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A Finite Volume SOFC Model for Coal-Based Integrated Gasification Fuel Cell Systems Analysis

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1 Introduction

Solid oxide fuel cells (SOFCs) operating at elevated temperatures (873–1273 K) hold the promise of power generation with higher efficiency and lower pollution. Due to high efficiency, high temperature operation, solid state design, and the potential for internal reforming of gaseous fuels, SOFC are ideal for stationary applications. Integrated gasification fuel cell (IGFC) systems that combine SOFC with gasifiers are expected to provide more efficient and environmentally viable utilization of coal, the most abundant fossil fuel resource around the world. Systems analyses have been performed to investigate and optimize IGFC systems with various configurations [1–4]. Most of these analyses have employed “black box” modeling of the SOFC reactor based on thermodynamic analysis and global mass/energy balances. Such models, however, are not capable of revealing many intrinsic constraints to SOFC operation (for example, temperature and current density profiles) and challenges of integrating fuel cell stacks with the gasifier and balance of plant.

Various models [5–9] have been developed to provide more detailed insight into SOFC operation: finite difference and finite element are the most common modeling approaches employed. As an integral form of finite difference discretization, the finite volume method has reasonable accuracy and relatively lighter computational expense, which has also led to its use in SOFC modeling [10–12]. The lower computational expense of the finite volume method is critical to its selection in the current work that is aimed at model development for use in detailed systems analyses.

This work discusses the definition and development of a quasi-two-dimensional (2D) finite volume SOFC model that: (1) is based on detailed electrochemical analyses and internal heat transfer calculations; (2) can give not only fuel cell overall performance but also internal profiles of temperature, current density, flow compositions, etc., so that more detailed characteristics of SOFC under different system configurations can be investigated; (3) has short calculation time and the flexibility to be linked to power system analysis tools. Special attention was paid to making the model capable of reflecting some recent developments in the SOFC community such as direct internal reforming (DIR), anode-supported geometry, and the use of metallic interconnects. A planar SOFC geometry was considered due to its higher current/power density and lower fabrication cost, but the approach can also be adapted for tubular or other geometries.

2 Model Description

2.1 Model Features. Only the two parallel-flow configurations (coflow and counterflow) were considered in this work because: (1) the two configurations are sufficiently representative for the purposes of system analysis; (2) the cross-flow configuration requires at least a full-2D model to resolve the geometry, while the parallel-flow configurations can be analyzed through a quasi-2D model, which is more computationally economic. Thus, the finite volume SOFC model represents the most centered channel in the centered cell layer in a fuel cell stack. The structures of
fuel flow channel, air flow channel, positive-electrolyte-negative structure (PEN) (which includes the two porous electrodes and the dense solid electrolyte layer), air- and fuel-side interconnects (including rib structures) are resolved. The geometric configuration of the model is shown in Fig. 1.

Figure 2 shows the discretization of the fuel cell channel into a user-defined number of control volumes. Each control volume contains separate temperatures for the fuel channel, air channel, PEN, and interconnects (by applying symmetric boundary conditions, the temperatures of fuel- and air-side interconnects are assumed the same). Campanari and Iora [12], in similar finite volume modeling work, investigated the differences between a “coarse” grid (where the PEN temperature and interconnect temperatures were lumped together as a solid temperature) and a “refined” grid (where fuel- and air-side interconnects were further divided into three control volumes of different temperatures, respectively) and concluded that for parallel-flow configurations, the two different approaches yielded very similar thermal profiles and the differences in terms of total cell balances were within 0.3%. Although it seems well justified to adopt the “coarse” grid in this work to save computational expense, further investigation reveals that at least one independent interconnect temperature should be retained to account for metallic interconnects, which have thermal conductivities at least one order of magnitude greater than that of the PEN.

The model requires the following input information:

1. cell geometry parameters (fuel and air channel dimensions, solid layer thickness, interconnect rib width, etc.)
2. inlet fuel and air thermodynamic properties (temperature and pressure) and chemical compositions
3. desired working voltage or desired average working current density (depending on the calculation option chosen)

The model generates the following information:

1. overall cell performance: fuel and air utilization, total power output, heat loss by radiation at the edges, average working current density, or working voltage (depending on the calculation option chosen), etc.
2. internal profiles of various properties: temperature, local current density, power density, local chemical species mole fractions, local electrochemical loss terms, etc.

Two calculation options are available for the model.

1. The desired working voltage of the fuel cell is given and the model will calculate the average working current density in a straightforward manner.
2. The desired average working current density is given and the model will calculate iteratively based upon trial working voltage values until a value that satisfies the working current density requirement is found.

2.2 Simplifications and Assumptions. The following simplifications and assumptions are made for the model.

1. Steady state.
2. The fuel may contain any combination of H₂, CH₄, CO,
CO₂, H₂O, N₂, and Ar, while air is considered to be comprised of O₂, N₂, CO₂, H₂O, and Ar. Contaminants generally present in coal gasification products, such as tars, particulate matter, nitrogen-containing compounds, and sulfur, are expected to be reduced to sufficiently low concentrations in the syngas that they do not affect the SOFC performance [13].

(3) Each control volume has uniform species concentrations within the fuel and air channels.

(4) Interconnects are treated as equipotential plates due to their high electrical conductivity.

(5) The water gas shift reaction occurs inside the fuel flow channel and is always in an equilibrium state. The equilibrium constant is determined by the local fuel temperature.

(6) Electrochemical oxidation of H₂ occurs at the anode-electrolyte interface, with the reaction kinetics controlled by the local PEN temperature.

(7) The kinetics of CO oxidation at the fuel cell anode is slow compared with H₂ oxidation. Only H₂ participates in electrochemical reactions, while CO is oxidized through the water gas shift reaction. Li and Chyu [14] showed that electrochemical oxidation of both CO and H₂ at the anode yields the same Nernst potential in a SOFC, as long as chemical equilibrium of the shift reaction is attained.

(8) Internal reformation of CH₄ is kinetically limited and occurs at the fuel-anode interface, with the reaction kinetics determined by the local PEN temperature.

(9) 100% of the surface area under the interconnect rib is active for H₂ oxidation but inactive for CH₄ reformation [5].

(10) The Peclet number is large: thus it is reasonable to neglect axial diffusion effects (thermal and mass diffusion) in the gas phases [7].

(11) Radiation heat transfer by gas emission is assumed negligible. Radiation heat transfer between PEN and interconnect in a single control volume was found to be very small due to the small temperature difference. Radiation heat transfer among solids of different control volumes (which may have a larger temperature difference) are also neglected due to small view factors.

(12) Heat loss from the edges of the channel occurs only by radiation. The edge of the fuel cell stack is modeled as a gray surface positioned in a large cavity. The environment (stack chamber) temperature is an input parameter controlled by the model user.

3 Model Equations

3.1 Electrochemical Model. The fuel cell working voltage is calculated as a function of working current density by:

\[ V_{cell} = V_{Nernst} - \eta_{act} - \eta_{ohm} - \eta_{diff} = f(j) \]  

where \( V_{cell} \) is the fuel cell working voltage, \( V_{Nernst} \) is the Nernst potential, \( \eta \) is the loss term, and \( j \) is the local working current density.

3.1.1 Nernst Potential. The Nernst potential \( V_{Nernst} \) is calculated according to the Nernst equation [15]:

\[ V_{Nernst} = E^0 + \frac{R T_{PEN}}{2F} \left[ \ln \left( \frac{\rho_{H_2}(x_iOX_i)^{1/2}}{\rho_{H_2O}} \right) + 0.5 \ln \left( \frac{p_{cat}}{p_{amb}} \right) \right] \]  

where \( E^0 \) is the ideal potential of H₂ oxidation at ambient pressure, as a function of fuel cell reaction site temperature, \( T_{PEN} \) is the local PEN temperature, \( x_i \) is the local mole fraction of species \( i \), and \( p \) is pressure. The value of \( E^0 \) is related to the change in Gibbs free energy for H₂ reaction with O₂ to produce H₂O at the operating temperature. \( E^0 \) is calculated according to a linear fit of JANAF thermochemical table data [16] for Gibbs free energy in the temperature range of 800–1400 K, which is a typical operating temperature range for SOFC, as follows:

\[ E^0 = 1.28628053 \times 10^{-4} T_{PEN} \]  

3.1.2 Activation Polarization. The activation polarization is estimated as the sum of activation polarization at each electrode-electrolyte interface

\[ \eta_{act} = \eta_{act}^{an}(j) + \eta_{act}^{cat}(j) \]  

The governing equation for the activation polarization is the general Butler–Volmer (BV) equation

\[ j = j_0 \left[ \exp \left( \frac{\alpha F \eta_{act}}{R T_{PEN}} \right) - \exp \left( - \frac{(1 - \alpha) n F \eta_{act}}{R T_{PEN}} \right) \right] \]  

The full B-V equation must be solved implicitly for the activation polarization, whereas in modeling it is often desirable to have the polarization term expressed explicitly as a function of current density. Noren and Hoffman [17] compared several types of explicit approximations and concluded that the hyperbolic sine approximation is recommended:

\[ \eta_{act} = \frac{R T_{PEN}}{\alpha n F} \sinh^{-1} \left( \frac{j}{2j_0} \right) \]  

(6)

The exchange current density \( j_0 \) can be expressed as an Arrhenius law function of the composition of the reacting species:

\[ j_{0,an} = \gamma_{an} \left( \frac{p_{H_2}}{p_{amb}} \right)^{0.25} \exp \left( - \frac{E_{act,an}}{R T_{PEN}} \right) \]  

\[ j_{0,cat} = \gamma_{cat} \left( \frac{p_{O_2}}{p_{amb}} \right)^{0.25} \exp \left( - \frac{E_{act,cat}}{R T_{PEN}} \right) \]  

(7)

(8)

Various values for the pre-exponential factor and activation energy of Eqs. (7) and (8) are reported in the literature [8,11–13]. Values reported by Campanari and Iora [12] and Costamagna et al. [8] for simulating an electrolyte-supported SOFC are used in this work for model verification. Note that Hernández-Pacheco et al. [13] clarified that the value of \( n \) in Eq. (6) should be 1 (in terms of an individual electron transferred) rather than 2 (number of electrons transferred per oxygen ion).

3.1.3 Ohmic Polarization. It is assumed that the electric current flow path is perpendicular to the SOFC plane. Current flows across interconnects, anode, electrolyte, and cathode under the cell potential difference. The overall ohmic polarization is divided into losses due to resistance of the fuel-side interconnect, PEN, and air-side interconnect:

\[ \eta_{ohm} = i(R_{PEN} + R_{IC,fuel} + R_{IC,air}) \]  

The resistance of the PEN structure \( R_{PEN} \) is calculated by:

\[ R_{PEN} = \frac{\rho_{ele} \delta_k}{A_k} \]  

where \( A_k \) is the area of the section where current flows, \( \delta_k \) is the corresponding current flow length and is equal to the thickness of the corresponding layer based on the assumptions mentioned above. The temperature dependent material electrical resistivity, \( \rho_{ele} \), of anode, cathode, and electrolyte are calculated according to equations listed in Table 1, cited from the International Energy Agency (IEA) sponsored steady-state modeling benchmark for planar SOFC [18].

For ceramic interconnects whose electrical resistance is comparable to that of PEN, a method presented by Selimovic [10] is adopted in this work. The “L-shaped” interconnect is divided into three rectangular parts, I, II, and III, as shown in Fig. 3.

For part I and II, the electrical resistance values are calculated according to Ohm’s law:

\[ R_k = \frac{\rho_{ele} \delta_k}{A_k (c - b)} \]  

(11)
Table 1 Summary of model parameters

| Parameter                                | Value                                      |
|------------------------------------------|--------------------------------------------|
| Methane reformation reaction             |                                            |
| Pre-exponential factor $K_{\text{re}}$   | 4274 mol s$^{-1}$ m$^{-2}$ bar$^{-1}$       |
| Coefficient $\alpha$                     | 1                                          |
| Coefficient $\beta$                      | 0                                          |
| Activation energy $E_{\text{act,cath}}$  | 82,000 J mol$^{-1}$                        |
| Pre-exponential factor for anode $\gamma_{\text{an}}$ | 5.5 x 10$^8$ A m$^{-2}$                 |
| Activation energy for anode $E_{\text{act,an}}$ | 100,000 J mol$^{-1}$                   |
| Pre-exponential factor for cathode $\gamma_{\text{cat}}$ | 7 x 10$^8$ A m$^{-2}$                |
| Activation energy for cathode $E_{\text{act,cat}}$ | 120,000 J mol$^{-1}$, 117,000 J mol$^{-1}$ |
| Specific resistivity of anode $\rho_{\text{an}}$ | 2 W m$^{-1}$ K$^{-1}$               |
| Specific resistivity of cathode $\rho_{\text{cat}}$ | 2 W m$^{-1}$ K$^{-1}$               |
| Specific resistivity of electrolyte $\rho_{\text{el}}$ | 2 W m$^{-1}$ K$^{-1}$               |
| Specific resistivity of interconnect $\rho_{\text{IC}}$ | 2 W m$^{-1}$ K$^{-1}$               |
| Thermal conductivity $\lambda_{\text{PEN}}$ | 2 W m$^{-1}$ K$^{-1}$                |
| Thermal conductivity $\lambda_{\text{IC}}$ | 2 W m$^{-1}$ K$^{-1}$                |
| Thermal conductivity $\lambda_{\text{metal}}$ | 25 W m$^{-1}$ K$^{-1}$               |

\[
f\left(\frac{b}{d-a}\right) = \frac{1}{0.41 \left[1 - \exp\left(-1.2 \frac{b}{d-a}\right)\right]}
\]  

(14)

The effective electrical resistance of fuel-side interconnects can be expressed as

\[
R_{\text{IC,fuel}} = 0.5 \left(\frac{R_{\text{I}1} + R_{\text{I}2}R_{\text{I}3}}{R_{\text{I}1} + R_{\text{I}3}}\right)
\]  

(15)

The effective electrical resistance of the air-side interconnect can be calculated in a similar manner.

For metallic interconnects, the electrical resistance of the material itself is so small that it can be neglected. However, it is necessary to take into account the electrical resistance of the oxide scale that grows on these interconnects. In this work, data for Crofer 22 APU are used [19].

3.1.4 Diffusion Polarization. In close proximity to the PEN reaction sites, the concentrations of reactants and products participating in the electrochemical reactions can differ significantly from bulk gas stream concentrations. This effect is related to mass transport by diffusion through the electrodes and results in diffusion polarization, which can be estimated as:

\[
\eta_{\text{diff}} = \eta_{\text{diff}}^{\text{an}} + \eta_{\text{diff}}^{\text{cat}} = \frac{R_{\text{IC}}T_{\text{PEN}}}{2F} \left[\frac{x_{\text{H}_2} \delta x_{\text{H}_2}}{x_{\text{H}_2} \delta x_{\text{H}_2}^{\text{an}} x_{\text{H}_2}^{\text{cat}} \delta x_{\text{H}_2}} + \frac{R_{\text{IC}}T_{\text{PEN}}}{4F} \left[\frac{x_{\text{O}_2} \delta x_{\text{O}_2}}{x_{\text{O}_2} \delta x_{\text{O}_2}^{\text{an}} x_{\text{O}_2}^{\text{cat}} \delta x_{\text{O}_2}}\right]\right]
\]  

(16)

where $b$ and $r$ represent bulk and reaction site concentrations, respectively.

By relating the diffusive flow of $H_2$, $H_2O$, and $O_2$ to the electric current density $j$ through the Faraday’s law and assimilating multicomponent diffusion to binary diffusion where necessary, the mole fractions of $H_2$, $H_2O$, and $O_2$ at the reaction sites can be calculated by the following equations [8,11,20].

\[
x_{\text{H}_2} = x_{\text{H}_2}^{\text{an}} - \frac{jR_{\text{IC}}T_{\text{PEN}}}{2F \rho_{\text{IC}}} \delta x_{\text{an}}
\]  

(17)

\[
x_{\text{H}_2O} = x_{\text{H}_2O}^{\text{an}} + \frac{jR_{\text{IC}}T_{\text{PEN}}}{2F \rho_{\text{IC}}} \delta x_{\text{an}}
\]  

(18)

\[
x_{\text{O}_2} = 1 + (x_{\text{O}_2}^{\text{an}} - 1) \exp\left[\frac{jR_{\text{IC}}T_{\text{PEN}}}{4F \rho_{\text{IC}}} \delta x_{\text{an}}\right]
\]  

(19)

The effective diffusivities at the anode and cathode sides are [20]:

\[
D_{\text{an,eff}} = D_{\text{IC}} = \frac{P_{\text{H}_2O}}{P_{\text{an}}} D_{\text{H}_2O,eff} + \frac{P_{\text{H}_2}}{P_{\text{an}}} D_{\text{H}_2,eff}
\]  

(20)

\[
D_{\text{cat,eff}} = D_{\text{O}_2,eff}
\]  

(21)

Since both ordinary diffusion and Knudsen diffusion occur simultaneously, the overall effective diffusivity for $H_2$, $H_2O$, and $O_2$ in porous electrodes can be determined from [8,20]:

\[
D_{12,eff} = \frac{\varepsilon}{\tau} \left(\frac{1}{D_{12}} + \frac{1}{D_{11}}\right)^{-1}
\]  

(22)

where $\varepsilon$ and $\tau$ are the porosity and tortuosity of the electrode materials, respectively.

The binary diffusivity $D_{12}$ is estimated using the Fuller equation [21]:

\[
D_{12} = \frac{0.001437 T_{\text{PEN}}^{1.75}}{p M_{12}^{1/2} \left[\left(\Sigma v_i\right)^{1/3} + \left(\Sigma c_i\right)^{1/3}\right]}
\]  

(23)

where $M_{12} = 2[(1/M_1) + (1/M_2)]^{-1}$ and $M_i$ is the molecular weight of species $i$; $(\Sigma v_i)$ is the diffusion volume of species $i$.

The Knudsen diffusivity [22] is estimated as:

Fig. 3 Electrical resistance of ceramic interconnects
where $d_{\text{pore}}$ is the diameter of the pore structure and $M_i$ is the molecular weight of the species.

3.1.5 Water Gas Shift Reaction. The CO in the fuel gas is converted into $H_2$ by the water gas shift reaction

$$CO + H_2O = H_2 + CO_2$$

(25)

This reaction is assumed fast, such that the species are always in local equilibrium, with the equilibrium constant depending only on the local fuel temperature [11]:

$$K_{\text{p, shift}} = \frac{p_{H_2}p_{CO_2}}{p_{H_2O}p_{CO}} = \frac{x_{H_2}x_{CO_2}}{x_{H_2O}x_{CO}} = \exp \left( \frac{4276}{T_{\text{fuel}}} - 3.961 \right)$$

(26)

3.1.6 Methane Reformation Kinetics. In the SOFC, $CH_4$ is converted to $H_2$ and CO in the SOFC by steam reformation, an endothermic reaction that is catalyzed by the nickel/zirconia cermet anode materials.

$$CH_4 + H_2O \rightarrow 3H_2 + CO$$

(27)

Modeling of this methane reformation reaction is based on a chemical kinetic approach. The expression for the molar reaction rate of $CH_4$ (mol s$^{-1}$) follows the empirical approach of Auchenbach [23]:

$$r_{CH_4} = \gamma_{CH_4} p_{CH_4}^\theta p_{H_2O}^\rho \exp \left( - \frac{E_{\text{act,rx}}}{R_{\text{PEN}}} \right) A_{rx}$$

(28)

where $A_{rx}$ is the reformation reaction surface of the discretized control volume (not including the surface area under the rib); the values of other parameters are listed in Table 1.

3.2 Species Conservation. The overall mole balances for the $i$th control volume in the fuel and air channels are:

$$n_{H_2}(i+1) = n_{H_2}(i) - r_{ele} + 3r_{rx} + \Delta n_{H_2}^{\text{shift}}$$

(29a)

$$n_{H_2O}(i+1) = n_{H_2O}(i) + r_{ele} - r_{rx} - \Delta n_{H_2O}^{\text{shift}}$$

(29b)

$$n_{CO}(i+1) = n_{CO}(i) + r_{rx} - \Delta n_{CO}^{\text{shift}}$$

(29c)

$$n_{CO_2}(i+1) = n_{CO_2}(i) + \Delta n_{CO_2}^{\text{shift}}$$

(29d)

$$n_{CH_4}(i+1) = n_{CH_4}(i) - r_{rx}$$

(29e)

$$n_{O_2}(i+1) = n_{O_2}(i) + \frac{1}{2} r_{ele} \quad \text{"+" for coflow, "-" for counterflow}$$

(29f)

$$n_{N_2(i+1)} = n_{N_2}(i)$$

(29g)

$$n_{Ar(i+1)} = n_{Ar}(i)$$

(29h)

where $r_{ele}$ stands for the rate of electrochemical oxidation of $H_2$ and can be related to electric current density through Faraday’s law:

$$r_{ele} = j A_{ele} / 2F$$

(30)

$r_{rx}$ is the methane reformation reaction rate given by Eq. (28), and $\Delta n_{\text{shift}}^{\text{shift}}$ represents the molar change of species due to the water gas shift reaction.

All reaction rates are estimated based on the flow compositions at the fuel inlet edge of each control volume. In a coflow configuration, the flow compositions at all control volumes can be calculated node by node explicitly; while in a counterflow configuration, iteration is required to determine the correct $O_2$ outlet flow rates that satisfy mass conservation.

3.3 Energy Conservation. Fuel flow energy conservation takes into account the convective heat transfer with the PEN and the interconnect, as well as the heat exchange with the PEN due to electrochemical and reformation reactions. The following integral form of the energy conservation equation can be obtained:

$$\sum_k n_k(i)h_k(i+1) - \sum_k n_k(i+1)h_k(i) + K_{\text{fuel}}A_{\text{fuel-PEN}}(T_{\text{PEN}})$$

$$= T_{\text{fuel}}(i) + K_{\text{fuel}}A_{\text{fuel-IC}}(T_{\text{IC}}(i) - T_{\text{fuel}}(i)) - r_{rx}h_{CH_4}(i)$$

$$- r_{rx}h_{H_2O}(i) + r_{rx}h'_{CO}(i) + 3 r_{rx}h'_{H_2}(i) - r_{re}h_{H_2}(i) + r_{re}h'_{H_2O}(i)$$

$$= 0$$

(31)

where $k$ is $H_2$, $CH_4$, $CO$, $CO_2$, $H_2O$, $N_2$, and $Ar$.

It is assumed that for both reformation of $CH_4$ and electrochemical oxidation of $H_2$, the reactants are at the fuel (or air) temperature while the products are at the PEN temperature; thus, $h$ is determined based on local fuel (or air) temperature, while $h'$ is determined based on local PEN temperature.

Similarly, the air flow energy conservation equation is:

$$\sum_k n_k(i)h_k(i+1) - \sum_k n_k(i+1)h_k(i) + K_{\text{air}}A_{\text{air-PEN}}(T_{\text{PEN}})$$

$$= T_{\text{air}}(i) + K_{\text{air}}A_{\text{air-IC}}(T_{\text{IC}}(i) - T_{\text{air}}(i)) - \frac{1}{2} r_{re}h_{O_2}(i) = 0$$

(32)

where $k$ is $O_2$, $N_2$, $CO_2$, $H_2O$, and $Ar$.

$K_{\text{fuel}}$ and $K_{\text{air}}$ are the convective heat transfer coefficients, calculated from local Nusselt numbers obtained from empirical expressions, and $A$ is the area involved in the convective heat transfer process.

The energy conservation equation for the PEN accounts for heat conduction in axial direction (along the cell length), as well as between the PEN and interconnects (modeled by Fourier’s law), convective heat transfer between the PEN and the fuel and air flows, heat generation (positive or negative) due to electrochemical and reformation reactions, as well as the electric work produced by the cell:

$$T_{\text{PEN}}(i+1) - T_{\text{PEN}}(i) - T_{\text{PEN}}(i+1) + T_{\text{IC}}(i) - T_{\text{PEN}}(i)$$

$$= R_{\text{PEN}} \left( T_{\text{PEN}}(i) - T_{\text{PEN}}(i+1) + T_{\text{IC}}(i) - T_{\text{PEN}}(i) \right) + K_{\text{fuel}}A_{\text{fuel-PEN}}(T_{\text{fuel}}(i) - T_{\text{PEN}}(i))$$

$$+ K_{\text{fuel}}A_{\text{fuel-IC}}(T_{\text{IC}}(i) - T_{\text{fuel}}(i)) + K_{\text{air}}A_{\text{air-PEN}}(T_{\text{air}}(i) - T_{\text{PEN}}(i))$$

$$= - r_{rx}h_{CH_4}(i) - r_{re}h_{H_2}(i) + \frac{1}{2} r_{ele}h_{O_2}(i) - r_{re}h'_{H_2O}(i) = 0$$

(33)

The energy conservation equation for the interconnect accounts for axial heat conduction, as well as conduction between interconnect and PEN, and convective heat transfer between interconnect and fuel and air flows:

$$T_{\text{IC}}(i+1) - T_{\text{IC}}(i) - T_{\text{IC}}(i+1) + T_{\text{PEN}}(i) - T_{\text{IC}}(i)$$

$$= R_{\text{IC}} \left( T_{\text{IC}}(i) - T_{\text{IC}}(i+1) + T_{\text{PEN}}(i) - T_{\text{IC}}(i) \right) + K_{\text{fuel}}A_{\text{fuel-IC}}(T_{\text{fuel}}(i) - T_{\text{IC}}(i)) + K_{\text{air}}A_{\text{air-IC}}(T_{\text{air}}(i) - T_{\text{IC}}(i))$$

$$= 0$$

(34)

The fuel and air inlet temperature constitute boundary conditions for the fuel and air energy conservation equations. The boundary conditions for the PEN and interconnect can be either adiabatic or controlled by radiation heat transfer to a chamber environment of fixed temperature, as described in Sec. 2.2.

3.4 Solution Scheme. The fuel cell model consists of two interacting modules: the “species conservation” (SC) module (de-
described in Sec. 3.2) and the “energy conservation” (EC) module (described in Sec. 3.3). The SC module calculates the chemical species profiles and current density distribution in the fuel cell. These data are then passed as inputs to the EC module, which calculates temperature distribution, heat transfer, and heat loss throughout the fuel cell. The calculation results from the EC module are then passed back as inputs to the SC module for an update. This iterative calculation process repeats until the temperature field difference between two consecutive iterations is smaller than a predefined residual error, at which point the calculation is considered converged.

To improve calculation speed, so that the model can be called within systems analysis tasks, the specific enthalpies of the species, which are typically characterized by high order functions of temperature, are linearized as follows:

\[ h_1 = a + bT \quad (k = H_2, CH_4, CO, CO_2, H_2O, O_2, N_2, Ar) \]  

This simplification is only valid for a reasonably narrow range of temperatures consistent with SOFC operation. With this simplification, the energy conservation equations can be written into four tridiagonal matrices, which can be solved very efficiently by the tridiagonal matrix algorithm (TDMA) [24].

The model can work as a standalone SOFC model or as an integrated user-defined block in chemical flow sheet software (e.g., ASPEN PLUS®). The results presented here were produced by the standalone SOFC model running in MATLAB®. The same model has been implemented in FORTRAN code and successfully linked to ASPEN PLUS® through a user-defined communication interface.

### 4 Model Verification

The model was verified using the planar SOFC modeling benchmark developed by the IEA [18]. The benchmark contains two cases of SOFC operation: (1) one-cell operation with humidified H\(_2\) fuel and ambient air feed and (2) one-cell operation with direct internal steam reformation of CH\(_4\) and air. The two cases are designated “Benchmark 1” and “Benchmark 2,” respectively; and the operating conditions for the two cases are listed in Table 2.

It is important to clarify the way air ratio is defined in order to make a consistent comparison. In this work, the air ratio is defined as the ratio of actual air molar flow rate to the stoichiometric air molar flow rate that is required to consume all the incoming fuel; the original number based on this definition is 7. The number here is converted to be consistent with the air ratio definition used in this work.

Some sources (including the IEA Benchmark) use a different definition, and these values have been converted appropriately here.

As stated previously, the parameters for activation polarization vary among different literature sources. For verification, the data sets used by Campanari and Iora [11] (Calculation I) and Costamagna et al. [8] (Calculation II) were both tested and the simulation results from these tests are listed in Tables 3 and 4.

For Benchmark 1, the performance predicted by this model closely agrees with the benchmark performance. For Benchmark 2 the model predicts a slightly lower voltage than the benchmark results. The discrepancy is likely related to activation and diffusion polarization parameters that differ from those used in the IEA Benchmark.

#### Table 2 IEA Benchmark parameters and conditions (cited from Ref. [18])

| Parameter                          | Benchmark 1 | Calculation I | Calculation II |
|------------------------------------|-------------|---------------|---------------|
| Inlet gas composition              |             |               |               |
| Fuel: 26.26% H\(_2\), 10% H\(_2\)O |             |               |               |
| Air: 21% O\(_2\), 79% N\(_2\)      |             |               |               |
| Fuel utilization                   | 85%         |               |               |
| Mean current density               | 3000 A m\(^{-2}\) |   |               |
| Inlet gas composition (Benchmark 1)| Fuel: 90% H\(_2\), 10% H\(_2\)O (mole fraction) |   |               |
|                                    | Air: 21% O\(_2\), 79% N\(_2\) (mole fraction) |   |               |
| Inlet gas composition (Benchmark 2)| Fuel: 26.26% H\(_2\), 17.1% CH\(_4\), 2.94% CO, 4.36% CO\(_2\), 49.34% H\(_2\)O (mole fraction) |   |               |
|                                    | Air: 21% O\(_2\), 79% N\(_2\) (mole fraction) |   |               |

#### Table 3 Model verification results for IEA Benchmark 1

| Parameter           | Benchmark I | Calculation I | Calculation II |
|---------------------|-------------|---------------|---------------|
| Voltage (V)         | High/low    | 0.722/0.702   | 0.715/0.704   |
| Current density (A m\(^{-2}\)) | High/low    | 3957/3725    | 3961/3780    |
| Max                  | 1366/1020   | 977/1190     |               |
| Min                  | 1371/1321   | 1333/1337    |               |
| PEN temperature (K)  | High/low    | 1203/1182    | 1190/1189    |
| Max                  | 1340/1321   | 1332/1335    |               |
| Min                  | 1341/1321   | 1333/1337    |               |

| Outlet gas temperature (K) | High/low | 1340/1321 | 1332/1335 |
| Fuel                 |          | 1341/1321 | 1333/1337 |
5 Model Results

5.1 Intermediate Temperature Anode-Supported SOFC Performance. Many developers are now focusing on SOFC that operate at reduced temperatures (823–1123 K), enabling the use of a wider range of materials (especially metallic interconnects) and more cost-effective fabrication. Also, anode-supported SOFCs that minimize ohmic losses through use of a very thin electrolyte are commonly used. In this work, the SOFC model is applied to an intermediate temperature, anode-supported SOFC with metallic interconnects. The fuel cell geometry and operating conditions are listed in Table 5. The cell working voltage is set to 0.7 V (which is reasonable for comparison to recent literature results for SOFCs), and the resulting current and power density distributions are calculated.

Results are listed in Table 6. At these operating conditions, cell performance is poor despite the reduced ohmic resistance. Activation polarizations dominate the losses and far outweigh the ohmic loss, a result that is not consistent with observations of modern SOFCs (e.g., those recently reported by Solid State Energy Conversion Alliance industry teams [25]), where ohmic loss is smaller than activation loss and overall performance is much greater. This indicates that the activation loss parameters used by Campanari and Iora [12] and Costamagna et al. [8] are not appropriate for state-of-the-art SOFCs operating at intermediate temperatures. Updated activation loss parameters are therefore required. However, such parameters are difficult to obtain because most of the detailed information on materials, microstructure, and properties are proprietary to developers and very rarely can be found in the published literature. Thus, a sensitivity analysis of the activation parameters was conducted to determine appropriate parameters for predicting state-of-the-art intermediate temperature SOFC performance.

SOFC developers have recently shown significant performance improvements compared with literature values. For example, GE has reported a 0.480 W cm⁻² power density at 0.8 V and 84% fuel utilization operating on simulated high H₂ syngas in a single cell at a uniform temperature of 1073 K [25]. Delphi has demonstrated a 0.725 W cm⁻² power density at 0.8 V for a five-cell stack with fuel containing 48.5% H₂ and 3% H₂O (balanced by N₂) at 1023 K [26]. It is expected that recent developments in SOFC technology would significantly reduce the activation energy for electrode-electrolyte interface charge transfer. Thus the sensitivity analyses vary the activation energies in Eqs. (7) and (8) to identify parameters that can produce performance consistent with recent data from state-of-the-art SOFC. The geometry and operation conditions are the same as those listed in Table 5, except that the operating voltage is increased from 0.7 V to 0.8 V, so that

| Parameter | Benchmark 2 | Calculation I | Calculation II |
|-----------|-------------|---------------|---------------|
| Voltage (V) | High/low | 0.64/0.633 | 0.626 | 0.607 |
| Current density (A m⁻²) | High/low | 3665/3040 | 3686 | 3718 |
| Max PEN temperature (K) | High/low | 1307/1294 | 1298 | 1304 |
| Min Outlet gas temperature (K) | High/low | 1135/1120 | 1120 | 1120 |
| Air | 1299/1289 | 1296 | 1301 |
| Fuel | 1299/1294 | 1298 | 1304 |

Table 4 Model verification results for IEA Benchmark 2

| Parameter | Benchmark 2 | Calculation I | Calculation II |
|-----------|-------------|---------------|---------------|
| Voltage (V) | High/low | 0.64/0.633 | 0.626 | 0.607 |
| Current density (A m⁻²) | High/low | 3665/3040 | 3686 | 3718 |
| Max PEN temperature (K) | High/low | 1307/1294 | 1298 | 1304 |
| Min Outlet gas temperature (K) | High/low | 1135/1120 | 1120 | 1120 |
| Air | 1299/1289 | 1296 | 1301 |
| Fuel | 1299/1294 | 1298 | 1304 |

Table 5 Parameters and operation conditions for intermediate temperature anode-supported SOFC test

| Cell single channel geometry |
|------------------------------|
| Channel length | 300 mm |
| Channel width | 3 mm |
| Fuel channel height | 1 mm |
| Air channel height | 2 mm |
| Anode thickness | 1 mm |
| Cathode thickness | 0.05 mm |
| Electrolyte thickness | 0.01 mm |
| Bipolar plate thickness | 3.5 mm |
| Rib width | 2.42 mm |

| Material properties |
|---------------------|
| Thermal conductivity of PEN | 2 W m⁻¹ K⁻¹ |
| Thermal conductivity of interconnect | 25 W m⁻¹ K⁻¹ |
| Anode, cathode, electrolyte conductivities | Same as listed in Table 1 |
| Interconnect resistivity | Negligible |

| Operation conditions |
|----------------------|
| System pressure | 1 bar |
| Periphery conditions | Adiabatic |
| Inlet temperature (air and fuel) | 973 K |
| Air ratio (O₂ basis) | 7 |
| Fuel utilization | 85% |
| Working voltage | 0.7 V |
| Inlet gas composition | Fuel: 90% H₂; 10% H₂O (mole fraction) |
| | Air: 21% O₂; 79% N₂ (mole fraction) |
recent improvements can be better simulated.

For modern SOFC, very small anode side activation losses, on the order of several mV, are expected. Cathode side activation losses, on the other hand, are generally higher and are expected to be on the order of 100 mV, while ohmic losses lie somewhere between. The results of the sensitivity analyses are listed in Table 7. Note that Test 3 achieves reasonable SOFC performance with the various loss terms in the expected range. Thus, the Test 3 parameters have been used in all subsequent analyses.

5.2 SOFC Performance on Humidified H2 Fuel. Using parameters obtained from the sensitivity analysis, an anode-supported SOFC operated at intermediate temperature was simulated. The model is designed to be used for coal-based IGFC system analyses; however, syngas compositions can vary significantly depending upon the various gasification and gas cleanup system analyses; however, syngas compositions can vary significantly depending upon the various gasification and gas cleanup processes that can be employed. Fortunately, the two gas compositions used in the IEA benchmark (Table 2) can be thought of as representative of the two categories that are of great interest to IGFC operation with CO2 separation and thus can still be employed here for consistency and simplicity. The humidified H2 case is representative of syngas after water gas shift reaction followed by CO2 capture; the second case, containing about 17% (mole fraction) CH4, is consistent with recent growing interest in employing direct internal reforming in SOFC operation coupled with lower temperature gasification for better heat integration.

The model predicts profiles of species mole fractions, temperatures, and all electrochemistry-related variables. Figures 4 and 5 present results for a SOFC operating on humidified H2 (the Benchmark 1 composition indicated in Table 2) in a coflow configuration.

Figure 4(a) presents the mole fraction profiles along the cell length of the gas species in the fuel channel. As expected, the H2 mole fraction decreases and H2O mole fraction increases along the flow direction. Figure 4(b) shows the temperature distribution along the cell length. All four temperatures increase monotonically along the flow direction. Fuel, PEN, and interconnect temperatures are very close to each other, while the air temperature is consistently lower. This is reasonable since, in this case, the air is the major sink for the heat generated by the electrochemical reactions. The slope of temperature increase is smaller at the fuel and air exit due to the slower hydrogen electrochemical oxidation (smaller local current density).

Profiles of current density, the Nernst potential, and various electrochemical loss terms are presented in Fig. 5. The Nernst potential decreases monotonically along the flow direction due to the temperature increase and reactant consumption. The current density peaks at about 1/3 of the channel length from the fuel inlet edge. This is because although the Nernst potential decreases monotonically along the cell length, the increasing temperature improves reaction rates and reduces some polarization terms (e.g., activation and ohmic polarization). Further downstream the reduction of polarization is not sufficient to compensate for the loss in Nernst potential, and the local current density begins to drop. The local current density is significantly lower at the fuel exit than at the inlet. As expected, activation polarization is the dominant loss term, followed by ohmic polarization. These results provide insights that are helpful for cell design, for example, by estimating the usefulness of the latter part of the channel or determining whether or not it is cost-effective to push the fuel utilization in a single pass to a very high level given the very low local current density near the fuel exit edge.

For the coflow geometry and H2 operation, the minimum and maximum fluid temperatures occur at the inlet and outlet of the SOFC, respectively. As a result, the insights provided by a dimensional SOFC model compared with a nondimensional thermodynamic model are useful, but not as consequential. On the other
hand, for operation on fuels that contain significant CH$_4$ concentrations where internal reformation is active, the internal profiles become much more complicated and a thermodynamic model will not generally be sufficient to resolve the conditions. Dimensional models may also be required when considering other cell configurations, like counter- or cross-flow.

For the H$_2$ fuel counterflow configuration, the predicted trends of H$_2$ and H$_2$O mole fractions along the cell length are similar to those of the coflow case. The internal peak temperature is again observed very close to the air outlet, which in this case is the fuel inlet. In the counterflow configuration, the fuel outlet temperature is low (approximating the air inlet temperature), due to the fact that fuel flow does not contribute significantly to heat removal from the cell. The result is a slightly higher air outlet temperature than that predicted for the coflow case (1143 K versus 1129 K in the current example). Also, because the temperatures are highest at the fuel inlet, where the fuel concentration is also greatest, this case results in a steeper current density distribution and a slightly higher overall power density at a constant cell voltage (0.435 W cm$^{-2}$ versus 0.416 W cm$^{-2}$). The cell performances for both coflow and counterflow hydrogen cases are listed in Table 8.

5.3 SOFC Performance on CH$_4$ Containing Fuel With Internal Reformation. Figures 6 and 7 present results for the SOFC operating on CH$_4$ containing syngas in a coflow configuration. Figure 6(a) presents the mole fraction distributions. Because of methane reforming and water gas shift reaction, the H$_2$ concentration first increases while the H$_2$O concentration decreases. CH$_4$ is completely consumed by about 2/3 of the flow channel. Due to the endothermic methane reforming reaction, there is a temperature dip near the fuel inlet edge, as can be seen in Fig. 6(b). Still the temperature of the fuel, PEN, and interconnect are very close to one another. The air temperature is higher than the PEN (and

![Fig. 5 Predicted working voltage, current density, and contribution of all the various polarization terms along the cell length for humidified H$_2$, coflow operation](image)

![Fig. 7 Predicted working voltage, current density and contribution of all the various polarization terms along the cell length for CH$_4$ containing syngas, coflow operation](image)

![Fig. 6 Fuel channel species mole fractions (a) and temperature distributions (b) along the cell length for CH$_4$ containing fuel with internal reformation, coflow operation](image)

|                   | Coflow | Counterflow |
|-------------------|--------|-------------|
|                   | Humidified H$_2$ | CH$_4$ containing | Humidified H$_2$ | CH$_4$ containing |
| Average current density (A cm$^{-2}$) | 0.52 | 0.30 | 0.54 | 0.42 |
| Average power density (W cm$^{-2}$) | 0.42 | 0.24 | 0.43 | 0.33 |
| Peak PEN temperature (K) | 1132 | 1062 | 1155 | 1105 |
| Lowest PEN temperature (K) | 1001 | 960 | 984 | 987 |
| Fuel outlet temperature (K) | 1132 | 1062 | 984 | 987 |
| Air outlet temperature (K) | 1129 | 1061 | 1143 | 1071 |

Table 8 Summary of SOFC performances using new parameters (cell operating at 0.8 V, fuel utilization=0.85, and air utilization=0.14)
fuel and interconnect) temperature near the inlet because the endothermic reaction reformation causes the PEN to serve as the heat sink in this region.

The current density peaks at a point further down the channel than in the H2 case, largely because of the cooling and additional H2 production that results from methane reforming (see Fig. 7). The activation polarization is more significant than that observed for the H2 case primarily because of the diluting impact of CH4 and other components in the fuel channel, which reduces the local Nernst potential.

Figures 8 and 9 present SOFC performance with CH4 containing syngas for the counterflow configuration. The species concentration distributions shown in Fig. 8(a) exhibit trends similar to those of the coflow case, except that CH4 is consumed faster in the counterflow configuration due to the higher temperatures near the fuel inlet. All CH4 is consumed in the first 1/3 of the cell length; while in the coflow case, CH4 is more gradually reformed along the fuel channel length, Figure 8(b) presents the internal temperature profiles, which are very different from any of the previous cases. The peak temperature position has moved inside the cell, away from the edges, and its magnitude is much greater than that of the inlet and outlet temperatures. The local current density exhibits a distribution that tracks the temperature profile, as can be seen in Fig. 9.

The high internal temperature in the counterflow case results from rapid methane reforming at the fuel inlet producing a very high local H2 concentration. The H2 is in-turn consumed very rapidly at the high local temperatures, causing the local current density to spike to nearly 0.7 A cm\(^{-2}\), approximately double the peak current density observed in the coflow case. The resulting cell power density is 39% greater than in the coflow case, which goes hand in hand with the higher average and peak SOFC temperatures and steep temperature gradients. In the counterflow configuration there are steeper local temperature gradients, either with humidified H2 or CH4 containing syngas than in the coflow case. Further, the maximum local cell temperatures can be significantly higher than those observed at either the inlet or the outlet. Aguiar et al. [7] observed similar modeling results using a finite difference model. Steep temperature gradients can lead to thermally induced fractures of SOFC ceramic components, and excessive local temperatures are associated with increased degradation rates. Therefore, it is important to understand and control internal temperature profiles, which are difficult to assess experimentally.

From the viewpoint of overall heat balance, CH4 containing fuel is capable of chemically recovering the heat generated inside the fuel cell channel and has the potential to cool the cell without high air flow. But results obtained in this work reveal that the concurrent processes of endothermic methane reforming and exothermic hydrogen electrochemical oxidation under SOFC operating conditions and with current SOFC materials sets do not necessarily counterbalance locally. The imbalanced local rates of reformation chemistry and electrochemistry lead to temperatures and gradients that are important to resolve and understand and that cannot be observed with a thermodynamic model.

A dimensional model is also needed to clarify the effects of SOFC design on performance. The cell performance for the modeled cases is listed in Table 8. For CH4 containing fuel, the performance improvement in the counterflow configuration is quite significant and related to the higher average cell temperature. From this point of view, it is preferable to use a counterflow configuration when operating with CH4 containing syngas, but the internal temperature profiles must be carefully monitored and controlled if this is to be enabled.

6 Summary and Conclusions

A finite volume SOFC model has been developed for coal-based IGFC systems analysis. The model solves species conservation and energy conservation equations, and contains an electrochemical model that accounts for various polarization mechanisms for SOFC operation. The developed model was first verified using IEA benchmark data showing that results well-matched the benchmarks. To overcome the problem that activation loss parameters available in literature cannot well simulate recent SOFC performance in the intermediate temperature range, a sensitivity analysis was conducted to identify a set of parameters that can match modern SOFC performance expectations. The model with new parameters was then used to investigate SOFC performance operating on two types of coal syngas (humidified H2 and CH4 containing syngas), under coflow or counterflow configurations. The counterflow configuration can generally produce higher current/power density, but has steeper local temperature gradients that have to be monitored and handled carefully. Except for the
relatively simple coflow humidified H₂ operation, SOFC operation generally results in complicated internal temperature, species, and current density profiles, which are beyond the resolving ability of a thermodynamic model. These results demonstrate the necessity of employing a detailed dimensional SOFC model in systems analysis to avoid erroneous or misleading conclusions. Future work will apply the developed model in detailed coal-based IGFC systems analysis work to better address the intrinsic constraints for SOFC under various system configurations.

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Nomenclature

\[
\begin{align*}
A &= \text{area, m}^2 \\
D_{i,\text{eff}} &= \text{effective diffusivity of species } i \text{ in porous materials, m}^2 \text{s}^{-1} \\
D_{i,j} &= \text{binary diffusivity of species } i \text{ in species } j, \text{ m}^2 \text{s}^{-1} \\
D_{K,i} &= \text{Knudsen diffusivity of species } i \text{ in porous materials, m}^2 \text{s}^{-1} \\
E^0 &= \text{ideal potential of H}_2 \text{ oxidation at ambient pressure, V} \\
E_{\text{act}} &= \text{activation energy, J mol}^{-1} \\
F &= \text{Faraday’s constant, 96,485.34 C mol}^{-1} \\
K &= \text{convective heat transfer coefficient, W m}^{-2} \text{K}^{-1} \\
K_e &= \text{equilibrium constant} \\
M &= \text{molecular weight, kg mol}^{-1} \\
R &= \text{ohmic resistance, } \Omega, \text{ or heat conduction resistence, W K}^{-1} \\
\rho_u &= \text{universal gas constant, 8.314 J mol}^{-1} \text{K}^{-1} \\
T &= \text{temperature, K} \\
V &= \text{voltage, V} \\
h &= \text{specific enthalpy of species, J mol}^{-1} \\
i &= \text{electric current, A} \\
j &= \text{electric current density, A m}^{-2} \\
e_0 &= \text{exchange current density, A m}^{-2} \\
k &= \text{thermal conductivity, W m}^{-1} \text{K}^{-1} \\
n &= \text{number of electrons transferred per reaction or mol flow rate, mol s}^{-1} \\
p &= \text{pressure, bar} \\
r_{\text{rx}} &= \text{rate of methane reforming reaction, mol s}^{-1} \\
r_{\text{ele}} &= \text{rate of electrochemical oxidation of H}_2, \text{ mol s}^{-1} \\
u_f &= \text{fuel utilization factor} \\
x_i &= \text{molar fraction of species } i
\end{align*}
\]

Greek Symbols

\[
\begin{align*}
\alpha &= \text{electron transfer coefficient or parameter in the methane reforming reaction rate expression} \\
\beta &= \text{parameter in the methane reforming reaction rate expression} \\
\gamma &= \text{pre-exponential factor in exchange current density calculation} \\
\delta &= \text{thickness, m} \\
\eta &= \text{polarization loss, V}
\end{align*}
\]

Subscripts

\[
\begin{align*}
\text{air} &= \text{air or air-side} \\
\text{amb} &= \text{ambient conditions} \\
an &= \text{anode} \\
cat &= \text{cathode} \\
ele &= \text{electrolyte or related to electrochemical oxidation of H}_2, \text{ mol s}^{-1} \\
fuel &= \text{fuel or fuel-side} \\
IC &= \text{interconnect} \\
PEN &= \text{positive-electrolyte-negative structure} \\
rx &= \text{methane reforming reaction} \\
\text{shift} &= \text{water gas shift reaction}
\end{align*}
\]

Superscripts

\[
\begin{align*}
an &= \text{anode} \\
b &= \text{bulk flow} \\
cat &= \text{cathode} \\
r &= \text{reaction site}
\end{align*}
\]

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