Lepton flavor violating Higgs decays induced by massive unparticle

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Abstract

We predict the branching ratios of the lepton flavor violating Higgs decays $H^0 \to e^\pm \mu^\mp$, $H^0 \to e^\pm \tau^\pm$ and $H^0 \to \mu^\pm \tau^\pm$ with the assumption that lepton flavor violation is due to the unparticle mediation. Here, we consider an effective interaction which breaks the conformal invariance after the electroweak symmetry breaking and causes that unparticle becomes massive. The new interaction results in a modification of the mediating unparticle propagator and brings additional contribution to the branching ratios of the lepton flavor violating decays with the new vertex including Higgs field and two unparticle fields. We observe that the branching ratios of the decays under consideration lie in the range of $10^{-6} - 10^{-4}$.

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The standard model (SM) electroweak symmetry breaking mechanism which can explain the production of the masses of fundamental particles will be tested at the Large Hadron Collider (LHC) and, hopefully, the Higgs boson $H^0$, which is responsible for this mechanism will be found soon. The possible decays of the Higgs boson to the SM particles are worthwhile to study and, among them, the lepton flavor violating (LFV) decays reach great interest \[1, 2, 4, 3, 5\] since the LF violation mechanism is sensitive to the physics beyond the SM. The addition of the new Higgs doublet to the SM particle spectrum is one of the possibilities to switch on the LFV interactions, arising from the tree level LFV couplings. In \[1, 2, 3\], $H^0 \rightarrow \tau \mu$ decay has been analyzed and the branching ratio (BR) at the order of magnitude of $0.001 - 0.1$ has been estimated. In \[4\], the observable BRs of LF changing $H^0$ decays have been obtained in the SM with right handed neutrinos. Another possibility to switch on the LF violation is to introduce the intermediate scalar unparticle ($U$) with the effective $U$-lepton-lepton vertex in the loop level. In \[5\], the BRs of the LFV Higgs decays $H^0 \rightarrow e^\pm \mu^\pm$, $H^0 \rightarrow e^\pm \tau^\pm$ and $H^0 \rightarrow \mu^\pm \tau^\pm$ have been estimated, by respecting the unparticle idea. Unparticles, introduced by Georgi \[6, 7\], come out as new degrees of freedom due to the SM-ultraviolet sector interaction; they are massless and have non integral scaling dimension $d_u$, around, $\Lambda_U \sim 1 \text{TeV}$.

In the present work we study the LFV SM Higgs decays by considering that the LF violation exists in the one loop level and it is carried by the effective $U$-lepton-lepton vertex. The effective interaction lagrangian, which is responsible for the LFV decays, is

$$L_{\text{LFV}} = \frac{1}{\Lambda_{du}^{-1}} \left( \lambda_{ij}^S \bar{l}_i l_j + \lambda_{ij}^P \bar{l}_i i\gamma_5 l_j \right) U,$$  

(1)

with the lepton field $l$ and scalar (pseudoscalar) coupling $\lambda_{ij}^S$ ($\lambda_{ij}^P$). Here we consider the operators with the lowest possible dimension since their contributions are dominant in the low energy effective theory (see \[8\]). Furthermore, we consider that there exists an additional interaction which ensures a non-zero mass to unparticle after the electroweak symmetry breaking \[9\] as

$$L_U = -\frac{\lambda}{\Lambda_{du}^{2}} U^2 H^1 H,$$  

(2)

and we get

$$L_U = -\frac{1}{2} \frac{\lambda}{\Lambda_{du}^{2}} U^2 \left( H^{02} + 2v H^0 + v^2 \right),$$  

(3)

when the Higgs doublet develops the vacuum expectation value. The interaction in eq.\[3\] leads to the lagrangian

$$L'_U = -\frac{m_{H^0}^{4-2du}}{v} U^2 H^0,$$  

(4)
with the unparticle mass

\[ m_U = \left( \frac{\sqrt{\Lambda} v}{\Lambda^{d_u-1}} \right)^{\frac{1}{2-d_u}}, \]  

(5)

and this term results in an additional diagram driving the LFV decays with the help of \( U \)-lepton-lepton vertices (see Fig.1-(d)). Here, the non-zero unparticle mass \( m_U \) is the sign of the broken conformal invariance and one expects that the unparticle propagator is modified. The propagator is model dependent (see [10]) and we consider the one in the simple model [11, 12]

\[
\int d^4x e^{ipx} < 0 | T(U(x) U(0)) 0 > = \frac{i A_{d_u}}{2\pi} \int_0^\infty ds \frac{s^{d_u-2}}{p^2 - \mu^2 - s + i\epsilon},
\]

(6)

with

\[
A_{d_u} = \frac{16 \pi^{5/2}}{(2\pi)^{2d_u}} \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1) \Gamma(2d_u)},
\]

(7)

and the scale \( \mu \) where unparticle sector becomes a particle sector. This choice has clues about the unparticle nature of the hidden sector, it carries the information on the effects of the broken scale invariance and ensures a possibility to estimate the scale invariance breaking effects. In our calculations we choose \( \mu = m_U \) and \( d_u \sim 1.0 \) which is the case that unparticle behaves as if it is almost gauge singlet scalar [2].

Now, we are ready to present the BR for \( H^0 \rightarrow l_1^- l_2^+ \) decay,

\[ BR(H^0 \rightarrow l_1^- l_2^+) = \frac{1}{16 \pi m_{H^0}} \frac{|M|^2}{\Gamma_{H^0}}; \]

(8)

where \( M \) is the matrix element of the LFV \( H^0 \rightarrow l_1^- l_2^+ \) decay (see Fig.1) and \( \Gamma_{H^0} \) is the Higgs total decay width. The square of the matrix element \( |M|^2 \) reads

\[
|M|^2 = 2\left(m_{H^0}^2 - (m_{l_1^-} + m_{l_2^+})^2\right) |A|^2 + 2\left(m_{H^0}^2 - (m_{l_1^-} - m_{l_2^+})^2\right) |A'|^2,
\]

(9)

with the amplitudes

\[
A' = \int_0^1 dx f_{self}^S + \int_0^1 dx \int_0^{1-x} dy f_{vert}^S,
\]

\[
A = \int_0^1 dx f_{self}^S + \int_0^1 dx \int_0^{1-x} dy f_{vert}^S.
\]

(10)

1Notice that the modification in the propagator needs a further analysis in order to understand whether it is based on a consistent quantum field theory and this is beyond the scope of the present manuscript.

2 This is the case that \( m_U \) lies near the electroweak scale [9].
The functions $f^S_{\text{self}}$, $f'_{\text{self}}$, $f^S_{\text{vert}}$, $f'_{\text{vert}}$ are

$$f^S_{\text{self}} = \frac{-i c_1 (1 - x)^{1-d_u}}{16\pi^2 (m_{t_2}^2 - m_{t_1}^2) (1 - d_u)} \sum_{i=1}^3 \left\{ (\lambda^S_{d_1} \lambda^S_{d_2} + \lambda^S_{d_1} \lambda^S_{d_2}) m_{t_1} m_{t_2} (1-x) \right\} \left( L_{\text{self}}^{d_a-1} L_{\text{self}}^{d_a-1} \right),$$

$$f'_{\text{self}} = \frac{i c_1 (1 - x)^{1-d_u}}{16\pi^2 (m_{t_2}^2 + m_{t_1}^2) (1 - d_u)} \sum_{i=1}^3 \left\{ (\lambda^P_{d_1} \lambda^P_{d_2} + \lambda^S_{d_1} \lambda^P_{d_2}) m_{t_1} m_{t_2} (1-x) \right\} \left( L_{\text{self}}^{d_a-1} L_{\text{self}}^{d_a-1} \right),$$

$$f^S_{\text{vert}} = \frac{i c_1 m_i (1 - x - y)^{1-d_u}}{16\pi^2} \sum_{i=1}^3 \frac{1}{L_{\text{vert}}^{2-d_u}} \left\{ (\lambda^P_{d_1} \lambda^P_{d_2} - \lambda^S_{d_1} \lambda^S_{d_2}) m_i \left( \frac{1}{1-d_u} \right) \right\} (1-x-y) \left( m_{t_2}^2 x + m_{t_2}^2 y + m_{t_2}^2 y \right),$$

$$f'_{\text{vert}} = \frac{i c_1 m_i (1 - x - y)^{1-d_u}}{16\pi^2} \sum_{i=1}^3 \frac{1}{L_{\text{vert}}^{2-d_u}} \left\{ (\lambda^P_{d_1} \lambda^P_{d_2} - \lambda^S_{d_1} \lambda^S_{d_2}) m_i \left( \frac{1}{1-d_u} \right) \right\} (1-x-y) \left( m_{t_2}^2 x + m_{t_2}^2 y + m_{t_2}^2 y \right),$$

where $L_{\text{self}}$, $L'_{\text{self}}$, $L_{\text{vert}}$, and $L_{2\text{vert}}$ are

$$L_{\text{self}} = x \left( m_{t_1}^2 (1-x) - m_{t_2}^2 \right) + m_U^2 (x-1),$$

$$L'_{\text{self}} = x \left( m_{t_2}^2 (1-x) - m_{t_1}^2 \right) + m_U^2 (x-1),$$

$$L_{\text{vert}} = (m_{t_1}^2 x + m_{t_2}^2 y) (1-x-y) = m_U^2 (x+y) + m_U^2 y x - m_U^2 (1-x-y),$$

$f^S_{\text{self}}$, $f'_{\text{self}}$ are the same as the functions presented in [5] except that the propagators $L_{\text{self}}$ and $L'_{\text{self}}$ contain the unparticle mass term $m_U$. On the other hand $f^S_{\text{vert}}$, $f'_{\text{vert}}$ include additional part proportional to the parameter $c_2$ which comes from the new interaction (see eq. (11)) leading to the vertex given in Fig.4d.
\[ L_{2\text{vert}} = (m_{l_i}^2 x + m_{l_2}^2 y) (1 - x - y) - m_l^2 (1 - x - y) + m_{H^0}^2 x y - m_U^2 (x + y), \quad (12) \]

with

\[
\begin{align*}
  c_1 &= \frac{g A_{d_u} e^{-i \pi d_u}}{4 m_W \sin \left(d_u \pi \right) A_u^{2 \left(d_u - 1\right)}}, \\
  c_2 &= \frac{A_{d_u}^2 m_U^{4 - 2 d_u} e^{-2 i \pi d_u}}{4 v \sin^2 \left(d_u \pi \right) A_u^{2 \left(d_u - 1\right)}}.
\end{align*}
\quad (13)
\]

Here \( \lambda_{ij}^{S,P} \) are the scalar and pseudoscalar couplings related to the \( U - l_i^- (l_2^+) \) interaction where \( i, (i = e, \mu, \tau) \) is the internal lepton and \( l_i^- (l_2^+) \) the outgoing lepton (anti lepton). Notice that, in the numerical calculations, we consider the BR due to the production of sum of charged states, namely,

\[
BR(H^0 \rightarrow l_1^\pm l_2^\mp) = \frac{\Gamma(H^0 \rightarrow (\bar{l}_1 l_2 + \bar{l}_2 l_1))}{\Gamma_{H^0}}.
\quad (14)
\]

**Discussion**

This section is devoted to the analysis of the BRs of the LFV \( H^0 \rightarrow l_1^- l_2^+ \) decays in the case that the LF violation is carried by the \( U^- \) lepton-lepton vertex. The LFV decays exist at least in the loop level with the help of the internal unparticle mediation. The interaction Lagrangian given in eq.(2) results in a nonzero mass for unparticle after the electroweak symmetry breaking and the propagator of unparticle existing in the loop should be modified. In the present work we take the propagator as (see eq.(6))

\[
P(p^2) = \frac{i A_{d_u}}{2 \sin \pi d_u} \frac{e^{-i d_u \pi}}{(p^2 - m_U^2)^2 - d_u},
\quad (15)
\]

which becomes a massive scalar propagator for \( d_u = 1 \).

The LF violation is carried by single unparticle mediation and two unparticles mediation in the loop (see Fig[1]). The possible two unparticles mediation brings an additional contribution to the LFV decays with the strength which is a function of unparticle mass \( m_U \), reaching 246 GeV when \( d_u \sim 1.0 \) for the coupling \( \lambda \sim 1.0 \). In our numerical calculations we take the scaling parameter \( d_u \) not far from 1.0, namely \( 1.0 \leq d_u \leq 1.2 \). On the other hand we take the coupling \( \lambda \) as \( \lambda \leq 1.0 \) in order to guarantee that the calculations are perturbative in case of \( d_u \sim 1.0 \) and we choose the energy scale \( \Lambda_u \) as \( \Lambda_u \sim 1.0 \) (TeV). The FV \( U^- \) lepton-lepton couplings, the scalar \( \lambda_{ij}^S \) and pseudo scalar \( \lambda_{ij}^P \), are among the free parameters which we choose
\(\lambda_{ij}^S = \lambda_{ij}^P = \lambda_{ij}\). Furthermore, we first consider that the diagonal \(\lambda_{ii} = \lambda_0\) and off diagonal \(\lambda_{ij} = \kappa \lambda_0, i \neq j\) couplings are family blind with \(\kappa < 1\). Second we assume that the the diagonal couplings \(\lambda_{ii}\) carry the lepton family hierarchy, namely, \(\lambda_{\tau\tau} > \lambda_{\mu\mu} > \lambda_{ee}\), on the other hand, the off-diagonal couplings, \(\lambda_{ij}\) are family blind, universal and \(\lambda_{ij} = \kappa \lambda_{ee}\). In our numerical calculations, we choose \(\kappa = 0.5\) and we take the magnitude of the FV coupling(s) at most 1.0 in order to ensure that the calculations are the perturbative for \(d_u = 1.0\).

In order to estimate the BR of the LFV decays under consideration one needs the Higgs mass and its total decay width. The theoretical upper and lower bounds of Higgs mass read 1.0\( TeV\) and 0.1\( TeV\) \[13\], respectively. This is due to the fact that one does not meet the unitarity problem and the instability of the Higgs potential both. Furthermore, the electroweak measurements predict the range of the Higgs mass as \(m_{H^0} = 129^{+74}_{-49}\) \[14\] which is not in contradiction with the theoretical results. The total Higgs decay width is another parameter which should be restricted and it is estimated by using the possible decays for the chosen Higgs mass\[1\]. Notice that throughout our calculations we choose \(m_{H^0} = 120\ (GeV)\) and we use the input values given in Table (1).

| Parameter | Value       |
|-----------|-------------|
| \(m_e\)  | 0.0005 (GeV) |
| \(m_\mu\) | 0.106 (GeV)  |
| \(m_\tau\) | 1.780 (GeV)  |
| \(\Gamma(H^0)|_{m_{H^0}=120\text{GeV}}\) | 0.0029 (GeV) |
| \(G_F\)  | \(1.1663710^{-5}(GeV^{-2})\) |

Table 1: The values of the input parameters used in the numerical calculations.

In Fig.2 we present the \(\text{BR}(H^0 \rightarrow \mu^\pm e^\pm)\) with respect to the scale parameter \(d_u\) for the flavor blind (FB) couplings \(\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0\). Here, the solid (long dashed-short dashed-dotted) line represents the BR for \(\lambda = 0.0\ (0.2-0.5-1.0)\). The possible interaction of unparticle with the Higgs scalar leads to a nonzero mass for unparticle after the spontaneous symmetry breaking and the mass term leads to a suppression in the BR. The additional term coming from the \(U - U - H^0\) vertex does not result is an enhancement in the BR. The BR reaches to the values of the order of 10\(^{-4}\) for \(\lambda = 0\) and \(d_u \sim 1.0\). For \(\lambda \sim 1.0\) and near \(d_u \sim 1.0\) the BR is of the order of 10\(^{-6}\). Fig.3 represents the \(\text{BR}(H^0 \rightarrow \mu^\pm e^\pm)\) with respect to \(\lambda\) for the scale parameter \(d_u = 1.0\). Here, the solid (long dashed-short dashed) line represents the BR

\[4\text{For the light (heavy) Higgs boson, } m_{H^0} \leq 130\text{GeV (} m_{H^0} \sim 180\text{GeV), the leading decay mode is } b\bar{b} \text{ pair } [15, 16, 17] (H^0 \rightarrow WW \rightarrow \ell^+\ell^-\nu\bar{\nu}, [18, 19, 20]).\]

\[5\text{This is the case that unparticle mass is near the vacuum expectation value, namely } m_U \sim 246\text{GeV}.\]
for $\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0$ ($\lambda_{ee} = 0.1$, $\lambda_{\mu\mu} = 0.5$, $\lambda_{\tau\tau} = 1.0$-$\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1.0$).

This figure shows the strong sensitivity of the BR to the $U - U - H^0$ interaction strength $\lambda$, especially for $\lambda < 0.3$.

Fig. 4 (5) shows the BR($H^0 \rightarrow \tau^\pm e^\pm (\tau^\pm \mu^\pm)$) with respect to the scale parameter $d_u$, for the FB couplings $\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0$. Here, the solid-long dashed-short dashed-dotted lines represent the BR for $\lambda = 0.0 - 0.2 - 0.5 - 1.0$. In the case of $d_u \sim 1.0$, the BR is almost $5.0 \times 10^{-6}$ ($6.0 \times 10^{-6}$) for $\lambda \sim 1.0$ and enhances up to $4.0 \times 10^{-4}$ for $\lambda = 0$ and $d_u \sim 1.0$. Similar to the previous decay the mass term leads to a suppression in the BR and the additional term coming from the $U - U - H^0$ vertex is not enough to enhance the BR over the numerical values which is obtained for the massless unparticle case.

In Fig. 6 (7) we present the BR($H^0 \rightarrow \tau^\pm e^\pm (\tau^\pm \mu^\pm)$) with respect to $\lambda$ for the scale parameter $d_u = 1$. Here, the solid (long dashed-short dashed) line represents the BR for $\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0$ ($\lambda_{ee} = 0.1$, $\lambda_{\mu\mu} = 0.5$, $\lambda_{\tau\tau} = 1.0$-$\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1.0$). It is observed that the BR is suppressed more than one order in the range $0.0 < \lambda < 1.0$ and this suppression is strong for $\lambda < 0.3$.

**Conclusion**

As a summary, the mass of unparticle which arises with unparticle Higgs scalar interaction results in that the BRs of the LFV $H^0 \rightarrow l_1^\pm l_2^\pm$ decays are suppressed. The BRs are of the order of $10^{-6}$ for $\lambda \sim 1.0$ and $d_u \sim 1.0$. If the unparticle-Higgs scalar interaction is switched off unparticle remains massless and the BRs of the decays studied reach to the values of the order of $10^{-4}$ for FB $U$-lepton-lepton couplings. With the possible production of the Higgs boson $H^0$ at the LHC the theoretical results of the BRs of the LFV Higgs decays will be tested and the new physics which drives the flavor violation, including the unparticle sector will be searched.
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Figure 1: One loop diagrams contribute to $H^0 \rightarrow l_1^- l_2^+$ decay with scalar unparticle mediator. Solid line represents the lepton field: $i$ represents the internal lepton, $l_1^-$ ($l_2^+$) outgoing lepton (anti lepton), dashed line the Higgs field, double dashed line unparticle field.
Figure 2: $d_u$ dependence of the BR ($H^0 \rightarrow \mu^\pm e^\pm$) for $\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0$. Here, the solid (long dashed-short dashed-dotted) line represents the BR for $\lambda = 0.0 (0.2 - 0.5 - 1.0)$.

Figure 3: $\lambda$ dependence of the BR ($H^0 \rightarrow \mu^\pm e^\pm$) for $d_u = 1$. Here, the solid (long dashed-short dashed) line represents the BR for $\lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau} = 1.0$ ($\lambda_{ee} = 0.1$, $\lambda_{\mu\mu} = 0.5$, $\lambda_{\tau\tau} = 1.0$-$\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1.0$).
Figure 4: The same as Fig. 2 but for $H^0 \rightarrow \tau^\pm e^\pm$ decay.

Figure 5: The same as Fig. 2 but for $H^0 \rightarrow \tau^\pm \mu^\pm$ decay.
Figure 6: The same as Fig. 5 but for $H^0 \to \tau^\pm e^\pm$ decay.

Figure 7: The same as Fig. 5 but for $H^0 \to \tau^\pm \mu^\pm$ decay.