Effect of Inhomogeneity of the Universe on a Gravitationally Bound Local System: A No-Go Result for Explaining the Secular Increase in the Astronomical Unit

Hideyoshi Arakida

Graduate School of Education, Iwate University, Iwate, Japan
e-mail: arakida@iwate-u.ac.jp

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Abstract. We will investigate the influence of the inhomogeneity of the Universe, especially that of the Lemaître–Tolman–Bondi (LTB) model, on a gravitationally bound local system such as the solar system. We concentrate on the dynamical perturbation to the planetary motion and derive the leading order effect generated from the LTB model. It will be shown that there appear not only a well-known cosmological effect arisen from the homogeneous and isotropic model, such as the Robertson–Walker (RW) model, but also the additional terms due to the radial inhomogeneity of the LTB model. We will also apply the obtained results to the problem of secular increase in the astronomical unit, reported by Krasinsky and Brumberg (2004), and imply that the inhomogeneity of the Universe cannot have a significant effect for explaining the observed \( \frac{d \text{AU}}{dt} = 15 \pm 4 \text{ [m/century]} \).

Key words. Celestial mechanics—gravitation—cosmology—LTB model—ephemerides—astronomical unit.

1. Introduction

The advancements in astronomical and astrophysical measurement techniques, particularly those involving the solar system, has been achieved remarkable accuracy of up to 9 to 11 digits level. These technical advancements have drastically improved the accuracy of planetary ephemerides such as DE (Standish 2003), EPM (Pitjeva 2005), VSOP (Bretagnon & Francou 1988) and INPOP (Fienga et al. 2008) and that of various astronomical constants. With increasingly improved measurement techniques, observational models are also required to be more accurate and rigorous; for details, refer to Soffel et al. (2003) and the references therein.

High-precision observational data also play a crucial role in experimental relativity (Will 1993, 2006). Presently, the main parameters of parametrized post-Newtonian (PPN) approximation, \( \beta \) and \( \gamma \) are tightly constrained to the value of general relativity, i.e., \( \beta = \gamma = 1 \). For a more accurate verification of gravity,
space tests such as LISA (Danzmann 2000), LATOR (Turyshhev et al. 2004), and ASTROD/ASTROD i (Ni 2008) have been planned.

Thus far, theoretical developments in studies of the solar system have pertained to slow motion, slow rotation and weak field approximation, \[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1, \]
where \( \eta_{\mu\nu} \) is the static Minkowski metric and \( h_{\mu\nu} \) is the perturbation (see Damour et al. 1991, 1992, 1993, 1994; Brumberg & Kopeikin 1989a, b). However, it is well known that our universe is expanding at an accelerated rate (Perlmutter et al. 1999). Therefore, it is natural to consider the situation that the metric tensor, instead of the Minkowskian metric, asymptotically reaches for the expanding space-time or the background Minkowskian metric \( \eta_{\mu\nu} \) is replaced by the cosmological type. Several investigations have been conducted, which combine the local metric, e.g., the Schwarzschild space-time or the barycentric celestial reference system adopted by IAU, with the global cosmological comoving coordinates (McVittie 1933; Järnefelt 1940a, b, 1942; Einstein & Straus 1945, 1946; Schücking 1954; Noerdlinger & Petrosian 1972; Cooperstock et al. 1998; Klioner & Soffel 2004; Faraoni & Jacques 2006; Sereno & Jetzer 2007; Adkins & McDonnell 2007; Kopeikin 2007; Carrera & Giulini 2010).

The cosmological contribution to the local system has thus far been discussed based on the homogeneous and isotropic cosmological model, i.e., the Robertson–Walker (RW) model. However, inhomogeneous cosmological models have recently attracted considerable attention since these models can provide a possibility to explanation for the observed accelerated cosmic expansion without introducing the concept of dark energy. For instance, the luminosity-distance was investigated in Tomita (2000a, b, 2001a, b, c) based on the local void model and in Iguchi et al. (2002) using the Lemaître–Tolman–Bondi (LTB) model (Lemaître 1933; Tolman 1934; Bondi 1947; Plebański & Krasiński 2006). Moreover, Kasai (2007) re-analyzed the observed Type Ia supernovae data and proposed a phenomenological method to describe the large-scale inhomogeneity of the Universe.

Therefore, it would be significant and interesting to investigate the influence of the inhomogeneity of the Universe on the gravitationally bound local system. As far as we know, this issue has previously been examined by Gautreau (1984) and Mashhoon et al. (2007). Gautreau studied the special case of the LTB model, \( \mathcal{E}(r) = 0 \), (see (19)) with respect to his cosmological theory of the curvature coordinates\(^1\), while Mashhoon et al. investigated the cosmological contribution due to the LTB model as the tidal dynamics in the Fermi normal coordinate system.

With remarkable improvements in the observations, it has been found that there exist the unexplained phenomena in theory within the solar system; the pioneer anomaly (Anderson et al. 1998), the Earth fly-by anomaly (Anderson et al. 2008), the secular increase in the astronomical unit (Krasinsky & Brumberg 2004), and the anomalous perihelion precession of Saturn (Iorio 2009). Presently, the origins of these anomalies are far from clear. Nonetheless, they may be attributable to some fundamental properties of gravitation (see Lämmerzahl et al. 2008 and the references therein).

\(^1\)Gautreau does not start from the original form of the LTB model. However, Krasiński (1997) suggested that the model by Gautreau corresponds to the sub case of the LTB model, \( \mathcal{E}(r) = 0 \).