Computer processing of distorted video sequences obtained by mobile road laboratories

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Abstract. One of the most promising road TPC diagnosis areas is computer processing of data obtained by mobile road laboratories equipped with global positioning systems (GPS, GLONASS), inertial navigation systems (INS), complex video and other devices. The aim of the research is to develop new technologies used for computer processing of video sequences obtained by mobile road laboratories equipped with video-recording systems providing minimization of energy and labor costs. The article presents the results of research on the development of software used for processing video sequences obtained by mobile road laboratories equipped with video-recording systems. Works on the use of video processing software published for the last five years were reviews.

1. Introduction

One of the promising areas of application of artificial intelligence methods is image processing which is widely used in various fields (safety and traffic control, analysis of satellite images, etc.). Computer laboratory processing of video sequences obtained by mobile road laboratories is required as the “human factor” (“digitization”) causes errors and requires much time and money.

The most popular image analysis algorithms implemented on the basis of neural networks [1] do not have scientific-based performance estimates which challenges their application for processing often distorted video sequences of mobile road laboratories.

The relevance of this work is due to the following facts:

1. Currently, it is necessary to apply artificial intelligence methods for processing video sequences, but the methods should be scientifically justified.
2. Cost, execution time and quality of laboratory image analysis (digitization) do not meet modern requirements. These problems can be solved by using artificial intelligence methods whose efficiency has been justified.

Currently, the efficiency of artificial intelligence algorithms is due to information technologies based on the BigTable [2] and BigData [3] methods. The methods described in this paper develop the BigTable methods [2] and complement the BigData approaches [3] to image analysis, ensuring efficient operation for an unlimited number of recognizable images.

The research purpose is

1. to assess complexity of the partial embedding of samples into the forefront objects of the image.
2. to assess complexity of the partial embedding of samples into other objects of the image.

The solution to these problems can be:
1. Development of software for controlling the partial embedding of samples into the forefront objects of the image.
2. Development of software for controlling the partial embedding of samples into other objects of the image.

The scientific novelty of the work is as follows:
1. Assessment of complexity of the partial embedding of samples into the forefront objects of the distorted image.
2. Assessment of complexity of the partial embedding of samples into other objects of the distorted image.

2. Materials and methods

The article [4] analyzes connected contour images (assesses complexity of the search for samples) represented by arcs and arc links with a degree measure. The article [5] assesses complexity of the search for connected contour images, where their relative dimensions are added to the degree measure of arcs (other quantitative characteristics can be used).

Introduction of relative sizes of the arcs raises the issue of scale series of representation of samples and images (for using the results of analysis of compressed images in order to improve the efficiency of analysis of original images) [6].

The article [13] analyzes distorted contour images, i.e. it analyzes foreground objects displayed on the image without any distortions, as well as objects of the second and other planes which are more or less covered by objects which are close to the survey point.

This work continues the research in [13], namely, it expands the range of tasks to be solved, since it assumes possible distortion of objects due to interference and partial covering of objects.

Remark 1. The term “object” denotes a part of the analyzed image which the algorithm recognizes as a sample from the database; “subjects” are parts of the image which the algorithm does not recognize as a sample from the database.

Complexity of the search for the partial (rather than isomorphic as in [13]!) embedding into the image of objects (samples) etc. will be assessed. It provides the ability to search for distorted objects.

The embeddings are based on more complex technical schemes which makes them more complex.

The partial embedding of a sample into the objects of the first plan will be considered successful (i.e., the sample was revealed in the image) if this embedding is almost isomorphic for the basis (a selected part of the sample)

The scheme for searching for second-plan samples involves checking of partially displayed parts of some samples are added to a selected part of the sample by arcs of the decision tree (universe) lying inside the union of samples of the first plan.

The scheme for searching for second-plan samples involves control over partially displayed parts of samples added to a selected part of the sample by arcs of the decision tree (universe) lying inside the union of foreground samples.

The third plan samples use the concept: to lie inside a set of samples of the first and second plans, etc.

The issue of complexity of testing “to lie inside the union of samples” can be solved on the basis of geometric representation of the universe on a Cartesian plane in the polar coordinate system, where complexity of testing “to lie inside the region” will have a mathematical solution based on the constructions and algorithms suggested in [13].

In the present article (similar to [13]), the geometric representation of the universe on a Cartesian plane in the polar coordinate system is not reduced to rigorous mathematical formalizations, since this
will increase the volume of the article, and cumbersome formulas will make it difficult to understand the main ideas and results.

Therefore, geometrical interpretation of arcs, arc links and the universe on a Cartesian plane in the polar coordinate system will be provided in the comments.

The results are based on the assumption that all the objects in the image, as well as samples, have the same scale. In addition, the shooting point does not distort objects due to the volume of objects (3d) and the shooting angle does not decrease the values of the results.

2.1. Formalization. Mathematical models of samples \( S_1, S_2, \ldots, S_n \) and image \( P \) will be represented by five-basic algebraic systems (a.c.) \([7, 8]\)

\[
P = \langle A, B, R, V, M; Se, Ag, Me, Re \rangle \tag{1}
\]

where \( A \) is the set of arcs; \( B \) is the subset of the set of arcs \( A \) (basis); \( V \) is the set of arc links; \( R \) is the set of permissible angles or sectors of the circle (e.g., from 0 to 180 degrees or fractions of degrees, represented by the initial segment of natural numbers from 0 to D); \( M \) is the set of relative measures of the length of arcs represented by the initial segment of natural numbers from 1 to E; the single function \( Se: A \rightarrow V \) that determines the number of degrees (or fractions of a degree) of the arc as a sector of the circle; the single function \( Ag: A \rightarrow M \) that determines the number of degrees (or fractions of a degree in accordance with the task of set \( V \)) of the angle of intersection of the arcs; the single function \( Me: A \rightarrow M \) that compares each arc with its relative value; the triple relation \( Re \) connects the connection of the arcs \( rel \) with the corresponding arcs, i.e. \( Re \) is the subset of the Cartesian product \( R \times A \times A \), if \( Re (rel, a, b) \) and \( Re (rel, a_1, b_1) \), \( a = a_1 \), \( b = b_1 \).

Remark 2. The arcs on a Cartesian plane will be represented by sectors of circles of certain radii. The arc links correspond to two arcs having a common point for their ends.

Let the samples be represented by the aggregate a.c.

\[
S_1 = \langle A_1, B_1, R_1, V, M; Se, Ag, Me, Re \rangle
\]

\[
S_2 = \langle A_2, B_2, R_2, V, M; Se, Ag, Me, Re \rangle
\]

\[
S_m = \langle A_m, B_m, R_m, V, M; Se, Ag, Me, Re \rangle
\]

(2)

Image \( P \) is represented by a.c. (1), where the base set \( B \) (basis) is empty.

We assume that a.c. \( P, S_1, S_2, \ldots, S_m \) represent connected images.

Remark 3. The external contour of the sample or the set of forming arcs is used as a basis.

Following [5], let us construct the universe \( Y \) for images with no more than \( n \) arcs and no more than \( k \) links of arcs. Without loss of generality, we can assume that \( n \) is equal to \( k \), and the maximum number of arcs and links of arcs in a.c. (1), (2) do not exceed \( n \).

Let us denote the universe as a.c.

\[
Y = \langle AA, RR, V, M; Se, Ag, Me, Re \rangle \tag{3}
\]

Where the set of arcs \( AA = \{ aa_{\alpha} |, \) arc indices \( \alpha \in U = U_0 \cup U_1 \cup \ldots \cup U_n, \) integer parameter \( k \) take the values from 0 to \( n \) (if \( \alpha \in U_i, i \leq k \) ); the set of arc links \( RR = \{ rr_{\alpha, \beta} \ | \alpha, \beta \in U \}, \) constraints on arc indices \( \alpha, \beta \) will be provided below; the definitions of functions \( Se, Ag, Me, \) and relation \( Re \) will be provided below.

The induction basis, The set

\[
U_0 = \{ (v, m, r, d) | v \in V, m \in M, r \in V, d \in D \}
\]

\( m = 1, \) the set of directions of arc transversal \( D = \{0,1\}, \) where 0 corresponds to the clock-wise transversal, 1 corresponds to the counterclockwise transversal.

Remark 4. Arcs having indices from set \( U_0 \) are \( AA_{0,0} = \{ aa_{\alpha} |, \alpha \in U_0 \} \)

When geometrically interpreted on a Cartesian plane of arcs, all of them are connected with the center of the polar coordinate system, and the parameter \( r \) for the arc \( aa(v, m, r, d) \) determines the angle between the x-axis and the tangent to the arc. 
The parameter \( d \) for the arc \( aa_{(v, m, r, d)} \) determines its concavity at \( d = 0 \) or convexity (at \( d = 1 \)). In particular, if the abscissa axis is the tangent of the arc (i.e., the parameter \( r \) is equal to zero), at \( d = 0 \) the arc \( aa_{(v, m, r, d)} \) is located below the abscissa axis, and at \( d = 1 \), it is located above abscissa axis.

**Induction step.** Let \( i, j > 0 \).

\[
U_i = \{ (v_0, m_0, r_0, d_0) \ldots (v_i, m_i, r_i, d_i) \mid m_j = 1, \\
v_j \in V, m_j \in M, r_j \in V, d_j \in D \}
\]

Then

\[
U_{i+1} = \{ (v_0, m_0, r_0, d_0) \ldots (v_{i+1}, m_{i+1}, r_{i+1}, d_{i+1}) \mid m_j = 1, \\
v_j \in V, m_j \in M, r_j \in V, d_j \in D \}
\]  

(4)

**Remark 5.** Arcs with indices from set \( U_{i+1} \) are \( AA_{i+1,i+1} \).

Geometrically interpreted on a Cartesian plane of arc \( aa_0 \) of rank \( i + 1 \), where \( \alpha = \beta \) (\( v, m, r, d \)), it is connected by its beginning with the end of arc \( aa_0 \) from \( AA_{i,i} \), the parameter \( r \) for arc \( aa_0 \) determines the angle between the tangents of these arcs.

All the arcs from the set \( BB = AA_{0,0} \cup AA_{1,1} \cup \ldots \cup AA_{n,n} \) have length \( l \). Arcs of arbitrary length are represented in sets \( AA_{i,j} \) where \( i < j \) and \( n \geq j \), which is determined as follows

\[
AA_{i,j} = \{ aa_{a,b} \mid \alpha \in U_i, \beta \in U_j, \\
\alpha = \alpha_t(v, m, r, d), \\
\beta = \alpha(v_i, m_i, r_i, d_i) \ldots (v_s, m_s, r_s, d_s) \\
s = j - i \\
v_i = \ldots = v_s, r_i = \ldots = r_s = 0 \\
d_i = \ldots = d_s = d \}
\]

(5)

Arcs \( aa_{a,b} \) have length \( j - i + 1 \), i.e. \( Me(aa_{a,b}) = j - i + 1 \), the sector of the circle of arc \( aa_{a,b} \) is equal to \( v^*(j - i + 1) \), i.e. \( Se(aa_{a,b}) = v^*(j - i + 1) \).

According to (4) and (5), the set of arcs \( AA \) of universe \( Y (3) \) can be determined by formula

\[
AA = \bigcup_{i,j} AA_{i,j}, \text{ where } i \leq j \text{ and } n \geq j , \text{ single functions } Se, Me \text{ are determined above.}
\]

To determine the sets of connections of arcs \( RR \), let us simplify the method for denoting arcs from the sets \( AA_{i,j} \), namely, for the arc of length \( l \) (\( i = j \)), we will write \( aa_{a_1} \) instead of \( aa_{a_i} \); at \( k > 1 \), instead of \( aa_{a_k} \), we will write \( aa_{a_{n_k}} \).

To reduce the cumbersome designations and solve technical problems, we introduced a set of arcs \( AA_{0, AA_{1}, \ldots, AA_{n}} \) where

\[
AA_i = \{ aa_{a,k} \mid \alpha \in U_i, k = \overline{1,n} \}.
\]

The arcs from set \( AA_i \) are of rank \( i \). It is evident that the set of arcs do not intersect in pairs and \( AA = AA_{0} \cup AA_{1} \cup \ldots \cup AA_{n} \). The need to introduce sets \( AA_i \) is due to the fact that otherwise, it is impossible to embed arcs of length more than \( 1 \) into the set of arcs \( AA_0 \) when forming an induction basis. Sometimes we will use sets of arcs \( AA_{i,j} \).

Let the set of arc links be

\[
RR = \{ rr_{a_1,a_2} \mid a_1 = aa_{a_{1,k_1}}, a_2 = aa_{a_{2,k_2}} \}
\]

(6)

\[
\alpha_1 = \beta(v_1, m_1, r_1, d_1), \\
\alpha_2 = \beta(v_2, m_2, r_2, d_2)
\]

Then \( Re(rr_{a_1,a_2}, a_1, a_2) \). \( Ag \ (rr_{a_1,a_2}) = r_1 + r_2 \).
Thus, universe \( Y (3) \) has been determined.
3. Results

Theorem 1. [13] Any a.c. of the form (1), having no more than n arcs and arc links can be isomorphically embedded into universe Y (3).

The proof follows from the construction of universe Y (3) by formulas (4), (5) and (6).

Let a.c. be
\[ \zeta_1: S_1 \rightarrow Y, \zeta_2: S_2 \rightarrow Y, \ldots, \zeta_m: S_m \rightarrow Y \]

are isomorphic embeddings whose existence is provided by Theorem 1, and the images of some arcs of samples S_1, S_2, \ldots, S_m belong to the set of arcs of rank 0 AA_0, i.e.
\[ \zeta_1(S_1) \cap AA_0 \neq \emptyset, \zeta_2(S_2) \cap AA_0 \neq \emptyset, \]
\[ \ldots, \zeta_m(S_m) \cap AA_0 \neq \emptyset \]

(7)

Let us assume that the set of arcs of image P (3) is \( A = \{a_1, a_2, \ldots, a_n\} \) and
\[ \rho_1: P \rightarrow Y, \rho_2: P \rightarrow Y, \ldots, \rho_m: P \rightarrow Y \]
isomorphic embeddings, where arcs \( a_1, a_2, \ldots, a_n \) are mapped into the set of arcs AA_0, i.e.
\[ \rho_1(a_1) \in AA_0, \rho_2(a_2) \in AA_0, \ldots, \rho_m(a_m) \in AA_0 \]

(8)

Let us consider the set \( \Sigma \) of partial injective mappings
\[ \Sigma = \{ \zeta_{\rho_1}, \zeta_{\rho_2}, \ldots, \zeta_{\rho_m} : S_i \rightarrow P \} \]

(9)

Theorem 2. [13] Let \( \xi \) isomorphic embedding of sample \( S_i \) be mapped into image P (1). Then in \( \Sigma \) there is such \( \zeta_{\rho_i} \) that for any arc \( a \in S_i \), \( \xi(a) = \zeta_{\rho_i}(a) \). Thus, all isomorphic embeddings of samples \( S_i \) into P (1) image are represented in set \( \Sigma \).

The proof follows from the assumption that any arc can be isomorphically embedded into AA_0. Let \( \zeta_i: S_i \rightarrow Y \) be isomorphic embedding whose existence is provided by Theorem 1. The arc \( a \in S_i \) such that \( \zeta_i(a) = b \) \( \in \) \( AA_0 \). Let \( \xi(a) = c \), where \( c \in P (1) \).

Let us assume (according to formulas (7) and (8)) that isomorphic embedding \( p_i: P \rightarrow Y \) is such that \( p_i(c) = b \). The \( \zeta_i: S_i \rightarrow Y, (p_i)^{-1}: Y \rightarrow P \) will coincide with isomorphic embedding \( \xi \) by the rules of the universe Y construction ((4), (5) and (6)). The theorem is proved.

The following definition will be basic. Partial injective image \( \zeta_{\rho_i} \) from set \( \Sigma \) is an embedding of sample \( S_i \) into image P (1) with admissible distortions, if restriction on image \( \zeta_{\rho_i} \) on basis \( B_i \) is an isomorphic embedding.

Remark 4. The isomorphic embedding of sample \( S_i \) into image P (1) is an embedding of sample \( S_i \) into image P (1) with admissible distortion.

One of the main results of this work is as follows.

Theorem 3. Let \( \xi \) be the partial embedding of sample \( S_i \) into image P (1) with permissible distortions. Then in set \( \Sigma \), there is \( \zeta_{\rho_i} \) for which \( \xi(a) = \zeta_{\rho_i}(a) \) is true for any arc a \( \in S_i \). Thus, all partial embeddings of \( S_i \) in P (1) with permissible distortions are presented in set \( \Sigma \).

The proof follows from the proof of Theorem 2 and the inductive definition of the totality of partial embeddings.

The direct consequence of Theorem 2 is as follows.

Theorem 4. [13] Complexity of the search for foreground samples does not exceed \( O((w + t) * \) w + m), where w is the number of arcs (t is the number of arc links) of image P (1), m is the number of samples.
Proof. The universe (scheme 2) of article [2] is no different from universe Y which is built on a much more rigorous mathematical level. Therefore, this theorem is a complete analog of Theorem 3 from article [2]. The theorem is proved.

According to the constraints formed above, parameters t and w are less than or equal to n, therefore the constraints on the upper limit of complexity do not exceed

\[ O((n + n) \cdot n + m). \]  \tag{10} 

To search for patterns with permissible distortions, Theorem 5 can be used.

Theorem 5. Complexity of searching for foreground samples with permissible distortions has an upper limit of complexity not exceeding \( O((w + t + w) \cdot w + m) \), where w is the number of arcs (t is the number of arc links) of image P (1), m is the number of samples.

The proof follows from the proof of Theorem 4 and the need to test the embedding of all arcs of the basis whose number does not exceed w.

To search for foreground samples with permissible distortions, the constraints on the upper limit of complexity are as follows

\[ O((n + n + n) \cdot n + m). \]  \tag{11} 

Let us assess search complexity for image of P (1) of samples of the second and further plans which enhances complexity estimates (10) and (11).

When searching for isomorphic embeddings, set \( \Sigma \) of partial injective images has been constructed.

Let \( \Sigma = \Sigma_1 \cup \Sigma_2 \) where \( \Sigma_1 = \{ \lambda_1, \lambda_2, ..., \lambda_n \} \), \( \Sigma_2 = \{ \theta_1, \theta_2, ..., \theta_r \} \), consists of all isomorphic embeddings, \( \Sigma \) is determined by formula (9).

Without loss of generality, we can assume that an isomorphic embedding has been constructed \( \chi : P \rightarrow Y \) and images of partial injective embeddings \( \{ \theta_1, \theta_2, ..., \theta_r \} \) and isomorphic embedding \( \chi \) are completed to full images of multiple arcs \( A_1, A_2, ..., A_m \) of samples \( S_1, S_2, ..., S_m \). Let us assume that partial injective embedding and the set of arcs \( A_j = \{ a_1, a_2, ..., a_p \} \) then the image of mappings will be \( \theta_i \) and \( \chi \). Let us assume that \( A_j = A_{j_1} \cup A_{j_2} \), where \( A_{j_1} = \{ b_1, b_2, ..., b_z \} \) arcs have no partial images due to partial mapping \( \theta_i \).

Thus, complexity of searching for the second plan samples depends on

a) construction of the set of arcs \( A_{j_1} = \{ b_1, b_2, ..., b_z \} \);

b) checking of the location of arcs inside the images \( \Sigma_1 = \{ \lambda_1, \lambda_2, ..., \lambda_n \} \).

Theorem 6. [13] Complexity of searching for second plan samples has an upper complexity limit not exceeding \( O(n + \Psi \cdot n \cdot m) \), where \( \Psi \) is the constant corresponding to the complexity of checking the location of arcs from \( A_{j_2} = \{ b_1, b_2, ..., b_z \} \) inside the images of the samples built by isomorphic embeddings \( \Sigma_1 = \{ \lambda_1, \lambda_2, ..., \lambda_n \} \), where n is the upper constraint on the number of arcs and arc links, m is the number of samples \( S_1, S_2, ..., S_m \).

Proof. Complexity of building sets of arcs \( A_{j_2} = \{ b_1, b_2, ..., b_z \} \) of a) does not exceed n.

The total number of arcs from the set \( A_{j_2} = \{ b_1, b_2, ..., b_z \} \) of b) does not exceed \( n^* \cdot m \).

If \( \Psi \) is the constant corresponding to search complexity for arcs from \( A_{j_2} = \{ b_1, b_2, ..., b_z \} \) inside the samples, then \( O(n + \Psi \cdot n \cdot m) \) is the upper complexity limit for b). The theorem is proved.

To search for patterns with permissible distortions, theorem 7 is used.

Theorem 7. Complexity of searching for second plan samples with permissible distortions has an upper complexity limit not exceeding \( O(n + n + n + \Psi \cdot n \cdot m) \), where \( \Psi \) is the constant corresponding search complexity for arcs from \( A_{j_2} = \{ b_1, b_2, ..., b_z \} \) inside the images of samples built by partial injective embeddings.
\[ \Sigma_i = \{ \lambda_1, \lambda_2, \ldots, \lambda_k \} , \] where \( n \) is the upper constraint on the number of arcs and arc links, \( m \) is the number of samples \( S_1, S_2, \ldots, S_m \).

The proof follows from the proof of Theorem 6 and the need to check the embedding of all arcs of the basis whose number does not exceed \( n \).

Remark 5. The estimate of Theorem 7 are not worse than that of Theorem 6 but it was worse than other estimates of [4, 5, 6], since there is a multiplicative dependence on the number of samples \( m \) (in estimates of [4, 5, 6], the number of samples \( m \) enhanced the complexity estimate only additively). The number of samples \( m \) increases complexity estimates only linearly which allows for solution of tasks of large dimension.

The results of Theorems 6 and 7 make it possible to assess complexity of the search in the analyzed image of samples of the first, second and further plans, if we take into account that during the transition to an arbitrary \( i+1 \)-th plan, the unification of sets \( A_{h_i} = \{ b_1, b_2, \ldots, b_r \} \) decreases.

Theorem 8. [13] Complexity of searching for samples of the second and further plans has an upper complexity limit not exceeding \( O((n + n) \cdot n + m + n + (\Psi \cdot n \cdot m) \cdot n) \)

where \( \Psi \) is the constant corresponding to the search complexity checking inside the images of samples (built by isomorphic embeddings \( \Sigma_i = \{ \lambda_1, \lambda_2, \ldots, \lambda_k \} \), where \( n \) is the upper limit on the number of arcs and links of arcs in the image and samples, \( m \) is the number of samples \( S_1, S_2, \ldots, S_m \).

Proof. The first part of estimate \( O((n + n) \cdot n + m) \) corresponds to the search for foreground samples (theorem 1).

The second part of estimate \( O(n) \) corresponds to the complexity estimate for building the set of arcs \( A_{h_i} = \{ b_1, b_2, \ldots, b_r \} \) (theorem 4). These sets are sufficient for searching for objects of the arbitrary \( i+1 \)-th plan.

The third part of estimate \( O((\Psi \cdot n \cdot m) \cdot n) \) corresponds to the iterations of checks for searching for arcs from sets \( A_{h_i} = \{ b_1, b_2, \ldots, b_r \} \) inside images \( \Sigma = \Sigma_1 \cup \Sigma_2 \cup \ldots \cup \Sigma_i \) (theorem 4), where \( \Sigma_i \) are added at the expense of found samples of \( i \)-th plan.

The number of iterations should no exceed \( n \). The theorem is proved.

Theorem 9. The search complexity for samples of the first, second and other plans with permissible distortions has an upper limit of complexity not exceeding

\[ O((n + n + n) \cdot n + m + n + (\Psi \cdot n \cdot m) \cdot n) \]

where \( \Psi \) is the constant corresponding to the arc search complexity inside the images of the samples found built by isomorphic embeddings \( \Sigma_i = \{ \lambda_1, \lambda_2, \ldots, \lambda_k \} \), where \( n \) is the upper limit on the number of arcs and arc links, \( m \) is the number of samples \( S_1, S_2, \ldots, S_m \).

The proof follows from the proof of Theorem 8 and the need to test embeddings of all arcs of the basis whose number does not exceed \( n \).

Remark 6. Estimates of Theorem 9 are not worse than those of Theorem 8 which shows that there is no complication in the search for samples of \( i \)-th plans which is natural, since it is necessary to consider partial embeddings of images which are generated after the embeddings of the foreground samples have been built.

4. Conclusion

1. The research develops and specifies the approach described in [4, 5, 6], suggests methods for solving image analysis tasks in real conditions, when the image is distorted. These distortions can be caused by imposition of the samples or other factors. The technical solutions can be used for more complex cases, for example, for hardware failures or distortions when transmitting video sequences through communication lines, etc.
2. A significant gap is caused by the lack of analysis of superposition.
3. Geometric interpretation of the universe should be an object of technical solutions.
4. Remark 6 states that, in contrast to the results of [4, 5, 6], in Theorem 4 (Theorem 5 and formula (11)), the number of samples \( m \) worsens the complexity estimate relative to general restriction \( n \) on the number of arcs and arc links. One should take into account applicability of the brute force optimization methods suggested in [6, 7] which can solve NP-difficult problems [11, 12] (e.g., network planning).
5. Assessment of complexity obtained in Theorem 9 shows that software developed on the basis of the suggested approach can analyze video sequences obtained by mobile road laboratories.

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