Topological Aspects of High Temperature Superconductivity and Berry Phase

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Abstract

We propose a mechanism of high temperature superconductivity from the viewpoint of chirality and Berry phase. It is observed that spin pairing and charge pairing is caused by a gauge force generated by magnetic flux quanta attached to them. From the renormalization group equation involving the Berry phase factor $\mu$ it is found that there are two crossovers above the superconducting temperature $T_c$, one corresponds to the glass phase and the other represents the spin gap phase. Actually, in this topological framework each charge carrier is dressed with a magnetic flux quantum and represents a skyrmion. The skyrmion-skyrmion bound state leads to the $d$-wave Cooper pair formation. We have also discussed the Magnus force acting on the vortices of this system.

PACS numbers: 74.20.Mn, 74.25.Dw, 12.39.Dc, 11.15.-q, 03.65.Vf

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I. INTRODUCTION

Since high temperature superconductivity was discovered in 1986, the investigation of this phenomenon has become one of the most exciting frontiers of condensed matter theory. A common feature among the high temperature superconducting compounds is that the mobility of the charge carriers is almost confined to planes of Copper-Oxygen atoms, with the off-plane atoms providing only as a reservoir of carriers. These compounds exhibit properties of strongly correlated electron systems, being antiferromagnetic insulators when undoped. It is clear that the well established BCS theory of low energy superconductivity cannot provide an adequate description of these new compounds and that some new approach is needed. Theoretical attempts to explain the phenomenon have therefore concentrated on strongly correlated electron models on a 2D lattice.

Immediately after the discovery of high temperature superconductivity Anderson proposed a spin liquid or resonating valence bond (RVB) state as the theory of this new phenomenon. Originally the proposal of this RVB state was for quantum antiferromagnets on a triangular (or similarly frustrated) lattice. Following this proposal Kivelson, Rokhsar and Sethna showed that a consequence of the existence of such a spin liquid is that there exist quasiparticles with reversed charge spin relations: charge 0 spin 1/2 \( \text{spinons} \) and charge \( e \) spin 0 \( \text{holons} \). These quasiparticles have topological character analogous to that of the quasiparticles in the quantum Hall effect. Recently, there is a proposal of algebraic spin liquid (ASL) state with spin-charge recombination picture to explain the unusual properties of underdoped high \( T_c \) superconductors.

Static or dynamical charge inhomogeneity or topological doping is a common feature for doped correlated insulators. In \( d \)-dimensions the charge forms one-dimensional arrays of (d-1) dimensional structures that are also antiphase domain walls for the background spins. In \( d = 1 \) there is an array of charged solitons whereas in \( d = 2 \) there are linear rivers of charge (stripes) threading through the antiferromagnetic background. In \( d = 3 \) there are arrays of charged planes. It is observed in \( \mu SR \) experiments that there exists a phase in which superconductivity coexists with a cluster spin glass. It is difficult to see how these two phases could coexist unless there is a glass of metallic stripes dividing the \( CuO_2 \) planes into randomly coupled antiferromagnetic regions. In fact, it appears that the phase diagram of a high-\( T_c \) superconductor suggests the evolution of the system in three stages. Above the superconducting transition temperature there are two crossovers. The upper crossover is indicated by the onset of a stripe glass phase. The lower crossover is where a spin gap or pseudogap (which is essentially the amplitude of the superconducting order parameter) is formed. Finally, superconducting phase order is established at \( T_c \).

Emery and Kivelson have argued that these experimental findings support the idea that these self-organized structures are designed to lower the zero-point kinetic energy. Lee observed that the spin-charge separation associated with the resonating valence bond (RVB) state accounts for all the qualitative features of the spin gap state. The spins form RVB singlets so that it costs energy (spin gap) to make triplet excitations.

Though the spin-charge separation naturally accounts for the qualitative features of the spin gap state, it has been realized that there actually exists a strong coupling among spinons and holons through a gauge interaction and such a gauge force plays a role essentially to confine spinon and holon together. Indeed, spinons and holons are decoupled in 1D and behave just like free particles. However, in 2D the gauge force plays a crucial role for spin-charge confinement. In the strong coupling (large \( U \)) regime, a correct spin-charge separation description has been established in a path integral formalism where an electron is described as a composite particle of a spinon and a holon together with a nonlocal phase-shift field. It is this phase shift field that helps to recover the right Fermi surface position.

In this article, we shall study these features from the point of view of the analysis of high-\( T_c \) superconductivity in the framework of Berry phase. In this scheme the three dimensional spinons and holons reduce to \( \frac{1}{2} \) fractional statistics when the motion is confined to equatorial planes. It is pointed out that though the spin-charge separation associated with RVB state can explain well the spin gap state, the superconducting phase is established when there is spin-charge recombination. Indeed, the magnetic flux associated with the Berry phase gives rise to a gauge interaction between spinons and holons which effectively confines them together. We have investigated the phase associated with high-\( T_c \) superconductivity using renormalization group fixed point theorem involving the Berry phase factor \( \mu \) when the Berry phase is given by \( e^{i2\pi \mu} \). It is found that there are three distinct phases: upper phase is associated with the glass phase at \( T_1^* \) and the lower one above the superconducting transition temperature \( T_c \) gives rise...
to the pseudogap (spin gap) at a temperature $T^*_2$. Finally, superconducting phase is established at $T_c$. It is noted that the spin gap phase is not independent of the superconducting phase owing to the manifest presence of the coupling between spin and charge degrees of freedom. The phase diagrams of different high $T_c$ cuprates display the universal behavior of $T^*_2/T_c$ as a function of the hole doping $\delta/\delta_0$ with $\delta_0$ being the optimal doping rate.

In Sec. 2 we shall formulate the basic ideas of the topological framework of high temperature superconductivity from the viewpoint of chirality and Berry phase. In Sec. 3 we intend to discuss the features associated with spin-charge separation and spinon-holon recombination. In the next section (Sec. 4) with the help of the remormalization group analysis we shall study the different phases associated with high temperature superconductivity. In Sec. 5, we shall show how the skyrmion-skyrmion bound state leads to the d-wave pairing in this framework. In Sec.6, we shall discuss the Magnus force acting on the vortices of high temperature superconductors.

II. TOPOLOGICAL ASPECTS OF FRUSTRATED ANTIFERROMAGNETS, RVB STATES AND BERRY PHASE

It is known that in the strong coupling limit and at half filling the system of correlated electrons on a lattice which is governed by the Hubbard model can be mapped onto an antiferromagnetic Heisenberg model with nearest neighbor interaction and is represented by the Hamiltonian

$$H = J \sum (S^x_i S^x_j + S^y_i S^y_j + S^z_i S^z_j)$$

with $J > 0$. The antiferromagnetic model on a triangular lattice emerges as a frustrated spin system when the ground state corresponds to the RVB state. For an antiferromagnetic spin system the existence of RVB states on a given lattice depends crucially on the type of lattice which allows frustration to occur. The two characteristic operators of the ground state of an antiferromagnet, namely density of energy

$$\epsilon_{ij} = \left( \frac{1}{4} + S^z_i S^z_j \right)$$

and chirality

$$W(C) = Tr \prod_{i \in C} \left( \frac{1}{2} + \sigma \cdot \vec{S}_i \right)$$

($\sigma$ are Pauli matrices and $C$ is a lattice contour) are related with the amplitude and phase $\Delta_{ij}$ of Anderson’s RVB through

$$\epsilon_{ij} = |\Delta_{ij}|^2$$

$$W(C) = \prod_C \Delta_{ij}$$

This suggests that $\Delta_{ij}$ is a gauge field. The topological order parameter $W(C)$ acquires the form of a lattice Wilson loop

$$W(C) = e^{i\phi(c)}$$

This is associated with the flux of the RVB field through

$$e^{i\phi(c)} = \prod_C e^{iA_{ij}}$$

where $A_{ij}$ represents a magnetic flux which penetrates through a surface enclosed by the contour $C$. This is essentially the Berry phase related to chiral anomaly when we describe the system in three dimensions through the relation

$$W(C) = e^{i2\pi \mu}$$
where $\mu$ appears to be a monopole strength. In view of this, when a two dimensional frustrated spin system on a lattice is taken to reside on the surface of a three dimensional sphere of a large radius in a radial (monopole) magnetic field we can associate the chirality with the Berry phase \cite{2}. In fact, to take the effect of spin chirality in the RVB theory of high temperature superconductivity we consider a two dimensional frustrated system in the spherical geometry with a monopole at the center.

To study this frustrated spin system leading to RVB state characterized by the chirality associated with it we consider a generalized Heisenberg-Ising Hamiltonian with nearest neighbor interaction

$$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$  \hspace{1cm} (9)$$

where $J > 0$ and the anisotropy parameter $\Delta = \frac{2\mu+1}{2\mu}$. The Berry phase factor $\mu$ can take the values $\mu = 0, \pm 1/2, \pm 1, \pm 3/2, \ldots$. It is noted that $\Delta = 1$ corresponds to $\mu = 1/2$. Indeed, the Ising part of the Hamiltonian corresponds to the near neighbor repulsion caused by free fermions and as $\mu = 1/2$ is related to a free fermion, we have the isotropic Hamiltonian which is $SU(2)$ invariant. For $\Delta \rightarrow \infty$, it corresponds to an Ising system. When $\Delta = 0 (\mu = -1/2)$ we have the $XX$ model. For a frustrated spin system, this corresponds to the singlets of spin pairs which eventually represents the RVB state giving rise to a non-degenerate quantum liquid. The ground state of antiferromagnetic Heisenberg model on a triangular lattice which allows frustration to occur is represented by $\mu = -1/2$ suggesting $\Delta = \frac{2\mu+1}{2} = 0$ in the Hamiltonian (9). Indeed, with $\Delta = 0$, the Hamiltonian effectively corresponds to a bosonic system represented by singlets of spin pairs which eventually leads to a resonating valence bond state (RVB).

To study the spinon and holon excitations in this frustrated spin system, let us consider a single spin down electron at a site $j$ surrounded by an otherwise featureless spin liquid representing a RVB state. As a result, the state characterized by $|\mu| = 1$ is formed by the single spin state ($\mu = -1/2$) in the spin liquid and the orbital spin caused by the monopole represented by $\mu = -1/2$ characteristic of a frustrated spin system leading to RVB ground state. Thus for the neutral spin $\frac{1}{2}$ excitation, the spinon characterized by $|\mu| = 1$ may be split into two parts: one spin $\frac{1}{2}$ excitation with $|\mu| = 1/2$ in the bulk and the other part is due to the orbital spin by $|\mu| = 1$ in the background characterized by the chirality of a frustrated spin system. This is analogous to the idea of Laughlin \cite{33} that spinons obey $\frac{1}{2}$ fractional statistics. It may be noted that such a spinon will be characterized by non-Abelian Berry phase.

It may be mentioned here that the RVB spin singlet state forming the quantum liquid are equivalent to FQH liquid with filling factor $\nu = 1/2$ \cite{34,35}. Indeed, in earlier papers \cite{34,35} we have pointed out that in QHE the external magnetic field causes the chiral symmetry breaking of the fermions (Hall particles) and an anomaly is realized in association with the quantization of Hall conductivity. This helps us to study the behavior of a quantum Hall fluid from the viewpoint of the Berry phase which is linked with chiral anomaly when we consider a 2D electron gas of N-particles on the surface of a three dimensional sphere in a radial (monopole) strong magnetic field. For the FQH liquid with even denominator filling factor \textit{i.e.} for the state with $\nu = 1/2$, the Dirac quantization condition $e\mu = 1/2$ suggests that $\mu = 1$.

In the angular momentum relation for the motion of a charged particle in the field of a magnetic monopole

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} - \mu \mathbf{\hat{r}}$$  \hspace{1cm} (10)$$

we note that for $\mu = 1$ ( or an integer) we can use a transformation which effectively suggests that we can have a dynamical relation of the form

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} - \mu \mathbf{\hat{r}} = \mathbf{r}' \times \mathbf{p}'$$  \hspace{1cm} (11)$$

This indicates that the Berry phase which is associated with $\mu$ may be unitarily removed to the dynamical phase. This implies that the average magnetic field may be taken to be vanishing in these states. However, the effect of the Berry phase may be observed when the state is split into a pair of electrons where each electron in the pair is spin polarized with the constraint of representing the state $\mu = \pm 1/2$. These pairs will give rise to the $SU(2)$ symmetry as we can consider the state of these two electrons as a $SU(2)$ doublet. This doublet of Hall particles for $\nu = 1/2$ FQH fluid may be taken to be equivalent to RVB singlets.

As a hole is introduced into the system by doping, this may combine with the spinon giving rise to a spinless charged excitation called holons. Thus holons may also be represented by $|\mu| = 1$ which eventually form a pair characterized by a flux $\phi_0 = \hbar c/2e$. This corroborates with the idea of Laughlin \cite{2} that a gas of such particles might actually be a superconductor with charge 2 order parameter. Evidently, just like spinons, holons will also be characterized by non-Abelian Berry phase.
III. SPIN AND CHARGE PAIRING AND SPINON-HOLON INTERACTION

When a hole is introduced in the concerned system, the spinon with magnetic flux characterized by $|\mu_{\text{eff}}| = 1$ will interact with the hole through the propagation of the magnetic flux and eventually this coupling will lead to the creation of the holon attached with magnetic flux corresponding to $|\mu_{\text{eff}}| = 1$. Hence, the residual spinon will correspond to $\mu_{\text{eff}} = 0$. This is realized when the unit of magnetic flux $\mu = -\frac{1}{2}$ associated with the single down spin in the RVB liquid will form a pair with another up spin having $\mu = +\frac{1}{2}$ associated with the hole. Indeed, a spin pair is formed when the isolated spin in the RVB liquid will be combined with the spin associated to the hole. Again, the holon having the effective Berry phase factor $|\mu_{\text{eff}}| = 1$ will also eventually form a pair of holes. As we see from eqn.(11), for any integer $\mu$ the Berry phase may be removed to the dynamical phase and the Berry phase is observed when the system forms a pair such that the units of magnetic flux are distributed among the pair. It is noted that the bosonic holon having $|\mu_{\text{eff}}| = 1$ and the residual bosonic spinon having $|\mu_{\text{eff}}| = 0$ (which eventually represents a pair) cannot give the correct statistics for electron when these two form a composite state. The correct statistics is only achieved when we introduce a phase associated with a unit of magnetic flux corresponding to $\mu = 1/2$ in this composite system. Thus the spinon holon recombination along with a phase shift only gives rise to an electron. This corroborates with the spin-charge separation description in a path integral formalism [26] where an electron is described as a composite particle of a spinon and holon together with a nonlocal phase-shift field.

It is now observed that the spin pairing as well as charge pairing in this scheme occurs through a gauge interaction. In case of spin pairing, we note that when the units of magnetic flux associated with the spinon having $|\mu_{\text{eff}}| = 1$ are transferred to the hole, the residual spinon having $|\mu_{\text{eff}}| = 0$ eventually forms a pair of spins having $\mu = 1/2$ and $-1/2$. The magnetic flux associated with each spin will give rise to a gauge force operating between them. Indeed, we can associate a chiral current with a spin. When a chiral current interacts with a gauge field, we have the anomaly which is related to the Berry phase through the relation [37]

$$q = 2\mu = -\frac{1}{2} \int \partial_\alpha J^5_\alpha d^4x = \frac{1}{16\pi^2} Tr \int *F_{\alpha\beta}F_{\alpha\beta} d^4x$$

(12)

where $J^5_\alpha$ is the axial vector current $\bar{\psi}\gamma_\alpha\gamma_5\psi$. $F_{\alpha\beta}$ is the field strength tensor and $*F_{\alpha\beta}$ represents the Hodge dual. Evidently $q = 2\mu$ represents the Pontryagin index and the field $F_{ij}(i,j = 1,2,3)$ is associated with the background magnetic field given by

$$B = -\frac{1}{2} \epsilon^{ij} F_{ij}$$

(13)

Thus we may consider that this gauge field is responsible for the spin-pairing observed in high-$T_c$ superconductivity. The same view will also be valid for a pair of holes which is eventually formed when the holon gets its share of magnetic flux having $|\mu_{\text{eff}}| = 1$ from the spinon. This magnetic interaction is responsible for the hole pairing which is strong enough to overcome the bare Coulomb repulsion. This leads to the suggestion that the superconducting phase order will be established when a spin pair each having unit magnetic flux and a pair of holes each having unit magnetic flux interacts with each other through a gauge force. That is, the pair of holes will be attached to the spin pair such that spin-charge recombination occurs when each hole is attached to a spin site of the spin pair. This ensures that the pseudogap is roughly of the same size as the superconducting gap.

Mathematically, the spin-charge recombination is formulated in the spirit of Weng, Sheng and Ting [36]. Indeed, the units of magnetic flux associated with the Berry phase factor $\mu$ may be represented through a phase

$$e^{i2\pi\mu} = \prod_C e^{iA_{ij}}$$

(14)

where the magnetic flux is associated with the gauge field $A_{ij}$. We can write the effective Hamiltonian for the system as

$$H_{\text{eff}} = H_s + H_h$$
where

\[
H_s = -J_s \sum_{<ij>} (e^{i\sigma A_{ij}}) b^\dagger_{i\sigma} b_{j\sigma} + h.c
\]  

(15)

with \(b_{i\sigma}(b^\dagger_{i\sigma})\) as the spinon annihilation (creation) operator and \(A_{ij}\) represents the magnetic flux penetrating through a surface enclosed by a contour \(C\) and is given by eqn.(14). Similarly, the Hamiltonian for the holon may be written as

\[
H_h = -t_h \sum_{<ij>} e^{i(-\phi_{ij}^0 + A_{ij})} h^\dagger_i h_j + h.c
\]  

(16)

where \(h_i(h^\dagger_i)\) is the holon annihilation (creation) operators respectively. Here \(\phi_{ij}^0\) represents flux quanta threading through each plaquette. The interaction between spinons and holons are then mediated through these gauge fields \(A_{ij}\) as represented in eqn.(14) [27, 28]. It appears that superconductivity and magnetism are closely related. Indeed, spinon-holon interaction as well as the pair interaction is found to be of magnetic origin as the magnetic flux associated with the Berry phase is responsible for these features.

In fact, the spin gap or pseudogap essentially corresponds to the superconducting order parameter with the onset of coherence in the charge degrees of freedom. It is noted that the temperature \(T_2^*\) at which the pseudogap is formed is a bit higher than the superconducting transition temperature \(T_c\). The superconducting state is characterized by spin-charge recombination which is responsible for the coherent motion of the pair of holes thus establishing phase coherence.

IV. RENORMALIZATION GROUP ANALYSIS TOWARDS DIFFERENT PHASES

We note that the study of different phases associated with high-\(T_c\) superconductivity indicates that above the superconducting transition temperature there are two crossovers. The upper crossover is indicated by the establishment of a cluster spin glass which suggests the existence of a stripe glass phase. The lower crossover is where a spin gap or pseudogap is formed. Finally, superconducting phase order is established at \(T_c\). We shall study these crossovers from the viewpoint of renormalization group (RG) analysis involving the Berry phase factor \(\mu\).

Indeed, we know that there is a relationship between the central charge \(c\) in conformal field theory and the Berry phase factor \(\mu\) [34]. This relation suggests the generalization of Zamolodchikov’s \(c\)-theorem in \(3 + 1\) dimension involving \(\mu\) and formulate \(\mu\)-theorem. It is noted from the Hamiltonian (9) that the Ising part has the effective coupling constant \(J' = J_0 \mu^\pm\). To study the different crossovers in the system we may consider the coupling constant as a function of temperature. We may take \(\mu\) not to be a fixed value but dependent on a parameter. Thus, we can consider a function \(\mu(\lambda)\) which satisfies

1) \(\mu\) is stationary at fixed points \(\mu^*\) of the RG flow i.e. \(\nabla \mu(\lambda^*) = 0\)
2) at the fixed points \(\mu(\lambda^*)\) is equal to the Berry phase factor \(\mu^*\) of the theory
3) \(\mu\) is decreasing along the infrared (IR) RG flows i.e. \(L \frac{d\mu}{dL} \leq 0\) where \(L\) is a length scale. This implies that there is a RG trajectory which flows from an ultraviolet (UV) fixed point \(\lambda_U^*\) to an IR fixed point \(\lambda_R^*\) then one must have \(\mu_U > \mu_R\).

Now let us consider magnetic flux quanta passing through a domain \(D\) characterizing a length scale \(L\) and let a three dimensional smearing density function \(f(a)\) be a positive decreasing function such that \(\frac{\partial f}{\partial a} \leq 0\). We now write the expression for the field strength

\[
[F_{\alpha\beta}(x)]_D = \int_D d^3 a f(a) \tilde{F}_{\alpha\beta}(x, a)
\]  

(17)

So from the expression which relates the Berry phase factor \(\mu\) with the chiral anomaly given by [37]

\[
2\mu = -\frac{1}{16\pi^2} \int \ast F_{\alpha\beta}(x) F_{\alpha\beta}(x) d^4 x
\]  

(18)

We can write for the flux density
At this point, the Ising part coupling constant is \( \mu \). The \( \mu \)-function defined above is a pure number but now explicitly depends on the length scale \( L \) characterizing the size of the domain. Now noting that a global change of scale \( L \) for the off-critical model amounts to a change of the coupling constant \( \lambda^i \rightarrow \lambda^i(L) \), the renormalization group flux equations can be written as

\[
L \frac{\partial}{\partial L} \lambda^i = -\beta^i
\]

which suggests that

\[
-\beta^i \frac{\partial \mu}{\partial \lambda^i} = L \frac{\partial \mu}{\partial L} \leq 0
\]

It is noted that \( \mu \) takes the usual discrete values of \( 0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2} \ldots \) at fixed points of the RG flows where \( \mu \) is stationary and represents the Berry phase factor \( \mu^* \) of the theory. In terms of energy scale, this suggests that as energy increases (decreases) \( \mu \) also increases (decreases). So to study a critical phenomena, we can associate a critical temperature such that a standard discrete value of \( \mu \) corresponding to the Berry phase factor \( \mu^* \) represents a fixed point of the RG flows.

Now to study the crossovers associated with high-\( T \) superconductivity, we consider the 3\( D \) Heisenberg anisotropic Hamiltonian representing nearest neighbor interaction given by eqn.(9)

\[
H = J \sum (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)
\]

It is noted that the 1\( D \) relative of this Hamiltonian is given by

\[
H = J \sum (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \frac{2\mu + 1}{2} \sigma_i^z \sigma_{i+1}^z)
\]

where the anisotropy parameter \( \Delta \) is written in terms of the Berry phase factor \( \mu \) corresponding to the 3 \( + 1 \) dimensional system.

We note that for \( 0 \leq |\Delta| < 1 \) we will have three critical values corresponding to \( \mu = 0, \mu = -\frac{1}{2}, \mu = -1 \) which represent the fixed points of the RG flows. We associate three critical temperatures \( T_1^*, T_2^* \) and \( T_c \) with fixed values of \( \mu = 0, \mu = 1/2 \) and \( \mu = -1 \) respectively. However, in a frustrated spin system, the chirality demands that \( \mu \) should be non-zero. So the critical value \( \mu = 0 \) is not achieved and as such there will be random coupling around the value \( \mu = 0 \). This will then represent the cluster glass phase at this critical temperature \( T_1^* \). Indeed, in this situation, after doping, holes will form a glass of stripes which may be regarded as a finite piece of electron gas dividing the \( CuO_2 \) planes into randomly-coupled antiferromagnetic regions. When the dopants are immobile, these holes will form arrays of metallic stripes which are topological, as they are antiphase domain walls for the antiferromagnetic background.

The next crossover will be at \( \mu = -\frac{1}{2} \) corresponding to the pseudogap (spin gap) phase. Indeed, \( \mu = -\frac{1}{2} \) suggests that the anisotropy parameter \( \Delta = 0 \) and the Hamiltonian corresponds to the XX model which for a frustrated spin system corresponds to a bosonic system represented by singlets of spin pairs. This effectively leads to the RVB state and the spin-charge separation accounts for the qualitative features of the spin gap state. The pseudogap temperature \( T_2^* \) depends on the doping rate \( \delta \) and displays nearly a linear decrease with \( \delta \).

Finally, we have the superconducting transition temperature \( T_c \) at \( \mu = -1 \) corresponding to \( \Delta = -\frac{1}{2} \). At this point, the Ising part coupling constant is \( -\frac{1}{2} J \) with a sign change, though the bosonic part of the Hamiltonian still dominates with the coupling constant \( J > 0 \). The sign change of the Ising part is caused by the presence of the magnetic flux quanta which are responsible for the interaction between holes in the pair which is strong enough to overcome the Coulomb repulsion and generates an attractive...
force. Indeed, prior to spin-charge recombination, a spinless holon having \( \mu = -1 \) may be viewed as if a spinless hole is moving in the background of a monopole characterized by the strength \( \mu = -1 \). This eventually causes the hole pair formation each having a magnetic flux quantum characterized by \( \mu = -\frac{1}{2} \). When the spin-charge recombination occurs a spin pair each having unit magnetic flux and a pair of holes each having unit magnetic flux interacts with each other through a gauge force and a phase coherence is established. It may be noted that the doping dependence of the spin gap temperature \( T^*_2 \) and the superconducting temperature \( T_c \) is such that they show interdependence. The behavior of \( \frac{T^*_2}{T_c} \) as a function of \( \frac{\delta}{\delta_0} \) where \( \delta_0 \) is the optimal doping rate shows a universal behavior. Indeed, we can derive a relationship between these two quantities from the following consideration.

From the renormalization group equation (21)

\[
L \frac{d \mu}{dL} \leq 0,
\]

let us specify

\[
L \frac{d \mu}{dL} = -a
\]

with \( a \geq 0 \).

Solving this, we find

\[
\mu = -a \ln L + c
\]

where \( c \) is an arbitrary constant.

Now changing the length scale to the temperature \( L \sim \frac{1}{T} \), we have

\[
\mu = a \ln(T) + c
\]

From this for \( \mu = -\frac{1}{2} \) and \( \mu = -1 \), we get

\[
\frac{T^*_2}{T_c} = e^{\frac{1}{a}}
\]

Taking \( a \) to be a function of \( \frac{\delta}{\delta_0} \), this gives a universal behavior of the dependence of \( \frac{T^*_2}{T_c} \) on \( \frac{\delta}{\delta_0} \). Indeed taking a simple ansatz \( a = k \frac{\delta}{\delta_0} \) with \( k \) a constant parameter, we can compute the respective values. We
have found that with $k = 0.85$, the result is consistent with the experimental values obtained for the high $T_c$ cuprates $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ and $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \text{[38]}$.

We have obtained three crossovers corresponding to critical points related to the glass phase, spin gap phase and the superconducting phase. The temperature $T^*_2 (> T_c)$ at which the spin gap appears is found to be dependent on the superconducting transition temperature $T_c$. This tacitly manifests the presence of coupling between spin and charge degrees of freedom and the superconducting phase is characterized by spin-charge recombination.

V. SKYRMIONS IN HIGH $T_c$ SUPERCONDUCTIVITY AND BERRY PHASE

In the present framework, superconductivity arises with the charge spin recombination when a phase coherence is established. Indeed, prior to spin-charge recombination, a spinless holon may be viewed as if a spinless hole is moving in the background of a monopole. This eventually causes the hole pair formation each having a magnetic flux quantum characterized by $|\mu| = 1/2$. When the spin charge recombination occurs a spin pair each having unit magnetic flux interact with each other through a gauge force and a phase coherence is established.

Now it is noted that when a spinless hole is dressed with a magnetic flux quantum given by $|\mu| = 1/2$, this will represent a skyrmion. Indeed, the magnetic flux quantum has its origin in the background chirality which is associated with the chiral anomaly and Berry phase. Indeed, from eqn. [12], we note that the Berry phase factor $\mu$ is associated with

$$\Omega^\sigma = -\frac{1}{16\pi^2} \varepsilon^{\sigma \nu \alpha \beta} Tr(A_\nu F_{\alpha \beta} + \frac{2}{3} A_\nu A_\alpha A_\beta)$$

(28)

is the Chern-Simons secondary characteristic class. In case we have $F_{\alpha \beta} = 0$ we can write

$$A_\sigma = g^{-1} \partial_\sigma g, \quad g \in SU(2) \quad (29)$$

$\Omega_\sigma$ will represent a topological current $J_\sigma$ given by

$$J_\sigma = \frac{1}{24\pi^2} \varepsilon^{\sigma \nu \alpha \beta} Tr(g^{-1} \partial_\nu g)(g^{-1} \partial_\alpha g)(g^{-1} \partial_\beta g)$$

(30)

This may eventually be written in terms of chiral fields $\pi_\alpha (a = 0, 1, 2, 3)$.

$$J_\sigma = \frac{1}{12\pi^2} \varepsilon^{\sigma \nu \alpha \beta} \varepsilon^{\alpha \beta \nu \delta} \partial_\nu \pi_\alpha \partial_\beta \pi_\delta$$

(31)

Now representing a hole by a Dirac fermion field $\psi$ we may consider the doped hole coupling with the magnetic flux associated with the chirality in terms of the interaction given by the Lagrangian

$$L = \bar{\psi} (i \hat{D} + im(\pi_0 + i\gamma_5 \vec{\pi})) \psi$$

(32)

where $\hat{D} = \gamma_\sigma (\partial_\sigma - iA_\sigma)$ following the constraint $\vec{\pi}_0^2 + \vec{\pi}^2 = 1$

The Dirac fermion may be viewed as if it has flavor $N$ so that for polarized and unpolarized state we have $N = 1$ and 2 respectively. Now integrating for fermions, we can write the action

$$W = -N \ln \int \exp(-L d^4x) D\psi \ D\bar{\psi}$$

$$= -N \ln \text{Det}(i \hat{D} + im^\nu)$$

$$= iN \int d^4x A_\sigma J_\sigma + i\pi N H_3$$

$$+ NM^2 \int d^4x Tr(\partial_\sigma g^{-1} \partial_\sigma g)$$

(33)
Here \( g^{\gamma_5} = \frac{1+i}{2} g + \frac{1-i}{2} g^{-1} \). \( M \) is a coupling constant having dimension of mass. \( H_3 \) is a topological invariant of the map of the space-time into the target space \( S^3 \). There are only two homotopy classes \( \pi_1(S^3) = \mathbb{Z}_2 \), so that \( H_3 = 0 \) or \( 1 \). In fact the term \( i\gamma_5 H_3 \) is the geometric phase and represents the \( \theta \)-term. Thus we see that the charge carriers dressed with magnetic flux can be represented by a nonlinear \( \sigma \)-model and may be treated as skyrmions [39].

This helps us to view the superconducting pair as a skyrmion-skyrmion bound state. Indeed, the skyrmion excitations are created at each position of the carriers and they play a role of magnetic field for the carriers. Because of the magnetic field around a carrier, the Lorentz force acts on another carrier. Due to this Lorentz force an attractive interaction is induced between carriers and leads to Cooper pair formation.

It is noted that the mechanism suggests a d-wave pairing. As already pointed out by Kotliar and Liu [40] that in the RVB theory spinons form the d-wave pairing. Now in the superconducting pair, the spin charge recombination occurring through spinon-holon interaction along with the phase coherence suggests the charge carriers also have d-wave pairing. Indeed, the fact that superconductivity occurs in the vicinity of antiferromagnetic long range order, the Cooper pair is d-wave.

To study the underdoped region of cuprates in this framework, we note that spinon-holon interaction through the gauge force effectively leads to a spin pair characterized by \( \mu_{eff} = 0 \) where the isolated down spin in the background with \( \mu = -1/2 \) forms the pair with the up spin of the hole with \( \mu = +1/2 \). Indeed this may be taken to represent as a spinon-antispinon bound state. This essentially corresponds to the SF flux phase as suggested by Ranter and Wen [8]. Indeed we can visualize a spin as a massless fermion and this picture of spinon-holon interaction may correspond to a massless fermion coupled to \( U(1) \) gauge field along with the holons coupled with the gauge field. The pair formed by massless fermions (spins) dressed with magnetic flux may be viewed as a spinon-antispinon bound state. This spinon-antispinon bound state present in the nearly antiferromagnetic chain will enhance the antiferromagnetic correlation of the system. The simultaneous presence of spin singlet state will lead to the pseudogap (spin gap). Thus in the underdoped region we will have the enhancement of the antiferromagnetic correlation along with the pseudogap. As mentioned earlier, as doping increases, the antiferromagnetic long range order is destroyed.

It is known that skyrmion topological defects which are introduced by doping are responsible for the destruction of the antiferromagnetic order parameter and their energy may be used as an order parameter [41, 42]. Indeed, in two spatial dimensions the nonlinear sigma field \( n^a \) may be expressed in the \( CP^1 \) language in terms of a doublet of complex scalar fields \( z_i, \ i = 1, 2 \) with the component \( z_i z_i = 1 \) as

\[
n^a = z_i^a \sigma^a_{ij} z_j
\]

where \( \sigma^a \) are Pauli matrices. In this language the continuous field theory corresponding to the Heisenberg antiferromagnet is described by the Lagrangian density in \( 2 + 1 \) dimensions

\[
L_{ns} = (D_\mu z_i)\Gamma(D_\mu z_i)
\]

where \( D_\mu = \partial_\mu + i A_\mu \) and \( A_\mu = i z_i^\dagger \partial_\mu z_i \). Evidently this possesses solitonic solutions called skyrmions and charge is defined as

\[
Q = \int d^3x J^0
\]

where \( J^0 \) is the zero-th component of the topological current \( J^\mu = \frac{1}{2\pi} \epsilon^{\mu\alpha\beta} \partial_\mu A_\beta \). It is noted that \( Q \) is nothing but the magnetic flux of the field \( A_\mu \) indicating that skyrmions are vortices and represent defects in the ordered Neel state.

Now the following Lagrangian density may be proposed for describing the dopants and their interaction with the background lattice in \( 2 + 1 \) dimensions with the topological \( \theta \)-term

\[
L_{\psi} = (D_\mu z_i)\Gamma(D_\mu z_i) + i \bar{\psi}_a \partial_\mu \gamma_\mu \psi_a - m^* v_F \bar{\psi}_a \psi_a - \bar{\psi}_a \partial_\mu \psi_a A_\mu + L_H
\]

where the hole dopants are represented by a two-component Dirac field \( \psi_a \), \( m^* \) and \( v_F \) are respectively the effective mass and Fermi velocity of dopants. Here \( L_H \) is the Hopf term given by

\[
L_H = \frac{\theta}{2} \epsilon^{\mu\alpha\beta} A_\mu \partial_\alpha A_\beta
\]
It is noted that the dopant dispersion relation is given by

\[ \epsilon(k) = \sqrt{k^2 v_F^2 + (m^* v_F^2)^2} \]  (39)

which is valid for \( YBCO \ (YBa_2Cu_3O_{6+\delta}) \) where the Fermi surface has an almost circular shape which is centered at \( k = 0 \). For \( LSCO \ (La_{2-\delta}Sr_{\delta}CuO_4) \) the Fermi surface is different [42] which corresponds to a dispersion relation of the form

\[ \epsilon(k) = \sqrt{\left( k_x \pm \frac{\pi}{2} \right)^2 + \left( k_y \pm \frac{\pi}{2} \right)^2 v_F^2 + (m^* v_F^2)^2} \]  (40)

Now following Marino [42] the doping parameter \( \delta \) is introduced by means of a constraint in the fermion integration measure

\[ D[\bar{\psi}_a, \psi_a] = D\bar{\psi}_a D\psi_a \delta(\bar{\psi}_a \gamma_\mu \psi_a - \Delta^\mu) \]  (41)

where \( \Delta^\mu = 4\delta \int_{x,L}^\infty d\xi^\mu \delta^3(z - \xi) \) for a dopant at the position \( x \) and varying along the line \( L \). Here the factor 4 corresponds to the degeneracy of the representation (4-component) for the Fermi fields. This yields the partition function

\[ Z = \int D(\bar{\psi}_a, \psi_a) \delta(\bar{\psi}_a \gamma_\mu \psi_a - \Delta^\mu) \]

\[ \times \exp \left\{ \int_0^\infty d^3x \left[ 2\rho_s (D_\mu \bar{z}_i \gamma_\mu z_i) + \bar{\psi}(i\partial_\mu \gamma_\mu - \frac{m^* v_F}{\hbar} - \gamma_\mu A_\mu) \psi + L_H \right] \right\} \]  (42)

where \( \rho_s \) is the spin stiffness and \( L_H \) is the Hopf term.

Upon integration over the fields \( \bar{z}, \bar{\psi}, \bar{\psi}, \psi \) the resulting equation of motion for the zero-th component \( A_0 \) yields the result

\[ \theta \epsilon^{ij} \partial_i A_j = 4\delta^2 \delta(z - x(t)) \]  (43)

where \( x(t) \) is the dopant position at a time \( t \). If \( B \) is the magnetic flux or vorticity of \( A_\mu \) then this equation becomes

\[ \theta B = 4\delta^2 \delta(z - x(t)) \]  (44)

For the skyrmion, \( B = \delta^2 (z - x(t)) \) indicates that the skyrmion topological defect configuration coincides with the dopant position at any time. We see that at zero doping the Hopf term vanishes. When we translate this result in the 3 + 1 dimensional formalism where the 2D spin system is considered to reside on the surface of a 3D sphere with a monopole at the centre, we note that in the Lagrangian [33], apart from \( \mu \) being a 4 dimensional index, we have to replace the Hopf term by the topological Pontryagin term given by

\[ P = -\frac{1}{16\pi^2} \epsilon^{\alpha\beta} F_{\alpha\beta} \]  (45)

where

\[ \epsilon^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\sigma\nu} \]  (46)

It is noted that in the partition function [42] when \( \int L_H d^3x \) is replaced by \( \int P d^4x \), the latter integral just represents the Pontryagin index \( q \) related to the monopole strength \( \mu \) through the relation \( q = 2\mu \) as given by eqn. [34].

From dimensional hierarchy, the relation between topological terms suggests that in 3 + 1 dimensions, when \( L_H \) is replaced by \( L_P \), the coefficient \( \theta \) is related to \( \mu \).

Indeed replacing \( L_H \) by the Chern-Simons Lagrangian

\[ L_{cs} = \frac{k}{4\pi} \epsilon^{\alpha\beta} A_\sigma \partial_\alpha A_\beta \]  (47)
We note that the current is given by

$$J_\sigma = \frac{k}{2\pi} e^{\sigma\alpha\beta} \partial_\alpha A_\beta$$  \hspace{1cm} (48)

and the zeroth component corresponds to

$$J_0 = k \frac{B}{2\pi}$$  \hspace{1cm} (49)

So from the relation (48) and (49) and we find

$$\pi \theta = \frac{k}{2} = 2\delta$$  \hspace{1cm} (50)

It has been shown in ref. [39] that the Chern-Simons coefficient $k$ is related to the monopole strength $\mu$ in $3+1$ dimensions by the relation $k = 2\mu$. This implies $\mu = 2\delta$. As in the previous section we have noted that each charge carrier in the superconducting pair is associated with the skyrmion topological defect which is caused by the magnetic flux quantum having $|\mu| = 1/2$, superconductivity occurs at $T = 0$ for the critical doping parameter $\delta_{sc}$ given by $|\mu| = 1/2 = 2\delta_{sc}$ yielding $\delta_{sc} = .25$ for $YBCO$. When the doping parameter $\delta$ is connected with the oxygen stoichiometry parameter $x$ we have the relation $\delta = x - .18$ so that we have $x_{sc} = .43$ [39], which is in good agreement with the experimental value $x_{sc} = .41 \pm .02$. For $LSCO$, the Fermi surface has four branches and this yields $\delta_{sc} = x_{sc} = .06$ [39] which is to be compared with the experimental result $x_{sc} = .02$. It is noted that $\delta_{sc}$ is a universal constant depending only on the nature of the Fermi surface.

We have pointed out earlier that in $3+1$ dimensions chiral anomaly leads to the realization of fermions represented by doped holes interacting with chiral boson fields $\pi_i$, with the constraint $\pi_0^2 + \pi^2 = 1$. The mapping of the space-time manifold on the target space leads to the homotopy $\pi_4(S^3) = Z_2$ which takes the values 0 or 1 and leads to the $\theta$-term representing the geometric phase. The third term in eqn. (33) gives rise to the solitonic solution such that the charge carrier appears as a skyrmion. However in $3+1$ dimensions, the stability of the soliton is not generated by this term alone as rescaling of the scale variable $x \to \lambda x$ may lead to shrinking it to zero size. However, in the present framework, the attachment of magnetic field with the charge carrier will prevent it from shrinking it to zero size.

Indeed this gives rise to a gauge theoretic extension of the extended body so that the position variable may be written as

$$Q_\sigma = q_\sigma + i A_\sigma$$  \hspace{1cm} (51)

where $q_\sigma$ is the mean position. As $\mu = -1/2$ and $+1/2$ corresponds to vortices in the opposite direction we may consider $A_\sigma$ as $SU(2)$ gauge field when the field strength is given by

$$F_{\sigma\nu} = \partial_\sigma A_\nu - \partial_\nu A_\sigma + [A_\sigma, A_\nu]$$  \hspace{1cm} (52)

When $F_{\sigma\nu}$ is taken to be vanishing at all points on the boundary $S^3$ of a certain volume $V^4$ inside which $F_{\sigma\nu} \neq 0$, in the limiting case towards the boundary, we can take

$$A_\sigma = g^{-1} \partial_\sigma g, \hspace{1cm} g \in SU(2)$$  \hspace{1cm} (53)

This helps us to write the action incorporating the $\theta$-term as

$$S = \frac{M^2}{16} \int \text{Tr}(\partial_\mu g^{-1} \partial_\mu g) d^4x + \frac{1}{32\eta^2} \int \text{Tr}[(\partial_\mu g^{-1} \partial_\mu g^{-1})] d^4x$$

$$+ \frac{i\pi}{24\pi^2} \int_{S^3} dS \epsilon^{\mu\nu\lambda\sigma} \text{Tr}[(g^{-1} \partial_\mu g)(g^{-1} \partial_\nu g)(g^{-1} \partial_\lambda g)]$$  \hspace{1cm} (54)

where $M$ is a constant having the dimension of mass and $\eta$ is a dimensionless coupling constant. Here the first term is related to the gauge noninvariant term $M^2 A_\mu A^\mu$, the second term (Skyrme term) is the stability term which arises from the term $F_{\mu\nu} F^{\mu\nu}$ and the third term is the $\theta$-term given by $* F_{\mu\nu} F^{\mu\nu}$ which is related to the chiral anomaly and Berry phase.
Marino and Neto [42] have pointed out that at the critical doping $\delta_{sc}$, the energy of the skyrmion vanishes. When we compute the energy of the skyrmion from the action (54), we find the expression for the minimum energy [43] as

$$E_{\text{min}} = \frac{12\pi^2 M}{\eta}$$

and the size for $E_{\text{min}}$ as

$$R_0 = \frac{1}{2M\eta}$$

Taking $M$ and $\eta$ as a function of $\delta$, we note that for the vanishing energy we have $M(\delta_{sc}) = 0$ which corresponds to the fact that the spin stiffness vanishes. From the relation for $R_0$, it indicates that the skyrmion size is infinite. However, we can have the vanishing energy for finite nonzero $M(\delta)$ when $\eta$ is infinite. This suggests that at this point $R_0 = 0$. This implies that for finite $M$, the vanishing energy suggests that the skyrmion shrinks to the zero size. So apart from energy, we can take the size of the skyrmion also as an order parameter.

VI. MAGNUS FORCE

In this section we shall study the Magnus force in the vortex dynamics of high $T_c$ superconductors.

It is known that a vortex line is topologically equivalent to a magnetic flux. Thus in a cuprate superconductor the charge carriers having magnetic flux associated with them may be viewed as quantized vortex lines attached to each of them. These vortex lines lie along the $\hat{z}$ axis. While studying this vortex dynamics, we assume $T = 0$ and low magnetic field so that vortex-vortex interaction can be ignored. To move a vortex with respect to the superconducting flow requires a transverse lift force which is known as the Magnus force. The Magnus force acting on a vortex is proportional to the vector product of the velocity of the vortex relative to the superconducting system and a vector directed along the vortex core. Ao, and Thouless [44] calculated the Berry phase for the adiabatic motion around a closed loop at zero temperature and showed the existence of the Magnus force associated with Berry phase, as a general property of vortex line in a superconductor.

In some recent papers [27, 28] we have shown that due to certain features in the background lattice how Berry’s topological phase plays an important role in describing high $T_c$ superconductivity. Within this framework, we can also study the Magnus force required for a vortex to move.

In earlier section we have shown that a gauge field is responsible for the spin pairing and also for the hole pairing. Due to this interacting magnetic fluxoid the hole pair can overcome the bare Coulomb repulsion in high $T_c$ superconductivity. The superconducting phase is established when spin charge recombination comes into play i.e. a spin pair with each having unit magnetic flux and a pair of holes with each hole having unit magnetic flux interacts with each other through a gauge force. This gauge field when coupled with the vortex current will lead to the transverse force responsible for the motion of the vortices.

In our present formalism, we note that in the hole pair the associated flux quantum corresponding to $|\mu| = 1/2$ is derived from the bulk whereas the other flux quantum with $|\mu| = 1/2$ is due to the background related to the chirality of the frustrated spin system. In our model, we may assume that with the movement of the hole pair, the associated vortex line corresponding to the contribution from the bulk moves along with the centre of mass of the paired charge carriers and the condensate will experience an interaction with the background magnetic field. To study this interaction, we have to introduce the $\theta–\text{term}$ (last term in the Lagrangian (57)) as this corresponds to the vortex line representing magnetic flux quantum associated with the background magnetic field. The Lagrangian density of the model in spherical geometry, where the 2D surface is residing on the surface of a 3D sphere of large radius with a monopole at the centre, may be written as

$$L = \frac{1}{2}\phi^* (\partial_\theta - ieA_\theta) \phi - \phi (\partial_\theta + ieA_\theta) \phi^* + \frac{1}{2m} (\partial_a - ieA_a) \phi |^2 + \frac{\lambda}{2} (|\phi|^2 - \rho_0)^2 + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{4} \tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta}$$

(57)
Here $\rho_0$ corresponds to the stationary configuration with $|\phi|^2 = \rho_0$. The term $F_{\alpha\beta}$ corresponds to the electromagnetic field strength and $\tilde{F}_{\alpha\beta}$ corresponds to the background magnetic field. $^*\tilde{F}_{\alpha\beta}$ is the Hodge dual

$$^*\tilde{F}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\lambda\sigma} F_{\lambda\sigma}$$

It is noted that the P and T violating term $^*\tilde{F}_{\alpha\beta} \tilde{F}_{\alpha\beta}$ takes care of the chirality of the system. It is a four divergence and hence does not contribute to the equation of motion but quantum mechanically it contributes to the action. It is noted that there is a singularity at the z-axis and hence we can take the two dimensional formalism. To study the vortex dynamics, being inspired by Stone [15], we set $\phi = f e^{i\theta}$ so that we may write

$$L = i f^2 (\partial_0 \theta - i e A_0) + \frac{f^2}{2m} (\partial_\alpha \theta - i e A_\alpha)^2 + \frac{\lambda}{2} (f^2 - \rho_0)^2 + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{k}{4\pi} \epsilon_{\alpha\beta\lambda\sigma} B_\alpha \partial_\beta B_\lambda$$  (58)

It is observed that the dimensional reduction suggests that the anomalous term $^*\tilde{F}_{\alpha\beta} \tilde{F}_{\alpha\beta}$ in 3+1 dimensions corresponds to the Chern Simons term $\epsilon_{\alpha\beta\lambda} B_\alpha \partial_\beta B_\lambda$ in 2+1 dimensions. We now introduce Hubbard-Stratonovich fields $\vec{J}$ with the relation $J_0 = f^2$ to obtain

$$L \rightarrow L' = i J_\alpha (\partial_\alpha \theta - i e A_\alpha) + \frac{1}{8m J_0} \partial_\alpha (J_0)^2 + \frac{m}{2J_0} J_\alpha^2 + \frac{\lambda}{2} (f^2 - \rho_0)^2 + \text{gauge field terms}$$  (59)

We set the vortex part of the phase $\theta = \bar{\theta} + \eta$ where $\bar{\theta} = \arg(\vec{r} - \vec{r}_i(t))$ is the singular part of the phase due to vortices at $\vec{r}_i$ and $\eta$ is the non-singular part. Integration over $\eta$ suggests the conservation equation $\partial_\alpha J_\alpha = 0$ indicating $J_\alpha$ as a current. So we can identify

$$J_\alpha = \epsilon_{\alpha\beta\lambda} \partial_\beta B_\lambda = \frac{1}{2} \epsilon_{\alpha\beta\lambda} \tilde{F}_{\beta\lambda}$$  (60)

such that the first term in expression (59) corresponds to the interaction with the background magnetic field. Indeed defining the vortex current

$$K_\alpha = \epsilon_{\alpha\beta\lambda} \partial_\beta \bar{\theta}$$  (61)

we note that the first term in equation (60) can be written as

$$i B_\alpha (K_\alpha - e \epsilon_{\alpha\beta\lambda} \partial_\beta A_\lambda)$$

This shows that the vortex current is coupled to the background gauge potential $B_\alpha$. It is noted that $J_0$ has an equilibrium value $\rho_0$ even when the vortex is at rest. Motion with respect to this background field gives rise to a Lorentz force which is here just the Magnus force. So the Magnus force is generated by the background magnetic field when it interacts with the vortex current. In other words, the Magnus force is generated by the background magnetic field associated with the chirality of the system.

To calculate this Magnus force we may take resort to the Berry phase approach [14]. When the vortex moves round a closed loop, we can express the Berry phase $e^{i\phi}$ with

$$\phi = 2\pi N \mu$$  (62)

where $N$ is the total number of flux quantum enclosed by the loop. In our approach each flux quantum in the background is associated with a hole pair and so the number of flux quanta trapped is identical with the number of hole pairs enclosed by the loop. Thus we can identify $N$ as the number of hole pairs and we can express $\phi$ as

$$\phi = 2\pi \mu \frac{n_s}{2}$$  (63)

where $n_s$ is the charged superfluid number density far from the vortex core. The Magnus force is given by the vector product of the vorticity and the motion relative to the superconducting velocity

$$F_m = \pm 2\pi \frac{n_s}{2} \mu \hat{c} \times \vec{V}_{\text{vortex}}$$  (64)
Here +(-) corresponds to vortex parallel (antiparallel) to \( \hat{c} \) axis and \( \vec{V}_{vortex} \) is the velocity of the vortex with respect to the superconducting velocity. It is to be noted that the Magnus force explicitly depends on the number of carriers instead of their mass. This supports the Ao, Thouless theory of the origin of the Magnus force. As high \( T_c \) superconductors are type-II superconductors, in the presence of an external magnetic field, when some magnetic flux quanta penetrates the material, the number density \( n_s \) should be replaced by \( n \), the total density of the fluid when the radius of the integration contour is much larger than the London penetration depth. This is a consequence of the Meissner effect.

It is known that the Aharonov- Casher phase is generated when the flux moves through the mobile fluid charges. In the present situation, the phase arising out of the flux moving through the fluid charges will be cancelled by that coming from the flux motion through the static background ion charges. As the net charge in the macroscopic region is zero, the two Aharonov- Casher phases will cancel each other.

Actually, the renewed interest on the problem of Magnus force generated two conflicting points of view on the theory of transverse force. Volovik [47] has shown that the motion of the vortex with respect to the stationary condensate induces a spectral flow. A momentum transfer from the vortex system to a heat bath system is caused by a relaxation of the quasiparticles of the vortex bound states (i.e., the electronic states inside a vortex core). Therefore the vortex can apparently be moved without any external source of transverse momentum. In this spectral flow theory the coefficient of the transverse force \( k \) essentially depends on the electronic states inside a vortex core in combination of the relaxation time \( \tau \) and the quasiparticles. On the contrary, Ao and Thouless showed that the transverse force on a moving vortex is a robust quantity which does not depend on the details of the vortex bound states inside a vortex core but only on the superfluid density far from the core. The study of Magnus force in high temperature superconductivity in Berry phase approach supports the Ao Thouless theory of robust Magnus force.

VII. DISCUSSION

We have shown above that some characteristic features of high-\( T_c \) superconductivity can be analyzed in the framework of Berry phase and the different phase structures associated with it can be well interpreted in this scheme. Emery and Kivelson [19] have argued that the experimental findings support the idea that the local electronic structures are designed to lower the zero-point kinetic energy. In view of this, we note that the Berry phase analysis implicitly determines this condition of lowering the zero-point kinetic energy.

The spin-charge separation associated with the RVB states accounts for all the features of spin gap state. However, the superconducting phase is characterized by spin-charge recombination. Indeed, there exists a strong coupling among spinons and holons mediated through a gauge interaction and this gauge force effectively confines spinons and holons together. We have noted above that the magnetic flux associated with the Berry phase provides this gauge force and for this no ad-hoc mechanism is necessary.

The crossovers observed above the superconducting transition temperature \( T_c \) can be well interpreted when we analyze the anisotropic Heisenberg Hamiltonian using renormalization group fixed point theorem involving the Berry phase factor \( \mu \). In our present formalism it appears that this is a natural consequence when we consider the formation of spinons and holons in a RVB state and their interactions in the framework of Berry phase analysis. In fact, the observed phase diagram in the plane of temperature \( T \) vs. hole doping rate \( \delta \) shows the Bose condensation (superconducting temperature) curve of an arch shape rather than the linear increase which manifests the presence of an optimal doping [38, 48]. However, the pseudogap temperature displays nearly a linear decrease with \( \delta \). This is in conformity with the experimental findings which shows an universal behavior of \( T_2^* / T_c \) as a function of hole doping \( \delta / \delta_0 \) with \( \delta_0 \), the optimal doping rate. This universal behavior is also manifested in the relationship between \( T^* / T_c^{\text{max}} \) where \( T_c^{\text{max}} \) is the maximum superconducting transition temperature at optimal doping [38, 48]. These observations suggest the presence of a relationship between the spin gap crossover and superconducting phase. Thus the spin gap phase and the superconducting phase are not independent which also manifests the presence of coupling between spin and charge degrees of freedom [40].

In our formalism, we can make a remark on the mysterious sign reversal of the Hall resistivity (conductivity) effect in the underdoped region in cuprate superconductors [50]. It is noted that in the underdoped region there will not be sufficient number of holes to form superconducting pairs. So, in this case, a holon characterized by \( | \mu | = 1 \) will not be able to share the magnetic flux with another hole and form the requisite pair. The integral value of \( \mu \) will lead to the removal of the Berry phase to the dynamical phase.
as given by eqn.(11). Hence the Magnus force will be decreased. Besides, this in combination with the magnetic flux lines induced by the external magnetic field within the penetration depth may change the orientation of the vortices. Indeed, the interaction of this single holon with $\mu = 1$ with a magnetic flux line having $\mu = -1/2$ (due to the external magnetic field) will correspond to $\mu = 1/2$ and as a result we will get a magnetic flux line with opposite orientation. This change in orientation of the magnetic flux line will change the sign of the Hall conductivity. The change of the electronic state due to doping could be related to the internal electronic structure inside vortex core so that it affects the dynamic property of vortices. Actually, some people \cite{51} have considered this many body effect between vortices and got results to support the Ao-Thouless theory. In our field theoretical analysis through Berry phase we got the same result by calculating the interaction of the background magnetic field with the vortex current.

The attachment of the magnetic flux of the charge carrier suggest that this may be viewed as a skyrmion. The interaction of a massless fermion representing a neutral spin with a gauge field along with the interaction of a spinless hole with the gauge field enhances the antiferromagnetic correlation along with the pseudogap at the underdoped region. The superconducting pairing may be viewed as caused by skyrmion-skyrmion bound states. This effectively leads to topological superconductivity.

Abanov and Wiegman \cite{52, 53} have pointed out that topological superconductivity in 3 + 1 dimensions and 2 + 1 dimensions has its roots in the 1D Peierls-Fr"ohlich model which suggests that the $2\pi$ phase solitons of the Fr"ohlich model \cite{54} are charged and move freely through the system making it an ideal conductor. In spatial dimension greater than one this corresponds to superconductivity when the solitonic feature of a charge carrier is attributed to the attachment of a magnetic flux to it. It may be remarked here that in 1 + 1 dimensions we will have a nonlinear sigma model with the Wess-Zumino term when the target space is $S^3$ which is the $O(4)$ nonlinear sigma model. In the Euclidean framework however, this geometrically corresponds to the attachment of a vortex line to the two dimensional sheet which is topologically equivalent to the attachment of a magnetic flux \cite{55}. This suggests that the topological feature of ideal conductivity visualized by Fr"ohlich in 1 + 1 dimensions and that of superconductivity in 2 + 1 and 3 + 1 dimensions have a common origin.
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