LORENTZ INVARIANCE RELATIONS
AND WANDZURA-WILCZEK APPROXIMATION

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Received 14 October 2009

A complete list of the so-called Lorentz invariance relations between parton distribution functions is given and some of their consequences are discussed, such as the Burkhardt-Cottingham sum rule. The violation of these relations is considered in a model independent way. It is shown that several Lorentz invariance relations are not violated in a generalized Wandzura-Wilczek approximation, indicating that numerically their violation may be small.

Keywords: Parton distribution functions; Lorentz invariance relations; Burkhardt-Cottingham sum rule; Wandzura-Wilczek approximation.

PACS Nos.: 13.88.+e, 13.85.Ni, 13.60.-r

1. Introduction

Higher twist parton distribution functions (PDFs) and transverse parton momentum dependent ($p_T$-dependent) distribution functions (TMDs) contain important information on the partonic structure of the nucleon. They become more important because of the increasing accuracy of recent and planned high energy scattering experiments. The forward twist-3 PDFs are accessible through certain spin asymmetries in polarized inclusive deep inelastic lepton nucleon scattering (DIS) and Drell-Yan processes (integrated upon the transverse momentum of the dilepton pair). On the other hand the TMDs typically give rise to spin and azimuthal

*Contribution to the proceedings of the workshop “Recent Advances in Perturbative QCD and Hadronic Physics”, July 20 – 24, 2009, at ECT*, Trento (Italy).
asymmetries in, for instance, semi-inclusive DIS\textsuperscript{9–13} and Drell-Yan\textsuperscript{14–17} and significant effort has already been devoted to measure such observables.\textsuperscript{18–30}

A systematic account of partonic $p_T$-effects leads to the introduction of many novel functions, making the interpretation of especially twist-3 observables difficult. It was therefore appealing, when the so-called 'Lorentz invariance relations' (LIRs) were proposed.\textsuperscript{2,9,10} These relations were derived from the Lorentz decomposition of the fully unintegrated correlator of two quark-fields, where the fields are located at arbitrary space-time positions. However, in the original treatment\textsuperscript{2,9,10} certain Lorentz-structures related solely to the Wilson line in the correlator were not taken into consideration. After hints from model calculations\textsuperscript{31,32} this was shown in a model-independent way.\textsuperscript{33}

In this paper, in Sec. 2, we first provide a complete list of LIRs and discuss some of their consequences. Then, in Sec. 3 we review previous work\textsuperscript{34} and show that, although they are in general incorrect in QCD, several LIRs hold in an approximation which consists in systematically neglecting quark-gluon-quark correlations and current quark mass terms. This is similar in spirit to the Wandzura-Wilczek-approximation. We discuss the utility of such approximate relations among TMDs. Section 4 contains the summary and conclusions.

2. Lorentz invariance relations

In order to discuss the LIRs we have to study the fully unintegrated quark-quark correlation function $\Phi$ for a spin-$\frac{1}{2}$ hadron\textsuperscript{35} (for spin-0 hadrons see Refs. 33, 36). The general decomposition of the correlator consists of 32 matrix structures multiplied by 12 scalar functions $A_i$ and 20 scalar functions $B_i$, the latter being associated solely with matrix structures induced by the Wilson line in the correlator.\textsuperscript{34} The TMDs are defined through Dirac traces of the $p_T$-dependent correlator\textsuperscript{9,12,34,35} integrated over $p^-$. Consequently the TMDs can be expressed through $p^-$-integrals upon specific linear combinations of the scalar functions $A_i$ and $B_i$.\textsuperscript{34}

In total there are 32 quark TMDs up to twist-4 which exactly agrees with the number of independent amplitudes $A_i$ and $B_i$. If one neglects the structures $B_i$ induced by the Wilson line, the correlator $\Phi$ merely consists of 12 matrix structures, namely those related to the functions $A_i$. In that case the number of TMDs is larger than the number of the amplitudes $A_i$. This feature gives rise to LIRs. Some of them were discussed in literature,\textsuperscript{2,9,10} but so far no complete list of LIRs has been presented. How many independent LIRs do we obtain? There are 24 TMDs up to twist-3 and 12 amplitudes $A_i$. Thus, there must be $24 - 12 = 12$ LIRs in total. Let us distinguish time-reversal even (T-even) and T-odd LIRs.

There are six twist-2 and eight twist-3 T-even TMDs but nine T-even amplitudes. Thus there must be five T-even LIRs, namely

$$g_T(x) \overset{\text{LIR}}{=} g_1(x) + \frac{d}{dx} g_1^{(1)}(x), \quad (1)$$
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\[ h_L(x) \overset{\text{LIR}}{=} h_1(x) - \frac{d}{dx} h_{1L}^{(1)}(x), \]
\[ h_T(x) \overset{\text{LIR}}{=} -\frac{d}{dx} h_{1T}^{(1)}(x), \]
\[ g_{L}^{\perp}(x) + \frac{d}{dx} g_{T}^{\perp}(x) \overset{\text{LIR}}{=} 0, \]
\[ h_T(x, p_T^2) - h_T^+(x, p_T^2) \overset{\text{LIR}}{=} h_{1L}(x, p_T^2). \]

There are two twist-2 and eight twist-3 T-odd TMDs but three T-odd amplitudes, to recall \( A_4, A_5, A_{12} \). Thus there must be seven T-odd LIRs, namely

\[ f_{T}(x) \overset{\text{LIR}}{=} -\frac{d}{dx} f_{1T}^{\perp}(x), \]
\[ h(x) \overset{\text{LIR}}{=} -\frac{d}{dx} h_{1}^{\perp}(x), \]
\[ e_{L}(x) + \frac{d}{dx} e_{T}^{(1)}(x) \overset{\text{LIR}}{=} 0, \]
\[ f_{L}^{\perp}(x, p_T^2) \overset{\text{LIR}}{=} -f_{1T}^{\perp}(x, p_T^2), \]
\[ e_{T}^{(1)}(x, p_T^2) \overset{\text{LIR}}{=} 0, \]
\[ f_{T}^{(1)}(x, p_T^2) \overset{\text{LIR}}{=} 0, \]
\[ g_{T}^{(1)}(x, p_T^2) \overset{\text{LIR}}{=} 0. \]

Notice that (5) and (9)–(12) hold unintegrated over \( p_T \). The functions on the \( \text{lhs} \) of the LIRs are subleading, those on the \( \text{rhs} \) leading twist. The ‘trivial’ LIRs (10)–(12) arise because the respective TMDs are due to the \( B_i \) amplitudes only. Finally, the \( 1) \)-\( p_T \)-moments of TMDs are defined as

\[ g_{1T}^{(1)}(x) = \int d^2 p_T \frac{p_T^2}{2M^2} g_{1T}(x, p_T^2), \quad \text{etc.} \]

Now let us discuss some consequences of the LIRs which follow if we integrate those relations over \( x \) in the region of \(-1 \leq x \leq 1\). Here TMDs at negative \( x \) are understood as \( (\pm 1) \)-TMDs of anti-quarks, depending on the \( C \)-parity of the operator.\(^9\) For example, with \( x \) positive: \( g_1^{(1)}(-x) = g_1^{(1)}(x), \ g_{1T}^{(1)}(-x) = -g_{1T}^{(1)}(x), \) etc. Defining \( g_2(x) = g_T(x) - g_1(x), \) we obtain in this way from LIR (11)

\[ \int_{-1}^{1} dx \ g_2(x) \overset{\text{LIR}}{=} C, \]

\(^9\) In case of the LIRs (1)–(4) and (6)–(8), where only \( x \)-dependent functions enter, corresponding relations involving higher \( p_T \)-moments of the same functions can be derived, provided that these moments exist.
where the constant $C$ is defined, with an $\varepsilon > 0$, assuming the limes exists, as

$$C \equiv \lim_{\varepsilon \to 0} \left[ g^{(1)q}_{1T}(\varepsilon) - g^{(1)\bar{q}}_{1T}(\varepsilon) \right].$$

(15)

Similar sum rules hold for $g^T_L(x)$, $h_T(x)$, $f_T(x)$, $e_L(x)$, $h^T_{-}(x) + h^T_L(x)$, and $h_L(x) - h_T(x) \equiv h_2(x)$. For higher Mellin moments $n \geq 1$ one obtains, for example,

$$\int_{-1}^{1} dx \ x^n g_2(x) \overset{\text{LIR}}{=} -n \int_{-1}^{1} dx \ x^{n-1} g^{(1)1}_{1T}(x), \text{ etc.}$$

(16)

Interestingly, LIRs and the additional assumption that $g^{(1)1}_{1T}(x)$ is continuous at $x = 0$ imply the validity of the Burkhardt-Cottingham sum rule (BC). Under similar assumptions one recovers also a sum rule for $h_2(x)$ which is analog to BC.

However, there is a priori no reason why $\lim_{\varepsilon \to 0} g^{(1)q}_{1T}(\varepsilon) \neq -\lim_{\varepsilon \to 0} g^{(1)\bar{q}}_{1T}(\varepsilon)$ should hold, implying that BC could be violated. At this point it is worthwhile remarking that we arrived at this conclusion (always assuming that LIRs are valid) only by carefully treating the integral in the vicinity of $x = 0$. The total derivative $\frac{d}{dx} g^{(1)1}_{1T}(x)$ gives no contribution at $x = \pm 1$ where TMDs vanish. But there are two more implicit boundaries, hidden by the compact notation, at $x \to \pm 0$. This is of importance for the lowest Mellin moment (14) but not for higher moments (16).

In fact, a discontinuity in $g^{(1)1}_{1T}(x)$ at $x = 0$ means that $g_2(x)$ has a singularity of the type $C \delta(x)$ at $x = 0$ with the coefficient $C$ as defined in (14) due to Eq. (1). A $\delta(x)$-function in $g_2(x)$ corresponds to a real constant term in a spin flip Compton amplitude which persists to high energy. This phenomenon is known in Regge theory as “a $J = 0$ fixed pole with non-polynomial residue”. The possibility that BC is violated, and more generally, that PDFs may have $\delta$-function type singularities at $x = 0$ attract continuous interest in literature.

We stress that these conclusions follow here from LIRs, i.e. in a not complete treatment of TMDs (due to the omission of $B_i$ amplitudes). Interestingly, this framework though incomplete contains sufficiently rich features such as to allow for a violation of BC or the analog sum rule involving $h_2(x)$.

In QCD, which is a gauge theory where the Wilson line is mandatory, neither the LIRs nor their consequences are expected to hold. However, in relativistic quark models without gauge field degrees of freedom the T-even LIRs must hold.

### 3. Lorentz invariance relations in a generalized Wandzura-Wilczek approximation

Knowing that the LIRs are violated it is now natural to ask to what extent they are violated numerically. If one gets an indication that the violation of the LIRs should be small, these relations and their consequences can still serve as a useful tool — at least for qualitative studies of the partonic nucleon structure. In what follows we consider certain LIRs and their violation in a model independent way using a special approximation.
3.1. T-even case

For the discussion of the two T-even LIRs \( \mathbb{I} \) and \( \mathbb{2} \) we recall the following relations between p\(_T\)-integrated T-even PDFs\(^{\,1,48}\):

\[
g_{T}(x) = \int_{x}^{1} \frac{dy}{y} g_{1}(y) + \tilde{g}_{T}(x), \quad h_{L}(x) = 2x \int_{x}^{1} \frac{dy}{y^{2}} h_{1}(y) + \tilde{h}_{L}'(x),
\]

(17)

where \( \tilde{g}_{T}(x) \) and \( \tilde{h}_{L}'(x) \) denote (purely interaction dependent) quark-gluon-quark correlations and terms proportional to current quark masses. An explicit representation of these terms can be found, e.g., in Ref. 49 and partly also in Ref. 47. The relations (17) isolate “pure twist-3 terms” in the PDFs \( g_{T}(x) \) and \( h_{L}(x) \).\(^{\,48}\)

The remarkable experimental observation is that \( \tilde{g}_{T}(x) \) is consistent with zero within the error bars\(^{\,3–8}\) and to good accuracy one has

\[
g_{T}(x) \overset{\text{WW}}{\approx} \int_{x}^{1} \frac{dy}{y} g_{1}(y),
\]

(18)

which is the Wandzura-Wilczek (WW) approximation. In Ref. 47 on the basis of the present data for the DIS structure function \( g_{2} \) it was found that the WW-relation (18) works with an accuracy of the order 15 – 40%, though more data are needed to ultimately settle the situation. For our purpose this accuracy is quite satisfactory. Lattice QCD\(^{\,50,51}\) and the instanton model of the QCD vacuum\(^{\,52}\) support the approximation (18). Interestingly, the latter predicts also \( \tilde{h}_{L}'(x) \) to be small\(^{\,53}\) such that

\[
h_{L}(x) \overset{\text{WW}}{\approx} 2x \int_{x}^{1} \frac{dy}{y^{2}} h_{1}(y).
\]

(19)

This approximation is not yet tested in experiment.\(^{\,54}\) Further discussions of the WW approximation in related and other contexts can be found in the literature.\(^{\,46,55–60}\)

Now it is possible to show that the LIRs \( \mathbb{I} \) and \( \mathbb{2} \) are not violated if one generalizes the WW approximation. For this purpose we consider the following exact relations\(^{\,9,12}\) originating from the QCD equations of motion (EOM):

\[
g_{T}^{(1)}(x) \overset{\text{EOM}}{=} x (g_{T}(x) - \tilde{g}_{T}(x)) - \frac{m}{M} h_{1}(x),
\]

(20)

\[
h_{L}^{(1)}(x) \overset{\text{EOM}}{=} -\frac{x}{2} (h_{L}(x) - \tilde{h}_{L}(x)) + \frac{m}{2M} g_{1L}(x),
\]

(21)

with the (1)-moments of the TMDs as defined in Eq. \( \text{(13)} \). The functions \( \tilde{g}_{T}(x) \) and \( \tilde{h}_{L}'(x) \) denote twist-3 quark-gluon-quark correlations. In lightcone gauge these objects, like \( \tilde{g}_{T} \) and \( \tilde{h}_{L}' \), represent matrix elements of the type \( \langle \bar{\Psi} A_{T} \Psi \rangle \). One therefore can assume that these functions are small as well, although the explicit

\[^{\,\text{b}}\text{Here the underlying “working definition” of twist}\(^{\,41}\) (a PDF is of “twist } t} \text{ if its contribution to the cross section is suppressed, in addition to kinematic factors, by } 1/Q^{t-2} \text{ with } Q \text{ denoting the hard scale of the process) differs from the strict definition of twist (mass dimension of the operator minus its spin).} \]
form of $\tilde{g}_T$ ($\tilde{h}_L$) differs from the one of $\tilde{g}^T$ ($\tilde{h}^L$). In the following we denote as “WW-type approximation” the neglect of the tilde-functions (and quark mass terms) in the EOM-relations. Below we will also address the phenomenological justification of the WW-type approximation.

In order to proceed we introduce the measures $\Delta g(x)$ and $\Delta h(x)$ for the violation of the respective LIRs (see for instance Refs. 49, 47 for explicit forms of these terms) according to

$$g_T(x) = g_1(x) + \frac{d}{dx} g_T^{(1)}(x) + \Delta g(x), \quad (22)$$
$$h_L(x) = h_1(x) - \frac{d}{dx} h_L^{(1)}(x) + \Delta h(x). \quad (23)$$

If one substitutes the (1)-moments $g_T^{(1)}$ and $h_L^{(1)}$ from (20, 21) in Eqs. (22, 23), and uses both the WW approximation and the WW-type approximation one finds

$$\Delta g(x)^{\text{WW,WW-type}} \approx -g_1(x) - x \frac{d}{dx} \int_x^1 \frac{dy}{y} g_1(y) = 0, \quad (24)$$
$$\Delta h(x)^{\text{WW,WW-type}} \approx -h_1(x) - x^2 \frac{d}{dx} \int_x^1 \frac{dy}{y^2} h_1(y) = 0. \quad (25)$$

Eqs. (24, 25) show that the LIRs (1) and (2) are not violated if a generalized WW approximation is applied. This result is not entirely surprising keeping in mind that the violation of the LIRs is related to the amplitudes $B_i$ which are associated with the gauge link of the fully unintegrated correlator. Therefore, the $B_i$’s are necessarily related to quark-ghon-quark correlations which are neglected in the (generalized) WW approximation. This is also in line with the fact that in relativistic nucleon models without gluonic degrees of freedom the LIRs must be fulfilled. 61, 62

We also point out that in the generalized WW approximation instead of having the three functions $g_1$, $g_T$, and $g_T^{(1)}$, there is only one independent PDF. The same applies to the $h$-functions. In particular, one can immediately write\(^6\)

$$g_T^{(1)}(x)^{\text{WW,WW-type}} \approx x \int_x^1 \frac{dy}{y} g_1(y), \quad (26)$$
$$h_L^{(1)}(x)^{\text{WW,WW-type}} \approx -x^2 \int_x^1 \frac{dy}{y^2} h_1(y). \quad (27)$$

Phenomenological work on the basis of those relations was, for instance, carried out in Refs. 63, 64, 65. In Ref. 65, Eq. (27) was used in order to describe data for a certain longitudinal single spin asymmetry in semi-inclusive DIS. This investigation shows that the approximation (27) is not excluded, although more precise data would be helpful for having a stronger test.

\(^6\)In Ref. 12 for instance this approximation (for brevity) was just called “WW approximation” because, like Eqs. (18, 19), it also corresponds to neglecting purely interaction dependent terms.
3.2. T-odd case

In case of the T-odd LIRs (6)–(12) the situation is slightly different and in principle even simpler. Due to time-reversal invariance the $p_T$-integrated T-odd PDFs $f_T(x)$, $h(x)$, and $e_L(x)$ vanish,\textsuperscript{12,35}

$$f_T(x) = \int d^2p_T \ f_T(x, p_T^2) = 0, \quad h(x) = \int d^2p_T \ h(x, p_T^2) = 0, \quad e_L(x) = \int d^2p_T \ e_L(x, p_T^2) = 0,$$

given by (28), (29), and (30), respectively. This means that $f_{1T}^{(1)}(x)$, $h_1^{(1)}(x)$, and $e_T^{(1)}(x)$ are constants. In fact, since these moments have to vanish for $x = 1$, one can conclude that they should vanish for the entire $x$-range.

Moreover, the Sivers effect studied at COMPASS is compatible with zero both for a deuteron as well as a proton target.\textsuperscript{21,23,25,26} Therefore, the current experimental situation is not in conflict with a rather small $\tilde{f}_T$.

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$$\frac{d}{dx} f_{1T}^{(1)}(x) = 0, \quad \frac{d}{dx} h_1^{(1)}(x) = 0, \quad \frac{d}{dx} e_T^{(1)}(x) = 0. \quad (31)$$

In the WW-type approximation the tilde-functions and quark mass terms are set to zero. It then follows directly from (32) that $f_{1T}^{(1)}(x)$, $h_1^{(1)}(x)$, $e_T^{(1)}(x)$, $f_T^{(1)}(x)$, $f_T^{(1)}(x)$, $e_T^{(1)}(x, p_T^2)$, and $g^{(1)}(x, p_T^2)$ are zero.\textsuperscript{12} This is consistent with the results following from the LIRs in (31) and from the LIRs (6)–(12). So the T-odd LIRs (6)–(12), or (x-times) (1)-moments formed from them, are also not violated in the WW-type approximation.

Also for the T-odd functions we already have some phenomenological input on the status of the WW-type approximation. Since a nonzero asymmetry, typically attributed to the Sivers effect, was found in the HERMES experiment,\textsuperscript{20,22,24} in the case of T-odd PDFs this approximation seems to be violated. On the other hand the observed effect is not very large (of the order of few percent), and one should not expect WW-type approximations to work to a much better accuracy than that. Moreover, the Sivers effect studied at COMPASS is compatible with zero both for a deuteron as well as a proton target.\textsuperscript{21,23,25,26} Therefore, the current experimental situation is not in conflict with a rather small $\tilde{f}_T$.\textsuperscript{12,35}
4. Summary

We have given a complete list of LIRs between parton distributions and have discussed some of their consequences. We have studied the LIRs, known to be violated in general, with the aim to understand how strong this violation might be. It was found that several LIRs are satisfied in a generalized WW approximation in which one systematically neglects certain quark-gluon-quark correlations as well as quark mass terms. If such terms were small which is sometimes assumed, that would mean that LIRs could provide useful approximations for unknown PDFs and TMDs whenever applicable. Our approximation goes beyond the successful “standard WW approximation” quoted in Eqs. [18] and [19]. In particular, we also neglected purely interaction dependent terms which show up in relations originating from the QCD equations of motion (see also Ref. 12). We argued that there exists experimental evidence for the validity of the generalized WW approximation. On the other hand more (precise) data and tests are needed before a final conclusion can be reached. Only forthcoming data analyses and experiments at COMPASS, HERMES, and Jefferson Lab can ultimately reveal to what extent the generalized WW approximation, the LIRs, and their consequences provide useful approximations. Eventually, it is likely that the quality of the approximation depends on the particular case (function) under consideration.

Acknowledgments

This paper is dedicated to our respected colleague and friend Anatoli V. Efremov on the occasion of his 75th birthday. The authors thank the organizers of workshop on “Recent Advances in Perturbative QCD and Hadronic Physics”, ECT*, Trento (Italy) for the opportunity to report our work at this memorable event. The work is partially supported by the Verbundforschung “Hadronen und Kerne” of the BMBF. A. M. acknowledges the support of the NSF under Grant No. PHY-0855501. P. S. is supported by DOE contract No. DE-AC05-06OR23177, under which Jefferson Science Associates, LLC operates Jefferson Lab. T. T. is supported by the Cusanuswerk.

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