Realizing the relaxion from multiple axions
and its UV completion with high scale supersymmetry

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Abstract

We discuss a scheme to implement the relaxion solution to the hierarchy problem with multiple
axions, and present a UV-completed model realizing the scheme. All of the $N$ axions in our
model are periodic with a similar decay constant $f$ well below the Planck scale. In the limit
$N \gg 1$, the relaxion $\phi$ corresponds to an exponentially long multi-helical flat direction which
is shaped by a series of mass mixing between nearby axions in the compact field space of $N$
axions. With the length of flat direction given by $\Delta \phi = 2\pi f_{\text{eff}} \sim e^{\xi N} f$ for $\xi = \mathcal{O}(1)$, both the
scalar potential driving the evolution of $\phi$ during the inflationary epoch and the $\phi$-dependent
Higgs boson mass vary with an exponentially large periodicity of $\mathcal{O}(f_{\text{eff}})$, while the back reaction
potential stabilizing the relaxion has a periodicity of $\mathcal{O}(f)$. A natural UV completion of our
scheme can be found in high scale or (mini) split supersymmetry (SUSY) scenario with the axion
scales generated by SUSY breaking as $f \sim \sqrt{m_{\text{SUSY}} M_*}$, where the soft SUSY breaking scalar
mass $m_{\text{SUSY}}$ can be well above the weak scale, and the fundamental scale $M_*$ can be identified
as the Planck scale or the GUT scale.

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I. INTRODUCTION

Recently a new approach to address the hierarchy problem has been proposed in [1]. The scheme introduces a scalar degree of freedom, the relaxion $\phi$, making the Higgs boson mass a dynamical field depending on $\phi$. During the inflationary epoch, the Higgs boson mass-square $\mu_h^2(\phi)$ is scanned by the rolling $\phi$ from a large positive initial value to zero. Right after the relaxion crosses the point $\mu_h(\phi) = 0$, so that $\mu_h^2(\phi)$ becomes negative, a nonzero Higgs vacuum expectation value (VEV) is developed and a Higgs-dependent back reaction potential begins to operate to stabilize the relaxion\(^1\). One can then arrange the model parameters in a technically natural way to result in the relaxion stabilized at a point where the corresponding Higgs VEV is much smaller than the initial Higgs boson mass.

An intriguing feature of the relaxion mechanism is that the relaxion potential involves two very different scales. One is the period of the back reaction potential, and the other is the excursion range of the relaxion necessary to scan $\mu_h(\phi)$ from a large initial value to zero. To see this, let us consider the relaxion potential given by

$$V(\phi, h) = V_0(\phi) + \mu_h^2(\phi)|h|^2 + V_{br}(\phi, h)$$

(1)

where $V_0$ is the potential driving the rolling of $\phi$ during the inflationary epoch and $V_{br}$ is the periodic back reaction potential stabilizing $\phi$ right after it crosses $\mu_h(\phi) = 0$. In fact, the key feature of the mechanism can be read off from the following form of potential:

$$V_0 = \epsilon_0 f^3 \phi + \ldots, \quad \mu_h^2 = M_h^2 + \epsilon_h f \phi + \ldots, \quad V_{br} = \Lambda_{br}^4(h) \cos \left( \frac{\phi}{f} \right),$$

(2)

where $M_h$ denotes the initial Higgs boson mass, $\epsilon_0$ and $\epsilon_h$ are small dimensionless parameters describing the explicit breaking of the relaxion shift symmetry in $V_0$ and $\mu_h^2$, respectively, and finally $f$ is the relaxion decay constant in the back reaction potential. In non-supersymmetric theory, the Higgs mass parameter $M_h$ is naturally of the order of

\(^1\) A mechanism to cosmologically relax the Higgs boson mass down to a small value through a nucleation of domain wall bubbles has been discussed in [2].
the cutoff scale of the model. On the other hand, in supersymmetric theory, it corresponds to the scale of soft supersymmetry (SUSY) breaking mass which can be well below the cutoff scale of the model. In any case, we are interested in the case that $M_h$ is much larger than the weak scale:

$$M_h \gg v \equiv \langle h \rangle = 174 \text{ GeV},$$

which might be explained by the relaxion mechanism.

Let us now list the conditions for the relaxion mechanism to work. First of all, in order for the rolling relaxion to cross $\mu_h(\phi) = 0$ without a fine tuning of the initial condition, it should experience a field excursion

$$\frac{\Delta \phi}{f} \gtrsim \frac{M_h^2}{\epsilon_h f^2}.$$  \hspace{1cm} (4)

In order for the scalar potential to be technically natural under radiative corrections, the symmetry breaking parameters $\epsilon_0$ and $\epsilon_h$ should obey

$$\epsilon_0 \gtrsim \epsilon_h \frac{M_h^2}{f^2}.$$  \hspace{1cm} (5)

On the other hand, from the stability condition $\partial_\phi V = 0$, one finds

$$\epsilon_0 \sim \frac{\Lambda_{\text{br}}^4}{f^4},$$  \hspace{1cm} (6)

and therefore

$$\frac{\Delta \phi}{f} \gtrsim \frac{M_h^4}{\Lambda_{\text{br}}^4}.$$  \hspace{1cm} (7)

As for the back reaction potential, generically $\Lambda_{\text{br}}(h = 0)$ may not be vanishing, and then one needs

$$\Lambda_{\text{br}}^4(h = v) \gg \Lambda_{\text{br}}^4(h = 0).$$  \hspace{1cm} (8)

Also, in order not to destabilize the weak scale size of the Higgs VEV, its magnitude should be bounded as

$$\Lambda_{\text{br}}(h = v) \lesssim \mathcal{O}(v).$$  \hspace{1cm} (9)
An immediate consequence of the above conditions is that the relaxion should experience a field excursion much bigger than \( f \) in the limit \( M_h \gg v \):

\[
\frac{\Delta \phi}{f} \gtrsim \frac{M_h^4}{v^4}.
\]  

(10)

The required excursion is huge in the case that the back reaction potential is generated by the QCD anomaly, in which \( \Lambda_{\text{br}}^4 \sim f^2 m_\pi^2 \) and therefore

\[
\frac{\Delta \phi}{f} \gtrsim \mathcal{O}\left( \frac{M_h^4}{f^2 m_\pi^2} \right) \sim 10^{12} \left( \frac{M_h}{v} \right)^4.
\]  

(11)

Even when the scale of the back reaction potential saturates the bound (9), the required relaxion excursion is still much larger than \( f \) as long as \( M_h \) is higher than the weak scale by more than a few orders of magnitudes. Note that the natural size of \( M_h \) is the cutoff scale of the model for non-SUSY case, while it is the soft SUSY breaking scalar mass for SUSY case.

Therefore, in the relaxion scenario, the hierarchy \( M_h/v \gg 1 \) is replaced with a much bigger hierarchy \( \Delta \phi/f \gtrsim M_h^4/v^4 \). Although \( \Delta \phi \gg f \) might be stable against radiative corrections, it is still crying for an explanation with a sensible UV completion. To incorporate a huge relaxion excursion, one may simply assume that the relaxion is a non-compact field variable. See [3–10] for recent discussions of the related issues. In this paper, we discuss an alternative scenario in which the relaxion corresponds to an exponentially long multi-helical flat direction in the compact field space spanned by \( N \) sub-Planckian periodic axions:

\[
\phi_i \equiv \phi_i + 2\pi f_i \quad (i = 1, 2, ..., N)
\]

with \( f_i \ll M_{\text{Planck}} \). Such a long flat direction is formed by a series of mass mixing between nearby axions, producing a multiplicative sequence of helical windings of flat direction, which results in

\[
\frac{\Delta \phi}{f_i} = \mathcal{O}(e^{\xi N})
\]

for \( \xi = \mathcal{O}(1) \). Our scenario is inspired by the recent generalization of the axion alignment mechanism for natural inflation [11] to the case of \( N \) axions [12]. Although it requires a
rather specific form of axion mass mixings, our scheme does not involve any fine tuning of continuous parameters, nor an unreasonably large discrete parameter.

As we will see, our scheme finds a natural UV completion in high scale or (mini) split supersymmetry (SUSY) scenario with soft SUSY breaking scalar mass $m_{\text{SUSY}} \gg v$. In the UV completed model, the axion scales are generated by SUSY breaking $[13-15]$ as

$$f_i \sim \sqrt{m_{\text{SUSY}}} M^*,$$

where $M^*$ can be identified as the Planck scale or the GUT scale. With the $(N-1)$ hidden Yang-Mills gauge sectors which confine at scales below $f_i$ to generate the desired axion mass mixings, the canonically normalized relaxion has a field range

$$\Delta \phi \equiv 2\pi f_{\text{eff}} \sim 2\pi f_i \left( \prod_{j=1}^{N-1} n_j \right),$$

where $n_j > 1$ corresponds to the number of flavors of the gauge-charged fermions in the $j$-th hidden sector. One can then arrange the microscopic parameters in a technically natural way to make the resulting relaxion potential $V_0(\phi)$ and the $\phi$-dependent Higgs boson mass $\mu_h^2(\phi)$ vary with an exponentially large periodicity of $\mathcal{O}(f_{\text{eff}})$, while the back reaction potential $V_{\text{br}}(h, \phi)$ has a periodicity of $\mathcal{O}(f_i)$. An interesting feature of our model is that the desired $V_0(\phi)$ and $\mu_h^2(\phi)$ arise as a natural consequence of the solution of the MSSM $\mu$-problem advocated in $[13-15]$.

The outline of the paper is as follows. In the next section, we describe the basic idea with a simple toy model and discuss the scheme within the framework of an effective theory of $N$ axions. In section 3, we present a UV model with high scale SUSY, realizing our scheme in the low energy limit. Section 4 is the conclusion.

II. EXPONENTIALLY LONG RELAXION FROM MULTIPLE AXIONS

To illustrate the basic idea, let us begin with a simple two axion model. The lagrangian density of the model is given by

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - \left( V_0 + V_0 + \mu_h^2 |h|^2 + V_{\text{br}} + \ldots \right),$$

(12)
where $h$ is the SM Higgs doublet and $\phi_i$ are the periodic axions:

$$\phi_i \equiv \phi_i + 2\pi f_i,$$

with a scalar potential

$$\tilde{V}_0 = -\Lambda_4 \cos \left( \frac{\phi_1}{f_1} + n \frac{\phi_2}{f_2} \right),$$

$$V_0 = -\epsilon f_2^4 \cos \left( \frac{\phi_2}{f_2} + \delta_2 \right),$$

$$\mu_h^2 = M_h^2 - \epsilon' f_2^2 \cos \left( \frac{\phi_2}{f_2} + \delta'_2 \right),$$

$$V_{\text{br}} = -\Lambda_{\text{br}}^4 (h) \cos \left( \frac{\phi_1}{f_1} + \delta_1 \right),$$

where

$$\Lambda_4 \gg \epsilon f_2^4 \gg \Lambda_{\text{br}}^4.$$  \hspace{1cm} (15)

Here $M_h$ is an \textit{axion-independent} mass parameter which is comparable to the cutoff scale of the above effective lagrangian, and $n > 1$ is an integer which will be determined by the underlying UV completion. We assume

$$\epsilon f_2^2 \gtrsim O(\epsilon' M_h^2), \quad \epsilon' f_2^2 \gtrsim O(M_h^2),$$

and therefore the model is stable against the radiative corrections which replace the Higgs operator $|h|^2$ with the cutoff-square of $O(M_h^2)$, while allowing $\mu_h^2 = 0$ for certain value of $\phi_2$.

As for the back reaction potential, one can consider two different possibilities. One option is to generate it by the coupling of $\phi_1$ to the QCD anomaly, yielding

$$\Lambda_{\text{br}}^4 (h) \sim y_u \Lambda_{\text{QCD}}^3 h,$$

where $y_u$ denotes the up quark Yukawa coupling to the SM Higgs field $h$, and $\Lambda_{\text{QCD}}$ is the QCD scale. This option corresponds to the minimal model, however generically is in conflict with the axion solution to the strong CP problem. Alternative option is
to introduce a new hidden gauge interaction which confines around the weak scale and generates a back reaction potential given by \[ \Lambda_{\text{br}}^4 = m_1^2 |h|^2 + m_2^4 \] (18)

with

\[ m_2^4 < m_1^2 v^2 \lesssim \mathcal{O}(v^4). \] (19)

In order for the model to be technically natural, the underlying dynamics to generate the back reaction potential should be arranged to make sure that the above conditions on \( m_1 \) and \( m_2 \) are stable against radiative corrections.

The above two axion model involves the shift symmetries

\[ U(1)_i : \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + c_i \quad (i = 1, 2), \] (20)

which are broken by \( \tilde{V}_0 \) down to the relaxion shift symmetry

\[ U(1)_\phi : \frac{\phi_1}{f_1} \rightarrow \frac{\phi_1}{f_1} + nc, \quad \frac{\phi_2}{f_2} \rightarrow \frac{\phi_2}{f_2} - c. \] (21)

The flat direction associated with \( U(1)_\phi \) has a helical winding structure in the compact 2-dim field space of \( \phi_i \) as depicted in Fig. (1). Then the periodicity of the flat direction is enlarged as

\[ \Delta \phi = 2\pi \sqrt{n^2 f_1^2 + f_2^2} \equiv 2\pi f_{\text{eff}}, \] (22)

which is larger than the original axion periodicities \( 2\pi f_1 \sim 2\pi f_2 \) by the winding number \( n \).

The relaxion shift symmetry \( U(1)_\phi \) is slightly broken by small nonzero values of \( \epsilon, \epsilon' \) and \( \Lambda_{\text{br}} \). Note that this particular form of \( U(1)_\phi \) breaking is technically natural as long as the first condition of (16) is satisfied. To find the effective potential of the flat relaxion direction, one can rewrite the model in terms of the canonically normalized heavy and light axions \[ \[11, 12\] \] :

\[ \phi_H = \frac{f_2 \phi_1 + n f_1 \phi_2}{f_{\text{eff}}}, \quad \phi = \frac{n f_1 \phi_1 - f_2 \phi_2}{f_{\text{eff}}}, \] (23)
FIG. 1: Flat relaxion direction in the two axion model.

for which

\[
\begin{align*}
\frac{\phi_1}{f_1} &= n \frac{\phi}{f_{\text{eff}}} + \frac{f_2^2}{n^2 f_1^2 + f_2^2} \frac{\phi_H}{f_H}, \\
\frac{\phi_2}{f_2} &= -\frac{\phi}{f_{\text{eff}}} + \frac{n f_1^2}{n^2 f_1^2 + f_2^2} \frac{\phi_H}{f_H},
\end{align*}
\]

(24)

where \( f_H = f_1 f_2 / f_{\text{eff}} \). In the limit \( \Lambda^4 \gg \epsilon f_2^4 \gg \Lambda_{\text{br}}^4 \), it is straightforward to integrate out the heavy axion \( \phi_H \) to derive the low energy effective lagrangian of the light axion \( \phi \). The resulting effective potential of the canonically normalized \( \phi \) is given by

\[
V_{\text{eff}} = -\epsilon f_2^4 \cos \left( \frac{\phi}{f_{\text{eff}}} - \delta_2 \right) + \left( M_h^2 - \epsilon f_2^2 \cos \left( \frac{\phi}{f_{\text{eff}}} - \delta_2 \right) \right) |h|^2
\]

\[
- \Lambda_{\text{br}}^4(h) \cos \left( \frac{\phi}{f} + \delta_1 \right),
\]

(25)

where

\[
f_{\text{eff}} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf.
\]

(26)

We can now generalize the above two axion model to the case of \( N > 2 \) axions to enlarge the effective axion scale further \cite{12}. The lagrangian density is given by

\[
\mathcal{L} = \frac{1}{2} \sum_i (\partial_\mu \phi_i)^2 - \left( \bar{V}_0 + V_0 + \mu_h^2 |h|^2 + V_{\text{br}} + \ldots \right),
\]

(27)
where
\[ \tilde{V}_0 = - \sum_{i=1}^{N-1} \Lambda_i^4 \cos \left( \frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \]
\[ V_0 = -\epsilon f_N^4 \cos \left( \frac{\phi_N}{f_N} + \delta_N \right), \]
\[ \mu_h^2 = M_h^2 - \epsilon' f_N^2 \cos \left( \frac{\phi_N}{f_N} + \delta_N' \right), \]
\[ V_{br} = -\Lambda_{br}^4 (h) \cos \left( \frac{\phi_1}{f_1} + \delta_1 \right), \]
(28)

with \( \Lambda_i^4 \gg \epsilon f_N^4 \gg \Lambda_{br}^4 \). The model involves the \( N \) axionic shift symmetries:
\[ U(1)_i : \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + c_i \]
(29)

which are broken by \( \tilde{V}_0 \) down to the relaxion shift symmetry:
\[ U(1)_\phi : \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + \gamma_i c \quad (\gamma_i = -n_i \gamma_{i+1}), \]
(30)

with the corresponding flat direction given by
\[ \phi \propto \sum_{i=1}^{N} (-1)^{i-1} \left( \prod_{j=i}^{N-1} n_j \right) f_i \phi_i. \]
(31)

Turing on small nonzero values of \( \epsilon, \epsilon' \) and \( \Lambda_{br} \), the relaxion shift symmetry (30) is slightly broken and nontrivial potential of \( \phi \) is developed. Although our way to break \( U(1)_\phi \) is rather specific, it is technically natural as the model involves many continuous or discrete axionic shift symmetries which are distinguishing our particular way of symmetry breaking from other possibilities. It is again straightforward to integrate out the \((N-1)\) heavy axions which receive a large mass from \( \tilde{V}_0 \) [12]. For the canonically normalized \( \phi \), the resulting effective potential is given by
\[ V_{eff} = V_0(\phi) + \mu_h^2(\phi)|h|^2 + V_{br}(h, \phi) \]
\[ = -\epsilon f_N^4 \cos \left( \frac{\phi}{f_{\text{eff}}} + (-)^{N-1} \delta_N \right) + \left( M_h^2 - \epsilon' f_N^2 \cos \left( \frac{\phi}{f_{\text{eff}}} + (-)^{N-1} \delta_N \right) \right) |h|^2 \]
\[ - \Lambda_{br}^4(h) \cos \left( \frac{\phi}{f} + \delta_1 \right), \]
(32)
where

\[ f_{\text{eff}} = \sqrt{\sum_{i=1}^{N} \left( \prod_{j=i}^{N-1} n_j^2 \right) f_i^2} \sim \left( \prod_{j=1}^{N-1} n_j \right) f_1 \sim e^{\xi N} f_1 \quad (\xi = \mathcal{O}(1)), \]

\[ f = \frac{f_{\text{eff}}}{\left( \prod_{j=1}^{N-1} n_j \right)} \sim f_1. \quad (33) \]

For simplicity here we assumed that all \( f_i \) are comparable to each other, or \( f_1 \) is the biggest among \( \{ f_i \} \).

Obviously, in the limit \( N \gg 1 \) the above relaxion potential involves two very different axion scales, an exponentially enhanced effective decay constant \( f_{\text{eff}} \) and another effective decay constant \( f \) which is comparable to the original decay constants \( f_i \). Such a big difference between \( f_{\text{eff}} \) and \( f \) can be understood by noting that in order for the \( N \)-th axion \( \phi_N \) to travel one period along the relaxion direction, i.e. \( \Delta \phi_N = 2\pi f_N \), the other axions \( \phi_i \) (\( i = 1, 2, ..., N - 1 \)) should experience a multiple windings as \( \Delta \phi_i = 2\pi \left( \prod_{j=i}^{N-1} n_j \right) f_i \). This results in

\[ \frac{\phi_i}{f_i} = (-1)^{i-1} \left( \prod_{j=i}^{N-1} n_j \right) \frac{\phi}{f_{\text{eff}}} + ..., \quad (34) \]

where the ellipsis stands for the \( (N-1) \) heavy axions receiving a large mass from \( \tilde{V}_0 \). For an illustration of this feature, we depict in Fig. 2 the relaxion field direction for the case of \( N = 3, n_1 = 2, n_2 = 4 \).

The effective potential (32) can easily realize the relaxion mechanism under several consistency conditions. First of all, like (16) of the two axion model, we need

\[ \epsilon f_N^2 \gtrsim \mathcal{O}(\epsilon' M_h^2), \quad \epsilon' f_N^2 \gtrsim \mathcal{O}(M_h^2), \quad (35) \]

in order for the model to be stable against radiative corrections, while allowing \( \mu_h = 0 \) for certain value of \( \phi \). Without invoking any fine tuning, there is always a certain range of \( \delta_N \) and \( \delta'_N \) for which the relaxion rolls down toward the minimum of \( V_0(\phi) \) starting from a generic initial value \( \phi_0 \) with \( \mu_h^2(\phi_0) = \mathcal{O}(M_h^2) > 0 \). After a field excursion \( \Delta \phi = \mathcal{O}(f_{\text{eff}}) \), the relaxion is crossing \( \mu_h(\phi) = 0 \), and then a nonzero Higgs VEV is developed together.
with the back reaction potential stabilizing the relaxion at the value giving $\langle h \rangle = v$. The stabilization condition leads to

$$\frac{\epsilon f_N^4}{f_{\text{eff}}} \sim \frac{\Lambda_{\text{br}}^4(h = v)}{f}.$$  \hspace{1cm} (36)

From [35], this then yields a lower bound on $f_{\text{eff}}$:

$$\frac{f_{\text{eff}}}{f} \gtrsim \frac{M_h^4}{\Lambda_{\text{br}}^4(h = v)} = \left( \frac{M_h}{v} \right)^4 \frac{v^4}{\Lambda_{\text{br}}^4(h = v)},$$ \hspace{1cm} (37)

where $v^4/\Lambda_{\text{br}}^4(h = v) \sim 10^{12}$ when $V_{\text{br}}$ is generated by the QCD anomaly, or $v^4/\Lambda_{\text{br}}^4(h = v)$ has a model-dependent value not exceeding $\mathcal{O}(1)$ when $V_{\text{br}}$ is generated by the hidden color dynamics which confines around the weak scale.

To summarize, in our scheme for the relaxion mechanism, $v \ll M_h$ can be technically natural with an exponential hierarchy between the two effective axion scales:

$$\frac{f_{\text{eff}}}{f} = \mathcal{O}(e^{\xi N}) \quad (\xi = \mathcal{O}(1))$$ \hspace{1cm} (38)
which is arising as a consequence of a series of mass mixing between nearby axions in
the compact field space of $N$ axions. Although it relies on a rather specific form of axion
mass mixings, the scheme does not involve any fine tuning of continuous parameters, nor
an unreasonably large discrete parameter.

III. A UV MODEL WITH HIGH SCALE SUPERSYMMETRY

In this section, we construct an explicit UV completion of the $N$ axion model discussed
in the previous section. We first note that our scheme requires that the axion potential
$\tilde{V}_0$ should dominate over the other part of the potential in (28) as it determines the key
feature of the model, i.e. an exponentially long flat direction in the compact field space
of $N$ axions. Specifically we need

$$ f_i^4 \gg |\tilde{V}_0| \gg |V_0| \gtrsim M_h^4. \tag{39} $$

On the other hand, we wish to have an explicit UV model providing the full part of the
axion potential in (28), as well as a mechanism to generate the axion scales $f_i$. This
implies that our UV model should allow the natural size of the Higgs boson mass, i.e.
$M_h$, to be much lower than its cutoff scale. As SUSY provides a natural framework for
this purpose, in the following we present a supersymmetric UV completion of the low
energy effective potential (32).

First of all, to have $N$ axions with the decay constants $f_i \ll M_{\text{Planck}}$, we introduce $N$
global $U(1)$ symmetries under which

$$ U(1)_i : \quad X_i \to e^{i\beta_i} X_i, \quad Y_i \to e^{-3i\beta_i} Y_i \quad (i = 1, 2, ..., N), \tag{40} $$

where $X_i$ and $Y_i$ are gauge-singlet chiral superfields with the $U(1)_i$-invariant superpoten-
tial

$$ W_1 = \sum_i \frac{X_i^3 Y_i}{M_*}, \tag{41} $$

where $M_*$ corresponds to the cutoff scale of the model, which might be identified as the
GUT scale or the Planck scale. Here and in the following, we ignore the dimensionless
coefficients of order unity in the lagrangian. We assume that SUSY is softly broken with SUSY breaking soft masses

$$m_{\text{SUSY}} \sim M_h \ll M_*.$$  \hspace{1cm} (42)

In particular, the model involves the soft SUSY breaking terms of $X_i$ and $Y_i$, given by

$$L_{\text{soft}} = -m_{X_i}^2 |X_i|^2 - m_{Y_i}^2 |Y_i|^2 + \left( A_i \frac{X_i^{\dag}Y_i}{M_*} + \text{h.c} \right),$$  \hspace{1cm} (43)

where

$$m_{X_i} \sim m_{Y_i} \sim A_i \sim m_{\text{SUSY}}.$$  

To achieve the $N$ axions in the low energy limit, we need all $m_{X_i}^2$ are tachyonic, which results in

$$\langle X_i \rangle \equiv x_i \sim \sqrt{m_{\text{SUSY}} M_*}, \quad \langle Y_i \rangle \equiv y_i \sim \sqrt{m_{\text{SUSY}} M_*}.$$  \hspace{1cm} (44)

Then the canonically normalized axion components $\phi_i$ can be identified as

$$X_i \propto e^{i\phi_i/f_i}, \quad Y_i \propto e^{-3i\phi_i/f_i}$$  \hspace{1cm} (45)

with

$$f_i = \sqrt{2(x_i^2 + 9y_i^2)} \sim \sqrt{m_{\text{SUSY}} M_*}.$$  \hspace{1cm} (46)

Now we need a dynamics to generate the axion potential $\tilde{V}_0$ in $[28]$, developing an exponentially long flat direction as described in the previous section. For this purpose, we introduce $(N - 1)$ hidden Yang-Mills sectors associated with the gauge group $G = \prod_{i=1}^{N-1} SU(k_i)$, including also the charged matter fields

$$\Psi_i + \Psi_i^c, \quad \Phi_{ia} + \Phi_{ia}^c \quad (i = 1, 2, ..., N - 1; a = 1, 2, ..., n_i),$$  \hspace{1cm} (47)

where $\Psi_i$ and $\Phi_{ia}$ are the fundamental representation of $SU(k_i)$, while $\Psi_i^c$ and $\Phi_{ia}^c$ are anti-fundamentals. These gauged charged matter fields couple to the $U(1)_i$-breaking fields $X_i$ through the superpotential

$$W_2 = \sum_{i=1}^{N-1} (X_i \Psi_i \Phi_{i1}^c + X_{i+1} \Phi_{ia} \Phi_{ia}^c).$$  \hspace{1cm} (48)
Note that $X_i$ couples to a single flavor of the $SU(k_i)$-charged hidden quark $\Psi_i + \Psi_i^c$, while $X_{i+1}$ couples to $n_i$ flavors of the $SU(k_i)$-charged hidden quarks $\Phi_{ia} + \Phi_{ia}^c$. With this form of hidden Yang-Mills sectors, the $N$ global $U(1)$ symmetries are explicitly broken down to a single $U(1)$ by the $U(1)_i \times SU(k_j) \times SU(k_j)$ anomalies. The charged matter fields $\Psi_i + \Psi_i^c$ and $\Phi_{ia} + \Phi_{ia}^c$ get a heavy mass of $\mathcal{O}(f_i)$, so can be integrated out at scales below $f_i$. This yields an axion-dependent threshold correction to the holomorphic gauge kinetic function $\tau_i$ of $SU(k_i)$ at scales below $f_i$:

$$
\tau_i = \frac{1}{g_i^2} + \frac{i}{8\pi^2} \left( \frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) + \theta^2 M_{\lambda}, \tag{49}
$$

where we ignored the dependence on $|X_i|$, while including the soft SUSY breaking by the gaugino masses $M_{\lambda} \sim m_{\text{SUSY}}$. As a consequence, at scales below $f_i$, the global symmetry breaking by the $U(1)_i \times SU(k_j) \times SU(k_j)$ anomalies is described by the following axion effective interactions:

$$
\sum_{i=1}^{N-1} \frac{1}{32\pi^2} \left( \frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \left( F \tilde{F} \right)_{SU(k_i)}, \tag{50}
$$

where $(F)_{SU(k_i)}$ denotes the gauge field strength of $SU(k_i)$ and $(\tilde{F})_{SU(k_i)}$ is its dual. As we wish to generate the axion potential $|\tilde{V}_0| \gg M_h^4 \sim m_{\text{SUSY}}^4$ from the above axion couplings, we assume

$$
\tilde{\Lambda}_i \gg m_{\text{SUSY}}, \tag{51}
$$

where $\tilde{\Lambda}_i$ denotes the confining scale of the hidden gauge group $SU(k_i)$. In such case, the resulting axion potential is determined by the non-perturbative effective superpotential describing the formation of the $SU(k_i)$ gaugino condensation [17]:

$$
W_{\text{np}} \sim \langle \lambda_i \lambda_i \rangle \propto \left( e^{-8\pi^2 \tau_i} \right)^{1/k_i}, \tag{52}
$$

yielding

$$
\tilde{V}_0 = -\sum_{i=1}^{N-1} \tilde{\Lambda}_i^4 \cos \left( \frac{1}{k_i} \left( \frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \right) \tag{53}
$$
with
\[ \Lambda_i^4 \sim \frac{8\pi^2}{k_i} M_{\lambda_i} \Lambda_i^3. \] (54)

Our next mission is to generate the axion potential \( V_0 \) and the axion-dependent Higgs mass-square \( \mu^2_h \) in (28), driving the evolution of the relaxion during the inflationary epoch, while scanning the Higgs mass-square from an initial value of \( \mathcal{O}(m^2_{\text{SUSY}}) \) to zero. This can be done by introducing a superpotential term given by
\[ W_3 = \left( \frac{X^2_{N-1}}{M^*_s} + \frac{X^2_N}{M^*_s} \right) H_u H_d, \] (55)
together with the associated Kähler potential term:
\[ \Delta K = \frac{X^2_{N-1}X^2_N}{M^2_s} + \text{h.c.} \] (56)
Here we ignore the irrelevant terms such as \( |X_N|^4 \) or \( |X_{N-1}|^4 \) in the Kähler potential. Note that the couplings in \( W_3 \) leads to a logarithmically divergent radiative correction to \( \Delta K \)[18], and our model is stable against such radiative correction as long as the coefficient of \( \Delta K \) is of order unity. Note also that \( W_3 \) provides a solution to the MSSM \( \mu \) problem as it yields naturally the Higgsino mass \( \mu_{\text{eff}} \sim m^4_{\text{SUSY}} \)[13][15].

After integrating out the \((N-1)\) axions which receive a heavy mass from \( \tilde{V}_0 \), while leaving the light relaxion \( \phi \) as described in the previous section, the Kähler potential term \( \Delta K \) gives rise to
\[ V_0 = -m^4_0 \cos \left( 2(n_{N-1} + 1) \frac{\phi}{f_{\text{eff}}} + \delta \right), \] (57)
where
\[ m^4_0 \sim \frac{f^2_{N-1}f^2_N}{M^2_s} m^2_{\text{SUSY}} \sim m^4_{\text{SUSY}}, \]
\[ f_{\text{eff}} = \sqrt{\sum_{i=1}^{N} \left( \prod_{j=i}^{N-1} n_j \right) f^2_i} \sim \left( \prod_{j=1}^{N-1} n_j \right) f_1, \] (58)
and \( \delta \) is a phase angle which is generically of order unity. In our scheme, the MSSM Higgsino mass \( \mu_{\text{eff}} \) originates from \( W_3 \), and therefore is naturally of the order of \( m_{\text{SUSY}} \).
Again, after integrating out the \((N - 1)\) heavy axions, we find the MSSM Higgs parameters \(\mu_{\text{eff}}\) and \(B\mu_{\text{eff}}\) depend on the relaxion \(\phi\) as

\[
\mu_{\text{eff}} = \mu_{N-1} \exp(-i2n_{N-1}\phi/f_{\text{eff}}) + \mu_N \exp(i2\phi/f_{\text{eff}}),
\]

\[
B\mu_{\text{eff}} = b_{N-1} \exp(-i2n_{N-1}\phi/f_{\text{eff}}) + b_N \exp(i2\phi/f_{\text{eff}}),
\]

(59)

where

\[
|\mu_N| \sim |\mu_{N-1}| \sim \frac{f^2}{M_*} \sim m_{\text{SUSY}},\quad |b_N| \sim |b_{N-1}| \sim m^2_{\text{SUSY}}.
\]

(60)

Then the determinant of the MSSM Higgs mass matrix

\[
D = (m^2_{H_u} + |\mu_{\text{eff}}|^2)(m^2_{H_d} + |\mu_{\text{eff}}|^2) - |B\mu_{\text{eff}}|^2
\]

(61)

also depends on \(\phi\) via

\[
|\mu_{\text{eff}}|^2 = |\mu_N|^2 + |\mu_{N-1}|^2 + 2|\mu_N\mu_{N-1}| \cos \left(2(n_{N-1} + 1)\frac{\phi}{f_{\text{eff}}} + \delta_\mu - \delta_{\mu_{N-1}}\right),
\]

\[
|B\mu_{\text{eff}}|^2 = |b_N|^2 + |b_{N-1}|^2 + 2|b_Nb_{N-1}| \cos \left(2(n_{N-1} + 1)\frac{\phi}{f_{\text{eff}}} + \delta_b - \delta_{b_{N-1}}\right),
\]

(62)

where \(\delta_\mu\) and \(\delta_b\) are the phases of \(\mu\) and \(b\), respectively. Obviously, for an appropriate range of \(\delta_\mu\) and \(\delta_b\), the determinant \(D\) can flip its sign from positive to negative as the relaxion experiences an excursion of \(O(f_{\text{eff}})\). Once the relaxion is stabilized near the point of \(D = 0\), the MSSM Higgs doublets \(H_u\) and \(H_d\) can be decomposed into the light SM Higgs \(h\) with a mass of \(O(v)\) and the other heavy Higgs bosons having a mass of the order of \(m_{\text{SUSY}} \gg v\).

To complete the model, we need to generate the back reaction potential \(V_{\text{br}}\). In regard to this, we simply adopt the schemes suggested in [1]. One option is to generate \(V_{\text{br}}\) through the QCD anomaly. For this, one can introduce

\[
W_{\text{br}} = X_1QQ^c,
\]

(63)

where \(Q + Q^c\) is an exotic quark which receive a heavy mass by \(\langle X_1 \rangle \sim f_1\). Once this heavy quark is integrated out, the axion \(\phi_1\) couples to the gluons as

\[
\frac{1}{32\pi^2} \frac{\phi_1}{f_1} \left(F\tilde{F}\right)_{\text{QCD}}.
\]

(64)
After the \((N-1)\) heavy axions are integrated out, this leads to the relaxion-gluon coupling

\[
\frac{1}{32\pi^2} \frac{\phi}{f} \left( \langle F \rangle \right)_{QCD},
\]

where

\[
f = \frac{f_{\text{eff}}}{\left(\prod_{j=1}^{N-1} n_j\right)} \sim f_1.
\]

Then the resulting back reaction potential is obtained to be

\[
V_{\text{br}}(h, \phi) \approx -y_u \Lambda_{QCD}^2 h \cos \left( \frac{\phi}{f} + \delta_{\text{br}} \right),
\]

where \(y_u\) is the up quark Yukawa coupling to the SM Higgs field \(h\), and \(\delta_{\text{br}}\) is a phase angle of order unity.

Alternatively, we can consider a back reaction potential generated by an \(SU(n_{HC})\) hidden color gauge interaction which confines at scales around the weak scale \([1, 16]\). For this, one can introduce the hidden colored matter superfields

\[
L + L^c, \quad N + N^c
\]

with the superpotential couplings

\[
W_{\text{br}} = \kappa_1 \frac{X_1^2}{M_s} LL^c + \kappa_u H_u L N^c + \kappa_d H_d L^c N,
\]

where \(L\) is an \(SU(n_{HC})\)-fundamental and \(SU(2)_L\)-doublet with the \(U(1)_Y\) charge \(1/2\), \(L^c\) is its conjugate representation, \(N\) is an \(SU(n_{HC})\)-fundamental but \(SU(2)_L \times U(1)_Y\) singlet, and \(N^c\) is its conjugate representation. At scales below \(m_{\text{SUSY}}\), all superpartners can be integrated out, leaving the following Yukawa interactions between the relevant light degrees of freedom:

\[
\mathcal{L}_{\text{br}} = m_L e^{2i\phi_1/f_1} LL^c + \kappa_u \sin \beta h L N^c + \kappa_d \cos \beta h^+ L^c N + m_N N^c N^c,
\]

where \(L + L^c\) and \(N + N^c\) denote the fermion components of the original superfields, \(\tan \beta = \langle H_u \rangle / \langle H_d \rangle\), and

\[
m_L \sim \kappa_1 \frac{f_1^2}{M_s} \sim \kappa_1 m_{\text{SUSY}}
\]
is presumed to be lighter than $m_{\text{SUSY}}$. Note that a nonzero Dirac mass of $N + Nc$ is induced by radiative corrections below $m_{\text{SUSY}}$, giving

$$m_N \sim \frac{1}{16\pi^2} \sin(2\beta)\kappa_u\kappa_d m_L^* e^{-2i\phi_1/f_1} \ln \left( \frac{m_{\text{SUSY}}}{m_L} \right).$$

(72)

Now this effective theory at scales below $m_{\text{SUSY}}$ corresponds to the non-QCD model proposed in [1, 16], yielding a back reaction potential of the form

$$V_{\text{br}} = m_1^2 h h^\dagger \cos \left( 2\frac{\phi}{f} + \delta_1 \right) + m_2^4 \cos \left( 2\frac{\phi}{f} + \delta_2 \right),$$

(73)

where we have expressed the axion component $\phi_1$ in terms of the light relaxion field $\phi$, and

$$m_1^2 \sim \frac{\kappa_u\kappa_d\sin(2\beta)}{m_L} \Lambda_{\text{HC}}^3, \quad m_2^4 \sim m_N\Lambda_{\text{HC}}^3$$

(74)

for the $SU(n_{\text{HC}})$ confinement scale $\Lambda_{\text{HC}}$. If $m_2^4 < m_1^2 v^2$ with $m_1^2 \lesssim \mathcal{O}(v^2)$, which can be achieved for $m_L < 4\pi v$ [1], this back reaction potential can successfully stabilize the relaxion at a value giving $v = \langle h \rangle \ll m_{\text{SUSY}}$.

### IV. CONCLUSION

In this paper, we have addressed the problem of huge scale hierarchy in the relaxion mechanism, i.e. a relaxion excursion $\Delta \phi \sim 2\pi f_{\text{eff}}$ which is bigger than the period $2\pi f$ of the back reaction potential by many orders of magnitudes. We proposed a scheme to yield an exponentially long relaxion direction within the compact field space of $N$ periodic axions with decay constants well below the Planck scale, giving $f_{\text{eff}}/f \sim e^{\xi N}$ with $\xi = \mathcal{O}(1)$. Although it relies on a specific form of the mass mixing between nearby axions, our scheme does not involve any fine tuning of continuous parameters, nor an unreasonably large discrete parameter. Furthermore, our scheme finds a natural UV completion in high scale or (mini) split SUSY scenario, in which all decay constants of the $N$ periodic axions are generated by SUSY breaking as $f_i \sim \sqrt{m_{\text{SUSY}} M_*}$, where $m_{\text{SUSY}}$ denotes the soft SUSY breaking scalar masses and $M_*$ is the fundamental scale such as the Planck scale or the GUT scale. In our model, the required relaxion potential and
the relaxion-dependent Higgs boson mass are generated through a superpotential term providing a natural solution to the MSSM $\mu$-problem.

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