The Effects of Element Diffusion on the Pulsational Properties of Variable DA White Dwarf Stars

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ABSTRACT

We explore the effects of element diffusion due to gravitational settling and thermal and chemical diffusion on the pulsational properties of DA white dwarfs. To this end, we employ an updated evolutionary code coupled with a pulsational, finite difference code for computing the linear, non-radial g-modes in the adiabatic approximation. We follow the evolution of a 0.55 M⊙ white dwarf model in a self-consistent way with the evolution of chemical abundance distribution as given by time dependent diffusion processes. Results are compared with the standard treatment of diffusive equilibrium in the trace element approximation. Appreciable differences are found between the two employed treatments. We conclude that time dependent element diffusion plays an important role in determining the whole oscillation pattern and the temporal derivative of the periods in DAV white dwarfs.

In addition, we discuss the plausibility of the standard description employed in accounting for diffusion in most of white dwarf asteroseismological studies.

Key words: stars: evolution - stars: interiors - stars: white dwarfs - stars: oscillations

1 INTRODUCTION

Asteroseismology is a method to extract information about the internal structure and evolution of stars by means of the study of their oscillatory pattern. This technique, very sophisticated in the case of the Sun, has also undergone a strong development in other stars, in particular the pulsating white dwarfs (WDs) (for reviews, see, e.g., Brown & Gilliland 1994; Gautschy & Saio 1995; 1996).

Pulsating WDs show multi-periodic luminosity variations in three ranges of effective temperatures (T_{eff}) corresponding to the currently called DOV, DBV and DAV (see, e.g., the reviews by Winget 1988 and Kepler & Bradley 1995). Of interest for this work are the DAVs (hydrogen-dominated atmospheres), or ZZ Ceti, that pulsate in the instability strip corresponding to 12500 K ≳ T_{eff} ≳ 10700 K. The periodicities in the light curves of pulsating WDs are naturally explained in terms of non-radial g-modes of low harmonic degree (ℓ ≤ 2), driven by the “κ mechanism” working in a partial ionization region near the stellar surface (Dolez & Vauclair 1981, Winget et al. 1982). Other physically plausible mechanism for overstability of g-modes in ZZ Ceti stars is the “convective driving mechanism” (see Brickhill 1991 and Goldreich & Wu 1999 for details). The periods (P) are found within a range of 100 s ≤ P ≤ 1200 s and photometric amplitudes reach up to 0.30 magnitudes.

Asteroseismology of WDs has recently reached important success, supplying independent constraints to several structural quantities. As a few examples we mention the cases of DOV PG 1159-035 (Winget et al. 1991), DBV GD 358 (Bradley & Winget 1994), and the DAVs G117-B15A and R548 (Bradley 1996, 1998).

The main observable feature in WD pulsations is the period pattern, which can be accurately measured. Another important quantity is the temporal derivative of the period (P), that allows to measure the cooling time-scale of WDs and to provide constraints on the chemical composition of the core. In this sense, the star DAV G117-B15A is particularly noteworthy. Its observed periods are 215.2, 271 and 304.4 s. For the 215.2 s mode it has been possible to find its temporal derivative to be ˙P = (2.3 ± 1.4) × 10^{-15} s s^{-1} (Kepler et al. 2000).

As mentioned, one of the main purposes of asteroseismology of WDs is to disentangle the observed periodic sig-
nals in terms of the internal structure and the evolution of such objects. In view of the detailed available observations, it is very important to study the pulsational properties of DAVs in the frame of evolutionary models as physically sound as possible. In this regard, most of existing calculations treat the chemical profile at the hydrogen-helium interface (the most relevant one in the context of ZZ Ceti pulsations) on the basis of the equilibrium diffusion in the trace element approximation, hereafter EDTE approximation, (Tassoul, Fontaine & Winget 1990, TFW; Bradley, Winget & Wood 1992; Bradley 1996; see also Appendix). In the treatment used by these authors, the chemical profile may change solely as a result of changes in the state of ionization in the plasma (see Appendix). Thus, if the compositional transition region occurs at thermodynamical conditions at which the plasma is fully ionized, then such a treatment predicts fixed profiles. However, even in the case of thick hydrogen envelopes (when both hydrogen and helium are completely ionized deep in the star) element diffusion modifies the chemical abundance distribution within the star, and this is true even during evolutionary stages corresponding to the ZZ Ceti domain (see Iben & McDonald 1985, particularly their Fig. 4).

It is the aim of this work to perform new pulsation calculations in DA WDs by relaxing both the trace element approximation and the diffusive equilibrium assumption. To this end, we carry out time-dependent diffusion calculations for a multicomponent plasma in a self-consistent way with stellar evolution. Detailed diffusion calculations consistent with stellar evolution have recently been performed by McDonald, Hernanz & José (1998) to study the problem of carbon pollution in cool WDs and by Althaus, Serenelli & Benvenuto (2001) to assess the role played by diffusion in the occurrence of hydrogen thermonuclear flashes in low-mass, helium-core WDs. In addition, Dehner & Kawaler (1995) have considered time-dependent diffusion in evolving hot WDs in the interest of exploring the possibility of an evolutionary link between DO PG 1159 stars and the much cooler DB WDs. In the context of pulsations, the change in the chemical composition (particularly at the hydrogen-helium interface) induced by diffusion processes is expected to affect the shape of the Ledoux term $B$ and hence the Brunt-Väisälä frequency (see Brassard et al. 1991 for a discussion of the calculation of the Brunt-Väisälä frequency in the context of WDs).

In order to gauge the actual importance of time-dependent diffusion in the computation of theoretical $P$ and $P$ we have to calculate the WD cooling considering a full evolutionary code considering diffusion coupled to a pulsational code. To our knowledge, this is the first time that such kind of calculation has been undertaken in the context of DA WDs. Note that in such kind of treatment the internal chemical profile is the consequence of realistic evolutionary models. Here, we present calculations of linear, adiabatic, non-radial pulsations of DAV model with a mass of 0.55 $M_\odot$ (which is representative of the mass of G117-B15A). In particular, we shall calculate two evolutionary evolutionary sequences, one considering time-dependent element diffusion, and another one in the frame of the standard EDTE approximation.

The remainder of this paper is organized as follows. In Section 2 we describe our evolutionary-pulsational computer code, paying special attention to the method for simulating the diffusion of elements in a time-dependent approach. Section 3 is devoted to presenting the calculations we performed. Finally, in Section 4 we discuss our results and make some concluding remarks.

2 OUR COMPUTER CODE

2.1 Evolutionary code and diffusion equations

The evolutionary code we employed is detailed in Althaus & Benvenuto (1997, 1998). This code is based on a very detailed and up-to-date physical description such as OPAL radiative (Iglesias & Rogers 1996) and molecular (Alexander & Ferguson 1994) opacities. The equation of state is an updated version of that of Magni & Mazzitelli (1979). High-density conductive opacity and neutrino emission rates are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997 for details). Also, a complete network of thermonuclear reaction rates corresponding to the proton-proton chain and the CNO bi-cycle is included. Nuclear reaction rates are from Caughlan & Fowler (1988) and electron screening is treated as in Wallace, Woosley & Weaver (1982).

Gravitational settling, and chemical and thermal diffusion have been fully taken into account following the treatment for multicomponent gases presented by Burgers (1969). Thus, we avoid the trace element approximation usually invoked in most WD studies. Radiative levitation, which is important for determining photospheric composition of hot WDs (Fontaine & Michaud 1979) has been neglected.

As a result of gravity, partial pressure, thermal gradients and induced electric fields (we neglect stellar rotation and magnetic fields) the diffusion velocities in a multicomponent plasma satisfy the set of $N - 1$ independent linear equations (Burgers 1969)

$$\frac{dp_i}{dr} - \frac{\rho_i dp}{dr} - n_i Z_i c E = \sum_{j \neq i}^N K_{ij} (w_j - w_i)$$

$$+ \sum_{j \neq i}^N K_{ij} \frac{m_j r_i}{m_i + m_j} (n_i - n_j),$$

(1)

and heat flow equation (N equations)

$$\frac{5}{2} n_i k_B \nabla T = \frac{5}{2} \sum_{j \neq i}^N K_{ij} z_{ij} \frac{m_j}{m_i + m_j} (w_j - w_i) - \frac{2}{5} K_{ii} z_{ii} r_i$$

$$- \sum_{j \neq i}^N \frac{K_{ij}}{(m_i + m_j)^2} \left(3m_i^2 + m_j^2 z_{ij} + 0.8m_i m_j z_{ij}^2 \right) r_i$$

$$+ \sum_{j \neq i}^N \frac{K_{ij} m_i m_j}{(m_i + m_j)^2} \left(3 + z_{ij} - 0.8z_{ij}^2 \right) r_j.$$  

(2)

In these equations, $p_i$, $\rho_i$, $n_i$, $Z_i$ and $m_i$ means, respectively, the partial pressure, mass density, number density, mean charge and mass for species $i$ ($N$ means the number of ionic species plus electron). The quantities $T$ and $k_B$ are the temperature and Boltzmann constant. The unknown variables are the diffusion velocities with respect to the centre of mass, $w_i$, and the residual heat flows $r_i$ (for ions and
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The set of equations is completed by using the conditions for no net mass flow with respect to the center of mass
\[ \sum_i A_i n_i w_i = 0, \]
and no electrical current
\[ \sum_i Z_i n_i w_i = 0. \]

In terms of the gradient in the number density we can transform Eq. (1) to
\[ \frac{1}{n_i} \left[ \sum_{j \neq i} K_{ij} (w_i - w_j) + \sum_{j \neq i} K_{ij} z_i \frac{m_i r_j - m_j r_i}{m_i + m_j} \right] - Z_i eE = \alpha_i - k_B T \frac{d \ln n_i}{dr}, \]
where
\[ \alpha_i = -A_i m H g - k_B T \frac{d \ln T}{dr}, \]
being \( A_i, m_H, \) and \( g \) the atomic mass number, hydrogen atom mass and gravity, respectively. Let us write the unknowns \( w_i, r_i \) and \( E \) in terms of the gradient of ion densities in the form
\[ w_i = w_i^q + \sum_{\alpha \in z \alpha(j)} \sigma_{ij} \frac{d \ln n_i}{dr}, \]
where \( w_i^q \) means the velocity component due to gravitational settling and thermal diffusion. With Eqs. (2) and (5) together with (3) and (4) we can easily find the components \( w_i^q \) and \( \sigma_{ij} \) by matrix inversions (LU decomposition). The evolution of the abundance distribution throughout the star is found by solving the continuity equation. In particular, we follow the evolution of the isotopes \( ^1\text{H}, ^4\text{He}, ^{12}\text{C}, ^{14}\text{N} \) and \( ^{16}\text{O} \). To calculate the dependence of the structure of our WD models on the evolving abundances self-consistently, the diffusion equations have been coupled to the evolutionary code.

2.2 The pulsational code

In order to compute the g-modes of the WD models we have coupled our evolutionary code to our new, finite difference, pulsational code described in Còrsico & Benvenuto (2002), which solves the equations for linear, adiabatic, non-radial pulsations (Unno et al. 1989).

We describe now how these codes work together. To begin with, an interval in \( P \) and \( T_{\text{eff}} \) (\( T_{\text{eff}} \)-strip) is chosen. The evolutionary code computes the model cooling until the hot edge of the \( T_{\text{eff}} \)-strip is reached, then, the program calls the pulsation routine beginning the scan for modes. When a mode is found, the code generates an approximate solution which is iteratively improved to convergence and stored. This procedure is repeated until the period interval is covered. Then, the evolutionary code generates the next stellar model and calls pulsation routines again. Now, the previously stored modes are taken as initial approximation to the modes of the new stellar model and iterated to convergence. Such a procedure is automatically repeated for all evolutionary models inside the \( T_{\text{eff}} \)-strip. The computational strategy described above has been successfully applied in fitting the observed periods of G117-B15A to impose constraints on the mass of axions (Còrsico et al. 2001a) and in computing the period structure of low mass, helium WDs (Còrsico & Benvenuto 2002).

We have tested our pulsational code with two carbon-oxygen DA WD models of 0.5 \( M_\odot \) and 0.85 \( M_\odot \), the structure of which was computed with the WDEC evolutionary code. The vibrational properties of such models were previously analyzed by Bradley (1996). In the interests of a detailed comparison, we have considered a large amount of modes and we found that the differences between the two sets of modes remain below \( \approx 0.1\% \).

3 COMPUTATIONS

We have evolved a 0.55 \( M_\odot \) WD model with an internal carbon - oxygen chemical profile corresponding to that calculated by Salaris et al. (1997). Such a model has hydrogen and helium mass fractions of \( M_\text{H}/M_\ast = 10^{-4} \) and \( M_\text{He}/M_\ast = 10^{-2} \) respectively. These values are in good agreement with evolutionary predictions and are also very similar to those found by Bradley (1998) for the case of G117-B15A. The internal chemical profile of our model is shown in Fig. 1. It is important to mention that at the bottom of the hydrogen envelope of our model, hydrogen and helium are fully ionized and this is so throughout the entire evolutionary stages we study in the present paper. Thus, the chemical abundance profile predicted by the trace element approach remains fixed during evolution. In computing radiative opacities, we have assumed \( Z = 0 \). We have treated convective transport in the frame of the ML3 version of the mixing length theory. The ML3 prescription, characterized by a high convective efficiency, assumes the mixing length to be two times the local pressure scale height (see Tassoul et al. 1990).

A realistic starting model for our evolutionary sequences was obtained by artificially brightening an initial WD configuration (see Benvenuto & Althaus 1998) up to \( \log L/L_\odot = 2 \). Such a procedure is known to produce an initial sequence of some unphysical models, but then relaxes to the correct cooling sequence (see Althaus & Benvenuto 2000 for further discussion) far before reaching the DAV instability strip. Since then on element diffusion is incorporated. When the model reaches \( T_{\text{eff}} = 14000 \text{ K} \) we start pulsational calculations. Specifically we have calculated dipolar (\( \ell = 1 \)) modes (which are usually encountered in ZZ Ceti light curves) with radial orders \( k = 1, \cdots, 21 \) which cover a period interval \( 100 \text{ s} \lesssim P \lesssim 1000 \text{ s} \). Calculations are stopped at \( T_{\text{eff}} = 10000 \text{ K} \), thus, the \( T_{\text{eff}} \)-strip amply embraces the observed DAV instability strip. For the modes we have found to fulfill such conditions, we have computed periods and eigenfunctions. For computing the Brunt - Väisälä frequency, we have employed the appropriate prescription for degenerate models, given in Brassard et al. (1991). After
period assessment we compute $\dot{P}$ by numerical differentiation.

4 RESULTS AND IMPLICATIONS

We begin by examining Fig. 2, in which we show the evolution of the chemical profile for helium resulting from time dependent element diffusion together with the chemical profile arising from the EDTE approximation. Note that in the latter case, the profile remains unchanged throughout the evolution because, as we mentioned, ionization is complete at such deep layers. By contrast, in the case with time-dependent diffusion, the chemical abundance distribution evolves appreciably during the ZZ Ceti evolutionary stages. Note that the shape of the profile in both treatments turns out to be markedly different particularly at the centre of the transition. As we shall show below, this will have an appreciable influence on the $P$ and $\dot{P}$ values for some of the modes.

In Fig. 3, we depict the resulting Ledoux term (panel A) and the squared Brunt-Väisälä frequency (panel B) at two selected $T_{\text{eff}}$ values. Let us remind the reader that the term $B$ depends not only on the shape of the chemical profile but also on the thermal and mechanical structure of the star (see Brassard et al. 1991). Thus, even in the EDTE approach the term $B$ changes with cooling. Because of the fact that in the Ledoux term there appears the derivative of the chemical profile (see Eq. 35 of TFW), a slight change in the slope of the hydrogen-helium interface translates into a noticeable change in $B$. Thus, it is not surprising the $B$ term exhibits a sharp peak in the case of the trace element treatment, in contrast with the more physical treatment as given by non-equilibrium diffusion. In turn, this feature is reflected in the Brunt-Väisälä frequency. As it has been exhaustively shown by Brassard et al. (1992ab), the shape of the Brunt-Väisälä frequency at the chemical interfaces plays a key role in fixing the structure of the period pattern (e.g., mode trapping) of ZZ Ceti stars.

Now let us turn our attention to the computed pulsational modes. In Figs. 4 and 5 we show $P$ and $\dot{P}$ corresponding to modes with $\ell = 1, k = 1, \cdots, 6$ for models with time-dependent element diffusion and with the EDTE approximation at the hydrogen-helium interface as a function of $T_{\text{eff}}$. From a close inspection of these figures it can be realized that the effects of time-dependent element diffusion are indeed non-negligible in $P$ and $\dot{P}$, although for some modes results are very similar. We want to mention that the same trend has been found in modes of higher radial order (not shown here for brevity). Note that for the modes analyzed in these figures the greatest relative differences encountered are $\approx 20\%$ for $P$ and $\approx 5\%$ for $\dot{P}$.

It is worth mentioning that the differences cited above arise mainly from the very different shape of the interface profile resulting from the two treatments of diffusion investigated here. In particular, the differences in the helium profile for $X_{\text{He}} \gtrsim 0.5$ (see Fig. 2) in these treatments are the main reason why the periods and period derivatives become different. In addition, there is a small contribution to the differences in $P$ and $\dot{P}$ due to the evolution of the profiles of each chemical interface in response to non-equilibrium diffusion.

Thus, as models with time dependent element diffusion are more physically plausible, these should be taken into account when an asteroseismological fit to observed periods is performed. Also, as $P$ is modified, this approach should also be taken into account at using observed $\dot{P}$ values to infer the composition of the WD core.

Before closing the paper, we would like to discuss at some length a major issue raised by our referee. Indeed, in his (her) report, our referee asked us to look for the underlying physical reasons for the differences between the standard treatment of EDTE approximation and our full treatment of time dependent element diffusion. To be specific, the referee asked us whether such differences are due mostly to the relaxation of the trace element approximation or the equilibrium hypothesis.

In order to find the answer, we decided to perform a simple numerical experiment: we simulate the equilibrium diffusion conditions with our full code. Equilibrium diffusion would be a good approximation if the diffusion timescale were much shorter than the evolutionary one. Then, in order to simulate this situation, we simply assumed the diffusion time step to be several times the evolutionary one. This is equivalent to assuming that the whole diffusive process occurs several times faster. We computed the evolution of our WD model under this hypothesis. The result was that the WD did not evolve along the cooling branch but instead suffered from a hydrogen thermonuclear flash. Physically, the reason for this behaviour is that, if diffusion were plenty of time to evolve to equilibrium profiles, then the tail of the hydrogen profile would be able to reach hot enough layers to be ignited in a flash fashion. The fact that at imposing equilibrium diffusion conditions the star undergoes a hydrogen thermonuclear flash, while in the detailed, self-consistent time dependent diffusive treatment the star cools down quiescently, clearly shows the incorrectness of the hypothesis of equilibrium. In view of this, we are forced to conclude that there is no way other than to abandon the idea that the shape of the internal profiles in the WD is determined by equilibrium diffusion. This conclusion is valid at least for massive hydrogen envelopes. We think that the only physically sound way to compute such profiles, a key ingredient in asteroseismological studies, is to calculate the WD evolution in a self-consistent way with time dependent element diffusion and nuclear burning.

In addition, some words are in order about the standard treatment of equilibrium diffusion, which is based on the work of Arcoragi & Fontaine (1980). Recent asteroseismological studies of DAV WDs (Clemens 1994, Bradley 1998, 2001) seem to favour large values for the thickness of the hydrogen envelope ($M_{\text{H}}/M_\odot \approx 10^{-3}$). At the thermodynamical conditions relevant to DAV WD models, we find that most of the hydrogen-helium interface occurs at degenerate conditions. Because the Arcoragi & Fontaine (1980) equations...
are valid for non-degenerate conditions, we should remark that this treatment cannot be applied to the modeling of the hydrogen-helium interfaces in DA WDs with massive hydrogen envelopes.

We should also remark that, in the case of the trace element approximation, we have found that the object does not undergo any thermonuclear flash. This is another artifact of the approximation, due to the ad hoc truncation of the profile at some low density (see Appendix). Indeed, in our numerical experiments, we have found that the stellar model evolves along the WD cooling track if we truncate the hydrogen profile at $X_H = 10^{-3}$. However, if we allow the equilibrium hydrogen profile to extend to slightly lower abundances (e.g., $X_H = 10^{-4}$) the model experiences a thermonuclear flash!

The results presented in this paper indicate that a more extensive and systematic exploration of asteroseismology of DAV in the frame of detailed evolutionary models considering time dependent element diffusion is worth being done and it will be the subject of further papers. While the present paper was in process of reviewing, some interesting results about the effects of diffusion on mode trapping have been presented in Córnsico et al. (2001b).

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APPENDIX

For the sake of completeness, we describe here the equations employed to include diffusion processes in most of WD pulsational studies. This approximation is based on the work of Arcoragi & Fontaine (1980) (see also Tassoul et al. 1990). Here we limit ourselves to comment on the most important aspects involved in this treatment.

To begin with, Arcoragi & Fontaine (1980) assume a stellar plasma made up of two-ionic species with average charge $Z_1$ and $Z_2$ and atomic weight $A_1$ and $A_2$. In addition, thermal diffusion is neglected and an ideal gas equation of state is considered under the assumption that the plasma is sufficiently diluted. Under these approximations, the diffusion velocity $w_{12}$ reads

$$w_{12} = D_{12}(1 + \gamma) \left[ -\frac{\partial \ln c_2}{\partial r} + \frac{A_2 - A_1}{A_1 + \gamma A_2} \frac{\partial \ln p}{\partial r} + \frac{A_2 Z_1 - A_1 Z_2}{A_1 + \gamma A_2} \frac{c E}{k_B T} \right].$$

(D.8)

$D_{12}$ is the diffusion coefficient, and $c_i$, the number concentration of ions of species $i$, is defined by

$$c_i = \frac{n_i}{n_1 + n_2} = \frac{p_i}{p_1 + p_2}.$$

(9)
being \( p_i \) the partial pressure. \( E \) is the electric field, given by

\[
eE = m_p g \frac{A_1 Z_1 + A_2 Z_2 \gamma}{Z_1 (Z_1 + 1) + Z_2 (Z_2 + 1) \gamma},
\]

and \( \gamma \) is defined as

\[
\gamma \equiv \frac{n_2}{n_1} = \frac{p_2}{p_1} = \frac{c_2}{c_1}.
\]

The remainder of the symbols have the usual meaning. Notice that Eqs. (9) and (11) are valid in an isothermal medium, i.e., we are neglecting temperature gradients.

Now, we impose equilibrium diffusion, by assuming \( \nu_{12} = 0 \), and from Eqs. (8 - 11) we get the ordinary differential equation for the equilibrium profile (Eq. A5 of Arcoragi & Fontaine 1980). In the trace element approximation \( (\gamma \ll 1) \), for the specie 2 considered as a trace, we get

\[
\frac{\partial \ln c_2}{\partial r} = \alpha_2 \frac{\partial \ln \gamma}{\partial r},
\]

where

\[
\alpha_2 = \frac{A_2}{A_1} (1 + Z_1) - Z_2 - 1.
\]

For the purpose of application, Tassoul et al. (1990) divide the transition zone into two parts: an upper one in which element 1 is dominant and element 2 is a trace, and a lower one in which the roles of the respective elements is reversed. For the upper region the abundance profile of element 2 considered as a trace is given by Eq. (12) and for the lower part of the transition zone the abundance of element 1 (considered as a trace) is given by

\[
\frac{\partial \ln c_1}{\partial r} = \alpha_1 \frac{\partial \ln \gamma}{\partial r},
\]

where

\[
\alpha_1 = \frac{A_1}{A_2} (1 + Z_2) - Z_1 - 1.
\]

\( \gamma \) is the mass fraction \( (1 - M_r/M_*) \). The integration of Eqs. (12) and (14) gives the equilibrium abundance profiles:

\[
c_2 = k_2 q^{\alpha_2} \quad \text{(upper region of interface)}
\]

and

\[
c_1 = k_1 q^{\alpha_1} \quad \text{(lower region of interface)}
\]

By invoking the condition of continuity in the middle point of the interface, we obtain the relation

\[
k_2 q_m^{\alpha_2} = k_1 q_m^{\alpha_1} = \frac{1}{2},
\]

where \( q_m \) is the mass fraction where the abundances of two species are equal; the \( q_m \) value is obtained by forcing the mass conservation of element 1. Thus, in the case of the hydrogen-helium transition region, the outer mass fraction of hydrogen \( (q_H = M_H/M_*) \) is employed for computing \( q_m \). Note that possible changes in the equilibrium profiles result only from slight changes in the ionization states of the elements present at the interfaces, i.e., variations in the exponents \( \alpha_1 \) and \( \alpha_2 \) (Tassoul et al. 1990).

To implement this approach to modeling of the hydrogen-helium transition zone, it is necessary to set small abundances to zero in order to avoid having a tail of hydrogen in regions deep enough where carbon is abundant (for this case we should generalize the above treatment for three species). Moreover, if we do not do so, hydrogen would be present at layers hot enough to force the star to undergo a thermonuclear flash. This is the case, at least for thick hydrogen envelopes like those we have treated here which, in turn, are the favoured ones by current asteroseismological studies (see the main text).
Figure 1. The internal chemical profiles of the 0.55 $M_\odot$ carbon-oxygen WD model for hydrogen, helium, carbon and oxygen at an effective temperature of 14000K. In the case of the EDTE approximation, the fixed profiles are represented by dotted lines. Profiles for models in which time dependent element diffusion has been considered are represented by solid lines. $q$ is the outer mass fraction defined by $q = 1 - M_r/M_\ast$. 

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Figure 2. Panel A: The internal chemical profile for helium in terms of the outer mass fraction $q$ at the hydrogen-helium interface. Solid line corresponds to a model at a temperature of 14000K in which time-dependent diffusion is considered. Long dashed line means the same treatment but for a model at 10000K. Finally dotted line corresponds to the EDTE approximation prediction. Panel B: The chemical profile for helium as given by the EDTE approximation according to a 0.50 $M_\odot$ model calculated by Bradley (private communication). Note that the shape of the profile in both models with diffusive equilibrium is the same.
Figure 3. In the upper panel it is shown the Ledoux term $B$ at the hydrogen-helium interface in the case of time-dependent diffusion at the $T_{\text{eff}}$ values of 14000K and 10000K as given by solid and long dashed lines, respectively. Dotted and dot-dashed lines correspond to the same temperature values but for the EDTE approximation. In the lower panel the square of the Brunt-Väisälä frequency is shown for the same cases analyzed in panel A. For details, see text.
Figure 4. Period and period derivative for $\ell = 1$, $k = 1, 2, 3$ modes for a 0.55 $M_\odot$ carbon-oxygen WD model in a $T_{\text{eff}}$ interval containing the DAV instability strip. Solid lines (filled squares) correspond to periods (period derivatives) computed considering non-equilibrium diffusion while dotted lines (empty squares) depict periods (period derivatives) computed according to the EDTE approximation at the hydrogen-helium interface. For discussion of the results, see text.
Figure 5. Same as figure 4 but for modes with $\ell = 1, k = 4, 5, 6$. 