Dual Feedback Control in Solid State Lasers: Discrete Maps and Experiments

M.V. Gorbunkov, Yu.Ya. Maslova, Yu.V. Shabalin
P.N. Lebedev Physics Institute, Leninskii prospect 53, Moscow, 119991 Russia

V.G. Tunkin
International Laser Center, Faculty of Physics, Moscow State University, Vorobevy gory, Moscow, 119992 Russia
E-mail: jmaslova@mail.ru

Abstract. Combinations of negative and positive feedback loops were investigated on the basis of discrete map analysis, numerical simulation and experimentally with lamp and diode pumped Nd:YAG lasers. Cases with relative feedback delay in two loops equal to one laser cavity round trip time were considered. Dual feedback control allows to achieve: 1) stable operation of both continuous and pulsed lasers in a wide range of active media gain, 2) laser pulse shortening down to several tens of picoseconds and 3) regular burst mode with periods up to hundred laser cavity round trip times.

1. Introduction
The stabilization of laser operation by single negative feedback loop is well known [1, 2]. Fast inertial negative feedback control allows one to generate short laser pulses via self mode-locking [3, 4]. However sufficient drawback of such a control is a fundamental instability at high active-media gain [5]. Moreover, short pulse duration in self mode-locking regime of Nd:YAG laser is not less than 100 ps [6]. Dual feedback control is a more powerful tool to control the generation dynamics of solid state lasers. In this paper we summarize our experience in dual feedback control of solid state lasers both inside the stability region and above its upper boundary. We focused on the problems of stable operation, laser pulse shortening, and regular bursting with large periods. The advantages of proposed dual feedback control were manifested by discrete map analysis and were verified by numerical simulation and experiments with lamp and diode pumped Nd:YAG lasers. Such control can be widely applied in laser design [7, 8] as well as other areas dealing with optimal control.

2. Highly stable dual feedback controlled laser
For a mode-locked laser controlled by one negative non-inertial (quickly relaxed) feedback loop the recurrent equation is the following:

\[ x_{n+1} = rx_n(1 - x_n). \]  \hspace{1cm} (1)

Here \( x_n \) is the laser pulse energy in the cavity at \( n \)-th round-trip, \( r \) is the overall gain including active medium gain and passive losses, and the term in brackets represents the feedback loop...
action which delay is less than a laser cavity round trip time $T_r$. In steady state $x_{n+1} = x_n = x_{st}$, and we have

$$x_{st} = \frac{r - 1}{r} , \quad \frac{dx_{st}}{dr} = \frac{1}{r^2}.$$  \hfill (2)

Good operation stability requires the low sensitivity of laser output $x$ to gain variation. From equation (2) we conclude that the sensitivity value $dx_{st}/dr$ decreases when $r$ is increased. Therefore the possibility of steady-state operation at large gain $r$ is the necessary condition for high-level laser stabilization. However, according to [5] maximum acceptable gain for steady operation of a laser controlled by negative feedback with delay $\ll T_r$ equals 3, and for a laser controlled by negative feedback with delay $\sim T_r$ acceptable gain is 2. We have found that dual negative feedback control allows the remarkable broadening of stability region [9]. For a laser controlled by dual non-inertial feedback with relative delay equal to $T_r$, a recurrence equation is the following:

$$x_{n+1} = r x_n (1 - \alpha x_n - x_{n-1}).$$ \hfill (3)

Now the term in brackets represents the dual feedback action, $\alpha$ is feedback loops sensitivities ratio and the relative feedback delay is $T_r$. The regions of stability were found analytically using linear analysis in the neighborhood of $x_{st}$. The values of multipliers $\mu$ [10] were calculated. The stability region covers the region where greater absolute value of multiplier is $< 1$. The upper boundary of stability region $r_2$ (referred to as the second threshold in literature) corresponds to $\mu = 1$. Figure 1a shows the stability region for map (3) which reveals the point $\alpha_{opt} = 3$ with maximum gain $r_{max} = 5$:

$$r_2(\alpha) = \begin{cases} \alpha + 2, & \alpha \leq 3 \\ 3 + \frac{4}{\alpha - 1}, & \alpha > 3 \end{cases}$$ \hfill (4)

This result is in a contradiction with common opinion [5] that greater delay in a feedback loop always leads to stability decrease.

**Figure 1.** Stability regions for dual feedback controlled systems [9]: a — map (3) controlled by dual non-inertial negative feedback, b — map (5) controlled by negative and delayed positive non-inertial feedback, c — map (6) controlled by positive and delayed negative non-inertial feedback.

It is important to note that combinations of non-inertial positive and negative feedback loops (with negative feedback delay $\ll T_r$ and with positive delay equal to $T_r$, or with negative feedback delay $T_r$ and with positive delay $\ll T_r$) give no stability increase (figures 1b, c):

$$x_{n+1} = r x_n (1 - \alpha_1 x_n + x_{n-1}), \quad r_2(\alpha_1) = 3 - \frac{4}{\alpha_1 + 1}.$$ \hfill (5)

$$x_{n+1} = r x_n (1 + x_n - \alpha_2 x_{n-1}), \quad r_2(\alpha_2) = 2 - \frac{1}{\alpha_2}.$$ \hfill (6)
The recurrence equation of map (3) with two negative feedback loops is close to a system with inertial feedback:

\[ x_{n+1} = r x_n \left( 1 - \sum_{m=0}^{\infty} x_{n-m} \gamma^m \right). \]  

(7)

Inertial feedback action is presented by a sum of pulse energies at preceding cavity round-trips taking into account a damping factor \( \gamma \). Damping factor dependence on feedback memory time (later referred to as \( RC \)) is given by \( \gamma = \exp(-1/\tau) \), where \( \tau \) is measured in \( T_r \) units. Since we discovered stability region broadening for map (3), the stability of systems with inertial control was examined in details. For a laser controlled by two inertial negative feedback loops with relative delay of \( T_r \), the recurrence equation is as follows:

\[ x_{n+1} = r x_n \left( 1 - \sum_{m=0}^{\infty} x_{n-m} \gamma^m - \alpha_3 \sum_{m=0}^{\infty} x_{n-m-1} \gamma^m \right), \]  

(8)

where \( \alpha_3 \) is the relative sensitivity of feedback loops. To demonstrate stability properties of this two-parametrical (parameters \( \alpha_3 \) and \( \gamma \)) system we used multiplier \( \mu \) diagrams (figures 2–3). Figure 2 shows stability diagrams for map (8). Figure 2a (\( \alpha_3 = 0 \)) corresponds to a laser controlled by a single negative feedback delayed by \( \ll T_r \). This system has a narrow stability region (maximum gain is \( r_{\text{max}} = 2 \)) and multiplier values close to 1 (which results in long steady-state build-up time).

![Stability regions](image)

**Figure 2.** Stability regions with contour plots of greater absolute value of multiplier \( \mu = 1, 0.9, \ldots, 0.1 \): a, b — map (8) controlled by negative feedback (a — case of one inertial negative feedback with delay \( \ll T_r \), b — case of one inertial negative feedback delayed by \( T_r \); c — map (9) controlled by negative feedback with delay \( \ll T_r \) and positive feedback delayed by \( T_r \).

Figure 2b corresponds to the laser controlled by single-loop negative feedback delayed by \( T_r \). One can see that giant stability region width is achieved at greater \( RC \) values. Unfortunately, the absolute value of multipliers increases with \( RC \) increase (\( \mu \sim 1 \)). Thus, steady-state build-up time is increased, which results in worse suppression of possible deviations of \( x \) from \( x_{\text{stat}} \) that may occur at gain variation.

In contrast to non-inertial systems, combination of positive and negative inertial feedback

\[ x_{n+1} = r x_n \left( 1 - \alpha_4 \sum_{m=0}^{\infty} x_{n-m} \gamma^m + \sum_{m=0}^{\infty} x_{n-m-1} \gamma^m \right). \]  

(9)

shows an improvement both in stability region width and multiplier values (figure 2c). Though the maximum acceptable gain value is slightly lower than in the case of two negative feedback loops...
loops, the values of $\mu$ are sufficiently smaller, providing deeper stabilization and shorter steady-state build-up times. It is worth to note that low-multiplier region ($\mu < 0.4$) spreads along $RC$ axis for indefinitely great $RC$ values. Our model imposes no restrictions on $RC$ (and $r_{\text{max}}$) increase. Best $\alpha_5$ values providing optimal stabilization are from 3 to 5. The feedback loops combination described above gives unique capabilities for stabilization. For comparison, the systems controlled by the combination of positive feedback delayed by $\ll T_r$ and negative feedback delayed by $T_r$, have very narrow stability regions at any $\alpha_5$. The conclusion is that for the real stabilization less delayed inertial feedback should be always negative. In this case, the second positive feedback delayed by $T_r$ compensates the memory effect of the first negative feedback providing greater stabilization.

Keeping in mind self-mode-locking regime, combination of inertial negative and non-inertial positive feedback loops was also investigated (figure 3):

\begin{align}
  x_{n+1} &= r x_n \left( 1 - \alpha_5 \sum_{m=0}^{\infty} x_{n-m} \gamma^m + x_{n-1} \right), \\
  x_{n+1} &= r x_n \left( 1 - \alpha_6 \sum_{m=0}^{\infty} x_{n-m-1} \gamma^m + x_n \right).
\end{align}

Similarly to the case of map (9) when inertial negative feedback was combined with inertial positive feedback, the stability region is broadened and $\mu$ remains moderate if a system is controlled by negative feedback with delay $\ll T_r$ and positive non-inertial feedback delayed by $T_r$ (map (10), figure 3a). Combination of negative feedback delayed by $T_r$ and positive feedback with delay $\ll T_r$ gives extremely narrow stability region (map (11), figure 3b).

3. Self-mode-locking in dual feedback controlled laser

Self-mode-locking [3] in a negative feedback-controlled laser by means of electrooptic modulator (EOM) is a well-known technique for stable ultrashort pulses generation. The laser mode-locked by a fast time-shifted negative optoelectronic feedback [11] (figure 4) is a very simple and reliable source of light pulses with several-hundred picosecond duration [4]. In such a laser the losses caused by an intracavity Pockels cell look like a periodic “saw” with a long front and abrupt decay (figure 4c). The decay is formed due to the fast charge of EOM capacity $C_m$ by photocurrent...
generated in optoelectronic element (OE) under ultrashort light pulse action (figure 4b). The long front is formed by slow discharge of intracavity Pockels cell capacity $C_m$ through a ballast resistor $R_0$ (figure 4a). This behavior is referred to as a simple relaxator [6].

![Figure 4. Electrooptic control circuit and signals: a — equivalent scheme, $R_0$ — ballast resistor, $C_m$ — electrooptic modulator (EOM) capacity, $U_m$ — EOM bias voltage, OE — optoelectronic element, $U_{oe}$ — optoelectronic element bias voltage, $C_1, C_2$ — capacitors ($C_1 \gg C_2, C_2 > C_m$); b — photocurrent pulse; c — EOM control voltage, 5 ns/div.](image)

For self-mode-locking, the discharge time of the EOM capacity should be about $T_r$ [6]. The evolution of the laser field and corresponding control voltage was simulated numerically starting from the spontaneous emission to the ultrashort pulse formation. Finally, the steady-state parameters were calculated. The best conditions for stable self-mode-locking occur when temporal delay in feedback loop corresponds to light pulse passage through the Pockels cell at the moment of the lowest intracavity losses and the maximum EOM transmission. However, this is not the case when the laser is mode-locked by feedback with delay $\ll T_r$ (figure 5a): light pulses are formed on a long front of EOM transmission by slow discharge of the EOM capacity and their duration is inversely proportional to $T_1 = 2T_r$ [12, 6, 13]. Therefore, despite fast electronics (i.e. short photocurrent response time) the great value of $T_r$ complicates short pulses generation in lasers controlled by highly stable negative feedback with delay $\ll T_r$.

Some pulse shortening occurs when the pulse passes through the Pockels cell at the moment of the maximum transmission (optimal mode-locking). To achieve this, the delay of a single negative feedback is to be increased up to $T_r$ and the width of the stability region is decreased. We proposed dual feedback combination which is able to overcome the drawbacks of single negative feedback control. It is based on inertial negative feedback with delay $\ll T_r$ and positive quickly relaxed feedback delayed by $T_r$ (figure 6). Positive feedback based on a simple relaxator with fast discharge leads to significant pulse shortening. Figure 7 gives the optical scheme of dual feedback controlled laser system with negative feedback delay $\ll T_r$ and positive feedback delayed by $T_r$.

The scheme of the picosecond flash-lamp pumped Nd:YAG laser based on the same dual feedback combination is shown in figure 8 [14]. In contrast to the previous scheme, signals reflected from AM and intracavity polarizer were used in feedback loops. Intracavity EOMs were based on LiTaO$_3$ crystals. They were controlled directly by photocurrent of high-voltage subnanosecond silica mesa-structures. Discharge time of the negative feedback loop EOM capacity was as large as $1.5T_r$. Positive feedback response time was not greater than 0.5 ns. For short pulse generation with high stability, optical delay in negative feedback was set as small as possible. Optimal optical delay in positive feedback was about $T_r$. At the optimal negative and positive feedback sensitivities, the train of short pulses with equal amplitudes and total duration of $>180$ $\mu$s was generated (figure 9a). Flat top of the train proves the high stabilization properties of the combination of inertial negative feedback loop with delay $\ll T_r$ and fast positive feedback loop delayed by $T_r$ predicted earlier in [9]. Single pulse duration measured by streak-camera was $43.5 \pm 1$ ps (see figure 9b).
Figure 5. Self-mode-locking regime in a laser controlled by negative feedback with delay $\ll T_r$: $I(t)$ — laser intensity, $P(t)$ — EOM transmission. Cavity round-trip time $T_r = 10$ ns, feedback delay $= T_r/10$, $RC = 2.5 T_r$, feedback response time $\tau_i = 0.02 T_r$, gain is 100% over the threshold, passive losses coefficient = 0.1, EOM bias voltage $U_0 = 0.3 U_\lambda/2$.

Figure 6. Self-mode-locking regime in a laser controlled by negative feedback with delay $\ll T_r$ and positive feedback delayed by $T_r$: $I(t)$ — laser intensity, $P(t)$ — EOM transmission. Positive feedback delay is about $T_r$, $RC = T_r/100$, relative feedback sensitivity is 2.2, feedback response time $\tau_i = 0.02 T_r$, other parameters are the same as in figure 5.

Figure 7. Highly stable dual feedback controlled self-mode-locked laser: $M_1$, $M_2$ — cavity mirrors; AM — active medium; BS$_1$, BS$_2$ — beam splitters; OD — variable optical delay line for the positive feedback loop; CS$_p$, CS$_n$ — optoelectronic control circuits of positive and negative feedback loops; EOM$_p$, EOM$_n$ — electrooptic modulators; P — polarizer. BS$_2$ provides the optimal positive to negative feedback sensitivity ratio. Optical delay in negative feedback loop (from BS$_1$ to CS$_n$) should be as small as possible.
Figure 8. Flash lamp pumped picosecond Nd:YAG laser controlled by positive and negative feedback loops [14]: AM — active medium; M₁, M₂ — cavity mirrors; P — polarizer; D — aperture; CCₚ, CCₚ — positive and negative feedback control circuits; OD — positive feedback optical delay line; F — neutral density filter.

Figure 9. Flash lamp pumped laser output under dual feedback control: a — train of picosecond pulses; b — short laser pulse measured by streak-camera and its fit (duration 43.5 ps) [14].

Figure 10. Diode end-pumped picosecond Nd:YAG laser controlled by dual feedback [7]: a — optical scheme: M₁–M₃ — mirrors, BS — beamsplitter, P — polariser; b — laser output radiation; c — negative feedback voltage.

Later this approach was implemented in a diode pumped Nd:YAG laser (figure 10) [7]. The laser cavity of length 1.2 m consisted of highly reflecting mirror M₁ deposited on the flat end face of the AM, two highly reflecting mirrors (spherical mirror M₂ with a radius of curvature 2 m and flat mirror M₃). AM was 15 mm long and 5 mm in diameter, a low-voltage bisectonal EOM was based on LiTaO₃ crystals. Radiation to control laser operation was deflected from the cavity using beam splitter BS. After optical delay line, the radiation was directed onto the fast semiconductor structures in the negative and positive feedback circuits. In spite of the
considerable variation of AM gain in the course of pulsed diode pumping, effective stabilization of radiation led to the formation of picosecond pulse trains with nearly constant amplitude of duration exceeding 200 $\mu$s (figure 10b). The time dependence of the negative feedback voltage is shown in figure 10c. The duration of picosecond pulses measured by AGAT SF-3 streak camera was 30 ps. The laser pulse generated by dual feedback-controlled laser system proved to be more than 3 times shorter than in single-loop feedback-controlled laser. This improvement is paid by the cost of self-mode-locking region narrowing.

4. Regular nonlinear dynamics with large period
In spite of significant shrinking of the stability region width in the case when positive feedback delay is shorter than the negative feedback delay (figures 1c, 3b), this combination is useful in generation of regular bursts with large periods compared to $T_r$. The period of regular dynamics in map (3) calculated at the stability region boundary depends on the relative sensitivity of feedback loops $\alpha$, and in the region of positive and negative feedback control (dashed line in figure 1, $0 \leq \alpha < 1$) is approximated by [16]:

$$T(r_2(\alpha)) = \frac{2\pi}{\sqrt{\alpha + 1}}.$$ (12)

To realize the idea, a laser system was assembled in which a signal reflected from an intracavity Pockels cell polarizer is directed to an optoelectronic negative feedback circuit. In such laser the control scenario corresponds to the combination of positive and delayed negative feedback loops (see figure 11) [16]:

$$y_{n+1} = y_n r \left(1 - y_{n-1} + \frac{P_0}{r} y_n \right),$$ (13)

where $P_0$ is the initial electrooptic modulator transmission. We take into account two-pass Pockels cell transmission

$$P_0(U_m) = \cos^2 \left( \frac{U_m \pi}{U_{\lambda/4} 2} \right),$$ (14)

where $U_m$ is a static bias voltage (figure 4) and $U_{\lambda/4}$ is a cell quarter-wave voltage. Applying (12) we estimate the nonlinear dynamics development threshold gain $r_2(U_m)$ and oscillation period $T(U_m)$ in two limits. If $U_m \rightarrow U_{\lambda/4}$ then $T(U_m) \rightarrow 6$ and $r_2(U_m) \rightarrow 2$. When $U_n$ is far less than $U_{\lambda/4}$,

$$T(U_m) \approx 2 \sqrt{\frac{2\pi U_{\lambda/4}}{U_m}}.$$ (15)

Therefore, in a laser controlled by optoelectronic negative feedback with the control signal obtained from the Pockels cell polarizer, regular bursts with periods starting from 6 up to hundreds $T_r$ can be obtained above the stability region boundary. Such regime can be applied in master oscillator of optical unit in a pulsed laser-electron Thomson X-ray source [8]. This idea was implemented using a flash-lamp pumped Nd:YAG laser ($\phi 6.3 \times 60$ mm rod was used) [16]. A PC-controlled laser pumping unit based on the incomplete discharge of large capacity allowed us to vary pump duration up to 3.9 milliseconds and pump energy up to 250 J. Laser cavity consisted of flat wedge-shaped mirrors of 98 and 35% reflection ($\lambda = 1064$ nm). An intracavity mirror telescope 3:1 allowed to enlarge a laser mode volume in AM and thus to raise the laser output. Total cavity length was 150 cm. An iris aperture was used for mode selection. A two-sectional temperature-compensated Pockels cell based on two $3 \times 3 \times 15$ mm$^3$ LiTaO$_3$ crystals ($U_{\lambda/4} \sim 200$ V for $\lambda = 1064$ nm) and a multilayer polarizer (Brewster angle) was placed close to the rear laser mirror. Variable bias voltage $U_m$ was applied to the Pockels cell. A high voltage (1200–1300 V) sub-nanosecond photocurrent response silicon mesa-structure [6] was used as a
Figure 11. Millisecond lamp pumped Nd:YAG laser designed for submicrosecond scale bursts [16]: AM — active laser medium; P polarizer; M₁, M₂ — cavity mirrors; M₃, M₄ — optical delay mirrors; Pr — prism; IA — iris aperture; MT — mirror telescope; EOM — electrooptic modulator based on LiTaO₃ crystals; CC — feedback control circuit; G — AM gain, R — mirror reflectivity, yₙ — normalized laser pulse energy at the n-th round-trip.

Figure 12. Oscilloscope traces of laser output, signal is proportional to picosecond pulse energy [16]. The development of regular 0.5 microsecond bursts: a — harmonic modulation, b — developed bursts.

control element of the optoelectronic system. Discharge time of intracavity Pockels cell capacity was set to 20 ns (2Tᵣ). The control signal was taken from an intracavity Pockels cell polarizer (figure 11). Setting the optimal feedback time delay and the diameter of intracavity iris aperture we obtained a train of 100-picosecond pulses with slowly decreasing amplitude and total number of pulses up to 350000. By increasing pump power we observed regimes of regular bursts with controlled period from 25 up to 75Tᵣ as Uₘ was decreased from 0.3Uₗ/₄ to 0.05Uₗ/₄. An example of 0.5 μs period bursting development is shown in figure 12a, b (microsecond time scale). Period of the developed bursts (figure 12b) is ≤ 25% greater than the period of harmonic modulation under the same Uₘ. The idea to use the control signal reflected from the polarizer was realized in diode-pumped laser as well [17].

5. Conclusions
The investigation of laser operation inside the stability region showed that dual feedback control provides stabilization improvement. To achieve this, inertial negative feedback with delay ≪ Tᵣ is combined with positive feedback delayed by Tᵣ. In such cases stabilization improvement is determined by: 1) non-sensitivity to gain variation owing to the broadening of the stability region and 2) realization of suitable multiplier absolute values. Low values of multipliers provide deep stabilization and shorter steady-state build-up time. Optimal positive to negative feedback sensitivities ratio is 10–50%. In self-mode-locked laser, positive feedback provides not only greater stability but also about 3 times pulse shortening due to the abrupt loss modulation.
Above the upper boundary of the stability region, the period of regular bursts can be increased by means of feedback combination when the positive feedback acts at the time $T_r$ earlier than the negative one. Such combined action was implemented in the laser system controlled by optoelectronic feedback which uses the signal from the intracavity Pockels cell polarizer. The period is increased when the Pockels cell bias voltage is decreased. Burst periods up to $90T_r$ were obtained.

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