Abstract

The calculations of the elementary atom (the Coulomb bound state of elementary particles) interaction with the atom of matter, which are performed in the Born approximation, are reviewed. We first discuss the nonrelativistic approach and then its relativistic generalization. The cross section of the elementary atom excitation and ionization as well as the total cross section are considered. A specific selection rule, which applies for the atom formed as positronium by particle-antiparticle pair, is analyzed.

The aim of my lecture is to discuss how the elementary atom, which is the Coulomb bound states of elementary particles, interacts with the atom of matter. The problem is interesting by itself but the main motivation comes from the experimental studies of $A_{\pi\mu}$, $A_{2e}$, $A_{2\pi}$ and $A_{2\mu}$. $A_{ab}$ denotes the atom of the positively charged particle $a$ and the negative particle $b$. The atom built of the particle $a$ and its antiparticle $\bar{a}$ is written as $A_{2a}$. It should be said that I am going to review my calculations which are over ten years old. Let me note however that the results have never been presented at a conference. When the series of my papers was published, very few people were interested in the topic – it was apparently too early. Now, after a decade it is somewhat too late since there have appeared studies beyond the Born approximation which I used. Nevertheless, the results obtained at the Born level remain a reference point for more elaborated approaches and are still of interest. Since my calculations were all published long ago, I will present only the results and stress the most important points. I would like to start however with a few personal recollections.

It was about 15 years ago when I got to know Leonid Nemenov. We met not on a professional ground but due to our children who played together. We were often walking along the Volga river and talking about politics and physics. Leonid introduced me the physics of elementary atoms. He also suggested me to perform systematic calculations of the elementary atom interactions with matter. At that time I worked as a Ph. D. student in Dubna. I was experimentalist involved in the relativistic heavy-ion physics.

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had already published a few phenomenological papers but wanted to be a real theorist and dreamt to work with the field theory. The elementary atom calculations seemed to be a good practical course of QED. At the beginning I reviewed the existing literature. It was not difficult because by that time there had been published all together only four papers [12, 13, 14, 15], where the cross sections of the elementary atom interaction with matter atom were calculated. I realized soon that the two earlier publications [12, 13] were erroneous while the latter ones [14, 15] rather unsatisfactory [8]. So, the systematic calculations were indeed needed. I started with the nonrelativistic calculations and then worked out the relativistic generalization. Let me briefly present the results.

Within the nonrelativistic Born approximation one easily finds the cross section of the elementary atom excitation from the state \((nlm)\) to \((n'l'm')\) as

\[
d\sigma^*_{nlm} = \frac{1}{2\pi v^2} |U(q)|^2 |F^*_{nlm}(\eta q) - F_{nlm}(\zeta q)|^2 q dq ,
\]

where \(v\) is the atom relative velocity; \(q\) is the momentum transfer and \(q \equiv |q|\); \(U(q)\) represents the potential generated by the atom of matter; \(\zeta \equiv m_1/M\) and \(\eta \equiv -m_2/M\) with \(m_1, m_2\) and \(M\) being the masses of, respectively, the atom components and the atom itself. Due to the smallness of the atom binding energy \(\zeta - \eta = 1\). The transition form factor is defined as

\[
F_{nlm}^* = \int d^3 r e^{iqr} \phi_{nlm}^*(r) \phi_{nlm}(r) ,
\]

where \(\phi_{nlm}(r)\) is the wave function of the elementary atom internal motion.

Except the paper [11], where the elementary atom interaction with hydrogen was studied, I treated the atom of matter as a structureless source of the electromagnetic potential of the Yukawa or Thomas-Fermi-Molier form. Therefore, the so-called incoherent interactions which lead to the target atom excitations were not taken into account. The target recoil was neglected as well. Such an approximation is justified for a sufficiently heavy matter atom [8]. The role of the incoherent interactions was studied by other authors [16, 17].

When the atom is composed, as positronium, of particle and antiparticle, the cross section (1) is nonzero if the atom state parities at the initial and final states differ from each other i.e. if

\[
(-1)^l = -(-1)^{l'} .
\]

This happens because of the relation

\[
F_{nlm}^{n'l'm'}(-q) = (-1)^{l-l'} F_{nlm}^{n'l'm'}(q) ,
\]

which follows from the parity properties of the hydrogen-like atom wave functions. When the masses of the atom constituents are close to each other, as in the case of \(A_{\pi\mu}\), the transitions, which break the selection rule (2), are strongly damped.

The ionization cross section is analogous to the excitation one (1) and reads

\[
 d\sigma^k_{nlm} = \frac{1}{(2\pi)^4 v^2} |U(q)|^2 |F^k_{nlm}(\eta q) - F^k_{nlm}(\zeta q)|^2 q dq d^3 k ,
\]
with the transition from-factor defined as

\[ F_{nlm}^k(q) = \int d^3r \ e^{iqr} \phi_k^*(r) \phi_{nlm}(r), \]

(4)

where \( \phi_k(r) \) is the wave function of the ionized elementary atom with \( k \) being the relative momentum of the atom components.

The minimal and maximum momentum transfer are determined by the reaction kinematics. However, one can take \( q_{min} = 0 \) and \( q_{max} = \infty \) as long as the elementary atom is sufficiently energetic in the initial state [8, 9]. When the minimal and maximal values of \( q \) are assumed to be the same for all final states, the total cross section can be easily computed due to the sum rule

\[ \sum_f |F_{nlm}^f(\eta q) - F_{nlm}^f(\zeta q)|^2 = 2 - 2F_{nlm}^{nlm}(q), \]

(5)

where the summation runs over the complete set of the quantum states. Then, the total cross section reads

\[ \sigma_{nlm}^{tot} = \frac{1}{\pi v^2} \int_0^\infty dq \ q |U(q)|^2 [1 - F_{nlm}^{nlm}(q)]. \]

(6)

When the ionization cross section is computed, one is tempted to substitute the plane-wave function into the form factor \( [3] \). In this case however, the cross section which is obtained by integration of the expression \( [3] \) equals the total cross section not the ionization one. The point is that the integration over the plane wave momentum corresponds to the summation over the complete set of quantum states as in eq. \( [3] \). Since using the exact scattering Coulomb wave function is rather cumbersome, Pak and Tarasov [16] computed the ionization cross section subtracting the elastic and excitation contributions from the total cross section.

The relativistic generalization of the results presented above is far not straightforward due to the well known difficulties of the relativistic treatment of the bound states. In particular, the bound state internal motion cannot be factorized from the motion of the center of mass. However, in the case of the elementary atoms, which are loosely bounded, the difficulties can be circumvented to a large extent when the interaction is studied in the reference frame where the elementary atom is initially at rest. Then, the atom internal motion in the initial as well as in the final state is basically nonrelativistic. The point is that the characteristic momentum transfer to the atom is of order of the inverse Bohr radius \( [3] \).

Within the relativistic approach it is desirable to distinguish between the spin–\( 1/2 \) and spinless atom components. Then, we have spin–\( 1/2 \)–1/2 atoms such as \( A_{2e}, A_{e\mu} \), the spin–0–1/2 as \( A_{\pi e}, A_{\pi K} \) and finally the spin–0–0 as \( A_{2\pi} \) or \( A_{\pi K} \). In the case of the spin–0–0 atoms, one finds the excitation cross section as

\[ d\sigma_{nlm}^{n'l'm'} = \frac{Z^2 e^4}{(2\pi)^2 v^2} |\Delta(Q)|^2 \left| \left( 1 + \frac{qv}{2M} \right) \left[ F_{nlm}^{n'l'm'}(\eta q) - F_{nlm}^{n'l'm'}(\zeta q) \right] \right|^2 qdq d\phi , \]

(7)
where $Ze$ is the electric charge of the matter atom nucleus and $\Delta(q)$ is the photon propagator in the Lorentz gauge which takes into account the effect of screening; $Q$ is the four-momentum transfer and $\phi$ is the azimuthal angle of $v$ with respect to the quantization axis; $G_{nlm}^{nl'm'}$ is the magnetic form factor defined as

$$G_{nlm}^{nl'm'}(q) = i \int d^3r \ e^{iqr} \phi^{*}_{n'l'm'}(r) \nabla \phi_{nlm}(r).$$

The main difference between the relativistic formula (8) and its nonrelativistic counterpart (7) is the appearance of the magnetic contribution which can be sizeable.

The formula (8) also holds for the spin $0-\frac{1}{2}$ and spin $\frac{1}{2}-\frac{1}{2}$ atoms when the interaction does not change the atomic spin or spin projection. The cross section of the spin flip process ($s \rightarrow -s$) of the spin $0-\frac{1}{2}$ atom is

$$d\sigma_{nlms}^{nl'm'-s} = \frac{Z^2 e^4}{8\pi v^2} |\Delta(Q)|^2 |F_{nlm}^{nl'm'}(\zeta q)|^2 \frac{q^3 \sin^2 \alpha}{m^2} dq,$$

where $\alpha$ is the angle between the quantization axis, which coincides with the vector $q$, and $v$. Finally, one finds the cross section of the spin $\frac{1}{2}-\frac{1}{2}$ atom interaction with the change of the atomic spin ($\sigma$) and/or spin projection ($\sigma'$). The cross section vanishes if (1) $\sigma \neq \sigma'$ and $\sigma_3 = \sigma'_3$, (2) $|\sigma'_3 - \sigma_3| > 1$. In all other cases

$$d\sigma_{nlms'\sigma'\sigma_3} = \frac{Z^2 e^4}{16\pi v^2} |\Delta(Q)|^2 \frac{1}{\zeta} F_{nlm}^{nl'm'}(\eta q) + (-1)^{\sigma - \sigma'} \frac{1}{\eta} F_{nlm}^{nl'm'}(\zeta q) \frac{2q^3 \sin^2 \alpha}{M^2} dq.$$

This formula can be used to compute, in particular, the transition from ortho- to para-positronium. The total cross sections can be found from eqs. (7), (9), and (10) by means of the sum rules analogous to (8).

In the relativistic approach the selection rule (2), which applies for the atoms composed by particle-antiparticle pairs, gets a more general form: The transition is allowed if the charge parity of the atom changes in the course of interaction i.e.

$$(-1)^{l+\sigma} = -(-1)^{l'+\sigma'}.$$ The reason is the following. The atom of particle and antiparticle is an eigenstate of the charge parity operator with the eigenvalue $(-1)^{l+\sigma}$. The photon is also the parity eigenstate with the eigenvalue $-1$. Since the Born approximation corresponds to the one-photon exchange the charge parity conservation leads to the selection rule of interest.

The numerical results of the cross sections were collected in the papers [8, 10]. Interaction of eight elementary atoms ($A_{2e}$, $A_{e\mu}$, $A_{e\pi}$, $A_{2\mu}$, $A_{\mu\pi}$, $A_{2\pi}$, $A_{\pi\pi}$, and $A_{2K}$) with five targets (C, Al, Cu, Ag, Pb) was studied. The excitation and total cross sections were calculated. The electric, magnetic, spin and para-ortho transitions were analysed separately.

The calculations of the elementary atom cross sections presented here were improved by several authors. I have already mentioned the works [16, 17], where the role of the incoherent interactions was studied. In the papers [18, 19] the multiphoton exchanges within
the eikonal approach were taken into account. Afanasyev and Tarasov [20] discussed the interaction of the excited atoms. Finally, I should mention very valuable calculations presented at this Workshop by Afanasyev, Cugnon, Tarasov and Trautmann.

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