Fuzzy Modal Control Applied to Smart Composite Structure

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Abstract. This paper proposes an active vibration control technique, which is based on Fuzzy Modal Control, as applied to a piezoelectric actuator bonded to a composite structure forming a so-called smart composite structure. Fuzzy Modal Controllers were found to be well adapted for controlling structures with nonlinear behavior, whose characteristics change considerably with respect to time. The smart composite structure was modelled by using a so called mixed theory. This theory uses a single equivalent layer for the discretization of the mechanical displacement field and a layerwise representation of the electrical field. Temperature effects are neglected. Due to numerical reasons it was necessary to reduce the size of the model of the smart composite structure so that the design of the controllers and the estimator could be performed. The role of the Kalman Estimator in the present contribution is to estimate the modal states of the system, which are used by the Fuzzy Modal controllers. Simulation results illustrate the effectiveness of the proposed vibration control methodology for composite structures.

1. Introduction
The recent years have seen the appearance of innovative materials, such as the so-called composite materials, particularly in aerospace applications. The structures constructed by this innovated arrangement are characterized by lightness, mechanical resistance, and the possibility of optimization for a specific working condition. Unlike the regular materials (steel, aluminum, etc), the composites are formed by various layers with different fiber orientations, which allows them to be adequate for particular applications [1]. Aircraft, aerospace and automotive industries are examples for which composite materials have been increasingly used.

Generally, piezoelectric layers (PZT sensor/actuator patches) are incorporated to the composite materials in order to offer potential benefits in a wide range of applications such as structural health
monitoring, noise suppression, precision positioning and active vibration control [2]. Thus, the set
encompassing the composite material, piezoelectric layers and monitoring and control systems, is
known as Smart Composite Structure.

Currently, it is observed an increasing number of research works in engineering devoted to the
development of new active vibration control techniques (AVC). Among the well-known techniques
for AVC, nowadays the Active Vibration Modal Control (AVMC) is a highlight and presents
successful applications in several areas [3-8].

In the present contribution the active vibration modal control is tested only numerically. The
control approach now studied is the Fuzzy Logic Controller (FLC). In this case, the modal
displacement and modal velocities are used by the controllers to determine the control force, and the
controllers considered were the Fuzzy Modal Controllers (FMCs). The FMCs were found to be well
adapted for controlling structures with nonlinear behavior, whose characteristics change considerably
with respect to time. Since the modal states cannot be accessed directly during experimentation, they
have to be rebuilt by using an estimator algorithm. Consequently, for determining the modal states the
Kalman estimator was chosen. As mentioned above, numerical results are presented to illustrate
the proposed methodology. Additionally, the balanced realization method is used to reduce the
complexity of the computational model for only the most relevant vibration modes of the structure in
order to increase the efficiency of the controller. The simulations are performed so that the procedure
is as similar as possible to the experimental conditions.

2. System Modeling

The coupling between the composite structure and the piezoelectric element is made through the
Hamilton's variational principle that incorporates all energy contributions presented in the structure.

According to Chee [9], the coupling elementary matrices using Mixed Theory are the following:

\[ [M^e] = \int_{V_e} \rho [N_a]^T [A_n] [N_a] dV_e \]  
(1)

\[ [K_{uu}^e] = \sum_{k=1}^{nc} \int_{\zeta_k}^{\zeta_{k+1}} \int_{\eta_k}^{\eta_{k+1}} \int_{z_k}^{z_{k+1}} (B_a)^T [c] B_n) dJ d\eta d\zeta \]  
(2)

\[ [K_{up}^e] = \sum_{k=1}^{nc} \int_{\zeta_k}^{\zeta_{k+1}} \int_{\eta_k}^{\eta_{k+1}} \int_{z_k}^{z_{k+1}} (B_a)^T [\epsilon] B_n) dJ d\eta d\zeta \]  
(3)

\[ [K_{pp}^e] = \sum_{k=1}^{nc} \int_{\zeta_k}^{\zeta_{k+1}} \int_{\eta_k}^{\eta_{k+1}} \int_{z_k}^{z_{k+1}} (B_a)^T [\chi] B_n) dJ d\eta d\zeta \]  
(4)

where \( \rho \) is the material density, \([M^e] \) is the elementary mass matrix and \([K_{uu}^e] \) is the elementary
matrix of elastic stiffness. \([K_{up}^e] \) and \([K_{pp}^e] \) are stiffness elementary matrices of electromechanical
coupling and \([K_{pp}^e] \) is known as dielectric elementary matrix. \([c], [\epsilon] \) and \([\chi] \) are, respectively, the
elastic stiffness, piezoelectric stress and electric permittivity matrices of constant values. \([B_a] \) and
\([B_n] \) are input matrices. \( V_e \) is the elementary volume. \( J \) is the Jacobian of the transformation [1].

Equation (5) shows the global matrices of the model constructed through the standard procedure, in
which the subscript \( g \) indicates global quantities.
where \( \{ F_g \} \) and \( \{ Q_g \} \) are, respectively, the generalized force and nodal charge (elementary).

2.1. Balanced Realization

Balanced realization consists in describing the model of the system in the space state form and combines the controllability and observability matrices for each state of the system by using the Gramians of controllability and observability of the system. The linear transformation that leads the system to this representation is called balanced transformation. In this method, the reduced model is obtained by neglecting the states associated with the small singular values [10-11].

A balanced realization is an asymptotically stable minimal realization in which the controllability and observability Gramians are equal and diagonal [12]. Consider a stable linear-time invariant system defined by the state space equations below:

\[
\dot{x}(t) = [A]x(t) + [B]u(t) \\
\{y(t)\} = [C]\{x(t)\}
\]

where \( x(t) \) is the state vector, \([A]\) is the \( nxn \) dynamic matrix, \([B]\) is the \( nxm \) input matrix, \([C]\) is the \( sxn \) output matrix, \{\{u(t)\}\} is the input force and \{\{y(t)\}\} is the output vector, \( n \) is the order of the system, \( m \) is the number of inputs and \( s \) is the number of outputs. The system is called balanced if the solutions to the following Lyapunov equations (8) and (9) are

\[
[A][P] + [P][A]^\top + [B][B]^\top = 0
\]

\[
[A]^\top [Q] + [Q][A] + [C]^\top [C] = 0
\]

where \([P]\) and \([Q]\) are, respectively, the controllability and observability Gramians and \( \sigma_i, i = 1,2,...,n \), are the singular values of the system (\( \sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n \geq 0 \)).

Every \( \sigma_i \) is associated with a state \( x_i \) of the balanced system. Its value quantifies the contribution that \( x_i \) makes to the input-output behavior of the system. As \( \sigma_i \geq \sigma_j \) then \( x_i \) affects the behavior of the system more than \( x_j \), due to the fact that the singular values are ranked from the most important to the least important one. In the balanced realization the fidelity of the reduced model with respect to the full model \((A, B, C)\) depends on the relation \( \sigma_r \geq \sigma_{r+i} \), where \( r \) is the reduced model order. Thus, the reduced state space system is given by:

\[
\dot{x}_r(t) = [A_r]\{x_r(t)\} + [B_r]\{u_r(t)\}
\]

\[
\{y_r(t)\} = [C_r]\{x_r(t)\}
\]

where \( \{x_r(t)\} \) is the reduced state vector, \([A_r]\) is the \( rxr \) reduced dynamic matrix, \([B_r]\) is the \( rxm \) reduced input matrix and \([C_r]\) is the \( srx \) reduced output matrix.

3. Control Approach

Active modal control is used as control strategy for a smart composite structure as shown in figure 1. The advantage of using active modal control is that this technique is very effective for flexible
structure applications, requiring a reduced number of actuators and sensors. The estimator is responsible for determining the modal states required by the controllers. The Kalman Estimator is able to estimate the states by using noise contaminated measurement signals. More details regarding the Kalman Estimator can be found in the literature [13-15]. The states are then used by the controllers to determine the control force.

Figure 1. Active modal control based on modal state feedback control (adapted from [8]).

Figure 1 shows that in the modal state feedback control a number of controllers are necessary. The method requires the modal displacements and modal velocities to determine the control effort of the controllers. For this purpose, the Fuzzy Modal Controllers were used to determine the control effort. This method permits to take into account slight nonlinearities and uncertainties in the model.

4. Fuzzy Modal Controllers

The first step of this approach consists in the fuzzification of each controlled mode $X_i(t)$ in the fuzzy input. In this work, two membership GBELL Matlab functions “Positive” and “Negative” are used. For these membership functions the relevance of each variable is given by:

$$
\mu_A(X_i(t)) = \frac{1}{1 + \left| \frac{X_i(t) - c_i}{a_i} \right|^{b_i}}
$$

(12)

where $a_i$, $b_i$ and $c_i$ are the parameters of the GBELL function.

The inference engine implements the “minimum” function $w_j$. Mahfoud et al [16] presented Fuzzy controller rules applied to flexible structures. These rules are used in the present contribution and they are presented in table 1.

Table 1. Fuzzy controller rules [16-17].

| Rule | Condition | Decision |
|------|-----------|----------|
| 1    | IF positive displacement AND positive velocity | Action |
| 2    | IF positive displacement AND negative velocity | No action |
| 3    | IF negative displacement AND positive velocity | No action |
| 4    | IF negative displacement AND negative velocity | Action |

Finally, the control is obtained after defuzzification. This defuzzification requires the knowledge of the fuzzy output variables corresponding to the rules and the aggregation of these rules as well as
the output membership functions. In this paper the Takagi-Sugeno method is used due to its good adaptation to controller real time computation. The membership functions for the four modes are similar and are presented in equation (13) ($z_1$ corresponds to “no action” and $z_2$ to “action”):

\[ z_1 = 0 \text{ and } z_2 = \alpha_i q_i + \beta_i \dot{q}_i \] (13)

where, $\alpha_i$ and $\beta_i$ are, respectively, the weights assigned to the displacement and velocity of the mode $i$, respectively.

The driving force $F_i$, for mode $i$, is given by the output of the controller and can be written as:

\[ F_i = \sum_{j=1}^{2} w_j z_j \] (14)

5. Numerical Simulations

The studied laminated composite beam, illustrated in figure 2, is such that it has 306 mm length ($L$), 25.5 mm width ($b$) and 1 mm thickness ($h$). A piezoelectric ceramic actuator of dimensions 45.9 x 25.5 mm$^2$ is bonded to the beam top surface, 1 mm away from the clamp. The composite laminate has a total of five layers made of graphite/epoxy and oriented as [45° / 0° / 45° / 0° / 45°]. The layers oriented at 0° are parallel to the x axis. The thicknesses of the layers are 0.2 mm and the thickness of the piezoceramic actuator is 1 mm.

The constants of elastic stiffness of the beam made of AS4/3501 carbon/epoxy composite are the following (given in GPa): $C_{11} = 173.6$; $C_{22} = C_{33} = 7.61$; $C_{12} = C_{13} = 2.48$; $C_{23} = 2.31$; $C_{44} = 1.38$; $C_{55} = C_{66} = 3.45$. The piezoelectric constants of the PZT patch are: $C_{11} = C_{22} = C_{33} = 102.23$; $C_{12} = C_{13} = C_{23} = 5.035$; $C_{44} = C_{55} = C_{66} = 2.594$. The piezoceramic actuator is considered in the matrices $\mathbf{c}$, $\mathbf{e}$ and $\mathbf{\xi}$, respectively, in equations (2), (3) and (4). The mass densities, in kg/m$^3$, are 1578 for the composite laminated material and 7700 for the PZT patch.

![Figure 2. Composite cantilever beam with active vibration control.](image)

The FE model has been derived by using a 10x1 uniform mesh. The excitation force (1N) was applied at point II and the time domain structure responses were captured at point I (see figure 2). The piezoelectric actuator is connected to an active control system, and the vibration amplitudes are to be minimized over time. Two cases of active vibration control considering non-parametric uncertainties were analyzed. The samples of the uncertainty term $\Delta$ are determined by using Monte Carlo simulation associated with Latin Hypercube. The number of samples was 100.

(a) Robustness analyses considering variation in the model of smart composite structure so that the uncertainties were considered in the dynamical matrix $[\mathbf{A}] + [\mathbf{D}]$. 

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(b) Robustness analyses considering variation in the dynamical matrix of Kalman Estimator $[A_r] + [\Delta A_r]$.

For the first two modes, the system was observable and controllable. The balanced realization was used to reduce the model for the first two modes of the structure. For these two modes, 4 uncertain modes result. The table 2 presents the 4 uncertain model configurations (matrix $A_i$). These models were used to determine the robust controllers.

| Table 2: Uncertain Model (Matrix $A_i$). |
|-----------------------------------------|
| $A_i$ | Mode 1 | Mode 2 |
|------|--------|--------|
|      | -10%   | +10%   |
| 1 X  | --     | X      |
| 2 X  | --     | --     |
| 3 -- | X      | X      |
| 4 -- | X      | --     |

6. Numerical Results

6.1. Case (a)

The figures 3, 4 and 5 present the results considering uncertainties in the dynamic matrix of the model of the smart composite structure.

In terms of impact response, it is possible to see that the system’s response was completely attenuated with the use of FMCs, as seen in figure 3, where the green line represents the uncontrolled system. Another result presented in the figure 3 is the robustness of the FMCs that is demonstrated by the small envelope (blue lines). The same trends were observed in terms of the control effort (see figure 5).
Finally, the FRF shows that the two modes considered in the design of the FMCs were completely attenuated in the deterministic case (see the red line in figure 4). Therefore, in terms of robustness, the FMCs were only robust for the first mode (see the blue line in figure 4).

6.2. Case (b)
The figures 6, 7 and 8 present results for the case (b), which considers uncertainty in the dynamical matrix used by the Kalman Estimator.

The case (b) presents envelopes smaller than those associated with the case (a) since when variation is considered in the structure’s model, the natural frequency varies according to this variation, once the dynamic matrix changes with the value of Δ. The system’s response were complete attenuated as shown both in time and frequency domains. Figure 7 demonstrates that the first two modes were attenuated for the considered uncertainty models; it is shown that the FMCs were more robust in case (b).

Another important result observed in the second case is the energy consumption (figure 8). The FMCs were more robust in case (b) than in case (a), which demonstrates the same trends previously observed in the responses of the system (figures 6 and 7).

7. Conclusions
The present contribution was dedicated to the study of active vibration control in smart composite structures by using Fuzzy Modal Controllers. The results demonstrated the efficiency of the FMCs in the AVC. The results have revealed that the number of considered modes (2 modes) was sufficient to achieve satisfactory control. It should be emphasized the importance of performing balanced realization in this stage, since this technique ranks the modes in order of relevance, regarding the dynamic behavior of the system. This means that the considered modes are the most important to the
system response. The robustness analysis demonstrated that the FMCs were robust mainly for the second case presented above.

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