Production of the X(3872) in B Meson Decay
by the Coalescence of Charm Mesons

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The recent unexpected discovery of a narrow charmonium resonance near 3.87 GeV challenges our understanding of heavy quarks and QCD. This mysterious state X(3872) was discovered by the Belle collaboration in electron-positron collisions in the B-meson decay $B^\pm \rightarrow K^\pm X$ followed by the decay $X \rightarrow J/\psi \pi^+\pi^-$. The discovery was confirmed by the CDF collaboration using proton-antiproton collisions $\bar{p}p$. The X is much narrower than all other charmonium states above the threshold for decay into a pair of charm mesons. Its mass is also extremely close to the threshold for decay into the charmed mesons $D^0\bar{D}^{*0}$ or $\bar{D}^0D^0$.

The proposed interpretations of the X(3872) include a D-wave charmonium state with quantum numbers $J^{PC} = 2^{--}$ or $2^{-+}$, an excited P-wave charmonium state with $J^{PC} = 1^{++}$ or $1^{-+}$, a “hybrid charmonium” state in which a gluonic mode has been excited, and a $D^0\bar{D}^{*0}/\bar{D}^{0}D^{0}$ molecule. The possibility that charm mesons might form molecular states was considered some time ago. If the binding is due to pion exchange, the most favorable channels are S-wave with quantum numbers $J^{PC} = 1^{++}$ or P-wave with $0^{-+}$. The proximity of the mass of X to the $D^0\bar{D}^{*0}$ threshold indicates that it is extremely loosely bound. If X is an S-wave $D^0\bar{D}^{*0}/\bar{D}^{0}D^{0}$ molecule, the tiny binding energy introduces a new length scale, the $D\bar{D}^{*}$ scattering length $a$, that is much larger than other QCD length scales. As a consequence, certain properties of the $X/D^0\bar{D}^{*0}/\bar{D}^{0}D^{0}$ system are determined by $a$ and are insensitive to the shorter distance scales of QCD. This phenomenon is called low-energy universality.

A challenge for any interpretation of the X(3872) is to explain its production rate. This could be problematic for the identification of X as an S-wave $D^0\bar{D}^{*0}/\bar{D}^{0}D^{0}$ molecule, because it can readily dissociate due to its tiny binding energy. One way to produce X is to produce $D^0$ and $\bar{D}^{*0}$ with small enough relative momentum that they can coalesce into X. An example is the decay $\Upsilon(4S) \rightarrow X hh'$, where h and h' are light hadrons, which can proceed through the coalescence into X of charm mesons from the 2-body decays of a virtual B and a virtual $\bar{B}$. Remarkably, low-energy universality determines the decay rate for this process in terms of hadron masses and the width $\Gamma_B$ of the B meson. Unfortunately, the rate is supressed by a factor of $(\Gamma_B/m_B)^2$ and is many orders of magnitude too small to be observed.

In this paper, we apply low-energy universality to the discovery mode $B^+ \rightarrow X K^+$ and to the process $B^+ \rightarrow D^0\bar{D}^{*0}K^+$. We point out that the interpretation of X as an S-wave $D^0\bar{D}^{*0}/\bar{D}^{0}D^{0}$ molecule can be confirmed by observing a peak in the $D^0\bar{D}^{*0}$ invariant mass distribution near the $D^0\bar{D}^{*0}$ threshold in the decay $B^+ \rightarrow D^0\bar{D}^{*0}K^+$. We also estimate the branching fraction for $B^+ \rightarrow X K^+$. The estimate is compatible with observations if X has quantum numbers $J^{PC} = 1^{++}$ and if $J/\psi \pi^+\pi^-$ is one of its major decay modes.

The mass of the X has been measured to be $m_X = 3872.0 \pm 0.6 \pm 0.5$ MeV by Belle and $3871.4 \pm 0.7 \pm 0.4$ MeV by CDF. It is extremely close to the $D^0\bar{D}^{*0}$ threshold 3871.2 ± 0.7 MeV. The binding energy is $E_b = -0.5 \pm 0.9$ MeV. If the state is bound, $E_b$ is positive, so it is likely to be less than 0.4 MeV. This is the smallest binding energy of any S-wave two-hadron bound state. The next smallest is the deuteron, a proton-neutron state with binding energy 2.2 MeV. For two hadrons whose low-energy interactions are mediated by pion exchange,
versality implies that the $X$ is at least an order of magnitude smaller than the natural space wavefunction at relative momentum $\mu$ where $E$ as $B$.

For a $\bar{D}^0 D^{*0}$ molecule, this scale is about 10 MeV, so $E_b$ is at least an order of magnitude smaller than the natural low-energy scale.

If the binding energy of $X$ is so small, low-energy universality implies that the $X/D^0 D^{*0}/D^0 D^{*0}$ system has properties that are determined by the $D^0 D^{*0}$ scattering length $a$ and are insensitive to the shorter distance scales of QCD. The universal binding energy of the molecule is

$$E_b \equiv m_D + m_{D^*} - m_X \simeq (2m^2)^{-1},$$

where $\mu = m_{D} m_{D^*} / (m_{D} + m_{D^*})$ is the reduced mass of the $D^0$ and $D^{*0}$. The universal normalized momentum-space wavefunction at relative momentum $k \ll m_\pi$,

$$\psi(k) \simeq \left(8\pi/a\right)^{1/2}(k^2 + 1/a^2)^{-1},$$

was used by Voloshin to calculate the momentum distributions for the decays $X \to D^0 D^{*0}$ and $X \to D^0 D^{*0} \gamma$.

The universal $D^0 D^{*0}$ elastic scattering amplitude at relative momentum $k_{\text{cm}} \ll m_\pi$ is

$$A[\bar{D}^0 D^{*0} \to D^0 D^{*0}] \simeq \frac{8\pi m_{D^*} m_{D^*}}{\mu (1/a - i k_{\text{cm}})} A[D^0 D^{*0} \to D^0 D^{*0}],$$

where $k_{\text{cm}} \approx \frac{1}{2}(E - m_D - m_{D^*})^{1/2}$ and $E$ is the total energy in the center-of-momentum frame. The amplitude $A[\bar{D}^0 D^0 \to D^0 D^{*0}]$ for scattering to the CP conjugate state differs by the charge conjugation $C = \pm$ of the channel with the large scattering length. Another consequence of low-energy universality is that as the binding energy $E_b$ decreases, the probabilities for components of the wavefunction other than $\bar{D}^0 D^0$ and $D^{*0} D^0$ decrease as $E_b^{1/2}$. In the limit $E_b \to 0$, the state becomes $|\bar{D}^0 D^0 \rangle \pm |D^{*0} D^{*0} \rangle / \sqrt{2}$ if $C = \pm$. The rates for decays that do not correspond to the decay of a constituent $D^{*0}$ or $D^{*0}$ also decrease as $E_b^{1/2}$. This suppression may explain the surprisingly narrow width of $X$.

The decay $B^+ \rightarrow X K^+$ proceeds through the weak decay $b \to \bar{c}c \bar{s}$ at very short distances. The subsequent formation of $X K^+$ is a QCD process that involves momenta $k$ as low as $1/a$. The contributions from $k \sim 1/a$ are constrained by low-energy universality, but those from $k \gtrsim m_\pi$ involve the full complications of low-energy QCD.

We analyze the decay $B^+ \rightarrow X K^+$ by separating short-distance effects involving $k \gtrsim m_\pi$ from long-distance effects involving $k \sim 1/a$. The decay can proceed via the short-distance 3-body decay $B^+ \rightarrow D^0 D^{*0} K^+$ followed by the long-distance coalescence process $\bar{D}^0 D^{*0} \rightarrow X$. It can also proceed through a second pathway consisting of $B^+ \rightarrow D^0 D^{*0} K^+$ followed by $\bar{D}^0 D^{*0} \rightarrow X$. The amplitude for the first pathway can be expressed as

$$A_1[B^+ \rightarrow X K^+] = -i \int \frac{d\ell}{(2\pi)^3} A[B^+ \rightarrow \bar{D}^0 D^{*0} K^+]$$

$$\times D(q + \ell, m_D) D(q_\ast - \ell, m_{D^*}) A[\bar{D}^0 D^{*0} \rightarrow X],$$

where $q = (m_D/m_X)Q$ and $q_\ast = (m_{D^*}/m_X)Q$ are 4-momenta that add up to the 4-momentum $Q$ of $X$ and $D(p, m) = (p^2 - m^2 + i\epsilon)^{-1}$. The sum is over the spin states of the $D^{*0}$. This amplitude can be represented by the Feynman diagram with meson lines shown in Fig. 1. We constrain the loop integral to the small-momentum region by imposing a cutoff $|\ell| < \Lambda$ in the rest frame of the virtual $D^0$ and $D^{*0}$. The natural scale for the cutoff is $\Lambda \approx m_\pi$. The amplitude for $D^0 D^{*0}$ to coalesce into $X$ is determined by the $D^0 D^{*0}$ scattering length $a$:

$$A[D^0 D^{*0} \rightarrow X] = (16\pi Z m_X m_D m_{D^*} / \mu^2 a)^{1/2} \epsilon_X \cdot \epsilon,$$

where $\epsilon_X$ and $\epsilon$ are the polarization vectors of $X$ and $D^{*0}$ and $Z$ is the probability for the $X$ to be in a $D^0 D^{*0} / D^{*0} D^0$ state. At the $D^0 D^{*0}$ threshold, the amplitude for $B^+ \rightarrow D^0 D^{*0} K^+$ is constrained by Lorentz invariance to have the form

$$A[B^+ \rightarrow D^0 D^{*0} K^+] = c_1 P \cdot \epsilon^*,$$

where $P$ is the 4-momentum of the $B$ meson and $c_1$ is a constant. The amplitude for $B^+ \rightarrow D^0 D^0 K^+$ has the same form with $c_1$ replaced by a constant $c_2$. In the $D^0 D^{*0}$ rest frame, the integral over $\ell_0$ of the two propagators in (4) is proportional to the momentum-space wavefunction of $X$. The subsequent integral over $\ell$ is linear in the ultraviolet cutoff $\Lambda$ for the low-momentum region:

$$\int \frac{d\ell}{(2\pi)^3} D(q + \ell, m_D) D(q_\ast - \ell, m_{D^*}) \frac{i\mu\Lambda}{4\pi^2 m_D m_{D^*}}.$$}

The total amplitude from the two pathways is

$$A[B^+ \rightarrow X K^+] = - (Z m_X / \pi m_D m_{D^*} a)^{1/2} \times (c_1 \pm c_2) \Lambda P \cdot \epsilon_X^*.$$
corrections suppressed by a factor $\Lambda_{QCD}/m_B$. The interference is constructive if $C = +$ and destructive if $C = −$. The dependence of the loop amplitude on $\Lambda$ is cancelled by a tree diagram with a $B − X K$ contact interaction whose coefficient therefore depends linearly on $\Lambda$. If the $X$ is predominantly a $D D^*$ molecule, there must be some value $\Lambda_1$ of the ultraviolet cutoff for which the loop amplitude dominates over the tree amplitude. Squaring the amplitude, summing over spins, and integrating over phase space, the final result for the decay rate is

$$\Gamma[B^+ \rightarrow X K^+] = \frac{Z \lambda^{3/2}(m_B, m_X, m_K)}{64\pi^3 m_B m_X^2} |c_1 \pm c_2|^2 \Lambda_{1}^2,$$  \eqref{eq:decay_rate}

where $\lambda(x, y, z) = x^4 + y^4 + z^4 - 2(x^2y^2 + y^2z^2 + z^2x^2)$. Due to the factor $1/a$, the decay rate scales like $E_b^{1/2}$ as $E_b \rightarrow 0$.

If another hadronic state $H$ is close enough to the $D^0 D^{*0}$ threshold that $X$ has a nonnegligible probability $Z_H$ of being in the state $H$, the decay can also proceed through a short-distance 2-body decay $B^+ \rightarrow HK^+$. In this case, there is an additional term $|A[B^+ \rightarrow HK^+]|Z_H^{1/2}$ in $\mathcal{S}$. Its contribution to the decay rate also scales like $E_b^{1/2}$ as $E_b \rightarrow 0$, because $Z_H$ scales like $E_b^{1/2}$ \cite{footnote:threshold}. If $C = +$, one possibility for such a state is the excited P-wave charmonium state $\chi_{c1}(2P)$. Recent coupled-channel calculations of the charmonium spectrum suggest that $\chi_{c1}(2P)$ is likely to be well above the $D^0 D^{*0}$ threshold \cite{footnote:threshold}. We will henceforth assume that $D^0 D^{*0}/D^{*0} D^0$ is the only important component of the wavefunction and set $Z \approx 1$.

We can calculate the differential decay rate for $B^+ \rightarrow \overline{D}^0 D^{*0} K^+$ in the same way. There are again two pathways: the short-distance decay $B^+ \rightarrow \overline{D}^0 D^{*0} K^+$ followed by the long-distance scattering $\overline{D}^0 D^{*0} \rightarrow \overline{D}^0 D^{*0}$ and $B^+ \rightarrow D^{*0} D^{0} K^+$ followed by $D^{*0} D^0 \rightarrow D^0 D^{*0}$. The amplitude for the first pathway can be represented by the Feynman diagram with meson lines shown in Fig. 2. The calculation of the amplitude is similar to that for $B^+ \rightarrow X K^+$ except that it involves the scattering amplitude \cite{footnote:threshold} instead of the coalescence amplitude \cite{footnote:threshold}.

In the loop amplitude for $B^+ \rightarrow \overline{D}^0 D^{*0} K^+$, we keep only the term \cite{footnote:threshold} that is nonzero at the $\overline{D}^0 D^{*0}$ threshold. There must be some value $\Lambda_2$ of the ultraviolet cutoff for which the loop amplitude dominates over the tree amplitude. The factor $\Lambda_1 \Lambda_2$ cancels in the ratio between the amplitudes for $B^+ \rightarrow \overline{D}^0 D^{*0} K^+$ and $B^+ \rightarrow X K^+$. Our final expression for the differential decay rate is

$$\frac{d\Gamma}{dM_{DD'}} [B^+ \rightarrow \overline{D}^0 D^{*0} K^+] = \Gamma[B^+ \rightarrow X K^+] \Lambda_2^2 \frac{\mu a^3 k_{cm}}{\pi(1 + a^2 k_{cm})},$$  \eqref{eq:differential_decay_rate}

where $M_{DD'}$ is the $\overline{D}^0 D^{*0}$ invariant mass and $k_{cm}$ is the relative momentum in the $\overline{D}^0 D^{*0}$ rest frame:

$$k_{cm} = \lambda^{1/2}(M_{DD'}, m_D, m_{D'})/(2M_{DD'}).$$  \eqref{eq:relative_momentum}

In \ref{footnote:threshold}, we have neglected terms suppressed by $k_{cm}^2/m_D^2$. The invariant mass distribution is illustrated in Fig. \ref{fig:invariant_mass_distribution} for several values of the binding energy $E_b$. The distributions are normalized to 1 at $k_{cm} = m$. As the binding energy is tuned toward 0, the peak value scales like $E_b^{-1/2}$ and the position of the peak in $M_{DD'} = (m_D + m_{D'})$ scales like $E_b$. The observation of such an enhancement near the $\overline{D}^0 D^{*0}$ threshold would confirm the interpretation of $X$ as a $\overline{D}^0 D^{*0}/D^{*0} D^0$ molecule.

The Babar collaboration has recently measured the branching fractions for $B^+$ to decay into $\overline{D}^0 D^{*0} K^+$, $D^0 D^{*0} K^+$, and $D^{*0} D^{0} K^+$ to be $(0.19 \pm 0.03)\%, (0.47 \pm 0.07)\%, (0.18 \pm 0.07)\%$, and $(0.53 \pm 0.11)\%$, respectively \cite{footnote:experimental}. We use these measurements to estimate the branching fraction for $B^+ \rightarrow X K^+$. We make the simplifying assumption that the decay amplitude factors into currents $\bar{c}\gamma^\mu(1 - \gamma_5)b$ and $\bar{s}\gamma^\mu(1 - \gamma_5)c$. Heavy
quark symmetry can then be used to express the 3-body double-charm decay amplitudes in terms of two functions $G_1(q^2)$ and $G_2(q^2)$, where $q^2$ is the invariant mass of the hadrons produced by the $s\bar{s}^\mu(1 - \gamma_5) c\bar{c}$ current [19]. For example, the amplitudes for decays into $D^0\bar{D}^{*0} K^+$ and $D^0\bar{D}^{*0} K^+$ are

$$A[B^+ \to D^0\bar{D}^{*0} K^+] = -i G_1 e^\star \cdot (V + v) - i(G_2/m_B) \epsilon^\star \cdot (V + v) - \epsilon^\mu \epsilon^\nu - (V + v) \mu \nu v^\star k^\star ,$$

and

$$A[B^+ \to D^{*0}\bar{D}^{*0} K^+] = i(G_1 v_\mu + G_2 k_\mu/m_B) \epsilon^\star \cdot [1 + V \cdot k] \epsilon^\star \cdot v^\star k^\star \epsilon^\mu v^\nu - \epsilon^\mu \epsilon^\nu v^\star k^\star ,$$

where $k$ is the 4-momentum of the $K^+$ and $V$, $v_\mu$, and $v$ are the 4-velocities of the $B^+$, $D^0$ or $D^{*0}$, and $D^0$ or $D^{*0}$, respectively. As a further simplification, we approximate $G_1$ and $G_2$ by constants. The resulting expressions for the 3-body double-charm decay rates are

$$\Gamma[B^+ \to D^0\bar{D}^{*0} K^+] = 10^{-3}\text{MeV} \times (178.9 |G_1|^2 + 51.8 \text{Re}(G_1 G_2) + 4.37 |G_2|^2) ,$$

and

$$\Gamma[B^+ \to D^{*0}\bar{D}^{*0} K^+] = 10^{-3}\text{MeV} \times (49.6 |G_1|^2 + 2.61 \text{Re}(G_1 G_2) + 3.49 |G_2|^2) .$$

We obtain a good fit to the Babar branching fractions with $G_1 = 3.2 \times 10^{-6}$ and $G_2 = (-14.6 + 9.6i) \times 10^{-6}$. In the corner of phase space where the 4-velocities of $D^0$ and $D^{*0}$ are equal, the amplitudes [12] and [13] reduce to the form on the right side of [15] with coefficients $c_1 = c_2 = -i G_1/m_B + i G_2 (m_D + m_{D'})/m_B$. If $X$ has charm conjugation $C = +$, the estimate [16] reduces to

$$\mathcal{B}[B^+ \to X K^+] \approx (2.6 \times 10^{-5}) \frac{\Lambda^2}{m^2} \left( \frac{E_b}{0.4\text{MeV}} \right)^{1/2} .$$

If $C = -$, the branching fraction would be significantly smaller because of destructive interference between $c_1$ and $c_2$. We could get a more reliable result for the numerical prefactor in [15] by relaxing the factorization assumption and carrying out a Dalitz plot analysis of the 3-body decays. Since the result depends quadratically on the ultraviolet cutoff $\Lambda_1$, the best we can do is obtain an order-of-magnitude estimate of the branching fraction by setting $\Lambda_1 \approx m_\pi$.

The Belle collaboration measured the product of the branching fractions $\mathcal{B}[B^+ \to X K^+]$ and $\mathcal{B}[X \to J/\psi \pi^+ \pi^-]$ to be $(1.3 \pm 0.3) \times 10^{-5}$ [17]. Our estimate of $\mathcal{B}[B^+ \to X K^+]$ is compatible with this result if $J/\psi \pi^+ \pi^-$ is one of the major decay modes of $X$. The experimental upper bound on the width of $X(3872)$ is $\Gamma_X < 2300$ keV. The sum of the widths for decay into $D^0 D^{*0} \pi^0$ and $D^0 D^{*0} \eta$ approaches $\Gamma[D^{*0}] \approx 50$ keV in the limit $E_b \to 0$ [18]. The remaining partial widths scale as $E_b^{1/2}$. Using a coupled-channel calculation in a model in which $X$ mixes with $J/\psi \rho$, the decay rate for $J/\psi \pi^+ \pi^-$ has been estimated to be 1290 keV for $E_b = 0.7$ MeV [19]. Thus it is at least plausible that $J/\psi \pi^+ \pi^-$ is one of the major decay modes. Other possible decay channels are $\eta, \pi \pi$, radiative transitions to charmonium states, and $cc$ annihilation decays.

We have calculated the decay rate for $B^+ \to XK^+$ and the differential decay rate for $B^+ \to D^0 D^{*0} K^+$ near the $D^0 D^{*0}$ threshold under the assumption that $X(3872)$ is a loosely-bound $S$-wave $D^0 D^{*0}/D^{*0} D^{*0}$ molecule and that its production rate is dominated by the coalescence of charm mesons. Observation of a sharp peak in the $D^0 D^{*0}$ invariant mass distribution near threshold in the decay $B^+ \to D^0 D^{*0} K^+$ would confirm the interpretation of $X$ as a $D^0 D^{*0}$ molecule. Our order-of-magnitude estimate of the branching fraction for $B^+ \to X K^+$ is compatible with observations if $X(3872)$ has quantum numbers $J^{PC} = 1^{++}$ and if $J/\psi \pi^+ \pi^-$ is one of its major decay modes.

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