IMPACT OF CAP-AND-TRADE REGULATION ON COORDINATING PERISHABLE PRODUCTS SUPPLY CHAIN WITH COST LEARNING

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(Communicated by Jie Sun)

ABSTRACT. This paper incorporates carbon emission regulation and cost learning effects to examine a manufacturer-retailer supply chain for deteriorating items over a multi-period planning horizon. We investigate their impacts on supply chain coordination under the assumption that the product demand is affected by the selling price, promotional effort and inventory level. We first propose two algorithms for determining optimal solutions of the centralized and decentralized models. We show that the decentralized system can be coordinated perfectly with a two-part tariff contract. Further, we study necessary conditions under which members of the supply chain can accept this contract. At last, we conduct numerical experiment to illustrate the obtained theoretical results in impact analysis and the robustness of the coordinated model.

1. Introduction. In recent decades, global warming has received increasing attention. It is generally understood that the emissions of anthropogenic greenhouse gases (GHG), with carbon dioxide (CO₂) being the most important, are largely responsible for global warming. As concerns about climate change continue to rise, carbon emission reduction is becoming significantly important and many national and international authorities have accordingly passed various regulations for carbon emission control and reduction. In particular, cap-and-trade, as a market-based approach, is designed to achieve certain target of carbon emission reduction. Under
this regulation, a central authority allocates a company a limit or cap on the amount of its carbon emissions. If the amount of the company’s carbon emissions is higher than the carbon cap, the company should buy extra carbon emission permits in a carbon trading market while the company can sell its surplus carbon emission permits if the amount of emissions is lower than the carbon cap. Cap-and-trade regulatory schemes have been implemented in many countries. Today there are more than 20 platforms for trading carbon emission permits in the world, amongst European Emissions Trading System (EUETS) is the first and the largest in the market. Additionally, EUETS covers more than 11,000 power stations and industrial plants in 31 countries, accounting for approximately 45% of carbon dioxide emissions in the European Union [11].

Carbon emission regulations have significant impacts on the supply chain structures of many manufacturing and service companies and their operational decision making. Firms in such supply chains seeking to maximize their profits would inevitably face new constraints, which in turn would force them to reduce carbon emissions. On the other hand, certain firms have taken measures to address issues arising from sustainable supply chain management under various carbon emission regulations. For instance, Walmart has made much efforts to achieve environmental sustainability in managing its supply chains. Its suppliers are required to provide the green labels on the packaging of certain products [32]. Deterioration of stored products is common in traditional and green supply chain management practice. For example, many dairy products, vegetables and fruits deteriorate rapidly over time. These perishable products may cause more damages to the environment than other products due to demanding transportation and storage conditions to preserve product quality. It has been shown in [2] that the Lekkerland group adopts a multi-temperature distribution strategy with special multi-temperature and multi-chamber facilities to distribute perishable products. The implementation of this distribution strategy saves the loss of product deterioration significantly and produces more carbon emissions compared with other products. Under the pressure of carbon emission reduction, coordination strategies may also be affected when perishable products are involved in a supply chain system. The presence of perishable products thus poses new challenges for the management of sustainable supply chain systems.

Motivated by the above, in this paper we consider a manufacturer-retailer supply chain with perishable products over a multi-period planning horizon. Since the production process is one of the most significant sources of greenhouse gas emissions in a supply chain, we incorporate the carbon footprint into the manufacturing process. To model the two-party supply chain, we assume that cap-and-trade regulation is imposed on a manufacturer, which is termed the emission-dependent manufacturer [9]. It has been widely verified that with learning-by-doing, the operational activity of the firm yields accumulate experience over time[25]. As to the retailer of the supply chain, when multiple ordering happens, a learning effect on the fixed ordering cost is considered in analysis because of the cumulative nature of operational experience. In this study we seek to address the following questions of interest. Namely, (i) What is the difference in profits between a decentralized system and a centralized system when cap-and-trade regulation and learning effects are incorporated into the supply chain with perishable products? (ii) Does there exist an effective contract to coordinate such a supply chain system? (iii) What effect will
carbon cap-and-trade regulation have on decision making for such perishable product supply chain management? In analysis, we assume the product demand might be affected by the selling price, promotional effort, and current inventory level. For the problem under consideration, we first formulate and compare a centralized and a decentralized model. Then, we propose a two-part tariff (TT) contract to coordinate the decentralized model. Finally, we use several numerical examples with sensitivity analysis to illustrate the obtained theoretical results and obtain several managerial insights.

Main contributions of this study are as follows. First, we incorporate carbon emission regulation and learning effects into a two-party manufacturer-retailer system with perishable products over a multi-period planning horizon, which extends traditional supply chain management by incorporating the constraint of carbon emission regulation. Second, if the demand is a linear function in the current inventory level, we show that it is optimal to replenish the inventory when the inventory level reaches zero. We derive the optimal joint inventory, pricing and promotional policies in closed form for both centralized and decentralized models together and analyze the difference in their corresponding profits. Third, we show that the TT contract could lead to perfect coordination and derive the necessary condition for each member of the sustainable supply chain to accept this contract. Finally, the effects of cap-and-trade regulation on supply chain coordination are further demonstrated through numerical experiments, and the robustness of the coordination is verified by performing sensitivity analysis.

The remainder of the paper is organized as follows. Section 2 reviews relevant research work in literature. Section 3 presents problem description, notations and assumptions. Section 4 states model development for the decentralized and centralized systems and the respective optimal replenishment policy, selling price and promotional effort. In Section 5, we propose the TT contract to coordinate the supply chain system and necessary conditions for accepting the contract. Section 6 illustrates the effectiveness of the theoretical results through numerical examples. Section 7 concludes.

2. Literature review. The following two streams of research are closely related to our study, that is, impacts of learning effects on supply chain systems and impacts of carbon regulations on perishable supply chain coordination. In this section, we review the literature in this regard.

2.1. Impacts of learning effects on supply chain systems. Learning effects refer to workers’ ability to improve their productivity after performing repetitive operations and exist in many industries. Many researchers have used various types of learning curves in modeling operations management problems including production planning, remanufacturing and inventory control. Related literature includes Jaber and Guiffrida [19], Jaber and EI Saadany [20], Teng et al. [36], Kazemi et al. [22], Kazemi et al. [21], and Kogan et al. [25]. Several researchers have investigated learning effects on supply chain coordination. For example, Tsao and Sheen [37] considered the impacts of learning effects on a retailer’s promotional efforts and sales and developed a model with multiple competing retailers. They also analyzed the optimal channel decisions. Khan et al. [24] proposed a single-vendor, single-buyer supply chain with a buyer that makes quality inspection errors and a vendor that experiences learning effects on production. The authors theoretically and numerically analyzed an optimal replenishment policy. Li et al. [26] considered a dynamic
one-manufacturer, one-buyer model with price-dependent demand. In their model, the products are sold by the manufacturer to the buyer over two time periods, and a random learning effect is imposed on the manufacturer’s unit production cost. They proposed a contract mechanism to coordinate this dynamic supply chain. Khan et al.[23] analyzed the impacts of production learning on a vendor-buyer supply chain with defective items. With stochastic lead time demands, they formulated a mathematical model to determine the optimal operational decisions of the supply chain. Giri and Glock[15] developed a manufacturer-retailer, closed-loop supply chain with stochastic product returns. For the case in which demand is linearly dependent on the retail price, they analyzed the impacts of learning and forgetting in production on the operational decisions of the system. Feng and Chan[12] considered a two-level trade credit in an inventory system with new products that the production cost of the manufacturer has a learning curve effect. The authors further solved the joint optimal production and pricing strategies. Other recent publications include Zanoni et al.[42], Glock and Jaber[16], Chen and Tsao[7], and Biel and Glock[4], Wu and Zhao[40], and Zhang and Zhang[45].

In contrast to the above-mentioned research papers, in this paper, we assume that multiple ordering may lead to a learning effect on fixed ordering costs. We further investigate the impacts of carbon emission regulation and learning on the coordination of a supply chain for deteriorating items.

2.2. Impacts of carbon regulations on perishable supply chain coordination. Supply chain coordination with perishable products is a common real-world problem and has thus received considerable attention. Recently, several researchers have proposed more specific and effective contracts to coordinate supply chains with perishable products for improving the system’s profit. For example, Chen and Wei[6] developed a single-manufacturer, single-vendor model with perishable products over a multi-period planning horizon. When the product demand depends on selling price and time, they proposed three contracts to coordinate the system. Giri and Bardhan[14] considered a two-party “manufacturer-retailer” system over a multi-period planning horizon. Assuming that the cycle length of replenishment is fixed and that demand depends on the selling price and inventory level, they proposed a contract mechanism to coordinate the supply chain. In the context of the model proposed by Giri and Bardhan[14], Bai et al.[3] assumed that demand depends on the selling price, time and advertising effort. They showed that a revised revenue-sharing contract can yield a higher profit than the revenue-sharing contract when these two contracts are used to coordinate the decentralized system. Zhang et al.[44] developed a one-manufacturer, one-buyer model with perishable products with a linear price-dependent demand. Under the assumption that the manufacturer cooperates with the buyer to invest in preservation technology to reduce deterioration, they coordinated the system with a revenue-sharing and cooperative investment contract. Dye[10] considered psychic stock effect in an inventory system with perishable products and solved the joint optimal operational strategies. Additional research articles on coordinating supply chain systems with perishable products include Yu[41], Chung and Kwon[8], and Zhang et al.[43].

In the above-mentioned literature, different contracts were proposed to coordinate traditional supply chain systems with perishable products. The effects of carbon emissions on supply chain coordination were not analyzed. As discussed above, when the carbon footprint is considered in supply chain management, the coordination model must be modified. Recently, several researchers have analyzed
the impacts of carbon regulations on operations management of the supply chain
systems with perishable products. Bai et al.\cite{1} considered a manufacturer-retailer
supply chain with perishable products under cap-and-trade regulation. The authors
proved that the two-part tariff contract has more robustness than other contracts
when the system is coordinated. Bai et al.\cite{2} studied the impacts of carbon emis-
sion reduction on the coordination of a single-manufacturer and two-retailer supply
chain with vendor-managed perishable product inventory. Wang et al.\cite{38} proposed
a single-supplier and multi-retailer fresh food supply chain under a carbon cap-and-
trade policy. Using the bargaining game approach, the authors solved the optimal
replenishment strategy for the three scenarios of the supply chain. A common fea-
ture of the three researches mentioned above is that the system is operated in a fixed
single period. Moreover, the three researches did not study how the time-changing
inventory of the perishable products affect the system coordination.

Unlike the research papers mentioned above, in this paper, we model a multi-
variance demand function of the current inventory level when the supply chain is
operated in a multi-period planning horizon. We focus on studying the impacts of
cap-and-trade regulation and cost learning on the coordination of the supply chain
with perishable products.

3. Problem description and assumptions.

3.1. Problem description and notations. We consider a two-party supply chain
comprising one emission-dependent manufacturer and one retailer for perishable
products over a multi-period planning horizon. When the inventory level of the
retailer falls to zero due to market demand and product deterioration, the products
ordered from the upstream manufacturer will arrive, and the lead time is assumed to
be zero in this deterministic environment. Shortages for the retailer are not allowed,
and product demand depends on the selling price, promotional effort and current
inventory level. We assume that learning effects exist since multiple orderings will,
in general, reduce the fixed ordering cost. In this regard, we incorporate learning
effects on fixed ordering costs into this inventory control model. The objective of
the retailer is to determine the optimal replenishment time, replenishment number,
selling price and promotional effort to maximize his/her profit. Greenhouse gases
are emitted in the manufacturing process, and cap-and-trade regulation is imposed
on the emission-dependent manufacturer. The emission-dependent manufacturer
adopts a lot-for-lot policy to provide the products to the retailer. The objective of
the emission-dependent manufacturer is to determine the optimal wholesale price
to maximize his/her profit. In addition, we have provided all proofs of theoretical
results in the Appendices, and the parameters and notations used throughout the
paper are as follows:

Decision variables

- \( n \) total number of replenishment cycles
- \( t_i \) time point of the \( i \)-th replenishment cycle, \( i = 1, 2, \ldots, n \), with
  \( t_0 = 0 \) and \( t_n = H \)
- \( s \) retailer’s promotional effort (advertisement, servicing, etc.) to en-
hance demand
- \( p \) unit selling price
- \( w \) unit wholesale price
- \( F \) fixed costs charged by the emission-dependent manufacturer in the
  TT contract
**Objective functions**

- \( \Pi_r(n, s, p) \) retailer’s profit over a multi-period planning horizon in the decentralized model
- \( \Pi_{r/\text{TT}}(n, s, p) \) retailer’s profit over a multi-period planning horizon in the TT contract
- \( \Pi_m(w) \) emission-dependent manufacturer’s profit over a multi-period planning horizon in the decentralized model
- \( \Pi_{m/\text{TT}}(w) \) emission-dependent manufacturer’s profit over a multi-period planning horizon in the TT contract
- \( \Pi_c(n, s, p) \) supply chain’s profit over a multi-period planning horizon in the centralized model
- \( J^j_m \) total emissions amount over a multi-period planning horizon in the \( j \) model, where \( j \) is model index and \( j = d \) for decentralized model, \( j = c \) for centralized model, and \( j = \text{TT} \) for decentralized model with the TT contract

**Parameters**

- \( H \) length of the multi-period planning horizon under consideration
- \( I(t) \) inventory level of the retailer at time \( t \)
- \( I_i \) total amount of inventory carried in the \( i \)-th replenishment cycle, \( i = 1, 2, \ldots, n \)
- \( Q_i \) ordered quantity of the retailer in the \( i \)-th replenishment cycle, \( i = 1, 2, \ldots, n \)
- \( A_i \) fixed ordering cost of the retailer in the \( i \)-th replenishment cycle, \( A_1 \geq A_2 \geq \cdots \geq A_n \)
- \( \phi \) learning rate. In real-life situations, various industries were found to experience learning rates ranging from 0.7 to 1.0 ([18])
- \( b \) learning index, \( b = \frac{\log \phi}{\log 2} \) and \( 0.7 < \phi < 1 \) yield \(-0.515 < b < 0\)
- \( \theta \) deterioration rate of the product, \( 0 < \theta < 1 \)
- \( f(\cdot) \) market demand rate
- \( h_r \) unit holding cost at the retailer
- \( h_d \) unit deterioration cost at the retailer
- \( C \) carbon emissions cap
- \( \hat{c} \) amount of carbon emissions per unit produced
- \( c_p \) unit carbon emission permit trading price
- \( S_m \) fixed cost per production of the emission-dependent manufacturer
- \( c_m \) unit production cost of the emission-dependent manufacturer

3.2. **Assumptions.** Throughout, we make the following assumptions.

1. There is no replacement for deteriorated items.
2. A formulation of the learning effect proposed by Wright [39] is utilized to characterize the learning phenomenon for the fixed ordering cost of the retailer [18].
3. In practice, promotional effort and the current inventory level usually have positive impacts on market demand, while the selling price has a negative impact [2, 5]. A linear formulation of the market demand has been widely adopted in the literature, e.g., Lus and Muriel [30] and Luo et al. [29]. Similarly, we formulate the demand rate function as \( f(\cdot) = f(s, p, I(t)) = D_0 + \alpha s - \beta p + \gamma I(t) \) for tractability, where \( D_0(> 0) \) is the market scale parameter, and \( \alpha(> 0), \beta(> 0) \) and \( \gamma(> 0) \) reflect the elasticities of demand with respect to the retailer’s effort, selling price
and current inventory level. In addition, $D_0 + \alpha s - \beta p \geq 0$ is assumed to ensure the non-negativity of the demand function.

4. The cost of the promotional effort of the retailer in each replenishment cycle is formulated as $\frac{1}{2} \eta s^2$, where $\eta (>0)$ is the cost efficiency coefficient\textsuperscript{17}.

4. **Mathematical models.** From the above problem description, the behavior of the retailer’s inventory system is depicted in Figure 1. The $i$-th replenishment cycle with zero stock begins at time $t_{i-1}$ and ends at time $t_i$. At $t_{i-1}$, the retailer receives the $i$-th order from the emission-dependent manufacturer, and the inventory is at its highest level. Thereafter, the inventory level gradually decreases and falls to zero at time $t_i$ due to meeting demand and deterioration. Hence, the differential equation describing the inventory level at time $t$ is given by

$$
\frac{dI(t)}{dt} = -f(s,p,I(t)) - \theta I(t), \quad t_{i-1} \leq t \leq t_i.
$$

(1)

Using the boundary condition $I(t_i) = 0$, Eq.(1) is solved as follows:

$$
I(t) = \frac{(D_0 + \alpha s - \beta p)}{\theta + \gamma} [e^{(\theta + \gamma)(t_i - t)} - 1].
$$

(2)

Hence, the cumulative inventories during $[t_{i-1}, t_i]$ and the quantity ordered by the retailer in the $i$-th replenishment cycle will be

$$
I_i = \int_{t_{i-1}}^{t_i} I(t)dt = \frac{(D_0 + \alpha s - \beta p)}{\theta + \gamma} \left\{ \frac{1}{\theta + \gamma} [e^{(\theta + \gamma)(t_i - t_{i-1})} - 1] - (t_i - t_{i-1}) \right\}
$$

(3)

and

$$
Q_i = I(t_{i-1}) = \frac{(D_0 + \alpha s - \beta p)}{\theta + \gamma} [e^{(\theta + \gamma)(t_i - t_{i-1})} - 1],
$$

(4)

respectively.

Since the number of perishable products during $[t_{i-1}, t_i]$ is $\theta I_i$, the quantity of the product sold by the retailer in the $i$-th replenishment cycle is

$$
Q_i - \theta I_i = \frac{(D_0 + \alpha s - \beta p)}{\theta + \gamma} \left\{ \frac{\gamma}{\theta + \gamma} [e^{(\theta + \gamma)(t_i - t_{i-1})} - 1] + \theta(t_i - t_{i-1}) \right\}.
$$

(5)

The total amount of carbon emissions generated in the production process over a multi-period horizon is given by

$$
\hat{c} \sum_{i=1}^{n} Q_i = \frac{\hat{c}(D_0 + \alpha s - \beta p)}{\theta + \gamma} \sum_{i=1}^{n} [e^{(\theta + \gamma)(t_i - t_{i-1})} - 1].
$$

(6)
4.1. Centralized system. In the centralized model, the emission-dependent manufacturer and the retailer operate within the same firm and jointly determine the optimal replenishment time, replenishment number, promotional effort and selling price to maximize the system’s profit. Under the constraint of carbon emission regulation, the system’s profit will be

\[
\Pi_c(n, s, p) = p \sum_{i=1}^{n} (Q_i - \theta I_i - (h_r + \theta h_d) \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} (c_m Q_i + A_i) - \frac{1}{2} n^2 \eta s^2 - n S_m + c_p (C - \hat{c} \sum_{i=1}^{n} Q_i).
\]

The first term in Eq. (7) is the total sales revenue of the system, the second term is the total cost of inventory holding and deterioration, the third term is the total ordering cost on the retailer’s side including variable cost and fixed cost, the fourth term is the promotional cost, the fifth term is the total fixed cost incurred by the manufacturer, and the last term is the revenue or cost from trading carbon emission permits.

Let \( \Delta p = \gamma p - (c_m + c_p \hat{c})(\theta + \gamma) - (h_r + \theta h_d). \) From Eq. (7), we have the following lemma.

**Lemma 4.1.** For any given values of \( n, s, p \) and \( \Delta p \neq 0 \), the necessary conditions for \( \Pi_c(n, s, p) \) to be maximized are \( t_i-t_{i-1} = \frac{H}{n}, i = 1, 2, \ldots, n \).

Using Lemma 4.1 yields the following conclusion.

**Theorem 4.2.** For any given values of \( n, s, p \) and \( \Delta p < 0 \), the necessary and sufficient conditions for \( \Pi_c(n, s, p) \) to be maximized are \( t_i-t_{i-1} = \frac{H}{n}, i = 1, 2, \ldots, n \).

Theorem 4.2 shows that an \( n \)-dimensional problem of finding the optimal value of \( \{t_i\} \) can be reduced to a one-dimensional problem. When the values of \( n, s \) and \( p \) are given and \( \Delta p < 0 \), from \( t_0 = 0 \) or \( t_n = H \), we can easily find the optimal value of \( \{t_i\} \) by maximizing \( \Pi_c(n, s, p) \) by using a one-dimensional search technique.

Next, we have the following conclusion.

**Theorem 4.3.** For any given values of \( s \) and \( p \), the following holds:

(i) When \( \Delta p \geq 0 \), the optimal value of \( n \) will be 1 such that \( \Pi_c(n, s, p) \) is maximized.

(ii) When \( \Delta p < 0 \), \( \Pi_c(n, s, p) \) is concave in \( n \).

Theorem 4.3 guarantees the existence of a unique optimal replenishment number. We conclude that there exists a threshold \( \frac{(c_m + c_p \hat{c})(\theta + \gamma) + (h_r + \theta h_d)}{\gamma} \) such that if the selling price is greater than or equal to the threshold, the optimal replenishment number will be 1. Otherwise, the retailer should adopt multiple ordering. On the other hand, since \( n \) is an integer, \( \Pi_c(n, s, p) \geq \Pi_c(n-1, s, p) \) and \( \Pi_c(n, s, p) \geq \Pi_c(n+1, s, p) \) are used to find the optimal replenishment number when \( \Delta p < 0 \). In the following, we let \( n^*_c, s^*_c \) and \( p^*_c \) be the optimal replenishment number, promotional effort and selling price of the centralized system, respectively.
For later convenience, we also let
\[
\Phi_1(n) = \frac{n}{\theta + \gamma} \left[ e^{(\theta + \gamma)\frac{n}{H}} - 1 \right],
\]
\[
\Phi_2(n) = \frac{n}{\theta + \gamma} \left\{ \frac{1}{\theta + \gamma} \left[ e^{(\theta + \gamma)\frac{n}{H}} - 1 \right] - \frac{H}{n} \right\},
\]
\[
\Phi_3(n) = \frac{n}{\theta + \gamma} \left\{ \frac{\gamma}{\theta + \gamma} \left[ e^{(\theta + \gamma)\frac{n}{H}} - 1 \right] + \frac{\theta H}{n} \right\}.
\]
It is easy to prove that \( \Phi_1(n) > 0, \Phi_2(n) > 0, \Phi_3(n) > 0 \) and \( \Phi_3(n) = \Phi_1(n) - \theta \Phi_2(n) \).
Substituting Eqs. (8)-(10) into Eq. (7), we have the following conclusion.

**Theorem 4.4.** For any given value of \( n \), if \( 2\beta m - \alpha^2 \Phi_3(n) > 0 \), then \( \Pi_c(n, s, p) \) is jointly concave in \( s \) and \( p \). Moreover, when the optimal replenishment number is \( n_c^* \), the corresponding optimal selling price and promotional effort are

\[
p_c^* = \frac{D_0 n_c^* \eta}{2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)} + \frac{[\beta n_c^* \eta - \alpha^2 \Phi_3(n^*_c)] [(c_m + c_p \tilde{c}) \Phi_1(n_c^*) + (h_r + \theta d) \Phi_2(n_c^*)]}{2\beta n_c^* \eta - \alpha^2 \Phi_3(n_c^*)} \tag{11}
\]

and

\[
s_c^* = \frac{\alpha [D_0 \Phi_3(n_c^*) - \beta (c_m + c_p \tilde{c}) \Phi_1(n_c^*) - \beta (h_r + \theta d) \Phi_2(n_c^*)]}{2\beta n_c^* \eta - \alpha^2 \Phi_3(n_c^*)}, \tag{12}
\]
respectively.

Based on the above results, we design the following algorithm to find the optimal solution \((n_c^*, s_c^*, p_c^*)\) of this model.

**Algorithm A**

**Step 1.** Set \( n = 1 \). Compute the value of \( 2\beta \eta - \alpha^2 \Phi_3(1) \). If \( 2\beta \eta - \alpha^2 \Phi_3(1) > 0 \), go to step 2. Otherwise, stop.

**Step 2.** Using Eqs. (11) and (12), compute the values of \( p_c^{(1)} \) and \( s_c^{(1)} \). Let \( (\Delta p)^{(1)} = \gamma p_c^{(1)} - (c_m + c_p \tilde{c})(\theta + \gamma) - (h_r + \theta d) \). If \( (\Delta p)^{(1)} \geq 0 \), then set \( n_c^* = 1 \) and \( (s_c^*, p_c^*) = (s_c^{(1)}, p_c^{(1)}) \); stop. Otherwise, go to step 3.

**Step 3.** Set \( n = k \). Compute the value of \( 2\beta \eta - \alpha^2 \Phi_3(k) \). If \( 2\beta n^k \eta - \alpha^2 \Phi_3(n^k) \leq 0 \), stop. Otherwise, using Eqs. (11) and (12), compute the values of \( p_c^{(k)} \) and \( s_c^{(k)} \). From Eq. (7), if \( \Pi_c(k, s_c^{(k)}, p_c^{(k)}) \geq \Pi_c(k - 1, s_c^{(k)}, p_c^{(k)}) \) and \( \Pi_c(k, s_c^{(k)}, p_c^{(k)}) \geq \Pi_c(k + 1, s_c^{(k)}, p_c^{(k)}) \), then set \( (n_c^*, s_c^*, p_c^*) = (k, s_c^{(k)}, p_c^{(k)}) \) and stop. Otherwise, go to step 4.

**Step 4.** Set \( n = k + 1 \); go to step 3.

From Eqs. (11) and (12), we have the system’s profit and total emission amount in the centralized model as

\[
\Pi_c(n_c^*, s_c^*, p_c^*) = \frac{n_c^* [2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{2\alpha^2 \Phi_3(n^*_c)} (s_c^*)^2 - \sum_{i=1}^{n_c^*} A_i - n_c^* S_m + c_p \tilde{c}\tag{13}
\]

and

\[
J_c^c = \frac{\hat{c} \beta n^*_c \Phi_1(n^*_c)}{\alpha \Phi_3(n^*_c)} s_c^* \tag{14}
\]
4.2. Decentralized model. In a decentralized model, these two members of the supply chain make decisions separately to maximize their own profits. We consider a Stackelberg game between the emission-dependent manufacturer and the retailer in which the retailer is the Stackelberg follower. Following the sequence of the game, the emission-dependent manufacturer first sets the wholesale price, and the retailer then determines the replenishment time, replenishment number, promotional effort and selling price to maximize his/her profit. The objective of the emission-dependent manufacturer is to determine the optimal wholesale price by maximizing his/her profit after considering all decisions made by the retailer. In this scenario, the retailer’s profit is expressed as

\[ \Pi_r(n, s, p) = \sum_{i=1}^{n} (Q_i - \theta I_i) - (h_r + \theta h_d) \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} (wQ_i + A_i) - \frac{n}{2} \eta s^2. \]  (15)

The first term in Eq.(15) is the total sales revenue of the retailer, the second term is the total cost of inventory holding and deterioration for the retailer, the third term is the total ordering cost on the retailer’s side including variable cost and fixed cost, and the last term is the total promotional cost of the retailer.

Since the emission-dependent manufacturer produces products generating carbon emissions and adopts a lot-for-lot policy to provide the products to the retailer, the emission-dependent manufacturer’s profit in the decentralized model is expressed by maximizing his profit after considering all decisions made by the retailer. In a decentralized model, these two members of the supply chain make decisions separately to maximize their own profits. We consider a Stackelberg game between the emission-dependent manufacturer and the retailer in which the retailer is the Stackelberg follower. Following the sequence of the game, the emission-dependent manufacturer first sets the wholesale price, and the retailer then determines the replenishment time, replenishment number, promotional effort and selling price to maximize his/her profit. The objective of the emission-dependent manufacturer is to determine the optimal wholesale price by maximizing his/her profit after considering all decisions made by the retailer. In this scenario, the retailer’s profit is expressed as

\[ \Pi_r(n, s, p) = p \sum_{i=1}^{n} (Q_i - \theta I_i) - (h_r + \theta h_d) \sum_{i=1}^{n} I_i - \sum_{i=1}^{n} (wQ_i + A_i) - \frac{n}{2} \eta s^2. \]  (15)

The first term in Eq.(15) is the total sales revenue of the retailer, the second term is the total cost of inventory holding and deterioration for the retailer, the third term is the total ordering cost on the retailer’s side including variable cost and fixed cost, and the last term is the total promotional cost of the retailer.

Since the emission-dependent manufacturer produces products generating carbon emissions and adopts a lot-for-lot policy to provide the products to the retailer, the emission-dependent manufacturer’s profit in the decentralized model is expressed as

\[ \Pi_m(w) = w \sum_{i=1}^{n} Q_i - \sum_{i=1}^{n} c_m Q_i + nS_m] + c_p(C - \bar{c} \sum_{i=1}^{n} Q_i). \]  (16)

The first term in Eq.(16) is the total sales revenue of the manufacturer, the second term is the total production cost including variable cost and fixed cost incurred by the manufacturer, and the last term is the revenue or cost from trading carbon emission permits. To find the optimal solution of the decentralized model, we can prove that Theorems 4.2 and 4.3 hold for Eq.(15) with \( \Delta p = \gamma p - w(\theta + \gamma) - (h_r + \theta h_d) \). This implies that for any given values of \( s, p \) and \( w \), there exist an optimal replenishment time and replenishment number such that \( \Pi_r(n, s, p) \) is maximized. Let \( n^*_d, s^*_d, p^*_d \) and \( w^*_d \) be the optimal replenishment number, promotional effort, selling price and wholesale price of the decentralized model, respectively. From Eqs.(15) and (16), we have the following conclusion.

**Theorem 4.5.** In the decentralized model, for any given value of \( n \), if \( 2\beta \gamma \eta - \alpha^2 \Phi_3(n) > 0 \), then the following holds.

1. The profit of the retailer, \( \Pi_r(n, s, p) \), is jointly concave in \( s \) and \( p \). Moreover, when the optimal replenishment number is \( n^*_d \), the corresponding promotional effort, \( s^*_d \), and selling price, \( p^*_d \), are

\[ p^*_d = \frac{D_0 \Phi_3(n^*_d)[3\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}{2\beta[2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)] \Phi_3(n^*_d)} + \frac{[\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)][(c_m + c_p \bar{c}) \Phi_1(n^*_d) + (h_r + \theta h_d) \Phi_2(n^*_d)]}{2[2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)] \Phi_3(n^*_d)}, \]  (17)

and

\[ s^*_d = \frac{\alpha[D_0 \Phi_3(n^*_d) - \beta(c_m + c_p \bar{c}) \Phi_1(n^*_d) - \beta(h_r + \theta h_d) \Phi_2(n^*_d)]}{2[2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}, \]  (18)

respectively.
(2) The profit of the emission-dependent manufacturer, $\Pi_m(w)$, is concave in $w$ and the optimal wholesale price, $w^*_d$, is
\[ w^*_d = c_m + c_p \hat{c} + \frac{[2\beta n_d \eta - \alpha^2 \Phi_3(n^*_d)]}{\alpha \beta \Phi_1(n^*_d)} s^*_d. \]

Similar to Algorithm A, we design the following algorithm to solve the decentralized model.

**Algorithm B**

**Step 1.** Set $n = 1$. Compute the value of $2\beta \eta - \alpha^2 \Phi_3(1)$. If $2\beta \eta - \alpha^2 \Phi_3(1) > 0$, go to step 2. Otherwise, stop.

**Step 2.** Using Eqs. (17), (18) and (19), compute the values of $p^{(1)}_d$, $s^{(1)}_d$ and $w^{(1)}_d$. Let $(\Delta p)^{(1)} = \gamma p^{(1)}_d - w^{(1)}_d (\theta + \gamma) - (h_r + \theta h_d)$. If $(\Delta p)^{(1)} \geq 0$, then set $n^*_d = 1$ and $(s^*_d, p^*_d, w^*_d) = (s^{(1)}_d, p^{(1)}_d, w^{(1)}_d)$; stop. Otherwise, go to step 3.

**Step 3.** Set $n = k$. Compute the value of $2\beta \eta - \alpha^2 \Phi_3(k)$. If $2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d) \leq 0$, stop. Otherwise, using Eqs. (17), (18) and (19), compute the values of $p^{(k)}_d$, $s^{(k)}_d$ and $w^{(k)}_d$. From Eq. (15), if $\Pi_r(k, s^{(k)}_d, p^{(k)}_d) \geq \Pi_r(k-1, s^{(k)}_d, p^{(k)}_d)$ and $\Pi_r(k, s^{(k)}_d, p^{(k)}_d) \geq \Pi_r(k+1, s^{(k)}_d, p^{(k)}_d)$, then set $(n^*_d, s^*_d, p^*_d, w^*_d) = (k, s^{(k)}_d, p^{(k)}_d, w^{(k)}_d)$ and stop. Otherwise, go to step 4.

**Step 4.** Set $n = k + 1$; go to step 3.

Using Eqs. (17)-(19), we simplify the profits of the retailer and the emission-dependent manufacturer and the total emissions amount in the decentralized model as
\[
\Pi_r(n^*_d, s^*_d, p^*_d) = \frac{n^*_d \eta [2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}{2\alpha^2 \Phi_3(n^*_d)} (s^*_d)^2 - n^*_d S_m + c_p C,
\]
\[
\Pi_m(w^*_d) = \frac{n^*_d \eta [2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}{\alpha \Phi_3(n^*_d)} (s^*_d)^2 - n^*_d \hat{C},
\]

and
\[
J_m^d = \frac{c_p \hat{c} \eta \Phi_1(n^*_d)}{\alpha \Phi_3(n^*_d)} s^*_d.
\]

Eqs. (13), (14), (20), (21) and (22) yield the following conclusion.

**Theorem 4.6.** For the centralized and decentralized models, the following holds:

(i) There exists a threshold $\Delta$ such that if $s^*_d < \Delta s^*_d$, then $J_m^d < J_m^d$; otherwise, $J_m^d \geq J_m^d$, where $\Delta = \frac{\Phi_1(n^*_d) \Phi_3(n^*_d)}{\Phi_3(n^*_d)}$.

(ii) The difference between the profits from the centralized model and the decentralized model is at least $\frac{n^*_d \eta [2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}{2\alpha^2 \Phi_3(n^*_d)} (s^*_d)^2$, i.e., $\Pi_c(n^*_c, s^*_c, p^*_c) - \Pi_r(n^*_d, s^*_d, p^*_d) + \Pi_m(w^*_d) \geq \frac{n^*_d \eta [2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]}{2\alpha^2 \Phi_3(n^*_d)} (s^*_d)^2$.

Theorem 4.6 compares the total emissions amount and profit in the centralized model with those in the decentralized model. Theorem 4.6(i) shows that when the ratio between the promotional effort level provided by the retailer in the centralized system and that in the decentralized system is less than the threshold, the
total emissions amount generated in the centralized system is less than that generated in the decentralized system. This implies that cooperation between the manufacturer and the retailer may lead to a decrease in carbon emissions. In this scenario, the emission-dependent manufacturer is more willing to cooperate with the retailer. Theorem 4.6(ii) shows that the system’s profit increases by at least
\[
\frac{n^*_d(n^*_d - n^*_c)}{2a^2\Phi_3(n^*_d)}(s^*_d)^2
\]
when the emission-dependent manufacturer cooperates with the retailer. In particular, when \(c_pC \leq n^*_dS_m + \sum_{i=1}^{n^*_d} A_i\), it is easy to verify that the system’s profit in the centralized model increases by at least \(\frac{1}{3}\). As the leader, the emission-dependent manufacturer typically expects that the retailer will agree to cooperate. However, the retailer may refuse to do so due to the possibility that he/she will realize lower profits. For example, from Theorems 4.4 and 4.5, we have that if \(n^*_d = n^*_c\), then \(s^*_d = 2s^*_d\) and \(p^*_d = 2p^*_d - \frac{D_0}{\beta}\). From Eq.(4), we have that the order quantity of the decentralized model is less than that of the centralized model. In addition, if \(p^*_d < \frac{D_0}{\beta}\), then \(p^*_d < p^*_d\) holds. In this scenario, the following arises: (i) cooperation leads to an increased order quantity, which incurs a higher holding cost and ordering cost; (ii) cooperation leads to an increased promotional effort, which incurs higher promotion costs and lower revenue; and (iii) a lower selling price and increased promotional efforts may result in higher customer demand, but the retailer’s profit may be increased. Hence, to cooperate with the retailer, the emission-dependent manufacturer needs to negotiate with the retailer and offer certain incentives. These incentives are designed as effective contract mechanisms. A two-part tariff contract has been widely used to coordinate different types of supply chains and it has the robustness compared with other contracts [1]. In the next section, we will propose the TT contract to coordinate the supply chain system considered in this paper. In addition, let \(n^*_tt, s^*_tt, w^*_tt, p^*_tt\) be the optimal replenishment number, promotional effort, wholesale price and product selling price in the TT contract, respectively.

5. Coordination by two-part tariff contract. In a TT contract, the emission-dependent manufacturer will charge a lower wholesale price, \(w\), and a fixed cost, \(F\), such that the retailer can implement ordering and selling decisions consistent with those in the centralized model. The profits of the retailer and the emission-dependent manufacturer are

\[
\Pi_{r/\text{tt}}(n, s, p) = (D_0 + \alpha s - \beta p)[p\Phi_3(n) - w\Phi_1(n) - (h_r + \theta h_d)\Phi_2(n)] - \sum_{i=1}^{n} A_i - \frac{n}{2}n\eta s^2 - F \tag{23}
\]

and

\[
\Pi_{m/\text{tt}}(w) = (D_0 + \alpha s - \beta p)(w - c_m - c_p\hat{c})\Phi_1(n) + c_pC + F \tag{24}
\]

respectively.

From Eq.(23) and using the coordination conditions of the TT contract, we have the following result.

**Theorem 5.1.** When \(n^*_tt = n^*_d\) and \(p^*_tt = p^*_c\), the TT contract can coordinate the supply chain. Moreover, the corresponding wholesale price and promotional effort are
and Figure 1 and Figure 2, the amount in the decentralized model are 20,000 more emissions because the retailer increases its promotional effort level in the fixed cost $F = 12$. Numerical example.

6.1. Numerical analysis. We will perform a numerical analysis to illustrate the above theoretical results and to gain some managerial insights from the analysis.

Comparing Eq.(12) with Eq.(26) yields $s^*_c = s^*_m$. Using the coordination conditions, we conclude that the optimal decisions of the retailer under the TT contract are the same as those in the centralized model. From Theorem 5.1, we further observe that the corresponding wholesale price is equal to the sum of the unit production cost and the unit cost of trading carbon permits. In this scenario, the TT contract coordinates the supply chain. We further analyze the conditions under which these two members of the supply chain accept the TT contract and obtain the following result.

**Theorem 5.2.** Under the TT contract, the following holds.

(i) The system’s total emissions amount is equal to that in the centralized model, i.e., $J^*_m = J^*_m$.

(ii) The system’s profit is equal to that in the centralized model, i.e., $\Pi(c_m, s^*_c, p^*_c) = \Pi(c_m, s^*_m, p^*_m)$.

(iii) When these two members of the system accept the TT contract, the value of the fixed cost $F$ satisfies $\frac{n^*_c n^*_m n^*_c n^*_m - \alpha^2 \Phi_3(n^*_c)}{4\sigma^2 \Phi_1(n^*_c)} (s^*_c)^2 \leq F \leq \frac{3n^*_c n^*_m n^*_c n^*_m - \alpha^2 \Phi_3(n^*_c)}{8\sigma^2 \Phi_1(n^*_c)} (s^*_c)^2$.

From Theorem 5.2, we observe that the total emissions amount and the profit of the decentralized supply chain under the TT contract are equal to those in the centralized model. This implies that the system’s total emissions amount may decrease and its profit may increase when the decentralized system is coordinated with the TT contract. Moreover, there always exists a fixed cost $F$ for both the emission-dependent manufacturer and the retailer to accept the contract. Based on the conditions for the existence of $F$ in Theorem 5.2, we conclude that the TT contract may improve each member of the decentralized supply chain’s profits.

6. Numerical analysis. We will perform a numerical analysis to illustrate the above theoretical results and to gain some managerial insights from the analysis.

6.1. Numerical example. The parameter values for this example are as follows: $H = 12, A_1 = 400, D_0 = 900, \alpha = 1, \beta = 3, \gamma = 0.8, h_r = 6, h_d = 1.5, \theta = 0.06, \eta = 25, \phi = 0.75, C = 3000, S_m = 600, c_m = 20, c_p = 3$ and $\bar{c} = 10$. The computational results are summarized in Table 1 and Figure 2. From Table 1 and Figure 2, we have the following observations:

(1) The system’s profit and total emissions amount in the centralized model are 25,589.00 and 285,270, respectively, while the system’s profit and total emissions amount in the decentralized model are 20,667.10 and 142,640, respectively. In the decentralized and centralized models, the emission-dependent manufacturer has to buy carbon emission permits because the total emissions amount in each model is higher than the carbon cap of 3000. Moreover, when the emission-dependent manufacturer cooperates with the retailer, the system’s profit and total emissions amount are increased by 23.82% and 99.99%, respectively. The system generates more emissions because the retailer increases its promotional effort level in the centralized system such that the ratio of the promotional effort level between the
Table 1. The optimal solution of the example

| Model       | w  | s   | p   | n   | Retailer’s profit | Manufacturer’s profit | Total profit | Total emissions amount |
|-------------|----|-----|-----|-----|-------------------|-----------------------|--------------|-----------------------|
| Centralized | -  | 12.15 | 213.75 | 3   | -                 | -                      | 25589.00     | 285270                |
| Decentralized | 56.76 | 6.08 | 256.88 | 3   | 3622.10           | 17045.00              | 20667.10    | 142640                |
| TT contract |     |     |     |     |                   |                       |              |                       |
| F = 9850    | 50 | 12.15 | 213.75 | 3   | 8539.00           | 17050.00              | 25589.00     | 285270                |
| F = 12000   | 50 | 12.15 | 213.75 | 3   | 6389.00           | 19200.00              | 25589.00     | 285270                |
| F = 14750   | 50 | 12.15 | 213.75 | 3   | 3639.00           | 21950.00              | 25589.00     | 285270                |

Figure 2. Intervals of contract parameters to attain the win-win outcome

centralized model and the decentralized model of 1.9984 is higher than the threshold value of 1.0. Hence, cooperation between the emission-dependent manufacturer and the retailer in this example leads to increases in the system’s profit and total emissions amount.

(2) Under the TT contract, the wholesale price remains unchanged when the fixed cost charged by the emission-dependent manufacturer increases and it is less than that in the decentralized model. This means that the emission-dependent manufacturer offers incentives to the retailer for cooperation. On the other hand, the optimal decision variables of the retailer are equal to those in the centralized model. This means that the system’s profit and total emissions amount in the decentralized supply chain under the TT contract are always equal to those in the centralized model when F varies. This implies that the decentralized supply chain under cap-and-trade regulation is coordinated by the TT contract.

(3) Figure 2 shows the conditions under which the emission-dependent manufacturer and the retailer accept the TT contract. From Figure 2, we have the following observations: Under the TT contract, the emission-dependent manufacturer’s profit is higher than that in the decentralized model when the fraction F exceeds 9845.00. However, the retailer’s profit under the TT contract is less than that in the decentralized model when F is larger than 14,767.00. In other words, the TT contract is acceptable for the emission-dependent manufacturer and the retailer when F is in the range of [9845.00, 14,767.00].
6.2. Sensitivity analysis. Using above numerical example, we investigate the impacts of cap-and-trade regulation and cost learning on decisions in the centralized and decentralized systems. First, we let $S.I.P$ be the ratio of $\Pi_c(n^*_c, s^*_c, p^*_c) - [\Pi_r(n^*_d, s^*_d, p^*_d) + \Pi_m(w^*_d)]$ over $\Pi_r(n^*_d, s^*_d, p^*_d) + \Pi_m(w^*_d)$. The formulation of $S.I.P$ shows the penalty for noncooperation between the emission-dependent manufacturer and the retailer. We vary the values of the carbon cap, $C$, unit emission permit trading price, $c_p$, and learning rate, $\phi$. The corresponding results are summarized in Figures 3-5. From Figures 3-5, we have the following observations:

Figure 3. Impacts of $C$ on profit and total emissions amount

(1) As carbon emissions cap, $C$, increases, the total emissions amount generated in the centralized or decentralized system and the retailer’s profit in the decentralized system remain unchanged, the centralized system’s profit and the manufacturer’s profit in the decentralized system increase, while $S.I.P$ decreases. This observation means that (i) under cap-and-trade regulation, the total emissions amount generated in the centralized system or decentralized system is not affected by varying the value of the carbon emission cap; (ii) increasing the value of the carbon emission cap leads to increases in the centralized system’s profit and the manufacturer’s profit in the decentralized system because this regulation provides several economic incentives for the emitter; and (iii) increasing the value of the carbon emission cap makes the decentralized system’s profit close to the centralized system’s profit.
(2) As the unit carbon emission permit trading price, $c_p$, increases, the total emissions amount generated in the centralized system or decentralized system, the retailer’s profit in the decentralized system, and $S.I.P$ decrease, while the centralized system’s profit and the manufacturer’s profit in the decentralized system increase. These observations mean that (i) under cap-and-trade regulation, increasing the value of $c_p$ encourages the emitter to generate less carbon emissions so that the cost of buying the carbon emissions permits is reduced, and the centralized system’s profit and the manufacturer’s profit in the decentralized system increase. (ii) Increasing the value of $c_p$ leads to an increase in the decentralized system’s profit and makes the decentralized system’s profit close to the centralized system’s profit.

(3) As the learning rate, $\phi$, increases, the total emissions generated in the centralized system or decentralized system, and the manufacturer’s profit in the decentralized system remain unchanged, the centralized system’s profit and the retailer’s profit in the decentralized system increase, while $S.I.P$ decreases. This observation means that increasing the value of the learning rate plays a positive role in increasing the retailer’s profit in the decentralized system and the centralized system’s profit, while it does not affect the manufacturer’s profit in the decentralized system or the total emissions amount generated in the centralized system. The main reason for this result is the learning effect on the retailer’s ordering cost. In
addition, decreasing the value of $\phi$ may lead to significant profit differences between the centralized and decentralized systems.

In the following, we perform a robust sensitivity analysis for the above coordination model using experimental design techniques as discussed in [34, 35] and [27]. Following the Taguchi method, we take the TT contract with $F = 12,000$ as an example to create the experimental design for the coordination model. First, we set five control factors including unit holding cost for the retailer, product deterioration rate, learning rate on the fixed ordering cost, carbon emission cap and unit carbon emission permit trading price. Second, we use levels 1 and 2 to represent $+30\%$ and $-30\%$ of the initial values for these control factors and design the corresponding $L_8(2^5)$ orthogonal table. Table 2 summarizes the results of the total profit, total emissions amount and their SN(signal-to-noise) ratios for the coordination model under the TT contract. The SN ratio is an indicator of the robustness of a system. A larger SN ratio means a more robust system. The computational formulation of the SN ratio can be found in [34]. Table 2 yields the following observations:

1. Of the eight experiments, the maximal profit and corresponding SN ratio appear in the first experiment, where the level of each parameter is 1, which means that when the values of all five parameters increase by 30%, the total profit and its robustness are maximized.
Table 2. Calculation results of $L_8$ orthogonal array experiment for the coordination model

| No. | A | B | C | D | E | Total profit | SN ratio of total profit | Total emissions amount | SN ratio of total emissions amount |
|-----|---|---|---|---|---|-------------|------------------------|----------------------|----------------------------------|
| 1   | 1 | 1 | 1 | 1 | 1 | 43394       | 92.75                  | 284600               | -109.085                       |
| 2   | 1 | 1 | 1 | 2 | 2 | 38191       | 91.64                  | 309080               | -109.801                       |
| 3   | 1 | 2 | 2 | 1 | 1 | 15727       | 83.93                  | 259150               | -108.271                       |
| 4   | 1 | 2 | 2 | 2 | 2 | 6371        | 76.08                  | 280390               | -108.955                       |
| 5   | 2 | 1 | 2 | 1 | 2 | 39551       | 91.94                  | 314060               | -109.940                       |
| 6   | 2 | 1 | 2 | 2 | 1 | 32531       | 90.25                  | 289570               | -109.235                       |
| 7   | 2 | 2 | 1 | 1 | 2 | 15409       | 83.76                  | 284830               | -109.092                       |
| 8   | 2 | 2 | 1 | 2 | 1 | 13943       | 82.89                  | 263590               | -108.419                       |

(2) Comparing the first experiment with the second experiment, we see that when the carbon emission parameters $C$ and $c_p$ are reduced with the other parameters being left unchanged, the total profit and its SN ratio decline by 11.99% and 1.20%, respectively, which indicates that the carbon emission parameters have the greatest impact on the total profit.

(3) Of the eight experiments, the lowest carbon emissions and a maximal SN ratio of the total emissions amount appear in the third experiment, which means that when $\theta, \phi$ decrease by 30% and the other three parameters increase by 30%, the total emissions amount is minimized, and its robustness is maximized.

(4) Comparing the third experiment with the fourth experiment, we see that when the carbon emission parameters $C$ and $c_p$ are reduced with other parameters being left unchanged, the total emissions amount increases by 8.20%, and its SN ratio decreases by 0.63%, which indicates that the carbon emission parameters have the greatest impact on the total emissions amount.

(5) When $h_r$ increases and $\theta, \phi, C, c_p$ decrease, the total profit is minimized, being 85.32% less than the maximal profit and its SN ratio is also minimized, being 17.97% less than the maximal SN ratio. When $h_r, \phi, c_p$ decrease and $\theta, C$ increase, the total emissions amount is maximized, being 21.19% greater than the minimal emission amount and its SN ratio is minimized, being 1.54% less than the maximal SN ratio. This demonstrates that changing multiple parameters can lead to a larger float in the total profit and a smaller float in the total emissions amount.

To compare the impacts on total profit and carbon emissions among the five parameters, we provide the main effects plot for the SN ratios and an analysis of variance for the orthogonal array in Figure 6 and Tables 3-4. From Figure 6, we can conclude the following.

(1) In Figure 6(a), the difference in the SN ratio of total profit between two levels of parameter $\theta$ is the largest. This suggests that the order of impact on the SN ratio of total profit among the five parameters is $\theta > C > \phi > c_p > h_r$. Furthermore, the difference in the SN ratio of total profit between the two levels of parameter $\theta$ is 9.98, which is 2.47 times more than that of $C$. This finding indicates that there is a substantial difference between the impacts of $\theta$ and $C$ on the robustness of total profit.

(2) In Figure 6(b), the difference in the SN ratio of total emissions amount between the two levels of parameter $\theta$ is the largest. The order of impact of the SN ratio of the total emissions amount among five parameters is $\theta > c_p > h_r > C > \phi$. Furthermore, the difference in the SN ratio of the total emissions amount between
two levels of parameter \( \theta \) is 0.8, while that of \( c_p \) is 0.7. This finding indicates that there is little difference between the impacts of \( \theta \) and \( c_p \) on the robustness of carbon emissions.

(3) The best combination of the five parameters in terms of obtaining greater robustness of total profits is \( h_r \) at level 2, \( \theta \) at level 1, \( \phi \) at level 1, \( C \) at level 1 and \( c_p \) at level 1. The best combination of the five parameters in terms of obtaining greater robustness of carbon emissions is \( h_r \) at level 1, \( \theta \) at level 2, \( \phi \) at level 1, \( C \) at level 1 and \( c_p \) at level 1. These findings indicate that decreasing the value of \( h_r \) and increasing the values of \( \theta, \phi, C, c_p \) can lead to a more robust total profit, while decreasing the value of \( \theta \) and increasing \( h_r, \phi, C, c_p \) can lead to more robust carbon emissions.

Tables 3-4 verify the results obtained from Table 2 and Figure 6 and provide the following observations.
Table 4. Analysis of variance for the carbon emissions and its SN ratio

(a) Analysis of variance for the carbon emissions

| Source | df | SS(×10^8) | MS(×10^8) | F    | P    |
|-------|----|------------|------------|------|------|
| A     | 1  | 0.4432     | 0.4432     | 16.84| 0.055|
| B     | 1  | 14.9468    | 14.9468    | 567.78| 0.002|
| C     | 1  | 0.0014     | 0.0014     | 0.05 | 0.837|
| D     | 1  | 0.0000     | 0.0000     | 0.00 | 0.998|
| E     | 1  | 10.4539    | 10.4539    | 397.11| 0.003|
| Error | 2  | 0.0527     | 0.0263     | -    | -    |
| Total | 7  | 25.8979    | -          | -    | -    |

(b) Analysis of variance for the SN ratio of the carbon emissions

| Source | df | SS      | MS      | F    | P    |
|-------|----|---------|---------|------|------|
| A     | 1  | 0.0411  | 0.04107 | 157.77| 0.006|
| B     | 1  | 1.3819  | 1.3819  | 5307.76| 0.000|
| C     | 1  | 0.0000  | 0.0000  | 0.01 | 0.920|
| D     | 1  | 0.0001  | 0.0001  | 0.25 | 0.669|
| E     | 1  | 0.9655  | 0.9655  | 3708.69| 0.000|
| Error | 2  | 0.0005  | 0.0003  | -    | -    |
| Total | 7  | 2.3891  | -       | -    | -    |

1. Table 3(a) shows that if the significance level is less than 10%, the impacts of the parameters \(\theta\) and \(C\) on total profit are statistically significant, while the impacts of \(h_r\), \(c_p\) and \(\phi\) are not. Table 3(b) shows that if the significance level is less than 10%, the impact of \(\theta\) on the SN ratio of total profit is statistically significant, while the impacts of other four parameters are not. This finding indicates that a manager should focus on the values of the parameters \(\theta\) and \(C\) when the goal of the system is to increase total profit and only focus on the value of the parameter \(\theta\) when the goal of the system is the robustness of total profit.

2. Table 4(a) shows that if the significance level is less than 10%, the impacts of parameters \(\theta\) and \(c_p\) on the total emissions amount are statistically significant, while the impacts of \(h_r\), \(C\) and \(\phi\) are not. Table 4(b) shows that if the significance level is less than 10%, the impacts of the parameters \(\theta, c_p, h_r\) on the SN ratio of carbon emissions are statistically significant, while the impacts of other two parameters are not. This findings indicates that a manager should focus on the values of the parameters \(\theta\) and \(c_p\) when the goal of the system is to decrease carbon emissions and focus on the values of the parameters \(\theta, c_p, h_r\) when the goal of the system is the robustness of carbon emissions.

7. Conclusions. Carbon cap-and-trade regulation has been implemented to control carbon emissions generated in manufacturing, storage, transportation and other supply chain activities. In addition, product deterioration and other practical issues create new challenges for operational decision making and supply chain management. This motivates us to study a manufacturer-retailer supply chain with perishable products over a multi-period planning horizon under the constraint of carbon emission regulation. In the system we consider, cap-and-trade regulation is imposed on an emission-dependent manufacturer and learning effects on the fixed ordering cost of the retailer are also considered. We coordinate the decentralized supply chain through a two-part tariff contract. By formulating and analyzing a
centralized model and a decentralized model under the TT contract, we obtain the following conclusions: (i) A lower bound on the difference in profits between the centralized and decentralized models can be derived. (ii) The TT contract may lead to perfect coordination and the necessary condition for the two members of the supply chain to accept the TT contract can be derived. (iii) Increasing the carbon emission cap or the unit carbon emission permit trading price or the learning rate leads to a decrease in the penalty for noncooperation between the emission-dependent manufacturer and the retailer. (iv) Increasing the carbon emission cap or the learning rate does not affect the total emission amount generated from the centralized or decentralized system, while increasing the unit carbon emission permit trading price leads to decreases in the total emission amount generated from the two supply chain systems. (v) Numerical analysis using the Taguchi method is provided to show that the impact of the deterioration rate is statistically significant for the robustness of both total profit and carbon emissions in the coordinated system. The results derived in this paper will be useful for supply chain management in determining coordination strategies under practical challenges and government environmental regulations.

In addition, the proposed model can be extended in several ways. First, in practice, many new technologies have been adopted to curb carbon emissions while customer demand may be affected by public preferences over the extent to which such sustainable technologies are applied. New technology and demand correlation can be incorporated into the supply chain and its impacts on system coordination should be studied. Second, we plan to extend our model to a multi-manufacturer, one-retailer supply chain under different carbon emission regulations. Finally, it will be a more challenging task to investigate the impacts of various carbon emission regulations on supply chain coordination models with stochastic demand.

Acknowledgments. The authors would like to thank the two anonymous referees for their constructive comments and suggestions which helped improve the presentation of this paper.

Appendices

Appendix A. Proof of Lemma 4.1. Substituting Eqs.(3)-(6) into Eq.(7), we have

$$\Pi_c(n, s, p) = \frac{(D_0 + \alpha s - \beta p)}{\theta + \gamma^2} \sum_{i=1}^{n} \{\Delta p e^{(\theta+\gamma)(t_i-t_{i-1})} - 1\} + (\theta + \gamma^(p\theta + h_r$$

$$+\theta h_d)(t_i - t_{i-1})\} - \frac{n}{2} \eta s^2 - \sum_{i=1}^{n} A_i - nS_m + c_p C. \quad (A.1)$$

For any given values of n, s and p, the necessary conditions for \(\Pi_c(n, s, p)\) to be maximized are \(\frac{\partial \Pi_c(n, s, p)}{\partial t_i} = 0, i = 1, 2, \ldots, n\). Then, using Eq.(A.1), we simplify \(\frac{\partial \Pi_c(n, s, p)}{\partial t_i} = 0\) as

$$\frac{\partial \Pi_c(n, s, p)}{\partial t_i} = \frac{(D_0 + \alpha s - \beta p)\Delta p [e^{(\theta+\gamma)(t_i-t_{i-1})} - e^{(\theta+\gamma)(t_{i+1}-t_i)}]}{\theta + \gamma} = 0. \quad (A.2)$$

Using the monotonicity of the function \(e^{\theta x}\), and solving Eq.(A.2), we have \(t_i - t_{i-1} = t_{i+1} - t_i, i = 1, 2, \ldots, n - 1\). This means that the time intervals between the
two successive inventory replenishments are equal. With \( t_0 = 0 \) and \( t_n = H \), we have \( t_i - t_{i-1} = \frac{H}{n}, i = 1, 2, \ldots, n \).

**Appendix B. Proof of Theorem 4.2.** For any given values of \( n, s \) and \( p \), when \( \Delta p < 0 \), from Lemma 4.1, we have that the necessary conditions for \( \Pi_c(n, s, p) \) to be maximized are \( t_i - t_{i-1} = \frac{H}{n}, i = 1, 2, \ldots, n \).

On the other hand, from Eq.(7) and taking the second partial derivatives of \( \Pi_c(n, s, p) \) with respect to \( t_i, i = 1, 2, \ldots, n \), we have

\[
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_i^2} = (D_0 + \alpha s - \beta p)\Delta p[e^{(\theta+\gamma)(t_i-t_{i-1})} e^{(\theta+\gamma)(t_{i+1}-t_i)}],
\]

(B.1)

\[
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_i \partial t_{i-1}} = -(D_0 + \alpha s - \beta p)\Delta p e^{(\theta+\gamma)(t_i-t_{i-1})}
\]

(B.2)

and

\[
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_i \partial t_{i+1}} = -(D_0 + \alpha s - \beta p)\Delta p e^{(\theta+\gamma)(t_{i+1}-t_i)}.
\]

(B.3)

Moreover, when \( j < i - 1 \) or \( j > i + 1 \), we easily have \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_i \partial t_j} = 0 \).

Let \( u(t_i) = (D_0 + \alpha s - \beta p)e^{(\theta+\gamma)(t_i-t_{i-1})} \). Using \( t_i - t_{i-1} = \frac{H}{n}, i = 1, 2, \ldots, n \), we have \( u(t_i) = u(t_{i+1}), i = 1, 2, \ldots, n \). For convenience, we further let \( u(t_i) = u \) and have \( u > 0 \). To ensure that \( t_i - t_{i-1} = \frac{H}{n} \) yields a maximum of \( \Pi_c(n, s, p) \), we will prove that the corresponding Hessian matrix is negative definite. Let \( M_k \) be the leading principal minor of order \( k \). When \( \Delta p < 0 \), using Eqs.(B.1)-(B.3), we have

\[
M_1 = \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1^2} = 2u\Delta p < 0,
\]

(B.4)

\[
M_2 = \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1^2} \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_2^2} - \left( \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1 \partial t_2} \right)^2 = 3u^2(\Delta p)^2 > 0
\]

(B.5)

and for \( k > 2 \),

\[
M_k = \begin{bmatrix}
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1^2} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1 \partial t_2} & \cdots & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1 \partial t_{k-1}} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_1 \partial t_k} \\
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_2 \partial t_1} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_2^2} & \cdots & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_2 \partial t_{k-1}} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_2 \partial t_k} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_{k-1} \partial t_1} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_{k-1} \partial t_2} & \cdots & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_{k-1} \partial t_{k-1}} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_{k-1} \partial t_k} \\
\frac{\partial^2 \Pi_c(n, s, p)}{\partial t_k \partial t_1} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_k \partial t_2} & \cdots & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_k \partial t_{k-1}} & \frac{\partial^2 \Pi_c(n, s, p)}{\partial t_k \partial t_k}
\end{bmatrix}_{k \times k}
\]

\[
= (k + 1)u^k(\Delta p)^k
\]

(B.6)

It is easy to find that if \( k \) is an odd number, then \( M_k > 0 \), otherwise, \( M_k < 0 \). This implies that the Hessian matrix of \( \Pi_c(n, s, p) \) is negative definite, which completes the proof.
Appendix C. Proof of Theorem 4.3. Since $n$ is an integer variable, we prove that the conclusion holds when $n$ is treated as a continuous variable. Using the learning curve equation proposed by [39], the fixed ordering cost of the $i$-th replenishment, $A_i$, can be characterized by the following expression: $A_i = A_1 i^b$, $i = 1, 2, \ldots, n$, where $A_1$ is the fixed ordering cost for the 1st replenishment and $b$ is the learning index. We further have the following approximation for the fixed ordering cost of the retailer:

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} A_1 i^b \approx \int_{0}^{n} A_1 t^b dt = \frac{A_1}{b+1} n^{b+1}. \quad (C.1)$$

Using Lemma 4.1 and substituting Eq. (C.1) into Eq. (A.1), the system’s profit function in the centralized model is simplified as

$$\Pi_c(n, s, p) = \frac{(D_0 + \alpha s - \beta p)}{(\theta + \gamma)^2} \Delta p \left[ e^{(\theta+\gamma)\frac{\mu}{\nu}} - 1 \right] - \frac{1}{2} \eta s^2 + (\theta + \gamma) \left( p \theta + h_r + \theta h_d \right) - A_1 \frac{n^{b+1}}{b+1} n - n s m + c_p C. \quad (C.2)$$

Solving $\frac{\partial \Pi_c(n, s, p)}{\partial n}$ yields the following:

$$\frac{\partial \Pi_c(n, s, p)}{\partial n} = \frac{(D_0 + \alpha s - \beta p) \Delta p}{(\theta + \gamma)^2} \left[ e^{(\theta+\gamma)\frac{\mu}{\nu}} - 1 - (\theta + \gamma) \frac{H}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} \right] - A_1 n^b - \frac{1}{2} \eta s^2 - S_m. \quad (C.3)$$

Let $x = (\theta + \gamma) \frac{\mu}{\nu}$ and $f(x) = e^x - 1 - xe^x$. It is easy to prove that $f(x)$ is a strictly decreasing function. Therefore, with $f(0) = 0$, we have $f(x) < 0$, i.e., $e^{(\theta+\gamma)\frac{\mu}{\nu}} - 1 - (\theta + \gamma) \frac{H}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} < 0$. When $\Delta p \geq 0$, we obtain $\frac{\partial \Pi_c(n, s, p)}{\partial n} < 0$ which implies that $\Pi_c(n, s, p)$ is a strictly decreasing function with respect to $n$. For this case, the optimal value of replenishment number should be equal to 1 when $\Pi_c(n, s, p)$ is maximized. This completes the proof of Theorem 4.3(i).

When $\Delta p < 0$, we set $\frac{\partial \Pi_c(n, s, p)}{\partial n} = 0$ and rearrange Eq. (C.3) as

$$\frac{(D_0 + \alpha s - \beta p)}{(\theta + \gamma)^2} \Delta p \left[ e^{(\theta+\gamma)\frac{\mu}{\nu}} - 1 - (\theta + \gamma) \frac{H}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} \right] - \frac{\eta s^2}{2} - S_m = A_1 n^b. \quad (C.4)$$

Solving $\frac{\partial^2 \Pi_c(n, s, p)}{\partial n^2}$ and using Eq. (C.4), we have

$$\frac{\partial^2 \Pi_c(n, s, p)}{\partial n^2} = \frac{(D_0 + \alpha s - \beta p) \Delta p}{(\theta + \gamma)^2} \frac{H^2}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} - A_1 bn^{b-1} \quad (C.5)$$

$$= \frac{(D_0 + \alpha s - \beta p) \Delta p}{(\theta + \gamma)^2 n} \left[ (\theta + \gamma)^2 H^2 \frac{e^{(\theta+\gamma)\frac{\mu}{\nu}}}{n} + \frac{b(\theta + \gamma) H}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} \right]$$

$$- b e^{(\theta+\gamma)\frac{\mu}{\nu}} + b \frac{b}{2n} \eta s^2 + b S_m. \quad (C.5)$$

With $x = (\theta + \gamma) \frac{\mu}{\nu}$, we let $g(x) = x^2 e^x + bxe^x - be^x + b$. Using $-0.515 < b < 0$, we can prove that $g(x)$ is a strictly increasing function because of $g'(x) > 0$. We further have $g(x) > g(0) = 0$, i.e., $\left( \frac{(\theta+\gamma)^2 H^2}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} + \frac{b(\theta + \gamma) H}{n} e^{(\theta+\gamma)\frac{\mu}{\nu}} \right) - b e^{(\theta+\gamma)\frac{\mu}{\nu}} + b > 0$. Consequently, when $\Delta p < 0$, we have $\frac{\partial^2 \Pi_c(n, s, p)}{\partial n^2} < 0$ which implies $\Pi_c(n, s, p)$ is concave in $n$. This completes the proof of Theorem 4.3(ii).
Appendix D. Proof of Theorem 4.4. Substituting Eqs.(8)-(10) into Eq.(C.2), and after simplification, we have
\[ \Pi_c(n, s, p) = (D_0 + as - \beta p)[p\Phi_3(n) - (c_m + c_p\hat{c})\Phi_1(n) - (h_r + \theta h_d)\Phi_2(n)] - \frac{1}{2}n\eta s^2 - \sum_{i=1}^{n} A_i - nS_m + c_pC. \]

(D.1)

For any given \( n \), \( \frac{\partial \Pi_c(n, s, p)}{\partial s} \) and \( \frac{\partial \Pi_c(n, s, p)}{\partial p} \) can be expressed as
\[ \frac{\partial \Pi_c(n, s, p)}{\partial s} = \alpha[p\Phi_3(n) - (c_m + c_p\hat{c})\Phi_1(n) - (h_r + \theta h_d)\Phi_2(n)] - n\eta s \]
and
\[ \frac{\partial \Pi_c(n, s, p)}{\partial p} = \beta[-2p\Phi_3(n) + (c_m + c_p\hat{c})\Phi_1(n) + (h_r + \theta h_d)\Phi_2(n)] + D_0\Phi_3(n) + \alpha s\Phi_3(n), \]
respectively.

From Eqs.(D.2) and (D.3), we further have \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s^2} = -n\eta, \frac{\partial^2 \Pi_c(n, s, p)}{\partial p^2} = -2\beta\Phi_3(n) \) and \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s \partial p} = \alpha\Phi_3(n) \), respectively. Using \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s^2}, \frac{\partial^2 \Pi_c(n, s, p)}{\partial p^2} - (\frac{\partial^2 \Pi_c(n, s, p)}{\partial s \partial p})^2 = \Phi_3(n)[2\beta n\eta - \alpha^2 \Phi_3(n)] > 0 \), we have that the system’s profit in the centralized model is jointly concave in \( s \) and \( p \), which implies that there exist unique promotional effort and selling price such that \( \Pi_c(n, s, p) \) is maximized. Therefore, when the optimal replenishment number is \( n^*_c \), solving \( \frac{\partial \Pi_c(n^*_c, s, p)}{\partial s} = 0 \) and \( \frac{\partial \Pi_c(n^*_c, s, p)}{\partial p} = 0 \) yields Eqs. (11) and (12).

Appendix E. Proof of Theorem 4.5. By backward sequential decision-making approach, we first analyze the optimal decisions of the retailer, and then derive the equilibrium strategies of the emission-dependent manufacturer. Using Theorems 2 and 3, we express Eqs.(15) and (16) as
\[ \Pi_c(n, s, p) = (D_0 + as - \beta p)[p\Phi_3(n) - w\Phi_1(n) - (h_r + \theta h_d)\Phi_2(n)] - \sum_{i=1}^{n} A_i - \frac{1}{2}n\eta s^2 \]

(E.1)
and
\[ \Pi_m(w) = (w - c_m - c_p\hat{c})(D_0 + as - \beta p)\Phi_1(n) + c_pC, \]

(E.2)
respectively.

For any values of \( n \) and \( w \), \( \frac{\partial \Pi_c(n, s, p)}{\partial s} \) and \( \frac{\partial \Pi_c(n, s, p)}{\partial p} \) are simplified as
\[ \frac{\partial \Pi_c(n, s, p)}{\partial s} = \alpha[p\Phi_3(n) - w\Phi_1(n) - (h_r + \theta h_d)\Phi_2(n)] - n\eta s \]

(E.3)
and
\[ \frac{\partial \Pi_c(n, s, p)}{\partial p} = -2\beta p\Phi_3(n) + \beta w\Phi_1(n) + \beta(h_r + \theta h_d)\Phi_2(n) + D_0\Phi_3(n) + \alpha s\Phi_3(n), \]

(E.4)
respectively.

From Eqs.(E.3) and (E.4), we have \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s^2} = -n\eta, \frac{\partial^2 \Pi_c(n, s, p)}{\partial p^2} = -2\beta\Phi_3(n) \) and \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s \partial p} = \alpha\Phi_3(n) \). We further have \( \frac{\partial^2 \Pi_c(n, s, p)}{\partial s^2} - \frac{\partial^2 \Pi_c(n, s, p)}{\partial s \partial p} - (\frac{\partial^2 \Pi_c(n, s, p)}{\partial s \partial p})^2 = \).
\( \Phi_3(n)[2\beta n\eta - \alpha^2 \Phi_3(n)] \). This implies that \( \Pi_r(n, s, p) \) is jointly concave in \( s \) and \( p \) when \( 2\beta n\eta - \alpha^2 \Phi_3(n) > 0 \).

Using Algorithm B, we can find the optimal replenishment number of the decentralized model, \( n^*_d \). Solving \( \frac{\partial \Pi_r(n^*_d, s, p)}{\partial n} = 0 \) and \( \frac{\partial \Pi_r(n^*_d, s, p)}{\partial p} = 0 \) yields the following:

\[
p^*_d = \frac{D_0n^*_d n\eta \Phi_3(n^*_d) + [\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)][w\Phi_1(n^*_d) + (h_r + \theta h_d)\Phi_2(n^*_d)]}{2[\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)]\Phi_3(n^*_d)} \tag{E.5}
\]

and

\[
s^*_d = \frac{\alpha[D_0 \Phi_3(n^*_d) - \beta w\Phi_1(n^*_d) - \beta(h_r + \theta h_d)\Phi_2(n^*_d)]}{2\beta n^*_d \eta - \alpha^2 \Phi_3(n^*_d)} \tag{E.6}
\]

Using Eqs. (E.5) and (E.6), we also have

\[
D_0 + \alpha s^*_d - \beta p^*_d = \frac{\beta n^*_d \eta}{\alpha \Phi_3(n^*_d)} s^*_d. \tag{E.7}
\]

Substituting Eq. (E.7) into Eq. (E.2), we have

\[
\Pi_m(w) = (w - c_m - c_p) \frac{\beta n^*_d \eta}{\alpha \Phi_3(n^*_d)} s^*_d + c_p C. \tag{E.8}
\]

The necessary condition \( \frac{\partial \Pi_m(w)}{\partial w} = 0 \) for maximizing \( \Pi_m(w) \) yields Eq. (19). Moreover, taking the second partial derivative of \( \Pi_m(w) \) with respect to \( w \), we have

\[
\frac{\partial^2 \Pi_m(n)}{\partial w^2} = -\frac{2\beta^2 n^*_d \eta \Phi_1(n^*_d)}{25n^*_d \eta - \alpha^2 \Phi_2(n^*_d)}. \tag{E.9}
\]

This means that \( \Pi_m(w) \) is concave in \( w \) for any given \( n \) with \( 2\beta n\eta - \alpha^2 \Phi_3(n) < 0 \).

Furthermore, substituting Eq. (19) into Eq. (E.6), we have Eq. (18). From Eqs. (18) and (19), we have

\[
w^*_d \Phi_1(n^*_d) + (h_r + \theta h_d)\Phi_2(n^*_d) = \frac{1}{2\beta}[D_0 \Phi_3(n^*_d) + \beta(c_m + c_p) \Phi_1(n^*_d)] + \beta(h_r + \theta h_d)\Phi_2(n^*_d)]. \tag{E.10}
\]

Substituting Eqs. (18), (19) and (E.9) into Eq. (E.5), we have Eq. (17). This completes the proof of Theorem 4.5 (i) and (ii).

**Appendix F. Proof of Theorem 4.6.** First, we find a feasible solution of the centralized model. When \( n^*_c = n^*_d \), comparing Eqs. (11) and (12) with Eqs. (17) and (18), we have \( s^*_c = 2s^*_d \) and \( p^*_c = 2p^*_d + \frac{D_0}{\beta} \). It is easy to obtain that \((n^*_d, 2s^*_d, 2p^*_d + \frac{D_0}{\beta})\) is a feasible solution of the centralized model. Using the optimal property of \((n^*_c, s^*_c, p^*_c)\), we have \( \Pi_c(n^*_c, s^*_c, p^*_c) \geq \Pi_c(n^*_d, 2s^*_d, 2p^*_d + \frac{D_0}{\beta}) \). From Eq. (13), we have

\[
\Pi_c(n^*_c, s^*_c, p^*_c) \geq \Pi_c(n^*_d, 2s^*_d, 2p^*_d + \frac{D_0}{\beta}) \tag{F.1}
\]

\[
= \frac{2n^*_c \eta[2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{\alpha \Phi_3(n^*_c)} (s^*_d)^2 - \sum_{i=1}^{n^*_c} A_i - n^*_d s^*_m + c_p C.
\]

Using Eq. (F.1), comparing Eq. (13) with Eqs. (20) and (21) yields

\[
\Pi_c(n^*_c, s^*_c, p^*_c) - [\Pi_c(n^*_d, s^*_d, p^*_d) + \Pi_m(w^*_d)] \\
\geq \Pi_c(n^*_d, 2s^*_d, 2p^*_d + \frac{D_0}{\beta}) - [\Pi_r(n^*_d, s^*_d, p^*_d) + \Pi_m(w^*_d)] \\
= \frac{n^*_c \eta[2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{\alpha \Phi_3(n^*_c)} (s^*_d)^2. \tag{F.2}
\]
Appendix G. Proof of Theorem 5.1. Similar to the proof of Theorem 4.5, we can prove that $\Pi_{r/tt}(n, s, p)$ is a jointly concave function of $s$ and $p$ for any given $n$ with $2\beta n^2 - \alpha^2 \Phi_3(n) > 0$. Therefore, for the optimal replenishment number $n^*_tt$, from $\frac{\partial \Pi_{r/tt}(n^*_tt, s^*_t, p^*_t)}{\partial s} = 0$ and $\frac{\partial \Pi_{r/tt}(n^*_tt, s^*_t, p^*_t)}{\partial p} = 0$, we have

$$p^*_t = \frac{D_0 n^*_t \Phi_3(n^*_t) + [\beta n^*_t \eta - \alpha^2 \Phi_3(n^*_t)][w \Phi_1(n^*_t) + (h_r + \theta h_d) \Phi_2(n^*_t)]}{2[\beta n^*_t \eta - \alpha^2 \Phi_3(n^*_t)] \Phi_3(n^*_t)} \quad (G.1)$$

and

$$s^*_t = \alpha [D_0 \Phi_3(n^*_t) - \beta w \Phi_1(n^*_t) - (h_r + \theta h_d) \Phi_2(n^*_t)] \frac{2[\beta n^*_t \eta - \alpha^2 \Phi_3(n^*_t)] \Phi_3(n^*_t)}{2[\beta n^*_t \eta - \alpha^2 \Phi_3(n^*_t)] \Phi_3(n^*_t)} \quad (G.2)$$

The key conditions for coordination are $n^*_tt = n^*_t$ and $p^*_t = p^*_t$ which occur when $w^*_tt = c_m + c_p \hat{c}$. Substituting it into Eq. (G.2) yields Eq. (26).

Appendix H. Proof of Theorem 5.2. (i) Using Theorem 5.1 and from Eq. (6), we have $J_m^0 = J_m^c$.

(ii) Using Theorem 5.1, we simplify the optimal profits of these two members in the TT contract as

$$\Pi_{r/tt}(n^*_tt, s^*_t, p^*_t) = \Pi_c(n^*_c, s^*_c, p^*_c) - c_p C - F \quad (H.1)$$

and

$$\Pi_{m/tt}(w^*_t) = c_p C + F, \quad (H.2)$$

respectively.

From Eqs. (H.1) and (H.2), we have $\Pi_{r/tt}(n^*_tt, s^*_t, p^*_t) + \Pi_{m/tt}(w^*_t) = \Pi_c(n^*_c, s^*_c, p^*_c)$.

(iii) For the decentralized model, we can find a feasible solution $(n^*_c, w^*_t, \frac{1}{2}s^*_c, \frac{1}{2}p^*_c + \frac{D_0}{2\beta})$ with $w^*_t = c_m + c_p \hat{c} + \frac{[2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)] \Phi_3(n^*_c)}{2[\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)] \Phi_3(n^*_c)} s^*_c$. Substituting this feasible solution into Eqs. (15) and (16), we have

$$\Pi_r(n^*_c, \frac{1}{2}s^*_c, \frac{1}{2}p^*_c + \frac{D_0}{2\beta}) = \frac{n^*_c \eta [2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{8 \alpha^2 \Phi_3(n^*_c)} (s^*_c)^2 - \sum_{i=1}^{n^*_c} A_i \quad (H.3)$$

and

$$\Pi_m(w^*_c) = \frac{n^*_c \eta [2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{4 \alpha^2 \Phi_3(n^*_c)} (s^*_c)^2 - n^*_c S_m + c_p C, \quad (H.4)$$

respectively.

Using the optimal property of $(n^*_tt, w^*_t, s^*_t, p^*_t)$ in the decentralized model, we have $\Pi_r(n^*_tt, s^*_t, p^*_t) \geq \Pi_c(n^*_c, \frac{1}{2}s^*_c, \frac{1}{2}p^*_c + \frac{D_0}{2\beta})$ and $\Pi_m(w^*_t) \geq \Pi_m(w^*_c)$. When the TT contract is accepted by these two members of the supply chain, we have $\Pi_{r/tt}(n^*_tt, s^*_t, p^*_t) \geq \Pi_r(n^*_tt, s^*_t, p^*_t)$ and $\Pi_{m/tt}(w^*_t) \geq \Pi_m(w^*_t)$. Using the optimal property of $(n^*_tt, w^*_t, s^*_t, p^*_t)$ for the decentralized model, we have $\Pi_r(n^*_tt, s^*_t, p^*_t) \geq \Pi_r(n^*_tt, \frac{1}{2}s^*_t, \frac{1}{2}p^*_t + \frac{D_0}{2\beta})$ and $\Pi_m(w^*_t) \geq \Pi_m(w^*_c)$. We further have $\Pi_{r/tt}(n^*_tt, s^*_t, p^*_t) \geq \Pi_r(n^*_tt, \frac{1}{2}s^*_t, \frac{1}{2}p^*_t + \frac{D_0}{2\beta})$ and $\Pi_{m/tt}(w^*_t) \geq \Pi_m(w^*_t)$.

Comparing Eqs. (H.3) and (H.4) with Eqs. (H.1) and (H.2), we have

$$\frac{n^*_c \eta [2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{4 \alpha^2 \Phi_3(n^*_c)} (s^*_c)^2 \leq F \leq \frac{3n^*_c \eta [2\beta n^*_c \eta - \alpha^2 \Phi_3(n^*_c)]}{8 \alpha^2 \Phi_3(n^*_c)} (s^*_c)^2. \quad (H.5)$$
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