Giant dynamic light-matter entanglement from driving neither too fast nor too slow

O. L. Acevedo, L. Quiroga, F. J. Rodríguez, and N. F. Johnson

1 Departamento de Física, Universidad de los Andes, A.A. 4976, Bogotá D. C., Colombia
2 Department of Physics, University of Miami, Coral Gables, Miami, FL 33124, USA

A significant problem facing next-generation quantum technologies is how to generate and manipulate macroscopic entanglement in light and matter systems. Here we report a new regime of dynamical light-matter behavior in which a giant, system-wide entanglement is generated by varying the light-matter coupling at intermediate velocities. This enhancement is far larger and broader-ranged than that occurring near the quantum phase transition of the same model under adiabatic conditions. By appropriate choices of the coupling within this intermediate regime, the enhanced entanglement can be made to spread system-wide or to reside in each subsystem separately.

Many-body quantum dynamics lie at the core of many natural phenomena and proposed quantum technologies, including information processing through schemes such as adiabatic quantum computing [1]. Achieving the controllable generation and manipulation of entanglement over many qubits is a key challenge, while doing so in light-matter systems is highly desirable for optoelectronic implementations. The ground state, and hence entanglement, of a quantum system can be varied in a controlled way through adiabatic perturbations, though this is in principle an infinitely slow process. Quantum Phase Transitions (QPTs) can provide a naturally occurring entangled state and it is known that the entanglement can be enhanced at the critical point [2]. Recent studies have focused on time-dependent perturbations around QPTs that are either very slow (adiabatic) [3] or very fast (sudden quench) [4]; or small dynamic oscillations around a phase space region [5]; or static coupling after a sudden quench [6–9].

Here we consider, by contrast, the regime of intermediate perturbation velocities that has so far been overlooked. We consider the experimentally realized light-matter system of the Dicke Model (DM) [10], which has been realized in a variety of systems (e.g. circuit QED [11] and cold atom settings [12]). We uncover a level of quantum complexity that is far richer than either the adiabatic or fast-quench regimes. The system-wide entanglement is dramatically enhanced over the static or adiabatic QPT values. Our results extend current understanding of coupled light-matter systems beyond the equilibrium ground state [2, 13–16], and also beyond more recent studies of out-of-equilibrium critical behavior [3, 5, 9]. Moreover, our fully quantum analysis covers all dynamical regimes from very slow adiabatic through to sudden quench, capturing at each stage the emergent non-linear self interactions and correlations within and between each subsystem.

Our calculations employ the DM Hamiltonian [10]:

\[
\hat{H} = \epsilon J_z + \omega \hat{a}^\dagger \hat{a} + 2 \frac{\lambda(t)}{\sqrt{N}} J_x (\hat{a}^\dagger + \hat{a}),
\]

where \(N\) is the number of matter qubits, the operators \(\hat{J}_z = \frac{1}{2} \sum_{j=1}^{N_q} \hat{a}_j^{(q)} \) denote collective operators of the qubits, and operator \(\hat{a}^\dagger (\hat{a})\) is the creation (annihilation) operator of the radiation field. In the thermodynamic limit (\(N \to \infty\)), the critical value of the light-matter coupling parameter \(\lambda = \sqrt{\omega \epsilon}/2\) while its finite-\(N\) equivalent is slightly different [13, 17]. We treat the Hamiltonian exactly using a large basis set [18] and avoid com-

FIG. 1. (color online) Dynamic evolution of the Von-Neumann entropy \(S_N\) (time varies from left to right) for the Dicke Model (DM) with \(N = 81\) qubits. For simplicity we set \(\epsilon = \omega = 1\) in Eq. 1 for this figure, as well as for Figs. 3 and 4. The velocity range spans all regimes: from adiabatic (bottom) to sudden quench (top). Roman numerals mark where the field (Wigner) and matter (Agarwal-Wigner) distributions are depicted in Fig. 4. The dotted line is a guide to the eye, denoting a region of novel dynamical light-matter behavior with greatly enhanced entanglement (purple) as compared to the QPT. It arises for intermediate velocities and lies well inside the coupling regime for the conventional ordered phase (\(\lambda > \lambda_c = 0.5\)). The QPT corresponds to \(\lambda \to 0.5\) as \(v \to 0\) and hence as \(\log v \to -\infty\) (i.e. it tends toward \(\lambda = 0.5\) on the horizontal axis of the diagram). Inset: a schematic representation of the DM which mimics various experimental realizations.
mon simplifications such as rotating-wave or semiclassical approximations. The total system evolves unitarily under Hamiltonian $\hat{H}$, starting at $t = 0$ from the instantaneous ground state at $\lambda = 0$. The light-matter coupling parameter is characterized by an annealing velocity (AV) $\nu$ under a linear ramping $\lambda(t) = \nu t$. More complicated time-dependencies can be treated but further complicate the understanding. The interval of interest in this paper is $\lambda \in [0, 2]$, meaning that the driving passes across a broad range of coupling strengths below and above the QPT. While the evolution of the total system $S$ is described by a pure state $|\Psi(t)\rangle$, any subsystem $A$ is described by a density matrix $\rho_A$ defined as the trace with respect to the other degrees of freedom not present in $A$:

$$\rho_A(t) = \text{tr}_{S-A} (|\Psi(t)\rangle \langle \Psi(t)|).$$

Due to Schmidt decomposition, and the fact that the total composite system is always in a pure state, both radiation and matter have the same value of the Von-Neumann entropy $S_N$. Because of the global unitary condition on the total system’s pure state, this entropy $S_N$ provides a quantitative measure of the degree of entanglement between the matter and light subsystems [19].

Figure 1 summarizes our main finding: A novel, dynamical light-matter regime with greatly enhanced system-wide properties including entanglement, when the light-matter coupling is driven at intermediate velocities. The peak entanglement value (purple) is far larger than the known equilibrium critical maximum [2], i.e. much larger than what can be achieved under adiabatic conditions. Also, this enhanced entanglement extends over a far broader range, well across the $\lambda > \lambda_c$ region. As the annealing velocity increases, the critical onset point of light-matter entanglement is pushed toward larger $\lambda$ values and is no longer represented by a sharp peak, but instead a wavy plateau. At much higher velocities beyond the giant entanglement regime, $\lambda$ varies so fast that a sudden quench condition is achieved. Now the system is not quick enough to respond to the light-matter coupling, at least not in the $\lambda \in [0, 2]$ interval of Fig. 1.

The origin of this giant enhancement of the entanglement at intermediate driving velocities, lies in the effective nonlinear interactions within each sub-system generated by the driving and mediated by the other components [13-16]. Matter-light interaction is in principle linear and generates little entanglement in the adiabatic limit, since the system has enough time to continuously stabilize in response to the perturbation. The intermediate regime is the only one in which these non-linearities can develop significantly, thereby generating enhanced internal entanglement in both the matter and light subsystems that is subsequently transformed into a system-wide giant light-matter entanglement.

We now develop a deeper theoretical understanding of the results in Fig. 1, by analyzing the underlying quantum state in the three main dynamical regimes, as illustrated in Fig. 2. We start by rewriting the Dicke Hamiltonian exactly as

$$\hat{H} = \omega \hat{b} \hat{b}^\dagger - \frac{4\lambda^2}{\omega N} \hat{J}_x^2 + \epsilon \hat{J}_z,$$

where $\hat{b} = \hat{a} + \frac{2\lambda}{\omega \sqrt{N}} \hat{J}_x$. In the $\lambda > \lambda_c$ range, the last term becomes less and less relevant and the Dicke Hamiltonian
can be seen as a radiation mode that feels a displaced harmonic potential whose values depend on the eigenstate \( |m_x \rangle \) of \( \hat{J}_x \) in which the matter system sits. Specifically, if \( \lambda \gg \lambda_c \), then

\[
\hat{H} \approx \sum_{m_x} \left( \frac{1}{2} \omega \left[ \hat{p}^2 + \left( \hat{x} - \frac{2\lambda}{\omega \sqrt{N}} m_x \right)^2 \right] - \frac{4\lambda^2}{N} m_x^2 \right) |m_x \rangle \langle m_x| ,
\]

where we have used the quadrature operators of the radiation mode. Importantly, the energy potential felt by the radiation mode is symmetrical with respect to a change in sign in \( m_x \), which is a source of degeneracy. In addition, as \( |m_x| \) gets bigger, the minimum value of the harmonic potential becomes lower. The ground state of this approximate Hamiltonian is any superposition of the form

\[
|\psi_0\rangle = \cos \theta |N/2, m_x \rangle |\beta\rangle + e^{i\varphi} \sin \theta |-N/2, m_x \rangle |\beta\rangle ,
\]

i.e. the two ground states corresponding to the two minimum parabolae. Hence the symmetry of the ground state is spontaneously broken. The field state \( |\beta\rangle \) is a coherent state with \( \beta = \frac{\lambda}{\omega \sqrt{N}} m_x \). As parity is preserved during the ramping, the projection of \( |\psi_0\rangle \) onto the even parity sub-space is the state achieved by an adiabatic evolution. Hence

\[
|\psi_0\rangle = \frac{1}{\sqrt{2}} \left( |N/2, m_x \rangle |\beta\rangle - |N/2, m_x \rangle |\beta\rangle \right).
\]

The entropy for each subsystem is \( S_N = \log 2 \). In the intermediate regime, the dynamical state will be a complicated superposition of states for each potential, heated up with respect to the ground state.

This means that for \( \lambda \) values well inside the conventional ordered phase, the interaction term in Eq. 1 prevails and the radiation mode feels a different confining potential depending on the eigenvalue of \( \hat{J}_x \), as depicted by the different parabolae in Fig. 2. The adiabatic evolution is characterized by the absence of heating, i.e., the energy is always kept in the lowest possible value. In this equilibrium ordered phase regime, the lowest energy is achieved by populating the lowest double-well parabolae, in a symmetric superposition that preserves parity. Both the symmetry breaking and the adiabatic asymptotic value of entropy in each subsystem can be explained by this double-well, since two coherent states are needed to describe each system. As the AV increases, the process generates a relative heating with respect to the ground state. For high enough AV (sudden quench), the system stays essentially in its starting condition, and the heating is just the consequence of the initial state being very different from the instantaneous ground state. Despite this sudden heating being very high, the simplicity of the initial state (i.e. confined in the central energy potential) leads to no matter-light entanglement (i.e. one coherent state describes each subsystem). In the novel intermediate regime, by contrast, all confining potentials simultaneously perturb the system. A complicated superposition of non-trivial states for each parabola is generated, with complex and chaotic features. Since almost every eigenvalue of \( \hat{J}_x \) has a non-zero probability, the entropy of each subsystem is significantly higher than in the other two regimes.

Figure 3 demonstrates how the range of velocities that classify as ‘intermediate’ actually increases with increasing number of qubits \( N \), meaning that the enhanced entanglement regime (EER) begins to dominate the space of behaviors as opposed to becoming a small niche. The EER can be imagined as lying between a lower bound AV \( v_{min} \) which marks the adiabatic evolution, and an upper bound one \( v_{max} \) defining the AV at which the sudden quench approximation starts to be valid. Specifically, Fig. 3 shows the dynamical phase diagram of the intermediate regime in which the giant entanglement occurs, including its scaling behavior (dependence on system size \( N \)). The adiabatic evolution is more difficult to achieve as the number of atoms increases. The other main variable is the value of \( \lambda \) reached by the annealing. The sudden quench condition requires higher AVs as this \( \lambda \) gets bigger. The oscillatory behavior near the adiabatic regime has been smoothed out in order to make the phase boundary visually clearer.

These distinct regime behaviors can also be interpreted as a momentary realization of a process of One-Axis Spin Squeezing with a Transverse Field [20, 21]. As the light-matter coupling increases in time from zero into the
λ > λc range, the system moves out of its frozen initial state. The interaction term in the Dicke Hamiltonian begins to dominate and the radiation mode works as a mediator of the interaction, realizing an effective non-linear interaction among the qubits. High values of squeezing in the matter qubit system are now generated [18]. However this process does not last indefinitely: the radiation mode starts to retain quantum information by itself, which is the moment when the entropy grows. The effective interaction is broken, leaving the qubits entangled with the radiation mode but not among themselves. Despite being a single radiation mode, the field acts as a reservoir that dissipates the quantum correlations present in the squeezed states of each subsystem. The quantum correlations developing in the field system can be represented by the Wigner quasi-probability distribution in Figs. 4(a) and (b). This shows that the distribution becomes highly fragmented yet retains some order, reflecting the complexity of the light subsystem in the intermediate regime. Round-tailed interference patterns as in Fig. 4 have been obtained in light with a non-linear Kerr-like interaction following a Fokker-Planck equation [22]. This confirms that the field experiences an effective non-linear interaction. Similar signatures of complexity arise in the matter subsystem, specifically the matter density matrices, as shown in the spherical Agarwal-Wigner functions in Figs. 4(c) and (d) [23].

In addition to the giant entanglement, the intermediate driving velocity regime also generates squeezing effects that are much stronger than those which can be achieved under near-adiabatic evolution. As well as enhancing current understanding of nonlinear interactions in composite quantum many-body systems, our work therefore informs understanding of separate squeezing effects in light and matter systems [18, 21, 22, 24, 25]. Other properties that are simultaneously also dynamically enhanced include superradiance, phase order, and sensitivity to weak forces.

Finally we note that by moving around the parameter space in time in Fig. 1, the enhanced squeezing and entanglement can be altered within the matter and light subsystems separately, and then transferred by means of the light-matter coupling. Potential applications include high precision quantum metrology and a range of quantum information processing technologies [26–28].

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* ol.acevedo53@uniandes.edu.co

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