Rapid training of quantum recurrent neural network

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Abstract. Time series prediction is the crucial task for many human activities e.g. weather forecasts or predicting stock prices. One solution to this problem is to use Recurrent Neural Networks (RNNs). Although they can yield accurate predictions, their learning process is slow and complex. Here we propose a Quantum Recurrent Neural Network (QRNN) to address these obstacles. The design of the network is based on the continuous-variable quantum computing paradigm. We demonstrate that the network is capable of learning time dependence of a few types of temporal data. Our numerical simulations show that the QRNN converges to optimal weights in fewer epochs than the classical network. Furthermore, for a small number of trainable parameters it can achieve lower loss than the latter.

Keywords: quantum machine learning, time series, continuous variables

1 Introduction

Quantum Machine Learning (QML) is a new and rapidly developing field. It aims to integrate quantum computers with machine learning in the hope that this will speed up training process or give better results than classical network. Recent papers show benefits of using this approach for satellite image classification \([6]\) or modeling joint probability distributions \([8]\). QML was also used to analyse time series \([1,7,2]\). In this paper we propose new approach to the task of time series prediction using Quantum Recurrent Neural Networks in continuous variable paradigm.

2 Architecture

The algorithm works on \(n = n_a + n_b\) qnodes (see fig.\([1]\)). The first \(n_a\) qnodes serve as a memory of the network and are not measured during the training process. The remaining \(n_b\) qnodes are used to encode the data, and are finally measured to obtain the output of the network. For each training data point \(x_t\) we apply to all qnodes the layer \(L(x_t, \theta)\), where \(\theta\) are parameters of the
network, randomly initialized before first run. The values for initialization are taken from the Gaussian distribution $N(0,0.01)$. Each layer $L(x_t, \theta)$ consist of three parts: 1) encoding, where the data point $x_t$ is encoded as a quantum state; 2) interaction, which depends on the parameters $\theta$ and is capable of performing any unitary operation [3]; 3) measurement of an observable $O$ on the remaining $n_b$ qmodes. The classical output of this measurement can then be processed, in particular we can set the output of the layer as any function acting on the results of all measurements i.e. $\tilde{x}_t = g \{\langle O_i \rangle\}_{i=1}^{n_a+1}$ [7]. All measured qmodes are set to $|0\rangle$ afterwards and, together with $n_a$ unmeasured modes, are fed to the next layer $L(x_{t+1}, \theta)$. Since all the qmodes are in general entangled with each other, the measurements collapse the remaining modes, inducing non-linearity without the need of any explicit non-linear gate such as the Kerr gate [5].

This process is repeated $T$ times, where $T$ is the length of the time series in the training set. After the last point $x_T$ is processed, the output $\tilde{x}_T$ of the layer is taken to be the prediction of the network. The subsequent output is produced by processing the output of the previous layer i.e. the point $\tilde{x}_{T+i}$ is an output of the layer $L(\tilde{x}_{T+i-1}, \theta)$.

\[ L(x, \theta) = D(x) S(\theta) I(\theta) \]

![Architecture of QRNN](image)

**Fig. 1: Architecture of QRNN;** (a) One layer of the QRNN; $x$ is the input data and $\theta$ are the trainable parameters of the network; $D$ is the displacement gate, $S$ is squeezing gate and $I$ is an interferometer consisting of rotation gates and beam-splitters; red dashed lines split the layer into three parts (from left to right): encoding, interaction and measurement. (b) processing the data of length $T$.

One layer of the quantum algorithm $L$, consisting of $n$ wires, has $n(n-1) + \max(1,n-1)$ parameters coming from each interferometer, $n$ parameters for each squeezing gate and $n$ for each displacement gate. One can also train the function $g$, which takes the measurement of the observable and returns the prediction of the network. In this case the parameters of this function should also be counted in the total number of parameters of the network. In our case we choose the function $g$ to be just multiplication by a number, which is learnt throughout the training process. In total the quantum layer has $2n(n+2) + \max(1,n-1) + 1$ trainable parameters.
Rapid training of quantum recurrent neural network

Fig. 2: Progress of training on data generated with the Bessel function of the first kind and order 0, for quantum (top row) and classical algorithm (bottom row). Blue points are true data, yellow points are predictions based on last 4 data points and red dashed line shows, how the data was split for training (on the left) and validation (on the right) set. Red lines connecting yellow and blue dots shows the difference between predicted and true value.

3 Results and discussion

Results. For small networks with 14 adjustable parameters the QRNN requires less epochs to achieve the same or better result than the classical LSTM with the same number of parameters. This holds for all tested functions, here one is presented – the Bessel function of the 1st kind (see fig. 2). Looking at fig. 3 one can see that the quantum circuit minimizes loss significantly faster in the first few epochs in comparison to the classical network. The loss curve is also more smooth in the quantum case, as was also noticed in [2].

For bigger networks (about 30 parameters), we observe that the loss at the end of the training is lower than for the smaller network, demonstrating that the networks predictions improve with increasing numbers of qmodes and parameters. However, we found that using more layers than just one per input data point (as was done in [7]) does not noticeably improve the results.

Discussion. We have seen that replacing a classical LSTM with a QRNN allows for a dramatic reduction in the training time needed to predict time series data. The results obtained in this work were achieved on only simple data sets, and the next step will be to try the same architecture on a more complicated data like e.g. hurricanes intensity [3]. The proposed architecture can outperform its classical counterpart in the domain of low number of parameters. Future works will check if such trend is visible also for larger numbers of parameters.
Fig. 3: Loss comparison between quantum and classical algorithm predicting Bessel function. Shaded region represents the standard deviation of several runs and solid line is an average of all the simulations.

References

1. Bausch, J.: Recurrent Quantum Neural Networks. In: Advances in Neural Information Processing Systems. vol. 33, pp. 1368–1379. Curran Associates, Inc. (2020)
2. Chen, S.Y.C., Yoo, S., Fang, Y.L.L.: Quantum Long Short-Term Memory. In: ICASSP 2022 - 2022 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). pp. 8622–8626 (May 2022). https://doi.org/10.1109/ICASSP43922.2022.9747369
3. Giffard-Roisin, S., Gagne, D., Boucaud, A., Kégl, B., yang, M., Charpiat, G., Monteleoni, C.: The 2018 Climate Informatics Hackathon: Hurricane Intensity Forecast. In: 8th International Workshop on Climate Informatics. p. 4. Proceedings of the 8th International Workshop on Climate Informatics: CI 2018, Boulder, CO, United States (Sep 2018)
4. Killoran, N., Bromley, T.R., Arrazola, J.M., Schuld, M., Quesada, N., Lloyd, S.: Continuous-variable quantum neural networks. Physical Review Research 1(3) (Oct 2019). https://doi.org/10.1103/PhysRevResearch.1.033063
5. Mitarai, K., Negoro, M., Kitagawa, M., Fuji, K.: Quantum Circuit Learning. Physical Review A 98(3) (Mar 2018). https://doi.org/10.1103/PhysRevA.98.032309
6. Sebastianelli, A., Zaidenberg, D.A., Spiller, D., Saux, B.L., Ullo, S.L.: On Circuit-Based Hybrid Quantum Neural Networks for Remote Sensing Imagery Classification. IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing 15, 565–580 (2022). https://doi.org/10.1109/JSTARS.2021.3134785
7. Takaki, Y., Mitarai, K., Negoro, M., Fujii, K., Kitagawa, M.: Learning temporal data with variational quantum recurrent neural network. Physical Review A 103(5) (Dec 2020). https://doi.org/10.1103/PhysRevA.103.052414
8. Zhu, E.Y., Johri, S., Bacon, D., Esencan, M., Kim, J., Muir, M., Murgai, N., Nguyen, J., Pisenti, N., Schouela, A., Sosnova, K., Wright, K.: Generative Quantum Learning of Joint Probability Distribution Functions (Sep 2021)