The Analysis Of Scattering For Transverse Electric By Perfectly Conducting Connected Strips At Node

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Abstract. This paper analyzes the scattering for transverse electric (TE) incidence by perfectly conducting connected strips. The configuration of the problem has multi-strips connected at a node. The proposed method is current-based hybrid method which reducing the number of unknowns. In the proposed procedure very few expansion functions are required to achieve the result. Hence this is very efficient method. The proposed method based on Galerkin scheme using expansion functions

Keywords: Connecting strips, hybrid method, conducting scatterer, Galerkin scheme

1. INTRODUCTION
For the analysis of edge behavior of objects with edges [1] requires appropriate functional space. Mexiner [2] gave electromagnetic field conditions for finding the solutions.

For this many problems based on expansion of the unknown current is studied and correct edge behavior was observed [3-12]. For the analysis of connected conducting strips, there need a high accuracy with quickly convergent procedure. In order to analyze we have to solve magnetic field integral equation (MFIE) and electric field integral equation (EFIE) for transverse electric (TE) incidence and transverse magnetic (TM) incidence respectively[4-5]. For this Galerkin scheme [6-8] is suitable in the spectral domains it is stable at frequencies up to approximate resonance frequencies. Hence this method is proposed in the spectral domain.

But for very thin strip scatterer magnetic field integral equation (MFIE) is not used[2], therefore the analysis for transverse electric (TE) incidence becomes limited for the closed cylinder scatterers. If we reduce the integral-differential equation to electric field integral equation (EFIE), then this limitation can be removed.

In this paper we have converted integral-differential equation of the object into electric field integral equation (EFIE).

2. THEORETICAL FORMULATION
In the present paper transverse electric (TE) incidence scattered from two dimensional (2-D) perfectly conducting scatterers is analyzed.

In this communication we have used simple Cartesian coordinate system (x,y,z) for the main
system and local coordinate system \((x, y, z)\) for the large number of strips of scatterers such that axes of all scatterers are parallel to the \(z\) axis. Thus the \(i\)th surface of multi-strip scatterer can be represented as \((\bar{x}_i, \bar{y}_i, 0)\).

And the angle of incidence for plane wave on this surface with \(x\) axis is denoted by \(\phi_i\) as shown in the fig 1.1.

**Fig.-1.1:** Different structure of conducting connected strips.

Now the electric field transverse component for transverse electric (TE) incidence along \(x_i\) is given by Maxwell equation as

\[
E_y(x, y) = \frac{1}{j \omega \varepsilon} \frac{\partial}{\partial y} H_z(x, y) = \frac{1}{2 \omega \varepsilon} \frac{\partial}{\partial y} \sum_{i=1}^{\infty} \left( \frac{\partial}{\partial y} \int_{-\infty}^{\infty} \tilde{J}_j(u) e^{-j|k|u-y} e^{-j\omega t} du \right) \quad \text{(1.1)}
\]

where \(\tilde{J}_j(u)\) = Fourier transform of the induced current on \(i\)th side.

Now we apply the total tangential component of the electric field to find the integral differential equation for this electric field.

As we are focused about analysis of edge behavior of connected strips so we need to study the distribution of the currents on strips and accordingly the edge behavior.

The advantages of fast convergence with high accuracy have out lined in many papers [4-12].

For the \(i\)th connected strip scatterer, correct edge behavior is [2] given by

\[
\frac{\partial}{\partial a_i} J_j(x_i) \sim w_i^0 \left( \frac{x_i}{a_i} \right) \quad \text{(1.2)}
\]

where \(\psi^r_i\) is the angle at point \(x_i = \pm a_i\) according to the dominant edge behavior.

In this case current on scatterer is taken as the superposition of the all currents and then among such two currents the dominant edge behavior is chosen. Also the current at node is continuous.

According to the edge behavior (1.2) the asymptotic Fourier transform for current components [2] is given as:

\[
\tilde{J}_j(u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} J_j(x_i) e^{jux_i} dx_i \sim \tilde{J}_j^0 (u) + \frac{\eta_i e^{jux_i}}{u |(x-x_i)|} + \frac{\eta_i e^{jux_i}}{u |(x+x_i)|} \quad \text{(1.3)}
\]

Where \(\eta^H_i\) depends on the angle of incidence as well as cross-section of object, also

\[
\tilde{J}_j^0(u) = \frac{-C_i e^{jux_i} + C_i e^{-jux_i}}{2\pi j u} \quad \text{..........(1.4)}
\]

where, \(C_i\) is the current at right edge of \(i\)th side and \(C_i^-\) is the current at left edge of \(i\)th side.

**Case [I]: Single Strip:**

In case of single strip, \(C_i = C_i^- = 0\)
and hence the asymptotic behavior is depending on other terms in (1.3). It is clear that in this case (1.4) is nothing but the decaying of the integrand of (1.1). Thus it is possible to invert one integration and derivative and hence (1.1) can be written as

\[ E_j(x, y) = \frac{1}{2 \omega e} \frac{\partial}{\partial y_j} \sum_{i=1}^{n} \text{sgn}(y_i) \int_{-\infty}^{\infty} \hat{J}_i(u)e^{-i|u|^2-y_j^2}e^{-jux} du \]

But for \( y_i = 0 \) direct inversion is not possible. Hence, we can write the above equation as

\[ E_j(x, y) = \frac{1}{2 \omega e} \frac{\partial}{\partial y_j} \sum_{i=1}^{n} \text{sgn}(y_i) \int_{-\infty}^{\infty} \hat{J}_i(u)e^{-i|u|^2-y_j^2} - \hat{J}_i^*(u)e^{i|u|^2} du \]

\[ + \frac{\partial}{\partial y_j} \sum_{i=1}^{n} \text{sgn}(y_i) \int_{-\infty}^{\infty} \hat{J}_i^*(u)e^{-i|u|^2-y_j^2} du \]

Now first integrand can be inverted and last term is zero. Thus we have

\[ E_j(x, y) = \frac{1}{2 \omega e} \frac{\partial}{\partial y_j} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \hat{J}_i(u) \hat{G}_i(u) e^{-i|u|^2+y_j^2} du \]

\[ = \frac{1}{2 \omega e} \frac{\partial}{\partial y_j} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \hat{J}_i(u) \hat{G}_i(u) e^{-i|u|^2+y_j^2} du \]

\[ G_i(u) = \sqrt{k^2 - u^2} \cos(\phi_i - \phi_j) + u \text{sgn}(y_j) \sin(\phi_i - \phi_j) \]

\[ G_i^*(u) = -j|u| \cos(\phi_i - \phi_j) + u \text{sgn}(y_j) \sin(\phi_i - \phi_j) \]

Here equation (1.7) is required electric field integral equation (EFIE) in which current derivative shows the edge behavior and is continuous at nodes.

**Case [II] : Multi-strips connected at node:**

In this case more than two strips are connected at node. That means strips are intersecting at each node. So, for the continuous current at the node, the current on side itself and on the connected sides expanded using appropriate functions. Although, the Fourier transform of each expansion function is analytical [13-14].

Now when we use Fourier transform into the electric field integral equation (EFIE) and proceeding by same functions then we get a set linear algebraic equations. Also due to reciprocity scattering matrix of such problem is symmetric [10], and hence scattering matrix elements may be converted into to single integrals.

Then this integrals computed very fast because such integrals are either exponentially decaying, or analytically accelerated [14].

**3. NUMERICAL RESULTS**

The method used is very fast and also accuracy is high. For testing this method so many geometries has studied.

i) The normalized truncation error for the connected strip is given by

\[ e(N) = \frac{\|J_{N+1} - J_N\|}{\|J_N\|} \]

where \( J_N \) and \( J_{N+1} \) are the vectors of all the expansion coefficient of the currents on all the sides evaluated with \( N \) and \( N+1 \) terms respectively on each side.

The normalized truncation error for the “tree-shaped” connected strip is shown in Fig.-1.2.

Here we can seen that the error decaying is almost exponential and hence error less than \( 10^{-2} \) can be obtained only by 5 terms.
ii) The proposed method also tested for single conducting strip. In this current at the strip center and far field value $F(\phi)$ is obtained by separation of variables and this result is compared with the result by domain-product technique [14] as shown in Table-1.1

| $d/\lambda = 0.7$ | $F(\pi/2)$ | Hz | $\arg Hz$ |
|------------------|-----------|----|----------|
| Proposed Method  | 1.27(5)   | 2.07 (5) | 0.24(5) |
| [14]             | 1.27      | 2.07  | 0.24     |

In this table the number of expansion terms is shown in parentheses. From the table it is clear that by a few terms one can obtain an accurate result as in [14].

When we analyze the current behavior at the center of one side we find that there is a slower convergence by numerical methods but the advantage of this method is only few expansion functions are needed for this as shown in Fig.-1.3.

4. COMPARISION

The work done in the previous paper for diffraction problems of TE and TM modes, mathematical theory of diffraction is used which is complicated task [26]. It takes more computational time [25]. It requires computational complexity and very expensive in handling large scale problems.

In the present paper all the elements of the scattering matrix can be reduced to single integrals, so that computed efficiently. In this way, quick convergence and effective error control can be achieved, even when high accuracy is required.
Table 1.2

| d/λ | N | EFIE | MFIE |
|-----|---|------|------|
|     | e(N) < 10-2 | e(N) < 10-3 | e(N) < 10-2 | e(N) < 10-3 |
| 0.7 | 5 | 8 | 6 | 10 |
| 0.71 | 5 | 8 | 6 | 10 |
| 0.707 | 5 | 8 | 6 | 10 |
| 0.7071 | 5 | 8 | 8 | 12 |
| 0.707107 | 5 | 8 | 12 | 16 |
| 0.7071068 | 5 | 11 | 18 | 24 |

As can be seen, more terms are needed in present case for MFIE with respect to EFIE.

Finally, the bistatic radar cross section has been computed for the scatterer sketched in the inset of Fig.-1.4 by means of 12 expansion terms. For the sake of comparison, the results reported in [25] and obtained by means of CST Microwave Studio also shown.

Fig.-1.4: Bistatic radar cross section of the scatterer sketched in the inset. Dimensions: kd = 20, N=12. Simulation times 29 s

5. CONCLUSIONS

In this paper the scattering by perfectly conducting connected strips scatterers for transverse electric (TE) incidence has been designed in the form of electric field integral equation (EFIE). For the quick convergence with high accuracy an effective method has been developed. This method is developed by using Galerkin method. All the scattering matrix elements in this method is reduced to a single integral. The method is therefore computing very efficiently and hence very attractive. This accelerating procedure may be extended to future work to the radar cross section, rectangular cavity, dielectric scatterers etc.

In this paper, the optical properties of 1D ternary photonic crystal are analyzed by using TMM method. Theoretical result shows that the proposed structure works as a tunable optical filter when the temperature of superconductor increases with wavelength. In this work, the defect mode layer shifted towards the higher wavelength range of spectrum and efficiency of the layer also increases with increases temperature of the superconductor. At a certain temperature (92K) it approaches to 100%. By tuning the wavelength, we design a wavelength division multiplexer or optical filter which is used in optical fiber communication. So, the designed structure can be used as a tunable multichannel filters at second transmission window in optical fiber communication.

REFERENCES

[1] Bouwkamp,C. “A note on singularities occurring at sharp edges in electromagnetic diffraction theory,” Physica, volume 12, p. 467, 1946.
[2] Meixner, J. “The behavior of electromagnetic fields at edges”, IEEE Trans. Antennas Propag., volume 20, pp. 442-446, Jul. 1972).
[3] Meixner, J. “Die Kantenbedingung in der Theorie der BeugungselektromagnetischerWellen an vollkommenleitendenebenenSchirmen”, Antennas Physics. volume 6, p. 1-9, 1949.
[4] Ofluoglu, A. E., Ciftci, T., and Ergul, O. (2015). “Magnetic-Field Integral Equation”. IEEE Antennas and Propagation Magazine, Volume 57, Issue 4, p. 134–142, 2015.

[5] Reuster, D. D., Thiele, G. A., and Eloe, P. W. “The convergence of iterative solutions to the Electric Field Integral Equation”. Applied Mathematics Letters, Volume 8, Issue 6, p. 43–49, 1995.

[6] Lucido, M., Panariello, G. and Schettino, F. “Analytically regularized evaluation of the scattering by perfectly conducting cylinders”, Microwave Optical Technology Letters, volume 41, p. 410–414, 2004.

[7] Lucido, M., G. Panariello, G., and F. Schettino F., “Accurate and efficient analysis of stripline structures”, Microwave Optical Technology Letters, volume 43, p. 14–21, 2004.

[8] Araneo, R. S. Celozzi, S. G. Panariello, G. F. Schettino, F. and Verolino, L. W. Ross Stone, W. “Analysis of Microstrip Antennas by means of Regularization via Neumann Series”, in Review of Radio Science 1999-2002 Piscataway, NJ/New York: IEEE Press/Wiley Inter-science, 2002, p. 111-124, 2002.

[9] Hodges, R. E., and Rahmat-Samii, Y. (1997). “An iterative current-based hybrid method for complex structures. IEEE Transactions on Antennas and Propagation, Volume 45, Issue 2, p. 265–276, 1997.

[10] Tie Jun Cui, Wiesbeck, W., and Herschlein, A. “Electromagnetic scattering by multiple three-dimensional scatterers buried under multilayered media. I. Theory”. IEEE Transactions on Geoscience and Remote Sensing, Volume 36, Issue 2, p. 526–534, 1998.

[11] Arıkoglu, A., Ozkol, I. “Solutions of integral and integro-differential equation systems by using differential transform method”. Computers and Mathematics with Applications, Volume 56, Issue 9, p. 2411–2417, 2008.

[12] Jones, D.S., The Theory of Electromagnetism London, U.K.: Pergamon Press, 1964.

[13] Lucido, M., Panariello, G. and Schettino, F. “Electromagnetic scattering by multiple perfectly conducting arbitrary polygonal cylinders”, IEEE Transactions on Antennas and Propagation, volume 56, p. 425–436, Feb. 2008.

[14] Chumachenko, V.P. "Domain-product technique solution for the problem of electromagnetic scattering from multi-angular composite cylinders,"IEEE Transaction on Antennas Propagation, volume 51, p. 2845-2851, 2003.

[15] Nosich, A.I., “The method of analytical regularization in wave scattering and eigenvalue problems: Foundations and review of solutions", IEEE Antennas Propagation Mag., volume 41, p. 34–49, 1999.

[16] Nesvit K. V., “Diffraction problem of scattering and propagation TM wave on pre-fractal impedance strips above shielded dielectric layer”, Science Publishing Corporation, Bremen, Germany: International Journal of Applied Mathematical Research, Vol. 3, No 1, pp. 7-14, 2014.

[17] Eswaran, K. “On the solutions of a class of dual integral equations occurring in diffraction problems", Proc. Royal Society London, ser. A. p. 399–427, 1990.

[18] Harrington, R. "Boundary integral formulations for homogeneous material bodies,"Journal Electromagnetic Waves Application, Volume 3 , p. 1-15, 1989.

[19] Veliev, E.I., Vereme, V.V., Hashimoto M., Idemen M., and Tretyakov, O.A., “Numerical-analytical approach for the solution to the wave scattering by polygonal cylinders and flat strip structures", in Analytical and Numerical Methods in Electromagnetic Wave Theory. Tokyo, Japan: Science House, 1993.

[20] Tranter, C.J., Bessel Functions with Some Physical Applications. Bath, U.K. English Univ., Press, 1968.

[21] Li M., Ma, K. P., Hockanson, D. M., Drewniak, J.L., Hubing, T.H., and Van Doren T. P., "Numerical and experimental corroboration of an FDTD thin-slot model for slots near comers of shielding enclosures,"IEEE Transaction on Antennas Propagation, volume. 39, p. 225-232, 1997.

[22] Ohnuki S., Ohtaka N., and Hinata T., "Electromagnetic scattering from rectangular cylinders with wedge cavities," in proc international symposium antennas and propagation society, p. 2925-2928, 2006.
[23] Wang, T. M., Cuevas A. and Ling H. "RCS of a partially open rectangular box in the resonant region," IEEE Transaction on Antennas Propagation, volume 38, p. 1498-1504, 1990.

[24] Lucido Mario, Panariello Gaetano, Schettino Fulvio. “TE Scattering by Arbitrary Connected Conducting Strips”. IEEE Transaction on Antennas Propagation, volume 57, Issue 7, p. 2212-2216, 2009.

[25] Yadav Vimal, “A Multiregion Model to the EM Scattering from a PEC rough surface”. in International Conference on Research and Innovation in Engineering (ICRIE-2016).

[26] Nesvit Kateryna, Kharkiv Karazin, “Scattering and Propagation of the TE/TM Waves on Pre-Fractal Impedance Grating in Numerical Results”. The 8th European Conference on Antennas and Propagation (EuCAP 2014).