On the Limits of Byzantine-tolerant Spanning Tree Construction in Route-Restricted Overlay Networks

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Abstract—All nodes in route-restricted overlays have an immutable set of neighbors, explicitly specified by their users. Popular examples include payment networks such as the Lightning network as well as social overlays such as the Dark Freenet. Routing algorithms are central to such overlays as they enable communication between nodes that are not directly connected. Recent results show that algorithms based on BFS spanning trees are the most promising provably efficient choice. All suggested solutions, however, fail to address whether and how distributed spanning tree algorithms can deal with Byzantine nodes.

Our contribution is twofold. First, we show that there is no protocol for route-restricted networks that achieves global spanning tree algorithms can deal with Byzantine nodes. Second, we design a novel spanning tree construction algorithm based on cryptographic signatures that provably reduces the set of nodes affected by Byzantine attacks. Our simulations substantiate this theoretical result with concrete values based on real-world data sets. In particular, our results indicate that our algorithm reduces the impact of an attack to affect as much as 90% less honest nodes.

I. INTRODUCTION

Payment or state channel networks like Lightning [28] are the most promising approach to scaling blockchains, i.e., enabling blockchain-based payment systems to process tens of thousands of transactions per second with nearly instant confirmation. Participants in such payment networks establish channels for trading assets such as digital coins. As establishing channels requires use of the blockchain, which is both time- and cost-intensive, only nodes that frequently trade with each other establish payment channels [19]. All other payments pass from a sender to the receiver via multi-hop paths of channels. It is essential to find these paths in an effective, efficient, and privacy-preserving manner [30].

Similarly, social overlays require finding paths between two peers in a network consisting only of connections between trusted pairs of nodes for realizing scalable and privacy-preserving distributed services [9], [29].

Both payment channel networks and social overlays hence share the need for a routing algorithm. A number of promising routing algorithms for both networks rely on rooted spanning trees [24], [29], [30], or more generally Breadth-First-Search (BFS) spanning trees as these offer the shortest paths and hence the most efficient communication. Those routing algorithms find a path between two nodes if there is a path of non-compromised nodes in the spanning tree. Thus, it is key that the algorithm for constructing the spanning tree in a decentralized setting indeed results in a spanning tree that offers such paths between as many pairs of nodes as possible.

There are three goals that a spanning algorithm has to fulfill in the context of route-restricted networks: (1) enable efficient communication by providing short paths between non-faulty nodes in the spanning tree, (2) efficiently adapt to changes of the network’s structure, and (3) maintain high availability in the presence of Byzantine nodes that deliberately deviate from the construction protocol in order keep the network from converging.

Yet, the existing work does not evaluate these three aspects jointly. Indeed, it is not even clear if there exist dynamic spanning tree algorithms that can achieve convergence as neither payment channel networks nor social overlays satisfy any of the assumptions commonly made in traditional consensus algorithms, such as restrictions on the number of Byzantine nodes [21], [22] or that all nodes have the same degree [4].

In this work, we focus on achieving all three requirements jointly, giving rise to three key contributions:

• We prove that self-stabilizing Byzantine leader election protocols for arbitrary network topologies inherently cannot guarantee convergence to a global consensus on a single leader, even in the absence of Byzantine nodes.

• We present a self-stabilizing algorithm for the computation of a BFS spanning tree that uses cryptographic signatures to check the integrity of statements about the distance to the root node. We prove that the fraction of nodes reaching a stable, non-compromised state is higher than in state-of-the-art protocols.

• We present results from an extensive simulation study based on real-world data sets. The results demonstrate that the construction of BFS spanning trees without cryptographic measures is highly vulnerable to attacks, even if the adversary establishes just a handful of connections to honest nodes. Furthermore, we show that our algorithm significantly raises the necessary number of such attack connections to mislead a comparable number of nodes.

II. RELATED WORK

We shortly discuss Byzantine leader election in the context of choosing a root node in a spanning tree, followed by a discussion of attack-resistant spanning tree algorithms.
A. Byzantine Leader Election

Classical Byzantine leader election aims at the consensual choice of a single leader amongst all non-Byzantine nodes. Yet, this is only possible if the number of Byzantine nodes is less than half of the number of node-disjoint paths between every pair of non-Byzantine nodes and strictly less than a third of all nodes \([11]\). The open membership in route-restricted overlays enables Sybil attacks \([13]\), i.e., inserting an arbitrary number of Byzantine nodes. Hence, route-restricted overlays cannot realistically meet these requirements.

For Byzantine leader election in networks with weaker connectivity assumptions, Dwork et al. \([18]\) introduced the concept of almost-everywhere Byzantine agreement, meaning that all but a small fraction of poorly connected non-Byzantine nodes are guaranteed to decide on a common value. So far, all the work in the area of almost-everywhere Byzantine leader election \([4, 22, 33]\) assumes that the network is a regular graph or a complete graph \([22]\). Because route-restricted overlays are based on social relationships, they are likely to have an irregular, heavy-tailed degree distribution \([25]\). Consequently, the assumptions of existing work on almost-everywhere Byzantine leader election are also too strong.

Furthermore, all of the previously cited work assumes that the total number of nodes in the network is fixed and known to all nodes. The work of Augustine et al. \([4]\) considers dynamic networks but assumes that the network size always remains stable, such that a new node joins whenever another node leaves. Recent work weakens this requirement, but either ignores Byzantine nodes \([3]\) or assumes a dedicated certificate authority or secure key exchange out of band for all pairs of nodes \([1, 2]\).

In summary, all of the current work on Byzantine leader election relies on assumptions unsuited to route-restricted overlays.

B. Construction of Rooted Spanning Trees

We categorize the existing work on the distributed construction of rooted spanning trees in the presence of Byzantine nodes into two classes. The first class are algorithms for networks with a distinguished root node given a priori, leaving open how that root node is chosen. The second class are algorithms for networks without a given root node.

Dubois, Mazusawa, and Tixeuil proposed a spanning tree algorithm for networks with a distinguished, non-Byzantine root node. In particular, they proposed an algorithm so that those nodes whose hop distance (or other distance metrics \([17]\)) to the root node is lower than or equal to any Byzantine node will eventually reach and remain in a correct state \([16]\). However, it remains unclear which fraction of nodes actually has such a shorter path in real-world networks. Roos et al. \([29]\) designed algorithms to increase the success ratio of message delivery in tree-based routing for sources or sinks with malicious ancestors in the tree. Hence, their work is orthogonal to ours as it focuses on protecting against attacks performed after the construction of the spanning tree rather than during the construction.

Considering networks without a given root node, all existing work focuses on the time and message complexity \([3, 5, 10]\) of leader election and tree construction in the absence of Byzantine nodes. In summary, none of the existing work fully addresses Byzantine-resilient construction of rooted spanning trees in networks.

III. Model and Notation

We now formalize route-restricted overlays as well as their security and performance. In particular, we introduce concepts that grasp the relatively weak assumptions made in such overlays, e.g., restrictions on the number of connections between Byzantine and honest nodes rather than restrictions on the number of Byzantine participants. Our definitions rely on the foundation of self-stabilization, which inherently models network dynamics.

A. System model

We model a route-restricted overlay \(S = (V, E)\) as a finite set \(V\) of \(n\) nodes and a set of bidirectional communication links \(E \subseteq V \times V\). For each node \(u\), the set \(N(u) = \{v \mid (u, v) \in E\}\) denotes the neighbors of \(u\). Similar to \([16]\), we assume the shared memory model where each pair of nodes \((u, v) \in E\) can communicate via shared registers \(r_{uv}\) and \(r_{vu}\), where \(u\) is only allowed to write into \(r_{uv}\) and read from \(r_{vu}\). We thus call \(r_{uv}\)’s output register and \(r_{vu}\)’s input register. Please note that we use the shared memory model solely to simplify formal analysis, as it omits the modeling of message transmission, which we do include in our simulation models. We consider this to be reasonable, as we focus on Byzantine node behavior and neither link failures nor delays. Furthermore, there exist simple algorithms to transform distributed systems with message passing to the shared memory \([13]\) model, and back \([14]\).

We express the combined state information of a system, i.e., the internal state of nodes and the register contents, at one point in time as a configuration. Nodes may change their internal state and register contents over time. The corresponding sequence of configurations is called a computation. More formally, a configuration \(\gamma\) is a vector containing the current internal state as well as the content of the registers of every node in \(S\). The internal state of a node consists of the values currently stored within its variables. A computation \(\Gamma\) is a sequence of configurations \(\gamma_0, \gamma_1, \ldots\). The transition from \(\gamma_t\) to \(\gamma_{t+1}\) is called a step and corresponds to one or more nodes atomically executing a sequence of instructions, including one or more read and write operations. In other words, a transition includes at least one node processing the data in its input register and writing corresponding data into its output register.

We assume that the computation is always fair in the sense that all nodes will eventually read or write. This means that none of the participating nodes gets stuck in an infinite loop or deadlock.

Route-restricted overlays are dynamic such that nodes may join and leave the system and connections between nodes are established or torn down over time. During such an event, an overlay \(S = (V, E)\) changes into an overlay \(S′ = (V′, E′)\)
with a potentially different network size. We call such a change a *churn event*. In order to account for the fact that computations are by default defined on a fixed system $S$, a churn event interrupts a computation on $S$ and starts a new computation on $S'$. At the beginning of the new computation, all nodes in $V \cap V'$ have the same internal state as at the end of the computation on $S$, reflecting the fact that they cannot detect the change until they read from their registers. The remaining nodes in $V'$ may start in an arbitrary initial state. In route-restricted networks, the initial state includes information about the register of neighbors, which the new node will eventually write to.

The actual computation performed in an overlay is determined by a distributed algorithm that is designed to eventually satisfy a *problem specification*, such as spanning tree construction or routing a message to a node without a direct connection. Formally, a problem specification consists of one or more boolean expressions over the variables of each node. All variables that are included in the specification are called *output variables* and all other variables are called *local variables*. In the context of spanning tree construction, an output variable might be the address of the parent node while supplemental connection. Formally, a problem specification consists of one or more boolean expressions over the variables of each node. All variables that are included in the specification are called *output variables* and all other variables are called *local variables*. In the context of spanning tree construction, an output variable might be the address of the parent node while supplemental data such as a node-internal ordering of its neighbors is a local variable. If the state of a node satisfies the specification, we call this state a *legitimate state*. A configuration in which every non-Byzantine node has a legitimate state is called a *legitimate configuration*. An example for a legitimate configuration could be a spanning tree where all honest node only have honest ancestors.

To be able to reason about the time complexity that a distributed algorithm requires to reach a legitimate state, we use the concept of *asynchronous rounds*. The first *asynchronous round* of a computation $\Gamma$ is the shortest prefix $\Gamma'$ of $\Gamma$ such that each node has read from and wrote to all of its registers at least once. The second asynchronous round then is the first asynchronous round of the computation following $\Gamma'$ and so on. In other words, the length of an asynchronous round corresponds to the maximum amount of time needed for the slowest node (regarding computational speed) to process its inputs and write the corresponding outputs.

**B. Attacker model**

In this work, we consider an adversary that aims to perform a large-scale denial of service attack. For payment networks, the adversary might be a competing payment network operator that wants to attract more users by rendering other networks unusable. For social overlays, the adversary might leverage to weaken the privacy or degrade utility so that users move to communication services with weaker privacy protection.

We assume that the adversary controls a set $B$ of Byzantine nodes and is able to set up a bounded number of connections between Byzantine nodes $B$ and non-Byzantine (also called *honest*) nodes $H$. The motivation for these bounds is the difficulty of large-scale social engineering that will only be successful for a subset of participants. During an attack, each Byzantine node may report incorrect data to the adjacent honest nodes in order to keep them from reaching or remaining in a legitimate state. Thus, Byzantine nodes may report false topological information by simulating the arrival of a new node or the failure of an old one. Because honest nodes only have local knowledge about the network, they cannot clearly distinguish simulated churn events from real churn events.

However, we assume that the adversary does not know all honest nodes and their connections a priori and hence cannot optimize its attack in a global manner. In particular, the adversary cannot choose which nodes will be Byzantine or connect to Byzantine nodes. Given that social overlay and payment networks are large-scale and dynamic distributed systems with participants from a multitude of countries, we consider this assumption to be realistic.

For all practical purposes, the Dolev-Yao model, which assumes an adversary with full control over the medium who nevertheless is limited to polynomial-time attacks – and hence unable to break secure cryptographic primitives – has been accepted as realistic [12]. Hence, we aim for computationally secure algorithms, i.e., algorithms that protect against adversaries that are polynomially bounded.

**C. Topology-aware self-stabilization**

To formalize the convergence of a distributed algorithm in the presence of Byzantine behavior, we adopt the notion of *topology-aware (TA) strong stabilization* proposed by Dubois et al. [16], as it does not require any assumptions about the number of malicious entities or the structure of the network. Similarly to almost-everywhere Byzantine agreement, topology-aware stabilization implies that a subset of nodes achieve consensus. In contrast to almost-everywhere Byzantine agreement, the subset is usually clearly defined in terms of structure, e.g., all nodes within a certain distance of a Byzantine node.

More formally, TA strong stabilization for a set $S_B \subset H$ requires that every honest node $u$ except those in the set $S_B$ eventually reaches a legitimate state $\gamma^+(u)$. The key idea is that nodes outside of $S_B$ might initially act based on incorrect information provided by Byzantine nodes but will eventually disregard such information. For instance, $u \notin S_B$ might initially accept a Byzantine node’s claims to be the root node in a distributed spanning tree. However, once it receives a message from the correct root node, it will change accordingly. Note that we do allow for the node to alternate between legitimate and illegitimate states initially as long as it reaches a final legitimate state after a bounded number of changes. For example, Dubois et al. [16] present a protocol for the computation of a BFS spanning tree that, given a dedicated root node, is TA strongly stabilizing for $S_B = \{ v \in H \mid d_S(v,r) \leq \min_{b \in B} d_S(v,b) \}$ with $d_S(u,v)$ being the hop distance between $u$ and $v$ in overlay $S$. We call the set $S_B$ the *containment area of $S$*, because $S_B$ represents the part of the network in which the adversary can keep the state of nodes from converging, whereas all nodes outside of $S_B$ will eventually reach and remain in a legitimate state. We
thus call the honest nodes in $S_B \equiv$ lost nodes. All honest nodes that are not lost are called safe nodes.

The above discussion leads to the following notion of a $S_B$-legitimate configuration, i.e., a configuration that is legitimate for all nodes in $S_B$.

**Definition 1.** (S-B-legitimate/S-B-stable configuration) A configuration $\gamma$ is $S_B$-legitimate for spec if the state of every safe node is legitimate for spec. Furthermore, it is $S_B$-stable if no safe node ever changes its output variables as long as no Byzantine node performs any action.

As stated above, we allow Byzantine nodes to disrupt legitimate configurations if the safe nodes afterwards return to a legitimate configuration.

**Definition 2.** (S_B-TA-disruption) A subsequence $e = \gamma_0, \gamma_1, \ldots, \gamma_l$ with finite $l > 0$ of a computation $\Gamma$ is a $S_B$-TA-disruption if and only if i) at least one safe node changes one of its output variables and ii) only $\gamma_0$ and $\gamma_l$ are $S_B$-legitimate for spec and $S_B$-stable.

Not only do we expect the overlay to recover from a disruption, we also want to limit the number of such disruptions.

**Definition 3.** ((t,k,f)-TA-time-contained configuration) A configuration $\gamma$ is $(t,k,S_B,f)$-TA-time-contained for spec if given at most $f$ Byzantine nodes, the following properties are satisfied:

1) $\gamma$ is $S_B$-legitimate for spec and $S_B$-stable,
2) every execution starting from $\gamma$ contains a $S_B$-legitimate configuration for spec after which the output variables of all safe nodes do not change anymore (irrespective of the actions of Byzantine nodes),
3) every execution starting from $\gamma$ contains at most $t$ $S_B$-TA-disruptions,
4) in every execution starting from $\gamma$, every safe node performs at most $k$ changes to its output variables.

Whereas the parameter $t$ denotes the number of $S_B$-TA-disruptions, the parameter $k$ denotes the maximal number of times each node outside of $S_B$ is affected. Given these definitions, we define a $(t,S_B,f)$-topology-aware-strongly stabilizing algorithm as follows:

**Definition 4.** ((t,S_B,f)-TA-strongly-stabilizing algorithm) A distributed algorithm $A$ is $(t,S_B,f)$-TA-strongly-stabilizing if and only if starting from any arbitrary configuration, every execution involving at most $f$ Byzantine processes contains a $(t,k,S_B,f)$-TA-time-contained configuration for a finite $k$.

A $(t,S_B,f)$-TA-strongly-stabilizing algorithm thus achieves eventual convergence to a legitimate configuration for all nodes in $S_B$. While the notion is weaker than traditional self-stabilization, it requires less assumptions. It is hence particularly useful for route-restricted overlays.

**IV. LEADER ELECTION**

In the following, we investigate under which conditions Byzantine leader election can be performed in arbitrarily connected networks.

We formalize Byzantine-tolerant leader election as follows:

**Definition 5.** (Leader Election Problem) Let $S = (V,E)$ be a connected route-restricted overlay, where a subset $B \subseteq V$ of size $f$ is Byzantine and $H = V \setminus B$ is the set of honest nodes. Every node $u \in V$ has a unique $ID_u$ and a variable $leaderID_u$, each holding a value from a set $ID$ with $|ID|$ much larger than $|V|$. We say that a configuration $\gamma$ is a solution to the Leader Election Problem for a subset $S_B \subseteq H$ of honest nodes if there is a value $ID^* \in ID$ such that the following properties are satisfied:

1) **Agreement:** $\forall u \in V \setminus S_B : leaderID_u = ID^*$
2) **Validity:** $\exists u \in H : ID_u = ID^*$

The node $u$ with $ID_u = ID^*$ is called leader.

While agreement and validity are analogous to the notions of leader election from classical algorithms, we consider convergence instead of termination, because self-stabilizing algorithms do not terminate.

Certain self-stabilizing algorithms for leader election, such as those based on an ordering among the IDs, have a putsch-resistant $ID$-value $ID_{PR}$. If there is an honest node $l$ with $ID_l = ID_{PR}$, then $l$ will always become and stay the leader node, irrespective of the initial configuration of the system. In the case of non-negative integer IDs, the $ID$-value 0 is putsch-resistant, since there is no lower value that a Byzantine node can claim as its $ID$-value. Observe that the definition of a putsch-resistant $ID$-value implies that there can be at most one putsch-resistant $ID$-value for any leader election algorithm. Otherwise, if two honest nodes with a putsch-resistant $ID$-value are present, agreement is impossible.

For route-restricted overlays, we consider the existence of a putsch-resistant leader to be extremely unlikely for the following two reasons:

- Without a centralized entity that assigns an $ID$-value to each node, every node needs to select its $ID$-value autonomously by sampling uniformly from a set of possible $ID$ values. The latter set needs to be reasonably large such that the probability that two nodes choose the same $ID$-value is negligible as is the probability to choose the one putsch-resistant $ID$-value.
- Route-restricted overlays are dynamic, such that a putsch-resistant leader may leave the system at any time.

Recalling the example of BFS spanning trees with a predefined root node (leader) in Sec. III-C, the set $S_B$ for a specific system $S$ typically depends on the network structure of $S$ and the position of Byzantine nodes. Ideally, we want the set $S_B$ to be adaptive in the sense that if no Byzantine node is present, then all nodes reach agreement on a single leader ID. More formally, we say that $S_B$ is adversary-adaptive if $|B| = 0 \implies S_B = \emptyset$.

However, it turns out that the leader election problem cannot be solved in most dynamic networks if $S_B$ is adversary-adaptive. Formally, our first main result is:
Theorem 1. There is no \((t, S_B, f)\)-TA-strongly-stabilizing algorithm with \(f > 0\) for the leader election problem on any dynamic system \(S\) and adversary-adaptive containment area \(S_B\) in the absence of a putsch-resistant leader.

Proof. We prove that no \((t, S_B, f)\)-TA-strongly-stabilizing algorithm exists for any adversary-adaptive containment area \(S_B\) by showing that agreement, validity and stabilization cannot be satisfied simultaneously if the containment area is adversary-adaptive and no honest node with a putsch-resistant ID-value is present. The main reason for this is that without additional assumptions on the dynamics of \(S\), none of the honest nodes in \(S\) can distinguish between the scenario that the system is under attack and the scenario that there is no attack but a change in the network structure has occurred.

First, observe that if there is no \((t, S_B, 1)\)-TA-strongly-stabilizing algorithm, then there also is no \((t, S_B, f)\)-TA-strongly-stabilizing algorithm for \(f > 1\), as all but one Byzantine node can behave like honest nodes.

Let \(S = (V, E)\) be a connected system with \(n > 1\) nodes of which exactly one node \(b\) is Byzantine. Assume that there exists a \((t, S_B, 1)\)-TA-strongly-stabilizing leader election algorithm \(\mathcal{A}\) for \(S\) and an adversary-adaptive containment area \(S_B\) with \(|S_B| > 0\). Starting from an arbitrary initial configuration \(\gamma_0\), the Byzantine node first behaves like an honest node with \(ID_b\) as its ID-value. Let \(K_H = \{ID_u | u \in H\}\) be the set of IDs belonging to honest nodes. By the convergence of \(\mathcal{A}\), \(S\) reaches a \((t, S_B, 1)\)-TA-contained configuration \(\gamma_1\) after a finite number of rounds. Because \(b\) currently behaves like an honest node, \(S_B\) is adversary-adaptive and \(\gamma_1\) is \(S_B\)-legitimate, all honest nodes have set their leaderID-variable to a value \(ID_i \in K_H \cup \{ID_b\}\).

After \(\gamma_1\) has been reached, the Byzantine node \(b\) pretends that a connection into a formerly separated network \(S' = (V', E')\) has been established, as illustrated by Figure 1. Due to the absence of a putsch-resistant leader, the probability that a node \(u_{S'}\) in \(S'\) is adopted as new leader by all honest nodes is greater than zero. \(b\) can choose \(S'\) arbitrarily in a way that maximizes the latter probability. Furthermore, \(b\) may repeatedly simulate the setup of new connections until a simulated node is adopted as leader. Because \(S_B\) is adversary-adaptive, all safe nodes also need to change their leaderID-value accordingly. Otherwise, we can easily find a system \(S_F = (V_F, E_F)\) and initial configuration \(\gamma_F\) for which \(\mathcal{A}\) does not reach agreement as follows: set \(V_F = V \cup V'\), \(E_F = E \cup E' \cup \{u, b | u \in V'\}\) and replace \(b\) and all nodes in \(V'\) with honest nodes. Then, set \(\gamma_F\) such that every node from the set \(V\) has the same internal state as in \(\gamma_1\) and every node from \(V'\) has the same internal state as the network simulated by the Byzantine node. Due to the construction of \(S'\) by \(b\), one node \(v\) from \(V'\) will eventually be elected as leader by at least one node in \(S\). Because there is no Byzantine node in \(S_F\) and \(S_B\) is adversary-adaptive, all nodes in \(S\) have to set their leaderID-value \(ID_s\) eventually in order to fulfill agreement.

By the convergence of \(\mathcal{A}\), all nodes in \(S\) will thus eventually reach a configuration \(\gamma_2\), where all nodes in \(S\) have set their leaderID-value to an ID-value \(ID_s\) of a simulated node. Now, \(b\) again starts to behave like an honest node that does not have any additional connections besides those originally in \(S\). Because the current configuration of system \(S\) then violates the validity property, \(S\) thus needs to reach a configuration \(\gamma_3\), in which all nodes have set their leaderID-variable to a value \(ID'_f\) from \(K_H \cup \{ID_B\}\). Again, because \(S_B\) is adversary-adaptive, all safe nodes also need to adapt their leaderID-variable in order to reach a legitimate configuration.

Once the system reached \(\gamma_3\), the Byzantine node again changes its behavior and simulates a connection to a formerly separated network, forcing all nodes in \(S\) to eventually change their leaderID-value. Note that we have shown that every time after \(b\) changed its behavior, all honest nodes (including all safe nodes) need to change their leaderID-variable at least once in order to preserve agreement and validity. Since \(b\) can change its behavior an infinite number of times, the number of changes to the leaderID-variable of each safe process cannot be bounded by a constant \(t\). Thus, \(\mathcal{A}\) does not fulfill the convergence property and hence does not solve \(S_B\)-Contained-Leader-Election-Problem for an adversary-adaptive containment area \(S_B\). □

V. BYZANTINE-TOLERANT COMPUTATION OF BFS SPANNING TREES

As shown in the previous section, a Byzantine-tolerant leader election algorithm is a necessary precondition for efficient routing based on BFS spanning trees. However, even in the presence of an honest root of the tree, which henceforth we define the leader, Byzantine nodes can pretend to have an arbitrarily low distance to the leader and thus cause honest nodes to adopt them as parents. In the following, we first formalize the problem of computing a BFS spanning tree for a given leader and afterwards present a cryptography-based algorithm that makes attacks based on false distance values provably less effective.

A. Problem Formalization

Definition 6. (BFS Spanning Tree Problem) Let \(S = (V, E)\) be a route-restricted overlay where each node \(u\) has a unique ID \(ID_u\) from a set \(ID\) that is much larger than \(|V|\). Furthermore, each node has a non-negative integer variable level\(_u\) as well as a variable parentID\(_u\), holding a value from...
We say that a configuration $\gamma$ is a solution to the BFS spanning tree problem for a given leader $l \in V$ with $ID$-value $ID^* \in ID$ if every non-Byzantine process $u$ fulfills the following conditions:

1) $level_u = 0$ iff $ID_u = ID^*$
2) $level_u = L_{\text{min}} + 1$
3) parent$ID_u = ID_u$ iff $ID_u = ID^*$
4) parent$ID_u \in \{ID_v \mid v \in N(u), level_v = L_{\text{min}}(u)\}$ iff $ID_u \neq ID^*$

where $N(u)$ denotes the neighbors of $u$ and $L_{\text{min}} = \min\{level_v \mid v \in N(u)\}$.

The spanning tree encoded in the node states is the graph $T = (V, E, r)$, where $E = \{(u, v) \mid \text{parent}ID_u = ID_v\}$ and $r \in V$ as root is the leader, where parent$ID_r = ID_r$.

An algorithm for the BFS Spanning Tree Problem thus needs to adapt the output variables $level_u$ and parent$ID_u$ of each honest node $u$ in the presence of Byzantine nodes. Figure 2 shows an example, where a Byzantine node lies about its level such that multiple honest nodes adopt it as parent, thus keeping those nodes from being able to route messages to other honest nodes.

We measure the influence of Byzantine nodes by the number of ill-directed (honest) nodes:

**Definition 7.** (Root-directed path) Given a route-restricted overlay $S$ and a solution $\gamma$ to the rooted spanning tree problem on $S$, then the root-directed path $P_u$ of a node $u$ is a finite sequence $v_1, v_2, \ldots, v_{n+1}$ of nodes such that $v_{n+1} = u$ and parent$ID_{v_{i+1}} = ID_{v_i}$ for all $1 \leq i \leq n$ and either parent$ID_{v_1} = ID_{v_0}$ (the legitimate root) or $v_1$ is a Byzantine node. We call $u$ ill-directed if $v_1$ is Byzantine for any $1 \leq i \leq n$, and well-directed otherwise.

As long as a node is ill-directed, it is subject to changes in the level-value reported by the Byzantine node on its root-directed path. Thus, it is not guaranteed to remain in a legitimate state.

**B. Spanning Tree Construction using Level Attestations**

Existing self-stabilizing algorithms that construct BFS spanning trees do not make assumptions on computational bounds of attackers. Those protocols naturally ensure that all honest nodes, whose distance to the closest Byzantine node is strictly higher than their distance to the root, will eventually be well-directed. More formally, given that $d_S(u, v)$ is the length of the shortest path between two nodes $u, v \in V$, then all honest nodes $H$ except the subset

$$S_B = \{u \in H \mid \exists b \in B : d_S(u, b) \leq d_S(u, r)\}$$

will eventually be well-directed [16].

To show that a significantly stronger containment can be achieved by means of cryptography, we first need to extend our model such that each node $u$ holds a public/private key pair $p_u, s_u$ of an asymmetric cryptosystem. The public key $p_u$ of each node $u$ is stored in the $ID$-register and the secret $s_u$ is stored in a new register called $secret_u$. Consequently, the given leader ID $ID^*$ then is the public key of the corresponding root node, implicitly choosing it as leader.

Second, we assume that all nodes have loosely synchronized clocks (e.g., using NTP) such that the clocks of any pair of nodes differs at most by a constant $\Delta_C$. Furthermore, we assume that the delay between two consecutive writes on any register of a node is at most $\Delta_D$.

In the following, we introduce additional cryptographic building blocks leveraged by our own algorithm.

**a) Level attestation:** In aiming to keep Byzantine nodes from lying about their distance to the root, we add a $levelAtt$-variable to each node $u$. This $levelAtt$-variable holds a finite sequence $P = (p_1, t_1, \text{sig}_1), (p_2, t_2, \text{sig}_2), \ldots, (p_n, t_n, \text{sig}_n)$ of tuples called a level attestation. The elements $p_i, t_i$, and $\text{sig}_i$ denote a public key, a timestamp, and a cryptographic signature, respectively.

Given a threshold $\Delta \geq \Delta_C + \Delta_D$, we say that such a sequence is valid for node $u$ at time $t$ if the following conditions are satisfied:

1) $p_1 = ID^*$,
2) $\forall 1 \leq i \leq n : |t - t_i| < \Delta$,
3) $\forall 1 \leq i < n : \text{sig}_i$ is a signature over $p_{i+1}||t_i$ that is valid for $p_i$,
4) $\text{sig}_n$ holds a signature over $ID_u||t_n$ that is valid for $p_n$, where $a||b$ denotes the concatenation of $a$ and $b$.

**b) Link signatures:** Additional to the level attestation, each node assigns a randomly chosen neighbor ID $nID_v$ to each neighbor $v$ once in the beginning of the algorithm. During the computation, every honest node tells each neighbor its respective neighbor ID. Whenever a neighbor of a node $u$ transmits a new level attestation, it also has to send a corresponding neighbor signature that includes its neighbor ID assigned by $u$. Given a valid level attestation $P$ with the last element $(p, t, \text{sig})$ and a cryptographic hash function $h$, a neighbor signature $s$ is valid for node $u$ and neighbor $v$ if $s$ is a valid signature over $nID_v||h(P)$ for $p$. This addendum keeps Byzantine nodes from proposing back a shortened version of an attestation received by an honest neighbor.

**c) BFS Spanning Tree Algorithm:** Algorithm [1] displays the pseudocode for our spanning tree construction algorithm. Each output register of every node $u$ holds 5 elements, namely the $ID$- and $level$-value of $u$ as well as the $levelAtt$- and $nID$-value together with the neighbor signature $\text{sig}_\text{neigh}$ for the corresponding neighbor. The algorithm leverages the following cryptographic functions: The $\text{sign}_u$-function uses the key stored in the $secret$-register to compute a signature $\text{sig}_u$. The function $h$ is a cryptographic hash function.

Every node periodically reads the content of each input register, processes the content, and writes corresponding out-
puts to output registers. As long as the neighborhood $N(u)$ of a node $u$ does not change, the foreach-statements in Algorithm 1 always traverse the neighbors in the same order.

The current leader node generates a level attestation for each neighbor and writes its own ID and level-value together with the respective nID-value, level attestation, and neighbor signature into the corresponding output register (Line 7–10). The isValAtt function checks if a given level attestation is valid, as defined above. If the given level attenuation is valid, isValAtt further checks whether the length of the attestation equals the given level value and returns false in case of a mismatch. Given this check succeeds, the isValLink-function checks if a given sigNeigh-value is a valid for the corresponding neighbor. If a parent node with a valid level attestation has been chosen, the node generates a corresponding level attestation for each neighbor and writes it into the respective output register (Line 7–10).

During the processing stage (Line 12–19), an honest leader node generates a level attestation for each neighbor and writes its own ID, level attenuation, and neighbor signature into the corresponding output register (Line 7–10). The current leader node generates a level attestation for each neighbor and writes its own ID, level attenuation, and neighbor signature into the corresponding output register (Line 7–10).

The foreach loop in Line 13 iterates over each neighbor $i$ in the node’s neighborhood $N(u)$, and for each neighbor, it reads the value of $r_{ui}$, gets the current time, and calculates the level of the neighbor to be $min(|r_{ui}|, nID, |r_{ui}|, sigNeigh)$. If the neighbor found is not the parent found, it assigns the parent ID as $r_{ui}$, and if the level of the neighbor is equal to the level of the output register, it updates the parent found to true. Otherwise, it iterates over each node in the neighborhood and checks if the output register is valid, and if so, it appends the level attenuation, exAtt, and sigNeigh into the output register (Line 14–16).

Algorithm 1: BFS spanning tree with level attestation

```plaintext
while true do
    foreach i in N(u) do
        lr_{ui} := read(r_{ui})
        ts := getCurrentTime()
        if ID = ID^u then
            parentID := ID
        foreach i in N(u) do
            sig_{i} := sign(lr_{ui}.ID, ts)
            sigNeigh_{i} := sign(lr_{ui}.nID, [ID, ts, sig_{i}])
            write(r_{ui}) := (ID, 0, (ID, ts, sig_{i}), nID, sigNeigh_{i})
        end
        parentFound := false
        N_valid := {i ∈ N(u) | isValidAtt(lr_{ui}.levelAtt, lr_{ui}.level) ∧ isValLink(lr_{ui}.levelAtt, lr_{ui}.sigNeigh)
        level := min(|lr_{ui}.level | i ∈ N_valid | + 1
        foreach i in N_valid do
            if not parentFound and lr_{ui}.level = level - 1 then
                parentID := lr_{ui}.ID
                levelAtt := lr_{ui}.levelAtt
                parentFound := true
            end
        end
        foreach i in N_valid do
            sig_{i} := sign(lr_{ui}.ID, ts)
            exAtt_{i} := append(levelAtt, (ID, ts, sig_{i})).
            sigNeigh_{i} := sign(lr_{ui}.nID, [ID, (exAtt)])
            write(r_{ui}) := (ID, level, exAtt, nID, sigNeigh_{i})
        end
    end
end
```

and that all nodes in $H \setminus S_B'$ will eventually be well-directed. The additional “-1” after $l_{min}^B$ results from the fact that a Byzantine node can copy the outputs of an honest neighbor into its output registers (hence pretending to be its own predecessor) and thus cheat by one level.

**Proof of self-stabilization:** We start by establishing some properties of level attestation to later leverage in the proof. In a nutshell, we first show that Byzantine nodes can only influence keys that are used after the $l_{min}^B$-th element of a valid level attestation $P$ but before the $|P| - l_{min}^B$-th element with $l_{min}^B = \min_{b \in B}(d_S(u, b))$ for any node $u$. Based on this result, we can then show that nodes are well-directed if their levelAtt-value is of length less than $l_{min}^B + l_{min}^B - 1$. Self-stabilization follows from the fact that the system will at some point reach a state when all nodes outside of $S_B'$ have a valid levelAtt-value of minimal length and hence will not change parents anymore.

**Lemma 1.** Let $P = (p_1, t_1, sig_1), \ldots, (p_n, t_n, sig_n)$ be a level attestation. At time $t$, we have $|t - t_i| \leq \Delta$ for all $1 \leq i \leq n$ and the computation has started at least $\Delta$ time units before. Consider a node $u$ such that sig$_u$ is a signature over $ID_u[|t_n|]$. If $P$ is valid, then the following two statements hold:

1. For $j \leq l_{min}^B$, $p_j$ is the public key of an honest node $v$ and $d_S(v, r) < j$.
2. For $j > n - l_{min}^B + 1$, $p_j$ is the public key of an honest node $v$ and $d_S(v, u) \leq n - j + 1$.

**Proof.** We show the first claim by induction on $j$. As $p_1$ always needs to be the public key of the leader and the leader is honest by assumption, the claim holds for $j = 1$. Let $1 < j \leq l_{min}^B$ and assume the claim holds for $j - 1$. Then sig$_{j-1}$ is a signature over $p_{j-1}[|t_{j-1}]$ using the secret key s$_{j-1}$ associated with $p_{j-1}$. By induction hypothesis, $p_{j-1}$ is the public key of an honest node $w$ with distance $d_S(w, r) < j - 1 \leq l_{min}^B - 1$. $d_S(w, r) < l_{min}^B - 1$ implies that $w$ has only honest neighbors, which only write their own keys to its output register for $w$ to sign.

Furthermore, because $w$ itself is honest, $w$ only signs keys and timestamps that it reads from its input registers. Thus, for $p_{j-1}[|t_{j-1}]$ to be signed by $w$, $p_j$ needs to be the key of an honest neighbor $v$ of $w$. Given that $w$’s distance to the root is less than $j - 1$ by induction hypothesis, we also have $d_S(v, r) \leq d_S(w, r) + 1 < j$. This proves the first claim. Similarly, we show the second claim by induction on $j' = n - j + 1$. Note that if $l_{min}^B = 1$, i.e., $u$ is the neighbor of a Byzantine node, then there is nothing to show as there is no $p_j$ such that $j > n - l_{min}^B + 1$. So, we assume $l_{min}^B > 1$. For $j' = 1$, we only have to consider the key $p_n$. As $u$ is honest, it will only write its own key into output registers to be signed by neighbors. If $l_{min}^B > 1$, all of $u$’s neighbors are honest. They would hence only sign $u$’s key concatenated with a timestamp with their own, meaning that any node $v$ with public key $p_n$ is indeed an honest node and $d_S(v, u) = 1$. Consider $1 < j' < l_{min}^B$, and assume the claim holds for $j' - 1$. Hence, $p_n[|t_{j' - 1}]$ is the public key of an honest node $w$ with $d_S(w, u) \leq j' - 1$. $w$ writes its public key and a
timestamp to the registers that will be read by its neighbors. As \( j' - 1 < t_{\text{min},u}^B - 1 \), these neighbors are honest and will sign the key and timestamp with their own keys. Hence, any public key \( p_{n-j'+1} \) whose corresponding secret key has been used to sign \( p_{n-(j'+1)-1} \) belongs to an honest neighbor \( v \) of \( w \) with \( d_S(v,u) = d_S(w,u) + 1 = j' \). So, the second claim follows by induction as well.

**Lemma 2.** Let the computation have started a least \( \Delta \) timeunits before and \( u \in V \setminus S_B^* \) be a node with a \( \text{levelAtt} \)-value of length \( n < l_{\text{min}}^B + l_{\text{u,min}}^B - 1 \). Then \( u \) is well-directed.

**Proof.** By Lemma 1, the first \( l_{\text{min}}^B \) public keys have to belong to honest neighbors and the last \( l_{\text{u,min}}^B - 1 \) keys have to belong to honest nodes as well. Hence, if \( n < l_{\text{min}}^B + l_{\text{u,min}}^B - 1 \), all keys \( p_j \) have to belong to an honest node \( v_j \) for \( 1 \leq j \leq n \).

Set \( v_{n+1} = u \).

\( u \) can only be ill-directed if at least one \( v_j \) has their parentID-value set to a key provided by a Byzantine node. First, consider the case that \( j < l_{\text{min}}^B \). By Lemma 1, \( d_S(v_j,u) < l_{\text{min}}^B - 1 \), meaning that \( v_j \) only has honest neighbors. Honest nodes only write their own keys in the register of their neighbors, so that \( v_j \) can hence only set its parentID-value to one of their keys. Now, consider \( j > l_{\text{min}}^B \), i.e., \( n - j + 1 < n - l_{\text{min}}^B + 1 \leq l_{\text{u,min}}^B - 1 \). According to Lemma 1, \( d_S(v_j,u) \leq n - j + 1 < l_{\text{u,min}}^B - 1 \). Again, \( v_j \) has only honest neighbors and hence can only set its parentID-value to one of their keys.

It remains to consider the case \( j = l_{\text{min}}^B \). By the first part of the proof, \( v_j \) is the only node that can have Byzantine neighbors. Assume that \( v_j \) has set its parentID to a Byzantine neighbor \( b \). For \( u \)'s \( \text{levelAtt} \) to correspond to a valid attestation, \( v_{j-1} \) has to sign \( p_j || \) \( t_{j-1} \) resulting in \( \text{sig}_{j-1} \), append \( (p_{j-1}, t_{j-1}, \text{sig}_{j-1}) \) to the attestation, and write the attestation to the register corresponding to the neighbor that wrote \( p_j \) to the register. Because \( v_{j-1} \) has only honest neighbors, the respective neighbor has to be \( v_j \), the only honest node that would claim \( p_j \) as its key. So, in order for \( u \)'s \( \text{levelAtt} \)-value to include \( (p_{j-1}, t_{j-1}, \text{sig}_{j-1}) \), \( v_j \) must have read the register and disseminated \( (p_{j-1}, t_{j-1}, \text{sig}_{j-1}) \) as part of a level attestation. Consequently, \( v_j \) is aware that \( v_{j-1} \) offers a root-directed path of supposed length \( j - 2 \leq l_{\text{min}}^B - 2 \). For \( v_j \) to choose a different parent, \( b \) has to produce a valid attestation \( \tilde{P} = (\tilde{p}_1, \tilde{t}_1, \tilde{\text{sig}}_1), \ldots, (\tilde{p}_l, \tilde{t}_l, \tilde{\text{sig}}_l) \) with \( l \leq j - 1 \) and \( \tilde{\text{sig}}_l \) being a signature over \( ID_{v_j}[t_l] \). Furthermore, \( b \) has to ensure that the isValidLink-function returns true. The neighbor-related signature has to be signed by the secret key \( \tilde{s}_l \) corresponding to \( \tilde{p}_l \). As \( b \) can not forge signatures, \( \tilde{P} \) has to be a (potentially shortened) attestation that \( b \) has read from one of its input registers. For such an attestation, \( \tilde{p}_l \) belongs to an honest node \( w \) at distance at most \( l - 1 \) from the root by Lemma 1. Due to \( d(w,r) \leq l - 1 < l_{\text{min}}^B - 1 \), \( w \) has no Byzantine neighbors. By Algorithm 1, \( w \) only writes signatures over \( nID_w[h(L)] \) for some \( L \) to registers of neighbors. Being honest, these neighbors do not disseminate the respective signatures. As a consequence, \( b \) can not obtain the required neighbor signature and hence \( v_j \) does not accept any attestation from \( b \) as its \( \text{levelAtt} \)-value.

In summary, none of the nodes \( v_j \) has its parentID-value set to a key provided by a Byzantine node and hence \( u \) is indeed well-directed.

**Theorem 2.** Given any route-restricted overlay network \( S \) with diameter \( \text{diam}(S) \), a computation of Algorithm 1 starting from an arbitrary configuration will reach a \((0,0,S_B^*,f)\)-time-contained configuration \( \gamma^* \) for the BFS Spanning Tree Problem within at most \( \text{diam}(S) + 1 \) asynchronous rounds. Furthermore, all nodes \( u \in H \setminus S_B^* \) are well-directed in \( \gamma^* \). Thus, Algorithm 1 is \((0, S_B^*, f)\)-TA-strongly-self-stabilizing.

**Proof.** We claim that after \( l + 1 \) rounds, all nodes \( u \in V \setminus S_B^* \) within distance \( l \) of the root are well-directed and have levelAtt-values of length \( l \). The properties from Definition 6 follow. Last, we show that the computation will indeed remain in a legitimate state.

After the first round, the root has written its information to all registers. After the second round, the neighbors of the root have processed these registers. Hence, each such neighbor \( u \) will set its levelAtt-value to an attestation of length 1. If \( u \in V \setminus S_B^* \), the distance \( d_S(u,b) \geq 2 \) for any Byzantine node \( b \) and hence by Lemma 2, \( u \) is well-directed. So, the claim holds for \( l = 1 \).

Assume the claim holds for \( l \), i.e., after \( l + 1 \) rounds, all nodes \( v \in V \setminus S_B^* \) within distance \( l \) of the root are well-directed and have levelAtt-values of length \( l \). They know the IDs their neighbors have assigned to them as \( l > 1 \) indicates that they have read it from the register at least once. As a consequence, they can construct a valid attestation of length \( l + 1 \) for each neighbor \( w \) as well as the necessary signature over the neighbor ID \( nID_w \). They write this information to the register \( r_{uw} \). After \( l + 1 \) rounds, any node \( u \in V \setminus S_B^* \) at distance \( l + 1 \) from the root has read the register corresponding to its neighbors at distance \( l \) to the root. As a consequence, \( u \)'s levelAtt-value is of length \( l + 1 \). As \( u \in V \setminus S_B^* \), Lemma 2 shows that \( u \) is well-directed.

It follows by induction that within \( \text{diam}(S) \) rounds, all nodes \( u \in V \setminus S_B^* \) are well-directed. We now argue that this constitutes a solution to the BFS Spanning Tree Problem as defined in Definition 6. For all nodes but the root, their level is equal to the length of levelAtt-value, whose length corresponds to their distance to the root and is hence 1 added to the distance of a neighbor closest to the root. So, property 1 and 2 of Definition 6 hold. Property 3 holds as being well-directed implies in particular that a node has a parentID-value of a distinct honest node unless it is the root. The fourth property follows as nodes \( u \) can only have an attestation of length \( l = d_S(u,r) \) if they choose a parent closer to the root, which is any neighbor with the minimal distance to the root.

It remains to prove that the computation remains in a legitimate state for all \( v \in V \setminus S_B^* \). In order to reach an illegitimate state, a node has to change its parentID-value. Let \( u \) be the first node to change its parentID-value. According to Algorithm 1, \( u \) selects the parent from those
neighbors that provide the shortest valid attestation and a valid neighbor signature. By assumption, $u$ breaks ties consistently, meaning $u$ only changes its parent if either i) $u$’s previous parent does not provide any valid attestation of the shortest length or provides an invalid neighbor signature, or ii) a neighbor that is not the current parent writes an attestation of a shorter length than $u$’s level-$\text{Att}$-value to its register and the content of the register passes the two validity checks.

In order to conclude that neither i) or ii) are possible, consider the following: Let $v$ be $u$’s parent and note that $v \in V \setminus S_B'$ by the definition of $S_B'$ as $d_S(v, r) = d_S(u, r) - 1$ and $d_S(v, b) \geq d_S(u, b) - 1$ for all Byzantine nodes $b$. It follows recursively that all nodes on a root-directed path of $u$ are in $V \setminus S_B'$. Case i) would imply that a node on the root-directed path changed its parent, as honest nodes do not write invalid attestations or neighbor signatures to registers. However, such a parent change contradicts the definition of $u$ as the first node in $V \setminus S_B'$ to change its parent. If case ii) holds, by Lemma~1 $u$ has to be well-directed after its parent change. Hence, its new parent $w$ is an honest node. By the above, $w$ and all nodes on the new root-directed path are in $V \setminus S_B'$ and at least one of them has to have changed its parent for $w$ to write an attestation of a different length. Again, such a change in parent is a contradiction to the definition of $u$. Consequently, nodes $u \in V \setminus S_B'$ do not change their parentID-value for the rest of the computation and remain in a legitimate state.

VI. Evaluation

Using OMNeT++~[34], we implemented a discrete, event-based simulation to evaluate the impact of our attestation-based construction algorithm on the number of ill-directed nodes in comparison to traditional non-cryptographic algorithms. Furthermore, we investigated the impact of the network structure, the position of the leader, and the placement of edges between honest nodes and Byzantine nodes.

A. Metrics, Data Sets, and System Parameters

Given a distributed system $S = (V, E)$ with a subset $H$ of honest nodes, we measured the ill-direction ratio (IDR) $|H_{II}|/|H|$, where $H_{II}$ is the set of ill-directed honest nodes and $H$ is the set of all honest nodes. A low ill-direction ratio thus corresponds to high Byzantine fault tolerance.

Route-restricted overlays include both social overlays and payment networks. We hence utilized a real-world graph for each of them and compare the results with synthetic graphs for the purpose of characterizing the impact of various topological features. Facebook denotes a real-world graph of Facebook [35], as used in several prior studies [26], [27], [29]. Ripple denotes a real-world graph from the Ripple payment network [30]. Ripple has a low number of edges and a heavily skewed degree distribution: 95% of all nodes have a degree less or equal than the average degree of approximately 3. Our synthetic data sets are i) a random synthetic network (denoted randomized Facebook) with the same degree distribution as the Facebook graph (generated using igraph [32]) and ii) an Erdős and Rényi graph (ER) (generated using GTNA [31]) with approximately the same number of nodes and edges as Facebook but normal distributed degrees [20]. Comparing Facebook with randomized Facebook enabled us to characterize the impact of clustering while the comparison of randomized Facebook and ER revealed the impact of degree distribution.

We considered the number of Byzantine nodes and the time of their presence to be unbounded but limit the number $g$ of connections between honest nodes and Byzantine nodes. To model that all Byzantine nodes are colluding, we represented them as a single node with $g$ edges.

B. Set-up

We investigated the resistance of spanning tree algorithms to Byzantine nodes given structural differences of the networks and a varying number $g$ of attack edges.

We assumed the adversary knows all nodes but can only establish a connection to a subset with limited size. Following [7] we also assumed that users with many contacts are more likely to accept new requests and thus connect with a Byzantine node. We hence added a single adversary $m$ to the graph and choose the $g$ honest neighbors at random, with a probability proportional to their degree. Afterwards, a root node $r$ was chosen uniformly at random from all honest nodes and the leader ID of each honest node was set accordingly. We then ran a standard BFS spanning tree protocol, where each node chose its neighbor closest to the source as parent, updates its level accordingly, and broadcasts the update to its other neighbors. Ties were broken according to the order of neighbors in the nodes’ local state. The execution proceeded until all honest nodes have reached their final state.

We distinguished between spanning trees without and with attestation. The first scenario modelled the case that state-of-the-art protocols, such as the one by Dubois et al. [16], are used. In this case, honest nodes cannot distinguish $m$ from the actual root node $r$. As the Byzantine node would always report the lowest level-value possible in order to maximize the number of nodes that accept it as their parent, $m$ thus propagated 0 as its level-value to all its neighbors. Investigating the effect of attestation, we changed the behavior of $m$ to always report the lowest level-value received from any of its neighbors. This behavior respected the fact that $m$ can cheat by one level, as described in Sec. V-B. To evaluate how strongly this deception affects the IDR, we additionally performed a simulation where $m$ always incremented the lowest received level-value by one before propagating it, i.e., $m$ follows the construction protocol honestly.
We performed 100 runs of each scenario on each graph to obtain statistically significant results. Figure 3 shows the obtained mean IDR with 99% confidence intervals. Especially for the Facebook graph, its randomized version, and the ER graph, the use of level attestation considerably reduced the ratio of ill-directed nodes compared to traditional tree construction. Without level attestation, an attack with 25 edges resulted in a mean IDR of 0.8, 0.68, 0.5, and 0.9 for the Facebook graph, the randomized Facebook graph, the ER graph and the Ripple graph, respectively. When level attestation was used, the mean IDR at 25 attack edges dropped down to 0.002, 0.0003, 0.0008, and 0.36 for the Facebook graph, the randomized Facebook graph, the ER graph, and the Ripple graph, respectively. Even for 1000 attack edges, the mean IDR for the Facebook graph, its randomized variant, and the ER graph significantly decreased from 0.98, 0.98, and 0.98 to 0.4, 0.21, and 0.27, respectively.

In the scenario with an honestly behaving adversary, the mean IDR was considerably lower than in the scenario with level attestation, especially with 1000 and 5000 attack edges. This is because all graphs used in our study have a very low average shortest path length as shown in Table I, such that all nodes have a very low hop distance to the root node. Consequently, an increase of the adversary’s reported level value by 1 already posed a significant advantage and causes many nodes to become well-directed.

For the Ripple graph, the improvement regarding mean IDR was considerably lower than for the other graphs. In the following, we describe the impact of distances between honest nodes, malicious nodes, and root node on the IDR to explain this stark difference.

D. Impact of Network Structure

The concrete IDR values obtained in our simulation study varied considerably between the different runs, especially in those configurations with a mean IDR near or between 0.8 and 0.2. On the Facebook graph for example, the standard deviation for the IDR amounted to 0.2 in the configuration with traditional tree construction and 25 attack edges. In the configuration with 1000 attack edges and level attestation, the standard deviation was 0.26.
the root node than $D(m)$, they should have been well-directed and otherwise not.

In all runs with 25 attack edges, $D(m)$ was considerably lower than $\overline{d}(r)$ such that only a very small number of nodes became ill-directed. Figure 5 thus shows our more distinct results for an adversary with 1000 attack edges, ordered by IDR. The results indeed validated the expected correlation. However, due to the high number of attack edges, the $\overline{d}(m)$ value of each run only differed slightly from the corresponding mean value of 2.75, 2.52, 2.63, and 2.02 for the Facebook graph, the randomized Facebook graph, the ER graph, and the Ripple graph, respectively. Thus, the values of $D(m)$ mainly depended on $d(m,r)$ and hence differed by integer values.

Again, the degree of correlation between $\overline{d}(r) - D(m)$ and the IDR varied between graphs. The Facebook graph generally had a longer average shortest path length and hence varied in $\overline{d}(r)$ considerably. In contrast, the value of $\overline{d}(r)$ was more stable for the randomized Facebook graph and the ER graph, so that $d(m,r)$ is indeed the main impact factor.

Here, we also find the explanation for the strong difference between the mean IDR values for the simulations with level attestation and those with an honestly behaving adversary on the Ripple graph. It stems from the fact that the $d(m,r)$ value decreased very slowly as the number of attack edges increases. Concretely, the mean value of $\overline{d}(r)$ was roughly 3.8, irrespective of the number of attack edges and the construction algorithm. In the case of 25 attack edges, the mean value for $D(m)$ was also 3.8 and in the case of 5000 edges, it was 2.9, such that the level value propagated by $m$ was still low enough to cause a high number of nodes to become ill-directed. As the value of $D(m)$ was increased by 1 if the adversary behaves honestly, it was higher than $\overline{d}(r)$ for any considered number of attack edges, resulting in a negative $\overline{d}(r) - D(m) - 1$ and hence a low impact of the attack. In contrast, $\overline{d}(r) - D(m)$ was positive, corresponding to an attack of high impact.

c) Summary of Results: The first part of our evaluation showed that our protocol based on cryptographic signatures is significantly more robust to Byzantine behavior and attacks than state-of-the-art solutions. Indeed, to compromise a similar number of nodes, the adversary needs to establish up to 200 times as many attack edges compared to the algorithm by Dubois [16]. The second part of our evaluation revealed that the resistance to attacks is highly correlated with the degree of the root node, indicating the need for leader election algorithms that preferably select high-degree nodes.

VII. Conclusion

In this paper, we have proven the impossibility of globally consensual leader election for BFS spanning trees with Byzantine nodes. Leveraging signatures, we designed a spanning tree algorithm that greatly reduces the number of nodes affected by Byzantine behavior. Our evaluation based on real-world scenarios demonstrates its security and indicates that the choice of high-degree nodes as leaders greatly benefits resilience. In conclusion, we identify the problem of electing such leaders securely as particularly promising future work.

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