Reconfigurable Origami-inspired Acoustic Waveguides

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ABSTRACT
We combine numerical simulations and experiments to design a new class of reconfigurable waveguides based on 3D origami-inspired metamaterials. Our strategy builds on the fact that the rigid plates and hinges forming these structures define networks of tubes that can be easily reconfigured. As such, they provide an ideal platform to actively control and redirect the propagation of sound. Interestingly, we design reconfigurable systems that, depending on the externally applied deformation, can act as networks of waveguides oriented along either one or two or three preferential directions. Moreover, we demonstrate that the capability of the structure to guide and radiate acoustic energy along well predefined directions can be easily switched on and off, as the networks of tubes are reversibly formed and disrupted. The proposed designs expand the ability of existing acoustic metamaterials and exploit complex waveguiding to enhance the control over propagation and radiation of acoustic energy, opening avenues for the design of a new class of tunable acoustic functional systems.

acoustic waveguide | origami | reconfigurable | sound | metamaterial

INTRODUCTION
Acoustic waveguides designed to direct sound are ubiquitous and can be found in cars, buildings, jet engines, medical devices and musical instruments, just to name a few. While most of the proposed acoustic waveguides consist of a single duct, it is well known that carefully connected tubes can result in significant transmitted noise reduction [1, 2]. Moreover, it has also been shown that the propagation of acoustic waves in tubes arranged to form a square lattice can be successfully described with tools from solid state physics and provides opportunities to control sound through dispersion and band gaps [3]. Finally, three-dimensional networks of waveguides have been used to study sound propagation in regular urban areas [4]. Although these examples illustrate the potential of acoustic waveguides with more complex geometry, they only cover a small region of the available design space and a natural question to ask is how the geometry of the three-dimensional networks of tubes affects the propagation of sound.

Origami [5] - the ancient art of paper folding - not only results in intricate and aesthetically pleasant designs, but also provides an ideal platform for the design of transformable mechanical metamaterials. In particular, two-dimensional sheets folded along pre-defined creases have enabled the design of multistable structures [6–9], materials with negative Poisson’s ratio [10–12] and tunable stiffness [13] and topological metamaterials [14]. While most of the proposed origami-like metamaterials are based on two-dimensional folding patterns, snapology [15, 16] - a modular origami technique - has recently inspired the design of highly reconfigurable three-dimensional metamaterials assembled from extruded polyhedra [17, 18]. Importantly, these designs also result in interconnected and reconfigurable networks of tubes defined by the assembly of rigid plates and elastic hinges. In this article we combine experiments and simulations to demonstrate that such three-dimensional networks of tubes can be exploited to design reconfigurable acoustic waveguides capable of efficiently controlling and redirecting the propagation of sound.

RESULTS
Reconfigurable acoustic waveguides based on extruded cubes. We start by considering a three-dimensional mechanical metamaterial consisting of a cubic array of connected extruded cubes (see Fig. 1). If we assume that all the faces are rigid and the structure can only fold along the edges, this periodic structure will have three degrees of freedom identified by the angles α1, α2, and α3 [17]. Importantly, changing these three angles not only deforms the assembly of plates into numerous specific shapes, but also significantly alters the network of channels defined by them (see Fig. 1 and movie S1), providing an ideal platform for the design of reconfigurable acoustic waveguides.

More specifically, for (α1) = (π/2, π/2, π/2) the system is fully expanded and comprises a three-dimensional network of interconnected channels oriented in three perpendicular directions (Fig. 1A). In this configuration the tubes are acoustically coupled, and we expect the radiation by the structure to take place in all three directions, covering the entire surrounding space without a specific directivity. Differently, for (α1, α2, α3) = (π/2, π/2, 0) and (π/3, 2π/3, π/3) the channels are all parallel to each other (Fig. 1, B and C), so that sound waves can only propagate in one direction. However, for (α1, α2, α3) = (π/2, π/2, 0) the channels are disjointed, while for (α1, α2, α3) = (π/3, 2π/3, π/3) they are all connected. Therefore, for (α1, α2, α3) = (π/2, π/2, 0) we expect the system to behave as a classical single-tube waveguide, while for (α1, α2, α3) = (π/2, 2π/3, π/3) we expect the additional wave interferences in the structure to result in a more complex frequency response and a variety of radiation patterns.

To demonstrate our ideas, we investigate both numerically and experimentally the propagation of sound waves through the proposed reconfigurable metamaterial. In the simulations we construct three-dimensional models of the metamaterial comprising a 4 × 4 × 4 cubic array of extruded cubes and deform them into the three different configurations shown in Fig. 1. The plates forming the structure are considered as reflective rigid boundaries and the air inside and around the resulting tubes is meshed using acoustic elements (Abaqus element type AC3D10M). Moreover, non-reflecting boundary conditions are imposed on the outer boundaries of the acoustic medium to avoid the reflection of energy back into it. Finally, acoustic waves are excited
by applying a harmonically varying pressure to one of the openings (highlighted in red in Fig. 1) and the steady-state dynamic linearized response of the system is calculated for a wide range of frequencies using the commercial Finite Element package Abaqus.

Furthermore, we fabricate a centimeter-scale prototype of the metamaterial from polymeric sheets (PET) and double-sided tape, using a stepwise layering and laser-cutting technique [17, 19] (see Materials and Methods for more details). To measure the transmission response of the three-dimensional waveguide, acoustic waves are excited through air inside the tubes using a loudspeaker placed at one end of one of the tubes (depicted by the red S in Fig. 2). Then the amplitudes of both the excited and scattered pressure waves are recorded using two microphones (model 378B02, PCB Piezotronics) mounted near the input loudspeaker (point S) and outlets (point A). Finally, the transmittance is computed in dB as the ratio between the output and input amplitude signals (i.e., $20 \log_{10} \left( \frac{A_A(f)}{A_S(f)} \right)$). Note that during the tests the sample is surrounded by sound absorb-ERS, structure and mimic free field conditions. These foam layers are re-sonance frequencies are given by $f_{\text{res}} = \frac{c_0}{2L} \sqrt{\left( \frac{n\pi}{a} \right)^2 + \left( \frac{m\pi}{a} \right)^2 + \left( \frac{\pi}{L} \right)^2}$, (1) where $c_0 = 343.2 \text{ m/s}$ is the speed of sound in air and $L$, $m$, and $n$ are three integers characterizing the waveguide modes. Since in our system $L = 14.2 \text{ cm}$, its lowest modes are characterized by $l = m = 0$ and correspond to standing plane waves along the tube. In fact, for $a = 3 \text{ cm}$ as in our prototype, non-planar mode across the waveguide section can propagate only for $f > f_{100} = f_{010} = c_0/(2a) = 5.7 \text{ kHz}$. We also note that, while Eq. (1) provides a good qualitative estimation of the resonances of the tube, it is insufficient for quantitative comparison with experiments [22]. In fact, the wave radiation at the tube open-ends results in a deviation from the zero pressure condition considered in deriving Eq. (1). Classically, this effect can be accounted for by adding a correction length $\delta$ to the physical length $L$ of the tube, which depends on the details of the tube geometry. Here, by comparing analytical and numerical results, we find a good agreement for $\delta = 0.3 \text{ cm}$, so that the resonance frequencies of the planar modes are given by $f_{\text{res}} = c_0/2(L + 0.6a)$. Finally, we find a very good agreement between our analytical, numerical and experimental results (see Fig. 3C), with only a few dB level discrepancy between simulated and experimental curves. This discrepancy is mainly due to the exact placement of the speaker and microphone at the tube opening in the experiments as well as to the small influence of the speaker/microphone presence on the wave field.

Next, in Fig. 4 we present results for the configuration defined by $(\alpha_1, \alpha_2, \alpha_3) = (\pi/3, 2\pi/3, \pi/3)$. Although in this case the channels are all also oriented along the same direction, they have a rhombic cross section and are interconnected inside the structure. Conse-quentlly, when deformed into this state the metamaterial is characterized by a totally different acoustic response. This is clearly demonstrated in Fig. 4, A and B, where we show the transmittance calculated using two different detection points (denoted as A and B in Fig. 4C). Both curves exhibit a strong and complex frequency dependence, originating from the interferences that occur inside the system as the waves can follow a myriad of different paths when traveling from the source to the receiver due to the multiple interconnections. We also find that the connectivity of the tubes reduces the average level of the transmittance over the studied frequency range to around -20 dB, significantly lower than that measured for the configuration of Fig. 3. Moreover, it should be noted that for this case quantitative agreement between simulated and experimental transmittances is not reached. This is because the geometric imperfections (sub-millimeter holes in the corners and narrow gaps between the folded rigid faces, inherent to the origami design - see fig. S6 in the Supplementary Materials) play a much bigger role than in the case of a single tube waveguide, as they significantly alter the path followed by the traveling waves and, consequently, the wave interferences. Finally, we find that not only the transmittance, but also the radiation patterns are strongly frequency dependent (movie S3). For example, at $2 \text{ kHz}$ the wave radiation by the structure gives rise to forward quasi-plane waves, while at $4.8 \text{ kHz}$ the wave fronts are more curved and show complex spatial patterns (Fig. 4C). Interestingly, we find that at $4.8 \text{ kHz}$ the modes propagating in the tubes are also non-planar, although $f < 5.7 \text{ kHz}$. This is due to the interconnections between air channels that increase the effective width of the waveguide structure. Consequently, we expect a multi-modal propagation in the structure to start at lower frequencies (typically $\sim 2 \text{ kHz}$) than those calculated in the case of the independent single tubes (i.e., for $(\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, 0)$).

Finally, in Fig. 5 we focus on the expanded state of the system defined by $(\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, \pi/2)$, for which the wave-guides are interconnected and oriented in three orthogonal directions. Similar to the case of Fig. 4, A and B, the transmittance curves reported in Fig. 5, A and B have an average value of $\sim -20 \text{ dB}$ and show a complex frequency dependence. Due to the waveguide interconnections, numerous interferences from the possible paths come into play even in the case of detection at point A which is aligned with the source. Moreover, as the waveguides are oriented in three different directions, the wave radiation by the structure covers the entire surrounding space (Fig. 4, C and D and movie S4) and we observe a much smaller radiated amplitude behind the structure (opposites to the source) than for the previous configurations of aligned waveguides.

Reconfigurable acoustic waveguides based on different extruded polyhedra.

While so far we have focused on a metamaterial comprising a cubic array of extruded cubes, the proposed strategy to design reconfigurable acoustic waveguides is not restricted to this specific geometry. In fact, a wealth of three-dimensional reconfigurable networks of tubes capable of qualitatively different deformation can be realized by taking space-filling tessellations of convex polyhedra as templates, and extruding arbitrary combinations of their polygonal faces [18]. As an example, in Fig. 6 we consider a metamaterial based on a tessellation of truncated octahedra. While the resulting structure is rigid if all the faces of the truncated octahedra are extruded [18], here we construct a metamaterial with a single degree of freedom (denoted by $\theta$ in Fig. 6A) by extruding the 8 green hexagonal faces (highlighted in green in Fig. 6A), removing 4 of the square faces (highlighted in yellow in Fig. 6A) and making the two remaining ones rigid (highlighted in blue in Fig. 6A). As for the case of the metamaterial based on the extruded cubes, by changing $\theta$ between 0 and $\pi/2$ the architecture of the system can be transformed (see Fig. 6, fig. S3 and movie S5). However, in this case for $0 < \theta < \pi/2$ the metamaterial does not act as an acoustic waveguide, since it does not...
not comprise a network of interconnected tubes (Fig. 6A). Only for \( \theta = 0 \) and \( \theta = \pi/2 \) the plates defining the structures form interconnected channels that can be used to guide acoustic waves. More specifically, for \( \theta = 0 \) all tubes are parallel and disconnected (Fig. 6B), so that the system behaves as a single-tube waveguide and has identical response as the extruded cube waveguide for \( (\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, 0) \) (see Fig. 3). Differently, for \( \theta = \pi/2 \) the folded structure functions as a two-dimensional waveguide (see the red arrows in Fig. 6C). Importantly, this example highlights another interesting feature of our design platform: the ability of switching on and off the guiding of acoustic energy, as for certain configurations the networks of tubes can be reversibly formed and disrupted.

Finally, we use extruded hexagonal prisms to construct a system that can act either as a one-dimensional, a two-dimensional, or a three-dimensional waveguide (Fig. 7). More specifically, the building block of this structure is an hexagonal prism with all its faces extruded except for two square faces that are kept rigid (highlighted in blue in Fig. 7A). The metamaterial formed by connecting these extruded unit cells is highly deformable and characterized by two degrees of freedom, denoted by \( \alpha \) and \( \gamma \) in Fig. 7A. Differently from the structures considered in Fig. 6, here for any admissible combination of \( (\alpha, \gamma) \) the assembly of plates and hinges forms a network of tubes that can be exploited as an acoustic waveguide. In the expanded configuration defined by \( (\alpha, \gamma) = (0, 0) \) the structure acts as a three-dimensional waveguide as the excited waves (red arrow in Fig. 7A) can propagate along three different directions (green arrow in Fig. 7A). However, we can transform the metamaterial into either a two-dimensional (for \( (\alpha, \gamma) = (\pi/4, -\pi/4) \) - Fig. 7C), a one-dimensional (for \( (\alpha, \gamma) = (\pi/4, \pi/4) \)- Fig. 7D), or a three-dimensional waveguide with mutually perpendicular channels (for \( (\alpha, \gamma) = (\pi/4, -\pi/4) \)- Fig. 7B) (see fig. S5 and movie S6).

DISCUSSION

In summary, we propose a new type of reconfigurable acoustic waveguides based on origami principles. Our results indicate that the reconfigurable tubular networks defined by the origami structures can be exploited to achieve very different acoustic responses and wave radiation patterns. Remarkably, the involved acoustic mechanisms are broadband, show a rich behavior in frequency and can be easily reproduced at different scales, from millimeter to meter scale structures.

It should be noted that our strategy to control wave propagation and radiation is based on networks of tubes supporting acoustic waves. Interestingly, these simple systems have not been widely studied yet in this context, as in recent years the control of sound wave propagation has been mostly achieved by structuring waveguides with periodic scatterers [23–25] or resonators [26–28]. While in most of the reported metamaterials the functionality is locked into place once they are fabricated, tunability has also been demonstrated by rotation of the scatterers [29–32], thermal radiation [33], adjustable resonating components [34, 35] and applied mechanical deformation [36]. However, in all these systems the tunability is limited to a relatively narrow frequency range. In contrast, the dramatic reconfiguration of the waveguides proposed here for audible acoustics is broadband and represents a new way of tuning and even switching the propagation of sound. All together, the large number of origami waveguide structures that can be designed, the richness of their deformation modes, as well as the possibility of using both single unit cells and complex ensembles leave many opportunities to guide and control the propagation of waves, with possible applications going well beyond the first demonstration of wave guiding and radiation reported here.

MATERIALS AND METHODS

Fabrication. The metamaterial comprising an array of extruded cubes is fabricated from thin polymeric sheets using an efficient stepwise layering and laser-cutting technique. To fabricate each of the six extruded rhombi that together form a building block (an extruded cube), we use a nearly inextensible polyethylene terephthalate sheet with thickness of \( t = 0.25 \) mm, covered with a double-sided tape layer (3M VHB Adhesive Transfer Tapes F9460PC) with a thickness of \( t = 0.05 \) mm and bonded to a second, thinner polyethylene terephthalate layer (\( t = 0.05 \) mm). Cutting slits are introduced into using a CO2 laser system (VLS 2.3, Universal Laser Systems) and the extruded rhombi could then be formed by removing the parts from the layered sheet, and bonding their ends together.

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SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at ....

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**Fig. 1.** Reconfigurable origami-inspired acoustic waveguides. Experimental and models of the building block (the extruded cube) and the corresponding reconfigurable acoustic metamaterial deformed into three different configurations: (A) \((\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, \pi/2)\); (B) \((\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, 0)\); (C) \((\alpha_1, \alpha_2, \alpha_3) = (\pi/3, 2\pi/3, \pi/3)\). The red arrows and shaded areas indicate the excited waves, while the green arrows and shaded areas highlight the points from which the structure radiates.

| Building block | 4X4X4 acoustic metamaterial |
|----------------|-----------------------------|
| A \(\alpha_1 = \alpha_2 = \alpha_3 = \pi/2\) | ![Central unit](image1) |
| B \(\alpha_1 = \alpha_2 = \pi/2, \alpha_3 = 0\) | ![Model 1](image2) |
| C \(\alpha_1 = \alpha_3 = \pi/3, \alpha_2 = 2\pi/3\) | ![Model 2](image3) |

**Fig. 2.** Experimental setup. Experimental setup without the sound absorbing foams surrounding the sample.
Fig. 3. Propagation of sound waves for \((\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, 0)\). (A) Model of the metamaterial. (B) Top cross-sectional view of the pressure field distribution at \(f = 3.5\, \text{kHz}\). The cutting plate is shown in (A) and the color indicates the pressure amplitude normalized by the input signal amplitude \(p_0\). (C) Frequency-dependent transmittance for the sample. Both experimental (red lines), numerical (blue line) and analytical (dashed black lines) results are shown.

Fig. 4. Propagation of sound waves for \((\alpha_1, \alpha_2, \alpha_3) = (\pi/3, 2\pi/3, \pi/3)\). (A) and (B) Frequency-dependent transmittances for the sample calculated considering two different detection points. Both experimental (red lines) and numerical (blue line) results are shown. (C) Model of the metamaterial, and top cross-sectional view of the pressure field distribution at \(f = 2\, \text{kHz}\) and \(f = 4.8\, \text{kHz}\). The cutting plate is shown in grey (left) and the color map indicates the pressure amplitude normalized by the input signal amplitude \(p_0\) (right).
Fig. 5. Propagation of sound waves for \( (\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, \pi/2) \). (A) and (B) Frequency-dependent transmittances for the sample calculated considering two different detection points. Both experimental (red lines) and numerical (blue line) results are shown. (C) Model of the metamaterial, and cross-sectional view of the pressure field distribution at \( f = 2 \text{kHz} \) and \( f = 4.8 \text{kHz} \). The cutting plate is shown in grey (left) and the color map indicates the pressure amplitude normalized by the input signal amplitude \( p_0 \) (right).

Fig. 6. Reconfigurable acoustic waveguide based on a tessellation of truncated octahedra. Models of the building block and the corresponding reconfigurable acoustic metamaterial deformed into three different configurations: (A) \( \theta = \pi/4 \); (B) \( \theta = 0 \); (C) \( \theta = \pi/2 \).
Fig. 7. Reconfigurable acoustic waveguide based on a tessellation of hexagonal prisms. Models of the building block and the corresponding reconfigurable acoustic metamaterial deformed into four different configurations: (A) \((\alpha, \gamma) = (0, 0)\); (B) \((\alpha, \gamma) = \left(\frac{\pi}{4}, -\frac{\pi}{4}\right)\); (C) \((\alpha, \gamma) = \left(-\frac{\pi}{4}, -\frac{\pi}{4}\right)\); (D) \((\alpha, \gamma) = \left(\frac{\pi}{4}, \frac{\pi}{4}\right)\).