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While the hydraulic fracturing technology, aka fracking (or fraccing, frac), has become highly developed and astonishingly successful, a consistent formulation of the associated fracture mechanics that would not conflict with some observations is still unavailable. It is attempted here. Classical fracture mechanics, as well as the current commercial softwares, predict vertical cracks to propagate without branching from the perforations of the horizontal well casing, which are typically spaced at 10 m or more. However, to explain the gas production rate at the wellhead, the crack spacing would have to be only about 0.1 m, which would increase the overall gas permeability of shale mass about 10,000×. This permeability increase has generally been attributed to a preexisting system of orthogonal natural cracks, whose spacing is about 0.1 m. But their average age is about 100 million years, and a recent analysis indicated that these cracks must have been completely closed by secondary creep of shale in less than a million years. Here it is considered that the tectonic events that produced the natural cracks in shale must have also created weak layers with nano- or micro-cracking damage. It is numerically demonstrated that a greatly enhanced permeability along the weak layers, with a greatly increased transverse Biot coefficient, must cause the fracking to engender lateral branching and the opening of hydraulic cracks along the weak layers, even if these cracks are initially almost closed. A finite element crack band model, based on recently developed anisotropic spherocylindrical microplane constitutive law, demonstrates these findings.

Permeability | Biot Coefficient | Seepage forces | Damage

Significant advances have been made in fracture mechanics of propagation of a single hydraulic crack in elastic rock under tectonic stress (1–11). They include characterization of the stress singularity at the tip of a water-filled advancing crack, flow of water of controlled viscosity along the crack, with or without proppant grains, and water leak-off into the shale. Interactions of parallel cracks, their stability, closing, and stress-shadow effect, have also been clarified (12–15). Discrete element models used in most commercial softwares, in which the hydraulic crack was simulated by a band of inter-element separations (16, 17), led to similar results.

These studies, however, predicted no branching of the hydraulic cracks, originally spaced at cca 10 m. This presented a dilemma since branching is the only way to reduce the crack spacing to about 0.1 m, which is necessary to explain the gas production rate. Consequently, it has been universally hypothesized that the preexisting natural cracks, spaced at cca 0.1 m, would somehow increase the overall permeability of the shale mass. A 10,000-fold increase of permeability would be necessary to match the gas production rate. But recent analysis (18, 19) showed that the natural, tectonically produced, cracks, which are on the average about 100 million years old, must have been closed by secondary creep (or viscous flow) of shale under tectonic stress within 10,000 to 1 million years (if not filled earlier by calcite deposit). This invalidated the hypothesis.

It might be objected that water in the cracks could have prevented crack closing. But the open spaces in shear cracks, created (due to shear dilatancy) by a tectonic event, could not have been filled by water immediately. If the water had to seep in from the ground surface, it would take about 10 million years, if from a nearby water-filled rock formation, certainly over a million years. This must have left plenty of time for the creep closing to proceed uninhibited.

A recent paper (20) presented a new model which, by contrast with all the previous studies, took into account: 1) the seepage forces (i.e., the body forces due gradients of pore pressure in Darcy diffusion of water into porous shale), and 2) the variation of effective Biot coefficient for the water pressure on the crack plane, caused by gradually vanishing bridges between the opposite faces of a widening bridged crack (another difference from the previous studies was to abandon the assumption of incompressibility of water in the cracks, since water is about 20-times more compressible than shale). This model (20) did predict extensive lateral crack branching.

Later analysis, however, showed that the branching indicated by the computer program in (20) was, in fact, triggered by the unintended coding of a sudden change of Biot coefficient for transverse water pressure on the crack. This change abruptly increased the water pressure on the solid phase and

Significance Statement

Development of a realistic model of fracturing would allow better control. It should make it possible to optimize various parameters such as the history of pumping, its rate or cycles, changes of viscosity, etc. This could lead to an increase of the percentage of gas extraction from the deep shale strata, which currently stands at about 5% and rarely exceeds 15%.

Rahimi-Aghdam did the simulations and wrote the first draft, Bažant designed the research and edited the draft. All the other coauthors contributed by discussions.

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where \( K \) is a datum. However, the permeability \( K \) potential calculated as, \( q = (q_x, q_y, q_z) \) may be written as

\[ q = -\mu^{-1} K \cdot \nabla \psi \]  

[1]

where \( \nabla \) is the vector of gradient operator; and \( K \) is the \( 3 \times 3 \) permeability matrix, which is diagonal if (and only if) the cartesian axes \( x, y, z \) are chosen to be parallel and normal to the bedding planes.

If the change in Biot coefficient is made gradual, the model from the previous study (20) would predict no crack branching, although the branching must occur to explain the observed gas production rate. This study will show that if the previous model (20) is enhanced by introducing, into the shale mass, significant heterogeneity due to damaged weak layers along preexisting natural cracks, then an extensive and dense crack branching is predicted.

It may be noted that the fracking companies are aware of the necessity of branched cracks running along preexisting natural fractures. Fig. 1 shows a picture similar to what is found on the websites of some companies. However, this awareness seems to be merely intuitive and empirical.

Fluid flow in porous solid, without or with cracking damage. Two types of flow play a role in hydraulic fracturing: 1) The fluid flow in porous solid, without or with cracking damage. The widths of cracks normal to \( x, y, z \) directions along with the cracks may be calculated as

\[ Q_x = -\frac{h_x^2}{12\mu} \nabla_x p, \quad Q_y = -\frac{h_y^2}{12\mu} \nabla_y p, \quad Q_z = -\frac{h_z^2 + h_y^2}{12\mu} \nabla_z p \]  

[2]

where \( \nabla_x = \partial / \partial x, \ldots \); \( h_x, h_y \) = opening widths of vertical cracks normal to axes \( x \) and \( y \) that are positioned into the bedding plane.

An effective way to simulate the hydraulic cracks numerically is the crack band model (25–27), in which cracking deformation is considered meared over the band (or element) width. The widths of cracks normal to \( x \) and \( y \) are:

\[ h_x = l_x e^{xx}, \quad h_y = l_y e^{yy} \]  

[3]

(Fig.2) where \( e^{xx}, e^{yy} \) = damage parts of normal strains due to smeared cracking normal to \( x \) and \( y \) directions; \( l_x, l_y \) = crack band widths, assumed equal to the minimum possible spacing of adjacent parallel hydraulic cracks (\( l_x, l_y \) must be treated as a material property, related to fracture energy \( G_f \) of shale; here \( l_x, l_y \) are not changed but if they were the postpeak would have to be adjusted to preserve \( G_f \)). Furthermore,

\[ e^{ij} = \epsilon^{ij} - \epsilon^{el}, \quad \epsilon^{el} = C_{ijkl} \sigma_{kl} \]  

[4]

where \( C_{ijkl} = \) transversely isotropic elastic compliance tensor of shale (for unloading); \( \sigma_{ij}, \epsilon_{ij} \) are the stress and strain tensors in the rock, calculated from a constitutive model for smeared cracking damage (with a localization limiter (25)), for which the spherocylindrical microplane constitutive model (28) has been used. The coordinates are Cartesian, \( x, i = 1, 2, 3 \) (\( x_1 \equiv x, x_2 \equiv y, x_3 \equiv z \)). Note that, the same as in (20), water is
considered as compressible. It is, in fact, about 20-times more compressible than concrete, and the water pressure during fracking can be high (up to 70 MPa).

**Equilibrium in two-phase solid and Biot coefficient.** The shale may be modeled as a two-phase medium with water-saturated pores, for which the classical Biot-type relations for the equilibrium of the phases apply. For undamaged shale, they read:

\[ S_{ij} = \sigma_{ij} - \delta_{ij}b_0p \]  

where \( p \) = pore pressure, \( b_0 \) = Biot coefficient of undamaged shale; \( S_{ij} \) = total stress tensor; \( \sigma_{ij} \) = stress tensor in the solid phase, and \( \delta_{ij} \) = Kronecker delta. As a special case, \( S_V = \sigma_V - b_0p \) where \( S_V = S_{kk}/3 = \) volumetric total stress, and \( \sigma_{ij} = \sigma_{kk}/3 = \) volumetric stress in the solid phase.

While typically \( \nu = 0.1 \), the Biot coefficient of shales can vary between 0.2 and 0.7. Test results (29-34) show that it increases with the cracking damage and depends on the load direction. This requires generalizing the Biot coefficient as a tensor, \( b_{ij} \) (35). The following, tensorially consistent, empirical relation, which appears to match test data, is proposed:

\[ b_{ij} = \min \left\{ b_0 + \beta \epsilon''_{ij} (\epsilon''_{kk}/3)^{-2/3}, 1 \right\} \quad (\varphi \leq b_0 \leq 1) \]  

Here \( b_0 \) refers to undamaged material, \( \beta = \) empirical parameter, \( \epsilon''_{ij} \) is the inelastic damage strain tensor, and \( \varphi = \) natural porosity of the shale. For the Biot coefficient in the direction of unit vector \( \nu_i \), this equation gives \( b_{ij} = \nu_i \nu_j b_{ij} = b_0 + \beta \epsilon''_{ij} (\epsilon''_{kk}/3)^{-2/3} \) (but \( \leq 1 \)), where \( \epsilon''_V = \epsilon''_{kk}/3 = \) inelastic relative volume expansion, and \( \epsilon''_{ij} = \nu_i \nu_j \epsilon''_{ij} = \) inelastic normal strain component in direction of vector \( \nu_i \).

For the special case of micro- or nano-cracking normal to \( \sigma_1 \) direction only, one has \( \epsilon''_V = \epsilon''_{11}/3 \) and \( b_{ij} = b_0 + \beta (9\epsilon''_{11})^{1/3} \) (but \( \leq 1 \)). This equation can be interpreted graphically as seen in Fig. 2b, which shows section A-A of a band of preexisting, mostly aligned, microcracks and the compressive stresses applied by the pore fluid onto the microcrack faces, resisted by tensile stresses in the ligaments of the solid between the microcrack tips.

The viscous drag of water flowing through a soil imposes a seepage force on the soil in the direction of flow. The seepage forces are body forces defined as

\[ f_s = \nu \nabla p \]  

They are applied on the porous solid and must be balanced by stresses in the solid. Seepage in an upward direction reduces the effective stress within the soil. When the water pressure at a point in the soil is equal to the total vertical stress at that point, the effective stress is zero and the soil has no frictional resistance to deformation (36, 37). They have long been considered in geotechnical engineering to assess the risk of sand liquefaction in cofferdams (38, 39) or under dams. However (except for (20)) they have been ignored in previous studies of hydraulic fracturing, although they do play a crucial role in crack branching. A poromechanical finite element (FE) code for a two-phase solid automatically takes the seepage forces into account in the form of nodal forces.

**Two-phase finite element (FE) simulations for a single damage band.** To clarify the role of nano- or micro-cracking, consider first a horizontal two-dimensional (2D) square block of shale of dimensions 1.1 m × 1.1 m, supported at the sides by springs approximately equivalent to an infinite medium, as shown in Fig. 3a. Water is injected at the center of south side at the constant rate of 2 m³/s. The anisotropic spherocylindrical microplane model, with the default parameters of shale given in (28), is used as the constitutive model; and \( l_x = l_y = 2.1 \) m. The initial Biot coefficient is \( b_0 = 0.4 \). The tectonic stresses are \( T_x = -30 \) MPa, \( T_y = -30 \) MPa.

Consider that there is a **single preexisting band of nano- or micro-cracks predominantly aligned with axis y, represented by the two red elements in Fig. 3a** (which is what remains after a crack was closed by up to a million years of secondary creep, or viscous flow). These cracks cause the vertical permeability in these two elements to increase cca 1000-times compared to undamaged shale, while the Biot coefficient increases up to 1 and the initial strength decreases to 10% of intact shale.

Figs. 3b,c show how damage and pressure propagate after water injection. For this case, the crack band with high water pressure is seen to propagate straight forward, without branching. Now look at stress variation. Fig. 3d shows the stress evolution within in the solid part of the first element above the initial damaged elements. Obviously, the damage during post-peak softening is captured in a stable manner. Finally, consider how the Biot coefficient and permeability vary in one cracked element (the first above the initial damaged elements). Fig. 3e contrasts the evolution of Biot coefficient in the transverse direction with its constancy in the forward direction, which agrees with experimental observations.

**Do they seepage forces suffice to induce crack branching?**? It is well known in classical fracture mechanics that pressurizing a crack cannot produce tension along the crack faces, and thus cannot initiate lateral crack branching (branching is possible only at the tip of a crack propagating at nearly the Raleigh wave speed). In a preceding study (20), it was surmised, under
various simplifications, that the seepage forces (Eq. 7) would suffice for produce tension along the crack face and thus initiate lateral crack branching. Let us examine it more rigorously. Consider again a horizontal 2D square domain 2.5 m × 2.5 m, containing one line crack (Fig. 4a). By virtue of symmetry, only a half-domain is simulated (Fig. 4b). The water pressure in the line crack is gradually ramped up to reach the maximum of 50 MPa. Water diffusion from the pressurized crack into the shale is simulated via Darcy law. First we neglect the increase of Biot coefficient due to damage (β = 0). Fig. 4c shows that the damage, as well as the crack, propagates only in the direct extension of the initial line crack, i.e., there is no branching. Fig. 4d shows the evolution of stress in the solid part, σ_{xx}, along the crack face. The results show that the Biot coefficient can have a major effect and cannot be ignored.

Lateral crack branching would happen if the stress in the solid phase became positive (tensile) and attained the tensile strength of shale. The results show that this cannot happen, regardless of the tectonic stress value (even if vanishing).

Nevertheless, the seepage forces reduce the magnitude of compressive stress along the crack face significantly.

We must thus conclude (as an update of (18)) that the seepage forces alone do not suffice to explain and model lateral branching of hydraulic cracks. So, what other phenomena could explain the lateral branching? Not surprisingly, the explanation is the natural (preexisting) fractures even though they must have been completely closed due to millions of years of secondary creep, or flow. We demonstrate it next.

**Hydraulic crack branching in two-phase porous solid with closed natural fractures.** In Fig. 5a we now consider the same 2D domain of two-phase porous solid as before, except that now there are two natural weak layers (or preexisting damage bands) in both x and y directions. The crack is uniformly pressurized and water diffuses out. The transverse Biot coefficient within the weak layers that represent the closed natural fractures is b_{nat} = 1 because the weak layer (or natural fracture) may consist of separate original crack faces in contact (uncemented by limestone deposit), while in the intact shale the b_{ij}-values increase according to Eq. (6) from the initial value b_0 = 0.4 (Fig. 5d) or 0.2 (Fig. 5d).

Fig. 5b reveals that the hydraulic crack tends to propagate simultaneously along the initial crack and along the weak layer. This confirms that branching can occur if transverse weak layers exist. Further, consider the normal stress parallel to the crack in one element of the weak layer. If this stress attains the tensile strength, a lateral crack branch can initiate and shale branching can happen. Fig. 5b shows the spreading of high and lower transverse strains along both weak layers for the case of Biot coefficient b = 0.4, with permeability K_{weak} along the weak band 5-times bigger than K_0 for intact shale.

The computed effect of ratio K_{weak}/K_0 on the σ_{xx} evolution in the first element of the weak layer above initial crack is plotted in Fig. 5d for the initial Biot coefficients, b_0 = 0.4 and 0.2. As water diffuses into the shale, the stress in the weak layer increases from negative to tensile values until it finally reaches the tensile strength of the weak layer. Evidently, a greater difference in Biot coefficient between the weak layer and the shale facilitates, and speeds up the crack branching.

Finally, to clarify the effect of the transverse tensile strength of the weak layer, three relative strength S_{rel} values are considered in Fig. 5e (here S_{rel} is the damaged-to-intact strength ratio of shale). As seen, a smaller S_{rel} leads to smaller stress, but generally, the effect of S_{rel} is almost negligible. Hence, whether or not the natural cracks are cemented by limestone is almost irrelevant.

It is instructive to see the evolution of the seepage force vectors acting on the mesh nodes, as portrayed in Fig. 5. Fig. 5f shows schematically the seepage forces acting on an ellipse around the crack. Fig. 5g-i illustrates the evolution of seepage forces. Their orientations make it intuitively clear that they must produce in the porous solid a biaxial tension.

From all these observations, it transpires that a major stimulus for crack branching is the difference in the Biot coefficient and in the permeability between the weak layers and the intact shale, as well as the shale mass heterogeneity due to the alternation of weak layers and intact porous solid.

It is worth mentioning that the expansion of solid due to the effect of Biot coefficient has been thought to prevent any tension parallel to the crack face, and thus cause the closing of any lateral crack. The preceding results show that this
Fig. 5. a) Schematic line crack with weak layer and spring boundaries; b) Strain and damage evolution; c) stress in solid part of one element in weak layer considering a shale with $b_0 = 0.4$ and different permeabilities for the weak layer; d) stress in solid part of one element in weak layer considering $b_0 = 0.2$ and different permeabilities for the weak layer; e) stress in solid part of one element in weak layer considering $b_0 = 0.4$ and different relative strength values for weak layer; f) schematic of seepage forces; and g-i) evolution of seepage forces.

Fig. 6. FE simulation of hydraulic crack branching in a small domain of shale with several orthogonal weak layers; (a) Ring of elastic elements providing elastic support of the boundaries; b) FE mesh, preexisting natural weak layers, and fracking water inlet; c-f) evolution of pressure in a shale with weak layers.

1. The natural fractures have a major effect on hydraulic fracturing and are crucial for its success (although they are currently neglected by the commercial fracking softwares).
2. Even though the natural fractures must have been closed by millions of years of creep, or sealed by mineral deposits, a weak layer of nano- and micro-cracks along these fractures must be expected to facilitate water diffusion.
3. Poromechanics with Biot coefficient depending on the damage of the solid phase must be used in the analysis of fracking.
4. Increase of the Biot coefficient in the transverse direction, caused by oriented cracking damage inflicted by fracking, is essential to achieve crack branching.
5. The typical spacing between natural fractures is roughly 0.1 m. This value matches the spacing of hydraulic cracks that is necessary to explain the typical gas production rate at the wellhead.
6. The widespread opinion that preexisting natural fractures somehow explain why the overall permeability of shale mass, inferred from the gas production rate, appears to be about 10,000 times higher than what is measured on
shale cores in the laboratory, has been basically correct. But these fractures are completely closed, do not convey any gas and their role is indirect.

7. a) No porosity ⇒ no branching. b) No seepage forces ⇒ no branching. c) No weak layers ⇒ no branching. d) Constant Biot coefficient ⇒ no branching. (Note: consequently, branching in granite is impossible)

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