Optimal Placement of PV Smart Inverters With Volt-VAr Control in Electric Distribution Systems

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Abstract—The high R/X ratio of typical distribution systems makes the system voltage vulnerable to the active power injection from distributed energy resources (DERs). Moreover, the intermittent and uncertain nature of the DER generation brings new challenges to the voltage control. This article proposes a two-stage stochastic optimization strategy to optimally place the photovoltaic (PV) smart inverters with Volt-VAr capability for distribution systems with high PV penetration to mitigate voltage violation issues. The proposed optimization strategy enables a planning-stage guide for upgrading the existing PV inverters while considering the operation-stage characteristics of the Volt-VAr control. One advantage of this planning strategy is that it utilizes the local control capability of the smart inverter that requires no communication, thus avoiding issues related to communication delays and failures. Another advantage is that the Volt-VAr control characteristic is internally integrated into the optimization model as a set of constraints, making placement decisions more accurate. The objective of the optimization is to minimize the upgrading cost and the number of the smart inverters required while maintaining the voltage profile within the acceptable range. Case studies on an actual 12.47 kV, 9-km-long Arizona utility feeder have been conducted using OpenDSS to validate the effectiveness of the proposed placement strategy in both static and dynamic simulations.

Index Terms—Dynamic model, stochastic optimization, unbalanced distribution system, Volt-VAr control, voltage violation mitigation.

NOMENCLATURE

Sets and Indices

| Symbol | Description |
|--------|-------------|
| \( \varphi \) | Set of phase indices \( p \). |
| \( \varphi(\ell) \) | Subset of phase at distribution line \( \ell \). |
| \( \Omega_B \) | Set of bus indices \( (i, p) \). |
| \( \Omega_B \subset \Omega_G \) | Subset of bus with load indices \( (d, p) \). |
| \( \Omega_L \) | Set of distribution lines indices \( (\ell, p) \). |

Parameters

| Symbol | Description |
|--------|-------------|
| \( c_{a}^{pv} \) | Allocation cost of the PV smart inverter. |
| \( c_{p}^{pv} \) | PV active power curtailment cost. |
| \( R_{\ell, p, m} \) | Self-resistance \( (p = m) \) and mutual resistance of line \( \ell \) between phases \( p \) and \( m (p \neq m) \). |
| \( X_{\ell, p, m} \) | Self-reactance \( (p = m) \) and mutual reactance of line \( \ell \) between phases \( p \) and \( m (p \neq m) \). |
| \( Z_{\ell, p, m} \) | Self-impedance \( (p = m) \) and mutual impedance of line \( \ell \) between phases \( p \) and \( m (p \neq m) \). |
| \( V_{\ell, m, k} \) | Susceptance of line \( \ell \) between phases \( m \) and \( k \). |
| \( E_{\ell, g, s}^{pv} \) | Maximum power point (MPP) of PV at bus \( g \) phase \( p \) at scenario \( s \). |
| \( E_{g, p, max}^{pv} \) | Rating active power value of PV inverter at bus \( g \) phase \( p \). |
| \( \tilde{E}_{g, p}^{pv} \) | Rating reactive power value of PV inverter at bus \( g \) phase \( p \). |
| \( \tilde{E}_{g, p}^{app} \) | Rating apparent power value of PV inverter at phase \( p \). |
| \( V_{g}^{(1)}, \ldots, V_{g}^{(6)} \) | Voltage magnitude breakpoints of the linear piecewise Q–V curve. |
| \( V_{\min}^{\text{max}} \) | Minimum/maximum voltage magnitude for the normal operation. |
| \( V_{n, p}^{\text{rms}} / V_{n, p}^{\text{im}} \) | Real/imaginary part of voltage measurement at substation before enabling Volt-VAr control \( n \) phase \( p \) at scenario \( s \). |
| \( p_{v}(s) \) | Scenario probability. |
| \( w_{c}, w_{a}, w_{e} \) | Weight coefficients in the objective function. |

Digital Object Identifier 10.1109/JSYST.2023.3256121
The rapid growth of distributed energy resources (DERs), especially the solar photovoltaic (PV) generators in distribution systems, is transforming the systems from passive networks to active ones [1]. The large R/X ratio of the active distribution feeders makes the voltages sensitive to the intermittent active power injection from the PVs, which may lead to unexpected voltage violations and voltage fluctuations [2].

There are four types of voltage control strategies applied in active distribution systems: local control, distributed control, decentralized control, and centralized control [3]. The local control is an autonomous control strategy that does not require any communication among different controllers [4], [5]. The centralized control can provide more flexibility by allowing the optimal utilization of the control devices among the entire system. However, a robust communication system is required to provide the real-time measurements for the central controller [6], [7]. Distributed control is a strategy that does not need a central controller. Only the communication between the neighboring controllers is required [8]. The decentralized control coordinates various control components to optimize the operation for a specific area, which means the system can be divided into different “centralized” zones [9]. These different control strategies are all based on the system operation stage with possible use of optimization techniques to coordinate the operation of different existing devices. However, the existing control devices in some distribution systems may not be adequate for handling the voltage issues caused by the high penetration of the PVs. Even worse, the bidirectional power flow of the active distribution network can mislead traditional control devices, such as the tap changers, and cause unexpected regulating or protecting actions [1].

To further mitigate voltage violation issues, the interconnection standard IEEE 1547-2003 has been thoroughly revised and published as 1547-2018 to allow smart inverter-based generators to participate in the distribution feeder voltage regulation by providing sufficient active and reactive power support [10]. This amendment enables the DERs with smart inverters to control and optimize the local voltage by injecting or absorbing reactive power based on the local operating condition. In order to better utilize the local control capability of the PV smart inverters in the distribution system, various optimal Volt-Var control strategies have been studied by the previous research efforts [11], [12], [13], [14], [15], [16]. These articles focus on the operational or short-term planning aspect of the reactive power optimization in multiple timescales, such as the day ahead prediction stage, hourly prediction stage, and real-time stage. By using multi-timescale optimization, centrally coordinated control between the PV smart inverters and traditional voltage control devices, such as the capacitor banks and the on-load tap changers, can be achieved by considering the different response speeds of them. These operation studies all have the premise that the inverters in the distribution system are already equipped with smart inverters.

The PV technologies have been evolving for many years with the previous IEEE 1547-2003 standard declaring that the DERs shall not actively regulate the voltage at the point of common coupling [17] before the new standard [10] was published. Most of the existing PV systems in the real distribution networks are still equipped with the conventional inverters that only allow the PV systems to operate at unity power factor to generate active power. These existing PV systems, especially the residential roof-top PV systems, are mostly customer owned. Retrofitting all or a large number of the existing PV inverters to have Volt-Var capability can be cost prohibitive for the utilities due to the related allocation costs and the human resources needed to negotiate with the customers to get fully reactive power control over their PV systems. So, it is cost effective to develop a long-term planning strategy to selectively upgrade the existing conventional PV inverters to smart inverters with local control capability to mitigate the voltage issues caused by the intermittent power injection from the distributed PVs.
by high penetration of PVs. Many approaches on distribution system planning associated with active voltage support have been investigated [18], [19], [20], [21], [22]. However, these planning strategies mainly focused on optimizing the locations and sizes of the DERs without considering the real-time operational characteristic of the DERs. Few planning strategies have been proposed in the literature that consider the operation strategies in the active distribution system. A joint planning and operation optimization algorithm was presented in [23] and [24] to upgrade traditional expansion measures and consider the voltage regulation impact of DERs with smart inverters in the operation stage. However, the inverter Q–V characteristics were discretized and implemented through a lookup table [24], and iterative procedure was used to ensure voltage and reactive power convergence. They did not implement the actual operational Q–V curves of the smart inverters in the planning process, which is more accurate. Moreover, the authors assumed that the existing inverters should be retrofitted for the desired control characteristics, which is not realistic.

This article proposes a novel long-term planning strategy to optimally place a minimum number of PV smart inverters with Volt-VAr control among the existing conventional PV systems to mitigate the possible voltage issues using only the local control capability of the newly placed smart inverters in the distribution systems. The problem is formulated as a two-stage stochastic mixed-integer programming optimization model considering the operation scenarios with the worst voltage violation. The first stage is to place the minimum number of PV smart inverters with Volt-VAr control. The second stage considers the detailed operational characteristics of the smart inverters and minimizes the PVs’ active power curtailments. The placed PV smart inverters work in the VAr priority mode and follow their own predefined Q–V curve to autonomously control the local voltage without communicating with other devices. To validate the system voltage stability with the autonomous inverter control, a detailed dynamic model of the PV inverter is developed as a dynamic link library (DLL) in OpenDSS. The key contribution of this work can be summarized as follows.

1) A stochastic decision process is proposed to optimally place the PV smart inverters with Volt-VAr control to upgrade the existing conventional PV inverters considering the uncertainties of PV generation and load demand. With the optimally placed smart inverters, the PVs’ local control can optimally mitigate the system-wide voltage violations without any centralized communication mechanism in real-time operation.

2) A set of analytical constraints are formulated in the operation stage (second stage of the planning problem) to model the impact of the Volt-VAr control with VAr priority on the voltage profile. The operational characteristics of the Q–V curve are defined as a piecewise-linear function and transformed into a block of variables and constraints to enforce the relationship between the reactive power output of the smart inverter and the local voltage magnitude. With the consideration of the Q–V operational characteristics of the PV smart inverters in the planning problem, the optimization can precisely resolve the voltage violation issues by taking the covariation of the voltage and the reactive power into account and achieve optimal placement of smart inverters without voltage violation in all scenarios analyzed, including the worst over and undervoltage scenarios for a given system.

3) A detailed dynamic model of the PV inverter with Volt-VAr control is developed as a DLL in OpenDSS to verify the optimization results and ensure system voltage stability.

4) The optimization has been applied on an actual distribution feeder model provided by the local utility with instantaneous penetration levels as high as 225% with significant overvoltage issues. The results show that a small subset of inverters upgraded with Volt-VAr control capability is sufficient to remove all voltage violations. The proposed optimization program can provide the utility with a decision-making guideline to upgrade the existing conventional PV inverters in the distribution system with minimal allocation cost and human resources to mitigate all the voltage violations caused by high penetration of PV.

The rest of this article is organized as follows. Section II describes the problem to be solved in this article. Section III presents the detailed mathematical formulation. Section IV provides the simulation results and the results verification. Finally, Section V concludes this article.

II. PROBLEM STATEMENT

The increased penetration of PVs in distribution systems may lead to severe voltage violation problems or reactive power problems [25]. PVs with smart inverters can control and optimize the local voltage by injecting or absorbing reactive power based on the local voltage. However, many of the PVs were not equipped with smart inverters. Retrofitting all or a large number of existing inverters to have Volt-VAr capability can be cost prohibitive for the utilities. On the other hand, to ensure that the smart inverter can successfully control the distribution system voltage profile, especially during periods of peak solar output, the smart inverter may need to operate under VAr priority mode. It indicates that the reactive power support is prioritized, the active power output may need to be curtailed due to the lack of sufficient headroom in the inverter rating. However, curtailing the active power output of PVs negates the economic benefit to PV owners and other environmental benefits. An optimal placement strategy to place the minimum number of PV smart inverters with Volt-VAr control considering the uncertainties of PV output and load needs to be developed to solve these problems.

As depicted in Fig. 1, the PV smart inverter placement problem is modeled as a two-stage stochastic decision process, given as follows.

1) The planner makes smart inverter placement decisions for the rooftop PV in the first stage.
2) The operational uncertainties are resolved in the worst-case voltage conditions, including (a) power demand and (b) the maximum power point of the PV output. The operator invokes recourse decisions (i.e., Volt-VAr control) to minimize voltage violation in the second stage.

Here, we use the historical data from a utility to construct the uncertainty, considering the following two worst voltage scenarios:

1) The worst overvoltage scenario occurs under the maximum generation condition.
2) The worst undervoltage scenario considers the maximum load condition.

III. MATHEMATICAL FORMULATION

This section presents a two-stage stochastic mixed-integer linear program (SMILP) formulation to place the minimum number of PV smart inverters with Volt-VAr control to meet the voltage requirements and mitigate under/overvoltage conditions. The first stage minimizes the number of PV smart inverters. The second stage minimizes the norm of PVs’ active power curtailment while maintaining the substation voltage at a certain level given the worst voltage scenario. Equation (1) presents the objective of the proposed SMILP problem, which minimizes the allocation cost of PV smart inverters and the expected operation cost of the second stage.

\[
\min w_c \sum_{(g,p) \in \Omega_{PV}} c_{pv}^{g,p} x_{pv}^{g,p} + \sum_{s \in S} p_r(s) \phi(x^{pv}, s).
\]  

(1)

The second stage models the unbalanced distribution system operation under worst-case voltage scenarios. There are two modeling challenges. First, if a PV smart inverter works in the Volt-VAr control mode with VAr priority, the generated reactive power should follow a specific Q–V curve, as shown in Fig. 2. The inverter operates in different modes generating or absorbing reactive power to support local voltage. This Volt-VAr control function can be analytically achieved using the piecewise linear Q–V curve constraint. At the same time, the squared norm of active power curtailment is minimized in the objective to encourage as many PV smart inverters as needed to participate in voltage mitigation through reactive power support.

Second, a multiphase optimal power flow formulation that models the unbalanced distribution system accurately is needed to obtain the optimal location of PV smart inverters with Volt-VAr control based on actual system requirements. This article makes full use of the unbalanced distribution system linearized ac power flow formulation proposed in [26] to model all details of a distribution network. This linearized power flow model is based on the rectangular current–voltage (I–V) formulation and uses the first-order approximation of the Taylor’s series expansion to linearize the nonlinear product of current and voltage in the node power balance constraint. In order to construct the Q–V curve constraints for modeling the reactive power of PV smart inverters with Volt-VAr control, the voltage magnitude is needed, which can be obtained using a nonlinear function of real and imaginary parts of voltage in the I–V formulation. We use a similar first-order approximation of the Taylor’s series expansion to get the linear expression of the voltage magnitude.

The detailed mathematical formulation of the second-stage problem is described as follows.

1) Second-Stage Formulation:

\[
\phi(x^{pv}, s) = \min_{(g,p) \in \Omega_{PV}} w_a \sum_{(g,p) \in \Omega_{PV}} e_{pv}^{g,p} (p^{pv,s} - p^{g,p})^2 \\
+ w_v \sum_{(n,p) \in \Omega_{an}} (V^{r,s}_{n,p} - V^{r,s}_{n,p})^2 + (V^{im,s}_{n,p} - V^{im,s}_{n,p})^2.
\]  

(2)

The first part of the detailed second-stage formulation in (2) minimizes the weighted least squares of the active power curtailment of PV with the smart Volt-VAr control. The second set of terms minimizes the weighted least squares of the feeder-head voltage difference between the optimization model and the base case without PV Volt-VAr control.

2) Smart Inverter Volt-VAr Control Constraints:

   a) PV Volt-VAr control enabling constraint:

\[
x_{pv}^{g,p} \leq \bar{x}_{pv}^{g,p} \forall (g,p) \in \Omega_{PVV}, s \in S.
\]  

(3)

Constraint (3) indicates that only if a PV smart inverter is placed at the selected PV bus node g phase p, the Volt-VAr control function can be enabled; otherwise, it will not be activated.
Here, $\Omega_{PV}$ is a pool of candidate PVs in the optimization that can be selected to install a PV smart inverter.

**b) PV output disjunctive constraints:**

\[
\begin{bmatrix}
0 \\ Q^\text{pv,s}\_\forall \in \bar{S}
\end{bmatrix}
\begin{bmatrix}
\delta^g,p,s \\ \lambda^g,p,s
\end{bmatrix}
\begin{bmatrix}
\bar{P}^\text{pv,s}
\bar{Q}^\text{pv,s}
\lambda^g,p,s \geq 0
\end{bmatrix}
\forall (g,p) \in \Omega_{PV}, \forall s \in S.
\]

The disjunction constraint (4) describes the logical relationship of the PV output with a binary decision, whether to enable the Volt-VAR control function or not. Here $\delta^g,p,s$ and $\lambda^g,p,s$ are used as binary variables to select between different groups of PV output constraints. These binary variables must satisfy the relationship $\delta^g,p,s + \lambda^g,p,s = 1$. If $\delta^g,p,s = 1$, it indicates that the PV at bus $g$ can perform Volt-VAR control: the active power output of the PV can be curtailed; the reactive power output of the PV needs to follow the Q–V curve, as shown in Fig. 2, which varies with the voltage magnitude $V^g$ and the apparent power output of PV should be less than the rated apparent power value $\bar{S}^g,p$. If $\lambda^g,p,s = 1$, it indicates that the PV at bus $g$ does not perform Volt-VAR control: the active power output of the PV is equal to the maximum power point for the PV, and its reactive power output is zero.

**c) Piecewise-Linear Q–V Curve Constraint:** The Q–V curve of Volt-VAR control shown in Fig. 2 can be expressed as a continuous piecewise-linear function

\[
Q^\text{pv,s}\_g,p =
\begin{cases}
Q^\text{pv,\max}\_g,p \\
\frac{Q^\text{pv,\max}\_g,p - Q^\text{pv,\min}\_g,p}{V^g_{(1)} - V^g_{(2)}} (V^g - V^g_{(2)}) + Q^\text{pv,\min}\_g,p & \text{if } V^g_{(1)} \\ 0 & \text{if } V^g_{(2)} \\ \frac{Q^\text{pv,\max}\_g,p - Q^\text{pv,\min}\_g,p}{V^g_{(3)} - V^g_{(2)}} (V^g - V^g_{(2)}) + Q^\text{pv,\min}\_g,p & \text{if } V^g_{(3)} \\ -Q^\text{pv,\max}\_g,p & \text{if } V^g_{(5)} \\ \frac{Q^\text{pv,\max}\_g,p - Q^\text{pv,\min}\_g,p}{V^g_{(4)} - V^g_{(3)}} (V^g - V^g_{(3)}) + Q^\text{pv,\min}\_g,p & \text{if } V^g_{(4)} \\ \frac{Q^\text{pv,\max}\_g,p - Q^\text{pv,\min}\_g,p}{V^g_{(5)} - V^g_{(4)}} (V^g - V^g_{(4)}) + Q^\text{pv,\min}\_g,p & \text{if } V^g_{(5)} \\
\end{cases}
\forall (g,p) \in \Omega_{PV}, \forall s \in S.
\]

Here, we use the disaggregated convex combination model to transform this function into a block of variables and constraints that enforces a piecewise linear relationship between the voltage magnitude variable (i.e., $V^g$) and the PV reactive power output variable (i.e., $Q^\text{PV}$). Let $Q^g_j \in \{Q^\text{PV,\max}\_g,p, Q^\text{PV,\max}\_g,p, 0, 0, -Q^\text{PV,\max}\_g,p, -Q^\text{PV,\max}\_g,p\}$ represent the corresponding reactive output at the voltage magnitude breakpoints of the Q–V curve. The reformulated constraints of (5) can be written as follows:

\[
\begin{align}
V^g_{(1)} & = V^g_{m,s} \\
Q^\text{PV}\_g,p & = \sum_{j \in J} (Q^g_j)_{\lambda^g,p,s} \forall (g,p) \in \Omega_{PV}, \forall s \in S.
\end{align}
\]

**3) Linearized I–V-Based AC Power Flow Constraints:** The objective of mitigating voltage violations under the worst voltage scenarios with the minimum number of PV smart inverters with Volt-VAR control makes the placement decisions very sensitive to the voltage change. As a result, it is necessary to accurately model the three-phase unbalanced distribution system power flow considering the impact of lines’ mutual impedances and shunt admittances on the bus voltage. Compared with the widely used linearized DistFlow model, which approximates the unbalanced voltage as balanced [27], [28], the I–V-based ac power flow model proposed in [26] has been validated to be more accurate to capture the operation characteristics of the unbalanced distribution systems with a high penetration level of PVs.

\[
\begin{align}
V^r\_g,p - V^r\_s,p & = \sum_{m \in \ell(p)} R^\ell,p,m \left( I^\ell\_m,s + \sum_{k \in \ell(p)} y^\ell,m,k V^m\_k \right) \\
& - \sum_{m \in \ell(p)} X^\ell,p,m \left( \frac{I^\ell\_m,s}{y^\ell,m,k} \right) \\
& \forall (g,p) \in \Omega_{PV}, j \in J, s \in S.
\end{align}
\]

\[
\begin{align}
V^m\_g,p - V^m\_s,p & = \sum_{m \in \ell(p)} R^\ell,p,m \left( I^\ell\_m,s + \sum_{k \in \ell(p)} y^\ell,m,k V^m\_k \right) \\
& - \sum_{m \in \ell(p)} X^\ell,p,m \left( \frac{I^\ell\_m,s}{y^\ell,m,k} \right) \\
& \forall (g,p) \in \Omega_{PV}, s \in S.
\end{align}
\]
line connecting bus \( \ell_o \) to bus \( \ell_e \), considering the impact of the line’s self and mutual impedances and admittances.

b) Current injection constraints:

\[
\begin{align*}
I_{i,p}^{m,s} &= \sum_{(\ell,p)\in\mathcal{L}_O(i)} I_{\ell,p}^{m,s} - \sum_{(\ell,p)\in\mathcal{L}_E(i)} I_{\ell,p}^{m,s} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S} \\
I_{i,p}^{\text{im},s} &= \sum_{(\ell,p)\in\mathcal{L}_O(i)} I_{\ell,p}^{\text{im},s} - \sum_{(\ell,p)\in\mathcal{L}_E(i)} I_{\ell,p}^{\text{im},s} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S}.
\end{align*}
\]

The injected current at each node and scenario is obtained using (16) and (17).

e) Linearized power balance constraints: In the rectangular \( I-V \) formulation, the power balance constraints contain nonlinear elements due to the product of voltage and injected current. To linearize the power balance constraints, an iterative first-order approximation of the Taylor’s series expansion developed in [26] is used and the linearized power balance constraints around an operating point for each phase (i.e., \( V_{i,p}^{r,s}, \hat{V}_{i,p}^{\text{im},s}, \hat{I}_{i,p}^{r,s} \), and \( \hat{I}_{i,p}^{\text{im},s} \)) can be expressed as follows:

\[
\begin{align*}
&\sum_{\mathcal{N}(p,s)\in\Omega_{\text{Dpa}}} P_{n,p}^{G,s} + \sum_{g=1}^{\mathcal{N}(p,s)\in\Omega_{\text{Dpa}}} P_{g,p}^{G,s} + \sum_{g=1}^{\mathcal{N}(p,s)\in\Omega_{\text{Dpa}}} \tilde{P}_{g,p}^{G,s} - \sum_{d=1}^{\mathcal{N}(d,p)\in\Omega_{\text{Dpa}}} P_{d,p}^{D,s} \\
&- \sum_{(m,p)\in\Omega_{\text{inv}}} P_{m,p}^{\text{Tr},s} + \hat{V}_{i,p}^{r,s} \hat{I}_{i,p}^{r,s} + \hat{I}_{i,p}^{\text{im},s} \hat{V}_{i,p}^{r,s} + \hat{V}_{i,p}^{\text{im},s} \hat{I}_{i,p}^{\text{im},s} + \hat{V}_{i,p}^{\text{im},s} \hat{I}_{i,p}^{\text{im},s} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S}
\end{align*}
\]

Two PV operating points are considered in the proposed SMILP problem: 1) without smart inverter and 2) with smart inverter. Since the objective of this placement problem is to minimize the number of PV smart inverters to mitigate the voltage violations in the worst-case scenarios, the PV operating condition without placing smart inverters here is considered to be the base case.

In Constraints (18) and (19), \( (\hat{V}_{i,p}^{r,s}, \hat{V}_{i,p}^{\text{im},s}, \hat{I}_{i,p}^{r,s}, \hat{I}_{i,p}^{\text{im},s}) \) are the real and imaginary parts of bus voltage and injected current at the operating point without smart inverter, which are used as the first-order approximation parameters of Taylor’s series expansion and can be updated iteratively for getting the voltage and injected current at the operating point with smart inverter.

Note that in Constraint (18), the active power output of the residential PVs that are not in the PV smart inverter candidate pool is assumed to follow the respective maximum power point value.

d) Voltage magnitude constraints: The voltage magnitude can be expressed by a nonlinear function of the real part and the imaginary part of voltage, given as follows:

\[
V_{i,p}^{m,s} = \sqrt{V_{i,p}^{r,s} + V_{i,p}^{r,s}^2} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S}.
\]

The linear approximation of (20) can be reformulated by the similar first-order Taylor-series expansion method

\[
\begin{align*}
V_{i,p}^{m,s} &= \sqrt{V_{i,p}^{r,s} + V_{i,p}^{r,s}^2} + \frac{\partial V_{i,p}^{m,s}}{\partial \hat{V}_{i,p}^{r,s}} (\hat{V}_{i,p}^{r,s} - \hat{V}_{i,p}^{r,s}) \\
&+ \frac{\partial V_{i,p}^{m,s}}{\partial \hat{V}_{i,p}^{\text{im},s}} (\hat{V}_{i,p}^{\text{im},s} - \hat{V}_{i,p}^{\text{im},s}) \\
&= \hat{V}_{i,p}^{r,s} + \hat{V}_{i,p}^{r,s} + \hat{V}_{i,p}^{\text{im},s} + \hat{V}_{i,p}^{\text{im},s} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S},
\end{align*}
\]

where \( (\hat{V}_{i,p}^{r,s}, \hat{V}_{i,p}^{\text{im},s}) \) represents the operating point without smart inverter for the first iteration of Taylor’s series expansion at bus \( i \), phase \( p \), and scenario \( s \). Constraint (22) bounds the voltage magnitudes in the normal operating range.

\[
V_{\text{min}} \leq V_{i,p}^{m,s} \leq V_{\text{max}} \quad \forall (i,p) \in \Omega_B, s \in \mathcal{S}.
\]

IV. SIMULATION RESULTS AND VALIDATION

This section presents the optimal results of the proposed PV smart inverter placement problem corresponding to an actual 12.47 kV, 9-km-long Arizona utility feeder that serves residential customers. This feeder has 7864 buses, 1790 primary sections, 5782 secondary sections, 371 distribution transformers, 1373 loads, and 766 residential rooftop PV units. These 766 residential rooftop PV units only operate at unity power factor following the maximum power point tracking to generate active power, which results in overvoltages during spring days when excessive power is generated with light load conditions. However, when PV generation is not enough to cover the heavy load consumption in the summer days, there are undervoltage issues. The detailed information of this feeder can be found in [29].

A. Scenario Generation and Solution Algorithm

As mentioned in Section II, two days corresponding to the actual historical feeder data—the maximum generation condition on 03/15/2019 (high PV) and load peak on 07/15/2019 (high load and relatively low PV)—were chosen for constructing the worst over and undervoltage scenarios. Each worst case is assigned a probability of 0.5. In the worst overvoltage scenario, the power generated by the installed residential rooftop PV is 3.6 MW, which corresponds to a penetration level of 225% (3.6 MW/1.6 MW) compared with the feeder’s corresponding total gross load. Fig. 3(a) and 3(b) presents the load node locations and the voltage profile with the overvoltage issue, and the total number of overvoltage load nodes is 439. For the worst undervoltage scenario, Fig. 3(c) and 3(d) presents the load node locations and the voltage profile, and the total number of undervoltage load nodes is 14.

As only two voltage scenarios are considered, the extensive form of the proposed two-stage SMIP model for the optimal
placement of PV smart inverters is directly solved using the PySP package in Pyomo (Version 5.7.3) with Gurobi 9.03 mixed-integer solver. The simulations were performed on a computer with a 3.6 GHz 8-Core Intel i9-9900 K CPU and 36 GB of RAM.

B. Operation Results Comparison of the SMIP Optimization With OpenDSS

The six parameters of the Q–V curves of Volt–Var control are set as $\bar{V}_g^{(1)} = 0.0, \bar{V}_g^{(2)} = 0.94, \bar{V}_g^{(3)} = 0.98, \bar{V}_g^{(4)} = 1.02, \bar{V}_g^{(5)} = 1.06, \bar{V}_g^{(6)} = 1.1 \quad \forall g \in \Omega_{pv}$. The power factor range of the PV smart inverter is $[-0.8, 0.8]$. The optimal location of the PV smart inverters given by the proposed SMIP model is shown in Fig. 4, and the total number is 99. There are eight PV smart inverters in Phase A, 69 in Phase B, and 22 in Phase C. OpenDSS is a commonly used distribution system simulator for static and quasi-static time-series power flow analysis. The selected PV smart inverters with Volt–Var control are implemented in OpenDSS to validate the accuracy and effectiveness of the proposed SMIP model. By comparing Figs. 3(a) and 4, it can be seen that the optimization typically favors the locations with overvoltages initially for the placement of smart inverters. The voltage profiles after enabling the optimally selected Volt–Var control in the OpenDSS simulation under the two worst voltage scenarios are shown in Fig. 5. It can be seen that the optimally placed PV smart inverters with Volt–Var control can successfully mitigate the voltage violation issues for both overvoltage and undervoltage scenarios, which validates the effectiveness of the proposed SMIP model.

To validate the accuracy of the proposed SMIP model, we compare the power flow solution from the SMIP model with OpenDSS after enabling Volt–Var control in the selected PV smart inverters. Fig. 6(a) and 6(b) presents the PV smart inverters’ reactive power comparison between the SMIP model and OpenDSS under both scenarios. The average squared difference of the PV smart inverter’s reactive power output between the
SMIP model and OpenDSS is 0.548% and 0.089%, respectively, for the overand undervoltage scenarios, which are relatively small values. The differences are caused by the linearization of the power flow in our optimization program. The difference between the active power output of the smart inverters in the SMIP model and the OpenDSS in both scenarios is zero. It indicates that the optimally placed PV smart inverter with Volt-Var control can guarantee the customers’ economic benefit of maximizing their PV units’ active power output even under the worst voltage scenarios. At the same time, we compare the voltage magnitude difference of each bus node in both the SMIP model and OpenDSS in Fig. 7. As shown in Fig. 7, the maximum voltage magnitude difference between the power flow solution given by the optimization program and OpenDSS is less than 1%, whereas the error is less than 0.1% for 99.9% of the nodes. The differences are caused by the linearization of the power flow as mentioned, but there are two exceptions with voltage errors greater than 0.005 per unit. These two nodes have the largest load demand in the system and are connected to a transformer with a different voltage level from the other transformers in the system, which has resulted in the largest error between the SMIP model and OpenDSS. These results validate the accuracy and effectiveness of the proposed SMIP model in modeling the unbalanced distribution system operation considering the impact of PV smart inverter with Volt-Var control on the system voltage profile.

As only the worst voltage violation scenarios are considered, it is necessary to validate whether the optimal placement decisions work for other time instants. In total, two 24-h time-series power flow studies disabling and enabling Volt-Var control at the selected 99 PV smart inverters are conducted in the OpenDSS.
Fig. 8. 24-h time-series voltage magnitude, active power output, and reactive power output comparison for the 45th PV smart inverter disabling and enabling Volt-VAr control.

Fig. 8 shows the hourly time-series voltage magnitude, active power output, and reactive power output comparison for the bus node with the 44th placed PV smart inverter. This specific PV bus has the maximum voltage violation in the overvoltage scenario. It is found that this bus node, when Volt-VAr control is disabled, violates the normal operation voltage limit from $t = 11 \text{h}$ and reaches its maximum voltage magnitude at $t = 14 \text{h}$, which causes the worst overvoltage problem. With Volt-VAr enabled, there is no voltage violation issue during the entire 24-h operation as the reactive power absorption of the PV smart inverters helps to reduce the voltage.

C. Dynamic Voltage Stability Validation

To validate the voltage stability of the Volt-VAr control operation, a detailed dynamic model of the PV smart inverter with Volt-VAr control capability is developed based on previous work [30]. Fig. 9 shows the flowchart of the entire validation process, which can be described as follows:

1) model the dynamics of the PV smart inverter with Volt-VAr control in the real-valued time domain;
2) adopt the dynamic phasor transformation to transform the real-valued inverter model to a phasor-based model implemented in a DLL;
3) test the system stability in both static and dynamic simulations to validate the effectiveness of the optimization results.

Fig. 10(a) shows the simplified block diagram of the inverter power stage used for validation, which is numerically implemented in the DLL. The PLL is used to extract the reference angle $\theta_v(t)$ from the terminal voltage $v_t$. The proportional-resonant (PR) current controller can force the grid-side inductor current, that is, the terminal current $i_t$ of the inverter, to follow the current reference $i^r$ provided by the Volt-VAr control block. The inverter’s active and reactive power output can be controlled to conduct various grid support functions by adjusting the active current and the reactive current, separately. The detailed algorithm for Volt-VAr control is shown in Fig. 10(b). The Volt-VAr control block controls the power output by automatically adjusting the current reference $i^r$ according to the terminal voltage.
\[ V_1 < \theta_{el} \]. The scheme corresponds to reactive power priority mode, and hence, active power output may be curtailed if the inverter does not have enough capacity. The block diagram for the PR current controller is shown in Fig. 10(c), in which \( K_p = 0.7 \) is the proportional gain, and \( K_r = 200 \) is the resonant gain. \( V_{dc} \) is the dc link voltage and \( d \) is the duty cycle that is fed into the pulse-width modulation block. The PR current controller can ensure the terminal current \( i_t \) follows the command \( i^\ast \) generated from the Volt-VAr control block.

The optimally placed PV smart inverters disable the Volt-VAr control at time \( t = 0.0 \) s, and Volt-VAr control is enabled at \( t = t_1 \) under both scenarios in the dynamic studies. Fig. 11 shows the dynamic results of the 44th PV smart inverter in the overvoltage scenario, which has one of the largest voltage violations before enabling Volt-VAr control. It can be observed that a significant voltage drop is induced by the reactive power absorption of the PV smart inverter at time \( t = t_1 \). After the transient period, the active power can be maintained at its original value. The reactive power is kept at a higher value corresponding to the Q–V curve settings to maintain the voltage at a lower value. At \( t = t_2 \), the active power generations of all the 766 PVs in the system are reduced to 65% of their original output, which simulates a sudden cloud cover over the area. At \( t = t_3 \), the original outputs are recovered for all the PV systems. In this overvoltage scenario, active power reduction helps further reduce the voltages along the feeder, and as a result, the reactive power absorption reduces following the voltage reduction. Similarly, Fig. 12 shows the dynamic results of the first PV smart inverter in the undervoltage scenario. In the undervoltage scenario, however, the active power reduction tends to make the voltage even lower, and the presence of the Volt-VAr control helps maintain the voltage at a higher level by further injecting reactive power into the grid. These results show that even in a feeder, such as the utility partner’s feeder used in this study with PV penetration exceeding 100\%, it is possible to manage the feeder voltage profile and keep the system stable using only a relatively small number (99 of 767) of optimally placed PV smart inverters to provide Volt-VAr support.

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V. CONCLUSION

In this article, a two-stage stochastic mixed-integer linear programming model is proposed to determine optimal numbers and locations of PV smart inverters with Volt-VAr control to mitigate under/overvoltage conditions while minimizing the active power curtailment of PV units in active unbalanced distribution networks. In the first stage, the upgrading cost of PV smart inverter with Volt-VAr control is minimized, whereas the second stage minimizes the expected cost of active power curtailment of PV units and considers the detailed model of Q–V curve characteristics of PV smart inverter according to IEEE 1547 standard. With the optimally upgraded PV smart inverters, the system voltage profile can be maintained within the allowable range using only the local control capability of the PV systems without any other centralized communication mechanism. In addition, a detailed dynamic model of PV smart inverters is developed using DLL in OpenDSS to evaluate the distribution system’s steady-state performance and dynamic stability with the obtained optimal locations of the PV smart inverters under different voltage, load, and PV output scenarios. The results illustrate that the optimal location of PV smart inverters with Volt-VAr control can mitigate the worst over and undervoltage conditions of the utility feeder network within the allowable voltage requirement of the system without any PV active power curtailment. The proposed model utilizes the local control capability of the smart inverter without requiring communication between the control center and different PV smart inverters, which avoids possible adverse impacts of communication delays or failures. Also, the simulation results of the dynamic transition from inactive Volt-VAr control mode to active Volt-VAr control mode in the optimally located PV smart inverters at the worst voltage condition illustrate that the system remains stable while reactive power output of PV units is adjusted to maintain the system voltage at the normal operating range.

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