THE GALAXY KINEMATICS FROM THE CEPHEIDS WITH THE PROPER MOTIONS FROM THE GAIA DR1 CATALOG

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The sample of classic Cepheids with known distances and line-of-sight velocities is supplemented by the proper motions from the Gaia DR1 catalog. From spatial velocities of 260 stars the components of the peculiar Solar velocity: $(U, V, W)_\odot = (7.90, 11.73, 7.39) \pm (0.65, 0.77, 0.62)$ km s$^{-1}$, parameters of the Galactic rotation curve: $\Omega_0 = 28.84 \pm 0.33$ km s$^{-1}$ kpc$^{-1}$, $\Omega'_0 = -4.05 \pm 0.10$ km s$^{-1}$ kpc$^{-2}$, $\Omega''_0 = 0.805 \pm 0.067$ km s$^{-1}$ kpc$^{-3}$ are obtained. For the adopted Galactocentric Solar distance $R_0 = 8$ kpc the linear circular velocity of the Local Standard of Rest is found as $V_0 = 231 \pm 6$ km s$^{-1}$.

Key words: classic Cepheids, proper motions, Gaia DR1, the Galaxy kinematics.

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INTRODUCTION

Cepheids are the most important data source for studying structure and kinematics of the Galaxy. Thanks to the reliably established “period-luminosity” relation, the distances to the Cepheids can be determined with high accuracy (the error is of about 10%) (Berdnikov, et al., 2000; Sandage & Tammann, 2006). Using distances in combination with high-precision proper motions of about 220 classic Cepheids from the HIPPARCOS catalog (1997) the Galaxy rotation parameters (Feast & Whitelock, 1997; Mel’nik, et al., 2015), the parameters of the spiral structure (Mel’nik, et al., 1999; Bobylev & Bajkova, 2012; Dambis, et al., 2015) and the parameters of the warp of the Galactic disk (Bobylev, 2013a; 2013b) were revised. From data on older Cepheids of II type the parameters of the central bulge and the distance to the Galactic center (Majaess, et al., 2009) were re-determined.

On 14 September 2016 the Gaia DR1 catalog was published. This catalog was created as a combination of Tycho-2 catalog (Høg, et al., 2000) with observational data of a space satellite Gaia (Prusti, et al., 2016; Brown, et al., 2016) received during the first year of operation. This version is designated as TGAS (Tycho–Gaia Astrometric Solution)(Michalik, et al., 2015; Brown, et al., 2016; Lindegren, et al., 2016). This catalog contains about 2 million brightest stars (up to ∼11.′′5). Random errors of the parameters included in the Gaia DR1 catalog are comparable or less than those given in the HIPPARCOS and Tycho-2 catalogs. An average error of the parallaxes is about 0.3 mas (milliarcseconds). This means that the distances with errors of about 10% cover the Solar neighborhood with radius of about 300 pc. However, in case of Cepheids the situation may be more favorable. In the opinion of Casertano, et al. (2016), who performed the analysis of the distances of 212 nearby (r < 2 kpc) Cepheids from the Gaia DR1 catalog, the errors of their trigonometric parallaxes are possibly overestimated by 20%.

At present for studying the structure and kinematics of the Galaxy at large distances from the Sun (3 kpc, and more) the use of the distance scale of classic Cepheids based on the relation “period–luminosity” remains as a topical problem. But the situation with proper motions is different. For the majority part of the stars of the TGAS version, the average error of proper motions is about 1 mas yr$^{-1}$ (milliarcseconds per year), but for (∼94000) stars, common with the HIPPARCOS catalog, this error is much smaller, being about 0.06 mas/yr (Brown, et al., 2016). So, the kinematic analysis of these stars with high-precision proper motions is of great interest.

The aim of this work is the revision of the Galactic rotation parameters on the basis of the Cepheids with the proper motions from the Gaia DR1 catalog. For this we analyze both full spatial velocities and velocities calculated from only proper motions.

METHOD

From observations we know three projections of the stellar velocity: the line-of-sight velocity $V_r$ as well as the two velocities $V_l = 4.74 r \mu_l \cos b$ and $V_b = 4.74 r \mu_b$ directed along the Galactic longitude $l$ and latitude $b$ and expressed in km s$^{-1}$. Here, the coefficient 4.74 is the ratio of the number of kilometers in an astronomical unit to the number of seconds in a tropical year, and $r$ is the heliocentric distance of the star $r$ in kpc. The proper motion components $\mu_l \cos b$ and $\mu_b$ are expressed in milliarcseconds per year (mas yr$^{-1}$). The velocities $U$, $V$, and $W$ directed along the rectangular Galactic coordinate axes are calculated via the components
\[ V_r, V_l, \text{ and } V_b : \]
\[
U = V_r \cos l \cos b - V_l \sin l - V_b \cos l \sin b, \\
V = V_r \sin l \cos b + V_l \cos l - V_b \sin l \sin b, \\
W = V_r \sin b + V_b \cos b, \\
\]
where \( U \) is directed from the Sun to the Galactic center, \( V \) is in the direction of Galactic rotation, and \( W \) is directed toward the north Galactic pole. We can find two velocities, \( V_R \) directed radially away from the Galactic center and \( V_{circ} \) orthogonal to it and pointing in the direction of Galactic rotation, based on the following relations:
\[
V_{circ} = U \sin \theta + (V_0 + V) \cos \theta, \\
V_R = -U \cos \theta + (V_0 + V) \sin \theta, \\
\]
where the position angle \( \theta \) is calculated as \( \tan \theta = y/(R_0 - x) \), while \( x \) and \( y \) are the rectangular Galactic coordinates of the star (velocities \( U, V, W \) are directed along the respective axes \( x, y, z \)).

To determine the parameters of the Galactic rotation curve, we use the equations derived from Bottlinger’s formulas in which the angular velocity \( \Omega \) was expanded in a series to terms of the second order of smallness in \( r/R_0 \):
\[
V_r = -U_\odot \cos b \cos l - V_\odot \cos b \sin l \\
- W_\odot \sin b + R_0 (R - R_0) \sin l \cos b \Omega'_0 \\
+ 0.5 R_0 (R - R_0)^2 \sin l \cos b \Omega''_0, \\
\]
\[
V_l = U_\odot \sin l - V_\odot \cos l - r \Omega_0 \cos b \\
+ (R - R_0)(R_0 \cos l - r \cos b) \Omega'_0 \\
+ 0.5 (R - R_0)^2 (R_0 \cos l - r \cos b) \Omega''_0, \\
\]
\[
V_b = U_\odot \cos l \sin b + V_\odot \sin l \sin b \\
- W_\odot \cos b - R_0 (R - R_0) \sin l \sin b \Omega'_0 \\
- 0.5 R_0 (R - R_0)^2 \sin l \sin b \Omega''_0, \\
\]
where \( R \) is the distance from the star to the Galactic rotation axis,
\[
R^2 = r^2 \cos^2 b - 2 R_0 r \cos b \cos l + R_0^2, \\
\]
\( \Omega_0 \) is the angular velocity of Galactic rotation at the Galactocentric Solar distance \( R_0 \), the parameters \( \Omega'_0 \) and \( \Omega''_0 \) are the corresponding derivatives of the angular velocity, and \( V_0 = |R_0 \Omega_0| \).

It is necessary to adopt a certain value of the distance \( R_0 \). One of the most reliable estimates of this value \( R_0 = 8.28 \pm 0.29 \) kpc obtained by Gillessen, et al. (2009) from the analysis of the orbits of the stars, moving around the supermassive black hole at the center of the Galaxy. From masers with trigonometric parallaxes Reid, et al. (2014) found \( R_0 = 8.34 \pm 0.16 \) kpc. From analysis of kinematics of the masers Bobylev & Bajkova (2014) estimated \( R_0 = 8.3 \pm 0.2 \) kpc, in work by Bajkova & Bobylev (2015) it is found \( R_0 = 8.03 \pm 0.12 \) kpc, Rastorguev, et al.(2016) obtained \( R_0 = 8.40 \pm 0.12 \) kpc. Recent analysis of the orbits of stars moving around the supermassive black hole at the center of the Galaxy gave an estimate \( R_0 = 7.86 \pm 0.2 \) kpc (Boehle, et al., 2016). In the present work the adopted value \( R_0 = 8.0 \pm 0.2 \) kpc.
**DATA**

In the present work the catalog of classic Cepheids, described by Mel’nik, et al. (2015), is used as a basis. This catalog contains 674 stars with the estimates of their heliocentric distances. For a large number of stars the system heliocentric line-of-sight velocities and proper motions are specified. The distances to the Cepheids were determined on the basis of the “period–luminosity” relation using the modern calibrations obtained from the infrared photometric observational data. The original version of the catalog contains the proper motion from the HIPPARCOS catalog (1997), which were taken from the version revised by van Leeuwen (2007).

Note that in the process of the identification of the catalogs, the indexes from the HIPPARCOS catalog were added to about a dozen of the Cepheids from the catalog of Mel’nik, et al. (2015). The line-of-sight velocities were added to two dozens of the Cepheids found using the database SIMBAD (http://simbad.u-strasbg.fr/simbad/).

In the catalog of Mel’nik, et al. (2015) the distances are known for all Cepheids. We identified 290 Cepheids with the proper motions from the Gaia DR1 catalog. 249 stars of them have the system line-of-sight velocities. All of this allows to calculate full spatial velocities of the stars and analyze their three-dimensional movement. To estimate the individual age of the Cepheids (\( t \)) from the period of the pulsations (\( P \)) we used the calibration of Efremov (2003) \( \log t = 8.5 - 0.65 \log P \), received by him from the Cepheids belonging to the open star clusters of the Large Magellanic Cloud.

As it is noted in the work of Mel’nik, et al. (2015) the Cepheid distance scale used in the catalog was built for the Galactocentric distance of the Sun \( R_0 = 7.1 \) kpc. We have adopted the value \( R_0 = 8.0 \) kpc. Therefore, for obtaining self-consistent solutions the distances of all the Cepheids were multiplied by factor of \( 8.0/7.1 = 1.127 \).

**RESULTS AND DISCUSSION**

The system of conditional equations (3)–(5) was solved by the least squares method with the following weights: 
\[
\begin{align*}
    w_r &= \frac{S_0}{\sqrt{S_0^2 + \sigma_{V_r}^2}}, \\
    w_l &= \frac{S_0}{\sqrt{S_0^2 + \sigma_{V_l}^2}}, \\
    w_b &= \frac{S_0}{\sqrt{S_0^2 + \sigma_{V_b}^2}},
\end{align*}
\]
where \( S_0 \) is the “cosmic” variance, \( \sigma_{V_r}, \sigma_{V_l}, \sigma_{V_b} \) are the error variances of respective observed
velocities. The value of $S_0$ is comparable to the standard root mean square residual $\sigma_0$ (unit weight error) in the solution of conditional equations (3)–(5). The value of this quantity depends on the age of Cepheids and lies in the interval from 10 km/s for the youngest to 14 km/s for the oldest ones. In the present work the value $S_0 = 14$ km/s is adopted. It was assumed that the error of the distances to the Cepheids is 10%.

A total number of the Cepheids with distances, line-of-sight velocities and proper motions from the Gaia DR1 catalog is equal to 249. In Fig. 1 the residual $UV$-, $UW$- and $VW$-velocities of all 249 stars without any exceptions are shown. From the figure we can see that there are a few Cepheids at high velocities, especially on the $UW$ plane. These stars can spoil a solution, so they should be excluded from the consideration. In total there are 239 Cepheids, which satisfy the following restrictions:

$$\sqrt{U^2 + V^2 + W^2} < 120 \text{ km s}^{-1},$$
$$|W| < 60 \text{ km s}^{-1},$$

where the velocities $U, V, W$ are residual, they are exempt from the differential rotation of the Galaxy using previously found rotation parameters (or already known). Note that the equations (3)–(5) we solved in two iteration with the exception of stars with large residuals in accordance with the $3\sigma$ criterion. In addition, we used 20 Cepheids for which only the proper motion from the Gaia DR1 catalog are available. These stars give the equations of the form (3)–(4).

Solving the conditional equations (3)–(5) for 260 Cepheids gave the following estimates for the kinematic parameters:

$$(U_\odot, V_\odot, W_\odot) =
(7.90, 11.73, 7.39) \pm (0.65, 0.77, 0.62) \text{ km s}^{-1},$$
$$\Omega_0 = 28.84 \pm 0.33 \text{ km s}^{-1} \text{ kpc}^{-1},$$
$$\Omega'_0 = -4.05 \pm 0.10 \text{ km s}^{-1} \text{ kpc}^{-2},$$
$$\Omega''_0 = 0.805 \pm 0.067 \text{ km s}^{-1} \text{ kpc}^{-3}.$$  

In this solution, unit weight error $\sigma_0 = 10.22$ km s$^{-1}$. The rotation velocity of the Local Standard of Rest $V_0 = 231 \pm 6$ km s$^{-1}$ (for an adopted value $R_0 = 8.0 \pm 0.2$ kpc), and the Oort constants: $A = -16.20 \pm 0.38$ km s$^{-1}$ kpc$^{-1}$ and $B = 12.64 \pm 0.51$ km s$^{-1}$ kpc$^{-1}$. In table 1 the kinematic parameters found by this method for three samples of different age are given.

In Fig. 2 the circular rotation velocities $V_{circ}$ versus distance $R$ for three samples of Cepheids of different age groups are given. On each panel of this figure the same rotation curve in accordance with the solution of (8) is shown. From the figure we can see that the Cepheids of different age groups have small deviations from the specified rotation curve, that is, they similarly take part in the Galactic rotation. This conclusion is also confirmed by the closeness of the found values of $\Omega_0, \Omega'_0, \Omega''_0$, specified in table 1. On the other hand, in Fig. 2 the velocities $V_{circ}$ do not show visible periodic variations caused by the Galactic spiral density wave.

In Fig. 3 the radial velocities $V_R$, versus distance $R$, for three samples of Cepheids of different age groups are shown. In the velocities of young Cepheids (panel a)), periodic variations caused by the Galactic spiral density wave are clearly visible. There is also a noticeable gradient of ages across the Carina-Sagittarius spiral arm ($R \approx 7$ kpc): old Cepheids are located closer to the Solar circle, while the younger Cepheids are farther from the Solar circle (closer to the center of the Galaxy).
Table 1: Kinematic parameters found for three samples of Cepheids of different age groups using full spatial velocities

| Parameters | $\mathcal{T} = 55$ Myr $P > 9^d$ | $\mathcal{T} = 95$ Myr $5^d < P \leq 9^d$ | $\mathcal{T} = 132$ Myr $P < 5^d$ |
|------------|----------------------------------|-----------------------------------|----------------------------------|
| $U_\odot$, km s$^{-1}$ | $5.7 \pm 1.2$ | $8.2 \pm 1.0$ | $9.9 \pm 1.2$ |
| $V_\odot$, km s$^{-1}$ | $13.8 \pm 1.6$ | $11.3 \pm 1.2$ | $11.2 \pm 1.5$ |
| $W_\odot$, km s$^{-1}$ | $6.6 \pm 1.2$ | $7.5 \pm 1.0$ | $7.9 \pm 1.1$ |
| $\Omega_0$, km s$^{-1}$ kpc$^{-1}$ | $28.43 \pm 0.49$ | $28.85 \pm 0.55$ | $29.44 \pm 0.83$ |
| $\Omega'_0$, km s$^{-1}$ kpc$^{-2}$ | $-4.10 \pm 0.15$ | $-4.11 \pm 0.16$ | $-4.21 \pm 0.24$ |
| $\Omega''_0$, km s$^{-1}$ kpc$^{-3}$ | $0.909 \pm 0.116$ | $0.795 \pm 0.106$ | $0.890 \pm 0.154$ |
| $\sigma_0$, km s$^{-1}$ | $10.2$ | $9.9$ | $10.6$ |
| $N_*$ | $75$ | $99$ | $86$ |
| $A$, km s$^{-1}$ kpc$^{-1}$ | $-16.40 \pm 0.58$ | $-16.45 \pm 0.63$ | $-16.84 \pm 0.97$ |
| $B$, km s$^{-1}$ kpc$^{-1}$ | $12.03 \pm 0.76$ | $12.40 \pm 0.83$ | $12.60 \pm 1.28$ |

It is interesting to compare the proper motion from the HIPPARCOS and Gaia DR1 catalogs. Following the Feast & Whitelock (1997), let’s rewrite equation (4) in the following way

$$V_l/r = (U_\odot \sin l - V_\odot \cos l)/r + (2A \cos^2 l - \Omega_0) \cos b,$$

where $A = 0.5\Omega''_0 R_0$ is the Oort constant.

Moving components of the Solar peculiar velocity to the left part of equation (9), we obtain, in the left part, the velocity, freed from the Solar peculiar velocity, and in the right part — members describing the rotation of a Galaxy:

$$V^* = [V_l - (U_\odot \sin l - V_\odot \cos l)]/r =$$

$$= (2A \cos^2 l - \Omega_0) \cos b.$$  \hspace{1cm} (10)

The velocity $V^*$ has the dimension of angular velocity, km s$^{-1}$ kpc$^{-1}$. As it was shown by Feast & Whitelock (1997) when analyzing Cepheids, the dependence of the velocity $V^*$ on the longitude $l$ is a good illustration of the Galactic rotation.

In Fig. 4 the velocities $V^*$ versus Galactic longitude $l$ for each of the two proper motions, taken from the HIPPARCOS and Gaia DR1 catalogues, are shown in panels a) and b) respectively. In both panels the sine waves, reflecting the effect of the rotation of the Galaxy, are perfectly visible. Meanwhile, the points on panel b) have lower scatter relative to the sine wave as compared to the points at panel a). So we can conclude that the proper motions of the Cepheids from the Gaia DR1 catalog are rather than the proper motions of the same stars from the HIPPARCOS catalog.

In table 2 the kinematic parameters, for three samples of Cepheids of different age groups, found from only spatial velocities $V_l$ are specified. These velocities were calculated using the proper motions taken from the Gaia DR1 catalog. To find the kinematic parameters the system of conditional equations (4) with five unknowns was solved using the least squares method. The Solar peculiar velocity component $W_\odot$ was fixed as $W_\odot = 7$ km/s, because it can not be determined only from equations of type (4). Two constraints (7) were used.
Figure 2: Circular rotation velocities $V_{\text{circ}}$ versus distance $R$ (from the Cepheid to the Galactic rotation axis) of young Cepheids a), middle-aged Cepheids b), old Cepheids c); thick line represents the rotation curve in accordance with the solution (11), thin lines mark the boundaries of the confidence regions corresponding to a level of $1\sigma$.

From 260 Cepheids we found the following estimates of the kinematic parameters:

$$
(U_\odot, V_\odot) = (8.38, 7.63) \pm (0.89, 1.43) \text{ km s}^{-1}, \\
\Omega_0 = 29.04 \pm 0.71 \text{ km s}^{-1} \text{kpc}^{-1}, \\
\Omega_0' = -4.05 \pm 0.18 \text{ km s}^{-1} \text{kpc}^{-2}, \\
\Omega_0'' = 0.778 \pm 0.117 \text{ km s}^{-1} \text{kpc}^{-3}.
$$

In this solution, unit weight error $\sigma_0 = 10.84 \text{ km s}^{-1}$, the Oort constants: $A = -16.20 \pm 0.71 \text{ km s}^{-1} \text{kpc}^{-1}$ and $B = 12.84 \pm 1.00 \text{ km s}^{-1} \text{kpc}^{-1}$, the rotation velocity of the Galaxy $V_0 = 232 \pm 8 \text{ km/s}$ for adopted value $R_0 = 8.0 \pm 0.2 \text{kpc}$.

It is interesting to compare our solution (11) and the parameters listed in table 2 with the results of Feast & Whitelock (1997) obtained using the same approach and the proper motions from the HIPPARCOS catalog: $(U_\odot, V_\odot) = (9.3, 11.2) \pm (1.5, 1.5) \text{ km s}^{-1}, \\
\Omega_0 = 27.19 \pm 0.87 \text{ km s}^{-1} \text{kpc}^{-1}, \\
A = -14.82 \pm 0.84 \text{ km s}^{-1} \text{kpc}^{-1}$ and $B = 12.37 \pm 0.64 \text{ km s}^{-1} \text{kpc}^{-1}$. We can see that the most noticeable differences there are in the estimates of the velocities $U_\odot$ and especially $V_\odot$. On the other hand, the results of Feast & Whitelock (1997)
are in good agreement with solution (S), but errors of parameters $\Omega_0$, $A$ and $B$ in (S) are two times smaller.

From the analysis of the spatial velocities of 257 Cepheids with proper motions from the HIPPARCOS catalog Mel’nik, et al. (2015) found the following solution: $(U_\odot, V_\odot) = (8.1, 12.7) \pm (0.8, 1.0)$ km s$^{-1}$, $\Omega_0 = 28.8 \pm 0.8$ km s$^{-1}$ kpc$^{-1}$, $\Omega'_0 = -4.88 \pm 0.14$ km s$^{-1}$ kpc$^{-2}$, $\Omega''_0 = 1.07 \pm 0.17$ km s$^{-1}$ kpc$^{-3}$, $\sigma_0 = 10.84$ km s$^{-1}$ kpc$^{-1}$, $A = -18.3 \pm 0.6$ km s$^{-1}$ kpc$^{-1}$. All these values are in good agreement with solution (S).

The most reliable estimates of the Galactic rotation parameters were obtained by Rastorguev, et al. (2016) from data on 130 masers with trigonometric parallaxes. For instance, the values for two components of the peculiar velocity of the Sun are: $(U_\odot, V_\odot) = (11.40, 17.23) \pm (1.33, 1.09)$ km s$^{-1}$, which are intrinsic for very young stars, and the following parameters for the rotation curve of the Galaxy: $\Omega_0 = 28.93 \pm 0.53$ km s$^{-1}$ kpc$^{-1}$, $\Omega'_0 = -3.96 \pm 0.07$ km s$^{-1}$ kpc$^{-2}$ and $\Omega''_0 = 0.87 \pm 0.03$ km s$^{-1}$ kpc$^{-3}$, the velocity of the Local Standard of Rest $V_0 = 243 \pm 10$ km s$^{-1}$ for $R_0 = 8.40 \pm 0.12$ kpc. As it is seen, these results and the solution (S) are in good agreement.
CONCLUSIONS

The classic Cepheids with the proper motions contained in the Gaia DR1 catalog are considered. We used the distances to the Cepheids found by Mel’nik, et al. (2015) on the basis of the “period–luminosity” relation using photometric data in infrared band.

The sample of the 239 Cepheids for which it is possible to calculate the spatial velocities was formed. In addition, 20 stars without line-of-sight velocities, but with proper motions in the Gaia DR1 catalog were used. From data of all these stars we found components of the peculiar velocity of the Sun \((U_\odot, V_\odot, W_\odot) = (7.90, 11.73, 7.39) \pm (0.65, 0.77, 0.62) \text{ km s}^{-1}\) and the following parameters of the rotation curve of the Galaxy: \(\Omega_0 = 28.84 \pm 0.33 \text{ km s}^{-1} \text{ kpc}^{-1}\), \(\Omega'_0 = -4.05 \pm 0.10 \text{ km s}^{-1} \text{ kpc}^{-2}\) and \(\Omega''_0 = 0.805 \pm 0.067 \text{ km s}^{-1} \text{ kpc}^{-1}\) for the adopted value of the Galactocentric distance of the Sun \(R_0 = 8.0 \pm 0.2 \text{ kpc}\). The circular velocity of the Local Standard of Rest is \(V_0 = 231 \pm 6 \text{ km s}^{-1}\), the Oort constants \(A = -16.20 \pm 0.38 \text{ km s}^{-1} \text{ kpc}^{-1}\) and \(B = 12.64 \pm 0.51 \text{ km s}^{-1} \text{ kpc}^{-1}\).

This sample was divided into three groups depending on the age of the Cepheids. It is shown that: a) there are no significant differences from the age of the groups when we determine the parameters of the Galactic rotation velocity \(\Omega_0, \Omega'_0\) and \(\Omega''_0\), though previously it was mentioned (Bobylev & Bajkova, 2012; Mel’nik, et al. 2015) that the most young Cepheids, by unknown reasons, rotate at a slower velocity around the Galactic center; b) velocities \(U_\odot\) and \(V_\odot\), determined from the sample of young Cepheids, differ significantly from ones obtained from the samples of older Cepheids.
Table 2: Kinematic parameters found for three samples of Cepheids of different age groups using only velocities $V_i$

| Parameters | $\bar{t} = 55$ Myr $P > 9^d$ | $\bar{t} = 95$ Myr $5^d < P \leq 9^d$ | $\bar{t} = 132$ Myr $P < 5^d$ |
|------------|-------------------------------|-------------------------------|-------------------------------|
| $U_\odot$, km s$^{-1}$ | 6.0 ± 1.9 | 7.9 ± 1.4 | 11.2 ± 1.5 |
| $V_\odot$, km s$^{-1}$ | 11.5 ± 3.4 | 7.9 ± 2.3 | 5.8 ± 2.2 |
| $\Omega_0$, km s$^{-1}$ kpc$^{-1}$ | 28.1 ± 1.4 | 28.4 ± 1.1 | 32.5 ± 1.6 |
| $\Omega'_0$, km s$^{-1}$ kpc$^{-2}$ | −4.11 ± 0.32 | −4.23 ± 0.28 | −4.01 ± 0.34 |
| $\Omega''_0$, km s$^{-1}$ kpc$^{-3}$ | 0.844 ± 0.218 | 0.970 ± 0.191 | 0.579 ± 0.210 |
| $\sigma_0$, km s$^{-1}$ | 10.0 | 10.2 | 9.8 |
| $N_\sigma$ | 75 | 99 | 86 |
| $A$, km s$^{-1}$ kpc$^{-1}$ | −16.4 ± 1.3 | −16.9 ± 1.1 | −16.0 ± 1.4 |
| $B$, km s$^{-1}$ kpc$^{-1}$ | 11.7 ± 1.9 | 11.4 ± 1.6 | 16.5 ± 2.1 |

There was also obtained the solution from only the proper motions from the Gaia DR1 catalog: $(U_\odot, V_\odot) = (8.38, 7.63) \pm (0.89, 1.43)$ km s$^{-1}$, $\Omega_0 = 29.04 \pm 0.71$ km s$^{-1}$ kpc$^{-1}$, $\Omega'_0 = −4.05 \pm 0.18$ km s$^{-1}$ kpc$^{-2}$, $\Omega''_0 = 0.778 \pm 0.117$ km s$^{-1}$ kpc$^{-3}$. Here $V_0 = 232 \pm 8$ km s$^{-1}$ for $R_0 = 8.0 \pm 0.2$ kpc, the Oort constants: $A = −16.20 \pm 0.71$ km s$^{-1}$ kpc$^{-1}$ and $B = 12.84 \pm 1.00$ km s$^{-1}$ kpc$^{-1}$. In this case there is a difference between our estimates of $U_\odot$ and $V_\odot$ and the estimates obtained by other authors, but it does not exceed the error level of 3$\sigma$.

The astrometric observations from a space satellite Gaia are continuing. The measurements of the star proper motions and parallaxes will be significantly improved by the end of the mission. Therefore, in the nearest future we can expect significant refinement of the Galaxy kinematic parameters on the basis of new Gaia data.

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