Analysis of firing after shortest distance between target and anti-aircraft gun

J Wang, S F Chang, X C Fan and X J Li

Nanjing University of Science and Technology, Advanced Launch Collaborative, Innovation Center, Nanjing, China, 210000

Email: wangj1125@163.com

Abstract. In order to make full use of the performance of high-muzzle-velocity anti-aircraft weapon, make full use of the target firing segments on the entire route of the target, fully grasp all the firing opportunities, and reduce the possibility of a target breaking through the defense, the original pre-navigation firing (firing before shortest distance between target and anti-aircraft gun) method based on a muzzle velocity below 3ma cannot meet the needs mentioned above. Based on the establishment of high-muzzle velocity projectiles, this paper aims at the current situation of less post-navigation firing (firing after shortest distance between target and anti-aircraft gun), verifies the feasibility of post-navigation firings by analyzing the relative remained velocity required to penetrate the target, and verifies that with the gradual increase in projectile muzzle velocity, its firing range and timing can increase simultaneously. It provides a theoretical basis for post-navigation firing, increases the firing opportunities for the target on the entire route, and provides a reference for the demonstration, design and testing of the new anti-aircraft fire control system.

1. Introduction
According to the pre-navigation firing principles stipulated in the firing tutorial, the relevant reasons are as follows [1]. The relative velocity of the principles shells and the target is relatively large [2]. Since most antiaircraft projectiles use collision instead of explosion to destroy the target, it is necessary to ensure that the relative remained velocity of the projectile and the target can penetrate the target. When the solution step size is determined, the firing elements are densely spaced, the future point interval is small, and the rotation follow-up angle of anti-aircraft body tube is small, which can ensure that the anti-aircraft follow-up error meets the requirements to a greater extent [3]. For these reasons, pre-navigation firing is usually used. With the increase of the projectile exit velocity, the relative remained velocity of the projectile and the target can be increased to a greater extent, and in the segment of post-navigation, the turning range of the follow-up system remains small. In this section, the reasons of post-navigation firing will be explained through specific example analysis, and the firing method will be demonstrated.

2. Post-navigation firing analysis

2.1. Anti-aircraft artillery damage analysis
Anti-aircraft projectiles include shelled armor-piercing, fragmentation and grenades. Taking the armor-piercing projectile as an example, the armor-piercing projectile is a kind of kinetic energy projectile [4].
It uses the long artillery tube and the barrel of the gun to reach a larger kinetic energy launching in order to achieve a larger armor-piercing capability [5]. From the perspective of mechanics analysis, the process of hitting the transient armor is affected by four types of stress: ductile extrusion, annular shear, tensile stress rupture, and circumferential. Because the material and size of the projectile and target armor are different [6], it is difficult to quantitatively analyze the magnitude of these four forces. Therefore, when researching the ability of the projectile to penetrate the armor, mathematical analysis is usually used.

According to the kinetic energy formula, the kinetic energy of the flying armor-piercing projectile can be obtained:

\[ W = \frac{1}{2} m v_f^2 \]  

Where \( v_f \) (m/s) represents the ultimate penetration velocity of the projectile; \( m \) (kg) represents the initial mass of the projectile.

Because the projectile only relies on the initial velocity and inertia, during the flight of the projectile, the kinetic energy continuously decreases, and the projectile itself deforms. However, the factors mentioned above are negligible compared to the kinetic energy consumed by the collision.

The armor-piercing formulas commonly used include Krupp's formula, Demar's formula, and Upolnikov's formula. And this paper uses Demar's formula which is given as follows:

\[ v_e = K \frac{d^{0.7} b^{0.7}}{m^{0.5} \cos(\alpha)} \]  

Where \( v_e \) (m/s) represents the minimum target velocity that can penetrate the target plate; \( K \) represents the armor ballistic resistance coefficient; \( d \) (dm) represents the projectile diameter; \( b \) (dm) represents the target thickness; \( m \) (kg) represents the total projectile mass; \( \alpha \) represents the angle of incidence.

Armor ballistic resistance coefficient \( K \) is a comprehensive coefficient reflecting the anti-impact physical ability of the armor material. It is usually obtained by a large number of firing experiments. The armor coefficient of some materials is shown in the table below.

| Armor material               | Low carbon steel plate | Pinch steel plate | General homogeneous armor | Treated armor |
|-----------------------------|------------------------|-------------------|---------------------------|--------------|
| Coefficient of resistance to artillery shells | 1530                   | 1900              | 2000-2400                 | 2400-2600    |

Take \( K = 1530 \) and \( K = 1900 \) to roughly estimate the relationship between the incident penetration angle and the minimum velocity of penetrating the target, as shown in the figure 1.

![Figure 1. Relation between incident penetration angle and target velocity.](image-url)
2.2. Future hit point moving velocity  
Consider a straight line of constant velocity from far to near and then from near to far [7]. When a point \( (x, y, z) \) on the route satisfies the firing conditions, the corresponding future hit point \( (x_1, y_1, z_1) \) can be obtained. Define the nearest point on the route to the origin of the coordinate as the route short-circuit, and the distance between this point and the origin is route \( d_s \). The oblique distance and horizontal distance corresponding to the advance point are as follows:

\[
d_i = \sqrt{x_i^2 + y_i^2} \quad (3)
\]

\[
D_i = \sqrt{x_i^2 + y_i^2 + z_i^2} \quad (4)
\]

For a constant-velocity straight route, the velocity at the point shooting ahead (under standard trajectory conditions, and ignore the influences of drift and high angles) is as follows:

\[
v_i = \frac{1}{v_i} \frac{1}{1 - \cos(\theta) \sqrt{d_i^2 + d_s^2}} \quad (5)
\]

Where \( v_i \) represents the flight velocity of target, \( v_i \) represents the remained velocity of the warhead at advance point, \( \theta \) represents the ballistic tangent tilt angle at advance point.

The angular velocity in advance position and in high and low directions at the advance point are as follows:

\[
\omega_{hi} = \frac{v_i d_i}{d_i^2} \quad (6)
\]

\[
\omega_{zi} = \frac{v_i H}{D_i^2 d_i} \quad (7)
\]

2.3. Future hit point interval  
Taking a certain caliber anti-aircraft gun of as an example [8], it is assumed that the target moves in a straight line with constant velocity, and the target motion equation is as follows:

\[
X(k+1) = \Phi(k+1, k) X(k) + U(k) \quad (9)
\]

Where \( U(k) \) is the random error term caused by atmospheric disturbance and other factors, and it is assumed to be a Gaussian white noise process with a mean of zero and a variance of \( Q(k) \).

\[
X(k) = \begin{bmatrix} x(k) \ v_x(k) \ y(k) \ v_y(k) \ z(k) \ v_z(k) \end{bmatrix}^T \quad (10)
\]

\[
\Phi(k) = \begin{bmatrix}
1 & \Delta T & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \Delta T & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \Delta T & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \quad (11)
\]

Where \( x(k), y(k), z(k), v_x(k), v_y(k), v_z(k) \) respectively represent the position and velocity components of the target in the rectangular coordinate system at time \( k \); \( \Delta T \) is the sampling interval.

Take a straight route with a velocity of about 200m/s as an example, and

\[
X(k) = \begin{bmatrix} -1000 & 200 & 2000 & 0 & 1000 & 0 \end{bmatrix} \quad (12)
\]

The sampling interval \( \Delta T \) for this route is 0.02s.

The dichotomy or secant method combined with firing table fitting function is used to solve the firing elements. The firing elements at different times are shown in the following figure 2.
The future hit points corresponding to different firing elements are given in figure 3 and figure 4.

![Figure 2. Firing elements.](image)

As can be seen from the above figures, as the short-circuit point approaches, the flying time of the shell changes from long to short, and the interval between firing elements at each moment is getting smaller and smaller. At the same time, the future hit points corresponding to each firing element are getting denser. As the short-circuit point is farther away, the flying time of the shell changes from short to long, and the firing elements in high and low directions change rapidly. As the target moves away, the distance between artillery and target becomes larger. The interval between future hit points calculated at each moment is getting larger and larger owing to the influence of drift and other factors.

2.4. Remained velocity of future hit point

The projectile velocity can be calculated by the following methods, first, the corresponding position velocity can be obtained by using the firing table fitting function. The same method as firing table fitting mentioned above can be used in the fitting. If it is necessary to obtain the partial velocity in each direction during analysis, the velocity can be decomposed according to the firing elements and the ballistic flatness theory. Second, the ballistic equation method can be used to calculate the corresponding position and velocity, which can obtain projectile status information more accurately. Third, in actual systems, information such as the position and velocity of the shell can be obtained according to the radar. However, due to the small size, high velocity, and low RCS value of the anti-aircraft shell, the detection method is difficult and lacks relevant data, so this method is not desirable.
In order to simplify the calculation process and use the firing table fitting conclusions, the remained velocity of projectile at each future hit point obtained by the firing table fitting function method is shown in the figure below.

![Figure 5](image5.png)

**Figure 5.** Remained velocity at different future point.

From the figure 2 and figure 5 above, it can be analyzed that the flying time of when the distance between target and anti-aircraft gun is minimum appears at about 4 second, with the remained velocity at future point is around the maximum value. When the future point is after the flight, the remained velocity at future point decreases faster and faster, but is still above 600m/s at 10 second. So it can be known that the remained velocity is still in a relatively large range when firing after the shortest distance between anti-aircraft gun and target.

As the trajectory of the high muzzle velocity projectile is nearly flat within its range, it is assumed that the trajectory of the projectile is flat to simplify the calculation, so that the influence of the projectile's attitude angle on the angle of the projectile during penetration is ignored. The same assumed route is also used to obtain the relative remained velocity at different future points, and the relationship between the relative remained velocity and the included angle of the artillery line at future points. From the figure below, it can be seen that the projectile velocity remains at a high level at different future points. From the relationship between the projectile declination and the remained velocity, it can be seen that the velocity at the short-circuit point is the largest, and becomes smaller as the short-circuit point is far away, but through the comparison between figure 5 and figure 6, it shows that the relative remained velocity is still within a sufficient range to penetrate the target.

![Figure 6](image6.png)

**Figure 6.** Velocity of future points.

3. **Firing area problem**

When the artillery conducts air defense firing on a mobile target, detection equipment such as tracking radar or tracking photoelectric continuously detects and provides current coordinates of the target to the fire control system.

Let $\Omega_0$ be the reachable area of the warhead, as shown in the figure below, where $h_p$ and $d_e$ are the projection coordinates of the target track in the vertical and horizontal directions, and $D(t)$ is the target track. $t_m$ is the instant when the target enters the shootable area $\Omega_m$. Usually when the target is in approaching flight, there is a certain non-fireable area after the target reaches the shootable area,
because the target route is assumed to be completed during the track processing stage, the error in the fire control solution is big, or the angular velocity of the anti-aircraft gun is too large, resulting in large deviations. The approach to the non-shootable segment that is just within the range of the flying target usually adopts the blocking firing scheme. The basic idea is to use the virtual extension technology of the firing table to obtain the rough values of the firing elements when the target enters the firing area and make the body tube of anti-aircraft points to this position. After reaching the firing area, the new target route information is used to make small corrections to the firing elements and the tube direction to complete the shooting. However, the use of the shootable area by this method is also limited to the pre-flight segment, which wastes the shootable area from the zenith dead zone.

![Figure 7. Firing area diagram.](image)

The curve of warhead flight time for a straight horizontal and constant velocity route is shown as follows.

![Figure 8. Curve of the flight time of projectile.](image)

As shown in the figure 8, \( t_0 \) is the flight time required for the warhead to reach the farthest effective firing area. A straight line with a slope of 1 is drawn on the left half plane of the time axis and the point passing \( t_0 \), and the point of intersection is \( t_a \). The projectile fired at this moment passes the time \( t_0 \) to reach the future hit point. At time \( t \), the straight line with a slope of 1 and the curve of ballistic time have only one intersection point, which indicates that there is only one solution when calculating the flight time, and at time \( t_0 \), the straight line with a slope of 1 and the curve of ballistic time have two intersection points. At this time, the problem of competitive solution occurs, that is, two different future hit points. Although there are different firing azimuth angles and firing height angles, the flight time of firing elements is indeed the same, competing for the same firing moment. Therefore, in the time period \( [t_a, t_f] \), there is only one future hit point.

If the target flight is a constant-velocity straight horizontal route, the flight time curve is left-right symmetrical with \( t_f \) as the axis of symmetry, otherwise the curve loses symmetry. When the target keeps the flight path unchanged and only increases the flight velocity by \( n \) times, the flight time curve will be reduced to \( 1/n \) on the horizontal axis. Similarly, when the target only reduces the flight velocity by \( n \) times, the flight time curve will expand \( n \) times to the original on the horizontal axis.
From the analysis mentioned above, the firing conditions are given as (13)

\[
\begin{align*}
\left[D_x(t) \right] & \leq D_n \\
\frac{\text{d} D_x(t)}{\text{d} t} & \leq 1
\end{align*}
\]

(13)

Where \( D_x(t) \) is the flight route of target, \( D_n \) is the effective range of fire.

Considering that the target has a limited range of fireable areas in approaching flight, in order to maximize the firing time, combined with the effectiveness of the post-navigation firing problem described in the previous chapter, this section mainly analyzes the fireable areas of firing in post-navigation.

Assume that the target partial velocity in the northeast celestial coordinate system is \([v_{nx}, v_{ny}, v_{nz}]\), and assume that the trajectory is flat, the projectile partial velocity is \([v_{mx}, v_{my}, v_{mz}] = [v_0 \cos \alpha \cos \beta, v_0 \cos \alpha \sin \beta, v_0 \sin \alpha]\) where \( v_0 \) is the projectile velocity, \( \alpha \) is height angle, \( \beta \) is the azimuth. The remained velocity of projectile can be expressed as follows.

\[
v_v = \sqrt{(v_{nx} - v_{mx})^2 + (v_{ny} - v_{my})^2 + (v_{nz} - v_{mz})^2}
\]

(14)

the projectile velocity \( v_0 \) is usually fixed, and the partial derivatives of \( \alpha \) and \( \beta \) for the above formula are as follows.

\[
\frac{\text{d} v_v}{\text{d} \alpha} = \frac{v_v \sin(\alpha) \cos(\beta) v_{mx} + v_v \sin(\alpha) \sin(\beta) v_{my} - v_v \cos(\alpha) v_{mz}}{\sqrt{(v_{nx} - v_{mx})^2 + (v_{ny} - v_{my})^2 + (v_{nz} - v_{mz})^2}}
\]

(15)

\[
\frac{\text{d} v_v}{\text{d} \beta} = \frac{v_v \cos(\alpha) \sin(\beta) v_{mx} - v_v \cos(\alpha) \cos(\beta) v_{my} + v_v \sin(\alpha) v_{mz}}{\sqrt{(v_{nx} - v_{mx})^2 + (v_{ny} - v_{my})^2 + (v_{nz} - v_{mz})^2}}
\]

(16)

It is known from (15) and (16) that, when

\[
\alpha = \arctan \frac{v_{my}}{v_{nx} \cos(\beta) v_{mx} + \sin(\beta) v_{mz}}
\]

(17)

\( v_v \) gets extreme value.

when

\[
\beta = \arctan \frac{v_{mx}}{v_{ny}}
\]

(18)

\( v_v \) gets extreme value too.

4. Analysis of examples

To simplify the analysis of the example, it is assumed that the target flight trajectory is within the firing plane of the anti-aircraft gun, that is, the azimuth angle of the firing elements is ignored, and the height angle problem is analyzed in a two-dimensional plane.

\[ \text{Figure 9. Schematic diagram of target flight trajectory.} \]
As shown in the figure above, the target is flying in a direction parallel to the x-axis. There is a curve of relative remained velocity as shown below.

![Figure 10](image-url) Relationship between target velocity and relative remained velocity.

As shown in the figure above, with the increase of distance between target and anti-aircraft gun, the remained speed and relative remained speed of projectile is decrease. With the increase of distance between target and anti-aircraft gun, the storage speed of projectile decreases, but the decline process is not obvious.

![Figure 11](image-url) Relationship between slope distance and remained velocity at target velocity of 200m/s.

Three kinds of antiaircraft guns with different caliber are selected for analysis (the data has been decrypted). The three caliber are classified as caliber a, caliber b and caliber c, among which the muzzle velocity of projectile is a<b<c. According to the different caliber, the relationship between the projectile remained speed and the projectile flying time is shown in figure 11, in which the target altitude is 400m and the flight speed is 280m/s. It can be seen that with the increase of the distance between anti-aircraft gun and targets, the relative velocity of projectiles decreases gradually. When the distance between anti-aircraft gun and target is increased to about 2500m, the remained speed of projectiles of caliber b and c is about 200m/s, the flying time is about 2s, the remained speed of projectiles of caliber a is still about 200m/s. When the distance between anti-aircraft gun and target is 4000m, and the flying time of caliber c is below 6s. It can be seen that with the increase of projectile initial velocity, the firing area and the firing opportunity increase simultaneously.

5. Summary

By analyzing the firing method that firing only before the shortest distance between anti-aircraft gun and target, the reason for choosing the new method that firing after the shortest distance between anti-aircraft gun and target is gotten, and then the possibility and feasibility of the new method are analyzed by calculating the velocity required to penetrate the armor and the relative speed of the projectile and target. A example is used to verify that with the increase of the projectile exit velocity, the projectile's remained velocity can still penetrate the target under certain conditions. When the anti-aircraft gun's muzzle velocity is above 1000m/s, the height of target is about 400m, and its velocity is below 300m/s, the firing area of the post-navigation can reach more than 4000m. The research mentioned above.
provides a reference for increasing the firing timing in the actual system design. When the software is implemented, the existing fire control solution model can be extended by modifying the method of sending fireable instructions just before the shortest distance between anti-aircraft gun and target to achieve this new firing method.

References
[1] Bo Yuming, Guo Zhi, Qian Longjun and Wang Jun. 2002 Acta Armamentarii 23 164-66
[2] Zhu Senyuan. 2008 Ordnance Industry Automation (06) 1-4
[3] Chen Xilin, Ji Xinyuan, Luo Xiaojun and Yang Xiaoliang 2014 Modern Defence Technology 42(06) 8-13
[4] Zha Q, Rui X, Liu F and Yu H. 2017 7th International Conference on Mechatronics, Computer and Education Informationization (MCEI 2017) 75 385-93
[5] Chen C, Jic C and Juan Z. 2007 2007 Chinese Control Conference 561-64
[6] Xu J, Lei Y, Liu B and Ji C. 2018 Prognostics and System Health Management Conference (PHM-Chongqing) 249-53
[7] Wang Y, Yang G. 2019 International Conference on Modeling, Simulation, Optimization and Numerical Techniques (SMONT 2019) 165 60-4
[8] Zhang An and Liu Wei. 2001 Computer Simulation 28(05) 21-5