$K^+$ production in $p-C$-collisions at a beam energy 1.2 GeV

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Abstract

The isobar model and the resonance model are applied for the first analysis of the subthreshold $K^+$-meson production in proton-carbon collisions, which was performed at GSI at an emission angle of 40 degrees and a bombarding energy of 1.2 GeV. In this study, we focus on the role of the secondary processes $\pi N \rightarrow K^+ Y$ in normal nuclear matter density. It turns out that the present approach can reproduce very well both the $\pi^+$- and $K^+$- meson spectra. It is also demonstrated that the different kinds of descriptions for the $\pi N \rightarrow K^+ \Lambda$ reactions substantially differentiate the calculated results for the $pA \rightarrow K^+ X$ differential cross sections.

It has been proposed for more than one decade that $K^+$-mesons are one of the most promising probes for hot and dense nuclear matter formed in heavy ion collisions [1]. However, in refs. [2, 3, 4, 5] a quite different scenario for kaon production in heavy ion collisions was suggested that kaons may be produced mainly through secondary processes in the pure hadronic phase with nuclear matter density (0.17$fm^{-3}$) obtained in nucleus-nucleus collisions, which is far away from the highly compressed and hot phase.

In order to clarify these arguments, the $KaoS$ collaboration of GSI have measured recently the spectra of $\pi^+$- and $K^+$- mesons in proton-nucleus collisions at an angle of 40 degrees and for a beam energy of 1.2 GeV [6]. It was found that the $K^+$-differential cross sections are substantially underestimated according to the calculations based on only the first step processes $pN \rightarrow K^+ \Lambda N$, even together with an inclusion of the high momentum component of the nuclear spectral function [7]. Thus, it has been claimed that the kaons might be produced by either the secondary $\Delta - N$- or the multi-nucleon collisions.

One of the purposes of the present study is to analyse those data theoretically for the first time, with particular focusing on the role of the secondary process.

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\( \pi N \rightarrow KY \) in the normal hadronic phase. We will compare our results with the GSI data for the spectra of \( K^+ \)-mesons as well as those for \( \pi^+ \)-mesons, which will turn out to show the validity of the present approach.

For the calculations of the \( \pi^+ \)-meson spectra at a beam energy of 1.2 GeV, we adopt the isobar model with the assumptions that the \( s \)-wave \( \Delta \)-production is dominant for the \( pN \rightarrow \Delta N \) reaction, and that the \( \Delta \)'s decay isotropically in the \( \Delta \)-isobar rest frame at the \( \Delta \)-isobar rest system. As was demonstrated by Cugnon et al. [8], the isobar model can reproduce the experimental data on pion production reasonably well for beam energies below 2 GeV/u, both in proton-nucleus and nucleus-nucleus collisions. We use the \( \Delta \)-production cross section as,

\[
\sigma(pN \rightarrow \Delta N) = \frac{Z}{A} \sigma_{in}(pp) + \frac{A - Z}{A} \sigma_{in}(pn),
\]

where \( A \) and \( Z \) denote the target mass and charge, respectively. The proton-proton and proton-neutron inelastic cross sections \( \sigma_{in}(pp) \) and \( \sigma_{in}(pn) \) are taken from ref. [9].

In order to take into account the internal momentum of the target, we average over the \( \Delta \)-differential cross section by convoluting the spectral function \( \Phi(q) \) of the target nucleus. Then, the differential pion production cross section in proton-nucleus collisions can be calculated as

\[
E_\pi \frac{d^3 \sigma}{d^3 p_\pi}(pA \rightarrow \pi X) = N_{eff}(p_\pi) \int \Phi(q) E_\Delta \frac{d^3 \sigma(\sqrt{s}, q)}{d^3 p_\Delta} \text{Lo}_{\Delta \rightarrow \pi N(\sqrt{s}, p_\pi)} \, dq, \tag{2}
\]

where \( E_\Delta \frac{d^3 \sigma(\sqrt{s}, q)}{d^3 p_\Delta} \) stands for the \( \Delta \)-differential cross section in the center-of-mass system of the incident proton and the target nucleus carrying the momentum \( q \) and the corresponding invariant mass \( \sqrt{s} \).

In eq. (2) \( \text{Lo}_{\Delta \rightarrow \pi N(\sqrt{s}, p_\pi)} \) stands for the Lorentz transformation of the pion momentum \( p_\pi \) from the \( \Delta \)-rest system to the laboratory system.

Here, the spectral function of the target carbon \( \Phi(q) \) necessary for the present calculations is parametrized by the experimental data \((e, e' C)\) and \((\gamma, \gamma' C)\) scatterings as follows [10];

\[
\Phi(q) = \frac{1}{\alpha^3} \exp \left( -\frac{q^2}{2\alpha^2} \right),
\]

where \( q = |\vec{q}| \) and the slope parameter \( \alpha = 82 \text{ MeV}/c \). It is assumed that the distribution of the vector \( q \) is isotropic.

The factor \( N_{eff}(p_\pi) \) appearing in eq. (2) is accounting for the mass number \( A \) dependence of the \( \Delta \) production and the final stage of the pion absorption. It should be mentioned that the energy dependence of the effective collision number \( N_{eff}(p_\pi) \) must be taken into account correctly, because there is a strong dependence of the \( \pi N \) cross sections \( \sigma(\pi N) \) on the produced pion momentum \( p_\pi \) [1], which can be understood clearly from Fig. 1a). Based on the approach used in refs. [11, 12], this \( N_{eff}(p_\pi) \) can be calculated as

\[
N_{eff}(p_\pi) = \int_0^{+\infty} db \int_{-\infty}^{+\infty} \rho(b, z)dz \int_0^{2\pi} d\phi \times \left[ \exp \left( -\sigma_{tot}(pN) \int_{-\infty}^{z} \rho(b, \xi)d\xi - \sigma_{tot}(\pi N, p_\pi) \int_0^{+\infty} \rho(\mathbf{r}[\zeta])d\zeta \right) \right]. \tag{4}
\]
where \( \rho(\mathbf{r}[\zeta]) \) is the one-particle density distribution which is taken as a harmonic oscillator \([13]\) and normalized to the target mass number \( A \). \( \mathbf{r}[\zeta] \) appearing in eq.(4) is defined as

\[
\mathbf{r}[\zeta] = \mathbf{r}_0(b, 0, z) + \zeta \hat{e},
\]

where \( b \) and \( z \) stand for the impact parameter and the \( z \)-component of the coordinate along the beam-axis, respectively. \( \hat{e} \) is a unit vector in coordinate space defined by \((\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\) with \( \theta \) being the detection angle. In order to obtain the collision number \( N_{\text{eff}} \) for the other processes, one may just replace the \( \sigma_{\text{tot}}(\pi N) \) in eq.(4) to the corresponding cross sections.

The effective collision number \((4)\) calculated for the carbon target is shown in Fig. 1b) as a function of total cross section \( \sigma \). Note that in the calculations performed in refs. \([3, 5]\), the factor \( N_{\text{eff}} \) was obtained by the Glauber approach, and parametrized by

\[
N_{\text{eff}} = A^{0.74 \pm 0.01}.
\]

Because the total cross section for the \( K^+N \)-interaction is very small \( \sigma(KN) \simeq 12 mb \), one can treat it pertubatively and can get the effective collision number \( N_{\text{eff}} \) for the curve plotted in Fig. 1b). Similar argument holds also for \( K^+ \)-meson production through the secondary \( \pi N \)-collisions, because the reaction threshold corresponds to the pion momenta is above 1 GeV/c and \( \sigma(\pi N) \) is also small.

However, for the calculations of the \( \pi^+ \)-meson spectra in proton-nucleus and nucleus-nucleus collisions one should take into account the pion momentum \( p_\pi \) dependence of \( N_{\text{eff}} \), since the spectra reflect directly the contributions of pion absorption.

At first we will discuss the results for \( \pi^+ \)-meson production. The solid line in Fig. 2a) shows the calculated spectrum for \( \pi^+ \)-mesons in proton-carbon collisions obtained by using eq. (2). The experimental results are represented by the circles. The dashed line shows the results obtained by the phase-space calculations for the \( \pi N \rightarrow NN\pi^+ \) reactions and averaged over by convoluting the spectral function (eq. (3)) and normalized to \( N_{\text{eff}} \). Here the \( \pi^+ \) production cross section is given by VerWest and Arndt \([14]\) as follows;

\[
\sigma(pN \rightarrow NN\pi^+) = \frac{Z}{A} \sigma(pp \rightarrow pn\pi^+) + \frac{A-Z}{A} \sigma(pm \rightarrow nn\pi^+).
\]

One can see that the isobar model fits the \( \pi^+ \)-meson spectrum quite satisfactory. Because the high momentum tail of the pion spectrum is reproduced pretty well and it implies that sufficient amount of high momentum pions exist, thus one can expect that \( K^+ \)-meson production through the \( \pi N \rightarrow K^+Y \) reactions is quite possible.

Next, we will discuss about \( K^+ \)-meson production. The differential kaon production cross section in proton-nucleus collisions is given by

\[
E_K \frac{d^3\sigma}{d^3p_K}(pA \rightarrow K^+X) = \int \frac{\xi(p_\pi)}{\sigma_{\text{tot}}(pN)} \left[ E_{\pi} \frac{d^3\sigma}{d^3p_\pi} \right] \Phi(q) E_K \frac{d^3\sigma(\sqrt{s})}{d^3p_K} d\mathbf{q},
\]

where the differential pion production cross section \( E_{\pi} \frac{d^3\sigma}{d^3p_\pi} \) is taken the same form as that is given by eq. (2), and \( E_K \frac{d^3\sigma}{d^3p_K} \) stands for the differential kaon production cross sections for the \( \pi N \rightarrow K^+Y \) reactions in the center-of-mass of the pion and the nucleon with the corresponding invariant mass \( \sqrt{s} \) of the system. Here,
we may take into account only the $\Lambda$-hyperon production reaction channel, which is reasonable for the small values of $\sqrt{s}$.

In eq. (8) the factor $\xi(p_\pi)$ stands for the probability of the $\pi$-meson interaction with nucleons inside the target, and it is given by

$$\xi(p_\pi) = 1 - \frac{N_{\text{eff}}(p_\pi)}{N_{\text{eff}}(\sigma(\pi N) = 0)}.$$ (9)

It was found in ref. [15] that the angular distribution of $K^+$-mesons induced by the $\pi N \to K^+\Lambda$ reactions is not isotropical at all even at energies close to the threshold. This does not have a large influence on the calculated results for the total kaon production cross sections [3, 5], however, this anisotropic effects should be taken into account adequately when the calculations for the differential $K^+$-meson spectrum are performed.

The angular spectra of $K^+$-mesons induced by the reaction $\pi^0p \to K^+\Lambda$ in the center-of-mass system are given in Fig. 3. The results represented by the circles, squares, triangles and stars are obtained by adopting the resonance model [16, 17], while the solid-, dashed-, dotted- and dashed-dotted- lines represent the results obtained by the use of the parametrization of refs. [15, 18] for the range of $\sqrt{s}$ from 1.62 to 1.7 GeV. The explicit parametrization is given by,

$$\frac{d\sigma}{d\Omega} \sim 1 + \eta(\sqrt{s})\cos^2\theta_K,$$ (10)

with $\eta(\sqrt{s}) = 10.9\sqrt{s} - 17.6$ and $\eta = 1$, corresponding to the ranges $\sqrt{s} < 1.7$ GeV and $\sqrt{s} > 1.7$ GeV, respectively.

It is worth noting that the resonance model shows a strong anisotropy for $K^+$-meson production, whereas the function represented by eq. (10) is quite flat as a function of $\cos\theta$ with $\theta$ being the angle between the kaon and the pion momenta.

Now we discuss the results for $K^+$-mesons. The results are given in Fig. 2b) for the kaon spectra in proton-carbon collisions at a kaon angle of 40 degrees and a proton bombarding energy of 1.2 GeV by applying eq. (8). The solid line shows the results obtained by adopting the resonance model, while the dashed line shows the results obtained with eq. (10). The experimental data are given by the circles. The calculations preformed by adopting the resonance model can reproduce the experimental data quite satisfactorily. This fact indicates that the importance of the precise description for the $\pi N \to K\Lambda$ reactions. Furthermore, this leads support to the assumption in the present calculations, that $K^+$-mesons may be produced in a pure hadronic phase under the normal conditions.

To summarize, we have performed the first theoretical analysis for $K^+$-meson production in proton-carbon collisions which is measured at GSI [5]. In the calculations, the isobar model and the resonance model were adopted with normal nuclear matter conditions. By taking into account the experimental spectral function for the carbon target, we could reproduce the experimental spectra quite well for both the $\pi^+$- and $K^+$- mesons. It was demonstrated that the secondary $\pi N$-collision processes give dominant contributions for $K^+$-meson production in proton-nucleus collisions below the reaction threshold in free space as was discussed by Cassing et al. [19]. The present results support the scenario that the $K^+$-mesons may be produced mainly in the pure hadronic phase under the normal conditions through the secondary processes $\pi N \to KY$. Furthermore, it turned out that the
angular distribution of the $K^+$-meson spectrum in proton-nucleus collisions is quite sensitive to the $\pi N \rightarrow K\Lambda$ differential cross sections, and can serve to differentiate between the two models. We therefore urge the experimental group at GSI to measure such an angular distribution of the kaons. For this purpose we give in figure 4 the angular distribution of the kaons integrated from the kaon momentum $p_K = 0.5$ to $0.7$ Gev/c for the reaction $^{12}\text{C}(p, K^+)$ at 1.2 GeV proton bombarding energy.

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Figure captions

Figure 1: Total $\pi^+ p$ cross section as a function of pion momentum $p_\pi$ a), and the calculated effective collision number for $^{12}C$ as a function of total cross section $\sigma$ b).

Figure 2: The spectra of $\pi^+$- and $K^+$- mesons in proton-carbon collisions at an angle of 40 degrees and a beam energy of 1.2 GeV. The circles in Figs. stand for the experimental data. The solid- and the dashed- lines in Fig. a) show the results obtained by the isobar model and the phase space distributions, respectively. The solid- and the dashed- lines in Fig. b) represent the results obtained by adopting the resonance model [16] and eq. (10), respectively.

Figure 3: The angular spectra of $K^+$-mesons for the $\pi^0 p \rightarrow K^+\Lambda$ reaction. The different curves are plotted as a function of the total center of mass energy $\sqrt{s}$. They represent the following theoretical results: The circles and the solid line give the results for $\sqrt{s} = 1.62$ GeV, the squares and the dashed line for $\sqrt{s} = 1.64$ GeV, the triangles and the dotted line for $\sqrt{s} = 1.66$ GeV, and the stars and the dashed-dotted line for $\sqrt{s} = 1.70$ GeV, respectively. Here, the circles, squares, triangles and stars show the results obtained by applying the resonance model, while the solid-, dashed-, dotted- and dashed-dotted- lines show the results obtained with eq. (10).

Figure 4: The angular distribution of kaons with the experimental data for the $^{12}C(p, K^+)$ reaction at 1.2 GeV proton bombarding energy. The results shown in figure are obtained by integrating the kaon momentum from $p_K = 0.5$ to 0.7 GeV/c. The solid- and the dashed- lines show the results obtained by adopting the resonance model [16] and eq. (10), respectively.
