The Deep-Inelastic Structure Functions of $\pi$ and $\rho$ Mesons

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We compute the lower moments of the structure functions of $\pi$ and $\rho$. Of particular interest are the spin-dependent structure functions of the $\rho$ as they give new information about quark binding effects.

1. INTRODUCTION

Much of our knowledge about QCD and the structure of hadrons has been derived from deep-inelastic scattering experiments. While most work to date has been on the nucleon, there are proposals to investigate the structure functions of mesons [1]. In this talk we shall consider $\pi$ and $\rho$ targets.

No polarization is possible for the $\pi$, so we have two structure functions: $F_1$ and $F_2$. The $\rho$, being a spin 1 particle, has eight structure functions [2]: $F_1$, $F_2$, $g_1$, $g_2$, $b_1$, $b_2$, $b_3$ and $b_4$. The structure functions $b_3$ and $b_4$ receive contributions from twist-four (and higher) operators only and so will not concern us here. We shall also not consider the structure function $g_2$, which is least likely that it will be measured.

Writing

$$M_n(f) = \int_0^1 dx x^{n-1} f(x),$$

we find for the moments of the $\pi$ structure functions

$$2M_n(F_1) = c_n^{(1)} v_n,$$
$$M_{n-1}(F_2) = c_n^{(2)} v_n,$$

while the moments of the $\rho$ structure functions are given by

$$2M_n(F_1) = c_n^{(1)} a_n,$$
$$M_{n-1}(F_2) = c_n^{(2)} a_n,$$
$$2M_n(b_1) = c_n^{(1)} d_n,$$
$$M_{n-1}(b_2) = c_n^{(2)} d_n,$$
$$2M_n(g_1) = c_n^{(3)} r_n,$$

where the $c_n$’s are the Wilson coefficients, and $v_n$, $a_n$, $d_n$ and $r_n$ are the operator matrix elements. The latter derive from two operators,

$$O_{\mu_1\cdots\mu_n} = \frac{1}{2^{n-1}} \bar{\psi} \gamma_{\mu_1} i \not{D}_{\mu_2} \cdots i \not{D}_{\mu_n} \psi - \text{Tr},$$
$$O_{\mu_1\cdots\mu_n}^5 = \frac{1}{2^{n-1}} \bar{\psi} \gamma_{\mu_1} \gamma_5 i \not{D}_{\mu_2} \cdots i \not{D}_{\mu_n} \psi - \text{Tr},$$

and are given by

$$\langle \pi | O_{\{\mu_1\cdots\mu_n\}} | \pi \rangle = 2v_n p_{\mu_1} \cdots p_{\mu_n} - \text{Tr}$$
and

\[ \langle \rho, \lambda | \mathcal{O}_{\mu_3 \ldots \mu_n} | \rho, \lambda' \rangle = 2 S [ a_n p_{\mu_3} \cdots p_{\mu_n} \delta_{\lambda\lambda'}^{\sigma} \]

\[ + d_n (\epsilon_{\mu_3}^{\lambda'} \epsilon_{\mu_2}^{\lambda} - \frac{1}{3} p_{\mu_3} p_{\mu_2} \delta_{\lambda\lambda'}^{\sigma}) p_{\mu_1} \cdots p_{\mu_n} ] , \]

\[ \langle \rho, \lambda | \mathcal{O}_{\mu_3 \ldots \mu_n}^{\ast} | \rho, \lambda' \rangle = \frac{2i}{m^{2}} S [ \tau_{n} \epsilon_{\rho\sigma \tau_{1}}^{\lambda} \epsilon_{\rho}^{\lambda'} \epsilon_{\sigma}^{\lambda'} \]

\[ \times p_{\tau} p_{\mu_2} \cdots p_{\mu_n} ] , \]

where \( S \) symmetrizes and subtracts traces.

In parton model language

\[ b_{1}(x) = \frac{1}{2} \left[ q^{0}(x) - q^{1}(x) \right] , \]

where \( q^{m}(x) \) is the probability to find a quark of fractional momentum \( x \) in a \( \rho \) with spin projection \( m \) relative to its momentum. If the quarks are in a pure \( s \)-wave state, we would expect \( b_{1} \) to vanish. The other structure functions have more or less the same interpretation as in case of the nucleon.

2. LATTICE CALCULATION

We have generated 500 quenched gauge field configurations on a \( 16^{3} \times 32 \) lattice at \( \beta = 6.0 \). On these configurations we have computed the matrix elements \( v_{n}, a_{n}, d_{n} \) and \( r_{n} \), using Wilson fermions. The calculations are done for three hopping parameters, \( \kappa = 0.1515, 0.1530 \) and 0.1550, corresponding roughly to quark masses of 190, 130 and 70 MeV, respectively.

The matrix elements can be obtained from the ratio of three- to two-point functions

\[ \frac{\langle \eta(t) \mathcal{O}(x) \eta^{\dagger}(0) \rangle}{\langle \eta(t) \eta^{\dagger}(0) \rangle} = R \]

for \( t \gg \tau \gg 0 \), where \( \eta = \pi, \rho \) is the source/sink. For the \( \pi \) we have

\[ R \propto \langle \pi | \mathcal{O} | \pi \rangle , \]

while for the \( \rho \) we have

\[ R_{ij} \propto \sum_{\lambda, \lambda'} \epsilon_{\lambda}^{\dagger} \epsilon_{\lambda'}^{\dagger} \langle \rho, \lambda | \mathcal{O} | \rho, \lambda' \rangle . \]

A typical such ratio is shown in Fig. 1.

The operators will give in general divergent results as the lattice spacing goes to zero. They must be renormalized in the same way the Wilson coefficients must be renormalized. We define finite operators renormalized at the scale \( \mu \) by (\( a \) being the lattice constant)

\[ \mathcal{O}(\mu) = Z_{\mathcal{O}}((a \mu)^{2}, g(a)) \mathcal{O}(a) . \]

The renormalization constants \( Z \) have been computed to one loop order in perturbation theory.

3. RESULTS

We work at a scale of \( \mu^{2} = (1/a)^{2} \approx 4 \text{GeV}^{2} \). We shall present our results as a series of plots. In Fig. 2 we show the moments of the \( \pi \) structure function \( F_{1} \). For the lowest moment, \( < x > \), we find that it is somewhat larger than the phenomenological result. The higher moments are in better agreement. This is similar to what has been found for the nucleon structure functions.

The unpolarized \( \rho \) structure function looks very similar to the \( \pi \) structure function, so that we will not discuss it here. In Fig. 3 we show the moments of the polarized \( \rho \) structure function \( g_{1} \). The lowest moment \( r_{1} \) indicates that the valence quarks carry about 60\% of the total spin of the \( \rho \). A similar calculation for the nucleon gave a quark spin fraction of the same value.
Figure 2. The moments of the $\pi$ structure function. The subscripts $a, b, bp$ stand for different representations [4]. For each moment the heaviest quark mass is on the right. The leftmost value is the extrapolation to the chiral limit. The heavy quark mass limit is given by the dotted lines, while the experimental value [6] is given by the dashed lines.

In Fig. 4 we show the moments of the polarized $\rho$ structure function $b_1$. The lowest moment turns out to be positive and surprisingly large, albeit with large statistical errors. Perhaps this indicates that the valence quarks have a substantial orbital angular momentum.

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