Density dependence of the $s$-wave repulsion in pionic atoms

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Abstract

Several mechanisms of density dependence of the $s$-wave repulsion in pionic atoms, beyond the conventional model, are tested by parameter fits to a large (106 points) set of data from $^{16}$O to $^{238}$U, including ‘deeply bound’ states in $^{205}$Pb. Special attention is paid to the proper choice of nuclear density distributions. A density-dependent isovector scattering amplitude suggested recently by Weise to result from a density dependence of the pion decay constant is introduced and found to account for most of the so-called anomalous repulsion. The presence of such an effect might indicate partial chiral symmetry restoration in dense matter. The anomalous repulsion is fully accounted for when an additional relativistic impulse approximation term is included in the potential.

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I. INTRODUCTION

Strong interaction effects in pionic atoms have been studied for several decades both experimentally and theoretically [1]. A theoretically-motivated phenomenological pion-nucleus potential [2] has been quite successful in reproducing the experimental values of strong interaction level shifts and widths throughout the periodic table. This potential has a local part which is effective over the whole of the nuclear volume and a $p$-wave part which is effective mostly in the surface region of the nucleus. Both components have terms linear in the nuclear densities which are closely related to the free pion-nucleon interaction, and quadratic (complex) terms which originate from pion absorption on pairs of nucleons. The latter are mostly determined empirically from fits to data whereas the former, although determined empirically, may also be calculated from the pion-nucleon interaction in free space. The $p$-wave component of the potential turns out to be fairly well understood, but the $s$-wave part of the potential had turned out too repulsive compared with expectations [3–5].

The renewed interest in pionic atoms in general and in the $s$-wave part of the potential in particular stems from two recent developments. The first is the experimental observation of ‘deeply bound’ pionic atom states in the $(d,^3\text{He})$ reaction [6,7] the existence of which was predicted a decade earlier [8–10]. The second is the very accurate measurement of the shift and width of the $1s$ level in pionic hydrogen [11] which leads to precise values of the $s$-wave scattering lengths.

Very recent attempts to explain the above mentioned $s$-wave repulsion in terms of a density dependence of the pion decay constant were made by Weise and Kaiser [12,13]. The proposed mechanism was implemented in fits to a large set of pionic atom data [14] and indeed found to remove most of the ‘anomaly’ in the $s$-wave term. A relativistic impulse approximation (RIA) term that was proposed earlier following Birbrair [15–19] was also shown [18,14] to remove part of the anomaly. Combining the two effects [14] removed the anomaly completely.

The present work is an extention of Ref. [14] in several respects. First, the data base of the present work contains 106 data points compared to 60 points in the earlier work. The additional data are mostly from the work of the Amsterdam group (see [20] and references therein) which includes several sequences of isotopes, of particular importance in the present context where most of the effect is due to an isovector term (see below). The second difference compared to the earlier work is in the nuclear density distributions, where in addition to ‘macroscopic’ densities [1] we have used ‘single particle’ densities constrained by results of relativistic mean field calculations. The third difference is that in the present work more flexibility was allowed in the RIA model and in the $\chi^2$ fits. The general context of the present work is the use of several models for the pion-nucleus interaction, each resulting in a well-defined functional of the local nuclear densities. Fits to a large set of experimental data provide the parameters of these various functionals.

The paper is organized as follows. Section II describes the pion-nucleus potential, including discussion of the nucleon densities. Section III summarizes the data base and the fit procedures. The results are given in Section IV and Section V is a summary.
II. THEORETICAL BACKGROUND

The interaction of pions at threshold with the nucleus is described by the Klein-Gordon (KG) equation of the form:

\[
\left[ \nabla^2 - 2\mu(B + V_{\text{opt}} + V_c) + (V_c + B)^2 \right] \psi = 0 \quad (\hbar = c = 1)
\]  

(1)

where \( \mu \) is the pion-nucleus reduced mass, \( B \) is the complex binding energy and \( V_c \) is the finite-size Coulomb interaction of the pion with the nucleus, including vacuum-polarization terms. Equation (1) assumes that the strong interaction potential \( V_{\text{opt}} \) behaves as a Lorentz scalar. The potential is usually taken as suggested by Kisslinger [21] and modified by Ericson and Ericson [2] to include absorption of pions on pairs of nucleons. The form used in the present work is

\[
2\mu V_{\text{opt}}(r) = q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla}
\]

(2)

with

\[
q(r) = -4\pi(1 + \frac{\mu}{M})\left\{ \bar{b}_0(r)[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)] \right\}
- 4\pi(1 + \frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r),
\]

(3)

\[
\alpha(r) = \frac{\alpha_1(r)}{1 + \frac{1}{3}\xi\alpha_1(r)} + \alpha_2(r),
\]

(4)

where

\[
\alpha_1(r) = 4\pi(1 + \frac{\mu}{M})^{-1}\{c_0[\rho_n(r) + \rho_p(r)] + c_1[\rho_n(r) - \rho_p(r)]\},
\]

(5)

\[
\alpha_2(r) = 4\pi(1 + \frac{\mu}{2M})^{-1}4C_0\rho_n(r)\rho_p(r).
\]

(6)

In these expressions \( \rho_n \) and \( \rho_p \) are the neutron and proton density distributions normalized to the number of neutrons \( N \) and number of protons \( Z \), respectively, and \( M \) is the mass of the nucleon. In this potential \( q(r) \) is referred to as the \( s \)-wave potential term and \( \alpha(r) \) is referred to as the \( p \)-wave potential term. The function \( \bar{b}_0(r) \) is given in terms of the local Fermi momentum \( k_F \)

\[
\bar{b}_0(r) = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)k_F(r),
\]

(7)

where \( b_0 \) and \( b_1 \) are minus the pion-nucleon isoscalar and isovector scattering lengths, respectively. The coefficients \( c_0 \) and \( c_1 \) are the pion-nucleon isoscalar and isovector \( p \)-wave scattering volumes, respectively. The parameters \( B_0 \) and \( C_0 \) represent \( s \)-wave and \( p \)-wave absorptions, respectively, and as such they have imaginary parts. Dispersive real parts are found to play an important role in pionic atom potentials. The parameter \( \xi \) is the usual Ericson-Ericson Lorentz-Lorenz coefficient (EELL) [3]. The terms with \( 4\rho_n\rho_p \) were originally
written as \((\rho_n + \rho_p)^2\) (see Ref. [2]), but the results hardly depend on which form is used. An additional relatively small term, known as the ‘angle-transformation’ term (see Eq.(24) of [1]), is also included. The local \(k_F(r)\) was taken from the Fermi gas model. There is no risk of using this model for extremely low densities because the major contributions to strong interaction effects come from regions near the half-density point, regardless of where the peak of the atomic wavefunction is. The position of that peak relative to the nuclear surface determines only the scale of the effects.

The potential as described above had been used extensively to analyze pionic atom data [1,20]. It will be referred to in the following as the conventional potential (C). Very good fits to the data are obtained with this potential but the combined repulsion due to the resulting values of the parameters \(b_1\) and \(\text{Re}B_0\) is about twice as large as is expected from the very well known values of the free pion-nucleon scattering lengths [11] and from the well-determined parameter \(\text{Im}B_0\) [1,20]. For that reason the \(s\)-wave interaction in the nuclear medium continued to be a topic of interest. Very recently Weise [12] showed that if the pion decay constant in nuclear matter \(f_\pi^2\) is given, in leading order, as a function of the local density \(\rho\)

\[
f_\pi^2 = f_\pi^2 - \frac{\sigma_N}{m_\pi^2} \rho
\]

(8)

where \(f_\pi\) is the decay constant of the pion in free space and \(\sigma_N\) is the pion-nucleon sigma term, then the \(s\)-wave scattering amplitude becomes a function of the local density as follows

\[
b_1(\rho) = \frac{b_1(0)}{1 - 2.3\rho}
\]

(9)

for \(\sigma_N=50\) MeV and with \(\rho\) in units of \(\text{fm}^{-3}\). Note that expanding this expression in powers of the density leads naturally to a repulsive \(\rho^2\) term. The parameter \(b_1(0)\) refers to the experimental free pion-nucleon interaction at threshold [11]. Introducing this function of the local density into the above potential, in Eq.(3) and (7), leads to the ‘W’ potential of Ref. [14]. If \(b_1(0)\) is taken as the first order chiral perturbation result of Weinberg, then a third order correction of about 15% brings it in line with the experimental value of [11]. In that case there is an additional density dependence in Eq.(9) which, however, does not affect the results for pionic atoms beyond what is obtained when Eq.(9) is used with the empirical pion-nucleon value. We therefore refer to the experimental free pion-nucleon value throughout, denoting this potential by ‘W’.

At this point it is useful to state clearly the philosophy behind the present work and the interplay between empirical quantities and more fundamental quantities. The potential of Eq.(2) is said to respect the low density limit when the coefficients of the linear terms \((b_0, b_1, c_0, c_1)\) are those obtained from the free pion-nucleon interaction and this can be easily achieved (see below) for the \(p\)-wave part of the potential. However, this is not the case with the parameters \(b_0\) and \(b_1\), with the latter found to be too repulsive. The coefficient \(\text{Re}B_0\), although purely empirical, is also somewhat constrained theoretically, e.g. it was shown [4] that it is very unlikely to be repulsive and larger in absolute value than \(\text{Im}B_0\). In practice the empirical values of \(\text{Re}B_0\) are at variance with this expectation, and together with the repulsive \(b_1\) lead to the so-called ‘anomalous’ repulsion. Because the \(b_0\) parameter is very small Weise [12] considered medium effects only on the parameter \(b_1\) suggesting that the
empirical extra repulsion compared to the free pion-nucleon interaction is due to medium modification of the pion decay constant. In Ref. [14] and in the present work we check that approach by large scale fits to data, while treating $B_0$ (and $C_0$) as purely phenomenological. For that reason no additional effects due to medium modification of the pion decay constant are considered in the present work. Note, however, that a recent preprint [22] reports failure when additional medium modification effects are considered.

Among several previous attempts to account for the anomalous $s$-wave repulsion a relativistic impulse approximation approach showed [15–19] that a specific version of the RIA was able to provide a significant part of the anomalous repulsion through the modification of the nucleon mass in the nuclear medium. This version of the RIA was considered as an additional option in [14] and is included in the present work. The additional term of the potential, to be added to Eq. (2), is

$$2\mu\Delta V_{opt}(r) = -4\pi m d_0 \left( \frac{M^2}{M^2(\rho)} - 1 \right) \rho_k(r)$$

where $M(\rho)$ is the effective mass of the nucleon in the medium, $\rho$ is the total density and $\rho_k(r)$ is the squared nucleon momentum distribution given in the local density approximation by

$$\rho_k(r) = \frac{3}{5} \left( \frac{3}{2\pi^2} \right)^{2/3} \rho^{5/3}(r).$$

The coefficient $d_0 = -0.190m_\pi^{-3}$ originates in the spin-dependent interaction [2]. For the effective nucleon mass the following parameterization was used [17]

$$\frac{M(\rho)}{M(0)} = \frac{1}{1 + a\rho}.$$  (12)

With $a = 2.7$ fm$^3$, we find $M(\rho)/M(0) = 0.7$ for the nucleon mass ratio at normal nuclear density. With $a = 1.56$ fm$^3$ this ratio is 0.8. Both values have been used in the present work. Note, however, that this version of the RIA is not unique [18].

The nuclear densities $\rho_p$ and $\rho_n$ are essential ingredients of the pion-nucleus potential, and have been discussed at some length in [1]. Two questions are relevant in connection with nuclear densities: (i) the model used to generate the densities and (ii) the radial extent of the neutron densities. The root mean square (rms) radii of the proton densities are obtained from the experimentally determined charge distributions [23] by unfolding the finite size of the proton. In the present work we have used both the macroscopic (MAC) densities and the single particle (SP) densities discussed in [1]. In previous analyses [20,14] the rms radii of the neutron densities ($r_n$) were assumed equal to the corresponding rms radii for the protons ($r_p$) in the case of light ($N = Z$) nuclei, and were taken to be slightly larger than the corresponding radii for the protons for $N > Z$ nuclei. Here we have used those previous values and in addition we used rms radii for the neutron densities as obtained from recent relativistic mean field (RMF) predictions [24] for the differences $r_n - r_p$. Moderate sensitivity to values of $r_n$ for given sets of potential parameters is observed, e.g. 100 and 50 keV for the binding energies of the $1s$ and $2p$ states in $^{205}$Pb, respectively, for a decrease of $r_n$ of 0.1 fm. However, once best fits to the data have been made, the sensitivity of the
results to the precise values of \( r_n \) is small. Nevertheless, attempting to use neutron densities with \( r_n = r_p \) resulted in major deterioration in the quality of fits to the data. Since the deeply bound pionic atom states of \(^{205}\text{Pb} \) \(^{[23]}\) may arise some interest we quote the rms radii used, namely 5.45 fm for the protons and 5.63 or 5.71 fm for the neutrons in the two sets of fits performed for each type of density as described above.

Finally we remark that the strong interaction shifts are always taken relative to the corresponding electromagnetic values calculated from the finite size charge of the nucleus and including vacuum polarization terms. This is the conventional definition and actually the only one meaningful for \( 1s \) states in heavy nuclei \((Z > 137/2)\).

### III. DATA BASE AND FIT PROCEDURES

The data base for the present work of 106 data points was made by adding 46 points to the 60 points used in Ref. \([14]\). Those 60 points were the 54 points of Ref. \([1]\) to which were added the shift and width of the \(4f\) level in \(^{208}\text{Pb} \) \(^{[20]}\) and the recently obtained results \(^{[25]}\) for the ‘deeply bound’ \(1s\) and \(2p\) states in pionic atoms of \(^{205}\text{Pb}\). In the case of the deeply bound states the shifts, relative to the finite size and vacuum polarization values discussed in the previous section, were simply obtained from the experimental binding energies \(^{[25]}\). The shift of the \(4f\) level as well as the 23 additional shifts \(^{[21]}\) were transformed to our normal basis of the finite size plus vacuum polarization reference using the parameters of the charge distributions listed in Ref. \(^{[20]}\).

The parameters of the pion-nucleus potential were varied in \( \chi^2 \) minimization, where \( \chi^2 \) was defined in the usual way. With so many data points it was possible to vary simultaneously all 9 parameters of the potential, but obviously some were determined better than others and, moreover, correlations exist between some of the parameters. Parameter values in the 9-dimensional space were chosen according to the following criteria, in descending order of importance:

1. The lowest possible total \( \chi^2 \) was required to within the natural unit for this problem which is \( \chi^2 \) per degree of freedom.
2. Respecting the low density limit for the \( p \)-wave part of the potential.
3. Requiring theoretically acceptable values for the ratios of real to imaginary parts of the empirical quadratic terms.

No \textit{a priori} restrictions were placed on the \( s \)-wave part of the potential as it is the topic being studied in the present work.

The final results were obtained by usually varying only 6 or 7 parameters, as is discussed in the next section. All fits were repeated for the four sets of density distributions mentioned in the previous section, namely for two sets of neutron radii using MAC densities and for the same two sets of neutron radii using SP densities. The whole process was repeated six times, using the conventional (C) potential, the Weise (W) potential, two RIA potentials with different values of the parameter \( a \) (Eq.\((12)) \) and two potentials where both the RIA and the W mechanisms were included.
IV. RESULTS

Starting with the conventional potential it was easy to obtain fits with $\chi^2/F$, the $\chi^2$ per degree of freedom, of about 1.9. This represents very good fits to pionic atom data from $^{16}$O to $^{238}$U, covering atomic states from $1s$ to $4f$. The fits to the deeply bound $1s$ and $2p$ states in $^{205}$Pb were better than the average. In an earlier work [26] full consistency was found between deeply bound states in $^{207}$Pb and normal pionic atom states. It means that, so far, the deeply bound states do not have any special weight in determining the parameters of the pion-nucleus potential. The results of these large scale fits are in general agreement with previous works [1,20]. The EELL parameter $\xi$ was not well determined and a shallow minimum was obtained for values around 2. However, the scattering volume parameters $c_0$ and $c_1$ turned out to be close to the free pion-nucleon values only when $\xi$ was close to 1 and we chose to keep the value of $\xi$ fixed at 1. The parameter $c_1$ was then found to be consistent with the free pion-nucleon value and consequently it was kept fixed at that value in the final fits. The parameter $c_0$ too was kept fixed at the free pion-nucleon value in part of the fits, as is shown in Table I although marginal improvements in the fits could be obtained when it was varied slightly. Comparing the present results with previous fits, we note that in Ref. [1] we showed results for $\xi=0$ because that was the only way to compare with Ref. [20], due to the different structure of the EELL effect in the two approaches. However, the results for $\xi=0$ were not the best possible. Nevertheless, in order to check the sensitivity of our conclusions regarding the $s$-wave potential to the $p$-wave potentials used, we show in Table I results for $c_0, c_1$ and $\xi$ fixed at the values of [1].

Turning to the present work, each fit was repeated 8 times: for the two sets of rms radii for neutron density distributions for each of the MAC and SP models used to generate the density distributions and then for $c_0$ fixed or variable. All the fits for the conventional (C) potential produced almost the same values of $\chi^2/F$ which makes it impossible to prefer any one of them on the basis of quality of fit. Introducing additional absorption terms proportional to $\rho^2$ [20] produced insignificant improvements, e.g. $\chi^2$ changed by less than 1.9 (out of 200) with the addition of such a term in the $p$-wave part and by 0.1 with the addition of such a term in the $s$-wave part. One difference compared to the earlier analysis [14] was that Re$C_0$ was not consistent with zero and therefore it was varied in all fits.

The other five potentials, each with additional dependence on the nuclear density relative to the conventional potential, were used in fits to the data in much the same way. Typical results are summarized in Table I. In this table the Weise model (Eq. (8)) is labelled by W and the two RIA models with $a=2.7$ and $a=1.56$ fm$^3$ (Eq.(12)) are labelled by B1 and B2, respectively. The rows WB1 and WB2 are for fits when the Weise and the RIA models were combined. It is seen from the table that the quality of the fits is essentially the same for all the potentials and therefore other criteria must be applied in order to prefer one potential or the other.

As was stated above, the present work is focused on the empirical values of $b_0$ and $b_1$ and their relation to the corresponding values for the free pion-nucleon interaction. Because the parameter $b_0$ is extremely small, we assess the various potentials essentially by the empirical values of $b_1$ with emphasis on the effects due to the Weise prescription (Eq.(8)). In this context it is particularly important to mention the weak coupling between the $s$-wave part and the $p$-wave part of the best-fit potentials. This is demonstrated by comparing the results...
in Table I with those in Table II, where the latter contains results of similar calculations as in the former but with the linear p-wave part of the potential taken from Ref. [1]. All three parameters \((c_0, c_1, \xi)\) are very different in the two sets of calculations yet the corresponding empirical parameters of the s-wave part are essentially the same. Note in particular the corresponding values of \(b_1\) and their uncertainties. Such small uncertainties can be achieved only in large scale fits and cannot possibly be achieved when studying isolated pieces of data [27,28]. The variation of \(b_1\) between the different models does not depend on the choice of the p-wave part, provided best fit to the data has been achieved. Comparing the ratios \(\text{Re}C_0/\text{Im}C_0\) in the two tables, Table I is preferred. In what follows we will refer only to the results where the p-wave part respects the low density limit, with \(\xi=1\), as in Table I.

The anomalous repulsion in the s-wave part of the potential can be seen in Table I through values of \(b_1\) and \(\text{Re}B_0\). For the C potential the former is significantly more repulsive than the free pion-nucleon value and the latter, which is the dispersive part of the absorptive term \(\text{Im}B_0\), is repulsive with a magnitude 3-4 times larger than the absorption. This is quite unacceptable as various theoretical approaches [4,5,29,30] suggest that the real part must have about the same magnitude as the imaginary part. The other rows in the table show various degrees of reduced repulsion in \(b_1\) and in \(\text{Re}B_0\). This is achieved through the density dependence of \(b_1\) in the Weise model which makes \(b_1\) progressively more repulsive with increasing nuclear density and through the repulsion generated by the RIA term. In both cases the phenomenological repulsion required by the data is obtained while keeping \(b_1\) closer to the free nucleon value and keeping \(\text{Re}B_0\) closer to expectations.

Figures 1 and 2 show the values of \(b_0\), \(b_1\) and \(\text{Re}B_0\) for all the 48 different fits. The six groups of eight points each are according to the potential (see Table I), the type of nuclear density used and for both fixed \(c_0\) and variable \(c_0\). Figure 1 includes, between horizontal dashed lines, the experimental values of the free pion-nucleon \(b_0\) and \(b_1\) parameters. It is seen that the WB1 and WB2 models yield the best agreement with experiment. The horizontal dashed lines in Figure 2 show the range of expected values of \(\text{Re}B_0\) between \(-\text{Im}B_0\) and \(\text{Im}B_0\). Within each potential one can see a fairly small dependence on details such as type of density and whether \(c_0\) was varied or not. The anomalous repulsion is clearly seen for the conventional potential. It essentially disappears for potentials WB1 and WB2.

Figure 3 shows values of \(\chi\) for all the data points, for fits based on the C and W potentials with MAC densities. It illustrates that there are no systematic differences between the different potentials from the point of view of fits to the data. Very similar results are obtained when different nuclear densities are used. It seems that whatever systematics is observed in Fig. 3 it is peculiar to the targets and not to differences between the models used here. Obviously it could indicate deficiencies in the models.

Another topic of interest is the way the real s-wave potential varies from the nuclear surface towards the nuclear interior. As an example Fig. 4 shows the real part of the \(\pi^-\)\(^{208}\)Pb potential for the six models of the present work, based on MAC densities. All six potentials produce almost identical fits to the data (see Table I) and indeed they are almost identical throughout the nucleus. Note that the potentials do not follow the nuclear density distribution because of the various non-linear terms. The almost unique potential all over the nucleus is a result of the featureless shape of the MAC densities. Figure 5 shows, again for \(^{208}\)Pb, the six potentials this time based on SP densities. The C potential for MAC densities is also included as a dashed line. Here we see that all the potentials essentially
agree in the surface region near 6.2 fm, which presumably is the region best determined by the data. The potential at that point is about 25 MeV whereas the point where the density is half of the central density (shown on the figures) is at 6.9 fm and the potential there is about 11.8 MeV. The potentials due to the various models, based on the SP densities, differ in the interior because these models extrapolate differently into the nuclear interior due to the structure of the densities and the variety of non linear effects. It is probably safe to conclude that the real potential in the nuclear interior is close to 30–35 MeV, in contrast with smaller values [31,27] advocated earlier but in reasonable agreement with the value of 28±3 MeV of Friedman and Gal [20] and with the latest result of Geissel et al [25].

V. SUMMARY, CONCLUSIONS AND OUTLOOK

The prime motivation behind the present work was the recent suggestion [12,13] that the anomalous s-wave repulsion of pions in nuclear matter has its origins in a density dependence of the pion decay constant which reflects the change of QCD vacuum structure in dense matter. The consequences for pionic atoms are evident then through the isovector parameter $b_1$ of the potential, which is quite important because of the extremely small value of the isoscalar parameter $b_0$. The density dependence of the pion decay constant (Eq.(8)) leads to an additional well-defined density dependence in the pion-nucleus potential which can be tested by fits to pionic atom data. The idea was to check the consequences of prescription (8) by applying it to modern and extensive pionic atom data. Possible modifications of $B_0$ have not been considered because we treat this parameter all along as purely phenomenological without any experimental or theoretical reference values, except for the expectation that $|\text{Re} B_0|$ cannot exceed much the value of $\text{Im} B_0$. Also included was a RIA term, which in some sense is equivalent to imparting a density dependence to the underlying nucleon mass and was shown [18] to explain part of the repulsion, although this term is not unique. A comment on this renormalization and on that of $f_\pi$ (Eq.(8)) has been made recently by Brown and Rho [32]. A very broad data base had been used and extra care was taken in choosing the nuclear densities which are essential ingredients of the potential. Radii of neutron distributions were varied slightly, and were constrained by recent RMF calculations. If one assumes that radii of neutron density distributions are equal to the corresponding radii for proton distributions, then the fits become totally unacceptable.

The results of the present work show that all the potentials produce rather equivalent fits to the data, displaying small sensitivity to the type of potential, to the type of density or to the precise values of the rms radii of the neutron distributions. The differences between the various potentials are mostly in the value of $b_1$ and how close it is to the free pion-nucleon value. The prescription Eq.(9) removes most of the ‘anomaly’ and when an RIA term is also included then the parameters $b_0$, $b_1$ and $\text{Re} B_0$ have most acceptable values. These conclusions provide support to the validity of the correction suggested by Weise [12] to the conventional pion-nucleus interaction.

Finally, it is interesting to study the above mentioned features at energies just above threshold through the elastic scattering of low energy pions by nuclei, thus testing further the validity of the chirally motivated approach [12]. Indeed it has been shown [33,34] that pion-nucleus potentials develop smoothly from the bound states regime to the elastic scattering regime. An experiment to measure the elastic scattering of 20 MeV $\pi^\pm$ by several
nuclei with that aim in mind has been approved recently at the Paul Scherrer Institute (PSI) [36].

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TABLE I. Parameter values from fits to 106 pionic atom data points. Other $p$-wave parameters were held fixed at $c_0=0.22m^{-3}_\pi$, $c_1=0.18m^{-3}_\pi$ and $\xi=1$. The free pion-nucleon values \(^{11}\) are $b_0=-0.0001^{+0.0009}_{-0.0021}m^{-1}_\pi$ and $b_1=-0.0885^{+0.0010}_{-0.0021}m^{-1}_\pi$

| potl. | $\chi^2/F$ | $b_0 (m^{-1}_\pi)$ | $b_1 (m^{-1}_\pi)$ | $ReB_0 (m^{-4}_\pi)$ | $ImB_0 (m^{-4}_\pi)$ | $ReC_0 (m^{-6}_\pi)$ | $ImC_0 (m^{-6}_\pi)$ |
|-------|------------|------------------|------------------|----------------|----------------|----------------|----------------|
| C     | 1.94       | 0.030±0.010      | -0.113±0.004     | -0.21±0.04     | 0.055±0.002    | -0.029±0.009   | 0.064±0.004    |
| W     | 1.93       | 0.018±0.010      | -0.095±0.003     | -0.14±0.05     | 0.054±0.002    | -0.026±0.009   | 0.063±0.004    |
| B1    | 1.95       | 0.016±0.010      | -0.102±0.004     | -0.06±0.05     | 0.054±0.002    | -0.026±0.009   | 0.064±0.004    |
| B2    | 1.94       | 0.022±0.008      | -0.107±0.004     | -0.12±0.03     | 0.054±0.002    | -0.028±0.008   | 0.064±0.003    |
| WB1   | 1.96       | 0.006±0.010      | -0.086±0.003     | 0.00±0.05      | 0.053±0.002    | -0.024±0.009   | 0.063±0.003    |
| WB2   | 1.94       | 0.011±0.010      | -0.090±0.003     | -0.06±0.04     | 0.054±0.002    | -0.025±0.009   | 0.063±0.003    |

TABLE II. Parameter values from fits to 106 pionic atom data points. Other $p$-wave parameters were held fixed at $c_0=0.261m^{-3}_\pi$, $c_1=0.104m^{-3}_\pi$ and $\xi=0$. The free pion-nucleon values \(^{11}\) are $b_0=-0.0001^{+0.0009}_{-0.0021}m^{-1}_\pi$ and $b_1=-0.0885^{+0.0010}_{-0.0021}m^{-1}_\pi$

| potl. | $\chi^2/F$ | $b_0 (m^{-1}_\pi)$ | $b_1 (m^{-1}_\pi)$ | $ReB_0 (m^{-4}_\pi)$ | $ImB_0 (m^{-4}_\pi)$ | $ReC_0 (m^{-6}_\pi)$ | $ImC_0 (m^{-6}_\pi)$ |
|-------|------------|------------------|------------------|----------------|----------------|----------------|----------------|
| C'    | 1.93       | 0.020±0.009      | -0.114±0.004     | -0.15±0.04     | 0.054±0.002    | -0.28±0.01     | 0.062±0.003    |
| W'    | 1.90       | 0.008±0.009      | -0.097±0.004     | -0.07±0.04     | 0.053±0.002    | -0.28±0.01     | 0.067±0.003    |
| B1'   | 1.94       | 0.006±0.009      | -0.104±0.004     | 0.00±0.04      | 0.053±0.002    | -0.28±0.01     | 0.066±0.003    |
| B2'   | 1.93       | 0.011±0.009      | -0.108±0.004     | -0.07±0.04     | 0.054±0.002    | -0.28±0.01     | 0.067±0.003    |
| WB1'  | 1.94       | -0.005±0.009     | -0.088±0.003     | 0.06±0.04      | 0.053±0.002    | -0.27±0.01     | 0.066±0.003    |
| WB2'  | 1.91       | 0.001±0.009      | -0.092±0.003     | 0.00±0.04      | 0.053±0.002    | -0.28±0.01     | 0.066±0.003    |
FIG. 1. Values of $b_0$ and $b_1$ obtained from fits to pionic atom data for the six potentials indicated and the various densities and fit procedures (see text). The horizontal bands indicate the experimental values for the free pion-nucleon interaction [1].
FIG. 2. Values of \( \text{Re} B_0 \) obtained from fits to pionic atom data for the six potentials indicated and the various densities and fit procedures (see text). The horizontal bands indicate the theoretically expected range of values.
FIG. 3. Values of $\chi$ for the best fit C potential (open circles) and W potential (+ signs) using MAC densities.
FIG. 4. The real part of the $s$-wave $\pi^-{^{208}}$Pb potential for the six models using MAC densities. The ‘half-density’ point is indicated at 6.9 fm.
FIG. 5. The real part of the s-wave $\pi^{-}^{208}$Pb potential for the six models using SP densities. Also shown (dashed) is the C potential based on MAC densities. The 'half-density' point is indicated at 6.9 fm.