Numerical study on the boundary effect of rigid model boxes in shaking table tests in underground engineering

X Wang¹, J C Li*¹

¹School of Civil Engineering, Southeast University, Nanjing 211189, China

*Corresponding author: jcli@seu.edu.cn

Abstract: The boundary effect of rigid model boxes in shaking table tests in underground engineering directly affects the reliability of test data. The lining of flexible materials in the side walls of model boxes has become an important method to reduce the boundary effect. However, the arrangement of flexible materials is mainly based on experience, and its rationality is unclear. Therefore, the reasonable design of flexible layers is important for reducing the boundary effect of rigid model boxes. In this work, the influencing factors of the lateral boundary effect of rigid model boxes under the action of S waves were studied. A flexible layer with different parameters was set on the boundary of a model box, and the reduction degree of the flexible layer was established relative to the boundary effect. The results can provide a reference for the selection and design of flexible boundary materials for rigid model box tests.

1 Introduction

Given the frequent occurrence of earthquakes and the resulting damage in underground engineering structures, an increasing number of studies have focused on the failure mechanisms and seismic performance of such structures. The shaking table test is one of the most important methods in the study of the structural response of underground engineering structures. According to their structural forms, main model boxes can be divided into three categories, namely, rigid model boxes, flexible model boxes, and laminar shear model boxes [1-5]. Flexible and laminar shear model boxes can effectively reduce the reflection of seismic waves on model boundaries, but their application is restricted because of the limited load-bearing capacity of flexible boxes and the complex fabrication and high cost of layered shear boxes [6]. Rigid model boxes are widely used in shaking table model tests because of their simple design, strong adaptability, and low cost. However, rigid model boxes...
have high rigidity and small deformation; hence, boundary reflection waves occur when the input ground vibration reaches the side walls of these model boxes. In addition, the horizontal deformation of rocks or soil mass is limited.

To improve the boundary conditions of rigid model boxes, scholars usually set a certain thickness for the flexible material on the side wall of the model box perpendicular to the vibration direction of the shaking table. For example, Shen et al. [7], Li et al. [8], Ding et al. [9], and others arranged expanded polystyrene (EPS) foam layers with thicknesses ranging from 100 to 200 mm at the boundaries of model boxes. Li et al. [10] used a flexible sponge with unknown thickness, Sun et al. [11] utilized organic rubber with a thickness of 4 mm), and Zhou et al. [12] combined 100 mm-thick EPS foam and 50 mm-thick rubber to improve the boundary conditions. However, the arrangement of these flexible materials is mainly based on experience, and its rationality is rarely discussed. Bathurst et al. [13] tested three different geofoam buffer materials retaining sand soil under idealized dynamic loading conditions and found that a reduction in dynamic load increases with decreasing seismic buffer density. Through numerical simulation, Lou et al. [14] concluded that adding a rubber layer to the boundaries can effectively reduce the boundary effect of model boxes and that the reasonable elastic modulus ratio of rubber to soil layer is 2.5 times. Huang et al. [15] concluded that foam thickness exerts a great influence on the boundary effect and that a foam cushion offers the best effect at a thickness of 2%-4% of the model box width. On the basis of the principle of vibration mode superposition, Zhang et al. [16] established a concentrated mass model of soil–box–flexible material and analyzed the influence of the elastic modulus, thickness, Poisson’s ratio, density, and damping ratio of the flexible material on the boundary effect of the model box. Their result showed that the influence of elastic modulus (0.58 MPa is the best) and thickness on the boundary effect of flexible materials is obvious and that the two parameters should be considered first in the design of flexible materials, followed by Poisson’s ratio. Meanwhile, density has little influence on the boundary effect of model boxes. Materials with a similar damping ratio to the soil should be selected to coordinate the deformation of the two. However, only a few studies have explored the influencing factors of the boundary effect of rigid model boxes and the conditions under which flexible material layers should be added.

Referring to existing research, the current work investigated the influence of the different relationships between the mechanical properties of model materials and model boxes on the lateral boundary effect of rigid models under the action of S waves. A flexible layer with different parameters was set on the lateral boundary of a model box, and the reduction degree of the flexible layer relative to the boundary effect of the model box was established. This research can provide a reference for the selection and design of flexible boundary materials for rigid model box tests.

2 Evaluation method of rigid model box boundary effect

At present, four methods are commonly used to evaluate rigid model boxes [15,17]: the first one is to compare the acceleration time histories of the boundary and center points of the rock or soil model directly, i.e., the closer they are, the smaller the boundary effect of the model box will be; the second is
to compare the ratios of the peak acceleration of the boundary and center points of the model, i.e., the closer they are to 1, the smaller the boundary effect will be; the third is to subject the acceleration time histories of the boundary and center points to Fourier transform and compare the frequencies, i.e., the closer they are, the smaller the boundary effect of the model box will be; the fourth one is to compare the 2-norm deviations of the response signals between the boundary point and the center point, i.e., the smaller the value is, the smaller the boundary effect will be. Among these methods, the first one is the most intuitive and easiest to understand, but it lacks a quantitative analysis. The second method allows quantitative characterization, but it is not comprehensive enough to evaluate the boundary effect on the basis of the information of the peak acceleration. The third method is a quantitative analysis method in the frequency domain, but it only reflects the differences in signals’ Fourier amplitude spectra, and it does not directly include the characteristics of amplitude and duration in the time domain. The fourth method is a comprehensive and quantitative method for studying the boundary effect of model boxes because it can reflect the response characteristics of particles in multiple parts of model boxes [16]. Therefore, the fourth method was used in the current work to study the influencing factors of the boundary effect of rigid model boxes and to evaluate the influence of flexible material parameters on the boundary effect.

The evaluation formula of the 2-norm deviation method is

\[ u_1 = \frac{\|x_{IM} - x_{FM}\|}{\|x_{FM}\|} = \sqrt{\frac{\sum (x_{IM} - x_{FM})^2}{\sum (x_{FM})^2}} \]  

(1)

where \(x_{IM}\) is the reference signal of the vibration response of the rock or soil mass at the center point of the model box and \(x_{FM}\) is the contrast signal of the vibration response of the rock or soil mass at the boundary point of the model box. \(x_{IM}\) and \(x_{FM}\) can be a time history of acceleration; a spectral signal; or peak acceleration, velocity, displacement, strain, etc. In this work, \(x_{IM}\) and \(x_{FM}\) are the peak accelerations of the center and boundary points of the rock or soil mass in the model box.

To make the results increasingly accurate, this study also analyzes the vibration response characteristics of the rock or soil mass in the model box and the vibration response in the free field by using the two-norm deviation method. The evaluation formula [16] is

\[ u_2 = \frac{\|x_{IF} - x_{FM}\|}{\|x_{IF}\|} = \sqrt{\frac{\sum (x_{IF} - x_{FM})^2}{\sum (x_{FM})^2}} \]  

(2)

where the \(x_{IF}\) is the reference signal of the vibration response of the rock or soil mass at the center point in the free field. In this work, \(x_{IF}\) is the peak acceleration of the center point of the rock or soil mass in the free field.

### 3 Influencing factors of boundary effect of rigid model box

The main reason behind the lateral boundary effect of rigid model boxes lies in the differences in the elastic moduli, Poisson’s ratios, densities, and damping values between rigid model boxes and the model materials. In this section, the influence of different model material parameters on the boundary
effect of rigid model boxes is discussed. The finite element software FLAC3D is used to establish the numerical models, as shown in Figure 1 (rock or soil mass box model) and Figure 2 (free field model of rock or soil mass). The model box material and the rock or soil material are assumed to be elastic models. Moreover, the elastic modulus $E_{\text{box}}$, Poisson’s ratio $v_{\text{box}}$, density $\rho_{\text{box}}$, and damping ratio $d_{\text{box}}$ of the model box are 209 GPa, 0.269, 7,890 kg/m$^3$, and 0.05, respectively. The scale of the model box is 3 m (length) $\times$ 3 m (width) $\times$ 2.5 m (height). Meanwhile, the elastic modulus ratio, Poisson’s ratio, density ratio, and damping ratio between the model material and the rigid model box are respectively defined as $K_E = E_{\text{mod}}/E_{\text{box}}$, $K_v = v_{\text{mod}}/v_{\text{box}}$, $K_\rho = \rho_{\text{mod}}/\rho_{\text{box}}$, and $K_d = d_{\text{mod}}/d_{\text{box}}$, where $E_{\text{mod}}$, $v_{\text{mod}}$, $\rho_{\text{mod}}$, and $d_{\text{mod}}$ are the elastic modulus, Poisson’s ratio, density, and damping ratio of the model materials, respectively. For the convenience of research, the acceleration peak values of the center and boundary points with the buried depths of 0.1, 0.5, 1.0, 1.5, and 2.0 m are closely monitored to calculate the 2-norm deviation values $u_1$ and $u_2$. The applied synthetic seismic S wave with a frequency of 20 Hz is shown in Figure 3.
When the material model is not affected by the elastic modulus, the influence of the Poisson’s ratio, density, and damping parameters on the boundary effect of the model box is minimal. When the elastic modulus, density, and damping of the material model are respectively fixed to 2.09 GPa, 2,000 kg/m³, and 0.05, the values of $u_1$ and $u_2$ are less than 0.6% and 3%, respectively, as $K_v$ increases from 0.5 to 1.7 (the Poisson’s ratio of the material model ranges from 0.1345 to 0.4573). When the elastic modulus, Poisson’s ratio, and damping of the material model are respectively fixed to 2.09 GPa, 0.25, and 0.05, the values of $u_1$ and $u_2$ are less than 0.7% and 5%, respectively, as $K_\rho$ increases from 0.15 to 0.45 (the density of the material model ranges from 1,183.5 kg/m³ to 3,550.5 kg/m³). When the elastic modulus, Poisson’s ratio, and density of the material model are respectively fixed to 2.09 GPa, 0.25, and 2,000 kg/m³, the values of $u_1$ and $u_2$ are less than 0.7% and 3%, respectively, as $K_d$ increases from 1 to 4 (the damping of the material model ranges from 0.05 to 0.2).

**Figure 3.** Applied synthetic seismic S wave (20 Hz)

**Figure 4.** Influence of $K_E$

**Figure 5.** Influence of $K_v$
4 Release effect of flexible material parameters on boundary effect

The analysis in Section 3 indicates that under the action of the S wave, the lateral boundary effect of the material model with high elastic modulus is small, whereas that of the model material with low elastic modulus is large and cannot be ignored in the experiment. However, the influence of the Poisson’s ratio, density, and damping on the boundary effect of the model box are relatively small. Therefore, in the analysis of the release effect of different flexible materials on the boundary effect of the model box, a test model with a relatively small elastic modulus was selected, and only the elastic modulus and thickness of the flexible materials were considered (Figure 8). The elastic modulus, Poisson’s ratio, density, and damping of the test model are set to 20.9 MPa, 0.25, 2,000 kg/m³, and 0.05, respectively. Meanwhile, the Poisson’s ratio, density, and damping of the flexible materials are set to 0.45, 600 kg/m³, and 0.05, respectively. The models of the flexible materials are also assumed to be elastic models. The ratio of the elastic modulus of the flexible material to that of the model material is defined as $K_{E*} = \frac{E_{fle}}{E_{mod}}$, where $E_{fle}$ is the elastic modulus of the flexible material. The ratio of the thickness of the flexible material to the length of the material model in the vibration direction is defined as $K_t = \frac{T_{fle}}{L_{mod}}$, where $T_{fle}$ is the thickness of the flexible material and $L_{mod}$ is the length of the material model in the vibration direction.
Figure 8. Diagram of rigid model box with flexible material

Figure 9. Influence of $K_{E^*}$

As shown in Figures 9–10, the elastic modulus and thickness of the flexible material exert a great influence on the boundary effect of the rigid model box. When the thickness of the flexible material is fixed to 0.2 m, the values of $u_1$ and $u_2$ decrease first and then increase with the increase of $K_{E^*}$. When $K_{E^*}$ is equal to 0.05 (the elastic modulus of the flexible material is 1.045 MPa), the boundary effect of the model box is the smallest. When the elastic modulus of the flexible material is fixed to 1.045 MPa, the values of $u_1$ and $u_2$ also show a trend of decreasing first and then increasing as $K_t$ increases. When $K_t$ is 0.067 (the thickness of the flexible material is 0.2 m), the boundary effect of the model box is the smallest. This result shows that reasonable values can be set for the elastic modulus and thickness of flexible materials to minimize the boundary effect of rigid model boxes. Such values should be determined in actual shaking table tests. In this simulation tests herein, the best values of the elastic modulus and thickness of the flexible material are 1.045 MPa and 0.2 m, respectively.

5 Discussion

In this work, the 2-norm method was used to study the influencing factors of the boundary effect of rigid model boxes and the release effect of flexible material parameters. The FLAC numerical
simulation revealed that the ratio of the elastic modulus between the material model and the model box exerts the greatest influence on the model box’s boundary effect while the Poisson’s ratio, density ratio, and damping ratio exert little influence. The elastic modulus and thickness of the flexible material can release the boundary effect, and reasonable values can be set for the elastic modulus and thickness of flexible materials to minimize the boundary effect.

Given the limitation in the number of pages of the manuscript, the research only considered one type of seismic wave frequency (20 Hz) and one type of model box scale. Therefore, the characteristic conditions of seismic waves and the scale conditions of model boxes should be further discussed. In addition, this study mainly involved a numerical simulation, and thus, some deviations with real shaking table test results are inevitable. Readers can refer to this method to prepare experiments, but the results of this work should not be directly used in actual experiments.

6 Conclusions
The changes of the elastic moduli of model materials exert the greatest influence on the boundary effect of model boxes. For a model material with a low elastic modulus, the lateral boundary effect of the model box must be considered during the S wave model test. For a material with a relatively large elastic modulus, the lateral boundary effect of the model box is relatively small. When the material model is not affected by the elastic modulus, the influence of the Poisson’s ratio, density, and damping parameters on the lateral boundary effect of the model box is minimal.

The elastic modulus and thickness of flexible materials greatly influence the boundary effect of rigid model boxes. With the increase of the elastic modulus or the thickness of the flexible material, the boundary effect of the model box shows a trend of decreasing first and then increasing. Reasonable values can be set for the elastic modulus and thickness of flexible materials to minimize the boundary effect of rigid model boxes, and they should be determined in actual shaking table tests.

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