Induced Superconducting Pairing in Integer Quantum Hall Edge States

Mehdi Hatefipour, Joseph J. Cuozzo, Jesse Kanter, William M. Strickland, Christopher R. Allemand, Tzu-Ming Lu, Enrico Rossi, and Javad Shabani*

ABSTRACT: Indium arsenide (InAs) near surface quantum wells (QWs) are promising for the fabrication of semiconductor−superconductor heterostructures given that they allow for a strong hybridization between the two-dimensional states in the quantum well and the ones in the superconductor. In this work, we present results for InAs QWs in the quantum Hall regime placed in proximity of superconducting NbTiN. We observe a negative downstream resistance with a corresponding reduction of Hall (upstream) resistance, consistent with a very high Andreev conversion. We analyze the experimental data using the Landauer-Büttiker formalism, generalized to allow for Andreev reflection processes. We attribute the high efficiency of Andreev conversion in our devices to the large transparency of the InAs/NbTiN interface and the consequent strong hybridization of the QH edge modes with the states in the superconductor.

KEYWORDS: Integer quantum hall effect, Andreev reflection, Superconductivity

Anyons with non-Abelian statistics are of great fundamental interest\(^1\) and can be used to realize topologically protected, and therefore intrinsically fault-tolerant qubits.\(^2\)–\(^4\) Non-Abelian anyons are expected to be realized in few fractional quantum Hall (QH) states\(^5\)–\(^9\) such as the QH states with filling factor \(\nu = 5/2\)\(^10\)–\(^12\) and possibly \(\nu = 12/5\).\(^13\) However, so far, no unambiguous experimental confirmation exists of the presence of non-Abelian anyons in such QH states.

An alternative route to realize non-Abelian anyons relies on inducing superconducting pairing between counter-propagating edge modes of QH states that intrinsically support only Abelian anyons.\(^14\)–\(^17\) These theoretical proposals build on an earlier proposal for creating Majorana zero modes, that is, the anyons with the simplest non-Abelian statistics, using 1D modes at the edge of a 2D topological insulator (TI) in contact with a superconductor (SC).\(^18\) In contrast to TIs, in two-dimensional electron gases (2DEGs) in the QH regime, by varying filling factor \(\nu\) states can be realized with a variety of topological orders. This allows access to more exotic edge states needed for engineering anyons with richer non-Abelian statistics. Key in all these theoretical proposals is the ability to induce robust superconducting correlations into the edge modes of a QH state.

In this work, we show that in high quality InAs/NbTiN heterostructures, very strong superconducting correlations can be induced in the edge modes of integer QH states realized in the InAs-based quantum wells (QWs). Such correlations appear to be robust, showing no oscillations as a function of doping, for gate voltages within the QH plateaus. We analyze the experimental data in conjunction with a microscopic model to extract the details of the processes determining the transport properties of the QH−SC interface.

Received: April 7, 2022
Revised: July 18, 2022
Published: July 22, 2022
Figure 1a shows a cross sectional schematic of the fabricated device used in this work. The QW is formed by a 4 nm layer of In$_{0.81}$Ga$_{0.19}$As layer, a 7 nm layer of InAs, and a 10 nm top layer of In$_{0.81}$Ga$_{0.19}$As. The QW is grown on In$_x$Al$_{1-x}$As buffer where the indium content is step-graded from $x = 0.52$ to 0.81 to help with dislocations originating from lattice mismatch between InP and quantum well in higher levels. A delta-doped Si layer with electron doping $n \sim 1 \times 10^{12}$ cm$^{-2}$ is placed 6 nm below the QW. This epitaxial structure has been used in previous studies on mesoscopic superconductivity in the development of tunable qubits and in studies aimed at realizing and detecting topological superconducting states.

A Hall bar, Figure 1b, is fabricated by electron beam lithography. In order to study the 2DEG/SC interface, a 90 nm thick layer of NbTiN was sputtered as the superconducting contacts with a 150 µm-wide interface after performing wet etch surface cleaning (Device A). We also fabricated a similar device with intentional no surface cleaning step before NbTiN sputtering (Device B). A metallic top gate is created by depositing a layer of Al oxide followed by an Al layer to control the QW electron density.

The mobility of the QW is determined to be $\mu \sim 12,000$ cm$^2$/V·s at $n \sim 8.51 \times 10^{11}$ cm$^{-2}$ corresponding to an electron mean free path of $l_e \sim 180$ nm. All data reported here were taken at $T \sim 30$ mK. We have provided more information on transport properties of the sample in the SI. We note that while we focus mainly on one device (Device A) in the main text, we have studied a few other similar devices which their data have been shown in SI.

When the sample is placed in a magnetic field, in the classical picture electrons and holes will alternate their skipping orbits across the interface of the superconductor and 2DEG. In the full quantum-mechanical analysis the electron and hole edge states hybridize due to the proximity of the SC and form a coherent chiral Andreev edge state (CAES) extended along the QH–SC interface. A schematic of CAES propagation along the QH–SC interface is shown in Figure 1c. In this picture, if more holes than electrons reach the normal lead downstream from the superconducting electrode (lead 5), then a negative potential difference ($V_5 - V_4'$) develops. In Figure 1d, we show the local density of states (LDOS) of a CAES obtained with a tight binding (TB) calculation performed using the python package Kwant. In the TB model, the
presence of the magnetic field is taken into account via a Peierls phase, and the superconductivity of the QW proximitized by NbTiN via a mean field s-wave pairing term of strength $\Delta$. The details of the TB model can be found in the SI.

Figure 1e,f shows the results for the downstream resistance, $R_{D}^{\text{IL}}$, measured between the voltage contacts 5 and 4’, as a function of gate voltage $V_{g}$ and magnetic field $B$. Hall resistance data measured between contacts 2 and 5 allow us to determine the filling factor of the different regions of Figure 1e,f. Figure 2a shows the horizontal cut at $B = 11$ T of Figure 1f, $R_{x}^{\text{IL}}$, and the corresponding longitudinal resistance $R_{x}$. From the $R_{x}$ measurements, we see that we have well developed integer QH states (IQHS). From Figures 1f and 2a, we clearly observe that $R_{D}$ is negative for IQHS, a fact that strongly suggests the presence of Andreev processes at the QH–SC interface for these IQHS. The similarity of $R_{x}^{\text{IL}}$ and $R_{D}$ also is a hint of IQHS being dissipationless states; therefore, going farther from the interface does not affect the magnitude of the negative resistance value. We notice the importance of a clean InAs/NbTiN interface by comparing the magnitude of negative resistance in Figure 1e,f. The clean interface on device A has been achieved by etching the surface of defined NbTiN pattern area by buffered oxide etchant (BOE) for 2 s immediately followed by loading into sputtering tool’s chamber in order to minimize the time for the native oxide growth at the interface. On the other hand, for device B this cleaning step has been skipped and NbTiN sputtered on the defined region after its exposure to air. For the rest of the paper, we focus only on device A results. The upstream resistance $R_{U}^{\text{IL}}$ (measured between contacts 3 and 4) exhibits plateaus corresponding to the $R_{x}$ plateaus in the magnetic field but with resistance values lower than $R_{x}$ (Figure 2b). Moreover, $R_{D}^{\text{IL}} - R_{U}^{\text{IL}}$ recovers the quantized Hall value, $R_{x}$, as shown in Figure 2c. Note that this difference does not necessarily match the $R_{x}$ data outside the QH regime.

These results can be understood within the Landauer–Buttiker (LB) theory, generalized to allow for the presence of a superconducting lead.\(^{33,44}\)

We start with the six-terminal setup shown in Figure 1b (see also the SI). We assume the terminal 1, 2, 3, 5, 6 to be ideal metallic leads, and contact 4 to be a superconducting lead. We first consider the limit in which no normal reflection or transmission processes take place at the superconducting lead. $I_{1}$ and $V_{1}$, the currents and voltages, respectively, are allowed at the terminals $i = (1, 2, 3, 4, 5, 6)$. Without loss of generality, we can set $V_{4} = 0$. We can use the charge conservation equation $\sum I_{i} = 0$ to express $I_{4}$ in terms of the currents at the other leads. With these considerations, the LB equations reduce to the following system of linear equations

$$\begin{align*}
\begin{pmatrix}
I_{1} \\
I_{2} \\
I_{3} \\
I_{4} \\
I_{5} \\
I_{6}
\end{pmatrix}
&=
\begin{pmatrix}
1 & 0 & 0 & 0 & -1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 2A - 1 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
V_{1} \\
V_{2} \\
V_{3} \\
V_{5} \\
V_{6}
\end{pmatrix}
\end{align*}$$

(1)

where $\nu$ is the number of edge states, $R_{ij}$ is the Hall resistance, and $A$ is the average probability, per edge mode, of Andreev reflection. Considering that no current flows into leads 2, 3, 5, 6, so that $I_{2} = I_{3} = I_{5} = I_{6} = 0$, $V_{1} = V_{2} = V_{5} = V_{6}$ and setting $I \equiv I_{4}$, it is straightforward to solve eq 1 to obtain

$$\begin{align*}
R_{U}^{\text{IL}} &= \frac{V_{2}}{I} = \frac{R_{H}^{-1}}{\nu} \\
R_{D}^{\text{IL}} &= \frac{V_{4}}{I} = \frac{R_{H}}{\nu} \left( \frac{1}{2A - 1} - 1 \right)
\end{align*}$$

(2)

(3)

Figure 3a shows the scaling of $R_{D}$ with respect to $1/\nu$ for different values of $B$. From the slope of the fits to the experimental data shown in Figure 3a, we obtain the value of $A_{\text{exp}} = 55\%$, independent, to very good approximation, on the value of $V_{x}$ within the QH plateaus (Figure 3b). It is instructive to compare our work with two other classes of experiments. First, studies based on graphene-MoRe, see ref 26, where the value of $R_{D}$ oscillates significantly with $B$ and gate voltage, and therefore there is no well-defined value of $A$. Second, studies based on graphene-NbTiN, see ref 23, in which a narrow and long superconducting electrode is placed perpendicular to the vacuum edge of the QH system. This geometry allows for additional processes, cross-Andreev-reflection (CAR) processes that are not present in our setup and the setup in ref 26. From the value of $R_{D}$ at $\nu = 1$ in ref 23, we extract a value of $A_{\text{exp}} = 50.5\%$ but the comparison to our setup is not straightforward due to the presence of CAR processes. We emphasize that in addition to the large value of $A$, a unique feature of our devices is the lack of oscillations as a function of $V_{x}$ and $B$ despite a QH–SC interface that is more than an order of magnitude longer than in the experiments in refs 23 and 26, and that this suggests that in QH–SC based on InAs/SC the CAES might have an extremely long coherence length.

Figure 3c shows the consistency of the measured values of $R_{D}^{\text{IL}}$ and $R_{D}^{\text{SI}}$ with the LB predictions by plotting the ratio $(R_{D}^{\text{IL}} - R_{U}^{\text{IL}})/R_{x}$ as a function of $B$ that, according to eqs 2 and 3, is expected to be equal to 1.

To understand qualitatively how such values of $A$ can arise, we consider an effective 1D Bogoliubov de Gennes
leading to a dephasing of the electron-like and hole-like modes along the QH–SC interface.\textsuperscript{28} In this case, the effective $A_{\text{eff}}$ can be obtained by averaging over $L_{sc}$ on the right-hand side of eq 4 to obtain $A_{\text{eff}} = \langle A \rangle = 1/[2(1 + (\nu_{k_F}/\Delta)^2)]$. Considering that $(\nu_{k_F}/\Delta)^2 > 0$, we see that in this case we cannot recover the value of $A$ extracted from the experimental results, $A = 0.55 > 0$.

To explain the large value of $A$, accompanied by the lack of oscillations as a function of $V_g$, we are led to two possibilities. The first possibility is that $\delta_{\text{h}}$ does not change appreciably as a function of $V_g$. From eq 4, considering that $0 < \sin^2(\delta \phi) < 1$, we can see that to have $A = 0.55$ we must have $\nu_{k_F}/\Delta < 0.9$. In the limit when $\delta \phi$ is such that $\sin^2(\delta \phi) \approx 0.55$, we must have $\nu_{k_F}/\Delta \ll 1$. In this limit we can write $\delta k \approx \Delta/\nu_{k_F}$. In good approximation, $\Delta$ and $\nu_{k_F}$ are independent of $V_g$ and we recover the observed values of $R_{U}$ and $R_{D}$ with no oscillations in the QH plateaus. Notice that the condition $\nu_{k_F}/\Delta \ll 1$ is equivalent to the condition $\delta k \xi \approx 1$, where $\xi \equiv \nu_{k_F}/\Delta$ can be interpreted as the superconducting coherence length of the edge modes in proximity of the SC. The other possibility is that dephasing processes are accompanied by a finite probability of single electron tunneling into the superconductor and breaking of particle-hole (p-h) symmetry. This would allow to have a situation in which electron-like states are more likely than h-like states to tunnel into the superconductor and therefore contribute less to the downstream current explaining a negative $R_D$ even when $A \leq 1.2$. If we denote by $T$ the probability, per edge mode, of an electron-like state to tunnel in the SC, in eqs 2 and 3 we would replace $2A$ with $2A + T$. In this case, from the measurements of $R_{U}$ and $R_{D}$ we recover $2A + T$. Assuming $(A) = 1/2$ would imply $T = 0.2$. The smallest value of $(A)$, consistent with particle conservation, is 15% to which it would correspond $T = 0.8$, a value that implies a very strong breaking of particle-hole symmetry at the QH–SC interface. It is difficult to distinguish between these two possibilities given that we cannot measure separately the quasiparticle and supercurrent contributions to the charge current flowing from the QH region into the superconducting lead.

In conclusion, we have fabricated a quantum Hall-superconductor (QH–SC) epitaxial heterostructure based on InAs and NbTiN and characterized the transport properties of its QH edge modes propagating along a superconducting interface. We have observed negative values for the downstream resistance $R_D$ between a normal lead and the superconducting lead and a corresponding suppression of the upstream resistance $R_U$ such that in the QH plateaus the difference $R_U - R_D$ is equal to the Hall resistance $R_H$. The negative values of $R_D$ are an unambiguous sign that at the QH–SC interface there is a very large electron–hole conversion probability, $A$. Using a Landauer-Büttiker analysis, we were able to explain the relation between $R_D$ and $R_U$ and express both resistances in terms of a single effective probability for Andreev reflections at the QH–SC interface. Our analysis led us to the conclusion that either the edge modes propagate along the QH–SC interface with negligible dephasing resulting in an electron–hole conversion close to 55%, or if dephasing processes dominate that a strong breaking of particle-hole symmetry at the QH–SC interface must occur. The interface could be further improved by hybrid methods of epitaxial superconductors and semiconductors.

Even the lower bounds’ estimates for $A$ that we extract from our measurements are remarkable, larger than any published

\begin{align}
A &= \frac{\sin^2(\delta \phi)}{[1 + (\nu_{k_F}/\Delta)^2]} \tag{4}
\end{align}

In eq 4, $\delta \phi$ is the difference of the phases accumulated by the electron-like and hole-like edge modes along the length of the QH–SC interface. Let $k_F^{(\text{e})}$, $k_F^{(\text{h})}$ be the Fermi wave vector of the electron-like and hole-like edge modes, and $\delta k \equiv |k_F^{(\text{e})} - k_F^{(\text{h})}| = [\Delta^2 + (\nu_{k_F})^2]^{1/2}/\nu_{k_F}$. We then write $\delta \phi = L_{sc} \delta k$.

Considering that $L_{sc} = 150 \mu m$ is quite large, any small change of $\delta k$ induced, for example, by changes in $V_g$ should result in a significant change of $A$ and therefore of $R_D$ and $R_U$. However, in the experiment within the QH plateaus, $R_D$ and $R_U$ do not show any oscillation as a function of $V_g$. It is natural to conclude that this might be due to scattering processes

Figure 3. (a) $R_D^{(\text{sc})}$ versus $1/\nu$ for different IQHSs and values of $B$, and linear fits corresponding to each magnetic field. (b) A obtained from the slope of linear fits to $R_D$ and $R_U$ data versus $1/\nu$ with their corresponding error bars. (c) $(R_U^{(\text{sc})} - R_D^{(\text{sc})})/R_{\text{sc}}$ for different $\nu$ versus and values of $B$. 

Hamiltonian, $H_{BdG}$, for the 1D chiral edge modes: $H_{BdG} = \int dx \psi^\dagger(x)H(x)\psi(x)$ where $\psi(x) = (c_{\uparrow}, c_{\downarrow})$ for a Fermion at position $x$ and spin up (down) and $H(x) = v_F(-i\partial_x)\tau_0 - v_{k_F}\tau_2 + \Delta \tau_3$. Here and in the remainder of this Letter, we set $\hbar = 1$. In the equation for $H(x)$, $v_F$ is the drift velocity of the edge modes, $\tau_i$ values are the Pauli matrices in Nambu space, $k_F$ is the edge modes Fermi wave vector number (measured with respect to the QH–SC interface), and $\Delta$ is the superconducting superconductor pairing induced via the proximity effect by of the superconducting lead. Using the expression for $H_i$, we can obtain the transfer matrix $M$ relating $\psi(x)$ at the two ends of the length $L_{sc}$ of the QH–SC interface of length $L_{sc}$ (see SI),\textsuperscript{40,45} and then the expression for the electron–hole conversion probability will be
results for QH–SC devices. This shows that in our InAs devices very strong superconducting correlations can be induced into the QH edge modes, an essential prerequisite to use QH–SC heterojunctions to realize non-Abelian anyons and topologically protected qubits and quantum gates based on such unusual quantum states.

- **ASSOCIATED CONTENT**
  
  - **Supporting Information**
     The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.nanolett.2c01413.

  2DEG characterization of device A, nanofabrication details, data for other fabricated devices for this study, Landauer-Büttiker edge state model, derivation of Andreev reflection probability, effect of electron tunneling into the superconductor, and tight binding model details (PDF)

- **AUTHOR INFORMATION**

  **Corresponding Author**
  Javad Shabani — Center for Quantum Phenomena, Department of Physics, New York University, New York, New York 10003, United States; orcid.org/0000-0002-0812-2809; Email: jshabani@nyu.edu

  **Authors**
  Mehdi Hatefipour — Center for Quantum Phenomena, Department of Physics, New York University, New York, New York 10003, United States
  Joseph J. Cucuzzo — Department of Physics, William & Mary, Williamsburg, Virginia 23187, United States
  Jesse Kanter — Center for Quantum Phenomena, Department of Physics, New York University, New York, New York 10003, United States
  William M. Strickland — Center for Quantum Phenomena, Department of Physics, New York University, New York, New York 10003, United States
  Christopher R. Allemand — Sandia National Laboratories, Albuquerque, New Mexico 87185, United States
  Tzu-Ming Lu — Sandia National Laboratories, Albuquerque, New Mexico 87185, United States; Center for Integrated Nanotechnologies, Sandia National Laboratories, Albuquerque, New Mexico 87123, United States; orcid.org/0000-0002-3363-1226
  Enrico Rossi — Department of Physics, William & Mary, Williamsburg, Virginia 23187, United States

  Complete contact information is available at: https://pubs.acs.org/10.1021/acs.nanolett.2c01413

- **Notes**

  The authors declare no competing financial interest.

- **ACKNOWLEDGMENTS**

  We thank Dr. Shashank Misra for his insightful comment and feedback. The NYU team acknowledge partial support from DARPA Grant DP18AP0900007 and ONR Grant N00014-21-1-2450. J.J.C. and E.R. acknowledge support from ARO Grant W911NF-18-1-0290. E.R. acknowledges the Aspen Center for Physics, which is supported by National Science Foundation grant PHY-1607611. J.J.C. also acknowledges support from the Graduate Research Fellowship awarded by the Virginia Space Grant Consortium (VSGC). Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia LLC, a wholly owned subsidiary of Honeywell International Inc. for the U.S. DOE’s National Nuclear Security Administration under contract DE-NA0003525. This work was performed, in part, at the Center for Integrated Nanotechnologies, a U.S. DOE, Office of BESs, user facility. This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. DOE or the United States Government.

- **REFERENCES**

  1. Moore, G.; Read, N. Nonabelions in the fractional quantum hall effect. *Nuclear Physics B* 1991, 360, 362–396.
  2. Kitaev, A. Fault-tolerant quantum computation by anyons. *Annals of Physics* 2003, 303, 2–30.
  3. Das Sarma, S.; Freedman, M.; Nayak, C. Topologically Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State. *Phys. Rev. Lett.* 2005, 94, 166802.
  4. Nayak, C.; Simon, S. H.; Stern, A.; Freedman, M.; Das Sarma, S. Non-Abelian anyons and topological quantum computation. *Rev. Mod. Phys.* 2008, 80, 1083–1159.
  5. Read, N.; Green, D. Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect. *Phys. Rev. B* 2000, 61, 10267–10297.
  6. Cheng, M. Superconducting proximity effect on the edge of fractional topological insulators. *Phys. Rev. B* 2012, 86, 195126.
  7. Alicea, J.; Fendley, P. Topological Phases with Parafermions: Theory and Blueprints. *Annual Review of Condensed Matter Physics* 2016, 7, 119–139.
  8. Vaezi, A. Fractional topological superconductor with fractionali-zed Majorana fermions. *Phys. Rev. B* 2013, 87, 035132.
  9. Vaezi, A. Superconducting Analogue of the Parafermion Fractional Quantum Hall States. *Phys. Rev. X* 2014, 4, 031009.
  10. Willert, R.; Eisenstein, J. P.; Störmer, H. L.; Tsui, D. C.; Gossard, A. C.; English, J. H. Observation of an even-denominator quantum number in the fractional quantum Hall effect. *Phys. Rev. Lett.* 1987, 59, 1776–1779.
  11. Miller, J. B.; Rudu, I. P.; Zumbühl, D. M.; Levenson-Falk, E. M.; Kastner, M. A.; Marcus, C. M.; Pfeiffer, L. N.; West, K. W. Fractional quantum Hall effect in a quantum point contact at filling fraction 5/2. *Nat. Phys.* 2007, 3, 561–565.
  12. Rudu, I. P.; Miller, J. B.; Marcus, C. M.; Kastner, M. A.; Pfeiffer, L. N.; West, K. W. Quasi-Particle Properties from Tunneling in the ν = 5/2 Fractional Quantum Hall State. *Science* 2008, 320, 899–902.
  13. Zhu, W.; Gong, S. S.; Haldane, F. D. M.; Sheng, D. N. Fractional Quantum Hall States at ν = 13/5 and 12/5 and Their Non-Abelian Nature. *Phys. Rev. Lett.* 2015, 115, 126805.
  14. Qi, X.-L.; Hughes, T. L.; Zhang, S.-C. Chiral topological superconductor from the quantum Hall state. *Phys. Rev. B* 2010, 82, 184516.
  15. Lindner, N. H.; Berg, E.; Refael, G.; Stern, A. Fractionalizing Majorana Fermions: Non-Abelian Statistics on the Edges of Abelian Quantum Hall States. *Phys. Rev. X* 2012, 2, 041002.
  16. Clarke, D. J.; Alicea, J.; Shtengel, K. Exotic circuit elements from zero-modes in hybrid superconductor–quantum-Hall systems. *Nat. Phys.* 2014, 10, 877–882.
  17. Mong, R. S. K.; Clarke, D. J.; Alicea, J.; Lindner, N. H.; Fendley, P.; Nayak, C.; Oreg, Y.; Stern, A.; Berg, E.; Shtengel, K.; Fisher, M. P. A. Universal Topological Quantum Computation from a Superconductor-Abelian Quantum Hall Heterostructure. *Phys. Rev. X* 2014, 4, 011036.
  18. Fu, L.; Kane, C. L. Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator. *Phys. Rev. Lett.* 2008, 100, 096407.
Mechanisms of Andreev reflection in quantum Hall graphene.

T. F. Q.; Draelos, A. W.; Li, H.; Watanabe, K.; Taniguchi, T.; Amet, structure.

Nat. Commun. 2015, 6, 7426.

(21) Rickhaus, P.; Weiss, M.; Marot, L.; Schönenberger, C. Quantum Hall Effect in Graphene with Superconducting Electrodes. Nano Lett. 2012, 12, 1942.

(22) Park, G.-H.; Kim, M.; Watanabe, K.; Taniguchi, T.; Lee, H.-J. Propagation of superconducting coherence via chiral quantum-Hall edge channels. Sci. Rep. 2017, 7, 10953.

(23) Gill, O.; Ronen, Y.; Lee, S. Y.; Shapourian, H.; Zauberman, J.; Lee, Y. H.; Watanabe, K.; Taniguchi, T.; Vishwanath, A.; Yacoby, A.; Kim, P. Andreev Reflection in the Fractional Quantum Hall State. Phys. Rev. X 2012, 12, 021057.

(24) Lee, G.-H.; Huang, K.-F.; Efetov, D. K.; Wei, D. S.; Hart, S.; Taniguchi, T.; Watanabe, K.; Yacoby, A.; Kim, P. Inducing superconducting correlation in quantum Hall edge states. Nat. Phys. 2017, 13, 693–698.

(25) Amet, F.; Ke, C. T.; Borzenets, I. V.; Wang, J.; Watanabe, K.; Taniguchi, T.; Deacon, R. S.; Yamamoto, M.; Bonze, Y.; Tarucha, S.; Finkelstein, G. Supercurrent in the quantum Hall regime. Science 2016, 352, 966–969.

(26) Zhao, L.; Arnauld, E. G.; Bondarev, A.; Seredinski, A.; Larson, T. F. Q.; Draelos, A. W.; Li, H.; Watanabe, K.; Taniguchi, T.; Deacon, R. S.; Yamamoto, M.; Bonze, Y.; Tarucha, S.; Finkelstein, G. Supercurrent in the quantum Hall edge states. Nat. Phys. 2020, 16, 862–867.

(27) Hoppe, H.; Zülicke, U.; Schön, G. Andreev Reflection in Strong Magnetic Fields. Phys. Rev. Lett. 2000, 84, 1804–1807.

(28) Kurilovich, V. D.; Raines, Z. M.; Glazman, L. I. Disorder in Andreev reflection of a quantum Hall edge. arXiv, 2022, 2201.00273 (accessed Jan. 2, 2022) https://arxiv.org/abs/2201.00273.

(29) Manesco, A. L. R.; Flór, I. M.; Liu, C.-X.; Akhmerov, A. R. Mechanisms of Andreev reflection in quantum Hall graphene. arXiv, 2021, 2103.06722 (accessed Feb. 15, 2022) https://arxiv.org/abs/2103.06722.

(30) Kjaergaard, M.; nichele, F.; Suominen, H.; Nowak, M.; Wimmer, M.; Akhmerov, A.; Folk, J.; Flensberg, K.; Shabani, J.; Palzmrom, C.; Marcus, C. Quantized conductance doubling and hard gap in a two-dimensional semiconductor-superconductor heterostructure. Nat. Commun. 2016, 7, 12841.

(31) Bet啾, C. G. L.; nichele, F.; Kjaergaard, M.; Suominen, H. J.; Shabani, J.; Palzmrom, C. J.; Marcus, C. M. Superconducting, insulating and anomalous metallic regimes in a gated two-dimensional semiconductor-superconductor array. Nat. Phys. 2018, 14, 1138–1144.

(32) Mayer, W.; Yuan, J.; Wickramasinghe, K. S.; Nguyen, T.; Dartiall, M. C.; Shabani, J. Superconducting proximity effect in epitaxial Al-InAs heterostructures. Appl. Phys. Lett. 2019, 114, 103104.

(33) Mayer, W.; Dartiall, M. C.; Yuan, J.; Wickramasinghe, K. S.; Rossi, E.; Shabani, J. Gate controlled anomalous phase shift in Al/InAs Josephson junctions. Nat. Commun. 2020, 11, 212.

(34) Casparis, L.; Pearson, N. J.; Kringlehaj, A.; Larsen, T. W.; Kuummeth, F.; Nygaard, J.; Krogstrup, P.; Petersson, K. D.; Marcus, C. M. Voltage-controlled superconducting quantum bus. Phys. Rev. B 2019, 99, 085434.

(35) Suominen, H. J.; Kjaergaard, M.; Hamilton, A. R.; Shabani, J.; Palzmrom, C. J.; Marcus, C. M.; nichele, F. Zero-Energy Modes from Coalescing Andreev States in a Two-Dimensional Semiconductor-Superconductor Hybrid Platform. Phys. Rev. Lett. 2017, 119, 176805.

(36) Formieri, A.; et al. Evidence of topological superconductivity in planar Josephson junctions. Nature 2019, 569, 89–92.

(37) Dartiall, M. C.; Cuzzo, J. J.; Efleky, B. H.; Mayer, W.; Yuan, J.; Wickramasinghe, K. S.; Rossi, E.; Shabani, J. Missing Shapiro steps in topologically trivial Josephson junction on InAs quantum well. Nat. Commun. 2021, 12, 78.

(38) Shabani, J.; Kjaergaard, M.; Suominen, H. J.; Kim, Y.; nichele, F.; Pakrouski, K.; Stankevic, T.; Lutchyn, R. M.; Krogstrup, P.; Feidenhansl, R.; Kraemer, S.; Nayak, C.; Troyer, M.; Marcus, C. M.; Palzmrom, C. J. Two-dimensional epitaxial superconductor-semiconductor heterostructures: A platform for topological superconducting networks. Phys. Rev. B 2016, 93, 155402.

(39) Chicholkatchev, N. M.; Burmistrov, I. S. Conductance oscillations with magnetic field of a two-dimensional electron gas-superconductor junction. Phys. Rev. B 2007, 75, 214510.

(40) van Ostaay, J. A. M.; Akhmerov, A. R.; Beenakker, C. W. J. Spin-triplet supercurrent carried by quantum Hall edge states through a Josephson junction. Phys. Rev. B 2011, 83, 195441.

(41) Khaymovich, I. M.; Chicholkatchev, N. M.; Shereshevski, I. A.; Melnikov, A. S. Andreev transport in two-dimensional normal-superconducting systems in strong magnetic fields. EPL (Europhysics Letters) 2010, 91, 17005.

(42) Groth, C. W.; Wimmer, M.; Akhmerov, A. R.; Waintal, X. Qwnt: a software package for quantum transport. New J. Phys. 2014, 16, 063065.

(43) Datta, S.; Bagwell, P. F.; Anantram, M. P. Scattering Theory for Transport in Mesoscopic Superconductors, ECE Technical Reports, Paper 107; Purdue University, 1996.

(44) Beconcini, M.; Polini, M.; Taddei, F. Nonlocal superconducting correlations in graphene in the quantum Hall regime. Phys. Rev. B 2018, 97, 201403.

(45) Gao, H.; Xue, H.; Wang, Q.; Gu, Z.; Liu, T.; Zhu, J.; Zhang, B. Observation of topological edge states induced solely by non-Hermiticity in an acoustic crystal. Phys. Rev. B 2020, 101, 180303.

(46) Drachmann, A. C. C.; Suominen, H. J.; Kjaergaard, M.; Shojaei, B.; Palzmrom, C. J.; Marcus, C. M.; nichele, F. Proximity Effect Transfer from NbTi to a Semiconductor Heterostructure via Epitaxial Aluminum. Nano Lett. 2017, 17, 1200–1203.