DYNAMICS OF THE STAR S0-1 AND THE NATURE OF THE COMPACT DARK OBJECT AT THE GALACTIC CENTER

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ABSTRACT

It has recently been shown that analysis of the orbits of the fast-moving stars close to Sgr A* provides a valuable tool to probe the gravitational potential near the Galactic center. As an example, we present here the results of a calculation of possible orbits of the star S0-1 in both the black hole and the degenerate neutrino ball scenarios of the central mass, based on the recent measurements of stellar proper motions at the Galactic center by Ghez et al. Taking into account the error bars of their analysis, it is shown that within a few years, the orbit of S0-1 may indeed reveal the nature of the supermassive compact dark object at the Galactic center.

Subject headings: black hole physics — celestial mechanics, stellar dynamics — dark matter — elementary particles — Galaxy: center

1. INTRODUCTION

The determination of the mass distribution near the center of our Galaxy and the question of whether it harbors a supermassive black hole (BH) are long-standing issues (Oort 1977; Genzel & Townes 1987; Genzel, Hollenbach, & Townes 1994; see Ho 1998 for a recent review). Various techniques have been used to find the mass of this supermassive compact dark object, which is usually identified with the radio source Sagittarius A* (Sgr A*) at or near the Galactic center. The most detailed information to date comes from statistical analysis of the dynamics of stars moving in the gravitational field of the central mass distribution (Sellgren et al. 1987, 1990; Rieke & Rieke 1988; McGinn et al. 1989; Lindqvist, Habing, & Winnberg 1992; Haller et al. 1996; Eckart & Genzel 1996; Genzel et al. 1996, 1997; Eckart & Genzel 1997; Ghez et al. 1998). Genzel et al. (1997) have established that the central dark object has a mass of $(2.61 \pm 0.76) \times 10^6 \, M_\odot$, concentrated within a radius of 0.016 pc and located very close to Sgr A*. In the most recent observations, Ghez et al. (1998) confirm a mass of $(2.6 \pm 0.2) \times 10^6 \, M_\odot$, enclosed within a radius of 0.015 pc. In the latter observations, the accuracy of the velocity measurements in the central arcsec$^2$ has been improved considerably, and thus the error bar on the central mass has been reduced by about a factor of 4. In both data sets, the presence of a supermassive compact dark object is revealed by the fact that several stars are moving within a projected distance of less than 0.01 pc from the central radio source Sgr A* at projected velocities in excess of 100 km s$^{-1}$.

For completeness, we mention here that the mass distribution at the Galactic center could also be studied through the motion of gas clouds and streamers (Lacy, Townes, & Hollenbach 1980; Genzel & Townes 1987; Lacy, Achtermann, & Serabyn 1991). However, gas flows may be easily perturbed by nongravitational forces such as shocks, radiation pressure, winds, magnetic fields, etc., and hence this probe is considered to be less reliable for determining the mass of the compact dark object at the Galactic center.

The nonthermal spectrum of Sgr A* (Serabyn et al. 1997), which has been shown to originate from a very compact source (Rogers et al. 1994; Genzel et al. 1997; Ghez et al. 1998), and the low proper motion of Sgr A* (Backer 1996) have led many (e.g., Lynden-Bell & Rees 1971) to suggest that Sgr A* may be a supermassive BH of mass $\sim 2.6 \times 10^6 \, M_\odot$. Supermassive BHs have also been inferred for several other galaxies, such as M87 (Ford et al. 1994; Harms et al. 1994; Macchetto et al. 1997) and NGC 4258 (Greenhill et al. 1995; Myoshi et al. 1995). Taking this suggestion seriously, one is immediately faced with fundamental issues such as the prevalence of supermassive BHs in the nuclei of normal galaxies and the nature of the accretion mechanism that makes Sgr A* so much fainter than typical active galactic nuclei (Melia 1994; Narayan, Yi, & Mahadevan 1995). However, since the best current observations probe the gravitational potential at radii $4 \times 10^4$ larger than the Schwarzschild radius of a BH of mass $2.6 \times 10^6 \, M_\odot$ (Ghez et al. 1998), it is perhaps prudent not to focus too much on the BH scenario without having explored alternative scenarios for the supermassive compact dark object.

One alternative to the BH scenario is a very compact stellar cluster (Haller et al. 1996; Sanders 1992). However, based on the evaporation and collision time stability criteria, it is doubtful that such clusters could have survived up to the present time (see Moffat 1997 for an alternative point of view). Indeed, in the case of our Galaxy and NGC 4258, Maoz (1995, 1998) has found that even the lower limits to the half-mass densities of such compact clusters ($1 \times 10^{12} \, M_\odot \, pc^{-3}$ for NGC 4258 and $6 \times 10^{11} \, M_\odot \, pc^{-3}$ for our Galaxy) are too large to be due to stable clusters of stellar or substellar remnants. The estimated maximal lifetimes for such dense clusters are about $10^8$ yr for our Galaxy and a few $\times 10^6$ yr for the NGC 4258, i.e., much shorter than the age of the universe. This seems to rule out the existence of dense clusters at the centers of the above-mentioned galaxies, unless we are prepared to believe that we happen to live in a privileged epoch of the lifetime of the universe. Note, however, that for other galaxies, such as M31, M32, M87, NGC 3115, NGC 3377, NGC 4261, NGC 4342, NGC 4486B, and NGC 4594, maximal lifetimes of dense stellar clusters are in excess of $10^{11}$ yr. Moreover, it should be acknowledged that the uncertainties in current understanding of the core collapse process of such dense clusters still leave some room for speculation about a possible interpretation of the supermassive compact dark objects at the...
centers of galaxies (including both our Galaxy and NGC 4258) in terms of, e.g., core-collapsed clusters (Maoz 1998). However, apart from a cluster of very low mass BHs that is free of stability problems, the most attractive alternative to a dense stellar cluster is a cluster of elementary particles.

In fact, in the recent past, an alternative model for the supermassive compact dark objects in galactic centers has been developed (Viollier et al. 1992, 1993; Viollier 1994; Tsiklauri & Viollier 1996, 1998a, 1998b, 1999; Bilić et al. 1998, 1999). The cornerstone of this model is that the dark matter at the center of galaxies is made of nonbaryonic matter in the form of massive neutrinos that interact gravitationally, forming supermassive neutrino balls in which the degeneracy pressure of the neutrinos balances their self-gravity. Such neutrino balls could have been formed in the early universe during a first-order gravitational phase transition (Bilić & Viollier 1997, 1998, 1999a, 1999b). In fact, it has recently been shown that the dark matter concentration observed through stellar motion at the Galactic center (Eckart & Genzel 1997; Genzel et al. 1996) is consistent with a supermassive object of $2.5 \times 10^6 M_\odot$ made of self-gravitating, degenerate heavy neutrino matter (Tsiklauri & Viollier 1998a). Moreover, it has been shown that an acceptable fit to the infrared and radio spectrum above 20 GHz, which is presumably emitted by the compact dark object, can be produced in the framework of standard accretion disk theory (Tsiklauri & Viollier 1999; Bilić et al. 1998), in terms of a baryonic disk imbedded in the shallow potential of the degenerate neutrino ball of $2.5 \times 10^6 M_\odot$.

The purpose of this paper is to compare the predictions of these two models for the supermassive compact dark object at the center of our Galaxy, i.e., (1) the black hole scenario and (2) the degenerate neutrino ball scenario as an example of an extended object. Neither of these models contradicts the technologically challenging proper motion observations and their statistical interpretation (Genzel et al. 1997; Ghez et al. 1998). It is therefore desirable to have an additional independent dynamical test, in order to distinguish between these two possible scenarios for describing the compact dark object at the center of our Galaxy. In the recent past, mainly statistical arguments involving many stars have been used to determine the gravitational potential at the Galactic center. However, in this paper we would like to demonstrate that it is also possible to draw definite conclusions from the motion of individual stars, in particular in the immediate vicinity of the Galactic center, where statistical arguments cannot be easily applied due to the low density of stars. To this end, we have recently calculated the orbits (Munyaneza, Tsiklauri, & Viollier 1998) of the fastest moving infrared source, S1, using the Genzel et al. (1997) data for a supermassive BH or a neutrino ball mass of $2.61 \times 10^6 M_\odot$. We have shown that tracking the orbits of S1 offers a good opportunity to distinguish in a few years time between the two scenarios for the supermassive compact dark object. Here we perform a full analysis of the orbits of the same star, S0-1, based on the most recent Ghez et al. (1998) data, including all the error bars of the measurements. A distance to the Galactic center of 8 kpc has been assumed throughout.

This paper is organized as follows. In § 2, we present the equations that describe degenerate neutrino balls and we establish some constraints on the neutrino mass based on the Ghez et al. (1998) data. In § 3 we study the dynamics of S0-1, and we conclude with the discussion in § 4.

2. THE COMPACT DARK OBJECT AS A NEUTRINO BALL

Dark matter at the Galactic center can be described by the gravitational potential, $\Phi(r)$, of the neutrinos and antineutrinos that satisfies Poisson's equation,

$$\Delta \Phi = 4\pi G \rho_v ,$$

(1)

where $G$ is Newton's gravitational constant and $\rho_v$ is the mass density of the neutrinos and antineutrinos. Neutrino matter will interact gravitationally to form supermassive neutrino balls in which the self-gravity of the neutrinos is balanced by their degeneracy pressure, $P_v(r)$, according to the equation of hydrostatic equilibrium,

$$\frac{dP_v}{dr} = - \frac{d\Phi}{dr} .$$

(2)

In order to solve equation (1), one needs a relation between the pressure, $P_v$, and the density, $\rho_v$. To this end, we choose the polytropic equation of state of degenerate neutrino matter, i.e.,

$$P_v = K \rho_v^{5/3} ,$$

(3)

where the polytropic constant $K$ is given by (Viollier 1994)

$$K = \left( \frac{6}{g} \right)^{2/3} \frac{\pi^{4/3} h_0^2}{5m_\nu^{8/3}} .$$

(4)

Here $m_\nu$ denotes the neutrino mass, $g$ is the spin degeneracy factor of the neutrinos and antineutrinos, i.e., $g = -2$ for Majorana and $g = 4$ for Dirac neutrinos and antineutrinos. We now introduce the dimensionless potential and radial variable, $v$ and $x$, by

$$\Phi(r) = \frac{GM_\odot}{a_v} \left[ v(x_0) - \frac{v(x)}{x} \right] ,$$

(5)

$$r = a_v x ,$$

(6)

where $x_0$ is the dimensionless radius of the neutrino ball, and the scale factor $a_v$, which here plays the role of a length unit, is given by

$$a_v = 2.1376 \text{ lt-yr} \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^{8/3} g_\nu^{-2/3} .$$

(7)

Assuming spherical symmetry, we finally arrive at the nonlinear Lane-Emden equation

$$\frac{d^2v}{dx^2} = - \frac{v^{3/2}}{x^{1/2}} ,$$

(8)

with polytropic index 3/2. The boundary conditions are chosen in such a way that $v$ vanishes at the boundary $x_0$ of the neutrino ball. The mass $M_B$ of a (pointlike) baryonic star at the center of the neutrino ball is fixed by $v(0) = M_B/M_\odot$. The case of $M_B = 0$ corresponds to a pure neutrino ball without a pointlike source at the center. The mass enclosed within a radius $r$ in a pure neutrino ball can be written in terms of $v(x)$ and its derivative $\dot{v}(x)$ as

$$M(r) = \int_0^r 4\pi \rho_v r^2 \, dr = M_\odot[v(x) - v(x)] .$$

(9)
Using equations (5)–(9), for a pure neutrino ball we obtain the scaling relation
\[
MR^3 = \frac{91.869 \hbar^6}{G^3 m_\nu^6} \left( \frac{2}{g_\nu} \right)^2 
= 6.3003 \times 10^{10} M_\odot \text{ light-day}^3 \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^8 g_\nu^{-2}.
\] (10)

Here \( M = M(R) \) and \( R \) denote the mass and radius of the neutrino ball, respectively. Thus, as the mass of the neutrino ball increases, its radius decreases until the degeneracy pressure is no longer able to support the gravitational pressure, because the neutrinos now obey the relativistic equation of state. The maximum value of the mass of a neutrino ball, i.e., the Oppenheimer-Volkoff (OV) limit, is given by (Bilic et al. 1999)
\[
M_{OV} = 0.54195 \left( \frac{\hbar c}{G} \right)^{3/2} m_\nu^{-2} g_\nu^{-1/2} 
= 2.9924 \times 10^9 M_\odot \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^2 g_\nu^{-1/2}.
\] (11)

The radius of such a neutrino ball that is close to being a black hole is
\[
R_{OV} = 4.8329 \left( \frac{\hbar c}{G} \right)^{1/2} m_\nu^{-2} g_\nu^{-1/2} 
= 1.5171 \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^2 g_\nu^{-1/2} \text{ light-day} 
= 4.4466 R_\odot,
\] (12)
where \( R_{OV} \) is the Schwarzschild radius of the mass \( M_{OV} \).

In order to describe the compact dark object at the Galactic center as a neutrino ball and constrain its physical parameters appropriately, it is worthwhile to use the most recent observational data from Ghez et al. (1998), who established that the mass enclosed within 0.015 pc at the Galactic center is \((2.6 \pm 0.2) \times 10^6 M_\odot\). Following the analysis of Tsiklauri & Viollier (1998a), we choose the minimal neutrino mass, \( m_\nu \), to reproduce the observed matter distribution, as can be seen from Figure 1, where we have added the Ghez et al. (1998) and Genzel et al. (1997) data points with error bars. In Figure 1 we include only the neutrino ball contribution to the enclosed mass, since the stellar cluster contribution is negligible by orders of magnitude at these radii. For a \( M = 2.4 \times 10^6 M_\odot \) neutrino ball, the constraints on the neutrino mass are \( m_\nu \geq 17.50 \text{ keV}/c^2 \) for \( g_\nu = 2 \) and \( m_\nu \geq 14.72 \text{ keV}/c^2 \) for \( g_\nu = 4 \), and the radius of the neutrino ball is \( R \leq 1.50 \times 10^{-2} \text{ pc} \). Using the value of \( M = 2.6 \times 10^6 M_\odot \), the bounds on the neutrino mass are \( m_\nu \geq 15.92 \text{ keV}/c^2 \) for \( g_\nu = 2 \) or \( m_\nu \geq 13.39 \text{ keV}/c^2 \) for \( g_\nu = 4 \), and the radius of the neutrino ball turns out to be \( R \leq 1.88 \times 10^{-2} \text{ pc} \). Finally, for a \( M = 2.8 \times 10^6 M_\odot \) neutrino ball, the range of neutrino mass is \( m_\nu \geq 15.31 \text{ keV}/c^2 \) for \( g_\nu = 2 \) and \( m_\nu \geq 12.87 \text{ keV}/c^2 \) for \( g_\nu = 4 \), and the corresponding neutrino ball radius \( R \leq 2.04 \times 10^{-2} \text{ pc} \).

In the following discussion, we restrict our calculations to the minimal neutrino mass or the largest neutrino ball size that is consistent with the Ghez et al. (1998) data, in order to emphasize the differences between the neutrino ball and black hole scenarios. By varying the neutrino mass between the minimal and maximal values given by the OV limit, i.e., \( m_\nu = 491 \text{ keV}/c^2 \) for \( g_\nu = 2 \) and \( M_{OV} = 2.60 \times 10^6 M_\odot \), one can interpolate smoothly between the extended neutrino ball and the “almost black hole” scenarios. However, it is important to note that the duration of the neutrino burst of SN 1987A restricts the possible Dirac neutrino mass to \( m_\nu \lesssim 30 \text{ keV}/c^2 \) (Raffelt 1999). This restriction is based on the fact that trapped Dirac neutrinos eventually produce their sterile components in nuclear collisions in the nascent neutron star. The sterile neutrinos would escape too early to be consistent with the duration of the neutrino burst of SN 1987A for \( m_\nu \lesssim 30 \text{ keV}/c^2 \). Moreover, if we want to explain the most massive compact dark objects at the centers of galaxies, e.g., at the center of M87 with \( M = (3.2 \pm 0.9) \times 10^9 M_\odot \), as a stable degenerate neutrino star at or below the OV limit, the neutrino mass must be smaller than \( 16.5 \text{ keV}/c^2 \) for \( g_\nu = 2 \) (Bilic et al. 1999).

3. DYNAMICS OF S0-1

We investigate the motion of S0-1, which is the star closest to the Galactic center, and at the same time also the fastest of the 15 stars in the central arcsec\(^2\) around Sgr A*. We study the motion of S0-1 in the gravitational potential near Sgr A*, assuming that the central object is either a BH of mass \( M \) or a spatially extended object represented by a neutrino ball of mass \( M \) that consists of self-gravitating degenerate heavy neutrino matter. The BH or neutrino ball mass \( M \) is taken to be \( 2.4, 2.6, \) or \( 2.8 \times 10^6 M_\odot \), which corresponds to the range allowed by the Ghez et al. (1998) data. We use Newtonian dynamics, since the problem is essentially nonrelativistic, because the mass of the neutrino ball is much less than the Oppenheimer-Volkoff limit corresponding to this particular neutrino mass (Bilic et al. 1999). Consequently, we can write Newton’s equations of...
where $x$ position of Sgr A*. The dot denotes the derivative with that the center of the neutrino ball or the BH is at the $r$ vector of the star $S_0-1$, and $\nu$ is the neutrino ball, $M$ when the coordinates of $S_0-1$ were and $R.A.$ velocity are km s$^{-1}$. The filled squares denote the time labels. The period of $S_0-1$ in the BH scenario is bounded by minimal and maximal distances from Sgr A* of 1.49 and 7.18 light-days, respectively. The orbit of $S_0-1$ in the neutrino ball scenario is bounded by minimal and maximal distances from Sgr A* of 3.98 and 42.07 light-days, respectively.

motion as

$$\ddot{x} = -\frac{GM(r)}{(x^2 + y^2 + z^2)^{3/2}} x,$$

(13)

$$\ddot{y} = -\frac{GM(r)}{(x^2 + y^2 + z^2)^{3/2}} y,$$

(14)

$$\ddot{z} = -\frac{GM(r)}{(x^2 + y^2 + z^2)^{3/2}} z,$$

(15)

where $x$, $y$, and $z$ denote the components of the radius vector of the star $S_0-1$, and $r = (x^2 + y^2 + z^2)^{1/2}$, Sgr A* being the origin of the coordinate system. We thus assume that the center of the neutrino ball or the BH is at the position of Sgr A*. The dot denotes the derivative with respect to time. In the case of a BH, $M(r) = M$ is independent of $r$, while in the neutrino ball scenario, $M(r)$ is given by equation (9), reaching $M(R) = M$ at the radius of the neutrino ball, $R$. The initial positions and velocities for this system of equations are taken to be those of $S_0-1$ in 1995.4, when the coordinates of $S_0-1$ were R.A. $= -0^\circ 107$ and decl. $= 0^\circ 039$. The $x$- and $y$-components of the projected velocity are $v_x = 470 \pm 130$ km s$^{-1}$ and $v_y = -1330 \pm 140$ km s$^{-1}$ (Ghez et al. 1998), respectively. Here $x$ is opposite to the R.A. direction, and $y$ is in the decl. direction.

In Figure 2 we plot two typical orbits of $S_0-1$, corresponding to a BH and neutrino ball mass of $M = 2.6 \times 10^6 M_\odot$. The input values for $v_x$ and $v_y$ are 470 and $-1330$ km s$^{-1}$, respectively. The $z$-coordinate of the star $S_0-1$ is assumed to be zero, and the velocity component in the line of sight of the star $S_0-1$, $v_z$, has also been set equal to zero in this graph. The filled square labels denote the time in yr from 1990 to 2015. In the case of a BH, the orbit of $S_0-1$ is an ellipse, with Sgr A* being located at one focus (denoted by the star in the figure). The period of $S_0-1$ is 12.7 yr, and the minimal and maximal distances from Sgr A* are 1.49 and 7.18 light-days, respectively. In the case of a neutrino ball, the orbit will be bound but not closed, with minimal and maximal distances from Sgr A* of 3.98 and 42.07 light-days, respectively. It can be seen from Figure 2 that in the case of a neutrino ball, $S_0-1$ is deflected much less than for a BH, since the gravitational force at a given distance from Sgr A* is determined by the mass enclosed within this distance. Using equation (9), we can estimate the mass enclosed within a radius corresponding to the projected distance of $S_0-1$ from Sgr A* (4.41 $\times 10^{-3}$ pc) to be $\sim 1.8 \times 10^5 M_\odot$. Thus, in the case of a neutrino ball, the force acting on $S_0-1$ is about 14 times less than in the case of a BH. This graph can serve to establish whether Sgr A* is a BH or an extended object, as a consequence of the fact that the positions of $S_0-1$ will differ as time goes on in the two scenarios. However, this conclusion is perhaps too optimistic, since we have not yet considered (1) the uncertainties in $v_x$ and $v_y$; (2) the error bars in R.A. direction, and (3) the complete lack of information on $z$ and $v_z$.

As a next step, we investigate the dependence of the orbits on the uncertainties in the velocity components. The results of this calculation are presented in Figure 3, where we have set $z = v_z = 0$. In the case of a BH, the orbits of $S_0-1$ are ellipses, while the other five thick lines are bound orbits of $S_0-1$ for the neutrino ball scenario. The spread of the orbits induced by the error bars in $v_x$ and $v_y$ is small compared to that of the recent analysis based on the Genzel et al. (1997) data (Munyaneza et al. 1998). The time labels, represented...
Projected orbits of the star S0-1 in the case of a BH or neutrino ball with $M = 2.4 \times 10^6 M_\odot$. In this graph we explore how the orbits are affected by the uncertainty in the mass of the BH or neutrino ball. The orbits are calculated for $z = v_z = 0$. Orbits are the same as in Fig. 3. The periods of S0-1 in the BH scenario vary between 11 and 20 yr, and all the orbits are bound in both scenarios.

In Figures 4 and 5 we present the results of our calculations for both scenarios, with central masses of $M = 2.4 \times 10^6 M_\odot$ and $M = 2.8 \times 10^6 M_\odot$, respectively. The neutrino masses consistent with the Ghez et al. (1998) data are $m_\nu \geq 17.50$ keV$/c^2$ for a $M = 2.4 \times 10^6 M_\odot$ neutrino ball and $m_\nu \geq 15.31$ keV$/c^2$ for a $M = 2.8 \times 10^6 M_\odot$ neutrino ball. The filled squares represent the time labels, spaced at 5 yr intervals, as in Figure 3. In the BH scenario, the periods of S0-1 with $M = 2.4 \times 10^6 M_\odot$ vary between 11 and 20 yr, while in the case of $M = 2.8 \times 10^6 M_\odot$, the periods vary between 9.5 and 15 yr. Comparing the orbits of S0-1 in Figures 4 and 5 with those in Figure 3, we conclude that the errors bars in the total mass of the BH or neutrino ball make no qualitative difference for the motion of S0-1. In both scenarios of the supermassive compact dark object, all the orbits considered for three different values of the BH or neutrino ball mass are bound for $z = v_z = 0$, as can be seen from Figures 6 and 7, where the escape and circular velocities are plotted as functions of the distance from Sgr A*.

In these graphs, we have also included the Ghez et al. (1998) data with error bars for the 15 stars in the central arcsec, assuming that the velocity component and distance from Sgr A* in the line of sight are both zero, i.e., $v_x = 0$ and $z = 0$. Thus, the data points are lower bounds on the true circular or escape velocity and radius, and the real values lie in the quarter-plane to the right of and above the measured data point. For instance, the innermost data point describing the star S0-1 is in both scenarios consistent with a bound orbit if $|z|$ and $|v_z|$ are not too large, as can be seen from the escape velocity in Figure 6. However, S0-1 cannot be interpreted as a virialized star in the neutrino ball scenario, as is evident from the plot of the circular velocity in Figure 7; it thus would have to be an intruder star. If the projected velocity of a star at a given projected distance from Sgr A* is larger than the escape velocity at the same distance (assuming $z = 0$), the neutrino ball scenario is virtually ruled out, since the kinetic energy of the star would have to be very large at infinity.
Fig. 7.—Circular velocity as a function of the distance from Sgr A* for BH and neutrino ball scenarios. The mass of the central object is varied as indicated on the graph. The data points with error bars of 15 stars in the central arcsec are taken from Ghez et al. (1998), assuming that the projected velocity and distance from Sgr A* are equal to the true velocity and distance, respectively, i.e., \( z = 0 \) and \( v_z = 0 \). This graph shows that the orbits of S0-1 are almost circular in the case of the BH scenario (see text for discussion concerning the innermost data point).

We now turn to an investigation of the dependence of the orbits on the \( z \)-coordinate and \( z \)-component of the velocity of the star S0-1. The two quantities, \( z \) and \( v_z \), are the major source of uncertainty in determining the exact orbit of the star S0-1. However, this shortcoming will not substantially affect the predictive power of our model, as we demonstrate below. In Figure 8 we show the results of a calculation of the dependence of the orbit on \( z \) for a mass \( M = 2.6 \times 10^6 M_\odot \) neutrino ball or BH. The input values for \( v_x \) and \( v_y \) are fixed at \( 470 \text{ km s}^{-1} \) and \( -1330 \text{ km s}^{-1} \), respectively, and \( v_z \) is assumed to be zero. The \( z \)-coordinate is varied from zero up to the radius of the neutrino ball, i.e., the distance from Sgr A* beyond which there is obviously no difference between the BH and the neutrino ball scenarios. In this case, the radius of the neutrino ball is \( 1.88 \times 10^{-2} \text{ pc} \), or 0.485. The top panel of Figure 8 represents the orbits in the case of a BH, for different values of \( z \), while the bottom panel describes the dependence of the orbit on \( z \) in the neutrino ball scenario. We conclude from this plot that increasing \( |z| \) has the effect of shifting the orbits toward the lower right corner of the graph. This is obviously due to the fact that increasing \( |z| \) means going farther away from the scattering center, thus yielding less deflection of the orbit. Moreover, in the neutrino ball scenario, the dependence on \( z \) is relatively insignificant, as long as \( |z| \) is smaller than the radius of the neutrino ball. This is in accordance with the fact that for small distances from the center, the potential of a neutrino ball can be approximated by a harmonic oscillator–type potential, where the Newtonian equations of motion decouple in Cartesian coordinates. The dependence of the orbits of S0-1 on \( v_z \) has a similar effect as in the previous graph. Here we have fixed \( z \) to zero, and \( v_z \) has been varied as an input parameter. Increasing \( |v_z| \) yields a greater velocity of the star, and obviously a fast-moving star will be deflected less than a star with smaller \( |v_z| \). The results of this calculation are summarized in Figure 9.

4. CONCLUSION AND DISCUSSION

We have demonstrated that the orbits of S0-1 differ substantially for the BH and neutrino ball scenarios of the Galactic center, especially with the new Ghez et al. (1998) data. We have shown that using these data, the error bars on velocities of S0-1 and the mass of the central object do not change the pattern of the orbits of S0-1. The orbit of S0-1 is much more curved in the case of a BH than in the neutrino ball scenario, as long as \( |z| \) is smaller than the radius of the neutrino ball. Increasing \( |z| \) and \( |v_z| \) shifts the orbits to the lower right corner of the graph, and this gives us a key to establish the allowed regions of S0-1, depending on whether it is a BH or a neutrino ball, and...
Projected orbits of the star S0-1 in the case of a supermassive BH (top) and in the case of a neutrino ball (bottom) of $M = 2.6 \times 10^6 M_\odot$. In this graph we explore how the orbits are affected by the uncertainty in $v_x$. The labels for the orbits are given in the graph. Here $v_x = 470 \text{ km s}^{-1}$, $v_y = -1330 \text{ km s}^{-1}$, and $z = 0$.

Irrespective of the values of the parameters $z$ and $v_z$, Figure 9 three orbits are plotted: the upper leftmost orbit of S0-1, corresponding to the neutrino ball scenario (line 9 of Fig. 4), and two orbits in a BH scenario with the smallest and maximal distances from Sgr A* (ellipses 2 and 4 from Fig. 5). This figure serves as a test to distinguish the supermassive BH scenario from the neutrino ball model of the Galactic center. It is clear that as the observations proceed within the next year, one might be able to tell the difference between the two models of the supermassive compact dark object at the center of our Galaxy.

Figure 10.—Prediction regions for the supermassive central object. This graph combines line 9 from Fig. 4 and lines 2 and 4 from Fig. 5. If the star S0-1 is found inside the ellipses (region F), this will rule out both the BH and the neutrino ball models. If the star S0-1 is eventually found in the upper left zone of the graph, i.e., above and left of the thick orbit, this will rule out the neutrino ball interpretation for the chosen neutrino mass. Finally, if S0-1 is found to the right of and below the thick line, then the supermassive central object should be interpreted either as a BH with large $z$ or as a neutrino ball.

If the star is found in the region $F$ inside the ellipses, this will rule out both the BH and the neutrino ball scenario of Sgr A*, as seen in Figure 10. We can estimate the minimal distance of approach to Sgr A* to be 0.909 light-days. If the orbit of S0-I ends up in the upper left zone of the thick line, this will clearly rule out the neutrino ball scenario for the chosen lower limit of the neutrino mass. However, if S0-1 is found in the lower right corner of the same line (i.e., below the thick line), then the supermassive object can be interpreted as either a neutrino ball or a BH with a large $z$ or $v_z$ parameter. One can, of course, repeat this analysis for several stars in the central arcsec$^2$ and use some statistical arguments: should there be no stars in the black hole zone, and many stars found in the zone for black holes and neutrino balls, the black hole interpretation of the supermassive compact dark object at the Galactic center would become less attractive, since some of the stars should be moving close to the plane perpendicular to the line of sight, i.e., they should have small $|v_z|$ and $|z|$.

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