Investigation into Joint Stiffness for Finite Element Modelling of the Dynamic Behaviour of a Structure with Bolted Joints

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Abstract. Accurate representation of the joint stiffness of bolted joints in the finite element (FE) model of a jointed structure is vital to ensure the accuracy and reliability of the predicted dynamic behaviour of the structure. The aim of this work was to put forward an efficient FE modelling method embodying the representation of the joint stiffness for the investigation of an assembled structure with bolted joints. The predicted FE results were compared with those obtained from the experimental modal analysis (EMA). EMA was performed to measure the natural frequencies and mode shapes of the structure with bolted joints. The calculated total error from the comparison of the natural frequencies between the FE and EMA were recorded. The results of the investigation showed that there is a noticeable contribution of the joint stiffness in the representation of the structure with bolted joints in developing the efficient FE modelling method.

1. Introduction

Bolted joint is one of the common elements used in an assembled structure. Typically, a structure with bolted joints consists of several structural components or members jointed with a series of bolts and nuts. However, to the best of authors knowledge, there has been very little discussion about the appropriate representation of the joint stiffness for the investigation of the dynamic behaviour of the structure with bolted joints.

There are two typical components in structures with bolted joints which are the members and bolts. The member stiffness is determined by considering the effective spring effect of each structural member. Structure with multiple members shall has the overall member stiffness represented by several springs connected in series \[1, 2\]. In this work, the bolted joints consisting of 2 members, hence the overall member stiffness, \( k_m \), is calculated by using equation (1) where \( k_1 \) and \( k_2 \) represent stiffness for the 1\textsuperscript{st} member and the 2\textsuperscript{nd} member respectively.

\[
\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}
\]  

Motosh \[2\], \[3\] proposed the conical pressure distribution envelopes for the highly stressed portion of the joint as a technique to estimate the member stiffness of the bolted joint model. This stiffness...
estimating method was also used by other researchers to determine the stiffness of jointed members of a structure [1], [2], [4]. As shown in Figure 1, the jointed region of the assembled structure of this work is divided into 2 frustums which fall off transversely from the bolt centreline, replicating the proposed method used in previous researches.

An analytical equation was originally proposed by Lehnhoff et al. [1], [2], [4] to calculate the stiffness of the member with the assumption of uniform pressure with conical envelope under the bolt’s head. Subsequently, Shigley and Mischke modified this equation with a fixed standard cone angle (α) at 30° to obtain a simpler expression for the calculation of the member stiffness as in equation (2) [1], [2], [5]. In this work, equation (2) was used to calculate the stiffness of each member of the structure with bolted joints which denoted by $k_1$ for frustum 1 and $k_2$ for frustum 2.

$$k = \frac{0.577\pi Ed}{\ln\frac{(1.15r + D - d)(D + d)}{(1.15r + D + d)(D - d)}}$$

(2)

The stiffness values of the overall members and bolts are used in the FE model of the assembled structure with bolted joints for the investigation of its dynamic behaviour. The predicted results are compared with the counterparts obtained from experimental modal analysis. This paper proposes an efficient FE modelling method embodying the representation of the joint stiffness for the investigation of the dynamic behaviour of an assembled structure with bolted joints.

2. Modelling of Joints Stiffnesses

2.1. Modelling of Bolted Joints

A simple lap joint is used to study the dynamic behaviour of the structure with bolted joints. The lap joint comprises 2 identical steel plates with stainless-steel bolts and nuts. Each plate comes with the length of 380 mm, width of 45 mm and thickness of 6 mm. The 3D CAD models of the components were developed and designated as Plate 1 and Plate 2. Both plates are assembled using bolts and nuts in a configuration shown in Figure 2. For the purpose of reducing the assembly contact zones of the assembled plates, two cylindrical bosses were introduced in each plate with the diameter of 18 mm and thickness of 2 mm as shown in Figure 3 [6].
The material properties of Plate 1, Plate 2, bolts and nuts [7] are tabulated in Table 1. The values of the material properties of each plate are slightly different from the standard values. The difference is because those values have been updated using the FE model updating procedures [8], [9]. The procedures provide an accurate modelling of the Plate 1 and Plate 2 before assembling the plates to form an assembled structure.

| Component       | Property              | Value       | Unit   |
|-----------------|-----------------------|-------------|--------|
| Plate 1         | Young's Modulus       | 2.064×10¹¹  | N/m²   |
|                 | Poisson's Ratio       | 0.302       | Unitless|
|                 | Mass Density          | 7.458×10³   | kg/m³  |
| Plate 2         | Young's Modulus       | 2.094×10¹¹  | N/m²   |
|                 | Poisson's Ratio       | 0.316       | Unitless|
|                 | Mass Density          | 7.45×10³    | kg/m³  |
| Bolts and nuts  | Young's Modulus       | 1.93×10¹¹   | N/m²   |
|                 | Shear Modulus         | 0.75×10¹¹   | N/m²   |
|                 | Poisson's Ratio       | 0.27        | Unitless|
|                 | Mass Density          | 7.86×10³    | kg/m³  |

2.2. Stiffness of the Members

Stiffness components of the 1st member and the 2nd member, which are Plate 1 and Plate 2 respectively, were calculated using equation (2). These stiffness values, named as $k_1$ and $k_2$ correspondingly, were used as the input to calculate the overall member stiffness, $k_m$, by using equation (1). Table 2 shows the calculated stiffness values of $k_1$, $k_2$ and $k_m$. The $k_m$ value was then used as the translational stiffness component of the CELAS element representing the jointed members in the FE modelling of the bolted joints.

| Stiffness components | Value (N/m) |
|----------------------|-------------|
| 1st member stiffness, $k_1$ | 6.642×10⁷ |
| 2nd member stiffness, $k_2$ | 6.954×10⁷ |
| Overall member stiffness, $k_m$ | 3.397×10⁹ |

2.3. Stiffness of the Bolts

In this work, CBUSH element was used to represent the bolts in the FE model of the assembled structure. The CBUSH stiffness components which consist of axial stiffness ($K_1$), shear stiffness ($K_2$, $K_3$) and
rotational stiffness \((K_4, K_5, K_6)\) were defined by using the stiffness with Swift’s flexibility formulae as used in the previous studies [10], [11]. The calculated stiffness components for CBUSH element are tabulated in Table 3.

| Stiffness components | Value (N/m) |
|----------------------|-------------|
| Axial stiffness, \(K_1\) | \(9.474 \times 10^8\) |
| Shear Stiffness, \(K_{2,3}\) | \(5.403 \times 10^8\) |
| Rotational Stiffness, \(K_4\) | \(1.000 \times 10^5\) |
| Rotational Stiffness, \(K_{5,6}\) | \(3.458 \times 10^9\) |

3. FE Modelling and Analysis of the Bolted Joints

The FE model of the structure with bolted joints was developed as shown in Figure 4 and Figure 5. The model was discretised into a combination of 1977 CQUAD8 and 30 CTRI6 elements with the size of 5 mm. The size was chosen based on the results of convergent tests carried out [12]. Total number of nodes is 6745 with a total number of 40470 degrees of freedom. In each bolted joint model, CELAS elements were used to represent the jointed members’ stiffness, CBUSH elements were used to represent the bolts, while RBE3 elements were used to represent the bolts’ heads and nuts.

![Figure 4](image1.png)  
**Figure 4.** FE model of the assembled structure with bolted joints.

![Figure 5](image2.png)  
**Figure 5.** Representation of the members and bolts stiffnesses by CELAS and CBUSH respectively.

In this work, normal modes analysis for the developed FE model of the assembled structure with bolted joints was performed using the NX11 software packages. The analysis was performed to predict the first ten natural frequencies and mode shapes of the assembled structure. The predicted results were then compared with the measured results of the assembled structure obtained from experimental modal analysis.

4. Experimental Modal Analysis (EMA)

The modal parameters, which are the natural frequencies and mode shapes, of the structure with bolted joints were characterised by performing EMA. The equipment packages used to characterise the modal parameters were the LMS system, LMS Test.Lab software, impact hammer & accelerometers as shown in Figure 6. The assembled structure consisting of 2 identical plates namely Plate 1 and Plate 2, was made from steel beams. The experimental setup for the assembled structure is shown in Figure 7. Plate 1 and Plate 2 were joined by using the stainless steel M10 bolts and nuts. The assembled structure under the test was suspended from the test rig by using rubber bands to simulate free-free boundary conditions.
5. Results and Discussion
The predicted natural frequencies were compared with the measured counterparts to compute discrepancies in terms of total error. The comparison of the first 10 natural frequencies between the predicted and measured results of the assembled structure is tabulated in Table 4. The comparison of the results show that the total error recorded is 14.573%, while the modal assurance criterion (MAC) values show a very good correlation between the predicted and measured mode shapes. The results also revealed that the proposed FE modelling method embodying the representation of the members’ and bolts’ stiffnesses has contributed significantly towards an accurate modelling of the assembled structure with bolted joints, especially for the first 3 modes.

Table 4. Comparison between the measured and predicted results for the assembled structure.

| Mode | EMA (Hz) | FE (Hz) | Error between I & II (%) | MAC | Mode of vibration |
|------|----------|---------|--------------------------|-----|--------------------|
| 1    | 75.08    | 75.28   | 0.266                    | 0.982 | Bending z-direction |
| 2    | 200.77   | 202.44  | 0.832                    | 0.940 | Bending z-direction |
| 3    | 404.92   | 401.63  | 0.813                    | 0.992 | Bending z-direction |
| 4    | 469.30   | 446.93  | 4.767                    | 0.945 | Bending y-direction & torsion |
| 5    | 630.77   | 632.62  | 0.293                    | 0.973 | Bending, z-direction |
| 6    | 674.39   | 661.27  | 1.945                    | 0.960 | Torsion |
| 7    | 1034.64  | 1019.16 | 1.496                    | 0.944 | Bending z-direction |
| 8    | 1053.23  | 1067.47 | 1.352                    | 0.910 | Torsion |
| 9    | 1276.95  | 1278.41 | 0.114                    | 0.957 | Bending z-direction |
| 10   | 1435.09  | 1473.76 | 2.695                    | 0.937 | Bending y-direction & torsion |
|      | Total Error | 14.573 |                  |     |                   |

6. Conclusions
The proposed FE modelling method incorporating the contribution of the joints stiffnesses into the investigation of the dynamic behaviour of the assembled structure with bolted joints is presented. The comparison between the predicted and measured modal parameters revealed that the members’ and bolts’ stiffness values have played a vital role in improving the accuracy of the prediction of the dynamic behaviour of the assembled structure with bolted joints. The accuracy and reliability of the predicted results of the FE model could be further improved by using the FE model updating method.
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