Lepton number violation at the LHC with leptoquark and diquark

Masaya Kohda,\textsuperscript{1}*, Hiroaki Sugiyama,\textsuperscript{2}† and Koji Tsumura\textsuperscript{3}‡

\textsuperscript{1} Department of Physics, National Taiwan University, Taipei 10617, Taiwan
\textsuperscript{2} Department of Physics, University of Toyama, Toyama 930-8555, Japan
\textsuperscript{3} Department of Physics, Graduate School of Science, Nagoya University, Nagoya 464-8602, Japan

Abstract

We investigate a model in which tiny neutrino masses are generated at the two-loop level by using scalar leptoquark and diquark multiplets. The diquark can be singly produced at the LHC, and it can decay into a pair of leptoquarks through the lepton number violating interaction. Subsequent decays of the two leptoquarks can provide a clear signature of the lepton number violation, namely two QCD jets and a pair of same-signed charged leptons without missing energy. We show that the signal process is not suppressed while neutrino masses are appropriately suppressed.

PACS numbers: 11.30.Fs, 14.60.Pq, 14.80.Fd, 14.80.Sv

Keywords: neutrino, lepton number violation, diquark, leptoquark

*Electronic address: mkohda@hep1.phys.ntu.edu.tw
†Electronic address: sugiyama@sci.u-toyama.ac.jp
‡Electronic address: ko2@eken.phys.nagoya-u.ac.jp
1. INTRODUCTION

The Standard Model (SM) gauge symmetry of elementary particles based on the SU(3)$_C \times$ SU(2)$_L \times U(1)_Y$ has been tested very accurately. On the other hand, the existence of the neutrino masses has been established [1–6]. This is clear evidence of the new physics beyond the SM because neutrinos are massless in the SM. Since neutrinos are electrically neutral, they can be Majorana particles unlike the other SM fermions [7]. The reason why neutrino masses are very different from those of the other SM fermions might be the Majorana property of neutrinos.

The most familiar utilization of the Majorana property to generate tiny neutrino masses is the so-called Type-I Seesaw mechanism in which SU(2)$_L$-singlet right-handed neutrinos mediate in the tree diagram [8]. Because of the suppression by mass scales of new heavy particles, naturally light neutrinos can arise. Another typical prescription to obtain tiny Majorana neutrino masses is the so-called radiative seesaw mechanism [9–13], where neutrino masses are induced at the loop level. In these models, the suppression of neutrino masses can be achieved by the loop suppression factor and/or a combination of new coupling constants even if new particles are not very heavy. The masses of charged leptons involved in the chirality flipping loop provide further suppression of the neutrino masses in some of such models [9–11].

Although the lepton number is conserved in the SM, the addition of the Majorana mass term of neutrinos breaks the lepton number conservation by two units. The measurement of the lepton number violating (L#V) processes such as the neutrinoless double beta decay [14, 15] is extremely important because it gives evidence that neutrinos are Majorana particles. Such processes are naively expected to be very rare because neutrino masses are very small. This is true for the Type-I seesaw model with very heavy right-handed neutrinos because light Majorana neutrino masses are unique lepton number breaking parameters at the energy scale which is experimentally accessible. However, in radiative seesaw models, a trilinear coupling constant for light (e.g. TeV-scale) scalars can be more fundamental than light neutrino masses as the L#V parameter at the accessible energy scale. Then, L#V processes via the trilinear coupling constant can be significant at the TeV-scale even if the neutrino masses are suppressed enough.

New particles related to the neutrino mass generation are usually produced via the electroweak interaction, and therefore the production cross sections are not so significant at the LHC. However, new particles in the loop of the radiative seesaw models can be charged under the SU(3)$_C$ [16–18]. Such a colored particle can easily be produced at hadron colliders. In these models, decay patterns of new colored particles could be related to the form of the neutrino mass matrix constrained by
the neutrino oscillation data [16–19] (See also [20]).

In this paper, we investigate a radiative seesaw model with a scalar leptoquark multiplet and a scalar diquark multiplet. Majorana masses of neutrinos are induced via the two-loop diagram where colored particles are involved in the loop. The lepton number violation is caused by the trilinear coupling constant of the leptoquarks and diquark, which can produce a characteristic signature at the LHC. The signature consists of two QCD jets and a pair of same-signed charged leptons without missing energy, which would be easily observed at the LHC.

This paper is organized as follows. In Sec. II, we present the model. Section III is devoted to discussion on the collider phenomenology and the low energy constraints for the leptoquark and the diquark in the model. Conclusions are given in Section IV.

II. THE MODEL

The particle contents of the colored radiative seesaw model are shown in Table I. The model is briefly mentioned in Ref. [16]. The model includes a scalar leptoquark multiplet \( S_{LQ} \) whose lepton number and baryon number are 1 and 1/3, respectively. Under the SM gauge group, the \( S_{LQ} \) is assigned to the same representation of right-handed down-type quarks; a \( 3 \) representation of SU(3)\(_C\), a singlet under SU(2)\(_L\), and hypercharge \( Y = -1/3 \). We also introduce a scalar diquark multiplet \( S_{DQ} \) which has a baryon number 2/3. We take \( S_{DQ} \) as a \( 6 \) representation of SU(3)\(_C\), a singlet under SU(2)\(_L\), and a \( Y = -2/3 \) field. The diquark of a \( 6 \) representation can be expressed

| SU(3)\(_C\) | SU(2)\(_L\) | U(1)\(_Y\) | Spin | \( L\# \) | \( B\# \) |
|-------------|-------------|-------------|-------|-----|-----|
| 1           | 2           | -1/2        | 1/2   | 1   | 0   |
| 3           | 2           | 1/6         | 1/2   | 1   | 0   |
| 1           | 1           | 2/3         | 1/2   | 0   | 1/3 |
| 3           | 1           | -1/3        | 1/2   | 0   | 1/3 |
| 3           | 3           | -1/3        | 2/3   | 1   | 0   |
| 3           | 6           | -2/3        | 0     | 0   | 2/3 |

TABLE I: List of particle contents of the model.
in a symmetric matrix form as

\[
S_{DQ} = \begin{pmatrix}
S_{DQ1} & S_{DQ4}/\sqrt{2} & S_{DQ5}/\sqrt{2} \\
S_{DQ4}/\sqrt{2} & S_{DQ2} & S_{DQ6}/\sqrt{2} \\
S_{DQ5}/\sqrt{2} & S_{DQ6}/\sqrt{2} & S_{DQ3}
\end{pmatrix}.
\] (1)

The baryon number conservation is imposed to the model such that the proton decay is forbidden. We introduce the soft-breaking term (see the next paragraph) of the lepton number conservation to the scalar potential in order to generate Majorana neutrino masses. The Yukawa interactions with the leptoquark and diquark, which preserve both of the lepton number and the baryon number, are given by

\[
L_{\text{Yukawa}} = -\left\{\mathcal{T}_i (Y_L)_{\ell i} i \sigma_2 Q_i^\alpha + (\ell R)^{\alpha} (Y_R)_{\ell i} u_{iR}^\alpha \right\} \left(S_{LQ}^\alpha\right)^* - \left(d_{iR}^\alpha\right)^T (Y_s)_{ij} d_{jR}^\beta \left(S_{DQ}^{\alpha\beta}\right)^* + \text{H.c.},
\] (2)

where \(\sigma_a\) (\(a = 1–3\)) are the Pauli matrices, \(\alpha\) and \(\beta\) (\(= r, g, b\)) denote the color indices; for example, \(S_{rr}DQ\) corresponds to \(S_{DQ1}\) in Eq. (1). We choose the diagonal bases of mass matrices for the charged leptons and down-type quarks. Then, the SU(2)\(_L\) partner of \(d_iL\) is described as \(u_i'_{L} = (V_{\text{CKM}})^{ij} u_jL\), where \(V_{\text{CKM}}\) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix and \(u_j = (u_{jR}, u_{jL})^T\) are mass eigenstates of up-type quarks. Mass eigenstates \(\nu_iL\) of neutrinos are given by \(\nu_iL = (U_{\text{MNS}}^\dagger)_{\ell i} \nu_{\ell L}\), where \(U_{\text{MNS}}\) is the Maki-Nakagawa-Sakata (MNS) matrix. The Yukawa matrices \((Y_L, Y_R, \text{and} Y_s)\) are 3 \(\times\) 3 matrices under the lepton flavor \((\ell = e, \mu, \tau)\) and the down-type quark flavor \((i, j = 1–3)\). While \(Y_L\) and \(Y_R\) are general complex matrices, \(Y_s\) is a symmetric matrix \((Y_s^T = Y_s)\). Note that neutrinos interact with the leptoquark only through \(Y_L\), and we will see later that \(Y_R\) is irrelevant to the neutrino mass at the leading order.

In the scalar potential of the model, we introduce the following three-point interaction:

\[
\mu \left(S_{LQ}^\alpha\right)^* \left(S_{LQ}^\beta\right)^* S_{DQ}^{\alpha\beta} + \text{H.c.}
\] (3)

The coupling constant \(\mu\) softly breaks the lepton number conservation by two units while the baryon number is conserved. There is no other possible soft-breaking term of the lepton number and/or the baryon number. We can take the \(\mu\) parameter as a real positive value by using the rephasing of \(S_{DQ}\). Considering radiative corrections to \(m_{LQ}\) and \(m_{DQ}\) via the \(\mu\) parameter, perturbativity requires \(\mu \lesssim \min(m_{LQ}, m_{DQ})\) as discussed in Ref. [21] for the Zee-Babu model (ZBM) [10].

The neutrino mass term \(\frac{1}{2} (M_\nu)_{\ell e} \overline{\nu}_{e\ell} (\nu_{e\ell})^T\) in the flavor basis is generated by a two-loop diagram in FIG. 1 including the leptoquark and the diquark. The mass matrix is calculated as

\[
(M_\nu)_{\ell e} = +24\mu (Y_L^\alpha)_{\ell i} m_{d_i} (Y_s)_{ij} I_{ij} m_{d_j} (Y_L^+)_{j\ell e},
\] (4)
FIG. 1: The two-loop diagram for the neutrino mass generation in the model.

where the loop function $I_{ij}$ is defined as

$$I_{ij} = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{k_1^2 - m_{d_i}^2} \frac{1}{k_2^2 - m_{d_j}^2} \frac{1}{k_1^2 - m_{L_Q}^2} \frac{1}{k_2^2 - m_{L_Q}^2} \frac{1}{(k_1 + k_2)^2 - m_{D_Q}^2}. \quad (5)$$

The diagram is similar to the one in the ZBM although SU(3)$_C$-singlet particles in the loop are replaced with colored particles. Thus, we refer to this model as the colored Zee-Babu model (cZBM). See, e.g., Refs. [21–23] for studies about the ZBM for comparison with the cZBM. Note that $Y_R$ does not contribute to the two-loop diagram.$^1$ In the ZBM, at least one massless neutrino is predicted because of the antisymmetric Yukawa coupling matrix. In contrast, all of three neutrino masses can be non-zero in the cZBM because $Y_L$ is not an antisymmetric matrix. Since new colored scalars should be much heavier than the SM fermions, the loop function can be reduced to $^2$

$$I_{ij} \simeq I_0 \equiv \frac{1}{(4\pi)^4} \frac{1}{\max[m_{L_Q}, m_{D_Q}]^2} \frac{\pi^2}{3} \tilde{I}(m_{D_Q}^2/m_{L_Q}^2), \quad (6)$$

where

$$\tilde{I}(r) = \begin{cases} 1 + \frac{3}{\pi^2} ((\ln r)^2 - 1) & \text{for } r \gg 1 \\ 1 & \text{for } r \ll 1 \end{cases} \quad (7)$$

Hereafter, we restrict ourselves to the simplest scenario where $Y_R$ is small enough to be ignored.$^2$

A benchmark point in the parameter space of the cZBM is shown in Appendix A.

$^1$ The $Y_R$ contributes to Majorana neutrino masses at the higher loop level. The four-loop contribution is utilized in the model of Ref. [24] where $Y_L$ is ignored.

$^2$ If we extend the model as a two-Higgs-doublet model, we can eliminate the $Y_R$ term by using a softly-broken $Z_2$ symmetry (e.g., $u_{iR}$ (or $\ell_{R}$) and the second Higgs doublet are $Z_2$-odd fields) which is also required to avoid the flavor changing neutral current at the tree level. Another example to eliminate the $Y_R$ term is the case where the leptoquark is not an SU(2)$_L$-singlet but a triplet.
III. NEW COLORED SCALARS AT THE LHC

A. Leptoquark

The main production channel of leptoquarks at hadron colliders would be the pair-creation from $gg$ and $qq$ annihilation. The associated production of $S_{\text{LQ}}$ with a lepton from $qq$ coannihilation could also be possible. The pair-production cross section is determined only by QCD interaction at the leading order, while the associated production mechanism highly depends on the Yukawa coupling constant of the leptoquark. The associated production mechanism is negligible at a benchmark point shown in the Appendix A because of tiny $(Y_L)_{\ell 1}$. The leptoquarks have been searched at the Tevatron and the LHC. The most stringent lower bound on the leptoquark mass at 95\% confidence level is set as 830 GeV (840 GeV) by the recent CMS result at $\sqrt{s} = 7$ TeV with 5.0 fb$^{-1}$ integrated luminosity; the pair-production of scalar leptoquarks is assumed as well as a hundred percent decay branching ratio into the first (second) generation quarks and leptons. See also Refs. [28, 29] for the ATLAS results with 1.03 fb$^{-1}$ integrated luminosity. The analysis of the decay into third generation fermions would be performed in near future. The search strategies for the third generation leptoquarks have been studied in Ref. [30].

The leptoquark induces various LFV processes. At the tree level, four-fermion operators (two left-handed leptons and two left-handed quarks) are generated by integrating leptoquarks out. The constraints on such operators have been extensively studied in Ref. [31]. Tables 3, 4, 12, and 13 in Ref. [31] are relevant to the cZBM. Especially, operators $(\overline{e_L} \gamma^\mu \mu_L)(\overline{u_L} \gamma^\mu u_L)$ and $(\overline{\nu_L} \gamma^\mu \nu_{\ell L})(\overline{\ell_L} \gamma^{\mu} s_{L})$ are strongly constrained by the $\mu$-e conversion search and the $K$ meson decay measurement, respectively. For the benchmark point given in Appendix A we have $|\langle Y_L \rangle_{\ell 1}(Y_{\text{L}}^{\ell})_{\mu 1}|/(4\sqrt{2} G_F m^{2}_{\text{LQ}}) = 6.1 \times 10^{-11}$ and $|\langle Y_L \rangle_{\ell 1}(Y_{\text{L}}^{\ell})_{\nu 2}|/(4\sqrt{2} G_F m^{2}_{\text{LQ}}) \lesssim 10^{-7}$ which satisfy constraints shown in Ref. [31], where $G_F = 1.17 \times 10^{-5}$ GeV$^{-2}$.

At the loop level, effects of leptoquarks on charged lepton transitions, i.e., $\ell_i \to \ell_j \gamma$, have also been studied. Since we assume that $S_{\text{LQ}}$ has the Yukawa interaction only with the left-handed quarks (namely $Y_R = 0$), the contribution from the top quark loop does not give a large enhancement of $\ell_i \to \ell_j \gamma$. Then, the branching ratio of $\mu \to e \gamma$ is calculated as

$$\text{BR}(\mu \to e \gamma) = \frac{3 \alpha_{\text{EM}}}{256 \pi G_F^2 m_{LQ}^{12}} \left| \langle Y_L \rangle_{\ell 1} \langle Y_{\text{L}}^{\ell} \rangle_{e \mu} \right|^2,$$  

It is known that the similar process $b \to s \gamma$ (induced by the uncolored charged Higgs boson) in the Type-II two-Higgs-doublet model is enhanced by the top quark loop.
where $\alpha_{EM} = 1/137$. For example, a benchmark point shown in Appendix A gives $\text{BR}(\mu \to e\gamma) = 6.5 \times 10^{-13}$ which satisfies the current upper bound ($2.4 \times 10^{-12}$ at 90% confidence level) in the MEG experiment.

Since we take $Y_R = 0$, the sign of the leptoquark contribution to the leptonic $g - 2$ cannot be changed. It is worth to mention that the contribution of the leptoquark has an appropriate sign (the plus sign) to compensate the difference between the measured value and the SM prediction for the muon $g - 2$. The preferred size of $Y_L$ is $(Y_L Y_L^\dagger)_{\mu\mu} \sim 1$ for $m_{LQ} \sim 1$ TeV. In order to satisfy LFV constraints with this size of $Y_L$, a simple ansatz is that $Y_L$ is a diagonal matrix. Note that we must take care about the constraint on $(\nu_{eL} \gamma_{\mu} \nu_{\mu L}) (d_{L} \gamma_{\mu} s_{L})$ (see Table 12 in Ref. [31]) even if $Y_L$ is diagonal; the constraint on $(Y_L)_{e1} (Y_L^\dagger)_{\mu2}$ is difficult to be satisfied with $(Y_s)_{12} \lesssim 1$ because $Y_L$ is related to $Y_s$ through the neutrino mass matrix. We could not find any viable example of such a parameter set although it might exist with more complicated structures of $Y_L$ and $Y_s$.

B. Diquark

At the LHC, the diquark $S_{DQ}$ in the cZBM would be singly produced by the annihilation of two down-type quarks. The single production mechanism has an advantage to search for the relatively heavy diquark due to the $s$-channel resonance. The single production cross section is determined by $(Y_s)_{11}$, which is evaluated in Ref. [36] as a function of the diquark mass with a fixed Yukawa coupling constant. The $(Y_s)_{11}$ in the cZBM is less constrained by the neutrino oscillation data because its contribution to neutrino masses is suppressed by $m_d^2/m_{DQ}^2$. If we assume $(Y_s)_{11} = 0.1$ and $m_{DQ} = 4$ TeV, the single production cross section $\sigma(dd \to S_{DQ})$ is about 5 fb at the LHC with $\sqrt{s} = 14$ TeV [36]. Note that the CMS experiment at $\sqrt{s} = 7$ TeV with 1 fb$^{-1}$ integrated luminosity excludes diquark masses between 1 TeV and 3.52 TeV at 95% confidence level by assuming the diquark decay into two QCD jets for the $E_6$ diquark which couples with an up-type quark and a down-type quark [37]. See also Refs. [38–40].

The diquark induces flavor changing neutral current processes in the down-type quark sector. Especially, it gives tree-level contributions to $K^0 - \overline{K^0}$, $B^0_d - \overline{B^0_d}$ and $B^0_s - \overline{B^0_s}$ mixings, resulting in strong constraints on $Y_s$. By using the notations in Ref. [41], the benchmark point in Eqs. (A1) gives

$$\tilde{C}_K^1 = -(Y_s^{*})_{11}(Y_s)_{22}/(2m_{DQ}^2) = 0.$$  

Similarly, we have $\tilde{C}_{B_d}^1 = +1.2 \times 10^{-12}$ GeV$^{-2}$ and $\tilde{C}_{B_s}^1 = 0$.

---

4 For a heavy scalar $\phi$ which interacts with $\mu_L$ and a light fermion $\psi_L$ as $\overline{\mu_L}(\psi_L)^c \phi$, its contribution to the muon $g - 2$ has the plus sign if the electric charge of $\phi$ is greater than $-2/3$. See also Ref. [22].

5 The diquark can also be created in pair via the gluon-gluon annihilation.
These values satisfy the constraints obtained in Ref. [41] (see also Refs. [42, 43]).

The diquark in the cZBM decays into not only a pair of the down-type quarks but also a pair of leptoquarks. The fraction of fermionic and bosonic decay modes is calculated as

$$\frac{\sum_{i,j} \Gamma(S_{DQ1} \rightarrow d_i^r d_j^r)}{\Gamma(S_{DQ1} \rightarrow S_{LQ} S_{LQ}^r)} \approx \frac{m_{DQ}^2 \text{tr}(Y_s Y_s^\dagger)}{\mu^2 \sqrt{1 - \frac{4m_{LQ}^2}{m_{DQ}^2}}} \cdot (9)$$

This formula is the same for the other diquarks because of the SU(3)$_C$ symmetry. We focus on the case where the ratio in Eq. (9) is less than about unity such that the branching ratio for $S_{DQ} \rightarrow S_{LQ} S_{LQ}$ becomes $O(10)$%. Subsequently, 50% of each leptoquark decays into an up-type quark (a down-type quark) and a charged lepton (a neutrino). Then, the model provides a characteristic signature in FIG. 2 whose final state consists of two QCD jets and the same-signed charged lepton pair without missing energy. The decay chain of the diquark can be fully reconstructed at the LHC. This signature can be a smoking gun for the lepton number violation because no lepton number is taken away by invisible particles. There is no SM background in principle because the SM conserves the lepton number. It would be also very rare that the SM process mimics the signal process because the leptons in the signal events are too energetic to be produced in the SM process.

It should be emphasized that the event rate of the process is not necessarily suppressed though the full process picks up all new coupling constants relevant to the small neutrino masses (namely, $Y_s$, $\mu$, and $Y_L$). One reason for that is because the process in FIG. 2 does not have suppressions with the two-loop factor $1/(16\pi^2)^2$ and with down-type quark masses, which are used for tiny neutrino masses. The other reason is that on-shell productions of a diquark and leptoquarks are utilized as $\sigma(dd \rightarrow S_{DQ}) \text{BR}(S_{DQ} \rightarrow S_{LQ} S_{LQ}) \left[ \sum_{\ell,i} \text{BR}(S_{LQ} \rightarrow \ell_L u_{iL}) \right]^2$; even if a partial decay width is controlled by a small coupling constant (e.g., $S_{LQ} \rightarrow \ell_L u_{iL}$ via $(Y_L)_{\ell i}$), its branching ratio becomes sizable when the total decay width is also controlled by small coupling constants. In this scenario, the cZBM seems the new physics model which is the most easily probed at the LHC and takes us to the top of the energy frontier.

For the benchmark point shown in the Appendix A, the ratio in Eq. (9) is 0.18 for which 85% of $S_{DQ}$ decays into $S_{LQ} S_{LQ}$. Then, 15% (7%) of $S_{LQ}$ decays into a charm quark (a top quark) associated with an electron or a muon. Decays into an up quark are negligible for the benchmark point. The decay into a tau lepton might not be reliable because it gives missing neutrinos. Even if $S_{LQ}$ decays into a top quark, hadronic decays (68%) of $W^\pm$ from the top quark decay have no missing energy. As a result, the cross section for L#V events without missing energy is about
FIG. 2: The same-signed charged lepton signature without missing energy at the LHC.

0.18 fb at the LHC with \( \sqrt{s} = 14 \text{ TeV} \) for the benchmark point.

In the energy scale which is much below the new scalar masses, the diagram in FIG. 2 becomes a dimension-9 operator of six fermions. Such L#V operators up to dimension-11 have been studied in Refs. [16, 44]. The dimension-9 operator in the cZBM is highly suppressed by the inverse power of mass scales of new colored particles as well as by new Yukawa coupling constants. Therefore the collider signature in FIG. 2 does not conflict with the stringent constraints from the other lepton number violating observable such as the neutrinoless double beta decay [14], and lepton number violating rare decays \( \tau^\pm \rightarrow \ell^\mp M^\pm M^\pm (M = \pi, K) \) [45], \( M^\pm \rightarrow M^\mp \ell^\pm \ell'^\pm (M = B, K, D) \) [46] and \( t \rightarrow b\ell^+\ell^-W^- \) [47, 48].

IV. CONCLUSIONS

We have studied a model for neutrino mass generation with the scalar leptoquark \( S_{LQ} \) and the scalar diquark \( S_{DQ} \). Tiny Majorana neutrino masses are induced at the two-loop level where the colored particles are involved in the loop. The trilinear scalar coupling constant between two leptoquarks and a diquark is the only parameter of the lepton number violation in this model. The diquark can be singly produced at the LHC via the resonance mechanism, and it can decay into a pair of leptoquarks through the lepton number violating coupling. The leptoquarks can further decay into a charged lepton and an up-type quark. Thus, the model gives a distinctive signature at the LHC, namely \( pp \rightarrow S_{DQ} \rightarrow S_{LQ} S_{LQ} \rightarrow \ell^-\ell^-jj \) without missing energy, which would be a clear evidence of the lepton number violation. We have shown that the lepton number violating process is not suppressed because of on-shell productions and decays of the diquark and the leptoquarks, while Majorana neutrino masses are highly suppressed.
Acknowledgments

We thank Joe Sato, Eibun Senaha, and Hiroshi Yokoya for valuable comments. M.K. is supported by the NTU Grant No. 101R7701 and the Laurel program. The work of H.S. was supported in part by the Grant-in-Aid for Young Scientists (B) No. 23740210. K.T. was supported, in part, by the Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 23104011.

Appendix A: A benchmark point

Here, we show a benchmark point of the model:

\[
Y_L = \begin{pmatrix}
8.1 \times 10^{-5} & 4.0 \times 10^{-2} & -7.0 \times 10^{-3} \\
-4.9 \times 10^{-5} & 5.3 \times 10^{-2} & 4.4 \times 10^{-2} \\
3.1 \times 10^{-5} & -2.3 \times 10^{-2} & 8.9 \times 10^{-2}
\end{pmatrix}, \quad (A1a)
\]

\[
Y_s = \begin{pmatrix}
1.0 \times 10^{-1} & 0 & 0 \\
0 & 0 & -1.2 \times 10^{-2} \\
0 & -1.2 \times 10^{-2} & -3.8 \times 10^{-4}
\end{pmatrix}, \quad (A1b)
\]

\[
\mu = 1 \text{ TeV}, \quad m_{LQ} = 1 \text{ TeV}, \quad m_{DQ} = 4 \text{ TeV}. \quad (A1c)
\]

We define an overall constant of neutrino masses as \( C \equiv 24 \mu I_0 \simeq 6.0 \times 10^{-7} \text{ GeV}^{-1} \).

The neutrino mass matrix is diagonalized as \( U_{\text{MNS}}^\dagger \bar{M}_\nu U_{\text{MNS}}^* = \text{diag}(m_1, m_2 e^{i \alpha_{21}}, m_3 e^{i \alpha_{31}}) \) with \( U_{\text{MNS}} \) which can be parametrized as

\[
U_{\text{MNS}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i \delta} \\
0 & 1 & 0 \\
-s_{13} e^{i \delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (A2)
\]

where \( c_{ij} (s_{ij}) \) denotes \( \cos \theta_{ij} (\sin \theta_{ij}) \). We use the following values: \( \sin^2 2\theta_{23} = 1, \sin^2 2\theta_{13} = 0.1, \sin^2 2\theta_{12} = 0.87, \delta = 0, \Delta m^2_{31} = 2.4 \times 10^{-3} \text{ eV}^2 > 0, \) and \( \Delta m^2_{21} = 7.6 \times 10^{-5} \text{ eV}^2 \). Matrices \( Y_L \) and
$Y_s$ in Eqs. (A1) are constructed by assuming the following structures:

$$Y_L = U_{\text{MNS}}^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{m_3}{m_2 + m_3}} & -\sqrt{\frac{m_2}{m_2 + m_3}} \\ 0 & \sqrt{\frac{m_2}{m_2 + m_3}} & \sqrt{\frac{m_3}{m_2 + m_3}} \end{pmatrix} \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix},$$  
(A3a)

$$Y_s = \begin{pmatrix} (Y_s)_{11} & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{m_2 m_3}{yz m_s m_b C}} \\ 0 & -\sqrt{\frac{m_2 m_3}{yz m_s m_b C}} & -\frac{m_3 - m_2}{z^2 m_b^2 C} \end{pmatrix}. $$  
(A3b)

It is easy to see that $M_\nu$ with these matrices results in $m_1 = x^2 m_d^2 C (Y_s)_{11}$, $\alpha_{21} = 0$, and $\alpha_{31} = \pi$.

We use $x = 10^{-4}$, $y = 0.07$, $z = 0.1$, $m_d = 5 \times 10^{-3}$ GeV, $m_s = 0.1$ GeV, $m_b = 4.2$ GeV and $(Y_s)_{11} = 0.1$. Note that the benchmark point gives $(M_\nu)_{ee} \simeq 1.5 \times 10^{-3}$ eV, which is the effective mass relevant for the neutrinoless double beta decay.

[1] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998); W. Hampel et al. [GALLEX Collaboration], Phys. Lett. B 447, 127 (1999); B. Aharmim et al. [SNO Collaboration], Phys. Rev. Lett. 101, 111301 (2008); J. N. Abdurashitov et al. [SAGE Collaboration], Phys. Rev. C 80, 015807 (2009); K. Abe et al. [Super-Kamiokande Collaboration], Phys. Rev. D 83, 052010 (2011); G. Bellini et al. [Borexino Collaboration], Phys. Rev. Lett. 107, 141302 (2011).

[2] R. Wendell et al. [Kamiokande Collaboration], Phys. Rev. D 81, 092004 (2010).

[3] M. H. Ahn et al. [K2K Collaboration], Phys. Rev. D 74, 072003 (2006); P. Adamson et al. [The MINOS Collaboration], Phys. Rev. Lett. 106, 181801 (2011).

[4] K. Abe et al. [T2K Collaboration], Phys. Rev. Lett. 107, 041801 (2011).

[5] M. Apollonio et al. [CHOOZ Collaboration], Eur. Phys. J. C 27, 331 (2003); Y. Abe et al. [DOUBLE-CHOOZ Collaboration], Phys. Rev. Lett. 108, 131801 (2012); F. P. An et al. [DAYA-BAY Collaboration], Phys. Rev. Lett. 108, 171803 (2012); J. K. Ahn et al. [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012).

[6] A. Gando et al. [The KamLAND Collaboration], Phys. Rev. D 83, 052002 (2011).

[7] E. Majorana, Nuovo Cim. 14, 171 (1937).

[8] P. Minkowski, Phys. Lett. B 67, 421 (1977); T. Yanagida, Conf. Proc. C 7902131, 95 (1979); Prog. Theor. Phys. 64, 1103 (1980); M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C 790927, 315 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[9] A. Zee, Phys. Lett. B 93, 389 (1980) [Erratum-ibid. B 95, 461 (1980)].
[10] A. Zee, Nucl. Phys. B 264, 99 (1986); K. S. Babu, Phys. Lett. B 203, 132 (1988).
[11] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D 67, 085002 (2003).
[12] E. Ma, Phys. Rev. D 73, 077301 (2006).
[13] M. Aoki, S. Kanemura and O. Seto, Phys. Rev. Lett. 102, 051805 (2009).
[14] W. H. Furry, Phys. Rev. 56, 1184 (1939).
[15] J. J. Gomez-Cadenas, J. Martin-Albo, M. Mezzetto, F. Monrabal and M. Sorel, Riv. Nuovo Cim. 35, 29 (2012).
[16] K. S. Babu and C. N. Leung, Nucl. Phys. B 619, 667 (2001).
[17] D. Aristizabal Sierra, M. Hirsch and S. G. Kovalenko, Phys. Rev. D 77, 055011 (2008).
[18] P. Fileviez Perez and M. B. Wise, Phys. Rev. D 80, 053006 (2009).
[19] P. Fileviez Perez, T. Han, S. Spinner and M. K. Trenkel, JHEP 1101, 046 (2011).
[20] S. Choubey, M. Duerr, M. Mitra and W. Rodejohann, JHEP 1205, 017 (2012).
[21] M. Nebot, J. F. Oliver, D. Palao and A. Santamaria, Phys. Rev. D 77, 093013 (2008).
[22] K. S. Babu and C. Macesanu, Phys. Rev. D 67, 073010 (2003).
[23] D. Aristizabal Sierra and M. Hirsch, JHEP 0612, 052 (2006).
[24] P. -H. Gu, Phys. Rev. D 85, 093016 (2012).
[25] M. Kramer, T. Plehn, M. Spira and P. M. Zerwas, Phys. Rev. Lett. 79, 341 (1997); Phys. Rev. D 71, 057503 (2005).
[26] A. Belyaev, C. Leroy, R. Mehdiyev and A. Pukhov, JHEP 0509, 005 (2005).
[27] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. D 86, 052013 (2012).
[28] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 709, 158 (2012) [Erratum-ibid. 711, 442 (2012)].
[29] G. Aad et al. [ATLAS Collaboration], Eur. Phys. J. C 72, 2151 (2012).
[30] S. Davidson and P. Verdier, Phys. Rev. D 83, 115016 (2011); B. Grippaios, A. Papaefstathiou, K. Sakurai and B. Webber, JHEP 1101, 156 (2011).
[31] M. Carpentier and S. Davidson, Eur. Phys. J. C 70, 1071 (2010).
[32] R. Benbrik and C. -K. Chua, Phys. Rev. D 78, 075025 (2008).
[33] T. G. Rizzo, Phys. Rev. D 38, 820 (1988); B. Grinstein and M. B. Wise, Phys. Lett. B 201, 274 (1988); R. Grigjanis, P. J. O’Donnell, M. Sutherland and H. Navelet, Phys. Lett. B 213, 355 (1988) [Erratum-ibid. B 286, 413 (1992)]; W. -S. Hou and R. S. Willey, Phys. Lett. B 202, 591 (1988); T. D. Nguyen and G. C. Joshi, Phys. Rev. D 37, 3220 (1988).
[34] J. Adam et al. [MEG Collaboration], Phys. Rev. Lett. 107, 171801 (2011).
[35] S. Kanemitsu and K. Tobe, arXiv:1207.1313 [hep-ph].
[36] O. Cakir and M. Sahin, Phys. Rev. D 72, 115011 (2005); T. Han, I. Lewis and Z. Liu, JHEP 1012, 085 (2010).
[37] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 704, 123 (2011).
[38] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 79, 112002 (2009).
[39] G. Aad et al. [ATLAS Collaboration], arXiv:1210.1718 [hep-ex].
[40] R. M. Harris and K. Kousouris, Int. J. Mod. Phys. A 26, 5005 (2011).
[41] M. Bona et al. [UTfit Collaboration], JHEP 0803, 049 (2008).
[42] D. Chakraverty and D. Choudhury, Phys. Rev. D 63, 112002 (2001).
[43] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. 102, 211802 (2009).
[44] A. de Gouvea and J. Jenkins, Phys. Rev. D 77, 013008 (2008).
[45] A. Ilakovac, Phys. Rev. D 54, 5653 (1996).
[46] L. S. Littenberg and R. E. Shrock, Phys. Rev. Lett. 68, 443 (1992).
[47] S. Bar-Shalom, N. G. Deshpande, G. Eilam, J. Jiang and A. Soni, Phys. Lett. B 643, 342 (2006).
[48] N. Quintero, G. Lopez Castro and D. Delepine, Phys. Rev. D 84, 096011 (2011) [Erratum-ibid. D 86, 079905 (2012)].