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Recent Advances in Redshift Surveys of the Local Universe

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Abstract

I review progress in the past few years in studying the large-scale structure of the universe through redshift surveys of galaxies. Of the many statistical methods used to describe the galaxy distribution, I concentrate here on the power spectrum, and go into some detail about the factors which complicate (and make interesting!) its interpretation, such as redshift space distortions, non-linear effects, and the relative bias of galaxies and dark matter. I also discuss two large redshift surveys which are just starting, the Sloan Digital Sky Survey, and the Two Degree Field Redshift Survey, which promise to increase the number of redshifts measured of galaxies in uniform surveys by more than an order of magnitude.

1.1 Introduction

The past two decades have seen an explosion of our knowledge in many areas of observational cosmology. One of the most significant has been increases in our understanding of the distribution of galaxies in the nearby universe (defined in the context of the present review as $z \lesssim 0.1$, where cosmological and evolutionary corrections can be neglected, for the most part). In addition to allowing us to do cosmography, whereby the structures and forms the galaxies find themselves in are mapped and catalogued, the several orders of magnitude increase in the number of galaxies with measured redshifts over this time period allows us to do quantitative cosmology, whereby we put specific constraints on models for structure formation and the various parameters which are input to the Friedman-Robertson-Walker metric. In addition to the much larger number of measured redshifts in complete samples of galaxies available now, there have been great advances in our theoretical understanding of the various statistics that have been measured from redshift surveys.

We shall not attempt in this review to give a complete and thorough summary of the entire subject of what can be learned from redshift surveys. Jeff Willick and I reviewed the field in a comprehensive article which covers material through the end of 1994 (Strauss & Willick 1995, hereafter SW), and I will try to avoid duplicating too much of the discussion found therein. Other recent reviews that discuss redshift surveys include those of Giovanelli & Haynes (1991), Dekel (1994), Borgani (1995), Efstathiou (1995, 1996), and Guzzo (1996). The emphasis in this review will be on developments which have occurred since the writing of SW, with special emphasis on future redshift surveys and what they will be able to measure.
1.2 Varieties of Redshift Surveys

In order to be useful for any sort of statistical work, a redshift survey must have a well-defined selection function in the most general sense of the term. That is, the selection criteria of the galaxies whose redshifts are included must be objective and quantifiable. This is not the same as saying that the redshift survey must be complete in some sense, just that its incompleteness follows some known rule(s), such that the fraction of galaxies with redshifts is known, at least statistically, as a function of the observational properties of a galaxy. In practice, this means that the following must be known for any redshift survey, and the galaxies contained therein, if it is to be useful:

- The region of sky covered by the survey.
- The observational criterion or criteria by which galaxies have been selected for redshifts, such as apparent magnitude in a given band, diameter, surface brightness cuts, emission-line strength†, etc.
- The value of the relevant quantity or quantities for all objects in the sample.
- Limits in the above quantity or quantities. This may very well be a function of position on the sky, such as in the case of an apparent magnitude-limited survey in optical bands, in the presence of Galactic extinction. These limits determine the depth of the survey, which can be quantified in terms of the expected number distribution of galaxies as a function of redshift.
- The fraction of galaxies for which redshifts are actually obtained. Again, this could be a function of position on the sky, or could be a fixed fraction over the survey area. If the fraction is close to 100%, we call this a complete redshift survey.
- The positions and redshifts of the individual galaxies.

Speaking loosely, then, a survey is characterized by its solid angle coverage, its depth, its sampling rate, and the method of selection of the galaxies. Comprehensive lists of redshift surveys are given in the various reviews listed in the Introduction, including SW; we will not repeat this here. However, we briefly summarize some of the redshift surveys we will find ourselves referring to through this review, with apologies to those whose surveys we do not have space to discuss here.

There have been a few surveys which have covered close to the entire celestial sphere; as we will see, these have been very important both for cosmography, and for making dynamical predictions of the peculiar velocity field in the nearby universe. Early work in this direction was done by Yahil, Sandage, & Tammann (1980) with the Revised Shapley-Ames catalog of galaxies (cf., Sandage & Tammann 1981), but this sample only extended to redshifts of 4000 km s\(^{-1}\), and was affected strongly by the Galactic zone of avoidance. The Infrared Astronomical Satellite, or IRAS, flew in 1983, and scanned the entire sky at \(\sim 1'\) resolution in four broad bands centered at 12, 25, 60, and 100\(\mu m\) (cf., IRAS Point Source Catalog Explanatory Supplement 1988 for details). An ordinary spiral galaxy with a moderate amount of star formation emits strongly at 60\(\mu m\) as thermal emission from dust heated by the interstellar radiation field. Because of the all-sky nature of the IRAS survey, and the transparency of the dust of the Milky Way to 60\(\mu m\) radiation, a galaxy sample selected at 60\(\mu m\) has uniform and full sky coverage. Two groups have carried out extensive full-sky redshift surveys based on these data: one centered in Berkeley, doing redshift surveys complete at 60\(\mu m\) to 1.936 Jy and subsequently to 1.2 Jy (Strauss et al. 1990, 1992c; Fisher et al. 1995a), and the other a mostly British collaboration which obtained redshifts of 1 in 6 galaxies to a flux limit of 0.6 Jy (Rowan-Robinson et al. 1990; Lawrence et al. 1996). This latter collaboration, referred to as QDOT for the initials of the institutions of the investigators, is currently extending their effort to measure redshifts for a complete sample of galaxies.

† relevant, e.g., in objective prism surveys for emission-line galaxies or quasars.
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Recent advances in redshift surveys of the Local Universe have led to a deeper understanding of the structure of the universe. The IRAS surveys, for example, have provided valuable data on galaxies. The IRAS surveys were limited by Galactic extinction at very low latitudes and systematic effects in the IRAS Point Source Catalog in regions of very high source density. These surveys thus cover between 80 and 90% of the sky.

Elliptical galaxies are very faint in the infrared bands and are therefore essentially absent from the IRAS surveys. Given that the cores of clusters of galaxies are dominated by elliptical galaxies, the number density of galaxies in the cores of clusters is systematically underestimated in IRAS, as is in fact borne out in direct comparisons of IRAS and optically selected samples. This, together with the sparse sampling of the galaxy distribution by the IRAS satellite, motivated the compilation of the Optical Redshift Survey (ORS), which selects galaxies from the Uppsala Galaxy Catalogue, the ESO Galaxy Catalogue, and its photometric counterpart, and the Extension to the Southern Galaxy Catalogue. Galactic extinction restricts the survey to Galactic latitudes $|b| > 20^\circ$. The depth is comparable to that of the original CfA survey but with 4.5 times the sky coverage. Santiago et al. (1996) detail the effort required to tie the three parts of the survey together, and the corrections made for Galactic extinction.

The Local Group is a member of the Local Supercluster, a highly flattened structure which extends at least to 3000 km s$^{-1}$ from us. Indeed, many of the other dramatic superclusters in the nearby universe are found in or at least intersected by the plane defined by the Local Supercluster, including the Perseus-Pisces Supercluster, the Hydra-Centaurus and Pavo-Indus-Telescopium Superclusters (almost certainly a single structure bisected by the zone of avoidance) and the Coma-A 1367 Supercluster. Fig. 1 shows isodensity contours of the galaxy distribution in the Supergalactic plane, in the IRAS 1.2 Jy (left) and ORS (right). The smoothing is Gaussian with $\sigma = 400$ km s$^{-1}$ at low redshift, increasing like the IRAS mean interparticle spacing at greater distances (thus the smoothing is the same in the two surveys). Mean density is indicated with a heavy line; contours above mean density are spaced logarithmically, with every third contour representing a factor of two in density. Dotted contours are at 0.66 and 0.33 of the mean density. Fingers of God associated with rich clusters have been collapsed to a common redshift; otherwise the galaxies are placed at the distances indicated by their redshifts in the Local Group frame (i.e., no correction for peculiar velocities). The low latitude regions of the ORS map are masked out, because of the absence of galaxies there. It is perhaps not surprising that the large-scale distribution of galaxies in these two surveys is similar; there is of course a great deal of overlap in the two samples, and of course the majority of galaxies in an optically magnitude limited sample are spirals.

The smoothing here is not uniform, but increases with distance from the origin as the samples become sparser. Therefore, the relative strength of features seen at different distances from the origin can be misleading. We could avoid this by choosing a single large smoothing scale. Alternatively, one can smooth with a noise-suppressing filter; a Wiener filter (cf., Hoffman, these proceedings) still gives an effective smoothing that increases with distance from the origin, but a simple variant on that, called the power-preserving filter (cf., Yahil, these proceedings) gives a constant smoothing with distance. See SW for maps of the IRAS density field with this smoothing scheme.

There are two major recent surveys which go somewhat deeper than the surveys discussed above, although they are far from full-sky surveys. The CfA2 survey (cf., Geller & Huchra 1988; 1989 for early reviews) covers 2.95 ster in the Northern Hemisphere, and includes all galaxies from the Zwicky et al. (1961-1968)
Fig. 1.1. Galaxy isodensity contours in the Supergalactic Plane for the ORS (left) and IRAS 1.2 Jy redshift surveys (right). The local group is at the center of each map. The smoothing in the two cases is the same, and increases with distance from the center, therefore relative strength of features at different distances from the center can be misleading. The heavy contour is at the mean density; dotted contours are underdense relative to the mean. The zone of avoidance is indicated in the case of the ORS sample.

catalog with $m_Z \leq 15.5$. A parallel effort (da Costa et al. 1994a) in the Southern Hemisphere is covering 1.13 ster to the same depth of the CfA2 survey, based on scans of sky survey plates. The two surveys have been analyzed together to determine the power spectrum of galaxies (da Costa et al. 1994b) and the small-scale velocity dispersion of galaxies (Marzke et al. 1995).

On smaller angular scales, but going deeper, are several surveys. Shectman et al. (1996) have used a multi-object spectrograph on the Las Campanas 2.5m Du Pont Telescope to obtain redshifts for 26,418 galaxies to $r \approx 17.5$ (the Las Campanas Redshift Survey, or LCRS). The photometry is based on CCD drift-scan data obtained by the same workers on the Swope 40” telescope at Las Campanas. This sample is not complete: fields in which to do spectroscopy were laid down on a regular grid in a series of six $\sim 90^\circ \times 1.5^\circ$ wide stripes across the sky, and the sampling rate was simply the ratio of the number of galaxies available to their magnitude limit, to the number of fibers of the spectrograph, averaging roughly 70% over their fields. This survey, covering a total of $\sim 700$ square degrees, has roughly a factor of two more redshifts than any other single redshift survey of galaxies.

These surveys are best viewed not as contour plots as in Fig. 1.1, but rather in the form of pie diagrams. Fig. 1.2 shows the redshift distribution in the LCRS survey; in each segment of the pie, three of the $1.5^\circ$ slices are plotted on top of one another. The angular coordinate is right ascension, and the radial component is redshift. Perhaps the most striking feature of this map is the fact that one does not see
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Fig. 1.2. A redshift pie diagram of galaxies in the LCRS, kindly supplied by Huan Lin. The sky distribution of this survey is shown below in Fig. 1.7. The three slices in the Northern and Southern Galactic caps are each shown projected on top of one another.

coherent structures stretching across the survey volume (compare, for example, with the famous CfA2 slice of de Lapparent, Geller, & Huchra 1986). Bob Kirshner of the LCRS team has called this “the end of greatness”, in the sense that surveys are now probing a volume appreciably larger than the largest structures in the universe. We discuss below quantitative measures of structure on the largest scales with this survey.

A similar survey has been carried out by Zucca et al. (1996) (cf., Guzzo 1996), using a fiber optic spectrograph on the ESO 3.6m telescope; galaxies are selected from the Edinburgh-Durham Southern Galaxy Catalogue (EDSGC; e.g., Heydon-Dumbleton, Collins, & MacGillivray 1989), and redshifts have
been obtained for 3348 galaxies to $b_J = 19.4$ over 36 square degrees. This survey is called the ESO Slice Project, or ESP.

Outside the scope of this review are the very exciting redshift surveys being done over small areas but going very deep, which attempt to look for evolution in the properties of the galaxies themselves and the large-scale structure they trace. Lilly et al. (1995) have carried out perhaps the most extensive redshift survey of faint galaxies; their survey contains redshifts for 591 galaxies over five small fields with $I < 22.5$; the median redshift is 0.56. Still deeper surveys have been done by Cowie, Broadhurst, Ellis and their collaborators (cf., the review by Ellis 1996).

Finally, there are two redshift surveys currently in the advanced planning stages: the Sloan Digital Sky Survey (SDSS), which plans to obtain redshifts for $10^6$ galaxies and $10^5$ quasar candidates over $\approx 3.2$ ster in the Northern Galactic Cap to $r' \approx 18$, and the Two-Degree Field Survey (2dF), which will go slightly deeper to measure 250,000 galaxies over a more limited region of sky. We will discuss these surveys in some detail below in §1.7.

1.3 The Luminosity and Selection Function

As we emphasized in the previous section, in order to do quantitative work with redshift surveys, one needs to know the selection function of the sample. As is apparent in Fig. 1.2, in a flux-limited sample, the number density of galaxies drops off as a function of distance from the observer, which is an effect we need to correct for. Let us understand the issues for the simplest case, that of a sample of galaxies limited to flux $f_{\text{min}}$ completely and uniformly selected over a given region (cf., Sandage, Tammann, & Yahil 1979; Efstathiou, Ellis, & Peterson 1988; Yahil et al. 1991; SW). We define the luminosity function $\Phi(L)$ of galaxies such that $\Phi(L) \, dL$ is the average number density of galaxies with luminosities (in some given band) between $L$ and $L + dL$. Let us make the assumption of a universal luminosity function; that is, the number density of galaxies at position $r$ with luminosity between $L$ and $L + dL$ is a separable function of $r$ and $L$: $\delta(r) \Phi(L) \, dL$. Thus, we assume that the luminosity function is independent of environment (we’ll discuss briefly in §1.4 observational evidence that this does not hold true in great detail).

With this assumption, it is straightforward to write down an expression for the expected number density of galaxies in a redshift survey at distance $r$, $n(r)$, in a universe without clustering:

$$n(r) = \int_{4\pi r^2 f_{\text{min}}}^{\infty} \Phi(L) \, dL \quad (1.1)$$

This suggests a simple estimator for the observed fractional overdensity of galaxies $\delta(r) \equiv (\rho(r) - \langle \rho \rangle) / \langle \rho \rangle$ smoothed with a window $W(x)$:

$$\delta(r) = \frac{1}{f d^3 r W(r)} \sum_{\text{galaxies}} \frac{W(|r - r_i|)}{n(r)} - 1 \quad (1.2)$$

where $n(r)$ is given by Eq. (1.1) and $r_i \equiv |r_i|$ of course. What this means, in effect, is that for the purposes of deriving the density field $\delta$, each galaxy in the sample is assigned a weight given by the inverse of $n(r)$ at the distance of that galaxy.

What is the best way to calculate $n(r)$? It is straightforward given a fit to the luminosity function. There is a lengthy literature of determinations of the luminosity function, which is reviewed in the various articles listed at the beginning of this section. The classic method, reviewed in, e.g., Felten (1977), simply involves binning the galaxies in a redshift survey by absolute luminosity and dividing each by the effective
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volume probed at that luminosity. However, this method gives unbiased results only in the limit that the number of galaxies per unit redshift is unaffected by clustering, which is exactly the quantity we ultimately wish to measure. Here we present a method of determining the luminosity function which does not require any assumption about the large-scale homogeneity of galaxies; its history can be traced through Sandage et al. (1979), Nicoll & Segal (1983), Efstathiou et al. (1988), Saunders et al. (1990), and Yahil et al. (1991).

We will take a maximum likelihood approach and ask, given that we know that a given galaxy $i$ has a distance $r_i$, what is the likelihood that it have its observed flux between $f_i$ and $f_i + df$? The answer clearly depends on the luminosity function, thus this approach will give us a handle on the luminosity function itself. Indeed, this likelihood $L_i$ is given by the luminosity function at $L_i = 4\pi r_i^2 f_i$, normalized by the integral of the luminosity function over all luminosities it could have, given the flux limit. Formally:

$$L_i = \frac{\Phi(L_i) dL}{\int_{4\pi r_i^2 f_{\text{min}}}^{\infty} \Phi(L_i) dL} \propto \frac{dn(r)/dr}{n(r_i)} \bigg|_{r=r_i(f_i/f_{\text{min}})^{1/2}}, \quad (1.3)$$

where the proportionality follows directly from Eq. (1.1). The constant of proportionality is just $dL/(8 \pi r f_i)$, which is independent of the parameters of the selection function, and therefore does not concern us as we maximize the likelihood below. Given the likelihood for observing any one galaxy, the likelihood for an entire sample is given by the product of this expression for all galaxies in the sample. The maximum likelihood method then consists of the following: choose a parametric form for $n(r)$. Calculate the likelihood function over all the galaxies in the sample as a function of the parameters, and find its maximum (as is often done in this sort of exercise, the quantity actually maximized in practice is the logarithm of the likelihood function). This determines $n(r)$, from which the luminosity function follows as a simple derivative. A variant of this approach involves specifying the luminosity function not in terms of a smooth functional form, but as a series of constants in bins. Nicoll & Segal (1983), Efstathiou et al. (1988), and Koranyi & Strauss (1996) show how the likelihood can be maximized with respect to the values of the constants through a straightforward iterative procedure.

It is clear how the likelihood procedure outlined above is independent of density inhomogeneities: the positions of galaxies are taken as a prior, and one asks for the likelihood of observing their fluxes. Thus the results are not biased by the distribution of the positions of the galaxies. But a consequence of this is that the normalization of $\Phi(L)$ (or equivalently, of $n(r)$) is not determined. With this in mind, we explicitly drop the normalization of $n(r)$ to define the selection function:

$$\phi(r) \equiv \begin{cases} n(r), & r > r_s \\ 1, & r \leq r_s \end{cases}, \quad (1.4)$$

where $r_s$ is some small fiducial distance, typically 500 km s$^{-1}$, below which the selection function is set to unity. That is, the selection function quantifies the fraction of the luminosity function seen at a distance $r$, relative to the numbers at $r_s$. This of course begs the question of how to normalize the selection function itself in order to calculate $\delta(r)$. The usual assumption is that the entire volume covered by a given redshift survey is a fair sample of the universe; that is, the mean density of galaxies in this volume differs negligibly from the true mean density. Given this, there are several ways one can calculate the mean density as a weighted sum over the galaxies in the sample (Davis & Huchra 1982). The simplest is to define

$$n(r_s) = \frac{1}{V} \sum_{\text{galaxies}} \frac{1}{\phi(r_i)}, \quad (1.5)$$
where the sum is over all galaxies in the sample between \( r_s \) and some outer radius beyond which the sample gets too sparse to be useful, and \( V \) is the volume enclosed. Davis & Huchra (1982) derive a minimum variance version of Eq. (1.3) that takes the known clustering of galaxies into account, although in practice, it requires knowing the strength of the correlation function on the largest scales, where it is most poorly understood. Another approach is that adopted by Saunders et al. (1990), who note that in the approximation that the sample is uniform, the total number of galaxies in the sample \( N \) should be given by \( \int d^3r n(r) \) (where the integral is over the solid angle and depth of the survey), which allows one to normalize \( n(r) \).

It is actually quite straightforward to modify the maximum likelihood method sketched out here to the more general case of selection depending on more complicated criteria, such as both a magnitude and diameter cut, or extinction as a function of direction; some of these complications are described in Santiago et al. (1996). Another interesting issue discussed in that paper, and also in Mancinelli (1996), is the effect of flux errors on the derived luminosity function, selection function, and density field. The derived luminosity function is given by the true luminosity function convolved with the flux errors, not surprisingly. This gives a selection function which falls less rapidly with distance than in the case without flux errors. One might think that this could cause the derived density field to be systematically biased as a function of distance, but there is a competing effect which goes in the opposite direction, namely Malmquist bias. Flux errors scatter galaxies over the flux limit of the sample. Because the number of galaxies is a monotonically decreasing function of flux, more galaxies scatter into the sample than out of it. If the flux errors are proportional to the flux itself (such as one gets with magnitude errors which are constant with magnitude), these two effects cancel exactly; the derived density field is unbiased, even though the luminosity function is biased.

### 1.4 Clustering Statistics

As Figs. 1.1 and 1.2 make clear, galaxies are not uniformly distributed in space; indeed, one of the main motivations for doing redshift surveys is to quantify the observed clustering and from it, draw inferences about larger cosmological questions such as are addressed throughout this volume: what is the nature of dark matter? What is the value of the Cosmological Density Parameter? How did galaxies form? We will be able to address only some of these issues here. But let us ask how best to quantify in a statistical way the clustering that is seen here.

The current dominant paradigm for the formation of large-scale structure postulates that it grows by the process of gravitational instability from an early epoch when the fluctuations were of very low amplitude (recent textbooks treating this include Kolb & Turner 1990; Peebles 1993; Padmanabhan 1993; Coles & Lucchin 1995). This basic picture was given great support by the fact that the fluctuations in the Cosmic Microwave Background (CMB) as detected by the Cosmic Background Explorer (COBE; Smoot et al. 1991) are within a factor of two of those expected from extrapolation of the present-day observed clustering of galaxies (e.g., Wright et al. 1992; cf., the contribution from Silk in this volume). Moreover, if the fluctuations arise from inflationary processes in the very early universe (e.g., Kolb & Turner 1990), they turn out to be random phase. This means the following. Given the density field \( \delta(r) \) at some early time, one can take its Fourier Transform to obtain the complex quantity \( \hat{\delta}(k) = |\hat{\delta}(k)| e^{i \theta} \), where \( \theta \) is the

\[\text{†} \] The caveat that the flux errors must be proportional to flux is not made clear in Santiago et al. (1996). However, there is no condition on the Gaussianity of the error distribution.
phase of each mode. Inflationary models predict the quantity $\theta$ to be uniformly distributed between 0 and $2\pi$. In any case, if the phases are random, then the Central Limit Theorem implies that the distribution function of the density field is Gaussian. What this means is that all its reduced moments of third order and higher vanish (after all, they are defined relative to a Gaussian). That is, one can give a full statistical description of the density field by specifying its second moment with respect to the only relevant independent variable, the scale (the Cosmological Principle says that space is isotropic, which is confirmed to fantastic accuracy with the COBE data, so there is no directional dependence to the clustering in a sufficiently large sample). There are a number of ways we might quantify this. One way is simply to calculate the statistic implied by this discussion: the second moment of the density distribution as a function of scale. That is, one asks for the variance in $\delta$, averaged over spheres of a given radius $r$; this quantity will be referred to as $\sigma^2 (r)$. Alternatively, one can calculate the correlation function of the density field: $\xi (r) \equiv \langle \delta (x) \delta (x + r) \rangle$, where the averaging is over position $x$ and over direction of $r$. Finally, one can define the power spectrum of the density field, which is the modulus squared of $\tilde{\delta}$:

$$\langle \tilde{\delta} (k) \tilde{\delta}^* (k') \rangle = (2\pi)^3 P(k) \delta_D (k - k'),$$

(1.6)

where the averaging is over directions of $k$, and $\delta_D$ is the Dirac $\delta$ function.

These three statistics are related to each other in straightforward ways:

$$\xi (r) = \frac{1}{2\pi^2} \int dk k^2 P(k) \frac{\sin kr}{kr},$$

(1.7)

and

$$\sigma^2 (r) = \frac{1}{2\pi^2} \int dk k^2 P(k) \tilde{W}^2 (kr),$$

(1.8)

where $\tilde{W} (x)$ is given by a spherical Bessel function for a spherical window:

$$\tilde{W} (x) = \frac{3 (\sin x - x \cos x)}{x^3}.$$

(1.9)

How does this relate to the galaxy distribution? As we said before, we believe (although do not have definitive proof) that clustering grew through the process of gravitational instability. In linear perturbation theory it is straightforwardly shown (cf., the textbooks referenced above) that the amplitude of perturbations grows as a function of time, independent of the wavelength of the perturbations. What this means is that the power spectrum and related statistics change in amplitude, but not in shape, as perturbations grow. Therefore, measurements of the shape of the power spectrum as a function of $k$ today are a direct measure of its shape in the past. That prospect is very exciting, because the shape of the power spectrum is commonly modelled to be due to two components. One is the primordial power spectrum, i.e., that laid down, perhaps during inflation, when the universe was very young. This is modified by the differing growth of perturbations of super-horizon and sub-horizon scales before and after the epoch of matter-radiation equality (cf., Kolb & Turner 1990; Efstathiou 1991 for reviews). The details of this

† Examples of models in which the phases are not random include those with gravitating seeds (such as cosmic string loops or textures) or explosion models. A general discussion of the large-scale structure implications of these models can be found in Weinberg & Cole (1992).

‡ However, the converse is not true: a Gaussian distribution function for the density field does not imply random phases.

† The normalization, and even the definition of $P(k)$ depends on one’s Fourier Transform convention in defining $\delta$; compare Eq. (1.6) with that in Peebles (1980), for example.
are determined by the nature of the dark matter (i.e., hot or cold) and how much of it there is (as this determines the matter-radiation equality epoch).

Thus in principle, measurements of the galaxy power spectrum can tell us a great deal about the early universe and the dark matter. However, there are some real complications that come in. The first is that we are observing galaxies, while it is the distribution of dark matter that the theories predict. Thus we need a model for the relative distribution of galaxies and dark matter. The simplest assumption is that the distribution of galaxies and dark matter are the same (i.e., \( \delta_{\text{galaxies}}(r) = \delta_{\text{dark matter}}(r) \)), but it was realized in the mid-1980’s (Kaiser 1984; Davis et al. 1985; Bardeen et al. 1986; Dekel & Rees 1987) that there is no a priori reason that this might be true. Indeed, one could explain a number of observations (such as the relative strength of the cluster and galaxy correlation functions, and the pairwise velocity dispersion of galaxies on small scales) if it were false. The simplest model of biasing has that the galaxy and dark matter density fields differ by a constant factor \( b \). That is,

\[
\delta_{\text{galaxies}}(r) = b \delta_{\text{dark matter}}(r),
\]

independent of smoothing scale, although reality is almost certainly more complicated than this. In any case, to the extent that this linear biasing holds, the shape of the derived galaxy power spectrum of galaxies will still be the same of that of the dark matter.

The next complication is due to non-linear evolution of the power spectrum. The statement that density perturbations grow at a rate which is independent of scale holds only when the perturbations are in the linear regime, i.e., \( \delta \ll 1 \). When this condition no longer holds, all modes no longer grow independently, and the growth rate does indeed become a function of scale, meaning that the non-linear power spectrum no longer keeps the same shape as its linear progenitor. In practice, because density perturbations are generically an increasing function of \( k \) (at least for power spectra \( P(k) \propto k^n, n \geq -3 \), cf., Eq. 1.8), this means that the power spectrum is modified by non-linear effects on small scales. To a certain degree the growth of the power spectrum on small scales can be calculated analytically (e.g., Jain & Bertschinger 1994) or phenomenologically (Hamilton et al. 1991; Jain, Mo, & White 1995), and indeed there is a quite extensive literature on extensions of linear theory for the growth of perturbations into the non-linear regime (cf., the review by Sahni & Coles 1995).

As we will see below, the power spectrum measured within the necessarily finite volume of any given redshift survey is not identical to the theoretical ideal of that measured in an infinite volume, essentially because of the difficulties of defining the continuous Fourier Transform in a finite volume. Indeed, the measured power spectrum is a convolution of the “true” power spectrum with the Fourier Transform of the observing volume, which tends to depress the power spectrum on large scales.

Finally, what we observe for each galaxy in a redshift survey is a redshift, not a distance. Only in the approximation that peculiar velocities are negligible are the two the same (cf., Eq. 1.27). Peculiar velocities have two effects on the power spectrum as measured in redshift space. On small scales, the pairwise velocity dispersion of galaxies spreads galaxies out in redshift space relative to their distribution in real space (think of a cluster of galaxies stretched out into a “Finger of God” in redshift space), thereby decreasing the apparent amplitude of clustering on small scales. On large scales, the dominant effect is due to coherent streaming of galaxies towards overdensities, giving a compression in redshift space, and therefore amplifying the apparent clustering (Kaiser 1987).

With all these effects acting, the interpretation of the observed power spectrum is thus quite non-trivial, and the next section of this review concentrates on the details of how people have tried to take these
various effects into account, and indeed, to take advantage of them to get additional information out of the available data.

1.5 Measurements of the Power Spectrum

1.5.1 Techniques

For two decades, the standard way to quantify the clustering seen in galaxy surveys was through the use of the N-point correlation functions, especially the two-point correlation function $\xi(r)$ (Peebles 1980; SW). However, much of the recent developments in the field have been on analyses of its Fourier Transform $P(k)$ (cf., Eq. 1.7), and we will emphasize this in this review.

The Fourier Transform of the galaxy density field (Eq. 1.2) is

$$\tilde{\delta}(k) = \frac{1}{nV} \sum_i \frac{1}{\phi(r_i)} e^{ik\cdot r_i} - W(k),$$

(1.11)

where

$$W(k) \equiv \frac{1}{V} \int_V d^3 r e^{ik\cdot r}$$

(1.12)

is the Fourier Transform of the survey volume. Our estimator of the power spectrum is then

$$\Pi(k) \equiv V\tilde{\delta}(k)\tilde{\delta}(k)^*,$$

(1.13)

where the factor of $V$ on the right hand side gets the units right. Several lines of algebra (e.g., Fisher et al. 1993; Vogeley 1995) show that the expectation value of this estimator is given by

$$\langle \Pi(k) \rangle = \int d^3 k' P(k')G(k - k') + \frac{1}{nV} \int d^3 r \frac{1}{\phi(r)},$$

(1.14)

where

$$G(k - k') \equiv \frac{V}{(2\pi)^3} |W(k - k')|^2.$$

(1.15)

In the limit of an infinitely large volume, $G$ approaches a Dirac delta function, as it must. Thus the power spectrum estimator is given by the true power spectrum convolved with an expression involving the Fourier Transform of the volume, plus a shot noise term. Because one normalizes the density field assuming the mean density inside the surveyed volume is equal to the true global mean, there is an additional correction factor to Eq. (1.14) to compensate for the resulting loss of power; this term is important for measurements of the power spectrum on scales approaching that of the survey itself.

The quantity in Eq. (1.14) is still a function of $k$, and thus must be averaged over solid angle in $k$-space in order to calculate a quantity dependent only on $k$. This is a straightforward procedure for values of $k$ probing scales appreciably smaller than the survey dimensions. However, when $1/k$ becomes comparable to the smallest dimension of the survey, this averaging can mix together modes with very different convolutions with the survey window (simply because $G$ in Eq. (1.13) becomes quite anisotropic for surveys with restricted geometries). A related problem is that an appreciable covariance can develop between determinations of the power spectrum for different values of $k$. Different workers have found different ways of dealing with these problems. Fisher et al. (1993), working with the IRAS 1.2 Jy survey, have close to full sky coverage, and therefore are not affected much by the anisotropy of the window function. They measure $\Pi(k)$ within cylinders embedded within the survey volume, whose long axis of
length $2R$ is parallel to the vector $k$. Choosing $kR$ to be an integral multiple of $\pi$ means that $\delta(k)$ now scales exactly with the mean density, and thus errors in the mean density affect only the amplitude, and not the shape, of $P(k)$.

Feldman, Kaiser, & Peacock (1994) approach the problem from a different viewpoint, by asking for a weighting to Eq. (1.11) that allows them to measure $P(k)$ with the minimum variance. For the case of a full-sky sample, and assuming that the different Fourier modes have random phases, they derive the weights:

$$w_i = \frac{1}{1 + n\phi(r_i)P(k)}.$$  \hspace{1cm} (1.16)

With this weight function, the variance in the estimate of the power spectrum is given by

$$\sigma^2[P(k)] = \frac{(2\pi)^3}{V_k \int d^3r [n\omega\phi(r)]^2},$$  \hspace{1cm} (1.17)

where $V_k$ is the volume in $k$-space occupied by the bin in question. This expression assumes that the bins are spaced far enough apart that the covariance is negligible (this happens roughly for separations $\Delta k > 2\pi/R$, where $R$ is the characteristic dimension of the volume surveyed).

Tegmark (1995) has carried this type of analysis further, by asking for the optimal weighting of $\Pi(k)$ as a function of amplitude and direction of $k$, given the survey geometry. One wants to measure the power spectrum with as much resolution in $k$ as possible, without introducing large amounts of covariance between adjacent values. Tegmark shows that this is done with a weighting function $w(r)$, which is the ground-state solution to the Schrödinger equation with potential given by the inverse of the selection function:

$$\left[-\frac{1}{2} \nabla^2 + \frac{\gamma}{\phi(r)}\right] w(r) = E w(r),$$  \hspace{1cm} (1.18)

where $\gamma$ is a parameter which determines the resolution in $k$ of the determination of the power spectrum (at the expense of signal-to-noise ratio).

Vogeley & Szalay (1996) and Tegmark, Taylor, & Heavens (1996) have addressed this problem at one further level of sophistication. While the Fourier modes are the ideal basis in which to expand the density field in the ideal case of an infinite survey, this is not the case in the realistic case of a survey covering a finite area of sky, with a radial selection function. In particular, the Fourier modes are not orthonormal over the survey volume. Thus the above authors expand the observed density field in orthonormal eigenmodes which maximize the signal-to-noise ratio, given the survey geometry and selection function, using the Karhunen-Loève (K-L) transform. These modes are linear combinations of the standard Fourier modes which enter the power spectrum.

Following Vogeley & Szalay, divide a survey volume into a series of $M$ volume elements centered at positions $x_i$ with volumes $V_i$. Let $f(x_i)$ be the counts of galaxies observed within cell $i$. We expand the $f(x_i)$ in a series of orthonormal basis vectors $\Psi_j$, i.e.,

$$f(x_i) = \sum_j \Psi_j(x_i) B_j.$$  \hspace{1cm} (1.19)

The K-L transform uses the basis vectors which satisfy the eigenvalue problem:

$$R\Psi_j = \lambda_j \Psi_j,$$  \hspace{1cm} (1.20)

where $R$ is the correlation matrix of the $f$'s: $R_{ij} = \langle f(x_i) f(x_j) \rangle$, and the eigenvalues are $\lambda_j = \langle B_j^2 \rangle$. The
relation to the power spectrum is clear: the matrix $R_{ij}$ has elements given by the sum of the correlation function between volume $V_i$ and $V_j$, and contributions from shot noise. Given the K-L expansion of an observed density field, the best-fit power spectrum can be found by the methods of maximum likelihood.

Vogeley & Szalay point out that the K-L transform is a maximally efficient representation of the data, in the sense that if the eigenmodes are ordered in decreasing order of their eigenvalues, the truncation of the eigenvectors to the first $N$ gives a representation of the data that differs from the truth by as small an amount as possible. Another way to say this is that this gives an expansion of the data in modes of decreasing signal-to-noise ratio. A dramatic demonstration of the power of the K-L transform to compress data, in quite a different astronomical context, is that of Connolly et al. (1995), who show that the optical spectral energy distributions of galaxies can be well represented by their first three eigenvectors.

This approach to analysis of redshift surveys promises the greatest advantage over the standard power spectrum analysis in the case of surveys with sharp boundaries, especially those covering a fraction of the sky with anisotropic sky coverage, such as slice surveys or pencil beam surveys. The LCRS is an example of a slice survey. It has not yet been analyzed using the methods of the K-L Transform (although I understand that Vogeley & Szalay intend to do so), but Landy et al. (1996) do an analysis of the power spectrum of this survey in the same spirit. Their survey consists of six narrow slices, and thus recognizing that there is little information on the large-scale power spectrum in a direction perpendicular to the slice, they calculate in effect the two-dimensional analogue of the power spectrum on the density field of the slices, collapsed along this narrowest direction.

### 1.5.2 Results

The power spectrum of various redshift surveys of galaxies has been measured by a large number of groups (Baumgart & Fry 1991; Peacock & Nicholson 1991; Park, Gott, & da Costa 1992, Vogeley et al. 1992; Fisher et al. 1993, Feldman et al. 1994; Park et al. 1994; da Costa et al. 1994b; Tadros & Efstathiou 1995, 1996; Vogeley 1995; Landy et al. 1996; Lin et al. 1996). A summary of results through 1994 is given by Peacock & Dodds (1994), and is reviewed in SW. For the most part, this large range of determinations of $P(k)$ is remarkable for the uniformity of the results they give. The amplitude of the power spectrum determined from different surveys often differs significantly, even for galaxies selected in the same way (compare Fisher et al. 1993, Feldman et al. 1994, and Tadros & Efstathiou 1995 for IRAS-selected galaxies), but there seems to be broad agreement between groups on the shape of the power spectrum, at least within the rather large error bars. This is especially remarkable, given the very different geometries, and therefore window functions, of the different surveys (cf., Eq. 1.14). The observed power spectrum is a power law, $P(k) \propto k^n$, $n \approx -2$ on small scales, changing to $n \approx -1$ on scales above $\lambda \equiv 2\pi/k \sim 30 h^{-1}$ Mpc. There is tentative evidence for a flattening of $P(k)$ on the largest scales on which it has been measured, $\lambda \sim 100 - 200 h^{-1}$ Mpc. The best evidence for this comes not from a redshift survey, but rather the APM photometric survey of $2 \times 10^6$ galaxies over the Southern Galactic Cap (Maddox et al. 1990abc, 1996). Baugh & Efstathiou (1993, 1994; cf., Gaztañaga 1995) discuss the determination of the spatial power spectrum from the angular correlation function; the evidence for a flattening of the power spectrum is fairly unambiguous.

The power spectra derived from the surveys described above are shown in Fig. 1.3, which corrects the published power spectra in redshift space for distortions as predicted in linear theory using Eq. (1.22) below, following Kolatt & Dekel (1995). With the exception of the CfA2+SSRS2 curve, all curves shown are fits of the power spectra to simple functional forms. Thus this figure does not give a sense of the size
Fig. 1.3. A real space correlation functions of some of the surveys discussed in the text, after a similar figure in Kolatt & Dekel (1995). The CfA2+SSRS2 line is drawn from interpolation between datapoints given in da Costa et al. 1994b; the others are parameterized fits to the observed power spectra. The APM curve is from Baugh & Efstathiou (1993), QDOT from Feldman et al. (1994), LCRS from Lin et al. (1996), and IRAS 1.2 from Fisher et al. (1993). Each curve extends to the largest scales on which the power spectrum was determined.

of the error bars (compare with Fig. 1.6 below). The amplitudes of the different power spectra are indeed quite different from one another† varying by almost a factor of 3 at $k = 0.1h\text{Mpc}^{-1}$, but as remarked above, the shapes are remarkably the same.

Before 1992, theoretical predictions for the form of the power spectrum had an important freedom. Current models for the origin of density fluctuations invoking quantum processes in the early universe do

† Tadros & Efstathiou (1995) go a long ways towards reconciling the amplitudes of the two IRAS surveys in Fig. 1.3.
not constrain the amplitude of these fluctuations (essentially because we do not yet have a detailed enough model of the relevant particle physics). Thus the normalizations of the power spectra were essentially unconstrained. However, now with observations of the fluctuations of the CMB with COBE (cf., Bennett et al. 1996; Górski et al. 1996 for the latest results), this normalization is tied down to 10% accuracy for any given model\footnote{This is true only to the extent that the fluctuations are interpreted as being due solely to the Sachs-Wolfe (1967) effect on large scales. However, there exist models in which a substantial contribution to the fluctuations comes in the form of gravitational waves (e.g., Davis et al. 1992), making the relationship between the COBE fluctuations and the normalization of the power spectrum ambiguous.} (cf., Bunn & White 1996 and references therein).

In any case, we can now make definite predictions, including normalization, for the power spectra of different models. If the biasing of galaxies relative to mass is independent of scale on large scales, then the measured galaxy power spectrum definitely rules out the standard Cold Dark Matter model, as defined, e.g., in Davis et al. (1985): adiabatic fluctuations with primordial spectral index $n = 1$, $\Omega = 1$, and $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$. If biasing is local, i.e., if the probability that a galaxy form at a given place is a function of the physical properties of that region of space within a few Mpc, then the bias is indeed probably independent of scale (Weinberg 1995; Kauffmann, Nusser, & Steinmetz 1996); if this assumption does not hold, one can find models in which standard CDM fits the observed power spectrum on large scales (Bower et al. 1993). Leaving this last possibility aside, the observed power spectrum with COBE normalization can be fit by a range of variants on the standard CDM model, all designed to give more power on large scales than small: moving the epoch of matter-radiation equality later by changing the value of $\Gamma = \Omega h$, by decreasing the index of the primordial power spectrum by a few tenths, and/or by replacing some of the CDM with Hot Dark Matter (cf., the contributions from Primack and Ostriker in this volume).

As mentioned above, there is rough agreement between different workers as to the shape of the galaxy power spectrum on intermediate and large scales. There is one dramatic exception to this. Landy et al. (1996), in their analysis of the two-dimensional power spectrum from the LCRS, find evidence for a strong peak in the power spectrum on the largest scales probed ($2 \pi/k \approx 100 h^{-1}$ Mpc), rising a factor of several above the best-fit CDM-like power spectrum as measured on smaller scales. This result, if true, is very important, and may point to baryon-dominated isocurvature models (Peebles 1987) which are designed to have a bump in the power spectrum on these scales. However, it remains unclear how such a bump could exist, and yet not appear in the angular power spectrum of Baugh & Efstathiou (1993, 1994).

\subsection*{1.5.3 Redshift-Space Distortions}

\subsubsection*{1.5.3.1 Linear Scales}

We remarked above that measurements of the power spectrum in redshift space differ systematically from those in real space, due to the distorting effects of peculiar velocities. On large scales (i.e., those on which density perturbations are small), a linear perturbation expansion of the equations of gravitational instability yields a simple relation between the peculiar velocity and density fields (Peebles 1980):

$$\nabla \cdot \mathbf{v}(r) = -\Omega^{0.6}\delta(r)$$

(cf., the contribution by Dekel in this volume). Given this, one can calculate that the effect of coherent infall into overdensities is to multiply the Fourier modes by a fixed operator independent of scale (Kaiser 1987):

$$\tilde{\delta}(k) \rightarrow (1 + \beta \mu^2)\tilde{\delta}(k)$$

(1.22)

† This is true only to the extent that the fluctuations are interpreted as being due solely to the Sachs-Wolfe (1967) effect on large scales. However, there exist models in which a substantial contribution to the fluctuations comes in the form of gravitational waves (e.g., Davis et al. 1992), making the relationship between the COBE fluctuations and the normalization of the power spectrum ambiguous.
in real space, this becomes the operator equation:

\[ \delta(r) \Rightarrow \left[ 1 + \beta \left( \frac{\partial}{\partial z} \right) \nabla^{-2} \right] \delta(r) , \tag{1.23} \]

where \( \beta \equiv \Omega^{0.6}/b \) is the proportionality constant between the galaxy density field and the divergence of the peculiar velocity field (cf., Eq. 1.10), and \( \mu \) is the cosine of the angle between the wavevector \( k \) and the line of sight. It is a straightforward calculation to propagate the effects of Eq. (1.22) through to the power spectrum; the power spectrum gets multiplied by a factor:

\[ K(\beta) = \left( 1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) , \tag{1.24} \]

which is an appreciable correction; \( K(\beta = 1) = 1.87 \).

On small scales, linear theory breaks down, and peculiar velocities are dominated by the pairwise velocities of galaxies in groups and clusters. The effect of this, as we saw above, is to reduce the amplitude of clustering in redshift space relative to real space. As we will see in a moment, the amplitude of the small-scale velocity dispersion of galaxy pairs remains quite unclear, making it difficult to predict \textit{a priori} on what scales the redshift distortions make the transition from nonlinear to linear behavior (cf., the discussion in Brainerd & Villumsen 1993; Gramann, Cen, & Bahcall 1993; Fisher \textit{et al.} 1994b). These effects can be measured by the fact that they make the clustering in redshift space anisotropic; that is, the radial direction is the only one affected by distortions. This can be seen directly by measuring the correlation function, not as a function simply of redshift space separation, as is usually done, but rather as a function of the separation both perpendicular and along the line of sight; redshift space distortions will make the contours of \( \xi \) in this space anisotropic. This has been carried out by a number of workers with various datasets (Hamilton 1993, Fisher \textit{et al.} 1994b; Loveday \textit{et al.} 1996). One can carry out similar analyses on the power spectrum (Cole, Fisher, & Weinberg 1994, 1995; Fisher & Nusser 1995; Taylor & Hamilton 1996; Hamilton 1996), or on a spherical expansion of the density field (Fisher, Scharf, & Lahav 1994c; Heavens & Taylor 1995); these various approaches are reviewed in SW. All these results have been limited by the fact that existing datasets still do not cover enough volume that the underlying real-space clustering on large scales can be adequately modelled as isotropic; the surveys cover only a relatively small number of superclusters, whose orientations relative to the line of sight do not necessarily average out. This is reflected in the range of values of \( \beta \) and the errors that people quote in their detection of this effect; most workers detect the distortions due to peculiar velocities on large scales at only the 3-\( \sigma \) level or so. Results from the correlation function and power spectrum redshift anisotropies give \( \beta = 0.4 \pm 0.7 \), with large error bars 1/3 as large as the signal; interestingly, the spherical expansion method quoted above gives appreciably larger values, \( \beta = 1 \) for the same data. In any case, the surveys discussed in § 1.7 promise to survey a large enough volume to give much smaller error bars, and should be among the most exciting science done with these samples.

There is another technical problem in these analyses. On the face of it, Eq. (1.22) seems nonsensical; what does \( \mu \), the cosine of the angle between the line of sight (defined in real space, of course) and the vector \( k \) mean? One is using the “distant observer” approximation, wherein the sample is approximated to be far from the observer, allowing the quantity \( \mu \) to be defined (cf., the discussion in Cole \textit{et al.} 1994). This means that in practice, one must either work with a fraction of the galaxy pairs available (especially in a full-sky redshift survey, where the distant observer approximation is most grossly violated), or work with a quantity which is related to the theorist’s ideal in a very complicated way. Zaroubi & Hoffman (1996) show that even in the limit of an infinite universe, the Fourier modes of the density field measured
in redshift space are coupled. Recently, Hamilton & Culhane (1996) have suggested a generalization of Eq. (1.23) that allows not having to invoke the distant observer approximation:
\[
\delta(r) \Rightarrow \delta(r) \left[ 1 + \beta \left( \frac{\partial^2}{\partial r^2} + \frac{\partial \ln r^2}{\partial \ln r} \frac{\partial}{\partial r} \frac{\partial}{\partial r} \right) \nabla^{-2} \right] \delta(r),
\]
which is an operator equation both in real space (as shown here) and \( k \)-space (Zaroubi & Hoffman 1996). This approach has yet to be applied to real data.

1.5.3.2 Non-linear Scales

On small scales, it is the pairwise velocity dispersion of galaxies, especially in virialized systems, that dominates the redshift space distortions. The small-scale velocity dispersion of galaxies is interesting for two reasons: first, given various assumptions about the stability of galaxy pairs on small scales, it can be related to the two-point and three-point correlation functions and to the value of \( \Omega \) (the Cosmic Virial Theorem) (Peebles 1976ab, 1980; cf., the discussion in Fisher et al. 1994b; Bartlett & Blanchard 1995; Kepner, Summers & Strauss 1996). Second, it is a quantity that can be predicted for various cosmological models from \( N \)-body simulations, and has been used in the literature in the past as a strong, although not completely unambiguous, discriminator between models (for the standard CDM model alone, which seems to overpredict the small-scale velocity dispersion by a large factor, one can follow the controversy through Davis et al. 1985; Couchman & Carlberg 1992; Cen & Ostriker 1993; Brainerd & Villumsen 1994; Zurek et al. 1994; and Brainerd et al. 1996).

Because the correlation function is strong on small scales, the approximation of isotropy is a good one on these scales, and the effects of peculiar velocities are large, the measurement of this small-scale pairwise velocity dispersion is quite straightforward, and indeed, measurements of this quantity have been done from the anisotropy of the correlation function for many surveys (cf., Davis & Peebles 1983b; Mo, Jing & Börner 1993; Fisher et al. 1994b; Marzke et al. 1995; Guzzo et al. 1995; Somerville, Davis, & Primack 1996). However, the results these workers have found have ranged over a large factor. The problem is that this statistic, as its name implies, is pair-weighted, which means that the densest regions (the rarest, richest clusters) where the velocity dispersion is the highest, tend to dominate the measurement. Therefore, a given measurement of the velocity dispersion is very sensitive to the presence or absence of the richest clusters in the survey volume. There have been several attempts to invent variants on the statistic that are less sensitive to this problem, and thus measure the velocity dispersion in the field. The point is that there is a fair amount of evidence from observations of the peculiar velocity field of galaxies (cf., the contribution by Willick to this volume) that indicates that the velocity field outside of clusters is very quiet (e.g., Sandage 1986; Brown & Peebles 1987; Groth, Juszkiewicz, & Ostriker 1989; Burstein 1990; Ostriker & Suto 1990; Strauss, Cen & Ostriker 1993), and that a measure of this should be a sharper discriminator of models than the classic velocity dispersion measure.

1.5.3.3 Correcting the Density Field for Peculiar Velocities

We have discussed the relationship between the clustering of galaxies in real space and redshift space, and found that statistical measures of clustering such as the power spectrum differ systematically in the two cases. However, there are situations in which we want to do more than simply ask for corrections to statistical quantities measured in redshift space. In particular, it is quite straightforward to measure the density field \( \delta(s) \) of galaxies in redshift space\(^{\dagger} \) but how might we correct the resulting map for peculiar velocities?\(^\dagger\)

\(^{\dagger} s \) is the standard notation to refer to redshift space, in contradistinction to the real space \( r \).
velocities? In linear theory, we have an answer to this question, due to the direct relation between the density and peculiar velocity field, in the form of Eq. (1.21) or its integral equivalent:

\[
v(r) = \frac{\beta}{4\pi} \int \frac{\delta(r')(r' - r)d^3r'}{|r' - r|^3},
\]

(1.26)

where we have assumed linear biasing as before. This equation offers an approach to correcting the distribution of galaxies explicitly for peculiar velocities if one has a moderately deep, full-sky redshift survey, at least on scales large enough that linear theory is likely to hold (Yahil et al. 1991). One measures the quantity \(\delta\) of the galaxies, and solves for the resulting velocity field using Eq. (1.26). One then corrects the redshifts of each galaxy accordingly:

\[
r = cz\hat{r} \cdot v(r),
\]

(1.27)

where \(\hat{r}\) is the unit vector in the direction of the galaxy in question, and \(v\) is the peculiar velocity predicted by Eq. (1.26) at that position. Of course, this changes the position of each galaxy, and therefore the density field, and thus this process must be done iteratively until convergence. There are a number of points to be made here:

- The corrections clearly depend on an assumed value of \(\beta\), and this analysis in and of itself yields no estimate of \(\beta\). Therefore, in practice, one produces a separate density field solution for each value of \(\beta\) one might be interested in.
- Eq. (1.26) is only valid on linear scales, and therefore one must in practice smooth the density field on small scales in applying this equation. A typical smoothing that is used is a Gaussian with \(\sigma = 500\ \text{km s}^{-1}\). A related problem is that clusters of galaxies typically have quite large velocity dispersions, which are very far from being describable by linear theory. In practice, one collapses the galaxies associated with the prominent clusters to a single redshift, to suppress this behavior.
- In a flux-limited redshift survey, the shot noise in the density field necessarily increases as a function of distance from the Local Group. Therefore one needs to carry out some sort of adaptive smoothing as a function of distance to suppress the shot noise. One possibility is to simply have a smoothing length that increases as the mean intergalaxy separation. Another is to expand the density field not in Cartesian coordinates, but rather in spherical harmonics (cf., Nusser & Davis 1994; Fisher et al. 1994c, 1995b). A third possibility is to filter the density field with a Wiener or related filter. As described in Hoffman’s contribution to this volume, these filters are optimal in the sense of suppressing the shot noise while giving the minimum variance difference between the derived and true density fields.
- In principle, the integral in Eq. (1.26) extends to infinity, while redshift surveys are clearly finite. This is not as serious a problem as it may seem. Redshifts are measured in the rest frame of the Earth, and are easily corrected to the heliocentric frame, and to the rest frame of the barycenter of the Local Group (cf., Yahil, Tammann, & Sandage 1977). We of course know that the Local Group is moving with respect to the rest frame of the CMB at \(\approx 620\ \text{km s}^{-1}\) (e.g., Kogut et al. 1993), but we can use a full-sky redshift survey and Eq. (1.26) to predict this motion (e.g., Strauss et al. 1992b; SW). With this predicted peculiar velocity at \(r = 0\), Eq. (1.27) is modified to:

\[
r = cz\hat{r} \cdot (v(r) - v(0)).
\]

(1.28)

Here it is apparent that any bulk flow induced by density fluctuations outside the volume surveyed cancel.

\[†\] Keep in mind that \(H_0 \equiv 1\) with our units.
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out; all that is relevant are higher-order (and therefore intrinsically weaker) multipoles of the large-scale velocity field.

- The redshift-distance relation along any line of sight (as given by Eq. 1.28) is not necessarily monotonic. In particular, in the vicinity of clusters of galaxies, one can get triple-valued zones, in which a single redshift can correspond to three distinct distances. This is illustrated in Fig. 1.4, which shows the relation between the redshift and distance (Eq. 1.28) along a line of sight that passes close to a large mass concentration. Infall into the cluster causes the redshift to be greater than the distance on the near side of the cluster, and further than the distance on the far side. Thus a galaxy at $cz = 1200 \text{ km s}^{-1}$ can be at three distinct distances, as indicated by stars. Methods for dealing with this are described in Yahil et al. (1991) and SW.

- There are other methods than the iterative method outlined here for correcting the density field for peculiar velocities. In particular, Nusser & Davis (1994) and Fisher et al. (1995b) describe non-iterative methods which involve expanding the density field in spherical harmonics and correcting each mode individually for peculiar velocities. The latter authors compare various different methods by means of $N$-body simulations, and conclude that all do roughly equally well.

Although these techniques indeed do correct the galaxy density field for redshift distortions, they simultaneously make a prediction for the velocity field at every point in space. This is very interesting, for it allows a direct comparison with the observed peculiar velocity field, which allows gravitational instability theory (in the form of Eq. 1.21 or 1.26) to be tested, and to constrain the value of $\beta$. This is reviewed in detail in Dekel’s contribution to this volume (cf., Dekel 1994; SW).

1.6 The Relative Distribution of Galaxies and Dark Matter

In this review, I have concentrated on the power spectrum as a measure of the clustering of galaxies. There is quite a variety of other useful statistics with which to measure the clustering properties of galaxies, as reviewed in SW and Borgani (1995). The power spectrum is a complete statistical description of the distribution of galaxies to the extent that the phases of the Fourier modes are randomly distributed, which, as we saw above, implies that the one-point distribution function of densities smoothed on a given scale is Gaussian. This may not hold on large scales due to non-Gaussian initial conditions in the density field (cf., the discussion of Weinberg & Cole 1992) or the generation of non-Gaussian terms by gravitational instability (cf., the review by Juszkiewicz & Bouchet 1996). On small scales, the distribution is manifestly non-Gaussian (as it must be on scales on which $\sigma^2 \rightarrow 1$ (cf., Eq. 1.8) because $\delta \geq -1$ by definition). Higher-order versions of the power-spectrum and correlation function statistics, counts-in-cell statistics, topological statistics, multi-fractals, and many others have been invented to measure these; again, see the reviews cited above.

As mentioned above, one of the big uncertainties in using the observed distribution of galaxies to infer cosmological quantities is the unknown relative distribution of galaxies and the dark matter. As Dekel discusses in this volume, Eq. (1.27) means that measurements of the galaxy velocity field yields a measure of the distribution of all gravitating matter, not just that component which is apparent in galaxies. To the extent that linear biasing holds (Eq. 1.11), this means that density-velocity comparisons in the linear regime are able to constrain the quantity $\beta = \Omega^{0.6}/b$, and not $\Omega$ and $b$ separately. If one goes beyond linear perturbation theory, and can successfully measure second-order effects in the velocity-density relation, then this degeneracy can be broken in principle. However, second-order effects are important when the density
Fig. 1.4. The redshift-distance diagram in the vicinity of a cluster. Infall into the cluster causes a region in which the redshift does not climb monotonically with distance. A galaxy with redshift 1200 km s$^{-1}$ in this direction could therefore be at three distinct distances, as indicated by the stars.

contrast becomes comparable to unity. On the smoothing scales where this is true, there will be regions of space where $\delta_{\text{dark matter}}$ approaches $-1$, that is, true voids. It is clear that when this happens, the linear biasing model, Eq. (1.11), must break down (at least for $b > 1$, as is usually assumed); it would predict $\delta_{\text{galaxies}} < -1$, a physical impossibility. Therefore, if we are to allow ourselves the generalization to a non-linear relation between velocity and density, we must also allow ourselves a non-linear relation between dark matter and galaxy density contrasts, with the resultant increase in the number of parameters, and the degeneracy between $\Omega$ and the biasing parameter(s) remains. The results quoted above in § 1.5.2 should be kept in mind, however, when considering these complications: to the extent that the biasing process is
local, that is, that the process of galaxy formation is a function of the physical properties of matter within a few Mpc of the galaxy in question, the bias parameter, as measured by the ratio of the power spectra of the galaxies and the dark matter, appears to be independent of scale (Weinberg 1995; Kauffmann et al. 1996).

Another interesting handle on bias comes through measurements of higher-order correlation functions; their scaling with scale, it turns out, depends non-trivially on the bias factor(s) (Fry & Gaztañaga 1993; Juszkiewicz et al. 1995; Fry 1994; Gaztañaga 1995; Mo, Jing, & White 1996). The observed amplitudes of the high-order correlations as a function of scale are sufficiently close to those predicted from gravitational instability theory without the complications of bias as to make several of the above papers argue for the bias being quite small (i.e., $|b - 1| \approx 0$).

There is a more indirect approach to the bias question, however. Although we scarcely understand the process of galaxy formation in any detail (cf., the contribution by White in this volume), we imagine that there existed astrophysical processes in the early universe which caused the relative distribution of galaxies and dark matter to differ from one another. If this is the case, we would expect generically that because the astrophysics of formation of galaxies of different types should be different, these various galaxies should be distributed differently with respect to each other. That is, there should exist a relative bias of galaxies of different types. This is of course straightforward to measure from redshift surveys.

There is one such bias which has been known about since the time of Hubble: although elliptical galaxies make up only 10-20% of the galaxies in the field, they dominate completely in the cores of clusters of galaxies. That is, the density fields of elliptical and spiral galaxies are quite different in the densest regions (cf., Dressler 1980, 1984; Postman & Geller 1984; Whitmore et al. 1993), something that is quite apparent in redshift space maps of spirals and ellipticals separately showing the relative distribution of each (e.g., Giovanelli, Haynes & Chincarini 1986; Huchra et al. 1990). It remains quite unclear how this relative bias continues into the field.

There are two ways one might imagine measuring the relative bias of two samples of objects. The first is to compare them statistically: compute the power spectrum, the correlation function or higher-order statistics for each, and determine the bias accordingly. Thus, for linear biasing, the ratio of the two power spectra is proportional to the relative bias parameter squared, while the $S_3$ parameter (defined as $\langle \delta^3 \rangle / \langle \delta^2 \rangle^2$ smoothed on some scale) should scale as $1/b$. SW give a brief review of the extensive literature on determinations of relative bias between samples using this approach. However, the statement $P(k)_{\text{galaxies}} = b^2 P(k)_{\text{dark matter}}$ is clearly a much weaker statement than is Eq. (1.10) (for the latter to hold, one not only needs a specific relation between the amplitudes of the Fourier modes of $\delta$, but also that the phases of the modes agree), and if two samples are contained in the same volume, it is a much more powerful tool simply to compare the density fields of the two samples directly. There is much work that has gone in this direction in the guise of looking for a population of objects that “fills the voids” defined by the distribution of ordinary galaxies. One elegant statistical approach to quantifying relative bias which takes the phase information into account is the cross-correlation statistic: one asks for the mean number of galaxies of sample 2 in excess of random a given distance from galaxies of sample 1. Another approach is to take Eq. (1.10) literally, and compare the density fields of the two samples point by point (cf., Strauss et al. 1992a; Santiago & Strauss 1992).

Despite a great deal of work on this question using a large variety of samples, the results can be summarized in a few sentences. On scales larger than $5 h^{-1}$ Mpc or so, there is no direct evidence for relative
biasing that is non-linear or a function of scale, although the limits on such effects are rather weak†. Late-type galaxies show a weaker clustering signal than do early type galaxies, by a factor of 1.5 to 2.0 in the two-point correlation statistics. This manifests itself in a number of ways: one can find papers in the literature that show that clustering strength increases with the luminosity, central surface brightness, redness, radio power, and mass of the population of galaxies, and decreases with their infrared emission and emission-line strength. Of course, all these quantities are correlated with morphological type, and so it remains unclear whether these trends are separate from the correlation of clustering strength with Hubble type. Moreover, there has yet to be a sample which has been shown clearly to fill the voids which are so prominent in the galaxy distribution.

If clustering is indeed a function of galaxy luminosity, the assumption of the universal luminosity function which went into the definition and use of the selection function (§ 1.3) is clearly not valid. For this reason, as the data get better and the precision with which we measure statistics such as the power spectrum improve, we will either have to include elaborate models for the dependence of clustering strength on luminosity in our analyses, or we will have to do analyses on subsamples of our data over narrow slices in luminosity.

1.7 Surveys for the Future

In reviewing this field, I am struck by the progress that has been made over the last decade or so. We are now measuring, or at least constraining, quantities such as the large-scale bias, the power spectrum on the largest scales, and the value of \( \Omega \), which we had very little hope of getting a handle on at the time people started carrying out large-scale redshift surveys at the end of the 1970’s. Part of this progress has been a realization, as is so common in astrophysics, that as we learn more about the systems we study, we realize how naïve and over-simplified our analyses and assumptions have been, and how much more complicated (and interesting!) reality is. In the early 1980’s, for example, people were modelling infall into the Virgo cluster (cf., Davis & Peebles 1983a) as if it were an isolated overdensity in a uniformly distributed sea of galaxies. A glance at Figs. 1.1 and 1.2 tells us that this is far from a valid model†.

That having been said, we are really only starting to be able to measure cosmological quantities of interest. For example, the values of \( \beta \) determined from redshift space distortions, analyses of the peculiar velocity field (cf., Dekel, this volume; Shaya, Peebles & Tully 1995), and the mass-to-light ratio of virialized systems (e.g., Bahcall, Lubin & Dorman 1995) vary from 0.3 to over unity, with little convergence between methods in sight at the moment (cf., Fig. 20 of SW). The amplitude of the galaxy power spectrum on large scales is uncertain by at least a factor of two, and probably more, and the relative biasing of galaxies of different types (§ 1.6) is only partially to blame for the ambiguity. In other words, we are only just starting to do real quantitative cosmology with large-scale surveys, and I imagine that there is much exciting, and unanticipated, science waiting to be discovered when we start measuring quantities to 10% accuracy (cf., the flurry of theoretical and observational activity prompted by the recent convergence of measurements of \( H_0 \) to within 25%, as summarized in the contributions of Tammann and Freedman to Turok 1996). To beat down the errors further requires much more massive redshift surveys, with:

- Much larger volume surveyed.
- Much larger number of galaxies with redshifts.

† On small scales, at least, the slopes of the two-point correlation functions of ellipticals and spirals are different (Davis & Geller 1976; Giovanelli et al. 1986).

† although Virgocentric infall remains a useful cosmological probe; see the contribution by Peebles in this proceedings.
Much tighter control of statistical and especially systematic errors of the photometric quantities by which galaxies are selected.

Here I discuss two such surveys in the advanced planning stages: the Sloan Digital Sky Survey (SDSS) and the Two-Degree Field Survey (2dF). As I am involved with the former, and thus much more familiar with it, I will put the greater emphasis here on it.

1.7.1 The Sloan Digital Sky Survey

The SDSS is a collaboration between Princeton University, the Institute for Advanced Study, the University of Chicago, the Fermi National Accelerator Laboratory, the University of Washington, Johns Hopkins University, the United States Naval Observatory, and the Japan Promotion Group. We are building a dedicated 2.5m large-field optical telescope at Apache Point in South-Eastern New Mexico, which should see first light late this fall (1996). The telescope will have two main instruments: a photometric camera with 30 2048 × 2048, and 24 2048 × 400 SItC CCD chips on its focal plane, and a pair of double multi-object spectrographs, together taking 640 fibers of 3″ aperture. The purpose of this survey is several-fold. With the photometric camera, the one-quarter of the sky (roughly 10,000 square degrees) centered on the Northern Galactic Cap will be surveyed in drift-scan mode through five broad-band filters (u′, g′, r′, i′, z′, a new photometric system (cf., Fukugita et al. 1996) with effective wavelengths of 3540 Å, 4760 Å, 6280 Å, 7690 Å, and 9250 Å, respectively) almost simultaneously, with an effective exposure time of 55 sec for each. Stellar objects will be detected at 5σ to r′ ≈ 23. From the resulting list of objects detected in the photometric survey, galaxies will be selected to a photometric limit of r′ ≈ 18, and quasar candidates will be selected from the stellar objects by their distinctive colors to a magnitude and a half fainter. These objects, with a density of roughly 120 per square degree, will be observed with the multi-object spectrograph to obtain spectra from 3900-9100 Å with a resolution of 2000. Over the course of the survey, which is expected to take five years, we will thus obtain spectra of roughly 10^6 galaxies and 1.5 × 10^5 quasar candidates, of which we hope 60-70% will be bona fide quasars. Finally, during the Fall months, when the Northern Galactic Cap is unreachable, we plan to repeatedly scan an equatorial stripe e 2.5° wide and 90° long in the South Galactic Cap, centered on δ = 0°. This will allow us to do photometry over 225 square degrees to a photometric limit roughly two magnitudes fainter than in the North. Further photometry (only a single pass) will be done on two “outrigger” great circle stripes centered roughly at δ = +15° and δ = −10° (cf., Fig. 1.7) to maximize the number of baselines for measurements of the power spectrum on the very largest scales.

This is a very quick overview of the plans for the SDSS. Reviews can be found by Gunn & Knapp (1993) and Gunn & Weinberg (1995), and a great deal of technical detail can be found in the text of our proposal to the NSF at http://www.astro.princeton.edu/GBOOK/ on the World-Wide Web. Here let us summarize the goals and prospects for large-scale structure studies with the data from the SDSS. We have space here to touch upon only a few scientific issues which the SDSS will impact; for many more (both in large-scale structure, and in many other fields of extragalactic and Galactic astronomy), see the Web site above.

The SDSS redshift survey will survey to a depth that has been probed previously; the depth is comparable to that of the LCRS and ESP surveys discussed above in § 1.2. Moreover, like the LCRS, the survey galaxies will be selected from photometric CCD data. However, the the volume covered by the SDSS redshift survey will be enormously greater; the solid angle coverage of the SDSS will be 14 times that of the LCRS.

Large-scale structure studies are a major goal of the SDSS. Thus a great deal of emphasis has been
put on controlling systematic errors in the survey. The focal plane of the photometric camera is shown in Fig. 1.5. The camera will draft-scan at sidereal rate along great circles in the sky and thus observe six parallel strips of the sky simultaneously; two passes separated by roughly $13'$ will cover a stripe $2.5^\circ$ wide, with roughly $2'$ overlap between strips. Astrometric calibration is done with the astrometric chips, as described in the caption to Fig. 1.5. Photometric calibration will be done with a separate dedicated 24-inch telescope at the Apache Point site, which will spend the night obtaining photometry of a series of roughly 30 primary standards through the SDSS filters to measure the extinction and photometricity of the sky every hour, and then tie this solution to the photometry of the 2.5m telescope by observing secondary patches in regions of the sky covered by the stripes. The photometric calibration will be checked a posteriori with the overlaps between survey stripes, and probably also with a series of stripes taken perpendicular to the main great circle scans of the sky.

We of course want a sample of galaxies with accurate photometry as they would be seen from outside our Galaxy. Therefore we plan to measure Galactic extinction using our multi-color data. In particular, we will use the colors of distant hot halo subdwarfs (selected by their colors, and confirmed spectroscopically), as well as number counts and color distributions of the faint galaxy populations we find, possibly supplemented by HI maps from Stark et al. (1992) and Burton & Hartmann (1994), and the long-wavelength DIRBE maps from Schlegel, Finkbeiner & Davis (cf., Schlegel 1995).

Galaxies will be selected for spectroscopy from the photometrically calibrated images after an a priori correction for reddening either from Burstein & Heiles (1982) or Schlegel (1995). We wish to select galaxies in as uniform a way as possible, minimizing the effects of redshift. Ideal would be to measure “total” fluxes for galaxies, but because galaxies are extended objects, without sharp edges, any “total” flux one actually measures for them is either a function of the sky level and the depth of the image, or requires a model-dependent extrapolation. Isophotal fluxes have the drawback of being affected by extinction and cosmological surface-brightness dimming in complicated ways, as well as being ill-defined for low surface brightness galaxies. We have thus opted for selecting our galaxy sample based on Petrosian (1976) fluxes in the $r'$ band. Let $I(\theta)$ be the azimuthally averaged surface brightness profile of a galaxy in the $r'$ band.

We define the Petrosian ratio $R_P$ as the ratio between the local and integrated surface brightness profile at radius $\theta$; in practice:

$$R_P(\theta) = \frac{\int_{0}^{1.25\theta} I(\theta') 2\pi \theta' \, d\theta'/[\pi (1.25^2 - 0.8^2) \theta^2]}{\int_{0}^{\theta} I(\theta') 2\pi \theta' \, d\theta'/[\pi \theta^2]}.$$  \hspace{1cm} (1.29)

We then define the Petrosian radius $\theta_P$ as that radius at which the Petrosian ratio falls to some specified level, say $1/8$:

$$R_P(\theta_P) = 0.125.$$  \hspace{1cm} (1.30)

The Petrosian flux $f_P$ is then that measured within a fixed number (say 2) of Petrosian radii:

$$f_P = \int_{0}^{2\theta_P} I(\theta') 2\pi \theta' \, d\theta'.$$  \hspace{1cm} (1.31)

It is this latter quantity (suitably turned into magnitudes) on which we will select our galaxies (with

† Given the subtleties of the determination of reddening using the methods mentioned above, we will be unable to create an improved reddening map “on the fly”, thus our final large-scale structure analyses will require correction for the difference between our a priori and final reddening maps.

‡ We have looked into the possibility of a joint selection in $r'$ and one of the bluer bands. However, the larger extinction in the atmosphere and the lowered throughput at $u'$ makes this band impractical, and simulations have shown that the galaxy populations selected in $g'$ would be almost indistinguishable from those at $r'$, both in redshift distribution and morphological mix.
Fig. 1.5. A schematic of the focal plane of the photometric camera of the SDSS. The camera works in drift scan mode, scanning in the vertical direction, and thus traces out six parallel strips on the sky. The radius of the focal plane (as indicated by the dashed circle) is roughly 2.5 degrees in diameter. Each of the large CCD’s is 2048 × 2048, with 0.4” pixel size. Each of the five rows of CCD’s has a different photometric filter in front of it, so that a given area of sky is scanned in r’, i’, u’, z’, and g’, in order, over a span of roughly 8 minutes. There are 22 2048 × 400 CCD’s with r’ filters and 4.2 mag neutral density filters, at the leading and trailing edges of the arrays. These chips will saturate at r’ = 6.6 (rather than r’ ≈ 14 for the large chips), allowing stars in astrometric catalogs (especially HIPPARCOS, cf., Kovalevsky et al. 1995) to be tied to the SDSS images. Finally, the two 2048 × 400 CCD’s at the top and bottom of the array are used to measure and adjust the focus.

perhaps some further adjustments in the values of the constants 0.125 and 2 in Eqs. (1.30) and (1.31). Because the definition of Petrosian quantities is based on the surface brightness profile of the galaxy itself, the Petrosian radius will be the same metric radius on a galaxy, independent of its distance to us or
foreground extinction, and tests have shown that at least to our spectroscopic limit, it depends only very weakly on effects of seeing or noise. It also has the advantage of being definable for galaxies of all types, including those of very low surface brightness. This last point has a drawback; we cannot obtain spectra of galaxies of arbitrarily faint central surface brightness, no matter how bright their Petrosian fluxes are. We could avoid this problem by cutting on the amount of light entering the 3'' aperture of the fibers. However, such a cut would be very difficult to model as a function of redshift. Therefore, we instead include a secondary cut on Petrosian surface brightness at $\mu_P \leq 22$ in $r'$. This quantity is defined as follows: we define a Petrosian half-light radius $\theta_{r50}$, such that:

$$\int_0^{\theta_{r50}} I(\theta') 2\pi \theta' d\theta' = 0.5 f_P \,, \quad (1.32)$$

the Petrosian surface brightness is then:

$$\mu_P = \frac{0.5 f_P}{\pi \theta_{r50}^2} \, . \quad (1.33)$$

Simulations show that the resulting galaxy sample has a distribution of 3'' aperture magnitudes with a sharp cut-off at $r' = 19.5$, at which point the signal-to-noise ratio is still adequate to obtain redshifts for our exposure time ($\sim 45$ min). It is unfortunate that the limitations of exposure time and the size of our aperture do not allow us to obtain redshifts for the very low surface brightness population of our galaxies, which show an intriguingly different large-scale distribution from those of “ordinary” galaxies (cf., Mo, McGaugh, & Bothun 1994).

There is another class of galaxies to be targeted spectroscopically in the SDSS. Luminous red elliptical galaxies tend to be metal-rich, and thus have strong and prominent metal absorption lines. This means that redshifts can be measured for them at a lower signal-to-noise ratio than for typical absorption-line galaxies. These objects are interesting in their own right, because they are often associated with clusters; indeed, brightest cluster galaxies make up the most luminous and red population of galaxies, and thus targeting the luminous red ellipticals allows us to obtain redshifts for a deep sample of clusters. We will estimate redshifts photometrically from the five-color data for all galaxies using the methods of Connolly et al. (1995) (which should be especially accurate for quiescent red ellipticals), and thereby determine a luminosity. Cuts will be made in luminosity and K-corrected color (the most luminous elliptical galaxies are very uniform in color, which is why the photometric redshift determination is so accurate for this class of objects), yielding a spectroscopic sample of $\sim 10^5$ objects volume-limited roughly to a redshift of 0.45.

I have gone into quite a bit of detail about the selection criteria for our galaxies in order to emphasize the care that is being taken to obtain a uniform sample, as free as possible of biases and systematic errors. The scientific goals of the project range from large-scale structure studies, which are the subject of this review, through quasars and the global properties of galaxies, to Galactic structure and interstellar extinction. Again, see our NSF proposal at the URL quoted above for more detail.

The large-scale structure goals of the survey are manifold. The most obvious is of course the measurement of the power spectrum on very large scales. Fig. 4.6 shows the measured power spectrum of IRAS galaxies from Fisher et al. (1993), together with a prediction of what the SDSS will see ($\Gamma = 0.3$ CDM), with

§ One important exception to this last statement is galaxies with power-law surface brightness profiles, as many cD galaxies are (e.g., Postman & Lauer 1995, and references therein); for such galaxies, the Petrosian ratio asymptotes to a constant that can be above the level at which the Petrosian radius is defined. In such cases, we will switch over to an aperture magnitude. Another potential problem is galaxies with hierarchical structure (e.g., a Seyfert nucleus in a disk galaxy) for which the Petrosian ratio may not be monotonic. Given our chosen value of the value of Petrosian ratio at which the Petrosian radius is defined, this is a problem only for a very small fraction of galaxies.
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Fig. 1.6. The observed power spectrum of IRAS galaxies from Fisher et al. (1993) (open circles) is shown on top of the linear theory $\Gamma \equiv \Omega h = 0.3$ CDM power spectrum, with error bars at selected points expected for the SDSS volume, using the formulae of Feldman et al. (1994). The physical scale corresponding to the $10^6$ resolution of the COBE satellite is indicated.

Error bars following the formalism of Feldman et al. (1994). Thus the SDSS will measure the galaxy power spectrum on scales on which the Sachs-Wolfe effect is directly measured by COBE. Indeed, it will measure $P(k)$ with great precision on smaller scales in the CMB which are now just starting to be probed by balloon and ground-based missions, and which will be studied in detail by the next generation of satellites (cf., Bennett et al. 1995; Mandolesi et al. 1995). If we have sufficient control on systematic errors, the red luminous elliptical sample described above should be able to probe $P(k)$ on the larger scales with appreciably smaller error bars than are shown in Fig. 1.6.
The redshift-space distortions discussed above in §1.5.3 have allowed determinations of $\beta$ to an accuracy of 50% or so, and even that has required a great deal of modelling (cf., the discussion in Fisher et al. 1994b). As we have mentioned above, the principal limitation of these analyses is the smallness of the volume probed; for this statistic, we are far from a fair sample.[†] The SDSS will probe a very large number of superclusters at (presumably) random inclinations, greatly reducing the statistical errors in this analysis. Moreover, one can probe for more subtle effects which could only be set a priori in the analysis of Fisher et al. (1994b), such as the amplitude of the mean streaming as a function of scale, and the detailed shape of the pairwise velocity distribution function on small scales. There is even the possibility of measuring the second moment of the velocity distribution function on linear scales (cf., Fisher 1995), which has the possibility of breaking the degeneracy between $\Omega$ and $b$.

But perhaps the most exciting large-scale structure studies to be done with the SDSS will involve measurements of the distribution of galaxies as a function of the physical properties of the galaxies (such as morphological type, color, luminosity, strength of emission or absorption lines, surface brightness, and so on). As we have discussed in §1.6, it is now clear that there exists a relative bias between galaxies of different types, but we as yet cannot characterize its nature as a function of scale or its universality. It is now becoming clear that the simple linear biasing model of Eq. (1.10) cannot hold true in detail, and that therefore different analyses of large-scale structure in a sense are sensitive to different moments of the relationship between the galaxy and dark matter distribution. The analyses mentioned above, as well as many others, will be done not only on the full sample of galaxies in the SDSS, but also on subsamples divided by the physical properties of galaxies (the sample will be large enough to allow us to do this!); comparison of the results will give us strong constraints on the relative bias of different populations of galaxies.

1.7.2 The Two Degree Field

The Two Degree Field (2dF) refers to an instrument that is undergoing final commissioning at this writing, mounted on the Prime Focus of the Anglo-Australian Telescope. It is a fiber-fed multi-object spectrograph with 400 fibers and a two-degree diameter field, as its name implies. The fiber coupler is doubled, so that one field can be prepared with a robot arm while a second one is observed, minimizing overhead. This is a general purpose observatory instrument which is to be used for a variety of surveys, but a collaboration of British and Australian astronomers is planning to use it for a redshift survey of galaxies selected from the APM galaxy catalog (cf., Maddox et al. 1990abc); thus, unlike the SDSS, the 2dF survey does not attempt to create a galaxy catalog from scratch. The survey will cover 1700 square degrees, and will obtain redshifts of galaxies to $B = 19.7^\dagger$ as measured by the APM. It will consist of three parts, contiguous strips of $75^\circ \times 12.5^\circ$ and $65^\circ \times 7.5^\circ$ in the Southern and Northern Galactic Caps, respectively, and 100 random circular fields of radius $2^\circ$ over the Southern Galactic Cap. The survey will contain roughly 250,000 galaxies, and will take roughly 90 nights of AAT dark time. The survey geometry is chosen to maximize sensitivity to large-scale structure; indeed, the 100 random fields give a variety of baselines to probe the power spectrum on the largest scales. The science goals of the 2dF survey are similar to those of the large-scale structure goals of the SDSS, although the two surveys differ on the approach to galaxy catalogs and target selection, the sky coverage, the depth of the survey, and other details.

† As this example makes clear, the definition of a “fair sample” of the universe depends very much on the sort of statistic one is trying to measure. Thus current samples are more than large enough to define the mean number density of luminous galaxies, but are certainly not adequate to measure the power spectrum on the largest scales.

† This is slightly deeper than the SDSS, given typical galaxy colors of $r^\prime - B \approx -1$ (Frei & Gunn 1994).
1.8 Conclusions

I have reviewed recent progress in measurements of large-scale structure with redshift surveys of galaxies in the nearby universe, with emphasis on the power spectrum of the galaxy density field. There are a number of complicating factors which separate the ideal linear power spectrum of the dark matter from the quantity measured, such as redshift space distortions, non-linear effects, and the relative biasing of galaxies and dark matter, but in each case, these complications include in themselves valuable cosmological information. I conclude with Fig. 1.7 modified from a similar figure in Guzzo (1996), which shows the coverage on the sky of the various surveys we have discussed. This is certainly not complete; the IRAS surveys are not shown here, nor are the CfA2 and SSRS2 surveys (cf., da Costa et al. 1994a) or the Perseus-Pisces redshift survey (Haynes & Giovanelli 1989). However, all the surveys shown here are probing or will probe the large-scale distribution of galaxies to redshifts of 0.1 over substantial solid angles, with more careful and
uniform sample selection than has been possible before now. We are just starting to measure cosmologically important parameters with redshift surveys with errors of 50% or less; this next generation of surveys hold out great hope to reduce these errors substantially, with the potential of new and unexpected insights into our picture about the formation of large-scale structure and galaxies.

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References
Bahcall, N.A., Lubin, L.M., & Dorman, V. 1995, ApJ, 447, L81
Bardeen, J., Bond, J. R., Kaiser, N., & Szalay, A. 1986, ApJ, 304, 15
Bartlett, J.G. & Blanchard, A. 1996, A&A, 307, 1
Baugh, C. M., & Efstathiou, G. 1993, MNRAS, 265, 145
Baugh, C. M., & Efstathiou, G. 1994, MNRAS, 267, 323
Baumgart, D. J., & Fry, J. N. 1991, ApJ, 375, 25
Bennett, C.L. et al. 1995, BAAS, 187, #71.09
Bennett, C.L. et al. 1996, ApJ, 464, L1
Borgani, S. 1995, Physics Reports, 251, 1
Bower, R. G., Coles, P., Frenk, C. S., & White, S. D. M. 1993, ApJ, 405, 403
Brainerd, T. G., Bromley, B.C., Warren, M.S., & Zurek, W. 1996, ApJ, 464, L103
Brainerd, T. G. & Villumsen, J. V. 1993, ApJ, 415, L67
Brainerd, T. G. & Villumsen, J. V. 1994, ApJ, 436, 528
Brown, M. E., & Peebles, P. J. E. 1987, ApJ, 317, 588
Bunn, E.F., & White, M. 1996, astro-ph/9607060
Burstein, D. 1990, Rep Prog. Phys., 53, 421
Burstein, D., & Heiles, C. 1982, AJ, 87, 1165
Burton, W.B., & Hartmann, D. 1994, ApJ, 217, 189
Cen, R. Y., & Ostriker, J. P. 1993, ApJ, 417, 415
Cole, S., Fisher, K. B., & Weinberg, D. 1994, MNRAS, 267, 785
Cole, S., Fisher, K. B., & Weinberg, D. 1995, MNRAS, 275, 515
Coles, P., & Lecchi, F. 1995, The Origin and Evolution of Cosmic Structure (New York: John Wiley and Sons)
Connolly, A.J., Csabai, I., Szalay, A.S., Koo, D.C., Kron, R.G., & Munn, J.A. 1995, AJ, 110, 1071
Corwin, H. G, & Skiff, B. A. 1996, Extension to the Southern Galaxies Catalogue, in preparation
Couchman, H. M. P., & Carlberg, R. G. 1992, ApJ, 389, 453
da Costa, L. N. et al. 1994a, ApJ, 424, L1
da Costa, L. N., Vogeley, M.S., Geller, M.J., Huchra, J.P., & Park, C. 1994b, ApJ, 437, L1
Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
Davis, M., & Geller, M. J. 1976, ApJ, 208, 13
Davis, M., & Huchra, J. P. 1982, ApJ, 254, 437
Davis, M., & Peebles, P. J. E. 1983a, ARA&A, 21, 109
Davis, M., & Peebles, P. J. E. 1983b, ApJ, 267, 465
Davis, M., Tonry, J., Huchra, J., & Latham, D. W. 1980, ApJ, 238, L113
Davis, R. L., Hodges, H. M., Smoot, G. F., Steinhardt, P. J., & Turner, M. S. 1992, PRL, 69, 1856
Dekel, A. 1994, ARA&A, 32, 371
Dekel, A., & Rees, M. J. 1987, Nature, 326, 455
de Lapparent, V., Geller, M. J., & Huchra, J. P. 1986, ApJ, 302, L1
Dressler, A. 1980, ApJ, 236, 351
Dressler, A. 1984, ARA&A, 22, 185
Recent Advances in Redshift Surveys of the Local Universe

Efstathiou, G. 1991, in Physics of the Early Universe, eds. J. A. Peacock, A. F. Heavens, & A. T. Davies (Edinburgh: SUSSP), p. 361

Efstathiou, G. 1995, Les Houches Lectures, ed. R. Schaefer (Netherlands: Elsevier Science Publishers), in press

Efstathiou, G. 1996, in Critical Dialogues in Cosmology, ed. N. Turok (Cambridge: Cambridge University Press), in press

Efstathiou, G., Ellis, R. S., & Peterson, B. S. 1988, MNRAS, 232, 431

Ellis, R.S. 1996, in Critical Dialogues in Cosmology, ed. N. Turok (Cambridge: Cambridge University Press), in press

Feldman, H., Kaiser, N., & Peacock, J. 1994, ApJ, 426, 23

Felten, J. E. 1977, AJ, 82, 861

Fisher, K. B. 1995, ApJ, 448, 494

Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1993, ApJ, 402, 42

Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994a, MNRAS, 266, 50

Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994b, MNRAS, 267, 927

Fisher, K. B., Huchra, J. P., Davis, M., Strauss, M. A., Yahil, A., & Schlegel, D. 1995a, ApJS, 100, 69

Fisher, K. B., Lahav, O., Hoffman, Y., Lynden-Bell, D., & Zaroubi, S. 1995b, MNRAS, 272, 885

Fisher, K.B. & Nusser, A. 1995, MNRAS, 279, L1

Fisher, K. B., Scharf, C. A., & Lahav, O. 1994c, MNRAS, 266, 219

Frei, Z. & Gunn, J.E. 1994, AJ, 108, 1476

Fry, J. N. 1994, PRL, 73, 215

Fry, J.N. & Gaztanaga, E. 1993, ApJ, 413, 447

Fukugita, M., Ichikawa, T., Gunn, J.E., Doi, M., Shimasaku, K., Schneider, D.P. 1996, AJ, 111, 1748

Gaztanaga, E. 1995, ApJ, 454, 561

Geller, M. J., & Huchra, J. P. 1988, in Large-Scale Motions in the Universe, ed. V. C. Rubin & G. V. Coyne (Princeton: Princeton University Press), p. 3

Geller, M. J., & Huchra, J. P. 1989, Science, 246, 897

Giovanelli, R., & Haynes, M. P. 1991, ARA&A, 29, 499

Giovanelli, R., Haynes, M. P., & Chincarini, G. L. 1986, ApJ, 300, 77

Górski, K.M. et al. 1996, ApJ, 464, L5

Gramann, M., Cen, R., & Bullock, N.A. 1993, ApJ, 419, 440

Groth, E. J., Juszkiewicz, R., & Ostriker, J. P. 1989, ApJ, 346, 558

Gunn, J. E., & Weinberg, D. H. 1995, in Wide-Field Spectroscopy and the Distant Universe, ed. S. J. Maddox and A. Aragón-Salamanca (Singapore: World Scientific), 3

Guazzo, L. 1996, in Mapping, Measuring, and Modelling the Universe, eds. P. Coles & V. Martinez, in press

Guazzo, L., Fisher, K.B., Strauss, M.A., Giovanelli, R., & Haynes, M.P. 1996, Astrophysics Letters and Communications, 33, 231

Hamilton, A. J. S. 1993, ApJ, 406, L47

Hamilton, A.J.S. 1996, in Clustering in the Universe, Proc. 30th Rencontres de Moriond, ed. S. Maurogordato, in press

Haynes, M. P., & Giovanelli, R. 1989, in Large Scale Motions in the Universe, eds. V. C. Rubin & G. V. Coyne (Princeton: Princeton University Press), p. 31

Heavens, A.F., & Taylor, A.N. 1995, MNRAS, 275, 483

Hermit, S., Lahav, O., Santiago, B.X., Strauss, M.A., Davis, M., Dressler, A., & Huchra, J.P. 1996, MNRAS, in press

Heydon-Dumbleton, N.H., Collins, C. A., & MacGillivray, H.T. 1989, MNRAS, 238, 379

Huchra, J. P., Davis, M., Latham, D., & Tonry, J. 1983, ApJS, 52, 89

Huchra J. P., Geller, M. J., de Lapparent, V., & Corwin, H. G. 1990, ApJS, 72, 433

IRAS Catalogs & Atlases, Explanatory Supplement 1988, ed. C. A. Beichman, G. Neugebauer, H. J. Habing, P. E Clegg, & T. J. Chester (Washington D. C.: U.S. Government Printing Office)

Jain, B., & Bertschinger, E. 1994, ApJ, 431, 495

Jain, B., Mo, H.J., & White, S. D. M. 1995, MNRAS, 276, L25
Juszkiewicz, R., & Bouchet, F.R. 1996, in Clustering in the Universe, Proc. 30th Rencontres de Moriond, ed. S. Maurogordato, in press
Juszkiewicz, R., Weinberg, D., Amsterdamski, P., Chodorowski, M., & Bouchet, F. 1995, ApJ, 442, 39
Kaiser, N. 1984, ApJ, 284, L9
Kaiser, N. 1987, MNRAS, 227, 1
Kauffmann, G., Nusser, A., & Steinmetz, M. 1996, MNRAS, in press (astro-ph/9512009)
Kepner, J.P., Summers, F J, & Strauss, M.A. 1996, ApJ, submitted
Kogut, A. et al. 1993, ApJ, 419, 1
Kolb, E. W., & Turner, M. S. 1990, The Early Universe (Redwood City: Addison-Wesley)
Kolatt, T., & Dekel, A. 1995, astro-ph/9512132
Koranyi, D.M., & Strauss, M.A. 1996, ApJ, submitted
Kovalevsky, J. et al. 1995, A&A, 304, 34
Landy, D.S., Shectman, S.A., Lin, H., Kirshner, R.P., Oemler Jr., A.A., & Tucker, D. 1996, ApJ, 456, L1
Lauberts, A. 1982, The ESO/Uppsala Survey of the ESO(B) Atlas (München: European Southern Observatory)
Lauberts, A., & Valențijn, E. A. 1989, The Surface Photometry Catalogue of the ESO-Uppsala Galaxies (München: European Southern Observatory)
Lawrence, A. et al. 1996, in preparation
Lilly, S.J., Le Fevre, O., Crampton, D., Hammer, F., & Tresse, L. 1995, ApJ, 455, 50
Lin, H., Kirshner, R.P., Shectman, S.A., Landy, S.D., Oemler Jr., A., Tucker, D.L., & Schechter, P.L. 1996, ApJ, in press
Loveday, J., Efstathiou, G., Maddox, S.J., & Peterson, B.A. 1996, ApJ, in press (astro-ph/9505099)
Maddox, S. J., Efstathiou, G., & Sutherland, W. J. 1990c, MNRAS, 246, 433
Maddox, S.J., Efstathiou, G., & Sutherland, W.J. 1996, MNRAS, in press (astro-ph/9601103)
Maddox, S. J., Efstathiou, G., Sutherland, W. J., & Loveday, J. 1990a, MNRAS, 242, 43P
Maddox, S. J., Efstathiou, G., Sutherland, W. J., & Loveday, J. 1990b, MNRAS, 243, 692
Mancinelli, P. 1996, PhD Thesis (SUNY Stony Brook)
Mandolesi, N. 1995, Planetary & Space Science, 43, 1459
Marzke, R.O., Geller, M.J., da Costa, L.N., & Huchra, J.P. 1995, AJ, 110, 477
Mo, H. J., Jing, Y. P., & Börner, G. 1993, MNRAS, 264, 825
Mo, H.J., Jing, Y.P., & White, S.D.M. 1996, astro-ph/9603033
Mo, H.J., McGaugh, S.S., & Bothun, G.D. 1994, MNRAS, 267, 129
Nicoll, J. F., & Segal, I. E. 1983, A&A, 118, 180
Nilson, P. 1973, The Uppsala General Catalogue of Galaxies, Ann. Uppsala Astron. Obs. Band 6, Ser. V:A. Vol. 1
Noordermeer, E. J., & Ensinge, T. P. 1994, ApJ, 421, L1
Ostriker, J. P., & Suto, Y. 1990, ApJ, 348, 378
Padmanabhan, T. 1993, Structure Formation in the Universe (Cambridge: Cambridge University Press)
Park, C., Gott, J.R., & da Costa, L. N. 1992, ApJ, 392, L51
Park, C., Vogeley, M. S., Geller, M. J., & Huchra, J. P. 1994, ApJ, 431, 569
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Peacock, J. A., & Nicholson, D. 1991, MNRAS, 253, 307
Peebles, P. J. E. 1976a, ApJ, 205, L109
Peebles, P. J. E. 1976b, A&SS, 45, 3
Peebles, P. J. E. 1980, The Large Scale Structure of the Universe (Princeton: Princeton University Press)
Peebles, P. J. E. 1987, Nature, 327, 210
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton University Press)
Petrosian, V. 1976, ApJ, 209, L1
Postman, M., & Geller, M. J. 1984, ApJ, 281, 95
Postman, M., & Lauer, T. R. 1995, ApJ, 440, 28
Rowan-Robinson, M., Lawrence, A., Saunders, W., Crawford, J., Ellis, R., Frenk, C. S., Parry, I., Xiaoyang, X., Allington-Smith, J., Efstathiou, G., & Kaiser, N. 1990, MNRAS, 247, 1
Sachs, R. K., & Wolfe, A. M. 1967, ApJ, 147, 73
Sahni, V., & Coles, P. 1995, Physics Reports, 262, 1
Sandage, A. 1986, ApJ, 307, 1
Sandage, A., & Tammann, G. A. 1981, A Revised Shapley-Ames Catalogue of Bright Galaxies (Washington DC: Carnegie Institute of Washington)
