Invisible $K_L$ decays in the SM extensions

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Abstract

In the Standard Model (SM) the branching ratio for the decay $K_L \to \nu \bar{\nu}$ into two neutrinos is helicity suppressed and predicted to be very small $\leq O(10^{-17})$. We consider two natural extensions of the SM, such as two-Higgs-doublet model (2HDM) and the $\nu$MSM with additional singlet scalar, those main features is that they can lead to an enhanced $\text{Br}(K_L \to \text{invisible})$. In the 2HDM the smallness of the neutrino mass is explained due to the smallness of the second Higgs doublet vacuum expectation value. Moreover, the $\nu$MSM extension with additional singlet field can explain the $(g - 2)$ muon anomaly. The considered models demonstrate that the $K_L \to \text{invisible}$ decay is a clean probe of new physics scales well above 100 TeV, that is complementary to rare $K \to \pi + \text{invisible}$ decay, and provide a strong motivation for its sensitive search in a near future low-energy experiment.

Key words: Neutral kaon, invisible decays, New Physics

1 Introduction

The discovery of the neutrino oscillations [12] means that at least two neutrino have nonzero masses. The minimal extension of the SM with nonzero neutrino masses is the $\nu$MSM model [34]. In this model one adds to the SM three additional massive Majorana(sterile) fermions $\nu_{Ri}$, $i = 1, 2, 3$. Due to the seesaw mechanism [35] after the spontaneous $SU_L(2) \otimes U(1)$ electroweak symmetry breaking the neutrinos acquire masses $m_{\nu_i} = \frac{m_{D_i}^2}{M_{Ri}}$. Here $m_{D_i}$ are the Dirac neutrino masses and $M_{R_i}$ are the Majorana masses of the sterile $\nu_{Ri}$ neutrinos. The $\nu$MSM with relatively light neutrino Majorana masses $M_{\nu_{Ri}} \leq O(5)$ GeV has a candidate - the lightest neutrino Majorana with a mass $M_{\nu_R} \leq O(50)$ KeV - for dark matter. Besides, the model with light Majorana neutrino can solve the problem of the baryon asymmetry in our Universe [4].

If masses of Majorana neutrinos are lying in sub-GeV region, the decays of neutral mesons into neutrino pairs, such e.g. as $\pi^0, \eta, \eta', K_S, K_L \to \nu \bar{\nu}$ decays
can occur. Since the neutrinos are weekly interacting particles and do not interact in the detector, such decays are invisible, thus making their searches extremely difficult task. In particular, the decay $K_L \rightarrow invisible$ has never been experimentally tested. The branching ratio of the $K_L \rightarrow invisible$ decay in the SM is predicted to be very small $\leq O(10^{-17})$ for $\nu$ masses laying in the sub-eV region favored by the observations of $\nu$ oscillations \cite{12}. Indeed, the $K_L$ has zero spin, and it cannot decay into two massless neutrinos, as it contradicts to momentum and angular momentum conservation simultaneously. Therefore, an observed $Br(K_L \rightarrow invisible) \geq 10^{-10}$ would unambiguously signal the presence of the BSM physics. Recently, an approach for performing such kind of experiments by using the $K^+ n \rightarrow K^0 p$ (or $K^- p \rightarrow \bar{K}^0 n$ ) charge-exchange reaction as a source of well tagged $K^0$'s has been reported and the first experimental bound $Br(K_L \rightarrow invisible) \lesssim 6.3 \cdot 10^{-4}$ has been set from existing experimental data \cite{6}. It has been shown, that compared to this limit, the expected sensitivity of the proposed search is at least two orders of magnitude higher - $Br(K_L \rightarrow invisible) \lesssim 10^{-6}$ per $\simeq 10^{12}$ incident kaons. This limit can be further improved by utilizing a more detailed design of the experiment, thus making the region $Br(K_L \rightarrow invisible) \simeq 10^{-8} - 10^{-6}$, and even below, experimentally accessible \cite{6}. In Ref.\cite{7} we considered several natural extensions of the SM, such as two-Higgs-doublet (2HDM), 2HDM and light scalar, and mirror dark matter models, those main feature is that they allow to avoid the helicity suppression factor for the previously mentioned pseudoscalar mesons decays into neutrino and lead to an enhanced $Br(K_L \rightarrow invisible)$.

In this note, which is a continuation of the work of Ref. \cite{7}, we consider two SM extensions that can lead to invisible $K_L$ decays at an experimentally interesting level $Br(K_L \rightarrow invisible) \geq O(10^{-8})$. Namely, we consider the 2HDM and the $\nu$MSM with additional scalar isosinglet field, those main feature is that they can lead to an enhanced $Br(K_L \rightarrow invisible)$. In the 2HDM the smallness of the neutrino mass is explained due to the smallness of the second Higgs doublet vacuum expectation value. The nonzero and very small value of the second Higgs doublet can arise as a consequence of nonzero quark condensate. Moreover, the $\nu$MSM extension with additional singlet field is able to explain the observed $(g - 2)$ muon anomaly. The considered models demonstrate that the $K_L \rightarrow invisible$ decay is a clean probe of new physics scales well above 100 TeV, that is complementary to rare $K \rightarrow \pi + invisible$ decay, and provide a strong motivation for its sensitive search in a near future experiment.

2 $K_L \rightarrow \nu \bar{\nu}$ decay in the two-Higgs-doublet model

Consider the $K_L \rightarrow \nu \bar{\nu}$ decay in the two-Higgs-doublet model (2HDM). The 2HDM can have tree level flavor-changing neutral currents, provide explanation of the origin of Dark Matter and $CP$ violation , see e.g. ref.\cite{8}. The
The decay rate $K_L \rightarrow \nu_{R1}\bar{\nu}_{R1}, \nu_{L1}\bar{\nu}_{L1}$ is determined by the formula

$$\Gamma(K_L \rightarrow \nu_{L1}\bar{\nu}_{L1}, \nu_{R1}\bar{\nu}_{R1}) = \frac{M_{K_L}^5}{16\pi M_X^4} \frac{F_K}{2(m_d + m_s)^2} K\left(\frac{m_{R1}^2}{M_{K_L}^2}\right),$$

where

$$\frac{1}{M_X^2} = \frac{|(h_{Qd,12}^2 + h_{Qd,21})h_{L1}|^2}{M_{H_2}^4}$$

and $K(x) = (1 - x)^2$ for Majorana neutrino $\nu_{R1}$ with a mass $m_{R1}$ and massless $\nu_{L1}$ neutrino. Here $F_K \approx 160 \text{ MeV}$ is kaon decay constant and $m_s, m_d$ are the masses of $s$- and $d$-quarks.\footnote{The quark masses $m_d, m_s$ and the effective mass $M_X$ implicitly depend on the

The Lagrangian of our variant of the 2HDM has the form

$$L_{tot} = L_{SM} + L_{H^2\nu} + L_{HH},$$

where

$$L_{H^2\nu} = -h_{Qd,ij}\bar{Q}_L H_2 d_{Rj} + H.c.,$$

$$L_{HH} = \Delta^\mu H^*_2 \Delta^\mu H_2 - M_{H_2}^2 H_2^* H_2 - \lambda_2 (H^*_2 H_2)^2 + (\delta m^2_{H_2 H_2} H^*_2 H_2 + H.c.)$$

and $L_{SM}$ is the SM Lagrangian. Here $Q_{L1} = (u_L, d_L)$, $Q_{L2} = (c_L, s_L)$, $Q_{L3} = (t_L, b_L)$, $d_{R1} = d_R$, $d_{R2} = s_R$, $d_{R3} = b_R$, $H_2 = (H^0_2, H^+_2, H^0_2, H^+_2)$. The smallness of the Dirac neutrino masses is a consequence of the $< H_2 >$ smallness. We can choose the basis in which the Majorana mass matrix and the Yukawa coupling constants $h_{L\nu,ij}$ are diagonal: $M_{\nu,ij} = M_{Ri} \delta_{ij}$, $h_{L\nu,ij} = h_{L\nu,ij} \delta_{ij}$ and $M_{R1}$ has the minimal value. The value $M_{R1} = 0$ corresponds to the case of Dirac neutrino. Since we are interested mainly in the $K_L \rightarrow invisible$ decay we assume that the decay $K_L \rightarrow \nu_{L1}\bar{\nu}_{R1}, \nu_{R1}\bar{\nu}_{L1}$ is kinematically allowed.

The effective four fermion Lagrangian describing the decay $K_L \rightarrow \nu_{R1}\bar{\nu}_{L1}, \nu_{L1}\bar{\nu}_{R1}$ has the form

$$L_{eff} = \frac{1}{M_{H_2}^2} \left[ h_{Qd,12} h_{L1} \bar{d}_L s_R \bar{\nu}_{L1}\nu_{R1} + h_{Qd,21}^* h_{L1}^* \bar{d}_R s_L \bar{\nu}_{R1}\nu_{L1}\right] + H.c..$$

The decay rate $K_L \rightarrow \nu_{R1}\bar{\nu}_{L1}, \nu_{L1}\bar{\nu}_{R1}$ is determined by the formula

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where

$$\frac{1}{M_X^2} = \frac{|(h_{Qd,12}^2 + h_{Qd,21})h_{L1}|^2}{M_{H_2}^4}$$

and $K(x) = (1 - x)^2$ for Majorana neutrino $\nu_{R1}$ with a mass $m_{R1}$ and massless $\nu_{L1}$ neutrino. Here $F_K \approx 160 \text{ MeV}$ is kaon decay constant and $m_s, m_d$ are the masses of $s$- and $d$-quarks.\footnote{The quark masses $m_d, m_s$ and the effective mass $M_X$ implicitly depend on the

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the value of $M_X$ up to $^2$

$$M_X \lesssim 0.6 \cdot 10^5 \text{ GeV}$$  \hspace{1cm} (8)

for small neutrino mass $m_{R1} \ll M_{KL}$. The Yukawa interaction (2) leads to the effective tree level flavour changing $\Delta S = 2$ effective interaction

$$L_{\Delta S=2} = \frac{1}{\Lambda^2_{\Delta S=2}} \bar{d}_L s_R \bar{d}_R s_L + H.c.,$$  \hspace{1cm} (9)

where

$$\frac{1}{\Lambda^2_{\Delta S=2}} = \frac{h_{Qd,12} h^*_{Qd,21}}{M^2_{H2}}.$$  \hspace{1cm} (10)

The measured $K_L - K_S$ mass difference and the CP-violation parameter $\epsilon_K$ strongly restrict $^9$ the effective $\Delta S = 2$ interaction (9), namely $^9$

$$|Re(\Lambda_{\Delta S=2})| \geq 1.8 \cdot 10^7 \text{ GeV},$$  \hspace{1cm} (11)

$$|Im(\Lambda_{\Delta S=2})| \geq 3.2 \cdot 10^8 \text{ GeV}.$$  \hspace{1cm} (12)

We shall consider the case of the CP-conserving interaction (2), i.e $h_{Qd,12}$ and $h_{Qd,21}$ are real. As a consequence of the inequality (11) and the formula (10) we find that

$$M_{H2} \geq 1.8(|h_{Qd,21} h_{Qd,12}|)^{1/2} \cdot 10^7 \text{ GeV}.$$  \hspace{1cm} (13)

It should be noted that the bound (13) restricts rather strongly but not excludes the phenomenologically interesting values of the $M_X$. Really, we can simultaneously avoid the $\Delta S = 2$ bound (13) and obtain phenomenologically interesting values $Br(K_L \rightarrow \nu_{L1} \bar{\nu}_{R1}, \nu_{R1} \bar{\nu}_{L1})$ for small quark Yukawa coupling constants $h_{Qd,12}, h_{Qd,21}$, relatively light second Higgs doublet and not small lepton Yukawa coupling constant $h_{L,1}$. For instance, for $h_{Qd,12} = h_{Qd,21} = (1/300)^2$ $h_{Qd,12} = 9 h_{Qd,21} = 0.5 \cdot 10^{-4}$, $h_{L,1} = 1$ and $M_{H2} = 300 \text{ GeV}$ we find that $\Lambda_{\Delta s = 2} = 2.7 \cdot 10^7 \text{ GeV}(1.8 \cdot 10^7 \text{ GeV})$ and $Br(K_L \rightarrow \nu \bar{\nu}) = 0.4 \cdot 10^{-6}(5 \cdot 10^{-6})$.

The existence of relatively light with a mass $M_{H2} = 300 \text{ GeV}$ second Higgs doublet does not contradict the LHC data. The best way to look for the second Higgs isodoublet at the LHC is the use of the reaction $pp \rightarrow Z^*/\gamma^* \rightarrow H^+_2 H^-_2 \rightarrow l^+ l^- \nu \bar{\nu}$ ($l^- = e, \mu, \tau$). So the signature is two $l^+ l^-$ leptons plus renormalization point $\mu$ but their combination $M^2_{\mu}(m_d + m_s)$ and hence the decay width (6) is renormalization group invariant and does not depend on the renormalization point $\mu$.

$^2$ In our estimate (8) we used the values $\tau(K_L) = 5.17 \cdot 10^{-8} \text{ sec}$ and $(m_d + m_s) = 160 \text{ MeV}$
nonzero $E_{\text{miss}}^T$ in final state that coincides with the signature used for the search for direct production of sleptons at the LHC.

In considered model the neutrino $\nu_1$ acquires nonzero Dirac mass $m_{\nu_1} = h_{L,1} < H_2 >$ due to nonzero vacuum expectation value of the second Higgs isodoublet $< H_2 > \approx \frac{\delta m^2_{HH_2}}{M^2_{H_2}} < H >$ ($< H > = 174$ GeV). The smallness of the Dirac neutrino mass is a consequence of the $< H_2 >$ smallness. The smallness of $< H_2 >$ is due to the small value of $\delta m^2_{HH}$\textsuperscript{3}. For instance, for $m_{\nu_1} = 0.1$ eV, $h_{L,1} = 1$, $M_{H_2} = 300$ GeV, $M_{R_1} = 100$ MeV we find $\frac{\delta m^2_{HH_2}}{M^2_{H_2}} = 1.9 \cdot 10^{-8}$ and $\delta m^2_{HH_2} = 1.7 \cdot 10^{-3}$ GeV\textsuperscript{2}. It is interesting to note that for $\delta m^2_{HH_2} = 0$ the second Higgs isodoublet vacuum expectation value $< H_2 > = 0$ at classical level but the spontaneous symmetry breaking of $SU_L(3) \otimes SU_R(3)$ chiral symmetry in QCD leads to nonzero vacuum expectation values for the Higgs fields \textsuperscript{11}. Really, for nonzero Yukawa interaction $L_{H_2Q,d} = h_{Qd,11} \bar{Q}_{1L} H_2 d_R + H.c.$ due to nonzero vacuum expectation value of quark condensate $< \bar{d}d > = - \frac{f^2 m^2}{(m_u + m_d)}$ ($f_\pi = 93$ MeV) the field $< H_2 >$ acquires nonzero vacuum expectation value $< H_2 > = \frac{h_{Qd,11} < \bar{d}d >}{M^2_{H_2}}$. For example, for $h_{L,1} = 1$, $M_{H_2} = 300$ GeV and $h_{Qd,11} = 10^{-3}$ we find\textsuperscript{4} the Dirac neutrino mass $m_{\nu_1} \approx 0.1$ eV. So for the model with $\delta m^2_{HH_2} = 0$ the vacuum expectation value $< H_2 > = 0$ at classical level but nonzero quark condensate leads to the appearance of small vacuum expectation value $< H_2 > \neq 0$ for the second Higgs isodoublet that explains the smallness of the neutrino masses.

It should be noted that the existence of $\Delta S = 1$ neutral flavour changing interaction (2) leads to additional contribution to rare decays $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$. In ref.\textsuperscript{7} the ratio $\beta \equiv \frac{Br_{BSM}(K^+ \to \pi^+ \nu \bar{\nu})}{Br_{BSM}(K_\pi \to \nu \bar{\nu})}$ has been calculated. Here “BSM” means the corresponding contribution beyond the SM. Note that the ratio $\beta$ does not depend on unknown value of $M_X$ and on the values of quark masses $m_d$, $m_s$. Numerically, for small Majorana neutrino mass $\beta \approx 2 \cdot 10^{-3}$\textsuperscript{7}. From the experimental value of $Br(K^+ \to \pi^+ \nu \bar{\nu})$\textsuperscript{12,13} and its theoretical predictions in the SM\textsuperscript{14} one can deduce that for very light $m_{R_1} \ll M_{K_L}$ Majorana neutrino $\nu_1$

$$Br(K_L \to \nu_{L_1} \bar{\nu}_{R_1}, \nu_{R_1} \bar{\nu}_{L_1}) \lesssim 10^{-7}. \quad (14)$$

For higher Majorana mass $m_{R_1}$ the limit (14) is more weak and for the case $M_{K_L} \geq m_{R_1} \geq M_{K^+} - M_{\pi^+}$ when the decay $K^+ \to \pi^+ \nu_{L_1} \bar{\nu}_{R_1}, \pi^+ \nu_{R_1} \bar{\nu}_{L_1}$ is

\textsuperscript{3} In ref.\textsuperscript{10} a model with additional Higgs isodoublet interacting only with lepton fields was proposed. In this model the neutrinos acquire nonzero Dirac masses due to nonzero vacuum expectation value of the second Higgs isodoublet that allows to decrease the seesaw scale from $O(10^{15})$ GeV to $O(10^{3})$ GeV or less.

\textsuperscript{4} In our estimate we use the value $m_d + m_u = 11$ MeV.
kinematically prohibited but the decay $K_L \rightarrow \nu_{L1} \bar{p}_{R1}$, $\nu_{R1} \bar{p}_{L1}$ is still allowed, the restriction from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay does not work.

3. $K_L \rightarrow \nu \bar{\nu}$ decay in the $\nu$MSM extension with additional scalar isosinglet

In this section we discuss the $K_L$ decay into neutrinos in the $\nu$MSM extension with additional scalar isosinglet field $\phi$.

The Lagrangian of the model has the form

$$L_{tot} = L_{SM} + L_{Qd\phi} + L_{\nu\phi} + L_{\nu R}.$$  \hspace{1cm} (15)

Here

$$L_{Qd\phi} = -\frac{h_{Qd\phi ij}}{M} \bar{Q}_{Li} \hat{\bar{H}} \phi d_{Rj} + H.c.,$$  \hspace{1cm} (16)

$$L_{\nu\phi} = \frac{1}{2} \bar{\partial} \mu \phi \partial^\mu \phi - \frac{M^2 \phi^2}{2} - \lambda \phi^4 - (\frac{\kappa_i}{2} \phi \nu_{Ri} \nu_{Ri} + H.c.),$$  \hspace{1cm} (17)

$$L_{\nu R} = i \bar{\nu}_{Rj} \hat{\partial} \nu_{Rj} - (\frac{M_{Rj}}{2} \nu_{Rj} \nu_{Rj} + h_{Lij} \bar{L}_i H \nu_{Rj} + H.c.),$$  \hspace{1cm} (18)

where $L_{SM}$ is the SM Lagrangian and $L_1 = (\nu_{eL}, e_L)$, $L_2 = (\nu_{\mu L}, \mu_L)$, $L_3 = (\nu_{\tau L}, \tau_L)$, $H = -(H^-)^*, (H^0)^*$.

After the spontaneous $SU_L(2) \otimes U(1)$ gauge symmetry breaking the Yukawa interaction of quarks with singlet field $\phi$ is

$$L_{d\phi} = -\bar{h}_{Qd\phi ij} \bar{Q}_{Li} \hat{\bar{H}} \phi d_{Rj} + H.c.,$$  \hspace{1cm} (19)

where

$$\bar{h}_{Qd\phi ij} = h_{Qd\phi ij} \times \frac{<H>}{M}$$  \hspace{1cm} (20)

and $<H> = 174$ GeV.

Note that the interaction (16) is nonrenormalizable. We can consider it as some effective interaction. For instance, the effective interaction (16) can be realized in renormalizable extension of the SM model with additional scalar field $\phi$ and new massive quark $SU(2)_L$ singlet fields $D_R$, $D_L$ with a mass $M_D$ and $U(1)$ hypercharges $Y_{D_L} = Y_{D_R} = -\frac{1}{3}$. The interaction of new quark fields $D_R$, $D_L$ with ordinary quarks and the neutral scalar field $\phi$ is

$$L_{qD\phi} = -c_i \bar{Q}_{Li} \bar{H} D_R - k_j \bar{D}_L d_{Rj} \phi + H.c..$$  \hspace{1cm} (21)

\footnote{Here $H = (H^0, H^-)$ is the SM Higgs doublet.}
In the heavy $D$-quark mass limit $M_D \to \infty$ we obtain the effective interaction (16) with
\[
\frac{h_{Qd,ij}}{M} = \frac{c_i k_j}{M_D}.
\]
(22)

Since we are interested in the $K_L \to$ invisible decays we assume that at least one Majorana neutrino mass $M_{R1}$ is lighter than $\frac{M_{KL}}{2}$. The effective Lagrangian describing the decay $K_L \to \nu_{R1}\bar{\nu}_{R1}$ is
\[
L_{\text{dsnu}} = \frac{\kappa_1}{2M_\phi^2}(\tilde{h}_{Q\phi,12}\bar{d}_L s_R + \tilde{h}_{Q\phi,21}^* \bar{d}_R s_L + H.c.)(\nu_{R1}\nu_{R1} + \bar{\nu}_{R1}\bar{\nu}_{R1}).
\]
(23)

The invisible decay $K_L \to \nu_{R1}\bar{\nu}_{R1}$ width is determined by formula
\[
\Gamma(K_L \to \nu_{R1}\bar{\nu}_{R1}) = \frac{M_{K_L}^5}{16\pi M_X^4} \left( \frac{F_K}{2(m_d + m_s)} \right)^2 K(M_{R1}^2/M_{KL}^2),
\]
where $K(x) = (1 - 4x)^{1/2}$ and
\[
\frac{1}{M_X^4} = \left| \frac{\kappa_1^2 (\tilde{h}_{Q\phi,12} + \tilde{h}_{Q\phi,21}^* - \tilde{h}_{Q\phi,12}^* - \tilde{h}_{Q\phi,21})^2}{2M_\phi^4} \right|.
\]
(24)

For $\tilde{h}_{Q\phi,12} = -\tilde{h}_{Q\phi,21}^*$ (the matrix $\tilde{h}_{Qd,ij}$ is antihermitean) the Lagrangian (23) takes the form
\[
L_{\text{dsnu}} = \frac{\kappa_1}{2M_\phi^2}(\tilde{h}_{Q\phi,12}\bar{d}_{\gamma_5}s - \tilde{h}_{Q\phi,12}^* \bar{s}_{\gamma_5}d)(\nu_{R1}\nu_{R1} + \bar{\nu}_{R1}\bar{\nu}_{R1})
\]
(26)

The Lagrangian (26) does not contain scalar quark bilinear terms $(\bar{d}s + \bar{s}d)(\nu_{R1}\nu_{R1} + \bar{\nu}_{R1}\bar{\nu}_{R1})$. As a consequence the decays of the $K$ mesons into pions and sterile neutrinos $K^+ \to \pi^+\nu_R\bar{\nu}_R$, $K_L \to \pi^0\nu_R\bar{\nu}_R$ are absent at least in the leading order on $|\tilde{h}_{Q\phi,12}|^2$. Therefore the kaon decays $K^+ \to \pi^+\nu_R\nu_R$, $K_L \to \pi^0\nu_R\nu_R$ don’t restrict the invisible $K_L \to \nu_R\bar{\nu}_R$ decay.

For $\tilde{h}_{Q\phi,12} = -\tilde{h}_{Q\phi,21}^*$ the exchange of singlet scalar field leads to the tree level $\Delta S = 2$ interaction
\[
L_{\Delta S=2} = \frac{1}{4\Lambda_{\Delta S=2}^2} \bar{d}_{\gamma_5}s\bar{s}_{\gamma_5}s + H.c.,
\]
(27)
where
\[
\frac{1}{4\Lambda_{\Delta S=2}^2} = \frac{\tilde{h}_{Q\phi,12}^2}{2M_\phi^2}.
\]
(28)
The $\Delta S = 2$ bound (11) restricts rather strongly but not excludes the phenomenologically interesting values of the $M_X$. For instance, for $M_\phi = 100 \text{ GeV}$, $M_{R_1} = 50 \text{ KeV}$, $h_{Q,d_{12}} = 0.25i \cdot 10^{-5}$ and $\kappa_1 = 1$ we find that $|\Lambda_{\Delta S=2}| = 2.8 \cdot 10^7 \text{ GeV}$ and $Br(K_L \rightarrow \nu \nu) \approx 6.5 \cdot 10^{-6}$. So we have demonstrated that the extension of the $\nu$MSM with additional isosinglet scalar field can lead to the existence of the $K_L \rightarrow \text{invisible}$ decay with the phenomenologically interesting values of the $Br(K_L \rightarrow \text{invisible}) \geq 10^{-8}$ without contradiction with $\Delta S = 2$ bound (11).

In this section we have considered the $K_L$ decay into sterile neutrino. It is possible instead of sterile neutrino to introduce light dark matter (fermionic or scalar) and consider the $K_L$ decay into dark matter particles. For instance, for $h_{L_{ij}} = 0$ in formula (18) we can consider the $\nu_{R_1}$ as stable dark matter particle not directly related with left handed neutrinos $\nu_L$. Instead of sterile neutrino $\nu_R$ we can introduce additional light scalar field $\chi$. For the interaction

$$L_{\phi \chi} = \lambda_{\phi \chi} \phi \chi^2$$

the invisible $K_L \rightarrow \chi \chi$ decay can occur if $M_\chi < \frac{M_{K_L}}{2}$. If the $\chi$ is stable, it can play the role of light dark matter.

Note that the $\nu$MSM with additional scalar field can explain the observed $(g-2)$ anomaly [17] if we assume the existence of additional nonzero interaction of the scalar $\phi$ with charged leptons, namely:

$$L_{l \phi H} = -\bar{h}_{L e, i, i} \bar{e}_L \phi e_{R_i} + H.c.$$  

Here $e_{R_1} = e_R$, $e_{R_2} = \mu_R$, $e_{R_3} = \tau_R$. After the spontaneous $SU(2)_L \otimes U(1)$ electroweak symmetry breaking the Yukawa interaction of the scalar field with charged leptons takes the form

$$L_{l \phi H} = -\bar{h}_{L e, i, i} \bar{e}_L \phi e_{R_i} + H.c,$$

where

$$\bar{h}_{L e, i, i} = h_{L e, i, i} \frac{H}{M_L}$$

and $e_{R_1} = e_L$, $e_{R_2} = \mu_L$, $e_{R_3} = \tau_L$. Note that in the SM the renormalizable lepton-Higgs Yukawa interaction

$$L_{lH} = -h_{L_{ij}} \bar{L}_i H e_{R_j} + H.c.$$  

lead to nonzero lepton masses due to nonzero Higgs doublet vacuum expectation value $<H> \neq 0$. Consider the model with zero $h_{L_{ij}} = 0$ renormalizable Yukawa couplings.\footnote{We can impose the discrete symmetry $e_{R_i} \rightarrow -e_{R_i}$, $\Phi \rightarrow -\Phi$ to suppress the renormalizable interaction (33).} For such model non zero vacuum expectation of the real
field Φ produce nonzero lepton masses, namely

\[ m_{Li} = \bar{h}_{Le,ii} < \Phi > \]  

(34)

Consider the case of real \( \bar{h}_{Le,ii} \) coupling constants. The additional one loop contribution to muon magnetic moment due to \( \phi \) scalar exchange is [18]

\[ \Delta a_\mu = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_\Phi^2} \bar{h}_{Le,22}^2 \left[ \ln \left( \frac{M_\Phi}{m_\mu} \right) - \frac{7}{12} \right], \quad (M_\Phi >> m_\mu). \]  

(35)

The precise measurement of the anomalous magnetic moment of the positive muon from the Brookhaven AGS experiment [17] gives a result which is 3.6σ higher than the Standard Model (SM) prediction, namely

\[ a_\mu^{exp} - a_\mu^{SM} = (288 \pm 80) \cdot 10^{-11}, \]  

(36)

where \( a_\mu \equiv \frac{g_\mu - 2}{2} \). Using the formulae (35, 36) we find that for \( m_\Phi = 100 \text{ GeV} \) (1 GeV) the muon \( g - 2 \) anomaly can be explained if \( \bar{h}_{Le,22}^2 \neq 0 \), namely

\[ \bar{h}_{Le,22}^2 = (1.6 \pm 0.44) \cdot 10^{-2} \quad \text{for} \quad m_\Phi = 100 \text{ GeV}, \]  

(37)

\[ \bar{h}_{Le,22}^2 = (5.9 \pm 1.6) \cdot 10^{-6} \quad \text{for} \quad m_\Phi = 1 \text{ GeV}. \]  

(38)

As in the SM the Yukawa couplings \( h_{Le,ii} \) are proportional to the lepton masses. As a consequence the interaction of the \( \Phi \) scalar with electrons is weaker than the interaction of the \( \Phi \) scalar with muons by factor \( m_\mu/m_e \approx 200 \) and the contribution of the \( \Phi \) scalar to the electron magnetic moment is suppressed at least by factor \( (m_e/m_\mu)^2 \) in comparison to the muon magnetic moment even for superlight \( m_\Phi \ll m_e \) scalar. So the search for light \( \Phi \) scalar in electron fixed target experiments or \( e^+e^- \) experiments is very problematic. Light scalar particle \( \Phi \) with a mass \( m_\Phi \lesssim 1 \text{ GeV} \) decaying into muon pair can be searched for at CERN SPS secondary muon beam in full analogy with the search for new light vector boson \( Z' \) [19].

4 Conclusion

The observation of the \( K_L \rightarrow invisible \) decay would unambiguously signal the presence of the BSM physics. In this note we considered the \( K_L \rightarrow \nu\bar{\nu} \) decay in the simplest extensions of the SM, such as the 2HDM and the νMSM with additional scalar isosinglet. Using constraints from the \( \Delta S = 2 \) flavour changing interactions and experimental value for the \( Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) \) we find that the \( K_L \rightarrow invisible \) decay branching ratio could be in the region \( Br(K_L \rightarrow invisible) \approx 10^{-8} - 10^{-6} \), which is experimentally accessible, allowing to test new physics scales well above 100 TeV, which is not accessible.
at present accelerators. In some scenarios the bound from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay can be avoided, as in the model with the massive sterile neutrino. This makes the $K_L \rightarrow invisible$ decay a powerful clean probe of new physics, that is complementary to other rare $K$ decay channels. We have also demonstrated that the $\nu$MSM with additional scalar field can explain the observed muon $(g - 2)$ anomaly. The obtained results provide a strong motivation for a sensitive search for this process in a near future $K$ decay experiment proposed in [6]. It should be noted that in full analogy with the case of $K_L$ invisible decay we can expect the existence of invisible decays of $B_d$ and $B_s$ mesons, see e.g. [15][16], with the branchings similar to those discussed above.

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