Supersymmetric Leptogenesis

Chee Sheng Fong

C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, NY 11794-3840, USA.
E-mail: fong@insti.physics.sunysb.edu

M. C. Gonzalez-Garcia

Institució Catalana de Recerca i Estudis Avançats (ICREA),
Departament d’Estructura i Constituents de la Matèria and ICC-UB,
Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain.
and:
C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, NY 11794-3840, USA.
E-mail: concha@insti.physics.sunysb.edu

Enrico Nardi

INFN, Laboratori Nazionali di Frascati,
Via E. Fermi 40, I-00044 Frascati, Italy
E-mail: enrico.nardi@lnf.infn.it

J. Racker

Departament d’Estructura i Constituents de la Matèria and ICC-UB,
 Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain.
E-mail: racker@ecm.ub.es

Abstract: We study leptogenesis in the supersymmetric standard model plus the seesaw. We identify important qualitative differences that characterize supersymmetric leptogenesis with respect to the non-supersymmetric case. The lepton number asymmetries in fermions and scalars do not equilibrate, and are related via a non-vanishing gaugino chemical potential. Due to the presence of new anomalous symmetries, electroweak sphalerons couple to winos and higgsinos, and QCD sphalerons couple to gluinos, thus modifying the corresponding chemical equilibrium conditions. A new constraint on particles chemical potentials corresponding to an exactly conserved $R$-charge, that also involves the number density asymmetry of the heavy sneutrinos, appears. These new ingredients determine the $3 \times 4$ matrices that mix up the density asymmetries of the lepton flavours and of the heavy sneutrinos. We explain why in all temperature ranges the particle thermodynamic system is characterized by the same number of independent quantities. Numerical differences with respect to usual treatment remain at the $O(1)$ level.

Keywords: [Leptogenesis, Supersymmetry, Neutrino Physics, Beyond Standard Model]
1. Introduction

Leptogenesis \[1,2\] is a theoretical mechanism that can explain the observed matter-antimatter asymmetry of the Universe. An initial lepton asymmetry is generated in the out-of-equilibrium decays of heavy singlet Majorana neutrinos, and is then partially converted to a baryon asymmetry by anomalous sphaleron interactions \[3\]. Heavy Majorana singlet neutrinos are also a fundamental ingredient of the seesaw model \[4\] that explains in an elegant way the suppression of the neutrino mass scale with respect to all other particle masses of the Standard Model (SM).

The discovery of neutrino oscillations has promoted leptogenesis to an utmost attractive scenario to explain the origin of the cosmic baryon asymmetry. This is because
without any fine-tuning of the seesaw parameters, a neutrino mass scale naturally compatible with the solar and atmospheric neutrino mass square differences would be optimal to yield the correct value of the baryon asymmetry. The possibility of explaining two apparently unrelated experimental facts (neutrino oscillations and the baryon asymmetry) within a single framework has boosted the interest in leptogenesis studies, leading to important developments in the field, as for example the inclusion of thermal corrections [5], spectator processes [6,7], flavour effects [8–12], CP asymmetries in scatterings [13], lepton asymmetries from the decays of the heavier Majorana neutrinos [14,15], and many more.

In spite of all these advancements, we believe that a proper treatment of leptogenesis in the supersymmetric case is still lacking. Supersymmetric leptogenesis constitutes a theoretically appealing generalization of leptogenesis for the following reason: while the SM equipped with the seesaw provides the simplest way to realize leptogenesis, such a framework is plagued by an unpleasant fine-tuning problem. For a non-degenerate spectrum of heavy Majorana neutrinos, successful leptogenesis requires generically a scale for the singlet neutrino masses that is much larger than the electroweak (EW) scale [16] but at the quantum level the gap between these two scales becomes unstable. Low-energy supersymmetry (SUSY) can naturally stabilize the required hierarchy, and this provides a sounded motivation for studying leptogenesis in the framework of the supersymmetrized version of the seesaw mechanism. Supersymmetric leptogenesis has been studied in several places, both in dedicated studies [18] or in conjunction with SM leptogenesis [5]. However, several features that are specific of supersymmetry in the high temperature regime relevant for leptogenesis, in which soft supersymmetry breaking parameters can be effectively set to zero, have been overlooked or neglected in these studies. When the new ingredients are left out, in spite of the large amount of new reactions, the differences between SM and supersymmetric leptogenesis can be resumed by means of simple counting of a few numerical factors [2,19,20], like for example the number of relativistic degrees of freedom in the thermal bath, the number of loop diagrams contributing to the CP asymmetries, the multiplicities of the final states in the decays of the heavy neutrinos and sneutrinos.

In this paper we show that, in contrast to the naive picture, supersymmetric leptogenesis is rich of new and non-trivial features, and genuinely different from the simpler realization within the SM. A first important effect that follows from the requirement that the supersymmetry breaking scale should not exceed by much the 1 TeV scale, is that above a temperature $T \sim 5 \times 10^7$ GeV the particle and superparticle leptonic density asymmetries do not equilibrate. It is then mandatory to account in the Boltzmann equations for the differences in the number density asymmetries of the boson and fermion degrees of freedom, that can be given in terms of a non-vanishing gaugino chemical potential. A second feature is that when soft supersymmetry breaking parameters are neglected, additional anomalous global symmetries that involve both $SU(2)$ and $SU(3)$ fermion representations emerge [21]. As a consequence, the EW and QCD sphaleron equilibrium conditions are modified with respect to the SM, and this yields a different pattern of sphaleron induced lepton-flavour asymmetries.

In turn, supersymmetric leptogenesis introduces a certain conflict between the gravitino bound on the reheat temperature and the thermal production of the heavy singlets neutrinos [17]. In this paper we will not be concerned with the gravitino problem, nor with its possible ways out.
mixing [8,10,11]. In addition, a new anomaly-free $R$-symmetry can be defined and the corresponding charge, being exactly conserved, provides a constraint on the particles density asymmetries that is not present in the SM. Finally, a careful counting of the number of constraining conditions versus the overall number of particle density asymmetries reveals that four independent quantities, rather than the three of the SM case, are required to give a complete description of the various particle asymmetries in the thermal bath, with the additional quantity corresponding to the number density asymmetry of the heavy scalar neutrinos. In spite of all these qualitative differences, numerical corrections with respect to the case when all new effects are neglected remain at the $\mathcal{O}(1)$ level. This is because only spectator processes get affected, while the overall amount of CP violation driving leptogenesis remains the same than in previous treatments.

In Section 2 we start with a general description of the consequences of having different reactions dropping out of equilibrium as we imagine to raise the thermal bath temperature. We will next discuss in Section 2.2 the superequilibration regime, in which leptogenesis would proceed much alike in the SM case, but that in fact can be realized only at temperatures much below the lower limit for successful leptogenesis. In Section 2.3 we discuss the most relevant theoretical issues, that are particle-particle non-superequilibration, the new global symmetries, the modified sphaleron conditions, and the relevant conservation laws that constrain the particle density asymmetries. We also analyze specific temperature ranges in which leptogenesis can be successful, and for each range we present results for the matrices that control the lepton flavour mixing induced by the fast sphaleron processes and that, differently from the SM case, now have four columns. In Section 3 we discuss the numerical relevance of our study with respect to previous treatments. Finally, in Section 4 we recap our main results and draw the conclusions.

2. Chemical equilibrium conditions and conservation laws

At each specific temperature, particle reactions in the early Universe can be classified according to their (thermally averaged) rates as:

I Much faster than the Universe expansion rate. In the whole temperature range that we will consider, the top-quark Yukawa interactions and the $SU(3) \times SU(2) \times U(1)$ gauge interactions are examples of reactions of this type.

II Much slower than the expansion. For example, at temperature $T \gtrsim (1+\tan^2 \beta)10^5$ GeV the rates of processes that involve the electron Yukawa coupling are completely negligible with respect to the Universe expansion rate.

III Of the order of the expansion. This class includes the neutrino Yukawa interactions responsible for leptogenesis when the temperature is of the order of the heavy Majorana neutrino mass $M$.

Reactions of type I enforce specific conditions on the chemical potentials of the particles in the thermal bath. Reactions of type II imply that in the relevant temperature interval,
some Lagrangian parameters can be effectively set to zero. In most cases (but not in all
cases) this results in exact global symmetries that correspond to conservation laws for the
 correspond ing charges. These conservation laws constrain the asymmetries in the number
densities of the particles that carry the conserved charges. For particles that are in chemical
equilibrium, there is a statistical correspondence between the boson \((b)\) and fermion \((f)\) number
density asymmetries \(\Delta n_{b,f} \equiv n_{b,f} - \bar{n}_{b,f}\) and the corresponding chemical potentials
\(\mu_{b,f}\). In the relativistic limit \(m_{b,f} \ll T\) and at first order in \(\mu_{b,f} \ll T\) this correspondence
has the particularly simple form:
\[
\Delta n_b = \frac{g_b}{3} T^2 \mu_b, \quad \Delta n_f = \frac{g_f}{6} T^2 \mu_f,
\]
where \(g_{b,f}\) are the number of degrees of freedom of the corresponding particles. Therefore,
if chemical equilibrium is enforced for all particles entering a conservation law condition,
the constraints on number density asymmetries implied by reactions of type II can be
directly translated into constraints for the particles chemical potentials. However, when
a charge conservation condition involves particle species that in general remain out of
chemical equilibrium, as is the case for states that participate only in reactions of type III,
such a correspondence does not exist, and the constraints must be formulated in terms of
number density asymmetries.

Conservation conditions related to gauge symmetries, like hypercharge conservation
(see Eqs. (2.7)–(2.8) below), are somewhat different from the conservation laws stemming
from reactions of type II. This is because EW symmetry restoration at high temperature
occurs when the order parameter \(v\) is dynamically driven to zero, which is different from
the cases when some Lagrangian parameters can be neglected. However, for our purposes
this difference has no relevance, and we will treat hypercharge conservation in the same
way than other global conservation laws.

Finally, reactions of type III violate some symmetries, and thus spoil the corresponding
conservation conditions, without being fast enough to enforce chemical equilibrium. To this
class belong, by assumption, all the reactions that involve the neutrinos \(N\), the sneutrinos
\(\tilde{N}\) and the anti-sneutrinos \(\tilde{N}^*\). The evolution of their number densities and, eventually,
of the corresponding non-conserved charges, must then be tracked by means of Boltzmann
equations.

In the following we will analyze the set of conditions that constrain the particle number
density asymmetries. Whenever possible we will express these constraints in terms of
particles chemical potentials. This is the appropriate language for the fast reactions of
type I, and in most cases can be consistently used also for conditions related to reactions
of type II. In principle there are as many chemical potentials, or more precisely as many
density asymmetries as there are particles in the thermal bath. However, there is also a
large set of conditions enforced by processes in chemical equilibrium and by conservation
laws which ensure that just a few independent quantities are sufficient to determine all the
others. In carrying out our analysis we imagine to start from relatively low temperatures,
that is, from temperatures well below the lower limit for successful leptogenesis. As we
raise the temperature, both the expansion rate and the particle reactions rates vary, and
this changes the way particle reactions are assigned to the previous three classes. More
specifically, we will only consider what happens when some $B-L$ conserving reaction moves *instantaneously* from class I to class II, since for these reactions the regime in which the rates become of the order of the expansion (class III) is in general a short-lasting transient with no relevant effects associated to it. Since we are interested in leptogenesis, we also assume that, for each temperature regime, the heavy Majorana neutrino masses and couplings are such that $B-L$ violating processes always belong to the third class. † According to the classification scheme described above, we would expect that at each temperature the thermodynamic system of the particle soup can be described in terms of the same number of independent quantities. This is because when we move one reaction from the first class to the second one, we lose one constraint from chemical equilibrium dynamics, but we gain one constraint from a new conservation law. While this is in general true, as we will see the detailed way in which this ‘switching’ of conditions is implemented has some subtleties, since in some cases a non trivial interplay between redundant conditions and new global symmetries that are anomalous, and thus do not correspond to conservation laws, is involved.

This section is organized in the following way: first we list in Section 2.1 the constraints that hold independently of assuming a regime in which particles and sparticles chemical potentials equilibrate (superequilibration (SE) regime) or do not equilibrate (non-superequilibration (NSE) regime). In Section 2.2 we list the constraints that hold only in the SE regime, and in Section 2.3 the ones that hold in the temperature range relevant for supersymmetric leptogenesis, when the temperature is sufficiently high that particles and sparticles chemical potentials do not equilibrate.

2.1 General constraints

The supersymmetric seesaw model is described by the superpotential of the supersymmetric SM with the additional terms:

$$W = \frac{1}{2} M_{pq} N_p^c N_q^c + \lambda_{pq} N_p^c \ell_\alpha H_u,$$

where $p, q = 1, 2, \ldots$ label the heavy singlet states in order of increasing mass, and $\alpha = e, \mu, \tau$ labels the lepton flavour. In Eq. (2.2) $\ell$, $H_u$ and $N^c$, are respectively the chiral superfields for the lepton and the up-type Higgs $SU(2)$ doublets and for the heavy $SU(2)$ singlet neutrinos defined according to usual conventions in terms of their left-handed Weyl spinor components (for example the $N^c$ supermultiplet has scalar component $\tilde{N}^c$ and fermion component $N^c_\ell$). Finally the $SU(2)$ index contraction is defined as $\ell_\alpha H_u = \epsilon_{\rho\sigma} \ell^\rho_H H_u^\sigma$ with $\epsilon_{12} = +1$.

Let us start at $T \gtrsim \text{few TeV}$, that is well above the EW or SUSY breaking scale, but low enough that all the SM and SUSY reactions can be considered in thermal equilibrium. We first list in items 1., 2. and 3. the conditions that hold in the whole temperature range that we will consider ($M_W \ll T \lesssim 10^{14}$ GeV). Conversely, some of the Yukawa coupling

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†Assigning $B-L$ violating processes to class I means that they are in thermal equilibrium and no $B-L$ asymmetry can be generated. Assigning them to class II means that $B-L$ is conserved, and baryogenesis cannot occur.
conditions given in items 4. and 5. will have to be dropped as the temperature is increased and the corresponding reactions go out of equilibrium. For simplicity of notations, in the following we denote the chemical potentials with the same notation that labels the corresponding field: \( \phi \equiv \mu_\phi \).

1. At scales much higher than \( M_W \), gauge fields have vanishing chemical potential \( W = B = g = 0 \) [22]. This also implies that all the particles belonging to the same \( SU(2) \) or \( SU(3) \) multiplets have the same chemical potential. For example \( \phi(I_3 = +\frac{1}{2}) = \phi(I_3 = -\frac{1}{2}) \) for a field \( \phi \) that is a doublet of weak isospin \( \vec{I} \), and similarly for color.

2. Fast gluino, wino and bino interactions relate the difference between the chemical potentials of the members of a given supermultiplet and the corresponding gaugino chemical potential. Furthermore \( \tilde{Q} + \tilde{g}_R \rightarrow Q, \tilde{Q} + \tilde{W}_R \rightarrow Q, \tilde{\ell} + \tilde{W}_R \rightarrow \ell, \tilde{\ell} + \tilde{B}_R \rightarrow \ell, \tilde{\ell} + \tilde{W}_R \rightarrow \ell, \tilde{\ell} + \tilde{B}_R \rightarrow \ell, \) where \( \ell, Q (\tilde{\ell}, \tilde{Q}) \) denote the (s)lepton and (s)quarks left-handed doublets, and \( \tilde{W}_R, \tilde{B}_R \) and \( \tilde{g}_R \) are respectively right-handed winos, binos and gluinos, enforce chemical equilibrium conditions which imply that all gauginos have the same chemical potential:

\[
-\tilde{g} = Q - \tilde{Q} = -\tilde{W} = \ell - \tilde{\ell} = -\tilde{B},
\]

where we have introduced \( \tilde{W}, \tilde{B} \) and \( \tilde{g} \) to denote the chemical potential of the left-handed gauginos. It follows that the chemical potentials of the SM particles are related to the chemical potential of their respective superpartners as

\[
\tilde{Q}, \tilde{\ell} = Q, \ell + \tilde{g} \quad \text{(2.4)}
\]

\[
H_{u,d} = \tilde{H}_{u,d} + \tilde{g} \quad \text{(2.5)}
\]

\[
\tilde{u}, \tilde{d}, \tilde{e} = u, d, e - \tilde{g}. \quad \text{(2.6)}
\]

The last relation, in which \( u, d, e \equiv u_R, d_R, e_R \) denote the R-handed \( SU(2) \) singlets, follows e.g. from \( \tilde{u}_L^c = u_L^c + \tilde{g} \) for the corresponding \( L \)-handed fields, together with \( u_i^c = -u_R \) and from the analogous relation for the \( SU(2) \) singlet squarks. Eqs.(2.4)–(2.6) allow us to express all the chemical equilibrium relations in terms of the 18 chemical potentials of the fermions (SM quarks and leptons, higgsinos and gauginos).

3. Before EW symmetry breaking hypercharge is an exactly conserved quantity. Therefore for the total hypercharge of the Universe we have

\[
\mathcal{Y}_{\text{tot}} = \sum_b \Delta n_b y_b + \sum_f \Delta n_f y_f = \text{const},
\]

where \( y_{b,f} \) denotes the hypercharge of the \( b \)-bosons or \( f \)-fermions. Given that leptogenesis aims to explain the origin of the matter-antimatter asymmetry, it is reasonable to assume as initial condition that at sufficiently high temperatures all particle-antiparticle density asymmetries vanish \( \Delta n_{b,f}|_{T \rightarrow \infty} = 0 \) which implies \( \mathcal{Y}_{\text{tot}} = 0 \). By using (2.3) the hypercharge conservation condition can be rewritten as:

\[
\frac{1}{3} \sum_b \mu_b g_b y_b + \frac{1}{6} \sum_f \mu_f g_f y_f = \sum_i (Q_i + 2u_i - d_i) - \sum_\alpha (\ell_\alpha + e_\alpha) + \tilde{H}_u - \tilde{H}_d = 0,
\]

(2.8)
where $Q_i$ denote the three quark doublets, $u_i = u, c, t$ and $d_i = d, s, b$. In writing down Eq.(2.8) we have used Eqs.(2.4)–(2.6) to express the chemical potentials of the scalars in terms of those of their fermion partners. Given that Eqs.(2.4)–(2.6) hold in the whole temperature range that we will analyze, the same is true also for Eq.(2.8).

Note that the hypercharge neutrality condition involves only the 17 chemical potentials of the fermionic components of the matter superfields since the gaugino chemical potential, that is involved in the substitutions (2.4)-(2.6), eventually cancels out. This is expected, since all gauginos have vanishing hypercharge; however, in counting the number of chemical potentials $\tilde{g}$ must be included, yielding a total number of 18.

4. When the reactions mediated by the leptons Yukawa couplings are faster than the Universe expansion rate $H \sim 1.66\sqrt{g_*}T^2/M_P$ (where $M_P$ is the Planck mass and $g_* = 228.75$ in the MSSM), the following chemical equilibrium conditions are enforced:

$$\ell_\alpha - e_\alpha + \tilde{H}_d + \tilde{g} = 0, \quad (\alpha = e, \mu, \tau). \quad (2.9)$$

For $\alpha = e$ the corresponding Yukawa condition holds only as long as

$$T \lesssim 10^5 (1 + \tan^2 \beta) \text{GeV}, \quad (2.10)$$

when Yukawa reactions between the first generation left-handed $SU(2)$ lepton doublets $\ell_\alpha$ and the right-handed singlets $e$ are faster than the expansion [23]. Note also that, as is discussed in refs. [24, 25], if the temperature is not too low lepton flavour equilibration induced by off-diagonal slepton soft masses will not occur. We assume that this is the case, and thus we take the three $\ell_\alpha$ to be independent quantities.

5. Reactions mediated by the quarks Yukawa couplings enforce the following six chemical equilibrium conditions:

$$Q_i - u_i + \tilde{H}_u + \tilde{g} = 0, \quad (u_i = u, c, t), \quad (2.11)$$

$$Q_i - d_i + \tilde{H}_d + \tilde{g} = 0, \quad (d_i = d, s, b). \quad (2.12)$$

The up-quark Yukawa coupling maintains chemical equilibrium between the left and right handed up-type quarks up to $T \sim 2 \cdot 10^6 \text{GeV}$. Note that when the Yukawa reactions of at least two families of quarks are in equilibrium, the mass basis is fixed for all the quarks and squarks. Intergeneration mixing then implies that family-changing charged-current transitions are also in equilibrium: $b_L \rightarrow c_L$ and $t_L \rightarrow s_L$ imply $Q_2 = Q_3$; $s_L \rightarrow u_L$ and $c_L \rightarrow d_L$ imply $Q_1 = Q_2$. Thus, up to temperatures $T \lesssim 10^{11} \text{GeV}$, that are of the order of the charm Yukawa coupling equilibration temperature, the three quark doublets have the same chemical potential:

$$Q \equiv Q_3 = Q_2 = Q_1. \quad (2.13)$$

At higher temperatures, when only the third family is in equilibrium, we will have instead $Q \equiv Q_3 = Q_2 \neq Q_1$. Above $T \sim 10^{13}$ when (for moderate values of $\tan \beta$) also the $\tau$ and $b$-quark $SU(2)$ singlets decouple from their Yukawa reactions, all intergeneration mixings are also negligible and $Q_3 \neq Q_2 \neq Q_1$. 

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2.2 Superequilibration regime

At relatively low temperatures, additional conditions from reactions in chemical equilibrium hold. Since the constraints below apply only in the SE regime, we number them including this label.

6SE. Equilibration of the particle-sparticle chemical potentials $\mu_\phi = \tilde{\mu}_\phi$ (generally referred as superequilibration [26]) is ensured when reactions like $\tilde{\ell} \tilde{\ell} \rightarrow \ell \ell$ are faster than the Universe expansion rate. These reactions are induced by gaugino interactions, but since they require a gaugino chirality flip they turn out to be proportional to its mass $m_{\tilde{g}}$, and can be neglected in the limit $m_{\tilde{g}} \rightarrow 0$.

Furthermore, since the $\mu$ parameter of the $H_uH_d$ superpotential term is expected to be of the order of the soft gaugino masses, it is reasonable to consider in the same temperature range also the effect of the higgsino mixing term, which implies that the sum of the up- and down- higgsino chemical potentials vanishes. The rates of the corresponding reactions, given approximately by $\Gamma_{\tilde{g}} \sim m_{\tilde{g}}^2/T$ and $\Gamma_{\mu} \sim \mu^2/T$, are faster than the Universe expansion rate up to temperatures

$$T \lesssim 5 \cdot 10^7 \left( \frac{m_{\tilde{g}}, \mu}{500 \text{ GeV}} \right)^{2/3} \text{ GeV}. \quad (2.14)$$

The corresponding chemical equilibrium relations enforce the conditions:

$$\tilde{g} = 0, \quad (2.15)$$

$$\tilde{H}_u + \tilde{H}_d = 0. \quad (2.16)$$

7SE. Up to temperatures given by (2.14) the MSSM has the same global anomalies than the SM, that are the EW $SU(2)-U(1)_{B+L}$ mixed anomaly and the QCD chiral anomaly. They generate the effective operators $O_{EW} = \Pi_\alpha(QQQ\ell_\alpha)$ and $O_{QCD} = \Pi_i(QQu_i^c d_i^c L_i)$. Above the EW phase transition reactions induced by these operators are in thermal equilibrium, and the corresponding conditions read:

$$9 Q + \sum_\alpha \ell_\alpha = 0 \quad (2.17)$$

$$6 Q - \sum_i (u_i + d_i) = 0, \quad (2.18)$$

where we have used the same chemical potential for the three quark doublets (Eq.(2.13)), which is always appropriate in the SE regime below the limit (2.14).

2.2.1 Flavour charges

Eqs.(2.9) and (2.11)–(2.13), together with the SE conditions (2.15)–(2.16), the two anomaly conditions (2.17)–(2.18) and the hypercharge neutrality condition (2.8), give $11 + 2 + 2 + 1 = 16$ constraints for the 18 chemical potentials. Note however that there is one redundant constraint, that we take to be the QCD sphaleron condition, since by summing up Eqs.(2.11)
and (2.12) and taking into account (2.13), (2.15), and (2.16) we obtain precisely Eq.(2.18). Therefore, like in the SM, we have three independent chemical potentials, that could be taken to be the ones corresponding to the leptons doublets. Another choice, that is more useful in leptogenesis, is to define three linear combinations of the chemical potentials corresponding to the $SU(2)$ anomaly free flavour charges $\Delta_\alpha \equiv B/3 - L_\alpha$. The reason is that these charges, being anomaly free and perturbatively conserved by the low energy MSSM Lagrangian, evolve slowly because the corresponding symmetries are violated only by the heavy Majorana neutrino dynamics. Their evolution is thus determined by reactions belonging to class III, and needs to be computed by means of three independent Boltzmann equations. We define the number densities of particles per degree of freedom, normalized to the entropy density $s$, as $Y_{\Delta b,f} \equiv \frac{n_{\Delta b,f}}{s}$. In terms of these quantities the density of the $\Delta_\alpha$ charges normalized to the entropy density can be written as:

$$Y_{\Delta\alpha} = 3 \left[ \frac{1}{3} \sum_i (2Y_{\Delta Q_i} + Y_{\Delta u_i} + Y_{\Delta d_i}) - (2Y_{\Delta\ell_\alpha} + Y_{\Delta e_\alpha}) - \frac{2}{3} Y_{\Delta\tilde{g}} \right]. \quad (2.19)$$

The expression above is completely general and holds in all temperature regimes, including the NSE regime (see Section 2.3). Note that $\tilde{g}$ in the equation above cancels for the quarks but not for the leptons, and thus in the NSE, in which the gaugino chemical potential does not vanish, when the $Y_{\Delta_\alpha}$ charges are expressed just in terms of the number density asymmetries of the fermions, $Y_{\Delta\tilde{g}}$ also contributes.

In Eq.(2.19) we have left in clear some numerical factors: the overall factor of 3 adds the contributions of scalars (that is twice that of fermions), the factor of 2 in front of the $Y_{\Delta Q_i}$ and $Y_{\Delta\ell_\alpha}$ accounts for the $SU(2)$ gauge multiplicity, while the color factor compensates against the the quark baryon number $B = 1/3$.

The density asymmetries of the doublet leptons and higgsinos, that weight the washout terms in the Boltzmann equations, can now be expressed in terms of the anomaly free charges by means of the $A$ matrix and $C$ vectors introduced respectively in ref. [8] and ref. [11] that are defined as:

$$Y_{\Delta\ell_\alpha} = A^\ell_{\alpha\beta} Y_{\Delta_\beta}, \quad Y_{\Delta H_{u,d}} = C_{\alpha}^{H_{u,d}} Y_{\Delta_\alpha}. \quad (2.20)$$

Here and in the following we will give results for the $A$ and $C$ matrices for the fermion states. We recall that in the SE regime the density asymmetry of a scalar boson that is in chemical equilibrium with its fermionic partner is given simply by $Y_{\Delta b} = 2Y_{\Delta f}$ with the factor of 2 from statistics.

### 2.2.2 All Yukawa reactions in equilibrium

Assuming moderate values of $\tan \beta$, at temperatures below the limit in Eq.(2.10) standard leptogenesis cannot be successful. However, this range can correspond to a low temperature windows in which soft leptogenesis [27–29] can successfully proceed, and therefore it is worth giving the results for $A$ and $C$. They are:

$$A^\ell = \frac{1}{9 \times 237} \begin{pmatrix} -221 & 16 & 16 \\ 16 & -221 & 16 \\ 16 & 16 & -221 \end{pmatrix}, \quad C^{H_{u,d}} = -C^{H_{d}} = \frac{-4}{237} (1, 1, 1). \quad (2.21)$$
Note that since in this regime the chemical potentials for the scalars and leptons degrees of freedom of each chiral multiplet equilibrate, the analogous results for $Y_{\Delta \ell_\alpha} + Y_{\Delta \tilde{\ell}_\alpha}$ can be obtained by simply multiplying the $A$ matrix in Eq.(2.21) by a factor of 3. This gives the same $A$ matrix obtained in the non-supersymmetric case in the same regime (see e.g. eq.(4.13) in ref. [11]). The $C$ matrix (multiplied by the same factor of 3) differs from the non-supersymmetric result by a factor $1/2$. This is because after substituting $\tilde{H}_d = -\tilde{H}_u$ (see Eq.(2.16)) all the chemical potential conditions are formally the same than in the SM with $\tilde{H}_u$ identified with the chemical potential of the scalar Higgs, but since $C$ expresses the result for number densities, in the SM a factor of 2 from boson statistics appears for the SM Higgs. This agrees with the analysis in ref. [30], and is a general result that holds for supersymmetry within the SE regime.

### 2.2.3 Electron and up-quark Yukawa reactions out of equilibrium

Raising the temperature above $10^5(1 + \tan^2 \beta)$ GeV the interactions mediated by the electron Yukawa $h_e$ are not able to maintain equilibrium, and one condition in Eq.(2.9) for $\alpha = e$ is lost. However, since in the effective theory at this temperature one can set $h_e \to 0$, one global symmetry is gained. This corresponds in the fermion sector to chiral symmetry for the R-handed electron, that in the present case translates into a symmetry under phase rotations of the $e$ chiral multiplet that holds in the limit of unbroken supersymmetry. Conservation of the corresponding charge ensures that $\Delta n_e + \Delta n_{\tilde{e}} = 3\Delta n_e$ is constant, and since leptogenesis aims to explain dynamically the generation of a lepton asymmetries we set this constant to zero, so that the R-handed electron chemical potential is $e = 0$. In this way the chemical equilibrium condition that is lost is replaced by a new condition implied by the conservation of a global charge, and three independent chemical potentials (or alternatively the three non-anomalous charges (2.19)) are still sufficient to describe all the density asymmetries of the thermodynamic system. At temperatures above $T \sim 2 \cdot 10^6$ GeV interactions mediated by the up-quark Yukawa coupling $h_u$ drop out of equilibrium. In this case however, by setting $h_u \to 0$ no new symmetry is obtained, since chiral symmetry for the R-handed quarks is anomalous and the corresponding charge is not conserved by fast QCD sphaleron interactions. However, after dropping the first condition in Eq.(2.11) for $u_i = u$, the QCD sphaleron condition Eq.(2.18) ceases to be a redundant constraint, with the result that also in this case no new chemical potentials are needed to determine all the particle density asymmetries. The corresponding results, that can be again relevant for soft leptogenesis, read:

$$A^\ell = \frac{1}{3 \times 2886} \begin{pmatrix} -1221 & 156 & 156 \\ 111 & -910 & 52 \\ 111 & 52 & -910 \end{pmatrix}, \quad C^\tilde{H}_u = -C^\tilde{H}_d = \frac{-1}{2886} (37, 52, 52) \cdot (2.22)$$

### 2.2.4 First generation Yukawa reactions out of equilibrium (SE regime)

Let us now consider what happens at temperatures $T \gtrsim 4 \cdot 10^6(1 + \tan^2 \beta)$ GeV, when also the $d$-quark Yukawa coupling can be set to zero (in order to remain within the SE regime we assume $\tan \beta \sim 1$). In this case the equilibrium dynamics is symmetric under
the exchange $u \leftrightarrow d$ (both chemical potentials enter only the QCD sphaleron condition Eq. (2.18) with equal weights) and so must be any physical solution of the set of constraints. Thus, the first condition in Eq. (2.12) can be replaced by the condition $d = u$, and again three independent quantities suffice to determine all the particle density asymmetries. The corresponding result is:

$$A^\ell = \frac{1}{3 \times 2148} \begin{pmatrix} -906 & 120 & 120 \\ 75 & -688 & 28 \\ 75 & 28 & -688 \end{pmatrix} , \quad C\hat{H}_u = -C\hat{H}_d = -\frac{1}{2148} (37, 52, 52) , \quad (2.23)$$

that agree with what is obtained in non-supersymmetric leptogenesis (see eq. (4.12) of ref. [11]) after the factor 1/2 for the higgsinos discussed below Eq. (2.21) is accounted for.

In the numerical analysis in Section 3 we will take this case as a benchmark to confront the results obtained in the SE regime with the corresponding results in the NSE regime.

### 2.3 Non-superequilibration Regime

At temperatures above the limit given in Eq. (2.14) the Universe expansion is fast enough that reactions induced by $m_{\tilde{g}}$ and $\mu$ do not occur. Setting to zero in the high temperature effective theory these two parameters has the following consequences:

(i) Condition (2.15) has to be dropped, and gauginos acquire a non-vanishing chemical potential $\tilde{g} \neq 0$ (corresponding to the difference between the number of L and R helicity states). The chemical potentials of the members of the same matter supermultiplets are no more equal (non-superequilibration) but related as in Eqs. (2.4)–(2.6).

(ii) Condition (2.16) also has to be dropped, and the chemical potentials of the up- and down-type Higgs and higgsinos do not necessarily sum up to zero.

(iii) The MSSM gains two new global symmetries: $m_{\tilde{g}} \to 0$ yields a global $R$-symmetry, while $\mu \to 0$ corresponds to a global symmetry of the Peccei-Quinn (PQ) type.†

#### 2.3.1 Anomalous and non-anomalous symmetries

The charges of the various states under the $R$ and $PQ$ symmetries together with the values of the other two global symmetries $B$ and $L$ are given in Table 3. Like $L$, also $R$ and $PQ$ are not symmetries of the seesaw superpotential terms $M N^c N^c + \lambda N^c \ell H_u$. In Table 3 we have fixed the charges of the heavy $N^c$ supermultiplets in such a way that the mass term $M N^c N^c$ is invariant under all symmetries and in particular has $R$ charge equal to 2.‡

†We assume that, similarly to $m_{\tilde{g}}$, in this regime also the other soft supersymmetry breaking terms can be neglected, and in particular the $B$-term for the heavy sneutrinos. This is always the case in the high temperature regime for successful leptogenesis. However, at the lower temperatures required in soft leptogenesis this is not necessarily true. Then the $R$-charge is not perturbatively conserved, as is clear from the last entry in Table 3. This can have quite interesting consequences for soft leptogenesis that are analyzed in a companion paper [31].

‡Under $R$-symmetry the superspace Grassmann parameter transform as $\theta \to e^{i\alpha} \theta$. Invariance of $\int d\theta$ then requires $R(d\theta) = -1$. Then the chiral superspace integral of the superpotential $\int d\theta^2 W$ is invariant if $R(W) = 2$. By expanding a chiral supermultiplet in powers of $\theta$ it is also clear that its $R$ charge equals the charge of the scalar boson $R(h) = R(f) + 1$. 

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Table 1: $B$, $L$, $PQ$ and $R$ charges for the particle supermultiplets that are labeled in the top row by their L-handed fermion component. Note that we use chemical potentials for the R-handed $SU(2)$ singlet fields $u, d, e$ that have opposite charges with respect to the ones for $u^c, d^c, e^c$ given in the table. The $R$-charges for bosons are determined by $R(b) = R(f) + 1$.

|   | $\tilde{g}$ | $Q$  | $u^c$ | $d^c$ | $\ell$ | $e^c$ | $\tilde{H}_d$ | $\tilde{H}_u$ | $N^c$ |
|---|-------------|------|-------|-------|-------|------|-------------|-------------|-------|
| $B$ | 0           | $\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | 0     | 0    | 0           | 0           | 0     |
| $L$ | 0           | 0    | 0     | 0     | 1     | $-1$ | 0           | 0           | 0     |
| $PQ$ | 0          | 0    | 0     | 1     | $-1$  | 2    | $-1$       | 2           | 0     |
| $R$ | $f$       | 1    | $-2$  | 1     | $-1$  | 2    | $-1$       | 3           | 0     |
|    | $b$       | 2    | 0     | $-2$  | 2     | 0    | 2           | 4           | 1     |

contrast, the Yukawa term $N^c L H_u$ violates $L, PQ$ and $R$. All the four global symmetries $B, L, PQ$ and $R$ have mixed gauge anomalies with $SU(2)$, and $R$ and $PQ$ have also mixed gauge anomalies with $SU(3)$. Two linear combinations $R_2$ and $R_3$ of $R$ and $PQ$, having respectively only $SU(2)$ and $SU(3)$ mixed anomalies have been identified in ref. [21]. They are:

\begin{align*}
R_2 &= R - 2PQ \\
R_3 &= R - 3PQ.
\end{align*}

The values of $R_{2,3}$ for the different states are given in Table 2. The authors of ref. [21] have also constructed the effective multi-fermions operators generated by the mixed anomalies:

\begin{align*}
\tilde{O}_{EW} &= \Pi_\alpha (QQ\ell_\alpha) \tilde{H}_u \tilde{H}_d \tilde{W}^4, \\
\tilde{O}_{QCD} &= \Pi_i (QQ^c d^c) \tilde{g}^6.
\end{align*}

Given that three global symmetries $B, L$ and $R_2$ have mixed $SU(2)$ anomalies (but are free of $SU(3)$ anomalies) it is clear that we can construct 2 anomaly free combinations, the first one of which we chose to be, as in the SM, $B - L$. Given that only one term, that is $N^c \ell H_u$, violates perturbatively the global symmetries, a second anomaly free combination $R$ can be chosen in such a way that it is respected also by the corresponding interactions, and thus it is an exact symmetry of the MSSM+seesaw in the NSE regime. The corresponding charge reads

\begin{equation}
R = \frac{5}{3}B - L + R_2,
\end{equation}

and is exactly conserved. In the $SU(3)$ sector, besides the chiral anomaly we now have also $R_3$ mixed anomalies. Thus also in this case anomaly free combinations can be constructed,

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*For definiteness we restrict ourselves to the case of three generations $N_g = 3$ and one pair of Higgs doublets $N_h = 1$. We also normalize $R_{2,3}$ in such a way that $R_{2,3}(b) = R_{2,3}(f) + 1$. 
and in particular we can define one combination for each quark superfield. Assigning to the L-handed supermultiplets chiral charge $\chi = -1$ these combinations have the form:

$$\chi_{qL} + \kappa_{qL} R_3$$

(2.29)

where, for example, $\kappa_{u_L} = \kappa_{d_L} = 1/3$ and $\kappa_{Q_L} = 2/3$. Note that since $R_3$ is perturbatively conserved by the complete MSSM+seesaw Lagrangian, when the Yukawa coupling of one quark is set to zero the corresponding charge Eq. (2.29) will be exactly conserved.

### 2.3.2 Constraints in the non-superequilibration regime

As explained in the introduction to this section, conditions corresponding to conservation laws constrain particle number density-asymmetries, while conditions from reactions that are in chemical equilibrium constrain particle chemical potentials. Eventually, to get a closed form solution to the set of constraining equations one needs to use a single set of variables. In the following, we will then express the number density-asymmetries of relativistic particles in terms of their chemical potentials by means of eq. (2.1) that holds in the ultra-relativistic limit and at first order in $\mu_{f,b}/T$.

In the NSE regime, the conditions listed in items 6$_{SE}$ and 7$_{SE}$ of the previous section have to be dropped, but new conditions arise.

6$_{NSE}$. The conservation law for the $\mathcal{R}$ charge yields the following global neutrality condition:

$$\mathcal{R}_{\text{tot}} = \sum_f \Delta n_f \mathcal{R}_f + \sum_b \Delta n_b \mathcal{R}_b + \Delta n_{\tilde{N}_1} \mathcal{R}_{\tilde{N}_1}$$

$$= \frac{T^2}{6} \left( \sum_f \mu_f g_f \mathcal{R}_f + 2 \sum_b \mu_b g_b \mathcal{R}_b \right) - \Delta n_{\tilde{N}_1} = 0,$$  

(2.30)

with the term in parenthesis given by:

$$\sum_f \mu_f g_f \mathcal{R}_f + 2 \sum_b \mu_b g_b \mathcal{R}_b =$$

$$2 \left( \sum_i (2Q_i - 5u_i + 4d_i) + 2 \sum_\alpha (\ell_\alpha + e_\alpha) + 5 \tilde{H}_d - \tilde{H}_u + 31 \tilde{g} \right).$$  

(2.31)

The last terms in both lines of Eq. (2.30) correspond to the contribution to $\mathcal{R}$-neutrality from the lightest sneutrino asymmetry $\Delta n_{\tilde{N}_1} = n_{\tilde{N}_1} - n_{\tilde{N}_1}^*$ with charge $\mathcal{R}_{\tilde{N}_1} = -\mathcal{R}_{N} = -1$. All the other heavier neutrinos are assumed to have already decayed at the time $N_1$ leptogenesis takes place, and do not contribute. Note that since in general $\tilde{N}_1$ is not in chemical equilibrium, no chemical potential can be associated to it, and hence this constraint needs to be formulated in terms of its number density asymmetry that has to be evaluated by solving a Boltzmann equation for $Y_{\tilde{N}_1} \equiv Y_{\tilde{N}_1} - Y_{\tilde{N}_1}^*$ (see Section 3). Note also that since the final lepton asymmetry in leptogenesis is generally determined by the dynamics at $T < M$, where the sneutrino abundance gets exponentially suppressed, we can expect that neglecting $\Delta n_{\tilde{N}_1}$ in Eq. (2.30) could still give a good approximation. This is indeed confirmed by the numerical analysis presented in Section 3.
7NSE. The operators in Eqs.(2.26)–(2.27) induce transitions that in the NSE regime are in chemical equilibrium. This enforces the generalized EW and QCD sphaleron equilibrium conditions [21]:

\[
3 \sum_i Q_i + \sum_\alpha \ell_\alpha + \bar{H}_u + \bar{H}_d + 4 \tilde{g} = 0,
\]

(2.32)

\[
2 \sum_i Q_i - \sum_i (u_i + d_i) + 6 \tilde{g} = 0,
\]

(2.33)

that replace (2.17) and (2.18).

8NSE. The chiral-R\textsubscript{3} charges in Eq.(2.29) are anomaly free, but clearly they are not conserved by the quarks Yukawa interactions. However, when a quark supermultiplet decouples from its Yukawa interactions an exact conservation law arises. (Note that \(h_{u,d} \to 0\) implies \(u\) and \(d\) decoupling, but \(Q_1\) decoupling is ensured only if also \(h_{c,s} \to 0\).) The conservation laws corresponding to these symmetries read:

\[
\frac{T^2}{6} \left[3 q_R + 6 (q_R - \tilde{g})\right] + \frac{1}{3} R_{3 \text{tot}} = 0 \tag{2.34}
\]

\[
\frac{T^2}{6} \left[3 Q_L + 6 (Q_L + \tilde{g})\right] - \frac{2}{3} R_{3 \text{tot}} = 0 \tag{2.35}
\]

and hold for \(q_R = u_i, d_i\) and \(Q_L = Q_i\) in the regimes when the appropriate Yukawa reactions are negligible. Note the factor of 2 for the \(Q_L\) chiral charge in front of the first square bracket in Eq. (2.33) that is due to SU(2) gauge multiplicity. In terms of chemical potentials and of the sneutrino number density asymmetry, the total \(R_3\) charge in Eqs. (2.34)-(2.35) reads:

\[
R_{3 \text{tot}} = \frac{T^2}{6} \left(\sum_f \mu_f g_f R_{3f} + 2 \sum_b \mu_b g_b R_{3b}\right) + \Delta n_{\tilde{N}_1} R_{3 \tilde{N}_1} \tag{2.36}
\]

where \(R_{3 \tilde{N}_1} = -1\), and the quantity in parenthesis is given by:

\[
\sum_f \mu_f g_f R_{3f} + 2 \sum_b \mu_b g_b R_{3b} = 82 \tilde{g} - 3 \sum_i (2 Q_i + 11 u_i - 4 d_i) + \sum_\alpha (16 \ell_\alpha + 13 e_\alpha) + 16 \bar{H}_d - 14 \bar{H}_u. \tag{2.37}
\]

As regards the leptons, since they do not couple to the QCD anomaly, by setting \(h_e \to 0\) a symmetry under chiral supermultiplet rotations is directly gained for the R-handed leptons implying \(\Delta n_e + \Delta \bar{n}_e = 0\) and giving the condition:

\[
e - \frac{2}{3} \tilde{g} = 0. \tag{2.38}
\]

No analogous condition arises for the lepton doublets relevant for leptogenesis, since by assumption they remain coupled via Yukawa couplings to the heavy \(N\)'s.
Table 2: Charges for the fermionic and bosonic components of the SUSY multiplets under the \( R \)-symmetries defined in Eqs.\((2.24), (2.25) \) and \((2.28) \). Supermultiplets are labeled in the top row by their L-handed fermion component. We use chemical potentials for the R-handed \( SU(2) \) singlet fields \( u, d, e \) that have opposite charges with respect to the ones for \( u^c, d^c, e^c \) given in the table.

|   | \( g \) | \( Q \) | \( u^c \) | \( d^c \) | \( \ell \) | \( e^c \) | \( \tilde{H}_d \) | \( \tilde{H}_u \) | \( N^c \) |
|---|---|---|---|---|---|---|---|---|---|
| \( R_2 \) | \( f \) | 1 | -1 | 1 | -1 | 1 | -3 | 1 | -1 | 0 |
|   | \( b \) | 2 | 0 | 2 | 0 | 2 | -2 | 2 | 0 | 1 |
| \( R_3 \) | \( f \) | 1 | -1 | 3 | -2 | 2 | -5 | 2 | -3 | 0 |
|   | \( b \) | 2 | 0 | 4 | -1 | 3 | -4 | 3 | -2 | 1 |
| \( \mathcal{R} \) | \( f \) | 1 | -\( \frac{4}{9} \) | \( \frac{4}{9} \) | -\( \frac{14}{9} \) | 0 | -2 | 1 | -1 | 0 |
|   | \( b \) | 2 | \( \frac{5}{9} \) | \( \frac{13}{9} \) | -\( \frac{5}{9} \) | 1 | -1 | 2 | 0 | 1 |

Let us now evaluate what is the number of independent chemical potentials in the NSE regime. Since we already know that this number should not change when some constraints from chemical equilibrium are dropped and replaced by conservation conditions, let us start by assuming \( \tan \beta \) large enough that the Yukawa couplings \( h_\ell \) and \( h_d \) enforce the equilibrium conditions \((2.9) \) and \((2.12) \), and also keep for the time being the condition \((2.11) \) for the up-quark. We have 9 conditions from the Yukawa couplings plus 2 from quark doublets equilibration \((2.13) \), the generalized EW sphaleron condition \((2.32) \) and the QCD sphaleron condition \((2.33) \) that now must be counted since it is independent from other constraints, the global hypercharge \((2.8) \) and \( \mathcal{R} \) \((2.31) \) neutrality conditions, for a total of 15 constraints for the 18 chemical potentials or corresponding particle density asymmetries considered in the previous section. However, in the NSE regime we have that the set of constraining conditions involve one additional independent quantity that is the sneutrinos density asymmetry \( Y_{\Delta \tilde{N}} \), and therefore the total number of relevant quantities is 19. We thus conclude that in the NSE regime the three flavour charges \( Y_{\Delta \alpha} \) in Eq.\((2.19) \) do not suffice to describe the density asymmetries of all MSSM particles and superparticles, and that \( Y_{\Delta \tilde{N}} \) is also required.

To proceed, we can now set \( h_u \rightarrow 0 \) to match the actual number of equilibrium constrains in the NSE regime. We lose one Yukawa condition, but we gain the corresponding chiral-\( R_3 \) conservation condition \((2.34) \) and, as expected, the overall number of independent chemical potentials or number density asymmetries remains the same.

In the following we give some results corresponding to temperature ranges relevant for successful supersymmetric leptogenesis, that always occurs within the NSE regime. Given that the Yukawa conditions for the leptons and for the down-type quarks depend on the value of \( \tan \beta \), the temperature ranges we refer to is only indicative (and correspond to moderate values of \( \tan \beta \)), but we will always specify in a clear way which conditions are used to obtain the corresponding results. In the non-superequilibration regime there are different flavour mixing matrices for the scalar and fermion components of the leptons.
and Higgs supermultiplets. To express more concisely all the results, it is convenient to introduce a new $C$ vector to describe the gaugino number density asymmetry per degree of freedom in terms of the relevant charges:

$$Y_{\Delta\tilde{g}} = C^\tilde{g}_a Y_{\Delta_a} , \quad \text{with} \quad \Delta_a = (\Delta_\alpha, \Delta_{\tilde{N}}).$$ (2.39)

### 2.3.3 First generation Yukawa reactions out of equilibrium (NSE regime).

In the temperature range between $10^8$ and $10^{11}$ GeV, and for moderate values of $\tan\beta$, all the first generation Yukawa couplings can be set to zero. Using for $u, d$ conditions (2.34) and for $e$ condition (2.38) as are implied by $h_{u,d}, h_e \to 0$ we obtain:

$$A^\ell = \frac{1}{9 \times 162332} \begin{pmatrix}
-198117 & 33987 & 33987 & -8253 \\
26634 & -147571 & 14761 & -8055 \\
26634 & 14761 & -147571 & -8055
\end{pmatrix},$$

$$C^\tilde{g} = \frac{-11}{162332} (163, 165, 165, -255),$$

$$C^{H_u} = \frac{-1}{162332} (3918, 4713, 4713, 95),$$

$$C^{H_d} = \frac{1}{3 \times 162332} (5413, 9712, 9712, -252),$$ (2.40)

where the rows correspond to $(Y_{\Delta_e}, Y_{\Delta_u}, Y_{\Delta_\tau}, Y_{\Delta_{\tilde{N}}})$. For completeness, in Eq. (2.40) we have also given the results for $C^{H_d}$ even if only the up-type Higgs density asymmetry is relevant for the leptogenesis processes. Note that neglecting the contribution of $\Delta n_{\tilde{N}_1}$ to the global charges $R_{tot}$ in Eq. (2.30) and $R_{3tot}$ in Eq. (2.36) corresponds precisely to set to zero the fourth column in all the previous matrices. Then, analogously with the SE and SM cases, within this ‘3-columns approximation’ all particle density asymmetries can be expressed just in terms of the three $Y_{\Delta_a}$ charge densities.

In the numerical analysis carried out in the next section we will confront the results obtained with the full matrices (2.40) with the results obtained for the analogous case discussed in Section 2.2.4 of first generation Yukawas out of equilibrium, but within the SE regime, that yielded the matrices in (2.23).

### 2.3.4 Second generation Yukawa reactions out of equilibrium: two flavour regime.

In the temperature range between $10^{11}$ and $10^{12}$ GeV we set $h_{\mu} \to 0$. This condition defines the two flavour regime for leptogenesis [8,10,11]. In this regime we have also $h_s, h_c \to 0$ that result in conserved $\chi_{Q_1} + \kappa_{Q_1} R_3$ charges ($q = s, c$) yielding the corresponding conditions (2.34). Since the first generation quark doublet does not mix with the other generations, $Q_1$ is not equal to $Q = Q_2 = Q_3$, but conservation of $\chi_{Q_1} + \kappa_{Q_1} R_3$ then provides the required new condition (2.35). Conservation laws stemming from $h_{e,\mu,u,d,s,c} \to 0$ then add up to a total of 7 constraints that replace the 6 Yukawa equilibrium conditions plus the condition $Q_1 = Q$.

In the two flavour regime only the lepton doublet $\ell_\tau$ is identified, while the $\ell_e$ and $\ell_\mu$ doublets, both of which have no Yukawa interactions, are not distinguished. In general,
one combination $\ell_{e\mu}$ of these two doublets remains coupled to lightest heavy singlet $N_1$, while the orthogonal combination decouples, so that $N_1$ decays do not generate directly a lepton asymmetry in this particular flavour direction. The corresponding non-anomalous $B/3 - L_\alpha$ charge is thus exactly conserved, and we can set its value to zero. This condition, together with the third generation Yukawa conditions, the two sphaleron conditions, and $Y$ and $R$ global neutrality, give 8 constraints for the remaining 10 chemical potentials $Q, b, t, \ell_\alpha, \tau, \tilde{H}_{u,d}, \tilde{g}$. Therefore, in this case the two flavour charge densities $Y_{\Delta e\mu}$ and $Y_{\Delta \tau}$ together with $Y_{\Delta \tilde{N}}$ suffice to express the density asymmetries for all the other particles. In the basis $(Y_{\Delta e\mu}, Y_{\Delta \tau}, Y_{\Delta \tilde{N}})$ we obtain:

$$A^\ell = \frac{1}{6 \times 580163} \begin{pmatrix} -460047 & 88166 & -17229 \\ 80783 & -349740 & -14094 \end{pmatrix},$$

$$C^{\tilde{g}} = \frac{1}{2 \times 580163} (-11056, -11786, 20307),$$

$$C^{\tilde{H}_{u,d}} = \frac{-1}{2 \times 580163} (69131, 70652, 4579),$$

$$C^{\tilde{B}_{u,d}} = \frac{1}{6 \times 580163} (8939, 77012, -12483).$$

Note that since EW sphaleron flavour mixing involves all the three lepton doublets, a non-vanishing lepton asymmetry (equal to $B/3$) is induced also in the decoupled flavour direction orthogonal to $\ell_{e\mu}$. However, since the corresponding doublets do not participate in the leptogenesis dynamics, this asymmetry is irrelevant for baryogenesis, and thus giving just the two entries corresponding to $\ell_{\tau}$ and $\ell_{e\mu}$ in the matrices $A$ and $C$ above is sufficient.

### 2.3.5 Above the EW sphaleron equilibration temperature: the $B = 0$ regime.

EW sphaleron processes take place at a rate per unit volume $\Gamma/V \propto T^4 \alpha_W^5 \log(1/\alpha_W)$ [32–35], and are expected to be in equilibrium up to temperatures of about $\sim 10^{12}$ GeV. Even for moderate values of $\tan \beta$, the $b$ and $\tau$ Yukawa interactions are still in equilibrium at this temperature. We will then consider here the temperature regime $10^{12} - 10^{13}$ GeV in which supersymmetric leptogenesis still occurs in the two flavour regime, but with EW sphalerons switched off. The related chemical equilibrium condition (2.32) is then replaced by baryon number conservation. We set $B = 0$ as initial condition since a dynamical generation of the baryon asymmetry is precisely the goal of leptogenesis. This also implies that $Y_{\Delta \alpha} = -Y_{\Delta L_\alpha}$ and therefore the total lepton asymmetry in the direction orthogonal
to $\ell_{e\mu}$ vanishes exactly. For this regime we obtain:

$$A_\ell = \frac{1}{6 \times 73327} \begin{pmatrix} -72807 & 1480 & -5676 \\ -77 & -50984 & -4236 \end{pmatrix},$$

$$C_\tilde{g} = \frac{1}{73327} (-130, -370, 1419),$$

$$C_{\tilde{H}_u} = \frac{-1}{2 \times 73327} (9187, 9226, 686),$$

$$C_{\tilde{H}_d} = \frac{1}{6 \times 73327} (1531, 9998, -1482).$$

(2.42) (2.43) (2.44)

2.3.6 Above the temperature for $\tau$ Yukawa equilibration: the one flavour regime.

For moderate values of $\tan \beta$, at temperatures above $T \sim 10^{13}$ GeV we can set $h_\tau \rightarrow 0$. This condition defines the one flavour regime, in which only the dynamics of the lepton doublet $\ell_1$ to which $N_1$ couples is important. This allows us to set to zero the two lepton density asymmetries in the direction orthogonal to $\ell_1$: $Y_{\Delta L_1} = Y_{\Delta L'_1} = 0$ and we are left with just one non-vanishing lepton flavour asymmetry that is $Y_{\Delta L_1}$. On the other hand, approximate $b - \tau$ Yukawa unification suggests that we should also set $h_b \rightarrow 0$, so that only the $h_t$ Yukawa condition (2.11) remains. Since quark mixing between the second and third generation is, for small $\tan \beta$, of the same order than the ratio $h_b/h_t \sim \mathcal{O}(10^{-2})$, we assume that $Q_2$ is also decoupled from Yukawa interactions and thus we replace the condition $Q_2 = Q_3$ by the conservation of the corresponding charge $\chi_{Q_2} + \kappa Q_2 R_3$, that yields the constraint (2.35).

Note that even if in this regime all lepton flavour effects can be neglected [8, 10, 11], not all spectator effects can [7]. For example, QCD sphaleron equilibration is maintained up to higher temperatures than the corresponding electroweak processes because they have larger rates: $\Gamma_{QCD}/\Gamma_{EW} \simeq 12(\alpha_s/\alpha_W)^{5}$, and are likely to be still in equilibrium at $T \sim 10^{13}$ GeV [35–37]. Since $Y_{\Delta \tilde{g}}$ remains coupled to the lepton flavour dynamics because of its contribution to $R_{tot}$ and $R_{3tot}$ we still have matrices with two columns ($-Y_{\Delta L_1}, Y_{\Delta \tilde{g}}$) even in the one flavour regime. After imposing all the relevant conservation conditions, including the vanishing of lepton number in two flavour directions, we obtain:

$$A_\ell = -\frac{1}{295} (49, 4), \quad C_{\tilde{g}} = \frac{1}{4 \times 295} (-1, 24),$$

$$C_{\tilde{H}_u} = \frac{-7}{16 \times 295} (49, 4), \quad C_{\tilde{H}_d} = \frac{1}{6 \times 295} (1, -24).$$

(2.45)

Before concluding this section, it is worth reminding that the conversion of the $B - L$ asymmetry generated by the leptogenesis dynamics into a baryon asymmetry should eventually be computed at a temperature $\mathcal{O}(100 \text{GeV})$, that is right before EW sphaleron transitions are switched off. At this temperature presumably all the supersymmetric particles have already decayed, and the particle content of the thermal bath is then the same than in the
SM with two Higgs doublets. Depending if this temperature is higher or lower than the temperature $T_{\text{EWPT}}$ of the EW phase transition, the conversion factors are [38]:

$$Y_{\Delta B} = Y_{\Delta B - L} \times \begin{cases} \frac{8}{10} & T > T_{\text{EWPT}} \\ \frac{3\Delta}{31} & T < T_{\text{EWPT}} \end{cases}$$

(2.46)

3. Numerical Results

As we have seen in the previous section, supersymmetric leptogenesis is characterized by important qualitative differences with respect to SM leptogenesis. These differences arise because in the NSE regime, which is the relevant one to ensure that a sufficient amount of baryon asymmetry can be generated, supersymmetric leptogenesis cannot be treated by simply accounting for the new degrees of freedom in the leptogenesis dynamics. In this section we analyze the quantitative relevance of the new effects we have been discussing.

They are all related with washout effects that in the NSE regime are controlled by the $3 \times 4$ matrix $W_{\text{NSE}} = A_{\text{NSE}} + C_{\tilde{N}}$, while in the SE regime the corresponding matrix $W_{\text{SE}} = A_{\text{SE}} + C_{\text{SE}}$ is $3 \times 3$. We can distinguish three types of effects:

(i) The overall strength of the washout processes, that is mainly controlled by the diagonal entries of the $3 \times 3$ submatrix $W_{\text{NSE}}$, has slightly larger weights.

(ii) Lepton flavour mixing effects, that are controlled by the off diagonal entries in $W_{\text{NSE}}$ are sizeably larger in the NSE regime.

(iii) New contributions to the washout arise from mixing with $\Delta_{\tilde{N}}$, given by the entries in the fourth column $W_{\text{NSE}}^{4\times3}$.

One can get an intuitive handle about the quantitative impact of these effects on the final result for the baryon asymmetry, by comparing the overall washout coefficients evaluated in the SE regime for the case of the first generation Yukawa reactions out of equilibrium given in Eq. (2.23), to the corresponding coefficients in the NSE regime given in Eq. (2.40).

We find, approximately independently of the particular entry:

$$[W_{\text{NSE}}^{3\times3} - W_{\text{SE}}]_{\alpha\beta} \approx -W_{\text{NSE}}^{\alpha4} \approx -0.01.$$  

(3.1)

For the (larger) diagonal entries this difference remains somewhat below 10%; for the (smaller) off-diagonal entries in $W_{\text{NSE}}^{3\times3}$ this represents an $O(1)$ difference, while the $W_{\text{NSE}}^{4\times3}$ entries are specific of the NSE regime. We have found that in the regimes in which the washouts are rather strong, and depending on the specific flavour configuration, the first two effects can produce $O(1)$ changes in the final value of the baryon asymmetry. The last correction, although potentially of a similar size, gives some effects on the evolution of the flavour density asymmetries only for $z \lesssim 1$, that is when the $\tilde{N}$’s approach an equilibrium distribution. However, at $z > 1$, $Y_{\Delta_{\tilde{N}}}$ gets exponentially suppressed implying that basically no effects are left in the final result.

The results of our numerical analysis are summarized by the plots in figures I and II, that are obtained by integrating numerically the complete set of Boltzmann equations given...
in the Appendix. They include decays, inverse decays and scatterings with top-quarks, and hold under the assumption that the $N$'s are hierarchical and that the lepton asymmetries only result from the decays of the lightest heavy singlet states $N \equiv N_1$. However, in order to illustrate how the new effects described above modify the structure of the equations, here we write much simpler expressions in which only decays and inverse decays are included:

$$
\dot{Y}_N = - \left( \frac{Y_N}{Y_{eq}^N} - 1 \right) \gamma_N, \quad (3.2)
$$

$$
\dot{Y}_{N_+} = - \left( \frac{Y_{N_+}}{Y_{eq}^N} - 2 \right) \gamma_{N_+}, \quad (3.3)
$$

$$
\dot{Y}_{\Delta_N} = - \frac{Y_{\Delta_N}}{Y_{eq}^N} \gamma_{\Delta_N} - \frac{3}{2} \gamma_{\Delta_N} \sum_a C_g^a \frac{Y_{\Delta_a}}{Y_{eq}^{\Delta a}} + \ldots, \quad (3.4)
$$

$$
\dot{Y}_{\Delta_a} = - \epsilon_a \left[ \left( \frac{Y_N}{Y_{eq}^N} - 1 \right) \gamma_N + \left( \frac{Y_{N_+}}{Y_{eq}^{N_+}} - 2 \right) \gamma_{N_+} \right] + \left( \frac{\gamma_{\Delta_N}}{Y_{eq}^N} \right) \gamma_{\Delta_N} \sum_a \left( A_\alpha^a + C_{H_a}^a + C_{g_a}^a \right) \frac{Y_{\Delta_a}}{Y_{eq}^{\Delta a}}. \quad (3.5)
$$

In the equations above the time derivative is defined as $\dot{Y} \equiv zsH dY/dz$ with $s$ the entropy density, $z = M/T$ and $H \equiv H(M)$ the expansion parameter at $T = M$. In Eq. $(3.3)$ we have introduced the overall sneutrino abundance $Y_{N_+} = Y_N + Y_{N_+}$, while $Y_{\Delta_N} \equiv Y_N - Y_{N_+}$, in Eq. $(3.4)$ is the sneutrino density asymmetry that was already introduced in Section 2.3.2. In the washout terms we have normalized the charge densities $Y_{\Delta_a} = (Y_{\Delta_a}, Y_{\Delta_N})$ to the equilibrium density of a fermion with one degree of freedom $Y_\ell$, and we refer to the Appendix for detailed definitions of the reaction densities and associated quantities. In Eqs. $(3.2)-(3.5)$ we have also neglected for simplicity all finite temperature effects. Taking these effects into account would imply for example that the CP asymmetry for $\tilde{N}$ decays into fermions is different from the one for decays into scalars, while we describe both CP asymmetries with $\epsilon_\alpha$. A few remarks regarding Eq. $(3.4)$ are in order. In the SE regime $\tilde{g} = 0$ and thus it would seem that the sneutrino density asymmetry $Y_{\Delta_N}$ vanishes. However, this only happens for decays and inverse decays, and it is no more true when additional terms related to scattering processes, that are represented in the equation by the dots, are also included (see ref. [18] and the Appendix). Therefore, also in the SE regime $Y_N$ and $Y_{\tilde{N}}$, in general differ. However, in this case recasting their equations in terms of two equations for $Y_{N_+}$ and $Y_{\Delta_N}$ is just a convenient parametrization. On the contrary, in the NSE regime this is mandatory, because the sneutrinos carry a globally conserved $R$-charge and $Y_{\Delta_N}$ is required to formulate properly the corresponding conservation law. As we have seen, this eventually results in $Y_{\Delta_N}$ contributing to the expressions of the lepton flavour density asymmetries in terms of slowly varying quantities.

The results for the final baryon asymmetry (normalized to the total CP asymmetry $\epsilon = \sum_\alpha \epsilon_\alpha$) are depicted in Figure I. We have fixed the value of the $N$ mass to $M = 10^{10}$ GeV that is, we work in the three flavour regime in which all the Yukawa couplings of the fermions of the first generation can be set to zero and the results of Section 2.3.3
\[ P_e = P_\mu = P_\tau = \frac{1}{3}, \quad \epsilon_\alpha = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \]

(a) \( M = 10^{10} \text{ GeV} \)
\[ \tilde{m}_1 = 0.1 \text{ eV} \]

NSE

SE

\[ \frac{|Y_{\Delta B}/\epsilon|}{z} \]

| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |
| --- | --- | --- | --- |
| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |

(b) \( M = 10^{10} \text{ GeV} \)
\[ \tilde{m}_1 = 0.1 \text{ eV} \]

NSE

SE

\[ \frac{|Y_{\Delta B}/\epsilon|}{z} \]

| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |
| --- | --- | --- | --- |
| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |

(c) \( M = 10^{10} \text{ GeV} \)
\[ \tilde{m}_1 = 5 \times 10^{-4} \text{ eV} \]

NSE

SE

\[ \frac{|Y_{\Delta B}/\epsilon|}{z} \]

| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |
| --- | --- | --- | --- |
| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |

(d) \( M = 10^{10} \text{ GeV} \)
\[ \tilde{m}_1 = 5 \times 10^{-4} \text{ eV} \]

NSE

SE

\[ \frac{|Y_{\Delta B}/\epsilon|}{z} \]

| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |
| --- | --- | --- | --- |
| \( z \) | \( 10^{-2} \) | \( 1 \) | \( 10 \) |

**Figure I:** The baryon asymmetry normalized to the total CP asymmetry \(|Y_{\Delta B}/\epsilon|\) computed in the three flavour NSE regime with the \( A \) and \( C \) matrices in (2.40) (solid red lines) confronted with what is obtained in the SE regime (2.23) (dashed blue lines). The two upper panels (a) and (b) correspond to strong washouts with \( \tilde{m}_1 = 0.1 \text{ eV} \), the two lower panels (c) and (d) to weak washouts with \( \tilde{m}_1 = 5 \times 10^{-4} \text{ eV} \). In the two panels (a) and (c) on the left a flavour equipartition configuration has been assumed, with flavoured parameters \( \epsilon_\alpha/\epsilon = \frac{P_\alpha}{1} = 1/3 \). In the two panels (b) and (d) on the right a strongly misaligned flavour configuration has been used, with \( \epsilon_e = 0, \epsilon_\mu = -2\epsilon_\tau = \epsilon \) and \( P_e = 1/100, P_\mu = 9/100, P_\tau = 90/100 \).

\[
\tilde{m}_1 = \sum_\alpha \frac{|\lambda_{1\alpha}|^2 v_u^2}{M} \equiv \sum_\alpha \tilde{m}_{1\alpha} \equiv \sum_\alpha P_\alpha \tilde{m}_1, \quad (3.6)
\]

where \( v_u \) is the vacuum expectation value of the up-type Higgs doublet, \( v_u = v \sin \beta \) (\( v=174 \text{ GeV} \)).\(^{\text{I}}\) For the single lepton flavours, the corresponding parameters are defined in terms of the tree level \( N \) and \( \tilde{N} \) decays branching ratios \( P_\alpha = |\lambda_{1\alpha}|^2/(\lambda \lambda^\dagger)_{11} \) according to:

\[
\tilde{m}_{1\alpha} \equiv P_\alpha \tilde{m}_1, \quad \sum_\alpha P_\alpha = 1. \quad (3.7)
\]

\(^{\text{I}}\)Once the set of Yukawa conditions used to derive the \( A \) and \( C \) matrices is established, the dominant dependence on \( \tan \beta \) in the numerical results arises via \( v_u \) in Eq. (3.6) and therefore it is very mild.
In Figure I, the solid red lines represent the results obtained with the NSE $A$ and $C$ matrices in Eq. (2.40), while the dashed blue lines give the corresponding results obtained assuming SE and using $A$ and $C$ in Eq. (2.23). In the two upper panels (a) and (b) we give the results obtained assuming a particularly strong washout regime corresponding to $\tilde{m}_1 = 0.1 \text{ eV}$, while the two lower panels (c) and (d) correspond to a weak washout regime with $\tilde{m}_1 = 5 \times 10^{-4} \text{ eV}$. In the attempt to disentangle the differences between NSE and SE due to the changes in the overall washout strength [effects of type (i)] from those due to the changes in the flavour mixing pattern [effects of type (ii)] we have assumed for the two panels (a) and (c) on the left a flavour equipartition configuration $[\frac{\epsilon}{\epsilon} = \frac{1}{3}; P_{\alpha} = \frac{1}{3}]$ while for the two panels (b) and (d) on the right we have assumed a strongly misaligned flavour configuration $[\frac{\epsilon}{\epsilon} = (0, 2, -1); P_{\alpha} = (\frac{1}{100}, \frac{9}{100}, \frac{90}{100})]$. In the first case the differences in the flavour mixing patterns produce irrelevant effects, and the changes that can be seen are essentially due to the different flavour diagonal washouts. In the second case the differences in the patterns of flavour mixing also contribute. As it is apparent from figure I, the numerical differences between the NSE and SE cases remain typically at the $\mathcal{O}(1)$ level. As regards the effects of type (iii) that are related to $Y_{\Delta N}$, we have verified that in all the plots in figure I any visible difference would appear only around $z \lesssim 1$, leaving the final results unchanged. However, for completeness, we present in figure II the detailed evolution of the single flavour charge densities $Y_{\Delta \alpha}$ and also of $Y_{\Delta N}$ corresponding to the flavour/washout configuration of figure I(b). As in the previous figure, the NSE results are depicted with solid lines [from up to down and around $z = 0.5$: $Y_{\Delta \mu}$ (blue), $Y_{\Delta \tau}$ (magenta), $Y_{\Delta N}$ (black), $Y_{\Delta e}$ (red)] and the SE results are depicted with dashed lines. The thicker portion of each line corresponds to positive values of the corresponding asymmetry, while the thinner portion to negative values. A couple of interesting features can be identified: firstly, the largest differences occur for $Y_{\Delta e}$ and this is because given that $\epsilon_e = 0$, this asymmetry evolves solely because of flavour mixing effects; secondly, it can be seen how $Y_{\Delta N}$ gets strongly suppressed around the time when the flavour asymmetries ‘freeze out’ ($z \sim 5$), and this explains the irrelevance of its effects on the final result. One can also notice that curiously in the NSE regime $Y_{\Delta N}$ changes sign twice, while only once in SE. This is, of course, due to the related mixing effects that are absent in the SE case.

4. Discussion and Conclusions

Motivated by all the recent advancements in leptogenesis studies, we have revisited the theory of supersymmetric leptogenesis in the attempt to put it on a more firm theoretical ground. By digging in some depth into the analysis of the various conditions that constrain the density asymmetries of the different particles in the thermal bath, we have found that important qualitative differences exist with respect to SM leptogenesis. We have first clarified the reasons why, even if the specific constraining conditions are different for the different temperature ranges in which leptogenesis can occur, the whole set of particle density asymmetries can be always expressed in terms of the same number of independent quantities. This fact is explained in the following way: whenever it happens that by raising the temperature a chemical equilibrium condition ceases to hold because
the particle reaction enforcing it goes out of equilibrium, then one Lagrangian parameter related to this out-of-equilibrium reaction can be set to zero. This generically results in a new global symmetry, and the associated conservation law then enforces a new condition for the particle number densities that replaces the one from chemical equilibrium. Following this scheme, we have identified new symmetries that are specific of the supersymmetric leptogenesis case. New chemical potentials associated to the superpartners of the SM particles are also a generic feature of supersymmetric leptogenesis, and a quite crucial example is provided by the gauginos. At the typical high temperatures relevant for leptogenesis, chirality flipping transitions for the gauginos are completely out of thermal equilibrium and basically do not occur. Gauginos thus develop a chemical potential that can be thought of as the difference between the number of L- and R-handed states. Three main consequences follow from this. The first one is that particle and sparticle chemical potentials are different during leptogenesis. This is because only at relatively low temperatures their chemical potentials are equilibrated by scatterings. However, the rates of these
scatterings vanish in the limit of vanishing gaugino masses, and are completely negligible at the high temperatures relevant for leptogenesis. This modifies the weights of the scalars and fermions-related washouts. The second one is that by setting to zero the gaugino mass, a new $R$-symmetry arises. While this symmetry is anomalous, a suitable anomaly free combination can be constructed and this corresponds to an exactly conserved global charge. The related conservation constraint, that is specific to supersymmetry, modifies the $A$ and $C$ matrices that describe the mixing between lepton flavours induced by the EW sphalerons. The third one is that since sneutrinos carry a unit charge under the $R$ symmetry, their number density asymmetry enters the corresponding conservation law. The result is that through the $A$ and $C$ matrices the lepton flavour asymmetries are now related to four independent quantities rather than the usual three flavour charges of the SM case, being the sneutrino density asymmetry the fourth one.

While we believe that the results presented in this paper are quite interesting from the theoretical point of view, by comparing the baryon asymmetry yield of leptogenesis within our framework with the results obtained by neglecting all the new effects, we have concluded that quantitatively we are dealing with $\mathcal{O}(1)$ corrections. This is hardly surprising since, quite in general, sizable changes in the baryon asymmetry generated through leptogenesis are related with the identification of new sources of CP violation or by spoiling some exact cancellation. While this is the case e.g. for flavoured leptogenesis [8,10,11] that requires the inclusion of the flavour CP asymmetries $\epsilon_\alpha$, or for soft leptogenesis [27–29] where new CP violating phases play a crucial role and thermal effects spoil an exact zero-temperature cancellation between the CP asymmetries for decays into scalars and fermions, in our case there are no changes in the amount of CP violation that drives leptogenesis nor cancellations that could be spoiled.

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A. Boltzmann equations for supersymmetric leptogenesis

The Boltzmann equations that we have used to derive the numerical results read:

\[
\begin{align*}
\dot{Y}_N &= -\left( \frac{Y_N}{Y_{eq}^N} - 1 \right) \left( \gamma_N + 4\gamma_t(0) + 4\gamma_t(1) + 4\gamma_t(2) + 2\gamma_t(3) + 4\gamma_t(4) \right), \\
\dot{Y}_{\tilde{N}_+} &= -\left( \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} - 2 \right) \left( \gamma_{\tilde{N}} + 3\gamma_{22} + 2\gamma_t(5) + 2\gamma_t(6) + 2\gamma_t(7) + \gamma_t(8) + 2\gamma_t(9) \right), \\
\dot{Y}_{\Delta_N} &= -\frac{Y_{\Delta_N}^2}{Y_{eq}^N} \left( \gamma_{\tilde{N}} + 3\gamma_{22} + 2\gamma_t(5) + 2\gamma_t(6) + 2\gamma_t(7) + \gamma_t(8) + 2\gamma_t(9) \right) + \frac{1}{2} \left( \frac{Y_{\Delta N}^2}{Y_{eq}^N} - \frac{Y_{\Delta t}^2}{Y_{eq}^N} \right) \gamma_{\tilde{N}} \\
&\quad + \frac{Y_{\Delta t}^2}{Y_{eq}^N} \left[ 2 + \frac{1}{2} \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \right] \gamma_{22} + \frac{1}{2} \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \gamma_t(5) + 1 \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \gamma_t(6) + \gamma_t(7) \\
&\quad + 2 \frac{Y_{\Delta H_u}^2}{Y_{eq}^N} \left[ 2 + \frac{1}{2} \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \right] \gamma_{22} + \frac{1}{2} \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \gamma_t(5) + 1 \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} \gamma_t(6) + \gamma_t(7) \\
&\quad - \frac{\epsilon_\alpha}{2} \left( \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} - 2 \right) \left( \gamma_N + 4\gamma_t(0) + 4\gamma_t(1) + 4\gamma_t(2) + 2\gamma_t(3) + 4\gamma_t(4) \right) \\
&\quad + \epsilon_\alpha \left( \frac{Y_{\tilde{N}_+}^1}{Y_{eq}^N} - 2 \right) \left( \gamma_N + 3\gamma_{22} + 2\gamma_t(5) + 2\gamma_t(6) + 2\gamma_t(7) + \gamma_t(8) + 2\gamma_t(9) \right), \\
\end{align*}
\]

Besides the decays and inverse decays included in Eqs. (3.3) - (3.5), these equations also
include scatterings with the top-quark, both in the washout and in the CP asymmetries. In the limit in which neutrinos are sufficiently hierarchical, as we are assuming here, the CP asymmetries in scatterings with top quarks are the same than the CP asymmetries in decays \cite{12, 13} and then can be easily included. We have not included gauge boson scatterings nor the corresponding CP asymmetries. We have neglected three body decays since their contribution is phase space suppressed, and also because we have found that in ref. \cite{18}, on which our equations are based, some diagrams related to these processes have been overlooked. $\Delta L = 2$ scatterings mediated by off-shell singlet neutrinos are also left out since in the temperature range $T \sim 10^{10}$ GeV in which our results are obtained they are completely irrelevant. We have also neglected all finite temperature effects except for the Higgs thermal mass that is kept to regulate the infrared divergences in scatterings with the top-quark.

We define the equilibrium densities per degree of freedom normalized to the entropy density as:

$$Y_{\ell}^{eq} = \frac{1}{2} Y_{\ell}^{eq} = \frac{1}{2} Y_{H_u}^{eq} = Y_{H_u}^{eq} = \frac{15}{8\pi^2 g_*},$$  \hspace{1cm} (A.5)

with the MSSM number of effective degrees of freedom $g_* = 228.75$. The number density asymmetries are defined according to $Y_{\Delta L} = Y_{\ell} - Y_{\ell}$. Density asymmetries and reaction densities without an index are understood to be summed over flavours: $Y_{\Delta L} = \sum_\alpha Y_{\Delta L,\alpha}$ and $\gamma_N = \sum_\alpha \gamma_N^\alpha$. In Eqs. \eqref{eq:3.2}-\eqref{eq:3.3} the density asymmetries for scalars $Y_{\Delta\tilde{\alpha},u}$ and $Y_{\Delta\tilde{\alpha},H_u}$ can be expressed in terms of the corresponding asymmetries for fermions $Y_{\Delta\tilde{\alpha},\ell}$ and $Y_{\Delta\tilde{\alpha},H_u}$ according to:

$$\frac{Y_{\Delta\tilde{\alpha},u}}{Y_{\ell}^{eq}} = \frac{Y_{\Delta\tilde{\alpha},H_u}}{Y_{H_u}^{eq}} = \frac{Y_{\Delta\tilde{\alpha}}}{Y_{\ell}^{eq}} + \frac{Y_{\Delta\tilde{\alpha}}}{Y_{H_u}^{eq}},$$ \hspace{1cm} (A.6)

$$\frac{Y_{\Delta\tilde{\alpha},H_u}}{Y_{H_u}^{eq}} = \frac{Y_{\Delta\tilde{\alpha},\ell}}{Y_{\ell}^{eq}} + \frac{Y_{\Delta\tilde{\alpha}}}{Y_{H_u}^{eq}},$$ \hspace{1cm} (A.7)

where $Y_{\ell}^{eq} = Y_{\ell}^{eq} = Y_{\ell}^{eq}$. In the SE regime, $Y_{\Delta L} = 0$ and thus $\frac{Y_{\Delta\tilde{\alpha},\ell}}{Y_{\ell}^{eq}} = \frac{Y_{\Delta\tilde{\alpha},\ell}}{Y_{\ell}^{eq}}$ and $\frac{Y_{\Delta\tilde{\alpha},H_u}}{Y_{H_u}^{eq}} = \frac{Y_{\Delta\tilde{\alpha},H_u}}{Y_{H_u}^{eq}}$ follow. The reaction densities entering Eqs. \eqref{eq:A.1}-\eqref{eq:A.4} are:

$$\gamma_N^\alpha \equiv \gamma \left( N \leftrightarrow \tilde{\ell}_\alpha H_u \right) + \gamma \left( N \leftrightarrow \tilde{\ell}_\alpha H_u \right) + \gamma \left( N \leftrightarrow \ell_\alpha H_u \right) + \gamma \left( N \leftrightarrow \tilde{\ell}_\alpha H_u \right),$$

$$\gamma_N^\alpha \equiv \gamma \left( N \leftrightarrow \tilde{\ell}_\alpha H_u \right) + \gamma \left( N \leftrightarrow \tilde{\ell}_\alpha H_u \right) + \gamma \left( N \leftrightarrow \ell_\alpha H_u \right) + \gamma \left( N \leftrightarrow \ell_\alpha H_u \right),$$

$$\gamma_{\tilde{\alpha},Q}^\alpha \equiv \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow \tilde{Q} \right) + \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow \tilde{Q} \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow \tilde{Q} \right),$$

$$\gamma_{\tilde{\alpha}^{(0)}\alpha} \equiv \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right),$$

$$\gamma_{\tilde{\alpha}^{(1)}\alpha} \equiv \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right),$$

$$\gamma_{\tilde{\alpha}^{(2)}\alpha} \equiv \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right),$$

$$\gamma_{\tilde{\alpha}^{(3)}\alpha} \equiv \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right) = \gamma \left( N \tilde{\ell}_\alpha \leftrightarrow Q \right).$$
\[ \gamma_t^{(4)} \equiv \gamma \left( Nu \leftrightarrow \ell_\alpha Q \right) = \gamma \left( N \overline{Q} \leftrightarrow \ell_\alpha \overline{\nu} \right), \]
\[ \gamma_t^{(5)} \equiv \gamma \left( \tilde{N} \ell_\alpha \leftrightarrow Q \overline{\nu}^* \right) = \gamma \left( \tilde{N} \ell_\alpha \leftrightarrow Q \overline{\nu} \right), \]
\[ \gamma_t^{(6)} \equiv \gamma \left( \tilde{N} \overline{u} \leftrightarrow \ell_\alpha Q \right) = \gamma \left( \tilde{N} \overline{u}^* \leftrightarrow \ell_\alpha Q \right), \]
\[ \gamma_t^{(7)} \equiv \gamma \left( \tilde{N} Q \leftrightarrow \ell_\alpha \overline{u}^* \right) = \gamma \left( \tilde{N} u \leftrightarrow \ell_\alpha \overline{\nu} \right), \]
\[ \gamma_t^{(8)} \equiv \gamma \left( \tilde{N} \ell_\alpha^* \leftrightarrow Q \nu \right), \]
\[ \gamma_t^{(9)} \equiv \gamma \left( \tilde{N} Q \leftrightarrow \ell_\alpha u \right) = \gamma \left( \tilde{N} \nu \leftrightarrow \ell_\alpha Q \right). \]  

(A.8)

The reduced cross sections for the processes listed above can be found in ref. [18].

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