Nucleosynthesis Bounds on a Time–varying Cosmological “Constant”

Michael Birkel and Subir Sarkar

Abstract

We constrain proposed phenomenological models for a vacuum energy which decays with the expansion of the universe from considerations of standard big bang nucleosynthesis. Several such models which attempt to solve the cosmological age problem are disfavoured or even ruled out.

26.35, 98.80.Cq, 98.80.Ft
Published in Astroparticle Physics 6 (1997) 197
I. INTRODUCTION

Several recent cosmological observations have revived the debate on the possibility of a cosmological constant $\Lambda$ [1]. The most discussed problem in this connection is the possible conflict between the age of the universe in the standard Friedmann-Robertson-Walker (FRW) cosmology, as inferred from recent measurements of the Hubble constant $H_0$ [2], versus the age of the oldest stars in globular clusters [3]. Constraints from the formation of large-scale structure in the universe together with dynamical estimates of its matter content, $\Omega_0 \approx 0.2$ [4], have also inspired suggestions of a non-vanishing cosmological constant, in order to make up $\Omega_0 + \Lambda_0/3H_0^2 = 1$ [5]. (Such a low value of $\Omega_0$ may also be required to reconcile the observed high nucleonic content of clusters of galaxies, revealed through their X-ray emission, with the upper limit on the nucleon density from big bang nucleosynthesis (BBN) [6].) These arguments limit the present value of the cosmological constant and the associated vacuum energy density, $\rho_v \equiv \Lambda M_p^2/8\pi$, to be at most $\sim (3 \times 10^{-12} \text{ GeV})^4$, whereas quantum field theories and the inflationary paradigm generally demand a value for the vacuum energy before the start of the FRW cosmological evolution, which is larger by at least 100 orders of magnitude [7]. While there is, as yet, no theoretical understanding of this problem [8], a phenomenological approach is to invoke a time-varying cosmological “constant”. In such a scenario, the vacuum energy density decays from its original high value during inflation to the small value allowed for at the present epoch. Many such models have been proposed in recent years [9]-[16].

An important constraint on models in which the vacuum energy decays into radiation is provided by nucleosynthesis arguments, as was noted by Freese et al. [9]. Since their analysis, however, there have been several developments in this area [7]. Input parameters such as nuclear cross-sections, the neutron lifetime and the number of light neutrinos are now better known, while the theoretical calculation of elemental abundances has become more accurate. Moreover there have been major developments on the observational front, requiring revision of the abundance bounds adopted in earlier work. In this paper we correct and update this nucleosynthesis constraint and test the viability of phenomenological proposals for a decaying vacuum energy.

II. BOUNDS FROM PRIMORDIAL NUCLEOSYNTHESIS

In considering the possible form of the time variation of $\Lambda(t)$, it is reasonable to assume that the vacuum energy is released as radiation, the energy density of which redshifts with the expansion scale factor as $R^{-4}$. (If the decaying vacuum energy creates massive particles instead, the present day energy density in matter will increase, exacerbating the cosmological age problem referred to above.) Hence Freese et al. [9] considered that the vacuum energy density decreases proportionally as the energy density of radiation which they took to include $e^+e^-$ pairs and $N_\nu = 3, 4, 5$ species of neutrinos in addition to photons, i.e. $\rho_r = \rho_v + \rho_e + \rho_\nu \equiv \frac{\pi^2 g_{\text{eff}}}{30} T^4$. To conveniently track the changing vacuum energy density they introduced the parameter

$$x \equiv \frac{\rho_v}{\rho_r + \rho_v},$$  \hspace{1cm} (1)
which they supposed remains constant with the expansion, even as $\rho_v$ decays increasing $\rho_r$. Although apparently plausible, this assumption is in fact physically inconsistent during the nucleosynthesis era for the following reasons. Since the weak interactions freeze out at a few MeV, the neutrinos generated by the decaying vacuum energy cannot be thermalized at lower temperatures. Further, $e^+e^-$ pairs turn non-relativistic and annihilate soon thereafter. Consequently, to have $\rho_v$ decrease proportionally to $T^4$ as desired for phenomenological reasons, we must assume that the vacuum energy decays into photons alone and redefine

$$x \equiv \frac{\rho_v}{\rho_r + \rho_v}.$$  \hspace{1cm} (2)

This does remain constant throughout the BBN epoch, since the photons produced by the decaying vacuum energy do get thermalized (within a Hubble time) down to a temperature of about 10 keV.

To quantitatively study the effects on nucleosynthesis, two important changes have to be made to the BBN computer code [18]. First, we add a term to the Friedmann equation to account for the vacuum energy density:

$$H^2 = \frac{8\pi}{3M_P^2}(\rho_r + \rho_v).$$  \hspace{1cm} (3)

(We have neglected the energy density of non-relativistic matter, $\rho_m$, as is appropriate in the radiation-dominated era.) Secondly, the equation of local energy conservation now has a term $\dot{\rho}_v$ describing the production of energy by the decaying vacuum energy:

$$\dot{\rho}_v + \dot{\rho} + 4\frac{\dot{R}}{R}\rho = 0.$$  \hspace{1cm} (4)

Freese et al. [9] had stated that this leads to the modified time-temperature relationship

$$T(t) \propto [g_{\text{eff}}(1-x)]^{-1/4}t^{-1/2};$$  \hspace{1cm} (5)

this, however, obtains only if the effects of $e^+e^-$ annihilation as well as the relative decrease of the neutrino temperature due to vacuum decay are neglected. (Because of the entropy generated by vacuum energy decay, the temperature of the background neutrinos (as well as the nucleon-to-photon ratio $\eta$) continually decreases in ratio to the photon temperature even at $T \ll m_e$ and drops below its standard asymptotic value of $T_\nu = (4/11)^{1/3}T$.) We discuss this in the Appendix and implement these effects in the Wagoneer code as updated by Kawano [18]. We assume 3 light neutrino species, use the latest value of the neutron lifetime $\tau_n = 887 \pm 2$ s [19], and incorporate small corrections to the helium abundance as reviewed in ref. [17].

The overall effect of a vacuum energy decaying to produce entropy is a decrease in the synthesized abundance of $^4$He. This is because the effect of the increased expansion rate, which causes earlier neutron-proton freeze-out, is sub-dominant relative to the effect of the dilution of $\eta$, which delays the onset of nuclear reactions. Consequently, although the $n/p$ ratio at freeze-out is higher, it decreases by $\beta$-decay to a smaller value before the neutrons
can be synthesized in light nuclei. Thus the magnitude of the decaying vacuum energy during nucleosynthesis can be bounded by requiring that the abundances of the synthesized elements be within observational limits. The relevant bounds considered in earlier work \[9\] were

\[
0.22 \leq Y_p(^4\text{He}) \leq 0.26, \quad 10^{-5} \leq (\text{D}/\text{H})_p \leq 10^{-4},
\]

for the primordial mass fraction of helium and the primordial abundance by number of deuterium, respectively. We adopt these bounds as well in order to illustrate the difference in results due to our use of an improved code and compare the constraints obtained on \(x\) as a function of \(\eta\) in Figure 1(a), having rescaled the previous result \[9\] to correspond to our redefinition (2) of \(x\), taking \(\rho_t = \frac{43}{8} \rho_\gamma\), for \(N_\nu = 3\). (Note that the constraints corresponding to the upper bound on \(Y_p\) are out of range, to the left of the y-axis). The constraint quoted earlier \[9\] corresponds to a bound on the vacuum energy which is less restrictive than our result by a factor of about 2.

Next we impose the updated bounds on the element abundances inferred from recent observations of helium in metal-poor extragalactic HII regions \[20\] and of deuterium in high redshift clouds along the line of sight to quasars \[21\]:

\[
0.23 \leq Y_p(^4\text{He}) \leq 0.25, \quad 1.1 \times 10^{-5} \lesssim (\text{D}/\text{H})_p \lesssim 2.5 \times 10^{-4}.
\]

(We do not consider the bound on \(^7\text{Li}\) because it does not provide a useful constraint.) We emphasize that these are conservative bounds based upon consideration of a variety of data which are critically discussed in ref. \[17\]. Because of the uncertainties in the input nuclear cross-sections and the neutron lifetime, as well as residual numerical errors in the computer code, the yield of an element for some specified value of \(\eta\) and \(x\) is not uniquely determined but has a spread which is about \(\pm 0.5\%\) for helium and as much as \(\pm 50\%\) for deuterium. Using a Monte Carlo method in which the BBN code is run many times while the input parameters are sampled at random from their known distributions, the “95%” C.L. bound can be identified as the value at which 5% of the runs pass the imposed observational bounds \[22\]. On the basis of these results we find a maximum value of

\[
x_{\text{max}} = 0.13
\]

for \(\eta \approx 3.7 \times 10^{-10}\) as shown in Figure 1(b). This corresponds to the bound

\[
\rho_\nu < 4.5 \times 10^{-12} \text{ GeV}^4,
\]

evaluated at a temperature \(T = 2.6\) MeV. This is about 3 times more stringent than the often quoted constraint calculated by Freese et al. \[9\]. For comparison, a constant vacuum energy is constrained by nucleosynthesis to be \(\rho_\nu < 4.4 \times 10^{-17} \text{ GeV}^4\) for \(\eta\) in the range \(10^{-11} - 10^{-9}\). (Although the bound on a time-varying cosmological “constant” appears weaker, it should be noted that it would decay to a much smaller value by the present epoch than one which is truly constant.)

\[\text{Note that if the decays do create (electron) neutrinos as assumed by Freese et al. \[9\], then there will be further direct effects on neutron-proton interconversions; such effects, however, were not considered by these authors.}\]
FIG. 1. Bounds on the decaying vacuum energy parameter $x \equiv \rho_v / (\rho_\gamma + \rho_v)$ as a function of the nucleon-to-photon ratio $\eta$ (evaluated at $T = 10^8$ K). The shaded regions are allowed. In panel (a) we compare previous results (dashed lines) with those obtained using the improved nucleosynthesis code, adopting the same bounds on the elemental abundances. In panel (b) we adopt updated observational bounds and also show (dashed lines) the “95%” C.L. limits corresponding to the known uncertainties in input parameters.
Many of the proposed models in the literature assume that
\[ \Lambda \propto R^{-m}, \]  
(10)
where \(1 \leq m \leq 3\) \cite{10}. Since the associated vacuum energy density \(\rho_v\) then decreases more slowly with time than the energy density of radiation, even a \(\rho_v\) having its maximum allowed value today would have been negligible at the epoch of nucleosynthesis. However, it would be physically more consistent to normalize the vacuum energy density at some high energy scale instead of at the present epoch. For example Gasperini \cite{11} interprets the cosmological constant as a measure of the Hawking temperature of the De Sitter vacuum, \(T_v = (\Lambda/12\pi^2)^{1/2}\); he assumes that the vacuum temperature equals the radiation temperature close to the Planck era, but that the subsequent expansion causes \(T_v\) to decay faster than \(T\) so as to be consistent with the upper bound on the cosmological constant today. However, the value of \(\rho_v\) during nucleosynthesis would still be substantial; in order to obey the constraint \(\mathcal{R}\), the exponent in eq.\( (10)\) is now required to be
\[ m > 3.5. \]  
(11)
Thus Gasperini’s suggestion is inconsistent with many models \cite{10} which assume \(m\) to be less than 3. (Note that it does not matter that the constraint \(\mathcal{R}\) was derived assuming \(m = 4\) since it would be even more stringent for smaller \(m\).)

Some other models \cite{12} have too many free parameters and/or are not specified sufficiently explicitly to be confronted with BBN. Specific models motivated by physical considerations \cite{13,19} can however be constrained, either by the bound \(\mathcal{R}\), or by the related bounds on the expansion rate or the values of fundamental constants during BBN. In some cases the authors have already considered such constraints but commented that the associated uncertainties preclude a definitive test. We reexamine the question taking into account all such sources of error and imposing conservative constraints from BBN \cite{17}.

Lima and collaborators \cite{13} consider nonsingular deflationary cosmology models with a decaying vacuum energy density. The cosmological history starts with the decay of a De Sitter vacuum and evolves smoothly to a quasi–FRW stage at late times. Irrespective of the spatial curvature, the models are characterized by the time scale \(H_t^{-1}\) which determines the initial temperature of the universe as well as the largest value of the vacuum energy density. The present matter and radiation content of the universe is generated by the slow decay of the vacuum energy density; for its time dependence, the authors use the phenomenological ansatz
\[ \rho_v = \frac{\Lambda M_p^2}{8\pi} = \beta \rho_{\text{tot}} \left(1 + \frac{1 - \beta}{\beta} \frac{H}{H_t}\right), \]  
(12)
where \(\rho_{\text{tot}} = \rho_v + \rho_m + \rho_r\) and \(\beta\) is a dimensionless parameter of order unity. This automatically generates a primordial inflationary scenario at the time \(H_t^{-1}\). The model thus represents a generalization of ref. \cite{9} since it does not fix the spatial curvature to be zero and introduces a time dependence in the parameter
\[ x' \equiv \frac{\rho_v}{(\rho_v + \rho_m + \rho_r)} = \beta + (1 - \beta) \frac{H}{H_1}, \]

(13)

At late times, when the term \( H/H_1 \) is negligible, we recover the model by Freese et al. \[9\] and find that \( x' \approx \beta \). In the physically preferred case, the deflationary process begins at the Planck time, \( H_1^{-1} \approx M_{Pl}^{-1} \), while the latest epoch at which it can occur is electroweak symmetry breaking (to permit the subsequent generation of a baryon asymmetry), i.e. \( H_1^{-1} \lesssim M_W^{-1} \), so \( H/H_1 \) will necessarily be negligible during nucleosynthesis. Thus the nucleosynthesis constraint \((8)\) translates into the requirement

\[ \beta < 0.13 \]; (14)

the inequality is reinforced by the fact that \( x \) as defined by us (eq.(2)) is always bigger than \( x' \). This conflicts with the lower limit on \( \beta \) \([13]\) following from the requirement of a sufficiently long age for the universe:

\[ \beta \geq 0.21 \], (15)

thus excluding this model \([13]\).

The model proposed by Wetterich \([14]\) involves a scalar field \( \phi \) with the potential

\[ V \propto \exp \left( -\frac{4\sqrt{\pi}}{M_{Pl}} a\phi \right), \]

(16)

where \( a > 0 \) is a parameter. Such exponential potentials may well arise in models of unification with gravity such as Kaluza-Klein theories, supergravity theories or string theories. The scalar field couples to gravity and its energy-momentum tensor induces a time-dependent cosmological “constant”. The model further contains a coupling of the scalar field to matter \( \propto \beta \rho \), where \( \beta \) is another model parameter which may, in principle, have different values during the matter-dominated and radiation-dominated eras. Wetterich assumes that \( \beta_r \) is zero and finds that \( \beta_m \leq 0 \) from considerations of the stability of condensed objects with a static energy density. After some critical transition time \( t_{tr} \), the cosmological “constant” adjusts itself dynamically to become proportional to the energy density \( \rho \) (of radiation plus matter), and all energy densities (\( \rho, V, \dot{\phi}^2/2 \)) decay \( \propto t^{-2} \). Wetterich considers two possible scenarios for the initial epoch before this transition. In the first, the universe is initially \( \rho \)-dominated and the scalar field contributions do not influence the time evolution of the scale factor. In the second, the universe starts off being \( \phi \)-dominated; this would lead to an exponential increase of the scale factor as in vacuum energy driven inflation, although without any subsequent entropy production. As an alternative to the above possibilities, a purely scalar-gravity system is also considered; this necessitates additional fine-tuning to evade the severe experimental bounds on such a theory. Thus only the first case, viz. initial \( \rho \)-domination, appears to be well motivated.

\[ ^2 \text{This is unappealing since the horizon and flatness problems of the universe cannot be solved if there is no such entropy production [28].} \]
The parameter $\beta$ can now be constrained as follows. In the matter-dominated era for distances much smaller than $H^{-1}$, the coupled system of gravity and small scalar fluctuations around the cosmological value $\phi(t)$ is identical to the standard Brans-Dicke (BD) theory. Experimental bounds on the Brans-Dicke parameter $\omega$ then constrain $\beta_m$ to be small \[14\]:

$$|\beta_m| < 0.016 .$$

If in fact $\beta_m$ is identically zero, then the contribution of the scalar field today is

$$\Omega_{\phi_0} = \frac{3}{2a^2} = 1 - \Omega_{m_0} .$$

Thus $\Omega_{\phi_0} > 0$ corresponds to $a^2 > 3/2$. We can obtain a stronger bound on $a$ if the transition time $t_{\text{tr}}$ is smaller than the BBN epoch so that the increased total energy density during BBN due to the scalar contributions speeds up the expansion as $t \to t' = \xi^{-1}t$. The recently revised \[17\] upper limit on such a speed-up factor $\xi$ is

$$\xi < 1.12 ,$$

corresponding to the conservative abundance bounds in eq.(7).\[4\] This translates into the lower bound

$$a^2 > 9.9 ,$$

where we have used the relation $\xi^2 - 1 = 2/|a^2 - 2|$. In turn this requires

$$\Omega_{m_0} > 0.85 (21)$$

at the present epoch, i.e. the present cosmological constant contribution must be quite small. A possible escape route is to assume $\beta_m \neq 0$, but this leads to a time dependence of the nucleon mass and, possibly, other fundamental constants (which can also be constrained by nucleosynthesis if the model is made explicit enough). Even so the modification to the age of the universe, $t_0 = \frac{2}{3}H_0^{-1}(1 - \frac{\beta}{a})$, is negligibly small, removing the major motivation for the scenario. This conclusion may however be evaded by assuming that the parameter $a$ is not constant but varies with the curvature scalar or the value of the scalar field. Another escape route would be to assume that $t_{\text{tr}}$ is larger than the BBN epoch, so the expansion rate at this time is unaltered. It is evident that the requirement of successful nucleosynthesis imposes interesting constraints on the possibilities considered in ref. \[14\].

Moffat \[15\] considers a model in the context of a BD theory where the non-minimal coupling of a scalar field to gravity allows for a negative pressure associated with the kinetic energy of the field. The cosmological “constant” is assumed to be space-time dependent and is equated to the kinetic energy in the BD theory. As a result the evolutionary equation for the scalar field in a FRW universe produces an attractor mechanism which drives $\Lambda$ towards a minimum of the potential and leads to a small constant value for $\Lambda$ today. In this model Moffat finds a solution to the problem of the age of the universe with $\Omega_{\nu}$ several times

\[3\]Wetterich \[14\] takes this limit to be $\xi^2 - 1 < 0.1$, which then requires $a^2 > 22$. 

8
larger than $\Omega_m$ at present. However, the constraint (19) on the speed-up factor $\xi$ during nucleosynthesis requires $\Omega_v \lesssim 0.1$, thus removing the motivation for this model.

Berman [16] assumes $\Lambda \propto t^{-2}$ in a BD theory, leading to a possible time variation in the gravitational constant $G_N$. If, in fact, $G_N$ is constant, the vacuum energy density equals the energy density in radiation, so that $x$ as defined in eq.(1) is 0.5 and thus in conflict with the nucleosynthesis constraint (8). On the other hand, if $G_N$ varies with time, it must do so in this model as $G_N \propto t^{-1}$ during matter-domination and as $G_N \propto t^2$ during radiation-domination [16]. Such a strong time variation is ruled out given that the value of $G_N$ during nucleosynthesis is required to be within about 25% of its present value [24].

IV. CONCLUSIONS

We have examined proposed models of a time-varying cosmological “constant” which can significantly increase the expansion age of the universe, even for a large Hubble parameter today, thus evading the potential age problem. However, these models also imply a deviation from the standard expansion history during primordial nucleosynthesis. We find that even conservative observational limits on the abundances of the synthesized light elements imply severe constraints on such models, demonstrating the power of this cosmological probe.

ACKNOWLEDGMENTS

M.B. gratefully acknowledges financial support from the Fellowship HSP II/AUFE of the German Academic Exchange Service (DAAD). S.S. acknowledges a PPARC Advanced Fellowship and support from the EC Theoretical Astroparticle Network.

APPENDIX:

As mentioned earlier, the ratio of neutrino to photon temperature continues to decrease even after $e^+ e^-$ annihilation. To study this, we follow the standard procedure [25] of combining the equations describing the change in entropy and energy conservation to obtain

$$S(T) - S(T_i) = \frac{4\pi^2 x}{15(x - 1)} \int_{T_i}^{T} d\tilde{T} \tilde{T}^3 \tilde{T}^2 ,$$

where $\rho_v = x \rho_\gamma/(1 - x)$ and $\rho_\gamma = \pi^2 T^4/15$. Inserting the expressions for $S(T)$ and $S(T_i)$,

$$S(T) = R^3(\rho + p)/T,$$

we can rewrite the above equation to obtain

$$T_\nu^3 = T^3 \left( \frac{4}{11} \right) \left[ 1 + \frac{45}{2\pi^4} \left( T^{21} + \frac{1}{3} I^{03} \right) \right] + \frac{12x}{11(1 - x)R^3} \int_{T_i}^{T} d\tilde{T} \tilde{T}^3 \tilde{T}^2 ,$$

where $I^{nn} \equiv \int_{m_e/T}^{\infty} dy y^n \left[ y^2 - \left( \frac{m_e}{T} \right)^2 \right]^{n/2} \frac{1}{e^y + 1}$. The first term in the sum above tracks the standard behaviour of the neutrino temperature during $e^+ e^-$ annihilation. The numerical solution of this equation is shown in Figure 2 for
several values of the parameter $x$; note that the curves converge at the neutrino decoupling temperature.

Freese et al. did not take into account the differing neutrino and photon temperatures or the change in the number of degrees of freedom due to $e^+e^-$ annihilation, hence their relation for the nucleon-to-photon ratio, $\eta = \frac{n_N}{n_\gamma} \propto T^{3x/(1-x)}$, only holds after this epoch. Between neutrino decoupling and $e^+e^-$ annihilation, the correct relation should be

$$\eta \propto T^{12x/11(1-x)}.$$  \hspace{1cm} (A4)

Similarly, the equation for $R(T)$ as given by Freese et al., viz. $R \propto T^{1/(x-1)}$, which holds only at $T \ll m_e$, is modified to

$$R \propto T^{(11-7x)/11(x-1)}$$ \hspace{1cm} (A5)

between neutrino decoupling and $e^+e^-$ annihilation.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2}
\caption{The evolution of the neutrino temperature after decoupling from kinetic equilibrium for various assumed values of the decaying vacuum energy parameter $x \equiv \rho_\nu/(\rho_\gamma + \rho_\nu)$.}
\end{figure}
REFERENCES

[1] L.M. Krauss and M.S. Turner, Preprint FERMILAB-Pub-95/063-A.
[2] M. Fukugita, C. Hogan and P.J.E. Peebles, Nature 366 (1993) 309;
   R.C. Kennicutt, W.L. Freedman and J.R. Mould, Astron. J. 110 (1995) 1476.
[3] M. Bolte and C.J. Hogan, Nature 376 (1995) 399;
   B. Chaboyer, P. Demarque, P.J. Kernan and L.M. Krauss, Science 271 (1996) 957.
[4] P.J.E. Peebles, Physical Cosmology (Princeton University Press, 1993).
[5] G.P. Efstathiou, W.J. Sutherland and S.J. Maddox, Nature 348 (1990) 705.
[6] S.D.M. White, J.F. Navarro, A.E. Evrad and C.S. Frenk, Nature 366 (1993) 429.
[7] A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic, 1990).
[8] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1;
   S. Carroll, W.H. Press and E.L. Turner, Annu. Rev. Astron. Astrophys. 30 (1992) 499.
[9] K. Freese, F.C. Adams, J.A. Frieman and E. Mottola, Nucl. Phys. B287 (1987) 797.
[10] M. ¨Ozer and M.O. Taha, Phys. Lett. B171 (1986) 363, Nucl. Phys. B287 (1987) 776;
     P.J.E. Peebles and B. Ratra, Astrophys. J. 325 (1988) L17;
     B. Ratra and P.J.E. Peebles, Phys. Rev. D37 (1988) 3406;
     W. Chen and Y.-S. Wu, Phys. Rev. D41 (1990) 695;
     A.-M. Abdel-Rahman, Phys. Rev. D45 (1992) 3497;
     V. Siveira and I. Waga, Phys. Rev. D50 (1994) 4890;
     J.L. Lopez and D.V. Nanopoulos, Mod. Phys. Lett. A11 (1996) 1.
[11] M. Gasperini, Phys. Lett. B194 (1987) 347.
[12] M. Reuter and C. Wetterich, Phys. Lett. B188 (1987) 38;
     Y. Fujii and T. Nishioka, Phys. Rev. D42 (1990) 361, Phys. Lett. B254 (1991) 347;
     M.O. Calvao, H.P. de Oliveira, D. Pavon and J.M. Salim, Phys. Rev. D45 (1992) 3869;
     J.C. Carvalho, J.A.S. Lima and I. Waga, Phys. Rev. D46 (1992) 2404;
     I. Waga, Astrophys. J. 414 (1993) 436;
     T. Damour and K. Nordtvedt, Phys. Rev. D48 (1993) 3436;
     J. Matygasek, Phys. Rev. D51 (1995) 4154;
     Y. Fujii, Preprint [gr-qc/9508029, gr-qc/9602030].
[13] J.A.S. Lima and J.M.F. Maia, Phys. Rev. D49 (1994) 5597;
     J.A.S. Lima and M. Trodden, Phys. Rev. D53 (1996) 4280.
[14] C. Wetterich, Astron. Astrophys. 301 (1995) 321.
[15] J.W. Moffat, Phys. Lett. B357 (1995) 526.
[16] M.S. Berman, Phys. Rev. D43 (1991) 1075.
[17] S. Sarkar, Rep. Prog. Phys. (in press) hep-ph/9602260.
[18] R.V. Wagoner, Astrophys. J. 179 (1973) 343;
     L. Kawano, Preprint FERMILAB-Pub-92/04-A.
[19] Particle Data Group, Phys. Rev. D50 (1994) 1173.
[20] Y.I. Izotov, T.X. Thuan and V.A. Lipovetsky, Astrophys. J. 435 (1994) 647.
[21] C.J. Hogan, Preprint astro-ph/9512003.
[22] P.J. Kernan and L.M. Krauss, Phys. Rev. Lett. 72 (1994) 3309.
[23] Y. Hu, M.S. Turner and E.J. Weinberg, Phys. Rev. D49 (1994) 3830.
[24] E.W. Kolb, M.J. Perry and T.P. Walker, Phys. Rev. D33 (1986) 869;
     F.S. Accetta, L.M. Krauss and P. Romanelli, Phys. Lett. B248 (1990) 146.
[25] S. Weinberg, Gravitation and Cosmology (John Wiley, New York, 1972).