Simulation and Experimental Studies of MPC for Level Control of Modified Quadruple Tank System

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Abstract. Model Predictive Control (MPC) has been a powerful control strategy as it has already been proved with simulation and experimental studies. In this study, MPC has been explored for the level control of a nonlinear benchmark quadruple tank system. MPC has been applied as a real-time control strategy for set point control and disturbance rejection. Rigorous simulations and experiments have been done and fairly compared with those from the use of decentralized PI controller. To validate the results, real-time experimental studies have also been carried out for two possible configurations. The application offers a new look at the performance of the MPC for complex systems control.

1. Introduction

Mainly industrial real practices are multivariable in nature and have multi inputs and multi outputs. Such systems are likely to have interactions among process variables and control signals [1]. Because of these interactions among the variables, it is complex to design appropriate controller for multivariable systems. To overcome this issue a number of methods have been proposed and accepted in order to control the multivariable and MIMO systems, e.g., Relative Gain Array (RGA), decentralized techniques, decoupled control [1-5], mainly because these methods exhibit flexibility in operation, simplified design and tuning to improve the performance. The advantage of these methods lies in the fact that simpler linear SISO theory and methods can be applied on complex MIMO systems [6-7].

Decentralized control is simplest technique to deal MIMO systems in which a control for every input-output pair is designed as like SISO approach [7]. However, this mechanism is efficient only in presence of weak interactions between the loops [8]. If the system is highly interacted, a small change in the set point in one loop significantly affects the output in the other loop. Consequently, response may become unacceptable. Whereas in decoupling control strategy, the coupling between loops are considered and controller compensates the interactions which yields non-interacting SISO systems, where the set-point changes affect only the desired control variables [8]. However, applicability of these approximations depends on the neglected interaction dynamics, which can be viewed as modeling errors. The advantage of applying SISO methods on MIMO systems by using aforesaid methods, is that the analysis becomes easier, but one must compromise with the system dynamics which in many cases may change the system behavior. Moreover, the actual performance of the system may become significantly different from that of the expected one [9-10]. Despite the benefits of these methods, they cannot suppress the interactions among the variables in MIMO systems [10]. To overcome these limitations advanced controller structures are developed in which Model Predictive Controllers (MPC) are most popular because it is capable of handling multivariable systems online more effectively, while satisfying the constraints [11]. This has been the motivation for the author to use MPC. MPC is a collection of methods for optimal control, characterized by using a model to predict the behavior of the system being controlled and optimize control over a moving time horizon...
The essence of MPC is to optimize forecasts of future behavior. The forecasting is accomplished with a system model and therefore the model is the essential element of the strategy.

In the present work, real-time experimental and simulation studies have been carried out on a benchmark control problem of Quadruple Tank System (QTS) and its modified version. The results are fairly compared with a decentralized Proportional-Integral (PI) controller. The rest of the paper is organized as follows, Section 2 illustrates the system dynamics, Section 3 describes the fundamentals and schematic of MPC, in section 4 results of experimental and simulation studies have been shown and fairly compared and finally conclusions are presented.

2. System description

The water level control of QTS is a challenging benchmark control problem owing to its multivariable nature, having well-built interaction among the variables, extremely nonlinear and non-minimum phase characteristics. QTS consists of four interconnected tanks and two pumps. The controlled inputs are voltages to the pumps and levels of lower two tanks are considered outputs. The outflows of every pump are separated into two tanks by using valves 1& 2. Pump-1 feeds Tank-1 & 4 whereas pump-2 feeds Tank-2&3. Tanks 3 & 4 discharge into Tanks-1 & 2 respectively while Tanks-1 & 2 discharges straight into the reservoir placed at the base, as given in Fig.1(a). The main objective is here to control the level of lower two tanks with a minimum mean squared error (MSE).

To test performance of MPC controller, author modified the structure of QTS by introducing an additional inter-connection valve between lower two tanks, as shown in Fig.1(b), which increases the complexity of system. This configuration is considered as Modified Quadruple Tank System (mQTS).

![Figure 1: Schematic of Quadruple Tank System](image)

2.1. Mathematical Modeling for simulation study

2.1.1. Mathematical modeling of QTS. The dynamics of the system are described by the following equations [9].
System parameters are tabulated in Table 1.

| Symbol | Parameters                      | Nominal values       |
|--------|---------------------------------|----------------------|
| Ai     | Area of i^{th} tank in cm^2 i=1,2,3,4 | 28,32,28,32          |
| ai     | Area of drain in cm^2 i=1,2,3,4    | 0.071,0.057,0.071,0.057 |
| a12    | Area of additional interconnection cm^2 | 0.064                |
| hi     | Maximum height if i^{th} tank in cm, i=1,2,3,4 | 25                   |
| g      | Gravitational constant           | 981                  |
| ki     | Valve proportionality constant   | 3.14, 3.15           |
| ui     | Voltage ranges of i^{th} control valve | [0-5 volts]         |

2.1.2. Mathematical modeling of mQTS. As there an interaction has been introduced in between Tank-1 and Tank-2 only, therefore the dynamics of Tank-3 and Tank-4 remains the same as of QTS and dynamics of Tank-1 and Tank-2 are modified as below:

\[
\frac{dh_i}{dt} = \gamma_i k_i u_i + a_i \sqrt{2gh_i} + a_i \frac{\sqrt{2gh_i}}{A_i} \\
\frac{dh_2}{dt} = \gamma_2 k_2 u_2 + a_2 \frac{\sqrt{2gh_2}}{A_2} - \frac{a_1 \sqrt{2gh_1}}{A_1} \text{sign}(h_1 - h_2) - \frac{a_1 \sqrt{2gh_1}}{A_1} \\
\frac{dh_3}{dt} = \frac{(1-\gamma_2)k_2 u_2}{A_2} - \frac{a_1 \sqrt{2gh_1}}{A_1} \\
\frac{dh_4}{dt} = \frac{(1-\gamma_1)k_1 u_1}{A_1} - \frac{a_1 \sqrt{2gh_1}}{A_1}
\]

(1)

2.1.3. Linearization of QTS and mQTS. If the nonlinear process is operated near a specified operating point then linearized model of the process may be sufficiently accurate. Using Taylor series and truncating after first order terms a linear approximation of the differential equations can be easily obtained. The linearization is performed around the normal steady-state points. Eq. (1) for QTS can be expressed in state-space model.

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

(3)
where

\[
A = \begin{bmatrix}
-a_1 \sqrt{\frac{g}{2h_{i0}}} & 0 & a_2 \sqrt{\frac{g}{2h_{i0}}} & 0 \\
0 & -a_1 \sqrt{\frac{g}{2h_{i0}}} & 0 & a_4 \sqrt{\frac{g}{2h_{i0}}} \\
0 & 0 & -a_1 \sqrt{\frac{g}{2h_{i0}}} & 0 \\
0 & 0 & 0 & -a_4 \sqrt{\frac{g}{2h_{i0}}}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\gamma_k \frac{k_i}{A_i} & 0 \\
0 & \gamma_k \frac{k_i}{A_i} \\
0 & (1-\gamma_k)k_i \\
(1-\gamma_k)k_i & 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix} k_v & 0 & 0 & 0 \end{bmatrix}, \quad D = [0] \quad \dot{x} = \begin{bmatrix} \frac{d\Delta h_1}{dt} & \frac{d\Delta h_2}{dt} & \frac{d\Delta h_3}{dt} & \frac{d\Delta h_4}{dt} \end{bmatrix}
\]

\[X = [\Delta h_1 \Delta h_2 \Delta h_3 \Delta h_4] \quad U = [\Delta u_1 \Delta u_2] \quad Y = [\Delta y_1 \Delta y_2]
\]

Similarly, A, B, C, D matrices for mQTS can also be derived and it is observed that the dynamics of mQTS differs from QTS only in the composition of matrix A. A matrix for mQTS is shown below.

\[
A = \begin{bmatrix}
\frac{a_1}{A_i} \sqrt{\frac{g}{2h_{i0}}} - \frac{a_2}{A_i} \sqrt{\frac{g}{2h_{i0}}} * \text{sign}(h_i - h_{i0}) & 0 & \frac{a_2}{A_i} \sqrt{\frac{g}{2h_{i0}}} * \text{sign}(h_i - h_{i0}) & 0 \\
0 & -\frac{a_1}{A_i} \sqrt{\frac{g}{2h_{i0}}} - \frac{a_4}{A_i} \sqrt{\frac{g}{2h_{i0}}} * \text{sign}(h_i - h_{i0}) & 0 & \frac{a_4}{A_i} \sqrt{\frac{g}{2h_{i0}}} \\
0 & 0 & -\frac{a_1}{A_i} \sqrt{\frac{g}{2h_{i0}}} & 0 \\
0 & 0 & 0 & -\frac{a_4}{A_i} \sqrt{\frac{g}{2h_{i0}}}
\end{bmatrix}
\]

3. Model Predictive Controller

Model predictive control represents a class of control algorithm that utilizes an explicit process model to forecast the expected response of a system. At each control horizon, algorithm optimizes future process behavior by calculating a sequence of expected manipulated variable manipulation. The first input in the optimal sequence is fed to the process, and the entire practice is repeated at succeeding control horizon. Ability to handle constraints, online process optimization, simplicity in design formulation are the major key factor that make it attractive tool to researchers and practitioners [17-20]. To design MPC it is mandate to define possible constraints, cost function, predictive model and actual process model. The control scheme is shown in Fig.2.
Let a state-space model of linear dynamic system

\[ X_{k+1} = AX_k + Bu_k \]  \hspace{1cm} (6)

\[ Y_k = DX_k \]  \hspace{1cm} (7)

for \( k = k + 1 \), \( Y_{k+1} = DX_{k+1} \)

Substituting the value of \( X_{k+1} \) from Eq. (6), we get,

\[ Y_{k+1} = D(AX_k + Bu_k) \]  \hspace{1cm} (8)

\[ Y_{k+1} = DAX_k + DBu_k \]  \hspace{1cm} (9)

for \( k = k + 2 \), \( Y_{k+2} = D(AX_{k+1} + Bu_{k+1}) \)

\[ Y_{k+2} = DA^2X_k + DBu_k + DBu_{k+1} \]  \hspace{1cm} (10)

for \( k = k + 3 \), \( Y_{k+3} = D(AX_{k+2} + Bu_{k+2}) \)

\[ Y_{k+3} = DA^3X_k + DA^2Bu_k + DABu_{k+1} + DBu_{k+2} \]  \hspace{1cm} (11)

\[
\begin{bmatrix}
  Y_1 \\
  Y_{k+1} \\
  Y_{k+2} \\
  Y_{k+3}
\end{bmatrix} =
\begin{bmatrix}
  D & 0 & 0 & 0 \\
  DA & DB & 0 & 0 \\
  DA^2 & DAB & DB & 0 \\
  DA^3 & DA^2B & DAB & DB
\end{bmatrix}
\begin{bmatrix}
  u_k \\
  u_{k+1} \\
  u_{k+2} \\
  u_{k+3}
\end{bmatrix}
\]  \hspace{1cm} (12)

where,

\( Y_{k|4} \) = output variables matrix, \( u_{k|3} \) = input variables matrix

\( O_4 \) = extended observability matrix for pair \((D, A)\)

\( H^T_4 \) = lower block triangular Toeplitz matrix for \( D, A, B \) matrices. Subsequently Eq. (12) becomes

\[ Y_{k|4} = O_4X_k + H^T_4u_{k|3} \]  \hspace{1cm} (13)

The above equation when the prediction horizon is equal to L becomes,

\[ Y_{k|L} = O_LX_k + H^T_Lu_{k+L-L} \]  \hspace{1cm} (14)

Eq. (14) when \( k = k + 1 \), becomes

\[ Y_{k+1|L} = O_LX_{k+1} + H^T_Lu_{k+L-L} \]  \hspace{1cm} (15)

From Eq. (7), \( X_{k+1} = AX_k + Bu_k \), substituting in the above equation,

\[ Y_{k+1|L} = O_LAx_k + O_LBu_k + H^T_Lu_{k+L-L} \]  \hspace{1cm} (16)

Writing the Eq. (16) in matrix form gives,

\[ Y_{k+1|L} = O_LAx_k + (O_LB, H^T_L) \begin{bmatrix}
  u_k \\
  u_{k+L-L}
\end{bmatrix} \]  \hspace{1cm} (17)

where, \( p_L = O_LAx_kF_L, F_L = (O_LB, H^T_L) \), \( u_{k|L} = \begin{bmatrix}
  u_k \\
  u_{k+L-L}
\end{bmatrix} \)

\[ Y_{k+1|L} = p_L + F_Lu_{k|L} \]

Eq. (17) is the prediction model and MPC calculates the predicted output by using this model. Substituting prediction model into the cost function,
where, 

\[
\begin{align*}
J_k &= (Y_{k+1L} - r_{k+1L})^T Q (Y_{k+1L} - r_{k+1L}) + u_{k|L}^T Ru_{k|L} \\
J_k &= (p_L + F_r u_{k|L} - r_{k+1L})^T Q (p_L + F_r u_{k|L} - r_{k+1L}) + u_{k|L}^T Ru_{k|L} \\
J_k &= u_{k|L}^T (F_r^T Q F_r + R) u_{k|L} + 2F_r^T Q (p_L - r_{k+1L}) + u_{k|L}^T Ru_{k|L} \\
J_k &= u_{k|L}^T H u_{k|L} + 2F_r^T Q (p_L - r_{k+1L}) + (p_L - r_{k+1L})^T Q (p_L - r_{k+1L}) \\
J_0 &= (p_L - r_{k+1L})^T Q (p_L - r_{k+1L})
\end{align*}
\] 

Minimizing the cost function with respect to \(u_{k|L}\),

\[
\frac{\delta J_k}{\delta u_{k|L}} = u_{k|L}^T H u_{k|L} + 2f^T u_{k|L} + J_0
\]

\[
u^*_{k|L} = -H^{-1} f
\]  

The Eq. (22) becomes the future optimal control, where simply the first element of the vector is used for further process..

4. Experimental and simulation results
A simple negative feedback control topology has been implemented which minimizes the objective function as in Eq. (18) for both set point changes and disturbance rejection. To test the effectiveness of MPC, simulation studies have been carried out on MATLAB R2017a with Intel Pentium dual CPU E2220 @ 2.40 GHz.

4.1. Simulation results & discussion. Step responses of Tank-1 & 2 are shown in Fig. 3 & 4. The sampling period is 0.1 second and the length of the prediction horizon for MPC is chosen 15. Control efforts are also shown. The results are fairly compared with decentralized PI controllers whose parameters are chosen by trial and error method e.g. Kp1 =11, Ti1= 2.35 sec-1, Kp2 =11 and Ti2 = 2.5sec-1. From the simulation results it is clear that while satisfying input constraint (0 to 5 V), MPC is capable to track the set point changes efficiently and offers lesser steady state errors, peak overshoot and settling time. The Mean Squared Error (MSE) of 0.0057 and 0.0022 has been calculated for PI and MPC based scheme respectively which shows MPC has 61.4 % lesser MSE than that of PI controller.
With the same objective and simulation parameters as for QTS, the whole scheme has been simulated for mQTS.

The step responses of Tank-1, Tank-2 and corresponding control efforts are shown in Fig. 5 & 6. From the simulation results it is clear that with the permissible inputs (0 to 5 V), the MPC is able to track the set point changes faster. MSE of 0.0056 and 0.0019 have been calculated for PI and MPC respectively which points towards the fact that MPC has 66.07% lesser MSE as compared to PI.

4.2. Experimental results & discussion. The experimental study has been carried out at Control laboratory of National Institute of Technology, Calicut, India. The real-time implementation of controller was carried out on LabVIEW platform which was interfaced by LTD_DAQ_013. The snapshot of QTS is depicted in Fig.7.

The parameters of PI controller and MPC have been chosen as adopted in simulation studies. Experiments for both set-point changes and disturbance rejection have been carried out. The responses
of PI controller and MPC are shown in Fig. 8 to 11. From the experimental results, it is noted that MPC is able to track set-point changes more smoothly. For the PI control scheme, the disturbances have been introduced by opening of outlet valves (more than 25% of the previous opening) for Tank-1 at instant of 920 second for the duration of 40 seconds and for Tank-2 at instant of 1290 second for the duration of 50 seconds. Whereas in MPC scheme disturbances have been introduced by closing the outlet valve for Tank-2 and by opening the valve for Tank-1, for the period of 30 secs starting at instant of 850 seconds and 750 seconds in Tank-1 & 2 respectively.

![Waveform Chart](image1.png)

**Fig 8.** Experimental results of PI controller for QTS

![Waveform Chart](image2.png)

**Fig 9.** Experimental results of MPC controller for QTS

It is important to note that MPC shows better results in trajectory tracking and disturbance rejection because the changes in level makes changes in dynamics of process, and a equivalent linearized model of the tank system can be utilized in real time by the MPC. However, the PI controller needs to have its parameters adjusted for optimal performance for every distinct case or when the dynamics of the process are changed by the changes in level.

Furthermore, this study has been extended to mQTS. The experiment results of PI controllers and MPC are shown in Fig. 10 & 11. All results are anticipated as discussed for QTS. In spite of strong coupling between Tank-1 and Tank-2, MPC is still capable of tracking the set point changes rapidly as compared to PI controller because PI controllers are not able to handle the strong coupling among the variables as shown in Fig. 10 at 425 seconds. In PI controller based scheme, disturbances are introduced at instant of 550 second for the duration of 30 seconds in Tank-1 and at the instant of 560 seconds for Tank-2 by opening the outlet valves of both tanks whereas the disturbances are introduced at instant of 620 second for the duration of 30 seconds in Tank-1 and at the instant of 1150 seconds for Tank-2 by opening the outlet valves of both tanks in MPC based scheme.
From the results, it is clearly observed that whenever a change in the set points of either of tanks occurs, the other tank gets significantly affected due to strong coupling among the variables. This effect is remarkably reduced in case of MPC.

5. Conclusions
In this work MPC based scheme has been formulated and implemented for nonlinear MIMO systems. Firstly, mathematical model of QTS has been derived and then it’s linear state space model is developed. The illustrated scheme has been simulated for level control of benchmark problem of QTS. Thereafter, to test more effectiveness of the scheme, author has introduced complexity into the QTS by connecting a valve in between lower two tanks which leads to a strong interaction among the variables. The scheme has been simulated for this modified system referred as mQTS. The simulation results have been validated with real time experimental studies. The simulation results are compared with decentralized PI controller and it has been established that MPC is capable to track the set-points faster and also able to reject disturbances efficiently while satisfying the input constraints. MSE for MPC based scheme are found to be significantly lower than with PI controller. MPC offers an efficient technique because of it’s ease use for wide application in process industry. To deal with the more complex situations, MPC strategy can be improved by using nonlinear process models and multiple objective functions.

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