Optimal strategy for supplier selection problem integrated with optimal control problem of single product inventory system with piecewise holding cost

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Abstract. In this paper, we formulate a hybrid mathematical model of supplier selection problem integrated with inventory control problem of a single product inventory system with piecewise holding cost. This model will be formulated in a piecewise affine (PWA) form that can be converted into mixed logical dynamic (MLD) form. By using this MLD model, we solve the supplier selection problem and control this inventory system so that the stock level tracks a desired level as the reference trajectory as closed as possible with minimal total cost. We use model predictive control for hybrid system to solve the problem. From the numerical experiment results, the optimal supplier was selected at each time period and the evolution of the stock level tracks the desired level well.

1. Introduction
Management on supply chain tries to minimize the operational cost and improve the profit. It focuses in relationships in order to maximize outcome for all parties in the chain [1]. Supply chain network consisting of all parties that are manufacturer, suppliers, carriers or transporters, warehouses, retailers and customers [2]. The optimal strategy in supply chain management is made to minimize the operational cost with some service level constraint and determine the optimal quantity, optimal locations and optimal time of a product to be produced and distributed [3]. Supply chain contains inventory system that has to be optimized so that it gives the minimal holding cost. Inventory problem can appear in the several forms like raw material inventory, work-in-process inventory and finished product inventory and each of them needs to be controlled in order to minimize the holding cost [3]. Most of researchers were developed a mathematical model to optimize supply chain management. The basic mathematical model in a linear state space equation for periodic review inventory system was developed in [4] and the optimal strategy was determined by using linear quadratic control method. Another approach was used a particle swarm optimization to optimize the basic inventory system considering discount [5].

Supply chain management is also contains supplier selection problem. Supplier selection problem is appeared if we have several supplier alternatives to supply the product and we have to decide how many the product that will be purchased from each supplier. Several approaches were used to find the optimal strategy for supplier selection problem solving. The most approach is mathematical model [6]. In control theory, there is a mathematical model called hybrid...
dynamical system where the state value is depends on the current location (or event) which is called as piecewise-affine (PWA) model [7]. The PWA model can be converted into mixed logical dynamical system (MLD) by writing the PWA model in hybrid systems description language (HYSDEL) and the equivalent MLD model can be generated by using mld function in hybrid toolbox for MATLAB [8, 9]. For an inventory system with piecewise holding cost where the holding cost is different for several stock level intervals, [10] was formulated a hybrid mathematical model in piecewise-affine (PWA) model and controlled the stock level to track some desired level by using model predictive control for hybrid system. Predictive control is an optimal control method that can be used to solve a trajectory tracking control problem. The basic MPC for non-hybrid system was developed in [11] and the MPC for hybrid system was developed in [12, 13]. This optimal control method solves the problem i.e. determine the optimal input by defining an objective function and optimizing the corresponding optimization by using some optimization method.

In this paper, we integrate the supplier selection problem with inventory control problem of a single product inventory system with piecewise holding cost by formulating a PWA model and determine the optimal strategy by using model predictive control for hybrid system. We formulate the integrated supplier selection problem and inventory control problem in a PWA model, convert it into MLD model and determine the optimal strategy by using predictive control for hybrid system.

2. Mathematical Model
Let $u_s(k)$ be the amount of arriving shipment of the product from supplier $s$ that arrived at time/review period $k$ and $n_p$ be the lead time delay. Then, the dynamic of stock level of the product $y(k) \geq 0$ for any $k \geq 0$ can be stated as follows

$$y(k) = u_1(0) + u_2(0) + \cdots + u_S(0) + u_1(1) + u_2(1) + \cdots + u_S(1) + \cdots + u_1(k-n_p-1) + u_2(k-n_p-1) + \cdots + u_m(k-n_p-1) - d(0) - \cdots - d(k-1)$$

$$= \sum_{j=0}^{k-n_p-1} \sum_{s=1}^{S} u_s(j) - \sum_{j=0}^{k-1} d(j).$$

where $S$ denotes the number of the suppliers and $d(k)$ denotes the demand of the product at review period $k$. We develop the model for inventory system with piecewise holding cost (or holding cost considering discount) as follows. Let $q_j, j = 1, 2, \ldots, m$ denotes the holding cost where

$$\text{holding cost} = \begin{cases} 
q_1, & \text{if } \hat{y}_0 \leq y(k) \leq \hat{y}_1 \\
q_2, & \text{if } \hat{y}_1 < y(k) \leq \hat{y}_2 \\
\quad \vdots \\
q_m, & \text{if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m.
\end{cases}$$

(2)
Let $x_1(k) = y(k)$, $x_i(k) = q_j u(k - n_p + i)$ for $i = 2, 3, ..., n_p + 1$ and

$$
\begin{align*}
&x_1(k + 1) = x_1(k) + x_2(k) - q_j d(k) \\
&x_2(k + 1) = x_3(k) \\
&x_3(k + 1) = x_4(k) \\
&\vdots \\
&x_{(n-1)}(k + 1) = x_n(k) \\
&x_n(k + 1) = q_j u(k)
\end{align*}
$$

where $n = n_p + 1$, $x(k) = [x_1(k), x_2(k), ..., x_n(k)]^T$, then the dynamic of stock level of the product can be written as a PWA model as follows

$$
x(k + 1) = \begin{cases}
Ax(k) + B_1 u(k) + D_1 d(k) & \text{if } \hat{y}_0 \leq y(k) \leq \hat{y}_1 \\
Ax(k) + B_2 u(k) + D_2 d(k) & \text{if } \hat{y}_1 < y(k) \leq \hat{y}_2 \\
\vdots \\
Ax(k) + B_m u(k) + D_m d(k) & \text{if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m,
\end{cases}
$$

(3)

$$
y(k) = \begin{cases}
\frac{1}{q_1} x(k), & \text{if } \hat{y}_0 < y(k) \leq \hat{y}_1 \\
\vdots \\
\frac{1}{q_j} x(k), & \text{if } \hat{y}_{j-1} < y(k) \leq \hat{y}_j \\
\vdots \\
\frac{1}{q_m} x(k), & \text{if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m.
\end{cases}
$$

(4)

where

\[
A = \begin{bmatrix}
1 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix},
B_j = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 & \cdots & 1 \\
q_j
\end{bmatrix},
\]

and

\[
D = \begin{bmatrix}
-q_j \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}.
\]

This PWA model can be controlled by using predictive control for hybrid system by converting it into equivalent MLD model then determining the optimal input by using mixed-integer quadratic programming.

3. Numerical Experiment

Suppose that there are four suppliers namely $s_1, s_2, s_3$ and $s_4$ to supply a product where the purchasing cost per unit product for each review period 1, 2, 3, ..., 80 from these suppliers is given by Table 1. The maximum capacity of each supplier to supply the product is given by Table 2.
Suppose that the holding cost per unit is $25 if the product volume is 0 to 100 units and it will be $20 if the product volume is more than 100 units, hence $q_1 = 25$ and $q_2 = 20$, the maximum capacity of the storage is 300 units and the lead time delay $n_p = 2$. We solved this problem in Windows 8 with AMD A6 2.7 GHz of processor and 4 GB of Memory by using software MATLAB R2013a with hybrid system toolbox. By writing PWA model (3)-(4) in HYSDEL (available in Appendix 1) and using mld function in hybrid system toolbox, we convert the PWA model (3)-(4) into equivalent MLD model by adding auxiliary states $\delta$ and $z$ in the form of

$$x(k + 1) = Ax(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k)$$

$$y(k) = Cx(k) + D_1 u(k) + D_2 \delta(k) + D_3 z(k)$$

$$E_2 \delta(k) + E_3 z(k) \leq E_1 u(k) + E_4 x(k) + E_5$$

where the values of the matrices $A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4$ and $E_5$ are available in the Appendix 2. Let $H_p$ be the length of the horizon prediction for controller designing by using model predictive control for hybrid system given by [9, 12], the objective function for this optimal control problem is defined as the gain of the actual stock level from the desired stock level with weighting matrix $Q_y$ and the input cost with weighting matrix $Q_u$ and this optimal control problem is defined as follows

$$\min_{[u, \delta, z]_{0:H_p-1}} \sum_{k=0}^{H_p-1} \left[ ||y(k) - y_d||^2_{Q_y} + ||u(k)||^2_{Q_u} \right]$$

subject to:

$$x(k + 1) = Ax(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k)$$

$$y(k) = Cx(k) + D_1 u(k) + D_2 \delta(k) + D_3 z(k)$$

$$-E_4 x(k) - E_1 u(k) + E_2 \delta(k) + E_3 z(k) \leq E_5$$

$$u_{\min} \leq u(k) \leq u_{\max}$$

$$x_{\min} \leq x(k) \leq x_{\max}$$

$$y_{\min} \leq y(k) \leq y_{\max}$$

where $k = 0, 1, ..., H_p - 1$, notations $v_{\min}$ and $v_{\max}$ mean the lower bound and upper bound for $v$ respectively and the notation $||v||_Q^2$ means $v^T Q v$. The value of the desired stock level $y_d$ can

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**Table 1. Purchasing cost per unit ($\$\)**

| Time period (k) | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|----------------|-------|-------|-------|-------|
| 1, 2, ..., 20   | 9     | 9.5   | 10    | 10.5  |
| 21, 22, 23, ..., 40 | 11    | 9     | 8     | 8     |
| 41, 42, 43, ..., 60 | 10    | 11    | 12    | 8     |
| 61, 62, 63, ..., 80 | 9     | 12    | 11    | 10    |

**Table 2. Supplier capacity (unit)**

| Time period (k) | Supplier capacity |
|----------------|-------------------|
|                | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
| 1, 2, 3, ..., 80 | 150   | 200   | 250   | 300   |
be found in the simulation result. The demand for this simulation is assumed to be constant that is \( d(k) = 20 \) for all \( k > 0 \). Assume that the initial product stock level is 70 units and the other parameters are given by Table 3 and Table 4.

### Table 3. Parameter value

| Parameter | \( k \) | \( Q_e \) | \( H_e \) | \( u_{\min} \) | \( u_{\max} \) | \( y_{\min} \) | \( y_{\max} \) |
|-----------|--------|--------|--------|--------|--------|--------|--------|
| Value     | any    | 1      | 10     | 0      | 150    | 0      | 300    |

### Table 4. Weighting matrices for input

| Time period \((k)\)  | \( Q_e \)          |
|-----------------------|--------------------|
| 1,2,...,30            | \( \text{diag}(9,9,10,10.5) \) |
| 21,22,...,40          | \( \text{diag}(11,9,8,8) \)   |
| 41,42,...,60          | \( \text{diag}(10,11,12,8) \)  |
| 61,62,...,80          | \( \text{diag}(9,12,11,10) \)  |

By solving (8) using mixed-integer quadratic programming, we have the simulation results shown by Fig. 1-2. The optimal input is shown by Fig. 1. The actual stock level \( y(k) \) and the desired stock level \( y_d(k) \) are shown by Fig. 2. Fig. 2 shows the optimal input \( u \) i.e. the amount of the arriving shipment of the product for each time or review period \( k \). From this figure, it can be seen that all suppliers were selected to supply the product with different volume. This optimal input shown by Fig. 1 is applied to the system to generate the output i.e. the actual stock level \( y(k) \) shown by Fig. 2. Fig. 2 shows the actual stock level and the desired stock level decided by decision maker. At time period 1 to 5, the product stock level was fluctuated due to the lead time delay is 2 periods and it has to satisfy the demand at time period 1 and 2. For time periods 5 to 80, the product stock level is following the desired level well although after the desired level is changed it needs 2 to 3 time periods to follow the desired level.
4. Conclusion and Future Research

In this paper, the integrated supplier selection problem and tracking control problem of a single product inventory system with piecewise holding cost was considered. The problem was solved by formulating a piecewise affine model and determining the optimal amount of the product shipment from the optimal supplier by using model predictive control method for hybrid system. From the numerical experiment result, it can be conclude that the supplier selection problem was solved i.e. the optimal product volume from each supplier was determined and the stock level was controlled and followed the reference level well. In the future work, we will develop the model for multi-product inventory system and nonstationary demand. Furthermore, we will develop the model in deterministic and probabilistic environments.

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Appendix A. Code in HYSDEL for PWA model (3)-(4)

```plaintext
SYSTEM pwa_hybrid_inventory {
  INTERFACE {
    STATE { REAL x1 [0,4000];
           REAL x2 [0,4000];
           REAL x3 [0,4000];
           REAL y1 [0,200]; }
    INPUT { REAL u1 [0,150];
            REAL u2 [0,200];
            REAL u3 [0,250];
            REAL u4 [0,300]; }
    OUTPUT{ REAL y; }
    PARAMETER { REAL y_hat_1, d, q1, q2; }
  IMPLEMENTATION {
    AUX { REAL z1,z2,z3,z4; BOOL d1; }
    AD { d1 = y1 >= y_hat_1; }
    DA { z1 = {IF d1 THEN x1 + x2 - q2*d ELSE x1 + x2 - q1*d};
         z2 = {IF d1 THEN x3 ELSE x3 }; }
```
\[ \begin{align*}
  z_3 &= \{ \text{IF } d_1 \ \text{THEN } q_2 u_1 + q_2 u_2 + q_2 u_3 + q_2 u_4 \ \text{ELSE } q_1 u_1 + q_1 u_2 + q_1 u_3 + q_1 u_4 \} ; \\
  z_4 &= \{ \text{IF } d_1 \ \text{THEN } x_1/q_2 \ \text{ELSE } x_1/q_1 \} ; \\
  \text{CONTINUOUS} &= \{ x_1 = z_1 ; \ x_2 = z_2 ; \ x_3 = z_3 ; \ y_1 = z_4 ; \} \\
  \text{OUTPUT} &= \{ y = y_1 ; \} 
\end{align*} \]

Appendix 2. Matrices for system (5)-(7)

\[ A = \begin{bmatrix}
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  -1 & 0 & 0 & 0 \\
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 0 & 0
\end{bmatrix}, \quad
E_3 = \begin{bmatrix}
  0 & 0 & 0 & -1 \\
  -1 & -1 & 0 & 0 \\
  -1 & -1 & 0 & 0 \\
  1 & 1 & 0 & 0 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0
\end{bmatrix}, \quad
E_4 = \begin{bmatrix}
  0 & 0 & 0 & 1 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  -0.04 & 0 & 0 & 0 \\
  0.04 & 0 & 0 & 0 
\end{bmatrix} \]

References

[1] Christopher M 2011 Logistics and supply chain management (Great Britain: Pearson Education Limited)
[2] Chopra S and Meindl P 2007 Supply chain management, strategy, planning, and operation (New Jersey: Prentice Hall)
[3] Levi D S 2000 Designing and managing the supply chain (USA: McGraw-Hill Companies)
[4] Ignaciuk P and Bartoszewica A 2010 Linear-quadratic optimal control strategy for periodic-review perishable systems IEEE Trans. on Con. Sys. Tech. 20 1400
[5] Mousavi S M Niaki S Bahreininejad A and Musa S N 2014 Multi-Item multiperiodic inventory control problem with variable demand and discounts: A particle swarm optimization algorithm The Sci. World Journal 2014 1
[6] Kara S S 2011 Supplier selection with an integrated methodology in unknown environment Exp. Sys. with App. 38 2133
[7] Branicky M Introduction to hybrid systems (USA: Department of Electrical Engineering and Computer Science, Case Western Reserve University)
[8] Bemporad A 2004 Efficient conversion of mixed logical dynamical systems into an equivalent piecewise affine form IEEE Trans. Automat. Control 49 832
[9] Bemporad A 2012 Hybrid toolbox, users guide
[10] Sutrivo, Widowati and Munawaroh D A 2015 Hybrid mathematical model of inventory system with piecewise holding cost and its optimal strategy IEEE International conference on advanced mechatronics, intelligent manufacture, and industrial automation 29
[11] Maciejowski J M 2001 Predictive control with constraints (USA: Prentice Hall)
[12] Borrelli F, Bemporad A and Morari M 2011 Predictive control for linear and hybrid systems
[13] Lazar M, Heemels W, Weiland W and Bemporad A 2006 Stabilizing model predictive control of hybrid systems IEEE Trans. Automat. Control 51 1813