Concerning the vacuum velocity of gravitational waves

Vicente Pleitez

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900– São Paulo, SP
Brazil

Abstract

It is pointed out that if gravitational interactions among ordinary bodies propagate in extra space-time dimensions the velocity of gravitational waves in vacuum could be different from the speed of light $c$.

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One of the main issues to be set by the detection programs of gravitational waves (like LIGO and VIRGO projects) is the vacuum velocity of gravitational waves \((c^*)\), and also their polarization. In the context of general relativity this velocity is the same as the light velocity \((c)\). For instance, if this velocity were found to be smaller than \(c\) it would imply, at first sight, that gravitons are massive. However, it has been shown, by Van Dam and Veltman \([1]\), through the measurement of the perihelion movement of Mercury and the bending of a ray of light passing near the Sun, that there is a discrete difference between the zero-mass theories and the very small but non-zero mass theories. Hence, consistency with the data imply that the graviton must be rigorously massless. As a consequence, if the measured velocity of the gravitational waves were different from the velocity of light, a problem would come out for theorists.

In this work we shall put forward a non-orthodox idea: gravitons are massless and although they are generated by ordinary matter, they propagate through a different space-time (which we will call \(x^*\)-world). In this world the speed limit in vacuum is \(c^*\), and it is, in principle, different from \(c\), the speed limit in the \(x\)-world. Connection between both \(x\)- and \(x^*\)-worlds is possible by assuming appropriate transformation properties for matter and radiation fields under an extended Lorentz symmetry \([2]\).

In order to implement this idea, let us assume that the Lorentz symmetry of an eight-dimensional world is \(L \otimes L^*\) being \(L\) and \(L^*\) Lorentz groups with different limit velocities \(c\) and \(c^*\), respectively. We denote \(x\) and \(x^*\) the space-time which transform as \((4, 1)\) and \((1, 4)\) under \(L \otimes L^*\) respectively. Our own world is identified with the \(x\)-world and the transformation properties of all known spin \(0, 1/2, 1\) fields are defined with respect to the \(L\) group but all of them transform as scalars under \(L^*\). From the point of view of quantum field theory, gravitation is caused by the exchange of a particle of spin-2 \([3]\). Then, the graviton is assumed to be described by a second rank symmetric tensor in the Minkowski space related with \(L^*\) but it is a scalar under \(L\).

The interaction between two material objects (denoted by \(a\) and \(b\)) caused by the exchange of a graviton in the Born approximation is \([\P\i\P]\)
\[ g^2 T_{\mu^* \nu^*} P_{\mu^* \alpha^* \beta^*} T_{\alpha^* \beta^*}^b, \]  

\(1\)

where \(P_{\mu^* \nu^* \alpha^* \beta^*}\) is the usual graviton propagator but now the propagation occurs in the \(x^*-\)world. The traceless tensors \(T\) in Eq. (1) can be formed with the scalar part (under \(L^*\)) of fields with different transformation properties under \(L\).

The coupling constant \(g\) is fixed by the requirement that Eq. (1) contains the Newton law for non-relativistic bodies. Usually the coupling with dimension of \((\text{mass})^{-2}\) is written as \(g^2 \to G_N/\hbar c\). In our context, however, we have

\[ g^2 \to g^*^2 = F^2 \frac{c}{c^*} \frac{G_N}{\hbar c} \equiv \frac{G_{\text{eff}}^N}{\hbar c}. \]

\(2\)

The dimensionless factor \(F^2\) appearing in Eq. (2) arises as follows. The interaction in Eq. (1) occurs in the \(x^*-\)world, i.e., the tensors \(T_{\mu^* \nu^*}^{a,b}\) are built with the usual fields which transform as scalars under \(L^*\). However, since we are interested in the gravitational effects observed in the \(x\)-world where we live, we can integrate over a finite volume of the \(x\)-world, so it is possible to calculate the dimensionless \(F\). There is a factor \(F\) in each vertex. Its value, in principle, depends on the process under consideration since the volume of integration may differ from one process to another. Similarly, a factor \(F\) will appear in the usual interactions among fields in the \(x\)-world. However, in this case we can integrate over the whole volume of the \(x^*-\)world. Thus, in this case \(F = 1\) and we obtain the usual interaction, say, between spin-\(\frac{1}{2}\) and photon fields. The interactions among gravitons are the same as the usual ones but they happen only in the \(x^*-\)world.

When \(T^a\) represents a fixed source, like the Sun, only \(T^a_{00}\) is non zero. If \(T^b\) is associated with a massless particle the only relevant part of the propagator gives

\[ g^*^2 T^a_{00} T^b_{00} \frac{1}{p^2 - i\epsilon}. \]

\(3\)

We do not know the value of \(c^*\), however \(g^*\) must have a value consistent with the data concerning the bending of a ray passing near the Sun, or the perihelion of the movement of Mercury. However, once we have admitted that \(c^* \neq c\) we can define a new energy scale related to the Planck scale as

3
\begin{equation}
E_{\text{new}} = \sqrt{\frac{\hbar c^5}{G_N}} \equiv \sqrt{\frac{\hbar c^5}{G_N}} \left( \frac{c^*}{c} \right)^{\frac{3}{2}} \approx 10^{19} \left( \frac{c^*}{c} \right)^{\frac{3}{2}} \text{GeV}.
\end{equation}

A possible implication of our approach is related to one of the main problems of the standard cosmological model. This is the difficulty for explaining the large-scale uniformity of the observed universe. As information cannot propagate faster than a light signal, in the standard cosmological model there is not enough time for this uniformity to be created by any physical process. This is usually called the horizon problem.

The most popular way to solve this problem is by considering the inflationary model. Notwithstanding, in our context it is possible to solve this problem if \( c^* > c \), by charging the gravitational waves for the transmission of the information through the universe.

From the point of view of classical general relativity, the weak gravitational field equations in the present context are

\begin{equation}
\Box_x \cdot \Box_{x^*} h_{\mu\nu}(x, x^*) = 16\pi G_N T_{\mu\nu}(x, x^*),
\end{equation}

where we have defined

\begin{equation}
\Box_{x^*} \equiv \nabla^{x^2} - \frac{1}{c^{x^2}} \frac{\partial^2}{\partial t^{x^2}},
\end{equation}

being \( \nabla^{x^2} \) the Laplacian operator with derivatives with respect to the \( \vec{x}^* \) space coordinates, and \( \Box_x \) denoting the usual d’Alembertian with respect to the \( x \)-world space-time coordinates; \( G_N^{\text{eff}} \) is the effective Newton constant defined in Eq. (2). On the other hand, the equation of motion of a Dirac fermion is

\begin{equation}
\left( i \gamma^\mu \cdot \partial_\mu - mc^2 \right)_{\alpha\beta} \left( \Box_{x^*} - m^2 c^{x^4} \right) \psi_{\beta}(x, x^*) = 0,
\end{equation}

\( \alpha \) and \( \beta \) are spinor indices in the \( x \)-world. (We have assumed that \( \hbar \) is the same in both worlds.) Similar modifications will appear in the case of other particles, scalar and vector ones. Although this formalism introduce form factors in all the known interactions it is not necessary to compactify the extra dimensions as was also pointed out in Ref. [2].

Thus, the right-hand side of Eq. (3) involves the scalar part of all fields, including the photon. Hence, since light is also affected only through its scalar part, the limiting velocity
in Eq. (5) is, in general, different from the local physical velocity of light. So gravitational waves may have a vacuum velocity different from the velocity of light. Besides the case $c^* = c$, both possibilities $c^* > c$ and $c^* < c$ are allowed in this context. From Eq. (2) Newton’s law can be written as usual with $G_N^{\text{eff}}$. So, although it is possible that $G_N^{\text{eff}} \approx G_N$ for all the situations considered up to now, obtaining the usual gravitational interactions, in our context it is also possible that $G_N^{\text{eff}}$ can change with the space-time point.

We would like to emphasize that although the gravitational interactions propagate through a different space-time they are capable of imparting momentum to the ordinary bodies.

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REFERENCES

[1] Van Dam H and Veltman M 1970 Nucl. Phys. B22 397

[2] An eight-dimensional space different from Kaluza-Klein-like theories in the context of non-gravitational interactions were proposed some years ago by Bryan D 1986 Phys. Rev. D 34 1184 and recently by Pleitez V 1995 Extra dimensions and color confinement, preprint IFT-P.019/95 [hep-th/9506009].

[3] Veltman M 1976 in Methods in Field Theories, edited by R. Balian and J. Zinn-Justin, North-Holland, and references therein.

[4] Kolb E W and Turner M S 1988 The Early Universe: Reprints, Addison-Wesley, 1988.

[5] Dicke R H and Peebles P J E 1979 in General Relativity, edited by S.W. Hawking and W. Israel, Cambridge University Press

[6] Ellis J 1994 Nuovo Cimento, 107 A 1091

[7] Guth A 1081 Phys. Rev. D23 347