Finite-element method for obtaining the regularized photon Green function in lossy material

MENG TIAN, YONG-GANG HUANG(a), SHA-SHA WEN, HONG YANG, XIAO-YUN WANG, JIN-ZHANG PENG and HE-PING ZHAO

College of Physical Science and Mechanical Engineering, Jishou University - Jishou 416000, China

received 21 December 2018; accepted in final form 17 April 2019
published online 22 May 2019

PACS 32.70.Jz - Line shapes, widths, and shifts
PACS 02.70.Dh - Finite-element and Galerkin methods
PACS 42.50.-p - Quantum optics

Abstract – The photon Green function (GF) is a vital and decisive factor in the field of quantum light-matter interaction. It is divergent with two equal space arguments in lossy structure and should be regularized. We introduce a finite-element method (FEM) for calculating the regularized GF in arbitrary-shaped lossy structure, which is expressed by the averaged radiation electric field over the finite size of a photon emitter. For an emitter located in a homogeneous lossy material, excellent agreement with the analytical results is found for both real and virtual cavity models. For an emitter located in a metal nano-sphere, the regularized scattered GF, which is the difference between the regularized GF and the analytical regularized one in homogeneous space, agrees well with the analytical scattered GF. Applying this method for an emitter located in a metal nano-rod where there is no analytical solution, we find that the scattering contribution to the enhancement of the spontaneous emission rate is nearly unrelated to the cavity radius while the homogeneous part depends heavily. Our results should be significant for novel photon sources within lossy structures.

Introduction. – According to quantum electrodynamics, the modifications of the spontaneous emission rate and energy level shift of a quantum emitter in arbitrary structure can be expressed in terms of the classical photon Green function (GF) [1–13]. However, both the real part and imaginary part of the GF with two equal space arguments in lossy structure are divergent [14], which lead to an unphysical divergent spontaneous emission rate and energy level shift [15]. A real cavity model or a virtual cavity model is usually adopted (see ref. [16] and references therein), where the emitter is assumed to be in a small lossless cavity. Thus, the local field seen by the emitter is different from the macroscopic field. For the real cavity model, the lossless small cavity introduces a dielectric mismatch, which leads to additional scattering. In this case, the scattered GF can be used to express the spontaneous emission rate and the energy level shift of a point emitter. But for larger photon emitters such as quantum dots and macromolecules, regularized GF, which is the averaged GF over the photon emitter, may be more proper [17–21]. For the virtual cavity model, it is assumed that the field outside the fictitious cavity is the same as there is no cavity. The local field is the sum of the average macroscopic field and the internal field, where the internal field is the difference between the actual contribution and the averaged one of the molecules in the virtual cavity. For the Clausius-Mossotti type, the actual contribution is thought to be zero and the local field is expressed by the average macroscopic field [14,16]. So, singular behavior remains and regularization is also required [16].

According to the field theory [1], GF is expressed by the electric field of a radiating electric point dipole and can be obtained by a number of ways. For a nano-structure with high symmetry, such as homogeneous media, sphere, planar layered structure and cylinder waveguide, an analytical solution is available. But for a nano-structure with arbitrary shape, there is no analytical solution and numerical methods should be adopted (for example, see a recent review [22]). For arbitrary-shaped nano-structure, the finite difference time domain (FDTD) method and finite-element method (FEM) are two popular numerical methods. It has been shown that the FDTD method can be used to calculate the regularized GF [15]. The size of the mesh grid should be smaller than that of the regularization volume, which is nearly the size of the photon emitter and is usually in the scale of sub-nanometer.
According to the FDTD method, the smaller the mesh grid size is, the longer the computation time is. In addition, the staircase effect cannot be avoided. But for FEM, it is a general numerical method to solve differential equations (see refs. [22–26]). The flexible discretization strategy and the use of basis functions enable one to account for an extremely complex and sophisticated nano-structure, which is designed to manipulate the light-matter interaction [27,28]. Recently, FEM is used to calculate the scattered GF [29] and the regularized GF [17] for an emitter located around a plasmonic nano-structure. However, its performance for an emitter located in a lossy material is not clear.

In this work, we demonstrate that FEM can be accurately applied to compute the regularized GF in a lossy material. To check its accuracy, analytical results for an emitter located in a homogeneous lossy material and nano-sphere are presented as a reference. Both a real cavity model and a virtual cavity model are taken into account. We first describe the model and present the numerical method. Next, we make a comparison between the results by FEM and the analytical ones. Applying this method for an emitter located at the center of a metal nano-rod for which no analytical solution is available, we then investigate the role of the emitter size on the enhancement of the spontaneous emission rate and find that the scattering contribution is nearly unrelated to the cavity radius while the homogeneous part depends heavily.

Model and method. – To demonstrate the efficiency and accuracy of our method, we first investigate the case for an emitter located in lossy homogeneous material and nano-sphere, where analytical results are available. Models and parameters are the same as those in ref. [15]. Figure 1 shows the schematics of the geometries. The lossy material is chosen to be silver (Ag) with a permittivity given by the Drude model \( \varepsilon_2 = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2} \), where \( \varepsilon_\infty = 6.0 \), \( \omega_p = 7.89 \text{ eV} \) and \( \gamma = 0.051 \text{ eV} \). For an emitter located in a homogeneous lossy material, a virtual cavity model and a real cavity model are shown in fig. 1(a) and (b), respectively. For nonhomogeneous nano-structures, we choose an Ag nano-sphere with radius \( r_0 \) shown in fig. 1(c) and (d) for virtual and real cavity models, respectively. The regularization volume is chosen to be a small sphere around the emitter with radius \( a \) and permittivity \( \varepsilon_1 \). In these cases, analytical regularized GF can be obtained. For simplicity, we set \( \varepsilon_1 = \varepsilon_2 \) and \( \varepsilon_1 = 1 \) for the virtual cavity model and the real cavity model, respectively, although \( \varepsilon_1 \) should be the permittivity of the emitter. It should be noted that the regularization volume can be of any other shape, such as cubic or pyramid and permittivity in the small cavity \( \varepsilon_1 \) can be of any realistic value for quantum emitter.

Theoretically, the photon GF satisfies [30]

\[
\nabla \times \nabla \times G(r; r_0; \omega) - \varepsilon(r, \omega) \frac{\omega^2}{c^2} G(r; r_0; \omega) = \frac{\omega^2}{c^2} \delta(r - r_0),
\]

where \( I \) is a unit dyadic. Its component can be expressed by the radiation electric field at \( r \) of a radiating point dipole \( d \) at \( r_0 \). Explicitly, it is \( E(r) = G(r; r_0; \omega) \cdot d / \varepsilon_0 \). However, for \( r \to r_0 \) in the source region, the electric field \( E(r) \) is divergent and regularization is required. The regularized electric field can be written as

\[
E_{\text{reg}}(r_0) = \frac{1}{\Delta V} \int_{\Delta V} E(r) dr,
\]

where \( \Delta V \) is the volume around \( r_0 \) and \( \varepsilon_0 \) is the permittivity of the medium. The self-term \( G_s \) and the regularized GF \( G_{\text{reg}} \) for an emitter located in a homogeneous material is defined as [27,28]

\[
G_{\text{reg}}(r_0; \omega) = G_s(r_0; \omega) + \frac{M}{\Delta V} - \frac{L}{\Delta V}.
\]

Here the regularized scattered GF is \( G_{\text{reg}}(r; r_0; \omega) = \int_{\Delta V} G_s(r; r_0; \omega) dr / \Delta V \). For a cubic or sphere principal volume, the source term is \( L = I / 3 \varepsilon_0 \). The self-term \( M \) can be numerically calculated for a cubic volume [20]. But for a sphere principal volume of radius \( a \), it can be analytically obtained and reads \( M = 2(1 - i k a)e^{ika} - 1 / 3 \varepsilon_0 \), where \( k = \sqrt{\varepsilon_0 \omega / c} \) and \( \varepsilon_0 \) is the relative permittivity of the quantum emitter.
Numerically, the regularized GF $G^{reg}(r_0; r_0; \omega)$ can be obtained from eq. (2), where the regularized electric field $E^{reg}(r_0)$ is calculated by first evaluating the radiation field $E(r)$ of a point dipole located at $r_0$ and then averaging it over the regularization volume $\Delta V$ (see eq. (1)). Thus, any modeling approach that can calculate the radiation field $E(r)$ of a point dipole can be applied. FEM is one of the most proper methods for this problem, especially for an emitter located in a nano-structure with arbitrary shape. The wave equation is transformed into its variational form, in the full-wave three-dimensional simulations based on FEM (COMSOL Multiphysics), the radiating electric field $E(r)$ of a point dipole has been calculated $[17, 29, 32]$. Although for arbitrary nano-structure, the regularized GF are independent and agree well with the analytical for the emitter located in a non-lossy material $[17, 29]$. For arbitary nano-structure, the procedure presented in refs. $[17, 29]$ can be used. But for the models considered in this work, an axial symmetry can be used to reduce the computational cost with a so-called 2.5D implementation in this work, an axial symmetry can be used to reduce the computation burden. For our models, a point dipole is located on the z-axis with $p = dz$. The current density is $J = -i \omega \varepsilon \delta(z - z_0)$ which only contains the $m = 0$ term in the cylindrical harmonics decomposition. Thus, the 2.5D method (eq. (6) with $m = 0$) can be applied for our system.

To conclude this section, we first solve eq. (6) for a point dipole source to obtain the radiating electric field $E(r)$. Then, $E^{reg}(r_0)$ can be evaluated from eq. (1). By eq. (2), the component of the regularized GF can be obtained easily. For arbitrary nano-structure, the regularized scattered GF $G^{sreg}_s(r_0; r_0; \omega)$ can be obtained by subtracting the analytical regularized homogeneous GF $G^{reg}_0(r_0; r_0; \omega) = \frac{\varepsilon_0}{\mu_0} - \frac{\varepsilon}{\mu}$ (the last two terms in eq. (3)) from the regularized GF $G^{reg}(r_0; r_0; \omega)$.

**Numerical results.** We first investigate the homogeneous case (models shown in fig. 1(a) and fig. 1(b)), where analytical regularized GF can be found. For the virtual cavity model (fig. 1(a)), the analytical regularized GF is $G^{reg}_0(r_0; r_0; \omega) = \frac{M}{L}$ which can be obtained from eq. (3) with $G^{reg}_s(r_0; r_0; \omega) = 0$. But for the real cavity model (fig. 1(b)), the regularized scattered GF $G^{sreg}_s(r_0; r_0; \omega)$ is replaced by the scattered GF $G_{sreg}(r_0; r_0; \omega)$ with the assumption that $G_{sreg}(r; r_0; \omega) \approx G_s(r; r_0; \omega)$ for $r$ around $r_0$. In addition, $G_{sreg}(r_0; r_0; \omega)$ can be rigorously obtained from the Mie theory $[30, 34, 35]$.

Results based on FEM and the analytic for the regularized GF are shown in fig. 2. Here, the radius for the regularized sphere is $a = 1$ nm. We find that numerical results agree well with the analytical solutions, both for the virtual cavity model (imaginary part shown in fig. 2(a) and the real part in fig. 2(b)) and for the real cavity model (fig. 2(c) and fig. 2(d) for the imaginary part and real part, respectively). The relative errors for the imaginary part (see the insets in fig. 2(a) and fig. 2(c)) are less than 0.5% over a wide frequency range. In fig. 2(a), the large peak
Fig. 2: Regularized GF for homogeneous lossy material with $a = 1$ nm. (a) and (b) are the imaginary part and the real part for the virtual cavity model. (c) and (d) correspond to the real cavity model. Red circles are results from the FEM and black lines are for the analytic. Their differences are shown in the insets. The relative errors are defined by the ratio of the differences to the analytical ones.

At 3.22098 eV corresponds to the real part of the root for $\varepsilon = 0$. This is slightly different from 3.23 eV obtained by FDTD in ref. [15]. But for fig. 2(c), the large peak changes to 3.09459 eV, which corresponds to the real part of the zeros for the denominator $C_2^N$ with $n = 1$ (see eq. (26b) in ref. [34]). For the real part, errors defined as the difference between the numerical results and the analytical ones are two orders lower than their values (see the insets in fig. 2(b) and fig. 2(d)). These results demonstrate that FEM is accurate and can be applied to investigate the renormalized GF in a homogeneous lossy material for both virtual and real cavity models.

The above results are for $a = 1$ nm. In fig. 3, we investigate the effect of the size for the emitter on the regularized GF. Here, the frequency is set to $\omega = 3$ eV for simplicity. Figures 3(a) and (b) are the results for the virtual cavity model and figs. 3(c) and (d) are for the real cavity model. We observe that numerical results also agree well with the analytic (to within less than 0.2%). In addition, all the results decrease as a function proportional to the radius of the small sphere $a^{-3}$. For the virtual cavity model, this can be clearly seen from eq. (3) where the first term is zero and the self-term $M$ is about five orders lower than the source term $L$ for the radius considered. For the real cavity model, this is consistent with the result in ref. [16] (for example, see eq. (52) therein for extremely small $a$), which is attributed to the character of the dipole-dipole interaction between the atom and the medium. Then, we turn to the case for an emitter located at the center of a silver nano-sphere (shown in fig. 1(c) for the virtual cavity model and fig. 1(d) for the real cavity model). For both the virtual cavity model and the real cavity model, the analytical results are obtained from eq. (3) with the assumption that the scattered GF $G_s(r; r_0; \omega)$ varies slowly with the space argument $r$. Thus, the regularized scattered GF $G_{reg}^s(r_0; r_0; \omega)$ is replaced by the scattered GF $G_s(r_0; r_0; \omega)$, which can be calculated by the technique in ref. [34]. From fig. 4, we find that numerical results also agree well with the analytical ones and their differences shown in the insets are also very small.

In addition, the regularized GF for the emitter at the center of the nano-sphere are similar to those for the emitter in a homogeneous lossy material. This can be seen by comparing the results in fig. 4 with those in fig. 2.
Finite-element method for obtaining the regularized photon Green function in lossy material

Gones. Here, the solid line is the analytical scattered GF that numerical results also agree well with the analytical. In fig. 5, we show the results for the scattered GF. We find the sphere is far weaker than the homogeneous part. In fig. 5(c) and (d) correspond to the real cavity model. (c) and (d) represent the differences between the case for nano-sphere and the homogeneous case. Red circles are results from the FEM and black lines are for the analytic. Their differences are shown in the insets in (a) and (b).

Thus, the scattering part from the surface of the nano-sphere is far weaker than the homogeneous part. In fig. 5, we show the results for the scattered GF. We find that numerical results also agree well with the analytical ones. Here, the solid line is the analytical scattered GF $G_{\text{a}}(r_0; r_0; \omega)$ and the red circles are for $G_{\text{a}}^{\text{reg}}(r_0; r_0; \omega) = G_{\text{a}}^{\text{reg}}(r_0; r_0; \omega) - G_{\text{a}}^{\text{reg}}(r_0; r_0; \omega)$ where the regularized GF $G_{\text{a}}^{\text{reg}}(r_0; r_0; \omega)$ is obtained by FEM as those in fig. 4 and the regularized homogeneous GF is analytically obtained by $G_{\text{a}}^{\text{reg}}(r_0; r_0; \omega) = \frac{M_{\text{a}}}{M_{\text{a}} - 3\omega}$. Compared to the homogeneous results shown in figs. 2(a) and (b), the magnitudes for the regularized scattered GF are about four orders weaker for the virtual cavity model shown in figs. 5(a) and (b). But for the real cavity model, the regularized scattered GF shown in fig. 5(c) and fig. 5(d) looks similar to those shown in figs. 2(c) and (d). Their differences are shown in fig. 5(e) and fig. 5(f). Its magnitude is also about three orders lower than the total scattered GF. Thus, for both the virtual cavity model and the real cavity model, the scattering at the surface of the nano-sphere has little effect. In addition, we find that numerical results agree well with the analytical ones. These results clearly demonstrate that FEM can be able to exactly calculate the regularized scattered GF, although it is the small difference between the total GF and the homogeneous GF.

From the above results for emitter located in a homogeneous lossy material and a metal nano-sphere, we can conclude that FEM is an efficient and accurate method for calculating the regularized total GF and scattered GF for both the virtual cavity model and the real cavity model. In the following of this section, we apply this method to the case for an emitter located at the center of a metal nano-rod, where there is no exact analytical solution and a numerical method should be adopted. The model and parameters shown in fig. 6(a) are the same as those in ref. [36]. A gold nano-rod with a length of $L = 100$ nm and radius $R = 15$ nm is located in oil.

As demonstrated in ref. [36], regularized GF can be written as $G^{\text{reg}} = G_0^{\text{reg}} + G_0^{\text{reg}}$ and its imaginary part can be used to express the total enhancement of the spontaneous emission rate $F = F_0 + 2s$, where $F_0/s = \frac{\Im[n \cdot G_0^{\text{reg}} \cdot n]}{\Im[n \cdot G_0^{\text{reg}} \cdot n]}$ with $\Im[n \cdot G_0^{\text{reg}} \cdot n] = \omega^3/(6\pi c^3)$. The homogeneous part $G_0^{\text{reg}}$ can be obtained with the two-layer sphere model [35] and its imaginary part is $\Im[n \cdot G_0^{\text{reg}} \cdot n] \approx \frac{9\omega c}{2\pi}$. There is no analytical solution for the scattering part. It has been demonstrated by FDTD that the larger the real cavity radius is, the larger the ratio of $F_0$ to $F_0$ is (shown in fig. 1 in ref. [36]). Here, we apply our FEM method to show the enhancement of the spontaneous emission rate $F$ for two different cavity radiuses.

Figures 6(b) and (c) are for $a = 0.62$ nm and $a = 2$ nm, respectively. The red circles are for the total enhancement $F$ by our FEM and the black solid line is for the homogeneous approximate solution $F_0$. For a small cavity radius

Fig. 5: Regularized scattered GF for emitter located at the center of a silver nano-sphere with $a = 1$ nm. (a) and (b) are the imaginary part and the real part for the virtual cavity model. (c) and (d) correspond to the real cavity model. (e) and (f) represent the differences between the case for nano-sphere and the homogeneous case. Red circles are results from the FEM and black lines are for the analytic.
(fig. 6(b)), the difference between the total enhancement $F$ and the approximate homogeneous contribution $F_0$ is small, which means that the scattering contribution is negligible. The reason is that $3|n \cdot G_0^\text{reg} \cdot n|$ shows a $1/a^3$ dependence, which is extremely large for a small cavity radius $a$. But for a larger cavity radius (fig. 6(c)), the difference between the total $F$ and the homogeneous approximate $F_0$ is large. Thus, we cannot ignore the scattering contribution in this case. Comparing in figs. 6(b) and (c), we observe a large difference for the homogeneous contribution $F_0$ between the above two different cavity radiuses. Figure 6(d) shows the scattering contribution $F_s = F - F_0$ for the above two radiuses. We find that they look almost the same. This is consistent with the usual assumption that the scattering GF varies slowly with the space argument and with the statement in ref. [36] that $F_s$ stems from the localized surface plasmons of the Au nano-rod for the emitter located at the center. In addition, $F_s$ shown in fig. 6(d) looks the same as that shown in fig. 2(b) of ref. [36].

Conclusions. – In conclusion, we have shown that FEM is an efficient and accurate tool for calculating the regularized total GF and scattered GF for both the virtual cavity model and real cavity model in a lossy material with arbitrary shape. The regularized total GF is expressed by the averaged electric field, which can be exactly obtained by solving the wave equation through FEM. For the regularized scattered GF, it is expressed by the difference between the total regularized GF and the analytical ones in homogeneous cases. We have confirmed that numerical results from our method agree well with the analytic ones when the emitter is located in a homogeneous lossy material and a metal nano-sphere. By our method for the emitter located at the center of a Au nano-rod for which no analytical solution is available, we have found that the enhancement of the spontaneous emission rate is very large due to the homogeneous contribution for a small cavity radius. The scattering contribution from the localized surface plasmon is nearly unrelated to the cavity radius. For the flexible discretization strategy and the use of basis functions in FEM, our method can be exactly applied in nano-photonic structure with any shape and material, for example a lithium niobate waveguide with large nonlinearity, anisotropy and electro-optical effect [37]. As is well known, GF plays an important role as a theoretical tool in standard physics subjects such as perturbation and scattering theory, bound-state formation, etc. [38], which have found applications not only in classical wave physics in both periodic and random media, photonic and phononic crystals but also in condensed-matter physics and quantum physics [39–45]. Our FEM method for calculating the regularized GF can be used to investigate the enhancement of the spontaneous emission rate, energy level shift (Lamb shift) and dispersion force of a quantum emitter due to its interaction with the electromagnetic environment [5, 6, 11, 46, 47].

**REFERENCES**

1. Novotny L. and Hecht B., Principles of Nano-Optics (Cambridge University Press) 2006.
2. Agarwal G. S., Quantum Statistical Theories of Spontaneous Emission and Their Relation to Other Approaches (Springer) 1974.
3. Jung H. T., Buhmann S. Y., Knöll L., Welsch D.-G., Scheel S. and Kästel J., Phys. Rev. A, 68 (2003) 043816.
4. Wubs M., Suttrop L. G. and Lagendijk A., Phys. Rev. A, 70 (2004) 053823.
5. Buhmann S. Y., Dispersion Forces I (Springer, Berlin, Heidelberg) 2012.
6. Buhmann S. Y., Dispersion Forces II (Springer, Berlin, Heidelberg) 2012.
7. Fuchs S. and Buhmann S. Y., EPL, 124 (2018) 34003.
8. Chen X., Yu W., Yue W., Yao P. and Liu W., EPL, 111 (2015) 17004.
9. Hughes S., Richter M. and Knorr A., Opt. Lett., 43 (2018) 1834.
10. Lu Y. W., Li L. Y. and Liu J. F., Sci. Rep., 8 (2018) 7115.
11. Liu R., Zhou Z.-K., Yu Y.-C., Zhang T., Wang H., Liu G., Wei Y., Chen H. and Wang X.-H., Phys. Rev. Lett., 118 (2017) 237401.
12. Zhang Y., Meng Q. S., Zhang L., Luo Y., Yu Y. J., Yang B., Zhang Y., Esteban R., Aizpurua J. and Luo Y., Nat. Commun., 8 (2017) 15225.
13. Chen B.-Q., Zhang C., Li J., Li Z.-Y. and Xia Y., Nanoscale, 8 (2016) 15730.
14. Jackson J. D., Classical Electrodynamics (John Wiley & Sons) 2012.
15. Van C. and Hughes S., Opt. Lett., 37 (2012) 2880.
16. Scheel S., Knöll L. and Welsch D.-G., Phys. Rev. A, 60 (1999) 4094.
17. Yun-Jin Z., Meng T., Yong-Gang H., Xiao-Yun W., Hong Y. and Xian-Wu M., Acta Phys. Sin., 19 (2018) 193102.
18. Yaghjian A. D., Proc IEEE, 68 (1980) 248.
19. Martin O. J. F., Phys. Rev. E, 58 (1998) 3999.
20. Chaumet P. C., Sentenac A. and Rahmani A., Phys. Rev. E, 70 (2004) 036606.
21. Scheel S., Knöll L., Welsch D.-G. and Barnett S. M., Phys. Rev. A, 60 (1999) 1590.
22. Gallinet B., Butet J. and Martin O. J. F., Laser Photon. Rev., 9 (2015) 577.
23. Zienkiewicz O. C., Taylor R. L., Nithiarasu P. and Zhu J., The Finite Element Method, Vol. 3 (McGraw-Hill, London) 1977.

This work was financially supported by the National Natural Science Foundation of China (Grants Nos. 11464014, 11347215, 11564013, 11464013), Hunan Provincial Innovation Foundation For Postgraduate (Grants No. CX2018B706) and Natural Science Foundation of Hunan, China (Grant No. 2016JJ4073).
Finite-element method for obtaining the regularized photon Green function in lossy material

[24] Liu H.-X., Liu K.-M. and Niu Y.-X., EPL, 119 (2017) 24003.
[25] Liu L., Li D. and Dong L., EPL, 123 (2018) 16002.
[26] Zhao Y., Huang M., Tang J., Ouyang X. and Morita C., Nucl. Eng. Des., 342 (2019) 115.
[27] Li Z. Y., EPL, 110 (2015) 14001.
[28] Burresi M., Pratesi F., Riboli F. and Wiersma D. S., Adv. Opt. Mater., 3 (2015) 722.
[29] Zhao Y.-J., Tian M., Wang X.-Y., Yang H., Zhao H. and Huang Y.-G., Opt. Express, 26 (2018) 1390.
[30] Van Vlack C., Kristensen P. T. and Hughes S., Phys. Rev. B, 85 (2012) 075303.
[31] Chen Y., Nielsen T. R., Gregersen N., Lodahl P. and Mørk J., Phys. Rev. B, 81 (2010) 125431.
[32] Bai Q., Perrin M., Sauvan C., Hugonin J.-P. and Lalanne P., Opt. Express, 21 (2013) 27371.
[33] Ciraci C., Urzhumov Y. and Smith D. R., Opt. Express, 21 (2013) 9397.
[34] Li L. W., Kooi P. S., Leong M. S. and Yee T. S., IEEE Trans. Microwave Theory Tech., 42 (1994) 2302.
[35] Tomaš M. S., Phys. Rev. A, 63 (2001) 053811.
[36] Ge R.-C., Young J. F. and Hughes S., Optica, 2 (2015) 246.
[37] Wang M., Wu R., Lin J., Zhang J., Fang Z., Chai Z. and Cheng Y., Quantum Eng., 1 (2019) e9.
[38] Economou E. N., Green’s Functions in Quantum Physics, Vol. 3 (Springer) 1983.
[39] Wubs M., Suttrop L. and Lagendijk A., Phys. Rev. A, 70 (2004) 053823.
[40] Zhou L., Gong Z. R., Liu Y.-X., Sun C. P. and Nori F., Phys. Rev. Lett., 101 (2008) 100501.
[41] Yang C.-J. and An J.-H., Phys. Rev. B, 95 (2017) 161408.
[42] Agram N., Yeyati A. L. and Van Ruttenbeek J. M., Phys. Rep., 377 (2003) 81.
[43] Wang X.-H., Gu B.-Y., Wang R. and Xu H.-Q., Phys. Rev. Lett., 91 (2003) 113904.
[44] Liu D. E., Quantum Eng., 1 (2019) e10.
[45] Dung H. T., Buhmann S. Y., Knöll L., Welsch D.-G., Scheel S. and Kästel J., Phys. Rev. A, 68 (2003) 043816.
[46] Huang Y.-G., Chen G., Jin C.-J., Liu W. M. and Wang X.-H., Phys. Rev. A, 85 (2012) 053827.
[47] Tian M., Huang Y.-G., Wen S.-S., Wang X.-Y., Yang H., Peng J.-Z. and Zhao H.-P., arXiv preprint, arXiv:1902.06387 (2019).