Quantum cryptography and long distance Bell experiments: How to control decoherence

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Abstract
Several mechanisms that affect one and two photon coherence in optical fibers and their remedies are discussed. The results are illustrated on quantum cryptography experiments and on long distance Bell inequality tests.

1 Introduction
The implementation of 1- and 2-photon quantum communication protocols in km long optical fibers suffers from several decoherence mechanisms. In this contribution we review the main ones and illustrate how one can control them.

Photons are characterized by three (non independent) parameters: their temporal coherence, their polarization and their frequency spectrum. In the next three sections decoherence affecting each of these parameters are presented, together with counter-measures. The first one, in the time domain, leads to a useful measurement method of polarization mode dispersion. Mastering the second one, depolarization, leads to a practical implementation of quantum cryptography. Finally, the phenomenon of two-photon chromatic dispersion cancelling opens the route to long distance Bell experiments.

2 Polarization Mode Dispersion: Decoherence in the time domain
Real fibers are not perfectly circular. Consequently, the two polarization modes are not degenerate and propagate at different phase and group velocities. The
difference in group velocities results in Polarization Mode Dispersion (PMD). The phenomenon of PMD is presently a very severe limitation to high speed optical communication. In addition to the presence of two group velocities, PMD is characterized by random polarization mode coupling: some energy of the fast mode couples to the slow mode and vice-versa. The locations where such couplings take place and their extend are very sensitive to thermal and mechanical variations. Hence, in practice, the coupling is described as a random phenomenon. The magnitude of the dispersion ranges from a few tenths of a picosecond up to tens of picoseconds. Because of its stochastic nature, PMD is measured in units of $ps/\sqrt{km}$.

Direct measurement of PMD is a non trivial task. When light with a short coherence time (typically light from a LED with $\tau_c \approx 0.05$ ps) propagates down a fiber, the dispersion is larger than the coherence, producing decoherence. However, coherence can be recovered by connecting an interferometer at the end of the fiber, see figure 1. When the interferometer is unbalanced, light that went out of coherence in the fiber by precisely the amount of imbalance of the interferometer can be brought back into coherence. This leads to interference fringes even when the interferometer’s imbalance is larger than the source coherence, see figure 2. This simple technique to recoalher light is widely used by the telecom industry to measure PMD.

An interesting generalization using 2-photon interferometry was demonstrated by A. Sergienko and A. Muller, see [4, 5].

3 Depolarization: Decoherence in the polarization domain

A single photon state $|\Psi_z\rangle$ at position $z$ along the fiber can be described as follows:

$$|\Psi_z\rangle = \int_0^\infty \psi_z(\omega)|1_\omega\rangle d\omega$$

where $|1_\omega\rangle$ denotes the 1-photon state at frequency $\omega$ and $\psi_z(\omega) \in \mathbb{C}^2$ is a (non normalized) Jones vector describing the polarization of the frequency component $\omega$ with the square norm $|\psi_z(\omega)|^2$ the corresponding intensity. Let us introduce the Poincaré vectors:

$$\vec{m}_z(\omega) = \frac{\langle \psi_z(\omega)|\vec{\sigma}|\psi_z(\omega)\rangle}{\langle \psi_z(\omega)|\psi_z(\omega)\rangle}$$

where $\vec{\sigma}$ are the Pauli matrices. Note that these vectors are normalized, $|\vec{m}_z(\omega)| = 1$, indicating that individual frequency components are always fully polarized. However, the polarization of the photon, given by

$$\vec{M}_z = \int_0^\infty |\psi_z(\omega)|^2 \vec{m}_z(\omega) d\omega,$$
can be partially ($|\vec{M}_z| < 1$) or even totally ($|\vec{M}_z| = 0$) depolarized.

If fully polarized light, e.g. from a laser diode, is launched into a fiber (position $z = 0$) one has: $\vec{m}_0(\omega) = \vec{m}_{\text{laser}}$ for all $\omega$, hence the photons are totally polarized: $|\vec{M}_0| = |\vec{m}_{\text{laser}}| = 1$. Let us model the optical fiber as a concatenation of trunks of length $\ell_j$, and birefringence $\vec{\beta}_j$ \cite{6}. Accordingly (neglecting losses) the photon states evolves to:

$$\psi_\ell(\omega) = e^{i\omega\ell_1}\vec{\beta}_1\vec{\sigma}/2 ... e^{i\omega\ell_2}\vec{\beta}_2\vec{\sigma}/2 e^{i\omega\ell_1}\vec{\beta}_1\vec{\sigma}/2 \psi_0(\omega) \quad (4)$$

where $\ell = \sum_j \ell_j$ is the total length of the fiber. In long fibers the $\vec{\beta}_j$, in particular their orientations, are random \cite{6}, hence the output light is depolarized: $\vec{M}_\ell \approx 0$.

Depolarization, i.e. decoherence in the polarization domain, severely limits potential applications of quantum cryptography \cite{7}. Indeed, coding the qubit in polarization becomes clearly unpractical, while coding the qubit in the phase \cite{9} is no better because phase decoding requires interferometers and interferences are sensitive to polarization. One possible way out is to limit the width of the optical spectrum, so that the integral in (3) is dominated by the central frequency and the output light remains polarized. But even so, polarization fluctuations would impose active feedbacks. A more elegant and practical solution exploit the feature of Faraday Mirrors (FM) \cite{8}. A FM consists of a $\lambda/4$ Faraday rotator followed by an ordinary mirror (with normal incidence). The effect of such a FM is to turn any incoming polarization state to its orthogonal state, as illustrated on figure 3. This non-unitary transformation is possible because one uses a description in which one switches from a right handed reference frame before the reflection to a left handed one after the reflection. This is quite convenient (though not necessary), as doing so the polarization transformations during propagation back up the fiber are precisely the inverse of those the photon underwent on the way to the FM:

$$\psi_{2\ell}(\omega) = e^{-i\omega\ell_1}\vec{\beta}_1\vec{\sigma}/2 ... e^{-i\omega\ell_2}\vec{\beta}_2\vec{\sigma}/2 ... e^{-i\omega\ell_1}\vec{\beta}_1\vec{\sigma}/2 T_{FM} \psi_{\ell}(\omega) \quad (5)$$

where $T_{FM}$ denote the transformation due to the FM and $\psi_{2\ell}(\omega)$ is the polarization state after a go-and-return through the fiber. The effect of this is easier analyzed using the Poincaré vectors $\vec{m}_{\ell}(\omega)$ for which the FM transformation simply reverses the orientation: $\tilde{T}_{FM} \vec{m} = -\vec{m}$ (where $\tilde{T}_{FM}$ is the corresponding $T_{FM}$ operator but acting on the Poincaré vectors). Accordingly, $\vec{m}_{2\ell}(\omega) = -\vec{m}_{\text{laser}}$ for all $\omega$, hence $\vec{M}_{2\ell} = -\vec{m}_{\text{laser}}$ and the return light is again fully polarized. Moreover, the state of polarization is fixed (relative to the source). This result holds as long as the fiber can be considered as fixed during the time of a go-and-return, typically some micro-seconds. That this is indeed the case for km long installed telecom fibers was first demonstrated in \cite{10}: more than 99.8% of repolarization was achieved on a 23 km long fiber below lake Geneva.

This way of "polarization recoherence" is exploited in our "Plug & Play" implementation of quantum cryptography \cite{10, 11, 12}, see figure 4.
4 Chromatic Dispersion: Decoherence in the frequency domain

For long distance Bell experiments, the use of polarization correlation is unpractical because of the depolarization mechanism described in the previous section (see however [3] where the distance and the photon spectrum were reduced to limit depolarization). Moreover the use of Faraday Mirrors is incompatible with the requirement that the two detectors and the source should be at three widely separated locations. In 1989 Jim Franson [4] proposed an elegant two-photon interferometer free of the depolarization problem and suitable for tests of the Bell inequality over long distance, see figure 5 (actually, in this scheme polarization has to be controlled inside the two distant interferometers, but depolarization in the long fibers connecting the source and interferometers is irrelevant [3, 14]). However, chromatic dispersion, the fact that different optical frequencies (wavelengths) propagate at different speeds, imposes severe limitations to the fringe visibility in Franson interferometers. Indeed, the two photons, emitted precisely at the same time by spontaneous parametric down-conversion in a nonlinear crystal, must be detected in coincidence, within a time window of typically 300 ps. This time window must be short enough so that one can distinguish the cases when the two photons took both the short or both the long arm of their interferometer, from the cases when they took different arms. For stability reasons it is reasonable to have arm length differences of some tens of cm, corresponding to a few ns. But if chromatic dispersion (or any other cause of dispersion) reduces the time correlation between the photons, then a coincidence detection no longer guaranties that both photons took the same path. Hence chromatic dispersion severely reduces the 2-photon interference visibility.

In a dispersive media, like the silica of optical fibers, the chromatic dispersion vanishes for a wavelength close to 1310 nm (the exact value depends on details of the manufacture). In our long distance Bell experiment, the single photons had a spectral width of about ±35 nm. Hence, for a fiber length of 17 km [3] the chromatic dispersion is of the order of 500 ps, large enough to reduce the fringe visibility down below the threshold set by Bell inequality (Bell inequality is violated for visibilities larger than 1/√2 ≈ 71%). One way around this decoherence mechanism is the following. In good approximation chromatic dispersion is a linear function of the wavelength \( \lambda \) (this approximation is valid over several tens of nm). Hence the differential group delay is a quadratic function of \( \lambda \), with its minimum at \( \lambda_0 \), the wavelength of zero chromatic dispersion. Accordingly, if the central wavelength of the photon pair is precisely at \( \lambda_0 \), when, thanks to the frequency correlation of the two photons, both photons are at wavelengths symmetrically above and below \( \lambda_0 \). Both photons undergo thus

\[^{1}\text{In our experiment the analyzers were separated by slightly more then 10 km, but the connecting fibers were quite longer.}\]
the same differential group delay, hence arrive at the analyzer in perfect coincidence. This phenomenon, called 2-photon chromatic dispersion cancellation [17], is essential for long distance Bell experiments using optical fibers. Note that we made two approximations: first that chromatic dispersion is approximately a linear function of wavelength, next that the the frequency correlation \(\nu_1 + \nu_2 = \nu_{pump}\), due to energy conservation, implies the approximate wavelength correlation \(\lambda_1 + \lambda_2 \approx \lambda_{pump}\) (this second approximation is not necessary, as all the discourse could be phrased in terms of frequency, but traditionally chromatic dispersion is expressed in wavelengths). These approximations are in excellent agreement with our experimental results [18, 19, 16].

5 Conclusion

Decoherence affects already systems of one and two-photon, setting limits to the viability of the corresponding communication protocols. However, we have illustrated how one can deal with decoherence for these relatively simple systems. The generalization to larger, more complex, systems is not straightforward. Nevertheless, there is hope that some of the ideas presented in this contribution may guide the research for improved "decoherence management".

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7 Figure Captions

1. Schematic of an instrument for Polarization Mode Dispersion measurements using the so-called "interferometric method": the interferometer re-coheres photons that went out of decoherence due to the polarization dispersion in the fiber under test.

2. Typical result obtained with the interferometric measurement method (see figure 1) for a standard telecom fiber.

3. Poincaré sphere picture of the polarization state transformation of a photon when reflected by a Faraday mirror. Each state P undergoes first a 1/4 tour around the vertical axis, next a reflection due to the standard mirror, finally a second 1/4 tour. As both rotations are due to the Faraday effect, they rotate in the same direction, despite that the photons are travelling in opposite directions. The final state P' is always opposite to the initial state P, i.e. P and P' represent orthogonal states.

4. Schematic of our "Plug & Play" quantum cryptography system. For details see [10, 11, 12].

5. Principle of a Franson test of Bell inequality.
Experimental setup for the interferometric measurement method.
Mirror displacement [µ]

Delay [pico seconds]  Wavelength: 1.55 [µm]

Intensity [AU]

-2000 -1500 -1000 -500 0 500 1000 1500 2000

PM Delay = 5.96 [ps]
Dispersion = 0.698 [ps/(Km^{1/2})]
Franson interferometer

Two unbalanced interferometers $\Rightarrow$ no first order interferences
photon pairs $\Rightarrow$ possibility to measure coincidences

One can not distinguish between "long-long" and "short-short"

Hence, according to QM, one should add the probability amplitudes

$\Rightarrow$ interferences (of second order)