Multicast Transmissions With Full-Duplex Amplify-and-Forward User Cooperation

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ABSTRACT In order to improve the multicast transmissions, a full-duplex (FD) user cooperation scheme is proposed. In this paper, the transmitter sends the common messages to two FD users, and each user forwards its received signals to its counterpart by amplify-and-forward (AF) scheme. Considering the imperfect self-interference (SI) cancellation at the users and the AF scheme, the residual self-interference and the additive noise (RIAN) is shown to be accumulated over time. This paper first analyzes the equivalent channel model of the considered system, as well as the asymptotic statistics of the accumulated RIAN. Then, with the forward and backward decoding schemes, the corresponding achievable rates are derived, and the optimal power allocation is obtained via solving a max-min problem. In particular, one-side cooperation scheme (i.e., only one user forwards its received signal to its counterpart) is shown to be optimal to achieve the best system performance.

INDEX TERMS Multicast, cooperative communication, full-duplex (FD), amplify-and-forward (AF), optimal power allocation.

I. INTRODUCTION

Data services in cellular networks, such as music sharing, video streaming, and downloading pictures, are growing explosively in recent years, and will continue to increase in the future [1]. One of the key features for those data services is that the different users may request to download the same information from the base station [2], [3]. Compared with the conventional unicast transmissions, i.e., the base station only serves one request during one-time slot, multicast transmissions, i.e., the base station serves multiple identical requests simultaneously, own the potential to dramatically reduce the energy consumption [2] and save the channel resources in the cellular systems [3].

Multicast transmissions have received widespread interest in wireless communications. The authors in [4] studied the capacity of the multicast channel, which is determined by the minimum received signal-to-noise ratio (SNR) among all the transmitter-user links. Thus, if any one of the links between the transmitter and the users suffers from a bad channel condition, the capacity of the considered multicast channel will become very low [4].

In cellular networks, the users may locate closely to each other [5], [6], and thus it is possible for them to help the transmissions of the user with worse channel condition.\(^1\) Therefore, the user cooperation [8]–[10], i.e., the user forwards its received signal to its counterpart, provides a new opportunity to improve the performance of multicast transmissions. The authors in [11]–[20] studied the half-duplex (HD) user cooperation schemes, where the two users can exchange received signals over two orthogonal channels. In [11], the multicast channel with HD decode-and-forward (DF) user cooperation

\(^1\)The channel conditions of the different users are uncorrelated, when antenna separation of half to one carrier wavelength is sufficient [7]. Thus, even if the users locate closely, their channel conditions are uncorrelated.
(i.e., the user first decodes the received signal, and re-encodes the data and forwards to its counterpart [21]), had been considered, and the achievable rate of the considered channel was obtained. In [12]–[14], the authors investigated the power allocation for multicast transmissions with the DF user cooperation scheme. The diversity order [14], [15] and the coding gain brought by user cooperation [15] was further discussed. In [16], the capacity of the multicast channels with DF user cooperation was derived for the cases with both the causally and the non-causally known channel state information, respectively. In [17], [22], the authors minimized the outage probability of the multicast transmission with the DF user cooperation scheme. The authors in [18]–[20] investigated the energy efficiency of the multicast transmissions with the DF user cooperation scheme, and the optimal power allocation was obtained for the users [18] and the transmitter [19], respectively.

By utilizing the full-duplex (FD) technique for user cooperation, the spectrum efficiency of the multicast communication can be potentially improved [23]–[27]. The authors in [23] investigated the capacity of the multicast channel with the FD user cooperation under the assumption that the self-interference (SI) is perfectly cancelled. Thus, the authors in [24]–[27] considered a more practical scenario that the FD communicating suffering from residual SI. The authors in [24] studied the cooperation schemes between two FD users by combining the conventional DF and compress-and-forward (CF) schemes. The authors in [25] proposed a non-orthogonal user cooperation scheme for the multicast channel to improve the previous work in [24], and showed that the new scheme owns better performance. In [26], the authors proposed the three FD user cooperation scheme based on the superpositions of the CF and DF schemes at the users, and further obtained the closed-form expressions for the information-theoretic upper and lower bounds on the achievable rate. The authors in [27] studied the user cooperation in cellular networks. The minimum SNR user was cooperated by a selected FD user with the DF scheme and proposed an algorithm to select the best FD user.

Compared with the DF and CF schemes in [24]–[27], the amplify-and-forward (AF) scheme (i.e., the user simply amplifies and forwards the received signal to its counterpart [21]) was shown to achieve the full diversity order and low-complexity [28]; however, by utilizing AF, the users also forward their received noises to each other, which brings a new challenge on the system design. To our best knowledge, there is no work discussing the FD user cooperation with AF scheme to improve the multicast transmissions.

Motivated by the above reasons, this paper considers the FD user cooperation with the AF scheme for multicast transmissions. In particular, the transmitter sends common messages to the two users, and each FD user forwards its received signals to its counterpart by the AF scheme at the next time slot. Thus, each user simultaneously receives the signals from the transmitter, its counterpart user, and itself. Notice that due to the imperfect SI cancellation at the users and the AF scheme, it will cause the accumulation of the residual self-interference and additive noise (RIAN) over time at the two users. It might degrade system performance. The main contributions are summarized as follows:

- First, the transmission and reception signal model is derived for the FD user cooperation with the AF scheme, and it is shown to be equivalent to a two-tap channel model at each user. Then, we compute the accumulated RIAN, which for each user forms a Markov process, and calculate its asymptotic statistics.
- Then, we obtain the achievable rate for the considered channel with both the forward and backward decoding schemes, respectively, and formulate the achievable rate maximization problem. We prove that one-side cooperation (i.e., only one user forwards its received signal to its counterpart) is optimal. Based on this result, the original rate maximization problem is solved by using the max-min technique and fractional programming. We obtain the optimal power allocation under one-side cooperation.

The structure of this paper is shown as follows. The system model is introduced in Section II. Section III calculates the auto-correlations of the accumulated RIAN, and formulates the achievable rate maximization problem. Section IV computes the optimal power allocation for the considered problem. Section V shows the numerical results. Section VI concludes this work.

II. SYSTEM MODEL

This paper studies a user cooperation scheme for the multicast transmissions, as shown in Fig. 1. User $j, j \in \{1, 2\}$, is denoted as the counterpart of user $j$ (i.e., when $j = 1$, it follows $j = 2$; and when $j = 2$, it follows $j = 1$). The transmitter sends the common messages to the two users. User $j, j \in \{1, 2\}$, first forwards its received signal to its counterpart, i.e., user $j$, by the AF scheme. The user cooperation is established via in-band FD communication between the two users: The users are capable to simultaneously transmit and receive at the same frequency band, and each user can receive the signal from its counterpart as well as that from itself. The link between the two users shares the same frequency band as the other links between the transmitter and users. Moreover, the channel coefficients among the transmitter and users are fixed after channel realizations and exactly known to the transmitters. After receiving the signals from both the transmitter and the counterpart user, user $j$ decodes its desired message. Obviously, there exists one-time slot delay at the users due to the AF relaying operation. Thus, $N – 1$ common messages will cost in a total of $N$ time slots to be transmitted. Our goal is to study the average achievable rate for the considered system, as $N$ goes to infinity.

Under the above setup, the signal transmission process is introduced as follows. At the first time slot, the transmitter sends signal $\sqrt{P_x}(1)$ to user $j$, where $x(1)$ is a common message that need to send at the first time slot, $\sqrt{P_x}$ is the power...
allocated to $x(1)$. User $j$ receives $r_j(1) = h_j \sqrt{p_s} x(1) + z_j(1)$, where $z_j(1)$ is the circularly symmetric complex Gaussian (CSCG) noise, and $h_j$ is the channel coefficient from transmitter to user $j$ [29].

At the 2-nd time slot, the transmitter sends $\sqrt{p_s} x(2)$ to user $j$, and user $j$ forwards $t_j(2) = \alpha_j [h_j \sqrt{p_s} x(1) + z_j(1)]$ to its counterpart, where $x(2)$ is a new common message, and $\alpha_j$ is a factor to control the transmission power of user $j$. Meanwhile, user $j$ receives $\sqrt{p_s} x(2)$ from the transmitter, $t_j(2)$ from its counterpart, and $t_j(2)$ from itself, where $t_j(2)$ is treated as SI and can be at least partially cancelled [30]. However, the part of $t_j(2)$ will remain in the received signal due to the imperfection of the SI cancellation. After the imperfect SI cancellation, the residual SI and noise $z_j(2)$ will be forwarded to the counterpart user, which will cause the accumulation of these signals in the rest of the $N$ time slots.

The detailed descriptions for the transmissions and receptions in the $i$-th time slot at the transmitter and user $j$, $j \in \{1, 2\}$, are introduced in the following three subsections.

A. TRANSMISSIONS AT TRANSMITTER AND USERS

In the $i$-th time slot, the transmitter sends $\sqrt{p_s} x(i)$, where $x(i)$ is a common message that need to send at the $i$-th time slot with zero mean and unit variance, i.e., $\mathbb{E}[|x(i)|^2] = 1$.

The transmit signal $t_j(i)$ at user $j$ is given as

$$t_j(i) = \sqrt{\alpha_j} y_j(i - 1),$$

where $y_j(i - 1)$ is the signal after the SI cancellation from the received signal $r_j(i - 1)$ given in (3). We will derive the expressions of $y_j(i)$ and $r_j(i)$ later in subsection II-B.

Since we consider the long term average achievable rate of the multicast channel, the average transmission power $p_j$ at user $j$ is defined as

$$\lim_{i \to \infty} \mathbb{E}[|\sqrt{\alpha_j} y_j(i)|^2] = p_j \leq p_{\text{max}},$$

where $p_{\text{max}}$ is the power budget.

B. RECEPTION AT USERS

The received signal $r_j(i)$ at user $j$ is composed of $x(i)$ from the transmitter, $t_j(i)$ from user $j$, SI $t_j(i)$ from itself, and CSCG noise $z_j(i)$, i.e.,

$$r_j(i) = h_j \sqrt{p_s} x(i) + g_{jj} t_j(i) + h_t j (i) + z_j(i),$$

where $h_j$ is the channel coefficient from the transmitter to user $j$, $g_{jj}$ is the cooperative channel coefficient from user $j$ to user $j$, $h_t j$ is the SI channel coefficient of user $j$, and $z_j(i)$ is the CSCG noise with zero mean and variance at user $j$. The channel coefficients $h_j$, $g_{jj}$, and $h_t j$ are constants, and they have been estimated before transmission.

Then, by substituting (1) and (6) into (3), it follows

$$r_j(i) = h_j \sqrt{p_s} x(i) + g_{jj} t_j(i) + h_t j (i) + z_j(i),$$

where $\hat{y}_j(i - 1)$ is the accumulated RIAN for user $j$ at the $(i - 1)$-th time slot and will be rigorously defined later in this section. Here, for $i \leq 0$, we obtain $x(i) = 0$ and $\hat{y}_j(i) = 0$.

C. SI CANCELLATION AT USERS

At user $j$, SI $h_t j (i)$ needs to be cancelled from the received signal $r_j(i)$. Due to the imperfection of the SI cancellation, residual SI $\hat{r}_j(i)$ will be remained after the SI cancellation. According to [30], [31], residual SI $\hat{r}_j(i)$ is modeled by additive white Gaussian noise with zero mean and variance $\eta_j p_j$, and it is independent of SI $t_j(i)$, where $\eta_j$ is the SI cancellation ratio. Thus, after the SI cancellation, the residual signal $y_j(i)$ of user $j$ at $i$-th time slot is given as

$$y_j(i) = h_j \sqrt{p_s} x(i) + g_{jj} \sqrt{\alpha_j} h_j \sqrt{p_s} x(i - 1) + g_{jj} \sqrt{\alpha_j} h_j \sqrt{p_s} x(i - 2) + g_{jj} \sqrt{\alpha_j} \hat{y}_j(i - 1) + h_t j (i) + z_j(i),$$

where the accumulated RIAN $\hat{y}_j(i)$ is defined as

$$\hat{y}_j(i) = g_{jj} \sqrt{\alpha_j} h_j \sqrt{p_s} x(i - 2) + g_{jj} \sqrt{\alpha_j} \hat{y}_j(i - 1) + \hat{r}_j(i) + z_j(i).$$
Remark 1: From (6) and (7), we observe that:
1) Equivalent Channel: The equivalent channel model for user $j$ is similar to the two-tap channel [32]. Due to the one-time slot delay introduced by the AF scheme, $x(i)$ will be interfered by the previous transmit signal $x(i-1)$, which may cause some trouble in the decoding at the users.
2) Transmission Power at Users: According to (6), the transmission power $p_j$, $j \in \{1, 2\}$, given in (2) can be rewritten as
\[
\lim_{i \to \infty} \mathbb{E} \left[ \left| \sqrt{\sigma_j} \hat{y}_j(i) \right|^2 \right] = \alpha_j \left[ |h_j|^2 p_s + |g_{jj}^\alpha|^2 |h_j|^2 p_s \right] + \lim_{i \to \infty} \mathbb{E} \left[ \left( \hat{y}_j(i) \right)^2 \right] = p_j, \tag{8}
\]
where the detailed descriptions of $\lim_{i \to \infty} \mathbb{E} \left[ \left( \hat{y}_j(i) \right)^2 \right]$ will be given in the next section.
3) Temporal Correlation of Accumulated RIAN: From (7), it is observed that the accumulated RIAN process $\{\hat{y}_j(i)\}$ is a Markov process [33]. The general formula of $\hat{y}_j(i)$ can be recursively computed from (7), and we obtain (9), as shown at the bottom of the page, where $\hat{y}_j(0) = 0$ and $z_i(i) = 0$, for $i \leq 0$, and $\lfloor \cdot \rfloor$ is the rounding down operator.

At the $N$-th time slot, the transmitter does not send any signal, and user $j$ receives the transmit signal $t_j(i)$ from user $j$ and the SI $\hat{y}_j(f(i))$ from itself.

III. ACHIEVABLE RATES

In this section, we characterize the corresponding achievable rates of the multicast transmissions with FD user cooperation for both the forward and backward decoding schemes. First, the statistics of the accumulated RIAN is analyzed.

A. ASYMPTOTIC STATISTICS OF ACCUMULATED RIAN

We compute the auto-correlation of the accumulated RIAN of user $j$, which is given in the following proposition.

Proposition 2: When
\[
|g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}| < 1, \tag{10}
\]
and $i$ goes to infinite, the auto-correlation of $\hat{y}_j(i)$ is computed under the following three cases.

- At even time slots, i.e., $i = 2m + 1$ or $i = 2k + 1$, $m, k \in \{0, 1, \ldots\}$, the auto-correlation of $\{\hat{y}_j(i)\}$ is given as
\[
\mathbb{E} \left[ \hat{y}_j(2m + 1)\hat{y}_j(2k + 1) \right] = \frac{\hat{p}_j}{1 - |a|^2}. \tag{11}
\]
where $(\cdot)^+ \triangleq \max(\cdot, 0)$, $a = g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}$, and $\hat{p}_j$ is given as
\[
\hat{p}_j = |\xi_j|^2 + |\kappa_j|^2 + |\xi_j|^2|\eta_j|p_j + |\eta_j p_j + |\xi_j|^2 + 1, \tag{12}
\]
with
\[
\xi_j = g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}h_j^{\sqrt{\sigma_j}}/\sqrt{P_s}, \tag{13}
\]
\[
\kappa_j = g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}h_j^{\sqrt{\sigma_j}}/\sqrt{P_s}, \tag{14}
\]
\[
\eta_j = g_{jj}^\sqrt{\sigma_j}. \tag{15}
\]
- At odd time slots, i.e., $i = 2m + 1$ or $i = 2k$, the auto-correlation of $\{\hat{y}_j(i)\}$ is given as
\[
\mathbb{E} \left[ \hat{y}_j(2m + 1)\hat{y}_j(2k) \right] = 0. \tag{16}
\]

Proof: Please see Appendix VI.

Then, by letting $i = k$ in Proposition 2, the following corollary is obtained.

Corollary 3: When $|g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}| < 1$ and $i$ goes to infinite, the power of the accumulated RIAN is given as
\[
\lim_{i \to \infty} \mathbb{E} \left( |\hat{y}_j(i)|^2 \right) = \frac{\hat{p}_j}{1 - |a|^2}. \tag{17}
\]

Remark 4: From the proof of Proposition 2, it is observed that when $|a| \geq 1$, i.e., $|g_{jj}^\sqrt{\sigma_j}g_{jj}^\sqrt{\sigma_j}| \geq 1$, $\frac{1 - |\alpha|^2}{1 - |a|^2}$ given in (10) goes to infinity, as $i \to \infty$, and it follows that the accumulated RIAN power $\lim_{i \to \infty} \mathbb{E} \left( |\hat{y}_j(i)|^2 \right)$ also goes to infinity. In this case, the users cannot decode any messages from the received signals, since the power of the accumulated RIAN is infinite. Thus, condition (10) must be satisfied to guarantee non-zero transmission rate.
B. FORWARD AND BACKWARD DECODING SCHEMES

For the two-hop channel in (7), it is optimal to simultaneously decode all the messages sent in the whole $N$ time slots. However, under this optimal scheme, the users should buffer all the received signals and the decoding complexity increases exponentially as $N$ increases, which are too complex to be realized. Thus, we consider some empirical decoding schemes, i.e., the forward and backward decoding schemes. The two decoding schemes only require to decode one message at one time. More specifically, the users decode message $x(i)$ based on the signals $y_j(i)$ and $y_j(i+1)$.

1) FORWARD DECODING SCHEME

User $j$ decodes message $x(i)$ sequentially from the first time slot to the $N$-th time slot, and at the $i$-th time slot, the transmitted messages $x(n)$, $n = 1, 2, \ldots, i-1$, are assumed to be successfully decoded before decoding $x(i)$. Thus, at the $i$-th time slot, signal $y_j(i)$ (obtained after SI cancellation) contains message $x(i-1)$, which is already known to user $j$ and can be completely cancelled. Besides, at the $(i+1)$-th time slot, signal $y_j(i+1)$ contains message $x(i+1)$, which is unknown and regarded as noise. Therefore, the channel input-output relationship between the transmitter and the users 1 and 2 are given as

$$
\begin{bmatrix}
  y_1(i+1) \\
  y_1(i) \\
  y_2(i+1) \\
  y_2(i)
\end{bmatrix} =
\begin{bmatrix}
  b_1 \\
  a_1 \\
  b_2 \\
  a_2
\end{bmatrix}
\begin{bmatrix}
  x(i) \\
  x(i+1) \\
  x(i+1) \\
  x(i)
\end{bmatrix} +
\begin{bmatrix}
  \hat{y}_1(i+1) \\
  \hat{y}_1(i) \\
  \hat{y}_2(i+1) \\
  \hat{y}_2(i)
\end{bmatrix},
$$

(19)

and the users 1 and 2 are given as

$$
\begin{bmatrix}
  y_1(i+1) \\
  y_1(i) \\
  y_2(i+1) \\
  y_2(i)
\end{bmatrix} =
\begin{bmatrix}
  b_1 \\
  a_1 \\
  b_2 \\
  a_2
\end{bmatrix}
\begin{bmatrix}
  y(i) \\
  y_1(i) + y_1(i+1) \\
  y_2(i) + y_2(i+1)
\end{bmatrix} +
\begin{bmatrix}
  \hat{y}_1(i+1) \\
  \hat{y}_1(i) \\
  \hat{y}_2(i+1) \\
  \hat{y}_2(i)
\end{bmatrix}.
$$

(21)

2) BACKWARD DECODING SCHEME

For the backward decoding scheme, user $j$ first saves all the received signals $y_j(1), \ldots, y_j(N)$, and then decodes the messages from the $N$-th to the 1-st time slots. For the backward decoding scheme, the users also need to buffer all the received signals.

At the $(i+1)$-th time slot, the transmitted messages $x(n)$, $n = i+1, i+2, \ldots, N-1$ are assumed to be successfully decoded before decoding $x(i)$. Thus, at the $(i+1)$-th time slot, the signal $y_j(i+1)$ contains message $x(i+1)$, which is already known to user $j$ and can be cancelled. Besides, at the $i$-th time slot, signal $y_j(i)$ contains message $x(i-1)$, which is unknown and regarded as the noise. Therefore, the channel input-output relationship between the transmitter and the users 1 and 2 are given as

$$
\begin{bmatrix}
  y_1(i+1) \\
  y_1(i) \\
  y_2(i+1) \\
  y_2(i)
\end{bmatrix} =
\begin{bmatrix}
  b_1 \\
  a_1 \\
  b_2 \\
  a_2
\end{bmatrix}
\begin{bmatrix}
  y(i) \\
  y_1(i) + y_1(i+1) \\
  y_2(i) + y_2(i+1)
\end{bmatrix} +
\begin{bmatrix}
  \hat{y}_1(i+1) \\
  \hat{y}_1(i) \\
  \hat{y}_2(i+1) \\
  \hat{y}_2(i)
\end{bmatrix},
$$

(20)

where $a_j = h_j \sqrt{p_s}$ and $b_j = g_{ji} \sqrt{\alpha_j} h_j \sqrt{p_s}$.

C. ACHIEVABLE RATE MAXIMIZATION

The achievable rate of the considered multicast transmissions with FD user cooperation is given in the following proposition.

Proposition 5: The achievable rate $R$ of the multicast transmissions with FD user cooperation is given as

$$
R = \min \{ \log (1 + \beta_1), \log (1 + \beta_2) \}.
$$

(23)

For the forward decoding scheme, $\beta_j$ is given as

$$
\beta_j = \beta_F = \frac{|b_j|^2}{A_j + |a_j|^2} + \frac{|a_j|^2}{A_j},
$$

(24)

with $A_j = \frac{\bar{p}_j}{1 - |\alpha_j|^2}$; and for the backward decoding scheme, $\beta_j$ is given as

$$
\beta_j = \beta_B = \frac{|b_j|^2}{A_j} + \frac{|a_j|^2}{A_j + |b_j|^2}.
$$

(25)

Proof: Please see Appendix VI.

Then, the rate maximization problem for the multicast transmissions with FD user cooperation is formulated as

$$
(P1) \quad \max_{a_1 \geq 0, a_2 \geq 0, p_1, p_2} \min_{\beta_1, \beta_2}
\begin{bmatrix}
  \beta_1F(a_2) \\
  \beta_1B(a_2)
\end{bmatrix},
$$

(26)

s.t. (2), (8), (10).

It is easy to see that inequality constraint (10) of Problem (P1) is non-convex, and thus Problem (P1) is also non-convex [34].

IV. OPTIMAL SOLUTION

In this section, we first show that Problem (P1) with only one design variable can be simplified as a new problem, and then we compute the optimal solution for the simplified problem. Without loss of generality, we set $|h_1|^2 \leq |h_2|^2$, and the obtained results can be easily applied to the case of $|h_1|^2 \geq |h_2|^2$.

First, a property of the optimal solution is shown in the following proposition, which can be used to simplify Problem (P1).

Since the functions $\log(1 + \beta_j), j = 1, 2$, are monotonically increasing over $\beta_j$, we only need to optimize $\beta_j$.

$$
\beta_{1F}(a_2) = \frac{|g_{21}|^2 \alpha_2 h_2^2 |p_s|}{1 + |g_{21}|^2 \alpha_2 |g_{21}|^2 \alpha_2 |p_s|} + \frac{|h_1|^2 |p_s|}{1 + |g_{21}|^2 \alpha_2 + |g_{21}|^2 \alpha_2 |p_s|} + \frac{|h_1|^2 |p_s|}{1 + |g_{21}|^2 \alpha_2 + |g_{21}|^2 \alpha_2 |p_s|}
$$

(27)

$$
\beta_{1B}(a_2) = \frac{|g_{21}|^2 \alpha_2 h_2^2 |p_s|}{1 + |g_{21}|^2 \alpha_2 + |g_{21}|^2 \alpha_2 |p_s|} + \frac{|h_1|^2 |p_s|}{1 + |g_{21}|^2 \alpha_2 + |g_{21}|^2 \alpha_2 |p_s|}
$$

(28)
Proposition 6: Under the case of $|h_1|^2 \leq |h_2|^2$, one-side cooperation is optimal, i.e., the optimal solution satisfies $\alpha_1^* = 0$ and $p_1^* = 0$ for Problem (P1).

Proof: Please see Appendix VI.

Then, based on Proposition 6, the following corollary can be obtained.

Corollary 7: When $|h_1|^2 < |h_2|^2$, Problem (P1) can be simplified as the following problem,

$$\text{max} \quad \min \{\beta_1 (\alpha_2), \beta_2 (\alpha_2)\}, \quad \text{subject to} \quad 0 \leq \alpha_2 \leq L,$$

where $L = \frac{1}{|h_1|^2 p_s + |h_2|^2 p_s + 1}$. For the forward decoding scheme, $\beta_1 (\alpha_2)$ and $\beta_2 (\alpha_2)$ are given as (27), shown at the bottom of the previous page, and

$$\beta_{2F} (\alpha_2) = \frac{|h_2|^2 p_s}{1 + \eta_2 \frac{|h_2|^2}{1 - |h_2|^2}}, \quad \beta_{2B} (\alpha_2) = \frac{|h_2|^2 p_s}{1 + \eta_2 \frac{|h_2|^2}{1 - |h_2|^2}},$$

respectively; and for the backward decoding scheme, $\beta_1$ and $\beta_2$ are given as (28), shown at the bottom of the previous page, and

$$\beta_{2B} (\alpha_2) = \frac{|h_2|^2 p_s}{1 + \eta_2 \frac{|h_2|^2}{1 - |h_2|^2}}.$$

respectively. The relationship between $\alpha_2$ and transmission power $p_2$ is given in

$$p_2 = \alpha_2 (|h_2|^2 p_s + 1) - \frac{\alpha_2}{1 - \alpha_2 \eta_2}.$$

Proof: Please see Appendix VI.

Next, we show how to solve Problem (P2).

A. MAX-MIN TECHNIQUE

Since Problem (P2) is a max-min problem [34], we adopt the max-min technique [35], which is utilized to find the max-min solution detection rule between two hypotheses [36]. In this subsection, we briefly review the max-min technique.

First, we define

$$\beta_1 (\alpha_2) = \max_{0 \leq \alpha_2 \leq L} \beta_1 (\alpha_2), \quad \beta_2 (\alpha_2) = \max_{0 \leq \alpha_2 \leq L} \beta_2 (\alpha_2),$$

where $\alpha_2^*$ and $\alpha_2^*$ maximize $\beta_1 (\alpha_2)$ and $\beta_2 (\alpha_2)$ over $0 \leq \alpha_2 \leq L$, respectively.

Then, based on [35], [36], the following lemma is obtained.

Lemma 8: For the optimal solution $\alpha_2^*$ of Problem (P2), the relationship between $\beta_1 (\alpha_2^*)$ and $\beta_2 (\alpha_2^*)$ can be summarized as the following three cases:

- Case 1: If $\max_{0 \leq \alpha_2 \leq L} \min \{\beta_1 (\alpha_2), \beta_2 (\alpha_2)\} = \beta_2 (\alpha_2^*), i.e., \alpha_2^* = \alpha_2^*$, it follows $\beta_1 (\alpha_2^*) \geq \beta_2 (\alpha_2^*)$;
- Case 2: If $\max_{0 \leq \alpha_2 \leq L} \min \{\beta_1 (\alpha_2), \beta_2 (\alpha_2)\} = \beta_1 (\alpha_2^*)$, i.e., $\alpha_2^* = \alpha_2^*$, it follows $\beta_1 (\alpha_2^*) \leq \beta_2 (\alpha_2^*)$;
- Case 3: The optimal value of Problem (P2) is the maximum intersection point between the curves $\beta_1 (\alpha_2)$ and $\beta_2 (\alpha_2)$, i.e., $\beta_1 (\alpha_2^*) = \beta_2 (\alpha_2^*)$.

B. CASE 1

For Case 1, we obtain the following proposition.

Proposition 9: For problem (P2), Case 1 never occurs.

Proof: Suppose $\alpha_2^* = \alpha_2^*$ and it follows

$$\beta_2 (\alpha_2^*) = \max_{0 \leq \alpha_2 \leq L} \beta_2 (\alpha_2) = \max_{0 \leq \alpha_2 \leq L} \frac{|h_2|^2 p_s}{1 + \eta_2 \frac{|h_2|^2}{1 - |h_2|^2}},$$

where (36) is due to the fact that $\beta_2 (\alpha_2)$ is monotonically decreasing over $[0, L]$. Thus, it is easy to see that $\alpha_2^* = 0$ is optimal to Case 1. However, when $\alpha_2^* = 0$, it follows $\beta_1 (0) = |h_1|^2 p_s$, and due to $|h_1|^2 < |h_2|^2$, we obtain

$$\beta_1 (0) = |h_1|^2 p_s < |h_2|^2 p_s = \beta_2 (0),$$

which does not satisfy condition $\beta_1 (\alpha_2^*) \geq \beta_2 (\alpha_2^*)$, and thus Case 1 never occurs.

Therefore, Case 1 does not need to be discussed, and we only focus on Cases 2 and 3 in the sequel.

C. CASE 2

If Case 2 occurs, i.e., $\alpha_2^* = \alpha_2^*$, it follows

$$\max_{0 \leq \alpha_2 \leq L} \min \{\beta_1 (\alpha_2), \beta_2 (\alpha_2)\} = \max_{0 \leq \alpha_2 \leq L} \beta_1 (\alpha_2) = \beta_1 (\alpha_2^*),$$

and condition $\beta_1 (\alpha_2^*) \leq \beta_2 (\alpha_2^*)$ must be satisfied.

Inspired by the above results, we give the main idea on how to verify Case 2. We first compute $\alpha_2^*$ to maximize $\beta_1 (\alpha_2)$. Then, we verify condition $\beta_1 (\alpha_2^*) \leq \beta_2 (\alpha_2^*)$ with the obtained $\alpha_2^*$: If $\beta_1 (\alpha_2^*) \leq \beta_2 (\alpha_2^*)$ is satisfied, it follows $\alpha_2^* = \alpha_2^*$; otherwise, Case 2 does not occur.

First, $\alpha_2^*$ and $\beta_1 (\alpha_2^*)$ can be obtained by solving the following problem:

$$\text{max} \quad \min \{\beta_1 (\alpha_2), \beta_2 (\alpha_2)\} = \max_{0 \leq \alpha_2 \leq L} \beta_1 (\alpha_2),$$

where $\lambda^*$ is the optimal value of Problem (P2.1a). For the forward decoding scheme, $\beta_1 (\alpha_2)$ is computed by (27) as

$$\beta_1 (\alpha_2) = \beta_{1F} (\alpha_2) = \frac{F_F (\alpha_2)}{A_D + A_C + BDF + BC},$$

with

$$A = |g_{21}|^2 \alpha_2 \frac{|h_2|^2}{1 - \alpha_2 \eta_2}, \quad B = |h_1|^2 p_s (1 - \alpha_2 \eta_2), \quad C = |g_{21}|^2 \alpha_2 \eta_2 (\eta_2 p_s + 1), \quad D = (1 + |g_{21}|^2 \alpha_2 (1 - \alpha_2 \eta_2)), \quad D_F = (1 + |g_{21}|^2 \alpha_2 + |h_1|^2 p_s (1 - \alpha_2 \eta_2)).$$
For the backward decoding scheme, $\beta_1(\alpha_2)$ is computed by (28) as

$$
\beta_1(\alpha_2) = \beta_{1B}(\alpha_2) = \frac{F_B(\alpha_2)}{G_B(\alpha_2)} = \frac{AD_B + AC + BD + BC}{(D + C)(D_B + C)},
$$

(47)

with

$$
D_B = (1 + |g_{21}|^2 \alpha_2 + |g_{21}|^2 \alpha_2 |h_2|^2 p_s) (1 - \alpha_2 \eta_2).
$$

(48)

Next, we adopt fraction program [37] to solve Problem (P2.1a). First, define a new problem as

$$
(P2.1b) \quad \Phi(\lambda) = \max_{0 \leq \alpha_2 \leq L} \hat{\beta}_1(\alpha_2),
$$

(49)

where for the forward decoding scheme, $\hat{\beta}_1$ is given as

$$
\hat{\beta}_1 = \hat{\beta}_{1F} = F_F(\alpha_2) - \lambda G_F(\alpha_2)
= AD + AC + BD_F + BC - \lambda \left[ (D_F + C)(D + C) \right],
$$

(50)

and for the backward decoding scheme, $\hat{\beta}_1$ is given as

$$
\hat{\beta}_1 = \hat{\beta}_{1B} = F_B(\alpha_2) - \lambda G_B(\alpha_2)
= AD_B + AC + BD + BC - \lambda \left[ (D + C)(D_B + C) \right].
$$

(51)

Since $C, D, D_F,$ and $D_B$ are greater than zero over $0 \leq \alpha_2 \leq L,$ the products of them are also greater than zero, i.e., $G_F(\alpha_2) \geq 0$ and $G_B(\alpha_2) \geq 0$ over $0 \leq \alpha_2 \leq L.$ From [37], we know that when the conditions $G_F(\alpha_2) \geq 0$ and $G_B(\alpha_2) \geq 0$ are satisfied, the optimal value $\lambda^*$ of Problem (P2.1a) is the root of equation $\Phi(\lambda^*) = 0,$ and when $\lambda = \lambda^*,$ Problems (P2.1a) and (P2.1b) own the same optimal point $\alpha_2^*.$ Since function $\Phi(\lambda)$ is monotonically decreasing for Problem (P2.1b) over $\lambda,$ Problem (P2.1a) can be solved by bisection search over $\lambda.$ In each iteration, we solve Problem (P2.1b) with the fixed $\lambda.$

**Proposition 10:** The bisection research range of the optimal value $\lambda^*$ for Problem (P2.1a) is given as

$$
0 \leq \lambda^* \leq |h_2|^2 p_s.
$$

(52)

**Proof:** This proof is similar to that of Proposition 6, and thus is omitted for brevity. ■

Then, we show how to solve Problem (P2.1b).

**Proposition 11:** The optimal value $\hat{\beta}_1(\alpha_2)_{1B}$ for Problem (P2.1b) with the fixed $\lambda$ is given as

$$
\hat{\beta}_1(\alpha_2)_{1B} = \max \left\{ \hat{\beta}_1(\alpha_2) : \alpha_2 \in Q \cup [0, L] \right\},
$$

(53)

where $\alpha_2$ maximizes the function $\hat{\beta}_1(\alpha_2)$ with the fixed $\lambda$ over $0 \leq \alpha_2 \leq L,$ and the set $Q$ is given as

$$
Q = \left\{ \alpha_2 : 0 \leq \alpha_2 \leq L, \hat{\beta}_1(\alpha_2) = 0 \right\}.
$$

(54)

Here, $\hat{\beta}_1(\alpha_2)$ is the first-order derivative of $\hat{\beta}_1(\alpha_2)$, and $\hat{\beta}_1'(\alpha_2) = 0$ is a cubic equation, which has a closed-form solution.

**Proof:** Since $\hat{\beta}_1(\alpha_2)$ is a quartic function, and thus the optimal solution $\alpha_2$ of Problem (P2.1b) is one of the roots of equation $\hat{\beta}_1(\alpha_2) = 0$ or the upper and lower bounds for $\alpha_2$ (i.e., 0 and $L$) [38], and thus it follows that $\alpha_2$ is one of the elements in the set $Q$ given in (54) or $[0, L].$ Since $\hat{\beta}_1(\alpha_2)$ is the optimal value of function $\hat{\beta}_1(\alpha_2)$, we obtain (53). ■

**D. CASE 3**

If Case 2 does not occur (i.e., condition $\beta_1(\alpha_2^1) \leq \beta_2(\alpha_2^1)$ is not satisfied), we conclude that Case 3 must occur, and the optimal solution $\alpha_2$ of Problem (P2) must satisfy condition $\beta_1(\alpha_2^1) = \beta_2(\alpha_2^1).$ Inspired by condition $\beta_1(\alpha_2^1) = \beta_2(\alpha_2^1),$ we obtain the following proposition to compute the optimal solution $\alpha_2$ of Problem (P2).

**Proposition 12:** When Case 3 occurs, the optimal solution $\alpha_2$ of Problem (P2) is given as

$$
\alpha_2 = \min \left\{ 0 \leq \alpha_2 \leq L : \Psi(\alpha_2) = 0 \right\},
$$

(55)

where for the forward decoding scheme, $\Psi(\alpha_2)$ is given as

$$
\Psi(\alpha_2) = \Psi_F(\alpha_2) = (\tilde{A}D + \tilde{A}C + \tilde{A}D_F + \tilde{B}C) F
- (D_F + C)(D + C) |h_2|^2 p_s,
$$

(56)

with

$$
\tilde{A} = |g_{21}|^2 \alpha_2 |h_2|^2 p_s,
\tilde{B} = |h_2|^2 p_s,
F = (1 - \alpha_2 \eta_2) + \eta_2 \alpha_2 \left( |h_2|^2 p_s + 1 \right),
$$

(57)-(59)

and for the backward decoding scheme, $\Psi(\alpha_2)$ is given as

$$
\Psi(\alpha_2) = \Psi_B(\alpha_2) = (\tilde{A}D_B + \tilde{A}C + \tilde{B}D + \tilde{B}C) F
- (D + C)(D_B + C) |h_2|^2 p_s.
$$

(60)

Here, $\Psi(\alpha_2) = 0$ is a quartic equation.

**Proof:** Please see Appendix VI. ■

**Remark 13:** Based on the above results, we propose a numerical algorithm to obtain $\alpha_2$ and $p_2^*$ for Cases 2 and 3, which is summarized in Algorithm 1. First, compute $\alpha_2$ by bisection search on $\lambda$, where $\Phi(\lambda)$ with the fixed $\lambda$ is obtained by Proposition 11. Then, verify condition $\beta_1(\alpha_2^1) \leq \beta_2(\alpha_2^1)$: If condition $\beta_1(\alpha_2^1) \leq \beta_2(\alpha_2^1)$ is satisfied, it follows $\alpha_2 = \alpha_2^1$; otherwise, it is obtained that $\alpha_2$ is given in (55). Finally, based on (32), we compute $p_2^*$ by $\alpha_2^*$.

**E. SYMMETRIC CASE**

In the previous subsections, a numerical algorithm was proposed to solve Problem (P2) for the general case of $|h_1|^2 \leq |h_2|^2.$ In this subsection, we consider a special symmetric case, i.e., $|h_1|^2 = |h_2|^2.$
Algorithm 1: Compute the Optimal Solution of Problem (P2)

Input: \( p_s, |h_1|^2, |h_2|^2, \) and \( \eta_2. \)

Output: \( \alpha_2^*, p_2^*. \)

1. Initialize \( \lambda_{\text{low}} = 0, \lambda_{\text{up}} = |h_2|^2 p_s, \) and the error tolerance \( \beta > 0. \) \( \alpha \) represents the output of the \( i \)-th iteration.
2. while \( |\lambda_i - \lambda_{i-1}| > \beta \) do
3. Compute \( \alpha_2^{1,i} \) and corresponding \( \Phi(\lambda_i) \) by Proposition 11.
4. if \( \Phi(\lambda_i) > 0 \) then
5. \( \lambda_{i+1} = \frac{\lambda_{i} + \lambda_{\text{low}}}{2} \) and \( \lambda_{\text{low}} = \lambda_{i}; \)
6. else
7. \( \lambda_{i+1} = \frac{\lambda_{i} + \lambda_{\text{up}}}{2} \) and \( \lambda_{\text{up}} = \lambda_{i+1}. \)
8. end if
9. end while
10. Let \( \alpha_1^1 = \alpha_2^{1,i}. \)
11. if \( \beta_1(\alpha_1^1) \leq \beta_2(\alpha_1^1) \) then
12. let \( \alpha_2^* = \alpha_1^1; \)
13. else
14. let \( \alpha_2^* = \min\{0 \leq \alpha_2 \leq L : \Psi(\alpha_2) = 0\}. \)
15. end if
16. compute \( p_2^* \) by (32) with \( \alpha_2^*. \)

From (12) and \( A_j = \frac{\tilde{P}_j}{|\text{low}|} \), it is obtained that \( A_1 \geq |\text{low}|^2 + 1, \) and thus we compute

\[
\beta_1F = \frac{|b_1|^2}{A_1 + |a_1|^2} A_1 \geq \frac{|b_1|^2}{|\text{low}|^2 + 1} + \frac{|a_1|^2}{|\text{low}|^2 + 1} \geq \frac{|\text{low}|^2 + 1}{|\text{low}|^2 + 1} |h_1|^2 p_s = |h_1|^2 p_s, \tag{61}
\]

where
- (62) is due to \( |\text{low}|^2 + 1 \leq \tilde{P}_1 \leq A_1; \)
- (63) is due to \( |h_1|^2 = |h_2|^2. \)

In a similar way, we can obtain

\[
\beta_2F = \frac{|b_2|^2}{A_2 + |a_2|^2} A_2 \geq \frac{|b_2|^2}{|\text{low}|^2 + 1} \frac{|a_2|^2}{|\text{low}|^2 + 1} \geq \frac{|\text{low}|^2 + 1}{|\text{low}|^2 + 1} |h_2|^2 p_s = |h_2|^2 p_s, \tag{64}
\]

and thus it follows

\[
\min(\beta_{1F}, \beta_{2F}) \leq \min\left( |h_1|^2 p_s, |h_2|^2 p_s \right). \tag{65}
\]

Proposition 14: Under the case of \( |h_1|^2 = |h_2|^2, \) the optimal solution for Problem (P1) is given as \( \alpha_1^* = 0, \) \( \alpha_2^* = 0, p_1^* = 0, p_2^* = 0, \) i.e., no cooperation at the users is optimal.

Proof: Without loss of generality, we only complete this proof for the forward decoding scheme, and the proof for the backward decoding scheme is similar.

The achievable rate of the multicast transmissions under the different cooperation schemes as a function of \( p_s. \)
V. NUMERICAL RESULT

In this section, we show the numerical results for the maximum achievable rate under different schemes. Please notice that since the one-side cooperation is optimal for the FD user cooperation scheme, it is obvious that when $|h_2|^2 > |h_1|^2$, the achievable rates for the FD user cooperation scheme are independent of $\eta_1$ and $|g_{12}|^2$. Thus, we do not need to present the influences of $\eta_1$ and $|g_{12}|^2$.

Fig. 2 compares the maximum achievable rates among the FD user cooperation scheme with the forward and backward decoding schemes, the HD user cooperation scheme [11], and the noncooperation scheme over different $p_s$, with $\eta_2 = -70$ dB, and $|g_{21}|^2 = 1$, where the channel gains between the transmitter and users 1 and 2, are set as $|h_1|^2 = 0.2$ and $|h_2|^2 = 0.4$, respectively. From Fig. 2, it is observed that the maximum achievable rate for the FD user cooperation scheme is around 0.9 bps/Hz better than that for the noncooperation, and the maximum achievable rate for the backward decoding scheme is around 0.08 bps/Hz better than that for the forward decoding scheme. Moreover, the maximum achievable rate for the HD user cooperation scheme is nearly half of that for the FD user cooperation scheme.
Fig. 3 presents the maximum achievable rates as a function of $|h_1|^2$, with $p_s = 30$ dB, $|g_{21}|^2 = 1$, $\eta_2 = -70$ dB, and $|h_2|^2 = 40$ dB. From the curves in Fig. 3, it is observed that the maximum achievable rates for the FD and HD user cooperation schemes remain constant, and the gap between the curves of the forward and backward decoding schemes steadily decreases from about 0.01 bps/Hz to 0 bps/Hz. In contrast, the maximum achievable rate for the noncooperation scheme increases as $|h_1|^2$ increases and then overtakes that for the HD user cooperation scheme.

Fig. 4 shows the relationship between the maximum achievable rates and $|h_2|^2$, with $p_s = 30$ dB, $|g_{21}|^2 = 1$, $\eta_2 = -70$ dB, and $|h_1|^2 = 1$. From the curves in Fig. 4, it is observed that the maximum achievable rates for both the FD and HD user cooperation schemes increase as $|h_2|^2$ increases, and the maximum achievable rate for the HD user cooperation scheme overtakes that for the noncooperation scheme at $|h_2|^2 = 30$ dB. Meanwhile, the gap between the curves of the FD and HD user cooperation schemes increases from around 5 bps/Hz to around 12 bps/Hz, and the gap between curves of
the backward and forward decoding schemes also increases from nearly 0 bps/Hz to around 0.5 bps/Hz.

Fig. 5 shows the relationship between the maximum achievable rates and $\eta_2$, with $p_s = 30$ dB, $|h_1|^2 = 0.2$, $|h_2|^2 = 0.4$, and $|g_{21}|^2 = 1$. From the curves in Fig. 5, it can be seen that the maximum achievable rates for the FD AF user cooperation scheme decrease as $|h_2|^2$ increases and the backward decoding scheme is larger than the forward decoding scheme. The gap between the curves of the forward and backward decoding schemes decreases from nearly 0 bps/Hz to around 0.5 bps/Hz. Moreover, as $\eta_2$ increases, the maximum achievable rate for the FD AF user cooperation scheme is shown to be smaller than that for the noncooperation scheme, and the forward decoding scheme is even smaller than the HD user cooperation scheme.

Fig. 6 plots the maximum achievable rates as a function of $|g_{21}|^2$, with $p_s = 30$ dB, $|h_1|^2 = 0.2$, $|h_2|^2 = 0.4$, and $|g_{21}|^2 = 1$. From the curves in Fig. 6, it is observed that the maximum achievable rates for the forward and backward decoding schemes first slightly increase and then remain stable as $|g_{21}|^2$ increases, and the gap between the curves of the forward and backward decoding schemes decreases from around 0.12 bps/Hz to 0.01 bps/Hz.

VI. CONCLUSION

In this paper, an FD user cooperation with the AF scheme was proposed for the multicast transmissions. First, the equivalent channel of the multicast transmission with the FD user cooperation was obtained. Next, the achievable rate for the equivalent channel model with forward and backward decoding schemes was derived, and an achievable rate maximization problem was formulated. Last, it was proved that the one-side cooperation is optimal for the maximization problem, and a numerical algorithm was proposed by using the max-min technique and fractional programming to compute the optimal power allocation.

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PROOF OF PROPOSITION 3.1

In this proof, we only focus on showing how to obtain (11), since (16) and (17) can be obtained with the same method. The main idea of this proof is given as follows: First, $\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right]$ is decomposed into the sum of the variances of the interference and noise; and then, the variances can be computed by the sum of geometric progression [38].

For $k > i$, as $k \to \infty$, we obtain

$$\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right] = (a^*)^{-k} \sum_{n=0}^{k} \left| a_{2n} \right|^2 \left( |\xi_j|^2 + |\kappa_j|^2 + |\gamma_j|^2 p_j \right) + \eta_j p_j + |\xi_j|^2 + 1$$

(68)

$$\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right] = (a^*)^{-k} \sum_{n=0}^{k} \left| a_{2n} \right|^2 \left( |\xi_j|^2 + |\kappa_j|^2 + |\gamma_j|^2 p_j \right) + \eta_j p_j + |\xi_j|^2 + 1$$

(69)

$$\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right] = (a^*)^{-k} \sum_{n=0}^{k} \left| a_{2n} \right|^2 \left( |\xi_j|^2 + |\kappa_j|^2 + |\gamma_j|^2 p_j \right) + \eta_j p_j + |\xi_j|^2 + 1$$

(70)

where

- (68) is due to the fact that the sequences $\{\xi_j(n)\}$, $\{\bar{\xi}_j(n)\}$, $\{\tilde{\xi}_j(n)\}$, $\{\bar{\gamma}_j(n)\}$, and $\{\tilde{\gamma}_j(n)\}$ are i.i.d. processes and are independent of each other;
- (69) is due to the fact that $\sum_{n=0}^{k} |a_{2n}|^2$ is the sum of the geometric progression [38];
- (70) is due to $|a_{2k}|^2 \to 0$, as $k \to \infty$, with $|a_{2k}|^2 < 1$.

Similarly, for $k > i$, we obtain

$$\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right] = a^{k-i} \left| \frac{1}{1 - |a|^2} \left( |\xi_j|^2 + |\kappa_j|^2 + |\gamma_j|^2 p_j + \eta_j p_j + |\xi_j|^2 + 1 \right) \right.$$

$$\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right] = a^{k-i} \left| \frac{1}{1 - |a|^2} \left( |\xi_j|^2 + |\kappa_j|^2 + |\gamma_j|^2 p_j + \eta_j p_j + |\xi_j|^2 + 1 \right) \right.$$

(71)

as $i \to \infty$. Based on (70) and (71), $\mathbb{E} \left[ \tilde{y}_j^*(2i) \tilde{y}_j (2k) \right]$ can be written as (11), which completes the proof.

PROOF OF PROPOSITION 3.2

For the forward decoding scheme, the signal model given in (19) and (20) is equivalent to a single-input multiple-output (SIMO) multicast channel [40]. The achievable rate of the SIMO multicast channel under the forward decoding scheme is given as

$$R = \log \left\{ 1 + \min \left\{ (b_1, a_1) Q^{-1}_{1F} (b_1, a_1)^H, \right. \right.$$

$$\left. \left( b_2, a_2 \right) Q^{-1}_{2F} (b_2, a_2)^H \right\} \right.$$

(72)

$$R = \log \left\{ 1 + \min \left\{ (b_1, a_1) \left[ A + |a_1|^2 0 \right]^{-1} \left( b_1, a_1 \right)^H, \right. \right.$$

$$\left. \left( b_2, a_2 \right) \left[ A + |a_2|^2 0 \right]^{-1} \left( b_2, a_2 \right)^H \right\} \right.$$
(74) is due to the fact that $Q_{jF}$, $j \in \{1, 2\}$, is the correlation matrix of the accumulated RIAN, i.e.,

$$Q_{jF} = \begin{pmatrix}
\hat{y}_j(i + 1) + x(i + 1)^2 & \hat{y}_j(i + 1) \hat{y}_j^*(i) \\
\hat{y}_j(i + 1) \hat{y}_j(i) & |\hat{y}_j(i)|^2
\end{pmatrix}$$

(77)

and (85) can be transformed as

$$p_2 \geq \frac{\alpha_2 \left( |h_2|^2 p_s + 1 \right)}{1 - \alpha_2 \eta_2}.$$  (86)

Then, in the same way, it is obtained that $\beta_{2F}(\alpha_1, \alpha_2) = \beta_{2F}(0, \alpha_2)$. Thus, we can derive

$$\min \{\beta_{1F}(\alpha_1, \alpha_2), \beta_{2F}(\alpha_1, \alpha_2)\} \leq \min \{\beta_{1F}(0, \alpha_2), \beta_{2F}(0, \alpha_2)\}. \quad (87)$$

From (87), it is obtained that $\alpha_1 = 0$ is optimal, i.e., $\alpha_1^* = 0$ for the forward decoding scheme.

Based on (8), it can be obtained that when $\alpha_1^* = 0$, we can compute $p_1^*$, which completes the proof.

**PROOF OF COROLLARY 4.1**

First, due to $\alpha_1^* = 0$ and $p_1^* = 0$ for case of $|h_1|^2 < |h_2|^2$, except constraint (8) under $j = 2$, all other constraints for Problem (P1) have been satisfied, and thus they can be removed from Problem (P1). Then, constraint (8) under $j = 2$ can be rewritten as

$$\alpha_2 \left( |h_2|^2 p_s + \eta_2 p_2 + 1 \right) = p_2.$$  (88)

and formula (88) can be transformed as (32). Thus, when $\alpha_2^*$ is obtained, we can compute $p_2^*$ by (32).

Next, due to (32), the constraint $0 \leq p_2 \leq p_{\text{max}}$ in Problem (P1) can be written as

$$0 \leq \frac{\alpha_2 \left( |h_2|^2 p_s + 1 \right)}{1 - \alpha_2 \eta_2} \leq p_{\text{max}}. \quad (89)$$

It is observed that the inequality (89) can be transformed as

$$0 \leq \alpha_2 \leq \frac{p_{\text{max}}}{\left( |h_2|^2 p_s + \eta_2 p_{\text{max}} + 1 \right)}, \quad (90)$$

which is the feasible region of $\alpha_2$. Here, for the sake of convenience in writing, we denote the upper bound of $\alpha_2$ as $L = \frac{p_{\text{max}}}{\left( |h_2|^2 p_s + \eta_2 p_{\text{max}} + 1 \right)}$.

Last, due to (32), $\alpha_1^* = 0$ and $p_1^* = 0$, (24) can be rewritten as (27) for $j = 1$ and (41) for $j = 2$, respectively, and (25) can be rewritten as (28) for $j = 1$ and (31) for $j = 2$, respectively, which completes the proof.

**PROOF OF PROPOSITION 4.5**

When Case 3 occurs, the optimal solution $\alpha_2^*$ for Problem (P2) is the root of equation $\beta_1(\alpha_2) = \beta_2(\alpha_2)$ over $0 \leq \alpha_2 \leq L$, and it follows

$$\beta_j(\alpha_2^*) = \max \{\beta_j(\alpha_2) : \beta_1(\alpha_2) = \beta_2(\alpha_2)\}, \quad 0 \leq \alpha_2 \leq L. \quad (91)$$

Notice that function $\beta_2(\alpha_2)$ is monotonically decreasing over $0 \leq \alpha_2 \leq L$. Thus, the least of all $\alpha_2$, which satisfies condition $\beta_1(\alpha_2) = \beta_2(\alpha_2)$, is the optimal solution $\alpha_2^*$ of Problem (P2). Therefore, (91) can be rewritten as

$$\alpha_2^* = \min \{0 \leq \alpha_2 \leq L : \beta_1(\alpha_2) = \beta_2(\alpha_2)\}. \quad (92)$$
Then, we show that equation $\beta_1 (\alpha_2) = \beta_2 (\alpha_2)$ can be simplified as $\Psi (\alpha_2) = 0$. For the forward decoding scheme, equation $\beta_1 (\alpha_2) = \beta_2 (\alpha_2)$ is written as $\beta_{1F} (\alpha_2) = \beta_{2F} (\alpha_2)$, and we obtain

$$0 = \beta_{2F} (\alpha_2) - \beta_{1F} (\alpha_2)$$

$$= \frac{AD + AC + BD_F + BC}{(D_F + C)(D + C)} - |h_2|^2 p_s (1 - \alpha_2 \eta_2)$$

$$= \frac{AD + AC + BD_F + BC}{(D_F + C)(D + C)} - |h_2|^2 p_s$$

$$= \frac{(BD_F + BC)(D + C) +\beta F}{(D_F + C)(D + C) +\beta F}$$

$$(93)$$

where (94) is due to $1 - \alpha_2 \eta_2 \neq 0$ over $0 \leq \alpha_2 \leq L$; and (95) is due to the reduction of fractions in (94) to a common denominator. By taking out the denominator of (95), we can obtain

$$\Psi_F (\alpha_2) = \left(AD + AC + BD_F + BC\right)F$$

$$- |(D_F + C)(D + C)| |h_2|^2 p_s.$$  

(96)

In the same way, for the backward decoding scheme, $\Psi_B (\alpha_2) = 0$ can be obtained by $\beta_{1B} (\alpha_2) = \beta_{2B} (\alpha_2)$, which completes the proof.

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