Experimental analysis of the measurement strength dependence of superconducting qubit readout using a Josephson bifurcation readout method

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Abstract. We succeeded in controlling the measurement strength of a macroscopic quantum system, namely a persistent current quantum bit (qubit), by using a transmission line type Josephson bifurcation amplifier (JBA: an ac-driven superconducting quantum interferometer device) as a probe. By employing a special pulse sequence, we found that the weighted average of the Ramsey fringe visibility ($\alpha$-value) can be used as an indicator of quantum state projection. The sudden change in the $\alpha$-value magnitude at around the threshold suggests that an entangled state of the qubit–JBA composite system ($|g\rangle_{\text{qubit}}|E\rangle_{\text{JBA}} + |e\rangle_{\text{qubit}}|G\rangle_{\text{JBA}}$) settles in one of the two possible classically correlated qubit–JBA states caused by strong decoherence. This is an important result in terms of understanding the mechanisms of quantum state measurement.

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1. Introduction

Quantum mechanics became well established in the last century, and it is now used to develop new technologies. In particular, in the field of quantum information science and related engineering research, quantum mechanics is the core concept that guides us to the new world of information technology [1]. Among many unconventional properties of physical systems that quantum mechanics reveals, measurement is the most subtle and plays one of the most important roles in quantum information technology. A (projective) measurement on a quantum mechanical superposition state selects one of the eigenstates of the measured physical quantity (observable) and the rest of the possibilities disappear after the measurement [2]. A projective measurement is also very important for quantum information processing applications, for example, measurement-based quantum computing [3].

A perfect understanding of the counterintuitive properties of quantum measurement is not within the reach of current theory [4]. However, we can calculate and design a measurement process quantitatively using the theory of quantum open systems if we have sufficient information on the measured physical system, the measurement apparatus (detector) and the interaction between them. In particular, the concept of ‘decoherence’ [5–7] is very useful for analyzing an actual measurement process, when it is applied to a system-detector composite system [8–11].

In this paper, we investigate a process for measuring superconducting qubits [12, 13]. Recently, it has become possible to fabricate controllable artificial quantum systems with superconducting micro-circuits [14–18]. Techniques for measuring the quantum state of a solid-state qubit have progressed significantly. For example, a partial measurement [19], weak continuous measurements [20] and successful quantum non-demolition measurements [21–23] have been reported. In those studies multiple quantum measurements were performed during the coherence time of the qubit and it was confirmed that the post-measurement state was maintained when a Josephson bifurcation amplifier (JBA) [24, 25] was used as the detector. The JBA technique uses a nonlinear resonator as a sensitive probe of the qubit quantum state. By choosing the optimum bias condition for the JBA, the bifurcation phenomenon in a strongly driven nonlinear resonator becomes sensitive to the qubit state coupling to the JBA. The JBA shows a macroscopically different oscillation mode (low- or high-amplitude oscillation states) depending on the qubit state. Therefore, we can detect the quantum state of a qubit.

A detector is often a macroscopic system with many degrees of freedom. When a detector has many degrees of freedom, it is very difficult and sometimes impossible to trace the dynamics during the measurement. A JBA readout, however, is an almost ideal example of
qubit measurement to analyze because JBA behavior is very simple regardless of its many degrees of freedom, and the interaction between the JBA and a superconducting qubit is well known. Recently, we have made some progress on a theoretical analysis of the JBA readout process [26]. We are interested in determining the condition that induces the projection in a real JBA measurement. In our system, it is possible to program the strength of the quantum detection. So we carried out a readout experiment in which we changed the measurement strength and clarified the projection condition.

The rest of this paper is organized as follows. In section 2, we introduce our experimental setup consisting of a superconducting flux qubit and a coplanar waveguide resonator (CWR) with a superconducting quantum interferometer device (SQUID), which we used in our qubit readout experiments. In addition, we describe the qubit state readout procedure where we use the CWR as a JBA. In section 3, we introduce our theoretical interpretation of the qubit readout process with a JBA as the time evolution of the qubit–JBA composite system. In particular, we emphasize the instance when the measurement-induced dephasing (MID) corresponds to an effective ‘projection’. The experiment involves systematically changing the driving strength of the JBA and is described in section 4. This constitutes the core of this paper. In section 5, we discuss the experimental results with the aid of the theoretical readout scenario described in the previous section. Then, we claim that our theoretical understanding and the experimental results consistently support our interpretation of the time evolution and the MID during the superconducting qubit readout process with a JBA. Our conclusions are presented in section 6.

2. Experimental setup for superconducting qubit readout

Our setup for the flux qubit readout is shown schematically in figure 1. A long coplanar type superconducting transmission line is fabricated on a substrate by Al evaporation. Both ends of the line are terminated with capacitances. The length between the terminals corresponds to almost half of the microwave wavelength along the line. This transmission line works as a resonator with a finite quality factor $Q$. It is called a CWR. A SQUID is embedded as a center conductor in the CWR. It provides the CWR with a strong nonlinearity. When we drive the resonator under appropriate operating conditions, it exhibits bistable behavior. Near the threshold condition, the state of the resonator becomes very sensitive to small changes in the operating conditions [25]. So, we can use the resonator as an amplifier of any small changes in the condition (input). The output is a signal that changes macroscopically depending on the state of the resonator, which is called a JBA [24].

When we use a JBA to detect a flux qubit state, the qubit is placed inside the SQUID. So the operating conditions differ slightly depending on the qubit state. A JBA set in the vicinity of the threshold condition between two stable states amplifies the difference in the qubit state as the difference in the realized JBA state, resulting in a macroscopically distinguishable change in the output signal [24, 26].

We call the two stable states of a JBA $|G\rangle$ and $|E\rangle$. $|G\rangle$ is a low-amplitude state that appears even in a linear resonator. The amplitude of the state increases monotonically with the driving strength. This state disappears when the driving strength becomes too large. $|E\rangle$ is a high-amplitude state that appears only in a nonlinear resonator, and it exists only when the driving strength is sufficiently large. Under appropriate operating conditions both of these states can appear. The JBA then becomes bistable. The transition from the monostable condition to the bistable condition is called bifurcation [25]. This is the origin of the name ‘Josephson
Figure 1. (a) Image of a sample for JBA readout. We designed a SQUID structure at the center of a half wavelength CWR. The flux qubit is magnetically coupled to the SQUID. The mutual inductance of this coupling is \( \sim 14 \) pH. We installed an RF line near the qubit to control the qubit state. The mutual inductance between the qubit and the RF line is \( \sim 0.1 \) pH. (b) Schematic diagram of measurement setup. Panel (c) shows the qubit spectrum of the sample. The qubit gap frequency is \( \Delta = 8.1 \) GHz and a persistent supercurrent of \( I_p = 130 \) nA is obtained away from the degeneracy point.

bifurcation amplifier’. Strictly speaking, neither \( |G\rangle \) nor \( |E\rangle \) is a quantum energy eigenstate of the JBA. They are kinds of macroscopic stable states that might be a superposition of the quantum states in a driven JBA. However, they are pure states, and they can join to form an entangled state, as described later.

On the other hand, we express the ground and excited states of the qubit as \( |g\rangle \) and \( |e\rangle \), respectively. When we change the driving strength, the effective Hamiltonian for the qubit changes slightly because of a finite interaction between the qubit and the JBA. In this paper, \( |g\rangle \) and \( |e\rangle \) are used as the effective ‘instantaneous’ energy eigenstates of the qubit for a given driving strength. Since every operation is adiabatically slow as regards the qubit energy, the instantaneous energy eigenstates are directly connected to them before the driving starts.

To read out the qubit state, we employ a short JBA pulse (hereafter referred to as a readout pulse). This is the RF pulse for driving the JBA. It excites an oscillation in the JBA resulting in the generation of an interaction between the JBA and the qubit. The shape of the pulse is shown schematically in figure 2(a). The pulse height is fixed so that it is above the JBA threshold of the switching from \( |G\rangle \) to \( |E\rangle \) when the qubit is in the ground state, and it is below the threshold when the qubit is in the excited state. So, the applied readout pulse induces JBA
π pulse (for qubit)  Readout pulse (for JBA)

(a)

(b)

Figure 2. (a) Typical pulse sequence for a qubit readout with a JBA. The qubit state before the measurement is set by applying an RF pulse/pulses to the qubit. For example, when we prepare $|e\rangle$, we apply a $\pi$ pulse to the qubit after waiting long enough to get the qubit into the thermal equilibrium state, that is almost $|g\rangle$. When we measure the qubit state, we apply the readout pulse to the JBA, which excites it. The pulse height is set between the thresholds of the JBA, each of which corresponds to the switching threshold when the qubit is in $|e\rangle, |g\rangle$. Then, we can distinguish the state the qubit is in by judging whether the JBA switched or not. (b) JBA switching probability as a function of the readout pulse height $h_{pl}$ (arbitrary unit). Filled circles show the probabilities when a $\pi$ pulse is applied to the qubit before the readout pulse, that is, the probabilities when the qubit is in $|e\rangle$. Filled squares are probabilities without a $\pi$ pulse, that is, the qubit is $|g\rangle$. The triangles are the differences between the two of them. They correspond to the visibilities, thus showing the ease of distinguishing $|g\rangle$ or $|e\rangle$.

switching only for a $|g\rangle$ qubit and not for an $|e\rangle$ qubit. Then we can distinguish to which state the qubit is projected by measuring the JBA state with a classical approach, such as homodyne detection. Unfortunately, in an actual readout, whether or not the JBA switches is not perfectly correlated with the qubit state. The switching probability distributions have an overlap, as shown in figure 2(b). This causes imperfect distinguishability (‘visibility’ in figure 2(b)) of the qubit state. Usually we use a readout pulse with the optimum height to give the maximum distinguishability.

This JBA method has been used successfully in the field of superconducting qubit experiments [21, 22]. However, when the initial qubit state is a superposition ($|\psi\rangle_q = a|g\rangle + b|e\rangle$),

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the details of the readout process are not trivial. In particular, it has been unclear as to how the excitation of the JBA transforms the superposition into $|g\rangle$ or $|e\rangle$.

### 3. Scenario of the qubit readout process

Here, we briefly introduce a simple scenario for the ‘projection’ during a qubit measurement with a detector. This is not a unique scenario, but it provides an intuitive understanding of what happens in our qubit readout with a detector.

The initial state of the qubit–detector composite system is

$$|\psi_{\text{ini}}\rangle = (a|g\rangle + b|e\rangle) |G\rangle$$

with $|a|^2 + |b|^2 = 1$, that is, a separable state consisting of a superposition qubit and a ground state detector. When we switch on the interaction between the qubit and the detector, the unitary evolution caused by the interaction forms an entangled state

$$|\psi_{\text{ent}}\rangle = a|g\rangle|E\rangle + b|e\rangle|G\rangle.$$  

Since the detector is an open system weakly interacting with an environment, it suffers from decoherence, resulting in the destruction of the entanglement. Then, the composite system becomes a mixed state consisting of $|g\rangle|E\rangle$ and $|e\rangle|G\rangle$. The weights for these two states are given by $|a|^2$ and $|b|^2$, respectively [5, 7, 11]. This process effectively works as a qubit projection measurement [2] because the interaction between the qubit and the detector and the decoherence causes a stochastic jump from a superposition state to one of $|g\rangle|E\rangle$ and $|e\rangle|G\rangle$, with the probability $|a|^2$ and $|b|^2$. Moreover, by measuring the detector state after the above process, we can know whether the qubit state is $|g\rangle$ or $|e\rangle$. Of course, this scenario does not explain why only one of two possibilities remains and the other disappears. So, this cannot provide an answer to the measurement problem in quantum mechanics. However, this scenario enables us to calculate and design a quantum measurement quantitatively for some actual cases.

For simplicity, hereafter, we call this process ‘projection’ (with quotation marks) although we are well aware that this is not a true projection but a MID [9].

Recently, we theoretically analyzed the qubit readout process using a JBA as a detector when the initial qubit state was a superposition of two energy eigenstates [26]. In that study, we showed the time evolution of a qubit–JBA composite system during the readout process. The system clearly exhibits the following time evolution. The excitation of the JBA forms an entanglement between the qubit and the JBA. Decoherence in the JBA suddenly destroys the superposition and a mixture appears consisting of classically correlated qubit–JBA pairs ($|g\rangle|E\rangle$ and $|e\rangle|G\rangle$). This time evolution of the density operator of the composite system is expressed schematically by

$$\rho(\tau_e) = \frac{1}{2} |G\rangle \langle E| \left( \frac{1}{\sqrt{2}} |g\rangle + \frac{1}{\sqrt{2}} |e\rangle \right) \left( \frac{1}{\sqrt{2}} \langle g| + \frac{1}{\sqrt{2}} \langle e| \right)$$

$$\rightarrow \rho(\tau_{e1}) = \frac{1}{2} (|G\rangle |e\rangle + |G^\prime\rangle |g\rangle) (\langle G| |e\rangle + \langle G^\prime| |g\rangle)$$

$$\rightarrow \rho(\tau_{e2}) = \frac{1}{2} |G\rangle |e\rangle \langle G| \langle e| + \frac{1}{2} |G^\prime\rangle |g\rangle \langle G^\prime| \langle g|$$

$$\rightarrow \rho(\tau_{e3}) = \frac{1}{2} |G\rangle |e\rangle \langle G| \langle e| + \frac{1}{2} |E\rangle |g\rangle \langle E| \langle g|.$$  

(3)
Figure 3. Theoretical calculation of JBA behavior. The horizontal axis is time normalized by the JBA linear resonance frequency $\Omega$. Times $\tau_{ei}$ ($i = 1, 2, 3$) in the figure are the same as those in equation (3). The red line shows the time variation of the driving strength (the amplitude of the readout pulse). The blue line corresponds to the oscillation amplitude of the JBA induced by the driving. The black line is the strength of the entanglement formed between the qubit and the JBA.

Figure 3 shows the time evolution of the strength of the entanglement obtained in [26]. Here, for later discussion, we add the time variations of the driving strength (envelope of the readout pulse) and the excited JBA amplitude (schematically). The times $\tau_{ei}$ ($i = 1, 2, 3$) in the figure are the same as those in equation (3). The driving strength (corresponding to the amplitude of the readout pulse in the previous section) is drawn with a solid line. The driving strength increases linearly with time to the maximum value, and is kept constant (corresponding to the pulse height in the previous section).

As the driving strength increases, the interaction between the qubit and the JBA forms an entanglement between them as a unitary evolution (from time 0 to $\tau_{e1}$). The entanglement strength $E$ suddenly disappears just after $\tau_{e1}$. This time point agrees with the almost divergent behavior of the JBA amplitude, where the JBA in the partial wavefunction $|G\rangle|g\rangle$ just begins switching to $|E\rangle$, and becomes $|G'\rangle$, which is slightly (microscopically) different from the original $|G\rangle$. Then, the environment (the linear loss in the JBA oscillator, in this calculation) becomes capable of distinguishing $|G\rangle$ from $|G'\rangle$ because the linear loss gives a different stochastic phase shift depending on the state. This causes the destruction of the entanglement, and the composite system becomes a classical mixture consisting of classically correlated qubit–JBA pairs: $|e\rangle|G\rangle$ and $|g\rangle|E\rangle$. When we measure and find the JBA state in a classical manner after $\tau_{e3}$, we can obtain one of the two possibilities. That completes the JBA measurement of a qubit state.

The destruction of the superposition between $\tau_{e1}$ and $\tau_{e2}$ corresponds to the MID (‘projection’ in the terminology used in this paper) of the qubit–JBA coupled system. Moreover,
in this expression the dephasing of the detector (JBA) is accompanied by the dephasing of the qubit. In $\rho(\tau_{e2})$ above, $|G\prime\rangle$ appears instead of $|E\rangle$. $|G\prime\rangle$ is a state orthogonal to $|G\rangle$. Therefore, measurement-induced correlated dephasing is finished at the instance $\tau_{e2}$. However, macroscopically $|G\prime\rangle$ and $|G\rangle$ are very similar to each other, so they are not distinguishable with a usual classical measurement. Only $|G\prime\rangle$ transits to $|E\rangle$ after $\tau_{e2}$. So, the process from $\tau_{e2}$ to $\tau_{e3}$ corresponds to classical amplification where the microscopic difference between $|G\prime\rangle$ and $|G\rangle$ is transformed into a macroscopic difference between $|E\rangle$ and $|G\rangle$.

What is most important is that the decoherence (linear loss) causes the destruction of the entanglement, and makes a classical mixture. This is quite different from a ‘fake’ quantum measurement where decoherence causes dephasing on the qubit before the quantum correlation (entanglement) is constructed between the qubit and the detector, and we carry out a classical measurement of the qubit state, which is already determined, with the detector. Our analysis above shows that the decoherence itself plays an important role in transforming the quantum correlation (entanglement) into a classical correlation. This information conversion from quantum to classical is discussed in [27]. We can also apply this analysis to a different kind of qubit coupled to a JBA resonator. We consider it is possible to apply our theory to a measurement system that has nonlinearity and a clear threshold such as a JBA system.

4. Experiment on driving strength dependence of dephasing

It should be noted that the qubit dephasing in the above process is not a simple dephasing, but is accompanied by the splitting of the qubit–detector entanglement. Therefore, the qubit state and the JBA state are determined classically at the same time, and their states are almost perfectly correlated. This is the core of our qubit readout scenario and should be confirmed experimentally.

If we could carry out a quantum state tomography experiment [28] on the qubit–JBA composite system and obtain the time evolution of the density operator, we would be able to reproduce the scenario described above. However, it is not possible, because the JBA is the detector in our experiment, and we only know the JBA state after the interaction with the qubit. Moreover, we also only postulate the qubit state after the interaction (that is, the measurement) from the obtained JBA state.

To confirm the scenario we described above, from our limited information about the composite system, we investigated the dephasing behavior by changing the driving strength of the JBA. All we can measure is the qubit state (postulated from the JBA state). Our policy for investigating the readout process is as follows.

If we observe sudden MID when we continuously change the driving strength, it provides important support for our scenario because such sudden dephasing is caused by the JBA starting to switch from $|G\rangle$ to $|E\rangle$. If the observed qubit dephasing is not related to the JBA switching, there should be no threshold behavior in the dephasing as a function of the driving strength. In other words, we have to check that the qubit is not significantly dephasing in the absence of JBA switching although the qubit is always interacting with the JBA.

The driving strength is controlled by the height of the readout pulse.

In the theoretical calculation mentioned above, the RF driving amplitude is increased and kept between two thresholds corresponding to two of the qubit states ($|e\rangle$ and $|g\rangle$). The envelope shape of our actual readout pulse is slightly more complex. The pulse consists of two parts. In the
Figure 4. (a) Pulse sequence of the projection measurement. (b) Schematic diagram of the qubit state during the measurement sequence. Panels (c)–(e) show the calculated \( \langle \sigma_z \rangle \) patterns that correspond to the pulse sequences for three different \( \alpha \) values.

front part, the amplitude is increased in a period of a few tens of nanoseconds and kept between the two thresholds for a similar length of time. We call this amplitude the ‘pulse height’, and this is the end of the front part. After that, the pulse is succeeded by a trailing plateau with a constant amplitude that is approximately 95% of the height during a time period exceeding 100 ns. This part of the pulse is used to maintain the switching triggered by the front part when the qubit state is \(|g\rangle\). Without this trailing plateau, an error often occurs whereby the invoked switching is accidentally canceled.

The front part of the readout pulse, of course, plays an essential role in the qubit readout process. So, to examine how the JBA readout process occurs, we observed the changes in the resulting qubit state when we employed various projection pulses with the same shape as the front part of our readout pulse. To examine the ‘projection’ conditions of the superposition state, we employed a pulse sequence consisting of two successive qubit control pulses with a short projection pulse between them (figure 4(a)). The qubit is initialized in the ground state simply by allowing it to relax. Applying the first control pulse, we generated a superposition state of our qubit (\(|\Psi\rangle = \sin \frac{\theta}{2} |e\rangle + \cos \frac{\theta}{2} |g\rangle\)). Then, we applied the projection pulse to the JBA. After that, to observe the state change it caused, we applied a rotation operation to this qubit state using
the second control pulse. If the projection pulse does not induce a ‘projection’, the first control pulse should induce a qubit Bloch vector rotation by $\theta_1$, and in addition the second control pulse should rotate it by $\theta_2$. If we can neglect the phase relaxation in the qubit, the expectation value of $\sigma_z$ becomes $\langle \sigma_z \rangle = \cos(\theta_1 + \theta_2)$ (figure 4(b) top), where $\sigma_z = |g\rangle \langle g| - |e\rangle \langle e|$ is the Pauli operator corresponding to the qubit energy eigenstate, and $\langle \cdots \rangle$ means taking an average over huge numbers of experimental trials with the same measurement condition. Namely, for example, when the switching of the JBA corresponds to the qubit $|g\rangle$, and we obtained the $m$ of switching from among $M$ measurements, $\langle \sigma_z \rangle$ becomes $(2m - M)/M$.

On the other hand, if a ‘projection’ occurs through the application of the projection pulse, immediately after the ‘projection’ $\langle \sigma_z \rangle$ becomes $\cos \theta_1$. After applying the second control pulse to this state, $\langle \sigma_z \rangle$ reads $\cos \theta_1 \cos \theta_2$ (figure 4(b) bottom). We can represent $\langle \sigma_z \rangle$ in more detail as

$$\langle \sigma_z \rangle = \cos \theta_1 \cos \theta_2 - \alpha \sin \theta_1 \sin \theta_2. \quad (4)$$

Here $\alpha$ is defined as a coherence reduction that is equivalent to the Ramsey fringe visibility. Theoretically, $\alpha$ is the reduction of the off-diagonal elements in the qubit density operator. It is constructed of two types of dephasing. One is a simple dephasing that is not related to the measurement. The other is the coherence reduction induced by the measurement itself. The latter is called MID and corresponds to the destruction of the entanglement between the qubit and the detector (JBA). This is the ‘projection’ we are discussing.

This situation can be approximately expressed by

$$\alpha = \alpha_0 e^{-\frac{\tau}{T_2}} \cos \left((\omega - \omega_0) \tau\right). \quad (5)$$

This expression contains three factors. (i) $\alpha$ is described as a product of a coherence reduction by MID $\alpha_0$. (ii) The exponential decay with the exponent $-\tau/T_2$ is a simple dephasing. (iii) Cosine oscillation gives a false reduction of coherence. The energy shift caused by the readout pulse application induces a small error in the phase of the second control pulse [29]. $\omega$ is the control pulse frequency and $\hbar \omega_0$ is the qubit energy. In the absence of dephasing and detuning (that is, $T_2 \to \infty$, $\omega \to \omega_0$), $\alpha$ is identical to $\alpha_0$.

When no ‘projection’ occurs, $\alpha$ takes a finite value. On the other hand, when ‘projection’ occurs, $\alpha$ becomes 0 regardless of dephasing or detuning. By applying a projection pulse within the qubit coherence time, we can detect an effective ‘wave packet reduction’ caused by the projection pulse. Figures 4(c)–(e) show the calculated $\langle \sigma_z \rangle$. In the absence of any ‘projection’ and any dephasing, the $\alpha$ value of equation (4) becomes 1. Therefore, we obtain a stripe pattern in the $\langle \sigma_z(\theta_1, \theta_2) \rangle = \cos(\theta_1 + \theta_2)$ plot (figure 4(c)). When a ‘projection’ is performed by the applied projection pulse, $\alpha$ becomes 0, so the checkerboard pattern shown in figure 4(e) is expected.

In our experiment, the resonance frequency of the JBA is approximately 7.5 GHz. We kept our flux qubit under an external magnetic field of $\Delta \Phi_{\text{qubit}} = \Phi_{\text{qubit}} - (n + \frac{1}{2}) \Phi_0 = -8.0 \times 10^{-3} \Phi_0$, where $\Phi_0 = \frac{\hbar}{2e}$ is the flux quantum. With this magnetic field, the resonance frequency of the qubit is 10.3 GHz. We used a dilution refrigerator, and measured the sample

$^6$ To carry out this type of interference experiment on a qubit, we always have to follow the Larmor rotation of the qubit to give the proper reversing condition (by the second control pulse). When the application of the readout pulse changes the qubit energy effectively, the frequency of the Larmor rotation is shifted from our experimental setting. Then it causes an error in the phase of the second control pulse. This type of technique for following the superconducting qubit Larmor rotation in an actual experiment is discussed in detail in [29].
at temperatures below 50 mK. Figure 1(b) is also a schematic of our measurement setup even for this experiment. The JBA readout pulse and the projection pulse propagate along the CWR in the refrigerator, and are amplified by a low-noise cold amplifier. The resonance state of the JBA that is realized by the readout pulse depends on the qubit state, so we can read out the qubit state by measuring the amplitude and phase of the obtained output signal of the amplifier using a homodyne detection technique. By applying a qubit resonant microwave pulse to the control line, we can control the qubit state before the projection pulse.

We employed the sequence shown in figure 4(a) and observed the way in which the ‘projection’ of a superposition state takes place. When we read out the qubit state, the JBA resonator bifurcates into a high- or low-amplitude state, and these two states correspond to the qubit ground state and the qubit excited state, respectively. To obtain \( \langle \sigma_z \rangle \), we observed the probability of the JBA low-amplitude state \( P \). The relationship between \( \langle \sigma_z \rangle \) and \( P \) can be described as follows: \( \langle \sigma_z \rangle = 2 \left( \frac{(P_0 - P)}{V + P_0} \right) - 1 \). In this formula \( P_0 \) is an offset and \( V \) is the visibility in the measured probability. We chose the time distance between the two control pulses \( \tau = 12 \) ns to reduce the influence of the phase relaxation (figure 4(a)). At this flux bias, the energy relaxation time of the qubit is \( T_1 = 58 \) ns, and the phase relaxation time is \( T_2 = 13 \) ns. Because both the ‘projection’ process and the phase relaxation contribute to the reduction of the \( \sin \theta_1 \sin \theta_2 \) term in equation (4), a short \( \tau \) is better for this projection measurement. The rise time of the projection pulse was approximately 10 ns, and it was maintained at a constant height for 10 ns. Under this condition, there is a small overlap between the projection pulse and the control pulse. However, the projection pulse height of the overlapping region is sufficiently low, so the pulse overlap is not a serious problem. We varied the pulse lengths of both control pulses from 0.5 to 10 ns. After the second control pulse, we detected the qubit state using the usual JBA readout pulse (with a trailing plateau). By averaging typically over 20000 times, we obtained probability \( P \).

5. Results and discussion

Figure 5 shows the \( P (\tau_1, \tau_2) \) pattern we observed when we employed projection pulses with different heights. \( \tau_1 \) and \( \tau_2 \) are the pulse lengths of the first and second control pulses, respectively. So, \( \theta_1 \propto \tau_1 \) and \( \theta_2 \propto \tau_2 \). \( h_{pl} \) is the pulse height normalized by that of the readout pulse that we usually use in our JBA measurements. When we employed an \( h_{pl} = 0.8 \) pulse, we observed the same pattern as without a projection pulse (\( h_{pl} = 0.0 \)), namely a stripe pattern. Then, no ‘projection’ occurred. Figure 5 (a) and (b) roughly correspond to figure 4(d). Even in figure 5(a) for \( h_{pl} = 0.0 \), there is unwanted weak dephasing. It is important that when \( h_{pl} = 0.8 \) (figure 5(b)), the dephasing is not stronger than \( h_{pl} = 0.0 \). The application of a pulse lower than the threshold does not increase dephasing. The almost similar dephasing we observed in figures 5(a) and (b) is background dephasing that is unrelated to the pulse application. It should be noted that even when the pulse height is 80% of its optimum value for the measurement, the dephasing is not very significant.

However, when we used an \( h_{pl} = 1.0 \) pulse, the \( \sin \theta_1 \sin \theta_2 \) component of equation (4) vanished and a clear checkerboard pattern appeared (figure 5(c)). This corresponds to figure 4(e) showing an almost complete dephasing. This sudden change from a stripe pattern to a checkerboard pattern suggests that the ‘projection’ was induced by the applied pulse although the pattern contrast was not as good as the calculated result because of the energy relaxation.
When we further increase the pulse height ($h_{pl}=1.2$), the observed pattern does not depend on the first control pulse ($\tau_1$), because the quantum state just after the ‘projection’ is destroyed by the over-strong projection pulse (figure 5(d)). Rather, we observed a small Rabi oscillation caused by the second control pulse. This is due to the energy relaxation that occurs after the randomization of the qubit state, which induces a bias in the post-pulse state to the ground state.

To discuss the pulse height dependence quantitatively, we evaluate $\alpha$ as follows:

$$\alpha = \frac{1}{4} \left\{ \langle \sigma_z \left( \frac{3\pi}{2}, \frac{\pi}{2} \right) \rangle - \langle \sigma_z \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \rangle \right\} + \left\{ \langle \sigma_z \left( \frac{\pi}{2}, \frac{3\pi}{2} \right) \rangle - \langle \sigma_z \left( \frac{3\pi}{2}, \frac{3\pi}{2} \right) \rangle \right\} .$$

\[ (6) \]
This is an approximate estimation of equation (4) for the experiments. We postulate the value of $\alpha$ from experimental values taken from the data for four sets of $(\theta_1, \theta_2)$. Figure 6 shows the dependence of $\alpha$ on the projection pulse height $h_{pl}$. The top panel shows S-curves, which reveal that our readout has a finite visibility in the $0.98 < h_{pl} < 1.03$ region. This means that our readout can provide qubit state information only in this region. Below $h_{pl} = 0.8$, $\alpha$ has a positive value of 0.65. Because of the small dephasing that occurs during the pulse sequences, $\alpha$ becomes less than 1. No ‘projection’ occurs in the region. Since we tuned the frequency of the applied control pulse to the energy of the flux qubit without pulse application ($\omega = \omega_0$ in equation (5)), $\alpha$ should have a positive value at every height $h_{pl}$. However, above $h_{pl} = 0.8$, $\alpha$ decreases and takes negative values.

This data is measured using the same sample, but the thermal cycling process slightly changes the characteristics of the sample (for example visibility, coherence time and gap energy).

Figure 6. (a) Pulse height dependence of S-curves (wider range behavior of switching probabilities shown in figure 2), and pulse height ($h_{pl}$) dependence of $\alpha$ (indicator of the dephasing). The filled (red) circles show the projection pulse height dependence of the $\alpha$. (b) Precise behavior of $\alpha$ around $h_{pl} \sim 1.7$. Cos and sin components (left panel), and the absolute value of $\alpha$ (right panel). They are obtained by changing the second control pulse phase.
To solve this problem, we carried out more precise measurements around $h_{pl} \sim 1.0$ (figure 6(b)), and confirmed that no ‘projection’ occurs in this region. Because of the energy shift in the qubit caused by the application of the projection pulse, the phase of the qubit state rotates during the period of the projection pulse. This leads to an error in the phase of the second control pulse applied to the qubit. So, the obtained $\alpha$ contains a $\cos(\omega_0 - \omega)\tau$ oscillation (equation (5)). Then it provides a fake negative value for $\alpha$ in figure 6(a) around $h_{pl} \sim 1.0$. The true absolute amplitude of $\alpha$ at $h_{pl} = 0.92$ remains almost the same as that below $h_{pl} = 0.8$ (figure 6(b) right panel). This means that the application of the projection pulse itself does not induce significant decoherence (dephasing) at least below $h_{pl} = 0.92$. Above $h_{pl} = 0.98$, $\alpha$ becomes 0 and this suggests that the ‘projection’ of the qubit state has occurred. In the $0.98 < h_{pl} < 1.03$ region, the resonance state of the JBA bifurcates and different qubit states lead the JBA to different final states. So we can detect the qubit state by observing the state of the JBA resonator.

Whether the behavior observed in $\alpha$ is caused by simple dephasing or ‘projection’ might be impossible to determine exactly by experiments. However, based on the results of our theoretical analysis [26], we think that the vanishing of $\alpha$ observed in our experiments is caused by the ‘projection’. Our calculation can explain many important aspects of our present experiment although it actually revealed the behavior of a qubit–JBA coupled system with a higher quality factor $Q$ in a CWR. In particular, it clearly showed the difference between simple dephasing and measurement-induced ‘projection’. It is worth remembering here that we use the word ‘projection’ for the phenomenon where a superposed qubit state is split into two states that are mutually incoherent, and each of them is correlated with each of the two macroscopic states of the JBA. Figure 3 shows the time variation of the strength of the entanglement between the qubit and the JBA. The horizontal axis is the time from the beginning of the application of the readout pulse. So the graphs in figures 3 and 6 cannot be compared directly. But, the envelope of the readout pulse in figure 3 is increased as shown by the solid red line. Then the horizontal axis approximately corresponds to the pulse amplitude in figure 6, and we can obtain some important information by comparing these two graphs.

In figure 3, $\tau_{e1}$ and $\tau_{e2}$ are important characteristic times (pulse amplitudes). From 0 to $\tau_{e1}$, the entanglement grows via the unitary evolution induced by the interaction. At $\tau_{e1}$, the JBA in one of the entangled states starts switching from the low-amplitude state $|G\rangle$ to the high-amplitude state $|E\rangle$. At this time, strong decoherence appears and destroys the entanglement. The resulting state is a mixture consisting of classically correlated qubit–JBA pairs. This corresponds to our ‘projection’. On the other hand, in figure 6, when the pulse height $h_{pl}$ is less than 0.7, the qubit dephasing is constant regardless of the pulse amplitude. This means that during the interaction between the control pulses we can ignore the decoherence that affects the qubit directly. Then, we can concern ourselves solely with the decoherence that appears when the JBA is driven sufficiently strongly. Side effect dephasing caused by the application of the driving should increase with the pulse amplitude and the resulting JBA oscillation amplitude. We observe that the JBA oscillation increases nonlinearly with the pulse height from the rotation in figure 6(b) because the shift in the effective qubit frequency is proportional to the square of the JBA amplitude. This agrees with the fact that the JBA response is nonlinear with regard to the pulse height. So, our experiment shows that the JBA amplitude has already started to grow from $h_{pl} \sim 0.75$. However, the absolute value of $\alpha$ does not decrease with the JBA amplitude. Figure 6(b) clearly shows that the absolute value does not decrease even when the JBA amplitude becomes large, and it is very important to note that the absolute values suddenly vanish just before $h_{pl} \sim 0.98$. As our theoretical calculation shows, this strongly supports
the fact that what we observed is not simple dephasing but ‘projection’. Simple dephasing induced by the measurement should increase gradually with the JBA amplitude. However, our calculation shows that the dephasing of the ‘projection’ occurs suddenly in an almost critical manner with the JBA amplitude. This is because, at that amplitude, a partial wavefunction in the JBA correlated with the qubit ground state starts to switch to the $|E\rangle$ state and the other part correlated with the qubit excited state remains in the $|G\rangle$ state. This separation in the JBA wavefunction induces sudden strong destruction in the qubit–JBA entangled state and results in very sharp dephasing in the qubit. After this dephasing process, the qubit state and the JBA state are classically correlated. This is our ‘projection’. So, the sudden vanishing of the absolute value of $\alpha$ is strong evidence of the ‘projection’.

Even above $h_{\text{pl}} = 1.03$, we still observe the checkerboard pattern of $\langle \sigma_z \rangle$, which is the same as in figure 5(c). This shows that the ‘projection’ occurred but the JBA state evolved to the high-amplitude state independent of the final qubit state. Then, there is no visibility despite the completion of the ‘projection’. In a simple description of a quantum measurement, the projection event and the possibility of obtaining qubit state information has a one to one relationship. However, here ($1.03 < h_{\text{pl}} < 1.2$), the post-measurement state of the qubit is a projected state although we cannot obtain any information about the qubit from the JBA readout. This is a clear example of the fact that such a simple relationship is not valid in a real measurement where we have to take account of the dynamics of the detector.

In figure 6(a), we find that the region where the visibility has finite values (indicated by ‘Visible’ in the top panel of the figure) and that where ‘projection’ occurs (indicated by ‘Projection’ for the $\alpha$ value curve) do not agree with each other. Here, we should explain why we claim that the qubit dephasing is a ‘projection’ even when the visibility is negligible at a height slightly below or above $h_{\text{pl}} = 1$. Just below $h_{\text{pl}} = 1$, the qubit–JBA entangled state is destroyed when the JBA switching starts as described above. At this time point, the dephased qubit state and the JBA state are certainly correlated. So, this is a ‘projection’. However, since the pulse height is not large enough to make the switched JBA state stable, the switched part returns to the unswitched state after the pulse has passed. Then, we cannot find the switched JBA and the visibility is negligible. On the other hand, when the pulse height is slightly higher than $h_{\text{pl}} = 1$, the situation is as described below. After the ‘projection’, one part of the JBA wavefunction switches. However, since the pulse height is too large, the other part of the wavefunction switches with some finite time delay from the first switching. In the final state, the corresponding JBA states have switched for both qubit states, and so there is no visibility. We confirmed the phenomena described above in our theoretical calculations although the parameters we used were not exactly the same as in the actual experiment because of limitations related to numerical precision. Above $h_{\text{pl}} = 1.2$, the information of the first rotation (control) pulse is completely lost, so the $\theta_1$ dependence of $\langle \sigma_z \rangle$ vanishes (figure 5(d)). This behavior means that the qubit state is destroyed by an over-strong projection pulse.

This observed abrupt destruction of the entanglement is the key feature of dephasing into a correlated qubit–JBA pair state, that is, ‘projection’. We think that in our experiment, the abrupt vanishing of $\alpha$ in the vicinity of $h_{\text{pl}} = 0.9$ (see figure 6(b)) must correspond to the abrupt destruction of the entanglement. If we are right, the experimental behavior is consistent with the theory over the entire pulse amplitude range. Moreover, to eliminate the possibility that the abrupt experimental disappearance of $\alpha$ is a simple dephasing (no correlation with the JBA state) we attempted to check whether an increase in the pulse amplitude could cause such an
abrupt increase in a simple dephasing by employing calculations that took account of several mechanisms not considered in [26] (e.g. thermal excitation in the JBA, etc). We found that a certain mechanism can cause a simple dephasing but that it is not significant compared with the entanglement destruction.

The above results show that the normal JBA readout ($h_{pl} = 1.0$) method projects the qubit state to one of the energy eigenstates and maintains that state. Then, this JBA readout realizes a quantum non-demolition measurement, as long as the post-measurement state remains in a projected state against other disturbing effects.

Very recently, in our experiments, we obtained preliminary confirmation that even if finite driving is applied to the JBA, the qubit dephasing with a $|G\rangle$ JBA or with an $|E\rangle$ JBA is insignificant compared with the absence of driving. Moreover, when we keep the JBA driving in the vicinity of the switching, the qubit dephasing is almost complete and we obtain an ensemble consisting of a composite system of $|g\rangle|E\rangle$ and $|e\rangle|G\rangle$. This strongly supports the idea that the variation of $\alpha$ as a function of the driving strength is not simple dephasing but MID ('projection') caused by the destruction of the qubit–JBA entanglement.

6. Conclusion

We succeeded in observing the behavior in projection measurements of superconducting qubit states when we used a transmission line type JBA as the detector. When we start to drive the JBA with a readout pulse (its front is the projection pulse, examined above), the qubit and the JBA begin to interact and form correlations (entanglement). This is the superposition of two possible states of the qubit–JBA composite system. However, the application of a pulse smaller than the threshold ($h_{pl} \sim 0.98$) of the JBA switching does not cause a ‘projection’ or dephasing in the qubit. After that, when the pulse height becomes large and the excitation is strong enough to make the JBA states corresponding to the two possible qubit states distinguishable, ‘projection’ occurs and the qubit–JBA composite system becomes one of two possible classically correlated qubit–JBA states. Then, quantum measurement has been accomplished. When the pulse height is in a range where only one of the qubit–JBA states performs the JBA transition, we can detect the qubit state from the JBA measurement. Our results clarified experimentally the way in which the qubit state ‘projection’ to an eigenstate is caused by the JBA bifurcation phenomenon. This supports a previous theoretical analysis of the JBA readout process [26], and we expect this to be an important result as regards understanding the mechanisms of quantum state measurement.

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