Probability of detection model for the non-destructive inspection of steam generator tubes of PWRs

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Abstract. This study proposes a probability of detection (POD) model to discuss the capability of non-destructive testing methods for the detection of stress corrosion cracks appearing in the steam generator tubes of pressurized water reactors. Three-dimensional finite element simulations were conducted to evaluate eddy current signals due to stress corrosion cracks. The simulations consider an absolute type pancake probe and model a stress corrosion crack as a region with a certain electrical conductivity inside to account for eddy currents flowing across a flaw. The probabilistic nature of a non-destructive test is simulated by varying the electrical conductivity of the modelled stress corrosion cracking. A two-dimensional POD model, which provides the POD as a function of the depth and length of a flaw, is presented together with a conventional POD model characterizing a flaw using a single parameter. The effect of the number of the samples on the PODs is also discussed.

1. Introduction
At the present, the emergence of flaws accompanying plant ageing is one of the most important issues in the nuclear industry. Harmful flaws must be detected in their early stages using a periodical non-destructive inspection in order to take suitable actions following proper rules, such as ASME Sec. XI. In contrast, not all flaws are actually harmful. For example, a tiny flaw in a thick structure would not affect the integrity of the structure at all. This indicates the importance of quantitatively evaluating the capability of non-destructive testing methods not on the basis of a minimum detectable flaw size but depending on whether or not they can surely detect flaws that must be detected. Aerospace industries have demonstrated the effectiveness of a concept of probability of detection (POD) to evaluate the capability of non-destructive testing methods from this point of view[1,2], and recent studies have pointed out that the concept of POD would be effective in nuclear industries[3]. However, a quite limited number of studies have actually evaluated non-destructive testing methods for nuclear industries using POD[4].

One of the largest reasons for this is conventional POD characterized a flaw using a single parameter. Conventional PODs used in aerospace industries have targeted mainly fatigue cracks appearing at the rivet holes of airplanes. It would be reasonable to characterize the fatigue cracks using a single parameter because they would have almost the same aspect ratio. In contrast, various flaws would appear in various components in nuclear power plants, which indicates the difficulty in characterizing a flaw using a single parameter. To address this issue, earlier studies of the authors have proposed how to construct POD with multiple flaw parameters[5,6] without postulating a closed-form of signal response or a constant variance[7,8]. The studies demonstrated its effectiveness by evaluating
the POD from eddy current signals. However, what they considered was a relatively simple problem of detecting artificial slits introduced in austenitic stainless steel plates.

This study discusses the application of the POD to more practical inspection in nuclear industries, more specifically eddy current inspection of stress corrosion cracks appearing at the outer surface of steam generator tubes of pressurized water reactors. Signals due to the stress corrosion cracks are obtained by three-dimensional finite element simulations on the basis of earlier studies discussing how to model stress corrosion cracks from the viewpoint of electromagnetic non-destructive testing. This study also describes a conventional POD model characterizing a flaw using a single parameter to discuss the advantage of a POD model with multiple parameters. The number of samples necessary to construct a reliable POD is also evaluated.

2. Eddy current signals

Figure 1 illustrates the configuration of the numerical simulations this study conducted to obtain eddy current signals. An absolute pancake probe scans the inner surface of a steam generator tube to gather signals due to an axial flaw appearing on the inner surface of the tube. The probe has a height, inner and outer diameters of 0.8, 1.0 and 3.2 mm, respectively. The exciting frequency and lift-off are 100 kHz and 1 mm, respectively. The thickness, conductivity and relative permeability of the tube are 1.27 mm, 1 MS/m, and 1, respectively.

The simulations calculated signals due to 96 rectangular slits and those due to 96 stress corrosion cracks, which would be sufficient to discuss the dependency of the POD model on the number of flaws because conventional POD models requires 40 or 60 flaws to obtain reliable POD. Parameters used in the simulations are listed in Table 1. The rectangular slits were modeled as air region. In contrast, the stress corrosion cracks were modeled as a rectangular region with a constant width and uniform conductivity inside. The conductivity was randomly set so that it followed a normal distribution with a mean of 0.1 MS/m and standard deviation of 0.03 MS/m on the basis of earlier studies discussing how to model stress corrosion cracks in finite element simulations [9,10].

The simulations were conducted using finite element program on the basis of A-ϕ formulation and FEM-BEM coupling discretization [11]. All signals were calibrated so that the maximum signal due to an artificial slit with a length of 20 mm and a depth of 20% tube wall thickness becomes 1.

![Figure 1. Configuration of numerical simulation](image)

Table 1. Parameters of flaws considered in the numerical simulations

| Parameters | Value |
|------------|-------|
| Width      | 0.2 [mm] |
| Length     | 2, 4, 6, 8, 10, 12, 16, 20 [mm] |
| Depth      | 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 70, 80 [%wt*] |

*wt: tube wall thickness (=1.27 mm)
3. POD models
This study evaluates POD using two approaches: a conventional $\hat{\alpha}$-$\alpha$ approach and two-dimensional POD approach. Both the approaches do not censor any signals; they adopted the same decision threshold, $\hat{\alpha}_{0.5}$=1.0, to compare their results clearly. When the number of stress corrosion cracks used to construct POD is less than 96, signals used were randomly selected among the 96 simulated ones. All simulations to construct POD were conducted on R using software developed by the author.

3.1. Conventional POD model [1,2]
This model assumes that the maximum signal due to a flaw, $\hat{\alpha}$, is represented as
\[
\ln(\hat{\alpha}) = \beta_1 + \beta_2 \ln(\alpha) + N(0, \sigma^2),
\]
where $\alpha$ and $N(0, \sigma^2)$ denote the depth of the flaw and a normal distribution with a mean of 0 and standard deviation of $\sigma$. After $\beta_1$, $\beta_2$, and $\sigma$ are estimated using a maximum likelihood analysis, POD($\alpha$) is calculated as the probability that $\hat{\alpha}$ exceeds $\alpha_{0.5}$ as
\[
POD(\alpha) = \Phi((\hat{\alpha} - \alpha_{0.5})/\sigma)
\]
where $\Phi$ stands for the cumulative distribution function of the standard normal distribution. Confidence bounds of POD($\alpha$) are calculated using a bootstrap analysis [12]. Note that only signals due to stress corrosion cracks are used in this conventional POD analysis, and those due to artificial slits do not contribute to the construction of the POD.

3.2. Two-dimensional POD model [5,6]
A two-dimensional POD model evaluates POD as a function of the depth and the length of a flaw. Specifically, this model assumes that the maximum signal due to a flaw, $\hat{\alpha}$, is represented as
\[
\hat{\alpha} = N(\mu_1, \sigma_1^2)a_{\text{sim}}(d, l) + N(\mu_2, \sigma_2^2),
\]
where $a_{\text{sim}}(d, l)$ stands for the maximum signal due to an artificial slit with a depth of $d$ and a length of $l$ obtained by numerical simulations. The two normal distributions represent the variation of signals relevant and irrelevant to the amplitude of the signals. The four unknown parameters, $\mu_1$, $\sigma_1$, $\mu_2$, and $\sigma_2$, are estimated on the basis of signals due to stress corrosion cracks using a maximum likelihood analysis. The probability that a flaw with a depth of $d$ and a length of $l$ is detected, POD($d, l$), is calculated as
\[
POD(d, l) = \Phi((\mu_1 a_{\text{sim}}(d, l) + \mu_2) - a_{\text{th}})/(a_{\text{sim}}(d, l)^2 \sigma_1^2 + \sigma_2^2)^{1/2}).
\]
The confidence bounds are calculated using a bootstrap analysis.

4. Results and discussion
Figure 2 shows the POD curves obtained using the conventional POD model. The figure demonstrates that the number of samples, denoted as $N_{\text{data}}$, has a large effect on the POD curve. When the number of samples is less than 40, $a_{0.95}$ becomes more than 80% wall thickness. This indicates that either the detectability is very poor or the POD does not represent the detectability reasonably. Figure 3 presents the result of the maximum likelihood analysis to correlate $\hat{\alpha}$ with $\alpha$ when $N_{\text{data}}$=96 and 10. The scatter of the signals is quite large, which clearly indicates the limitation in characterizing flaws using only one parameter.

Figures 4 and 5 present the results of the analyses using the two-dimensional POD model. Because of the difficulty in presenting a three-dimensional surface quantitatively, the figures show the contour line of the POD surface on a two-dimensional plane. The figures illustrate that the POD depends both on the depth and length of a flaw. Specifically, it is difficult to detect a short, deep flaw, and the length of a flaw has little effect on signals when a flaw is longer than a certain value. These results agree well with the general characteristics of eddy current testing. The figures also reveal that the two-dimensional POD model requires a much lower number of samples compared with the conventional...
POD model. The results of the maximum likelihood analyses when $N_{\text{data}}=96$ and 10 are shown in figures 6 and 7, respectively. The figures do not differ significantly in spite of the large difference in the number of the samples used, which supports that the two-dimensional POD model does not require so large number of samples.

![Figure 2](image1.png)

**Figure 2.** POD curve (solid line) with lower 95% confidence bound (broken line) obtained using the conventional POD model.

![Figure 3](image2.png)

**Figure 3.** The result of the maximum likelihood analysis to correlate $\hat{a}$ and $a$ in order to obtain the result shown in figure 2. The circles indicate the signals used to construct the POD curves. The black solid and broken lines represent the mean and mean±standard deviation of $a$. The horizontal line shows $a=a_{\text{th}}=1$. 
Figure 4. Contour lines of POD=0.1, 0.5 and 0.9 obtained using the two-dimensional POD model with different number of samples.

(a) $N_{\text{data}}=96$
(b) $N_{\text{data}}=60$
(c) $N_{\text{data}}=40$
(d) $N_{\text{data}}=30$
(e) $N_{\text{data}}=20$
(f) $N_{\text{data}}=10$

Figure 5. POD=0.9 (solid line) with their lower 95% confidence bounds (broken line) obtained using the two-dimensional POD model with different number of samples.

(a) $N_{\text{data}}=96$
(b) $N_{\text{data}}=60$
(c) $N_{\text{data}}=40$
(d) $N_{\text{data}}=30$
(e) $N_{\text{data}}=20$
(f) $N_{\text{data}}=10$
Figure 6. The results of the maximum likelihood analyses to correlate eddy current signals with flaw parameters in two-dimensional POD model with $N_{\text{data}}=96$. Results concerning flaw lengths of 6 and 10 mm are excluded to avoid redundancy. The horizontal line shows $a=a_{\text{th}}=1$.

Figure 7. The results of the maximum likelihood analyses to correlate eddy current signals with flaw parameters in two-dimensional POD model with $N_{\text{data}}=10$. The horizontal line shows $a=a_{\text{th}}=1$. 
5. Conclusion
This study evaluated the POD of the eddy current inspection of stress corrosion cracks appearing on the outer surface of steam generator tubes of PWRs. Finite element simulations were conducted to evaluate signals due to cracks; accounting for both the depth and length of a crack explicitly to evaluate POD quantitatively. Furthermore, combinational use of numerical simulations to evaluate the general dependency of signals on flaw parameters enables the construction of a reliable POD using fewer samples compared with a conventional POD analysis using only experimental data [13,14]. Whereas this study considered the eddy current inspection of steam generator tubes of PWR, the two-dimensional POD model used in this study should be applicable to discuss POD of any non-destructive testing inspections, regardless of the target is either PWR or BWR, so long as signals obtained by the inspections are given as a quantitative one and numerical simulations can calculate signals due to a flaw with a known profile.

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