The reflective abstraction profile of junior high school students in solving mathematical problems based on cognitive style of field independent and field dependent

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Abstract. In solving a problem, students have different characteristics. This research describes students’ abstraction profile in solving mathematical problems based on cognitive style. The qualitative method was applied to analyze data taken from one student who has field-independent’s cognitive style and one student who has field-dependent’s cognitive style. The research finding shows that students’ abstraction profile of field-independent’s cognitive style is different from students of field-dependent’s cognitive style in solving a mathematical problem. The implication is that the teacher should know and understand each student’s cognitive style to apply appropriate learning approach.

1. Introduction

Abstraction is the basic process in mathematics. Mathematical thinking is a mental activity that in the process always uses abstraction or generalization. Mathematical thinking is defined as “a mental activity that involves abstraction and generalization of mathematical ideas” [1]. This statement means that abstraction is always used in each of mathematical thinking.

Abstraction is defined as “an organization activity of mathematical vertical concept which is constructed previously and becomes a new mathematical structure” [2]. The new mathematical objects are constructed through such formatting relation until finding generalization as a new proof or strategy for solving problems.

Abstraction theory is divided into three parts. Firstly, empirical abstraction that focuses on the way children construct the meaning of object characteristics (focusing on objects and their properties). Secondly, pseudo-empirical abstraction that focuses on the way children construct the meaning of characteristics that involves an object (focusing on action on objects and the properties). Thirdly, reflective abstraction that focuses on the idea of action and operation (focusing on mental objects) [3]. The result of reflective abstraction is the scheme of knowledge for each phase of development and reflective abstraction to extract the scheme from the related form of action.

Reflective abstraction refers to a subject's ability to project and reorganize structure created from activities and the subject’s interpretation of the new situation. In line with this, reflective abstraction refers to the subject’s ability to project on a new level and organize a structure created from the subject's activities and interpretation” [4].

Levels of reflective abstraction activities were defined as follows: the first level is recognition; the second level is representation; the third level is structural abstraction; and the fourth level or the highest level is structural awareness [4]. The researcher uses the level of reflective abstraction activity
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that is proposed [4] to describe the abstraction process of junior high school students in solving a mathematical problem.

At the recognition level, the problem solver encounters a new situation and recalls or identifies activity from previous situations as being appropriate. The ability to identify mathematical structure is when one realizes the structure existed and has the ability to solve the problem faced. To solve the problem, they have to recall previous knowledge that has been gained from previous activity and use it for further activity.

The second level is representation. The problem solver utilizes a diagram in resolving a problematic situation to aid reflection. The problem solver is operating at this level if more control over the solution activity is demonstrated or, more precisely, if the solver re-presents this solution activity. This reflective level requires the individual to demonstrate a certain degree of flexibility and control over prior activity in the sense that the activity could mentally be “run through”.

At the third level, structural abstraction, students are able to project and reorganize mathematical structure has been created from previous activity and student’s interpretation into a new situation. The existing mathematical structure is projected and reorganized so students could gain new knowledge from it.

The fourth level or the highest one is structural awareness. It refers to student’s metacognition about activity and organization of its cognitive structure. The problem structure that students comprehend for solving the problem becomes the object in the reflection. Students are able to think such structure as the objects and able to decide without trying or using physical forming or mentality present completion method. When problem takers gain the higher level of reflective abstraction, their thought becomes more flexible.

In the solving problem, the students have different characteristics. Psychology with its branches has indicated many various variables which demonstrated individual differences as well as encourages the learning process such as intelligence, talent, cognitive style, thinking style, adoption style, persistent and commencement ability.

Individual differences in this research are the differences based on the cognitive style. Identifying a student's cognitive style, the educator is helped to understand how somebody organizes and presents information [5]. Psychological dimensions that represent the consistencies of individual’s manner in acquiring and processing information [6].

The student who has field-independent’s cognitive style is able to abstract the context elements or its background. He/she tends to solve the problem with the analytical approach. Meanwhile, students of field-dependent’s cognitive style use problem solving approach globally, he/she thinks all the pictures in the contexts given and has difficulties to separate one item from whole context [7].

Problem solving is defined as the process done by the individual in combining prior knowledge to face a new situation. It means that problem solving is the process that is done by somebody in combining the prior knowledge to solve the problem that has not yet known. The ways of solving problem consisting four steps, they are; (1) understand the problem, (2) devise a plan, (3) carry out the plan, and (4) recheck the result [8].

From these explanations, it can be concluded that solving problems of mathematics is the process of finding mathematics problem which included understanding the problem, planning of solving the problem (devise a plan), solving the problem (carry out the plan), and recheck the result.

Based on the previous background, this research intends to answer the research question “how students’ abstraction profile who has field-independent’s cognitive style and who has field-dependent’s cognitive style in solving mathematical problem?”

2. Method
It is explorative research uses a qualitative approach. The participants were the grade 8 students in one of Indonesian Province. The selective subject is one student who has field-independent’s (FI) cognitive style and one student who has field-dependent’s (FD) cognitive style. FI and FD subject selection is done in the following; The Group Embedded Figures Test (GEFT) is given to the students, set
the students group of FI and FD according to the scores obtained by the students within the time limit available. Students with scores 0-9 are categorized as groups FD and students who score 10-18 are categorized as FI student groups.

Out of 40 students who attended the GEFT test, there were 26 (65%) students who had afield independent cognitive style and 14 people (35%) who had field dependent cognitive style. From the group of field independent cognitive style, one student is selected, who has the highest GEFT test score, and from the group of cognitive style field dependent, one student is selected who has the lowest GEFT test score.

The instruments in this research are GEFT and problem solving task. The problem is "a fond fish in rectangle form has 18 meters in length and 8 meters in width. Around of beyond of it will be built a path of 1-meter scale, which is made of gravels. For each meter quadrate is needed 2 kilograms of gravels. If the price of the gravels is 4,500.00 rupiahs per kilogram, so how much is the cost of buying gravels to cover the scale?"

The recorded data through a deep interview is analyzed by a qualitative method that involved data reduction, displaying data, and taking conclusion. Data reduction phase is the activity of selection, the concentration of attention, simplification, abstraction, and transformation of raw data. The code is then generated so it is known to come from which source. The valid data is analyzed. In data presentation stage, the researcher collected data and organized it; so, it is easier to draw the conclusion from the data. The conclusion step, the researcher draws the conclusions from the data collected and verified the conclusion. Verification is done by reviewing field notes, discussing with colleagues or with teachers from research subjects. This analysis is related to student’s appearance of abstraction process in solving a mathematical problem.

3. Results and discussion

This following is found the analysis of the abstraction process of two students in solving mathematical problems.

3.1. Analysis of student’ abstraction profile of field independent (subject FI)

To know the abstraction process appear in solving the problem, so the interview is done to Subject 1 (FI). Subject FI has read the item/problem; he gave attention to the problem, so he thought quickly or done abstraction to understand it. After asking of what has been known and asked, he was able to explain rightly in the form of words. It indicates a reflective abstraction level of recognition and representation.

Subject FI state that the information of the item that is the path will be built in around beyond the pond fish is 1 meter. It means that there is a new rectangle whose 2 meters in length and 2 meters in width so will make the planning of new situation (rectangle of 20 meters length and 10 meters width) rather than previous situation (remembering the rectangle of 18 meters length and 10 meters width), recognizing previous activity. The planning has been done before in the form of a rectangle to help reflection. It shows the reflective abstraction level of recognition and representation.

Then the researcher wants to know whether still there is another idea to answer the question of the problem. Subject S1 has gained the insight to measure the width of the path that will be built by gravels. It the whole width or the path width increased by fishpond then decrease by fishpond width.

Subject FI believes that his steps are right, so he completes the idea directly by reflecting previous activity for the new situation; it is to count the path width by decreasing the whole width with the width of pond fish, which is drawn by a rectangle. It shows the abstraction level of representation and structural abstraction.

The ways to recheck the obtained result, subject FI checks the obtained result with the steps done previously by adding the width of each rectangle (there are four rectangles) at around of the pond fish. In this case, the subject FI develops a new strategy or method to count the width of the path. In his work exists crossing which indicates that the subject FI makes the wrongness. Subject FI immediately corrects his work as much as required. Subject FI can give the argument or the true reason for the decisions made, and to summarize his activities while doing the task. It shows the reflective abstraction level of structural abstraction and structural awareness.
3.2. Analysis student’s abstraction profile of field dependent (subject FD)

After giving the test sheet to subject FD, he reads the test instruction and writes down his name on the answer sheet provided. To find out the increasing of the abstraction process in mathematical problems, so the researcher (R) did interview to the subject of FD (S2).

Subject FD has read the test/problem. He has a lack of attention to what has been read before, so he thought in a long time to understand it. It can be seen when the subject FD was asked what is known in the test; he is silent. The researcher attempts to understand what has been being thought by the subject by review the same question. Then he answered it incompletely. Meanwhile, in the end, his answer is correct. Besides that, he can describe what is known in the test in the form of words. It shows the reflective abstraction level of recognition and representation.

Subject FD state the information related to the test is the pond fish circumference. He has difficulty to make the plan; he does not understand the previous information because he does not know to differentiate the meaning of “around” and “circumference.” The researcher hopes the subject FD can distinguish the around of pond fish and its circumference so the subject can understand the information related to the test.

From the question addressed by the researcher, subject FD is able to count the width of the path by applying one way. Then, the researcher wants to know whether any other ways or not. Subjects FD state his idea doubtfully. But, through the questions given by the researcher, he can state his idea. Finally, he is able to remember the way to measure the width of the path in another method. He states that the left and right side of each border of the pond fish is 10 meters quadrate. It is said because he has a glance the whole circumference of the pond fish and the path is two times 20 meters and add two times 10 meters. It shows that subject FD still cannot differentiate the meaning of words "around" and "circumference." He relates the new situation to the previous situation, so the plan is gotten which is stated in form of words to help reflection. It shows the reflective abstraction level of recognition and representation.

Subject FD writes directly the result of the calculation around of left, right, up, and the bottom side, so he is sure that the steps which are done are the right one. He states that the calculation of the path in the form of words. It shows the reflective abstraction level of representation.

Subject FD checks the result of the path width which will be built and review the wrongness through writing another way of solving the problem then he reviews the necessary one. The review is stated in form of words. It shows the reflective abstraction level of representation.

Subject Field Dependent difficult to interpret the word or information that is in the problem. This is in accordance with the opinion that the subject of the field dependent is difficult to provide a structure of ambiguous information, besides the subject only sees the problem globally [5]. In contrast to the Field Independent subject, before solving the problem, first analyzes the problem and then abstracts it, according to the opinion that individuals who are field independent tend to think analytically [9].

4. Conclusion

Based on the analysis and finding it can be concluded that profile abstraction process of the third class of junior high school in solving the mathematical problems as follows:

4.1. Reflective abstraction profile belongs to a student’s cognitive style of field independent

In the step of understanding the problem, the student describes the necessary information based on the problem; he states what has been known and what is asked in the item in form of words. In this case appears reflective abstraction level of recognition and representation. In the step of planning solving problem, the student understands the information at new situation and understand the new information firstly at once he is able to relate the information quickly and uses pictures to help reflection in order to get right planning solving the problem. In this case appears reflective abstraction level of recognition and representation. In the step of solving problem-based on the preparation, the students carry out the plan. He reflects previous activity into a new situation and uses picture representation. In this case appears reflective abstraction level of representation and structural abstraction. In the step of
review, the gained result the student uses a new strategy to check the result obtained previously. Giving argument or valid reason for decided decisions and summarizing all activities while solving the problem. In this case appears reflective abstraction level of structural abstraction and structural awareness.

4.2. Reflective abstraction profile belongs to a student’s cognitive style of field dependent

In the step of understanding the problem, the student states necessary information based on the problem, stated what is known and asked in the item test in the form of words. In this case appears reflective abstraction level of recognition and representation. In the step of planning solving problem, the student does not understand new information in a new situation and does not remember previous details yet, so he needs longer time to relate the data to the beginning he has faulty in completing the test. In this case appears reflective abstraction level of recognition and representation. In carrying out the planning, the student only does it based on the plan without developing new strategy or method. In this case appears reflective abstraction level of representation. In the step of re-checking the gained result, the student reviews his wrong as much as required or writing another way of completing the test. He is not able to differentiate the result from a different way. In this case appears reflective abstraction level of recognition and representation.

Based on the conclusion the recommendation or implication that the teacher should know and understand each student’s cognitive style in order to adapt to the appropriate learning approach. In completing the test, the teacher should be able to create questions which to reach knowledge or prior information to bring he/she up to reflective abstraction level of structural awareness.

References

[1] Wood T, Williams G & Mc Neal B 2006 Children’s mathematical thinking in different classroom cultures Journal for Research in Mathematics Education 37 222
[2] Mitchelmore M and White P 2004 Abstraction in mathematics and mathematics learning In M J & A B Fuglestad (Eds.) Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education 3 329
[3] Gray E and Tall D 2007 Abstraction as a natural process of mental compression Mathematics Education Research Journal 19 2
[4] Goodson-Espy T 2005 Why Reflective Abstraction Remains Relevant In Mathematics Education Research In Lloyd G M, Wilson M, Wilkins J L M & Behm S L (Eds) Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Appalachian State University
[5] Altun A and Cakan M 2006 Undergraduate students’ academic achievement, field dependent/independent cognitive styles and attitude toward computers Journal of Educational Technology and Society 9 289
[6] Ausburn L J & Ausburn F B 1978 Cognitive styles some information and implications for instructional design Educational Communication and Technology 26 337
[7] Riding R and Rayner S 1998 Cognitive styles and learning strategies: Understanding style differences in learning and behavior (London: David Fulton Publishers)
[8] Polya G 1973 How To Solve It Second Edition (New Jersey: Princeton University Press)
[9] Witkin H, Moore C, Goodenough D & Cox P 1977 Field-dependent and field-independent cognitive style and their educational implications Review of Educational Research 47 1