Exploring the phase diagram of QCD with complex Langevin simulations

Benjamin Jäger

Swansea University
Prifysgol Abertawe

Lattice 2014

In collaboration with G. Aarts, F. Attanasio, E. Seiler, D. Sexty and I.-O. Stamatescu
Phase diagram of QCD

- Important input for heavy-ion collisions experiments.
- Still a conjecture (only very little is known)
- Direct Monte Carlo simulations not possible
The problem

Complex weight

- Simulations with a finite chemical potential typically lead to a severe sign problem

\[
(\det D(\mu))^* = \det D(-\mu^*) \rightarrow \det D(\mu \neq 0) \in \mathbb{C}.
\]

- Importance Sampling based Monte Carlo methods cannot be applied to path-integrals with a complex weight

\[
\langle A \rangle = \frac{1}{Z} \int A(x) \left| \det(D(x)) \right| e^{i\phi(x)} e^{-S_G(x)} \, dx.
\]
The solution

Complex Langevin simulations

- The expectation value of the operator $A$ can be obtained by integrating along a path of the so-called Langevin time $\tau$

$$\langle A \rangle = \int A(x(\tau)) \, d\tau.$$ 

- The Langevin evolution is achieved by a stochastic process in the degrees of freedom, generically denoted as $x$

$$\frac{d x}{d \tau} = - \frac{\partial S}{\partial x} + \eta(\tau).$$

where the Gaussian random noise $\eta(\tau)$ has to fulfil

$$\langle \eta(\tau) \rangle = 0 \text{ and } \langle \eta(\tau) \eta(\tau') \rangle = 2\delta(\tau - \tau').$$
The solution

Complex Langevin simulations

- Real Langevin simulations converge to the expected result.
- Simulations with complexified degrees of freedom are no longer in a compact group,
  \[
  \text{SU}(3) \to \text{SL}(3, \mathbb{C}),
  \]
  and convergence is no longer guaranteed.
- Empirical studies show that convergence is achieved, if the distribution is compact in the imaginary part.
- We use gauge-invariance to get gauge-links to the proximity of the SU(3) manifold, \textit{gauge cooling}
  \[
  U_\mu(x) \to V(x)U_\mu(x)V^{-1}(x + a\hat{\mu}) \quad U, V \in \text{SL}(3, \mathbb{C}).
  \]
QCD in the limit of heavy quarks

Complex Langevin simulations

- Here we present results for QCD in the limit of heavy quarks.
- The fermion determinant can be written in terms of the (conjugate) Polyakov loops $P_\vec{x}$ and $P_{\vec{x}}^{-1}$ as

$$
\det D(\mu) = \prod_{\vec{x}} \det (1 + C P_\vec{x})^2 \det \left(1 + C' P_{\vec{x}}^{-1}\right)^2,
$$

where the coefficients $C$ and $C'$ are defined as

$$
C = (2 \kappa e^{\mu})^{N_t} \quad \text{and} \quad C' = (2 \kappa e^{-\mu})^{N_t}.
$$
- For the gluonic part we use the full Wilson gauge action.
Observables

8 \cdot 8^3, \beta = 5.80, \mu = 1.55

- Quantities such as the Polyakov loop $P_{\vec{x}}$ can be extracted from the Langevin evolution.
A measure for the distance of the gauge links from the SU(3) manifold is given by the unitarity norm:

\[ \text{Tr} \left( \mathbf{U} \mathbf{U}^\dagger - \mathbf{I} \right)^2 \]
Strategy

- Scan in $\mu$ for different $N_t$ (temperatures)
- Determine $\mu$-transition in Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$
Strategy

- Scan in $N_t$ for different $\mu$
- Determine $T$-transition in Polyakov loop
Strategy

- Determine $\mu$-transition in Fermion density
- Determine $T$-transition in Polyakov loop
**Strategy**

**Simulation Setup**

- $\beta = 5.8$, $\kappa = 0.12$  \((a \sim 0.15 \text{ fm}, \mu_c \sim -\ln(2\kappa) = 1.43)\)
- Volume: $8^3 \times N_t$
- $N_t = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 22, 24, 26, 28$
- $\mu = 0.0, 0.1, 0.2, \ldots, 2.4$
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

$8^3$ HDQCD $N_f = 2, \kappa = 0.12$

- $N_t = 0.3$

Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ 8^3 \text{ HDQCD } N_f = 2, \kappa = 0.12 \]

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.

\[ 8^3 \text{ HDQCD} \quad N_f = 2, \kappa = 0.12 \]

\[ N_t = 18 \]
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density

\[ n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu} \]

- High to low temperatures.
- Transition in \( \mu \) visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density
Fermion density

T [MeV]

μ

0.0 0.5 1.0 1.5 2.0

400 600 200 5 10
Fermion density (imaginary part)
Strategy

- Determine $\mu$-transition in Fermion density ✓
- Determine $T$-transition in Polyakov loop
Polyakov loop
Polyakov loop

![Graph of Polyakov loop with axes T [MeV] and μ]
Strategy

- Determine $\mu$-transition in Fermion density ✓
- Determine $T$-transition in Polyakov loop ✓
- Determine the order of phase transition: susceptibilities
Polyakov loop susceptibility
Conclusion and Outlook

Conclusion

- Complex Langevin simulation can be used to study the phase diagram of QCD.
- Thermal transition is visible in the polyakov loop $P_{\bar{x}}$.
- Transition in $\mu$ is studied in the Fermion density $n$.

Outlook

- Extend simulations to different $\beta$ values and improve the thermal transition.
- Determine the order of the transition.
- Include fully dynamical fermions (Staggered or Wilson)
Fermion density - Dynamical fermions
Number of iterations - Dynamical fermions
Thank you for your attention!
Backup - Simulation time in 24h run (HDQCD)