Constancy of any signal velocity in all inertial frames

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Abstract

If isotropy of space and homogeneity of space and time are the valid laws of nature then one can show that velocity of any signal (whether it is light in a vacuum or in a medium or something having constant velocity in the rest frame of the observer and by using which we can perform all (but not partial) measurements of all physical quantities related with the particular experiment) is constant in all inertial frames. To verify this constancy for other signals apart from light in vacuum (which is well-known) we have proposed particularly one experiment in which all measurements can be done using light in water as signal. If we perform experiment in a medium then the signal connecting the measurements of an event in two inertial frames can not be light in vacuum rather it is light in the medium. In such cases where signal used for measurement is not light in vacuum, relativistic relationships of various physical quantities in Special Theory of Relativity (in which light in vacuum has been used as signal) require modifications. This work might be verified from the analysis related with the number of cosmic muons reaching earth through atmosphere or that related with the amount of oscillation of atmospheric neutrinos as in both these cases signal for our measurement can not be light in vacuum but it is light in air.

PACS NO. 03.30.+p

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Einstein’s Special theory of relativity (STR) takes into account the important fact that signal velocity is finite and the measurement of physical quantities in two inertial frames depends upon this velocity. There are two axioms in Einstein’s STR. One is that the laws of physics are same in all inertial frames and another is the constancy of velocity of light (in vacuum) in all inertial frames. Light in vacuum is the signal providing the link between the measurements of physical quantities in inertial frames. However, if we perform experiment in a medium then the signal connecting the measurements in two inertial frames can not be light in vacuum rather it is light in the medium. Then two questions arise. Firstly how to modify the relativistic relationships when signal is not light in vacuum and secondly whether only light in vacuum has a fundamental property that the velocity of it will be constant in all inertial frames or this property is valid for other signals also when signal velocity is measured by using the signal itself for the measurements of length and time associated with the measurement of velocity of the signal.

It may be noted that if one uses another signal instead of light in vacuum (which is considered as signal in special theory of relativity) still clocks in the rest frame of the observer measure standard time. Synchronization of two clocks in the observer’s rest frame is same whatever signal we use. This is because any signal one may use is supposed to have some constant velocity in the rest frame of the observer and one can appropriately adjust the time of two clocks placed at two different places in the rest frame by considering the time taken by that signal to travel from one clock to the other. In relating measurements of an event done in two inertial frames light in vacuum has been used in special theory of relativity. But there are situations when light in vacuum can not be the available signal. In that case it has been discussed in this letter, what should be the relationship between the measurements in two inertial frames. We have discussed here that although clocks will measure standard time in the rest frame of the observer but the relationship of two measurements of the same event in two inertial frames will depend upon the signal which has been used depending upon the experiment. Although measurements in the moving frame using different signals are different but those can be easily related provided that we know the velocity of different signals in the rest frame of the observer. This has been discussed later.

We have shown that any signal velocity remains constant in different inertial frames where signal is that using which we can perform all (not partial 1) measurements of all physical quantities corresponding to a particular experiment. This has been derived by assuming that isotropy of space and homogeneity of space and time are fundamental laws of physics for all inertial frames2. General transformation of space and time coordinates for two inertial frames for any signal which is obtained is same as those in special theory of relativity with only replacement of velocity of light in vacuum in the transformation equations by the velocity of any signal considered for all measurements of an experiment. At the end, to verify the constancy of velocity of the signal (other than light in vacuum) we have proposed one experiment in which light in water has been considered as signal which is to be used for all measurements in that experiment. If velocity of light in water does not change in another inertial frame (which we expect from our analysis) then there will be no shift in the interference fringe in this experiment. Experiments on cosmic muon decay in the atmosphere or experiments on atmospheric neutrino oscillation (where signal can not

1Suppose in part of the experiment light in vacuum is the signal and in another part of the same experiment light in air is the signal. In this case if one considers two inertial frames then according to our work velocity of the signal which is higher will remain constant whereas the velocity of the signal which is lower i.e., velocity of light in air will be found to change in different inertial frames.

2It has also been assumed that the relative velocity of two inertial frames is less than the signal velocity which is discussed later.
be light in vacuum rather it is light in air) might give some evidence in support of the work presented here.

Without considering the constancy of velocity of light some authors \[1\] have obtained the relativistic transformations of space-time coordinates in which they have obtained a constant which on dimensionality ground have been related with the velocity of light in vacuum which is known experimentally as constant. In our approach in obtaining transformation rules for space-time in different inertial frames we also have not consider the constancy of velocity of light in vacuum. But we first prove that not only light in vacuum but any signal velocity is constant in all inertial frames. Then we obtain the transformation rules which are valid for any signal velocity.

An event in one inertial frame \(S\) is characterized by the coordinates \(x, y, z, t\) and the same event as observed in another inertial frame \(S'\) is characterized by the coordinates \(x', y', z', t'\). We consider that space and time are homogeneous and space is isotropic. Let \(S'\) is moving with velocity \(v\) with respect to \(S\) along the positive \(x\) direction and we assume that the relative velocity \(v\) of the two frames are along \(x\) and \(x'\) axes. Let us consider that the points \(x = 0\) and \(x' = 0\) coincide when \(t = 0\) and also consider that \(t' = 0\) then. As space, time are considered as homogeneous so the transformation equations are linear. As the \(x\) axis coincides always with the \(x'\) axis and as the length of the same rod at rest in any inertial frames should be same, one can show that \(y' = y\) and \(z' = z\). Now we have to find only \(x'\) and \(t'\) in terms of the coordinates \(x\) and \(t\) of the event measured in the \(S\) frame. Apart from considering \(x = 0\) we may consider \(y = 0\) and \(z = 0\) when \(t = 0\) without losing any generality in our approach to find the transformation equations.

Let us analyse at some non-zero value \(t = T\) what is the value of \(x\) in the stationary frame corresponding to moving frame point \(x' = 0\). According to our consideration at \(t = 0\), point \(x = 0\) of \(S\) frame coincides with \(x' = 0\) of \(S'\) frame. Also according to our initial consideration the way \(S'\) is moving for that \(x' = 0\) point will move with velocity \(v\) with respect to \(x = 0\) point in the \(S\) frame. So \(x' = 0\) corresponds to \(x = v T\). Such correspondence is true for any value of \(t\). Considering this and also taking into account that the transformation should be linear we can relate \(x'\) with any value of \(x\) and \(t\) in the stationary frame as

\[
x' = a \ (x - vt) \tag{1}
\]

where \(a\) is independent of \(x\) and \(t\).

Next we like to analyse at some non-zero value \(x = X\) what is the value of \(t\) in the stationary frame corresponding to moving frame time \(t' = 0\). According to our consideration for an event occurring at \(x = 0\), the corresponding times in the \(S\) frame and \(S'\) frame are \(t = 0\) and \(t' = 0\) respectively. To represent the distance \(X\) from point \(x = 0\) where event has occurred we imagine that two rods \(AB\) and \(A'B'\) are placed in the \(S\) frame and \(S'\) frame respectively as shown in Fig 1 (a). As observed in the \(S\) frame at \(t = 0\) both are of equal length \(X\) and the end points of these rods \(A\) and \(A'\) coincide with \(x = 0\) and \(B\) and \(B'\) coincides with \(x = X\). To find time at a distance \(X\) corresponding to the event occurred at \(x = 0\) and \(t = 0\) at first we have to get common time for both these points or to synchronize the two clocks placed at these two points. For this we have to send signal from \(x = 0\) to \(x = X\). To synchronize the clocks we can set the time of the clock at \(x = X\) to \(X/v_s\) when the signal reaches \(x = X\). Here \(v_s\) is the signal velocity in the rest frame \(^3\). According to our imagination it is like sending signal from one end \(A\) to the other end \(B\) of the rod in the stationary frame to know \(t = 0\) and sending signal from one end \(A'\) to

\(^3\)As stated earlier the synchronization of two clocks in the rest frame of the observer is independent of the signal which has been used.
the other end $B'$ of other rod placed in the moving frame to know $t$ value corresponding to $t' = 0$ of the moving frame. Signal starts at $t = 0$ which is same as telling signal starts at $t' = 0$. When the signal reaches $x = X$ i.e, $B$ of the other end of the rod the stationary frame observer finds that his/her signal is yet to cover $X$ distance in the moving frame at that time i.e, yet to reach the other end $B'$ of the rod placed in the moving frame. The time taken by the signal to reach $x = X$ is $X/v_s$. So when signal reaches $B$ the distance between $B$ and $B'$ is $vX/v_s$ which is shown in Fig 1 (b). The signal in the stationary frame will need extra time $vX/v_s^2$ to cover this distance. So although $t = 0$ and $t' = 0$ are simultaneous but the signal reaching $X$ distance in two frames as observed by the stationary observer is not simultaneous and differs by $vX/v_s^2$ time. So at distance $X$, $t' = 0$ correspond to $t = vX/v_s$ in the stationary frame. Such correspondence is true for any other values of $x$ also. Considering this and and also taking into account that the transformation should be linear we can relate $t'$ with any value of $t$ and $x$ in the stationary frame as

$$t' = b \left( t - \frac{vx}{v_s^2} \right)$$

(2)

where $b$ is independent of $x$ and $t$.

Using eq. (1) & (2) we can write $x$ in terms of $x'$ and $t'$ as

$$x = \frac{1}{a} \left( 1 - \frac{v^2}{v_s^2} \right) \left( x' + \frac{a}{b} vt' \right)$$

(3)

If we consider the isotropy of space and consider $S$ as the moving frame then it is moving with $-v$ velocity with respect to $S'$ frame [2]. In that case similar to equation (1) we can write

$$x = a' \left( x' + vt' \right)$$

(4)

where $a'$ is independent of $x'$ and $t'$. Comparing this with eq. (3) we find that

$$a = b \quad a' = \frac{1}{a} \left( 1 - \frac{v^2}{v_s^2} \right)$$

(5)

Using eq. (1) & (2) we can write $t$ in terms of $t'$ and $x'$ as

$$t = \frac{1}{a} \left( 1 - \frac{v^2}{v_s^2} \right) \left( t' + \frac{b}{a} \frac{vx'}{v_s^2} \right)$$

(6)

Considering $S$ as the moving frame, similar to equation (2) we can write

$$t = b' \left( t' + \frac{vx'}{v_s^2} \right)$$

(7)

where $b'$ is independent of $x'$ and $t'$. Comparing this with eq. (6) we find that

$$a = b \quad b' = \frac{1}{b} \left( 1 - \frac{v^2}{v_s^2} \right)$$

(8)

4It is clear that for $v = v_s$, signal in the rest frame which starts at $A'$ at $t = 0$ will never reach $B'$. In other words, we have assumed that $v < v_s$.  

4
We show here that the velocities of signal as measured in both $S$ and $S'$ frame are same. Let us consider that something is moving with velocity $u'$ as measured in the $S'$ frame and $u$ as measured in the $S$ frame. Then we can write

$$x' = u't'$$  \hspace{1cm} (9)

and using eqs. (1) and (2)

$$x = \frac{v + \frac{b}{a}u'}{1 + \frac{bu'}{av^2}} t$$  \hspace{1cm} (10)

But as $x = ut$ and $b = a$ so

$$u = \frac{v + u'}{1 + \frac{u'v}{v^2}}$$  \hspace{1cm} (11)

which is the velocity addition rule. If $u'$ corresponds to signal velocity $v_s$ then

$$u = v_s$$  \hspace{1cm} (12)

So signal velocity is same in $S$ and $S'$ frames and it can be shown that they are same in all inertial frames. If $u' = v_s$ then $u$ is also $v_s$ but not higher than that. Suppose two signal velocities $v_{s1}$ and $v_{s2}$ are available for our experiments. If $v_{s1} < v_{s2}$ then one can see the change of $v_{s1}$ in two inertial frames when one uses signal with velocity $v_{s2}$ although there will be no change in $v_{s1}$ when signal with velocity $v_{s1}$ is used. So far apart from light in vacuum no other signal velocity has been found to be constant in different inertial frames because in measuring the velocity of that signal ($v_{s1}$ in this discussion) all related physical quantities have not been measured by the signal itself but by some other signal which is in general light in vacuum ($v_{s2}$ in this discussion). If in future we find experimentally any signal velocity $v_{sh}$ which is higher than velocity of light in vacuum in the observer’s rest frame then we shall find the change in the velocity of light in vacuum in different inertial frames provided that we do the measurement of it by using the signal with $v_{sh}$.

Here we obtain the transformation rules for the space and time coordinates in two inertial frames corresponding to an event for any signal. For that we consider that to make things physically equivalent the length of the rod which is at rest in $S$ frame and measured in $S'$ frame should be equal to the length of the same rod which is at rest in the $S'$ frame and measured in the $S$ frame. The two ends of the rod is measured at the same time. Say the length of the rod is of one unit when measured in frame in which it is at rest. Then from equation (1) corresponding to the first measurement it follows that length is $a$ and from equation (4) corresponding to the second measurement it follows that the length is $a'$. Considering the isotropy of space these measurements should be physically equivalent and then

$$a = a'.$$  \hspace{1cm} (13)

Using this in eq.(5) we obtain

$$a = \frac{1}{\sqrt{1 - \frac{u^2}{v^2}}}$$  \hspace{1cm} (14)

Using eqs. (5), (8) and (13)

$$b' = a' = a = b$$  \hspace{1cm} (15)
Using this in eq. (8) we obtain

\[ b = \frac{1}{\sqrt{1 - \frac{v^2}{v_s^2}}} \]  

(16)

So from eqs. (1), (2), (14) and (16) we obtain the general space-time transformation rules for any signal as

\[ x' = \frac{1}{\sqrt{1 - \frac{v^2}{v_s^2}}} (x - vt) ; \quad t' = \frac{1}{\sqrt{1 - \frac{v^2}{v_s^2}}} \left( t - \frac{vx}{v_s^2} \right) \]  

(17)

These transformation rules are like STR with only replacement of the velocity of light in vacuum by the signal velocity \( v_s \).

We define the position-time four vector \( x^\mu(v_s) \), \( \mu = 0, 1, 2, 3 \) corresponding to signal \( v_s \) as \( x^0 = v_s t, x^1 = x, x^2 = y, x^3 = z \) then the quantity which remains invariant in different inertial frame is

\[ g_{\mu\nu} x^\mu(v_s) x^\nu(v_s) = g_{\mu\nu} x^\mu(v_s) x^\nu(v_s). \]  

(18)

Let us consider that there are two signals having constant velocity \( v_{s1} \) and \( v_{s2} \) respectively in the observer’s rest frame. If the signal with velocity \( v_{s2} \) is used for measurement in a particular experiment then the Lorentz symmetry associated with \( v_{s1} \) is broken in that experiment. Here the Lorentz group is given by the transformations \( x^2 = x'^2, x^3 = x'^3 \) and those given by eqs. (17) and the invariance in (18). Origin of the violation of Lorentz invariance in this work is different from other works on this violation [3]. We differ here in the sense that although Lorentz invariance corresponding to one signal (say light in vacuum) is violated in the above-mentioned experiment but the invariance corresponding to another signal will be seen experimentally provided that all measurements are done using that signal. Possible such experiments have been discussed later.

The measurements of physical quantities in \( S \) and \( S' \) frames as obtained by using different signals can be related. The measurement of relative velocity between two different inertial frames is independent of the signal which we use. Furthermore, according to our earlier discussion this relative velocity is supposed to be less than all the velocities of different signals which might be used. If the observed event is at rest with respect to say \( S' \) frame then the measurement of \( x' \) and \( t' \) is independent of the signal. So if \( x_{v_{s1}} \) and \( t_{v_{s1}} \) are the space and time coordinates of an event occurred in \( S' \) frame and measured in \( S' \) frame by using signal \( v_{s1} \) and if \( x_{v_{s2}} \) and \( t_{v_{s2}} \) are the space and time coordinates of same event measured in the \( S \) frame by using signal \( v_{s2} \) then from eqs. (17) it follows that

\[ x_{v_{s1}} = \gamma_{v_{s1}} \frac{v_{s1}}{\gamma_{v_{s2}}} x_{v_{s2}} ; \quad t_{v_{s1}} = \frac{v_{s1}}{\gamma_{v_{s2}}} \left[ \frac{1}{\gamma_{v_{s1}}} - \frac{1}{\gamma_{v_{s2}}} v_{s1} \right] x_{v_{s2}} + \frac{\gamma_{v_{s2}}}{\gamma_{v_{s1}}} t_{v_{s2}} \]

where \( \gamma_{v_{s1}} = 1/\sqrt{1 - v^2/v_{s1}^2} \). If \( v_{s1} > v_{s2} \) then \( \gamma_{v_{s1}} > \gamma_{v_{s2}} \) and it follows from above equations that \( x_{v_{s1}} > x_{v_{s2}} \) for \( x \neq 0 \) and \( t_{v_{s2}} > t_{v_{s1}} \) when \( x_{v_{s2}} = 0 \).

Maxwell’s equations for electromagnetic fields remain invariant under the space time transformations for the signal - light in vacuum. One can show that these equations in a medium remain invariant in which case light in the medium is the signal. It is because in this case the force transformation rules between two inertial frames are similar with the force transformation rules of STR with only replacement of velocity of light in vacuum by the velocity of light in the medium in the transformation rules. Hence the same replacement occurs in the transformation rules for electric and magnetic fields. Then there are space
time coordinates in the Maxwell’s equations in the transformation rules of which also same replacement occurs. Considering these transformations of space, time and electric and magnetic fields it can be shown in a similar way like STR that Maxwell’s equations in a medium remain invariant in other inertial frames when light in that medium is used as signal.

We discuss here how one may experimentally verify whether the velocity of other signal apart from light in vacuum - say light in water is constant in different inertial frames. The experimental set-up has been shown diagrammatically in Figure 2. There are four mirrors \(M_1, M_2, M_3\) and \(M_4\) among which \(M_1\) is partially silvered and all are kept inside water. Monochromatic light coming from the light source falls on mirror \(M_1\) and is then split into reflected and transmitted parts and their directions are shown by arrows. The transmitted and reflected light follows some path where water is flowing and some path where water is not flowing with respect to the apparatus as shown in the figure. We may think that the water which is flowing with respect to apparatus as the moving frame \(S'\) and the water which is not flowing with respect to apparatus and the apparatus as rest frame \(S\). Waters in different regions of the apparatus are of same refractive index. Part of light reflected at mirror \(M_1\) again reflects at mirror \(M_4\), then mirror \(M_3\) and then mirror \(M_2\) and finally reflects at \(M_1\) again and go to telescope. The telescope and the observer/detector are both inside water. Another part of light after being transmitted at \(M_1\) is reflected at \(M_2\), then \(M_3\) after that \(M_4\) and finally is transmitted through \(M_1\) and go to telescope. So there will be interference due to optical path difference of the transmitted and reflected beams and fringe will be observed. If there is change in the velocity of light in water which is flowing with respect to the velocity of light in water which is not flowing with respect to the apparatus then shift of fringe will be observed with the variation of the velocity of water flowing. However, according to our work as only the light in water of particular refractive index is used in the apparatus as the signal there should not be any change in the velocity of light in water which is flowing and so no shift in fringe should be observed. If from the apparatus we remove all water which is not flowing then the experiment corresponds to the famous experiment of Fizeau in which fringe shift was observed and is given by the formula

\[
\Delta N \approx 4\ln^2 v_w \left( n^2 - 1 \right) / \left( \lambda c n^2 \right)
\]

where \(2l\) is the length of the path in which water is flowing, \(n\) is the refractive index of water, \(v_w\) is the velocity of water and \(c\) is velocity of light in vacuum and \(\lambda\) is the wavelength of light. Similar to light in a medium if one use sound as signal for all measurements in a particular experiment then the velocity of sound would be same in all inertial frames.

Our work shows that the property that the velocity of light in vacuum remains constant in all inertial frames is not the property of it only rather in a general way one can tell that any signal velocity is constant in different inertial frames. When we use different signal other than light in vacuum then in all transformation rules of special theory of relativity, velocity of light \(c\) in vacuum is replaced by \(v_s\). So it shows that depending on the signal used whose velocity is \(v_s\) the energy, momentum relation and mass, energy relation are given respectively by

\[
E = v_s \sqrt{p^2 + m_0^2 v_s^2} ; \quad E = m v_s^2
\]

where \(m_0\) is the rest mass of the particle. So if we perform experiment of mass, energy relation and if whole experimental set-up is fully covered in a medium where the velocity of light is less than that in vacuum we shall find lesser amount of energy is released from a certain amount of mass in comparison to the energy released from the same amount of mass in an experiment in vacuum or in an experiment whose set up is fully covered with a medium where velocity of light is higher than the earlier medium. This might be verified if
experiment on $e^+e^-$ annihilation in a medium could be performed and the energy of $\gamma$-rays is measured.

Half life for muon (moving in air) measured in air should be longer than that predicted by STR as half life of muon moving at velocity $v$ will be $T(v) = (1 - v^2/v^2_s)^{-1/2} T_0$ where $T_0$ is half life of muon at rest. Measuring the attenuation of the cosmic ray muon beam as it proceeds down the atmosphere one may verify this at different elevations. If one considers the velocity of muon to be about 0.995 $c$ and $v_s$ - the velocity of light in air about $c/1.0002$ then half life $T(v)$ of muon in the atmosphere should be found experimentally to be about 10.22 $T_0$. But this value according to STR is about 10.01 $T_0$. Also for the atmospheric neutrinos one should consider light in air (instead of light in vacuum) as the signal in the relativistic relationships. Then there will be modifications in the decay width of pions and kaons which decay to $\mu^+ + \nu_\mu (\bar{\nu}_\mu)$ and that of muon which decays to $e^+ + \nu_e (\bar{\nu}_e)$ as pion, kaon & muon pass through the atmosphere. Analysing the different neutrino flux on the basis of this might give some evidence supporting this work.

If various experiments proposed in this work show negative results then that would indicate nature does not support the constancy of any signal velocity in all inertial frames. In that case in the relativistic transformations $c$ is entering as some fundamental constant but not as signal velocity. Then the concept of measurement in Special Theory of relativity by using signal is required to be changed.

Acknowledgment

Author would like to thank Palash B. Pal for comments and for reading the manuscript and theory division of Saha Institute of Nuclear Physics for kind hospitality during visit. Discussion with G. Rajasekaran has helped in making further clarifications. Author would like to thank Shilpi Ghosh for drawing the schematic diagram and Papiya Nandy for granting academic leave.

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Figure 1: In (a) two rods $AB$ and $A'B'$ are shown as observed in the $S$ frame when signal is sent from $x = 0$ at $t = 0$ when $t' = 0$ also. In (b) two rods $AB$ and $A'B'$ are shown as observed in the $S$ frame when signal has reached $B$. 
Figure 2: Schematic experimental set-up to verify the constancy of the velocity of signal (light inside water).