Set-theoretical approach to finding the range of possible values of the size of a group of objects in a situation of multiple indistinguishability

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Abstract. In situations where it is not possible to timely use new methods for assessing hazardous objects, traditional methods of risk assessment can be supplemented with new methods based on possible assessments of objective primary information. Evaluation of the destructive capacity and consequences of the use of high-risk systems becomes increasingly important as their structural and functional complexity grows. Despite the existence of the considered works on certain aspects, the known approaches to the consideration of the problem of indistinguishability of influencing factors are characterized by limited functionality. These capabilities do not fully provide the required level of reliability when making management decisions. In view of this, the problem of predicting equipment failure caused by an unfavourable coincidence of circumstances due to the indistinguishability of the initial data is one of the urgent tasks of risk management. Its solution is of significant theoretical and practical interest for many complex heterogeneous dynamical systems. The paper deals with a problem from the field of probabilistic modeling. The relevance of the performed research lies in the fact that the existing methods for predicting equipment failure in systems of potentially hazardous objects do not take into account the causes of accidents that occurred due to unfavourable circumstances.

The aim of the work is to develop a method for the set-theoretic modeling of the occurrence of problems with equipment due to an unfavourable coincidence of circumstances, which provides a range of possible values for the size of a group with a given composite very dangerous sign. To achieve this goal, the following tasks are solved:

- finding the range of possible values of the number of groups of objects modeled by the intersection of indistinguishable sets and finding the range of possible values of the number of groups of objects modeled by the union and the difference of indistinguishable sets;
- finding possible values of the number of groups of objects modeled by an arbitrary composite property and generalization of the problem, features of the set-theoretic apparatus of its solution.

In the course of the study, it was found that a set of objects with a compound very dangerous feature of an arbitrary type can be associated with a set representing the union of intersections (intersection of unions) of sets of objects with simple features and their complements. Moreover, the operations of intersection, union, and complement constitute a complete set of operations for the most common version of Boolean algebra of sets.

1. Introduction

The efficiency and the possibility of using technically complex systems containing damaging factors directly depends on ensuring a high level of their safety during operation. The defeat of a person or a
material object in the event of an emergency can be considered as a result of the joint action of many random independent factors: the moment of occurrence, the degree of impact, the damaging nature, etc. In this case, the same emergency can be described by different models in view of different degrees of detail and knowledge about it.

At the same time, as before, one of the problems in science and technology is decision support in cases of incomplete knowledge about a certain object, in the presence of uncertainty and unclear conditions and emerging risks \[1\]. In a traditional high-risk environment, it is assumed that the technical states of the system are distinguishable. However, in practice, the technical states of complex systems, often sharply contrasting in terms of the caused consequences, are indistinguishable for a long time due to the complexity and high cost of monitoring.

An important task of mathematical modelling remains the establishment of space-time dependencies. They can be specified using the probability distribution functions for hazardous events or the probability distribution function of random values of damaging factors. Thanks to mathematical modelling, it becomes possible to obtain an objective risk assessment necessary for making informed decisions on the prevention of emergencies in complex control systems. If, under the current conditions of the situation, the probabilistic characteristic under consideration takes on a value greater than some established one, then the appropriate decision is selected. In the case when the identification of the values of the required characteristic due to the limited time is not possible, the characteristic is considered as an indiscernible hazardous factor. At the same time, the person responsible for making management decisions has to face a variety of situations of uncertainty.

Modern equipment used in technically complex heterogeneous systems is inevitably subject to wear and tear, obsolescence and failure. From the point of view of practical assessment of the reliability of the components of complex equipment, the task of developing its mathematical model is urgent. Model building is essential to improve equipment utilization, reduce costs and extend equipment life.

Due to the inevitable influence of extraneous factors, it cannot have absolute reliability in determining the data and has a probabilistic nature, since detection technology can provide correct and incorrect solutions.

There are many works on the problems of forecasting and risk assessment. So, in works \[2\]-\[3\] it is assumed that the assessment of each of the indistinguishable criteria is the arithmetic mean of these numbers.

If there are multiple indistinguishable outcomes in the test results, the problem of constructing interval estimates for an unknown probability is considered \[4\]. The authors proposed two methods of solution: the first one takes into account all indistinguishable outcomes, while causing a distortion of the result, or the second one rejects indistinguishable outcomes, which also leads to errors. The authors give recommendations on choosing a situation for applying the proposed methods.

The study \[5\] clarifies the uncertainty and indistinguishability in the field of diagnostics of the states of power plants. In this case, indistinguishability is understood as the uncertainty of the state of the controlled object. The authors propose to introduce a threshold value. In this case, if the square of the difference does not exceed the specified threshold, then the solutions are considered indistinguishable.

In \[6\], the requirements for the creation of decision rules "if-then" are established. The author specifies that on the basis of the theory of approximate sets \[7\], this decision-making method is being developed in the field of multiple criteria optimization.

Also in the study \[8\], issues in the field of safe operation of cranes are considered and the use of the method of expert assessment of the frequency of an adverse event is proposed to develop recommendations for reducing the risk.

In work \[9\], the situation is considered when the search engines are identical and in the course of performing the activity - indistinguishable.

And in the article \[10\], the substantiation of the connection between indistinguishability and fuzzy subsets is presented.

Despite the existence of the considered works on certain aspects, the known approaches to the consideration of the problem of indistinguishability of influencing factors are characterized by limited
functionality. These capabilities do not fully provide the required level of reliability when making management decisions. In view of this, the problem of predicting equipment failure caused by an unfavourable coincidence of circumstances due to the indistinguishability of the initial data is one of the urgent tasks of risk management. Its solution is of significant theoretical and practical interest for many complex heterogeneous dynamical systems.

The relevance of the performed research lies in the fact that the existing methods for predicting equipment failure in systems of potentially hazardous objects do not take into account the causes of accidents that occurred due to unfavourable circumstances. Therefore, the development of a mathematical apparatus that takes into account the indistinguishability factor is an urgent scientific task.

The aim of the work is to develop a method for the set-theoretic modelling of the occurrence of problems with equipment due to an unfavourable coincidence of circumstances, which provides a range of possible values for the size of a group with a given composite very dangerous sign. To achieve this goal, it is necessary to solve the following tasks:

- finding the range of possible values of the number of groups of objects modelled by the intersection of indistinguishable sets;
- finding the range of possible values of the number of groups of objects modelled by the union and the difference of indistinguishable sets;
- finding possible values of the number of groups of objects modelled by an arbitrary composite property;
- generalization of the problem, features of the set-theoretic apparatus of its solution.

In view of this, the task of predicting damage caused by an unfavourable coincidence of circumstances in view of the indistinguishability of the initial data is one of the urgent tasks of risk management.

2. Formulation of the problem

Uncertainties can be exogenous (determined by external influences) and endogenous (determined by internal changes) [11]. Each of the types of uncertainty can significantly degrade the accuracy of the decision.

In modern works, uncertainty is understood as a situation associated with the absence, incompleteness and insufficiency of information about an object, process or phenomenon, regarding which a decision is made, as well as with a person's limitations in obtaining information and its constant variability [12].

Consider the features of the application of optimization methods in conditions of uncertainty. A necessary condition is a clear mathematical formulation of the problem, taking into account the type of situation and the emerging control actions on the system under consideration. The means of mathematical modeling of situations arising under an unfavorable coincidence of circumstances are easiest to represent in terms of the traditional (Boolean) algebra of sets.

Let us designate \( A_i, i = 1 \ldots k \) - sets corresponding to groups of objects with simple attributes: \( n_i \) - their known (according to the initial data) number (cardinality). Let \( U = \{u_1, u_2, \ldots, u_n\}, n = 1 \ldots N \) - a set representing the totality of all potentially dangerous objects of the same type in the system under consideration, \( n \) - its cardinality. Within the framework of a practical problem, let us assume that all sets \( A_i \) have a non-constant composition of elements, i.e. it is not known what he really is. We will call such sets indistinguishable.

For the sake of clarity in what follows, we assume first that the composition of \( A_i \) for each \( i \) is known. We will call such sets explicit (distinguishable). In this case, we will find the intersection, which will be an explicit set, its elements will represent objects with a compound danger sign.

\[
A_\cap = \bigcap_{i=1}^{k} A_i
\]
When modeling the problem, it is assumed that although the composition of any group with simple characteristics is unknown, it is the same. Therefore, there is a single factual list of objects with a composite highly dangerous sign.

Thus, this problem arises when the composition of a group with simple attributes is unknown (multiple indistinguishability occurs) and is unchanged. The main result is the minimum in specific cases of indistinguishability, the range of possible numbers of a group of objects with a very dangerous composite sign.

3. Theoretical part
Example 1. Let there be two groups of objects with simple attributes: \( U = \{1, 2, \ldots, 10\} \), \( k = 2 \) and \( A_1 = \{2, 3, 5, 6\}, A_2 = \{2, 4, 6, 9\} \), where the \( U \) elements represent the object numbers. Then their intersection will include the elements available simultaneously in \( A_1 \) and \( A_2 \): \( A_1 \cap A_2 = \{2, 6\} \).

The intersection operation simulates the most common adverse event. However, in practice, it is also necessary to know the number of objects that have at least one simple feature. It often turns out that the larger the number of such objects, the more real the occurrence of unfavorable coincidences of circumstances in the system.

The corresponding set represents union (2), which will be explicit if all \( A_i \) are explicit:

\[
A_U = \bigcup_{i=1}^{k} A_i
\]

Considering further the first example with explicit \( A_1, A_2 \), we will establish that the union \( A_1 \cup A_2 = \{2, 3, 4, 5, 6, 9\} \), since it contains elements that are contained in both \( A_1 \) and \( A_2 \). Of practical interest may be groups consisting of objects with simple attributes and modeled by the differences of the sets: \( A_1 \setminus A_2, A_2 \setminus A_1 \). They include objects that have simple attributes of the first group, which do not have a simple attribute of the second group, and vice versa. For the example under consideration, \( A_1 \setminus A_2 = \{3, 5\}, A_2 \setminus A_1 = \{9, 4\} \).

In addition to these operations of the algebra of sets used separately, when modeling situations with an unfavorable coincidence, other operations may be of interest, for example, the addition of \( U \) to \( A_1 \), that is, the difference \( \overline{A_1} = U \setminus A_1 \), as well as various combinations of operations of the type \( (A_1 \cap A_2) A_3 \cap A_4 \). In this case, no matter what operations are used, if \( A_i \) are explicit \( i = 1 \ldots k \), then the resulting sets will also be explicit. In this case, the powers are directly determined by their explicit composition.

All other operations on sets can be expressed without loss of generality in terms of intersection, union, and complement; therefore, it is sufficient to restrict ourselves to considering the resulting sets of their initial ones with their application. In this case, the main attention will be paid (in most situations to the most unfavorable) coincidence of circumstances, when a composite very dangerous sign includes all simple signs that appear together at the object.

Now let's move on to situations when, for a known number, the unchanging composition of each group of objects with simple attributes is unknown. Then the sets \( A_i, i = 1 \ldots k \) corresponding to these groups, will be indistinguishable, that is, have an unknown composition. Their powers \( n_1, n_2, \ldots, n_k \) are given. It is clear that, using operations on indistinguishable initial sets, the resulting sets of the form (1), (2) will be indistinguishable. moreover, in principle it is impossible to find their capacities. For each of them, only a range of values can be determined, any of which may turn out to be its power. However, it cannot be outside the mentioned range. The corresponding computational procedure is discussed below.

4. The range of possible values of the number of groups of objects modeled by the intersection of indistinguishable sets
Consider a method for finding the range of possible values of the number of groups of objects modeled by the intersection of indistinguishable sets. Let indistinguishable sets \( A_1, A_2, \ldots, A_k \) with the same cardinalities as the number of groups correspond to the original groups of objects with simple attributes,
unknown composition and certain numbers \( n_1, n_2, \ldots, n_k \). The number of system objects is, as before, \( n \), i.e., it corresponds to the set \( U \) with cardinality \( n \). Let us consider an unfavorable combination of circumstances at potentially dangerous objects of the system as a result of the manifestation of a composite very dangerous sign on them, representing a combination of all simple signs. As a result, it is required to extract from the available data as much information as possible about the group of objects with a compound highly dangerous feature, represented by the intersection \( W_i = \cap A_i \). If the composition of the original sets were explicit, then the set \( W_i \) would also be explicit, as demonstrated in Example 1. Exhaustive information about \( W_k \) would then represent its cardinality \( r_k \) and the elementary composition of this set. Due to the indistinguishability of \( A_i \), \( i = 1 \ldots k \), \( W_k \) turns out to be indistinguishable and its cardinality is unknown. You can only find a range of possible values for \( r_k \). It is revealed by defining two intersections of sets of type \( A_1, A_2, \ldots, A_k \). The first intersection has the lowest possible cardinality \( m_k^m \), the second has the highest possible cardinality \( m_k^b \).

To clarify the essence, we find the intersection with the cardinalities \( m_k^m \), \( m_k^b \) in the simplest situation, when \( k = 2 \) and, as usual, specific cardinalities \( n_1, n_2 \) of the sets \( A_1, A_2 \) are known.

Example 2: Let \( n_1 = 4 \), \( n_2 = 8 \) be the cardinalities of indistinguishable sets \( A_1, A_2 \), cardinality of \( U \) equal to \( n = 10 \). The value \( m_2^m \) will have the intersection of such sets of type \( A_1, A_3 \), the number of common elements of which is as small as possible. This property is possessed by the sets \( A_{1,1}, A_{2,1} \), the elements of which are represented by the filled cells of the second and third rows of Table 1. Moreover, in its first row, the elements \( U \) are designated by numbers \( 1 \ldots 10 \).

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | \( A_{1,1} \) | | | | | | | | |
| 2 | \( A_{2,1} \) | | | | | | | | |
| 3 | \( A_{1,1} \cap A_{2,1} \) | | | | | | | | |
| 4 | \( A_{1,2} \) | | | | | | | | |
| 5 | \( A_{2,2} \) | | | | | | | | |
| 6 | \( A_{1,2} \cap A_{2,1} \) | | | | | | | | |

As can be seen from line 3 of the table, the cardinality \( m_2^m \) of the set \( A_{1,1} \cap A_{2,1} \) is 2. It follows from the table that if \( n_1 + n_2 \geq n \), then \( m_2^m = 0 \), since in this case sets of type \( A_1, A_2 \) may not have elements in common.

For arbitrary \( n_1, n_2, n \) taking into account what has been said in the case \( k = 2 \), we obtain:

\[
m_2^m = \max(0, n_1 + n_2 - n).
\]  

By increasing the number \( k \), it can be shown that in general form (3) represents the dependence

\[
m_k^m = \max(0, \sum_{i=1}^{k} n_i - (k - 1)n)
\]  

So, if \( k = 3, n_1 = 6, n_2 = 8, n_3 = 7 \) and \( n = 10 \), then \( m_3^m = \max(0, 6 + 8 + 7 - 20) = 1 \). Under the same conditions, if \( n \geq 11 \), the power is \( m_3^m = 0 \). In cases common in practice, \( n \) can significantly exceed \( n_1, n_2, \ldots, n_k \). Then \( m_k^m = 0 \).

This indicates the possibility of avoiding an unfavorable coincidence of circumstances, since it may be that the intersection \( A \cap = \emptyset \).
The largest possible value \( m_2^b \) of cardinality \( r_2 \) will take place if the role of sets of type \( A_1, A_2 \) is, for example, the sets \( A_{1.2}, A_{2.2} \) given in rows 5 and 6 of Table 1 and having the maximum number of common elements. Wherein:

\[
m_2^b = \min(n_1, n_2).
\] (5)

In general case:

\[
m_2^b = \min(n_1, n_2, ..., n_k).
\] (6)

Thus, according to the available information on groups of objects with simple attributes, the boundaries of the range of possible values of the number of objects with a composite very dangerous attribute have been identified. It corresponds to the intersection \( W_k = \bigcap A_i \). The size of this group, that is, the cardinality \( r_k \) of the set \( W_k \), cannot take less than according to (4) and more than according to (5) value. However, it can be any defined value within the range \( \{ m_k^m, m_k^m + 1, ..., m_k^b \} \), that is

\[
\exists! r_k \in \{ m_k^m, m_k^m + 1, ..., m_k^b \}.
\] (7)

In (7), the quantifier \( \exists!x \) is a shorthand notation for the expression "there is a single value for \( x \)."

Here, the role of \( x \) is played by the actual size of the \( r_k \) group with a composite highly dangerous sign [13].

5. The range of possible values of the number of groups of objects modeled by the union and the difference of indistinguishable sets

Consider a method for finding the range of values of the number of groups of objects modeled by the union and the difference of indistinguishable sets. Let it be necessary to determine the composition of groups of objects that have at least one simple feature. Then such a group of objects corresponds to the set \( A_\cup = A_1 \cup A_2 \cup ... \cup A_k \).

The smallest possible power \( A_\cup \) at \( k = 2 \), as can be seen from rows 5, 6 of Table 1, is

\[
l_2^m = \max(n_1, n_2).
\] (8)

For arbitrary \( k \), dependence (3.8) takes the form

\[
l_2^m = \max(n_1, n_2, ..., n_k)
\] (9)

The large possible cardinality, as follows from the comparison of rows 2, 3 of Table 1, for \( k = 2 \) and arbitrary \( k \), respectively, will be

\[
l_2^b = \min(n_1 + n_2, n).
\] (10)

Consequently, the cardinality \( l_k \) of a real set \( A_\cup \) of a group of objects possessing at least one simple feature can be any one of the values of the interval (12).
$\exists! d_{12} \in \{\max(0, n_1 - n_2), \max(0, n_1 - n_2) + 1, \ldots, \min(n_1, n - n_2)\}$.  

(13)

By analogy, for the difference $A1 \setminus A2$ we find:

$\exists! d_{21} \in \{\max(0, n_2 - n_1), \max(0, n_2 - n_1) + 1, \ldots, \min(n_2, n - n_1)\}$.  

(14)

Note that, in practice, the groups of objects modeling $A_i$ тем are the more dangerous the larger their number, so that of the two distributed systems, the one whose number of objects with at least one simple feature is large is in the worst position. Note also that the corresponding sets $A_1 \setminus A_2$ and $A_2 \setminus A_1$ of the group of objects allow us to consider the original objects not only with simple features.

Thus, the practical problem has been solved for situations common in practice, when it is necessary to determine the groups of objects modeled by the resulting sets, reducible to multiphase suppression or unions of the original (representing groups of objects with simple attributes) sets and their complements.

Now let us characterize its solution in a more general case, when the modeling set of groups of objects with a compound very dangerous feature represents combined operations, including the above-mentioned unions, intersections and complements.

6. The range of possible values of the number of groups of objects modeled by an arbitrary composite property

A method for finding the range of possible values of the number of groups of objects modeled by an arbitrary composite property. If the groups of objects with a composite very dangerous feature correspond to generators with a more complex structure, then they turn out to be not reducible to a multiphase union of type $A_u$ or an intersection of type $A_u$, such as:

$$A_1 \cup (A_2 \setminus A_3) = A_1 \cap A_2 \cap A_3.$$

(15)

So, the expression:

$$(A_1 \cup A_2) \cap (A_3 \cup A_4)$$

(16)

not representable in the form (3.15) or in a form similar to $A_u$. To determine the possible options for the cardinality of such sets (the number of corresponding groups of objects), the computational methods developed above must be supplemented with interval analysis operations as applied to reliable intervals of the type (7), (12), (13). Let us illustrate this with an example.

Let the ranges $\{4,5,6\}$ and $\{3,4,5\}$, respectively, be defined for the union of $A1 \setminus A2$ and $A1 \cup A4$ in (3.16) according to formulas (8), (9). Let the cardinality $U$ in this case $n = 15$. Possible intersections of these unions correspond to the pairs: $(4,3), (4,4), (4,5), (5,3), (5,4), (5,5), (6,3), (6,4), (6,5)$. For each pair, according to (3), (5), we find the range of possible power options: $\{0, \ldots, 3\}, \{0, \ldots, 4\}, \{0, \ldots, 4\}, \{0, \ldots, 3\}, \{0, \ldots, 4\}, \{0, \ldots, 5\}, \{0, \ldots, 3\}, \{0, \ldots, 4\}, \{0, \ldots, 5\}, \{0, \ldots, 4\}$. From their analysis it can be seen that the range of possible variants of the cardinality of the set (16) is limited to the numbers 0 and 5. It is defined as the sum of integer intervals representing the previously obtained intervals. It can be shown that for the intersection (16) of two unions, the range boundaries are determined by the dependencies:

$$m^m_n = \max(0, \min a_i + \min b_j - n),$$

(17)

$$m^m_n = \min(\max a_i, \max b_j),$$

(18)

where $a_i$ and $b_j$ are arbitrary elements of the ranges of possible cardinality of unions in (16).

As can be seen, dependences (17), (18) have a structure similar to formulas (3) and (5). They can be generalized without any special complications to the intersection of an arbitrary number of unions. Any number of sets can be in one or another union. The latter will be either sets corresponding to groups of
objects with simple attributes, or their additions to the set \( U \), which simulates the totality of all objects in the system.

Since, as is known from set theory [13], sets modeling groups of objects with any composite highly dangerous feature can be reduced to this kind of intersection, the solution to practical problem 1 obtained here can be considered complete.

Similarly, the arbitrary form of the sets formed from the original sets, as is known, can be reduced to the union of intersections and for them ranges of possible cardinality can be determined. So, if there is a union of two intersections, then, similarly to (8), (10), we obtain

\[
I_m^U = \max(\min a_i, \min b_i) \quad (19)
\]

\[
I_n^U = m \max(\max a_i, \max b_i, n) \quad (20)
\]

Thus, a set of objects with a very dangerous composite feature of an arbitrary type can be associated with a set representing the union of intersections (intersection of unions) of sets of objects with simple features and their complements. Note that in this case the operations of intersection, union, and complement constitute a complete set of operations for the most common version of the Boolean algebra of sets [13]. Let's give a practical example that requires appropriate transformations.

Example 3: Let \( A, B \) and \( C \), respectively, be the initial sets representing groups of the same type of potentially dangerous objects, the structural elements of which are exposed to a variety of simple signs: \( A \) - abnormal loads, in \( B \) - violation of operating rules and in \( C \) - having a manufacturing defect, and \( D \) is a set corresponding to a group of objects on which constant diagnostics and prompt elimination of defects are carried out. All potentially dangerous objects are associated with the set \( U \). In practice, it may turn out that the resulting set will have a very dangerous composite feature that causes an unfavorable coincidence of circumstances:

\[
W = (A \cup B) \cup (C \setminus D) \quad (21)
\]

Let us show that \( W \) transforms to the following union of intersections:

\[
W = (U \cap \overline{A} \cap \overline{B}) \cup (C \cap \overline{D}). \quad (22)
\]

For this purpose, we first consistently apply the law of double negation, the law of duality, the complement operation and the law of associativity to the expression \( A \cup B \):

\[
A \cup B = A \cup B = A \cap B = U \cap (A \cap B) = U \cap A \cap B \quad (23)
\]

Now we represent the difference \( C \setminus D \) in the form of intersections

\[
C \setminus D = C \cap \overline{D}. \quad (24)
\]

Finding the union of the right-hand sides of the last two identities, we obtain the required result.

Interval analysis tools are also required when considering practical problem 1 in relation to two or more systems of potentially dangerous objects, when an unfavorable combination of circumstances may take place on each of them. It may also be required when forecasting economic damage. Consider the following example.

Example 4: It has been established that the number \( m \) of objects with a composite highly hazardous feature in the coming season of operation is in the reliable range \( \{5, 6, 7\} \). Let the amount of costs to prevent an unfavorable degree of circumstances at any object have, like \( m \), elemental indistinguishability and the range of its possible options is \( \{10, 11, 12\} \) cost units. Let's denote \( m_i \) and \( s_j \) - an arbitrary element of the first and second ranges. Then the boundaries of the set of possible options for the value of total costs will be found using the operation of decreasing intervals:
\[ m_i \cdot m_s = 5 \cdot 10 = 50 \]  \hspace{1cm} (25) \\
\[ m_i \cdot m_s = 7 \cdot 12 = 84 \]  \hspace{1cm} (26)

By direct calculation, one can make sure that such a set has the form \{50, 55, 60, 66, 70, 72, 77, 84\}.

7. Generalized problems, features of the set-theoretic apparatus of its solution

In practice, when objects represent complexes that are rather complex in structure and functioning, a solution to a generalized practical problem 1 is required. The initial data in the model can be not only simple but also complex indistinguishable sets. The powers of some of them may have elementary indistinguishability. The solution of the generalized practical problem 1 requires the formulation of the following problem in terms of the set-theoretic apparatus.

Given:
1) not necessarily indistinguishable initial sets \( A_i \), \( i = 1, \ldots, k \) (multiple indistinguishability takes place). Elemental indistinguishability is inherent in the power of some of them;
2) a structure is known that simulates a group of objects with a compound very dangerous feature, built (according to the rules of Boolean algebra of sets) from all the original sets and by means of identical transformations, reduced to the set \( W_k \) only with the operations of union, intersection and complement.

It is required to find a range of possible variants of the cardinality \( W_k \), in other words, the number of a group of objects with a very basic composite feature.

It can be shown that, as is known, to solve this problem, it is sufficient to apply the above dependences in a certain sequence. Let us illustrate this with a specific example.

Example 5: You are given indistinguishable sets \( A_1, A_2, \ldots, A_5 \). The powers \( A_1, A_2, A_3, A_4 \) are respectively equal to 4, 3, 5, 3 and it is known that the power of \( A_5 \) is in the range \{3, 4, 5\}. The cardinality \( U \) (the number of objects in the system) is 10. As a result of a meaningful analysis, a resulting set was found that corresponds to a group with a compound very dangerous feature. It is converted to its equivalent form:

\[ W_5 = (A_1 \cap A_2) \cup [(A_3 \setminus A_4) \cap A_5] \]  \hspace{1cm} (27)

First, replace in (21) the variety \( A_3 \setminus A_4 \) by \( A_3 \setminus A_4 \) and, using dependences (3) and (5), we find that the cardinality of the set \( A_1 \cap A_2 \) belongs to the range \{0,1,2,3\}. Then, according to (3.3), (3.5), we obtain the range of possible power options \( A_3 \setminus A_4 \), equal to \{2,3,4,5\}. Now let us find the range \{0,1,2,3,4,5\} for the cardinality of the set \( (A_3 \setminus A_4) \cap A_5 \) using dependences (17), (18) by the last range specified for \( A_5 \). Finally, for the cardinality of the set \( W \), which has the form (27), according to (19), (20) in relation to the range \{0,1,2,3\} for \( A_1 \cap A_2 \) and the range \{0,1,2,3,4, 5\} for the set in square brackets (21), we obtain that it belongs to the range \{0,1, \ldots, 8\}.

Thus, the solution of practical problem 1 in general form is reduced to the construction, if necessary, of the union of the intersections or the intersection of the unions of the original sets and their complements. If all the original sets are explicit, then for merging intersections and intersecting unions are used to obtain the corresponding ranges of dependences (4), (6) and (9), (11), respectively. If among the initial sets there are indistinguishable sets, then to obtain a range of possible values of the size of a group with a given compound very dangerous sign, in addition to the indicated dependences, the corresponding dependences of interval analysis (17) - (20) are used.
8. **Practical significance**
The results obtained are focused on the construction of analytical algorithms for establishing indistinguishability in the process of monitoring, modeling and forecasting processes associated with the state and complex heterogeneous dynamic systems.

9. **Discussion and conclusions**
When working with real data, when the number of features becomes more than a certain threshold, it becomes necessary to develop automated information systems that implement the proposed mathematical methods.

Note that by now the mathematical apparatus of the algebra of indistinguishability subsets is rather poorly developed, especially in the part representing analogs of the indistinguishability intervals considered above and which are subsets of a finite set of states of the system.

Under deterministic conditions, the traditional approach to accounting for uncertainty is limited by the unreliability of the results, which is dangerous when applied in the real world. The proposed approach provides true information and its use in conjunction with the traditional approach becomes preferable in the task of monitoring dangerous situations.

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