Damping Identification of an Operational Offshore Wind Turbine using Enhanced Kalman filter-based Subspace Identification.

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Abstract

Operational Modal Analysis (OMA) provides essential insights into the structural dynamics of an Offshore Wind Turbine (OWT). In these dynamics, damping is considered an especially important parameter as it governs the magnitude of the response at the natural frequencies. Violation of the stationary white noise excitation requirement of classical OMA algorithms has troubled the identification of operational OWTs due to harmonic excitation caused by rotor rotation. Recently, a novel algorithm was presented that mitigates harmonics by estimating a harmonic subsignal using a Kalman filter and orthogonally removing this signal from the response signal, after which the Stochastic Subspace Identification algorithm is used to identify the system. Although promising results are achieved using this novel algorithm, several shortcomings are still present like the numerical instability of the conventional Kalman filter and the inability to use large or multiple datasets. This paper addresses these shortcomings and applies an enhanced version to a multi-megawatt operational OWT using an economical sensor setup with two accelerometer levels. The algorithm yielded excellent results for the first three tower bending modes with low variance. A comparison of these results against the established time-domain harmonics-mitigating algorithm, Modified LSCE, and the frequency-domain PolyMAX algorithm demonstrated strong agreement in results.

Keywords: Operational Modal Analysis, Stochastic Subspace Identification, Offshore Wind Turbine, Kalman filter, Harmonics, Damping

1. Introduction

Damping identification of operational vibrating structures is commonly achieved by employing Operational Modal Analysis (OMA), a sub-field in system identification that does not require knowledge of the exciting force. A well-known limitation of this practice is the stationary white noise constraint that is placed on the excitation force. Although most excitations can be assumed stationary white noise, such as the traffic passing over a bridge or the wind blowing against it, several applications exist that are also excited by harmonic loading, such as Offshore Wind Turbines (OWTs). These harmonics originate from the rotation of the rotor and their presence renders the application of conventional OMA techniques nontrivial. For instance, structural modes might not be identified due to disturbance of the response signal caused by harmonic loading. Furthermore, harmonics in the response spectrum might be mistaken for structural modes, or inaccuracies might develop in the identification results due to merging of the vibration response data generated by harmonic loading and ambient loading \cite{1}.

Several attempts have been made to develop methods that deal with harmonic presence in OMA. An elementary method involves recognising harmonic components in response spectra as zero-damped modes \cite{2}.

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This technique generally does not apply to OWTs as variations in rotor velocity over a measurement length cause non-stationary harmonic components which are non-zero damped.

The Least-squares Complex Exponential algorithm (LSCE) is a classical time-domain OMA algorithm and was modified by Mohanty et al. (2004a) to incorporate harmonics in the identification steps [1, 2]. This extension was added to other classical algorithms as well [3–5]. The authors caution that exact knowledge of the harmonic frequency is required to use these techniques to obtain accurate results. In continuation, this methodology was also incorporated in a Stochastic Subspace Identification (SSI) approach by Dong et al. (2014) [6].

Besides modified classical algorithms, some preprocessing techniques attempt to filter out harmonics. Time-synchronous averaging (TSA) is a popular technique in gearbox modal analysis [7, 8], where harmonics are generally stationary. For non-stationary harmonics, Cepstrum editing has been proposed as a more suitable technique [9–10]. However, this method requires careful application and can affect the damping [11]. Both preprocessing approaches have been applied to OWTs with inconclusive results. In Manzato et al. (2014a) [11], the frequency content was affected after applying Cepstrum editing, but the TSA approach yielded good results. Contradicting, in Manzato et al. (2014b) [12], the results were not satisfying with TSA, most likely due to non-stationary harmonics, and a slightly different application of Cepstrum yielded good results.

Other techniques involve harmonic localisation utilising statistical indicators, such as the Probability Density Function (PDF) [13], Kurtosis [14, 15] and Entropy [15], whose performance depends on the peakedness of the distribution of the harmonic component.

One of the latest categories of algorithms is independent of the input spectrum. Transmissibility functions contain modal information unaffected by harmonic excitation [16]. However, they are not yet optimal for this application, as important limitations are still present. For instance, several of the approaches require different loading conditions or many output sensors, making applications complicated [16–18]. Other implementations require the acting forces to be uncorrelated [19–21], which can be problematic for identification with correlated harmonics induced by the turbine rotor [22].

Recently, a novel approach called Kalman filter-based Stochastic Subspace Identification (KF-SSI) was developed using the classical SSI algorithm [23] that involves state reconstruction using a Kalman filter and subsequent orthogonal removal of the harmonic subsignal from the original signal [24]. This method was reviewed together with other state-of-the-art OMA algorithms on applicability to operational OWTs in Van Vondelen et al. (2021) [25] and was rated favourably due to its effective harmonic mitigating properties. Although the algorithm in this novel approach is based on the numerically robust and stable SSI algorithm, the combination with the conventional Kalman filter can cause numerical instability, complicating identification [26]. Furthermore, identifying large datasets with SSI entails increasingly large memory requirements and increases computation time notably. The KF-SSI approach is currently lacking the option to split these large datasets into smaller sets or use multiple measured datasets to overcome this issue and increase the identification fidelity.

For identification of OWTs, the installed sensors required by the IEC 61400 International Standard [27] are usually not enough to perform modal analysis. Although it is possible to identify the first-order mode as the most deviation for this mode takes place in the tower top where the IEC 61400 sensors are installed, higher-order modes are much more difficult to identify [28]. Additional equipment is required to also capture higher-order modes [11], but this is often a costly endeavour. Özbeck et al. provide a comprehensive overview of different sensor setups for OMA [29]. In identifying the structural dynamics of a multi-megawatt OWT in this paper, we will attempt to identify higher-order modes in an economical setup, where only two accelerometer levels will be used. This, however, does not allow distinguishing mode shapes, as at least three accelerometers are required.

The contribution of this paper is hence fourfold: 1) enhancing the numerical stability of KF-SSI and incorporating the use of multiple datasets from different periods in time, 2) localising the harmonics of an OWT and coupling this to the KF-SSI algorithm, 3) applying the identification framework to an industrial problem using limited instrumentation and identifying the damping, 4) comparing identification results from well-established OMA methods.

The following section presents the field setup and how data was acquired. Section 3 recalls some theory
2. Acquiring data from an operational wind farm

The field measurements were taken from a 6 MW Siemens Gamesa SWT-6.0-154 offshore wind turbine in the Dudgeon wind farm located in the United Kingdom 32 km off the coast of Cromer, a town in North Norfolk. The site has a water depth between 18 and 25 m, an average wind speed of 9.8 m/s and a mean wave height of 1.1 m. The turbines have monopile foundations, a rotor diameter of 154 m and a hub height of 110 m. The layout of the wind farm is illustrated in Figure 1.

Several turbines in the Dudgeon wind farm are instrumented with 4 accelerometers in addition to the standard sensor setup of the controller. These accelerometers are distributed over 2 levels across the turbine tower (See Figure 2a). The sensors are placed optimally for modal analysis but their locations are limited by the mounting options on the tower. Besides these sensors, the yaw and rotor velocity signals were used for the identification procedure. The yaw signal measures the angle of the rotor around the z-direction. This signal was used to transform the x and y directions of the accelerometers to the rotor coordinate system such that the transformed sensor outputs align with the direction of the Fore-Aft (FA) and Side-Side (SS) bending modes (see Figure 2b for an illustration of the first FA and SS modes). The rotor velocity was used to determine the location of the harmonics.

The datasets were sampled at a rate of 25 Hz, such that vibrations up to 12.5 Hz could be reconstructed. For confidentiality reasons, all results have been normalised to fall between the values 0 and 1. The data was collected in 10-min batches. This dataset length can provide good estimates of the modal parameters. However, in order to enhance identification results even more, it is often desired to concatenate several measurements.

3. Theory of the Enhanced KF-SSI Algorithm

Having acquired the tower vibrations through measurements, the modal parameters defining the structural dynamics will be identified using the Enhanced KF-SSI algorithm. This section presents some of the original theory and extends on this algorithm by proposing enhancements to the original version.
3.1. State-Space Model in the Presence of Harmonics

Herein, a state-space model that includes the influence of the unmeasured harmonics in its model as derived in [24] is presented. First, the general formulation for the system part is given for a Linear Time-Invariant (LTI) system with $m$ degrees of freedom and $r$ measured outputs:

$$\dot{x}_{\text{sys}}(t) = A_{\text{sys}} x_{\text{sys}}(t) + Bu(t) + w(t), \quad (1)$$
$$y(t) = C_{\text{sys}} x_{\text{sys}}(t) + Du(t) + v(t), \quad (2)$$

where $y(t) \in \mathbb{R}^r$ is the output vector, $x(t) \in \mathbb{R}^{2m}$ is the state vector, $A_{\text{sys}} \in \mathbb{R}^{2m \times 2m}$ is the state matrix, $C_{\text{sys}} \in \mathbb{R}^{r \times 2m}$ is the observation matrix, $w(t) \in \mathbb{R}^{2m}$ is the process noise, $v(t) \in \mathbb{R}^r$ is the measurement noise, and $B$ and $D$ are of consistent dimensions and are matrices that determine the effect of the input $u(t)$ on the state and output vector, respectively.

The goal is to formulate the unmeasured harmonics as a periodic forcing input $u(t)$ and then eliminate it from the model by including its effects in the system matrices and state vector. The harmonics are assumed to have the following shape with $h$ frequencies:

$$u(t) = \sum_{i=1}^{h} a_i \sin(\omega_i t + g_i), \quad (3)$$

where $a_i, \omega_i$ and $g_i \in \mathbb{R}$ are the amplitude, circular frequency and shift. The harmonic effects can then be included by defining $u(t) = s_h x_{\text{per}}(t)$, where $s_h = [1 \; 0 \; \ldots \; 1 \; 0] \in \mathbb{R}^{1 \times 2h}$ and:

$$x_{\text{per}}(t) = \begin{bmatrix} a_1 \sin(\omega_1 t + g_1) \\ a_1 \cos(\omega_1 t + g_1) \\ \vdots \\ a_h \sin(\omega_h t + g_h) \\ a_h \cos(\omega_h t + g_h) \end{bmatrix} \in \mathbb{R}^{2h}, \text{ and therefore } \dot{x}_{\text{per}}(t) = \begin{bmatrix} \omega_1 a_1 \cos(\omega_1 t + g_1) \\ -\omega_1 a_1 \sin(\omega_1 t + g_1) \\ \vdots \\ \omega_h a_h \cos(\omega_h t + g_h) \\ -\omega_h a_h \sin(\omega_h t + g_h) \end{bmatrix}, \quad (4)$$

such that $\dot{x}_{\text{per}}(t) = A_{\text{per}} x_{\text{per}}(t)$, \quad (5)
where $A_{\text{per}}^{\text{eq}} = \text{diag}(H_1, \ldots, H_h)$ and $H_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$. Now, by defining matrices $A_{\text{per}}^{B} = B_{\text{sh}} \in \mathbb{R}^{2m \times 2h}$ and $C_{\text{per}} = D_{\text{sh}} \in \mathbb{R}^{r \times 2h}$, a combined state-space model with order $n = 2(m + h)$ is obtained that includes the effects of the unknown harmonic input $u(t)$:

$$
\begin{bmatrix}
    x_{\text{sys}}(t) \\
    x_{\text{per}}(t)
\end{bmatrix} =
\begin{bmatrix}
    A_{\text{sys}} & A_{\text{per}}^{B} \\
    0 & A_{\text{per}}^{C}
\end{bmatrix}
\begin{bmatrix}
    x_{\text{sys}}(t) \\
    x_{\text{per}}(t)
\end{bmatrix}
+ 
\begin{bmatrix}
    w(t) \\
    0
\end{bmatrix},
$$

(6)

$$
y(t) =
\begin{bmatrix}
    C_{\text{sys}} & C_{\text{per}}
\end{bmatrix}
\begin{bmatrix}
    x_{\text{sys}}(t) \\
    x_{\text{per}}(t)
\end{bmatrix}
+ v(t).
$$

(7)

The discrete-time model is obtained by sampling Equations 6 and 7 at instants $t = k\tau$:

$$
\begin{bmatrix}
    A_{\text{sys}} & A_{\text{per}}^{C} \\
    0 & A_{\text{per}}^{C}
\end{bmatrix} =
\exp\left(\begin{bmatrix}
    A_{\text{sys}} & A_{\text{per}}^{B} \\
    0 & A_{\text{per}}^{C}
\end{bmatrix} \tau\right),
$$

(8)

$$
\begin{bmatrix}
    x_{\text{sys}}_{k+1} \\
    x_{\text{per}}_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    A_{\text{sys}} & A_{\text{per}}^{B} \\
    0 & A_{\text{per}}^{C}
\end{bmatrix}
\begin{bmatrix}
    x_{\text{sys}}_{k} \\
    x_{\text{per}}_{k}
\end{bmatrix}
+ 
\begin{bmatrix}
    w_{k} \\
    0
\end{bmatrix},
$$

(9)

$$
y_{k} =
\begin{bmatrix}
    C_{\text{sys}} & C_{\text{per}}
\end{bmatrix}
\begin{bmatrix}
    x_{\text{sys}}_{k} \\
    x_{\text{per}}_{k}
\end{bmatrix}
+ v_{k},
$$

(10)

The process and measurement noise are assumed uncorrelated and have zero mean:

$$
E[w_{k}] = E[v_{k}] = E[w_{k}v_{j}^{T}] = 0,
$$

(11)

with known covariance matrices:

$$
E[w_{j}w_{k}^{T}] = Q_{jk},
E[v_{j}v_{k}^{T}] = R_{jk},
$$

(12)

where,

$$
\delta_{jk} = \begin{cases}
0, & \text{if } k \neq j \\
1, & \text{if } k = j
\end{cases}.
$$

(13)

Here, $w_{k}$ results from the stochastic white noise input to the system. The harmonic is deterministic and hence does not cause process noise in the states $x_{\text{per}}$. The pairs of complex conjugate eigenvalues of the structural part of the system are noted as $\lambda_{\text{sys}}^{i}, \overline{\lambda}_{\text{sys}}^{i}$ for $i = 1, \ldots, m$ and the eigenvalues of the harmonic part of the system are noted as $\lambda_{\text{per}}^{i}, \overline{\lambda}_{\text{per}}^{i}$ for $i = 1, \ldots, h$. Likewise, $\phi_{\text{sys}}^{i}, \overline{\phi}_{\text{sys}}^{i}$ and $\phi_{\text{per}}^{i}, \overline{\phi}_{\text{per}}^{i}$ are the pairs of conjugate complex eigenvectors of the structural and harmonic system, respectively.

3.2. Application of the Square-Root Covariance Filter to Estimate Harmonic Subsignal

The first step in KF-SSI involves the estimation of the harmonic subsignal which is incorporated in the state-space model derived in Section 3.1 to permit high-accuracy estimation of the underlying system dynamics. The conventional Kalman filter has been demonstrated to have numerical stability issues [26], which was confirmed by the authors during application to OWT field data. The Square-Root Covariance filter (SRCF) is therefore proposed in the Enhanced KF-SSI algorithm to overcome this issue. This method uses a numerically stable LQ decomposition, which improves the reliability considerably compared to the conventional version.
The objective of the SRCF is to derive an unbiased and minimum variance estimate $\hat{x}_{k+1}$ of the state, where $(\hat{\cdot})$ denotes an estimate. A non-steady-state filter will be used to account for the correlation between the process noise and measurement noise. The initial condition of the state estimate is taken as $\hat{x}_0 = 0 \in \mathbb{R}^n$, and the initial condition of the error covariance matrix is taken as $P_0 = I_n \in \mathbb{R}^{n \times n}$.

The SRCF uses Cholesky factors of the error covariance matrix $P_{k+1}$ to obtain the filtered states of the combined system $\{A,C,Q,R\}$ of Equations 9 and 10:

$$P_{k+1} = W_k W_k^T, \quad Q = Q^{1/2}[Q^{1/2}]^T, \quad R = R^{1/2}[R^{1/2}]^T,$$

with $W_k$ lower triangular, and $Q^{1/2}$ and $R^{1/2}$ arbitrarily upper or lower triangular. The SRCF method is summarised in the following LQ decomposition (see [30] for a complete derivation):

$$\begin{bmatrix} R^{1/2} & CW_k & 0 \\ 0 & AW_k & Q^{1/2} \end{bmatrix} = L_{sr} Q_{sr}^T = \begin{bmatrix} R_{sr}^{1/2} & 0 \\ G_k & W_{k+1} & 0 \end{bmatrix} \begin{bmatrix} Q_{sr}^T \\ Q_{sr}^T \end{bmatrix},$$

where $Q_{sr}^T$ is an orthogonal matrix and $R_{sr}^{1/2} = R + C P_k C^T$. The following state-space system is found by rewriting the state estimates and system outputs using the innovation signal $e_k = y_k - C \hat{x}_k \in \mathbb{R}^r$ and taking $K_{sr}^* = G_k R_{sr}^{1/2}$:

$$\hat{x}_{k+1} = A \hat{x}_k - G_k R_{sr}^{1/2} (C \hat{x}_k - y_k), \quad y_k = C \hat{x}_k + e_k.$$

As shown below, these states can be transformed by any transformation matrix that is invertable without affecting the outputs, eigenvalues, or mode shapes:

$$\hat{x}_{k+1} = A^V \hat{x}_k + K_{sr}^V e_k, \quad y_k = C^V \hat{x}_k + e_k.$$

Only a transformed version of the original state-space model can be derived from the data. This transformed state-space model does not affect the eigenfrequencies and damping as they are invariant. Consequently, these transformations are included in the state-space model:

$$\hat{x}_{k+1}^V = V^{-1} \hat{x}_k, \quad A^V = V^{-1} AV, \quad C^V = CV, \quad K_{sr}^{V^1} = V^{-1} K_{sr}^r.$$

The transformation matrix is chosen such that the modal basis is obtained [24]. This basis is advantageous because it allows distinguishing harmonic states from structural ones. The transformation matrix is constructed as follows:

$$V = \begin{bmatrix} \text{Re}(\Psi) & \text{Im}(\Psi) \end{bmatrix},$$

where $\Psi = \begin{bmatrix} \phi_1^{\text{sys}} & \cdots & \phi_m^{\text{sys}} & \phi_1^{\text{per}} & \cdots & \phi_h^{\text{per}} \end{bmatrix} \in \mathbb{C}^{2(m+h) \times (m+h)}$. 


Now, let \( \Lambda = \text{diag}(\lambda_{sys}^1, \ldots, \lambda_{sys}^m, \lambda_{per}^1, \ldots, \lambda_{per}^h) \) and \( \Phi = C\Psi = [\varphi_{sys}^1 \ldots \varphi_{sys}^m \varphi_{per}^1 \ldots \varphi_{per}^h] \). By using the eigenvalues and mode shapes, respectively, the real-valued system matrices in modal basis are found:

\[
A^V = \begin{bmatrix} \text{Re}(\Lambda) & \text{Im}(\Lambda) \\ -\text{Im}(\Lambda) & \text{Re}(\Lambda) \end{bmatrix}, \quad C^V = [\text{Re}(\Phi) \quad \text{Im}(\Phi)].
\] (25)

As mentioned, the modal basis allows extracting the entries of \( \hat{x}_V^k \) that belong to the harmonic part. This extraction can, in its turn, be used to estimate the harmonic output signal, which is caused by the unknown harmonic input. For this, a selection matrix is defined:

\[
S = \begin{bmatrix} 0_{m \times m} & I_h \\ I_h & 0_{m \times m} \end{bmatrix}.
\] (26)

The identity matrices, \( I_h \), select the entries of the estimated state vector \( \hat{x}_V^k \) that belong to the harmonic components. Finally, the estimated harmonic output signal is given by:

\[
\hat{y}_{per}^k = C^V S \hat{x}_V^k.
\] (27)

### 3.3. Removal of Harmonic Subsignal

Now, it remains to remove the determined estimate of the harmonic subsignal of Equation 27 from the output data. This removal is done by performing a projection of the raw output data onto the orthogonal complement of the estimated harmonic subsignal. The resulting signal will only contain a representation of the structural system.

Hankel matrices will be used as defined by:

\[
A_{i\mid j} = \begin{bmatrix} a_i & a_{i+1} & \ldots & a_{i+N-1} \\ a_{i+1} & a_{i+2} & \ldots & a_{i+N} \\ \vdots & \vdots & \ddots & \vdots \\ a_j & a_{j+1} & \ldots & a_{j+N-1} \end{bmatrix} \in \mathbb{R}^{(j-i+1)b \times N},
\] (28)

where \( k = i, \ldots, j + N - 1 \) are the samples of the discrete signal \( a_k \in \mathbb{R}^b \) with \( i \leq j \). The Hankel matrices constructed for the output data are ‘past’ and ‘future’ matrices as commonly done in SSI algorithms [31–33]. These matrices contain the output information of the past and future time horizon, respectively. The parameter \( p \) is defined as the length between these two time horizons. These two horizons may have different lengths but have been chosen to be equal for convenience. For the raw output data \( y_k \) of Equation 23, the past and future Hankel matrices become:

\[
Y^-_{raw} = \frac{1}{\sqrt{N}} Y_{0\mid p-1}^-, \quad Y^+_{raw} = \frac{1}{\sqrt{N}} Y_{p\mid 2p-1}^+,
\] (29)

where \((\bullet)^-\) and \((\bullet)^+\) denote the past and future Hankel matrix, respectively. In the same manner, the past and future Hankel matrices of the estimated harmonic subsignal \( \hat{y}_{per}^k \) from Equation 27 are given as:
\[
Y^-_{\text{per}} = \frac{1}{\sqrt{N}} \hat{\phi}^-_{\text{per}, p_{i-1}}, \quad Y^+_{\text{per}} = \frac{1}{\sqrt{N}} \hat{\phi}^+_{\text{per}, p_{i-1}}.
\] (30)

These matrices can directly be used to estimate the observability matrix in the numerically efficient implementation presented in the next section. The complete derivation of this procedure can be found in Greš et al. (2020) [24].

3.4. Numerically Efficient Implementation and Concatenation of Datasets

The KF-SSI algorithm can be implemented in a numerically efficient way such that computation time is reduced and numerical robustness is achieved using an LQ decomposition [24]. Improving identification results can be achieved by using more data. However, multiple distinct datasets cannot simply be assembled, as there will be a state mismatch between the final entry of the first data sequence and the first entry of the second data sequence, which could yield incorrect results. Especially for wind turbines, where the dynamics change with wind speed, it may be required to assemble multiple datasets from distinct periods, where the environmental and operational conditions were found to be similar. A method for achieving this desire is proposed in Theorem 1 by the authors that involves adding datasets in the LQ decomposition step, where no state irregularity arises.

**Theorem 1.** Consider the LQ decomposition of dataset 1

\[
\begin{bmatrix}
Y^-_{\text{per}} \\
Y^+_{\text{per}} \\
Y^-_{\text{raw}} \\
Y^+_{\text{raw}}
\end{bmatrix}_1 = L_1 Q_1^T.
\] (31)

One can concatenate the next dataset 2 horizontally with dataset 1 as long as there is no state-discontinuity.

\[
\begin{bmatrix}
Y^-_{\text{per}} \\
Y^+_{\text{per}} \\
Y^-_{\text{raw}} \\
Y^+_{\text{raw}}
\end{bmatrix}_1 \begin{bmatrix}
Y^-_{\text{per}} \\
Y^+_{\text{per}} \\
Y^-_{\text{raw}} \\
Y^+_{\text{raw}}
\end{bmatrix}_2 = L_2 Q_2^T.
\] (32)

Alternatively, the two datasets can be concatenated irrespective of state-accordance, which yields the same \(L\) matrix but a different \(Q\) matrix:

\[
\begin{bmatrix}
L_1 \\
Y^-_{\text{per}} \\
Y^+_{\text{per}} \\
Y^-_{\text{raw}} \\
Y^+_{\text{raw}}
\end{bmatrix}_2 = L_2 Q_3^T.
\] (33)

**Proof of Theorem 1.** See Appendix A.

The concatenation proposed in Theorem 1 can be iteratively completed for an arbitrary number of datasets \(d = 2, 3, ..., D\):

\[
\begin{bmatrix}
L_{d-1} \\
Y^-_{\text{per}} \\
Y^+_{\text{per}} \\
Y^-_{\text{raw}} \\
Y^+_{\text{raw}}
\end{bmatrix}_d = L_d Q_d^T.
\] (34)

The resulting decomposition is then partitioned as:
\[
\begin{bmatrix}
L_{D-1} & Y_{\text{per}}^- & Y_{\text{per}}^+ \\
Y_{\text{raw}}^- & L_{D-1}D & Y_{\text{raw}}^+
\end{bmatrix} = L_{D}Q_{D}^T = \begin{bmatrix}
L_{11} & 0 & 0 & 0 \\
L_{21} & L_{22} & 0 & 0 \\
L_{31} & L_{32} & L_{33} & 0 \\
L_{41} & L_{42} & L_{43} & L_{44}
\end{bmatrix}
\begin{bmatrix}
Q_{11}^T \\
Q_{21}^T \\
Q_{31}^T \\
Q_{41}^T
\end{bmatrix} = \begin{bmatrix}
L_{12,12} & 0 \\
L_{34,12} & L_{34,34}
\end{bmatrix}
\begin{bmatrix}
Q_{12}^T \\
Q_{34}^T
\end{bmatrix}.
\] (35)

Subsequently, the orthogonally projected matrices are found as:
\[
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{pro}}^+
\end{bmatrix}_D = \begin{bmatrix}
L_{11} & 0 & 0 & 0 \\
L_{21} & L_{22} & 0 & 0 \\
L_{31} & L_{32} & L_{33} & 0 \\
L_{41} & L_{42} & L_{43} & L_{44}
\end{bmatrix}
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{pro}}^+
\end{bmatrix}_D
\]
(36)

Now, inserting Equation 35, this becomes:
\[
\begin{bmatrix}
Y_{\text{pro}}^- \\
Y_{\text{pro}}^+
\end{bmatrix} = \begin{bmatrix}
L_{11} & 0 & 0 & 0 \\
L_{21} & L_{22} & 0 & 0 \\
L_{31} & L_{32} & L_{33} & 0 \\
L_{41} & L_{42} & L_{43} & L_{44}
\end{bmatrix}
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{pro}}^+
\end{bmatrix}_D
\]

The observability matrix \( \Gamma_{\text{sys}} \) relates to matrix \( H \) defined as follows:
\[
H = \frac{Y_{\text{pro}}^+}{Y_{\text{pro}}^-} = \frac{Y_{\text{sys}}^+}{Y_{\text{sys}}^-} + o(1) = \Gamma_{\text{sys}}X_{\text{sys}}^+/Y_{\text{sys}}^- + o(1),
\] (38)

where \( o(1) \) is a term that converges to zero for \( N \to \infty \). For proof of this relation, please read Appendix D in [24]. A numerically efficient way to compute this matrix is finally derived from Equation 37 as follows:
\[
H = \frac{Y_{\text{pro}}^+}{Y_{\text{pro}}^-} = (L_{43,12}Q_{12}^T + L_{34,34}Q_{34}^T)(I - Q_{12}L_{12,12}^{-1}L_{12,12}Q_{12})^{-1}L_{12,12}Q_{12}^T
\]

This efficient implementation enormously decreases the computational cost, as the observability matrix \( \Gamma_{\text{sys}} \) can now directly be estimated using only \( L_{43} \) due to the orthogonality of \( Q_{3} \). Furthermore, the novel data concatenation step allows assembling an arbitrary amount of datasets for identification, increasing accuracy significantly.

3.5. Reducing Estimation Variance using Leave-one-out Cross-Validation

Leave-one-out (LOO) cross-validation is a model validation technique to assess the generalisation of a model to a dataset [34]. Several rounds of cross-validation can be used to reduce the variability of estimates, where a different dataset is ‘left out’ of the concatenation in each identification round. The resulting estimates can then statistically be presented in box plots. Subsequently, the median result can be chosen as the most representative estimate of system parameters based on the entire collated dataset. The procedure is summarised in the following pseudo-code:

4. Localising the Harmonics

Removing harmonics using the KF-SSI algorithm requires knowledge of their frequency location. The Kurtosis indicator, as used in Jacobsen et al. (2007) [35], was initially suggested to distinguish harmonics
Algorithm 1 Variance reduction

1: % Reduce variance in dataset concatenation step
2: Datasets ← {1, 2, ..., D}
3: for d ← Datasets do
4: kcount ← 0
5: for k ← Datasetsk (Datasets ! = d) do
6: kcount ← kcount + 1
7: ybatch(kcount) ← y(k)
8: [frequency(d), damping(d)] = KF-SSI(ybatch)

from structural modes by Greš et al. (2020) [24]. Besides Kurtosis, other approaches will be considered in this section to determine the location of the harmonics, such as the Entropy approach, as used in Agneni et al. (2012) [15]. Finally, the rotor velocity method is proposed by the authors. Note that the rotor velocity method only works for systems where the rotating components causing the disturbing harmonics are measured (i.e., helicopter, OWT). The best-suited approach for an OWT will be selected to localise the harmonics.

4.1. Harmonic Localisation using Statistical Indicators

The first approach type is the use of statistical indicators, Kurtosis and Entropy, which use properties of the distribution of the response signal to determine the location of a harmonic.

The Kurtosis $\gamma$ can be computed at each frequency of the response signal by shifting a narrow bandpass filter over the entire frequency band of interest. This computation results in a value of $\gamma = 3$ for a signal that has a standard normal distribution, and a value of $\gamma \approx 1.5$ for harmonic signals that consist of sinusoids with zero mean $\mu$ and unit variance $\sigma^2$. Based on the difference in value, a distinction can be made between the two types of distributions.

Entropy attains a zero value if and only if the occurrence of a value is certain. Otherwise, it has a positive value. This property can be exploited to distinguish deterministic values, which have a certain occurrence, from random processes. The Entropy value is non-maximum in deterministic components and maximum in regions of stochastic components.

Both statistical indicators have been tested on the simple 3 DoF system illustrated in Figure 3 and a field dataset (Figure 4). Each signal is bandpass filtered by passing a Butterworth filter over the entire frequency range. The vertical dashed red lines indicate where the harmonics are located. The Kurtosis and Entropy statistical indicators are compared for different filter orders. It can be observed that the local minima in the Entropy plot indicate the harmonic locations. However, other local minima can be observed that do not indicate a harmonic, complicating localisation without prior knowledge. In the Kurtosis plot, the results are accurate for all harmonics, as the Kurtosis value attains 1.5 at each harmonic frequency.

![Figure 3: Illustration of the 3 degree-of-freedom system where $m_i$, $k_i$ and $c_i$ for $i = 1, 2, 3$ are the mass, stiffness and damping, respectively.](image)

Remarkably, both indicators show poor performance when applied to field data (Section 2) as can be seen in Figure 5. Here, the 1P, 3P, 9P and 12P harmonics are indicated by the red dashed lines. No consistent correspondence to the expected harmonics is visible in the local minima of the Entropy plots. Likewise for Kurtosis, no attainment of the 1.5 value can be observed at the expected locations of the harmonics.

Poor performance might be attributed to the broader harmonic peaks in the response spectra (see e.g. Figure 6) of field data from an OWT. The peaks are not as sharp as the peaks caused by the ideal harmonics.
in the response of the 3 DoF system, and therefore, a more comprehensive filter might be required to capture the entire harmonic. However, a wider filter could result in less accurate localisation of the harmonic and undermine its purpose. In a subsequent experiment, large filter widths were taken, which did not result in better localisation.

From this analysis, it can be concluded that the statistical indicators are not directly suitable for the localisation of harmonics in the identification problem of the OWT.

4.2. Harmonic Localisation using the Rotor Velocity Signal

In OWT measurements, the rotor velocity is generally available. This signal can be used for harmonic localisation but it is not constant over time. Therefore, the average value over the entire sample can be used to estimate the location of the base frequency of the rotor and its harmonics. This method is expected to work best for approximately stationary rotational velocities, as the harmonic peak will be concentrated around a single frequency value. For less stationary rotational velocities, the peak will be less sharp, and estimation accuracy might deteriorate.

Data from an OWT grid drop event is used to investigate whether this method provides accurate estimations. During a grid drop event, the power of the turbine drops to zero within a few milliseconds, which gives a massive thrust impulse to the OWT and sets up long-lasting tower oscillations. The speed reduction also causes a reduction in damping, leading to longer-lasting oscillations. The turbine then is in ‘idling’ state, where no electricity is generated. Because the turbine tower is still vibrating from the operational
state and is excited by wind and wave loading, the acceleration sensors provide the tower response from these loads only and no longer from the periodic loading caused by the rotor.

When comparing the time series from both states, idling and operational, different response spectra are found. In Figure 6, the red vertical dashed lines indicate the harmonics 1P, 3P, 6P, 9P, and 12P estimated based on the rotational velocity. These lines agree with distinct peaks in the spectrum of the full time series, where the operational state is considered. When inspecting the response spectrum of the idling state, the indicated peaks have vanished, which suggests that they are associated with harmonics. Several modes that were clouded by the harmonics also become visible now.

![Figure 6: Comparison of the response spectra of the tower sensors using idling-only and operational data.](image)

4.3. Conclusion

Based on the above analyses of harmonic localisation methods, the rotor velocity approach is a suitable method for identifying the locations of the harmonics of an operational OWT. Therefore, this method is selected for the KF-SSI algorithm instead of the Kurtosis method, originally suggested in Gres et al. (2020) [24].

5. Application to an Operational Offshore Wind Turbine

The damping and natural frequency of a 6 MW OWT (Section 2) are estimated in this section. There are only two accelerometer levels available at the tower top and tower bottom, which is a minimal setup for higher mode identification. Mode shape estimation is not possible as a minimum of three accelerometer levels is required. Multiple responses are measured for 10 minutes and sampled at 25 Hz for wind speeds ranging between 5 and 26 m/s. At each wind speed, ten datasets were used in this application.

5.1. Single field-measured dataset

First, the Enhanced KF-SSI algorithm is applied to a single dataset at an above-rated wind speed of 13 m/s for the Side-Side direction and compared against classical KF-SSI and SSI. The 3P, 6P and 9P harmonics are removed using the (Enhanced) KF-SSI algorithms. Consequently, stabilisation diagrams are generated and displayed in Figure 7 to allow analysis of the results.

The stabilization diagram provides the analyst with an overview of identified poles for a range of orders, such that an optimal order can be found in a heuristic approach. The power spectrum of the identified signal is plotted and the order is incremented by two with each identification cycle. If the algorithm finds a pole at an identification order that is within a user-defined tolerance of the previous order, an ‘S’ is plotted at the identified natural frequency in the figure on the horizontal line belonging to the order, which is indicated on the right vertical axis. The damping is indicated by the coloured dots at the top of the figure and the harmonics by the red dashed lines.

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An immediate difference can be observed when comparing the power spectra in the stabilisation diagrams of SSI and KF-SSI. Several peaks indicated by the red dashed lines have vanished, which are the spurious harmonics 3P, 6P, and 9P. The 3P harmonic was reasonably spaced away from the first mode and the SSI algorithm was able to identify the first mode properly at several identification orders. However, the harmonic presence can still affect the estimated modal parameters [24]. Removal of the 3P harmonic allows easier identification across multiple orders, as can be seen in the KF-SSI plot.

After removing the 6P harmonic, there is still a peak visible in the spectrum. As no modes are expected to be found in that region, it might an artefact of the removal of the 6P harmonic. Another reason might be that two modes were assigned to the 6P harmonic, while only one was removed. Strikingly, this phenomenon is observed in several datasets for different wind speeds. Further investigation is required to properly assess the origin of this remaining peak.

In the SSI stabilisation diagram, it can be seen that the 9P harmonic is right on top of the second mode, which is usually problematic for damping identification using classical OMA algorithms. However, the SSI algorithm is able to provide estimates. This might be the damping of the 9P harmonic or a mix of the harmonic and structural mode. Removing this harmonic allows easier estimation of the second mode, as can be seen in the KF-SSI stabilisation diagram. Also, due to the harmonic removal, estimates were found
for the third bending mode at around 0.8 Hz.

All three modes were also found by the Enhanced KF-SSI. Note that unlike with regular KF-SSI the edited time signal cannot be reconstructed from the Hankel matrices of the LQ decomposition due to the different $Q$ matrix (Theorem 1). Hence, for illustrative purposes, the power spectrum given in Figure 7c was created by averaging the individual power spectra obtained from each dataset after separate LQ decompositions. The estimated modal parameters are also given in Table 1. A notable difference is found between the estimates of KF-SSI and Enhanced KF-SSI. This difference demonstrates the desire for a method that allows the assembly of multiple datasets, such that estimation variance can be reduced. The next section will cover a more extensive comparison between KF-SSI and Enhanced KF-SSI.

Table 1: Comparison of the modal parameters obtained in the field data experiment for KF-SSI.

| Mode | Natural frequency (Hz) | | Damping $\delta$ (%) | |
|------|------------------------|----------------|-----------------------|----------------|
|      | Classical | KF-SSI | Enhanced | Classical | KF-SSI | Enhanced |
| 1    | .0663         | .0668 | .0667       | .500      | .943   | .431     |
| 2    | .351          | .342  | .354        | .254      | .170   | .297     |
| 3    | -             | .803  | .812        | -         | .243   | .373     |

5.2. Multiple field-measured datasets

A thorough investigation for robustness and accuracy of the Enhanced KF-SSI algorithm can be done by analysing the results for multiple datasets at different wind speeds using both the original and Enhanced KF-SSI. To prevent the requirements of extensive manual analysis, a method was developed to interpret the stabilisation diagrams automatically. After identifying for a range of system orders, all unique poles within a tolerance that occur at least $n$ times are selected. Subsequently, the minimum order is found at which the maximum of these unique poles is identified. This order is then selected as the optimal order. A pseudo-code is given below using the MATLAB functions `ismembertol` and `uniquetol`.

Box plots are constructed for the Side-Side direction for both the KF-SSI and Enhanced KF-SSI algorithms and are displayed in Figure 8. A notable reduction in the spread is visible for the enhanced version compared to the original version, which improves identification precision strongly. The first mode is identified persistently by both algorithms. There is a low variance for the estimated natural frequency and a more extensive variance for the damping estimates. Furthermore, there is an increasing trend visible for the damping of the first mode at higher wind speeds, which can be attributed to the increasing effect of aerodynamic damping for higher wind speeds. For the second mode, the spread is more prominent with the original algorithm at the natural frequencies and damping values. The damping value for the second mode is approximately constant across all wind speeds. Careful manual analysis could improve the results even more, but it is time-consuming as 220 datasets need to be interpreted and processed. The third mode shows an even larger spread, also for the enhanced algorithm. However, this is expected as higher-order modes are more difficult to estimate due to lower energy. Again for the third mode, the damping value remains approximately constant across all wind speeds.

1https://www.mathworks.com/help/matlab/ref/ismembertol.html
2https://www.mathworks.com/help/matlab/ref/uniquetol.html
Algorithm 2 Automatic interpretation Stabilisation diagram

1: % Run OMA algorithm for different orders \( n \) and store found modal parameters 
2: for all \( n \) do 
3: \[ [f_n, d_n] \leftarrow OMA(n) \]
4: \( F(n) \leftarrow f_n \)
5: \( D(n) \leftarrow d_n \)
6: % Find the unique natural frequencies within a tolerance \( tol \)
7: \( F_{\text{unique}} \leftarrow \text{uniquetol}(F, tol) \)
8: % Select unique values that appear at least \( m \) times 
9: \( idx2 \leftarrow [ ] \) % Pre-allocate 
10: for \( i \leftarrow 1 : \text{length}(F_{\text{unique}}) \) do 
11: \( idx \leftarrow \sum(\sum(\text{ismembertol}(F, F_{\text{unique}}, tol))) \)
12: if \( idx \geq m \) then 
13: \( idx2(\text{end} + 1) \leftarrow i \)
14: \( F_m \leftarrow F_{\text{unique}}(idx2) \)
15: % Find the lowest order at which most of these unique values exist 
16: \( \text{order} \leftarrow \max(\sum(\text{ismembertol}(F_m, tol)), 2) \)
17: \( F_{\text{order}} \leftarrow F(\text{order}, :) \)
18: \( D_{\text{order}} \leftarrow D(\text{order}, :) \)
19: % Remove spurious poles 
20: \( idx3 = \text{ismembertol}(F_{\text{stable}}, F_m, tol) \)
21: \( F_{\text{stable}} \leftarrow F_{\text{order}}(idx3) \)
22: \( D_{\text{stable}} \leftarrow D_{\text{order}}(idx3) \)

Field data, Side-Side direction

Figure 8: Comparison of Enhanced KF-SSI against original KF-SSI.
Figure 9: Comparison of Enhanced KF-SSI against PolyMAX and Modified LSCE.

5.3. Comparison against other algorithms

To validate the identification precision, the results obtained using the Enhanced KF-SSI algorithm are compared against two other algorithms. The PolyMAX algorithm is a well-established frequency-domain OMA algorithm due to its fast convergence and clear stabilisation diagrams [36]. It does not employ harmonics-mitigating steps in its identification procedure, and therefore careful selection is required, as harmonics might be identified as false modes. The second algorithm for comparison is the Modified Least-squares Complex Exponential (LSCE) algorithm [1]. This time-domain algorithm incorporates the harmonics in the identification steps and uses the same a priori information as the Enhanced KF-SSI algorithm based on the rotational velocity.

A comparison is made of the median, first and third quantiles of each algorithm on the same datasets as in the previous section. The resulting box plots are shown in Figure 9 and the median values are displayed in Tables 2 and 3. Here, the small spread of the Enhanced KF-SSI compared to other algorithms is especially notable, and the median values correspond well with the other algorithms. More importantly, the overall trend is correctly captured by Enhanced KF-SSI. Another observation is that Enhanced KF-SSI shows persistency in identification for higher-order modes, whereas other algorithms fail to estimate at some wind speeds.

Table 2: Comparison of the median natural frequency estimates of the Enhanced KF-SSI, PolyMAX, and Modified LSCE algorithms at different wind speeds.

| Mode         | Algorithm | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  |
|--------------|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| First        | Enhanced KF-SSI | 0.0669 | 0.0666 | 0.0659 | 0.0668 | 0.0667 | 0.0665 | 0.0660 | 0.0665 | 0.0666 | 0.0666 | 0.0664 | 0.0665 | 0.0664 | 0.0662 | 0.0664 | 0.0663 | 0.0665 | 0.0664 | 0.0665 | 0.0664 | 0.0663 | 0.0664 | 0.0665 | 0.0664 | 0.0663 |
| PolyMAX      |           | 0.0668 | 0.0664 | 0.0667 | 0.0669 | 0.0668 | 0.0667 | 0.0671 | 0.0669 | 0.0666 | 0.0667 | 0.0666 | 0.0670 | 0.0667 | 0.0668 | 0.0665 | 0.0664 | 0.0663 | 0.0665 | 0.0664 | 0.0663 | 0.0664 | 0.0663 | 0.0664 | 0.0663 | 0.0664 | 0.0663 |
| Modified LSCE|           | 0.0672 | 0.0665 | 0.0667 | 0.0668 | 0.0668 | 0.0662 | 0.0670 | 0.0668 | 0.0664 | 0.0665 | 0.0666 | 0.0670 | 0.0667 | 0.0668 | 0.0664 | 0.0663 | 0.0662 | 0.0664 | 0.0663 | 0.0664 | 0.0663 | 0.0664 | 0.0663 | 0.0664 | 0.0663 | 0.0664 |
| Second       | Enhanced KF-SSI | 0.358 | 0.354 | 0.353 | 0.345 | 0.352 | 0.352 | 0.354 | 0.356 | 0.355 | 0.353 | 0.354 | 0.353 | 0.354 | 0.352 | 0.352 | 0.352 | 0.351 | 0.345 | 0.346 | 0.348 | 0.352 | 0.354 | 0.354 | 0.356 | 0.354 | 0.356 | 0.354 | 0.354 | 0.356 |
| PolyMAX      |           | 0.359 | 0.358 | 0.352 | 0.354 | 0.352 | 0.350 | 0.356 | 0.353 | 0.353 | 0.352 | 0.351 | 0.352 | 0.352 | 0.351 | 0.347 | 0.350 | 0.353 | 0.350 | 0.347 | 0.349 | 0.350 | 0.352 | 0.352 | 0.347 | 0.350 | 0.352 | 0.347 | 0.347 | 0.350 |
| Modified LSCE|           | 0.358 | 0.358 | 0.354 | 0.355 | 0.353 | 0.352 | 0.355 | 0.357 | 0.355 | 0.354 | 0.353 | 0.353 | 0.352 | 0.354 | 0.347 | 0.351 | 0.352 | 0.354 | 0.347 | 0.349 | 0.350 | 0.352 | 0.352 | 0.347 | 0.350 | 0.352 | 0.347 | 0.347 | 0.350 |
| Third        | Enhanced KF-SSI | -    | -    | -    | -    | 0.26 | 0.28 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| PolyMAX      |           | -    | -    | -    | -    | -    | -    | 0.38 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| Modified LSCE|           | -    | -    | -    | -    | -    | -    | 0.38 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
Table 3: Comparison of the median damping estimates of the Enhanced KF-SSI, PolyMAX, and Modified LSCE algorithms at different wind speeds.

| Mode   | Algorithm          | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  |
|--------|--------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| First  | Enhanced KF-SSI    | 0.234 | 0.241 | 0.279 | 0.316 | 0.279 | 0.285 | 0.335 | 0.483 | 0.484 | 0.297 | 0.498 | 0.393 | 0.502 | 0.545 | 0.679 | 0.635 | 0.583 | 0.575 | 0.666 | 0.670 |
|        | PolyMAX            | 0.339 | 0.319 | 0.334 | 0.271 | 0.438 | 0.495 | 0.463 | 0.614 | 0.396 | 0.502 | 0.611 | 0.599 | 0.525 | 0.683 | 0.705 | 0.692 | 0.688 | 0.697 | 0.618 | 0.619 |
|        | Modified LSCE      | 0.561 | 0.364 | 0.338 | 0.367 | 0.489 | 0.502 | 0.501 | 0.677 | 0.596 | 0.515 | 0.537 | 0.365 | 0.516 | 0.511 | 0.702 | 0.746 | 0.795 | 0.736 | 0.837 | 0.651 |
| Second | Enhanced KF-SSI    | 0.213 | 0.267 | 0.288 | 0.411 | 0.243 | 0.386 | 0.318 | 0.361 | 0.298 | 0.261 | 0.221 | 0.306 | 0.251 | 0.267 | 0.317 | 0.260 | 0.357 | 0.338 | 0.345 | 0.393 |
|        | PolyMAX            | 0.313 | 0.345 | 0.468 | 0.315 | 0.321 | 0.405 | 0.492 | 0.425 | 0.297 | 0.121 | 0.301 | 0.327 | 0.377 | 0.345 | 0.383 | 0.385 |
|        | Modified LSCE      | 0.210 | 0.341 | 0.203 | 0.278 | 0.267 | 0.300 | 0.303 | 0.320 | 0.223 | 0.229 | 0.351 | 0.411 | 0.258 | 0.364 | 0.311 | 0.349 | 0.467 | 0.361 | 0.431 |
| Third  | Enhanced KF-SSI    | 0.343 | 0.403 | 0.361 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
|        | PolyMAX            | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
|        | Modified LSCE      | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

6. Conclusions

This study enhanced the recently proposed Kalman filter-based Stochastic Subspace Identification (KF-SSI) algorithm. The KF-SSI algorithm mitigates harmonic disturbances and is therefore attractive for application to operational Offshore Wind Turbines (OWTs). Although promising results were shown using this method, the conventional Kalman filter used in the original version was not appropriate due to its numerical instability. Therefore, the authors proposed to instead use the Square-Root Covariance Filter, such that numerical stability is achieved.

Furthermore, the authors enhanced the algorithm such that long or multiple datasets can be used that have been measured at different moments in time. This allows significant enhancements of the estimate precision through the use of more data. By subsequently applying the Leave-One-Out methodology, the median can be taken as a representative modal estimate.

Excellent results were found for the damping and frequency of the first three tower bending modes using an economically attractive setup of only two accelerometer levels. More detailed analysis can be done when more accelerometers are installed on the turbine, such as mode shape estimation. Finally, the results were compared against the established PolyMAX and Modified Least-squares Complex Exponential methods from which it could be deduced that the Enhanced KF-SSI method correctly captured the overall trend.

Additionally, a heuristic method was developed to interpret stabilisation diagrams in an automated fashion, relieving the analyst of tedious manual work. Now, multiple analyses can be done in considerably less time.

Lastly, the (Enhanced) KF-SSI algorithms require a priori information on the harmonic locations before they can be mitigated. It was found that conventional statistical methods, such as the Kurtosis and Entropy, are not suited for OWT datasets given that the harmonics must be ideal. Alternatively, it was proposed to use the rotational velocity to compute the harmonic location.

Appendix A. Proof of Theorem 1

For ease of notation, denote from Equation 31:

\[
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{per}}^+ \\
Y_{\text{raw}}^- \\
Y_{\text{raw}}^+
\end{bmatrix}_d = \begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{per}}^+ \\
Y_{\text{raw}}^- \\
Y_{\text{raw}}^+
\end{bmatrix}.
\]  

\text{(A.1)}

The LQ decomposition of batch 1 is given as follows:

\[
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{raw}}^-
\end{bmatrix}_1 = L_1 Q_1 = \begin{bmatrix} L_{111} & 0 \\ L_{121} & L_{122} \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{12} \end{bmatrix}.
\]  

\text{(A.2)}

Now consider the LQ decomposition of the concatenated batch 1 and batch 2:

\[
\begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{raw}}^-
\end{bmatrix}_1 \begin{bmatrix}
Y_{\text{per}}^- \\
Y_{\text{raw}}^-
\end{bmatrix}_2 = L_2 Q_2 = \begin{bmatrix} L_{211} & 0 \\ L_{221} & L_{222} \end{bmatrix} \begin{bmatrix} Q_{21} \\ Q_{22} \end{bmatrix}.
\]  

\text{(A.3)}
Next, consider the LQ decomposition of batch 1 and batch 2 using instead the \( L_1 \) of the LQ decomposition of batch 1 data (Equation A.2) as follows:

\[
L_1 = \begin{bmatrix}
Y_{\text{per}} \\
Y_{\text{raw}}
\end{bmatrix}
\]

\[
L_2 = \begin{bmatrix}
[ L_{111} & 0 ] \\
L_{121} & L_{122}
\end{bmatrix}
\]

\[
Q_3 = \begin{bmatrix}
0 \\
[ L_{211} & 0 ]
\end{bmatrix}
\]

\[
Q_{12} = \begin{bmatrix}
0 \\
Q_{13}
\end{bmatrix}
\]

(A.4)

Observe that the \( L_2 \) matrix will be the same for both approaches, but the orthogonal \( Q \) matrix will be different. This can be verified by calculating the squares of all LQ decompositions:

\[
L_1 Q_1 Q_1^T L_1^T = L_1 L_1^T = \begin{bmatrix}
L_{111} & 0 \\
L_{121} & L_{122}
\end{bmatrix}
\]

\[
L_1 L_1^T = \begin{bmatrix}
L_{111} & 0 \\
L_{121} & L_{122}
\end{bmatrix}
\]

\[
L_1 L_1^T = \begin{bmatrix}
L_{111} & [ L_{111} L_{112} ] \\
L_{121} & [ L_{112} L_{122} ]
\end{bmatrix}
\]

\[
L_1 L_1^T = \begin{bmatrix}
L_{111} & [ L_{111} L_{112} ] \\
L_{121} & [ L_{112} L_{122} ]
\end{bmatrix}
\]

\[
L_1 L_1^T = \begin{bmatrix}
L_{111} & [ L_{111} L_{112} ] \\
L_{121} & [ L_{112} L_{122} ]
\end{bmatrix}
\]

(A.5)

\[
L_2 Q_2 Q_2^T L_2^T = L_2 L_2^T = \begin{bmatrix}
Y_{\text{per}} \\
Y_{\text{raw}}
\end{bmatrix}
\]

\[
L_2 L_2^T = \begin{bmatrix}
Y_{\text{per}} \\
Y_{\text{raw}}
\end{bmatrix}
\]

\[
Y_{\text{per}} Y_{\text{per}}^T + Y_{\text{raw}} Y_{\text{per}}^T + Y_{\text{per}} Y_{\text{raw}}^T + Y_{\text{per}} Y_{\text{raw}}^T
\]

(A.6)

\[
L_2 Q_3 Q_3^T L_2^T = L_2 L_2^T = \begin{bmatrix}
L_{111} & 0 \\
L_{121} & L_{122}
\end{bmatrix}
\]

\[
L_2 L_2^T = \begin{bmatrix}
L_{111} & 0 \\
L_{121} & L_{122}
\end{bmatrix}
\]

\[
L_2 L_2^T = \begin{bmatrix}
L_{111} & [ L_{111} L_{112} ] \\
L_{121} & [ L_{112} L_{122} ]
\end{bmatrix}
\]

\[
L_2 L_2^T = \begin{bmatrix}
L_{111} & [ L_{111} L_{112} ] \\
L_{121} & [ L_{112} L_{122} ]
\end{bmatrix}
\]

(A.7)

The result of Equation A.5 can now be substituted in the result of Equation A.7 and will yield the same result as Equation A.6.

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