A SIMPLE MODEL LINKING GALAXY AND DARK MATTER EVOLUTION

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ABSTRACT

We construct a simple phenomenological model for the evolving galaxy population by incorporating predefined baryonic prescriptions into a dark matter hierarchical merger tree. The model is based on the simple gas-regulator model introduced by Lilly et al., coupled with the empirical quenching rules of Peng et al. The simplest model already does quite well in reproducing, without re-adjusting the input parameters, many observables, including the main sequence sSFR–mass relation, the faint end slope of the galaxy mass function, and the shape of the star forming and passive mass functions. Similar to observations and/or the recent phenomenological model of Behroozi et al., which was based on epoch-dependent abundance-matching, our model also qualitatively reproduces the evolution of the main sequence sSFR(z) and SFRD(z) star formation rate density relations, the $M_* - M_h$ stellar-to-halo mass relation, and the SFR – $M_h$ relation. Quantitatively the evolution of sSFR(z) and SFRD(z) is not steep enough, the $M_* - M_h$ relation is not quite peaked enough, and, surprisingly, the ratio of quenched to star forming galaxies around $M^*$ is not quite high enough. We show that these deficiencies can simultaneously be solved by ad hoc allowing galaxies to re-ingest some of the gas previously expelled in winds, provided that this is done in a mass-dependent and epoch-dependent way. These allow the model galaxies to reduce an inherent tendency to saturate their star formation efficiency, which emphasizes how efficient galaxies around $M^*$ are in converting baryons into stars and highlights the fact that quenching occurs at the point when galaxies are rapidly approaching the maximum possible efficiency of converting baryons into stars.

Key words: dark matter – galaxies: abundances – galaxies: evolution – galaxies: high-redshift – galaxies: luminosity function, mass function

Online-only material: color figures

1. INTRODUCTION

Galaxy evolution is a field where cosmological structure formation needs to be enriched with astrophysical processes (i.e., astrophysics has to be embedded into a cosmological model). It is the largest scale where astrophysical models have to succeed and the smallest scales where the cosmological structure formation model has to prove its validity. Therefore, galaxies, and the galaxy population, offer tests for both astrophysics and cosmology.

Several approaches have been taken to understand the link between galaxies and dark matter halos. Usually, the dark matter component is assumed to be well understood on the basis of both analytic and numerical models that are based on input parameters derived from cosmological observations (e.g., the cosmic microwave background, CMB). Small collapsed objects (i.e., “halos”) form earlier and subsequently merge together to form more massive objects. Numerical $N$-body simulations provide an accurate description of the evolution of the population of dark matter halos in the cosmological context (e.g., Springel & Hernquist 2003a; Klypin et al. 2011). Much of the difficulty in galaxy formation and evolution arises in understanding the actions of baryonic physics within these halos.

A major theoretical effort has been made using so-called semi-analytic techniques to follow the evolution of baryons in the halos. In semi-analytic models (or SAMs), simple parametric descriptions of the most important baryonic physics are combined with a dark matter merger tree that is usually obtained from a large volume $N$-body simulation. The treatment of the relevant baryonic processes is necessarily simplified (e.g., Lacey & Silk 1991; White & Frenk 1991; Kauffmann et al. 1993; Somerville & Primack 1999; Kauffmann et al. 1999; Springel et al. 2001; Helly et al. 2003; Hatton et al. 2003; Springel et al. 2005). Some or all of the parameters describing these processes can be adjusted to match particular observational properties of galaxies or of the galaxy population, either at a single epoch or at many. Although much progress has been made and the range of output quantities can be large, the total number of parameters in such models is often quite large and as a result, the uniqueness and predictive power of SAMs is limited. In addition, the apparent complexity of the SAMs can often hide underlying links between different aspects of galaxy evolution.

Much progress has also been made using the alternative approach of ab initio simulations in which the baryonic physics is directly incorporated into hydrodynamic codes. However, due to the very large dynamical range that must be covered, such simulations are currently unable to resolve star formation and associated feedback processes, and so cannot describe these processes from first principles. Simulation codes therefore include these as “sub-grid” physics, which leads to the emergence of a number of alternative approaches (e.g., Springel et al. 2005; Croton et al. 2006).

Partly in response to these difficulties, other, more phenomenological, approaches have been developed. One has been to study the statistical connection between galaxies and dark matter halos in terms of the conditional luminosity function (CLF; Yang et al. 2003) or the halo occupation distribution (HOD; e.g., Peacock & Smith 2000; Seljak 2000). These methods are anchored on our good understanding of the statistical properties of dark matter halos in the current $\Lambda$CDM model, as well as the hypothesis that galaxy properties should be closely
linked to the properties (and especially the masses) of dark matter halos. A variety of statistical tools can then be used to constrain the galaxy dark matter connection: galaxy clustering (e.g., Zehavi et al. 2011), galaxy–galaxy lensing (e.g., Brainerd 1996; Sheldon et al. 2004; Leauthaud et al. 2010), galaxy group catalogs (e.g., Berlind et al. 2006; Yang et al. 2007), abundance matching (recently e.g., Leauthaud et al. 2012b; Hearin et al. 2013; Reddick et al. 2013; Tinker et al. 2013), and satellite kinematics (e.g., More et al. 2009).

In recent years, large-scale surveys of the distant universe have yielded sufficient data to apply similar approaches at significant look-back times. The differential effects with redshift then allow a phenomenological description of the evolving galaxy population using simple parametric descriptions. The parameters of these are matched to the evolving statistical descriptions of the stellar-to-halo mass relation (e.g., Firmani & Avila-Reese 2010; Yang et al. 2012; Behroozi et al. 2013a; Lu et al. 2014). Such models can provide consistency checks within several data sets and observables. As an example, when compiling different data sets, Behroozi et al. (2013a) finds a disagreement between galaxy abundances for high redshift surveys and high systematic errors in the stellar mass and star formation rate (SFR) estimates.

The increasingly good observational data on the evolving galaxy population has also opened up other phenomenological approaches that instead focus on the baryonic processes. A successful approach has been to broadly classify galaxies as either actively forming stars or quiescent. Most star forming galaxies exhibit a rather tight relation between their SFR and stellar masses, producing the so-called main sequence (Brinchmann et al. 2004; Noeske et al. 2007; Daddi et al. 2007; Peng et al. 2010; Rodighiero et al. 2011). The quiescent galaxies have specific star formation rate (sSFR) that are one to two orders of magnitude lower, and these galaxies are not forming stars at a cosmologically significant rate. We will henceforth refer to these passive galaxies as “quenched.” A few underlying simplicities in the galaxy population can be then identified (e.g., the observed constancy of the Schechter $M^*$ of star forming galaxies or the separability of the fraction of galaxies that are quenched—colloquially the “red fraction”). The analytic consequences of these can then be explored using the most basic continuity equations (Peng et al. 2010, 2012, hereafter P10 and P12). This has proved very successful in describing the evolution of the galaxy population and, in particular, in deriving the simple empirical “laws” for the quenching of star formation in galaxies as a function of stellar mass (even if other parameters are involved or are the main causal drivers). This approach has also yielded new insights into the relationships between the mass functions of active and passive galaxies, and the relative importance of mass and environment in the quenching of star formation in galaxies.

Several papers have also developed simple toy analytic models for the SFR in galaxies (e.g., Bouché et al. 2010; Davé et al. 2011a; Krumholz & Dekel 2012; Dekel & Mandelker 2014; Dayal et al. 2013; Lilly et al. 2013a, hereafter L13). These have been motivated by the small dispersion in the specific star formation rate (sSFR, SFR/stellar mass) of actively star forming galaxies, and by the strong evolution of this characteristic sSFR with time. In terms of the CLF a phenomenological approach has been chosen by Tacchella et al. (2013). Dekel et al. (2013) developed a toy analytic model when comparing to hydrodynamical simulations. These models have tried to boil down the complexity arising in numerical simulations and detailed semi-analytic models into simple analytic models that are motivated by either simulation results or observational constraints. The aim was to provide a simple picture of how galaxies evolve in the cosmological context and to highlight the connections between different aspects of galaxy evolution. In particular, L13 developed a toy analytic model in which the SFR is regulated via the variable mass of gas in the gas reservoir feeding the star formation. Such a model links the sSFR to the specific accretion rate onto the regulator system. The self-regulation by the gas reservoir naturally introduces the SFR as a second parameter in the mass–metallicity relation $Z(m, SFR)$ and also naturally explains why the $Z(m, SFR)$ relation should be more or less independent of time. This model also links the different slopes of the mass functions of galaxies and halos.

By construction, the phenomenological analytic models in P10, P12, and L13 have only been tangentially linked to the dark matter halos and not at all to the overall evolving population of halos that is produced by hierarchical assembly in the cosmological context. The whole approach, and in particular the derivation of the numerical values of the few parameters in the models, was based on a comparison with baryonic systems. This has been a strength and a weakness of these analyses.

The aim of this paper is to explore how far we can get by taking these simple baryonic prescriptions and combining them with a dark matter structure formation formalism. Specifically, we will take the self-regulation model of L13 and the quenching “laws” of P10 and P12, and couple them with a Monte Carlo realization of dark matter halo merger trees. Our goal is to present a phenomenological model whose few parameters are taken from the earlier papers, and are not adjusted in the combined model. The parameters are well constrained and therefore considered to be non-adjustable in this paper. We can then explore how well these predictions match the observed universe, and identify where and how the model needs further improvement. In a second step, we propose two changes in the model and show their impact on the predictions.

Our approach is rather different to the one in Lu et al. (2014) or Behroozi et al. (2013a), as we do not explore a parameter space but instead develop a physical picture without further tuning within the combined model. We stress that the current model is not intended to replace more complex SAMs, as their greater sophistication will no doubt be required to account for a more multi-dimensional view of galaxies.

The current paper is structured as follow: In Section 2 we review the key concepts that were introduced in the earlier papers P10, P12, and L13, which we use to establish the characteristics of the baryonic processes. We define our notation and parameterization of these independent models and describe the dark matter structure formation formalism we apply. In Section 3 we describe how these are then combined into the dark matter merger tree, what further assumptions have to be added, and how the model is then run. In Section 4, we present our results in terms of the most basic observables of the galaxy population, such as the overall star formation rate density (SFRD), the sSFR–mass relation of star forming galaxies, the mass function of active and passive galaxies, and the form of the stellar mass versus halo mass relation for star forming and passive galaxies, as well as compare them with other work. In Section 5 we discuss the implications of the model, explore how one could modify it, and show where we are more restricted by the linkages between different parts of the model. Finally, in Section 6 we summarize our conclusions.
Throughout this paper, we assume a flat cosmology with $h = 0.7$ (i.e., $H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1}$), $\Omega_m = 0.045$, $\Omega_r = 0.3$, $\Omega_k = 0.7$, $\sigma_8 = 0.8$, and $n_s = 1.0$ consistent with Komatsu et al. (2011) WMAP7 results. We use the BBKS (Bardeen et al. 1986) transfer function to calculate the matter power spectrum. We define a halo as having a mean overdensity $\Delta \equiv 3M_\ln/4\pi\Omega_m\rho_{\text{crit}}R_\ln^3 = 170$ to be consistent with the merger tree we use in this paper. We use “dex” to refer to the antilogarithm, so that 0.3 dex represents a factor of 2.

2. MODEL INGREDIENTS

In this section we review the concepts and descriptions used in our model. We start with the differential equations that control the regulator system from L13 (Section 2.1). We then quote the mass- and satellite-quenching expressions from P10 and P12 (Section 2.2). In Section 2.3 we describe the dark matter structure formation formalism that we apply to our model. These ingredients are completely independent of each other and do not rely on mechanisms described in other sections.

2.1. Galaxies as Gas-regulated Systems

We adopt the model proposed in L13. Several similar models have been proposed in the literature (e.g., Bouché et al. 2010; Davé et al. 2011a; Krumholz & Dekel 2012; Dekel & Mandelker 2014; Dayal et al. 2013) although there are significant differences in both concept and detail. We identify a galaxy as a gas-regulated system sitting in a dark matter halo. The SFR in the galaxy is set by the gas mass $M_{\text{gas}}$ within a reservoir in the galaxy via a star formation efficiency, $\epsilon$. There is also mass loss from the reservoir in the form of a wind that is parameterized by a mass-loading factor, $\lambda$, such that the outflow is $\lambda \cdot \text{SFR}$. The $\epsilon$ and $\lambda$ parameters are allowed to vary with the stellar mass $M_\ast$ of the galaxy (and possibly the epoch, or redshift). In L13, the baryonic infall rate into the regulator $\Phi_b$, which replenishes the reservoir, was assumed to be some fixed fraction $f_{\text{gal}}$ of the baryonic infall onto the surrounding halo. Two obvious simplifications of the L13 model were that gas expelled from the galaxy in the wind was assumed to be lost forever (i.e., it does not mix with any surrounding gas in the halo), and that substructure within a halo was neglected (i.e., there was only one regulator in each halo). These issues will be discussed later in this paper.

As in L13, the stellar mass is defined as the long-lived stellar population, assuming that a fraction, $R$, of newly formed stellar mass is promptly returned to the gas reservoir. The remaining stars will have a lifetime that is longer than the universe. As in L13, we set the mass-return factor $R = 0.4$, motivated by stellar population models (e.g., Bruzual & Charlot 2003). The “stellar masses” used throughout this paper are these “long-lived” stellar masses, which are of an order of 0.2 dex smaller than the stellar masses that are obtained by integrating the SFR, which are sometimes quoted in the literature.

The build up in stellar mass $M_\ast$ is then given by

$$M_\ast = \text{SFR} \cdot (1 - R).$$

Following L13, the differential equations of the regulator in differential form can be written as

$$\text{SFR} = \epsilon \cdot M_{\text{gas}},$$

$$M_{\text{gas, outflow}} = \lambda \cdot \text{SFR},$$

$$M_{\text{gas, in}} = \Phi_b - M_\ast - M_{\text{gas, outflow}} = \Phi_b - (\epsilon + (1 - R)(1 - \lambda))M_{\text{gas}},$$

$$M_{\text{gas}} = \Phi_b - M_\ast - M_{\text{gas, outflow}} = \Phi_b - (1 - R + \lambda)M_{\text{gas}}.$$

We will not go into detail into the analytic solution of these differential equations, for more information, see L13.

The efficiency, $\epsilon$, and the outflow load, $\lambda$, are intended to cover, albeit simplistically, all the baryonic processes within the galaxy system. L13 considered a power law parameterization for both these quantities as a function of the stellar mass, $M_\ast$, in order to match the observed $Z(M_\ast, \text{SFR})$ relation in Mannucci et al. (2010). The parameterization as a function of stellar mass is a convenience and is still valid even if other quantities (e.g., halo mass) are responsible for the physical effect. The parameterization is

$$\epsilon(M_\ast, z) = \epsilon_{10} \cdot \left(\frac{M_\ast}{10^{10} M_\odot}\right)^b \cdot \left(\frac{H(z)}{H_0}\right)$$

$$\lambda(M_\ast) = \lambda_{10} \cdot \left(\frac{M_\ast}{10^{10} M_\odot}\right)^a.$$

$H(z)$ is the Hubble rate at redshift $z$ and $H_0$ the present-day Hubble constant. L13 assumed, following Mo et al. (1998), that the star formation efficiency would scale as the inverse dynamical time of the galaxies and halos, which should scale as the Hubble rate, and we do the same until revisiting this issue toward the end of the paper. In this section we review the concepts and descriptions used in our model. We start with the differential equations that control the regulator system from L13 (Section 2.1). We then quote the mass- and satellite-quenching expressions from P10 and P12 (Section 2.2). In Section 2.3 we describe the dark matter structure formation formalism that we apply to our model. These ingredients are completely independent of each other and do not rely on mechanisms described in other sections.

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Sloan Digital Sky Survey (SDSS) given by Mannucci et al. (2010), L13 derived nominal values for the parameters $\epsilon_{10}$, $b$, $\lambda_{10}$, and $a$ in Equations (5) and (6) above. Given the extreme simplicity of the model, the resulting values for $\epsilon(M_*)$ and $\lambda(M_*)$—which are quoted in Table 1 in L13 and included in Table 1 of this paper—are surprisingly reasonable, giving gas depletion timescales ($\epsilon^{-1}$) at $M_* \sim 10^{10} M_\odot$ of about 2 Gyr and mass-loading factors of the order of unity. The gas depletion timescale and the outflow mass loading both decrease with increasing stellar mass, resulting in more and more efficient conversion of inflowing baryons into stars as the stellar mass of the system increases. The fraction of incoming baryons that are converted to stars is denoted as $f_{\text{star}}$ in L13. In the context of the simple analysis of L13, this “saturation” of $f_{\text{star}}$ can be traced to the pronounced flattening of the $Z(M_*)$ relation at high masses. We will return to this later in the paper.

The processes associated with star formation in galaxies are thus represented in our model by the four parameters (Equations (5) and (6)) describing $\epsilon(M_*)$ and $\lambda(M_*)$, and taken straight from L13. As noted above, we will initially assume $\epsilon$ increases as $H(z)/H_0$, although we will revisit this assumption later.

Work by Springel & Hernquist (2003b); Benson et al. (2003); Lucia et al. (2004); Governato et al. (2007); Oppenheimer & Davé (2008); Scannapieco et al. (2008); Bower et al. (2012) have emphasized the importance of supernova feedback. In L13, outflows of material represent an inefficiency in the production of stars, but do not regulate the level of star formation, which is instead defined by the gas mass.

2.2. Quenching of Star Formation in Galaxies

In this paper, we apply the phenomenological quenching prescriptions derived by P10 and P12. This is distinct from introducing a turnover in the efficiency parameter as done by Behroozi et al. (2013a) and Lu et al. (2014), or cutting off the supply of gas, as done by Bouché et al. (2010), although the...
outcomes may be similar. There are many physical mechanisms that have been proposed for quenching. One popular approach is active galactic nucleus (AGN) feedback (see, e.g., Governato et al. 2004; Croton et al. 2006; Bower et al. 2006; Booth & Schaye 2009). The AGN feedback also presents a viable solution to the cooling flow problem (see, e.g., Fabian et al. 1994; Böhringer et al. 2002; Ishibashi & Fabian 2012), hence its popularity. The P10 approach comes from the continuity in the two distinct galaxy populations, and is not based on a particular physical mechanism, but rather seeks to define the characteristics that any viable mechanism must satisfy.

We will assume that star formation within a galaxy stops as soon as it is quenched and that no significant star formation occurs afterward. As a shorthand (and on plots) we will denote the actively star forming galaxies as blue and those that are quenched as red, although we will not consider the colors of galaxies per se. The “red fraction” will then be the fraction of galaxies of a given mass that have been quenched.

P10 showed that the red fraction of galaxies as a function of mass and local projected overdensity is separable in the two variables, suggesting that there are two dominant processes: one that depends on mass but not density (so-called mass-quenching), and a second environment-related process that should be independent of stellar mass. The mass-quenching process is then the only one that depends on mass, and therefore is the one that determines the shape of the mass function of the surviving star forming galaxies and, via the continuity equation, the shape of the mass function of the resulting (mass-quenched) population of passive galaxies. The observed constancy of the shape of the mass function of star forming galaxies over a wide redshift range up to \( z \sim 2 \) (or even higher) imposes a strong requirement on the form of mass quenching (see P10 and below).

Subsequently, P12 showed that the environment quenching in the overall population could be fully accounted for by a satellite quenching process that applies only to satellite galaxies. The probability that a previously star forming central galaxy is quenched when it becomes the satellite of another galaxy is about 50%, independent of its stellar mass. There are many possible suggestions for an environment-dependent quenching mechanism (see, e.g., Gunn & Gott 1972; McCarthy et al. 2008; Font et al. 2008).

The P10 prescription for mass quenching can be written either as a quenching rate (i.e., the probability that a given star forming galaxy will be quenched per unit time), or as a survival probability to reach a certain mass without being quenched. The probability, \( p_{\text{quench}} \), for a galaxy becoming quenched when increasing its stellar content by \( \Delta M \), is given by

\[
dp_{\text{quench}} = \mu \, \text{d}M, \tag{7}
\]

for an infinitesimal \( \text{d}M \). For a finite increase, \( \Delta M \), one gets

\[
p_{\text{quench}} = 1 - \exp\left[-\mu \Delta M\right]. \tag{8}
\]

The constant \( \mu \) is required (see P10) to be \( M^{*-1} \), where \( M^* = 10^{10.68} M_\odot \) is the value of the characteristic stellar mass of the (single) Schechter stellar mass function (SMF) of the blue star forming population. Following P10, we take \( \mu \) to be constant with time because \( M^* \) is observed to be more or less constant.

We will assume that the mass-quenching process acts on all galaxies (i.e., both centrals and satellites). This is motivated by the observational fact that \( M^* \) is the same for central and satellite star forming galaxies (P12). Because of the close coupling of stellar mass (and even black hole mass) and halo mass for central galaxies, the action of a mass quenching that is driven by stellar mass (as in the equation above) is hard to distinguish from one driven by halo mass for centrals, but again our point is that the outcome must be well represented by the empirical P10 quenching “laws.”

For satellites, we apply an additional stochastic quenching process. When a central galaxy becomes the satellite of another galaxy because its own halo merges with another more massive halo, the chance of it being (instantly) quenched is set to \( p_{\text{sat}} = 0.5 \). This additional quenching probability is only applied once to any particular galaxy when it first becomes a satellite. Because we do not, in the current paper, consider the radial distribution of galaxies within halos (e.g., Prescott et al. 2011), or try to compute the local overdensity as in P12 or Kovač et al. (2014), we do not consider the density dependence of \( p_{\text{sat}} \) instead adopting a mean value. This mean value of \( p_{\text{sat}} = 0.5 \) is assumed to be constant with the epoch, as shown in the zCOSMOS group catalogue to \( z \sim 0.7 \) (Knebel et al. 2013; Kovač et al. 2014).

To summarize, the quenching of galaxies in this model is accounted by just two constants: \( \mu = 10^{-10.6} M_\odot^{-1} \) for mass quenching and \( p_{\text{sat}} = 0.5 \) for satellite quenching.

2.3. Dark Matter Structure Formation

To describe the hierarchical structure formation process, we take a simple model that is far below the complexity of N-body simulation, but aim to account for most of the features of those simulations. The descriptions we apply have been incorporated by many authors in one or another way (recently, e.g., by Lu et al. 2014). We use the dark matter merger tree generator from Parkinson et al. (2008), which is based on the excursion set theory (e.g, Press & Schechter 1974; Epstein 1983; Bond et al. 1991; Lacey & Cole 1993) tuned to match the Millennium simulation (Springel et al. 2005). Parkinson et al. (2008) showed that the tuned merger tree generator matches the overall halo mass function and the progenitor mass function for different halo masses very well, back to redshift \( z = 4 \). The merger tree uses a Monte Carlo method. Given a halo mass \( M_h \) at redshift \( z \) it generates the progenitors at \( z + \Delta z \) for small time steps \( \Delta z \) (backward process). In addition to a smoothed component growth, there is also a probability of having a binary split in the merger tree with a host and satellite halo:

\[
M_h \xrightarrow{\Delta z} M_{\text{host}} + M_{\text{sat}} + M_{\text{smoothed}}, \tag{9}
\]

where \( M_{\text{host}} \) is the most massive progenitor of \( M_h \). The tree naturally divides the progenitors into a smooth component (all progenitors below a mass threshold \( M_{\text{thresh}} \)) and a merger component (growth due to accretion of mergers above \( M_{\text{thresh}} \)). We express the growth of a halo as

\[
\dot{M}_h = M_{h,\text{smoothed}} + M_{h,\text{merger}}. \tag{10}
\]

For the subhalo evolution we apply the formalism from Boylan-Kolchin et al. (2008). They used high resolution dark matter simulations with one host and one satellite halo to invert the dynamical friction timescale \( t_{\text{df}} \) and provide a fitting formula for \( t_{\text{df}} \) (Equations (5) and (6) in their paper, with the further assumption that the last factor in their Equation (5) is equal to unity):

\[
\frac{t_{\text{df}}}{t_{\text{dyn}}} = 0.216 \frac{(M_{\text{host}}/M_{\text{sat}})^{1.3}}{\ln(1 + M_{\text{host}}/M_{\text{sat}})} e^{1.97} \tag{11}
\]
This formula depends on the host-to-satellite mass ratio \(M_{\text{host}}/M_{\text{sat}}\) and orbital circularity \(\eta\). Boylan-Kolchin et al. (2008) noted in their analysis that including the effect of baryonic bulges gives an approximately 10% shorter \(t_{\text{df}}\). This fitting formula has been tested for \(0.025 \leq M_{\text{sat}}/M_{\text{host}} \leq 0.3\) and is applicable for \(\eta \geq 0.2\). Note that the dynamical time \(t_{\text{dyn}} \approx 0.1H^{-1}\) with \(H\) being the Hubble parameter. The inverted dynamical friction timescale can be several times larger than the dynamical timescale \(t_{\text{dyn}}\). From numerical simulations, Zentner et al. (2005) have shown that the probability distribution of the orbital circularity, \(\eta\), of dark matter subhalos can be approximated by

\[
P(\eta) \propto \eta^{1.2}(1-\eta)^{1.2}.
\]

For every merger event in our merger tree we therefore draw \(\eta\) from this distribution, thereby introducing some scatter in the dark matter structure formation process.

So far, we have an expression for the survival time \(t_{\text{df}}\) of a subhalo. For the subhalo mass evolution \(M_{\text{subhalo}}(t)\), we implement a step function following Yang et al. (2012)

\[
M_{\text{subhalo}}(t) = \begin{cases} 
M_{\text{sat}}(t = t_a) & t - t_a < t_{\text{df}} \\
0 & t - t_a > t_{\text{df}},
\end{cases}
\]

where \(t_a\) is the time of accretion.

In this paper, \(M_h\) refers to the total halo mass. The halo mass associated with the central galaxy is then given by

\[
M_{\text{central}} = M_h - \sum_i M_{\text{subhalo},i},
\]

where the sum is over all surviving subhalos above a certain mass threshold \(M_{\text{thresh}}\). We thereby identify all substructure above \(M_{\text{thresh}}\), and trace its evolution.

Our dark matter formalism clearly consists of some simplifications. The merger tree is tuned to a dark matter only simulation, whereas our model also contains baryonic matter. One implicit simplification is that the baryonic matter component will not deviate from the behavior of dark matter. In other words, the gravitational forces from the dark matter are the dominant driver of baryonic structure formation, and pressure terms are ignored. Likewise, there is no reverse effect from the baryons on the dark matter (see, for example, Borgani et al. 2006 for a more detailed description).

It should be noted that the merger tree is tuned to a slightly different cosmology. However, the tuned parameters are dimensionless and as the excursion-set approach is formulated for arbitrary power spectra, Parkinson et al. (2008) argued that their merger tree could also be applied to different cosmologies. See also Jiang & van den Bosch (2014) for discussion on the accuracy. For the substructure evolution, we applied a very simple description, especially for the time evolution of the substructure. Despite these simplifications, our chosen description provides us with a good picture of what is going on in the dark matter structure formation process. It does not, however, contain the detailed and accurate descriptions that would be needed for doing precision cosmology.

To summarize, we introduced one arbitrary parameter \(M_{\text{thresh}}\) in our structure formation model and take \(t_{\text{df}}\) from Boylan-Kolchin et al. (2008). The remaining parameters for the dark matter are taken from the standard cosmology.

3. THE MODEL

In this section we describe how we combine all the ingredients given in Section 2. In particular, we describe in Section 3.1 how we link the baryonic infall rate onto the regulator system to the dark matter structure formation process. In Section 3.2 we describe and discuss what happens in a galaxy–galaxy merging event in our model framework. In Section 3.3 we describe how the regulator at very low stellar masses can be described. The procedure to predict the cosmic abundances of galaxies and their properties is described in Section 3.4. Finally, we emphasize in Section 3.5 how our model differs from others parametric approaches.

3.1. Link Between Baryonic and Dark Matter Infall Rate

To consistently integrate our regulator and quenching models into the dark matter framework, some further assumptions must be made. First, only dark matter halos and subhalos above \(M_{\text{thresh}}\) in Section 2.3 will contain a regulator system. In other words, we ignore star formation in halos that are so small that we considered their infall as part of the smooth dark matter inflow. This is because they will be mostly gaseous. We set \(M_{\text{thresh}} = 1.4 \times 10^6 M_\odot\). This is somewhat arbitrary, but is consistent with photoionization heating suppressing cooling and star formation below a certain halo mass \(M_f\). \(M_f \sim 10^8 M_\odot\) during reionization to \(M_f \sim 10^9 M_\odot\) (Gnedin 2000; Okamoto et al. 2008). For a more realistic model aiming to make predictions of low-mass galaxies back to the epoch of reionization, one would need to account for a change in the mass threshold. We explore the effect of changing \(M_{\text{thresh}}\) in Appendix A.3 and show that it is small for the galaxy mass scales of interest.

In order to trace the gaseous baryons through the build-up of halos, the following simple scheme was used. We will later refer to this model as Model A.

1. First, all gaseous baryons in a given halo are associated at all times with one of the regulator systems (i.e., “galaxies”) within that halo, except for those baryons that have been processed through a regulator and ejected from the galaxy through the wind described by \(\lambda\) in Section 2.1. These ejected baryons are assumed to be “lost” (we will revisit this assumption later in the paper) and are no longer tracked. However, aside from this, all gaseous baryons are found within the reservoirs of the regulator systems.

2. Second, when two halos merge, the baryons that are within each of the regulator systems in the halos stay within those regulators, unless the (sub)halo subsequently decays and is disrupted (see below).

3. Last, smooth accretion of gas onto halos (i.e., the baryonic inflow associated with the merging of halos below \(M_{\text{thresh}}\)), is split between the subhalos as follows:

\[
\Phi_{b,i} = f_b \dot{M}_{\text{h,smoothed}} \frac{M_{\text{subhalo},i}}{M_h}.
\]

This scheme ensures that every baryon that has not flown into some regulator in the past will be assigned to a regulator when coming into a halo above \(M_{\text{thresh}}\). It also ensures that when a regulator becomes a satellite, its infall rate and thus its SFR will not dramatically change, as observed (see P12). We note that when a galaxy is quenched, the gas inflow associated with it will not be redirected to other active regulators. In our discussion later in the paper, we introduce a different assignment of the
in a given halo and introduced provides far more freedom in assigning gas to regulator systems. As noted previously, L13 only considered a single regulator in a given halo and introduced \( f_{\text{gal}} \) as the fraction of inflowing baryons that penetrate down and enter the regulator system at the center of the halo. L13 concluded that \( f_{\text{gal}} \sim 0.5 \) was required to reproduce the stellar to dark mass ratio of typical galaxies. By associating all gaseous baryons to regulator systems, we are effectively setting \( f_{\text{gal}} \) to unity (i.e., eliminating this parameter) in Models A and B in the present paper. However, because we now include multiple regulators (associated with the subhalos) in a given halo and a two component growth (mergers and smoothed accretion), the net effect for the central regulator will be similar because only a fraction \( (M_{\text{h,smooth}}/M_h) \) of the halo growth is associated with gas accretion and will only receive a fraction \( (M_{\text{central}}/M_h) \) of the incoming gas. In other words, we would now understand that the adoption of the lower \( f_{\text{gal}} \sim 0.5 \) in L13 simply accounted for the two component growth of a halo, which was neglected in their treatment of regulator systems.

3.2. Subhalo Disruption/Galaxy–Galaxy Merging

We now turn to what happens when a subhalo decays according to Equation (13), and specifically what happens to the gas and stars within the regulator associated with that sub-halo. The two extreme cases would be adding all the stars and gas to the central galaxy or distributing them into the inter-cluster medium, which for the gas would involve redistributing the gas among the surviving regulators according to Equation (15). In reality, it is likely to be in between these extremes. For concreteness and convenience, we set the fraction of stars and gas that are given to the central galaxy \( f_{\text{merge}} = 0.5 \), but show in Appendix A.1 that the output of the model is insensitive to this parameter. When the gas and stellar components from the two different regulator systems are merged in this way, the new state of the regulator will likely not be in equilibrium with the gas infall rate. Galaxy–galaxy merging can thus lead to some scatter in the regulator properties. As discussed in L13 and illustrated in their Figure 3, the regulators adapt quickly to the new conditions and rapidly settle to the new equilibrium state.

3.3. Break Down of the Regulator Description at Low \( M_\ast \)

In L13, the parameters of the regulator (Equations (5) and (6) in this paper) were tuned to match the metallicities of galaxies with stellar masses above \( 10^8 M_\odot \) and this parameterization must break down at lower stellar masses—not least, the mass loading cannot increase without limit, simply on energetic grounds. However, we need to include such low-mass galaxies in our model so as to have larger galaxies later on, so we introduce a maximal outflow load and set \( \lambda_{\text{max}} = 50 \). This value is far off the regime where L13 tuned their parameters and therefore will not affect the validity of the tuning in L13. It also does not significantly affect the output of the model for galaxies above \( M_\ast = 10^8 M_\odot \), which is the mass range of primary interest. Further discussion of this parameter can be found in Appendix A.2.

3.4. Implementation

The input galaxy data going into the model was derived independently of the number of galaxies (i.e., it was the mean mass–SFR–metallicity relation (L13), the shape of the star forming mass function parameterized by \( M^* \) (P10), and the red fractions of satellites (P12)). A primary output of the model will be the expected number density of galaxies.

We therefore need to create a representative sample of the universe. Merger trees derived from \( N \)-body simulations are sampled according to the halo mass function and produce far more low-mass halo trees than for high-mass halos. As we want to achieve the same statistical power over a wide range in halo mass, we want to equally sample the halo masses, and weight their abundances in a second step. The merger tree generator makes this possible. The procedure is as follows. We sample 10,000 halos at redshift \( z = 0 \), which are chosen randomly from a flat distribution in logarithmic halo mass, from \( 7.1 \times 10^9 M_\odot \) to \( 1.4 \times 10^{15} M_\odot \). We then weight their abundance according to the halo mass function of Sheth & Tormen (1999) at \( z = 0 \). By construction, the weighted abundance of our halos is in perfect agreement with the input halo mass function at \( z = 0 \). We then let these halos run backward in cosmic time by applying the merger tree description. We stop when our resolution limit \( M_{\text{thresh}} \) is reached or at \( z = 15 \). At that point, we identify our regulator systems, put in some initial stellar and gas mass, and solve the differential equations for every single tree component. In parallel we apply the subhalo evolution model in the forward process. We thereby keep track of every satellite halo with its own regulator system. The model is not sensitive to the initial state of the regulators, as described in Appendix A.4.

This description has no spatial resolution, either within galaxies, within halos, or to follow the large-scale distribution of halos. The last of these would be relatively easy to implement, and will be the subject of a future paper. The other two would take us deeper into details that we wish to avoid.

3.5. A Model Without Re-adjusting the Parameters

Table 1 lists all the parameters of our model, with a short description and reference to the input data from which they are based. These are mainly taken from the three papers (P10, P12, and L13) and from cosmology and computational simplifications in the dark matter sector. The effects of the three additional parameters that we have introduced in this paper (i.e., \( M_{\text{thresh}}, f_{\text{merge}}, \) and \( \lambda_{\text{max}} \)) are investigated in the Appendices A.1–A.3. We conclude that any reasonable variation within these parameters do not invalidate our conclusions. In essence, these parameters are introduced for practical reasons to make the model operable and the output does not depend very much on their precise values.

Within our chosen gas inflow description we therefore have virtually no freedom in changing our predictions—the model either matches observations or produces a discrepancy from which we may hope to learn. The goal is therefore not to produce a model that fits all available data, or to observationally determine parameters. Rather, and in the spirit of the previous papers (P10, P12, and L13), we aim to provide insights into how well the ideas presented in those papers perform in the global context of a dark matter hierarchy, and to see where we encounter limitations.

4. RESULTS

In Section 2, we reviewed the different and independent inputs that were combined in Section 3 to produce a single model of star formation and quenching in galaxies within a dark matter hierarchical framework. In this section, we compare the output of the default model A with both observations directly, and with
the outputs of other phenomenological approaches to galaxy evolution, most notably that of Behroozi et al. (2013a).

As discussed previously we will not vary any pre-adjusted parameter in our model beyond the three parameters introduced to allow the model to be computed (the values of which do not much affect the outcome), and so we can examine these comparisons one at a time. Throughout this section, we refer to the same output sample that was generated with the parameters given in Table 1 with the inflow description of Equation (15), referred as our fiducial Model A.

It should be noted that the observational data used to determine these parameters were (a) gas metallicity data (as in L13) from Mannucci et al. (2010) SDSS, specifically the $Z(M_\text{s}, \text{SFR})$–relation; (b) the red fraction of satellites (as in P12 from Abazajian et al. (2009) SDSS DR7); and (c) the value of $M^\beta$ of star forming galaxies (as in P10, also from SDSS). Any predictions of these particular quantities must therefore match observations, by construction, but the predictions of all other quantities are bona fide and can be meaningfully compared with other data.

Comparing these predictions with other data enables us to draw several interesting conclusions. Some of the successes of these “predictions” mirror the conclusions that were drawn in the original papers on which our new model is based (e.g., the discussions of mass functions and red fractions in P10 and P12, and the link between sSFR and specific accretion rate in L13). It is reassuring to see these predictions holding up in the context of a more realistic treatment of the halos, including substructure and merging. However, none of the predictions based on the population of dark matter halos were made in the earlier works. These include the normalization of the mass functions and the computation of the SFRD. We can also predict the scatter in various relations coming from different halo assembly histories.

Finally, we make explicit comparisons with the output from the orthogonal phenomenological approach of Behroozi et al. (2013a). The Behroozi et al. (2013a) approach is anchored in the dark matter hierarchy and derives a very general description of the effect of baryonic processes within these halos. In that work, a general $M_\epsilon/M_\text{b}$ relation is assumed. The epoch-dependent form of this is then derived by simultaneously applying statistical tools, such as abundance matching of the mass functions at different redshifts, coupled with a comparison of the consequent information on star formation with a variety of observational data, including the sSFR($M_\epsilon, t$) and the global SFRD. Our own approach is orthogonal to this, as it is based on a prior determination of the purely baryonic phenomenology, which is then imported into the dark matter structure. Despite the different approaches, and the obvious limitations of each, we find that a very similar picture emerges.

### 4.1. Stellar Mass Dependence of the Main Sequence sSFR at the Present-day

We first plot in Figure 1 the sSFR of all blue (i.e., star forming) central galaxies of the output sample at $z = 0$ as a function of their stellar masses. The model successfully recovers the tight correlation between sSFR and mass that is known as the main sequence (e.g., Brinchmann et al. 2004; Noeske et al. 2007) and an almost constant sSFR with a scatter about this relation of about 0.2 dex.

For comparison with the data, we overplot an sSFR($M_\epsilon, z$) relation of the form

$$\text{sSFR} \propto M^\beta,$$  

Observational estimates of $\beta$ range between $-0.4 < \beta < 0.0$ at stellar masses above $10^9 M_\odot$, with most estimates $\beta \sim -0.1$ (e.g., Brinchmann et al. 2004; Noeske et al. 2007; Elbaz et al. 2007; Daddi et al. 2007; Pannella et al. 2009; Stark et al. 2013; Peng et al. 2010). In Figure 1 the red line illustrates the data compilation in the form

$$\text{sSFR}(M_\epsilon, z) = 0.12 \left( \frac{M_\epsilon}{10^{10.5} M_\odot} \right)^\beta (1 + z)^3 (z < 2),$$  

with $\beta = -0.1$ evaluated at $z = 0$ (see L13, and references therein). The observed scatter among real galaxies is about 0.3 dex once outliers with much higher sSFR are excluded (e.g., Rodighiero et al. 2011; Sargent et al. 2012). These are associated with star bursts, probably induced by mergers.

The mean sSFR($M_\epsilon$) at $z = 0$ is well reproduced by the model. As noted in L13 and discussed earlier in this paper, a key feature of the kind of gas regulation considered in this paper is that it sets the sSFR close to the specific mass accretion rate of the system, independent of the values of the parameters, $\epsilon$ and $\lambda$, controlling the regulator. There is a modest “boost” to the sSFR if an individual regulator system is increasingly efficient at producing stars as time passes (as would be expected if the efficiency increases with mass). This boost at $z = 0$ is expected to be of order 0.3 dex for typical galaxies. It increases to lower masses, potentially reversing the slope of the sSFR($M_\epsilon$) relation relative to that of the specific accretion rate, which is defined as $\text{sSFR} = \psi_\epsilon/M_\text{b}$. L13 took the approximation for the sMIR provided by Neistein & Dekel (2008). Despite our model using a more complex description for the baryonic infall rate $\Phi_\epsilon$, we would expect to have the same underlying link between the sMIR and sSFR. The good agreement with the mean $z = 0$ sSFR($M_\epsilon$) relation in the current model, which contains a wide variety of individual halos, is therefore reassuring but not unexpected given the discussion in L13 (see their Figure 9).
The scatter in sSFR($M_\star$) in our model is caused by the different halo formation histories, i.e., by the variation in the gas inflow rate caused by variations in the merger tree (green dots in Figure 1), and by the effects of galaxy–galaxy merging (see Section 3.2). Our model does not include any further stochastic time variation in the gas infall $\Phi_b$, such as might be caused by other baryonic processes, and also neglects any stochastic scatter in the baryonic processes controlling star formation within the galaxy regulator systems. Both of these could further increase the scatter (in our model there is almost no scatter occurring in the SFR–$\Phi_b$ relation). Our predicted scatter can therefore be interpreted as a lower boundary in the expected sSFR($M_\star$) scatter. The fact that it is already two-thirds of the observed scatter suggests that these two further contributors to the scatter (stochastic infall variability and variation in the regulator) can contribute only of an order of 0.2 dex in normal main sequence galaxies.

4.2. Epoch Dependence of the Main Sequence sSFR and the Star Formation Rate Density

In Figure 2, we show the evolution in the sSFR for galaxies in the mass range $10^{10} M_\odot$–$10^{10.5} M_\odot$ back to $z = 5$, compared with data from Stark et al. (2013) and a highly parameterized model of Behroozi et al. (2013a) adopted to our definition of sSFR = SFR/$M_\star$. We also show for comparison the mean sMIR and the specific gas infall rate. As expected, the sSFR tracks the increase in sMIR with redshift. While this broadly matches the data, the rise with redshift is not steep enough. As discussed in L13, this discrepancy is roughly the same as for the sSFR($z$) evolution. We return to this later.

4.3. The Evolution of the Gas Fraction in Galaxies

In Figure 4, we plot the gas-to-star ratio $\mu = M_{\text{gas}}/M_\star$ as a function of stellar mass for different redshifts. We get about a factor of six higher gas-to-star ratio at $z \sim 4$ compared to $z = 0$. From the definition of the regulator quantities in L13, the gas ratio is simply given by the ratio of the sSFR and the star modification of the accretion rate of baryons onto the regulator systems (i.e., breaking the link between the baryonic accretion rate onto the galaxy and the specific growth rate of the dark matter halo) or a rather dramatic adjustment of the efficiency with which the inflowing gas is converted to stars (i.e., the $f_{\text{star}}$ parameter of L13) so as to increase the boost factor associated with temporal changes in this quantity (see L13). We will return to this discrepancy in Models B and C, but note here that it is not inconceivable that some of the offset of 0.3 dex could reflect observational difficulties in determining stellar mass and SFRs at high redshifts.

Our model naturally produces a deviation of the baryonic increase rate to the dark matter growth rate at very high redshifts as the dynamical friction timescale cannot catch up the halo growth rate, resulting in far more substructure surrounding the central at high redshifts. More substructure means that within our model less baryonic infall will be assigned to the central, as described in Equation (15).

In Figure 3, the overall SFRD is plotted over the whole range of cosmic time, compared with data from the compilation by Hopkins & Beacom (2006) and the phenomenological model by Behroozi et al. (2013a). The gray region is the 1$\sigma$ inter-publication scatter noted by Behroozi et al. (2013a).

The broad features of the evolving SFRD of the universe are reproduced and our predicted value at $z = 0$ matches well the observational data of the nearby universe. We again see a tension in the model that the SFRD is too low at $z = 2$. The size of the discrepancy is roughly the same as for the sSFR($z$) evolution. We return to this later.

Figure 2. Prediction of the mean sSFR for blue galaxies within a stellar mass range of $10^{10} M_\odot$–$10^{10.5} M_\odot$ of Model A as a function of redshift (red curve). These are compared with data points (in black without error bars) from Stark et al. (2013) and a model based on a data compilation from Behroozi et al. (2013a) adjusted to our definition of SFR. The gray region reflects the 1$\sigma$ scatter between different measurements in the literature given by Behroozi et al. (2013a). The specific gas infall rate of the same galaxy sample of our model is over-plotted in green. The sSFR follows this quantity with an offset (boost) as discussed in L13. Furthermore, the specific mass increase rate of the halo (sMIR) is overplotted.
(A color version of this figure is available in the online journal.)
present our results for the evolution of the SMFs for different redshifts for Model A. With our default regulator parameters, this ratio increases at $z = 4$ by about a factor of six relative to locality.

(A color version of this figure is available in the online journal.)

The galaxy SMF is a well measured quantity at low redshifts (e.g., Baldry et al. 2008; Pozzetti et al. 2010; Peng et al. 2010; Baldry et al. 2012). Our model provides predictions for the overall SMF from our Model A. The blue curves are for the blue population, the red curves for the red population (including centrals and satellites). The output is compared to the data of Baldry et al. (2012). Dashed lines correspond to Schechter fits to the blue and red population in their paper. (A color version of this figure is available in the online journal.)

4.4 Stellar Mass Function (SMF)

The galaxy SMF is a well measured quantity at low redshifts (e.g., Baldry et al. 2008; Pozzetti et al. 2010; Peng et al. 2010; Baldry et al. 2012). Our model provides predictions for the overall SMF and for the the population split into blue and red galaxies (i.e., star forming and quiescent), and into centrals and satellites. As noted above, the model is constructed to reproduce the characteristic Schechter cutoff of the blue population at $M^* \sim 10^{10.68} M_\odot$ and for this to be constant with time, but we have not introduced any other parameter that is based on the faint end slope of the blue and red population, or the red fraction at $M^*$, for example. The mass quenching law of P10 can directly predict the relative faint end slopes of the blue and red population, but the absolute slope $\alpha_{b,\text{blue}}$ of the blue population had to be assumed. The red fraction at $M^*$ also follows from the input $\alpha_{b,\text{blue}}$.

In Figure 5 the model prediction for the blue, red, and total population at $z = 0$ is plotted, while in Figure 6, we present our results for the evolution of the SMFs for different galaxy types (split into red and blue, as well as central and satellite) over cosmic time. The Schechter parameters for the SMFs of the red and blue centrals and satellites are given in Appendix B. The red satellite population can be better described by a double Schechter function. As shown in P12, this is due to the superposition of mass- and satellite-quenching (more about the fits in Appendix B).

The model successfully reproduces the correct faint end slope of the mass function. This is a reflection of the link between the slope of the mass–metallicity relation and the faint-end slope, $\alpha$, of the mass function (see L13 for discussion). The relations between the Schechter parameters ($M^*$ and $\alpha$) of the different populations in Figure 6 are also as observed. The universality of $M^*$ (all populations have very similar $M^*$) and the change in faint end slope, $\Delta \alpha \sim 1.0$, between blue and red centrals, are
also successfully reproduced. These follow from the forms of the quenching laws derived in P10 and P12.

Less trivial is the overall normalization of the SMFs of the different populations. The $\phi^*$ describes the normalization at $M^*$ in the Schechter function fits. The SMF is the convolution of the stellar-to-halo mass relation (SHMR), including its scatter, with the underlying halo mass function. We note that the underlying halo mass function is Press–Schechter like and not Schechter like. If we do not apply the mass quenching description, the SMF prediction would look Press–Schechter like and would have a rapidly evolving characteristic mass. At very high redshift, where the galaxy population could not build up a significant fraction of galaxies with stellar masses above $M^*$, we predict a Press–Schechter like SMF. In our model we see that the transition from a Press–Schechter to a “vertically evolving” Schechter-like SMF happens between $z = 6$ and $z = 4$ (from Figure 6). It is the moment when the stellar mass function breaks away from the halo mass function. Lilly et al. (2013b) referred to this as the Phase 1 to Phase 2 transition. We can also clearly see that the satellite population grows more rapidly with cosmic time than that of the centrals in Figure 6, also indicated by the Schechter fits in the Appendix B. This means that the special role of the quenching of satellite galaxies becomes more and more important with cosmic time. The satellite quenching leads to the double-Schechter component in the SMF of the red population. The differential rate of quenching of the two populations, and the fact that the quenched satellites dominate at lower masses, leads to the appearance of “downsizing,” i.e., a more gradual buildup of the stellar mass function at lower masses.

The biggest problem with the mass functions is surprising. Although the shape of the mass function of the passive galaxies is right, the overall number density is too low. This also produces a weaker bump in the “double” Schechter function, which is caused by the superposition of the red and blue SMF (which have different faint end slopes $\alpha$). This is surprising because one of the great successes of the P10/P12 quenching formalism was to explain, via the continuity equation, the ratio of these two components, which is given simply as $(1 + \alpha)^{-1}$, where $\alpha$ is the faint end slope of the star forming mass function. For $\alpha \sim -1.4$ this would predict a ratio of about 2.5, close to what is observed, whereas our model prediction is closer to 1.5. However, we clearly note that with $\alpha \sim -1.5$ (our Schechter fit) the ratio goes down to about 2.0. We discuss this interesting question further in Section 5.

4.5. Star Formation Rate History in Different Mass Halos and the Evolution of the Star Formation Rate Density

We now turn to comparisons with the phenomenological model of Behroozi et al. (2013a). In Figure 7, we show our prediction for the SFR in halos (including centrals and satellites) as a function of cosmic time and halo mass. This may be compared with the similar Figure 4 from Behroozi et al. (2013b), which was derived from their completely different, but similarly phenomenological approach.

Behroozi et al. (2013a) concluded that most stars were formed around $z = 2$ in halos of about $10^{12} M_\odot$. This is a natural output of our model as the regulator is highly inefficient in producing stars at low stellar masses and (mass-) quenching is most effective above $M_* = M^*$, which corresponds to about $10^{12} M_\odot$ in halo mass.

The fact that these two orthogonal approaches produce a similar phenomenological picture is very reassuring. It furthermore emphasizes the operational difficulty of distinguishing, for central galaxies, whether the dark matter mass or the (baryonic) stellar mass is driving the variable efficiency with which halos convert baryons into stars, simply because these two quantities are tightly linked.

4.6. Stellar-to-halo Mass Relation (SHMR)

One of the central properties of galaxies is the SHMR, both for centrals and for satellite galaxies. The SHMR represents the overall efficiency with which halos convert baryons into stars. This quantity has been extensively studied using abundance matching and other statistical techniques, such as HODs, which are based on the conviction that the SHMR should be well behaved. Observations using weak-lensing can be used to directly test these, generally with success (e.g., Leauthaud et al. 2012a).

The SHMR for our output sample at the present epoch is plotted in Figure 8 and compared with the zero-redshift relation from Behroozi et al. (2013a). As would be expected, the increase in the $M_\star/M_{h}$ ratio at low masses simply reflects the increasing efficiency of converting baryons to stars (i.e., $f_{\text{star}}$ in L13) in more massive regulators, whereas the turnover and subsequent decline is due to the mass quenching of galaxies, which becomes progressively more important at masses around and above $M^*$, corresponding to about $10^{13} M_\odot$ in halo mass.

The 1$\sigma$ scatter in the SHMR of the blue population in the model is about 0.21 dex. This mostly comes from the different halo assembly histories (e.g., the time when the last major merger happened). The scatter in the red population is larger and is about 0.36 dex. This ultimately reflects the quite broad range in stellar (or halo) mass over which central galaxies have been mass quenched and the continued growth of halos after the star formation has been quenched.

Red galaxies have systematically lower $M_\star/M_{h}$ than blue ones at a given $M_h$ because their stellar masses are frozen at quenching (apart from mass growth due to merging), whereas their dark matter halos continue to grow. They may scatter down...
terms of observational data and in terms of the independent and orthogonal phenomenological model of Behroozi et al. (2013a).

In particular, there is no reason for the total number density of galaxies to come out right. The models and the parameters taken from the previous papers (P10, P12, L13) did not have any information about the abundance of dark matter halos, nor were they designed to match the number density of galaxies in the universe. This is a remarkable success of our model. It is simple, but still reproduces a wide range of non-trivial results. In this section, we take a closer look at those areas where our model produces discrepancies that may give clues as to where additional features could be added, or which may highlight more fundamental tensions.

First we look at the specific SFR evolution and note how we can, in principle, achieve a better agreement with the data compilation of Stark et al. (2013) and Behroozi et al. (2013a) in Section 5.1. Then, relating the sSFR evolution to the SFRD evolution, we argue that we cannot easily bring these two observations in agreement with each other, independent of our model assumptions (Section 5.2). We then turn our attention to the missing red galaxies. We discuss how this is linked to the form of the SHMR in Section 5.3 and discuss its relation to the saturation feature of the L13 regulator model. In Section 5.4 we propose two other ways of assigning the gas inflow to the galaxies within the halo and see that we get a further improvement in matching the SMF, sSFR, and SFRD history, our Models B and C. In Section 5.5 we discuss a very specific feature of our models, and finally in Section 5.6 we relate our results to abundance matching methods.

5.1. Modification to Match the sSFR at \( z = 2 \)

In Section 4.2 and in Figure 2 we noted a deviation of the sSFR evolution at \( z = 2 \) between our predictions and the data compilation of Stark et al. (2013) and Behroozi et al. (2013a). One possible way of modifying our model in an attempt to get a better match is to change the star formation efficiency, \( \epsilon \),

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**Figure 8.** SHMR at \( z = 0 \) of Model A is plotted as a function of the total halo mass, \( M_h \), for the set of central galaxies, separated into red and blue. The blue (red) continuous line is the mean value of the blue (red) population in our model and the black line is the mean SHMR of the overall sample for centrals (i.e., a suitably weighted average of the red and blue lines). The thick dotted black line is the contribution of satellites to the SHMR, while the green thin dotted line indicates the cosmic baryonic fraction. The global turnover of the star formation efficiency can fully be accounted for by quenching galaxies around \( M^* \), corresponding to about \( 10^{12} M_\odot \) in halo mass. The agreement with the abundance matching reconstruction of Behroozi et al. (2013a) is quite impressive, although there is a systematic reduction in \( M_*/M_h \) above \( M_h \sim 10^{13.5} M_\odot \), which may be traced to the saturation of the efficiency with which the regulator in L13 converts baryons to stars that is in turn linked to the flattening of the \( Z(M_*) \) mass–metallicity relation.

(A color version of this figure is available in the online journal.)

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**Figure 9.** Average SHMR of Model A for different galaxy types as a function of total halo mass. Upper left: considering all galaxies within a halo and all types of galaxies. Upper right: considering only the central galaxy (red and blue). Lower left: considering only blue central galaxies. Lower right: considering only red central galaxies. The scatter in this relation can be deduced from Figure 8. The SHMR is predicted to be an increasing function in cosmic time (i.e., decreasing with increasing redshift) because regulators at high redshift are more gas rich.

(A color version of this figure is available in the online journal.)

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the \( M_* \sim M^* \) locus, which explains the observation (e.g., Woo et al. 2013) that at a given stellar mass, red galaxies are found in higher mass halos (e.g., with more satellites). As the overall population of central galaxies changes from predominantly blue at low halo masses to predominantly red at higher halo masses, the mean SHMR shifts from that of the blue galaxies to that of the red. The overall scatter is expected to be 0.32 dex at the peak, but deviates from being a log-normal distribution in stellar mass.

Overall, the agreement between the output of our model and the reconstruction from Behroozi et al. (2013a) is very good. Our curves for the overall population are slightly lower around the peak, by up to about 0.2 dex at halo masses above \( 10^{11.5} M_\odot \), and this can be traced to the saturation of \( f_{\text{sat}} \) in L13, which itself was driven by the saturation in the adopted \( Z(M_*) \) mass metallicity relation. We will return to this point later and show that it is closely linked to the issue of the deficit of quenched galaxies noted in Section 4.4.

Our model has a slight redshift evolution in the SHMR (see Figure 9). Within our model, this is due to the fact, that regulators (i.e., galaxies) at higher redshifts contain proportionally more gas and thus less stellar mass as discussed in Section 4.3. However, the general behavior remains the same at all redshifts. At very low halo masses, the stellar content remains dominated by the maximum outflow load \( \lambda_{\text{max}} \) and the saturation feature occurs at every redshift at roughly the same halo mass. The nominal drop in the SHMR at \( z = 4 \) is about a factor of two.

5. DISCUSSION

In Section 4 we recovered a number of encouraging agreements of various predictions compared to the literature, both in...
at high redshift. Several authors have had detailed discussions about the link between star formation and gas reservoir (recently e.g., Feldmann 2013). However, because the link between sSFR and the sMIR (specific mass accretion rate of the system) is independent of $\epsilon$ and $\lambda$ (see L13, and thus also of $f_{\text{star}}$), modifications of $\epsilon(z)$ would only change the sSFR through the "boost" effect on sMIR that is associated with a change in $f_{\text{star}}$ with time, and so the effect of this change should be quite weak. It turns out that a higher $\epsilon$ at high redshift leads to a drop in the offset of the sSFR compared with the sMIR. To explore this, we modify the parameterization of $\epsilon$ to

$$\epsilon(z) \propto (1 + z)^c,$$  

(19)

with $c$ being the additional model parameter. In our default Model A (as well as Models B and C), the efficiency scales as the Hubble rate. In Figure 10 we plot three different models with $c = 0, 1, 2.35$ (i.e., assuming no redshift evolution), with one coming close to the fiducial model and one in which the efficiency scales as the sMIR according to Neistein & Dekel (2008). We note that at fixed redshift, the efficiency is parameterized as a function of $M_*$. This parameterization is fitted at $z = 0$ and may not provide a direct link to the physical process that actually sets the efficiency.

We see that lowering the star formation efficiency at higher redshifts actually boosts the sSFR. Because it lowers $f_{\text{star}}$ at high redshifts and therefore increases the boost term in Equation (36) of L13. On the other hand if the efficiency increases with redshift as fast as the specific infall rate, we reduce the sSFR. In both cases, the effect of the change in the sSFR is spread out over a wide range of redshifts (because of the smooth evolution in $\epsilon$) and we cannot get a peak at one particular redshift, or drastically change the overall slope.

An alternative approach is to decouple the specific accretion rate onto the regulator systems from the specific growth rate of the surrounding dark matter halos. A redshift-dependent cold gas accretion efficiency (e.g., Bouché et al. 2010) could do this, or some other scheme to limit the baryonic accretion onto the regulators. In Section 5.4 we will explore some modifications by introducing Models B and C.

5.2. The Link Between sSFR and SFRD

Staying with the same expansion in our model as in Section 5.1 we turn our attention to the SFRD. We plot in Figure 11 the SFRD history for the same three models as for Figure 10. The figure shows that lowering the efficiency at high redshift shifts star formation to later times. The redshift dependence of the efficiency, $\epsilon$, does not have a significant influence on the outcome at $z = 0$. However, it does have a slight effect on where the stellar mass is formed. As the model has a smoothed evolution in $\epsilon$, significant deviations in the sSFR history from our default model cannot be made.

5.3. Matching the Red Fraction at $M^*$

As mentioned in 4.4, our model under-predicts the abundance of red galaxies around $M^*$. In other words, the relative fraction of red to blue galaxies is too low.

The number density of red galaxies around $M^*$ is directly related to the number of dark matter halos between $M_h(M^*)$ and infinity. As the halo mass function is a very steeply decreasing function of halo mass, the number of red galaxies around $M^*$ is very sensitive to the halo mass $M_h(M^*)$ that corresponds to the quenching mass $M^*$.

However, simply changing the parameter $M^*$ (i.e., $\mu^{-1}$) will have a severe impact on the blue population that we match very well. Boosting the SHMR (e.g., by just letting more gas flow in the regulator) is also not satisfactory. By doing so, we boost the number density of blue and red satellites by the same amount. We would be able to get the needed number density in the red population around $M^*$ (as we are lowering the halo mass corresponding to $M^*$) but at the same time we would end up with too many blue galaxies at the same stellar mass range. So, how can one change the red fraction without changing either the number density of the blue population or $M^*$? The fraction between blue and red galaxies around $M^*$ is dependent on how fast the galaxies are approaching $M^*$.

We have to elevate the sSFR at $M^*$, or in terms of the SHMR, the power law parameter for the main sequence, $\gamma$, defined as

$$M_* \propto M_h^\gamma$$

(20)

must be steeper around $M^*$ than our model prediction.
Our model produces a flattening of the SHMR around $M^*$ (see Figure 8 for $z = 0$ and Figure 9 for the redshift evolution). This is an intrinsic feature of the regulator model and independent of quenching. The overall fraction of baryons in stars cannot exceed the cosmic fraction, and indeed can only asymptotically approach it. In fact, because of the “loss” of outflowing gas in this first Model A, it will saturate at an even lower value. The regulator $f_{\text{star}}$ saturates when the gas within the halo is nearly used up.

We note that our model, even without any quenching mechanism, therefore has a saturation feature coming from the regulator because $f_{\text{star}}$ is limited to some value. Our model predicts just at the stellar mass when quenching happens a flattening of $\gamma$ due to the saturation. In contrast, we get a better match to the red population when abandoning the saturation feature or invoking an even steeper $\gamma$ at $M^*$. This might provide a hidden link between the quenching process and the running out of gas of the galaxy. We return to this later.

5.4. Changing Gas In-flow Description

One of the weaknesses of our models is that we do not trace the outflowing gas. The need for gas reincorporation in a cosmological context was initially analyzed in (Benson et al. 2003; Lucia et al. 2004). Other recent works include (Oppenheimer & Davé 2008; Oppenheimer et al. 2010; Henriques et al. 2013). In our simple model, we do not allow the expelled gas to get back into the same regulator or transfer it to another regulator sitting in the same dark matter halo. Letting some or all of this gas back into the regulator at the time when saturation occurs. This process can, in principle, be accomplished by setting an appropriate recycling time (of the order of several dynamical times). Such a behavior can consistently be applied to our model. The only worry is that this new type of metal-enriched inflow will significantly change the metallicity-fitted parameters inferred in L13 and used in our combined model. This might indicate that the metallicity modeling is unrealistic.

Some gain in the direction can be achieved by simply modifying how gas is assigned to the regulators. In combining the different models of Section 2, we have the freedom to assign the gas inflow to the different galaxies (central or satellites). So far in our Model A we have assigned the gas according to the weights of the subhalos (Equation (15)), with the weight of the central given in Equation (14). The substructure fraction is increasing with halo mass and therefore the second term in Equation (14) assigns a smaller proportion of the infalled gas to the central galaxy as it grows in stellar mass. This can also contribute to the flattening of the SHMR.

Model A assumed no domination of the central galaxy over its satellites. The other extreme would be for the central galaxy to dominate completely and get all the gas inflow, in which case the satellites would not get any gas inflow. Our Model B, presented here, is identical to our Model A, except that Equation (15) is changed so that all the incoming gas is given to the central galaxy:

$$\Phi_{h,i} = \begin{cases} 
    f_b M_{h,\text{smoothed}} & \text{central} \\
    0 & \text{satellite}.
\end{cases} \quad (21)$$

The result in terms of the SFRD is plotted in Figure 12. We clearly see an additional boost in the SFRD around $z = 2$ and even at higher redshift. This brings the model closer to what is required by the data. The reason for the difference between the two proposed models is that at high redshift the halo merger rate is very high compared to the subhalo decay rate. This leads to more substructure within a halo at high redshift. In our Model A this leads to less gas inflow onto the central galaxy, which is avoided in Model B. Furthermore the gas infalled onto the central galaxy is turned into stars more efficiently than in (lower mass) satellites. However, despite this improvement, Model B still under-predicts the SFRD at $z = 2$.

In terms of the sSFR history we do not get any change in the predictions from Model A to Model B, as presented in Figure 13. To match the sSFR history, we have to change the model further.
Looking at the SMF at $z = 0$ predicted by Model B in Figure 14, we can also partially improve matching the red fraction around $M^*$. A discrepancy remains, however, coming from the regulator description as discussed in Section 5.3. The SHMR of Model B (Figure 15) for central galaxies is similar to Model A and also comes close to the Model of Behroozi et al. (2013a).

This discussion also demonstrates the importance of how one assigns the gas inflow to the different galaxies within a halo. However, we want to emphasize that no complicated description (e.g., recycling of outflow gas, decoupling of baryonic inflow, and dark matter growth, and so on) is needed to achieve the level of agreement that is already presented in Models A and B. In terms of the quenching “laws,” they are intended to be purely descriptive. These laws would likely be more complicated if they were formulated in terms of physical mechanisms, which are still unclear.

Having said that, the red fraction problem and the sSFR and SFRD at $z = 2$ still do not match perfectly. Our next approach is an “effective SAM.” From our earlier discussion, we concluded that the gas inflow description is crucial in perturbing our model and, if done in the right way, matching the observables. For Model C we introduce a redshift and halo mass-dependent gas inflow. We change Equation (15) to

$$\Phi_{b,i} = \begin{cases} f_b M_b, \text{smoothed} & \text{central} \\ 0 & \text{satellite}, \end{cases}$$

with

$$f_b(M_h) = 1 + 30 \times \left( \frac{M_h}{10^{12} M_\odot} \right)^{2.5}$$

and

$$f_z(z) = \begin{cases} -1.25 \times (1 + z)^{-1} + 1.4 & z < 2 \\ 0.25 + 6.75 \times (1 + z)^{-3} & z \geq 2. \end{cases}$$

The functions $f_b$ and $f_z$ are arbitrary and designed to have four desirable features.

1. $f_z$ is a decreasing function between $z = 2$ and $z = 0$ accounting for the steep decline in the SFRD.

2. $f_z$ is a rapidly increasing function approaching $z = 2$ accounting for the boost in the sSFR around $z = 0$.

3. $f_b$ has an additional term such that there is significantly more gas inflow onto massive galaxies around $M_b(M^*)$ to counteract the saturation feature of the regulator.

4. $f_b \cdot f_b$ is normalized such that the baryonic mass within the regulator never exceeds the cosmic baryonic fraction of the universe.

The functional form of $f_z$ and $f_b$ are completely arbitrary. The functions and values are chosen to match the four criteria just mentioned. We want to emphasize that a priori no physical argument was chosen to justify our approach, except their result on the observables mentioned previously. Recently (Oppenheimer & Davé 2008; Oppenheimer et al. 2010;
Henriques et al. (2013) provided physical pictures or reincorporation of gas and (e.g., Schaye et al. 2010) discussed extensively the impact of different physical processes on the evolution of the SFRD. The SFRD of Model C is plotted in Figure 12 in red. We get about a factor of 10 difference in the SFRD at $z = 2$ and at $z = 0$. The sSFR gets an additional boost at $z = 2$ (red line in Figure 13) and the SMF at $z = 0$ matches very well all the different galaxy populations in shape and amplitude (Figure 16). The resulting SHMR plotted in Figure 17 looks very different. The blue population approaches the cosmic baryonic fraction very rapidly, but gets quenched just before exceeding the limit (in stellar mass).

This extension cannot be considered as a “best fit” model. The aim is just to indicate the power of this specific extension for future model buildings. Other predictions, such as the gas-to-star ratio are only marginally affected by this extension. We will not break the degeneracy between recycled and newly infallen gas components with this extension of our model. Metallicity and H I data (see, e.g., model of Davé et al. 2013) might give further insights into this processes.

5.5. The Coincidence of Getting Quenched When Approaching the Baryonic Fraction

We notice from our analysis in Sections 5.3 and 5.4, the SHMR is far below the cosmic baryonic fraction $f_b$ at low $M_*$ and is coming closer to $f_b$ when approaching $M^*$. By coincidence, quenching occurs in our model just when the stellar baryonic fraction approaches the cosmic fraction $f_b$. In our model, the regulator is not allowed to get more baryons in than the baryonic fraction (see Equation (15)) and so will automatically saturate. It no longer follows the power law description of Section 5.3 and will flatten. In our model this saturation feature is completely independent of the quenching formalism with its crucial parameter $M^*$.

However, apparently as a coincidence, these two completely different features arise at the same point in the evolution history of a star forming galaxy. It is ultimately the simultaneous appearance of these two features that led to the under-prediction of the red population around $M^*$. In Model C, we see that to match the SMF we have to steepen the SHMR of the blue population around $M^*$ such that the blue population must approach the cosmic baryonic limit even faster, without apparently noticing it, but suddenly quench just before reaching the ultimate limit.

If one has one mechanism suppressing star formation in low-mass galaxies and quenching at high masses, a peak is inevitable. However, the peak in $M_*/M_h$ that is caused by quenching could have occurred at any mass (e.g., if it was driven by AGN feedback, morphological effects, and so on). The fact that it appears to occur just when the overall efficiency of the conversion of baryons into stars is maximal, in our view, is noteworthy and probably tells us that it is not a coincidence.

5.6. Abundance Matching

We note from Figure 8 (for Model A) and from Figure 15 and 17 (for Models B and C, respectively) that, at halo masses around $10^{12} M_\odot$, the mean value of the SHMR of the blue population is elevated by about 0.2 dex compared to the mean value of the red population. The 1σ dispersion in the blue population alone is about 0.2 dex, and the overall scatter in the combined red and blue populations at $10^{12} M_\odot$ is larger, 0.35 dex, and the distribution is not Gaussian in log $M_*/M_h$ (i.e., log-normal in the ratio). Simple abundance matching techniques usually do not take this possible variation into account.

Behroozi et al. (2013a) noted that the range in SFRDs that is implicit in a star forming and a passive population, is only a problem if it results in a distribution of stellar masses at fixed halo mass that cannot be reasonably modeled by a log-normal distribution (the main assumption in their work). In our model, we produce a clearly different distribution in stellar mass around the peak $M_*/M_h$. The SHMR of Model C in Figure 17 is substantially different to the one of Behroozi et al. (2013a), but still reproduces the SMF at the same accuracy. In other words, the SHMR from our Model C is effectively a kind of abundance matching, as it is specifically tuned to match the abundance properties of the galaxy population, but with a different assumption (motivated by our quenching laws) of how blue and red galaxies will populate the dark matter halos.

Tinker et al. (2013) uses measurements of the SMF, galaxy clustering, and galaxy–galaxy lensing within the COSMOS survey to constrain the SHMR of blue and red galaxies over the redshift range $z = [0.2, 1]$. Their underlining assumption on the functional form of the blue and red galaxy SHMR is very different to our output (e.g., their blue population is described with a turnover in the SHMR).

6. CONCLUSION

We have presented a simple model of the evolving galaxy population that is based on importing pre-formulated baryonic prescriptions for the control of star formation in galaxies into a dark matter halo merger tree. Specifically, the model is based on the gas-regulation model of star forming galaxies from L13, and the empirical quenching formulae of P10 and P12.

The parameters for these baryonic prescriptions are taken directly from these earlier works and are not adjusted according to the output of the current model. A very limited number of additional a priori assumptions are however required to ensure the model can operate, but these do not greatly affect the output of the current model.
the outcome. The model deleted... allows us to make predictions about the numbers and properties of galaxies that are independent of the observation inputs used to determine the model prescriptions in the previous papers and which can therefore be used to test the model. The observational inputs to the previously tuned parameters were: The exponential cutoff scale $M^*$ of the main sequence galaxies at $z = 0$, $Z(M, SFR)$ data at $z = 0$, and the averaged enhanced fraction of red galaxies in groups and clusters. The only input from the dark matter picture in the previous papers (namely L13) was the average halo growth rate of Neistein & Dekel (2008).

The output of this model is compared with independent observational data, as well as other recent phenomenological models Behroozi et al. (2013a), for the evolving galaxy populations that have been based on epoch-dependent abundance matching of halos and galaxies. The output quantities examined include (a) the Main Sequence sSFR–mass relation, (b) the integrated SFRD, (c) the SMFs of star forming and quenched galaxies, and (d) the $M_*$ versus $M_h$ relation and SFR $- M_h$ relations, as well as the epoch dependence of these over the whole redshift interval $0 < z < 5$. The predicted gas content of galaxies is also presented.

The goal of this work has been to see how far we can get with this simple model and to explore how it may need to be adjusted so as to rectify any failings in reproducing the real universe. We have drawn the following conclusions from this work.

1. The attractive features of the input baryonic prescriptions that were highlighted in the original papers, including the mass dependence of the main sequence sSFR, the faint end slope of the galaxy mass function, the relative Schechter $M^*$ and $\alpha$ parameters of the blue and red (star forming and quiescent) galaxy populations are certainly all preserved when transplanted into a realistic dark matter structure. The argument of L13 in relating the faint end slope, $\alpha$, from the regulator scaling law does not suffer from the limitations of a single mean sMIR. The mass function of star forming galaxies is also well reproduced and the general form of the SFR $- M_h$ and $M_*$ $- M_h$ relations are very similar to those constructed by Behroozi et al. (2013a), and arise from the competition between the increased efficiency of turning baryons into stars as the mass increases (due to lower mass loss in winds) and the quenching of star formation in galaxies. The overall forms of the sSFR(z) and SFRD(z) are also qualitatively produced by the model. These are major and rather striking successes from a simple model, which are very largely independent of the original observational inputs used previously to define our baryonic prescriptions.

2. As with other models in the literature, our simplest model has quantitative difficulty in reproducing the steep increase back to $z \sim 2$ in both the sSFR(z) and SFRD(z). This cannot be solved by simple adjustments to the adopted star formation efficiencies. We also find that the peak in the $M_*$ $- M_h$ relation is a little softer than in the Behroozi et al. (2013a) representation and, surprisingly, that the ratio of quenched to star forming galaxies around $M^*$ is lower than observed (and than can be predicted from the original P10 formalism). We show that the latter two issues are closely related and are due to saturation in the efficiency with which halos form stars, which is inherent in the adopted regulator model, especially as the cosmic baryon limit is approached.

3. All four of these quantitative deficiencies can be simultaneously solved by adjusting the specific infall rate of material onto galaxies by allowing them to re-ingest material previously expelled by winds, provided that this occurs in a redshift- and mass-dependent way, being most effective at masses around $M^*$ and at redshifts $z \sim 2$.

4. Our models allow us to predict the $M_*$ $- M_h$ relation for star forming and quiescent galaxies separately. Red galaxies always have a higher $M_*$ at given $M_h$ because of the continued growth of halos after star formation ceases, and there is a $1\sigma$ scatter in stellar mass of 0.36 dex for halos of mass $M_h = 10^{12} M_{\odot}$, with two clearly distinguishable populations. There is significantly less scatter in the blue population than in the red one. The SHMR around $M^*$, where mass quenching happens, has to be steeper than predicted from our original model to match the blue and red galaxy abundances at the same time. Such a qualitative behavior brings a simple regulator model to its limits, as one expects the SHMR to flatten when approaching the baryonic limit.

5. While others have emphasized the “inefficiency” of star formation in halos, we stress instead the efficiency of $M^*$ galaxies in forming stars. Further, we note the coincidence that quenching happens in our model at the same time that the regulators are rapidly approaching the maximum possible efficiency in covering baryons into stars, even though these two description are completely independent of each other in the model.

Our analysis emphasizes the continued importance of pinning down as reliably as possible the bulk characteristics of the evolving galaxy population over a wide span of cosmic time. One crucial factor in our model is the gas infall onto galaxies, and it will be of great importance to trace the gas in the universe in a more observationally comprehensive way.

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APPENDIX A
MODEL SENSITIVITY ON ADDITIONAL PARAMETERS AND INITIAL CONDITIONS

A.1. Model Dependence on $f_{\text{merge}}$

The parameter $f_{\text{merge}}$ in the model describes the fraction of the stellar and gas mass of a satellite that enters the central galaxy when the satellite is disrupted. In the main text this is set to $f_{\text{merge}} = 0.5$. To understand the sensitivity to this parameter we present three models that have the same parameters as in Table 1, except that $f_{\text{merge}}$ is set to $f_{\text{merge}} = 0, 0.5, 1$. There is a small change in the blue population. Even with $f_{\text{merge}} = 1$, a blue central galaxy almost never has more than 20% of its mass above $10^{13} M_{\odot}$, however, which are generally located in massive halos (e.g., above $10^{13} M_{\odot}$) merging is the primary channel for mass growth and there is therefore a significant effect of $f_{\text{merge}}$ on the mass function and the SHMR for these most massive galaxies. The effect on the SHMR is shown in Figure 18. If we want to predict this quantity of the SMF at these high masses, then we would...
have to constrain $f_{\text{merge}}$ (or vice versa). However this regime is not a central consideration of this paper.

A.2. Model Dependence on $\lambda_{\text{max}}$

In the model, $\lambda_{\text{max}}$ gives the maximum mass loading of the wind that is required to limit the extrapolation of the $\lambda$, which varies inversely with mass at higher masses. In Figure 19, three models with different $\lambda_{\text{max}}$ are plotted. As would be expected, there is a significant dependence for the lowest mass galaxies, corresponding to halos below $10^{11} \, M_\odot$, where the SHMR scales linearly with $\lambda_{\text{max}}$ at the very low stellar mass end. We chose $\lambda_{\text{max}}$ in such a way that the regulator above $10^9 \, M_\odot$ in stellar mass is not affected by the floor value. This gives us a prior of $\lambda_{\text{max}} \geq 20$. Values between 20 and 200 only marginally affect the intermediate range. We chose $\lambda_{\text{max}} = 50$ and note that our model is not tuned to predict the SHMR below $10^{11} \, M_\odot$ in halo mass or $10^8 \, M_\odot$ in stellar mass, respectively.

A.3. Model Dependence on $M_{\text{thresh}}$

The parameter $M_{\text{thresh}}$ controls the threshold above which a subhalo contains a regulator system, which in turn affects the way in which baryons are brought into the larger halos. The dependence of our model on the $M_{\text{thresh}}$ parameter is a little more complicated. First, when lowering $M_{\text{thresh}}$ we increase the merging component, $M_{\text{merger}}$, and lower the smoothed accretion component, $M_{\text{smoothed}}$. Second, the very high host-to-satellite ratio makes those low-mass substructures survive very long (often longer than the age of the universe). The result is that $M_{\text{central}}$ defined by Equation (14) is lowered compared to $M_{\text{h}}$. From Equation (15), a lowered $M_{\text{central}}$ leads to a reduction of the infall rate in Model A. In Figure 20 four models with different $M_{\text{thresh}}$ are plotted. We see a scaling difference in the SHMR. When changing $M_{\text{thresh}}$ by four orders of magnitude, we change the SHMR by less than one order of magnitude. L13 introduced in their paper a parameter $f_{\text{gal}}$ to account for the fact that if they let all the accreted baryons into their regulator, they would end up with too high an SHMR. In our model, we do not let all the baryons fall in the central because of the sub-structure. The parameter $f_{\text{gal}}$ is therefore effectively absorbed into the (more physical) parameter $M_{\text{thresh}}$. We get an equivalent of $f_{\text{gal}} = 0.5$ with $M_{\text{thresh}} = 10^7 h^{-1} M_\odot$. This mass is consistent with photoionization heating operating at low masses and suppress cooling and star formation below a certain halo mass $M_{\text{yr}}$. This halo mass scale increases from $M_{\text{yr}} \sim 10^8 \, M_\odot$ during reionization to $M_{\text{yr}} \sim \text{few} \times 10^9 \, M_\odot$ (Gnedin 2000; Okamoto et al. 2008). For a more realistic model aiming to make predictions back to the epoch of reionization, one has to account for a change in the mass threshold.

A.4. Initial Conditions

When we stop expanding our merger tree (in a backward process) at either redshift $z = 15$ or at halo masses of $10^9 h^{-1} M_\odot < M_{\text{h}} < 2 \times 10^9 h^{-1} M_\odot$ we have to initialize the baryonic component of the halo. To start the forward process of the regulator system, we have to put in some initial values for $M_\text{gas}$ and $M_{\text{gas}}$. In principle we should start with $M_{\text{init}} = 0$. However, with this initial condition the star formation efficiency
is zero and so the differential equation we want to solve has the solution \( M_g(t) = 0 \) for all times. Whether we start with \( M_{g,\text{init}} = 1 M_\odot \) or \( M_{g,\text{init}} = 10^3 M_\odot \) does not really matter when predicting the quantities in \( M_g = 10^8 M_\odot \) galaxies. The time to form these first \( 10^3 M_\odot \) is rather short when considering a gas reservoir of order \( M_{gas} \approx 10^9 M_\odot \). For the initial condition of the gas content in the regulator \( M_{g,\text{init}} \) we have the freedom of \( 0 < M_{g,\text{init}} < f_b M_h,\text{init} \). This has not more than a 1% effect on the total amount of gas that comes into a halo of mass \( M_h = 10^{11} M_\odot \). We conclude that for merger trees that reach \( M_{\text{thresh}, \text{init}} \) our freedom in the initial condition does not affect our predictions by more than 1%.

The situation for starting the forward process at \( z = 15 \) with a halo more massive than \( M_{\text{thresh}} \) is slightly different. This case
only happens for halos of present-day mass $M_h > 10^{14} M_\odot$. We cannot a priori predict the stellar or gas content of a halo of $M_h = 10^{12} M_\odot$ at $z = 15$. However, we only studied the output of the model at $z < 8$, by which time these halos have grown in mass by an order of magnitude. Whatever initial conditions we put in, it affects predictions at $z = 8$ by only about 10%, and even less at later epochs.

### APPENDIX B

#### TABLES AND FITS

In this Appendix we provide tables with functional fits for the mass functions from the simple models presented in the paper and for the SHMR at $z = 0$. As discussed in the main text, the goal of this work is not to perfectly match observational data,

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**Table 5**

Schechter Fits of the Sample Plotted in Figure 6 for Red Satellites of Model A

| $z$  | $\log_{10}(\phi_*)$ (Mpc$^{-3}$) | $\log_{10}(\phi_z)$ (Mpc$^{-3}$) Error | $\log_{10}(M_*/M_\odot)$ | $\log_{10}(M^*/M_\odot)$ Error | $\alpha_s$ | $\alpha_z$ Error |
|------|---------------------------------|--------------------------------------|--------------------------|---------------------------------|------------|-----------------|
| 0.0  | $-4.24$                         | 0.13                                 | 10.64                    | 0.038                           | $-1.54$   | 0.067           |
| 0.5  | $-4.17$                         | 0.05                                 | 10.76                    | 0.064                           | $-1.46$   | 0.029           |
| 1.0  | $-4.38$                         | 0.05                                 | 10.74                    | 0.071                           | $-1.54$   | 0.03            |
| 1.5  | $-4.64$                         | 0.04                                 | 10.75                    | 0.039                           | $-1.6$    | 0.02            |
| 2.0  | $-4.85$                         | 0.04                                 | 10.64                    | 0.040                           | $-1.66$   | 0.027           |
| 2.5  | $-5.19$                         | 0.07                                 | 10.59                    | 0.035                           | $-1.75$   | 0.041           |
| 3.0  | $-5.95$                         | 0.80                                 | 10.62                    | 0.027                           | $-1.98$   | 0.229           |
| 3.5  | $-6.00$                         | 0.72                                 | 10.40                    | 0.049                           | $-2.0$    | 0.058           |
| 4.0  | $-6.60$                         | 0.89                                 | 10.38                    | 0.046                           | $-2.17$   | 0.106           |
| 4.5  | $-6.55$                         | 0.51                                 | 10.16                    | 0.045                           | $-2.17$   | 0.057           |
| 5.0  | $-8.57$                         | 5.27                                 | 10.32                    | 0.109                           | $-2.65$   | 0.147           |
| 5.5  | $-10.06$                        | 4.34                                 | 10.8                     | 0.280                           | $-2.64$   | 0.187           |
| 6.0  | $-11.16$                        | 1.88                                 | 10.9                     | 0.277                           | $-2.95$   | 0.238           |

**Note.** Parameterization is according to Equation (B1).

**Table 6**

Schechter Fits of the Sample Plotted of Model B for Blue Centrals

| $z$  | $\log_{10}(\phi_*)$ (Mpc$^{-3}$) | $\log_{10}(\phi_z)$ (Mpc$^{-3}$) Error | $\log_{10}(M_*/M_\odot)$ | $\log_{10}(M^*/M_\odot)$ Error | $\alpha_s$ | $\alpha_z$ Error |
|------|---------------------------------|--------------------------------------|--------------------------|---------------------------------|------------|-----------------|
| 0.0  | $-2.91$                         | 0.04                                 | 10.68                    | 0.023                           | $-1.47$   | 0.018           |
| 0.5  | $-2.94$                         | 0.04                                 | 10.64                    | 0.022                           | $-1.47$   | 0.019           |
| 1.0  | $-3.03$                         | 0.04                                 | 10.61                    | 0.019                           | $-1.50$   | 0.019           |
| 1.5  | $-3.18$                         | 0.04                                 | 10.63                    | 0.020                           | $-1.54$   | 0.017           |
| 2.0  | $-3.28$                         | 0.04                                 | 10.59                    | 0.018                           | $-1.55$   | 0.017           |
| 2.5  | $-3.47$                         | 0.04                                 | 10.58                    | 0.019                           | $-1.61$   | 0.018           |
| 3.0  | $-3.66$                         | 0.04                                 | 10.55                    | 0.017                           | $-1.65$   | 0.018           |
| 3.5  | $-3.96$                         | 0.04                                 | 10.56                    | 0.017                           | $-1.74$   | 0.018           |
| 4.0  | $-4.15$                         | 0.04                                 | 10.48                    | 0.014                           | $-1.79$   | 0.017           |
| 4.5  | $-4.32$                         | 0.04                                 | 10.39                    | 0.013                           | $-1.83$   | 0.019           |
| 5.0  | $-4.75$                         | 0.03                                 | 10.38                    | 0.011                           | $-1.97$   | 0.016           |
| 5.5  | $-5.16$                         | 0.04                                 | 10.36                    | 0.016                           | $-2.08$   | 0.020           |
| 6.0  | $-4.83$                         | 0.03                                 | 9.957                    | 0.006                           | $-1.97$   | 0.021           |

**Note.** Parameterization is according to Equation (B1).

**Table 7**

Schechter Fits of the Sample of Model B for Blue Satellites

| $z$  | $\log_{10}(\phi_*)$ (Mpc$^{-3}$) | $\log_{10}(\phi_z)$ (Mpc$^{-3}$) Error | $\log_{10}(M_*/M_\odot)$ | $\log_{10}(M^*/M_\odot)$ Error | $\alpha_s$ | $\alpha_z$ Error |
|------|---------------------------------|--------------------------------------|--------------------------|---------------------------------|------------|-----------------|
| 0.0  | $-3.98$                         | 0.04                                 | 10.61                    | 0.022                           | $-1.45$   | 0.020           |
| 0.5  | $-4.07$                         | 0.04                                 | 10.61                    | 0.021                           | $-1.49$   | 0.019           |
| 1.0  | $-4.09$                         | 0.03                                 | 10.52                    | 0.015                           | $-1.5$    | 0.017           |
| 1.5  | $-4.35$                         | 0.04                                 | 10.56                    | 0.019                           | $-1.55$   | 0.019           |
| 2.0  | $-4.50$                         | 0.04                                 | 10.5                     | 0.016                           | $-1.59$   | 0.019           |
| 2.5  | $-4.64$                         | 0.03                                 | 10.4                     | 0.011                           | $-1.6$    | 0.018           |
| 3.0  | $-5.22$                         | 0.04                                 | 10.47                    | 0.014                           | $-1.82$   | 0.018           |
| 3.5  | $-5.28$                         | 0.04                                 | 10.31                    | 0.014                           | $-1.78$   | 0.02            |
| 4.0  | $-5.71$                         | 0.04                                 | 10.25                    | 0.013                           | $-1.92$   | 0.022           |
| 4.5  | $-6.04$                         | 0.41                                 | 10.16                    | 0.177                           | $-2.0$    | 0.064           |
| 5.0  | $-7.07$                         | 0.1                                  | 10.35                    | 0.047                           | $-2.33$   | 0.029           |
| 5.5  | $-6.45$                         | 0.2                                  | 9.73                     | 0.093                           | $-2.24$   | 0.048           |
| 6.0  | $-6.65$                         | 0.07                                 | 9.687                    | 0.026                           | $-2.02$   | 0.037           |

**Note.** Parameterization is according to Equation (B1).
but rather to explore the consequences of particularly simple representations of galaxy evolution.

### B.1. Stellar Mass Function

We have fitted Schechter functions to the blue central, red central, blue satellite, and red satellite galaxy populations. Fits are made assuming a fixed 10% error in log space for each binning point of the sample of Section 4 plotted in Figure 6. We define our parameters according to

$$\phi(m) dm = \phi_\ast \left( \frac{m}{M_\ast} \right)^{\alpha_\ast} e^{-m/M_\ast} \frac{dm}{M_\ast},$$  \hspace{1cm} \text{(B1)}$$

with $\phi$ being the number density in units of Mpc$^{-3}$/dm and $M_\ast$ in units of $M_\odot$. Fits are made over all stellar masses above $10^8 M_\odot$. The fitted functions were integrated through the bins to compare with the number of galaxies in the model. The fitted

| $z$ | $\log_{10}(\phi_\ast)$ (Mpc$^{-3}$) | Error | $\log_{10}(M^\ast/M_\odot)$ | Error | $\alpha_\ast$ | Error |
|-----|-------------------------------|-------|----------------|-------|----------------|-------|
| 0.0 | $-2.71$                       | 0.02  | 10.66          | 0.02  | $-0.48$        | 0.018 |
| 0.5 | $-2.86$                       | 0.03  | 10.67          | 0.022 | $-0.58$        | 0.019 |
| 1.0 | $-2.97$                       | 0.02  | 10.62          | 0.019 | $-0.58$        | 0.018 |
| 1.5 | $-3.16$                       | 0.02  | 10.61          | 0.019 | $-0.66$        | 0.018 |
| 2.0 | $-3.36$                       | 0.03  | 10.59          | 0.020 | $-0.71$        | 0.019 |
| 2.5 | $-3.56$                       | 0.02  | 10.53          | 0.015 | $-0.71$        | 0.019 |
| 3.0 | $-3.89$                       | 0.03  | 10.54          | 0.017 | $-0.81$        | 0.019 |
| 3.5 | $-4.19$                       | 0.03  | 10.48          | 0.014 | $-0.9$         | 0.019 |
| 4.0 | $-4.54$                       | 0.03  | 10.44          | 0.014 | $-0.98$        | 0.019 |
| 4.5 | $-4.93$                       | 0.03  | 10.41          | 0.013 | $-1.07$        | 0.02  |
| 5.0 | $-5.19$                       | 0.03  | 10.26          | 0.011 | $-1.07$        | 0.019 |
| 5.5 | $-5.43$                       | 0.03  | 10.05          | 0.008 | $-1.06$        | 0.024 |
| 6.0 | $-5.74$                       | 0.03  | 9.83           | 0.013 | $-1.08$        | 0.027 |

**Note.** Parameterization is according to Equation (B1).

| $z$ | $\log_{10}(\phi_\ast)$ (Mpc$^{-3}$) | Error | $\log_{10}(M^\ast/M_\odot)$ | Error | $\alpha_\ast$ | Error |
|-----|-------------------------------|-------|----------------|-------|----------------|-------|
| 0.0 | $-3.99$                       | 0.08  | 10.70          | 0.054 | $-1.44$        | 0.046 |
| 0.5 | $-4.04$                       | 0.07  | 10.69          | 0.053 | $-1.45$        | 0.041 |
| 1.0 | $-4.20$                       | 0.07  | 10.63          | 0.041 | $-1.5$         | 0.039 |
| 1.5 | $-4.37$                       | 0.05  | 10.69          | 0.067 | $-1.52$        | 0.031 |
| 2.0 | $-4.68$                       | 0.04  | 10.75          | 0.048 | $-1.59$        | 0.022 |
| 2.5 | $-4.91$                       | 0.04  | 10.64          | 0.039 | $-1.65$        | 0.021 |
| 3.0 | $-5.29$                       | 0.45  | 10.61          | 0.030 | $-1.74$        | 0.107 |
| 3.5 | $-5.67$                       | 0.30  | 10.55          | 0.034 | $-1.84$        | 0.058 |
| 4.0 | $-6.42$                       | 1.28  | 10.52          | 0.045 | $-2.06$        | 0.099 |
| 4.5 | $-6.80$                       | 1.00  | 10.39          | 0.062 | $-2.19$        | 0.177 |
| 5.0 | $-6.57$                       | 1.24  | 9.945          | 0.042 | $-2.11$        | 0.155 |
| 5.5 | $-6.76$                       | 1.33  | 9.81           | 0.111 | $-2.19$        | 0.163 |
| 6.0 | $-5.94$                       | 1.38  | 9.135          | 0.63  | $-1.53$        | 0.452 |

**Note.** Parameterization is according to Equation (B1).

| $z$ | $\log_{10}(\phi_\ast)$ (Mpc$^{-3}$) | Error | $\log_{10}(M^\ast/M_\odot)$ | Error | $\alpha_\ast$ | Error |
|-----|-------------------------------|-------|----------------|-------|----------------|-------|
| 0.0 | $-3.21$                       | 0.04  | 10.71          | 0.025 | $-1.49$        | 0.019 |
| 0.5 | $-3.1$                        | 0.04  | 10.59          | 0.019 | $-1.45$        | 0.019 |
| 1.0 | $-3.26$                       | 0.04  | 10.66          | 0.024 | $-1.47$        | 0.02  |
| 1.5 | $-3.48$                       | 0.04  | 10.74          | 0.026 | $-1.51$        | 0.019 |
| 2.0 | $-3.61$                       | 0.04  | 10.7          | 0.022 | $-1.51$        | 0.018 |
| 2.5 | $-3.95$                       | 0.04  | 10.8           | 0.029 | $-1.55$        | 0.016 |
| 3.0 | $-4.23$                       | 0.05  | 10.83          | 0.03  | $-1.58$        | 0.018 |
| 3.5 | $-4.45$                       | 0.05  | 10.76          | 0.029 | $-1.6$         | 0.019 |
| 4.0 | $-4.62$                       | 0.04  | 10.68          | 0.023 | $-1.59$        | 0.019 |
| 4.5 | $-5.01$                       | 0.05  | 10.72          | 0.027 | $-1.65$        | 0.019 |
| 5.0 | $-5.12$                       | 0.04  | 10.55          | 0.018 | $-1.6$         | 0.021 |
| 5.5 | $-5.93$                       | 0.07  | 10.76          | 0.041 | $-1.8$         | 0.019 |
| 6.0 | $-6.48$                       | 0.06  | 10.69          | 0.032 | $-1.94$        | 0.021 |
Schechter parameters and their errors are given in Table 2 for blue centrals, Table 3 for the blue satellites, Table 4 for the red centrals, and Table 5 for the red satellite population for Model A. Fits for Model B are provided in Table 6–9, and for Model C in Table 10–13.

It should be noted that the fitted $M^*$ from the model output(s) are not equivalent to the model parameter $M^*$ that is used in the paper. Merging after quenching will result in a higher $M^*$ fit. Also, at high redshifts, the exponential cutoff of the Press–Schechter like dark matter halo mass function results in a lower value of the Schechter function $M^*$ during Phase 1 in the parlance of Lilly et al. (2013b).

In fact, our fits at redshifts above $z ∼ 3$ are influenced by the Press–Schechter shape of the SMF, and the fits might be not as good as those at lower redshifts. According to P12, the red satellite population has the form of a double-Schechter function. When fitting a double-Schechter function to our red satellite population, the mass-quenched part of the Schechter function cannot be well constrained and a single-Schechter function provides a reasonable fit to our sample of red satellites for a lower value of the Schechter function $M^*$ during Phase 1 in the parlance of Lilly et al. (2013b).

### Table 11
Schechter Fits of the Sample of Model C for Red Centrals

| $z$ | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) Error | $\operatorname{log}_{10}(M^*/M_\odot)$ | $\operatorname{log}_{10}(M^*/M_\odot)$ Error | $\alpha_1$ | $\alpha_1$ Error |
|-----|---------------------------------|---------------------------|---------------------------------|---------------------------|---------|----------------|
| 0.0 | −2.47                           | 0.02                      | 10.61                           | 0.02                      | −0.12   | 0.018          |
| 0.5 | −2.55                           | 0.02                      | 10.65                           | 0.019                     | −0.12   | 0.018          |
| 1.0 | −2.61                           | 0.02                      | 10.62                           | 0.018                     | −0.13   | 0.018          |
| 1.5 | −2.86                           | 0.02                      | 10.73                           | 0.023                     | −0.34   | 0.017          |
| 2.0 | −3.07                           | 0.02                      | 10.73                           | 0.026                     | −0.37   | 0.02           |
| 2.5 | −3.35                           | 0.02                      | 10.75                           | 0.026                     | −0.43   | 0.019          |
| 3.0 | −3.64                           | 0.02                      | 10.75                           | 0.026                     | −0.46   | 0.018          |
| 3.5 | −3.98                           | 0.02                      | 10.77                           | 0.025                     | −0.53   | 0.018          |
| 4.0 | −4.32                           | 0.02                      | 10.74                           | 0.026                     | −0.57   | 0.02           |
| 4.5 | −4.81                           | 0.03                      | 10.84                           | 0.032                     | −0.73   | 0.018          |
| 5.0 | −5.32                           | 0.03                      | 10.82                           | 0.032                     | −0.85   | 0.019          |
| 5.5 | −5.96                           | 0.04                      | 10.87                           | 0.036                     | −1.03   | 0.019          |
| 6.0 | −6.61                           | 0.06                      | 10.93                           | 0.763                     | −1.12   | 0.02           |

**Note.** Parameterization is according to Equation (B1).

### Table 12
Schechter Fits of the Sample of Model C for Blue Satellites

| $z$ | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) Error | $\operatorname{log}_{10}(M^*/M_\odot)$ | $\operatorname{log}_{10}(M^*/M_\odot)$ Error | $\alpha_1$ | $\alpha_1$ Error |
|-----|---------------------------------|---------------------------|---------------------------------|---------------------------|---------|----------------|
| 0.0 | −4.27                           | 0.05                      | 10.76                           | 0.03                      | −1.46   | 0.02           |
| 0.5 | −4.53                           | 0.05                      | 10.85                           | 0.031                     | −1.55   | 0.018          |
| 1.0 | −4.52                           | 0.04                      | 10.76                           | 0.026                     | −1.52   | 0.018          |
| 1.5 | −4.79                           | 0.24                      | 10.89                           | 0.423                     | −1.56   | 0.022          |
| 2.0 | −5.21                           | 0.05                      | 10.93                           | 0.036                     | −1.61   | 0.018          |
| 2.5 | −5.25                           | 0.04                      | 10.62                           | 0.021                     | −1.58   | 0.02           |
| 3.0 | −5.61                           | 0.05                      | 10.67                           | 0.028                     | −1.6    | 0.019          |
| 3.5 | −6.03                           | 0.05                      | 10.63                           | 0.022                     | −1.68   | 0.019          |
| 4.0 | −6.02                           | 0.04                      | 10.33                           | 0.016                     | −1.61   | 0.021          |
| 4.5 | −6.09                           | 0.03                      | 10.14                           | 0.009                     | −1.52   | 0.021          |
| 5.0 | −7.27                           | 0.09                      | 10.45                           | 0.051                     | −1.83   | 0.03           |
| 5.5 | −7.54                           | 0.41                      | 10.08                           | 0.426                     | −1.88   | 0.059          |
| 6.0 | −9.3                            | 0.18                      | 11.02                           | 0.458                     | −1.96   | 0.076          |

**Note.** Parameterization is according to Equation (B1).

### Table 13
Schechter Fits of the Sample of Model C for Red Satellites

| $z$ | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) | $\operatorname{log}_{10}(\phi_*)$ (Mpc$^{-3}$) Error | $\operatorname{log}_{10}(M^*/M_\odot)$ | $\operatorname{log}_{10}(M^*/M_\odot)$ Error | $\alpha_1$ | $\alpha_1$ Error |
|-----|---------------------------------|---------------------------|---------------------------------|---------------------------|---------|----------------|
| 0.0 | −4.93                           | 0.27                      | 10.88                           | 0.04                      | −1.69   | 0.103          |
| 0.5 | −4.85                           | 0.28                      | 10.86                           | 0.037                     | −1.67   | 0.111          |
| 1.0 | −5.14                           | 0.26                      | 10.86                           | 0.038                     | −1.71   | 0.104          |
| 1.5 | −5.23                           | 0.23                      | 10.85                           | 0.033                     | −1.69   | 0.09           |
| 2.0 | −5.48                           | 0.25                      | 10.82                           | 0.032                     | −1.71   | 0.094          |
| 2.5 | −5.98                           | 0.18                      | 10.82                           | 0.032                     | −1.81   | 0.066          |
| 3.0 | −6.95                           | 0.29                      | 10.93                           | 0.041                     | −2.04   | 0.094          |
| 3.5 | −6.94                           | 0.25                      | 10.81                           | 0.039                     | −1.94   | 0.103          |
| 4.0 | −7.26                           | 0.25                      | 10.7                            | 0.046                     | −1.99   | 0.111          |
| 4.5 | −8.02                           | 0.39                      | 10.81                           | 0.054                     | −2.14   | 0.147          |
| 5.0 | −8.3                            | 1.05                      | 10.57                           | 0.084                     | −2.15   | 0.164          |

**Note.** Parameterization is according to Equation (B1).
most redshifts. Note however that this does not invalidate the explanation of P12 and the implied difference in $\alpha$. We provide in Table 5 and 9 these single-Schechter fits.

B.2. SHMR at $z = 0$

In Table 14 the values of the SHMR are plotted for different mass bins in the range of $10^{10} M_{\odot}$ and $10^{14} M_{\odot}$ in halo mass. These values have been calculated by binning over 0.5 dex in halo mass. The halo mass given in the table is the mean halo mass of the sample being binned over. Bins without values did not consist of at least two galaxies of the specific type within our sample.

| $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ |
|-------------------------|-------------------------------|-------------------------|-------------------------------|-------------------------|-------------------------------|-------------------------|-------------------------------|
| 10                      | -2.98                         | 0.09                    | -2.98                         | 0.09                    | -2.98                         | 0.09                    | -2.98                         | 0.09                    |
| 10.5                    | -2.9                           | 0.14                    | -2.9                           | 0.14                    | -2.9                           | 0.14                    | -2.9                           | 0.14                    |
| 11                      | -2.59                          | 0.24                    | -2.59                          | 0.24                    | -2.59                          | 0.24                    | -2.59                          | 0.24                    |
| 11.5                    | -2.12                          | 0.27                    | -2.11                          | 0.26                    | -2.11                          | 0.26                    | -2.11                          | 0.26                    |
| 12                      | -2.09                          | 0.20                    | -2.09                          | 0.20                    | -2.09                          | 0.20                    | -2.09                          | 0.20                    |
| 12.5                    | -2.01                          | 0.36                    | -1.68                          | 0.15                    | -1.68                          | 0.15                    | -1.68                          | 0.15                    |
| 13                      | -2.30                          | 0.33                    | -1.72                          | 0.14                    | -1.72                          | 0.14                    | -1.72                          | 0.14                    |
| 13.5                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    |
| 14                      | -2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    |

Table 14

Values of the SHMR at $z = 0$ for all Centrals, Red Centrals, and Blue Centrals, and Their Corresponding Scatter for Model A

| $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ | $\log_{10}(M_s/M_{\odot})$ | $\log_{10}(M_{*}/M_{\odot})$ |
|-------------------------|-------------------------------|-------------------------|-------------------------------|-------------------------|-------------------------------|-------------------------|-------------------------------|
| 10                      | 2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    |
| 10.5                    | 2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    |
| 11                      | 2.31                          | 0.33                    | -2.31                          | 0.33                    | -2.31                          | 0.33                    | -2.31                          | 0.33                    |
| 11.5                    | 2.34                          | 0.38                    | -2.34                          | 0.38                    | -2.34                          | 0.38                    | -2.34                          | 0.38                    |
| 12                      | 2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    | -2.56                          | 0.24                    |
| 12.5                    | 2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    | -2.84                          | 0.19                    |

Notes. The figure with all individual points is given in Figure 8. (-) symbols are indicating that our sample did not generate any objects of the specific kind in the specified mass range.
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