The starting-up dynamics of the pumping station

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Abstract
A mathematical model of the starting-up of the centrifugal pump rotor with an asynchronous electric motor has been built as part of a pumping station operating on the grid. The described method takes into account such factors as the pressure loss in the pipeline, inertial pressure occurring during the operation of the pump, the torque of the impeller and the torque of the engine.

The obtained method of dynamic analysis makes it possible to estimate the overheating of the electric motor of a centrifugal pump depending on various factors and, as a consequence, to predict its possible failure.

Introduction
Centrifugal pumps are widely used in industrial enterprises, in agriculture and for urban water supply, as a transport system for various working fluids.

The issues related to the centrifugal pumps analysis are widely reported in the literature [1–15]. The works [16]-[22] are of particular interest. However, the issues related to the temperature of the motor during the pump starting-up require separate consideration.

Let's consider a pump set, a schematic diagram of which is shown in figure 1.

A method for analyzing starting phenomena in the pumps has been developed in the course of work on the mathematical model of the considered pump. The correctness of this technique would be checked on an experienced pump with the following parameters: working fluid density $\rho = 1000 \text{ kg/m}^3$; total efficiency of the pump $\eta_\Sigma = 0.7$; radius of the impeller at the output $R_2 = 0.24 \text{ m}$; pipe diameter $d = 0.142 \text{ m}$; pipeline length $l = 20 \text{ m}$; flow coefficient $\mu = 0.5 \cdot 10^{-4}$; front axial clearance between the impeller and the housing $a = 0.001 \text{ m}$; the coefficient of torque-mechanical characteristics of the electric drive $K_1 = 35 \text{ N} \cdot \text{m} \cdot \text{s}$; the coefficient of the torque-mechanical characteristics of the motor $K = 5000 \text{ N} \cdot \text{m}$; winding mass $m = 2 \text{ kg}$; motor operating voltage $U = 220 \text{ V}$; starting factor $K_p = 20$; specific heat capacity $C_{\text{medi}} = 500$; friction resistance coefficient $l = 0.04$; coefficients $K_2 = 200$ and $K_3 = 0$; rotor inertia moment $J = 132 \cdot 10^{-4} \text{kg} \cdot \text{m}^2$.

All graphic dependencies are for the certain pump.
Mathematical model

Based on the momentum theorem, we'll compose the moment equation:

$$J \frac{d\omega}{dt} = M_{db} (t) - \alpha M_{rk} (t).$$  \hspace{1cm} (1)

Where $J$ — the rotor moment of inertia about the axis; $\omega$ is the angular speed of rotation of the pump shaft; $M_{db}$ — the moment of the drive without load; $M_{rk}$ — the moment of the electrodrive at the time of launch; $\alpha$ — losses on the coupling, bearings, pump seals during torque transmission, $\alpha > 1$.

Let’s consider the terms of the equation (1).

The torque on the impeller can be calculated like this

$$M_{rk} = M_c + M_{dt},$$ \hspace{1cm} (2)

where $M_c$ — centrifugal moment, $M_{dt}$ — frictional disk torque.

Centrifugal moment:

$$M_c (t) = \rho Q R^2 \omega(t).$$ \hspace{1cm} (3)
\[ M_{dt}(t) = \frac{\omega(t) \mu \pi R_2^2}{a}. \]  

(4)

where \( a \) — the axial clearance between the impeller and the pump housing.

Thus, the torque of the impeller

\[ M_{rk}(t) = \frac{\omega(t) \mu \pi R_2^2}{a} + \rho QR_2^2 \omega(t). \]  

(5)

The torque, generated by drive, is described by formula [8]:

\[ M_d(t) = K - K_1 \omega(t), \]  

(6)

where \( K \) and \( K_1 \) — coefficients of torque-mechanical characteristics of the electric drive.

Let’s derive the balance equation for the required head:

\[ H_{Ht} = H_{st} + H_{tr} + h_{in}, \]  

(7)

where \( H_{Ht} \) — pump head required to overcome losses; \( H_{st} \) — static pressure between the tanks of the feeder and receiver; \( H_{tr} \) — pressure loss in the pipeline; \( h_{in} \) — inertial head.

Let’s consider the terms of the equation (7).

We’ll use the dynamic similarity formulas for a pump with a constant linear size for recalculation from different frequencies:

\[ H_{Ht}(Q, \omega) = H_0 \left( \frac{\omega_0}{\omega} \right)^2, \]  

(8)

\( 0 \) — the initial values \( \omega \) — head and angular velocity of the pump at starting-up; \( H_0, \omega \) where \( H_{Ht} \), of the pump head and angular velocity.

The inertial head is caused by the acceleration or deceleration of the fluid flow, therefore, to find it, we’ll use the second Newton law for the element of the stream of an ideal incompressible fluid, and as a result the equation would be obtained:

\[ h_{in} = \frac{1}{g} \int_{h_1}^{h_2} \frac{\partial V}{\partial t} \, dl = \frac{j}{g} \cdot l, \]  

(9)

where \( j \) — the acceleration of the fluid flow; \( l \) — length of the pipeline.

The acceleration of fluid flow \( j \) will be obtained after differentiating the flow formulas bytime \( t \):

\[ \frac{dV}{dt} = j = \frac{1}{F} Q'. \]  

(10)

The pressure losses in the pipeline are the sum of friction losses along the length and losses in local resistance, expressed by flow rate. With this in mind, we’ll write down the formula of general loss:

\[ H_{tr} = \left( \frac{\lambda}{d} + \frac{\zeta(t)}{2gF^2} \right) Q^2(t), \]  

(11)

where \( \lambda \) is the friction resistance coefficient; \( \zeta(t) \) — the total coefficient of local resistance; \( l, d \) — the length and diameter of the pipeline; \( F = \frac{\pi d^2}{4} \) — the cross-section area of the pipeline; \( Q(t) \) — the flow rate through the section.
The initial conditions for the task:

\( \omega(0) = 0; \) \( Q(0) = 0. \) \( (12) \) \( (13) \)

So, the mathematical model of the starting-up process of the considered pump is:

\[
\begin{align*}
H_0 \left( \frac{\omega_0}{\omega} \right)^2 &= H_{st} + \left\{ \frac{\lambda l}{d g F^2} + \frac{\zeta(t)}{2gF^2} \right\} Q^2(t) + \frac{Q(t)}{F_g} \\
J \frac{d\omega}{dt} &= K - K_1 \omega(t) - \alpha \left( \frac{\omega(t) \mu \pi R^2}{a} + \rho QR^2 \omega(t) \right) \\
\omega(0) &= 0 \\
Q(0) &= 0
\end{align*}
\]  \( (14) \)

Let’s find the solution of the system of equations (14). To do this, we’ll rewrite it as follows:

\[
\begin{align*}
Q(t) &= \frac{F_g}{l} \left[ H_0 \left( \frac{\omega_0}{\omega} \right)^2 - H_{st} - \left\{ \frac{\lambda l}{d g F^2} + \frac{\zeta(t)}{2gF^2} \right\} Q^2(t) \right] \\
J \frac{d\omega}{dt} &= \frac{K - K_1 \omega(t) - \alpha \left( \frac{\omega(t) \mu \pi R^2}{a} + \rho QR^2 \omega(t) \right)}{J} \\
\omega(0) &= 0 \\
Q(0) &= 0
\end{align*}
\]  \( (15) \)

**Results**

The system of equations (15) will be solved by the Mathcad software package using the Runge-Kutta method of the 4-th order. The graphical dependence of the angular velocity on time will have the form shown in Fig. 2, and the dependence of the flow on time is shown in Fig. 3.

![Fig. 2. The dependence of the angular velocity of the rotor on time](image-url)
The dependence of the flow on time:

![Flow versus time graph](image)

**Fig. 3.** The flow versus time graph

The Joule - Lenz law for the amount of heat:

\[ I_{\text{nom}} K_p U t_{\text{pp}} = C_{\text{medi}} m_{\text{prov}} \Delta T, \]  

where \( I_{\text{nom}} \) — current intensity; \( K_p \) — start factor; \( t_{\text{pp}} \) — transient time; \( U \) — voltage; \( C_{\text{medi}} \) — copper specific heat capacity; \( m_{\text{prov}} \) — the mass of the conductor; \( \Delta T \) — drive over heating temperature.

Power of electric current in the drive winding:

\[ P = I_{\text{nom}} U = \frac{\rho g H Q}{\eta}, \]  

where \( \eta \) is the pump efficiency.

Let's express the value of the current \( I_{\text{nom}} \) from the equation (17)

\[ I_{\text{nom}} = \frac{\rho g Q H}{\eta U}, \]  

A general equation for temperature based on the equations (16) and (18):

\[ \Delta T = \frac{t_{\text{pp}} I_{\text{nom}} K_p U}{C_{\text{medi}} m_{\text{prov}}} = \frac{t_{\text{pp}} \rho g Q H K_p}{C_{\text{medi}} m_{\text{prov}} \eta}. \]

So, we obtain the calculation of the temperature of the drive overheating during starting-up.

**Conclusion**

A mathematical model of starting-up of a centrifugal pump rotor with an asynchronous electric drive has been constructed. The described method takes into account such factors as the pressure loss in the pipeline \( H_n \), inertial pressure \( h_{in} \) arising during pump operation, the moment of the impeller \( M_{rk} \) and the moment of the engine \( M_d \).

The obtained method of dynamic calculation makes it possible to estimate the over heating of the electric motor of a centrifugal pump depending on various factors and, as a result, to predict its possible failure.
A number of assumptions has been made during the construction of the model, in particular, it has been assumed that during the transition process the starting current is considered to be constant and a multiply greater than the nominal with the same parameters; also, the cooling process was not taken in to account during starting up because of the smallness of the drive starting time.

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