Monte Carlo Simulation Approach to Determination of Oil Spill Domains at Port and Sea Water Areas

E. Dąbrowska & K. Kołowrocki
Gdynia Maritime University, Gdynia, Poland

ABSTRACT: Monte Carlo simulation method of oil spill domains determination based on the probabilistic approach to the solution of this problem is proposed. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed and its parameters are defined. The general stochastic model of oil spill domain movement for various hydro-meteorological conditions is described. Monte Carlo simulation procedure is created and applied to generating the process of changing hydro-meteorological conditions and the prediction of the oil spill domain movement impacted by these changes conditions.

1 INTRODUCTION

The probabilistic approach to determination of oil spill domains at port and sea water areas is proposed in (Dąbrowska & Kołowrocki 2019B). In this paper, the stochastic approach is supplemented by the Monte Carlo simulation approach (Dąbrowska 2019, Law & Kelton 2000, Zio & Marseguerra 2002) to the oil spill domain movement in changing hydro-meteorological conditions (Dąbrowska & Kołowrocki 2019A, 2019B). First, the model of the process of changing hydro-meteorological conditions is defined and its parameters are introduced. The identification methods of the unknown parameters of process of changing hydro-meteorological conditions is described in (Dąbrowska & Soszyńska-Budny 2018, 2019A). Under these assumptions, the process of changing hydro-meteorological conditions is completely described by the following parameters (Dąbrowska & Soszyńska-Budny 2018, Kołowrocki & Soszyńska-Budny 2011):

- the vector of probabilities of its initial states at the moment $t = 0$

$$[p(0)] = [p_1(0), p_2(0), ..., p_m(0)]$$

2 MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

We denote by $A(t)$ the process of changing hydro-meteorological conditions at the sea water areas where the oil spill happened and distinguish $m$ its states from the set $A = \{1, 2, ..., m\}$ in which it may stay at the moment $t$, $t \in <0, T>$, where $T > 0$. Further, we assume a semi-Markov model of the process $A(t)$ and denote by $\theta_{ij}$ its conditional sojourn time in the state $i$ while its next transition will be done to the state $j$, where $i, j = 1, 2, ..., m$, $i \neq j$ (Dąbrowska & Soszyńska-Budny 2018, 2019A). Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska & Soszyńska-Budny 2018, Kołowrocki & Soszyńska-Budny 2011):
the matrix of probabilities of its transitions between the particular states

\[
[p_k] = \begin{bmatrix}
  p_{11} & p_{12} & \cdots & p_{1m} \\
  p_{21} & p_{22} & \cdots & p_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  p_{m1} & p_{m2} & \cdots & p_{mn}
\end{bmatrix},
\]

(2)

where

\[ p_n = 0, \; i = 1,2,\ldots,m; \]

- the matrix of distribution functions of its conditional sojourn times \(0j\) at the particular states

\[
[W_0(t)] = \begin{bmatrix}
  W_{11}(t) & W_{12}(t) & \cdots & W_{1n}(t) \\
  W_{21}(t) & W_{22}(t) & \cdots & W_{2n}(t) \\
  \vdots & \vdots & \ddots & \vdots \\
  W_{m1}(t) & W_{m2}(t) & \cdots & W_{mn}(t)
\end{bmatrix},
\]

(3)

where

\[ W_0(t) = 0, \; i = 1,2,\ldots,m. \]

### 3 MODELLING OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

We assume that the process of changing hydro-meteorological conditions \(A(t)\) in succession takes the states

\[ k_1, k_2, \ldots, k_{n+1} \in \{1,2,\ldots,m\}, \; i = 1,2,\ldots,n+1. \]

For a fixed step of time \(\Delta t\), after multiple applying sequentially the procedure from Section 4.1 in (Dąbrowska & Kolowrocki 2019B):

- for

\[ t = 1\Delta t, 2\Delta t, \ldots, s_1\Delta t, \]

- at the process \(A(t)\) state \(k_i;\)

- for

\[ t = (s_i + 1)\Delta t, (s_i + 2)\Delta t, \ldots, s_2\Delta t, \]

- at the process \(A(t)\) state \(k_j;\)

\ldots

- for

\[ t = (s_{n+1} + 1)\Delta t, (s_{n+1} + 2)\Delta t, \ldots, s_n\Delta t, \]

- at the process \(A(t)\) state \(k;\)

we receive the following sequence of oil spill domains:

\[ \overline{D}^{k_1}(1\Delta t), \overline{D}^{k_2}(2\Delta t), \ldots, \overline{D}^{k_n}(s_1\Delta t), \]

(7)

\[ \overline{D}^{k_2}((s_1 + 1)\Delta t), \overline{D}^{k_2}((s_1 + 2)\Delta t), \ldots, \overline{D}^{k_n}(s_2\Delta t), \]

(8)

\ldots

\[ \overline{D}^{k_n}((s_{n-1} + 1)\Delta t), \overline{D}^{k_n}((s_{n-1} + 2)\Delta t), \ldots, \overline{D}^{k_n}(s_n\Delta t), \]

(9)

where \(s_i, i = 1,2,\ldots,n\), are such that

\[ (s_i - 1)\Delta t \leq \sum_{j=1}^{i} \overline{\theta}_{s_{ij}}, \; s_i\Delta t \leq T, \]

(10)

and

\[ \overline{\theta}_{s_{ij}}, \; j = 1,2,\ldots,n, \]

(11)

are the realizations of the process \(A(t), \; t \in (0,T)\), conditional sojourn times \(\theta_{s_{ij}}, j = 1,2,\ldots,n\) at the states \(k_i\) upon the next state is \(k_{i+1}\); \(k_i \in \{1,2,\ldots,m\}, i = 1,2,\ldots,n\) introduced in Section 2 and in (Dąbrowska & Kolowrocki 2019B).

Hence, the oil spill domain

\[ \overline{D}^{k_1,k_2,\ldots,k_n}, \; k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\}, \]

is described by the sum of determined domains of the sequences (7)-(9), given by

\[ \overline{D}^{k_1,k_2,\ldots,k_n} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{n-i} \overline{D}^{k_i}((s_i + j)\Delta t) \]

\[ \bigcup [\overline{D}^{k_2}((s_1 + 1)\Delta t) \cup \overline{D}^{k_2}((s_1 + 2)\Delta t) \cup \ldots \cup \overline{D}^{k_n}(s_2\Delta t)] \]

\ldots

\[ \bigcup [\overline{D}^{k_n}((s_{n-1} + 1)\Delta t) \cup \overline{D}^{k_n}((s_{n-1} + 2)\Delta t) \cup \ldots \cup \overline{D}^{k_n}(s_n\Delta t)] \]

for \(k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\}, s_0 = 0 \)

(12)

The oil spill domain \(\overline{D}^{k_1,k_2,\ldots,k_n}\) defined by (12) is determined for constant radiiuses

\[ r^{k_i}(t) = r^{k_i}, \; t \in (0,T), k_i \in \{1,2,\ldots,m\}, i = 1,2,\ldots,n. \]

If the radiiuses are not constant, we define the sequence of domains for each state \(k_i, k_i \in \{1,2,\ldots,m\}, i = 1,2,\ldots,n\) in a way similar to that described in Remark 1 in (Dąbrowska & Kolowrocki 2019B), i.e. we define the sequence of domains

\[ \overline{D}^{k_1,k_2,\ldots,k_n}(b_i\Delta t) = \bigcup_{i=1}^{n} \bigcup_{j=1}^{b_i} \overline{D}^{k_i}((s_i + j)\Delta t) \]

\[ \bigcup [\overline{D}^{k_2}((s_1 + 1)\Delta t) \cup \overline{D}^{k_2}((s_1 + 2)\Delta t) \cup \ldots \cup \overline{D}^{k_n}(s_2\Delta t)] \]

\ldots

\[ \bigcup [\overline{D}^{k_n}((s_{n-1} + 1)\Delta t) \cup \overline{D}^{k_n}((s_{n-1} + 2)\Delta t) \cup \ldots \cup \overline{D}^{k_n}(s_n\Delta t)] \]

\ldots
\[
\bigcup (\mathbb{D}^+((s_{i-1} + 1)\Delta t) \cup \mathbb{D}^+((s_{i-1} + 2)\Delta t) \cup \ldots \cup \mathbb{D}^+((s_{\tau} + \Delta t)))
\]
for \( b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, \)

\[ i = 1, 2, \ldots, n, \] (13)

where

\[
\mathbb{D}^+((s_{i-1} + a_i\Delta t)) \approx \mathbb{D}^+((s_{i-1} + a_i\Delta t)),
\]

\( a_i = 1, 2, \ldots, b_i, \ b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, \)

\( i = 1, 2, \ldots, n, \)

with the following, modified slightly in comparison that defined in (Dąbrowska & Kołowrocki 2019B), substitutions:

\[
m^h_i(t) := m_i^{h_i}(s_{i-1}, a_i\Delta t),
\]

\[
m^l_i(t) := m_i^{l_i}(s_{i-1}, a_i\Delta t),
\]

\[
\bar{\sigma}_\nu^l(t) := \overline{\sigma}_\nu^l(s_{i-1} + a_i\Delta t)
\]

\[
= \sigma_i^l(s_{i-1} + a_i\Delta t) + \sum_{j=1}^i r^l_j(b_j, \Delta t),
\]

\[
\bar{\sigma}_\nu^l(t) := \overline{\sigma}_\nu^l(s_{i-1} + a_i\Delta t)
\]

\[
= \sigma_i^l(s_{i-1} + a_i\Delta t) + \sum_{j=1}^i r^l_j(b_j, \Delta t),
\]

where \( m_i^{h_i}(s_{i-1}, \Delta t) = 0, \ m_i^{l_i}(s_{i-1}, \Delta t) = 0, \) for

\( a = 1, 2, \ldots, b_i, \ b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, \)

\( i = 1, 2, \ldots, n, \)

\( s_0 = 0. \)

4 MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

4.1 Generating process of changing hydro-meteorological conditions at oil spill area

We denote by \( k_i = k(q), \ i \in \{1, 2, \ldots, m\}, \) the realization of the process’ \( A(t) \) initial state at the moment \( t = 0. \) Further, we select this initial state by generating realizations from the distribution defined by the vector \( [p(0)]_{m\times m} \), according to the formula

\[
k(q) = k(q) = k_i \sum_{\xi=1}^q p_{i\xi}(0) \leq q < \sum_{\xi=1}^\psi p_{i\xi}(0),
\]

\( \psi \in \{1, 2, \ldots, m\}, \) (14)

where \( q \) is a randomly generated number from the uniform distribution on the interval \((0, 1)\) and \( p(0) \) for \( \xi = 0 \) equals \( 0. \)

After selecting the initial state \( k_i, \ i \in \{1, 2, \ldots, m\}, \) we can fix the next operation state of the process of changing hydro-meteorological conditions at oil spill area. We denote by \( k_i = k(q) \), \( j \in \{1, 2, \ldots, m\}, \ i \neq j, \) the sequence of the realizations of the operation process’ consecutive states generated from the distribution defined by the matrix \([p_{ij}]_{m\times m}\). Those realizations are generated for a fixed \( i, \ i \in \{1, 2, \ldots, m\}, \) according to the formula

\[
k_i(q) = k(q) \sum_{\xi=1}^{\psi} p_{ij\xi}(0) \leq q < \sum_{\xi=1}^{\psi} p_{ij\xi}(0),
\]

\( \psi \in \{1, 2, \ldots, m\}, \psi \neq i, \) (15)

where \( g \) is a randomly generated number from the uniform distribution on the interval \((0, 1)\) and \( p_{i0} = 0. \)

We can use several methods generating draws from a given probability distribution, e.g. an inverse transform method, a Box-Muller transform method, Marsaglia and Tsang’s rejection sampling method (Dąbrowska 2019). The inverse transform method (also known as inversion sampling method) is convenient if it is possible to determine the inverse distribution function (Grabski & Jażwiński 2009). This section will consider only this one sampling method, but the other methods are discussed in (Law & Kelton 2000, Rao & Naikan 2016, Zio & Marseguerra 2002).

We denote by \( t_{\nu ij}^{(\nu \psi) \nu} \), \( i, j \in \{1, 2, \ldots, m\}, \ i \neq j, \)

\( \nu = 1, 2, \ldots, n, \) the realization of the conditional sojourn times \( \theta_{ij} \) of the process \( A(t) \), \( t < 0, T >, \) generated from the distribution function \( W_{ij}(t) \), where \( \nu \) denotes the subsequent number of the sojourn times realizations and \( n \) is the number of those sojourn time realizations during the experiment time \( T \). Thus, using the inverse transform method, the realization \( t_{\nu ij}^{(\nu \psi) \nu} \) is generated from

\[
t_{\nu ij} = W_{\nu ij}^{-1}(h), \ i, j \in \{1, 2, \ldots, m\}, \ i \neq j,
\]

where \( W_{\nu ij}^{-1}(h) \) is the inverse function of the conditional distribution function \( W_{ij}(t) \) and \( h \) is a randomly generated number from the interval \((0, 1)\).

Having the realizations \( t_{\nu ij}^{(\nu \psi) \nu} \), \( i, j \in \{1, 2, \ldots, m\}, \ i \neq j, \)

\( \nu = 1, 2, \ldots, n, \) of the process \( A(t) \), it is possible to determine approximately the entire sojourn time as the sum of all sojourn time realizations during the experiment time \( T \), applying the formula

\[
t_n = \sum_{\nu=1}^n t_{\nu ij}^{(\nu \psi) \nu}, \ i, j \in \{1, 2, \ldots, m\}, \ n = 1, 2, \ldots.
\]

The exemplary realisation of the process \( A(t) \) and the entire sojourn time is presented in a figure below.
characteristics is formed as follows. meteorological conditions at oil spill area parameters of the process of changing hydro-

\[ \tau = t_y^{(1)} + t_y^{(2)}. \]

Further, we compare it with time \( T \). If the sum \( \tau \) is less than the experiment time \( T \), we draw the sequence of domains using formula (13).

We repeat the procedure above until the sum \( \tau \) of all generated realizations \( t_y^{(v)}, v = 1, 2, \ldots, n \), reach a fixed experiment time \( T \). Consequently, we calculate the entire sojourn time \( \tau_0 \) according to (17) and draw the sequence of domains using formula (13).

Finally, we put together all the sequences of domains drawn before and we get the oil spill domain movement (Figure 2). In the interval \((0, \tau)\) the number of ellipses is \( S_1 - s_0 = S_1 \) in the next intervals \((\tau - \tau, \tau)\), \( \ldots, (\tau_0 - \tau, \tau)\) the number of ellipses are respectively \( S_2 - S_1, S_3 - S_2, \ldots, S_n - S_{n-1}\), where \( S_0, I = 1, 2, \ldots, n \), are defined by (10).

The general Monte Carlo simulation flowchart for generating and determination of a process of changing hydro-meteorological conditions at oil spill area is illustrated in Figure 3.

4.3 Monte Carlo simulation prediction of the oil spill domain in varying hydro-meteorological conditions

Using the procedures of the process of changing hydro-meteorological conditions at oil spill area prediction described in Sections 4.1-4.2 and the modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) the Monte Carlo simulation oil spill domain prediction can be done.

The modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) depends on changing the procedure (4)-(12) by replacing the conditions (10)-(12) by conditions:

The \( s_0, i = 1, 2, \ldots, n \), existing in (4)-(9), according to (10), are such that

\[ (s_i - 1)\Delta t < \sum_{j=1}^{i} t_{ij}, i = 1, 2, \ldots, n, s_i \Delta t \leq T, \]  \( \text{ (18)} \)

and

\[ t_{ij}, j = 1, 2, \ldots, n - 1, \]  \( \text{ (19)} \)

are the realizations of the process \( A(t), t \in <0,T> \), conditioned sojourn times

\[ \theta_{ij}, j = 1, 2, \ldots, n - 1 \]

at the states \( k_i \) upon the next state is \( k_{ij}, j = 1, 2, \ldots, n - 1, \)

\( k_{ij}, k_{j+1} \in [1,2,\ldots,m], j = 1,2,\ldots,n - 1 \), defined in Section 4.1.
5 CONCLUSIONS

The proposed Monte Carlo simulation approach allows us for the determination of oil spill domains at port and sea water areas in changing hydro-meteorological conditions. It is a new supplementary method to the probabilistic methods of the oil spill domains determination in the varying hydro-meteorological states presented in (Dąbrowska & Kołowrocki 2019A, 2019B), (Kim et.al. 2013) and (Chen, Li & Li 2007). The comparison of results of these methods’ applications in real conditions should lead to the selection one of them with the best accuracy and develop in the future research.

REFERENCES

Al-Rabeh A. H., Cekirge H. M. & Gunay N. A. 1989. Stochastic simulation model of oil spill fate and transport, Applied Mathematical Modelling, p. 322-329.

Bogalecka, M. & Kolowrocki, K. 2018. Minimization of critical infrastructure accident losses of chemical releases impacted by climate-weather change, Proc. International Conference on Industrial Engineering and Engineering Management - IEEM, Bangkok, Thailand.

Chen, H, Li, D. & Li, X. 2007. Mathematical modeling of oil spill on the sea and application of the modeling in Daya Bay, Journal of Hydrodynamics, Ser. B: Elsevier, (19)3, 282-291.

Dąbrowska, E. 2019. Monte Carlo simulation approach to reliability analysis of complex systems, PhD Thesis, System Research Institute, Polish Academy of Science, Warsaw, Poland (under examination).
Dąbrowska, E. & Kołowrocki, K. 2019A. Modelling, identification and prediction of oil spill domains at port and sea water areas, Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 10(1), 43-58.

Dąbrowska, E. & Kołowrocki, K. 2019B. Probabilistic Approach to Determination of Oil Spill Domains at Port and Sea Water Areas, Proc. of TransNav Conference 2019.

Dąbrowska, E. & Soszyńska-Budny, J. 2018. Monte Carlo simulation forecasting of maritime ferry safety and resilience, Proc. International Conference on Industrial Engineering and Engineering Management - IEEM, Bangkok, Thailand.

Fay J. A. 1971. Physical Processes in the Spread of Oil on a Water Surface. Proceedings of Joint Conference on Prevention and Control of Oil Spills, sponsored by American Petroleum Industry, Environmental Protection Agency, and United States Coast Guard.

Fingas, M. 2016. Oil Spill Science and Technology, 2nd Edition, Elsevier.

Grabski, F., Jaźwiński, J. 2009. Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki. Wydawnictwa Komunikacji i Łączności: Warszawa.

Guze, S., Kołowrocki, K. & Mazurek, J. 2017. Modelling spread limitations of oil spills at sea. Proc. The 17th Conference of the Applied Stochastic Models and Data Analysis – ASMDA, London, UK.

Guze S., Mazurek J. & Smolarek L. 2016. Use of random walk in two-dimensional lattice graphs to describe influence of wind and sea currents on oil slick movement. Journal of KONES Powertrain and Transport, Vol. 23, No. 2.

Huang J. C. 1983. A review of the state-of-the-art of oil spill fate/behavior models. International Oil Spill Conference Proceedings: February 1983, Vol. 1983, No. 1, p. 313-322.

Kim, T., Yang, C.-S., Ouchi, K. & Oh, Y. 2013. Application of the method of moment and Monte-Carlo simulation to extract oil spill areas from synthetic aperture radar images, OCEANS - San Diego, 1-4.

Kołowrocki, K. 2014. Reliability of large and complex systems, Elsevier.

Kołowrocki, K. & Soszyńska-Budny, J. 2011. Reliability and Safety of Complex Technical Systems and Processes: Modelling - Identification - Prediction - Optimization, London, Dordrecht, Heidelberg, New York: Springer.

Kuligowska, E. 2017. Monte Carlo Simulation of Complex System Reliability Evaluation. Chapter in Monograph: Current research in mathematical and computer sciences. Red. A. Lecko, Wydawnictwo UWM: Olsztyn, 139-152.

Kuligowska, E. 2018. Monte Carlo simulation of climate-weather change process at maritime ferry operating area, Technical Sciences, University of Warmia and Mazury in Olsztyn, 1(21), 5-17.

Law, A. M. & Kelton, W. D. 2000. Simulation Modeling and Analysis, 3rd ed., McGraw Hill.

NOAA. Trajectory Analysis Handbook. NOAA Hazardous Material Response Division. Seattle: WA, http://www.response.restoration.noaa.gov/.

Rao, M. S., & Naikan, V. N. A. 2016. Review of simulation approaches in reliability and availability modeling, International Journal of Performability Engineering, 12 (4), 369-388.

Zio, E. & Marseguerra, M. 2002. Basics of the Monte Carlo Method with Application to System Reliability, LiLoLe.

Spaulding M. L. 1988. A state-of-the-art review of oil spill trajectory and fate modeling. Oil and Chemical Pollution, Vol. 4, Issue 1, 39-55.