Transport observables for a FNF mesoscopic system in the extreme quantum limit regime

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Abstract. We study the current-induced torque for electrons on ferromagnet-normal metal-ferromagnet (FNF) trilayer with non-collinear magnetizations. Our system is composed of a chaotic cavity and two junctions, coupled with ideal contacts to ferromagnetic reservoirs in local equilibrium by means of a small number of scattering channels. We assume that the normal part is disordered. Each junction is related to the direction of the reservoirs magnetization. The system is taken out of equilibrium causing a spin accumulation. We suppose that all elements have a length smaller than the coherence phase and the spin flip lengths. This consideration lets us neglect decoherence and spin flip processes. The conductance and torque were found numerically by employing the random-matrix theory and scattering matrix formalism for the spin-flux. The observables are presented as a function of the number of propagating channels in the extreme quantum limit regime and of the angle between the magnetizations of the different ferromagnet layers.

Introduction

Transport properties in magnetic mesoscopic systems are sensitive to the spin dependent band structures [1-3]. The production of hybrid devices formed by ferromagnet/normal-metal layers has generated notable electric properties. An example of them is the phenomenon of giant magnetoresistance [4]. Classic models of two channels can be applied to hybrid systems when the magnetic components have collinear magnetizations [5]. However, these models fail when the magnetizations are not parallel. Currently, two theories have been applied to systems with non-collinear magnetizations. These theories allow calculating the conductance and the current-induced torque on the magnetic components. One of them, based on circuit theory, was developed by Brataas et al. [1]. Its base is the Keldysh Green function formalism in the semi-classical regime. The other one is based on the random matrices theory and was constructed by Waintal et al. [2]. It allows the attainment of the extreme quantum limit regime.

A good level of interest has been focused on the special case of spin-polarized current passing through a ferromagnet/normal-metal/ferromagnet (FNF) trilayer system, particularly motivated on the transfer of spin angular momentum which exerts a torque on the magnetic moment [1,2]. A question of fundamental importance is to study the transport properties in connection with few opened conduction channels. This system brought the possibility of improving our understanding of the extreme quantum regime and of making inference about qualitative and quantitative features of the semiclassical limit.
This is the goal of this work. We study the extreme quantum limit regime for the FNF trilayer with non-collinear magnetizations. The conductance and torque are found numerically using the random matrix theory and scattering matrix formalism for the spin-flux.

This paper is organized as follows. In section 2 we introduce the scattering approach. The results are presented in section 3. Finally, in section 4 we conclude.

**Scattering approach**

Figure 1 shows the system studied. We consider a FNF trilayer system consisting of two ferromagnetic layers $F_a$ and $F_b$ with a normal disordered metal (N) spacer. The system is composed of a chaotic cavity and two junctions, coupled with ideal contacts to ferromagnetic reservoirs in local equilibrium by means of a small number of scattering channels. We suppose that all elements have a length smaller than the coherence phase and the spin flip lengths. This consideration lets us neglect decoherence and spin flip processes.

![Figure 1. Schematic picture of the FNF system.](image)

We follow the approach developed by Waintal et al. [2]. We define a scattering matrix that connects the reservoir and each other through perfect guides. The electron wave function, in the representation of transverse modes, can be defined as a $2N$-dimensional column vector $\Psi_{i\alpha}$, where $i$ ($i=0,1,2,3$) labels the guide and $\alpha \in \{R, L\}$ labels the right and left directions of propagation. $\Psi_{i\alpha}$ indicates the amplitudes of the $N$ up and $N$ down channels. Considering the ferromagnetic layer $F_a$, the amplitudes of outgoing modes $\Psi_{1E}$ and $\Psi_{0D}$ are related with the amplitude of incoming modes $\Psi_{1D}$ and $\Psi_{0E}$ by the relation

$$
\begin{pmatrix}
\Psi_{1E} \\
\Psi_{0D}
\end{pmatrix} = S_a
\begin{pmatrix}
\Psi_{1D} \\
\Psi_{0E}
\end{pmatrix}.
$$

(1)

Following the same reasoning we can define $S_b$ and $S_N$. The $S$ matrices are the $4N \times 4N$ scattering matrices represented as

$$S = \begin{pmatrix}
r & r' \\
t & r'
\end{pmatrix},
$$

(2)

where the elements $r, r', t$ and $t'$ are $2N \times 2N$ reflection and transmission matrices, in order to write the $N \times N$ matrices as

$$r = \begin{pmatrix}
r_{\uparrow\uparrow} & r_{\downarrow\downarrow} \\
r_{\uparrow\downarrow} & r_{\downarrow\uparrow}
\end{pmatrix}.
$$

(3)

The indices $\uparrow$ and $\downarrow$ correspond to the up and down spin. The $r', t$ and $t'$ matrices have a similar structure. Also, there is dependence between the elements of $S_a$ and $S_b$ and the direction of magnetization $m_a$ and $m_b$. Considering the magnetization of the reservoirs in the x-z plane as showed
in Figure 1 we see that $m_a$ forms an angle $\theta$ with the z-axis. We observe that the elements of $S_a$ have to be rotated in the spin space. The magnetic layers are spin filtered, and we write $r_{aj\uparrow} = r_{aj\uparrow}(0) = r_{aj\uparrow}(0) = 0$ [5]. Non-conservation of the spin current in $F_a$ and $F_b$ results in a torque $\tau_a$ exerted on the magnetic moment of $F_a$. In order to find the torque $\tau$ with respect to the voltage $V_0$ generated by the applied current $I$, we use [2]

$$\frac{\partial \tau_a}{\partial V_0} = -\frac{e}{4\pi} \text{Re} \left[ \text{Tr} \left( \bar{\sigma} \Omega \sigma \Omega^\dagger \sigma \Omega - \bar{\sigma} \Omega \sigma \Omega^\dagger \sigma \Omega \right) \right],$$

(4)

where $\bar{\sigma}$ is the vector of the Pauli matrices and $\Omega = \frac{1}{1-r_{aj\uparrow}^* r_{aj\uparrow}^\dagger (1-r_{aj\uparrow}^* r_{aj\uparrow}^\dagger)^{-1} r_{aj\uparrow}^* r_{aj\uparrow}^\dagger}. (5)$

This implies that, after some calculation, the torque along the x direction can be written as $\frac{\tau^x_a}{I} = \frac{1}{G \frac{\partial V_0}{\partial \Omega}}$, measured in unit of $h/2\pi e$. $G$ is the adimensional conductance matrix defined by $G = \text{Tr}(t't')$ where $t' = t'_{ab} \Omega$. As the normal layer is disordered, we study the average conductance and average torque.

**Results**

The parameters that characterize the barrier are constructed from the following quantities[1,2]:

$$G_j^\dagger = \frac{e^2}{h} \sum_{n,m} |t_{j,n,m}|^2 g_j = \frac{e^2}{h} \sum_{n,m} |t_{j,n,m}|^2 \text{ and } G_j^\dagger = \frac{e^2}{h} \left( N - \sum_{n,m} |t_{j,n,m}|^2 \right),$$

(6)

where $N$ is the total number of open channels, $j \in \{a,b\}$ labels the magnetic layer, and $n,m$ labels the open channels. In the following we will consider $|t_{a1}|^2 + |t_{a2}|^2 = 1$ and $3G_{j2}^\dagger = 0$. For convenience we define the independent parameters

$$R_a = \frac{|t_{a1}|^2}{|t_{a2}|^2} \text{ and } R_b = \frac{|t_{b1}|^2}{|t_{b2}|^2}.$$  

(7)

We perform numerical calculations of transport quantities (conductance and torque) of a FNF system having a number of propagating modes $N=1, 2$ and $3$ for all layers at the Fermi energy. The metal normal layer is described by a Gaussian random matrix $H$ describing the states in a cavity with a scattering matrix given by the Mahaux-Weidenmuller formula [6]. The torque per unit current for $R_a=9$ and $R_b=1, 3, 9, 1/3, 1/9$ is illustrated in Figure 2 for $N=1$ versus $\theta$. Figure 3 presents the conductance versus $\theta$ for the same parameter of Figure 1. There is a reflection symmetry between $R_b$ and $1/R_b$ for $\theta=\pi/2$. Observe that $\tau_b=0$ for $R_b=1$, because in this case the barrier $b$ is not active. For $\tau_b \neq 0$ it is necessary that the two barriers be active. The conductance $G$ decreases if the $R_b>0$ and it increases for $R_b<0$.

We can alternatively fix $R_b$ and vary $N$ in order to cover the other number of propagating modes. Figure 4 shows the torque per unit current as a function of $\theta$ for symmetric barrier $R_a=R_b=9$ for $N=1, 2, 4$. For intermediate angle ($1 < \theta < 3$) torque has a greater value than average and for small ($\theta < 1$) and large angle ($\theta > 3$) torque have smaller values than average. Figure 5 presents the conductance. The conductance decreases if the angle increases. Fixing $\theta$ and increasing $N$ the conductance increases.
Conclusions

In summary, we study numerically the conductance and the spin transfer torque with respect to the transport of electrons through mesoscopic devices formed by the attachment in series of two ferromagnetic metal layers separated by a quantum dot, and connected to wave guides. Here, the torque is written as a function of the angle between the magnetic moments of the layers. The results reproduce the analytical expressions available in the literature. The most notable feature of our data is the difference between the transport properties for systems with a small number of channels and those of the semiclassical regime. This is relevant since it reveals singular properties of these regimes. For instance, in normal-metal mesoscopic systems, the conductance tends to Ohm’s law in the semiclassical regime while in the extreme quantum limit the conductance has quite unusual law results [6]. A more detailed analysis, comparing the two regimes will be one of our future works.

Acknowledgments

We thank F. A. G. Almeida for comments. This work was supported by CNPq and FAPITEC/SE.

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