Is it possible to improve the present LAGEOS–LAGEOS II Lense–Thirring experiment?

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Abstract

The Lense-Thirring effect is currently being measured by means of a combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. The claimed total error should be of the order of 20%. The most insidious systematic error is due to the mismodelled even zonal harmonics of the geopotential and amounts to 12.9%, according to the full covariance matrix of the EGM96 model up to degree $l = 20$. The role and the importance of the LAGEOS–LAGEOS II Lense–Thirring experiment is investigated. Using other suitable combinations with orbital elements of the other existing laser-ranged satellites does not yield significative improvements except, perhaps, for one combination including the nodes of LAGEOS, LAGEOS II and Ajisai and the perigee of LAGEOS II. The related systematic error due to the mismodelled even zonal part of the geopotential reduces to almost 10.7%, according to the full covariance matrix of EGM96 up to degree $l = 20$. 
1 Introduction

In regard to General Relativity (GR), the experience shows that designing and effectively performing experiments aimed to test it is very difficult and often expensive. This is particularly true for its linearized weak-field and slow-motion approximation which is adequate in the Solar System. The relativistic effects quite often fall below or lie just at the edge of the experimental sensitivity of various techniques which could be used. Moreover, there are lots of biases and aliasing effects induced by a host of classical phenomena which pose severe constraints to the obtainable accuracy in space-based experiments. In many cases the systematic errors are larger than the standard statistical errors resulting from the least-squares procedures with which the relativistic effects are commonly extracted from the data.

So, it is of the utmost importance

- To design experiments capable of effectively testing some post-Newtonian effects not yet directly checked at relatively low expense of time and money
- To assess as more reliably and clearly as possible the error budget so to single out the various aliasing competing forces and to evaluate the impact of their systematic errors
- To investigate more thoroughly and reanalyze the role played by the already existing or proposed experiments in order to correctly evaluate their importance

1.1 The importance of the present Lense-Thirring LAGEOS experiment

Nowadays, a first attempt to measure the Lense-Thirring drag of the orbit of a test body [Ciufolini and Wheeler, 1995] in the gravitational field of the Earth, by using the following linear combination of orbital residuals of the rates of the node $\delta \dot{\Omega}$ of LAGEOS and LAGEOS II and the perigee $\delta \dot{\omega}$ of LAGEOS II [Ciufolini, 1996]

$$\delta \dot{\Omega}^I + c_1 \delta \dot{\Omega}^H + c_2 \delta \dot{\omega}^H \sim \mu_{LT}60.2,$$  \hspace{1cm} (1)

has been reported in [Ciufolini et al., 1998; Ciufolini, 2002]. In eq. (1) $c_1 \sim 0.295$, $c_2 \sim -0.35$ are coefficients suitably designed in order to cancel out every mismodelled contributions of the first
two even \((l = 2, 4)\) zonal \((m = 0)\) harmonics \(J_2\) and \(J_4\) of the multipolar expansion of the non-spherical Earth’s gravitational field and are built up with the orbital parameters of LAGEOS and LAGEOS II (see section 3 for further details). Finally, \(\mu_{LT}\) is the parameter in terms of which the Lense-Thirring drag is expressed: it is 1 in GR and 0 in Newtonian mechanics. GR predicts for eq.(1) a linear trend with a slope of 60.2 milliarcseconds per year \((\text{mas/yr})\). Eq.(1) would allow to obtain a claimed total experimental error of the order of 20% \([\text{Ciufolini et al., 1998; Ciufolini, 2002}]\). The important GP–B mission \([\text{Everitt et al., 2001}]\), which is aimed to the detection of a complementary gravitomagnetic effect on freely falling gyroscopes and which is scheduled to fly at the beginning of 2003, would reach a claimed accuracy level of 1%. It is worth noting that the strategy of the combined residuals of LAGEOS and LAGEOS II could allow to constraining strongly the hypothesis of a Yukawa–like fifth force \([\text{Iorio, 2002a}]\) and to measure in the field of the Earth the gravitoelectric relativistic perigee advance \([\text{Iorio et al., 2002a}]\).

What is the real importance of the observable of eq.(1)? Is it the only possibility we have at present in order to measure the Lense-Thirring drag of the orbit of a test body in the terrestrial space environment \([\text{Casotto et al., 1990}]\)? Would it be possible to adopt alternative combinations involving orbital elements of other SLR satellites? We will try to answer these questions in the following.

In Table 1 we quote the orbital parameters of the existing SLR geodetic satellites and of the proposed LARES. In it \(a, e, i\) and \(n\) are the semimajor axis, the eccentricity, the inclination and the Keplerian mean motion, respectively.

The paper is organized as follows. In section 2 we analyze the systematic errors affecting such measurement. In section 3 we look for alternative combinations including the orbital residuals of other existing SLR satellites. Section 4 is devoted to the conclusions.

## 2 The systematic errors

The systematic errors affecting such measurement are of two types

- Long-periodic aliasing harmonics of gravitational (solid Earth and ocean tides \([\text{Iorio, 2001; Iorio and Pavlis, 2001; Pavlis and Iorio, 2002}]\)) and non-gravitational (direct solar radia-
Table 1: Orbital parameters of the existing spherical passive geodetic laser-ranged satellites and of LARES. \( \text{Aj}=\text{Ajisai}, \text{Stl}=\text{Stella}, \text{Str}=\text{Starlette}, \text{WS}=\text{WESTPAC–1}, \text{E1}=\text{ETALON–1}, \text{E2}=\text{ETALON–2}, \text{L1}=\text{LAGEOS}, \text{L2}=\text{LAGEOS II}, \text{LR}=\text{LARES.} \) \( a \) is in km, \( i \) in deg and \( n \) in \( s^{-1} \).

|     | \( a \)  | \( e \)  | \( i \)  | \( n \)  |
|-----|--------|--------|--------|--------|
| Aj  | 7,870  | 0.001  | 50     | 0.0009 |
| Stl | 7,193  | 0      | 98.6   | 0.001  |
| Str | 7,331  | 0.0204 | 49.8   | 0.001  |
| WS  | 7,213  | 0      | 98     | 0.001  |
| E1  | 25,498 | 0.00061| 64.9   | 0.00015|
| E2  | 25,498 | 0.00066| 65.5   | 0.00015|
| L1  | 12,270 | 0.0045 | 110    | 0.00015|
| L2  | 12,163 | 0.014  | 52.65  | 0.00046|
| LR  | 12,270 | 0.04   | 70     | 0.00047|

1Notice that the period of the 18.6–year tide depends only on the luni-solar variables, while the periods of the \( K_1 \) tide and of the direct solar radiation pressure harmonic depend on the orbital geometry of the satellite.
Eq. (1) cancels out the first two even zonal harmonics: the effect of the remaining higher degree mismodelled harmonics $\delta J_6$, $\delta J_8$, ... amounts to 12.9% according to the covariance matrix of EGM96 Earth gravity model [Lemoine et al., 1998] up to degree $l = 20$.

3 The search for alternative combinations

A possible strategy for improving the accuracy of the present–day Lense–Thirring LAGEOS–LAGEOS II experiment consists of suitable combinations of the orbital residuals $\delta \dot{\Omega}$ and $\delta \dot{\omega}$ of the rates of the nodes and the perigees of different SLR satellites. Such combinations can be written in the form

$$\sum_{i=1}^{N} c_i f_i = X_{GR} \mu_{GR}, \quad (2)$$

in which the coefficients $c_i$ are, in general, suitably built up with the orbital parameters of the satellites entering the combinations, the $f_i$ are the residuals $\delta \dot{\Omega}$, $\delta \dot{\omega}$ of the rates of the nodes and the perigees of the satellites entering the combination, $X_{GR}$ is the slope, in mas/y, of the general relativistic trend of interest and $\mu_{GR}$ is the solve–for parameter, to be determined by means of usual least–square procedures, which accounts for the general relativistic effect. For example, in the case of the Lense–Thirring LAGEOS–LAGEOS II experiment [Ciufolini, 1996] $X_{LT} = 60.2$ mas/y. More precisely, the combinations of eq.(2) are obtained in the following way. The equations for the residuals of the rates of the $N$ chosen orbital elements are written down, so to obtain a non homogeneous algebraic linear system of $N$ equations in $N$ unknowns. They are $\mu_{GR}$ and the first $N – 1$ mismodelled spherical harmonics coefficients $\delta J_l$ in terms of which the residual rates are expressed [Iorio, 2002c]. The coefficients $c_i$ and, consequently, $X_{GR}$ are obtained by solving for $\mu_{GR}$ the system of equations. So, the coefficients $c_i$ are calculated in order to cancel out the contributions of the first $N – 1$ even zonal mismodelled harmonics which

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2In regard to this point, it should be mentioned that, at present, there is no full consensus in the scientific community on the reliability of such estimate which might be rather optimistic [Ries et al., 1998]. Indeed, it would not be entirely correct to automatically extend the validity of the covariance matrix of EGM96, which is based on a multi–year average that spans the 1970, 1980 and early 1990 decades, to any particular time span like that, e.g., of the LAGEOS–LAGEOS II analysis which extends from the middle to the end of the 1990 decade. Indeed, there would not be assurance that the errors in the even zonal harmonics of the geopotential during the time of the LAGEOS–LAGEOS II experiment remained correlated exactly as in the EGM96 covariance matrix, in view of the various secular, seasonal and stochastic variations that we know occur in the terrestrial gravitational field and that have been neglected in the EGM96 solution.
represent the major source of uncertainty in the Lense–Thirring precessions. The coefficients $c_i$ can be either constant or depend on the orbital elements of the satellites entering the combinations.

Now we expose how to calculate the systematic error due to the mismodelled even zonal harmonics of the geopotential for the combinations involving the residuals of the nodes and the perigees of various satellites.

In general, if we have an observable $q$ which is a function $q = q(x_j)$, $j = 1, 2,...M$ of $M$ correlated parameters $x_j$, the error in it is given by

$$\delta q = \left[ \sum_{j=1}^{M} \left( \frac{\partial q}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^{M} \left( \frac{\partial q}{\partial x_h} \right) \left( \frac{\partial q}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}}$$

(3)
in which $\sigma_j^2 \equiv C_{jj}$ and $\sigma_{hk}^2 \equiv C_{hk}$ where $\{C_{hk}\}$ is the square matrix of covariance of the parameters $x_j$.

In our case the observable $q$ is any residuals’ combination

$$q = \sum_{i=1}^{N} c_i f_i(x_j), \quad j = 1, 2...10,$$

(4)

where $x_j$, $j = 1, 2...10$ are the even zonal geopotential’s coefficients $J_2$, $J_4$...$J_{20}$. Since

$$\frac{\partial q}{\partial x_j} = \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_j}, \quad j = 1, 2...10,$$

(5)

by putting eq.(4) in eq.(3) one obtains, in mas/y

$$\delta q = \left[ \sum_{j=1}^{10} \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^{10} \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_h} \right) \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}}.$$

(6)

The percent error, for a given general relativistic trend and for a given combination, is obtained by taking the ratio of eq.(6) to the slope in mas yr$^{-1}$ of the general relativistic trend for the residual combination considered.

The validity of eq.(6) has been checked by calculating with it and the covariance matrix of the EGM96 gravity model up to degree $l = 20$ the systematic error due to the even zonal

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In general, the coefficient of the first orbital element entering a given combination is equal to 1, as for the combinations in [Ciufolini, 1996; Iorio, 2002b; Iorio et al., 2002a; 2002b].
harmonics of the geopotential of the Lense–Thirring LAGEOS–LAGEOS II experiment; indeed the result
\[ \delta \mu_{LT} = 13\% \mu_{LT} \]  
(7)
claimed in [Ciufolini et al., 1998] has been obtained again\(^4\). For the systematic errors due to the even zonal harmonics of the geopotential of alternative proposed gravitomagnetic and gravitoelectric experiments, according to EGM96, see [Iorio, 2002b; Iorio et al., 2002a; 2002b]. It is worth noticing that, since the orbits of the LAGEOS satellites are almost insensitive to the geopotential’s terms of degree higher than \( l = 20 \), the estimates based on the covariance matrix of the EGM96 gravity model up to degree \( l = 20 \) should be considered rather reliable although the higher degree terms of EGM96 might be determined with a low accuracy.

It should also be pointed out that the evaluations of the systematic error due to geopotential based on the approach of the combined orbital residuals should be free from some uncertainties due to possible secular, seasonal and stochastic effects. Indeed, even putting aside the fact that most of the secular and seasonal variations of the geopotential are concentrated just in the first even zonal harmonics, if we cancel out as many even zonal harmonics as possible, the uncertainties in the evaluation of the error based on the remaining correlated even zonal harmonics of higher degree should be greatly reduced irrespectively of the chosen time span.

A very important point to stress is that the forthcoming new data on the Earth’s gravitational field by CHAMP, which has been launched in July 2000, and GRACE, which has been launched in March 2002, will have a great impact on the reduction of the systematic error due to the mismodelled part of geopotential and on the reliability of the estimates based on it.

3.1 The combinations without the perigee of LAGEOS II

In regard to alternative combinations, we first explore the possibility of reducing \( \delta \mu_{LT} \) by adopting different residual combinations obtained by substituting the perigee of LAGEOS II\(^5\).

\(^4\)It may be interesting to notice that if, with a more conservative approach, we consider only the diagonal part of the EGM96 covariance matrix, up to degree \( l = 20 \), the systematic error due to the mismodelled even zonal harmonics of the geopotential for the Lense–Thirring LAGEOS–LAGEOS II experiment amounts to 46.5% [Iorio, 2002c].

\(^5\)The orbits of the other SLR satellites are far less elliptical than that of LAGEOS II (except for Starlette), so that their perigees are not good observables. It turns out that the inclusion of the perigee of Starlette would not yield any improvements.
with one or more nodes of Ajisai, Starlette, Stella, WESTSPAC-1, ETALON-1 and ETALON-2. The advantages of using only the nodes would be

- No odd zonal geopotential harmonics
- No odd degree ocean tidal perturbations
- Non-gravitational perturbations of smaller magnitude and fairly well modelled
- Atmospheric drag negligible not only for the LAGEOS and the ETALON satellites, but also for Starlette and Ajisai orbiting at lower altitudes
- Smaller rms on the experimental residuals

The exclusion of the perigee of LAGEOS II leads to two classes of residual combinations.

- Combinations without the ETALON satellites. They should be discarded because the error due to the uncancelled higher degree even zonal harmonics of the geopotential, according to EGM96, is itself far larger than 20%, which is the estimated total error in the LAGEOS–LAGEOS II Lense–Thirring measurement. It is so because of the fact that the satellites orbiting at lower altitudes than LAGEOS and LAGEOS II are more sensitive to the higher degree even zonal harmonics of the geopotential.

- Combinations with the ETALON satellites. Due to their higher altitude with respect to the LAGEOS satellites \( \delta \mu_{LT} \) falls, in some cases, below 1%. However, they should not be considered because the coefficients of the nodes of the ETALON satellites are, in these cases, of the order of \( 10^3 \) so that they greatly amplify every perturbations acting on such elements. This is particularly true for the solid Earth tesserel \( K_1 \) tide. Indeed, it turns out that its \( l = 2, \ m = 1, \ p = 1, \ q = 0 \) constituent induces on ETALON–1 a perturbation with a nominal amplitude of 1,216.57 mas and period of 10,880.8 days while for ETALON-2 we have a nominal amplitude of 1,269.35 mas and a period of 11,130.1 days. Even if the solid Earth tides are known at a 0.5% level [Iorio, 2001], the effect of the too large weighing coefficients and the very long periods of such tidal perturbations would induce insidious mismodelled linear trends over reasonable observational time spans.
of few years. However, it must pointed out that a practical useful use of the SLR data of the ETALON satellites would be problematic mainly because the Russian satellites are, at present, very poorly tracked. Moreover, they wobble and their center of mass is not well defined. Last but not least, the accuracy with which they were constructed and the prelaunch ground controls are not comparable to those of LAGEOS satellites.

In conclusion, even if very appealing, the possibility of using residual combinations involving only the nodes of the existing geodetic SLR satellites should be rejected.

3.2 The combinations with the perigee of LAGEOS II

It turns out that it is possible to obtain admissible results only by involving the perigee of LAGEOS II.

- Combinations with ETALON-1. We obtained seventeen combinations which, in principle, could be considered because \( \delta\mu_{LT} < 20\% \) for them. However, it turns out that the combinations including the perigee of LAGEOS II and the node of ETALON–1 cannot represent a genuine improvement with respect to the present day LAGEOS–LAGEOS II Lense–Thirring experiment because \( \delta\mu_{LT} \geq 13\% \) for them. The systematic errors due to the time–dependent non–gravitational perturbations would make the total error budget unfavorable with respect to the current experiment.

- Combinations without ETALON-1. We obtained eight combinations involving only the LAGEOS satellites and the lower altitude satellites. It turns out that the combination

\[ \delta\hat{\Omega}^I + c_1\delta\hat{\Omega}^II + c_2\delta\hat{\Omega}^A + c_3\delta\dot{\omega}^I \sim \mu_{LT}61.2, \]  

with \( c_1 = 0.443, c_2 = -0.0275, c_3 = -0.341 \) is the only one that could lead to a real, slight improvement of the error budget\(^6\) [Iorio, 2002b]. Indeed

The systematic error due to the even zonal harmonics amount to \( \delta\mu_{LT} \sim 10.7\% \) since eq.\(^8\) cancels out the effect of \( \delta J_6 \) as well.

\(^6\)It may be interesting to notice that if, with a more conservative approach, we consider the diagonal part only of the EGM96 covariance matrix, up to degree \( l = 20 \), the systematic error due to the mismodelled even zonal harmonics of the geopotential for such alternative combination amounts to 64.2\% [Iorio, 2002c].
– The estimates relative to the periodic time–varying perturbations for eq.(11) would be valid also in this case because the coefficients of the node and the perigee of LAGEOS II are almost the same
– The additional perturbations induced by Ajisai are damped since the coefficient of its node amounts only to -0.0275
– It turns out that the tidal perturbations on the node of Ajisai have periods not longer than few years and the non-gravitational perturbations are fairly well studied [Sengoku et al., 1995; 1997]
– Ajisai, contrary to the ETALON satellites, is a well known and accurately tracked geodetic satellite

4 Conclusions

The combination of eq.(11) plays a really important role in the context of the efforts aimed to the detection of the general relativistic Lense-Thirring drag in the gravitational field of the Earth. Indeed, at present, it represents a relatively fast, cheap, reliable and almost unique way for measuring this effect in the terrestrial space environment by means of the analysis of the orbital data of the existing laser–tracked satellites. This should also justify and encourage the efforts aimed to improve the error budget and the knowledge of the aliasing classical forces affecting such measurement. However, according to the covariance matrix of the EGM96 model up to degree $l = 20$, the alternative combination of eq.(8) might enforce and slightly improve the experimental accuracy of the measurement.

At the present–day level of knowledge of the perturbing forces of the terrestrial space environment, only the not yet approved LAGEOS–LARES mission, whose originally proposed configuration has been recently modified [Iorio et al., 2002b], could yield a notable, genuine improvement in measuring the Lense–Thirring effect with SLR.

Moreover, within few years the new data on the geopotential from the CHAMP and GRACE missions should be available. They should improve our knowledge, among other things, of the spherical harmonics coefficients of the geopotential, and, consequently, the systematic gravitational errors in all the proposed Lense–Thirring measurements should reduce as well.
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