Magnetic Moments of Negative Parity Baryons from Effective Hamiltonian Approach to QCD

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Submitted 9 December 2013

Magnetic moments of \( S_{11}(1535) \) and \( S_{11}(1650) \) baryons are studied in the framework of the relativistic three-quark Hamiltonian derived in the Field Correlation Method. The baryon magnetic moments are expressed via the average kinetic quark energies which are defined by the fundamental QCD parameters: the string tension \( \sigma \), the current quark masses, and the strong coupling constant \( \alpha_s \). Resulting magnetic moments for the \( J^P = 1/2^- \) nucleons are compared both to model calculations and to those from lattice QCD.

1. Introduction. Magnetic moments encode information about the leading-order response of a bound system to a soft external magnetic field. In particular, baryon magnetic moments are dynamical characteristics which provide valuable insights into baryon internal structure in terms of quark and gluon degrees of freedom. In this paper we shall explore the magnetic moments of negative parity resonances employing the QCD dynamics of a baryon in the form of the three-quark Effective Hamiltonian (EH). The EH is derived from the QCD path integral, (see e.g. [1]), and was already used in the studies of baryon spectra without external fields [2] - [5]. Within this method the magnetic moments of the \( 1/2^+ \) octet baryons have been studied analytically in Ref. [7]. The model was shown to agree with experiment within 10% accuracy. The same accuracy was achieved for the baryon magnetic moments in Ref. [8], where the QCD string dynamics was investigated from another point of view.

Although the magnetic moments of the \( 1/2^+ \) baryon octet are well-known both experimentally and theoretically, little is known about their \( 1/2^- \) counterparts. Experimentally, magnetic moments of these states can be extracted through bremsstrahlung processes in photo- and electro-production of mesons at intermediate energies. For \( N(1535) \) a similar process \( \gamma p \rightarrow \gamma np \) can be used [9], but to date no such measurements have been made.

There exist limited number of theoretical studies of the magnetic moments of negative parity baryons based on constituent quark model [9], unitarized chiral perturbation theory (U\( \chi \)PT) [10], chiral constituent quark model (\( \chi \)CQM) [11], Bethe-Salpeter approach (BSA) [12] and on the lattice [13]. Comparison study of magnetic moments for positive- and negative-parity states offers insight into underlying quark-gluon dynamics. Given that the mass spectrum of the \( 1/2^- \) states has been reasonably well established from the EH, it is instructive to investigate the magnetic moments of these states. In this paper we extend the results of Ref. [7] to the magnetic moments of the negative parity \( S_{11}(1535) \) and \( S_{11}(1650) \) resonances. The paper builds on the previous work presented in Ref. [4] where the employed EH contained the three quark string junction interaction and the Coulomb potential with the fixed strong coupling constant.

In Sections 2.3 we briefly discuss the theoretical formalism of EH method for baryons, including the techniques required to extract the average quark energies \( \omega_i \) which are cornerstone of the present calculation. As a result one obtains the resonance magnetic moments without introduction of any fitting parameters. Details of calculation of the magnetic moments for excited \( 1/2^- \) nucleons and the results are given in Section 4. For the sake of completeness we also report in this Section the magnetic moments of the \( 1/2^- \) octet baryons and \( 3/2^- \) nucleon resonances. Section 5 contains a summary of the obtained results.

2. Effective Hamiltonian for Baryons. The key ingredient of the EH method is the use of the auxiliary fields (AF) initially introduced in order to get rid of the square roots appearing in the relativistic Hamiltonian [14]. Using the AF formalism allows one to derive a sim-
ple nonrelativistic form of the EH for the three-quark system which comprises both confinement and relativistic effects, and contains only universal parameters: the string tension \( \sigma \), the strong coupling constant \( \alpha_s \), and the bare (current) quark masses \( m_a \), \( a = 1, 2, 3 \),

\[
H = \frac{3}{2} \sum_{a=1}^{3} \left( \frac{m_a^2}{2\omega_a} + \frac{\omega_a}{2} \right) + H_0 + V.
\]  
(1)

In Eq. (1) \( H_0 \) is the nonrelativistic kinetic energy operator for the constant AF \( \omega_a \), the spin-independent potential \( V \) is the sum of the string potential

\[
V_Y(r_1, r_2, r_3) = \sigma r_{\text{min}},
\]  
(2)

with \( r_{\text{min}} \) being the minimal string length corresponding to the Y-shaped configuration, and a Coulomb interaction term

\[
V_C(r_1, r_2, r_3) = -C_F \sum_{a < b} \frac{\alpha_s}{r_{ab}}
\]  
(3)

arising from the one-gluon exchange. In Eq. \( C_F = \frac{3}{2} \) is the color factor. The constant \( \alpha_s \) is treated either as a fixed parameter, \( \alpha_s = 0.39 \) [4] or as the running coupling constant with the freezing value \( \sim 0.5 \) [5]. The results for \( \omega_a \) for these two cases coincide with the accuracy better than 1% (compare Tables 1 and 2 of Refs. [4] and [5], respectively). In what follows we use \( \omega_a \) taken from Ref. [4].

3. The auxiliary field formalism The EH (1) depends explicitly on both bare quark masses \( m_a \) and the constants AF \( \omega_a \) that finally acquire the meaning of the average kinetic energies of the current quarks [4]

\[
\omega_a = < \sqrt{p_a^2 + m_a^2} >.
\]  
(4)

Although being formally simpler the EH is equivalent to the relativistic Hamiltonian up to elimination of AF. The confinement Hamiltonian contains three parameters: the current quark masses \( m_q \) and \( m_s \) and the string tension \( \sigma \). Let us stress that they are not the fitting parameters. In our calculations we used \( \sigma = 0.15 \text{GeV}^2 \) found in the SU(3) QCD lattice simulations [19]. We employed the current light quark masses \( m_u = m_d = 9 \text{ MeV} \) and the bare strange quark mass \( m_s = 175 \text{ MeV} \).

4. Magnetic moments of \( S_{11}(1535) \) and \( S_{11}(1650) \) resonances. To calculate the nucleon magnetic moment one introduces a vector potential \( A \) and calculate the energy shift \( \Delta M_B \) due the Hamiltonian \( H = H(A) + H_\sigma \) where \( H \) is defined by Eq. (1) with the substitution \( p_a \rightarrow p_a - e_a A_a \)

\[
H_\sigma = -\sum_a \frac{e_a \sigma_a}{2\omega_a} B,
\]  
(8)

where \( B \) is an external magnetic field. The magnetic moment operator consists of contributions from both intrinsic spins of the constituent quarks that make up the bound state \( \mu_S \) and angular momentum of the three-quark system \( \mu_L \) with the center of mass motion removed. Straightforward calculation using the London gauge \( A = \frac{1}{2}(B \times r) \) yields

\[
\hat{\mu} = \hat{\mu}_S + \hat{\mu}_L,
\]  
(9)

Taking the constituent quarks to be Dirac point particles the spin contribution in Eq. (9) is determined by the effective quark masses \( \omega_i \)

\[
\mu_S = \sum_a e_a \sigma_a \omega_a.
\]  
(10)

The orbital contribution in Eq. (9) reads

\[
\mu_L = \sum_a \frac{e_a}{\omega_a} r_a \times p_a
\]  
(11)
In what follows instead the usual prescription which is to symmetrize the nucleon wave function between all three quarks we symmetrize only between equal-charge (up or down) quarks. In other words for the proton we use the \( uud \) basis in which the \( d \) quark is singled out as quark 3 but in which the quarks \( uu \) are still antisymmetrized. In the same way, for the neutron we use the basis in which the \( uud \) quarks we symmetrize only between equal-charge (up or down) quarks. In other words for the proton we use the basis in which the quarks \( uu \) are still antisymmetrized.

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By definition, the magnetic moment \( \mu \) of the baryon with the spin \( J \) is the expectation value of the operator \( \hat{\mu}^2 \) for the state with \( M_L = J \)

\[
\mu = < \hat{\mu}^2 > = < JJ | \hat{\mu}_3^2 + \hat{\mu}_5^2 | JJ >
\] (17)

In particular, for baryons with \( L = 0 \) where \( L \) the angular momentum of the three-quark system with the correct center of mass motion removed

\[
\mu = \mu_{\text{spin}} = < \frac{1}{2} | \sum a \frac{e_a}{2a_0} | \frac{1}{2} > = < \chi_{\frac{3}{2}}^1(12; 3)| \sum a \frac{e_a}{2a_0} | \chi_{\frac{3}{2}}^1(12; 3) > = \frac{1}{3}(2\mu_1 + 2\mu_2) - \frac{1}{3}\mu_3
\] (18)

where \( \chi_{\frac{3}{2}}^1 \) is the doublet spin function symmetric under interchange \( 1 \leftrightarrow 2 \). Eq. (18) is standard result of the additive quark model for the \( \frac{1}{2}^+ \) baryons [21].

In Table 1 we show the parameters \( \omega_q \) and \( \omega_s \) calculated for the different baryons with \( L = 0 \) in Ref. [4] using the constant value of \( \alpha_s = 0.39 \). The symbol \( q \) denotes the light quarks \( u \) or \( d \) and we use the notation \( \omega_1 = \omega_2 = \omega_q \), \( \omega_3 = \omega_s \) for the \( qqq \) and \( qqs \) baryons and \( \omega_1 = \omega_2 = \omega_s \), \( \omega_3 = \omega_q \) for the \( ssq \) baryons. These parameters have been calculated for the string tension \( \sigma = 0.15 \text{ GeV} \) and the strong coupling constant \( \alpha_s = 0.39 \) with the values of the current light quark masses, \( m_u = m_d = 9 \text{ MeV} \), \( m_s = 175 \text{ MeV} \). The very similar values of \( \omega_i \) have been calculated in Ref. [5] where instead the constant \( \alpha_s \) the running coupling constant \( \alpha_s(r) \) has been used with \( \alpha_s(\infty) \sim 0.5 \).

There is no good theoretical reason why \( \omega_s \) need to be the same in different mesons and baryons. However from the results of Table 1 we conclude that \( \omega_s \) are increased by \( \sim 10 \text{ MeV} \) when going from the nucleon to \( \Xi \). This variation is marginal and is within the accuracy of calculations. For ground states of \( \Lambda \) and \( \Sigma \) hyperons we obtain \( \omega_q = 0.414 \text{ MeV} \), \( \omega_s = 0.453 \text{ MeV} \) that agrees with the corresponding values for the ground state of \( K \) meson [16].

The magnetic moments for the \( \frac{1}{2}^+ \) baryons with \( L = 0 \) are presented in Table 2. For the \( \frac{3}{2}^+ \) baryons one obtains \( \mu_{\Delta^+} = 3\mu_u = 4.575\mu_N \), other moments are \( \mu_{\Delta^+} = 2\mu_u + \mu_d = 3\mu_{\Delta^++} \), \( \mu_{\Delta^0} = 0 \), \( \mu_{\Delta^-} = 3\mu_d = -\frac{1}{3}\mu_{\Delta^++} \). Recall that so far, only the magnetic moment of \( \Delta^{++}(1232) \) has been studied in the reaction \( \pi^+ p \rightarrow \gamma_{\gamma^p} p \) with the result \( \mu_{\Delta^{++}} \sim 3.7 - 7.5\mu_N \) [20]. The uncertainty in the number arises from the ambiguity in the theoretical analysis of the reaction.
The wave function of an $\frac{1}{2}^-$ resonance is given as a superposition of two spin $(S = 1/2$ and $3/2$) states in the $l = 1$ 70-dimensional representation of SU(6):

$$|S_{11}(1535)\rangle = \cos \theta |P_{1/2}\rangle - \sin \theta |4P_{1/2}\rangle$$

$$|S_{11}(1650)\rangle = \sin \theta |P_{1/2}\rangle + \cos \theta |4P_{1/2}\rangle,$$

where mixing angle $\theta$ depends on the hyperfine spin interaction between the quarks and the standard spectroscopic notations $|2S + 1P_{1/2}\rangle$ are used to indicate the total quark spin $S = 1/2, 3/2$, orbital angular momentum $L = 1$, and total angular momentum $J = 1/2$. The corresponding spin-angular functions are given by

$$|2P_{1/2}\rangle =$$

$$\frac{1}{\sqrt{2}} \left( \sqrt{\frac{2}{3}} Y_{11}(\lambda) \chi^+_{\frac{1}{2}+} - \sqrt{\frac{1}{3}} Y_{10}(\lambda) \chi^0_{\frac{1}{2}+} \right) +$$

$$\frac{1}{\sqrt{2}} \left( \sqrt{\frac{2}{3}} Y_{11}(\rho) \chi^0_{\frac{1}{2}+} - \sqrt{\frac{1}{3}} Y_{10}(\rho) \chi^0_{\frac{1}{2}+} \right), \quad (21)$$

where $\chi^z_{\frac{1}{2}m_z}$ and $\chi^0_{\frac{1}{2}m_z}$ are the two spin functions symmetric and antisymmetric under interchange $1 \leftrightarrow 2$, and

$$|4P_{1/2}\rangle =$$

$$\sqrt{\frac{1}{2}} Y_{1-1}(\lambda) \chi^1_{\frac{1}{2}+} - \sqrt{\frac{1}{6}} Y_{10}(\lambda) \chi^1_{\frac{1}{2}+} + \sqrt{\frac{1}{6}} Y_{11}(\lambda) \chi^1_{\frac{1}{2}+}$$

$$\sqrt{\frac{1}{6}} Y_{10}(\lambda) \chi^1_{\frac{1}{2}+} + \sqrt{\frac{1}{6}} Y_{11}(\lambda) \chi^1_{\frac{1}{2}+} \quad (22)$$

Note that parameters $\omega$ for the $1/2^-$ nucleons depend also on the type of excitation. However, the difference is marginal and does not exceed 2%, see Table 2 of Ref. [5]. In what follows we use the common value $\omega = 0.457$ GeV both for $\rho$ and $\lambda$ excitations.

Straightforward calculation yields [2]

$$\mu(S_{11}^z(1535)) = \mu((2P_{1/2})^z) \cos^2 \vartheta + \mu((4P_{1/2})^z) \sin^2 \vartheta$$

$$-2 <^2 P_{1/2}^z |\lambda S_{11}^z |4P_{1/2}^z > \sin \vartheta \cos \vartheta = 1.24 \mu_N \quad (23)$$

$$\mu(S_{11}^z(1650)) = \mu((2P_{1/2})^z) \sin^2 \vartheta + \mu((4P_{1/2})^z) \cos^2 \vartheta$$

$$+2 <^2 P_{1/2}^z |\lambda S_{11}^z |4P_{1/2}^z > \sin \vartheta \cos \vartheta = -0.33 \mu_N \quad (24)$$

$$\mu(S_{11}^z(1535)) = \mu((2P_{1/2})^z) \cos^2 \vartheta + \mu((4P_{1/2})^z) \sin^2 \vartheta$$

$$-2 <^2 P_{1/2}^z |\lambda S_{11}^z |4P_{1/2}^z > \sin \vartheta \cos \vartheta = -0.84 \mu_N \quad (25)$$

$$\mu(S_{11}^z(1650)) = \mu((2P_{1/2})^z) \cos^2 \vartheta + \mu((4P_{1/2})^z) \sin^2 \vartheta$$

$$-2 <^2 P_{1/2}^z |\lambda S_{11}^z |4P_{1/2}^z > \sin \vartheta \cos \vartheta = 0.744 \mu_N \quad (26)$$

3) The indexes +, 0 refer to the charge of the nucleon $\frac{1}{2}^-$ states.
Table 2. Baryon magnetic moments of $J^P = \frac{1}{2}^-$ baryons.

| Baryon | $\omega_q$ | $\omega_s$ | $\mu_u$ | $\mu_d$ | $\mu_s$ | $\mu$ | Expt. |
|--------|------------|------------|---------|---------|---------|-------|-------|
| $p$    | 0.408      | 1.53       | -0.77   | 2.29    | 2.79    |       |       |
| $n$    | 0.408      | 1.53       | -0.77   | -1.53   | -1.91   |       |       |
| $\Lambda$ | 0.414  | 0.453      | 1.51    | -0.76   | -0.69   | -0.61 |       |
| $\Sigma^+$ | 0.414 | 0.453      | 1.51    | -0.76   | -0.69   | 2.23  | 2.46  |
| $\Sigma^0$ | 0.414  | 0.453      | 1.51    | -0.76   | -0.69   | 0.80  | 0.83  |
| $\Sigma^-$ | 0.414  | 0.453      | 1.51    | -0.76   | -0.69   | -0.91 | -1.16 |       |
| $\Xi^0$ | 0.419      | 0.458      | 1.485   | -0.742  | -0.75   | -0.50 | -0.65 |       |
| $\Xi^-$ | 0.419      | 0.458      | 1.689   | -0.845  | -0.75   | -0.50 | -0.65 |       |
| $\Omega^-$ | 0.463 |       | -0.671  | -2.01   | -2.02   |       |       |

where

$$\mu(2P_{1/2}^+ - P_{1/2}^+) = \frac{2}{9} \mu_u + \frac{1}{9} \mu_d = 0.23 \mu_N, \quad (27)$$

$$\mu(4P_{1/2}^0 = \mu_u + \frac{1}{3} \mu_d = 1.14 \mu_N \quad (28)$$

and

$$<4P_{1/2}^+ |\mu_z| 2P_{1/2}^+ > = \frac{4}{9}(\mu_u - \mu_d) = 0.91 \mu_N \quad (29)$$

Eqs. (27) - (29) are written for the positive charge resonances. For the neutral resonances one should interchage $\mu_u$ and $\mu_d$

$$\mu(2P_{1/2}^0 = \frac{1}{9} \mu_u + \frac{2}{9} \mu_d = 0, \quad (30)$$

$$\mu(4P_{1/2}^0 = \frac{1}{3} \mu_u + \mu_d = -0.232 \mu_N, \quad (31)$$

and

$$<4P_{1/2}^0 |\mu_z| 2P_{1/2}^0 >= \frac{4}{9}(\mu_d - \mu_u) = -0.926 \mu_N \quad (32)$$

Assuming a phenomenological value $\theta = -\frac{\pi}{6}$, we obtain the results summarized in Table 3. In this Table we also quote the magnetic moments obtained using other theoretical models.

5. Conclusions. To summarize, we have carried out a calculation of the magnetic moments of the low-lying negative parity $S_{11}(1535)$ and $S_{11}(1650)$ resonances. In the framework of the quark model these resonances are configuration mixtures of two $SU(6)$ states with excited orbital wavefunctions. Calculating both the quark spin and orbital angular momentum contribution for the magnetic moment, the cross terms due to the configuration mixing, and using the average value of the quark kinetic energy $\omega = 0.457$ GeV obtained from the variational solution for the einbein field in the EH method we obtain the values of magnetic moments of the $S_{11}(1535)$...
Table 3. Magnetic moments of $J^P = \frac{1}{2}^-$ nucleons. $\omega_q = 0.457$ GeV

| State   | CQM [9] | $\chi PT$ [10] | $\chi CQM$ [11] | BSA [12] | LQCD [13] | This work |
|---------|---------|----------------|-----------------|----------|-----------|-----------|
| $S_{11}^+(1535)$ | 1.894   | 1.1            | 2.085           | 0.37     | -1.8      | 1.24      |
| $S_{01}^-(1535)$ | -1.284  | -0.25          | -1.57           | -0.1     | -1.0      | -0.84     |
| $S_{11}^+(1650)$ | 0.11    | 1.85           |                 |          |           | 0.12      |
| $S_{11}^-(1650)$ | 0.951   | -0.69          |                 |          |           | 0.74      |

and $S_{11}^-(1650)$ listed in Table 3. The results differ from the magnetic moments of the low-lying $J^P = \frac{1}{2}^-$ nucleon calculated both in hadronic and quark models and from lattice QCD. In particular, the lattice results are different, even by sign. Any future measurement of the magnetic moment would have important implications in understanding the nature of parity partners of the nucleon.

Finally we note that the magnetic moments of the other $\frac{1}{2}^-$ low-lying baryon resonances can similarly be calculated using EH approach. The results will be published elsewhere.

Acknowledgements. The authors thank Yu. A. Simonov for valuable discussions.

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