Modeling of viscoelastic properties of composite springs in wheeled vehicles suspension systems

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Abstract. The article provides a review of models allowing to take into account the viscoelastic properties of springs made using polymer composite materials. The features of solving equations with fractional derivatives that describe the considered models are considered. The equation of the stress-strain state of the composite spring is obtained, which takes into account the rheological properties of the material. A model has been developed in MATLAB / Simulink, which allows calculating composite springs. A composite spring was made and tested, which made it possible to compare the elastic characteristics of the spring obtained as a result of modeling and experiment. The authors of the article determined the scope of such springs in the automotive industry and the direction of improvement of composite springs used in wheeled vehicles.

Introduction

The analysis of application of polymeric composite materials (PCM) in the automotive industry establishes the trends for increasing use of PCM parts in vehicle parts and components every year [1–2]. One of the perspective directions of use of the parts made with the use of PCM is their application in wheeled vehicle suspension systems. PCM is most commonly used in wheeled vehicle suspension systems to create elastic elements and guiding system elements. Nowadays PCM is actively used in leaf spring design, besides, recently there have been tendencies to use helical springs made with PCM as elastic elements of suspension systems.

A topical issue is the reduction of the weight of the vehicle as a whole, as well as unsprung masses. Fiberglass springs allow to achieve not only reduction of masses, but also a lot of other advantages in comparison with steel ones: provision of large suspension strokes, high durability, absence of corrosion, cost-effective production, good fatigue resistance, low creep, good vibration and noise insulation properties, possibility to provide interchangeability with steel springs, possibility to obtain complex shapes and progressive loading characteristics, availability of damping properties [3–7]. Therefore, the use of springs made with the use of PCM as elastic elements of the vehicle is a logical and actual direction of research. The wide application of composite springs in the automotive industry requires the study of new design methods, as such elements should take into account the internal hysteresis losses that result from the viscoelastic properties of the material, so the actual issue is the creation of a method for calculating such springs.

To take into account the properties of viscoelasticity, different models were used, which were developed by Maxwell, Voigt and Kelvin [8]. These models can be used not only to take into account the properties of solid substances (dense solutions, elastic bodies and slurries), but also to describing and taking into account the properties of PCM.
One of the simplest models to describe the properties of viscoelasticity is Maxwell model, which consists of a combination of elastic and viscous elements (Fig. 1). Hooke's Law for the elastic medium and Newton's Law for the viscous medium are combined into one viscoelastic medium equation that has the form:

\[ \sigma(t) + \tau \, D_t \sigma(t) = E \tau \, D_t \varepsilon(t), \]

where \( \sigma(t) \) — stresses; \( \varepsilon(t) \) — deformations; \( E \) — Young’s modulus.

Another classic model for describing a viscoelastic medium is the Kelvin-Feugt model, which consists of a spring and damper connected in parallel (Fig. 2). The structural equation for this model of viscoelastic medium is:

\[ \sigma(t) = E \left[ \tau \, D_t \varepsilon(t) + \tau \, D_t \varepsilon(t) \right]. \]

Both above classic models do not give the most accurate description of viscoelastic properties, so for a more complete description of the properties of the model of Zener (Fig. 3), fractional-differential form of this model (three-parameter model) is described by the equation:

\[ \sigma(t) + \tau D_t \sigma(t) = E \left[ \varepsilon(t) + \theta D_t \varepsilon(t) \right]. \]

For the most accurate description of the material properties, a generalized model is used. The equations of state of the material have the following form:
The above equations with integer derivatives describing different models have insufficient adequacy in terms of the quality of the model or have a large number of components, so the apparatus of fractional derivatives is used to describe the models under consideration [8–9]. With this approach, the fractional-differential generalization of a standard three-parameter model with integer derivatives is the four-parameter Zener model, which is described by the equation:

\[ \sigma(t) + \tau^a_0 D^\alpha_x \sigma(t) = E \left[ \varepsilon(t) + \theta^a_0 D^\beta_x \varepsilon(t) \right], \]

where \( D^\alpha_x f(x) \) — fractional derivative \( x(t) \).

There are several different methods to generalize the definition of a fractional derivative, but all of them are the same as an usual derivative in the case of a natural order [10]. One of the ways to find a fractional derivative is a function called a Riemann-Liouville derivative, for which the expression of a fractional derivative of the order \( 0 \leq \nu < 1 \) is written as follows:

\[ D^\nu f(x) = \frac{1}{\Gamma(1-\nu)} \int_a^x (x-\xi)^{-\nu} f(\xi) d\xi, \]

where \( \Gamma(z) \) — gamma function.

In addition to the Riemann-Liouville fractional derivative, another definition, called the Caputo derivative, is often used. The fractional derivatives of Riemann-Liouville and Caputo generally do not match, as they have mathematical principles and are used as a tool for the most accurate description of the physical processes in the considered objects, as well as for the matching of calculation and experiment results. In addition, there is a form of fractional derivative of Marchaux, which is used to differentiate smooth functions without jumps and differentiable.

As a result of the numerical methods used in the research, another definition of a fractional derivative is used, which is called a fractional derivative of Grunwald-Letnikov with a finite limit. The fractional derivatives of Riemann-Liouville, Caputo and Marchaux reviewed above are applicable for known functions \( f(x) \), so their application to random functions and in the case of real-time calculations and simulations is limited or impossible. The definition of Grunwald-Letnikov's fractional derivative is obtained by considering finite differences, i.e. the definition is obtained from considering...
the increment of the function and the argument, and the type of the function may be random. In the MATLAB Simulink environment, calculations are performed discretely with a constant or variable step. In addition, the external power or kinematic disturbance in real objects is often random, so the definition of a fractional derivative of Grunwald-Letnikov, best suited to solve this type of problems. Grunwald-Letnikov's derivative can be written down as follows:

$$\alpha D^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{t/a} (-1)^j \frac{\Gamma(\alpha+1)}{j!\Gamma(\alpha-j+1)} f(t - jh).$$

To solve a fractional derivative in MATLAB Simulink environment, an S-function was written. It consists of a subsystem inside which there is an S-function with two inputs: an input for the function to be differentiated and an input of the order of the fractional derivative. At the output of the S-function, a fractional derivative of the input function of the corresponding order is determined.

On the example of the sin(x) function, we debugged the model under consideration. The results of the numerical calculations using the model described above fully coincide with the results obtained in the analytical calculation. Figure 4 shows the plots of the initial function sin(x) and its fractional derivative of the order of 0.5.

![Figure 4. Plots of the sin(x) function and its fractional derivative of the order 0.5](image)

If the order of the fractional derivative tends to zero, the function we are looking for will fully repeat the original function, and if the order of the fractional derivative tends to be one, the function we are looking for will tend to the integer derivative of the original function — cos(x).

According to the Voltaire principle, the problem solution for linear system elasticity can be used to solve problems with regard to viscoelastic properties by replacing the elastic parameters of the system with the parameters of the system with regard to viscoelasticity. For this purpose, the integral Laplace transform is used.

To take into account the properties of viscoelasticity, an equation of the stress-strain state in the linear statement was made, according to Hooke's law, then a transition to the operator's representation was made, the elastic parameters of the system were replaced by their operator's representation, and the equation in the fractional-differential statement, which describes the viscoelastic properties of the system under consideration, was returned to the originals.

We will consider in detail the approach described above on the example of composite springs. Figure 5 shows the spring, which is affected by an external disturbance in the form of force $F(t)$.
The deflection of such a spring can be determined using Mora integral:

\[
\lambda = \int_l^l \frac{M_k M_{kl}}{G J_k} dz = \frac{8FD^3n}{Gd^4},
\]

where \( D \) — mean diameter spring;
\( n \) — number of active coils;
\( d \) — spring wire diameter;
\( G \) — shear modulus of material.

The four-parameter model [10] of a standard linear body for a torsion case is described by the equation:

\[
\tau(t) + \zeta^\alpha D^\alpha [\tau(t)] = G\left[\gamma(t) + \theta^\alpha D^\alpha [\gamma(t)]\right].
\]

Operator polynomials that correspond to the four-parameter viscoelastic body model under consideration:

\[
Q(s) = \zeta^\alpha s^\alpha + 1,
\]

\[
R(s) = \theta^\alpha s^\alpha + 1.
\]

The elastic operator, which corresponds to the shear modulus when taking into account the viscoelastic properties, will look like:

\[
G(s) = \frac{GR(s)}{Q(s)}.
\]

Using the Laplace transformations and replacing the elastic parameters with the corresponding parameters when taking into account the viscoelastic properties, we obtain a differential equation with fractional derivatives that connects the spring deflection \( \lambda(t) \) with a force acting on the spring \( F(t) \):

\[
\lambda(t) + \theta^\alpha D^\alpha [\lambda(t)] = \frac{8D^3n}{Gd^4} \left[ F(t) + \zeta^\alpha D^\alpha [F(t)] \right].
\]

Using this equation, a model in the MATLAB Simulink environment, shown in Figure 6, was created to describe the elastic properties of the spring taking into account rheological features. The

**Figure 5.** General parameters of a helical spring and its loading scheme
model is designed in the form of a subsystem whose input parameter is the spring deflection, i.e. the kinematic effect, and the output parameter is force.

![Composite spring model describing its viscoelastic properties](image)

**Figure 6.** Composite spring model describing its viscoelastic properties

To check the adequacy of the model under consideration, a composite spring made of fiberglass was made as shown in Figure 7 (a). The spring was mounted on the test bench and the loading and unloading characteristics of the spring were obtained, which are shown in Figure 7 (b). At a force of 2500 N, the spring deflection is 100 mm. Figure 7 (b) shows the hysteresis loop, which is further studied.

![Composite spring](image)
As the compression spring wire operates on torsion [11], the optimum winding angles of the composite spring wire are $+45^\circ$ and $-45^\circ$. The main geometrical parameters of such a spring are given in Table 1.

Table 1. Main geometrical parameters of the fiberglass spring

| Spring parameter            | Value    |
|-----------------------------|----------|
| Spring stroke               | 160 mm   |
| Spring freelength           | 340 mm   |
| Spring innerdiameter        | 56 mm    |
| Spring wire diameter        | 16 mm    |
| Number of active coils      | 8 pcs.   |
| Spring stiffness            | 25 N/mm  |

The kinematic perturbation in the form of a sinusoid with a frequency of 3Hz and an amplitude of 100 mm in the form of an input parameter acting on the composite spring. The loading characteristic obtained by modeling such a spring is shown in Figure 8. The figure shows that the maximum deflection (abscissa axis) of such a spring force (ordinate axis), arising in the spring is the same as the experimentally removed load characteristic. Of particular interest is the hysteresis loop obtained by modeling with the use of fractional differentiation and observed in the experiment with the use of springs made with the use of PCM. This loop requires an additional series of experimental research, on the base of which it is possible to specify the parameters included in the four-parameter model and the fractional-differential equation describing
The composite spring model can be used for simulation purposes. For a wheeled vehicle, the suspension force depends on two components: the relative deflection and the suspension deflection rate. The expression for determining the force in the independent suspension can be written in the following form [12,13]:

\[ F_{ij} = P_{y_{spr}}(h_{ij}) + P_{y_{dempij}} \left( \frac{dh_{ij}}{dt} \right) , \]

where \( P_{y_{spr}}(h_{ij}) \) — is the force in the \( i \)-th elastic element of the \( j \)-th side;

\( P_{y_{dempij}} \left( \frac{dh_{ij}}{dt} \right) \) — force in the \( i \)-th damping element of the \( j \)-th side.

Figure 9 shows such a simple model of the suspension system. In such a suspension system, the damping characteristic is determined by the dependence of the deflection velocity on the force in the shock absorber, and the elastic characteristic of the composite spring is calculated with the use of equations with derivatives of fractional order.
Figure 9. Model of the suspension system with composite spring

The composite spring model developed allows to predict the rheological properties of PCM. To predict such properties it is necessary to realize a number of experiments which will allow to specify and systematize the parameters which are a part of the four-parameter model of a composite spring. A perspective direction is the use of springs with progressive load characteristic. The technology of composite springs production and the use of PCM in their manufacture allow to obtain progressive loading characteristics of composite springs, which are impossible or difficult to obtain when using steel springs. Such loading characteristics can be obtained by creating springs with different reinforcement angles and variable wire diameter. The method considered in the article can be applied in determining the elastic properties of composite springs with progressive characteristics. The obtained model of composite springs can be used in the simulation of the vehicle [14–16] as a whole, as well as to assess the impact of such a spring on the operational parameters of the vehicle.

It should be noted that this work was carried out at the Bauman Moscow State Technical University, with financial support from the government in the face of the Russian Ministry of Education under the project: № 14.577.21.0287. (Identification number: RFMEFI57718X0287)

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