Dynamic Response of Functionally Graded Carbon Nanotube Reinforced Sandwich Plate

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Abstract: In this article, the dynamic response of the carbon nanotube-reinforced functionally graded sandwich composite plate has been studied numerically with the help of finite element method. The face sheets of the sandwich composite plate are made of carbon nanotube-reinforced composite for two different grading patterns whereas the core phase is taken as isotropic material. The final properties of the structure are calculated using the rule of mixture. The geometrical model of the sandwich plate is developed and discretized suitably with the help of available shell element in ANSYS library. Subsequently, the corresponding numerical dynamic responses computed via batch input technique (parametric design language code in ANSYS) including Newmark’s integration scheme. The stability of the sandwich structural numerical model is established through the proper convergence study. Further, the reliability of the sandwich model is checked by comparison study between present and available results from references. As a final point, some numerical problems have been solved to examine the effect of different design constraints (carbon nanotube distribution pattern, core to face thickness ratio, volume fractions of the nanotube, length to thickness ratio, aspect ratio and constraints at edges) on the time-responses of sandwich plate.

Keywords: Carbon nanotube; Dynamic response; Finite element method; ANSYS

1. Introduction
Functionally graded carbon nanotube-reinforced composite (FG-CNTRC) is a new kind of composite material. In functionally graded composite, material properties varies in one or more coordinate directions by some defined grading pattern. From last one decade, many researchers have been attracted to work on the FG-CNTRC structures. Because, CNT possesses extraordinary mechanical, electrical and thermal properties. In 2009, first time Shen [1] used the functionally graded concept to examine the nonlinear bending responses of the FG-CNTRC plate with functionally graded concept based on the higher-order shear deformation theory (HOSDT). Later, Zhu et al. [2] implemented the first-order shear deformation theory (FOSDT) to examine the natural frequency and mid-point displacement of the FG-CNTRC plate. Dastjerdi et al. [3] implemented a refine shear deformation theory to examine the free vibration behaviour of FG-CNTRC plate resting in an elastic medium. They found that increasing the CNT volume fraction within the matrix lead to increase the natural frequency. It is also found that the increasing of thickness ratio and aspect ratio lead to increase the fundamental frequency. Mehar et al. [4] computed the linear fundamental frequency of the FG-CNTRC plate using the HOSDT mid-plane kinematics and found the similar trend of results as discussed in [3]. Further, the concept of functionally graded concept implemented on the CNT reinforced sandwich structure [5–7]. It is well known that sandwich structure consists minimum three faces, two stiffer face sheets to wear the external load and one lightweight core to transfer the shearing force between both face sheets. The static and free vibration
behaviours of the CNT reinforced sandwich plate are examined by Natarajan et al. [5] using shear deformation theories with different degree of freedom (DOF). The results indicate that increases in DOF of nodes lead to increase the flexibility of the plate. In another word central deflection is increases and the natural frequency decreases with increase the DOF of nodes. Wang and Shen [6] examined the nonlinear mechanical characteristics of the CNT-reinforced sandwich plate using HOSDT. The ratio of nonlinear to linear frequency increases with the higher value of amplitude ratio (ratio of maximum central deflection to thickness of the plate). Lei et al. [8] illustrated the dynamic behaviour of the FG-CNTRC plate using an element-free kp-Ritz method based the FOSDT kinematic model.

From the literature review, it is found that dynamic response of the CNT-reinforced sandwich plate with functionally graded concept is not presented in open sources. In the present paper, the dynamic response of the FG-CNT-reinforced sandwich plate using ANSYS environment based on the FOSDT has been examined. Extended ROM has implemented to examine the material properties of the CNT/epoxy composite and Newmark’s time integration method to solve the final governing equation. Finally, effect of different design constraints on the dynamic response of the sandwich plate has been computed and illustrated in details.

2. Theory and formulation
In this paper, two different type CNT-reinforced sandwich plate are used namely, UD-CNT and FG-CNT. UD and FG represent for uniformly distributed and functionally graded, respectively. The geometrical dimensions of the sandwich plate are taken as length \((a)\), width \((b)\), the thickness of the core \((h_c)\), the thickness of each face sheet \((h_f)\) and the total thickness of the sandwich plate as \((h = h_c + 2h_f)\) as shown in Figure 1. In the face sheets, CNT volume fraction is assumed to be distributed using predefined grading patterns in \(Z\)-direction and align along the length of the plate. The effective volume fraction \((V_{CNT})\) with respect to \(Z\)-direction can be calculated using the following formulae:

For FG-CNT-reinforced sandwich
\[
V_{CNT} = 2 \left( \frac{h - z}{h - h_f} \right) V_{CNT}^* \quad \text{for top bottom sheet} \\
V_{CNT} = 0 \quad \text{for core} \\
V_{CNT} = 2 \left( \frac{z - h_f}{h - h_f} \right) V_{CNT}^* \quad \text{for top face sheet}
\]

(1)

For UD-CNT-reinforced sandwich
\[
V_{CNT} = V_{CNT}^* \quad \text{for top and bottom face sheet} \\
V_{CNT} = 0 \quad \text{for core}
\]

(2)

where, \(V_{CNT}^*\) is the total CNT volume fraction within each face sheet.

![Figure 1. Geometry of CNT-reinforced sandwich plate.](image)
The core of the sandwich is made of by the pure epoxy an isotropic homogeneous material. To the convenience, the final elastic and shear modulus of the face sheets are calculated by the extended rule of mixture (ROM) as follows [2,9]:

\[
E_{11} = \frac{V_w}{E_w} + \eta_1 V_{CNT} E_{11}^{CNT}
\]

(3)

\[
\frac{\eta_2}{E_{22}} = \frac{V_w}{E_w} + \frac{V_{CNT}}{E_{22}^{CNT}}
\]

(4)

\[
\frac{\eta_3}{G_{12}} = \frac{V_w}{G_w} + \frac{V_{CNT}}{G_{12}^{CNT}}
\]

(5)

where, \(\eta_1, \eta_2\) and \(\eta_3\) are the effectiveness parameters of CNT within the composite. The values of effectiveness parameters taken as same as [2]. Similarly, the density and Poisson’s ratio of the composite are calculated using the ROM [2].

To examine the dynamic behaviour of the CNT-reinforced sandwich plate, a simulation model is developed using ANSYS parametric design language (APDL) code. In ANSYS, various kind of elements are available for modelling of the sandwich structure. In the present study, shell 281 has been implemented to develop the sandwich plate model. Shell 281 possesses eight nodes on each element and each node possesses six DOF (three translations along the X, Y and Z-directions and three rotations about X, Y and Z-directions). It is based on the FOSDT and the kinematic model is as follows:

\[
\begin{align*}
\mathbf{u}_x &= \mathbf{u}_{0x} + Z \theta_x \\
\mathbf{u}_y &= \mathbf{u}_{0y} + Z \theta_y \\
\mathbf{u}_z &= \mathbf{u}_{0z} + Z \theta_z
\end{align*}
\]

(6)

where, \(\mathbf{u}_{0x}, \mathbf{u}_{0y}, \mathbf{u}_{0z}\) and \(\theta_x, \theta_y, \theta_z\) are the displacement of any point along the X, Y and Z-directions, respectively. Similarly, \(\theta_x, \theta_y, \theta_z\) are representing the rotation of mid-plane about X, Y and Z-directions, respectively. “0” in \(\mathbf{u}_{0x}, \mathbf{u}_{0y}\) and \(\mathbf{u}_{0z}\) is represented for mid-plane.

The final governing equation for dynamic analysis can be written in the following form:

\[
[M]\{\ddot{\mathbf{\delta}}\} + [K]\{\mathbf{\delta}\} = \{\mathbf{f}\}
\]

(7)

where, \([M]\) and \([K]\) are global mass and stiffness matrix, respectively. \{\mathbf{\delta}\}, \{\delta\} and \{\mathbf{f}\} are acceleration, displacement and force vector, respectively.

Finally, the dynamic response is computed by solving the final governing equation using Newmark’s time integration method with time steps “\(\Delta t\)” and total time “\(T\)”.

3. Result and discussion

In the present analysis, single-walled carbon nanotube and the epoxy are used as the reinforcement and matrix material, respectively. The material properties of the CNT are taken as same as [8] \(E_{11}^{CNT} = 5.6466\) GPa, \(E_{22}^{CNT} = 7.08\) GPa, \(G_{12}^{CNT} = 1.9445\) GPa, \(v_{w} = 0.17\) and \(\rho_{w} = 1400\) kg/m³. The material properties of the isotropic epoxy are considered as \(E_{w} = 2.1\) GPa, \(v_{w} = 0.34\) and \(\rho_{w} = 1150\) kg/m³. In general material and geometrical properties are taken as \(V_{CNT} = 0.17, a/h=10, a/b=1, h_c/h_f=2, q_0=0.1\) MPa, \(\Delta t=1\) μ-sec and all edges simply supported condition (SSSS), if not explained. To provide the restriction at free edges, SSSS type supported condition is used such that:

\[
\begin{align*}
\mathbf{u}_{0x} &= \mathbf{u}_{0y} = \theta_x = \theta_z = 0 \text{ at } X=0 \text{ and } a \\
\mathbf{u}_{0x} &= \mathbf{u}_{0y} = \theta_x = \theta_z = 0 \text{ at } Y=0 \text{ and } b
\end{align*}
\]

Convergence and validation

In this section, the accuracy and reliability of the developed model have been checked by performing the convergence and validation studies. The convergence study is performed for different mesh size with other design parameters as FG-CNT, SSSS, \(V_{CNT} = 0.11, a/h=10, a/b=1, h_c/h_f=2, q_0=0.1\) MPa and time step \(\Delta t=1\) μ-sec. Convergence study shows that developed model is converging well with mesh refinement at 10×10 mesh size as illustrated in Figure 2 and this size used for the further study.
Figure 2. Convergence with mesh refinement.

Figure 3. Comparison of dynamic response of isotropic composite plate.

To illustrate the reliability of developed model for dynamic analysis, two comparison studies have been performed. First comparison study conducted for isotropic plate by taking the design parameters as same as Kant et al. [10] such as $E_2=2.1\times10^6\text{N/cm}^2$, $E_1=25E_2$, $G_{12}=G_{13}=G_{23}=0.5E_2$, $v_{12}=v_{23}=0.25$, density $=8\times10^3 \text{ N sec}^2/\text{cm}^4$, $h=5\text{cm}$, $a=b=25\text{cm}$, $q_0=10\text{N/cm}^2$ and $\Delta t=2\mu\text{-sec}$ with all edges simply supported and illustrated in Figure 3.
The second comparison study performed for the UD-CNTRC plate using the same material properties as those in [8] and presented in Figure 4. Both comparison studies show that the developed model is in agreement well with previously published results.

**Parametric study**

In this section, the effect of different design parameters on the dynamic behavior of the CNT-reinforced sandwich plate is examined as shown in Figures 5-8. Figure 5 illustrates that the central point transverse displacement of the FG-CNT-reinforced sandwich plate decreases with an increase in CNT volume fraction within the composite. Because CNT possesses higher stiffness compared to the matrix, the concentration of CNT within the composite leads to an increase in the total stiffness of the composite.

Figure 6 illustrates that an increase in thickness ratio ($a/h$) leads to an increase in central point transverse displacement and cycle time of displacement with higher values of thickness ratio. Further, the influence of aspect ratio for three different values ($a/b = 1, 1.5$ and $2$) and effect of CNT grading within the matrix are examined as shown in Figure 7. From the figure, it is clear that the transverse displacement of the central point and cycle time of displacement decreases with a higher value of aspect ratio ($a/b$). Figure 7 also illustrates that the central deflection and cycle time of the FG-CNT are less as compared to the UD-CNT-reinforced sandwich plate.
4. Conclusion
In the present study, the dynamic responses of the FG-CNT-reinforced composite plate based on the FOSDT is examined for two different types of CNT grading patterns. The effective material properties are computed using the extended ROM and solved using Newmark’s time integration method. Finally effect of different design constraints on the dynamic behaviour is computed and the major conclusions discussed in the following points.

- The transverse displacements of the central point of plate decreased for the higher values of aspect ratios and volume fractions of CNT within the composite, however, it follows a reverse trend for the thickness ratio.
- The cycle time of response decreases for the higher values of the aspect ratios and volume fractions of CNT within the composite, however, it shows a converse trend for the thickness ratio.
- The results reveal that the FG-CNT-reinforced sandwich plate is the stiffer configuration than that of the UD-CNT-reinforced sandwich plate.

Nomenclature

$a$, $b$ and $h$ are dimension of the plate in X, Y and Z-direction, respectively.

$h_c$ and $h_f$ = thickness of the core and face sheets, respectively

$E_m$, $G_m$ and $v_m$ = Young’s modulus, shear modulus and Poisson’s ratio of the epoxy, respectively.

$E_{11}^{CNT}$, $E_{22}^{CNT}$ and $E_{33}^{CNT}$ = Young’s modulus of CNT in X, Y and Z-directions, respectively.

$G_{12}^{CNT}$ = Shear modulus and Poisson’s ratio of the CNT

$E_{11}$, $E_{22}$ and $E_{33}$ = Young’s modulus of the composite in X, Y and Z-directions, respectively.

$V_m$ and $V_{CNT}^{total}$ = total volume fraction of epoxy and CNT in face sheets

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