The $1P$ quarkonium fine splittings at NLO

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Abstract

We calculate the $1P$ heavy quarkonium fine splittings at NLO and discuss the impact of the calculation on the $\chi_b(1P)$ splittings.

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I. INTRODUCTION

The calculation of the heavy quarkonium potentials and spectra in perturbation theory has a long history \[1, 2, 3, 4, 5, 6, 7\]. Early calculations had to face two main problems: (1) the consistent inclusion of non-perturbative effects and (2) the poor convergence of the perturbative series. Recently remarkable progress has been made in both. Non-relativistic effective field theories of QCD, have provided a systematic way to factorize non-perturbative effects in heavy quarkonium observables. Moreover, they have proved useful in organizing and cancelling renormalon singularities and in resumming logs with renormalization group techniques: these being the main sources of large corrections in the perturbative series. For a recent review on effective field theories for heavy quarkonium we refer to \[8\].

This progress has triggered in the last few years a renewed interest in perturbative calculations of the heavy quarkonium spectrum. In \[9, 10\] the bottomonium spectrum was calculated in a fully analytical NNLO calculation that implemented the leading order renormalon cancellation. In \[11, 12\] a numerical calculation of the spectrum was done that included NLO spin-dependent potentials. This calculation also relies (weakly, according to the authors) on some assumption about the long-range behaviour of the static potential. In \[13, 14\] the hyperfine splittings of the 1$S$ bottomonium, charmonium and $\bar{b}c$ system were calculated at NLL accuracy. In general these calculations show rather stable (in the normalization scale) and convergent (in the perturbative series) results.

In this work we calculate the $n = 2$, $l = 1$ quarkonium fine splittings at NLO. Some contributions were calculated in \[4\], but a complete analytical calculation was missing up to now. The calculation may be relevant for heavy quarkonium states for which the momentum transfer scale $p$ is much larger than the scale of non-perturbative physics $\Lambda_{QCD}$. Two situations may occur under this condition \[15\]. Let us call $E$ the typical kinetic energy of the heavy quark and antiquark in the centre-of-mass reference frame: in a non-relativistic bound state $p \gg E$. The first situation corresponds to quarkonium states for which $E \gtrsim \Lambda_{QCD}$. Under this circumstance the heavy quarkonium potential is purely perturbative and non-perturbative contributions are parametrically suppressed with respect to the NLO corrections. The second situation corresponds to quarkonium states for which $p \gg \Lambda_{QCD} \gg E$. Under this circumstance the potential contains a perturbative part and short-range non-perturbative contributions. A NLO calculation may help in this case to constrain the size of...
the non-perturbative contributions affecting the spin-dependent potentials, which have been since long the object of intense study [16]. Moreover, due to Poincaré invariance, they are related by some exact relation to the non-perturbative contributions in the static potential [17].

The paper is organized as follows. In Sec. II we will calculate the fine splittings for \( n = 2, l = 1 \) quarkonium states at NLO. Our main result is Eq. (26). In Sec. III we will apply our result to the fine splittings of the \( \chi_b(1P) \) states.

II. \( n = 2, l = 1 \) FINE SPLITTINGS AT NLO

We want to calculate the energy splitting

\[
E(1^3P_j) - E(1^3P_j'),
\]

at NLO accuracy. In order to do this we need the relevant spin-dependent potentials and the wave function at NLO accuracy.

The spin-orbit, \( V_{LS} \), and tensor, \( V_T \), potentials at NLO can be found, for instance, in [3]; they read:

\[
V_{LS}(r) = V_{LS}^{(0)}(r) + V_{LS}^{(1)}(r),
\]

\[
V_{LS}^{(0)}(r) = \frac{3C_F\alpha_s(\mu)}{2m^2r^3} \mathbf{L} \cdot \mathbf{S},
\]

\[
V_{LS}^{(1)}(r) = V_{LS}^{(0)}(r) \frac{\alpha_s}{\pi} \left[ \frac{\beta_0}{2} (\ln m r + \gamma_E) - \frac{2}{3} C_A (\ln m r + \gamma_E) - \frac{11}{36} C_A + \frac{2}{3} C_F + \frac{1}{9} T_F n_f \right],
\]

\[
V_T(r) = V_T^{(0)}(r) + V_T^{(1)}(r),
\]

\[
V_T^{(0)}(r) = \frac{C_F\alpha_s(\mu)}{4m^2r^3} \mathbf{S}_{12},
\]

\[
V_T^{(1)}(r) = V_T^{(0)}(r) \frac{\alpha_s}{\pi} \left[ \frac{\beta_0}{2} (\ln m r + \gamma_E) - \frac{1}{4} C_A + C_F + \frac{1}{3} T_F n_f \right],
\]

where \( \mathbf{L} = \mathbf{r} \times \mathbf{p}, \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2, \mathbf{S}_i = \sigma_i / 2, \mathbf{S}_{12} = 3 \mathbf{r} \cdot \mathbf{S}_1 \mathbf{r} \cdot \mathbf{S}_2 - \mathbf{S}_1 \cdot \mathbf{S}_2, C_A = N_c = 3, C_F = 4/3, T_F = 1/2, \beta_0 = 11 N_c/3 - 4 T_F n_f / 3, \) \( n_f \) is the number of active massless flavours, \( \alpha_s \) is the strong coupling constant in the \( \overline{\text{MS}} \) scheme and \( \gamma_E \simeq 0.577216 \) is the Euler constant.

In order to calculate the wave function at NLO we have to consider the heavy quarkonium Hamiltonian at NLO. In the centre-of-mass reference frame it reads [18]:

\[
H = \frac{\mathbf{p}^2}{m} + V^{(0)}(r) + \delta V^{(0)}(r),
\]
\[ V^{(0)}(r) = -\frac{C_F \alpha_s(\mu)}{r}, \]  
\[ \delta V^{(0)}(r) = V^{(0)}(r) \frac{\alpha_s}{\pi} \left[ \frac{\beta_0}{2}(\ln r \mu + \gamma_E) + \frac{31}{36} C_A - \frac{5}{9} T_F n_f \right]. \]

We call \(|nljs\rangle\) the eigenstates common to \(H\), \(L^2\), \(J^2\) (\(J = L + S\)) and \(S^2\). At NLO \(|nljs\rangle\) is given by

\[ |nljs\rangle = |nljs\rangle^{(0)} + |nljs\rangle^{(1)}, \]

where

\[ \left( \frac{\mathbf{p}^2 + V^{(0)}(r)}{m} \right) |nljs\rangle^{(0)} = E_n^{(0)} |nljs\rangle^{(0)}, \quad \text{with} \quad E_n^{(0)} = -m \left( \frac{C_F \alpha_s}{2n} \right)^2, \]

\[ |nljs\rangle^{(1)} = \sum_{(k \neq n), j'j's'} |k'l'j's'\rangle^{(0)} \langle k'l'j's' | \delta V^{(0)}(r) |nljs\rangle^{(0)} \frac{E_n^{(0)} - E_k^{(0)}}{E_n^{(0)}}. \]

For later use we notice that

\[ \langle nljs | f(r) \mathbf{L} \cdot \mathbf{S} | n'l'j's' \rangle = \delta_{l'j'} \delta_{s's} \frac{j(j+1) - l(l+1) - s(s+1)}{2} \langle nljs | f(r) | n'l'js \rangle, \]

\[ \langle nljs | f(r) S_{12} | n'l'j's' \rangle = \delta_{l'j'} \delta_{s's} \langle S_{12} \rangle_{ljs} \langle nljs | f(r) | n'l'js \rangle, \]

where

\[ \langle S_{12} \rangle_{ljs} = \begin{cases} -\frac{2l+2}{2l+1} & , \quad j = l - 1 \\ +2 & , \quad j = l \\ -\frac{2l}{2l+3} & , \quad j = l + 1 \end{cases}, \quad \text{for} \ l \neq 0. \]

\[ \langle S_{12} \rangle_{lj0} = \langle S_{12} \rangle_{0j0} = 0, \]

and that

\[^{(0)}\langle nljs | f(r) | n'l'j's' \rangle^{(0)} = \delta_{l'j'} \delta_{s's} \langle nl | f(r) | n'l \rangle^{(0)}, \]

where \(|nl\rangle^{(0)}\) are the eigenstates common to \([p^2/m + V^{(0)}(r)]\) and \(L^2\).

The fine splitting \(E(1^3P_j) - E(1^3P_{j'})\) at NLO is given by

\[ E(1^3P_j) - E(1^3P_{j'}) = \langle 21j1 | V_{LS} + V_T | 21j1 \rangle - \langle 21j'1 | V_{LS} + V_T | 21j'1 \rangle, \]

where

\[ \langle 21j1 | V_{LS,T} | 21j1 \rangle =^{(0)} \langle 21j1 | V_{LS,T} | 21j1 \rangle^{(0)} +^{(1)} \langle 21j1 | V_{LS}^{(0)} | 21j1 \rangle^{(0)} +^{(0)} \langle 21j1 | V_{LS,T}^{(0)} | 21j1 \rangle^{(1)}. \]
Using the explicit expressions of the potentials, Eqs. (2)–(7), we obtain:

\begin{align}
(0) \langle 21j|V_{1S}^{(0)}|21j \rangle^{(0)} &= m \frac{(C_F \alpha_s(\mu))^4}{256} \left[ j(j + 1) - 4 \right], \\
(0) \langle 21j|V_{1T}^{(1)}|21j \rangle^{(0)} &= (0) \langle 21j|V_{1S}^{(0)}|21j \rangle^{(0)} \\
&\times \frac{\alpha_s}{\pi} \left[ \frac{\beta_0}{2} \left( \ln \frac{2\mu}{mC_F\alpha_s} + 1 \right) - \frac{2}{3} C_A \ln \frac{2}{C_F\alpha_s} - \frac{35}{36} C_A + \frac{2}{3} C_F + \frac{1}{9} T_F n_f \right], \\
(0) \langle 21j|V_{2S}^{(0)}|21j \rangle^{(0)} &= m \frac{(C_F \alpha_s(\mu))^4}{768} (\mathbf{S}_{12})_{1j1}, \\
(0) \langle 21j|V_{2T}^{(1)}|21j \rangle^{(0)} &= (0) \langle 21j|V_{2S}^{(0)}|21j \rangle^{(0)} \\
&\times \frac{\alpha_s}{\pi} \left[ \frac{\beta_0}{2} \left( \ln \frac{2\mu}{mC_F\alpha_s} + 1 \right) - C_A \ln \frac{2}{C_F\alpha_s} - \frac{3}{4} C_A + \frac{1}{3} C_F + \frac{1}{9} T_F n_f \right].
\end{align}

Equations (20) and (22) correct the analogous expressions that may be found in [4, 5].

To calculate \( (1) \langle 21j|V_{1S}^{(0)}|21j \rangle^{(0)} \) we use Eqs. (14), (15), (16) and the momentum-space representation given in [20]:

\[
\sum_{(k \neq 2), l} \frac{\langle p|kl \rangle^{(0)} \langle kl|q \rangle^{(0)}}{E_2^{(0)} - E_k^{(0)}} = \frac{(2\pi)^3 \delta^3(p - q)}{\gamma_2^2/m - p^2/m} - \frac{1}{\gamma_2^2/m - p^2/m} \frac{1}{4\pi C_F \alpha_s} \frac{1}{|p - q|^2} - \frac{\gamma_2^2/m - q^2/m}{\gamma_2^2/m - q^2/m}
\]

They should read (for \( l \neq 0 \)):

\begin{align}
(0) \langle n|ls|V_{1S}^{(0)}|nljs \rangle^{(0)} &= m \frac{3(C_F \alpha_s(\mu))^4}{16n^3(l + 1)(2l + 1)} \left[ j(j + 1) - l(l + 1) - s(s + 1) \right] \\
&\times \left\{ 1 + \frac{\alpha_s}{\pi} \right\} \left[ \frac{\beta_0}{2} \left( \ln \frac{n\mu}{mC_F\alpha_s} - \psi(n + l + 1) + \psi(2l + 3) + \psi(2l + \gamma_E - \frac{n - l - 1/2}{n}) \right) \\
&- \frac{2}{3} C_A \left( \ln \frac{n}{C_F\alpha_s} - \psi(n + l + 1) + \psi(2l + 3) + \psi(2l + \gamma_E - \frac{n - l - 1/2}{n}) \right) \\
&- \frac{11}{36} C_A + \frac{2}{3} C_F + \frac{1}{9} T_F n_f \} \},
\end{align}

\begin{align}
(0) \langle n|ls|V_{1T}^{(1)}|nljs \rangle^{(0)} &= m \frac{(C_F \alpha_s(\mu))^4}{16n^3(l + 1)(2l + 1)} (\mathbf{S}_{12})_{lj} \\
&\times \left\{ 1 + \frac{\alpha_s}{\pi} \right\} \left[ \frac{\beta_0}{2} \left( \ln \frac{n\mu}{mC_F\alpha_s} - \psi(n + l + 1) + \psi(2l + 3) + \psi(2l + \gamma_E - \frac{n - l - 1/2}{n}) \right) \\
&- C_A \left( \ln \frac{n}{C_F\alpha_s} - \psi(n + l + 1) + \psi(2l + 3) + \psi(2l + \gamma_E - \frac{n - l - 1/2}{n}) \right) \\
&+ \frac{1}{4} C_A + \frac{1}{3} C_F + \frac{1}{9} T_F n_f \} \},
\end{align}

where \( \psi \) is the derivative of the logarithm of the Gamma function. We thank F. J. Yndurain for communications on this point.
Equation (26) is the main result presented here.

\[
- \frac{128 \pi m \gamma_2^3}{(p^2 + \gamma_2^2)(q^2 + \gamma_2^2)^2} \left\{ \frac{2 \gamma_2^2 (p - q)^2}{(p^2 + \gamma_2^2)(q^2 + \gamma_2^2)} \left( \frac{9}{2} + \frac{6 \gamma_2^2}{p^2 + \gamma_2^2} + \frac{6 \gamma_2^2}{q^2 + \gamma_2^2} \right) \\
+ \frac{3}{2} \frac{4 \gamma_2^2}{p^2 + \gamma_2^2} - \frac{4 \gamma_2^2}{q^2 + \gamma_2^2} \right\} \left( \frac{1}{2C_2 - 1} \ln C_2 + \frac{2C_2 - 4 + 1/C_2}{\sqrt{4C_2 - 1}} \frac{\text{arctan}}{\sqrt{4C_2 - 1}} \right),
\]

(23)

with

\[ C_2 = \frac{(p^2 + \gamma_2^2)(q^2 + \gamma_2^2)}{4 \gamma_2^2(p - q)^2}, \quad \text{and} \quad \gamma_2 = m C_F \alpha_s / 4. \]

Eventually we obtain:

\[
(1) \langle 21j1 | V_{LS}^{(0)} | 21j1 \rangle^{(0)} = \langle 0 \rangle \langle 21j1 | V_{LS}^{(0)} | 21j1 \rangle^{(0)} \left(1\right) = \langle 0 \rangle \langle 21j1 | V_{LS}^{(0)} | 21j1 \rangle^{(0)} = \langle 0 \rangle \langle 21j1 | V_{LS}^{(0)} | 21j1 \rangle^{(0)}
\]

\[ \times \frac{3}{4} \frac{\alpha_s}{\pi} \left[ \beta_0 \left( \frac{155}{54} - \frac{2}{9} \pi^2 + \ln \frac{2 \mu}{m C_F \alpha_s} \right) + \frac{31}{24} C_A - \frac{5}{6} T_F n_f \right], \quad (24) \]

\[
(1) \langle 21j1 | V_T^{(0)} | 21j1 \rangle^{(0)} = \langle 0 \rangle \langle 21j1 | V_T^{(0)} | 21j1 \rangle^{(1)} = \langle 0 \rangle \langle 21j1 | V_T^{(0)} | 21j1 \rangle^{(0)}
\]

\[ \times \frac{3}{4} \frac{\alpha_s}{\pi} \left[ \beta_0 \left( \frac{155}{54} - \frac{2}{9} \pi^2 + \ln \frac{2 \mu}{m C_F \alpha_s} \right) + \frac{31}{24} C_A - \frac{5}{6} T_F n_f \right]. \quad (25) \]

In Eqs. (24) and (25) the logs agree with what calculated in [4] using a variational method and the last two terms proportional to \( C_A \) and \( T_F n_f \) may be also derived from Eqs. (10), (19) and (21). The other terms are the new contributions to the fine splittings at NLO provided by this work.

Summing up all contributions, the fine splittings of the \( n = 2, l = 1 \) quarkonium states are given at NLO accuracy by

\[
E(1^3P_j) - E(1^3P_{j'}) = \frac{m(C_F \alpha_s)^4}{768} \left\{ \\
3 \left[ j(j + 1) - j'(j' + 1) \right] \left[ 1 + \frac{\alpha_s}{\pi} \left[ \beta_0 \left( 2 \ln \frac{2 \mu}{m C_F \alpha_s} + \frac{173}{36} - \frac{\pi^2}{3} \right) \right. \right. \\
- \frac{2}{3} C_A \ln \frac{2}{C_F \alpha_s} + \frac{29}{18} C_A + \frac{2}{3} C_F - \frac{14}{9} T_F n_f \left] \right. \\
\left. \left. + \left[ (S_{12})_{1j1} - (S_{12})_{1j'1} \right] \left[ 1 + \frac{\alpha_s}{\pi} \left[ \beta_0 \left( 2 \ln \frac{2 \mu}{m C_F \alpha_s} + \frac{173}{36} - \frac{\pi^2}{3} \right) \right. \right. \right. \\
- C_A \ln \frac{2}{C_F \alpha_s} + \frac{11}{6} C_A + C_F - \frac{4}{3} T_F n_f \right] \right\}. \quad (26)
\]

Equation (26) is the main result presented here.

The energy splitting between the centre-of-gravity energy \( E(1^3P)_{c.o.g.} = \left[ 5 E(1^3P_2) + 3 E(1^3P_1) + E(1^3P_0) \right] / 9 \) and the \( 1^1P_1 \) state is entirely given at order \( (C_F \alpha_s)^4 \alpha_s / \pi \) by the
spin-spin potential at NLO and is, therefore, not affected by our analysis. It has been considered in [4, 5, 6] and we reproduce it here for completeness:

\[ E(1^3P_{\text{c.o.g.}}) - E(1^1P_1) = \frac{m(C_F \alpha_s)^4 \alpha_s}{288 \pi} \left[ \frac{\beta_0}{2} - \frac{7}{4} C_A \right] . \]  

**III. APPLICATION TO THE \( \chi_b(1P) \) SPLITTINGS**

In this section we apply Eq. (26) to the fine splittings of the bottomonium \( 1P \) states, \( \chi_b(1P) \). As argued in Sec. I a necessary condition for this to make sense is that \( p \gg \Lambda_{\text{QCD}} \). In order to have an idea of the size of the scale \( p \), we may follow [4] and solve the (perturbative) self-consistency equation \( mC_F \alpha_s(p)/(2n) = p \) for \( n = 2 \). We obtain \( p \approx 0.9 \) GeV for a choice of the mass of 4.73 GeV (i.e. half of the \( \Upsilon(1S) \) mass). For comparison, in the charmonium ground state case we obtain \( p \approx 0.8 \) GeV for a choice of the mass of 1.55 GeV (i.e. half of the \( J/\psi \) mass). We see that it is reasonable to expect the typical momentum transfer scale for \( n = 2 \) bottomonium states to be larger than the scale of non-perturbative QCD (\( \lesssim 0.5 \) GeV). Noteworthy it is also larger than the typical momentum transfer scale of the charmonium ground state, which has been often assumed to be in the perturbative regime [1, 4, 5, 6, 7, 9, 13]. Indeed, \( p \) has been assumed larger than \( \Lambda_{\text{QCD}} \) for \( \chi_b(1P) \) states in [4, 5, 6, 7, 9, 10].

In Figs. 1 and 2 we show \( E(1^3P_1) - E(1^3P_0) \) and \( E(1^3P_2) - E(1^3P_1) \) respectively as a function of \( \mu \). The light and dark bands show the LO and NLO expectations respectively as from Eq. (26). The widths of the bands account for the uncertainty in \( \alpha_s(M_Z) \) only. The mass has been chosen to be \( m = 4.73 \) GeV. The correct number of massless flavours in the bottomonium case is \( n_f = 3 \) [10]. We note that in the momentum region around the physically motivated momentum transfer of about 0.9 GeV the LO and NLO bands overlap with each other and with the experimental value. This means that the perturbative series shows convergence inside the physically motivated momentum transfer region and reproduces, inside the uncertainties, the experimental values. Note that the NLO curves have a relative maximum, i.e. a minimal sensitivity point, that goes from 0.82 GeV to 0.90 GeV for \( E(1^3P_1) - E(1^3P_0) \) and from 0.83 GeV to 0.91 GeV for \( E(1^3P_2) - E(1^3P_1) \) as \( \alpha_s(M_Z) \) goes from 0.1167 to 0.1207. Therefore, the physically motivated momentum transfer region overlaps also with the region covered by the minimal sensitivity points of the NLO curves.

The agreement between the fine splittings calculated at NLO and the experimental data
FIG. 1: $E(1^3P_1) - E(1^3P_0)$ versus the normalization scale $\mu$. The light band shows the LO expectation, the dark one the NLO one as from Eq. (26). The widths of the bands account for the uncertainty in $\alpha_s(M_Z) = 0.1187 \pm 0.002$. The horizontal band shows the experimental value $32.8 \pm 1.2$ MeV [21].

FIG. 2: $E(1^3P_2) - E(1^3P_1)$ versus the normalization scale $\mu$. The horizontal band shows the experimental value $19.9 \pm 0.8$ MeV [21]. All the rest is like in Fig. 1.

appears reasonably good. Moreover it happens in a region where the physical momentum transfer is expected to be, the perturbative series shows convergence and the NLO curve is less dependent on the scale $\mu$. However, the uncertainties are large so that no conclusive statement can be made at this point. A quantity less sensitive to $\alpha_s$ and to correlated
FIG. 3: $\rho$ versus the normalization scale $\mu$. The horizontal line at 0.8 corresponds to the LO expectation, the curve to the NLO one. The band accounts for the uncertainty in $\alpha_s(M_Z) = 0.1187 \pm 0.002$ [21]. The horizontal band at 0.607 $\pm$ 0.038 shows the experimental value [21].

uncertainties in the splittings is the ratio

$$\rho = \frac{E(1^3P_2) - E(1^3P_1)}{E(1^3P_1) - E(1^3P_0)}. \quad (28)$$

It has been suggested long ago that this observable may be rather sensitive to non-perturbative contributions in the spin-dependent potentials [16]. The ratio $\rho$ is equal to 0.8 at LO in perturbation theory, while it is measured to be 0.607 $\pm$ 0.038 for the $1P$ bottomonium state [21]. The dark band in Fig. 3 shows the NLO expectation. We see that the NLO corrections go toward the data and may explain from about 15% to 65% of the difference between them and the LO value in the scale range 0.8 $\div$ 1 GeV. At the scale $\mu = 0.9$ GeV the NLO calculation seems to account for 20% $\div$ 40% of the difference between 0.8 and the experimental value. At this stage we cannot say to what extent the remaining difference is due to non-perturbative corrections, since higher-order perturbative corrections not included in this analysis can still be large, as the scale dependence of $\rho$ and the case of the $n = 1$ hyperfine splitting may indicate. In this respect it would be important to perform a NLL analysis similar to those done in [13, 14].

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