Measuring Given Partial Information about Intuitionistic Fuzzy Sets

Priya Arora¹*, V. P. Tomar²

¹Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonepat, 131039, India
²Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonepat, India

Received June 27, 2020; Revised October 12, 2020; Accepted October 24, 2020

Cite This Paper in the following Citation Styles

(a): [1] Priya Arora, V. P. Tomar , "Measuring Given Partial Information about Intuitionistic Fuzzy Sets," Mathematics and Statistics, Vol. 8, No. 6, pp. 665 - 670, 2020. DOI: 10.13189/ms.2020.080606.

(b): Priya Arora, V. P. Tomar (2020). Measuring Given Partial Information about Intuitionistic Fuzzy Sets. Mathematics and Statistics, 8(6), 665 - 670. DOI: 10.13189/ms.2020.080606.

Copyright©2020 by authors, all rights reserved. Authors agree that this article remains permanently open access under the terms of the Creative Commons Attribution License 4.0 International License

Abstract

Background: Measuring the information and removal of uncertainty are the essential nature of human thinking and many world objectives. Information is well used and beneficial if it is free from uncertainty and fuzziness. Shannon was the primitive who coined the term entropy for measure of uncertainty. He also gave an expression of entropy based on probability distribution. Zadeh used the idea of Shannon to develop the concept of fuzzy sets. Later on, Atanassov generalized the concept of fuzzy set and developed intuitionistic fuzzy sets. Purpose: Sometimes we do not have complete information about fuzzy set or intuitionistic fuzzy sets. Some partial information is known about them i.e either only few values of membership function \( \mu_A(x_i) \) or non membership function \( \upsilon_A(x_i) \) are known or a relationship between them is known or some inequalities governing these parameters are known. Kapur has measured the partial information given by a fuzzy set. In this paper, we have attempted to quantify partial information given by intuitionistic fuzzy sets by considering all the cases. Methodologies: We analyze some well-known definitions and axioms used in the field of fuzzy theory. Principal Results: We have devised methods to measure the incomplete information given about intuitionistic fuzzy sets. Major Conclusions: By devising the methods of measuring partial information about IFS, we can use this information to get an idea about the given set and use this information wisely to make a good decision.

Keywords

Fuzzy Sets, Intuitionistic Fuzzy Sets, Entropy

1. Introduction

Information theory was developed by Shannon [1] in 1948. Shannon [1] was the first to utilize the term entropy for measuring the information. Let \( X \) be a discrete random variable with probability distribution \( P = (p_1, p_2, ..., p_n) \) in an experiment. The information contained in this experiment is given by \( H(P) = - \sum_{i=1}^{n} p_i \ln(p_i) \) [1] which is well known as Shannon’s entropy. Then this notion of entropy was extended and used to measure the information provided by Fuzzy Set. De Luca and Termini [2] suggested the following measure of fuzzy entropy:

\[
H(A) = - \sum_{i=1}^{n} \left[ \mu_A(x_i) \ln \mu_A(x_i) + \left(1 - \mu_A(x_i)\right) \ln\left(1 - \mu_A(x_i)\right) \right]
\]

(1)

where \( A \) is the fuzzy set defined on universe of discourse \( X = \{x_1, x_2, ..., x_n\} \). Kaufman [3] defined measure of entropy of fuzzy set as

\[
H_{KF} = - \frac{1}{\ln n} \sum_{i=1}^{n} \phi_A(x_i) \ln \phi_A(x_i)
\]

where \( \phi_A(x_i) = \frac{\mu_A(x_i)}{\sum_{i=1}^{n} \mu_A(x_i)} \), \( i = 1, 2, ..., n \).

Bhandari and Pal [4] defined the measure of fuzzy entropy as

\[
H_{BP} = \frac{1}{1 - \alpha} \sum_{i=1}^{n} \left[ \ln \left( \frac{\mu_A(x_i)}{1 - \alpha} \right) + \left(1 - \mu_A(x_i)\right)^{\alpha} \right],
\]

where \( \alpha \neq 1, \alpha > 0 \).

Likewise, numerous other entropy measures were developed for fuzzy sets. Fuzzy sets theory has become...
very useful because of its wide applications. In recent times, Rana [5] studied fuzzy models and did a comparison study for crop production forecasting. Radzi and Ahmed [6] worked on fuzzy soft sets and found its application in TOPSIS. Anuradha, mehra et al. [7] worked on Archimedean Fuzzy M-Metric Space and Fixed Point Theorems. Atanassov [8] developed the concept of Intuitionistic Fuzzy sets (IFS). IFS are actually a generalization of fuzzy sets. IFS are characterized by two functions: membership function and non membership function. Atanassov [8] defined a new term hesitancy degree for the intuitionistic fuzzy sets. Entropy measures were also developed for intuitionistic fuzzy sets. Burillo and Bustince [9] were first to characterize the entropy on IFS. Szmidt and Kacprzyk [10] used the axioms of De luca and Termi to characterize non probabilistic intuitionistic fuzzy entropy. Hung and Yang [11] suggested the intuitionistic fuzzy entropy based on the distance measure between IFSs. Vlachos and Sergiagis [12] developed a mathematical connection between fuzzy entropy and intuitionistic fuzzy entropy and proposed a measure of intuitionistic fuzzy entropy. Afterwards, Zhang and Jiang [13] generalized the De Luca Termi [2] logarithmic fuzzy entropy to define a measure of intuitionistic fuzzy entropy. Ye, Wei et al. [14, 15] gave other types of intuitionistic entropy to define a measure of intuitionistic fuzzy entropy. [13] generalized the De Luca Termini [2] logarithmic fuzzy intuitionistic fuzzy entropy and proposed a measure of mathematical connection between fuzzy entropy and intuitionistic fuzzy sets. Buillo and Bustince [9] were first to characterize the entropy on IFS. Szmidt and Kacprzyk [10] used the axioms of De luca and Termi to characterize non probabilistic intuitionistic fuzzy entropy. Hung and Yang [11] suggested the intuitionistic fuzzy entropy based on the distance measure between IFSs. Vlachos and Sergiagis [12] developed a mathematical connection between fuzzy entropy and intuitionistic fuzzy entropy and proposed a measure of intuitionistic fuzzy entropy. Recently, Wei and Li et.al [21] have devised a novel generalized exponential entropy for IFS. Information theory is concerned with quantitatively measuring the information. But sometimes, complete information is not given. Only a partial knowledge about the variables is given. In 1997, Kapur [22] tried to measure that partial information provided by the fuzzy set using the entropy measure given by De luca and Termi.

He used the term fuzziness gap for the difference between maximum and minimum value of the entropy. He came to know that this fuzziness gap gets reduced if we know some information about the IFS. So, he used this fuzziness gap to quantify the partial information given by IFS. Then in 2017, N.Singh [23] discussed the various methods to measure the partial information using the entropy formula given by De luca and termi, Kaufmann and Bhandari and Pal.

In this paper, we will generalize the ideas of Kapur [22] from fuzzy sets to intuitionistic fuzzy sets so as to measure the partial information given about them.

2. Methodology

Now, we study some well-known definitions and concepts related to fuzzy sets and intuitionistic fuzzy sets given by various researchers.

2.1. Basic Definitions and Preliminaries

Zadeh [24] introduced the concept of fuzzy set.

Definition 1: Fuzzy set: Let \( X = \{ x_1, x_2, \ldots, x_n \} \) be the universe of discourse, then a fuzzy set \( A \) defined on \( X \) is

\[
A = \{ < x, \mu_A(x) > | x \in X \}
\]

where \( \mu_A(x): X \rightarrow [0,1] \) is the degree of membership of \( x \) in \( A \). [11]

Definition 2: Intuitionistic fuzzy set: Intuitionistic fuzzy set \( A \) on \( X \) is

\[
A = \{ < x, \mu_A(x), \nu_A(x) > | x \in X \}
\]

where \( \mu_A(x): X \rightarrow [0,1] \) is the degree of membership and \( \nu_A(x): X \rightarrow [0,1] \) is the degree of non membership where \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).

\( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) is called the hesitancy degree or the degree of indeterminacy [8]

Definition 3: A real function \( e: \text{FS}(X) \rightarrow \mathbb{R}^+ \) is called entropy on \( \text{FS}(X) \) if \( e \) satisfies the following properties:

\begin{enumerate}
  
  \item \( (e1) \ e(A)=0 \) if \( A \) is a crisp set.
  
  \item \( (e2) \ e(A) \) achieves its maximum at \( \mu_A = 0.5 \)
  
  \item \( (e3) \ e(A) \leq e(B) \) if \( A \) is more crisp than \( B \) i.e. \( \mu_A \leq \mu_B \) for \( \nu_A \leq 0.5 \) and \( \mu_A \geq \mu_B \) for \( \mu_B \geq 0.5 \).
  
  \item \( (e4) \ e(A) = e(A^c) \) where \( A^c \) denotes the complement of \( A \). [2]
\end{enumerate}

Definition 4: A real function \( E: \text{IFS}(X) \rightarrow \mathbb{R}^+ \) is called an entropy on \( \text{IFS}(X) \) if \( E \) satisfies the following properties:

\begin{enumerate}
  
  \item \( (E1) \ E(A)=0 \) if \( A \) is a crisp set.
  
  \item \( (E2) \ E(A) \) achieves its maximum at \( \mu_A = \pi_A = 1/3 \)
  
  \item \( (E3) \ E(A) \leq E(B) \) if \( A \) is more crisp than \( B \) i.e. \( \mu_A \leq \mu_B \) and \( \nu_A \leq \nu_B \) for \( \max \{ \mu_B, \nu_B \} \leq 1/3 \) and \( \mu_A \geq \mu_B \) and \( \nu_A \geq \nu_B \) for \( \min \{ \mu_B, \nu_B \} \geq 1/3 \)
  
  \item \( (E4) \ E(A) = E(A^c) \) where \( A^c \) denotes the complement of \( A \). [11]
\end{enumerate}

Langrange’s Method of Multipliers:

Let \( f(x,y,z) \) be a function of variables \( x,y \) and \( z \). We want to find maximum or minimum value of \( g(x,y,z) \) subjected to the constraint \( h(x,y,z)=k \) where \( k \) is a real number.

\begin{enumerate}
  
  \item For stationary points, put \( \frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0, \frac{\partial f}{\partial z} = 0 \)
  
\end{enumerate}
Solving, the above three equations, we get the value of \( x, y, z \) for which the value of \( f(x, y, z) \) is extreme.

2.2. Fuzziness Gap

Hung and Yang [11] defined entropy for IFS as
\[
E_{HV} = -\sum_{i=1}^{n} \mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) + \pi_A(x_i) \ln \pi_A(x_i)
\]

This entropy defined by Hung and Yang achieves maximum at \( \mu_A(x_i) = \nu_A(x_i) = \pi_A(x_i) = 1/3 \). We will use the above measure of entropy in the rest of our paper.

Let \( A \) be an IFS. Suppose, we have only partial information about \( \mu_A(x) \) and \( \nu_A(x) \) i.e. we know only few of \( \mu_A 's \) and \( \nu_A 's \) or we know some relations between them. We know that, the information provided by such an IFS is incomplete. Now, we try to quantitatively measure the information provided about such an IFS by knowing only a few values of \( \mu_A 's \) and \( \nu_A 's \).

As we know, entropy for IFS given by Hung and Yang is
\[
G(A) = -\sum_{i=1}^{n} \mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) + \pi_A(x_i) \ln \pi_A(x_i)
\]

This (1) measures the fuzziness associated with intuitionistic fuzzy sets.

Suppose we don’t know anything about IFS \( A \) i.e none of the values of \( \mu_A(x_i) \) and \( \nu_A(x_i) \) are known. Then the maximum and minimum value of \( G \) is:
\[
G_{\text{max}} = \ln n3 \quad \text{for IFS } (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})
\]
\[
G_{\text{min}} = \text{for crisp set.}
\]

The difference between the values of \( G_{\text{max}} \) and \( G_{\text{min}} \) is called Fuzziness Gap.

So, in this case, Fuzziness Gap = \( G_{\text{max}} - G_{\text{min}} \)
\[
= \ln n3 - 0
\]
\[
= \ln n3
\]

3. Results

If we know something about an IFS like values of some of the \( \mu_A 's \) and \( \nu_A 's \) or some relationships governing them, then that information is known as partial information about the IFS. The fuzziness gap can increase or decrease depending upon the information given by the partial knowledge. Hence,

Information given by partial knowledge = Fuzziness Gap before we use the knowledge − Fuzziness Gap after we use the knowledge.

Now, we illustrate the methods of measuring given partial information in different cases.

3.1. Methods of Measuring Given Partial Information in Different Cases

Case I: When none of the values of \( \mu_A 's \) and \( \nu_A 's \) are known

Then the maximum and minimum value of \( G \) is:
\[
G_{\text{max}} = \ln n3 \quad \text{for IFS } (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})
\]
\[
G_{\text{min}} = 0 \quad \text{for crisp set.}
\]

Fuzziness Gap = \( G_{\text{max}} - G_{\text{min}} = (n-1)\ln n3 \) (discussed above)

Case II: When one or more values of \( \mu_A, \nu_A \) are given

Suppose \( (\mu_A(x_k), \nu_A(x_k)) \) is known

Then the term containing \( \mu_A(x_k), \nu_A(x_k) \) in \( G_{\text{max}} \) got fixed and remaining \((n-1)\) terms are completely unknown.

\[
G_{\text{max}} = -\mu_A(x_k) \ln \mu_A(x_k) - \nu_A(x_k) \ln \nu_A(x_k) - \pi_A(x_k) \ln \pi_A(x_k) - (n-1)\ln n3
\]

\[
G_{\text{min}} = -\mu_A(x_k) \ln \mu_A(x_k) - \nu_A(x_k) \ln \nu_A(x_k) - \pi_A(x_k) \ln \pi_A(x_k) - 0
\]

Fuzziness gap = \( G_{\text{max}} - G_{\text{min}} = (n-1)\ln n3 \)

We have seen that the fuzziness gap before we know this knowledge is \( \ln n3 \). So, by knowing \( (\mu_A(x_k), \nu_A(x_k)) \), there is a reduction in fuzziness gap being equal to \((n-1)\ln n3 \). Therefore, the information provided by \( (\mu_A(x_k), \nu_A(x_k)) \) is \( \ln n3 \). If we know \( r \) values of \( \mu_A \) and \( \nu_A \) then the corresponding fuzziness gap will be \((n-r)\ln n3 \) and information provided by them will be \( \ln n3 \).

Hence, the fuzziness gap is getting reduced if we are getting information about the IFS.

Case III: When one or more relationships among \( \mu_A 's \) and \( \nu_A 's \) are given.

Suppose we are given that
\[
\mu_A(x_1) + \mu_A(x_2) + \ldots \ldots + \mu_A(x_n) = k_1
\]
\[
\nu_A(x_1) + \nu_A(x_2) + \ldots \ldots + \nu_A(x_n) = k_2
\]

For determining \( G_{\text{max}} \), we use Lagrange’s Method of Multipliers.

Let \( f = -\sum_{i=1}^{n} \left[ \mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) + \pi_A(x_i) \ln \pi_A(x_i) \right] + \lambda_1 \left( \mu_A(x_1) + \mu_A(x_2) + \ldots \ldots + \mu_A(x_n) - k_1 \right) + \lambda_2 \left( \nu_A(x_1) + \nu_A(x_2) + \ldots \ldots + \nu_A(x_n) - k_2 \right) = 0
\]

For \( G_{\text{max}} \), we have
\[
\frac{\partial f}{\partial \mu_A(x_i)} = 0, \quad \frac{\partial f}{\partial \nu_A(x_i)} = 0
\]
\[
\frac{\partial f}{\partial \mu_A(x_i)} = 0 \Rightarrow -\ln \mu_A(x_i) - 1 + \ln \pi_A(x_i) + \lambda_1 = 0
\]
\[
\Rightarrow \ln \frac{\mu_A(x_i)}{\pi_A(x_i)} = \lambda_1 \quad (9)
\]
\[
\Rightarrow \ln \frac{\mu_A(x_i)}{\nu_A(x_i)} = \lambda_1 \quad (10)
\]
\[
\Rightarrow \frac{\mu_A(x_i)}{\nu_A(x_i)} = e^{\lambda_1} \quad (11)
\]
\[
\Rightarrow \frac{e^{\lambda_1} - \nu_A(x_i) e^{\lambda_1}}{1 - \mu_A(x_i) \nu_A(x_i)} = e^{\lambda_1}
\]
\[
\Rightarrow (1 + e^{\lambda_1}) \mu_A(x_i) = e^{\lambda_1} - \nu_A(x_i) e^{\lambda_1} \quad (13)
\]
Similarly, from \( \frac{\partial f}{\partial v_A(x_i)} = 0 \), we get
\[
(1 + e^{\lambda_2}) v_A(x_i) = e^{\lambda_2} - \mu_A(x_i) e^{\lambda_2}
\] (14)
Solving equation (13) and (14) simultaneously, we get
\[
\Rightarrow \mu_A(x_i) = \frac{e^{\lambda_2}}{1 + e^{\lambda_1} + e^{\lambda_2}}
\] (15)
But \( \sum_{i=1}^{n} \mu_A(x_i) = k_1 \)
\[
\Rightarrow n \cdot \frac{e^{\lambda_1}}{1 + e^{\lambda_1} + e^{\lambda_2}} = k_1 \Rightarrow \frac{e^{\lambda_1}}{1 + e^{\lambda_1} + e^{\lambda_2}} = \frac{k_1}{n}
\]
\[
\Rightarrow \mu_A(x_i) = \frac{k_1}{n} \forall i
\] (16)
Similarly, \( v_A(x_i) = \frac{k_2}{n} \forall i \) (17)

If \( k_1 \) and \( k_2 \) are positive integers
Fuzziness Gap = \( G_{\max} - G_{\min} = -n \frac{k_1}{n} \ln \frac{k_1}{n} + \frac{k_2}{n} \ln \frac{k_2}{n} + (1 - \frac{k_1}{n} - \frac{k_2}{n}) \ln (1 - \frac{k_1}{n} - \frac{k_2}{n}) \)
\[
= -[k_1 \ln k_1 n + k_2 \ln k_2 n + (n - k_1 - k_2) \ln (1 - \frac{k_1}{n} - \frac{k_2}{n})]
\]
Subtract it from \( n \ln 3 \) to get the information provided by the constraint.

If \( k_1 \) and \( k_2 \) are not integers
Fuzziness Gap = \( G_{\max} - G_{\min} \)
\[
G_{\max} = -[k_1 \ln \frac{k_1}{n} + k_2 \ln \frac{k_2}{n} + (n - k_1 - k_2) \ln (1 - \frac{k_1}{n} - \frac{k_2}{n})]
\]
For \( G_{\min} \), \( \mu_A(x_i) = k_1 - [k_1] \) for some \( x_i \)
\[
v_A(x_i) = k_2 - [k_2] \) for some \( x_j \)
where \( i, j \) are chosen such that \( G_{\min} \) is minimum.

If \( k_1 - [k_1] + k_2 - [k_2] \leq 1 \)
Then,
\[
G_{\min} = [(k_1 - [k_1]) \ln (k_1 - [k_1]) + (k_2 - [k_2]) \ln (k_2 - [k_2]) + (1 - (k_1 - [k_1]) - (k_2 - [k_2])) \ln (1 - (k_1 - [k_1]) - (k_2 - [k_2]))]
\]
Subtracting it from \( n \ln 3 \) will give information given by the constraint.

If \( k_1 \) is a positive integer but \( k_2 \) is not an integer
Fuzziness Gap = \( G_{\max} - G_{\min} \)
\[
= -(k_1 \ln \frac{k_1}{n} + k_2 \ln \frac{k_2}{n} + (n - k_1 - k_2) \ln (1 - \frac{k_1}{n} - \frac{k_2}{n}) + [(k_2 - [k_2]) \ln (k_2 - [k_2]) + (1 - (k_2 - [k_2])) \ln (1 - (k_2 - [k_2]))]
\]
And subtract it from \( n \ln 3 \) to get the information provided by the constraint.
Similarly, it can be done for \( k_2 \) being positive integer but not \( k_1 \)

Case IV: When equality constraints are given
Suppose, for an IFS it is given that
\[
c_1 \mu_A(x_1) + c_2 \mu_A(x_2) + \ldots + c_n \mu_A(x_n) = k_1 \] (18)
\[
c_1' v_A(x_1) + c_2' v_A(x_2) + \ldots + c_n' v_A(x_n) = k_2 \] (19)
For finding \( G_{\max} \) and \( G_{\min} \), we again use Langrange's method of multipliers, Let
\[
f = -\sum_{i=1}^{n} (\mu_A(x_i) \ln \mu_A(x_i) + v_A(x_i) \ln v_A(x_i) + \pi_A(x_i) \ln \pi_A(x_i)) + \lambda_1 (c_1 \mu_A(x_1) + c_2 \mu_A(x_2) + \ldots + c_n \mu_A(x_n) - k_1) + \lambda_2 (c_1' v_A(x_1) + c_2' v_A(x_2) + \ldots + c_n' v_A(x_n) - k_2) = 0 \] (20)
\[
\frac{\partial f}{\partial \mu_A(x_i)} = 0 \Rightarrow -n \mu_A(x_i) - 1 + \ln \mu_A(x_i) + 1 + \lambda_1 c_i = 0 \] (21)
\[
\Rightarrow \ln 1 - \mu_A(x_i) - v_A(x_i) = -\lambda_1 c_i \] (22)
\[
\Rightarrow \frac{1 - \mu_A(x_i) - v_A(x_i)}{\mu_A(x_i)} = e^{-\lambda_1 c_i} \] (23)
\[
\Rightarrow \mu_A(x_i) = e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - v_A(x_i)) \] (24)
Similarly, \( \frac{\partial f}{\partial v_A(x_i)} = 0 \Rightarrow v_A(x_i) = e^{\lambda_1 c_i} (1 - \mu_A(x_i)) \] (25)
Solving equations (23) and (25) simultaneously, we get
\[
\mu_A(x_i) = e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - \mu_A(x_i))
\]
\[
= e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - \mu_A(x_i)))
\] (26)
\[
\Rightarrow \mu_A(x_i) = e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - \mu_A(x_i)))}
\] (27)
\[
\Rightarrow \mu_A(x_i) = e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - \mu_A(x_i)))}
\] (28)
Similarly, \( v_A(x_i) = e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - e^{\lambda_1 c_i} \frac{1}{1 + e^{\lambda_1 c_i}} (1 - \mu_A(x_i)))}
\] (29)
Where \( \lambda_1, \lambda_2 \) can be determined from the equations
\[
c_1 \mu_A(x_1) + c_2 \mu_A(x_2) + \ldots + c_n \mu_A(x_n) = k_1 \]
\[
c_1' v_A(x_1) + c_2' v_A(x_2) + \ldots + c_n' v_A(x_n) = k_2 \]
\( G_{\min} \) can be found out by taking \( n-1 \) \( \lambda_A \)'s zero or one and remaining one is determined by the constraints (18) and (19).

Case V: When inequality constraints are given
Suppose we are given that
\[
t_1 \leq c_1 \mu_A(x_1) + c_2 \mu_A(x_2) + \ldots + c_n \mu_A(x_n) \leq t_2
\] (30)
\[
t_1' \leq c_1' v_A(x_1) + c_2' v_A(x_2) + \ldots + c_n' v_A(x_n) \leq t_2'
\] (31)
In this case, we first find the maximum value of \( G \) corresponding to:
\[
c_1 \mu_A(x_1) + c_2 \mu_A(x_2) + \ldots + c_n \mu_A(x_n) = t
\] (32)
By following up the method of Case 4, we will get

$$G_{\text{max}} \text{ corresponding to } t = \tilde{t} \text{ and } t' = \tilde{t}' .$$

Now, if the values of \( t \) and \( t' \) lies between \( t_1 \) & \( t_2 \) and \( t_1' \) and \( t_2' \) respectively then we are done.

If not, then we have to put \( t = t_1 \) or \( t_2 \) and \( t' = t_1' \) or \( t_2' \) accordingly depending upon the value which gives maximum value of \( G \).

For minimum value of \( G \), we will keep one of \( \mu_A \)’s and

\( (n-r) \ln 3 = \frac{k_2}{n} + \ln \left( \frac{k_1}{n} \right) \frac{k_2}{n} + (n - k_1 - k_2)\ln \left( 1 - \frac{k_1}{n} - \frac{k_2}{n} \right) \) \[ \] \((k_1 - [k_2])\ln (k_1 - [k_2]) + (k_2 - [k_2])\ln (k_2 - [k_2]) + (1 - (k_1 - [k_2]) - (k_2 - [k_2])\ln (1 - (k_1 - [k_2]) - (k_2 - [k_2])) \]

Putting the values of \( k_1, k_2 \) and \( n \), we get

Fuzziness Gap of A after using the information = 2.566

Therefore, the partial information provided by the constraint about company A=4ln3 -2.566 =1.828

In case of B, \( k_1 = 2, k_2 = 1.8 \) and \( n = 4 \)

By Case III, Fuzziness Gap of B after using the information

\[ = -[k_1 \frac{k_1}{n} + \frac{k_2}{n} (n - k_1 - k_2)\ln (1 - \frac{k_1}{n} - \frac{k_2}{n}) + ((k_2 - [k_2])\ln (k_2 - [k_2]) + (1 - (k_1 - [k_2]) - (k_2 - [k_2])\ln (1 - (k_1 - [k_2]) - (k_2 - [k_2]))] \]

Putting the values of \( k_1, k_2 \) and \( n \), we get

Fuzziness Gap of B after using the information = 2.9221

Therefore, the partial information provided by the constraint about company B=4ln3-2.9221=1.4723

So, company A is more transparent in its policies.

3.2. Numerical Examples

Example: Suppose there are two companies A and B. A person wants to invest in one of the companies depending upon the transparency in their policies given by \( x_1, x_2, x_3, x_4 \). Then \( \mu_A(x_i) \) denotes the clarity in policy \( x_i \) and \( v_A(x_i) \) denotes the opacity in policy \( x_i \). Information about company A is given by

\[ (a) \quad A=\{ (x_1, 0.7,0.2), (x_2, 0.5,0.3), (x_3, \mu_A(x_3), v_A(x_3)), (x_4, \mu_A(x_4), v_A(x_4)) \} \]

where \( \mu_A(x_1), v_A(x_1), \mu_A(x_2), v_A(x_2) \) are unknown and

\[ B=\{ (x_1, 0.8, 0.1), (x_2, \mu_A(x_2), v_A(x_2)), (x_3, \mu_A(x_3), v_A(x_3)), (x_4, \mu_A(x_4), v_A(x_4)) \} \]

where \( \mu_A(x_2), v_A(x_2), \mu_A(x_3), v_A(x_3), \mu_A(x_4), v_A(x_4) \) are unknown. Which company is more transparent in its policies?

(a) \( \mu_A(x_1) + \mu_A(x_2) + \mu_A(x_3) + \mu_A(x_4) = 2 \)

and \( v_A(x_1) + v_A(x_2) + v_A(x_3) + v_A(x_4) = 1.3 \)

(b) \( \mu_B(x_1) + \mu_B(x_2) + \mu_B(x_3) + \mu_B(x_4) = 2 \)

and \( v_B(x_1) + v_B(x_2) + v_B(x_3) + v_B(x_4) = 1.8 \).

Which company is more transparent in its policies?

Answer: (a) As value of membership and non membership function are known only for two values. By Case II, Fuzziness gap for company A after using the knowledge = (n-r) ln3 = (4-2)ln3 = 2ln3.

Fuzziness gap before using the knowledge for company A = 4ln3

Partial information for company A given by the knowledge = 4 ln3 - 2 ln3 = 2ln3

Fuzziness gap for company B after using the knowledge = (n-r) ln3 = (4-1)ln3 = 3ln3.

Fuzziness gap before using the knowledge for company B = 4ln3

Partial information for company B given by the knowledge = 4 ln3 - 3 ln3 = ln3

As information provided by company A is more than the information provided by the company B, company A is more transparent in its policies.

(b) Here in case of A, \( k_1 = 2.3, k_2 = 1.3 \) and \( n = 4 \)

By Case III, Fuzziness Gap of A after using the information

\[ = \left[ k_1 \ln \frac{k_1}{n} + k_2 \ln \frac{k_2}{n} + (n - k_1 - k_2)\ln \left( 1 - \frac{k_1}{n} - \frac{k_2}{n} \right) \right] \]

REFERENCES

[1] C.E Shannon, “The Mathematical theory of Communication”, Bell Syst. Tech. Journal, vol. 27, pp. 425 - 467, 1948.
[2] De Luca, S. Termini, “A definition of a non probabilistic entropy in the setting of fuzzy sets theory”. Information Control., vol. 20, pp. 301–312, 1972.

[3] A. Kaufmann, “Introduction to the Theory of Fuzzy Subsets”, Academic–Press, New York 1975.

[4] D. Bhandari and N. R. Pal, “Some new information measures for fuzzy sets,” Information Sciences, vol. 67, no. 3, pp. 209–228, 1993.

[5] Amit Kumar Rana, "Comparative Study on Fuzzy Models for Crop Production Forecasting," Mathematics and Statistics, Vol. 8, No. 4, pp. 451 - 457, 2020. DOI: 10.13189/ms.2020.080412.

[6] Zahari Md Rodzi, Abd Ghafur Ahmad, "Fuzzy Parameterized Dual Hesitant Fuzzy Soft Sets and Its Application in TOPSIS," Mathematics and Statistics, Vol. 8, No. 1, pp. 32 - 41, 2020. DOI: 10.13189/ms.2020.080104.

[7] Anuradha, Seema Mehra, Said Broumi, "Non-Archimedean Fuzzy M-Metric Space and Fixed Point Theorems Endowed with a Reflexive Digraph," Mathematics and Statistics, Vol. 7, No. 5, pp. 229 - 238, 2019. DOI:10.13189/ms.2019.070509.

[8] K.T Atanassov, “Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems”, vol.20, pp. 87-96, 1986.

[9] P. Burillo, H. Bustince, “Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets”, Fuzzy Sets and Systems, vol. 78, pp. 305-316, 1996.

[10] E. Szmidt, J. Kacprzyk, “Entropy for intuitionistic fuzzy sets”, Fuzzy Sets and Systems vol.118, pp. 467-477, 2001.

[11] W. L. Hung and M.S Yang, “Fuzzy entropy on Intuitionistic Fuzzy Sets”, Int J Intell Syst, vol. 21. pp. 443-451, 2006.

[12] I. K. Vlachos and G. D. Sergiagis, “Pattern Recognition Letters”, vol. 28, pp. 197–206, 2007.

[13] Q. S. Zhang and S. Y. Jiang: “A note on information entropy measure for vague sets.” Inform. Sci., vol. 178, no. 21, pp. 4184–4191, 2008.

[14] J. Ye, “Two effective measures of intuitionistic fuzzy entropy” Computing vol.87, pp. 55–62, 2010.

[15] C. Wei, Z. Gao, T. Guo, “An intuitionistic fuzzy entropy measure based on trigonometric function.” Control Decision. vol.27, pp. 571–574, 2012.

[16] R. Verma, B.D Sharma, “Exponential entropy on intuitionistic fuzzy sets”, Kybernetika vol.49, pp. 114–127, 2013

[17] T. Chen, C. Li, “Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis”, Information Sciences, vol. 180, pp. 4207–4222, 2010.

[18] R. Joshi, S. Kumar, “A new parametric intuitionistic fuzzy entropy and its applications in multiple attribute decision making”, International Journal of applied and computational mathematics, vol. 4, no. 52, 2018.

[19] A. Alusfyani, H.B Owny, “Exponential intuitionistic fuzzy entropy measure based image edge detection”, International Journal of Applied Engineering Research, vol. 13, no. 10, pp. 8518-8524, 2018.

[20] S. Yin, Z. Yang, S.Y Chen, “Interval-valued intuitionistic fuzzy multiple attribute decision making based on the improved fuzzy entropy”. Syst. Eng. Electron. vol.40, no.5, 1079–1084, 2018.

[21] A. Wei, D. Li et al., “The Novel Generalized Exponential Entropy for Intuitionistic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets”, vol. 21, 2327–2339, 2019.

[22] J.N Kapur, “Measures of Fuzzy Information”, Mathematical Science Trust Society, New Delhi, 1997.

[23] N Singh, “On information measure for fuzzy set”, Advances in Fuzzy Mathematics, vol 12, no. 2, pp. 255-261, 2017.

[24] L.A. Zadeh, “Fuzzy Sets”, Information and Control, vol.8, pp. 338-353, 1965.