Twister quintessence scenario

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Abstract

We study generic solutions in a non-minimally coupled to gravity scalar field cosmology. It is shown that dynamics for both canonical and phantom scalar fields with the potential can be reduced to the dynamical system from which the exact forms for an equation of the state parameter is derived. We have found the stationary solutions of the system and discussed their stability. Within the large class of admissible solutions we have found a non-degenerate critical points and we pointed out multiple attractor type of trajectory travelling in neighborhood of three critical points at which we have the radiation dominating universe, the barotropic matter dominating state and finally the deSitter attractor. We have demonstrated the stability of this trajectory which we call the twister solution. Discovered evoluntional path is only realized if there exist the non-minimal coupling constant. We have found simple duality relations between twister solutions in phantom and canonical scalar fields in the radiation domination phase. For the twister trajectory we have found an oscillating regime of approaching the deSitter attractor.

Key words: modified gravity, dark energy theory, scalar field, non-minimal coupling

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Observational evidences\textsuperscript{[1, 2]} indicated that the current universe is dominated by an exotic form of matter with negative pressure called dark energy. The properties of dark energy component of the universe can be characterized by the equation of the state coefficient $w_\phi = p_\phi/\rho_\phi$, where $p_\phi$ and $\rho_\phi$ are pressure and energy density of dark energy component, respectively. In modern cosmology the scalar field plays the major role in modelling of dark energy and in explanation of the accelerated expansion of the current universe (for review see\textsuperscript{[3]}). In the simplest case the scalar field minimally coupled to gravity is assumed (so-called the quintessence idea\textsuperscript{[4, 5, 6, 7]}). In such a case the equation of the state parameter is $w_\phi > -1$, but as it was noted by Caldwell\textsuperscript{[8]} the observational data also admit the possibility that $w_\phi < -1$. Such a “phantom” form of a scalar field describing the dark energy component has many peculiar properties such as, for example, a big-rip singularity (energy density becomes infinite in finite time), the Lorentz invariance condition is then violated etc.\textsuperscript{[9, 10]}. 

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In this contribution we investigate dynamics of the cosmological model with the scalar field non-minimally coupled to the gravity with the positive and negative kinetic energy forms (i.e., canonical and phantom scalar fields) in the background of the flat Friedmann-Robertson-Walker (FRW) geometry. We point out interesting properties of a three phase model obtained within these class of solutions. They are interesting because they are generic and interpolate three physically important phases of the evolution of the universe, namely, radiation, matter and dark energy domination in the evolution of the universe. Therefore, his solutions can be treated as a natural extension of the quartessence idea [11, 12, 13, 14].

In our investigations we apply the dynamical systems methods in exploring stationary states represented by critical points in the phase space as well as their stability [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. We characterize all generic scenarios appearing in the case of the constant non-minimal coupling for both canonical and phantom scalar fields. In our dynamical study we relax the choice of the potential function. The presented approach to study the dynamics with the dynamical form of the equation of the state parameter is adifferent form the most popular one mainly used in the confrontation of the assumed model with dynamical dark energy with the observational data [27, 28]. While the authors who estimate parameters from the observational data postulate at the very beginning the form of the parameterization of the equation of state parameter $w(z)$ as a function of the redshift $z$, in the presented approach such a form is directly derived from the closed dynamics of the FRW model filled by the non-minimally coupled scalar field. Moreover, basing on the twister solution one can derive approximated forms of the effective equation of the state parameter $w(z)$ in three characteristic phases of the evolution of the universe, namely during the radiation, the baryonic matter and the dark energy domination.

In the model under consideration we assume the spatially flat FRW universe filled with the non-minimally coupled scalar field and barotropic fluid with the equation of the state coefficient $w_m$. The action assumes following form

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left( \frac{1}{\kappa^2} R - \varepsilon \left( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi R \phi^2 \right) - 2U(\phi) \right) + S_m, \quad (1)$$

where $\kappa^2 = 8\pi G$, $\varepsilon = +1, -1$ corresponds to canonical and phantom scalar field, respectively, the metric signature is $(-, +, +, +)$, $R = 6 \left( \frac{\dot{a}}{a} + \frac{a}{a} \right)$ is the Ricci scalar, $a$ is the scale factor and a dot denotes differentiation with respect to the cosmological time and $U(\phi)$ is the scalar field potential function. $S_m$ is the action for the barotropic matter part.

The dynamical equation for the scalar field we can obtain from the variation $\delta S / \delta \phi = 0$

$$\ddot{\phi} + 3H \dot{\phi} + \xi R \phi + \varepsilon U'(\phi) = 0, \quad (2)$$

and energy conservation condition from the variation $\delta S / \delta g = 0$

$$\mathcal{E} = \frac{1}{2} \dot{\phi}^2 + \varepsilon \xi H^2 \phi^2 + \varepsilon \xi H(\dot{\phi}^2) + U(\phi) + \rho_m - \frac{3}{\kappa^2} H^2. \quad (3)$$

Then conservation conditions read

$$\frac{3}{\kappa^2} H^2 = \rho_\phi + \rho_m, \quad (4)$$

$$\dot{H} = -\frac{\kappa^2}{2} \left[ (\rho_\phi + p_\phi) + \rho_m(1 + w_m) \right] \quad (5)$$
where the energy density and the pressure of the scalar field are

$$\rho_\phi = \frac{1}{2} \varepsilon \dot{\phi}^2 + U(\phi) + \varepsilon 3 \xi H^2 \phi^2 + \varepsilon 3 \xi H (\phi^2),$$

$$p_\phi = \frac{1}{2} (1 - 4 \xi) \dot{\phi}^2 - U(\phi) + \varepsilon \xi H (\phi^2) - \varepsilon 2 \xi (1 - 6 \xi) \dot{H} \phi^2 - \varepsilon 3 \xi (1 - 8 \xi) H^2 \phi^2 + 2 \xi \phi U'(\phi).$$

(6)

(7)

In what follows we introduce the energy phase space variables

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6} H}, \quad y \equiv \frac{\kappa \sqrt{U(\phi)}}{\sqrt{3} H}, \quad z \equiv \frac{\kappa}{\sqrt{6}} \phi,$$

(8)

which are suggested by the conservation condition

$$\frac{\kappa^2}{3H^2} \rho_\phi + \frac{\kappa^2}{3H^2} \rho_m = \Omega_\phi + \Omega_m = 1$$

(9)

or in terms of the newly introduced variables

$$\Omega_\phi = y^2 + \varepsilon \left[ (1 - 6 \xi) x^2 + 6 \xi (x + z)^2 \right] = 1 - \Omega_m.$$

(10)

The acceleration equation can be rewritten to the form

$$\dot{H} = -\frac{\kappa^2}{2} (\rho_{\text{eff}} + p_{\text{eff}}) = -\frac{3}{2} H^2 (1 + w_{\text{eff}})$$

(11)

where the effective equation of the state parameter reads

$$w_{\text{eff}} = \frac{1}{1 - \varepsilon 6 \xi (1 - 6 \xi) z^2} \left[ -1 + \varepsilon (1 - 6 \xi)(1 - w_m)x^2 + \varepsilon 2 \xi (1 - 3 w_m)(x + z)^2 + (1 + w_m)(1 - y^2) - \varepsilon 2 \xi (1 - 6 \xi) z^2 - 2 \xi \lambda y^2 z \right]$$

(12)

where $\lambda = -\frac{\varepsilon}{\kappa} \frac{dU(\phi)}{d\phi}$. 

The dynamical system of the model under considerations is in the form [16]

$$x' = -3x - 12 \xi z + \varepsilon \frac{1}{2} \lambda y^2 \left[ 1 - \varepsilon 6 \xi z (x + z) \right] + \varepsilon 6 \xi (1 - 6 \xi) x z^2 +$$

$$+ \frac{3}{2} (x + 6 \xi z) \left[ -\varepsilon (1 - 6 \xi)(1 - w_m)x^2 + \varepsilon 2 \xi (1 - 3 w_m)(x + z)^2 + (1 + w_m)(1 - y^2) \right],$$

(13)

$$y' = -\frac{1}{2} \lambda \left\{ x \left[ 1 - \varepsilon 6 \xi (1 - 6 \xi) z^2 \right] + 6 \xi y^2 z \right\} - \varepsilon 12 \xi (1 - 6 \xi) y z^2 +$$

$$+ \frac{3}{2} y \left[ -\varepsilon (1 - 6 \xi)(1 - w_m)x^2 + \varepsilon 2 \xi (1 - 3 w_m)(x + z)^2 + (1 + w_m)(1 - y^2) \right],$$

(14)

$$z' = x \left[ 1 - \varepsilon 6 \xi (1 - 6 \xi) z^2 \right],$$

(15)

$$\lambda' = -\lambda^2 (\Gamma - 1) x \left[ 1 - \varepsilon 6 \xi (1 - 6 \xi) z^2 \right],$$

(16)
Table 1: The location and eigenvalues of the critical points in twister quintessence scenario

|\( w_{\text{eff}} \) | location | eigenvalues |
|---|---|---|
|\( w_{\text{eff}} = \frac{1}{3} \) | \( x_1^* = 0, y_1^* = 0, (\lambda_1^*)^2 = \frac{\alpha^2}{6\xi} \) | \( l_1 = -6\xi, l_2 = 12\xi, l_3 = 6\xi(1 - 3w_m) \) |
|\( w_m \) | \( x_2^* = 0, y_2^* = 0, \lambda_2^* = 0 \) | \( l_{1,3} = -\frac{4}{3}(1 - w_m)(1 \pm \sqrt{1 - \frac{16}{3}\xi\frac{1 - 3w_m}{(1 - w_m)^2}}), l_2 = \frac{3}{2}(1 + w_m) \) |
|\( -1 \) | \( x_3^* = 0, (y_3^*)^2 = 1, \lambda_3^* = 0 \) | \( l_{1,3} = \frac{1}{2}(3 \pm \sqrt{9 + 6\alpha - 48\xi}), l_2 = -3(1 + w_m) \) |

where \( \Gamma = \frac{U''(\phi)U(\phi)}{U'(\phi)^2} \) and prime denotes differentiation with respect to time \( \tau \) defined as

\[
\frac{d}{d\tau} = [1 - 6\xi(1 - 6\xi)z^2]\frac{d}{d\ln a}.
\] (17)

If \( \lambda \) is constant then we obtain the scaling potential \( \exp(\lambda\phi) \) and the basic system reduces to the 3-dimensional autonomous dynamical system in the case of the model with the barotropic matter. In the case without the matter the dynamical system is a 2-dimensional autonomous one.

In the rest of the paper we will assume the following form of the function \( \Gamma(\lambda) \)

\[
\Gamma(\lambda) = 1 - \frac{\alpha}{\lambda^2}
\] (18)

where \( \alpha \) is an arbitrary constant and for which we can simply eliminate one of the variables namely \( z \) given by the relation

\[
z(\lambda) = -\int \frac{d\lambda}{\lambda^2(\Gamma(\lambda) - 1)} = \frac{\lambda}{\alpha} + \text{const}
\] (19)

where in the rest of the paper we take the integration constant as equal to zero.

From equation (18) and the definition of the function \( \Gamma \) we can simply calculate the form of the potential function

\[
U(\phi) = U_0 \exp \left[ -\frac{\alpha^2}{6}(\phi^2 + \beta\phi) \right] = \bar{U}_0 \exp \left[ -(\phi + \gamma)^2 \right]
\] (20)

where \( \beta \) is the integration constant. As we can see the dynamics of the model does not depend on the value of this parameter. In such a case we are exploring the solutions in the very rich family of potential functions.

Following the Hartman-Grobman theorem [29] the system can be well approximated by the linear part of the system around a non-degenerate critical point. Then stability of the critical point is determined by eigenvalues of a linearization matrix only. In Table I we have gathered critical points appearing in twister scenario together with the eigenvalues of the linearization matrix calculated at those points.

The critical point of a saddle type which represents the radiation dominated universe \( w_{\text{eff}} = 1/3 \)
In the case of canonical scalar field $\varepsilon = 0$, its character depends on the value of $\lambda$.

The matter dominated universe where $w_{\text{eff}} = w_m$ is represented by the critical point ($x_2^* = 0, y_2^* = 0, \lambda_2^* = 0$) which character depends on the value of the parameter $d$

$$d = 1 - \frac{16 \xi}{3} \frac{1 - 3w_m}{(1 - w_m)^2}. $$

For $d > 0$ the critical point is of a saddle type and the linearized solutions are in the form

$$x_2(a) = \frac{a^{\frac{1}{2}(1-w_m)}}{2 \sqrt{d}} \left\{ (1 + \sqrt{d}) x_2^{(i)} + \frac{1}{\alpha} (1 - w_m) (1 - \sqrt{d}) \lambda_2^{(i)} a^{\frac{1}{2}(1-w_m) \sqrt{d}} - (1 - \sqrt{d}) x_2^{(i)} + \frac{1}{\alpha} (1 - w_m) (1 + \sqrt{d}) \lambda_2^{(i)} a^{\frac{1}{2}(1-w_m) \sqrt{d}} \right\},$$

$$y_2(a) = y_2^{(i)} a^{\frac{1}{2}(1+w_m)},$$

$$\lambda_2(a) = -\frac{2\alpha a^{\frac{1}{2}(1-w_m)}}{3(1-w_m) \sqrt{d}} \left\{ x_2^{(i)} + \frac{1}{\alpha} (1 - w_m)(1 - \sqrt{d}) \lambda_2^{(i)} a^{\frac{1}{2}(1-w_m) \sqrt{d}} - [x_2^{(i)} + \frac{1}{\alpha} (1 - w_m)(1 + \sqrt{d}) \lambda_2^{(i)} a^{\frac{1}{2}(1-w_m) \sqrt{d}} \right\}. $$

For $d < 0$ the critical point is of an unstable focus type

$$x_2(a) = -\frac{1}{\sqrt{|d|}} a^{\frac{1}{2}(1-w_m)} \left\{ x_2^{(i)} + \frac{1}{\alpha} (1 - w_m) (1 - |d|) \lambda_2^{(i)} \sin \left( \frac{3}{4}(1 - w_m) \sqrt{|d|} \ln a \right) - \sqrt{|d|} x_2^{(i)} \cos \left( \frac{3}{4}(1 - w_m) \sqrt{|d|} \ln a \right) \right\},$$

$$y_2(a) = y_2^{(i)} a^{\frac{1}{2}(1+w_m)},$$

$$\lambda_2(a) = \frac{4}{3 (1-w_m) \sqrt{|d|}} a^{\frac{1}{2}(1-w_m)} \left\{ x_2^{(i)} + \frac{1}{\alpha} (1 - w_m) \lambda_2^{(i)} \sin \left( \frac{3}{4}(1 - w_m) \sqrt{|d|} \ln a \right) + \frac{1}{\alpha} (1 - w_m) \lambda_2^{(i)} \cos \left( \frac{3}{4}(1 - w_m) \sqrt{|d|} \ln a \right) \right\}. $$

The final critical point represents the deSitter universe with $w_{\text{eff}} = -1$ is ($x_3^* = 0, (y_3^*)^2 = 1, \lambda_3^* = 0$) its character depends on the value of $\Delta = 9 + \varepsilon 2 \alpha - 48 \xi$.  


For $\Delta > 0$ the critical point is of a stable node type and the linearized solutions in the vicinity of this type critical points are

$$x_3(a) = \frac{a^{-\frac{\alpha}{2}}}{\sqrt{|\Delta|}} \left\{ (3 + \sqrt{\Delta})[x_3(i) + \frac{1}{2\alpha}(3 - \sqrt{\Delta})l_3(i)]a^{-\frac{\sqrt{\Delta}}{2\alpha}} - (3 - \sqrt{\Delta})[x_3(i) + \frac{1}{2\alpha}(3 + \sqrt{\Delta})l_3(i)]a^{-\frac{\sqrt{\Delta}}{2\alpha}} \right\} (30)$$

$$y_3(a) = y_3^* + (y_3^* - y_3^{})a^{-3(1+w_m)},$$

$$\lambda_3(a) = -\frac{aa^{-\frac{\alpha}{2}}}{\sqrt{|\Delta|}} \left\{ \left[ x_3(i) + \frac{1}{2\alpha}(3 - \sqrt{\Delta})l_3(i) \right] a^{-\frac{\sqrt{\Delta}}{2\alpha}} - \left[ x_3(i) + \frac{1}{2\alpha}(3 + \sqrt{\Delta})l_3(i) \right] a^{-\frac{\sqrt{\Delta}}{2\alpha}} \right\}. \quad (31)$$

For $\Delta < 0$ the critical point is of a stable focus type and the linearized solutions are

$$x_3(a) = \frac{a^{-\frac{\alpha}{2}}}{\sqrt{|\Delta|}} \left\{ - \left[ 3x_3(i) + \frac{1}{\alpha}(9 + \varepsilon \alpha - 24x)l_3(i) \right] \sin \left( \frac{\sqrt{|\Delta|}}{2} \ln a \right) + x_3(i) \sqrt{|\Delta|} \cos \left( \frac{\sqrt{|\Delta|}}{2} \ln a \right) \right\}, \quad (32)$$

$$y_3(a) = y_3^* + (y_3^* - y_3^{})a^{-3(1+w_m)},$$

$$\lambda_3(a) = \frac{2aa^{-\frac{\alpha}{2}}}{\sqrt{|\Delta|}} \left\{ \left[ x_3(i) + \frac{3}{2\alpha}l_3(i) \right] \sin \left( \frac{\sqrt{|\Delta|}}{2} \ln a \right) + \frac{\sqrt{|\Delta|}}{2\alpha} \lambda_3(i) \cos \left( \frac{\sqrt{|\Delta|}}{2} \ln a \right) \right\}. \quad (33)$$

The solutions of the linearized system in the vicinity of each critical point $x_i(a)$, $y_i(a)$ and $\lambda_i(a)$ can be used to constrain the model parameters thorough the cosmological data from various cosmological epochs. For example, the parameters for the solution describing radiation dominated universe (1) can be constrained from CMB data, and the solutions (3) describing the current accelerating expansion of the universe through the SN Ia data. Therefore one can estimate the parameters of the variability with redshift of true $w(a)$ (see Fig[2]). It is possible because we have the linearization of exact formula in different epochs.

The presented possibility of appearing twister type quintessence scenario is not restricted to the considered case of the $\Gamma(\lambda)$ function [13]. One can easily show that such a scenario will be always possible if only the following functions calculated at the critical points [16]

$$f(\lambda^*) = (\lambda^*)^2(\Gamma(\lambda^*) - 1) = \text{const}, \quad \frac{df(\lambda)}{d\lambda} |_{\lambda^*} = f'(\lambda^*) = \text{const}$$

are finite.

In this letter we pointed out the new interesting solution for the non-minimally coupled scalar field cosmology which we called the twister solution (because of the shape of the corresponding trajectory in the phase space, see Fig[1]). This type of the solution is very interesting because in the phase space it represents the 3-dimensional trajectory which interpolates different stages of evolution of the universe, namely, the radiation dominated, dust filled and accelerating universe. We found linearized solutions around all these intermediate phases as well as the solution of the separatrices which play an important role in the organization of the phase space structure. In is interesting that the presented structure of the phase space is allowed only for non-zero value of coupling constant, therefore it is a specific feature of the non-minimally coupled scalar field cosmology.
Figure 1: Three-dimensional phase portrait of the dynamical system under consideration. Trajectories represent a twister type solution which interpolates between the radiation dominated universe (a saddle type critical point), the matter dominated universe (an unstable focus critical point) and the accelerating universe (a stable focus critical point).

Figure 2: The evolution of $w_{\text{eff}}$ given by the relation (12) for the non-minimally coupled canonical scalar field $\varepsilon = +1$ and the positive coupling constant $\xi$. The sample trajectory used to plot this relation starts its evolution at $\tau_0 = 0$ near the saddle type critical point ($w_{\text{eff}} = 1/3$) and then approaches an unstable focus critical point $w_{\text{eff}} = w_m = 0$ and next escapes to the stable deSitter state with $w_{\text{eff}} = -1$. The existence of a short time interval during which $w_{\text{eff}} \approx \frac{1}{3}$ is the effect of the nonzero coupling constant $\xi$. 
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