Double $J/\psi$ Production at Photon Colliders

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Abstract

The double $J/\psi$(DJ) production in direct photon-photon collision is investigated. It is found that the $J/\psi$ production rate in this process is of the same order of magnitude as those of previously discussed ones, which hints the dominant $J/\psi$ inclusive production process in the direct photon-photon collision is still not being touched. As DJ production process has a clear distinction from the single $J/\psi$ inclusive processes in the final states, experimental study on it would be helpful to clarify the validity of color-singlet model.

PACS Number(s)12.38.Bx, 13.65.-i, 14.40.Lb

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The quarkonium production and decays have long been taken as an ideal means in investigating the nature of QCD and other new phenomena. Hence, to establish a proper theory which can precisely describe the heavy quarkonium production and decays is very necessary. The novel effective theory, the non-relativistic QCD(NRQCD) [1], is possibly the one to this aim, which is formulated from the first principles. To make a precise prediction for quarkonium production, however, at now, with only the NRQCD is not enough since the magnitude of the non-perturbative parameters in the theory are still unknown. Or in other words, results are always sensitive to the values of the input non-perturbative parameters, whereas the NRQCD can at most give the relative weights of these parameters in order of $v^2$ based on the ”velocity scaling rules”. The present situation in the Onium physics is that on one side the color-octet model [2] stands as the most plausible approach, till now, in explaining the large transverse momentum $\psi(\psi')$ production ”anomaly”; on the other hand, the model meets some difficulties in confronting with other phenomena [3]. Therefore, to what degree the color-octet mechanism plays the role in quarkonium production is still not clear and interesting. Either to find distinctive signatures of Color-octet or more strong arguments against it is currently an urgent task in this research realm. To this end lots of works have been done, however, unfortunately up to now there is still no definite answer.

In recent years several new concepts on linear colliders aiming at providing collisions at the center-of-mass energy from hundreds GeV to multi-TeV with high luminosity are proposed and the feasibilities are pre-tested, such as JLC at KEK, TESLA at DESY and CLIC at CERN, etc. Theoretically, high-intensity photon beams may be obtained by the Compton back-scattering of laser light off the linac electron beams and realize the photon-photon collision with approximately the same luminosity as that of the $e^+e^-$ beams. Such a photon linear collider can have high energy up to the TeV order. Since the first discovery of $J/\psi$ in 1974, researches on quarkonium production at $e^+e^-$ colliders at various energies have been carried out in details [4–8]. However, studies on the photon-photon scattering are very limited and just begin [9–11]. One of the difficulties one may meet in doing a complete calculation for two photon process beyond LO in $\alpha$, the electromagnetic coupling constant, or $\alpha_s$, the strong coupling constant, is how to simplify the lengthy expressions in order to present them analytically. In this brief report we study the double quarkonium, the $J/\psi$, production in direct photon-photon collision(of the resolved case see, e.g., ref. [8]) at the order $\alpha^2\alpha_s^2$. From physical point of view, in this process one may get rid of the inaccuracy induced by the uncertainties of the color-octet matrix elements and by the contributions from higher excited states as well, because they are much suppressed relative to the direct contribution in color-singlet.

In $\gamma\gamma$ scattering, at leading order in $\alpha$ the $J/\psi$ is produced via the process
\[ \gamma + \gamma \rightarrow J/\psi + \gamma. \] (1)

However, since at the scale of heavy quark mass, the strong coupling constant is not too small, the process
\[ \gamma + \gamma \rightarrow J/\psi^{(8)} + g \] (2)

may compete with the pure electromagnetic process (1) through the Color-Octet mechanism \[9\]. Here, we schematically use the \( J/\psi^{(8)} \) to denote those evolved from the Color-Octet states.

To go one order up in \( \alpha_s \), one may still expect to get the same order of magnitude in \( J/\psi \) production rate, because in this case the \( J/\psi \) may be produced in color-singlet and therefore will get compensation for the \( \alpha_s \) suppression from the non-perturbative sector relative to the octet process (2). What we proceed here, the DJ production in \( \gamma\gamma \) collision, is a sub-category of the inclusive \( J/\psi \) production process at order \( \alpha_s^2\alpha_s^2 \). That is
\[ \gamma(k_1) + \gamma(k_2) \rightarrow J/\psi(P) + J/\psi(P'), \] (3)

as shown in Figure 1, which consists of twenty Feynman diagrams. Here, the momenta of the related particles are shown in the parentheses explicitly. Similar as the situation of \( B_c \) production in photon-photon collision \[12\], these twenty diagrams involved here can also be classified as two sub-groups, (A) and (B), and each of them are gauge invariant.

The differential cross-section of process (3) is obtained as
\[
\frac{d\sigma}{dt} = \frac{262144\alpha_s^2\alpha_s^2\pi|R(0)|^4}{729m^2s^6(m^2 - t)^4(m^2 - u)^4} \times \\
[296m^{20} - 1112m^{18}(t + u) + t^4u^4(t + u)^2 - 8m^2t^3u^3(t + u)^3 \\
+ m^{16}(1917t^2 + 3578tu + 1917u^2) - 8m^{14}(242t^3 + 657t^2u + 657tu^2 + 242u^3) \\
+ 2m^{12}(623t^4 + 2204t^3u + 3298t^2u^2 + 2204tu^3 + 623u^4) \\
- 4m^{10}(131t^5 + 563t^4u + 1156t^3u^2 + 1156t^2u^3 + 563tu^4 + 131u^5) \\
+ m^8(143t^6 + 690t^5u + 1899t^4u^2 + 2640t^3u^3 + 1899t^2u^4 + 690tu^5 + 143u^6) \\
- 4m^6(6t^7 + 29t^6u + 107t^5u^2 + 212t^4u^3 + 212t^3u^4 + 107t^2u^5 + 29tu^6 + 6u^7) \\
+ 2m^4(t^8 + 4t^7u + 21t^6u^2 + 68t^5u^3 + 104t^4u^4 + 68t^3u^5 + 21t^2u^6 + 4tu^7 + u^8)].
\] (4)

In obtaining the above analytical expression, we start from the general Feynman rules and project the Charm-anti-Charm pair into the S-wave vector Onium state in color-singlet. To manipulate the trace and matrix element square of those twenty diagrams, the computer algebra system MATHEMATICA is used with the help of the package FEYNCALC \[13\].
The total cross section of the direct photon-photon collision can be obtained by convoluting the differential cross-section with the photon distribution functions, like

\[ \sigma_{\text{total}} = \int dtdx_1 dx_2 f_\gamma(x_1) f_\gamma(x_2) \frac{d\sigma_{\gamma\gamma}}{dt}(x_1, x_2), \] (5)

where \( \frac{d\sigma_{\gamma\gamma}}{dt}(x_1, x_2) \) is the differential cross-section given in (4); the \( f_\gamma(x_i) \), \( i = 1, 2 \), is the photon distribution with the fraction \( x_i \) of the beam energy,

\[ f_\gamma(x) = \frac{1}{N} \left[ 1 - x + \frac{1}{1-x} - 4r(1-r) \right]. \] (6)

Here, \( r \equiv x/(x_m(1-x)) \), the normalization

\[ N = (1 - \frac{4}{x_m} - \frac{8}{x_m^2}) \log(1 + x_m) + \frac{1}{2} + \frac{8}{x_m} - \frac{1}{2(1 + m_x)^2} \] (7)

and

\[ x_m = \frac{4E_bE_l}{m_c^2 \cos^2 \theta} \] (8)

where \( E_b \) and \( E_l \) are the energies of electron beam and laser photon, respectively, and the \( \theta \) is the angle between them. The energy fraction \( x \) of the photon is restricted in

\[ 0 \leq x \leq \frac{x_m}{1 + x_m}, \] (9)

and the maximum energy of the obtained photon beam depends on \( x_m \), i.e.,

\[ \omega_{\text{max}} = \frac{x_m}{1 + x_m} E_b. \] (10)

In our numerical calculation we take the optimum value of \( x_m = 4.83 \) in avoiding the background \( e^+e^- \) pair production from the laser and backscattered photon collision [14].

Other values of input parameters used in our numerical calculation are:

\[ m_c = 1.5 \text{ GeV}, \ \alpha = 1/128, \ \alpha_s(m_c) = 0.3, \ |R(0)|^2 = 0.8 \text{ GeV}^3, \ <\mathcal{O}^{J/\psi}_{8}\ > = 0.01\text{GeV}^3. \] (11)

With the above formulas and input parameters one can immediately get the total cross-sections. For instance, the \( J/\psi \) production cross-sections of the processes (1), (2) and (3) are 2.54 fb, 1.92 fb and 2.15 fb, respectively with colliding energy being of 500 GeV. Since the projected linear colliders with luminosity of hundreds fb\(^{-1}\) per year, for example the TESLA, and the integrated total cross-sections increase with the colliding energy decreasing as shown
in Figure 2, we may have hundreds of events being observed in one year at colliding energy 500 GeV or less. However, when the colliding energy goes up to 1TeV the cross-sections for processes (1) – (3) would be three orders less than those of them at 500 GeV, respectively, hence, all of them would be unobservable. In Figure 2 the energy dependences of the total cross-sections are shown from about the energy of LEP II to the next generation of linear collider energy. It can be seen that the color-octet process is the smallest one all over this energy scope in the concerned three processes, though they are in the same order.

Another thing we want to point out here is that the DJ production discussed in this work can not be simply explained by the quark fragmentation. With more details: we have the differential cross-section of $\gamma \gamma \rightarrow Q\bar{Q}$ as

$$\frac{d\sigma}{dt} = \frac{-2\pi\alpha^2 e_Q^4}{s^2(m_Q^2 - t)(m_Q^2 - u)^2} \left[ 6m_Q^8 - tu(t^2 + u^2) - m_Q^4(3t^2 + 14tu + 3u^2) 
+ m_Q^2(t^3 + 7t^2u + 7tu^2 + u^3) \right],$$

and know the probability of Charm quark evolving to $J/\psi$ is of $1.6 \times 10^{-4}$ via fragmentation [15]. To a certain degree of approximation, one can get the DJ production rate through fragmentation mechanism by simply combining the Charm production rate with the universal fragmentation probability. Therefore, we have a cross-section of $1.1 \times 10^{-3}$fb at energy 500 GeV, which is much smaller than that of the complete calculation of the DJ process. The reason of this is that the fragmentation mechanism only applicable when the fragmenting quark possesses large momentum and small off-shellness. In the double Onium production this condition can not be fulfilled for both fragmenting quark at the same time. On the other hand, one can know from the above discussion that the cross-section for single $J/\psi$ production via fragmentation mechanism would be about one order larger than those of the processes (1) – (3) at 500 GeV. Furthermore, we get know that the process at order $\alpha^2 \alpha_s^2$ for single $J/\psi$ inclusive production should be the dominant one in photon-photon collision, which seems to be overlooked.

Considering there are remaining disagreements among three independent calculations on the leading order processes ([1] and [2], [3], [4]), we have done a check over them and find an agreement with refs. [2] and [4] on the expression for octet process, i.e., the eq.(7) of ref. [4]. And our replacement for obtaining the singlet process agrees with that in ref [3].

In conclusion, we have calculated the DJ production at photon colliders. It is found that at moderate energy of the next generation linear colliders there would be hundreds of events to be detected per year with the high projected luminosity. In addition, since the production rates of $J/\psi$ via the color-octet mechanism, electromagnetic process, and what discussed here are almost the same in the full scope of the colliding energy, to differentiate
the color-octet mechanism from the color-singlet one in these processes, experimentally one should detect not only the \( J/\psi \) but also other final states. Relatively the processes (1) and (3) are more liable to be discerned than (2), because the signal of the latter is easily to be entangled with that of the single \( J/\psi \) production process at the order \( \alpha^2 \alpha_s^2 \) \cite{16}.

Like in the case of double \( \pi \) production at photon colliders, where the Pion wavefunction and the photon-to-pion transition form factor may be detected \cite{17}, the DJ process may provide more information on the color-singlet description of quarkonium production considering that it gets less influences from the color-octet uncertainties and the higher excited states feeddown.

In the end, with the expression (4) similar process in the soft bremsstrahlung photon collision can be easily discussed with the Weizäcker-Williams approximation.

**ACKNOWLEDGEMENTS**

This work is supported by the Grant-in-Aid aid of JSPS committee and the author would like to express his gratitude to NSF of China for support, ITP for hospitality, and J.P. Ma for kind invitation for the visit, while this work initiated, and also to Stanley J. Brodsky for helpful comments and discussions.
REFERENCES

[1] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D51 (1995) 1125.

[2] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D46 (1992) R3703; E. Braaten and S. Fleming, Phys. Rev. Lett. 74 (1995) 3327.

[3] For recent review see: I. Rothstein, hep-ph/9911276.

[4] J.H. Kühn and H. Schneider, Phys. Rev. D24 (1981) 2996; Z. Phys. C11 (1981) 263; W.Y. Keung, Phys. Rev. D23 (1981) 2072.

[5] Feng Yuan, Cong-Feng Qiao, Kuang-Ta Chao, Phys. Rev. D56 (1997) 321; Phys. Rev. D56 (1997) 1663.

[6] E. Braaten and Y.-Q. Chen, Phys. Rev. Lett. 76 (1996) 730; Chao-Hsi Chang, Cong-Feng Qiao, Jian-Xiong Wang, Phys. Rev. D56 (1997) R1363; Phys. Rev. D57 (1998) 4035.

[7] G.A. Schuler and M. Vänttinen, Phys. Rev. D58 (1998) 017502; C. Glenn Boyd, Adam K. Leibovich, and I.Z. Rothstein, Phys. Rev. D59 (1999) 054016.

[8] R.M. Godbole, D. Indumathi, and M. Krämer, hep-ph/0101333.

[9] J.P. Ma, B.H.J. McKellar and C.B. Paranavitabe, Phys. Rev. D57 (1998) 606.

[10] G. Japaridze and A. Tkabladze, Phys. Lett. B433 (1998) 139.

[11] M. Klasen, B.A. Kniehl, L. Mihaila, and M. Steinhauser, hep-ph/0104044.

[12] K. Kolodziej, A. Leike, and R. Rückl, Nucl. Phys. B348 (1995) 219.

[13] R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345.

[14] V.I. Telnov, Nucl. Instrum. Methods in Phys. Res. A355 (1995) 3.

[15] E. Braaten, K. Cheung, and T.C. Yuan, Phys. Rev. D48 (1993) 4230.

[16] C.F. Qiao, in progress.

[17] See: Stanley J. Brodsky, hep-ph/0102051, and references therein.
FIG. 1. The Feynman diagrams of the double $J/\psi$ production in $\gamma\gamma$ collision.

FIG. 2. The energy dependence of the cross-sections. DJ: the double $J/\psi$ process; CS: the color-singlet process (1); CO: the color-octet process (2).