HADRON PROPERTIES IN THE VICINITY OF $T_c$ †

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ABSTRACT

Modifications of hadron masses and some of their basic properties with temperature or nuclear density are considered as one possible signatures for the formation of dense hadronic matter in nuclear collisions. We discuss here some basic results on the temperature dependence of hadron properties obtained from calculations of hadron correlation functions, the chiral condensate as well as the equation of state in finite temperature lattice QCD.

1. Introduction

The theory of strongly interacting particles – Quantumchromodynamics (QCD) – has many unusual non-perturbative features, which are under intensive theoretical and experimental investigation. It is the hope that some of the low energy characteristics of QCD like confinement and chiral symmetry breaking may become better understood through studies of strongly interacting matter, i.e. large systems of hadrons at high temperature and/or high baryon number density. This is, of course, also of relevance for our understanding of the evolution of the early universe.

Properties of strongly interacting matter are studied in ultra-relativistic heavy ion experiments. Through an analysis of the final state hadrons one tries to extract information about the dense thermal medium, created in these collisions. Basic properties of hadrons like their masses, widths and decay constants reflect many features of the complex, non-perturbative structure of the QCD vacuum. Any modifications of these hadronic properties at high temperature and/or density may lead to modifications in the particle spectra, which will lead to experimentally observable effects. It thus is important to understand theoretically the equilibrium properties of hadrons in dense/hot nuclear media, although the detailed description of heavy

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ion experiments will still involve further assumptions about non-equilibrium effects and the thermal evolution in a heavy ion collision\(^\dagger\).

Effective theories, deduced from QCD in order to describe low energy properties of hadrons, establish a close link between hadronic properties and the non-perturbative structure of the QCD vacuum, which is described by various non-vanishing condensates. This is, for instance, used in the operator product expansion (OPE) for correlation functions of hadronic currents\(^2\), which allows to relate hadron masses to condensates of the quark and gluon fields. The temperature and density dependence of the latter has been discussed within the context of chiral perturbation theory\(^3\). Through an extension of the OPE to finite temperature it is then possible to discuss the temperature dependence of hadron masses and other hadronic properties in a nuclear medium. We should, however, mention that this approach is not at all straightforward and led to some controversial discussion about the relevant states which should be used in the finite temperature expansion\(^4\).

We will concentrate here on a discussion of the influence of non-zero temperatures on hadronic properties\(^\dagger\). The applicability of the OPE as well as chiral perturbation theory is limited to low temperatures. In the vicinity of the QCD phase transition from hadronic matter to the quark gluon plasma phase, however, the hadronic medium becomes so dense that the influence of many particle states and resonances can no longer be neglected. In this regime an entirely non-perturbative approach becomes mandatory. This is the regime where lattice QCD might provide the only reliable approach to study the temperature dependence of condensates, hadrons and their properties.

We will discuss here some of the recent results of lattice calculations, which aim at an analysis of hadronic properties close to the QCD phase transition. In the next section we briefly review results about the thermodynamics of QCD, focusing on the equation of state and chiral properties in the vicinity of \(T_c\). Section 3 is devoted to a discussion of lattice calculations of hadron masses and decay constants at finite temperature within the quenched approximation. We briefly comment about the situation in two-flavour QCD in Section 4. Finally we give our conclusions in Section 5.

\section{2. Basic Thermodynamics from Lattice QCD}

Thanks to the rapid development of computer technology and the improvement\(^\dagger\) for a discussion of the influence of a non-vanishing baryon density we refer to the contribution of W. Weise to these proceedings.
of numerical algorithms for Monte Carlo calculations the studies of lattice QCD at finite temperature lead to a steadily improving understanding of details of the QCD phase transition. In the absence of dynamical quark degrees of freedom (pure gauge theory/quenched QCD) we have obtained quantitative results for the order of the transition, the transition temperature and equation of state on lattices of different size. This allows with some confidence an extrapolation to the continuum limit\textsuperscript{5,6}. Although simulations with light quarks did not yet reach a similar accuracy we know also here a lot about the qualitative behaviour of QCD at finite temperature.

As discussed in the introduction we expect that the properties of hadrons at finite temperature are closely related to changes in non-perturbative condensates in the QCD vacuum. With increasing temperature more and more hadronic states get excited and populate the vacuum. This is expected to result in decreasing values for the condensates. Finally, when the density of hadrons becomes too large the whole space is filled by hadronic bubbles which completely suppress the non-perturbative chiral condensate – the phase transition to the quark gluon plasma phase occurs and the chiral symmetry is restored above $T_c$.

This simplified picture of the QCD phase transition suggests a similarity with a percolation phase transition. In fact, the close relation between the density of the hadronic medium, a percolation threshold, and the occurrence of the QCD phase transition finds support in the numerical simulations of QCD with varying number of partonic degrees of freedom. With increasing number of light (nearly massless) quark flavours the phase transition has been found to occur at lower temperatures. This seems to be natural for a phase transition controlled be a percolation threshold: QCD with $n_f$ massless quarks describes a world with $(n_f^2 - 1)$ Goldstone particles (pions). At a given temperature the density of the hadronic medium thus will be proportional to $(n_f^2 - 1)$ and a critical density at which these hadrons start overlapping occurs earlier with larger $n_f$. While the change in the critical temperature is relatively small when changing the number of flavours from, in general, two to four, there is a large change between $n_f = 0$ (pure gauge theory) and $n_f = 2$. The transition temperature in QCD with two light quarks is found to occur at $T_c(n_f = 2) \approx 150\text{MeV}$, while in the purely gluonic theory, $n_f \equiv 0$, it is found to be substantially higher, $T_c(n_f = 0) \approx 230\text{MeV}$. This may also easily be understood in terms of a percolation threshold: In the absence of quarks there are no light hadrons, the lightest states are glueballs with a mass of $O(1 \text{ GeV})$. One thus needs a rather large temperature to excite enough of these heavy glueball states to reach a critical density.
Besides this change in the temperature scale and details of the phase transition itself – the transition is first order for $n_f = 0$ and likely to be second order for $n_f = 2$ – the temperature dependence of bulk thermodynamic quantities is very similar. In Fig.1 we show recent results for the energy density, $\epsilon$, and the pressure, $P$, in the $SU(3)$ gauge theory\textsuperscript{5} ($n_f = 0$) as well as two-flavour QCD\textsuperscript{7}. As can be seen, in both cases a substantial change in the effective number of degrees of freedom occurs only in the vicinity of $T_c$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Energy density and pressure for the pure $SU(3)$ gauge theory (a) and two-flavour QCD (b) as a function of $T/T_c$. The pure gauge theory results have been obtained from simulations on a large $32^3 \times 6$ lattice\textsuperscript{6} while those for two-flavour QCD so far exist only for a rather small lattice\textsuperscript{7} ($8^3 \times 4$). For more details on the inherent finite size effects in these lattice calculations we refer to the above papers as well as one of the reviews on finite temperature lattice QCD\textsuperscript{5}.}
\end{figure}

The numerical results for the energy density suggest that up to temperatures of about 0.9$T_c$ the hadronic medium is well described by a gas of hadrons, in which only the lightest states contribute. In the case of a $SU(2)$ gauge theory the critical energy density at $T_c$ has been studied in great detail\textsuperscript{8}. Here it is found that the deconfinement transition occurs at $\epsilon_c/T_c^4 \simeq 0.25$, which is only a factor five larger than the energy density of a glueball gas. Already below 0.9$T_c$ higher excited states play no role and one is left with a rather dilute glueball. A rough estimate yields values of about (0.1-0.2) for the number of glueballs per hadronic volume.
The slow variation of the energy density and pressure below 0.9\(T_c\) shows that also the gluon condensate will vary only little with temperature. The gluon condensate is directly related to the difference \(\epsilon - 3P\),

\[
\langle G^2 \rangle_T = \langle G^2 \rangle_0 - (\epsilon - 3P) ,
\]

where \(\langle G^2 \rangle_0\) denotes the gluon condensate at \(T = 0\). The small variation with \(T\), which can be deduced from Fig.1, is in accordance with chiral perturbation theory, which suggests that the temperature dependence of \(\langle G^2 \rangle_T\) starts only at \(O(T^8)\) (Ref. 3).

For the chiral condensate \(\langle \bar{\psi}\psi \rangle\) chiral perturbation theory suggests a stronger dependence on temperature, A

\[
\langle \bar{\psi}\psi \rangle(T) = \langle \bar{\psi}\psi \rangle(0) \left[1 - \frac{n_f^2 - 1}{n_f} \left(\frac{T^2}{12f_\pi^2}\right) - \frac{n_f^2 - 1}{2n_f^2} \left(\frac{T^2}{12f_\pi^2}\right)^2 + O(T^6)\right] .
\]

**Figure 2.** The chiral condensate at finite temperature extrapolated to the chiral limit \((m_q \equiv 0)\) and normalized to the corresponding zero temperature value for various values of the number of flavours (Fig.2a). In Fig.2b we show results from a simulation in quenched QCD \((n_f=0)\) for various values of the quark mass. The simulations have been performed at fixed value of the gauge coupling \(g^2\), \(6/g^2=6.0\), on a \(32^3\times8\) lattice, corresponding to \(T\lesssim 0.92T_c\) and on various low temperature lattices with sizes ranging from \(24^3\times32\) to \(32^3\times64\) (Ref.10). The condensates \(\langle \bar{\psi}\psi \rangle_1\) and \(\langle \bar{\psi}\psi \rangle_2\) are defined in Section 4. No temperature dependence is visible.
In Fig. 2 we show a collection of results obtained for QCD with various number of flavours. They suggest at most a weak dependence of the chiral condensate on temperature. However, in particular in the case of QCD simulations with light quarks this behaviour has to be confirmed in simulations on larger lattices. In the case of pure $SU(3)$ gauge theory simulations on large lattices have been performed and confirm that there is no significant temperature dependence of the chiral condensate up to $T \simeq 0.92T_c$. It thus seems to be conceivable that significant changes in the QCD condensates occur only quite close to $T_c$.

3. Hadronic Properties close to $T_c$

Information about hadron masses and decay constants can be extracted in lattice simulations from the long-distance behaviour of correlation functions of hadron operators

$$G_H(x) = \langle H(x)H^+(0) \rangle \to e^{-m_H|x|}, \quad x \equiv (\tau, \vec{x}),$$

with $H(x)$ denoting an operator with the appropriate quantum numbers of the hadronic state under consideration, for instance $H(x) = \bar{\psi}_u(x)\gamma_5\psi_d(x)$ for the pion. At zero temperature one studies the behaviour of the correlation function at large Euclidean times $\tau$. From the exponential decay of $G_H$ one deduces the hadron mass of the lightest state in this channel, whereas the amplitude is related to the decay constant of this hadronic state. At finite temperature the Euclidean extent is limited, $0 \leq \tau \leq 1/T$, and one thus studies the behaviour of $G_H$ for large spatial separations, $|\vec{x}| \to \infty$. This yields information about the finite temperature screening masses which are related to pole masses as long as there is a bound state in the quantum number channel under consideration.

Finite temperature screening masses have been studied in lattice simulations of QCD for quite some time. A very drastic qualitative change in the screening masses is seen when one crosses the QCD transition temperature. Parity partners become degenerate above $T_c$, the pseudo-scalar mass becomes massive and approaches the free quark/anti-quark value, $m_{meson} = 2\pi T$, at large temperatures. These features do not seem to depend much on the number of quark flavours. In particular, they also have been found in quenched QCD simulations. It thus seems to be meaningful to first study the behaviour of hadronic properties in the quenched approximation where results on large lattices with high accuracy can be obtained. In the following we will describe the results of such an investigation performed on
a rather large lattice \((32^3 \times 8)\) close to the phase transition temperature.

### 3.1. The GMOR Relation and the Pion Decay Constant

The GMOR relation relates the chiral condensate to the pion mass and pion decay constant,

\[
\frac{f_\pi^2 m_\pi^2}{m_q} = m_q \langle \bar{\psi}\psi \rangle_{m_q=0} .
\]

Below \(T_c\) the pion is a Goldstone particle, its mass squared depends linearly on the quark mass,

\[
m_\pi^2 = a_\pi m_q .
\]

A calculation of the chiral condensate and the pion mass at different values of the quark mass allows the determination of the pion slope, \(a_\pi\), and the zero quark mass limit of the condensate, \(\langle \bar{\psi}\psi \rangle_{m_q=0}\).

The pion decay constant \(f_\pi\) can be determined directly from the relevant matrix element

\[
\sqrt{2} f_\pi m_\pi^2 = \langle 0 | \bar{\psi} u \gamma_5 \psi d | \pi^+ \rangle .
\]

The square of the matrix element appearing on the right hand side of Eq.(6) is proportional to the amplitude of the pion correlation function and can be determined in a Monte Carlo calculation. A comparison of \(f_\pi\) determined this way with the ratio \(\langle \bar{\psi}\psi \rangle_{m_q=0}/a_\pi\) thus provides a direct test of the GMOR relation at finite temperature. This is shown in Fig.3a at \(T \simeq 0.92 T_c\). Moreover, we can compare the value of \(f_\pi\) calculated at finite temperature with corresponding zero temperature results. This is shown in Fig.3b. Clearly we do not have any evidence for violations of the GMOR relation nor for a significant change of \(f_\pi\) with temperature below \(T_c\). The sudden change above \(T_c\) reflects the drastic change in the structure of the pseudo-scalar correlation function. It does no longer have a pole corresponding to a Goldstone-particle (pion). The existence of a pseudo-scalar bound state above \(T_c\) thus is questionable. Certainly for large temperatures such a state does not exist, the correlation function is dominated by a quark/anti-quark cut. The notion of \(f_\pi\) used for the square root of the amplitude of the correlation function should thus be used with caution above \(T_c\).

### 3.2. Vector Meson Mass and Nucleon Mass

The possibility of a variation of the \(\rho\) meson mass with temperature has been discussed a lot as this might lead to modifications of dilepton spectra\(^1\), which are experimentally detectable. The temperature dependence of the meson masses has
Figure 3. Test of the GMOR relation at $T=0.92T_c$ in quenched QCD (a) and the temperature dependence of the pion decay constant (b). In Fig.3a we compare the results obtained from the amplitude of the pion correlation function through Eq.(6) (circles) and extrapolated to zero quark mass with the result obtained from the GMOR relation (star). Also shown is the zero temperature result at this value of the gauge coupling (square).

been discussed within the framework of the OPE. Arguments have been given that the meson masses are temperature independent up to $O(T^2)$ (Ref. 4). The Monte Carlo calculations of the vector meson correlation function at finite temperature also show no significant temperature dependence of the mass even close to $T_c$. In Fig.4 we show the result of our calculation of the screening mass in the vector channel correlation function at $T \simeq 0.92T_c$ and compare this with zero temperature calculations at the same value of the gauge coupling. There is no evidence for any temperature dependence. The same holds true for the nucleon mass, although the details are more subtle in this case. As can be seen in Fig.4b there is a clear difference between the local masses, $m_N(z) \sim \ln G_N(z)/G_N(z + 1)$, extracted on a large zero temperature lattice and those extracted at finite temperature on a lattice of size $32^3 \times 8$. However, as the nucleon is a fermion, there is a non-negligible contribution from the non-zero Matsubara mode to the nucleon screening mass,

$$m_N = \sqrt{\tilde{m}_N^2 + (\pi T)^2}$$

(7)

After removing the thermal contribution, $\pi T$, the nucleon mass, $\tilde{m}_N$, agrees with
Figure 4. The $\rho$-meson mass in quenched QCD for various values of the quark mass (a) and local masses from the nucleon correlation function (b). The simulations have been performed at fixed value of the gauge coupling, $6/g^2=6.0$, on a $32^3 \times 8$ lattice corresponding to $T \simeq 0.92 T_c$ and on various low temperature lattices (see Fig.2). In Fig.4b we show estimates for the nucleon mass obtained from the nucleon correlation function at distance $z$. These local masses converge to the nucleon mass in the limit $z \to \infty$. The horizontal band indicates the corresponding extrapolation at zero temperature. No temperature dependence is visible in the $\rho$-meson mass as well as the nucleon mass (see text for appropriate subtraction of the finite temperature Matsubara mode).

the zero temperature result within statistical errors.

4. Two-Flavour QCD

The discussion of hadron masses and decay constants in quenched QCD shows that there are no visible deviations from the zero temperature values up to temperatures $T \simeq 0.9 T_c$. One may expect that this is partly due to the fact that the phase transition in pure $SU(3)$ gauge theory is first order. The change from the hadron phase to the quark gluon plasma phase is more abrupt than in the case of a second order transition and might lead to less significant changes in condensates and hadronic properties.

The presently available simulations of QCD with two light quark flavours, indeed, suggest that the transition is smoother. In fact, the analysis of the scaling behaviour of various thermodynamic quantities with the quark mass seems to be
consistent with the behaviour expected for a second order chiral phase transition\textsuperscript{12}. We thus may wonder whether stronger modifications of hadron properties can be expected in this case.

\textbf{Figure 5.} The chiral condensate (a) and the chiral susceptibility (b) in two flavour QCD obtained from simulations on an $8^3 \times 4$ lattice\textsuperscript{12}.

In Fig.5 we show the behaviour of the chiral condensate and its derivative with respect to the quark mass – the chiral susceptibility $\chi_m$,

\begin{align}
\langle \bar{\psi}\psi \rangle_1 &= \frac{n_f}{4} \frac{T}{V} \frac{\partial}{\partial m_q} \ln Z, \\
\chi_m &= \frac{T}{V} \frac{\partial^2}{\partial m_q^2} \ln Z. \tag{8}
\end{align}

Also shown there is an improved estimator for the zero quark mass chiral condensate in which the $O(m_q)$ contribution to $\langle \bar{\psi}\psi \rangle_1$ is removed,

\begin{align}
\langle \bar{\psi}\psi \rangle_2 &= \langle \bar{\psi}\psi \rangle_1 - m_q \chi_m. \tag{9}
\end{align}

As can be seen $\langle \bar{\psi}\psi \rangle_1$ deviates from $\langle \bar{\psi}\psi \rangle_2$ strongly only in a narrow region around $T_c$. This indicates that only in this region contributions from the singular part of the free energy, which is responsible for the occurrence of a phase transition, is
dominant. Also the chiral susceptibility, $\chi_m$, has a very narrow peak at the pseudo-critical point. We thus expect that also in the case of two-flavour QCD substantial changes in hadronic properties will occur only in a narrow temperature regime close to $T_c(m_q) \equiv T_{\text{peak}}$, 
\[
\Delta_{\text{crit}} = \left| \frac{T_{\text{peak}}/2 - T_{\text{peak}}}{T_{\text{peak}}} \right| \lesssim 0.1 .
\] (10)

5. Conclusions

We have discussed some of the basic properties of QCD thermodynamics, which are relevant for a discussion of the temperature dependence of hadronic properties in the hadronic phase of QCD. There are strong indications that the gluon as well as the chiral condensates show at most a weak dependence on temperature for $T \lesssim 0.9 T_c$. So far no statistically significant temperature dependence could be observed in lattice QCD calculations. The same is true for hadron masses and decay constants calculated in quenched QCD.

At least for the chiral condensate one expects, however, a strong variation with temperature for $0.9T_c < T < T_c$. This is supported by the currently available lattice calculation in two-flavour QCD. The temperature variation of the gluon condensate, on the other hand, might not be that strong as indicated by the behaviour of the equation of state shown in Fig.1. The influence of this on the behaviour of hadronic parameters will certainly be investigated in the near future.

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