Precoded Turbo Equalizer for Power Line Communication Systems

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Abstract—Power line communication continues to draw increasing interest by promising a wide range of applications including cost-free last-mile communication solution. However, signal transmitted through the power lines deteriorates badly due to the presence of severe inter-symbol interference (ISI) and harsh random pulse noise. This work proposes a new precoded turbo equalization scheme specifically designed for the PLC channels. By introducing useful precoding to reshape ISI, optimizing maximum a posteriori (MAP) detection to address the non-Gaussian pulse noise, and performing soft iterative decision refinement, the new equalizer demonstrates a gain significantly better than the existing turbo equalizers.

I. INTRODUCTION

The inception of power line communication (PLC) has lead to many useful applications from remote voltage monitoring, to meter readings of power line systems, broadband Internet access, and more recently, indoor wired local area networks. The ubiquity of power line makes it a promising candidate for providing an cost-free last-mile communication solution.

However, the power line system was not originally designed to transmit (data) signal. Data transmission over power lines generally suffers from harsh random pulse noise and serious multipath fading caused by the impedance mismatch, as branches of the power line network reflect signal back and create several signal paths from a transmitter to a receiver. Equalization techniques are therefore actively exploited to cope with the severe multipath fading, or, the inter-symbol interference (ISI) effect, in PLC channels.

Traditional channel equalizers, including transversal equalizers, decision feedback equalizers (DFE) and maximum likelihood sequence equalizers (MLSE), are optimized under Gaussian noise. In PLC systems, nonlinear equalizers such as radial basis function (RBF) networks and fuzzy equalizers [5–8] are also favorable choices, and several studies have demonstrated their good performances in the presence of serious uncertainty posed by the pulse noise.

Another important branch of equalization technique combines channel coding and channel equalization, known as turbo equalization. Incepted in 1995 [11], it borrows principles from turbo codes, and is capable of achieving a remarkable interleave gain by jointly performing maximum a posteriori (MAP) detection [14] or near-MAP detection for the ISI channel (inner code) and soft-decoding the channel code (outer code) through an iterative process. The considerable success of turbo equalizers in radio frequency (RF) wireless communication systems has also promoted a serious investigation of its applicability and performance in PLC. Specifically, a pioneering work in [4] proposed to complement the conventional turbo equalizer with a front-end myriad filter (MMyF) for efficient baseband filtering in impulsive channels. The use of MMyF was shown to bring encouraging gains than otherwise, but the capacity potential of turbo equalization was still not fully exploited in the system discussed in [4]. The reason is two-fold: 1) with the pulse noise processed outside the turbo equalizer, the turbo equalizer was not optimized with the pulse noise taken into consideration; and 2) the PLC ISI channel was not precoded. The latter is particularly pitiful, because from the coding theory, we know that the interleaving gain of a serially concatenated turbo system is attainable only when the inner code is recursive. In the case of turbo equalization, as an ISI channel (e.g. the PLC channel) acts as an inner code, it is by nature non-recursive, and must therefore be precoded in order to appear recursive.

This work proposes a new turbo equalizer scheme to overcome these two defects. In the new scheme, a precoder is carefully applied to re-structure the PLC channel to a recursive ISI channel, and the conventional MAP detector is re-designed and tailored to the precoded ISI channel and especially to the presence of random pulse noise. Additionally, we propose a modified extrinsic information transfer chart (EXIT) approach to simplify the design of precoder. Simulation results demonstrate that the new turbo equalizer scheme noticeably outperforms the existing ones and that the modified EXIT chart provides a useful tool for selecting appropriate precoders.

II. CHANNEL MODEL

Data communication over power lines is subject to a number of destructive interferences and impairments including signal attenuation caused by cable loss, ISI caused by multipath propagation, additive white Gaussian noise, and intermittent strong pulse noise.

Consider feeding a binary data sequence $w_i$ into a transmit filter with pulse shape $g(t)$. The shaped signal $x(t)$ takes the form of

$$x(t) = \sum_{i=1}^{\infty} w_i g(t - iT).$$  \hspace{1cm} (1)

After the signal passes through the PLC channel with
frequency-domain impulse response $H_C(f)$, time-domain impulse response $h_c(t)$ and additive noise $z(t)$, the receiver gets
\[ r(t) = \sum_{i=1}^{\infty} w_i c h(t - i T) + z(t) \] (2)
where
\[ c h(t) = \int_{-\infty}^{\infty} g(\tau) h_c(t - \tau) d\tau. \] (3)

This PLC channel model can also be expressed in a discrete form:
\[ y[n] = \sum_{k=0}^{L_f - 1} h[k] w[n - k] + \text{Noise}[n], \] (4)
where $h[k]$ is the sampled sequence of the channel response $ch[t]$, $L_f$ is the number of (multi) paths, $\text{Noise}[n]$ is the sampled additive noise and $y[n]$ is the discrete-form received signal at the $n$-th time slot.

The multi-path fading of a PLC channel results from the tree-like topology of the PLC network with multiple branches. These branches have different lengths and are loaded with various impedance. The transmitted wave suffers from reflections at every impedance mismatch point caused by the difference between characteristic impedance of the cables. Consequently, instead of propagating along a single path, the signal reflects along different branches and forms a multi-path channel. Generally, the signal attenuation along the PLC channel increases with the distance.

The most widely known and cited frequency-domain PLC channel model [2] approximates the overall channel response $H_c(f)$ by emphasizing the most dominant set of paths that exist over the frequency range of 500 kHz to 20 MHz:
\[ H_C(f) = \sum_{i=1}^{L_f} \xi_i e^{-(a_0 + a_1 f^\kappa) d_i} e^{-i2\pi f(d_i / v_p)} \] (5)
where $L_f$ is the total number of paths, whose typical value ranges from 3 to 5. The attenuation of the $i$-th path increases exponentially with the distance $d_i$ where the attenuation power factor is determined by some parameters $\{a_0, a_1\}$ and $\kappa$, where $\kappa$ is typically in the interval of $[0.2, 1]$. All the $L_f$ paths accumulate like a weighted sum with weight $\xi_i$ for the $i$-th path. The last term $e^{-i2\pi f(d_i / v_p)}$ represents the propagation delay with the velocity of propagation parameter $v_p$, which can be calculated by
\[ v_p = \frac{C_0}{\sqrt{\varepsilon_f}} \] (6)
where $C_0$ is the speed of light, and $\varepsilon_f$ represents the dielectric constant of the insulating material.

Besides ISI, a PLC channel also surfs from a harsh non-Gaussian additive noise, which is commonly assumed to be a composition of five sources [3]: background noise, narrow band interference noise coming from the surrounding radio signals, and pulse noise caused by the fundamental component of the power system, by the switching power supplies, and by the random switching transients in the power line network.

The prevailing statistical model to characterize the cumulative effect of all this additive noise is a two-term Gaussian mixture model [9] [7]:
\[ \text{Noise} = (1 - \varepsilon)\mathcal{N}(0, \sigma^2) + \varepsilon\mathcal{N}(0, K\sigma^2), \] (7)
where $\varepsilon$ represents the probability of impulses, and $\mathcal{N}(0, \sigma^2)$ and $\mathcal{N}(0, K\sigma^2)$ are Gaussian distributions with zero mean and variances of $\sigma^2$ and $K\sigma^2$, respectively. In the model of (7), the channel is for the most time dominated by the background noise and the narrowband interference, which are collectively represented by an Gaussian distribution $\mathcal{N}(0, \sigma^2)$; and with a small probability of $\varepsilon$, it is overwhelmed by the pulse noise, which is modeled by another independent Gaussian distribution $\mathcal{N}(0, K\sigma^2)$ with a much larger variance.

III. DESIGN OF TURBO EQUALIZERS FOR PLC CHANNELS

A. System Model

The remarkable performance of turbo equalizers in combating ISI has been demonstrated in a rich variety of wireless systems and applications [11]. At the transmitter side, by serially connecting the ISI channel and an error correction code (ECC) through an interleaver, turbo equalizer can treat the ISI channel as the inner code and the ECC as the outer code of a serially concatenated system. At the receiver side, both of the component codes are softly-decoded and the soft extrinsic information is exchanged back and forth between them to iteratively refine the decision (i.e. iterative processing instead of the conventional sequential one-way processing).

It is widely recognized that the inner code of an interleaved serially concatenated system must be recursive, in order to effectively achieve the spectrum thinning effect and hence attain the so-called interleafing gain [12]. In a PLC system, the interleafing gain becomes particularly important and desirable, largely due to the long delay spread of the PLC channel. However, an ISI channel is by nature non-recursive, that is, the output from the channel is the result of a non-recursive convolution between the input signal and the ISI channel. Hence, to obtain the interleafing gain, it is necessary to reshape the channel by adding a rate-1 recursive precoder before the ISI channel.

Figure 1 shows the system model. A binary sequence $\vec{u}$ with length of $N$ is encoded by an outer code, which, in this specific example, is a recursive systematic convolutional (RSC) code $C_1$. The coded sequence from the outer code $\vec{v}$ is then interleaved, and subsequently passed through a rate-1 recursive convolutional code $C_2$ (the precoder), and finally sent through the PLC channel. The combination of the ISI channel and the recursive precoder acts like a recursive convolutional inner code, whose output sequence is denoted by $\vec{x}$. At the receiver side, the received sequence $\vec{y}$ is fed into an iterative decoder consisting of two sub-decoders, the maximum a posteriori (MAP) equalizer and the BCJR decoder which are matched, respectively, to the precoded ISI channel (the inner code) and the outer convolutional code. Soft extrinsic information from the these sub-decoders, $\vec{e}_1$ and $\vec{e}_2$, are exchanged to iteratively refine the detection and decoding decisions.
corresponding to \( v_k = 1 \) and \( v_k = 0 \), respectively, and \( \alpha_k \), \( \beta_k \) and \( \gamma_k \) are the forward path metric, the backward path metric, and the branch metric, respectively.

The branch metric \( \gamma_k \) must be modified to reflect the specific channel condition. When the noise in the PLC is specified as in (7), \( \gamma_k \) takes the form of:

\[
\gamma_k(s_k = s, s_{k-1} = s') = P(y_k, s_k | s_{k-1}) = P(y_k | x_k)P(u_k) = P(u_k) \sum_i P_i(\sigma_i | x_k)P(y_k | x_k, \sigma_i) = P(u_k) \sum_i P_i^{ini}(\sigma_i) \frac{1}{\sigma_i} e^{\frac{(x_k - \gamma_k)^2}{\sigma_i^2}}.
\]

The forward and backward path metrics capture the PLC channel characteristics through the branch metric \( \gamma_k \). Their mathematical forms follow a recursive way of computing [14]:

\[
\alpha_k(s) = P(s_{k-1} = s, y_0^{k-1}) = \sum_{s'} \alpha_{k-1}(s') \gamma(s', s),
\]

\[
\beta_k(s) = P(y_{k+1}^{N-1} | s_k = s) = \sum_{s'} \beta_{k+1}(s') \gamma(s, s').
\]

Inserting (9)–(11) in (8) leads to a modified soft-input soft-output MAP algorithm tailored for the specific channel model of the PLC.

IV. PRECODER DESIGN THROUGH EXIT ANALYSIS

We now discuss the issue of precoder design. The general principle for precoding is to have the precoder to be a rate-1 (i.e. no rate loss) recursive convolutional code whose memory does not exceed that of the ISI channel (i.e. no increase of equalizer complexity due to the precoder). Different precoders can make a difference in the performance of the entire coded equalization system. Direct simulations can be used to guide the choice of the precoder, but a more efficient way can make use of the extrinsic information transfer (EXIT) charts.

EXIT charts [13] are a powerful tool for the analysis and evaluation of an iterative decoder by tracking the evolution of mutual information between the extrinsic information and the source sequence as the number of iterations increases. Its ability to visualize the trajectory of the probabilistic evolution as well as its elegant properties (such as the area property) make it extremely popular. It is also employed to predict and analyze the performance of turbo-like codes to reduce the simulation complexity.

Again, just like the MAP equalizer, the computation of the EXIT curves here must also be tailored to match to the PLC channel characteristics (i.e. precoded ISI channel with non-Gaussian additive noise).

Consider the outer ECC. Let \( u \in \{0, 1\} \) be an information bit. Let \( L_u^n \) and \( L_e^n \) be the input and the output LLR associated with \( u \), whose probabilistic density function (pdf) is given by \( p_{L_u}^n(L_u) \) and \( p_{L_e}^n(L_e) \), respectively. For ease of presentation, below we neglect the superscript \( a \) and \( e \), since the derivations apply to both quantities.
Since the channel is symmetric, we have $p_L(L_u|u = 0) = p_L(-L_u|u = 0)$. The mutual information between $u$ and $L_u$ can be computed using

$$I(u; L_u) = \int_{-\infty}^{\infty} p_L(L_u|u = 0) \log_2\left(\frac{2p_L(L_u|u = 0)}{p_L(L_u|u = 0) + p_L(-L_u|u = 0)}\right) dL_u. \quad (12)$$

Now following the conventional assumption that the message $L_u$ follows a Gaussian distribution with mean $\mu$ and variance $\sigma^2 = 2\mu$, the mutual information can be simplified to

$$I(u; L_u) = I_{\mu,\sigma}(\mu, \sigma) \triangleq 1 - \frac{1}{\sqrt{2\pi}\sigma^2} \int_{-\infty}^{\infty} e^{-(L_u-\mu)^2/2\sigma^2} \log_2(1+e^{-L_u}) dL_u \quad \text{(bit).} \quad (13)$$

Similarly, for the inner code (the precoded ISI channel), let $v \in \{0,1\}$ be an information bit, and $L_v^a$ and $L_v^e$ be the a priori (input) and the extrinsic (output) LLRs, associated with $v$, whose pdf’s are given by $p_L^a(L_v^a)$ and $p_L^e(L_v^e)$ respectively. Since the precoded ISI channel suffers from a non-Gaussian noise, the conventional Gaussian assumption will not apply to the inner code with accuracy. Hence, in stead of Gaussian-based analytical forms, we resort to Monte Carlo simulations.

An interesting discovery we made in our simulation study is that, given the mixed Gaussian noise model in (7) for PLC, the output LLRs at the MAP equalizer can be modeled by a mixed Gaussian distribution as shown in Fig. 2. Since each component of the mixed-Gaussian output LLR corresponds well to the respective component in the mixed-Gaussian noise, following a similar form in (7), it is possible to categorize the output extrinsic pdf $p_L(L_v^e)$ by the collection of $D = 2$ components, $p_L(L_v^e(i))$, $0 \leq i \leq D$, each being approximated by a single Gaussian distribution. The output mutual information from the equalizer can therefore be expressed as

$$I(v; L_v^e) = \sum_{i=0}^{D-1} P_i^{ini} I(v; L_v^e(i))$$

$$= \sum_{i=0}^{D-1} P_i^{ini} \int_{-\infty}^{\infty} p_L(L_u|u = 0) \log_2\left(\frac{2p_L(L_u|u = 0)}{p_L(L_u|u = 0) + p_L(-L_u|u = 0)}\right) dL_u$$

$$= \sum_{i=0}^{D-1} P_i^{ini} \left(1 - \frac{1}{\sqrt{2\pi}\sigma^2} \int_{-\infty}^{\infty} e^{-(L_u-\mu)^2/2\sigma^2} \log_2(1+e^{-L_u}) dL_u\right)$$

V. SIMULATION OF TURBO EQUALIZERS

We simulate the proposed precoded turbo equalization scheme on a 4 - path power line network model with VVF (Vinyl insulation, Vinyl sheath, Flat) cable ($\varepsilon_r = 3.17$). The channel impulse response in the frequency domain is modeled by (5), where the weight factors and the distances for the $i$th path are $\xi_1 = 0.64$, $\xi_2 = 0.38$, $\xi_3 = -0.15$, $\xi_4 = 0.05$, and $d_1 = 200m$, $d_2 = 222.4m$, $d_3 = 244.8m$, $d_4 = 267.5m$. The attenuation factors are $\kappa = 1$, $a_0 = 0$, and $a_1 = 7.8 \times 10^{-10} s/m$. The frequency response of this PLC channel model is demonstrated in Fig. 3. The pulse is shaped by a raised-cosine filter with $\beta = 0.7$. The impulse response $h(t)$ of the equivalent channel (combining the PLC channel and the pulse shaping filter) in the time domain is shown in Fig. 4. Suppose that an impulse sequence with a transmission rate of $1/0.15\mu s$ travels along the channel. We sample the impulse response in the time domain and get a normalized 4-tap discrete channel model, which is used in all the simulations shown here:

$$h(t) = 0.8709 + 0.4758D - 0.1153D^2 + 0.0435D^3.$$
We consider the outer ECC code in use as a convolutional code with generator polynomial $[1, \frac{1+D+D^2+D^3}{1+D+D^2}]$, and look for a precoder that best matches this ECC code and the PLC channel in (14). We do so by studying the EXIT chart. The best precoder, when applied to the PLC channel, must exhibit an EXIT curve that matched best, in shape and in position, with that of the outer ECC. Following the analysis in Section IV, the EXIT curves for the precoded PLC channel with four different precoders are evaluated, and plotted together with the EXIT curve of the outer ECC code in Figure 5. At a signal-to-noise ratio (SNR) of $-5dB$, we see that most of the precoders have EXIT curves either touching or crossing that of the ECC code. The only exception is precoder $\frac{1}{1+D^3}$, whose EXIT curve still leaves a desirable open tunnel, which will allow the soft information to be iteratively changed and continually improved without hitting a fixed point. Hence, we can conclude that precoder $\frac{1}{1+D^3}$ fits the PLC channel the best. Simulation results of the actual bit error rate performance in Fig. 6 confirms the prediction, demonstrating a gain of 0.6dB over other choices of precoders.

To further demonstrate the efficiency of the proposed turbo equalizer, and especially the importance of the right precoder and the right MAP algorithm, we compare our performance with two reference systems in Fig. 6. Both reference models are turbo equalizers. The first uses the optimized precoder $\frac{1}{1+D^3}$ but the traditional MAP algorithm (not modified for the PLC channel), and the second reference uses the modified MAP equalizer discussed in Section III-B but no precoders. The simulation results clearly demonstrate that the proposed system gains from both accounts: some 0.7dB gain from matching the MAP equalizer to the PLC channel, and more than 2.5dB gain from employing a precoder (evaluated at the BER of $10^{-4}$).

VI. CONCLUSION

We have proposed a precoded turbo equalizer scheme for PLC systems characterized by severe inter-symbol interference and strong pulse noise. The new scheme transforms the non-recursive ISI channel to one that is recursive by precoding it with an appropriate rate-1 recursive precoder, hence enabling the renowned interleaving gain. The new scheme also includes a modified MAP algorithm specifically designed to address the non-Gaussian pulse noise. EXIT charts (with modified algorithm to compute the EXIT curve) are exploited to facilitate the selection of the right precoder, and extensive simulations confirm the effectiveness of the proposed scheme.

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Fig. 5. EXIT charts for the precoded turbo equalizer. Inner EXIT curve corresponds to precoded PLC channel; outer EXIT curve corresponds to convolutional code $[1, \frac{1+D+D^2+D^3}{1+D+D^2}]$.

Fig. 6. The bit error rate of turbo equalizer.