Pilot design for underwater MIMO cosparse channel estimation based on compressed sensing

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Abstract
In multiple-input multiple-output–orthogonal frequency-division multiplexing underwater acoustic communication systems, the correlation of the sampling matrix is the key of the channel estimation algorithm based on compressed sensing. To reduce the cross-correlation of the sampling matrix and improve the channel estimation performance, a pilot design algorithm for co-sparse channel estimation based on compressed sensing is proposed in this article. Based on the time-domain correlation of the channel, the channel estimation is modeled as a common sparse signal reconstruction problem. When replacing each pilot indices position, the algorithm selects multiple pilot indices with the least cross-correlation from the alternative positions to replace the current pilot indices position, and it uses the inner and outer two-layer loops to realize the bit-by-bit optimal replacement of the pilot. The simulation results show that the channel estimation mean squared error of pilot design algorithm for co-sparse channel estimation based on compressed sensing can be reduced by approximately 18 dB compared with the least square algorithm. Compared with the genetic algorithm and search space size methods, the structural sequence search proposed by pilot design algorithm for co-sparse channel estimation based on compressed sensing is used to design the pilot to complete the channel estimation. Thus, the mean squared error of the channel estimation can be reduced by 2 dB. At the same bit error rate of 0.03, the signal-to-noise ratio can be decreased by approximately 7 dB.

Keywords
Compressed sensing, pilot design, multiple-input multiple-output–orthogonal frequency-division multiplexing, underwater channel estimation, sparsity

Date received: 18 July 2020; accepted: 23 April 2021

Handling Editor: Lyudmila Mihaylova

Introduction
There are many problems in the ocean channel, such as a strong multipath effect and narrow available bandwidth.¹–³ The single carrier modulation of an underwater acoustic (UWA) communication system has difficulties meeting the communication requirements for a high transmission rate. Multiple-input multiple-output (MIMO)–orthogonal frequency-division multiplexing (OFDM) technology based on multi-input and multi-output has the advantages of multi-carrier high-
frequency spectrum utilization and can effectively suppress the multi-path interference caused by the UWA channel. Therefore, multiple-input multiple-output (MIMO) technology has attracted great attention in the underwater communications field.

The CSI (Channel State Information) acquisition, namely, channel estimation, is one of the key technologies in MIMO–OFDM UWA communication system. In an MIMO UWA system, in order to achieve stable communication between the receiver and the transmitter, the receiver must accurately obtain the CSI to eliminate the influence of the channel on the transmission signal and realize the correct demodulation of the receiver. The accuracy of CSI has an important influence on beamforming, signal detection, signal decoding, channel equalization, and so on. Therefore, channel estimation, or the acquisition of CSI, is one of the key technologies in MIMO–OFDM UWA communication. For the channel estimation method using a training sequence, the pilot cost increases linearly as the number of antennas increases, which reduces the transmission rate and the advantages of an MIMO system. Therefore, how to accurately estimate the channel with fewer pilots becomes a challenge in an MIMO–OFDM system. At present, although there are many traditional methods to obtain CSI, these methods need a large pilot cost to accurately estimate the channel.

Compressed sensing (CS) algorithm has been widely used in sparse channel estimation. In recent years, Candes et al. proposed a CS theory that can reconstruct the original sparse signal with the high probability of sparse signal. The theory showed that the original sparse signal can be reconstructed with the high probability sparse signal using the CS method, in which the measurement value of a sparse signal is far lower than the original signal dimension. CS technology is widely used for sparse channel estimation. Compared with traditional channel estimation methods such as the least square (LS), the minimum mean square error (MMSE), and so on, CS methods, such as the orthogonal matching pursuit (OMP) and the basis pursuit (BP), can reconstruct the sparse channel with a high probability with only a few pilots, which greatly reduces the pilot cost.

In an MIMO–OFDM UWA communication system, regarding the correlation of continuous time channels, it can be considered that the same non-zero tap position of the channel is only the difference of the channel fading coefficient, which is named the common sparsity. Therefore, we can use the common sparsity of a UWA channel to reconstruct the common sparsity channel and obtain better channel estimation performance than channel reconstruction alone. In addition, different from the traditional channel estimation method, the uniform pilot distribution achieves the optimal channel estimation performance, and the channel estimation performance of the CS algorithm depends on the cross-correlation of the sampling matrix. The smaller the correlation of the sampling matrix is, the better the reconstruction performance of the CS algorithm. To improve the sparse channel estimation performance, it is necessary to design the sampling matrix. In Choi et al. it is proposed that the smaller the cross-correlation of the sampling matrix is, the better the channel estimation algorithm based on the pilot design. However, this method in land cannot directly applied into the UWA communication. The main reason is that in the above estimation channel algorithm, the pilot of each antenna is pseudorandomly generated and it varies with the orthogonal frequency-division multiplexing (OFDM) symbols. The pseudorandomly generated pilot will cause the sampling matrix to have greater cross-correlation and then reduce the channel estimation performance. Furthermore, the pseudorandomly generated pilot will increase the search time to obtain the pilot position, which is not suitable for UWA communication. In UWA communication, there are several challenges, such as limited power due to the inability to recharge, time–frequency–space variations in the underwater channel, and a lower transmission rate underwater than on land. Thus, the pilot algorithm for land is directly used for underwater. In other words, it is impracticable in real underwater MIMO–OFDM due to the huge amount of computations caused by the large number of system subcarriers and pilot symbols.

Related work

In the past few years, the pilot design algorithm based on CS has been studied. In Yun et al., the problem of pilot design in an MIMO–OFDM system is studied and a genetic algorithm (GA) is proposed to search for pilots. In this article, the common sparse channel reconstruction of multiple antennas is considered. The running time of the GA is difficult to control and the performance of the algorithm is greatly affected by the parameter settings of the algorithm. The pilot design depends on a better pilot search algorithm. In reference an extended scheme based on a statistical random search is proposed to search for multiple antenna pilots; however, the number of pilot combinations considered in this method is limited by the pilot position replacement method.

Due to the sparsity of the UWA channel, the pilot design method based on CS is also suitable for the underwater environment. The typical representatives of these algorithms include the GA, particle swarm optimization (PSO), artificial bee colony (ABC), bat-inspired algorithm (BA), gray wolf optimizer
and the more recent whale optimization algorithm (WOA). However, almost all these studies are based on an LS algorithm that minimizes the estimation error of the mean squared error (MSE) criterion. Inevitably, these algorithms do not make full use of the significant sparsity of UWA multi-path channels to improve the spectral efficiency of the communication system. In addition, the aforementioned methods may suffer from difficulties related to the convergence time and convergence accuracy when searching the optimal pilot allocation. Considering the underwater communication conditions, such as limited power, the complexity of the underwater channel, and the low transmission rate, the pilot design algorithm cannot be too large and the pilot position needs to be fixed and then can be meet the requirements the amount of calculations. Therefore, it is necessary to optimize the pilot position to improve the bandwidth utilization. In view of the characteristics of the UWA channel, in order to reduce the search speed and convergence in the pilot design, this article proposes a pilot design algorithm for compressed sensing (PDACS) algorithm.

**Our contribution**
As stated previously, the design of the underwater pilot based on CS has been studied to some extent. However, it is still rare to find studies that improve the channel estimation performance by reducing the cross-correlation of the sampling matrix. To improve the channel estimation performance and reduce the pilot cost, the pilot is designed to reduce the cross-correlation of the sampling matrix:

1. To reduce the complexity of the design and calculation of the underwater pilot and the randomness of the pilot selection, this article proposes a method to optimize the fixed pilot placement in the pilot selection process.
2. According to the common sparsity of an MIMO–OFDM UWA channel, this article constructs a common sparsity signal sampling matrix, taking the correlation of the sampling matrix as the objective function. By reducing the cross-correlation of the sampling matrix, the probability of signal reconstruction is improved.
3. To optimize the cross-correlation of the sampling matrix, the tree structure sequence replacement search algorithm is proposed to solve the multi-antenna pilot. Compared with the GA, the tree structure sequence method is less affected by the random selection of parameters, the convergence time is guaranteed, and the cross-correlation of the sampling matrix is at its minimum.

According to the simulation, the PDACS algorithm is verified to have a better optimization capability than other algorithms. Regarding the channel estimation performance, the robustness and effectiveness of the proposed PDACS algorithm are demonstrated by maintaining consistently superior performance with respect to the bit error rate (BER) and MSE for different system subcarriers and channel models. In addition, simulation results confirm that our proposal retains almost the same computational complexity but yields better convergence performance than the GA, PSO, and LS-based methods in optimizing pilot allocations.

The remaining portion of this article is organized as follows. In section “System model,” the MIMO–OFDM underwater system model is described briefly. In section “Pilot design algorithms,” the pilot design algorithm is presented in detail. In section “Simulation result and analysis,” the simulations and results of the considered methods are given and analyzed. Finally, section “Conclusion” is devoted to the conclusions.

**System model**
For the MIMO–OFDM UWA communication system with one transmitting antenna and one receiving antenna, it is assumed that the channel between each pair of transmitting antennas is a frequency selective channel and the coherence time of the channel is longer than the duration of the OFDM symbol, that is, the channel is a quasi-static channel, and the channel impulse response is constant in one OFDM symbol period.

It is assumed that the OFDM symbols contain sub-carriers, in which the numbers of data subcarriers and pilot subcarriers are\( N_D \) and \( N_P \), respectively. To avoid interference, the common frequency-domain orthogonal pilot is used for \( N_T \) transmitting antennas, and the set of antenna \( i_T \) is set as \( \Lambda_{iT} = \{ k_{i_T,1}, k_{i_T,2}, \ldots, k_{i_T,N_T} \} \). Then, \( \Lambda_{iT} \cap \Lambda_{jT} = \emptyset, (i \neq j) \) and \( \Lambda_{iT} \subset \Omega = \{ 1, 2, \ldots, N \} \). Therefore, \( N_T \times N_R \) MIMO channel estimation can be decomposed into the \( N_T \times N_R \) channel estimation of a single-input single-output OFDM system. If the cyclic prefix of the OFDM symbol is larger than the maximum delay spread of the channel, the frequency-domain OFDM symbol \( i_R \) received by antenna \( r^i_R \) can be expressed as

\[
r^{i_R} = \sum_{i_T=1}^{N_T} X^{i_T} W^{N_D \times L} h^{i_R,i_T} + \eta^{i_R,i_T} \tag{1}
\]
where \( X^i = \text{diag}(X^{i1}(1), X^{i2}(2), \ldots, X^{i(N-1)}), i_T = 1, 2, \ldots, N_T \) is the diagonal matrix of the frequency-domain symbols transmitted by the \( i_T \)th antenna. \( W_{N \times L} \) is the \( N \times L \) matrix composed of the first \( L \) columns of the \( N \times N \) Fourier transform matrix, and \( \mathbf{n}^{k,i_T} \) is the complex Gaussian white noise vector obeying a \( CN(0, \sigma^2) \) distribution. \( h^{k,i_T} \) is the discrete channel pulse response with a length of \( L \) between the \( i_T \)th transmitting antenna and the \( i_T \)th receiving antenna. For \( K \) sparse multi-path channels, the number of non-zero elements in the channel vector \( \mathbf{h}^{k,i_T} \) is far less than the channel length, and \( K = L \).

To estimate the channel impulse response, the received pilot is extracted from the received signal. Now, considering the channel estimation of the \( i_T \)th receiving antenna, let the pilot vector received by the \( i_T \)th user and transmitted by the \( i_T \)th antenna be \( \mathbf{p}^{k,i_T} \). Then, \( \mathbf{p}^{k,i_T} \) can be expressed as follows

\[
\mathbf{p}^{k,i_T} = E^i X^i (E^i)^T E^i W_{N \times L} h^{k,i_T} + E^i \mathbf{n}^{k,i_T} \tag{2}
\]

where \( E^i \) is the \( N_p \times N \) matrix composed of \( \Lambda_M \) lines of an \( N \times N \) unit matrix, and \( E^i X^i (E^i)^T \) is the diagonal matrix of the pilot transmitted by the \( i_T \)th antenna. Assume the following

\[
A^i = E^i X^i (E^i)^T E^i W_{N \times L} \tag{3}
\]

Then, equation (2) can be expressed as follows

\[
\mathbf{p}^{k,i_T} = A^i h^{k,i_T} + \mathbf{n}^{k,i_T} \tag{4}
\]

Considering the continuous \( s \)-correlation channels of the \( i_T \)th transmitting antenna and the \( i_T \)th receiving antenna, the subscript \( t + i \) in \( A^i_{t+i} \), \( h^{k,i_T}_{t+i} \), and \( \mathbf{n}^{k,i_T}_{t+i} \) represents the \( i \)th OFDM symbol sequence number starting from time \( t \). Then, the pilot of the \( i_T \)th antenna received by the continuous \( s \)th OFDM symbols is superposed and constructed as \( \mathbf{p}^s = [(\mathbf{p}^{k,i_T}_{t+s})_{t+1}, (\mathbf{p}^{k,i_T}_{t+s})_{t+2}, \ldots, (\mathbf{p}^{k,i_T}_{t+s})_{t+L}]^T \). \( \mathbf{p}^s \) can be expressed as follows

\[
\mathbf{p}^s = \begin{bmatrix}
A_{t+1}^{i_T, 1} & 0_{s \times L} & \cdots & 0_{s \times L} \\
0_{s \times L} & A_{t+2}^{i_T, 2} & \cdots & 0_{s \times L} \\
\vdots & \vdots & \ddots & \vdots \\
0_{s \times L} & 0_{s \times L} & \cdots & A_{t+s}^{i_T, s} \\
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{n}_{t+1}^{k,i_T} \\
\mathbf{n}_{t+2}^{k,i_T} \\
\vdots \\
\mathbf{n}_{t+s}^{k,i_T} \\
\end{bmatrix}
\tag{5}
\]

To use the common sparse CS algorithm, vector \( \mathbf{h} \) and matrix \( \mathbf{A} \) are rearranged so that \( \mathbf{Q} = [\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_L] \), where \( \mathbf{Q}_i = [\mathbf{e}_1, \mathbf{e}_{i+1}, \ldots, \mathbf{e}_{i+(L-i)\times L}] \) is the matrix formed by extracting the \( i \)th, \((i + L)\)th, \((i + (s-1)L)\)th column from the \( sL \times sL \) unit matrix \( \mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{sL}] \). Formula (5) can be expressed as follows

\[
\mathbf{p}^s = \begin{bmatrix}
A_{t+1}^{i_T, 1} & 0_{s \times L} & \cdots & 0_{s \times L} \\
0_{s \times L} & A_{t+2}^{i_T, 2} & \cdots & 0_{s \times L} \\
\vdots & \vdots & \ddots & \vdots \\
0_{s \times L} & 0_{s \times L} & \cdots & A_{t+s}^{i_T, s} \\
\end{bmatrix} \mathbf{Q} \mathbf{Q}^T + \begin{bmatrix}
\mathbf{n}_{t+1}^{k,i_T} \\
\mathbf{n}_{t+2}^{k,i_T} \\
\vdots \\
\mathbf{n}_{t+s}^{k,i_T} \\
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{b}_1 \\
\mathbf{b}_2 \\
\vdots \\
\mathbf{b}_K \\
\end{bmatrix} \tag{6}
\]

where \( \mathbf{b}_i = [h^{k,i_T}_{t+i}(1), h^{k,i_T}_{t+i}(2), \ldots, h^{k,i_T}_{t+i}(L)]^T \). Therefore, \( \mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_2^T, \ldots, \mathbf{b}_K^T]^T \) is only the \( K \)th non-zero \( \mathbf{b}_i \), and vector \( \mathbf{b} \) is \( k \)th sparse vector.

Due to sparse vector \( \mathbf{b} \), the receiver can obtain the channel impulse response \( \mathbf{h} \) under the known conditions and sampling matrix \( \mathbf{T} \). The expression is as follows

\[
\min \| \mathbf{b} \|_0 \text{ s.t. } \| \mathbf{p}^s - \mathbf{T} \mathbf{b} \|_2 \leq \alpha \tag{7}
\]

\( \alpha (\alpha \geq 0) \) is the noise tolerance. For an \( N_T \times N_R \) MIMO system, the whole system channel in each antenna can be solved by expression (7).

### Pilot design algorithms

#### Optimized fixed pilot

In Zhu et al.,\(^7\) the smaller the restricted isometric property (RIP) is, the better the sparse signal reconstruction performance. Different pilot distributions make up different sampling matrices, and so the pilot directly determines the channel estimation accuracy. However, there is no known method to calculate whether the given matrix satisfies the RIP. Another alternative method to evaluate the sample matrix reconstruction performance is the cross-correlation of the sample matrix, and the cross-correlation condition is better than the RIP.\(^23,24\) The theory of CS shows that the smaller the correlation of the sampling matrix is, the smaller the reconstruction deviation of a sparse signal.\(^24\) Therefore, the pilot can be designed to reduce the cross-correlation of the sampling matrix to improve the channel estimation performance.

Equation (7) is a block sparse signal reconstruction problem. The block sparse signal reconstruction accuracy is also related to the sampling matrix.

The cross-correlation of matrix \( \mu(\mathbf{A}) \) is defined as the maximum normalized absolute value inner product of any two columns of the matrix

\[
\mu(\mathbf{A}) = \max_{1 \leq m < n \leq L} \frac{|\mathbf{A}_m^T \mathbf{A}_n|}{\|\mathbf{A}_m\|_2 \|\mathbf{A}_n\|_2} \tag{8}
\]
where \( A_i, 1 \leq i \leq sL \) represents \( i \)th column of the matrix. Because matrix \( T \) is a column rearrangement of matrix \( A \), the cross-correlation of the minimization matrix \( T \) is equivalent to the cross-correlation of the minimization matrix \( A \). Therefore, the pilot design problem becomes

\[
\min \left( \max_{1 \leq m < n \leq sL} \frac{|A^T_{m} A_n|}{\|A_m\|_2 \|A_n\|_2} \right)
\]

(9)

The cross-correlation of the \( i \)th matrix \( A^T_{i} + v \) on the diagonal of matrix \( A \) is

\[
\max_{0 \leq m < n \leq sL - 1} \left| \sum_{l=1}^{N_p} |X^T_{i + v}(k_{l}r_{i}, l)| e^{-j2\pi k_{l} r_{i} (m-n) / N} \right| \]

(10)

Because the pilot with equal amplitude and zero autocorrelation is often used in a practical system, assume that the pilot symbol modulus is equal to \( |X^T_{i + v}(k_{l}r_{i}, 1)| = |X^T_{i + v}(k_{l}r_{i}, 2)| = \cdots = |X^T_{i + v}(k_{l}r_{i}, N_e)| \). According to expression (5), sampling matrix \( A \) is a diagonal matrix, and so \( \mu(A) \) is equal to the maximum value of \( \mu(A^T_{i + v}), 1 \leq i \leq s \). The reason is that the OFDM symbols of the same antenna have the same pilot indices set and pilot amplitude. Then, the cross-correlation of diagonal matrix \( A \), \( \mu(A) \), is the cross-correlation of any diagonal unit. Minimizing \( \mu(A) \) equals minimizing \( \mu(A^T_{i + v}), 1 \leq i \leq s \). Expression (10) is as follows

\[
g(\Lambda_i) = \frac{1}{N_p} \max_{0 \leq m < n \leq sL - 1} \left| \sum_{l=1}^{N_p} e^{-j2\pi k_{l} r_{i} (m-n) / N} \right|
\]

(11)

Under the known \( L, N, \) and \( N_p \), the value of \( g(\Lambda_i) \) is uniquely determined by the pilot set \( \Lambda_i \). For an underwater MIMO system with \( N_T \) transmission antennas, the value of \( g(\Lambda_i) \) is uniquely determined by the pilot set. The \( r_{i} (r_{i} \geq 2) \) pilot design is the selected pilot set \( \Lambda_i \) in the set \( \Omega^i(1) \cup \ldots \cup \Omega^i(r_{i}) \). To minimize expression (11) (when \( r_{i} = 1 \), the set is \( \Omega \)) and then obtain the pilot \( \Lambda_T \), expression (11) is set as follows

\[
\min g(\Lambda_i), \quad g(\Lambda_i) = \frac{1}{N_p} \max_{0 \leq m < n \leq sL - 1} \left| \sum_{l=1}^{N_p} e^{-j2\pi k_{l} r_{i} (m-n) / N} \right|
\]

(12)

For the cosparse channel estimation, Figure 1(a) shows the location of the pseudorandom pilot in Qi et al.8 However, the pseudorandom pilot cannot guarantee the small cross-correlation of the sampling matrix. Therefore, in this article, a fixed pilot distribution is proposed, as shown in Figure 1(b), and the cross-correlation of the sampling matrix is reduced by the pilot design. Figure 1 shows the transmission diagram of the OFDM system, where \( N = 16, N_{T} = 3, N_{p} = 3, s = 3 \). In Figure 1, each square represents a time-frequency resource, the shadow represents the antenna transmitting the pilot, and the dotted box represents the OFDM symbols corresponding to the three consecutive correlation channels of antenna 1.

Because expression (12) is a combinatorial optimization problem, the most direct method is to exhaust all possible results. However, for OFDM systems with more subcarriers, the exhaustion algorithm has too much of a computation load, and so it cannot effectively solve the pilot.

**Search algorithm of tree structure order replacement**

In this section, a PDACS algorithm is proposed to search for the suboptimal pilot position. The algorithm consists of three steps: initialization, obtaining a single antenna pilot, and result output.

Step 1 initialization: the number of external cycles is \( M_{1} \), the number of internal cycles is \( M_{2} \), the branch tree of each node is \( T_{1} \), and the number of reserved nodes is \( \Omega_{1} = \{1, 2, \ldots, N \} \). The pilot carrier \( h \) is available for the first antenna, and use steps (1)–(4) in step 2 to obtain the pilot of the first antenna \( P_{1} \).

Step 2 obtaining a single antenna pilot: for \( i = 2, \ldots, N_{T} \), update \( \Omega = \Omega^{-1} / P^{-1} \), run steps (1)–(4) and search \( P_{1} \) in \( \Omega \):

1. Set \( P_{U} \) as matrix \( M_{1} \times N_{p} \), which is used to store the pilot searched by \( M_{1} \) external loops. For the \( u = 1, 2, \ldots, M_{1} \)th outer loop, \( P_{U} \) positions are randomly selected from all available carrier sets \( \Omega \) and then optimized bit-by-bit according to step (2).

2. In the \( l = 1, 2, \ldots, M_{2} \) internal cycle, \( P_{U}^{0} = P_{0}^{0} = P_{U}^{0} \), update each element of \( P_{U}^{l-1} \) according to step (3), and the pilot obtained after the \( h \)th internal cycle is \( P_{U}^{h} \). If the pilot \( P_{U}^{h} \) is obtained after \( M_{2} \) internal cycles, save \( P_{U}^{h} \) to line \( u \) of \( P_{U} \), exit the internal cycle and turn to step (1); otherwise, \( P_{U}^{h} = P_{U}^{h-1} \) in order to avoid invalid optimization. If \( P_{U}^{h} = P_{U}^{h-1} \), end the internal cycle and save the result.

3. The pilot to be optimized in this step is \( P_{U}^{0} = P_{U}^{0}_{l-1} = (p_{1}, p_{2}, \ldots, p_{N_{p}}) \). When replacing the \( k = 1, 2, \ldots, N_{p} \) pilot position, replace the \( k - 1 \) position of pilot \( T_{2} (P_{U}^{l-1})_{k-1} \) obtained by replacing position \( k \), select the optimal node \( T_{2} \) from the nodes \( N - N_{p} + 1 \), and then select the
optimal node $T_2 \{p^k_{v_{i-1}}\}$ from the nodes $T_1 T_2$ as the result of the replacement of $k$. Specifically, pilot node $v$ replaced by pilot $k$ $p^k_{v_{i-1}} = \{p^k_{v_{v,1}}, p^k_{v,2}, \ldots, p^k_{v,k-1}, p^k_{v,k}, p^k_{v,k+1}, \ldots, p^k_{v_{N_p}}\}$, $1 \leq v \leq T_2$ remains unchanged as $\tilde{p}^{k-1}_v = \{p^k_{v,1}, p^k_{v,2}, \ldots, p^k_{v,k-1}, p^k_{v,k+1}, \ldots, p^k_{v_{N_p}}\}$. Add each element in the set $\Lambda = \{\Omega \backslash \tilde{p}^{k-1}_v\} \cup p^{k-1}_v$ to position $k$ of $p^{k-1}_v$, get the pilot set $N - N_p$ + 1, calculate the pilot cross-correlation after replacement according to equation (11), and select the pilot nodes $T_1$ corresponding to the minimum cross-correlation. Each node in node $T_2 \{p^k_{v_{i-1}}\}$ generates $T_1$ branches, and node $T_2$ generates $T_1 T_2$ nodes in total. Select the nodes $T_2$ with the minimum cross-correlation from nodes $T_1 T_2$ as the replacement result, which is recorded as $\{p^k_{v_i}\}$ $T_1$. Then, replace it for time $k + 1$.

4. After $M_1$ cycles, $M_1$ pilots are generated in total. The line output with the minimum cross-correlation is selected from $P_U$.

Step 3 result output: output the pilot $\{P^*\}$ $T_1$ of $N_{ri}$ antennas.

The key of the above algorithm is step (3) in step 2. In the bit-by-bit optimization process, the first node only generates $T_1$ branches, and the last pilot position replacement only generates one pilot node. Set $T_1 = T_2 = 2$. When $N_r = 3$, step (3) in step 2 completes the internal loop node selection process, as shown in Figure 2, where the shadow node represents node $T_2 = 2$ selected in the replacement.

**Simulation result and analysis**

To simulate the authenticity of an underwater communication environment, the BELLHOP ray model is applied to establish the multi-path channel of real ocean data. As shown in Figure 3(a), the latitude and longitude coordinates of the ocean area are 20.373 and 113.875, respectively; the SSP (sound speed profile), seafloor, and sea surface refraction and scattering are as shown. The transmitter and the receiver are placed in a water area with a depth of 100 m. They are placed at a distance 1000 m apart and a depth of 30 m. According to the parameters in Jiang et al., when the sound velocity changes from 1540 m/s on the water surface to 1512 m/s on the water bottom, the SSP is as shown in Figure 3(b). After all, these environmental profiles are input to BELLHOP model, and it produces a variety of useful outputs such as the transmission loss, the arrival time-series and amplitudes, and so on.

Figure 3(c) predicts the transmission loss with respect to different depths and ranges. In Figure 3(d), the CIR (Channel Impulse Response) is clustered by six taps with dominated powers at the receiver.

Assume that the subcarrier in the OFDM symbol is $N = 512$, the channel length is $L = 60$, and $N_p = 14$. Then, the number of non-zero multi-paths is $K = 6$, and the sampling value $L - K = 54$ is 0. As mentioned above, Qi et al. proposed a classical pilot design algorithm on land. To reflect the superiority of the algorithm proposed in this article, the simulation has been done and compared with the search space size (SSS) algorithm in this article. Set $N_T = 3$, $M_1 = 10$, $M_2 = 20$, and $T_1 = T_2 = 3$. When each externally circulates, there is the same initial value $P_u^*$. Furthermore, in order to show the superiority of this algorithm underwater, the simulation compares the PDACS algorithm with the...
underwater GA algorithm. The change curve of the sum of the cross-correlation sum values of the orthogonal pilots obtained in turn as time increased is shown in Figure 4(a). As time increased, the cross-correlation sums also increased. The cross-correlation sums in the SSS algorithm are 0.3039, 0.3060, and 0.3146; the cross-correlation sums in the underwater GA algorithm are 0.3045, 0.3110, and 0.3249; and the cross-correlation sums in the PDACS are 0.2880, 0.2951, and 0.2971. Obviously, the cross-correlation sums in the PDACS outperform those of the underwater GA and SSS. The reason is that the PDACS algorithm proposed in this article uses a tree structure to select the optimal nodes when replacing each element of the pilot. Compared with the SSS algorithm, the PDACS algorithm can search for pilots using more pilot combinations; in addition, the PDACS algorithm fixes the pilot distribution and reduces the cross-correlation of the sampling matrix according to the pilot design. Therefore, the PDACS algorithm proposed in this article can continue to converge and search for pilots with the minimum cross-correlation sum. Meanwhile, the simulation shows that compared with the underwater GA algorithm, the GA algorithm used in Jiang et al.\textsuperscript{16} is for a single antenna pilot design, which does not consider the problem of the cosparsity channel in the case of underwater MIMO. Moreover, the GA algorithm is greatly affected by the parameter settings of the algorithm itself, and the running time is difficult to control. Therefore, the cross-correlation sum of the GA algorithm is worse than that of other algorithms. Although the running time of the PDACS algorithm is 600 s, which is approximately four times that of the SSS algorithm, the pilot is designed to run offline before the communication system is designed, and so the running time of the algorithm can be ignored within the acceptable range. Especially in an underwater MIMO system, by obtaining the optimized fixed pilot information in

Figure 3. BELLHOP ray model of a UWA multi-path channel. (a) Schematic diagram of ray propagation. (b) Measured SSP values versus the depths. (c) Predicted transmission loss versus the depths and ranges with the source at a depth of 30 m. (d) Deterministic CIR for a multi-path sparse channel.
advance, the pilot search time can be greatly reduced, energy consumption can be saved, and the channel estimation accuracy can be improved at the same time.

The underwater MIMO with $N = 256$ subcarriers is simulated to verify the cross-correlation sum of various pilot allocation methods, as shown in Figure 4(b). As with the simulation of $N = 512$ subcarriers, the cross-correlation sum of $N = 256$ in the PDACS algorithm is better than that of other algorithms. The reason is as mentioned above.

To verify the performance of the proposed optimized pilot time-domain correlation channel estimation, this article compares the four pilot schemes, including the LS, GA, SSS and PDACS algorithm, using the channel estimation MSE and BER. In the simulation, the sparse channel is randomly generated, the non-zero tap position of the channel is uniformly distributed, and the non-zero fading coefficient is normally distributed as $N(0, 1)$. The MSE of the MIMO system is defined as expression (13). It uses the estimated channel value $\hat{h}_{n}^{(a)}(i_T, i_R)$, the real channel value $h_{n}^{(a)}(i_T, i_R)$, and the simulation times $N_{\text{sim}}$ of each signal-to-noise ratio (SNR) of the $n$th simulation of the $i_T$th transmitting antenna and the $i_R$th receiving antenna, respectively

$$\text{MSE} = \frac{1}{N_{\text{sim}} N_T N_R} \sum_{n=1}^{N_{\text{sim}}} \left( \sum_{i_T=1}^{N_T} \sum_{i_R=1}^{N_R} \left\| \hat{h}_{n}^{(a)}(i_T, i_R) - h_{n}^{(a)}(i_T, i_R) \right\|_2^2 \right)^{1/2}$$

The simulation sets the number of time-domain correlation channels as 2, each SNR is simulated 600 times, and zero forcing equalization detection is adopted. The BERs are as shown in the figure, where Figure 5(a) is the BER with 256 subcarriers, and Figure 5(b) is the BER of 512 subcarriers. The simulations show that the BER of the proposed PDACS algorithm outperforms the LS, GA, and SSS with a lower BER when the maximum SNR is reached. SSS follows the PDACS, and GA is inferior to SSS but superior to LS. The pilot design method in this article has a smaller channel estimation error rate, and as the SNR increases, the performance advantage of the PDACS in this article is increasingly more obvious. When the SNR is 30 dB, the SNR is reduced by approximately 2.3 dB, and when the

![Figure 4](image-url-1)

(a) Comparison of pilot searching algorithms ($N = 256$). (b) Comparison of pilot searching algorithms ($N = 512$).

![Figure 5](image-url-2)

(a) Mean squared error versus SNR ($N = 256$, $N_T = 14$, and $L = 60$). (b) Mean squared error versus SNR ($N = 512$, $N_T = 14$, and $L = 60$).
same 0.03 error rate is achieved, the SNR is approximately 7 dB lower. This is because the sampling matrix obtained by the proposed PDACS in this article has a smaller cross-correlation. The distance of the different sparse signals in measurement space is ensured, and the sparse channel can be estimated more accurately. The simulation shows that the traditional LS channel estimation method cannot accurately get the channel because of the small number of pilots. Compared with the SSS and GA, the proposed PDACS algorithm uses a method with a fixed pilot position and tree search, and so it has less cross-correlation and achieves a low channel estimation error rate. When the number of pilot subcarriers is 256, the proposed PDACS algorithm has the lowest BER, as shown in Figure 5(a).

As shown in Figure 6(a) and (b), respectively, as the number of subcarriers changes from 256 to 512, the maximum MSE gain of the proposed PDACS algorithm in this article is 18 dB.

Considering the pilot cost, the proposed PDACS algorithm in this article has a smaller channel estimation BER and MSE. Therefore, under the condition of requiring the same channel estimation performance, compared with the LS, GA and SSS, the method proposed in this article can further reduce the pilot cost and reduce the complexity, energy loss and hardware cost of the receiver and transmitter design.

Conclusion

In this article, time-domain correlation channel estimation based on CS in an underwater MIMO system is studied. To solve the problem of poor sparse signal reconstruction performance caused by the random distribution of pilots, an optimized fixed pilot distribution and pilot design are proposed. Based on the CS theory, a common sparse channel sampling matrix is constructed. To minimize the cross-correlation sum, a tree structure sequential replacement search algorithm is proposed to solve the pilot position. The simulation results show that compared with the LS algorithm, the proposed PDACS algorithm can obtain an 18-dB channel MSE gain at most. Compared with the LS, GA, and SSS algorithms, when the SNR is 30 dB, the SNR is reduced by approximately 2.3 dB, and when the BER is 0.03, the SNR is reduced by approximately 7 dB.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported, in part, by the following projects: the National Natural Science Foundation of China through the grants 61861014, the Guangxi Nature Science Fund (2015GXNSF AA139298 and 2016GXNSFAA380226), the Guangxi University high-level innovation team and outstanding scholar program, the Guangxi Science and Technology Project (AC16380094, AA17204086, and 1598008-29), the Guangxi Nature Science Fund Key Project (2016 GXNSFDA380031), and the Guangxi University Science Research Project (ZD 2014146).

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References

1. Radosevic A, Duman TM, Proakis JG, et al. Selective decision directed channel estimation for UWA OFDM systems. In: Proceedings of the 49th annual Allerton conference on communication, control, and computing (Allerton), Monticello, IL, 28-30 Sep 2011, pp.647–653. New York: IEEE.
2. Yun LY, Su Y, Zhigang J, et al. The partial power control algorithm of underwater acoustic sensor networks based on outage probability minimization. *Int J Distrib Sens Netw* 2016; 12(7): 5363724.

3. Wen M, Xiang C, Yang L, et al. Index modulated OFDM for underwater acoustic communications. *IEEE Commun Magazine* 2016; 54(5): 132–137.

4. Yang Z and Zheng YR. Iterative channel estimation and turbo equalization for multiple-input multiple-output underwater acoustic communications. *IEEE J Ocean Eng* 2016; 41(1): 232–242.

5. Tao J and Zheng YR. Turbo detection for MIMO-OFDM underwater acoustic communications. *Int J Wireless Inf Ntw* 2013; 20(1): 27–38.

6. Rusek F, Persson D, Lau BK, et al. Scaling up MIMO: opportunities and challenges with very large arrays. *IEEE Signal Pr Magazine* 2013; 30(1): 40–60.

7. Zhu XD, Wang JT, Dai LL, et al. Sparsity-aware adaptive channel estimation based on SNR detection. *IEEE T Broadcast* 2015; 61(1): 119–126.

8. Qi C, Wu Y and Zhu P. Sparse channel estimation and pilot optimization for cognitive radio. *J Electr Inform Tech* 2014; 36(4): 763–768.

9. Qi C, Yue G, Wu L, et al. Pilot design schemes for sparse channel estimation in OFDM systems. *IEEE Trans Veh Technol* 2015; 64(4): 1493–1505.

10. Candes EJ, Romberg J and Tao T. Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. *IEEE Trans Inform Theory* 2006; 52(2): 489–509.

11. Candes J, Romberg JK and Tao T. Stable signal recovery from incomplete and inaccurate measurements. *Commun Pure Appl Math* 2007; 19(5): 410–412.

12. Qi CH and Wu LN. Uplink channel estimation for massive MIMO systems exploring joint channel sparsity. *Electron Lett* 2014; 50(23): 1770–1772.

13. Dai LL, Wang JT, Wang ZC, et al. Spectrum- and energy-efficient OFDM based on simultaneous multi-channel reconstruction. *IEEE Trans Signal Pr* 2013; 61(23): 6047–6059.

14. Choi JW, Shim B and Chang S. Downlink pilot reduction for massive MIMO systems via compressed sensing. *IEEE Commun Lett* 2015; 19(11): 1889–1892.

15. Yun HX, Fang SR and Ping ZW. Pilot allocation for distributed compressed sensing based sparse channel estimation in MIMO-OFDM systems. *IEEE Trans Veh Tech* 2016; 65(5): 2990–3004.

16. Jiang R, Wang X, Cao S, et al. Joint compressed sensing and enhanced whale optimization algorithm for pilot allocation in underwater acoustic OFDM systems. *IEEE Access* 2019; 7(1): 95779–95796.

17. Kennedy J and Eberhart R. Particle swarm optimization. In: *Proceedings of ICNN’95—international conference on neural networks*, Perth, WA, 27 November–1 December 1995, pp.1942–1948. New York: IEEE.

18. Karaboga D and Basturk B. A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (ABC) algorithm. *J Global Optim* 2007; 39(3): 459–471.

19. Yang XS. A new metaheuristic bat-inspired algorithm. *Nature Inspired Cooperat. Strategies Optim* 2010; 284: 65–74.

20. Mirjalili S, Mirjalili SM and Lewis A. Grey wolf optimizer. *Adv Eng Softw* 2014; 69: 46–61.

21. Mirjalili S and Lewis A. The whale optimization algorithm. *Adv Eng Softw* 2016; 95: 51–67.

22. Wang H, Guo Q, Zhang GX, et al. Pilot pattern optimization for sparse channel estimation in OFDM systems. *IEEE Commun Lett* 2015; 19(7): 1233–1236.

23. Tropp JA. Greed is good: algorithmic results for sparse approximation. *IEEE Trans Inform Theory* 2004; 50(10): 2231–2242.

24. Holland JH. Genetic algorithms. *Sci Amer* 1992; 267(1): 66–73.