Implementation of a numerical holding furnace model in foundry and construction of a reduced model

Thomas LOUSSOUARN¹,², Denis MAILLET¹, Benjamin REMY¹, Diane DAN²
¹LEMTA, UMR 7563, CNRS, Vandoeuvre-Lès-Nancy, 54504, France
²ITKMM Foundry modelling, SNECMA Gennevilliers, Colombes, 92700, France

Abstract. Vacuum holding induction furnaces are used for the manufacturing of turbine blades by loss wax foundry process. The control of solidification parameters is a key factor for the manufacturing of these parts in according to geometrical and structural expectations. The definition of a reduced heat transfer model with experimental identification through an estimation of its parameters is required here. In a further stage this model will be used to characterize heat exchanges using internal sensors through inverse techniques to optimize the furnace command and the optimization of its design. Here, an axisymmetric furnace and its load have been numerically modelled using FlexPDE, a finite elements code. A detailed model allows the calculation of the internal induction heat source as well as transient radiative transfer inside the furnace. A reduced lumped body model has been defined to represent the numerical furnace. The model reduction and the estimation of the parameters of the lumped body have been made using a Levenberg-Marquardt least squares minimization algorithm with Matlab, using two synthetic temperature signals with a further validation test.

1. Introduction
The manufacturing of single crystal turbine blades by lost wax foundry process requires a control of the heat exchanges in the process. Most of the research in investment casting are focused on the study of the heat exchange between the metal and the mould [1] [2]. There is no well-known systemic approach. The furnaces used for this process are equipped with few sensors. The only information required for the control of the furnace during a manufacturing cycle are the regulation parameters. So, it is usual to find only one thermocouple by heating element of the furnace. In our configuration, the furnace has two heating elements. The goal of this study is the understanding of the interactions between the electrical power spent by the inductor, the heating power generated in the susceptor, the heating load of the mould and the heating losses to the outside environment. The design of a detailed furnace model and its numerical implementation through the commercial finite element code FlexPDE [3] will allow to get reference simulations of the heat transfer field in the furnace first. We will then define a reduced model that aims at describing the furnace state at all times by the transient response of its sensors.

The detailed numerical model of an axisymmetric furnace consists of two separate sub-models. The first one describes the induction phenomenon. It uses the electrical power spent by the unit as input, and the effective heating power distribution in the system as output. The second one describes heat transfer in the system: the heating power density in the susceptors is its input with temperature at any point in the furnace as its output. These two 2D sub-models are weakly coupled since the electromagnetic characteristic times are very low with respect to the characteristic times of heat conduction: the induction model will be run first and its output will be used as input of the heat transfer model, to generate the temperature field output of the detailed model. This two-step procedure will allow to simulate heating and cooling by controlling the electrical power consumed by the furnace. The detailed model outputs will be compared with those of a reduced 0D model that has to be designed in order to be representative of heat generation and transfer in the induction furnace.
2. Furnace presentation

2.1. The used furnace configuration

The heating module of the furnace is presented in Figure 1. The whole unit is under vacuum conditions, which means that no convection occurs. This module has two parts called upper part and lower part. Each part has its own inductor, susceptor, insulator and thermocouple. Inductors have different properties and their surface temperature is 473 K. The radiation exchanges between insulators and inductors is characterized by a coefficient $X_{\text{induc}}$. The heating power density is generated in the susceptors. Thermocouples are not represented in the figure: they will be considered as measurement points near the inner side of the susceptors here. In the heating module, the cylindrical mould base rests on a cooling copper part, which is called the hearth. The surface temperature of the hearth is 473 K. Thermal contact between mould and hearth is modelled by a low exchange coefficient $h_{\text{hearth}}$ corresponding to poor contact conditions. The upper and lower closings of the heating chamber are considered as perfect insulators. The system has a rotational symmetry relative to the vertical axis that is why it will be always represented in a 2D axisymmetric.

![Figure 1. Heating module configuration studied with the load (mould) inserted](image)

2.2. Electromagnetic modelisation and heating power density calculation

The modelling is based on the equation which governs the magnetic vector potential $\vec{A}$ (Wb.m$^{-2}$). The working frequency is 3 kHz, a magneto quasi-static approximation is used [4]. This approximation greatly simplifies the Maxwell equations. Then, we get the diffusion equation of the magnetic potential vector $\vec{A}$ projected onto the unit tangential vector $\vec{e}_\theta$:

$$
-\frac{1}{\mu} \left( \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} \right) + \Sigma \frac{\partial A}{\partial t} = J_s
$$

with $J_s$ the norm of the current density source (A.m$^{-2}$), $\Sigma$ the electric conductivity ($\Omega^{-1}.m^{-1}$) and $\mu$ the magnetic permeability (H.m$^{-2}$). We have chosen a current source formulation for the induction model [5] and the expression of the current density source is given by:
\[ J_s = \frac{I_s}{S_{eff}} \]  

where \( I_s \) is the current in the inductor in A and \( S_{eff} \) the effective area of the current cross section caused by the skin effect in m\(^2\). \( I_s \) corresponds to the square root of the electrical power furnished at the inductor in W divided by the inner resistance of the inductors with the complete heating module in Ω. Solution of the diffusion equation is sought in the complex space with an harmonic approximation [4]. Solving is implemented through FlexPDE code.

The heating power density \( P_V \) (W.m\(^{-3}\)) is connected to the magnetic vector potential by the following equation:

\[ P_V = \frac{1}{2} \sum |\frac{\partial A}{\partial t}|^2 \]  

Therefore, once the magnetic vector potential has been calculated, the heating power density at any point of the different materials in the module is known.

2.3. Heat transfer in the heating module and radiation

Thermal modelling of the furnace is applied to the domain made up of the mould, of the vacuum in the heating chamber, of the susceptors and insulators. Conduction (in solid parts) and radiation (in vacuum) transfers are conjugated in the module. So the transient heat equation (4) and the equation of radiative transfer in a non-diffusive and weakly absorbing grey body (5) should be simultaneously solved. There is a coupling by the boundaries. The emissivity of the mould is equal to 0.6 and the susceptor’s one is 0.8. We are considering the radiative flux as a constant in the vacuum and the directional luminance field \( L' \) in W.m\(^2\).sr\(^{-1}\) gradient in any \( \vec{n} \) direction is equal to 0. We get the following system of equations for temperature \( T \) (K) in each solid part of volumetric capacity \( \rho c_p \) (J.m\(^{-3}\).K\(^{-1}\)), for volumetric power densities \( P_{up} \) and \( P_{low} \):

\[ \text{div} \left( \lambda \text{grad}(T(r,z,t)) \right) + P_{up}(r,z,t) + P_{low}(r,z,t) = \rho c_p \frac{\partial T(r,z,t)}{\partial t} \]  

\[ \frac{dL'(s,\vec{n})}{ds} = 0 \]  

Solution of the radiative transfer equation has been made with the P\(_1\) method in a non-diffusive and weakly absorbing medium [6]. This method uses the decomposition of the directional luminance field \( L' \) in a spherical harmonics complete basis. The first order solution consists in the approximation of the directional luminance field by the zero and first order terms on this basis. We get the following expression for the 2D axisymmetric case, where \( (\theta, \phi) \) are the two angles of the optical path:

\[ L'(r,z,\theta,\phi) = \frac{1}{4\pi} \left[ L_0 + 3L_z \cos(\theta) + 3L_r \sin(\theta) \cos(\phi) \right] \]  

Finally, it yields the moment equations system (7), (8) and (9). The two components of the radiative flux are the solution of the system of partial differential equations of the mean luminance \( L_0 \) with \( k \) (m\(^{-1}\)) the absorption coefficient.

\[ \text{div} \left( \frac{1}{3k} \text{grad}(L_0) \right) = 0 \]  

\[ L_r = -\frac{1}{3k} \frac{\partial L_0}{\partial r} \]  

\[ L_z = -\frac{1}{3k} \frac{\partial L_0}{\partial z} \]
The expression of the radiative flux is then:

\[ Q_r = L_r e_r + L_z e_z \]  

(10)

The radiative boundary conditions are written as Marshak’s boundary conditions[6].

3. Generating a furnace heating and cooling behavior and output of the detailed model
In FlexPDE, the inputs are the electrical power time profiles \( P_{up}(t) \) and \( P_{low}(t) \) consumed by the upper and lower inductors. They are shown in Figure 2.

![Figure 2. Time distribution of electrical power spent in the upper and lower parts inductors](image)

The electromagnetic model provides a heating power density map in the different parts of the furnace (Figure 3 and Figure 4).

![Figure 3. Display of the heating power density due to the upper inductor](image)

![Figure 4. Display of the heating power density due to the lower inductor](image)

The heat transfer model, implemented under FlexPDE, gives the temperature field, see Figure 5. We can extract temperature time distribution at any point of interest, see Figure 6, or a mean temperature time profiles in a given region.

4. Parameters estimation and model reduction
4.1. The 0D reduced model and its assumptions
A 0D lumped body model coupled by the power source is chosen. The lumped bodies are the upper and lower susceptors. The heat loss with outside are modelled by an equivalent conducto-radiative resistance \( R_{out} (K.W^{-1}) \) corresponding to the exchange between susceptor and inductor surface through the insulator.
The heat exchange with the mould are modelled by an equivalent radiative resistance $X_{\text{in}}$ (m$^{-2}$) and the Stefan-Boltzmann constant $\sigma$ (W.m$^{-2}$.K$^{-4}$). There is no heat transfer (conduction, radiation) between the upper and the lower parts. The reduced model is defined by the following system of equations (11) and (12):

$$\rho C_p V_u \frac{dT_u}{dt} = \eta_{uu} P_{up}(t) + \eta_{ul} P_{low}(t) - \frac{1}{R_{\text{out}u}} (T_u - T_{\text{out}}) - \frac{\sigma}{X_{\text{in}u}} (T_u^4 - T_{\text{mould}}^u)$$ (11)

$$\rho C_p V_l \frac{dT_l}{dt} = \eta_{ll} P_{low}(t) + \eta_{lu} P_{up}(t) - \frac{1}{R_{\text{out}l}} (T_l - T_{\text{out}}) - \frac{\sigma}{X_{\text{in}l}} (T_l^4 - T_{\text{mould}}^l)$$ (12)

The efficiencies $\eta_{uu}$ and $\eta_{ll}$, the cross efficiencies $\eta_{ul}$ and $\eta_{lu}$, the equivalent resistance for the exchange with outside $R_{\text{out}u}$ and $R_{\text{out}l}$, and the equivalent radiative resistance with the mould $X_{\text{in}u}$ and $X_{\text{in}l}$ are the parameters to estimate. We know the electrical power time profiles consumed by the inductors $P_{up}(t)$ and $P_{low}(t)$, and the outside temperature $T_{\text{out}}$ (inputs of the detailed model), the temperature profiles in susceptors $T_u$ and $T_l$, and in the mould $T_{\text{mould}}^u$ and $T_{\text{mould}}^l$ (exits of the detailed model).

### 4.2. Identification of the 0D reduced model with the integrated numerical model and definition of a parameters list $\beta_{\text{exact}}$

In this part, we are going to estimate the parameters using least squares minimization through the Levenberg-Marquardt algorithm [7]. The aim is to define the values of the 8 parameters in the case.
where the temperatures are the output temperatures of the detailed model that have been averaged over parts of the whole solid domain (upper susceptor, lower susceptor, upper mould and lower mould). The electrical power sources correspond with the power profiles presented in Figure 2. The recalculated temperature profiles for the upper and lower susceptors are shown in Figure 7. The RMS (Root Mean Square) residual is equal to 3.6 K which is lower than the 5 K of measurement uncertainty of the sensors in the physical furnace. So, the reduced model is able of reproducing the output from the FlexPDE model with a good accuracy for the input used in this case. The estimated parameters are given in the Table 1. They are regarded as references from here on.

![Figure 7. Mean temperature profiles of upper and lower susceptors from FlexPDE model and 0D reduced model with the parameters estimated by Levenberg-Marquardt](image)

**Table 1.** Values of the 8 estimated parameters in the case of mean temperatures with no noise

| \( \eta_{hh} \) | \( \eta_{hl} \) | \( \eta_{li} \) | \( \eta_{lh} \) | \( R_{out}^h \) | \( R_{out}^l \) | \( X_{in}^h \) | \( X_{in}^l \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.168          | 0.539          | 0.671          | 0.215          | 0.168          | 1.022          | 1.389          | 5.643          |

4.3. **Parameters estimation for local noised temperatures**

In this case, in order to correspond to the thermocouples measurement outputs, we take the noised temperatures presented in Figure 6 as data for parameter estimation of the reduced model. The noise is a white Gaussian noise with a 5 K standard deviation.

The recalculated temperature profiles of susceptors reproduce the local temperature profiles from detailed model, Figure 8, with a good fit. The level of the RMS residual is 6.9 K which is close to the 5 K standard deviation of the added white noise. The values of the estimated parameters are given in the Table 2. The errors of the estimated parameters could not be justified by a bad condition number of the scaled sensitivity matrix of the system (at convergence) because in the two estimation cases (no noise or white noise) the condition number is lower than 35. The stochastic standard deviation of each of the estimated parameters (derived from their variance-covariance matrix, which can be calculated using the sensitivity matrix of the reduced model at convergence) is lower than 5 % in this configuration. These differences show that our 0D reduced model is not able of representing the furnace in its globality. However, a set of parameters can characterized a specific local or mean model reduction.

Let us note that, in the estimations presented in Tables 1 and 2, the sum of the estimated efficiencies (high and low) for each inductor, is higher than unity, which shows that the structure of the reduced is not intrinsic, even if its parameters can be estimated with a good fit for a given specific input pair \((P_{up}(t), P_{low}(t))\).
4.4. Test of the intrinsic values of the parameters estimated on a complex input and comparison of the profiles from FlexPDE and the 0D reduced model

We are now going to verify the set of parameters estimated previously. The aim is to get the same local temperature profiles with the 0D reduced model and with the detailed model for a complex input. We take identical power profiles for the upper and lower parts (see Figure 9). The temperature profiles obtained by the 0D reduced model (with the parameters given in Table 2) and the detailed model, for these power sources, are presented in Figure 10.

![Figure 9. Power source profiles for upper and lower inductors in order to verify the reduction model in the local case](image)

The difference between 0D and FlexPDE temperature profiles is quiet low for the upper part with a RMS residual equal to 9.4 K. On the contrary, the RMS level in the lower part is equal to 29 K. The error for the lower part is due to the fact that we have considered also the mould as a lumped body. In fact the lower part of the mould is significantly impacted by the cooling hearth, so there is a temperature gradient in the lower part of the mould.

5. Conclusion

Modelling of heat transfer in the furnace equipped with a mould, using a detailed electromagnetic/thermal model (solved in the FlexPDE environment) has allowed us to get reference data for the identification of a reduced model structure and for estimation of its parameters.
Figure 10. Local temperature profiles in upper and lower susceptors from FlexPDE and 0D reduced model parametrised by the Levenberg-Marquardt estimation in 4.3

This transient reduced model is based on 0D heat balances with a coupling by the sources (electrical power of the inductors). The results were quiet good, in terms of output fits, for the upper part of the module, but for the lower part, the 0D model is not sufficient to describe the physics of the furnace. This is due to the significant thermal gradient in the lower part of the mould.

Future works will concentrate on the construction of a 0.5D model. This model should take into account the gradient in the lower part of the mould (1D model) and keep the lumped body modelling for the other elements. The other strategies would be to use model identification with autoregressive models like ARX [8].

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