Effects of Large CP-violating Soft Phases on Supersymmetric
Electroweak Baryogenesis

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(March 27, 2022)

Abstract

We revisit the results of recent electroweak baryogenesis calculations and include all allowed large CP-violating supersymmetric phases. If the phases are large, the resulting baryon asymmetry can be considerably larger than the observed value $n_B/s \sim 4 \times 10^{-11}$. Much of the asymmetry must therefore be washed out, and we argue that the upper bound on the light Higgs mass is larger than the value reported in previous work.
I. INTRODUCTION

The universe is not matter-antimatter symmetric; as shown by primordial nucleosynthesis measurements, the baryon density to entropy density ratio $n_B/s$ is constrained to be about $4 \times 10^{-11}$ \cite{1}. The necessary ingredients for a theory to explain this asymmetry are the Sakharov criteria: baryon number violation, C and CP violation and non-equilibrium conditions \cite{2}. Electroweak baryogenesis is an attractive mechanism with the best chance of verification through traditional collider experiments \cite{3}.

The central feature of electroweak baryogenesis is spontaneous symmetry breaking. At high temperatures, the vacuum expectation value of the Higgs field is zero but as the universe cools down over time, a second minimum appears in the potential at $v \neq 0$. The universe undergoes a phase transition, from a “symmetric” phase where $v = 0$ to the “broken” phase where $v \neq 0$. If the phase transition is second order, which means that no potential barrier exists, then the universe is approximately in equilibrium throughout the transition and no baryogenesis can result (since the third Sakharov criterion is violated). If the phase transition is first order, then the phase transition occurs through quantum tunneling. As the universe cools, the probability of making a transition from symmetric to broken phase grows and when the transition occurs at a certain point in space, a bubble of broken phase forms and expands. Because of its first order nature, the phase transition does not occur throughout the whole universe at the same time, resulting in non-equilibrium conditions.

The baryon asymmetry is generated as the wall of the expanding bubble passes through points in space. Particles in the unbroken phase close to the wall interact with the changing Higgs field profile in the wall and the presence of CP-violating couplings produces source terms for participating particles. Different chiralities couple with different strength when CP is violated and a difference occurs in the reflection and transmission probabilities for the two different chiralities. Due to rapid gauge, Yukawa and strong sphaleron interactions, the CP-violating source terms are translated into a net left handed weak doublet quark density which is finally converted into a baryon asymmetry by weak sphaleron decays. The asymmetry then diffuses through the bubble wall into the broken phase where the weak sphaleron interactions are exponentially suppressed. Subsequent washout of the baryon asymmetry can therefore be kept under control provided the first order phase transition is strong enough.

The Standard Model satisfies the three Sakharov criteria, but it can not generate the required value of $n_B/s$. This is because the only source of CP violation in the SM comes from the phase $\delta$ in the CKM quark mixing matrix. The baryon asymmetry is $\sim \sin \delta$, but this value is suppressed by flavor mixing factors \cite{4,5} and the resulting asymmetry cannot be too large. Therefore, the phase transition has to be strongly first order to avoid washing out the produced baryon number, which translates into the criterion $v(T_c)/T_c \gtrsim 1$ \cite{6}. However, in the Standard Model, the phase transition is too weakly first order to prevent washout, unless the SM Higgs mass is less than 50 GeV, far below the present experimental limit \cite{7}.

The situation can be improved in the Minimal Supersymmetric Standard Model (MSSM) as the light right handed top squark contribution to the temperature dependent effective potential can push the value of $v(T_c)/T_c$ up to acceptable values even for light Higgs masses allowed by experimental searches \cite{8,9,10,11,12}. Also, supersymmetric extensions of the SM include additional sources, which in general can further enhance the possibility of baryon
asymmetry production during the electroweak phase transition. As a result, a specific region in the MSSM parameter space corresponding to a heavy CP-odd Higgs boson, a light CP-even Higgs boson, a light right-handed stop and a heavy left-handed stop can provide a plausible framework for electroweak baryogenesis.

The CP-violating interactions in the MSSM arise in the complex phases of the soft supersymmetry breaking terms in the Lagrangian and in the phase of \( \mu \). Since the largest contributions come from charginos and neutralinos \([27,30]\) and less importantly from the right handed stops, only the gaugino mass phases \( \varphi_1, \varphi_2 \) and \( \varphi_3 \), the phase of \( \mu \) and the phase of the stop trilinear parameter \( A_t \) are relevant for baryogenesis. These phases have to be small, typically \( \lesssim 10^{-2} \), if they are considered individually with sparticle masses \( O(\text{TeV}) \), otherwise they induce contributions to the electric dipole moments (EDMs) of the neutron and electron exceeding experimental limits \([21–23]\). However, these constraints can be avoided if relations among soft breaking parameters ensure cancellations of individual contributions to the EDMs \([24,35]\). In the most general case this possibility leads to models with light superpartner spectra and CP-violating phases of \( O(1) \) \([33]\). Remarkably, string motivated scenarios with non-universal gaugino masses can be described where such EDM cancellations occur naturally as a result of the SM embedding on five-branes \([26]\).

Different methods of calculating the baryon asymmetry, using both classical and quantum Boltzmann equations, have produced the required asymmetry, provided that \( \sin \varphi_\mu \sim 10^{-2} – 10^{-4} \) \([27,30,34]\) in agreement with the experimental limits on the EDMs of the electron and neutron, when the other MSSM phases are taken to be zero. In this paper we assume that large phases are allowed by the cancellation mechanism and at the same time the right handed stop is light enough to significantly modify the finite temperature Higgs potential. All other sfermions are heavier and their contribution to the Higgs potential is Boltzmann suppressed.

We briefly review the generation of the baryon asymmetry and the phase dependence factorization emphasizing the significance of the \( \varphi_2 + \varphi_\mu \) phase combination. In the large phase scenario it is then possible that the amount of baryon asymmetry generated at the phase transition is \( 10^2 – 10^4 \) larger than what is observed.

We solve the full quantum Boltzmann equations to extract the CP violating source terms which also provides enhancement compared to classical Boltzmann equations due to quantum memory effects \([33]\).

It is necessary to wash out some of the asymmetry; therefore the constraint \( v(T_c)/T_c \gtrsim 1 \) can be relaxed. We reevaluate the washout calculation using the effects of requiring some washout to occur and derive our limits for the light Higgs mass and the right handed stop based on electroweak baryogenesis. We are able to show that even in the one-loop approximation for the temperature dependent effective Higgs potential the light Higgs mass upper limit can be pushed up to 115 GeV without invoking negative values of the right handed stop soft breaking mass parameter \( m^2 \), which can lead to color-breaking global minima and scalar potential instability \([9]\).
II. BARYON ASYMMETRY CALCULATION

In order to calculate the baryon asymmetry of the universe, one has to start with a self-consistent computation of the CP-violating sources resulting from particle interactions with the changing Higgs profile in the bubble wall. We follow the procedure of reference [33], which uses the closed-time-path formalism of finite-temperature field theory to derive quantum Boltzmann equations for Higgsinos and stops. Quantum memory effects resulting from correct treatment of particle propagation in the plasma strengthen the non-equilibrium character of the the particle scattering off the bubble wall and produce larger source terms.

The dominant sources of CP-violation that are relevant to the electroweak baryogenesis scenario comes from the interactions of the Higgs fields with charginos and neutralinos. These interactions couple the Higgsino and gaugino components of charginos and neutralinos and involve potentially large CP-violating phases originating from the mixing. For the charginos, the CP-violating Lagrangian in the symmetric phase is

$$\mathcal{L} = -gH_1^0 \tilde{\tilde{H}} P_L \tilde{W} - gH_2^0 \tilde{\tilde{W}} P_L \tilde{H} + h.c. \quad (1)$$

The CP-violating phases $\varphi_\mu$ and $\varphi_2$ are introduced by switching to the basis of mass eigenstates in the broken phase (see Appendix). Following the steps in [33] to generate the CP-violating source term in the Boltzmann equation, we obtain for any point $X$ inside the bubble wall

$$S_C = 2g^2 \text{Im}(M_2\mu)v^2(X)\dot{\beta}(X)\mathcal{I}_W = \gamma_W \sin(\varphi_\mu + \varphi_2), \quad (2)$$

where $v^2 = v_1^2 + v_2^2$ and $\dot{\beta}(X) = d\beta(X)/dt$ characterizes the temporal variation of the Higgs profile $\tan\beta(X) = v_2(X)/v_1(X)$ as the wall passes through point $X$. $\mathcal{I}_W$ is a temperature dependent phase-space integral (the explicit form can be found in [33]) and includes information about thermal behavior of the winos and Higgsinos in the high temperature plasma.

The neutralino CP-violating interactions are

$$\mathcal{L} = -\frac{1}{2}[H_1^0 \tilde{\tilde{H}} P_L(g_2 \tilde{W}_3 - g_1 \tilde{B}) + H_2^0(g_2 \tilde{W}_3 - g_1 \tilde{B})P_L \tilde{H}_2] + h.c. \quad (3)$$

Following the same steps as in the chargino case, we find the source term

$$S_N = g_2^2 \text{Im}(M_2\mu)v^2(X)\dot{\beta}(X)\mathcal{I}_W + g_1^2 \text{Im}(M_1\mu)v^2(X)\dot{\beta}(X)\mathcal{I}_B$$

$$= \gamma_W \sin(\varphi_\mu + \varphi_2) + \gamma_B \sin(\varphi_\mu + \varphi_1). \quad (4)$$

We can combine the source terms for the charginos and neutralinos to obtain the total source term for Higgsinos

$$S_H = 3\gamma_W \sin(\varphi_2 + \varphi_\mu) + \gamma_B \sin(\varphi_1 + \varphi_\mu) \quad (5)$$

where

$$\gamma_W = |\mu||M_2|g_2^2v^2(X)\dot{\beta}(X)\mathcal{I}_W, \quad (6)$$

$$\gamma_B = |\mu||M_1|g_1^2v^2(X)\dot{\beta}(X)\mathcal{I}_B. \quad (7)$$
Here we emphasize that the source depends on the full physical (reparametrization invariant) combinations of the phases $\varphi_1 + \varphi_\mu$ and $\varphi_2 + \varphi_\mu$ which factorize from the rest of the source term. In this sense our considerations are independent of the particular details going into the calculation of the Higgsino thermal production rate.

The phase space integrals $I_\tilde{W}$ and $I_\tilde{B}$ exhibit strong resonant behavior leading to a maximum for $m_\tilde{H} \sim m_\tilde{W}$ ($m_\tilde{H} \sim m_\tilde{B}$) \[33\]. In terms of the soft breaking parameters the enhancement occurs when the gaugino masses $M_1$ or $M_2$ are close in value to the Higgsino mass parameter $\mu$. For similar reasons it is easy to understand why the right handed stop contribution to the CP-violating source is always subdominant. The corresponding phase-space integral $I_{\tilde{t}_R}$ is typically far away from its maximum as $m_\tilde{t}_L \gg m_\tilde{t}_R$ in our framework and the stop source can therefore be neglected.

The baryon asymmetry can be directly related to the Higgsino source by solving a set of coupled diffusion equations for the Higgs and Higgsino, top and stop, and the first two generation quark and squark densities \[27\]. We assume that strong sphaleron transitions and interactions induced by the top Yukawa coupling are very fast, allowing us to reduce the number of relevant equations to the diffusion equations for Higgses and Higgsinos and baryon number. They can be solved analytically, with the result

$$\frac{n_B}{s} = -\frac{81 A \tilde{D} \Gamma_{ws}}{82 v_w^2 s},$$

where $\tilde{D}$ is an effective diffusion constant, $\Gamma_{ws}$ is the weak sphaleron transition rate, and $v_w$ is the velocity of the bubble wall. The coefficient $A$ is determined by boundary conditions to be

$$A = \frac{1}{D \lambda_+} \int_0^\infty du \, S_H \, e^{-\lambda_+ u},$$

where

$$\lambda_+ = v_w + \sqrt{v_w^2 + 4 \tilde{\Gamma} \tilde{D}} \frac{2 D}{2 D},$$

and $\tilde{\Gamma}$ is an effective decay constant.

III. CALCULATION OF WASHOUT

Once the phase transition has occurred, weak sphaleron transitions tend to erase any asymmetry that has been created. In the broken phase, the weak sphaleron rate is \[28\]

$$\Gamma_{ws} \simeq 2.8 \times 10^5 T \left(\frac{\alpha_W}{4 \pi}\right)^4 \kappa_T \left(\frac{E_{sp}}{B T}\right)^7 e^{-\frac{E_{sp}}{T}},$$

where

$$E_{sp} = \frac{g v(T) B}{\alpha_W},$$

5
\begin{equation}
B = B\left(\frac{\lambda}{g^2}\right) \simeq 1.87. \tag{13}
\end{equation}

κ is a constant in the range \(10^{-4} \leq \kappa \leq 10^{-1}\). The differential equation satisfied by \(n_B\) in the broken phase is

\begin{equation}
\frac{dn_B}{dt} = -C n_f \Gamma_{ws} n_B, \tag{14}
\end{equation}

where \(C\) is a number of \(\mathcal{O}(1)\) \[29\] and we take \(C \simeq 1\) absorbing the uncertainty into the uncertainty of \(\kappa\) and \(B\). Solving (14) we get

\begin{equation}
B(t) = B(t_c) \exp(-\int_{t_c}^t dt' n_f \Gamma_{ws}). \tag{15}
\end{equation}

The traditional bound is found by assuming that all of the washout happens at \(T = T_c\), which amounts to saying that the integral in equation (3.5) is equal to the value of the integrand at \(T = T_c\). However, we want to evaluate the integral more carefully, and in the process derive a new bound.

Since we do not have an analytic formula for \(v(T)\), we need to examine the integrand to see if we can make any simplifying assumptions. The sphaleron rate is proportional to \((v(T)/T)^7 \exp(-36v(T)/T)\), and we know that \(1 \lesssim v(T)/T < \infty\). If we approximate that \(v(T) = v(T_c)\) throughout this period, then the washout rate is negligible when \(T \simeq 0.75 T_c \simeq 75\) GeV. Since \(v(T)\) changes by about a factor of 2.5 in the temperature range from \((0 - 100)\) GeV, treating \(v(T)\) as a constant is a good approximation; it is also conservative, because it will actually overestimate the washout slightly.

We can make a change of variables to

\begin{equation}
\zeta = \frac{E_{sp}(T_c)}{T} \tag{16}
\end{equation}

by using the relation between time and temperature,

\begin{equation}
t \simeq (3 \times 10^{-2}) \frac{M_{Pl}}{T^2}. \tag{17}
\end{equation}

The integral we now have to do is

\begin{equation}
I = (3.4 \times 10^{-8}) \frac{M_{Pl}}{E_{sp}(T_c)} \kappa \int_{\zeta_c}^{\infty} \zeta^7 e^{-\zeta} d\zeta
= (3.4 \times 10^{-8}) \frac{M_{Pl}}{E_{sp}(T_c)} \kappa e^{-\zeta_c} \times \sum_{n=0}^{7} \frac{7!}{n!} \zeta_c^n \tag{18}
\end{equation}

Because \(\zeta_c = 4\pi B v(T_c)/g T_c \sim 36\), the \(\zeta_c^7\) term will dominate all of the other terms. (Again, dropping the smaller terms is a conservative approximation.) Then

\begin{equation}
I \simeq (3.4 \times 10^{-8}) \frac{M_{Pl}}{T_c} \kappa e^{-\zeta_c} \zeta_c^6
\simeq (4.1 \times 10^9) \kappa \zeta_c^6 e^{-\zeta_c} \tag{19}
\end{equation}
Inserting this expression into equation (15), we now have an approximate expression for the observed $n_B/s$:

$$n_B(T \sim 0)/s = (n_B(T = T_c)/s) \exp(-(4.1 \times 10^9)\kappa_6 e^{-\zeta_c}) \gtrsim 4 \times 10^{-11},$$

from which we can obtain

$$\zeta_c - 6 \log \zeta_c - \log \kappa - 9 \log 10 - \log(\log \frac{n_B/s(T_c)}{4 \times 10^{-11}}) \gtrsim 0.$$  

By choosing a certain value for $\kappa$ and $n_B/s(T_c)$, this equation can be solved numerically. We will do this in the next section after we have presented our numerical result for $n_B/s(T_c)$. Once this is done, we can use the relation

$$\zeta_c \simeq 36 \frac{v(T_c)}{T_c}$$

(22)

to find the lower bound on $v(T_c)/T_c$.

This lower bound on $v(T_c)/T_c$ can be translated into an upper bound on the light Higgs mass. The one-loop finite-temperature effective potential has a form

$$V(\varphi) = -\frac{1}{2} m^2(T)\varphi^2 - T[E_{SM}\varphi^3 + F_{MSSM}(\varphi, T)] + \frac{1}{8} \lambda(T)\varphi^4.$$  

(23)

For the MSSM with heavy decoupled stops, the potential becomes SM-like and one has

$$\frac{v(T_c)}{T_c} \simeq \frac{2E_{SM}}{\lambda},$$

(24)

where

$$E_{SM} = \frac{2M_W^2 + M_Z^2}{4\pi \tan^2 \beta}.$$  

(25)

Using $m_h^2 = 2\lambda v^2$, equation becomes

$$m_h^2 \lesssim \frac{4E_{SM} v^2}{v(T_c)/T_c}.$$  

(26)

The other extreme is a light right-handed stop whose temperature dependent self energy, responsible for screening of the stop interactions in the plasma, is balanced by a negative soft squared mass term $-m_\tilde{t}^2 \sim \Pi_{\tilde{t}} = (\frac{4}{9} g_3^2 + \frac{1}{9} g'^2) T^2 + \frac{1}{9} h_t^2 [1 + \sin^2 \beta(1 - \frac{\tilde{A}_t^2}{m_Q^2})] T^2$. In such a case $F_{MSSM}$ is

$$F_{MSSM}(\varphi, T) = \varphi^3 E_{MSSM} + \varphi^3 [\frac{m_\tilde{t}^2 (1 - \frac{\tilde{A}_t^2}{m_Q^2})^2}{2\pi v^3}],$$

(27)

(28)

where $\tilde{A}_t = A_t - \mu/\tan \beta$ is the stop left-right mixing parameter. Then

$$\frac{v(T_c)}{T_c} \simeq \frac{2(E_{SM} + E_{MSSM})}{\lambda},$$

(29)

$$m_h^2 \lesssim \frac{4(E_{SM} + E_{MSSM}) v^2}{v(T_c)/T_c}.$$  

(30)

When screening is present ($m_\tilde{t}^2 + \Pi_{\tilde{t}} > 0$) the temperature dependent Higgs potential can be analyzed numerically.
IV. NUMERICAL RESULTS

For the calculation of the baryon asymmetry, we have to adopt a model for the electroweak phase transition and evaluate \( v^2(X) \) and \( \beta(X) \). These functions have been calculated numerically using the two-loop finite-temperature effective potential in \[30\]. However, we have instead chosen to use the model of \[30\] which lends itself more easily to our computation:

\[
v(X) = v(L_w)[1 - \cos\left(\frac{X\pi}{L_w}\right)][\theta(X) - \theta(X - L_w)] + v(L_w)\theta(X - L_w),
\]

\[
\beta(X) = \Delta\beta[1 - \cos\left(\frac{X\pi}{L_w}\right)][\theta(X) - \theta(X - L_w)] + \beta(X = 0) + \Delta\beta\theta(X - L_w).
\]

We have tested that the answer only weakly depends on the model used for the phase transition, so the results should be of at least the right order of magnitude. We chose the value \( \Delta\beta = .001 \) which is suggested by the results of \[30\], and a value \( v(L_w) \sim 100 \text{ GeV} \).

The width of the wall is chosen to be \( L_w \sim 25/T \) \[30\] and the typical velocity of the bubble wall is expected to be \( v_w \sim 0.1 \). For the averaged diffusion coefficient we use \( \bar{D} = 0.8 \text{ GeV}^{-1} \) and \( \bar{\Gamma} = 1.7 \text{ GeV} \) as in \[33\]. All of these parameters enter the calculation of the baryon asymmetry, however, the scaling dependence of \( n_B \) on these quantities is straightforward and does not interfere with CP-violation effects.

Figure 1 shows the baryon asymmetry \( n_B/s \) generated as a function of \(|\mu|\) for several combinations of \( \varphi_\mu \) and \( \varphi_1 \) and \( \varphi_2 = 0 \). \(|M_1| = 140 \text{ GeV}, |M_2| = 250 \text{ GeV}, \tan\beta \sim 3 \). The asymmetry \( n_B/s \) can be as large as \( 10^{-7} \) for \(|\mu| \sim |M_2|\).

Let us note in passing that the signs of \( \varphi_1 \) and most importantly of \( \varphi_\mu \) (as it appears in both chargino and neutralino matrices) determines the sign of \( n_B \). This fact is also demonstrated in Fig. 1 since \( n_B \) is positive for \( \sin \varphi_\mu > 0 \) with the exception of case e and \( \mu \sim M_1 \) where \( n_B \) turns negative being dominated by the neutralino contribution to the Higgsino source. Experimental determination of the magnitude and sign of the soft phases is therefore essential if the feasibility of the supersymmetric electroweak baryogenesis is to be verified. Alternatively, the observed sign of \( n_B \) could be used to determine the sign of \( \varphi_\mu \) until it can be measured other ways.

In order to calculate the lower bound on \( \zeta_c \), we have to specify the the sphaleron parameters \( B \) and \( \kappa \). The non-perturbative scaling factor \( B \) comes from numerical minimization of the sphaleron energy and was evaluated for the MSSM in \[14\]. The usual range depending on the coupling strengths is \( 1.5 < B(\Delta v) < 2.7 \) with a typical median of 1.87. The value of \( \kappa \) is obtained as a functional determinant associated with fluctuations about the sphaleron and was estimated to be \( \kappa \sim 0.1 \) \[13\]. However, when the Higgs propagator uncertainty is absorbed the allowed range for kappa is \( 10^{-4} < \kappa < 10^{-1} \).

The required value of \( v(T_c)/T_c \) depends on the amount of dilution of the baryon number produced during the electroweak phase transition that can be allowed in order to explain the observed value of \( n_B/s \). Inclusion of both large CP-violating soft phases and quantum memory effects leads to a substantial enhancement in the value of the produced asymmetry \( n_B/s(T_c) \). Consequently, the constraints on the strength of the first order phase transition can be softened as a function of \( n_B(T_c)/n_B(0) \). This decrease of \( v(T_c)/T_c \) is illustrated in Fig. 2 for several values of \( \kappa \). The enhancement in \( n_B(T_c) \) coming from quantum effects
FIG. 1. Plot of the baryon asymmetry produced at the electroweak phase transition for $|M_1| = 140 \text{ GeV}$, $|M_2| = 250 \text{ GeV}$, $\tan \beta \sim 3$ and $\varphi_2 = 0$. The five cases correspond to some typical cases, with 
\begin{itemize}
    \item[a] $\varphi_\mu = \frac{\pi}{4}$, $\varphi_1 = \frac{\pi}{10}$,
    \item[b] $\varphi_\mu = \frac{\pi}{4}$, $\varphi_1 = \frac{\pi}{4}$,
    \item[c] $\varphi_\mu = \frac{\pi}{4}$, $\varphi_1 = \frac{\pi}{16}$,
    \item[d] $\varphi_\mu = \frac{\pi}{4}$, $\varphi_1 = \frac{\pi}{10}$,
    \item[e] $\varphi_\mu = \frac{\pi}{4}$, $\varphi_1 = \frac{\pi}{8}$,
    \item[f] $\varphi_\mu = 5 \times 10^{-3}$, $\varphi_1 = 5 \times 10^{-3}$.
\end{itemize}

(about a factor of $10^2$ [33]) for typical order of magnitude values of $\sin \varphi_\mu$ is demonstrated by the intervals with the position of the left (right) arrow corresponding to no (full) quantum enhancement respectively. The minimal value of $\sin \varphi_\mu$ required in order to generate $n_B/s(T_c) \sim n_B/s(0)$ (negligible washout) without including the quantum effects in the sources is about $5 \times 10^{-2}$ which is in agreement with the values obtained in Ref. [9].

Since we are looking for an absolute lower bound on $v(T_c)/T_c$, we will take $n_B/s(T_c)$ to have its maximum value, $10^{-7}$. For $\kappa = 10^{-1}$, the numerical solution gives

$$\zeta_c \gtrsim 39.6,$$

while the solution for $\kappa = 10^{-4}$ yields

$$\zeta_c \gtrsim 31.3.$$

This translates into a bound on $v(T_c)/T_c$ of

$$\frac{v(T_c)}{T_c} \gtrsim 1.1, \quad \kappa = 10^{-1},$$

$$\frac{v(T_c)}{T_c} \gtrsim 0.87, \quad \kappa = 10^{-4}.$$
FIG. 2. Plot of required \( v(T_c)/T_c \) ratio as a function of the actual amount of baryon asymmetry produced at the electroweak phase transition. The intervals for different values of \( \sin \varphi_\mu \) illustrate the enhancement resulting from quantum memory effects with the left (right) arrow corresponding to no (full) quantum enhancement.

Using the bound for \( \kappa = 10^{-4} \), we can find the upper bound on the light Higgs mass. For heavy stops, the upper bound is

\[
m_h \lesssim 51.7 \, \text{GeV}. \quad (37)
\]

For light stops with no thermal screening and negligible mixing (\( \tilde{A}_t^2 << m_Q^2 \)) the upper bound is

\[
m_h \lesssim 138.1 \, \text{GeV}. \quad (38)
\]

In Figure 3 we show a plot of the upper Higgs mass limit as a function of the right handed stop mass calculated in our framework. The stop mass range corresponds to variation of the right handed stop soft mass parameter \( m_{\tilde{t}}^2 \) from \(-\Pi_{rR}\) (no thermal screening) to \( \Pi_{rR} \) (strong thermal screening). The wide band corresponds to variation of \( B \) and \( \kappa \) within their full range and the narrow central band shows the set of curves resulting from taking \( B = 1.87 \) and varying \( \kappa \) in the full range.

It is important to stress that the role of large CP-violating phases is crucial in this context. If the baryon asymmetry is overproduced during the electroweak phase transition due to \( \mathcal{O}(1) \) values of \( \varphi_\mu \) and/or \( \varphi_1 \) the washout conditions (35) and (36) are less stringent then if the phases are constrained to be \( 10^{-2} \). As a result, the upper bound on the Higgs
mass can be as much as 15 GeV higher compared to the situation where $n_B/s(T_c) \sim n_B/s(0)$ and no washout is allowed.

It is obvious from our results that inclusion of large CP-violating phases in the calculation of the baryon asymmetry relaxes the stringent constraints on the strength of the first order transition and consequently the light Higgs masses can be pushed towards larger values. Even for stop masses $m_{\tilde{t}_R} \gtrsim 170$ GeV resulting from positive values of the soft breaking parameter $m_U^2$ the upper bound on the Higgs mass can be as high as 115 GeV.

Our considerations are based on the one loop temperature dependent effective Higgs potential evaluation. Two loop corrections are known to significantly enhance the strength of the first order phase transition \cite{18} and further relax the upper bound on the Higgs mass. In this respect our results represent a conservative estimate of the upper Higgs mass limit and it is likely to be moved upward when two loop corrections are included.

V. SUMMARY AND CONCLUSIONS

The upper bound on the Higgs mass which still allows electroweak baryogenesis is an important issue which is becoming relevant as the Higgs mass experimental lower limits are approaching 100 GeV. The baryon asymmetry produced during the electroweak phase
transition can successfully explain the observed value provided there are additional degrees of freedom contributing to the finite temperature Higgs potential. Supersymmetric models with a light right handed stop are a very good candidate for a theory that can provide these degrees of freedom. Also, the CP-violating phases appearing in the soft breaking terms of the Lagrangian can supplement (or entirely replace) the effects of a CP-violating phase in the CKM matrix.

Previously it has been thought that based on the electron and neutron EDM experimental limits the supersymmetric CP-violating phases have to be small and consequently there is no room for washout of the produced baryon asymmetry. This translates into stringent constraints on the Higgs and right handed stop masses, often leading to problems with color breaking minima and potential stability.

We have shown that once there is a possibility of cancellations among individual contributions to the EDMs and the CP-violating phases are allowed to be $\mathcal{O}(1)$, the produced baryon asymmetry exceeds the observed value and the baryon density can be allowed to be diluted by a factor of $10^{-3}$ or more. In such a case the upper Higgs mass bound is increased and depending on the right handed squark mass it can go beyond 115-120 GeV while at the same time the scalar potential is stable.

This scenario opens new possible implications for supersymmetric phenomenology. As pointed out in [37] it is important to independently measure the CP-violating phases to correctly interpret experimental observables. If the phases are small the window for supersymmetric electroweak baryogenesis will get increasingly smaller as the LEP and Tevatron experiments will be pushing up the lower Higgs mass limit. It is natural to expect that in the case of small phases the experimentally determined Higgs mass should not be too far above 100 GeV if electroweak baryogenesis is expected to work. On the other hand, if the Higgs is not discovered at LEP nor at the Tevatron, one can expect that the CP-violating phases of the MSSM are of $\mathcal{O}(1)$ if baryogenesis occurs at the electroweak phase transition, and they should be measurable if the superpartners are discovered. Of course, finding a light Higgs boson at LEP or Fermilab is consistent with having large supersymmetric soft phases.

VI. ACKNOWLEDGMENTS

We thank Jim Cline for valuable discussions, A. Riotto for helpful clarifications, and L. Everett for suggestions and comments on the manuscript. We also thank M. Quiros for correspondence.

VII. APPENDIX

Above the electroweak scale, the chargino mass matrix is

$$\mathcal{M}_C \simeq \begin{pmatrix} |M_2| e^{i\varphi_2} & 0 \\ 0 & |\mu| e^{i\varphi_\mu} \end{pmatrix}. \tag{A1}$$

This matrix is made real and diagonal by two complex matrices,
\[ \mathcal{M}_C^{\text{diag}} = U^* \mathcal{M}_C V^{-1}, \]  
(A2)

where we can take
\[ U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad V = \begin{pmatrix} e^{i\varphi_1} & 0 \\ 0 & e^{i\varphi_\mu} \end{pmatrix}. \]  
(A3)

If we switch to a basis of two-component Weyl spinors,
\[ P_L \tilde{W} = P_L V_{j_1}^* \tilde{\psi}_j, \quad P_R \tilde{W} = P_R U_{j_1} \tilde{\psi}_j, \]  
(A4)
\[ P_L \tilde{H} = P_L V_{j_2}^* \tilde{\psi}_j, \quad P_R \tilde{H} = P_R U_{j_2} \tilde{\psi}_j, \]  
(A5)

where \( \tilde{\psi}_j \) are two-component spinors. In terms of \( \tilde{\psi}_j \), the interaction becomes
\[ \mathcal{L} = -gH_1^0 e^{-i\varphi_2} \tilde{\psi}_2 P_L \psi_1 - gH_2^0 e^{-i\varphi_\mu} \tilde{\psi}_1 P_L \psi_2 + \text{h.c.} \]  
(A6)

Above the electroweak scale, the neutralino mass matrix is
\[ \mathcal{M}_N \simeq \begin{pmatrix} |M_1|e^{i\varphi_1} & 0 & 0 & 0 \\ 0 & |M_2|e^{i\varphi_2} & 0 & 0 \\ 0 & 0 & 0 & -|\mu|e^{i\varphi_\mu} \\ 0 & 0 & -|\mu|e^{i\varphi_\mu} & 0 \end{pmatrix}. \]  
(A7)

It is made real and diagonal by the complex matrix \( N \),
\[ \mathcal{M}_N^{\text{diag}} = N^\dagger \mathcal{M}_N N, \]  
(A8)

where we can take
\[ N = \begin{pmatrix} e^{i\varphi_1} & 0 & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{\sqrt{1+\sqrt{2}} e^{i\varphi_\mu}} & \frac{1}{\sqrt{1+\sqrt{2}}} e^{i\varphi_\mu} \\ 0 & 0 & \frac{1}{\sqrt{1+\sqrt{2}}} e^{i\varphi_\mu} & \frac{\sqrt{2}}{\sqrt{1+\sqrt{2}} e^{i\varphi_\mu}} \end{pmatrix}. \]  
(A9)

Switching to the two-component spinor basis, we have
\[ P_L \tilde{H}_1 = P_L N_{j_3}^* \tilde{\psi}_0^0, \quad P_R \tilde{H}_1 = P_R N_{j_3} \tilde{\psi}_0^0, \]  
(A10)
\[ P_L \tilde{H}_2 = P_L N_{j_4}^* \tilde{\psi}_0^0, \quad P_R \tilde{H}_2 = P_R N_{j_4} \tilde{\psi}_0^0, \]  
(A11)
\[ P_L \tilde{W}_3 = P_L N_{j_2}^* \tilde{\psi}_0^0, \quad P_R \tilde{W}_3 = P_R N_{j_2} \tilde{\psi}_0^0, \]  
(A12)
\[ P_L \tilde{B} = P_L N_{j_1}^* \tilde{\psi}_0^0, \quad P_R \tilde{B} = P_R N_{j_1} \tilde{\psi}_0^0, \]  
(A13)

resulting in the interaction
\[ \mathcal{L} = -\frac{g_2^2}{2\sqrt{2}} e^{-i(\varphi_2+\varphi_\mu)} \left[ -iH_1^0 \tilde{\psi}_3 P_L \tilde{\psi}_2^0 + H_1^0 \tilde{\psi}_2 P_L \tilde{\psi}_3^0 - H_2^0 \tilde{\psi}_2 P_L \tilde{\psi}_4^0 \right] + \text{h.c.} \]  
(A14)
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