Temperature variation of ultraslow light in a cold gas

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A model is developed to explain the temperature dependence of the group velocity as observed in the experiments of Hau et al. [Nature (London) 397, 594 (1999)]. The group velocity is quite sensitive to the change in the spatial density. The inhomogeneity in the density and its temperature dependence are primarily responsible for the observed behavior.

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I. INTRODUCTION

The phenomenon of Bose-Einstein condensation in atomic gases [1,2] lends itself to the study of many fundamental effects. Among them, one aspect presently being investigated both theoretically and experimentally is the interaction of light with atoms in the quantum degeneracy regime [2,3]. In this context, the propagation of light inside a cold gas is still an open problem. Because of the optical density, it is well known that the transmission of resonant light through a condensate is almost zero [3]. However, electromagnetically induced transparency (EIT) [4] was found to allow the propagation of light by means of quantum coherence between different internal atomic levels [5,6]. In this context, Hau et al. discovered a remarkable property of pulse propagation in a Bose condensate. These authors demonstrated the slowing down of the group velocity of the pulse to 17 m/sec [7]. Furthermore, they have shown a definite dependence of the group velocity on the temperature of the ultracold sample. One would like to understand the observed temperature dependence from first principles. For this purpose, it is necessary to extend the standard theory of EIT to a cold gas at finite temperature. However, a theoretical description of this problem is rather complex. Complexities arise when one attempts a systematic treatment of interactions, finite temperature effects, and dynamics. Most studies treat these aspects as disjoint: interactions are included in the zero-temperature effects, and dynamics. Most studies treat these aspects together. However, a complete theory of the interaction of light and interacting particles is still unavailable, and a full numerical treatment is a rather hard task. Here, we present approximate but plausible arguments to explain the experimental observations in [7]. The simplicity of our model allows for an analytical expression for the group velocity in the following cases: atoms confined in a box and by a harmonic potential. We obtain results that reproduce the ones in [7] for $T>T_c$. In particular, the treatment brings out the factors playing key roles in the phenomenon. Here we show that the variation of spatial density of atoms with temperature is the major factor responsible for the temperature dependence of the group velocity.

The paper is organized as follows: In Sec. II the model is introduced. In Sec. III we derive the group velocity of a pulse propagating in an ideal gas confined inside a box, extend the calculation to the case of an ideal gas in a harmonic-oscillator potential, and present and discuss the results in relation to the experiment of Hau et al. In Sec. IV we present estimates for the group velocity in the interacting case and in the limit of zero temperature.

II. MODEL

In this section we introduce the model used throughout this article. Here we write the Maxwell-Bloch equations that describe the dynamics of the system consisting of light field and atoms. We derive the linear response of the medium to a weak probe field, taking into account the quantum statistics of the atoms. The group velocity is then defined in the standard manner [12].

1. Maxwell-Bloch equations

We consider a gas of $N$ noninteracting bosons. The relevant internal structure corresponds to a three-level atom, with internal levels $|g\rangle$ (stable state), $|r\rangle$ (metastable state), and $|e\rangle$ excited state, whose energies are $\omega_g$, $\omega_r$, and $\omega_e$, respectively (see Fig. 1). The radiative decay rate of the excited state is $\gamma=\gamma_g+\gamma_r$, with $\gamma_g(\gamma_r)$ the rate of decay on the transition $|e\rangle\rightarrow|g\rangle(|e\rangle\rightarrow|r\rangle)$. Laser light with frequency $\omega_{le}$ and wave vector $k_e$ drives the transition $|g\rangle\rightarrow|e\rangle$, whereas the transition $|r\rangle\rightarrow|e\rangle$ is driven by a field of frequency $\omega_{lr}$ and wave vector $k_r$. The dynamics of the whole system is given by the Maxwell equation for the electric field vector $\mathbf{E}$.

![FIG. 1. Level scheme.](image-url)