Abstract It is possible to define and calculate in a gauge-invariant manner the chiral as well as the partial wave content of the quark–antiquark Fock component of a meson in the infrared, where mass is generated. Using the variational method and a set of interpolators that span a complete chiral basis we extract in a lattice QCD Monte Carlo simulation with $n_f = 2$ dynamical light quarks the orbital angular momentum and spin content of the $\rho$-meson. We obtain in the infrared a simple $^3S_1$ component as a leading component of the $\rho$-meson with a small admixture of the $^3D_1$ partial wave, in agreement with the $SU(6)$ flavor–spin symmetry.

1 Introduction

The $SU(6)_{FS}$ flavor–spin symmetry for the low-lying hadrons in the light flavor sector [1] and its roots (as due to the nonrelativistic quark model [2]) predated QCD and had numerous phenomenological successes. When QCD was established as the fundamental theory of strong interactions it has soon become clear that the current quarks of QCD in the $u, d$ sector have tiny masses, of the order of a few MeV at the renormalization scale of 1–2 GeV. They are very far away from the rather heavy constituent quarks of the quark model. In view of this the QCD Lagrangian has approximate chiral symmetry.

At the same time it was clear that this approximate chiral symmetry is dynamically broken in the vacuum and this breaking is a source of mass of the low-lying hadrons. Different microscopical models (with varying definitions of the quark mass) exist for chiral symmetry breaking in the vacuum and they indicate that indeed at large space-like momenta the quark mass matches its bare values of a few MeV, while at low momenta it runs to the value of the constituent quark mass in the definitions given by the models. There also exist lattice determinations of such masses at low momenta, see, e.g., [3,4], though such determinations are manifestly gauge dependent. By fixing a gauge and considering a single quark propagator in the background gluonic field it is not clear a priori how the confining gluodynamics (that drives the dynamics in the color-singlet hadrons) is taken into account. Then it is interesting to see how it would be possible to provide a bridge from QCD in the infrared to the language of the quark model in a model-independent and gauge-invariant manner.

There is a systematic method to study the hadron composition on the lattice—the variational method [5,6]. In that approach one selects a set of interpolating operators \{${O}_1$, ${O}_2$, \ldots , ${O}_N$\} with the proper quantum numbers that couple to a given hadron. One computes the cross-correlation matrix

$$C_{i,j}(t) = \langle O_i(t)O_j^\dagger(0) \rangle. \quad (1)$$

Masses of the ground and excited states of hadrons with fixed quantum numbers can be extracted from the $t$-dependence of the eigenvalues of this matrix at large Euclidean times $t$. If the set of operators $O_i$ is complete
enough, then the eigenvectors represent the “wave function” of the hadron (see the cautionary remarks at the begin of Sect. 4 and the definition in (19)). Of course, hadrons contain many different Fock components. Our task is to reconstruct the leading one, the quark–antiquark component of the low-lying mesons. For this one needs a set of operators that allows one to define uniquely such a component. This set of operators must be complete in the $\bar{q}q$ space with regard to the chiral basis.

All possible $\bar{q}q$ interpolators for non-exotic mesons in the $u, d$ sector have been classified according to transformation properties with respect to $SU(2)_L \times SU(2)_R$ and $U(1)_A$ chiral groups [7,8]. If one assumes that there is no explicit excitation of the gluonic field with the non-vacuum quantum numbers, which is certainly true for low-lying hadrons, then the $SU(2)_L \times SU(2)_R$ representations for the quark–antiquark system specify a complete and orthogonal basis. Consequently, a set of interpolators that is in one-to-one correspondence with all possible chiral representations of $SU(2)_L \times SU(2)_R$ is a complete one and can be used to define the $\bar{q}q$ component of a meson. The cross-correlation matrix with such a set can be used to reconstruct the $\bar{q}q$ Fock component. The eigenvectors of this correlation matrix represent the $\bar{q}q$ content in terms of different chiral representations. Observing a superposition of different chiral representations implies that chiral symmetry is broken.

It turns out that it is also possible to reconstruct a composition of the $\bar{q}q$ component in terms of the $2S+1L_J$ basis, where $J = L + S$ are standard angular momenta. Indeed, the complete and orthogonal chiral basis can be related, through a unitary transformation, to the complete and orthogonal $2S+1L_J$ basis in the center-of-momentum frame [9]. Then diagonalizing the cross-correlation matrix with interpolators that span a complete set of chiral representations and using this unitary transformation to the $2S+1L_J$ basis one can obtain from the eigenvectors of the correlation matrix a partial wave decomposition of the $\bar{q}q$ component.

In QCD the decomposition of a hadron should depend on the scale at which we probe this hadron. In other words, what we see in our microscope depends on the resolution. In our case “the microscope” is our interpolating operator $O_I$ that creates the hadron from the vacuum. A true point-like source would correspond to the point-like lattice interpolator in the limit of the lattice spacing approaching 0, $a \to 0$. The point-like interpolator applied on the lattice with the spacing $a$ probes the hadron at the scale specified by $a$. In the continuum limit it becomes the true point-like operator and probes the hadron at the scale $\mu^2 \to \infty$. Here we want to study the hadron structure at the infrared scale, where the mass is generated. This scale is determined by the hadron size, of the order 0.3–1 fm. In lattice simulations we cannot use such a large $a$, because then the lattice artifacts are too large and matching to continuum theory is lost. However, such a low scale can be fixed by the gauge-invariant smearing of the interpolators. If we use the interpolating operator smeared over the size $R$ in the physical units such that $R/a \gg 1$, then even in the continuum limit $a \to 0$ we probe the hadron structure at the scale $R$. Changing the smearing size $R$ we can study the hadron content at different scales of the continuum theory at $a \to 0$.

In this paper, we expand our results on the chiral and partial wave decomposition of the $\rho$-meson in the infrared, presented in a recent letter [10], discuss all required details of the formalism and physical interpretation as well as give some additional numerical results.

2 Chiral Classification of the Quark–Antiquark Interpolators

The chiral classification of some of the $\bar{q}q$ interpolators was done in [11]. A complete classification was performed in [7,8] and is summarized here.

We consider the two-flavor mesons. All quark–antiquark bilinear operators in the chiral limit can be classified according to the representations of the $SU(2)_L \times SU(2)_R$ and $U(1)_A$ chiral groups. Consider, as example, local interpolators of $J = 0$ mesons, built from quark isodublets $q$:

$$O_{\pi}(x) = i \bar{q}(x) \tau \gamma_5 q(x),$$

$$O_{f_0}(x) = \bar{q}(x) q(x),$$

$$O_{\eta}(x) = i \bar{q}(x) \gamma_5 q(x),$$

$$O_{\eta_0}(x) = \bar{q}(x) \tau q(x),$$

where $\tau$ denotes the vector of isospin Pauli matrices. The $SU(2)_L \times SU(2)_R$ transformations consist of vectorial and axial transformations in the isospin space. The axial transformation mixes the currents of opposite parity:

$$O_\pi(x) \leftrightarrow O_{f_0}(x)$$