Propagation of slow electromagnetic disturbances in plasma

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Abstract

Electromagnetic (EM) waves/disturbances are typically the best means to understand and analyze an ionized medium like plasma. However, the propagation of electromagnetic waves with frequency lower than the plasma frequency is prohibited by the freely moving charges of the plasma. In dense plasmas though the plasma frequency can be typically quite high, EM sources at such higher frequency are not easily available. It is, therefore, of interest to seek possibilities wherein a low frequency (lower than the plasma frequency) EM disturbance propagates inside a plasma. This is possible in the context of magnetized plasmas. However, in order to have a magnetized plasma response one requires a strong external magnetic field. In this manuscript we demonstrate that the nonlinearity of the plasma medium can also aid the propagation of a slow EM wave inside plasma. Certain interesting applications of the propagation of such slow electromagnetic pulse through plasma is also discussed.
I. INTRODUCTION

The electromagnetic disturbances can get shielded by the moving free charges in the plasma \[ \text{[1]} \]. The plasma contribution in the Maxwell’s equations arises in the form of conduction current which tries to neutralize the displacement current and shields the EM wave propagation through the plasma. The conduction current in the plasma, however, has an upper limit of \( |J| < enc \) (where \( e \), \( n \), and \( c \) are electronic charge, electron number density and the speed of light respectively) which cannot be exceeded under any circumstances. The displacement current on the other hand would keep increasing with the frequency of EM disturbance. Thus a plasma system of a given density would be able to shield the EM disturbance only up to a certain frequency. This exact condition translates to the well known fact that plasma with a given density can shield radiation only up to plasma frequency. Thus the EM radiation with frequencies less than the plasma frequency get reflected from the plasma. This property played a crucial role in radio wave communication during pre-satellite era. The radio wave reception was possible even when transmitters and receivers where not along the line of sight merely because the radio waves got reflected from the ionospheric plasma region. Nowadays, for satellite which hover at higher altitudes than ionosphere, microwaves are used for communication. Microwaves have a higher frequency compared to the plasma frequency associated with the ionospheric plasma layer and hence can propagate through the region without reflection, to reach the satellites \[ \text{[1]} \]. As one gets towards plasmas which are denser, seeking EM sources with frequencies higher than the plasma frequency for propagation becomes difficult. For instance, in the fast ignition concept \[ \text{[2]} \] of laser fusion the ignitor laser pulse is unable to propagate beyond a density of \( 10^{21}/cc \) to deposit energy at the core with densities in the range of \( \sim 10^{25}/cc \) for creating an ignition spark. It is, therefore, of interest to seek possibilities by which a low frequency EM disturbance can be made to propagate inside a dense plasma region. One clear solution is to have a magnetized plasma. For instance the Alfven wave \[ \text{[24]} \] and the Whistler waves \[ \text{[6] [20]} \] with frequencies lower than the plasma frequency do propagate inside the plasma. These EM waves, however, require the presence of an externally applied magnetic field in the medium.

In this paper we show that the nonlinearity of the plasma medium can be exploited for the propagation of slow EM pulses in dense plasmas. In addition we also present certain
interesting application of such EM disturbances. This includes a novel mechanism of guiding, collimating and trapping of the EM pulse by appropriate tailoring of the local plasma density profile. The possibility of splitting a single EM pulse and transporting them individually to distinct desired locations in plasma has also been shown. These EM pulses are also associated with certain electron current configurations in the plasma. Thus electron transportation also occurs along with the propagation of these pulses. This mechanism of electron transport can be viewed as an alternative to other recently proposed schemes which utilize specially structured targets prepared of different materials having varying resistivity transverse to the propagation direction, for efficient transport [4, 5].

The manuscript has been organized as follows. Section II contains the description of such nonlinear EM disturbances which can be made to propagate inside the dense plasma medium. Section III discusses the possibility of exploitation of these structures for novel applications by suitably tailoring the plasma density profile. Section IV contains the summary and discussion.

II. PROPAGATION OF SLOW ELECTROMAGNETIC DISTURBANCES IN HIGH DENSITY PLASMAS

It is well known that the electromagnetic wave with frequencies greater than the plasma frequency only can propagate inside plasma if the plasma is unmagnetized [1]. In the presence of external magnetic field the charged particles motion is unable to shield the displacement current. This is because the charges cannot simply get accelerated along the electric field, when an ambient magnetic field perpendicular to the electric field is present. The presence of magnetic field curves the trajectories of the charge particles and makes them drift along the $\vec{E} \times \vec{B}_0$ direction. Where $\vec{B}_0$ is the external magnetic field. Thus, even in the low frequency domain the displacement current does not get screened. This is responsible for the existence and propagation of low frequency electromagnetic Alfven and whistler wave in magnetized plasmas [6, 24]. The Alfven wave corresponds to frequencies lower than the ion gyrofrequencies making both ion and electron species as magnetized. The Whistler wave on the other hand has frequencies between ion and electron gyrofrequency, and in this case only the electrons species is magnetized. The possibility of shielding by unmagnetized ions
in this case is possible provided the frequency of the EM wave is lower than the ion plasma frequency.

The application of external magnetic field in plasmas is often not feasible. In this manuscript, therefore, we look for alternative mechanism aided by the nonlinearity of the plasma medium to promote the propagation of slow EM disturbance through the plasma medium. We restrict here to that regime of time variations in which the response from the heavy ion species can be ignored. They merely provide a static neutralizing background charge for the plasma medium. In other words we confine our discussions here to the time scale regime of Electron Magnetohydrodynamics (EMHD) model \[6, 12, 14, 16, 18\]. The model also neglects the displacement current under the assumption that the plasma is over-
dense \((\omega_p/\omega > 1 \text{ and } \omega^2_p/\omega_c > 1)\) and the conduction current density far exceeds the displacement current. Under these conditions the electron motion is incompressible and its vorticity equations can be cast in terms of a set of equations for the magnetic field \(B\).

These equations posses exact nonlinear solutions describing current pulse structures and its associated magnetic field in a plasma \[11\]. Two kinds of structure are of special interest. They are two dimensional structures having magnetic fields with monopolar and dipolar symmetries depicted in Fig.\([1]\). The top three subplots (a), (b) and (c) show the contour plot of the associated magnetic field in 2-D plane, the profile of magnetic field and the electron flow at the mid \(y = 0\) section of the structure respectively for the monopolar electron current pulse. These are radially symmetric rotating electron current flow patterns which are non - propagating in a homogeneous plasma. The subplots (d), (e) and (f) of Fig.\([1]\) corresponds to the same features for the dipolar solutions which move with uniform axial speed \(U\) in a homogeneous plasma. The speed \(U\) typically increases with the maximum amplitude of \(|b|\) shown by the peak value in subplot(e) and it also increases with the increasing proximity of the two lobes. This dipolar solution can thus be considered as a model for the finite propagating electron current pulse in the plasma for our studies. It may also be viewed as a propagating magnetic field disturbance in an overdense plasma which is screened in the radial and axial directions by typical scale length of the order of electron skin depth. For these dipolar structures the central region (subplot(f)) shows a forward (along the propagation direction) current flow which bifurcates and returns along both sides as a
return current. As stated earlier this entire structure propagates with a constant speed $U$ in the plasma. The Electric field lines in the plane associated with these monopolar and dipolar structures have also been shown in Fig. Fig.\(^2\) and Fig.\(^3\) respectively. In the first subplot of these figures the line out of the two component of electric fields as a function of .. coordinate has been shown. The second on the other hand shown the electric field lines in the 2-D plane.

While the monopolar structures remain static in the medium, the dipolar structures are of great interest. To a stationary observer this propagating dipole would appear as a time dependent Electromagnetic pulse having a Doppler shifted frequency of $kU$, where $k = 2\pi/d$, $d$ being the scale of the dipolar structure. The scale length of such dipolar structures are typically of the order of electron skin depth $d \sim c/\omega_{pe}$. Thus the associated frequency $kU = \omega_{pe}U/c << \omega_{pe}$. These dipoles thus may be viewed as slowly moving exact nonlinear EM disturbances. EMHD simulations have shown their steady and stable propagation \(^7\).

A natural question that arise in this context is whether such slow EM structures can be made to propagate inside an even higher density plasma region. In a recent study it was shown that it is indeed possible \(^8\) for these structures to penetrate and propagate in higher density regions of plasmas. A generalization of the EMHD model (G-EMHD) which incorporates the spatially inhomogeneous plasma densities \(^8\) was used to carry out the simulations. A simplified 2-D case when the symmetry axis is along $\hat{z}$ and the electron flow is confined to the 2-D $x – y$ plane, the G-EMHD model can be cast in terms of a single scalar field $b$ representing the sole component of magnetic field along the symmetry direction due to the 2-D electron current pulse. The evolution equation for $b$ can then be written as \(^8\)\(^\text{[10]}\):\(^{\text{[10]}}\)

$$\frac{\partial}{\partial t} \left\{ b - \nabla \cdot \left( \frac{\nabla b}{n} \right) \right\} + \hat{z} \times \nabla b \cdot \nabla \left[ \frac{1}{n} \left\{ b - \nabla \cdot \left( \frac{\nabla b}{n} \right) \right\} \right] = 0 \quad (1)$$

The equation has been written in normalized variables. The magnetic field and the electron density have been normalized by some typical values $B_{00}$ and $n_{00}$ respectively. Time is normalized by the corresponding electron gyroperiod $\omega_{ce0}^{-1} = 1/(eB_{00}/mc)$, length by the electron skin depth $d_{e0} = c/\omega_{pe0}$ (where $\omega_{pe0} = 4\pi n_{00}e^2/m$). When the density $n$ is uniform, the Eq.\(^1\)) reduces to a simpler form of EMHD model.
In the presence of a density inhomogeneity the magnetic field patterns associated with these current pulses acquire an additional drift velocity \( \vec{v}_d = \vec{B} \times \nabla n/n^2 \) which is clearly transverse to the density gradient as well as the direction of magnetic field \( \vec{B} = b\hat{z}. \) The monopolar current pulses which are otherwise non-propagating in a plasma can thus be made to move along the contours of constant plasma density. By altering the sign of \( \nabla n \) the direction of propagation of the monopolar pulse can be reversed. However, the monopolar current profiles are pretty restricted in terms of their maneuverability. They cannot move across the density gradient. The dipolar form of current profiles, which mimic the combination of forward and return currents allows for a far superior maneuverability. The drift associated with the density gradient in conjunction with the intrinsic axial drift of the dipoles allows easy penetration in higher density regions. This has been clearly illustrated in the simulations carried out by Sharad et al., [8, 26] using Eq. (1). It has been shown that the EMHD dipole solution travels through the region of increasing density and can easily enter a high density region where it continues its steady propagation. The structure scale length is observed to typically adjust to the skin depth of the high density region.

An interesting aspect of these structures are that the energy content of the pulse is equipartitioned between the field and the electron kinetic energy. In the context of linear EM waves propagating in the plasma it is typically partitioned in the ratio of \( \omega_{pe}^2/\omega^2. \) Thus the propagating pulse also carries a significant fraction of energetic electrons along with it. Thus, the pulses provide for the propagation of not merely EM energy but energetic electrons also along with it. In the next section we discuss some other interesting and novel features of such pulses as they propagate in an inhomogeneous plasma medium.

### III. APPLICATIONS

We now discuss some other interesting aspects of the propagation of these dipolar EM pulses through an appropriately tailored inhomogeneous electron density profile. It has been shown in simulations by Sharad et. al. [8][10] that once the pulse enters a high density region remains trapped inside it. Thus, an appropriate choice of plasma density profile can be utilized to trap the EM pulse inside it. In fact the high density plasma can act as a tweezer by confining the pulse inside it. This has been illustrated in Fig. (4(i)), where we...
have chosen a high density circular hump of plasma density. The electromagnetic pulse enters inside it and then gets trapped within the high density structure. By changing the density profile and making it narrower in one dimension the EM pulse gets collimated as has been shown in Fig.(4(ii)).

This is a very attractive proposition as a simple choice of a narrow high density plasma can suitably focus a divergent flow of EM disturbance as well as the electrons associated with the current in such pulses. In the context of many experiments for the focused transport of electrons this attribute would be very desirable.

The observed features can be easily interpreted in terms of an interplay of the two drifts associated with the dipolar current pulse. The drift associated with the density gradient brings the two lobes with opposite polarity of the magnetic field together as the dipole approaches the high density region. This results in the collimation of the current pulse structure. The collimated structure moves with greater axial speed and penetrates the high density region of the plasma. Once inside the high density region the current pulse propagates along it to reach the other end through the axial dipolar drift.

In the case of Fig.5 we had shown a straight propagation of the current pulse towards the other end of the narrow high density plasma region. We have also carried out simulations for a curved high density region as indicated by the contours in Fig.(5(i)). The current pulse trajectory in this case gets suitably altered to get guided along the path defined by the high density narrow plasma region. We thus see that by choosing appropriately tailored and different forms of the plasma density the dipole current pulse can be guided and sent to a desired destination. In Fig.(5(i)) in fact one has been able to reverse the propagation direction of the original pulse.

The question as to how narrow should the density inhomogeneity be for the guiding to occur is settled from the considerations of the fact that the electron current pulse should be able to feel the inhomogeneity of the density region. If the high density region to which the dipole pulse enters is considerably broader than the structure size of the pulse, the pulse will not feel the edges and will not experience the density inhomogeneity related drift. The
pulse size on the other hand gets suitably adjusted to the local skin depth of the associated plasma. Thus the width of the path along which the pulse is guided should be smaller compared to the typical skin depth size.

The observed collimated propagation of current pulse through G-EMHD simulations along the direction defined by the narrow high density plasma region forms the underlying physical basis of some experimental observations. For instance, in an experiment reported by Kodama et al. [3] the fast electrons get generated by impinging ultra intense laser pulse on a target in the shape of a gold cone. A fine carbon wire was attached at the tip of the cone. It was shown in the experiment that the electrons followed the path defined by the direction of the solid carbon wire. When the wire was tilted with respect to the cone axis the electrons hit the imaging plate target at an off axis location defined by the tilt of the wire. The experiment can be understood on the basis of our mechanism. The wire gets ionized by the front of the energetic electron pulse, creating a narrow high density plasma region of the shape of the wire. The subsequent part of the electron pulse then gets guided along this inhomogeneous plasma as proposed by us.

We now provide another possibility in connection with maneuvering the propagation of the current pulse path. We show that one can also bifurcate a current pulse which arise from the same source. The two parts can then be made to propagate and reach altogether different destinations. This has been shown by the snapshots at various times for the current pulse structure through a density inhomogeneity of the kind shown by the thick black lines of Fig. (5(ii)). The lines show the contours of the high density region. As the pulse enters the high density region, it gets separated in two parts which then the two propagate along different directions.

IV. SUMMARY AND DISCUSSION

It is well known that low frequency (lower than plasma frequency) EM waves cannot propagate inside an unmagnetized plasma [1]. We show that the dipolar current solutions of the EMHD equations can in fact be treated as a low frequency EM disturbances which can propagate inside a high density plasma aided by the inherent nonlinearity. These pulses
can easily be pushed in the domain of even higher plasma densities. The energy content of these pulses are typically equipartitioned between the electron kinetic and field energies. Thus the structure can be an effective tool for the transport of energy in denser regions of the plasma.

A host of frontline physics experiments today are associated with generation and/or the utilization of energetic electrons. One is either chasing the possibility and devising new schemes for the generation of electrons with desirable energy range (electron acceleration) or is interesting in utilizing them for a variety of purposes. For instance, the energetic electrons are used for the purpose of localized heating in hot dense plasma targets as in the fast ignition experiments [13, 19, 21–23], or as a diagnostic tool for hot dense plasma studies etc. For such diverse applications one often encounters the question of keeping the electron pulse collimated and also its controlled guided propagation along a desired path. This question has been addressed by a number of authors recently. For instance, techniques which rely on structured target designs of different materials with varying resistivity has been used for the purpose of electron pulse guiding [4]. The strong magnetic field generated at the interface of materials having different resistivities is important for guiding the current pulse in this case. The guiding due to such structured targets has also been experimentally verified in the work by Kar et al. [5]. The preparation of structured targets, however, may not be convenient and/or feasible for use in all kinds of experiments. The recent work by Kodama et al. [3] reported in Nature provides a simpler scheme wherein they are able to show that the electron pulse gets guided along the direction of solid wire placed in their path. The effects shown here, however, have been for low non relativistic electrons and need to be generalized for the relativistic case for it to be suitably applied for the applications mentioned above. such a generalization will be carried out and presented in a future publication. Here, we confine ourselves to some interesting physical insights on the nonlinear EM pulse propagation in plasmas aided by the electron flow in the medium.

Just as plasma photonics promises a novel technique for guiding photons, we show here that the plasma medium with a tailored density profile offers a simple and novel scheme for guided electron transport. The physics of electron current pulse propagation through plasmas is analyzed and then with the help of numerical simulations it is shown that the
introduction of an appropriate density inhomogeneity leads to the guiding of electron current pulses. We specifically show here that electron current pulse can be collimated and guided along a desired path by a suitable choice of plasma density profile. We also demonstrate that a single electron current pulse can be bifurcated and send to distinct locations in a plasma for dual usage similar to the splitting of a photon beam. The proposed technique is practically viable as the appropriate plasma inhomogeneity in question can easily be created by the ionizing a solid target of appropriate density profile. In fact in the experiments by Kodama et al. [3] the energetic electron itself generates the requisite plasma inhomogeneity profile by ionizing the wire through which it propagates.

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FIG. 1: Schematic diagram of monopole and dipole magnetic field structures. The subplots (a) and (d) show the magnetic field contours and the color plot for monopoles and dipoles respectively. The subplot (b) and (c) show the line out profiles for the magnetic field $\mathbf{b}$ and the electron current flow respectively at $y = 0$ as a function of $x$ for monopole magnetic field structures. The subplot (e) and (f) show the line out profiles for the magnetic field $\mathbf{b}$ and the electron current flow respectively at $y = 0$ as a function of $x$ for dipole magnetic field structures.
FIG. 2: Schematic diagram of (a) monopole magnetic field structure (b) corresponding electric field structure, and (c) line out profiles of $x$ and $y$ component of electric field.

FIG. 3: Schematic diagram of (a) dipole magnetic field structure (b) corresponding electric field structure, and (c) line out profiles of $x$ and $y$ component of electric field.
FIG. 4: Propagation of magnetic dipole structure (i) in a circular shaped density hump profile (ii) in an elliptic shaped density hump profile, at different times.
FIG. 5: Propagation of magnetic dipole structure in a curved shaped density hump profile for (i) guiding the pulse (ii) bifurcation of the pulse, at different times.