

T-Duality For String in Hořava-Lifshitz Gravity

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ABSTRACT: We continue our study of the Lorentz breaking string theories. These theories are defined as string theory with modified Hamiltonian constraint which breaks the Lorentz symmetry of target space-time. We analyze properties of this theory in the target space-time that possesses isometry along one direction. We also derive the T-duality rules for Lorentz breaking string theories and show that they are the same as that of Buscher’s T-duality for the relativistic strings.

KEYWORDS: Bosonic String, Hořava-Lifshitz Background.
1. Introduction and Summary

The recent proposal of Hořava [1, 2, 3] for a candidate theory of gravity that is symmetric under the Lifshitz type of anisotropy scaling of space-time coordinates \( t \rightarrow l^z t, \ x^i \rightarrow l x^i, \) where \( z \) being the scaling exponent, has been a very interesting area of research since the last year \(^1\). This theory is constructed as a UV completion of Hilbert-Einstein gravity so that it is perturbatively renormalizable. This modification is possible only when we sacrifice Lorentz symmetry at high energy. However it is again observable at low energy. Among various versions of Hořava-Lifshitz theories, only the class of projectable theories where the so called lapse function depends only on time, is the consistent choice. It is often, as in the present case, interesting to study theories with broken general covariance. The so called Lorentz symmetry breaking Hamiltonian formalism has been used to study the point particles and strings in Hořava gravity. The basic idea of this Lorentz breaking Hamiltonian formalism is that time and spatial components of momenta are treated differently. Indeed in [51] the construction of a new string theory, called Lorentz-breaking string theory (LBS) has been studied extensively by generalizing the point particle dynamics [52, 56, 57, 58, 59, 60] in Hořava-Lifshitz gravity. The basic idea of the construction of the LBS theory is following. We start with the Hamiltonian formulation of two dimensional theory when the Hamiltonian is linear combination of two constraints: the spatial diffeomorphism constraint and Hamiltonian constraint. As opposite to the Hamiltonian formulation of Polyakov string we consider the Hamiltonian constraint that breaks the Lorentz invariance of the target space-time in the similar way as in the point particle case [52, 56, 57, 58, 59, 60]. However as opposite to the point particle case now the world-sheet modes depend on spatial coordinate of world-sheet theory so that it is possible to define many LBS theories that reduce to the point particle Hamiltonian constraint [52, 56, 57, 58, 59, 60] in case of the world-sheet dimensional reduction. In doing so, the consistency of the LBS theory demands that the spatial component of the world-sheet metric has to be dynamical. As a by product, the

\(^1\)Some aspects of Hořava-Lifshitz theory has been discussed in for example [4]-[50]
world-sheet theory is no more invariant under full two dimensional diffeomorphism but only under the world-sheet foliation preserving transformation. Furthermore, the consistency of the Hamiltonian dynamics of LBS theory implies that the world-sheet lapse has to obey the projectability condition consistent with the of Hořava-Lifshitz gravity. On the other hand it is natural to demand that the Hamiltonian constraint of LBS theory reduces to the Hamiltonian constraint of the relativistic string in the case when the target Hořava-Lifshitz gravity reduces to General Relativity in low energy regime. This requirement now implies that we should consider LBS theory where the world-sheet mode $x^0$ depends on the world-sheet spatial coordinate $\sigma$ as well which is more general situation than was consider in the paper [51]. Then we will be able to show that the Hamiltonian constraint reduces to the Hamiltonian constraint for the relativistic string in the limit which has been used to recover General Theory of Relativity from Hořava gravity. However at this place we should stress one important point that makes the construction of LBS theory as intricate as the construction of Hořava-Lifshitz gravity. Explicitly, we argue in [51] that the consistency of the Hamiltonian formulation of LBS theory forces us to consider the world-sheet lapse function that depends on the world-sheet time coordinate $\tau$ only. As a result the LBS theory reduces to the Polyakov action when however the lapse function does not depend on the world-sheet spatial coordinate. In other words LBS theory does not reduce to the Polyakov string in the IR limit of target Hořava-Lifshitz gravity.

Despite of this fact, we feel that it is interesting to study LBS theory further as a toy model of the theory with broken Lorentz invariance that is more general then the corresponding point particle action. We discussed the symmetries of the action and have shown that the action is invariant under the target space foliation preserving diffeomorphism and under world-sheet foliation preserving diffeomorphism [51]. We further derive the T-duality rules for LBS string and show that they are same as that of Buscher’s T-duality for the relativistic strings [53, 54, 55]. It would be interesting to study other extended objects like D-branes in LBS theory. In particular it will be desired to look for D-brane action in Hořava-Lifshitz background and examine their fate by using the T-duality transformation.

The rest of the paper is organized as follows. In section-2, we generalize the construction of LBS theory of [51] to include other auxiliary fields and more general world sheet modes. In section-3, we present the symmetries of the LBS theory action and show they are invariant under the target space foliation preserving diffeomorphism transformation. Finally, section-4 is devoted to the study of T-duality transformation of the LBS theory.

2. Review of LBS Theory

In this section we review and slightly generalize the construction of LBS theory given in [51] where more details of motivation for this construction can be found. As in [51] we begin with the following Hamiltonian formulation of LBS

$$H = \int_\Sigma d\sigma \mathcal{H}(\sigma) , \quad \mathcal{H}(\sigma) = n_\tau(\tau)\mathcal{H}_\tau(\sigma) + n^\sigma(\sigma)\mathcal{H}_\sigma(\sigma) +$$

$$+ \lambda_\tau(\tau)\pi^\tau(\tau) + \lambda_\sigma(\sigma)\pi^\sigma(\sigma) + v_A(\sigma)P_A(\sigma) + v_B(\sigma)P_B(\sigma) ,$$

(2.1)
where

\[
\mathcal{H}_\tau = -\pi\alpha' \frac{1}{\sqrt{\omega N^2}} (p_0 - N^i p_i) + \sqrt{\omega} G \left( -\frac{1}{4\pi\omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \\
+ B \left( \frac{\pi\alpha'}{\omega} p_0 p_j + \frac{(z-1)}{2\sqrt{\omega}} \pi^2 \omega^2 \pi^\omega + \\
+ \frac{1}{4\pi\omega} \left( \partial_\sigma x^i + N^i \partial_\sigma x^0 \right) h_{ij} \left( \partial_\sigma x^j + N^j \partial_\sigma x^0 \right) - A \right) + \sqrt{\omega} F(A) ,
\]

\[
\mathcal{H}_\sigma = p_M \partial_\sigma x^M - 2\omega \nabla_\sigma \pi^\sigma ,
\] (2.2)

where \(x^M, M, N = 0, \ldots, D\) are world-sheet modes that parameterize the embedding of the string into target space-time \(M\) with general metric \(g_{MN}\) and where \(p_M\) are conjugate momenta with following non-zero Poisson brackets

\[
\{x^M(\sigma), p_N(\sigma')\} = \delta_M^N \delta(\sigma - \sigma') .
\] (2.3)

Further, we introduced two dimensional metric \(\gamma_{\mu\nu}\) in \(1+1\) formalism

\[
\gamma_{\alpha\beta} = \begin{pmatrix} -n_\tau^2 + \frac{1}{\omega} n_\sigma^2 & n_\sigma \\ n_\sigma & \omega \end{pmatrix} ,
\] (2.4)

where \(n_\tau\) is a world-sheet lapse, \(n_\sigma\) is a world-sheet shift, \(n^\sigma \equiv \frac{n_\sigma}{n_\tau}\) and \(\omega\) is a spatial part of world-sheet metric and where \(\pi^\tau, \pi^\sigma\) and \(\pi^\omega\) are corresponding conjugate momenta with following non-zero Poisson brackets

\[
\{n_\tau, \pi^\tau\} = 1 , \quad \{n_\sigma(\sigma), \pi^\sigma(\sigma')\} = \delta(\sigma - \sigma') , \quad \{\omega(\sigma), \pi^\omega(\sigma')\} = \delta(\sigma - \sigma') .
\] (2.5)

We further defined world-sheet covariant derivative

\[
\nabla_\sigma n_\sigma = \partial_\sigma n_\sigma - \Gamma n_\sigma , \quad \nabla_\sigma \pi^\omega = \partial_\sigma \pi^\omega + \Gamma \pi^\omega , \quad \Gamma = \frac{1}{2\omega} \partial_\sigma \omega .
\] (2.6)

We consider target space-time \(M\) labeled with coordinates \(t = x^0, x = (x^1, \ldots, x^D)\) with the metric in \(1+1\) form

\[
g_{00} = -N^2 + N_i h^{ij} N_j , \quad g_{0i} = N_i , \quad g_{ij} = h_{ij} , \quad \text{det } g = -N^2 \text{det } h
\] (2.7)

with inverse

\[
g^{00} = -\frac{1}{N^2} , \quad g^{0i} = \frac{N^i}{N} , \quad g^{ij} = h^{ij} - \frac{N^i N^j}{N^2} .
\] (2.8)

Note that the dynamics of target space metric \(g_{MN}\) is governed by Hořava-Lifshitz gravity action. Further, \(A, B\) and corresponding conjugate momenta \(P_A, P_B\) are auxiliary modes with Poisson brackets

\[
\{A(\sigma), P_A(\sigma')\} = \delta(\sigma - \sigma') , \quad \{B(\sigma), P_B(\sigma')\} = \delta(\sigma - \sigma') .
\] (2.9)

Finally \(\lambda_\tau, \lambda_\sigma, v_A, v_B\) are Lagrange multipliers that ensure that \(\pi^\tau \approx 0, \pi^\sigma(\sigma) \approx 0, P_A(\sigma) \approx 0, P_B(\sigma) \approx 0\) are primary constraints of the theory. Note that following arguments given
in [51] we presume that \( n_\tau \) depends on \( \tau \) only. Then the requirement of the preservation of the primary constraints implies following secondary ones

\[
\partial_\tau \pi_\tau = \{ \pi_\tau, H \} = - \int d\sigma \mathcal{H}_\tau \approx 0 , \\
\partial_\tau \pi^\sigma = \{ \pi^\sigma, H \} = - \frac{1}{\omega} \mathcal{H}_\sigma \approx 0 , \\
\partial_\tau P_A = \{ P_A, H \} = B - \sqrt{\omega} F'(A) \equiv G_A \approx 0 , \\
\partial_\tau P_B = \{ P_B, H \} = - \left( \pi_\alpha' \frac{1}{\omega} p_i h_{ij} p_j + \frac{(z-1)}{2\sqrt{\omega}} \pi^\omega \omega^2 \pi^\omega + \right. \\
\left. + \frac{1}{4\pi_\alpha' \omega} (\partial_\sigma x^i + N^i \partial_\sigma x^0)h_{ij}(\partial_\sigma x^j + N^j \partial_\sigma x^0) - A \right) \equiv G_B \approx 0 .
\]

(2.10)

It is easy to show that the secondary constraints \( G_A \approx 0, G_B \approx 0 \) together with the primary ones \( P_A \approx 0, P_B \approx 0 \) form the collection of the second class constraints. Solving the system of the second class constraints \( (P_A, P_B, G_A, G_B) \) we express \( A, B \) as functions of the canonical variables \( x^M, p_M \) and we find non-linear form of the Hamiltonian constraint. This procedure was extensively studied in [51] so that we skip all details and recommend an interesting reader to look at this reference.

As opposite to the case of the Hamiltonian formulation of the relativistic string we see that the Hamiltonian constraint (2.2) contains kinetic term for \( \pi^\omega \). Arguments why there is non-trivial dynamics the spatial part of the metric \( \omega \) were given in [51] and we briefly recapitulate here. Let us imagine for the time being that \( \pi^\omega \) is a primary constraint of the theory. However we see from (2.1) that the Hamiltonian constraint depends on \( \omega \) in non-trivial way. Then if \( \pi^\omega \approx 0 \) were be the primary constraint of the theory we would find overconstrained theory due to the requirement of the consistency of this constraint with the time evolution of the system. Then in order to avoid imposing additional constraint on the system we demand that \( \omega \) is a dynamical mode with kinetic term in the action.

We see that the Hamiltonian constraint (2.2) is characterized by the presence of two functions \( F \) and \( G \). As in [52, 56, 57, 58, 59, 60] we presume that \( F(A) \) has the form \( F(A) = A + \sum_{n=2}^z \lambda_n A^n \) with \( z \) being the critical exponent of the Hořava-Lifshitz gravity. It is believed that in the IR limit the Hořava-Lifshitz gravity reduces to the ordinary General Relativity when \( z = 1 \). Let us now study properties of the Hamiltonian constraint (2.2) in this limit. Firstly we see that the kinetic term for the spatial part of the metric vanishes for \( z \rightarrow 1 \). Since we consider more general case than in [51] when \( x^0 \) depends on \( \sigma \) we mean that it is natural to add into (2.2) the term \( G(- \frac{1}{4\pi_\alpha' \omega^2} N^2 \partial_\sigma x^0 \partial_\sigma x^0) \). We assume that \( G(A) = A + \sum_{n=2}^z \omega_n A^n \) where \( \omega_n \) are constants. Then it is easy to see that in the limit \( z \rightarrow 1 \) we have \( F(A) \rightarrow 1 \), \( G(A) \rightarrow 1 \) and hence we find that for \( z \rightarrow 1 \) the Hamiltonian constraint given in (2.2) takes the following form

\[
\mathcal{H}_\tau = -\pi_\alpha' \frac{1}{\sqrt{\omega N^2}} (p_0 - N^i p_i)^2 - \frac{1}{4\pi_\alpha' \sqrt{\omega}} N^2 \partial_\sigma x^0 \partial_\sigma x^0 + \\
+ B \left( \pi_\alpha' \frac{1}{\omega} p_i h_{ij} p_j + \frac{1}{4\pi_\alpha' \omega} (\partial_\sigma x^i + N^i \partial_\sigma x^0)h_{ij}(\partial_\sigma x^j + N^j \partial_\sigma x^0) - A \right) + \sqrt{\omega} A .
\]
Solving now the second class constraints $P_A, P_B, G_A, G_B$ is equivalent to the integration out $A$ and $B$ from (2.11) and we find that the Hamiltonian constraint (2.11) reduces to
\[
\mathcal{H}_\tau = -\pi' \frac{1}{\sqrt{\omega}}(p_0 - N^i p_i)^2 + \frac{1}{\sqrt{\omega}} p_i h^{ij} p_j - \frac{1}{4\pi\alpha'} \sqrt{\omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 + \frac{1}{4\pi\alpha'} \sqrt{\omega} (\partial_\sigma x^i + N^i \partial_\sigma x^0) h_{ij} (\partial_\sigma x^j + N^j \partial_\sigma x^0) = -\pi' \frac{1}{\sqrt{\omega}} p_N g^{MN} p_N - \frac{1}{4\pi\alpha' \sqrt{\omega}} \partial_\tau x^M g_{MN} \partial_\sigma x^N .
\]

This is clearly the Hamiltonian constraint of the Polyakov action. We see that the LBS action reduces to the Polyakov in the IR limit of target space-time. However we should stress the crucial point in the formulation of LBS theory that is the same way resembles the same problem as in the formulation of the Ho\v{r}ava-Lifshitz gravity. Explicitly, we argued in [51] that LBS theory is well defined only when the lapse function $n$ depends on $\tau$ only i.e. $n = n(\tau)$. However then we see that in the IR limit of the Ho\v{r}ava-Lifshitz gravity the LBS theory reduces to the Polyakov action where $\gamma_{00} = -n_\tau$ depends on $\tau$ only and hence the full diffeomorphism invariance is not restored.

The next step is to find Lagrangian corresponding to Hamiltonian (2.1). To do this we determine the time derivatives of $x^M, A, B, \omega$
\[
\partial_\tau x^0 = \{x^0, H\} = -\frac{2\pi\alpha'}{N^2 \sqrt{\omega}} (p_0 - N^i p_i) n_\tau + n^\sigma \partial_\sigma x^0 , \\
\partial_\tau x^i = \frac{2\pi\alpha'}{N^2 \sqrt{\omega}} N^i (p_0 - N^i p_i) n_\tau + 2\pi\alpha' B \frac{1}{\omega} h^{ij} p_j n_\tau + n^\sigma \partial_\sigma x^i , \\
\partial_\tau A = \{A, H\} = v_A , \quad \partial_\tau B = \{B, H\} = v_B , \\
\partial_\tau \omega = \{\omega, H\} = n_\tau B (z - 1) \frac{\omega^2}{\sqrt{\omega}} \pi^\omega + 2\nabla_\sigma n_\sigma .
\]

It is convenient to introduce following object
\[
K_\sigma = \frac{1}{n_\tau} (\partial_\tau \omega - 2\nabla_\sigma n_\sigma) .
\]

Then it is simple task to find corresponding Lagrangian
\[
\mathcal{L} = p_M \partial_\tau x^M + \partial_\tau A p_A + \partial_\tau B p_B + \partial_\tau \omega p^\omega - \mathcal{H} = \\
= -\frac{\sqrt{\omega}}{4\pi\alpha'} \frac{1}{n_\tau} (\partial_\tau x^0 - n^\sigma \partial_\sigma x^0)^2 - n_\tau \sqrt{\omega} G \left( -\frac{1}{2\pi\alpha' \omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \\
+ \omega n_\tau \frac{1}{B} \left( \frac{1}{4\pi\alpha' n^2} (V_i^j - n^\sigma V_i^j) h_{ij} (V_j^i - n^\sigma V_j^i) + \frac{1}{2(z - 1)} K_\sigma^2 \frac{1}{\omega^2} K_\sigma \right) - \\
- B n_\tau \left( \frac{1}{2\pi\alpha' \omega} V_i^i h_{ij} V_j^i - A \right) - \sqrt{\omega} n_\tau F(A) ,
\]

(2.15)
\[ V_i^\tau = \partial_\tau x^i + N^i \partial_\tau x^0, \quad V_\sigma^i = \partial_\sigma x^i + N^i \partial_\sigma x^0. \]  

(2.16)

Observe that this theory is well defined in case when \( z \to 1 \) on condition that \( K_\sigma = 0 \) which is in agreement with our requirement that this theory reduces to Polyakov action in this limit with exception that \( \gamma_{00} \) depends on \( \tau \) only.

Finally we integrate out \( A \) and \( B \) from (2.15). The equation of motion for \( A \) implies

\[ B - \sqrt{\omega} F'(A) = 0 \]  

(2.17)

while the equation of motion for \( B \) implies

\[
- \frac{\omega}{B^2} \left( \frac{1}{4\pi\alpha'n_\tau^2} (V_i^\tau - n^\sigma V_\sigma^i) h_{ij} (V_j^\tau - n^\sigma V_\sigma^j) + \frac{1}{2(z-1)} K_\sigma \frac{1}{\omega^2} K_\sigma \right) - \\
- \left( \frac{1}{2\pi\alpha'\omega} V_\sigma^i h_{ij} V_\sigma^j - A \right) = 0 .
\]  

(2.18)

Inserting (2.17) into (2.18) we find the equation for \( A \) in the form

\[
\frac{1}{F'^2(A)} \left( \frac{1}{4\pi\alpha'n_\tau^2} (V_i^\tau - n^\sigma V_\sigma^i) h_{ij} (V_j^\tau - n^\sigma V_\sigma^j) + \frac{1}{2(z-1)} K_\sigma \frac{1}{\omega^2} K_\sigma \right) - \\
- \left( \frac{1}{2\pi\alpha'\omega} V_\sigma^i h_{ij} V_\sigma^j - A \right) = 0
\]  

(2.19)

that in principle allows as to find \( A \) as

\[
A = \Psi \left( \frac{1}{4\pi\alpha'n_\tau^2} (V_i^\tau - n^\sigma V_\sigma^i) h_{ij} (V_j^\tau - n^\sigma V_\sigma^j) + \frac{1}{2(z-1)} K_\sigma \frac{1}{\omega^2} K_\sigma, \frac{1}{2\pi\alpha'\omega} V_\sigma^i h_{ij} V_\sigma^j \right) .
\]  

(2.20)

Collecting all these results together we find the Lagrangian density in the form

\[
\mathcal{L} = \sqrt{\omega n_\tau} \left[ - \frac{1}{4\pi\alpha'} \frac{1}{n_\tau^2} (\partial_\tau x^0 - n^\sigma \partial_\sigma x^0)^2 - G \left( \frac{1}{2\pi\alpha'\omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) - \\
- F'(\Psi) \left( \frac{1}{2\pi\alpha'\omega} V_\sigma^i h_{ij} V_\sigma^j - A \right) - 2F(\Psi) \right] .
\]  

(2.21)

This it the final form of the Lorentz breaking string theory Lagrangian. In the next section we study invariance of the action \( S = \int d\tau d\sigma \mathcal{L} \) under local and global world-sheet symmetries.

3. Symmetries of the LBS Action

We start with the global transformations from the point of view of the string world-sheet theory. These transformations correspond to the foliation preserving diffeomorphism of the
target space-time

\[ x^0(τ, σ) = x^0(τ, σ) + f(x^0(τ, σ)) , \]
\[ x^i(τ, σ) = x^i(τ, σ) + ζ^i(τ, σ) , \]

(3.1)

where \( f(x^0), ζ^i(x^0, x) \) are infinitesimal parameters. Note that under these transformations the metric component transform as

\[
N'_i(x^0, x') = N_i(x^0, x) - N_i(x^0, x) ̇f(x^0) - N_j(x^0, x) \partial_i \xi^j(x^0, x) - g_{ij}(x^0, x) ̇ζ^j(t, x) ,
\]
\[
N'^i(x^0, x') = N'_i(x^0, x) + N^j(x^0, x) \partial_j \xi^i(x^0, x) - N^i(x^0, x) ̇f - ̇ζ^i(x^0, x) ,
\]
\[
N'(x^0) = N(x^0) - N(x^0) ̇f(x^0)
\]

(3.2)

and

\[
g'_{ij}(x^0, x') = g_{ij}(x^0, x) - g_{ij}(x^0, x) \partial_j \xi^i(x^0, x) - \partial_i \xi^k(x^0, x)g_{kj}(x^0, x) ,
\]
\[
g'^{ij}(x^0, x') = g^{ij}(x^0, x) + \partial_i \xi^j(x^0, x)g^{nj}(x^0, x) + g^{ij}(x^0, x) \partial_n \xi^j(x^0, x) .
\]

(3.3)

Then it is easy to see that \( V^i_α \) transform under (3.1) as

\[
V^i_α(τ, σ) = V^i_α(τ, σ) + \partial_j \xi^i(τ, σ) V^j_α(τ, σ) .
\]
\[
V'^i_α(τ, σ) = V^i_α(τ, σ) + \partial_j \xi^i(τ, σ) V'^j_α(τ, σ) .
\]

(3.4)

Performing the same analysis as in [5] we find that the Lagrangian density (2.21) is invariant under target-space foliation preserving diffeomorphism (3.1).

As the next step we check the invariance of the action under world-sheet foliation preserving diffeomorphism that we define as the world-sheet transformation

\[
τ' = τ + f(τ) , \quad σ' = σ + ε(τ, σ) .
\]

(3.5)

where \( f, ε \) are infinitesimal parameters. In the same way as in [2] we find that the world-sheet metric components transform under (3.3) as

\[
n'_σ(τ', σ') = n_σ(τ, σ) - n_σ(τ, σ) \partial_σ ε(τ, σ) - \partial_σ f(τ)n_σ(τ, σ) - \partial_σ ε(τ, σ)ω(τ, σ) ,
\]
\[
n'_τ(τ', σ') = n_τ(τ, σ) - n_τ(τ, σ) \partial_τ f(τ) ,
\]
\[
ω'(τ', σ') = ω(τ, σ) - 2\partial_σ ε(τ, σ)ω(τ, σ) ,
\]
\[
n'^{σ}(τ, σ') = n^{σ}(τ, σ) + n^{σ}(τ, σ) \partial_σ ε(τ, σ) - n^{σ}(τ, σ) \partial_σ f(τ) - \partial_σ ε(τ, σ) .
\]

(3.6)

Then it is easy to see that

\[
dτ' dσ' n'_τ \sqrt{ω'} = dτ dσ n_τ \sqrt{ω} .
\]

(3.7)
Note that $\Gamma$ transforms under the world-sheet foliation preserving diffeomorphism (3.5) as
\[ \Gamma'(\tau', \sigma') = \Gamma(\tau, \sigma) - \Gamma(\tau, \sigma)\partial_\sigma \epsilon(\tau, \sigma) - \partial_\sigma^2 \epsilon(\tau, \sigma). \] (3.8)

Then after some algebra we find that $K_\sigma$ transforms as
\[ K_\sigma'(\tau', \sigma') = K_\sigma(\tau, \sigma) - 2K_\sigma(\tau, \sigma)\partial_\sigma \epsilon(\tau, \sigma). \] (3.9)

Clearly, the world-sheet modes $x^M$ are scalars under (3.5)
\[ x^M(\tau', \sigma') = x^M(\tau, \sigma). \] (3.10)

Collecting all these results together and performing the same analysis as in [51] we can show that the Lagrangian density (2.21) is invariant under world-sheet foliation preserving diffeomorphism (3.5).

4. T-duality for LBS theory

In this section we analyze properties of LBS theory under T-duality transformations. In other words we would like to see whether this theory shares the same properties as ordinary string theory action. For further purposes we again write the Lagrangian density for LBS theory
\[ L = -\frac{\sqrt{\omega}}{4\pi \alpha'} \frac{1}{n_\tau}(\partial_\tau x^0 - n^\tau \partial_\sigma x^0)^2 - n_\tau \sqrt{\omega} G \left(-\frac{1}{2\pi \alpha'} n_\tau^2 N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \right. \]
\[ + \omega n_\tau \frac{1}{B} \left( \frac{1}{4\pi \alpha'} \frac{1}{n_\tau^2} (V^i_\tau - n^\tau V^i_\sigma) h_{ij}(V^j_\tau - n^\tau V^j_\sigma) + \frac{1}{2(z-1)} K_\sigma \frac{1}{\omega^2} K_\sigma \right) - \]
\[ - B n_\tau \left( \frac{1}{2\pi \alpha' \omega} V^i_\sigma h_{ij} V^j_\sigma - A \right) - \sqrt{\omega} n_\tau F(A), \] (4.1)

where
\[ V^i_\tau = \partial_\tau x^i + N^i \partial_\tau x^0, \quad V^i_\sigma = \partial_\sigma x^i + N^i \partial_\sigma x^0. \] (4.2)

Let us now presume that that the background possesses isometry along $\phi$ direction where we performed the splitting of the target space coordinates $x^i = (x^\alpha, \phi), \alpha, \beta = 1, \ldots, D - 1$. The fact that there is an isometry of the background along $\phi$ direction implies that the action is invariant under the shift
\[ \phi'(\tau, \sigma) = \phi(\tau, \sigma) + \epsilon, \] (4.3)

where $\epsilon = \text{const}$. The invariance of the action implies an existence of the conserved current
\[ J^\alpha = \frac{\delta S}{\delta \phi^\alpha}, \quad \partial_\alpha J^\alpha = 0. \] (4.4)
explicitly, we find
\[ J_\tau = \frac{\omega}{2\pi \alpha' n_\tau B} h_{\phi i} (V^i_\tau - n^\sigma V^i_\sigma) , \]
\[ J_\sigma = -\frac{n_\sigma}{2\pi \alpha' n_\tau B} h_{\phi i} (V^i_\tau - n^\sigma V^i_\sigma) - B \frac{n_\tau}{\pi \alpha' \omega} h_{\phi i} V^i_\sigma . \]
(4.5)

Let us now try to implement the T-duality rules as in standard bosonic theory. We gauge the shift symmetry so that \( \epsilon \rightarrow \epsilon(\tau, \sigma) \). Then in order to ensure the invariance of the Lagrangian [4.1] we have to introduce the gauge field \( a_\alpha \) and replace
\[ \partial_\alpha \phi \rightarrow D_\alpha \phi = \partial_\alpha \phi + a_\alpha . \]
(4.6)

Note that under \( a_\alpha \) transforms for non-constant \( \epsilon \) as
\[ a'_\alpha(\tau, \sigma) = a_\alpha(\tau, \sigma) - \partial_\alpha \epsilon(\tau, \sigma) . \]
(4.7)

Then it is easy to see that
\[ (D_\alpha \phi)' = D_\alpha \phi . \]
(4.8)

In the same way we perform the replacement
\[ V^\phi_\tau = \partial_\tau \phi + N^\phi \partial_\tau x^0 \rightarrow D_\tau \phi + N^\phi \partial_\tau x^0 \equiv \tilde{V}^\phi_\tau , \]
\[ V^\phi_\sigma = \partial_\sigma \phi + N^\phi \partial_\sigma x^0 \rightarrow D_\sigma \phi + N^\phi \partial_\sigma x^0 \equiv \tilde{V}^\phi_\sigma . \]
(4.9)

However we have to also check that terms containing \( a_\alpha \) are invariant under world-sheet foliation preserving diffeomorphism [3.3]. To do this we presume that \( a_\alpha \) transform under world-sheet foliation preserving diffeomorphism [3.3] as
\[ a'_\tau(\tau', \sigma') = a_\tau(\tau, \sigma) - a_\tau(\tau, \sigma) f(\tau) - a_\sigma(\tau, \sigma) \partial_\tau \xi(\tau, \sigma) , \]
\[ a'_\sigma(\tau', \sigma') = a_\sigma(\tau, \sigma) - a_\sigma(\tau, \sigma) \partial_\sigma \xi(\tau, \sigma) . \]
(4.10)

Then it is easy to see that the covariant derivatives transform as
\[ D'_\tau \phi(\tau', \sigma') = D_\tau \phi(\tau, \sigma) - D_\tau \phi(\tau, \sigma) \hat{f}(\tau) - D_\sigma \phi(\tau, \sigma) \partial_\tau \xi(\tau, \sigma) , \]
\[ D'_\sigma \phi(\tau', \sigma') = D_\sigma \phi(\tau, \sigma) - D_\sigma \phi(\tau, \sigma) \partial_\sigma \xi(\tau, \sigma) . \]
(4.11)

As the next step we introduce \( f_{\alpha\beta} \) defined as
\[ f_{\tau\sigma} = \partial_\tau a_\sigma - \partial_\sigma a_\tau \]
(4.12)

that transform under foliation preserving diffeomorphism as
\[ f'_{\tau\sigma}(\tau', \sigma') = f_{\tau\sigma}(\tau, \sigma) - f_{\tau\sigma}(\tau, \sigma) \hat{f}(\tau) - f_{\tau\sigma}(\tau, \sigma) \partial_\tau \xi(\tau, \sigma) . \]
(4.13)
Then it is easy to see that $dr d\sigma f_{\alpha\beta}$ is invariant under (3.5). Collecting all these terms together we find following Lagrangian density invariant under the foliation preserving diffeomorphism (3.3)

\[
\mathcal{L} = -\frac{\sqrt{\omega}}{4\pi \alpha' n^2_r} (\partial_\tau x^0 - n^\sigma \partial_\sigma x^0)^2 - n_\tau \sqrt{\omega} G \left( -\frac{1}{2\pi \alpha' \omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \\
+ \omega n^\tau \frac{1}{B} \left[ \frac{1}{4\pi \alpha' n^2_r} (\hat{\nabla}_\tau^\phi - n^\sigma \hat{\nabla}_\sigma^\phi) h_{\phi\phi} (\hat{V}_\tau^\phi - n^\sigma \hat{V}_\sigma^\phi) + 2 (\hat{V}_\tau^\phi - n^\sigma \hat{V}_\sigma^\phi) h_{\phi\alpha} (V_\tau^\alpha - n^\sigma V_\sigma^\alpha) + \\
+ \frac{1}{4\pi \alpha' n^2_r} (V_\tau^\alpha - n^\sigma V_\sigma^\alpha) h_{\alpha\beta} (V_\tau^\beta - n^\sigma V_\sigma^\beta) + \frac{1}{2(\alpha - 1)} K_\sigma \frac{1}{2} K_\sigma \right] - \\
- Bn_\tau \left[ \frac{1}{2\pi \alpha' \omega} [\hat{V}_\sigma^\phi h_{\phi\phi} \hat{V}_\sigma^\phi + 2 \hat{V}_\sigma^\phi h_{\phi\beta} V_\sigma^\beta + V_\sigma^\alpha h_{\alpha\beta} V_\sigma^\beta] - A \right] - \sqrt{\omega} n_\tau F(A) + \hat{\phi} f_{\tau\sigma} ,
\]

(4.14)

where $\hat{\phi}$ is the Lagrange multiplier that ensures that $a_\alpha$ is non-dynamical field. Note that $\hat{\phi}$ transforms as scalar under (3.5). The gauge invariance of the Lagrangian density (4.14) can be fixed by imposing the condition $\hat{\phi} = 0$ that implies

\[
D_\tau \phi = a_\tau , \quad D_\sigma \phi = a_\sigma .
\]

(4.15)

Inserting (4.15) into (4.14) we obtain

\[
\mathcal{L} = -\frac{\sqrt{\omega}}{4\pi \alpha' n^2_r} (\partial_\tau x^0 - n^\sigma \partial_\sigma x^0)^2 - n_\tau \sqrt{\omega} G \left( -\frac{1}{2\pi \alpha' \omega} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \\
+ \omega n^\tau \frac{1}{B} \left[ \frac{1}{4\pi \alpha' n^2_r} (\hat{\nabla}_\tau^\phi - n^\sigma \hat{\nabla}_\sigma^\phi) h_{\phi\phi} (\hat{V}_\tau^\phi - n^\sigma \hat{V}_\sigma^\phi) + 2 (\hat{V}_\tau^\phi - n^\sigma \hat{V}_\sigma^\phi) h_{\phi\alpha} (V_\tau^\alpha - n^\sigma V_\sigma^\alpha) + \\
+ \frac{1}{4\pi \alpha' n^2_r} (V_\tau^\alpha - n^\sigma V_\sigma^\alpha) h_{\alpha\beta} (V_\tau^\beta - n^\sigma V_\sigma^\beta) + \frac{1}{2(\alpha - 1)} K_\sigma \frac{1}{2} K_\sigma \right] - \\
- Bn_\tau \left[ \frac{1}{2\pi \alpha' \omega} [\hat{V}_\sigma^\phi h_{\phi\phi} \hat{V}_\sigma^\phi + 2 \hat{V}_\sigma^\phi h_{\phi\beta} V_\sigma^\beta + V_\sigma^\alpha h_{\alpha\beta} V_\sigma^\beta] - A \right] - \sqrt{\omega} n_\tau F(A) + \hat{\phi} f_{\tau\sigma} ,
\]

(4.16)

where

\[
\hat{V}_\tau^\phi = a_\tau + N^\phi \partial_\tau x^0 , \quad \hat{V}_\sigma^\phi = a_\sigma + N^\phi \partial_\sigma x^0 .
\]

(4.17)

Now it is easy to see that the equation of motion for $\hat{\phi}$ implies

\[
f_{\tau\sigma} = 0
\]

(4.18)

that can be solved as

\[
a_\tau = \partial_\tau \theta , \quad a_\sigma = \partial_\sigma \theta .
\]

(4.19)

Inserting this result into (4.16) we recover the original Lagrangian density after replacement $\theta \rightarrow \phi$. However the equations of motion for $a_\alpha$ that follow from (4.16) take more complicated form

\[
\frac{\omega}{2\pi \alpha' B n^\tau} h_{\phi\phi} (\hat{V}_\tau^\phi - n^\sigma \hat{V}_\sigma^\phi) + \frac{\omega}{2\pi \alpha' B n^\tau} h_{\phi\alpha} (\hat{V}_\tau^\alpha - n^\sigma \hat{V}_\sigma^\alpha) + \partial_\sigma \tilde{\phi} = 0 ,
\]

(4.18)
\[-\frac{\omega n^\sigma}{2\pi\alpha' Bn_\tau} h_{\phi\phi}(V^\phi_{\tau} - n^\sigma \hat{V}^\phi_{\sigma}) - \frac{\omega n^\sigma}{2\pi\alpha' Bn_\tau} h_{\phi\alpha}(\hat{V}^\alpha_{\alpha} - n^\sigma \hat{V}^\alpha_{\sigma}) - \frac{Bn_\tau}{\pi\alpha' \omega} (h_{\phi\phi} \hat{V}^\phi_{\alpha} + h_{\phi\alpha} V^\alpha_{\alpha}) + \partial_t \tilde{\phi} = 0.\] (4.20)

Solving these equations for \(a_\alpha\) we find
\[
a_\sigma = -\frac{\pi\alpha' \omega}{h_{\phi\phi} Bn_\tau} (\partial_\tau \tilde{\phi} - n^\sigma \partial_\sigma \tilde{\phi}) - \frac{1}{h_{\phi\phi}} (h_{\phi\phi} N^\phi \partial_\sigma x^0 + h_{\phi\alpha} V^\alpha_{\tau}) ,
\]
\[
a_\tau = -\frac{n_\sigma \pi\alpha'}{B h_{\phi\phi} n_\tau} (\partial_\tau \tilde{\phi} - n^\sigma \partial_\sigma \tilde{\phi}) - \frac{2\pi\alpha' B}{\omega h_{\phi\phi}} n_\tau \partial_\sigma \tilde{\phi} - N^\phi \partial_\tau x^0 - \frac{h_{\phi\alpha} V^\alpha_{\tau}}{h_{\phi\alpha}} .\] (4.21)

Inserting these results into the Lagrangian density (4.16) we obtain the Lagrangian density for T-dual theory
\[
\mathcal{L} = -\frac{\sqrt{\omega n_\tau}}{4\pi\alpha' n_\tau^2} (\partial_\tau x^0 - n^\sigma \partial_\sigma x^0)^2 - n_\tau \sqrt{\omega G} \left( -\frac{1}{2\pi\alpha' \omega^2} N^2 \partial_\sigma x^0 \partial_\sigma x^0 \right) + \frac{\omega n_\tau}{B} \left[ \frac{1}{4\pi\alpha' n_\tau^2} (V^i_{\tau} - n^\sigma V^i_{\sigma} \hat{h}_{ij} (V^j_{\tau} - n^\sigma V^j_{\sigma}) + \frac{1}{2(z - 1)} K_{\sigma} \frac{1}{\omega^2} K_{\sigma} - B n_\tau \left( \frac{1}{2\pi\alpha' \omega} V^i_{\alpha} \tilde{h}_{ij} V^j_{\sigma} - A \right) - \sqrt{\omega n_\tau} F(A) + \frac{1}{\sqrt{2\pi\alpha'}} N^\phi (\partial_\tau x^0 \partial_\tau \tilde{\phi} - \partial_\tau x^0 \partial_\sigma \tilde{\phi}) + \frac{1}{\sqrt{2\pi\alpha'}} h_{\phi\phi} (V^\alpha_{\sigma} \partial_\tau \tilde{\phi} - V^\alpha_{\sigma} \partial_\sigma \tilde{\phi}) ,\right.
\] (4.22)

where
\[
V^\phi_{\alpha} = \partial_\alpha \tilde{\phi} + \hat{N}^\phi \partial_\sigma x^0 , \quad V^\alpha_{\alpha} = \partial_\alpha x^\alpha + \hat{N}^\alpha \partial_\alpha x^\alpha .\] (4.23)

Note that T-dual lapse, shift and metric components take the form
\[
\hat{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{h_{\alpha\phi} h_{\phi\beta}}{h_{\phi\phi}} , \quad \hat{h}_{\phi\alpha} = 0 , \quad \hat{N} = N , \quad \hat{N}^\phi = 0 , \quad \hat{N}^\alpha = N^\alpha .\] (4.24)

Note that T-dual metric written in \(D + 1\) formalism takes the form
\[
\hat{g}_{00} = -\hat{N}^2 + \hat{N}_i \hat{h}^{ij} \hat{N}_j , \quad \hat{g}_{0i} = \hat{N}_i , \quad \hat{g}_{ij} = \hat{h}_{ij} ,\] (4.25)

with inverse
\[
\hat{g}^{00} = -\frac{1}{\hat{N}^2} , \quad \hat{g}^{0i} = \frac{\hat{N}_i}{\hat{N}} , \quad \hat{g}^{ij} = \hat{h}^{ij} - \frac{\hat{N}_i \hat{N}_j}{\hat{N}^2} .\] (4.26)
From (4.24) we see that in T-dual theory $\hat{g}_{\phi\phi} = \hat{g}_{\phi\alpha} = 0$ and also

$$\hat{h} = \frac{1}{\hat{h}_{\phi\phi}}, \quad \hat{h}_{\phi\alpha} = 0 \quad (4.27)$$

Then with the help of (4.24) we find

$$\hat{g}_{00} = -\hat{N}^2 + \hat{N}^\alpha \hat{h}_{\alpha\beta} \hat{h}^{\beta} + \hat{N}^\phi \hat{h}_{\phi\phi} \hat{N}^\phi = g_{00} - \frac{g_{0\phi} g_{\phi\alpha}}{g_{\phi\phi}}. \quad (4.28)$$

Finally we consider the last term in (4.22)

$$\frac{1}{\sqrt{2\pi\alpha'}} N^\phi (\partial_\sigma x^0 \partial_\tau \tilde{\phi} - \partial_\tau x^0 \partial_\sigma \tilde{\phi}) + \frac{h_{\phi\alpha}}{\sqrt{2\pi\alpha'} h_{\phi\phi}} (V^\alpha_\sigma \partial_\tau \tilde{\phi} - V^\tau_\sigma \partial_\sigma \tilde{\phi}) =$$

$$= -\frac{1}{\sqrt{2\pi\alpha'}} N^\phi (\partial_\tau x^0 \partial_\sigma \tilde{\phi} - \partial_\sigma x^0 \partial_\tau \tilde{\phi}) - \frac{h_{\phi\alpha}}{\sqrt{2\pi\alpha'} h_{\phi\phi}} (\partial_\tau x^0 \partial_\sigma \tilde{\phi} - \partial_\sigma x^0_\alpha \partial_\tau \tilde{\phi}) =$$

$$= \frac{1}{\sqrt{2\pi\alpha'}} \hat{b}_{0\phi} (\partial_\tau x^0 \partial_\sigma \tilde{\phi} - \partial_\sigma x^0 \partial_\tau \tilde{\phi}) + \frac{1}{\sqrt{2\pi\alpha'}} \hat{b}_{\alpha\phi} (\partial_\tau x^0 \partial_\sigma \tilde{\phi} - \partial_\sigma x^0_\alpha \partial_\tau \tilde{\phi}) \quad (4.29)$$

where

$$\hat{b}_{0\phi} = -\frac{N^\phi}{h_{\phi\phi}} = -\frac{g_{0\phi}}{g_{\phi\phi}}, \quad \hat{b}_{\alpha\phi} = -\frac{h_{\phi\alpha}}{h_{\phi\phi}} = -\frac{g_{\phi\alpha}}{g_{\phi\phi}}. \quad (4.30)$$

We see that the relations between original and T-dual background fields given in (4.24), (4.28) and (4.30) exactly coincide with the standard Buscher’s rules [53, 54, 55] between original and T-dual metric components (see also [51, 52])

$$\hat{g}_{\phi\phi} = \frac{1}{g_{\phi\phi}}, \quad \hat{g}_{\phi0} = \frac{b_{0\phi}}{g_{\phi\phi}}, \quad \hat{g}_{\phi\alpha} = \frac{b_{\alpha\phi}}{g_{\phi\phi}},$$

$$\hat{g}_{00} = g_{00} - \frac{g_{0\phi} g_{\phi\alpha}}{g_{\phi\phi}}, \quad \hat{h}_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{\alpha\phi} g_{\phi\beta}}{g_{\phi\phi}},$$

$$\hat{b}_{0\phi} = \frac{g_{0\phi}}{g_{\phi\phi}}, \quad \hat{b}_{0\phi} = -\frac{g_{0\phi}}{g_{\phi\phi}}, \quad \hat{b}_{\alpha\phi} = \frac{g_{\alpha\phi}}{g_{\phi\phi}}, \quad \hat{b}_{\alpha\phi} = -\frac{g_{\alpha\phi}}{g_{\phi\phi}}. \quad (4.31)$$

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