that we propose for specifying the publication of the final decisions, has been used in the past as synonym for dynamism [Silaghi and Faltings 2002b]. The term “Utilitarian” DCOP has been used for specifying explicit representation of Privacy, but also to express utilitarian maximization of social welfare [Walras 1896]. However, the possible acronyms in the new nomenclature will not conflict with prior notations (that did not already have conflict, like DCSP).

Conclusion

We have identified a new type of privacy relevant to Distributed Constraint Optimization, namely privacy of existence of secrets, leading to the formal introduction of the Steganographic DCOPs. The new framework allows for disambiguating between problems where privacy is public but quantified and when privacy requirements are hidden (steganography). We review existing frameworks offering privacy in constraint optimization
to position the new type of privacy in the literature, and propose a new nomenclature system that, if adopted, can help disambiguate all privacy frameworks found in use. Further, the obtained classification can be used to identify gaps in the literature as well as valuable new frameworks to study.

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