The Standard Model, Dark Matter, and Dark Energy: From the Sublime to the Ridiculous

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The Standard Model of cosmology of the 1980’s was based on a remarkable interplay of ideas from particle theory, experiment and astrophysical observations. That model is now dead, and has been replaced by something far more bizarre. Interestingly, the aspect that has survived involves perhaps the most exotic component: dark matter that dominates the gravitational dynamics of all galaxies, and appears to be composed of a sea of new weakly interacting elementary particles. But this sea of dark matter appears to play second fiddle to an unknown energy density that appears to permeate all of space, causing the expansion of the Universe to accelerate. We are left with many more questions than answers, and our vision of the future of the Universe has completely changed.

(Lectures Given at the XIV Canary Islands Winter School in Astrophysics 2002: Dark Matter and Dark Energy in the Universe. Nov 2002 To Appear in the Proceedings)

1. Introduction: Why Cosmology?

Astrophysics and Cosmology involve observations, not experiments, and thus are intrinsically suspect. Nevertheless, we have learned over the past 30 years that the universe provides a laboratory for exploring fundamental physics that in many cases exceeds the reach of any terrestrial laboratory.

Consider the following:

(i) Energy: The center of mass energy of accelerators on Earth has increased by a factor of 1000, from 1 GeV to 1 TeV in the thirty years since 1975. As a result, we have been able to probe the nature of quantum chromodynamics, and the electroweak theories with unprecedented sensitivity, and we hope we are on the threshold of discovering experimental evidence that will shed light on the origin of mass. Many theorists expect that this may involve the discovery of supersymmetry. At the same time, we have learned that studying the light element abundances in the Universe sheds insight into Big Bang Nucleosynthesis in a way that directly constrains particle physics up to regime of the QCD phase transition, while the physics of supernovae is dominated by the weak interactions of neutrinos, and may even be sensitive to particles that cannot be probed in accelerators, such as axions. The highest energy cosmic rays impacting upon the Earth interact with center of mass energies that exceed those that will be accessible at the LHC. The physics of dark matter may reveal evidence of supersymmetry before it is probed in accelerators, while understanding the origin of baryons in the universe has constrained both the electroweak theory, and possible grand unified theories.

(ii) Cross Section and Target Mass: Accelerators probe cross sections in the range down to about $10^{-40} \text{cm}^2$, with targets that are generally less than a kilometer across. Astrophysics on the other hand allows us to probe processes that involve cross sections well below $10^{-42} \text{cm}^2$, and with targets in excess of 10,000 kilometers across.

As a result, astrophysics and cosmology provide great discovery potential. Because of the limitations of observational uncertainties, however, terrestrial experiments are required if we are to move beyond discovery to exploring the fundamental details of
nature at the smallest scales. In these lectures, I will describe the current observational situation in cosmology, and the theoretical questions that may shed light on unraveling the nature of dark matter, and dark energy.

2. The Standard Model and Cosmological Observables

As early as a decade ago, the uncertainties in the measurement of cosmological parameters was such that few definitive statements could be made regarding cosmological models. That situation has changed completely. Instead all cosmological observables have now converged on a single cosmological model. In this section I will review our present picture.

First, some basic background. The Universe as we observe it is isotropic, homogeneous, and expanding. I shall assume that readers are familiar with these basic features, which are determined observationally in reverse order, by measuring redshift-distance relations, by examining the number counts of galaxies, and by observations of the cosmic microwave background radiation. I shall discuss the measurement of the expansion rate, and of the cosmic microwave background in some detail shortly. I will briefly mention here that one can show, and you can find in any basic textbook on cosmology, that in a flat Euclidean Universe measuring the number of galaxies as a function of magnitude $m$ that if $N(< m) \approx 10^{9.6m}$ then the underlying distribution is basically homogeneous. Remarkably this relation holds out to significant distances even in our expanding universe today.

I am also not going to review here the basic features of FRW cosmology, and will assume the reader is familiar with Einstein’s Equations for the evolution of the scale factor an expanding isotropic homogeneous universe, which relate the expansion rate, given by the Hubble Constant, $H$ to the density and curvature, where the ratio of the actual matter density today to that required for a flat universe (given by $\rho_c = 3H^2/8\pi G$) is given by the parameter $\Omega_m$.

Within the framework of an isotropic homogeneous expanding universe, there are a finite set of fundamental observables. It seems reasonable to divide this into three subsections, Space, Time, and Matter. Specifically, I shall concentrate on the following:

Space:
- Expansion Rate
- Geometry

Time:
- Age of the Universe

Matter:
- Baryon Density
- Large Scale Structure
- Matter Density
- Equation of State

2.1. Space: The Final Frontier:

2.1.1. The Hubble Constant

Arguably the most important single parameter describing the physical universe today is the Hubble Constant. Since the discovery in 1929 that the Universe is expanding, the determination of the rate of expansion dominated observational cosmology for much of the rest of the 20th century. The expansion rate, given by the Hubble Constant, sets the overall scale for most other observables in cosmology.
The big news, if any, is that by the end of the 20th century, almost all measurements have converged on a single range for this all important quantity. (I say almost all, because to my knowledge Alan Sandage still believes the claimed limits are incorrect (Parodi et al (2000)).)

Recently, the Hubble Space Telescope Key Project has announced its final results. This is the largest scale endeavor carried out over the past decade with a goal of achieving a 10% absolute uncertainty in the Hubble constant. The goal of the project has been to use Cepheid luminosity distances to 25 different galaxies located within 25 Megaparsecs in order to calibrate a variety of secondary distance indicators, which in turn can be used to determine the distance to far further objects of known redshift. This in principle allows a measurement of the distance-redshift relation and thus the Hubble constant on scales where local peculiar velocities are insignificant. The five distance indicators so constrained are: (1) the Tully Fisher relation, appropriate for spirals, (2) the Fundamental plane, appropriate for ellipticals, (3) surface brightness fluctuations, and (4) Supernova Type Ia distance measures, and (5) Supernovae Type II distance measures.

The Cepheid distances obtained from the HST project include a larger LMC sample to calibrate the period-luminosity relation, a new photometric calibration, and corrections for metallicity. As a result they determined a new LMC distance modulus, of $\mu_o = 18.50 \pm 0.10$ mag. The number of Cepheid calibrators used for the secondary measures include 21 for the Tully-Fisher relation, and 6 for each of the Type Ia and surface fluctuation measures.

The HST-Key project reported measurements for each of these methods is presented below (Freedman et al (2001)). (While I shall adopt these as quoted, it is worth pointing out that some critics have stressed that this involves utilizing data obtained by other groups, who themselves sometimes report different values of $H_0$). The first quoted uncertainty is statistical, the second is systematic (coming from such things as LMC zero point measurements, photometry, metallicity uncertainties, and remnant bulk flows).

$$H_{TF}^O = 71 \pm 3 \pm 7$$
$$H_{FP}^O = 82 \pm 6 \pm 9$$
$$H_{SBF}^O = 70 \pm 5 \pm 6$$
$$H_{SN1a}^O = 71 \pm 2 \pm 6$$
$$H_{SNII}^O = 72 \pm 9 \pm 7$$

On the basis of these results, the Key Project reports a weighted average value:

$$H_{WA}^O = 72 \pm 3 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1} (1\sigma)$$

and a final combined average of

$$H_{WA}^O = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1} (1\sigma)$$

The Hubble Diagram obtained from the HST project (Freedman et al (2001)) is reproduced as figure 1.

In the weighted average quoted above, the dominant contribution to the 11% one sigma error comes from an overall uncertainty in the distance to the Large Magellanic Cloud. If the Cepheid Metallicity were shifted within its allowed 4% uncertainty range, the best fit mean value for the Hubble Constant from the HST-Key project would shift downward to $68 \pm 6$. 
S-Z Effect:

The Sunyaev-Zeldovich effect results from a shift in the spectrum of the Cosmic Microwave Background radiation due to scattering of the radiation by electrons as the radiation passes through intervening galaxy clusters on the way to our receivers on Earth. Because the electron temperature in Clusters exceeds that in the CMB, the radiation is systematically shifted to higher frequencies, producing a deficit in the intensity below some characteristic frequency, and an excess above it. The amplitude of the effect depends upon the Thompson scattering across section, and the electron density, integrated over the photon’s path:

\[ SZ \approx \int \sigma_T n_e dl \]

At the same time the electrons in the hot gas that dominates the baryonic matter in galaxy clusters also emits X-Rays, and the overall X-Ray intensity is proportional to the square of the electron density integrated along the line of sight through the cluster:

\[ X - \text{Ray} \approx \int n_e^2 dl \]

Using models of the cluster density profile one can then use the differing dependence on \( n_e \) in the two integrals above to extract the physical path-length through the cluster. Assuming the radial extension of the cluster is approximately equal to the extension across the line of sight one can compare the physical size of the cluster to the angular size to determine its distance. Clearly, since this assumption is only good in a statistical sense, the use of S-Z and X-Ray observations to determine the Hubble constant cannot be done reliably on the basis of a single cluster observation, but rather on an ensemble.

A recent preliminary analysis of several clusters (Birkinshaw (1999)) yields:

\[ H_0^{SZ} = 60 \pm 10 \text{ km s}^{-1} \text{ Mpc}^{-1} \]
Type 1a SN  (non-Key Project):
One of the HST Key Project distance estimators involves the use of Type 1a SN as standard candles. As previously emphasized, the Key Project does not perform direct measurements of Type 1a supernovae but rather uses data obtained by other groups. When these groups perform an independent analysis to derive a value for the Hubble constant they arrive at a smaller value than that quoted by the Key Project. Their recent quoted value is (Jha et al (1999)):

$$H_{0}^{1a} = 64^{+8}_{-6} \text{ ks}^{-1} \text{ Mpc}^{-1}$$

At the same time, Sandage and collaborators have performed an independent analysis of SNe Ia distances and obtain (Parodi et al (2000)):

$$H_{0}^{1a} = 58 \pm 6 \text{ ks}^{-1} \text{ Mpc}^{-1}$$

Surface Brightness Fluctuations and The Galaxy Density Field:
Another recently used distance estimator involves the measurement of fluctuations in the galaxy surface brightness, which correspond to density fluctuations allowing an estimate of the physical size of a galaxy. This measure yields a slightly higher value for the Hubble constant (Blakeslee et al (1999)):

$$H_{0}^{SBF} = 74 \pm 4 \text{ ks}^{-1} \text{ Mpc}^{-1}$$

Time Delays in Gravitational Lensing:
One of the most remarkable observations associated with observations of multiple images of distant quasars due to gravitational lensing intervening galaxies has been the measurement of the time delay in the two images of quasar $Q0957+561$. This time delay, measured quite accurately to be $417 \pm 3$ days is due to two factors: The path-length difference between the quasar and the earth for the light from the two different images, and the Shapiro gravitational time delay for the light rays traveling in slightly different gravitational potential wells. If it were not for this second factor, a measurement of the time delay could be directly used to determine the distance of the intervening galaxy. This latter factor however, implies that a model of both the galaxy, and the cluster in which it is embedded must be used to estimate the Shapiro time delay. This introduces an additional model-dependent uncertainty into the analysis. Two different analyses yield values (Chae (1999)):

$$H_{0}^{TD1} = 69^{+18}_{-12} (1 - \kappa) \text{ ks}^{-1} \text{ Mpc}^{-1}$$

$$H_{0}^{TD2} = 74^{+18}_{-10} (1 - \kappa) \text{ ks}^{-1} \text{ Mpc}^{-1}$$

where $\kappa$ is a parameter which accounts for a possible deviation in cluster parameters governing the overall induced gravitational time delay of the two signals from that assumed in the best fit. It is assumed in the analysis that $\kappa$ is small.

Summary:
It is difficult to know how to best incorporate all of the quoted estimates into a single estimate, given their separate systematic and statistical uncertainties. Assuming large
number statistics, where large here includes the quoted values presented here, I perform a simple weighted average of the individual estimates, and find an approximate average value:

\[ H_0^{Av} \approx 70 \pm 5 \, \text{km/s/Mpc} \]  

(2.1)

2.1.2. Geometry:

Again, for much of the 20th century the effort to determine the geometry of the Universe involved a very indirect route. Einstein’s Equations yield a relationship between the Hubble constant, the energy density, and the curvature of the Universe. By attempting to determine the first two quantities, one hoped to constrain the third. The problem is that until the past decade the uncertainty in the Hubble constant was at least 20-30% and the uncertainty in the average energy density of the universe was even greater. As a result, almost any value for the net curvature of the universe remained viable.

It has remained a dream of observational cosmologists to be able to directly measure the geometry of space-time rather than infer the curvature of the universe by comparing the expansion rate to the mean mass density. While several such tests, based on measuring galaxy counts as a function of redshift, or the variation of angular diameter distance with redshift, have been attempted in the past, these have all been stymied by the achilles heel of many observational measurements in cosmology, evolutionary effects.

Recently, however, measurements of the cosmic microwave background have finally brought us to the threshold of a direct measurement of geometry, independent of traditional astrophysical uncertainties. The idea behind this measurement is, in principle, quite simple. As shown in figure 2, the CMB originates from a spherical shell located at the surface of last scattering (SLS), at a redshift of roughly \( z \approx 1000 \):

If a fiducial length could unambiguously be distinguished on this surface, then a determination of the angular size associated with this length would allow a determination of the intervening geometry, as shown in figure 3.

Fortunately, nature has provided such a fiducial length, which corresponds roughly to the horizon size at the time the surface of last scattering existed (in this case the length is the ”sound horizon”, but since the medium in question is relativistic, the speed of sound is close to the speed of light.) The reason for this is also straightforward. This is the largest scale over which causal effects at the time of the creation of the surface of last scattering could have left an imprint. Density fluctuations on such scales would result in acoustic oscillations of the matter-radiation fluid, and the doppler motion of electrons moving along with this fluid which scatter on photons emerging from the SLS produces a characteristic peak in the power spectrum of fluctuations of the CMB at a wavenumber corresponding to the angular scale spanned by this physical scale. These fluctuations should also be visually distinguishable in an image map of the CMB, provided a resolution on degree scales is possible.

A number of different ground-based balloon experiments, launched in places such Texas and Antarctica have resulted in maps with the required resolution (de Bernardis et al (2000), Hanany et al (2000), Scott et al (2003), Halverson et al (2002)). Shown in figure 4 is a comparison of the Boomerang map with several simulations based on a gaussian random spectrum of density fluctuations in a cold-dark matter universe, for open, closed, and flat cosmologies. Even at this qualitative level, it is clear that a flat universe provides better agreement to between the simulations and the data than either an open or closed universe.

Recently the Wilkinson Microwave Anisotropy Probe (WMAP) has produced a high resolution CMB map of the entire sky. Using this one can produce a quantitative con-
constraint by comparing the inferred power spectra with predicted spectra (Jaffe et al (2001)). Such comparisons for the most recent data (Spergel et al (2003)) yields a constraint on the density parameter:

\[ \Omega = 1.02 \pm 0.02 (68\% CL) \]  \hspace{1cm} (2.2)

For the first time, it appears that the longstanding prejudice of theorists, namely that we live in a flat universe, may have been vindicated by observation! However, theorists cannot be too self-satisfied by this result, because the source of this energy density appears to be completely unexpected, and largely inexplicable at the present time, as we will shortly see.

2.2. Time

2.2.1. Stellar Ages:

Ever since Kelvin and Helmholtz first estimated the age of the Sun to be less than 100 million years, assuming that gravitational contraction was its prime energy source, there has been a tension between stellar age estimates and estimates of the age of the universe. In the case of the Kelvin-Helmholtz case, the age of the sun appeared too short to accommodate an Earth which was several billion years old. Over much of the latter half of the 20th century, the opposite problem dominated the cosmological landscape. Stellar ages, based on nuclear reactions as measured in the laboratory, appeared to be too old to
accomodate even an open universe, based on estimates of the Hubble parameter. Again, as I shall outline in the next section, the observed expansion rate gives an upper limit on the age of the Universe which depends, to some degree, upon the equation of state, and the overall energy density of the dominant matter in the Universe.

There are several methods to attempt to determine stellar ages, but I will concentrate here on main sequence fitting techniques, because those are the ones I have been involved in. For a more general review, see Krauss and Chaboyer (2003).

The basic idea behind main sequence fitting is simple. A stellar model is constructed by solving the basic equations of stellar structure, including conservation of mass and energy and the assumption of hydrostatic equilibrium, and the equations of energy transport. Boundary conditions at the center of the star and at the surface are then used, and combined with assumed equation of state equations, opacities, and nuclear reaction rates in order to evolve a star of given mass, and elemental composition.

Globular clusters are compact stellar systems containing up to $10^5$ stars, with low heavy element abundance. Many are located in a spherical halo around the galactic center, suggesting they formed early in the history of our galaxy. By making a cut on those clusters with large halo velocities, and lowest metallicities (less than 1/100th the solar value), one attempts to observationally distinguish the oldest such systems. Because these systems are compact, one can safely assume that all the stars within them formed at approximately the same time.
Observers measure the color and luminosity of stars in such clusters, producing color-magnitude diagrams of the type shown in figure 5 (based on data from Durrell and Harris (1993)).

Next, using stellar models, one can attempt to evolve stars of differing mass for the metallicities appropriate to a given cluster, in order to fit observations. A point which is often conveniently chosen is the so-called main sequence-turnoff (MSTO) point, the point in which hydrogen burning (main sequence) stars have exhausted their supply of hydrogen in the core. After the MSTO, the stars quickly expand, become brighter, and are referred to as Red Giant Branch (RGB) stars. Higher mass stars develop a helium core that is so hot and dense that helium fusion begins. These form along the horizontal branch. Some stars along this branch are unstable to radial pulsations, the so-called RR Lyrae stars mentioned earlier, which are important distance indicators. While one in principle could attempt to fit theoretical isochrones (the locus of points on the predicted CM curve corresponding to different mass stars which have evolved to a specified age), to observations at any point, the main sequence turnoff is both sensitive to age, and involves minimal (though just how minimal remains to be seen) theoretical uncertainties.

Dimensional analysis tells us that the main sequence turnoff should be a sensitive function of age. The luminosity of upper main sequence stars is very roughly proportional to the third power of solar mass. Hence the time it takes to burn the hydrogen fuel is proportional to the total amount of fuel (proportional to the mass M), divided by the Luminosity — proportional to \( M^3 \). Hence the lifetime of stars on the main sequence is roughly proportional to the inverse square of the stellar mass.

Of course the ability to go beyond this rough approximation depends completely on the on the confidence one has in one’s stellar models. What is most important for the comparison of cosmological predictions with inferred age estimates is the uncertainties in stellar model parameters, and not merely their best fit values.

Over the course of the past several years, I and my collaborators have tried to in-
corporate stellar model uncertainties, along with observational uncertainties into a self consistent Monte Carlo analysis which might allow one to estimate a reliable range of globular cluster ages. Others have carried out independent, but similar studies, and at the present time, rough agreement has been obtained between the different groups (i.e. see Krauss (2000)).

I will not belabor the detailed history of all such efforts here. The most crucial insight has been that stellar model uncertainties are small in comparison to an overall observational uncertainty inherent in fitting predicted main sequence luminosities to observed turnoff magnitudes. This matching depends crucially on a determination of the distance to globular clusters. The uncertainty in this distance scale produces by far the largest uncertainty in the quoted age estimates.

In many studies, the distance to globular clusters can be parametrized in terms of the inferred magnitude of the horizontal branch stars. This magnitude can, in turn, be presented in terms of the inferred absolute magnitude, $M_v(\text{RR})$ of RR Lyrae variable stars located on the horizontal branch.

In 1997, the Hipparcos satellite produced its catalogue of parallaxes of nearby stars,
causing an apparent revision in distance estimates. The Hipparcos parallaxes seemed to be systematically smaller, for the smallest measured parallaxes, than previous terrestrially determined parallaxes. Could this represent the unanticipated systematic uncertainty that David has suspected? Since all the detailed analyses had been pre-Hipparcos, several groups scrambled to incorporate the Hipparcos catalogue into their analyses. The immediate result was a generally lower mean age estimate, reducing the mean value to 11.5-12 Gyr, and allowing ages of the oldest globular clusters as low as 9.5 Gyr. However, what is also clear is that there is now an explicit systematic uncertainty in the RR Lyrae distance modulus which dominates the results. Different measurements are no longer consistent. Depending upon which distance estimator is correct, and there is now better evidence that the distance estimators which disagree with Hipparcos-based main sequence fitting should not be dismissed out of hand, the best-fit globular cluster estimate could shift up perhaps 1σ, or about 1.5 Gyr, to about 13 Gyr.

Within the past two years, Brian Chaboyer and I have reanalyzed globular cluster ages, incorporating new nuclear reaction rates, cosmological estimates of the $^4$He abundance, and most importantly, several new estimates of $M_v$ (RR), shown below.

The result is that while systematic uncertainties clearly still dominate, we argue that the best fit age of globular clusters is now $12.6^{+3.4}_{-2.4}$ (95%) Gyr, with a 95% confidence range of about 11-16 Gyr (Krauss and Chaboyer (2003)).

If we are to turn this result into a lower limit on the age of the Universe we must add to this estimate the time after the Big Bang that it took for the first globular clusters in our galaxy to form. Here there is great uncertainty. However a robust lower limit comes from observations of structure formation in the Universe, which suggest that the first galaxies could not have formed much before a redshift of 6-7. Turning this redshift into an age depends upon the equation of state of the dominant energy density at that time (see
However, one can show that at such high redshifts, the effects of a possible dark energy component are minimal, leading to a minimum age of globular cluster formation of about .8 Gyr. The maximum age is much less certain, as it is possible for galaxies to form at redshifts as low as 1-2. Thus, one must add an age of perhaps 3.5-4 Gyr to the globular age estimate above to get an upper limit on the age of the Universe. Putting these factors together, one derives a 95% confidence age range for the Universe of 11.2-20 Gyr.

2.2.2. Hubble Age:

As alluded to earlier, in a Friedman-Robertson-Walker Universe, the age of the Universe is directly related to both the overall density of energy, and to the equation of state of the dominant component of this energy density. The equation of state is parameterized by the ratio \( \omega = p/\rho \), where \( p \) stands for pressure and \( \rho \) for energy density. It is this ratio which enters into the second order Friedman equation describing the change in Hubble parameter with time, which in turn determines the age of the Universe for a specific net total energy density.

The fact that this depends on two independent parameters has meant that one could reconcile possible conflicts with globular cluster age estimates by altering either the energy density, or the equation of state. An open universe, for example, is older for a given Hubble Constant, than is a flat universe, while a flat universe dominated by a cosmological constant can be older than an open matter dominated universe.

If, however, we incorporate the recent geometric determination which suggests we live in a flat Universe into our analysis, then our constraints on the possible equation of state on the dominant energy density of the universe become more severe. If, for existence, we allow for a diffuse component to the total energy density with the equation of state of a cosmological constant (\( \omega = -1 \)), then the age of the Universe for various combinations of matter and cosmological constant is given by:

\[
H_0 t_0 = \int_0^\infty \frac{dz}{(1 + z)(\Omega_m)(1 + z)^3 + (\Omega_X)(1 + z)^{3(1+w)}}^{1/2}
\]  

(2.3)
This leads to ages as shown in Table 1.

The existing limits on the age of the universe from globular clusters are thus already incompatible with a flat matter dominated universe. This is a very important result, as it implies that now all three classic tests of cosmology, including geometry, large scale structure, and age of the Universe now support the same cosmological model, which involves a universe dominated by dark energy (Indeed, before the direct supernova evidence for dark energy it was argued that these factors favored the existence of dark energy (Krauss and Turner (1995)) . We can provide limits on the equation of state for dark energy as well. Shown in figure 8, is the constraint on $w$, assuming a Hubble constant of 72 (Krauss and Chaboyer (2003)).

At the same time, it is worth noting that unfortunately the upper limit on the age of the universe coming from globular cluster ages cannot provide a useful limit on the equation of state parameter $w$, because there is an upper limit on the Hubble Age, independent of $w$, if the contribution of matter to the total density is greater than 20% (Krauss (2004)).

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### Table 1. Hubble Ages for a Flat Universe, $H_0 = 70 \pm 8$.

| $\Omega_M$ | $\Omega_x$ | $t_0$  |
|-----------|-----------|--------|
| 1         | 0         | 9.7 ± 1|
| 0.2       | 0.8       | 15.3 ± 1.5|
| 0.3       | 0.7       | 13.7 ± 1.4|
| 0.35      | 0.65      | 12.9 ± 1.3|

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**Figure 8.** Constraint on the equation of state parameter for dark energy as a function of the fraction of closure density in matter resulting from age constraint described here.
2.2.3. CMB, Hubble Age, Galaxy Formation and Equation of State

Perhaps not so remarkably, the CMB can also give a direct measure of the Hubble Age, in much the same way as one can use it to measure the geometry of the Universe. The physical distance to the last scattering surface depends upon the age of the Universe, so that measuring the physical angle of the first doppler peak can, provided one uses other observational information about $H_0$ and $\Omega_{\text{matter}}$, give a measure of the Hubble age. The first such estimates were obtained with Boomerang data, but once again the WMAP data gives the best current limit, which is quoted as (Spergel et al (2003)) $13.7 \pm 0.2$ Gyr.

By comparing WMAP observations with previous estimates of globular cluster ages, one can derive provide important new handles to probe the likely formation of the milky way galaxy, and in a broader sense the formation of large scale cosmic structures. The two key WMAP observations in this regard are the estimate of cosmic age ($13.7 \pm 0.2$ Gyr), and the redshift of reionization (Spergel et al (2003)).

Comparing the 68% lower confidence limit age of 11.2 Gyr (Krauss and Chaboyer (2003)) with the 68% upper limit on the age of the Universe from WMAP of 13.9 Gyr suggests an 90% upper limit $\approx 2.7$ Gyr as the time after the Big Bang that globular clusters in our galaxy first formed from the primordial halo of gas that ultimately collapsed to form the Milky Way. At the 95% confidence level the limit becomes approximately 3 Gyr. This not only improves upon previous estimates, it is the first direct constraint on this quantity.

Of somewhat more interest is a determination of the most probable time after the Big Bang at which our globular clusters formed. Now that WMAP has determined a surprisingly early time where the Universe reionized, corresponding to an age of about 200-300 Myr after the Big Bang, it is interesting to know whether this corresponds to an early period of star formation, and whether structures as large as globular clusters of stars also formed this early. Note that (Jimenez et al (2003)) have recently assumed this to be the case.

A variety of different methods have been used to determine the age of globular clusters in our galaxy. The Monte Carlo analysis referred to above involves dating these clusters using main-sequence turnoff luminosity, and yields an age estimate for the oldest clusters at the 95% confidence level of $12.6^{+3.1}_{-2.2}$ Gyr. The most likely age for these globular clusters is thus $\approx 800$ Myr younger than the WMAP lower limit on the age of the Universe. However, because the distribution is broad, the possibility that globular clusters formed before the period of cosmic reionization determined by WMAP to occur $\approx 200$ Myr after the Big Bang is certainly still viable (Jimenez et al (2003)). Nevertheless, examining the probability distribution in Krauss and Chaboyer (2003), as fit analytically in Jimenez et al (2003), one finds a 75% likelihood that the oldest globular clusters are in fact less than 13.5 Gyr old.

While not compelling, the possibility that globular clusters in our galaxy may have formed well after reionization could shed light on a number of issues, including whether reionization is due to a very early generation of massive stars and whether such systems formed before (and if so, how much before) larger structures such as globular clusters. This could probe the nature of possible hierarchical clustering. The likelihood of this possibility is increased when one recognizes that several other methods for determining the age of globular clusters, including using luminosity functions (Jimenez and Padoan (1998)), white dwarf cooling (Hansen et al (2002)) and eclipsing binaries (Chaboyer and Krauss (2002)) favor globular cluster ages in the range of 11-13 Gyr.

The existing uncertainty in globular cluster dating techniques is at present too large to
do more than hint that there may be a gap in time between reionization in the Universe and the formation of larger scale structures. However, this hint strongly motivates efforts to further reduce the absolute uncertainty in globular dating techniques.

Next, one can use the Hubble Age determination from the CMB to constrain the possibility that the equation of state parameter for dark energy is actually less than $-1$. While there are really no sensible models of this, there is also no understanding of the dark energy, so who knows?

Age estimates can in principle give strong constraints on values of $w$ less than $-1$, since the age of the Universe is a strongly varying function of $w$ for values of $w > -5$ (Krauss (2004)).

In Figs. 9 and 10, the predicted age of the Universe for various values of $w < -1$ as a function of the Hubble constant in comparison to the 2σ upper limit on the cosmic age from WMAP, for two different values of the assumed matter density today (corresponding to midpoint of the WMAP allowed range for matter density, and the 2σ upper limit) (Krauss (2003)). As is clear from these figures, for a flat universe the inferred bound on $w$ from the WMAP cosmic age limit depends sensitively on the assumed total matter density today.

It is important to realize however that one is not free to independently vary $\Omega_m$ and $H_0$ in deriving bounds using the WMAP data. These two quantities are themselves highly anti-correlated in the WMAP fit (Spergel et al (2003)). As can be seen from the WMAP fits as $w$ is decreased (for $w > -1$), the allowed range of $\Omega_m$ decreases roughly linearly, while the allowed range of Hubble constant increases roughly linearly. If we assume this behavior extrapolates to values of $w < -1$, then we can use this relation to derive a conservative lower bound on $w$. The most conservative bound on $w$ comes from assuming the largest allowed value of $\Omega_m$ for any value of $H_0$. We fit this value using the anti-correlation described above, and fitting to the WMAP plots to derive $\Omega_m^{\text{max}}h^2 = 0.309 - 0.243h$ within the allowed range of $H_0$. When we include this relation explicitly for $\Omega_m$ in the cosmic age relation, one derive limits on the age of the Universe shown in Fig. 11 (Krauss (2003)).

To derive a bound on $w$ it is necessary to note that lower bound on $H_0$ derived from WMAP is correlated with the inferred value of $w$. If we extrapolate the allowed range of $H_0$ to values of $w < -1$, we find a lower bound on $H_0$ as a function of $w$ shown by the thick solid line in this figure. If we use this lower bound on $H_0$, and compare the predicted age as a function of $w$ with the WMAP upper limit, we derive a bound $w > -1.22$. If we were instead to allow the full HST range for $H_0$ in deriving this limit, the lower bound would decrease slightly to $w > -1.27$.

The WMAP team has used a global fit to constrain $w$ and they find the lower bound $w > -1.2$. If one combines this result with the WMAP-derived upper bound on $w$, one thus finds an allowed region $-1.2 < w < -0.8$. It is interesting, but perhaps not surprising that the uncertainty is symmetric about the value $-1$. I shall have more to say about this later.

2.3. Matter

It has long been established that visible matter, namely matter associated with stars, planets, and luminous gas falls far short of the amount required to close the Universe. The best estimate today of the total visible matter density yields $\Omega_{\text{lum}} = .004_{-0.3}^{+0.4}$.

The problem is that this estimate is clearly a lower limit on the total baryon density in the universe, since matter need not shine. In order to directly determine the total baryon abundance we have been able to resort to calculations from the early universe of the light element abundances, and then comparing these to observations, as I describe below.
2.3.1. The Baryon Density: a re-occurring crisis?

The success of Big Bang Nucleosynthesis in predicting in the cosmic abundances of the light elements has been much heralded. The basis is quite simple: At $T = 10^9 - 10^{10} K$ nuclear reactions convert protons and neutrons to $^4He$ via intermediate reactions that produce $D$ and $^3He$. Since reaction rates depend upon the density of protons and neutrons, this ultimately depends upon $\Omega_B$. The final important fact is that production
Figure 11. Assuming a negative linear correlation between inferred value of the density parameter $\Omega_m h^2$ and the inferred value of $h$, based on the WMAP data, age estimates as a function of equation of state parameter $w$ can be determined. Because predicted age is a decreasing function of the density parameter, the most conservative limits on $w$ come from choosing the maximum allowed density parameter (at the $2\sigma$ level) from WMAP. A fit to WMAP yields ($\Omega_m^{\text{max}} h^2 = 0.309 - 0.243h$). Also shown (dotted line) is the WMAP upper cosmic age constraint, where it is assumed that the age limit is not correlated with the values of the Hubble constant within the range allowed by WMAP. Finally, also shown (solid curve) is the inferred lower bound on $h$ as a function of $w$ estimated by extrapolating WMAP plots. The constraints on $w$ derived with and without this extra constraint are obtained at the points B, and A respectively.

of $^4He$ cannot begin until sufficient $D$ is produced so that further reactions processing $D$ to $He$ can take place.

The greater the density of protons and neutrons, the more efficiently, and the earlier $^4He$ is produced. Thus the remnant $^4He$ abundance is a monotonically increasing function of $\Omega_B$. Similarly the more efficiently $^4He$ is produced, the more efficiently $D$ and $^3He$ are burned to produce it, and thus these remnant abundances are monotonically decreasing functions of the baryon density.

The finer the ability to empirically infer the primordial abundances on the basis of observations, the greater the ability to uncover some small deviation from the predictions. Over the past five years, two different sets of observations have threatened, at least in some people’s minds, to overturn the simplest BBN model predictions. I believe it is fair to say that most people have accepted that the first threat was overblown. The concerns about the second have only recently subsided.

i. Primordial Deuterium: As noted above, the production of primordial deuterium during BBN is a monotonically decreasing function of the baryon density simply because the greater this density the more efficiently protons and neutrons get processed to helium, and deuterium, as an intermediary in this reactions set, is thus also more efficiently processed at the same time. The problem with inferring the primordial deuterium abundance by using present day measurements of deuterium abundances in the solar system, for example, is that deuterium is highly processed (i.e. destroyed) in stars, and no one has a good enough model for galactic chemical evolution to work backwards from the observed abundances in order to adequately constrain deuterium at a level where this constraint could significantly test BBN estimates.
Five years ago, the situation regarding deuterium as a probe of BBN changed dramatically, when David Tytler and Scott Burles convincingly measured the deuterium fraction in high redshift hydrogen clouds that absorb light from even higher redshift quasars. Because these clouds are at high redshift, before significant star formation has occurred, little post BBN deuterium processing is thought to have taken place, and thus the measured value gives a reasonable handle on the primordial BBN abundance. The best measured system (Burles and Tytler (1998)) yields a deuterium to hydrogen fraction of

\[ \frac{D}{H} = (3.3 \pm 0.8) \times 10^{-5} \quad (2\sigma) \quad (2.4) \]

This, in turn, leads to a constraint on the baryon fraction of the Universe, via standard BBN,

\[ \Omega_B h^2 = 0.02 \pm 0.002 \quad (2\sigma) \quad (2.5) \]

where the quoted uncertainty is dominated by the observational uncertainty in the D/H ratio, and where \( H_0 = 100h \). Thus, taken at face value, we now know the baryon density in the universe today to an accuracy of about 10%!

When first quoted, this result sent shock waves through some of the BBN community, because this value of \( \Omega_B \) is only consistent if the primordial helium fraction (by mass) is greater than about 24.5%. However, a number of previous studies had claimed an upper limit well below this value. However, recent studies, for example, place an upper limit on the primordial helium fraction closer to 25%.

\[ \frac{^3\text{He}}{\text{H}} = (1.1 \pm 0.2) \times 10^{-5}, \quad \text{which in turn implies} \quad \Omega_B h^2 = 0.02 \]

Thus, all data is now consistent with the assumption that the Burles and Tytler limit on \( \Omega_B h^2 \) is correct, adding further confidence in the predictions of BBN. Taking the range for \( H_0 \) given earlier, one derives the constraint on \( \Omega_B \) of

\[ \Omega_B = 0.045 \pm 0.15 \quad (2.6) \]

Note that the measured baryon density is a factor of 5-10 greater than the measured density of luminous matter in the Universe. Clearly, most baryons are dark. The next question is, can these dark baryons account for all gravitating matter in the Universe?

2.3.2. \( \Omega_{\text{matter}} \)

If one is interested in measuring gravitating matter, the best way to measure it is using gravity. Indeed, one can measure the mass of the Sun by using Newton’s relation for the velocity of objects in roughly circular orbits, \( v^2 = GM/r \). In the 1970’s, this
technique was used to attempt to measure the mass of our galaxy. Our Sun orbits around the outer edge of our galaxy with a velocity of approximately 220 km/s, at a radius of approximately 8 kpc, implying a mass inside its orbit of approximately $10^8$ solar masses, in good agreement with mass estimates based on the total number of stars in our galaxy. However, when test objects, including globular clusters, gas clouds, and satellite galaxies that orbit at distances of up to 10 times the distance from the galactic center, far outside the luminous region, instead of falling off, velocities remain roughly constant. Unless gravity is changing, this implies a total mass that increases with radius from the center of the galaxy, thus implying at least 90% of the mass of our galaxy is dark. What is more remarkable is that a similar behavior is observed in almost all spiral galaxies.

A mass that grows linearly would derive from a density distribution (assuming sphericity) that falls like $1/r^2$. Interestingly enough, if one assumes a collisionless gas with isotropic initial velocity distribution $<v^2>\approx$ constant, then its equation of state is given by

$$p(r) = \rho(r)\sigma^2 = \rho(r) <(v_x - \bar{v}_x)^2>$$  \hspace{1cm} (2.7)

Then if one imposes the condition of hydrostatic equilibrium on the system, with pressure balancing gravity,

$$-\frac{dp}{dr} = \frac{GM(r)}{r^2} \rho(r)$$  \hspace{1cm} (2.8)

and solves this equation in the limit $r- \to \infty$ one finds

$$\rho = \frac{\sigma^2}{2\pi r^2 G}$$  \hspace{1cm} (2.9)

This configuration, called an isothermal sphere, involving gravitational collapse of collisionless particles strongly suggests that the dark matter does not interact strongly or electromagnetically. In addition, estimating the total dark matter around galaxies implies a lower bound of $\Omega_m > O(0.1)$, which exceeds the total baryonic matter density. It is for this reason that cosmologists were initially driven to consider exotic non-baryonic dark matter.

The next question, of course, is how much dark matter is there out there? Perhaps the second greatest change in cosmological prejudice in the past decade relates to the inferred total abundance of matter in the Universe. Because of the great intellectual attraction Inflation as a mechanism to solve the so-called Horizon and Flatness problems in the Universe, it is fair to say that most cosmologists, and essentially all particle theorists had implicitly assumed that the Universe is flat, and thus that the density of dark matter around galaxies and clusters of galaxies was sufficient to yield $\Omega = 1$. Over the past decade it became more and more difficult to defend this viewpoint against an increasing number of observations that suggested this was not, in fact, the case in the Universe in which we live.

The earliest holes in this picture arose from measurements of galaxy clustering on large scales. The growth of structure in the Universe, if gravity is responsible for such growth, provides an excellent probe of the universal mass density, based largely on issues associated with causality alone. The basic idea is the following: If primordial density fluctuations have no preferred scale, then one can express their Fourier transform as a simple power of the wavenumber $k$. At the same time, if this power is much greater than unity, density fluctuations will blow up for large wavenumber, or small wavelength,
and too many primordial black holes will be created. If the power is much less than unity, then fluctuations on large scales (small wavenumbers) will be inconsistent with the observed isotropy of the Cosmic Microwave Background radiation. Thus, we expect the exponent, \( n \), to be near one, and inflationary models happen to predict precisely this behavior.

The primordial power spectrum, however, is not what we observe today, as density fluctuations can be affected by causal microphysical processes once the scale of these fluctuations is inside the horizon scale—the distance over which light can have travelled between \( t=0 \) and the time in question. One can show that in an expanding universe, as long as the dominant form of energy resides in radiation, gravity is ineffective at causing the growth of density fluctuations. In fact, such primordial fluctuations in baryons will be damped out due to their coupling to the radiation gas. Once the universe becomes matter dominated, however, primordial fluctuations on scales smaller than the horizon size can begin to grow.

These arguments suggest that an initial power law spectrum of fluctuations will “turn over” as shown in Figure 12 for large wavenumbers which entered inside the causal horizon during the early period of radiation domination in the Universe. By exploring the nature of the clustering of galaxies today over different scales, including measurements of the two point correlation function of galaxies, the angular correlation of galaxies across the sky on different scales, etc, one can hope to probe the location of this turn-around, and from that probe the time, and thus the scale which first entered the horizon when the universe became matter dominated. Clearly this time will depend upon the ratio of matter to radiation in the Universe today (if this ratio is increased, then matter, whose density decreases at a slower rate than radiation as the universe expands, will begin to dominate the expansion at an earlier time, and vice versa. In turn, knowing this ratio today gives us a handle on \( \Omega_{\text{matter}} \).

Making the assumption that dark matter dominates on large scales, and moreover that the dark matter is cold (i.e. became non-relativistic when the temperature of the Universe was less than about a keV), fits to the two point correlation function of galaxies on large scales yielded (Peacock and Dodds (1996), Liddle et al (1996)):

\[
\Omega_M h = 0.2 - 0.3 \tag{2.10}
\]

Unless \( h \) was absurdly small, this would imply that \( \Omega_M \) is substantially less than 1.

New data from the Sloan and 2DF surveys refine this limit further, with reported values of (Hawkins et al (2003), Dodelson et al (2002))
\[ \Omega_M = 0.23 \pm 0.09 (2DF) \]  
\[ \Omega_M h \approx 0.14^{+0.11}_{-0.06} \text{ (Sloan)} \]  

The second nail in the coffin arose when observations of the evolution of large scale structure as a function of redshift began to be made. Bahcall and collaborators (Bahcall et al (1997)) argued strongly that evidence for any large clusters at high redshift would argue strongly against a flat cold dark matter dominated universe, because in such a universe structure continues to evolve with redshift up to the present time on large scales, so that in order to be consistent with the observed structures at low redshift, far less structure should be observed at high redshift. Claims were made that an upper limit \( \Omega_B \leq 0.5 \) could be obtained by such analyses.

A number of authors have questioned the systematics inherent in the early claims, but it is certainly clear that there appears to be more structure at high redshift than one would naively expect in a flat matter dominated universe. Future studies of X-ray clusters, and use of the Sunyaev-Zeldovich effect to measure cluster properties should be able to yield measurements which will allow a fine-scale distinction not just between models with different overall dark matter densities, but also models with the same overall value of \( \Omega \) and different equations of state for the dominant energy (Haiman et al (2001)).

One of the best overall constraints on the total density of clustered matter in the universe comes from the combination of X-Ray measurements of clusters with large hydrodynamic simulations. The idea is straightforward. A measurement of both the temperature and luminosity of the X-Rays coming from hot gas which dominates the total baryon fraction in clusters can be inverted, under the assumption of hydrostatic equilibrium of the gas in clusters, to obtain the underlying gravitational potential of these systems. In particular the ratio of baryon to total mass of these systems can be derived. Employing the constraint on the total baryon density of the Universe coming from BBN, and assuming that galaxy clusters provide a good mean estimate of the total clustered mass in the Universe, one can then arrive at an allowed range for the total mass density in the Universe (White et al (1993), Krauss (1998), Evrard (1997)).

Many of the initial systematic uncertainties in this analysis having to do with cluster modelling have now been dealt with by better observations, and better simulations (i.e. see Mohr et al (2000)), so that now a combination of BBN and cluster measurements yields:

\[ \Omega_M = 0.35 \pm 0.1 \text{ (2}\sigma) \]  

Combining these results, one derives the constraint:

\[ \Omega_M \approx 0.3 \pm 0.05 \text{ (2}\sigma) \]  

2.3.3. Equation of State of Dominant Energy:

The above estimate for \( \Omega_M \) brings the discussion of cosmological parameters full circle, with consistency obtained for a flat 13 billion year old universe, but not one dominated by matter. As noted previously, a cosmological constant dominated universe with \( \Omega_M = 0.3 \) has an age which nicely fits in the best-fit range. However, based on the data discussed thus far, except for the CMB data which is consistent with a flat universe dominated by dark energy, there was no direct evidence that the dark energy necessary to result in a flat universe actually has the equation of state appropriate for a vacuum energy. Direct motivation for the possibility that the dominant energy driving the expansion of the Universe violates the Strong Energy Condition actually came somewhat earlier, in 1998, from two
Table 2. Cosmological Parameters

| Parameter | Allowed range | Formal Conf. Level (where approp.) |
|-----------|---------------|-----------------------------------|
| $H_0$     | $70 \pm 5$    | $2\sigma$                         |
| $t_0$     | $13.7 \pm 0.2$ Gy$	au$ | $2\sigma$                         |
| $\Omega_B$ | $0.02 \pm 0.004$ | $2\sigma$                         |
| $\Omega_M$ | $0.3 \pm 0.1$ | $2\sigma$                         |
| $\Omega_{TOT}$ | $1.02 \pm 0.04$ | $2\sigma$                         |
| $\Omega_X$ | $0.7 \pm 0.1$ | $2\sigma$                         |
| $\omega$  | $-1.2 - -0.8$ | $2\sigma$                         |

different sets of observations of distant Type 1a Supernovae. In measuring the distance-redshift relation (Perlmutter et al (1999), Schmidt et al (1998)) these groups both came to the same, surprising conclusion: the expansion of the Universe seems to be accelerating. This is only possible if the dominant energy is ”cosmological-constant-like”, namely if $\omega < -0.3$ (recall that $\omega = -1$ for a cosmological constant).

Note, as I have described, that the CMB data combined with supernova measurements now favor a range $-1.2 < w < -0.8$. In order to try and determine if the dominant dark energy does in fact differ significantly from a static vacuum energy—as for example may occur if some background field that is dynamically evolving (see next section) is dominating the expansion energy at the moment—one can hope to search for deviations from the distance-redshift relation for a cosmological constant-dominated universe. To date, none have been observed. Either other measurements, such as galaxy cluster evolution observations, or space-based SN observations would be required to further tighten the constraint.

2.4. Summary

I list the overall constraints on cosmological parameters discussed above in the table below. It is worth stressing how completely remarkable the present situation is. After 20 years, we now have the first direct evidence that the Universe might be flat, but we also have definitive evidence that there is not enough matter, including dark matter, to make it so. We seem to be forced to accept the possibility that some weird form of dark energy is the dominant stuff in the Universe. It is fair to say that this situation is more mysterious, and thus more exciting, than anyone had a right to expect it to be.

3. Dark Matter and Dark Energy: A Particle Physics Perspective

3.1. Dark Matter

The possible existence of non-baryonic dark matter should not come as a surprise. After all, while normal matter is all we are familiar with, by number baryons are an almost insignificant fraction of the universe. There is one baryon per billion photons in the CMB, for example. Moreover, the CMB remained hidden until 1965, even though there is nothing more visible than electromagnetic radiation!

Thus, it is easy to imagine how some particles could have been created in the early universe with a remnant abundance far bigger than baryons, and which could still remain unobserved. In fact, virtually every single extension of the standard model of particle
physics predicts natural dark matter candidates, and as we shall see, even the standard model itself includes particles which could have been fine dark matter candidates!! The surprise, in this sense, would have been if no dark non-baryonic matter were discovered.

There are three different mechanisms by which elementary particle dark matter can be created. Some elementary particles are either:

1. Born Dark!

2. Achieve Dark Matter-dom!

3. Have Dark Matter-dom thrust upon them!

1. There are two examples of the first type: light neutrinos and monopoles. Neutrinos were the first non-baryonic dark matter candidate proposed because we know there is a cosmic neutrino background of about 100 neutrinos/cc permeating space, just as there is a cosmic microwave background of electromagnetic radiation. Neutrinos were present in thermal equilibrium in the early hot-dense phase of the universal expansion with an abundance, per helicity state of:

$$\zeta(3)\frac{37^3}{4\pi^2}$$

where \(\zeta\) is the Riemann zeta function.

One can solve Boltzmann’s equation for neutrinos to determine when they go out of thermal equilibrium. A rough approximation is when their weak interaction rate is smaller than the expansion rate. Performing such a calculation for light neutrinos yields a decoupling temperature of 2 MeV. Since electrons and positrons annihilate with each other when the universe drops below this temperature, and these particles dump their energy and entropy into photons, the remnant neutrino temperature is reduced compared to photons by a factor of \((4/11)^{1/3}\), which can be calculated by considering the number of helicity states in equilibrium in the radiation gas both before this entropy is dumped and afterwards. Plugging in this number density today, one finds that if light neutrinos have a mass of approximately 10 eV, they could account for all the dark matter in the universe.

As you can see, almost none of this depends upon detailed interactions of neutrinos—they just have to be initially part of the heat bath, then decouple. Then, just by existing in the early universe, and then having a small mass, they would dominate the universe today! The same is true for magnetic monopoles, which can be produced as topological defects in the early universe. If a phase transition occurs in which Grand Unified gauge group breaks to yield a U(1) subgroup, then roughly one defect will form per horizon volume at that time. Again, for the appropriate mass range, and assuming inflation does not occur after this transition, then monopoles could ultimately dominate the mass density of the universe today.

2. Neutrinos also are the prototypes for the second kind of dark matter, the dark matter achievers. In this case, the particles in question would be ultra-heavy neutrinos, greater than about 1 GeV. These were the prototypical Weakly Interacting Dark Matter (WIMP) candidates.

In the case of heavy neutrinos, the weak interactions of these particles causes them to decouple from the heat bath only once the temperature has fallen far below their
During the intervening period, neutrinos and antineutrinos annihilate, suppressing their number density relative to photons by a factor $\exp(-M/T)$. In this case the details of their interactions are extremely important, because these details determine their decoupling temperature and therefore their remnant abundance.

For WIMPs one finds a relatively general relation based on incomplete annihilations in an initial thermal population:

$$\Omega_X h^2 \approx 10^{-37} \text{cm}^2 < \sigma_{\text{ann}} v >$$

Considering a heavy Dirac neutrino with standard weak interactions, one finds that a 2 GeV neutrino would just close the Universe today. Interestingly, since the annihilation cross section increases with neutrino mass, heavier neutrinos would contribute a smaller relic density today.

As a result of experiments at LEP, and also searches for heavy cosmic neutrinos we know that no such heavy WIMP neutrinos exist. Moreover, there was no reason to expect them to, as there is nothing special about the GeV mass range for neutrinos in the standard model. However, if we extend the standard model to incorporate low energy supersymmetry breaking in order to attempt to resolve the hierarchy problem, then one expects the masses of supersymmetric partners of ordinary matter will lie near the weak scale. The lightest supersymmetric partner (LSP) is generally stable in most such models, and since it interacts with ordinary matter via the exchange of supersymmetric particles whose mass is on the order of the weak scale, the LSP is a natural version of a heavy neutrino WIMP! Hundreds of calculations have been performed over the years, and there is significant phase space in supersymmetric models for WIMP dark matter, having densities $\Omega \approx 0.1 - 0.3$.

What makes WIMPs so interesting is that because their remnant abundance is determined by their annihilation cross section, one can use crossing symmetry to get a direct relation between this quantity, which determines their relic abundance, and their scattering cross section with matter. As a result of this, one can determine that these WIMPs may be detectable in direct detection experiments, as you shall read in this volume. But also because of this, it is actually good news that the density of WIMPs is smaller than we had previously imagined it to be. For both WIMPs, and for the other well motivated cold dark matter candidate that I have not yet discussed, axions, one can write down a general relation:

$$\sigma_{\text{detection}} \approx \frac{1}{\Omega_{\text{DM}}}$$

The reasons for this are different for each candidate. For WIMPs it is obvious. Because remnant abundance decreases as the annihilation cross section increases, and because of the crossing symmetry relation between annihilation and scattering cross sections, as the WIMP abundance decreases, its scattering cross section generally increases.

Astute experimentalists may argue that this is a scam, because as the WIMP (axion) density decreases, the flux on Earth also decreases, so even if there are larger cross sections, the event rate will not change! However, this is wrong. Until the density decreases to the point (below about $\Omega_x < 0.1$ ) when WIMPs (axions) do not have sufficient densities to account for all galactic halo dark matter, it is natural to assume that their galactic density is given by the halo density. Just because their overall cosmic density is insufficient to close the universe, this need not imply that their flux on earth is reduced.
3. As advertised, axions are an example of the third class of ‘forced’ dark matter candidates. Axions are pseudo-goldstone bosons that get a very small mass due to QCD effects in a way which is associated with the solution of the Strong CP Problem. Because they are goldstone bosons, axion fields can be represented as an angular field. In spite of their very small mass, which would mean that any axions that are thermally produced in the early universe would provide a negligibly small contribution to the energy density today, a non-thermal production mechanism changes everything.

At early times their potential (considered as a function of an angular variable which can be taken to go from $-\pi$ to $\pi$) changes, as seen in figure 13.

In the former case, no energy is stored in the axion field. However, once the axion gets a mass, energy is stored in the axion field, which then dynamically rolls to the bottom of its potential. However, the time it takes to begin rolling is inversely proportional to the curvature of its potential, and is thus inversely proportional to the axion mass. Thus, the smaller the axion mass, the longer the energy gets stored before it begins to redshift and the greater the remnant axion density. One finds that for an axion with mass approx. $10^{-5}$ eV, the axion density can be naturally in the range of $\Omega = 1$.

Axions too are detectable, in principle, because they have a coupling to two photons. Thus, even though their coupling is extremely small if the symmetry breaking scale associated with their existence is large, they could coherently couple to a large magnetic field within some volume, converting into photons with a frequency equal to the axion mass. Experiments designed to detect such cosmic axion-photon conversion are currently underway.

By the way, because of this non-thermal production mechanism, axions too share with WIMPs the fact that the smaller their cosmic density today, as long as it is larger than the galactic halo density, the larger the cosmic axion interaction rate in detectors would be. This is because axion couplings are inversely proportional to the axion mass.

### 3.2. Dark Energy

The equation of state for dark energy that appears to be favored by the existing data is $w = -1$, which is precisely that predicted for ‘vacuum’ energy, which is in turn precisely that of a cosmological constant originally introduced by Einstein into his equations when he thought the universe was static.

Once relativity and quantum mechanics were combined, the existence of vacuum energy was in some sense inevitable, as zero point energy is inevitably associated with quantum mechanical ground state configurations. Indeed, in the current picture magnitude of the vacuum energy associated with a cosmological constant which would be required by the present data is remarkably small. After all, if it is quantum gravity at the Planck scale that cuts off the magnitude of quantum fluctuations, then the natural magnitude of the vacuum energy density would be, by dimensional analysis, $M_P^4$.

To get a sense of experimentally how remarkably small the energy density associated with the dark energy today is by comparison, consider the following experimental proposal:
"To see what is in front of one’s nose requires a constant struggle"

George Orwell

What, you may ask, does this have to do with the topic at hand. Plenty, I claim. For it reminds us that we can put remarkably stringent limits on certain quantities by using macroscopic amounts of material. In particular, it harkens back to another famous quotation, this time from Maurice Goldhaber, who put one of the first limits on proton decay by declaring that if the proton had a lifetime less than about $10^{17}$ years, “You could feel it in your bones!”. By this he meant that proton decays in our body would be so frequent that we would die from the radiation exposure.

In this spirit we can perform a similar experiment. Look at the end of your nose. Now, in a universe dominated by a cosmological constant, space begins to expand exponentially. One can calculate than for distances separated by larger than an amount $R > M_{Pl}/3\Lambda^{1/2}$, points will have a relative velocity exceeding that of light, and thus will remain out of causal contact. Thus, the fact that you can see the end of your nose implies a bound $\Lambda < 10^{-68} M_{Pl}^4$.

Of course, the fact that we can see distant galaxies gives us an even stronger bound. And, the fact that the cosmological constant affects dynamics on larger scales no more than it is claimed to by the present observations gives a bound $\Lambda < 10^{-123} M_{Pl}^4$. What makes this small number so hard to understand, in a cosmological context is not merely the “naturalness” problem of which particle physicists are aware, but rather, if this has been constant over cosmological time, this is the first time in the history of the universe when the energy density in a cosmological constant is comparable to the energy density of matter and radiation! It is for this reason that some cosmologists are driven to the idea that what is being observed is not really a cosmological constant, but something perhaps more exotic (e.g. Caldwell et al (1998)).

There is a problem, from my point of view, however, with all of these proposals, which is why I have publicly bet my house on the fact that observers are bound to measure $w = 1$ when all is said and done. The reasoning is quite simple. One can imagine a background scalar field, $\phi$, rolling down to the minimum of its potential. If it has not yet settled at its minimum, then the equation of state for the field is given by:

$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \quad (3.18)$$

Since the kinetic energy of the scalar field as it rolls in the potential gives a positive contribution to the pressure, any rolling implies $w > -1$.

At some level, most models of a non-cosmological constant type of dark energy rely on such a mechanism to produce equations of state close to, but not equal to that of a cosmological constant. The problem is that in order to have a curvature so small so that the field has not yet reached the minimum of its potential, one generally is required to have extremely small masses associated with the field. But beyond this fine tuning problem, there are three problems I see with this:

(i) if the field potential can be fine tuned to take $10^{10}$ years to begin to roll down its potential, then why not imagine a field that takes $10^{100}$ years to do so? Indeed, it seems to me that for such a field to just begin rolling now is HIGHLY coincidental and contrived. If this were not the case, the field, stuck at a non-zero value of its potential now, would be observationally indistinguishable from a cosmological constant.

(ii) What about the cosmological constant problem? If this wierd field is to dominate the energy of empty space today, then somehow vacuum fluctuations must give a yet
much smaller, or zero contribution. But this requires solving the cosmological constant problem anyway.

(iii) Finally, we should remember that almost all quantum field theories PREDICT a non-zero cosmological constant. The only problem is making it small enough to agree with observations. Thus, a cosmological constant is, in a sense, the most natural candidate for dark energy. All we have to do is figure out why...

4. Geometry, Destiny, and the Future

Once we accept that we live in a universe dominated by dark energy, everything about the way we think about cosmology changes. In the first place, Geometry and Destiny are no longer linked. Previously, the holy grail of cosmology involved determining the density parameter \( \Omega \), because this was tantamount to determining the ultimate future of our universe. Now, once we accept the possibility of a non-zero cosmological constant, we must also accept the fact that any universe, open, closed, or flat, can either expand forever, or reverse the present expansion and end in a big crunch (Krauss and Turner (1999)).

The mathematical basis of this is described simply. Einstein’s equations imply, for an isotropic and homogeneous Universe, the following evolution equations for the cosmic scale factor, \( R(t) \):

\[
H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho_{\text{TOT}} - \frac{k}{R^2} \tag{4.19}
\]

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i) \tag{4.20}
\]

where \( k \) is the signature of the 3-curvature, the pressure in component \( i \) is related to the energy density by \( p_i = w_i \rho_i \) and the total energy density \( \rho_{\text{TOT}} = \sum_i \rho_i \). The evolution of the energy density in component \( i \) is determined by

\[
\frac{d\rho_i}{\rho_i} = -3(1 + w_i) \frac{dR}{R} \Rightarrow \rho_i \propto R^{-3(1+w_i)} \tag{4.21}
\]

All forms of normal matter satisfy the strong-energy condition, \((\rho_i + 3p_i) = \rho_i (1 + 3w_i) > 0\), and so if the Universe is comprised of normal matter, the expansion of the Universe always decelerates, cf. Eq. (4.20). Also, since \( \rho \) is positive for normal matter, the first equation implies that \( \dot{R}/R \) remains positive and non-zero if \( k \leq 0 \), and thus the Universe expands forever. Equation (4.21) and the strong-energy condition imply that \( \rho_i \) decreases more rapidly than \( R^{-2} \). Thus, for \( k > 0 \) there is necessarily a turning point with \( H = 0 \) and \( \dot{R} < 0 \), and the Universe must ultimately recollapse. Geometry determines destiny.

However, a cosmological constant violates the strong-energy condition, completely obviating the logic of the above argument. Recalling that \( \rho_{\Lambda} = -\rho_{\Lambda} \) for a cosmological term, and that \( \rho_{M} = 0 \) for matter, the above equations become,

\[
H^2 = \frac{8\pi G}{3} (\rho_{M} + \rho_{\Lambda}) - \frac{k}{R^2} \tag{4.22}
\]

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} (\rho_{M} - 2\rho_{\Lambda}) \tag{4.23}
\]

Since \( \rho_{\Lambda} = \text{constant} \), while \( \rho_{M} \propto R^{-3} \), even if \( k > 0 \), as long as \( H > 0 \) when \( \rho_{\Lambda} \) comes to dominate the expansion, it will remain positive forever, and as is well known, the expansion will ultimately accelerate, \( R(t) \to e^{Ht} \) with \( H = \sqrt{8\pi G \rho_{\Lambda}/3} \).
One conventionally defines the scaled energy density $\Omega \equiv \rho_{\text{TOT}}/\rho_{\text{crit}} = 8\pi G \rho/3H^2$, so that $\Omega - 1 = k/H^2R^2$. Thus the sign of $k$ is determined by whether $\Omega$ is greater than or less than 1. In this way, a measurement of $\Omega$ at any epoch—including the present—determines the geometry of the Universe. However, we can no longer claim that the magnitude of $\Omega$ uniquely determines the fate of the Universe.

It is interesting to determine how small a cosmological constant could be at the present time and still stop the eventual collapse of a closed Universe. For a closed, matter-only Universe, the scale factor at turnaround is

$$R/R_0 = \frac{\Omega_0}{\Omega_0 - 1} \quad \text{(4.24)}$$

While all the evidence today suggests that $\Omega_0 \leq 1$, existing uncertainties could allow $\Omega_0$ to be as large say as 1.1. For $\Omega_0 = 1.1$ the scale factor at turnaround is $11R_0$. Since the density of matter decreases as $R^{-3}$, this means that an energy density in a cosmological term as small as $1/10000th$ the present matter density will come to dominate the expansion before turnaround and prevent forever recollapse. A cosmological constant this small, corresponding to $\Omega_\Lambda \sim 0.001$, is completely undetectable by present, or foreseeable observational probes.

Alternatively, it may seem that if we can unambiguously determine that $k < 0$ then we are assured the Universe will expand forever. However, this is the case only as long as the cosmological constant is positive. Since we have no theory for a cosmological constant, there is no reason to suppose that this must be the case. When the cosmological constant is negative, the energy density associated with the vacuum is constant and negative. In this case, from Eqs. (4.22,4.23), one can see that not only is the ultimate expansion guaranteed to decelerate, but recollapse is also inevitable, no matter how small the absolute value of $\Omega_\Lambda$ is.

Finally, what if we indeed ultimately verify $w = 1$ at the present time, as current observations suggest? Even in this case we are not guaranteed an eternal expansion. As I have described scalar field which is not at the minimum of its potential will, as long as the age of the Universe is small compared to the characteristic time it takes for the field to evolve in its potential, mimic a cosmological term in Einstein’s equations. Until the field evolves to its ultimate minimum, we cannot derive the asymptotic solution of these equations in order to determine our destiny.

As Michael Turner and I have demonstrated, there is no set of cosmological measurements, no matter how precise, that will allow us to determine the ultimate future of the Universe. In order to do so, we would require a theory of everything.

On the other hand, if our universe is in fact dominated by a cosmological constant, the future for life is rather bleak (Krauss and Starkman (2000)). Distant galaxies will soon blink out of sight, and the Universe will become cold and dark, and uninhabitable.

This bleak picture may seem depressing, but the flip side of all the above is that we live in exciting times now, when mysteries abound. We should enjoy our brief moment in the Sun.

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