Simulating The Residual Three-body Force in a Small System

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Abstract. The effect of residual three-body force on the small system was investigated in this paper. A one-level and two-level system were chosen as the model to study the feature of three-body force due to their simplicity and lower symmetry. In both systems, we found a consistent result, that the inclusion of three-body force will enhance and accelerate the pairing condensation, which is shown by a higher pairing gap and lower energy as compared to the system with only two-body pairing force. Some interesting results on the behavior of the residual three-body force were also presented here.

Keywords: three-body force, pairing gap, small system, BCS theory

1. Introduction

Pairing correlations are one of the most common phenomena in the universe. It occurs (or predicted to occur naturally) everywhere, ranging from tiny quarks, nucleus, atom, material, or even in the astronomical body like neutron stars. It is just natural that interest to study the pairing correlation and all aspect related to that phenomena is always high. Since its first publication in 1957, BCS theory of superconductivity [1] continuously become the main reference for the scientist working in superconductivity, superfluidity, and pairing phenomena. Similar theory but using other approaches like Bogouliubov transformation and Landau theory of liquid also enrich our understanding in the pairing related research. The BCS theory initially was designed to explain the pairing phenomena namely electron’s pairing in condensed matter/liquid also enrich our understanding in the pairing related research. The BCS theory initially was designed to explain the pairing phenomena namely electron’s pairing in condensed matter/liquid also enrich our understanding in the pairing related research. The BCS theory initially was designed to explain the pairing phenomena namely electron’s pairing in condensed matter/liquid also enrich our understanding in the pairing related research. The BCS theory initially was designed to explain the pairing phenomena namely electron’s pairing in condensed matter/liquid also enrich our understanding in the pairing related research. The BCS theory initially was designed to explain the pairing phenomena namely electron’s pairing in condensed matter/liquid also enrich our understanding in the pairing related research.

As long as we have a high-density state and lower temperature, the pairing condensation phenomena must naturally (spontaneously) be occurred. The idea of using BCS type equation into a three-body force in the nuclear and mesoscopic system, for example can be found in reference [3]. Some applications in the realistic nuclear problem can be seen in reference [4]. Some mathematical/spin algebra reviews and critics have also been done by other [5].

The mesoscopic or small system is a system that comprised of few particles up to thousands of particles. This small system is a perfect playground for the scientist to test and play around with the theory since it is small enough for quantum phenomena to occur yet adequate enough to be treated statistically.

In this paper, the effect of residual three-body force (R3BF) in the small system was investigated by using a BCS framework. The one-level and two-level systems were chosen as a
model to study the general features and properties of the R3BF due to their simplicity and a straightforward demonstration of the effect.

2. Methods
The pairing interaction between two bodies resulted in the formation of pairing state which has lower energy compared to the normal (unpaired) state. This difference of energy expressed in a famous BCS pairing gap equation [1] as shown in Equation 1.

\[ \Delta_k = \frac{1}{2} \sum_{k' > 0} \frac{\Delta_k}{\sqrt{\epsilon_k^2 + \Delta_k^2}} V_{kk'} \]  

(1)

where \( \Delta_k \) denotes the pairing gap at state \( k \), and \( \epsilon_k \) denotes the single particle energy at state \( k \) and \( V_{kk'} \) denotes the interaction matrix between state \( k \) and \( k' \). This equation can be solved numerically using a self-consistency method. In the BCS framework, the R3BF is formulated as:

\[ \Delta_1 = \Omega_2 \frac{\Delta_2}{4E_2} \left( G_{12} - g_{12;3} \langle n_3 \rangle \right) \]  

(2)

With \( \Delta_1 \) denotes the pairing gap of three-body force, while \( \Omega_2 \) denotes the occupation number of the state, \( G_{12} \) denotes the two-body pairing constant, while \( g_{12;3} \) denotes the three-body pairing constant. Detail of the above formula derivation can be found in [6]. The main features of the R3BF are (1) it is a sequential pairing in which the two-body interaction occurred first and create a single condensate before making another interaction with the incoming third particle, (2) the R3BF is a density-dependent interaction and becomes more significant at very high-density state.

The most important parameter in this study is the pairing gap enhancement which is expressed by :

\[ \Delta_{enh} \left( g_{12;3} \right) = \left| \Delta_{3B} \left( g_{12;3} \right) - \Delta_{2B} \right| \]  

(3)

while the pairing energy enhancement is calculated by :

\[ E_{enh} \left( g_{12;3} \right) = \left| E_{3B} \left( g_{12;3} \right) - E_{2B} \right| \]  

(4)

where \( \Delta_{2B} \) and \( E_{2B} \) is the two-body pairing gap and the total energy of two-body pairing system (at \( g_{12;3} = 0 \)) respectively. The parameters of \( \Delta_{3B} \left( g_{12;3} \right) \) and \( E_{3B} \left( g_{12;3} \right) \) denote the three-body pairing gap and the total energy of the three-body force system at a particular three-body constant \( g_{12;3} \). The effect of R3BF in this paper will be shown in terms of pairing enhancement parameter, such as a pairing gap enhancement and a pairing energy enhancement. All calculations were performed using the FORTRAN77 program under a Fedora 14 Linux platform.

3. Result and Discussions
In the one-level model, we assume that all particles have the same energy and lying on a single degenerate system. The half occupied system was chosen in this paper since it has the maximum pairing gap compared to other configurations. In Fig.1a, the pairing gap as a function of a number of particles (N) for even N is shown. The calculation was started from \( N = 4 \) since there is no three-body
force working on $N = 2$. It was found that when $g_{12;3}$ is increased (black line is indicating the absence of three-body force, while the red line indicating three-body constant $g_{12;3} = 0.0050$) the pairing gap is increased as well. A bigger pairing gap means a more stable pairing phase is formed separated from the normal phase. In the absence of three-body force, the maximum pairing gap occurs when the system is half occupied ($\omega = \Omega/2$), where $\Omega$ is the total occupation number and $\omega$ is the half occupation number of the energy level. The introduction of R3BF will shift the maximum pairing gap to the right side ($N > \omega$), depending on the value of three-body constant $g_{12;3}$.

![Figure 1](image1.png)

Figure 1. (a) Pairing gap vs Number of Particle (N) of one level system for $g_{12;3}=0.000$ (without three-body force) and $g_{12;3}=0.0050$. (b) The total energy of the system for $g_{12;3}=0.000$ and $g_{12;3}=0.0050$.

For the energy of the system, the presence of three-body force is again enhancing the pairing interaction and condensation as indicated by a lower energy of the system. As an illustration, let’s take a look at $N = 20$ particles; when there is no three-body force present, the total energy of the system is -220.216 a.u (in arbitrary unit), but when $g_{12;3}$ is 0.0050 the energy becomes -226.513 a.u. The behavior of the total energy of the system as a function of a number of particles is shown in Fig 1b. Similar to pairing gap, the effect of the three-body force also vanishes when the system is fully occupied. The addition of R3BF to a one-level system will increase the energy gap and enhance the stability of pairing state by lowering its energy.

It is also interesting to study the effect of a number of particles to the pairing (gap) enhancement due to a three-body constant. In Fig. 2, the pairing enhancement as a function of three-body constant were shown for two different half occupied systems, $N = 16$ and $N = 50$. The pairing (gap) enhancement of R3BF as compared to a regular two-body pairing shows a linear dependent to the number of particles $N$ and this signifies the importance of R3BF at a higher density as seen in Fig.2.
Figure 2. The pairing gap enhancement of R3BF as compared to regular two-body pairing gap for half occupied one level systems with N=16 and N=50 particles.

In the case of the two-level system, there is a critical two-body pairing constant \( G_c \) in which the pairing effect start to occur. The introduction of the three-body constant will lower the critical pairing strength and accelerates the occurrence of pairing condensate as can be seen in Fig.3a, where the critical two-body pairing constant \( G_c \) which initially 0.47 (no three-body constant presence) now shifted to 0.08 when \( g_{12;3}=0.002 \). At \( g_{12;3}=0.003 \), \( G_c \) is even vanishes which means that without any two-body pairing strength, the pairing condensation will still occur at particular three-body constant. The same effect can also be seen in term of the total energy of the system as shown in Fig 3b. In general, the presence of R3BF accelerates the pairing condensation by shifting the critical two-body pairing strength into lower values or even annihilate the two-body pairing constant.

Figure 3. The pairing acceleration process due to the introduction of R3BF which lowering the critical two-body pairing constant as seen from the (a) pairing gap (b) total energy of N=20 particles of a half occupied two-level system.
The effect of R3BF on the two-level system is also shown in Fig. 4. The pairing gap enhancement is behaving non-linearly when the applied two-body pairing constant below its critical point \( G_c \) (\( G < G_c \)), and tend to be constant at the region above its critical point (\( G > G_c \)). The application of small three-body constant at the \( G > G_c \) region does not introduce any new behavior as shown by its monotonic behavior of pairing gap and total energy enhancement versus \( G_{12} \) in Fig 4a and 4b.

Figure 4. (a) The non-linear behavior of pairing gap enhancement, and (b) the total energy of the system at various three-body constant in the two-level system.

4. Conclusion
The effect of the residual three-body force in the small system has been investigated using a BCS framework by introducing a three-body constant into a pairing gap equation. The results have shown that the R3BF, in general, enhances the pairing condensation by widening the gap and lowering the energy of the system. The pairing enhancement is density dependent and increases as the number of particles in the half occupied degenerate system increased. In a two-level system, the effect of R3BF is non-linear for \( G < G_c \) and tend to be constant at \( G > G_c \). The monotonic behavior (constant pairing enhancement) of R3BF at \( G > G_c \) shows that the three-body constant seems to act as an “amplifier” for the two-body pairing interaction as shown in the pairing gap and energy enhancement and does not introduce any new-type of interaction.

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