WHICH SCALAR MESON IS THE GLUE-STATE?

M. BOGLIONE

Dipartimento di Fisica Teorica, Universita’ di Torino,
I.N.F.N, Sezione di Torino, Via P Giuria 1, I-10125 Torino, Italy
and
Centre for Particle Theory, University of Durham,
Durham DH1 3LE, U.K.

Preliminary results of a work in collaboration with M.R. Pennington are presented. Extending a scheme introduced by Tornqvist, we investigate a dynamical model in which the spectrum of scalar mesons can be derived, with the aim of locating the lightest glue-state. Adding hadronic interaction contributions to the bare propagator, to ‘dress’ the bare quark-model $q\bar{q}$ states, we are able to write the amplitudes and the phase shifts in the approximation in which scalar resonances decay only into two pseudoscalar channels. The fit of these quantities to experimental data gives a satisfactory understanding of how hadronic interactions modify the underlying ‘bare’ spectrum. In particular, we examine the case in which a glue-state is introduced into the model.

1 Introduction

Gluons carry colour charge which means they interact together. Consequently it is possible for them to cluster and form objects which are colourless overall. These would be just like conventional hadrons, but with constituents that are massless gauge bosons. These are known as glueballs.

Interest in glueballs has increased since new candidates for gluonic states have emerged from experimental results especially in the 1.5 – 2.0 GeV energy region. Moreover, Lattice QCD calculations (in quenched approximation) have recently suggested the presence of light scalar glue-states in this same energy region. From the experimental point of view, it is now clear that there are too many confirmed scalar $(0^{++})$ mesons to form one $q\bar{q}$ meson nonet. The problem is particularly pronounced for the I=0 sector, since at least four $f_0$’s now appear in the particle data listing. As a consequence, we can infer that some of them have to be extra states.

The quark model gives a reasonably good description of the vector and tensor meson spectra and properties, but its predictions for the scalar sector are very disappointing. To understand how and why scalars are so different from vectors and tensors, we consider a simple model in which all bare meson states belong to ideally mixed quark multiplets. We call $n\pi$ the nonstrange light state and suppose that substituting a strange quark for a light one increases the mass of the state by $\Delta m_s \simeq 100$ MeV, as illustrated in Fig. 1.
The bare propagator for each of these bound states will be of the form
\[ P(s) = \frac{1}{M_0^2 - s}, \]  
(1)
with a pole on the real axis, corresponding to a non decaying state, for example
\[ |\phi\rangle_0 = |s\rangle \]
for the vector \( I = 0 \) state, and
\[ |f'_0\rangle_0 = |s\rangle \]
for the scalar \( I = 0 \) state. If we now assume that the experimentally observed hadrons are obtained from the bare states \( (n\pi, s\pi, ss, \ldots) \) by dressing them with hadronic interactions, the propagator becomes
\[ P(s) = \frac{1}{M^2(s) - s - iM(s)\Gamma(s)}, \]  
(2)
and the pole moves into the complex \( s \)-plane. The corresponding state can be decomposed as
\[ |\phi\rangle = \sqrt{1 - \epsilon^2}|s\rangle + \epsilon_1|K\rangle + \epsilon_2|\rho\rangle + \ldots \]  
(3)
where calculation would give \( \epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \ldots \ll 1 \). The hadronic loop contributions allow the bare states \( (s\pi, ss) \) in this example to communicate with all hadronic channels permitted by quantum numbers, and this enables the \( \phi \) meson to decay. A similar picture works for the tensors.
For scalars the situation is different because the dominant decays are just into two pseudoscalars, the couplings are bigger and they couple strongly to more than one channel, creating overlapping and interfering resonance structures. Furthermore, being S-waves, the opening at thresholds produces a more drastic \( s \)-dependence in the propagator. As a consequence, the \( f_0(980) \), for example, turns out to be predominantly a \( |K\rangle \) state, and not an \( |ss\rangle \) one:
\[ |f_0(980)\rangle = \sqrt{1 - \epsilon^2}|K\rangle + \epsilon_1|s\rangle + \ldots. \]  
(4)
This does not mean that the \( f_0(980) \) is a \( K\bar{K} \) molecule, because the seeds of the model are conventional \( q\bar{q} \) states and the binding forces are not due to inter-hadron interactions alone. The crucial point here is the fact that hadronic interactions allow the \( q\bar{q} \) bare states to communicate with all possible hadronic channels and these channels to communicate with each other, giving rise to a mixing which means, for example, that the \( f_0(980) \) spend most of its time in a \( K\bar{K} \) state and not in an \( s\bar{s} \) one.

2 A closer look at the model

Let us now examine the model in more detail starting, for simplicity, from the case in which just one resonance is produced. If we define a vacuum polarization function \( \Pi(s) \) which takes into account all the possible two pseudoscalar loop contributions to the propagator \( P(s) \), we can easily write its imaginary part:

\[
\text{Im}\Pi(s) = - \sum_i G_i^2(s) = - \sum_i g_i^2 \frac{k_i(s)}{\sqrt{s}} (s - s_{A,i}) F_i^2(s) \theta(s - s_{th,i})
\]  

(5)

where the index \( i \) runs over the pseudoscalar channels, the \( g_i \)'s are the \( SU(3) \) flavour couplings, the \( k_i \)'s are the c.m. momenta and \( (s - s_{A,i}) \) are the Adler zeros. \( F_i(s) \) are the form factors, which take into account the fact that the interaction is not pointlike but has a spatial extension. These are parametrized by

\[
F_i = \exp \left( -\frac{k_i^2(s)}{2 k_0^2} \right),
\]

(6)

where the momentum \( k_0 \) is inversely proportional to the range of the interaction. The real part of the vacuum polarization function can be found from the dispersion relation

\[
\text{Re}\Pi(s) = \frac{1}{\pi} P \int_{s_{th,1}}^{\infty} ds' \frac{\text{Im}\Pi(s')}{s' - s}
\]

(7)

No subtraction is needed, since the form factors decrease fast enough when \( |s| \to \infty \). At this point we are able to write the propagator as

\[
P(s) = \frac{1}{m_0^2 + \Pi(s) - s},
\]

(8)

and the contribution to the \( i \to j \) amplitude as

\[
R_{ij}(s) = \frac{G_i(s) G_j(s)}{m_0^2 + \Pi(s) - s}.
\]

(9)
Notice that $R_{ij}(s)$, having its numerator and denominator related to each other by the same $G_i$ couplings, is not the most general amplitude satisfying unitarity. This would look like:

$$A_{ij}(s) = R_{ij}(s) e^{2i\alpha(s)} + \sin\alpha(s) e^{i\alpha(s)}$$

where $\alpha(s)$ is an unknown function of $s$ real along the right hand cut, and the second term in Eq. (8) accounts for the background contribution. To avoid an increasing number of parameters in the model, we will consider just the approximate amplitude $R_{ij}$ of Eq. (9), having checked that a constant value of $\alpha$ of about $15^\circ$ in Eq. (10) would typically work just as well.

Fig. 2 shows the behaviour of the real and imaginary part of the running complex mass function $m^2(s) = m^2_0 + \Pi(s)$, for the $I = 1/2$, $K^*_0(1430)$, and the $I = 1$, $a_0(980)$. Here the parameters of the model (the mass of the bare $n\pi$ state, the form factor cut-off $k_0$, an overall coupling $\gamma$ and the position of the Adler zero) are determined by fitting the amplitudes and the phase shifts to the experimental data from LASS. As we can see, the opening of each threshold gives an extra contribution (square root cusp) to the imaginary part of the vacuum polarization function. This is reflected in the shape of its real part, which presents a very strong $s$-dependence at every threshold. The intersection of $\text{Re} m^2(s)$ with the curve $s$ represents the square of the Breit Wigner mass, and the value of $\text{Im} \Pi(s)$ at this point is related to the Breit Wigner width by

$$\Gamma_{BW} = \frac{-\text{Im} \Pi(m_{BW})}{m_{BW}}.$$ 

Notice how the negative contribution of $\text{Re} \Pi(s)$ shifts down the actual mass of the resonance with respect to the value of the corresponding bare mass. This effect is particularly pronounced for the $a_0(980)$ because the mass of the dressed bound state happens to coincide with the first of two thresholds, the $K\bar{K}$ and the $\pi\eta'$ ones, which are very close to each other and have similar couplings: despite the bare mass $m_0$ being fixed at 1420 MeV, the Breit Wigner mass of the $a_0$ is found to be as low as 987 MeV.
To analyze the more controversial $I = 0$ scalar sector, we need to examine the case in which more than one resonance is produced. Now, the polarization function has to incorporate the features (couplings, masses, form factors, ...) of each of the intermediate particles created in the process:

$$\text{Im}\Pi_{\alpha\beta}(s) = -\sum_i G_{\alpha,i}(s) G_{i,\beta}(s),$$  \hspace{1cm} (12)

$$\text{Re}\Pi_{\alpha\beta}(s) = \frac{1}{\pi} P \int_{s_{th,1}}^{\infty} ds' \frac{\text{Im}\Pi_{\alpha\beta}(s')}{s' - s},$$  \hspace{1cm} (13)

where the indices $\alpha$ and $\beta$ run over the $N$ different resonances. As a consequence, the propagator

$$P_{\alpha\beta}(s) = \frac{1}{(m_0^2 - s)\delta_{\alpha\beta} + \Pi_{\alpha\beta}(s)}$$  \hspace{1cm} (14)

becomes an $N \times N$ complex matrix, and the couplings are $N$-dimensional column vectors. The amplitudes are determined using a diagonalization procedure and can be written as

$$R_{ij}(s) = \sum_{\alpha} \frac{G'_{\alpha,i}(s) G'_{j,\alpha}(s)}{m_{\alpha,\text{diag}}^2(s) - s}$$  \hspace{1cm} (15)

where the diagonalized masses $m_{\alpha,\text{diag}}(s)$ are admixtures of all the bare masses and the vacuum polarization matrix elements. They are, like the new couplings

Figure 2: Real and imaginary parts of the complex running mass functions for the (a) $K^0_\ell(1430)$, and (b) $a_0(980)$, illustrating the effect of thresholds.
$G'_{a,i}(s)$, complex and $s$-dependent.

What is the interpretation of this more complicated picture? The physical observed hadrons are the states we obtain after diagonalization, an admixture of the seed states of the model. The mixing among them is embodied in the diagonalization procedure, which allow all the channels to communicate with each other and to create new physical states with masses, couplings and widths different from the ones of the primitive states.

We now want to consider explicitly the case in which not only the two conventional $f_0$ and $f'_0$ are present in the $I = 0$ sector, but also a third state arises, thanks to the presence of a glue $gg$ seed. Since, in principle, we do not know the mass of the bare glue-state, $m_{gg}$ is assumed to be an extra parameter of the model. To avoid a further increase in the number of parameters we will also assume that the $gg$ state cannot mix with the $q\bar{q}$ states at the bare level: all the mixing will occur via hadron interactions. The bare couplings for the glueball to the two pseudoscalar channels are the ones of an $SU(3)$ flavour singlet. Fig. 3 presents the results we obtain if we choose a $gg$ bare mass of 1.8 GeV and readjust the other parameters to fit the data from $\pi\pi$ scattering\cite{10,11}, in such a way that the agreement with the LASS data on $K^*_0(1439)$\cite{9} remains satisfactory. The two lower states have the same features that would be obtained if only two resonances were considered: a very broad state with a mass of about 1 GeV and a narrow one with a mass of about 1.2 GeV. The higher glue-state almost decouples from the lowest channels, but shows relatively strong couplings not only to the heaviest $\eta'\eta'$, but also to the $\eta\eta'$ channel, to which it had a zero underlying bare coupling. Again, the Breit Wigner mass is considerably lower than the input one: for these values of the parameters our glueball mass is about 1.57 GeV and its width is about 200 MeV. Notice how the masses of all the scalar $I = 0$ mesons are related to the positions of the two pseudoscalar thresholds. The amplitude presents two dips: the former corresponds to the presence of a very narrow resonance sitting on top of a very broad one, the second is related to the third heavier resonance, in agreement with the experimental data from Crystal Ball\cite{12}, GAMS\cite{13}, and the analysis from Bugg et al.\cite{14} and Anisovich et al.\cite{15}. 
3 Conclusions

In this talk I have presented some preliminary results of a work performed in collaboration with M.R. Pennington, which is still in progress. In particular, a proper fit of the more recent experimental data up to about 1.8 GeV has still to be completed.

The model we are using requires approximations, to allow the calculation to be performed. The amplitudes are assumed to be pole dominated and so the unitarization chosen is not the most general one. Furthermore, only decays into two pseudoscalars are considered, whereas other thresholds like multipion, vector-vector or axial-pseudoscalar ones are neglected.

Despite the assumptions, this scheme has many great advantages: first of all it gives a dynamical description of the particularly complex mechanism which leads to the creation of scalar resonances, naturally taking into account the mixing amongst different states, and explaining why the scalars differ from vector and tensor mesons. Moreover, it is simple and with very few parameters.

Consequently, we believe this model is not a machine from which one can blindly extract numbers from a fitting program, but an efficient schematic way to approach the dynamics of the non-perturbative hadronic world: a world in which so much remains to be understood.
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