Shock propagation and attenuation in Green River oil shale

D E Grady
Associate and Principal Scientist, Applied Research Associates, Southwest Division, 4300 San Mateo Blvd NE, Albuquerque, NM 87110, USA
E-mail: dgrady@ara.com

Abstract. Shock waves produced by planar impact of thin plates onto samples of oil shale are monitored with time-resolved velocity interferometer diagnostics. Peak shock stresses are below the Hugoniot elastic limit. Stress wave measurements at successive sample thickness are analysed to determine the experimental shock energy attenuation with propagation distance. Shock attenuation is attributed to stress wave scattering at planes of oil shale kerogen within the shale matrix. Wave scattering from planar defects are evaluated from a shock physics perspective and a scattering model is constructed that sensibly reproduces the experimental observation of shock energy attenuation.

1. Introduction
The test material is oil shale from the U.S. state of Wyoming region of the Green River Formation. This specific oil shale has a kerogen content of about 40 gallons/ton. Kerogen is the organic component from which oil is extracted from the shale. Kerogen in the oil shale tested is not distributed uniformly but rather resides in thin and parallel planes separated by regions relatively lean in kerogen content. Matrix material consists of fine-grained carbonate (dolomitic) marlstone mineral matter. An image of the oil shale surface is illustrated in figure 1.

![Figure 1. An image of oil shale surface (approximately 1 cm²) illustrating laminar character kerogen planes. Stress wave propagation in tests is from left to right.](image-url)

2. Experimental method and data
In the experiments performed thin sheets of PMMA (plexiglas) are mounted on projectiles and undergo planar impact on samples of oil shale [1]. Planar stress wave pulses propagate through the oil shale samples in the direction of the normal to the planes of kerogen. Impact velocities of about 70 m/s resulted in impulsive pressure loading of oil shale samples with a nearly square pressure pulse to approximately 150 MPa stress amplitude and 1.4 µs time duration.
Hugoniot data for Green River oil shale are provided by Carter [2]. Shock profile experiments on oil shale indicated a Hugoniot elastic limit and measure of the dynamic strength of slightly in excess of 200 MPa [3]. Accordingly, impact at the somewhat reduced shock pressure amplitude is expected to be nominally elastic in stress wave response. The velocity histories shown in figure 2 are measured using the velocity interferometer method within PMMA window material backing increasing thicknesses of the oil shale test samples. Sample thicknesses and corresponding wave pulse propagation distances are indicated on the left in the figure.

Figure 2. Measured particle velocity histories resulting from stress pulse propagation through increasing thicknesses of 40 gallons/ton oil shale [1]. Stress pulse is produced by planar impact of a 2 mm thickness PMMA plate at approximately 70 m/s velocity. Particle velocity history is measured by interferometry methods within a PMMA window material directly behind the oil shale sample. Peak input pressure in the oil shale is approximately 150 MPa. The same velocity profiles overlay on the right hand side to illustrate change in wave structure with propagation distance.

Present analysis of the stress-wave data is focused on the energy of the shock pulse. A reasonable measure of wave energy is provided by the work applied by the pulse at the recording plane; namely. \( \int \sigma u dt = \int \sigma^2 / z \, dt \), where \( z = \rho c \), is the wave impedance of the oil shale with \( \rho = 2.15 \, \text{g/cc} \) and \( c = 3.0 \, \text{km/s} \). Input shock pulse energy provided by the impacting plate is 5.0 kJ/m². Subsequent energy of the wave pulse at increasing propagation distance is calculated from the integrated time history of the measured pulse accounting for the impedance mismatch between the oil shale sample and the PMMA window material. Wave energy as a function of propagation distance for the four tests is plotted in figure 3.

3. Mechanism for shock attenuation
Several mechanisms are candidates for dissipation in the shock propagation through the oil shale that leads to energy loss with propagation distance. Such mechanisms include modest plasticity, viscosity, or elastic scattering in the heterogeneous media. Here the latter method of elastic scattering is assumed for the dissipation and energy attenuation. The thin planes of organic kerogen have lower wave impedance than the predominately carbonate mineral matrix. This wave reflection (scattering) occurs on passage of the shock wave through the kerogen planes and is assumed the principal contribution to energy loss from the shock pulse.
4. Scattering at a planar defect

Accordingly consider wave interaction at a planar defect. The problem of interest is abstracted in the figure 4. A planar step shock wave of amplitude $\sigma_o$, propagates in a medium with shock impedance $z_1$ and is normal incident on a thin plane of material with thickness $a$ and shock impedance $z_2$. The impedance $z_2 < z_1$ and both material are considered linear elastic. A reduced amplitude stress wave is transmitted into the lower impedance medium. The stress wave echoes between the two interfaces as successively higher stress levels are achieved, ultimately equilibrating to the incident stress level.

Analysis of the stress wave reverberation process leads to the following relation for the stress steps as is illustrated by the pressure versus particle velocity plot on the right in the figure 5. The stress amplitude of the $n$th step is provided by,

$$\sigma_n = (1 - (-R)^n)\sigma_0, \quad (1)$$
where the acoustic reflection coefficient is provided by \( R = \frac{z_2 - z_i}{z_2 + z_i} \). A continuous approximation in time to the stress amplitude is provided by \( n = ct / a \) with \( a \) the thickness and \( c \) the acoustic speed of the defect medium,
\[
(R)_{\text{eff}} = (-R)^{1/a} = e^{(c/a)\ln(-R)},
\]
yielding the corresponding continuous stress history,
\[
\sigma(t) = \left(1 - e^{-t/\tau}\right)\sigma_o,
\]
with \( \tau = a / c \ln(-R) \).

The reflected or back scattered energy resulting from passage of the step stress wave through the defect follows from the energy integral using the stress history provided by equation (3),
\[
E_r = \int (1 - \sigma) \sigma_0 dt = \frac{1}{2z_i} \sigma_0^2 \tau.
\]

![Wave interactions within the defect medium are shown in the stress versus particle velocity diagram on the right. Stress history of the midpoint of the defect region is shown on the left. An impedance ration \( z_1 / z_2 \) of five is used for the calculation.](image)

**Figure 5.** Wave interactions within the defect medium are shown in the stress versus particle velocity diagram on the right. Stress history of the midpoint of the defect region is shown on the left. An impedance ration \( z_1 / z_2 \) of five is used for the calculation.

### 5. Model for shock attenuation

The objective is to assess the energy removed from a stress pulse as it propagates through a random collection of parallel and planar defects. Consider a square pulse with stress amplitude \( \sigma \) and pulse duration \( T \) incident on one defect. The pulse duration is considered to be large compared to the characteristic defect equilibration time \( \tau \) identified above. As the front and back of the square pulse passes over the defect, energy from the pulse is lost through reflection from the planar defect. Twice the energy provided by equation (4) is removed from the pulse on passage of each defect.

If many such planar defects are distributed through the body with an average spacing \( b \) then the energy loss from the pulse over a propagation distance \( dh \) is,
\[
dE = -2E_r dN = -2 \left( \frac{1}{2z_i} \sigma_0^2 \tau \right) \left( \frac{dh}{b} \right).
\]

The energy and impulse associated with a square pulse of amplitude \( \sigma \) and pulse duration \( T \) passing over a Lagrangian position \( h \) is provided by,
Particle velocity following passage of the stress pulse is zero (or nearly zero) and impulse of the stress pulse is conserved (or nearly conserved) with \( I(0) = I_0 \). Joining equations (5), (6) and (7) yields a differential equation for the rate of energy loss from the stress pulse from scattering as a function of propagation distance,

\[
\frac{dE}{dh} = -\frac{z_1 \tau}{b \sigma_o} E^2. 
\]

The differential equation is solved yielding,

\[
E(h) = \frac{E_0}{1 + h/h_o},
\]

where,

\[
h_o = \frac{L_o}{\sigma_o \tau} = \frac{T_o - b}{\tau},
\]

while \( E_0, I_0, \sigma_o \) and \( T_o \) are input properties of the pulse.

Comparison of equation (9) with the experimental data in figure 3 reveals reasonable agreement. The sensible agreement of theory and experiment, and the intuitively reasonable argument that planes of low impedance organic media in otherwise higher impedance rock media should lead to stress wave scattering, supports scattering as a mechanism for stress wave attenuation. A fit to the experimental data in figure 3 yields \( h_o = 40 \text{ mm} \). Impedance differences of different kerogen content oil shale (principally density difference) suggest impedance difference between defect regions and matrix material of about a factor of two. Thus, the logarithmic term in the characteristic time \( \tau = -a/c \ln(-R) \) is close to unity. With \( T_o \) approximately 1.5 \( \mu \text{s} \) and \( c \) approximately 3 km/s, equation (10) leads to a defect spacing to defect thickness ratio \( b/a \) of about ten. The one available image of the test oil shale shown in figure 1 reveals considerable variation in spacing of kerogen rich regions. Nonetheless, the ratio required by the attenuation data is not unreasonable. Whether scattering is the only, or even the dominant, mechanism governing stress-wave energy dissipation is, however, open to argument.

6. Viscous attenuation

Stress-wave viscosity might alternatively explain the measured experimental energy attenuation with propagation distance. Of interest is the form of the viscous constitutive relation necessary to describe the measured experimental attenuation. Consider a viscous constitutive relation of the form,

\[
\sigma(\varepsilon, \dot{\varepsilon}) = E \varepsilon + \eta \dot{\varepsilon}^m.
\]

Again assume an input pulse of amplitude \( \sigma \) and duration \( T \), resulting in a differential equation comparable to equation (8) that when solved provides the solution,

\[
E(h) = \frac{E_0}{(1 + h/h_o)^{1/2m}}.
\]

A viscous exponent of \( m = 1/2 \) in equation (12) is required to reproduce the dependence of attenuation on propagation distance predicted by the scattering model in equation (9) and suggested by
the experimental data. This result is curious. A viscosity coefficient $\eta$ that decreases as the square root of the viscosity has found application in describing other aspects of shock physics such as structuring of steady shock waves [4]. Wave scattering has been suggested as a contributor to viscosity responsible for the spatial thickening and structuring of large amplitude shock waves [5].

7. Summary

Experimental stress wave profiles with the ultimate purpose of predicting wave propagation, fracture and fragmentation in oil shale [6] are obtained that illustrate attenuation and dispersion of stress pulse waves with propagation distance. Planar impact loading induces stress waves with peak stress levels that are below the Hugoniot elastic limit of oil shale. Analysis of the data reveals that energy and peak stress in the wave will asymptote to attenuation proportional to the inverse first power of the propagation distance. Stress wave scattering from thin planes of kerogen in otherwise silicate-carbonate rock material is assumed responsible for the measured wave dissipation and attenuation with propagation distance. A continuum model of shock pulse propagation is developed that is based on shock-physics-based scattering from planar defects. The model sensibly reproduces the experimental attenuation of wave energy with propagation distance. Similar analysis assuming viscous dissipation of the oil shale medium requires a viscosity coefficient that decreases as the inverse square root of the strain rate to provide comparable agreement with the experimental data.

References

[1] Grady D E 1983 Stress Wave Studies on Selected Crustal Rocks and Minerals Sandia National Laboratory Tech. Rep. SAND83-1898
[2] Carter W J 1976 Explosively Produced Fracture of Oil Shale Los Alamos National Laboratory Tech. Rep. LA-06594-PR
[3] Munson D 1977 Analysis of High Pressure Oil Shale Hugoniots Sandia National Laboratory Tech. Rep. SAND77-0118
[4] Swegle J W and Grady D E 1985 J. Appl. Phys. 58 692
[5] Grady D E 2010 J. Appl. Phys. 107 013506
[6] Grady D E and Kipp M E 1979 Proc. 20th U.S. Symp. on Rock Mechanics (Austin TX) p 403