A CFD model of the wake of an offshore wind farm: using a prescribed wake inflow

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Abstract. An CFD model of the wake of an offshore wind farm, expanding existing measurements is proposed. The method is based on solving the Navier Stokes equation in a large domain downstream an offshore wind farm. The inflow of the domain is estimated using existing met mast measurements from both free stream and directly in-wake conditions. A comparison between the simulation results and measurements from a met mast are presented and the shortcomings of the methods are discussed.

1. Introduction

The extension of two existing offshore wind farms, Horns Rev and Nysted, are currently planned in Denmark. These new extensions might be built at a relatively short distance from the original ones (less than 20km), which causes two problems. What will be the wind power resources available for the new wind farm, and how will the new wind farm affect the wind power resources of the original wind farm? To address these questions, there is an urgent need for a large offshore wind farm wake model applicable in those situations.

In this context several approaches are simultaneously investigated at Risø/DTU [1]. This article focuses on a method extending the data available from the existing wind farms, using a CFD analysis. The outcome of this method is meant to give extra information to calibrate engineering models, which can then be used in a systematic way.

The basic idea of the method presented here is to estimate the wind properties at the exit of a wind farm and to model the development of the wake downstream of the wind farm. The key element is how to specify the wind farm wake correctly at the computational inlet. The procedure followed is to use the measurements of two met masts placed at a relatively short distance from the farm, one in the free stream; and one directly downstream of the park. The free stream mast is used to define the region of the inlet, where the wind is undisturbed by the wind farm, and the downstream met mast is used to model the wake region of the inlet.

The two offshore wind farms Horns Rev and Nysted were used as test cases as they both present a similar cluster and measurement mast setup. Because of confidential restrictions, only the results from Horns Rev are presented.
2. Method

2.1. Methodology
The method is meant to extend the available information from the existing wind farms to calibrate offshore wind farm downstream-wake engineering models. The data set available includes 3 meteorological masts surrounding the wind farm (one at a corner, and two aligned with a row of turbines, see Figure 1). The two aligned masts give an idea on how the wind is recovering from the influence of the wind farm, but with only two locations, no trends can be seen. The idea is to use these two met masts to “extrapolate” a trend of the wind speed recovery after the wind park.

A steady Computational Fluid Dynamics (CFD) code is used to model the wind exiting the wind farm. The domain modeled is beginning at the location of the first met mast downstream the park and is encompassing a large area downstream the wind farm, including the second met mast (see Figure 1). The turbulence model used is the \( k-\varepsilon \) model, which implies that the inputs needed at the inlet are the mean wind speed \( U_{\text{mean}} \), turbulent kinetic energy \( k \), and dissipation distribution \( \varepsilon \), the free stream friction velocity \( u^* \), and the roughness coefficient of the sea \( z_0 \). All these parameters are estimated from the met masts measurements whenever it is possible, or, otherwise, derived from physical considerations.

![Figure 1. Model Setup.](image)

2.2. Navier-Stokes Solver
The Navier-Stokes solver used is EllipSys, an in-house (Risø/DTU) CFD code under development for the last 15 years [4]. The EllipSys3D code is a multiblock finite volume discretization of the incompressible Reynolds Averaged Navier-Stokes (RANS) equations. The approximations of the Reynolds terms are done using the first-order two-equations \( k-\varepsilon \) model originally introduced by Jones and Launder [2]. The set of constants used in atmospheric conditions are given by Panofsky and Dutton [3]. The solver is used in steady state mode.

The main advantage of using such turbulence model is that it does not require a very fine mesh to resolve the flow near a wall. On the other hand, this model is also known to have a limited accuracy in
boundary layers cases with adverse pressure gradient [6]. This aspect was nonetheless assumed to be negligible compared to the scale of the domain. Another potential limitation of this model is coming from the assumption that the eddy viscosity is proportional with height in sheared flows. The effect of the eddy viscosity on the development of a wake needs further investigation.

2.3. Computational Mesh
Two types of meshes are considered, a 3D mesh of 30km x 30km x 1.2km (around 4,000,000 points), and a 2D mesh of 30km x 1.2km (around 300,000 points). In order to obtain a satisfying convergence speed, the mesh was stretched to obtain the highest precision at the location of the highest gradients. One of the key restrictions concerning the design of the mesh is the size of the cells close to the wall boundary, which needs to be of the same order of magnitude as the roughness length of the sea $z_0$.

2.4. Boundary Conditions
The side boundary conditions are taken as symmetric, while the top boundary condition is taken as an inlet boundary, and the bottom as a wall boundary with a no-slip condition.

As previously mentioned, the inlet boundary is composed of two main regions, a free stream region, where the flow is assumed to be undisturbed by the wind farm, and a wake region (see Figure 2).

The wake region is defined as a rectangle of 5km of width and 200m of height. In addition a linear transition region of 100m around the wake region is applied to smooth the resulting shear forces generating by the difference of wind speed from one region to another.

![Figure 2. Inlet specification](image-url)
The free stream region is defined in terms of mean flow velocity, turbulent kinetic energy and dissipation. It is based on the requirements necessary for obtaining a balanced logarithmic profile using the $k$-$\varepsilon$ turbulence model. The mean velocity is therefore a logarithmic profile, while the turbulent kinetic energy is kept constant and the turbulent dissipation is inversely dependent on the height. Following the formulas (1)

$$U_{\text{mean}} = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right), \quad k = \frac{u_*^2}{\sqrt{\frac{C_\mu}{\kappa}}}, \quad \varepsilon = \frac{u_*^3}{\kappa z}$$

where $U_{\text{mean}}$ is the mean axial velocity, $\kappa$ is the von Karman constant, $z$ is the elevation, $z_0$ is the roughness length of the sea, $u_*$ is the friction velocity, $k$ is the turbulent kinetic energy, $\varepsilon$ is the turbulent dissipation and $C_\mu$ is a constant of the $k$-$\varepsilon$ turbulence model.

In order to determine the appropriate roughness length $z_0$ and the friction velocity $u_*$, measurement data from the free stream mast is fitted for a prescribed wind speed bin.

In the case of the wind farm wake region, this inflow definition is modified in order to account for the wake inflow. The mean velocity at the downstream mast is found to be logarithmic, and a new set of friction velocity $u_*$ and roughness length applied in the mean velocity formula.

The turbulent kinetic energy profile can be estimated from the met mast measurements of the wind speed fluctuation (i.e. the standard deviation of the wind speed during a 10-minute period). This is done under an assumption of neutral boundary layer stratification, and using the similarity relationship lists given by Stull [4], which gives an empirical relationship between the different components of the wind fluctuations $u'$, $v'$, and $w'$ and the friction velocity $u_*$.

$$\frac{u'^2}{u_*^2} = 6.1, \quad \frac{v'^2}{u_*^2} = 2.9, \quad \frac{w'^2}{u_*^2} = 1.7$$

which gives a relationship between $u'$, $v'$ and $w'$.

$$v'^2 = 0.48u'^2, \quad w'^2 = 0.28u'^2$$

The available measurements are recorded using standard cup anemometers which are mainly influenced by the horizontal components of the wind velocity $u$ and $v$. It is therefore assumed that the measured standard deviation $\sigma_m$ can be expressed as

$$\sigma_m = \sqrt{u'^2 + v'^2}$$

Using the relationship given by Stull between $u'$ and $v'$, the measured wind fluctuations are then

$$u'^2 = 0.68\sigma_m^2$$

Finally, the turbulent kinetic energy $k$ can be expressed as

$$k = \frac{1}{2} (u'^2 + v'^2 + w'^2) = 0.88u'^2 = 0.60\sigma_m^2$$

This relation is only valid in an undisturbed atmospheric flow, where the vertical fluctuations are relatively small compared with the two other components. In a wake situation this balance is changed and the turbulence is believed to be more isotropic. It is nonetheless difficult to say how this turbulence distribution is evolving in the wake, which makes it difficult to compare wind speed fluctuation measurements with the CFD results that only give an estimate of the turbulent kinetic energy $k$. Ideally one would need more precise measurements of all the components of the wind speed fluctuation at different heights to have a more reliable comparison data set.

As the met masts are only giving measurement until hub height, some extra assumptions have to be taken considering how to match the free stream and the wake profile. The assumption taken was to fit a Gaussian distribution, centered around hub height, over the turbulent kinetic energy profile, derived from the measurements. This solution is not based on any physical evidence and is therefore expected to have a significant influence on the validity of the results. Finally the turbulent dissipation was kept to the same as in the free stream region. Again, this choice is not expected to be realistic, but without
any proper measurement of turbulence, or a physical model of the wind farm wake region, it was found to be the most appropriate solution.

2.5. Measurements
Horns Rev is an offshore wind farm located on the west coast of Denmark. It is composed of 80 Vestas V80 2MW turbines arranged in a cluster of 10x8 (see Figure 1).

The measurements available are recorded at three locations around the wind farm. The met mast MM2 is used as the free stream reference wind speed, while the met mast MM6 is used as the wake region reference, for constituting the inlet profile. The met mast MM7, 4km downstream MM6, is used to compare with the results of the method.

The reference wind direction chosen was 270°+/-3°, which is the direction angle of the row of turbines for the west wind direction. The two downstream masts are also aligned on this direction, but not directly in line with a row of turbine. The reference hub height wind speed chosen is 8.5+/-0.5 m/s.

The measurements from the top anemometer in each 3 masts deviate with the rest of the anemometers. This effect is believed to come from the met mast geometry. These anemometers are therefore excluded from the curve fitting. The mean wind speed profiles are fitted using the logarithmic law (Equation 1) in order to extract the roughness length $z_0$ and the friction velocity $u^*$.

The values found for the free stream profile are in agreement with the logarithmic law defined in the $k$-$\varepsilon$ model, (see Equation 1) with an atmospheric set of constants ($C_\mu=0.03$).

3. Results discussion
The 2D results are similar to the 3D results in the center plane of the domain. This indicates that most of the recovery of the wind farm wake is done from a transfer of momentum from the top. There is nonetheless also a horizontal recovery done on the side of the wake area, which seems of smaller influence. Therefore, the spreading of the wake cross sectional profile are not following a Gaussian shape as it is usually seen in wake situations, with rectangular inflow profile (see Figure 5).

The vertical mean wind speed distribution 4km inside the domain seems in good agreement with measurements (see Figure 6).

![Figure 3. Mean velocity horizontal profile in the center plane at hub height (70m).](image)

![Figure 4. Turbulent Kinetic Energy horizontal profile in the center plane at hub height (70m).](image)

On the other hand the turbulence profile is largely different from the measurements (see Figure 7). In addition the expected trend of the turbulence would be to decrease constantly instead of increasing as in Figure 4. This turbulence plot clearly shows the weakness of this model. As there is no physical model of the balance between mean wind speed profile and turbulence profile at the inlet, the arbitrary
wind shear of the transition area yields a dramatic increase of the turbulent kinetic energy and dissipation until they reach a balance, and begin to decrease.

![Figure 5. Mean wind speed horizontal profile at hub height (70m)](image)

The horizontal evolution of the mean wind speed at hub height is converging asymptotically to a constant value (see Figure 3). This value, significantly lower than the free stream hub height wind speed, is linked with the ratio between the height of the domain considered, and the wake area height, because of the conservation of mass and momentum in the domain considered. Increasing the height of the domain would also increase this value, making it converge to the free stream value. The height of the domain is nonetheless limited by the height of the planetary boundary layer (around 1000-1500m).

![Figure 6. Mean wind vertical profile in the center plane of the domain](image)

![Figure 7. Turbulent kinetic energy vertical profile in the center plane of the domain](image)

4. Shortcomings and limitations of the method

The rate of the wake recovery is directly dependent on the prescribed turbulent kinetic energy and dissipation. While the first one can be partly estimated from the available measurements, the second is totally unknown and requires a more detailed description.

Similarly, the transition area, defined as linear, is also unphysical. This high velocity gradient generates a high shear directly responsible for the unrealistic increase of turbulent kinetic energy that can be seen on Figure 4. In order to avoid this jump, this transition area also necessitates a better specification of the turbulence profile.
For these reasons, without a proper physical wake definition of a wind farm, or more detailed information on the flow leaving a wind farm, the method still needs further investigation to obtain reliable results.

5. Conclusion
A method to extend the measurement downstream an offshore wind farm is presented. Nonetheless the information needed as inputs are far too demanding compared to the available data sets. In consequence, without a more detailed setup, and without a proper wind farm/boundary layer model, this method cannot be used to give satisfying results.

In order to palliate this issue, a CFD model of an offshore wind farm wake, including the wind farm, is currently in development. This method, modeling the wind turbines as actuator discs, will hopefully reduce the amount of input necessary to extend the available measurements. The end goal of this method is to assist the development of an analytical expression of an offshore wind farm wake.

References
[1] Frandsen S 2005 *Turbulence and turbulence-generated structural loading in wind turbine clusters* (Roskilde, Denmark: Risø National Laboratory), Risø-R-1188(EN).
[2] Jones W P and Launder B E 1972 The Prediction of Laminarization with a Two-Equation Model of Turbulence, *International Journal of Heat and Mass Transfer*, Vol. 15, pp. 301-314
[3] Panofsky H A and Dutton J A 1984 *Atmospheric Turbulence Models and Methods for Engineering Applications* (New York: John Wiley & Sons)
[4] Stull R B 1988 *An Introduction to Boundary Layer Meteorology* (Amsterdam: Kluwer Academic Publishers) pp. 366
[5] Sørensen N N 2003 *General Purpose Flow Solver Applied to Flow over Hills* (Roskilde, Denmark: Risø National Laboratory) Risø-R-827(EN)
[6] Wilcox D C 2006 *Turbulence Modeling for CFD* (San Diego: DCW Industries) Third Ed. pp. 129

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