Coherence length of magnetic field in the mixed state of type-II superconductors

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Abstract. Influence of impurities on coherence length $\xi_h$ in the mixed state of $s$-wave superconductors is investigated in framework of quasiclassical Eilenberger theory. The increasing of impurity scattering rate results in decreasing of $\xi_h$. The obtained field dependence of $\xi_h$ for clean superconductors has a minimum and it is similar to that in Hao-Clem and Miranović-Ichioka-Machida theories for order parameter of coherence length. It is found that growing behavior of $\xi_h$ with magnetic field in dirty superconductors is different from order parameter coherence length determining by pairing potential near with vortex core. The magnetic field dependence of coherence length in normalized units, $\xi_h/\xi_2(B/B_{c2})$, is nonuniversal and depends on impurity scattering potential.

Last ten years much attention has been paid to the investigation of the field distribution in high-$\kappa$ superconductors [1–3]. On the theoretical level, there are four widely used methods: solving of the Bogoliubov-de Geennes (BdG) equations [4], the quasiclassical nonlocal Eilenberger theory [5–7] (this is the quasiclassical limit of the BdG theory for $k_F\xi_{BCS} \gg 1$), solving of the Usadel theory [8] (this is the dirty local limit of the Eilenberger equations with the strong impurity scattering rates ($\Gamma/T_c \gg 1$)) and the phenomenological Ginzburg-Landau (GL) theory [9–13] which is valid near $T_c$. Because BdG method is very time consuming for the self-consistent numerical calculation [4], the Eilenberger or Usadel theories are used in the microscopical consideration. In analysis of the experimental data, the analytical GL model (AGL) with penetration depth $\lambda$ and cutoff parameter $\xi_h$ as a fitting parameters is used very often. The cutoff parameter $\xi_h$ (in the notation of the AGL $\xi_v$) is connected with GL coherence length $\xi_2$, determined by the relation $B_{c2} = \Phi_0/2\pi\xi_2^2$. From theoretical reasons (see discussion in Ref. [14]), $\lambda$ can not be taken arbitrarily and should be taken as a differential operator $L_{ij}$ [15], for the description of the nonlocal effects giving the additional field dependence in the mixed state, or its local limit obtained from microscopical consideration of the Meissner state [16] independent on the magnetic field.

There is no consensus about the meaning of $\xi_h$, the problem was discussed originally by de Geennes group [17]. There are several proposes for the value of $\xi_h$: it can be taken as a coherence length $\xi_2$ with some numerical coefficient [18], or as the order parameter coherence length $\xi_1$, or as a proportional to the superconducting current coherence length $\xi_2$. Characteristic length $\xi_1$ is determined as $1/\xi_1 = (\partial|\Delta(r)|/\partial r)|_{r=0}/|\Delta_{NN}|$, where $|\Delta_{NN}|$ is the maximum value of the order parameter along the nearest-neighbor direction which is the direction of taking the derivative [19] and $\xi_2$ is determined by maximum of screening current around the vortex [20].
But connections between $\xi_h$, $\xi_1$, and $\xi_2$ is not investigated in detail yet. The microscopical model allowing to obtain analytical solution for $\xi_h(B)$ has been suggested in Ref. [21] (the KZ model). In this model linearized Eilenberger equation has been solved and uniform magnetic field has been suggested. It means that Kramer-Pesch effect is not included in the consideration. The exact form equation of $\xi_h(B)$ for the zero-$T$ clean case for both Fermi sphere and cylinder has been obtained. The result can be represented as $\xi_{KZ}(B)/\xi_{A}(B) = U(B)/\xi_{A}(B)$ with $U$ being an universal function. The most important features of the KZ model and $\xi(B)$ dependence are as follows: (i) this dependence is weakened by scattering and disappears in the dirty limit; (ii) the $B$ dependence of $\xi$ vanishes as $T \to T_c$; (iii) in reduced variables, the dimensionless coherence length $\xi^* = \xi/\xi_n$ should be nearly universal function of the reduced field $b = B/\xi_n$ for clean materials in high fields and low temperatures; and (iv) for materials on the clean side ($\Gamma < 1$) the low-$T$ slope $d\xi^*/db^{(-1/2)}$ is nearly universal in high fields ($b \to 1$). It is found that the microscopical calculations of $\xi_1$ [5] do not agree with the KZ theory. In particular, these calculations don’t confirm the KZ assertion about weakening of the field dependence of the core size with the increasing scattering. As noted in Ref. [14] the question still remains: which of these two theoretical approaches, Ref. [5] or Ref. [21], describes better various data on $\xi_h(B)$? It is important to note also that the GL theory predictions is not reproduced by the KZ theory.

Recently, an effective London model with the magnetic coherence length $\xi_h(B)$ as a fitting parameter has been obtained for clean [22] and dirty [23] superconductors, using self-consistent solution of quasiclassical nonlocal Eilenberger equations. Such theory looks appropriate for the description of the vortex core where strong nonlinear and nonlocal effects are expected. In this approach the coherence length obtained from the Ginzburg-Landau model is extended over the whole field and temperature range. The Fourier components of magnetic field in this model are described by London equation with GL type cutoff function

$$h_{EGL}(r) = \frac{\phi_0}{S} \sum G \frac{F(G)e^{iGr}}{1 + \lambda^2 G^2}, \quad (1)$$

where $F(G) = uK_1(u)$, $u = \xi_h G$. It is important to note that $\xi_h$ in Eq. (1) is obtained by solving the Eilenberger equations and $\xi_h$ doesn’t coincide with the variational parameter of the AGL model. We will call obtained field distribution as an Eilenberger - Ginzburg-Landau field distribution $h_{EGL}(r)$. In Eq. (1) $\lambda(T)$ is calculated from microscopical theory for the Meissner state and renormalized by impurity scattering [24]. In dirty superconductors the value of $\lambda$ increases considerable and gives the main effect of impurities in the field distribution (Eq. (1)) suppressing deviation of the field from the mean value $B$. Thus, in this model there is only one fitting parameter for the description of the vortex state, $\xi_h$, similar to Ref. [14].

The aim of our paper is to calculate $\xi_h(B)$ in the framework of the Eilenberger theory and to study the applicability of the above mentioned theories in wide temperature range and at different impurity scattering rates. In particular, we are interested in looking for possible predicted universal behavior. With the Riccati transformation of the Eilenberger equations, quasiclassical Green functions $f$ and $g$ can be parameterized via functions $a$ and $b$ [5]

$$\bar{f} = \frac{2a}{1 + ab}, \quad \bar{f} = \frac{2b}{1 + ab}, \quad \frac{g}{1 + ab}, \quad (2)$$

satisfying the nonlinear Riccati equations. In Born approximation for impurity scattering we have

$$u \cdot \nabla a = -a[2(\omega_n + G) + iu \cdot A] + (\Delta + F) - a^2(\Delta^* + F^*), \quad (3)$$

$$u \cdot \nabla b = b[2(\omega_n + G) + iu \cdot A] - (\Delta^* + F^*) + b^2(\Delta + F), \quad (4)$$
where $\omega_n = \pi T (2n + 1)$, $F = 2\pi \langle f \rangle \cdot \Gamma$ and $G = 2\pi \langle g \rangle \cdot \Gamma$. Here, $\Gamma = \pi n_F |u|^2$ is the impurity scattering rate, $u$ is impurity scattering amplitude and $\mathbf{u}$ is a unit vector of the Fermi velocity. The FLL create the anisotropy of the electron spectrum [19]. Therefore the impurity impurity scattering rate, $\Gamma = 0$ (b) with impurity scattering $\Gamma = 0$ (a). The renormalization correction in Eq. (3) and (4) are averaged over Fermi surface and can be reduced to averages over the polar angle $\theta$, i.e. $\langle \ldots \rangle = (1/2\pi) \int \ldots d\theta$.

To take into account the influence of screening the vector potential $A(r)$ in Eqs. (3) and (4) is obtained from the equation $\nabla \times \nabla \times A_E = \frac{1}{\kappa^2} \mathbf{J}$, where the supercurrent $\mathbf{J}(r)$ is given in terms of $g(\omega_n, \theta, \mathbf{r})$ by

$$\mathbf{J}(r) = 2\pi T \sum_{\omega_n > 0} \int_{0}^{2\pi} \frac{d\theta}{2\pi} \frac{\hat{k}}{i} g(\omega_n, \theta, \mathbf{r}).$$

Here $A$ and $\mathbf{J}$ are measured in units of $\phi_0/2\pi \xi_0$ and $2e v_F N_0 T_c$, respectively. The self-consistent condition for the pairing potential $\Delta(r)$ is given by

$$\Delta(r) = V^{SC} 2\pi T \sum_{\omega_n > 0} \int_{0}^{2\pi} \frac{d\theta}{2\pi} f(\omega_n, \theta, \mathbf{r}),$$

where $V^{SC}$ is the superconducting coupling constant and $\omega_c$ is the ultraviolet cutoff determining $T_{c0}$[23]. All over this paper the energy, the temperature, and the length are measured in units of $T_{c0}$ and the coherence length $\xi_0 = v_F / T_{c0} = \xi_{BCS} \pi \Delta_0 / T_{c0}$. Here $\xi_{BCS} = v_F / \pi \Delta_0$, where $v_F$ is the Fermi velocity and $\Delta_0$ is temperature dependent uniform gap. The magnetic field $\mathbf{h}$ is given in units of $\phi_0/2\pi \xi_0^2$. The impurity scattering rates are in units of $2\pi T_{c0}$.

Calculations in the present paper are based on the Eilenberger equations (Eq. 3 and 4) are solved by the Fast Fourier Transform (FFT) method [23]. This method is reasonable for dense FLL discussed in this paper. In high field the pinning effects are weak and they are not considered in our paper. After solving the Eilenberger equations the obtained magnetic field distribution $h_E(r)$ is fitted with the London field distribution $h_{EGL}(r)$ (Eq. (1)). To study high field regime we should calculate upper critical field $B_{c2}(T)$ [25].

Our calculations show that in clean superconductors $\xi_h(B)$ dependence has minimum which disappears at low temperatures. The absolute values of $\xi_h$ are smaller than those of the AGL theory predictions, with increasing temperature $\xi_h$ dependences move to higher values.
These effects can be seen in Fig. 1 (a), where $\xi_h(B/B_{c2})$ are presented with $\Gamma = 0$ at $T/T_c = 0.2, 0.4, 0.6, 0.8, 0.95$. The same tendency is also visible in the presence of impurity scattering, but shifting of $\xi_0$ to direction of AGL curve is slower. Fig. 1 (b) presents $\xi_h(B/B_{c2})$ dependence with $\Gamma = 0.5$ at $T/T_c = 0.2, 0.4, 0.6, 0.8, 0.95$. Strong decreasing of $\xi_h$ with decreasing of temperature can be explained by Kramer-Pesch effect [7]. The change of the shape of $\xi_h(B)$ curve in low fields with increasing of scattering rate $\Gamma$ is shown in details at Fig. 2 (a) at $T/T_c = 0.5$. At high scattering rate, a flat dependence is clearly visible.

We also calculate magnetic field dependence of mean square deviation of $h_{EGL}$ distribution of the magnetic field from the Eilenberger distribution normalized by the variance of the Eilenberger distribution, $\varepsilon$, for $T/T_c = 0.5$ with different impurity scattering $\Gamma$.

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We also calculate magnetic field dependence of mean square deviation of $h_{EGL}$ distribution of the magnetic field from the Eilenberger distribution normalized by the variance of the Eilenberger distribution $\varepsilon = \sqrt{(h_{EGL} - h_{EGL})^2 / (h_{E} - B)^2}$, where $\overline{\cdots}$ is average over unit vortex cell. Fig. 2 (b) shows $\varepsilon(B)$ dependence for $T = 0.5$ with different impurity scattering $\Gamma$. It can be seen from this picture that accuracy of EGL model is getting better with increasing impurity scattering and saturates at the $\Gamma \approx 1.5$.

We should note that the the similarity of our results to the AGL theory (which is suppose to be quantitatively incorrect [11]) can be considered only as a coincidence. First, in our methods there is only one fitting parameter $\xi_h$, while in the AGL theory there are two of them, $\xi_e$ and $f_\infty$ ($f_\infty = 1$ in our case). Because of the boundary condition of the field distribution ($h(r) \to B$ at $B \to B_{c2}$) $\xi_h$ is growing function of $B$ at the high fields resulting in the appearance of the minimum. In the AGL theory (and ”improved” analytical GL theory [11]) the boundary condition is satisfied by the limit $f_\infty \to 0$ at $B \to B_{c2}$, so the behavior of $\xi_e(B)$ dependence is not predetermined. For example, in the ”improved” analytical GL theory there is no minimum in $\xi_e(B)$ [11]. Absence of the minimum in $\xi_h(B)$ results also from local Usadel theory for $\xi_1(B)$ [26] and $\xi_2(B)$ [27] dependences. Second, there is clear impurity dependence of the $\xi_h/\xi_{c2}$ value even at high temperatures (compare Fig. 1 (a) and Fig. 1 (b)), which can not be explained by the local Usadel or ”improved” analytical GL theories, where scaling $\xi_h/\xi_{c2} = Const$ (independent on $\Gamma$) is expected.

In Fig. 1 the normalization constant $\xi_{c2}$ is dependent on impurity scattering rate $\Gamma$. It is well known that at high $\Gamma$ $\xi_{c2} \sim \sqrt{1/\Gamma} \sim \sqrt{l}$, where $l$ is the mean-free path. Therefore, the

![Figure 2](image-url)
visible from Fig. 2 (a) that dependence resulting in nonmonotonous behavior of $\xi$ found that at low temperatures impurity scattering suppresses Kramer-Pesch effect in $\xi$ of superconductors and different temperatures are presented. Strong suppression of the Kramer-Pesch effect there. This can be seen from Fig. 3, where predictions of the various theories for clean superconductors and different temperatures are compared with the behavior of the another characteristic length $h$. It has been found that at low temperatures impurity scattering suppresses Kramer-Pesch effect in $\xi_1(T)$ dependence resulting in nonmonotonous behavior of $\xi_1(\Gamma)$. On the another hand, it is clearly visible from Fig. 2 (a) that $\xi_h$ monotonously decreases with $\Gamma$, where normalization constant $\xi_{BCS}$ is used ($\xi_{BCS}$ is not dependent on $\Gamma$).

The data in Figs. 1 and 2 demonstrate nearly universal behavior near $T_c$ and small scattering rates: (i) the nonmonotonous field dependence with a minimum and (ii) the similar slope $d(\xi_h/\xi_{c2})/db$ at $b = 1$ which is weakly dependent on temperature and scattering rate. But the results are very different from the prediction of the KZ theory because of negligence of the Kramer-Pesch effect there. This can be seen from Fig. 3, where predictions of the various theories for clean superconductors and different temperatures are presented. Strong suppression of $\xi_h$ (the $\xi_h$ curves) and $\xi_1$ (the $\xi_1$ curves) with temperature lowering is visible in contrast to the increasing of $\xi_{KZ}$ (the KZ curves). But impurity induced behavior is similar for $\xi_h/\xi_{c2}$ and $\xi_{KZ}/\xi_{c2}$: both decreases with increasing impurity rates.

To conclude, the magnetic coherence length $\xi_h$ (cutoff parameter) in the mixed state of high-$\kappa$ s-wave superconductors is investigated in framework of quasiclassical Eilenberger theory. Nearly universal field dependence with a minimum is found near critical temperature in clean superconductors. A similar slope $d(\xi_h/\xi_{c2})/d(B/B_{c2})$ at $B/B_{c2} = 1$ weakly dependent on temperature and scattering rate is discovered. Quasiparticle scattering by impurities and lowering of the temperature reduce the value of $\xi_h$ shifting it considerably downward from the AGL curve and at low temperatures strong influence of the Kramer-Pesch effect is found. It

\begin{equation}
\frac{\xi_h(B, T, \tau)}{1 + \frac{\tau_0(B, T)}{\tau}} = \frac{\xi_{pure}(B, T)}{1 + \frac{\tau_0(B, T)}{\tau}}
\end{equation}

where $\xi_{pure}(B, T)$ is the effective coherence length in clean superconductors [22] and $\tau_0$ is a characteristic relaxation time. It results in $\xi_h \sim l$ dependence at high $\Gamma$ similar to the behavior of nonlocality radius [18] resulting in decreasing of $\xi_h/\xi_{c2}$ at high $\Gamma$. Such fast decreasing of $\xi_h$ can be compared with the behavior of the another characteristic length $\xi_1$. It has been found that at low temperatures impurity scattering suppresses Kramer-Pesch effect in $\xi_1(T)$ dependence resulting in nonmonotonous behavior of $\xi_1(\Gamma)$. On the another hand, it is clearly visible from Fig. 2 (a) that $\xi_h$ monotonously decreases with $\Gamma$, where normalization constant $\xi_{BCS}$ is used ($\xi_{BCS}$ is not dependent on $\Gamma$).
can explain muon spin rotation experimental results in some low temperature superconductors, where the ratio $\xi_h/\xi_c^2 \ll 1$ [3] is observed in intermediate fields. A comparison with the behavior of the order parameter coherence length $\xi_1$ and another theories is done. It is found that impurities influence by different way on $\xi_h$ and $\xi_1$.

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