Survival-Supervised Topic Modeling with Anchor Words: Characterizing Pancreatitis Outcomes

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Abstract
We introduce a new approach for topic modeling that is supervised by survival analysis. Specifically, we build on recent work on unsupervised topic modeling with so-called anchor words by providing supervision through an elastic-net regularized Cox proportional hazards model. In short, an anchor word being present in a document provides strong indication that the document is partially about a specific topic. For example, by seeing “gallstones” in a document, we are fairly certain that the document is partially about medicine. Our proposed method alternates between learning a topic model and learning a survival model to find a local minimum of a block convex optimization problem. We apply our proposed approach to predicting how long patients with pancreatitis admitted to an intensive care unit (ICU) will stay in the ICU. Our approach is as accurate as the best of a variety of baselines while being more interpretable than any of the baselines.

1 Introduction
Health care is replete with problems of understanding how much time there is before a critical event happens, such as how long a patient has to live or how long a patient will stay in a hospital intensive care unit (ICU). These time durations can be modeled by numerous survival analysis techniques. Especially as survival analysis models learned can inform costly interventions, ensuring that they are clinically interpretable is essential. At the same time, these models now have to cope with an enormous number of measurements collected per patient in electronic health records for which we do not fully understand how all the measurements relate. In this paper, we aim to address the twin objectives of learning how measurements relate in the form of a topic model, and learning how the topics can make survival predictions at the patient level.

We phrase the problem setup in terms of predicting how long pancreatitis patients admitted to an ICU will stay in the ICU. We have training data of $n$ patients. For each patient, we know how many times each of $d$ words appears. The dictionary of words is pre-specified based on clinical events derived from electronic health record data. We denote $X_{w,i}$ to be the number of times word $w \in \{1, \ldots, d\}$ appears for patient $i \in \{1, \ldots, n\}$. Viewing $X$ as a $d$-by-$n$ matrix, the $i$-th column of $X$ can be thought of as the feature vector for the $i$-th patient. As for the training label for the $i$-th patient, we have two recordings: $R_i \in \{0, 1\}$ which indicates whether we know the ICU length of stay for the patient, and $Y_i \in \mathbb{R}_+$ is the ICU length of stay for the patient if $R_i = 1$, or the “censoring time” if $R_i = 0$. The censoring time gives a lower bound on how long the $i$-th patient was in the ICU.

Our goal is to discover topics for the $d$ words that help predict how long a pancreatitis patient newly admitted into the ICU will stay in the ICU. Note that an unsupervised topic model would not use any of the training labels, instead learning topics using only the word count matrix $X$. Meanwhile, in survival analysis, a standard approach would learn a survival model using all the patients’ feature vectors and labels but would not learn thematic structure in the different features, e.g., topics.

Our contributions are twofold:
- To jointly learn both a topic and a survival model, we extend a recently proposed unsupervised topic modeling approach using so-called anchor words [3, 4] to be supervised by an elastic-net regularized Cox proportional hazards model [11] (which generalizes the standard Cox proportional hazards model [6]). Anchor words, to be described in detail shortly, act as exemplar words...
for the topics. Our proposed method finds anchor words and then alternates between learning a topic model and learning a survival model, corresponding to finding a local minimum of a block convex optimization problem. We remark that how we combine topic modeling and survival analysis is similar to an existing approach \cite{8} that combines Latent Dirichlet Allocation (LDA) \cite{5} with the standard Cox proportional hazards model; however, the resulting algorithms are syntactically very different. We explain why topic modeling with anchor words is preferable to LDA.

- We apply our proposed method to predicting how long pancreatitis patients admitted to an ICU will stay in the ICU. Our method is as accurate as the best of a variety of baseline methods yet comes with new interpretative advantages. In particular, the best performing baselines either do not learn how different measurements relate, or do not learn clinically interpretable topics. In contrast, our proposed method simultaneously learns topics corresponding to different collections of events likely to have happened to patients, how presence of a topic contributes to lengthening or shortening a patient’s stay in the ICU, and what anchor words are for the topics. The anchor words found turn out to correspond to catheters used by physicians. Although our method is only competitive with and does not necessarily outperform the best of the baselines considered, we believe that among methods with similar performance, we should opt for the most interpretable.

2 Background: Topic Modeling with Anchor Words

The intuition behind anchor words is that some words are very strong indicators of specific topics in a document. For example, for text documents, if the word “gallstones” appears in a document, we can be fairly certain that the document is partially about medicine. Similarly, “401k” can be an indicator for finance, and “Oscar-winning” for movies. These are examples of anchor words. Importantly, not every document about medicine has to have the anchor word “gallstones” in it. In fact, anchor words could occur infrequently. In the context of characterizing outcomes of a specific disease like pancreatitis, the words correspond to specific measurements or events such as a patient’s lab measurements quantized to discrete values, whether a patient was previously put on a specific kind of intravenous fluid, etc. The topics consist of frequently co-occurring measurements or events.

Anchor word topic modeling has three properties that altogether make it preferable over a more standard topic modeling approach like LDA. First, we can find anchor words quickly. In particular, there is a fast, provably correct algorithm that finds an accurate approximation of the anchor words (see Algorithm 4 and Theorem 4.3 by Arora et al. \cite{2}). Second, we can check whether the anchor words have enough expressive power to explain the full dictionary of words. This property results from a key interpretation of anchor words: anchor words turn out to, in some sense, be proxies for their respective topics. Under the assumption that every topic we aim to learn has an anchor word, then there is a way to represent every word as a convex combination of the anchor words. As a result, we can efficiently check whether a word is representable by the anchor words found (the problem reduces to checking whether a point is inside a convex hull). If not, then it means that more anchor words are needed to explain the full dictionary, i.e., more topics are needed under the assumption that every topic has an anchor word. Alternatively it means that we can quickly identify words to remove from the dictionary if we do not want to use more topics. Lastly, anchor word topic modeling makes no assumption on the topics being uncorrelated, in contrast to LDA.

To build up to the formal definition of an anchor word, we establish some notation used in the rest of the paper. Topic models generally assume that each document (in our survival context, each document corresponds to a patient) contains a mixture of topics, and each topic has a distribution over words. If we assume that there are \( k \) topics, then words in document \( i \in \{1, \ldots, n\} \) are assumed to be sampled independently from the following distribution over words \( w \in \{1, \ldots, d\} \):

\[
M_{w,i} \equiv \mathbb{P}(\text{word } w | \text{ document } i) = \sum_{g=1}^{k} \mathbb{P}(\text{word } w | \text{ topic } g) \mathbb{P}(\text{topic } g | \text{ document } i).
\]

As the notation suggests, we can view topic modeling in terms of nonnegative matrices \( M \in \mathbb{R}_{+}^{d \times n} \), \( A \in \mathbb{R}_{+}^{k \times d} \), and \( W \in \mathbb{R}_{+}^{k \times n} \). The \( i \)-th column of \( M \) is document \( i \)'s word distribution. The \( g \)-th column of \( A \) is topic \( g \)'s word distribution. The \( i \)-th column of \( W \) is document \( i \)'s topic distribution. Then the equation above can be written as the nonnegative matrix factorization \( M = AW \). We do not observe \( M \). What we instead have is the word count matrix \( X \in \mathbb{R}_{+}^{d \times n} \) described in Section 1 where \( X_{w,i} \) is the number of times word \( w \) appears in document \( i \). Denoting \( m_i \) to be the number of words in the \( i \)-th document, then \( X_{w,i} \sim \text{Binomial}(m_i, M_{w,i}) \). Different topic models place different assumptions on matrices \( A \) and \( W \).
We now state our proposed algorithm. Given a pre-specified number of topics \( k \), we run Algorithm 4 by Arora et al. [2]. Since this algorithm is randomized, to make the result more stable (in fact, seemingly deterministic), we run it a pre-specified number of times, and pick whichever specific run has anchor words that have appeared the most across all the runs. Details are in the longer version of this paper.

**Definition 2.1.** An *anchor word* for a topic is defined as a word that has positive probability (within word-topic matrix \( A \)) of appearing only for that one specific topic. (For example, if topic 3 has word 5 as an anchor word, then in the row 5 of \( A \), the only nonzero entry appears in column 3.)

Anchor word topic modeling learns word-topic matrix \( A \) given word count matrix \( X \) under the assumption that \( A \) is deterministic and unknown, and columns of \( W \) are stochastically generated. The key assumption used in learning an anchor word topic model is that every topic that we learn has an anchor word. Supposing that there are \( k \) topics, then every topic \( g \in \{1, \ldots, k\} \) has an associated anchor word \( a_g \in \{1, \ldots, d\} \) (if a topic has multiple anchor words, we can arbitrarily choose one of them to use). Anchor word topic model learning proceeds by first finding what the anchor words are given word count matrix \( X \) using a greedy (albeit provably accurate) combinatorial search. Next, we determine how to represent every word in the dictionary in terms of the anchor words, which amounts to finding how to represent each word as a specific convex combination of the anchor words.

Lastly, a Bayes’ rule calculation can be used to estimate \( A \). For details, see the paper by Arora et al. [2].

3 Method

Following how Dawson and Kendziorski combine LDA with the standard Cox proportional hazards model to form the SURV-LDA model [8], we set the \( i \)-th training patient’s feature vector in the Cox proportional hazards model to be the estimated probabilities of each of the \( k \) topics appearing in the \( i \)-th patient/document. Put another way, this is a layered composition: the input word count matrix \( X \) goes through an anchor word topic modeling layer that outputs each patient’s topic proportions. These topic proportions across all patients are then treated as a single input to an elastic-net regularized Cox proportional hazards model layer that outputs the labels (\( R_i \)'s and \( Y_i \)'s for whether censoring happened and the survival/censoring times)\(^\dagger\).

We now state our proposed algorithm. Given a pre-specified number of topics \( k \), we first compute an estimate \( \hat{a}_1, \ldots, \hat{a}_k \) of the anchor words using the same algorithm as in unsupervised anchor word topic modeling\(^\dagger\). To describe the optimization problem to be numerically optimized next, we introduce some variables that we can compute based on the word count matrix \( X \). First, the \( i \)-th column of \( X \) corresponds to the \( i \)-th patient’s word counts. Let \( \bar{X}^{(i)} \in [0, 1]^d \) denote the \( i \)-th patient’s word counts normalized to sum to 1. From \( X \), we also compute the \( d \)-by-\( d \) word co-occurrence matrix \( Q \). Let \( Q_w \in [0, 1]^d \) denote the \( w \)-th row of \( Q \) normalized to sum to 1. Let \( \theta_{w,g} = \mathbb{P}(\text{topic } g \mid \text{word } w) \) and \( \beta \in \mathbb{R}^k \) be Cox regression coefficients. We numerically solve:

\[
\minimize_{\theta \in \mathbb{R}^d, \beta \in \mathbb{R}^k} \sum_{w=1}^d D_{Kt}(\bar{Q}_w) \sum_{g=1}^k \theta_{w,g} \bar{Q}_{a_g} + \sum_{i=1}^n R_i \left( -\beta^T \left( \bar{X}^{(i)} \right)^T \theta + \log \sum_{j=1}^n \exp(\beta^T \left( \bar{X}^{(j)} \right)^T \theta) \right) + \lambda \|\beta\|_1 + \frac{\lambda}{2} (1 - \alpha) \|\theta\|_2^2
\]

subject to: \( \sum_{g=1}^k \theta_{w,g} = 1 \) for each \( w \in \{1, \ldots, d\} \), \( \theta_{a_g,h} = 1 \) if \( g = h \) for each \( g, h \in \{1, \ldots, k\} \),

where \( 1 \{\cdot\} \) is the indicator function that is 1 when its argument holds and 0 otherwise, and constants \( \lambda > 0 \) and \( \alpha > 0 \) are hyperparameters controlling the elastic-net regularization on Cox regression coefficients \( \beta \). The first term in the objective function finds a representation of each word \( \bar{Q}_w \) as a convex combination \( \theta_w \) of the anchor words \( \bar{Q}_{a_1}, \ldots, \bar{Q}_{a_k} \). The second and third terms in the objective function correspond to the elastic-net regularized Cox partial log likelihood, where the \( i \)-th patient’s feature vector is set to the patient’s estimated proportion of words coming from each topic \( \sum_{g=1}^k \theta_{w,g} \bar{X}^{(i)} \theta \in [0, 1]^k \). Removal of these two survival-related terms from the objective function

\(^\dagger\)Despite this layered composition explanation, we remark that anchor word topic modeling does not readily fit into the framework of standard black box inference (e.g., autoencoding variational Bayes [10]) approaches due to its multiple-stage learning involving first solving a combinatorial optimization greedily. Separately, black box inference struggles to learn topic models; only recently has there been a method [1] developed that uses black box inference to learn LDA with carefully chosen optimization parameters and a Laplace approximation. Topic models compatible with black box inference are not yet as theoretically well-understood as anchor word topic modeling. Meanwhile SURV-LDA uses standard (non-black-box) variational EM [8].

\(^\dagger\)We run Algorithm 4 by Arora et al. [2]. Since this algorithm is randomized, to make the result more stable (in fact, seemingly deterministic), we run it a pre-specified number of times, and pick whichever specific run has anchor words that have appeared the most across all the runs. Details are in the longer version of this paper.

\(^\dagger\)Technical details for how to count word self co-occurrences and how to account for varying document lengths in forming \( Q \) are in the supplementary material Section 4.1 of Arora et al. [2].
With respect to interpretability, the results illustrate that SAW identifies clusters of outcome-relevant, clinical events and provides a level of risk associated with those clusters. Among thousands of highly co-linear events, SAW produces a digestible output that both matches clinical knowledge, e.g., catheter size and transfer as indicators of severity, and offers hypotheses about unforeseen relationships and substitutability. Clinical concepts of organ failure and hemoconcentration are known predictors of pancreatitis severity and are closely related to the anchor words with nonzero coefficients.

### 4 Experimental Results

For our experiments, we looked at patients with pancreatitis in the MIMIC III dataset [9] who required admission to the ICU, amounting to a total of 371 individuals. Features extracted include demographics, medications, billing codes, procedures, laboratory measurements, events recorded into charts, and vitals. Each record was placed into a 4-column format: patient id, time, event, and event value. Preprocessing details such as how words are chosen are given in the longer version of this paper. At a high-level, we use patient data up until they first enter ICU; any data afterward is not present in the word counts X. Words are derived from pairs (event, event value), with the event value discretized if it is continuous. Words too similar in frequency across different ICU lengths of stay are filtered out. We split the data into 75% training and 25% test. For the methods that we compare, 3-fold cross-validation within the training set is used to select the best algorithm parameters as to minimize root-mean-square error (RMSE) in predicted median ICU length of stay compared to the true length of stay. After finding these parameters we train each method on the full training set.

We compared our proposed supervised anchor word topic modeling method (abbreviated “SAW”) in Figure 1(a) with a two-stage approach that learns an unsupervised anchor word topic model first and then learns an elastic-net regularized Cox model without joint optimization (“USA W”). We also compare against survival analysis baselines that do not use anchor words: Kaplan Meier (“KM”), Cox proportional hazards (“Cox”), Aalen additive models (“Aalen”), and accelerated failure time models with a Weibull distribution (“AFT”). Because our task has many more features than patients (d \(\gg\) n), we used singular value decomposition (SVD) as a preprocessing step to produce a low-dimensional feature space for the last three methods (we remark that this low-dimensional representation can be thought of as a topic model that is not clinically interpretable). Separately for the Cox proportional hazards model, we also compare with both its lasso-regularized (“Lasso Cox”) and, more generally, elastic-net regularized (“EN Cox”) variants, both of which are clinically interpretable but do not learn topics. We were unable to acquire the survLDA code of Dawson and Kendziorski. We report test set RMSE’s, mean absolute errors (MAE’s), and c-indices in Figure 1(left), and anchor words (found by our proposed method SAW) for all three topics with nonzero \(\beta\) coefficient (out of a total of \(k = 5\) topics; as with other parameters, \(k\) was selected via cross-validation) along with a few of each topic’s most frequently occurring words in Figure 1(right).

The anchor words focus on catheters physicians used, with the ones associated with more severe illness, e.g., arterial lines and lower-gauge catheters, being part of the topic with a negative \(\beta\) coefficient, indicative of longer ICU stays. The five topics learned roughly typify: severe, non-severe, demographic indicators, laboratory indicators, and heart-related concepts. While these last two topics are interpretable, they were not deemed (sufficiently) predictive with \(\beta = 0\) coefficients. In terms of prediction accuracy metrics, our proposed method is competitive with (although not necessarily better than) the best-performing baselines (SVD AFT, Lasso Cox, EN Cox).

With respect to interpretability, the results illustrate that SAW identifies clusters of outcome-relevant, clinical events and provides a level of risk associated with those clusters. Among thousands of highly co-linear events, SAW produces a digestible output that both matches clinical knowledge, e.g., catheter size and transfer as indicators of severity, and offers hypotheses about unforeseen relationships and substitutability. Clinical concepts of organ failure and hemoconcentration are known predictors of pancreatitis severity and are closely related to the anchor words with nonzero \(\beta\).

| METHOD    | RMSE (DAYS) | MAE (DAYS) | c-INDEX |
|-----------|-------------|------------|---------|
| KM        | 14.4        | 7.4        |         |
| SVD COX   | 14.4        | 7.2        | 0.59    |
| SVD AALEN | 13.6        | 7.4        | 0.59    |
| SVD AFT   | 13.5        | 6.9        | 0.59    |
| LASSO COX | 13.5        | 6.8        | 0.63    |
| EN COX    | 13.5        | 6.8        | 0.63    |
| USAW      | 13.3        | 7.3        | 0.59    |
| SAW       | 13.3        | 6.8        | 0.59    |

Figure 1: (Left) Test set ICU length of stay RMSE and MAE, and c-index by method. (Right) A few top words in topics with nonzero \(\beta\) coefficients.
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