Scattering of Hot Excitons in Quantum Wells

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Abstract

The scattering probabilities of hot excitons in narrow quantum wells (QWs) are obtained. The exciton-phonon matrix element is considered by using an envelope function Hamiltonian approach in the strong quantization limit where the QW width is smaller than the exciton bulk Bohr radius. The Fröhlich-like interaction is taken into account and the contribution of the confined and interface modes to the scattering probability are calculated as a function of quantum well width, electron and hole effective masses, and in-plane center-of-mass kinetic energy. Inter- and intra-subband excitonic transitions are discussed in term of the phonon scalar potential selection rules. It is shown that even parity electrostatic potential states for confined and interface modes give the main contribution to the excitonic scattering rate. The consequences of exciton relaxation and scattering probability present for the multiphonon resonance Raman scattering in narrow QWs are briefly discussed.
I. INTRODUCTION

In this work we investigate the hot exciton scattering rates in semiconductor narrow quantum wells (QWs). An exciton with kinetic energy higher than the thermal temperature of the medium is called a hot exciton. This concept has been introduced in the past when multiphonon LO-lines have been observed in resonant Raman scattering in bulk semiconductor materials (see [1] and references therein). For polar semiconductors when the exciton energy is greater than a longitudinal optical phonon, the scattering rate is governed by the exciton-LO phonon interaction and the Fröhlich like mechanism is the stronger one playing the main role in the exciton relaxation time.

In the past and for the bulk semiconductor case, transition probabilities of hot excitons interacting with longitudinal and acoustic phonons have been carried out in Refs. [2-4]. The dependence on exciton kinetic energy, effective electron and hole masses, and on the excitonic quantum numbers were obtained, showing the conditions that favored the hot excitons contribution to the multiphonon Raman process in semiconductors.

The importance of hot exciton relaxation via optical phonon emission in QWs has been stressed by photoluminescence experiments [5] and by time-resolved Raman scattering measurements [6]. Also, the relative role of confined phonons and interface modes in time resolved resonant Raman spectroscopy experiments is pointed out in Ref. [7]. Today, there are several theoretical studies of carrier relaxation times in QWs considering the main mechanism due to electron-LO-phonon interaction (e.g. Ref. [8] and references therein). Nevertheless, a direct calculation of the hot exciton scattering rate due to the confined optical phonons in a QW is lacking in the literature.

In this paper we will focus on the excitonic scattering rate calculation and on its contribution to the multiphonon resonant Raman process (MPR) in narrow QWs. Mowbray et al [9] reported a multiphonon process in GaAs/AlAs superlattices. Raman spectra for the second and third order processes from GaAs phonons show structures involving the combination of confined and interface LO phonons. The relative intensities of the second and
third order processes due to the combination of $L_2$ confined phonon are weaker than that obtained with $L_2$ and interface phonons for samples of 6-GaAs monolayer. The enumeration of phonon symmetries in the QW follows Ref. [10]. For the case of 10-GaAs monolayer samples, the combined intensities of $L_2$ and $L_4$ phonons are comparable to the intensities due to $2L_2$ or $3L_2$ ones. These results show that the relative intensities of a MPR scattering process are very sensitive to the quantum well width. The connection of the MPR process with excitonic transitions followed when the excitonic cascade model is invoked to explain the main features of the scattered light [1]. According to this model, the scattering cross section with an emission of one emitted phonon is proportional to the exciton scattering rate $W(E)$ with kinetic energy $E$. Thus, an explicit calculation of $W(E)$ as a function of the QW width and phonon mode contribution will bring an further understanding of MPR process in QWs as we will discuss bellow.

II. SCATTERING PROBABILITY

The scattering probability per unit time of an exciton in a QW is given by the Fermi “golden rule”

$$W = \frac{2\pi}{\hbar} \sum_{f} |M|^2 \delta(E_f - E_i),$$

(1)

where $E_i$ and $E_f$ are the energies at the initial and final states, $M$ is the matrix element describing the scattering of the hot exciton. For the case we are dealing with, the transitions are due to the interaction between excitons and LO-phonons. In a QW the interaction Hamiltonian is given by

$$H_{int} = \sum_{n,p,q} C_F \left[ \phi_{n,p,q}(z_e) e^{i\mathbf{q} \cdot \rho_e} - \phi_{n,p,q}(z_h) e^{i\mathbf{q} \cdot \rho_h} \right] \hat{b}_{n,p,q} + H.c.,$$

(2)

where $\hat{b}_{n,p,q}$ is the LO-phonon creation operator with in-plane wavevector $\mathbf{q}$ and frequency $\omega_{n,p,q}$, $\rho_e(\rho_h)$ and $z_e(z_h)$ are the electron (hole) coordinates in the X-Y plane and along the growth direction, respectively, and $C_F$ is the Fröhlich constant. To describe the confined phonons and interface modes we follow the phenomenological treatment given in Refs.
The function $\phi_{n_p, q}$ shows a definite parity and the corresponding analytical expression for phonon dispersion relation for odd and even potential states have been considered in Ref. [11]. The phonon dispersion for a structure of GaAs/AlAs with 2.0 nm and 1.7 nm well width are shown in Fig. 1. The branches are labeled as $L_N$ and $T_N$ ($N=1,2,...$) according to their longitudinal and transverse character at $q = 0$. The modes which show a strong dispersion as a function of the in-plane wavevector $q$ are those with the strongest electric field component and also the strongest interface character. For the 2.0 nm QW the two interface branches are $L_1$ and $L_4$ modes, while in the case of Fig. 1(b) these corresponds to $L_1$ and $T_1$ modes. The upper mode is purely antisymmetric ($L_1$) while the lower interface mode ($L_4$ and $T_1$ in Figs. 1(a) and (b), respectively) is symmetric. The antisymmetric part of the potential ($L_1$, $L_3$ modes) couples to the exciton via inter-subband transitions with exciton wavefunctions having different symmetry with respect to the center of the well. The symmetric part of the function $\phi_{n_p, q}$ ($L_2$, $L_4$ modes) couples to excitons via intra-subband transitions since the exciton wave function have the same symmetry respect to Z-axis. The exciton inter-subband transition is also possible through the symmetric phonon potential part, if the initial and final excitonic states have the same parity with respect to the center of the QW.

For the excitonic states we assume a Wannier-Mott model in the quantum well in the strong quantization limit (very narrow QWs). In this case, the QW width is smaller than the exciton Bohr radius $a_B$ ($d << a_B$) and the three-dimensional Wannier exciton motion can be reduced to two-dimensional Wannier exciton and one-dimensional electron and hole motion along the growth direction [13]. Hence, the wave function $\Psi_{exc}(r_e, r_h)$ can be cast in the form

$$\Psi_{exc}(r_e, r_h) = \Psi_{N,m}(R, \rho) \Phi_{n_e}(z_e) \Phi_{n_h}(z_h),$$

(3)

where $R$ is the electron-hole pair center-of-mass coordinates on the X-Y plane, $\rho$ is the in-plane relative coordinates. The electron and hole motion along the Z-axis is described by $\Phi_{n_e}(z_e)$ and $\Phi_{n_h}(z_h)$ wavefunctions ($n_e, n_h = 1, 2, ...$). The function $\Psi_{N,m}(R, \rho)$ corresponds
to the two-dimensional exciton wavefunction with radial quantum number \( N \) and angular quantum number \( m = 0, \pm 1, \pm 2, \ldots \) with the bound state energies equal to

\[
\epsilon_N = -\frac{R_y}{(N + 1/2)^2} \quad N = 0, 1, \ldots
\]  

(4)

where \( R_y \) is the effective Rydberg constant. The total exciton energy is

\[
E = E_g + E_{n_e} + E_{n_h} + \epsilon_N + \frac{\hbar^2 K^2}{2M}. 
\]

(5)

\( E_g \) being the bulk gap energy, \( E_{n_e}(E_{n_h}) \) is the electron (hole) energy in the QW along the Z-direction, and \( \frac{\hbar^2 K^2}{2M} \) is the exciton kinetic energy of the center-of-mass in the X-Y plane. In our case where \( d << a_B \) the exciton motion can be decoupled from the Z-component and the three-dimensional Wannier exciton is taken in separable form according to Eq. (3) by the product of the wavefunction in the plane \( \Psi(R, \rho) \) and the electron and hole subbands \( \Phi_n(z_i) \) \( (i = e, h) \) wavefunctions. The 3D Coulomb interaction effect on the total exciton energy (5) could be treated by perturbation theory with the Hamiltonian

\[
H_p = \frac{e^2}{\epsilon} \left[ \frac{1}{|\rho_e - \rho_h|} - \frac{1}{|r_e - r_h|} \right] 
\]

(see Ref. [14]).

Substituting Eqs. (3) and (2) in Eq. (1), one can find that the transition probability due to the interaction with the phonon branch \( n_p \) as a function of the exciton wave number \( K \) is given by

\[
W^{(n_p)}_{N,N',m,m';n_e,n_h} = \alpha_0 \int_0^\infty dq \int_0^{2\pi} d\theta_1 \times
\]

\[
|I_{N,m}^{N',m'}(Q_h) < \Phi_{n_e} |\phi_{n_p,q} | \Phi_{n_e'} > \delta_{n_e,n_e'} - I_{N,m}^{N',m'}(Q_e) < \Phi_{n_h} |\phi_{n_p,q} | \Phi_{n_h'} > \delta_{n_h,n_h'}|^2 \times
\]

\[
\delta \left( \frac{\hbar^2 q^2}{2M} - \frac{\hbar^2 K q}{M} \cos \theta_1 + \hbar \omega_{n_p,q} + E_{n_e'} - E_{n_e} + E_{n_h'} - E_{n_h} + \epsilon_{N'} - \epsilon_N \right),
\]

(6)

where \( \theta_1 \) is the angle between \( \mathbf{K} \) and \( \mathbf{q} \), \( Q_{h(e)} = m_h(m_e)a_B q/M \) with \( m_h(m_e) \) being the hole (electron) mass, \( \alpha_0 = e^2 \omega_L(\epsilon_{1,\infty}^{-1} - \epsilon_0^{-1})/d, \) \( \omega_L \) is the LO bulk phonon frequency, and \( \epsilon_{1,\infty}(\epsilon_0) \) is the high (static) frequency dielectric constant. In the above expression only the
phonon emission Stokes process has been considered \((T \rightarrow 0K)\). It is clear that obtaining scattering probability considering the phonon absorption process is straightforward if the phonon occupation number \(N(q) = \left[ e^{\frac{\hbar \omega_{np,q}}{k_B T} - 1} \right]^{-1} \) is introduced in the r.h.s. of Eq. (1) and \(\hbar \omega_{np,q} \) is substituted by \(-\hbar \omega_{np,q} \). The first term in the r.h.s of Eq. (1) is due to the phonon emission by the electron with matrix elements \(I^{N',m'}_{N,m}(Qh) \) in the X-Y plane and \(\Phi_{n_e} | \phi_{np,q} | \Phi_{n'_e} \rangle \) along the Z-axis, while the second term is the hole contribution to the scattering probability. The transition matrix elements \(I^{N',m'}_{N,m} \) and \(\Phi_{n} | \phi_{np,q} | \Phi_{n'} \rangle \) are given elsewhere [13]. The energy and quasi-momentum conservation laws impose restrictions on the wave number \(q\) within the range \(q_{min} \leq q \leq q_{max}\), where \(q_{min}\) and \(q_{max}\) are solutions of the equation

\[
f_{\pm}(E_K) = q^2 + 2 \frac{M}{a_B} \sqrt{\frac{M}{\mu} \sqrt{E_K q}} + \frac{M}{a_B^2 \mu R_y} (\hbar \omega_{np,q} + E_{n'_e} - E_{n_e} + E_{n'_h} - E_{n_h} + \epsilon_{N'} - \epsilon_N) = 0.
\]

Here, \(\mu\) is the reduced mass, \(E_K = \hbar^2 K^2 / 2M\) is the in-plane exciton kinetic energy and the condition \(q_{min} \geq 0\) must be fulfilled.

Integrating with respect to \(\theta_1\) and summing over \(m\) and \(m'\), we obtain from Eq. (6) the transition probability

\[
W^{(np)}_{N,n_e,n_h \rightarrow N',n'_e,n'_h}(E_K) = \alpha_0 \frac{2M}{\hbar^2} \int_{q_{min}}^{q_{max}} \frac{dq q}{\sqrt{f_{+}(E_K) f_{-}(E_K)}} \times
\]

\[
\sum_{m,m'} | e^{im-m'| \theta_0 | I^{N',m'}_{N,m}(Qh) < \Phi_{n_e} | \phi_{np,q} | \Phi_{n'_e} \rangle < \delta_{n'_e,n_h} - I^{N',m'}_{N,m}(Qh) < \Phi_{n_h} | \phi_{np,q} | \Phi_{n'_h} \rangle < \delta_{n'_h,n_e} |^2
\]

where

\[
\tan(\theta_0) = 2 \sqrt{f_{+}(E_K) f_{-}(E_K) / (f_{+}(E_K) + f_{-}(E_K))}.
\]

From the energy and quasi-momentum conservation laws it follows that the scattering probability is different from zero if the exciton kinetic energy \(E_K > \hbar \omega_{np,q} + E_{n'_e} - E_{n_e} + E_{n'_h} - E_{n_h} + \epsilon_{N'} - \epsilon_N\).
In Fig. 2, we illustrate the content of Eq. (8), showing the types of one phonon Stokes scattering processes included in the scattering rate. Each exciton band is characterized by single electron and hole quantum numbers \( (n_e, n_h) \) and the total quantum number \( N \): two excitonic bands are shown including dispersion due to center-of-mass motion as in Eq. (5).

For an exciton excited to the initial state labeled A in Fig. 2 three processes are allowed in principle:

(a) intra-subband scattering, with phonon potential state of even parity \( n_p = 2, 4 \) and \( n_e = n'_e, n_h = n'_h \),

(b) inter-subband transition where \( n_e \neq n'_e \) and \( n_h \neq n'_h \) with even parity phonon potential if \( |n_e - n'_e| = \text{even} \) or/and \( |n_h - n'_h| = \text{even} \), and for the odd parity potential case \( (n_p = 1, 3) \) if \( |n_e - n'_h| = \text{odd} \) or/and \( |n_e - n'_h| = \text{odd} \), finally

(c) inter- intra-subband scattering with \( n_e = n'_e \) but \( n_h \neq n'_h \) and viceversa. Here, if \( |n_h - n'_h| \) is even the phonon potential is an even function, while for \( |n_h - n'_h| \) equal to an odd number, the phonon potential states need to be odd giving a zero electron contribution to the scattering probability.

However, in the approximation used in this paper, only two processes will occur: (a) and (c), due to the separation of motion in the X-Y plane and the orthogonal Z direction. This is illustrated in Fig. 2, where single phonon inter- intra-subband scattering are shown by arrows \( A \rightarrow B', A \rightarrow C' \) defining the corresponding \( q_{\text{min}} \) and \( q_{\text{max}} \) for these processes.

The exciton intra-subband transitions are coupled to the symmetric part of the phonon potential, while the exciton inter-subband scattering (with mixed character or not) couples to the symmetric or antisymmetric part of the Fröhlich Hamiltonian interaction.

Hence, the total inverse exciton lifetime characterized by the quantum numbers \( N; n_e, n_h; E_K \) and due to the interaction with an optical phonon \( n_p \) can be written as

\[
W^{(n_p)}_{N; n_e, n_h} (E_K) = W^{(n_p)}_{N; n_e, n_h \rightarrow N; n_e, n_h} (E_K) + \sum_{N' \neq N} W^{(n_p)}_{N; n_e, n_h \rightarrow N'; n_e, n_h} (E_K) \\
+ \sum_{n_e \neq n'_e, n_h \neq n'_h; N'} W^{(n_p)}_{N; n_e, n_h \rightarrow N'; n_e, n_h} (E_K)
\]  

The first term in the r.h.s of Eq. (10) represents the intra-band scattering probability
without changes in the internal state of the exciton branches. The second term is the probability of the intra-band scattering accompanied by transitions from $N$ to other internal quantum states $N'$, while the last term gives the inter-subband contribution to the exciton lifetime in the branch $n_e, n_h$ with internal quantum number $N$ and in-plane kinetic energy $E_K$.

In the next section we study the dependence of the scattering rate on the in-plane exciton kinetic energy, on the phonon state $n_p$, and on the electron and hole effective masses.

### III. DISCUSSION AND CONCLUSIONS

For the numerical calculation of the scattering rate given by Eq. (8) we selected the GaAs/AlAs parameters given in [11]. We used for the masses the in-plane mass $m_{h\perp} = m_0/(\gamma_1 \pm \gamma_2)$ and along Z-direction $m_{hz} = m_0/(\gamma_1 \mp 2\gamma_2)$, where $\gamma_1$ and $\gamma_2$ are the Luttinger parameters [15]. The sign (+) or (-) corresponds to the light or heavy character of the mass, respectively. In Fig. 3 we show the dependence of the scattering rate on $E_K$ in the $n_e = 1, n_h = 1$ excitonic branch and for the excitonic ground state $N = 0 \rightarrow N = 0$, $W_{0\rightarrow0}^{(n)}$. Fig. 3(a) corresponds to a QW with a well width $d=2$ nm while Fig. 3(b) is for $d=1.7$ nm. The calculation represents the heavy hole contribution along the quantum well growth direction (light hole character in the X-Y plane) due to the GaAs-like phonon modes $L_2$ and $L_4$ ($n_p = 2$ and $n_p = 4$). Since we are dealing with intra-band transitions ($n_e = n_h = 1$ for the final and initial states) the exciton-phonon interaction with odd modes is forbidden. It can be seen in Fig. 3(a) ($d=2$ nm) that the main contribution corresponds to the $L_4$ mode and it is almost 3 times stronger than the $L_2$ mode. Figure 3(b) shows opposite effect where the $L_4$ phonon mode is one order of magnitude weaker than the $L_2$ one. To explain the above result we must to go back to the phonon dispersion relation of Fig. 1. For the $d=2$ nm QW the $L_2$ confined mode is almost flat while the $L_4$ phonon has a stronger dispersion as a function of the phonon wavenumber $q$ and it is in addition the symmetric QW phonon which has a predominantly interface character. The electrostatic potential associated with
the $L_4$ mode in Fig. 1(a) gives strong coupling with excitons by the Fröhlich interaction in intra-subband transitions. The modes with largest $\phi_{n_p,q}$ component in Eq. (8) are those which have the largest interface contribution, explaining why the scattering rate assisted by the $L_4$ phonon in Fig. 3(a) is stronger than that due to the $L_2$ confined mode. In the case of $d=1.7$ nm the $L_4$ GaAs-type phonon is more confined and it moves to lower frequency with an almost flat dispersion relation (see Fig. 1(b)). For the range of $q$ values where the phonons are flat the Fröhlich interaction is proportional to $1/\sqrt{q^2 + q_z^2}$ with $q_z = n_p \pi d / d (n_p = 1, 2, ...)$ \[16\]. Hence, the contribution to the intra-band scattering rate of the $n_p = 2$ mode is four times stronger than for the $n_p = 4$.

Let us comment on the effect of the carrier effective masses on the scattering rate. First, the confinement is a function of the hole masses along the Z-axis; Second, the hole-phonon matrix element depends on the hole masses through the $Q_h = m_h/M qa_B$ factor and the range of values $q_{min} \leq q \leq q_{max}$ is a function of the electron-hole mass (in plane along the Z-axis); Third, the scattering probability value strongly depends on the in-plane density of states. Fig. 3(c) shows the intraband exciton scattering rate for the case of the light-hole mass along the Z-direction (heavy-hole character in X-Y plane) and $d=2$ nm. As in Fig. 3(a) the $L_4$ mode contribution is stronger than the $L_2$, showing that the relative intensity between several phonon branches is only a function of the phonon dispersion which itself depends on the QW width. The scattering rate values, in units of the constant $W_0$, for the light hole exciton contribution is smaller than the heavy-hole exciton one (see Figs. 3(a) and 3(c)). It is important to note that for the case we consider of isotropic effective masses and $m_e = m_h$ the intra-band transition $W_{0 \to 0}^{(n_p)}$ is identically zero \[2\].

In conclusion we have performed an analysis of the exciton scattering rate in narrow QWs. The relaxation of hot excitons accounts for the emission of confined and interface optical longitudinal phonons due to the Fröhlich-like exciton-phonon interaction. The results obtained above show a different behavior of the scattering probability as a function of the well width and in-plane exciton kinetic energy. The value of $W_{0 \to 0}^{(n_p)}$ is not equal to zero for $E_K > \hbar \omega_{n_p,0}$ (for the case of intra-subband transition), increases with increasing $E_K$.
reaching a maximum and beginning to fall according to the law $E_K^{-1/2}$ [17]. The relative strength of the different phonon modes is proportional to the ratio of the square of their overlap integral of the function $\phi_{n,p,q}$ with initial and final electron and hole subband states in the GaAs/AlAs QW. For a given phonon state, inter-subband transitions are less efficient processes on the exciton relaxation time in comparison with the intraband probability.

The intra-subband scattering rate is accompanied only by symmetric phonon states ($n_p = 2, 4, ...$) in correspondence with the observed multiphonon Raman spectra in a series of short-period GaAs/AlAs superlattices [9]. The reported multiphonon peaks involving pure overtones of GaAs phonons are combinations of the even confined modes (see Fig. 6 in Ref. [9]). For samples with 10 GaAs monolayer width ($d \approx 2$ nm) combination of $L_2$ and $L_4$ (such as $2L_2$, $2L_4$, $3L_2$, $L_2 + 2L_4$, etc) are observed in the GaAs phonon spectra. In the case of samples with the 6 GaAs monolayer well width ($d \approx 1.7$ nm) only the combination of $L_2$ and interface GaAs modes ($2L_2$, $3L_2$, $L_2 + I$) are reported. Following the idea of the cascade model, where the iterated exciton-one-phonon interaction is the dominant mechanism, the MPR cross-section will be proportional to the $W_{0 \rightarrow 0}(E_K)$ factor [18] (the inter-band exciton transitions give very small contribution and have been neglected). The obtained results shown on Fig. 3(b) predict that in a 6 monolayers sample, the MPR spectra needs to be composed mostly by combinations of $L_2$ confined modes while the combinations with $L_4$ must be almost forbidden. The opposite conclusion is obtained from Fig. 3(a) ($d=2$ nm) when combinations of $L_2$ and $L_4$ GaAs phonon modes in the multiphonon Raman spectra should present similar relative intensities.

Thus, the experimental observation in GaAs/AlAs superlattices on the relative intensities of multiphonon scattering can be explained in the framework of a MPR cascade model taking into account the relative role of the confined and interface phonons on the hot exciton scattering rate. Future work on the present issue is in progress.
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Figure Caption

Figure 1. Dispersion of the GaAs optical phonons of the 2.0 nm (a) and 1.7 nm (b) GaAs/AlAs quantum well. The phonons are labeled as longitudinal (L) and transverse (T) according to its character at $q = 0$. Symmetric phonon potentials correspond to $L_2$ and $L_4$, and antisymmetric phonon potential to $L_1$ and $L_3$. The interface modes are those showing strong dispersion as a function of $q$. The anticrossing of $L_4$ and $T_1$ modes near $q = 0.5 \times 10^6$ 1/cm is clearly seen in figure (a). $L_4$ and $T_1$ modes present the same parity respect to center of the QW.

Figure 2. Schematic representation of allowed scattering processes. See text, section II, for discussion.

Figure 3. Exciton intra-subband scattering rate probability in units of $W_0 = 2\omega_L(\epsilon_0/\epsilon_\infty - 1)dM/(a_B\mu)$ in a narrow QW as a function of the in-plane center-of-mass exciton kinetic energy. (a) and (b) correspond to the heavy-hole mass along the QW growth direction with $d=2$ nm ($W_0 = 1.415 \times 10^{12}$ 1/sec) and 1.7 nm ($W_0 = 1.203 \times 10^{12}$ 1/sec), respectively. (c) represents the light-hole mass along Z-direction for $d=2$ nm ($W_0 = 2.199 \times 10^{12}$ 1/sec). The $L_2$ and $L_4$ phonon modes according to the notation followed in Fig. 1 have been considered for the one-phonon assisted exciton scattering rate.
(a) GaAs/AlAs
2.0nm
(b) 1.7nm
Exciton Scattering Rate ($W_{n' \to 0}/W_0$) vs. Exciton Kinetic Energy (meV) for GaAs/AlAs Heavy Hole Exciton with $d=2\text{nm}$.
Exciton Scattering Rate \( \frac{W_{0 \rightarrow 0}}{W_0} \)

(b) Heavy Hole Exciton
\( d=1.7 \text{nm} \)
Exciton Scattering Rate \( \left( \frac{\nu_{0 \rightarrow 0}}{W_0} \right) \)

\( L_2 \)

\( L_4 \)

(c) GaAs/AlAs

Light hole

Exciton

d=2nm