Galactic rotation curves and brane world models

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ABSTRACT

In the present investigation flat rotational curves of the galaxies are considered under the framework of brane-world models where the 4d effective Einstein equation has extra terms which arise from the embedding of the 3-brane in the 5d bulk. It has been shown here that these long range bulk gravitational degrees of freedom can act as a mechanism to yield the observed galactic rotation curves without the need for dark matter. The present model has the advantage that the observed rotation curves result solely from well-established non-local effects of gravitation, such as dark radiation and dark pressure under a direct use of the condition of flat rotation curves and does not invoke any exotic matter field.

Key words: gravitation - dark matter - galaxies: general - galaxies: peculiar - galaxies: structure.

1 INTRODUCTION

The presence of dark matter was suspected for the first time by Oort (1930) while carrying out his observations of stellar motions in the local galactic neighborhood. On the other hand, Zwicky (1933, 1937) discovered the presence of dark matter on a much larger scale through his studies of galactic clusters. He, by comparing the virial mass and luminous mass of galactic clusters, concluded that a large amount of matter remains hidden in the galactic haloes. Afterwards, observation of the flatness of galactic rotation curves (Roberts & Rots 1973; Ostriker, Peebles & Yahil 1974; Einasto, Kaasik & Saar 1974; Rubin, Thonnard & Ford 1978; Rubin, Roberts & Ford 1979) supported Zwicky’s suggestion regarding the presence of non-luminous matter. The flatness of the galactic rotation curves are indicative of the presence of much more matter within the galaxy than the visible material. Best estimates support a spherical distribution of matter thus leading to the picture of a visible galaxy surrounded by a dark matter halo. It can be observed that dark matter gravitational effects are more manifest at larger radius (Begeman 1989).

It is seen that inside the optical radius the shapes of the rotation curves probably correlate with the light distribution (Kent 1987) whereas some degree of velocity variation has been observed in the outer regions which might be related to the overall mass distribution (van Albada & Sancisi 1984). The velocity decrease may be even more than 50 km/s on both sides of some of the galaxies (Casertano & van Gorkom 1991). However, the measurements on rotation curves in spiral galaxies show that the coplanar orbital motion of gas in the outer parts of these galaxies keeps more or less a constant value up to several luminous radii (Persic, Salucci & Stel 1996). Another important feature is that the rotation curve profile of a spiral galaxy is flat outside a central galactic region (Rubin, Roberts & Ford 1979) and this flatness of the rotation curves might be due to the energy density of dark matter which varies as $1/r^2$ (Matos, Guzman & Nunez 2000).

In subsequent years, gravitational lensing of objects like bullet clusters and the temperature distribution of hot gas in galaxies and galactic clusters as well as some other approaches have further confirmed the existence of dark matter (Maoz 1994; Cheng & Krauss 1999; Weinberg & Kamionkowski 2002; Faber & Visser 2006; Metcalf & Silk 2007).

Now, observation of anisotropy in the CMB (Spergel 2003) suggests that the total energy-density of the Universe is $1.02 \pm 0.02$. On the other hand, observations of CMB and deuterium abundance (Sahni 2004) as well as theoretical predictions (Olive, Steigman & Walker 2000) confirm that the baryon density of the Universe cannot be more than 4% of the total energy-density. These results hinted at a huge discrepancy between $\Omega_{DE}$ and $\Omega_{Baryons}$. The introduction of Inflationary Theory by Guth (1981) and others (Linde 1982; Albrecht & Steinhardt 1982) enabled the theorists to conclude that the Universe must appear to be flat and the total energy-density of the Universe is close to the critical value.
Since it was well established that visible matter could not contribute more than 4 − 5% of the total energy-density, it was initially speculated that 90 − 96% of the total energy-density must be hidden matter, i.e. dark matter (Guzman, Matos, Nunez & Ramirez 2003). But, in spite of intense searches, the requisite amount of dark matter was not found. So, a crisis developed in the cosmological arena. However, the emergence of the idea of dark energy which is thought to be responsible for accelerating the Universe (Riess et al. 1998; Perlmutter et al. 1998) seems to have, ultimately, removed that crisis. It is now known that about two-thirds of the total energy-density comes in terms of this dark energy while the remaining one-third is contributed by matter, both visible and dark (Sahni 2004). Thus, at present, the dynamics of the Universe is governed by two components, the exact nature of both of which are unknown, one is dark matter and another is dark energy.

It has been argued that dark matter played a significant role during structure formation in sub-megaparsec scales of the early Universe. However, though the exact composition of dark matter is still unknown, it is thought that most probably this is of non-baryonic type. The reason behind this is that it is difficult to reconcile baryonic dark matter with the small density perturbations ($\Delta \rho / \rho \sim 10^{-5}$ at $z \simeq 1100$) measured by COBE and CMB experiments (Sahni 2004). Moreover, it is now quite established that the hot dark matter – like light neutrinos cannot contribute significantly to the large amount of dark matter residing in the Universe, because in that case galactic clusters with huge masses ($\sim 10^{15} M_{\odot}$) were the first objects to form. However, this is not supported by the observational constraint regarding neutrino mass and their density (Elgarov et al. 2003; Minakata & Sugiyama 2003; Spergel 2003; Ellis 2003). On the other hand, cold dark matter, which does not contain any appreciable internal thermal motion, can cluster on small scales (Sahni 2004). This supports the well known hierarchical structure formation and suggests that most of the dark matter must be cold and non-baryonic.

The previous cold dark matter models, introduced in the early 1980’s, assumed that $\Omega_{CDM} = 1$ and were known as Standard Cold Dark Matter (SCDM) models. In spite of initial success SCDM models have fallen out of grace (Efstathiou, Sutherland & Madox 1990; Pope et al. 2004). Particularly, after the introduction of the idea of an accelerating Universe, the SCDM model is replaced by $\Lambda$-CDM (LCDM) model for including dark energy as a part of the total energy-density of the Universe. This $\Lambda$-CDM model is found to be in nice agreement with various sets of observations (Tegmark et al. 2004).

However, it is to be noted that not only the neutrino, as mentioned earlier, but also many other candidates exist for dark matter. Literature surveys reveal that Guzman, Matos, Nunez & Ramirez (2003) considered quintessence-like dark matter in spiral galaxies whereas Nukamendi and others (Nukamendi, Salgado & Surasky 2000; Matos, Guzman & Lopez 2000; Matos, Guzman & Nunez 2000; Nukamendi, Salgado & Surasky 2001; Lee & Lee 2004) have suggested that monopoles could be the galactic dark matter whose energy density varies as $1/r^2$. However, Nukamendi, Salgado & Surasky (2000; Nukamendi, Salgado & Surasky 2001; Lee & Lee 2004; Rahaman, Mondal, Kalam & Raychaudhuri 2007) studied global monopoles as a candidate for galactic dark matter in the framework of the scalar tensor theory of gravity. In this line of investigations Matos, Guzman & Lopez (2000; Matos, Guzman & Nunez 2000) have examined the possibility and hence the type of dark matter that determines the geometry of a spacetime where the flat rotational curves could be explained. Cembranos, Dobado & Maroto (2003) have shown that in the context of brane-world scenarios with low tension massive brane fluctuations, i.e. branons, are natural dark matter candidates and they could make up the galactic halo. There are also examples where the neutralino (Nihei, Okada & Seto 2005) and axino (Panotopoulos 2005) dark matter have been studied in brane-world cosmology. Under the same brane-world scenario it has been concluded that the neutral hydrogen clouds at large distances from the galactic center may be explained by postulating the existence of dark matter (Harko & Mak 2003; Harko & Cheng 2006). On the other hand, it is found by Panotopoulos (2007) that the gravitino can play the role of dark matter in the Randall-Sundrum type II brane model (Randall & Sundrum 1999b) and determined what the gravitino mass should be for different values of the five-dimensional Planck mass. Interestingly, Viznyuk & Shtanov (2007) have discussed the subject in frames of the brane-world with induced curvature, which can explain not only dark matter in galaxies but also dark matter effects on the cosmological scale.

Therefore, it can be observed that in the framework of brane cosmology researchers have made several attempts to understand various features of galactic dark matter with different approaches (Mukohyama 2004; Cembranos, Dobado & Maroto 2003; Gumjudpai, Maartens & Gordon 2003; Harko & Mak 2005; Harko & Cheng 2006; Böhmer & Harko 2007; Panotopoulos 2007; Viznyuk & Shtanov 2007).

Here we will study a five dimensional brane-world model and show that the long range bulk gravitational degrees of freedom can act as a mechanism to yield the observed galactic rotation curves without the need for dark matter. This model has the advantage that the observed rotation curves result solely from well-established non-local gravitational effects, such as dark radiation and dark pressure, due to the $5d$ bulk and does not rely on the introduction of an unknown and unseen matter field.

The above result that the dark radiation and its associated mass, known as dark mass, being a linearly increasing function of the distance has a similar behaviour as the dark matter at the galactic scale have also been obtained by Böhmer & Harko (2007). However, the approach of Harko and his collaborators is quite different from us as their method is either involved in the conformally symmetric space-times (Mak & Harko 2004; Harko & Mak 2005) or although they also consider constant rotational velocity with different approach, their emphasis is on the qualitative analysis of varied physical parameters in terms of observable quantities (Harko & Cheng 2004; Böhmer & Harko 2007). Unlike the previous studies the motivation in the present study is primarily concerned with the condition for flat rotation curves (which arises from the demand that the considered test particles should have constant tangential velocity) which has been exploited as a key to our entire investigation. This simple supposition provides us the required connection between dark matter and brane-world model in a very straightforward way.

The scheme of the present investigation is as follows. In section 2 we present the relevant vacuum field equations for five dimensional brane-world models whereas solutions will be sought for and utilised to provide acceptable galactic rotation curves on the brane in section 3. We discuss in section 4 how different features of non-local effects of the brane-world scenarios are relevant for dark matter and hence may be responsible for the flatness of rotation curves which are in agreement with observations.
2 FIELD EQUATIONS FOR FIVE DIMENSIONAL BRANE-WORLD MODELS

In the simplest brane world models, a five dimensional space-time is governed by the five dimensional Einstein field equations with 5d cosmological constant, $\Lambda_5$, which are given by

$$ R^\mu_\nu - \frac{1}{2} R \delta^\mu_\nu - (k_5)^2 \Lambda_5 \delta^\mu_\nu = -(k_5)^2 T^\mu_\nu, \quad (1) $$

where Greek indices take the values (0, 1, 2, 3, 4), $(k_5)^2 = 8\pi G_5$ and $T^\mu_\nu$ is the stress-energy tensor. Since matter is confined to the four dimensional brane and only gravity permeates the bulk, $T^\mu_\nu$ can therefore be written as

$$ T^\mu_\nu \delta(Y) \left[ T^\mu_\nu_{\ (\text{matter})} + \lambda_b \delta^\mu_\nu \right], \quad (2) $$

where $Y = 0$ denotes the location of the brane in the bulk and $\lambda_b$ is the vacuum energy on the brane (related to the brane tension).

Since observations indicate that the dark matter distribution around galaxies is spherical, we shall consider a static spherically symmetric line element on the four-dimensional brane of the form

$$ ds^2 = -e^{\nu(r)} dt^2 + e^\lambda(r) dr^2 + r^2 d\Omega^2 \quad (3) $$

with $d\Omega^2 = 4d^2 \sin^2 \theta d\phi^2$. Here $\nu$ and $\lambda$ are the metric potentials and are function of the space coordinate $r$ only, such that $\nu = \nu(r)$ and $\lambda = \lambda(r)$. We are making the reasonable assumption that the rotation is not large enough to spoil the spherical symmetry sufficiently to invalidate the metric (3).

The Gauss equations yield the following effective four-dimensional gravitational field equations on the brane

$$ G^i_j = R^i_j - \frac{1}{2} g^i_j R - \Lambda g^i_j = -8\pi T^i_j + (k_5)^2 S^i_j + E^i_j, \quad (4) $$

where $S^i_j$ is the quintic function of stress-energy tensor (therefore $S^i_j = 0$ in vacuum) and $\Lambda = (k_5)^2 \left[ \Lambda_5 + (k_5)^2 (\lambda_b)^2 / 6 \right] / 2$ is the 4d cosmological constant which has been neglected in our calculations for simplicity. Here $E^i_j$ is due to the long range gravitational degrees of freedom and is a projection of the bulk Weyl tensor onto the brane via $E_{i\mu} = C_{ab\mu\nu \alpha \beta \gamma} n^a n^b n^\alpha n^\beta n^\gamma$ with $C$ the five dimensional Weyl tensor and the $n$ vectors are five dimensional unit normal vectors of the brane. Note from [4] that on the brane we experience standard 4d general relativity with the exception that the source of the 4d Einstein tensor is augmented by higher order stress-energy effects and 5d Weyl tensor gravitational terms.

In vacuum, for the metric (3), equations (4) can explicitly be written as (Mak & Harko 2004)

$$ e^{-\lambda} \left( \frac{\nu'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = \frac{48\pi G_5}{k^3 \lambda_b} U, \quad (5) $$

$$ e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = \frac{16\pi G_5}{k^3 \lambda_b} (U + 2P), \quad (6) $$

$$ e^{-\lambda} \left( \nu' + \frac{\nu'^2}{2} - \frac{\nu' \nu'}{2} + \frac{\nu' - \nu'}{r} \right) = \frac{32\pi G_5}{k^3 \lambda_b} (U - P), \quad (7) $$

$$ \nu' = \frac{1}{2U + P} \left( U' + 2P' + \frac{6P}{r} \right), \quad (8) $$

1 Here we are not concerned with the form of the full five dimensional metric and therefore it is not specified. Given a well behaved four dimensional metric, the bulk metric’s existence as a solution of the bulk field equations is guaranteed by the Campbell-Maagar embedding theorem (Seahra & Wesson 2003; Wesson 2005).

Figure 1. Plot for the variation of $e^\nu$ vs $r$. The upper and lower panels correspond to short and long $r$ behaviours. The dotted, dashed and long-dashed curves represent $v_\phi = 200, 250$ and 300 Kms/second, respectively. For all these, thin curves and thick curves represent $B = 0.9999995$ and $B = 1.0$, respectively. The chain, solid and thick solid curves respectively, represent $v_\phi = 200, 250$ and 300 Kms/second but for $B = 1.0000005$.

where $U$ and $P$ are the dark radiation energy density (or simply dark radiation) and dark pressure, respectively, of the bulk which are function of the space coordinate $r$ only. According to Mak & Harko (2004) these parameters $U = U(r)$ and $P = P(r)$ are the components of $f_{\omega\mu}$ which consists of the projection of the bulk Weyl tensor onto the brane. This projected Weyl tensor effectively serves as an additional matter source. Here a prime denotes derivative with respect to space coordinate $r$.

3 SOLUTIONS FOR DARK MATTER PARAMETERS ON BRANE-WORLD MODELS

Let us now consider the case of flat rotation curves for which the required condition (Rahaman, Mondal, Kalam & Raychaudhuri 2007) can be put as

$$ e^\nu = Br^l, \quad (9) $$

where $l = 2v_\phi^{-2}$ and $B$ is an integration constant.

The form of (9) arises from demanding that test particles, which obey the equation of motion (Chandrasekhar 1983)

$$ v_\phi = v_{\text{angular}} = \left[ \frac{r (e^\nu)^l}{2e^\nu} \right]^{1/2}, \quad (10) $$

have constant tangential velocity. It is to be noted that the observed rotational curve profile in the dark matter dominated region is such that the rotational velocity $v_{\phi}$ becomes more or less a constant with $v_{\phi} \sim 200 - 300 \text{ km/s}$ for a typical galaxy (Binney & Tremaine 1987; Persic, Salucci & Stil 1996; Matos, Guzman & Lopez 2003; Boriello & Salucci 2001).
One may generate an appropriate galactic rotation curve by fitting to available observational data. An example of such a curve is shown in figure 2. The key point of interest is that the curve must tend to an appropriate constant in the outer regions. Equivalently, the metric function $e^{\nu(r)}$ must asymptotically approach the form dictated by equation (9). A reasonably general function which produces suitable rotation curves for both large and small $r$ is given by

$$v_\phi = \alpha r \exp(-k_1 r) + \beta [1 - \exp(-k_2 r)],$$

where $\alpha$, $\beta$, $k_1$ and $k_2$ are constants determined by appropriate fitting to data. Although our analysis will concentrate on the outer regions, where the velocity is constant, a few comments are appropriate for the inner regions.

In the inner regions, it is expected that the mass due to the concentration of stars will dominate the dynamics. A typical galactic star will, however, be moving in what to a reasonable approximation is vacuum, so that the vacuum equations are valid. To make the analysis tractable we will assume a test-particle star, located at some radius $r_*$, moving under the influence of the spherically symmetric gravitational field due to stars located inside its orbit ($r < r_*$) in the brane-world scenario. Without detailed knowledge of the Weyl stresses one does not know the exact form of the spherically symmetric vacuum equations. However, given the form of the rotation curve as given by the equation (11), we can study the properties the metric must possess. Equation (10), utilising (11), may actually be integrated to yield an analytic expression for $\nu(r)$. The result is rather unwieldy and not very perspicuous so instead a plot of $e^{\nu(r)}$ is provided in figure 3 from which it may be verified that we have the asymptotic form of (9) for appropriate $l$.

Now, from the equations (5) to (7) and then using the simple proposition expressed in the equation (9) one obtains the following simplified form

$$\frac{2}{r} = \left(2 + l + \frac{l^2}{2}\right) \frac{e^{-\lambda}}{r} - \left(2 + \frac{l}{2}\right) \frac{\lambda'}{r} e^{-\lambda}.$$

(12)

If we substitute $e^{-\lambda} = z$, then the above equation (12) reduces to

$$z' + \frac{az}{r} = \frac{2}{(2 + \frac{l}{2})r},$$

where $a = (2 + l + \frac{l^2}{2})/(2 + \frac{l}{2})$.

This can be put in the following integral form

$$zr^a = \int \frac{2}{(2 + \frac{l}{2})} r^{a-1} dr + D,$$

(14)

where $D$ is an integration constant.

Therefore, after integration of the equation (14), we get the metric potential $e^{-\lambda}$ as

$$e^{-\lambda} = \frac{2}{(2 + \frac{l}{2})a} + \frac{D}{r^a}.$$

(15)

With the flat rotational curve condition, therefore, the metric (3) becomes

$$ds^2 = -B_0 r^f dt^2 + \left[\frac{2}{(2 + \frac{l}{2})a} + \frac{D}{r^a}\right]^{-1} dr^2 + r^2 d\Omega^2.$$
By the use of equation (15) in (5) one can easily obtain the following expression for the dark radiation

\[ U(r) = \frac{1}{b} \left[ \frac{D(a - 1)}{r^{a+2}} + \left( 1 - \frac{2}{a(2 + \frac{1}{a})} \right) \frac{1}{r^2} \right] \]  

(17)

with \( b = 48\pi G/\lambda_0^4 \).

Now, after differentiating equation (9) and equating it with equation (8) we get

\[ \frac{\nu'}{r} = -\frac{1}{2U + P} \left[ U' + 2P' + \frac{6P}{r} \right] \]  

(18)

which after substitution of equation (17) yields the following first order differential equation

\[ P' + \frac{eP}{r} = \frac{d}{r^{3+a}} + \frac{e}{r^3} \]  

(19)

with notations \( e = (6 + l)/2, d = D(a - 1)[a + 2(1 - l)]/2b \) and \( e = (1 - l) \left[ 1 - \frac{2}{a(2 + \frac{1}{a})} \right] /b \).

The above equation (19), after some manipulation, can be written in the integral form as

\[ P(r) = -\frac{d}{(2 + a - e)r^{2+a}} \]  

(21)

Here as \( r \) tends to infinity \( P \) will vanish. However, the third term goes as \( E/r^{3+1/2} \) so it seems to have the fall-off properties more rapid than the other two terms. In this connection it can also be observed that for large values of \( l \), which is proportional to square of the tangential velocity, both the \( d \) term and the \( E \) term possess similar fall-off properties. This automatically suggests that for small values of tangential velocity one can easily set \( E \) to zero. Moreover, integration constant \( D \rightarrow 0 \) implies \( d \rightarrow 0 \). With this, the first term of equation (21) sets to zero too and only the second term, \( P(r) = -e/(2 - c)r^2 \), defines the pressure. We plot it in Figure 6 as represented by black curves, wherein we observe that pressure is always negative. The first term starts to dominate even with a very small value of \( D \) making the pressure positive with positive value of \( D \) as denominator of this term is negative. The sensitivity of pressure with \( D \) is plotted in Figure 6. Curves for different rotational velocities are distinguishable with \( D = 0 \). However, they tend to overlap with increasing value of \( D \).

Let us now study an expression of active gravitational dark mass associated with the dark radiation which can be given by

\[ M(r) = \int_{r_{min}}^{r} \frac{2\pi r^2 C}{k^4 \lambda_0^4} U r^2 dr \]

Figure 5. Plot for the variation of \( U \) vs. \( r \). The upper and lower panels correspond to short and long \( r \) behaviours. The dotted, dashed and long-dashed curves represent \( \nu_0 = 200, 250 \) and \( 300 \) Kms/second, respectively. For all these, thin curves and thick curves correspond to \( D = 0 \) and \( D = 0.5 \), respectively. The chain, solid and thick solid curves respectively, represent \( \nu_0 = 200, 250 \) and \( 300 \) Kms/second but for \( D = 2.0 \). In the lower panel, plots represent the case for \( D = 1.0 \) only because results are not found significantly sensitive to \( D \) at long distances.

Figure 6. Variation of \( P \) vs. \( r \). The upper and lower panels correspond to short and long \( r \) behaviours. The black curves represent \( D = 0.0 \) for both the panel. For the upper panel, blue, orange and red colours represent \( D = 0.0000001, D = 0.000001 \) and \( D = 0.00001 \), and green, brown and pink colours represent \( D = -0.0000001, D = -0.000001 \) and \( D = -0.00001 \), respectively. For the lower panel, blue, orange and red colours represent \( D = 0.001, D = 0.01 \) and \( D = 0.1 \), and green, brown and pink colours represent \( D = -0.001, D = -0.01 \) and \( D = -0.1 \), respectively. For all the colours, dotted, dashed and long-dashed curves correspond to \( \nu_0 = 200, 250 \) and \( 300 \) Kms/second, respectively.
mass acts as the so-called dark matter which is thought to be responsible for the flatness of rotation curves around galaxies. From the equation (22) it can easily be seen that $M$ is linearly increasing with the radial distance of the galaxy. This is also clearly evident from the graphical plots (Figure 1 and Figures 4 - 7) of the different physical quantities. However, to get this connection we have employed a very simple supposition as expressed in the equation (2) which arises from the demand that test particles under consideration will have constant tangential velocity. Note that this constant tangential velocity actually refers to the flatness of the galactic rotation curves.

In this context it can be observed that there is a striking similarity between the Figures 1 and 4 of the present work under brane-world models with those of Rahaman, Mondal, Kalam & Raychaudhuri (2007) drawn for a global monopole field in the Brans-Dicke theory under the same equation (2). Actually, Rahaman, Mondal, Kalam & Raychaudhuri (2007) have shown by using Brans-Dicke theory that monopole could be the galactic dark matter in the spiral galaxies whereas, Guzman & Matos (2000) have proposed scalar fields as dark matter in the spiral galaxies. This immediately indicates the global monopole as one of the candidates for galactic dark matter irrespective of the underlying theories. At the same time it seems that we get confirmation of dark matter not only from the brane-world model but also from Brans-Dicke theory. In this connection it is asserted by Harko & Mak (2005) that “... despite more than 20 years of intense experimental and observational effort, up to now no non-gravitational evidence for dark matter has ever been found...”.

We would like to put some comments on the properties of Weyl tensor which might be relevant here. It is already mentioned in the field theoretical part that $E^i_j$ is due to the long range gravitational degrees of freedom and is a projection of the bulk Weyl tensor onto the brane via $E_{\mu\nu} = C_{\mu\alpha\nu\beta}n^\alpha n^\beta$ with $C$ the five dimensional Weyl tensor and the $n$ vectors are five dimensional unit normal vectors of the brane. It is also mentioned in connection to (2) that on the brane, we experience standard 4d general relativity with the exception that the source of the 4d Einstein tensor is augmented by higher order stress-energy effects and 5d Weyl tensor gravitational terms. It is therefore highly probable, that if Randall-Sundrum models (Randall & Sundrum 1999a,b) are correct, the physical effects we attribute to unseen material (for example, dark matter and dark energy) are actually due to these extra source terms. Another point regarding the Weyl tensor is that the presence of Weyl stresses means that the matching conditions do not have a unique solution on the brane and hence knowledge of 5d Weyl tensor is needed as a minimum condition for uniqueness. It is shown by Dadhich, Maartens, Papadopoulos & Rezania (2000) that in the 5d brane, the high energy corrections to the energy density, together with the Weyl stresses from bulk gravitons, imply that on the brane the exterior metric of a static source is no longer the Schwarzschild metric.

There have been several studies addressing the issue of acceptable spherically symmetric vacuum solutions in brane-world scenarios (Visser & Wittshire 2003; Harko & Mak 2004; Creek, Gregory, Kanti & Mistry 2006; Ponce de Leon 2007; Viznyuk & Shtanov 2007). It was explicitly demonstrated in

2 However, in this context we would like to make a point that global monopoles are a dark matter candidate (with very strong restrictions) different to scalar fields.

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these references how the brane-world equations provide a weaker requirement on the 4d brane metric than the Einstein equations in four dimensions. Therefore, a more general class of solutions may be admitted in the brane-world picture than the Schwarzschild solution. Ponce de Leon demonstrated that the non-static situation differs from the static one by the extra requirement that $T^{0}_{0} = T^{1}_{1}$ at the matter-vacuum boundary. This requirement is sufficient to close the system of equations, yielding a Schwarzschild or Reissner-Nordström-like metric (where the “charge” term arises from an integration constant) as possible exteriors for non-static systems. For the static system he showed that no such restriction occurs at the boundary and therefore the vacuum solutions admit a much wider (infinitely many) possible vacua.

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