Effects of three-nucleon forces and two-body currents on Gamow-Teller strengths

A. Ekström,1 G. R. Jansen,2,3 K. A. Wendt,3,2 G. Hagen,2,3 T. Papenbrock,3,2 S. Bacca,4 B. Carlsson,5 and D. Gazit6

1Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway
2Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA
3Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA
4TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada
5Department of Fundamental Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden
6Racah Institute of Physics, Hebrew University, 91904, Jerusalem

PACS numbers: 23.40.-s, 24.10.Cn, 21.10.-k, 21.30.-x

Introduction. — β decay is one of the most interesting processes and most useful tools in nuclear physics. On the one hand, searches for neutrino-less double-β decay probe physics beyond the standard model and basic properties of the neutrino, see Avignone et al. 11 for a recent review. If neutrinoless double-β decay is observed, an accurate nuclear-physics matrix element is needed to extract neutrino masses from the life time. On the other hand, β decay of rare isotopes populates states in exotic nuclei and thereby serves as a spectroscopic tool 2, 3. The theoretical calculation of electromagnetic transition matrix elements in atomic nuclei is a challenging task, because it requires an accurate description of the structure of the mother and daughter nuclei, and an employment of a transition operator that is consistent with the Hamiltonian.

For the transition operator, the focus is on the role of medium-energy currents 4 and two-body currents (2BCs) from chiral effective field theory (χEFT). Two-body currents are related to three-nucleon forces (3NFs) 5, 6 because the low energy constants (LECs) of the latter constrain the former within χEFT. Consistency of Hamiltonians and currents is one of the hallmarks of an EFT 7, and 2BCs are applied in electromagnetic processes of light nuclei, see Gazit et al. 8, Griesshammer et al. 9, and Pastore et al. 10. For weak decays, only the calculation of triton β decay 8, the related μ decay on 3He and the deuteron 11, and proton-proton fusion 12, exhibits the required consistency, while the very recent calculation of the neutral-current response in 12C employs phenomenological 3NFs together with chiral 2BCs 13.

The one-body operator $g_A \sum_{i=1}^{A} \sigma_i \tau_i^{±}$ induces Gamow-Teller transitions. Here $g_A$ is the axial-vector coupling, $\sigma$ denotes the spin, and $\tau_i^{±}$ changes the isospin. Gamow-Teller strength functions 14, 15 are of particular interest also because of their astrophysical relevance 16. Charge-exchange measurements on 90Zr and other medium mass nuclei have suggested that the total strength for β decay is quenched by a factor of $q^2 \approx 0.84 - 0.92$ when compared to the Ikeda sum rule 21. Similarly, shell-model calculations 22, 23 suggest that $g_A$ needs to be quenched by a factor $q \approx 0.75$ to match data. It is not clear whether renormalizations (including 2BCs) of the employed Gamow-Teller operator, missing correlations in the nuclear wave functions, or model-space truncations are the cause of this quenching.

Recent calculations 24, 25 show that chiral 2BCs yield an effective quenching of $g_A$. However, the Hamiltonians employed in these works are not consistent with the currents (and they contain no 3NFs), and/or the 2BCs are approximated by averaging the second nucleon over the Fermi sea of symmetric nuclear matter. The recent studies 26, 27 of electroweak transitions in light nuclei employ 3NFs but lack 2BCs. This gives urgency for a calculation of weak decays that employs 3NFs and consistent 2BCs.

In this Letter, we address the quenching of $g_A$ and employ 3NFs together with consistent 2BCs for the computation of β decays and the Ikeda sum rule. We study the β decays of 14C and 22,24O with interactions and currents from χEFT at next-to-next-to leading order (NNLO) for cutoffs $\Lambda_\chi = 450, 500, 550$ MeV. For the states of the daughter nuclei, we generalize a coupled-cluster technique and compute them as isospin-breaking excitations of the mother nuclei. We present predictions and spin assignments for the exotic isotopes 22,24F, and revisit the anomalously long half life of 14C 28, 29.

Hamiltonian and model space. — The chiral nucleon-nucleon (NN) interactions are optimized to the proton-proton and the proton-neutron scattering data for laboratory scattering energies below 125 MeV, and to deuteron observables. The $\chi^2$/datum varies between 1.33 for $\Lambda_\chi = 450$ MeV and 1.18 for $\Lambda_\chi = 550$ MeV. The $\chi^2$-
optimization employs the algorithm POUNDerS [32].

Table I shows the parameters of the NN interaction for the
cutoff $\Lambda_c = 500$ MeV; the parameters for the other
cutoffs are supplementary material. The parameters displayed
in Table I are close to those of the chiral interaction
NNLOopt [32].

| LEC   | Value       | LEC   | Value       | LEC   | Value       |
|-------|-------------|-------|-------------|-------|-------------|
| $c_1$ | -0.91940746 | $c_3$ | -3.83983848 | $c_4$ | 4.30736747  |
| $C_{0p}^{1S_0}$ | -0.15136364 | $C_{0p}^{1S_0}$ | -0.15215263 | $C_{0p}^{1S_0}$ | -0.15180482 |
| $C_{1S_0}$ | 2.40431235  | $C_{1S_1}$ | 0.9293712   | $C_{3S_1}$ | -0.1584125  |
| $C_{1P_3}$ | 0.41482908  | $C_{1P_0}$ | 1.62578978  | $C_{1P_1}$ | -0.77998484 |
| $C_{3S_1} - D_1$ | 0.61855040  | $C_{3P_0}$ | -0.67347042 |       |             |

Table I. Pion-nucleon LECs $c_i$ and partial-wave contact
LECs ($C_i, C$) for the chiral $NN$ interaction at NNLO using
$\Lambda_c = 500$ MeV and $\Lambda_{\text{SPR}} = 700$ MeV [33]. The $c_i, C_i,$ and
$C_i$ have units of GeV$^{-1}$, $10^4$ GeV$^{-2}$, and $10^4$ GeV$^{-4}$, respectively.

The 3NF is regularized with nonlocal cutoffs [34, 35]
(to mitigate the convergence problems documented
by Hagen et al. [36] for local cutoffs). Following Gazit
et al. [8], we optimize the two LECs ($c_D$ and $c_E$) of
the 3NF to the ground-state energies of $A = 3$ nuclei
and the triton lifetime. Figure 1 shows the transition matrix element $\langle E_T^A \rangle = \langle ^3\text{He} | | E_T^A | | ^3\text{H} \rangle$
as a function of $c_D$. Here $E_T^A$ is the $J = 1$ electric
multipole of the weak axial vector current at NNLO [8].

The leading-order (LO) contribution to $E_T^A$ is pro-
tional to the one-body Gamow-Teller operator,
$E_T^A|_{LO} = i g_A (6\pi)^{-1/2} \sum_{i=1}^{A} \sigma_i \tau_i^\pm$.
For the current we use the empirical value $g_A = 1.2695(29)$.
The 2BCs enter at NNLO and depend on the LECs $c_D, c_3, c_4$
of the chiral interaction [37, 38]. The triton half-life
yields an empirical value for $\langle E_T^A \rangle_{\text{emp}}$, which constrains
$c_D$ and $c_E$. For the three different chiral cutoffs
$\Lambda_c = 450, 500, 550$ the sets of $(c_D, c_E)$ that reproduce
the triton half-life and the $A = 3$ binding energies are
$(0.0004, -0.4231), (0.0431, -0.5013), (0.1488, -0.7475)$,
respectively. The vertical bands in Fig. 1 give the range
of $c_D$ that reproduce $\langle E_T^A \rangle_{\text{emp}}$ within the experimental
uncertainty.

We employ an $N = 12$ model space consisting of $N + 1$
oscillator shells with frequency $\hbar \Omega = 22$ MeV. The 3NFs
use an energy cutoff of $E_{3\text{max}} = N \hbar \Omega$, i.e., the sum of
the excitation energies of three nucleons does not exceed
$E_{3\text{max}}$. We employ the intrinsic Hamiltonian
$H = T - T_{\text{cm}} + V_{NN} + V_{3\text{NF}}$

to mitigate any spurious center-of-mass excitations [39, 40]. Here, $T$ and $T_{\text{cm}}$ are the kinetic energy and
the kinetic energy of the center-of-mass, while $V_{NN}$ and $V_{3\text{NF}}$
are the chiral $NN$ interaction and 3NF, respectively.

We perform a Hartree-Fock (HF) calculation and com-
pute the normal-ordered Hamiltonian $H_N$ with respect
to the resulting reference state $|HF\rangle$. We truncate $H_N$
at the normal-ordered two-body level, and note that 3NFs
contribute to the vacuum energy, and the normal-ordered
one-body and two-body terms. This approximation is ac-
curate in light and medium-mass nuclei [41, 42].

Formalism. — We compute the closed-subshell mother nuclei $^{14}\text{C}$ and $^{22,24}\text{O}$ with the coupled-cluster method [43, 50]. The similarity-transformed Hamiltonian
$H_{\text{opt}} = e^{-T} H_N e^T$
(2)
employs the cluster amplitudes
$T = \sum_{ia} t_{ia}^a N_i^\dagger a | N_i \rangle + \frac{1}{4} \sum_{ijab} t_{ijab}^{ab} N_i^\dagger a N_j^\dagger b | N_i N_j \rangle$
(3)
that create 1-particle – 1-hole (1p-1h) and 2-particle –
2-hole (2p-2h) excitations. Here, $i, j$ denote occupied or-
bitals of the HF reference while $a, b$ denote orbitals of
the valence space. The operators $N_i^\dagger a$ and $N_a$ create and
annihilate a nucleon in orbital $q$, respectively. It is un-
erstood that the cluster amplitudes $T$ do not change
the number of protons and neutrons, i.e. they conserve
the $z$-component $T_z$ of isospin. We note that $|HF\rangle$ is the
right ground state of the non-Hermitian Hamiltonian $H_{\text{opt}}$.
Its left ground state is not $|HF\rangle$ but $(|HF\rangle | 1 + \Lambda \rangle$, with $\Lambda$ being a linear combination of 1p-1h and 2p-2h
de-excitation operators [41, 50].

The daughter nuclei $^{14}\text{N}$ and $^{22,24}\text{F}$ are computed via
a novel generalization of the coupled-cluster equation-
of-motion approach [43, 55]. We view the states of the
daughter nuclei as isospin-breaking excitations $|R\rangle \equiv R |HF\rangle$ of the coupled-cluster ground state, with
$R = \sum_{ia} r_{ia}^a p_i^\dagger a n_i + \frac{1}{4} \sum_{ijab} r_{ijab}^{ab} N_i^\dagger a N_j^\dagger b n_i n_j$.
Here, $p^+_q$ and $p^a_q$ $(n^+_q$ and $n^a_q$) create and annihilate a proton (neutron) in orbital $q$. The combination $N^+_q N^a_q$ either involves neutrons $N^+_q N^a_q = n^+_q n^a_q$ or protons $N^+_q N^a_q = p^+_q p^a_q$. We note that $R$ lowers the isospin component $T_z$ of the HF reference by one unit and keeps the mass number unchanged.

The states of the daughter nucleus result from solving the eigenvalue problem $\mathcal{H} R_\alpha |\text{HF}\rangle = \omega_\beta R_\alpha |\text{HF}\rangle$. Here, $\omega_\beta$ is the excitation energy with respect to the HF reference, and $R_\alpha$ denotes a set of amplitudes $R_\alpha = (r^a_i(\alpha), r^b_{ij}(\alpha))$. We recall that the similarity-transformed Hamiltonian in Eq. (2) is not Hermitian. Therefore, we also introduce the left-acting de-excitation operator

$$L \equiv \sum_{ia} l^i_n p^i_a + \frac{1}{4} \sum_{ijab} l^i_{ab} n^i_j N^j_a n^a_j p^j_a , \tag{5}$$

and solve the left eigenvalue problem $\langle \text{HF} | L \phi = \omega_\beta |\text{HF}\rangle$. The left and right eigenvectors are biorthogonal, i.e., $\langle \text{HF} | L \phi = \sum_{ia} l^i_n(\alpha) r^a_i(\beta) + \frac{1}{4} \sum_{ijab} l^i_{ab}(\alpha) r^b_{ij}(\beta) = \delta_\alpha \beta$.

The operators $R$ and $L$ in Eqs. (4) and (5) excite states in the daughter nucleus that results from $\beta^-$ decay. If instead we were interested in $\beta^+$ decay, we would employ $R^\dagger$ and $L^\dagger$, and solve the corresponding eigenvalue problems. Our approach allows us to compute excited states in the daughter nucleus that are dominated by isospin-breaking 1p-1h excitations of the closed-shell reference $|\text{HF}\rangle$ (with 2p-2h excitations being smaller corrections).

Results. – The spectra for $^{14}$N and $^{22,24}$F are shown in Fig. 2 for $\Lambda = 500$ MeV and compared to data. Errorbars from variation of the chiral cutoff $\Lambda = 500$ MeV are shown for selected states. The odd-odd daughter nuclei $^{14}$N and $^{22,24}$F exhibit a higher level density than their mother nuclei. Overall, 3NFs increase the level densities slightly and yield a slightly improved comparison to experiment. For the neutron-rich isotopes of fluorine we make several predictions and spin assignments. In these isotopes, our spectra compare also well to shell-model calculations by Brown and Richter [54]. The ground state energies of the mother nuclei (obtained at $N = 12, \hbar \Omega = 22$ MeV and $\Lambda = 500$ MeV) are $-74.4$ MeV, $-104.6$ MeV, and $-105.7$ MeV for $^{14}$C, and $^{22,24}$O, respectively. Thus, these nuclei are significantly underbound compared to experiment. Our calculations employ the same nucleon mass for protons and neutrons, and we find the ground-state energies of the daughter nuclei are $0.54$ MeV, $-2.62$ MeV, and $-6.55$ MeV with respect to their corresponding mother nuclei, and in fair agreement with experiment.

Within the coupled-cluster framework we compute the total strengths

$$S_+ = \langle \Lambda |\hat{O}_\text{GT} | \hat{O}_\text{GT}^\dagger |\text{HF}\rangle , \quad S_- = \langle \Lambda |\hat{O}_\text{GT}^\dagger | \hat{O}_\text{GT} |\text{HF}\rangle$$

for $\beta^\pm$ decays. Here $\hat{O}_\text{GT}$ is the similarity-transformed Gamow-Teller operator

$$\hat{O}_\text{GT} \equiv \hat{O}_\text{GT}^{(1)} + \hat{O}_\text{GT}^{(2)} = g_1 \frac{1}{\sqrt{3}} \sqrt{\pi E_1^A} . \tag{6}$$

The one-body operator is $\hat{O}_\text{GT}^{(1)} = g_1 \frac{1}{\sqrt{3}} \sqrt{\pi E_1^A} |\text{LO}\rangle$, and the two-body operator $\hat{O}_\text{GT}^{(2)}$ is from the 2BC at NNLO [37, 38].

The Ikeda sum rule is $S_- - S_+ = 3(N - Z)$ for $\hat{O}_\text{GT} \approx \hat{O}_\text{GT}^{(1)}$. This identity served as a check of our calculations. Our interest, of course, is in the contribution of the 2BC operator $\hat{O}_\text{GT}^{(2)}$ to the total $\beta$ decay strengths $S_\pm$. We considered two approximations of this two-body operator. In the normal-ordered one-body approximation (NO1B), the second fermion of the 2BC is approximated by the chiral interaction with 3NF at NNLO [37, 38]. The dotted lines show the NO1B result. Thus, a major part of the quenching results from the NO1B approx.
proximation. The sensitivity of our results to the chiral cutoffs ($\Lambda_\chi = 450, 500, 550$ MeV) is shown as the gray band for values of $c_D$ and $c_E$ that reproduce the triton half-life. The quenching factor depends on the nucleus, with $q^2 \approx 0.84 - 0.92$ due to 2BCs for the studied nuclei. We recall that $q^2 \approx 0.88 - 0.92$, extracted from experiments on $^{90}$Zr [18, 29], are within our error band. We also computed the low-lying strengths for $\beta^-$ decay, and found that only 70% - 80% of the total strength $S_\pm$ is exhausted below 10 MeV of excitation energy.

Let us finally turn to the $\beta^-$ decay of $^{14}$C. The long half life of this decay, about 5700 s, is used in carbon dating of organic material. This half life is anomalously long in the sense that it exceeds the half lives of neighboring $\beta^-$ unstable nuclei by many orders of magnitude. Recently, several studies attributed the long half life of unstable nuclei by many orders of magnitude. Re- to this picture? To address this question, we compute the matrix element $\langle E_1^A \rangle \equiv \langle 14N|E_1^A|14C \rangle$ that governs the $\beta^-$ decay of $^{14}$C to the ground state of $^{14}$N, with $c_D$ and $c_E$ from the triton life time. Figure 4 shows the various contributions to the matrix element. In agreement with Maris et al. [28] and Holt et al. [30], 3NFs reduce the matrix element significantly in size, and our result is similar in magnitude as reported by Maris et al. [28]. However, 2BCs counter this reduction to some extent, with the NO1B approximation and the LO approximation both giving significant contributions. Our results for $\langle E_1^A \rangle$ from 2BCs and 3NFs are between $5 \times 10^{-3}$ and $2 \times 10^{-2}$. This is more than an order of magnitude larger than the empirical value $\langle E_1^A \rangle_{\text{emp}} \approx 6 \times 10^{-3}$ extracted from the 5700 s half life of $^{14}$C.

We also find that the matrix element $\langle E_1^A \rangle$ depends on the energy of the first excited $1^+$ state in $^{14}$N. For the three different cutoffs $\Lambda_\chi = 450, 500, 550$ MeV this excited $1^+$ state is at 5.69, 4.41, 3.35 MeV, respectively (compared to 3.95 MeV from experiment). As the value of $\langle E_1^A \rangle$ decreases strongly with decreasing excitation energy, a correct description of this state is important for the half-life in $^{14}$C.

Summary. — We studied $\beta^-$ decays of $^{14}$C, and $^{22,24}$O. Due to 2BCs we found a quenching factor $q^2 \approx 0.84 - 0.92$ from the difference in total $\beta$ decay strengths $S_- - S_+$ when compared to the Ikeda sum rule value $3(N - Z)$. To carry out this study, we optimized interactions from $\chi$EFT at NNLO to scattering observables for chiral cutoffs $\Lambda_\chi = 450, 500, 550$ MeV. We developed a novel coupled-cluster technique for the computation of spectra in the daughter nuclei and made several predictions and spin assignments in the exotic neutron-rich isotopes of fluorine. We find that 3NFs increase the level density in the daughter nuclei and thereby improve the comparison to data. The anomalously long half life for the $\beta^-$ decay of $^{14}$C depends in a complicated way on 3NFs and 2BCs. While the former increase the theoretical half life, the latter somewhat counter this effect. Taken together, the inclusion of 3NFs and 2BCs yield an increase in the computed half life.

We thank D. J. Dean, J. Engel, Y. Fujita, K. Hebeler, M. Hjorth-Jensen, M. Sasano, T. Uesaka, and A. Signoracci for useful discussions. We also thank E. Epelbaum for providing us with nonlocal 3NF matrix elements. This work was supported by the Office of Nuclear

FIG. 3. (Color online) The quenching factor $q^2$ for $^{14}$C (black line), $^{22}$O (red dashed line), and $^{24}$O (blue dashed-dotted line) for different $c_D$ values. The calculations used $NN$ and $3NF$ with consistent 2BCs. The gray area marks the region of $c_D$ that yields the triton half life and shows the cutoff dependence. The dotted lines show the NO1B result.

FIG. 4. (Color online) The squared transition matrix element for $\beta^-$ decay of $^{14}$C from increasingly sophisticated calculations (from left to right). $NN$, 1BC: $NN$ interactions and one-body currents (1BC) only. $NN + 3NF$, 1BC: addition of 3NF. $NN + 3NF$, 1BC + 2BC: addition of 2BC in the NO1B approximation. $NN + 3NF$, 1BC + 2BC: addition of leading-order 2BC.
Physics, U.S. Department of Energy (Oak Ridge National Laboratory), under DE-FG02-96ER40963 (University of Tennessee), DE-SC0008499 (NUCLEI SciDAC collaboration), NERRSC Grant No. 491045-2011, the Field Work Proposal ERKBP57 at Oak Ridge National Laboratory, and by the National Research Council and by the Nuclear Science and Engineering Research Council of Canada. DG’s work is supported by BMBF ARCHES. Computer time was provided by the Innovative and Novel Computational Impact on Theory and Experiment (INCITE) program. This research used resources of the Oak Ridge Leadership Computing Facility located in the Oak Ridge National Laboratory, which is supported by the Office of Science of the Department of Energy under Contract No. DE-AC05-00OR22725, and used computational resources of the National Center for Computational Sciences, the National Institute for Computational Sciences, and the Notur project in Norway.

Supplementary material. – The LECs for the NNLO interactions with cutoffs $\Lambda = 450, 550$ MeV can be found in Tables II[1] and II[3].

| LEC value | LEC value | LEC value |
|-----------|-----------|-----------|
| $c_1$ | -0.91029482 | $c_3$ | -3.88068766 | $c_4$ | 4.67092062 |
| $C_{\chi}^{S_0}$ | -0.15203546 | $C_{\chi}^{S_0}$ | -0.15228740 | $C_{\chi}^{S_0}$ | -0.15247585 |
| $C_{S_0}$ | 2.43109829 | $C_{S_0}$ | 0.98757436 | $C_{S_0}$ | -0.16953957 |
| $C_{S_0}$ | 0.46691821 | $C_{S_0}$ | 1.21516744 | $C_{S_0}$ | -0.85034985 |
| $C_{S_0}^{\chi} D_1$ | 0.68142133 | $C_{S_0}^{\chi}$ | -0.67318268 |

TABLE II. Pion-nucleon LECs ($C$, $\tilde{C}$) for the chiral NN interaction at NNLO using $\Lambda_X = 450$ MeV and $\Lambda_{S\chi} = 700$ MeV [33]. The $c_i$, $\tilde{C}_i$, and $C_i$ have units of GeV$^{-1}$, 10$^4$ GeV$^{-2}$, and 10$^4$ GeV$^{-2}$, respectively.

| LEC value | LEC value | LEC value |
|-----------|-----------|-----------|
| $c_1$ | -0.90630268 | $c_3$ | -3.89738533 | $c_4$ | 3.90628243 |
| $C_{\chi}^{S_0}$ | -0.15067278 | $C_{\chi}^{S_0}$ | -0.15162371 | $C_{\chi}^{S_0}$ | -0.15121579 |
| $C_{S_0}$ | 2.38956389 | $C_{S_0}$ | 0.83899578 | $C_{S_0}$ | -0.14677863 |
| $C_{S_0}$ | 0.38612051 | $C_{S_0}$ | 1.32532984 | $C_{S_0}$ | -0.68424744 |
| $C_{S_0}^{\chi} D_1$ | 0.56266120 | $C_{S_0}^{\chi}$ | -0.67444090 |

TABLE III. Pion-nucleon LECs ($C$, $\tilde{C}$) for the chiral NN interaction at NNLO using $\Lambda_X = 550$ MeV and $\Lambda_{S\chi} = 700$ MeV [33]. The $c_i$, $\tilde{C}_i$, and $C_i$ have units of GeV$^{-1}$, 10$^4$ GeV$^{-2}$, and 10$^4$ GeV$^{-2}$, respectively.

[1] F. T. Avignone, S. R. Elliott, and J. Engel, Rev. Mod. Phys. 80, 481 (2008), URL http://link.aps.org/doi/10.1103/RevModPhys.80.481.
[2] J. A. Winger, S. V. Ilyushkin, K. P. Ryckezewski, C. J. Gross, J. C. Batchelder, C. Goodin, R. Grzywacz, J. H. Hamilton, A. Korgul, W. Królas, et al., Phys. Rev. Lett. 102, 142502 (2009), URL http://link.aps.org/doi/10.1103/PhysRevLett.102.142502.
[3] K. Miernik, K. P. Ryckezewski, C. J. Gross, R. Grzywacz, M. Madurga, D. Miller, J. C. Batchelder, I. N. Borzov, N. T. Brewer, C. Jost, et al., Phys. Rev. Lett. 111, 132502 (2013), URL http://link.aps.org/doi/10.1103/PhysRevLett.111.132502.
[4] R. Schiavilla and R. B. Wiringa, Phys. Rev. C 65, 054302 (2002), URL http://link.aps.org/doi/10.1103/PhysRevC.65.054302.
[5] J. Fujita and H. Miyazawa, Progress of Theoretical Physics 17, 360 (1957), URL http://ptp.ips.jp/link?PTP/17/360/
[6] U. van Kolck, Phys. Rev. C 49, 2932 (1994), URL http://link.aps.org/doi/10.1103/PhysRevC.49.2932.
[7] E. Epelbaum, H.-W. Hammer, and U.-G. Meißner, Rev. Mod. Phys. 81, 1773 (2009), URL http://link.aps.org/doi/10.1103/RevModPhys.81.1773.
[8] D. Gazit, S. Quaglioni, and F. Navrátíl, Phys. Rev. Lett. 103, 102502 (2009), URL http://link.aps.org/doi/10.1103/PhysRevLett.103.102502.
[9] H. Grollhammer, J. McGovern, D. Phillips, and G. Feldman, Progress in Particle and Nuclear Physics 67, 841 (2012), URL http://dx.doi.org/10.1016/j.ppnp.2012.04.005.
[10] S. Pastore, S. C. Pieper, R. Schiavilla, and R. B. Wiringa, Phys. Rev. C 87, 035503 (2013), URL http://link.aps.org/doi/10.1103/PhysRevC.87.035503.
[11] L. E. Marcucci, A. Kievesky, S. Rosati, R. Schiavilla, and M. Viviani, Phys. Rev. Lett. 108, 052502 (2012), URL http://link.aps.org/doi/10.1103/PhysRevLett.108.052502.
[12] L. E. Marcucci, R. Schiavilla, and M. Viviani, Phys. Rev. Lett. 110, 192503 (2013), URL http://link.aps.org/doi/10.1103/PhysRevLett.110.192503.
[13] A. Lovato, S. Gandolfi, J. Carlson, S. C. Pieper, and R. Schiavilla, Phys. Rev. Lett. 112, 182502 (2014), URL http://link.aps.org/doi/10.1103/PhysRevLett.112.182502.
[14] R. G. T. Zegers, T. Adachi, H. Akimune, S. M. Austin, A. M. van den Berg, B. A. Brown, Y. Fujita, M. Fujiwara, S. Galès, C. J. Guess, et al., Phys. Rev. Lett. 99, 202501 (2007), URL http://link.aps.org/doi/10.1103/PhysRevLett.99.202501.
[15] Y. Fujita, H. Fujita, T. Adachi, C. L. Bai, A. Algora, G. P. A. Berg, P. von Brentano, G. Colò, M. Csátlós, J. M. Deaven, et al., Phys. Rev. Lett. 112, 112502 (2014), URL http://link.aps.org/doi/10.1103/PhysRevLett.112.112502.
[16] K. Langanke, G. Martínez-Pinedo, J. M. Sampaio, D. J. Dean, W. R. Hix, O. E. B. Messer, A. Mezzacappa, M. Liebendörfer, H.-T. Janka, and M. Rampp, Phys. Rev. Lett. 90, 241102 (2003), URL http://link.aps.org/doi/10.1103/PhysRevLett.90.241102.
[17] H. Sakai, T. Wakasa, H. Okamura, T. Nonaka, T. Ohnishi, K. Yako, K. Sekiguchi, S. Fujita, Y. Satou, H. Otsu, et al., Nuclear Physics A 649, 251 (1999), ISSN 0375-9474, giant Resonances, URL http://www.sciencedirect.com/science/article/pii/S037594749900069X.
[18] K. Yako, H. Sakai, M. Greenfield, K. Hatanaka, M. Hatano, J. Kamiya, H. Kato, Y. Kitamura, Y. Maeda, C. Morris, et al., Physics Letters B 615, 193 (2005),
van den Berg, G. P. A. Berg, P. von Brentano, D. Frekers, D. De Frenne, H. Fujita, et al., Phys. Rev. Lett. 97, 062502 (2006), URL http://link.aps.org/doi/10.1103/PhysRevLett.97.062502