ABSTRACT This paper considers the distributed tracking problem for leader-follower multiagent systems, where the followers are heterogeneous multi-input multi-output systems which are subject to linearly parameterizable nonlinear matching uncertainties, and the leader’s information is disturbed during transmission. These two specific considerations make the problem investigated in this paper unsolvable by the existing control approaches. To tackle this problem, a novel distributed control approach is proposed. In particular, in order to recover the true value of the leader’s information in the presence of disturbances, an additive observability lemma has been established which facilitates the design of a new kind of filtering distributed observer. Moreover, an invariance-like lemma has been established to analyze the stability of a time-varying perturbed system, which helps to prove the stability of the closed-loop system. A numerical example is presented to validate the effectiveness of the proposed control approach.

INDEX TERMS Distributed tracking, filtering distributed observer, leader-follower multiagent system, nonlinear matching uncertainty.

I. INTRODUCTION
Over the past few years, motivated by the potential applications in vehicle formation and robot coordination [1]–[4], the cooperative control of multiagent systems has attracted a lot of attention and has been addressed by various control approaches, such as the robust control approach [5]–[8], the neural network approximation approach [9]–[11], the data-driven model-free approach [12]–[14], and so on. Particularly, the distributed tracking of leader-follower multiagent systems with linearly parameterizable nonlinear matching uncertainties was investigated in [15]–[17] by the adaptive control approach, where the first order systems, the second order systems and the more general systems with identical linear structure were considered in [15], [16] and [17], respectively.

One of the systematic and effective approaches to dealing with the cooperative control problem for leader-follower multiagent systems is the distributed observer approach [18], which consists of two parts. First, a distributed observer is designed for each follower to recover the leader’s information. Second, based on the estimated information of the leader, a local tracking controller is synthesized to achieve the cooperative control objective. The distributed observer approach was first proposed in [19] for a linear leader system under fixed communication network, and then extended in [20] for jointly connected switching communication network. Since after, some other variants of the distributed observer have been studied in [21]–[23], which further reduced the information needed from the leader. Specifically, [21] estimated both the system matrices and the state of the leader system under fixed communication network, and [22] found that the same distributed observer in [21] also works for jointly connected switching communication network. [23] further considered the scenario of a fully unknown leader system, and the proposed distributed observer depends solely on the output of the leader without any structural information of the leader’s dynamics. There are also some results focusing on
distributed observer for the case of nonlinear leader system, such as the rigid body system [24], [25], and for the case of multiple leaders [26]–[28].

The distributed tracking problem can also be viewed as a special case of the cooperative output regulation problem [19]–[21], [29]–[32], where the leader is referred to as the exosystem. Roughly speaking, there are mainly two control frameworks for solving the cooperative output regulation problem, i.e., the distributed feedforward approach [19]–[21] and the distributed internal model approach [29]–[32]. The distributed feedforward approach has the advantage that it does not rely on the transmission zeros condition, and thus allows the system output dimension to be higher than the system input dimension. While, the distributed internal model approach is endowed with the property of robustness against uncertain system parameters.

In this paper, we further consider the distributed tracking problem for leader-follower multiagent systems, where the followers are heterogenous multi-input multi-output systems which are subject to linearly parameterizable nonlinear matching uncertainties, and the leader’s information is disturbed during transmission. These two specific considerations make the problem investigated in this paper unsolvable by the existing control approaches. To tackle this problem, a novel distributed control approach is proposed. In particular, in order to recover the true value of the leader’s information in the presence of disturbances, an additive observability lemma has been established which facilitates the design of a new kind of filterting distributed observer. Moreover, an invariance-like lemma has been established to analyze the stability of a time-varying perturbed system, which helps to prove the stability of the closed-loop system under the filtering distributed observer and the local tracking controller.

In contrast to the existing works, the key contributions of this paper are summarized as follows.

- In [5]–[10], the followers’ dynamics have identical linear structure. In this work, by taking advantage of output regulation theory, the followers’ dynamics can be heterogenous. Moreover, the coefficient for the control input is allowed to be unknown, which increases the robustness of the proposed control from the perspective of fault tolerance.

- In contrast to the distributed internal model approaches as in [29]–[31] which studied single-input single-output nonlinear systems, the proposed control approach in this work is capable of dealing with multi-input multi-output nonlinear systems.

- Different from most of the existing works studying leader-follower multiagent systems, in this paper, the leader’s information obtained by the followers is subject to disturbances in a way that the disturbances cannot be pre-rejected by the leader itself and thus will propagate over the communication network. To tackle this issue, each follower is equipped with a novel filtering distributed observer to reject the disturbances and recover the leader’s information locally.

The rest of this paper is organized as follows. Preliminaries are summarized in Section II. Problem formulation is given in Section III. Section IV presents the main results. A numerical example is shown in Section V. Section VI concludes the paper.

II. PRELIMINARIES

In this section, we will first introduce the notation adopted in this paper, and then establish three lemmas which will be used subsequently.

A. NOTATION

- \( \otimes \) denotes the Kronecker product of matrices. \( 1_N \) denotes an \( N \)-dimensional column vector whose components are all 1.
- \( |x| \) denotes the Euclidean norm of a vector \( x \) and \( |A| \) denotes the induced norm of a matrix \( A \) by the Euclidean norm.
- \( R \) and \( C \) denote the real and complex number sets, respectively. For any \( a \in R \), \( S(a) = \left( \begin{array}{cc} 0 & 1 \\ -a^T & 0 \end{array} \right) \). For \( x_i \in R^n \), \( i = 1, \ldots, m \), \( \text{col}(x_1, \ldots, x_m) = [x_1^T, \ldots, x_m^T]^T \).
- For any vector \( x \in R^n \), \( x[i] \) denotes the \( i \)-th element of \( x \).
- For any matrix \( A \in R^{m \times n} \), \( R(A) \) denotes the rank of \( A \).
- \( \text{vec}(A) = \text{col}(A_1, \ldots, A_n) \) with \( A_i \) being the \( i \)-th column of \( A \).
- For a square matrix \( A \in R^{n \times n} \), \( \sigma(A) = \{ \lambda_1(A), \ldots, \lambda_n(A) \} \), where \( \lambda_i(A) \) denotes the \( i \)-th eigenvalue of \( A \), \( A \) is called neutrally stable if all the eigenvalues of \( A \) are semi-simple with zero real part.
- \( g(a) = \min_{1 \leq i \leq n}[\Re(\lambda_i(A))] \), where \( \Re(a) \) denotes the real part of a complex number \( a \).
- For a set of matrices \( A_1, A_2, \ldots, A_n \), \( D(A_1, A_2, \ldots, A_n) = \text{block diag}(A_1, A_2, \ldots, A_n) \).
- For any column vector \( X \in R^q \) for some positive integers \( n \) and \( q \), \( M_n^q(X) = [X_1, \ldots, X_q] \), where \( X_i \in R^n \), \( i = 1, \ldots, q \), and \( X = \text{col}(X_1, \ldots, X_q) \).
- If a function \( f(t) : R \rightarrow R^{m \times n} \) satisfies \( ||f(t)|| \leq be^{-\alpha t} \) for some \( \alpha, b > 0 \), then \( f(t) \) is said to decay to zero exponentially at the rate of \( \alpha \).

- A digraph \( G \) is defined as \( G = (V, E) \) which consists of a node set \( V = \{ 1, \ldots, N \} \) and an edge set \( E = \{ (i,j), i,j \in V, i \neq j \} \). An edge from node \( i \) to node \( j \) is denoted by \( (i,j) \), and node \( i \) is called the neighbor of node \( j \). If the digraph \( G \) contains a sequence of edges of the form \( (i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1}) \), then the set \( \{(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})\} \) is called a path of \( G \) from node \( i_1 \) to node \( i_{k+1} \), and node \( i_{k+1} \) is said to be reachable from node \( i_1 \). A digraph is said to contain a spanning tree if there exists a node \( i \) such that any other node is reachable from it and node \( i \) is called the root of the spanning tree. The edge \( (i,j) \) is called undirected if \( (i,j) \in E \) implies \( (j,i) \in E \). The digraph \( G \) is called undirected if every edge in \( E \) is undirected.
- The weighted adjacency matrix \( A = [a_{ij}] \in R^{N \times N} \) of \( G \) is defined as \( a_{ij} = 0 \), and for \( i \neq j \), \( a_{ij} > 0 \iff (i,j) \in E \) and \( a_{ij} = 0 \) otherwise. Moreover, \( a_{ij} = a_{ji} \) if \( (i,j) \) is an undirected edge. The Laplacian matrix \( L = [l_{ij}] \in R^{N \times N} \) of \( G \) is defined as \( l_{ii} = \sum_{j=1}^{N} a_{ij} \) and \( l_{ij} = -a_{ij} \) if \( i \neq j \).
B. LEMMAS

Lemma 1: Consider two observable pairs \((C_1, A_1), (C_2, A_2)\), where \(C_1 \in \mathbb{R}^{r \times p}, A_1 \in \mathbb{R}^{p \times r}, C_2 \in \mathbb{R}^{r \times q}, A_2 \in \mathbb{R}^{q \times r}\). If \(\sigma(A_1) \cap \sigma(A_2) = \emptyset\), then the pair \((C, A)\) with \(C = [C_1, C_2], A = D(A_1, A_2)\) is also observable.

Proof: Let

\[
H(\lambda) = \begin{pmatrix} C_1 & C_2 \\ \lambda I_p - A_1 & 0_{p \times q} \\ 0_{q \times p} & \lambda I_q - A_2 \end{pmatrix}.
\]

By the well-known PBH test (Theorem 2.4.9, [33]), the pair \((C, A)\) is observable if and only if for all \(\lambda \in \mathbb{C}\), \(\mathcal{R}(H(\lambda)) = p + q\). Let

\[
H_1(\lambda) = \begin{pmatrix} C_1 \\ \lambda I_p - A_1 \\ 0_{q \times p} \end{pmatrix}, \quad H_2(\lambda) = \begin{pmatrix} C_2 \\ 0_{p \times q} \\ \lambda I_q - A_2 \end{pmatrix}.
\]

Since the pair \((C_1, A_1)\) is observable, for all \(\lambda \in \mathbb{C}\), the columns of \(H_1(\lambda)\) are linear independent. Similarly, the columns of \(H_2(\lambda)\) are also linear independent.

- If \(\lambda \not\in \sigma(A_2)\), then \(\lambda \not\in \sigma(A_1)\) since \(\sigma(A_1) \cap \sigma(A_2) = \emptyset\). As a result, \(\lambda I_p - A_1\) has full rank. Therefore, the columns of \(H_1(\lambda)\) and \(H_2(\lambda)\) are linear independent with respect to each other. Hence, \(\mathcal{R}(H(\lambda)) = p + q\).

- Write \(H(\lambda) = H_a(\lambda)H_b(\lambda)\) where

\[
H_a(\lambda) = \begin{pmatrix} I_r \\ 0_{r \times p} \\ 0_{q \times r} \end{pmatrix}, \quad H_b(\lambda) = \begin{pmatrix} C_1 \\ \lambda I_p - A_1 \\ 0_{q \times p} \end{pmatrix}.
\]

If \(\lambda \not\in \sigma(A_2)\), then \(\mathcal{R}(H_b(\lambda)) = r + p + q\). Since \((C_1, A_1)\) is observable, \(\mathcal{R}(H_b(\lambda)) = p + q\). Therefore, by the Sylvester’s inequality,\(^1\) we have

\[
(r + p + q) + (p + q) - (r + p + q) \leq \mathcal{R}(H(\lambda)) \leq p + q
\]
and thus we have \(\mathcal{R}(H(\lambda)) = p + q\).

Therefore, for all \(\lambda \in \mathbb{C}\), \(\mathcal{R}(H(\lambda)) = p + q\), and hence the pair \((C, A)\) is observable.

By invoking Lemma 1 recursively, the following result can be obtained immediately.

Corollary 1: Consider \(n\) observable pairs \((C_1, A_1), (C_2, A_2), \ldots, (C_n, A_n)\) where \(C_k \in \mathbb{R}^{r \times p_k}, A_k \in \mathbb{R}^{p_k \times r}, k = 1, \ldots, n\). If \(\sigma(A_1) \cap \sigma(A_2) \cap \cdots \sigma(A_n) = \emptyset\), then the pair \((C, A)\) with \(C = [C_1, C_2, \ldots, C_n], A = D(A_1, A_2, \ldots, A_n)\) is observable.

Remark 1: Lemma 1 presents an observability result for two additive systems, i.e., whether the state of the augmented system is observable from the outputs of the two subsystems. The sufficient condition \(\sigma(A_1) \cap \sigma(A_2) = \emptyset\) is easy to understand because if \(\sigma(A_1) \cap \sigma(A_2) = \emptyset\), then it is possible that the components of the outputs of the two subsystems having the same mode are mixed together, which, as a result, may disable the distinction of the original state. A simple example is the addition of two constant signals.

Lemma 2: Consider the following system

\[
\dot{x} = f(x, t) + \Delta(t)
\]

where \(x \in \mathbb{R}^n, f: \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n\) is locally Lipschitz in \(x\), piecewise continuous in \(t\), and uniformly bounded if \(x\) is bounded for all \(t \geq 0, \Delta: \mathbb{R} \rightarrow \mathbb{R}^n\) is bounded, piecewise continuous in \(t\), and satisfies \(\int_0^\infty ||\Delta(t)|| dt < \infty\). Suppose \(x = \text{col}(x_a, x_b)\) where \(x_a \in \mathbb{R}^{n_a}, x_b \in \mathbb{R}^{n_b}\) satisfying \(n_a > 0, n_b \geq 0\) with \(n_a + n_b = n\), and moreover, there exists \(V(x): \mathbb{R}^n \rightarrow \mathbb{R}\) being positive definite, radially unbounded, and satisfying

\[
\dot{V}(x) = -x_a^T K x_a + x_a^T \Sigma \Delta(t)
\]

where \(K \in \mathbb{R}^{n_a \times n_a}\) is positive definite and \(\Sigma \in \mathbb{R}^{n_a \times n_a}\). Then \(\lim_{t \to \infty} x_a(t) = 0\).

Proof: Let \(\kappa = \|K\|, \epsilon = ||\Sigma||\). By (7), we have

\[
\dot{V}(x) \leq -\kappa ||x_a||^2 + \epsilon ||x_a|| \cdot ||\Delta(t)||
\]

\[
= -\kappa ||x_a|| \left( ||x_a|| - \frac{\epsilon}{\kappa} ||\Delta(t)|| \right)
\]

For all \(t \geq 0\), the right hand side of (8) achieves the maximum value if and only if \(||x_a|| = \frac{\epsilon}{\kappa} ||\Delta(t)||\). Therefore,

\[
\dot{V}(x) \leq \frac{\epsilon^2 ||\Delta(t)||^2}{4\kappa}
\]

which leads to

\[
V(x(t)) - V(x(0)) \leq \int_0^t \epsilon^2 ||\Delta(t)||^2 d\tau.
\]

Since \(\Delta(t)\) is bounded and \(\int_0^\infty ||\Delta(t)|| dt < \infty\), \(V(x(t)) - V(x(0)) < \infty\) by (10). Thus, \(V(x(t))\) is bounded for all \(t \geq 0\), and so is \(x(t)\) since \(V(x(t))\) is radially unbounded. Let \(w(t) = x_a(t)^T \Sigma \Delta(t)\). Since \(x_a\) is bounded and \(\int_0^\infty ||\Delta(t)|| dt < \infty\), \(\int_0^\infty ||w(t)|| dt < \infty\). Therefore, \(\int_0^t ||w(t)|| d\tau < \infty\) for all \(t \geq 0\) and thus \(\int_0^t \dot{W}(t) d\tau\) is bounded for all \(t \geq 0\). Let \(W(t) = V(x(t)) - \int_0^t \dot{W}(t) d\tau\). Since \(V(x)\) is lower bounded and \(\int_0^t \dot{W}(t) d\tau\) is bounded, \(W\) is also lower bounded. By (7), we have

\[
\dot{W} = \dot{V}(x) - w = -x_a^T \Sigma x_a \leq 0.
\]

Since \(x\) is bounded, \(\dot{x}\) is bounded by (6) and so is \(\dot{W}(t)\). Thus, by Barbalat’s Lemma, \(\lim_{t \to \infty} \dot{W}(t) = 0\), which indicates that \(\lim_{t \to \infty} x_a(t) = 0\).

Remark 2: Lemma 2 presents an invariance-like result which can be viewed as a generalization of Theorem 8.4 of [34] in that the system is subject to external perturbation. Note that in Theorem 8.4 of [34], \(\dot{V}\) is negative semidefinite. While, in Lemma 2, \(\dot{V}\) is indefinite.

Lemma 3: Let \(A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m\), and \(\text{rank}(A) = \text{rank}(A, b) = k\) for some positive integers \(m, n, k \geq 1\), and let

\[
\mathcal{R}(A) + \mathcal{R}(B) - n \leq \mathcal{R}(AB) \leq \min\{\mathcal{R}(A), \mathcal{R}(B)\}\text{ for any matrices } A \in \mathbb{R}^{m \times n} \text{ and } B \in \mathbb{R}^{n \times p}.
\]
\( \dot{A}(t) \in \mathbb{R}^{n \times n} \) and \( \dot{b}(t) \in \mathbb{R}^m \) be any bounded and piecewise continuous functions of \( t \) such that \( \dot{A}(t) \triangleq (\dot{A}(t) - A) \rightarrow 0 \) and \( \dot{b}(t) \triangleq (\dot{b}(t) - b) \rightarrow 0 \) as \( t \rightarrow \infty \) exponentially at the rate of \( \alpha \). Then, for any \( x(t_0) \in \mathbb{R}^n \) and \( \varepsilon > 0 \), the following system

\[
\dot{x} = -\varepsilon \dot{A}(t)^T (\dot{A}(t)x - \dot{b}(t))
\]  

(12)

has a unique bounded solution \( x(t) \) for \( t \geq t_0 \) such that, for some \( x^* \in \mathbb{R}^n \) satisfying \( Ax^* = b \), \( \lim_{t \to \infty} (x(t) - x^*) = 0 \) exponentially. In particular, if \( \varepsilon \) is sufficiently large, then \( x(t) - x^* \to 0 \) as \( t \to \infty \) exponentially at the rate of \( \alpha \).

**Proof:** Let \( V_x = \frac{1}{2}x^T x \). Since \( \dot{A}(t) \) and \( \dot{b}(t) \) are bounded, along (12), we have

\[
\dot{V}_x = x^T \left( -\varepsilon \dot{A}(t)^T \dot{A}(t)x - \dot{b}(t) \right)
\]

\[
= x^T \left( -\varepsilon \dot{A}(t)^T \dot{A}(t)x + \varepsilon x^T (A)^T \dot{b}(t) \right)
\]

\[
= -\varepsilon x^T (A)^T \dot{A}(t)x + \dot{b}(t) x^T (A)^T \dot{b}(t)
\]

\[
\leq \varepsilon x^T (A)^T \dot{b}(t) \leq \varepsilon ||x|| \cdot ||\dot{A}(t)^T \dot{b}(t)||
\]

\[
\leq a_1 \sqrt{V_x}
\]  

(13)

for some \( a_1 > 0 \). Therefore, by the comparison lemma (Lemma 3.4 of [34]), \( V_x(t) \leq (a_1 t + a_2)^2/4 \) for some \( a_2 \in \mathbb{R} \), which in turn implies \( ||x(t)|| \leq |a_1 t + a_2|/\sqrt{2} \).

By singular value decomposition method, there exists an orthogonal matrix \( P \in \mathbb{R}^{n \times n} \) such that \( AP = \begin{bmatrix} \tilde{A} & 0_{m \times (n-k)} \end{bmatrix} \) where \( \tilde{A} \in \mathbb{R}^{m \times k} \). Then we have

\[
P^T A^T AP = \begin{bmatrix} \tilde{A}^T \tilde{A} & 0_{k \times (n-k)} \n 0_{(n-k) \times k} & 0_{(n-k) \times (n-k)} \end{bmatrix}
\]

(14)

and

\[
P^T A^T b = \begin{bmatrix} \tilde{A}^T b \\
0_{(n-k) \times 1} \end{bmatrix}.
\]

Since \( \tilde{A} \) has full column rank, there exists a unique \( x^* \in \mathbb{R}^k \) such that \( \tilde{A} x^* = b \). Let \( x^* \) be any column vector of dimension \((n-k), x^* = P \begin{bmatrix} \bar{x}^*_1 \\
\bar{x}^*_2 \end{bmatrix} \). Then

\[
Ax^* = APP^T x^* = \begin{bmatrix} 0 \\
\bar{x}^*_1 \bar{x}^*_2 \end{bmatrix} = b.
\]

(16)

Next, let \( \tilde{x} = P^T x \). Then,

\[
\dot{\tilde{x}} = P^T ( -\varepsilon \dot{A}(t)^T \dot{A}(t)x + \varepsilon x^T (A)^T \dot{b}(t) )
\]

\[
= -\varepsilon P^T A^T AX + \varepsilon P^T A^T x - \varepsilon P^T (A)^T b - \varepsilon P^T A^T b + \varepsilon P^T A^T b
\]

\[
= -\varepsilon P^T A^T AX + \varepsilon P^T (A)^T b - \varepsilon P^T A^T b + \varepsilon P^T A^T b
\]

\[
= -\varepsilon P^T A^T AX + \varepsilon P^T (A)^T b - \varepsilon P^T A^T b + \varepsilon P^T A^T b
\]

\[
= -\varepsilon P^T (A)^T \dot{b}(t) - A^T b + \varepsilon P^T (A)^T \dot{b}(t) - A^T b.
\]

(17)

Clearly, (17) is in the same form as equation (16) of [21], and \( \lim_{t \to \infty} d(t) = 0 \) exponentially at the rate of \( \alpha \). The rest of the proof is similar to the proof of Lemma 3 of [21] and thus is omitted.

**Remark 3:** Lemma 3 is a straightforward extension of Lemma 3 of [21] and thus the proof is similar.

### III. PROBLEM FORMULATION

Consider a leader-follower multiagent system consisting of one leader and \( N \) followers. The dynamics of the leader are given by

\[
\dot{y}_0 = \Phi_0 y_0
\]

(19a)

\[
y_0 = \Psi_0 \eta_0
\]

(19b)

where \( \eta_0 \in \mathbb{R}^{q_0}, y_0 \in \mathbb{R}^p \) are the state and output of the leader, respectively, and \( \Phi_0 \in \mathbb{R}^{q_0 \times q}, \Psi_0 \in \mathbb{R}^{p \times q} \) are constant matrices. For \( i = 1, \ldots, N \), the dynamics of the \( i \)th follower are given by

\[
\dot{x}_i = A_i x_i + B_i (\dot{u}_i + f_i(x_i, t) \theta_i)
\]

(20a)

\[
y_i = C_i x_i
\]

(20b)

where \( x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, y_i \in \mathbb{R}^p \) are the system state, control input and system output, respectively; \( b_i > 0 \) is some uncertain control input coefficient; \( A_i, B_i, C_i \) are known constant matrices of compatible dimensions; \( f_i : \mathbb{R}^{n_i} \times \mathbb{R} \to \mathbb{R}^{m_i} \times \mathbb{R}^n \) is a known regression function which is assumed to be locally Lipschitz in \( x_i \), piecewise continuous in \( t \), and uniformly bounded if \( x_i \) is bounded for all \( i \geq 0 \); \( \theta_i \in \mathbb{R}^{q_i} \) is a constant vector consisting of uncertain system parameters.

Some assumptions regarding systems (19) and (20) are listed as follows.

**Assumption 1:** \((A_i, B_i)\) is stabilizable.

**Assumption 2:** \((\Psi_0, \Phi_0)\) is observable.

**Remark 4:** Assumption 1 is a standard assumption for the control problem of systems embedded with linear structure. One may refer to [19]–[21], [27], [28], [32], [35], just to name a few. Regarding Assumption 2, in this paper, we consider a challenging case where only the output \( y_0 \) of the leader is available. In this scenario, Assumption 2 is necessary since without it, one can never recover the leader’s state \( y_0 \) solely from the output \( y_0 \). Note that for the case where the state \( y_0 \) is available, it turns out that \( \Psi_0 = I_q \) and hence Assumption 2 shall always hold. Therefore, it becomes a special case of the case considered in this paper.

The communication network for the leader-follower multiagent system composed of (19) and (20) is described by a digraph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) with \( \mathcal{V} = \{0, 1, \ldots, N\} \), where the node 0 is associated with the leader and the node \( i, i = 1, \ldots, N \), is associated with the \( i \)th follower. For \( i = 0, 1, \ldots, N, j = 1, \ldots, N, (i, j) \in \mathcal{E} \) if and only if follower \( j \) can access the information of follower \( i \) or the leader if \( i = 0 \).

We further define a subgraph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) of \( \mathcal{G} \) where \( \mathcal{V} = \{1, \ldots, N\} \) and \( \mathcal{E} = \mathcal{E} \cap (\mathcal{V} \times \mathcal{V}) \). Let \( \tilde{A} = [a_{ij}] \) denote the weighted adjacency matrix of \( \mathcal{G} \). \( \mathcal{L} \) denote the Laplacian
of $\mathcal{G}$ and $H = \mathcal{L} + \mathcal{D}(a_{10}, \ldots, a_{N0})$. To ensure that the leader’s information can pass to each follower through at least one communication path, the following standard assumption regarding the communication network is imposed.

**Assumption 3:** $\mathcal{G}$ contains a spanning tree with the node 0 as the root.

**Remark 5:** Under Assumption 3, by Lemma 1.6 of [4], $\sigma(H) > 0$. Let $U = \mathcal{D}(\mu_1, \ldots, \mu_N)$ with $\mu_i > 0$ and $H^U = UH$. Then, by Lemma 1 of [36], $\sigma(H^U) \geq \mu\sigma(H)$, where $\mu = \min\{\mu_1, \ldots, \mu_N\}$.

For $i = 1, \ldots, N$, define the tracking error as

$$ e_i = y_i - y_0. \quad (21) $$

Then the distributed tracking problem considered in this paper is formulated as follows.

**Problem 1:** Given systems (19), (20), and a digraph $\mathcal{G}$, for each follower, design a distributed control law $u_i$ which utilizes only local measurement and neighboring information over the communication network such that the state of the closed-loop system starting from any initial condition exists and is bounded for all $t \geq 0$, and for $i = 1, \ldots, N$,

$$ \lim_{t \to \infty} e_i(t) = 0. \quad (22) $$

In order to solve Problem 1, the following two assumptions are necessary.

**Assumption 4:** $\Phi_0$ is neutrally stable.

**Assumption 5:** The following regulator equations

$$ X_i\Phi_0 = A_iX_i + B_iu_i \quad (23a) $$

$$ 0 = C_iX_i - \Psi_0 \quad (23b) $$

admit a solution pair $(X_i, U_i)$ where $X_i \in \mathbb{R}^{n_i \times q}$, $U_i \in \mathbb{R}^{m_i \times q}$.

**Remark 6:** In practical applications, the boundedness of the state, and thus the output, of the closed-loop system for a nonlinear control system is often considered as a basic control objective\(^2\) as required by Problem 1. By the definition of the tracking error (21) and the control objective (22), it is obvious that if Problem 1 is solvable, then $y_i$ is bounded if and only if $y_0$ is bounded. Given this circumstance, Assumption 4 is necessary since given the linear leader system (19), under Assumption 2, $y_0$ is bounded, and, in the meanwhile, not decaying to zero, only if Assumption 4 holds. While, if we do not require the boundedness of the state or the output of the closed-loop system for some specific problem, then we no longer need Assumption 4.

**Remark 7:** Combine systems (19), (20) and (21) as

$$ \dot{\tilde{y}}_0 = \Phi_0\tilde{y}_0 \quad (24a) $$

$$ \dot{\bar{x}}_i = A_i\bar{x}_i + B_i\bar{u}_i \quad (24b) $$

$$ e_i = C_i\bar{x}_i - \Psi_0\tilde{y}_0, \quad i = 1, \ldots, N \quad (24c) $$

where $\bar{u}_i = b_iu_i + f_i(x_i, t)\theta_i$. Then, $u_i = b_i^{-1}\bar{u}_i - b_i^{-1}f_i(x_i, t)\theta_i$, which implies that there exists a control law $u_i$ solving Problem 1 with respect to systems (19), (20) and (21) only if there exists a control law $\bar{u}_i$ solving Problem 1 with respect to system (24). Furthermore, by Theorem 1.7 of [35], there exists a control law $\bar{u}_i$ solving Problem 1 with respect to system (24) only if the regulator equations (23) admit a solution pair $(X_i, U_i)$. As a result, Problem 1 is solvable only if Assumption 5 holds. Note that the existence of the solution to the regulator equations is a standard assumption in the literature of output regulation theory, and one may refer to [18]–[21], [28], [32], [35].

In this paper, different from the existing results on the cooperative control problem of leader-follower multiagent system which assume perfect leader, we consider the situation where the leader is subject to external disturbance. In general, there are two ways that the leader could get disturbed. First, the disturbance might impose directly on the leader as shown by case (a) in Fig. 1, which, mathematically, can be described as follows

$$ \dot{\hat{y}}_0 = \bar{y}_0d_0 \quad (25a) $$

$$ \dot{y}_0 = \bar{y}_0d_0 \quad (25b) $$

$$ \dot{\bar{y}}_0 = y_0 + \bar{y}_0 \quad (25c) $$

where $d_0 \in \mathbb{R}^d$, $\bar{y}_0 \in \mathbb{R}^p$, $\bar{y}_0 \in \mathbb{R}^{q \times n_d}$ and $\bar{y}_0 \in \mathbb{R}^{p \times n_d}$ are constant matrices, and $\bar{y}_0 \in \mathbb{R}^p$ denotes the disturbed output of the leader. Without loss of generality, it can be assumed that $(\bar{y}_0, \bar{y}_0)$ is observable. Let $\Gamma_0 = (\Psi_0, \bar{y}_0)$ and $\Delta_0 = \mathcal{D}(\Phi_0, \bar{y}_0)$. Then, according to Lemma 1, under Assumption 2, $(\Gamma_0, \Delta_0)$ is also observable if $\sigma(\Phi_0) \cap \sigma(\bar{y}_0) = \emptyset$. Then, by designing the following local Luenberger observer,

$$ \begin{align*}
\dot{\hat{y}}_0 &= \Delta_0(\hat{y}_0) + L_0(\bar{y}_0 - \bar{y}_0) \\
\end{align*} \quad (26a) $$

where $L_0$ is such that $\Delta_0 - L_0\Gamma_0$ is Hurwitz, it follows that

$$ \lim_{t \to \infty} (y_0(t) - \hat{y}_0(t)) = 0 \quad (27) $$

where $\hat{y}_0(t) = \Psi_0\hat{y}_0(t)$. Hence, instead of sending the disturbed output $\bar{y}_0(t)$, the leader can send the estimated output $\hat{y}_0(t)$ which will converge to the true value $y_0(t)$ exponentially without making great impact on the followers. In other words,

![FIGURE 1. Two ways the leader could get disturbed.](image-url)
the impact of the external disturbance can be annihilated locally at the leader, which makes this case easy to deal with. The second scenario, which will be the focus of this paper, is much more difficult to deal with than the first one. For simplicity, we use the generic symbol $\chi_0 \in \mathbb{R}^{a \times}$ to represent the leader’s information that will be passed to each follower through the communication network. Note that, as will be seen later in the next section, the leader’s information that will be passed to each follower of transmission as shown by case (b) in Fig. 1, which is no longer locally at the leader and thus has to be dealt with by each follower. Then, we consider the case that $\chi_0$ is disturbed on the way of transmission as shown by case (b) in Fig. 1, which is no longer locally at the leader and thus has to be dealt with by each follower.

IV. MAIN RESULTS

To solve Problem 1, we adopt the distributed observer approach, which consists of two parts. The first part aims to find a distributed observer for each follower to estimate the leader’s information, and the second part aims to design a local tracking controller based on the estimated leader’s information obtained by the distributed observer. In what follows, we will show the design process of these two parts.

A. FILTERING DISTRIBUTED OBSERVER

In our previous result [37], for the case of a perfect leader, the following distributed observer was proposed to recover the leader’s information. Under Assumption 2, by condition 2 of [38], there exists a unique positive definite matrix $P_0 \in \mathbb{R}^{q \times q}$ satisfying

$$P_0 \Phi_0 + \Phi_0 P_0^T \Psi_0 + \Psi_0 P_0 + Q_0 = 0$$

given some positive definite matrix $Q_0 \in \mathbb{R}^{q \times q}$. Let $\ell_0 = P_0 \Psi_0^T \in \mathbb{R}^{q \times p}$. For $i = 1, \ldots, N$, define

$$\dot{\Phi}_i = \mu_\Phi \sum_{j=0}^{N} a_{ij}(\Phi_j - \Phi_i)$$  \hspace{1cm} (29a)

$$\dot{\Psi}_i = \mu_\Psi \sum_{j=0}^{N} a_{ij}(\Psi_j - \Psi_i)$$  \hspace{1cm} (29b)

$$\dot{\ell}_i = \mu_\ell \sum_{j=0}^{N} a_{ij}(\ell_j - \ell_i)$$  \hspace{1cm} (29c)

$$\dot{\eta}_i = \Phi_i \eta_i + \mu_\eta \ell_i \sum_{j=0}^{N} a_{ij}(y_j - y_i)$$  \hspace{1cm} (29d)

$$y_i = \Psi_i \eta_i$$  \hspace{1cm} (29e)

where $\Phi_i \in \mathbb{R}^{q \times q}, \Psi_i \in \mathbb{R}^{p \times q}, \ell_i \in \mathbb{R}^{q \times p}, \eta_i \in \mathbb{R}^q$ are the estimation of $\Phi_0, \Psi_0, \ell_0$ and $\eta_0$ by the $i$th follower, respectively, and $\mu_\Phi, \mu_\Psi, \mu_\ell$ and $\mu_\eta$ are positive observer gains. The following result has been established in [37].

Lemma 4: Given systems (19), (29) and the communication graph $\bar{G}$, under Assumptions 2, 4, 3, for any $\eta_0(t) \in \mathbb{R}^q$, $t = 0, 1, \ldots, N$, $\Phi_i(0) \in \mathbb{R}^{q \times q}, \Psi_i(0) \in \mathbb{R}^{p \times q}, \ell_i(0) \in \mathbb{R}^{q \times p}$, $i = 1, \ldots, N$, and for any $\mu_\Phi, \mu_\Psi, \mu_\ell > 0, \mu_\eta > \sigma(H)^{-1}$, $\Phi_i(t), \Psi_i(t), \ell_i(t), \eta_i(t)$ exist for all $t \geq 0$ and satisfy, for $i = 1, \ldots, N$,

$$\lim_{t \to \infty} (\Phi_i(t) - \Phi_0) = 0$$  \hspace{1cm} (30a)

$$\lim_{t \to \infty} (\Psi_i(t) - \Psi_0) = 0$$  \hspace{1cm} (30b)

$$\lim_{t \to \infty} (\ell_i(t) - \ell_0) = 0$$  \hspace{1cm} (30c)

$$\lim_{t \to \infty} (\eta_i(t) - \eta_0(t)) = 0$$  \hspace{1cm} (30d)

exponentially.

Note that for the distributed observer (29), the leader’s information $\chi_0$ can be detailed as

$$\chi_0 = \text{col}(\text{vec}(\Phi_0), \text{vec}(\Psi_0), \text{vec}(\ell_0), y_0) \in \mathbb{R}^{q^2 + 2mp + p}.$$  \hspace{1cm} (31)

However, in contrast to [37], in this paper, the acquisition of $\chi_0$ cannot be made accurately, but is subject to equation (28). As a result, the distributed observer (29) is no longer feasible. In light of Lemma 1, in order to recover the leader’s information $\chi_0$ from the disturbed information $\chi_0^d$, we impose the following assumption on the external disturbance $\chi_0^d$.

Assumption 6: Suppose $\chi_0^d = \text{col}(\chi_{0,1}^d, \ldots, \chi_{0,N}^d)$, and

$$\chi_{0,i}^d = \sum_{k=1}^{\delta} a_{k,i} \sin(\omega_k t + \beta_{k,i}), \hspace{1cm} i = 1, \ldots, N$$  \hspace{1cm} (32a)

with known frequencies $\omega_k$ and unknown amplitudes $a_{k,i}$ and initial phases $\beta_{k,i}$. Moreover, $\sigma(\Phi_0) \cap \Omega = \emptyset$ where $\Omega = \{\pm \omega_k, k = 1, \ldots, \delta\}$ with $r^2 = -1$.  \hspace{1cm} (32b)

Remark 8: By Fourier series, any periodic disturbance satisfying the Dirichlet conditions with zero periodical integral can be approximately expressed in the form of (31) with arbitrarily prescribed accuracy. Thus, Assumption 6 can accommodate a large class of disturbances. While, the constant bias is not considered in (31) because it would make the estimation impossible. In particular, it is necessary for the distributed observer to estimate the system and output matrices of the leader, namely, $\Phi_0$ and $\Psi_0$, since they contain the key structure information of the leader’s dynamics. Note that the entries of $\Phi_0$ and $\Psi_0$ are all constants, and it is obviously impossible to separate two constants from their sum. Therefore, to make it possible to recover $\Phi_0$ and $\Psi_0$ from their disturbed values, constant bias in the disturbance should be ruled out.

Next, we show the design of the filtering distributed observer, which is able to filter out the external disturbance imposed on the leader’s information.

First, let

$$\nu_0 = D(S(\omega_1), \ldots, S(\omega_\delta)) \in \mathbb{R}^{2\delta \times 2\delta}$$  \hspace{1cm} (32a)

$$\xi_0 = (1 \ 0 \ 1 \ 0 \ \ldots \ 1 \ 0) \in \mathbb{R}^{1 \times 2\delta}. \hspace{1cm} (32b)$$
Under Assumption 6, \( y_k^d \) can be generated by (25a) and (25b) with
\[
Y_0 = I_p \otimes v_0, \quad \Xi_0 = I_p \otimes \xi_0.
\]
Let \( \psi_0 = \text{col}(\eta_0, d_0), \Lambda_0 = D(\Phi_0, Y_0) \in \mathbb{R}^{(q+2p\delta) \times (q+2p\delta)}, \Gamma_0 = (\Psi_0, \Xi_0) \in \mathbb{R}^{p \times (q+2p\delta)}, \) and then it follows that
\[
\dot{\psi}_0 = \Lambda_0 \psi_0, \quad \dot{\gamma}_0 = \Gamma_0 \psi_0.
\] (33a)
\[
\bar{\psi}_0 = \Lambda_0 \bar{\psi}_0.
\] (33b)

Noting that the pair \([L_0, \Phi_0] = (1, \ldots, \delta)\) is observable for any \( \omega_k \in \mathbb{R} \), and \( \omega_k \neq \omega_j \) if \( i \neq j \), by Corollary 1, the pair \((L_0, \Phi_0)\) is observable, and so is \((L_0, \Theta_0)\), and hence \((\Gamma_0, \Lambda_0)\) under Assumption 2. Thus, there exists a unique positive definite matrix \( P_0 \in \mathbb{R}^{(q+2p\delta) \times (q+2p\delta)} \) satisfying
\[
P_0 \Lambda_0^T + \Lambda_0 P_0 - P_0 \Gamma_0^T \Gamma_0 P_0 + Q_0 = 0 \]
given some positive definite matrix \( Q_0 \in \mathbb{R}^{(q+2p\delta) \times (q+2p\delta)} \). Let \( L_0 = P_0 \Gamma_0^T P_0 \in \mathbb{R}^{(q+2p\delta) \times p} \).

Furthermore, let
\[
\Lambda_\rho = D(0, v_0) \in \mathbb{R}^{(1+2\delta) \times (1+2\delta)}, \quad \Gamma_\rho = (1, \xi_0) \in \mathbb{R}^{1 \times (1+2\delta)}.
\] (35a)
\[
\Gamma_\rho = (1, \xi_0) \in \mathbb{R}^{1 \times (1+2\delta)}.
\] (35b)

Then by Lemma 1, \((\Gamma_\rho, \Lambda_\rho)\) is observable. Thus, there exists a unique positive definite matrix \( P_\rho \in \mathbb{R}^{(1+2\delta) \times (1+2\delta)} \) satisfying
\[
P_\rho \Lambda_\rho^T + \Lambda_\rho P_\rho - P_\rho \Gamma_\rho^T \Gamma_\rho P_\rho + Q_\rho = 0 \]
given some positive definite matrix \( Q_\rho \in \mathbb{R}^{(1+2\delta) \times (1+2\delta)} \). Let \( L_\rho = P_\rho \Gamma_\rho^T \rho \in \mathbb{R}^{(1+2\delta)} \).

Similar to (29a)-(29c), we will first estimate the constant matrices \( \Phi_0, \Psi_0 \) and \( L_0 \) based on their disturbed values \( \Phi^0_0, \Psi^0_0, L^0_0 \) in a pointwise way. To this end, we define
\[
\varphi_\Phi = \text{vec}(\Phi_0), \quad \varphi^0_\Phi = \text{vec}(\Phi^0_0), \quad \varphi_\Psi = \text{vec}(\Psi_0), \quad \varphi^0_\Psi = \text{vec}(\Psi^0_0), \quad \varphi_L = \text{vec}(L_0), \quad \varphi^0_L = \text{vec}(L^0_0)
\] (36a)
\[
\varphi_\Phi = \text{vec}(\Phi_0), \quad \varphi^0_\Phi = \text{vec}(\Phi^0_0), \quad \varphi_\Psi = \text{vec}(\Psi_0), \quad \varphi^0_\Psi = \text{vec}(\Psi^0_0), \quad \varphi_L = \text{vec}(L_0), \quad \varphi^0_L = \text{vec}(L^0_0)
\] (36b)
\[
\varphi_\Phi = \text{vec}(\Phi_0), \quad \varphi^0_\Phi = \text{vec}(\Phi^0_0), \quad \varphi_\Psi = \text{vec}(\Psi_0), \quad \varphi^0_\Psi = \text{vec}(\Psi^0_0), \quad \varphi_L = \text{vec}(L_0), \quad \varphi^0_L = \text{vec}(L^0_0)
\] (36c)
where \( \varphi_\Phi, \varphi^0_\Phi, \varphi_\Psi, \varphi^0_\Psi, \varphi_L, \varphi^0_L \in \mathbb{R}^{pq}, \varphi_\Phi, \varphi^0_\Phi, \varphi_\Psi, \varphi^0_\Psi, \varphi_L, \varphi^0_L \in \mathbb{R}^{(q+2p\delta)} \). Now, it is necessary to present the first step of the filtering distributed observer. For \( i = 1, \ldots, N \),
\[
\dot{\rho}_{ik}^\psi = \Lambda_\rho \rho_{ik}^\psi + \mu_1^\psi L_\rho \sum_{j=0}^N a_{ij} (\varphi_{ik}^j - \varphi_{ik}^*) \quad k = 1, \ldots, q^2
\] (37a)
\[
\dot{\rho}_{ik}^\psi = \Lambda_\rho \rho_{ik}^\psi + \mu_1^\psi L_\rho \sum_{j=0}^N a_{ij} (\varphi_{ik}^j - \varphi_{ik}^*) \quad k = 1, \ldots, p q
\] (37b)
\[
\dot{\rho}_{ik}^{1\delta} = \Lambda_\rho \rho_{ik}^{1\delta} + \mu_1^{1\delta} L_\rho \sum_{j=0}^N a_{ij} (\varphi_{ik}^{1\delta} - \varphi_{ik}^*) \quad k = 1, \ldots, p (q + 2p\delta)
\] (37c)
where \( \mu_1^\psi, \mu_1^\psi, \mu_1^{1\delta} \) are positive observer gains and \( \rho_{ik}^\psi = \text{col}(\rho_{ik}^\psi, \rho_{ik}^{1\delta}), \rho_{ik}^{1\delta} \in \mathbb{R}, \rho_{ik}^{1\delta} \in \mathbb{R}^{25} \).
Let $\vec{\rho}_k^\Phi = \text{col}(\vec{\rho}^\Phi_1, \ldots, \vec{\rho}^\Phi_{N_k})$, $U^\Phi = D(\mu^\Phi_1, \ldots, \mu^\Phi_N)$, $H^\Phi = U^\Phi H$. Then we have

$$\vec{\rho}_k^\Phi = (I_N \otimes \Lambda_{\rho} - (H^\Phi \otimes P_{\rho} \Gamma_{\rho})\vec{\rho}_k^\Phi) \triangleq A_\phi \vec{\rho}_k^\Phi \cdot (45)$$

Let $T \in \mathbb{C}^{N \times N}$ be such that

$$T^{-1} H^\Phi T = D(\lambda_1(H^\Phi), \ldots, \lambda_N(H^\Phi)) \quad (46)$$

and thus

$$(T^{-1} \otimes I_{1+2\beta}) A_\phi (T \otimes I_{1+2\beta}) = D(\Lambda_{\rho} - \lambda_1(H^\Phi)P_{\rho} \Gamma_{\rho} T \Gamma_{\rho}, \ldots, \Lambda_{\rho} - \lambda_N(H^\Phi)P_{\rho} \Gamma_{\rho} T \Gamma_{\rho}) . \quad (47)$$

Since $\mu^\Phi \sigma(H) \geq 1$, by Remark 5, $\mathfrak{N}(\lambda_i(H^\Phi)) \geq 1$ for all $i = 1, \ldots, N$. Therefore, by Lemma 1 of [39], $A_\phi$ is Hurwitz and thus $\lim_{t \to \infty} \vec{\rho}_k^\Phi(t) = 0$ exponentially. Note that $\rho_k^\Phi(\Phi_i)$ and $\vec{\rho}_k^\Phi(\Phi_{ij})$ as a result,

$$\lim_{t \to \infty} (\rho_k^\Phi(t) - \varphi_k(\Phi_i)) = 0 \quad (48)$$

exponentially, which by (36a) and (38) gives

$$\lim_{t \to \infty} (\Phi_i(t) - \Phi_0) = 0 \quad (49)$$

exponentially and hence the proof is complete. \hfill \Box

Based on (37), for $i = 1, \ldots, N$, further define

$$A_i = D(\Phi_i, I_{\rho} \otimes \xi_0) \in \mathbb{R}^{(q+2b\rho) \times (q+2b\rho)} \quad (50a)$$

$$\Gamma_i = (\Psi_i, I_{\rho} \otimes \xi_0) \in \mathbb{R}^{p \times (q+2b\rho)}. \quad (50b)$$

Then, the second part of the filtering distributed observer is designed as, for $i = 1, \ldots, N$,

$$\dot{\psi}_i = A_i \psi_i + \mu_i^\psi L_i \sum_{j=0}^N a_{ij}(\zeta_j - \zeta_i) \quad (51)$$

where $\mu_i^\psi$ is a positive observer gain, $\psi_i = \text{col}(\eta_i, d_i)$, $\eta_i \in \mathbb{R}^q$, $d_i \in \mathbb{R}^{2b\rho}$, and $\zeta_i = \Gamma_i \psi_i$. For $i = 1, \ldots, N$, let $\tilde{\eta}_i = \eta_i - \eta_0$. Moreover, let $\mu_i^\psi = \min(\mu_i^\psi, \ldots, \mu_N^\psi)$. Then we have the following result.

Lemma 6: Given system (19) and the communication network $\tilde{G}$, under Assumptions 2, 4, 3 and 6, if $\mu^\Phi, \mu^\psi, \mu^L, \mu^F \geq \sigma(H)^{-1}$, the solution of system (51) exists and is bounded for all $t \geq 0$, and satisfies, for $i = 1, \ldots, N$,

$$\lim_{t \to \infty} \tilde{\eta}_i(t) = 0 \quad (52)$$

exponentially.

Proof: For $i = 1, \ldots, N$, let $\tilde{\Lambda}_i = \Lambda_i - \Lambda_0$, $\tilde{\Gamma}_i = \Gamma_i - \Gamma_0$. Then, by Lemma 5, $\lim_{t \to \infty} \tilde{\Lambda}_i(t) = 0$ and $\lim_{t \to \infty} \tilde{\Gamma}_i(t) = 0$ exponentially.

For $i = 1, \ldots, N$, let $\tilde{\psi}_i = \psi_i - \psi_0$. Then we have

$$\dot{\tilde{\psi}}_i = \Lambda_i \tilde{\psi}_i + \mu_i^\psi L_i \sum_{j=0}^N a_{ij}((\tilde{\Gamma}_j + \Gamma_0)\psi_j - (\tilde{\Gamma}_i + \Gamma_0)\psi_i) + \Lambda_i \psi_i - \Lambda_0 \psi_i + \Lambda_0 \psi_i - \Lambda_0 \psi_0$$

$$= \mu_i^\psi L_i \sum_{j=0}^N a_{ij}((\tilde{\Gamma}_j + \Gamma_0)\psi_j - (\tilde{\Gamma}_i + \Gamma_0)\psi_i) + \Lambda_i \psi_i - \Lambda_0 \psi_i + \Lambda_0 \psi_i - \Lambda_0 \psi_0$$

Similarly to (46) and (47), since $\mu_i^\psi \geq \sigma(H)^{-1}$, it can be concluded that $(I_N \otimes \Lambda_0 - (H^\psi \otimes P_0 \Gamma_0)\tilde{\Gamma}_0)$ is Hurwitz. Noting that $L$ is bounded, by Lemma 5, both $(\tilde{\Lambda} - L(H^\psi \otimes I_\rho)\tilde{\Gamma})$ and $(\tilde{\Lambda} - L(H^\psi \otimes I_\rho)\tilde{\Gamma})$ decay to zero exponentially. Under Assumption 4, $\psi_0$ is bounded. Then, by Lemma 1 of [21], $\lim_{t \to \infty} \tilde{\psi}_i(t) = 0$ exponentially. As a result, $\lim_{t \to \infty} \tilde{\eta}_i(t) = 0$ exponentially and thus the proof is complete. \hfill \Box

Together, (37) and (51) constitute the filtering distributed observer which recovers $\Phi_0, \psi_0$ and $\eta_0(t)$ of the leader for each follower by $\Phi_i(t)$, $\psi_i(t)$ and $\eta_i(t)$, respectively.

Remark 9: In Lemmas 5 and 6, the selection of the observer gains $\mu_i^\Phi, \mu_i^\psi, \mu_i^L, \mu_i^F$ depends on the communication graph. While, for a communication graph with fixed number of nodes, there exists a maximal $\sigma(H)^{-1}$ for all possible network topologies. Consequently, the selection of the observer gains only depends on the number of the followers. It might also be interesting to develop adaptive gain techniques as in [5] to update the observer gain online. Moreover, in the filtering distributed observer design, all the
entries of $\Phi_0$, $\Psi_0$ and $L_0$ are estimated for completeness.
While, in practice, it suffices to estimate the unknown entries of these matrices to lower down the dimension of the distributed observer.

**B. LOCAL TRACKING CONTROL LAW AND STABILITY ANALYSIS**

In this part, we will show how to design a local tracking controller based on the estimated leader’s information $\Phi(t)$, $\Psi(t)$ and $\eta(t)$ obtained by the distributed observer (37) and (51).

Let
\[
\phi_i = \text{vec} \left( \begin{bmatrix} 0_{n_i \times q} \\ -\Psi_0 \end{bmatrix} \right) \quad (55a)
\]
\[
Z_i = \Phi_0^T \otimes \begin{bmatrix} I_{n_i} \\ 0 \\ 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i \\ B_i \\ C_i \\ 0 \end{bmatrix}. \quad (55b)
\]

Then, under Assumption 5, by Theorem 1.9 of [35], the solution of the regulator equation (23) can be expressed as
\[
Z_i \xi_i = \phi_i \quad (56a)
\]
\[
\left( \begin{array}{c} X_i \\ U_i \end{array} \right) = \mathcal{M}^q_{(n_i+m_i)}(\xi_i) \quad (56b)
\]
where $X_i \in \mathbb{R}^{n_i \times q}$, $U_i \in \mathbb{R}^{m_i \times q}$. Let
\[
\hat{\phi}_i = \text{vec} \left( \begin{bmatrix} 0_{n_i \times q} \\ -\Psi_i \end{bmatrix} \right) \quad (57a)
\]
\[
\hat{Z}_i = \Phi_i^T \otimes \begin{bmatrix} I_{n_i} \\ 0 \\ 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i \\ B_i \\ C_i \\ 0 \end{bmatrix}. \quad (57b)
\]

Then, by Lemma 5, $\hat{Z}_i = \hat{Z}_i - Z_i$ and $\hat{\phi}_i = \hat{\phi}_i - \phi_i$ would decay to zero exponentially. For $i = 1, \ldots, N$, design
\[
\dot{\xi}_i = -\mu_i \xi_i T \left( \hat{Z}_i \xi_i - \phi_i \right) \quad (58)
\]
where $\xi_i \in \mathbb{R}^{q(n_i+m_i)}$, $\mu_i > 0$. Let
\[
\left( \begin{array}{c} \hat{X}_i(t) \\ \hat{U}_i(t) \end{array} \right) = \mathcal{M}^q_{(n_i+m_i+m_i)}(\hat{\xi}_i(t)) \quad (59)
\]
where $\hat{\xi}_i(0) \in \mathbb{R}^{q(n_i+m_i)}$, and $\mu_i > 0$, the solution of (58) has a unique bounded solution over $t \geq t_0$ and satisfies
\[
\lim_{t \to \infty} (\hat{X}_i(t) - X_i) = 0, \quad \lim_{t \to \infty} (\hat{U}_i(t) - U_i) = 0
\]
exponentially.

Let
\[
\Pi_{1i} = A_i \hat{X}_i + B_i \hat{U}_i - \hat{X}_i \Phi_0 \quad (60a)
\]
\[
\Pi_{2i} = C_i \hat{X}_i - \Psi_i \quad (60b)
\]

Then by Lemma 7, it follows that $\lim_{t \to \infty} \Pi_{1i}(t) = 0$ and $\lim_{t \to \infty} \Pi_{2i}(t) = 0$ exponentially. Moreover,
\[
\dot{\hat{Z}}_i \xi_i - \phi_i = Z_i \xi_i - \phi_i + Z_i \xi_i - \phi_i \quad (61)
\]
which indicates that $\dot{\xi}_i$, and hence $\dot{\hat{X}}_i$ and $\dot{\hat{U}}_i$ will all decay to zero exponentially.

Under Assumption 1, by condition 5 of [38], there exists a unique positive definite matrix $P_i \in \mathbb{R}^{m_i \times m_i}$ satisfying
\[
P_i A_i + A_i^T P_i - P_i B_i B_i^T P_i + Q_i = 0
\]
given some positive definite matrix $Q_i \in \mathbb{R}^{m_i \times m_i}$. Let
\[
K_{xi} = -B_i^T P_i \quad (62a)
\]
\[
\hat{K}_{xi} = \hat{U}_i - K_{xi} \hat{X}_i \quad (62b)
\]
For $i = 1, \ldots, N$, design the following control law
\[
\dot{\theta}_i = \mu_i \eta_i^T (x_i, \eta_i, \hat{z}_i, t) B_i^T P_i \hat{X}_i \quad (63a)
\]
\[
u_i = -h_i(x_i, \eta_i, \hat{z}_i, t) \theta_i \quad (63b)
\]
where $\dot{\theta}_i \in \mathbb{R}^{r_i+1}$, $\hat{x}_i = x_i - \hat{X}_i \eta_i$, $\mu_i > 0$ and
\[
h_i(x_i, \eta_i, \hat{z}_i, t) = \left[ -(K_{xi} x_i + \hat{K}_{xi} \eta_i) f_i(x_i, t) \right] \in \mathbb{R}^{m_i \times (r_i+1)}.
\]
We have the following result.

**Theorem 1:** Given systems (19), (20) and the communication graph $\hat{G}$, under Assumptions 1-6, if $\mu_i^u, \mu_i^u, \mu_i^u, \mu_i^u \geq \sigma(H)^{-1}$, then for any $\mu_i^u, \mu_i^u > 0$, Problem 1 is solvable by the control law composed of (37), (51), (58) and (63).

**Proof:** For $i = 1, \ldots, N$, define
\[
\eta_{di} = (I_q - b_{ps}^\theta \times b_{ps}^\theta) \left[ \mu_i^u \psi_i \sum_{j=0}^{N} a_{ij}(\xi_j - \xi_i) \right].
\]
Then by (50a) and (51), it follows that
\[
\dot{\eta}_i = \Phi_i \eta_i + \eta_{di}
\]
and by Lemma (6), $\eta_{di}$ will decay to zero exponentially. Let $\delta_i = \text{col}(b_{ps}^\theta, b_{ps}^\theta) \in \mathbb{R}^{r_i+1}$, $\bar{\delta}_i = \delta_i - \dot{\delta}_i$, $\bar{A}_{ci} = A_i + B_i K_{xi} \in \mathbb{R}^{r_i+1}$, $\bar{A}_{ci} = A_i + B_i K_{xi}$. Then, we have
\[
\dot{\theta}_i = \mu_i^u \eta_i^T (x_i, \eta_i, \hat{z}_i, t) B_i^T P_i \hat{X}_i \quad (64)
\]
ad
\[
\hat{x}_i = A_i x_i + B_i (b_{ps}^u + f_i(x_i, t) \theta_i)
\]
\[
- \hat{X}_i \eta_i - \hat{X}_i (\Phi_i \eta_i + \eta_{di})
\]
\[
= A_i x_i + B_i (K_{xi} x_i + \hat{K}_{xi} \eta_i)
\]
\[
- \hat{X}_i \eta_i - \hat{X}_i (\Phi_i \eta_i + \eta_{di})
\]
\[
+ B_i (-K_{xi} x_i - \hat{K}_{xi} \eta_i) + b_{ps}^\theta + f_i(x_i, t) \theta_i
\]
\[
= A_i x_i + B_i \hat{U}_i - K_{xi} \hat{X}_i \eta_i - \hat{X}_i \eta_i + \hat{X}_i (\Phi_i \eta_i + \eta_{di})
\]
\[
- \hat{X}_i \eta_i - \hat{X}_i (\Phi_i \eta_i + \eta_{di}) - b_i B_i h_i(x_i, \eta_i, \hat{z}_i, t) \delta_i
\]
\[
= A_i x_i + \Pi_{1i} \eta_i - A_i \hat{X}_i \eta_i - \hat{X}_i \Phi_i \eta_i
\]
\[
- \hat{X}_i \eta_{di} - b_i B_i h_i(x_i, \eta_i, \hat{z}_i, t) \delta_i
\]
\[
= A_i x_i + \Pi_{1i} \eta_i - A_i \hat{X}_i \eta_i - \hat{X}_i \Phi_i \eta_i
\]
\[
- \hat{X}_i \eta_{di} - b_i B_i h_i(x_i, \eta_i, \hat{z}_i, t) \delta_i
\]
\[
= A_i x_i - b_i B_i h_i(x_i, \eta_i, \hat{z}_i, t) \delta_i + \Delta_i
\]
where \( \Delta_i = \Pi_i \bar{\eta}_i - \dot{\bar{X}}_i \bar{\eta}_i - \dot{\bar{X}}_i \bar{\eta}_{d_i} \). Since \( \eta_i, \dot{\bar{X}}_i \) are bounded and \( \Pi_i, \dot{\bar{X}}_i, \bar{\Phi}_i, \bar{\eta}_d \) all tend to zero exponentially, we have \( \Delta_i(t) \to 0 \) exponentially as \( t \to \infty \).

Regarding systems (64) and (65), for \( i = 1, \ldots, N \), let

\[
V_i(\bar{\theta}_i, \bar{x}_i) = \bar{x}_i^T P_i \bar{x}_i + (\mu_i^0)^{-1} b_i \bar{\theta}_i^T \bar{\theta}_i. \tag{66}
\]

Then along the trajectories of (64) and (65), we have

\[
V_i \dot{} = -\bar{x}_i^T (Q_i + P_i B_i \bar{\theta}_i^T P_i) \bar{x}_i + 2 \bar{x}_i^T P_i \Delta_i
- 2 b_i \bar{\theta}_i^T P_i B_i h_i(x_i, \bar{\eta}_i, \bar{\zeta}_i, t) \dot{\bar{\theta}}_i
+ 2 b_i \bar{\theta}_i^T h_i^T(x_i, \bar{\eta}_i, \bar{\zeta}_i, t) B_i^T P_i \dot{\bar{x}}_i
= -\bar{x}_i^T (Q_i + P_i B_i \bar{\theta}_i^T P_i) \bar{x}_i + 2 \bar{x}_i^T P_i \Delta_i. \tag{67}
\]

Since \( \bar{x}_i = \bar{\bar{x}}_i + \dot{\bar{X}}_i \bar{\eta}_i \) and \( \ddot{\bar{x}}_i, \dot{\bar{\eta}}_i \) are bounded, if \( \dot{\bar{x}}_i \) is bounded, then \( \bar{x}_i \) is bounded. As a result, if \( \bar{x}_i \) and \( \dot{\bar{\theta}}_i \) are bounded, the right hand sides of both (64) and (65) with \( \Delta_i = 0 \) are bounded. Moreover, \( V_i \) is positive definite and radially unbounded. Since \( \Delta_i(t) \to 0 \) exponentially, \( \Delta_i(t) \) is bounded and \( \int_0^\infty |\Delta_i(t)| dt < \infty \). Therefore, by Lemma 2, \( \bar{x}_i(t) \to 0 \) as \( t \to \infty \). Finally, noticing that

\[
e_i = C_i \bar{\theta}_i - \Psi_0 \bar{\theta}_0
= C_i \bar{x}_i + C_i \dot{\bar{X}}_i \bar{\eta}_i - \Psi_0 \bar{\eta}_i + \Psi_0 \bar{\eta}_i
= C_i \bar{x}_i + \Pi_i \bar{\eta}_i + \Psi_0 \bar{\eta}_i \tag{68}
\]

gives \( \lim_{t \to \infty} e_i(t) = 0 \) and the proof is complete. \( \square \)

Remark 10: There are two main differences between this work and the work of [21]. First, [21] considered linear follower systems, while this paper considers a class of nonlinear follower systems subject to uncertain system parameters, which makes the local control law design much more complicated. In order to conduct the stability analysis of the closed-loop system, a new analysis tool, i.e., Lemma 2, has been established. Second, in contrast to [21] which assumed an ideal leader system, in this paper, the leader’s information obtained by the followers is subject to disturbances in a way that the disturbances cannot be pre-rejected by the leader itself and thus will propagate over the communication network. To tackle with this issue, a new filtering distributed observer has been proposed which is capable of recovering the leader’s information locally by each follower.

V. EXAMPLE

In this section, we present a numerical example to illustrate the proposed control approach. Consider a leader-follower multiagent system consisting of one leader and six followers, whose communication network is depicted by Fig. 2. Clearly, Assumption 3 is satisfied.

The dynamics of the leader are given by (19) with

\[
\Phi_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \Psi_0 = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}. \tag{69}
\]

The dynamics of the \( i \)th follower are given by (20) with

\[
A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & a_i \\ 0 & -a_i & 0 \end{pmatrix}, \quad B_i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad f_i(x_i, t) = \begin{pmatrix} x_3^3 \\ x_3^3 \\ x_2 \end{pmatrix} \tag{70}
\]

where \( x_i = \text{col}(x_i(1), x_i(2), x_i(3)) \) and \( a_1 = 0.2, a_2 = 0.4, a_3 = 0.6, a_4 = 0.3, a_5 = 0.5, a_6 = 0.7 \). Moreover, the true values of the unknown parameters are given by

\[
b_1 = 0.1, \quad b_2 = 0.3, \quad b_3 = 1.5, \quad b_4 = 3, \quad b_5 = 2.3, \quad b_6 = 1.7
\]

\[
\theta_1 = 10, \quad \theta_2 = 20, \quad \theta_3 = 30, \quad \theta_4 = 40, \quad \theta_5 = 50, \quad \theta_6 = 60.
\]
Regarding the disturbance (31), we assume $\delta = 1$, $\alpha_i = 10$, $\alpha_{ij} \in [-1, 1]$, $\beta_i \in [-\pi, \pi]$.

The observer and control gains are selected as follows. Let $\mu_i^T = \mu_i^L = \mu_i^N = 1000$, $Q_0 = 100I_5$, $Q_p = 100I_3$, $\mu_i = 100$, $\mu_i^S = 0.01$, $Q_i = 100I_3$.

The outputs and the estimated unknown parameters are shown by Figs. 3 and 4, respectively. It can be seen that the outputs of the followers have successfully tracked the output of the leader and the estimated unknown parameters converge to constant values.

VI. CONCLUSION AND FUTURE WORK

In this paper, the distributed tracking problem for a leader-follower multiagent system is considered, where the follower dynamics are heterogeneous, multi-input, multi-output, and uncertain, and the leader’s information is subject to disturbances. A novel distributed control approach has been proposed combining a new kind of filtering distributed observer and the local tracking controller. In the future, it might also be interesting to consider the scenario where the disturbances not only occur to the information transmission from the leader to the followers, but also to the information transmission among the followers.

There are also some interesting issues which are considered as our future directions. First, the communication network considered in this paper is assumed to be static and delay-free. In the future, one may further consider the more practical scenario where the communication network is subject to switched topology and uncertain time-delay as in [40], [41]. Second, in this paper, the sign of $b_i$ is a prerequisite for the control law design. By utilizing the Nussbaum-type gain method [42], [43], it might be possible to deal with the case where the sign of $b_i$ is also unknown. Third, the non-linear uncertainties considered in this paper should satisfy the matching condition. It is also desirable to consider the distributed tracking problem of leader-follower multiagent systems with the followers being subject to mismatched non-linear uncertainties as in [44], [45].

REFERENCES

[1] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches. London, U.K.: Springer-Verlag, 2014.

[2] Z. Qu, Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles. London, U.K.: Springer-Verlag, 2009.

[3] W. Ren and R. W. Beard, Distributed Consensus in Multi-Vehicle Cooperative Control, Communications and Control Engineering Series. London, U.K.: Springer-Verlag, 2008.

[4] W. Ren and Y. Cao, Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues. London, U.K.: Springer-Verlag, 2011.

[5] Z. Li, Z. Duan, and F. L. Lewis, “Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties,” Automatica, vol. 50, no. 3, pp. 883–889, Mar. 2014.

[6] C. Sun, G. Hu, and L. Xie, “Robust consensus tracking for a class of high-order multi-agent systems,” Int. J. Robust Nonlinear Control, vol. 26, no. 3, pp. 578–598, Feb. 2016.

[7] Y. Zhao, G. Wen, Z. Duan, and G. Chen, “Adaptive consensus for multiple nonidentical matching nonlinear systems: An edge-based framework,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 62, no. 1, pp. 85–89, Jan. 2015.

[8] J. A. Colunga, H. M. Becerra, C. R. Vazquez, and D. Gomez-Gutierrez, “Robust leader-following consensus of high-order multi-agent systems in prescribed time,” IEEE Access, vol. 8, pp. 195170–195183, 2020, doi: 10.1109/ACCESS.2020.3033789.

[9] Z. Peng, H. Wang, D. Wang, G. Sun, and H. Zhang, “Distributed model reference adaptive control for cooperative tracking of uncertain dynamical multi-agent systems,” IET Control Theory Appl., vol. 7, no. 8, pp. 1079–1087, May 2013.

[10] J. Sun, Z. Geng, and Y. Lv, “Adaptive output feedback consensus tracking for heterogeneous multi-agent systems with unknown dynamics under directed graphs,” Syst. Control Lett., vol. 87, pp. 16–22, Jan. 2016.

[11] H. Zhang and F. L. Lewis, “Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics,” Automatica, vol. 48, no. 7, pp. 1432–1439, Jul. 2012.

[12] Y. Yang, H. Modares, D. C. Wunsch, and Y. Yin, “Leader—Follower output synchronization of linear heterogeneous systems with active leader using reinforcement learning,” IEEE Trans. Neural Netw. Learn. Syst., vol. 29, no. 6, pp. 2139–2153, Jun. 2018.

[13] W. Wang and X. Chen, “Model-free optimal containment control of multi-agent systems based on actor-critic framework,” Neurocomputing, vol. 314, pp. 242–250, Nov. 2018.

[14] X. Wang and H. Su, “Completely model-free RL-based consensus of continuous-time multi-agent systems,” Appl. Math. Comput., vol. 382, Oct. 2020, Art. no. 125312.

[15] H. Yu and X. Xia, “Adaptive consensus of multi-agents in networks with jointly connected topologies,” Automatica, vol. 48, no. 8, pp. 1783–1790, Aug. 2012.

[16] J. Hu and W. X. Zheng, “Adaptive tracking control of leader—follower systems with unknown dynamics and partial measurements,” Automatica, vol. 50, no. 5, pp. 1416–1423, May 2014.

[17] J. Sun and Z. Geng, “Adaptive consensus tracking for linear multi-agent systems with heterogeneous unknown nonlinear dynamics,” Int. J. Robust Nonlinear Control, vol. 26, no. 1, pp. 154–173, Jan. 2016.

[18] J. Huang, “Certainty equivalence, separation principle, and cooperative output regulation of multiagent systems by the distributed observer approach,” in Control of Complex Systems: Theory and Applications. Butterworth-Heinemann, 2016, pp. 421–449.

[19] Y. Su and J. Huang, “Cooperative output regulation of linear multi-agent systems,” IEEE Trans. Autom. Control, vol. 57, no. 4, pp. 1062–1066, Apr. 2012.

[20] Y. Su and J. Huang, “Cooperative output regulation with application to multi-agent consensus under switching network,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 42, no. 3, pp. 864–875, Jun. 2012.

[21] H. Cai, F. L. Lewis, G. Hu, and J. Huang, “The adaptive distributed observer approach to the cooperative output regulation of linear multi-agent systems,” Automatica, vol. 75, pp. 299–305, Jan. 2017.

[22] T. Liu and J. Huang, “Leader-following consensus with disturbance rejection for uncertain Euler–Lagrange systems over switching networks,” Int. J. Robust Nonlinear Control, vol. 29, no. 18, pp. 6638–6656, Dec. 2019.

[23] S. Wang and J. Huang, “Adaptive leader-following consensus for multiple Euler–Lagrange systems with an uncertain leader system,” IEEE Trans. Neural Netw. Learn. Syst., vol. 30, no. 7, pp. 2188–2196, Jul. 2019.

[24] H. Cai, J. Huang, “The leader-following attitude control of multiple rigid spacecraft systems,” Automatica, vol. 50, no. 4, pp. 1109–1115, Apr. 2014.

[25] T. Liu and J. Huang, “Leader-following attitude consensus of multiple rigid body systems subject to jointly connected switching networks,” Automatica, vol. 92, pp. 63–71, Jun. 2018.

[26] Y. Yang, H. Modares, D. C. Wunsch, and Y. Yin, “Optimal containment control of unknown heterogeneous systems with active leaders,” IEEE Trans. Control Syst. Technol., vol. 27, no. 3, pp. 1228–1236, May 2019.

[27] H. Liang, Y. Zhou, H. Ma, and Q. Zhou, “Adaptive distributed observer approach for cooperative containment control of nonidentical networks,” IEEE Trans. Syst., Man, Cybern. Syst., vol. 49, no. 2, pp. 299–307, Feb. 2019.

[28] W. Jiang, G. Wen, Z. Peng, T. Huang, and A. Rahmani, “Fully distributed formation-containment control of heterogeneous linear multiagent systems,” IEEE Trans. Autom. Control, vol. 64, no. 9, pp. 3889–3896, Sep. 2019.

[29] L. Liu, “Adaptive cooperative output regulation for a class of nonlinear multi-agent systems,” IEEE Trans. Autom. Control, vol. 60, no. 6, pp. 1677–1682, Jun. 2015.
S. Zuo, Y. Song, F. L. Lewis, and A. Davoudi, "Output containment, active disturbance rejection control, and networked control of linear heterogeneous multi-agent systems using internal model principle," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 2099–2109, Aug. 2017.

T. Kailath, *Linear Systems*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1980.

H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

J. Huang, *Nonlinear Output Regulation: Theory and Applications*. Philadelphia, PA, USA: SIAM, 2004.

H. Cai, G. Hu, F. L. Lewis, and A. Davoudi, "A distributed feedforward approach to cooperative control of AC microgrids," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 4057–4067, Sep. 2016.

W. Liu, B. Ma, J. Lu, and F. Xie, "Cooperative output regulation problem for linear time-delay multi-agent systems under switching network," *Neurocomputing*, vol. 190, pp. 132–139, May 2016.

M. Lu and J. Huang, "Leader-following consensus of multiple uncertain Euler–Lagrange systems subject to communication delays and switching networks," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2604–2611, Aug. 2018.

M. Guo, D. Xu, and L. Liu, "Cooperative output regulation of heterogeneous nonlinear multi-agent systems with unknown control directions," *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 3039–3045, Jun. 2017.

W. Liu, B. Ma, J. Lu, and F. Xie, "Fault-tolerant adaptive control for a class of nonlinear systems with uncertain parameters and unknown control directions," *IEEE Access*, vol. 7, pp. 8582–8590, 2019.

X. Wang, S. Li, X. Yu, and J. Yang, "Distributed active anti-disturbance consensus for leader-follower higher-order multi-agent systems with mismatched disturbances," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5795–5801, Nov. 2017.

X. Wang, S. Li, and M. Z. Q. Chen, "Composite backstepping consensus algorithms of leader-follower higher-order nonlinear multiagent systems subject to mismatched disturbances," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1935–1946, Jun. 2018.

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