Bifibrations of model categories

In this talk, I will explain how to endow the total category \( E \) of a well-behaved Grothendieck bifibration \( E \to B \) with a structure of a model category when both the basis \( B \) and all fibers \( E_b \) of the bifibration are model categories.

The motivating example is the well-known Reedy model structure on a diagram category \( [\mathcal{R}, \mathcal{M}] \). The crucial step in its construction by transfinite induction lies in the successor case, which is usually handled by reasoning on latching and matching functors. A first observation is that those functors define a Grothendieck bifibration on the restriction functor \( [\mathcal{R}_{\lambda+1}, \mathcal{M}] \to [\mathcal{R}_\lambda, \mathcal{M}] \) where \( \mathcal{R}_\lambda \) denotes the full subcategory of \( \mathcal{R} \) whose objects have degree less than \( \lambda \). Unfortunately, this bifibration fails to fulfill the conditions of application of existing theorems in the literature ([1], [2]), which would have allowed to lift the model structure from the base category \( B = [\mathcal{R}, \mathcal{M}] \) to the total category \( E = [\mathcal{R}_{\lambda+1}, \mathcal{M}] \).

I will explain how to relax the hypotheses appearing in [1] and [2] by focusing on (co)cartesian lifts over acyclic (co)fibrations rather than over weak equivalences. This idea leads us to a simple and elegant condition for our new construction: some commutative squares in the base category are required to satisfy a homotopical version of the Beck-Chevalley condition. To conclude, I will apply the result to the Reedy construction and its generalizations ([3], [4]).

[1] Stanculescu, A.E., Bifibrations and weak factorization systems, Applied Categorical Structures, 20(1):19-30, 2012
[2] Harpaz, Y, and Prasma, M., The Grothendieck construction for model categories, Advances in Mathematics, 218:1306-1363 (August 2015)
[3] Berger, C., and Moerdijk, I., On an extension of the notion of Reedy category, Mathematische Zeitschrift, 269(3):977-1004, December 2011
[4] Shulman, M., Reedy categories and their generalizations, arXiv preprint, arXiv:1507.01065 (2015)

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