Modeling the heat transfer processes in the pipe-soil system

V V Golik\textsuperscript{1,4}, Yu D Zemenkov\textsuperscript{1}, A A Gladenko\textsuperscript{2} and I V Seroshtanov\textsuperscript{3}

\textsuperscript{1}Industrial University of Tyumen, 38 Volodarskogo str., Tyumen, 625000, Russia
\textsuperscript{2}Omsk State Technical University, 11 Mira Ave., Omsk, 644050, Russia
\textsuperscript{3}Gazprom Transgaz Yugorsk, 15 Mira str., Yugorsk, 628260, Russia
\textsuperscript{4}darkpaint@mail.ru

Abstract. The paper presents a mathematical model and methodology for conducting thermotechnical calculations of buried multilayer pipelines. A section of the existing oil pipeline running under difficult geological conditions is considered. The methodology for calculating thermal processes and the main results for this section of the pipeline are described, and halos of thawing of permafrost are modeled on the basis of a mathematical model. Modeling was carried out using several options of insulating material with a constant internal diameter of the pipeline and pumping mode. Modeling was carried out in an all-purpose software system of finite element modeling - ANSYS and a TPS module developed by the authors for conducting thermotechnical calculations.

1. Introduction
The study of the force effect of frost heaving of soils on an existing pipeline is associated with freezing of soils - a complex process that depends on heat and mass transfer in frozen and thawed soil, the nature of the phase transition and chemical transformations, on a possible change in the stress-strain state of the soil massif (thawed and frozen) with subsequent structural change. Such multifactorial nature of the freezing process, the dependence of its course on many different physical and chemical parameters should be taken into account with an adequate mathematical description of the phenomenon under study. [1]

Even more difficult is the study of the mechanism of freezing of soils in the presence of thermal influence from a buried pipeline, which is explained by both a “deterioration” of the process geometry and a change in the energy carrier temperature along the route.

2. Materials and methods
The sections of the main pipelines are divided (according to the temperature of the transported energy carrier) as follows:
1) hot (the energy carrier temperature is positive at any time of the year);
2) warm (the average annual product temperature is positive, but may be negative for some time);
3) cold (the average annual energy product temperature is negative, but may be positive for some time).

The first include the entire length of oil pipelines since at low temperatures there is a significant increase in the viscosity of the oil, which requires large energy costs when pumping it.

Gas pipelines belong to the second or third type, with the exception of relatively short sections downstream compressor stations, which can be attributed to the first type. [2,3]
In general, under certain conditions, a main pipeline can have all three types of the indicated sections. The thermal impact of buried pipelines on the dynamics of seasonal freezing of soils is determined by a combination of average annual $t_{zh}$, maximum $t_{zh,\text{max}}$ and minimum $t_{zh,\text{min}}$ temperatures of the transported product (Figure 1).

![Diagram of soil freezing along the pipeline route](image)

**Figure 1.** Thermal interaction of underground pipelines with soils

As noted above, the formation of the temperature field of freezing soil is determined by a seasonal decrease in the ambient temperature and the thermal impact of the pipeline. In other words, the task set can be solved within the framework of a joint study of heat exchange processes inside the pipeline and in freezing soil with the conjugation of temperature and heat flux in the "pipe-soil" system. [4,5]

The scheme of soil freezing along the pipeline route is shown in Figure 2, the air temperature $t_N(\tau)$ is negative, i.e. $t_N(\tau) \leq 0$. Heat removal from frozen soil to the outside air (through the snow cover) is carried out by convection with a heat transfer coefficient $\alpha_N$. The depth of the pipeline laying to its axis is $H_0$, the mass flow rate $G_{zh}$ of the energy carrier and its temperature $t'_{zh}$ at the inlet of the pipeline are considered set. The following physical and thermophysical parameters of the transported product are also assumed to be known: density $\rho_{zh}$, kinematic viscosity $\nu_{zh}$, thermal conductivity $\lambda_{zh}$, specific heat $c_{zh}$ and thermal diffusivity $a_{zh}$. [1]

In addition to the laying depth $H_0$, the defining geometric parameters of the process include the internal $D_{vn}$ and external $D_{n}$ diameters of the pipeline, as well as the insulation thickness of $\delta$. If the insulation thickness practically does not affect the outer diameter of the pipeline when calculating its force interaction with the soil, then when calculating the heat transfer coefficient from the energy carrier to the soil, it makes a significant contribution due to the low thermal conductivity of the insulation. The thermal conductivity of the pipeline material and insulation are equal to $\lambda$ and $\lambda_{iz}$, respectively, the thermal conductivity of thawed $\lambda_{zh}$ and frozen $\lambda_F$ soils, as well as their volumetric heat capacities $c_{zh}$ and $c_F$ are determined using SP 25.13330.2012 based on data on the density of the soil skeleton and its moisture. [6]
The average energy carrier temperature depends on the $z$ coordinate, measured from the inlet of the pipeline and the current time $\tau$ (in fractions of a month):

$$t_{zh} = t_{zh}(z; \tau).$$  

(1)

The temperature fields of both thawed and frozen soils are determined by the coordinates $y$ (vertical), $x$ (horizontal), $z$ (along the length of the pipeline) and time $\tau$:

$$t_{th} = t_{th}(x; y; z; \tau) \text{ and } t_{f} = t_{f}(x; y; z; \tau).$$

(2)

The dependences for temperatures on the internal and external surfaces of the pipeline look similar:

$$t_{1tr} = t_{1tr}(x; y; z; \tau); \quad t_{2tr} = t_{2tr}(x; y; z; \tau).$$

(3)

**Figure 2.** Soil freezing along the pipeline route during the cold period ($t_N(\tau) \leq 0 ^\circ C$)

**Figure 3.** Calculation model of thermal interaction of an underground oil pipeline in permafrost soils: 1. steel pipe; 2. PU foam thermal insulation; 3. nano-modified concrete coating; 4 permafrost soil.
The most complete mathematical model of the processes of freezing and thawing of moist soils in the framework of a continuous medium is the Stefan problem. [7] The presence of moving interface between zones with different aggregate states of pore moisture in the soil, the law of movement of which is not known in advance, puts the Stefan problem on a par with the most complex nonlinear problems of mathematical physics. Given the above conditions, the calculation model of the heat distribution process around the underground oil pipeline is shown in Figure 3.

A mathematical model of the thermal interaction of a pipeline of radius $r$ laid to a depth $H$ in the soil describes the heat exchange in the soil [8]:

$$c_{ef} \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial t}{\partial y} \right),$$

(4)

where $t$ – soil temperature;
$\tau$ – calculation time.

Assuming that the phase transition of pore soil moisture occurs in a certain temperature range $\Delta t_i$, the effective heat capacity $C_{ef}$:

$$c_{ef} = \begin{cases} 
  c_{rh}, & \text{at } t > t_f + \Delta t; \\
  \rho_x c - \chi y \frac{dW(t)}{dt}, & \text{at } t \in [t_f - \Delta t, t_f + \Delta t]; \\
  c_F, & \text{at } t > t_f - \Delta t,
\end{cases}$$

(5)

where $t_f$ – moisture phase transition temperature;
$\chi$ – latent heat of the moisture phase transition of;
$W$ – unfrozen moisture content;
$\gamma$ – soil skeleton specific gravity;
$\rho_x$ – coefficient taking into account the density of the soil during the subsidence of the pipeline.

The indices T and M correspond to thawed and frozen soil conditions:

$$\lambda = \begin{cases} 
  \lambda_{rh}, & \text{при } t > t_f; \\
  \lambda_F, & \text{при } t \leq t_f.
\end{cases}$$

(6)

If the temperature on the surface of the pipe $T_s$ is set, then for a period of 5 years the steady-state field $T_n$ of the soil temperature is calculated:

$$T(x, y, \tau) \bigg| \tau = 0 = T_n,$$

(7)

On the surface of the ground, the equality of heat fluxes is set:

$$\frac{\partial t}{\partial y} \bigg|_{y=0} = \frac{h_{sr}}{\lambda_{gr}} \left[ t(x, y, \tau) \bigg|_{y=0} - t_{sr} \right],$$

(8)

where $t_{sr}$ – air temperature;
$\lambda_{gr}$ – surface thermal conductivity coefficient;
$h_{sr}$ – heat transfer coefficient of the soil surface.

The derived relation of symmetry with respect to the OY axis is the condition

$$\frac{\partial t}{\partial x} \bigg|_{x=0} = 0.$$

(9)

Boundary conditions at infinity:

$$\frac{\partial t}{\partial x} \bigg|_{x=\infty} = 0; \quad \frac{\partial t}{\partial y} \bigg|_{y=0} = 0; \quad \frac{\partial t}{\partial y} \bigg|_{y=\infty} = 0.$$  

(10)
In this statement, the shock-capturing method with the introduction of effective heat capacity makes it possible to solve one general equation for the entire studied area.

However, such an approach is more applicable to one-dimensional problems and encounters great computational difficulties already in the transition to two-dimensional temperature fields. An additional complicating factor is the fact that under certain conditions it is possible to split the freezing front, which makes the approach proposed above unacceptable. [9]

In this case, we restrict ourselves to the conditions:

\[
\lambda = \begin{cases} 
\lambda_{th}, & \text{при } t > t_f, \\
\lambda_F, & \text{при } t \leq t_f; 
\end{cases} \quad c_{eph} = \begin{cases} 
c_{th}, & \text{при } t > t_f; \\
c_F, & \text{при } t \leq t_f. 
\end{cases}
\]

(11)

We transform the basic equation by introducing a new value - thermal diffusivity \(a\):

\[
a = \frac{\lambda}{c \rho}.
\]

(12)

\[
\frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right).
\]

(13)

We expand the differential equation in a Taylor series. Let

\[
\frac{\partial t_i}{\partial \tau} = \frac{t_i^{i+1} - t_i^i}{\Delta \tau},
\]

(14)

then

\[
\frac{\partial^2 t}{\partial x^2} = \frac{t_i^{i+1} + t_i^{i-1} - 2t_i^i}{(\Delta x)^2}, \quad \frac{\partial^2 t}{\partial y^2} = \frac{t_j^{j+1} + t_j^{j-1} - 2t_j^j}{(\Delta y)^2}.
\]

(15)

Thus, expressing \(t_i^{i+1}\), we obtain a numerical method for solving the equation (1):

\[
t_i^{i+1} = t_i^i + a \cdot \Delta \tau \left( \frac{t_i^{i+1} + t_i^{i-1} - 2t_i^i}{(\Delta x)^2} + \frac{t_j^{j+1} + t_j^{j-1} - 2t_j^j}{(\Delta y)^2} \right).
\]

(16)

The software implementation of this model was carried out in Visual Basic for Applications (VBA) and ANSYS.

Initial calculations of the mathematical model were carried out in VBA, boundary conditions were set corresponding to the information obtained during the examination of the Yarudeyskoye oil field pipeline (Figure 4). In order to increase the accuracy of the calculation, an approximation is made of the function of the temperature change on the soil surface using the MATLAB software package (Figure 5). [10]
Soil thermal diffusivity
Frozen soil
Thawed soil
\( \lambda, \) W/m·°C
\( \lambda, \) W/m·°C
\( \lambda, \) W/m·°C
\( \lambda, \) W/m·°C
\( \lambda, \) W/m·°C
\( \rho, \) kg/m³
\( \rho, \) kg/m³
\( c, \) J/kg·°C
\( c, \) J/kg·°C
\( a, \) m²/h
\( a, \) m²/h

**Figure 4.** Entering initial conditions in the program interface

**Figure 5.** Approximation of the function of the ambient temperature in MATLAB

Table 1 presents the average monthly ambient temperature according to the weather station WMO ID 23256 in Tazovsky region.
Table 1. Average monthly ambient temperature in °C

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Jan |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2017 | 2018 |
| -25.6 | -23.3 | -7.1 | -12.5 | -5.2 | 9.8 | 10 | 15.7 | 3.3 | -3 | -15.8 | -12.9 | -22.5 |

3. Conclusion
The developed mathematical model includes a large number of variables, including coefficients developed by the team of authors under the guidance of Professor B.V. Moiseev, which allows for the most accurate description of the processes occurring in the "pipe-soil" system over a sufficiently long period of time. When conducting mathematical modeling according to the developed methodology, results were obtained that suggest that it is advisable to use multilayer pipelines in the Arctic. The developed TPS program based on the above mathematical model will allow design engineers at the design stage to have complete data on the behavior of the future structure for many years to come.

References
[1] Under general ed. of Grigoriev V A and Zorin V M 1982 Heat and mass transfer. Thermotechnical experiment. Reference book (Moscow: Energoizdat) 512
[2] Zemenkov Yu D, Moiseev B V, Bogatenkov Yu V, Nalobin N V and Dudin S M 2015 Energy-technological complexes in the design and operation of transport and storage facilities of hydrocarbon raw materials Under general ed. of Moiseev B V (Tyumen: TSOGU, TyumGASU) 256
[3] Moiseev B V, Kushakova N P and Nalobin N V 2002 Mathematical modeling of unsteady temperature interaction of heat conductors and seasonally freezing soils Composite building materials: Collection of scientific papers of the international scientific-practical conference. (Penza: PenzGASA) 261-4
[4] Zemenkova M Y, Shastunova U, Shabarov A, Kislitsyn A and Shuvaev A 2018 Physical and mathematical modeling of process of frozen ground thawing under hot tank IOP Conference Series: Materials Science and Engineering
[5] Zemenkova M Methodology of Monitoring of Hydrocarbon Transportation Hydraulic Reliability 2018 Advances in Intelligent Systems and Computing
[6] SP 25.13330.2012 Bases and foundations on permafrost soils. Updated edition of SNiP 2.02.04-88 (with changes 1, 2)
[7] Golik V V, Moiseev B V, Gulkova S G and Zemenkov Yu D 2018 Mathematical simulation of the effect of a buried oil pipeline on permafrost soils IOP Conference Series: Materials Science and Engineering
[8] GOST 25100-2011 Ground. Classification (Amended)
[9] GOST 26263-84 Ground. Laboratory method for determining the thermal conductivity of frozen grounds
[10] RD-75.180.00-KTN-198-09 2009 Unified technological calculations of facilities of main oil pipelines and oil product pipelines
[11] Fedorova O B and Chizhevskaya E L 2014 Adapting intellectual property evaluation methods to the region brand evaluation Middle - East Journal of Scientific Research DOI: 10.5829/idosi.mejsr.2014.19.1.12477
[12] Balikaeva M B, Chizhevskaya E L, Ya Grevtseva G, Kotlyarova I O and Volkova M A 2018 Innovative technologies as a means of the development of future engineers' professional mobility abroad IOP Conference Series: Materials Science and Engineering doi:10.1088/1757-899X/441/1/012007
[13] Chizhevskaya E L, Fedorova O B and Kot A D 2013 Regional branding as factor of achievement of competitiveness of the territory World Applied Sciences Journal
[14] Stepanov O, Moiseev B, Chekardovskiy M, Aksenov B and Shapoval A 2017 Physical and mathematical conditions of non-stationary thermal conditions of the underground air channels *International Journal of Applied Engineering Research*

[15] Moiseev B, Zemenkov Y, Nalobin N and Dudin S 2017 Thermal calculations of underground oil pipelines *MATEC Web of Conferences*