BRST Quantization, Strong CP Violation, the U(1) Problem and $\theta$ Vacua

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The non-perturbative implications of BRST quantization are examined for $\theta$ vacua and related issues. Strong CP violation is shown to be absent for QCD in the BRST formalism. Previous evidence for CP violation is reexamined, and much of it is found to be inconclusive. It is proposed that some lattice calculations be redone to clarify the situation. For the U(1) problem, difficulties are encountered for a conventional solution within the BRST framework. We also find problems with previous instanton and non-instanton approaches.

1. Introduction

BRST invariance has proven to be a powerful tool to establish the renormalizability of the standard model and to elucidate its formal structure. It has been subsequently extended to encompass general gauge theories and string theories.

The purpose of this paper is to examine the non-perturbative implications of BRST quantization for QCD. For QCD in the BRST formalism, specific conclusions can be reached regarding the nature of $\theta$ vacua, strong CP violation and the U(1) problem. Taken at face value, there appears to be some evidence against these conclusions. Upon closer scrutiny, however, this evidence is found to be inconclusive. As a result, we also find that the U(1) problem is still a problem. Previous instanton and non-instanton approaches have problems.

The organization of the paper is as follows. In section 2, we discuss the strong CP problem. We show that strong CP violation is absent with BRST quantization. Some other corollaries are also derived. In section 3, we discuss the U(1) problem. Difficulties are encountered in finding a conventional solution within the BRST framework. In section 4, previous evidence for CP violation is reanalyzed and much of it is found to be inconclusive. In particular, existing instanton calculations in singular gauges are shown to violate divergence identities, and chiral perturbation theory is found to break down away from $\theta = 0$. Problems with previous instanton and non-instanton approaches to the U(1) problem are also noted. In section 5, the validity of the BRST quantization itself is discussed, and its possible inequivalence with other quantization schemes is considered. Specific suggestions for new lattice calculations are suggested, that may shed light on the presently discussed issues.

In the discussion, we will occasionally need to switch between Minkowski space and Euclidean space. The notation $A_\mu B^\mu$ will be used for Minkowski space and $A_\mu B^\mu$ for Euclidean space, to distinguish between the two when necessary.

2. Strong CP Violation

The discovery of instantons has led to the problem of strong CP violation. Generally one has tried to eliminate the problem by some relaxation mechanism for the vacuum angle. If the relaxation mechanism is due to weak interactions, one has axions. However, empirical evidence does not favor this possibility so far. There have also been attempts to find a mechanism within QCD itself. However, decisive results are difficult to obtain, due to the intractability of QCD at long distances.

In this section, we show that such an explicit mechanism may be unnecessary. For QCD in the BRST formalism, there is no strong CP violation in the first place.

We recall that the topological charge density $\Xi = (g^2/32\pi^2)F^a_{\mu\nu}F^{\mu\nu a}$ is a total divergence $\Xi = \partial^\mu K_\mu$, where

$$K_\mu = \frac{g^2}{32\pi^2}f_{\mu\nu\rho\sigma}(A^\nu a F^{\rho\sigma a} - \frac{g}{3} f^{abc} A^\nu a A^{\rho b} A^{\sigma c})$$  \hspace{1cm} (1)

is the Loos-Chern-Simons current. It follows that the QCD Hamiltonian for vacuum angle $\theta = 0$ is related to that for $\theta = 0$ by a unitary transformation $H(\theta) = e^{i\theta \mathbf{X}} H(0) e^{-i\theta \mathbf{X}}$ with $\mathbf{X} = \int d^4 x K_\alpha$. $\mathbf{X}$ is invariant under infinitesimal gauge transformations, so it is invariant under BRST transformations. Hence, if $|0 >$ is the physical ground state of $H(0)$, \textit{i.e.}

$$H(0)|0 > = 0 \quad Q_{\text{BRST}}|0 > = 0$$  \hspace{1cm} (2)

then, $e^{i\theta \mathbf{X}}|0 >$ is the physical ground state of $H(\theta)$,

$$H(\theta)e^{i\theta \mathbf{X}}|0 > = 0 \quad Q_{\text{BRST}}e^{i\theta \mathbf{X}}|0 > = 0$$  \hspace{1cm} (3)

with the same energy. In general, $H(\theta)$ is physically equivalent to $H(0)$, and there is no CP violation in particular.

\footnote{Note that here $\mathbf{X}$ is single-valued over the physical subspace, while in the canonical formalism (see below) $\mathbf{X}$ is multi-valued.}
The point is that there seems to be no constraints corresponding to topologically non-trivial "large" gauge transformations. The BRST Hamiltonian is not invariant under ordinary "large" gauge transformations, and there are no "large" BRST transformations, since BRST transformations are global (rigid) [16].

The \( \theta \)-independence of the vacuum energy density \( \mathcal{E} \) in BRST quantization implies \( \langle \theta | \mathcal{E} | \theta \rangle = -\partial \mathcal{E} / \partial \theta = 0 \). This result also follows from translation invariance [12]. If the vacuum state is normalizable and translation invariant

\[
\langle \theta | K_\mu(x) | \theta \rangle = \langle \theta | e^{iP\cdot x} K_\mu(0) e^{-iP\cdot x} | \theta \rangle = \langle \theta | K_\mu(0) | \theta \rangle \tag{4}
\]

is constant, where \( P_\mu \) is the energy-momentum operator. Hence, \( \langle \theta | \Xi | \theta \rangle = \partial^\mu < \theta | K_\mu | \theta > = 0 \).

The above derivation fails to go through in the canonical formalism, where the vacuum state is not normalizable, so that \( \langle \theta | K_\mu | \theta \rangle \) may be ambiguous like \( \langle p|x|p \rangle \) in quantum mechanics. However, there is no difficulty in covariant gauge, where the vacuum state is normalizable.

It follows that the topological susceptibility at zero momentum

\[
\chi(\theta) = \frac{\partial < \theta | \Xi | \theta >}{\partial \theta} = -i \int d^4x \partial^\mu \partial^\nu < \theta | T^* K_\mu(x) K_\nu(0) | \theta >_{\text{con}} \tag{5}
\]

is also zero.

In the above, we have used the fact that derivatives come outside of the time-ordered product in a path integral formulation. This is necessary for the Fujikawa-Vergeles analysis [13] to be compatible with the Ward identities for the gauge-variant flavor-singlet axial current \( j_{\mu5}^{\text{sym}} \lbrack A,q \rbrack \). For an operator \( \mathcal{O} \) with chirality \( \alpha \), the former gives

\[
\int [dA][dq][d\bar{q}] e^{iS[A,q,\bar{q}]} (\partial^\mu j_{\mu5}^{\text{inv}}(x) \mathcal{O}(0) + \alpha \delta^4(x) \mathcal{O}(0) - 2N_f \Xi(x) \mathcal{O}(0)) = 0 \tag{6}
\]

in the chiral limit, where \( j_{\mu5}^{\text{inv}} = j_{\mu5}^{\text{sym}} + 2N_f K_5 \) is the gauge-invariant axial current and \( N_f \) is the number of flavors. On the other hand, the latter gives

\[
\partial^\mu < \theta | T^* j_{\mu5}^{\text{sym}}(x) \mathcal{O}(0) | \theta > = -\alpha \delta^4(x) < \theta | \mathcal{O}(0) | \theta > \tag{7}
\]

Hence, \( \delta \) should be interpreted as

\[
\partial^\mu < \theta | T^* j_{\mu5}^{\text{inv}}(x) \mathcal{O}(0) | \theta > + \alpha \delta^4(x) < \theta | \mathcal{O}(0) | \theta > = 2N_f \partial^\mu < \theta | T^* K_\mu(x) \mathcal{O}(0) | \theta > \tag{8}
\]

Another important check is that \( \delta \) is invariant under additive renormalization as it affects only the contact term in \( < \theta | T^* K_\mu(x) K_\nu(0) | \theta > \), which is annihilated by \( \int d^4x \partial^\mu \partial^\nu \). With this in mind, we note that at finite momentum, the correlation function

\[
- \int d^4x e^{i\theta \cdot x} \partial^\mu \partial^\nu < \theta | T^* K_\mu(x) K_\nu(0) | \theta >_{\text{con}} \tag{9}
\]

obeys a twice-subtracted dispersion relation. Therefore, the vanishing of the topological susceptibility \( \delta \) provides no useful information on the spectrum, apart from the prohibition of massless vector ghosts.

Another corollary is obtained by taking the vacuum expectation value of the anomaly,

\[
\partial^\mu < j_{\mu5}^{\text{inv}} >_\theta = 2m < \bar{q}i\gamma_5 q >_\theta + 2N_f < \Xi >_\theta \tag{10}
\]

where we have taken real and equal quark masses for simplicity. The vacuum expectation value of a gauge-invariant operator should be well defined and translation invariant in any gauge, so the left-hand side is zero. With \( < \Xi >_\theta > 0 \), we have

\[
< \theta | \Xi >_\theta = 0 \quad m \neq 0 \tag{11}
\]

3. The U(1) Problem

The axial U(1) symmetry of the QCD action is not apparent in nature, although a non-vanishing quark condensate suggests the existence of a ninth Nambu-Goldstone mode in the chiral limit [17]. This is the well-known U(1) problem. The absence of such a mode in the hadronic spectrum is believed to be related to the chiral anomaly [16]. 't Hooft [17] has suggested that instantons may provide a solution to the problem through the anomaly (see, however [18]). Witten [19] has proposed that the problem can be solved in the large \( N_c \) limit, where the anomaly can be treated as a perturbation. Witten's proposal was later interpreted by Veneziano [20] in terms of vector ghosts.

Note, however, that the large \( N_c \) analysis for QCD rests on the assumption that the topological susceptibility is non-zero in Yang-Mills theory, which is incompatible with BRST quantization as we have seen. In particular, Veneziano's analysis involves the assumption that translation invariance is broken for the one-point function of \( K_\mu \) but not for the two-point function.

A non-zero \( < \Xi >_\theta \) also tends to run counter to large \( N_c \) arguments. The Eguchi-Kawai reduction [21] implies that translation invariance should be maintained on the lattice for large \( N_c \). Also, the master field \( A_5^\mu \) (in the weaker sense of [22]) for large \( N_c \) Yang-Mills theory is
expected to be translation invariant \(2^{3}\). It would then follow that

\[
\langle \theta | \Xi | \theta \rangle = \Xi[A^M_\theta] = \partial^\mu (K_\mu [A^M_\theta]) = 0 \quad (12)
\]

Another piece of evidence is the vacuum energy density \(\mathcal{E}\). \(N_c\) counting arguments give \(\mathcal{E} = N_c^2 F(\theta/N_c)\) to leading order, where \(F\) is some function. For this to be true in \(\theta\) with a period independent of \(N_c\), \(F\) must be a constant. Hence \(\langle \Xi \rangle = -\partial \mathcal{E}/\partial \theta = 0\) to leading order.

To avoid this conclusion, Witten \(24\) has previously argued that \(F\) is a multi-valued function. In as much as the original premise of \(N_c\) counting relies on the fact that amplitudes are given by an infinite sum of Feynman diagrams which is necessarily single-valued, one is at a dilemma. On the other hand, Leutwyler and Smilga \(24\) have argued that the period of \(\theta\) is indeed \(2\pi N_c\) for Yang-Mills theory. However, such a result is contrary to the canonical formalism (see below) as well as the BRST formalism.

We therefore reexamine the U(1) problem within the BRST formalism. The absence of a physical massless U(1) boson for massive quarks gives

\[
0 = \int d^4x \partial^\mu <\theta | T^* j_{\mu}^{m\gamma} (x) T \bar{q} q(0) | \theta >
= -2i <\theta | \bar{q} q | \theta >
+ 2m \int d^4x <\theta | T^* T \bar{q} q(x) T \bar{q} q(0) | \theta >
+ 2N_f \int d^4x \partial^\mu <\theta | T^* K_\mu (x) T \bar{q} q(0) | \theta >
\quad (13)
\]

where we have used the massive analogue of \(8\). On the other hand, \(11\) gives

\[
\int d^4x \partial^\mu <\theta | T^* K_\mu (x) T \bar{q} q(0) | \theta >
= -i \frac{\partial}{\partial \theta} <\theta | \bar{q} q | \theta > = 0
\quad (14)
\]

Hence

\[
-2i <\theta | \bar{q} q | \theta >
+ 2m \int d^4x <\theta | T^* T \bar{q} q(x) T \bar{q} q(0) | \theta > = 0
\quad (15)
\]

However, this is just the type of equation we would obtain for the flavor non-singlet current

\[
0 = \int d^4x \partial^\mu <\theta | T^* \bar{q} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} q(x) T \bar{q} \gamma_\mu \gamma_5 \lambda^b q(0) | \theta >
= -i \frac{\partial}{\partial \theta} <\theta | \bar{q} \lambda^a, \lambda^b | q | \theta >
+ m \int d^4x <\theta | T^* \bar{q} \gamma_\mu \gamma_5 \lambda^a q(x) T \bar{q} \gamma_\mu \gamma_5 \lambda^b q(0) | \theta >
\quad (16)
\]

Therefore it appears that the same conventional assumptions which lead to \(m^2_\pi = \mathcal{O}(m)\) for the latter imply that

the mass squared of the flavor-singlet boson is also \(\mathcal{O}(m)\) \(3\).

In general, \(10\) implies that \(15\) is proportional to \(\chi\), so our result is consistent with a previous observation \(25\) of a trade-off between the solution of the strong CP problem and the solution of the U(1) problem. Samuel \(3\) has suggested that a simultaneous resolution may be possible, but this does not appear to be the case.

The point may be seen in another way. The difference between the flavor-singlet and non-singlet correlator causes a 1/m singularity which could have made a difference.

\[
<< \text{Tr}(i\gamma_5 S_A(x,y)) \text{Tr}(i\gamma_5 S_A(0,0)) >>_\theta
\quad (17)
\]

where \(S_A(x,y)\) denotes the quark propagator in a fixed gluon-background \(A\), the trace is over internal indices, and the averaging is over the gluon field only \(3\). Writing

\[
S_A(x,y) = \sum \psi_n(x) \psi_n^\dagger(y) - \frac{1}{m} \sum_{\lambda_n=0} \psi_n(x) \psi_n(y) + \mathcal{A}(x,y)
\quad (18)
\]

where \(\mathcal{A}\) is the non-zero mode part of \(S_A\), the index theorem yields

\[
\frac{1}{V} \int d^4x d^4y << \text{Tr}(i\gamma_5 \mathcal{A}(x,y)) \text{Tr}(i\gamma_5 \mathcal{A}(y,x)) >>_\theta = -\frac{N_f^2}{Vm} << \nu^2 >>_\theta
\]

\[
+ \frac{2iN_f}{Vm} \int d^4y << \nu \text{Tr}(i\gamma_5 \mathcal{A}(y,y)) >>_\theta
\]

\[
+ \frac{1}{V} \int d^4x d^4y << \text{Tr}(i\gamma_5 S_A(x,x)) \text{Tr}(i\gamma_5 S_A(y,y)) >>_\theta
\quad (19)
\]

where \(V\) is the spacetime volume and \(\nu = \int_V d^4x \Xi\) is the topological charge. Hence, the vanishing of the topological susceptibility \(8\) and \(13\) imply the absence of \(1/m^2\) and \(1/m\) singularities which could have made a difference.

4. Reexamination of Previous Arguments

\(3\) Since the isosinglet correlator obeys a twice-subtracted dispersion relation, it may be that one of the subtraction constant diverges in the chiral limit, thereby solving the U(1) problem. This behavior, however, is at odds with conventional QCD perturbation theory. Also, as the pionic thresholds move to zero in the chiral limit, they may pile up in the flavor-singlet correlator causing a \(1/m\) singularity, and thereby solving the U(1) problem. The new problem, however, is then why this does not happen for the flavor non-singlet correlator.

\(4\) We have suppressed the quark determinant, for convenience.
There are five reasons why CP violation was previously assumed to be present.

The first reason is instantons. Instanton calculations give the vacuum energy density as a non-trivial function of $\theta$. However, the results are not self-consistent, since they are based on ’t Hooft’s calculation which employs BRST quantization.

A possible source of this inconsistency is that the calculations in [24,27] adopt a singular gauge to evaluate the instanton-antiinstanton interaction [1]. To see this, let $D$ be a domain in Euclidean space bounded by two surfaces, one $S_1$ surrounding the singularity, and another $S_2$ at a large distance away, so that

$$\int_D d^4x \Xi(x) = \int_{S_2} d^3S_\mu K_\mu - \int_{S_1} d^3S_\mu K_\mu$$  \hspace{1cm} (20)

For an instanton in singular gauge, the fields fall off rapidly at infinity so that the integral over $S_2$ vanishes as $S_2$ is sent off to infinity. Shrinking $S_1$ then gives

$$\lim \int_{S_1} d^3S_\mu K_\mu = -1$$  \hspace{1cm} (21)

However, this means that $\partial_\mu K_\mu = \Xi - \delta^4$, thus violating the basic identity.

The second reason is canonical quantization. The Hamiltonian still obeys $H(\theta) = e^{i\theta X} H(0) e^{-i\theta X}$, but the subsidiary condition is altered to

$$U[\Omega] |\text{phys}>= |\text{phys}>$$  \hspace{1cm} (22)

where $U[\Omega]$ is the unitary operator implementing the gauge transformation $\Omega$ (unitarized form of Gauss law).

If $\Omega$ is topologically non-trivial, $e^{i\theta X}$ does not commute with the constraint [22] unless $\theta$ is an integer multiple of $2\pi$. Hence, Hamiltonians with $\theta \neq 0$ were regarded as physically inequivalent. However, the argument applies independently of the presence of quarks, whereas physics should be independent of $\theta$ if massless quarks are present so it is not generally reliable.

The third reason is an effective Lagrangian calculation [24], in which the vacuum energy density is given by

$$E_0 = -\Sigma \left( m_u^2 + m_d^2 + 2m_wm_d \cos \theta \right)^{1/2}$$  \hspace{1cm} (23)

where $m_u$ and $m_d$ are the up and down quark masses, and $\Sigma = -<0|\pi_u|0>$ in the chiral limit.

However, the effective Lagrangian approach relies on chiral perturbation theory, which is not valid for $\theta \neq 0$ if BRST quantization is valid. For example, the Fujikawa-Vergeles analysis [13] gives

$$\lim_{m \rightarrow 0^+} <\theta, \hat{\phi} e^{-i\theta \gamma_5/N_f} |\Gamma_{\theta} |\hat{\phi} e^{-i\theta \gamma_5/N_f} >=$$

$$= -N_f \cos \theta \frac{\theta}{N_f}$$

$$\lim_{m \rightarrow 0^+} <\theta, \hat{\phi} e^{-i\theta \gamma_5/N_f} |\hat{\Gamma}_{\theta} |\hat{\phi} e^{-i\theta \gamma_5/N_f} >=$$

$$= +N_f \sin \theta \frac{\theta}{N_f}$$  \hspace{1cm} (24)

where we have written out the quark mass matrix explicitly. According to conventional wisdom $\Sigma \neq 0$, so comparison with [13] implies

$$\lim_{m \rightarrow 0^+} <\theta, \hat{\phi} e^{-i\theta \gamma_5/N_f} |\Gamma_{\theta} |\hat{\phi} e^{-i\theta \gamma_5/N_f} >$$

$$\neq \lim_{m \rightarrow 0^+} <\theta, \hat{\phi} |\hat{\Gamma}_{\theta} |\hat{\phi} >$$  \hspace{1cm} (25)

in general. Hence, chiral perturbation theory breaks down for $\theta \neq 0$.

A weaker but gauge-independent argument may also be given. Eq. [11] implies that $<\Xi>_\theta$ must be $O(m)$ for chiral perturbation theory to hold. On the other hand, the $m$-dependence of $<\Xi>_\theta$ enters only through the determinant obtained after integrating out the quarks, i.e.

$$<\Xi>_\theta = \frac{<\det(\gamma_\mu \nabla_\mu - m)>^{N_f}}{<\det(\gamma_\mu \nabla_\mu - m)>^{N_f}}$$  \hspace{1cm} (26)

where $\det(\gamma_\mu \nabla_\mu - m)$ is the determinant for one flavor and $<\det(\gamma_\mu \nabla_\mu - m)>^{N_f}$ denotes the average over gluon fields only. However, it is seen that (26) is not generally $O(m)$. Indeed, it is $O(1)$ in the quenched approximation $N_f = 0$.

The fourth reason is the topological susceptibility. The BRST formalism gives $\chi = 0$ as we have seen. On the other hand, lattice calculations give a non-zero topological susceptibility in Yang-Mills theory [28] and QCD [24] for zero vacuum angle. However, almost all of them use periodic boundary conditions which is problematic. In the continuum limit, this would imply a sharp (zero) topological charge $\nu$ independently of $\theta$, and hence zero topological susceptibility.

There are two reasons why the lattice calculations yield a non-zero $\chi$. Most of the lattice calculations [28,29] employ a lattice topological density $\Xi_L$ [11] in a conventional Monte-Carlo simulation to compute $<0|T^* \Xi_L(x) \Xi_L(0)|0>$ [11]. However, such a calculation can differ from the correct result by contact terms. Specifically, in the continuum limit

$$-\partial^\mu \partial^\nu <0|T^* K_\mu(x) K_\nu(0)|0>$$

$$= <0|T^* \Xi_L(x) \Xi_L(0)|0> - \alpha_s \delta^4(x) - \beta_s \partial^2 \delta^4(x) - ...$$  \hspace{1cm} (27)
A vanishing $\chi(0)$ implies that

$$\chi_L(0) = \int_\mathcal{V} d^4x < 0|T^\alpha \Xi L(x)\Xi L(0)|0 > = \alpha.$$ (28)

We also note that unlike $\chi(0)$, the lattice result (28) is scheme dependent and can be renormalized to an arbitrary value.

To remedy this problem, some calculations have used cooling procedures to smooth out the lattice gauge configurations, washing them out of their ultraviolet content. After several cooling sweeps, bare topological susceptibilities were reported. However, the gauge field configurations with non-zero lattice topological charge must correspond to singular gauge configurations in the continuum limit. The latter carry $\Xi_L \neq \partial_\mu K_\mu$ in the continuum, but this is unacceptable since the operator identity $\Xi = \partial^\mu K_\mu$ requires all configurations in the path integral to obey the same identity. We note that the cooled lattice calculations, when rid of the specific gauge configurations for which $\nu_L = \int_\mathcal{V} d^4x < \Xi_L > \neq 0$ give $\chi_L = 0$.

The fifth reason is 1+1 dimensional models, the simplest example being free Maxwell theory. The analog of $\Xi$ is then the electric field which can be a non-zero constant. However, this is due to the fact that the analog of $K_\mu$ is ill-defined in 1+1 dimensions. The infrared divergence $\int d^2k/k^2$ which makes $K_\mu$ ill-defined is peculiar to 1+1 dimensions, and it is not clear whether the example is relevant to 3+1 dimensions.

To summarize, the current evidence for strong CP violation appears to be inconclusive. As far as the U(1) problem is concerned, ‘t Hooft’s calculation relies on BRST quantization. Since the existing instanton calculations of the vacuum energy are at odds with the BRST result, instantons cannot be said to solve the U(1) problem without further amendment. Given that Veneziano’s analysis assumes translation invariance for the two-point function $K_\mu$ but not for the one-point function, we conclude that the U(1) problem is still a problem.

4. Conclusions

We have assumed the validity of BRST quantization throughout. However, it is well-known that the Faddeev-Popov prescription suffers from the Gribov ambiguity at the non-perturbative level, so there is reason for concern about such an assumption. However, the BRST

Lagrangian for $Q_{BRST} = 0$ is equivalent to QCD, and gauge independence follows from the Fradkin-Vilkovisky theorem. Hence, BRST quantization should represent a legitimate quantization scheme for QCD. Indeed, ‘t Hooft’s instanton calculations require the validity of BRST quantization at the non-perturbative level.

It is also important that the BRST formalism holds outside of QCD. Indeed, if it were not the case, one may ask whether the vacuum expectation value of the Higgs field (another gauge-variant object) could break translation invariance, and if so, what happens to the W and Z mass.

The simplest possibility is that BRST quantization may be inequivalent to other quantization schemes at the non-perturbative level. If we turn to other models, it is reasonably certain that the 2+1 dimensional Polyakov model has no $\theta$ dependence in the physics, since the analog of $K_\mu$ is gauge invariant, and $e^{i\theta X}$ commutes with the constraint in both the BRST formalism and the canonical formalism. On the other hand, canonical reasoning implies $\theta$ dependence in the monopole sector of the Yang-Mills-Higgs system, so this is a case where a difference is expected with the BRST formalism. As we did note, however, the canonical formalism is inapplicable in the presence of quarks.

Given the above, it is desirable to perform new lattice calculations of the topological susceptibility to ascertain whether lattice quantization is equivalent (or not) with the BRST quantization for Yang-Mills theory and QCD. For reasons stated above, the calculations should employ free boundary conditions, and a large enough lattice to avoid edge effects. The lattice configurations should be explicitly checked against contaminations by singular gauge configurations (dislocations as well as divergence violating ones). The results should give us a better idea whether the problem massive QCD really has to face is strong CP violation or the U(1) problem.

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\footnote{Note that singular gauge transformations are also excluded in the canonical formalism.}

\footnote{Of course, the averaging over all gauge configurations give necessarily zero for the total $\nu_L$, since the vacuum for $\theta = 0$ is parity even.}

\footnote{A modified canonical formalism incorporating the anomaly has been discussed by Manton, for the massless Schwinger model on a circle.}
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