Monopoles in Abelian Polyakov gauge and projection (in)dependence of the dual superconductor mechanism of confinement

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Abstract

We discuss the effect of the choice of an Abelian projection on the dual superconductor mechanism of confinement in SU(2) gluodynamics. Using qualitative arguments we show that the dual superconductor Lagrangian corresponding to the Abelian Polyakov gauge has a different structure compared to the dual Lagrangian in the Maximal Abelian gauge. A difference between these Lagrangians reflects the fact that in continuum limit the monopoles should be static in the Abelian Polyakov gauge and therefore these monopoles cannot give rise to the confinement of static quarks. Using the Abelian Polyakov gauge as an example, we show that (i) the dual superconductor scenario of confinement may depend on the Abelian projection; (ii) the condensation of the Abelian monopoles – which may be realized in any Abelian projection – is necessary but not sufficient condition for confinement. These results do not exclude an existence of a class of Abelian gauges in which the dual superconductor scenario works well.

1. The confinement of color in QCD is one of the most interesting problems in quantum field theory. Numerical lattice simulations \[1, 2\] of non–Abelian gauge theories clearly indicate that the confinement of fundamental color charges is due to an appearance of the chromoelectric string spanned between them. Despite an analytical explanation of the string formation is still missing, the formation and properties of the chromoelectric string may be studied within effective models. For example, the dual superconductor mechanism \[3\] provides a natural scenario of the string formation in QCD. This mechanism suggests that the vacuum of QCD can be regarded as a media filled by condensed Abelian monopoles. One of the properties of the monopole condensate is the dual Meissner effect which squeezes the chromoelectric flux coming from the quarks into the flux tube. This flux tube is analogous to the Abrikosov vortex in an ordinary superconductor.

The basic element of the dual superconductor mechanism is the Abelian monopole. This object does not exist on the classical level in QCD. However, the monopoles can be identified with particular configurations of the gluon fields by the so–called Abelian projection formalism \[4\]. This formalism relies on a partial gauge fixing of the SU(N) gauge symmetry up to an Abelian subgroup. The Abelian monopoles appear naturally in the Abelian gauge as a result of the compactness of the residual Abelian group.
Various numerical simulations indicate that the Abelian monopoles are likely to be responsible for the confinement of quarks (for a review, see, e.g., Ref. \[5\]). The Abelian monopoles provide a dominant contribution to the tension of the fundamental chromoelectric string \[7, 8, 6\]. The monopole condensate is formed in the low temperature (confinement) phase and it disappears in the high temperature (deconfinement) phase \[9, 10\]. The energy profile of the chromoelectric string as well as the field distribution inside it can be described with a good accuracy by the dual superconductor model \[2\].

Most of the results supporting the dual superconductor scenario were obtained in the so called Maximal Abelian (MA) projection \[11\] of the SU(2) gauge theory. This gauge is defined by the maximization of the lattice functional (\(\sigma_i\) are the Pauli matrices),

\[
\max_\Omega R_{MA}[U^\Omega], \quad R_{MA}[U] = \sum_{s, \mu} \text{Tr} \left( \sigma_3 U(s, \mu) \sigma_3 U(s, \mu)^\dagger \right),
\]

with respect to the gauge transformations, \(U(s, \mu) \to U^\Omega(s, \mu) = \Omega(s) U(s, \mu) \Omega^\dagger(s + \hat{\mu})\). In the continuum limit a local condition of the maximization can be written in the form of the differential equation, \((\partial_\mu + igA_3^\mu)(A_1^\mu - iA_2^\mu) = 0\). Both the continuum gauge condition and the lattice functional \((1)\) are invariant under the residual U(1) gauge transformations,

\[
\Omega_{\text{Abel}}(\omega) = \text{diag}(e^{i\omega}, e^{-i\omega}),
\]

where \(\omega\) is an arbitrary function in continuum or on the lattice, respectively.

The MA gauge is a natural candidate for a realization of the dual superconductor scenario because the MA gauge makes the off–diagonal gluon fields of freedom as small as possible. Consequently, the diagonal (Abelian) components of the gluon field are expected to play a leading role compared to the off-diagonal gluons and the Abelian dominance \[7\] is a natural effect in the MA gauge. The MA gauge belong to the class of extremization gauges which also includes the Minimal Abelian gauge \[12\] defined by the minimization of the gauge functional \((1)\), and the Abelian gauges corresponding to minimization of the Abelian action and the Abelian monopole density \[13\].

There is also a class of the diagonalization gauges which contains Abelian gauges defined by the conditions of diagonalization of some functional \(X[U]\) with respect to gauge transformations \(\Omega\). To define an Abelian gauge the functional \(X[U]\) must belong to the adjoint representation of the SU(2) gauge group \[4\]:

\[
X[U] \to X[U^{(\Omega)}] = \Omega^\dagger X[U] \Omega.
\]

After the Abelian projection is fixed, the matrix \(X[U]\) becomes diagonal and the theory possesses the (residual) \(U(1)\) gauge symmetry \[2\]. The class of the diagonalization gauges includes, for example, the Abelian Polyakov (AP), the Abelian field strength and the Abelian butterfly gauges. These gauges correspond, respectively, to the diagonalization of the Polyakov line, the \(U_{1,2}\) plaquette variable and the butterfly operator \[14, 15\]. The butterfly operator is usually used in a simplest definition of the topological charge.

The success of the dual superconductor scenario in the MA gauge gives rise to the obvious question \[16\]: "Is the dual superconductor scenario gauge independent?". The positive answer
to this question would imply that the monopoles identified in any Abelian gauge are always associated with the confining configurations of the gluon field. The dual superconductor mechanism would get an additional support in this case because it is natural to think that the confinement – as a gauge–invariant phenomenon – can not be described by the gauge–dependent model. On the other hand the Abelian projection by itself can be considered as just a gauge–dependent tool to associate the confining gluon configurations with the Abelian monopoles. This tool may work well in one gauge and may not work in another gauge. Thus the negative answer to the above question would not be a setback for the dual superconductor hypothesis either. The negative answer would also imply that in one gauge the confining gluon configurations may be associated with the monopoles while in another gauge these configurations may be related to other objects. There are at least two examples in favor of this way of thinking: in the (indirect) Maximal Center gauge \[17\] the confining configurations are realized as center vortices, and in the Minimal Abelian gauge \[12\] the confining configurations are probably associated with the so-called ”minopoles”.

In the current literature there are conflicting opinions about the gauge independence of the dual superconductor mechanism. The important indication of the gauge independence is the observation of the Abelian and monopole dominance not only in the MA gauge \[7, 18\] but also in other Abelian gauges, for example in two Abelian projections corresponding to minimization of the Abelian action and Abelian monopole density \[13\]. In the AP gauge the Abelian dominance holds automatically \[19\]. A generalization of the MA gauge, called the Laplacian Abelian gauge, also possesses the Abelian and monopole dominance which are even stronger then in the MA gauge \[20\]. Lattice studies indicate that in the MA gauge, in the AP gauge and in the Abelian field strength gauge the vanishing of the Polyakov loop in the confinement phase of SU(2) and SU(3) theories is due to the Dirac string associated with the Abelian monopoles \[16\].

In Ref. \[14\] the monopole condensation in the SU(2) lattice gauge theory was numerically shown to be independent on the type of the Abelian projection. Moreover in various Abelian projections the monopole condensate vanishes at the temperature corresponding to the deconfinement phase transition. Similar effect was also observed for the SU(3) gauge model \[15\]. The Abelian gauges used in Ref. \[14, 15\] were the MA gauge, the AP, Abelian field strength and Abelian butterfly gauges. The gauge independence of the monopole condensate is also favored by the fact that in the SU(2) gluodynamics the London penetration length measured in the MA projection is the same as the one obtained without gauge fixing \[21\]. Note also that recently proposed gauge–invariant definition of the monopole \[22\] may also provide a gauge–independent description of the dual superconductor.

On the other hand there are indications that the monopole dynamics is affected by the choice of the Abelian projection. First of all, the length distribution of the monopole trajectories is a gauge dependent quantity \[19\]. In the MA gauge a typical monopole ensemble consists of one large infrared cluster and many short (ultraviolet) loops. The large monopole cluster (contrary to the ultraviolet clusters) is responsible for the dominant contribution to the string tension coming from the Abelian monopoles \[18\]. In the extremization gauges such as the MA gauge the infrared and ultraviolet clusters are clearly separable in histograms of the length distribution while in the diagonalization gauges this is no longer the case \[13\]. Moreover, the density of the monopoles in the diagonalization gauges is typically higher than in the
extremization gauges. However, these results can not be considered as a serious argument against the gauge independence since the above mentioned differences between extremization and diagonalization gauges are due to the short monopole loops which are irrelevant to the confinement of quarks [13]. Moreover, the monopole density is not an order parameter for the deconfining phase transition [23].

An evidence against the gauge independence was given in Ref. [24] where the effect of the choice of the Abelian projection on the properties of the Abelian projected chromoelectric string was studied. It was shown that the string in different projections looks differently: the correlation length (the inverse monopole mass) extracted from the string profile in the AP gauge is consistent with zero contrary to the MA gauge [24].

Another indication of the projection dependence is based on measurements of the chiral condensate in non–Abelian models with fermions. It was shown in Refs. [26, 25] that the chiral condensate is dominated by the contributions of the Abelian monopoles in the MA gauge of both SU(2) and SU(3) gauge theories. However, this is no longer the case in the Abelian field strength [26] and the AP [25] gauges. It was checked in Ref. [26] that in the field strength gauge the Abelian contribution to the chiral condensate does not scale properly towards continuum and chiral limits because it is insensitive to the lattice gauge coupling and to the quark mass.

Below we discuss the question of the projection (in)dependence of the dual superconductor scenario considering an effective dual superconductor model in the AP gauge. We restrict ourselves to the simplest case of the SU(2) gauge theory. We work in Euclidean space–time, and we use both lattice and continuum formulations of this theory.

2. The main objects in the dual superconductor scenario are the Abelian monopoles which are associated with singularities in the gauge fixing conditions. The easiest way to understand the appearance of the Abelian monopoles is to consider the diagonalization condition (3). If in some point \(x\) of the space–time the eigenvalues of the matrix \(X\) coincide,

\[
X[x, A] = \begin{pmatrix}
\lambda_1(x) & 0 \\
0 & \lambda_2(x)
\end{pmatrix}, \quad \lambda_1(x) = \lambda_2(x),
\]

(4)

then the gauge degrees of freedom can not be fixed completely. The Abelian fields are singular in such a point. In order for the two eigenvalues of the matrix \(X[U]\) to coincide, three independent equations must be satisfied [4]. Thus these singularities form closed loops in the four-dimensional space–time. It was argued in Ref. [4] that these loops correspond to trajectories of the Abelian monopoles which should be considered as additional degrees of freedom in the corresponding Abelian gauge. The closeness of the monopole loops indicates that the magnetic charge is a conserved quantity.

Now let us study explicitly the AP projection. This projection can be considered only in the case of non–zero temperatures when one of the directions of the Euclidean space–time is compactified (we call it ”time”, or, ”temperature”, direction). One can also study the zero–temperature case as a limit of arbitrarily small temperatures.

The AP projection is defined by the diagonalization condition (3) where the operator \(X[U]\) coincides with the matrix \(P[s, s_4; U]\) trace of which is equal to the Polyakov loop, \(P[s, U] =\)
\( \frac{1}{2} \text{Tr} P[\vec{s}, s_4; U] \). On the lattice this operator can be written as

\[
X[U] \equiv P[\vec{s}, s_4; U] = U_0(\vec{s}, s_4) \cdot U_0(\vec{s}, s_4 + 1) \cdots U_0(\vec{s}, s_4 - 1) \equiv \prod_{s_4'=s_4}^{s_4-1} U_0(\vec{s}, s_4') .
\]

(5)

where the product of the lattice gauge fields \( U_0 \) goes along the closed parallel to the time direction. The path starts and ends at the same point \((\vec{x}, x_4)\).

Let us show that in the continuum limit all the Abelian monopoles in the AP gauge must be static. Here we follow Refs. [27, 28]. In the continuum limit the analog of the matrix (5) to be diagonalized is

\[
P_x[x, A] = \mathcal{T} \exp \left\{ \frac{i}{2} \oint_{C_x} dx_0 \sigma_i A'_0(x) \right\} ,
\]

(6)

where the symbol \( \mathcal{T} \) means the path ordering and the integration goes over the same closed path which starts and ends in the same point \( x \).

Suppose, that a monopole trajectory passes through the point \( x \). According to the definition by ’t Hooft [4] in this point the eigenvalues of the Polyakov loop coincide with each other. This immediately implies that the matrix (6) – being an element of the SU(2) group – must belong to the center of the SU(2) group, \( \mathbb{Z}_2 \), \( P[x, A] = \pm \mathbb{I} \). Consequently, the Polyakov loop is \( P[\vec{x}] \equiv \frac{1}{2} \text{Tr} P[x, A] = \pm 1 \). To prove the static nature of the monopole current let us consider another point \( y \) which lies on the same Polyakov loop (in other words, \( y_i = x_i, \; i = 1, 2, 3 \) while \( y_4 \neq x_4 \)). Due to the cyclic nature of the trace operation, \( \frac{1}{2} \text{Tr} P[y, A] \equiv \frac{1}{2} \text{Tr} P[x, A] = \pm 1 \). Thus, the eigenvalues of the matrix \( P[y, A] \) in the point \( y \) must also belong to the center of the group, \( P[y, A] = \pm \mathbb{I} \). Thus, we conclude, that if an Abelian monopole passes through the point \( x = (\vec{x}, x_4) \) it must also pass through all points \( y \) with the same spatial coordinates: \( y = (\vec{x}, y_4) \) for all \( y_4 \). Thus in the Polyakov Abelian projection all Abelian monopole trajectories must be static\(^1\).

Similar derivation can be done on the lattice using the lattice definition of the diagonalization matrix (4) along with the ’t Hooft’s condition of the monopole positions (4). However, in real lattice simulations the determination of the monopole positions using Eq. (4) is problematic because an exact equality of the Polyakov matrices (5) to an element from the center of the SU(2) group is a very rare event. This is expected because the exact coincidence of a point–like monopole position with the lattice site is highly improbable. Therefore a widely accepted way of the monopole localization on the lattice uses a lattice version of the Gauss theorem known as the DeGrand–Toussaint (DGT) construction [29]. In this construction the monopole positions are identified in 3D cubes which are sources of the magnetic flux. The disadvantage of this construction is that in the AP gauge it produces many short monopole loops with the length of the order of the lattice spacing \( a \), as observed in Ref. [13]. These loops couple geometrically to each other and a large number of the short loops makes a practical observation of the static monopole trajectories impossible on the lattices with a moderate lattice spacing. However, one may naturally expect that in the continuum limit, \( a \rightarrow 0 \), the DGT and ’t Hooft definitions

\(^1\)Note that a spatially degenerate configuration containing infinitely closed points \( \vec{x}_1 \) and \( \vec{x}_2 \) such that \( P[\vec{x}_1] = P[\vec{x}_2] \in \mathbb{Z}_2 \) does not correspond to a monopole.

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of the monopole loops should coincide with each other giving the static monopole trajectories only.

Note that the Polyakov loops in the AP gauge are Abelian by definition and the Abelian dominance holds automatically in this gauge. However, the Abelian nature of these loops seems to be inconsistent with the dual superconductor scenario because the key objects in this scenario, the Abelian monopoles, are static in the AP gauge and therefore these monopoles can not contribute to the Polyakov loop correlators.

3. Now let us derive an effective dual superconductor model in the AP gauge. The standard dual superconductor model corresponding to the SU(2) gauge theory in the 4D Euclidean space is described by the dual Ginzburg–Landau (DGL) Lagrangian [30]:

\[ L_{DGL}[B, \Phi] = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |D_\mu(B) \Phi|^2 + V(\Phi) , \]  

(7)

where \( F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \) is the field strength of the dual gauge field \( B_\mu \), \( \Phi \) is the monopole field with the magnetic charge \( g_M \), and \( D_\mu = \partial_\mu + ig_M B_\mu \) is the covariant derivative. The gauge field \( B_\mu \) is dual to the third component of the gluon field, \( A_3^\mu \), in an Abelian gauge. The model possesses the dual \( U(1) \) gauge symmetry, \( B_\mu \rightarrow B_\mu - \partial_\mu \alpha \), \( \Phi \rightarrow e^{ig_M \alpha} \Phi \). The form of the potential

\[ V(\Phi) = \lambda \left( |\Phi|^2 - \eta^2 \right)^2 , \]  

(8)

implies the existence of the monopole condensate, \( \langle |\Phi| \rangle = \eta > 0 \).

To derive this effective model the authors of Ref. [30] started from the partition function of the monopole currents:

\[ Z_{\text{mon}} = \int \mathcal{D}k \exp \left\{ -\frac{g_M^2}{2} \int d^4x \int d^4y \ k_\mu(x)D(x-y)k_\mu(y) - S_{\text{int}}(k) \right\} , \]  

(9)

where the first term in the action corresponds to the Coulomb interaction between the monopoles (\( D(x) \) is the inverse Laplacian) and \( S_{\text{int}}(k) \) is the action of the closed monopole currents \( k \),

\[ k_\mu(x) = \int d\tau \frac{\partial \tilde{x}_\mu(\tau)}{\partial \tau} \delta^{(4)}[x - \tilde{x}(\tau)] . \]

Here the four–dimensional vector \( \tilde{x}_\mu(\tau) \) represents the trajectory of the monopole current \( k \).

Representing the Coulomb interaction in Eq. (9) as an integral over the vector \( B_\mu \), we get\(^2\)

\[ Z_{\text{mon}} = \int \mathcal{D}k \int \mathcal{D}B \exp \left\{ -\int d^4x \left[ \frac{1}{4} F_{\mu\nu}^2 + ig_M k_\mu(x) B_\mu(x) \right] - S_{\text{int}}(k) \right\} , \]  

(10)

A further integration over monopole trajectories \( k \) in Eq. (10) leads to the effective Lagrangian (7). The potential term (3) in Eq. (7) comes from the self–interaction of the monopole trajectories, \( S_{\text{int}}(k) \), Ref. [30].

\(^2\)Here and below we omit the constant pre–factors in the partition functions.
The DGL model \([7]\) was shown to correctly describe the profiles of the chromoelectric string \([2]\) as well as the monopole actions \([31]\) obtained in numerical simulations of the lattice SU(2) gauge theory in the MA gauge.

The easiest way to derive the form of an effective action in the Polyakov gauge is to use the formalism of differential forms in the lattice regularization described in Ref. \([32]\) and also in the second paper of Ref. \([5]\). Let us perform a lattice derivation of the Lagrangian \([7]\) from Eq. \([9]\). Assuming for simplicity the quadratic form of the self–interaction term, \(S_{\text{int}}(k)\), we get the lattice version of Eq. \([9]\):

\[
Z = \sum_{*k \in \mathbb{Z}^{(c_3)}} \exp\left\{ -\frac{1}{2\beta} (*k, \Delta^{-1} *k) - \mu^2 ||*k||^2 \right\}, \tag{11}
\]

where \(\Delta^{-1}\) is the inverse lattice Laplacian and \(*k\) is the closed (\(\delta *k = 0\)) monopole current defined on the dual lattice. The scalar product of two forms \(a\) and \(b\) belonging to the cells \(c\) (\(c = \text{sites, links}, \text{plaquettes} \text{ etc.}\) ) is denoted as \((a, b) = \sum_c a_c *b_c\) and \(||a||^2 \equiv (a, a)\). The parameter \(\beta\) is the lattice gauge coupling and the parameter \(\mu\) defines the strength of the self–interactions of the monopole currents.

Following Ref. \([33]\) we can represent the partition function \([11]\) in the form:

\[
Z = \int \mathcal{D}^* B \int \mathcal{D}^* \varphi \int \mathcal{D}^* G \sum_{*k \in \mathbb{Z}^{(c_3)}} \exp\left\{ -\frac{\beta}{2} ||d^* B||^2 - \frac{1}{4\mu^2} ||d^* B||^2 - \frac{1}{4\mu^2} ||*G||^2 \right\}
\]

\[
+ \left\{ i(*B + d^* \varphi, *k) + i(*G, *k) \right\}, \tag{12}
\]

where we have introduced two Gaussian integrations, represented a closeness condition as an integral over the compact scalar field \(\varphi\), and used the lattice integration by parts, \((d^* \varphi, *k) \equiv (*\varphi, \delta *k)\). Using the Poisson formula, \(\sum_{m \in \mathbb{Z}} e^{imx} \propto \sum_{m \in \mathbb{Z}} \delta(2\pi m - x)\), we get the constraint in Eq. \([12]\), \(*G = d^* \varphi + *B + 2\pi *l, *l \in \mathbb{Z}^{(c_3)}\). Integrating over the field \(*G\) we get the following partition function,

\[
Z = \int \mathcal{D}^* B \int \mathcal{D}^* \varphi \sum_{*l \in \mathbb{Z}^{(c_3)}} \exp\left\{ -\frac{\beta}{2} ||d^* B||^2 - \frac{1}{4\mu^2} ||d^* \varphi + *B + 2\pi *l||^2 \right\}. \tag{13}
\]

This is nothing but the lattice version of the dual Ginzburg–Landau model \([7]\) in the London limit corresponding to the infinitely deep, \(\lambda \to \infty\), potential \([5]\) on the monopole field \(*\Phi\). In this limit the radial part of the Higgs field is frozen and only the phase \(*\varphi\) of the monopole field is active. In Eq. \([13]\) the interaction of the phase \(*\varphi\) of the monopole field with the dual gauge field is written in the Villain form \([34]\).

In the continuum limit the monopoles must be static in the AP gauge. To derive the AP analog of the DGL model we impose the suppression of the spatial currents in the partition function \([14]\). To this end we introduce two parameters, \(\mu_t\) and \(\mu_{sp}\), to control temporal and spatial currents, respectively:

\[
Z(\mu_t, \mu_{sp}) = \sum_{*k \in \mathbb{Z}^{(c_3)}} \exp\left\{ -\frac{1}{2\beta} (*k, \Delta^{-1} *k) - \mu_t^2 ||*k||_4^2 - \mu_{sp}^2 ||*\vec{k}||^2 \right\}, \tag{14}
\]
Here $||\mathbf{k}||_4^2 = \sum s^* k_{s,t}^2$ and $||\mathbf{\tilde{k}}||^2 = \sum_{s,i}^3 k_{s,i}^2$ are the self-interactions of, respectively, the temporal and the spatial currents. The AP projection corresponds to suppressed spatial currents, $\mu_{sp} \to \infty$.

Applying to Eq. (14) transformations which led us from Eq. (11) to Eq. (13) we get:

$$Z = \int \mathcal{D}^* B \int \mathcal{D}^* \varphi \sum_{t \in \mathbb{Z}}^\infty \exp\left\{-\frac{\beta}{2} ||\mathbf{d}^* B||^2 - \frac{1}{4\mu_t^2} ||d^* \varphi + * B + 2\pi^* l||^2_4 - \frac{1}{4\mu_{sp}^2} ||\mathbf{d}^* \varphi + * \mathbf{B} + 2\pi^* \mathbf{l}||^2_4\right\}. \quad (15)$$

One can immediately observe, that the suppression of the monopole currents corresponds to the disappearance of the spatial part of the interactions between the monopole field, $* \varphi$, and the dual gauge field, $* B$. Coming back to the continuum formulation of the dual superconductor model we conclude that in the continuum AP gauge the dual model should have the form

$$L_{DGL}^{AP}[B, \Phi] = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |D_4(B) \Phi|^2 + V(\Phi) \equiv \frac{1}{4} F_{ij}^2 + \frac{1}{2} F_{4i}^2 + \frac{1}{2} |D_4(B) \Phi|^2 + V(\Phi). \quad (16)$$

The difference between this model and Eq. (7) is in the spatial interaction of the gauge and the monopole fields. Effectively, the spatial and temporal degrees of freedom decouple from each other in the AP gauge. The temporal component of the gauge field, $B_4$, is interacting with the monopole field while the spatial components, $B_i$, $i = 1, 2, 3$, are free. From the point of view of the original non–Abelian theory the space–time asymmetry in the dual model (16) does not come as a surprise because even in the limit of the infinitely large fourth dimension the AP gauge condition (i.e., the diagonalization of the matrix (6)) violates the Lorentz symmetry.

4. Let us now discuss the consistency of the dual model (16) with known numerical results in the AP gauge of the SU(2) lattice gauge theory. First, we evaluate the monopole contribution to the quark potential. The quantum average of the Abelian Wilson loop corresponds to the 't Hooft loop in the dual representation (16):

$$\mathcal{H}_C[B] = \exp\left\{-\frac{1}{4} \int d^4x \left[\left(F_{\mu\nu} - \frac{2\pi}{g_M} * \Sigma_{\mu\nu}\right)^2 - F_{\mu\nu}^2\right]\right\}, \quad (17)$$

where $* \Sigma_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} \Sigma_{\alpha\beta}^C$. The two–dimensional $\delta$–function,

$$\Sigma_{\mu\nu}^C(x) = \int d^2 \tau \frac{\partial_{[\mu} \mathbf{x}(\tau)}{\partial \tau_1 \partial \tau_2} \delta^{(4)}(x - \mathbf{x}(\tau)),$$

represents a surface spanned on the trajectory of the particles $j_{j}^C$, $\partial_{\mu} \Sigma_{\mu\nu}^C = j_{j}^C$. The surface is parameterized by the vector $\mathbf{x}(\tau_1, \tau_2)$. The quantum average of the 't Hooft operator (17) does not depend on a particular the shape of the surface $\Sigma^C$. 

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The ’t Hooft operator \((17)\) effectively shifts the field strength, \(F_{\mu\nu} \rightarrow F_{\mu\nu} - \frac{2\pi}{gM} \Sigma_{\mu\nu}\). For static quark and anti–quark located, respectively, at positions \((0, 0, 0)\) and \((R, 0, 0)\) of the three-dimensional space, the surface \(\Sigma^C\) can be taken static and flat:

\[
\Sigma^{C}_{41}(x) = -\Sigma^{C}_{14}(x) = \Theta(x_1) \Theta(R - x_1) \delta(x_2) \delta(x_3)
\]

(18)

All other components of the surface delta–function \(\Sigma_{\mu,\nu}\) are equal to zero. Therefore, the dual surface \(\ast\Sigma^C\) has only two non–zero elements, \(\ast\Sigma^C_{23} = \ast\Sigma^C_{32} = \Sigma^C_{01}\), which belong to the three-dimensional part of the dual model \((16)\). The quantum average of such ’t Hooft operator can easily be taken:

\[
\langle H_C \rangle = \text{const. exp}\{-TV(R)\}, \quad V(R) = \frac{g^2}{2} D_{3D}(R),
\]

(19)

where \(D_{3D}(R)\) is the three dimensional inverse Laplacian, \(T\) is the length of the quarks trajectories, \(T \gg R\), and the prefactor does not depend on distance between the quarks, \(R\). The electric charge of the quark, \(g\), is related to the magnetic charge of the monopole, \(g_M\), according to the Dirac condition, \(g \cdot g_M = 2\pi\).

Thus, in the dual model corresponding to the AP gauge the static quarks are not confined. This is expected, because the monopoles are static in this gauge as we have already discussed. In numerical simulation the string between the static color charges should be seen as an infinitely thin object in the AP gauge because in the Lagrangian \((16)\) the spatial components of the dual gauge field \(B_i\) do not couple to the monopole field, \(\Phi\), and, consequently, the monopole field does not feel the electric flux of the string. This prediction agrees nicely with the observation of Ref. \([24]\), where the thickness of the Abelian string in terms of the monopole field \((i.e.,\) the correlation length) was found to be consistent with zero in the AP gauge.

Another interesting observation is that the fact of the absence of the confinement for the static quarks in the AP gauge is correct for any form of the potential of the monopole field, \(V(\Phi)\). Indeed, the crucial fact of our derivation was the specific form of the coupling of the monopole field to the dual gauge field in the Lagrangian \((16)\) while a particular form of the potential \(V(\Phi)\) has not played any role. Thus, the presence of the monopole condensate does not mean by itself that the quarks must be confined. The interaction between the monopole field and the dual gauge fields are also essential for the dual superconductor scenario of the confinement.

5. Summarizing, we have proposed a form for an effective Lagrangian of the dual superconductor in the Abelian Polyakov gauge of the SU(2) gauge theory. Our derivation has a qualitative nature based on the fact that the monopoles in this gauge must be static in the continuum space–time. The monopole condensation – as it follows from our study – is necessary but not sufficient condition for the explanation of the confinement phenomena within the dual superconductor models.

Another consequence of our study is that the dual superconductor scenario of confinement is projection–dependent\(^3\). This statement does not contradict the observation of the independence

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\(^3\)Note that our conclusion on the projection dependence is based only on the comparison of the MA and AP gauges. Thus, our results do not exclude a possible existence of a class of Abelian gauges (including the Maximal Abelian gauge) in which the dual superconductor scenario works well.
of the monopole condensation on the choice of the Abelian projection \cite{10, 15, 14}, because – as we have shown – the monopole condensation by itself does not guarantee the confinement of color. On a technical level, the fact of existence of the monopole condensate relies on the form of the potential of the monopole field, while the quark–anti-quark potential is also dependent on other terms in the dual superconductor Lagrangian. It seems that the most complete information about the monopole Lagrangian in a particular Abelian projection can be extracted from the effective monopole action on the lattice \cite{35}. The monopole action may further be used \cite{31, 36} to determine the form and the parameters of the dual superconductor Lagrangian in the continuum limit.

These results do not exclude the existence of a wide class of Abelian gauges (including the Maximal Abelian gauge) in which the dual superconductor scenario works well.

Since the monopoles in the AP gauge are static they may contribute to the string tension corresponding to the spatial Wilson loops. Moreover, the Pontryagin index of the gauge field is related to the magnetic charges defined in an axial version of the AP gauge \cite{37}. Therefore, in this gauge the magnetic monopoles are related to the nontrivial topological structures of a non–Abelian gauge theory (a generalization of this statement to other Abelian gauges can be found in Ref. \cite{38}). Another interesting fact about the Abelian monopoles is that on a classical level they are correlated \cite{39} with the positions of the constituent monopoles in the van Baal–Kraan calorons \cite{40}. Thus, the fact that the monopoles in the Abelian Polyakov gauge are not relevant to the confinement of static quarks does not mean that these monopoles do not carry interesting information about non–perturbative physics.

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