Dispersion theoretic calculation of the $H \rightarrow Z+\gamma$ amplitude

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work done together with
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talk based on [1] - PRD 97 (2018) no.7, 073008 [arXiv:1711.07298]
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Motivation

- $H \rightarrow \gamma \gamma$ decay rate, W-loop contribution has become a subject of a controversy.
- Is a loop induced process, total amplitude is finite.
- Individual amplitudes UV divergent, thus most authors use dimensional regularisation of the loop integrals (DimReg)
- Direct computation within the unitary gauge is also possible.
- DimReg and unitary gauge results differ in general!
- $H \rightarrow \gamma \gamma$ automatically included in $H \rightarrow Z \gamma$ calculation
- Working with dispersion integral – no regularisation necessary
- SM has a broken SU(2) symmetry – massive vector bosons have three polarisations
We consider:

- $H \rightarrow Z + \gamma$
- The $W$-loop contribution
- we use the dispersion method
- we compare to the commonly used DimReg
Our dispersion method in 2 steps:

1. We calculate $\text{Im}(\text{amplitude})$ – in the unitary gauge
2. We calculate the amplitude by using the dispersion integral
The Feynman diagrams
in the unitary gauge

\( M_1 \)

\( M_2 \)

\( M_3 \)

The inclined lines indicate the cuts.

\( M_4 \)

\( M_5 \)

These two selfenergy graphs do not contributed to the imaginary part of the amplitude.
The amplitude in the unitary gauge:

\[ M = \mathcal{M}_{\mu\nu}(k_1, k_2) \epsilon_1^\mu \epsilon_2^\nu \]

\[ M_{1\mu\nu} = \frac{-ie g^2 \cos \theta_W M}{(2\pi)^4} \int d^4k \frac{V_{\mu\rho\beta}(-k_1, -P_2, P_1) V_{\nu\gamma\sigma}(-k_2, -P_3, P_2)}{D_1 D_2 D_3} \times \left( g^\alpha - \frac{P_1^\alpha P_1^\beta}{M^2} \right) \left( g^\rho - \frac{P_2^\rho P_2^\sigma}{M^2} \right) \left( g^\gamma - \frac{P_3^\alpha P_3^\beta}{M^2} \right) \]

\[ M_{3\mu\nu}(k \to -k) = M_{1\mu\nu} \]

\[ M_{2\mu\nu} = \frac{ie g^2 \cos \theta_W M}{(2\pi)^4} \int d^4k \frac{V_{\gamma\beta\mu
u}}{D_1 D_3} \left( g^\alpha - \frac{P_1^\alpha P_1^\beta}{M^2} \right) \left( g^\rho - \frac{P_3^\rho P_3^\sigma}{M^2} \right) \]

\[ M_{\mu\nu} = 2M_{1\mu\nu} + M_{2\mu\nu} \]

\[ M \] is the W-mass

\[ P_1 = k + \frac{p}{2}, \quad P_2 = k - \frac{v}{2}, \quad P_3 = k - \frac{p}{2}, \]

\[ D_i = P_i^2 - M^2 + i\epsilon, \quad (i = 1, 2, 3), \]

\[ \tilde{P}_2 = k + \frac{v}{2}, \quad \tilde{D}_2 = \tilde{P}_2^2 - M^2 + i\epsilon \]

\[ p = k_1 + k_2, \quad v = k_1 - k_2. \]
Absorptive part of the amplitude

Using Cutkosky rules - sets the momenta of the W’s on-shell:

\[ \frac{1}{p^2 - M^2 + i\epsilon} \rightarrow (2\pi i) \theta(\pm p_0) \delta(p^2 - M^2). \]

The invariant absorptive part \( A \) of the amplitude is defined by

\[ \Im M_{\mu\nu} = \frac{eg^2 \cos \theta_W}{8\pi M} A(\tau) \mathcal{P}_{\mu\nu} \quad \text{with} \quad A(\tau) \mathcal{P}_{\mu\nu} = \frac{M^2}{\pi} \int d^4 k \mathcal{I}_{\mu\nu} \theta(P_{10}) \theta(-P_{30}) \delta(D_1) \delta(D_3), \]

The transverse factor is \( \mathcal{P}_{\mu\nu} = k_{2\mu} k_{1\nu} - (k_1 \cdot k_2) g_{\mu\nu} \)

and

\[ \mathcal{I}_{\mu\nu} = \frac{8M_Z^2}{M^4 D_2} k^2 \left( k_{\mu} k_{\nu} + \frac{k_{2\mu} k_{2\nu}}{2} - \frac{k_{\mu} k_{1\nu}}{2} - \frac{k_{2\mu} k_{1\nu}}{4} \right) + \frac{-2M_Z^2}{M^4} k^2 g_{\mu\nu} \]

\[ + \frac{8M_Z^2}{M^2 D_2} \left[ -k_{\mu} k_{\nu} - \frac{k_{2\mu} k_{2\nu}}{2} + \frac{k_{\mu} k_{1\nu}}{2} - \frac{k_{2\mu} k_{1\nu}}{8} + \frac{1}{4} g_{\mu\nu} k_1 \cdot k_2 - \frac{1}{8} g_{\mu\nu} k \cdot (k_1 - k_2) \right] + \frac{M^2}{M^2} g_{\mu\nu} \]

\[ + \frac{2}{M^2 D_2} \left[ 4k_1 \cdot k_2 k_{\mu} k_{\nu} + 2k^2 k_{2\mu} k_{1\nu} - 4k_1 k_{2\mu} k_{1\nu} - 4k_1 k_{2\mu} k_{1\nu} \right. \]

\[ \left. + g_{\mu\nu} \left( 4k_1 k_1 k \cdot k_2 - 2k^2 k_1 \cdot k_2 \right) \right] \]

\[ + \frac{2}{D_2} \left[ \left( -3k^2 + 3k \cdot k_1 - 3k \cdot k_2 - \frac{9}{2} k_1 \cdot k_2 + 3M^2 - \frac{3}{4} M_Z^2 \right) g_{\mu\nu} \right. \]

\[ \left. + 12k_{\mu} k_{\nu} + 3k_{1\nu} k_{2\mu} - 6k_{\mu} k_{1\nu} + 6k_{2\mu} k_{1\nu} \right]. \]

\[ \tau = \frac{p^2}{4M^2}, \ a = \frac{M_Z^2}{4M^2}, \ p - \text{momentum of Higgs boson}, \ M = M_W \]
Using the integrals given in Appendix B of [1] we get the non-zero result

\[ A(\tau) = \frac{a}{\tau - a} \left\{ \left[ 1 + \frac{1}{\tau - a} \left( \frac{3}{2} - 2a\tau \right) \right] \beta 
- \left[ 1 - \frac{1}{2(\tau - a)} \left( 2a - \frac{3}{2\tau} \right) - \frac{3}{2a} \left( 1 - \frac{1}{2\tau} \right) \right] \ln \left( \frac{1 + \beta}{1 - \beta} \right) \right\}, \quad \tau > 1. \]

\[ \beta = \sqrt{1 - \tau^{-1}} \]
Real (dispersive) part of the amplitude

We define the full invariant amplitude $F(\tau, a)$ by

$$\mathcal{M}_{\mu\nu} = -\frac{e g^2 \cos \theta_W}{8\pi M} F(\tau, a) \mathcal{P}_{\mu\nu}$$

The invariant \textit{unsubtracted} amplitude $F_{un}(\tau, a)$ is defined by the convergent dispersion integral

$$F_{un}(\tau, a) = \frac{1}{\pi} \int_1^\infty \frac{A(y)}{y-\tau} \, dy, \quad \tau < 1.$$

$F_{un}(\tau, a)$ defines the full amplitude $F(\tau, a)$ up to an additive constant $C(a)$:

$$2\pi F(\tau, a) = 2\pi F_{un}(\tau, a) + C(a)$$

We fix $C(a)$ through the Goldstone Boson equivalence theorem (GBET), which fixes the behaviour of the amplitude at $\tau \to \infty$.

Using the integrals given in Appendix C of [1] we get the result for $F_{un}(\tau, a)$:

$$2\pi F_{un}(\tau, a) = \frac{3 - 4a^2}{\tau - a} + \left(6 - 4a - \frac{3 - 4a^2}{\tau - a}\right) F(\tau, a) - 2a \left(2 + \frac{3 - 4a\tau}{\tau - a}\right) G(\tau, a),$$

$F$ and $G$ denote loop integrals and can be found in [1].

$F_{un}(\tau, a)$ has the properties:

* is finite at threshold $\tau = a$
* it vanishes for $\tau \to \infty$ with fixed $a$
* for $a \to 0$ we get the corresponding amplitude for $H \to \gamma\gamma$
C(a) from GBET

We determine the subtraction constant $C(a)$ through the charged ghost contribution adopting the Goldstone Boson Equivalence Theorem which implies that at $M_W \to 0$, i.e. at $\tau \to \infty$, the $SU(2) \times U(1)$ symmetry of the SM is restored and the longitudinal components of the physical $W^{\pm}$-bosons are replaced by the physical Goldstone bosons $\phi^{\pm}$. In the following $\mathcal{M}_{\mu\nu}^\phi$ denotes the amplitude of $H \to Z + \gamma$ in which the $W^{\pm}$ are replaced by their Goldstone bosons $\phi^{\pm}$. The GBET implies

$$\lim_{\tau \to \infty} \mathcal{M}_{\mu\nu}(\tau, a) = \lim_{\tau \to \infty} \mathcal{M}_{\mu\nu}^\phi(\tau, a).$$

We calculate the charged ghost contribution in two different ways: through direct calculations and via the dispersion integral. Both calculations lead to the same result.

Again, applying Cutkosky rules to the amplitude we get

$$\Im m \mathcal{M}_{\mu\nu}^\phi(\tau, a) = -\frac{eg^2 \cos \theta_W}{8\pi M} \frac{M_H^2}{4M^2} A^\phi(\tau, a) \mathcal{P}_{\mu\nu} \quad \text{with} \quad A^\phi(\tau, a) = (1 - 2a) \frac{2a\beta - \ln \frac{1+\beta}{1-\beta}}{2(\tau - a)^2}.$$

The dispersion integral is

$$\mathcal{F}^\phi(\tau, a) = \frac{1}{\pi} \int_{1}^{\infty} \frac{A^\phi(y, a)}{y - \tau} dy \quad \text{and we obtain} \quad \lim_{\tau \to \infty} \mathcal{F}^\phi(\tau, a) = \frac{2(1 - 2a)}{2\pi(\tau - a)}$$

$$\lim_{\tau \to \infty} 2\pi \mathcal{F}(\tau, a) = \lim_{\tau \to \infty} 2\pi [\tau \mathcal{F}^\phi(\tau, a)] = C(a). \quad \text{Thus we determine} \quad C(a) = 2(1 - 2a).$$

**Important note:**

$M_H^2$ in the coupling - large-$\tau$ behavior $\mathcal{F}^\phi(\tau, a) \sim \mathcal{O}(\tau^{-x})$ with $x \geq 1.$

$\to (1/\pi) \int_{ARC} dy \mathcal{F}(y, a)/(y - \tau)$ over the infinite arc in the complex $y$-plane, is zero.

$+\no\text{physics reason as GBET for }\mathcal{F}(\tau, a)$
The inclined lines indicate the cuts.

These two selfenergy graphs do not contributed to the imaginary part of the amplitude.
The Feynman diagrams, in $R_\xi$ gauge

24 genuine vertex graphs and 10 with $Z - \gamma$ selfenergy transitions in the $R_\xi$ gauge.
Calculation in $R_\xi$ gauge

$R_\xi$ gauge calculation done with the Mathematica Packages FeynArts and FormCalc. 
$\xi$ independence of total amplitude checked.

Here dimensional regularization (DimReg) is used.

The result $F_{\text{DimReg}}(\tau, a)$ coincides with the "classical" one, see e.g. [6] L. Bergström and G. Hulth (1985)

In the limit $a \to 0$ we get the result for $H \to \gamma\gamma$ [2] J. Ellis, M. K. Gaillard, D. V. Nanopoulos (1976)

We get the relation

$$2\pi F_{\text{DimReg}}(\tau, a) = 2\pi F_{\text{un}}(\tau, a) + 2(1 - 2a) = 2\pi F(\tau, a)$$

We see that both calculations agree, obeying the GBET.
The decay width

Approximating the total width by top and W-boson loop we get

\[ \Gamma(H \to Z + \gamma) = \frac{M_H^3}{32\pi} \left(1 - \frac{M_Z^2}{M_H^2}\right)^3 \left[\frac{e g^2}{(4\pi)^2 M}\right]^2 \cos \theta_W [2\pi F_W(\tau)] + \frac{2(3 - 8\sin^2 \theta_W)}{3 \cos \theta_W} [2\pi F_t(\tau)] \right]^2 \]

\( F_t(\tau) \) stands for the sum of the t-quark one-loop diagrams and \( F_W(\tau) \) stands for the sum of the W-boson one-loop diagrams.

**correct result**

\[ \Gamma(H \to Z + \gamma) = 8.1 \text{ KeV using } F_W(\tau) = F(\tau, a) = F_{\text{DimReg}}(\tau, a) \]

\[ \Gamma(H \to Z + \gamma) = 6.6 \text{ KeV using } F_W(\tau) = F_{un}(\tau, a) \]

Note, for \( \Gamma(H \to \gamma\gamma) \) even 52% reduction!

**wrong result, 20% smaller**

Working with regularisation:

\[ I_{\mu\nu} = \int \frac{d^n k}{(2\pi)^n} \frac{4k_\mu k_\nu - k^2 g_{\mu\nu}}{(k^2 - M_W^2)^3} \]

\[ I_{\mu\nu} = i\frac{\pi^2}{2} g_{\mu\nu} \text{ for } n = D \]

\[ I_{\mu\nu} = 0 \text{ for } n = 4 \]
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$H \to \gamma \gamma$

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$H \rightarrow Z \gamma$
Concluding remarks

- $W$-boson induced corrections to the decay $H \rightarrow Z + \gamma$ in the Standard Model calculated in the unitary gauge using the dispersion-relation approach.

- Decay $H \rightarrow \gamma\gamma$ automatically included.

  **Plus:** – Only finite quantities and thus does not involve any uncertainties related to regularization.
  – simpler, working in the unitary gauge effectively we deal with only 2 Feynman diagrams, while in the $R_\xi$-gauge one has 24 graphs.

  **Minus:** The dispersion method determines the amplitude merely up to an additive subtraction constant.

- Subtraction constant fixed by using the Goldstone Boson Equivalence Theorem.

- As a cross-check we also calculated the amplitude in the commonly used $R_\xi$-gauge class with dimensional regularization as regularization scheme. **Same result** as in the dispersion method.
Thank you!