Dilatons in the Topological Soliton Model for the Hyperons

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Abstract

We show that the predicted hyperon masses in the topological soliton model are very sensitive to the value of the gluon condensate parameter that appears when the scale invariance and trace anomaly of QCD are taken into account by introduction of a dilaton field. This contrasts with the insensitivity of the soliton properties to the dilaton coupling. In order that the predicted strange and charmed hyperon spectra agree with the empirical ones the gluon condensate parameter has to be about $(400 \text{ MeV})^4$, which agrees with the result obtained from QCD sum rules. This implies that the bag formed by the scalar field must be very shallow.
1. Introduction

The extension to the heavy flavour sectors of the Skyrme model [1], in which the hyperons are described as bound states of heavy flavour isodoublet mesons and a soliton formed of the light (u, d) flavours [2, 3], leads to remarkably good predictions of the hyperon properties [4, 5]. The spectra and magnetic moments of the strange, charm and bottom hyperons are close to the available empirical values, and also to the corresponding quark model based predictions [5, 6]. This bound state model, which incorporates chiral symmetry for the light flavour (SU(2)) sector, also to a good approximation respects the constraints imposed by heavy quark symmetry in the baryon sector [7], although in its original form it does not include the heavy flavour vector meson fields.

The topological soliton model can also be modified so as to incorporate the scale invariance and trace anomaly of the underlying QCD, for which it is intended to describe in the large colour number approximation. The simplest way to achieve this is to introduce a scalar or dilaton field, which should model the vacuum fluctuations of the gluon field [8]. This leads to the formation of a ”bag” around the soliton, where the dilaton field is nonzero. The depth of this scalar bag will depend on the model used for the soliton Lagrangian. When the original Skyrme model Lagrangian is used it has been found that the sensitivity of the predicted nucleon properties to the dilaton coupling, and the two parameters that govern its strength - the gluon condensate parameter and the scalar meson (glueball) mass - is rather weak [8].

We here study the modification of the bound state model for the hyperons caused by the coupling to the dilaton field. As the dilaton field leads to a modification of both the soliton profile and the wave equation for the heavy flavour mesons, the predicted values for the hyperon masses are far more sensitive to the presence of the dilaton field than the nucleon properties, which only depend on the SU(2) soliton. We find that the main effect of the coupling to the dilaton field is to increase the binding energy of the heavy flavour mesons, and thus to lower the hyperon masses. This is mainly due to the reduction of the mass term in the meson wave equation at short range. Retention of the successful predictions of the hyperon phenomenology achieved in ref. [4-6] therefore requires that
the coupling to the dilaton field be weak and that the bag formed by the scalar field be very shallow. The corresponding value for the gluon condensate has to be of the order $(400 \text{ MeV})^4$, in agreement with QCD sum rules. The results are less sensitive to the glueball mass, but the nonobservation of any low lying glueball suggests that it has to be at least $1.5 \text{ GeV}$.

In section 2 of this paper we show how the bound state model for the hyperons is modified by the coupling to the dilaton field. In section 3 we show the numerical results for the soliton profile and the scalar field amplitude obtained with different values for the gluon condensate. In section 4 we show how the introduction of the dilaton field affects the predicted hyperon properties, and demonstrate the sensitivity to the gluon condensate value. Section 5 contains a concluding discussion.

### 2. Dilaton coupling in the bound state model

The bound state model for the hyperons is based on the Skyrme model [4, 5] or some extension thereof, to include the heavy flavours [10]. The Lagrangian density of this model does not have the scale invariance of the classical QCD Lagrangian density. The required scale invariance can be introduced by coupling of a scalar dilaton field to those terms in the Lagrangian density that break the scale invariance. The divergence of the classically conserved Noether current of the scale transformation is finally determined by the trace anomaly of QCD at the quantum level. [8,9].

We here consider the original Skyrme model extended to include the heavy flavour sectors. Denoting the soliton field $U$ and the scalar dilaton field $\sigma$, the scale invariant basic Lagrangian density is

$$L = e^{2\sigma} \left\{ \frac{1}{2} \Gamma^{2\sigma}_0 \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4} f^2 \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] \right\}$$

$$+ \frac{1}{32 e^2} \text{Tr}[L_\mu, L_\nu]^2 + \mathcal{L}_{CSB} - V(\sigma),$$

(2.1)

with $L_\mu = U^\dagger \partial_\mu U$. Here $\mathcal{L}_{CSB}$ is the charge symmetry breaking mass term, which for the case that $U$ is an SU(3) field takes the form [11].

The Lagrangian (2.1) has finally to be completed with the Wess-Zumino action

$$S = -i \frac{N_C}{240\pi^2} \int d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr [L_\mu L_\nu L_\alpha L_\beta L_\gamma],$$

(2.5)

which is scale invariant by itself. This term does not contribute to the energy of the system at the level of SU(2) but it does lead to an important contribution in the meson wave equation.

For the SU(3) field $U$ we adopt the form [12]
\[ U = \sqrt{U_M U_\pi \sqrt{U_M}}. \]

Here \( U_\pi \) is the soliton field

\[ U_\pi = \begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix} \]

(2.7)

with \( u \) being the SU(2) hedgehog field that describes the Skyrmion:

\[ u = e^{i\vec{\tau} \cdot \hat{r} \theta(r)}, \]

(2.8)

The chiral angle \( \theta(r) \) is determined by the Euler-Lagrange equation that corresponds to the Lagrangian density (2.1), (2.2). This coupled differential equation for the functions \( \theta(r) \) and \( \sigma(r) \) is given below.

The heavy flavour meson field \( U_M \) has the form

\[ U_M = \exp \left\{ \frac{i\sqrt{2}}{f} \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} \right\}. \]

(2.9)

Here \( M \) is one of the \( S = -1, C = +1 \) or \( B = -1 \) doublets

\[ M = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \left( \begin{array}{c} D^0 \\ D^- \end{array} \right), \left( \begin{array}{c} B^+ \\ B^0 \end{array} \right) \].

(2.10)

When the field expression (2.6) is inserted into the Lagrangian density (2.1), (2.2) (and the Wess-Zumino action (2.5)), and this is expanded to second order in the heavy flavour field \( U_M \), a linear wave equation for the meson field modes is obtained. Upon a rescaling of the meson field this wave equation takes the form [12]
\( a(r) \nabla^2 M + b(r) \hat{r} \cdot \vec{\nabla} M - c(r) \vec{L}^2 M \)

\[-[v_0(r) + v_{IL}(r) \vec{L} \cdot \vec{L}] M - e^{3\sigma} m^2 M + d(r) \omega^2 M + 2\omega \lambda(r) M = 0. \] (2.11)

Here \( \vec{I} \) is the effective meson spin operator, \( \vec{L} \) the orbital angular momentum operator and \( \omega \) the meson energy. The radial functions \( a(r) \ldots \lambda(r) \) are

\[ a(r) = e^{2\sigma} + \frac{1}{2e^2 f^2} \frac{\text{sin}^2 \theta}{r^2}, \quad b(r) = \frac{1}{2e^2 f^2} \frac{\text{sin}2\theta}{r} - 2\frac{\text{sin}^2 \theta}{r^2} + 2e^{2\sigma} \sigma', \]

\[ c(r) = \frac{1}{4e^2 f^2 r^2} (\theta^2' - \frac{\text{sin}^2 \theta}{r^2}), \quad d(r) = e^{2\sigma} + \frac{1}{4e^2 f^2} (\theta^2' + 2\frac{\text{sin}^2 \theta}{r^2}), \]

\[ \lambda(r) = -\frac{3}{8\pi^2 f^2} \frac{\text{sin}^2 \theta}{r^2} \theta'. \] (2.12)

The potential functions \( v_0(r) \) and \( v_{IL}(r) \) are

\[ v_0(r) = \frac{1}{2} (\theta'' \tan \frac{\theta}{2} + \frac{\theta'}{2}) (e^{2\sigma} + \frac{1}{2e^2 f^2} \frac{\text{sin}^2 \theta}{r^2}) \]

\[ -\frac{\theta'}{r} \tan \frac{\theta}{2} \left[ (1 + r \sigma') e^{2\sigma} + \frac{1}{4e^2 f^2} \frac{\theta' \text{sin}2\theta}{r} \right] \]

\[ \frac{1}{2} \left( 1 - \frac{f^2}{f^2} \right) e^{2\sigma} \tan \frac{\theta}{2} \left[ 2\sigma' \theta' + \frac{2\theta'}{r} - \frac{\text{sin}2\theta}{r^2} \right]. \] (2.13a)

\[ v_{IL}(r) = \frac{4\text{sin}^2 \frac{\theta}{r^2}}{e^2 f^2} \left[ e^{2\sigma} + \frac{1}{e^2 f^2} (\theta^2' + \frac{\text{sin}^2 \theta}{r^2}) \right] \]

\[ -\frac{3}{2e^2 f^2 r^2} \frac{\text{sin}^2 \theta}{r^2} - \theta^2 (1 - 4\text{sin}^2 \frac{\theta}{2}) - \theta'' \text{sin}\theta]. \] (2.13b)

These functions are the same as those derived in ref. [12], except for the additional factors that involve the scalar field \( \sigma \) in the terms in the radial functions that arise from the quadratic term in the Lagrangian density (2.1) and the factor \( e^{3\sigma} \) in the mass term.
The hyperon mass is obtained as the sum of the mass of the SU(2) soliton, the meson energy and hyperfine structure correction. The expression for the mass of the soliton coupled to the dilaton field is

\[
M = \pi \int dr \left[ 2 \Gamma_0^2 e^{2\sigma} r^2 \sigma'^2 + 2 f_\pi^2 e^{2\sigma} (r^2 \theta'^2 + 2 \sin^2 \theta) + \frac{2}{e^2} \sin^2 \theta (2 \theta'^2 + \frac{1}{r^2} \sin^2 \theta) \right. \\
\left. + 4 f_\pi^2 m_\pi^2 e^{3\sigma} r^2 (1 - \cos \theta) + C_G r^2 (e^{4\sigma} (4\sigma - 1) + 1) \right].
\]

(2.14)

The general expression for the hyperfine structure correction to the hyperon mass is given in ref. [5]; the only modification being the insertion of the factor \( e^{2\sigma} \) in all the terms in the expression for the hyperfine structure constant, which do not involve the factor \( e^{-2} \) - i.e. the terms that arise from the stabilizing term in the Lagrangian density (2.1).

By requiring the soliton mass (2.14) to be stationary one obtains the coupled equations of motion for the functions \( \theta \) and \( \sigma \) as:

\[
f_\pi^2 e^{2\sigma} (\sin 2\theta - 2r^2 \sigma' \theta' - 2r \theta' - r^2 \theta'') + \frac{1}{e^2} \left( \frac{1}{r^2} \sin^2 \theta \sin 2\theta - \theta'^2 \sin 2\theta - 2 \theta'' \sin^2 \theta \right) \\
+ f_\pi^2 m_\pi^2 e^{3\sigma} r^2 \sin \theta = 0,
\]

(2.15a)

\[
f_\pi^2 (r^2 \theta'^2 + 2 \sin^2 \theta) - \Gamma_0^2 (2r \sigma' + r^2 \sigma'' + r^2 \sigma''') + 3 f_\pi^2 m_\pi^2 e^\sigma r^2 (1 - \cos \theta) \\
+ 4 C_G e^{2\sigma} r^2 \sigma = 0.
\]

(2.15b)

The only parameters in the model are the pion and heavy meson decay constants \( f_\pi \) and \( f \), the inverse strength of the stabilizing term in the soliton Lagrangian \( e \) and the value for the gluon condensate \( C_G \) and the glueball mass \( m_G \).

3. The soliton parameters
In order that meaningful predictions for the hyperon spectra be obtained the parameters in the Lagrangian density (2.1) should be chosen so that the nucleon and the $\Delta_{33}$ resonance take their empirical values. This leaves two of the parameters free. It is natural to choose the value of the gluon condensate $C_G$ and the glueball mass to agree with available results from QCD sum rules and lattice gauge calculations and then to vary $f_\pi$ and $\epsilon$ until the correct nucleon and $\Delta_{33}$ mass values are obtained. The nonobservation of any low lying glueball suggests that the lowest possible value for the mass of the glueball is at least 1.5 GeV. In order that the dilaton coupling not be insignificantly small we shall choose this value for $m_G$. We have verified numerically that increasing this value to 2.0 GeV does not change the result significantly.

The average of the values for the gluon condensate parameter with the factor 9/8 obtained by QCD sum rules is $C_G \simeq (392 \text{ MeV})^4$ [13]. With these values for $m_G$ and $C_G$ we find that the remaining parameters $f_\pi$ and $\epsilon$ should be 53.63 MeV and 4.795 respectively, in order that the nucleon and the $\Delta_{33}$ resonance mass take their empirical values. These two values are close to those for the case of no dilaton field ($f_\pi=54 \text{ MeV}, \epsilon=4.84$) [14]. For these parameter values the scalar bag that is formed by the dilaton field is very shallow. The functions $\theta(r)$ and $\sigma(r)$ obtained using these parameter values are shown in Fig. 1.

Smaller values for the gluon condensate parameter leads to a deeper bag formation. The concomitant shrinkage of the soliton profile $\theta(r)$ leads to poorer values for the nuclear radii and axial coupling constant however [9], although the nucleon and $\Delta_{33}$ mass values may still have their empirical values. Results of lattice QCD calculations suggest somewhat smaller values for the gluon condensate ($C_G \simeq (291 \text{ MeV})^4$) [15], than the QCD sum rules. This still leads as a shallow bag. However, we shall below show that much lower values for $C_G$ would lead to very poor predictions for the hyperon masses. For example, choosing the small value $C_G=(180 \text{ MeV})^4$, which leads to a deep bag, and $m_G=1.5 \text{ GeV}$ we find that the nucleon and $\Delta_{33}$ masses agree with the empirical ones if $f_\pi=52.25 \text{ MeV}$ and $\epsilon=4.412$. The corresponding values for the chiral angle $\theta$ and scalar field $\sigma$ are also plotted in Fig. 1.
4. The influence of the scalar field on the hyperon masses

The parameter choice that leads to the shallow scalar bag implies that the coupling to the scalar meson field is very weak and that consequently the soliton profile is very close to that obtained in the case of no scalar field. As a consequence the solutions to the wave equation (2.11) are also very close to those obtained in the case \( \sigma = 0 \). Quantitatively the relative change in the meson energy caused by the weakly coupled \( \sigma \)-field is about twice that of the corresponding maximal relative change in the chiral angle. This proportionality however only holds when the absolute value of \( \sigma \) is less than about 0.1.

In the case of the strange pseudoscalar isodoublet, with \( f = f_K = 1.23 f_\pi \), and using the parameter values that correspond to the shallow bag case, the meson energy in the ground (P-) state is 205 MeV, as compared to the value 209 MeV that obtains when \( \sigma = 0 \) \cite{5}. In the case of the D-meson, with \( f_D = 2 f_\pi \) the meson energy is 1306 MeV as compared to the value 1342 MeV obtained in the absence of the dilaton. A similar small relative shift of the B-meson energy from the value 3773 MeV given in ref. \cite{5} to 3665 MeV is caused by the shallow scalar bag. It is therefore evident that a weak dilaton coupling of this type, which is obtained with values for the gluon condensate and the glueball mass obtained from QCD sum rules and phenomenological analyses, has no real numerical significance for the predicted hyperon masses.

A weak coupling of the dilaton field of this type has an almost negligible effect on the hyperfine correction to the hyperon mass, which is responsible for the isospin splitting of the hyperon masses. For the ground state the calculated hyperfine splitting constant \( c \) retains the value 0.39 that is obtained in the absence of the dilaton field \cite{5}. The predicted values for the masses of the \( \Lambda(1116) \) and \( \Sigma(1193) \) hyperons thus in the presence of a weakly coupled dilaton are 1082 MeV and 1201 MeV as compared to the values 1086 MeV and 1297 MeV in the absence of a dilaton \cite{5}.

In the case of the parameter choices that correspond to a deep scalar bag above the chiral angle remains close to the original values for the case \( \sigma = 0 \) (Fig. 1.). The large negative values for \( \sigma \) at short ranges do however have a disastrous influence on the
predicted hyperon masses. The main reason for this is the reduction of the strength of the meson mass term in the wave equation (2.11) that is caused at short ranges by the factor $e^{3\sigma}$. In the case of the kaons, with $f_K = 1.23 f_\pi$ as above, the meson energy obtained by solving (2.11) using the functions $\theta$ and $\sigma$ that correspond to the deep bag case, drops to only 76 MeV from the original value 209 MeV [5]. This would imply an underbinding of more than 100 MeV for the kaons. The deep bag model also leads to a poor value for the hyperfine splitting constant. In the ground state its value increases from 0.39 to 0.64.

The problem becomes even sharper in the case of the D-meson. With $f_D = 2f_\pi$ (as in ref. [2]), the D-meson energy obtained by solving the wave equation (2.11) for the deep bag case drops to 374 MeV. This large underbinding of 1 GeV is clearly unrealistic and rules out the deep bag parameters. The corresponding drop of the predicted B-meson energy would be from the original value of 3773 MeV to only 730 MeV. The deep bag model leads to too small values for the hyperfine structure constants for the heavy flavour mesons. In the case of the B-meson ground state it becomes negative.

5. Discussion

The results obtained above show that the hyperon masses predicted by the bound state version of the topological soliton model are very sensitive to the coupling to the dilaton field and in particular to the value for the gluon condensate. In contrast the soliton profile - and consequently also the predicted nucleon observables - are rather weakly dependent on the strength of the dilaton coupling and thus do not by themselves rule out a deep scalar bag at short range. The sensitivity of the predicted hyperon masses (or heavy flavour meson energies) to the parameters for the scalar field, and the very unrealistic mass values obtained with a deep bag at short range do however clearly rule out a deep bag model.

It is satisfying that the values for the gluon condensate and glueball mass required for the shallow bag formation agree with those obtained from QCD based analyses and particle phenomenology [9]. The bound state hyperon model [2, 3] is thus robust in this sense. The gain in the model that is achieved by introduction of the dilaton field in this
minimal way is however mainly decorative in the theoretical sense, as it is numerically insignificant although it is important in principle to build in the scale invariance of QCD.

The hyperon mass values predicted in refs. [4,5] remain almost unmodified by the presence of a weakly coupled dilaton field. The predicted masses of the strange and charmed hyperons thus remain in good agreement with the available empirical values. In the case of the bottom hyperons the predicted mass values are too low by $\simeq 600$ MeV. The reduction in binding energy required to overcome this problem can probably only by achieved by explicit coupling of the heavy flavour vector meson fields. Introduction of the heavy vector meson fields is important in principle as well, as heavy quark symmetry implies a degeneracy of the heavy pseudoscalar and vector mesons and that they be treated in a similar way, with the same coupling strength to baryonic matter.
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Figure Captions

Fig. 1. The chiral angle $\theta$ and scalar field $\sigma$ as functions of $r$ (in fm) for the case of a shallow (solid curve) and deep (dashed curve) scalar bag.