Optimization on effects of design parameter on displacement amplification ratio of 2 DOF working platform employing Bridge-type compliant mechanism flexure hinge using Taguchi method

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Abstract. High precision positioning systems was usually utilized in industrial applications. But the systems have workspace limits. Therefore, a new design for 2-DOF working platform employing two bride-type compliant mechanism flexure hinge was suggested in this paper. The INVENTOR 2018 software was used to design the mechanism and the output displacement, output stress were obtained by Finite element method in ANSYS. The simulation results were achieved maximum value for output displacement of 0.61 mm according x axis direction and y direction, and maximum value for output stress of 120 MPa while input displacement of 0.01 mm, input body length of 5 mm, fillet of radius of 0.2 mm, incline angle of 1.2 degree, width of flexure hinge of 2 mm, thickness of flexure hinge changes from 0.25 mm to 0.75 mm. The simulation results of two remain cases was also presented and discussed. In three cases, thickness of flexure hinge has important effected on output displacement and output stress. The maximum value of displacement amplification ratio was achieved 61.13 by simulation for two x and y axis directions. The S/N analysis demonstrated the design parameters have important effects on output displacement and output stress. The optimal total displacement amplification ratio was obtained 91.35 compared with the predicted value of 98.08 that was good agree with 6.86% deviation error and the optimal value of stress was obtained 73.3562 MPa compared with the predicted value of 80.0936 MPa that was good agree with deviation error 8.41%.

1. Introduction
The tradition joints always present clearance that cause vibration and friction, which lead to wear the joint quickly. In order to avoid these phenomena of traditional joints, the flexure hinge was created and was used in many mechanisms. This paper investigated and optimized influence of the design factors on output displacement and output stress of two DOF platform employing bridge-type compliant mechanism flexure hinge (BTCMFH) by using static structural in ANSYS. Many FHs with different profile were designed and developed such as right circular, right angle, parabolic, elliptic, V-shape, corner filleted, cycloidal, hyperbolic and hybrid FH. Xu and Li [1] compared displacement amplification ratio (DAR) obtained between some methods theory, FEA and experiment. Qi et al. [2]
applied the kinematics theories and elastic beam theory (EBT) determined (DAR) of BTCMFH. In 2016, Liu and Yan [3] stated a new analyzed method using EBT to estimate influence of external load on the DAR, and was compared with FEM in ANSYS. Ling et al. [4] applied the power preservation law and EBT to improve an approach for bridge-type, rhombus-type compliant mechanisms, and compared with FEM and experiment. Choi et al. [5] suggested a BTCMFH which was fully compliant with concentration force and distribution force. Their results were compared with experiment and published research. Ma et al. [6] stated that the DAR increases as the TOFH reduces, and this problem was obtained by using the FEM and a mathematic model. Ling et al. [7] stated an analytical approach towards for accuracy positioning platform. This method was confirmed by FEM and the published investigation. The stiffness matrix method was applied to calculated the DAR and was compared with FEM and experiment Bhoge and Deshmukh [8] pointed out difference between flexure joints using ANSYS.

T. P. Dao and S.-C. Huang [9-14], designed and optimized some compliant mechanism using Taguchi method, grey relational analysis and fuzzy logic method.

The goal of the paper investigated and optimized effects of design parameter on a 2-DOF working platform employing bridge-type compliant mechanism flexure hinge based on the simulation results of ANSYS.

2. Modeling and methodology

2.1. Modeling

The 2-DOF working platform employing BTCMFH as shown in Figure 1. Figure 1a presented 3D of the mechanism, Figure 1b presented projection of the mechanism on the oxy plane with total dimension of 80 mm x 80 mm x 10 mm. This mechanism used two BTCMFHs similar to each other. Therefore, we only need to compute for one BTCMFH.

![Figure 1](image)

**Figure 1.** Two DOF platform employing Bridge-type compliant mechanism flexure hinge ((a) 3D, (b) 2D).

The BTCMFH was presented in Figure. 2 with dimension of 70 mm x 25 mm x 10mm. Figure 1a is 3D model, Figure 2b is 2D model and Figure 2c is zoom in medium body and two FHs to obviously see incline angle θ between two FHs. The mechanism has 8 FHs with 4 mm length and thickness change, 4 medium bodies, 2 input bodies with input length L change, 1 fixed body and 1 output body of 8 mm length and of 10 mm width. The concentration force, distribution force or displacement can be used to input for mechanism. And then output body will be move downward reverse y axis direction and will be move to x axis direction.

2.2. Analysis finite element method

The first, the INVENTOR software was used to create the mechanism and then insert into Static structural of ANSYS. And then select material for the mechanism was aluminum AL-7075 as listed in
Table 1. The mechanism was meshed as shown in Figure 2a with number of nodes and number of elements equal to 130524 nodes, 73352 elements, respectively. The fixed support was utilized to fixed the A surface and the B surface, and two input bodies were inputted the displacement of 0.01 mm each on the C surface and the D surface according to x directions, the E surface and the F surface according to y directions as presented in Figure 2b.

![Figure 2](image)

**Figure 2.** Bridge-type compliant mechanism ((a) 3D, (b) 2D, (c) Large view medium body and two flexure hinges.

| Material  | Young's modulus (GPa) | Poisson's ratio | Yield (MPa) |
|-----------|----------------------|----------------|-------------|
| Aluminium | 72                   | 0.33           | 503         |

2.3. **Taguchi method**

In order to optimize output characteristics, the mathematical models must establish firstly, and then optimization methodology are used. However, if errors of the mathematic model are so large, the optimization result can’t accept. Therefore, in this study used Taguchi method as an effective optimization method to optimize displacement amplification ratio and stress of mechanism as presented in Figure 2. Advantages of the Taguchi method include fewer required experiments or simulations by utilizing orthogonal arrays, and its ability to minimize influences of parameters that cannot be controlled. Moreover, it is a simple method and easy to use. In this method, a loss function is used to determine the error between simulation or experiment and actual values. This function is the signal-to-noise (S/N) ratio [8, 9]. It has three categories, but only the “Smaller is the better” and “the larger the better” categories were used in this paper to optimize the output displacement and output stress, as follows.

“Smaller is the better” approach [9-15]:

\[
S / N = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} y_i^2 \right)
\]  

(1)

“Larger is the better” approach:

\[
S / N = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i} \right)
\]  

(2)

here \( y_i \) is the observed data average of the ith experiment, \( n \) is the number of experiments.

In this paper, the Minitab 18 software was used to design Taguchi method, S/N analysis. The analysis results are presented in results and discussion.
3. Results and discussion

3.1. Influence of thickness of flexure hinge

The deformation was presented according to x and y direction as illustrated in Figure 3 and Figure 4 that identify this deformation (or displacement) reduces from 0.61 mm to 0.138 mm when TOFH increase from 0.25 mm to 0.75 mm with input displacement is equal to 0.01 according to x and axis direction, with input body length of 5 mm, incline angle between two FHs of 0.7 degree, width of FH of 2 mm, fillet radius of FH of 0 mm. The simulation results are higher references [1, 2, 3, 4, 6]. The equivalent stress reduces from 120 MPa to 84.46 MPa as illustrated in Figure 5. The problem demonstrated displacement and stress have been significant effected by TOFH.

3.2. Influence of width of flexure hinge

The phenomenon revealed WOFH have been slightly effected on displacement and stress. Because the displacement reduced from 0.254 mm to 0.230 mm according to x and y axis direction as shown in Figure 6 and Figure 7 and the equivalent stress reduced from 130.87 MPa to 127.53 MPa as illustrated
in Figure 8. While input displacement of 0.01 mm, WOFH increase from 2 mm to 6 mm, input body length of 15 mm, TOFH of 0.5 mm, fillet radius of 0.2 mm, incline angle of 0.8 degree. The displacement amplification reduced from 25 to 23. This result is larger than the result in references [1, 2, 4, 8].

3.3. Influence of incline angle
Besides, the incline angle has been slightly effected on displacement and stress. Because the displacement increased from 0.119 mm to 0.178 mm according to x and y axis direction as presented in Figure 9 and Figure 10 and the equivalent stress reduced from 131.35 MPa to 134.9 MPa as pointed out in Figure 11. While input displacement of 0.01 mm, incline angle increase from 0.8 degree to 1.6 degree, input body length of 15 mm, TOFH of 0.75 mm, fillet radius of 0.2 mm, WOFH of 6 mm. The displacement amplification ratio was achieved from 11.9 to 17.8. The result is larger than the results stated in references [1, 2, 8].

Figure 6. FEM result the x direction displacement with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).

Figure 7. FEM result the y direction displacement with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).

Figure 8. FEM result stress with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).
Figure 9. FEM result the x direction displacement with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).

Figure 10. FEM result the y direction displacement with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).

Figure 11. FEM result stress with different width of flexure hinge ((a) Width of 2 mm, (b) Width of 4 mm, (c) Width of 6 mm).

3.4. Simulations design

The paper presented methodology to optimize displacement and stress of 2-DOF working platform employing bridge-type compliant mechanism flexure hinge and confirm the optimal combination of design parameters. The first, the parameters and theirs level were utilized to design orthogonal array L27 as listed in Table 2. Table 3 present orthogonal array was created by using Minitab 18, output total displacement, output stress and S/N for displacement and stress. In this investigation chooses larger-the-better for output displacement and smaller-the better for output stress. Five input parameters and 3 levels. x is input body length, y is TOFH, z is the fillet radius of FH, t is incline angle between two FHs is w is width of flexure hinge.

3.5. S/N analysis

Table 4 and Table 5 listed S/N for total displacement and stress, respectively. From the maximum values of S/N combine with Figure 12a and Figure 12b identified optimal combination parameters as x3y1z1t3w1 for total displacement and x1y3z3t2w2 for stress. In Table 4 also pointed out TOFH has significant effected on total displacement of this mechanism next to was WOFH, incline angle between two FHs, input body length and the final is fillet radius. And Table 5 revealed input body
length has important effect next to was WOFH, fillet radius, TOFH and the final was incline angle between two FHs.

Table 6 and Table 7 listed means value for total displacement and stress in which the bold value is optimal value.

3.6. Predicted optimization

In this section, the forecasted value is calculated and verified by the optimization models was simulation in ANSYS, as based the optimal combination parameters. First, the mean value of the first frequency modal were listed in Table 5, Second, $\mu_D$ $\mu_\sigma$ and were obtained by using Eq. (3), Eq. (4) and Table 6 and Table 7. The total mean value is equal to 0.4607 mm for displacement and 96.6491 MPa for stress and $\mu_D$ $\mu_\sigma$ were obtained, as follows:

$$\mu_D = D_m + \sum_{i=1}^{q} (D_0 - D_m) = x^3 + y^3 + z^3 + t^3 + w^3 - 4D_m$$

$$= 0.492 + 0.837 + 0.501 + 0.467 + 0.527 - 4 \times 0.4607 = 0.9808 \text{ (mm)}$$

$$\mu_\sigma = \sigma_m + \sum_{i=1}^{q} (\sigma_0 - \sigma_m) = x^3 + y^3 + z^3 + t^3 + w^3 - 4\sigma_m$$

$$= 90.7 + 93.87 + 93.82 + 93.84 + 93.96 - 4 \times 96.6491 = 80.0936 \text{ (MPa)}$$

while the optimal values were obtained with combination parameters $x_3y_1z_1t_3w_1$ for total displacement of 0.9135 mm, $x_1y_3z_3t_2w_2$ of 73.3562 MPa. The deviation error between the optimal values and the predicted values were 6.86% for total displacement and 8.41% for stress, respectively. The displacement value according to x and y direction were obtained 0.646 mm. The displacement amplification ratio was obtained 64.4 for two x and y axis direction.

![Figure 12](image)

Figure 12. S/N plot ((a) For total displacement and (b) For stress.

**Table 2.** Design parameters and their levels.

| Factor       | Unit | Levels |
|--------------|------|--------|
| Input body length | x    | 5 10 15 |
| TOFH         | y    | 0.25 0.5 0.75 |
| Fillet radius | z    | 0.8 1.2 1.6 |
| Incline angle | t    | 0.0 0.2 0.4 |
| WOFH         | w    | 2 4 6   |
### Table 3. Orthogonal arrays, simulation results and S/N ratio for the first frequency modal shape.

| Trial No. | x   | y   | z   | t   | w   | Output displacement (mm) | S/N for output displacement | Output stress (MPa) | S/N for output stress |
|-----------|-----|-----|-----|-----|-----|--------------------------|-----------------------------|---------------------|----------------------|
| 1         | 5   | 0.25| 0.8 | 2   | 0.90249 | -0.8912 | 102.24                  | -40.1924            |
| 2         | 5   | 0.25| 0.2 | 1.2 | 0.81073 | -1.8225 | 98.789                  | -39.8942            |
| 3         | 5   | 0.25| 1.6 | 6   | 0.70507 | -3.0354 | 95.048                  | -39.5589            |
| 4         | 5   | 0.5 | 0   | 1.2 | 0.29433 | -10.6233 | 80.808                 | -38.1491            |
| 5         | 5   | 0.5 | 0.2 | 1.6 | 0.41599 | -7.6183 | 104.45                 | -40.3782            |
| 6         | 5   | 0.5 | 0.4 | 0.8 | 0.24688 | -12.1503 | 80.995                | -38.1692            |
| 7         | 5   | 0.75| 0.8 | 1.6 | 0.18952 | -14.4469 | 77.99                  | -37.8408            |
| 8         | 5   | 0.75| 0.2 | 0.8 | 0.10081 | -19.9299 | 89.578                | -39.044             |
| 9         | 5   | 0.75| 0.4 | 1.2 | 0.18717 | -14.5553 | 86.366                | -38.7269            |
| 10        | 10  | 0.25| 0   | 0.8 | 0.93349 | -0.5978 | 100.77                | -40.0666            |
| 11        | 10  | 0.25| 0.2 | 1.2 | 0.84677 | -1.4447 | 92.266                 | -39.3008            |
| 12        | 10  | 0.25| 0.4 | 1.6 | 0.73869 | -2.6308 | 90.243                | -39.1083            |
| 13        | 10  | 0.5 | 0   | 1.2 | 0.34982 | 9.1231 | 92.694                | -39.341             |
| 14        | 10  | 0.5 | 0.2 | 1.6 | 0.44157 | 7.1000 | 111.12                | -40.9158            |
| 15        | 10  | 0.5 | 0.4 | 0.8 | 0.28695 | -10.8439 | 94.052           | -39.4674            |
| 16        | 10  | 0.75| 0   | 1.6 | 0.2236 | -13.0106 | 93.235              | -39.3916            |
| 17        | 10  | 0.75| 0.2 | 0.8 | 0.12594 | 17.9967 | 105.01               | -40.4246            |
| 18        | 10  | 0.75| 0.4 | 1.2 | 0.2095 | 13.5763 | 98.191                | -39.8414            |
| 19        | 15  | 0.25| 0   | 0.8 | 0.95969 | -0.3819 | 105.74                | -40.4848            |
| 20        | 15  | 0.25| 0.2 | 1.2 | 0.87465 | -1.633 | 95.132                | -39.5665            |
| 21        | 15  | 0.25| 0.4 | 1.6 | 0.76453 | -2.3321 | 90.978                | -39.1787            |
| 22        | 15  | 0.5 | 0   | 1.2 | 0.40407 | 7.8709 | 106.8                | -40.5714            |
| 23        | 15  | 0.5 | 0.2 | 1.6 | 0.46214 | 6.7045 | 112.11                | -40.9929            |
| 24        | 15  | 0.5 | 0.4 | 0.8 | 0.32527 | 9.7551 | 110.48                | -40.8657            |
| 25        | 15  | 0.75| 0   | 1.6 | 0.25742 | -11.7872 | 102.69              | -40.2306            |
| 26        | 15  | 0.75| 0.2 | 0.8 | 0.15158 | -16.1833 | 93.71              | -39.4357            |
| 27        | 15  | 0.75| 0.4 | 1.2 | 0.22986 | -12.7707 | 98.04                | -39.8281            |

### Table 4. Response Table for S/N Ratios for total displacement.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | -9.45 | -1.59 | -7.64 | -9.86 | -7.13 |
| 2     | -8.48 | -9.09 | -8.89 | -8.11 | -8.49 |
| 3     | -7.66 | -14.92 | -9.07 | -7.63 | -9.97 |
| Delta | 1.792 | 13.33 | 1.44 | 2.23 | 2.84 |
| Rank  | 4    | 1    | 5    | 3    | 2    |

### Table 5. Response Table for S/N Ratios stress.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | -39.11 | -39.71 | -39.59 | -39.79 | -40.16 |
| 2     | -39.76 | -39.87 | -39.99 | -39.47 | -39.41 |
| 3     | -40.13 | -39.42 | -39.42 | -39.73 | -39.42 |
| Delta | 1.02 | 0.45 | 0.58 | 0.33 | 0.74 |
| Rank  | 1    | 4    | 3    | 5    | 2    |

### Table 6. Response Table for Means for total displacement.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | 0.428 | 0.837 | 0.501 | 0.448 | 0.527 |
| 2     | 0.462 | 0.359 | 0.470 | 0.467 | 0.451 |
| 3     | 0.492 | 0.187 | 0.410 | 0.467 | 0.404 |
| Delta | 0.064 | 0.651 | 0.091 | 0.019 | 0.122 |
| Rank  | 4    | 1    | 3    | 5    | 2    |

### Table 7. Response Table for Means for stress.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | 90.70 | 96.80 | 95.89 | 98.06 | 102.11 |
| 2     | 97.51 | 99.28 | 100.24 | 94.34 | 93.96 |
| 3     | 101.74 | 93.87 | 93.82 | 97.54 | 93.87 |
| Delta | 11.05 | 5.41 | 6.42 | 3.72 | 8.24 |
| Rank  | 1    | 4    | 3    | 5    | 2    |
4. Conclusions
This paper analyzed and discussed effects of design parameters on displacement amplification ratio of two DOF working platform employing bridge-type compliant mechanism by using ANSYS. The simulation results demonstrated TOFH has important effected on output displacement and output stress and two remain have slightly effected. The results compared with the previous study. It is identified that the mechanism has displacement amplification ratio is larger than some investigation published. The simulation results were obtained 0.95698 mm for output displacement and 105.74 MPa for output stress. The optimal values were obtained 0.9135mm for output displacement and 73.3562 MPa and amplification ratio was obtained 64.6 for two x and y axis direction. The optimal results demonstrated the design parameters have significant effected on displacement amplification ratio, first is thickness of flexure hinge, second is width of flexure hinge, third is incline angle between two flexure hinges, fourth is input body length and the final is fillet radius. Besides, the design parameters also have strong influenced on output stress, first is length of input body, second is width of flexure hinge, third is fillet radius, fourth is thickness of flexure hinge and the final is incline angle between two flexure hinges.

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