Abstract
Lightweight and rigid honeycomb cores have a feature as deployable structures, i.e., their shapes can be changed as desired from the viewpoint of manufacturing processes. This paper presents a geometrical design method for cylindrical honeycomb cores that satisfy high rigidity and deployability, considering core orientation, core distortion, and applicable manufacturing processes. The proposed method is based on a conventional construction process of flat honeycomb cores and applied to a cylinder segmented with longitudinal and latitudinal lines. Cylindrical honeycomb cores with the radial core direction are possibly suitable for the application of radial forces; however, core walls are distorted and the shape cannot be maintained uniformly. To solve this problem, the core direction is revised based on the correction angle. As a result, it is observed that the obtained cores are not distorted and core directions can be designed approximately along radial directions. Geometrical restrictions and feasible design ranges are discussed in this paper. It is possible to fabricate the cores using the conventional expansion process. This implies that they are promising as deployable structures that can be constructed or deconstructed at workplaces.

Keywords: Structural design, Composite material, Core structures, Deployable structures, Origami

1. Introduction

Origami, which originated as a traditional craft, has been extensively studied by engineers, mathematicians, and physicists owing to its applicability to the production of three-dimensional shapes by folding flat sheets. The most famous applications are in the space industry, where it is used to produce components such as solar panels, solar sails, and parabolic antennas, which require compact folding on the ground before transportation, deployment, and reconstruction in space (Guest, et al., 1996, Freeland, et al., 1998, Johnson, et al., 2007, Tsuda, et al., 2011). In addition, such deployable structures are applied to robot designs that are originally flat, but fold themselves up, accomplish their assigned tasks, and disappear by unfolding at a user’s disposal (Miyashita, et al., 2015, Karras, et al., 2017).

Honeycomb cores, which are rigid and widely used to construct industrial structures such as airplanes and railway vehicles, also have a feature as deployable structures because they are originally flat strips and constructed by the extension process in that the strips are expanded outward to form a desired honeycomb shape. Core orientation is essential for achieving high rigidity and deployability. Load should be applied parallel to core height, and expanding (deploying) motion must be performed perpendicular to core height.

Curved honeycomb cores can be manufactured by cutting flat honeycomb cores to the desired shape. Regarding origami-inspired manufacturing methods, Saito et al. (2014, 2016) developed a new method of designing curved honeycomb cores, such as those with three-dimensional wing shapes, by folding a flat sheet. Both methods can design arbitrary contours of honeycomb cores. However, core orientation is uniform and structural rigidity against external forces is not considered. On the contrary, the core heights of SHAPEGRID honeycomb cores are aligned appropriately for users’ intended purposes so that no strength loss occurs (Alcore Inc., Sato, ed., 2002). Maleczek (2015) designed axisymmetric honeycomb cores by expanding cores rotationally so that the direction of core height was aligned to the radial directions from the axis. This method is possibly applicable because it is the simplest and does not disturb the
conventional manufacturing process of honeycomb cores. However, the open edges of the obtained honeycomb cores were warped largely owing to the anticlastic bending behavior of the cores because they were constructed from simple straight strips. Warping deformation can be prevented using non-hexagonal core shapes such as UltraFlex (Ultracor Inc.), which is flexible to bending deformation; however, core walls get distorted.

As mentioned above, it is difficult to find a suitable configuration that satisfies multiple requirements. This paper presents a geometrical design method for cylindrical honeycomb cores and discusses geometrical restrictions and feasible design ranges considering core orientation, core distortion, and applicable manufacturing processes. Practical applications in which radial forces are applied are considered, such as the reinforcement of vehicle tires and robotic rovers, which can be constructed or deconstructed at workplaces. Results show that the obtained cores are not distorted and core directions can be designed approximately along radial directions. Furthermore, the cores are fabricated using the conventional expansion process, which implies that they are promising as deployable structures.

2. Design approach

Figure 1 shows a schematic description of the corrugation process, which is one of the common manufacturing processes for conventional flat honeycomb cores. Straight strips are corrugated (Fig. 1(a)), and two adjacent strips are glued to construct hexagonal hollows (Figs. 1(b) and (c)). Because the adjacent strips are placed symmetrically, the cores are deployable and reversible in the direction perpendicular to the axis of symmetry as long as the material permits. The expansion process is another manufacturing method (Noguchi, ed., 2008). However, the geometry of honeycomb cores is identical for both processes.

As demonstrated by Maleczek (2015), cylindrical honeycomb cores cannot be formed by expanding assembled straight strips because they are anticlastic structures that are curved in opposite manners in two orthogonal directions. In other words, non-straight strips must be designed to form cylindrical honeycomb cores. Thus, in this study, desired cylindrical honeycomb cores are developed on a plane and required strip designs are explored following conventional manufacturing processes.

Fig. 1 Construction of flat honeycomb cores, (a): Top view of corrugated strip, (b): Assembling of strips and symmetric axes, (c): Overview of honeycomb cores.

Fig. 2 Hexagonal segmentation of cylindrical surface, (a): Segmentation of cylindrical surface, (b): Vertex shifting to the azimuth direction, (c): Construction of hexagons by corrugated longitudinal lines, (d): Hexagonal segmentation generated in the regular manner.
3. Cylindrical honeycomb cores with radial directions of core height

To consider a cylindrical surface, it is parametrized as a function of \((r, \theta, z)\) in the cylindrical coordinates, where \(r\) is the radius of the cylinder, \(\theta\) is the azimuth, and \(z\) is the altitude. Thus, a cylinder is segmented by longitudinal and latitudinal lines as shown in Fig. 2(a).

3.1 Construction of Model 1

As straight strips are corrugated in the conventional manufacturing process, straight longitudinal lines are modified to be zigzag lines on the cylindrical surface. The vertex located at \(i\)-th longitude and \(j\)-th latitude is denoted by \(V_{ij}(r, \theta, z_j)\). The interval of azimuth \(\theta_{i+1} - \theta_i\) is defined as a constant.

\[
\theta_{i+1} - \theta_i = \frac{2\pi}{N_t}
\]  

(1)

where \(N_t\) is the number of vertices on the circumference, and it is a multiple of two for the continuity of cores. The azimuth, \(\theta_i\), for vertex \(V_{ij}\) is shifted forward to \((\theta_i + \theta_{i+1})/2\) or backward to \((\theta_{i-1} + \theta_i)/2\) without changing \(r\) and \(z_j\) according to the regular pattern shown in Fig. 2(b). The shifted vertices are still on the cylindrical surface because radius \(r\) is fixed. Thus, corrugated longitudinal lines are generated by connecting the \(i\)-th vertices (Fig. 2(c)), which create hexagonal shapes on the cylindrical surface (Fig. 2(d)). To create an equilateral hexagon on the surface, the interval of altitude \(z_{j+1} - z_j\) must be defined by the following equation:

\[
z_{j+1} - z_j = 2r \sin \frac{\pi}{2N_t} \tan \frac{\pi}{6} \quad \text{or} \quad 2r \sin \frac{\pi}{2N_t} \cos^{-1} \frac{\pi}{6}
\]  

(2)

It is evident that the latitudinal lines are not necessary, and they are suppressed in the figure. Next, the corrugated longitudinal line is extracted from the cylindrical surface (Fig. 3(a)). The vertices on the extracted line are connected perpendicularly to the central axis of the cylinder (Fig. 3(b)) to generate the radial lines that are used to define the directions of core height on each vertex. The strip is obtained with a given core height, as shown in Fig. 3(c). As generated in the regular pattern shown in Fig. 2, the cores are symmetric and all strips are identical; hence, it is sufficient to consider a certain strip. The corrugated strip consists of a sequence of quadrilateral elements. The quadrilateral elements to be glued to the adjacent elements that are arranged in a vertical direction are flat. However, the quadrilateral elements not to be glued are distorted because the two radial fold lines of the element are not parallel each other and the four vertices of the quadrilateral element are not placed on the same plane in three-dimensional space. Thus, the quadrilateral elements are triangulated and developed on the flat plane (Fig. 3(d)). Finally, cylindrical honeycomb cores are generated by folding and assembling the strips. Figure 3(e) shows a physical model of handmade cores created using general copy paper; the thickness of the paper is negligible compared to core size. The open edges of the model are slightly warped inward.

![Fig. 3](image_url)

Fig. 3 Generation of corrugated strip, (a): Extracted corrugated line in space, (b): Radiation perpendicular to the axis of the cylinder, (c): Generated corrugated strip, (d): Development of the corrugated strip on plane, (e): Physical model of handmade cylindrical honeycomb cores (Model 1).
because the strip is elastically distorted and tends to return to the original flat but arc shape. To assemble the strips and construct a rotational cylinder, the first strip is glued to the last strip so that the model is fixed and not reversible once deployed. This rotational configuration is similar to that proposed by Maleczek (2015).

### 3.2 Construction of Model 2

Similar to the construction of Model 1, the altitude, $z_i$, for vertex $V_{ij}$ can be shifted upward to $(z_i + z_{i+1})/2$ or downward to $(z_{i-1} + z_i)/2$ by fixing $r$ and $\theta_i$, as shown in Fig. 4(b). For this case, corrugated lateral lines are generated by connecting the $j$-th vertices (Fig. 4(c)) and hexagonal shapes are obtained on the cylindrical surface (Fig. 4(d)). To create an equilateral hexagon on the surface, the intervals of azimuth $\theta_{i+1} - \theta_j$ must be given by angles $\theta_a$ and $\theta_b$ that satisfy

$$\frac{N_t}{2}(\theta_a + \theta_b) = 2\pi \quad \text{and} \quad \sin \frac{\pi}{6} \sin \frac{\theta_a}{2} = \sin \frac{\theta_b}{2}. \quad (3)$$

Then, the intervals of altitude $z_{i+1} - z_j$ must be defined as constants.

$$z_{i+1} - z_j = 2r \cos \frac{\pi}{6} \sin \frac{\theta_a}{2}. \quad (4)$$

The extracted corrugated lateral line forms a ring (Fig. 5(a)). The vertices on the extracted line are connected perpendicularly to the central axis of the cylinder (Fig. 5(b)) to generate a corrugated strip with a given core height, as shown in Fig. 5(c). Again, the quadrilateral elements to be glued to the adjacent elements are flat, but the quadrilateral elements not to be glued are distorted so that they are triangulated and developed on the plane, as shown in Fig. 5(e). Finally, cylindrical honeycomb cores are generated by folding and assembling the strips. Figure 5(f) shows a physical model of handmade cores created using the same material as that for Model 1 (i.e., general copy paper). The open edges of the model are slightly warped outward because the elastically distorted strips tend to return to the original flat shape. To create a ring of the corrugated strip, one edge is glued to the other so that the model is fixed and not deployable.

### 3.3 Structural restrictions and difficulty in manufacturing

In Model 1 (discussed in Section 3.1), the central angle of the developed strip, $\varphi$, is a function of the height of the cylinder. For a long cylinder, the developed strip can be obtained by repeating a series of identical quadrilateral elements because of the symmetricity of the design. However, central angle $\varphi$ cannot exceed $360^\circ$ to prevent the self-interference of the strip. Thus, the height of the cylinder is restricted as long as it is not allowed to glue separate strips into one. This is a fatal restriction in the design of cylindrical honeycomb cores for practical applications.

![Fig. 4 Hexagonal segmentation of cylindrical surface, (a): Segmentation of cylindrical surface, (b): Vertex shifting to the altitude direction, (c): Construction of hexagons by corrugated latitudinal lines, (d): Hexagonal segmentation generated in the regular manner.](image-url)
In contrast, in Model 2 (discussed in Section 3.2), the height of the cylinder can be determined by assembling multiple corrugated rings as desired because there is no geometrical restriction. However, the central angle of the developed strip is a function of the configuration of cores. Figure 6 shows the relationship between central angle $\phi$ and the design parameters of the honeycomb cores, i.e., radius $r$ of the cylinder, core size $c = 2(z_{j+1} - z_j)$, and core height $t$. Here, radius $r$ and core height $t$ can vary continuously; however, core size $c$ is a discrete parameter because it is defined by a distance between the two sides that face each other in an equilateral hexagon (Fig. 4(c)) and depends on the number of cores in the azimuth direction, which should be an integer. Thus, ratio $c/r$ represents the density of cores on the cylindrical surface. For all combinations of the three design parameters, central angle $\phi$ does not reach $360^\circ$. This implies that one edge of the developed strip must be glued to the other edge to form a ring of the corrugated strip. Thus, the model is not deployable and the manufacturing process becomes complicated.

The strips are elastically distorted in Models 1 and 2. As the strips tend to release the elastic deformation and return to the original flat shape, the obtained cylinders are not exactly accurate according to the requirements. The copy paper used in the study is a relatively soft material for bending. However, if a conventional metallic material, such as aluminum alloy, is used for honeycomb cores, the effect on the geometrical accuracy of the final shapes will be significant compared to the manufacturing errors due to the hand-making process employed in this study. The properties of Models 1 and 2 are summarized in Table 1.
4. Revised cylindrical honeycomb cores

As discussed in Section 3, as long as core height is fixed strictly to radial directions, the quadrilateral elements are distorted and the developed strip is arc shaped. To solve these problems, the correction angle is introduced and the direction of core height in Models 1 and 2 is slightly modified.

4.1 Construction of revised Model 1

The vertices on the inner and outer radii of the corrugated strip are denoted by $A_i$ and $B_i$, respectively (Fig. 3(b)). Vertex $A_1$ is rotated by correction angle $\psi_1$ around vertex $B_1$ from the original radial direction without changing azimuth $\theta_1$. Next, revised vertex $A_2$ is defined as a vertical projection of original vertex $A_2$ on plane $A_1B_1B_2$. The correction angle, $\psi_2$, for vertex $A_2$ is obtained numerically because it is a function of angle $\psi_1$ and design parameters $r$, $c$, and $t$. Thus, quadrilateral element $A_1B_1B_2A_2$ does not get distorted. Assuming that quadrilateral elements $A_2B_2B_3A_3$ and $A_1B_1B_2A_3$ are trapezoidal and successive elements are symmetrically designed in a similar manner, the corrugated strip of revised Model 1 is obtained. The extensions of fold lines $A_1B_1$ pass through the axis of the cylinder, but not perpendicularly, because the fold lines are not defined by the radiation from the axis of the cylinder. Even though all quadrilateral elements are untwisted, the developed strip is still arc shaped (Fig. 7(c)). Thus, the height of the cylinder is strictly restricted. Regarding possible manufacturing processes, revised Model 1 can be fabricated using the conventional corrugation process because the last strip must be fixed to the first strip to close the cylinder.

4.2 Construction of revised Model 2

The vertices on the inner and outer radii on the corrugated strip are denoted by $A_i$ and $B_i$, respectively (Fig. 3(a)). Vertex $A_1$ is rotated by correction angle $\psi$ around vertex $B_1$ from the original radial direction without changing altitude $z_1$ (Fig. 7(c)). Next, revised vertex $A_2$ is defined to keep fold line $A_2B_2$ parallel to line $A_1B_1$. Thus, quadrilateral element $A_1B_1B_2A_2$ does not twist. Again, assuming that quadrilateral elements $A_2B_2B_3A_3$ and $A_4B_4B_5A_5$ are trapezoidal and successive elements are symmetrically designed in a similar manner, the corrugated strip of revised Model 2 is obtained. The extensions of fold lines $A_1B_1$ do not pass through the axis of the cylinder because the fold lines are not defined by the radiation from the axis of the cylinder. However, it is remarkable that the developed corrugated strip forms a ring (Fig. 8(d)). This indicates that cylindrical honeycomb cores can be constructed by pulling laminated ring-shaped strips outwards. Thus, the manufacturing process can be simplified and the conventional extension process is likely applicable in addition to the corrugation process because the revised strip does not require a gluing process after deployment, unlike original Model 2.
4.3 Correction angle and feasible design ranges

In revised Model 1 (discussed in Section 4.1), when correction angle $\psi_1$ is relatively large, fold line $A_4B_4$ can intersect with line $A_5B_5$. For the case where trapezoidal element $A_4B_4B_5A_5$ is slender and ratio $t/r$ is small, fold line $A_4B_4$ can overlap with line $B_4B_5$. Both cases indicate that the desired honeycomb cores are not feasible on the given design configuration because trapezoidal element $A_4B_4B_5A_5$ is not formed. Similarly, for negative $\psi_1$, angle $\psi_2$ is also negative; hence, trapezoidal element $A_2B_2B_3A_3$ is affected. Thus, the upper and lower limits of correction angle $\psi_1$ must be considered. Figure 9 shows the feasible design ranges of revised Model 1; the ranges depend on design parameters $r$, $c$, and $t$. In the distorted models (i.e. original Models 1 and 2), it is difficult to increase core height because the twisted elements tend to return to the original flat shapes. On the contrary, ratio $t/r$ can be increased to up to 1.0 in revised Model 1; it forms a column without a central hollow.

Similar to revised Model 1, the geometrical constraints on trapezoidal elements $A_2B_2B_3A_3$ and $A_4B_4B_5A_5$ must be considered in revised Model 2 (discussed in Section 4.2). Thus, the upper and lower limits of correction angle $\psi$ depend on design parameters $r$, $c$, and $t$, and they are shown in Fig. 10. In this model, the maximum value of ratio $t/r$ is limited to 0.66 owing to the intersection of fold lines. Figure 11 shows a model with a configuration of $c/r = 0.45$, $t/r = 0.66$, and $\psi_1 = -3.7^\circ$. The quadrilateral elements and hexagonal hollows are reduced to triangles and quadrilaterals on an inner side of the cylinder, respectively.

The properties of revised Models 1 and 2 are summarized in Table 1. Comprehensively, revised Model 2 is the most appropriate design for the purpose of this study, which is to find a model that satisfies high rigidity and deployability. However, for any practical purpose, one can find a possible model and its combination of design parameters for cylindrical honeycomb cores according to the feasible design ranges (Figs. 9 and 10) and the properties of the models (Table 1).

Fig. 8 Generation of revised corrugated strip, (a): Extracted corrugated strip in space, (b): Top view of the corrugated strip in space, (c): Enlarged view of (b), (d): Development of the corrugated strip on plane, (e): Physical model of handmade cylindrical honeycomb cores (revised Model 2).
Fig. 9 Feasible design ranges of revised Model 1, (a): $c/r = 0.0281$, (b): $c/r = 0.4479$, (c): $c/r = 1.531$.

Fig. 10 Feasible design ranges of revised Model 2, (a): $c/r = 0.0324$, (b): $c/r = 0.5168$, (c): $c/r = 1.751$.

Fig. 11 Generation of thick cylindrical honeycomb cores of revised Model 2, (a): Extracted corrugated strip in space, (b): Top view of the corrugated strip in space, (c): Development of the corrugated strip on plane, (d): Physical model of handmade cylindrical honeycomb cores.

Table 1 Properties of obtained models.

| Models          | Core orientation | Distortion | Restriction to design | Deployability/Reversibility | Possible manufacturing process |
|-----------------|------------------|------------|-----------------------|-----------------------------|-------------------------------|
| Model 1         | Radial           | Distorted  | Geometrically restricted | Difficult once deployed      | -                             |
| Model 2         | Radial           | Distorted  | No restriction         | Not deployable or reversed   | -                             |
| Revised Model 1 | Not radial       | Not distorted | Geometrically restricted | No restriction               | Corrugation                   |
| Revised Model 2 | Not radial       | Not distorted | Geometrically restricted | Deployable and reversible    | -                             |
5. Conclusions and future work

In this study, a geometrical design method for cylindrical honeycomb cores is demonstrated. In original Models 1 and 2, core heights are arranged strictly in radial directions; however, core walls are distorted and the shape cannot be maintained uniformly. Revised Models 1 and 2 are proposed to solve these problems. These models do not have radial core heights but are untwisted. There is no restriction on core height in revised Model 1, but the height of the cylinder is geometrically restricted and it is not a reversible structure once deployed. On the contrary, core height is restricted in revised Model 2, but the height of the cylinder is not restricted. In particular, revised Model 2 is a deployable and reversible structure as long as the material permits. Thus, it is concluded that revised Model 2 is the most appropriate design for high rigidity and deployability.

As core height should be along the radial direction of the cylinder to undergo radial forces, the correction angle can be minimized within the feasible design range. The effects of the correction angle on structural rigidity will be verified in future.

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