Order statistics of high-intensity speckles in stimulated Brillouin scattering and plasma-induced laser beam smoothing

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\textbf{Abstract.} We have studied plasma-induced smoothing due to stimulated Brillouin scattering (SBS) under the aspect of the extremal statistics of smoothed laser beams. As pointed out in the work by Rose and DuBois (1994 \textit{Phys. Rev. Lett.} \textbf{72} 2883), scattered light can be subject to uncontrolled (or even ‘explosive’) behaviour, associated with a critical gain value for SBS. In this work we show how this critical behaviour can be predicted on the basis of the order statistics of laser speckle fields, and we analyse the transition to uncontrolled behaviour of the laser beam due to the dominance of high intensity speckles.

\textbf{S} Online supplementary data available from \texttt{stacks.iop.org/NJP/15/025003/mmedia}

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1. Introduction

The spreading of optically smoothed laser beams propagating through a plasma has been the subject of theoretical [1–5] and experimental studies [6–8] in recent years. The common interest of these studies was to understand the potential additional smoothing capacities of the plasma that can eventually improve the laser imprint on targets situated behind the plasma layer. That the interaction of the laser beam with the plasma can lead to strong self-focusing (SF) and ‘dancing filaments’ was demonstrated by simulations in [1]. In this case the strong angular spreading of the beam comes together with loss in temporal coherence due to the non-stationary behaviour of the transmitted light. At lower beam intensities, such that laser speckles at average intensity are not subject to SF, stimulated Brillouin forward scattering (FSBS) was understood as the principal mechanism leading to an additional loss in spatial and temporal coherence in the transmitted light. This loss of coherence in the plasma eventually has useful smoothing capacities. In experiments, mostly using gas jets, with both weakly and strongly ionized plasmas, the smoothing could be identified to be active [7]. In a further step towards higher electron densities, plasmas generated from low-density foam targets were used and the reduction of the laser imprint contrast could be clearly demonstrated [6]. The process leading to additional smoothing is most commonly known in the literature as ‘plasma-induced smoothing’.

A critical aspect of this smoothing technique is the potential transition towards uncontrolled behaviour in the scattered light with strong spreading. Uncontrolled characteristics of the beam are, of course, not desirable for the purpose of plasma-induced smoothing. In earlier work
by Rose and DuBois [9], the possible explosive behaviour of scattered light from stimulated scattering instabilities has been pointed out and has been associated with a critical gain value of the instability. For the case of stimulated Raman scattering, Rose and Mounaix [10] have recently shown the evidence for the explosive behaviour in spite of diffraction. Mounaix and Divol [11] have shown that the cumulative response of stimulated Brillouin scattering (SBS), based on the independence of speckles (the ‘independent hot spot model’), can spread strongly around a critical gain value.

In this paper we show the physical relevance of the high-intensity speckle order statistics for SBS and for the control of the plasma-induced smoothing of optically smoothed laser beams incident on the plasma layer. Since the onset of the latter mechanism is closely related to the amplification of FSBS, the effect of extreme speckles can lead to the poorly controlled spreading of the light beams exiting the plasma layer.

The paper is organized as follows: in section 2 we recall first the model equations for SBS following the model by Elisseev et al [12] to determine the solution for the scattered light field response from a single speckle as a function of the standard expression for the gain coefficient [13]. These results will be used to derive the total light field intensity in section 3. For a first layer of speckles (in the propagation direction) of a speckle pattern generated from a random phase plate (RPP), one can make use of a perturbative approach for the scattered light field. In section 5, the modification of the total light intensity due to the scattered light is then evaluated by computing the sum over speckles, using order statistics of speckle fields for both forward (FSBS) and backward SBS (BSBS). From the cumulative response of the scattered light from all speckles, we compute the speckle intensity range corresponding to the speckles which contribute dominantly to this response. Through evaluation of order statistics and by means of the numerical simulations in section 6, we demonstrate that the cumulative response of the scattered light critically depends on the average gain coefficient, associated with the spatial amplification in a single speckle, \( \langle G \rangle \). In both sections 5 and 6, a modified speckle distribution for the total field is determined that shows departure from an exponential-type decrease as a function of the speckle intensity around the expectation value of the highest speckle intensity.

2. Model equations

The system of coupled differential equations for SBS, in the approximation of slowly varying envelopes, can be written as

\[
\left( \partial_t + v_{\text{law}} + \frac{c_s^2}{\omega_{\text{aw}}} (\vec{k}_0 - \vec{k}_1) \cdot \nabla + i \frac{c_s^2}{2 \omega_{\text{aw}}} \Delta_\perp \right) \frac{n_1}{n_0} = i \frac{2k_0^2 \sin^2(\theta/2)}{\omega_{\text{aw}}} \beta_{\text{SBS}} E_0 E_s^* \tag{1}
\]

for the ion acoustic wave (IAW) density perturbation \( n_1 \) (with respect to the average electron density \( n_0 \)) where \( v_{\text{law}} \) stands for the IAW damping (Landau damping), \( c_s \) for the sound speed, \( \omega_{\text{aw}} \equiv 2k_0 c_s \sin \theta/2 \) for the IAW frequency, \( \beta_{\text{SBS}} \) for the coupling coefficient (defined below), \( \vec{k}_{0,1} \) for the wave vectors of the incident (0) and scattered (1) electromagnetic wave components, respectively, and with the angle \( \theta \) between \( \vec{k}_0 \) and \( \vec{k}_1 \) and

\[
\left( \partial_t + \vec{c}_{g,0} \cdot \nabla + \delta \epsilon_0 + i \frac{c_s^2}{2 \omega_0} \Delta_\perp \right) E_0 = -i \frac{\omega_0 n_0}{2 n_e} \frac{n_1 E_s}{n_0}, \tag{2}
\]

\[
\left( \partial_t + \vec{c}_{g,1} \cdot \nabla + \delta \epsilon_1 + i \frac{c_s^2}{2 \omega_1} \Delta_\perp \right) E_s = -i \frac{\omega_1 n_0}{2 n_e} \frac{n_1^* E_0}{n_0}. \tag{3}
\]
for the electromagnetic incident (pump) and scattered wave component with the (complex-valued) amplitudes $E_0$ and $E_s$ at frequencies $\omega_0$ and $\omega_1$, respectively, written in paraxial approximation. Here $\vec{c}_{e,0}$ and $\vec{c}_{e,1}$ are the group velocities and $\delta\epsilon_{0,1}$ stands for the difference of the complex dielectric constant $\delta\epsilon_{0,1} = (\epsilon^2 k^2_0/2\omega_0k_0)[1 - n_e(z)/[n_c(1 + iv_{col}/\omega_0,1)] - k^2/k^2_0]$ with respect to the dielectric constant used in the dispersion relation where $k = k_0(1 - n_{ref}/n_e)^{1/2}$ is the wave number for a reference density $n_{ref}$ (here $= n_0$) and $v_{col}$ the frequency for collisional absorption. The fields are normalized to a reference value $\vec{E}_0$ which is the peak field value associated with the incoming laser flux. The terms $(\vec{c}_e \cdot \nabla)$ can also be expressed as a function of the angle between $\vec{k}_0$ and $\vec{k}_1$ such that $(\vec{c}_e \cdot \nabla) \simeq c_e \cos \theta \partial_z$ holds in the ‘paraxial’ limit.

Note that in equation (1) the dominating term of $(\vec{k}_0 - \vec{k}_1) \cdot \nabla$ for small angles is given by $\equiv 2k_0 \sin(\theta/2)\nabla_z$. We define the dimensionless coupling constant at the peak value of the real field, related by $\tilde{E} = \tilde{E}_0/2$ to the peak field value of the pump (laser) field to which the quantities $E_0$ and $E_s$ are normalized to, as

$$\beta_{\text{SBS}} = \frac{e^2(\tilde{E}_0^2)}{m_e j_e^2 \omega_0^2} = \frac{1}{4} \frac{\epsilon_0 \tilde{E}_0^2}{n_c c k_B T}$$

with $m_e$ ($m_i$) denoting the electron (ion) mass, $T$ the effective ($\simeq$ electron) temperature, $k_B$ the Boltzmann constant and $\epsilon_0$ the elementary dielectric constant.

The laser flux (‘intensity’) is given by

$$I_L = \frac{c \epsilon_0 \tilde{E}_0^2}{2} = 2c \epsilon \tilde{E}^2 = 2\beta_{\text{SBS}} n_e c k T$$

which yields, in practical units, the relation

$$I_L = 9.6 \times 10^{15} \text{ (W cm}^{-2}\text{)} \beta_{\text{SBS}} \frac{n_e}{1.1 \times 10^{21} \text{cm}^{-3}} T_{\text{keV}} \text{ or } \beta_{\text{SBS}} \simeq 0.01 I_{14} \lambda_{\mu}^2 / T_{\text{keV}},$$

where $T_{\text{keV}}$ stands for $Z T_e/\bar{a}$ in units of keV, $\lambda_{\mu}$ for the laser wavelength ($\lambda = 2\pi/k_0$) in microns and $I_{14}$ for the laser intensity in $10^{14}$ W cm$^{-2}$.

3. Scattering amplitude

In [12] by Elisseev et al it has been shown that the scattered light field along an integration path can be expressed in the form

$$E_s(\theta, \nu t) = E_{\text{seed}} \left(1 + \int_0^{\nu t} d\tau \sqrt{\frac{G_i}{\nu}} I_1 \left(2\sqrt{\tau G_i} \right) e^{-\tau} \right),$$

which involves the Bessel function $I_1$ and the quantity $G_i$ representing the spatial SBS gain coefficient resulting from an integration path trespassing the laser pump wave profile with an angle $\theta$ with respect to the propagation direction, see appendix A,

$$G_i(\theta) \equiv \frac{\xi_i(\theta)}{\nu} = \int_{l = -l}^{l} \frac{G_i(l', \theta)}{c_e \nu^2} dl'.$$

The standard growth rate, $\gamma_0$, depends on the angle between the path and the principal propagation axis $l || z$. We recall Elisseev’s approach in appendix A and develop there the time-dependent solution in more detail. The time-dependent solution for $E_s$ has the simple and well-known asymptotic solution

$$E_s(\theta, \tau \to \infty) = E_{\text{seed}} \exp \left( G_i \right),$$
in which $E_{\text{seed}}$ stands for the noise level of the scattered light; it establishes for strongly damped IAWs a time interval $\tau > 2G_1$, see appendix A.

For an inhomogeneous pump wave profile, like in laser speckles, the angular dependence, $\theta$, of the integration path has to be taken into account. It is then evident that the expression (5) for a sufficiently long integration path with respect to the speckle correlation length, is nothing else but the SBS gain over a single speckle, which can be written as

$$ G_I(\theta) \equiv G_{l_R} F(\theta, 0) \sin(\theta/2) $$

with $F$ being a form factor that depends on the shape of the speckle, see appendix B. For the correlation length, one can assume $l_R \sin \theta/2$, with $l_R$ being the Rayleigh length. The peak value of the speckle gain $G_{l_R}$ is then defined as

$$ G_{l_R} \equiv \frac{\gamma_0^2}{c_g v_{\text{iaaw}}} 2l_R \sin(\theta/2)|_{\theta=\pi} = \frac{1}{4} \frac{\omega_1 \omega_{\text{iaaw}} n_0}{c_g} \frac{n_c}{v_{\text{iaaw}}} \beta_{\text{SBS}} $$

with $\gamma_0^2 = \max\{\gamma_0^2(l, \theta)\}$ in $l = [-l_R, l_R]$. Here enters the (normalized) peak laser intensity, $I_{sp} \propto \beta_{\text{SBS}}$, of the speckle and in which $\omega_{\text{iaaw}}(\theta) \equiv 2k_0 c_0 \sin \theta/2$ is the IAW frequency.

The damping for IAWs may similarly depend on the angle, so that the angular dependence of $(\omega/\nu)$ is typically very weak. The latter is the case when linear Landau damping is the dominant damping mechanism, because both the linear IAW Landau damping coefficient as well as the IAW frequency itself are proportional to the IAW wave vector. For viscous damping, usually proportional to the square of the wave number, the angular dependence of $(\omega/\nu)$ has to be considered.

The value of the gain $G_I$ hence depends on the length, $l_{\text{eff}}(\theta)$ of the efficient amplification along the path. We will use the expressions for $G_I$ for BSBS and FSBS for further evaluation in the following sections.

### 3.1. Backward stimulated Brillouin scattering

Expression (8) with $2l_R \sim 2k_0 l_{\perp}^2$ yields the SBS gain for backscattering in a speckle, which is

$$ G_{\text{BSBS}} \equiv G_{l_R} \equiv \left(\frac{\omega}{\nu}\right)_{\text{iaaw}} \frac{n_0}{n_c} \frac{\omega_1}{c_g} \frac{I_{sp} l_{\perp}^2}{\lambda_0 n_c c_0 k T} = \frac{\pi}{2} \left(\frac{\omega}{\nu}\right)_{\text{iaaw}} \frac{n_0}{n_c} \beta_{\text{SBS}} \frac{l_R}{\lambda} $$

for homogeneous plasmas in the strong damping limit. It is well known that the amplification of SBS in an inhomogeneous plasma can be expressed via Rosenbluth’s gain coefficient, replacing $l_R$ by the effective interaction length $\sim v_{\text{iaaw}} L_v/c_s$ in a plasma with flow gradient length $L_v$, see e.g. [14]. The latter is of interest when the contribution of speckles from subsequent plasma layers can be considered to be independent.

### 3.2. Near forward stimulated Brillouin scattering

For FSBS, $G_I$, being by the factor $l_{\text{eff}}/l_R < 1$ smaller than the value for backscattering, can be computed along the integration path for a given angle $\theta$ and the impact parameter $x_i$, see appendix B. There it is shown that the scattered field over one speckle should take into account all paths with a shift with respect to the central path. In appendix B we compute the angle-dependent form factor that comes into play when integrating over all paths. Following the analysis in appendix B, reporting equation (B.8), the relevant expression for the SBS gain along
the central path (setting the impact factor to $x_i = 0$) can be written as

$$G_{sp} = G_{lk} F(\theta, x_i = 0) \sin(\theta/2) \simeq G_{lk} \frac{l_\perp}{l_R} \frac{\sqrt{\pi}/\cos(\theta/2)}{1 + \sqrt{1 + (4/\pi)(l_\perp/l_R)^2 \cot^2 \theta}},$$

(10)

with $G_{sp} < G_{lk}(l_\perp/l_R)(\sqrt{\pi}/2 \cos(\theta/2))$ for $\theta < \pi/2$.

It is important to mention that expression (9) is by a numerical factor $f_{num}$ of the order of unity and by the factor relating IAW frequency and damping, $(\omega/v)_{iaw}$, equivalent to the ratio $P_{sp}/P_c$ between the laser power in a speckle, $P_{sp} = \pi l_{sp}^2$ and the SF critical power [15–17],

$$G_{FSBS} \equiv G_{sp} = f_{num} \frac{l_\perp(\theta)}{l_R} \frac{(\omega)}{(v)_{iaw}} \frac{P_{sp}}{P_c}$$

(11)

with $P_c = 2N_c(c\lambda_0/\omega_0)(n_c/n_0)n_c c k T (1 - n_0/2n_c)$, in which $N_c$ assumes the value of $N_c = 1.90$ for the example of a Gaussian beam (with a corresponding definition of $l_\perp$, see [18]). This is an important issue for discussion on the dominating process, namely between FSBS and SF, see section 6.

The analysis in appendix B also shows that we can, with good approximation, make use of equation (6) to express the SBS amplification in a speckle by using $G_{sp}$ from equation (10) as the gain value.

4. Total field: pump and scattered fields over numerous speckles

For the perturbative approach that allows separation of the pump and scattered light field contributions to the total field in the plasma, we establish in this section a hierarchy of the different terms contributing to the total light field intensity. In the total field, the complex field components of the pump field $E_0$ and the scattered light field $E_s$ are superposed. The sum over speckles (index $j$) then reads

$$E_{tot} = \sum_j E_{0j} e^{i\Phi_j} + \sum_j E_{sj} e^{i\Psi_j}.$$  (12)

This superposition in a nonlinear medium, such as the plasma, is valid as long as either the spectral components (in the frequency and/or wave vector) are well separated or the amplitude of the scattered field can be considered small with respect to the pump field.

The square field $|E_{tot}|^2$ has then components that can be ordered in powers of the smallness parameter $\varepsilon \propto |E_s/E_0|$ associated with a perturbative approach,

$$|E_{tot}|^2 = O(\varepsilon^0) + O(\varepsilon) + O(\varepsilon^2) + \ldots,$$

(13)

with $O(\varepsilon^0) = \sum_j |E_{0j}|^2 + \sum_j \sum_{k \neq j} E_{0j} E_{0k}^* e^{i(\Phi_j - \Phi_k)} + c.c.$,

$$O(\varepsilon) = \sum_j \sum_k E_{0j} E_{sk}^* e^{-i(\Phi_j - \Psi_k)} + c.c.,$$

$$O(\varepsilon^2) = \sum_j |E_{sj}|^2 + \sum_j \sum_{k \neq j} E_{sj} E_{sk}^* e^{i(\Phi_j - \Psi_k)} + c.c.$$
By using speckle statistics, the sums can be replaced by integrals over the speckle distribution
\[ \sum_j E_{0j} E_{sk}^* e^{i(\Phi_j - \Psi_k)} + \text{c.c.} = \sum_j E_{0j} E_{sj}^* e^{i(\Phi_j - \Psi_j)} \]
\[ + \sum_j \sum_{j \neq k} E_{0j} E_{sk}^* e^{i(\Phi_j - \Psi_k)} e^{-i2k_{0c} t \sin \theta/2} + \text{c.c.,} \quad (14) \]

wherein \( \Phi_j \) and \( \Psi_j \) are uniform random variables in the interval \((-\pi, \pi\).\]

Note, however, that for stimulated scattering the phases of the scattered light \( \Psi_j \) have to be considered enslaved to the pump field. Consequently, the phase \( \Psi_j \) of the scattered field excited in speckle \( j \) is no longer independent of the phase \( \Phi_j \) of the pump field in the same speckle.

The second order terms \( O(e^2) \) read analogously
\[ \sum_j \sum_k E_{sk} E_{sj}^* e^{i(\Psi_j - \Psi_k)} + \text{c.c.} = \sum_j |E_{sj}|^2 + \sum_j \sum_{j \neq k} E_{sj} E_{sk}^* e^{-i4k_{0c} t \sin \theta/2} e^{i(\Psi_j - \Psi_k)}. \quad (15) \]

Each of the terms can be ordered in powers of the average field intensity of the incident field, \( \langle I \rangle \propto \langle |\hat{E}_0|^2 \rangle^{1/2} \) and the relative (normalized) intensity \( M_j \) of a speckle, \( |E_{0j}(M)| \propto M_j^{1/2} \langle I \rangle \).

By using speckle statistics, the sums can be replaced by integrals over the speckle distribution
\[ dF(M) = f_{sp}(M) dM, \quad (16) \]
for which we specify \( f_{sp} \), the probability density function (pdf), below. This yields for the terms containing \( E_{sj} \) average contributions
\[ \left< \sum_j E_{0j} E_{sj}^* \right> = \sum_j \int dF(M) |E_{0j}(M)| |E_{s}(\theta, t, M)| e^{i(\Phi_j - \Psi_j)} e^{-i2k_{0c} t \sin \theta/2} \]
\[ = \langle I \rangle e^{-i2k_{0c} t \sin \theta/2} \sum_j \left< e^{i(\Phi_j - \Psi_j)} \right> \int dF(M) M^{1/2} |E_s(\theta, t, M)|, \quad (17) \]
\[ \left< \sum_j \sum_{j \neq k} E_{0j} E_{sk}^* \right> \]
\[ = \sum_j \sum_{j \neq k} \int dF(M) \int dF(M') |E_{0j}| |E_{s}(\theta, t, M')| \left< e^{i(\Phi_j - \Psi_j)} \right> e^{-i2k_{0c} t \sin \theta/2} \]
\[ = \langle I \rangle e^{-i2k_{0c} t \sin \theta/2} \sum_j \sum_{j \neq k} \left< e^{i(\Phi_j - \Psi_j)} \right> \int dF(M) M^{1/2} \int dF(M') |E_s(\theta, t, M')|, \quad (18) \]
\[ \left< |E_{sj}|^2 \right> = \int dF(M) |E_s(\theta, t, M)|^2 \quad (19) \]
and
\[ \left< \sum_j \sum_{j \neq k} E_{sj} E_{sk}^* \right> = \sum_j \sum_{j \neq k} \int dF(M) |E_s(\theta, t, M)| \int dF(M') |E_s(\theta, t, M')| \left< e^{i(\Phi_j - \Psi_j)} \right>. \quad (20) \]
We refer to [18] for the calculation of the distribution of the maxima $M_j$, $f_{sp}(M) = Z^{-1} \exp((1/2)e^{M-\log n_{sp} - M})$ with $Z = \int_1^{\log n_{sp} + \gamma_e} \exp((1/2)e^{M-\log n_{sp} - M})dM$. The value of $Z$ can be approximated by the integral $\int_0^{\gamma_e^2} \exp(u)u^{-2} du \simeq 0.736n_{sp}$ with $n_{sp}$ number of speckles (in finite volume) and the Euler Gamma constant $\gamma_e \simeq 0.5772$.

Note that we can assume that the term in equation (17), $\sum_{j=1}^{N}(e^{(\Phi_j - \Psi_j)})/N$, is of the order of unity, as the phases $\Phi_j$ and $\Psi_j$ are relative to the same speckle. The sum $\sum_{j,k}^{\neq} (e^{(\Phi_j - \Psi_k)})$ in equation (20), in which the phases $\Phi_j$ and $\Psi_k$ refer to different $j \neq k$, is a finite quantity that we need not compute here as it appears in negligible orders.

Indeed, all terms with $j \neq k$ in the preceding expressions and in equation (13) correspond to products between the scattered field originating from speckle $k$ and the pump field from speckle $j \neq k$, for which the correlation is in general weak and assuming a correlation length close to the typical speckle size—an assumption used for independent speckles. The sums over a single index should therefore, in general, be dominant. The terms of equations (17) and (19) can be evaluated in the following via the integral over the speckle distribution functions and powers of the speckle intensity $I$.  

4.1. Backward stimulated Brillouin scattering and the role of phase conjugation

For BSBS, in contrast to FSBS discussed below, the components $E_0$ and $E_s$ are spectrally well-separated in wave vector space and the terms $\sim E_{0j}E_{sk}$ (for any $k,j$) are rapidly oscillating functions in $z$ with $|2k_0 \sin \theta/2| \simeq 2k_0$ (corresponding to the IAW response). Due to this spectral separation, the backscattered light response is represented by $\propto \sum_{j,k} E_{sj}E_{sk}^*$, dominated by $\sum_j E_{sj}E_{sj}^*$; hence we can use equation (19) to evaluate the response. The requirement for the separation between $E_0$ and $E_s$ to be valid is thus essentially determined by flux conservation, $|c_{g,0}E_0E_0^*| - |c_{g,1}E_sE_s^*| = \text{const}$, the consequence of which is pump depletion in the case of strong backscattering.

In our evaluation of the contributions from scattering in speckles, equations (16)–(20), we have not accounted for phase conjugation (PC) [19], which is a phenomenon that has particular relevance in the case of backscattering. To account for it, one has to be aware that the scattered light may contain two components, $E_s = \sigma E_0^* + E_{s,n}$ (with $\sigma$ real and positive), one of them being proportional to the conjugated pump field $E_0^*$, while the other component, $E_{s,n}$, is the ‘noise’ excluding the PC component. The value of $\sigma$ indicates then the magnitude of the portion of the scattered light seed that is due to PC.

Using this ‘orthogonal’ decomposition, one can compute the contributions from speckles out of the ‘noise’ source seed term with the help of expression (19). For PC it is the spatial average of $E_0E_s$ over the dimensions transverse to the propagation direction $\langle E_0E_s \rangle = \sigma |E_0|^2 \propto \sigma \langle |I| \rangle$ that leads to a spatial amplification, with $\langle |I| \rangle$ being the average beam intensity. The ensemble average over speckle realizations, as in (19), is therefore unnecessary (for an $\langle |I| \rangle$ value independent of the RPP realization). The phase conjugated scattered light signal will be amplified at average intensity along $z$. The total amplification is then either limited by the length of the interaction region or by the finite spatial extent of the resonant coupling (for inhomogeneous plasmas). One can associate a gain value corresponding to PC, $G_{pc}(z) = 2z_{\text{R}}^2/(c_0v_{\text{Iaw}})$ wherein $z_{\text{R}}^2$ is evaluated at the average intensity of the beam, so that we can express it in units of the average speckle gain $G_{sp}(z) = 2G_{sp}|_{M=1} z/l_R = 2\langle G\rangle_{\text{BSBS}} z/l_R$. 

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Analogous to the asymptotic solution for scattered light in equation (6), the amplification due to PC hence results in

\[ |E_{s,pc}(L)|^2 = |E_0|^2 \sigma^2 e^{G_{pc}} = \sigma^2 \langle I \rangle e^{2\varepsilon_f(G)_{BSBS}}, \]

(21)

4.2. Forward stimulated Brillouin scattering

In the case of FSBS the pump and the scattered light fields are spectrally not strongly separated because the wave number and frequency shifts of the scattered light (∼2k_0\sin\theta/2) are small. In a perturbative approach, as used here for a layer limited to about one speckle length, the dominating modification to the pump field can be evaluated from equation (17), while terms from equation (19) should remain small. In section 5 we will hence determine the modification of the total field of speckles due to FSBS by evaluating equation (17).

5. Ordering contributions to the total field from speckles

In section 3 we have derived the response of the scattered light from a single speckle, which can be written in the asymptotic limit as equation (6), with the gain coefficient for convective amplification from equation (10). The result for the scattered light collecting the contributions from numerous, independent, single speckles brings about the problem of contributions from extreme speckles to the total field. For this reason we will express, from now onwards, the single speckle gain \( G_{sp} \) as a multiple \( M \) of the gain of an average speckle

\[ G_{sp} = M \langle G \rangle, \]

where \( \langle G \rangle \) refers to the gain in a speckle with the average beam intensity \( \langle I \rangle \).

In [18] order statistics of speckle fields have been investigated, which will be used in the following.

5.1. Backward stimulated Brillouin scattering from speckles

For BSBS, apart from contributions due to PC, the response function from a single speckle, indexed \( k \), that contributes to the total field via expression (19) can be written as

\[ R_{BSBS}(M_k) = \min(1, E_{seed}^2 e^{2\langle G \rangle_{BSBS} M_k}) = E_{seed}^2 \exp[2\langle G \rangle_{BSBS} \min(M_k, M_{*BSBS})] \]

(22)

where \( M_k \) is the speckle peak intensity normalized to the average pump field intensity, \( \langle G \rangle_{BSBS} \) is the gain of an ‘average intensity’ speckle and \( M_{*BSBS} = (G)_{BSBS}^{-1} \ln(E_{seed}^{-1}) \) denotes the ‘cutoff’ of exponential growth in \( M_k \) values after which the response saturates. The latter definition is an auxiliary way to account for pump depletion, which works sufficiently well, but can of course be described in an improved manner, usually leading to an expression which is no longer explicit for \( R(M_k) \) in BSBS [20].

In what follows we continue the evaluation of the cumulative response from speckles for the case of FSBS. The derivation for the case of BSBS, not explicitly shown here, is by analogy straightforward and will be discussed again in section 5.4.
5.2. Forward stimulated Brillouin scattering from speckles

For FSBS the response function from a single speckle, indexed \( k \), that contributes to the total field via expression (17) reads similarly to BSBS:

\[
R_{\text{FSBS}}(M_k) = \sqrt{M_k} \min(1, E_{\text{seed}} e^{(G)M_k}) = M_k \tilde{E}_{\text{seed}} \exp\{\langle G \rangle \min(M_k, M^*)\},
\]

where \( M_k \) is the speckle peak intensity normalized to the average pump field intensity and \( \langle G \rangle \) is the FSBS gain of an ‘average intensity’ speckle. The speckle gain defined in equation (11) is hence replaced by \( M_k \langle G \rangle \) from now on. Here we choose the seed amplitude as proportional to the speckle field amplitude, namely \( E_{\text{seed}} = \tilde{E}_{\text{seed}} \sqrt{M_k} \) (and not to the average field amplitude) and allow for pump depletion in order to limit the exponential amplification of the scattered light. Accordingly, the (normalized) speckle intensity and allow for pump depletion in order to limit the exponential amplification of the scattered light. Accordingly, the (normalized) speckle intensity \( M^* \) defined via \( E_{\text{seed}} \exp\{\langle G \rangle M^*\} = \tilde{E}_{\text{seed}} M^{*1/2} \exp\{\langle G \rangle M^*\} \equiv 1 \) corresponds to the intensity where the scattered light starts to be saturated by pump depletion, hence

\[
M^* = \langle G \rangle^{-1} \ln(E_{\text{seed}}^{-1}) = \langle G \rangle^{-1} \left( \ln(\tilde{E}_{\text{seed}}) - \frac{1}{2} \ln M^* \right),
\]

the latter being an implicit expression (that can be solved iteratively, \( M^* < \langle G \rangle^{-1}[\ln(\tilde{E}^{-1}) + \frac{1}{2} \ln(n_{sp} + \gamma_E)] \), with \( \gamma_E = 0.5772 \) standing for the Euler gamma constant).

The total (or cumulative) response from all \( n_{sp} \) speckles can be written as

\[
\mathcal{R}_{\text{tot}} = \sum_{k=1}^{n_{sp}} \mathcal{R}(M_k) = \sum_{k=1}^{n_{sp}} \tilde{R}(M_k) \mathcal{P}^{(G)}(M_k)
\]

with \( \tilde{R}(M_k) = Z E_{\text{seed}} M_k \) and \( Z = \sum_k \exp\{\langle G \rangle \min(M_k, M^*)\} \) so that

\[
\mathcal{P}^{(G)}(M_k) = Z^{-1} \exp\{\langle G \rangle \min(M_k, M^*)\},
\]

wherein the values of \( M_k \) correspond to the values of speckle maxima taken from a speckle field realization.

To the cumulative field response one can associate a typical speckle intensity

\[
\langle M \rangle \equiv \frac{\mathcal{R}_{\text{tot}}}{E_{\text{seed}} Z},
\]

that indicates the intensity of the speckle population which contributes most to the scattered light field \( \mathcal{R}_{\text{tot}} \) as discussed later.

By introducing a speckle order \( M_1, \ldots M_k \) following the hierarchy \( M_1 > M_2 > \cdots > M_k \), strong fluctuation in \( \mathcal{R}_{\text{tot}} \) can be expected when the value of \( \langle M \rangle \sim M_1 \sim \ln n_{sp} \).

To evaluate properly the cumulative response from the total field on the basis of statistics, one can make use of order statistics, which allows to take into account the intervals in \( M \) over which the successive speckle maxima can occur. By defining as \( f_k(M) \) the probability density of the \( k \)th order speckle, the cumulative response can be, in contrast to equation (25), defined as

\[
\tilde{\mathcal{R}}_{\text{tot}} = \sum_{k=1}^{n_{sp}} \tilde{E}_{\text{seed}} \int_0^{\infty} M f_k(M) e^{(G) \min(M, M^*)} \, dM,
\]
wherein the probability density follows the expression \[ f_k(M) = \frac{n_{sp}^k}{\Gamma(k)} e^{-n_{sp} \exp \{-M\} - kM} \] (29) which peaks close to \( M \sim \ln(n_{sp})/k \). It follows from order statistics that the values \( M_k \) of the most intense maxima cover a large interval in \( M \), while smaller, and in our notation higher-indexed maxima, can be well described by a shrinking interval in \( M \) around their expectation value \( \bar{M}_k \) from equation (29).

A technical remark concerning the evaluation of equation (28) using (29) and (31): the interval in \( M \) on which the function \( f_k(M) \) assumes non-negligible values becomes narrower with increasing orders of \( k \). For this reason it is not necessary to evaluate the integrals in equation (28) for high orders of \( k \), i.e. lower-intensity speckles. This applies for \( \langle G \rangle < [k \ln(n_{sp}/k)/2]^{1/2} \). Practically, for \( k > k_\ast \), the integral in equation (28) can be simply replaced by evaluating the response function for a single speckle at the expectation value \( M_k \equiv \int M f_k(M) \, dM \) of the \( k \)th order speckle from \( M_k^{(G)} \equiv \int M f_k^{(G)}(M) \, dM \),

\[
\tilde{R}_{tot} = \sum_{k=k_\ast}^{k-1} \int_0^\infty M f_k(M) e^{(G) \min(M,M^* \bar{M})} \, dM + \sum_{k=k_\ast}^{n_{sp}} \tilde{R}(\tilde{M}_k) P^{(G)}(\tilde{M}_k). \tag{30}
\]

5.3. Modified probability density of speckles

To find equivalence between equations (28) and (25) we can introduce a new probability density function \( f_k^{(G)}(M) \) in equation (28) such that

\[
f_k^{(G)}(M) = (1/N_k^{(G)}) f_k(M) e^{(G) \min(M,M^*)} \tag{31}\]

with the normalization coefficient \( N_k^{(G)} \equiv \int_0^\infty f_k(M) \exp\{(G) \min(M,M^*)\} \, dM \). This results in an alternative definition (to expression (27)) of the average speckle intensity,

\[
\langle \bar{M} \rangle = \tilde{R}_{tot}/E_{seed} \tilde{Z}, \tag{32}
\]

with \( \tilde{Z} \equiv \sum_{k} N_k^{(G)} \). It is important to note that \( f_k^{(G)} \) contains the factor \( \exp\{(G) - kM\} \) which changes from an exponential decrease to growth when the average gain \( \langle G \rangle \) exceeds \( k \), the speckle order index. For this reason, one can observe a critical change in the features of the scattered light in the vicinity of \( \langle G \rangle = 1 \), coming primarily from the most intense speckle \( M_1 \), as illustrated by figures 1–7.

Corresponding to \( f_k^{(G)}(M) \) we can define a complementary probability distribution function

\[
F^{(G)}(M) \equiv \sum_{k} \int_M^\infty f_k^{(G)}(M') \, dM' \tag{33}
\]

which is the equivalent to \( \int_M^\infty f_{sp}(M) \) in equation (16) for \( \langle G \rangle = 0 \).

To compare \( R_{tot} \) with \( R_{tot}^{(G)} \) in equation (16) for \( \langle G \rangle = 0 \),

\[
\bar{M}_k = \int_0^\infty M f_k \, dM = \bar{M}_{k-1} - \frac{1}{k} \ln n_{sp} - \ln(k-1) - \frac{1}{2(k-1)} \text{ for } k > 1 \tag{34}
\]
Figure 1. Speckle expectation values computed from equation (35) for speckle orders 1, . . . , 5, 10 and 50, as well as the value of \( M^* \) for \( n_{sp} = 3000 \) and \( \bar{E}_{seed} = 10^{-5} \). Values of \( M_k^{(G)} < M^* \) are affected by pump depletion.

Figure 2. Speckle expectation values computed from equation (35) for speckle orders 1, . . . , 5, 10 and 50, as well as the value of \( M^* \) for \( n_{sp} = 3000 \) and \( \bar{E}_{seed} = 10^{-15} \). Values of \( M_k^{(G)} < M^* \) are affected by pump depletion.

with \( \bar{M}_1 = \ln n_{sp} + \gamma_E \) or by (ii) from equation (31)

\[
\bar{M}_k^{(G)} = \int_0^\infty M f_k^{(G)}(M) \, dM = \frac{\bar{E}_{seed} \int_0^{M_k^*} M f_k^{(G)}(M) e^{(G)M} \, dM + \int_{M_k^*}^{\infty} M f_k^{(G)}(M) e^{(G)M} \, dM}{\bar{E}_{seed} \int_0^{M_k^*} f_k^{(G)}(M) e^{(G)M} \, dM + \int_{M_k^*}^{\infty} f_k^{(G)} \, dM}.
\]

Note that \( \bar{M}_k = \bar{M}_k^{(G)} \big|_{(G)=0} \).

The expression shows that due to the modification of the field from scattering, expectation values of the intense speckle maxima will shift \( \bar{M}_k \) to higher values \( \bar{M}_k^{(G)} > M_k^{(G)} \big|_{(G)=0} \) for gain
Figure 3. Histogram of the speckle distribution to linear-log scale. Symbols show the speckle number $k$ (ordered from the most intense to the least intense) as a function of its expectation value $\bar{M}_k^{(G)}$ computed from equation (35) for different values of the average gain $\langle G \rangle = 0.5$ (red), 1 (pink), 1.5 (blue), 2 (grey), 0 (black); lines show the speckle distribution $F^{(G)}$ as a function of the normalized speckle intensity $M$ computed from equation (33), in the same colour as for the symbols. Parameters: $n_{sp} = 3000$ and $\bar{E}_{seed} = 10^{-5}$.

Figure 4. Histogram of the speckle distribution as in figure 3 computed from equations (35) and (33) to linear-logarithmic scale. Symbol index for different values of the average gain $\langle G \rangle = 0.5$ (red cross), 1 (pink), 1.5 (blue), 2 (red triangle), 3 (grey), 0 (black). Parameters: $n_{sp} = 3000$ and $\bar{E}_{seed} = 10^{-15}$.

values around and beyond $\langle G \rangle \sim 1$. We illustrate this for two cases: one considering $n_{sp} = 3000$ speckles in the volume and with a seed level for the scattered field of $\bar{E}_{seed} = 10^{-5}$, and an extreme case (with the same $n_{sp}$) with a very low seed value, $\bar{E}_{seed} = 10^{-15}$, so that pump depletion of individual speckles practically does not arise. Speckle expectation values computed from equation (35) for speckle orders 1...5, 10 and 50 are shown in figures 1 and 2 together.
Figure 5. Average speckle intensity $\langle M \rangle$ over $n_{sp} = 3000$ speckles as a function of the gain value $\langle G \rangle$ computed from equation (27) for $E_{seed} = 10^{-5}$ using the values of $M_k^{(G)}$ of equation (35) (star symbols) and of $M_k$ of equation (34) in equation (25).

Figure 6. Average speckle intensity $\langle M \rangle$ over $n_{sp} = 3000$ speckles as a function of the gain value $\langle G \rangle$ computed from equation (27) for $E_{seed} = 10^{-15}$ using the values of $M_k^{(G)}$ of equation (35) (star symbols) and of $M_k$ of equation (34) in equation (25).

with the value of $M^*$ indicating the onset of pump depletion once $M_k^{(G)} > M^*$. For $E_{seed} = 10^{-5}$ pump depletion rapidly limits the influence of the most intense speckles for increasing values of $\langle G \rangle > 1$. For the extreme case with a very low seed level, many intense speckles remain unaffected by pump depletion up to average gain values $\langle G \rangle \sim 5$, so that the modification of the values $M_k^{(G)}$ as a function of $\langle G \rangle$, with respect to $\langle G \rangle = 0$, covers a wider range than for lower seed levels.
The expressions for the speckle distribution equation (33) and for the expectation values for the $k$th order speckle allow to construct histograms as a function of the speckle peak intensity, as shown in figure 3 for $n_{sp} = 3000$ speckles, $E_{seed} = 10^{-5}$ and (4) for the case with an extremely low seed value $E_{seed} = 10^{-15}$. The histograms from the expectation values are constructed by plotting the speckle order count (from the most to the least intense speckle) as a function of its expectation value $M_k^G$. The histogram for no amplification, $\langle G \rangle = 0$, follows an exponential with a slightly slower decrease for the most intense speckles, as derived in [18]. For the case of $E_{seed} = 10^{-5}$, a strong modification of the histogram in the tail for the most intense speckles is limited to the gain interval $0.5 < \langle G \rangle < 1.5$. In a limited interval of $M$ values, around $M \sim M_1 \sim \ln n_{sp}$, a departure from Gaussian statistics can be observed. The values computed via the expectation values and via equation (33) show good agreement and reproduce the same tendency. The extreme case with a very low seed value and much less pump depletion in intense speckles, figure 4, shows much stronger departure from $\langle G \rangle = 0$ and for all shown values $\langle G \rangle > 0.5$. The decrease in the tail of the distribution is considerably slower than an exponential one (close to a power-law up to a cutoff value) depending on $\langle G \rangle$.

The quantity $\langle M \rangle$ deduced from equation (27) or (32) of the transmitted light field is a good measure to understand which speckle population the scattered light field originates from. This is illustrated in figures 5 and 6. Three different curves for the value of $\langle M \rangle$, with similar behaviour, are shown. These follow three different methods to evaluate $\langle M \rangle$. In the first case (i) $\langle M \rangle$ results from equation (32) by evaluating equation (28), where the sum over the integrals of all speckle orders is computed; in the second and third cases, (ii) and (iii) respectively, one simply computes the sum according to expression (25) by evaluating $R(M_k)$ at discrete values, namely in case (ii) $M_k = M_k(G = 0)$ from equation (34) and in case (iii) $M_k = \tilde{M}_k(G)$ from equation (35). For all three cases, a rapid transition from $\langle M \rangle \simeq 1$ towards values close to the...
expectation value of the most intense speckle $\langle M \rangle \sim \bar{M}_1$ can be observed close to $\langle G \rangle \simeq 1$. The values evaluated following method (i) are the most precise, while methods (ii) and (iii) are easier to compute.

The steepness in the transition around $\langle G \rangle = 1$ is more pronounced when accounting properly for order statistics. This quite steep transition, with $d\langle M \rangle / d\langle G \rangle$ being positive and maximum around $\langle G \rangle = 1$, is a clear signature of critical behaviour as also discussed in [9, 11] and more recently shown in [10] for the cumulative backscatter reflectivity of stimulated Raman backscattering as a function of gain. The value of $\langle M \rangle$ shown as a function of the gain value $\langle G \rangle$ indicates which speckle population of the speckle field contributes most to the cumulative response of the scattered field $R_{\text{tot}}$. For the case with an extremely small seed, the average value $\langle M \rangle$ remains considerably greater than 1 up to $\langle G \rangle \sim 5$, while for the case with $E_{\text{seed}} = 10^{-5}$ due to pump depletion $\langle M \rangle$ converges towards 1 with $\langle G \rangle$.

The fact that for $\langle G \rangle \simeq 1$ the value of $\langle M \rangle$ is close to $\bar{M}_1(\langle G \rangle \simeq 1)$, has also the consequence that the spread of the response of the scattered field from all speckles depends also critically on the variance of $M_1$ itself. The standard deviation $\delta M = (\langle M^2 \rangle - \langle M \rangle^2)^{1/2}$ of the (normalized) speckle intensity $M$ as a function of $\langle G \rangle$ is shown in figure 7. It is evident that for the regime when the relatively low abundant intense speckles contribute most, i.e. around $\langle G \rangle \sim 1$, the cumulative response varies strongly for different realizations which is translated in an important spread $\delta M$ around the average value $\langle M \rangle$. This spread $\delta M$ can be of the order of $\langle M \rangle/2$. The expectation value of cumulative scattering response $R_{\text{tot}}$ has hence a very low reliability in this regime. As a function of $\delta M$ one can estimate the standard deviation in $R_{\text{tot}}$, namely $\Delta R_{\text{tot}}/R_{\text{tot}} \simeq \{\exp(\langle G \rangle \min(\langle M \rangle + \delta M, M^*) - \exp(\langle G \rangle \min(\langle M \rangle - \delta M, M^*))\}/2$ yielding $\Delta R_{\text{tot}}/R_{\text{tot}} \sim \exp(4)$ for $\langle G \rangle = 1$ and $\delta M \sim 4$, it follows that $R_{\text{tot}}$ may vary strongly from one to another speckle pattern realization.

5.4. Consequences of backward stimulated Brillouin scattering and the presence of phase conjugation

We recall here that the derivation to compute the expectation values $\langle M \rangle$ and the standard deviation $\delta M$, here done for the example of FSBS, can analogously be applied to BSBS. Mounaix and Divol [11] have shown in their work, based on simulations using numerous speckle field realizations accompanied by a concise analytical model, that the standard deviation to the average backscatter reflectivity from an ensemble of laser speckles strongly increases for $\langle G \rangle > 0.5$ and peaks at $\langle G \rangle \sim 1.5$ with $\Delta R_{\text{tot}}/R_{\text{tot}} \sim 30$. Our results support these findings based on numerical simulations, as illustrated in figure 7.

For BSBS, the role of PC has to be discussed with respect to the cumulative response of backscattered light from speckles. Rose and Mounaix [10] have investigated this problem for the case of stimulated Raman scattering. As a first measure one has to compare the gain values in a single speckle, $\langle G \rangle M$, to the equivalent gain for PC, $2\langle G \rangle z/l_R$, involved in equations (22) and (21), respectively. PC could then dominate the backscatter response if its amplification along $z$ is not limited (by a box length or a characteristic length) so that $2z/l_R > \alpha_{\text{pc}} + \langle M \rangle$, where $\langle M \rangle$, as defined above, stands for the typical intensity of speckle dominating the backscatter response and where $\alpha_{\text{pc}} = (G)^{-1}\ln(E_{\text{seed}}/\sigma)$ compares the levels between speckle and PC seed. As pointed out in [21], supposing $E_{\text{seed}} = \sigma$, PC will be dominant for small speckle gains $\langle G \rangle$ because $\langle M \rangle \sim 1$, as evident from figures 5 and 6. In homogeneous plasmas (media)
where hypothetically \( z/I_R \) can attain large values, pump depletion due to PC could occur, \( 2z/I_R > \ln(1/\alpha) \) which would of course question the contribution of the speckle ensemble in the pump-depleted zone.

6. Numerical simulations

We have carried out numerical simulations in two spatial dimensions (2D) with the paraxial laser–plasma coupling code HARMONY2D [15, 22]. While the role of intense speckles for gain values around unity for the case of BSBS has clearly been demonstrated via numerical simulations in [11], we concentrate here on the case of FSBS. To simulate this situation, we use a single equation for the electromagnetic light field propagating in the forward direction. Hence, for the simulations we do not separate the light field in a perturbative manner as in the preceding analysis. Equation (2) is hence solved for \( E = E_0 + E_\varepsilon \). The response of the IAWs, equivalent to equation (1), is computed via the fluid response of the plasma under the ponderomotive drive proportional to \( \nabla E E^* \) which contains the term the \( \propto E_0 E_\varepsilon^* \) driving the IAW resonant with the FSBS process, as well as the term \( \nabla \cdot E_0 E_\varepsilon^* \) responsible for SF and filamentation. The equations in the fluid code account for a wave number-dependent damping so that, as in our model, both the IAW frequency as well as the IAW damping follow the same wave number dependence. In these simulations, no BSBS occurs because the short-wavelength IAWs are not seeded. Both FSBS and SF are in direct competition because of the slower time scale and the longer IAW wavelength with respect to BSBS. This competition is the subject of substantial, and in our opinion not yet conclusive, discussion in the literature [1, 3, 4].

The onset of SF in 2D simulations has been determined in [3, 15], leading to the criterion for speckle power, \( P_{sp} \) to SF critical power, \( P_c \)

\[
P_{2D} \equiv \left( P_{sp}/P_c \right)_{2D} = I_{2D}^2 k_{opt}^2 \approx (\pi^2/2) f_g^2 (n_0/n_c) \beta_{SBS},
\]

where \( k_{opt} \) stands for the transversal wave number with optimum growth rate for filamentation and where \( I_{2D} \approx 0.5 f_g \lambda \) is the typical radius of a (Gaussian-type) RPP speckle (here in 2D), see [17] and the appendix in [18].

The speckle gain for FSBS, as discussed in the preceding sections, can be estimated using equation (11), \( G_{sp,2D} \approx (0.2/f_g) G_{l_R} \approx 0.25 \pi (\omega/\nu)_{iaw} (n_0/n_c) \beta_{SBS} f_g \), where we have assumed \( I_{2D} \approx 2.5 f_g^2 \lambda \) [18]. For an average speckle gain of \( G_{sp,2D} = \langle G \rangle \equiv 1 \), we have \( \beta_{SBS} = 0.01 I_{14} \lambda^2 \mu_e/TeV \approx 1.27 (\nu/\omega)_{iaw} (n_c/n_0) f_g^{-1} \) and for \( p_{2D} \equiv 1 \), \( \beta_{SBS} \approx 0.2 (n_c/n_0) f_g^{-2} \).

For \( f_g = 4 \), \( (\nu/\omega)_{iaw} = 0.03 \) and \( \beta_{SBS} \approx 0.025 \), both \( p_{2D} \) and \( G_{sp,2D} \) are close to 1. In this parameter regime the onset of SF should appear together with effects from FSBS due to intense speckles. For higher dumping \( (\nu/\omega)_{iaw} > 0.07 \), SF for an average speckle and thus for the entire beam, should be expected to be the dominant process.

We have chosen to run simulations for which \( p_{2D} \) stays below the SF threshold \( p_{2D} = 0.5 \). Indeed for the regime \( P_{sp}/P_c < 1 \) it was suggested to introduce a new criterion for the onset of beam spreading effects [4], depending on the IAW damping. To control the average speckle gain for FSBS and to discriminate the behaviour from SF \( (\langle G \rangle = G_{sp,2D} \approx 1 \) namely, \( G_{sp,2D} = 0.5, 1 \) and 1.5), we varied the IAW damping \( (\nu/\omega)_{iaw} = 0.03, 0.015 \) and 0.01, respectively. While the onset of SF should not directly depend on IAW damping, the change of the FSBS gain value

4 For low values of IAW damping, wave trains originating from the density evacuation out of intense speckles can eventually influence the density in neighbouring speckles; however, there is low likelihood that such wave trains arrive in other intense speckles.

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governs the behaviour of the plasma response. No additional noise for FSBS was imposed to seed the process. For this reason, both SF and FSBS grow from the ponderomotively-induced density perturbations and the IAW evolving out of these perturbations.

We have chosen a simulation box 100 laser wavelengths ($\lambda$) long, corresponding to $2.5 \lambda_{R,2D}$ for the optical beam aperture $f_\# = 4$, and 8200 wavelengths wide. To smooth the incident laser light corresponding via RPPs, the method proposed in [17] was used. For this configuration the simulation volume should contain about 2200–2400 speckles in width, which corresponds to $\log n_{sp} \sim 7.8$.

Figure 8 shows a snapshot of the light field patterns in a part of the interaction volume, for the mentioned cases (a) $\langle G \rangle = 0.5$, (b) $= 1$ and (c) $= 1.5$. The light intensity is displayed in units of the mean intensity. The most significant changes arise clearly from intense speckles, as illustrated in the supplementary movie 1 (available from stacks.iop.org/NJP/15/025003/mmedia). Both enhancement of the peak intensity value as well as lateral oscillations in the rear part can be observed.

From the resulting total field, we have computed a speckle distribution function by determining the local field intensity maxima and integrating around their neighbourhood ($\lambda_{R,2D}/2$ in $z$), while assuming that the speckle size has not changed substantially (which, of course, can no longer be assumed in the layers subsequent to this first layer of speckles in $z$). Note that by this method, integrating along the speckle axis, simple speckle intensity enhancement due to SF would not immediately lead to a change of the measured speckle intensity because SF, while enhancing the peak value, would shorten the effective Rayleigh length.

We show the histograms corresponding to this distribution function in figure 9, as a function of the speckle intensity $M$ with respect to the average speckle intensity, and the corresponding pdf in figure 10. The results, here for a single speckle field realization, show the same tendency as the histogram shown for the case of low seed noise in our model based on order statistics, figure 4:

- The value of the most intense speckles observed increases up to $M \sim 12$. In the simulation results, of course, this value depends not only on the gain value, as in the model, but also

Figure 8. Snapshot of total field intensity, normalized to the average beam intensity (see colour bar), in a part of the interaction volume in $x_\perp$ (out of 0–8200$\lambda$), for three cases of IAW damping, $(\nu/\omega)_{IAW} = 0.03$, 0.015 and 0.01, corresponding to (a) $\langle G \rangle = 0.5$, (b) 1 and (c) 1.5, respectively.
Figure 9. Histogram of the speckle distribution computed from numerical simulations with the code HARMONY2D, in linear–logarithmic scale. Symbol index for different values of the average gain: $\langle G \rangle = 0.5$ (red), 1 (pink), 1.5 (blue). Parameters: $n_{sp} \sim 2300$.

Figure 10. Probability density function of the speckle distribution computed from numerical simulations with the code HARMONY2D, in log–log scale. Symbol index for different values of the average gain: $\langle G \rangle = 0.5$ (red), 1 (pink), 1.5 (blue). Parameters: $n_{sp} \sim 2300$. In the interval $M = 5–10$, the cases for $\langle G \rangle > 1$ show a power law decrease instead of an exponential dependence as still visible for $\langle G \rangle = 0.5$.

on the particular realization. An error bar for the value $\langle G \rangle = 1$ indicates the range derived from four realizations. As the standard deviation $\delta M$ in figure 7 is largest around $\langle G \rangle \sim 1$, the spread of values in the distribution tail can be important.

- The decrease with the speckle intensity slows down in the vicinity of the expectation value of the most intense speckle associated with the (initial) pump field, $\sim \ln n_{sp} \approx 8$. The probability density shown in figure 10 in the double-logarithmic scale shows for $\langle G \rangle = 1$
and 1.5 a decrease that is, around $M \sim 8$, closer to a power law dependence (roughly $f_{sp}^{(G)} \sim M^{-3.5 \pm 0.2}$ in $5 < M < 10$) than an exponential law, indicating a departure from Gaussian statistics in this domain of the distribution function.

7. Discussion and conclusions

Our results show that the magnitude of (forward) scattered light through a plasma layer can exhibit critical behaviour when the gain coefficient $\langle G \rangle$ of FSBS, associated with average laser speckles, increases in the interval from $\langle G \rangle - \epsilon$ to $\langle G \rangle + \epsilon$. The light field can show explosive behaviour because of the dominance of high-intensity speckles. Explosive behaviour has been suggested in earlier work [9], and can be explained by the fact that an exponential increment for the amplification of stimulated scattering, $\sim \exp(\langle G \rangle M)$ (with $M \equiv I_{sp}/\langle I \rangle$), compensates the—usually exponential—decrease, $\sim \exp(-M)$ in the speckle distribution function [17] occurring for $\langle G \rangle \geq 1$. In more recent work [4] it had been shown that the observed strong spreading of transmitted light behind a plasma layer was related to a criterion different from the onset for SF [16], i.e. $P_{sp}/P_c$ with the ratio between laser power in an average speckle, $P_{sp}$, and the SF critical power $P_c$. The modified criterion relates the ratio $P_{sp}/P_c$ to the ratio between the IAW damping and its frequency, $(v/\omega)_{iaw}$. The resulting criterion $(P_{sp}/P_c)/(v/\omega)_{iaw} > O(1)$ is closely related to our criterion for the SBS gain $\langle G \rangle$.

Our results show that when $\langle G \rangle$ increases to $\langle G \rangle > 1$, the dominance of the high-intensity speckles is crucial: in the mentioned regime, they strongly contribute to the cumulative response in the scattered and in the transmitted light image, although their abundance is small with respect to the total number of speckles in the pattern. This has the consequence that changes in speckle patterns can yield a strongly diverging response of the scattered light. This means that the resulting light field, due to the contributions from scattering, is poorly controlled and not reliable, for example for the purposes of ‘plasma-induced smoothing’.

With respect to previous work [9, 11] using speckle distribution functions to compute the cumulative scattering response, we here make use of the order statistics [18] of intense speckles, which allows to take into account the whole intensity range and the order in which each speckle is likely to contribute. By using this method, the impact of the different populations of speckles on the critical behaviour in the cumulative scattering response becomes evident. A good measure for sensitivity of the scattering response on high-intensity speckles is the expectation value of the intensity of speckles that contribute most to the response.

As elaborated in the previous sections, it becomes evident that the closer the expectation value $\langle M \rangle$ approaches the intensity value of the most intense speckle, $\sim \ln n_{sp}$, the less controllable and predictable the light field behind the plasma may be. The standard deviation $\delta M$ shown in figure 7 is a measure of the interval, around $\langle M \rangle$, over which the intensity of the most contributing speckle population can vary from one realization of a speckle pattern to another. We find that for the regime when the most intense speckles contribute most, i.e. around $\langle G \rangle \sim 1$, the spread in intensity, $\delta M$, around $\langle M \rangle$ can be of the order of $\langle M \rangle/2$ itself. Consequently, the expectation value of the cumulative scattering response $R_{tot}$, as shown also by means of numerical simulations in [11], is therefore of low significance in this regime and has low reliability. The uncertainty in the cumulative response which can be estimated by $\Delta R_{tot}/R_{tot} \sim \exp(\langle G \rangle \delta M) - 1$ may assume large values ($\sim 30–60$ in the examples given).

While a plasma layer can have an extension of many speckles along the laser propagation axis, in the frame of this work, our analysis is restricted to a first layer of the order of one speckle

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length. We include the influence of pump depletion in our analysis in order to see the saturation of amplification-scattered light in speckles. The information computed by superposition of the incoming laser light with the scattered light from the first layer is already quite significant and important for further scattering in successive layers. It characterizes the light intensity pattern behind this zone. In particular, as shown in section 5, the statistics of the light field exiting the first layer can show a departure from an exponential decrease in speckle intensity, and slower than exponential in the tail of the speckle distribution.

For the purpose of ‘plasma-induced smoothing’, uncontrolled behaviour that originates already from the first layer of speckles has to be avoided. It is therefore preferential to operate in a regime with plasma parameters such that \( \langle G \rangle \) assumes values well beyond \( \langle G \rangle \sim 1 \) so that the highest intensity speckles do not dominate the scattered light response.

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Appendix A. Transient solution to the scattered light field

In [12] by Elisseev et al the temporal dependence of the scattered light wave amplitude \( E_s \) for the case of SBS has been derived by combining the equation of the scattered light field and a heavily damped IAW. In the regime of FSBS, one can neglect the temporal derivative for the scattered light wave in order to simplify the description, and replace the paraxial propagation operator \( \vec{c}_{g,1} \cdot \nabla - i \frac{c^2}{2\omega_1} \Delta_{1} \) by a derivative along the path with \( (c^2k_1/\omega_1)\partial_{\theta} \), where the path-length \( l \) diverges with the angle \( \theta \) from the principal propagation axis \( \parallel z \),

\[
c_{g,1} \partial_\theta E_s = -\frac{i\omega_1 n_0 n^*_1}{2n_c n_0} E_0, \tag{A.1}
\]

furthermore, the assumption of a sufficiently damped IAW allows equation (1) to be reduced to a first-order equation in time, reading

\[
(\partial_t + v_{iaw}) \frac{n_1^*}{n_0} = -ic_{g}k_{0}\beta_{SBS} \sin(\theta/2) E_0^* E_s, \tag{A.2}
\]

which eventually yields for \( a = (n_1^*/n_0) \exp(-v_{iaw}t) \) the partial differential equation [12]

\[
\frac{\partial^2 a}{\partial_t \partial_\theta} - \frac{\gamma_0^2(l, \theta, x_i)}{c_g} a = 0, \tag{A.3}
\]

and with \( \gamma_0(l, \theta, x_i) \) denoting the local SBS growth rate at the path-length \( l \), in which the additional impact parameter \( x_i \) indicates the shift in the integration path with respect to the path going exactly through the speckle peak. With an intermediate step using the method of Riemann functions, the scattered wave amplitude can be expressed as

\[
E_s(\theta, x_i, vt) = E_{seed} \left( 1 + \int_0^{vt} d\tau \sqrt{\frac{\xi_l}{vt}} I_1 \left( 2\sqrt{\frac{\tau \xi_l}{vt}} \right) e^{-\tau} \right) \tag{A.4}
\]

which involves the Bessel function \( I_1 \) and where \( E_{seed} \) stands for the noise level of the scattered light. The quantity \( \xi_l/v \) is the spatial amplification coefficient computed over the
Figure A.1. Solution of the transient solution of the scattered light field amplitude $E_s$ as a function of the normalized time $\tau = v_{\text{IAW}} t$ for the example of a speckle with the gain $G_{l,i} = 5$ from equation (A.5) (black line), from equation (A.7) with all terms (red line), from equation (A.7) with the blue line for the limit $\tau > \sqrt{G_{l,i}}$ and the green line for the limit $\tau < 1$. The dashed line indicates the asymptotic limit solution (A.6).

integration path length $l$ and we replace it, see equation (5), in the following by $G_{l,i} \equiv \xi_l/v_{\text{IAW}} \int_{l'}=-l (\gamma_0^i(l', \theta)/c_0^i) \, dl'$; we keep here the additional index ‘$i$’ to account for the fact that the gain depends on the choice of the integration path with respect to the impact parameter $x_i$. By partial integration one obtains

$$E_{s,i}(\theta, x_i, \tau) = E_{\text{seed}} \left[ 1 + \left[ e^{-\tau I_0(2\sqrt{G_{l,i} \tau})} \right]_{\tau=0}^{\tau=v_{\text{IAW}}} - \int_0^{\tau=v_{\text{IAW}}} d\tau' e^{-\tau' I_0(2\sqrt{G_{l,i} \tau'})} \right].$$  \hspace{1cm} (A.5)

The integral part of equation (A.5) dominates for $\tau = v_{\text{IAW}} t > 1$, while the term $e^{-\tau I_0(2\sqrt{G_{l,i} \tau})}$ dominates for the early time behaviour, $\tau = v_{\text{IAW}} t < 1$.

The asymptotic value of the integral, equation (A.4), for $v_{\text{IAW}} t \to \infty$, results in the simple expression for the exponential amplification of the seeding scattered field amplitude, $E_{\text{seed}}$,

$$E_{s,i}(\theta, x_i, \tau \to \infty) = E_{\text{seed}} \exp \left( G_{l,i} \right).$$  \hspace{1cm} (A.6)

A good approximation for all times (within the model of equations (A.1)–(A.2) not covering the very initial growth, and the strong coupling limit) can be derived applying the saddle point method to the integral part of equation (A.5) by assuming that $I_0(2\sqrt{G_{l,i} \tau})$ can be replaced by its asymptotic expression for $2\sqrt{G_{l,i} \tau} \gg 1$. This yields, see figure A.1,

$$E_{s,i}(\theta, x_i, \tau) = E_{\text{seed}} \left[ e^{-\tau I_0(2\sqrt{G_{l,i} \tau})} \frac{1 - e^{-\tau + 2\sqrt{G_{l,i} \tau}}}{2\sqrt{\pi G_{l,i}}} + \exp(G_{l,i}) \frac{\text{erf}(\sqrt{G_{l,i}}) - \text{erf}(\sqrt{G_{l,i}} - \sqrt{\tau})}{2} \right],$$  \hspace{1cm} (A.7)

where the last term asymptotically dominates once $\tau > 2G_{l,i}$ because $(1/2)\{\text{erf}(\sqrt{G_{l,i}}) - \text{erf}(\sqrt{G_{l,i}} - \sqrt{\tau})\}$ in equation (A.7) transits from $= 0$ for $t = 0$ to $= 2$ for $\tau \gg G_{l,i}$.
We have restricted our analysis in section 5 to the asymptotic regime in time, for \( V_{\text{aw}} t \gg 1 \) of convective amplification of scattered light, characterized by an exponential increment with the speckle gain \( G_i \). The transient regime, \( \tau < 2(G_i \tau)^{1/2} \), implies a transition function, see equation (A.7), depending in fact on the speckle intensity \( M\langle I \rangle \), via \( G_i = \langle G \rangle M \). The onset of the asymptotic regime is therefore retarded for speckles with increasing peak intensity, yielding the criterion \( \tau > 4M\langle G \rangle \), thus \( \tau > 4M \) for the most relevant regime discussed here, \( \langle G \rangle \sim 1 \).

Appendix B. Speckle shape and form factor

In equation (5) the SBS is expressed for a path through a speckle. For the scattered field from a speckle, all paths with impact parameter \( x_i \) contribute, while the contribution from the central one \( (x_i = 0) \) is likely to be dominant. For this reason, it is useful to express the SBS gain as a product of the gain value \( G_i \) depending essentially on the speckle peak intensity, and of a form factor \( F(\theta, x_i) \),

\[
G_i(\theta, x_i) \equiv G_i F(\theta, x_i) \sin(\theta/2),
\]

where \( F \) depends on the shape of the speckle. For a Gaussian-type speckle it reads

\[
F(\theta, x_i) = \int_{l'=-l/l}^{l/l} \frac{\exp\{-i(l/R/l_{\perp})(l' \sin \theta - x_i)^2\}}{1 + l'^2 \cos^2 \theta} \, dl', \tag{B.1}
\]

with \( l_{\perp} \) denoting the speckle (correlation) radius in the direction perpendicular to the propagation axis. The integration over \( l \) in equation (B.1) converges rapidly when the limits chosen around the speckle focus \( (l = 0) \) are chosen sufficiently large, \( l > 2l_R \). To simplify the analysis, we can extend the limits to infinity so that the form factor can be expressed as a function of the angle \( \theta \), and of \( x_i \),

\[
F(\theta, x_i) = \frac{\pi}{2 \cos \theta} \left\{ e^{(l_R/l_{\perp})^2 z^2} \text{erfc}(\sqrt{(l_R/l_{\perp})^2 z^2}) + e^{(l_R/l_{\perp})^2 z^2} \text{erfc}(\sqrt{(l_R/l_{\perp})^2 z^2}) \right\} \tag{B.2}
\]

with \( z = i x_i + \tan \theta \), \( \bar{z} = -i x_i + \tan \theta \) and \( \text{erfc}(\cdot) \equiv 1 - \text{erf}(\cdot) \) standing for the complementary error function. This expression was obtained using the Plancherel–Parseval theorem.

In order to evaluate the asymptotic expression for the scattered field, equation (A.6), it is useful to simplify the expression by developing around \( x_i = 0 \), which allows eventually a rapidly converging integration in \( x_i \) around the speckle peak. The series expansion of equation (B.2) in \( x_i \) only yields even orders,

\[
F(\theta, x_i) = F(\theta, 0) \left(1 - x_i^2/W^2(\theta) + O(x_i^4)\right) \tag{B.3}
\]

with

\[
\bar{F} = F(\theta, 0) = \frac{\pi}{\cos \theta} e^{(l_R/l_{\perp})^2 \tan^2 \theta} \text{erfc}((l_R/l_{\perp}) \tan \theta)
\]

and

\[
W(\theta) = l_{\perp} \tan^{-1} \theta \left[ 1 - 2 \frac{e^{-(l_R/l_{\perp})^2 \tan^2 \theta} (l_R/l_{\perp}) \tan \theta \sqrt{\pi}}{\text{erfc}(\sqrt{(l_R/l_{\perp})^2 \tan^2 \theta})} + 2 \left( \frac{l_R}{l_{\perp}} \right)^2 \tan^2 \theta \right]^{-1/2}. \tag{B.4}
\]

For \( (l_R/l_{\perp})^2 \tan^2 \theta \gg 1 \) this simplifies to \( W(\theta) \simeq (\tan \theta l_R/l_{\perp})^{-1} \). In a good approximation the function for the form factor is given by \( F(\theta, 0) \simeq 2\sqrt{\pi} \cos^{-1} \theta/[(l_R/l_{\perp}) \tan \theta + \sqrt{(l_R/l_{\perp})^2 \tan^2 \theta + 4/\pi}] \) behaving as \( F(\theta, 0) \simeq \sqrt{\pi} (l_{\perp}/l_R)/\sin \theta \) in the limit \( (\tan \theta l_R/l_{\perp})^2 \gg 1 \).
A good approximation that allows further simplification of the evaluation is

$$\mathcal{F}(\theta, x_i) = \mathcal{F}(\theta, 0) e^{-x_i^2/W^2(\theta)}. \quad (B.5)$$

An integration over all values of the impact parameter $x_i$ hence can be performed in this approximation. Expression (A.6) can hence be integrated over $x_i$ in the limit to a neighbouring speckle $x_i \simeq -2\rho_c \cdots +2\rho_c$ where $\rho_c \sim l_R W(\theta)$ is the typical correlation width in $x_i$. For rapidly converging correlation, the integration should not depend considerably on the integration limit beyond $\pm 2\rho_c$ so that it is convenient to choose $\pm \infty$ as the limit for the evaluation,

$$E_{\text{seed}} + E_{\text{seed}} \int_{x_i = -2\rho_c}^{+2\rho_c} \left( \exp\{G_{lk}\mathcal{F}(\theta, x_i) \sin(\theta/2)\} - 1 \right) \, dx_i \quad (B.6)$$

$$\simeq E_{\text{seed}} + E_{\text{seed}} \int_{x_i = -\infty}^{+\infty} \left( \exp\{G_{lk}\mathcal{F}(\theta, 0) \sin(\theta/2) e^{-x_i^2/W^2(\theta)}\} - 1 \right) \, dx_i$$

$$= E_{\text{seed}} + E_{\text{seed}} \sum_{n=1}^{\infty} \frac{G_{sp}^n}{n!} \int_{x_i = -\infty}^{+\infty} e^{-n x_i^2/W^2(\theta)} \, dx_i$$

$$= E_{\text{seed}} \left( 1 + \sqrt{\pi} W(\theta) \sum_{n=1}^{\infty} \frac{G_{sp}^n}{n! \sqrt{n}} \right). \quad (B.7)$$

with, see equation (10), the definition for the effective speckle gain

$$G_{sp} = G_{lk} \mathcal{F}(\theta, 0) \sin(\theta/2) \simeq G_{lk} \frac{l_\perp}{l_R} \frac{2\sqrt{\pi} \sin(\theta/2) / \sin \theta}{1 + \sqrt{1 + (4/\pi)(l_\perp/l_R)^2 \cot^2 \theta}}$$

$$< G_{lk} \frac{l_\perp}{l_R} \frac{\sqrt{\pi}/2}{\cos(\theta/2)} \quad \text{for } \theta < \pi/2. \quad (B.8)$$

This gain value of $G_{sp}$ is now the relevant value to use in equation (5) and it is used in the analysis starting in section 4.

The expression (B.7) can be approximated with a Bessel function $I_n$ of order $\alpha$ close to 1 (best fit with $\alpha \simeq 1.15$), or almost as well, by an exponential function $\sim \exp\{G_{sp}\}$ for not too small values of $G_{sp}$,

$$E_{\text{seed}} \left( 1 + \sqrt{\pi} W(\theta) \sum_{n=1}^{\infty} \frac{G_{sp}^n}{n! \sqrt{n}} \right) \simeq E_{\text{seed}} \left[ 1 + \pi W(\theta) I_{\alpha}(G_{sp}) \right] \simeq W(\theta) \exp\{G_{sp}\}/[G_{sp}]^{1/2}. \quad (B.9)$$

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