Trapping, detection and manipulation of single Rb atoms in an optical dipole trap using a long-focus objective lens

I I Beterov\,1,2, E A Yakshina\,1,2, D B Tret'yakov\,1,2, V M Entin\,1,2, U Singh\,2, Ya V Kudlaev\,1,2, K Yu Mityanin\,1,2, K A Panov\,1,2, N V Ayanova\,1,2, C Andreeva\,3,4 and I I Ryabtsev\,1,2*

1 Rzhanov Institute of Semiconductor Physics, Siberian Branch, Russian Academy of Sciences, 630090 Novosibirsk, Russia
2 Novosibirsk State University, Novosibirsk 630090, Russia
3 Institute of Electronics, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria
4 Faculty of Physics, St. Kliment Ohridski University of Sofia, 5 James Bourchier Blvd., 1164 Sofia, Bulgaria

*E-mail: ryabtsev@isp.nsc.ru

Abstract. Single alkali-metal atoms in arrays of optical dipole traps represent a quantum register that can be used for quantum computation and simulation based on short-term Rydberg excitations, which switch the interactions between qubits. To load single atoms into optical dipole traps and then detect them by resonance fluorescence, lenses with a large numerical aperture (NA > 0.5) are commonly used. We present our recent experimental results on demonstrating the trapping of single \(^{87}\)Rb atoms using a long-focus objective lens with a low numerical aperture (NA = 0.172) placed outside the vacuum chamber, and detecting single atoms with a low-cost sCMOS camera. We also present our current results on implementing a single-qubit gate based on optical pumping and subsequent microwave transition between two hyperfine sublevels of a single \(^{87}\)Rb atom with fidelity near 95%.

1. Introduction

Single alkali-metal atoms, being loaded in arrays of optical dipole traps [1, 2], represent a quantum register that can be used for quantum computation and simulation based on short-term Rydberg excitations, which switch the interactions between qubits [3, 4]. To load single atoms into optical dipole traps and then detect them by resonance fluorescence, lenses with a large numerical aperture (NA > 0.5) are commonly used. Such lenses provide focusing of a Gaussian laser beam into a spot of small radius (~1 \(\mu\)m), which ensures localization of the trapped atoms and provides conditions for collisional blockade [5, 6] when loading of more than one atom into a dipole trap becomes inhibited. In addition, the use of lenses with a large numerical aperture increases the efficiency of collecting scattered photons when detecting atoms by resonance fluorescence. In experiments on trapping single atoms, short-focus aspherical lenses are typically installed inside the vacuum chamber of a magneto-optical trap (MOT), while the low-intensity fluorescence of single atoms is typically detected by highly sensitive EMCCD cameras.

Despite the obvious advantages of optical schemes that employ lenses or objective lenses with a large numerical aperture, their principal feature is the placement of optical surfaces at a relatively short
distance from the cloud of cold atoms. For lenses placed inside a vacuum chamber, this distance usually does not exceed 5-10 mm, which leads to the potential occurrence of spurious electric fields in depositing alkali-metal atoms on dielectric surfaces. The uncontrolled electric fields arising in this case strongly affect the interactions of Rydberg atoms [7] and reduce the fidelity of quantum gates. In addition, EMCCD cameras are expensive devices that are not always available in laboratories.

In this paper, we present our recent experimental results [8] on demonstrating the trapping of single \(^{87}\)Rb atoms using a long-focus objective lens with a numerical aperture \(NA = 0.172\) placed outside the MOT vacuum chamber and detecting single atoms with a lower-cost sCMOS camera at a short exposure time (50-200 ms). We also present our current results on implementing a single-qubit gate based on optical pumping and subsequent microwave transition between two hyperfine sublevels of a single \(^{87}\)Rb atom.

2. Experimental setup

The optical scheme of the experimental setup is shown in figure 1. Initially, the Rb atoms are cooled and trapped in a magneto-optical trap (MOT), thus forming a cloud of cold atoms with a temperature \(\sim 100\ \mu\text{K}\) in the center of the vacuum chamber. A laser system with a wavelength of 850 nm is then used to capture the atoms from the MOT into the dipole trap. The laser beam is focused onto the Rb atomic cloud by an objective with a focal distance \(f = 119\ \text{mm}\) and numerical aperture \(0.172\). This objective was developed and first used in the work described in [9]. In order to expand the laser beam and thus tighten the focusing, a telescope with two lenses with \(f = 25\ \text{mm}\) and \(f = 150\ \text{mm}\) is placed before the objective. The optical system is adjusted to obtain a beam waist of less than 10 \(\mu\text{m}\) at the intensity level of \(e^{-2}\), in order to provide loading of predominantly single atoms due to the effect of light-assisted collisional blockade.

![Figure 1. Scheme of the experimental setup for trapping single Rb atoms in an optical dipole trap, their registration and realization of one-qubit quantum gates based on them.](image)

The first demonstration of a single-qubit quantum gate with a single \(^{87}\)Rb atom in a single dipole optical trap was based on the microwave transition \(5S_{1/2}(F = 2, M_F = 0) \rightarrow 5S_{1/2}(F = 1, M_F = 0)\), with a frequency of 6.834 GHz between the hyperfine levels of the ground state, which serve as the qubit working levels. For this, two additional lasers were used: a pump laser for pumping the atoms onto a certain Zeeman sublevel (qubit initialization), and a blow-away laser for identifying the qubit state. A microwave generator with a horn at the output induces Rabi oscillations between the two working levels of the qubit. AOMs modulate the laser fields in order to form pulses with the required length.

The time diagram of the experiment on trapping of a single \(^{87}\)Rb atom and performing a single-qubit gate is shown in figure 2. The experiment is carried out in pulsed regime. First the Rb atoms are loaded into the MOT for 0.1-1 s and at the same time – in the optical dipole trap. Then the cooling lasers and
the gradient magnetic field of the MOT are switched off. The atoms are held in the dipole trap for 100 ms, while the atoms in the MOT are released. Next the cooling lasers and the digital camera are turned on to register the fluorescence signal of the trapped atom. At that time the dipole trapping laser is modulated by rectangular pulses with a frequency of 1 MHz. Thus, in the absence of the trapping laser there is no light shift influence on the registration, pumping and blowing-away, while during the pulses the atoms are captured in the trap. After that, the cooling laser is turned off, and a dc magnetic field of 2-5 G is applied to remove the degeneracy of the magnetic sublevels and provide a quantization axis. At the next step, the linearly polarized pumping laser is turned on for 1-5 ms to pump the atoms into the Zeeman sublevel $5S_{1/2}(F = 2, M_F = 0)$ when the repump laser is on, or into the $5S_{1/2}(F = 1, M_F = 0)$ sublevel if the cooling laser is on.

**Figure 2.** Time diagram of the experiment on the trapping of a single $^{87}$Rb atom and realization of one-qubit quantum gate.

In the next step, the pumping and repumping lasers are turned off, while the microwave pulse is switched on. The microwave transitions correspond to a rotation of the qubit state vector, i.e. one-qubit gate. Following the rotation, the blow-away laser is turned on to probe the final atomic state. In the absence of the pumping laser it can be arbitrary, while during its action it should always be the same. For example, for $^{87}$Rb the state $5S_{1/2}(F = 2)$ corresponds to no signal, since the blow-away laser is tuned to the closed $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F = 3)$ transition and pushes away from the trap atoms in the $5S_{1/2}(F = 2)$ state. On the other hand, the state $5S_{1/2}(F = 1)$ would yield a maximal signal, since the blow-away laser does not interact with atoms in this state. The final registration of the end state of the single atom is realized by turning the cooling lasers on and obtaining a second fluorescence signal on the digital camera. The data processing includes a post-selection of those events when only a single atom was registered in the trap.

3. **Trapping and detection of a single rubidium atom**

The single Rb atom captured following the time sequence in figure 2 is imaged by means of resonance fluorescence induced by the cooling lasers with wavelength 780 nm (not shown in figure 1). The spontaneously emitted photons are collected by the same objective $f = 119$ mm and focused on the sCMOS camera FLIR Tau CNV by the $f = 25$ mm lens.
The capture of an atom in the optical dipole trap is seen as a small additional spot on the image of the cold atomic cloud. The recorded images are processed by a specialized software. The signal from the Rb atoms trapped in the dipole trap is seen on the background of the camera noise and parasitic scattered photons. The calibrated magnitude of the fluorescence signal is used to estimate the time-averaged number of atoms in the trap. An area of 10×10 pixels around the atoms is used to statistically analyze the loading of atoms in the dipole trap (figure 3 (a)) and subtract the space-averaged level of scattered photons.

Figure 3. (a) Image of 0, 1 and 2 atoms registered by the digital videocamera FLIR Tau CNV for exposure time of 100 ms.

The distribution of the videosignal amplitudes is studied in more detail by their representation as histograms, shown in figure 4. The sCMOS-videocamera FLIR Tau CNV used for our experiments is not designed for photon counting, thus the histogram (shown in figure 4) does not provide full resolution on the number of atoms. The histogram for the loading of a single atom consists of three peaks. The first one is the dark noise of the camera, the second one – the fluorescence from one atom, and the third one – from two atoms. The probability of loading a single atom can be estimated as the ratio of the amplitude of the single-atom peak to the sum of the amplitudes of the three peaks.

Figure 4. Histogram of the output signals of the videocamera FLIR Tau CNV for the registration of single Rb atoms trapped in the optical dipole trap for continuously working MOT. The sCMOS-camera is not designed for photon counting regime.

Despite the partial overlapping of the peaks in the histogram of figure 4, the uncertainty in the determination of the number of atoms is less than 5% (it corresponds to the area of overlapping of neighbouring peaks). Therefore, it can be concluded that the sCMOS-videocamera FLIR Tau CNV in our experimental setup can be used to register single atoms. A similar conclusion was drawn earlier in the work [10], where a sCMOS videocamera was used for the registration of single Cs atoms.

An important parameter of the trapped single atom is its lifetime in the trap, since it determines the timescale on which the single qubit gates are realized. The time dependence of the videosignal yields an average capture time of 5-10 s. In the pulsed regime with two-atom detections, and optimizing the parameters of the laser cooling and trapping, the lifetime increased to 30-50 s.

4. Optical pumping of a single rubidium atom
The energy level diagram of the optical pumping of $^{87}$Rb atoms into the state $5S_{1/2}(F = 1)$ with moment projection $M_F = 0$ for linear laser polarization is shown in figure 5 (a). The pumping laser with linear polarization along the external magnetic field induces transitions $5S_{1/2}(F = 1) \rightarrow 5P_{1/2}(F = 1)$ into the $5P_{1/2}(F = 1)$ excited state without change in the moment projection. Then, under the action of the cooling laser tuned to the $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F = 3)$ transition, but also inducing a partial transition at the wings
of the $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F = 2)$ absorption profile, all population is accumulated onto the Zeeman sublevel $5S_{1/2}(F = 1)$ with moment projection $M_F = 0$. This state serves as the initial qubit state of the single qubit quantum gate. The final qubit state is $5S_{1/2}(F = 2)$ with moment projection $M_F = 0$, detuned at a frequency of 6.834 GHz from the initial state. The transition into the final state can be induced by microwave field.

The pumping laser can be also tuned to the transition $5S_{1/2}(F = 2) \rightarrow 5P_{1/2}(F = 2)$, as shown in figure 5 (b). In the presence of the repumping laser tuned to the $5S_{1/2}(F = 1) \rightarrow 5P_{3/2}(F = 2)$ transition, due to the spontaneous decay of the exited state, the population is accumulated on the Zeeman sublevel $5S_{1/2}(F = 2)$ with moment projection $M_F = 0$. This state can also serve as the initial qubit state.

Figure 5. (a) Scheme of the transitions for optical pumping of $^{87}$Rb at the transition $5S_{1/2}(F = 1) \rightarrow 5P_{1/2}(F = 1)$ into the $5S_{1/2}(F = 1, M_F = 0)$ state. (b) Scheme of the transitions for optical pumping of $^{87}$Rb at the transition $5S_{1/2}(F = 2) \rightarrow 5P_{1/2}(F = 2)$ into the $5S_{1/2}(F = 2, M_F = 0)$ state for linear laser polarization.

Figure 6 presents the experimental results on the time dependence of the population of the $5S_{1/2}(F = 1)$ state after the optical pumping and blowing-away according to the time diagram of figure 2. The intensity of the pumping laser was 0.1-1 mW/cm$^2$, the total intensity of the repumping laser was 40 mW/cm$^2$, and that of the cooling laser, 55 mW/cm$^2$. The repumping and cooling laser beams were guided isotropically to the atom from six directions to avoid their light pressure on the atom. The pumping laser was perpendicular to the optical dipole trap beam and was linearly polarized in a horizontal plane, its electrical vector being collinear to the additional homogeneous magnetic field of 2-5 G. The intensity of the blow-away laser was above 100 W/cm$^2$ due to the tight focusing, and the efficiency of removal of the $5S_{1/2}(F = 2)$ atoms from the trap was better than 97%.

Figure 6. (a) Measured time dependence of the population of the $5S_{1/2}(F = 1)$ state for optical pumping of the transition $5S_{1/2}(F = 1) \rightarrow 5P_{3/2}(F = 1)$ and using the cooling laser as a repumper according to the scheme in figure 5 (a). (b) Measured time dependence of the population of the $5S_{1/2}(F = 1)$ state for optical pumping of the $5S_{1/2}(F = 2) \rightarrow 5P_{1/2}(F = 2)$ transition and using the repumping laser according to the scheme in figure 5 (b).
Figure 6 (a) shows the time dependence of the population of the 5S_{1/2}(F = 1) state for optical pumping at the 5S_{1/2}(F = 1) → 5P_{3/2}(F = 1) transition and using the cooling laser as a repumping laser according to the scheme in figure 5 (a). The dependence is a saturation curve with a time constant 0.6 ms. The characteristic pumping time is 2 ms, and the pumping precision reaches 93 ± 3%. Figure 6 (b) shows the time dependence of the population of the 5S_{1/2}(F = 1) state for optical pumping at the 5S_{1/2}(F = 2) → 5P_{3/2}(F = 2) transition and using the repumping laser according to the scheme in figure 5b. Since in the process of blowing-away atoms in the 5S_{1/2}(F = 2) state are removed, this dependence characterizes the rate of depletion of the 5S_{1/2}(F = 1) state by the pumping laser. It has the shape of an inverse saturation curve with a time constant 1.4 μs. The characteristic pumping time is 2-3 orders of magnitude less than in figure 6 (a), being only around 5 μs, with pumping precision of 91 ± 2%.

Thus, we were able to successfully realize optical pumping both on the 5S_{1/2}(F = 2, M_F = 0), and the 5S_{1/2}(F = 1, M_F = 0) magnetic sublevels, with a precision close to that required for single qubit gates with single Rb atoms in a single trap.

5. Microwave transitions and one-qubit gates in a single Rubidium atom

In the $^{87}$Rb atoms, a microwave field with a frequency of 6.834 GHz induces magnetodipole transitions between the Zeeman sublevels 5S_{1/2}(F = 2, M_F = 0) → 5S_{1/2}(F = 1, M_F = 0) of the ground-state hyperfine levels (figure 7 (a)). The microwave transitions correspond to rotation of the qubit state vector at a given angle, i.e. realization of a single qubit gate. The detection of a microwave transition is based on recording the time dependence of the population of the final state 5S_{1/2}(F = 1) (in fact, the dependence on the area of the microwave pulse). For incoherent transitions, it is a saturation curve with asymptotic stationary value of 1/2. For coherent transitions, Rabi oscillations are observed (figure 7 (b)). The microwave power is adjusted for obtaining maximal transition probability faster than 10 ms. The signal processing includes a post-selection of the events of capturing a single atom in the trap.

The microwave transitions were registered using the scheme of figure 1 following the time diagram presented in figure 2 with a microwave pulse. The signal from the generator is sent to a horn antenna positioned on one of the windows of the MOT vacuum chamber. The linearly polarized magnetic component of the microwave field of the antenna is along the homogeneous magnetic field, in order to induce microwave transitions without changing the atomic total moment projection.

The aim of the experiment was to observe the "clock" microwave transition 5S_{1/2}(F = 2, M_F = 0) ↔ 5S_{1/2}(F = 1, M_F = 0) in the presence of 2-5 G magnetic field. This magnetic field leads to a large Zeeman shift of the other transitions, making it possible to tune the RF field in resonance only with the "clock" transition, as seen from figure 7 (a). Under ideal conditions (in the absence of external fields), the "clock" transition is represented by a two-level system without population and phase relaxations. The microwave transition probability for such system is described by the formula:

$$
\rho(t) \approx \frac{\Omega^2/2}{\Omega^2 + \delta^2} \left[ 1 - \cos \left( tv \frac{\Omega^2}{\Omega^2 + \delta^2} \right) \right]
$$

(1)
where $\Omega$ is the Rabi frequency of the magnetodipole “clock” transition, and $\delta$ – the detuning from exact resonance with the “clock” transition taking into account the possible light shift due to the dipole trap laser (if the measurement is taken with working trap). For exact resonance ($\delta = 0$) the population oscillates between the initial and final state with a frequency $\Omega$ (Rabi oscillations). Equation (1) also gives the spectrum of the microwave transition when scanning $\delta$ for a fixed interaction time $t_0$. According to equation (1), the Rabi oscillations are infinite. In practice, there are always parasitic processes that lead to their decay and setting the populations to steady-state values. In this case, the time evolution of the populations is described by a more complex formula, obtained by us in the work [11] based on the solution of the density matrix equations in the presence of relaxation of the populations and phases with a rate $\gamma$:

$$
\rho(t) \approx \frac{\Omega^2}{2\Omega^2 + \gamma^2 + 4\delta^2} \left[ 1 - e^{-\frac{2\Omega^2 + 8\delta^2}{4\Omega^2 + 8\delta^2}t} \right] + \frac{\Omega^2/2}{\Omega^2 + \delta^2} \left[ e^{-\frac{2\Omega^2 + 8\delta^2}{4\Omega^2 + 8\delta^2}t} - e^{-\frac{6\Omega^2 + 8\delta^2}{4\Omega^2 + 8\delta^2}t/2} \cos\left(\frac{t\sqrt{\Omega^2 + \delta^2}}{2}\right) \right]
$$

(2)

This formula is valid for $\Omega > 3\gamma$, when the Rabi frequency is significantly higher than the relaxation rate. The coherence time (decay) of the Rabi oscillations can be defined as $\tau = 1/\gamma$ and it can be obtained by comparing the experimental results with the numerical ones using equation (2).

Figure 8 shows the Rabi oscillations registered at the “clock” transition for initial pumping of the atoms into the $5S_{1/2}(F = 2, M_F = 0)$ state with continuously operating dipole trap. For the maximal microwave generator power of 24 dBm and after additional alignment of the polarization of the microwave field, Rabi oscillations with a frequency of 4 kHz and amplitude of $94 \pm 2\%$ were measured, with a coherence time of 3 ms, as seen in figure 8. Such contrast is enough for the realization of single qubit gates with fidelity of 95%.

**Figure 8.** Experimentally recorded Rabi oscillations at the $5S_{1/2}(F = 2, M_F = 0) \rightarrow 5S_{1/2}(F = 1, M_F = 0)$ "clock" transition (blue circles) and their comparison with the theoretical calculations following equation (2) (green dashed curves) for generator power of 24 dBm without dipole trap modulation.

Beside registration of the Rabi oscillations at the microwave transition, the realization of single-qubit quantum gates requires the experimental building of truth tables when rotating the qubit state vector at an angle $\pi$ for the NOT gate and at an angle $\pi/2$ for the Hadamard gate for different initial states (logical «0» and «1»). The initial state $5S_{1/2}(F = 2, M_F = 0)$ can be chosen as the logical «0» and the logical «1» is the final state $5S_{1/2}(F = 1, M_F = 0)$. To prepare the initial state «1», first the NOT gate is applied, followed by the qubit rotation. The end of each single-qubit gate corresponds to the microwave pulse areas shown in figure 7 (b), where dots denote the final phases of the Rabi oscillations used for different qubit gates.

The Rabi oscillations shown in figure 8 have been analyzed to estimate the reachable fidelity of the single-qubit gates in our experiments. For initial state «0», the Hadamard gate corresponds to a rotation at $\pi/2$, and the microwave pulse ends at moment 1 in figure 7 (b) (a $\pi/2$-area pulse). After this rotation, the population in point 1 of figure 8 is $0.489 \pm 0.02$ instead of the ideal value of 0.5 in figure 7 (b). Thus, the fidelity of realization of the Hadamard gate from initial state «0» is $97.8 \pm 4\%$.

For initial state «0», the «NOT» gate corresponds to a rotation at an angle $\pi$, and the microwave pulse ends at moment 2 in figure 7 (b) (a $\pi$-area pulse). After this rotation, the population in point 2 of figure
8 is $0.935 \pm 0.02$ instead of the ideal value of 1 in figure 7 (b). Thus, the fidelity of realization of the «NOT» gate from initial state «0» is $93.5 \pm 2\%$.

To build the truth tables for the initial state «1», the state is prepared by qubit rotation at an angle $\pi$, which corresponds to point 2 in figure 7 (b). From the experiment in figure 8 this yields a fidelity of realization of the state «1» of $93.5 \pm 2\%$.

The Hadamard gate from state «1» is realized by a further rotation at an angle $\pi/2$, which corresponds to completing the microwave pulse at moment 3 in figure 7 (b). For such rotation at $\pi/2$ from point 2, the population in point 3 is $0.465 \pm 0.02$ (figure 8) instead of the ideal value of 0.5 (figure 7 (b)). Thus, the fidelity of realization of the Hadamard gate from initial state «1» is $93 \pm 4\%$.

For initial state «1», the «NOT» gate is realized by additional rotation at an angle $\pi$, corresponding to completing the microwave pulse at point 4 in figure 7 (b). From figure 8 it can be seen that after rotation from point 2 at an angle of $\pi$, the population in point 4 is $0.035 \pm 0.02$, instead of the ideal value of 0 in figure 7 (b). Thus, the fidelity of realization of the «NOT» gate from initial state «1» is $96.5 \pm 2\%$.

From the described measurement results it can be estimated that the average fidelity of realization of single-qubit quantum gates with a single Rb atom was $95.2 \pm 3\%$.

6. Conclusions

Our experiments with a low numerical aperture objective lens placed outside the MOT vacuum chamber have shown that it is possible to trap, detect and manipulate a single Rb atom in an optical dipole trap with the waist diameter of ~10 microns. This diameter ensures the collisional blockade mechanism to deterministically load single atoms. Moreover, we have shown that single atoms can be reliably observed and detected with a low-cost scmos video camera instead of expensive EMCCD cameras, which are typically used for this purpose. Using this camera, we have also observed the "clock" microwave transition between two hyperfine sublevels of a single $^{87}$Rb atom and demonstrated the Rabi population oscillations of these sublevels with contrast up to 95%. This is equivalent to implementing the single-qubit gates with fidelity up to 95%. Our next steps should be creation of an array of optical dipole traps and implementation of two-qubit gates between neighboring qubits aimed at the further development of a neutral-atom quantum computer.

Acknowledgments

This work was supported by the Russian Science Foundation Grant No 18-12-00313 in the part of experiments, the RFBR Grant No 19-52-15010 in the part of creating an optical dipole trap, and the Novosibirsk State University.

References

[1] Piotrowicz M J, Lichtman M, Maller K, Li G, Zhang S, Isenhower L and Saffman M 2013 Phys. Rev. A 88 013420
[2] Barredo D, de Léséleuc S, Lienhard V, Lahaye T and Browaeys A 2016 Science 354 1021
[3] Saffman M, Walker T and Molmer K 2010 Rev. Mod. Phys. 82 2313
[4] Ryabtsev I I, Beterov I I, Tretyakov D B, Entin V M and Yakshina E A 2016 Phys. Usp. 59 196
[5] Grimm R, Weidemuller M and Ovchinnikov Yu B 2020 Adv. At. Mol. Opt. Phys. 42 95
[6] Schlosser N, Reymond G and Grangier P 2002 Phys. Rev. Lett. 89 023005
[7] Yakshina E A, Tretyakov D B, Beterov I I, Entin V M, Andreeva C, Cinins A, Markovski A, Ifikhar Z, Ekers A and Ryabtsev I I 2016 Phys. Rev. A 94 043417
[8] Beterov I I, Yakshina E A, Tretyakov D B, Entin V M, Singh U, Kudlaev Ya V, Mityanin K Yu, Panov K A, Aryanova N V and Ryabtsev I I 2020 Quantum Electronics 50(6) 543–550
[9] Pritchard J D, Isaacs J A and Saffman M 2016 Rev. Sci. Instr. 87 073107
[10] Picken C J, Legaie R and Pritchard J D 2017 Appl. Phys. Lett. 111 164102
[11] Entin V M, Yakshina E A, Tretyakov D B, Beterov I I and Ryabtsev I I 2013 J. Exper. Theor. Phys. 116 721–731