Taming the Cross Entropy Loss

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Abstract. We present the Tamed Cross Entropy (TCE) loss function, a robust derivative of the standard Cross Entropy (CE) loss used in deep learning for classification tasks. However, unlike other robust losses, the TCE loss is designed to exhibit the same training properties than the CE loss in noiseless scenarios. Therefore, the TCE loss requires no modification on the training regime compared to the CE loss and, in consequence, can be applied in all applications where the CE loss is currently used. We evaluate the TCE loss using the ResNet architecture on four image datasets that we artificially contaminated with various levels of label noise. The TCE loss outperforms the CE loss in every tested scenario.

1 Introduction

The most common way to train Convolutional Neural Networks (CNNs) for classification problems is to use stochastic gradient descent coupled with the Cross Entropy (CE) loss. The CE loss is popular mainly due to its excellent convergence speeds, alongside its excellent performance in terms of Top-1 and Top-5 classification accuracy.

However, the CE loss is not without weaknesses. Theoretically, the CE is proven to be a calibrated loss [20], and thus should provide well-behaved probability estimates, however, in reality a different behavior is observed: the calibration of a classifier using the CE loss worsens as the classification accuracy improves [7]. As a consequence, many techniques have have been proposed to improve calibration (e.g., Bayesian Neural Networks [1]).

A related problem of the CE loss is its suboptimal performance when dealing with noisy data [5]. Although complex CNNs architectures have shown considerable robustness to noise in the training dataset [3,17,18], noisy labels and outliers are still a significant problem, particularly when dealing with weak labels. As a consequence, the problem of dealing with label noise when learning has been studied extensively [4].

In particular, there are loss functions for classification tasks that are more robust or have more discriminative power than the CE. For example, the pairwise loss [8] and the triplet loss [19] are effective ways to learn discriminative features between individual classes. Also, the OLE loss [15] explicitly maximizes intra-class similarity and inter-class margin, and thus, improves its discriminative power with respect to the CE. However, such losses are either slower or significantly more complex to apply than the CE.
Ghosh et al. [5] used a risk minimization framework to analyze the CE loss, the Mean Absolute Error (MAE) loss, and the Mean Squared Error (MSE) loss, for classification tasks under artificially added label noise. Their results show that the MAE is inherently robust to noise, while the CE is particularly vulnerable to label noise, and the MSE should perform better than the CE but worse than the MAE. Sadly, being an $\ell_1$ loss, the MAE has abysmal convergence properties and is not well suited for practical use.

We aim to offer a more convenient alternative to the currently available losses for robust classification. We follow the same spirit than Huber et al. [10] and Girshick et al. [6], who independently hand crafted a robust regression loss by fusing the MSE loss and the MAE loss together, and thus obtained a loss with the convergence properties of the MSE, and the robustness to noise of the MAE.

Our result is the Tamed Cross Entropy (TCE) loss, which is derived from the CE and thus it shares the same convergence properties, while, at the same time, it's more robust to noise. Instead of fusing two losses, we started from the CE and designed guidelines on how the gradient of our tentative TCE should behave in order to behave like the CE and be robust to outliers.

Finally, to design the actual TCE, we used a power normalization over the CE gradient to make it compatible with our previously designed guidelines. We choose this kind of regularization because power normalizations have already been used with great success to robustify features [12].

The gradient of the TCE is identical to the gradient of the CE if the predicted confidence with respect to the actual label is high, and tends to zero if the predicted confidence of with respect to the actual label is low. This way, training samples that produce low confidence values (ideally outliers or mislabeled data), generate a reduced feedback response.

To ensure that the TCE can be used as a drop-in replacement for the CE, we used the reference implementation for the ResNet [9] architecture and we replaced the CE with the TCE without altering any configuration parameters. We also tested the performance of the TCE against the CE, the MSE, and the MAE losses in the same scenario, and we also evaluated the robustness of the loss functions against uniformly distributed label noise.

In all tested cases, our TCE outperformed the CE while having almost the same convergence speed. Furthermore, with 80% of random labels, the TCE offers Top-1 accuracy improvements of 9.36%, 9.80%, and 4.94% in CIFAR10+, CIFAR100+, and VSHN respectively.
2 Taming the Negative Log Likelihood Loss

2.1 Background

The cross entropy loss is commonly used after a softmax layer that normalizes the output of the network, and is defined as:

\[
\text{Softmax}(o) = \frac{e^o}{\sum_{j=1}^{N} e^{o_j}},
\]

whereas the cross entropy between two \(N\) sized discrete distributions \(p \in [0,1]^N\) and \(q \in (0,1]^N\) is:

\[
H(p, q) = -\sum_{i=1}^{N} p_i \log q_i,
\]

where \(p\) corresponds to the classification target, and \(q\) the output of the softmax layer, i.e., the likelihood predicted per class.

\[\text{Is it important to note that the actual value of the loss function does not affect in any way the training procedure, as only its gradient is used during back propagation. We analyze the gradient of the cross entropy loss with respect to the log-likelihood, which is a commonly used trick. Using } p \in \{0,1\}^N, \text{ the partial derivatives of the CE loss with respect to the predicted log-likelihoods are:}\]

\[
\frac{\partial H(p, q)}{\partial \log q_i} = \begin{cases} 
0 & \text{if } p_i = 0, \\
-1 & \text{if } p_i = 1.
\end{cases}
\]

2.2 Design Goals

We define the following set of design goals in order to guide us in the design process towards a robust classification goal:

1. We want the gradient of the TCE loss \(\hat{H}\) to be proportional to the gradient of the CE loss. This way we expect that both losses will behave in a similar way. We aim to:

\[
\nabla \hat{H}(p, q) \propto \nabla H(p, q).
\]

2. If the network is confident about the predicted class, i.e., \(q_i \rightarrow 1\), we want the TCE to behave exactly like the CE.

\[
\nabla \hat{H}(p, q) = -1 \text{ if } p_i = 1 \text{ and } q_i \rightarrow 1.
\]

3. We aim to reduce the impact of outliers by reducing the feedback from the gradient when there is a large discrepancy between a prediction and its associated label:

\[
\nabla \hat{H}(p, q) = 0 \text{ if } p_i = 1 \text{ and } q_i \rightarrow 0.
\]
Fig. 1: The CE (case $\alpha = 0.0$) has a constant gradient when plotted against $\log q_i$, thus its response is independent of the confidence estimate of the prediction. On the other hand, TCE’s gradient gets smaller as the confidence estimate of the prediction decreases.

To summarize, we aim to design a function whose gradient behaves in the following way:

$$\frac{\partial \hat{H}(p, q)}{\partial \log q_i} \approx \begin{cases} 
0 & \text{if } p_i = 0, \\
-1 & \text{if } p_i = 1 \text{ and } q_i \to 1, \\
0 & \text{if } p_i = 1 \text{ and } q_i \to 0.
\end{cases} \quad (7)$$

2.3 The gradient of the Tamed Cross Entropy Loss

We suggest the following gradient that fulfills the requirements expressed in Eq. 7:

$$\frac{\partial \hat{H}_\alpha(p, q)}{\partial \log q_i} = \begin{cases} 
0 & \text{if } p_i = 0, \\
-(1 - \log q_i)^{-\alpha} & \text{if } p_i = 1.
\end{cases} \quad (8)$$

We based our regularization on the domain $[1, \infty)$ of the power function, which we applied to the $\log p_i$ term. And we control the regularization factor using the parameter $\alpha \in \mathbb{R}^+$. The loss function that corresponds with the gradient presented in Eq. 8 is:

$$\hat{H}_\alpha(p, q) = \frac{1}{1 - \alpha} \sum_{i=1}^{N} p_i \left( (1 - \log q_i)^{1-\alpha} - \frac{1}{1-\alpha} \right). \quad (9)$$

We can observe the behavior of both $\hat{H}_\alpha$ and $\nabla \hat{H}_\alpha$ in Fig. 1. Also, note that $\hat{H}_\alpha$ corresponds to $H$, when $\alpha$ equals 0.
3 Experiments

3.1 Experimental Setup

We evaluate the TCE against the CE loss and other baselines on four datasets: MNIST [14], CIFAR10 [13], CIFAR100 [13], and VSHN [16]. All datasets are well known, and consist of 32x32 pixel images. MNIST, CIFAR10, and VSHN contain 10 classes each, while CIFAR100 contains 100 different classes.

Our training setup is based on the reference implementation for the ResNet [9], implemented in Torch [2]. We train the same architecture (ResNet-20) for all datasets, and we use the default training strategy, which is optimized for the CE loss we aim to replace. For CIFAR10 and CIFAR100 we apply common data augmentation schemes (shifting and mirroring), thus we decorate both datasets with a "+" mark on the evaluation. On MNIST and VSHN we apply only shifting, as they depict numbers. We normalize the data using the channel means and standard deviation. We use an initial learning rate of 0.1, a Nesterov momentum of 0.9, batch size of 128, and weight decay of $1e^{-4}$. We train for 256 epochs, and we divide the learning rate by 10 at epoch 128, and again at epoch 192.

We hold out 5000 images from the training set of each dataset and we use them as a validation set. Such validation set is used only to determine at which epoch the lowest validation error is obtained. Then, we run 5 times each experiment (using the entire training data) and we report the mean and the standard deviation (when significant) of the test error captured at the epoch determined by the previous validation step.

All losses can be computed efficiently, hence there is no discernible difference in time when training using different loss functions.

3.2 Baselines

We compare our TCE loss to the CE loss we aim to replace, as well as the MSE and the MAE losses, both suggested by Ghosh et al. [5] as robust alternatives to the CE loss. The Huber loss [10], also known as SmoothL1 loss [3], is a well known loss used in robust regression, however there is no need to evaluate it as it is equivalent to the MSE loss when applied to the $[0, 1]$ domain used for classification.

As both MSE and MAE are losses designed for regression problems, we had to adapt them prior to use them for classification. We followed the Torch guidelines: we prefixed them with a Softmax later, and we scaled their gradient by the number of classes in the output.
Table 1: Evaluation of our TCE loss against alternatives using a ResNet-20 on CIFAR100+. We used the reference ResNet implementation and default parameters except for the loss function. A proportion of the training dataset (η) had its labels replaced randomly. CIFAR100+ is a complex dataset with 100 classes, and thus we can observe how neither MSE nor TCE with α = 2 converge when η = 0.8. Less extreme values of α are able to converge just as well as the CE loss while outperforming it. The MAE loss, as expected, failed to converge under the default training parameters.

3.3 Top-1 Accuracy under Uniform Label Noise

A common experiment to evaluate robustness in deep learning is to perform an experiment where we apply uniformly distributed random labels to a portion of the training dataset. In this setup, the noise ratio (η) determines the proportion of the training dataset corrupted with random labels, and we evaluated our losses on the four datasets using η ∈ {0.0, 0.2, 0.4, 0.6, 0.8}.

We group the full results of this experiment on the challenging CIFAR100+ dataset in Table. We group the results on the 10-class datasets in the Table where we only show the results for η ∈ {0.0, 0.4, 0.8} for space reasons.

We observe that, when using default training regimes, the MAE norm fails to converge, something expected from a pure ℓ1 norm loss. Although in [5] it is argued that the MSE should be more robust to noise than the CE, the improvement is small and only occurs on low noise factors (i.e., η ≤ 0.4). In general terms, both CE and MSE losses obtain similar performance.

On the other hand, the TCE losses achieve the best Top-1 accuracy in all but one case, where it is second best after MSE. For η = 0.8, the TCE improves Top-1 accuracy by 9.36% in CIFAR10+, 9.80% in CIFAR100+, and 4.94% in VSHN. Furthermore, the TCE loss shows little sensitivity to its regularization parameter, offering solid performances for α ∈ {0.5, 1.0, 1.5, 2.0}. 

| noise (η) | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|-----------|-----|-----|-----|-----|-----|
| CE        | 68.18 | 61.16 | 54.52 | 44.08 | 20.30 |
| MSE       | 67.78 | 62.81 | 55.98 | 42.48 | 15.41 |
| MAE       | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| TCE α = 0.5 | 68.25 | 63.58 | 57.88 | 48.10 | 25.13 |
| TCE α = 1.0 | 68.33 | 64.11 | 59.90 | 51.59 | 29.58 |
| TCE α = 1.5 | 68.45 | 65.10 | 61.07 | 53.76 | 30.10 |
| TCE α = 2.0 | 66.81 | 64.37 | 61.51 | 52.09 | 18.75 |
Table 2: Evaluation of our TCE loss against alternatives using a ResNet-20 on MNIST, CIFAR10+, and VSHN. We used the reference ResNet implementation and default parameters except for the loss function. A proportion of the training dataset ($\eta$) had its labels replaced randomly. Our TCE loss offers generally better performance than the CE and the MSE losses, in particular when the training labels are noisy. The MNIST dataset is not challenging anymore, and even when training with 80% of noisy labels the default configuration offers excellent performance.

### 3.4 Learning Behavior under Label Noise

In Fig. 2 we show the test set accuracy during training for the CIFAR100+ dataset with different levels of noise.

The first stage of training, with a learning rate of $10^{-1}$, correspond to the annealing stage, and it generally shows little overfitting behavior. In this stage, the CE loss converges the fastest, and the TCE with $\alpha = 1$ performs similarly. On the other hand, when using $\alpha = 2$, the convergence ratio of the TCE is similar to that of the MSE loss.

At the epoch 128, we reduce the learning rate to $10^{-2}$, and all losses experience a significant drop in the error rate. However, after this drop, the networks start to overfit on the mislabeled images, raising the error rate. Although this effect grows stronger together with the noise ($\eta$), it is still present even with $\eta = 0$, as can be seen in Fig. 2b.
Fig. 2: Test error curves during training ResNet20 on CIFAR100+ under different losses, and different noise ratios. Note that (b) is a detail of (a), where we can observe how the CE loss performance worsens even without label noise. The TCE and MSE losses are more robust to accuracy regressions.
3.5 Top-1 Accuracy vs. Convergence Speed

In Fig. 3 we evaluate Top-1 Accuracy against convergence speed. We measure convergence speed by counting how many epochs are necessary for the network to achieve an accuracy threshold. We observe a large variance when measuring convergence speed in 10-class datasets like CIFAR10+ (see Fig. 3a), thus our analysis will be based only on the CIFAR100+ results (Fig. 3b).

For $\alpha \leq 1$, the TCE loss shows approximately the same convergence speed than the CE loss, but better accuracy. Only for $\alpha > 1$ the TCE keeps growing slower, but never reaches the slow convergence speed of the MSE.

This results seem to indicate that TCE with $\alpha = 1.5$ offers the best trade-off between accuracy and convergence speed.

4 Conclusions

We have proposed a new loss function, named Tamed Cross Entropy (TCE), that behaves like the popular CE loss but is more robust to universal label noise. The TCE loss is friendly to use and can be applied to classification tasks where the CE loss is currently used without altering the training parameters. We found experimentally that the only regularization parameter of the TCE loss has a limited effective range $0 < \alpha < 2$, and changes in $\alpha$ do not have dramatic effects on the performance of the loss. We expect to extend this work in the future by applying the TCE loss to more classification tasks, as well as weakly supervised tasks.
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