Optimization of Travelling Salesman Problem on Single Valued Triangular Neutrosophic Number using Dhouib-Matrix-TSP1 Heuristic

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ABSTRACT

The Travelling Salesman Problem (TSP) is one of the fundamental operational research problems where the objective is to generate the cheapest route for a salesman starting from a given city, visiting all the other cities only once and finally returning to the starting city. In this paper, we study the Travelling Salesman Problem in uncertain environment. Particularly, the single valued triangular neutrosophic environment is considered viewing that it is more realistic and general in real-world industrial problems. Each element in the distance matrix of the Travelling Salesman Problem is presented as a single valued triangular neutrosophic number. To solve this problem, we enhance our novel column-row heuristic Dhouib-Matrix-TSP1 by the means of the center of gravity ranking function and the standard deviation metric. In fact, the center of gravity ranking function is applied for defuzzification in order to convert the single valued triangular neutrosophic number to crisp number. A stepwise application of several numerical Travelling Salesman Problems on the single valued triangular neutrosophic environment shows that the optimal or a near optimal solution can be easily reached; thanks to the Dhouib-Matrix-TSP1 heuristic enriched with the center of gravity ranking function and the standard deviation metric.

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1. INTRODUCTION

In 1995, Smarandache [1] introduced the philosophy of neutrosophic which covers wide concepts more than the intuitionistic (that can handle only the incomplete information); which consists of: set, probability, statistics, logic and theory. The neutrosophic number can handle three independent memberships: Truth (T), Indeterminacy (I) and Falsity (F), where T, I and F are subsets of [0, 1].

Several recent research papers deal with the application of neutrosophic concept. Ibraheem et al. [2] developed a neutrosophic analytical hierarchy process model to measure the degree of credit risk for a private bank. Khalifa Abd El-Wahed and Kumar [3] solved the neutrosophic assignment problem where the matrix elements; they presented as interval-valued trapezoidal neutrosophic number using the order relations technique. Moreover, Hamiden [4] used the weighting Tchebycheff technique to generate a relative weights and ideal targets for the multi-objective assignment problem in the neutrosophic trapezoidal fuzzy situation. Prabha and Vimala [5] optimized the triangular fuzzy neutrosophic assignment problem is using the branch and bound technique and the efficiency is illustrated on a real-world agricultural problem. Chakraborty et al. [6] designed a new score and accuracy technique is to convert the pentagonal neutrosophic fuzzy numbers into crisp numbers for the transportation problem in pentagonal neutrosophic environment.

Subasri and Selvakumari [7] solved the travelling salesman problem in neutrosophic environment with trapezoidal fuzzy distance via the branch and bound method. In addition, Subasri and Selvakumari [8] optimized the TSP in neutrosophic domain using triangular fuzzy distance through the ones assignment method.

Obviously, the TSP is an NP-complete problem [9].

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Its objective is to find the cheapest route of a salesman starting from a given city, visiting all the other cities only once and finally returning to the starting city. This problem has been formulated by Equation (1):

\[ \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} p_{ij} = 1, \quad i = 1,...,n \]

\[ \sum_{j=1}^{n} p_{ij} = 1, \quad j = 1,...,n \]

\[ p_{ij} = 0 \text{ or } 1, \quad i = 1,...,n, \quad j = 1,...,n \]

Where \( p_{ij} \) is a binary variable (if city \( i \) and city \( j \) are not connected then \( p_{ij} = 0 \) else \( p_{ij} = 1 \)) and \( d_{ij} \) denotes the distance between city \( i \) and city \( j \).

Our investigation shows that all of the research papers in the field of neutrosophic theory in operational research were intensive on the transportation problem and the assignment problem. Likewise, very limited number of research papers focused on solving the TSP in neutrosophic environment. Motivated by the above-mentioned problem, this paper proposes the first resolution of the TSP in single valued triangular neutrosophic environment with center of gravity score function. In fact, each element \( d_{ij} \) in the TSP distance matrix is considered as a single valued triangular neutrosophic distance and defuzzied to crisp number using the center of gravity score function. Hence, this TSP is optimized by our recently invented heuristic [10] entitled Dhouib-Matrix-TSP1 (DM-TSP1). Moreover, this paper presents the first application of the DM-TSP1 to the neutrosophic domains.

The rest of this paper is structured as follows. In section 2, we study the neutrosophic number concept. In section 3, we present the proposed DM-TSP1 heuristic enriched with the center of gravity ranking function. In section 4, we illustrate the resolution by applying the adapted DM-TSP1 technique on several numerical examples. Finally, in section 5 we present the conclusion and our further research work.

2. THE TRINAGULAR NEUTROSOPHIC NUMBER

This section gives a brief overview of the triangular neutrosophic concept [11, 12]. The triangular fuzzy number \( \tilde{Y} \) is denoted by \( \tilde{Y} = (y_a, y_b, y_c) \) where \( y_a, y_b \) and \( y_c \) are real numbers with \( y_a \leq y_b \leq y_c \). Let’s consider \( X \) a space of points where the generic elements in \( X \) are denoted by \( x \).

The single valued triangular neutrosophic number \( Y^N \) over \( X \) has the form of \( Y^N = \{(x: T^N(x), I^N(x), F^N(x)), x \in X\} \) where the functions \( T^N, I^N, F^N : X \rightarrow ]0,1[ \) with the condition \( 0 \leq T^N(x) + I^N(x) + F^N(x) \leq 3 \).

The truth \((T)\), indeterminacy \((I)\) and falsity \((F)\) membership functions for the single valued triangular neutrosophic number \( Y^N = \{\{y_a, y_b, y_c; (\mu_x, \nu_x, \lambda_x)\}\} \) are defined by Equation (2):

\[ T^N(x) = \begin{cases} 
\frac{(x - y_a)\mu_x}{y_b - y_a}, & y_a \leq x \leq y_b \\
\frac{(y_b - x)\mu_x}{y_b - y_a}, & y_b \leq x \leq y_c \\
0, & \text{otherwise}
\end{cases} \]

\[ I^N(x) = \begin{cases} 
\frac{y_a + (x - y_a)\nu_x}{y_b - y_a}, & y_a \leq x \leq y_b \\
\frac{y_b + (y_c - x)\nu_x}{y_b - y_a}, & y_b \leq x \leq y_c \\
0, & \text{otherwise}
\end{cases} \]

\[ F^N(x) = \begin{cases} 
\frac{y_a + (x - y_a)\lambda_x}{y_b - y_a}, & y_a \leq x \leq y_b \\
\frac{y_b + (y_c - x)\lambda_x}{y_b - y_a}, & y_b \leq x \leq y_c \\
0, & \text{otherwise}
\end{cases} \]

3. THE PROPOSED METHOD: Dhouib-Matrix-TSP1 (DM-TSP1)

We designed and developed a novel column-row method namely DM-TSP1 to solve the TSP [10]. Then, we adapted the DM-TSP1 heuristic for the case of: the trapezoidal fuzzy TSP [13] and the octagonal fuzzy TSP [14]. Moreover, a stochastic version of DM-TSP1 named DM-TSP2 was performed [15]. More recently, we invent a simple heuristic, entitled Dhouib-Matrix-TP1, in order to optimize the transportation problem [16]. This paper introduces the first resolution of the TSP in single valued triangular neutrosophic environment. Furthermore, this paper also presents the first application of our DM-TSP1 heuristic on the neutrosophic environment.

The DM-TSP1 heuristic starts by the defuzzification of the single valued triangular neutrosophic number to crisp value using the center of gravity (COG) ranking
function described by Broumi et al. [17]. For a triangular fuzzy distance \( \hat{d} = [y_a, y_s, y_c] \) with \( y_a \leq y_s \leq y_c \), the COG is computed as Equation (3):

\[
\text{COG}(\hat{d}) = \frac{1}{4} \left[ y_a + 2y_s + y_c \right]
\]  

(3)

Thus, the score and the accuracy functions for the single valued triangular neutrosophic number \( Y^N = ([y_a, y_s, y_c]; (\mu_y, \nu_y, \lambda_y)) \) are defined as follows:

\[
S(Y^N) = \text{COG}(Y^N) \times \frac{2 + \mu_y - \nu_y - \lambda_y}{3}
\]

(4)

\[
a(Y^N) = \text{COG}(Y^N) \times \frac{2 + \mu_y - \nu_y + \lambda_y}{3}
\]

Here is an example of single valued triangular neutrosophic number \( Y^N = ((3, 6, 9), 0.9, 0.5, 0.1) \) which can be converted into a crisp number (with the value equal to 4.60) by the means of Equations (3) and (4):

\[
\text{COG}(Y^N) = \frac{1}{4} \left[ 3 + 2 \times 6 + 9 \right] = 6
\]

And

\[
S(Y^N) = 6 \times \frac{2 + 0.9 - 0.5 - 0.1}{3} = 4.60
\]

Figure 1 depicts the graphical representation of the single valued triangular neutrosophic number \( Y^N = ((3, 6, 9), 0.9, 0.5, 0.1) \).

The DM-TSP1 is composed of four steps iterated in a sequential manner as described in Figure 2.

The DM-TSP1 heuristic is developed using the Python programing language in a sequential structure. However, we will look in a further research to insert the DM-TSP1 in a multi-agent structure as reported in literature [18, 19, 20] using different metric functions (Min, Max, Standard Deviation) in each agent.

4. COMPUTATIONAL RESULTS

Three numerical examples are used to prove the performance of the proposed DM-TSP1 heuristic.

4.1. Numerical Example 1

Consider the following symmetric TSP with single valued triangular neutrosophic distance (see Figure 3).

Where:

\[
\begin{align*}
d_{12} &= (4.6, 10); 0.8, 0.4, 0.2) \\
d_{13} &= (2.7, 9); 0.7, 0.6, 0.3) \\
d_{14} &= (4.6, 10); 0.8, 0.4, 0.2) \\
d_{21} &= (4.7, 9); 0.6, 0.6, 0.3) \\
d_{23} &= (4.6, 10); 0.8, 0.4, 0.2) \\
d_{24} &= (2.7, 9); 0.8, 0.5, 0.4) \\
d_{31} &= (1.5, 8); 0.7, 0.6, 0.3) \\
d_{32} &= (2.5, 9); 0.7, 0.6, 0.3) \\
d_{34} &= (1.5, 8); 0.8, 0.5, 0.4) \\
d_{41} &= (2.5, 9); 0.7, 0.6, 0.3) \\
d_{42} &= (1.5, 8); 0.8, 0.5, 0.4) \\
d_{43} &= (1.5, 8); 0.8, 0.5, 0.4)
\end{align*}
\]

Figure 3. The single valued triangular neutrosophic distance matrix
For all single valued triangular neutrosophic distance in the TSP distance matrix will be converted into crisp numbers by using Equations (3) and (4). Here is an example, the single valued triangular neutrosophic distance \((d_{12})\) in the element \(d_{12}\) is converted into the crisp distance equal to 4.76 by:

\[
\text{COG}(d_{12}) = \frac{1}{4} \left[ 4 + 2 \times 6 + 10 \right] = 6.50
\]

\[
S(d_{12}) = 6.50 \times \frac{2 + 0.8 - 0.4 - 0.2}{3} = 4.77
\]

Similarly proceeding for all single valued triangular neutrosophic distance we get the crisp distance matrix (see Figure 4).

Now, from the given crisp distance matrix the standard deviation for each row is computed and the smallest standard deviation value is selected which is 1.59 in row 3 (see Figure 5) and finding its minimal element (at position \(d_{32}\)).

Insert cities 3 and 1 in the List-cities \([3-1]\) and discard their columns. Next, select the smallest values for row 3 and row 1 (see Figure 6).

The smallest value is in row 3 and at position \(d_{32}\). Thus, insert city 2 at the left side (because it is generated from city 3) in the List-cities \([2-3-1]\) and discard column 2 (see Figure 7).

Similarly, select the smallest element in rows 1 and 2, which is 3.83 at position \(d_{12}\). Then, insert city 4 at the right side (because it is generated from city 1) in the List-cities \([2-3-1-4]\) and discard column 4 (see Figure 8).

Finally, if there is no more columns to select so the last step is to generate a cycle from the List-cities \([2-3-1-4]\). Starts by translating the city from left position to the right one until the city number 1 will be at the first position: so, translate city 2 to the last position to obtain \([3-1-4-2]\); hence, translate city 3 to the last position to get \([1-4-2-3]\). Finally, add city 1 to the last position to obtain the cycle \([1-4-2-3-1]\). Therefore, the crisp optimal solution found by the DM-TSP1 heuristic using the center of gravity ranking function is \([1-4-2-3-1]=3.83 + 3.96 + 3.15 = 14.27\).

### 4.2. Numerical Example 2

Here is a second example, let us consider the following 5x5 distance matrix with single valued triangular neutrosophic number. Where:

\[
d_u = \{(1, 9, 20); 0.9, 0.4, 0.1\}, \quad d_u = \{(2, 9, 25); 0.8, 0.5, 0.1\}
\]

\[
d_u = \{(5, 7, 9); 0.9, 0.7, 0.1\}, \quad d_u = \{(2, 9, 19); 0.4, 0.4, 0.3\}
\]

\[
d_u = \{(1, 9, 20); 0.9, 0.4, 0.1\}, \quad d_u = \{(3, 9, 14); 0.7, 0.3, 0.3\}
\]

\[
d_u = \{(5, 8, 13); 0.6, 0.2, 0.4\}, \quad d_u = \{(7, 9, 18); 0.4, 0.1, 0.1\}
\]

\[
d_u = \{(2, 9, 25); 0.8, 0.5, 0.1\}, \quad d_u = \{(3, 9, 14); 0.7, 0.3, 0.3\}
\]

\[
d_u = \{(4, 8, 17); 0.8, 0.5, 0.2\}, \quad d_u = \{(5, 9, 15); 0.9, 0.6, 0.1\}
\]

\[
d_u = \{(5, 7, 9); 0.9, 0.7, 0.1\}, \quad d_u = \{(5, 8, 13); 0.6, 0.2, 0.4\}
\]

\[
d_u = \{(4, 8, 17); 0.8, 0.5, 0.2\}, \quad d_u = \{(1, 9, 16); 0.7, 0.4, 0.3\}
\]
\[ d_u = \left\{ (2.8, 19) ; 0.4, 0.4, 0.3 \right\}, \ d_v = \left\{ (7.9, 18) ; 0.4, 0.1, 0.1 \right\} \]
\[ d_u = \left\{ (5.9, 15) ; 0.9, 0.6, 0.1 \right\}, \ d_v = \left\{ (1.9, 16) ; 0.7, 0.4, 0.3 \right\} \]

Figure 9 depicts the five steps needed to solve the 5x5 distance matrix with single valued triangular neutrosophic number.

4.3. Numerical Example 3

Let us consider the following 5x5 distance matrix where the distance \( d_{ij} \) are presented as follows:
\[ d_{ij} = \left\{ (2.8, 18) ; 0.8, 0.3, 0.2 \right\}, \ d_{ij} = \left\{ (1.9, 24) ; 0.9, 0.6, 0.2 \right\} \]
\[ d_{ij} = \left\{ (4.9, 15) ; 0.6, 0.5, 0.2 \right\}, \ d_{ij} = \left\{ (3.6, 13) ; 0.6, 0.3, 0.3 \right\} \]
\[ d_{ij} = \left\{ (2.8, 18) ; 0.8, 0.3, 0.2 \right\}, \ d_{ij} = \left\{ (6.9, 19) ; 0.9, 0.4, 0.1 \right\} \]
\[ d_{ij} = \left\{ (1.7, 12) ; 0.9, 0.1, 0.2 \right\}, \ d_{ij} = \left\{ (5.9, 18) ; 0.9, 0.8, 0.2 \right\} \]
\[ d_{ij} = \left\{ (1.9, 24) ; 0.9, 0.6, 0.2 \right\}, \ d_{ij} = \left\{ (6.9, 19) ; 0.9, 0.4, 0.1 \right\} \]
\[ d_{ij} = \left\{ (3.8, 23) ; 0.7, 0.1, 0.0 \right\}, \ d_{ij} = \left\{ (2.8, 32) ; 0.6, 0.5, 0.4 \right\} \]
\[ d_{ij} = \left\{ (4.9, 15) ; 0.6, 0.5, 0.2 \right\}, \ d_{ij} = \left\{ (1.7, 12) ; 0.9, 0.1, 0.2 \right\} \]
\[ d_{ij} = \left\{ (3.8, 23) ; 0.7, 0.1, 0.0 \right\}, \ d_{ij} = \left\{ (2.5, 11) ; 0.9, 0.3, 0.1 \right\} \]
\[ d_{ij} = \left\{ (3.6, 13) ; 0.6, 0.3, 0.3 \right\}, \ d_{ij} = \left\{ (5.9, 18) ; 0.9, 0.8, 0.2 \right\} \]
\[ d_{ij} = \left\{ (2.8, 32) ; 0.6, 0.5, 0.4 \right\}, \ d_{ij} = \left\{ (2.5, 11) ; 0.9, 0.3, 0.1 \right\} \]

Figure 10 depicts the five steps needed to solve the 5x5 distance matrix with single valued triangular neutrosophic number.

5. CONCLUSIONS

The Travelling Salesman Problem aims to generate the shortest cycle among all cities where each city is visited only once except the first city which should be revisited at the end.

Neutrosophic philosophy can be used to solve many real-life problems like the travelling salesman problem. In this research work, the Dhouib-Matrix-TSP1 heuristic is proposed for solving the neutrosophic triangular fuzzy travelling salesman problem using center of gravity ranking function. The efficiency of this heuristic was proved by solving several numerical case studies.

Our further research work will be focused on adapting the Dhouib-Matrix-TSP1 heuristic to solve other neutrosophic forms (trapezoidal, octagonal, etc.) using different score functions and on enhancing the Dhouib-Matrix-TSP1 to solve the neutrosophic environment in the multi-objective travelling salesman problem widely occurred in real life industrial situations.

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چکیده
مشکل فروشنده مسافری یکی از مشکلات اساسی تحقیقات عملیاتی است که هدف آن ایجاد ارتباط ترین مسیر برای فروشندگان است. به این معنی، یک مسیر از شرکت به شرکت دیگر در ماتریسی به‌صورت نوتوئروسی متغیرهای فروشندگان، فروشندگان و مرکز سوپرمارکت باید تعیین شود. لازم است ذکر شود که توانایی حل مشکل فروشنده مسافری به علت اندازه و پیچیدگی این مشکل، به راحتی حل نمی‌شود. به همین دلیل، در این مقاله، یک روش به‌نام Dhouib-Matrix-TP1 است که به راحتی و به‌طور مناسب به‌طور کلی و به‌طور تکمیلی می‌تواند به‌طور بازیابی مشکلات اساسی تحقیقات عملیاتی در عملکرد یک سیستم سوپرمارکت کمک کند.

Persian Abstract

چکیده
مشکل فروشنده مسافری یکی از مشکلات اساسی تحقیقات عملیاتی است که هدف آن ایجاد ارتباط ترین مسیر برای فروشندگان است. به این معنی، یک مسیر از شرکت به شرکت دیگر در ماتریسی به‌صورت نوتوئروسی متغیرهای فروشندگان، فروشندگان و مرکز سوپرمارکت باید تعیین شود. لازم است ذکر شود که توانایی حل مشکل فروشنده مسافری به علت اندازه و پیچیدگی این مشکل، به راحتی حل نمی‌شود. به همین دلیل، در این مقاله، یک روش به‌نام Dhouib-Matrix-TP1 است که به راحتی و به‌طور مناسب به‌طور کلی و به‌طور تکمیلی می‌تواند به‌طور بازیابی مشکلات اساسی تحقیقات عملیاتی در عملکرد یک سیستم سوپرمارکت کمک کند.