Inelasticity in hadron-nucleus collisions from emulsion chamber studies

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Abstract

The inelasticity of hadron-carbon nucleus collisions in the energy region exceeding 100 TeV is estimated from the carbon-emulsion chamber data at Pamirs to be $\langle K_C \rangle = 0.65 \pm 0.08$. When combined with the recently presented data on hadron-lead nucleus collisions taken at the same energy range it results in the $K \sim A^{0.086}$ mass number dependence of inelasticity. The evaluated partial inelasticity for secondary ($\nu > 1$) interactions, $K_{\nu>1} \simeq 0.2$, suggests that the second and higher interactions of the excited hadron inside the nucleus proceed with only slight energy losses.

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1 Introduction

The inelasticity of hadronic reactions, understood as the fraction of the incident beam energy not carried off by fragments of the projectile, is (next to the inelastic cross section) the most significant variable for all cosmic ray experiments involved in cascade developments [1, 2]. The low energy data (in the 100 − 200 GeV range) show that the stopping power of nuclei is rather low [3, 2]. At higher energies there is no accelerator data for inelasticity [4] and only rough indications from cosmic ray experiments are available [1, 2]. Recently [5] the inelasticity in hadron-lead collisions was estimated in the energy region exceeding 100 TeV. In the present paper we discuss hadron-carbon nucleus collisions observed by carbon emulsion chamber, which are exposed to cosmic rays at the Pamirs. In the next Section we present the experimental method used (which is similar to that used in [5] and more straightforward than the one explored in [3]). Section 3 contains our results, which, when combined with those of [5], allow us to deduce the mass number dependence of inelasticity directly from experimental data. In Section 4 we discuss the (model dependent) notion of partial inelasticity providing the information on the character of secondary interactions in the nuclei (albeit in a model dependent way). Last Section summarizes and concludes our presentation.

2 Experimental method - repeated registration of cascades

In the Pamir experiments, among others, a multi-layer X-ray film emulsion chambers (EC) with large area two-carbon-generators (the so-called hadronic (H) blocks) have been exposed [6]. The carbon chamber designed to observe hadrons consisted of: Γ-block of 6 cm Pb (corresponding to 0.35λ and 10.5 c.u.) and two H-blocks of carbon layer of 60 cm thickness each (66g/cm², 0.9λ, 2.5 c.u.) followed by 5 cm of lead-emulsion sandwiches, cf. Fig. 1. In EC (which is a shallow calorimeter) only the energy transferred to the electromagnetic component is measured:

\[ E_h^\gamma = K_{\gamma} \cdot E_h, \]

(here coefficient \( K_{\gamma} \) denotes the respective electromagnetic part of the inelasticity) and in the hadronic block a given nuclear-electromagnetic cascade (NEC) produces spots with optical density \( D \) on X-ray film. General methodical problem of hadronic block measurements of how to obtain the energy of the incoming hadron, \( E_h \), from data on the optical densities \( D \), i.e., the transition: \( D \longrightarrow E_h^\gamma \longrightarrow E_h \), was examined in [4].

This specific structure of the carbon-emulsion chamber allows for a relatively straightforward estimation of the total inelasticity for hadron-carbon nucleus interactions. Although such a possibility was pointed out already in Ref.[8] it was not utilised so far. We shall use it now to estimate inelasticity for hadron-carbon interactions at energies exceeding 100 TeV. The proposed method is connected with the repeated registration of the same NEC in the two subsequent hadronic blocks. If \( N_1 \) denotes the number of cascades registered in the first hadronic block (each cascade with visible energy greater than some threshold energy \( (E_h^\gamma)_1 \)) and \( N_2 \) denotes the number of cascades repeatedly
registered also in the second hadronic block (each cascade with visible energy above the threshold \( (E_h^\gamma)^2 \)), then it turns out that the quantity

\[
\eta = \frac{N_2}{N_1} \quad (2)
\]

is sensitive to the total (mean) inelasticity \( \langle K \rangle \). Similarly, for each event where NEC develops both in the upper and lower \( H \)-blocks depositing there energies \( E_1 \) and \( E_2 \), respectively, the ratio

\[
\epsilon = \frac{E_2}{E_1} \quad (3)
\]

also depends on \( \langle K \rangle \). The weak dependence of both quantities on the methodical errors (which to large extend cancel in the ratio) and ease with which the experimental data may by obtained render this method very useful and promising for possible future applications.

To illustrate sensitivity of both quantities, \( \epsilon \) and \( \eta \), on the inelasticity let us first consider simplified case of monochromatic beam of nucleons of energy \( E_0 \) entering our EC and let us neglect for a moment the NEC in the target. In this case for each event we have:

\[
\epsilon = \frac{E_2}{E_1} = \frac{(1 - \langle K \rangle) \cdot E_0 \cdot \langle K_{\gamma} \rangle}{\langle K_{\gamma} \rangle \cdot E_0} = 1 - \langle K \rangle, \quad (4)
\]

where \( \langle K \rangle \) is the (mean) total inelasticity. Notice that \( \langle K_{\gamma} \rangle \) from the eq.(1) drops out from the ratio \( \epsilon \). Similarly, the relative number \( \eta \) of hadrons repeatedly registered in the two subsequent \( H \)-blocks of thickness \( x/\lambda \) each is

\[
\eta = \frac{N_2}{N_1} = \frac{N_1}{N_1} \left(1 - e^{-x/\lambda}\right) \cdot \Phi (\langle K \rangle) = \left(1 - e^{-x/\lambda}\right) \cdot \Phi (\langle K \rangle), \quad (5)
\]

where \( \Phi = \int^{1}_{K_{\min}} \varphi(K) dK \) accounts for the energy thresholds \( E_1^{th} \) and \( E_2^{th} \), which lead to the fact that from the inelasticity distribution \( \varphi(K) \) only the inelasticities \( K > K_{\min} = E_2^{th} / (1 - \langle K \rangle) E_1^{th} \) are observed. In the case of \( \varphi(K) = \text{const} \) one gets \( \Phi (\langle K \rangle) = 1 - K_{\min} \).

However, in true event one has to account for the following facts:

(i) The incoming cosmic ray flux is not monochromatic but has typical energy spectrum \( N(E_0) \sim E_0^{-\gamma} \) and all energies should be considered. In the region of interest to us (i.e., at the mountain altitudes and energy region where data were collected) \( \gamma \simeq 3 \) [4, 4, 4]

(ii) Cosmic ray flux at mountain altitudes considered here contains not only nucleons but also mesons produced in previous cascading processes in the atmosphere [12].

(iii) In reality EC do not register individual hadrons but rather NEC developed by them. In Fig. 1 the incoming hadron originates in the upper \( H \)-block NEC, which then develops further. Its electromagnetic component is registered as visible energies \( E_1 \) and \( E_2 \) (cf. eq.(4)) released in the upper and lower \( H \)-blocks, respectively [14]. Each cascade is therefore recorded as single event with visible energies \( E_1 \) and \( E_2 \).

To account for these points one therefore has to resort to the Monte Carlo simulation calculations.
3 Inelasticity in hadron-carbon nucleus interactions

The experimental data collected from 110m² carbon EC contain $N_1 = 70$ cascades with energies $E_1 > 30$ TeV among which $N_2 = 24$ cascades have energies $E_2 > 2$ TeV). They give the value of $\eta = 0.27 \pm 0.06$ (at energy threshold $E_2 > 4$ TeV, being free from the detection bias) and the energy ratio $\epsilon = 0.24 \pm 0.07$. These data have been then recalculated by using the simulated $D(E_h)$ dependence [7]. The repeated registrations of cascades has been simulated by the standard SHOWERSIM Monte-Carlo event generator [15]. Primary hadrons (assumed to consist of 75% nucleons and 25% p ions [13, 12]) were sampled from the power spectrum representing distribution of the initial energy with a differential slope equal to $\gamma = 3$ [4, 11]. In each cascade gamma quanta and electrons above 0.01 TeV, reaching the detection level within the radius of 5 mm, were recorded and the corresponding optical densities were calculated within the radii utilized in the experiment. Only cascades with the energies above $E_1 = 30$ TeV and $E_2 = 2$ TeV were selected.

The ratio $\eta$ of the number of cascades repeatedly registered in two hadronic blocks and the number of all cascades registered in the first hadronic block is presented in Fig. 2 for different total inelasticities: $\langle K \rangle = 0.5, 0.65$ and 0.80. Note that the ratio $\eta$ is more sensitive to the mean value of inelasticity $\langle K \rangle$ than the energy ratio $\epsilon$, shown for illustration in Fig. 3. In Fig. 4 we show the $\chi^2$ per degree of freedom obtained for $\eta$ fits plotted as a function of the assumed inelasticity $K$. The comparison of experimental data with simulated dependences indicates that $\langle K_C \rangle = 0.65 \pm 0.08$ for hadron-carbon nucleus collisions at the hadron energies of above $\sim 100$ TeV is most probably choice for the mean value of inelasticity at this energy for hadron-carbon collisions. This consist the main result of our work.

Recently analysis of similar successive hadron interactions registered in other emulsion chambers exposed at Pamirs, in the so called thick-lead-emulsion chambers (60 cm Pb or 3.2 mean free paths $\lambda$ of inelastic collision of nucleon) have been reported [3]. The corresponding inelasticity distribution of hadron-lead collisions in the energy region exceeding 100 TeV was estimated by using distribution of the energy ratio $z = E_1/\sum E_i$ obtained from 74 events of hadron interactions. The resulting average value of the inelasticity is $\langle K_{Pb} \rangle = 0.83 \pm 0.17$. Comparing now this result with our estimation of inelasticity for hadron-carbon nucleus results in the following mass number dependence of inelasticity: $K \sim A^{0.086}$.

4 Partial inelasticity $K_\nu$

Following the work of Ref. [2] we shall now consider for hadron-nucleus collision the so called partial inelasticities $K_\nu$. This is model dependent quantity and in the framework of Glauber multiple scattering formalism [14] it is defined in the following way:

$$\langle 1 - K \rangle = \sum_{\nu=1} P_\nu \prod_{i=1}^{\nu} \langle 1 - K_i \rangle$$

(6)

where $P_\nu$ is the probability for encountering exactly $\nu$ wounded nucleons in a target of mass $A$ and $\langle 1 - K_i \rangle$ is the mean elasticity of the leading hadron in the encounter with
the \( i^{th} \) wounded nucleon. We assume now that partial inelasticity \( K_1 \) is determined by hadron-proton scattering and shall treat the remaining partial inelasticities \( K_{\nu>1} = K_2 \) as one free parameter \([2]\) constrained by fitting the \( h \)-nucleus data. The total elasticity can be now written as

\[
\langle 1 - K \rangle = (1 - K_1) \sum_{\nu=1}^{\nu<1} (1 - K_2)^{\nu-1} P_\nu.
\]

(7)

The ratio of elasticities in collisions on \( Pb \) and \( C \) targets,

\[
\kappa = \frac{\langle 1 - K_{Pb} \rangle}{\langle 1 - K_C \rangle},
\]

(8)

depends only on \( K_2 \) once the \( P_\nu \) is known. Assuming now, for simplicity, Poisson distribution for the number of repeated collisions,

\[
P_\nu = \frac{(\nu - 1)^{\nu-1}}{(\nu - 1)!} \exp(- <\nu - 1>) \quad \text{(for } \nu = 1, 2, \ldots, \text{)}
\]

(9)

we obtain that

\[
\kappa = \frac{\exp(-<\nu_{Pb} - 1>) K_2}{\exp(-<\nu_{C} - 1>) K_2} \quad \text{or} \quad K_2 = - \frac{\ln \kappa}{<\nu_{Pb} > - <\nu_{C}>}.
\]

(10)

In Fig. 5 we show, for different mass number dependence of mean number of wounded nucleons as provided by the exponent \( \alpha : <\nu> \sim A^\alpha \), the dependence of the partial inelasticity \( K_2 \) on the power index \( \alpha \) and for the value of \( \kappa = 0.5 \) which is obtained from the comparison of data on \( Pb \) and \( C \) nuclei. The value of \( K_2 \) for the expected mean number of wounded nucleons, \( <\nu> = A \sigma_{h-n}/\sigma_{hA} \sim A^{1/3} \), is therefore equal to \( K_2 \sim 0.2 \). Notice that there is tacit assumption made here behind this value of partial inelasticity \( K_2 \), namely that the ultimate identity of the final state nucleon is determined only once during the interaction with the nucleus (what in \([2]\) corresponds to the value \( \beta = 1 \) for the parameter specifying the fraction of isospin preserving reactions).

Our estimation of \( K_2 \) at energies above 100 TeV is consistent with low energy data (cf. Ref. \([4]\)). Note that inequality \( K_{\nu>1} < K_1 \) is characteristic to all string-type interaction models (cf. Quark-Gluon String model \([17]\) or Dual Parton Model \([18]\)). On the other hand the SIBYLL model \([2, 19]\) predict much smaller value of \( K_2 \) in the examined energy region. In DPM and QGS models, when only one target nucleon is wounded, a constituent quark (di-quark) belonging to the projectile hadron couples to a string that in turn connects to a di-quark (quark) belonging to the wounded nucleon. In the case where there are two or more wounded nucleons in the target, the additional nucleons can couple only to the sea quarks of the projectile. In this way the desired physics can be reproduced by the model. In particular, the excited hadron, being off mass-shell, does not interact repeatedly as a physical hadron inside the nucleus.

5 Summary

For hadron-carbon nucleus collisions in energy region exceeding 100 TeV the inelasticity is estimated to be equal to \( <K_C> = 0.65 \pm 0.08 \). This value, when compared with the value
\( \langle K_{Pb} \rangle = 0.83 \pm 0.17 \) obtained recently for hadron-lead collisions, results in the mass number dependence of inelasticity given by \( K \sim A^{0.086} \). Essentially the same \( A \)-dependence has been reported in [3] (the lower values of inelasticities obtained there can be attributed to the fact that in our case we estimate total inelasticity whereas in [3] inelasticity was estimated more indirectly from the production and distribution of charged secondary particles only). The evaluated partial inelasticity \( K_{\nu>1} = 0.2 \) leads to the (model dependent) conclusion that the second and higher interactions of the excited hadron inside the nucleus are relatively elastic. Our estimation of \( K_{\nu>1} \) at energies above 100 TeV [20] is consistent with the low energy data (\( \sim 100 \) GeV) and coincides with the string-type model predictions.
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[11] This fact means that experimentally selected inelasticity will be distorted and equal to the spectrum weighted one:

\[
⟨K_{\text{exp}}⟩ = ⟨Kγ−1⟩^{1/γ},
\]

where \(K\) is inelasticity from eqs.(4, 5) above and \(<\ldots\>)\ means averaging over the energy spectrum \(N(E_0)\).

[12] At the \(\sim 600 \text{ g/cm}^2\) of depth of atmosphere where Pamir experiments were performed all heavier (i.e., nuclear) components of incoming flux are already dissociated into nucleons in the first tens of \(\text{g/cm}^2\) of atmosphere. This all-nucleon flux is the enriched with mesonic (for all practical purposes - pionic) component produced in subsequent interactions in the atmosphere. The relative percentage of nucleons and pions can be fixed from other cosmic ray experiments [13].
For example, analysis of single to accompanying hadron ratio observed in EC experiments performed at Pamir leads to the primary hadron composition consisting of \( \sim 75\% \) of nucleons and \( \sim 25\% \) of pions. Cf., for example, Z.Włodarczyk, Proc. 5th Int. Symp. Very High Energy Cosmic Int., Lódź, Poland 1998; The Univ. of Lódź Publishers, Lódź 1998, p. 160.

Notice that in reality the transverse dimension of regions where energies \( E_1 \) and \( E_2 \) are deposited are both very small, of the order of 100 \( \mu \)m, and that particles in cascade are not separated experimentally.

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Although our results, as well as those of [5], are based on rather low statistics, they seem to be the only results for inelasticity available for nuclear targets for foreseeable future (at least for the energies above \( E_{LAB} > 100 \) TeV). The remarkable feature of both EC’s allows for the most direct estimation of \( K \) done so far.
Figure captions

Fig.1. The scheme of the carbon emulsion chamber with a typical nuclear-electromagnetic cascade (NEC). The incoming hadron initiates NEC in which leading particle and secondary hadrons (solid lines) interact repeatedly while the electromagnetic component, i.e., γ quanta from π⁰ decays (broken lines), are recorded as total energies \( E_1 \) and \( E_2 \) deposited in the two lead-emulsion sandwiches following, respectively, upper and lower \( H \)-blocks. Notice that in reality transverse dimensions of NEC are very small (of the order of 100 µm) and particles are not separated experimentally.

Fig.2. Dependence of \( \eta = N_2/N_1 \) on the energy threshold \( E_2^{th} \) in the second hadronic block for: \( \langle K \rangle = 0.65 \) (solid line), \( \langle K \rangle = 0.50 \) (dotted line) and for \( \langle K \rangle = 0.80 \) (black dots), compared with the experimental data.

Fig.3. Dependence of \( \epsilon = E_2/E_1 \) on the thickness \( H/\lambda \) of carbon target (the plotted curves correspond to different \( \langle K \rangle \) as in Fig. 2). The experimental point at \( H/\lambda = 1.1 \) corresponds to our specific carbon emulsion chamber (with inclusion of the averaging over zenith angle distribution of incoming hadron which shifts the value \( H/\lambda = 0.9 \) to 1.1).

Fig.4. The quality of \( \eta - E_2^{th} \) fit shown by the \( \chi^2 \) per degree of freedom (def) plotted versus the mean inelasticity of hadron-carbon nucleus collisions.

Fig.5. The dependence of partial inelasticity \( K_2 \) on the power index \( \alpha \) (in the formula \( <\nu> \sim A^\alpha \)) for the experimental value of \( \kappa = 0.5 \).
Figure 1
Figure 3
Figure 4
Figure 5

![Graph showing the relationship between $K_2$ and $\alpha$. The graph illustrates a decreasing curve as $\alpha$ increases, indicating a negative correlation.]