Logarithmic correction to the Cardy-Verlinde formula in Achucarro-Oritz Black Hole

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Abstract

In this paper we calculate leading order correction due to small statistical fluctuations around equilibrium, to the Bekenstein-Hawking entropy formula for the Achucarro-Oritz black hole, which is the most general two-dimensional black hole derived from the three-dimensional rotating Banados-Teitelboim-Zanelli black hole. Then we obtain the same correction to the Cardy-Verlinde entropy formula (which is supposed to be an entropy formula of conformal field theory in any dimension).

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1 Introduction

It is commonly believed that any valid theory of quantum gravity must necessarily incorporate the Bekenstein-Hawking definition of black hole entropy \([1, 2]\) into its conceptual framework. However, the microscopic origin of this entropy remains an enigma for two reasons. First of all although the various counting methods have pointed to the expected semi-classical result, there is still a lack of recognition as to what degrees of freedom are truly being counted. This ambiguity can be attributed to most of these methods being based on dualities with simpler theories, thus obscuring the physical interpretation from the perspective of the black hole in question. Secondly, the vast and varied number of successful counting techniques only serve to cloud up an already fuzzy picture.

de Sitter/Conformal Field Theory correspondence (dS/CFT) \([3]-[31]\) may hold the key to its microscopical interpretation. Naively, we would expect dS/CFT correspondence to proceed along the lines of Anti-de Sitter /Conformal Field Theory (AdS/CFT) correspondence \([32]\) because de Sitter spacetime can be obtained from anti-de Sitter spacetime by analytically continuing the cosmological constant to imaginary values. The Cardy-Verlinde formula proposed by Verlinde \([33]\), relates the entropy of a certain CFT with its total energy and its Casimir energy in arbitrary dimensions. Using the AdS\(_d/\text{CFT}_{d-1}\) and dS\(_d/\text{CFT}_{d-1}\) correspondences, this formula has been shown to hold exactly for different black holes. In previous paper \([28]\), by using the Cardy-Verlinde formula, we have obtained the entropy of the Achucarro-Ortiz black hole which is a two-dimensional black hole derived from the three-dimensional rotating BTZ black hole.

In 1992 Bañados, Teitelboim and Zanelli (BTZ) \([34, 35]\) showed that (2+1)-dimensional gravity has a black hole solution. This black hole is described by two (gravitational) parameters, the mass \(M\) and the angular momentum (spin) \(J\). It is locally AdS and thus it differs from Schwarzschild and Kerr solutions since it is asymptotically anti-de-Sitter instead of flat spacetime. Additionally, it has no curvature singularity at the origin. AdS black holes, are members of this two-parametric family of BTZ black holes and they are very interesting in the framework of string theory and black hole physics \([36, 37]\).

For systems that admit 2D CFTs as duals, the Cardy formula \([38]\) can be applied directly. This formula gives the entropy of a CFT in terms of the central charge \(c\) and the eigenvalue of the Virasoro operator \(l_0\). However, it should be pointed out that this evaluation is possible as soon as one has explicitly shown (e.g using the AdS\(_d/\text{CFT}_{d-1}\) correspondence) that the system under consideration is in correspondence with a 2D CFT \([39, 40]\).

In \([39]\) Cadoni and Mignemi, using Cardy formula have been calculated the statistical entropy of two-dimensional Jackiw-Teitelboim black hole, which can be considered as the dimensional reduction of the \(j = 0\) (zero angular momentum) BTZ black hole. Using a canonical realization of the asymptotic symmetry of two-dimensional anti-de Sitter space and Cardy’s formula they have been calculated the statistical entropy of 2D black hole. In this case this reference relate a two-dimensional black hole to a one-dimensional CFT, living on the boundary of AdS\(_2\). In fact the one-dimensional nature of the boundary CFT, implies that we are dealing with some kind of particle quantum mechanics, rather than quantum field theory. In the other hand as have been shown in second paper by Cadoni and Mignemi \([39]\) in the family of the AdS\(_d/\text{CFT}_{d-1}\) dualities, the \(d = 2\) case is very similar to the \(d = 3\) one, the conformal group being in both instances infinite dimensional. But a feature that is peculiar to the \(d = 2\) case is the complete equivalence of the diffeo-
morphisms and the conformal group in one dimension. The physical implication of this equivalence is that the usual difference between gauge symmetries and symmetries related to conserved charges disappears. If one accepts the $d=2$ case is not fundamental then the CFT$_1$ should be thought of just as (half) of CFT$_2$, in the way have been explained in [39], therefore in the $d=2$ context the general AdS$_d$/CFT$_{d-1}$ duality becomes a duality between two 2d conformal field theories. Although the possibility of describing 2D black holes by means of a CFT has been widely investigated [39, 41, 42, 43], it is not completely clear if it is always possible to mimic the gravitational dynamics of the 2D black hole through a CFT. However, in some cases and/or for generic black holes in particular regimes, CFTs have been shown to give a good description, this is in particular true for black holes in AdS space. In this paper I consider the two-dimensional Achucarro-Oritz black hole which is asymptotically AdS$_2$, then we show a 2D CFT give a good description in this case also. When the black hole is put in correspondence with a 2D CFT, the entropy of black hole horizon is reproduce using the Cardy-Verlinde formula.

There has been much recent interest in calculating the quantum corrections to $S_{BH}$ (the Bekenestein-Hawking entropy) [44]-[80]. The leading-order correction is proportional to $\ln S_{BH}$. There are, two distinct and separable sources for this logarithmic correction [71, 74] (see also recent paper by Gour and Medved [78]). Firstly, there should be a correction to the number of microstates that is a quantum correction to the microcanonical entropy, secondly, as any black hole will typically exchange heat or matter with its surrounding, there should also be a correction due to thermal fluctuations in the horizon area. In a recent work Carlip [46] has deduced the leading order quantum correction to the classical Cardy formula. The Cardy formula follows from a saddle-point approximation of the partition function for a two-dimensional conformal field theory. This leads to the theory’s density of states, which is related to the partition function by way of a Fourier transform [81]. In [66]Medved has been applied the Carlip’s formulation to the case of a generic model of two-dimensional gravity with coupling to a dilaton field.

In this paper we consider the Achucarro-Oritz black hole. In section 2 we calculate the corresponding thermodynamical quantities for black hole horizon. In section 3 we calculate leading order correction due to small statistical fluctuations around equilibrium, to the Bekenestein-Hawking entropy formula then we obtain the same correction to the Cardy-Verlinde entropy formula. In the other term we assume the equality of Bekenstein-Hawking and CFT entropy, then we uses the known statistical corrections to the Bekenstein-Hawking entropy to predict the corrections to the Cardy-Verlinde formula. Last section contain a summary of paper.

### 2 Thermodynamical quantities of Achucarro-Oritz black hole

The black hole solutions of Bañados, Teitelboim and Zanelli [34, 35] in $(2+1)$ spacetime dimensions are derived from a three dimensional theory of gravity

$$S = \int dx^3 \sqrt{-g} \left( R^{(3)} + 2\Lambda \right)$$

with a negative cosmological constant ($\Lambda = \frac{1}{l^2} > 0$).
The corresponding line element is
\[ ds^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)dt^2 + \frac{dr^2}{\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)} + r^2\left(d\theta - \frac{J}{2r^2}dt\right)^2 \quad (2) \]

There are many ways to reduce the three dimensional BTZ black hole solutions to the two dimensional charged and uncharged dilatonic black holes [82, 83]. The Kaluza-Klein reduction of the (2 + 1)-dimensional metric (2) yields a two-dimensional line element:
\[ ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2 \quad (3) \]

where
\[ g(r) = \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right) \quad (4) \]

with \( M \) the Arnowitt-Deser-Misner (ADM) mass, \( J \) the angular momentum (spin) of the BTZ black hole and \(-\infty < t < +\infty, 0 \leq r < +\infty, 0 \leq \theta < 2\pi\).

The outer and inner horizons, i.e. \( r_+ \) (henceforth simply black hole horizon) and \( r_- \) respectively, concerning the positive mass black hole spectrum with spin \((J \neq 0)\) of the line element (3) are given as
\[ r_{\pm}^2 = \frac{l^2}{2} \left(M \pm \sqrt{M^2 - \frac{J^2}{l^2}}\right) \quad (5) \]

and therefore, in terms of the inner and outer horizons, the black hole mass and the angular momentum are given, respectively, by
\[ M = \frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2} \quad (6) \]

and
\[ J = \frac{2r_+r_-}{l} \quad (7) \]

with the corresponding angular velocity to be
\[ \Omega = \frac{J}{2r_+^2} \quad (8) \]

The Hawking temperature \( T_H \) of the black hole horizon is [84]
\[ T_H = \frac{1}{2\pi r_+} \sqrt{\left(\frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2}\right)^2 - \frac{J^2}{l^2}} \]
\[ = \frac{1}{2\pi r_+} \left(\frac{r_+^2}{l^2} - \frac{J^2}{4r_+^2}\right) \quad (9) \]

In two spacetime dimensions we do not have an area law for the black hole entropy, however one can use thermodynamical reasoning to define the entropy [84]
\[ S_{bh} = 4\pi r_+ \quad (10) \]

The specific heat of the black hole is given by
\[ C = \frac{dE}{dT} = \frac{dM}{dT} = 4\pi r_+ \left(\frac{r_+^2 - r_-^2}{r_+^2 + 3r_-^2}\right) = S_{bh} \left(\frac{r_+^2 - r_-^2}{r_+^2 + 3r_-^2}\right), \quad (11) \]

\( r_+ > r_- \), then the above specific heat is positive. The stability condition is equivalent to the specific heat being positive, so that the corresponding canonical ensemble is stable.
3 Logarithmic correction to the Bekenstein-Hawking entropy and Cardy-Verlinde formula

There has been much recent interest in calculating the quantum corrections to $S_{BH}$ (the Bekenstein-Hawking entropy) [44]-[80]. The corrected formula takes the form

$$S = S_0 - \frac{1}{2} \ln C + \ldots$$

(12)

When $r_+ \gg r_-$, $C \simeq S_{bh} = S_0$, in this case we have

$$S = S_0 - \frac{1}{2} \ln S_0 + \ldots$$

(13)

It is now possible to drive the corresponding correction to Cardy-Verlinde formula. In a recent paper, Verlinde [33] propound a generalization of the Cardy formula which holds for the $(1+1)$ dimensional Conformal Field Theory (CFT), to $(n+1)$-dimensional spacetime described by the metric

$$ds^2 = -dt^2 + R^2 d\Omega_n$$

(14)

where $R$ is the radius of a $n$-dimensional sphere.

The generalized Cardy formula (hereafter named Cardy-Verlinde formula) is given by

$$S_{CFT} = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)}$$

(15)

where $E$ is the total energy, $E_C$ is the Casimir energy, $a$ and $b$ a priori arbitrary positive coefficients, independent of $R$ and $S$. The definition of the Casimir energy is derived by the violation of the Euler relation as

$$E_C \equiv n (E + pV - TS - J\Omega)$$

(16)

where the pressure of the CFT is defined as $p = E/nV$. The total energy may be written as the sum of two terms

$$E(S, V) = E_E(S, V) + \frac{1}{2} E_C(S, V)$$

(17)

where $E_E$ is the purely extensive part of the total energy $E$. The Casimir energy $E_C$ as well as the purely extensive part of energy $E_E$ expressed in terms of the radius $R$ and the entropy $S$ are written as

$$E_C = \frac{b}{2\pi R} S^{1-\frac{1}{n}}$$

(18)

$$E_E = \frac{a}{4\pi R} S^{1+\frac{1}{n}}.$$  

(19)

After the work of Witten on AdS$_d$/CFT$_{d-1}$ correspondence [85], Savonije and Verlinde proved that the Cardy-Verlinde formula (15) can be derived using the thermodynamics of AdS-Schwarzschild black holes in arbitrary dimension [86]. For the present discussion, the total entropy is assumed to be of the form Eq.(13), where the uncorrected entropy,
correspondence to that associated in Eq. (10). Since the two-dimensional Achúcarro-Oritz black hole is asymptotically anti-de-Sitter, the total energy is $E = M$. It then follows by employing Eqs.(6-9) that the Casimir energy Eq.(16) can be expressed in term of the uncorrected energy.

$$E_C = \frac{J^2}{2r_+^2} - \frac{1}{2} T_H \ln S_0,$$

(20)

Then by setting the above corrected Casimir energy in Eq.(15) and expanding in term of $\frac{1}{2} T_H \ln S_0$ we obtain

$$\frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} \simeq S_0 (1 + \frac{1}{2} T_H \ln S_0 \frac{E_C - E}{E_C (2E - E_C)}).$$

(21)

In the limit where the correction is small, the coefficient of the logarithmic term on the right-hand side of Eq.(21) can be expressed in terms of the energy and Casimir energy

$$\frac{(E_C - E)}{2E_C (2E - E_C)} T_H S_0 = \frac{(E_C - E)(2E - E_C - E_q)}{2E_C (2E - E_C)},$$

(22)

where

$$E_q = Q\phi = J\Omega,$$

(23)

is the electromagnetic energy, in our analysis the charge $Q$ is the angular momentum $J$ of the two-dimensional Achucarro-Oritz black hole, the corresponding electric potential $\phi$ is the angular velocity $\Omega$. We may conclude, therefore that in the limit where the logarithmic corrections are sub-dominant, Eq.(21) can be rewritten to express the entropy in terms of the energy, and Casimir energy.

$$S_0 = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} - \frac{(E_C - E)(2E - E_C - E_q)}{2E_C (2E - E_C)} \ln \left(\frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)}\right),$$

(24)

and consequently, the total entropy Eq.(13) to first order in the logarithmic term, is given by

$$S \simeq \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} - \frac{(E_C - E)(2E - E_C - E_q)}{2E_C (2E - E_C)} + \frac{1}{2} \ln \left(\frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)}\right).$$

(25)

Therefore taking into account thermal fluctuations defines the logarithmic corrections to the black hole entropies. As a result the Cardy-Verlinde formula receive logarithmic corrections in our interest Achucarro-Oritz black hole background in two dimension, in the way similar to the Cardy-Verlinde formula for the SAdS and SdS black holes in 5-dimension [77, 79] also for TRNdS black holes in any dimension[80]. It is easily seen that the logarithmic prefactor is negative and therefore the thermal corrections are also negative. Furthermore, the entropy of two-dimensional Achucarro-Oritz black hole described in the context of Das et al [47] analysis by the modified Cardy-Verlinde formula satisfy the holographic bound [87].

4 Conclusion

For a large class of black hole, the Bekenstein-Hawking entropy formula receives additive logarithmic corrections due to thermal fluctuations. On the basis of general thermodynamic arguments, Das et al [47] deduced that the black hole entropy can be expressed
as
\[ S = \ln \rho = S_0 - \frac{1}{2} \ln \left( C T^2 \right) + \cdots. \tag{26} \]

In this paper we have analyzed this correction of the entropy of Achucarro-Oritz black hole in two dimension in the light of AdS/CFT. We have obtain the logarithmic correction to black hole entropy. Then using the form of the logarithmic correction Eq.(13) we have derived the corresponding correction to the Cardy-Verlinde formula which relates the entropy of a certain CFT to its total energy and Casimir energy. The result of this paper is that the CFT entropy can be written in the form Eq.(25).

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