Abstract. The recently sharpened $H_0$ tension is argued not to be a result of data calibration or other systematic but an indication for the common nature of dark matter and dark energy. This conclusion is devised within modified weak-field General Relativity where the accelerated expansion of the Universe and the dynamics of galaxy groups and clusters are described by the same parameter, the cosmological constant. The common nature of the dark sector hence will result in intrinsic discrepancy/tension between the local and global determinations of values of the Hubble constant.

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1 Introduction

Recent measurements [1] increase the existing tension between the Hubble constant determinations from Planck satellite data [2] and lower redshift observations; the earlier studies and various approaches for resolving the tension are discussed in [1].

The $H_0$ tension we will consider within the approach of weak-field modified General Relativity (GR) which enabled the common description of the dark matter and dark energy by means of the same value of the cosmological constant $\Lambda$ [3,4,5]. That approach is based on the Newton’s theorem on the equivalency of the gravity of the sphere and of a point situated in its center and provides a natural way for the weak-field modification of GR, so that dark energy is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) equations while the dark matter in galaxy groups and clusters is described by the weak-field GR.

It is a principal fact that by now both the strong field GR has been tested by the discovery of gravitational waves, while the weak-field effects such as at the frame-dragging are traced by measurements of laser ranging satellites [6]. The weak-field modifications we are discussing below are by now far from being tested at satellite measurements and therefore the dynamical features of the local universe including of the galactic dark halos [7], galaxy groups [8,5], can serve as unique probes for such weak-field modifications of GR. Among other modified gravity tests are the accurate measurements of gravitational lenses [9], along with the effects in the Solar system [10] or traced from large scale matter distribution [11].

Thus, we show that if the cosmological constant $\Lambda$ describes both the accelerated expansion and dark matter at galaxy cluster scales, then it will lead to the intrinsic discrepancy in the global and local values of the Hubble constant.

2 Newton’s theorem and $\Lambda$

In [3] it is shown that the weak-field GR can involve the cosmological constant $\Lambda$, so that the metric tensor components have the form

\[
g_{00} = 1 - \frac{2Gm}{rc^2} - \frac{\Lambda r^2}{3}; \quad g_{rr} = \left(1 - \frac{2Gm}{rc^2} - \frac{\Lambda r^2}{3}\right)^{-1}.
\]

This follows from the consideration of the general function for the force satisfying Newton’s theorem on the identity of sphere’s gravity and that of a point situated in its center and crucially, then shell’s internal gravity is no more force-free [12]. That general function besides the $r^{-2}$ term contains also a second term

\[
f(r) = Ar^{-2} + Ar.
\]
When the modified Newtonian law (for the potential) is taken as weak-field GR, one has the constant $\Lambda$ as a second gravity constant along with the classical Newtonian constant $G$ \cite{3}. Namely, the second constant $\Lambda$, on the one hand, acts as the cosmological constant in the cosmological solutions of Einstein equations, on the other hand, enters in the low-energy limit of GR which hence is attributed to the Hamiltonian dynamics of galaxy groups and clusters \cite{5}, instead of commonly involving Newtonian potential.

Within isometry group representation the Lorentz group $O(1,3)$ acts as stabilizer subgroup of isometry group of 4D maximally symmetric Lorentzian geometries and depending on the sign of $\Lambda$ ($+,-,0$) one has the non-relativistic limits \cite{3}

\begin{align}
A > 0 & : O(1, 4) \rightarrow (O(3) \times O(1, 1)) \times R^6, \\
A = 0 & : IO(1, 3) \rightarrow (O(3) \times R) \times R^6, \\
A < 0 & : O(2, 3) \rightarrow (O(3) \times O(2)) \times R^6.
\end{align}

The $O(3)$ is the stabilizer group for the spatial geometry since for all three cases the spatial algebra is Euclidean

$$
E(3) = R^3 \rtimes O(3).
$$

Thus, the Newton’s theorem in the language of group theory can be formulated as each point of spatial geometry admitting the $O(3)$ symmetry.

The fact of no force-free inside a shell (in contrast to Newton’s law) fits the observational indications that the properties of galactic disks are determined by halos, see \cite{2}.

### 3 Local and global Hubble flows vs $\Lambda$

The Hubble-Lematre law as one of established pillars of modern cosmology is characterized by the Hubble constant $H_0$ which can be derived by various ways depending on the observational dataset. Namely, the Planck satellite provided the data on Cosmic Microwave Background (CMB) which within the $\Lambda$CDM model led to the following global value $H_0 = 67.66 \pm 0.42 \text{ km/s/Mpc}$, as well as $\Lambda = 1.11 \times 10^{-52} \text{ m}^{-2}$ \cite{13}. The recent analysis of Cepheid variables in Large Magellanic Cloud (LMC) by Hubble Space Telescope (HST) \cite{11} led to the local value $H = 74.03 \pm 1.42 \text{ km/s/Mpc}$. This discrepancy between the global and local values of the Hubble constant is the above mentioned tension.

Our Universe is considered to be described by FLRW metric

$$
\begin{equation}
\begin{aligned}
ds^2 &= -c^2 dt^2 + a^2(t) \left( \frac{1}{1 - kr^2} dr^2 + r^2 d\Omega^2 \right),
\end{aligned}
\end{equation}
$$

where depending on the sign of sectional curvature $k$, the spatial geometry can be spherical $k = 1$, Euclidean $k = 0$ or hyperbolic $k = -1$. Consequently, according to GR equations the 00-component of FLRW metric is

$$
\begin{equation}
\begin{aligned}
H^2 &= - \frac{k c^2}{a^2(t)} + \frac{\Lambda c^2}{3} + \frac{8\pi G \rho}{3},
\end{aligned}
\end{equation}
$$

where $H = \dot{a}(t)/a(t)$ is the Hubble constant.

Here an important point is the following. The Hubble-Lematre law originally was established for a sample of nearby galaxies which in contrast to principles of GR, do not move along the geodesic equations of FLRW. In other words, that law was observed at scales for which it should not be observed. Nevertheless, in spite of this apparent contradiction the local flow has been confirmed by observations: the detailed analysis of the nearby galaxy surveys reveal the local Hubble flow with $H_{loc} = 78 \pm 2 \text{ km/s/Mpc}$ \cite{14}.

We will now show that considering Eq.(1) as the weak-field limit of GR, it is possible to solve this tension. Namely, the global Hubble flow will be described by the cosmological constant of FLRW metric, while the local flow by the weak-field GR given by Eq.(1).

So, we are not allowed to use FLRW metric in local scales since the Local Supercluster galaxies do not move by FLRW geodesics, and while the pure Newtonian gravity cannot produce a repulsive force to cause the local Hubble flow, the modified gravity will produce an acceleration

$$
\begin{equation}
a = - \frac{GM}{r^2} + \frac{\Lambda c^2 r}{3}.
\end{equation}
$$

It is simple to find out the distance at which the acceleration of ordinary Newtonian term becomes subdominant with respect to the second term. In Table 1 the values for such distances are listed for different mass scales. For objects less massive than the Local Group (LG), that critical distance is located outside the object’s boundary, which means that
it cannot be observed. For LG, the critical distance is around 1.4 Mpc. Here, it is worth to mention that, since we have used Eq.(1) according to Newton’s theorem, this distance can be considered as the radius of a sphere which the whole mass of LG is concentrated at its center. Thus, we conclude that, for those objects located outside this radius we will be able to observe an outward acceleration. These results obtained based on Newton’s theorem are in agreement with other analysis [17].

Meantime considering Eq.(1), one can obtain the analogue of Eq.(6) in the weak-field limit

$$H^2 = \frac{\Lambda c^2}{3} + \frac{8\pi G \rho}{3}.$$  

(8)

However, in contrast to Eq.(6), in the above equation $\rho$ stands only for matter density. Thus, one can conclude that the $H$ observed by HST in local scales is not the one obtained via Eq.(6) by considering the FLRW metric. It is a local effect which can be described by Eq.(8). However, before considering the weak-field limit equations for local flow, first let us take a look at the Eq.(1) itself. According to principles of GR, the weak-field limit is defined when $\phi/c^2 \ll 1$, where $\phi$ is the weak-field potential. Now, by taking this into consideration, besides the Newtonian term a new limit is defined at large distances

$$\frac{Ar^2}{3} \ll 1, \quad r \simeq 1.46 \times 10^{26}m = 5.33 \text{Gpc}.$$  

(9)

Considering the fact that, the local Hubble flow is observed in few Mpc scales, we are allowed to use the Eq.(8) to describe that flow. By taking cosmological parameters [13], Eq.(6) confirms that the total matter density in our Universe is $\rho = 2.68 \times 10^{-27} kgm^{-3}$. However, by substituting $H = 74.03 \pm 1.42 km s^{-1} Mpc^{-1}$, the matter density which causes the observed local Hubble flow will be $\rho_{loc} = 4.37^{+0.39}_{-0.36} \times 10^{-27} kgm^{-3}$.

Now, in order to complete our justification we need to check the mean density of the local astrophysical structures. From hierarchical point of view the LG is located about 20 Mpc away from Virgo cluster [15]. The, Virgo cluster itself together with LG is in a larger Virgo supercluster [13], which itself is the part of Laniakea supercluster [16]. Considering the mass and their distances from LG, it is possible to find the distance where the density of these objects become exactly equal to $\rho_{loc}$. These results are exhibited in Table 2.

From these results it becomes clear that not only the error bars fully cover each other, but also the whole range of the local flow is covered by these values i.e. from 1.70 to 7.07 Mpc. Meantime, according to Eq.(7) the critical distance of Virgo supercluster from LG roughly is 7.27 Mpc, which means that the objects beyond that distance are gravitationally bounded to the supercluster. Considering the upper limit of Table 2 it turns out that there is no overlapping between the bounded objects and those who move away according to Eq.(8). Furthermore, these values exactly coincide with the density of Virgo cluster at distances in which Virgocentric flow changes to the FLRW linear Hubble-Lematre law [17].

Thus, the $H_0$ tension is not a calibration discrepancy but is a natural consequence of presence of $\Lambda$ in GR as well as weak-field limit equations. While for global value we have to consider the Eq.(6) as the immediate consequence of FLRW metric and the cosmological parameters defined as

$$\Omega_k = -\frac{k^2c^2}{a^2(t)H^2}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}, \quad \Omega_m = \frac{8\pi G \rho}{3H^2}.$$  

(10)

Table 1. Critical distance for different objects

| Central Object | Mass (Kg) | Radius (m) |
|----------------|-----------|------------|
| Earth          | $5.97 \times 10^2$ | $4.92 \times 10^2$ |
| Sun            | $2 \times 10^3 = M_\odot$ | $3.42 \times 10^8$ |
| Sgr A*         | $4.3 \times 10^8 M_\odot$ | $5.56 \times 10^9$ |
| Milky Way      | $1.5 \times 10^{10} M_\odot$ | $3.91 \times 10^{12}$ |
| Local Group    | $2 \times 10^{12} M_\odot$ | $4.31 \times 10^{14}$ |

Table 2. Distances of objects where the density is $\rho_{loc}$

| Object           | Mass (Kg) | Distance from LG (Mpc) |
|------------------|-----------|------------------------|
| Local Group      | $2 \times 10^{12} M_\odot$ | $1.95 \pm 0.06$ |
| Virgo cluster    | $1.2 \times 10^{10} M_\odot$ | $3.45 \pm 0.52$ |
| Virgo supercluster | $1.48 \times 10^{11} M_\odot$ | $2.26 \pm 0.56$ |
| Laniakea         | $10^{17} M_\odot$ | $5.00 \pm 0.25$ |
The local value of $H$ is obtained by weak-field limit equations and depends strictly on the local density of matter distribution.

Note that, besides the above mentioned two evaluations of $H$, other independent measurements also confirm this discrepancy. Among such measurements are those of the Dark Energy Survey (DES) Collaboration, where the so-called inverse distance ladder method based on baryon acoustic oscillations (BAO) is used [19]. Considering the BAO as a standard ruler in cosmology, it turns out that its scale is roughly equal to 150 Mpc which clearly exceeds the typical distance of our local structures (the Virgo cluster etc.). Namely, the relevant SNe Ia are located at redshifts $0.018 < z < 0.85$ [19], which means that according to the Planck data [13] such objects are located at distances $80\,\text{Mpc} < r < 3\,\text{Gpc}$. Thus, by comparing these scales with the typical distance to our local structures, one concludes that the measured $H$ for these observations should mainly be induced by cosmological parameters. This statement is justified by their measured value $H = 67.77 \pm 1.30\,\text{km/s/Mpc}^{-1}$.

Other measurements, again using BAO, are those of [20], where like the DES survey, the distances are $1.8\,\text{Gpc} < r < 6.2\,\text{Gpc}$ and yield $H = 67.6^{+0.8\,}_{-1.2}\,\text{km/s/Mpc}^{-1}$.

Thus, one can conclude that there are two different $H$s of two different scales, local and global ones. Consequently, the measurement of these two quantities will depend on scales attributed by the observations. Namely, for observations of local scales it is expected to get the local $H$, while moving to cosmological scales i.e. beyond the Virgo cluster, the measurements should yield the global $H$.

Note one more important point: although currently the numerical values of these two different $H$s are close to each other, their physical content is totally different. Namely, this semi-coincidence is due to the fact that, for the global case the density in Eq.(3) is the current mean density in the Universe. At earlier phases of the Universe the radiation density had a major contribution to the mean density

$$\Omega_\rho = \Omega_m + \Omega_r.$$  \hspace{1cm} (11)

Also, current observations [13] indicate close to zero curvature of the Universe, $k=0$, and hence Eq.(6) will be similar to the weak-field equation. In other words, while for the local flow - no matter in which era - the contribution of matter density would have been the dominant one, for the global flow the contribution to the density in Eq.(6) was different for other cosmological eras where the radiation and $k$ were not negligible.

Considering the FLRW metric’s Hubble constant i.e. the global $H$ for different eras, one has

$$H(t) = H_0[\Omega_m a^{-3}(t) + \Omega_r a^{-2}(t) + \Omega_k + \Omega_A]^{1/2},$$ \hspace{1cm} (12)

where $H_0$ is the current value of the global Hubble constant. In this sense, the above statement about the differences between $H$s will be also true as the Universe tends to de Sitter phase. In that case, all $\Omega$s except $\Omega_A$ will gradually tend to zero. But again, for the local measures one still will have the same non-zero matter density.

4 Conclusions

The $H_0$ tension can be not a result of data calibration/systematic but a genuine indication for the common nature of the dark matter and dark energy. This conclusion is argued above based on the Newton’s theorem and resulting weak-field limit of General Relativity which includes the $\Lambda$ constant. Within that approach while the Friedmannian equations with the $\Lambda$ term are describing the accelerated Universe, the same $\Lambda$ is responsible for the dynamics of galaxy groups and clusters. Correspondingly, the global Hubble constant derived from the CMB and the local one devised from the galaxy surveys, including within the Local Supercluster, have to differ.

Then, the long known so-called Local Hubble flow [14] i.e. when the galaxies within the Local Supercluster are fitting the Hubble-Lematre law while the galaxies themselves are not moving via geodesics of FLRW metric, finds its natural explanation within the metric Eq.(1).

Accurate studies of the dynamics of galactic halos, groups and galaxy clusters, the gravitational lensing, can be decisive for further probing of described weak-field GR and the common nature of the dark sector.

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References

1. Riess A.G. et al. [arXiv:1903.07603] (2019)
V.G. Gurzadyan, A. Stepanian: $H_0$ tension: clue to common nature of dark sector?

2. Ade P. A. R. et al, A&A, 594, A13 (2016)
3. Gurzadyan V.G., Stepanian A., Eur. Phys. J. C, 78, 632 (2018)
4. Gurzadyan V.G., Eur. Phys. J. Plus, 134, 98 (2019)
5. Gurzadyan V.G., Stepanian A., Eur. Phys. J. C, 79, 169 (2019)
6. Ciufolini I. et al, Eur. Phys. J. C 76, 120 (2016)
7. Gurzadyan V.G. et al, A&A, 609, A131 (2018)
8. Karachentsev I.D., Kashibadze O.G., Karachentseva V.E., Astrophys. Bull. 72, 111 (2017)
9. Gurzadyan V.G., Stepanian A., Eur. Phys. J. C, 78, 869 (2018)
10. S. Kopeikin, M. Erofeimsky, G. Kaplan, Relativistic celestial mechanics of the solar system, (Wiley, 2001)
11. M. Eingorn, ApJ, 825, 84 (2016)
12. Gurzadyan V.G., Observatory, 105, 42 (1985)
13. Planck Collaboration, arXiv:1807.06209 (2018)
14. Karachentsev I.D., et al, MNRAS, 393, 1265 (2009)
15. Fouque P. et al, A&A, 375, 3p (2001)
16. Tully R.B. et al, Nature, 513, 71 (2014)
17. Chernin A.D. et al, A&A, 520, A104 (2010)
18. Einasto M. et al, A&A, 476, 2 (2007)
19. DES Collaboration, arXiv:1811.02376 (2018)
20. Ryan J., Chen Y., Ratra B., arXiv:1902.03196 Submitted to MNRAS, (2019)