TESTING THE CORE OVERSHOOT MIXING DESCRIBED BY A TURBULENT
CONVOLUTION MODEL ON THE ECLIPSING BINARY STAR HY VIR

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Received 2012 April 10; accepted 2012 October 31; published 2012 December 4

ABSTRACT

Helioseismic investigation has suggested applying turbulent convection models (TCMs) to convective overshoot. Using the turbulent velocity in the overshoot region determined by a TCM, one can deal with overshoot mixing as a diffusion process, which leads to incomplete mixing. It has been found that this treatment can improve solar sound speed and Li depletion in open clusters. In order to investigate whether the TCM can be applied to overshoot mixing outside the stellar convective core, new observations of the eclipsing binary star HY Vir are adopted to calibrate the overshoot mixing parameter. The main conclusions are as follows: (1) the solar TCM parameters and overshoot mixing parameter are also suitable for the eclipsing binary system HY Vir, (2) the incomplete mixing results in a continuous profile of hydrogen abundance, and (3) the e-folding length of the region, in which the hydrogen abundance changes due to overshoot mixing, increases during stellar evolution.

Key words: convection – stars: individual (HY Vir) – turbulence

1. INTRODUCTION

Convective overshoot in the stellar interior is caused by turbulent flows moving across the standard convective boundary, which is defined by the Schwarzschild criterion (i.e., $\nabla_R \geq \nabla_{ad}$), into the dynamically stable region. This process leads to chemical mixing in the overshoot region. Based on the framework of non-local mixing length theories, the overshoot region is considered to be a fully mixed region with a length $L_{OV}$, where $H_P$ is the local pressure scale height. This description of the overshoot is based on the framework of the “ballistic” overshoot models, which are non-local convection theories based on hydrodynamic equations and modeling assumptions (Xiong 1981, 1985, 1989; Canuto 1997, 2011; Canuto & Dubovikov 1998; Xiong & Deng 2001; Li & Yang 2001, 2007; Deng et al. 2006; Li 2012). Using the turbulent velocity determined by TCMs, one can deal with overshoot mixing as a diffusion process that results in incomplete mixing in the overshoot region. Zhang & Li (2012a) applied a TCM (Li & Yang 2007) and incomplete overshoot mixing to the downward overshoot region of the solar convective envelope and found that the sound speed profile is significantly improved. Applying the same method to the stellar model also shows improvements in Li depletion in open clusters; thus the TCM has been considered to be generally applicable to the downward overshoot region of the convective envelope of low-mass stars (Zhang 2012).

It is interesting whether or not the incomplete overshoot mixing based on the TCM is suitable for convective core overshoot. Convective core overshoot refreshes hydrogen in the nuclear-burning core, deeply affecting the evolution of massive and intermediate-mass stars. The evolution track of a star with a convective core in the Hertzsprung–Russell diagram is sensitive to the efficiency of core overshoot mixing. Accordingly, it is a direct and efficient way of studying core overshoot mixing by calibrating the radius and effective temperature of the stellar model to fit the observations. Double-lined detached eclipsing binary systems in the main-sequence phase are the best candidates for studying core overshoot mixing because the mass, radius, and temperature of each component of a binary system can be obtained by observations. Ribas et al. (2000) and Claret (2007) studied core overshoot based on a set of double-lined eclipsing binary systems and found that the fully mixed overshoot region with a length $0.20–0.25H_P$ is the best overall. Recently, Lacy & Fekel (2011) observed the double-lined detached eclipsing binary system HY Vir and acquired very accurate masses and radii of the two components from the analysis of new light curves and radial velocity curves. The mass of each component of HY Vir is (Lacy & Fekel 2011): $M_1 = 1.838 \pm 0.009 M_\odot$ and $M_2 = 1.404 \pm 0.006 M_\odot$ for the primary and the secondary, respectively. These results indicate that both the primary and the secondary have a convective core. The accurate data make HY Vir a good candidate for studying the core overshoot mixing described by a TCM.

In this paper, I investigate the suitable parameter of overshoot mixing via calibrating the effective temperatures and the radii of the stellar models of HY Vir to fit the observations and study the effects of the application of a TCM (Li & Yang 2007). The method of applying a TCM to stellar models is described in Section 2. The calibrated results for the HY Vir system are shown in Section 3. The properties of stellar models based on a TCM are discussed in Section 4. The conclusions and discussions are presented in Section 5.

2. CALCULATIONS

2.1. Overshoot Mixing

Traditionally, overshoot is assumed to result in a completely mixed region with the length being proportional to the local pressure scale height. This description of the overshoot is based on the framework of the “ballistic” overshoot models, which are non-local mixing length theories (e.g., Shaviv & Salpeter 1973; Maeder 1975; Bressan et al. 1981; Zahn 1991). Those non-local mixing length theories result in a steep temperature gradient profile at the boundary of the overshoot region (e.g., Shaviv & Salpeter 1973; Zahn 1991). However, recent helioseismic investigations have found that the required temperature gradient in the overshoot region is not in agreement with the framework.
of “ballistic” overshoot models, but suggests that only TCMs are capable of providing the required temperature gradient profile (Christensen-Dalsgaard et al. 2011).

According to hydrodynamic equations, chemical mixing is determined by the turbulent-velocity–turbulent-concentration correlation (Xiong 1986; Canuto 1999, 2011). Unfortunately, there is a lack of knowledge about such a correlation in astrophysical cases. Besides full mixing, the incomplete mixing described by diffusion is at present popular for dealing with overshoot mixing (e.g., Deng et al. 1996a, 1996b; Freytag et al. 1996; Ventura et al. 1998; Herwig 2000; Paxton et al. 2011). In the diffusion description, the turbulent velocity in the overshoot region is required to calculate the diffusion coefficient. Turbulent velocity is usually set as an exponentially decreasing function in the overshoot region (e.g., Freytag et al. 1996; Ventura et al. 1998), and can be calculated by defining the exponential index and the initial value (i.e., the turbulent velocity at the convective boundary). An option for determining the initial value is extrapolation from the mixing length theory (MLT; e.g., Freytag et al. 1996; Ventura et al. 1998). However, the MLT is a local theory, while the non-local effect dominates near the convective boundary. Another option is to obtain the turbulent velocity by solving the TCM, which is a non-local turbulent convection theory.

The diffusion coefficient is proportional to the characteristic length of mixing and the characteristic velocity: \( D = flv \), where \( l \) is the characteristic length, \( v \) is the characteristic turbulent velocity, and \( f \) is a diffusion parameter. \( v \) is assumed to be \( \sqrt{k} \), where \( k \) is the turbulent kinetic energy. The diffusion parameter multiplying the characteristic length is assumed to be proportional to \( H_P \), i.e., \( f = C_X H_P \). Therefore, the diffusion coefficient of overshoot mixing is assumed to be:

\[
D_{OV} = C_X H_P \sqrt{k}. \tag{1}
\]

The characteristic length of mixing \( l \) is between the Kolmogorov scale and the largest eddy scale. In order to cause incomplete mixing in the overshoot region on a timescale comparable to the evolutionary timescale, the characteristic length of mixing should be much smaller than the largest eddy scale (Deng et al. 1996a). As in Equations (27) and (29) in Deng et al. (1996a), \( C_X \) in Equation (1) should be on the order of \( 10^{-10} \), which is suitable for the downward overshoot region of the convective envelope in a low-mass star (Zhang & Li 2012).

The diffusion equation of the hydrogen abundance in the stellar interior is as follows:

\[
\delta \frac{\partial X}{\partial t} = \frac{\partial}{\partial M^r_x} \left( \frac{4 \pi \rho r^2}{2} D \frac{\partial X}{\partial M^r_x} \right), \tag{2}
\]

where \( \delta \frac{\partial X}{\partial t} \) means the time derivative of hydrogen abundance caused by only convective and overshoot mixing, and \( D \) is the diffusion coefficient due to convection or overshoot:

\[
D = \begin{cases} 
D_{OV}: (\nabla_R < \nabla_{ad}) \\
D_{CZ}: (\nabla_R \geq \nabla_{ad}).
\end{cases} \tag{3}
\]

where \( D_{CZ} \) is the diffusion coefficient in the convection zone. In order to ensure full mixing in the convection zone, large \( D_{CZ} \) is required so that \( D_{CZ} = H_P \sqrt{k} \) is adopted in the calculations. It is not difficult to solve Equation (2) by using the tridiagonal matrix algorithm.

### 2.2. The Stellar Evolution Code

The stellar evolution code, which was originally developed by Paczynski (1969) and Kozlowski and updated by Sienkiewicz, is used to model the components of HY Vir. The OPAL equation of state (Rogers et al. 1996), the OPAL opacity tables for high temperatures (Iglesias & Rogers 1996), and Alexander’s opacity tables for low temperatures (Alexander & Ferguson 1994) are used. The composition mixture is assumed to be the same as that of the sun (Grevesse & Sauval 1998).

In this paper, I solve the complete structure equations comprising the stellar structure equations and TCM (Li & Yang 2007) equations. In the stellar structure equations, the temperature gradient \( \nabla \) is calculated as follows:

\[
\nabla = \nabla_R - \frac{H_P \rho c_P}{T} \frac{\eta w_T T'}{\lambda}, \tag{4}
\]

where \( w_T T' \) is the turbulent heat flux determined by the TCM.

The evolution code has been modified to take into account overshoot mixing. In the original code, the evolution of chemical composition in the stellar interior and the stellar structure are calculated separately. For each time step, the evolution of chemical composition (nuclear burning, complete mixing in the convection zone) is calculated before solving the stellar structure equations. In the present work, the first step is to solve Equation (2). After the evolution of chemical composition (nuclear burning and convective and overshoot mixing) during the time step is accomplished, the complete structure equations are solved, and the stellar structure variables and the turbulent variables are updated. The method of solving the complete structure equations is as follows.

1. Solve the TCM equations based on the current stellar structure variables (e.g., \( \rho, T, r, L \)). Calculate the temperature gradient \( \nabla \) at all mesh points according to Equation (4).
2. Solve the localized TCM in which the non-local terms (i.e., the diffusion terms) are ignored. Calculate the local temperature gradient \( \nabla_L \) at all mesh points. Discussions on the localized TCM can be found in Appendix A of Zhang & Li (2012b).
3. Calculate the ratio \( \eta = \nabla / \nabla_L \) at all mesh points. Use \( \eta \) to obtain \( \eta' \) at all mesh points based on the relaxation technique, i.e., \( \eta' = \eta_{pre} + \xi (\eta - \eta_{pre}) \), with a proper relaxation parameter \( \xi \), where \( \eta_{pre} \) and \( \eta_{pre} \) are the previous values of \( \eta' \) and \( \eta \).
4. Solve the stellar structure equations in which the temperature gradient is calculated as \( \nabla = \eta' \nabla_L \), and update the stellar structure variables (e.g., \( \rho, T, r, L \)). It should be noted in this process that \( \nabla_L \) is updated when the stellar structure variables are updated in the Newtonian iterations.
5. Check the differences \( |\eta - \eta'| \) and \( |\eta - \eta_{pre}| \). The calculations are thought to converge if \( |\eta - \eta'| \) and \( |\eta - \eta_{pre}| \) are less than the required accuracy at all mesh points; otherwise, return to step (1).

It is not difficult to understand that the turbulent variables and the stellar structure variables are consistent with each other when \( \eta_{pre} = \eta = \eta' \); thus, the scheme is reasonable. The scheme is an improved version of the previous one (e.g., Zhang & Li 2009). It is found that this scheme is stabler and more efficient than the previous one.

There are seven parameters in the TCM (Li & Yang 2007): \( \alpha, C_t, \) and \( C_e \) are dissipation parameters of turbulent kinetic energy \( k \), turbulent heat flux \( w_T T' \), and turbulent temperature...
fluctuation $T^\prime T'$, respectively; $C_\alpha$, $C_{\alpha 1}$, and $C_{\alpha 1}$ are diffusion parameters of $k$, $u_\alpha T'$, and $T^\prime T'$, respectively; and $C_k$ is the "return to isotropy" term.

The turbulent parameters adopted in this paper are listed in Table 1. $C_\alpha$, $C_k$, $C_{\alpha 1}$, and $C_{\alpha 1}$ are the same in the case of the solar model (Zhang & Li 2012a). The diffusion parameters $C_{\alpha 1}$ and $C_{\alpha 1}$ are zero to be sure because the calculation does not converge when the diffusions of $u_\alpha T'$ and $T^\prime T'$ are taken into account. The turbulent dissipation parameter $\alpha = 0.85$ is adopted. This value is due to the solar calibration based on the input physics and other TCM parameters.

The age and chemical composition of each component of HY Vir are assumed to be the same. Interactions between the primary and the secondary are ignored. The stellar models evolve from the zero-age main sequence (ZAMS) to the age when the radius of the primary stellar model fits the observation. The step time is no more than 0.5% of the age. All stellar models comprise more than 2000 mesh points.

3. CALIBRATIONS OF STELLAR MODELS OF HY VIR

The main aim of this paper is to test whether or not the solar turbulent parameters (i.e., the TCM parameters in Table 1 and the overshoot mixing parameter $C_X = 10^{-10}$) are suitable for the core overshoot region and reproduce the observations of HY Vir. Mathematically, only one turbulent parameter can be derived by calibrating the effective temperatures $T_{\text{eff}}$ and radii $R$ of stellar models for HY Vir. I choose the most sensitive parameter, i.e., $C_X$, to be adjustable. The strength of the overshoot mixing is determined by the diffusion coefficient, which is proportional to $C_X$ and $V_X$. Besides $C_X$, the TCM parameters indirectly affect the diffusion coefficient because of their effects on $k$. However, in an acceptable range of TCM parameters, $k$ does not change too much.

The adjustable parameters in modeling the components of HY Vir are as follows: the overshoot mixing parameter $C_X$, the initial hydrogen abundance $X$, and the metal abundance $Z$. Observations make it possible to fix the adjustable parameters ($C_X$, $X$, $Z$). The effective temperatures and the radii of the stars (i.e., $T_1$ and $R_1$ for the primary, $T_2$ and $R_2$ for the secondary) are functions of the adjustable parameters ($C_X$, $X$, $Z$) and the age $t$. There are four free independent variables and the same number of dependent variables. Because the effective temperatures and the radii should be consistent with observations, the adjustable parameters and the age can be solved.

The scheme of solving the adjustable parameters is as follows. (1) Use the trial parameter set ($C_X$, $X$, $Z$) to model the primary of HY Vir. (2) Obtain the effective temperature $T_1$ and the age $t$ when the radius of the primary $R_1$ evolves to the observational value. (3) Use the same parameter set ($C_X$, $X$, $Z$) to model the secondary of HY Vir. The stellar model evolves from the ZAMS to age $t$. Obtain the effective temperature and the radius, i.e., $T_2$ and $R_2$, of the stellar model of the secondary at age $t$. (4) Compare $\lg T_1$, $\lg T_2$, and $R_2$ with the observations shown in Table 2. Use the Newtonian iteration method to revise the parameters ($C_X$, $X$, $Z$), then go to step (1) until $\lg T_1$, $\lg T_2$, and $R_2$ are consistent with the observations within an accuracy of 0.001. $R_1$ is calibrated to fix the age $t$ because the radius of the primary is the most sensitive variable, i.e., $|d R_1/dt| > \max(|d R_2/dt|, |d \lg T_1/dt|, |d \lg T_2/dt|)$. Therefore, to derive $t$ by calibrating $R_1$ is the most accurate way to compare by calibrating $R_2$, $T_1$, and $T_2$.

Using the above scheme, the adjustable parameters and their standard errors are worked out as follows:

$$C_X = (0.8 \pm 0.5) \times 10^{-10}$$  \hfill (5)

$$X = 0.67 \pm 0.03$$  \hfill (6)

$$Z = 0.031 \pm 0.007.$$  \hfill (7)

For the calibrated models, the age of the system is $t = 1.4$ Gyr. This is consistent with Lacy & Fekel’s (2011) results, which show $Z = 0.027$, $\Delta Y/\Delta Z = 2.0$ ($Y$ is the initial helium abundance), and $t = (1.35 \pm 0.1)$ Gyr.

The standard errors $\sigma$ of the parameters ($C_X$, $X$, $Z$) are calculated according to the Gaussian distribution:

$$\sigma^2(y_i) = \sum_j \left( \frac{\partial y_i}{\partial x_j} \right)^2 \sigma^2(x_j),$$  \hfill (8)

where $(x_1, x_2, x_3, x_4, x_5, x_6) = (\lg T_A, \lg T_B, R_A, R_B, M_A, M_B)$ and $(y_1, y_2, y_3) = (C_X, X, Z)$. The values of $\sigma(x_j)$ are taken from Table 2. Although the observations of $\lg T$, $R$, and $M$ are very accurate, $C_X$ shows a relatively large $\sigma$. The standard errors of the parameters ($C_X$, $X$, $Z$) are mainly due to $\sigma(\lg T_B)$ because the contributions of $j = 2$ terms in Equation (8) are (64%, 44%, 85%) for $i = (1, 2, 3)$.

Figure 1 shows the evolutionary tracks of stellar models for both the primary and the secondary, with $X = 0.67$, $Z = 0.031$, and different $C_X$. Three cases are shown: $C_X = 0$ (no overshoot mixing), $C_X = 0.8 \times 10^{-10}$ (the result of calibration), and $C_X = 1.0 \times 10^{-10}$ (suitable for the downward overshoot region of the convective envelope in low-mass stars, Zhang & Li 2012b; Zhang 2012). It is found that the two cases including overshoot mixing reproduce the observational effective temperatures and radii in $1\sigma$. Larger $C_X$ values result in a higher effective temperature and higher luminosity at the termination of the evolutionary tracks. This is because the overshoot mixing refreshes the hydrogen content of the nuclear-burning core and boosts nuclear reaction in the core.

4. PROPERTIES OF THE STELLAR MODELS

4.1. Profile of Chemical Abundance

The main effect of diffusive mixing of convective overshoot is the modification of the profile of hydrogen abundance. Figure 2 shows hydrogen abundance profiles of the calibrated stellar models. Stellar models with $X_C = 0.50, 0.32$ for the primary and stellar models with $X_C = 0.60, 0.55$ for the secondary are
Figure 1. Evolutionary tracks of two components of HY Vir with different $C_X$ in the diagram of (a) radius and (b) luminosity vs. effective temperature. Note that the temperature, luminosity, and radius scales are different for the two components of HY Vir. Solid lines correspond to $C_X = 0.8 \times 10^{-10}$, the calibrated model. Dashed lines correspond to $C_X = 1.0 \times 10^{-10}$, the value for the solar case. Dotted lines correspond to the stellar models without overshoot mixing. The 1σ error bars denoted as “AO” and “BO” are based on Table 2.

shown, where $X_C$ is the hydrogen abundance of the convective core. The hydrogen abundance profiles of the stellar models with complete mixing in a 0.3$H_P$ overshoot region are also shown for comparison. The standard convective boundaries are denoted with arrows in the figure. In the stellar models with complete mixing, the profile is discontinuous at the convective boundary when the convective core expands (e.g., Figure 2(b)) or with a high gradient when the convective core slowly contracts (e.g., Figure 2(a)). In the stellar models with diffusive mixing, $X$ continuously changes from the convective boundary to the outer envelope.

The most important property is the length of the region, in which chemical abundance changes due to overshoot mixing, since the length indicates the strength of the hydrogen refreshing in the nuclear-burning core. Because the profile of hydrogen abundance continuously changes, it is helpful to discuss it using the concept of “e-folding” length. The initial hydrogen abundance can be assumed to be the surface hydrogen abundance $X_S$ and the equilibrium abundance is $X_C$. The $e$-folding length of the chemical abundance changing region is the distance from the convective boundary to the $e$-folding location defined as $X = X_S + (X_C - X_S)/e$ in which $X_S$ is the hydrogen abundance at the stellar surface. The solid lines correspond to the calibrated stellar models, and the dashed lines correspond to stellar models with 0.3$H_P$ full mixing in the overshoot region for comparison. The arrows indicate the convective boundary.

$e$-folding length of the chemical abundance changing region is the distance from the convective boundary to the $e$-folding location defined as $X = X_S + (X_C - X_S)/e$. Figure 2 shows the $e$-folding length of corresponding stellar models of two components. It is found that the $e$-folding length increases during stellar evolution. In order to understand the relation between the $e$-folding length and the age of the stellar model, I estimate the $e$-folding length. In the concerned evolutionary stage of the components of HY Vir, the stars are in the main-sequence phase, and the stellar structures are relatively stable (i.e., no rapid core contraction, expansion, ignition of new elements). The convective boundaries show only a little variation. Therefore, one can approximately study the diffusion in a “quiescent” scene. The time required by the matter diffusing from the convective boundary to the $e$-fold location should be comparable to the age of the stellar model:

$$t \sim \tau \approx \frac{l_E^2}{4D_{OV}},$$  

where $l_E$ is the $e$-folding length. The diffusion time $\tau \approx l_E^2/(4D_{OV})$ is based on the fundamental solution (i.e., the
The 

e
\text{-}
fold length parameter \( \alpha_E \) of the chemical abundance changing region during stellar evolution. The solid lines correspond to the stellar model, and the dashed lines correspond to the prediction from Equation (11).

Green’s function) of the basic diffusion equation. Defining \( l_E = \alpha_E H_P \), and according to the asymptotical solution of the TCM (Zhang & Li 2012b), one finds that the diffusion coefficient at the \( e \)-folding location is as follow:

\[
D_{OV} = C_X H_P \sqrt{k_C} \exp(-\theta \alpha_E / 2), \tag{10}
\]

where \( k_C \) is the turbulent kinetic energy at the boundary of the convective core and \( \theta = \frac{d \ln k}{d \ln P} \) is the exponential decreasing index of the turbulent kinetic energy in the overshoot region. According to the asymptotical solution, the parameters of the TCM adopted in this paper lead to \( \theta = 4.8 \). Therefore, Equation (9) can be rewritten as

\[
\alpha_E^2 \exp \left( \frac{\theta \alpha_E}{2} \right) \sim 12.6 \frac{C_{X,-10} W_{C,4} t_G}{H_{P,10}}, \tag{11}
\]

where \( C_{X,-10} = C_X / 10^{-10}, W_{C,4} = \sqrt{k_C} / (10^4 \text{ cm s}^{-1}), t_G = t / (1 \text{ Gyr}) \), and \( H_{P,10} = H_P / (10^{10} \text{ cm}) \). Equation (11) indicates that the \( e \)-folding length increases during the stellar evolution, which has been found in Figure 2.

Figure 3 shows the comparison between the \( e \)-folding length predicted by Equation (11) and the numerical results of stellar models. It is found that the predicted \( e \)-folding length is consistent with the numerical results. The predicted \( e \)-folding length is zero when \( t \to 0 \). However, the \( e \)-folding length in stellar models is not zero when \( t \to 0 \). This is because nuclear burning outside the convective core results in a chemical abundance changing region. Equation (11) takes into account only overshoot mixing, and ignores nuclear burning. It should be emphasized again that Equation (11) holds only if the stellar structure is relatively stable. It is invalid for stellar models after the end of the main-sequence phase, during which the convective core starts to contract rapidly.

4.2. Temperature Gradient in the Overshoot Region

In this paper, the stellar structure equations are solved together with the TCM. In the convection zone, the MLT, which is used in the standard stellar evolutionary calculations, is replaced by the TCM. Compared with the MLT, the localized TCM shows similar results for the temperature gradient in the convection zone. However, the TCM results in non-zero turbulent heat flux \( u_T' T' \) in the overshoot region. The temperature gradient in the overshoot region is modified according to Equation (4).

Figure 4 shows the temperature gradient near the convective boundaries. It can be seen in Figure 4 that \( \nabla > \nabla_R \) in the overshoot region. This is a common property of non-local
turbulent convection theories (Xiong 1985, 1989; Canuto 1997; Xiong & Deng 2001; Deng et al. 2006; Zhang & Li 2012b). These theories show $u' r T' < 0$ in the overshoot region because the buoyancy prevents turbulent motions, and thus $V > V_R$ according to Equation (4).

In Figure 4, in the case of a high Péclet number at the convective boundary, i.e., $P_e(C) \gg 1$, there is a small adiabatic overshoot region adjoining the convective boundary. This has been pointed out by Zhang & Li (2012b) in their theoretical analysis that, an overshoot region with $P_e \gg 1$, $C_{e1} = 0$ (adopted in this paper) results in an adiabatic overshoot region. The length of the adiabatic overshoot region $l_{ad}$ can be estimated using the parameters of the TCM (Zhang & Li 2012b). The parameters adopted in this paper show $l_{ad} \approx 0.01 R_F$. The diffusion of the turbulent temperature fluctuation $V = T' T'$ is absent because of some problems with numerical calculations. If diffusion is taken into account, there should be no adiabatic overshoot region. In most of the overshoot region, even with $P_e(C) \gg 1$, the temperature gradient is close to the radiative one. This may be because, in the overshoot region with $P_e(C) \gg 1$, the correlation of turbulent velocity and temperature is almost equal to zero (Xiong 1985; Xiong & Deng 2001; Deng et al. 2006; Zhang & Li 2012b), and thus turbulent motions can hardly transport heat.

Although the TCM modifies the temperature gradient in the overshoot region, the modification is small. Zhang & Li’s (2012a) results have indicated that, compared with overshoot mixing, the temperature gradient modification based on the adopted TCM parameters has a negligible effect on the stellar structure and evolution.

4.3. Turbulent Variables in the Overshoot Region

The turbulent variables involved in the TCM are as follows: $u' r u'$ is the radial turbulent kinetic energy, $k$ is total turbulent kinetic energy, $u' r T'$ is the turbulent heat flux, and $T' T'$ is the turbulent temperature fluctuation.

The overshoot region can be classified into two types: a high $P_e$ overshoot region with $P_e \gg 1$ and a low $P_e$ one. The core overshoot region is always the high $P_e$ one because of the short free path of photons in the stellar interior. The type of downward overshoot region in the convective envelope depends on the temperature at the convective boundary $T_{BCE}$. Commonly, it is a high $P_e$ overshoot region for $T_{BCE} > 5.5$ or a low $P_e$ one for $T_{BCE} < 5.5$. The properties of the two types of overshoot regions have been found to differ from each other (Xiong & Chen 1992; Xiong & Deng 2001; Zhang & Li 2012b). The properties of the high $P_e$ overshoot region in the framework of Li & Yang’s (2007) TCM were studied by Zhang & Li (2012b). However, the property of the low $P_e$ overshoot region is not as clear as the high $P_e$ one.

Figures 5 and 6 show the turbulent variables in the stellar interior. For the primary, the ZAMS model and the model with $X_C = 0.32$ are shown. For the secondary, only the stellar model with $X_C = 0.55$ is shown because the structure of the turbulent variables shows little variation during the concerned evolutionary range. At all convective boundaries, $u' r T'$ and $T' T'$ are equal to zero because their diffusion is ignored in the calculations.

As shown in Figures 5 and 6, $T' T'$ in the core overshoot region is larger than in the core convection zone. This is coincident
with Xiong (1985). The turbulent temperature fluctuation $\overline{T'T''}$ represents the difference in temperature between the turbulent flows and the environment, i.e., $\Delta T$. It is always $P_e \gg 1$ in the core convection zone. In the $P_e \gg 1$ convection zone, the turbulent heat transport is very efficient, leading to $\nabla \approx \nabla_{ad}$. Therefore, the temperature of the turbulent flows is almost equal to the temperature of the environment, i.e., $\Delta T \approx 0$, and thus $\overline{T'T''} \approx 0$ in the $P_e \gg 1$ convection zone. In the overshoot region, however, $\nabla$ is close to $\nabla_{ad}$ so that $\Delta T$ is significant.

In the high $P_e$ overshoot region (in the right side of Figures 5(a)-(b), and in Figure 6), the turbulent variables $k$, $\overline{u'u''}$, and $\overline{T'T''}$ decrease exponentially, and the anisotropic degree $\omega = \overline{u'u''} / (2k)$ almost does not change. This is consistent with the asymptotic analysis of the TCM (Zhang & Li 2012b) and Xiong’s non-local turbulent model (see, e.g., Xiong 1985, 1989; Xiong & Deng 2001; Deng & Xiong 2008). Table 3 compares the properties of the turbulent variables in the high $P_e$ overshoot region with the results from theoretical analysis (Zhang & Li 2012b). It is found that the numerical results are in agreement with the theoretical analysis. The high $P_e$ overshoot region extends to the location of $P_e \sim 1$ and then becomes the low $P_e$ region.

In the low $P_e$ overshoot region, thermal dissipation dominates; thus the turbulent variables $k$, $\overline{u'u''}$, and $\overline{T'T''}$ decrease super-exponentially and $\omega$ decreases to zero according to the properties of the TCM equations (Zhang & Li 2012b). Those properties can be found in Figure 5. Overshoot is finally cut off in the region $P_e \ll 1$. According to the definition of $P_e$, $P_e \propto \sqrt{k}$; thus $P_e$ exponentially decreases in the overshoot region with an index $\theta/2$. As a consequence, the distance (in $R_P$) from the convective boundary to the cutoff location depends on $P_e(C)$. A larger $P_e(C)$ leads to a longer distance to the cutoff location. This can be found in Figures (4–6) by comparing $P_e(C)$ and the distance from the convective boundary to the cutoff location of each overshoot region.

There is a thin overshoot region between two convective shells, which are caused by H and He ionization, respectively, in the ZAMS stellar model of the primary as shown in Figure 5(a). The convective shells merge during stellar evolution; thus, this overshoot region disappears in the $X_C = 0.32$ stellar model. The details of the turbulent variables in the thin overshoot region are shown in Figure 7. The turbulent velocity $\sqrt{k} \sim 10^5$ cm s$^{-1}$ in the region is larger than in the convective core, so that two convective envelopes can be considered to be dynamically connected. However, the minimum of the ratio of the convective flux to the total flux is about $|F_C/F| = 1 - \nabla / \nabla_{ad} \sim 10^{-5}$, where $F_C = \rho_c \overline{u'T'}$ is the convective flux. Moreover, $P_e < 1$ in the two convective shells and the overshoot region. These conditions prevent turbulent heat transport between two convective shells. The convective envelopes can therefore be considered to be thermally disconnected.

### 5. CONCLUSION AND DISCUSSION

In this work, I apply a TCM to stellar structure and evolution in order to study convective core overshoot in stellar models of the eclipsing binary star HY Vir. Using new accurate observations, I calibrate the stellar effective temperatures and radii of two components of HY Vir, and then obtain the required overshoot mixing parameter $C_X = (0.8 \pm 0.5) \times 10^{-10}$. The turbulent parameters (i.e., the TCM parameters and the overshoot mixing parameter $C_X$), which are proper for the downward overshoot region of the convective envelope in low-mass stars, can reproduce the observational radii and effective temperatures of the two components of HY Vir. This indicates that diffusive overshoot mixing based on the turbulent velocity described by the TCM can also be applied to core overshoot. This result encourages us to apply the TCM to convective overshoot.

Diffusive overshoot mixing causes a continuous profile of hydrogen abundance. The $e$-folding length of the region, in which the chemical abundance changes due to overshoot mixing, can be estimated using Equation (11). It is found that the $e$-folding length increases during stellar evolution.

The overshoot region can be classified into two types: a high $P_e$ overshoot region in which $P_e \gg 1$ and a low $P_e$ region. The properties of the high $P_e$ overshoot region are in agreement with theoretical analysis (Zhang & Li 2012b). The turbulent variables (turbulent kinetic energy $k$, turbulent heat flux $\overline{u'T'}$, and turbulent temperature fluctuation $\overline{T'T''}$) decrease exponentially in the high $P_e$ overshooting region. The properties of the low $P_e$ overshoot region are very different from those of the high $P_e$ one. In the low $P_e$ overshoot region, the turbulent variables decrease super-exponentially and are finally cut off.

### Table 3

| Item                  | C Figure 5(a) | C Figure 5(b) | C Figure 6 | E Figure 6 | Asy. |
|-----------------------|---------------|---------------|------------|------------|------|
| $\sqrt{\frac{C}{\sqrt{\tau}}}$ | 2060          | 2760          | 1410       | 16500      |      |
| $\sqrt{\frac{C}{\sqrt{T}(TMOD)}}$ | 2120          | 2510          | 1630       | 15200      |      |
| $d \ln k/d \ln P$     | 4.5           | 4.4           | 4.6        | -4.8       | ±4.8 |
| $d \ln(\overline{u'u''})/d \ln P$ | 6.8           | 7.1           | 6.7        | -7.2       | ±7.2 |
| $d \ln(\overline{T'T''})/d \ln P$ | 4.6           | 4.5           | 4.5        | -4.6       | ±4.8 |
| $\omega = \overline{u'u''} / (2k)$ | 0.28          | 0.28          | 0.28       | 0.28       | 0.28 |

**Notes.** $k_C$ is the turbulent kinetic energy at the convective boundary. The “Asy.” column shows the results predicted by the asymptotical analysis (Zhang & Li 2012b). “$\sqrt{\frac{C}{\sqrt{T}}}$ (TMOD)” is the value of $\sqrt{\frac{C}{\sqrt{T}}}$ predicted by the method called “the maximum of diffusion” (Zhang & Li 2012b). The “C Figure 5(a)” column is the core overshoot in Figure 5(a), and the “E Figure 6” column is the envelope overshoot in Figure 6.
The diffusions of $\nu T'$ and $T T'$ are not taken into account in this paper because of some problems with numerical calculations. However, $C_\chi$ for the calibrated stellar models of HY Vir should not change significantly if those diffusions are present. Those diffusions may affect the calibrated results mainly in two ways: (1) modifying the turbulent heat flux and (2) affecting the core overshoot mixing, which plays an important role in stellar evolution. Although this diffusion can redistribute $\nu T'$ in the stellar interior, it should not significantly change the integral value of $\nu T'$. The effective temperature of stellar models should be insensitive to such diffusion. The diffusion of $\nu T'$ is negligible in the region $P_r > 1$ (Zhang & Li 2012b), and thus it does not affect the properties of core convection and overshoot.

The diffusion of $T T'$ can affect the exponential index of the turbulent kinetic energy $\theta(=d \ln k/d \ln P)$. The solar value for the diffusion of $T T'$ is small, $C_{\chi 1} = 0.02$ (Zhang & Li 2012a) and should modify the turbulent properties in the core overshoot region slightly. $C_\chi$ for the calibrated model should change a little if the diffusion of $T T'$ is present. However, since the relative standard error of $C_\chi$ is about 60%, the solar value $C_\chi = 10^{-10}$ should also reproduce the required radius and effective temperature of HY Vir in 1σ.

In other treatments of overshoot with incomplete mixing mentioned in Section 2.1, there is no information on how the temperature gradient profile is modified near the convective boundary. The helioseismic investigation by Christensen-Dalsgaard et al. (2011) cannot judge which of them is suitable for the solar case. However, mixing can affect the sound speed profile. It is well known that, in the standard solar model, there is a bump of sound speed difference below the solar convection zone. In order to eliminate the bump, the proper diffusion coefficient at the base of the solar convection zone is on the order of $10^5$–$10^3$ (see, e.g., Christensen-Dalsgaard & Di Mauro 2007; Zhang & Li 2012a). The mixing should be effective at about 0.1$R$, which is the width of the bump below the base of convection zone. Helioseismic inversion requires the diffusion coefficient to satisfy those conditions in the solar case.

Many thanks to the referee for providing productive and valuable comments. This work is co-sponsored by the National Natural Science Foundation of China through grant No. 10973035, the Science Foundation of Yunnan Observatory under grants No. Y0ZX011009 and No. Y1ZX011007, the Science Foundation of Yunnan Province under grant No. 2010CD112, and the Chinese Academy of Sciences under grant No. KJCX2-YW-T24.

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