Regge Trajectories For All Flavors

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Abstract

Based on available data for mesonic resonances of light, medium and heavy flavors, we have performed a global analysis to construct the corresponding linear Regge trajectories. These have been supplemented by results from various phenomenological models presented in the literature. A satisfactory formula is found for the dependence of the intercept and the slope on quark masses. We find reasonable agreement with data on production of charmed hadrons through exchange of our charmed trajectories in the space-like region. When applied to mesons containing the top quark, our results suggest the impossibility of their formation as evidenced by other independent analyses.

Exactly how quarks are bound together to form hadronic bound states is not known. QCD - the underlying theory for strong interactions - is flavor independent and it has been found difficult to compute the dependence of mesonic bound states (say) on the quark masses. In consequence, quite different approximation schemes and models exist which try to obtain the mass spectra for different flavors. These may be roughly classified as follows.

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(i) light-light systems: mesons composed of u, d and s quarks \((m_{\text{quark}} \ll \Lambda)\) are described by ultrarelativistic Bag models or Chiral theories, where quarks are treated essentially as massless.

(ii) Charmonium and Bottomium systems: Since both quarks are heavy \((m_c, m_b \gg \Lambda)\), non-relativistic models with a central potential are invoked. A smooth function interpolating between the Coulomb term and a confining linear term gives the short and the long distance behaviors. Other potentials include a constant in addition to a small power in \(r\).

(iii) light-heavy systems \((B\text{ and } D\text{ mesons})\) are poorly known experimentally and phenomenologically: a heavy quark mass expansion has often been employed.

It is fair to conclude that there is as yet no overall picture providing an understanding of all the mesonic bound states.

In this work we describe a different approach based on the Regge trajectories to answer the generic question posed above: how do mesonic masses depend upon the quark masses. This formalism is particularly suited for strongly interacting systems since (i) Regge trajectories exist for all flavors, for large or small quark masses and (ii) angular momentum becomes a continuous variable facilitating interpolation. Redundancy of data allows for self consistency checks and the theoretical exercise is not merely curve fitting. Moreover, as we show later, the same Regge trajectories constructed in the time-like region can be employed in the space-like region (exchange processes) as predictive tools. Our results a posteriori confirm this endeavor.

Based on much trial and experimentation, we made the following two crucial simplifying assumptions to obtain the Regge trajectories:

(i) All trajectories were assumed linear in \((mass)^2\) of the hadronic state,

\[
\alpha(s) = \alpha(0) + s\alpha' \tag{1}
\]

so that a meson of angular momentum \(J\) has the \((mass)^2\) equal to \(sJ\). For light mesons (and baryons), we know this to be true experimentally.

(ii) The functional dependence of the two parameters \(\alpha(0)\) and \(\alpha'\) on the quark masses is via \((m_1 + m_2)\).

For light mesons, experimental data \([1]\) and some models \([2]\) give the value, for the light-light system

\[
\alpha' \approx 0.9\, GeV^{-2} \tag{2}
\]

We supplement this with experimental data \([1]\), theoretical models and phenomenological spectra for charmonium and bottomium systems \([3,4,5]\) to obtain the following simple formula giving a global description

\[
\alpha'(m_1 + m_2) = \frac{0.9\, GeV^{-2}}{\left[1 + 0.22 \left(\frac{m_1 + m_2}{GeV}\right)^{3/2}\right]} \tag{3}
\]

where \(m_1\) and \(m_2\) are the corresponding quark masses for that trajectory. We show in Fig.(1), how well (3) compares with available data and some models.
An analysis similar to the above was performed for the intercept \( \alpha_I(0) \), where the subscript \( I \) refers to the leading trajectory. In this work we limit ourselves to those mesonic systems for which the lowest physical state is at \( J = 1 \). A global description for these is given by

\[
\alpha_I(m_1 + m_2; 0) = 0.57 - \frac{(m_1 + m_2)}{\text{GeV}} \quad (4)
\]

A comparison of Eq.(4) with input data is shown in Fig(2). We note that only two points from a theoretical analysis [6] fall quite below our curve. These points refer to the \( B_c \) (composed of quarks \( b \) and \( c \)) system.

We next consider the secondary Regge trajectories and splitting between the energy levels for same \( J \) of a given system. Calling \( \alpha_I(0) \) the leading and \( \alpha_{II}(0) \) the secondary Regge trajectory intercept, we estimated the “distance”

\[
\Delta \alpha(0) = \alpha_I(0) - \alpha_{II}(0) \quad (5)
\]

from data (not very precise) and phenomenological models. We find a rather loose bound

\[
1.3 < \Delta \alpha(0) < 1.6. \quad (6)
\]

The result becomes more interesting when transposed in terms of a physically more amenable quantity, viz., the energy splitting

\[
\Delta E_{J}^{II-I} = (E_{J}^{II} - E_{J}^{I}) \quad (7)
\]

between the states of a given angular momentum of a given system. Quite strikingly, it is found to be a constant (between \( 0.5 \) – \( 0.8 \text{ GeV} \)) for all systems (composed of \( u, d, s; c \) and \( b \) quarks). It is shown in Fig(3). The approximate constancy of this energy difference for all \( J \) and all flavors, gives us confidence in the generality of this result and we expect it to be verified in all viable phenomenological models.

Combining (3) and (4), we are in a position to give a general expression for the leading mesonic Regge trajectory formed by any two quarks of masses \( m_1 \) and \( m_2 \)

\[
\alpha(m_1 + m_2; t) = 0.57 - \frac{(m_1 + m_2)}{\text{GeV}} + \frac{0.9 \text{ GeV}^{-2}}{1 + 0.22 \left( \frac{m_1 + m_2}{\text{GeV}} \right)^{3/2}} t \quad (8)
\]

In Fig(4), we show the leading Regge trajectories for different flavors for space- and time- like regions \((-500 \text{GeV}^2 < t < 500 \text{GeV}^2)\). In this figure, we have included the “top” and the “toponium” trajectories as well, even though, as we shall demonstrate later, top and toponium bound states cease to exist as

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1Leading trajectories whose ground states begin at \( J = 0 \) (corresponding to the pion, or \( \eta_c(1S) \), etc.), which have an intercept approximately 0.5 lower from the leading trajectory, follow a similar pattern, but they are not considered here.
physical states due to the fast weak decays of the top quark into a $W$ and a $b$ quark.

Eq. (8) allows us to make “predictions” (or consistency checks) about the energy spectra of excited mesons for the $D, D_s$ and $B, B_s$ systems, which were not used as input data. In table I we show our results for the states of the leading trajectory only. Whenever data exist, there is reasonable agreement. A more complete spectrum for mesons may be found in [7,8].

As another application, we employ the analytic continuation into the space-like region of these trajectories constructed through data in the time-like (resonance) region, to discuss the inclusive production of heavy flavour through the di-triple Regge formalism. Data exist for $D$ [9,10,11] and $\Lambda_c$ [12] production reactions,

$$\pi + N \rightarrow D + X \text{ and } \pi + N \rightarrow \Lambda_c + X$$

(9)

for a wide range of $x_F$. Assuming a factorized $p_t$ dependence, experimental data have been parametrized through

$$\frac{d\sigma}{dx_F} \approx (1 - x_F)^n$$

(10)

with $n \approx (3.7 \div 3.9)$ [9,10,11] and $\approx 3.5$ [12].

We invoke the di-triple Regge formula [13,14], valid for large $M_X^2$ and large $(s/M_X^2)$,

$$\frac{d^2\sigma}{dM_X^2 dt} \rightarrow \frac{\gamma(t)}{s} \left( \frac{M_X^2}{s} \right)^{1-2\alpha(t)}$$

(11)

where $\alpha(t)$ is the exchanged Regge trajectory. Neglecting the $t$-dependence, we have in this region

$$\frac{d\sigma}{dx_F} \approx (1 - x_F)^{1-2\alpha(0)}$$

(12)

For the reactions of Eq.(9), inserting a D- Regge trajectory exchange, we have $\alpha(0) = -1.35$ (see Table I), leading to the exponent $n \approx 3.7$, in satisfactory agreement with the data. We stress that this is an important test since the space-like extrapolation depends crucially on the slope parameter of the exchanged D trajectory (which is roughly a half of that for the light system).

As a third application, let us consider the top system. It is generally believed that the top quark cannot form bound states either with another $t$ quark or with a light one. The point being that the top quark would decay into a real $W$ and $b$ with a large width: the lifetime would be even smaller than the revolution time thereby precluding the formation of mesonic bound states [15].

Our analysis confirms the above physical picture quite nicely. The energy splitting between the ground state and that lying on the second trajectory are as follows. For the toponium, we find $1.3 \text{ GeV} < \Delta E_{Toponium} < 1.6 \text{ GeV}$, in good agreement with ref.[16]. For the top mesons, we find $1.1 \text{ GeV} < \Delta E_{Top} < 1.3 \text{ GeV}$. Neither can exist, since $\Delta E_{Toponium} < 2\Gamma_t$ and $\Delta E_{Top} < \Gamma_t$. 

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In conclusion, we have obtained a global unified description of linear Regge trajectories for all flavors, and their preliminary applications to space- as well as time-like regions seem encouraging. Further work in many directions is in progress. Inclusion of iso-spin, charge and other quantum numbers can be incorporated perturbatively. So far we have also ignored the imaginary parts of the Regge trajectories $\alpha(s)$. A knowledge of $\text{Im} \, \alpha(s)$ in the time-like region is useful since it is directly related to the resonance widths. Through unitarity, it is also connected to the residue function $\beta(s)$, which when continued to the space-like region is relevant for the Regge exchange amplitudes. Another interesting problem concerns the baryon Regge trajectories - a much more difficult task since three masses are involved. If further work confirms the viability of this approach, an effective interaction should be constructed which is able to produce these results dynamically. We shall return to this question elsewhere. Here we shall limit ourselves to making one general observation. It appears that an interaction capable of generating linear Regge trajectories requires a linear mass dependence in the confining term in order to reproduce the $(m_1 + m_2)^{-3/2}$ behavior in the slope.

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REFERENCES:
1. Review of Particle Physics, Phys. Rev. D 54 (1996), 1-720.
2. D. Flamm and F. Schoberl, Introduction to the quark model of elementary particles, Vol.1 (Gordon and Breach Publishers, 1982); N.A.Tornqvist, in Stockholm 1990, Proceedings, Low energy antiproton physics, p.287-303 and Helsinki University preprint HU-TFT-90-52.
3. J. L. Richardson, Phys. Lett. B 82 (1979) 272.
4. A. Martin, Phys. Lett. B 93 (1980) 338; B 100 (1981) 511.
5. A. K. Grant, J. L. Rosner and E. Rynes, Phys. Rev. D47 (1993) 1981.
6. J. Morishita, M. Kawaguchi and T. Morii, Phys. Lett. B 185 (1987) 159; Phys. Rev. D 37 (1988) 159.
7. S. Filipponi and Y. Srivastava, Invited Talk at Hadron97 (BNL, N.Y., August 1997), ed. Suh-Urk Chung, to be published.
8. S. Filipponi, G. Pancheri and Y. Srivastava, Invited Talk at XXVII Symposium on Multiparticle Dynamics (LNF Frascati, Italy, September 1997), to be published.
9. E769 Collab., G. A. Alves et al., Phys. Rev. Lett. 69 (1992) 3147; Phys. Rev. D 49 (1994) 4317.
10. LEBC-EHS Collab., M. Aguillar Benitez et al., Phys. Lett. 161B (1985), 400.
11. ACCMOR Collab., S. Berlag et al., Z. Phys. C 49 (1991), 555.
12. ACCMOR Collab., S. Berlag et al., Phys. Lett. B 247 (1990) 113.
13. A. H. Muller, Phys. Rev. D2 (1970), 2963.
14. G. Pancheri and Y. Srivastava, Lett. Nuovo Cimento Vol.II (1971) 381.
15. I. Bigi, Y. Dokshitzer, V. Khoze, J. Kuhn and P. Zerwas, Phys. Lett. B181 (1986) 157.
16. N. Fabiano, A. Grau and G. Pancheri, Il Nuovo Cimento A107 (1994) 2789.
Table I: Leading trajectory parameters and predicted masses for $D$ and $B$ mesons.

| $D$ mesons | $D_s$ mesons |
|------------|-------------|
| $\alpha'$ (GeV$^{-2}$ units) | 0.586 | 0.562 |
| $\alpha_I(0)$ | -1.35 | -1.51 |

States of the Leading trajectories (GeV units)

| $J=1$ | Predicted | Experimental |
|-------|------------|--------------|
| 2.01  | 2.0067±0.0005 | 2.11 2.1124±0.0007 |
| 2.39  | 2.4589±0.0020 | 2.50 | / |
| 2.73  | / | 2.83 | / |

| $B$ mesons | $B_s$ mesons |
|------------|-------------|
| $\alpha'$ (GeV$^{-2}$ units) | 0.228 | 0.223 |
| $\alpha_I(0)$ | -5.45 | -5.55 |

States of the Leading trajectories (GeV units)

| $J=1$ | Predicted | Experimental |
|-------|------------|--------------|
| 5.32  | 5.3248±0.0018 | 5.41 5.4163±0.0033 |
| 5.72  | / | 5.81 | / |
| 6.09  | / | 6.18 | / |
Figure 1: Slope parameter of the Regge trajectories as a function of the sum of the constituent quark masses; input data compared with our analytic result Eq.3.
Figure 2: The intercept parameter of the leading trajectory as a function of the sum of the constituent quark masses; comparison of Eq.4 with input data.
Figure 3. *Energy splitting for mesons of different flavors but same angular momentum; our predictions compared to available data and some theoretical models.*
Figure 4: Leading trajectories for different flavors for space- and time- like regions, as given by Eq.(8).