Beam Splitters, Interferometers and Hong-Ou-Mandel Effect for Interacting Bosonic and Fermionic Walkers in a Lattice

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Introduction:– Beam splitters (BS) acting on two modes are indispensable tools for both fundamental investigations and practical applications. They enable one to design simple two output interferometers such as the Mach-Zhender, and indeed can be composed together to perform arbitrary unitary operations on an arbitrary number of input modes. A 50-50 BS enables one to observe bosonic behavior of two incident particles in the most striking way through the Hong-Ou-Mandel effect where the probability of one photon in each output is completely suppressed. The same types of effects form the bedrock of linear optics quantum computation, which is currently performed with photons, though its implementation with free bosonic atoms has also been suggested. While a BS opens up unprecedented opportunities, it generally requires mobile entities to be incident on it, i.e., the modes between which it acts are generally distinct momenta modes. The tunnel coupling Hamiltonian between two potential wells is, of course, the same as that of a BS, but it requires the wells to be really adjacent (directly coupled). Can we create an effective BS between distant sites in a lattice? This would be really interesting as one would then be able to replicate the whole methodology of linear optics for distant trapped particles (stationary and measurable both “before” and “after” the linear optics operation), including interesting phenomena shown by mobile particles such as interferometry and two-mode bunching and anti bunching statistics. These phenomena can be spotted in generic two particle quantum walks in a dispersive “multi-mode” (multi-site) manner, whereas a neat two mode demonstration as in photonic quantum optics would be both foundationally interesting, topical and more adaptable to applications.

Recently, the dynamics of two interacting particles in a one dimension has been studied experimentally and interesting phenomena have been observed, also opening up new theory in interacting nonequilibrium dynamics. Inspired by these, in this paper we show how to implement linear optics via the dynamics of trapped neutral atoms interacting via the Bose-Hubbard Hamiltonian. Free space evolution is mapped into a hopping problem on the lattice. When a defect is introduced into the model, we find that it is possible to realise BS (see e.g. Fig. 1 for a scheme showing the implementation of a beam splitter) and phase shifters, which are the building blocks of linear optics. This enables us to study the two-mode Hong-Ou-Mandel effect between atoms at distant lattice sites. As in [11], our method is more suited for practical applications, being based on effective two-mode beam splitters. We show how to implement a Mach-Zehnder interferometer and how to overcome the wave-packet dispersion. Finally we discuss how our results may lead to a neutral atom quantum computer even without tweezers or interactions.

Effective 50/50 beam splitter in 1D:– As a paradigmatic model we use the Bose-Hubbard Hamiltonian which in one dimension reads

$$H = -\frac{J}{2} \sum_{j=1}^{L-1} \left[ a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j + h.c. \right] + \sum_{j=1}^{L} \left[ \frac{U}{2} n_j (n_j - 1) - \mu n_j \right],$$

being $a_j^\dagger$ ($a_j$) the boson creation (annihilation) operators, $n_j = a_j^\dagger a_j$ the number operator, $L$ the lattice sites, $J$ the tunneling rate, $U$ the on-site interaction and $\mu$ the chemical potential. In the fermionic case we model the system as in [11] with the operators $a_j$ substituted by their fermionic counterparts and $U=0$. By varying the ratio $U/J$ the system undergoes a quantum phase transition to the Mott insulator phase.
where the number of particles per lattice site is fixed to a constant value that depends on the parameters \((J, U, \mu)\). Once in the Mott insulating phase the system can be initialized via single site addressing \(^{13}\) in a state with a few localized particles, and the dynamics of a single traveling particle \(^{19}\) and of two (interacting) particles \(^{3}\) can be observed.

In this Letter we show how to encode linear optics in the dynamics of bosons or fermions in an optical lattice. All linear optics operations can be performed with beam splitters, phase shifters and mirrors \(^{1}\). A beam splitter can be implemented by introducing a localized optical potential in the chain that scatters an incoming particle (see Fig. \(1\)). We consider an odd chain \((L=2N+1)\) with a local potential on site \(N+1\) which leads to an impurity in the chemical potential \(^{20}\): 

\[
\mu_{j} = \mu + J\beta \delta_{N+1,j}.
\]

When the number of particles is fixed, the constant term \(\mu\) only gives rise to an irrelevant global phase. Therefore, for simplicity we set \(\mu=0\). On the other hand, the potential barrier \(\beta\) favors the splitting of an incoming particle into a transmitted and a reflected component. To define an effective beam splitter we consider the dynamics of a particle initially located on site 1 and we define the transmission and reflection coefficients \((T\) and \(R\) respectively) as 

\[
T(t) = \langle 0 | a_{1} e^{-i\mu t} | a_{1}^\dagger | 0 \rangle, \quad R(t) = \langle 0 | a_{1} e^{-i\mu t} | a_{1}^\dagger | 0 \rangle, \quad \text{being} \ | 0 \rangle \text{the vacuum state.} \]

\(R(t)\) represents the probability amplitude that the particle returns to site 1 after time \(t\); \(T(t)\) on the other hands is the probability amplitude that that particle reaches the opposite end site \(L\) on time \(t\). Due to the symmetries of the system, the same coefficients also describe the case of a particle initially located on the end site \(L\). These initial locations were chosen to force the particles to travel on a single direction, namely towards the defect and ultimately towards the other end. Analytical expressions for \(T\) and \(R\) are obtained (see Supplementary Material) by exploiting the theory presented in \(^{21}\). For the relevant values of \(\beta\) we have proved that the transmission time \(t^*\), where \(T(t)\) and/or \(R(t)\) displays their first maximum, do not depend on \(\beta\). Therefore \(t^*\) coincides with the transmission time of a traveling particle to the opposite end when \(\beta=0\), \(t^*=2(N+1)+1.02(N+1)^{1/3}/J\) (see e.g. \(^{22}\)). However, due to the non-linear dispersion relation of the model, the wavefunction is not perfectly reconstructed and there is some probability to find the particle far from the ends at \(t^*\).

Via the coefficients \(R\) and \(T\) we define an effective beam splitter operator whose input ports are sites 1 and \(L\) at time 0, and whose output ports are the same sites at time \(t^*\):

\[
S(t^*) = \begin{pmatrix} R(t^*) & T(t^*) \\ T(t^*) & R(t^*) \end{pmatrix} \approx \text{diag} \left( \frac{\beta}{\nu_{\text{eff}}} , \frac{\nu_{\text{eff}}}{\beta} \right) + \mathcal{O}(L^{-1}),
\]

where the second equality holds in the limit \(L\to\infty\). The effective beam splitter operator Eq. \((2)\) is the product of a damping factor \(D=\mathcal{O}(L^{-1/3})\) and a unitary matrix \(S=S(t^*)/D\). The operator \(S(t^*)\) is non-unitary because, due to wave-packet dispersion, there is a non-zero probability that the wave-packet at \(t^*\) is not perfectly reconstructed at the boundaries. As shown hereinafter the condition \(|D|=1\) can be obtained with a further engineering of the interactions which avoids wave-packet dispersion. Eq. \((2)\) quantifies the amount of the splitting of traveling particle into a transmitted and reflected component. For \(\beta\to0\) there is just the transmitted component, whereas for \(\beta\to\infty\) only the reflected component is non-zero. On the other hand, a 50/50 beam splitter is implemented when \(\beta=\beta^{(50/50)}=1\). For finite \(L\) there is a \(\mathcal{O}(L^{-2/3})\) correction \(^{23}\) to the value of \(\beta^{(50/50)}\) which is therefore obtained numerically by imposing \(|R(t^*)|=|T(t^*)|\). Typically \(\beta^{(50/50)} \ll 1\). For every \(\beta\geq0\) there is a phase shift \(-\pi/2\) between \(T(t^*)\) and \(R(t^*)\). The dynamics of \(R(t)\) and \(T(t)\) in the 50/50 regime is shown in Fig. \(2\) when \(L=51\). With these parameters, we observe that \(D=64\%\).

**Hong-Ou-Mandel effect:** We now consider the evolution generated by two traveling particles initially located on the boundary sites 1 and \(L\) when there is an optical defect as in Fig. \(1\). The evolution is described in the Schrödinger picture by the state \(|\Psi(t)\rangle = \sum_{j,k} A_{jk}(t) |a_{j}^\dagger a_{k}^\dagger | 0 \rangle\). The amplitudes \(A_{jk}(t)\) evolve through the Schrödinger equation as \(i\dot{A}_{jk}(t) = \sum_{m,n} K_{jk,mn} A_{mn}(t)\), where \(A_{jk}(0)=\delta_{j,1}\delta_{L,k}\) and \(K_{jk,mn}\) is obtained from \(^{1}\) via the algebra of commutation relations \(^{23}\). We consider \(A_{j,k}\) as a matrix where \(A_{j,k}=0\) for \(j>k\) (\(j\leq k\)) for bosons (fermions). In the limit \(U\to\infty\) the Bose-Hubbard Hamiltonian \(^{1}\) describes hard-core bosons and the system is equivalent to a spin \(1/2\) chain with XX interactions. In this limit we found the superoperator \(K\) to coincide with that of the fermionic chain. Therefore, the boson dynamics in the hard-core regime is indistinguishable from that of the fermions. We study the evolution of the observables \(P_{jk}^{b,f}(t) = \langle \Psi(t) | a_{j}^\dagger a_{k}^\dagger a_{j} a_{k} | \Psi(t) \rangle = |A_{jk}(t)|^{2}\), which represent the probability of the two particles being on sites \(j,k\) \((j\leq k)\). We use the superscript \(b/f\) to explicitly denote the bosonic/fermionic case. When \(U=0\) it is \(P_{jk}^{b}(t) = |T_{j}^{b}(t)|^{2} R^{b}(t)^{2}\), \(P_{11}^{f}(t) = P_{L,L}^{f}(t) = 4|T(0)R(t)|^{2}\). Therefore, owing to the explicit expression or the effective beam splitter operator \(^{2}\), one obtains perfect bunching (anti-bunching) with bosons (fermions) at time \(t^*\) when \(\beta=\beta^{(50/50)}\). In Fig. \(3\) we show that, although this effect is damped (via the factor \(D\)) because of the dispersive transmission along the chain, it is still observable when one is able to measure \(P_{j,k}(t)\) as \(^{3,19}\). To cancel possible boundary effects, \(P_{j,k}(t)\) is plotted for \(t=0.75t^*\). As shown in Fig. \(4\) bosons bunch while fermions and hard-

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**FIG. 2.** \(R(t)\) and \(T(t)\) as function of time for \(L=51\) and \(\beta=\beta^{(50/50)}=0.95\). The figure shows the amplitude of \(R\) and \(T\) while the inset shows their relative phase. The obtained transfer time is \(t^*=55\).
core bosons (HCB) anti-bunch. A finite $U$ reduces the probability to have two bosons on the same site and one expects a transition from bunching to anti-bunching when $U$ goes from 0 to $\infty$. We found that $P_{\beta}$ is almost indistinguishable from the hard-core boson case when $U \geq 10$.

To analyze in detail the transition from bosons to hard-core bosons when $U$ increases we study the probability amplitude to have two bosons in the last site of the lattice $|A_{LL}(t')\rangle$ for several values of $\beta$. For fixed $U$, $t'$ and $\beta_{opt}$, are found numerically by maximizing $|A_{LL}(t')\rangle$ around $t'-L/J$ and $\beta=1$. As shown in the inset of Fig. 4 the observed optimal $\beta$ for different $U$ is approximately equal to the value $\beta_{50/50}$ obtained when $U=0$. In Fig. 4 we report the results obtained for the maximum of $|A_{LL}\rangle$ for several values of $U$ and $L$, normalized with respect to the $U=0$ case. We see that for $U\leq 0.1$ the bunching effect is unaffected while for $U\geq 1$ a power law decay occurs and that there is a rapid change of behavior near $U=1$. The threshold value $U_c$ separating the two regions is obtained by fitting the data in the power law region and taking the intersection value with the unit constant line. For $L=51$ in particular we estimate $U_c \approx 1.33$. The estimated $U^c$ is surprisingly similar to the critical value of the Mott insulator transition $[24]$. This numerical evidence raises interesting questions on the possibility to detect the Mott phase transition via specific features of the chain’s non-equilibrium evolution.

**Mach-Zehnder output patterns:** All the three optical elements which form a Mach-Zehnder interferometer can be obtained in a lattice: the beam splitter has been discussed in this Letter, the mirrors are implemented by boundary reflections and a phase shift can be obtained by freezing the hopping so that the system acquires a phase because of the chemical/optical potential. However, in view of an experimental application, it is more compelling to devise a scheme which minimizes the dynamical control on the chain, and in particular which does not require to switch on and off some interactions to implement different effective operations.

Let us consider again the scheme of Fig. 1 where the effective optical operation is obtained via the unmodulated dynamics of the system and where the input and output ports of the optical operation are the sites 1 and $L$ at time 0 (input) and time $t'$ (output). The combined action of a beam splitter and a local phase-shifter can be achieved by applying a different optical potential to the right half of the chain: $\mu_j=J\gamma_R$ for $j>N+1$, being again $\mu_{N+1}=\beta\beta_{50/50}$, and $\mu_j=0$, $j\leq N$. The idea is that when $\gamma_R \neq 0$ a particle acquires a tunable phase $\phi \gamma_R$ when it travels in the right part of the chain. With some numerical experiments we found that an effective 50/50 beam splitter can be obtained by tuning $\beta$ also when $\gamma_R \neq 0$: it is $\beta_{50/50} \approx 1$ with some corrections with respect to the $\gamma_R=0$ case.

As in the $\gamma_R=0$ case, the scattering matrix can be approximately factorized as $S(t')=D \tilde{S}(t')$, being $D$ a damping factor and $\tilde{S}$ a unitary matrix. In the 50/50 case the latter is found to be $\tilde{S}_{\phi}=\frac{1}{\sqrt{2}}\left(\begin{array}{cc} 1 & -ie^{i\phi} \\ -ie^{-i\phi} & e^{i\phi} \end{array}\right)$ where $\phi=\gamma_R t'/2$. It turns out that $S_{\phi}$ is is a composition of a 50/50 beam splitter and two phase shifts. By applying the unitary matrix $\tilde{S}_\phi$ twice one obtains an optical operation which is equivalent (up to local phase shifts) to a Mach-Zehnder unitary operation $[23]$ (with phase $2\phi$). In other terms, the chain evolution after a time $2t'$ acts on the boundary sites as a Mach-Zehnder interferometer: $S(2t')=D_2 \tilde{S}_\phi^2$, where $D_2 \approx 1.2D$.

**Engineered chains for perfect operations:** In the results presented so far we consider an unmodulated chain with a constant hopping term. The drawback, for applications, is that one have to face dispersion as an unavoidable effect: due to the non-constant group velocity the particle is not perfectly transferred (say) from site 1 to site $L$ or back. This effect yields a damping factor which limits the applicability of our scheme as a building block for scalable linear optics in a lattice. However, we now show that the damping factor can be almost completely removed by exploiting perfect state transfer strategies (see $[26]$ for a review) where the dynamics is dispersionless. Here we consider the paradigm of perfect state transfer between the ends of the chain, i.e., the Hamiltonian with engineered hopping rates $J_n=2J \sqrt{n(L-n)}$ between sites $n$ and $n+1$ $[27]$. In principle, a coupling pattern $J_n$ can be obtained...
via suitably configured laser intensity profiles \cite{28}.

When an impurity $\beta$ is introduced in the central site of an engineered chain, an incoming wave-packet is split into a transmitted and reflected component. We found that the dependence on $\beta$ shown in Eq. (2) is recovered. Even though the chain’s perfect transmission capabilities are reduced by the impurity, we still obtain an almost dispersionless dynamics where $|D|=1$ (e.g. $D=0.989$ for $L=51$). In particular, when $\beta \approx 1$ an almost perfect 50/50 beam splitter is obtained. Similarly, when the right part of the chain is subject to a different optical potential $\gamma_R$, then the dynamics after time $2t^*$ implements an almost perfect Mach-Zehnder. An interesting application of the latter equivalence is that, as in the Mach-Zehnder interferometer \cite{25}, by properly tuning the phase $\phi$ via the parameter $\gamma_R$ one can vary the probability outcomes on the two output ports. Indeed, acting on $\phi$ one can tune the probability that a particle traveling from site 1 goes to site 1 or $L$ after time $2t^*$. This effect is shown in Fig. 5.

**Imperfections:** We discuss how possible imperfections in real experiments could affect our theoretical results. We focus on the implementation of a beam splitter with an unmodulated chain. Firstly we consider a non perfectly localized optical impurity in the form of a Gaussian field $\beta_j = \beta^{0/50} \exp[-(N+1-j)^2/\sigma^2]$ varying $\sigma$ for a chain with $L \in \{21, 31, 41, 51\}$. We find that $(|R|-|T|)/R$ is unaffected compared to the ideal case for FWHM$\leq 0.66a$, being $a$ the lattice spacing, while for $L=51$ we find small deviations (\leq 5\%) with small variations of $\beta^{0/50}$ when FWHM$\leq 8a$. An open-ended chain can be physically realized by considering an extended lattice and adding two strong localized fields $\beta_{\text{walls}}$ on two particular sites, say 0 and $L+1$. When $\beta_{\text{walls}}$ is sufficiently high, the chain composed by the sites between 1 and $L$ is expected to be decoupled from the rest of the lattice. We consider a large lattice (more than 3$L$ sites) and we define sites 0 and $L+1$ in two symmetric position from the center of the lattice. Studying $(|R|-|T|)/R$ for $L \in \{21, 31, 41, 51\}$ we find that the deviation from the ideal case is lower than the 5\% when $\beta_{\text{walls}} \geq 3$.

Finally we consider the effect of a non zero curvature of the optical lattice. Following \cite{29} we consider a harmonic correction yielding an effective chemical potential $\mu_{eff}^I = \mu - J \omega^2 f^2/2$. When $\omega < 0.03$ results do not deviate from the ideal case.

**Discussion towards KLM quantum computation on lattices:** Phase shifters and beam splitters are the building blocks for the linear optical Knill-Laflamme-Milburn (KLM) quantum computer \cite{2}, where the need for non-linear interactions is circumvented by the construction of a non-linear phase shift with linear optics and photo-detectors. A KLM-type quantum computer with bosonic atoms has been theorized in \cite{3} in order to overcome some limitations of current photonic technology. Indeed, single-photon detectors are generally inefficient, whereas single-photon sources rely on parametric down-conversion which is a probabilistic event and does not guarantee unit efficiency. On the other hand, the preparation and measurement of single atoms is believed to be much easier \cite{5}. However, in the atomic version of the KLM computer proposed in \cite{3} uses the atoms in the same way as photons, thereby requiring an exquisite momentum control and particularly long wavepackets to minimize unwanted scattering.

Compared to the latter atomic KLM computer, our scheme has some clear advantages. On general grounds, it requires localized atoms that are easier to manipulate and to use both for gates and memories. Moreover, because of current single-site addressing capabilities \cite{18} the initialization and readout should be quite easy: by combining a spatial light modulator and resonant microwave field, one should be able to initialize one and two particle states in the same way as single \cite{19} and two \cite{5} magnon states were created in recent experiments. Alternatively, $n_i$ can be set by entering the Mott phase with properly tuned local potentials $n_i - 1 < \mu_i / \sigma_n$. An arbitrary unitary operation on atoms in two arbitrary sites $i, j$ of a 2D lattice can be obtained by suddenly decoupling (in a time $t \ll J^{-1}$, a speed of control that has already been demonstrated \cite{5, 19}) a chain that connects $i$ and $j$ from the rest of the lattice (via a suitably high energy barrier), and simultaneously raising a suitable potential shape in the effective chain to implement the desired beam splitter operation. These operations are possible in view of the high degree of tunability of current holographic masks \cite{31} that allow a fine design of local optical potential to both decouple a particular chain of bosons and attach suitable optical impurities to implement the desired operation. Wave-packet dispersion, which is the main limiting issue of unengineered chain, can in principle be overcome with a suitable engineering of the interactions, as we have shown, or by applying quantum control techniques on local potentials \cite{32} or other state transfer strategies \cite{33}. Provided that perfect transfer strategies are used, the two bosons are again in the initial sites $i$ and $j$ after the desired unitary gate, so the procedure can be repeated multiple times to obtained the desired set of beam splitter operations on different sets of atoms. Because the latter protocol also implements an operation on the qubits connecting $i$ to $j$ one has to divide the lattice into register sites and bus sites, so that register sites are never used as a bus. For instance one can implement a register as a small chain embedded into a 2D lattice and use the rest of the lattice for connecting purposes. As there are no known theoretical limitation to this procedure, our results place on solid...
grounds the realization of a linear-optical quantum computer on optical lattices.

In conclusion, we have devised a strategy to obtain the fundamental building blocks of linear optics via the dynamics of on optical lattices. The strategy is based on the use of an unmodulated chain to obtain both the physical transmission of the boson and the desired linear optical operation by suitably inserted optical impurities. Several applications are considered; in particular, the achievement of a 50/50 beam splitter and the observation of the two-mode Hong-Ou-Mandel effect in a lattice setup. Moreover, we have studied how to implement simultaneously a beam splitter and a phase shifter and, as an application, we have studied the achievement of a Mach-Zehnder interferometer. Although our method has been tailored around current experimental capabilities it allows the observation of many-photon interference effects on an optical lattice and may pave the way for the realization of an atomic KLM quantum computer.

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