Inflow length and tripping effects in turbulent boundary layers

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Abstract. A recent assessment of available direct numerical simulation (DNS) data from turbulent boundary layer flows [Schlatter & Örlü, J. Fluid Mech. 659, 116 (2010)] showed surprisingly large differences not only in the skin friction coefficient or shape factor, but also in their predictions of mean and fluctuation profiles far into the sublayer. For the present paper the DNS of a zero pressure-gradient turbulent boundary layer flow by Schlatter et al. [Phys. Fluids 21, 051702 (2009)] serving as the baseline simulation, was re-simulated, however with physically different inflow conditions and tripping effects. The downstream evolution of integral and global quantities as well as mean and fluctuation profiles are presented and results indicate that different inflow conditions and tripping effects explain most of the differences observed when comparing available DNS. It is also found, that if transition is initiated at a low enough Reynolds number (based on the momentum-loss thickness) \( Re_\theta < 300 \), all data agree well for both inner and outer layer for \( Re_\theta > 2000 \); a result that gives a lower limit for meaningful comparisons between numerical and/or wind tunnel experiments.

1. Introduction and Motivation

Since the first resolved direct numerical simulation (DNS) of turbulent channel flows became available (Kim et al., 1987) more than two decades ago, a large number of DNSs have been performed for various flow cases, and steadily increased the Reynolds number (Re) extent, finally reaching those ranges put forward by well-resolved experiments. Hence, it was not further surprising that a detailed comparison between DNS and experiments in a turbulent channel flow showed good agreement for the first and second order moment as well as spectral distributions of the streamwise velocity component (Monty & Chong, 2009).

Although zero pressure-gradient (ZPG) turbulent boundary layers (TBLs) have been simulated from Spalart (1988) on, DNS of truly spatially developing TBLs have only recently become feasible reaching up to \( Re \) comparable to those of DNS from high Reynolds number channel flows. Furthermore, a recent assessment of published DNS data from ZPG TBL flows has revealed that there exists considerable scatter even in basic integral quantities, e.g. up to 5 and 20% in the shape factor \( H_{12} \) or skin friction coefficient \( c_f \), respectively, as well as first and second order statistics not only within the wake region, but throughout the entire boundary layer far into the buffer region (Schlatter & Örlü, 2010). Despite these differences good agreement between a recent large-scale DNS and experiment with similar boundary conditions at \( Re_\theta \approx 2500 \) has been reported in Schlatter et al. (2009) for the same quantities. Hence,
there is a need to address why integral quantities as well as higher order turbulence statistics between compiled DNS of the same flow differ substantially, even beyond the aforementioned Reynolds number at which the good agreement between experiment and DNS was documented (Schlatter et al., 2009). Some of the main players have already been conjectured in Schlatter & Örlü (2010), such as the inflow Reynolds number and turbulence generation, the settling length to reach a final turbulent state, as well as box dimensions and boundary conditions.

To what extend the differences due to the aforementioned causes, persist in statistical and structural quantities and whether the outer layer actually completely forgets its initial or upstream conditions and thereby converges (or tends within the investigated Re-range) to an asymptotic state has been the focus of various studies dating back to Klebanoff & Diehl (1954), who investigated artificially thickened ZPG TBL flows. They found that a fully developed TBL could be achieved free from any distortions introduced by the thickening process, i.e. independent of its initial conditions, beyond a certain development length depending on the thickening device itself. However, the question regarding the sufficient development length remains open up to today as emphasised in a recent review paper by Marusic et al. (2010). When it comes to the Reynolds stresses, it has even been questioned whether there exists an asymptotic state independent of upstream conditions at all (Castillo & Johansson, 2002; Seo et al., 2004).

In particular the classical assessment of experimental data sets by Coles (1954) states that “it is not obvious a priori that a state is ever reached in which the dependence of the turbulent boundary layer on its early history is no longer measurable in terms of the local mean properties of the flow”. Similarly, the review on ZPG TBL flows by Fernholz & Finley (1996) mentions the tripping devices as an important secondary factor that might explain differences between various data sets, while a recent re-assessment of experimental data from ZPG TBL flows by Chauhan et al. (2009) lists “three major local experimental conditions existing in the tunnel for various data sets, i.e. an insufficient or excessive transition trigger, a history of pressure gradient and inadequate development length” as the cause for differences in the outer part of the mean flow. One reason why apparent differences between past and present experimental data sets could not solely been traced back to initial or upstream conditions (in contrast to measurement technique related shortcomings) is the “fact that in many experiments these effects [i.e. the initial or upstream conditions] are not, or only incompletely, documented” (Marusic et al., 2010); a fact that is mainly related to the elaborate work associated with quantifying the full set of initial and upstream conditions in experiments. An exception in this respect and a seminal contribution is the work by Erm & Joubert (1991) who studied the effect of different tripping devices for various freestream velocities and concluded that an overall agreement independent of tripping device could be reached for $Re_\theta = 2175$, where $\theta$ denotes the momentum-loss thickness.

In the framework of the present work the DNS of a ZPG TBL flow by Schlatter et al. (2009), which here serves as the baseline simulation, was re-simulated, however with physically different inflow conditions and tripping effects. Hence, the present investigation can be seen as a direct continuation of Schlatter & Örlü (2010), since it enables the direct assessment of various turbulence generation mechanism and the possibility to define various measures to discern from where on a ZPG TBL flow can be considered independent of its turbulence generation mechanism. Although Jimenez et al. (2008) and Simens et al. (2009) investigated the effect of turbulent inflow conditions for their DNS of a ZPG TBL, their conclusion, i.e. that at least 300 initial momentum-loss thicknesses have to be discarded before the effect of the artificial inflow is forgotten, is not necessarily of general nature, but applied to the employed recycling method. The present work, on the other hand, can more closely be related to wind tunnel experiments, where the flow undergoes laminar-turbulent transition and various tripping devices are tested by studying the downstream evolution of the boundary layer. In this respect it can be seen as the numerical counterpart to the study by Erm & Joubert (1991) with the advantage of having access to the full set of initial and upstream conditions (a fact that is rarely accomplished in
experiments; cf. Marusic et al., 2010). To the authors’ knowledge there is no previous study investigating the effects of different turbulence generation mechanisms in spatially developing flows, so the current investigation is especially significant in this respect.

2. Existing and new direct numerical simulations

For the main part of the present investigation the DNS of a ZPG TBL by Schlatter et al. (2009), here serving as the baseline simulation, was re-simulated, however with different tripping parameters to examine these effects on the turbulent boundary layer developing downstream. In particular, simulations with tripping at a lower $Re$, with lower tripping amplitude, lower and higher tripping frequency, and “classical transition” via exponential growth of Tollmien-Schlichting (TS) waves were considered as the turbulence-generation mechanisms. Additionally, a simulation with tripping at a considerably higher $Re$ was considered, since this is a common way to reach cost-efficiently a high Reynolds number (Khujadze & Oberlack, 2007). All simulations are performed using a fully spectral method to solve the time-dependent, incompressible Navier-Stokes equations (Chevalier et al., 2007). In the wall-parallel directions, Fourier series with dealiasing are used whereas the wall-normal direction is discretised with Chebyshev polynomials. Periodic boundary conditions in the streamwise direction are combined with a spatially developing boundary layer by adding a “fringe region” at the end of the domain. Note that a truly spatially developing boundary layer is simulated, and the fringe region is purely a numerical method to achieve this with high accuracy. For further details on the simulations the reader is referred to e.g. Schlatter et al. (2009).

The various runs in this study are summarised in Table 1, together with their Reynolds-number range. The numerical resolution for cases LA to HF are exactly the same as the baseline case (Schlatter et al., 2009), i.e. $\Delta^+_x = 12$, $\Delta^+_y, \max = 8.6$, $\Delta^+_z = 6.4$ obtained with a grid of $3072 \times 301 \times 256$ spectral collocation points in the streamwise, wall-normal and spanwise direction, respectively. Cases LR and HR are large-eddy simulations (LESs), with slightly reduced resolutions similar to Schlatter & Örlü (2010).

The tripping is implemented as a weak random volume force acting in the wall-normal direction. The location of this forcing, attenuated by a Gaussian in both streamwise and wall-normal direction. The location of this forcing, attenuated by a Gaussian in both streamwise and wall-normal direction.

Table 1. List of employed DNS data, their Reynolds number ranges and the key characteristics of the turbulence generation mechanism. All simulations were performed in a single domain using a spectral method.

| Reference/Label          | $Re_\theta$     | Method                                  | Symbol |
|--------------------------|-----------------|-----------------------------------------|--------|
| Schlatter et al. (2009)  | 180 – 2500      | tripping at $Re_\theta = 180$           |        |
| “baseline”               |                 | optimised tripping parameters           |        |
| Schlatter & Örlü (2010) | 180 – 4300      | tripping at $Re_\theta = 180$ lower amplitude compared to above |        |
| LA                       | 180 – 2400      | low-amplitude tripping at $Re_\theta = 180$ |        |
| TS                       | 180 – 2000      | Tollmien-Schlichting waves ($F = 120$)  |        |
| LF                       | 180 – 2100      | low frequency tripping at $Re_\theta = 180$ |        |
| HF                       | 180 – 2500      | high frequency tripping at $Re_\theta = 180$ |        |
| LR                       | 55 – 2000       | tripping at $Re_\theta = 55$, LES       |        |
| HR                       | 750 – 2900      | tripping at $Re_\theta = 750$, LES      |        |
normal direction, is very close to the inlet at (laminar) $Re_\theta = 180$. The shape of the forcing is described by two parameters, the spanwise scale $z_s$ and the temporal scale $t_s$. In the (periodic) spanwise direction random harmonic fluctuations are included larger than $z_s$. The temporal fluctuations are implemented using a series of third-order Lagrange interpolants with time scale $t_s$ (Chevalier et al., 2007). In Fig. 1, the temporal spectrum is shown; the spectral energy is equipartitioned until the cutoff length is reached, which is followed by a steep decrease of the energy $\sim f^6$. For the baseline case, a temporal cutoff scale $t_s = 4\delta_0^\ast/U_\infty$ and a spanwise cutoff scale $z_s = 1.7\delta_0^\ast$ has been chosen. For case LA, the amplitude is reduced to 25%, case LF increases the temporal scale by a factor of 5, case HF increases the frequency by a factor of 4 and reduced $z_s$ to half. Cases LR and HR use similar scales as the baseline run. Case TS – instead of a random tripping – introduces harmonic disturbances with non-dimensional frequency $F = 120$ leading to exponentially growing Tollmien-Schlichting (TS) waves, with superimposed low-amplitude noise to trigger secondary instability close to Branch II.

The fluctuations introduced into the flow will lead to laminar-turbulent transition following different routes depending on the parameters. The baseline case leads to quick breakdown to turbulence via the lift-up effect and the subsequent formation of hairpin vortices on the streaks (Schlatter et al., 2010a). The other cases tend to form turbulent spots (similar as in bypass transition), which then grow while travelling downstream and eventually merge to produce a fully turbulent flow. An example is shown in Fig. 2, in which the intermittent character of the flow at low $Re_\theta$ becomes evident. Note that the individual turbulent spots share all characteristics of canonical spots.

**Figure 1.** Normalised temporal spectrum of the tripping amplitude. The cutoff time scale is denoted by $t_s = 1$.

**Figure 2.** Top view of the initial part of the computational domain pertaining to the low-amplitude (LA) tripping. The Reynolds number ranges from $Re_\theta = 180$ at the laminar inflow to $Re_\theta = 1140$ on the right-hand side. The middle of the shown domain corresponds to $Re_\theta \approx 670$. Isocontours of $\lambda_2^+ = -0.005$ are drawn, coloured with the streamwise velocity. Flow from left to right.
3. Results and Discussion

3.1. Integral and global quantities

The streamwise development of vortical structures emerging from a low amplitude tripping mechanism, shown in Fig. 2, indicates that a considerable portion of the domain is affected by the way the flow is triggered and consequently will yield a different \( Re_\theta \) at which laminar-turbulent transition is induced and concluded. A direct and overall assessment of the boundary layer development can be obtained through the boundary layer thickness \( \delta_{99} \) as well as its integral measures, i.e. the displacement \( \delta^* \) and momentum-loss \( \theta \) thicknesses. The ratio of the latter two integral quantities, the shape factor \( (H_{12}) \), as function of \( Re_\theta \) is depicted in Fig. 3(a) for all utilised simulations and is “a good indicator of the state of a ZPG boundary layer” (Chauhan et al., 2009). Starting from the laminar value of \( H_{12} \approx 2.59 \) the different cases reach a common trend line at around \( Re_\theta = 1000 \), where the difference between the various simulations is within 2% and further reduces to below 1% at around \( Re_\theta = 2000 \). The flow case tripped at a high \( Re \) (HR) is naturally outside the given margins, and it is interesting to note that even at \( Re_\theta = 2500 \) it is not only 2% below the value inherent by all other flow cases, but depicts still the opposite \( Re \)-trend, i.e. it increases with increasing \( Re \): something that can e.g. also be observed in the simulations by Khujadze & Oberlack (2007). This confirms the conclusion drawn in Schlatter & Örlü (2010) regarding this approach, viz. that “the necessary inflow length for developed turbulence at higher \( Re \) is longer” and emphasises that tripping at relatively high \( Re \) (while at first glance being a cost-efficient method to reach high \( Re_\theta \)) does not imply that the simulated flow case represents a canonical ZPG TBL flow, even not at a considerably high \( Re_\theta \approx 2900 \).

A more sensitive measure of the growth of the boundary layer can be obtained from the boundary layer thicknesses \( \delta^* \) and \( \theta \) themselves, as shown in Fig. 3(b). In particular, the simulation denoted as “baseline” turns indeed out to be a baseline, since all other simulations eventually converge onto the trend prescribed by it. Note that although the high \( Re \) DNS (Schlatter & Örlü, 2010) appears to be slightly below the baseline simulation, this is mainly caused by a different way of evaluating the integrals which yields differences when considerably different domain heights are used (cf. Schlatter & Örlü, 2010).

The skin-friction coefficient \( c_f \) for the same set of data is depicted in Fig. 4(a) and clearly shows that for all simulations the inflow is laminar (i.e. \( c_{f,\text{lamin}} = 0.441 Re_\theta^{-1} \)). Depending on

![Figure 3](image-url)  
**Figure 3.** Ratios of boundary layer thicknesses. *a*) Shape factor \( (H_{12}) \) and *b*) displacement thickness \( (\delta^*) \) and momentum-loss thickness \( (\theta) \) over boundary layer thickness \( (\delta_{99}) \) as function of Reynolds number \( (Re_\theta) \) for various trippings.
tripping, laminar-turbulent transition is induced at different $Re_\theta$, and all except the LF and LA cases exhibit typical overshoots of $c_f$ as a result of transition, after which they settle on a common $c_f$ distribution indicating a rather quick adaptation of the near-wall turbulence. The root mean square (rms) value of wall shear stress ($\tau_{w,rms}$) is shown in Fig. 4(b) and, irrespective of tripping mechanism, the curves also indicate a fast convergence towards the classical value of 0.4 (Alfredsson et al., 1988) with a small increase with Reynolds number, coming from the growing influence of the outer spectral peak as recently documented by Örlü & Schlatter (2011).

It is interesting to point out that even when including the HR case the difference between all employed cases in $\tau_{w,rms}$ is within 2%; thereby letting the DNSs by Wu & Moin (2009) and Wu & Moin (2010) with more than 10% higher values (irrespective of $Re$) appear quite different from all simulations presented here (cf. Örlü & Schlatter, 2011).

3.2. Mean velocity profiles

The log-law indicator function $\Xi = y^+ dU^+ / dy^+$, which is commonly employed to measure the von Kármán constant in the overlap region is presented in Fig. 5, clustered around four distinct Reynolds numbers $Re_\theta = 670$, 1100, 1550, and 2000. It is apparent that the curves for $\Xi$ within the inner region, i.e. the region below $y^+ \approx 0.25^+$ (indicated through the dashed vertical lines), show a good agreement from $Re_\theta = 1100$ on; while only the LF case needs a longer development length in accordance with the slower convergence already evinced in the integral quantities. Although the HR case exhibits a lower peak value at $y^+ \approx 10$, this is not related to its lower resolution, since the LR case agrees with the DNS cases. It should be noted that this discrepancy can more clearly be seen in the mean velocity profile when considering the deviation from a reference profile (such as the composite profile by Chauhan et al., 2009). This consequently implies that the HR case deviates in the mean streamwise velocity component within the buffer region over the entire $Re$-range shown here; an observation that is also valid for the profiles by Khujadze & Oberlack (2007) as demonstrated by Schlatter & Örlü (2010). Note also that the Reynolds numbers are far from being close to an asymptotical logarithmic region (Österlund et al., 2000). The profiles also underline the conclusions deduced from the integral quantities, i.e. that the near-wall turbulence convergences rapidly, and that with increasing $Re$ the curves within the inner layer not only agree with each other, but also collapse onto the same curve irrespective of the Reynolds number for a successively larger $Re$-range; thereby establishing the law of the wall.

The Reynolds shear stress and total shear stress profiles are given in Fig. 6 and are shown for two particular reasons. Work by Castillo & Johansson (2002) and Seo et al. (2004) suggests that
Figure 5. Profiles of the indicator function ($\Xi = y^+ dU^+/dy^+$ vs. $y^+$) in the inner region of the ZPG TBL flow. Dashed vertical lines indicate the wall-normal boundary of the overlap region, i.e. $y^+/Re_\tau = 0.2$. Profiles are grouped according to their Reynolds number, viz. a) $Re_\theta =670$, b) $Re_\theta =1100$, c) $Re_\theta =1550$, d) $Re_\theta =2000$. Horizontal lines indicate the level corresponding to $\kappa^{-1} = 0.38, 0.40, and 0.42.

the Reynolds shear stress and the wall-normal component of the Reynolds normal stress, show the strongest influence of upstream and history effects. However, as apparent from the Reynolds shear stress profiles, there is clear agreement for all but the HR case after transition. This can also be confirmed from the wall-normal component of the Reynolds normal stress (not shown here), hence there are no apparent differences due to different upstream effects for $Re_\theta \geq 1100$, which is in agreement with the global picture drawn by all previous depicted quantities. Another topic of recent interest is the observed overshoot of the total shear stress over the wall shear stress clearly visible for the TS and LF cases at $Re_\theta = 670$. Both of these tripping cases were also slower in converging to the general trend in all of the shown integral/global quantities highlighting that this feature is solely related to the delayed laminar-turbulent transition. This overshoot was noted in the DNS by Wu & Moin (2009) and the authors stated that the “observed total shear stress overshoot in the turbulent region is a diminishing effect of transition, and is unlikely to persist in fully developed turbulent boundary layers” (Wu & Moin, 2009). However, in a follow-up study by Wu (2010) the total stress overshoot present in the data by Wu & Moin (2010) was associated with a localised streamwise acceleration of the streamwise velocity component in the transitional region and deemed “valid, and presumably general to a class of...

Figure 6. Reynolds shear stress ($\nu^+ u^+$) and total shear stress ($dU^+/dy^+ - \nu^+ u^+$) denoted through thin and thick lines, respectively. The profiles are grouped according to their Reynolds number, a) $Re_\theta =670$, b) $Re_\theta =1100$, c) $Re_\theta =1550$, d) $Re_\theta =2000$. 
similar types of boundary layers” (Wu, 2010). In fact, the strength of the overshoot in the total shear stress of the data by Wu & Moin (2010) does not diminish but exhibits a 2% overshoot independent of Reynolds number from $Re_\theta = 900$ up to the highest $Re_\theta$ of 1840. For the simulated cases here, on the other hand, the overshoot is clearly associated solely with laminar-turbulent transition in accordance with previous transition studies (Brand et al., 2004), and all profiles (excluding the HR case) collapse nicely on a single curve and clearly do not exhibit an overshoot for $Re_\theta \geq 1100$.

All of the previous figures clearly indicate a consistent trend in terms of convergence towards general curves, which essentially follow the trend prescribed by the baseline simulation. On the other hand, as illustrated by Fig. 7, the outer layer convergence, quantified by contours of $u^+_{rms}$ in a streamwise/wall-normal plane takes considerably longer than the near-wall turbulence. For clarity only the TS and HF cases are shown here in addition to the baseline simulation. Focussing on the near-wall region (highlighted in the semi-logarithmic re-presentation) a collapse of the contour lines even beyond the near-wall peak of the rms profile $(y^+ \approx 15)$ can be observed for $Re_\theta \geq 670$. However, when it comes to the outer layer the TS case represents the slowest

Figure 7. Contours of constant root mean square values (in steps of $\Delta u^+_{rms} = 0.25$) of the streamwise velocity fluctuations as function of Reynolds number in inner scaling against a) linear and b) logarithmic ordinate. For clarity only the baseline, TS and HF cases are shown.

Figure 8. a) Streamwise mean velocity profile inner scaling for $Re_\theta = 1100$ and 2000 with inserts enlarging the wake region from $y^+ = 100$–2000. b) Streamwise velocity root mean square value for $Re_\theta = 1100$ and 2000 with insert depicting the $Re_\theta = 2000$ case against a logarithmic abscissa.
convergence and indicates that only for $Re_\theta \geq 2000$ the flow has forgotten about its history and the outer layer has reached a common trend prescribed by the baseline simulation and approached by all other (not shown) cases.

The reminiscence of upstream and history effects within the outer region of the TBL and the related slow convergence of the same can also be observed in the mean and rms profiles depicted in Fig. 8 for all simulations at $Re_\theta = 1100$ and 2000. While the law of the wall has clearly been established in the streamwise mean velocity profile and the rms profile collapses ($Re$-dependent) onto a single curve within the inner region, the outer region exhibits remarkable differences in the wake region and considerably differing fluctuation levels. It is, however, gratifying that if transition is initiated at a low enough $Re_\theta < 300$, all data agree well for both inner and outer layer for $Re_\theta > 2000$.

4. Summary and Conclusions

The recent assessment of available DNS data from TBL flows by Schlatter & Örlü (2010) showed surprisingly large differences not only in the skin friction coefficient or shape factor, but also in their predictions of mean and fluctuation profiles far into the sublayer. By re-simulating the DNS of a ZPG TBL flow by Schlatter et al. (2009), however with physically different inflow conditions and tripping effects, the downstream evolution of integral and global quantities as well as mean and fluctuation profiles have been presented. Based on the presented results it is reasonable to conclude that the reminiscence of transitional effects over the entire boundary layer height as well as the slow convergence of the outer layer are the main players explaining some of the differences observed while comparing DNS data of different origin. It remains an open question what conditions to apply to determine whether a boundary layer has reached a “fully-turbulent” or “canonical” state, but it is clear that the shape factor and skin friction is not sufficient. For a meaningful comparison between numerical and/or wind tunnel experiments it can be conjectured that $Re_\theta > 2000$ is a lower limit when transition is triggered at $Re_\theta < 300$ in order not to be biased by transitional and in particular (since these appear much more gradually) upstream effects. It can also be stated that the baseline simulation, based on the comparison given throughout the paper, constitutes a ZPG TBL from $Re_\theta \approx 500$ onwards.

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