Aspect ratio influence on the stability of Taylor-Couette flow

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Abstract. The study deals with the effect of the fluid height on the stability of the Taylor-Couette flow of a Newtonian fluid between two coaxial cylinders, the upper surface of the fluid system being free. The exploration of the flow regimes is carried out for different values of the Taylor number $Ta$ and of the aspect ratio $\Gamma$. By means of polarographic technique, we have determined the temporal mean value of the gradient velocity at the inner wall of the outer cylinder maintained at rest. We also performed the spectral analysis of the fluctuations of the velocity gradient. In order to complete wall measurements, the LDA technique was used to measure particularly the axial velocity component and rotation speed of the azimuthal wave. In this wavy-mode regime, the analysis of the results associated with the fluctuations of the fundamental frequency shows a change in the circumferential wave number. We established that the appearance of the azimuthal wave is delayed when the aspect ratio decreases. In addition we found that below the critical value $\Gamma_c$, the azimuthal wave regime is no longer observed.

Key words: Taylor-Couette flow, aspect ratio, instabilities

1. Introduction
The flow between two coaxial rotating cylinders continues to attract the attention of many researchers for the detailed examination of its structure and evolution, particularly in the laminar-turbulent transition regime. Because of its simplicity, the configuration of the flow offers several possibilities for fundamental studies of the theoretical and experimental aspects. Since G.I Taylor’s work [1], a considerable amount of work was devoted to this type of flow. In 1976, J.A Cole [2] explored the effect of a finite height on the transition phenomena from the movement of Taylor vortex flow to the azimuthal wave. He showed that the appearance of the cells occurs at the ends of the cylinders for a Reynolds number value $Re$ (very below the critical value $Re_1$). The latter corresponds to the classical case of the infinite height studied in 1965 by D.Coles [3].

Moreover J.A Cole found that the Reynolds number $Re_2$ characterizing the establishment of the azimuthal waves regime increases considerably when the flow height is reduced. Previously following the investigations [4] and [5] devoted to the effects of the height and the free surface on the flow stability, we showed the existence of a critical aspect ratio value $\Gamma_c = 10$. For $\Gamma < \Gamma_c$ the transition from stationary flow towards the chaos appears directly without observing the azimuthal wave regime. In that context the present study is intended to complete the previous work [5], using the polarographic
technique for evaluation of the mean value of velocity gradient $\bar{S}$ and a simultaneous analysis of the associated fluctuations of the velocity gradient $s'$. The LDA technique is used to measure the axial velocity component of the flow.

2. Measurement device and experimental technique

2.1. Measurement device.

The experimental device consists of two coaxial cylinders of radius $R_1$ and $R_2$, made of a transparent material (Plexiglas). The inner cylinder ($R_1 = 35$ mm) is driven by direct current motor with a velocity variator, the outer cylinder ($R_2 = 41$ mm) at rest. The annular gap size is $d = (R_2 - R_1) = 6$ mm, so the curvature $d/R_1 = 0.171$. The aspect ratio $\Gamma = H/d$ can vary from 0 to 20.

The various flow regimes are characterized by the Taylor-number $Ta = R_1 \cdot d/R_1$. The probes on platinum are fixed so as to touch slightly the internal wall of the fixed cylinder. The fluid used is an electrochemical solution that consists of a couple-redox (Ferri-Ferrocyanure of potassium) bathing in a chemically neutral electrolyte (sulphate of potassium in excess).

2.2. Experimental technique.

The local evaluation of the parietal gradient velocity $\bar{S}$ is carried out using the polarographic method [6] which makes it possible to relate the diffusion limit current $I$, measured on the probe, with the mean value of gradient $\bar{S}$:

$$\bar{S} = \frac{1}{(0.807 \cdot n \cdot F \cdot A \cdot C \cdot 0)^3} \cdot \frac{L}{D^2} \cdot (\bar{I})^3$$

The accuracy of $\bar{S}$ is estimated at less than 3%. The determination of the gradient $\bar{S}$ involves the evaluation of the local friction coefficient $f^*$. With the LDA, we have measured the axial and radial velocity components versus the Taylor number at different values of $\Gamma$.

3. Results and discussion

3.1. Friction factor analysis.

Usually the evaluation of the friction factor has played an important role on both theoretical and experimental aspects in order to validate the Taylor-Couette flow model. Presently, there are two methods for the friction evaluation:

- the measurement of the torque exerted by the fluid on the rotating inner cylinder ($r = R_1$),
- the polarographic method based on the parietal velocity gradient measurement at the external cylinder surface. This method is no disturbing one, it is more accurate than the previous. The first expressions of the friction coefficient based on the measurements of the torque are due to F.Wendt [7] and G.I.Taylor [8]. They were followed by those of R.J. Donnely [9].

Wendt proposed the following relation:

$$f_w = \frac{1}{2} \frac{\bar{\tau}}{\rho (R_1 \Omega)^2} = \frac{G}{\pi H \rho (R_1 \Omega)^2 R_1^2}$$

where $G$ is the developed torque, $H$ the height, $\bar{\tau}$ is the parietal stress at the surface of the inner cylinder ($r = R_1$). The knowledge of the diffusion limit current measured with the polarographic
method makes it possible to determine the average gradient of the velocity. In these conditions we calculate the dimensionless average friction coefficient \( f^* \) given by the following relation:

\[
f^* = \frac{\bar{\tau} (R_1 + R_2)}{2 \rho \Omega^2 R_1^2 \sqrt{d.R_1}}
\]

where \( \bar{\tau} = \mu \bar{S} \) represents the shear stress of the flow exerted on the external cylinder surface \( (r = R_2) \). The factor \( (R_1 + R_2)/2 \) represents the average radius of the annular space. The evolution of the coefficient \( f^* \) is represented in Figure 1 according to the regime and the various aspect ratio values \( \Gamma = 20 \) and \( \Gamma = 13.33 \). The results are compared with those of G.I.Taylor [8] and of R.J Donnelly’s [9], and the polarographic measurements of G.Cognet [10] and of A.Bouabdallah [11] carried out in an infinite geometry. Our results are in a good agreement with those of these authors for the aspect ratio value \( \Gamma = 20 \).

**Figure 1.** Evolution of friction factor \( f^* \) versus Taylor Number - case of large aspect ratio: \( \Gamma > 10 \)

**Figure 2.** Evolution of friction factor \( f^* \) versus Taylor number for \( \Gamma \leq 10 \). - comparison with the case of small aspect ratio and large aspect ratio.
Figure 2 shows the factor $f^*$ evolution in function of the flow regime for various aspect ratio $\Gamma = 10$ and $\Gamma = 7$.

First, in comparison with large aspect ratio we note that, for $\Gamma \leq 10$, the evolution of friction factor $f^*$ is considerably affected quantitatively and qualitatively for the values of $Ta$ in the range of $Tc1 < Ta < Tc2$.

Second, we point out the same behavior of $f^*$ if $Ta > Tc2$, $f^*$ is much greater than the value of the infinite geometry.

3.2. Evolution of the circumferential wave train.
In addition to our study on the evolution of the average parietal velocity gradient, we analyzed the associated fluctuations. By considering the power spectra of the fluctuations of the velocity gradient $s'$ we have determined the value of the fundamental frequency $F_0 = nf_0$, where $f_0$ denotes the oscillation frequency of the azimuthal waves and is the azimuthal wave number. Figure 3 represents the evolution of the frequency of the azimuthal wave train according to the aspect ratio for several values of the number of Taylor.

![Figure 3. Evolution of fundamental frequency $nf_0$ versus aspect ratio $\Gamma$.](image)

The value of the fundamental frequency depends on the conditions of speed setting of. For a given Taylor’s number, it remains constant at $\Gamma = 10$. Below $\Gamma = 10$, we measure only $N_0$ which is rotating speed of inner cylinder and therefore the vortex rotates at the same frequency as the inner cylinder and the azimuthal wave disappears completely.

3.3. Velocity components measure
Measurements were made in the middle of the annular space at the position corresponding to the center of Taylor cell. The choice of the center, makes it possible to avoid measuring the number of revolutions of Taylor’s vortices. We represent in Figure 4 the variation of the axial component $V_z$ in order to characterize eventually the azimuthal wave as a function of $Ta$. 

![Diagram](image)
For an aspect ratio \( \Gamma = 8.33 < 10 \), the axial velocity is practically zero compared to the other values corresponding to \( \Gamma = 16.7 \) and \( \Gamma = 11.7 \).

![Graph showing evolution of axial velocity \( V_z \) versus flow regime for different values of \( \Gamma \).]

**Figure 4.** Evolution of axial velocity \( V_z \) versus flow regime for different values of \( \Gamma \).

**4. Conclusion**

The experimental results, obtained by the polarography method, presented here are consistent with previous observations obtained by visualization. They confirm the existence of a critical height \( H_c \).

For an aspect ratio \( \Gamma < 10 \), the transition from stationary flow towards the chaos takes place directly without an azimuthal wave regime. The appearance of the secondary instability is delayed when the height decreases. This result is confirmed by measurements by LDA. The axial component velocity \( V_z \) which characterizes the azimuthal wave is practically null as \( \Gamma \) is decreasing below \( \Gamma = 10 \).

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