Effect of Decoherence on the Dynamics of Bose-Einstein Condensates in a Double-well Potential

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Bose-Einstein condensates (BECs) in a double-well potential exhibit many fascinating phenomena that are absent in thermal atomic ensembles, for example, quantum tunneling and self-trapping\cite{1,2,3,4,5,6}. Quantum tunneling through a barrier is a paradigm of quantum mechanics and usually takes place on a nanoscopic scale, such as in two superconductors separated by a thin insulator\cite{7} and two reservoirs of superfluid helium connected by nanoscopic apertures\cite{8,9}. Recently, tunneling on a macroscopic scale (\(\mu m\)) in two weakly linked Bose-Einstein condensates in a double-well potential has been observed\cite{10}. Similar to tunneling oscillations in superconducting and superfluid Josephson junctions, Josephson oscillations are observed when the initial population difference of the BECs is chosen to be below the critical value. When the initial population difference exceeds a critical value, the system undergoes a quantum transition from the superfluid phase to the normal phase. This phenomenon is known as macroscopic quantum self-trapping.

The interactions between the condensate and noncondensate atoms lead to decoherence. Describing decoherence by fully including the quantum effects requires sophisticated theoretical studies that include the effect of noncondensate atoms. Treating the noncondensate atoms as a Markovian reservoir, master equations that govern the dynamics of the condensate atoms might be derived\cite{11,12,13}. In fact, in the experiments on BECs, trapped atoms are evaporatively cooled and they continuously exchange particles with the noncondensate atoms. Thus standard procedure in quantum optics for open systems would naturally lead to master equations for treating atomic BECs. This Markovian treatment for the BECs also can be understood as the presence of lasers for trapping/detection of atoms, which will polarize the atoms and thus couple them to the vacuum modes of the electromagnetic field\cite{14}. On the other hand, due to the unavoidable interaction of the BECs with its environment, the decoherence is always there in BECs, hence the characterization of decoherence in this system becomes interesting. Because different decoherence may have different effects on the dynamics of the BECs, the character of decoherence in the BECs may be read out from the dynamics of the BECs. Indeed, as we shall show, different BEC-environment coupling leads to different final population imbalance of the BECs in the double-well potential. This may be used to characterize the decoherence in the double-well systems.

In this paper, we study the effect of decoherence on the dynamics of BEC in a double-well potential by studying the evolution of the master equation for the BEC within a mean-field framework, where the number of atoms in the condensates is supposed to be infinity and the quantum fluctuation is negligible. To derive the master equation, we need modeling the environment and BEC-environment coupling. However, this is not an easy task that we do not address at present. Instead, we write the master equation by analyzing the effects of environment-induced decoherence. When analyzing decoherence effects on the dynamics of BECs in a double-well potential, we are interested in answering two basic questions: (1) What effects are made by the decoherence on self-trapping in the BECs? And (2) how does the decoherence affect the quantum tunneling in BECs in a double-well potential?

Consider BECs in a double-well potential, the wave function of BECs can be expressed as the superposition of individual wave functions in each well,\footnote{Electronic address: yixx@dlut.edu.cn}

\begin{equation}
|\phi\rangle = a_R|R\rangle + a_L|L\rangle, \quad (1)
\end{equation}

where \(|R\rangle\) and \(|L\rangle\) denote the wavefunction of the right and left well, respectively. The coefficients \(a_R\) and \(a_L\) of the expansion satisfy the two-mode Gross-Pitaevskii
The Hamiltonian is given by

$$H = \left( \frac{\gamma}{2} + \frac{2}{V} (|a_R|^2 - |a_L|^2) \right) - \frac{\gamma}{2} - \frac{\Gamma}{2} (|a_R|^2 - |a_L|^2), \quad (3)$$

where $\gamma$ is the energy bias between the two wells, $c$ stands for the nonlinear parameter describing the condensate self-interaction, and $V$ depending on the height of the barrier, is the coupling constant between the two condensates. In this paper, we shall focus on $\gamma = 0$, i.e., the case of BECs in a symmetric double-well potential. This situation is interesting because the amplitude distributions of all eigenstates are symmetric, leading to Josephson oscillations in the absence of decoherence. With the Markov approximation, the master equation that results from the condensate-environment coupling takes the form,

$$i \frac{\partial}{\partial t} \rho = [H_{\rho}, \rho] + \mathcal{L}(\rho),$$

$$\mathcal{L}(\rho) = i \frac{\Gamma}{2} (2A\rho A^\dagger - \rho A^\dagger A - A^\dagger A \rho), \quad (4)$$

where $\Gamma$ denotes the decoherence rate of the condensates, and $A$ stands for an operator of the condensates. This master equation can be derived by assuming that the condensates-environment couplings take the form $H_{1} \sim \sum_{j} g_{j} (A_{j}^{\dagger} + h.c.)$, where $g_{j}$ denotes the constant of interaction between the condensates and the environmental mode $b_{j}$. The condensate operator $A$ in general is expressed as a linear superposition of three pauli operators, i.e., $A = \lambda_{x} \sigma_{x} + \lambda_{y} \sigma_{y} + \lambda_{z} \sigma_{z}$ with notations $\sigma_{x} = |R\rangle \langle R| - |L\rangle \langle L|$, $\sigma_{y} = |R\rangle \langle L| + |L\rangle \langle R|$, and it is similar for $\sigma_{y}$. The values of $\lambda_{y}(\alpha = x, y, z)$ depends on the source of decoherence and its couplings to the environment. For example, $\lambda_{y} = \lambda_{x} = 0$ is for the environment that dephasingly couples to the condensates, while $\lambda_{z} = 0$ is for the environment leading the BECs into dissipation. $H_{1}$ takes the same form as in Eq. (3), except a change $|a_{x}|^{2} \rightarrow \rho_{xx} = \langle x|\rho|x\rangle, x = R, L$.

To start with, we consider the case of $A = \sigma_{+} = \sigma_{x} + i\sigma_{y}$. This situation happens in the case where the double-well potential is formed by using a Raman scheme to couple two hyperfine states in a spinor BEC. The condensate in the upper hyperfine states decays into the lower one, reminiscent of atomic spontaneous emission. The dynamics of the master equation is studied by numerical simulations, the results are presented in Fig. 1 and Fig. 2. In Fig. 1-(a) and 1-(b), we have plotted the population of condensates in the Left well 1-(a) and Right well 1-(b). The initial state is all the condensate atoms in the Right well, and the decoherence rate has been set to be $\Gamma = 0.1V$. In contrast, the dynamics of the condensate in the Left well without decoherence ($\Gamma = 0$) is presented in Fig. 1-(c). Clearly, the decoherence increase the damping of the oscillations. When the nonlinearity characterized by $c$ is small compared to the tunneling $V$, the oscillations of the population are suppressed, and the condensate finally remains in the two wells with equal probability. If the nonlinearity is large with respect to the tunneling $V$ and the population imbalance exceeds a critical value, the condensate would be locked in one of
the wells, depending on the initial population. We would like to notice that the population change drastically in the vicinity of the critical value \( c = 2V \), this is due to the suppression of population oscillations by the decoherence. With decoherence increasing, the jump-like change near the critical value in the population becomes unclear, as Fig. 2 shows. That means the decoherence may determine the final population imbalance in the two wells. On the other hand, the nonlinear interaction together with the initial population and the relative phase can affect the decoherence effect, which may be characterized by \(|\rho_{12}|\), i.e., the norm of the off-diagonal element of the density matrix. This effect was shown in figure 3. We would like to note that the jump-like change in Fig. 1 might appear at different \( c \), depending on the fixed points around that the population imbalance and the relative phase oscillate. For example, our simulations show that the jump-like change could appear at \( c = V \) with initial relative phase \( \theta = \pi \), and non-zero population imbalance [6].

Next, we take \( A = \sigma_x \), corresponding to BECs in a spatial double-well potential. The tunneling is driven by an environment (or by fluctuational fields), leading to the decay in the quantum tunneling. An alternative BEC system can be formed by using a Raman scheme to couple two degenerate hyperfine states in a spinor BEC. The driving fields may fluctuate, resulting in decoherence in the quantum tunneling. We have performed extensive numerical simulations for the master equation Eq. (1). Selected results, divided into three regimes by the nonlinearity, are presented in Fig. 4, 5, and 6. Fig. 4 shows the dynamics of the condensate in the self-trapping regime. The decoherence clearly increases the amplitude of oscillations in the population first, then increases the damping of the amplitude of oscillations, meanwhile it averagely decreases the population imbalance, and finally spoils the self-trapping. In the self-trapping regime, the frequency of the oscillation depends on the nonlinear parameter \( c \), the initial population imbalance and relative phase, as well as the coupling constant \( V \) between the BECs. This can be found by comparing Fig. 4(a) and 5. With \( c \) and \( V \) fixed, the decoherence changes the population imbalance, this results in the frequency change as shown in Fig. 4(a). In the quantum tunneling regime (Fig. 5), the decoherence increases the damping of the oscillation as expected. And at last, in Fig. 6 we have plotted the dynamics of the condensate in the regime between the quantum tunneling and self-trapping. We see that the decoherence increases the tunneling at the beginning of evolution, and then destroys the quantum tunneling/self-trapping after a few cycle of evolution.

In summary, we have studied the dynamics of Bose-Einstein condensate in a double-well potential. The dynamics is govern by the master equations with the condensate-environment coupling. Two kinds of decoherence characterized by \( \sigma_z \) and \( \sigma_x \) are considered. By numerically solving the master equation, we show that there is a
FIG. 6: This figure shows the population of the condensate at the critical value $c = 2V$. (a) is plotted for $\Gamma = 0.01V$, while (b) is for $\Gamma = 0$. It confirms that the decoherence first leads the BEC from the self-trapping regime to the quantum tunneling regime, then spoils the quantum tunneling. This work was supported by EYTP of M.O.E, NSF of China under Grant No. 60578014.

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