Pre weakly generalized closed sets in topological spaces

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Abstract: In this paper, we introduce a new class of generalized closed sets namely Pre weakly generalized closed sets in topological space, which lies between Closed sets and Weakly Generalized Closed sets. We also investigate and study their fundamental properties and generalize the results with other defined classes of generalized closed sets.

1. Introduction

The modern topology was established by Georg Cantor in 19th century, which strongly depends upon the idea of Set theory. This branch of topology proved to be the backbone of Generalization of closed sets. In topology, applications of closure and interior of sets have given rise to a large number of generalization of closed sets. In topology, generalization of closed sets was first presented by Levine [1,3] in 19th century. He lead the foundation of generalization of closed sets and initiated with the introduction of generalized closed sets in topological space as: Consider a topological space \((X, \tau)\), and let \(A\) be any subset of space \(X\), then \(A\) is said to be Generalized closed(or g-closed) in \(X\) if closure of \(A\) is contained in some \(U\) whenever \(A\) is contained in \(U\) where \(U\) is open set in \(X\). He used g-closed sets to form new separation axioms known as \(T_{1/2}\) spaces, that are formed when g-closed sets and closed sets coincide and lies between \(T_0\) and \(T_1\) spaces. Then further contributions were made by other mathematicians like Kannan [6] introduced \(\beta\)-Generalized closed and open sets in topological space, Benchalli and Wali [2] presented one more class called rw closed sets in topological space. This development continued with time and resulted in a comparable number of generalizations in topological space. Moreover, the concept has now extended to continuity, connectedness and compactness also. With the use of properties like closure and interior, this field is being studied and investigated on regular basis. The arena of generalization of closed sets has not only given rise to a new bunch of classes but plays a considerable role in Fuzzy topology also. In this paper a new class of generalized closed set is introduced called Pre weakly generalized closed sets in topological space, which is properly placed between the class of closed sets and wg-closed sets in topological space.
Furthermore the fundamental properties of this new class of closed sets are investigated and studied. Moreover, we studied their properties regarding Compactness and one of the separation axioms also. Next, we define Open sets and Open sets also, complementary to originally defined closed sets and their relation with Pre weakly generalized Closed Sets in topological space.

Throughout this paper \((X, \tau)\) signifies a topological space on which separation axiom is assumed unless otherwise mentioned. And closure, interior and complements are being denoted as \(\text{cl}(A)\) or \(\overline{A}\), int\((A)\) and \(A^c\) respectively. Here, we establish a new class of generalized closed set called Pre weakly generalized Closed set in a Topological Space. This class lies in the middle of Closed sets and a class of \(Wg\)-closed sets. We study the fundamental properties of this class.

**Keywords**: rw-closed sets and open sets.

2. Preliminaries

**Definition 2.1**[1] A set \(P \subseteq (X, \tau)\) is known as generalized closed (concisely g-closed) set if \(\overline{P} \subseteq N\) whenever \(P \subseteq N\) and \(N\) is open.

**Definition 2.2**[16] A set \(P \subseteq (X, \tau)\) is known as regular open (concisely r-open) set if \(P = \text{int}(\overline{P})\) and regular closed (concisely r-closed) set if \(P = \overline{\text{int}(P)}\).

**Definition 2.3**[23] A set \(P \subseteq (X, \tau)\) is known as pre-open set for if \(P \subseteq \text{int}(\overline{P})\) and pre-closed set if \(P \subseteq (\text{int}(P))\).

**Definition 2.4** A set \(P \subseteq (X, \tau)\) is known as \(\alpha\)-open [26] set; if \(\text{int}(\overline{P}) \subseteq P\).

**Definition 2.5**[27] A set \(P \subseteq (X, \tau)\) is known as \(\theta\)-closed set; if \(P \subseteq \text{cl}(\overline{P})\), where \(\text{cl}(P) = \{x \in X : \overline{P} \cap P \neq \emptyset N \in \tau, x \in N\}\).

**Definition 2.6**[27] A set \(P \subseteq (X, \tau)\) is known as \(\delta\)-closed set; if \(P = \text{cl}(\overline{P})\), where \(\text{cl}(P) = \{x \in X : \text{int}(\overline{N}) \cap P \neq \emptyset N \in \tau, x \in N\}\).

**Definition 2.7**[6] A set \(P \subseteq (X, \tau)\) is known as regular semi-open set; if there exists a regular open set \(N \subset P \subset \overline{N}\). The collection of regular semi-open sets of \(X\) is denoted by \(\text{RSO}(X)\).

**Definition 2.8**[8] A set \(P \subseteq (X, \tau)\) is known as semi-regular open sets if it is both semi-open and semi-closed.

**Definition 2.9**[11] Consider \(X\) to be a topological space. We state \(\pi\)-open sets as the finite union of regular open sets in \(X\). In addition, their complement is known as \(\pi\)-closed.

Let \((X, \tau)\) be a topological space and and let \(P \subseteq (X, \tau)\), we define semi-closure (resp. semi-kernel) of set \(P\) as the intersection of all those semiclosed (resp. semipoen) subsets of \((X, \tau)\) which contain \(P\) in them, and is signified by \(\text{scl}(P)\) (resp. \(\text{sker}(P)\)). Likewise, pre-closure (resp. semi-pre-closure and \(\alpha\)-closure) of \(P\) is defined as the intersection of all those pre-closed (resp. semi-preclosed and \(\alpha\)-closed) subsets of \((X, \tau)\) which contain \(P\) in them, and is represented by \(\text{pcl}(P)\) (resp. \(\text{spcl}(P)\) and \(\alpha - \overline{P}\)).

**Theorem 2.10**[2] For if \(P\) is rw-open in \(X\), \(X \setminus P\) is rw-open.

**Lemma 2.11** Each set which is open is rw-open also, but converse does not hold.
Proof. Trails from the definition.

**Theorem 2.12** [17] All closed set are rw-closed, but conversely the result does not hold.

**Definition 2.13** A set $P \subseteq (X, \tau)$ is termed as:

1. Generalized closed (concisely g-closed)[1] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
2. Generalized semi-closed (concisely gs-closed)[5] if $\text{scl}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
3. Semi-generalized closed (concisely sg-closed) [4] if $\text{scl}(P) \subseteq N$ when $P \subseteq N$ and $N$ being semi-open in $X$.
4. Generalized a-closed (concisely ga-closed)[21] if $\alpha - \bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being $\alpha$-open.
5. $\alpha$-generalized closed (concisely ag-closed)[21] if $\alpha - \bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
6. Generalized semi-pre-closed (concisely gsp-closed)[9] if $\text{spcl}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
7. Regular generalized closed (concisely rg-closed)[25] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being regular open in $X$.
8. Generalized pre-closed (concisely g-closed) [20] if $\text{pcl}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
9. $\theta$-generalized closed (concisely $\theta$-g-closed) [12] if $\text{cl}_{\theta}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
10. Generalized pre-regular closed (concisely gpr-closed) [15] if $\text{pcl}(P) \subseteq N$ when $P \subseteq N$ and $N$ being regular open in $X$.
11. $\delta$-generalized closed (concisely $\delta$-g-closed) [10] if $\text{cl}_{\delta}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
12. Weakly generalized closed (concisely wg-closed) [24] if $\bar{\text{int}}(P) \subseteq N$ when $P \subseteq N$ and $N$ being open in $X$.
13. Strongly generalized closed (concisely g*-closed)[14] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $P$ being $g$-open in $X$.
14. $\Pi$-generalized closed (concisely $\pi$-g-closed) [11] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being $\pi$-open in $X$.
15. Weakly closed (concisely w-closed) [18] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being semi-open in $X$.
16. Semi weakly generalized closed (concisely swg-closed) [24] if $\bar{\text{int}}(P) \subseteq N$ when $P \subseteq N$ and $N$ being semi-open in $X$.
17. Mildly generalized closed (concisely mildly g-closed) [19] if $\bar{\text{int}}(P) \subseteq N$ whenever $P \subseteq N$ and $N$ being $g$-open in $X$.
18. Regular weakly closed (concisely rw-closed) [24] if $\bar{P} \subseteq N$ when $P \subseteq N$ and $N$ being regular semi-open in $X$. We represent the set of all rw-closed sets in $X$ by $\text{RWC}(X)$.
19. Regular weakly generalized closed (concisely rwg-closed) [24] if $\bar{\text{int}}(P) \subseteq N$ when $P \subseteq N$ and $N$ being regular open in $X$.

3. The Pre weakly generalized Closed Sets in Topological Space

**Definition 3.1** Consider $X$ to be a topological Space with topology $\tau$ defined on $X$. A set $P \subseteq X$ is called Pre weakly generalized Closed if $\bar{P} \subseteq N$ whenever $P \subseteq N$ and $N$ is $\text{rw}$-open in $(X, \tau)$.

First, we show that this class lies in the middle of closed sets and wg-closed sets.
Theorem 3.2 All closed sets are Pre weakly generalized Closed, but conversely the result does not hold.

Proof. The proof is clear as per the definition of closure is concerned and the fact being that all closed sets are contained in their closure. Therefore, all closed sets are Pre weakly generalized Closed.

Converse may not be true, as can be told from the following example:

Example 3.3 Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $C = \{a, d\}$ is Pre weakly generalized Closed but is not closed. Therefore, conversely the result does not hold.

Theorem 3.4 Every Pre weakly generalized closed set is wg-closed, but converse does not hold.

Proof. Assume $P$ to be a Pre weakly generalized closed set in $X$. Then $\overline{P} \subseteq N$ when $\overline{P} \subseteq N$, where $N$ is rw-open. Since $\text{int}(P) \subseteq P \subseteq P$. Therefore, $\overline{\text{int}(P)} \subseteq \overline{P} \subseteq N$. It suggests that $\overline{\text{int}(P)} \subseteq N$ when $P \subseteq N$ and $N$ is rw-open. But as we know, that all open sets are rw-open. So, we get $\overline{\text{int}(P)} \subseteq N$ whenever $P \subseteq N$ where $N$ is open. Therefore, $P$ is wg-closed. Henceforth every Pre weakly generalized Closed is wg-closed.

Converse of this result is not always true, as is clear from the example below:

Example 3.5 Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $C = \{c\}$ is wg-closed but not Pre weakly generalized Closed.

Corollary 3.6 All regular closed sets are Pre weakly generalized Closed.

Proof. As every regular closed set is closed[16], and by theorem 3.2, every regular closed sets is Pre weakly generalized closed.

Corollary 3.7 All $\theta$-closed sets are Pre weakly generalized Closed.

Proof. As every $\theta$-closed set is closed[27], and by theorem 3.2, every regular closed sets is Pre weakly generalized closed.

Corollary 3.8 All $\pi$-closed sets are Pre weakly generalized Closed.

Proof. As every $\pi$-closed is closed[11], and by theorem 3.2, every regular closed sets is Pre weakly generalized closed.

Corollary 3.9 All $\delta$-closed sets are Pre weakly generalized Closed.

Proof. As every $\delta$-closed set is closed[27], and by theorem 3.2, every regular closed sets is Pre weakly generalized closed.

Remark 3.10 The succeeding example confirms that Pre weakly generalized Closed sets have no relation with Semi-closed sets, Sg-closed sets, $g\alpha$-closed sets, $\beta$-Closed sets, $\alpha$-closed sets.

Example 3.11 Suppose $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.

(i) Closed sets in $(X, \tau)$ are $\{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.

(ii) Regular closed sets in $(X, \tau)$ are $\{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$.

(iii) Wg-closed sets in $(X, \tau)$ are $\{X, \varnothing, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

(iv) $\alpha$-closed sets in $(X, \tau)$ are $\{X, \varnothing, \{c\}, \{d\}, \{a, c, d\}, \{b, c, d\}\}$.
(v) \( \beta \)-closed sets in \((X, \tau)\) are \( \{X, \varphi, \{a\}, \{b\} \} \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\).

(vi) Pre weakly generalized Closed sets in \((X, \tau)\) are \( \{X, \varphi, \{d\}, \{a, d\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}\).

(vii) \( g\alpha \)-closed sets in \((X, \tau)\) are \( \{X, \varphi, \{c\}, \{d\}, \{b, c\}, \{a, c, d\}\}\).

(viii) \( S\alpha \)-closed sets in \((X, \tau)\) are \( \{X, \varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\}\).

(ix) Semiclosed sets in \((X, \tau)\) are \( \{X, \varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{a, b, d\}, \{a, d, c\}, \{b, c, d\}\}\).

(x) \( \pi \)-closed sets of \((X, \tau)\) are \( \{X, \varphi, \{c, d\}, \{a, c, d\}, \{a, d\}, \{b, c, d\}\}\).

Keeping in mind the above argument and known outcomes, the table of implications we get are:

| \( A \) | \( B \) | \( C \) | \( D \) | \( E \) | \( F \) | \( G \) | \( H \) | \( I \) | \( J \) | \( K \) | \( L \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |
| \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) | \( \Rightarrow \) |

For this given table 1, \( A, B, C, D, E, F, G, H, I, J, K, L \) symbolizes Regular closed sets, \( \theta \)-closed sets, Closed sets, Pre weakly generalized Closed Sets, \( \pi \)-Closed sets, \( \delta \)-Closed, \( Wg \)-Closed Sets, Semi-closed, \( \alpha \)-Closed, g\( \alpha \)-Closed, Sg-Closed, and \( \beta \)-Closed respectively. Where \( " \Rightarrow " \) denotes ‘implies’ and \( " \nRightarrow " \) denotes ‘doesnot imply’.

**Theorem 3.12** Union of two Pre weakly generalized Closed sets is Pre weakly generalized Closed.

Proof. Consider \( P \) and \( Q \) to be two Pre weakly generalized Closed sets in \( X \). Then \( \overline{P} \subseteq N \) when \( P \subseteq N \) where \( N \) is rw-open. And \( \overline{Q} \subseteq N \) when \( Q \subseteq N \) where \( N \) is rw-open. Assume \( P \cup Q \subseteq N \).

So \( P \subseteq N \) and \( Q \subseteq N \). As \( \overline{P} \subseteq N \) and \( \overline{Q} \subseteq N \). So, \( \overline{P \cup Q} \subseteq P \cup \overline{Q} \subseteq N \).That is \( \overline{P \cup Q} \subseteq N \) whenever \( P \cup Q \subseteq N \) and \( N \) is rw-open. Henceforth, \( P \cup Q \) is Pre weakly generalized Closed set in \( X \).

**Theorem 3.13** For any set \( P \subseteq X \) which is Pre weakly generalized Closed in \( X \), \( \overline{P \setminus P} \) doesnot contain any non-empty rw-open in \( X \).
Proof. Let \( P \) be Pre weakly generalized Closed set in \( X \). We will verify this theorem by contradiction method. Assume \( N \) to be rw-open set in \( X : \sim \mathcal{P} \mathcal{N} \Rightarrow N \neq \varnothing \). Now \( \mathcal{N} \subseteq \mathcal{P} \mathcal{N} \). It implies \( \mathcal{N} \subseteq X \mathcal{P} \mathcal{N} \). This shows \( P \subseteq X \mathcal{N} \mathcal{P} \). Meanwhile \( \mathcal{N} \subseteq \mathcal{P} \mathcal{N} \). It implies \( \mathcal{N} \). This shows \( \mathcal{N} \). Meanwhile \( \mathcal{N} \) is rw-open. Thus by using lemma 1.1 we get \( X \mathcal{N} \mathcal{P} \). As \( \mathcal{N} \) is Pre weakly generalized Closed in \( X \), then by the definition, we have \( \sim \mathcal{P} \mathcal{N} \subseteq X \mathcal{N} \mathcal{P} \). Likewise, \( \mathcal{N} \subseteq \mathcal{P} \mathcal{N} \). This shows \( \mathcal{N} \subseteq ( \mathcal{P} \mathcal{N} \cap (X \mathcal{P} \mathcal{N})) = \varnothing \Rightarrow N = \varnothing \). But that contradicts with our assumption. Hence \( \mathcal{P} \mathcal{N} \) does not contain any non-empty rw-open set.

**Corollary 3.14** For a set \( P \subseteq X \) which is Pre weakly generalized Closed in \( X \), \( \mathcal{P} \mathcal{N} \) does not contain any open set in \( X \), but the converse does not always hold.

Proof. This trails from lemma 2.11.

**Theorem 3.15** The set \( X \setminus \{y\} \) is Pre weakly generalized Closed or rw-open, for each element \( y \in X \).

Proof. Suppose \( X \setminus \{y\} \) is not rw-open. So, \( X \) is the only rw-open set : \( X \setminus \{y\} \subseteq X \). It implies \( (X \setminus \{y\}) \subseteq \mathcal{X} \Rightarrow (X \setminus \{y\}) \subseteq X \), which implies \( X \setminus \{y\} \) is Pre weakly generalized Closed.

**Theorem 3.16** If \( P \) is Pre weakly generalized Closed set of \( X : P \subseteq Q \subseteq \mathcal{P} \), then \( Q \subseteq X \) is Pre weakly generalized Closed.

Proof. Let \( P \) be Pre weakly generalized Closed set in : \( P \subseteq Q \subseteq \mathcal{P} \). Let \( \mathcal{N} \subseteq X \) is rw-open : \( Q \subseteq \mathcal{N} \). Then \( \mathcal{P} \subseteq \mathcal{N} \). Since \( P \) is Pre weakly generalized Closed, then we are left with \( \mathcal{P} \subseteq \mathcal{N} \). Now, \( \mathcal{N} \subseteq \mathcal{P} \mathcal{N} \). Therefore \( \mathcal{P} \mathcal{N} \) is Pre weakly generalized Closed set in \( X \).

**Remark 3.17** Conversely, the above theorem 3.16 need not to be true in general. Suppose \( X = \{a, b, c, d\} \) with topology \( \mathcal{T} = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \). Let \( P = \{d\} \) and \( Q = \{c, d\} \). Thus \( P \) and \( Q \) are Pre weakly generalized Closed sets in \( (X, \mathcal{T}) \), but \( P \not\subseteq \mathcal{P} \).

**Theorem 3.18** Suppose \( P \) is Pre weakly generalized Closed set in \((X, \mathcal{T})\). Then \( P \) is closed iff \( \mathcal{P} \mathcal{N} \) is rw-open.

Proof. Assume \( P \subseteq X \) is closed. Then \( \mathcal{P} \mathcal{N} = P \) and so \( \mathcal{P} \mathcal{N} = \varnothing \). Which is rw-open in \( X \).

Contrariwise, let us suppose \( \mathcal{P} \mathcal{N} \) is rw-open in \( X \). As \( P \) is Pre weakly generalized Closed, using theorem 3.14, we conclude that there is no rw-open set of \( X \) contained in \( \mathcal{P} \mathcal{N} \). Then \( \mathcal{P} \mathcal{N} = \varnothing \). Therefore, \( P \subseteq X \).

**Theorem 3.19** If a set \( P \subseteq (X, \mathcal{T}) \) is rw-open as well as Pre weakly generalized Closed, then the set \( P \) is closed also.

Proof. Consider a set \( P \subseteq X : P \) is rw-open as well as Pre weakly generalized Closed. Now \( P \subseteq P \), then \( \mathcal{P} \subseteq P \). Therefore \( P \) is closed.

**Corollary 3.20** Let \( P \) be rw-open and Pre weakly generalized Closed in \( X \). If \( F \) be closed in \( X \). Then \( P \cap F \) is Pre weakly generalized Closed in \( X \).

Proof. Assume \( P \) to be rw-open and Pre weakly generalized Closed in \( X \) and \( F \) is closed. Using theorem 3.19, \( P \) is closed. So \( P \cap F \) is closed thus, \( P \cap F \) is Pre weakly generalized Closed in \( X \).

**Theorem 3.21** If a set \( P \) is open and Pre weakly generalized Closed, then the set \( P \) is closed and thus Clopen.
Proof. Suppose $P$ is open and Pre weakly generalized Closed. As all open sets are rw-open and $P \subseteq \overline{P}$, and $P \subseteq \overline{P}$. Also $P \subseteq \overline{P}$.

\[\therefore \overline{P} = P\] Therefore, $P$ is Closed, and so is Clopen.

**Theorem 3.22** For if $P$ is open and wg-closed, then $P$ is Pre weakly generalized Closed in $X$.

Proof. Let $P$ is open and wg-closed in $X$. To prove that $P$ is Pre weakly generalized Closed in $X$, let $N$ be any rw-open set in $X : P \subseteq N$. Meanwhile, $P$ is open and wg-closed.

\[\therefore \overline{P} \subseteq P \Rightarrow \overline{P} \subseteq P \subseteq N \Rightarrow \overline{P} \subseteq N\]

Hence $P$ is Pre weakly generalized Closed in $X$.

**Theorem 3.23** Let $(X, \tau)$ be a topological space, if $RWO(X) = \{X, \varphi\}$. Then all the subsets of $X$ are Pre weakly generalized Closed.

Proof. Let $(X, \tau)$ be a topological space and $RWO(X) = \{X, \varphi\}$. Consider $P \subseteq X$. Assume that $P = \varphi$. Then $\varphi$ is Pre weakly generalized Closed in $X$. Now let $P \neq \varphi$, then there is a single rw-open set i.e. $X$ that contains $P$, so $\overline{P} \subseteq \overline{X}$. Thus $P$ is Pre weakly generalized Closed in $X$.

**Definition 3.24** A subset $P$ of $(X, \tau)$ is known as Pre weakly generalized open in $X$ if $P^C$ is Pre weakly generalized Closed in $X$.

**Theorem 3.25** Every set containing only one element of a topological space $(X, \tau)$ is either Pre weakly generalized open or rw-open.

Proof. Consider a topological space $(X, \tau)$. Let $z \in X$. We prove $\{z\}$ is either Pre weakly generalized open or it is rw-open. The result is clear from Theorem 3.15.

**Theorem 3.26** For if $X$ is a regular space in which every set which is rw-open is open also. If $P$ is compact subset of $X$, then $P$ is Pre weakly generalized Closed.

Proof. Supposing $P \subseteq N$ and $N$ is rw-open. Using hypothesis $N$ is open. But $N$ being compact subset of regular space $X$, so there exists an open set : $P \subseteq \overline{O} \subseteq \overline{O} \subseteq N$. This implies $P \subseteq \overline{O} \Rightarrow \overline{P} \subseteq \overline{\overline{O}} \subseteq \overline{O} \subseteq N$. This implies $\overline{P} \subseteq N$. Hence, $P$ is Pre weakly generalized Closed.

**Theorem 3.27** In a topological space $(X, \tau)$, $RWO(X, \tau) \subseteq \{S \subseteq X; S^C \in \tau\}$ if and only if each subset of $(X, \tau)$ is Pre weakly generalized Closed.

Proof. Suppose $RWO(X, \tau) \subseteq \{S \subseteq X; S^C \in \tau\}$. Let $P \subseteq (X, \tau) : P \subseteq N, N$ is regular weakly open set. At that time, $N \in RWO(X, \tau) \subseteq \{S \subseteq X; S \in \tau\}$. Which implies $N \in \{S \subseteq X; S^C \in \tau\}$. So, $N$ is closed, implies $\overline{N} = N$. Likewise, $\overline{P} \subseteq \overline{N} = N$. Therefore, $P$ is Pre weakly generalized Closed in $X$. Meanwhile, $P$ was arbitrary, therefore each subset of $(X, \tau)$ is Pre weakly generalized Closed.

Contrariwise, supposing that all the subsets of $(X, \tau)$ are Pre weakly generalized Closed. Assume $N \in RWO(X, \tau)$. Meanwhile, $N \subseteq N$, and $N$ is Pre weakly generalized Closed $\Rightarrow \overline{N} \subseteq N \Rightarrow \overline{N} = N$. It suggests $N \in \{S \subseteq X; S^C \in \tau\}$. As a result, $RWO(X, \tau) \subseteq \{S \subseteq X; S^C \in \tau\}$.

**Definition 3.28** The intersection of all rw-open subsets of $(X, \tau)$ which contain $P$, is known as regular weakly kernel of $P$. It is symbolized by $\text{rWker}(P)$.

**Lemma 3.29** Consider a topological space $X$ and $P \subseteq X$. If $P$ is regular weakly-open in $X$, then $\text{rWker}(P) = P$. But the converse does not hold.
Proof. Trails from Definition 3.28.

**Lemma 3.30** For any set $P \subseteq (X, \tau)$, $P \subset \text{rwker}(P)$.

Proof. Trails from Definition 3.28.

**Theorem 3.31** A set $P \subseteq (X, \tau)$ is Pre weakly generalized Closed iff $\bar{P} \subset \text{rwker}(P)$.

Proof. Suppose $P$ is Pre weakly generalized Closed set. So by definition, $\bar{P} \subset N$, whenever $P \subset N$ and $N$ is rw-open in $X$. Let $y \in \bar{P}$. Supposing $y \notin \text{rwker}(P)$, so a rw-open set $N$ which contains $P$ : $y \notin N$. As $P$ is Pre weakly generalized Closed. Thus $\bar{P} \subset N$, suggests $y \notin \bar{P}$. Which is a contradiction to our supposition. So $y$ is an element of $\text{rwker}(P)$. Henceforth, $\bar{P} \subset \text{rwker}(P)$.

Contrariwise, let $\bar{P} \subset \text{rwker}(P)$. For if $N$ is rw-open that contains $P \Rightarrow \text{rwker}(P) \subset N$. It suggests $\bar{P} \subset \text{rwker}(P) \subset N$, suggests $\bar{P} \subset N$. Hence, $P$ is Pre weakly generalized Closed in $X$.

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