Gamow-Teller strengths in $^{48}$Ca with the charge-exchange subtracted second random-phase approximation

D. Gambacurta,$^1$ M. Grasso,$^2$ and J. Engel$^3$

$^1$INFN-LNS, Laboratori Nazionali del Sud, 95123 Catania, Italy
$^2$Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France
$^3$Department of Physics and Astronomy, CB 3255, University of North Carolina, Chapel Hill, North Carolina 27599-3255, USA

We develop a fully self-consistent subtracted second random-phase approximation for charge-exchange processes with Skyrme energy-density functionals. As a first application, we study Gamow-Teller excitations in the doubly-magic nucleus $^{48}$Ca, which is the lightest double-$\beta$ emitter that could be used in an experiment. The amount of Gamow-Teller strength below 20 or 30 MeV is considerably smaller than in other energy-density-functional calculations and agrees better with experiment. This important result, obtained without ad hoc quenching factors, is due to the presence of two-particle–two-hole configurations. Their density progressively increases with excitation energy, leading to a long high-energy tail in the spectrum, a fact that may have implications for the computation of nuclear matrix elements for neutrinoless double-$\beta$ decay in the same framework.

Charge-exchange (CE) excitations$^{1,2}$ such as the Gamow-Teller (GT) resonance are closely linked to electron capture and $\beta$ decay, which play important roles in nuclear astrophysics$^{3,4}$. They aid the construction of nuclear effective interactions, for which they constrain couplings in the spin-isospin channel. Finally, they are relevant to the nuclear physics that affects neutrinoless double-$\beta$ decay, which play important roles in nuclear matrix elements even suggests that one could deduce the latter from double-CE experiments$^{10,11}$. Within chiral effective field theory, one can hope to compute both single- and double-$\beta$ matrix elements in an ab-initio way — see Ref. $^{12}$ for a discussion of the double-$\beta$ operator structure and, e.g., Refs. $^{13,14}$ for reviews of ab-initio many-body methods — but most double-$\beta$ emitters that could be used in experiments are still too complex to easily treat from first principles. Ref. $^{17}$ very recently presented the first non-perturbative ab-initio treatment of the double-$\beta$ decay of $^{48}$Ca, but heavier nuclei are more difficult. More phenomenological approaches, such as the shell model$^{15,21}$, the Interacting Boson Model$^{21}$, nonrelativistic$^{22,23}$ and relativistic$^{24,25}$ energy-density-functional (EDF) theory (often based on the generator-coordinate-method), and the quasiparticle-random-phase-approximation (QRPA)$^{26–28}$ are therefore still important, even necessary.

Unfortunately, these theoretical schemes do not correctly describe the available data for GT excitations and $\beta$-decay half-lives and must resort to ad hoc “quenching factors” to obtain reasonable results for GT strength below 20 or 30 MeV of excitation energy. In Ref. $^{29}$, for example, models based on the random-phase approximation (RPA) overestimate the strength significantly. This kind of over-prediction is usually ascribed to missing correlations$^{30,31}$. The results in Ref. $^{29}$ indicate that higher-order correlations are needed beyond those in the RPA, which is essentially a time-dependent version of mean-field theory.

Ab-initio work with operators and currents from chiral effective field theory has recently had some success in explaining the quenching in $\beta$ decay. Reference$^{32}$ showed that correlations omitted from the shell model and from mean-field-based calculations, together with two-body weak currents, account for most of that quenching. Reference$^{33}$ showed that the same effects quench the integrated $\beta$ strength function. But weak two-body currents play no obvious role in charge-exchange transitions, and so the implications of this last result for our work are not clear. Similarly, starting from realistic potentials, the authors of Ref. $^{34}$ used many-body perturbation theory to derive effective shell-model operators that implicitly include correlations from outside shell-model spaces, obtaining the correct quenching of GT strength in several nuclei of interest for double-$\beta$ experiments, but failing to do so in the lightest of these, $^{48}$Ca. They also had difficulty in that nucleus with two-neutrino double-$\beta$ decay, a very closely related process.

Finally, EDF-based models that go beyond mean-field theory have been proposed for CE excitations, for example in both relativistic (see Refs. $^{35,36}$ for the most re-
cent developments) and nonrelativistic (see for instance Ref. [37]) particle-vibration-coupling models. The predicted integrated strengths are always better than in the \((Q)\)RPA. Again, however, the improvement is minor for \(^{48}\text{Ca}\). Reference [37] shows that the GT strength below 20 MeV continues to be significantly overestimated in that nucleus, even when beyond-mean-field correlations are included.

In this paper a better description of the GT\(^{-}\) strength — measured in charge-exchange reactions by adding a proton and removing a neutron — in \(^{48}\text{Ca}\) is achieved with a more comprehensive extension of EDF theory, a CE version of the subtracted second RPA (SSRPA) [38]. In nuclei with a significant neutron excess, this strength is much larger than GT\(^{+}\) strength, measured by adding a neutron and removing a proton. The excitation operator for GT\(^{-}\) transitions can be written as

\[
\hat{O} = \sum_{i=1}^{A} \sum_{\mu} \sigma_{\mu}(i) \tau^{-(i)}
\]

where \(A\) is the number of nucleons, \(\tau^{-(i)}\) is the isospin-lowering operator \(\tau^{-} = t_{x} - it_{y}\) for the \(i^{th}\) nucleon, and \(\sigma_{\mu}(i)\) is the corresponding spin operator. The so-called Ikeda sum rule relates the integrated strengths \(S\) to the number of neutrons \(N\) and protons \(Z\) in the nucleus:

\[
S_{GT^{-}} - S_{GT^{+}} = 3(N - Z).
\]

Because \(S_{GT^{-}}\) is so much larger than \(S_{GT^{+}}\) this sum rule is essentially a measure of the total GT\(^{-}\) strength.

Reference [39] reports the results of a \(^{48}\text{Ca}\)(p, n) and \(^{48}\text{Ti}\)(n, p) experiments at a beam energy of 300 MeV at the Research Center for Nuclear Physics in Osaka. The total GT\(^{-}\) strength below 30 MeV (which probably includes some contributions from isovector spin-monopole excitations) is only 64 ± 9\% of that given by the Ikeda sum rule. The location of this “missing strength” has long been a mystery for nuclear physics.

Here we deploy for the first time a CE subtracted second RPA (CE-SSRPA), together with the Skyrme interaction [40, 42] SGII [43, 44]. The CE-SSRPA includes the “subtraction procedure” [45], which has been applied extensively in the past few years within the charge-conserving SSRPA [38, 40, 46]. The formalism underlying the second RPA (CE-SSRPA without the subtraction procedure) appears in Ref. [31], where one may find expressions for the Hamiltonian matrix. This matrix contains a one-particle — one-hole (1p1h) sector characterized by the matrices \(A_{11}\) and \(B_{11}\), a sector that mixes 1p1h and two-particle — two-hole (2p2h) configurations, with the matrices \(A_{12}\) and \(B_{12}\), and a pure 2p2h sector, with the matrices \(A_{22}\) and \(B_{22}\). The authors of Ref. [31] presented CE-SSRPA calculations in which the highly demanding numerical problem of diagonalizing the Hamiltonian matrix, intractable at that time, was simplified by neglecting the interaction between 2p2h configurations. That step allowed the full SRPA diagonalization to be replaced by an RPA-type computation with an energy-dependent Hamiltonian.

Our approach is to carry out a full diagonalization. The subtraction procedure requires in addition the inversion of the matrix \(A_{22}\) [38]. The procedure corrects for the possibility of over-counting correlations, cures the instabilities of the SRPA [47], and regularizes the ultraviolet divergence generated by the use of contact forces beyond mean-field theory [38]. We consistently cut off 2p2h configurations at 40 MeV, both in the diagonalization of \(A_{22}\) and its inversion, having verified that results do not change significantly when the cutoff is raised beyond that level.

Compared to other EDF approaches (both mean-field and beyond-mean-field) ours better predicts the strength distributions, so that we obtain a much more accurate value for the sum of the strength up to 20 or 30 MeV, without resorting to quenching factors. The 2p2h configurations, which increase in density with excitation energy, lead to a long high-energy tail that draws strength from lower energies. The “missing strength” is thus spread out over a large range at higher energies, making it hard to discriminate from background.

Figure 1 shows the experimental strength extracted from Ref. [39]. One of the most important features of the SSRPA is its ability to describe the width and fragmentation of excitation spectra. This asset is visible in the figure, in which both the RPA and SSRPA discrete-strength distributions are also included. Because the experimental strength is a continuous function of energy, it has different units from the discrete theoretical strengths, and the absolute strength values are thus not comparable. How-
ever, by plotting the discrete spectra one can compare the location and fragmentation of the main peaks, without generating any artificial spreading by folding. To better display the results in the figure, we rescale the RPA and SSRPA discrete strengths so that their respective highest peaks have approximately the same height as the corresponding experimental peak. To achieve this, we divide the RPA strength by nine and multiply the SSRPA strength by two.

The SSRPA strength is indeed quite fragmented, particularly in the region between 6 and 16 MeV, where three groups of peaks are concentrated around 8, 11, and 14 MeV, in accordance with the experimental distribution of peaks. One may also observe another group of much weaker peaks concentrated around 17 MeV, which corresponds to the location of the highest-energy experimental peak. Finally, a very dense high-energy SSRPA tail is visible in the insert, which focuses on the energy region between 20 and 30 MeV. Such a tail is completely absent from the RPA spectrum, which is composed of a few well separated peaks and misses the complex structure of the experimental strength. The long high-energy tail is indeed the explanation for the missing strength at lower energies.

The very lowest-energy part of the SSRPA spectrum is less satisfactory than the rest, with a main peak predicted at about 5 MeV; the lowest experimental peak, by contrast, is located at 3 MeV. The SSRPA does predict some fragmented strength is in the region around 3 MeV, however.

To more directly compare the theoretical and experimental strengths, we have folded our response functions together with a Lorentzian distribution of width 1 MeV. Panel (a) of Fig. 2 presents the folded RPA and SSRPA strength distributions along with the experimental distribution. Panel (b) shows the cumulative strength as a function of energy up to 30 MeV. As we have already seen in the discrete spectra, the SSRPA reproduces the GT distribution quite well, with the exception of the lowest-energy peak. The RPA, on the other hand, cannot reproduce the complex structure of the spectrum.

The most striking result is in panel (b). The SSRPA cumulative strength is greatly reduced from that of the RPA and is in much better agreement with the experimental value. Furthermore, the SSRPA curve is smooth and follows the experimental profile (owing to the physical description of widths and fragmentation, and to the subtraction procedure, which is needed to place centroids at the correct energies), except beyond 20 MeV, where the tail is a little too high. The RPA curve, by contrast, shows steps because of its very few well separated peaks. The improvement with respect to the RPA is more significant than in other beyond-mean-field approaches. In Ref. [37], for example, the same Skyrme interaction SGII produces more than 20 units of strength below 20 MeV.

The ratio between the experimental and the theoretical integrated strength below 20 MeV is 0.58 for the RPA and 0.93 for the SSRPA, showing that quenching factors are not needed in SSRPA. In the particle–vibration–coupling calculations of Ref. [37], this ratio is ≤ 0.68, the value 0.68 corresponding to an integrated strength of 20 units. The explicit inclusion of 2p2h configurations efficiently generates the high-energy tail that accounts for the missing strength.

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SSRPA strength below 20-30 MeV is much smaller than in other mean-field and beyond-mean-field EDF models, and in much better agreement with the corresponding experimental values, without the use of ad hoc quenching factors. By working with two additional Skyrme parametrizations, we showed that these successes are due primarily to our many-body method, the key ingredient of which is the explicit inclusion of 2p2h configurations. Their density strongly increases with the excitation energy, leading to a high-energy tail in the spectrum.

The ability to describe CE strength may have a strong impact in astrophysical scenarios where GT resonances play an important role. It also promises to improve EDF-based calculations of the nuclear matrix elements governing $0\nu\beta\beta$ decay, a process at the intersection of several scientific domains. We plan to apply our approach to open–shell nuclei by including pairing correlations of both the usual isovector type and the isoscalar type that are important for $\beta$ and double-$\beta$ decay.

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