Axial and pseudoscalar form factors from charged current quasielastic neutrino-nucleon scattering

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We study the scattering of neutrinos on polarized nucleons or detecting the polarization of recoil particles. In contrast to electromagnetic processes, the parity-violating weak interaction does not suppress spin asymmetries contributing sizably at leading order. The future measurements with polarized particles could provide independent access to the proton axial structure and allow us the first extraction of the pseudoscalar form factor from neutrino data without assumptions regarding its form. Limited by charged lepton mass suppression, the latter is possible scattering muon (anti)neutrinos with hundreds of MeV energy but requires a percent or even sub-percent measurement of spin asymmetries or scattering tau (anti)neutrinos. Axial form factor can be extracted from all energies of accelerator neutrinos.

After pioneering studies of polarization observables \cite{1–10}, the rapid development of neutrino physics and necessity for the improved phenomenology of neutrino interactions achieving precision in oscillation experiments have motivated a few groups to revisit polarization effects in neutrino-nucleon CCQE \cite{11–14}. Expressions for all possible single, double and triple spin asymmetries are collected in Ref. \cite{13}. Contribution of second class currents to polarization observables was considered in Ref. \cite{15}, polarization effects in inverse reactions $ep \rightarrow \mu n$ were investigated in Ref. \cite{16}. The discovery of tau neutrino \cite{17} and further experiments \cite{18–20} have studied CCQE observables with polarized recoil tau lepton \cite{21–30}. Induced nucleon polarization in neutrino-nucleus neutral-current scattering was investigated in Refs. \cite{31–34}.

Describing neutrino-nucleon interactions, pseudoscalar form factor is expressed in terms of axial form factor exploiting PCAC ansatz in the assumption of the pion-pole dominance which can be valid only at relatively low momentum transfers \cite{35}. Pseudoscalar coupling, form factor at momentum transfer $Q^2 \sim 0.88m_n^2$, is extracted from measurements of muon capture on the proton \cite{36–39}, see Refs. \cite{40,41} for a review. The pseudoscalar form factor at other values of momentum transfer was extracted only once from the pion electroproduction cross section data \cite{42,43}. Advances in lattice QCD provided us with ab-initio results for the axial and pseudoscalar form factors \cite{44–54}. Strong contradictions to the PCAC ansatz in the assumption of the pion-pole dominance \cite{55,56} were recently resolved to understand of nucleon dynamics at $Q^2 \lesssim 1−3$ GeV$^2$ and could be useful for modelling of neutrino interactions at DUNE \cite{64,65}, Hyper-K \cite{66}, and ESS/SEB \cite{67}.

In this work, we study the sensitivity of single-spin asymmetries in neutrino-nucleon charged current quasielastic scattering to axial and pseudoscalar form factors. We determine neutrino beam energies suitable for the simultaneous extraction of both form factors in one experiment and identify single-spin asymmetries sensitive to the axial contributions at GeV energies.

At energies of accelerator experiments, charged current neutrino-quark scattering is described by four-fermion interaction:

$$\mathcal{L}_{\text{eff}} = -\sum_{q \neq q'} c_{qq'} \bar{\ell} \gamma^\mu \nu \ell \bar{q} \gamma^{\mu} \gamma_5 P_L q',$$

(1)

with projection operator on the left-handed chiral states $P_L = \frac{1-\gamma_5}{2}$. At leading order, Wilson coefficients are determined by the Fermi coupling constant $G_F$ and CKM matrix elements $V_{qq'}$ as $c_{qq'} = 2\sqrt{2}G_FV_{qq'}$. More precise determination is given in Ref. \cite{68}.

The matrix element of the quark current inside the nucleon can be expressed in terms of Sachs electric $G_E^V$ and magnetic $G_M^V$ isovector, axial $F_A$ and pseudoscalar $F_P$ form factors as

$$\Gamma_\mu(Q^2) = \langle p(p')|\bar{u} \gamma_\mu P_L d|n(p)\rangle = \frac{1}{2}B \left[ \gamma_\mu G_M^V(Q^2) - \frac{p_\mu + p'_\mu}{2M} G_M^V(Q^2) + \gamma_\mu \gamma_5 F_A(Q^2) + \frac{q_\mu}{M} \gamma_5 F_P(Q^2) \right] n,$$

(2)

with $q = p' - p$, $Q^2 = -(p - p')^2$ and $\tau = Q^2/(4M^2)$. Assuming isospin symmetry, the isovector form factors are given by difference of the proton and neutron form factors $G_{E,M}^V = G_{E,M}^p - G_{E,M}^n$ and mass of both nucleons is $M$. Antineutrino-proton scattering is described by conjugated current.
We consider a few experimental observables in the following.

The unpolarized neutrino-nucleon scattering cross section is conveniently expressed as

$$
\frac{d\sigma}{dQ^2}(Q^2, E_\nu) = \frac{r_q^2}{16\pi} \frac{M^2}{E_\nu^2} \left[ \left( \tau + r^2 \right) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1 + \tau} C(Q^2) \right],
$$

(3)

with $r = m/(2M)$, $\tau = Q^2/(4M^2)$, incoming neutrino energy $E_\nu$ and variable $\nu = E_\nu/M - \tau - r^2$. The structure-dependent factors $A$, $B$, and $C$ are given by

$$
A = \tau \left( G_M^V \right)^2 - \left( G_E^V \right)^2 + (1 + \tau)F_A^2 - r^2 \left( \left( G_M^V \right)^2 + F_A^2 - 4\tau F_P^2 + 4F_AF_P \right),
$$

(4)

$$
B = 4\eta\tau F_A G_M^V,
$$

(5)

$$
C = \tau \left( G_M^V \right)^2 + \left( G_E^V \right)^2 + (1 + \tau)F_A^2,
$$

(6)

where $\eta = 1$ in the scattering on the neutron $\nu appliance \nu \to \ell^- p$ and $\eta = -1$ in the scattering on the proton $\bar{\nu} \to \ell^+ n$.

Target, recoil and lepton single-spin asymmetries $T, R, L$ respectively are defined from the cross section with a fixed spin direction $S$ of one incoming or outgoing particle:

$$
T, R, L = \frac{d\sigma(S) - d\sigma(-S)}{d\sigma(S) + d\sigma(-S)},
$$

(7)

and can be described by two independent components since all form factors at leading order are real functions. At leading order, asymmetries are conveniently expressed in terms of the structure-dependent functions as

$$
T, R, L = \frac{(\tau + r^2) A^{T,R,L}(Q^2) - \nu B^{T,R,L}(Q^2) + \frac{\nu^2}{1 + \tau} C^{T,R,L}(Q^2)}{(\tau + r^2) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1 + \tau} C(Q^2)}.
$$

(8)

The asymmetry $T$ in the neutrino scattering on the polarized nucleon target with the spin vector $S$ is determined by the following structure-dependent factors $A^T$, $B^T$, and $C^T$:

$$
A^T = G_M^V \left( F_A - \eta G_E^V \right) (p' \cdot S) - 2\eta G_M^V G_E^V (k' \cdot S) + 2\tau C_M^V \left( \frac{\eta G_E^V - F_A + 2\tau F_P}{\tau + r^2} (k \cdot S) - F_P (p' \cdot S) \right),
$$

(9)

$$
B^T = \left( \eta F_A^2 - F_A G_E^V + \eta \tau G_M^V G_M^V - G_E^V \right) \left( \frac{1}{1 + \tau} \right) (p' \cdot S) - 2F_A G_E^V (k' \cdot S)
$$

$$
- \tau^2 \left( F_A G_M^V - G_E^V \right) \left( \frac{1}{1 + \tau} \right) (p' \cdot S),
$$

(10)

$$
C^T = F_A \left( G_M^V - G_E^V \right) (p' \cdot S).
$$

(11)

To evaluate transverse to the beam asymmetry $T_1$ with spin direction in the scattering plane, we substitute $(p' \cdot S) = -(k' \cdot S) = 2\frac{M}{E_\nu} \sqrt{\tau \nu^2 - (1 + \tau)(\tau + r^2)^2}$. To evaluate longitudinal to the beam asymmetry $T_1$, we substitute $(p' \cdot S) = -2 \left( \tau + \frac{M}{E_\nu} (\tau + r^2) \right)$ and $(k' \cdot S) = -(p' \cdot S) - \frac{E_\nu}{M}$.

The asymmetry $R$ in the neutrino scattering with measurements of the recoil nucleon spin $S$ is determined by the following structure-dependent factors $A^R$, $B^R$, and $C^R$:

$$
A^R = G_M^V \left( F_A - \eta G_E^V \right) (p \cdot S) - 2\eta G_M^V G_E^V (k \cdot S) + 2\tau C_M^V \left( \frac{\eta G_E^V + F_A - 2\tau F_P}{\tau + r^2} (k \cdot S) - F_P (p \cdot S) \right),
$$

(12)

$$
B^R = \left( \eta F_A^2 - F_A G_E^V + \eta \tau G_M^V G_M^V - G_E^V \right) (p \cdot S) - 2F_A G_E^V (k \cdot S)
$$

$$
+ \tau^2 \left( F_A G_M^V - G_E^V \right) \left( \frac{1}{1 + \tau} \right) (p \cdot S),
$$

(13)

$$
C^R = F_A \left( G_M^V - G_E^V \right) (p \cdot S).
$$

(14)

For the simplicity of our expressions, we exploit an unconventional normalization for the spin vector: $S^2 = -1/M^2$.
To evaluate transverse to the recoil nucleon asymmetry \( R_t \) with spin direction in the scattering plane, we substitute \((p \cdot S) = 0\) and \((k \cdot S) = -\sqrt{\tau\nu^2 - (1 + \tau)(\tau + r^2)^2}/\sqrt{(1 + \tau)}\). To evaluate longitudinal to the recoil nucleon asymmetry \( R_l \) with spin direction in the scattering plane, we substitute \((p \cdot S) = 2\sqrt{\tau(1 + \tau)}\) and \((k \cdot S) = (\tau\nu - (1 + \tau)(\tau + r^2))/\sqrt{(1 + \tau)}\).

The asymmetry \( L \) in the neutrino scattering with determination of the recoil lepton spin \( S \) is determined by the following structure-dependent factors \( A^L, B^L, \) and \( C^L \):

\[
\begin{align*}
(\tau + r^2) A^L &= -A(k \cdot rS) + 2\eta(\tau + r^2) F_A G_M^V (k + 2p \cdot rS) - 2r^2 \left( (G_M^V)^2 + F_A^2 - 4\eta F_P^2 + 4F_A F_P \right) (k \cdot rS), \\
B^L &= -2\eta F_A G_M^V (k \cdot rS) + \frac{C}{1 + \tau} (k + 2p \cdot rS), \\
C^L &= 0.
\end{align*}
\]

To evaluate transverse to the recoil lepton asymmetry \( L_t \) with spin direction in the scattering plane, we substitute \((p \cdot rS) = 0\) and \((k \cdot rS) = 2\tau\sqrt{\tau\nu^2 - (1 + \tau)(\tau + r^2)^2}/\sqrt{(\nu + r^2 - \tau)^2 - 4r^2}\). To evaluate longitudinal to the recoil lepton asymmetry \( L_l \) with spin direction in the scattering plane, we substitute \(2(p \cdot rS) = \sqrt{(\nu + r^2 - \tau)^2 - 4r^2}\) and \((k \cdot rS) = -\left((r^2 - \tau)\nu + (\tau + r^2)^2\right)/\sqrt{(\nu + r^2 - \tau)^2 - 4r^2}\).

Asymmetries provide a complementary way of accessing the nucleon structure. Contrary to asymmetries in electromagnetic and strong interactions, spin-dependent contributions in weak interaction enter observables with similar to unpolarized cross-section weights. In the experiment, flux normalization errors and detector systematics can be significantly reduced on the level of asymmetry.

Pseudoscalar contribution in the scattering of \( \nu_e \) and \( \bar{\nu}_e \) is suppressed by factors \( m_e^2/E_\nu^2, \) \( m_e^2/M^2 \) and \( m_e^2/(ME_\nu) \) and therefore negligible at energies of accelerator experiments. Pseudoscalar contribution in the scattering of \( \nu_\mu \) and \( \bar{\nu}_\mu \) is negligible at energies above \( E_\nu \gtrsim M. \) However, it becomes sizable at neutrino beam energies of hundreds MeV and rises approaching the muon production threshold in asymmetries with polarized nucleons and unpolarized cross section while event rates decrease. In the following Figs. (1-6), we present all non-vanishing at leading order single-spin asymmetries in muon (anti)neutrino scattering substituting nucleon form factors from Refs. [69, 70] in the assumption of the partial conservation of the axial-vector current and pion-pole dominance \( (PCAC \text{ ansatz}) \) for pseudoscalar form factor: \( F_P(Q^2) = 2M^2/(m_\pi^2 + Q^2) F_A(Q^2) \) \( (\text{though PCAC ansatz can be valid only at } Q^2 \lesssim \Lambda_{QCD}^2) \). We also compare central values varying the axial form factor by 20 \% versus varying pseudoscalar form factor from PCAC value by 20 \%. Both axial and pseudoscalar form factors can be accessed with the muon (anti)neutrino beam of a few hundred MeV energy.

\[ \text{\textsuperscript{2}} \text{The normalization of axial and pseudoscalar form factors are known pretty well from neutron decay and muon capture rates on hydrogen, so our variations can represent deviations only away from } Q^2 = 0. \]
FIG. 1: Spin asymmetry $T_t$ in charged current quasielastic muon-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 200$ MeV, 300 MeV, 500 MeV.

FIG. 2: Spin asymmetry $T_l$ in charged current quasielastic muon-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 200$ MeV, 300 MeV, 500 MeV.
FIG. 3: Spin asymmetry $R_t$ in charged current quasielastic muon neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 200$ MeV, 300 MeV, 500 MeV.

FIG. 4: Spin asymmetry $R_l$ in charged current quasielastic muon-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 200$ MeV, 300 MeV, 500 MeV.
The contribution of axial form factor to unpolarized cross section and spin asymmetries is sizable at all beam energies. Though asymmetries at GeV energies require a few percent or sub-percent precision to add a complementary information regarding the axial structure, besides $T_l, R_l, R_\ell$ in $\bar{\nu}_p \rightarrow \ell^+ n$, see Fig. 7 with asymmetries of practical interest. Averaging over the anticipated flux profiles of the DUNE Near Detector [64, 71] at Fermilab and neglecting detector details, we present a closer to experiment result in Fig. 8. Adding high-energy flux components, the asymmetry $R_\ell$ loses sensitivity to the axial structure. However, $T_l$ and $R_l$ require just a factor of a few more statistics to be equally useful as the unpolarized cross section.
Contrary to the unpolarized cross sections, spin asymmetries in the scattering of tau (anti)neutrino are sensitive to the pseudoscalar form factor, see Figs. (9-14) for details. Above the τ-production threshold, recoil and target asymmetries at low $Q^2$ are more sensitive to pseudoscalar than to axial form factor. Asymmetries $L_l$ and $L_t$ are sensitive only to the axial form factor. An improved dataset with $\nu_\tau, \bar{\nu}_\tau$ could allow us to access the pseudoscalar form factor from the neutrino scattering data.

FIG. 7: Spin asymmetries $T_l, R_t, R_l$ in charged current quasielastic muon-antineutrino-proton scattering at neutrino beam energies $E_{\nu_{\mu}} = 1$ GeV. Asymmetries for electron (anti)neutrino scattering are indistinguishable from results on these figures.

FIG. 8: Spin asymmetries $T_l, R_t, R_l$ in charged current quasielastic antineutrino-proton scattering averaged over expected DUNE near-detector flux.

FIG. 9: Spin asymmetry $T_t$ in charged current quasielastic tau-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_{\nu} = 5$ GeV, 7 GeV, 10 GeV.
FIG. 10: Spin asymmetry $T_l$ in charged current quasielastic tau-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 5$ GeV, 7 GeV, 10 GeV.

FIG. 11: Spin asymmetry $R_t$ in charged current quasielastic tau-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 5$ GeV, 7 GeV, 10 GeV.
FIG. 12: Spin asymmetry $R_l$ in charged current quasielastic tau-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 5$ GeV, 7 GeV, 10 GeV.

FIG. 13: Spin asymmetry $L_l$ in charged current quasielastic tau-neutrino-neutron (upper panel) and antineutrino-proton (lower panel) scattering at neutrino beam energies $E_\nu = 5$ GeV, 7 GeV, 10 GeV.
In this work, we study the sensitivity of single-spin asymmetries in (anti)neutrino-nucleon charged current quasielastic scattering to axial and pseudoscalar form factors. Such asymmetries provide independent access to the nucleon axial structure. Pseudoscalar form factor can be accessed either from asymmetries scattering muon (anti)neutrino at hundreds of MeV energies or from asymmetries scattering tau (anti)neutrino above the tau production threshold $E_\nu \gtrsim 3.5$ GeV. Axial structure contributes significantly to recoil and transverse longitudinal electron and muon antineutrino-proton charged current quasielastic scattering at the GeV energy range. The first measurement of polarized observables in neutrino-nucleon scattering experiments could provide another confirmation of the Standard Model of particle physics, complementary information on the axial form factor, and the way to access pseudoscalar form factor independently.

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