Characterization of the capacitance variation of electrostatic vibration energy harvesters biased following rectangular charge-voltage diagrams

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Abstract. This paper presents for the first time a method to measure the capacitance variation of electrostatic vibration energy harvesters (e-VEHs) that employ conditioning circuits implementing a biasing scheme that can be represented by a rectangular charge-voltage diagram. Given the increasing number of e-VEHs using such complex conditioning circuits and the complex dynamics that are induced from this type of biasing, a mean to assess this measurement is of primary importance for the analysis of e-VEHs. The proposed method is based on the inspection of the voltage evolution across two simple conditioning circuits implementing a rectangular charge-voltage diagrams biasing scheme. After the method is presented, it is carried out for the characterization of a state-of-the-art MEMS e-VEH.

1. Introduction
Recent research on micro-machined electrostatic vibration energy harvesters (e-VEHs) have shown that the electrical conditioning and load interfacing part of such systems is critical to the performances in terms of harvested power. In the light of this consideration, researchers have started to investigate new types of conditioning circuits [1, 2, 3]. These new conditioning circuits have in common that their biasing of the transducer, across one cycle of its capacitance variation, can be summarized by a rectangular charge-voltage characteristic diagram (QV diagram) [4].

However, the way this type of biasing affects the device’s dynamics via the electromechanical coupling effect is still unclear: although semi-analytical approximations are possible for a limited range of input conditions [5], no measurement method allowed to measure the electromechanical coupling effect on the dynamics of a given e-VEH. In particular no method was reported for the measurement of the extremal values of capacitance variation for a device submitted to harmonic input excitation and biased following a rectangular QV diagram.

In this paper, a simple method is proposed to carry out such a measurement. This method only involves the use of characterization equipment that is typically found in MEMS e-VEHs characterization labs, i.e., generic electronics components, characterization gear, and a mechanical shaker. This method is based on the dynamics of two simple circuits implementing biasing schemes described by rectangular QV diagrams. After a brief presentation and analysis of these circuits, the method is presented and carried out as an example on a state-of-the-art micro-machined, symmetrical gap-closing geometry e-VEH.
2. Presentation of the circuits used for the characterization

![First conditioning circuit](image1.png)  ![Second conditioning circuit](image2.png)

**Figure 1.** First conditioning circuit used for the characterization of $C_{\text{max}}$ and $C_{\text{min}}$.  
**Figure 2.** Second conditioning circuit used for the characterization of $C_{\text{max}}$ and $C_{\text{min}}$.

**Figure 3.** QV diagram for the circuit depicted in Fig. 1.  
**Figure 4.** QV diagram for the circuit depicted in Fig. 2.

Consider the two conditioning circuits depicted in Fig. 1 and Fig. 2. The voltage source elements $E_1$ and $E_2$ model external voltage sources, eventually superposed with the effect of electret charging. The QV diagrams of the corresponding circuits are depicted in Fig. 3 and Fig. 4. The diodes are supposed to follow an ideal diode voltage-current law, with threshold voltage $V_T \geq 0$. Defining a variation cycle by the transducer’s variable capacitance varying from a maximal value $C_{\text{max}}$ to a minimal value $C_{\text{min}}$ and then from $C_{\text{min}}$ to $C_{\text{max}}$, it comes for the local evolution laws of each of the circuits that:

$$V_{\text{n+1}}^{\#1} = \frac{C_S V_{\text{nr}}^{\#1} + (C_{\text{max}} - C_{\text{min}})E_1 - (C_{\text{max}} + C_{\text{min}})V_T}{C_{\text{min}} + C_S},$$  
$$V_{\text{n+1}}^{\#2} = \frac{C_S V_{\text{nr}}^{\#H2} + (C_{\text{max}} - C_{\text{min}})E_2 - (C_{\text{max}} + C_{\text{min}})V_T}{C_{\text{max}} + C_S},$$

where $V_{\text{n}}^{\#1}$ and $V_{\text{n}}^{\#2}$ denote the value of the voltage across $C_S$ at cycle index $n$ for the circuits depicted in Fig. 1 and Fig. 2, respectively.

3. Presentation of the measurement method

From the preceding equations, denoting $\Delta_1 = V_{\text{n+1}}^{\#1} - V_{\text{n}}^{\#1}$, $\Delta_2 = V_{\text{n+1}}^{\#1} - V_{\text{n}}^{\#1}$, $V_1 = V_{\text{n}}^{\#1}$ and $V_2 = V_{\text{n}}^{\#2}$, it comes:

$$C_{\text{min}} = C_S \frac{\Delta_2(E_1 - V_T) - \Delta_1(E_2 - V_T) - \Delta_2 + \Delta_2}{(E_1 + \Delta_1 + V_1 + V_T)(E_2 - \Delta_2 - V_2 - V_T) - (E_1 - V_T)(E_2 + V_T)},$$

$$C_{\text{max}} = \frac{C_S \Delta_2 + C_{\text{min}}(E_2 + V_T)}{E_2 - V_2 - \Delta_2}.$$
Let’s consider an e-VEH for which the value of maximal capacitance $C_{\text{max}}$ is to be measured, when the device is submitted to an harmonic external excitation, and when its biasing across one cycle of variation is described by a rectangular charge-voltage diagram of extremal voltages $V_L$ and $V_R$ (see Fig. 6). Suppose the slope of the non-horizontal portions of the QV cycles of the two conditioning circuits is large compared to $C_{\text{max}}$, i.e., $C_S \gg C_{\text{max}}$ (see Fig. 3 and Fig. 4). By choosing the values of the external voltages sources and initial voltages across $C_S$ such that:

\begin{align}
E_1 &= V_R, \\
E_2 &= V_L, \\
V_1 &= V_2 = V_R - V_L,
\end{align}

it comes that evaluating (3) and (4) with the values of $\Delta_1$ and $\Delta_2$ measured with these parameters yields the values of $C_{\text{max}}$ and $C_{\text{min}}$ as affected by the electromechanical coupling induced by the rectangular biasing depicted in Fig. 6.

Note that the values of $C_{\text{max}}$ and $C_{\text{min}}$ result from a dynamical process, and hence the measurement method is accurate only if the electromechanical coupling impact on the e-VEH’s dynamics can be considered as a quasi-static process: otherwise, a systematic error is introduced in the measurement. This hypothesis is verified if $C_S$ is chosen such that the values of $\Delta_1$ and $\Delta_2$ do not change abruptly from one cycle to the next, i.e., that $C_S$ is chosen large enough. Also, this results in the QV diagrams in Fig. 3 and Fig. 4 to become more alike, which improves the measurement accuracy. However, a too large value of $C_S$ results in $\Delta_1$ and $\Delta_2$ becoming indiscernible one from another because of a non-null noise floor due to random noise sources. Thus, prior to the measurement, a study should be done to chose the optimal value for $C_S$ given those constraints.

4. Example of transducer characterization using the presented method
As an illustrating example, the following describes the application of the method to the measurement of the value of $C_{\text{max}}$ for a practical transducer, that has been previously reported in [6]. The schematic of the transducer is depicted in Fig. 5. The measurement is carried out for the transducer biased following a rectangular QV diagram, with $V_L = 10V$ and $V_R = 15V$, and mechanically forced by an external excitation of 1.5g amplitude, 150Hz frequency. Because of the symmetrical gap-closing geometry of the device, the value of $C_{\text{min}}$ is fixed throughout
the e-VEH’s operation, and is measured by standard capacitance measurement means, as $C_{\text{min}} = 55\, \text{pF}$. In both circuits, the fixed capacitor is chosen such that $C_S = 500\, \text{pF}$.

Because $C_{\text{min}}$ is fixed, only the circuit in Fig. 2 is necessary. The parameters for the measurements are $E_2 = 15\, \text{V}$, and $V_2 = 5\, \text{V}$ (see (6) and (7)). Several measurements are carried out for the characterization, with approximately equal values of $V_2$, to statistically estimate the uncertainty due to type A error sources on the measurements. With the obtained results for ($\Delta_2$, $V_2$), applying (4) yields a value of $C_{\text{max}} = 94.7\, \text{pF} \pm 2.8\, \text{pF}$.

Equation (3) can be used to check the results as $C_{\text{min}}$ is fixed, by carrying out the same measurement with the circuit in Fig. 1, choosing $E_1 = 10\, \text{V}$, and at $V_1 = 5\, \text{V}$ (see (5) and (7)). If the geometry of the device is such that $C_{\text{min}}$ is not unalterable, this latter measurement has to be carried out to measure the value of $C_{\text{min}}$ as impacted by the electromechanical coupling resulting from the bias depicted in Fig. 6.

5. Conclusion
This work presented the first method to measure the capacitance variation of electrostatic MEMS transducers of an e-VEHs, when electrically conditioned following a rectangular charge-voltage diagram. This enables more thorough studies on the effect of the electromechanical coupling on the dynamics of e-VEH using electrical interfaces implementing these types of conditionings schemes, such as charge-pump conditioning circuits.

6. References
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