Comments on CPT tests

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Abstract

What do we know about CPT symmetry? We conclude that the direct measurement of CPT violation leads to a result that CPT violating forces is limited to about 30% of CP violating forces. The best test of CPT symmetry is to see if theoretical prediction for the phases of $\eta^+\eta^-$ and $\eta^0\eta^0$ agrees with experiments. We discuss uncertainties associated with this prediction, and how to improve it. A new test of CPT is also discussed.
I. INTRODUCTION

The 'classical' tests of CPT symmetry concern the equality of masses and lifetimes for particles and antiparticles. The most dramatical way of stating the validity of CPT symmetry is to state the experimental upper bound [1]:

\[
\frac{|M_{K^0} - M_{K^0}|}{M_K} < 10^{-18} .
\] (1.1)

This is very misleading. The upperlimit on the right hand side depends on what we choose for the denominator. Why is \(M_K\) a reasonable choice? Bulk of K meson mass comes from strong interaction. So, if we compare the strength of CPT violating force with that of QCD, indeed, CPT violating force is very small. Since CP symmetry forces \(M_{K^0} = M_{K^0}\), CPT symmetry must be tested in the environment where CP symmetry is broken. Then it is perhaps more reasonable state:

\[
\frac{|M_{K^0} - M_{K^0}|}{\eta \Gamma_{K_S}} < 0.04
\] (1.2)

Suddenly, it is no longer so impressive.

In Sec. 2, we first discuss a direct test of CPT using recent experimental result from CPLEAR. In Sec. 3, we discuss theoretical prediction of \(\eta_{+-}\) and \(\eta_{00}\) phases - exposing all theoretical subtleties. In Sec. 4, we suggest a new CPT test which requires only studying \(2\pi\) and \(3\pi\) decays of \(K_L\).

II. DIRECT TEST OF CPT

The most direct test of CPT is to study time dependence of \(K_{l3}\) decays.

\[
A_T(t) = \frac{\Gamma(K^0(t) \rightarrow e^+) - \Gamma(K^0(t) \rightarrow e^-)}{\Gamma(K^0 \rightarrow e^+) + \Gamma(K^0 \rightarrow e^-)}
\]

\[
A_{CPT}(t) = \frac{\Gamma(K^0(t) \rightarrow e^-) - \Gamma(K^0(t) \rightarrow e^+)}{\Gamma(K^0 \rightarrow e^-) + \Gamma(K^0 \rightarrow e^+)}
\] (2.1)
For $t \gg \Gamma_S^{-1}$ we have

$$A_T(t) + A_{CPT}(t) \simeq 2(\text{Im } \phi + \text{Re } \cos \theta), \quad (2.2)$$

where $\cos \theta \neq 0$ and $\phi \neq 0$ imply CPT, and CP symmetries are violated, respectively. Since $\epsilon \sim \frac{i}{2} \phi$ we can extract $\text{Re } \cos \theta$. The most accurate measurement of $\text{Re } \cos \theta$ is given by CPLEAR collaboration [2]:

$$\text{Re } \cos \theta = (6.0 \pm 6.6_{\text{stat.}} \pm 1.2_{\text{syst.}}) \times 10^{-4}. \quad (2.3)$$

Comparing this number to $\epsilon \sim 2 \times 10^{-3}$, we see that CPT violating forces is limited to 30% of CP violating forces. While the new experimental result represents a major step toward understanding CPT symmetry, this is hardly illuminating.

III. CPT TESTS AND PHASES OF $\eta^{+ -}$ AND $\eta_{00}$

In this section, we show that the most stringent and legitimate test of CPT is provided by the phases of $\eta^{+ -}$ and $\eta_{00}$. It is well known that with CPT symmetry, $\phi^{+ -} = \phi_{00} = \phi_{SW}$, where $\phi_{SW} = \tan^{-1} \left( \frac{2\Delta M}{\Delta \Gamma} \right)$, to a very good approximation. We want to compute the correction to this relation. To first order in $\phi^{+ -} - \phi_{SW}$ and $\phi_{00} - \phi_{SW}$, it is possible to derive the following relation:

$$\left| \frac{\eta^{+ -}}{\sin \phi_{SW}} \right| \left( 2 \phi^{+ -} + 1 \phi_{00} - \phi_{SW} \right) = -ie^{-i\phi_{SW}} \frac{\sum f \neq 0 \epsilon(f)}{\sin \phi_{SW}} - \frac{1}{2} \text{Re } \Delta_0$$

$$\epsilon(f) = i e^{i\phi_{SW}} \frac{\text{Im } \Gamma_{12}(f)}{\Delta \Gamma} \cos \phi_{SW}, \quad (3.1)$$

where $\Delta_I = 1 - \frac{A_I}{A}$, and $A_I$ and $\overline{A}_I$ are $K \rightarrow (\pi \pi)_I$ and $\overline{K} \rightarrow (\pi \pi)_I$ amplitudes, respectively. Here $(\pi \pi)_I$ denotes $\pi \pi$ final state with isospin $I$. Finally, we emphasize that the sum over $f$ does not include $f = (2\pi)_0$ channel.

Let us first consider $\text{Re } \Delta_0$. Since we assume CPT symmetry, we can write

$$\frac{\overline{A}_0}{A_0} = e^{-2i\xi_0} \quad (3.2)$$
Then \( \text{Re} \ \Delta_0 = O(2\xi_0^2) \) and is second order in \( \text{CP} \) violating parameters. These terms are same order as neglected terms.

Now, we estimate \( \epsilon(f) \). First some preliminaries. For those final states \( f \) for which \( \text{CPT}|f\rangle = |f\rangle \) holds, we can write

\[
\Gamma_{12}^f \equiv 2\pi \rho_f \langle f|H_W|K^0\rangle^2, \tag{3.3}
\]

\[
\text{Im} \ \Gamma_{12}^f = -i\pi \rho_f \left( A_f^2 - A_f^*2 \right) = -i\pi \rho_f \left( A_f^2 - \overline{A}_f^2 \right)
\]

where \( A_f = \langle f|H_W|K^0\rangle \) and \( \overline{A}_f = \langle f|H_W|K^0\rangle \), and we have used the fact that \( \text{CPT} \) is an anti-linear operator. To first order in \( \text{CP} \) violation,

\[
\epsilon(f) = e^{i\phi_{sw}} \cos \phi_{sw} \frac{\Gamma(K \rightarrow f)}{\Delta \Gamma} \left( 1 - \text{CP} \frac{\overline{A}_f}{A_f} \right) \tag{3.4}
\]

Now we consider various intermediate states \( f \).

- \( f = (2\pi)_2 \)

Here \( \epsilon((2\pi)_2) \) can be estimated by noting that

\[
\epsilon' = \frac{1}{2\sqrt{2}} \omega \ e^{i(\delta_2 - \delta_0)} (\Delta_0 - \Delta_2) \tag{3.5}
\]

where \( \Delta_f = 1 - \frac{\overline{A}_f}{A_f} \). Baring unexpected cancellation between \( \Delta_0 \) and \( \Delta_2 \), we estimate that

\[
|\epsilon((2\pi)_2)| = 2\omega \epsilon' \sim 10^{-7}. \tag{3.6}
\]

- \( f = (\pi^+\pi^-\pi^0)_{\text{CP}=+} \)

\[
|\epsilon((\pi^+\pi^-\pi^0)_{\text{CP}=+})| = \frac{1}{2\sqrt{2}} \text{Br}(K_S \rightarrow (3\pi)_{\text{CP}+}) \left| 1 - \frac{\overline{A}((3\pi)_{\text{CP}+})}{A((3\pi)_{\text{CP}+})} \right| \leq \left( 1.2^{+0.4}_{-0.3} \right) \times 10^{-7} \tag{3.7}
\]

where we have used \( \text{Br}(K_S \rightarrow \pi^+\pi^-\pi^0) = \left( 3.4^{+1.1}_{-0.9} \right) \times 10^{-7} \).
\[ f = (\pi^+\pi^-\pi^0)_{\text{CP}=-} \]
\[ |\epsilon((\pi^+\pi^-\pi^0)_{\text{CP}=-})| = \frac{1}{2\sqrt{2}} \text{Br}(K_L \to (3\pi)_{\text{CP}=-}) \frac{\Gamma_L}{\Gamma_S} \left(1 + \frac{\overline{A}(3\pi)_{\text{CP}=-}}{A(3\pi)_{\text{CP}=-}}\right) \]

(3.8)

\[ \text{CP odd } \pi^+\pi^-\pi^0 \text{ with symmetrized } \pi^+\pi^- \text{ state is given by [3]:} \]
\[ \text{Re } \eta_{+-0} = (-2 \pm 7 \begin{pmatrix} +4 \\ -1 \end{pmatrix}) \times 10^{-3} \]
\[ \text{Im } \eta_{+-0} = (-9 \pm 9 \begin{pmatrix} +2 \\ -1 \end{pmatrix}) \times 10^{-3}. \]

(3.9)

(3.10)

where the first and the second errors are statistical error, and systematic error, respectively. Using \( \eta_{+-0} = \frac{1}{2} \left(1 + \frac{q_1 A(\pi^+\pi^-\pi^0)_{\text{CP}=-}}{p_1 A(\pi^+\pi^-\pi^0)_{\text{CP}=-}}\right) \), we are lead to a reasonable guess:
\[ \left(1 + \frac{\overline{A}(\pi^+\pi^-\pi^0)_{\text{CP}=-}}{A(\pi^+\pi^-\pi^0)_{\text{CP}=-}}\right) \sim \left(1 + \frac{q_1 A(\pi^+\pi^-\pi^0)_{\text{CP}=-}}{p_1 A(\pi^+\pi^-\pi^0)_{\text{CP}=-}}\right) < 2 \times 10^{-2} \]

(3.11)

which gives
\[ \epsilon(\pi^+\pi^-\pi^0)_{\text{CP}=-} \leq 1.4 \times 10^{-6}. \]

(3.12)

\[ f = 3\pi^0 \]

This can be estimated by noting that
\[ \eta_{000} = \frac{1}{2} \left(1 + \frac{q_1 A(3\pi^0)}{p_1 A(3\pi^0)}\right) \]

(3.13)

and the experimental value for \( \eta_{000} \) is given by [4]:
\[ \text{Re } \eta_{000} = 0.18 \pm 0.14_{\text{stat.}} \pm 0.06_{\text{syst.}}. \]

(3.14)

\[ \text{Im } \eta_{000} = -0.05 \pm 0.12_{\text{stat.}} \pm 0.05_{\text{syst.}}. \]

(3.15)

which leads to
\[ \left|1 + \frac{\overline{A}(3\pi^0)}{A(3\pi^0)}\right| \sim \left|1 + \frac{q_1 A(3\pi^0)}{p_1 A(3\pi^0)}\right| < 2|\eta_{000}| < .46 \]

(3.16)

Using Eq.(3.4), we obtain:
\[ |\epsilon(3\pi^0)| = \frac{\Gamma(K_L \to 3\pi^0)}{2\sqrt{2}\Gamma_S} \left(1 + \frac{\overline{A}(3\pi^0)}{A(3\pi^0)}\right) < 6 \times 10^{-5} \]

(3.17)
In considering \( f = \pi l \nu \) we have to allow for a violation of the \( \Delta Q = \Delta S \) rule as expressed by the complex parameter

\[
x = \frac{\langle l^+ \nu \pi^- | H_W | K \rangle}{\langle l^+ \nu \pi^- | H_W | K \rangle}.
\]  

(3.18)

Since

\[
\text{Im } \Gamma_{12}^{\pi l \nu} \simeq \text{Im } x \Gamma(K \rightarrow \pi \mu \nu),
\]

(3.19)

we find

\[
|\epsilon(\pi l \nu)| \leq 4 \cdot 10^{-7},
\]

(3.20)

where we have used the bound \( \text{Im } x = (0.5 \pm 2.5) \times 10^{-3} \) from \([2]\) and \( Br(K_S \rightarrow \pi^\mp \mu^\pm \nu) \sim 5 \times 10^{-4} \).

In summary: we can conclude that the upper limit is dominated by the \( 3\pi^0 \) channel, and we have

\[
|\sum_f \epsilon_f| \leq \sum_f |\epsilon_f| \leq 6 \times 10^{-5}.
\]

(3.21)

With our new evaluation of theoretical error we obtain \([5, 6]\):

\[
\frac{2}{3} \dot{\phi}_{++} + \frac{1}{3} \dot{\phi}_{00} - \phi_{SW} \simeq (0.17 \pm 0.81_{\text{exp}} + 1.7_{\text{theory}})^\circ
\]

(3.22)

So, we have about \( \frac{1}{\phi_{SW}} \sim 4\% \) test of CPT violation.

**IV. NEW TEST OF CPT SYMMETRY**

Define the rate asymmetry

\[
A(t) = \frac{\Gamma(K(t) \rightarrow \pi^+ \pi^- \pi^0) - \Gamma(K(t) \rightarrow \pi^+ \pi^- \pi^0)}{\Gamma(K(t) \rightarrow \pi^+ \pi^- \pi^0) + \Gamma(K(t) \rightarrow \pi^+ \pi^- \pi^0)}
\]

(4.1)

Noting that, in general,
\[ |K^0\rangle = \frac{1}{p_1 q_2 + q_2 p_1} [q_1 |K_L\rangle + q_2 |K_S\rangle] \]
\[ |\bar{K}^0\rangle = \frac{1}{p_1 q_2 + q_2 p_1} [p_1 |K_L\rangle + p_2 |K_S\rangle] \]

(4.2)

we see that decay \( K \to \pi^+\pi^-\pi^0 \) at \( t \to \infty \) leads to an asymmetry

\[ \lim_{t \to \infty} A(t) = 1 - \frac{|q_1|}{|p_1|}^2. \]

(4.3)

This is a curious result. If we look at \( K \to \pi^+\pi^- \) at \( t \to \infty \), which is \( K_L \to \pi^+\pi^- \), we obtain

\[ \epsilon \sim \left( 1 - \frac{q_2}{p_2} \right). \]

(4.4)

So, by comparing \( t \to \infty \) limit of \( K \to \pi^+\pi^- \) and \( K \to \pi^+\pi^-\pi^0 \) decays we have a test of \textbf{CPT} symmetry:

\[ \frac{q_2}{p_2} \simeq \frac{q_1}{p_1} \]

(4.5)

\textbf{V. SUMMARY}

We have examined existing tests of \textbf{CPT} symmetry. We found that strength of \textbf{CPT} violating interaction is bounder by 4% of the \textbf{CP} violating interaction. The majority of the theoretical error comes from the uncertainty in \( \eta_{000} \). We recommend some effort in improving this measurement. A new test of \textbf{CPT} comparing \textbf{CP} violation in \( K_L \to 3\pi \) and \( K_L \to 2\pi \) is suggested.

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