A simple experiment for quantum erasers and geometric phases

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We introduce a simple experiment involving a double-slit interferometer by which one can learn basic concepts of quantum interference such as which-path marking, quantum erasers, and geometric phases. Each of them exhibits seemingly mysterious phenomena in quantum physics. In our experiment, we use the double-slit interference of visible light with the polarization as an internal state to demonstrate the disappearance of fringes by which-path marking, recovery of interference using quantum erasers, and the rapid shifting of the fringe pattern induced by the geometric phase. We also present a simple theoretical analysis of an interferometer with an internal state.

I. INTRODUCTION

In the early 1800s, Thomas Young performed the well-known double-slit interference experiment; this experiment enabled the observation of the interference of light and the confirmation of its wave-like behavior. Over the course of the nineteenth century, physicists discovered that light consists of quantum particles called photons and each photon behaves like both a particle and a wave. This property is called wave-particle duality of light. In addition, the confirmation of its wave-like behavior. Over the course of the nineteenth century, physicists discovered that light consists of quantum particles called photons and each photon behaves like both a particle and a wave. This property is called wave-particle duality of light.

In the double-slit experiment, each photon creates a single spot on the screen according to the probability density, and the spots created by thousands of photons result in a definite interference fringe. This phenomenon is interpreted as the self-interference of the individual photons: a photon can pass through both the slits simultaneously and interfere with itself. The interference pattern is generated due to the superposition of the two path state wavefunctions followed by the photon.

However, assume that there exists a device to “mark” a photon according to the path followed by it. This operation, called as which-path marking, enables the two path states to be distinguished, and this destroys the superposition of the path states. Thus, the photons impacting the screen do not form fringes but only form a broad diffraction pattern.

Surprisingly, although which-path marking eliminates the interference, the interference fringe can be retrieved from the broad diffraction pattern by erasing the which-path information. This idea is called quantum eraser and it was first proposed by Scully and Drühl.

A simple demonstration of the path marking and the quantum eraser using the internal states of the photon can be described as follows. A photon is marked by the right and left circular polarization states according to the paths. Since the which-path information could be obtained by measuring the polarity of the circular polarization, no interference pattern is obtained. However, when a linear polarizer is placed in front of the double slit, the circular polarizations are projected into the same linear polarization and thus the polarization carries no which-path information. Therefore, the interference fringes are recovered.

In addition to the recovery of interference, due to the change in the polarization states, the quantum eraser also induces a displacement of the fringe or an additional phase shift. This phase shift is found to be related to the Pancharatnam phase and it is determined by three polarization states: two states due to the which-path marking and one due to the linear polarizer used for the quantum eraser. It is noteworthy that this Pancharatnam phase is invariant even if a phase shift is introduced in either of the paths (gauge invariance). Such a phase factor is called the geometric phase and it was first reported by Berry. The state space of the polarization can be represented by a sphere called the Poincaré sphere. The Pancharatnam phase is determined by the area of the spherical triangle connecting the three states on the Poincaré sphere.

It is noteworthy that the Pancharatnam phase can be very sensitive to a change in state for a certain arrangement. This phenomenon can be used to induce a large displacement of the wave packet. This is related to the weak measurement that was first proposed by Aharonov, Albert, and Vaidman.

In this study, we introduce a “kitchen-table” experiment for a double-slit interferometer with internal states. This simple setup can be used to confirm some important concepts in quantum mechanics such as which-path marking, quantum erasers, and geometric phases. By using the polarization of the photon as an internal state, we can demonstrate the disappearance of interference by which-path marking, recovery of interference using the quantum eraser, and nonlinear phase shift of the fringe pattern induced by the Pancharatnam phases. A simple theoretical analysis of the interferometer as a composite quantum system can be easily referred by a reader who is familiar with the foundation of quantum mechanics and electromagnetism.

The remainder of this paper is organized as follows. In Sec. III we introduce a theoretical model for a double-slit interferometer with the internal states and analyze the interference pattern in the process of which-path marking and the quantum eraser. Moreover, we confirm the nonlinear variation of the Pancharatnam phase in a certain arrangement. In Sec. III we describe our experimental
setup and results on the quantum eraser and the Pancharatnam phase. A summary is presented in Sec. [LV]

II. THEORETICAL ANALYSIS OF DOUBLE-SLIT INTERFEROMETERS WITH INTERNAL STATES

An interferometer with internal states can be analyzed as a quantum system composed of the path state and the internal state. In this section, we theoretically analyze the interference patterns in our double-slit experiment with attention to both the intensity and the phase.

A. Young’s double-slit interference

In a typical Young’s double-slit interferometer without an internal state, a photon is emitted from a source, passes through the double slit, and is recombined on the screen, as shown in Fig. (1a). The paths from each slit to the observation point on the screen are not equal and therefore a photon passing through each slit acquires different phases. Thus, an interference fringe is formed on the screen according to the phase difference.

Because of the large distance between the double slit and the screen, the state of the photon through slits A and B can be assumed to be an eigenstate of the transverse wave number on the screen, $k_\text{A}$ and $k_\text{B}$, respectively. Assuming that the photon has a 50:50 chance of passing through each slit, the path state of the photon on the screen can be represented as a superposition:

\[ |\Psi\rangle = C\left( |k_\text{A}\rangle + |k_\text{B}\rangle \right), \tag{1} \]

where $C$ is the normalization constant with the dimension $L^{-1/2}$, $L$ being the dimension of length. We introduce the operator $\hat{P}_x \equiv |x\rangle\langle x|$ that projects into the position state $|x\rangle$ on the screen. With the position representation of the wave-number eigenfunction, $\langle x|k\rangle = e^{ikx}/\sqrt{2\pi}$, the probability distribution is given by

\[ P(x) = \langle \Psi | \hat{P}_x | \Psi \rangle = \frac{C^2}{\pi} \left(1 + \cos kx\right), \tag{2} \]

where $k \equiv k_\text{B} - k_\text{A}$. In the double-slit apparatus, $k$ is calculated as $k = 2\pi d/\lambda L$, where $\lambda$ is the wavelength of light; $d$, the distance between two slits; $L$, the distance between the double slit and the screen. The second term in Eq. (2) indicates the interference between two paths.

B. Which-path marking

A photon is marked with the polarization states $|\psi_\text{A}\rangle$ and $|\psi_\text{B}\rangle$ according to the paths using the quarter-wave plates, QWP$_\text{A}$ and QWP$_\text{B}$ (see Fig. (1b)). In this case, the total state vector for the composite system is represented as

\[ |\Psi_m\rangle = C\left( |\psi_\text{A}\rangle|k_\text{A}\rangle + |\psi_\text{B}\rangle|k_\text{B}\rangle \right). \tag{3} \]

Here, the path states and two polarization states are correlated or entangled. Considering that $\hat{P}_x$ operates only on the path states, we have the probability distribution $P_m(x)$ as

\[ P_m(x) = \langle \Psi_m | \hat{P}_x | \Psi_m \rangle = \frac{C^2}{\pi} \left[1 + V_m \cos (kx - \delta_m)\right], \tag{4} \]

where $V_m$ and $\delta_m$ are given as

\[ V_m = \langle |\psi_\text{B}\rangle|\psi_\text{A}\rangle\rangle, \tag{5} \]
\[ \delta_m = \text{arg} (\langle \psi_\text{B}|\psi_\text{A}\rangle). \tag{6} \]

The coefficient of the interference term, $V_m$, can be experimentally obtained from the fringe pattern as

\[ V_m = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}. \tag{7} \]
where $P_{\text{max}}$ and $P_{\text{min}}$ are the maximum and minimum values of $P_m(x)$, respectively. Equation (7) indicates the fringe visibility, which quantifies the contrast of interference fringes if the visibility is unity, the interference is perfect, otherwise it is partial. In particular, zero visibility implies complete disappearance of the fringe.

The degradation of visibility is related to the performance of which-path marking, which depends on the inner product $\langle \psi_A | \psi_B \rangle$. The lesser the value of $| \langle \psi_A | \psi_B \rangle |$, the lesser is the visibility. When $\langle \psi_A | \psi_B \rangle = 0$, two states are perfectly distinguishable and the path followed by the photon is discriminated unambiguously. Therefore, the interference is completely eliminated.

C. Quantum eraser

Now, we erase the which-path information by the projection of polarization using the linear polarizer, LP$_2$, which projects the polarization state into $|\psi\rangle$ (see Fig. 1b). This operation can be represented by $|\psi\rangle \langle \psi|$. The state vector for the composite system after LP$_2$ is calculated as follows:

$$|\Psi_f\rangle = |\psi_2\rangle \langle \psi_2| \Psi_m\rangle = C|\psi_2\rangle \left( c_A |k_A\rangle + c_B |k_B\rangle \right),$$ (8)

where $c_A = \langle \psi_2 | \psi_A \rangle$ and $c_B = \langle \psi_2 | \psi_B \rangle$. Although the path state in Eq. (5) is represented as the superposition of two path states in a manner similar to Eq. (1), the weights and the relative phase between two states are different. The probability distribution $P_f(x)$ is given by

$$P_f(x) = \langle \Psi_f | P_2 | \Psi_f \rangle \propto 1 + V_f \cos (kx - \delta_f),$$ (9)

where the visibility $V_f$ and the phase shift $\delta_f$ are given as

$$V_f = \frac{2|c_A| - |c_B|}{|c_A|^2 + |c_B|^2},$$ (10)

$$\delta_f = \arg \langle \psi_B \rangle \langle \psi_2 \rangle |\psi_2\rangle \langle \psi_2| \psi_A \rangle.$$ (11)

Equation (11) shows that even when $|\psi_A\rangle$ is orthogonal to $|\psi_B\rangle$, the visibility is recovered completely provided that $|c_A| = |c_B|$. In this case, the states of the which-path marker, $|\psi_A\rangle$ and $|\psi_B\rangle$, are projected into the same polarization state $|\psi_2\rangle$ with the same probability, and it cannot be determined whether the photon came from slit A or B. This implies that LP$_2$ completely erases the which-path information, and the interference is recovered.

D. Geometrical interpretation of Pancharatnam phase

As shown in the previous sections, the evolution of the polarization state in which-path marking and in the quantum eraser induce the phase shifts $\delta_m$ and $\delta_f$, respectively. The net phase shift $\delta$ produced by the last projection is given by

$$\delta = \delta_f - \delta_m = \arg \langle \psi_B \rangle \langle \psi_2 \rangle \langle \psi_2| \psi_A \rangle.$$ (12)

Equation (12) is gauge invariant, i.e., independent of the choice of the phase factor of each state because the bra and ket vectors for each state appear in a pair. For example, even if $|\psi_A\rangle$ is substituted with $e^{i\phi_A}|\psi_A\rangle$, Eq. (12) does not change. This phase shift $\delta$ is identified with the Pancharatnam phase.

The Pancharatnam phase, which is a type of geometric phase, can be interpreted geometrically on the state space of the polarization, which is called the Poincaré sphere. The Poincaré sphere has poles that correspond to the right and left circular polarizations and an equator that corresponds to the linear polarization states, for example, the horizontal polarization $|H\rangle$, vertical polarization $|V\rangle$, 45$^\circ$ polarization $|D\rangle$, and 135$^\circ$ polarization $|X\rangle$.

FIG. 2: Spherical triangle on the Poincaré sphere formed by three states, $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$, and its solid angle $\Omega$. The poles on the Poincaré sphere correspond to the right and left circular polarization states, $|R\rangle$ and $|L\rangle$, and the equator corresponds to the linear polarization states, for example, the horizontal polarization $|H\rangle$, vertical polarization $|V\rangle$, 45$^\circ$ polarization $|D\rangle$, and 135$^\circ$ polarization $|X\rangle$.
E. Nonlinear variation of Pancharatnam phase

From Eq. (12), we can calculate the Pancharatnam phase in our experiments. We modify the conventional double-slit interferometer to include two linear polarizers and two quarter-wave plates, as shown in Fig. 1(b).

First, we prepare the initial polarization state $|\psi_1\rangle$ using the linear polarizer LP$_1$:

$$|\psi_1\rangle = \cos \theta_1 |H\rangle + \sin \theta_1 |V\rangle,$$

where $\theta_1$ is the angle between the horizontal line and the transmission axis of LP$_1$; $|H\rangle$, the horizontal polarization state; $|V\rangle$, the vertical polarization state. A quarter-wave plate induces a phase shift of $\pi/2$ in the slow-axis component of the polarization relative to the fast-axis component. In our setup, the fast axes of two quarter-wave plates, QWP$_A$ and QWP$_B$, are aligned to form angles of 0° and 90°, respectively, from the horizontal line. Thus, they induce phase shifts of $\pm \pi/2$ between the horizontal and the vertical components:

$$|\psi_A\rangle = \cos \theta_1 |H\rangle + i \sin \theta_1 |V\rangle,$$

$$|\psi_B\rangle = i \cos \theta_1 |H\rangle + \sin \theta_1 |V\rangle.$$

Here, the pair of quarter-wave plates serves as the which-path marker in Eq. (3). The final state of the polarization is expressed as $|\psi_2\rangle$:

$$|\psi_2\rangle = \cos \theta_2 |H\rangle + \sin \theta_2 |V\rangle,$$

where $\theta_2$ is the angle between the horizontal line and the transmission axis of LP$_2$.

Substituting Eqs. (15) and (16) into Eq. (12), we can obtain the Pancharatnam phase as

$$\delta = 2 \tan^{-1}(\tan \theta_1 \tan \theta_2).$$

Figure 3(a) shows the variation of $\delta$ with respect to $\theta_1$ and $\theta_2$. It is noteworthy that this variation exhibits strong nonlinearity for a certain condition. Rapid changes are observed around $(\theta_1, \theta_2) = (0, \pi/2)$, $\pi/2$, $(\pi/2, 0)$, $(\pi/2, \pi)$, and $(\pi, \pi/2)$. In Fig. 3(b), the variation of the Pancharatnam phase with $\theta_2$ is plotted for different values of $\theta_1$. The figure shows that the smaller the value of $\theta_1$, the faster is the change in the Pancharatnam phase with respect to $\theta_2$ near $\theta_2 = \pi/2$. We observe this nonlinear variation as a quick displacement of the fringes when we change $\theta_1$ or $\theta_2$ by rotating LP$_1$ or LP$_2$.

The nonlinear variation of the Pancharatnam phase can be explained by the spherical geometry on the Poincaré sphere, as shown in Fig. 4. In our experiment, $|\psi_A\rangle$ and $|\psi_B\rangle$, given by Eqs. (15) and (16), can be depicted at a latitude of $\pm 2\theta_1$ on the prime meridian, respectively, and the final state $|\psi_2\rangle$, given by Eq. (17), can be depicted on the equator at a longitude of $2\theta_2$. We assume that $|\psi_A\rangle$ and $|\psi_B\rangle$ are located near $|H\rangle$, that is, $0 < \theta_1 < \pi/4$ is satisfied, and $|\psi_2\rangle$ moves on the equator from $|H\rangle$. When the distance between $|\psi_2\rangle$ and $|V\rangle$ is greater than $2\theta_1$, the area of the spherical triangle spanned by $|\psi_A\rangle$, $|\psi_B\rangle$, and $|\psi_2\rangle$ remains small. However, when $|\psi_2\rangle$ approaches $|V\rangle$ and the distance between them becomes less than $2\theta_1$, the spherical triangle blows up.
FIG. 5: Experimental setup for double-slit quantum eraser. Light passing through the right and left of the wire interferes. Each path is marked by two film-type quarter-wave plates, QWP\textsubscript{A} and QWP\textsubscript{B}, whose fast axes F make angles of 0° and 90°, respectively. The interference fringe is captured using a CCD camera.

very quickly, and after traversing $|V\rangle$, the triangle covers most of the Poincaré sphere. In the same manner, when $|\psi_2\rangle$ is located near $|V\rangle$, the variation of the solid angle exhibits a strong nonlinearity with respect to the movement of $|\psi_A\rangle$ and $|\psi_B\rangle$ around $\theta_1 = 0$. This is the geometrical reasoning why the Pancharatnam phase changes rapidly in certain conditions.

III. EXPERIMENTS

The experimental setup is shown in Fig. 5. The light source is a 532-nm green laser with a 3-mm beam diameter (model DPGL-2200, SUWTECH). A thin opaque wire crossing the beam works as the double slit; the light passing through the right- and left-hand sides of the wire interferes due to diffraction. We attached two film-type linear polarizers, LP\textsubscript{1} and LP\textsubscript{2}, to the rotatable mounts with graduated scales for adjusting the angles $\theta_1$ and $\theta_2$. At a distance of approximately 1 m from the double slit, the recombined beam is captured using a charge-coupled device (CCD) camera (model LBP-2-USB, Newport) connected to a personal computer (PC). The CCD camera has a resolution of 640 $\times$ 480 pixels with size of 9 $\mu$m $\times$ 8 $\mu$m, and it is equipped with a gain controller.

A. Experimental results of quantum erasers

First, by setting $\theta_1 = \pi/4$ and removing the linear polarizer LP\textsubscript{2}, the initial state of polarization $|D\rangle$ is evolved into two orthogonal states through the quarter-wave plates, right circular polarization and left circular polarization according to the paths. Since we can determine which slit the photon has passed through by measuring the polarity of the circular polarization of the photon, no interference pattern is obtained. (This is mathematically confirmed from Eq. (5), which vanishes when $|\psi_A\rangle$ is orthogonal to $|\psi_B\rangle$.) Instead, we observed a typical diffraction pattern that only has broad peaks, as shown in Fig. 6(a).

By inserting LP\textsubscript{2}, the right and left circular polarizations are projected into the same linear polarization with the same probability and thus the polarization provides no which-path information. As a result, the interference fringe reappears. (Mathematically, this corresponds to the fact that Eq. (10) becomes unity when $|c_A\rangle = |c_B\rangle$.) Figure 6(b) shows the recovered interference fringe for $\theta_2 = 0$. Similarly, if we set $\theta_2 = \pi/2$, we can obtain the corresponding interference fringe, as shown in Fig. 6(c), which is out of phase with that observed for $\theta_2 = 0$ (see Fig. 7). The sum of these interference patterns reproduces the broad peak pattern, as shown in Fig. 6(a),
that is obtained when LP$_2$ is removed. Thus, the quantum eraser actually filters out one of these fringes and perfectly recovers the visibility.

**B. Observation of Pancharatnam phase and its nonlinearity**

In order to observe the variation of the Pancharatnam phase with respect to $\theta_1$ and $\theta_2$, we measured the displacement of the fringes. Figure 8 shows the fringe shift with respect to $\theta_1$. The light intensity of each fringe in Fig. 8 is normalized individually. When $\theta_2 = 9^\circ$, the fringe exhibits a quick displacement around $\theta_1 = 90^\circ$ that is obtained when LP$_2$ is removed. Thus, the quantum eraser actually filters out one of these fringes and perfectly recovers the visibility.

Figure 9 shows the variation of the Pancharatnam phase for several conditions. The points in Fig. 9 indicate the experimental results, and the solid lines indicate the theoretical lines calculated using Eq. (18). The vertical axis is the displacement of the fringe normalized by the spatial period of the fringe $\Delta x$. The origin of the vertical axis is determined by the position of the fringes when $\theta_1 = 0^\circ$ or $\theta_2 = 0^\circ$. Figure 9(a) shows the variation of the Pancharatnam phase with respect to $\theta_1$ for $\theta_2 = 45^\circ$, $30^\circ$, $18^\circ$, and $9^\circ$. All experimental results agree well with the theoretical ones. The gradient of the variation of the shift around $\theta_1 = 90^\circ$ becomes steeper as $\theta_2$ is decreased. This implies that the variation of the shift becomes more sensitive to the variation of the initial polarization state. Similarly, Fig. 9(b) shows the variation of the Pancharatnam phase with respect to $\theta_2$ for some values of $\theta_1$. This result also agrees well with the theoretical ones.

**IV. SUMMARY**

We have introduced a double slit interferometer with internal states to demonstrate which-path marking, quantum erasers, and geometric phases. Although each phenomenon has already been described in previous works, in our experiment we can observe these phenomena using a single setup. The laser and the CCD camera that we have used are for laboratory use; however, they can be replaced with a laser pointer and a webcam. The movement of fringes can be observed by the naked eyes while rotating the polarizers. Film-type linear polarizers and wave plates are easily available.

Recently, it has been shown that the nonlinearity of the Pancharatnam phase can be related to the unusually large shift of the wavepacket in the weak measurement.\[15]
which was first proposed by Aharonov, Albert, and Vaidman. This phenomenon can be interpreted as an extension of the double-slit interferometer and the amplification mechanism utilizes the steeper gradient of the Pancharatnam phase.

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