Measuring kinetic energy changes in the mesoscale with low acquisition rates

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(Dated: March 13, 2014)

We describe a new technique to estimate the mean square velocity of a Brownian particle from time series of the position of the particle sampled at frequencies several orders of magnitude smaller than the momentum relaxation frequency. We apply our technique to determine the mean square velocity of single optically trapped polystyrene microspheres immersed in water. The velocity is increased applying a noisy electric field that mimics a higher kinetic temperature. Therefore, we are able to measure the average kinetic energy change in isothermal and non-isothermal quasistatic processes. Moreover, we show that the dependence of the mean square time-averaged velocity on the sampling frequency can be used to quantify properties of the electrophoretic mobility of a charged colloid. Our method could be applied to detect temperature gradients in inhomogeneous media and to characterize the complete thermodynamics of microscopic heat engines.

PACS numbers:

I. INTRODUCTION

Colloidal particles suspended in fluids are subject to thermal fluctuations that produce a random motion of the particle, which was firstly observed by Brown [1] and described by Einstein’s theory [2]. Fast impacts from the molecules of the surrounding liquid induce an erratic motion of the particle, which returns momentum to the fluid at times \( \sim m/\gamma \), with \( m \) the mass of the particle and \( \gamma \) the friction coefficient [3], thus defining an inertial characteristic frequency \( f_p = \gamma/2\pi m \). The momentum relaxation time is of the order of nanoseconds for the case of microscopic dielectric beads immersed in water. Therefore, in order to accurately measure the instantaneous velocity of a Brownian particle, it is necessary to sample the position of the particle with sub-nanometer precision and at a sampling rate above MHz.

Optical tweezers constitute an excellent and versatile tool to manipulate and study the Brownian motion of colloidal particles individually [4,5]. Li et al. [7] were able to measure the instantaneous velocity of micrometer-sized beads of glass held in air by an optical tweezer, using sampling frequencies of the order of MHz. Similar capabilities allowed Huang et al. [8] to observe the transition from ballistic to diffusive Brownian motion in liquids at frequencies larger than \( f_p \). In both [7] and [8], the determination of the velocity of the Brownian particle is performed in equilibrium conditions.

Due to the thermal origin of Brownian motion, it is clear that the temperature plays a key role on it. However, its accurate and fast control in micro-manipulation experiments is difficult, and hence experimental studies of the effects of changing the temperature are scarce [9–12]. A useful strategy to analyze such changes is mimicking the actual temperature by an effective one, related to the motion of the center of mass of the particle. This can be controlled by applying an external force that induces a random motion of the particle with the same characteristics of the Brownian one. The equivalence of the actual temperature and the effective one has been recently demonstrated in Ref. [13], where the external force is controlled by an electric field via the dynamic electrophoretic response of the particle [14].

It would be interesting to find a technique that allows one to estimate the velocity of Brownian particles from data acquired at slower rates than MHz, i.e., with a less demanding technology. Such a technique would provide experimental access to averages of kinetic energy changes in processes that occur at the microscopic scale. Stochastic energetics [15,17], the notion of entropy at small scales [18,19] and fluctuation theorems [20–25] could be described in a more detailed framework where kinetic energy is measurable. A correct energetic description of micro and nano heat engines would only be possible taking into account the kinetic energy changes [12,17,26]. Applications on the microrheology of complex media [27], the study of hydrodynamic interactions between suspended colloids [28] and single colloid electrophoresis [29,33] would also benefit from the experimental evaluation of Brownian motion at short time scales.

In this contribution, we describe a new method to estimate the mean squared instantaneous velocity of a Brownian particle — or equivalently of its kinetic energy — from measurements of the average velocity of the particle at sampling rates far below the momentum relaxation
frequency. We apply our technique to determine experimentally the kinetic energy of an optically-trapped colloidal particle immersed in water whose effective kinetic temperature is controlled by means of a random electric field. We investigate the validity of our technique to ascertain the kinetic energy in equilibrium as well as along isothermal and non-isothermal quasistatic processes. A careful analysis of the experimental results also provides information of the dynamic electrophoretic response of our particle, showing evidences of a low frequency relaxation process.

II. METHOD

A. Experimental setup

Polystyrene beads (G. Kisker-Products fo Biotechnology, PPs-1.0, diameter (1.00 ± 0.05) µm) are diluted in Milli-Q water to a final concentration of a few microspheres per mL. The solution is injected into a custom-made fluid chamber. The chamber is placed in a holder whose position in the three axes can be controlled with picomotors (Newport, 8752). We add two aluminium electrodes at the two ends of the chamber to apply a controllable voltage ($V_T$) to the sample (See 34 for an exhaustive description).

Figure 1 shows a depiction of our optical setup. Based on a horizontal self-built inverted microscope, the sample is illuminated by a white lamp while the image is captured by a CCD camera. An infrared diode laser ($\lambda = 980$ nm, Lumics, 100 mW maximum power) coupled in a single-mode fiber (Avanex, 1998PLM 3CN00472AG HighPower 980 nm) is highly focused by a high NA immersion oil objective $O_1$ (Nikon, CFI PL FL 100× NA 1.30) to create the optical potential. Previously, the optical beam is expanded by lenses $L_1$ (focal length $-30$ cm) and $L_2$ (focal length $20$ cm) to overfill the input pupil of the objective. Laser controller (Arroyo Instruments 4210) allows the management of the optical power at a maximum rate of $250$ kHz using an external voltage $V_\kappa$. Hence, the stiffness of the trap, $\kappa$, can be controlled at the same rate 35.

The particle is tracked using an additional 532 nm laser collimated by a microscope objective ($\times 10$, NA 0.10) and sent through the trapping objective ($O_1$). The light scattered by the trapped object is collected by the objective $O_2$ (Olympus, 40×, NA 0.75) and projected into a quadrant photo detector (QPD, Newfocus 2911). The maximum acquisition frequency of the QPD is $200$ kHz. A 532 nm pass filter (F) blocks additional scattered light. The signal is transferred through an analog-to-digital conversion card (National Instruments PCI-6120) and recorded with LabView software.

Both $V_\kappa$ and $V_T$ are controlled by the same signal generator (Tabor electronics, WW5062) run by Labview software. In the case of $V_T$, the output signal of the signal generator is amplified 1000 times with a high-voltage power amplifier (TREK, 623B).

The potential created by an optical tweezers around its center is quadratic, $U(x) = \frac{1}{2} \kappa x^2$, $x$ being the position of the particle with respect to the trap center and $\kappa$ the stiffness of the trap. The calibration of the nanodetection is obtained from the analysis of the thermal fluctuations of the bead within a static trap. From the study of the power spectral density of the trajectories, both voltage-to-nanometers conversion factor, $S_{QPD}(\text{nm/V})$, and $\kappa$ are obtained 36.

The kinetic temperature is calibrated as described in 13. In equilibrium, the trapped object cannot distinguish between an increase of the medium temperature and an external Gaussian white noise. Hence, we can apply the equipartition theorem $\kappa \langle x^2 \rangle_{ss} = kT_{kin}$, where $\langle x^2 \rangle_{ss}$ is steady-state value of the mean square position, $k$ is the Boltzmann’s constant and $T_{kin}$ is the kinetic temperature of the particle 37. The kinetic temperature of the particle equals to

$$T_{kin} = T + \frac{\sigma^2}{2\gamma k_B},$$

where $T = 300$ K is the temperature of the water and $\sigma^2$ the noise intensity. The input voltage controls the noise intensity and can be linked to the effective temperature as $T_{kin} = T + S_{QPD}V_T^2$, where $S_{QPD}(K/V^2)$ is the calibration factor. All calibrations are repeated each time a new bead is trapped.

In the experiments presented here, $\kappa$ is of the order of tens of pN/µm, and noise amplitudes of the order of thousands of $V$ which led to values of $T_{kin}$ up to thousands of Kelvins. Note that, although we do not know the actual value of the electric field in our chambers, it is not needed for our calculations.

B. Time averaged velocity (TAV)

In order to observe the diffusive behavior of a Brownian particle, the sampling rate of its position, which we denote as $f$, has to be larger than the corner frequency of the trap $f_c = \kappa/2\pi\gamma$. Sampling the position of the particle at frequencies of the order of kHz, or every $\Delta t \sim \text{ms}$, one can observe the Brownian fluctuations of the position in a time series of $n$ data $\{x(i\Delta t)\}^n_{i=0}$. In this situation, one can access to the time averaged velocity (TAV) over the sampling rate $f$,

$$\overline{v_f}(t) = \frac{x(t + \Delta t) - x(t)}{\Delta t} = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} v(s) \, ds,$$

where $v(s)$ is the instantaneous velocity of the particle at time $s$, and $\Delta t = 1/2\pi f$. For a Brownian particle immersed in a fluid at temperature $T_{kin}$, equipartition theorem reads $m\langle v^2 \rangle = kT_{kin}$. Unlike the instantaneous velocity, the TAV does not satisfy equipartition theorem, $m\langle \overline{v_f}^2 \rangle \leq kT_{kin}$, and the bound is saturated in the limit $f \to \infty$. 
The average potential and kinetic energy changes are, by virtue of equipartition theorem, \( \langle \Delta U \rangle = \langle \Delta E_{\text{kin}} \rangle = \frac{k}{2}[T_{\text{kin}}(\tau) - T_{\text{kin}}(0)] \). Finally, the total energy change is \( \langle \Delta E_{\text{tot}} \rangle = \langle \Delta U \rangle + \langle \Delta E_{\text{kin}} \rangle = k[T_{\text{kin}}(\tau) - T_{\text{kin}}(0)] \).

### D. Data analysis

The calculation of thermodynamic parameters as work and heat is performed using experimental data of the position of the sphere \( x(t) \) in the trapping potential \( U(x,t) \). The work done on the bead from \( t \) to \( t + \Delta t \) is given by

\[
\delta W(t) = U[x(t + \Delta t), t + \Delta t] - U[x(t), t],
\]

while the the heat is

\[
\delta Q(t) = U[x(t + \Delta t), t] - U[x(t), t].
\]

These two expressions coincide with Sekimoto’s expressions of stochastic work and heat in the limit of \( \Delta t \) small, \( \delta W_{\text{Seki}}(t) = (\partial U/\partial t)dt \) and \( \delta Q_{\text{Seki}}(t) = (\partial U/\partial x) \circ dx \), where \( \circ \) denotes the product in the Stratonovich sense.

The work and heat transferred to the particle associated to a trajectory \( x_t = \{x(t)\}_{t=0}^\tau \) are calculated by summing the contributions from \( t = 0 \) to \( \tau \), i.e., \( W[x_t] = \sum_{t=0}^\tau \delta W(t) \) and \( Q[x_t] = \sum_{t=0}^\tau \delta Q(t) \).

### III. RESULTS AND DISCUSSION

#### A. Measurement of the TAV

Using the setup described above, we investigate how \( \langle \tau^2 \rangle \) changes with the kinetic temperature for a fixed value of the acquisition frequency, \( f \). In Fig. 2, we show the probability density functions (PDFs) of the position of the particle, obtained for the different values of \( T_{\text{kin}} \) indicated in the caption. In these experiments, we reached a \( T_{\text{kin}} \) close to 6000 K. Each PDF is obtained from a time series sampled at \( f = 5 \) kHz during \( \tau = 12 \) s. The PDFs are fitted to Gaussian distributions whose variances are consistent with the values of \( T_{\text{kin}} \) indicated in the figure. Figure 2 shows the distributions of the TAV for the same values of noise intensity. TAV distributions are also Gaussian and get wider the larger the noise amplitude. The width of the distribution, however, is in all cases significantly smaller than that expected by equipartition theorem \( v_{\text{ET}} = \sqrt{kT/m} \).

We now investigate the dependence of the TAV on \( f \). In Fig. 3, we plot the values of \( m\langle \tau^2 \rangle/kT \) as a function of the sampling rate, obtained from the very same time series but sampled at different frequencies ranging from \( 1 \) kHz to \( 200 \) kHz. In all cases, \( m\langle \tau^2 \rangle \) increases with the noise intensity, but all the curves collapse to the same value for high acquisition rates.

Our starting point to study the origin of the apparent violation of equipartition theorem from measurements of...
Probability density function

0.001
0.01
0.1

$x/x_{ET}$

0.1
1
10

-10 -5 0 5 10

Probability density function

-0.4 -0.2 0.0 0.2 0.4

$\bar{v}/v_{ET}$

-10 -5 0 5 10

FIG. 2: a. Probability density function of the position, $x$, normalized to the standard deviation predicted by equipartition theorem $x_{ET} = \sqrt{kT/m}$. The data were obtained from a single measurement of $\tau = 12s$, sampled at $f = 5$ kHz and for different values of the noise intensity, corresponding to the following kinetic temperatures: $T_{kin} = 300K$ (blue circles), $1060K$ (green squares), $3100K$ (red diamonds), $5680K$ (magenta triangles). b. Probability density function of the TAV (in units of the standard deviation predicted from equipartition, $v_{ET} = \sqrt{kT/m}$) measured from the same time series as those in panel a. Solid lines are fits to Gaussian distributions. In both panels, the arrows point towards larger values of $T_{kin}$. The variance of the TAV, is the underdamped Langevin equation for a Brownian particle trapped in a harmonic potential. The particle is subject both to the thermal and the random electrostatic noise. As shown in the Appendix, the Power Spectral Density (PSD) of the averaged and instantaneous velocity are related to each other through a function of the sampling frequency and the physical parameters of the system (stiffness, friction coefficient and mass of the particle). The variance of the TAV satisfies a modified equipartition theorem,

$$\frac{m\langle \vec{v}^2 \rangle}{kT} = L(f),$$

(8)

where

$$L(f) = 2f^2 \frac{T_{kin}}{T} \left[ \frac{1}{f_0^2} + \frac{e^{-f/f_1}}{f_1} \left( \frac{e^{-f/f}}{f_p + 2f_1} - \frac{e^{f/f}}{f_p - 2f_1} \right) \right],$$

(9)

$f_0 = \sqrt{f_p f_c}$ and $f_1 = \sqrt{f_p^2/4 - f_0^2}$. The function $L(f)$ is thus a measure of the deviation from the equipartition theorem. It satisfies $L(f) < 1$ for any sampling frequency below $f_p$, and its asymptotic behaviour is in accordance with equipartition theorem, i.e., $L(f) \rightarrow 1$ when $f \rightarrow \infty$. Equations (8) and (9) reproduce the observed experimental data without any fitting parameter when $f$ is sufficiently low, as shown by the dashed curves in Fig. 3. However, the measured mean square of the TAV departs from the predicted curves for $f \gtrsim 10$ kHz, except in the absence of electric noise. Our formulas were derived assuming a white spectrum for the random electric force. In the experimental setup, the noise spectrum is indeed flat but only up to some cutoff frequency where it decays to zero very fast. The computation of $\langle \vec{v}^2 \rangle$ can be modified to take into account this cutoff frequency, at least numerically, as shown in the Appendix. Fig. 3 also shows such calculation, using a cutoff frequency of $f_c = 3$ kHz, with excellent agreement between theory and experiment. Therefore, our technique is able to predict the behavior of the standard deviation of the TAV at any accessible value of the sampling frequency.

We measured the cutoff frequency of the amplifier, and

FIG. 3: Average kinetic energy measured from the TAV, $m\langle \vec{v}^2 \rangle/kT$, as a function of the sampling rate. Different symbols indicate values obtained from the same experiments as those shown in Fig. 2. The arrow points towards larger values of $T_{kin}$. The expected theoretical values of $m\langle \vec{v}^2 \rangle/kT$ for the case of an external field described by a Gaussian white noise are plotted with dashed curves. Solid curves are obtained numerically for the case of an external noise with flat spectrum up to a cutoff frequency of $f_c = 3$ kHz. Inset: Real part of the electrophoretic velocity as a function of the frequency $f_{ac}$ of a sinusoidal electric signal of amplitude $1.5V$. The solid line is a guide to the eye.
hence of the generated electric noise, which was found to be \( f_{\text{ac, amplifier}} = 10 \text{kHz} \). This result is in agreement with the specifications of our device, but significantly larger than the cutoff obtained from the fit of the data in Fig. 3. In order to elucidate the origin of this discrepancy, we evaluated the dynamic electrophoretic velocity of the trapped bead as a function of the frequency. For this purpose, we applied a sinusoidal electric field of frequency \( f_{\text{ac}} \) and measured the amplitude of the forced oscillations of the trapped bead [29, 30]. From this, we obtain the dynamic electrophoretic velocity \( \mu^* \), which is proportional to the amplitude of the applied electric field \( E_0 \) through \( \mu^*(f_{\text{ac}}) e^{-2\pi f_{\text{ac}} t} = \mu^*(f_{\text{ac}}) E_0 e^{-2\pi f_{\text{ac}} t} \). Both the electrophoretic velocity and the dynamic electrophoretic mobility \( \mu^*(f_{\text{ac}}) \) are complex quantities that depend on the frequency of the applied field. Moreover, \( \mu^*(f_{\text{ac}}) \) carries all the information about the dielectric relaxation phenomena determining the electrophoretic response. These results are shown in the inset in Fig. 3. There, we see that the electrophoretic response is almost flat up to the kHz region, where a strong decay above \( f_{\text{ac}} \approx 3 \text{kHz} \) is clearly seen. This decay at frequencies below that of the cutoff frequency of the amplifier is due to the well-known alpha or concentration-polarization process, which predicts a relaxation of the mobility with a characteristic frequency \( f_\alpha \approx D^2/4\pi a^2 \), \( D \) being the diffusion coefficient of the counterions in the electric double layer and \( a \) the radius of the bead. Under our experimental conditions, with no added salts in solution, the counterions are protons, and the expected characteristic frequency is \( f_\alpha \approx 3 \text{kHz} \) [14, 59]. This value is in perfect agreement with the cutoff frequency obtained from the temperature measurements and the observed decay in the electrophoretic response.

We can draw two main conclusions. Our results demonstrate that, at sufficiently low sampling rates, we cannot distinguish, neither in the position nor in the velocity of the bead, between the effect of a random force or an actual thermal bath at higher temperature. From this result, we claim that our setup can be used as a simulator of thermodynamic processes at very high temperatures, as we show in the next section.

**B. Energetics including the kinetic energy**

We now investigate the possibility of applying our method to ascertain the average kinetic energy change of a microscopic system in thermodynamic quasistatic processes. We are interested in measuring the kinetic energy change along a process, averaged over many realizations of the process, \( \langle \Delta E_{\text{kin}} \rangle = \frac{1}{N} \langle \frac{d^2}{dt^2} \rangle \). To do so, we first compute the value of the TAV along a trajectory \( x_t \) and then estimate the mean square instantaneous velocity using:

\[
\langle v^2(t) \rangle = L(f)^{-1} \langle \frac{d^2}{dt^2} \rangle. \tag{10}
\]

Equivalently, the average kinetic energy at any time \( t \) in a quasistatic process can be estimated as

\[
\langle \Delta E_{\text{kin}}(t) \rangle = \frac{1}{2} m \langle \frac{d^2}{dt^2} \rangle. \tag{11}
\]

We first implement a non-isothermal process in which the stiffness of the trap is held fixed at \( \kappa = (18.0 \pm 0.2) \text{pN/\pit} \) and the kinetic temperature of the particle is changed linearly with time. We modify the kinetic temperature of the particle from \( T_{\text{kin},1} = 300 \text{K} \) to \( T_{\text{kin},2} = 1300 \text{K} \). The duration of the process is \( \tau = 0.5 \text{s} \) and each process is repeated 900 times. Figure 4 shows the experimental and theoretical values of the averaged cumulative sums of the thermodynamic quantities obtained using an acquisition rate of 1kHz. Since the control parameter does not change, there is no work done on the particle along the process. The potential energy change satisfies equipartition theorem along the process, and both heat and potential energy are equal to

\[
\langle \Delta U_{\text{non-isot}}(t) \rangle = \langle Q_{\text{non-isot}}(t) \rangle = \frac{k}{2} \left[ T_{\text{kin}}(t) - T_{\text{kin}}(0) \right]. \tag{12}
\]

Our measurement of kinetic energy is in accordance with equipartition theorem as well, yielding \( \langle \Delta E_{\text{kin}}(t) \rangle = \frac{k}{2} \left[ T_{\text{kin}}(t) - T_{\text{kin}}(0) \right] \). Adding the kinetic and internal energies, we recover the total energy balance of the particle, \( \langle \Delta E_{\text{tot}}(t) \rangle = k[T_{\text{kin}}(t) - T_{\text{kin}}(0)] \).

In a second application, we realize an isothermal process, where the external noise is switched off (\( T_{\text{kin}} = T \)) and the stiffness of the trap is increased linearly from \( \kappa_1 = (5.0 \pm 0.2) \text{pN/\pit} \) to \( \kappa_2 = (32.0 \pm 0.2) \text{pN/\pit} \) in a process of duration \( \tau = 0.5 \text{s} \). The process is repeated
The potential energy change, $\langle \Delta U(t) \rangle = \langle W(t) \rangle + \langle Q(t) \rangle$ vanishes as expected for the isothermal case. We measure the kinetic energy change from the TAV using Eq. (11).

Our estimation of kinetic energy change vanishes, in accordance with equipartition theorem $\langle \Delta E_{\text{kin}}(t) \rangle = \frac{k}{2} \Delta T_{\text{kin}} = 0$. The variation of the total energy, $\langle \Delta E_{\text{tot}}(t) \rangle = \langle \Delta U(t) \rangle + \langle \Delta E_{\text{kin}}(t) \rangle$ vanishes as well. We remark that despite the TAV is obtained from position data, it captures the distinct behavior of the velocity with respect to the position in the isothermal case, where $\langle x^2(t) \rangle = kT_{\text{kin}}/\kappa(t)$ changes with time whereas $\langle v^2(t) \rangle = kT_{\text{kin}}/m$ does not.

Our technique could be extended straightforwardly to measure the energetic of nonequilibrium processes provided that the velocity degree of freedom relaxes faster than the position degree of freedom. To this extent, our method would be applicable to nonequilibrium processes whose duration $\tau$ is bounded as $\tau_0 \ll \tau \ll \tau_x$, $\tau_x, \tau_0$ being the position and velocity relaxation times, respectively.

IV. CONCLUSIONS

We have developed a method to measure the mean square instantaneous velocity of a Brownian particle from trajectories of its position sampled at low frequencies.

The technique is able to quantify properties of the electrophoretic mobility of the particle in water such as the alpha relaxation frequency. In addition, measuring the time averaged velocity we can estimate the kinetic energy change of the colloid with high accuracy, both in isothermal and non-isothermal quasistatic processes. In the latter case, measuring the heat transferred to the velocity degree of freedom would be of great importance for the design of novel micro and nano heat engines.

V. ACKNOWLEDGMENTS

ER, IM and RR acknowledge financial support from the Fundació Privada Cellex Barcelona, Generalitat de Catalunya grant 2009-SGR-159, and from the Spanish Ministry of Science and Innovation (MICINN FIS2011-24409). LD and ER acknowledge financial support from the Spanish Government (ENFASIS). LD acknowledges financial support from Comunidad de Madrid (MODELIC). We acknowledge enlightening discussions with Antonio Ortiz-Ambriz, Juan M. R. Parrondo and Félix Carrique.

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VI. APPENDIX

A. Derivation of Equations (8) and (9)

We consider a Brownian particle of mass \( m \) trapped with a harmonic potential of stiffness \( \kappa \) in a fluid at temperature \( T \) and with friction coefficient \( \gamma \). The particle moves in one dimension \( x \) and its dynamics is described by the underdamped Langevin equation. The PSD of the velocity in the underdamped limit is

\[
S_v(f') = \frac{\gamma kT}{2\pi m^2} \frac{f'^2}{[(f' - f)^2 + f_p^2 f'^2]}, \tag{14}
\]

where \( f_p = \frac{\gamma}{2\pi m} \) and \( f_0 = \sqrt{f_p\kappa} \). The mean square velocity can be calculated from the PSD

\[
\langle v^2 \rangle = \int_{-\infty}^{\infty} df' S_v(f'), \tag{15}
\]

which leads to the expected result from the equipartition theorem,

\[
\langle v^2 \rangle = \frac{kT}{m}. \tag{16}
\]

The TAV is defined as

\[
\overline{v}(t) = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} v(s) ds, \tag{17}
\]

where \( \Delta t \) is the inverse of the sampling (angular) frequency, \( \Delta f = 1/\omega = 1/2\pi f \). We now study the relationship between the correlation of the time-averaged velocity (TAV) \( \langle \overline{v}^2 \rangle \) and that of the instantaneous velocity, \( \langle v^2 \rangle \), as a function of the sampling frequency \( f \). The Fourier transform of the TAV is

\[
\overline{v}(f') = \frac{1}{\Delta t} \int_{-\infty}^{\infty} dt \int_{t}^{t+\Delta t} v(s) e^{-2\pi i f' t} ds \times v(s) \Pi \left[ \frac{1}{\Delta t} (t + \Delta t/2 - s) \right] e^{-2\pi i f' t} \times \int_{-\infty}^{\infty} dt \Pi \left[ \frac{1}{\Delta t} (t + \Delta t/2 - s) \right] e^{-2\pi i f' (t + \Delta t/2 - s)} \times \int_{-\infty}^{\infty} ds v(s) e^{-2\pi i f' s} e^{\pi i f' \Delta t} \times \int_{-\infty}^{\infty} ds \Pi \left[ \frac{1}{\Delta t} (t + \Delta t/2 - s) \right] e^{-2\pi i f' (t + \Delta t/2 - s)} \times \int_{-\infty}^{\infty} ds v(s) e^{-2\pi i f' s} e^{\pi i f' \Delta t} \int_{-\infty}^{\infty} ds v(s) e^{-2\pi i f' s} = e^{\pi i f' \Delta t} \operatorname{sinc} (f' \Delta t) \int_{-\infty}^{\infty} ds v(s) e^{-2\pi i f' s} = e^{\pi i f' \Delta t} \operatorname{sinc} (f' \Delta t) \overline{v}(f'), \tag{18}
\]

where \( \Pi \) is the rectangular function and \( \operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \).

The PSD of the TAV can be expressed in terms of the PSD of the instantaneous velocity,

\[
S_v(f') = \frac{\gamma kT}{2\pi m^2} S_v(f'), \tag{19}
\]

Therefore, the correlation of the TAV is [cf. Eq. (15)]

\[
\langle \overline{v}^2 \rangle = \int_{-\infty}^{\infty} df' \sin^2 \left( \frac{f'}{2\pi f} \right) S_v(f'). \tag{20}
\]

B. TAV correlation with cutoff in the electric signal

In this section we consider a flat noise PSD \( S_n \) up to a cutoff frequency \( f_c \) when it becomes zero, i.e.,

\[
S_n(f') = \sigma^2 \Pi \left( \frac{f'}{f_c} \right), \tag{22}
\]

where \( \sigma^2 \) is the noise intensity. We first adjust the noise intensity such that it gives the observed value of \( T_e = T_{\text{kin}} - T \) when including the cutoff frequency.

From the underdamped Langevin equation with both the actual thermal bath and the electric noise, we can obtain the spectrum \( S_x(f) \) which has two contributions,

\[
S_x^{\text{tot}}(f') = S_x(f') + S_x^{\text{c}}(f'), \tag{23}
\]

where

\[
S_x(f') = \frac{\gamma kT}{8\pi^4 m^2} \frac{1}{[(f_0^2 - f^2)^2 + f_p^2 f'^2]}, \tag{24}
\]

is the PSD of a Brownian particle in a bath at temperature \( T \) and

\[
S_x^{\text{c}}(f') = \frac{S_n(f')}{16\pi^4 m^2} \frac{1}{[(f_0^2 - f^2)^2 + f_p^2 f'^2]} \tag{25}
\]
the contribution of the electric field. As noted in the main text, in the steady state, fluctuations of $x$ can be used to define a kinetic temperature $T_{\text{kin}}$ which in this case is given by

$$\frac{kT_{\text{kin}}}{\kappa} = \frac{k}{\kappa}(T + T_e) = \langle x \rangle_{\text{ss}} = \int_{-\infty}^{\infty} df' S_x^{\text{tot}}(f')$$

$$= \int_{-\infty}^{\infty} df' S_x(f') + \int_{-\infty}^{\infty} df' S_x(f').$$

(26)

The first integral can be checked to give $kT/\kappa$ yielding for the electric temperature:

$$\frac{kT_e}{\kappa} = \frac{\sigma^2}{16\pi^4} \int_{-f_c}^{f_c} df' \frac{1}{(f_0^2 - f'^2)^2 + f_p^2 f'^2},$$

(27)

where we have substituted the expressions (22) and (25).

The integral

$$I_e(f_c) = \frac{1}{16\pi^4} \int_{-f_c}^{f_c} df' \frac{1}{(f_0^2 - f'^2)^2 + f_p^2 f'^2},$$

(28)

can be evaluated numerically, fixing then the value of noise intensity as a function of experimentally measured values and the cutoff frequency

$$\sigma^2 = \frac{k(T_{\text{kin}} - T)}{\kappa I_e(f_c)}.$$ 

(29)

Once the electric noise spectrum is known, the TAV correlation can be found using expression (20) with the correct PSD for the instantaneous velocity which is now equal to

$$S_v(f') = (2\pi f')^2 S_x^{\text{tot}}(f').$$

(30)

Hence, TAV correlation is finally given by

$$\langle \tau_1^2 \rangle = \frac{2kT_m f^2}{m} \left[ \frac{1}{f_0^2} + \frac{e^{-f_c/f_1}}{f_1} \left( \frac{e^{-f_c/f_1}}{f_p + 2f_1} - \frac{e^{f_c/f_1}}{f_p - 2f_1} \right) \right]$$

$$+ \frac{f^2}{4\pi^4 m^2} \frac{k(T_{\text{kin}} - T)}{\kappa I_e(f_c)} \int_{-f_c}^{f_c} df' \frac{\sin^2(f'/2f)}{[(f_0^2 - f'^2)^2 + f_p^2 f'^2]}.$$ 

(31)

The integral in the last expression has been evaluated numerically and the resulting $\langle \tau_1^2 \rangle$ used to fit the data in Fig.3, being $f_c$ the only fitting parameter.