THE FORMATION OF URANUS AND NEPTUNE IN SOLID-RICH FEEDING ZONES: CONNECTING CHEMISTRY AND DYNAMICS

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ABSTRACT

The core accretion theory of planet formation has at least two fundamental problems explaining the origins of Uranus and Neptune: (1) dynamical times in the trans-Saturnian solar nebula are so long that core growth can take \( > 15 \) Myr, and (2) the onset of runaway gas accretion that begins when cores reach \( \sim 10 M_{\oplus} \) necessitates a sudden gas accretion cutoff just as Uranus and Neptune’s cores reach critical mass. Both problems may be resolved by allowing the ice giants to migrate outward after their formation in solid-rich feeding zones with planetesimal surface densities well above the minimum-mass solar nebula. We present new simulations of the formation of Uranus and Neptune in the solid-rich disk of Dodson-Robinson et al. (2009) using the initial semimajor axis distribution of the Nice model (Gomes et al. 2005; Morbidelli et al. 2005; Tsiganis et al. 2005), with one ice giant forming at 12 AU and the other at 15 AU. The innermost ice giant reaches its present mass after 3.8–4.0 Myr and the outermost after 5.3–6 Myr, a considerable time decrease from previous one-dimensional simulations (e.g. Pollack et al. 1996). The core masses stay subcritical, eliminating the need for a sudden gas accretion cutoff.

Our calculated carbon mass fractions of 22\% are in excellent agreement with the ice giant interior models of Podolak et al. (1995) and Marley et al. (1997). Based on the requirement that the ice giant-forming planetesimals contain \( > 10\% \) mass fractions of methane ice, we can reject any solar system formation model that initially places Uranus and Neptune inside of Saturn’s orbit. We also demonstrate that a large population of planetesimals must be present in both ice giant feeding zones throughout the lifetime of the gaseous nebula. This research marks a substantial step forward in connecting both the dynamical and chemical aspects of planet formation. Although we cannot say that the solid-rich solar nebula model of Dodson-Robinson et al. (2009) gives exactly the appropriate initial conditions for planet formation, rigorous chemical and dynamical tests have at least revealed it to be a viable model of the early solar system.

Subject headings: planetary formation; Uranus; Neptune; origin, Solar System

1. INTRODUCTION: STATEMENT OF THE PROBLEM AND PREVIOUS WORK ON ICE GIANT FORMATION

The canonical core accretion theory of planet formation, in which planetesimals collide to form solid cores which then destabilize the surrounding gas to accrete an atmosphere (Safronov 1969; Pollack et al. 1996), has at least two fundamental problems explaining the origins of Uranus and Neptune. First, dynamical times in the trans-Saturnian solar nebula are so long and solid surface densities \( \Sigma \) are so low \(< 1 \text{ g cm}^{-2} \) according to the assumed \( \Sigma \propto R^{-2} \) mass distribution (Pollack et al. 1996) that planet growth takes \( > 15 \) Myr, far longer than both observed and theoretical protostellar disk lifetimes (Haisch et al. 2001; Alexander et al. 2006). Second, runaway gas accretion begins when solid cores reach 10 to 15 \( M_{\oplus} \), requiring a sudden and complete gas accretion cutoff just as Uranus and Neptune reach their current masses. Pollack et al. (1996) pointed out these problems in their seminal paper on the viability of the core accretion theory. More recently, Benvenuto et al. (2009) showed that Uranus and Neptune could grow within a few Myr in a population of planetesimals with a distribution of radii between 30 and 100 km. However, planetesimals as small as 30 km are not consistent with the prevailing theory of planetesimal formation, based on the streaming instability, which produces planetesimals around 100 km and in some cases up to the radius of Ceres (457 km; Johansen et al. 2007).

Uranus and Neptune’s total masses, 14.5 and 17.2 \( M_{\oplus} \) respectively, place them squarely in the predicted critical mass range for nucleating an instability in the surrounding gas and accreting Jupiter’s mass or more in under 1000 years (Mizuno 1980; Papaloizou and Nelson 2005). The first challenge for theorists is to find a combination of the parameters that control core accretion—feeding zone location, ice inventory and planetesimal surface density—that leads to solid planet cores of \( > 14 M_{\oplus} \) that form within observed protostellar disk lifetimes and are subcritical with respect to the surrounding gas density. An ice giant formation theory should also account for the planets’ bulk composition, particularly their 20–50 \times \) solar tropospheric \( \text{C/H} \) ratios (Encrenaz 2005).
Treating feeding zone location as a free parameter creates the further challenge of moving Uranus and Neptune into their current orbits.

Two previous theories attempted to explain both the timely formation and subsequent orbital evolution of the ice giants. Thommes et al. (1999, 2002) proposed that Uranus and Neptune are failed gas giants that formed between Jupiter and Saturn. Jupiter scattered the ice giants into orbits with semimajor axes \( a > 15 \) AU once it reached runaway gas accretion, while interactions with planetesimals further forced the ice giants slowly outward. The “collisional damping scenario” was put forth by Goldreich et al. (2004a, 2004b). According to Goldreich et al., Uranus and Neptune formed \textit{in situ} from a dynamically cold planetesimal disk that also produced three other proto-ice giants. The protoplanets formed quickly despite long dynamical times because the planetesimal disk scale height fit within the Hill sphere (the protoplanet’s zone of gravitational dominance), leading to high solid accretion rates. Dynamical friction could no longer damp the eccentricities of the \( \sim 5 \) trans-Saturnian oligarchs once they attained a surface density comparable to the surrounding planetesimal disk. The oligarchs suffered close encounters and the resulting instability ejected all proto-ice giants but Uranus and Neptune.

The assumptions underlying the order-of-magnitude analysis in Goldreich et al. (2004a, 2004b) have ultimately proven unreliable. Levison and Morbidelli (2007) demonstrated that the collisional damping scenario cannot reproduce the current Solar System: rather than ejecting three of five ice giants, the trans-Saturnian protoplanets simply spread out and all planets were retained. Furthermore, the collisional damping scenario requires that oligarchs grow while planetesimals fragment to sizes \( \ll 1 \) km. Since low-velocity particles \((v < 10 \text{ cm s}^{-1})\) in the COLLIDE-2 microgravity experiment burrowed into the target material without producing ejecta (Colwell 2003), there is no reason planetesimals should fragment in the dynamically cold planetesimal disk required to produce Uranus and Neptune \textit{in situ}.

Thommes et al. (1999, 2002) “failed gas giant” model has substantial success reproducing the current Solar System and does not require finely tuned planetesimal behavior. Studies of planet formation in the 5-10 AU region demonstrate the efficiency of growing ice giant-sized cores between Jupiter and Saturn [Hubickyj et al. (2005), Dodson-Robinson et al. (2008)]. However, the compositions of Uranus and Neptune strongly indicate an origin in the trans-Saturnian solar nebula. Tropospheric abundances of methane show carbon enrichments of 20-50 times solar (Encrenaz 2005), and interior models find methane mass fractions of \( \sim 20\% \) (Marley et al. 1995; Podolak et al. 1995). The combined dynamical and chemical model of the solar nebula calculated by Dodson-Robinson et al. (2009) shows that the methane condensation front is beyond Saturn’s orbit during the first \( 5 \times 10^5 \) years of solar nebula evolution. Without methane ice present during the planetesimal-building epoch—which lasts only \( 5 \times 10^4 \) years according to Johansen et al. 2007—neither planet could obtain its methane-rich composition.

The \textit{Nice} model of planetary dynamics [Tsiganis et al. (2005), Gomes et al. (2005), Morbidelli et al. (2005)] uses initial conditions that place Uranus and Neptune initially in the methane ice-rich regions beyond 10 AU. In the \textit{Nice} model, Neptune and Uranus assume initial semimajor axes of \( \sim 12 \) and \( \sim 15 \) to 17 AU. When planetesimal perturbations pull Jupiter and Saturn across their 1:2 mean motion resonance (MMR), their eccentricities suddenly increase, forcing close encounters between all possible pairs of giant planets except Jupiter and Saturn. In about half of the simulations, Neptune is scattered across Uranus’ orbit, leapingfrogging to \( \sim 23 \) AU within a few \( 10^5 \) years. Slow outward migration due to interaction with a planetesimal disk pulls Uranus and Neptune into their current orbits over the course of \( \sim 40 \) Myr.

Although the \textit{Nice} model explains the current orbits of the giant planets, it is incomplete without an assessment of the planets’ ability to form in their predicted initial orbits. With high solid surface-density, methane-rich planetesimals, the protostellar disk model of Dodson-Robinson et al. (2009) contains promising initial conditions for ice giant formation between 12 and 15 AU. None of the three dynamical theories discussed—the \textit{Nice} model, the failed gas giant theory and the collisional damping scenario—treats the growth of the ice giants’ envelopes, a gap in the literature that this work is partly intended to fill. Verifying that \( \sim 10 - 15 M_\oplus \) solid cores can form is an important step—one which N-body simulations show is extremely difficult even in the inner nebula (McNeil et al. 2005; Chambers 2008)—but one also has to verify that the ice giant atmospheres stay under \( 10\% \) of the total planet mass and do not experience runaway growth.

In this paper, we demonstrate that Uranus and Neptune can form by core accretion, in the feeding zone approximation, in the trans-Saturnian solar nebula using the \textit{Nice} model initial semimajor axis distribution. In \textsection 2 we describe the numerical methods used in our experiments. In \textsection 3 we discuss the results of our core accretion simulations, focusing on formation timescale, accretion efficiency and solid/gas ratio. In \textsection 4 we discuss the strengths and weaknesses of our model as a realistic descriptor of Uranus and Neptune’s formation. We present our conclusions in \textsection 5.

\section*{2. Core Accretion Model}

The contraction and buildup of protoplanetary cores and their gaseous envelopes embedded in our model evolving disk are computed with a Henyey-type code (Henyey et al. 1964), which solves the standard equations of stellar structure for the envelope. A detailed description of the core accretion-gas capture code is available in Pollack et al. (1996), Bodenheimer et al. (2000) and Hubickyj et al. (2005). Here we explain the initial conditions used for our experiments and give an overview of our numerical method, describing its strengths and weaknesses.

We use a core accretion rate of the form

\[ \frac{dM_{\text{core}}}{dt} = C_1 \pi \Sigma_{\text{solid}} R_c R_h \omega \]  

(Papaloizou and Terquem 1999), where \( \Sigma_{\text{solid}} \) is the surface density of solid material in the disk, \( \omega \) is the orbital frequency at the position of the planet, \( R_c \) is the effective capture radius of the protoplanet for solid particles, and \( R_h = a[M_{\text{planet}}/(3M_*)]^{1/3} \) is the tidal radius of
the protoplanet (where $a$ is the semimajor axis of the protoplanet’s orbit), and $C_1$ is a constant near unity.

Our numerical experiments are based on the feeding zone approximation, in which the growing embryo accretes planetesimals from an annulus extending ≈ 4$R_h$ on either side of its semimajor axis (Kary & Lissauer 1994). Planetesimals in the feeding zone exit only by being accreted; they are not scattered out of the system, and they encounter the core at the Hill velocity, $v_h = R_h \Omega$. As the protoplanet grows, its tidal radius expands to include previously undepleted regions of the solar nebula. New planetesimals enter the feeding zone as it expands, and the solid surface density evolves according to the competing effects of planetesimal loss by accretion and gain by feeding zone expansion. The fact that the feeding zone model does not include planetesimal loss by scattering limits its accuracy in the trans-Saturnian region, where planetesimals may be stirred to velocities near the local escape speed of the Solar System (Levison & Stewart 2001; Levison & Morbidelli 2007).

We view our core accretion experiments as a “first pass” at reconciling the Nice model with planet formation theory. Our simulations will tend to overestimate the solid accretion rate, so if we cannot form Uranus and Neptune on a reasonable timescale, the initial conditions in the Nice model cannot possibly represent Uranus and Neptune’s formation zones. If we can successfully produce planets with the proper ice giant mass and composition, the detailed planetesimal dynamics at 12–15 AU will warrant further investigation with N-body simulations. Simulations such as those of Chambers (2008) can establish whether or not the initial 1$M_⊕$ seed core can form and resist orbital decay between 12 and 15 AU, and whether our low-velocity planetesimals are an adequate approximation.

Once the protoplanet reaches ≈ 5$M_⊕$ it begins to accrete gas at the combined rate at which (1) its Bondi sphere is evacuated due to cooling and contraction, and (2) its Bondi sphere expands due to increasing mass. Gas flows freely from the nebula into the evacuated volume. The outer boundary conditions of the gaseous atmosphere include the decrease with time in the background nebular density and temperature, modeled by Dodson-Robinson et al. (2009). Since ice giants are not massive enough to limit their gas accretion by opening a gap in the gaseous disk, we carry the simulations beyond Uranus and Neptune’s present masses in order to assess the likelihood that they remain low-mass ice giants. Since grain settling in the protoplanetary envelope reduces envelope opacity where grains exist (Podolak 2003), we adopt grain opacities of ≈ 2% of the interstellar values used in Pollack et al. (1996). However, Dodson-Robinson et al. (2008) demonstrated that grain opacity has little effect on formation timescale and planet composition in the trans-Saturnian region of the solar nebula.

Since the ice giants swap orbits in ≈ 50% of the Nice model simulations, we generically simulate the formation of two ice giants in trans-Saturnian orbits without identifying which planet embryo is proto-Uranus and which is proto-Neptune. In accordance with the initial conditions for the Nice model, we place “Planet I”, the innermost ice giant during the formation epoch, at 12 AU. “Planet O”, the outermost ice giant, begins forming at 15 AU. Which embryo eventually becomes which planet depends on whether or not the ice giants swap orbits.

Solid surface densities available for planet formation are taken from Dodson-Robinson et al. (2009) and are based on a detailed chemical inventory of 211 gas and ice species. To allow for the time needed to build the 1$M_⊕$ seed core and the planetesimals, we adopt a 0.15 Myr offset between the initial formation of the solar nebula and the beginning of planet formation (Dodson-Robinson et al. 2008). Planetesimal formation models based on the streaming instability show that the timescale can be as short as 5 × 10^4 years and is not a strong function of location in the disk (Youdin and Goodman 2005; Johansen et al. 2007), so our 0.15 Myr planetesimal and seed embryo formation timescale is realistic. Planet I at 12 AU begins forming in a feeding zone with 8.4 g cm^−2 of solids while Planet O’s feeding zone at 15 AU contains 6.4 g cm^−2 of solids. For comparison, Pollack et al. (1996) assumed only 0.75 g cm^−2 of planetesimals were available in Uranus’ in situ feeding zone at 19 AU.

Since the goal of our simulations is to bridge the initial solar nebula inventory and the final planet composition—both bulk gas/solid ratio and measured atomic abundances—our numerical experiments are designed to be as deterministic as possible. The three “free parameters” in our core accretion model are semimajor axis, solid surface density of planetesimals, and atmospheric opacity. Initial planet semimajor axes are taken from the Nice model of planetary dynamics so as to be consistent with late-stage orbital evolution; solid surface densities come from a detailed chemical model whose few free parameters were constrained by observational and laboratory data; and Dodson-Robinson et al. (2008) convincingly demonstrated that atmospheric opacity has little effect on planet composition or formation timescale in the outer solar nebula, where isolation masses are high. Here we are testing whether or not one single disk model (Dodson-Robinson et al. 2009) can serve as a formation platform for all four giant planets. If it can, we will have made substantial progress toward determining the physical conditions in the true solar nebula.

3. RESULTS

Figure 1 shows the result of our core accretion simulations. Planet I reaches its present mass in 3.8 Myr while Planet O requires 5.3 Myr to form. However, there is enough solid material remaining in the nebula to allow the ice giants to continue growing beyond 17.2$M_⊕$ (Neptune) and 14.5$M_⊕$ (Uranus). One characteristic of planet formation in solid-rich feeding zones is that the solid cores do not reach isolation mass (Dodson-Robinson and Bodenheimer 2009). In the solid-rich solar nebula of Dodson-Robinson et al. (2009), Planet I’s isolation mass is 88$M_⊕$ while Planet O’s is 167$M_⊕$. Previous simulations of forming planets that don’t reach isolation mass (Dodson-Robinson et al. 2008, Dodson-Robinson and Bodenheimer 2009) show that $M$ monotonically increases with time. The canonical Phase II, in which the planet’s growth is limited by the rate at which gas can enter the Hill sphere (Pollack et al. 1996, Hubickyj et al. 2005), does not occur.

Upon attaining their current masses, both planet cores are still subcritical and have yet to trigger rapid gas accretion. Here our results agree with those of Rafikov
(2006), who showed that critical core mass is not simply a constant $10 M_\oplus$ (e.g. Mizuno 1980), but is a strong function of planetesimal accretion rate, semimajor axis and gas density. In our simulations, the continued fast accretion of planetesimals deposits kinetic energy that stabilizes the ice giant atmospheres against collapse even as the planet masses reach $15 M_\oplus$. In the trans-Neptunian nebula, dynamical times are so long and isolation masses are so large ($M_{\text{cap}} \propto a^3$) that is not possible for any protoplanet to cleanly sweep up the planetesimals in its feeding zone.

If allowed to grow beyond its present mass, Planet I would enter rapid gas accretion at 4.3 Myr with a total mass of $43 M_\oplus$. At 5.7 Myr, Planet O had not yet begun hydrodynamic gas accretion. We therefore cannot evaluate the critical core mass in the feeding zone at 15 AU, but suspect it is somewhat higher than on Planet I’s orbit at 12 AU because of the lower nebular gas density. We calculate that both planets contain 7% hydrogen and helium gas by mass and 93% heavy elements, in agreement with bulk compositions inferred from rotation rates (Podolak and Reynolds 1981, Guillot 2005).

Is it realistic that all but 10–15 $M_\oplus$ of planetesimals are lost from each feeding zone, without forming planet cores? If Uranus and Neptune are to remain at their present masses, the planetesimal-to-planet conversion efficiency in this model is extremely low at 17% for Planet I and 9% for Planet O. Fortunately, we can perform a sanity check on the planet formation efficiency by calculating the accretion-to-ejection probability ratio in the strongly scattering regime, where the protoplanet stirs planetesimals to of order the local solar system escape velocity. N-body simulations by Levison & Stewart (2001) and Levison & Morbidelli (2007) show that the strong-scattering assumption is appropriate for the trans-Neptunian solar nebula. Ida & Lin (2004) demonstrate that the accretion-to-ejection probability ratio of strongly scattered planetesimals is

$$f_{\text{cap}} \approx \left( \frac{2GM_\odot/a}{GM_{\text{core}}/R_{\text{core}}} \right)^2,$$

where the denominator is the square of the typical planetesimal velocity with respect to the core and the numerator is the square of the local escape velocity from the sun. In Eq. 2, $R_{\text{core}}$ is the physical radius of the core, not a gravitationally enhanced capture radius. The probability that a given planetesimal will be scattered, rather than accreted, is therefore

$$P_{\text{sca}} \approx \frac{1}{f_{\text{cap}} + 1}.$$  

For a core density of 3 g cm$^{-3}$ and mass $15 M_\oplus$, $f_{\text{cap}} \approx 0.20$ and $P_{\text{sca}} \approx 0.83$ at 12 AU. At 15 AU, $f_{\text{cap}} \approx 0.14$ and $P_{\text{sca}} \approx 0.88$. Once reaching ice giant mass, growing embryos can accrete less than 1/5 of the available planetesimals and should reach a stage of self-limited solid accretion. Scattering therefore limits the core masses of Uranus and Neptune and efficiently clears away the remaining planetesimals.

4. DISCUSSION: MODEL STRENGTHS AND WEAKNESSES

We have demonstrated that timely formation of Uranus and Neptune in the trans-Neptunian solar nebula, within the ~5 Myr disk lifetimes found by young cluster observations and photoevaporative models of T-Tauri disks (Alexander et al. 2006; Currie et al. 2009) and without extreme planetesimal eccentricity damping (Goldreich et al. 2004a, 2004b), is possible in the feeding zone approximation. In this section we assess the extent to which our core accretion simulations, which use the initial semimajor axes of the Nice model and the solid-rich disk of Dodson-Robinson et al. (2009), provide a realistic picture of the formation of Uranus and Neptune. Our model’s strengths include the ice giants’ predicted compositions and the fact that both are comfortably below critical mass. Our model’s weaknesses are lack of coevality—the outer ice giant O requires 1.5 Myr more for formation than the inner ice giant I—and the simplified description of planetesimal accretion provided by the feeding zone approximation.

4.1. Weakness: Ice giant coevality

Although our cores stay subcritical and we do not need to cut off hydrodynamic gas accretion to replicate Uranus and Neptune’s gas-poor composition, both planets reach their present masses in the midst of rapid solid accretion, well past the “elbow” of the $M(t)$ curve (Fig. 1). Why should they not continue to grow into massive cores analogous to HD 149026b (Sato et al. 2005)? From Fig. 1, we see that Planet I reaches $70 M_\oplus$ at the end of the simulation. In the feeding zone approximation, there is nothing to stop it growing to Saturn’s mass or greater while Planet O slowly moves toward $15 M_\oplus$.

The simplest explanation is self-limiting accretion due to planetesimal scattering, as discussed in Dodson-Robinson et al. Planet I would reach ~ $15 M_\oplus$, begin to scatter far more planetesimals than it accretes, and dramatically slow its core growth. Planet O would similarly halt its growth in due course. If a self-regulating mechanism impedes core growth in the trans-Neptunian nebula, there may be no need for Planet I and Planet O to be coeval. Based on an accretion-to-ejection probability ratio of 0.1 (Lin & Ida 1997), Ida & Lin (2004) derive a maximum planet core mass of

$$M_{\text{core}} \approx 1.4 \times 10^3 \left( \frac{a}{1 \text{ AU}} \right)^{-3/2} M_\oplus.$$  

Planet I’s core could not grow beyond $34 M_\oplus$ while Planet O’s core would be limited to $24 M_\oplus$. Furthermore, as demonstrated in Dodson-Robinson et al., the accretion rate reduction kicks in before the cores reach their absolute maximum mass. Strong scattering of planetesimals by the growing protoplanets could hold off rapid solid growth long enough for Planet O’s mass to catch up with Planet I’s.

To demonstrate the effects of planetesimal scattering on ice giant composition and growth rate, we simulated the growth of both planets, this time modifying the accretion rate to take into account planetesimal scattering. We first calculated the solid growth rate $\dot{M}$ according to Equation 1. Then, since $f_{\text{cap}}$ (Equation 2) is defined as the ratio of the probabilities of accretion and ejection for a given planetesimal, it follows that planetesimals are ejected from the feeding zone at a rate of

$$\dot{M}_{\text{eject}} = \frac{\dot{M}}{f_{\text{cap}}}.$$  

At each timestep, we subtracted the sum of the accreted and the ejected planetesimal mass, $M_{\Delta t} + M_{\text{eject}} \Delta t$, from the total solid feeding zone mass. Figure 2 shows the results of the simulations that include planetesimal ejection. As expected, both ice giants take longer to reach their current masses when their solid accretion is slowed by scattering: Planet I forms in 4.0 Myr and Planet O forms in 6 Myr.

With self-limiting accretion, however, we risk losing our solid-rich composition. Continual planetesimal accretion is what stabilizes the ice giant atmospheres against hydrodynamic collapse. As demonstrated by Hubickyj et al. (2005), suddenly cutting off accretion once a planet core reaches $10M_⊕$ hastens the onset of rapid gas accretion. We see this behavior in the left panel of Figure 2 as Planet I, within a few $10^5$ years after reaching ice giant mass, begins to accrete gas at an exponential rate. Since Planet I begins to halt clear its feeding zone and slow its solid accretion before Planet O finishes forming, it evolves toward Jupiter’s mass and composition. Even where planetesimal scattering is efficient, we still require a way of halting the innermost ice giant’s growth. Strong planetesimal scattering does not mitigate the need for Uranus and Neptune to form on similar timescales.

For Planet O, with its formation timescale of 6 Myr coming uncomfortably close to the upper limit of protostellar disk lifetimes (Haisch et al. 2001, Alexander et al. 2006), it is sensible to speculate that its growth was halted by the dissipation of the solar nebula. Even though Planet O is primarily accreting solids, rather than gas, between 5 and 6 Myr, nebular gas provides eccentricity damping to the planetesimals, helping keep them in the feeding zone. Without the gas, the planetesimal velocity dispersion rapidly increases, cutting off solid accretion except in the case of stochastic, high-velocity collisions (e.g. Fernandez and Ip 1984). Such a collision could be responsible for Uranus’ extreme rotation axis tilt. Postulating the dissipation of the Nebula as the mechanism for halting Planet O’s solid growth, however, does nothing to rectify the coevality problem.

A second possibility for keeping both ice giants at the same mass is that Planet I was scattered to a wider orbit—perhaps into the region of the solar nebula. In roughly half of the Nice model simulations, Neptune and Uranus cross orbits, with Neptune (corresponding to Planet I in this scenario) moving from ~12 to ~23 AU in about $10^5$ years. Given the long orbital timescale at 23 AU (110 years vs. 41 years at 12 AU), Neptune’s accretion would be nearly cut off, especially in light of its subsequent outward migration due to planetesimal scattering. Although the authors of the Nice model use the Jupiter–Saturn 1:2 MMR crossing to explain the Late Heavy Bombardment occurring 600 Myr after the birth of the solar system (Gomes et al. 2005), this resonance crossing could have occurred at any time (or multiple times) during the evolution of the solar nebula. If the Jupiter–Saturn 1:2 MMR crossing caused the ice giants to remain at similar masses by halting Planet I’s accretion, Neptune must have formed interior to Uranus.

Within the feeding zone approximation, there is no entirely satisfying way to explain why Uranus and Neptune have nearly the same mass. More detailed N-body simulations are needed to investigate the degree to which the ice giants limit their own accretion in the 12–15 AU region of the solar nebula.

4.2. Strength: Ice giant composition

Although traditional core accretion-gas capture simulations cannot entirely explain the coeval formation of the ice giants without requiring an external growth-limiting mechanism, they still provide strong constraints on the conditions required to produce ice giants. In this section, we discuss the characteristics of feeding zones that can produce solid-rich, gas-poor planets.

In Section 3 we showed that by keeping Uranus and Neptune’s cores subcritical, we reproduce their solid-rich compositions of $>90\%$ solid by mass (Podolak and Reynolds 1981) without needing to halt gas accretion during the extremely short hydrodynamic growth phase. We also bypass the long opacity-limited gas contraction phase (Phase II), which lasted ~4 Myr in the simulations of Pollack et al. (1996), because our ice giants never reach isolation mass. Protoplanetary envelope opacity should not substantially affect the formation timescale or bulk composition of Uranus or Neptune (Dodson-Robinson et al. 2008). The solid/gas ratios we calculate are therefore insensitive to the composition and size of grains in the ice giant envelopes.

Figures 2 suggests that continual planetesimal accretion is required to limit the mass of the protoplanet atmosphere. Rapid gas accretion begins as soon as Planet I’s solid growth rate begins to level off, at the inflection point of the $M_{\text{core}}(t)$ curve. The more massive the protoplanet, the more thermal energy is required to stabilize its atmosphere. Our results suggest that once a protoplanet core grows beyond the fiducial critical core mass of $10M_⊕$, it will trigger runaway gas accretion unless its solid accretion rate keeps increasing with time. Indeed, we conducted one experiment in which solid accretion was suddenly cut off after 3.5 Myr of planet growth with a core mass of $12M_⊕$. As expected from the results of Hubickyj et al. (2005), the onset of rapid gas accretion was immediate.

To keep the solid/gas mass ratio high and ensure the planetary composition does not become Jupiter-like, a large supply of available planetesimals is critical. Planetesimals could either come from the initial feeding zone or be accreted as the protoplanet migrates into undepleted regions of the disk (Alibert et al. 2005). We tested the effect of planetesimal availability on final planet composition by varying the initial solid surface density in the feeding zones and calculating the solid/gas ratio once the protoplanet reached ice giant mass ($14.5M_⊕$ for Uranus and $17.2M_⊕$ for Neptune). These tests included planetesimal scattering in the accretion rate calculation. High values of initial planetesimal surface density $\Sigma_0$ ensure that either the feeding zone will never be depleted of planetesimals, or in the case of migration, mimic a protoplanet always moving into areas of the solar nebula with a fresh planetesimal supply.

Figure 3 shows solid/gas ratio as a function of $\Sigma_0$. Since we do not know which ice giant formed in which feeding zone, we calculate pairs of solid/gas ratios for each initial value of $\Sigma_0$, one corresponding to the final mass of Uranus and the other to the final mass of...
Neptune. For both feeding zones, at distances of 12 and 15 AU, solid/gas ratios within 25% of the fiducial ice giant composition of ~ 90% solids can only be reached for a narrow range of initial surface densities between 6 and 11 g cm$^{-2}$. Note that while Pollack et al. (1996) reproduced Uranus’ composition by requiring the planet to form in situ in a 19 AU feeding zone with only 0.75 g cm$^{-2}$ of planetesimals, its formation timescale was 16 Myr—for far longer than observed protostellar disk lifetimes. For surface densities from 6 to 11 g cm$^{-2}$, we find formation timescales of 8.8–2.9 Myr at 12 AU and from 9.5–3.0 Myr at 15 AU. Solid-rich feeding zones with $6 < \Sigma_0 < 11$ g cm$^{-2}$ at positions consistent with the Nice model can give both appropriate formation timescales and bona fide ice giant compositions.

Even though we require that planetesimals always be present in the ice giant feeding zones as long as the gas disk lasts, our work does not necessarily conflict with the Nice model results. Gomes et al. (2005) invoke a sharp inner edge of the planetesimal distribution at 15.3 AU in order to delay the onset of Jupiter and Saturn’s 1:2 MMR crossing and allow the resulting dynamical shakeup to coincide with the Late Heavy Bombardment 700 Myr after the birth of the solar system. Their Figure 1b shows that the time of the 1:2 MMR crossing is a strong function of the location of the inner disk edge: the greater the distance between Jupiter and Saturn and the planetesimals that perturb their orbits, the smaller the perturbations to the gas giant orbits and the longer it takes to force them through the resonance. Gomes et al. chose 15.3 AU because planetesimals outside that radius have a dynamical lifetime, defined as the time required for the planetesimal to pass within the Hill sphere of a planet, longer than their assumed gas disk lifetime of 3 Myr.

However, in order to build the planets, the dynamical lifetime of planetesimals in their feeding zones cannot possibly be longer than the nebula lifetime—otherwise, no protoplanet could ever grow. For a planetesimal disk with an inner edge at 12 AU, coinciding with our innermost ice giant, we see from Figure 1 of Gomes et al. (2005) that the time till Jupiter and Saturn’s 1:2 MMR crossing is ~ 5 Myr. Tsiganis et al. (2005) found a 6 Myr lag time between the beginning of their simulations and the 1:2 MMR crossing. Our simulations demonstrate that the orbital evolution of the giant planets must take place shortly after Uranus and Neptune finish forming. Sculpting the planetesimal distribution to allow the giant planets’ orbital evolution to trigger the Late Heavy Bombardment does not allow Uranus and Neptune to retain their solid-rich composition.

In the Podolak et al. (1995) solid-rich disk model, the methane condensation front is at 10 AU at the beginning of planet formation. Any planetesimals beyond 10 AU contain 10% methane ice by mass. Podolak et al. calculated a series of ice giant interior models using three concentric layers: a rock+metal core, an ice shell with H$_2$O, CH$_4$, NH$_3$ and H$_2$S in solar proportions, and an atmosphere of H$_2$, He and small amounts of the aforementioned ices. For both planets, the models that best reproduced the gravitational moments measured by Voyager had ice shells of ~ 13M$_{\oplus}$. Podolak et al. found that total methane mass fraction in the ice giants is ~ 20%, whereas our simulations construct planets that are ~ 9% methane by mass. The reason for this discrepancy is that we include refractory CHON material in the planet cores, while Podolak et al. assume iron and silicate cores.

We calculate total carbon mass fractions of 22%, in agreement with those derived by Podolak et al. (1995) and Marley et al. (1995). We have thus created the first planet formation model to reproduce the observed carbon enrichment of both ice giants (Encrenaz 2005). We predict that the methane present on Triton’s surface (e.g. Hicks and Buratti 2004) and in the ice giant atmospheres was accreted as ice from the primordial solar nebula. The Saturn formation model of Dodson-Robinson et al. (2008) came to the same conclusion about the ammonia ice present in Saturnian satellites (Prentice 2007; Freeman et al. 2007).

The fact that we have reproduced Uranus and Neptune’s bulk composition and carbon enrichment so closely places an important constraint on their feeding zone location: the presence of methane ice in planetesimals is required during the ice giant formation epoch. Based on both chemically evolving models of the solar nebula (Dodson-Robinson et al. 2009) and the relatively carbon-poor compositions of Jupiter and Saturn (Encrenaz 2005)—which indicate that they did not form in methane ice-rich regions—we can reject any formation model in which the ice giants form inward of Saturn’s orbit.

5. CONCLUSIONS

We have shown that it is possible to assemble Uranus and Neptune from 1M$_{\oplus}$ seed cores in a 0.12M$_{\odot}$ solar nebula, with initial semimajor axes of 12 AU and 15 AU as predicted by the Nice model (Gomes et al. 2005; Morbidelli et al. 2005; Tsiganis et al. 2005). The resulting growth timescales are 3.8 Myr for the innermost ice giant and 5.3 Myr for the outermost ice giant. However, numerous details still remain to be fully resolved:

1. What is the migration history of the planets after formation and can we explain the current orbits?

2. If Planets I and O do not change places, how can Planet I’s lack of a massive gaseous atmosphere be explained?

3. If Planets I (Neptune) is scattered to a wide orbit while the disk is still gas-rich, what happens to Planet O (Uranus)?

4. How can the Late Heavy Bombardment be explained if the giant planets’ orbital evolution must take place soon after they finish forming?

Because we chose the initial semimajor axes of the Nice model, both ice giants’ post-formation orbital evolution may be explained according to that model. If Uranus and Neptune form between 12 and 15 AU in the feeding zone approximation, they end up with either different formation timescales or different masses. If the proto-ice giants begin limiting their own solid accretion rates by ejecting planetesimals, the outermost planet core can continue to grow after the innermost core has assembled the bulk of its mass, but the inner planet may evolve into a gas giant. One way to form Uranus and Neptune at 12 AU and 15 AU but keep their near-equal masses may...
be for Neptune (the innermost planet in this scenario) to get scattered out to a wide orbit upon reaching its present mass—but in this scenario we cannot be assured of Uranus' survival and continued growth.

One way our results differ from the scenario described by the Nice model suite of papers is that the orbital evolution of the giant planets must take place shortly after Uranus and Neptune form, as the mutual planetesimal distribution at any time during their growth can form planets with ~ 90% solids by mass. As stated in §3 we still lack a mechanism for checking Planet I's growth while Planet O finishes forming. If Neptune is the innermost ice giant, Jupiter and Saturn's 1:2 MMR crossing might perform such a function by suddenly sending Neptune into an extremely wide orbit.

The formation timescale of the ice giant forming at 15 AU is quite close to both the upper limit of observed protostellar disk lifetimes (Haisch et al. 2001; Currie et al. 2009) and the predicted onset of photoevaporation for the solar nebula (Alexander et al. 2006), indicating that quick ice giant formation in the trans-Saturnian nebula is only marginally possible even in the optimistic feeding zone approximation. Since the compositions of Uranus and Neptune strongly indicate that they originated in the trans-Saturnian solar nebula, however, we encourage more detailed N-body investigations of embryo growth rates between 12 and 15 AU.

Without knowing for sure whether Uranus and Neptune swapped orbits, we cannot pair each planet with its unique formation zone. However, the chemically evolving solar nebula simulations of Dodson-Robinson et al. (2009) show that the CO and N₂ ice lines fall between 12 and 15 AU. Detailed measurements of the carbon and nitrogen abundances of both planets and their satellites could resolve the question of which planet formed outermost in the solar nebula: the planet that formed near 15 AU should be slightly more carbon- and nitrogen-rich than the planet that formed near 12 AU. Such detailed measurements of Uranus and Neptune's atmospheric compositions await another space mission to the outer planets, but ground-based spectroscopy of their satellites (e.g. Stern & McKinnon 2000; Bauer et al. 2002) may make some progress on this question. Our formation model reproduces the ice giants' solid/gas ratios and carbon abundances inferred from gravitational moment measurements and spectroscopy.

Our results indicate that planet formation in the outer nebula is a fundamentally inefficient process. Even in the feeding zone approximation, where we calculate what is best viewed as an upper limit to accretion rate, building ice giants within the lifetime of the solar nebula requires substantially more solid and gas mass than the planet can ever accrete. The dynamical times in the massive Dodson-Robinson et al. (2009) disk (total mass 0.12M☉) beyond 15 AU are still too long to allow gas or ice giant formation. From the combined constraints of the methane ice line location and the upper limits to ice giant growth rates, we can constrain the ice giant formation zone to between 10 and 15 AU.

The fact that the solar nebula model of Dodson-Robinson et al. (2009) can form all the giant planets at locations at or near those predicted by the Nice model, as well as reproduce important features of the compositions of the planets and their satellites—such as ammonia enrichment in the Saturnian system (Dodson-Robinson et al. 2008)—indicates that substantial progress has been made at modeling the ice inventory and chemical evolution of the solar nebula. Here we have created the first successful progression (to the best of our knowledge) from primordial solar system ice inventory to detailed planet composition. This research marks a substantial step forward in connecting both the dynamical and chemical aspects of planet formation. Although we cannot say that the solid-rich solar nebula model (Dodson-Robinson et al. 2009) gives exactly the appropriate initial conditions for planet formation, rigorous chemical and dynamical tests have at least revealed it to be a viable model of the early solar system.

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Fig. 1.—Aided by a solid-rich solar nebula, Uranus and Neptune grow in the trans-Saturnian region in under 5.5 Myr. Left: Forming at 12 AU according to the predictions of the Nice model (Tsiganis et al. 2005), the innermost ice giant attains its current mass in only 3.8 Myr. Right: At 15 AU, the outer-forming ice giant reaches its present mass in 5.3 Myr. The dashed lines show the planets’ current masses. The solid-rich nebula contains enough planetesimals for the ice giants to grow well beyond their current sizes, given efficient accretion and enough time.

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Fig. 2.— Planetary ejection by the growing ice giant cores leads to self-limiting accretion, which removes the possibility of runaway solid growth and only slightly increases the accretion timescales for both planets. Left: The innermost ice giant, forming at 12 AU, attains its current mass in 4.0 Myr, as opposed to 3.8 Myr when planetesimal ejection is not included in the simulation. Rapid gas accretion begins as soon as Planet I's solid growth rate begins to level off, at the inflection point of the $M_{\text{core}}(t)$ curve. Right: At 15 AU, the outer-forming ice giant reaches its present mass in 6 Myr, vs. 5.3 Myr without planetesimal ejection. The dashed lines show the planets’ current masses.

Fig. 3.— Final solid/gas mass ratio in the planet is a strong function of initial solid surface density, $\Sigma_0$, in the solar nebula. Both Planet I and Planet O must have between 6 and 11 g cm$^{-2}$ of planetesimals in their feeding zones in order to reach the true ice giant solid/gas ratio of $\sim 9$ (dash-dot line). The final solid/gas ratio depends on whether the planet being formed is Uranus (14.5$M_\oplus$) or Neptune (17.2$M_\oplus$). High initial solid surface densities lead to high solid accretion rates throughout formation, which stabilize the protoplanet atmosphere against rapid contraction and lead to the bona fide ice giant composition.