Mathematical and visual models of asynchronous electric machines energy fields

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Abstract. The research considers the development of topological theory of asynchronous electric machines (AEM). The results of the development of energy aspects of the topological theory are presented in the article. The facts proving the actuality of the research are also given. The central notion giving the topological theory the characteristic of being systematic is the vector space notion. The definition of AEM vector space, its dimension, canonical basis, power and energy field of AEM are presented. Mathematical and visual models of AEM energy fields are analyzed. Energy fields of electric losses and magnetic field dissipation having the spherical symmetry are analyzed in particular. Energy and visual models of the main magnetic field are analyzed using AEM with a short circuited rotor and linear AEM as examples. It is established that this field obtains cylindrical or elliptical symmetry. In general the main magnetic field and the main matrix of winding make AEM vector space fully or partially heterogeneous. Energy efficiency is analyzed and its differences for vector space currents are revealed. The areas of vector spaces with high and low energy currents efficiency are defined. According to this indicator longitudinal and transvers subsets of current having, correspondingly, high and zero energy efficiency are distinguished in the vector space. The description of practical application spheres of AEM topological theory is given.

Keywords: asynchronous electric machine, topological theory, vector space, basis, the canonical basis, energy field, mathematical models, visual models, energy efficiency, a subset of currents

1. Introduction

One of the actual aim of the modern electromechanics is to create a theory reflecting systematic properties and systematic reactions of asynchronous electric machines (AEM) on external actions and changes of internal properties. There is a number of factors concerning AEM internal heterogeneous demonstration which are ignored or interpreted indefinitely by the existing theoretical models.
These factors are:

- the difference in energy efficiency of some types of systems of AEM phase current;
- the efficiency of mathematical methods for AEM with homogeneous internal properties and their inefficiency for AEM with heterogeneous properties;
- the appearance and energy consequence of compensating currents and interchange power in the heterogeneous electromechanical structures;
- the generation of not basic frequencies or additional mechanical power by AEM heterogeneous rotor.

Besides the number of problematic questions are connected with the correctness of mathematical models of heterogeneous AEM.

Among the earliest works affecting aspects of the systemic nature of electrical objects, the works of N. Hancock and G. Kron should be mentioned [1-3], referring to the second half of the last century. G. Kron introduced the term "topological theory". Among the late publications is the work of A.D. Artym [4]. Today the significant progress has been achieved in this study. The terminological apparatus of the theory [5-7] is formed, definitions of the vector space of its dimension, phase basis, canonical basis, longitudinal and transverse current subsets are given to determine the rules and properties of basis transformations [8 - 10]. Based on them, a method for analyzing vector space has been created that focuses on a multidimensional vector-matrix representation of electromagnetic states and energy fields of AEM [11,12], and it describes the phenomenon of excessive energy dissipation in AEM [13 - 15]. The study of the vector spaces of parametric winding matrices of the AEM [16 - 18] has been carried out and a number of mathematical models of the AEM have been developed taking into account their topological properties [19 - 22]. It should be noted that, when applying aspects of topological theory, work in the field of monitoring of energy indicators and technical condition of AEM [23 - 25] takes into account the specifics of their energy processes and characteristics [26, 27].

In this article the authors present the results of their developments of energy aspects of topological theory.

2. Main definitions

The central notion giving the topological theory the property of being systematic is the notion of AEM vector space. Let us introduce the definition of this term and some accompanying definitions.

The vector space of AEM polyphase winding is understood as a set of vectors of electromagnetic values (currents), voltages, electromotive force (EMF) and flux linkage corresponding all its possible electromagnetic conditions.

The dimension of the winding vector space fully corresponds to the number of its phases. The vector space having the greatest dimension is considered to be AEM vector space. However the electromagnetic transformation of energy in AEM is possible only in case the electromagnetic values of rotor and stator windings have a common vector subspace.

The basis of the vector subspace of three–phase AEM considers any of orthonormal three vectors belonging to this subspace. The vectors of AEM windings currents are physically given in the basis of phase axes. It means that their coordinates in these basis are the physically phase currents of windings. Every vector can be written in any of other vector subspace basis by the operation of basis transformation. This operation performing corresponds to the change of the real winding on the equivalent one situating along the axes of the new basis and also the change of the winding parametric matrix on the similar one. In this case the power of the real and equivalent windings are equal.

Among the basis of vector space canonic basis are distinguished. The canonic basis of the winding vector space considers the orthonormal three eigenvectors of its parametric matrix.

Not only the vector of winding current corresponds to every point of the vector space but the scalar value – the power of the winding vector space as well

\[ p = (\bar{r} Z \bar{T}) \]
where - winding parametric matrix connecting the vectors of its current and power (EMF).

In canonic basis matrix obtains a diagonal form

\[
Z = \begin{pmatrix}
\lambda_x & 0 & 0 \\
0 & \lambda_y & 0 \\
0 & 0 & \lambda_z
\end{pmatrix}
\]

where \(\lambda_x, \lambda_y, \lambda_z\) - the eigenvalues of the matrix along the axes of basis and quadratic form obtains the canonical form

\[
p = \lambda_x i_x^2 + \lambda_y i_y^2 + \lambda_z i_z^2
\]

where \(i_x, i_y, i_z\) - the coordinates of current vector in canonical basis.

Parametrical AEM matrixes, excepting the matrixes of mutual induction, are symmetric [5, 6]. The transformation of quadratic form to the canonical form is always defined for them. So the basis transformation is invariant with the respect to the power

\[
p = (\bar{\tau}, Z\bar{\tau}) = (\bar{\tau}, Z\bar{\tau})
\]

where index \(\nu\) shows that the object belongs to the canonical basis.

The scalar function of the vector argument \(p(\bar{\tau})\) or \(W(\bar{\tau})\) expressing the power or energy of some winding physical field is called the winding energy field. The energy field is usually represented by surfaces of the level in the form of \(p(\bar{\tau}) = C = \text{const}\) or \(W(\bar{\tau}) = C = \text{const}\).

3. Energy fields of asynchronous electric machines

Let us write down in accordance with [6] the balance of power of AEM stator three–phase winding.

\[
(t_x, u_s) + \frac{d(t_x, r_s E_x)}{dt} + (t_x, l_{ss} E) + \frac{d(t_x, M)}{dt} + (t_x, l_{ss} E) + \frac{d(t_x, M)}{dt}
\]

where \(u_s, i_s, l_{ss}, l_m\) - vectors of supply voltage, stator winding current, rotor winding current and magnetizing current in the canonical basis of stator winding \([\bar{x}, \bar{y}, \bar{z}]\), \(r_s E, l_{ss} E, M\) - diagonal matrixes of active phase resistance, inductances of phase dissipation, the main matrix of stator winding; \(E\) - unity matrix; \(r_s, l_{ss}\) - active resistance and inductance of stator winding dissipation phase.

In the left part (1) there are powers of external, for stator, source (drain) of energy: the power of the source of supply and electromagnetic power. In the right part (1) there are powers of stator winding energy fields: energy losses, magnetic field of dissipation and the main magnetic field.

The power field of electrical losses is a scalar field of the vector argument \(t_x\). In accordance with (1),

\[
p_{el} = p_{el}(t_x) = (t_x, r_s E_x) = r_s \left[\vec{E}_x\right]^2
\]

In vector space the surfaces of the field level are the surfaces of the fixed power \(C\).
\[ p_{el}(\tau) = r_s \left( t_{Sx}^2 + t_{Sy}^2 + t_{Sz}^2 \right) = C, \]

representing the spheres by the radius \( |\tau_s| \).

Two of them, for the values of power and electrical losses \( C \) and \( 4C \), in canonical coordinates are shown in figure 1. The vector space multiple currents \( \tau_s \) are shown here by the arrows.

The field of energy of the magnetic field of a stator winding dissipation is a scalar field of vector argument \( \tau_s \). In accordance with (1), the power of the field is equal to

\[ \left( \tau_s, t_{os} E \right) = \frac{d}{dt} \left( W_{os} \right) = \frac{d}{dt} \left( \frac{l_{os} |\tau_s|^2}{2} \right) \]

In vector spaces the surfaces of the field level \( W_{os}(\tau_s) \) are the surfaces of the fixed energy \( C \)

\[ W_{os}(\tau_s) = \frac{l_{os}}{2} \left( t_{Sx}^2 + t_{Sy}^2 + t_{Sz}^2 \right) = C, \]

representing the spheres of radius \( |\tau_s| \). Two of them, which are for the values of energy of the magnetic field dissipation \( C \) and \( 4C \), are shown in figure 1.

**Figure 1.** Energy field with spherical symmetry.

The field of energy of the main magnetic field is a scalar field of vector argument \( \tau_m \). In accordance with (1), the power of the field is equal to

\[ \left( \tau_m, M \right) = \frac{d}{dt} \left( W_m \right). \]

For the AEM with a short – circuit rotor the main matrix along the axis x is degenerated, and the plane yz is parametrically homogeneous [6]
With this connection its energy field has the cylindrical symmetry and the surfaces of the level

\[ W_m(\mathbf{r}_m) = \frac{\mu_0}{2} \left( r_m^2 + r_n^2 \right) = C \]  

represent a cylinder of the infinite length along the axis \( x \) having the radius of rolling circle \( \sqrt{2\times C/\mu_0} \). For the values of energy of the main field \( C \) and \( 4C \) they are shown in figure 2. The arrows show the multiple currents \( i_n \).

For linear AEM the main matrix is not degenerated and parametrically heterogeneous along all axes of basis [6]

\[ M = \begin{pmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{pmatrix}. \]

In accordance with this its energy field has elliptic symmetry and the surfaces of level

\[ W_m(\mathbf{r}_m) = \frac{1}{2} \left( \mu_0 i_m^2 + \mu_0 i_n^2 + \mu_0 i_n^2 \right) = C \]  

represent ellipsoids with semi-axes \( \sqrt{2C/\mu_0}, \sqrt{2C/\mu_0}, \sqrt{2C/\mu_0} \). For the values of energy of the main field \( C \) and \( 4C \) they are shown in figure 3. The arrows show the multiple currents \( i_n \).

Thus, the main matrix of the stator winding makes AEM vector space fully or partially heterogeneous.
The energy efficiency of vector space. In accordance with (2) and (3) eigenvalues of the matrix have the sense of specific energy of the main magnetic field for the squared current unit. They can be regarded as the characteristics of vectors current efficiency orienting along the axes of canonical basis. In accordance with such an interpretation and figure 2, figure 3 the area of the most energy efficiency of the vector space is the plane yz and the currents oriented along the axis x are less efficient from the point of their contribution to the energy of the main magnetic field.

In vector space two sets of current vectors having correspondingly maximum and zero efficiency are distinguished in general. The authors called them longitudinal and transvers subsets of currents.

In figure 4 the surface trace of level $W_m(t_m) = C$ along (3) in plane xz and three of its characteristic points a, b, c are shown. The figure corresponds to the general case of parametric heterogeneous of AEM vector space at which in plane xz the inequalities are valid

$$\lambda_{mx} \neq 0, \lambda_{mc} < \lambda_{mc}$$

The vectors of the main flux linkage $\vec{\psi}_m$ and currents $\vec{I}_m$ for each point are shown in the figure. Vectors of flux linkage being the gradient vectors of the energy field are orthogonal to the level surfaces.

**Figure 4.** Vectors of electromagnetic values in heterogeneous vector space.

Points a and c are on canonic axes z and x and point b is between them. The position provides the space coincidence of flux linkage vectors for points a and c:

- for point a and

  $$\vec{\psi}_m = \lambda_{mc} \vec{I}_m$$

- for point c. These equations express the fact that the space is homogeneous along the canonic axes but has different inductances.

In the direction of current $\vec{I}_m$ of point b the space is parametrically heterogeneous, that is why

$$\vec{\psi}_m = M \vec{I}_m = \begin{pmatrix} \lambda_{mx} & 0 & 0 \\ 0 & \lambda_{my} & 0 \\ 0 & 0 & \lambda_{mc} \end{pmatrix} \begin{pmatrix} I_{mx} \\ I_{my} \\ I_{mc} \end{pmatrix} = \begin{pmatrix} \lambda_{mc} I_{mc} \\ 0 \\ \lambda_{mc} I_{mc} \end{pmatrix}.$$

and between vectors $\vec{\psi}_m$ and $\vec{I}_m$ there is a space angle $\phi$.
In figure 4 collinear ($\tilde{t}_{md}$) and orthogonal ($\tilde{t}_{mq}$) to vector $\vec{\Psi}_m$ current components are given. The energy of the main magnetic field in point b is made only by current component $\tilde{t}_{md}$

$$W = \frac{(\tilde{t}_{md}, \vec{\Psi}_m)}{2} = \frac{(\tilde{t}_{md} + \tilde{t}_{mq}, \vec{\Psi}_m)}{2} = \frac{(\tilde{t}_{md}, \vec{\Psi}_m)}{2}.$$

The energy of current component $\tilde{t}_{mq}$ is equal to zero

$$\frac{(\tilde{t}_{mq}, \vec{\Psi}_m)}{2} = 0.$$

The last equation testifies that equivalent phases of AEM along axes x and z interchange the energy.

$$\frac{\lambda_{md} \tilde{t}_{md} \tilde{t}_{mx}}{2} = \frac{\lambda_{mq} \tilde{t}_{mq} \tilde{t}_{mx}}{2}.$$

Current vector $\tilde{t}_{mq}$ does not take part in making the energy of the main magnetic field of AEM. At the same time quite certain powers of dissipation correspond to it, they are equal to

$$r_{\lambda} \frac{\tilde{t}_{mq}^2}{2}$$

and

$$\frac{d}{dt} \left( \frac{1_{rs} \tilde{t}_{mq}}{2} \right)$$

These powers should be brought to the category of energy overdissipation. It is defined by the fact that they are not connected with doing effective work and caused only by the heterogeneity of AEM vector space.

The set of vectors $\tilde{t}_{md}$ of vector space forms the longitudinal subset of currents. The set of vectors $\tilde{t}_{mq}$ of vector space forms the transvers subsets of currents. The subsets of currents $\tilde{t}_{md}$ and $\tilde{t}_{mq}$ of heterogeneous vector space are shown in figure 5 and figure 6 by the arrows.

**Figure 5.** Vectors of longitudinal subsets of currents.

**Figure 6.** Vectors of transvers subsets of currents.
4. Some practical applications
Particular application of topological theory are the energy processes in asynchronous motors with the alternating form of a rotor bar. Rotor cages of such motors represent a parametrically heterogeneous structures and the question about their rational design is open at present.

Wider area of application is given by the problem of diagnostic of AEM and their operating. For such problems it is necessary to have information factors pointing authentically to the object’s saving or loosing parametrical homogeneity because of operational imperfection, machine aging, violation of technological and operating standards etc.

Minimization of overdissipation for linear AEM is one of the criteria of rational design together with such criterias as the highest efficiency and maximum starting force.

5. Conclusion
AEM vector space is partially or fully parametrically heterogeneous.

Parametrical heterogeneity is appeared in AEM energy fields configuration, in effects of internal energy interchange and energy overdissipation.

Different fields of AEM vector spaces have different energy efficiency. Thus, it is reasonable to supply power and operate AEM the way to concentrate its currents vectors in the area of the highest energy efficiency.

The apparatus of the energy fields gives visual images of energy conditions and energy processes of AEM.

Topological theory gives methods and tools for solving a number of problems of development and operation of heterogeneous AEM.

References
[1] Hancock N 1967 Matrichny’j analiz e’lektricheskix mashin [Matrix Analysis of Electric Machines] (Moscow: Energiya) [In Russian]
[2] Kron G 1972 Issledovaniye slozhnyh sistem po chastyam (diakoptika) [The development of complex systems by components (Dacoptics)] (Moscow: Nauka) [In Russian]
[3] Kron G 1978 Tenzorny analiz setey [Tensor analysis of circuits] (Moscow: Sovetskoje radio) [In Russian]
[4] Artym A D 1987 Matrichno-topologicheskiye metody analiza elektricheskih tsepei s primeneniyem EVM [Topological methods of electrical circuits with computer application] (St. Petersburg: Leningr.elektrotehn.inst-t svyazi im.M.A. Bonch-Bruevicha) [In Russian]
[5] Kurilin S P, Denisov V N and Kruglov V V 2008 Matrichnaya teoriya elektricheskikh mashyn [Matrix theory of electric machines ] (Moscow: Rossiiskiy un-t kooperatsii) [In Russian]
[6] Denisov V N and Kurilin S P 2011 Matrichnoye modelirovaniye elekromagnitnyh i energeticheskig protsessov v elektricheskih mashinah [Matrix modelling of electromagnetic and power processes in electric machines] (Smolensk: RIO filiala GOUVO “MEI(TU) v Smolenske”) [In Russian]
[7] Denisov V N and Kurilin S P 2007 Preobrazovaniye koordinat i analiz parametricheskikh svoistv elektricheskih mashin [Coordinate conversion and analysis of the parametric properties of electric machines] Elekrichesto 6 pp 45 – 50 [In Russian]
[8] Zheng J, Huang S, Rong F and Lye M 2018 Six-phase space vector PWM under stator one-phase open-circuit fault condition Energies 7 1796
[9] Kurilin S P and Denisov V N 2013 Matimaticheskiye osnovy modelirovaniya i ih prilozheniya v elektromehanike [Mathematical basis in modelling and their application in electromechanics] (Smolensk: Universum) [In Russian]
[10] Denisov V N and Kurilin S P 2014 Metody i prilozheniya matimaticeskogo modelirovaniya v elektrotehnike. Monografiya [Methods and applications of mathimatic modeling in electromechanics] (Smolensky filial Rossiiskogo universiteta kooperatsii) [In Russian]
[11] Kurilin S P and Denisov V N 2010 Visualnye modeli vektornykh prostranstv i problemy energosberezheniya [Visual models of vector spaces and energy saving problems] Sistemy komp'juternoj matematiki i ih prilozheniya: materialy XI mezhdunarodnoj nauchnoj konferencii, posvyashchennoj 70-letiju professora V.P. D'jakonova. vol 11 (Smolensk: Izdatel' stvovo SmolGU) pp 45-6 [In Russian]

[12] Denisov V N and Kurilin S P 2004 Visualnye modeli moshchnosti mnogofaznykh obmotok [Visual power models of multi-phase windings] Perspektivy i tendencii razvitiya jelektrotehnicheskogo oborudovaniya Trudy pjatogo mezhdunarodnogo simpoziuma JeLMASH vol 1 (Moscow) pp 105 – 11 [In Russian]

[13] Kurilin S P and Denisov V 2013 Izybytchnoye rasseyaniye v obmotkah nesemetricnykh elektrycheskich mashin [Overdissipation in windings of asynchronous electric machines] Shornik trudov 3 Mezhdunarodnoj nauchno-tekhnicheskoy konferencii "Jenergetika, informatika, innovacii - 2013" vol 1 (Smolensk: filial MEI v g. Smolenske) pp 215-20 [In Russian]

[14] Kurilin S P and Denisov V N 2014 Matematicheskaya model neyavnopolyusnoi elektricheskoj mashyny v matrichnoy forme [Mathematical model of a non-salient pole machine in the matrix form] Elektrichestvo 4 pp 43 – 9 [In Russian]

[15] Kurilin S P and Denisov V N 2015 Energeticheskiye polya i izbytchnoye rasseyaniye energii v neyavnopolyusnoy elektricheskoj mashine [Energy fields and excess energy dissipation in a Non-Salient Pole Machine] Elektrichestvo 3 pp 35-41 [In Russian]

[16] Zoric I, Jones M and Levi E 2017 Arbitrary power sharing among three-phase winding sets of multiphase machines IEEE Transactions on Industrial Electronics vol 65 2 pp 1128-1139

[17] Denisov V N and Kurilin S P 2006 Vidyi s svoystva parametricheskikh matrits matematicheskikh modeli elektricheskikh mashin [Types and properties of parametric matrices of mathematical models of electric machines] Vserossiiskaya nauchno-tekhnicheskaya konferentsiya (Tula: Izdatel'stvo TulGu) pp 115-7 [In Russian]

[18] Mamyedov F A, Denisov V N and Kurilin S P 2002 Parametricheskiye svoystva i osobennosti energetiki assimetricnykh obmotok elektricheskikh mashin [Parametric properties and energy features of asymmetric windings of electrical machines] Trudy chetvertogo mezhdunarodnogo simpoziuma “ELMASh – 2002” vol 2 (Moscow) pp 85-8 [In Russian]

[19] Kosmatov V I, Sarvarov A S and Danilov E I 2018 Mathematical model of energy-saving asynchronous motor in the rotating coordinates system. Proc.of the 17th International Ural Conference on AC Electric Drives (ACED-April,8341702) pp 1-5 [In Russian]

[20] Nos O V 2008 Matematicheskaya model asynyhnornogo dvigatelya v lineinyyh prostranstvah,svazannых so statorom i rotorom [Mathematical Model of Asynchronous Motor in Linear Spaces Connected with Stator and Rotor] Izv. vuzov. Electromehanika 2 pp 14-20 [In Russian]

[21] Denisov V N and Kurilin S P 2011 Inzhenernaya model lineinogo asynyhnornogo dvigatelya [Engineering model of a linear asynchronous motor] Elektrichestvo 3 pp 52-4 [In Russian]

[22] Mamedov F, Kurilin S and Tchutorov D 2001 Ideal no – load operation and asymmetry of phases of low speed linear induction machine Proc. of the 5th Int. Conf. on unconventional electromechanical and electrical systems vol 2 ( Poland:Szczecin) pp 323-8

[23] Smolin V I, Topolskaya I G and Volovich G I 2016 The Energy Method for Monitoring the Instantaneous State and the Formation of a Synchronous Motor Control Variables The 2nd Int. Conf. on Industrial Engineering, Applications and Manufacturing (ICIEAM) (Russia)

[24] Bitoleanu A, Popescu M, Linca M and Mitran A 2011 Energetical Performances Analysis of PWM Asynchronous Motor and Voltage Inverter Driving System The 7th International Symposium on Advanced Topics in Electrical Engineering (ATEE) (Romania)

[25] Farhani F, Zafouri A and Chaari A 2017 Real time induction motor efficiency optimization Journal of the Franklin Institute vol 354 8 pp 3289-304

[26] Mamyedov F A, Denisov V N and Kurilin S P 2002 Osobennosty energeticheskikh protsessov v
The Features of Power Processes in Electric Machines with Asymmetric Windings [In Russian]

Mamyedov F A and Kurilin S P 2001 Ob opredelenii aktivnoi i reaktivnoi moshchnosti dlya assimetrichnyh mashin [About the detection of active and reactive power for asymmetric electric machines] Elektrotehnika 4 pp 17 – 21 [In Russian]