Analysis of $B_s \to \phi \ell^+ \ell^-$ decay with new physics effects

U. O. Yilmaz

The J. Stefan Institute, Jamova 39, P. O. Box 300, 1001 Ljubljana, Slovenia
and
Physics Department, Mersin University 33343 Ciftlikkoy Mersin, Turkey

Abstract

The rare $B_s \to \phi \ell^+ \ell^-$ decay is investigated by using the most general model independent effective Hamiltonian for $\ell = \mu, \tau$. The calculated $Br(B_s \to \phi \mu^+ \mu^-) = 1.92 \times 10^{-6}$ is in consistent with the experimental upper bound. The dependencies of the branching ratios and polarization asymmetries of leptons and combined lepton-antilepton asymmetries on the new Wilson coefficients are presented. The analysis shows that the branching ratios and the lepton polarization asymmetries are very sensitive to the scalar and tensor type interactions. The results obtained in this work will be very useful in searching new physics beyond the standard model.

*e-mail: uoyilmaz@mersin.edu.tr
1 Introduction

In searching the new physics, the analysis of rare B decays induced by flavor changing neutral current (FCNC) can play an important role. Since the observation of exclusive $B \to K^*\gamma$\cite{1} decay, which stimulated the works in this area, there have been increasing number of investigations of new physics both in theoretical and experimental side, induced by FCNC $b \to s,d$\cite{2}-\cite{3} transitions. These processes occur at loop level in the SM and are very sensitive to the gauge structure and various extensions of the SM so they can give useful information on parameters of the SM, testing its predictions at loop level and also probing new physics. The new physics effects in rare B meson decays can indicate itself through new contributions to the Wilson coefficients that are already present in the SM, and through the new operators in the effective Hamiltonian which are absent in the SM. In this work we use a most general model-independent effective Hamiltonian that combines both approaches and contains the scalar and tensor type interactions as well as the vector types.

In this work we consider the $B_s \to \phi\ell^+\ell^-$ decay induced by FCNC $b \to s\ell^+\ell^-$ transition at quark level. This semileptonic decay is one of the suitable tools of investigating new physics, via calculating many observables since it occurs only at loop level in the SM. However, being an exclusive decay, its theoretical investigation may be more difficult than that of corresponding inclusive decays, although experimentally the situation is in contrary. This difficulty is because of requiring additional knowledge on form factors, the matrix elements of the Hamiltonian between initial and final meson states. This is related to the nonperturbative sector of QCD and can be solved in the framework of nonperturbative approaches. For $B_s \to \phi\ell^+\ell^-$ decay, the matrix elements of the effective Hamiltonian between the initial and final states have been calculated in the framework of different approaches, such as light cone sum rules\cite{4,5,6} and in different quark models; relativistic constitute quark model\cite{7}, constituent quark model\cite{8} and light front quark model\cite{9}.

In establishing new physics beyond the SM, measurement of the lepton polarization is an useful and efficient way. Besides different decay modes, polarization properties of exclusive semileptonic decay modes have been studied both in the SM\cite{10}-\cite{12} and beyond\cite{13}-\cite{18}. In this work we study the branching ratio and lepton polarization asymmetries in the exclusive $B_s \to \phi\ell^+\ell^-$ decay using the model independent general effective Hamiltonian. This decay has already been studied in the SM\cite{9}, two Higgs doublet model\cite{19} and in a universal extra dimension scenario\cite{20}.

On the experimental side, the first limit on branching fraction was stated by CDF Collaboration\cite{21}. DO Collaboration\cite{22} has reported an upper limit on the branching ratio for $Br(B_s \to \phi\mu^+\mu^-) < 4.1 \times 10^{-6}$ and recently, another upper bound $Br(B_s \to \phi\mu^+\mu^-) < 6.0(5.0) \times 10^{-6}$ at 95(90)% C.L. by CDF Collaboration\cite{23}. At LHCb for a nominal year of data taking ($2 fb^{-1}$) about $1340 \pm 250$ annual yield of $B_s \to \phi\mu^+\mu^-$ is expected over $10^{12}$ $b\bar{b}$ events\cite{24}-\cite{25}. At Atlas Experiment, $\sim 600$ $B_s \to \phi\mu^+\mu^-$ can be reconstructed over a three-year period with $30 fb^{-1}$. These experiments and also the running B factories encourage the study of rare B meson decays, in our case $B_s \to \phi\ell^+\ell^-$ decay.

This work is organized as follows. In section 2, we drive the model independent expressions for the longitudinal, transversal and normal polarizations of leptons and combined lepton-antilepton polarizations starting from the quark level process and using appropriate
dependence of branching ratios and polarizations on new Wilson coefficients for $\mu, \tau$.

## 2 Effective Hamiltonian and lepton polarizations

The quark level transition of the $B_s \rightarrow \phi \ell^+\ell^-$ decay is described by $b \rightarrow s\ell^+\ell^-$ in the standard effective Hamiltonian approach, and can be written in term of twelve four–Fermi interactions, including all possible terms calculated independent of any models, as follows [26],

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2} \pi} V_{ts} V_{tb}^* \left\{ C_{SL} \bar{s}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \ell \gamma^\mu \ell + C_{BR} \bar{s}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \ell \gamma^\mu \ell 
+ C_{LL} \bar{s}\gamma_\mu b_L \ell \gamma^\mu L_L + C_{LR}^{\text{tot}} \bar{s}\gamma_\mu b_L \ell \gamma^\mu R_L + C_{RL} \bar{s}\gamma_\mu b_R \ell \gamma^\mu L_L + C_{RR} \bar{s}\gamma_\mu b_R \ell \gamma^\mu R_L 
+ C_{LLR} \bar{s}L b_R \ell R_L + C_{RLR} \bar{s}L b_R \ell R_L + C_T \bar{s}\sigma_{\mu\nu} b \ell \sigma^{\mu\nu} \ell 
+ i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b \ell \sigma_{\alpha\beta} \ell \right\},
$$

where $L = 1 - \gamma_5/2$ and $R = 1 + \gamma_5/2$ are the chiral projection operators and $C_X$ are the coefficients of the four–Fermi interactions. The coefficients $C_{SL}$ and $C_{BR}$ are the nonlocal Fermi interactions and their correspondence in the SM are $-2m_s C_7^{\text{eff}}$ and $-2m_b C_7^{\text{eff}}$, respectively. The terms with coefficients $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$ are the vector interactions. Two of them are written as $C_{LL}^{\text{tot}} = C_9^{\text{eff}} - C_{10} + C_{LL}$ and $C_{LR}^{\text{tot}} = C_9^{\text{eff}} + C_{10} + C_{LR}$. So these terms describe the sum of the contributions from the SM and the new physics. The terms with coefficients $C_{LLR}$, $C_{RLR}$, $C_{LLR}$ and $C_{RLR}$ describe the scalar type and $C_T$ and $C_{TE}$ describe the tensor type interactions.

Having given the general form of four–Fermi interaction for the $b \rightarrow s\ell^+\ell^-$ transition, we now need to calculate the matrix element for the $B_s \rightarrow \phi \ell^+\ell^-$ decay in order to calculate the decay amplitude. These transition matrix elements can be written in terms of invariant form factors over $B_s$ and $\phi$ in the following form [1] together with [27],

$$
\langle \phi(p_\phi, \epsilon) | \bar{s} \gamma_\mu (1 \pm \gamma_5) b | B_s(p_{B_s}) \rangle = 
- \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_\phi^\alpha q^\beta \frac{2V(q^2)}{m_{B_s} + m_\phi} \pm i \epsilon^{*\nu}(m_{B_s} + m_\phi) A_1(q^2)
\mp i(p_{B_s} + p_\phi) \mu (\epsilon^* q) \frac{A_2(q^2)}{m_{B_s} + m_\phi} \mp i q_\mu (\epsilon^* q) \frac{2m_\phi}{q^2} [A_3(q^2) - A_0(q^2)],
$$

$$
\langle \phi(p_\phi, \epsilon) | \bar{s} i\sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B_s(p_{B_s}) \rangle = 
2 \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_\phi^\alpha q^\beta T_1(q^2) \pm i \left[ \epsilon^*_\mu (m_{B_s}^2 - m_\phi^2) - (p_{B_s} + p_\phi) \mu (\epsilon^* q) \right] T_2(q^2)
\pm i (\epsilon^* q) \left[ q_\mu - (p_{B_s} + p_\phi) \mu \frac{q^2}{m_{B_s}^2 - m_\phi^2} \right] T_3(q^2),
$$

$$
\langle \phi(p_\phi, \epsilon) | \bar{s} \sigma_{\mu\nu} b | B_s(p_{B_s}) \rangle =
$$

2
\[ i\epsilon_{\mu\nu\alpha\beta} \left[ -T_1(q^2)\varepsilon^{*\alpha}(p_{B_s} + p_{\phi})^\beta + \frac{1}{q^2}(m_{B_s}^2 - m_{\phi}^2)\varepsilon^{*\alpha}q^\beta[T_1(q^2) - T_2(q^2)] \right. \]
\[ -\frac{2}{q^2}\left( T_1(q^2) - T_2(q^2) - \frac{q^2}{m_{B_s}^2 - m_{\phi}^2}T_3(q^2) \right)(\varepsilon^*q)p_{\phi}^\beta q^\beta \],
\]

and
\[ \langle \phi(p_{\phi}, \varepsilon)|\bar{s}(1 \pm \gamma_5)b|B_s(p_{B_s})\rangle = \frac{1}{m_b}\left[ \mp 2im_{\phi}(\varepsilon^*q)A_0(q^2) \right], \tag{5} \]

where \( q = p_{B_s} - p_{\phi} \) is the momentum transfer and \( \varepsilon \) is the polarization vector of \( \phi \) meson. The matrix element in (3) is calculated by contracting both sides of (2) with \( q^\mu \), using equation of motion and the following relation \[27]\n
\[ A_3(q^2) = \frac{m_{B_s} + m_{\phi}}{2m_{\phi}}A_1(q^2) - \frac{m_{B_s} - m_{\phi}}{2m_{\phi}}A_2(q^2). \]

In order to avoid kinematical singularity in the matrix element at \( q^2 = 0 \), it is assumed that \( A_0(0) = A_3(0) = T_1(0) = T_2(0) \) \[4\].

After the definitions of the form factors, the matrix element of the \( B_s \to \phi \ell^+\ell^- \) decay can be written by using \(1 - 5\) as,
\[ \mathcal{M}(B_s \to \phi \ell^+\ell^-) = \frac{G_F}{4\sqrt{2}\pi}V_{tb}V_{ts}^* \]
\[ \times \left\{ \bar{\ell}\gamma^\nu(1 - \gamma_5)\ell \left[ -2A_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\alpha}p_{\phi}^\beta - iB_1\varepsilon_{\mu}^* + iB_2(\varepsilon^*q)(p_{B_s} + p_{\phi}) + iB_3(\varepsilon^*q)q_\mu \right] \right. \]
\[ + \bar{\ell}\gamma^\mu(1 + \gamma_5)\ell \left[ -2C_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_{\phi}^\beta - iD_1\varepsilon_{\mu}^* + iD_2(\varepsilon^*q)(p_{B_s} + p_{\phi}) + iD_3(\varepsilon^*q)q_\mu \right] \]
\[ + \bar{\ell}(1 - \gamma_5)(1 + \gamma_5)\gamma^\nu \left[ iB_4(\varepsilon^*q) \right] + \bar{\ell}(1 + \gamma_5)(1 - \gamma_5)\gamma^\nu \left[ iB_5(\varepsilon^*q) \right] \]
\[ + 4\bar{\ell}\sigma^{\mu\nu}(iC_{T}\epsilon_{\mu\nu\alpha\beta}) \left[ -2T_1\varepsilon^{*\alpha}(p_{B_s} + p_{\phi})^\beta + B_6\varepsilon^{*\alpha}q^\beta - B_7(\varepsilon^*q)p_{\phi}^\alpha q^\beta \right] \]
\[ + 16C_{T}\bar{\ell}\sigma_{\mu\nu}\ell \left[ -2T_1\varepsilon^{*\mu}(p_{B_s} + p_{\phi})^\nu + B_6\varepsilon^{*\mu}q^\nu - B_7(\varepsilon^*q)p_{\phi}^\mu q^\nu \right] \], \tag{6} \]

where
\[
A_1 = (C_{LL}^{tot} + C_{RL})\frac{V}{m_{B_s} + m_{\phi}} - (C_{BR} + C_{SL})\frac{T_1}{q^2},
\]
\[
B_1 = (C_{LL}^{tot} - C_{RL})(m_{B_s} - m_{\phi})A_1 - (C_{BR} - C_{SL})(m_{B_s}^2 - m_{\phi}^2)\frac{T_2}{q^2},
\]
\[
B_2 = \frac{C_{LL}^{tot} - C_{RL}}{m_{B_s} + m_{\phi}}A_2 - (C_{BR} - C_{SL})\frac{1}{q^2}\left( T_2 + \frac{q^2}{m_{B_s}^2 - m_{\phi}^2}T_3 \right),
\]
\[
B_3 = 2(C_{LL}^{tot} - C_{RL})\frac{m_{\phi}}{q^2}(A_3 - A_0) + (C_{BR} - C_{SL})\frac{T_3}{q^2},
\]
\[
C_1 = (C_{LL}^{tot} + C_{RR})\frac{V}{m_{B_s} + m_{\phi}} - (C_{BR} + C_{SL})\frac{T_1}{q^2},
\]

3
\[
D_1 = (C_{LR}^{\text{tot}} - C_{RR})(m_{B_s} - m_{\phi})A_1 - (C_{BR} - C_{SL})(m_{B_s}^2 - m_{\phi}^2)\frac{T_2}{q^2},
\]
\[
D_2 = \frac{C_{LR}^{\text{tot}} - C_{RR}}{m_{B_s} + m_{\phi}}A_2 - (C_{BR} - C_{SL})\frac{1}{q^2}\left(T_2 + \frac{q^2}{m_{B_s} - m_{\phi}^2}T_3\right),
\]
\[
D_3 = 2(C_{LR}^{\text{tot}})\frac{m_{\phi}}{q^2}(A_3 - A_0) + (C_{BR} - C_{SL})\frac{T_3}{q^2},
\]
\[
B_4 = -2(C_{LRRL} - C_{RLRL})\frac{m_{\phi}}{m_b}A_0,
\]
\[
B_5 = -2(C_{LRLR} - C_{RLLR})\frac{m_{\phi}}{m_b}A_0,
\]
\[
B_6 = (m_{B_s}^2 - m_{\phi}^2)\frac{T_1 - T_2}{q^2},
\]
\[
B_7 = \frac{2}{q^2}\left(T_1 - T_2 - \frac{q^2}{m_{B_s} - m_{\phi}^2}T_3\right).
\]

At this point, we would like to calculate the final lepton polarizations for the \(B_s \to \phi \ell^+ \ell^-\) decay. In order to do this, we define the orthogonal unit vector \(S_{-L}^\mu\) in the rest frame of \(\ell^-\) and \(S_{+L}^\mu\) in the rest frame of \(\ell^+\), for the polarization of the leptons along the longitudinal (\(L\)), transversal (\(T\)) and normal (\(N\)) directions. Using the convention of \cite{26, 28}, we can write

\[
S_{-L}^\mu \equiv (0, e_{-L}) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right),
\]
\[
S_{-N}^\mu \equiv (0, e_{-N}) = \left(0, \frac{\mathbf{p} \times \mathbf{p}_-}{|\mathbf{p} \times \mathbf{p}_-|}\right),
\]
\[
S_{-T}^\mu \equiv (0, e_{-T}) = \left(0, \mathbf{e}_{-N} \times \mathbf{e}_{-L}\right),
\]
\[
S_{+L}^\mu \equiv (0, e_{+L}) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right),
\]
\[
S_{+N}^\mu \equiv (0, e_{+N}) = \left(0, \frac{\mathbf{p} \times \mathbf{p}_+}{|\mathbf{p} \times \mathbf{p}_+|}\right),
\]
\[
S_{+T}^\mu \equiv (0, e_{+T}) = \left(0, \mathbf{e}_{+N} \times \mathbf{e}_{+L}\right),
\]

where \(\mathbf{p}_\pm\) and \(\mathbf{p}\) are the three momenta of \(\ell^\pm\) and \(\phi\) meson in the center of mass (CM) frame of the lepton pair system, respectively. The longitudinal unit vectors \(S_{L}^\pm\) are boosted to CM frame of the lepton pair by Lorentz transformation,

\[
S_{L,CM}^{-L} = \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell |\mathbf{p}_-|}{m_\ell |\mathbf{p}_-|}\right),
\]
\[
S_{L,CM}^{+L} = \left(\frac{|\mathbf{p}_-|}{m_\ell}, -\frac{E_\ell |\mathbf{p}_-|}{m_\ell |\mathbf{p}_-|}\right),
\]

while vectors of perpendicular directions are not changed by the boost.

The differential decay rate of the \(B_s \to \phi \ell^+ \ell^-\) can be written in any spin direction, as
\[ \frac{d\Gamma(n^\pm)}{ds} = \frac{1}{2} \left( \frac{d\Gamma}{ds} \right)_0 \left[ 1 + \left( P_{L}^{\pm} e_{L}^{\pm} + P_{N}^{\pm} e_{N}^{\pm} + P_{T}^{\pm} e_{T}^{\pm} \right) \cdot n^{\pm} \right], \]  

(10)

where \( s = q^2/m_{B^*}^2 \), the superscripts \( + \) and \( - \) respectively correspond to \( \ell^+ \) and \( \ell^- \) cases and \( (d\Gamma/ds)_0 \) corresponds to the unpolarized decay rate,

\[ \left( \frac{d\Gamma}{ds} \right)_0 = \frac{G^2\alpha^2m_{B^*}^2}{2^{11}\pi^5} |V_{tb}V_{ts}^*|^2 \sqrt{\lambda} \Delta \]  

(11)

where \( \Delta \) is given in Appendix A and \( \lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs - 2s^2 \) and lepton velocity is \( v = \sqrt{1 - 4m_{\ell}^2/sm_{B^*}^2} \).

The polarizations \( P_{L}^{\pm}, P_{T}^{\pm} \) and \( P_{N}^{\pm} \) in (10) are defined by the equation

\[ P_i^{\pm}(s) = \frac{d\Gamma}{ds}(n^{\pm} = e_i^{\pm}) - \frac{d\Gamma}{ds}(n^{\pm} = -e_i^{\pm}) \]

\[ \frac{d\Gamma}{ds}(n^{\pm} = e_i^{\pm}) + \frac{d\Gamma}{ds}(n^{\pm} = -e_i^{\pm}) \],

for \( i = L, N, T \). Here, longitudinal and transversal asymmetries of the charged leptons \( \ell^\pm \) in the decay plane are \( P_{L}^{\pm} \) and \( P_{T}^{\pm} \), respectively, and the normal component to both of them is \( P_{N}^{\pm} \).

The main contribution to \( P_{L}^{-} \) and \( P_{N}^{-} \) is due to tensor interactions. In case of \( P_{T}^{-} \), it also receives considerable scaler contribution in addition to tensor effects. The expressions for the lepton polarizations are given in the Appendix B.

From (B-1)-(B-5), it can be observed that for longitudinal and normal polarizations, the difference between \( \ell^+ \) and \( \ell^- \) lepton asymmetries results from the scalar and tensor type interactions. Similar situation takes place for transverse polarization asymmetries in the \( m_{\ell} \rightarrow 0 \) limit. From this, we can conclude that their experimental study may provide essential information about new physics.

In searching new physics, the combined analysis of the lepton and antilepton polarizations can be another useful source, since in the SM \( P_{L}^{-} + P_{L}^{+} = 0, P_{N}^{-} + P_{N}^{+} = 0 \) and \( P_{T}^{-} - P_{T}^{+} \approx 0 \) [26]. Using (B-1) we obtain combined longitudinal polarization, the combined transversal polarization which is the difference of the lepton and antilepton polarizations, from (B-2) and (B-3) and finally the combined normal polarization, from (B-4) and (B-5). The explicit form of these asymmetries are also given in the Appendix B.

One should note from (B-6) that in \( P_{L}^{-} + P_{L}^{+} \) the SM contribution coming with \( C_{BR}, C_{SL}, C_{LL}^{tot} \) and \( C_{LR}^{tot} \) terms, completely cancels. Any nonzero measurement of the value of \( P_{L}^{-} + P_{L}^{+} \) in future experiments, may be an evidence of the discovery of new physics beyond the SM.

### 3 Numerical analysis and discussion

We will present our numerical analysis of the branching ratios and polarizations and their dependencies on Wilson coefficients in a series of figures, but before doing this, let us remark on a few points.
are defined by the quantities. So, by taking the averaged forms over the allowed kinematical region, we eliminate the dependency of the lepton polarizations on $s$. The averaged lepton polarizations are defined by

$$
\langle P_i \rangle = \frac{\int_{(2m_{\ell}/m_{B_s})^2}^{(1-m_{\phi}/m_{B_s})^2} P_i \frac{dB}{ds} ds}{\int_{(2m_{\ell}/m_{B_s})^2}^{(1-m_{\phi}/m_{B_s})^2} \frac{dB}{ds} ds}.
$$

The input parameters we used in our numerical analysis are:

$$m_{B_s} = 5.367 \text{ GeV}, \ m_\phi = 1.019 \text{ GeV} \ m_\ell = 4.8 \text{ GeV}, \ m_\mu = 0.105 \text{ GeV}, \ m_\tau = 1.77 \text{ GeV}, \ |V_{tb}V_{ts}^*| = 0.0385, \ \alpha^{-1} = 129, \ G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}, \ \tau_{B_s} = 1.425 \times 10^{-12} \text{ s}.$$

The values of the individual Wilson coefficients that appear in the SM at $\mu \sim m_\ell$ are listed in Table 1 and the parameters that are not given here are taken from [29].

The given $C_{9}^{\text{eff}}$ value in Table 1 corresponds only to the short-distance contributions, but it should be noted that $C_{9}^{\text{eff}}$ also receives long-distance contributions due to conversion of the real $\bar{c}c$ into lepton pair $\ell^+\ell^-$ and are usually absorbed into a redefinition of the short-distance Wilson coefficients:

$$C_{9}^{\text{eff}}(\mu) = C_{9}(\mu) + Y(\mu),$$

where

$$Y(\mu) = Y_{\text{reson}} + h(y, s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)]$$

$$- \frac{1}{2} h(1, s) [4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)]$$

$$- \frac{1}{2} h(0, s) [C_3(\mu) + 3C_4(\mu)]$$

$$+ \frac{2}{9} (3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)).$$

with $y = m_\ell/m_\ell$, and the functions $h(y, s)$ arises from the one loop contributions of the four quark operators $O_1, ..., O_6$. The explicit forms of them can be found in [30]–[32]. Parametrization of the resonance $\bar{c}c$ contribution, $Y_{\text{reson}}(s)$, given in [34] can be done by using a Breit-Wigner shape with normalizations fixed by data given by [33]

$$Y_{\text{reson}}(s) = -\frac{3}{\alpha_{em}^2} \kappa \sum_{V_i=\psi_i} \frac{\pi \Gamma(V_i \rightarrow \ell^+\ell^-) m_{V_i}}{s m_{B_c}^2 - m_{V_i}^2 + i m_{V_i} \Gamma_{V_i}}$$

$$\times [3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)],$$

| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7^{\text{eff}}$ | $C_9$ | $C_{10}$ |
|------|------|------|------|------|------|-----------------|------|--------|
| −0.248 | +1.107 | +0.011 | −0.026 | +0.007 | −0.031 | −0.313 | +4.344 | −4.624 |

Table 1: Values of the SM Wilson coefficients at $\mu \sim m_{b}$ scale.
where the phenomenological parameter $\kappa$ is taken as 2.3.

The new Wilson coefficients are the free parameters in this work, but it is possible to establish ranges out of experimentally measured branching ratios of the semileptonic rare $B$-meson decays

\[
BR(B \to K \ell^+ \ell^-) = (4.8^{+1.0}_{-0.9} \pm 0.3) \times 10^{-7} \text{[34]},
\]
\[
BR(B \to K^* \mu^+ \mu^-) = (1.27^{+0.76}_{-0.61} \pm 0) \times 10^{-6} \text{[35]},
\]

and also the upper bound of pure leptonic rare $B$-decays in the $B^0 \to \mu^+ \mu^-$ mode [36):

\[
BR(B^0 \to \mu^+ \mu^-) \leq 1.5 \times 10^{-7}. 
\]

Compliant to this upper limit and the branching ratios for the semileptonic rare $B$-decays, in this work we take all new Wilson coefficients as real and varying in the region $-4 \leq C_X \leq 4$.

The numerical values of the form factors that we used in this work are the results of [4]. The form factors are calculated in light cone sum rule approach and include the radiative and higher twist corrections and SU(3) breaking effects. The $q^2$ dependencies of the form factors in three parameter fit are given as

\[
F(q^2) = \frac{F(0)}{1 - as + bs^2},
\]

where the values of parameters $F(0)$, $a$ and $b$ for the $B_s \to \phi$ decay are listed in Table 2.

|     | $A_0$ | $A_1$ | $A_2$ | $V$  | $T_1$ | $T_2$ | $T_3$ |
|-----|-------|-------|-------|------|-------|-------|-------|
| $F(0)$ | 0.382 | 0.296 | 0.255 | 0.433 | 0.348 | 0.348 | 0.254 |
| $a$   | 1.77  | 0.87  | 1.55  | 1.75  | 1.820 | 0.70  | 1.52  |
| $b$   | 0.856 | -0.061| 0.513 | 0.736 | 0.825 | -0.315| 0.377 |

Table 2: Light cone sum rule approach $B_s \to \phi$ meson decay form factors in a three parameters fit, including radiative and higher twist corrections and SU(3) breaking effects.

Before discussing the figures including the results of our analysis, we would like to give our SM predictions for the longitudinal, transverse and the normal components of the lepton polarizations for $B_s \to \phi \ell^+ \ell^-$ decay for the $\mu$ ($\tau$) channel for reference:

\[
<P_L> = 0.8373 (0.4299),
\]
\[
<P_T> = 0.0025 (0.0498),
\]
\[
<P_N> = 0.0013 (0.0214).
\]

The $<P_L>$ value is in consistent with $<P_L>_{\mu (\tau)} = -0.81 (-0.49)$, respectively [9]. (The opposite sign is caused by the definition of the form factors.)

In Figures (1) and (2), we give the dependence of the integrated branching ratio (BR) on the new Wilson coefficients for the $B_s \to \phi \mu^+ \mu^-$ and $B_s \to \phi \tau^+ \tau^-$ decays, respectively.
The SM predictions for the integrated branching ratios, which are comparable with [8, 19], are

\[ BR(B_s \rightarrow \phi \mu^+\mu^-) = 1.92 \times 10^{-6}, \]
\[ BR(B_s \rightarrow \phi \tau^+\tau^-) = 2.34 \times 10^{-7}. \]

The former one is also in the experimental range reported by [22]-[23].

In figures, the strong dependence of BR on the tensor interactions is clear. There is a weak dependence on vector interactions \( C_{LL} \) and \( C_{RL} \), while BR is completely insensitive to the scalar interactions for \( \mu \) case, negligibly sensitive for \( \tau \) case. It can also be seen from these figures that dependence of the BR on the new Wilson coefficients is symmetric with respect to the zero point for the muon final state, but such a symmetry is not observed for the tau final state for the tensor interactions.

In Figs. (3) and (4), we present the dependence of averaged longitudinal polarization \( < P_L^- > \) of \( \ell^- \) and the combined averaged \( < P_L^- + P_L^+ > \) for \( B_s \rightarrow \phi \mu^+\mu^- \) decay on the new Wilson coefficients. We observe that the contributions coming from all types of interactions to \( < P_L^- > \) are positive, and more sensitive to the existence of the tensor interactions. It is an increasing (decreasing) functions of both tensor interactions for their negative (positive) values and it should also be noted that \( < P_L^- > \) becomes substantially different from the SM value (at \( C_X = 0 \)) as \( C_X \) becomes different from zero. This indicates that measurement of the longitudinal lepton polarization in \( B_s \rightarrow \phi \mu^+\mu^- \) decay can be very useful to investigate new physics beyond the SM. On the other hand, the contributions to the combined average \( < P_L^- + P_L^+ > \) is a result of scalar and \( C_T \) tensor interactions. Vector type interactions are cancelled when the longitudinal polarization asymmetry of the lepton and antilepton are considered together. This expected result is also tested here since there is no vector contribution on \( < P_L^- + P_L^+ > \). Additionally, \( < P_L^- + P_L^+ > \) becomes zero at \( C_X = 0 \), which conforms the SM results. So, any nonzero measurement can be the signal of new physics beyond the SM. The dependence of \( < P_L^- + P_L^+ > \) on \( C_X \) is symmetric with respect to the zero value and is positive for all values of \( C_{LRLR} \) and \( C_{RLRL} \), while it is negative for remaining scalar type interactions. A last note on the \( C_T \) interaction. The \( C_T \) contribution is positive (negative) for \( C_X < 0 \) (\( C_X > 0 \)). This can also be useful.

Figures (5) and (6) are the same as Figs. (3) and (4), but for \( B_s \rightarrow \phi \tau^+\tau^- \). Similar to the muon case, \( < P_L^- > \) is more sensitive to the tensor interactions than others. Contributions to \( < P_L^- > \) from all type of interactions are positive for all values of \( C_X \) except for \( C_T \) and \( C_{TE} \). In the region \( 1.2 \lesssim C_T < 4 \) and \( 0.17 \lesssim C_{TE} < 4 \), \( < P_L^- > \) changes the sign and becomes negative. For \( < P_L^- + P_L^+ > \), although their values are bigger than that of the \( \mu \) case, the scalar and \( C_{TE} \) tensor contributions become less important as comparing the dominance of \( C_{TE} \). \( < P_L^- + P_L^+ > \) changes sign as individual Wilson coefficient changes its sign. Specifically speaking, the \( < P_L^- + P_L^+ > \) takes positive (negative) values for negative (positive) value of \( C_T \). Thus, one can provide essential information about new physics by determining the sign and the magnitude of \( < P_L^- + P_L^+ > \). In tau final state, \( < P_L^- + P_L^+ > \) also becomes zero at \( C_X = 0 \), a conforming result of the SM.

In Figs. (7) and (8), we present the dependence of averaged transverse polarization \( < P_T^- > \) of \( \ell^- \) and the combined averaged \( < P_T^- - P_T^+ > \) for \( B_s \rightarrow \phi \mu^+\mu^- \) decay on the new Wilson coefficients. As seen from the figure, for \( < P_T^- > \) the vector contributions are negligible but there appears strong dependence on tensor and scalar interactions. The
scalar terms $C_{LRRL}$, $C_{LRLR}$ and $C_{RLRL}$, $C_{RLRR}$ are approximately identical in pair. When
the formers are positive (negative), $<P_T^->$ is negative (positive) while it is opposite for
the others. On the other hand, the $C_{LRRL}$ and $C_{RLRL}$ components of scalar interactions
become less important in $<P_T^- - P_T^+>$, as comparing their effects in $<P_T^->$.

Figures (9) and (10) are the same as Figs. (7) and (8), but for $B_s \rightarrow \phi \tau^+ \tau^-$. We see
from these figures that the $<P_T^->$ and $<P_T^- - P_T^+>$ are quite sensitive to all types of
interactions. The dependence of vector interactions are also more sizable comparing with
the tensor interactions only. We observe that $<P_T^- + P_T^+>$ is negative (positive) when
$C_{TE} < 0$ ($C_{TE} > 0$) while the behavior of $C_T$ is opposite with respect to $C_{TE}$. In addition,
as expected in the SM, $<P_T^- + P_T^+>$ becomes zero at $C_X = 0$.

Figures (13) and (14) are the same as Figs. (11) and (12), but for $B_s \rightarrow \phi \tau^+ \tau^-$. In
opposite to the muon final state case, we should notice the dependence of $<P_T^->$ on vector
type interactions, too. $<P_T^- + P_T^+>$, as in the muon case, depends only on the tensor
interactions and their behaviors are similar.

Finally, a few words on the detectability of lepton polarization asymmetries to have an
idea of this possibility folowing [10]. Experimentally, the required number of events are
$N = n^2/(B<P_t>^2$ for a decay with the branching ration $B$ at $n \sigma$ level to be able to
measure an asymmetry $<P_t>$. Using our SM predictions for lepton polarizations given in
(10) we simply find that to observe $<P_T^->$, $<P_T^->$ and $<P_T^- >$ in $B_s \rightarrow \phi \mu^+ \mu^-$ at $1 \sigma$
level we need $N = (0.74; 8.33 \times 10^4; 3.08 \times 10^5) \times 10^6$ number of events, respectively. For the
$B_s \rightarrow \phi \tau^+ \tau^-$, the required number of events are $N = (2.31; 1.72 \times 10^2; 9.33 \times 10^3) \times 10^7$. The
number of $b \bar{b}$ events expected, at least at LHC-b, is $\sim 10^{12}$. So, comparing these numbers
we conclude that in principle measurement of these values could be possible.

In conclusion, starting the general model independent form of the effective Hamiltonian,
we present the most general analysis of the lepton polarization asymmetries in the
rare $B_s \rightarrow \phi \ell^+ \ell^-$ decay. The dependence of the longitudinal, transversal and normal polariza-
tion asymmetries of $\ell^-$ and their combined asymmetries on the new Wilson coefficients
are studied. The lepton polarization asymmetries are very sensitive to the existence of the
tensor type interactions and in some cases effect of scalar type interactions should be con-
sidered. The tensor $C_T$ term plays a significant role throughout this work. Additionally,
in the most cases, the value of polarization asymmetries change sign as the new Wilson
coefficients vary in the region of interest, which is useful to determine the sign in addition
magnitude of new physics effect. In the SM, in the limit $m_\ell \rightarrow 0$, the combined lepton
polarizations are $<P_T^- + P_T^+> = 0$, $<P_T^- + P_T^+> = 0$ and $<P_T^- - P_T^+> \simeq 0$. Therefore,
in the experimental searches, any nonzero measurement will be an effective tool in looking for
new physics beyond the SM.
Acknowledgments
The author would like to thank S. Fajfer for valuable contributions and critical comments, G. Turan for reading the manuscript and comments on it and B. Golob for sharing experimental experience. This work was supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under BIDEB-2219 program.
\[
\Delta = \frac{32}{3} m_B^4 \lambda \left( m_B^2 s - m_\ell^2 \right) \left( |A_1|^2 + |C_1|^2 \right) + 6 m_\ell^2 \text{Re}(A_1 C_1^*) \\
+ 96 m_\ell^2 \text{Re}(B_1 D_1^*) - \frac{4}{r} m_B^2 m_\ell \lambda \text{Re}(B_1 - D_1)(B_4^* - B_5^*) \\
+ \frac{8}{r} m_B^2 m_\ell^2 \lambda \left( \text{Re}(B_1(-B_3^* + D_2^* + D_3^*)) + \text{Re}(D_1(B_2^* + B_3^* - D_3^*)) - \text{Re}(B_4 B_5^*) \right) \\
+ \frac{4}{r} m_B^2 m_\ell (1-r) \lambda \left( \text{Re}((B_2 - D_2)(B_4^* - B_5^*)) + 2 m_\ell \text{Re}((B_2 - D_2)(B_3^* - D_3^*)) \right) \\
- \frac{8}{r} m_B^4 m_\ell^2 \lambda (2 + 2r - s) \text{Re}(B_2 D_2^*) + \frac{4}{r} m_B^4 m_\ell s \lambda \text{Re}((B_3 - D_3)(B_4^* - B_5^*)) \\
+ \frac{4}{r} m_B^4 m_\ell^2 \lambda (|B_3 - D_3|^2 + \frac{2}{r} m_B^2 m_\ell (m_B^2 s - 2 m_\ell^2) \lambda \left( |B_4|^2 + |B_5|^2 \right) \\
- \frac{8}{3r} m_B^2 \lambda \left( m_B^2 (2 - 2r + s) + m_B^2 s (1 - r - s) \right) \left[ \text{Re}(B_1 B_2^*) + \text{Re}(D_1 D_2^*) \right] \\
+ \frac{4}{3r} \left[ 2 m_B^2 (\lambda - 6rs) + m_B^2 s (\lambda + 12rs) \right] \left( |B_1|^2 + |D_1|^2 \right) \\
+ \frac{4}{3r} m_B^4 \lambda \left( m_B^2 s \lambda + m_\ell^2 [2 \lambda + 3s (2 + 2r - s)] \right) \left( |B_2|^2 + |D_2|^2 \right) \\
+ \frac{32}{r} m_B^6 m_\ell \lambda^2 \text{Re}((B_2 + D_2)(B_7 C_{TE})^*) \\
- \frac{32}{r} m_B^4 m_\ell \lambda (1 - r - s) \left( \text{Re}((B_1 + D_1)(B_7 C_{TE})^*) + 2 \text{Re}((B_2 + D_2)(B_6 C_{TE})^*) \right) \\
+ \frac{64}{r} (\lambda + 12rs) m_B^2 m_\ell \lambda \text{Re}((B_1 + D_1)(B_6 C_{TE})^*) \\
+ \frac{256}{3r} |T_1|^2 |C_T|^2 m_B^2 \left( 4 m_\ell^2 \lambda (8r - s) - 12rs (2 + 2r - s) \right) \\
+ m_B^2 s \left( \lambda (16r + s) + 12rs (2 + 2r - s) \right) \\
+ \frac{1024}{3r} m_B^2 |C_{TE}|^2 \left( 8 m_\ell^2 \lambda (4r + s) + 12rs (2 + 2r - s) \right) \\
+ m_B^2 s \left( \lambda (16r + s) + 12rs (2 + 2r - s) \right) \\
- \frac{128}{r} m_B^2 m_\ell \lambda (\lambda + 12r (1 - r)) \text{Re}((B_1 + D_1)(T_1 C_{TE})^*) \\
+ \frac{128}{r} m_B^4 m_\ell \lambda (1 + 3r - s) \text{Re}((B_2 + D_2)(T_1 C_{TE})^*) + 512 m_B^4 m_\ell \lambda \text{Re}((A_1 + C_1)(T_1 C_T)^*) \\
+ \frac{16}{3r} m_B^2 \left( 4 (m_B^2 s + 8 m_\ell^2) |C_{TE}|^2 + m_B^2 s v^2 |C_T|^2 \right) \times \left( 4 (\lambda + 12rs) |B_6|^2 \right) \\
+ m_B^4 \lambda^2 |B_{7r}^2| - 4 m_B^2 (1 - r - s) \lambda \text{Re}(B_6 B_7^*) - 16 [\lambda + 12r (1 - r)] \text{Re}(T_1 B_{7r}^*) \\
+ 8 m_B^2 \lambda (1 + 3r - s) \lambda \text{Re}(T_1 B_7^*), \tag{A-1}
\]

where \( \lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs \), \( r = m_\phi^2 / m_B^2 \) lepton velocity is \( v = \sqrt{1 - 4m_\ell^2 / sm_B^2} \).
Appendix B

The longitudinal polarization $P_L^\pm$ for the $\ell^\pm$;

\[
P_L^\pm = \frac{4}{\Delta} m_{B_*}^2 v \left\{ \pm \frac{1}{3r} \lambda^2 m_{B_*}^4 \left[ |B_2|^2 - |D_2|^2 \right] + \frac{1}{r} \lambda m_{\ell} \Re[(B_1 - D_1)(B_4^* + B_5^*)] 
- \frac{1}{r} \lambda m_{B_*}^2 m_{\ell}(1 - r) \Re[(B_2 - D_2)(B_1^* + B_5^*)] \mp \frac{8}{3} \lambda m_{B_*}^4 s \left[ |A_1|^2 - |C_1|^2 \right] 
- \frac{1}{2r} \lambda m_{B_*}^2 m_{\ell}s \left[ |B_4|^2 - |B_5|^2 \right] - \frac{1}{r} \lambda m_{B_*}^2 m_{\ell}s \Re[(B_3 - D_3)(B_4^* + B_5^*)] 
\pm \frac{2}{3r} \lambda m_{B_*}^2 (1 - r - s) \Re[(B_1 B_2^*) - \Re(D_1 D_2^*)] \mp \frac{1}{3r}(\lambda + 12rs) \left[ |B_1|^2 - |D_1|^2 \right] 
\pm \frac{256}{3} \lambda m_{B_*}^2 m_{\ell} \left( \Re[A_1^*(C_T + C_{TE})T_1] - \Re[C_1^*(C_T \mp C_{TE})T_1] \right) 
+ \frac{4}{3r} \lambda m_{B_*}^2 m_{\ell} \left( \Re[B_2^*(C_T \mp 4C_{TE})B_7] + \Re[D_2^*(C_T \mp 4C_{TE})B_7] \right) 
- \frac{8}{3r} \lambda m_{B_*}^2 m_{\ell}(1 - r - s) \left( \Re[B_2^*(C_T \mp 4C_{TE})B_6] + \Re[D_2^*(C_T \mp 4C_{TE})B_6] \right) 
- \frac{4}{3r} \lambda m_{B_*}^2 m_{\ell}(1 - r - s) \left( \Re[B_1^*(C_T \mp 4C_{TE})B_7] + \Re[D_1^*(C_T \mp 4C_{TE})B_7] \right) 
+ \frac{8}{3r}(\lambda + 12rs)m_{\ell}\left( \Re[B_1^*(C_T \mp 4C_{TE})B_6] + \Re[D_1^*(C_T \mp 4C_{TE})B_6] \right) 
- \frac{16}{3r} m_{\ell}[\lambda + 12r(1 - r)] \left( \Re[B_1^*(C_T \mp 4C_{TE})T_1] + \Re[D_1^*(C_T \mp 4C_{TE})T_1] \right) 
+ \frac{16}{3r} \lambda m_{B_*}^2 m_{\ell}(1 + 3r - s) \left( \Re[B_1^*(C_T \mp 4C_{TE})T_1] + \Re[D_1^*(C_T \mp 4C_{TE})T_1] \right) 
+ \frac{16}{3r} \lambda^2 m_{B_*}^6 s |B_7|^2 \Re(C_T C_{TE}^\ast) + \frac{64}{3r}(\lambda + 12rs)m_{B_*}^2 s |B_6|^2 \Re(C_T C_{TE}^\ast) 
- \frac{64}{3r} \lambda m_{B_*}^4 s(1 - r - s) \Re(B_6 B_7^*) \Re(C_T C_{TE}^\ast) 
+ \frac{128}{3r} \lambda m_{B_*}^4 s(1 + 3r - s) \Re(B_7 T_1^\ast) \Re(C_T C_{TE}^\ast) 
- \frac{256}{3r} m_{B_*}^2 s[\lambda + 12r(1 - r)] \Re(B_6 T_1^\ast) \Re(C_T C_{TE}^\ast) 
+ \frac{256}{3r} m_{B_*}^2 s[\lambda(4r + s) + 12r(1 - r)^2] |T_1|^2 \Re(C_T C_{TE}^\ast) \right\}, \tag{B-1}
\]

where $\Delta$ is given in (A-1).

The transverse polarization $P_T^\pm$ for $\ell^\pm$;

\[
P_T^\pm = \frac{\pi}{\Delta} m_{B_*} \sqrt{s} \lambda \left\{ -8m_{B_*}^2 m_{\ell} \Re[(A_1 + C_1)(B_1^* + D_1^*)] 
+ \frac{1}{r} m_{B_*}^2 m_{\ell}(1 + 3r - s) \left[ \Re(B_1 D_2^*) - \Re(B_2 D_1^*) \right] 
+ \frac{1}{rs} m_{\ell}(1 - r - s) \left[ |B_1|^2 - |D_1|^2 \right] 
+ \frac{1}{rs}(2m_{\ell}^2 - m_{B_*}^2 s)(1 - r - s) \left[ \Re(B_1 B_5^*) - \Re(D_1 B_1^*) \right] \right\},
\]

\[12\]
\[
- \frac{1}{r} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_1 + D_1)(B_3^*-D_3^*)]
\]
\[
- \frac{2}{rs} m_B^2 m_\ell^2 \lambda \left[ \text{Re}(B_2 B_2^*) - \text{Re}(D_2 D_2^*) \right]
\]
\[
+ \frac{1}{rs} m_B^4 m_\ell (1 - r) \lambda \left[ |B_2|^2 - |D_2|^2 \right] + \frac{1}{r} m_B^4 m_\ell \lambda \text{Re}[(B_2 + D_2)(B_3^*-D_3^*)]
\]
\[
- \frac{1}{rs} m_B^2 m_\ell [\lambda + (1 - r - s)(1 - r)] \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right]
\]
\[
+ \frac{1}{rs} (1 - r - s)(2m_\ell^2 - m_B^2 s) \left[ \text{Re}(B_1 B_1^*) - \text{Re}(D_1 D_1^*) \right]
\]
\[
+ \frac{1}{rs} m_B^2 \lambda (2m_\ell^2 - m_B^2 s) \left[ \text{Re}(D_2 B_5^*) - \text{Re}(B_2 B_4^*) \right]
\]
\[
- \frac{16}{rs} \lambda m_B^2 m_\ell^2 \text{Re}[(B_1 - D_1)(B_7 C_{TE})^*]
\]
\[
+ \frac{16}{rs} \lambda m_B^4 m_\ell^2 (1 - r) \text{Re}[(B_2 - D_2)(B_7 C_{TE})^*]
\]
\[
+ \frac{8}{r} \lambda m_B^4 m_\ell \text{Re}[(B_4 - B_5)(B_7 C_{TE})^*]
\]
\[
+ \frac{16}{r} \lambda m_B^4 m_\ell^2 \text{Re}[(B_3 - D_3)(B_7 C_{TE})^*]
\]
\[
+ \frac{32}{rs} m_\ell^2 (1 - r - s) \text{Re}[(B_1 - D_1)(B_6 C_{TE})^*]
\]
\[
- \frac{32}{rs} m_B^2 m_\ell^2 (1 - r)(1 - r - s) \text{Re}[(B_2 - D_2)(B_6 C_{TE})^*]
\]
\[
- \frac{16}{r} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_4 - B_5)(B_6 C_{TE})^*]
\]
\[
- \frac{32}{r} m_B^2 m_\ell^2 (1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_{TE})^*]
\]
\[
- 16m_B^2 \left( 4m_\ell^2 \text{Re}[A_1^*(C_T + 2C_{TE})B_6] - m_B^2 s \text{Re}[A_1^*(C_T - 2C_{TE})B_6] \right)
\]
\[
+ 16m_B^2 \left( 4m_\ell^2 \text{Re}[C_1^*(C_T + 2C_{TE})B_6] - m_B^2 s \text{Re}[C_1^*(C_T + 2C_{TE})B_6] \right)
\]
\[
+ \frac{32}{s} m_B^2 (1 - r) \left( 4m_\ell^2 \text{Re}[A_1^*(C_T + 2C_{TE})T_1] - m_B^2 s \text{Re}[A_1^*(C_T - 2C_{TE})T_1] \right)
\]
\[
- \frac{32}{s} m_B^2 (1 - r) \left( 4m_\ell^2 \text{Re}[C_1^*(C_T - 2C_{TE})T_1] - m_B^2 s \text{Re}[C_1^*(C_T + 2C_{TE})T_1] \right)
\]
\[
+ \frac{64}{rs} m_B^2 m_\ell^2 (1 - r)(1 + 3r - s) \text{Re}[(B_2 - D_2)(T_1 C_{TE})^*]
\]
\[
+ \frac{64}{rs} m_B^2 m_\ell^2 (1 + 3r - s) \text{Re}[(B_3 - D_3)(T_1 C_{TE})^*]
\]
\[
+ \frac{32}{r} m_B^2 m_\ell (1 + 3r - s) \text{Re}[(B_1 - B_5)(T_1 C_{TE})^*]
\]
\[
+ \frac{64}{rs} [m_B^2 rs - m_\ell^2 (1 + 7r - s)] \text{Re}[(B_1 - D_1)(T_1 C_{TE})^*]
\]
\[
- \frac{32}{s} m_\ell^2 + m_B^2 s \text{Re}[(B_1 + D_1)(T_1 C_T)^*]
\]
\[
- 2048 m_B^2 m_\ell \text{Re}[(C_T T_1)(B_6 C_{TE})^*]
\]
\[ + \frac{4096}{s} m_B^2 m_{\ell}(1 - r) |g|^2 \text{Re}(C_T C_{TE}^*) \] 

and \( P_T^+ \) for \( \ell^+ \)

\[ P_T^+ = \frac{\pi}{\Delta} m_B \sqrt{s \lambda} \left\{ -8m_B^2 m_{\ell} \text{Re}[(A_1 + C_1)(B_1^* + D_1^*)] - \frac{1}{r} m_B^2 m_{\ell}(1 + 3r - s) \left[ \text{Re}(B_1 D_2^*) - \text{Re}(B_2 D_1^*) \right] - \frac{1}{rs} m_{\ell}(1 - r - s) \left[ |B_1|^2 - |D_1|^2 \right] + \frac{1}{rs} (2m_B^2 - m_B s)(1 - r - s) \left[ \text{Re}(B_1 B_3^*) - \text{Re}(D_1 B_3^*) \right] + \frac{1}{r} m_B^2 m_{\ell}(1 - r - s) \text{Re}[(B_1 + D_1)(B_3^* - D_3^*)] - \frac{1}{rs} m_B^2 \lambda (2m_B^2 - m_B s) \left[ \text{Re}(B_2 B_3^*) - \text{Re}(D_2 B_4^*) \right] - \frac{1}{rs} m_B^4 m_{\ell}(1 - r) \lambda \left[ |B_2|^2 - |D_2|^2 \right] - \frac{1}{r} m_B^4 m_{\ell} \lambda \text{Re}[(B_2 + D_2)(B_3^* - D_3^*)] + \frac{1}{rs} m_B^2 m_{\ell}[\lambda + (1 - r - s)(1 - r)] \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right] + \frac{2}{rs} m_B^2(1 - r - s) \left[ \text{Re}(B_1 B_4^*) - \text{Re}(D_1 B_5^*) \right] + \frac{2}{rs} m_B^2 m_{\ell} \lambda \left[ \text{Re}(B_2 B_4^*) - \text{Re}(B_2 B_4^*) \right] + \frac{16}{rs} \lambda m_B^2 m_{\ell} \text{Re}[(B_1 - D_1)(B_1 C_{TE})^*] - \frac{16}{rs} \lambda m_B^2 m_{\ell}^2 (1 - r) \text{Re}[(B_2 - D_2)(B_2 C_{TE})^*] - \frac{8}{r} \lambda m_B^4 m_{\ell} \text{Re}[(B_4 - B_5)(B_3 C_{TE})^*] - \frac{16}{r} \lambda m_B^4 m_{\ell} \text{Re}[(B_3 - D_3)(B_3 C_{TE})^*] - \frac{32}{rs} m_B^2(1 - r - s) \text{Re}[(B_1 - D_1)(B_3 C_{TE})^*] + \frac{32}{rs} m_B^2 m_{\ell}(1 - r)(1 - r - s) \text{Re}[(B_2 - D_2)(B_3 C_{TE})^*] + \frac{16}{rs} m_B^2 m_{\ell}(1 - r - s) \text{Re}[(B_4 - B_5)(B_6 C_{TE})^*] + \frac{32}{rs} m_B^2 m_{\ell}(1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_{TE})^*] + 16m_B^2 \left( 4m_B^2 \text{Re}[A_1^*(C_T - 2C_{TE})B_6] - m_B^2 s \text{Re}[A_1^*(C_T + 2C_{TE})B_6] \right) - 16m_B^2 \left( 4m_B^2 \text{Re}[C_1^*(C_T + 2C_{TE})B_6] - m_B^2 s \text{Re}[C_1^*(C_T - 2C_{TE})B_6] \right) - \frac{32}{s} m_B^2(1 - r) \left( 4m_B^2 \text{Re}[A_1^*(C_T - 2C_{TE})T_1] - m_B^2 s \text{Re}[A_1^*(C_T + 2C_{TE})T_1] \right) \]
+ \frac{32}{s} m_{B_s}^2 (1 - r) \left( 4m_{T}^2 \text{Re}[C_1^*(C_T + 2C_{TE})T_1] - m_{B_s}^2 s \text{Re}[C_1^*(C_T - 2C_{TE})T_1] \right) \\
- \frac{64}{rs} m_{B_s}^2 m_{T}^2 (1 - r)(1 + 3r - s) \text{Re}[(B_2 - D_2)(T_1 C_{TE})^*] \\
- \frac{64}{r} m_{B_s}^2 m_{T}^2 (1 + 3r - s) \text{Re}[(B_3 - D_3)(T_1 C_{TE})^*] \\
- \frac{32}{r} m_{B_s}^2 m_{T} (1 + 3r - s) \text{Re}[(B_4 - B_5)(T_1 C_{TE})^*] \\
- \frac{64}{rs} m_{B_s}^2 m_{T}^2 (1 + 7r - s) \text{Re}[(B_1 - D_1)(T_1 C_{TE})^*] \\
- \frac{32}{s} (4m_{T}^2 + m_{B_s}^2 s) \text{Re}[(B_1 + D_1)(T_1 C_T)^*] \\
- 2048 m_{B_s}^2 m_{T} \text{Re}[(C_T g)(B_0 C_{TE})^*] \\
+ \frac{4096}{s} m_{B_s}^2 m_{T} (1 - r) |T_1|^2 \text{Re}(C_T C_{TE}^*) \right] . \quad (B-3)

The normal polarization $P^\perp_N$ for $\ell^-$

\[
P^\perp_N = \frac{1}{\Delta} \pi v m_{B_s}^3 \sqrt{s\lambda} \left\{ 8m_{\ell} \text{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
- \frac{1}{r} m_{B_s}^2 \lambda \text{Im}[(B_2^* B_4) + (D_2^* B_5)] \\
+ \frac{1}{r} m_{B_s}^2 m_{\ell} \lambda \text{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
- \frac{1}{r} m_{\ell} (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
+ \frac{1}{r} (1 - r - s) \text{Im}[(B_4^* B_4) + (D_4^* B_5)] \\
- \frac{1}{r} m_{\ell} (1 - r - s) \text{Im}[(B_1 + D_1)(B_2^* - D_2^*)] \\
- \frac{8}{r} m_{B_s}^2 m_{\ell} \lambda \text{Im}[(B_4 + B_5)(B_7 C_{TE})^*] \\
+ \frac{16}{r} m_{\ell} (1 - r - s) \text{Im}[(B_4 + B_5)(B_0 C_{TE})^*] \\
- \frac{32}{r} m_{\ell} (1 + 3r - s) \text{Im}[(B_4 + B_5)(T_1 C_{TE})^*] \\
- 16m_{B_s}^2 s \left( \text{Im}[A_1^*(C_T - 2C_{TE})B_6] + \text{Im}[C_1^*(C_T + 2C_{TE})B_6] \right) \\
+ 32m_{B_s}^2 (1 - r) \left( \text{Im}[A_1^*(C_T - 2C_{TE})T_1] + \text{Im}[C_1^*(C_T + 2C_{TE})T_1] \right) \\
+ 32 \left( \text{Im}[B_1^*(C_T - 2C_{TE})T_1] - \text{Im}[D_1^*(C_T + 2C_{TE})T_1] \right) \\
+ 512m_{\ell} \left( |C_T|^2 - 4|C_{TE}|^2 \right) \text{Im}(B_0^* T_1) \right\} , \quad (B-4)
\]

and $P^+_N$ for $\ell^+$

\[
P^+_N = \frac{1}{\Delta} \pi v m_{B_s}^3 \sqrt{s\lambda} \left\{ - 8m_{\ell} \text{Im}[(B_1^* C_1) + (A_1^* D_1)] \right\}
\]
The combined longitudinal polarization $P_L^+ + P_L^-$, from (3-1):

$$
P_L^+ + P_L^- = \frac{4}{\Delta} m_B^2 \lambda \text{Im}[(B_2^* B_4) + (D_2^* B_3)]
- \frac{2}{r} m_B^2 m_\ell \lambda (1 - r) \text{Re}[(B_2 - D_2)(B_4^* + B_3^*)]
- \frac{1}{r} m_\ell (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)]
- \frac{1}{r} (1 - r - s) \text{Im}[(B_2 - D_2)(B_1^* - D_1^*)]
- \frac{1}{r} m_\ell (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)]
+ \frac{8}{3r} m_B^4 m_\ell \lambda^2 \text{Re}[(B_2 + D_2)(B_7 C_T)]
+ \frac{32}{3r} m_B^6 \lambda^2 |B_T|^2 \text{Re}(C_T C_T^*)
- \frac{8}{3r} m_B^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_1 + D_1)(B_T C_T^*)]
- \frac{16}{3r} m_B^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_2 + D_2)(B_6 C_T^*)]
- \frac{128}{3r} m_B^4 m_\ell \lambda (1 - r - s) \text{Re}(B_6 B_7^*) \text{Re}(C_T C_T^*)
+ \frac{16}{3r} m_\ell (\lambda + 12rs) \text{Re}[(B_1 + D_1)(B_6 C_T^*)]
+ \frac{128}{3r} m_B^2 m_\ell \lambda (1 + 12rs) \text{Re}(B_6 C_T^*)
+ \frac{512}{3r} m_B^2 (\lambda(4r + s) + 12r(1 - r)^2) |T_1|^2 \text{Re}(C_T C_T^*)
$$

(B-5)

The combined longitudinal polarization $P_L^- + P_L^+$, from (3-1):

$$
P_L^- + P_L^+ = \frac{4}{\Delta} m_B^2 \lambda \text{Im}[(B_2^* B_4) + (D_2^* B_3)]
- \frac{2}{r} m_B^2 m_\ell \lambda (1 - r) \text{Re}[(B_2 - D_2)(B_4^* + B_3^*)]
- \frac{1}{r} m_\ell (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)]
- \frac{1}{r} (1 - r - s) \text{Im}[(B_2 - D_2)(B_1^* - D_1^*)]
- \frac{1}{r} m_\ell (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)]
+ \frac{8}{3r} m_B^4 m_\ell \lambda^2 \text{Re}[(B_2 + D_2)(B_7 C_T)]
+ \frac{32}{3r} m_B^6 \lambda^2 |B_T|^2 \text{Re}(C_T C_T^*)
- \frac{8}{3r} m_B^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_1 + D_1)(B_T C_T^*)]
- \frac{16}{3r} m_B^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_2 + D_2)(B_6 C_T^*)]
- \frac{128}{3r} m_B^4 m_\ell \lambda (1 - r - s) \text{Re}(B_6 B_7^*) \text{Re}(C_T C_T^*)
+ \frac{16}{3r} m_\ell (\lambda + 12rs) \text{Re}[(B_1 + D_1)(B_6 C_T^*)]
+ \frac{128}{3r} m_B^2 m_\ell \lambda (1 + 12rs) \text{Re}(B_6 C_T^*)
+ \frac{512}{3r} m_B^2 (\lambda(4r + s) + 12r(1 - r)^2) |T_1|^2 \text{Re}(C_T C_T^*)
$$

(B-5)
\(- \frac{512}{3r} m_B^2 s \left[ \lambda + 12r(1 - r) \right] \text{Re}(T_1 B_6^*) \text{Re}(C_T C_T^*) \)
\(+ \frac{256}{3r} m_B^2 s \lambda (1 + 3r - s) \text{Re}(T_1 B_7^*) \text{Re}(C_T C_T^*) \)
\(+ \frac{512}{3} m_B^2 m_\ell \lambda \text{Re}[(A_1 + C_1)(T_1 C_T)^*] \)
\(- \frac{32}{3r} m_\ell \lambda \text{Re}[(B_1 + D_1)(T_1 C_T)^*] \)
\(+ \frac{32}{3r} m_B^2 m_\ell (1 + 3r - s) \text{Re}[(B_2 + D_2)(T_1 C_T)^*] \) \right\}. \quad (B-6)

The combined transversal polarization \(P_T^- - P_T^+\), from (B-2) and (B-3):

\[
P_T^- - P_T^+ = \frac{\pi}{\Delta} m_B s \sqrt{s\lambda} \left( \frac{2}{rs} m_B^2 m_\ell (1 - r) \lambda \left[ |B_2|^2 - |D_2|^2 \right] \right.
\+ \frac{1}{r} m_B^4 \lambda \text{Re}[(B_2 + D_2)(B_4^* - B_5^*)] \)
\+ \frac{2}{r} m_B^4 m_\ell \lambda \text{Re}[(B_2 + D_2)(B_3^* - D_3^*)] \)
\+ \frac{2}{r} m_B^2 m_\ell (1 + 3r - s) \left[ \text{Re}(B_1 D_2^*) - \text{Re}(B_2 D_1^*) \right] \)
\+ \frac{2}{rs} m_\ell (1 - r - s) \left[ |B_1|^2 - |D_1|^2 \right] \)
\- \frac{1}{r} m_B^2 (1 - r - s) \text{Re}[(B_1 + D_1)(B_4^* - B_5^*)] \)
\- \frac{2}{r} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_1 + D_1)(B_3^* - D_3^*)] \)
\- \frac{2}{rs} m_B^2 m_\ell [(1 + r)(1 - r - s)] \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right] \)
\- \frac{32}{rs} m_B^2 m_\ell \lambda \text{Re}[(B_1 - D_1)(B_7 C_T)^*] \)
\+ \frac{32}{rs} m_B^4 m_\ell \lambda (1 - r) \text{Re}[(B_2 - D_2)(B_7 C_T)^*] \)
\+ \frac{16}{r} m_B^4 m_\ell \lambda \text{Re}[(B_4 - B_5)(B_7 C_T)^*] \)
\+ \frac{32}{r} m_B^4 m_\ell \lambda \text{Re}[(B_3 - D_3)(B_7 C_T)^*] \)
\+ \frac{64}{rs} m_B^2 (1 - r - s) \text{Re}[(B_1 - D_1)(B_6 C_T)^*] \)
\- \frac{64}{rs} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_2 - D_2)(B_6 C_T)^*] \)
\- \frac{32}{r} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_4 - B_5)(B_6 C_T)^*] \)
\- \frac{64}{r} m_B^2 m_\ell (1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_T)^*] \)
\+ 32 m_B^4 s v^2 \text{Re}[(A_1 - C_1)(B_6 C_T)^*] \)

17
\[ \begin{align*}
&+ \frac{64}{\ell} \ell (1 + 3r - s) \text{Re}[(B_4 - B_5)(T_1C_{TE})^*] \\
&- 64m_B^2(1 - r)v^2 \text{Re}[(A_1 - C_1)(T_1C_T)^*] \\
&+ \frac{128}{rs} [m_B^2 m(1 + 7r - s)] \text{Re}[(B_1 - D_1)(T_1C_{TE})^*] \\
&+ \frac{128}{rs} [m_B^2 m^2(1 - r)(1 + 3r - s) \text{Re}[(B_2 - D_2)(T_1C_{TE})^*] \\
&+ \frac{128}{r} m_B^2 m^2(1 + 3r - s) \text{Re}[(B_3 - D_3)(T_1C_{TE})^*] \bigg) \\
\end{align*} \] (B-7)

The combined normal polarization \( P_N^+ + P_N^- \), from (B-4) and (B-5):

\[ \begin{align*}
P_N^- + P_N^+ &= \frac{1}{\Delta} \pi v m_B^3 \sqrt{s \lambda} \bigg\{ - \frac{2}{\ell} \ell (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
&- \frac{2}{\ell} \ell (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
&- \frac{1}{\ell} (1 - r - s) \text{Im}[(B_1 - D_1)(B_4^* - B_5^*)] \\
&+ \frac{2}{\ell} m_B^2 \ell \lambda \text{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
&+ \frac{1}{\ell} m_B^2 \lambda \text{Im}[(B_2 - D_2)(B_4^* - B_5^*)] \\
&+ 32m_B^2 s \text{Im}[(A_1 + C_1)(B_6C_T)^*] \\
&+ 1024m_\ell \left[ |C_T|^2 - |4C_{TE}|^2 \right] \text{Im}(B_6^* T_1) \\
&- 64m_B^2 (1 - r) \text{Im}[(A_1 + C_1)(T_1C_T)^*] \\
&+ 128 \text{Im}[(B_1 + D_1)(T_1C_{TE})^*] \bigg\} \] (B-8)
References

[1] CLEO Collaboration, M. S. Alam, et. al., Phys. Rev. Lett. 74, 2885 (1995)

[2] M. Artuso, et. al. hep-ph/0801.1833v1

[3] A. Ali, Int. J. Mod. Phys. A20 (2005) 5080.

[4] P. Ball, W. M. Braun, Phys. Rev D58 (1998) 094016.

[5] P. Ball, R. Zwicky, Phys. Rev D71 (2005) 014029.

[6] Y. L. Wu, M. Zhong, Y.B. Zuo, Int. J. Mod. Phys. A21, (2006) 6125.

[7] D. Melikhov, B. Stech, Phys. Rev. D62 (2000) 014006.

[8] A. Deandrea, A. D. Polosa, Phys. Rev D64 (2001) 074012.

[9] C. Q. Geng, C. C. Liu, J. Phys. G29 (2003) 1103.

[10] C. Q. Geng, C. P. Kao, Phys. Rev. D54 (1996) 5636.

[11] D. Melikhov, N. Nikitin, S. Simula, Phys. Lett. B430 (1998) 332.

[12] T. M. Aliev, M. Savci, Phys. Lett. B481 (2000) 275.

[13] T. M. Aliev, M. K. Cakmak, A. Ozpineci, M. Savci, Phys. Rev. D64 (2001) 055007.

[14] T. M. Aliev, M. K. Cakmak, M. Savci, Phys. Nucl. Phys. B607 (2001) 3005.

[15] S. Rai Choudhury, N. Gaur, N. Mahajan, Phys. Rev D66 (2002) 054003.

[16] T. M. Aliev, A. Ozpineci, M. Savci, Phys. Rev D67 (2003) 035007

[17] A. S. Cornell, N. Gaur, JHEP, 0502:005 (2005).

[18] U. O. Yilmaz, G. Turan, Eur. Phys. J. C51, 63 (2007)

[19] G. Erkol, G. Turan, Eur. Phys. J. C25 (2002) 575.

[20] R. Mohanta, A. K. Giri, Phys. Rev D75 (2007) 035008.

[21] CDF Collaboratio, D. Acosta, et. al., Phys. Rev. D65 (2002) 111101.

[22] DO Collaboration, V. M. Abazov, et. al., Phys. Rev. D74 (2006) 031107.

[23] CDF Collaboration, T. Aaltonen, et. al., hep-ph/0804.3908v1.

[24] A. Tayduganov, V. Egorychev, A. Golutvin, I. Belyaev, CERN-LHCb-2007-154.

[25] M. P. Altarelli, F. Teubert, hep-ph/0802.1901.

[26] S. Fukae, C. S. Kim, T. Yoshikawa, Phys. Rev. D61 (2000) 074015.
[27] T. M. Aliev, V. Bashiry, M. Savci, JHEP, 0505:037 (2005).

[28] F. Krüger, L. M. Sehgal, Phys. Lett. B380 (1996) 199.

[29] W. -M. Yao, et. al. (Particle Data Group), J. Phys. G33 (2006) 1.

[30] A. J. Buras, M. Münz, Phys. Rev. D52 (1995) 186

[31] M. Misiak, Nucl. Phys., B393 (1993) 23.

[32] M. Misiak, Nucl. Phys. B439 (1995) 461 [Erratum].

[33] A. Ali, T. Mannel, T. Morozumi, Phys. Lett. B273 (1991) 505;

[34] BELLE Collaboration, A. Ishikawa, et al., Phys. Rev. Lett. 91, 261601 (2003)

[35] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 91, 221802 (2003)

[36] CDF Collaboration, B. Abulencia, et al., Phys. Rev. Lett. 95, 221805 (2005)
Figure 1: The dependence of the integrated branching ratio for the $B_s \rightarrow \phi \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 2: The dependence of the integrated branching ratio for the $B_s \rightarrow \phi \tau^+ \tau^-$ decay on the new Wilson coefficients.
Figure 3: The dependence of the averaged longitudinal polarization $< P^-_L >$ of $\ell^-$ for the $B_s \to \phi \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 4: The dependence of the combined averaged longitudinal lepton polarization $< P^-_L + P^+_L >$ for the $B_s \to \phi \mu^+ \mu^-$ decay on the new Wilson coefficients.
Figure 5: The same as Fig. (3), but for the $B_s \to \phi \tau^+ \tau^-$ decay.

Figure 6: The same as Fig. (4), but for the $B_s \to \phi \tau^+ \tau^-$ decay.
Figure 7: The dependence of the averaged transverse polarization $< P_T^- >$ of $\ell^-$ for the $B_s \rightarrow \phi \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 8: The dependence of the combined averaged transverse lepton polarization $< P_T^- - P_T^+ >$ for the $B_s \rightarrow \phi \mu^+ \mu^-$ decay on the new Wilson coefficients.
Figure 9: The same as Fig. (7), but for the $B_s \to \phi \tau^+ \tau^-$ decay.

Figure 10: The same as Fig. (8), but for the $B_s \to \phi \tau^+ \tau^-$ decay.
Figure 11: The dependence of the averaged normal polarization $<P_N^->$ of $\ell^-$ for the $B_s \to \phi \mu^+\mu^-$ decay on the new Wilson coefficients.

Figure 12: The dependence of the combined averaged normal lepton polarization $<P_N^+ + P_N^->$ for the $B_s \to \phi \mu^+\mu^-$ decay on the new Wilson coefficients.
Figure 13: The same as Fig. (11), but for the $B_s \to \phi \tau^+\tau^-$ decay.

Figure 14: The same as Fig. (12), but for the $B_s \to \phi \tau^+\tau^-$ decay.