Time Scales and Characteristics of Stock Markets in Different Investment Horizons

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Investors adopt varied investment strategies depending on the time scales ($\tau$) of short-term and long-term investment time horizons (ITH). The nature of the market is very different in various investment $\tau$. Empirical mode decomposition (EMD) based Hurst exponents ($H$) and normalized variance (NV) techniques have been applied to identify the $\tau$ and characteristics of the market in different time horizons. The values of $H$ and NV have been estimated for the decomposed intrinsic mode functions (IMF) of the stock price. We obtained $H_1 = 0.5 \pm 0.04$ and $H_1 \geq 0.75$ for the IMFs with $\tau$ ranging from a few days to 3 months and $\tau \geq 5$ months, respectively. Based on the value of $H_1$, two time series have been reconstructed from the IMFs: a) short-term time series [$X_{ST}(t)$] with $H_1 = 0.5 \pm 0.04$ and $\tau$ from a few days to 3 months; b) long-term time series [$X_{LT}(t)$] with $H_1 \geq 0.75$ and $\tau \geq 5$ months. The $X_{ST}(t)$ and $X_{LT}(t)$ show that market dynamics is random in short-term ITH and correlated in long-term ITH. We have also found that the NV is very small in the short-term ITH and gradually increases for long-term ITH. The results further show that the stock prices are correlated with the fundamental variables of the company in the long-term ITH. The finding may help the investors to design investment and trading strategies in both short-term and long-term investment horizons.

Keywords: empirical mode decomposition, Hurst exponent, short-term investment time horizon, long-term investment time horizon, time scale, normalized variance

1. INTRODUCTION

The stock market is a complex dynamical system where the evolution of the dynamics depends on the participation of different types of investors or traders [1–3]. Investors/traders participate in the stock market to gain profit implementing different investment and trading strategies depending on investment time horizons (ITH) [4, 5]. The participation of diversified investors, reaction to the information, and short-term and long-term investment approaches play crucial roles in the movement of stock prices [4].

In the stock markets, there are mainly two types of investors: short-term investors who invest for short-term gain and long-term investors who invest for long-term gain [6, 7]. Studies show that the ITH for short-term investors ranges from a single day to a few months, and for long-term investors, it usually ranges from a few months to several years [8, 9]. The fund managers and foreign exchange dealers of various countries use technical analysis for the short-term ITH and fundamental analysis for the long-term ITH [8, 10]. The time scales ($\tau$) of short-term and long-term ITH by the investors are generally chosen in an arbitrary manner based on the investment experience [8, 10]. So the
identification of \( \tau \) from stock price time series using a well-defined technique may be helpful for both the short-term and long-term investors.

As the market is mean reversing in short-term \( ITH \) [9], traders fail to generate significant returns using technical analysis [11]. On the other hand, in the long-term \( ITH \), an investor can generate significant return or help in making decisions whether to exit from that particular stock to avoid loss by determining the financial health of a company by using fundamental variables [12–14]. Fundamental analysis is an essential tool to find out the relation between stock price and fundamental variables such as book to market (B/M), sales to price, debt to equity, earnings to price, and cash flow of a stock [12–14]. The stock price is found to be positively correlated with the essential fundamental variables [14–20]. The study of the correlation in the short-term \( ITH \) and long-term \( ITH \) is essential to take a fruitful investment decision.

In the short-term \( ITH \), the market is generally considered to be governed by psychological behavior of the investors. However, the fundamental variables are the main determining crucial factors in the long-term \( ITH \). Usually, investors choose the \( \tau \) of short-term and long-term investment horizon in an arbitrary manner [8, 9]. Recently, we used structural break study to show that the \( \tau \) for the short-term is usually less than a few months [9]. The separation of the short- and long-term dynamics in terms of \( \tau \) plays a vital role in the prediction of future price movement. Hence, detailed studies are required to find the correlation of the stock price with fundamental variables and to identify the \( \tau \) of market dynamics in the short-term \( ITH \) and long-term \( ITH \).

In this article, we estimated the \( \tau \) of the stock price in the short-term and long-term \( ITH \) for twelve leading global stock indices and the stock price of some companies using empirical mode decomposition (EMD) based Hurst exponent (H) analysis. We have reconstructed short-term and long-term time series based on the H. Finally, we estimated the correlation coefficient between long-term time series and fundamental variables. Herein we establish that short-term \( ITH \) is normally less than 3 months and long-term \( ITH \) is more than 5 months. Correlation analysis shows that the long-term stock price is positively correlated with the fundamental variables.

The remaining part of this paper is organized as follows: In Section 2, we introduce the method of analysis, while Section 3 presents the data analyzed. Results and discussion and conclusion are delineated in Sections 4 and 5, respectively.

2. METHOD OF ANALYSIS

A nonlinear two-step technique—EMD followed by Hilbert–Huang Transform (HHT)—has been applied to analyze the stock data as it is nonlinear and nonstationary. Nonlinearity in the stock market appears due to the presence of market frictions and transaction costs, existence of bid–ask spread, and short selling, whereas nonstationarity appears due to different time scales present in the stock market [21, 22]. This approach helps us to identify the characteristic \( \tau \) and the important trends and components present in the data [3].

The EMD method decomposes the stock index and stock price into the intrinsic oscillatory modes of different \( \tau \) by preserving the nonstationarity and nonlinearity of the data. These oscillatory modes are termed intrinsic mode functions (IMF). The IMFs can be both amplitude and frequency modulated as well as nonstationary [23, 24]. The \( \tau \) of each IMFs has been identified by HHT. The HHT eliminates the spurious harmonic components generated due to the nonlinearity and nonstationarity of the data [23, 24].

The IMFs satisfy the following two conditions; i) the number of extrema and the number of zero crossing must be equal or differ by one; and ii) mean values of the envelope, defined by the local maxima and local minima, for each point are zero. The IMF is calculated in the following way [23, 25]:

- Lower envelope \( U(t) \) and upper envelope \( V(t) \) are drawn by connecting minima and maxima of the data, respectively, using spline fitting.
- Mean value of the envelope \( m = [U(t) + V(t)]/2 \) is subtracted from the original time series to get new data set \( h = X(t) - m \).
- Repeat the processes (a) and (b) by considering \( h \) as a new data set until the IMF conditions (i and ii) are satisfied.

Once the conditions are satisfied, the process terminates, and \( h \) is stored as the first IMF. The second IMF is calculated repeating the above steps (a)–(c) from the data set \( d(t) = X(t) - IMF_1 \). When the final residual is monotonic in nature, the steps (a)–(c) are terminated and the original time series can be written as a set of IMFs plus residue,

\[
X(t) = \sum_{i=1}^{n} IMF_i + \text{residue},
\]

where \( IMF_i \) represents the \( i \)th IMF, and residue represents the trend of the stock data.

IMFs are the signal with different \( \tau \). The \( IMF_1 \) is a signal with the smallest \( \tau \), the \( IMF_2 \) is the signal with the second smallest \( \tau \), and so on. Hence, EMD technique is useful to extract different \( \tau \) from the signal. The characteristic \( \tau \) of each IMF can be estimated from the frequency (\( \omega \)) by using Hilbert Transform, which is defined as

\[
Y(t) = \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{IMF(t)}{T - \tau} dt,
\]

where \( P \) is the Cauchy principle value, and \( \tau = \frac{1}{\omega} \) where \( \omega = \frac{d\theta(t)}{dt} \) and \( \theta(t) = \tan^{-1} \frac{Y(t)}{IMF(t)} \) [23]. Identification of important IMFs is essential to differentiate the market dynamics in short-term from long-term \( ITH \), and the differentiation can be done by evaluating the \( H \).

Rescaled-range (R/S) analysis is a technique to estimate the correlation present in a time series by calculating \( H \) [26–28]. Details of the R/S technique are described below. Let us consider a time series of length \( L \) and divided into \( p \) subseries of length \( l \). Each subseries is denoted as \( Y_{jl} \) where \( j = 1, 2, 3, \ldots, p \). Mean and standard deviation of the subseries \( Y_{jl} \) are defined as
and
\[
S_t = \sqrt{\frac{1}{I} \sum_{j=1}^{I} (Y_{jt} - D_t)^2},
\]
respectively. Mean adjusted series is calculated as
\[
Z_{jt} = Y_{jt} - D_t
\]
for \(j = 1, 2, 3, \ldots, I\). Cumulative time series is given by
\[
X_{jt} = \sum_{i=1}^{j} Z_{it}
\]
for \(j = 1, 2, 3, \ldots, I\).

Range of the series has been calculated as
\[
R_t = \max(X_{1t}, \ldots, X_{It}) - \min(X_{1t}, \ldots, X_{It}).
\]
Individual subseries range can be rescaled or normalized by dividing the standard deviation. So, R/S is written as
\[
(R/S)_t = \frac{1}{p} \sum_{i=1}^{p} R_i / S_i.
\]

The ratio of each subseries of length \(l\) is expressed as \((R/S)_t \propto l^H\), where \(H\) is the Hurst exponent. \(H\) can be estimated from the slope of \(\ln(R/S) vs. \ln(l)\). For a random time series, \(H\) is around 0.5, and for correlated and anticorrelated time series, \(H\) is greater than 0.5 and less than 0.5, respectively.

Normalized variance (NV) is another important statistical tool to identify the important IMFs based on the energy of the signal. The higher the NV value is, more significant the signal is. The technique estimates the energy of the \(i\)th IMFs by calculating variance \([29, 30]\), and NV of \(i\)th IMF is defined as
\[
NV_i = \frac{\sum_{T} \text{IMF}_i^2 (t)}{\sum_{i=1}^{I} \sum_{T} \text{IMF}_i^2 (t)}
\]
where \(q\) is the total number of IMFs.

4. RESULTS AND DISCUSSIONS

The stock market shows different behavior in different investment horizon. EMD based \(H\) and NV techniques have been applied to analyze the market dynamics as discussed below.

Figures 1A–J show the IMFs to IMFs and the residue of the S&P 500 index calculated using EMD technique as described in detail in Section 2. IMF1 in Figure 1A represents the mode with the lowest \(\tau\), and it gradually increases with the increase in IMF numbers. Figure 1J represents the residue of the signal, which indicates the overall trend of the index. Similarly, we have calculated all IMFs for all the indices and companies to analyze the market.

4.1. EMD Based \(H\) and NV Analysis

\(H\) has been calculated for all the IMFs. Figure 2A shows the typical plot of \(H\) versus \(\tau\) of all the indices and companies. We obtained single \(H\) from IMF1 – IMF3 and it is indicated as \(H_1\). Higher-order IMF shows two \(H\), namely, \(H_1\) and \(H_2\). We obtained \(H_1 = 0.5 \pm 0.04\) for IMF1 to IMF5 with \(\tau\) ranging from a few days (D) to 3 months (M). The value of \(H_1\) jumps to \(\geq 0.75\) for IMF6 with \(\tau \approx 5\) M. It gradually increases for IMF7 to IMF9 with \(\tau\) ranging from 1 year (Y) to 12 Y. \(H_1 = 0.5 \pm 0.04\) for IMF1 to IMF5 indicates that the nature of the first five IMFs is random. IMF6 to IMF9 show a long-range correlation up to one period lag. \(\tau\) of IMF1, IMF2, IMF3, IMF4, IMF5, IMF6, IMF7, IMF8, IMF9, and IMF5 of all the indices and companies stock data analyzed here are in the range of 3–4 D, 7–10 D, 15–18 D, 1–1.5 M, and 2.5–3 M, respectively.

To further validate the robustness of the proposed method, analysis of the decomposed time series has been carried out using NV technique. Figures 2B–D represent NV of all the IMFs of all the indices and companies, where plots have been arranged according to the order of higher NV of IMFs. Figures 2B–D show that the value of NV is very low for all the indices and companies up to IMF5, and NV increases significantly for IMF6 to IMF9. Hence, based on the value of NV the time series can also be decomposed into two time series with two distinct time horizons: short-term time series by adding IMF1 to IMF5 and long-term time series by adding IMF6 to IMF9 plus residue as described in Section 4.2. Further, NV technique can be used to find a time series with important time scale in the form of IMF. Figures 2B–D show that the value of NV is higher for IMF7 with 0.8 \(Y \leq \tau \leq 1.9\) Y, IMF8 with 2.0 \(Y \leq \tau \leq 4.4\) Y, and IMF9 with 4.5 \(Y \leq \tau \leq 12\) Y, respectively, for the companies mentioned in the plots. The decomposed time series with higher value of NV may play important role to predict long-term behavior of the market \([29]\). More such studies in detail can be pursued in future.
Figure 1: The plots (A)–(I) represent the IMF$_1$ to IMF$_9$, respectively, and (J) represents residue of the S&P 500 index.

Figure 2: (A) shows the Hurst exponents ($H_1$ and $H_2$) vs. $\tau$ of all the IMFs of all the indices and companies with 2$\sigma$ error bar. The first point represents the average value of $H_1$ of all the first IMFs of all stock data, the second point represents the average value of $H_1$ of all the second IMFs of all stock data, and so on. For IMF$_1$ to IMF$_5$ of all indices and companies $H_1 = 0.5 \pm 0.04$ with a maximum $\tau$ of around 3 M. The value of $H_1$ jumps to 0.75 $\pm$ 0.04 for IMF$_6$ (with a $\tau \approx 5$ M) and gradually increases for IMF$_7$ to IMF$_9$. $H_1$ value shows that nature of IMF$_1$ to IMF$_5$ is random and IMF$_6$ to IMF$_9$ have a long-range correlation. (D), (I), and (Y) in the x-axis represent the day, month, and year, respectively. (B)–(D) represent the normalized variance ($NV$) of IMFs of all the indices and company, respectively.
4.2. Reconstruction of Short-Term and Long-Term Time Series

In order to analyze the market dynamics in short-term ITH and long-term ITH, we have reconstructed two time series from the decomposed IMFs as discussed below.

We have reconstructed a time series \(X_{ST}(t)\) by adding the IMFs to \(\text{IMF}_j\) whose \(H_j = 0.5 \pm 0.04\), that is, \(X_{ST}(t) = \sum \text{IMF}_j\). The time scale of \(X_{ST}(t)\) ranges in \(3 \leq \tau \leq 3\). Figure 3B shows the reconstructed time series \(X_{ST}(t)\) obtained by decomposing the original time series of Apple Inc. which is shown in Figure 3A. \(H_1 = 0.5 \pm 0.04\) shows that the stock market is random when \(\tau\) ranging from a few days to 3 months. Hence, \(X_{ST}(t)\) represents the short-term time series in \(3 \leq \tau \leq 3\). The above analysis shows that the market behavior is random in the short-term ITH when \(\tau\) is in the range of a few days to 3 months.

Higher-order IMF shows two Hurst exponents (\(H_1\) and \(H_2\)). We have reconstructed another time series \(X_{LT}(t)\) by adding \(\text{IMF}_j\) to \(\text{IMF}_0\) whose \(H_0 = 0.75\) and residue, that is, \(X_{LT}(t) = \sum \text{IMF}_j\) residue to understand the market dynamics in long-term ITH. The time scales of \(X_{LT}(t)\) are \(\geq 5\). Figure 3C shows the reconstructed long-term time series \(X_{LT}(t)\) obtained by decomposing the original time series of Apple Inc., which is shown in Figure 3A. The present analysis yielded a \(H_1\) value of \(\geq 0.75\), which shows that the stock market has a long-range correlation with \(\tau \geq 5\). Hence, \(X_{LT}(t)\) represents the long-term time series with \(\tau \geq 5\). From the above analysis, it can be concluded that the market has a long-range correlation in the long-term ITH with \(\tau \geq 5\) and hence may be used to predict a future price. Further, it has been observed that the future price of a stock is actually much more dependent on the fundamental variables of a company. In order to understand such dependence, we have studied the correlation between \(X_{LT}(t)\) and the fundamental variables of the companies.

Table 1 shows that the correlation coefficient \(J\) between \(X_{LT}(t)\) and three fundamental variables: sale, net profit (NP), and cash from operating activity (COA) for some Indian and American companies which are listed in NSE, NYSE, and NASDAQ, from March 2007 to March 2018 in the annual price level. Fundamental variables data have been downloaded from screener and macro trend. We obtained a positive correlation between \(X_{LT}(t)\) and sale, NP, and COA for all the years. It implies that stock price is correlated with the sale, NP, and COA. We have obtained a small \(J\) for a few stocks. These stocks show a small \(J\) for the following two possible reasons: a) the stock price of a company with strong growth prospect increases even though sale or NP decreases temporarily; b) the stock price of a company with weak growth prospect decreases even though sale or NP increases temporarily. Hence, for long-term investment, the fundamental variables are the most crucial variables for the prediction of the future price. In the future, we would like to study the correlation between stock price and other fundamental variables of companies.

5. CONCLUSION

In this paper, we have studied the stock market using the empirical mode decomposition (EMD) based Hurst exponent (H) analysis and normalized variance (NV) technique. EMD
A detailed study of the market in the long-term \( ITH \) in terms of fundamental variables of a company is necessary to predict the future price. We believe that the outcome of the present study may help in making investment decisions in both short-term \( ITH \) and long-term \( ITH \).

**DATA AVAILABILITY STATEMENT**

Publicly available datasets were analyzed in this study. These data can be found here: https://in.finance.yahoo.com, https://www.screener.in, https://www.macrotrends.net

**AUTHOR CONTRIBUTIONS**

All the authors have equally contributed to preparing the manuscript.

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