A new method for electromagnetic parameters measurement with air coaxial line in low frequency band

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Abstract  A new measurement method is proposed in this letter to improve the accuracy of electromagnetic parameters measurement with air coaxial line, especially in low frequency band. The new method measures reflection parameters of the air coaxial line with terminal short-circuited and open-circuited, then the impedances from the specimen section to terminal are calculated. Combined with the impedance equations established based on the transmission line theory, the complex permittivity and complex permeability of the specimen are derived. Compared with the classic Nicolson-Ross-Weir (NRW) method, the method proposed achieves better accuracy in low frequencies, which is proved in this letter through comparative measurements.

key words: complex permittivity, complex permeability, air coaxial line, Nicolson-Ross-Weir (NRW) method, low frequencies

Classification: Electromagnetic theory

1. Introduction

Electromagnetic parameters of materials mainly include complex permittivity and complex permeability [1, 2, 3]. In the fields of electromagnetic compatibility and microwave, it is very important to obtain the electromagnetic parameters of materials [4, 5]. Frequency domain methods are generally used in electromagnetic parameters measurement, which can be roughly divided into three categories according to the measurement principles: lumped parameter method, network parameter method and resonant cavity method [6, 7, 8, 9, 10]. Among them, the network parameter method based on air coaxial line is widely used which has advantages of simple operation and wide bandwidth [11, 12, 13]. The measurement setup with air coaxial line is shown in Fig. 1. Based on the S parameters measured by the vector network analyzer (VNA), combined with the corresponding inversion algorithms, the electromagnetic parameters of the material can be obtained [6]. NRW method is one of the most classical and commonly used algorithms in that it can calculate both the complex permittivity and complex permeability with the transmission coefficient and reflection coefficient [14, 15, 16].

![Fig. 1. Electromagnetic parameters measurement setup with air coaxial line.](image)

However, there are some inherent problems with the NRW method [17, 18]. In general, the most appropriate specimen length for air coaxial line is a quarter wavelength [19], but it is difficult to achieve in the low frequency range, and the measurement with a shorter specimen may achieve results with large errors. The reasons mainly include following aspects:

First, the VNA has a certain error in the measurement of S parameters [20, 21]. Take the Keysight E5061B VNA as an example, set the IF bandwidth to 10 Hz, and take the calibration kit 85032F (Type-N, 50 Ω) for full two-port calibration. The measurement uncertainty of S parameters can be seen in Pages 4 to 11 of [22]: for S₂₁, the smaller the transmission coefficient, the greater error of amplitude and phase; for S₁₁, the larger the reflection coefficient, the smaller phase error while the amplitude error gets greater slightly. This problem is more obvious at low frequencies.

Algorithm is another reason [23]. The standard [24] gives Eqs. (1) and (2) based on the NRW method to calculate the complex permittivity and complex permeability:

\[
\mu_r = \frac{2\pi}{\Lambda \sqrt{\gamma_0^2 - \gamma_c^2}} \frac{1 + \Gamma}{1 - \Gamma} \quad (1)
\]

\[
\varepsilon_r = \frac{2\pi}{\mu_r \gamma_0^2} \frac{4\pi^2}{\Lambda^2} = \frac{\gamma_c^2}{\gamma_0^2} \quad (2)
\]

There are many symbols in Eqs. (1) and (2), where \(\gamma_0 = j2\pi f \sqrt{\varepsilon_0 \mu_0}\) is the propagation constant of the air section, \(\gamma_c = \gamma_0 \sqrt{\varepsilon_r \mu_r}\) is the propagation constant of the specimen section, \(\varepsilon_0 \) and \(\mu_0 \) are the relative permittivity and
relative permeability of air respectively, \(\varepsilon_r\) and \(\mu_r\) are the relative permittivity and relative permeability of the specimen respectively. \(\Gamma, X, T\) and \(\Lambda\) are expressions that \(\Gamma = X \pm \sqrt{X^2 - 1}, X = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}, T = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma}\) and \(\frac{1}{\Lambda} = -\left[\frac{1}{2\pi l} \ln(\Gamma)\right]^2\), where \(S_{11}\) and \(S_{21}\) are the reflection parameter and transmission parameter measured between port 1 and port 2 of the air coaxial line in Fig. 1 respectively, \(L\) is the length of the specimen.

It can be seen from Eqs. (1) and (2) that the NRW method takes both the transmission coefficient and reflection coefficient to calculate electromagnetic parameters. When the specimen exhibits high loss characteristics, the transmission coefficient will be small and the uncertainty will be large [22], which may lead to the calculation results with great errors [6, 14, 25].

This letter proposes a new measurement method based on air coaxial line. According to the characteristics of the measurement uncertainty of the \(S\) parameters of VNA, the calibration kits short and open are connected to the terminal of air coaxial line respectively to reduce the measurement errors. Then the input impedances of the test system can be derived from the measured reflection coefficients, and the complex permittivity and complex permeability of the specimen can be calculated. Compared with the NRW method, the method proposed in this letter only uses one port of VNA and achieve better accuracy in the low frequency band.

2. Method and principle

2.1 Measurement setup

To improve the measurement accuracy of the electromagnetic parameters in low frequency band with air coaxial line, it is necessary to minimize the measurement errors of the VNA. According to [22], the uncertainty of the \(S\) parameters measurement of the VNA is related to the signal-to-noise ratio of the port. When adopt the measurement setup in Fig. 1, both ports are almost matched, and the reflection coefficient will be small, which may cause a large deviation of the measurement results.

The measurement setup proposed in this letter is shown in Fig. 2, where plane 1 is the measurement reference plane of system after calibration, plane 2 is the left end of the specimen while plane 3 is the right end, plane 4 is the reference plane of calibration kit. The test process is very simple. First, connect the coaxial cable to port 1 of the VNA and calibrate the system with calibration kits, put the specimen into the air coaxial line, and connect the port 1 of the air coaxial line to the cable. The position of the specimen needs to be noted down. Then connect the calibration kits short and open to port 2 of the air coaxial line respectively, and record the reflection coefficient, both amplitude and phase. In the whole process, a torque wrench is needed, which is very important [26].

2.2 Calculation principle

In order to express clearly, the impedance and reflection coefficient are expressed in the form of \(X(a/b)\), where \(X\) represents impedance or reflection coefficient; \(a = s\) or \(o\), indicating that the port 2 of the air coaxial line is short-circuited or open-circuited; \(b = 1, 2, 3\), indicating the reference plane.

The equivalent circuit model of the measurement setup in Fig. 2 is shown in Fig. 3, where \(Z_0\) and \(Z_m\) are the characteristic impedance of the air coaxial line section and the specimen section respectively, \(l_1\) is the length from plane 1 to plane 2, \(d_1\) is the thickness of the specimen, and \(l_2\) is length from plane 3 to plane 4. The model of calibration kit short / open can be expressed by a coaxial line section (length \(d_2\), characteristic impedance \(Z_0\)) and inductance \(L\) / capacitance \(C\) [27]. In low frequency band, the effects of inductance or capacitance can be ignored where the terminal is considered to be an ideal short circuit or open circuit state, i.e., the load \(Z_L\) is equal to 0 and infinite respectively.

When the terminal of the air coaxial line is connected to the calibration kit short, the impedance from plane 3 to the terminal can be expressed as Eq. (3) according to [28].

\[
Z_{(s/o)} = \frac{Z_0 + Z_0 \tan \left[\gamma_0(l_2 + d_2)\right]}{Z_0 + Z_1 \tan \left[\gamma_0(l_2 + d_2)\right]} = Z_0 \tan \left[\gamma_0(l_2 + d_2)\right]
\]

(3)

when \(\gamma_0(l_2 + d_2)\) is small enough, Eq. (3) can be simplified to Eq. (4).

\[
Z_{(s/o)} \approx Z_0 \gamma_0(l_2 + d_2)
\]

(4)

The relationship between the impedance from plane 2 to the terminal and the impedance from plane 3 to the terminal
can be expressed as Eq. (5).

\[ Z_{(s/o)(2)} = \frac{Z_m Z_{(s/o)(3)} + Z_m \tanh(\gamma_c d_1)}{Z_m + Z_{(s/o)(3)} \tanh(\gamma_c d_1)} \]  

(5)

where \( Z_0 = Z_0 \sqrt{\varepsilon_r}. \)

By substituting Eq. (4) into Eq. (5), the impedance from plane 2 to the terminal under the state that the terminal is short-circuited can be obtained:

\[ Z_{(s)(2)} = Z_0 \frac{\gamma_0 (l_2 + d_2) + \gamma_0 d_1 \mu_r}{1 + \gamma_0^2 (l_2 + d_2)d_1 \varepsilon_r} \]  

(6)

Similarly, when the terminal is connected to the calibration kit open, the impedance from plane 3 to the terminal can be expressed as:

\[ Z_{(o)(3)} = \frac{Z_0 Z_t + Z_0 \tanh \left( \frac{\gamma_0 (l_2 + d_2)}{Z_0 + Z_t \tanh \left( \frac{\gamma_0 (l_2 + d_2)}{Z_0} \right)} \right)}{\tanh \left( \frac{\gamma_0 (l_2 + d_2)}{Z_0} \right)} \]  

(7)

when \( \gamma_0 (l_2 + d_2) \) is small enough, Eq. (7) can be simplified to Eq. (8).

\[ Z_{(o)(3)} \approx \frac{Z_0}{\gamma_0 (l_2 + d_2)} \]  

(8)

According to Eq. (5), the impedance from plane 2 to the terminal can be expressed as:

\[ Z_{(o)(2)} = \frac{Z_0 1 + \gamma_0^2 (l_2 + d_2)d_1 \mu_r}{\gamma_0 (l_2 + d_2) + \gamma_0 d_1 \varepsilon_r} \]  

(9)

There are 4 unknowns in Eqs. (6) and (9): \( Z_{(s)(2)}, Z_{(o)(2)}, \varepsilon_r \) and \( \mu_r \). To figure out \( \varepsilon_r \) and \( \mu_r \), it is necessary to know the value of \( Z_{(s)(2)} \) and \( Z_{(o)(2)} \). Assuming that the reflection coefficient measured by VNA is \( S_{11(s/o)(2)} \), impedance from plane 2 to the terminal can be obtained through Eq. (10) based on phase extension:

\[ Z_{(s/o)(2)} = Z_0 \frac{1 + S_{11(s/o)(2)} e^{2\gamma_0 d_1}}{1 - S_{11(s/o)(2)} e^{2\gamma_0 d_1}} \]  

(10)

Through the analysis and derivation above, the relative complex permittivity of the specimen can be obtained by combining Eqs. (6), (9) and (10):

\[ \varepsilon_r = \frac{Z_0 (\gamma_0 (l_2 + d_2))^2 + \gamma_0 (l_2 + d_2)(Z_{(o)(2)} - Z_{(s)(2)}) - Z_0}{Z_0 \gamma_0 (l_2 + d_2)(A - B)} \]  

(11)

where, \( A = \frac{Z_{(o)(2)} Z_0^2}{Z_0^2 (l_2 + d_2)^2} \) and \( B = \frac{Z_{(o)(2)} Z_0^2}{Z_0 (l_2 + d_2)^2} \).

The complex permeability can be obtained by substituting Eq. (11) into Eq. (6) or (9).

3. Experiment and analysis

In order to prove the effectiveness of the method proposed in this letter, the electromagnetic parameters of poly tetra fluoroethylene (PTFE) are tested by NRW method and the method proposed respectively in the frequency range of 10 MHz to 1 GHz with the air coaxial line. References [29, 30] point out that PTFE is a non-magnetic material with a relative permeability of 1 and the relative permittivity is \( 2.05 - j6.18 \times 10^{-6} @ 1.082 \, \text{GHz} @ 25 \, \text{°C} \) which changes with frequency and temperature. Actually, the relative permittivity, both the real and imaginary parts are a little bit smaller than the reference value above between 10 MHz and 1 GHz. The measurement setup is shown in Fig. 4 and the setting of VNA (Keysight E5061B) is as follows: frequency 10 MHz to 1 GHz, output power 0 dBm, IF bandwidth 700 Hz, 1601 sampling points and no averaging applied to data. R&S ZV - Z270 (Type N, 50 Ω) is adopted as the calibration kit. The size of the PTFE specimen meets the measurement requirements of the air coaxial line (outer diameter 7 mm, inner diameter 3.04 mm, length 10 mm). In order to facilitate the calculation and to make Eqs. (4) and (8) well established, the PTFE specimen is installed at port 2 of the air coaxial line, which means that \( l_2 = 0 \, \text{mm} \).

The calculation results of the NRW method and the method proposed are shown in Fig. 5. It can be seen that the results of two methods are basically consistent, which is also in accordance with the record in [29, 30]. However, it is easy to find that the calculation results of the method proposed in this letter are more stable in the range of 10 MHz to 1 GHz, especially from 10 MHz to 200 MHz. The fluctuation of the method proposed is significantly smaller than that calculated by the NRW method, and is also more close to the reference value, which is quite evident in the calculation of the imaginary parts.

However, the method proposed in this letter has frequency limits. At low frequency especially below 200 MHz, although the results are more stable than NRW method, the fluctuations are still inevitable. The reason is that in the low frequency band, both the length of the air coaxial line and the specimen are too short compared with wavelength, which leads to insufficient phase measurement accuracy of the VNA. At high frequency, since the measurement is carried out under the condition that the terminal of air coaxial line is short-circuited and open-circuited, resonance will occur.
Fig. 5. Calculation results of the NRW method and the method proposed in this letter.

when the length of the air coaxial line is equal to an integer multiple of half-wavelength [31], and it will result in incorrect calculation results. This is also the reason that the real parts of the relative permittivity and permeability in Fig. 5 show slight upward trends around 1 GHz. The upper frequency limit can be extended by using a shorter air coaxial line.

4. Conclusion

This letter presents a new method to measure and calculate the complex permittivity and complex permeability of materials with single port based on the air coaxial line. According to the uncertainty characteristics of \( S \) parameters measured by VNA, the calibration kits short and open are connected at the terminal of air coaxial line respectively to achieve results with smaller error. Then the impedance from specimen section to terminal is calculated according to the reflection coefficients measured, and the electromagnetic parameters of the materials are deduced by establishing a set of equations. The experiment results show that the method proposed has the advantages of less fluctuation and more accurate than traditional NRW method in low frequency band.

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