Impact of a clipped phase-modulated photodiode signal on the demodulated signal

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Abstract. Laser Doppler Vibrometers (LDV) are used as an optical measurement instrument for measuring vibrations with high measurement accuracy. Nowadays, the carrier-to-noise-density ratio ($C/N_0$) of the optical signal is limited with a maximum $C/N_0$ of 168 dB/Hz. However, due to the noise of electronic devices and the digitizer, the final digital signal has a lower carrier-to-noise ratio than the optical signal, therefore, the expected resolution of the demodulation results cannot be obtained. In order for the sampled signal to maintain the $C/N_0$ of the optical signal, the signal needs to be amplified. However, because of the saturation characteristics of the amplifier, choosing a large amplification will lead to clipping. To investigate the impact of a clipped LDV signals, this paper demonstrates mathematically that, for signals with small phase modulation, clipping does not affect the sidebands of the signal. Therefore, a large amplification can lead to a higher power of the sidebands in the digital signal and thus obtain a higher sideband-to-noise ratio. Meanwhile, the results of mathematical derivation prove that the amplitude of the carrier is lost after clipping. Therefore, the demodulation results of the clipped PM signal are incorrect for small phase modulation. In this paper, the simulation of the error of the demodulation results is given for different vibration amplitudes of the measured objects and different amplification.

1. Introduction

Laser Doppler Vibrometers (LDV) enable non-contact vibration measurements, which play an important role in the measurement of MEMS devices [1]. The vibration amplitude and frequency are extracted from the Doppler shift of the reflected laser beam [2]. Surface motions of the specimen generate the Doppler shift [3]. The photodetector in a heterodyne LDV receives a phase-modulated (PM) signal. Nowadays, the resolution of the detector signal is limited by the shot noise of the measured light. The carrier to shot noise density ratio $C/N_{\text{shot}}$ in 1 Hz bandwidth will improve with the power of the measuring light $P_s$ [4]. Therefore, the telecommunication wavelength of 1550 nm has recently become popular in LDV because laser radiation at wavelength above 1.4 µm and $P_s=10$ mW is classified as "Class 1", and it is safe for the eyes. With such a high power of measurement light, the shot-noise limited, maximal achievable $C/N_{\text{shot}}$ of LDV can reach 168 dB/Hz.

However, when the optical signal is converted to voltage by the transimpedance amplifier (TIA) and then sampled to obtain a digital signal, new noise is superimposed on the signal. For example, electrical and thermal noise, as well as quantization noise from the analog-to-digital converter (ADC) [5]. The carrier-to-noise density ratio $C/N_0$ of the signal may be reduced and as a result, the resolution
of the measurement is reduced [6]. Therefore, the application of measurement light with higher C/N\textsubscript{shot} places higher demands on the electronics and digitizers. For example, the maximum carrier-to-quantization noise density ratio (C\textsuperscript{QN}R) of an 12 bit, 160 MSps ADC is only 153 dB/Hz [7], which is significantly lower than the C/N\textsubscript{shot} of an LDV with 10 mW measurement light. Thus, the resolution of the demodulation results will be limited by the larger quantization noise rather than shot noise.

Although the purchase of low-noise electronics or better ADCs (higher bit count or higher sampling rate) can avoid the degradation of the signal C/N\textsubscript{o}, the expensive price and the limitations of ADC production technology make it difficult to apply in practice. In addition, a preamplifier with a large amplification factor is also meaningful to improve the noise insensitivity of the signal. It can increase the power of the signal to reduce the effects of subsequent noise. However, the determined supply voltage of the amplifier and the measurement range of the ADC will limit the amplification factor of the signal [8]. If the signal is amplified to exceed the amplifier threshold, it will cause clipping. In general, to maintain signal integrity and avoid information loss near the peak, the operator should select the appropriate amplification to prevent clipping [9]. However, the amplitude of the PM signal is a known fixed value and the information is mainly concentrated near the zero point, so the impact of clipping on the PM signal is different.

This paper focuses on the impact of clipping on PM signals and its demodulation. A mathematical analysis illustrates the different changes of the carrier and the sidebands in the spectrum after clipping. It is demonstrated that for PM signals with small phase modulation, clipping does not destroy the useful information carried by the sidebands. Therefore, the method for improving the sideband-to-noise ratio of the digital signal by using a large amplification and active strong clipping is feasible. This method can replace the more sensitive and expensive electronics and ADCs to reduce the interference of the electronics and digitization process on the sidebands. However, due to the loss of carrier amplitude caused by clipping, the signal is demodulated with errors. This paper shows the simulation results of sampling a clipped and unclipped PM signal at 168 dB/Hz with a common 12-bit ADC, and studies the variation of the demodulation results with the vibration amplitude of the measurement object and the amplification factor.

2. Principles and Modelling of a clipped PM signal

For a heterodyne LDV, the balanced photodetector [4] as a receiver will output a phase modulated current signal \(i_s(t)\):

\[
i_s(t) = I_s \sin\left(2\pi f_c t + \varphi(t)\right).
\]

Here \(I_s\) is the amplitude of the signal, \(f_c\) is the frequency of the carrier and \(\varphi(t) = M \cos(2\pi f_m t)\)

\[\text{(2)}\]

is the modulation angle with the modulation index \(M\) and the modulation frequency \(f_m\). In vibration measurements, \(M = \frac{4\pi}{\lambda}\) is determined by the wavelength \(\lambda\) of the laser and the vibration amplitude \(\delta\).

By applying the Jacobi–Anger expansion [10], equation (1) can be rewritten as

\[
i_s(t) = I_s \sin\left(2\pi f_c t \left(J_0(M)+2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(M) \cos(2n\pi f_m t)\right)\right) - I_s \cos(2\pi f_c t) \left(2 \sum_{n=1}^{\infty} (-1)^n J_{2n-1}(M) \cos((2n-1)2\pi f_m t)\right),
\]

\[\text{(3)}\]

where \(J_n(M)\) is the \(n\)-th Bessel function of the first kind.

When the modulated phase is very small, namely \(M \ll 1\), equation (3) can be approximated by

\[
i_s(t) = I_s \left(J_0(M) \sin 2\pi f_c t + 2J_1(M) \cos 2\pi f_c t \cos 2\pi f_m t\right).
\]

\[\text{(4)}\]

Because of the product-to-sum identities

\[
\cos a \cos b = \frac{1}{2} \left(\cos(a+b) + \cos(a-b)\right)
\]

\[\text{(5)}\]

equation (4) can further be expanded to

\[
i_s(t) = I_s J_0(M) \sin 2\pi f_c t + I_s J_1(M) \cos 2\pi f_c t \cos 2\pi f_m t + I_s J_1(M) \cos 2\pi f_c t \sin 2\pi f_m t.
\]

\[\text{(6)}\]
In order to apply digital signal processing (DSP) technology and digital demodulation, the current signal needs to pass through a limiting amplifier and A/D converter as shown in figure 1. Finally, the electrical signal becomes a digital signal. Here \( n_{adc}[n] \) represents the noise of the ADC.

The TIA is used to convert the current signal \( i_s(t) \) into a voltage signal \( u_s(t) \), and provides an amplification factor \( A \) so that the amplitude of the voltage signal \( U_s \) is

\[
U_s(t) = AI_s(t) = U_s J_0(M) \sin 2\pi f_c t + U_s J_1(M) \cos 2\pi (f_c + f_m) t + U_s J_1(M) 2\pi \cos (f_c - f_m) t.
\]

(7)

Although a larger amplification factor \( A \) may provide greater signal power relative to the ADC noise, i.e., a larger carrier to noise density ratio of the ADC, a larger \( A \) can also cause clipping. The voltage signal can only be amplified to the amplifiers supply voltage \( K \), after which the signal will no longer change, as shown in figure 2.

When no clipping occurs (\( U_s < K \)), the voltage signal at the output of the amplifier is

\[
u_s[n] = U_s J_0(M) \sin \frac{2\pi f_c n f_s}{k} + U_s J_1(M) \cos \frac{2\pi (f_c + f_m) n f_s}{k} + U_s J_1(M) \frac{2\pi \cos (f_c - f_m) n f_s}{k}.
\]

(8)

The one-sided amplitude spectrum \( U[k] \) of a sampled signal \( u_s[n] \) is given by the Discrete Fourier Transform (DFT):

\[
U[k]=\left\{ \begin{array}{ll}
\frac{1}{N} |\text{DFT} \{u_s[n]\}|=\frac{1}{N} \sum_{n=0}^{N-1} u_s[n] \exp(-i \frac{2\pi}{N} k n) & k=0 \\
\frac{2}{N} |\text{DFT} \{u_s[n]\}|=\frac{2}{N} \sum_{n=0}^{N-1} u_s[n] \exp(-i \frac{2\pi}{N} k n) & k=1, 2, \cdots, \frac{N}{2}
\end{array} \right.
\]

(11)

where \( i = \sqrt{-1} \). There is a correspondence between the discrete frequency \( f[k] \) and the index \( k \) through the relation \( k = f[k] f_s \). In order to avoid the spectrum leakage, we choose integer \( c = \frac{f_s N}{f_s} \) and integer \( m = \frac{f_s N}{f_s} \).
to match the carrier frequency and modulation frequency. It can be seen from equation (10) that when the modulation angle is very small, because of

\[ \text{DFT}\left\{ \sin \frac{2\pi cn}{N} \right\} = \frac{N}{2i} \delta[k-c] \]  \hspace{1cm} (12)

\[ \text{DFT}\left\{ \cos \frac{2\pi (c+m)n}{N} \right\} = \frac{N}{2} \delta[k-(c+m)] \]  \hspace{1cm} (13)

\[ \text{DFT}\left\{ \cos \frac{2\pi (c-m)n}{N} \right\} = \frac{N}{2} \delta[k-(c-m)] \]  \hspace{1cm} (14)

the spectrum is composed of the carrier with amplitude \( U_sJ_0(M) \) and the first pair of sidebands with amplitude \( U_sJ_1(M) \) (figure 3). Note that the DFT function here only retains the positive semi-axis. The function \( \delta[\cdot] \) is the unit impulse for discrete arguments.

![Figure 3](image_url)  

**Figure 3.** Sampling by \( f_s=160 \) MHz for an unclipped phase-modulated signal \( u_s(t) \) with \( U_s=1 \) V, \( f_c=40 \) MHz, \( M=0.05 \), \( f_m=400 \) kHz (a) and its spectrum after sampling (b).

When the amplification factor \( A \) is large enough \( (U_s>K) \) that clipping occurs,
the digital signal $u_s[n]$ can be obtained:

$$u_s[n] = \begin{cases} 
K & A_i(n) > K \\
A_i(n) & -K \leq A_i(n) \leq K \\
-K & A_i(n) < -K 
\end{cases}$$

When strong clipping occurs ($U_s \gg K$), only the information of the sampling points close to the zero is retained. Therefore, for small angle modulation ($M \ll 1$) and sampling frequency $f_s = \frac{N}{c}f_c = qf_c$ with $q \in \mathbb{N}$, only for all $n \in \{x | x = \frac{pq}{2}, x \in \mathbb{N} \text{ and } p \in \mathbb{N}\}$, i.e., $\sin \frac{2\pi cn}{N} = 0$, the voltage signal

$$u_s[n] = U_sJ_1(M) \cos \frac{2\pi (c+1)m}{N} + U_sJ_1(M) \cos \frac{2\pi (c-1)m}{N}$$

is small ($2U_sJ_1(M) < K$). It can also be seen from the simulation in figure 4 that only for $n \in \{x | x = \frac{pq}{2}, x \in \mathbb{N} \text{ and } p \in \mathbb{N}\}$, the sampling points $u_s[n]$ of the clipped signal are located on the signal $u_s(t)$ before clipping.

Therefore, equation (16) in the case of strong clipping, can also be expressed as

$$u_s[n] = \begin{cases} 
K & n \in \left\{x | x = \frac{pq+1}{2}, x \in \mathbb{N} \text{ and } p \in \mathbb{N}\right\} \\
U_sJ_1(M) \cos \frac{2\pi (c+1)m}{N} & n \in \{x | x = \frac{pq}{2}, x \in \mathbb{N} \text{ and } p \in \mathbb{N}\} \\
-K & n \in \left\{x | x = \frac{pq-1}{2}, x \in \mathbb{N} \text{ and } p \in \mathbb{N}\right\} 
\end{cases}$$

With

\begin{align*}
\text{Figure 4.} & \quad \text{Sampling by } f_s = 160 \text{ MHz for a clipped phase-modulated signal } u_s(t) \\
& \text{with } U_s = 10 \text{ V}, f_c = 40 \text{ MHz}, M = 0.05, f_m = 400 \text{ kHz and } K = 1 \text{ V.}
\end{align*}

When analyzing the spectrum of the clipped signal, because of the linearity of the DFT, the spectrum of the signal $u_s[n]$ can be divided into the sum of three parts according to equation (11):

$$U[k] = \frac{2}{N} \left| \text{DFT} \{u_s[n]\} \right| = \frac{2}{N} \left| \text{DFT} \{u_{i1}[n]+u_{i2}[n]+u_{i3}[n]\} \right|$$

$$= \frac{2}{N} \left| \text{DFT} \{u_{i1}[n]\} \right| + \frac{2}{N} \left| \text{DFT} \{u_{i2}[n]\} \right| + \frac{2}{N} \left| \text{DFT} \{u_{i3}[n]\} \right| = U_{i1}[k] + U_{i2}[k] + U_{i3}[k]$$

with
The signal $u_{s1}[n]$ can be regarded as equation (17) down-sampling to $f_c$ then up-sampling by factor $D = \frac{q}{2}$ for $q=\text{even}$ with $q=\frac{f_s}{f_c}$. After substituting equations (13-14), the sideband component in baseband can be obtained from the DFT of equation (20):

$$U_{s1}[k]=\frac{2}{N}\left|\text{DFT}\{u_{s1}[n]\}\right| = \frac{2U_sJ_1(M)}{D}\delta[k-(c+m)] + \frac{2U_sJ_1(M)}{D}\delta[k-(c-m)],$$

Equation (23) shows that the information-carrying sidebands remain after clipping and are proportional to the sidebands before clipping.

On the other hand, the parts $u_{s2}[n]$ and $u_{s3}[n]$ are respectively a periodic angular pulse signal with amplitude of $K$ and $-K$ and a frequency of $f_c$. They provide the carrier component in the spectrum baseband:

$$U_{s2}[k]+U_{s3}[k]=\frac{2}{N}\left|\text{DFT}\{u_{s2}[n]\}\right| + \frac{2}{N}\left|\text{DFT}\{u_{s3}[n]\}\right| = \frac{4K}{m}\frac{\sin(mdf)}{\sin(mq)}\delta[k-c],$$

where $d = \frac{q^2}{2q}$ for $q=\text{even}$ and $d = \frac{q^2}{2q}$ for $q=\text{odd}$ is the angular pulse duty cycle. Equation (24) indicates that the amplitude of the carrier component is only related to the factor $q=\frac{f_s}{f_c}$ and clipping threshold $K$, and is not dependent on the signal amplitude before clipping $U_sJ_0(M)$. Figure 5 illustrates the spectrum of a clipped PM signal.

![Figure 5](image)

Figure 5. Spectrum for a sampled by $f_s=160$ MHz clipped phase-modulated signal $u_s(t)$ with $U_s=10$ V, $f_c=40$ MHz, $M=0.05$, $f_m=400$ kHz and $K=1$ V.
Although the carrier amplitude in figure 5 is unchanged compared to figure 3, the sidebands are amplified by a factor of 10. Therefore, active clipping is meaningful for a small-angle modulated PM signal, because a greater sideband to ADC noise density ratio can be obtained after digitization.

However, because the sideband amplitude and the carrier amplitude are affected differently by clipping, the demodulation result of this signal will also be incorrect.

For example, when \( q = 4 \), the clipped PM signal according to equations (23) and (24)

\[
\begin{align*}
    u_s[n] &= K \sin \frac{2\pi cn}{N} + U_s J_1(M) \cos \frac{2\pi(c+m)n}{N} + U_s J_1(M) \cos \frac{2\pi(c-m)n}{N} \\
    u_q[n] &= K \sin \frac{2\pi cn}{N}
\end{align*}
\]

is demodulated by the method as shown in figure 6.

By substituting equation (25), we get

\[
\begin{align*}
    u_I[n] &= \text{LPF} \left( u_s[n] \cos \frac{2\pi cn}{N} \right) = U_s J_1(M) \cos \frac{2\pi n}{N}, \\
    u_Q[n] &= \text{LPF} \left( u_s[n] \sin \frac{2\pi cn}{N} \right) = \frac{K}{2}
\end{align*}
\]

Therefore, the amplitude \( \hat{s} \) of the displacement \( s[n] \) of the measurement object obtained by demodulation is

\[
\hat{s}_{\text{clipped}} = \arctan \left( \frac{2U_s J_1(M)}{K} \right) \frac{\lambda}{4\pi} \quad \hat{s}_{\text{uncropped}} = \arctan \left( \frac{2U_s J_1(M)}{K} \right) \frac{\lambda}{4\pi}
\]

LPF refers to low pass filter, which will filter out the high frequency components of the IQ signal. Compared with the unclipped signal,

\[
\hat{s}_{\text{Clipped}} = \arctan \left( \frac{2U_s J_1(M)}{K} \right) \frac{\lambda}{4\pi} \quad \hat{s}_{\text{Uncropped}} = \arctan \left( \frac{2U_s J_1(M)}{K} \right) \frac{\lambda}{4\pi}
\]

the demodulation result \( \hat{s}_{\text{clipped}} \) is amplified by a factor of approximately:

\[
\hat{s}_{\text{clipped}} \approx U_s J_0(M) \frac{K}{K} \frac{\lambda}{4\pi}
\]

However, when \( 2U_s J_1(M) > K \), the sideband signal (equation (17)) is also limited by the clipping threshold \( K \). Therefore, the demodulation result is a square wave with amplitude:

\[
\hat{s}_{\text{Clipped}} = \arctan(1) \frac{\lambda}{4\pi} \quad \hat{s}_{\text{Uncropped}} = \frac{\lambda}{4\pi}
\]

### 3. Simulation and discussion

If the amplified sidebands remain below the clipping threshold, the amplitude of the sidebands does not change by clipping. Therefore, the amplified sideband signal obtains better sideband to noise density ratio. However, at the same time, the changed carrier-to-sideband ratio has an impact on the demodulation results.
The following simulation results compare the sideband to noise density ratio of the unclipped and clipped PM signals after sampling through the ADC. The impact of a clipped PM signal on the demodulation results is also shown.

Here, we use a 12-bit 160 Msps ADC as an example to simulate a digitizing device.

The signal received by the balanced detector is
\[
i_s(t) = 2\kappa\sqrt{P_s P_r} \sin(2\pi f_c t + \phi(t)) + n_{\text{shot}}(t)
\]
with the shot noise \(n_{\text{shot}}(t)\) and the responsivity of the photodiode \(\kappa=1 \text{ A/W}\). The RMS (root mean square) of the shot noise for the bandwidth \(B=1\text{ Hz}\) is given by \(\sqrt{2q(P_s+P_r)}\). Assuming that the measuring light \(P_s\) is 10 mW and the reference light \(P_r\) is 100 mW. The modulated phase is \(\phi(t) = M\cos 2\pi f_m t\) with \(M=8\times10^{-8}\) and \(f_m=400\text{ kHz}\). The carrier frequency \(f_c\) is at 40 MHz. The power density spectrum of the signal with a duration of 1 s is given by \(PSD[k]\).

![Power spectral density of discrete current signal](image)

**Figure 7.** Power spectral density of discrete current signal \(i_s[n]\).

The power spectral density of the computer-generated current signal of the balanced photo detector is shown in figure 7. Because the modulation index \(M\) is very small, the signal only retains the first pair of sidebands, and the sideband amplitude is much smaller than the carrier amplitude. The \(C/N_0\) of the signal is 168 dB/Hz.

After TIA conversion and amplification, a voltage signal is obtained.
\[
u_s(t) = U_s \sin\left(2\pi f_c t + \phi(t)\right) + \frac{U_s}{2\kappa\sqrt{P_s P_r}} n_{\text{shot}}(t)
\]
(33)

Usually, the saturation threshold \(K\) of the amplifier is set to be equal to the measurement range of the ADC, here \(K=\pm 1\text{ V}\). When the signal is only amplified to \(U_s=0.9\text{ V}\), which does not exceed the measurement range of the ADC, the complete signal is digitized and converted to \(u_s[n]\). In the process of digitization, ADC thermal noise, electronic noise and quantization noise may interfere with the signal. In this simulation, only the quantization noise \(n_{\text{adc}}[n]\) of the ADC is considered.

The power spectral density of the digitized signal is shown in figure 8.

In figure 8, the discrete voltage signal without quantization noise is on the left and on the right is the digital signal with quantization noise. The noise floor of the signal with quantization noise is significantly higher than the noise floor of the signal without quantization noise. Obviously, during the digitization process, due to the quantization noise of the ADC, the \(C/N_0\) of the digitized signal is reduced from 168 dB/Hz to about 150 dB/Hz, which will reduce the resolution of the demodulation.
result. In addition, for small PM, the sideband amplitude is so small that the quantization noise almost completely covers the sidebands. In this case, the demodulation algorithm cannot recover the information carried by the signal.

If the active clipping is used, the signal is amplified to $U_s=20\text{ V}$ and then clipped by $K=1\text{ V}$. Figure 9 shows the power spectral density of a clipped signal.

![Figure 8](image1.png) ![Figure 9](image2.png)

**Figure 8.** Power spectral density of discrete voltage signal $u_s[n]$ (a) and $u_s[n]+n_{adc}[n]$ (b) with $U_s=0.9\text{ V}$.

**Figure 9.** Power spectral density of discrete clipped voltage signal $u_s[n]+n_{adc}[n]$ with $U_s=20\text{ V}$ and $K=1\text{ V}$.

It can be seen that, consistent with the mathematical analysis, the sideband amplitude increases compared to figure 8 (a), even though the clipping threshold limits the carrier amplitude. Thus, the quantization noise $n_{adc}[n]$ can no longer completely cover the sidebands. The sideband signal is not only readable, but maintains the same sideband-to-noise ratio as in figure 7.

However, due to the loss of carrier amplitude, the results obtained by the conventional demodulation method are incorrect. In figure 10 and 11, the curve of the ratio of the demodulated result $\hat{s}_{\text{clipped}}$ of the clipped PM signal to the vibration amplitude $\hat{s}$ of the measured object with $\hat{s}$ and $U_s$ is given.
In figure 10, when the vibration amplitude $\hat{s}$ is small ($M \ll 1$), the signal satisfies the condition of small-angle modulation and $J_0(M) \approx 1$. The demodulation result of the clipped signal $\hat{s}_{\text{clipped}}$ at this time is approximately $\frac{U_s J_0(M)}{k} \approx 10$ times the true value of $\hat{s}$ (equation (30)).

![Figure 10. Ratio of the demodulation result $\hat{s}_{\text{clipped}}$ to the vibration amplitude $\hat{s}$ with $\hat{s}$ from 1 fm to 1 $\mu$m for a clipped phase-modulated signal $u_s[n]$ with $U_s=10$ V, $f_c=40$ MHz, $f_m=400$ kHz, $f_s=160$ MHz and $K=1$ V.](image)

![Figure 11. Ratio of the demodulation result of the clipped PM signal $\hat{s}_{\text{clipped}}$ to the vibration amplitude $\hat{s}$ with $U_s$ from 0.1 V to 100 V for a clipped phase-modulated signal $u_s[n]$ with $\hat{s}=12$ nm, $f_c=40$ MHz, $f_m=400$ kHz, $f_s=160$ MHz and $K=1$ V.](image)

In figure 10, when the vibration amplitude $\hat{s}$ is small ($M \ll 1$), the signal satisfies the condition of small-angle modulation and $J_0(M) \approx 1$. The demodulation result of the clipped signal $\hat{s}_{\text{clipped}}$ at this time is approximately $\frac{U_s J_0(M)}{k} \approx 10$ times the true value of $\hat{s}$ (equation (30)).
As the vibration amplitude gradually increases, \( J_1(M) \) gradually increases and no longer satisfies \( 2U_s J_1(M) \ll K \). Under this condition, \( \arctan \left( \frac{2U_s J_1(M)}{K} \right) \) is no longer approximately equal to \( \frac{2U_s J_1(M)}{K} \) but less than \( \frac{2U_s J_1(M)}{K} \), therefore \( \frac{\Delta_{\text{clipped}}}{\delta} \) in figure 10 decreases with increasing \( \delta \).

When the vibration amplitude \( \delta > 12 \text{ nm} \) \((M > 0.1\) and \( J_1(M) > 0.05)\), the sidebands (equation (17)) exceed the clipping threshold and start to be clipped. Until all sampling points are equal to \( \pm K \), \( \frac{\Delta_{\text{clipped}}}{\delta} = \frac{2}{4\pi} \) according to equation (31). The ratio \( \frac{\Delta_{\text{clipped}}}{\delta} \) decreases according to the function \( \frac{\Delta_{\text{clipped}}}{\delta} = \frac{1}{4\pi K} \).

In addition, when \( \delta > 100 \text{ nm} \) \((M > 0.8)\), the signal \( u_s(n) \) cannot be considered as a small-angle modulated signal, and the demodulation results can no longer be explained by the previous mathematical analysis.

Figure 11 shows the effect of the amplifier's amplification on the demodulation results of the clipped signal. Here, \( J_0(M=0.1) = 1 \) and \( J_1(M=0.1) = 0.05 \). When \( U_s \) is less than \( K \), the signal is not clipped and the demodulation result is the true vibration amplitude \( \frac{\Delta_{\text{clipped}}}{\delta} = 1 \). When \( U_s \) is slightly higher than \( K \), the ratio is in accordance with \( \frac{\Delta_{\text{clipped}}}{\delta} = \frac{U_s J_0(0.1)}{K} \) (equation (30)). When \( U_s \) is amplified to \( 5 \text{ V} \) and above, the ratio \( \frac{\Delta_{\text{clipped}}}{\delta} < \frac{U_s J_0(0.1)}{K} \) because of the function characteristic of \( \arctan \). Finally, when the sideband is amplified to the clipping threshold \( K \), \( \frac{\Delta_{\text{clipped}}}{\delta} = \frac{1}{M} \approx 10 \) will remain constant as shown in equation (31).

4. Conclusion

In this paper we examined the impact of clipping on phase-modulated signal from LDV. This investigation is of great significance because the \( C/N_0 \) of the optical signal is often limited by the quantization noise during digitization and amplifying the signal to improve the \( C/N_0 \) will cause clipping.

Through the mathematical analysis and the simulation results, we have demonstrated in this paper that the behavior of active clipping of the PM signal does not alter the sidebands but the carrier if the clipping threshold is below the carrier amplitude but above the sideband amplitudes. For small phase modulated signals, even if the carrier amplitude is out of the measurement range, we can still increase the amplification factor to increase the power of the sideband, and small sideband signals will not be covered by ADC noise.

Furthermore, through the analysis and simulation of the demodulation results of the clipped signal, we found that clipping would lead to wrong demodulation results. However, the relative error of demodulation is almost constant for the small vibration amplitude of the measurement object and small amplification factor. Our future research addresses a new demodulation scheme to obtain the signal accurately from a clipped signal.

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