Cosmic Lensing (II) - Empirical evidences

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ABSTRACT

The foundations of the Universe Expansion theory were formulated close to a century ago. Since early thirties, the Tolman surface brightness test was proposed to differentiate between static and expanding universes. According to this test, the surface brightness of astronomical sources should decrease with redshift as $(1 + z)^{-4}$. Recently, Cosmic Lensing (CL) paradigm was presented. CL unveiled the flux focusing, an unnoticed feature of cosmological images that increase the received flux by $(1 + z)^2$ with respect to what is expected since the test was proposed a century ago. Based on this property, Cosmic Lensing predicts a different behaviour for the surface brightness on an expanding universe decreasing as $(1 + z)^{-2}$. In this paper, galaxy samples of two different surveys, SDSS and VIPERS, were submitted to the Tolman surface brightness test. The surface brightness $\mu$ was derived from the spectroscopic measurements in both cases. By averaging the surface brightness in redshift bins an estimation of $\mu(z)$ was obtained. $\mu(z)$ behaves close to CL predictions in a large redshift range about $0.47 < z < 0.8$ for a uniform sample of bulge-shape galaxies, with small fluctuations mainly due to passive evolution. On the contrary, surface brightness transits far from the Stardard Model prediction in the same redshift range, being this behaviour inexplicable neither by galaxy spectral evolution nor by galaxy merging, and pointing to a fault of the model.

Key words: cosmological parameters, dark energy, dark matter, distance scale, observation, theory – cosmology: large-scale structure of Universe.

1 INTRODUCTION

The last hundred years have provided a great impulse towards the knowledge of the history and fate of the Universe. The field equations of general relativity formulated by Einstein (1915) on one hand, the Friedmann (1922), Lemaître (1927, 1931), Robertson (1933) and Walker (1937) (FLRW) model on the other, along with several cosmological distances defined from luminosity and angles subtended by objects, constitute the basis of the Standard Model. The matching between theoretical predictions and observational data provides information about the different components of the Universe (i.e. curvature, radiation, matter and dark energy).

Since the discovery of the Universe expansion by Hubble (1929), many astronomical surveys have been performed to determine the evolution of the Universe. Different test have been proposed to confront expansion against other theories as the tired light (Zwicky 1929). The most conclusive test is the time dilation of Type Ia supernovae light curves that was suggested by Wilson (1939) and verified by Leibundgut et al. (1996) and Goldhaber et al. (2001). Another relevant test was proposed by Tolman in 1934 which predicts a surface brightness dropping as $\sim (1 + z)^{-4}$ with redshift $z$ for an expanding Universe. The values obtained for the exponent $n$ in different data analysis (Hoyle & Sandage (1956), Sandage (1961), Petrosian (1976), Meier (1976), Sandage & Perlmutter (1991), Pahre et al. (1996), Lubin & Sandage (2001) Sandage (2010)) differ from the predicted value of $n = -4$ and thus it is assumed the existence of a non negligible galaxy evolution effect.

Regarding the rate of expansion, the goal during the eighties and well into the nineties was to measure the Hubble constant and the deceleration parameter for a Cold Dark Matter (CDM) Universe model (Peebles (1982), Bond et al. (1982), Blumenthal et al. (1982), Blumenthal et al. (1984)). Surprisingly, in 1998 two independent groups (Riess et al. (1998), Perlmutter et al. (1999)) discovered the accelerated expansion of the Universe, compatible with a solution of Einstein’s field equations based on the cosmological constant
A. A new component of the Universe – dark energy – is assumed as responsible of the accelerated expansion and the model is currently known as $\Lambda CDM$ or Standard Model of cosmology.

Recently, “Cosmic Lensing (I) - A new paradigm for Universe expansion interpretation” was submitted to publication (De Vicente-Albendea (2020)). Cosmic Lensing (CL) predicts a new luminosity-angular distance relation $D_L = D_A(1+z)$ unlike the $D_L = D_A^2(1+z)^2$ assumed by the Standard Model. In the same sense, the Tolman surface brightness-redshift relation changes from $\mu \sim (1+z)^{-4}$ to $\mu \sim (1+z)^{-2}$. This new relations affect deeply to cosmology. The relative content between radiation, curvature, matter and dark energy of the Universe would change.

In this paper, the mean surface brightness of galaxy samples from SDSS (Blanton et al. (2017)) and VIPERS (Guzzo et al. (2014)) surveys have been computed from spectra. This amount allows one to submit Standard Model and Cosmic lensing paradigms to the Tolman’s surface brightness test. Relevant conclusions are extracted from this comparison.

The rest of the paper is organized as follows: Section 2 introduces some basic elements of the Standard Model of cosmology. In Section 3 the foundations of Cosmic Lensing paradigm are unveiled. Section 4 describes the surface brightness measurement from the spectra, and its comparison within the values predicted by the Standard Model and Cosmic Lensing paradigms. The conclusions are presented in Section 5.

## 2 STANDARD MODEL OF COSMOLOGY

The Standard Model of cosmology compiles the current knowledge related to the beginning, evolution and fate of the Universe. The model is based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric Eq. 1, which is a solution of Einstein’s field equation of General Relativity describing a homogeneous and isotropic expanding Universe,

$$-c^2dt^2 = -c^2dr^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2d\Omega^2 \right]$$ (1)

where $k$ describes the curvature and $a(t)$ is the scale factor responsible of the Universe expansion.

Along with the main equations of cosmology there are several distances defined to link the theory with the observational data. Let us to reproduce here a brief summary of the distances and its relation with cosmological models described by relative densities $\Omega_M, \Omega_r, \Omega_{\Lambda}, \Omega_k$ for matter, radiation, cosmological constant and curvature respectively (Hogg (1999)).

Let $E(z)$ be the function defined as:

$$E(z) = \sqrt{\Omega_k(1+z)^2 + \Omega_r + \Omega_M(1+z)^3 + \Omega_{\Lambda}(1+z)^4}$$ (2)

The Line of sight Comoving Distance $D_C$ is defined by

$$D_C(\Omega_i) = D_H \int_0^z \frac{dz'}{E(z')}$$ (3)

where $\Omega_i$ remarks the dependence from relative densities and where

$$D_H = c/H_0 = 3000h^{-1}Mpc$$ (4)

is the Hubble distance.

The Transverse Comoving Distance $D_M$ is defined by

$$D_M(\Omega_i) = \begin{cases} D_H \frac{1}{\sqrt{\Omega_k}} \sinh[\sqrt{\Omega_{\Lambda}D_C/D_H}] & \text{for } \Omega_k > 0 \\ D_M + \frac{1}{\sqrt{\Omega_{\Lambda}}} \sin[\sqrt{|\Omega_k|D_C/D_H}] & \text{for } \Omega_k < 0 \end{cases}$$ (5)

On the other hand, the Luminosity Distance defines the relation between the bolometric flux energy $f$ received at earth from an object to its bolometric luminosity $L$ by means of

$$f = \frac{L}{4\pi D_L^2}$$ (6)

being

$$D_L = D_M(1+z) \quad \text{(Standard Model)}$$ (7)

Eq. 4 provides the link between a measurable amount $D_L$ and the densities of the components of the Universe $(\Omega_M, \Omega_r, \Omega_{\Lambda}, \Omega_k)$.

The Angular Diameter Distance $D_A$ is defined as the ratio between the size of the object $S$ and its angular size $\theta$

$$D_A = \frac{S}{\theta}$$ (8)

The Angular Diameter Distance is related to the transverse comoving distance by

$$D_M = D_A(1+z)$$ (9)

and taking into account Eq. 4 we have

$$D_L = D_A(1+z)^2 \quad \text{(Standard Model)}$$ (10)

Finally the surface brightness ($\mu$) is given by

$$\mu = l_s(1+z)^{-4} \quad \text{(Standard Model)}$$ (11)

where $l_s$ is the luminosity of the source per surface unit and time unit. Note that the $\mu$ only depends on the redshift for $l_s = cte$.

Finding $l_s$ in Eq. 11 one has

$$l_s = \mu(1+z)^4 \quad \text{(Standard Model)}$$ (12)

## 3 COSMIC LENSING

Cosmic Lensing is a new paradigm for Universe expansion interpretation. It is based on a feature of the expanding Universe, the flux focusing, that has passed unnoticed for
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\[ I_S = \mu (1 + z)^2 \quad \text{(Cosmic Lensing)} \quad (15) \]

Finally, the relation between \( D_L \) and \( D_M \) becomes

\[ D_L = D_M \quad \text{(Cosmic Lensing)} \quad (16) \]

Thus, the relation between \( D_L \) and the component densities represented by \( D_M = f(\Omega_c) \) changes in Cosmic Lensing by a factor \((1+z)\) with respect to the Standard Model.

4 SURFACE BRIGHTNESS: STANDARD MODEL VS COSMIC LENSING

Few years after the discovery of the Universe expansion, Tolman proposed the *surface brightness* \( (\mu) \) test to differentiate between a static and expanding Universe. According to his prediction the *surface brightness* should decrease as Eq. 11 for an expanding Universe. This relation is assumed nowadays by the Standard Model. In a recent work, the Cosmic Lensing paradigm was presented predicting a new *surface brightness* relation in an expanding Universe following the Eq. 14. In this section the *surface brightness* of different galaxy samples is computed. The comparison of \( \mu \) with the predictions of both paradigms, Standard Model and Cosmic Lensing, provides constraints to galaxy evolution. The feasibility of such galaxy evolution can be then analysed.

4.1 Measuring the *surface brightness* from spectra

Let \( f_\lambda \) be the observed spectrum (e.g. the flux density measured in \( \text{erg/cm}^2/\text{s}/\lambda \)) of a galaxy. Since the spectrum is taken with a constant fiber aperture, it can be converted to mean surface brightness \( (\mu) \) by dividing by the aperture. In what follows, let us to equate \( f_\lambda \) to \( \mu_\lambda \) by introducing \((\text{in the spectrum units})\) a constant \( \beta \) representing the aperture normalization to \( \text{arcsec}^{-2} \). On the other hand, let \( z \) be its measured redshift. The surface brightness within a band \( b \) would be given by

\[ \mu = \int_b f_\lambda d\lambda \quad (17) \]

To compare the surface brightness of galaxies along different redshifts we have to blue-shift spectra \( (f_\lambda) \) to the common rest-frame emission band. The rest-frame spectrum \( f_{\lambda'} \) for each galaxy can be obtained by applying the equations

\[ \lambda = (1 + z)\lambda' \quad (18) \]

\[ d\lambda = (1 + z)d\lambda' \quad (19) \]

and since

\[ f_\lambda d\lambda = (1 + z)f_{\lambda'} d\lambda' \quad (20) \]

then

\[ f_{\lambda'} = (1 + z)f_\lambda \quad (21) \]
Eq. 18 and Eq. 21 allows one to obtain the rest-frame spectra in a common emission selected band.

To prevent the surface brightness-redshift relationship $\mu(z)$ from the effects of galaxy evolution, one needs to find a source with constant luminosity along a redshift period (i.e. a standard candle). The best known standard candle are supernovae Type Ia since they provide a very uniform luminosity. Thought they have been used extensively in the last decades, the technique is complex and there are some complications in their measurements associated to eventuality and other issues as is related in Riess et al. (1998) and Perlmutter et al. (1999).

Luminous Red Galaxies (LRGs) constitutes also a very uniform and homogeneous set of galaxies that provide high luminosity up to redshift of cosmological interest. Thought LRGs are not recognized standard candles, the study of the surface brightness on this sample allows one to constrain the galaxy evolution within the Standard Model and Cosmic Lensing paradigms.

4.2 Surface brightness on SDSS

The SDSS DR15 (Aguado et al. 2019) is a very complete survey that provides simultaneously spectrum and image measurements for about a million of galaxies. For cosmological studies on SDSS sample, one is interested on galaxies composed uniquely by bulge (mostly LRGs) since they represent a very luminous and homogeneous sample composed of very stable stars and hence foreseeable low mean evolution. It was started with an initial catalog obtained by joining spectrum and photometric samples ($\sim 900,000$ galaxies). From this it was selected a subcatalog by setting fracDeV selection parameter equal to one, which account for exclusive De Vaucouleurs profile and hence bulge-shape galaxies ($\sim 127,000$ galaxies).

4.2.1 $\mu(z)$-redshift relation

Let one to apply Eq. 18 and Eq. 21 to obtain the rest-frame spectrum on bulge-SDSS sample. After shifting the sample to rest frame, all galaxy spectra become aligned (Fig. 2 left). Such alignment can be better visualized by averaging spectra in redshift bins and histograming in wavelength axis (Fig. 2 right). $\mu(z)$ can be obtained by integrating the spectra in a common rest-frame emission band. The SDSS spectrum of galaxies was taken between (3650 − 10400)Å. The secure integration interval should not be larger than $\lambda_{\text{max}}/(1+z_{\text{max}})$ to ensure that all rest-frame spectra have valid measured data. Thus, it has been selected a conservative wavelength integration interval $b' = (3940 − 5200)$Å. Then, the surface brightness $\mu$ in this band can be obtained by

$$\mu = \int_{b(z)} f_\lambda d\lambda = \int_{b'} f'_\lambda d\lambda' = \int_{3940}^{5200} f'_\lambda d\lambda'$$

Averaging $\mu$ in redshift bins one obtains $\mu(z)$ (Fig. 3). In this plot it was also represented the surface brightness prediction for Standard Model (Eq. 11) and for Cosmic Lensing (Eq. 14) paradigms assuming in both cases a constant value of $l_S$. The difference between a prediction and $\mu(z)$ would correspond to $l_S$ variation and hence galaxy evolution. Four samples are considered depending on bulge-disk relation provided by fracDeV parameter: disk-SDSS ($\text{fracDeV} = 0$), disk-bulge-SDSS ($0.4 < \text{fracDeV} < 0.6$), SSDS (complete sample) and bulge-SDSS ($\text{fracDeV} = 1$). The shadow shows the galaxy dispersion in each sample. In all the cases there is a notable divergent behaviour of $\mu$ for $z < 0.47$ and $z > 0.47$. Let one to focus by now on $z > 0.47$. The case of major interest is the bulge-SDSS (down-right) that corresponds the most uniform sample where start formation is small (the other samples are shown for reference). In this case, $\mu(z)$ is very close to Cosmic Lensing prediction and far from Standard Model one. As we see below in Section 4.2.3 the closeness to Cosmic Lensing prediction indicates low passive evolution of bulge-SDSS as expected.

4.2.2 bulge-SDSS dry mergers

Nevertheless, low passive evolution of bulge-SDSS in the emission wavelength band studied (3960Å − 5200Å) does not explain the large large break observed in Fig. 3 (down-right) for $z < 0.47$. Thus we need an explanation different from spectral evolution for this break. Fig 4 shows the luminosity per surface unit $l_S$ for Cosmic Lensing (Eq. 24)
Figure 3. Surface brightness ($\mu$) in $3600\AA - 5700\AA$ emission wavelength bands for Standard Model vs Cosmic Lensing: (a) disk-SDSS ($\frac{DeV}{V}=0$) (top left) (b) bulge-disk ($0.4 < \frac{DeV}{V} < 0.6$) (top right) (c) SDSS (down left) (d) bulge-SDSS ($\frac{DeV}{V}=1$) (down right). Note how $\mu$ approximates to Cosmic Lensing prediction with the uniformity of the sample for $z > 0.47$.

Figure 4. Relation between the number of galaxies $N(z)$ and the luminosity per surface unit ($l_S$) in Cosmic Lensing: (a) disk-SDSS ($\frac{DeV}{V}=0$) (top left) (b) bulge-disk ($0.4 < \frac{DeV}{V} < 0.6$) (top right) (c) SDSS (down left) (d) bulge-SDSS ($\frac{DeV}{V}=1$) (down right). Dry mergers on bulge-SDSS sample for $z > 0.5$. Note the correlation between luminosity per surface unit growth and drop in the number density of galaxies.

The increase in luminosity slope of bulge-SDSS be due to dry mergers (i.e., gas-poor galaxies merging with low star formation but significant stellar mass growth) [Bell et al., 2006]. More clues about dry merging are given below. The other plots of Fig. 4 are shown for reference.
4.2.3 Luminosity per surface unit

Another evidence of the reality of Cosmic Lensing and the unfeasibility of Standard Model surface brightness prediction becomes patent by the luminosity per surface area $l_S$. This amount is not an observable but can be derived from the spectrum of galaxies for both paradigms. Let one consider $f_\lambda = \mu_\lambda$. Substituting in Eq. 11 and Eq. 14 one obtains

$$l_S(\lambda) = f_\lambda(1 + z)^4 \quad \text{(Standard Model)}$$

$$l_S(\lambda) = f_\lambda(1 + z)^2 \quad \text{(Cosmic Lensing)}$$

Fig. 5 shows the mean luminosity $l_S(\lambda)$ of bulge-SDSS sample for different redshift bins within the Cosmic Lensing (left) and Standard Model (right) paradigms. Left figure shows basically the luminosity evolving in a narrow band for all redshift bins, except for the lowest redshift bin where the mean luminosity grows. Note how this last growth maintains the spectrum shape, which is a clear clue of the merging of similar galaxies (i.e. dry merging of bulge-SDSS galaxies). On the contrary, for Standard Model luminosity drops from $z=0.85$ to $z=0.45$ and then grows. While the growth could be explained by dry merging, the previous parallel decay of the luminosity in all wavelengths discard the galaxy spectral evolution, pointing to some unknown factor of cosmological origin, i.e. the fault of the Standard Model to explain the observations.

4.3 Surface brightness on VIPERS

The VIMOS Public Extragalactic Redshift Survey (VIPERS) was conceived to study the large-scale distribution and evolution of galaxies at $0.5 < z < 1.2$. In this paper we focus on W1 field of VIPERS PDR-2 (Scodッグgio et al. 2018) that provides spectrum and redshift measurements for about $\sim 60,000$ galaxies to $iAB=22.5$.

Figure 6. Standard Model vs Cosmic Lensing: Surface Brightness ($\mu$) in common rest-frame emission wavelength band 3960 Å − 5200 Å for VIPERS sample. Note how $\mu(z)$ evolves close to Cosmic Lensing prediction for $z > 0.6$.

4.3.1 $\mu(z)$-redshift relation

The spectra were measured at the band $b = (5500 - 9500)$ Å. Since our selected rest-frame band is $b_0 = (3960 - 5200)$ Å, the minimum and maximum redshifts with valid data at this band are

$$z_{\text{min}} = \frac{5500}{3960} - 1 = 0.39$$

and

$$z_{\text{max}} = \frac{9500}{5200} - 1 = 0.83$$

Fig. 6 shows the surface brightness $\mu$ of the VIPERS sample as a function of redshift. Note that within small fluctuations due to possible mergers and residual spectral evolution, $\mu(z)$ follows the prediction of Cosmic Lensing for $z > 0.6$. On the contrary, as occurs with SDSS samples, VIPERS $\mu(z)$ transits far from surface brightness Standard Model prediction.
5 DISCUSSION

The surface brightness test provided in Section 4 clearly support Cosmic Lensing paradigm over the current Standard Model surface brightness prediction. The key of this result is Eq. 21. It can be verified that if one confuses the apostrophes in Eq. 21 assuming the equality in Eq. 27, the performed surface brightness test would support the misleading Standard Model surface brightness prediction. Operating in this way is equivalent to applying k-corrections for photometric methods in opposite way.

\[ f_{\lambda} \neq f_{\lambda'} (1 + z) \]

(27)

6 CONCLUSIONS

Early after the discovery of the Universe expansion, Tolman proposed a surface brightness test as a mean to differentiate an expanding from a non-expanding universe. The test predicts the relation \( \mu \sim (1 + z)^{-4} \) for an expanding universe, which is assumed by the current Standard Model. Recently, Cosmic Lensing – a novel cosmological paradigm – was presented providing a new assessment of the flux received from cosmological sources. Consequently, Cosmic Lensing predicts a different behaviour of the evolution of the surface brightness with redshift for an expanding universe given by \( \mu \sim (1 + z)^{-2} \).

In this paper, empirical evidences of the reality of the Cosmic Lensing paradigm are presented. The surface brightness-redshift relation has been derived and analysed from the public DR15 SDSS and PDR-2 VIPERS spectroscopic data releases. The results clearly support the Cosmic Lensing surface brightness predictions against the expected by the Standard Model.

Based on these results, a deep revision of cosmology should be performed within the Cosmic Lensing paradigm. The Hubble constant and the assumed dark components of the Universe (i.e. dark matter, dark energy) have to be reassessed.

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REFERENCES

Aguado D. S., Ahumada R., Almeida A., Anderson S. F., Andrews B. H., Anguiano B., Ortíz E. A., Aragón-Salamanca A., Argudo-Fernández M., Aubert M., et al., 2019, The Astrophysical Journal Supplement Series, 240, 23

Bell E. F., Naab T., McIntosh D. H., Somerville R. S., Caldwell J. A., Barden M., Wolf C., Rix H.-W., Beckwith S. V., Borch A., et al., 2006, The Astrophysical Journal, 640, 241

Blanton M. R., Bershady M. A., Abolfathi B., Albareti F. D., Prieto C. A., Almeida A., Alonso-García J., Anders F., Anderson S. F., Andrews B., et al., 2017, The Astronomical Journal, 154, 28

Blumenthal G. R., Faber S., Primack J. R., Rees M. J., 1984, Nature, 311, 517

Blumenthal G. R., Pagels H., Primack J. R., 1982, Nature, 299, 37

Bond J. R., Szalay A. S., Turner M. S., 1982, Physical Review Letters, 48, 1636

De Vicente-Albendea J., 2020, arXiv preprint astro-ph/2003.05307

Einstein A., 1915, Sitzung der physikalische-

matematischen Klasse, 25, 844

Friedmann A., 1922, Zeitschrift fur Physik, 10, 377

Goldhaber G., Groom D., Kim A., Aldering G., Astier P., Conley A., Deustua S., Ellis R., Fabbro S., Fruchter A., et al., 2001, The Astrophysical Journal, 558, 359

Guzzo L., Scodeggio M., Garilli B., Granett B., Fritz A., Aubas U., Adami C., Arnouts S., Bel J., Bolzonella M., et al., 2014, Astronomy & Astrophysics, 566, A108

Hogg D. W., 1999, arXiv preprint astro-ph/9905116

Hoyle F., Sandage A., 1956, Publications of the Astronomical Society of the Pacific, 68, 301

Hubble E., 1929, Proceedings of the National Academy of Sciences, 15, 168

Leibundgut B., Schommer R., Phillips M., Riess A., Schmidt B., Spyromilio J., Walsh J., Suntzeff N., Hamuy
M., Maza J., et al., 1996, The Astrophysical Journal Letters, 466, L21
Lemaître G., 1927, Ann. Soc. Sci. Bruxelles, Ser. 1, 47, 49
Lemaître G., 1931, Monthly Notices of the Royal Astronomical Society, 91, 483
Lubin L. M., Sandage A., 2001, The Astronomical Journal, 122, 1084
Meier D. L., 1976, The Astrophysical Journal, 207, 343
Pahre M. A., Djorgovski S., De Carvalho R., 1996, The Astrophysical Journal Letters, 456, L79
Peebles P., 1982
Perlmutter S., Aldering G., Goldhaber G., Knop R., Nugent P., Castro P., Deustua S., Fabbro S., Goobar A., Groom D., et al., 1999, The Astrophysical Journal, 517, 565
Petrosian V., 1976, The Astrophysical Journal, 209, L1
Riess A. G., Filippenko A. V., Challis P., Clocchiatti A., Diercks A., Garnavich P. M., Gilliland R. L., Hogan C. J., Jha S., Kirshner R. P., et al., 1998, The Astronomical Journal, 116, 1009
Robertson H. P., 1933, Reviews of modern Physics, 5, 62
Sandage A., 1961, The Astrophysical Journal, 133, 355
Sandage A., 2010, The Astronomical Journal, 139, 728
Sandage A., Perlmutter J.-M., 1991, The Astrophysical Journal, 370, 455
Scodeggio M., Guzzo L., Garilli B., Granett B., Bolzonella M., de La Torre S., Abbas U., Adami C., Arnouts S., Bottini D., et al., 2018, Astronomy & Astrophysics, 609, A84
Walker A. G., 1937, Proceedings of the London Mathematical Society, 2, 90
Wilson O., 1939, The Astrophysical Journal, 90, 634
Zwicky F., 1929, Proceedings of the National Academy of Sciences, 15, 773