Geometric Optimization of Thermo-electric Coolers Using Simulated Annealing

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Abstract. The field of thermo-electric coolers (TECs) has grown drastically in recent years. In an extreme environment as thermal energy and gas drilling operations, TEC is an effective cooling mechanism for instrument. However, limitations such as the relatively low energy conversion efficiency and ability to dissipate only a limited amount of heat flux may seriously damage the lifetime and performance of the instrument. Until now, many researches were conducted to expand the efficiency of TECs. The material parameters are the most significant, but they are restricted by currently available materials and module fabricating technologies. Therefore, the main objective of finding the optimal TECs design is to define a set of design parameters. In this paper, a new method of optimizing the dimension of TECs using simulated annealing (SA), to maximize the rate of refrigeration (ROR) was proposed. Equality constraint and inequality constraint were taken into consideration. This work reveals that SA shows better performance than Cheng’s work.

Nomenclature

| Symbol | Description                        | Unit |
|--------|------------------------------------|------|
| \textit{STEC} | Single-stage thermo-electric cooler |      |
| $L$    | Height of the confined volume       | mm   |
| $A$    | Cross-sectional area of STEC legs   | mm$^2$ |
| $N$    | Number of thermo-electric couple    | -    |
| $ROR$  | Rate of refrigeration               | W    |
| $COP$  | Coefficient of performance          | -    |
| $T_h, T_c$ | Hot side and cold side temperature | $^\circ$ K |
| $T_{ave}$ | Average of cold and hot side temperature | $^\circ$ K |

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α_n = α_p  Seebeck coefficient of n-type and p-type thermo-electric element  \(V/K\)

ρ_n = ρ_p  Electrical resistivity of n-type and p-type of thermo-electric element  \(Ωm\)

κ_n = κ_p  Thermal conductivity of n-type and p-type of thermo-electric element  \(W/m°K\)

r_c  Electrical contact resistance  \(Ωm^2\)

1. Introduction
Mud-Pulse High-Temperature Measurement While Drilling (MWD) is a system developed to perform drilling related measurements down-hole and transmit information to the surface while drilling a well [1]. In a typical down-hole environment, the temperature can reach 230°C. Therefore, maintaining optimal payload temperatures is very important and requires a cooling system which is capable of pumping a significant load and requires a low thermal resistance path on the heat rejection (hot side). It has been identified that this can be accomplished using thin-film thermo-electric cooling devices.

Thermo-electric coolers (TECs) (figure 1) are solid state cooling devices that use the Peltier effect through p-type and n-type semiconductor elements. These types of coolers are used to convert electrical energy into a temperature gradient. TECs use no refrigerant and have no dynamic parts which make these devices highly reliable and require low maintenance. However, for this application the most attractive feature of TECs is that they have the capacity for cooling instruments such as MWDs under extreme physical conditions [2]. Two parameters play crucial role in characterization of TECs are the ROR and COP. The main drawback of TECs is the poor COP and ROR [10]. Improvement on the performance of TECs can be achieved by means of developing new materials with a higher figure of merit (Z), optimizing module design and fabrication, improvement of the heat exchange efficiency and employment of multi-stages system [2]. Several intelligent techniques that can be used for engineering design optimization are discussed in Deb [3]. However, one of the most effective and non-traditional method used as an optimization technique for TECs is Non-dominated Sorting Genetic Algorithm [4]. Similar sophisticated techniques such as Simulated Annealing (SA), other evolutionary algorithms (Differential Evolution and Particle Swarm Optimization) can be used in their pure and hybrid form to enhance the effectiveness of the optimization of TECs [5, 6]. SA is a point-based meta-heuristic which has the ability to escape from local optima [7], flexibility and ability to approach global optimality [8]. SA is easy to code, even for complex system and can deal with highly nonlinear models with many constraints [9]. This study focuses on optimizing the design parameters of STEC using SA optimization technique, to create the maximum ROR under some defined constraints. The \(L, A, N\) of thermo-electric module were optimized.

2. Mathematical modelling of TECs
Operation of TECs is based on the Peltier effect. When a DC current passes through a pair of p- and n-type semiconductor materials, one side of the junctions is cooled and the other side is heated. The energy balance equations at the hot junction and the cold junction for TECs can be described as in equations (1) and (2) [13]. The input electrical power and coefficient of performance can be calculated.
using equations (3), (4). $\alpha, \rho, \kappa$ represent for thermo-electric material properties. $A, L, N$ are geometric properties. $COP$ is a common metric used to quantify the effectiveness of a heat engine [10, 11]. The material properties are considered to be dependent on the average temperature of the cold side and hot side temperatures of each stage and their values can be calculated using equations (5), (6), (7) [12].

\[
ROR = N \left[ \alpha I T_c - \frac{1}{2} I^2 \left( \frac{\rho_r}{A} + \frac{2r_c}{A} \right) - \frac{\kappa A(T_h - T_c)}{L} \right] 
\]

\[
Q_h = N \left[ \alpha I T_b + \frac{1}{2} I^2 \left( \frac{\rho_r}{A} + \frac{2r_c}{A} \right) - \frac{\kappa A(T_h - T_c)}{L} \right] 
\]

\[
P = Q_h - ROR \quad (3)
\]

\[
COP = \frac{ROR}{Q_h - ROR} \quad (4)
\]

\[
\alpha = \alpha_p = -\alpha_n = (-263.38 + 2.78 T_{ave} - 0.00406 T_{ave}^2) \times 10^{-6} \quad (5)
\]

\[
\rho = \rho_p = \rho_n = (22.39 - 0.13 T_{ave} + 0.00030625 T_{ave}^2) \times 10^{-6} \quad (6)
\]

\[
\kappa = \kappa_p = \kappa_n = 3.95 - 0.014 T_{ave} + 0.00001875 T_{ave}^2 \quad (7)
\]

3. **Optimization process**

This work used SA in optimizing the design parameters of STEC. The objective of the optimization calculation is to determine the optimal $L, A$ and $N$. One objective, namely maximizing $ROR$ of STEC is considered for single objective optimization. The design variables $A, L, N$ of STECs were put in an inequality constraint which was bound by upper and lower limits of the design variables and the total area $S$ of STEC. Additionally, STEC is put in some requirements which are the confined volume of STEC ($S$), the minimum requirement of $COP$ and the maximum cost of material [2]. Because optimization of STEC geometry may cause the reduction in the $COP$, the $COP$ is used as a constraint condition during the optimization in order to guarantee that the TECs with the optimal geometry has a relatively high $COP$ [10]. Referring to equations (1), (2), (3) and (4), the parameters $T_c, T_b, I$ are defined in the beginning of the calculation. The unknown term is material properties of TECs will be determined based on the equations (5), (6) and (7) with values of $T_b$ and $T_c$. SA is a method for solving constrained and bound-constrained optimization problems [11]. The method models the physical process of heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy.

4. **Simulation results**

Preliminary tests for finding the optimal value of geometric properties of STEC were conducted by using SA and run the system in MATLAB. Parameters selection of STEC were referred from [13] and was listed in Table 1. The purpose of this test is to understand the operation and benefits of SA in the optimization of STEC design, especially the stability and reliability of SA.

Table 1. Parameters setting of STEC.

| Group | Parameters setting       | Specific values               |
|-------|--------------------------|------------------------------|
| 1     | Objective function       | Maximize $ROR$               |
| 2     | Variables                | $0.05mm < L < 1mm$           |
|       |                          | $0.09mm^2 < A < 100mm^2$     |
1 < N < 1000

3 Fixed parameters

\[ S = 100\text{mm}^2 \]
\[ T_b = T_c = 323 \text{ °K} \]
\[ r_c = 10^8 \Omega \text{m}^2 \]

4 Constraints

- \( A.N < 100\text{mm}^2 \)
- Maximum cost USD 385
- Minimum requirement \( COP = 1 \)

| Parameters setting | Specific values |
|--------------------|-----------------|
| Initial temperature | 100             |
| Final temperature   | \(10^{10}\)      |
| Temperature reduction| 0.95            |
| Boltzmann annealing | 1               |
| Stopping criteria   | - Final stopping temperature |
|                     | - Function tolerance \(10^{-6}\) |

Table 2. Parameters setting of simulated annealing algorithm.

Table 3. Parameters setting of SA.

| Maximum fitness | Minimum fitness | Mean fitness | Standard deviation |
|-----------------|-----------------|--------------|--------------------|
| 9.975           | 8.663           | 9.589        | 0.386              |

SA code for programming was used and adjusted its parameters so that it can be used for finding the optimal design of TECs [14]. Choosing the good algorithm parameters is very important because it affects the whole optimization process [15]. Parameters setting of SA are listed in Table 2. Using this strategy, SA was run in 30 trials randomly on STEC system. The first case is for nonlinear inequality constraint \( A.N < 100\text{mm}^2 \), the second case is for equality constraint \( COP = 1 \). The best, average value and the lowest value of 30 trials were noted down to evaluate the performance of the algorithm such as reliability.

After testing, SA converged quickly in several seconds. As shown in figure 2, when running under nonlinear inequality constraint \( A.N < 100\text{mm}^2 \), SA converged quickly in several seconds and satisfied the constraint. The obtained results of 30 trials runs are nearly same. Maximum and minimum values of best fitness are collected from figure 2 and calculate the standard deviation (Table 3). A low standard deviation 0.386 indicates that data points tend to be very close to the mean. Therefore, the optimal results of SA are stable and reliable. SA is robust with nonlinear inequality constraint. Problem happened when running SA under nonlinear equality constraint \( COP = 1 \). As shown in figure 3, the maximum value of \( ROR \) is very small when compare with Cheng’s work [13]. The optimal dimensions of STEC were unable to satisfy the constraint, SA gets stuck at local optima. Finding optimal dimension of STEC when running SA under nonlinear equality constraint is not reliable. In the following step, STEC system was run with SA under the two nonlinear inequality constraints of total area \( A.N < 100\text{mm}^2 \), \( COP \) requirement was neglected. \( I \) and \( T_c \) of STEC were varied independently. Table 4 presents the optimal design variables for various input currents from 0.1\( A \) to 2\( A \). The maximum values of \( ROR \) seem unchanged when the input current is larger than 0.5\( A \). Table 4 demonstrates that TECs can reach its maximum \( ROR \) even under various inputs current. Table 5 presents the optimal dimension of STEC for various cold side temperatures from 293 °K to 330 °K. The maximum \( ROR \) increases when \( T_c \) is increased. The increasing of \( ROR \) is more obvious when cold side temperature exceeds hot side temperature. And as the cold side temperature is increased, the optimal leg area decreases and the number of thermo-element couple increases. In figures 4 and 5, the comparison with Cheng’s work [13] in same conditions are shown, SA obviously demonstrate better performance when SA can find good dimensions of STEC with bigger values of \( ROR \).
Figure 2. Run STEC using SA under nonlinear inequality constraint ($A, N < 100 \text{mm}^2$).

Figure 3. Run STEC using SA under nonlinear equality constraint ($COP = 1$).

Figure 4. Comparison between SA and GA [13] on maximum ROR under various input currents.

Figure 5. Comparison between SA and GA [13] on the maximum ROR under various cold side temperature.

Table 4. Optimal dimension of STECs under various input currents.

| $I$ (A) | $[A, L, N]$       | Max. ROR |
|--------|-------------------|---------|
| 0.1    | [0.099, 0.300, 1000] | 5.6     |
| 0.2    | [0.099, 0.300, 1000] | 8.86    |
| 0.5    | [0.183, 0.306, 546]  | 9.74    |
| 1      | [0.354, 0.300, 283]  | 9.79    |
| 2      | [0.700, 0.300, 143]  | 9.79    |
| 4      | [1.388, 0.300, 72]   | 9.79    |
| 6      | [2.081, 0.300, 48]   | 9.79    |
| 8      | [2.786, 0.300, 36]   | 9.79    |

Table 5. Optimal dimension of STECs under various input cold side temperature.

| $T_c$ ($^\circ\text{K}$) | $[A, L, N]$       | Max. ROR |
|-------------------------|-------------------|---------|
| 283                     | -                 | -       |
| 293                     | [0.517, 1.000, 194]| 1.65    |
5. Conclusion
The obtained results showed that simulated annealing performed very well when running system with nonlinear inequality constraint, but simulated annealing got stuck at local optimum when run the system with nonlinear equality constraint. Simulated annealing gave better optimal dimensions of STECs with bigger value of rate of refrigeration compared with genetic algorithm which was used in Cheng’s work.

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