The Combined Multi-objective Optimization Design for a Light Guide Rod

Yu-Sen Yang 1, Chun-Yao Shih 2, Hong-Yao Chien 3, Rong-Fong Fung 1,*
1 Department of Mechanical & Automation Engineering
National Kaohsiung First University of Science and Technology,
1 University Road, Yenchau, Kaohsiung County 824, TAIWAN
2 Graduate Institute of Electro-optical Engineering,
National Kaohsiung First University of Science and Technology,
1 University Road, Yenchau, Kaohsiung County 824, TAIWAN
3 Industrial Master Program on Advanced Headlamp Industry
National Kaohsiung First University of Science and Technology,
1 University Road, Yenchau, Kaohsiung County 824, TAIWAN

Abstract. The light guide rod (LGR) has been popularly used for the vehicles, and the automobile lamp industries need mass production to match this trend. This paper aims to develop a systemic way to find the best parameters’ combination for the LGR, and the parameters are usually restricted to some levels and random values. In this paper, the LGR example with two optical performances of illuminance flux and uniformity is to be optimized by use of the real-coded genetic algorithm (RGA) and grey relational analysis (GRA). The illuminance flux and uniformity of the best parameters’ combination are obtained and compared with the initial set. Comparisons with Taguchi-Grey can improve 5% of gain and comparisons with Pareto genetic algorithm (PaGA) can improve 1.7% of gain. The combined multi-objective optimization can saving 7% time and it is found that the new proposed method has positive gains in performances.

Keywords: Grey relational analysis (GRA), light guide rod, multi-objective optimization, real-coded genetic algorithm (RGA).

1. Introduction
The light guide rod (LGR) used in daily life is more and more than before, because it has the merit of uniform light and energy saving, and can be designed in any light shape freely. Moreover, it is easily designed in a restrained space and has good performance if one uses the light emitting diode (LED) to be the light source. On the other hand, the LED strip might not be uniform and the cold-cathode fluorescent lamp consumes more power, therefore more and more advanced, fashion vehicles use the LGR to take place of lamp eyebrows or other lightings.

However, material using and parameter choosing of the LGR for the automotive lamp factory are inexperienced so that they are usually not able to find a good setting for the manufacture and design. The LGR’s scattering in this study is made by Mie scattering material, which uses small particles in medium. The beam’s incident angle is greater than the critical angle of total internal reflection and

*Corresponding author: Professor Rong-Fong Fung; Tel.: +886-7-6011000 ext. 2238; Fax: 886-7-6011066
then the light will go out of the matter. The LED placing angle, viewing angle, distance between LEDs, transmittance percent of LGR’s outer layer, and density of scattering particles will affect the optical performances. Therefore, a systematical process is needed to deal with every type of parameters’ combination and to find out the best one.

In this paper, we propose a new modified of the real-coded genetic algorithm (RGA) and grey relational analysis (GRA). A method to deal with the LGR design problem and the main difference from the combined Taguchi-Grey method [1-2] is to permit some factors with random values within limited bounds. From our method, it is found that the effect of individual factor’s response is faster than the RGA with standard GRA. Therefore, our proposed method provides a valuable, credible, non-restricted and faster process to find out multi-objective optimization parameters, even for no reference or experience of the same type products before.

2. Experimental details

These LGR products must meet the characteristics of good performances and product standards. Figure 1 shows the flow chart of this paper, and the experimental details are described as follows.

![Flow chart of optimization illuminance simulation](image)

**Figure 1. The flow chart of optimization illuminance simulation.**

2.1. The 3D Model and standardization

We build up or import the three-dimensional (3D) geometrical model of the LGR by SolidWorks software and then link with LightTools software to perform illuminance simulations, where the position relationships among light source, receiver and the LGR are shown in Figure 2.

Because the LGR is a large-sized and complex shape, it will take long time to perform illuminance simulations. In this paper, the optimized results are expected to be applied to other kinds of LGR. With the design standardization of the LGR, it provides a useful basis model and can be applied directly. Therefore, we do not need to perform all the process over again for another new LGR. In this way, it can accelerate the development, and reduce the cost and timing of the LGR research.

To simplify and represent the LGR model adequately, we take a 2-LED part as a standard one from the origin model, which is designed for 64 LEDs. The width of the standard model is 23.5 mm and height is 9 mm. In this paper, the optimization design is performed first in the standard model, and then applies the results to deal with the origin model and validates this standard model effective. The position relationships among light source, receiver and the standard LGR are shown in Figure 3.
2.2. Simulation process and material
In this paper, the goal is to perform the multi-objective optimization for the LGR automatically. In order to design experiments and analyze these data, the RGA method written in the MS Office Excel VBA software connecting with LightTools software is used to change parameters’ setting, read and process illuminance simulations, perform optimization algorithm, and do the whole loop by default settings.

The LGR material is the MIE scattering material composed of the polycarbonate (PC) mixing with the \( \mu m \) size polymethylmethacrylate (PMMA) particles. Usually, the light cannot penetrate the inorganic molecules and its shape is often sphere. The refractive index of selected particles must be different and the added weight ratio is usually 1% to 5%. The MIE scattering particle’s parameters will affect the optical result.

2.3. Simulation convergence analysis
In this paper, the convergence analysis is estimated by the average value (AV) and average difference (AD) of illuminance flux. The convergence criteria is taken as the fluctuation percentage of the last four measured AV and AD values, which are all required under one percentage. The measured AV and AD for the LGR are shown in Figure 4, where the numbers of illuminance simulations are from 10,000 to 30,000. It is found that when the light number is larger than 24,000 the fluctuation percentage of the last four measured AV and AD value are less than one percentage and the numerical simulations are convergent.

2.4. The real-coded genetic algorithms
The genetic algorithms which make use of the real-coded chromosomes are termed as the real-coded genetic algorithm (RGA). In a continuous searching space, the RGA is more natural than binary-coded GA to perform the optimization process [3-4]. The optimization searching RGA is a simulating evolution mechanism on a computer-based platform in conjunction with natural selection and genetic mechanism. The RGA continuously searches for the better chromosomes in this way until the convergent index is satisfied. The reproduction procedure adopts the roulette wheel selection, and the crossover and uniform mutation adopt the methods in [5-6].

The RGA process [7-8] is briefly described by the following nine steps: setting the constraint specification, determining the fitness function, generating the initial population, evaluating the fitness
value, reproduction, crossover, mutation, evaluating the fitness value for offspring chromosomes, and constructing the new population.

Before executing the RGA process, the population size, maximum generation number, crossover probability, mutation probability, fitness function, range of each parameter must be assigned for the RGA by our expert knowledge and numerical experience. How to define the fitness function is the key point of the RGA, since the fitness function is a figure of merit, and is computed by using any domain knowledge. In this paper, we employ the GRD as the fitness function. This is the main difference from the traditional GA by using the specialized fitness function.

In this paper, there are three parameters including LED viewing angle, distance between LED, and scattering particles’ density, and the population size is set 40. The probability of the jth chromosome into the mating pool can use the following equation:

$$\text{fit ratio}_j = \frac{\text{fit value}_j}{\sum_i \text{fit value}_i}$$

where $k$ is the population size. Here, the arithmetic crossover operator is defined as follows:

$$x_{o1} = (1 - \alpha)x_{p1} + \alpha x_{p2}$$

$$x_{o2} = \alpha x_{p1} + (1 - \alpha)x_{p2}$$

where $x_{p1}$ and $x_{p2}$ are two genes in parent chromosomes, $x_{o1}$ and $x_{o2}$ are two children, and $\alpha$ is selected randomly between 0 and 1. The crossover probability is given between 0.8 and 1 by ordinary. In this paper, the crossover probability is set 0.8. Here, uniform mutation is used and is defined as follows:

$$x_{\text{new}} = LB + \beta(UB - LB)$$

where $x_{\text{new}}$ is the gene after mutation, $\beta$ is the mutation probability and is selected randomly between 0 and 1, $LB$ and $UB$ are the minimum and maximum values of the gene’s ranges, respectively. In general, the mutation probability is often given a low value, and set 0.01 in this paper.

Finally, repeating steps to search for the optimal solution until the end of the maximum generation. In this paper, the maximum generation number is 30.

2.5. Quality characteristic and factor combination
The performance indicators of the LGRs are the average value (AV) of output illuminance and uniformity. The average value of illuminance is the larger the better (LTB), but the average difference (AD) of uniformity is the smaller the better (STB).

In this paper, we consider three factors: LED viewing angle, distance between LED, and scattering particles’ density for the LGR. For these three factors of the LGR, one restricted level factor and two random level factors are shown in Table 1.

| Factor                      | Level A | Level B | Level C |
|-----------------------------|---------|---------|---------|
| LED viewing angle (°grove)  | 120     | 60      | 30      |
| Distance between LED (mm)   | 0       | –       | 17      |
| Scattering particles’ density (/mm^3) | 4,000   | –       | 5,000   |

2.6. Grey relational analysis (GRA)
Grey theory is the most used for the co-relational analysis and modeling in the incomplete information and its model is not clear. In this paper, the GRA is briefly described as follows. The pre-processing method is to normalize the data. Converting all parameters’ values to the numbers between 0 and 1,
and all targets of factors are adjusted to the larger-the-best (LTB), and all factors are then defined. This method has different equations according to various quality characteristics:

Larger-the-best:

$$x_i(k) = \frac{x_i^0(k) - \min_i x_i^0(k)}{\max_i x_i^0(k) - \min_i x_i^0(k)}$$  \hspace{1cm} (5)

Smaller-the-best:

$$x_i(k) = \frac{\max_i x_i^0(k) - x_i^0(k)}{\max_i x_i^0(k) - \min_i x_i^0(k)}$$  \hspace{1cm} (6)

where \(\max_i x_i^0(k)\) and \(\min_i x_i^0(k)\) are the maximum and minimum of sequence \(x_i^0(k)\), respectively. It can avoid adjusting to a single quality characteristic and ignoring the other one.

In the grey relational space, there is a sequence \(x_i(1), x_i(2), ..., x_i(k) \in \mathbb{R}^n\), and \(i = 0, 1, 2, ..., m\), \(k = 0, 1, 2, ..., n \in \mathbb{N}\), so that \(x_0 = (x_0(1), x_0(2), ..., x_0(n))\), \(x_i = (x_i(1), x_i(2), ..., x_i(n))\), \(i = 0, 1, 2, ..., m\). The GRC is defined by

$$\gamma(x_i(k), x_j(k)) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{ij}(k) + \zeta \Delta_{\max}}$$  \hspace{1cm} (7)

where \(i, j = 1, 2, 3, ..., m\), \(k = 1, 2, 3, ..., n\). The \(x_i(k)\) is the reference series, \(x_j(k)\) is the specific series, where \(\Delta_{ij}(k) = \| x_i(k) - x_j(k) \|\) is the absolute difference between \(x_i(k)\) and \(x_j(k)\). \(\Delta_{\min} = \min\{\Delta_{ij}(k), i = 1, 2, ..., m; j = 1, 2, ..., m\}\) and \(\Delta_{\max} = \max\{\Delta_{ij}(k), i = 1, 2, ..., m\}\) are the minimum and maximum absolute difference in the sequence, respectively. The \(\zeta\) is the distinguishing coefficient, and \(\zeta \in [0, 1]\). In general, the distinguishing coefficient is taken as 0.5, and can also be adjusted according to the actual needs.

Since the GRC is obtained, we can calculate GRD by averaging the GRCs as

$$\Gamma(x_i, x_j) = \frac{1}{n} \sum_{k=0}^{n} \gamma(x_i(k), x_j(k))$$  \hspace{1cm} (8)

where \(n\) is the number of performance characteristics. In this study, two output responses of the AV and AD illuminance are applied to calculate the integration. The normalization is only used for the LTB type of equation (5). Next, the GRC calculation will proceed in equation (7).

For these multi-objective cases, we try to combine this flow and integrate the quality characteristics into an equation. Using equation (5) or (6) to normalize the AV and AD values of illuminance flux, but not their S/N values, we calculate the absolute difference as

$$\Delta(k) = ||1 - x_{AV}(k)|| + ||1 - x_{AD}(k)||$$  \hspace{1cm} (9)

where \(x_{AV}(k)\) or \(x_{AD}(k)\) are the AV and AD values of illuminance flux, respectively. The absolute differences \(\Delta(k)\) is used to evaluate the illuminance effects with the STB, that means the two normalized results are better as closer to 1. Its merit is to avoid adjusting to a single quality characteristic and they are equally important. Comparing the proposed method (9) with the S/N ratio in Taguchi method, our method doesn’t need so many expressions and can have the same function of multi-objective optimization.

2.7. Combining the RGA and GRA

In this paper, we combine the RGA and GRA for the multi-objective optimization design. The procedure of the RGA-Grey is shown in Figure 5. The GRA can deal with the complex relations and multi-objective cases. The RGA generates the factor combination, and needs the fitness function to find the optimal solution. In this paper, the combination idea is proposed to take the absolute difference (9) as the fitness function for the RGA.
In our proposed process, after the illuminance simulation for factor combination, the AV and AD values of optical effects are normalized into the range between 0 and 1, which is also called the Grey relational generating. The GRC is then calculated from the normalized optical effects to express the relationship between the estimated factor combination and the previous combination for each optical effect. The value is the LTB and the survival chromosome probability is proportional to this value. The re-produced chromosomes then undergo a re-combination that consists of crossovers and mutations. The whole process will do loop until achieving the stop condition or the optimal solution is obtained.

3. Results and discussions
The results of our proposed RGA-Grey multi-objective optimization design for the standard model is found that the best factor combination is: LED viewing angle is $120^\circ$, distance between LED is 11.7mm, and scattering particles’ density is $4,687.132 / mm^3$. By using this factor combination, the illuminance flux is 0.456(lm) and AD is 0.162. The GRC of standard method deals with quality characteristics individually, and then takes their average. In our improved method, the GRC can process quality characteristics by using equation (9) at the same time. By this way, we can integrate into a single performance index even though don’t take their average. The results of the standard GRA are the best factor combination is: LED viewing angle is $120^\circ$, distance between LED is 11.9mm, and scattering particles’ density is $4,510.648 / mm^3$. By using this factor combination, the illuminance flux is 0.445(lm) and the AD is 0.166. As a result, the differences of illuminance flux and the AD are both about 2.5%, it is found that the results are very similar as shown in Table 2. However, in our proposed method, less equations are used and can reduce 58.2969 seconds for one generation of the RGA, which are performed in a computer with Windows XP Service Pack 3 operating system, Intel Pentium 3.4GHz processor and 1GB of RAM.

| Table 2. The results of the standard method and improved method |
|---------------------------------------------------------------|
| **Factor sets** | **Illuminance (lm)** | **Average difference** |
| Standard method | $120^\circ$ /11.9mm / $4,510.648 / mm^3$ | 0.445 | 0.166 |
| Improved method | $120^\circ$ /11.7mm / $4,687.132 / mm^3$ | 0.456 | 0.162 |
| Difference | 3.9% | 2.5% | 2.4% |
In the GRA method, the raw data is converted to the S/N ratio by a simplified calculation. The comparisons between the proposed method and the RGA method are shown in Figure 6. It is found that the GRCs are all required for the two methods, but the proposed one does not need the S/N calculation.

![Figure 6](image)

(a) RGA-Grey  
(b) GRA whose raw data is converted to the S/N ratio

**Figure 6.** The compared results of the RGA-Grey and the GRA whose raw data is converted to S/N ratio.

### 3.1. Comparison with Taguchi-Grey method

In order to compare with Taguchi-Grey method, where levels and the responses of the GRD are shown in Table 3. According to the responses of GRC which is shown in Figure 7, we select the larger value as the best factor set, which is a set of A1, B3, and C2. That is, the LED viewing angle is 120°, distance between LEDs is 12mm, scattering particles’ density is 4,500 / mm³.

| factor | A (degree) | B (mm) | C (distribution / mm³) |
|--------|------------|--------|------------------------|
| Level 1| 120        | 17     | 5,000                  |
| Level 2| 60         | 14.5   | 4,500                  |
| Level 3| 30         | 12     | 4,000                  |

**Table 3.** LGR Selected Taguchi factors and their levels

![Figure 7](image)

**Figure 7.** The response of Grey relational grade

From our analysis, it is known that the LED viewing angle and distance between LEDs are very similar in the RGA-Grey and Taguchi-Grey optimal solutions. However, the scattering particles’ density in Taguchi’s method is restricted in levels, but is a random value in the RGA method. Therefore, the result in the RGA method can be found a more precise value. The optimal solutions between the RGA-Grey and Taguchi-Grey methods are compared in Table 4.
Table 4. The gains of the RGA-Grey and Taguchi-Grey for LGR standard model

| Factor sets | Illuminance (lm) | Average difference |
|-------------|-----------------|-------------------|
| Taguchi-Grey | 120° / 12mm / 4,500 / mm³ | 0.434 / 0.168 |
| RGA-Grey | 120° / 11.7mm / 4,687.132 / mm³ | 0.456 / 0.162 |
| Gain | | 5.07% / 3.57% |

3.2. Comparison with Pareto genetic algorithm

The Pareto optimal solutions’ set can be provided for decision-makers’ reference and evaluation. The conclusion illustrated Pareto genetic algorithm (PaGA) can search across the Pareto optimal solution set [9]. The difference of the PaGA and RGA-Grey is that the PaGA result is a set of solutions and that of the RGA-Grey is only a solution. To confirm the RGA-Grey method can make equal quality characteristics, we use one simple way to combine multi-objective functions into a scalar fitness function for the PaGA. The weighted sum approach [10] is

\[ P(k) = w_1 * x_i(k) + w_2 * x_j(k) \]  

where \( x_i(k) \) or \( x_j(k) \) is the normalized one of illuminance simulations’ results, \( i=1, 2, 3, ..., m \), \( j=1, 2, 3, ..., m \), and \( w_1 \) or \( w_2 \) is a constant weight for \( x_i(k) \) or \( x_j(k) \). In this case, \( w_1 \) and \( w_2 \) are taken as 0.5 for the equal optical effects of the LGR. We take equation (10) in the place of \( \Delta(k) \) and the PaGA result is shown in Figure 8. According to this result, we choose three sets (A, B, and C) to compare with that decided by the RGA-Grey method, and the compared result is shown in Table 5. It is found that the optical effects of PaGA A set is very similar to the RGA-Grey method, and its mean is same as the \( \Delta(k) \).

Table 5. The gains of RGA-Grey and PaGA for LGR standard model

| Factor sets | Illuminance (lm) | Average difference |
|-------------|-----------------|-------------------|
| RGA-Grey | 120° / 11.7mm / 4,687.132 / mm³ | 0.456 / 0.162 |
| PaGA A | 120° / 10.9mm / 4,051.613 / mm³ | 0.450 / 0.170 |
| Gain | | -1.32% / -4.93% |
| PaGA B | 120° / 11.8mm / 4,137.943 / mm³ | 0.462 / 0.171 |
| Gain | | 1.32% / -5.56% |
| PaGA C | 60° / 11.9mm / 4,051.613 / mm³ | 0.344 / 0.134 |
| Gain | | -24.56% / 17.28% |

In order to confirm optimal results of the standard model’s are effective, we perform the same process with the same conditions for the origin model. The optimal result is: LED viewing angle is 120°, distance between LEDs is 11.9 mm, and scattering particles’ density is 4,694.867/mm³.
The illuminance simulations of the LGR by using the best set from the RGA-Grey method are compared with the initial set, and their results are shown in Table 6 and Figure 9.

### Table 6. The gains of the optimized and initial factor sets for LGR origin model.

| Factor sets | Illuminance (lm) | Average difference |
|-------------|-----------------|--------------------|
| Initial     | 120° / 17mm / 4,000 / mm³ | 22.543             |
|             |                  | 1.684              |
| Optimized   | 120° / 11.9mm / 4,694.867 / mm³ | 22.931             |
|             |                  | 1.195              |
| Gain        |                  | 1.72%              |
|             |                  | 29.04%             |

**Figure 9.** The compared results of the RGA-Grey and initial sets for LGR origin model.

Converting the lighting image of these real samples to 16 colors and grey values, the results are shown in Figure 10. It is seen that the chart line by the RGA-Grey method is the smoothest than the initial and Taguchi-Grey ones. It is known that the best set is found by the RGA-Grey method, because its interval of bright and dark is the smallest than the others. A comparison of the integrated performance index between the initial and optimal sets shows that the illuminance flux increases from 22.54(lm) to 22.93(lm) and the AD value decreases from 1.68 to 1.19. The positive gains for the total illuminance flux and the AD value, compared with the initial condition, are reported as 1.72% and 29.04%, respectively. These optical properties are obviously improved.

**Figure 10.** The compared results of real samples.

### 4. Conclusions

By the RGA method, the best factor combination is more ideal than Taguchi’s method, in which factors are restricted in levels. In our proposed examples, not all factors are random, but some of them have several specifications and can’t change freely. However, it is a real condition for product setting for manufacturers. The RGA-Grey method can obtain the same result as the process flow whose raw data is converted to the S/N ratio and save about 7.6% time for per generation of RGA. The GRA could combine with not only the RGA method but also Taguchi method to deal with the multi-
objective optimization problem. For this case, the method can make quality characteristics of both are as equally important as possible and don’t need to decide by decision-maker as Pareto. Our proposed method can also reduce the developing time and production cost from simulation samples to mass productions, and enhance the competitiveness for the industry manufacturer. Finally, the results of the optimized combination confirm the optical properties with AV and AD of illuminance flux are improved 3~5% of gain by using the proposed method in this paper.

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