A Novel Motion Compensation Method for Random Stepped Frequency Radar with M-sequence

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Abstract. The random stepped frequency radar is a new kind of synthetic wideband radar. In the research, it has been found that it possesses a thumbtack-like ambiguity function which is considered to be the ideal one. This also means that only a precise motion compensation could result in the correct high resolution range profile. In this paper, we will introduce the random stepped frequency radar coded by M-sequence firstly and briefly analyse the effect of relative motion between target and radar on the distance imaging, which is called defocusing problem. Then, a novel motion compensation method, named complementary code cancellation, will be put forward to solve this problem. Finally, the simulated experiments will demonstrate its validity and the computational analysis will show up its efficiency.

1. Introduction
The random stepped frequency (RSF) radar is a new kind of synthetic wideband radar which has been widely researched [1-2]. Its transmitted signal waveform looks a bit like the linear stepped frequency (LSF) signal [3]. However, its modulated frequencies of different transmitted pulses are randomly distributed over the given frequency band. The hopping frequencies make it possible to cover ultra-wide frequency band which is necessary for synthesizing the high resolution range profile (HRRP).

The frequency of RSF waveform is modulated by the pseudorandom code, a permutation of unique integer, such as Costas code [4] or Chaos code using Bernoulli mapping [5]. In our work, M-sequence which has been widely applied in communication and radar field to generate quasi-white noise will be employed. Different with the slant ridge ambiguity function (AF) of LSF signal, the AF of RSF signal looks more like a thumbtack. It means there is only one peak in the centre of echo delay and Doppler offset plane and other places are filled with the approximately uniform side-lobes. Meanwhile, it also indicates that, without a precise velocity measurement and motion compensation, the range profile of RSF radar will be destroyed, which is called de-focusing problem. In other words, the RSF signal is a very sensitive to Doppler frequency [6]. This is different with the case of LSF radar system [7], whose synthetic range profile will cyclically migrate and be slowly blurred when target is moving.

Therefore, it is necessary to carry out a precise motion compensation before the distance imaging for RSF radar. In terms of HRRP radar, there are several ways to achieve motion compensation, i.e., minimum waveform entropy (MWE) [8], range velocity iterative alternating project (RV-IAP) [9], cross-correlation between two adjacent pulse-trains [10] and a bank of Doppler filters [3]. They either need a lot of calculations or consume plenty of hardware resources. Besides, the RV-IAP algorithm is proposed to detect range and velocity parameters of multiple targets, however it requires the number of targets as a prior knowledge, which is unknown in most situations.
Taking all of these into account, we propose an effective and efficient motion compensation method, named complementary code cancellation (CCC), for the RSF radar with M-sequence. This paper is organized as follows. The complemented schematic diagram is provided in section 2 to show up the global framework of RSF radar system. In section 3, the defocusing problem is introduced. And in section 4, the motion compensation method based on CCC algorithm is presented. Then in section 5, the performance of the proposed method is characterized with the simulated experiments. Finally, some conclusions are drawn in the last section.

2. Schematic diagram

This section will introduce the brief implemented schematic diagram of RSF radar system based on CCC algorithm in Figure 1.

The pseudorandom code generation module is framed out in the red dash line at right side. Firstly, the M-sequence is generated by linear combination of shift registers. Then, the pseudorandom code could be obtained via decoding the binary flow of this sequence. For the convenience of motion compensation, its complementary code shall be employed in the next coherent processing interval (CPI). It can be seen that this schematic diagram is characterized by easy implementation.

Besides, the bottom part boxed in the red dash line shows the velocity detection method which will be emphatically presented in section 4. The velocity of target should be measured in front of distance imaging, because the range profile is very sensitive to the Doppler frequency, which will be uncovered in the next section.

![Schematic Diagram](image)

**Figure 1.** The implemented schematic diagram of RSF radar system based on CCC algorithm.

3. Defocusing problem

In this section, the effect of target movement on the range profile, named defocusing problem, will be briefly discussed. Firstly, the transmitted and received signal model of RSF radar is introduced. Then, the range profile synthesis method is presented as the groundwork of the defocusing problem. Finally, the defocusing problem would be formally explained in the last subsection to indicate the relationship between the target movement and the range profile.

3.1. Signal Model

3.1.1. Transmitted waveform

Consider a RSF radar using a train of $N$ monotone bursts, with a pulse repetition interval (PRI) of $T_r$ seconds and pulse duration of $T$ seconds. And the total synthetic bandwidth is $B$ Hz. In this way, the stepped frequency interval could be obtained as $\Delta f = B/N$. As well as the case of LSF signal [11], the
stepped frequency interval and the pulse duration must meet the following tight contrast to present the appearance of grating lobes.

\( \Delta f \cdot T \leq 1 \)  

The complex envelope of RSF signal in a train can be represented as

\[
x(t) = \sum_{n=0}^{N-1} \text{rect} \left( \frac{t-nT}{T} \right) e^{j2\pi A(t-nT)}.
\]  

(2)

Here, the function \( \text{rect}(x) \) denotes the unit rectangle function. When \( 0 \leq x \leq 1 \), it is equal to 1 and 0 otherwise. The frequency step coefficient \( c_n \) acquired by the M-sequence is a unique integer randomly chosen from 1 to \( N \).

Let \( A \) and \( f_c \) denote the transmitting gain and carrier frequency. Then, the transmitted signal can be written as

\[
x(t) = A \text{Re} \{ x(t)e^{j2\pi f_c t} \}.
\]  

(3)

3.1.2. Echo signal

Consider an ideal point target at the initial distance of \( R_0 \) meters with a radial velocity of \( V \) meters per second running away from the radar. Then, the echo delay of \( n^{th} \) monotone burst can be written as

\[
\tau_n = \frac{2(R_0 - nVT_c)}{c}.
\]  

(4)

Here, the parameter \( c \) denotes the propagation speed. The RSF echoes are always received in the narrow band. Therefore, based on the stop and jump model, the echo is a delay version of the transmitted signal.

\[
x_e(t) = x(t - \tau_n).
\]  

(5)

Substituting (3) into (5) and performing the quadrature digital down conversion processing, the complex echo signal at the sampling range bin where target locates will become

\[
x_e[n|R_0,V] = e^{-j\Delta f \tau_n c} e^{j\pi c n^2}.
\]  

(6)

It can be seen that the distance and velocity of target are coupled in the phase of echoes. In the next subsection, the range profile synthesis will be introduced to tell how to obtain the distance information of target. However, the velocity is bound to make an impact on the synthesis result, which will be studied in the subsection 3.2.

3.2. Range profile synthesis method

The echo model of RSF signal can be divided into two parts, i.e., distance item and velocity item. As for the distance item, it is coded by the coefficient \( c_n \). Yet, the coefficient \( c_n \) is a kind of unique integer from 1 to \( N \), which means it can be reordered to an ascending sequence. When reordering coefficient \( c_n \), the independent variable in the velocity item will change to the inverse map from \( n \) to \( c_n \). We note it as the coefficient \( \tilde{c}_n \) which is also a random unique integer from 1 to \( N \). In this way, the reordered echo will turn into the following form.

\[
x_e[n|R_0,V] = e^{-j\Delta f \tau_n \tilde{c}_n} e^{j\pi \tilde{c}_n n^2}.
\]  

(7)

After reordering processing, the distance item gets a linear phase. That is to say the range profile can be synthesized by the inverse discrete Fourier transform (IDFT) processing. However, the velocity item remains random. It is conceivable that its effect on the synthesis result depends on the size of
velocity. Figure 2 lists the synthetic range profiles of ideal point target at 5.1km under the conditions of different velocities.

![Figure 2](image.png)

**Figure 2.** The synthetic range profiles of the ideal point target at 5.1km with the velocity of 0, 0.1 and 1(m/s) for (a), (b) and (c) respectively. The parameters of RSF radar is set as follows: $T=1\mu s$, $T_r=100\mu s$, $f_c=35GHz$, $B=100MHz$, $N=128$.

### 3.3. Defocusing problem

The range profiles in figure 2 intuitively illustrate the defocusing problem of RSF radar. Firstly, when the target is stationary relative to the radar, the range profile could be accumulated very well, which has been shown in the red line of figure 2. However, even if the target moves with a small velocity, the peak of range profile will attenuate rapidly, because the carrier frequency $f_c$ is always very huge. When the velocity continues increasing, the peak will disappear completely. In summary, the defocusing problem of RSF radar reflects a more complex coupling relationship between distance and velocity.

Then, we will define a variable to quantify the defocusing problem, named velocity tolerance. It is defined as a velocity threshold that make the peak of range profile of ideal point target attenuate 3dB. It is difficult to directly deduce the velocity tolerance. But it can be seen that it is constrained by the carrier frequency $f_c$ and PRI $T_r$. Through fitting simulation, we get an approximate result:

$$v_r = \frac{0.24c}{f_cNT_r}$$

(8)

The velocity tolerance indicates the precision threshold of motion compensation. Only if the precision is less than velocity tolerance, the motion compensation method can be regard as being valid.

### 4. Motion compensation method

#### 4.1. Complementary code cancellation algorithm

In the transmitting stage, the waveform defined by (2) is employed during the first CPI. However, the waveform modulated by the complementary frequency hopping coefficient $c_n$ will be used during the next CPI. The frequency hopping coefficients in the adjacent two CPI meet the following relationship

$$c_{n+1} + c_n = N, n = 0, 1, \ldots, N-1.$$  

(9)

In the coherent receiver, after the same processing as the first CPI, the sampling echo signal of the second CPI will have the similar form

$$i_n[n|R_{s_n}, V] = e^{-j\frac{\pi}{2}(1 + c_n + Nc_n) (n + NT_r) - \frac{\pi}{2}(1 + Nc_n)NVT_r}, n = 0, 1, \ldots, N-1.$$  

(10)

Then, multiply each corresponding signals of the both CPI together.

$$\rho[n|R_{s_n}, V] = i_n[n|R_{s_n}, V] \cdot i_n[n|R_{s_n}, V]$$

$$= e^{-j\frac{\pi}{2}(1 + c_n + Nc_n) (n + NT_r) - \frac{\pi}{2}(1 + Nc_n)NVT_r} \cdot e^{-j\frac{\pi}{2}(1 + Nc_n)NVT_r}$$

(11)
It can be seen that the distance item is involved in the first factor which consists of the constant phase part. And the second factor is composed of the linear velocity item which makes up the most important part of velocity measure. At last, the third factor is the disturbing item. But, as long as the velocity satisfies the following relationship, it will not cause a significant interference on the result.

\[
\max_{\alpha \in [0, \pi - \gamma]} \left\{ \frac{4\pi}{c} \alpha, \Delta f V N T \right\} \leq \frac{\pi}{2} \Rightarrow V \leq \frac{c}{8\Delta f N^2 T_c}
\]  

(12)

Compared with the velocity tolerance \(v_T\), this is a quite rough condition for the most radar system. Therefore, the third factor can be omitted in the succeeding analysis. As a result, conducting the IDFT processing on the multiplying output, we will obtain the approximate velocity profile

\[
P[k|V] = \sin \left( \frac{\pi}{K} \left( (2f_c + N\Delta f)VT, K - k \right) \right)
\]

Here, the parameter \(K\) denotes the length of IDFT processing. Consequently, searching its maximal value, the measured velocity could be acquired.

\[
\bar{V} = \arg \max_{i\in[0, K]} \left\{ P[k|V] \right\}
\]

Then, the motion compensation factor can be generated via the measured velocity. Compensate the sampling echo in (7). Finally, the range profile could be synthesized by the method mentioned in the previous section. The detailed algorithm procedure has been displayed in the schematic diagram in the Figure 1.

4.2. Algorithm precision analysis

From the velocity profile, the 3dB velocity resolution rate of the presented method can be figured out.

\[
\Delta v = \frac{0.442c}{(2f_c + N\Delta f)NT_c}
\]

(15)

Compare the velocity resolution rate with the velocity tolerance.

\[
\Lambda = \frac{\Delta v}{v_T} = \frac{1}{1.086 + 0.543 \frac{B}{\Delta f}}
\]

(16)

It can be seen that the ratio is always less than 1, which means the precision of presented method could meet the demand of the motion compensation.

5. Simulated experiments

The simulated experiments will carried out on the basis of the following parameters: the pulse duration \(T=1\mu s\), PRI \(T_r=100\mu s\), the carrier frequency \(f_c=35GHz\), the synthetic bandwidth \(B=100MHz\), the number of bursts in a CPI \(N=128\). The simulated scene is a tank model at the distance of 5.1km with the velocity of 10m/s running away from the radar. The intermediate and final experiment results will be displayed in the Figure 3.

The picture of simulated tank is shown in subfigure (a) of Figure 3. Its velocity profile and range profile are shown by the blue line in subfigure (b) and (c). The red line in subfigure (b) indicates the velocity detecting threshold. It can be seen that the velocity of tank can be well measured. Besides, the red diamond in subfigure (c) also marks out the real RCS values of simulated tank. The peaks of range profile are very close to these values, which means the presented motion compensation method is valid.
Figure 3. The intermediate and final results of range profile synthesis processing of a tank (shown in (a)). The velocity profile and range profile is shown in (b) and (c), respectively.

Compared with the existing methods, the motion compensation method presented in the paper features lower computation overhead. The cross-correlation method needs $O(N^2)$ multiplication times. The MWE method needs $O(N^3)$ multiplication times. And the RV-IAP algorithm needs more, due to the iterative search. However, the CCC method just requires $O(N \log_2 N)$ multiplication times because of the fast Fourier transform (FFT). But, the unambiguous velocity interval of CCC method looks not large since the data of two CPI is employed. Therefore, in some application situations, the rough velocity measurement is needed to be performed in advance.

6. Conclusions

The RSF radar can be applied to synthesize the ultra-wide bandwidth to achieve HRRP. However, it suffers from the defocusing problem. In this paper, the schematic diagram of RSF radar based on M-sequence is designed, which benefits from sample structure. Then, the velocity tolerance is imported to indicate the threshold of motion compensation. And the CCC method is proposed to solve the defocusing problem. Finally, the simulated experiments demonstrate that the presented method is novel. It is believed that its effectiveness and efficiency will make it useful in the radar imaging.

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