Linear-response theory of spin Seebeck effect in ferromagnetic insulators

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We formulate a linear response theory of the spin Seebeck effect, i.e., a spin voltage generation from heat current flowing in a ferromagnet. Our approach focuses on the collective magnetic excitation of spins, i.e., magnons. We show that the linear-response formulation provides us with a qualitative as well as quantitative understanding of the spin Seebeck effect observed in a prototypical magnet, yttrium iron garnet.

I. INTRODUCTION

The generation of spin voltage, i.e., the potential for an electron’s spin to drive spin currents, by a temperature gradient in a ferromagnet is referred to as the spin Seebeck effect (SSE). Since the first observation of the SSE in a ferromagnetic metal Ni81Fe19,1 this phenomenon has attracted much attention as a new method of generating spin currents from heat energy and opened a new possibility of spintronics devices.2 The SSE triggered the emergence of the new field dubbed “spin caloritronics”3,4 in the rapidly growing spintronics community. Moreover, as the induced spin voltage can be converted into electric voltage through the inverse spin Hall effect5 at the attached nonmagnetic metal, this phenomenon put a new twist on the long and well-studied history of thermoelectric research.6

One of the canonical frameworks to describe nonequilibrium transport phenomena is linear-response theory.7 Having been applied to a number of transport phenomena, linear-response theory has been so successful because it is intimately related to the universal fluctuation-dissipation theorem. Up to now, however, the linear-response formulation of the SSE has not been known mainly because, unlike the charge current, the spin current is not a conserved quantity. Therefore, it is of great importance to formulate the SSE in terms of linear-response theory.

Concerning the SSE, a big mystery is now being established, which is, how can conduction electrons sustain the spin voltage over such a long range of several millimeters,8 in spite of the conduction electrons’ short spin-flip diffusion length, which is typically of several tens of nanometers? A key to resolve this puzzle was reported by a recent experiment on electric signal transmission through a ferromagnetic insulator9 which demonstrates that the spin current can be carried by the low-lying magnetic excitation of localized spins, i.e., the magnon excitations, and that it can transmit the spin current as far as several millimeters. Subsequently, the SSE was reported to be observed in the magnetic insulator LaY2Fe12O19 despite the absence of conduction electrons.9 These experiments suggest that contrary to the conventional wisdom over the last two decades that the spin current is carried by conduction electrons10 the magnon is a promising candidate as a carrier for the SSE.

The purpose of this paper is twofold. First, we analyze the SSE observed in LaY2Fe12O19 (hereafter referred to as YIG) in terms of magnon spin current, i.e., a spin current carried by magnon excitations. Second, we develop a framework for analyzing the SSE by means of the standard linear-response formalism which is amenable to the language of the magnetism community. This allows us to describe the spin transport phenomena systematically, and it can be easily generalized to a situation including degrees of freedom other than magnons, e.g., conduction electrons and phonons, to describe a more complicated process in the case of metallic systems.

The plan of this paper is as follows. In Sec. II we present a linear-response approach to the “local” spin injection by thermal magnons, in which the spin injection is driven by the temperature difference between the ferromagnet and the attached nonmagnetic metal. Next, in Sec. III we develop a linear-response theory of the “non-local” spin injection by thermal magnons, in which the spin injection is driven by the temperature gradient inside the ferromagnet. As one can see below, this process can explain the SSE observed in YIG. Finally, in Sec. IV we summarize and discuss our results.

II. “LOCAL” SPIN INJECTION BY THERMAL MAGNONS

We start by briefly reviewing the SSE experiment for YIG. Figure 1 shows the experimental setup where several Pt terminals are attached on top of a YIG film in a static magnetic field \( H_0 \hat{z} \) (\( \gg \) anisotropy field) which aligns the localized magnetic moment along \( \hat{z} \). A temperature gradient \( \nabla T \) is applied along the \( z \) axis, and it induces a spin voltage across the YIG/Pt interface. This spin voltage then injects a spin current \( I_s \) into the Pt terminal (or ejects it from the Pt terminal). A part of the injected/ejected spin current \( I_s \) is converted into a charge voltage through the so-called inverse spin Hall effect:9

\[
V_{\text{ISHE}} = \Theta_H (|e|I_s)(\rho/w),
\]  
(1)
where $|e|$, $\Theta_H$, $\rho$, and $w$ are the absolute value of electron charge, spin Hall angle, resistivity, and width of the Pt terminal (see Fig. 1), respectively. Hence, the observed charge voltage $V_{\text{ISHE}}$ is a measure of the injected/ejected spin current $I_s$.

To investigate the SSE observed in YIG, we consider a model shown in Fig. 2. While YIG is a ferrimagnet, we model it as a ferromagnet since we are interested in the low-energy properties. The key point in our model is that the temperature gradient is applied over the insulating ferromagnet, but there is locally no temperature difference between the ferromagnet and the attached non-magnetic metals, i.e., $T_{N_1} = T_{F_1} = T_1$, $T_{N_2} = T_{F_2} = T_2$, and $T_{N_0} = T_{F_0} = T_3$. We assume that each domain is initially in thermal equilibrium without interactions with the neighboring domains, and then calculate the nonequilibrium dynamics after we switch on the interactions. Note that this procedure is essentially equivalent to that used by Luttinger to realize the initial condition mentioned above.

Let us consider first the low-energy excitations in the ferromagnet. In the following, we focus on the spin-wave region where the magnetization $M(r)$ fluctuates only weakly around the ground state value $M_s \hat{z}$ with the saturation magnetization $M_s$, and we set $M/M_s = (1 - m^2/2)\hat{z} + m$ to separate the small fluctuation part $m$ (and $\hat{z}$) from the ground state value. Then, the low-energy excitations of $M$ are described by boson (magnon) operators $a^+_q$ and $a_q$ through the relations $m^+_q = \sqrt{1/S_0}a^+_q$ and $m^-_q = \sqrt{1/S_0}a_q$ where $m^q = (m^+ + im^-)/\sqrt{2}$, $S_0$ is the size of localized spins, and $m(q,r,t) = N_F^{-1/2} \sum_{q} m_q(t)e^{iqr}$ with $N_F$ being the number of localized spins in the ferromagnet. Consistent with this boson mapping, the magnetization dynamics is described by the following action:

$$S_F = \int dt \sum_{q} m^-_q(t)X_q(i\partial_t)^{-1}m^-_q(t),$$

where the integration is performed along the Keldysh contour $C$ and the bare magnon propagator is given by

$$X_q(\omega) = \left( \begin{array}{cc} X^{R}_q(\omega) & X^{K}_q(\omega) \\ 0 & X^{A}_q(\omega) \end{array} \right),$$

with the following equilibrium condition:

$$X^{A}_q(\omega) = [X^{R}_q(\omega)]^*, \quad X^{K}_q(\omega) = 2i \text{Im} X^{R}_q(\omega) \coth(\frac{\nu}{2k_B T})$$

The retarded component of $X_q(\omega)$ is given by $X^{R}_q(\omega) = S_0^{-1}(\omega - \omega_q + i\nu)^{-1}$ where $\alpha$ is the Gilbert damping constant, and $\omega_q = \gamma H_0 + \omega_q$ is the magnon frequency. Here, $\gamma$ is the gyromagnetic ratio and $\omega_q = D_{xx}q^2$, where $D_{xx} = 2S_0J_{xx}a_0^2$ is the spin-wave stiffness constant with $J_{xx}$ and $a_0^2$ being the exchange energy and the effective block spin volume.

In the nonmagnetic metal, the dynamics of the spin density $s$ can be described by the action:

$$S_N = \int dt \sum_{k} s^-_k(t)[(\chi_k(i\partial_t))^\dagger s^-_k(t),$$

where $s^-_k = (s^+ k \pm is^0 k)/2$ is defined by $s_k = N_N^{-1/2} \sum_{p} (\sigma^{+}_{p} + \sigma^{-}_{p} + \sigma^{+}_{p} + \sigma^{-}_{p})$, and $N_N$ being the Pauli matrices, the electron creation operator for spin projection $\uparrow$ and $\downarrow$, and the number of atoms in the nonmagnetic metal. The equilibrium spin-density propagator is given by

$$\chi_k(\omega) = \left( \begin{array}{cc} \chi^{R}_k(\omega) & \chi^{K}_k(\omega) \\ 0 & \chi^{A}_k(\omega) \end{array} \right),$$

with the following equilibrium condition:

$$\chi^{A}_k(\omega) = [\chi^{R}_k(\omega)]^*, \quad \chi^{K}_k(\omega) = 2i \text{Im} \chi^{R}_k(\omega) \coth(\frac{\nu}{2k_B T})$$

The retarded part of $\chi$ is given by $\chi^{R}_k(\omega) = \chi N(1 + \lambda^2 k^2 - i\omega \tau_d)^{-1}$ with $\chi N$, $\lambda$, and $\tau_d$ being the paramagnetic susceptibility, spin diffusion length, and spin relaxation time, the form of which is consistent with the corresponding diffusive Bloch equation [see Eq. (10) below].

Finally, the interaction between magnons and spin density at the interface is given by

$$S_{F,N} = \int dt \sum_{k,q} \frac{S_0 J_{sd}^{k-q}}{N_F N_N} m^-_q(t) \cdot s_k(t),$$

where $J_{sd}^{k-q}$ is the Fourier transform of $J_{sd}(r) = J_{sd} \xi_0(r)$ with $J_{sd}$ the s-d exchange interaction between conduction-electron spins and localized spins, and $\xi_0(r) = \sum_{r_0 \in N_{F}\mbox{-interface}} \delta(r - r_0)$.

It is instructive to point out that in the spin-wave region and in the classical limit with negligible quantum fluctuations, a system described by Eqs. (2), (3), and (4) is equivalent to a system described by the stochastic Landau-Lifshitz-Gilbert equation:

$$\partial_t M = [\gamma(H_{eff} + h) - \frac{4\alpha}{3} s] \times M + \frac{\alpha}{3} M \times \partial_t M,$$
deploying the equilibrium conditions [Eqs. (4) and (7)], we
the fluctuation-dissipation theorem.

magnon-mediated spin injection given in the Appendix A
into
satisfies
γM
3
\(\gamma M\)

\(\gamma M\)

where we have introduced the shorthand notation
\(\int_0^\infty \frac{d\omega}{2\pi} N_{int}\), and \(N_{int}\) is the number of localized spins at the \(N_1-F_1\) interface playing a role of the number of channels. The \(\omega\) integration can be performed by picking up only magnon poles under the condition \(\alpha \hbar \omega_q \ll k_B T_{N_1}, k_B T_{F_1}\) (always satisfied for YIG), giving

\[\int_0^\infty \text{Im} \chi_k(\omega) \text{Im} X_q(\omega) [\coth(\frac{\hbar \omega}{2 k_B T_{N_1}})] \approx -\frac{\text{Im} \chi_k(\omega_q)}{\hbar \omega_q} [\coth(\frac{\hbar \omega_q}{2 k_B T_{F_1}})].\]

By making the classical approximation \(\coth(\frac{\hbar \omega_q}{2 k_B T_{F_1}}) \approx \frac{2 k_B T_{F_1}}{\hbar \omega_q}\), we obtain

\[I_s^{N_1} = \frac{N_{int} J_{sd}^2 S_0^3 N_{sd}}{2 2^{\pi 3} \hbar^3 (\gamma M/n)^3} \frac{T_1}{k_B (T_{N_1} - T_{F_1})},\]  

where \(T_1 = \int_0^1 dx \int_0^1 dy \int_2 (x+y)^2 \frac{x^2 \sqrt{y}}{(x+y) \omega_0 S_0 \tau_{sd}/k_B T_{F_1}}\) with the dimensionless variables \(x = k \lambda \), and \(y = \hbar \omega_q/(2 J_{ex} S_0)\), and we used the relation \(N_F^{-1} \sum q = (2\pi)^{-2} \int \sqrt{y} dy\).  

III. MAGNON-MEDIATED SPIN SEEBECK EFFECT

Equation [13] means that, through the “local” process \(P_1\) shown in Fig. 3(a), the spin current is not injected into the nonmagnetic metal \(N_1\) when \(F_1\) and \(N_1\) have the same temperature. That is, the “local” process cannot explain the experiment where no temperature difference exists between the YIG film and the attached Pt film. A way to account for the experiment within the “local” picture is to invoke a difference between the phonon temperature and magnon temperature. In this paper, on the other hand, we take a different route and consider the effect of temperature gradient within the YIG film on the spin injection into the Pt terminal.

The basic idea of our approach is as follows. The above result [Eq. \[13\]] that the injected spin current vanishes when \(T_{F_1} = T_{N_1}\), originates from the equilibrium condition of the magnon propagator [Eq. \[1\]]. When magnons deviate from local thermal equilibrium by allowing the magnons to feel the temperature gradient inside the ferromagnet, the magnon propagator cannot be written in the equilibrium form, and it generates a nontrivial contribution to the thermal spin injection. The relevant “non-local” process \(P'_1\) is shown in Fig. 3(a) in which magnons feel the temperature difference between \(F_1\) and \(F_2\). The interaction between \(F_1\) and \(F_2\) is described by the action

\[S_{F-F} = \int_C dt \sum_{q,q'} \frac{2 J_{ex}^q q'}{N_F} m_q(t) \cdot m_{-q'}(t),\]

where \(J_{ex}^q q'\) is the Fourier transform of \(J_{ex}(r) = J_{ex} \xi_1(r) + \xi_1(r) = \sum_{q \in C \in F-F} \alpha_2 \delta(r - r_0)\).

We now regard the whole of the magnon lines appearing in the process \(P'_1\) as a single magnon propagator \(\delta \hat{X}_q(\omega)\), namely,

\[\delta \hat{X}_q(\omega) = \frac{1}{N_F^2} \sum q' \left[ J_{ex}^q q' \right] \hat{X}_q(\omega) \hat{X}_q(\omega) \hat{X}_q(\omega),\]
Then the propagator is decomposed into the local-equilibrium part and nonequilibrium part as

\[ \delta X_q(\omega) = \delta \tilde{X}_q^{\text{eq}}(\omega) + \delta \tilde{X}_q^{\text{neq}}(\omega), \tag{16} \]

where

\[ \delta \tilde{X}_q^{\text{eq}} = \begin{pmatrix} \delta X_q^{\text{eq},R} & \delta X_q^{\text{eq},K} \\ 0 & \delta X_q^{\text{eq},A} \end{pmatrix} \tag{17} \]

is the local-equilibrium propagator satisfying the local-equilibrium condition, i.e., \[ \delta X_q^{\text{eq},A} = [\delta X_q^{\text{eq},R}]^* \] and \[ \delta X_q^{\text{eq},K} = [\delta X_q^{\text{eq},R} - \delta X_q^{\text{eq},A}] \coth(\frac{\hbar \omega}{2k_B T}) \] with

\[ \delta X_q^{\text{eq},R}(\omega) = \frac{1}{N_F} \sum_{q'} |\tilde{\sigma}_{\text{ex}}^{-q'}|^2 \left[ X_q^{R}(\omega) \right]^2 X_q^{R}(\omega) \tag{18} \]

while

\[ \delta X_q^{\text{neq}} = \begin{pmatrix} 0 & \delta X_q^{\text{neq},K} \\ 0 & 0 \end{pmatrix} \tag{19} \]

is the nonequilibrium propagator with \[ \delta X_q^{\text{neq},K}(\omega) \] given by

\[ \delta X_q^{\text{neq},K}(\omega) = \sum_{q''} \frac{|\tilde{\sigma}_{\text{ex}}^{-q''}|^2}{N_F} \left[ X_q^{R}(\omega) - X_q^{A}(\omega) \right] \times \left[ X_q^{R}(\omega) \right]^2 \left[ \coth(\frac{\hbar \omega}{2k_B T_2}) - \coth(\frac{\hbar \omega}{2k_B T}) \right] \tag{20} \]

Note that the local equilibrium propagator [Eq. (17)] does not contribute to the “nonlocal” spin injection.

When we substitute Eq. (10) into Eq. (19) and use Eq. (11) with \[ X_q(\omega) \] being replaced by \[ \delta X_q(\omega) \], we obtain the following expression for the magnon-mediated thermal spin injection:

\[ I_s^{N_i} = \frac{N_{i\text{int}}^N}{\sqrt{2\hbar^2 N_p N_N}} \sum_{q', q''} \int \omega \left[ \Im \chi_k^R(\omega) \right. \times \left. |X_q^{R}(\omega)|^2 \Im X_{q'}^{R}(\omega) \coth(\frac{\hbar \omega}{2k_B T}) - \coth(\frac{\hbar \omega}{2k_B T_2}) \right], \tag{21} \]

where \[ N_{i\text{int}}^N \] is the number of localized spins at the \( F_1-F_2 \) interface, and we used \[ T_{N_i} = T_{F_i} = T_i \] for \( i = 1, 2 \). The \( \omega \) integration can be performed as before, giving

\[ \int \omega |\Im \chi_k^R(\omega)|^2 |X_q^{R}(\omega)|^2 |\Im X_{q'}^{R}(\omega)| \coth(\frac{\hbar \omega}{2k_B T}) - \coth(\frac{\hbar \omega}{2k_B T_2}) \]

(22)

where \[ \delta T = T_1 - T_2 \], \( \Lambda \) is the size of \( F_1 \) along the temperature gradient, and \[ T_2 \approx \frac{1}{2} (T_1 + \tau_T^e + \tau_T^f (2S_F J_{ex} \tau_{\text{int}}/\hbar)^2) \] is approximated as \[ T_2 \approx 0.1426 (T_2 \approx 0.337\hbar/2 S_F J_{ex} \tau_{\text{int}}) \] for \( 2 S_F J_{ex} \tau_{\text{int}}/\hbar \lesssim 1 \) (for \( 2 S_F J_{ex} \tau_{\text{int}}/\hbar \gg 1 \)).

The spin current \[ I_s^{N_2} \] injected into the right terminal \( N_3 \) can be calculated in the same manner by considering the process \( P_3 \), which gives \[ I_s^{N_3} = -I_s^{N_1} \] from the relation \( T_1 - T_2 = -(T_3 - T_2) \). The spin current \( I_s^{N_2} \) injected into the middle terminal \( N_2 \) vanishes because the two relevant processes \( (P_2 \text{ and } P'_2) \) cancel out. Therefore, we obtain the spatial profile of the injected spin current as shown in Fig. 3(b). Note that the effect of the spatial dependence of magnetization \( M[T(r)] \) through the local temperature \( T(r) \) is already taken into account in our treatment because the temperature dependence of \( M \) in the magnon region is automatically described by the number of thermal magnons discussed in this paper.

For an order of magnitude estimation, we compare Eq. (22) with the experiment.\(^2\) By using \[ T_H \approx 0.0037 \times 10^{-6} \, \text{ps}, \, w = 0.1 \, \text{mm}, \, \lambda_{\text{mag}} \approx 7 \, \text{nm}, \, \tau_{\text{sf}} \approx 1 \, \text{ps}, \, a = 2 \, \text{Å}, \, a_S = 12.3 \, \text{Å}, \, S_0 = 16, \, \alpha \approx 5 \times 10^{-5} \, \text{cm}^3/\text{g K}, \, \chi_0 = 1 \times 10^{-6} \, \text{cm}^3/\text{g K}, \, N_{i\text{int}} = 0.1 \times 4 \, \text{mm}^2/a_S^2, \, J_{ex} \text{ exchange coupling extracted from the previous ferromagnetic resonance experiments} \] \( (J_{ex} \approx 10 \, \text{meV}) \) can account for the spin Seebeck voltage \( V_{\text{SHE}}/\delta T \approx 0.1 \, \text{μV/K} \) observed at room temperature.

Finally, we comment on the issue of length scales associated with the SSE. In the original SSE experiment for a metallic ferromagnet, the signal maintained over several millimeters was a big surprise because the spin diffusion length for that system is much shorter than a millimeter. Concerning the magnon-mediated SSE in an insulating magnet, which we have discussed, it is of crucial importance to recognize that the length scale relevant to the SSE is related to magnon density fluctuations and is given by longitudinal fluctuations of magnons, while the magnon mean free path is related to magnon dephasing and is given by transverse fluctuations of magnons.\(^2\) It was shown by Mori and Kawasuki that these two length scales do not coincide with each other since they obey quite different dynamics, and it was demonstrated that in a certain situation the length scale of magnon density fluctuations (which is relevant to the SSE as well) is much
longer than the magnon mean free path [see Eq. (6.33) in Ref. 30 where the length scale of long-wavelength magnon density fluctuations is infinitely long].[31] The notion of these two different length scales is the key to understanding the length scales observed in the SSE experiment in an insulating magnet.  

IV. CONCLUSION

We have developed a theory of the magnon-mediated spin Seebeck effect in terms of the canonical framework of describing transport phenomena, i.e., the linear-response theory, and shown that it provides us with a qualitative as well as quantitative understanding of the spin Seebeck effect observed in a prototypical magnet, yttrium iron garnet. Because the carriers of spin current in this scenario are magnons, we can obtain a bigger signal for iron garnet.

The Gaussian action for conduction electrons in the density fluctuations is infinitely long.[30] Ref. 30 where the length scale of long-wavelength magnon interaction [Eq. (8)]. The spin current induced in the nonmagnetic metal $N_1$ is calculated as the rate of change of the spin accumulation in $N_1$, i.e., $I_{N_1}^s(t) = \sum_{r \in N_1} \langle \partial_t s^r(r, t) \rangle = \langle \partial_t \vec{s}_{\vec{k}0}(t) \rangle_{t \to 0}$, where $\langle \cdots \rangle$ means the statistical average at a given time $t$, and $\vec{s}_{\vec{k}} = \sqrt{N_N s_k}$ with $s$ being defined below Eq. (3).

The Heisenberg equation of motion for $s_{\vec{k}0}$ gives

$$\partial_t \vec{s}_{\vec{k}0} = \sum_{q, k} \frac{i J_{sd}^q - q S_0}{2N_F N_N \hbar} \left( m^+ \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} + m^- \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} \right)$$

$$= i \sum_{q, k} \frac{2 J_{sd}^q - q S_0}{2N_F N_N \hbar} \left( m^+ \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} - m^- \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} \right)$$

where we have used the relation $\vec{s}_{\vec{k}0} = \pm 2 \vec{s}_{\vec{k}0}^\perp$, and neglected a small correction term arising from the spin-orbit interaction assuming that the spin-orbit interaction is weak enough at the neighborhoods of the interface. Then, the statistical average of the above quantity gives the following spin current:

$$I_{N_1}^s(t) = \sum_{q, k} \frac{4 J_{sd}^q - q S_0}{2N_F N_N \hbar} \text{Re} C_{k, q}^< (t, t)$$

where $C_{k, q}^< (t, t') = -i \langle m^+ \vec{s}_{\vec{k}0} (t') \vec{s}_{\vec{k}0} (t) \rangle$ is the interface Green’s function. In the steady state, the Green’s function $C_{k, q}^< (t, t) \equiv \int_{-\infty}^{\infty} d\omega C_{k, q}^< (\omega) e^{-i\omega(t-t')}$. Adopting the representation $\tilde{C} = \left( \begin{array}{cc} C_R & C_K \\ C_K & C_A \end{array} \right)$ and using $C^< = \frac{1}{2} (C_R - C_K + C_A)$, we finally obtain

$$I_{N_1}^s = \sum_{q, k} \frac{2 J_{sd}^q - q S_0}{2N_F N_N \hbar} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \text{Re} C_{k, q}^R (\omega)$$

for the spin current $I_{N_1}^s$ in a steady state. As in the case of tunneling charge current driven by a voltage difference, the spin current $I_{N_1}^s$ can be calculated systematically.

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Appendix A: Linear-response expression of magnon-induced spin injection

The Gaussian action for conduction electrons in the nonmagnetic metal $N_i$ $(i = 1, 2, 3)$ is given by

$$\mathcal{S}_N = \int_C dt \sum_{p, p'} c_p^\dagger(t) \left\{ i \hbar \left( \epsilon_p \delta_{p, p'} + U_{p-p'} \left[ 1 + i \eta_{so} \sigma \times (p \times p') \right] \right) \right\} c_{p'}(t),$$

where $c_p^\dagger = (c_p^\dagger, c_{p+}^\dagger)$ is the electron creation operator for spin projection $\uparrow$ and $\downarrow$, $U_{p-p'}$ is the Fourier transform of the impurity potential $U_{\text{imp}} \sum_{\vec{n}_0} \delta (\vec{r} - \vec{r}_0)$, and $\eta_{so}$ measures the strength of the spin-orbit interaction.

At the ferromagnet/nonmagnetic-metal interface, the magnetic interaction between conduction-electron spin density and localized spin is described by the s-d interaction [Eq. (8)]. The spin current induced in the nonmagnetic metal $N_1$ can be calculated as the rate of change of the spin accumulation in $N_1$, i.e., $I_{N_1}^s(t) = \sum_{r \in N_1} \langle \partial_t s^r(r, t) \rangle = \langle \partial_t \vec{s}_{\vec{k}0}(t) \rangle_{t \to 0}$, where $\langle \cdots \rangle$ means the statistical average at a given time $t$, and $\vec{s}_{\vec{k}} = \sqrt{N_N s_k}$ with $s$ being defined below Eq. (3).

The Heisenberg equation of motion for $s_{\vec{k}0}$ gives

$$\partial_t \vec{s}_{\vec{k}0} = \sum_{q, k} \frac{i J_{sd}^q - q S_0}{2N_F N_N \hbar} \left( m^+ \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} + m^- \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} \right)$$

$$= i \sum_{q, k} \frac{2 J_{sd}^q - q S_0}{2N_F N_N \hbar} \left( m^+ \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} - m^- \vec{s}_{\vec{k}0} \cdot s_{\vec{k}0} \right)$$

where we have used the relation $\vec{s}_{\vec{k}0} = \pm 2 \vec{s}_{\vec{k}0}^\perp$, and neglected a small correction term arising from the spin-orbit interaction assuming that the spin-orbit interaction is weak enough at the neighborhoods of the interface. Then, the statistical average of the above quantity gives the following spin current:

$$I_{N_1}^s(t) = \sum_{q, k} \frac{4 J_{sd}^q - q S_0}{2N_F N_N \hbar} \text{Re} C_{k, q}^< (t, t)$$

where $C_{k, q}^< (t, t') = -i \langle m^+ \vec{s}_{\vec{k}0} (t') \vec{s}_{\vec{k}0} (t) \rangle$ is the interface Green’s function. In the steady state, the Green’s function $C_{k, q}^< (t, t) \equiv \int_{-\infty}^{\infty} d\omega C_{k, q}^< (\omega) e^{-i\omega(t-t')}$. Adopting the representation $\tilde{C} = \left( \begin{array}{cc} C_R & C_K \\ C_K & C_A \end{array} \right)$ and using $C^< = \frac{1}{2} (C_R - C_K + C_A)$, we finally obtain

$$I_{N_1}^s = \sum_{q, k} \frac{2 J_{sd}^q - q S_0}{2N_F N_N \hbar} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \text{Re} C_{k, q}^R (\omega)$$

for the spin current $I_{N_1}^s$ in a steady state. As in the case of tunneling charge current driven by a voltage difference, the spin current $I_{N_1}^s$ can be calculated systematically.
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