Modern Life-Care Tontines

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joint work with:
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We want to thank Prof. Michel Denuit (UC Louvain) for many comments and discussions.
Belgium: LTC spending (in terms of GDP) increased from 1.7% in 2000 to 2.3% in 2018 (source: Eurostat).

United Nations projections: The number of elderly people, i.e. older than 65, is projected to triple from 2020 to 2080 to reach 2.2 billion. The global share of the elderly population is expected to rise from 9.4% in 2020 to 20.6% in 2080.
Definition: A mutual insurance company (tontine) is an insurance company owned entirely by its policyholders.

▶ This usually avoids risk charges and reduces administration, regulation.
What is mutual insurance?
What is mutual insurance?
Mutual life/pension insurance?
Mutual life/pension insurance?
Mutual insurance schemes in past...

Historic tontines (17th-19th century)

- Plans to rise government money.
- Predefined income stream is paid to survivors of a pool.

Li, Y., & Rothschild, C. (2020). Selection and redistribution in the Irish tontines of 1773, 1775, and 1777. *Journal of Risk and Insurance*, 87(3), 719-750.
... start a revival today:

The New York Times

When Others Die, Tontine Investors Win

By Tom Verde
March 24, 2017

Living a long life is its own reward. But when you invest in a tontine, there's an added benefit: You collect money that would have gone to people who have died.

That is part of the macabre appeal of the tontine, a 350-year-old investment vehicle that fell into disfavor more than a century ago but is now getting fresh consideration as a way to help people receive steady income in retirement.
Modern tontines: Example Xianghubao

| Age group           | Mild critical illness | Severe critical illness |
|---------------------|-----------------------|-------------------------|
| 30 days to 39-year-old | 50,000 yuan           | 300,000 yuan            |
| 40- to 59-year-old  | 50,000 yuan           | 100,000 yuan            |

- **Disability insurance.**

- Based on an **app in China**, founded 2018.

- After 1 year, **100 million users**.

See also:

- Abdikerimova, S., & Feng, R. (2022). Peer-to-Peer multi-risk insurance and mutual aid. *European Journal of Operational Research*, 299(2), 735-749.
1. Motivation: Mutual insurance (tontines)

2. A fair, heterogeneous, modular mutual insurance scheme

3. Modern Life-Care Tontine
Related literature

Mutual (life) insurance schemes gain popularity in academic literature:

- **(Natural) tontines**: Milevsky, Salisbury [2015, 2016], Chen, Hieber, Klein [2019], Chen, Hieber, Rach [2020], Chen, Qian, Yang [2021], Bernhardt, Donnelly [2021], Denuit, Robert [2021], Bernhardt, Qu [2022], Winter, Planchet [2022], Denuit, Dhaene, Robert [2022]. (many more . . .)

- **Pooled annuities, P2P insurance, (tontines)**: (Sabin [2010]), Qiao, Sherris [2013], Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019]. (many more. . .)
This talk: Academic research

- Mutual risk-sharing schemes for **heterogeneous pools** (for example heterogeneous in age, health).

- **We pool mortality and morbidity** (long-term care) risks.

- Hieber, P., & Lucas, N. (2022). *Modern life-care tontines*. ASTIN Bulletin: The Journal of the IAA, 52(2), 563-589.

- Denuit, M., Hieber, P., & Robert, C. Y. (2022). *Mortality credits within large survivor funds*. ASTIN Bulletin: The Journal of the IAA, in press.

*Joint work with Nathalie Lucas (National Bank, Belgium), Michel Denuit (UC Louvain), Christian Y. Robert (ENSEA Paris)*
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Tontine products and surplus distribution

How do modern tontines work? Difference to pure financial investment?

We share insurance “gains”:

► **mortality credits**: As in traditional insurance, accounts of deceased are (maybe only partially) distributed to survivors.

► **morbidity credits** (see later slides): Long-term care risks (more dependent people, longer time in dependency) are shared. This can be a surplus or a deficit!

All this comes on top of the regular financial return.
Multi-period heterogeneous tontine: Sketch/example

Fixed payoff (gray line) = “individual account”
Some notation

- Pool members $L_0 = \{1, 2, \ldots, n\}$. Time in periods $t = 0, 1, 2, \ldots$.
- Individual $j \in L_0$ contributes single premium $c_j(0)$ at time 0.
- Deterministic, risk-free rate $\delta_t, t \geq 0$.
- Remaining lifetimes $T_j, j \in L_0$, are assumed to be independent.
- Death probability: $q_{x_j}$. Maximal age $\omega \in \mathbb{N}$.
- Individual account value, fixed payoff $s_j(t)$:

$$c_j(t) = \begin{cases} e^{\int_{t-1}^{t} \delta_s \, ds} c_j(t-1) - s_j(t), & j \in L_t \\ 0, & \text{otherwise} \end{cases}$$

(1) ($c_j(t)$ is the “individual account”, the gray line!)
In case of death, the pool shares the remaining account value

$$X(t) := \sum_{j=1}^{n} 1_{j \in D_{t}} \cdot e^{\int_{t-1}^{t} \delta_{s} ds} c_{j}(t - 1).$$

An individual $j \in L_{t-1}$ receives a payoff of:

$$W_{j}(t) = \begin{cases} 
    s_{j}(t) + \beta_{j}(X(t)), & \text{if } j \in L_{t} \\
    \beta_{j}(X(t)), & \text{if } j \in D_{t}
\end{cases}$$  \hspace{1cm} (2)

decomposed of

- $s_{j}(t)$: individual, fixed withdrawal amount,

- $\beta_{j}(X(t))$: collective part of the benefits, i.e. the mortality credits.
Examples: Sharing rules

Share linearly according to (1) amount invested and (2) death probability.

Example (Linear risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \frac{q_{x_j+t-1} \cdot c_j(t-1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j+t-1} \cdot c_j(t-1)} \cdot X(t). 
$$

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018])
Actuarial fairness: Insurer’s view

For each $t = 0, 1, \ldots$, the premium equivalence holds: (pool view)

$$
\sum_{j=1}^{n} c_j(t) = \sum_{j=1}^{n} \sum_{s=t+1}^{\omega-x_j} e^{-\int_{t}^{s} \delta u \, du} W_j(s).
$$

- **Right hand side:** random (big letter!)
- **Left hand side:** deterministic.
Actuarial fairness: Individual’s view

For each $t = 0, 1, \ldots$, the contract is fully-funded: (individual view)

\[
\underbrace{c_j(t)}_{\text{retrospective reserve}} = \mathbb{E}_t \left[ \sum_{s=t+1}^{\omega-x_j} e^{-\int_{t}^{s} \delta u \, du} W_j(s) \right].
\] (5)

The expected present value of future benefits equals the current account value.
2. A fair, heterogeneous, modular mutual insurance scheme

\[ s_j(t) \]

\[ W_j(t) : \text{average payoff} \]

\[ W_j(t) : 95\% \text{ confidence interval} \]

fixed payoff

mortality credits

payoff 65y−cohort

elapsed time t
1. Motivation: Mutual insurance (tontines)

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Why pool mortality and morbidity (long-term care) risks?

- People moving into dependency need more money but have a reduced life expectancy!
  \[\Rightarrow \text{Natural hedge, diversification!}\]

- Individuals in bad health cannot receive long-term care insurance!
  \[\Rightarrow \text{Combined product gives access to insurance for a larger share of the population!}\]

- Cost reduction due to \textit{reduced adverse selection}!
  \[\Rightarrow \text{Combined product is attractive for people in bad health...}\]
Life-Care Tontine: semi-Markov model

\[ 1 p_{x_j}^{aa} \]

\[ 1 p_{x_j}^{ai} \]

\[ 1 p_{x_j}^{ad} = q_{x_j}^{(a)} \]

\[ 1 p_{x_j;z}^{id} = q_{x_j;z}^{(i)} \]

\[ z: \text{time spent in dependency.} \]
Modern Life-Care Tontine

We move in two steps:

1. **A natural, actuarial fair increase** in payments in dependency:
   Higher payments “compensated” by lower life expectancy.

2. The **increase** in dependency is fixed a priori. Any gains / deficits are shared within the pool of active individual (“morbidity credits”).

We use the notation $\alpha(T^{(a)})$ where $T^{(a)}$ is the time where the individual moves into dependency to account for the increase in payments: $b_j(t)$ as an active; $\alpha(T^{(a)}) \cdot b_j(t)$ as a dependent person.
Modern Life-Care Tontine

“Natural increase”: French mortality/disability data shows actuarially fair values for $a(T^{(a)})$:

Mortality credits of a dependent person depend on the death probability

$$q_{x_j+t-1}^{(i)} > q_{x_j+t-1}^{(a)}.$$
3. Modern Life-Care Tontine

- Graph showing the payoff for a 65-year-old cohort with different types of credits: mortality, morbidity, and fixed.

- Lines representing different states and payoff values over time.

- Key terms and notation:
  - $s_j(t)$: male mortality
  - $W_j(t)$: average payoff
  - $W_j(t)$: 95% confidence interval
  - Fixed payoff
  - Morbidity credits
  - Mortality credits
  - $T^{(a)}$
Discussion and conclusion

- It is beneficial to pool mortality and long-term care (morbidity) risks.
- We show how this scheme can be adapted to a life-care tontine introducing the concept of morbidity credits.
- The scheme allows to pool different age cohorts.
- It is fully-funded at all times, allowing individuals to later join the scheme!
Thank you!

Hieber, P., & Lucas, N. (2022). *Modern life-care tontines*. **ASTIN Bulletin: The Journal of the IAA**, 52(2), 563-589.

Denuit, M., Hieber, P., & Robert, C. Y. (2022). *Mortality credits within large survivor funds*. **ASTIN Bulletin: The Journal of the IAA**, in press.

Denuit, M. (2019). *Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines*. **ASTIN Bulletin: The Journal of the IAA**, 49(3), 591-617.

Donnelly, C., Guillén, M., and Nielsen, J. P. (2014). *Bringing cost transparency to the life annuity market*. **Insurance: Mathematics and Economics**, 56, 14-27.

Milevsky, M. A., and Salisbury, T. S. (2015). *Optimal retirement income tontines*. **Insurance: Mathematics and Economics**, 64, 91-105.

Chen, A., Hieber, P., and Klein, J. K. (2019). *Tonuity: A novel individual-oriented retirement plan*. **ASTIN Bulletin: The Journal of the IAA**, 49(1), 5-30.
Definition (Fair distribution rule: mortality credits)

A fair distribution rule $\beta_j(X(t))$ satisfies:

- **Self-sufficiency property:** $\sum_{j \in \mathcal{L}_{t-1}} \beta_j(X(t)) = X(t)$.

- **Positivity property:** $\beta_j(X(t)) \geq 0$.

- **Fairness property:**

$$
\mathbb{E}_{t-1} \left[ \beta_j(X(t)) \right] = \mathbb{E}_{t-1} \left[ 1_{j \in \mathcal{D}_t} \right] \cdot \mathcal{E}^{t-1} \delta_{s \in \mathcal{D}_t} c_j(t - 1),
$$

where $\mathbb{E}_t := \mathbb{E} [ \cdot | \mathcal{F}_t ]$ is an expectation conditional on the information $\mathcal{F}_t := \sigma(\mathcal{L}_t)$. 

\[ (6) \]
Example (Linear risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \frac{q_{x_j+t-1} \cdot c_j(t - 1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j+t-1} \cdot c_j(t - 1)} \cdot X(t). 
$$  \hspace{1cm} (7)

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018])

Example (Linear regression rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \mathbb{E}_{t-1}[X_j(t)] + \frac{\text{Cov}_{t-1}[X_j(t), X(t)]}{\text{Var}_{t-1}[X(t)]} (X(t) - \mathbb{E}_{t-1}[X(t)]).
$$ \hspace{1cm} (8)
Example (Conditional mean risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \mathbb{E}_{t-1}[X_j(t) \mid X(t)].
$$

(see, e.g., Denuit and Dhaene [2012], Denuit [2019])
Individual $j \in \mathcal{L}_t$’s time-$t$ account value is given by:

$$c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s ds} s_j(u).$$ (10)

How do we choose $s_j(u)$, $u = 1, 2, \ldots, \omega - x_j$?

For example, choose the average payoff to be constant, equal to $b_j > 0$:

$$\mathbb{E}_{t-1}[W_j(t) \mid j \in \mathcal{L}_t] = \mathbb{E}_{t-1}[\mathbbm{1}_{j \in \mathcal{L}_t} \cdot s_j(t) + \mathbbm{1}_{j \in \mathcal{L}_{t-1}} \cdot \beta_j(X(t)) \mid j \in \mathcal{L}_t]$$

$$= s_j(t) + \mathbb{E}_{t-1}[\beta_j(X(t))]$$

$$= s_j(t) + q_{x_j+t-1} e^{\int_{t-1}^t \delta_s ds} c_j(t-1) \overset{!}{=} b_j.$$ (11)

((13) is a system of equations backwards in time!)
Theorem (Backwards iteration)

If an individual $j \in \mathcal{L}_t$ aims for an average payoff $b_j(t)$, the fixed payoff is given by:

$$s_j(t) = \begin{cases} 
\frac{b_j(t)}{1+q_{\omega-1}}, & \text{for } t = \omega - x_j \\
\frac{b_j(t) - q_{x_j+t-1} \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s ds} s_j(u)}{1+q_{x_j+t-1}}, & \text{for } t = \omega - x_j - 1, \omega - x_j - 2, \ldots, 1
\end{cases}$$

(12)

We derive the individual’s account value as

$$c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s ds} s_j(u)$$

(13)

and the initial single premium as $c_j(0)$. 