NNLO contributions to $\varepsilon_K$ and rare kaon decays

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We discuss the theory prediction of $\varepsilon_K$ and the rare $K \rightarrow \pi \nu \bar{\nu}$ decays and review the structure and current status of higher-order contributions to these flavour changing processes in the standard model in some detail. This includes the next-to-next-to-leading order QCD calculation to the charm quark contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and to the charm-top quark contribution to $\varepsilon_K$. Electroweak corrections to the rare kaon decays are also discussed.

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1. Introduction

Rare decays of K-mesons as well as $K^0$-$\bar{K}^0$ mixing continue to play an important role in fixing parameters of the standard model (SM) and in constraining models of new physics. In the future $\varepsilon_K$, the parameter describing indirect CP violation in kaon mixing, and the decays $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \pi^0\nu\bar{\nu}$ will provide a decisive test of the SM and its extensions: they are highly sensitive to new physics [1] and their theory prediction is under good control for $\varepsilon_K$ and remarkably clean for $K \rightarrow \pi\nu\bar{\nu}$. In the SM these modes are dominated by internal top quark contributions proportional to powers of $V_{ts}^*V_{td}$ and as such are suppressed with respect to generic new physics scenarios by the near diagonality of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Also these modes can be calculated precisely using an effective theory framework. The matrix elements are extracted from $K_{I3}$ decays for $K \rightarrow \pi\nu\bar{\nu}$ [2], and from the lattice for $\varepsilon_K$ [3].

This leads to an exceptionally clean prediction for the rare $K \rightarrow \pi\nu\bar{\nu}$ decays, while the recent and expected progress for the lattice calculations of $B_K$, the bag parameter for $\varepsilon_K$, is quite remarkable.

2. Structure of $K \rightarrow \pi\nu\bar{\nu}$ at NLO

The theoretical cleanness of the $K \rightarrow \pi\nu\bar{\nu}$ decays in the SM is related to the quadratic Glashow-Iliopoulos-Maiani (GIM) mechanism. Using $\lambda_t = V_{ts}^*V_{td}$ and $x_t = m_t^2/M_W^2$ we can write the amplitude of the $Z$-penguin and electroweak box diagrams (Fig. 1) as

$$\lambda_t (F(x_t) - F(x_u)) + \lambda_c (F(x_c) - F(x_u)) = \mathcal{O} \left( \lambda^5 \frac{m_t^2}{M_W^2} \right) + \mathcal{O} \left( \lambda \frac{m_c^2}{M_W^2} \right) \ln \frac{m_c}{M_W} + \mathcal{O} \left( \lambda \frac{\Lambda_{QCD}^2}{M_W^4} \right) \ln \frac{m_c}{M_W^2}. \quad (2.1)$$

Here $\lambda_t F(x_t)$ is the top quark contribution, which is suppressed by five powers of the Cabibbo angle $\lambda = |V_{ts}|$, and $\lambda_c F(x_c)$ is the charm quark contribution. The contribution of soft internal up quarks is suppressed by $\Lambda_{QCD}^2/M_W^4$.

Related to the quadratic GIM mechanism is the fact that the low-energy effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi\sin^2\theta_W} \sum_{i=e,\mu,\tau} \left( \lambda_c X^i(x_c) + \lambda_t X^i(x_t) \right) (\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L) \quad (2.2)$$

involves only one single operator $Q_\nu = (\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$. The hadronic matrix element of the low-energy effective Hamiltonian can be extracted from the well-measured $K_{I3}$ decays, including isospin breaking and long-distance QED radiative corrections.

The anomalous dimension of the operator in (2.1) has no large logarithm and is calculated in fixed-order perturbation theory. The matching of the electroweak box and $Z$-penguin diagrams in Fig. 1 with internal charm quarks gives the charm quark contribution to the Wilson coefficient of $Q_\nu$. Next, one matches Green’s functions with internal $W$- and $Z$-Bosons and dimension-six current-current operators. The bilocal mixing into the dimension-eight operator $m_t^2 Q_\nu$ – see Fig. 1 – resums the large logarithm in (2.1). The GIM mechanism cancels all loop contributions which do not carry an explicit charm mass dependence. Only when integrating out the charm quark higher-dimensional operators appear. The matching
Figure 1: Left column: The Z-penguin and electroweak box give the matching contribution to the charm and top sector. Middle column: The mixing of the current-current operators into $m^2 Q_v$ and the self-mixing of the current-current operators. Right column: Integrating out the charm quark produces $m^2 Q_v$ and subleading higher dimensional operators.

onto $Q_v$ in Fig. 1 gives the dominant contribution to the branching ratio, while the contribution of the higher-dimensional operators can be computed together with the matrix element containing soft up quarks with the help of chiral perturbation theory ($\chi$PT) [4].

After extracting the matrix element of $Q_v$ from $K_{23}$ decays and summation over the three neutrino flavours the resulting branching ratio for $K \to \pi \nu \bar{\nu}$ can be written as [4, 5, 6]

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = \kappa_+ (1 + \Delta_{\text{EM}}) \left[ \left( \frac{\text{Im} \lambda}{\lambda^5} X(x_i) \right)^2 + \left( \frac{\text{Re} \lambda}{\lambda^5} X(x_i) + \frac{\text{Re} \lambda_s}{\lambda} (P_c + \delta P_{c,u}) \right)^2 \right],$$

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im} \lambda_s}{\lambda^5} X(x_i) \right)^2.$$

(2.3)

Here $\kappa_+ = 0.5173(25) \times 10^{-10} (\lambda / 0.225)^8$ and $\kappa_L = 2.231(13) \times 10^{-10} (\lambda / 0.225)^8$ contain higher-order electroweak corrections for the normalisation to the $K_{23}$ decays, and $\Delta_{\text{EM}} \simeq -0.3\%$ denotes long distance QED corrections [6]. The top quark $X(x_i) = 1.464 \pm 0.041$, computed at two-loop in Ref. [7], gives the only contribution to $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$, while its contribution to $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ is 63%. The perturbative part of the charm quark contribution at NLO [7] is

$$P_c^{\text{NLO}} = 0.364 \pm 0.036_{\text{theory}} \pm 0.009_{m_c} \pm 0.009_{\alpha_s},$$

(2.4)

for $\lambda = 0.2255$. The parametric uncertainty – see Ref. [8] for input parameters – is small compared to the theoretical error, which results from higher order corrections. In a $\chi$PT calculation [4] the contribution of higher dimensional operators and soft up quarks has been calculated to $\delta P_{c,u} = 0.04 \pm 0.02$. Using Eq. (2.3), Eq. (2.4), and the input parameters of Ref. [8] results in:

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})^{\text{NLO}} = (8.5 \pm 0.5_{\text{sd}} \pm 0.2_{\text{id}} \pm 0.6_{\text{param}}) \times 10^{-11},$$

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})^{\text{NLO}} = (2.7 \pm 0.1_{\text{sd}} \pm 0.04_{\text{id}} \pm 0.4_{\text{param}}) \times 10^{-11}.$$

(2.5)
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The subscript “sd”, “ld”, and “param” labels the perturbative, long-distance, and parametric uncertainties respectively. The parametric error is dominated by the CKM parameters, while $\delta P_{c,u}$ gives the largest contribution to the long-distance uncertainty of the charged decay mode.

3. Structure of $\varepsilon_K$ at NLO

The parameter

$$\varepsilon_K = \frac{\mathcal{A}(K_L \rightarrow (\pi\pi)_{I=0})}{\mathcal{A}(K_S \rightarrow (\pi\pi)_{I=0})}$$

measures CP violation in $K_0 - \bar{K}_0$ mixing via the ratio of the respective decay amplitudes of a $K_L$ and a $K_S$ decaying into a two pion state of isospin zero. Its generic structure shares features with the previously discussed $K \rightarrow \pi\nu\bar{\nu}$ decays: only one operator $Q_{S2} = (\bar{s}\gamma^\mu d_L)(\bar{s}\gamma^\mu d_L)$ contributes dominantly below the charm quark mass scale and long-distance effects are power suppressed, this time because of CP violation.

For the theoretical prediction it is useful to express $\varepsilon_K$ in terms of $\langle K^0 | H_{\Delta S=2}^{\text{eff}} | \bar{K}^0 \rangle = 2m_KM_{12}^*$, the matrix element of the $\Delta S = 2$ effective Hamiltonian, and write:

$$\varepsilon_K = e^{i\phi_e} \sin\phi_e \left( \frac{\text{Im}(M_{12}^*)}{\Delta M_K} + \xi \right).$$

Here the phase of $\varepsilon_K$ is $\phi_e = 43.5(7)^\circ$ [9] and $\xi = \text{Im}A_0/\text{Re}A_0 \simeq 0$ is the imaginary part divided by the real part of the isospin zero amplitude $A_0 = \mathcal{A}(K_S \rightarrow (\pi\pi)_{I=0})$. The ratio $\kappa_e = |\varepsilon_K^\text{SM}/\varepsilon_K(\phi_e = 45^\circ, \xi = 0)|$ encompasses the change of $|\varepsilon_K|$ if the values $\phi_e = 45^\circ$ and $\xi = 0$ are used in (3.2), as has been done in most of the older analyses, instead of the exact values. The authors of Ref. [10] used $\varepsilon'/\varepsilon$ to extract the value of $\kappa_e = 0.92 \pm 0.02$ in the SM.
After the GIM mechanism has been used to eliminate top quark contribution, via the bilocal mixing in Fig. 3a, yet do not induce large logarithms times tree-level Wilson coefficients proportional to $\lambda_t^2$ and $\lambda_c^2$. QCD corrections do not change this picture but only induce the well known RGE effects for the $\Delta S = 1$ effective Hamiltonian [11] and for the $\Delta S = 2$ Operator $Q_{S2}$ (Fig. 3b). A LO analysis of the charm quark and top quark contribution to $\varepsilon_K$ then requires a one-loop calculation both for the matching at $\mu_W$, for the running, and for the matching of the charm quark contribution also for the matching at $\mu_c$ (Fig. 3a). This is contrary to the charm-top quark contribution where a tree-level matching at $\mu_W$ and $\mu_c$ is sufficient at LO.

After integrating out the charm quark the $\Delta S = 2$ effective Hamiltonian reads

$$\mathcal{H}_{\text{eff}}^\Delta S=2 = \frac{G_F^2}{4\pi^2} M_W^2 \left[ \lambda_t^2 \eta_{tt} S(x_t) + \lambda_c^2 \eta_{ct} S(x_c) + \lambda_c^2 \eta_{ct} S(x_c, x_t) \right] b(\mu) Q_{S2} + \text{h.c.} + \ldots$$

(3.5)
where the QCD and logarithmic corrections are known at NLO and parametrised by \(\eta_{cc} = 1.43(23)\) \cite{12}, \(\eta_{ct} = 0.47(4)\) \cite{12}, and \(\eta_{lt} = 0.5765(65)\) \cite{13}. The parameter \(b(\mu)\) is factored out such that

\[
\hat{B}_K = \frac{3}{2} b(\mu) \frac{\langle K^0|Q_{52}|\bar{K}^0\rangle}{f_K m_K^2}\tag{3.6}
\]

is a renormalisation group invariant quantity, which can be calculated on the lattice – see e.g. \cite{14} and Ref.\cite{3} of this conference. Using \(\hat{B}_K = 0.720 \pm 0.013 \pm 0.037\) one finds for \(\varepsilon_K\) at NLO \cite{10}:

\[
\varepsilon_K^{\text{NLO}} = (1.78 \pm 0.25),
\]

where \(\eta_{lt}, \eta_{ct}, \) and \(\eta_{cc}\) contribute with 75\%, 37\%, and \(-12\%\) respectively to the total value of \(\varepsilon_K\), while 60\% of the uncertainty is of parametric origin and 40\% is of theoretical origin. The parametric error is dominated by the uncertainty in the CKM parameters, while the perturbative and non-perturbative uncertainties are comparable in size for the theory uncertainty.

Finally note that \(\mathcal{H}_\text{eff}^{\Delta S=2}\) also contains higher dimensional Operators in \(\mathcal{H}_\text{eff,de=8}\) and current-current operators with up-quarks in \(\mathcal{H}_\text{eff}^{\Delta S=1}\), as indicated by the ellipses in Eq. (3.5). At leading order in the \(1/N_C\) expansion only one higher-dimensional operator is present and its matrix element is estimated in \cite{15} to result in a 0.5\% enhancement of \(\varepsilon_K\).

4. NNLO QCD corrections

The NNLO calculation of \(\varepsilon_K\) and \(K^+ \to \pi^+ \nu \bar{\nu}\) aims at resumming all \(\mathcal{O}(\alpha_s^n \ln^{n-1}(\mu_W^2/\mu_\text{m}^2))\) logarithms for \(P_c\) and \(\eta_{ct}\) and all \(\mathcal{O}(\alpha_s^3 \ln^{n-2}(\mu_W^2/\mu_\text{m}^2))\) for \(X(x_t), \eta_{lt}\), and \(\eta_{cc}\). The theory uncertainty of \(P_c\) and \(\eta_{ct}\) dominates the perturbative error for \(K^+ \to \pi^+ \nu \bar{\nu}\) and \(\varepsilon_K\) respectively at NLO, while the large theory uncertainty in \(\eta_{cc}\) is somewhat suppressed by the smallness of the charm contribution to \(\varepsilon_K\).

A NNLO analysis for \(P_c\) and \(\eta_{ct}\) will reduce the theory uncertainties and comprises (i) the \(\mathcal{O}(\alpha_s^2)\) matching corrections to the relevant Wilson coefficients arising at \(\mu_\text{m}\), (ii) the \(\mathcal{O}(\alpha_s^2)\) anomalous dimensions describing the mixing of the dimension-six and the \(Q_5\) and \(Q_{52}\) operators, (iii) the \(\mathcal{O}(\alpha_s^2)\) threshold corrections to the Wilson coefficients originating at \(\mu_\text{b}\), and (iv) the \(\mathcal{O}(\alpha_s^2)\) matrix elements of the operators emerging at \(\mu_\text{c}\). To determine the contributions of type (i), (iii), and (iv) one must calculate two-loop Green functions in the full SM and in effective theories with five or four flavours. Sample diagrams for steps (i) and (iv) are shown in the left and right columns of Fig. 4. The contributions (ii) are found by calculating three-loop Green functions with operator insertions. Sample diagrams with a double insertion of dimension-six operators are shown in the centre column of Fig. 4. The corresponding three-loop amplitudes are evaluated using the method that has been described in \cite{11, 16}. A comprehensive discussion of the technical details of the matching, the renormalisation of the effective theory and the actual calculation is given in \cite{5} for the calculation of \(P_c\) and will be given in \cite{17} for the calculation of \(\eta_{ct}\). The same techniques have also been used to reduce the uncertainties in the short-distance contribution to \(K_L \to \mu^+ \mu^-\) \cite{18}.

The aforementioned QCD calculation results for the input parameters of Ref. \cite{8} in the following value for \(P_c\) at NNLO:

\[
P_c^{\text{NNLO}} = 0.368 \pm 0.009_{\text{theory}} \pm 0.009_{m_\mu} \pm 0.009_{\alpha_s}.	ag{4.1}
\]
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Figure 4: Examples of Feynman diagrams arising in the full SM (left column), describing the mixing of operators (centre column) and the matrix elements (right column) for the $Z$-penguin sector of $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ (upper row) and for $\varepsilon_K$ (lower row). Only the divergent pieces of the diagrams displayed in the centre column have to be computed, while the Feynman graphs shown on the left- and right-hand side are needed including their finite parts.

Comparing these numbers with Eq. (2.4) we observe that the NNLO calculation reduces the theoretical uncertainty by a factor of 4. Because of this and the improvement in the calculation of long-distance effects [2], unknown electroweak corrections could potentially dominate the theory uncertainty of the rare $K \rightarrow \pi \nu\bar{\nu}$ decays. Even though a similar reduction of the error for $\eta_{K1}$ is expected at NNLO in QCD [17] no electroweak corrections are needed for the present theoretical status of $\varepsilon_K$.

5. Electroweak corrections

The NLO calculation of electroweak corrections for rare $K \rightarrow \pi \nu\bar{\nu}$ decays resums all LO and NLO logarithmic QED corrections and fixes the scheme electroweak input parameters, like $\sin^2 \theta_W$, by an electroweak matching calculation. The function $P_c$ depends on the charm quark $(\overline{\text{MS}})$ mass through the parameter $x_c$, conventionally defined as $x_c = m_c^2/M_W^2$. The point of fixing the input parameters can be exemplified by noting that the charm quark contribution is mediated by a double insertion of two dimension-six operators. This results in a contribution of $\mathit{O}(G_F^2)$ – the second power of $G_F$ resides in $x_c$ – plus electroweak corrections. Yet the leading result of Eq. (2.2) can only approximate the electroweak corrections for a specific choice of the renormalisation scheme for the prefactor of the charm quark contribution, expressed as $\alpha/\sin^2 \theta_W$. While it is expected that using $(\overline{\text{MS}})$ parameters renormalised at the electroweak scale would approximate the electroweak corrections best [19], only an explicit calculation can provide a definite result. We normalise all dimension-six operators to $G_F$ and replace the parameter $x_c$ with the definition

$$x_c = \sqrt{2} \frac{\sin^2 \theta_W}{\pi \alpha} G_F m_c^2(\mu_c),$$

(5.1)
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Figure 5: Left column: Example of a diagram describing the NLO matching to a dimension-six operator involving charm quarks and neutrinos (top) and of a diagram contributing to the NLO mixing of two dimension-six operators into \( Q_\nu \). Right column: \( R_c(X) \) as a function of \( \mu_c \) at NNLO QCD (dashed dotted line), including LO QED (dotted line), and NLO electroweak corrections (solid line). The dashed line shows \( P_c(X) \) at NNLO QCD where the definition \( x_c = m_c^2/M_W^2 \) is used.

which only at tree level equals the ratio \( m_c^2(\mu_c)/M_W^2 \).

The NLO analysis of electroweak effects of Ref. [8] involves the calculation of one-loop matching corrections for the dimensions-six operators (top left diagram of Fig. 5) and QED corrections to the LO QCD operator mixing (bottom left diagram of Fig. 5), and the inclusion of QED effects in the expansion of the matrix elements at \( \mu_c \). Note that the LO QED corrections start at \( \mathcal{O}(\alpha^2 \ln^2(\mu_W^2/\mu_c^2)) \) while the first NLO electroweak correction is \( \mathcal{O}(\alpha^2 \ln(\mu_W^2/\mu_c^2)) \). This explains why \( P_c(x) \), which is plotted on the right column of Fig. 5 as a function of the parameter \( \mu_c \), receives corrections of similar size. Also the cancellation of the scheme dependence between the LO QED and the NLO electroweak contribution is clearly visible and we see that including the full electroweak corrections, \( P_c(X) \) is mildly increased as compared to the pure NNLO QCD. The number for the branching ratio then reads [8]:

\[
\mathcal{B} \left( K^+ \rightarrow \pi^+ \nu \bar{\nu} \right) = (8.51^{+9.57}_{-6.2 \text{ CKM}} \pm 0.20_{m_c} \pm 0.36_{\alpha_s} \pm 0.20) \times 10^{-11}.
\]

The CKM parameters dominate the parametric uncertainty. The main contributions to the theory error stem from the uncertainty in \( \delta P_{c,a} \) and \( X_t \), where we used an error of 2%. In detail, the contributions to the theory error are \( (\kappa^+ : 6\%, X_0 : 38\%, P_c : 17\%, \delta P_{c,a} : 39\%) \), respectively. All errors have been added in quadrature.

6. Conclusions

Rare \( K \rightarrow \pi \nu \bar{\nu} \) decays and the CP violating parameter \( \varepsilon_K \) are extremely sensitive to flavour violating new physics. The good control of long-distance contribution to these observables makes the calculation of NNLO QCD and sometimes even NLO electroweak corrections mandatory. Results for NNLO QCD and NLO electroweak corrections for the charm quark contributions to rare
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K decays have been published in Ref. [5, 8], while the NNLO calculation of the charm-top quark contribution is finished [17]. This, together with current [20] and future [21] progress from the experimental side, will increase the new physics reach of these observables further.

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