Neutrino processes in the $K^0$ condensed phase of color flavor locked quark matter

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Abstract

We study weak interactions involving Goldstone bosons in the neutral kaon condensed phase of color flavor locked quark matter. We calculate the rates for the dominant processes that contribute to the neutrino mean free path and to neutrino production. A light $K^+$ state, with a mass $\tilde{m}_{K^+} \propto (\Delta/\mu) (\Delta/m_s)(m_d - m_u)$, where $\mu$ and $\Delta$ are the quark chemical potential and superconducting gap respectively, is shown to play an important role. We identify unique characteristics of weak interaction rates in this novel phase and discuss how they might influence neutrino emission in core collapse supernova and neutron stars.

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I. INTRODUCTION

At high baryon density, where the baryon chemical potential $\mu$ is very large compared to the strange quark mass $m_s$, three flavor quark matter is expected to be in a symmetric phase called the Color Flavor Locked (CFL) phase, in which BCS like pairing involves all nine quarks [1]. This, color superconducting, phase is characterized by a gap in the quark excitation spectrum. Model calculations indicate that the gap $\Delta \sim 100$ MeV for a quark chemical potential $\mu \sim 500$ MeV [2, 3]. In this phase, the $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B$ symmetry of QCD is broken down to the global diagonal $SU(3)$ symmetry. The lightest excitations in this phase are the nonet of pseudo-Goldstone bosons transforming under the unbroken, global diagonal $SU(3)$ as an octet plus a singlet and a massless mode associated with the breaking of the global $U(1)_B$ symmetry. At lower density, those that are of relevance to neutron stars, the baryon chemical potential $\mu$ is similar to $m_s$. At these densities, the strange quark mass induces a stress on the SU(3) flavor symmetric CFL state. Bedaque and Schafer [4] have shown that the stress induced by the strange quark mass can result in the condensation of neutral kaons in the CFL state. This less symmetric phase, which is called the CFLK$^0$ phase, breaks hypercharge $U(1)_Y$ and isospin symmetries and we can expect its low energy properties to be quite distinct from the CFL phase. For a detailed discussion of various meson condensed phases in CFL quark matter and their possible role in compact stars see Ref. [5].

In this article we extend earlier work (see Refs. [6, 7]) on neutrino emission and propagation rates in the CFL phase to the CFLK$^0$ phase. As discussed in detail in these aforementioned references, novel phases inside compact stars, if they should exist, influence the stars thermal evolution. In particular, neutrino production and propagation rates in these novel phases can directly influence observable aspects such as the supernova neutrino emission rate and longer term neutron star cooling rates [8].

II. THERMODYNAMICS OF THE CFLK$^0$ PHASE

The low energy excitations about the $SU(3)$ symmetric CFL ground state can be written in terms of the two fields: $B = H/(\sqrt{24}f_H)$ and $\Sigma = e^{2i(\pi/f_s + \eta'/f_A)}$, representing the Goldstone bosons of broken baryon number $H$, and of broken chiral symmetry, the pseudo-scalar
octet $\pi$, and the pseudo-Goldstone boson $\eta'$, arising from broken approximate $U(1)_A$ symmetry. The leading terms of the effective Lagrangian describing the octet Goldstone boson field $\pi$ is given by

\begin{equation}
\mathcal{L} = \frac{1}{4} f_\pi^2 \left[ \text{Tr} \nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \text{Tr} \nabla \Sigma \cdot \nabla \Sigma^\dagger \right] + f_\pi^2 \left[ \frac{a}{2} \text{Tr} \tilde{M} (\Sigma + \Sigma^\dagger) + \frac{\chi}{2} \text{Tr} M (\Sigma + \Sigma^\dagger) \right],
\end{equation}

where $\nabla_0 \Sigma = \partial_0 \Sigma - i (\mu_Q Q - X L) \Sigma - \Sigma (\mu_Q Q - X R)$.

The decay constant $f_\pi = 0.21 \mu$ has been computed previously and is proportional to the quark chemical potential. The quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$. $X_{L,R}$ are the Bedaque-Schafer terms: $X_L = \frac{M M^\dagger}{2\mu}$, $X_R = \frac{M^\dagger M}{2\mu}$, $\tilde{M} = |M|M^{-1}$ and $\mu_Q$ is the electric charge chemical potential associated with the unbroken $U(1)$ in the CFL phase.

A finite baryon chemical potential breaks Lorentz invariance of the effective theory. The temporal and spatial decay constants can thereby differ. This difference is encoded in the velocity factor $v$ being different from unity. An explicit calculation shows that $v = 1/\sqrt{3}$ and is common to all Goldstone bosons, including the massless $U(1)_B$ Goldstone boson.

At asymptotic densities, where the instanton induced interactions are highly suppressed and the $U(1)_A$ symmetry is restored, the leading contributions to meson masses arise from the $\text{Tr} \tilde{M} \Sigma$ operator whose coefficient $a$ has been computed and is given by $a = 3 \Delta^2 \pi^2 f_\pi^2$.

At densities of relevance to neutron stars the instanton interaction may become relevant. In this case a $\langle \bar{q} q \rangle$ condensate is induced. Consequently, the meson mass can receive a contribution from the operator $\text{Tr} M \Sigma$. Its coefficient, $\chi$, at low density is poorly known and is sensitive to the instanton size distribution and form factors. Although, conservative current estimates indicates that the instanton contribution to the $K^0$ mass for $\mu \sim 400$ MeV lies in the range $5 - 120$ MeV theory favors a value that is $\lesssim 30$ MeV. The $\eta'$ mass is also poorly known at moderate density. The contribution to the $\eta'$ mass due to the $U(1)_A$ anomaly, which does not vanish in the chiral limit, is not known. If this contribution is negligible, the neutral $\pi^0$, $\eta$ and $\eta'$ mix due to the explicit breaking of SU(3)$_{\text{flavor}}$ by the quark masses. In our investigation here we will assume that the anomalous contribution to the $\eta'$ mass is large and that the $\eta'$ decouples from the low energy theory. We will return to comment on how a light $\eta'$ will affect our results in §IV.

The meson masses, written in terms of the coefficients $a$ and $\chi$ are given by

$$m_{\pi^\pm}^2 = a(m_u + m_d)m_s + \chi(m_u + m_d)$$
The dispersion relations for Goldstone modes in the CFL phase are unusual, as will become clear from the following discussion. They are easily computed by expanding the effective Lagrangian to second order in meson fields

\[ E_{\pi^\pm}(p) = \sqrt{v^2 p^2 + m_{\pi^\pm}^2} \]
\[ E_{K^+}(p) = -X + \sqrt{v^2 p^2 + m_{K^+}^2} \]
\[ E_{K^-}(p) = X + \sqrt{v^2 p^2 + m_{K^-}^2} \]
\[ E_{K^0}(p) = -X + \sqrt{v^2 p^2 + m_{K^0}^2} , \]

(3)

where \( X = m_s^2/2\mu \). We note that in deriving the above relations we have neglected terms of order \( m_{\text{light}}^2/\mu \) since they are negligible compared to all other relevant scales in the problem, namely \( \mu, m_s \) and \( \Delta \). Meson dispersion relations violate Lorentz invariance and the induced effective chemical potential arising from the analysis of Bedaque and Schafer \[4\] breaks the energy degeneracy of the kaons and anti-kaons. In Ref. \[6\] we had chosen the instanton contribution to the \( K^0 \) mass is \( \sim 50 \text{ MeV} \) (corresponding to \( \chi \sim 15 \text{ MeV} \)) at \( \mu = 400 \text{ MeV} \) and \( \Delta = 100 \text{ MeV} \). For this choice the kaon mass is too large to allow for \( K^0 \) condensation.

If the instanton contribution is small at \( \mu \sim 400 \text{ MeV} \), the CFL phase becomes unstable to \( K^0 \) condensation when \( E_{K^0}(p = 0) < 0 \) or \( X \geq m_{K^0} \[4\] \). In this article, wherein we investigate weak interaction processes in the \( K^0 \) condensed phase we assume that the instanton contribution is negligible at \( \mu = 400 \text{ MeV} \) and set \( \chi = 0 \). Furthermore, the numerical results we present in subsequent sections will be for the specific choice of \( \mu = 400 \text{ MeV}, \Delta = 100 \text{ MeV}, m_u = 3.75 \text{ MeV}, m_d = 7.5 \text{ MeV} \) and \( m_s = 150 \text{ MeV} \).

In the CFL\( K^0 \) phase the ground state expectation value of \( \Sigma = \Sigma_0 \neq 1 \) (in the symmetric CFL phase the expectation value of \( \Sigma = 1 \)). As discussed in Ref. \[5\], the CFL\( K^0 \) phase is characterized by

\[ \Sigma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & i \sin \theta \\ 0 & i \sin \theta & \cos \theta \end{bmatrix} , \]

(4)

where \( \cos \theta = m_{K^0}^2/X^2 \) and the number density of condensate \( n_{K^0} = f_\pi^2 X \sin^2 \theta \).

To study the spectrum of low energy excitations in this phase we perturb about this new ground state \[13\]. We can incorporate the effect of the background kaon condensate by
replacing $\Sigma = \exp(2i\pi/f_\pi)$ by $\tilde{\Sigma} = \xi \Sigma \xi$ in Eq. (1), where $\xi = \sqrt{\Sigma_0}$. The matrix $\pi = \pi_a T_a$ where $T_{a=1..8}$ are the generators of SU(3) and $\pi_{a=1..8}$ are real scalar fields characterizing the excitations about the CFLK$^0$ phase. Replacing $\Sigma \rightarrow \tilde{\Sigma}$ in Eq. (1) and expanding to quadratic order in the fields $\pi_{a=1..8}$, we obtain the leading order Lagrangian

$$\mathcal{L} = \text{Tr}[\partial_t \pi \partial_t \pi] - v^2 \text{Tr}[\nabla \pi \cdot \nabla \pi] - i \text{Tr}[\partial_t \pi ([\mu_r, \pi] + [\mu_l, \pi])]$$

$$- \text{Tr}[[\mu_r, \pi] [\mu_l, \pi]] - a \text{Tr}[(\tilde{M} + \tilde{M})\pi^2] - \chi \text{Tr}[(M + M)\pi^2].$$

(5)

where

$$\mu_r = \xi (\mu_Q Q + X_R) \xi^\dagger \quad \mu_l = \xi^\dagger (\mu_Q Q + X_L) \xi$$

(6)

$$\tilde{M}_r = \xi^\dagger \det M M^{-1} \xi^\dagger \quad \tilde{M}_l = \xi \det M M^{-1} \xi$$

(7)

$$M_r = \xi^\dagger M \xi^\dagger \quad M_l = \xi M \xi.$$  

(8)

The equations of motion for the fields $\pi_{a=1..8}$ are given by

$$(\omega^2 - v^2 k^2) \pi_a = \kappa_{ab} \pi_b$$

where $\omega, k$ are the energy and momenta of the propagating modes.

Mixing arising due to non-diagonal components of $\kappa_{ab}$ is sparse. Spontaneous breaking of $U(1)_Y$ in the $K^0$ condensed phase leads to strong mixing between $K^0$ and $\bar{K}^0$ states. The dispersion relations for these neutral kaons are given by

$$\omega^2_{K_1} = v^2 p^2 + \frac{X^2}{2} \left[ (1 + 3 \cos^2 \theta) - \sqrt{(1 + 3 \cos^2 \theta)^2 + 16 \cos^2 \theta \frac{v^2 p^2}{X^2}} \right]$$

(10)

$$\omega^2_{K_2} = v^2 p^2 + \frac{X^2}{2} \left[ (1 + 3 \cos^2 \theta) + \sqrt{(1 + 3 \cos^2 \theta)^2 + 16 \cos^2 \theta \frac{v^2 p^2}{X^2}} \right].$$

(11)

For $v \, p \ll X$ they are given by

$$\omega^2_{K_1} = \frac{1 - \cos^2 \theta}{1 + 3 \cos^2 \theta} v^2 p^2 + \frac{16 \cos \theta^4}{(1 + 3 \cos^2 \theta)^3} \frac{v^4 p^2}{X^2} + O \left( \frac{v^6 p^6}{X^4} \right)$$

(12)

$$\omega^2_{K_2} = (1 + 3 \cos^2 \theta) X^2 + \frac{1 + 7 \cos^2 \theta}{1 + 3 \cos^2 \theta} v^2 p^2 + \frac{16 \cos \theta^4}{(1 + 3 \cos^2 \theta)^3} \frac{v^4 p^2}{X^2} + O \left( \frac{v^6 p^6}{X^4} \right).$$

(13)

This simple parameterization was originally suggested by David Kaplan, see also Ref. [14].
FIG. 1: Left panel: The excitation energy (see Eq. (15)) of the charged Goldstone modes in the $K^0$ phase. The variation with temperature arises because of the induced charge chemical potential. The square symbols on the y-axis correspond to the masses at $T = 0$. Right Panel: Charged particle densities normalized by the photon number density and the induced electric charge chemical potential normalized by the temperature in the CFLK$^0$ phase.

The $K_1$ mode is the massless Goldstone boson which we expected on general grounds since the ground state breaks hypercharge symmetry. As an aside, we note that when iso-spin is not explicitly broken by the quark masses, the $K^+$ is also massless mode and has a quadratic dispersion relation $\omega_{K^+} = v^2 p^2/2Y + (v^4 p^4)$. This latter mode, with its quadratic dispersion relation, has been shown to account for two broken generators and hence alters the number of expected Goldstone modes [13, 15].

In contrast, since $U(1)_Q$ remains unbroken the charged kaon (and pion) states do not mix. The dispersion relations for the charged kaons and pions are given by

$$\omega_{K^\pm} = \mp(Y + \mu_Q) + \sqrt{v^2 p^2 + Y^2 + \delta m^2} \quad (14)$$

$$\omega_{\pi^\pm} = \mp(Z + \mu_Q) + \sqrt{v^2 p^2 + Z^2 + m_{\pi^\pm}^2}, \quad (15)$$

where $Y = X(1+\cos \theta)/2$, $Z = X(1-\cos \theta)/2$ and $\delta m^2 = m_{K^+}^2 - m_{K^0}^2$. The mass of the $K^+$ mode in the CFLK$^0$ phase $\tilde{m}_{K^+} = \sqrt{Y^2 + \delta m^2} - Y$ and is the lightest charged excitation. When the condensate amplitude is large, i.e. $\cos \theta \simeq 0$, the $\pi^+$ is also a relatively light
excitation with a mass \( \tilde{m}_{\pi^+} = \sqrt{Z^2 + m^2_\pi} - Z \).

As discussed in Ref. [6], differences in the excitation energies of mesons with positive and negative charge will result in a net charge density in the meson gas at finite temperature. With increasing temperature a finite electric charge chemical potential is induced to ensure that matter is electrically neutral. The excess positive electric charge of the meson gas is compensated by electrons with chemical potential \( \mu_e = -\mu_Q \). Fig. 1 shows the excitation energy and the number density of charged mesons (normalized by the photon number density \( n_\gamma = 2\zeta(3)T^3/\pi^2 \)) and the electron chemical potential \( \mu_e \) (normalized by the temperature) as a function of the ambient temperature. The two massless modes, namely the \( H \) and \( K_1 \) are the most abundant species. The ratio \( n_H/n_\gamma = 1/(2v^3) \simeq 2.6 \), and for \( T \ll X \) the ratio \( n_{K_1}/n_\gamma = 1/v^3_{K_1} \), where \( v_{K_1} = \sqrt{(1 - \cos^2 \theta)/(1 + 3 \cos^2 \theta)} \) \( v \) is the velocity of the \( K_1 \) mode. For \( T \ll X \) and for the numerical values of the parameters chosen, we find that \( n_{K_1}/n_\gamma \simeq 4 \). This indicates that the thermodynamic properties of the CFLK\(^0\) phase is dominated by the \( H \) and \( K_1 \) modes. The low temperature specific heat of the \( K^0 \) phase is roughly twice as large as the CFL phase.

On general grounds we can expect the amplitude of the kaon condensate to decrease with increasing temperature. A precise calculation of the critical temperature at which the condensate melts is beyond the scope of this work. However, we can make a rough estimate by noting that \( X = m_s^2/2\mu \) is the important dimensional scale that characterizes the condensate at zero temperature (we assume a robust condensate that \( X \gg m_{K^0} \)). A weakly interacting Bose Einstein condensate melts when the thermal wavelength becomes comparable to the inter-particle distance in the ground state. The inter-particle distance in the ground state \( l = (1/n_{K^0})^{1/3} \) where \( n_{K^0} \simeq f_\pi^2/X \). Numerical values of the parameters employed, we can expect \( T_c \gtrsim 50 \) MeV. For \( T \) small compared to \( T_c \), it is reasonable to assume that finite temperature effects on the propagation of Goldstone modes in the CFLK\(^0\) phase can be ignored. Further, we note that corrections due to thermal loops are suppressed by the factor \( T/f_\pi \) due to the derivative coupling between the mesons. In the subsequent discussion of weak interaction rates we will restrict ourselves to these low temperatures where \( T \ll T_c \lesssim f_\pi \).
We now turn to the calculation of weak interaction rates in the CFLK\(^0\) phase. To compute the weak couplings of Goldstone modes in the \(K^0\) phase, which we represented by a \(3 \times 3\) matrix \(\Phi\), we gauge the chiral Lagrangian in the presence of the background field characterized by \(\xi\). We replace the derivative term of the original Lagrangian by the covariant derivative and this generates the leading order couplings. The gauged chiral Lagrangian is given by

\[
\mathcal{L} = \frac{f^2}{4} \text{Tr} \left[ D_\rho \Sigma_\Phi (D^\rho \Sigma_\Phi)^\dagger \right],
\]

where the covariant derivative

\[
D_\rho \Sigma_\Phi = \nabla_\rho \Sigma_\Phi - \frac{ig}{\sqrt{2}} (W_\rho^+ \tau^+ + W_\rho^- \tau^-) \Sigma_\Phi - \frac{ig}{\cos \theta_W} Z_\rho (\tau_3^W \Sigma_\Phi - \sin \theta_W [Q, \Sigma_\Phi]) - i \tilde{e} A_\rho [Q, \Sigma_\Phi],
\]

\[
\nabla \Sigma_\Phi = (\partial_0 \Sigma_\Phi + i [X_L \Sigma_\Phi - i \Sigma_\Phi X_R], \vec{\nabla} \Sigma_\Phi), \quad \text{and}
\]

\[
\Sigma_\Phi = \xi \exp \left( \frac{2i}{f_\pi} \Phi \right) \xi.
\]

In the above equation, \(\theta_W\) is weak mixing angle, \(Q\) is the quark charge matrix, \(\tau_3^W = 1/2 \text{ diag } (1, -1, -1)\) is the weak isospin matrix and \(\tau^\pm\) are usual charged current raising and lower operators which include the Cabibo mixing and are given by

\[
\tau^+ = \begin{bmatrix} 0 & \cos \alpha & \sin \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tau^- = \begin{bmatrix} 0 & 0 & 0 \\ \cos \alpha & 0 & 0 \\ \sin \alpha & 0 & 0 \end{bmatrix}.
\]

As discussed earlier the factor \(v = 1/\sqrt{3}\) breaks Lorentz invariance and Minkowski product is given by

\[
D_\rho \Sigma_\Phi (D^\rho \Sigma_\Phi)^\dagger = D_0 \Sigma_\Phi (D^0 \Sigma_\Phi)^\dagger - v^2 \vec{D} \Sigma_\Phi (\vec{D} \Sigma_\Phi)^\dagger.
\]

Expanding the kinetic term in inverse powers of \(f_\pi\) we can identify the leading order couplings of the Goldstone modes to the neutral and charged weak currents. At order \(f_\pi\), the charged current weak interactions is described by

\[
\mathcal{L}_{cc}^{(1)} = -\frac{g f_\pi}{\sqrt{2}} \text{Tr} \left[ \nabla^R \Phi \left[ \hat{\tau}^+ W^+ + \hat{\tau}^- W^- \right] \right],
\]

where

\[
\nabla^R \Phi = (\partial_0 \Phi - i [\Phi, \xi X_R \xi^\dagger], \vec{\nabla} \Phi),
\]

\[
\hat{\tau}^+ = \xi^\dagger \tau^+ \xi, \quad \hat{\tau}^- = \xi^\dagger \tau^- \xi.
\]
We note that the condensate, which introduces mixing between down and strange quarks, modifies the Cabibo suppression of charged current reactions. The *rotated* raising and lowering operators $\tilde{\tau}_\pm$ are given by

$$
\tilde{\tau}^+ = \begin{bmatrix}
0 & \cos \tilde{\alpha}^* & \sin \tilde{\alpha}^* \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \\
\tilde{\tau}^- = \begin{bmatrix}
0 & 0 & 0 \\
\cos \tilde{\alpha} & 0 & 0 \\
\sin \tilde{\alpha} & 0 & 0
\end{bmatrix},
$$

where

$$
\sin \tilde{\alpha} = \cos \frac{\theta}{2} \sin \alpha - i \sin \frac{\theta}{2} \cos \alpha, \hspace{1cm} (24)
$$

$$
\cos \tilde{\alpha} = \cos \frac{\theta}{2} \cos \alpha - i \sin \frac{\theta}{2} \sin \alpha. \hspace{1cm} (25)
$$

Note, although $\tilde{\alpha}$ is complex, $|\sin \tilde{\alpha}|^2 + |\cos \tilde{\alpha}|^2 = 1$.

The Lagrangian describing neutral current interactions at leading order can be obtained similarly from Eq. 17. Here, since $\xi$ commutes with both $\tau^W_3$ and $Q$, we find that

$$
\mathcal{L}^{(1)}_{nc} = -\frac{g_f}{\cos \Theta_W} Z^\sigma T_r \left[ \nabla^\sigma \Phi \tau^W_3 \right]. \hspace{1cm} (26)
$$

A. Neutrino Opacity

Two classes of processes involving mesons contribute to the neutrino opacity. They are $\nu \to e\phi$ and $\nu\phi \to e$ where $\phi = K^\pm, \pi^\pm$. These processes are kinematically forbidden in the vacuum since meson dispersion relations are restricted to be time-like with $\omega(p) > p$. In the medium, where Lorentz symmetry is broken, mesons on mass-shell can acquire a space-like dispersion relation. This allows for novel processes wherein a relativistic neutrino can radiate or absorb mesons. These processes dominate the opacity since they appear at leading order and are proportional to $f_\pi$. Other processes such as $\nu e \to \phi$ are also possible and their contribution is proportional to the electron/positron density. Further, since there are two relativistic particles in the initial state (typical electrons have momenta $p \approx 3T \gg m_e$) the kinematic constraints for producing a meson final state are restrictive. Consequently, the contribution of these reactions is found to be negligible.

The neutral current process involving the massless baryon number Goldstone boson which we label as $H$ is identical to that in the CFL phase. The contribution of reactions $\nu H \to \nu$
and $\nu \rightarrow H\nu$ to the neutrino opacity have been computed previously \cite{6} and are given by

$$
\frac{1}{\lambda_{\nu \rightarrow H\nu}(E_{\nu})} = \frac{256}{45\pi} \left[ \frac{v(1-v)^2(1+\frac{4}{3})}{(1+v)^2} \right] G_F^2 f_H^2 E_{\nu}^3 \tag{27}
$$

where \(\gamma = 2vE_{\nu}/(1-v)T\) and the integrals \(g_n(\gamma)\) are defined by the relation \(g_n(\gamma) = \int_0^1 dx x^n/(\exp(\gamma x) - 1)\). The decay constant for the $H$ mode has been derived earlier and is given by \(f_H^2 = 3\mu^2/(8\pi^2)\). As we shall find below these reactions continue to be the dominant source of opacity in the CFLK\(^0\) phase.

To begin, we consider processes involving the $K^+$ Goldstone mode since the $K^+$ is lightest charged mode. The leading order processes are $\nu_e \rightarrow e^- K^+$ and $\bar{\nu}_e K^+ \rightarrow e^+$. The matrix element for these processes is given by

$$
A_{K^+} = G f_{\pi} \sin \tilde{\alpha} \bar{p}_\mu \bar{e}(k_2)\gamma^\mu(1-\gamma_5)\nu(k_1), \tag{29}
$$

where $\bar{p}^\mu = (E_{K^+} + X t_K, v^2 \vec{p})$, $P^\mu = (E_\phi, \vec{p})$, \(t_K = \begin{bmatrix} \cos \theta \sin \alpha - i \sin \theta \cos \alpha \\ \sin \alpha \end{bmatrix} \cos \frac{\tilde{\alpha}}{2}\tag{30}\)

and $\tilde{\alpha}$ (see Eq. 26) is the effective Cabibo angle in the $K^0$ phase. As discussed earlier, since the $K^0$ condensate breaks $U(1)_Y$, it modifies the Cabibo suppression of charged current reactions involving kaons. When the condensate amplitude is large corresponding to $\cos \theta \simeq 0$, the rate of charged current weak reactions involving kaons becomes independent of the $\alpha$ and are not Cabibo suppressed.

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The opacity or inverse mean free path is defined to be the cross section per unit volume. For the process $\nu_e \rightarrow e^- K^+$ this is given by

$$
\frac{1}{\lambda_{\nu_e \rightarrow e^- K^+}} = \frac{1}{2E_1} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \sum_{\text{spin}} |A|^2(2\pi)^4(1-f_{e^-}(E_2))\delta(k_1 - P - k_2) \tag{32}
$$

where

$$
E_{K^+} = \sqrt{v^2p^2 + Y^2 + \delta m^2}, \tag{33}\)

Note that $f_{e^-}(E_2) = (1 + \exp((E_2 - \mu_e)/T))^{-1}$ is the Fermi - Dirac distribution function for the electrons and the factor $(1 - f_{e^-}(E_2))$ accounts for Pauli blocking of the final state electron. Since $K^+$ has a finite mass, neutrinos must have a threshold energy to radiate
them. Energy and momentum conservation restrict the initial neutrino energy to have an energy \( E_1 \geq E_{\text{th}} \) given by
\[
E_{\text{th}} = \frac{\sqrt{y + \delta m^2 (1 - v^2)} - y}{(1 - v^2)}
\]
\[
E_{\text{th}} \simeq \tilde{m}_{K^+} \quad \text{for} \quad y \ll \delta m^2.
\]
(34)

Similarly, the opacity for the process \( \bar{\nu}_e(k_1) K^+(P) \rightarrow e^+(k_2) \) is given by
\[
1 = \frac{1}{\lambda_{\bar{\nu}_e K^+ \rightarrow e^+}} = \frac{1}{2E_1} \int \frac{d^3p}{(2\pi)^3 2E_K^+} f(E_\phi) \int \frac{d^3k_2}{(2\pi)^3 2E_2^+} \sum_{\text{spin}} |A|^2 (2\pi)^4 (1 - f_{e^+}(E_2)) \delta(k_1 + P - k_2)
\]
(35)

where \( f_B(E_{K^+}) \) is a Bose distribution function for \( K^+ \) meson and \( f_{e^+}(E_2) = (1 + \exp((E_2 + \mu_e)/T))^{-1} \) is the Fermi - Dirac distribution function for the electrons.

Reactions involving other Goldstone bosons make relatively small contributions to the neutrino mean free path because they are heavy compared to the \( K^+ \). Nonetheless, it is straightforward to calculate their contribution to the neutrino mean free path. The amplitude for reactions involving charged pions is obtained by replacing \( \sin \tilde{\alpha} \rightarrow \cos \tilde{\alpha} \) and
\[
t_K \rightarrow t_\pi = \left[ \frac{\sin \theta \cos \alpha + i \cos \theta \sin \alpha}{\cos \tilde{\alpha}} \right] \sin \frac{\theta}{2}
\]
(36)

As noted earlier, for maximal condensation i.e., when \( \cos \theta \simeq 0 \), the effective Cabibo angle \( |\cos \tilde{\alpha}| = 1/\sqrt{2} \). Consequently, neutrino reactions involving charged pions are mildly suppressed in the \( K^0 \) phase.

Neutral current reactions involving \( \pi^0 \) and \( \eta \) also contribute to the neutrino opacity. Isospin breaking due to kaon condensation results in a mixing between these states. Like in the vacuum, the \( \pi^0 \) and \( \eta \) mix in the CFL phase due to isospin breaking arises from \( m_u \neq m_d \). However, since \( m_d - m_u \) is still small compared to other mass scales in the CFL phase, this mixing is small (albeit moderately large compared to mixing in the vacuum). In the CFLK\(^0\) phase, we can compute the mixing from Eq.\([\text{4}]\) The mass matrix for the fields \( \pi_3 \) and \( \pi_8 \) corresponding to the \( \pi_0 \) and \( \eta \) fields is given by
\[
\Pi = \begin{bmatrix} \kappa_{33} & \kappa_{38} \\ \kappa_{83} & \kappa_{88} \end{bmatrix}
\]
(37)
where

\[ \kappa_{33} = YZ + a \left( m_d m_s - m_u m_s \frac{Y}{X} + m_u m_d \frac{Z}{X} \right) \]  
(38)

\[ \kappa_{88} = 3YZ + \frac{a}{3} \left( m_d m_s (1 + 5 \frac{Y}{X}) + m_u m_s (1 + 5 \frac{Z}{X}) - m_u m_d \right) \]  
(39)

\[ \kappa_{38} = \kappa_{83} = -\sqrt{3}YZ + \frac{a}{\sqrt{3}} \left( m_d m_s + m_u m_s \frac{Y}{X} + m_u m_d \frac{Z}{X} \right) \]  
(40)

The unitary matrix

\[ U = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \]
that diagonalizes \( \Pi \) determines the masses and mixing. The masses \( m_{\tilde{\pi}_0} \) and \( m_{\tilde{\eta}} \) corresponding to the mass eigenstates \( \tilde{\pi}_0 \) and \( \tilde{\eta} \) are given

\[ \begin{bmatrix} m_{\tilde{\pi}_0}^2 & 0 \\ 0 & m_{\tilde{\eta}}^2 \end{bmatrix} = U \Pi U^{-1}, \]  
(41)

and the eigenstates are given by

\[ \begin{bmatrix} \tilde{\pi}_0 \\ \tilde{\eta} \end{bmatrix} = U \begin{bmatrix} \pi_0 \\ \eta \end{bmatrix}. \]  
(42)

For the numerical values of parameters chosen in this work, we find that in the CFLK\( ^0 \) phase the mixing angle \( \psi \simeq 25^\circ \) (in the CFL phase, \( \psi_{\text{CFL}} \simeq 15^\circ \)).

The amplitude for the tree level process such as \( \nu \to \nu \tilde{\pi}_0 \) and \( \nu \to \nu \tilde{\eta} \) are given by

\[ A_{\tilde{\pi}_0/\tilde{\eta}} = \frac{G_F C_{\tilde{\pi}_0/\tilde{\eta}}}{\sqrt{2}} \bar{\nu}(k_2)\gamma_\mu(1 - \gamma_5)\nu(k_1) \]  
(43)

where \( \bar{\nu}_\mu = (E_{\tilde{\pi}_0/\tilde{\eta}}, v^2 \vec{p}) \),

(44)

and neutral current coupling constants for the mass eigenstates are given by

\[ \begin{bmatrix} C_{\tilde{\pi}_0} & C_{\tilde{\eta}} \end{bmatrix} = \begin{bmatrix} C_{\pi_0} & C_{\eta} \end{bmatrix} U^{-1}, \]  
(45)

where \( C_{\pi_0} = 1 \) and \( C_{\eta} = 1/\sqrt{3} \) are the weak (isospin) charges of the \( \pi_0 \) and \( \eta \) respectively. Contribution to the neutrino mean free path from the emission and a bsorption of the neutral \( \tilde{\pi}_0 \) and \( \tilde{\eta} \) mesons can be computed using the charged current results in Eq\( ^{\text{33}} \) and Eq\( ^{\text{35}} \) respectively but with the following substitutions: \( |A|^2 \to |A_{\tilde{\pi}_0/\tilde{\eta}}|^2 \), \( f_\nu(E_2) \to f_\nu(E_2) = 1/(\exp(E_2/T) + 1) \), and \( \mathcal{E}_K \to E_{\tilde{\pi}_0/\tilde{\eta}} = \sqrt{v^2 p^2 + m_{\tilde{\pi}_0/\tilde{\eta}}^2}. \)

We present numerical results for neutrino mean free path in the CFLK\( ^0 \) phase at \( \mu = 400 \) MeV and for two different temperatures \( T = 5 \) MeV and \( T = 15 \) MeV. For \( T \gtrsim 15 \) MeV we
FIG. 2: Left Panel: Neutrino mean free path for a thermal neutrino with energy $E_{\nu} = \pi T$ arising from dominant reactions in the CFLK$^0$ phase. Right Panel: The average neutrino mean free path (defined in the text) in the CFLK$^0$ phase.

expect that finite temperature corrections to the meson dispersion relations would play a role. As discussed earlier, a rough dimension estimate of the critical temperature at which the $K^0$ condensate would melt is given by $T_c \sim 30 - 50$ MeV. The results we present employ the following numerical values: $m_u = 3.75$ MeV, $m_d = 7.5$ MeV, $m_s = 150$ MeV, $\Delta = 100$ MeV and $\chi = 0$. The neutrino mean free path for thermal neutrinos ($E_{\nu} = \pi T$) due to different charged and neutral current processes involving chiral Goldstone modes are given in the table below. The results indicate that reactions involving the $K^+$ are important.

| process $\rightarrow$ | $\lambda$(T=5 MeV) | $\lambda$(T=15 MeV) |
|----------------------|---------------------|---------------------|
| $\nu_e \rightarrow K^+e^-$ | 682 m | 42 m |
| $\bar{\nu}_e K^+ \rightarrow e^+$ | 848 m | 43 m |
| $\nu_e \rightarrow \pi^+e^-$ | $\infty$ | 81 m |
| $\bar{\nu}_e \pi^+ \rightarrow e^+$ | 8.3 km | 77 m |
| $\nu_e K^- \rightarrow e^-$ | $\infty$ | $\infty$ |
| $\nu_e K^- \rightarrow e^-$ | $\gtrsim 10$ km | 1.2 km |

| process $\rightarrow$ | $\lambda$(T=5 MeV) | $\lambda$(T=15 MeV) |
|----------------------|---------------------|---------------------|
| $\bar{\nu}_e \rightarrow \pi^-e^+$ | $\infty$ | $\infty$ |
| $\nu_e \pi^- \rightarrow e^-$ | $\gtrsim 10$ km | 200 m |
| $\nu \rightarrow \pi^0\nu$ | $\infty$ | 6.9 km |
| $\nu\bar{\pi}^0 \rightarrow \nu$ | $\gtrsim 10$ km | 350 m |
| $\nu\bar{\eta} \rightarrow \nu$ | $\gtrsim 10$ km | 146 m |

Since the $K^+$ meson is the lightest and hence the most abundant flavor Goldstone mode
reactions involving the $K^+$ dominate the neutrino opacity. In Fig. 2 we show the neutrino mean free path arising due to the dominant processes - those involving the $H$ and $K^+$ Goldstone modes. The left panel shows the contribution of individual reactions to the neutrino mean free path for thermal neutrinos whose typical energy is $E_\nu = \pi T$. The right panel shows the total Roseland averaged mean free paths for the different neutrino species. The Roseland mean free path is defined as

$$\frac{1}{\langle \lambda_\nu \rangle} = \frac{1}{\mathcal{N}} \int_0^\infty dE_\nu \frac{E_\nu^2}{\lambda(E_\nu)} f_\nu(E_\nu),$$

(46)

where $\mathcal{N} = \int_0^\infty dE_\nu E_\nu^2 f_\nu(E_\nu)$ and $f_\nu(E_\nu) = 1/(1 + \exp(E_\nu/T))$ is the distribution function for thermal neutrinos. The neutral current process involving the $H$ mode is common to all neutrino types, while the process $\nu_e \rightarrow K^+ e^-$ is specific to the electron neutrinos and the reaction $\bar{\nu}_e K^+ \rightarrow e^+$ is specific to the anti-electron neutrinos.

**B. Neutrino emissivities**

We can compute the rate for these processes as outlined in Ref. [6]. The emissivity is defined to be the rate at which energy is radiated in neutrinos per unit volume. To begin, we consider the emissivity arising due to the decay of charged mesons into electrons(positrons) and anti-neutrinos(neutrinos). The contribution arising due to the decay of charge kaons can be computed from Eq. 47 with appropriate substitutions for the amplitude and the distribution function. The inverse reactions such as $K^\pm + e^\mp \rightarrow \nu$ can also be computed by replacing the factor $1 - f_{e^\pm}$ by $f_{e^\mp}$ in the above equation. Emissivity due neutral current decays of $\tilde{\pi}^0$ and $\tilde{\eta}$ can be also be calculated using Eq. 47 by making the following substitutions: $(1 - f_{e^\pm}) \rightarrow 1$, $A_{K^\pm \rightarrow e^\pm \nu} \rightarrow A_{\tilde{\pi}^0/\tilde{\eta} \rightarrow e^\pm \nu}$ and $E_{K^\pm} \rightarrow E_{\tilde{\pi}^0/\tilde{\eta}} = \sqrt{v^2p^2 + m_{\tilde{\pi}^0/\tilde{\eta}}^2}$. 


We present results for the emissivities arising due to the various reactions in the CFLK\(^0\) phase. Numerical results are presented in units of ergs/cm\(^3\)/s and are for chosen values of physical parameters indicated earlier. As expected at low temperature, the lightest mesons dominate the emissivity. However, the decay of the \(K^+\) meson is not an efficient means of producing neutrinos. This is because the \(K^+\) has a time-like four momentum only for momenta \(p \lesssim m_{K^+}\). Thus, only a small fraction of the thermal \(K^+\) modes can decay. The inverse reaction \(K^+\overline{e} \rightarrow \nu\), on the other hand is more efficient, because in this case, a large fraction of the \(K^+\) mesons with a space-like dispersion relation contribute. This is also true for reactions involving the \(\pi^+\) mode. In contrast the more massive negatively charged modes have time-like dispersion relation over a wider range of momenta and for this reason their decays makes a relevant contribution to the total emissivity. Due to their larger masses these contributions become important at higher temperature. The decay of neutral modes, which have intermediate mass, also make an important contribution to the total emissivity.

### IV. CAVEATS

Prior to discussing how the results of this work will impact supernova and neutron stars we briefly mention a few caveats to the results presented in this work.

The size of the superconducting gap and the instanton induced \(\langle \overline{q}q \rangle\) at these relatively low densities is poorly known. If \(\langle \overline{q}q \rangle\) were large such that the \(Tr[M\Sigma]\) contribution to the masses become relevant, then it will affect the mass spectrum of Goldstone modes and thereby directly alter the neutrino rates. In particular, the strange mesons would be too heavy and would decouple from the low energy response.

In this work, we neglected the contribution of the \(\eta'\) Goldstone mode. This was based on our naive expectation that the anomalous contribution to its mass would be large. If this were not the case, the \(\eta'\) would be a very light state that would mix with the \(\pi^0\) and \(\eta\)
mesons [16]. To study how a light $\eta'$ Goldstone mode would affect weak interaction rates we have computed the masses and mixing of neutral mesons. In doing so we assumed that only $Tr[\hat{M}\Sigma]$ operator contributes to the masses. For the parameters used in this work we find that the eigenstates $\tilde{\pi}^0, \tilde{\eta}$ and $\tilde{\eta}'$ have masses given by $m_{\tilde{\pi}^0} = 31.5$ MeV, $m_{\tilde{\eta}} = 29.3$ MeV and $m_{\tilde{\eta}'} = 3.7$ MeV, respectively. Their neutral current couplings are given by $C_{\tilde{\pi}^0} = 0.71$, $C_{\tilde{\eta}} = 0.29$ and $C_{\tilde{\eta}'} = 0.96$, respectively. Due to its small mass and large neutral current coupling the $\tilde{\eta}'$ contribution to the neutrino mean free path due to the processes $\nu + \tilde{\eta}' \rightarrow \nu$ and $\nu \rightarrow \tilde{\eta}' + \nu$ are larger than similar reactions involving $\tilde{\pi}^0$ and $\tilde{\eta}$. At $T = 5$ MeV: $\lambda_{\nu + \tilde{\eta}' \rightarrow \nu} = 2.9$ km and $\lambda_{\nu \rightarrow \tilde{\eta}' + \nu} = 817$ m; and at $T = 15$ MeV: $\lambda_{\nu + \tilde{\eta}' \rightarrow \nu} = 101$ m and $\lambda_{\nu \rightarrow \tilde{\eta}' + \nu} = 177$ m. This is comparable to the $K^+$ contribution to the opacity. The emissivity due to the reaction $\eta' \rightarrow \nu\bar{\nu}$ is small compared to those due the other neutral mesons. At $T = 5$ MeV, $\dot{\epsilon}_{\tilde{\eta}' \rightarrow \nu\bar{\nu}} = 3.5 \times 10^{30}$ ergs/cm$^3$/s and at $T = 15$ MeV we find that it is $\dot{\epsilon}_{\tilde{\eta}' \rightarrow \nu\bar{\nu}} = 1.1 \times 10^{31}$ ergs/cm$^3$/s.

We have ignored the electromagnetic contribution to the mass of the charged Goldstone modes. Earlier estimates indicate that this could be of the order of a few MeV [5, 16, 17]. If $m_{el} \gtrsim \delta m$ it would alter our numerical results for the rates involving the $K^+$ mode. However, we note that this can be easily incorporated by substituting $\delta m^2 \rightarrow \delta m^2 + m_{el}^2$. Another source of concern is related to our use of asymptotic expressions for the coefficients of the effective theory.

V. DISCUSSION

We have studied neutrino reactions involving Goldstone bosons in the kaon condensed phase of CFL quark matter. A light and electrically charged $K^+$ mode is found to play an important role. The massless neutral kaon mode does not play a role because it does not couple to the neutrinos at leading order. In contrast, the massless mode associated with the breaking of baryon number in the CFL phase is unaffected by kaon condensation and continues to dominate the neutrino opacity. As in the CFL phase [6], space like propagation of Goldstone modes allows for novel, Cerenkov-like, processes in which neutrinos radiate or absorb Goldstone bosons as they propagate in the medium. The amplitude for these leading order processes is large since they are proportional $f_\pi \sim \mu$. These processes continue to dominate the neutrino opacity.
A. Comparison between CFL and CFLK\(^0\) phases:

The mean free path of neutrinos in the CFL and CFLK\(^0\) phases are nearly equal. This is one of our main findings. This is because the mean free path in the CFLK\(^0\) phase continues to be dominated by the neutral current process involving the U(1)\(_B\) Goldstone mode. Thus, the main conclusions drawn in Ref.\[6\] regarding the temporal aspects of the neutrino signal remain largely unchanged.

Neutrino emission processes arise mainly due to reactions involving massive pseudo-Goldstone modes. Reactions involving only one meson dominate because these amplitudes are proportional \(f_\pi \sim \mu\). These include decays such as \(K^\pm \rightarrow e^\pm \nu\), \(\tilde{\pi}^0 \rightarrow \nu \bar{\nu}\) and absorption reactions such as \(K^\pm e^\mp \rightarrow \nu\). Reactions involving the \(K^+\), and the neutral \(\tilde{\pi}^0\) and \(\tilde{\eta}\) are dominant at low temperature. We find the neutrino emissivity in the CFLK\(^0\) phase to be roughly 1-2 orders of magnitude larger than in the CFL phase for temperatures \(T \lesssim 15\) MeV. At \(T = 5\) MeV, the emissivity in the CFL phase \(\dot{\epsilon}_{\text{CFL}} \simeq 5 \times 10^{33}\) ergs/ cm\(^3\)/s, while in the CFLK\(^0\) phase the emissivity \(\dot{\epsilon}_{\text{CFLK}\,^0} \simeq 3 \times 10^{35}\) ergs/ cm\(^3\)/s. This enhancement in the CFLK\(^0\) phase arises because: (1) on average the Goldstone modes in the CFLK\(^0\) phase are lighter and (2) the Cabibo suppression of reactions involving kaons in the CFL phase is greatly alleviated in the CFLK\(^0\) phase.

B. Comparisons between CFL/CFLK\(^0\) and other phases

Neutrino mean free path in the CFL and CFLK\(^0\) phases are typically larger than those in nuclear phase. They are similar to those in the unpaired quark matter under similar ambient conditions. At \(T = 15\) MeV and baryon density of \(n_B = 5\) \(n_0\), the neutrino mean free path in the CFLK\(^0\) phases \(\lambda_{\text{CFLK}\,^0} \simeq 6\) m. The dominant neutrino reaction in the nuclear phase is \(\nu + n \rightarrow \nu + n\) results in a mean free path \(\lambda_{\text{nuclear}} \simeq 10\) cm. Under similar conditions the mean free path of neutrinos in unpaired quark matter arising due to neutral current scattering off quarks yields \(\lambda_{\text{unpaired}} \simeq 13\) m. We note that these estimates ignore the possible role of strong interaction correlations and account only for the Pauli blocking effects in these reactions. Model calculations have shown that correlations in the nuclear phase can greatly increase the neutrino mean free path (by a factor of 3-5) \[18, 19, 20\]. Comparing the mean free paths in these different phases we conclude that the mean free path in unpaired quark
and CFL/CFLK\(^0\) phases are similar (same order of magnitude) but are roughly about an order of magnitude larger than in the nuclear phase.

The neutrino emissivity in the CFL\(^4\) and CFLK\(^0\) phases are exponentially small compared to those in the nuclear phase for \(T \ll 1\) MeV. This is because the meson masses are of the order of a few MeV. The lightest excitation that can contribute to the emissivity is the \(K^+\) mode in the CFLK\(^0\) phase. At \(T = 5\) MeV, the emissivity in the CFLK\(^0\) phase \(\dot{\epsilon}_{\text{CFLK}^0} \simeq 3 \times 10^{35}\) ergs/cm\(^3\)/s which is only an order of magnitude smaller than in the unpaired quark phase where \(\dot{\epsilon}_{\text{unpaired}} \simeq 3 \times 2 \times 10^{36}\) ergs/cm\(^3\)/s. At higher temperature the rates become more similar. A general trend we see is that when the temperature become comparable to the mass of the lightest charged particle the rates in the CFL and CFLK\(^0\) phases become comparable to those in the unpaired quark phase.

### C. Astrophysical Implications

The primary motivation for studying the neutrino mean free paths in dense matter is core collapse supernova. The results obtained here will have little impact on the long term cooling of neutron stars which are characterized by temperatures \(T \ll 1\) MeV. This is because at these low temperatures, the neutrino mean free path is large compared to the size of the neutron star. Massive Goldstone bosons with masses of order a few MeV are negligible and their contribution to the emissivity is exponentially small. In contrast, during the first tens of seconds subsequent to the birth of the neutron star neutrinos carry almost all (99\%) of the Gravitational binding energy (~10\(^{53}\) ergs) stored inside the newly born hot "neutron" star, with \(T \sim 30 - 50\) MeV. The newly born star is also called the proto-neutron star (PNS). The rate at which neutrinos diffuse and the spectrum with which they decouple from the PNS can affect key aspects of core collapse supernova - the explosion mechanism and r-process nucleosynthesis. For a galactic supernova, the several thousand neutrino events predicted in detectors such as Super Kamiokande and SNO will provide information about the propagation of neutrinos inside the dense PNS. In the discussion that follows we speculate on how some the results obtained in this work might affect core collapse supernova.

Since neutrino mean free path is dominated by neutral current process in the CFL and CFLK\(^0\) phases, the mean free path for all six neutrino species are very nearly equal. This is in contrast to what is observed in the neutron-rich nuclear phase. Here, \(\lambda_{\nu_e} \lesssim \lambda_{\bar{\nu}_e} \lesssim \lambda_{\nu_{\mu/\tau}}\).
In the nuclear phase the charged current reaction $\nu_e + n \rightarrow e^- + p$ dominates and the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ makes a smaller but relevant contribution to the mean free path while $\mu/\tau$ neutrinos interact only through the neutral current reactions. As can be inferred from Fig. 2 the differences between the mean free path of the different neutrino types is small in the CFLK$^0$ phase because the charged current reactions involving the $K^+$ mode makes only a modest contribution to the total mean free path. Nonetheless, it is interesting to note that the Roseland mean free path show a trend that is similar to that of nuclear matter, i.e. $<\lambda_{\nu_e}> \lesssim <\lambda_{\bar{\nu}_e}> \lesssim <\lambda_{\nu_{\mu/\tau}>}$.

It is interesting to inquire if neutrinos decoupling from the quark phase could have (relative) spectra that are different from those that decouple from the nuclear phase. The spectra with which neutrinos emerge from the PNS impact several observable aspects of supernova such as the explosion mechanism, r-process nucleosynthesis and the number of detected neutrinos in the terrestrial detectors. The r-process in particular is sensitive to the relative spectra of the electron and anti-electron type neutrinos since this determines the neutron excess in the neutrino driven r-process wind in the supernova (for a recent review see Ref. [21]). If indeed, all six neutrinos types emerge from the CFL phase with similar spectra it could affect the neutron to proton ratio in the r-process wind. It is premature to make definitive statements regarding how our findings here would affect supernova observables since neutrino transport in the PNS depends on several micro and macroscopic inputs. Nonetheless, a robust finding of this work is that if neutrinos decouple from the CFL or CFLK$^0$ phase the spectra of all six neutrino types will be very similar.

The characteristic time that governs the rate of cooling by neutrinos diffusion is given by $\tau_D = \bar{C}_V R^2/c\bar{\lambda}$, where $\bar{C}_V$ is the average specific heat, $R$ is the size of the diffusion region and $\bar{\lambda}$ is the typical neutrino mean free path in the region [22]. In nuclear and unpaired quark phases the specific heat per unit volume is large $c_V \sim \mu^2 T$, while in the CFL and CFLK$^0$ phases it is very small with $c_V \sim T^3$. This difference is likely to be the dominant effect that distinguishes the cooling of PNS with CFL and CFLK$^0$ quark matter from the other conventional scenarios. As noted earlier, the presence of additional light degrees of freedom in the CFLK$^0$ leads to a specific heat that is roughly twice as larger than that in the CFL phase. This difference could also be a potentially important in distinguishing between the CFL and CFLK$^0$ phases.

Incorporating the findings of this work in astrophysical simulations of PNS evolution
warrants much further work. This will allow one to make quantitative predictions for the supernova neutrino signal and r-process nucleosynthesis. A crucial, future, step in theoretical efforts to constrain novel high density phases with supernova observations.

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