Research on Mechanism and Application of Coriolis Acceleration

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Abstract. Chongqing Bus falling into the river shocked the whole country. In order to find out whether the Coriolis illusion is one of the real causes of Chongqing Bus falling into the river, the mechanism of Coriolis acceleration is studied, and the corresponding analysis of the phenomenon caused by the Coriolis acceleration is carried out. The driver's head in Chongqing Bus falling into the river is modelled and calculated. It is concluded that the effect of Coriolis acceleration can be ignored when the object's motion speed is low, that is, the main cause of Chongqing Bus falling into the river is not Coriolis illusion.

1. Introduction
At 10:08 on October 28, 2018, a bus in Wanzhou District of Chongqing fell into the middle of the river after it collided with a car on the bridge deck of the Second Yangtze River Bridge in Wanzhou city, resulting in the death of all 15 drivers and passengers on board. On November 2, 2018, the reason for the bus falling into the river was announced. According to the monitoring video of the black box inside the bus, it was the fierce dispute between the passengers and the driver that caused the vehicle out of control [1]. As soon as the in car monitoring video of the last five seconds before falling into the river was released, it immediately triggered a heated discussion on the Internet. In addition to lamenting the fragility of life and warning everyone to be calm in case of trouble, there is also a discussion around the doubtful point of why the driver should slam the steering wheel to the left at the end. Many people think that when the bus driver and the female passenger argue, they turn around repeatedly, resulting in the "Coriolis acceleration illusion". They mistakenly think that the vehicle is turning right, so they turn left and cause tragedy.

In order to explore whether "Coriolis illusion" is the real cause of Chongqing Bus falling into the river, it is necessary to analyze the process, mechanism, result and influence of Coriolis acceleration in a general sense.

2. Coriolis acceleration and its mechanism
In 1835, French physicist Coriolis pointed out in his paper "equation of relative motion of object system", that if the object moves relative in the reference frame of uniform rotation, there is an inertial force acting on the object which is different from the usual centrifugal force. He called this force a compound centrifugal force, and its magnitude and direction can be expressed as

\[ -m a_C = 2 m v_r \times \omega \]  \hspace{1cm} (1)
Where, $m$ is the mass of the object, $v_r$ is the relative velocity, and $\omega$ is the angular velocity of the reference system. Later, this force is called Coriolis force, and the corresponding acceleration $a_C = 2\omega \times v_r$ is called Coriolis acceleration.

From the above, it is not difficult to see that if the particle moves relative to the rotating reference frame, the Coriolis acceleration will be generated. In this paper, using the relevant knowledge of kinematics, combined with mathematical derivation and physical analysis, how to generate the Coriolis acceleration is studied.

![Figure 1. Motion composition graph of points.](image)

Take $Oxyz$ as the definite system, $O'x'y'z'$ as the dynamic system, the vector diameter of the origin $O'$ in the determination is $r_o'$, the vector diameter of the moving point $M$ in the determination is $r$, the vector diameter in the dynamic system is $r'$, and the unit vectors of the dynamic system are respectively $i'$, $j'$ and $k'$, as shown in Figure 1. According to the relationship in the diagram,

$$r = r_o' + r'$$  \hspace{2cm} (2)

Where $r' = x'i' + y'j' + z'k'$. $x' = x'(t)$, $y' = y'(t)$ and $z' = z'(t)$ are the coordinates of the moving point $M$ in the moving system.

Let the motion system rotate on a fixed axis, and the axis passes through point $O'$, and its angular velocity vector is $\omega$, then

$$\frac{d}{dt}i' = \omega \times i', \quad \frac{d}{dt}j' = \omega \times j', \quad \frac{d}{dt}k' = \omega \times k'$$ \hspace{2cm} (3)

By definition, the relative velocity of moving point $M$ at instantaneous $t$ is

$$v_r = \frac{dx'}{dt}i' + \frac{dy'}{dt}j' + \frac{dz'}{dt}k' = \frac{\tilde{d}r'}{dt}$$ \hspace{2cm} (4)

Where $\frac{\tilde{d}}{dt}$ is the relative derivative, and

$$\frac{d}{dt}r' = \frac{dx'}{dt}i' + \frac{dy'}{dt}j' + \frac{dz'}{dt}k' + x'i' \frac{dx'}{dt} + y'j' \frac{dy'}{dt} + z'k' \frac{dz'}{dt} = \frac{\tilde{d}r'}{dt} + \omega \times (x'i' + y'j' + z'k') = v_r + \omega \times r'$$ \hspace{2cm} (5)

In the same way,
\[
\frac{dv_r}{dt} = \frac{\delta v_r}{\delta t} + \omega \times v_r \tag{6}
\]

By deriving the two sides of formula (2) to time \(t\), and noting \(\frac{dr_r}{dt} = 0\) and formula (5), we can obtain

\[
v_a = v_r + \omega \times r' \tag{7}
\]

Where \(v_a\) is the absolute velocity. By introducing the angular acceleration vector \(a = \frac{d\omega}{dt}\) of the implicated motion, deriving the two sides of formula (7) to time \(t\), and substituting formula (5) and (6), we obtain,

\[
a_a = \frac{\delta v_r}{\delta t} + \frac{d\omega}{dt} \times r' + \omega \times \frac{\delta r'}{\delta t} = \frac{\delta v_r}{\delta t} + \omega \times v_r + \alpha \times r' + \omega \times (v_r + \omega \times r')
\]

\[
= a_r + [a \times r' + \omega \times (\omega \times r')] + 2\omega \times v_r = a_r + a_e + 2\omega \times v_r \tag{8}
\]

Where \(a_r = \frac{\delta v_r}{\delta t}\) is the relative acceleration and \(a_e = a \times r' + \omega \times (\omega \times r')\) is the implicated acceleration.

Let

\[
a_c = 2\omega \times v_r \tag{9}
\]

Where \(a_c\) is the Coriolis acceleration. Therefore, the acceleration composition formula (8) can be written as

\[
a_a = a_r + a_e + a_c \tag{10}
\]

That is, when the motion system rotates on a fixed axis, the absolute acceleration of the moving point at a certain instant is equal to the vector sum of its implicated acceleration, relative acceleration and Coriolis acceleration. Coriolis acceleration is caused by the interaction of relative motion and implicated motion [2].

### 3. Application of Coriolis acceleration

From the previous reasoning, we can see that the condition of Coriolis acceleration is simple, and the resulting inertial force Coriolis force is widely used in nature and life. Some phenomena that are difficult to understand can be explained scientifically, such as: why do you often bend when walking with eyes closed? Why does water flow whirlpool when flushing toilet? Here, the influence of Coriolis acceleration is simply discussed from the two aspects of geostrophic force and Coriolis illusion.

#### 3.1 Geostrophic deflection force

The earth is regarded as a rotating non-inertial reference frame. When an object moves on the surface or inside of the earth at a certain speed, if its motion direction is not parallel to the rotation direction of the earth's axis, then the moving object will be affected by the Coriolis force, which is called geostrophic deflection force. Relative to the heading direction, the geostrophic deflection force is right in the northern hemisphere, left in the southern hemisphere and zero at the equator. Of course, in daily life, the geostrophic deflection force is very small and can be ignored. Anders Persson gave the difference between the derivation of Coriolis effect on rotating turntable and rotating earth [3].
The geostrophic force does not change the velocity of the moving object on the earth's surface, but can change the direction of the object's motion. Therefore, in the northern hemisphere, if people "walk forward" with their eyes closed, they will deflect to the right due to the geostrophic deflection force.

The geostrophic deflection force has obvious effects on the natural phenomena such as monsoon circulation, air mass movement, migration path of cyclones (typhoons) and anticyclones (cold air), ocean current direction and river motion. The geostrophic deflection force is also reflected in many aspects of life, as shown below.

1. The tracks on the right side of trains in the northern hemisphere are more easily worn.
2. In the northern hemisphere, the right bank of the river is seriously eroded, and the floating timber in high latitudes is concentrated to the right bank. For example, Chongming Island is the accumulation island formed by geostrophic force when the Yangtze River flows into the East China Sea in Shanghai.
3. Southeast monsoon and southwest monsoon are formed in summer and northwest monsoon in winter.
4. In the northern hemisphere, when flushing the toilet, the water does not flow directly to the drain hole, but in a counter clockwise whirlpool. This is because, in the process of water flow to the middle of the drain hole, the water deflects to the right under the action of geostrophic deflection force, and finally forms a vortex.

3.2 Coriolis illusion
In the field of Aeronautics and Astronautics, the effect of Coriolis acceleration cannot be ignored because of the high flying speed of aircraft. When the aircraft rotates around the axis of rotation (such as circling, rolling and spiraling) in flight, if the pilot's head moves or rotates at the same time in the cabin, or when the aircraft rotates around two or three axes at the same time, such as doing half somersault rolling and other stunts, the pilot's head will produce Coriolis acceleration. Coriolis illusion refers to the complex sensation of turning, rolling or rotation that pilots feel when they are subjected to this acceleration [4].

Coriolis illusion has been widely concerned since it was first put forward in 1932. It has a high incidence in flight. According to the survey, 50% of 105 pilots in the United States have experienced this illusion in flight. Kowalezuk, Krzysztof P et al. have shown that Coriolis illusion can affect the direction and vision of pilots. Under normal oxygen pressure, Coriolis illusion can increase the average saccade amplitude and duration by 55% and 31% respectively [5].

Coriolis illusion is a kind of spatial orientation disorder that pilots often encounter. This kind of illusion may lead to pilots' misjudgment and even lead to serious tragedy. In particular, the sustained high acceleration, high acceleration growth rate, high angular velocity and other characteristics of high-performance fighter are more likely to lead to the occurrence of serious Coriolis illusion [6]. At present, pilots basically carry out targeted training, which can greatly reduce the probability of accidents caused by Coriolis illusion. At present, the effective way to prevent and overcome Coriolis illusion is to simulate the psychological and physiological training of Coriolis illusion on the ground and in the air.

4. Mechanical analysis of Chongqing Bus falling into the river
It should be pointed out that the "Coriolis acceleration illusion" in the online review of Chongqing Bus falling into the river should be corrected as the "Coriolis illusion" in concept. The psychological and physiological effects caused by this illusion are mainly caused by Coriolis acceleration. The following is to analyze the Coriolis acceleration produced by the driver's head in the Chongqing bus crash from the perspective of mechanics.

According to the surveillance video of the black box in the car, from the top to the bottom, the driver's body rotates clockwise and his head rotates counterclockwise. Assuming that the rotation angular velocity is 60°/s and the diameter of the driver's head is 20cm, the simplified mechanical model of looking down at a certain moment is shown in Figure 2.
At this time, the Coriolis acceleration at the outer edge of the head is the largest. Now take point A of the head as the moving point and the driver's body as the dynamic system. Then

\[ \omega = \frac{60}{360} \times 2\pi = 1.047 \text{rad/s} \]  \hspace{1cm} (11)

\[ v_r = \omega r = 1.047 \times \frac{0.2}{2} = 0.1047 \text{m/s} \]  \hspace{1cm} (12)

Because the angular velocity vector \( \omega \) of the powertrain (i.e. the angular velocity of the driver's body rotation) is vertically downward at this moment, the relative velocity \( v_r \) points forward and is perpendicular to the \( \omega \) direction. According to the right-hand rule, the Coriolis acceleration \( a_c \) direction is right, and its value is

\[ a_c = 2\omega v_r = 2 \times 1.047 \times 0.1047 = 0.219 \text{m/s}^2 \]  \hspace{1cm} (13)

If the acceleration of gravity is \( g = 9.8 \text{m/s}^2 \), then

\[ \frac{a_c}{g} = \frac{0.219}{9.8} \approx \frac{1}{45} \]  \hspace{1cm} (14)

It can be seen that the maximum Coriolis acceleration produced by the head is only 1/45 times of the acceleration of gravity, and its effect can be completely ignored. Therefore, the reason why the "Coriolis illusion" caused the bus to fall into the river is untenable, and the biggest reason in this accident should be man-made disaster.

5. Conclusion

To sum up, Coriolis acceleration is caused by the interaction between the induced rotation and the relative motion. In other words, the Coriolis acceleration must satisfy two conditions at the same time:

(1) The moving coordinate system has rotation,

(2) The moving object has a relative motion to the moving coordinate system.

The magnitude of Coriolis acceleration depends on the angular velocity, relative velocity and the angle between the two directions. Typical effects such as geostrophic force and Coriolis illusion in aerospace field are discussed. However, in daily life, the influence of Coriolis acceleration can be ignored because of the low velocity of the object. The Chongqing Bus falling into the river also reminds us that we can't easily believe and blindly follow the dissemination of many lack of basic information on the Internet. The best way is to verify it through scientific means.
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