Using a block of three separated solid elements, a thermal source and drain together with a gate made of an insulator-metal transition material exchanging near-field thermal radiation, we introduce a nanoscale analog of a field-effect transistor which is able to control the flow of heat exchanged by evanescent thermal photons between two bodies. By changing the gate temperature around its critical value, the heat flux exchanged between the hot body (source) and the cold body (drain) can be reversibly switched, amplified, and modulated by a tiny action on the gate. Such a device could find important applications in the domain of nanoscale thermal management and it opens up new perspectives concerning the development of contactless thermal circuits intended for information processing using the photon current rather than the electric current.

PACS numbers: 44.05.+e, 12.20.-m, 44.40.+a, 78.67.-n

Figure 1: Electric and radiative thermal transistor.
Classical field-effect transistor (a): a three-terminal device known as source, gate and drain, which correspond to the emitter, base, and collector of electrons. The gate is used to actively control, by applying a potential on it, the apparent conductivity of the channel between the source and the drain. Radiative thermal transistor (b): a layer of IMT material (the gate) is placed at subwavelength distances from two thermal reservoirs (the source and the drain). The temperature $T_S$ of the gate can be modulated from the external environment around its steady-state temperature $T_{eq}$ which corresponds to the situation where the net flux $\Phi_G$ received by the gate vanishes so that the flux $\Phi_D$ received by the drain and lost by the source $\Phi_S$ can be tuned. When $T_{eq}$ is a little bit smaller than the critical temperature $T_c$ of IMT material a small amount of heat applied on the gate induces a strong switching of heat fluxes $\Phi_D$ and $\Phi_S$ owing to its metal-insulator phase transition.

We introduce here a thermal transistor (Fig. 1) based on the heat transport by evanescent photons rather than by acoustic waves or electrons. This near-field thermal transistor (NFTT) basically consists of a gate made of an insulator-metal transition (IMT) material which is able to qualitatively and quantitatively change its optical properties through a small change of its temperature around a critical temperature $T_c$. Vanadium dioxide ($\text{VO}_2$) is one of such materials which undergoes a first-order transition (Mott transition) from a high-temperature metallic phase to a low-temperature insu-
lating phase close to room-temperature \( T_c = 340 \text{ K} \). Different works have already shown that the heat-flux exchanged at close separation distances (i.e. in the near-field regime) between an IMT material and another medium, can be modulated by several orders of magnitude across the phase transition of IMT materials. Further radiative thermal diodes have been recently conceived allowing for rectification of heat flux using materials with thermally dependent refractive indices and IMT materials. But so far controlling the heat flow with contactless systems to get the same functionalities as a classical FET has remained a challenging problem. Here we show that IMT materials are very promising candidates for designing efficient gates to control (switch, modulate or amplify) the heat flux exchanged between two media.

To start, let us consider the system as illustrated in Fig. 1(b), where two media which we call by analogy with a FET the source and the drain labeled by the indices S and D are maintained at temperatures \( T_S \) and \( T_D \) with \( T_S > T_D \) by some thermostats. A thin layer of IMT material labeled by G having a thickness \( \delta \) is placed between both media at a distance \( d \) from the source and the drain. Without external excitation, the system reaches its steady state for which the net flux \( \Phi_G \) received by the intermediate medium, the gate, is zero. In this case its temperature \( T_G \) is set by the temperature of the surrounding media, i.e. the drain and the source. When a certain amount of heat is added to or removed from the gate (for example by applying a voltage difference through a couple of electrodes as illustrated in Fig. 1 or extracted from it using Peltier elements), its temperature can be either increased or reduced around its equilibrium temperature \( T^{eq}_G \). Hence, the heat flux \( \Phi_D \) received by the drain and the heat flux \( \Phi_S \) lost by the source can be tailored accordingly. These heat fluxes correspond to the heat fluxes of Poynting vector across any plane separating the gate and the drain (and the source and the gate). In a three body system the flux received by photon tunneling by the drain reads

\[
\Phi_D = \int_0^\infty \frac{d\omega}{2\pi} \phi_D(\omega, d), \tag{1}
\]

where the monochromatic heat flux is given by

\[
\phi_D = \hbar \omega \sum_{j=s,p} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left[ n_{SG}(\omega) T^{S/G}_j(\omega, \mathbf{k}; d) + n_{DG}(\omega) T^{G/D}_j(\omega, \mathbf{k}; d) \right]. \tag{2}
\]

Here \( T^{S/G}_j \) and \( T^{G/D}_j \) denote the efficiencies of coupling of each mode \((\omega, \mathbf{k})\) between the source and the gate and between the gate and the drain for both polarization states \( j = s, p; \mathbf{k} = (k_x, k_y)^T \) is the wavevector parallel to the surfaces of the multilayer system. In the above relation \( n_{ij} \) denotes the difference of Bose-distributions functions \( n_i = \{ e^{\hbar \omega/k_B T_i} - 1 \}^{-1} \) at the frequency \( \omega \); \( k_B \) is Boltzmann’s constant and \( 2\pi \) is Planck’s constant.

According to the N-body near-field heat transfer theory presented in Ref. [10], the transmission coefficients \( T^{S/G}_j \) and \( T^{G/D}_j \) of the energy carried by each mode written in terms of reflection coefficients \( \rho_{ES,j} = \rho_{SE,j} \) and transmission coefficients \( \tau_{ES,j} \) of each basic element of the system and in terms of reflection coefficients \( \rho_{EF,j} \) of couples of elementary elements [11]

\[
T^{S/G}_j(\omega, \mathbf{k}, d) = \frac{4 | \tau_{G,j} |^2 \Im[\rho_{SG,j} \rho_{D,j} e^{-2\gamma d}]|^2}{1 - |\rho_{SG,j} \rho_{D,j} e^{-2\gamma d}|^2}, \tag{3}
\]

\[
T^{G/D}_j(\omega, \mathbf{k}, d) = \frac{4 \Im[\rho_{SG,j} \rho_{D,j} e^{-2\gamma d}]^2}{1 - |\rho_{SG,j} \rho_{D,j} e^{-2\gamma d}|^2},
\]

introducing the imaginary part of the wavevector normal to the surfaces in the multilayer structure \( \gamma = \Im(k_z) = \sqrt{\kappa^2 - \omega^2/c^2}; c \) is the velocity of light in vacuum. Similarly the heat flux lost by the source reads

\[
\phi_S = \hbar \omega \sum_{j=s,p} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left[ n_{DG}(\omega) T^{D/G}_j(\omega, \mathbf{k}; d) + n_{DG}(\omega) T^{G/S}_j(\omega, \mathbf{k}; d) \right], \tag{4}
\]

where the transmission coefficients are analog to those defined in Eq. [3] and can be obtained making the substitution \( S \leftrightarrow D \).

At steady state, the net heat flux received/ emitted by the gate which is just given by the heat flux from the source to the gate minus the heat flux from the gate to the drain vanishes, i.e. \( \Phi_S = \Phi_D \) or \( \Phi_G = \Phi_S - \Phi_D = 0 \). \tag{5}

This relation allows us to identify the gate equilibrium temperature \( T^{eq}_G \) (which is not necessary unique) for given temperatures \( T_S \) and \( T_D \). Note that out of steady state the heat flux received/ emitted by the gate is \( \Phi_G = \Phi_S - \Phi_D \neq 0 \). If \( \Phi_G > 0 \) (\( \Phi_S < 0 \)) an external flux is added to (removed from) the gate by heating (cooling).

To illustrate the operating modes of NFTT we consider now a system composed by a source and a drain both made of silica (each coupled to a thermostat to maintain their local temperatures constant in time) and a gate made of vanadium dioxide VO \(_2\). When the gate temperature \( T_G \) is smaller than its critical temperature \( T_c \), then the gate is in its monoclinic phase and it behaves as an uniaxial crystal. On the other hand, when \( T_G = T_c \) the gate transits toward its amorphous metallic phase and remains in this state for greater temperatures. We consider here the case where the optical axis of VO \(_2\) film is orthogonal to its interfaces. The p-polarized transmission coefficients of the energy carried by the modes \((\omega, \mathbf{k})\) through such a system are plotted in Fig. 2. When the gate is in its crystalline state,
by the modes. Efficiency of coupling of modes ($\omega, \kappa$) in a SiO$_2$-VO$_2$-SiO$_2$ system ($\delta = 50$ nm and $d = 100$ nm). (a) $\tau_{p}^{S/G}$ and (b) $\tau_{p}^{G/D}$ with VO$_2$ in its crystalline state. (c) $\tau_{p}^{S/G}$ and (d) $\tau_{p}^{G/D}$ with VO$_2$ in its amorphous state. Wien’s frequency (where the transfer is maximum) at $T = 340$ K is $\omega_{\text{Wien}} \sim 1.3 \times 10^{14}$ rad/s. The dielectric permittivity of SiO$_2$ is taken from the database [17].

$\tau_{p}^{S/G}$, which represents the exchange between the source and the drain mediated by the presence of the gate [see Fig. 2(a)], and $\tau_{p}^{G/D}$, which corresponds to the exchange between the couple source-gate treated as a unique body and the drain [see Fig. 2(b)] shows an efficient coupling of modes between the different blocks of the system around the resonance frequencies $\omega_{\text{SPP1}} \sim 1 \times 10^{14}$ rad/s and $\omega_{\text{SPP2}} \sim 2 \times 10^{14}$ rad/s of surface waves (surfaces phonon-polaritons) supported by both the source and the drain. Below $T_c$ all parts of the system support surface waves in the same frequency range close to the thermal peak frequency $\omega_{\text{Wien}} \sim 1.3 \times 10^{14}$ rad/s. The anti-crossing curves which appear in Fig. 2(a) and (b) result from the strong coupling of silica surface phonon-polaritons (SPPs) and the surface waves (symmetric and antisymmetric ones) supported by the thin VO$_2$ layer. Beyond $T_c$ the gate becomes amorphous (metallic) and it does not support surface wave anymore. In this case $\tau_{p}^{S/G}$ [see Fig. 2(c)] vanishes owing to the field screening by the gate. Moreover, as is clearly shown in Fig. 2(d), the coupling of modes between the couple source-gate and the drain at the frequency of surface waves is less efficient for the large parallel values of $\kappa$ reducing so the transfer of heat towards the drain, i.e. the number of participating modes decreases [18 19]. By using different physical parameters (temperatures, sizes, separation distances...) several functions can be assigned to this system which can become either (i) a thermal switch, (ii) a thermal modulator or (iii) a thermal amplifier. We discuss below those operating modes. To do so we show Fig. 3 the net heat flux received by each part of the system in a particular configuration where $\delta = 50$ nm, $d = 100$ nm, $T_S = 360$ K and $T_D = 300$ K.

(i) Thermal switching:

In the situation depicted in Fig. 3 we have $T_G^{\text{eq}} = 332$ K. An increase of $T_G$ by about 10 degrees ($\Phi_G$ is increased by $\sim 10^{-8}W/\mu m^2$) leads, as clearly shown in Fig. 2 to a reduction of heat flux received by the drain and lost by the source by more than one order of magnitude. That means our NFTT can be used in two operating modes where $T_G$ is slightly below or above the critical temperature $T_c$, where in the case $T_G < T_c$ we are in the ‘on’ mode and for $T_G > T_c$ we are in the ‘off’ mode.

(ii) Thermal modulation:

Over the temperature region around $T_G^{\text{eq}}$ (gray shadow strip on Fig. 3) the heat current $\Phi_G$ over the gate remains quite small (i.e. $\Phi_S \sim \Phi_D$) while the flux received by the drain or lost by the source can be modulated from high to low values. The thermal inertia of the gate as well as its phase transition delay of IMT material define the timescale at which the modulator can operate. A much larger modulation of fluxes can be achieved with the NFTT when using $T_G^{\text{eq}} < T_c$. Then the flux can be modulated over one order of magnitude by a small temperature change of $T_G$.

(iii) Thermal amplification:

The most important feature of a transistor is its ability to amplify the current or electron flux towards the drain. In the region of phase transition around $T_c$ we see that an increase of $T_G$ leads to a drastic reduction of flux received by the drain. This corresponds to a negative differential thermal conductance as recently described for SiC in Ref. [13] (note that this behavior does not violate the second principle of thermodynamics because the heat flux continues to flow from the hot to the cold body). Having a negative differential thermal conductance is the key for having an amplification which is defined as (see for example Ref. [2])

$$\alpha = \left| \frac{\partial \Phi_D}{\partial \Phi_G} \right| = \frac{1}{1 - \phi^{S/D}}$$

where

$$\phi^{S/D} = \frac{\partial \Phi_{S/D}}{\partial T_G}.$$ 

It can be easily shown that $\alpha = 1/2$ for $T_G$ much smaller or larger than $T_c$ where the material properties of VO$_2$ are more or less independent of $T_G$, since $\phi^{S} = -\phi^{D}$. On
the other hand, inside the transition region of VO$_2$ that means for temperatures around $T_c$ the material properties of VO$_2$ change drastically showing a negative differential thermal resistance/conductance which leads to an amplification, i.e. $\alpha > 1$. This behaviour is qualitatively demonstrated in Figs. 3 and 4 where the dielectric permittivity of VO$_2$ in the transition region is modelled using a Bruggeman mixing rule as introduced in Ref. [5]. Hence, by chosing the temperatures such that $T^*_{eq} \approx T_c$ the NFTT works as an amplifier.

Figure 3: Operating regimes of near-field thermal transistor. When $T^*_{eq}$ is a little bit smaller than the critical temperature $T_c$ of the IMT material a small amount of heat applied on the gate induces a strong switching of heat fluxes $\Phi_D$ and $\Phi_s$ owing to its phase transition. By changing the flux $\Phi_G$ supplied to the gate different functions (thermal switching, thermal modulation and thermal amplification) can be performed. The fluxes plotted here correspond to a gate of VO$_2$ with thickness $\delta = 50$ nm located at a distance $d = 100$ nm from two massive silica samples maintained at $T_S = 360$ K and $T_D = 300$ K.

The ability to control the flow of heat at subwavelength scale in complex architectures of solids out of contact, opens up new opportunities for an active thermal management for dissipating systems. It also suggests the possibility to develop contactless thermal analogs of electronic devices such as thermal logic gates and thermal memories, for processing information by utilizing thermal photons rather than electrons. Unlike other schemes for creating thermal transistors which were so far based on the control of acoustic phonons, the present concept authorizes much higher operational speeds (speed of light) and should be very competitive compared to the previous ones. We think also that the near-field thermal transistors could find broad applications in MEMS/NEMS technologies and could be used to generate mechanical works by modulating the heat flux received by the drain, by using microresonators such as cantilevers in contact with it.

Figure 4: Amplification factor $\alpha$ of the NFTT. When $T^*_{eq} \ll T_c$ or $T^*_{eq} \gg T_c$ the slopes of $\Phi_s$ and $\Phi_D$ are almost identical (modulo the sign) so that $\alpha \approx 1/2$. On the contrary, in the close neighborhood of $T_c$ we have $\alpha > 1$ owing to the negative differential thermal resistance in the transition region. Here, the parameters of the NFTT are the same as in Fig. 3.

* Electronic address: pba@institutoptique.fr
† Electronic address: s.age.biehs@uni-oldenburg.de
[1] J. Bardeen and W. H. Brattain, Phys. Rev. 74, 230 (1948).
[2] B. Li, L. Wang and G. Casati, Appl. Phys. Lett. 88, 143501 (2006).
[3] L. Wang, B. Li, Phys. Rev. Lett. 99, 177208 (2007).
[4] N. Li, J. Ren, L. Wang G. Zhang, P. Hänggi and B. Li, Rev. Mod. Phys. 84, 1045 (2012).
[5] M. M. Qazilbash, M. Brehm, B. G. Chae, P.-C. Ho, G. O. Andreiev, B. J. Kim, S. J. Yun, A. V. Balatsky, M. B. Maple, F. Keilmann, H. T. Kim, D. N. Basov, Science, 318, 5857, 1750-1753 (2007).
[6] A. S. Barker, H. W. Verleur, and H. J. Guggenheim, Phys. Rev. Lett. 17, 1286 (1966).
[7] P. van Zwol, K. Joulain, P. Ben-Abdallah, J. J. Greffet and J. Chevrier, Phys. Rev. B (R), 83, 20, 201404 (2011).
[8] P. van Zwol, K. Joulain, P. Ben-Abdallah and J. Chevrier, Phys. Rev. B(R), 84, 161413 (2011).
[9] P. J. van Zwol, L. Ranno, and J. Chevrier, Phys. Rev. Lett. 108, 234301 (2012).
[10] C. Starr, J. Appl. Phys. 7, 15 (1936).
[11] N. A. Roberts and D. G. Walker, Int. J. thermal Sciences 50, 648 (2011).
[12] C. R. Otey, W. T. Lau, and S. Fan, Phys. Rev. Lett. 104, 154301 (2010).
[13] L. Zhu, C. R. Otey, and S. Fan, Appl. Phys. Lett. 100, 044104 (2012).
[14] P. Ben-Abdallah and S.-A. Biehs, arXiv:1307.3154 (2013).
[15] E. Nezzaoui, J. Drevillon, Y. Ezzahri, and K. Joulain, arXiv:1306.6209v1 (2013).
[16] R. Messina, M. Antezza and P. Ben-Abdallah, Phys. Rev. Lett. 109, 244302 (2012).
[17] Handbook of Optical Constants of Solids, edited by E. Palik (Academic Press, New York, 1998).

[18] S.-A. Biehs, E. Rousseau, and J.-J. Greffet, Phys. Rev. Lett. 105, 234301 (2010).

[19] P. Ben-Abdallah and K. Joulain, Phys. Rev. B 82, 121419(R) (2010).