A Mixed Objective Assignment Problem

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ABSTRACT - A general assignment problem includes N tasks that intend assign to N workers to what place each laborer has the sufficiency to do all tasks. The objective is to confront the optimum assignment of a number of tasks (jobs) to an enlarge number of services (or persons) at a minimum cost. In this freebie the objectives are proposed as mixed objective which is triangular fuzzy number. Yager’s ranking approach has been used for grade the fuzzy numbers. The Hungarian method is used to solve the problem. The results of the numerical example prove that a fine conciliation solution can be reached effectively.

Keywords
Assignment problem, triangular fuzzy number, fuzzy ranking

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1. INTRODUCTION
Assignment problem is used as a matter of fact frequently in solving problems of manufacturing and administration discipline. It plays a prominent role in trading and distinctive application. In an assignment problem, n jobs are subsequent performed by n persons restrictive their flexibility to do the job. In this problem, c_{ij} denotes the charge of assigning the jth service to the ith person. We imply that a well known person gave a pink slip be assigned sure as can be one service, by the same token each person gave a pink slip do at practically one job. The problem is to seize an finest task so that the most part jobs is minimum or the total prosperity is maximum.

Numerous researchers have developed contradictory techniques for solving generalized assignment problems. Ross and Soland [15], Cattrysse and Wassenhove [2], Amini and Racer [1], Lorena and Narciso [14], Chu and Beasley [5], Diaz and Fernandez [9] and Haddadi and Ouzia [12]. Zadeh [17] made a component of the work of genius of fuzzy sets to deal mutually imprecision and uncertainty in real continuance situations. Since by the time mentioned, consistent advances have been firm on the knowledge of myriad techniques and their application to different censure problems. Fuzzy assignment problems have confirmed large acknowledgment in different years [11,13]. In this article, a trendy scheme is considered to salute the optimal consolidation of mixed assignment problem by instead of the parameter as triangular fuzzy numbers.

We separate this freebie as follows: In article 1, we introduce the assignment problem. In article 2, we comprise the mathematical ideal for the mixed objective assignment problem. In article 3, methodology is explained. In article 4, numerical concrete illustration is subject to and absolutely we reach the problem.

2. MATHEMATICAL MODEL
In this article, we comprise a preferably practical problem that is the mixed objective assignment problem by all of fuzzy costs and non-fuzzy costs. Lying on the core of this point, the yager’s ranking method [16] has been utilized to reorganize the fuzzy objectives to crisp one in case the predictable resolution schemes manage be useful to respond our problem.

A. Problem formulation
The mixed objective assignment problem is therefore:
minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} (C_{ij} + c_{ij})x_{ij}$

subject to $\sum_{j=1}^{n} x_{ij} = 1$ for $i = 1, 2, \ldots, n$

$\sum_{i=1}^{n} x_{ij} = 1$ for $j = 1, 2, \ldots, n$

$x_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ person is assigned the } j^{th} \text{ job} \\ 0, & \text{Otherwise} \end{cases}$

3. METHODOLOGY

Since the model is complicated, it is unthinkable to apply the analytic ways to earn the excellent solution. In decision to solve uncertain programming model, the ranking stratagem is probably closely useful.

A. Representation of generalized fuzzy number

In general, a generalized fuzzy number $A$ is described as entire fuzzy subset of the real line $R$, whose membership function $\mu_A$ satisfies, the henceforth conditions:

1. $\mu_A$ is a incessant mapping from $R$ to the closed interval $[0,1]$
2. $\mu_A (x) = 0$, $-\infty < x \leq a_1$
3. $\mu_A (x) = L(x)$ is exactingly increasing on $[a_1, a_2]$
4. $\mu_A (x) = w$, $a_2 \leq x \leq a_3$
5. $\mu_A (x) = R(x)$ is exactingly decreasing on $[a_3, a_4]$
6. $\mu_A (x) = 0$, $a_4 \leq x \leq \infty$

We denote this type of generalized fuzzy number as $A = (a_1, a_2, a_3, a_4; w)_{LR}$

B. Triangular fuzzy number

A triangular fuzzy number $A$ is denoted by $(l, m, n)$ and its membership function $\mu_A (x)$ is given by

$\mu_{(A)}(x) = \begin{cases} \frac{x-l}{m-l} & l \leq x \leq m \\ \frac{x-n}{m-n} & m \leq x \leq n \\ 0, & \text{otherwise} \end{cases}$
Lin and Wen solved the assignment problem mutually fuzzy interim integer costs by a category algorithm [10]. Chen [6] proved a few theorems and expected a fuzzy task method. Dominance of fuzzy numbers can be explained by multiple ranking methods [5, 8, 9, 13, 16]. Yager’s ranking method [16] is a competent grade practice which executed the property of cooperation, linearity and additively. We serve yager’s grade technique [16] in this paper.

The yager’s ranking index is defined by

\[ Y(C) = \int_{0}^{1} 0.5(C_{a}^{L} + C_{a}^{U}) da \]

Where \( C_{a}^{L} \) is the \( \alpha \) cut of fuzzy number \( C \).

### 4. NUMERICAL EXAMPLE

Consider a matrix in which the rows denote 3 peoples A, B, C and columns denote the 3 works 1, 2, 3. The cost matrix \( \tilde{C} \) is given in triangular fuzzy numbers.

\[
\tilde{C}_{ij} = [(1,3,4) \ (5,8,11) \ (9,10,11)] \\
(7,8,10) \ (5,6,7) \ (8,10,12)] \\
(5,7,10) \ (8,11,13) \ (2,4,5)]
\]

We calculate \( Y(1,3,4) \) by applying the yager’s ranking method. The membership function of the triangular number (1,3,4) is

\[
\mu_{A}(x) = \begin{cases} 
\frac{x-1}{2}, & \text{if } 1 \leq x \leq 3 \\
\frac{x-4}{1}, & \text{if } 3 \leq x \leq 4 \\
0, & \text{if } x \geq 4 
\end{cases}
\]

The \( \alpha \)-cut of the fuzzy number (1,3,4) is \((c_{L}^{\alpha}, c_{U}^{\alpha}) = (2\alpha + 1, 4 - \alpha)\)

\[ Y(\tilde{C}_{11}) = Y(1,3,4) = \int_{0}^{1} 0.5(C_{a}^{L} + C_{a}^{U}) da \]
\[ = \int_{0}^{1} 0.5(2\alpha + 1 + 4 - \alpha) da \]
\[ = \int_{0}^{1} 0.5(\alpha + 5) da = 2.75 \]
Proceeding similarly, the Yager’s ranking indices for the fuzzy costs $C_{ij}$ are calculated as:

$Y(C_{12}) = 8, \quad Y(C_{13}) = 10,$
$Y(C_{23}) = 8.25, \quad Y(C_{32}) = 6, \quad Y(C_{33}) = 10,$
$Y(C_{3}) = 7.25, \quad Y(C_{1}) = 10.75, \quad Y(C_{3}) = 3.75,$

Therefore $C_{ij}$ becomes $c_{ij}$

\[
C_{ij} = \begin{bmatrix}
2.75 & 8 & 10 \\
8.25 & 6 & 10 \\
7.25 & 10.25 & 3.75
\end{bmatrix}
\]

As per on mathematical formulation, the crisp cost is

\[
c_{ij} = \begin{bmatrix}
4 & 8 & 12 \\
10 & 7 & 4 \\
9 & 8 & 5
\end{bmatrix}
\]

Adding these, we have defuzzified cost matrices

\[
C_{ij} = \begin{bmatrix}
6.75 & 16 & 22 \\
18.25 & 13 & 24 \\
16.25 & 18.25 & 8.75
\end{bmatrix}
\]

Applying the Hungarian method, the solution is

\[
C_{ij} = \begin{bmatrix}
6.75 & 16 & 22 \\
18.25 & 13 & 24 \\
16.25 & 18.25 & 8.75
\end{bmatrix}
\]

*Optimal assignment

Therefore, The minimum cost = 6.75 + 13 + 8.75

= 28.5

5. CONCLUSION

In this paper, mixed objective assignment problem solved. The assignment costs are triangular fuzzy numbers and crisp numbers. Yager’s ranking techniques are used to transform the triangular fuzzy number into crisp number. The numerical example shows that all jobs can be assigned effectively, the use of resources can be minimized and the assignment problem solved efficiently. We have revealed that the total cost obtain is optimal.

6. REFERENCES

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