The full electroweak Standard Model prediction for \((g - 2)\) of the muon and improvements on the MSSM prediction

H. G. Fargnoli\textsuperscript{a}, C. Gnendiger\textsuperscript{b}, S. Paßehr\textsuperscript{c}, D. Stöckinger\textsuperscript{b}, H. Stöckinger-Kim\textsuperscript{b}

\textsuperscript{a} Universidade Federal de Lavras, Lavras, Brazil
\textsuperscript{b} Institut für Kern- und Teilchenphysik, TU Dresden, Dresden, Germany
\textsuperscript{c} Max-Planck Institut für Physik, München, Germany

Recent progress on the \((g - 2)_\mu\) prediction is presented. In the SM, the Higgs-boson mass dependent contributions have been evaluated exactly up to the two-loop level and consistently combined with leading three-loop effects. Thus, the currently most accurate value including a detailed error analysis for the SM electroweak contributions has been obtained. The SUSY two-loop corrections from fermion/sfermion-loop insertions have been computed; they are generally large and logarithmically enhanced for heavy squarks.

1. Introduction

The deviation between the measurement\textsuperscript{1} of the muon anomalous magnetic moment \(a_\mu = (g - 2)_\mu/2\) and the SM prediction\textsuperscript{2} amounts to more than 3\(\sigma\). It is of utmost interest to further scrutinize this longstanding deviation.

In these proceedings we discuss a recent reevaluation\textsuperscript{3} of the SM electroweak contributions. These are the only SM contributions with a dependence on the Higgs boson mass. Earlier evaluations had an irreducible theory uncertainty since the Higgs boson mass used to be unknown. After the Higgs boson mass has been measured at the LHC, we are now in the position to obtain the final value of the Higgs-dependent part of the SM prediction for \(a_\mu\) (see Sec.\textsuperscript{2}).

We also briefly comment on complementary progress\textsuperscript{4,5} for the prediction of \(a_\mu\) in the minimal supersymmetric standard model (MSSM) (see Sec.\textsuperscript{3}).

2. The electroweak contributions after the Higgs boson mass measurement

Before the Higgs boson mass measurement, the most precise evaluation\textsuperscript{6} of the electroweak contributions obtained the result \(a_\mu^{\text{EW}} = (154 \pm 1 \pm 2) \times 10^{-11}\), where the first error is due to electroweak hadronic uncertainties, but the second, larger uncertainty is due to the unknown Higgs boson mass.

In the update\textsuperscript{7} of this result we take into account the following aspects:

\* Contribution to the Proceedings for the International Workshop on \(e^+e^-\) collisions from Phi to Psi, Rome, September 2013.
• All Higgs-boson mass dependent contributions up to the two-loop level are exactly evaluated, using the value $M_H = 125.6 \text{ GeV}$, with a conservative error of $\pm 1.5 \text{ GeV}$. As further input parameters, the masses of muon, Z-boson and top quark, the muon decay constant $G_F$ and the fine-structure constant $\alpha$ in the Thomson limit, are chosen.
• The W-boson mass is then unambiguously predicted by the SM, and we use the appropriate theory value $M_W = 80.363 \pm 0.013 \text{ GeV}$.
• An analysis of the theory uncertainty, in particular due to the uncertainty of the SM input parameters, is carried out.
• The exact evaluation of the Higgs-dependent contributions is consistently combined with the remaining contributions up to the three-loop level. In order to avoid double counting, the choice of $\alpha$ in the two-loop contributions has to match the one in the evaluation of leading three-loop contributions. We choose the Thomson-limit definition for $\alpha$ at the two-loop level; see Eq. (7) below for the consequences.

The most important new results are the exact results of the Higgs-dependent two-loop contributions. Fig. 1 shows them for a range of Higgs boson masses. The left panel shows the bosonic contributions $a_{\mu;\text{bos}}^{\text{EW}(2)}$ (i.e. without fermion loop), the right panel the Higgs-dependent fermionic contributions $a_{\mu;\text{f-rest},H}^{\text{EW}(2)}$. For the input parameters given above, these contributions, and the one-loop contribution amount to

$$
a_{\mu}^{\text{EW}(1)} = (194.80 \pm 0.01) \times 10^{-11},
$$

$$
a_{\mu}^{\text{EW}(2);\text{bos}} = (-19.97 \pm 0.03) \times 10^{-11},
$$

$$
a_{\mu}^{\text{EW}(2);\text{f-rest},H} = (-1.50 \pm 0.01) \times 10^{-11}.
$$

The results are exact up to the parametric uncertainties due to the uncertainty of the input parameters $M_W$, $m_t$, and $M_H$. The dominant uncertainty arises in $a_{\mu;\text{bos}}^{\text{EW}(2)}$ due to the uncertainty of the Higgs boson mass. Nevertheless, the overall uncertainty of these contributions is of the order $10^{-13}$ and thus extremely tiny.

We briefly comment on the difference between these results and earlier results in the literature.

The first computation\cite{10} of $a_{\mu;\text{bos}}^{\text{EW}(2)}$ was a milestone but employed an approximation assuming $M_H \gg M_W$. Refs.\cite{11,12} provided the full $M_H$-dependence of $a_{\mu;\text{bos}}^{\text{EW}(2)}$, however only in (semi)numerical form and for a particular, different set of input parameters than the one used here.

The first computation of the Higgs-dependent fermionic contributions $a_{\mu;\text{f-rest},H}^{\text{EW}(2)}$ was carried out in Ref.\cite{13} in three limiting cases, $M_H \ll m_t$, $M_H = m_t$, $M_H \gg m_t$. Furthermore, the approximation $s_W^2 = 1/4$ was used and diagrams with $\gamma-Z$–Higgs subdiagram were neglected. The results of Fig. 1 (right) show that the approximation of Ref.\cite{13} for large $M_H$ is surprisingly poor, compared to the exact result. There, the higher-order terms in $m_t^2/M_H^2$ are important.
Electroweak Standard Model prediction for \((g-2)_\mu\) and improvements on the MSSM prediction

The remaining two-loop contributions beyond \(a_{\mu,\text{bos}}^{\text{EW}(2)}\) and \(a_{\mu,\text{f-rest},H}^{\text{EW}(2)}\) are non-Higgs dependent fermion-loop contributions: 
\(a_{\mu,\text{f-loop}}^{\text{EW}(2)}(f)\), the diagrams with \(\gamma\gamma Z\) interaction generated by a fermion \(f\)-loop; 
\(a_{\mu,\text{f-rest},\text{no }H}^{\text{EW}(2)}\), the remaining two-loop contributions from fermion loops. These have been evaluated in Ref. 13 after earlier work13,14 particularly on \(a_{\mu,\text{f-loop}}^{\text{EW}(2)}(f)\). The leading three-loop contributions enhanced by large logarithms have been evaluated in Refs. 15, 6.

The results for these remaining, non-Higgs dependent electroweak two-loop and leading three-loop contributions, are

\[
a_{\mu}^{\text{EW}(2)}(e, \mu, u, c, d, s) = -(6.91 \pm 0.20 \pm 0.30) \times 10^{-11},
\]

(4)

\[
a_{\mu}^{\text{EW}(2)}(\tau, t, b) = -(8.21 \pm 0.10) \times 10^{-11},
\]

(5)

\[
a_{\mu,\text{f-rest},\text{no }H}^{\text{EW}(2)} = -(4.64 \pm 0.10) \times 10^{-11},
\]

(6)

\[
a_{\mu}^{\text{EW}(\geq 3)} = (0 \pm 0.20) \times 10^{-11}.
\]

(7)

Eq. (7) is correct for the parametrization of the two-loop result in terms of \(\alpha\), while the result for the alternative parametrization in terms of \(G_F\) would have been\[a_{\mu}^{\text{EW}(\geq 3)} = (0.40 \pm 0.20) \times 10^{-11}\]. The dominant theory error arises in the first two generations in \(a_{\mu}^{\text{EW}(2)}(e, \mu, u, c, d, s)\). It has been estimated\[6] by varying the hadronic input parameters and by estimating higher-order QCD corrections.

The final result for the electroweak SM contributions to \(a_\mu\) is the sum of all presented contributions,

\[
a_{\mu}^{\text{EW}} = (153.6 \pm 1.0) \times 10^{-11}.
\]

(8)

The final theory error of these contributions dominated by the electroweak hadronic and three-loop uncertainties of Eqs. (4-7). It is enlarged to the conservative value \(\pm 1.0 \times 10^{-11}\) in line with Ref. 6. The parametric uncertainty due to the input parameters \(M_W\), \(m_t\), and particularly \(M_H\) is negligible. The precision of the result is by far sufficient for the next generation of \(a_\mu\) measurements. Clearly, the full Standard Model theory error remains dominated by the non-electroweak hadronic contributions.
3. Non-decoupling two-loop contributions in the MSSM

Supersymmetry (SUSY) is a promising explanation of the $3\sigma$ deviation in $a_\mu$, although simple SUSY scenarios where all SUSY particles are light are already ruled out by the LHC. Ref. [3] has defined several benchmark parameter points which illustrate that already the one-loop SUSY contributions to $a_\mu$ have an intricate parameter dependence, if non-trivial SUSY mass patterns are allowed: SUSY contributions in the ballpark of the current $3\sigma$ deviation can be obtained if, e.g., the Higgsino mass $\mu$ is much larger than the bino mass $M_1$ and the smuon masses, or if the wino mass $M_2$ and the left-handed smuon mass are much larger than $\mu$, $M_1$ and the right-handed smuon mass. Parameter scenarios with large $\mu$ have been studied including leading higher-order corrections also in Refs. [16] recently.

Recently, a class of contributions has been computed [4, 5] which can become particularly important for such split spectra: two-loop contributions where a fermion/sfermion loop is inserted into a SUSY one-loop diagram, see Fig. 2 (left).

The most prominent features of these two-loop contributions (plus the associated counterterm contributions) are that

(i) they contain the large universal quantities $\Delta\alpha$ and $\Delta\rho$ from fermion loops, which make the contributions generically sizeable;

(ii) they are logarithmically enhanced if the sfermion masses in the inner loop become large.

These points allow for a very compact approximation formula [5, 19]. Fig. 2 (right) illustrates the non-decoupling, logarithmic enhancement of the contributions for large sfermion masses. We restrict ourselves to various motivated combinations of sfermion soft SUSY breaking parameters $M_{Q_i}, M_{U_i}, M_{D_i}, M_{L_i}, M_{E_i}$ (where the first index denotes the supermultiplet, the second the generation): either of a common third generation sfermion mass $M_{U3,D3,Q3,E3,L3} \equiv M$, or of a universal first and
second generation squark mass $M_{U_1,D_1,Q_1,U_2,D_2,Q_2} \equiv M$, or, as an example with particularly large corrections, purely as a function of $M_{Q_3}$ with $M_{U_3}$ fixed to 1 TeV. Each time, the non-varied sfermion masses are kept at standard values, which are 7 TeV for the squark masses and 3 TeV for the third generation slepton masses. The selectron masses are set to the smuon masses, and the trilinear $A$ parameters are set to zero.

As the figure shows, these new fermion/sfermion-loop contributions can be the largest MSSM two-loop contributions to $a_\mu$ — already for moderate inner sfermion masses. For different sets of input parameters than shown in the figure, up to 10% corrections for small and up to 30% corrections for large sfermion masses can be found. Fig. 2 also shows the other known two-loop contributions: the SUSY corrections to SM-one-loop diagrams (“class 2L(a)”$^{11}$, the photonic corrections$^{17}$, and the $(\tan \beta)^2$-correction$^{18}$, which are in the range $(-7, \ldots, +7)\%$.

References

1. G.W. Bennett, et al., (Muon ($g - 2$) Collaboration), Phys. Rev. D 73, 072003 (2006).
2. F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1; J. P. Miller, E. d. Rafael, B. L. Roberts and D. Stöckinger, Ann. Rev. Nucl. Part. Sci. 62 (2012) 237.
3. C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, Phys. Rev. D 88 (2013) 053005.
4. H. Fargnoli, C. Gnendiger, S. Passehr, D. Stöckinger and H. Stöckinger-Kim, arXiv:1309.0980 [hep-ph].
5. H. Fargnoli, C. Gnendiger, S. Passehr, D. Stöckinger and H. Stöckinger-Kim, arXiv:1311.1775 [hep-ph].
6. A. Czarnecki, W. J. Marciano, A. Vainshtein Phys.Rev.D 67 (2003) 073006, Erratum-ibid.D73 (2006) 119901.
7. [ATLAS Collaboration], ATLAS-CONF-2013-014; [CMS Collaboration], CMS-PAS-HIG-13-005.
8. J. Beringer et al. (Particle Data Group) Phys. Rev. D 86 (2012) 010001.
9. M. Awramik, M. Czakon, A. Freitas and G. Weiglein Phys. Rev. D 69 (2004) 053006.
10. A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76 (1996) 3267.
11. S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B 690 (2004) 62; Nucl. Phys. B 699 (2004) 103.
12. T. Gribouk and A. Czarnecki, Phys. Rev. D 72 (2005) 053016.
13. A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. D 52 (1995) 2619.
14. S. Peris, M. Perrottet and E. De Rafael, Phys. Lett. B 355 (1995) 523; M. Knecht, S. Peris, M. Perrottet and E. De Rafael, JHEP 0211 (2002) 003.
15. G. Degrassi and G. F. Giudice, Phys. Rev. D 58 (1998) 053007.
16. M. Endo, K. Hamaguchi, S. Iwamoto and T. Yoshinaga, arXiv:1303.4256 [hep-ph]; M. Endo, K. Hamaguchi, T. Kitahara and T. Yoshinaga, arXiv:1309.3065 [hep-ph]; M. Endo, K. Hamaguchi, S. Iwamoto, T. Kitahara and T. Moroi, arXiv:1310.4496 [hep-ph].
17. P. von Weitershausen, M. Schäfer, H. Stöckinger-Kim and D. Stöckinger, Phys. Rev. D 81 (2010) 093004.
18. S. Marchetti, S. Mertens, U. Nierste and D. Stöckinger, Phys. Rev. D 79, 013010 (2009).
19. The Mathematica implementation of the approximation formula can be obtained from http://iktp.tu-dresden.de/?id=theory-software.