Applying SVSSI sampling scheme on np-chart to Decrease the Time of Detecting Shifts - Markov chain approach, and Monte Carlo simulation

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Abstract

One of the main criteria for judging about the power of control charts is their ability in fast detection of deviations and shifts in the process. Average time to signal (ATS) or adjusted average time to signal (AATS) are among such criteria calculated under a certain state and assumption. Several studies have shown that using the idea of variable design for control charts, by separating their limits to the safe and the warning regions, can allow quick discovery of shifts and increase sensitivities to small changes. In this paper, a new variable sampling scheme with three sample sizes and two different sampling intervals, called SVSSI, is developed to increase the efficiency of the np control chart. Through various numerical examples, the performance of this scheme is evaluated by calculating ATS and AATS values using Markov chain method. Monte Carlo simulation method is used to validate the results of Markov chain method of SVSSI sampling scheme. In comparison with other schemes, better performance of applying SVSSI is proved in all conditions.

Keywords: SVSSI scheme, np control chart, Average time to signal, Adjusted average time to signal, Markov chain

1. Introduction
Control chart is the most important tool to control processes during production. Average run length (ARL), Average time to signal (ATS), and adjusted average time to signal (AATS) are well-known criteria to measure the performance of control charts. The ARL criterion is applied only in cases where the sampling interval is supposed to be fixed during the process, otherwise, ATS or AATS criteria must be used. Based on the ARL, several studies have been conducted on the VSS sampling method in which only the sample size varies. Comparing VSI method with fixed sampling method, by Teoh et al. [1], Muhammad et al. [2], Annadi et al. [3], Amiri et al. [4], Aparisi [5], and Yeong et al. [6], in different states for various control charts have been carried out, and accordingly, positive results have been achieved using VSS. Due to the performance of the VSS sampling method, designing variable control charts have attracted attention. Further, studies have mostly been conducted to reduce the time of signal by ATS and AATS criteria on other variable sampling methods such as VSI (with variable sampling interval), and VSSI (with variable sample size and sampling interval); see, e.g. Khaw et al. [7], Khoo et al. [8], Khatun et al. [9] and Shojaie et al. [10]. ATS and AATS are comprehensive and well-known criteria that can be used to evaluate any type of sampling methods for control charts. Ultimately, Costa and Rahim [11] presented the full adaptive (FA) or variable parameter (VP) sampling scheme. The main difference between the FA sampling scheme and other sampling schemes is that in FA, in addition to the variable values of the sample size and sampling interval, the control and warning limits are also considered variable. To study the details of designing this sampling scheme see e.g. Faraz et al. [12], Katebi and Moghadam [13], Abolmohammadi et al. [14], Salmasnia et al. [15] and Seif et al. [16].

ATS, which represents the mean time of detecting the change from the moment of occurring the shift in the process mean to the moment that control chart signals, was applied by Prabhu et al. [17] to evaluate the VSSI sampling method for X control chart. This criterion, calculated based on the concepts of the Markov chain and the transfer between different states, was also used by Aparisi and Haro [18] to measure the performance of the VSI sampling method for the $T^2$ Hotelling chart. In their study, for different shifts, the ATS values for VSI and fixed sampling methods were calculated and compared. Moreover, taking into account the same assumptions about the ATS calculation, Aparisi and Haro [19] evaluated VSSI and other sampling methods for the $T^2$ Hotelling chart based on the lowest amount of ATS. In order to monitor simple linear profiles, Kazemzadeh et al. [20] provided a complete study on the reduction of ATS and other statistical indices by fixed, VSS, and VSI sampling methods with different shift values and input parameters. Luo and Wu [21], Wu and Luo [22] explored in separate studies the effects of variable sampling methods on the np control chart by numerical examples and considering ATS in different states. They provided a statistical model aimed at reducing ATS to assess VSS, VSI, VSSI, and fix sampling methods. Similar researches in ATS-based evaluation of the results can be found in Abdella et al. [23], Chen and Hsieh [24], Lee and Khoo [25], Zhang et al. [26], Chen et al. [27] and Castagliola et al. [28].
ATS criterion is based on the assumption that the process starts from out of control state. However, Costa [29] proposed another approach to analyze the results assuming that the process begins from in control state, and the difference between the duration of a production cycle and in control time indicates the duration of change detection, i.e. AATS. They calculated AATS in different states for the variable $X$ control chart. Faraz et al. [30] presented the differences between ATS and AATS in their research, and then, conducted their assessments for VSSI-$T^2$ control chart based on AATS. Accordingly, although using ATS simplifies and reduces the volume of calculations, AATS can be more realistic for assessments since it is usually ensured of being in control before initiation of any process. Lin and Chou [31] considered both AATS and ATS for assessing the results. They analyzed the influence of using VSS, VSI, and VSSI sampling methods on the $X$ control chart under Normal and Non-Normal conditions based on the AATS and ATS. Katebi and Moghadam [13], taking into account the AATS criteria, implemented a comprehensive study on the performance of VSS, VSI, VSSI, FA, and fixed sampling schemes. According to the obtained results, the relative superiority of the FA scheme was evident compared to other schemes. In recent years, AATS has been of greater interest to researchers (e.g., refer to Celano et al. [32], Li et al. [33], and Lee [34]).

In all mentioned researches and other similar works, such as Faraz and Sanig [35], Mahadik [36], Khaw et al. [37], and Saha et al. [38], using VSSI and FA sampling method in designing of control charts has indicated better performance. An important point to notice is whether the performance of control charts can be further improved or not. Faraz and Parsian [39] proposed the idea of using double warning lines (DWL) for the $T^2$ control chart. The DWL sampling method is similar to the VSSI method, except for using two separate warning lines for sample sizes and sampling intervals to monitor the process. Despite being more complicated, it has a better performance comparing to VSSI. Since then, the economic and economic-statistical performance of using this sampling method were investigated by Faraz et al. [40] and Faraz and Saniga [41].

Mahadik and Shirke [42, 43] evaluated the SVSSI sampling method for $X$ and $T^2$ control charts in separate studies. Their proposed method, which is simpler and more practical in comparison to DWL, improved the performance of the $X$ and $T^2$ control charts and led to better results for it than VSSI and FA schemes. Furthermore, Katebi et al. [44], from a cost viewpoint, and taking into account the statistical criteria, assessed SVSSI sampling method for the $T^2$ control chart. Their results were satisfactory and reduced costs by using SVSSI method in comparison to other methods.

Despite the widespread use of np control chart in the literature, the current research introduces some contributions in the following paragraphs.

For the characteristics of np control charts, mostly two variables have been considered for their simplicity. However, in this paper, the SVSSI scheme presented for the np control chart provides three different sample sizes and two different sampling intervals depending
on the process situations. Besides being easy to understand and implement, this scheme is superior in performance to the other sampling schemes.

Since the ARL criterion is not applicable because of the time variability, two important criteria for checking the time of signal, namely AATS and ATS, are presented and fully discussed in this paper. Comprehensive evaluations based on ATS and AATS are provided. Accordingly, by improving these criteria using the proposed sampling algorithm, better performance of the control chart, and consequently, reduction in detection time, avoiding the production of further defective items and decrease in costs are expected.

In the next section, the method of sampling is introduced. Statistical criteria are reviewed in section 3. By providing various numerical examples in section 4, variable sampling methods are evaluated and compared. Finally, conclusions are made along with suggestions for the future.

2. Designing SVSSI-np control chart

In the SVSSI sampling method, three sample sizes \( n_1, n_2, \) and \( n_3 \) and two sampling intervals \( h_1 \) and \( h_2 \) are used assuming that \( n_1 < n_2 < n_3 \) and \( h_1 < h_2 \). To determine the time of change in sample size and sampling interval of subgroups, two warning limits \( W_i^1 \) and \( W_i^2 \), and one control limit \( K_i \) are used. Depending on the size of the sample, the values of these limits vary for different \( i \) values. The SVSSI sampling method is briefly described as follows:

\[
\begin{align*}
\begin{cases}
  n_1, h_1 & 0 \leq np < W_i^1 \\
  n_2, h_2 & W_i^1 \leq np < W_i^2 \\
  n_3, h_2 & W_i^2 \leq np < K_i
\end{cases}
\]

Therefore, if the value of the statistic for \((i-1)^{th}\) subgroup be in the first region \((0 \leq np < W_i^1)\), there is no reason that a change in the process has occurred. Thus, \(i^{th}\) subgroup with small sample size \( n_1 \) and longer sampling interval \( h_1 \) is plotted on the chart. If the value of the statistic be in the second region \(( W_i^1 \leq np < W_i^2)\), then it is likely that a change in the mean has occurred. Therefore, it is possible to signal a warning by the chart in the next subgroup. To have more control over the process, the \(i^{th}\) subgroup with the size of \( n_2 \) is taken after the interval of \( h_2 \). Finally, if the statistic be in the third region \(( W_i^2 \leq np < K_i)\), then it is more likely a change in the mean of the process occurs. Hence, the next sample is taken with an increased size of \( n_3 \) after the interval of \( h_2 \). In Figure 1, the SVSSI-np control chart is shown based on the description given above.

Figure 1 is around here

The limits of the warning and control of the SVSSI-np chart are calculated as \(np_0 + r(n; p_0 (1 - p_0))^{0.5}\), where \( r \) is the coefficient of control and warning limit and \( p_0 \) represented the mean of the process in the in-control state (The parameters are defined in
Appendix A). At each sampling time, the probability of occurring one of the following scenarios exists:

1. If $0 \leq np < W_i^1$, the next sample with a size of $n_1$ is taken after the interval of $h_1$,
2. If $W_i^1 \leq np < W_i^2$, the next sample with a size of $n_2$ is taken after the interval of $h_2$,
3. If $W_i^2 \leq np < K_i$, the next sample with a size of $n_3$ is taken after the interval of $h_2$.

Furthermore, the probability of occurrence of the fourth state should be considered in which $np \geq K_i$. In this state, the control chart signals that the process is out of control. Accordingly, the process is stopped for further investigations. If the process is identified in control, this signal is considered as incorrect. Otherwise, corrective actions are taken to find the cause of change, fix it, and thus, to restore the process. Since the design of the SVSSI-$np$ control chart is based on the concepts of the Markov chain, according to Fallaghnezhad et al. [45] and Faraz and Saniga [35], this state can be regarded as the absorbing state in Markov chains.

3. Calculation of Efficiency

In this section, statistical well-known criteria that can be used to evaluate any types of sampling schemes for control charts are reviewed.

3.1. AATS

As stated, the AATS criterion is based on the assumption that the process begins from in control state. According to the scenarios described in Section 2, all of them can be in control or out of control. Thus, the probability of occurring eight states exist at each sampling of the process. The change of state or the probability of transform between different states using the properties of the Markov chain is shown in this section with $p_{ij}$. According to Faraz and Saniga [35], $p_{ij}$ values can be defined as follows:

$$p_{11} = \Pr(1 \rightarrow 1) = \Pr(n_1,h_1,0 \leq np < W_i^1) = F(n_1,W_i^1,p_0) \times \exp(-\lambda h_1).$$

$$p_{16} = \Pr(1 \rightarrow 6) = \Pr(n_1,h_1,W_i^1 \leq np < W_i^2) = F(n_1,W_i^2,p_1) - F(n_1,W_i^1,p_1) \times (1 - \exp(-\lambda h_1)).$$

$$p_{28} = \Pr(2 \rightarrow 8) = \Pr(n_2,h_2,K_2 \leq np) = (1 - F(n_2,K_2,p_1)) \times (1 - \exp(-\lambda h_2)).$$

$$p_{32} = \Pr(3 \rightarrow 2) = \Pr(n_3,h_3,W_i^3 \leq np < W_i^3) = F(n_3,W_i^3,p_0) - F(n_3,W_i^3,p_0) \times \exp(-\lambda h_2).$$

The $np$ control chart, based on the Binomial distribution, plots the number of defective items in the sample. Therefore, $F$ represents the cumulative distribution function of Binomial in calculating each $p_{ij}$. One of the main assumptions of this research in obtaining AATS, which is based on Costa et al. [29] and Faraz et al. [30], is to consider the occurrence of the deviation according to Poisson distribution at a rate of $\lambda$ per time unit. Thus, the time interval between consecutive occurrences of deviations follows Exponential distributions with parameter $\lambda$. As another main assumption, only one assignable cause leads to shifting the mean of the process from $p_0$ to $p_1$. The value of $p_1$ for the shift size of $r$ is obtained as follows:
Accordingly, the calculations of all \( p_{ij} \)s for a Markov chain with seven transition states and one absorbing state are given below (The probability of transforming from an out of control state to in control states, and the probability of transforming from an absorbing state to other states is supposed to be zero):

\[
p_{11} = F(n_1, W_1^1, p_0) \times \exp(-\lambda h_1),
\]

\[
p_{12} = \left( F(n_1, W_1^2, p_0) - F(n_1, W_1^1, p_0) \right) \times \exp(-\lambda h_1),
\]

\[
p_{13} = \left( F(n_1, K_1, p_0) - F(n_1, W_1^2, p_0) \right) \times \exp(-\lambda h_1),
\]

\[
p_{14} = (1 - F(n_1, K_1, p_0)) \times \exp(-\lambda h_1),
\]

\[
p_{15} = F(n_1, W_1^1, p_1) \times (1 - \exp(-\lambda h_1)),
\]

\[
p_{16} = \left( F(n_1, W_1^2, p_1) - F(n_1, W_1^1, p_1) \right) \times (1 - \exp(-\lambda h_1)),
\]

\[
p_{17} = \left( F(n_1, K_1, p_1) - F(n_1, W_1^2, p_1) \right) \times (1 - \exp(-\lambda h_1)),
\]

\[
p_{18} = (1 - F(n_1, K_1, p_1)) \times (1 - \exp(-\lambda h_1)),
\]

\[
p_{21} = F(n_2, W_2^1, p_0) \times \exp(-\lambda h_2),
\]

\[
p_{22} = \left( F(n_2, W_2^2, p_0) - F(n_2, W_2^1, p_0) \right) \times \exp(-\lambda h_2),
\]

\[
p_{23} = \left( F(n_2, K_2, p_0) - F(n_2, W_2^2, p_0) \right) \times \exp(-\lambda h_2),
\]

\[
p_{24} = (1 - F(n_2, K_2, p_0)) \times \exp(-\lambda h_2),
\]

\[
p_{25} = F(n_2, W_2^1, p_1) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{26} = \left( F(n_2, W_2^2, p_1) - F(n_2, W_2^1, p_1) \right) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{27} = \left( F(n_2, K_2, p_1) - F(n_2, W_2^2, p_1) \right) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{28} = (1 - F(n_2, K_2, p_1)) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{41} = p_{31} = F(n_3, W_3^1, p_0) \times \exp(-\lambda h_2),
\]

\[
p_{42} = p_{32} = \left( F(n_3, W_3^2, p_0) - F(n_3, W_3^1, p_0) \right) \times \exp(-\lambda h_2),
\]

\[
p_{43} = p_{33} = \left( F(n_3, K_3, p_0) - F(n_3, W_3^2, p_0) \right) \times \exp(-\lambda h_2),
\]

\[
p_{44} = p_{34} = (1 - F(n_3, K_3, p_0)) \times \exp(-\lambda h_2),
\]

\[
p_{45} = p_{35} = F(n_3, W_3^1, p_1) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{46} = p_{36} = \left( F(n_3, W_3^2, p_1) - F(n_3, W_3^1, p_1) \right) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{47} = p_{37} = \left( F(n_3, K_3, p_1) - F(n_3, W_3^2, p_1) \right) \times (1 - \exp(-\lambda h_2)),
\]

\[
p_{48} = p_{38} = (1 - F(n_3, K_3, p_1)) \times (1 - \exp(-\lambda h_2)).
\]
\[ p_{55} = F \left( n_1, W_{1,1}, p_1 \right). \]
\[ p_{56} = F \left( n_1, W_{1,2}, p_1 \right) - F \left( n_1, W_{1,1}, p_1 \right). \]
\[ p_{57} = F \left( n_1, K_1, p_1 \right) - F \left( n_1, W_{1,2}, p_1 \right). \]
\[ p_{58} = 1 - F \left( n_1, K_1, p_1 \right). \]
\[ p_{65} = F \left( n_2, W_{1,1}, p_1 \right). \]
\[ p_{66} = F \left( n_2, W_{2,1}, p_1 \right) - F \left( n_2, W_{1,1}, p_1 \right). \]
\[ p_{67} = F \left( n_2, K_2, p_1 \right) - F \left( n_2, W_{2,1}, p_1 \right). \]
\[ p_{68} = 1 - F \left( n_2, K_2, p_1 \right). \]
\[ p_{75} = F \left( n_3, W_{1,1}, p_1 \right). \]
\[ p_{76} = F \left( n_3, W_{2,1}, p_1 \right) - F \left( n_3, W_{1,1}, p_1 \right). \]
\[ p_{77} = F \left( n_3, K_3, p_1 \right) - F \left( n_3, W_{2,1}, p_1 \right). \]
\[ p_{78} = 1 - F \left( n_3, K_3, p_1 \right). \]
\[ p_{88} = 1. \]

For more information on calculating transmission states for other variable control charts, such as VSS, VSI, VSSI, and FA the reader(s) can refer to Luo and Wu [21], Fallahnezhad et al. [45], Katebi and Moghadam [13], Faraz et al. [30] and Faraz and Saniga [35].

Therefore, the AATS, based on the Markov chain approach, is calculated as follows:

\[ AATS = B \times (I - Q)^{-1} \times (h_1, h_2, h_2, h_1, h_2, h_2) \cdot \frac{1}{\lambda}, \quad (2) \]

where \(1/\lambda\) is the mean of Exponential distribution which indicates the duration of in control state for the process. \(B\) is a vector of initial probabilities, which is recommended to avoid potential problems at the start, and to apply more control over the process, to start the sampling of the process from the state between the last warning threshold and the control limit (see Fallahnezhad et al. [45], Katebi et al. [44] and Seif et al. [46]). Therefore, the vector of \(B\) is considered as \((0,0,1,0,0,0,0)\). Moreover, \(I\) is an identity matrix of degree 7, and \(Q\) is calculated as follows:

\[
Q = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} \\
P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} \\
P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & P_{37} \\
P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & P_{46} & P_{47} \\
0 & 0 & 0 & 0 & P_{55} & P_{56} & P_{57} \\
0 & 0 & 0 & 0 & P_{65} & P_{66} & P_{67} \\
0 & 0 & 0 & 0 & P_{75} & P_{76} & P_{77}
\end{bmatrix}
\]

As can be seen, the matrix \(Q\) contains all states of transitions except for the transition to absorbing state.
3.2. ATS

The main assumption in the calculation of the ATS, and its difference with the AATS, is at the start of the process. In the calculations of this criterion, according to Faraz et al. [30], it is assumed that the process starts from out of control state \((d>0)\). Accordingly, only the probability of occurrence of four states in the out of control state can be defined for the process. In this section, the probability of transition between different states, shown by \(p_{ij}^d\), are obtained as follows:

\[
p_{11}^d = p_{55}, \quad p_{21}^d = p_{66}, \quad p_{31}^d = p_{75},
\]

\[
p_{12}^d = p_{56}, \quad p_{22}^d = p_{66}, \quad p_{32}^d = p_{76},
\]

\[
p_{13}^d = p_{57}, \quad p_{23}^d = p_{67}, \quad p_{33}^d = p_{77},
\]

\[
p_{14}^d = p_{58}, \quad p_{24}^d = p_{68}, \quad p_{34}^d = p_{78}, \quad p_{44}^d = p_{88},
\]

and

\[
ATS = B^d \times (I^d - Q^d)^{-1} \times (h_1, h_2, h_2)'.
\]  

(3)

\(B^d\) is the initial probability vector when beginning the process from out of control state, and is equal to \((0,0,1)\). Here, the start of the process is also considered to be between the last warning threshold and the control limit. \(I^d\) is an identity matrix of degree 3 and \(Q^d\) is described as follows:

\[
Q^d = \begin{bmatrix}
p_{11}^d & p_{12}^d & p_{13}^d \\
p_{21}^d & p_{22}^d & p_{23}^d \\
p_{31}^d & p_{32}^d & p_{33}^d
\end{bmatrix}.
\]

4. Numerical Examples and Comparisons

In this section, the performance of different sampling methods is studied according to both AATS and ATS. For each criterion, the results are shown in three separate Tables based on different values of the fix sample size \((n_0)\) and the fix sampling interval \((h_0)\). In each Table, the studies are performed based on five different values of 0.03, 0.05, 0.08, 0.12 and 0.18 for \(p_0\) and six different values of 0.05, 0.10, 0.30, 0.50, 0.70 and 0.90 for \(d\), respectively. Due to the facts that: 1) the shifts are usually considered up to 8 hours, and 2) the time intervals of less than 0.1 in practice can be problematic because the process should be allowed to produce \(n\) unit at a short time \(h_2\), the values of sampling intervals are ranged from 0.1 to 8. Moreover, the range of sample sizes is considered between 1 and 50. In Table 1, the approach of determining the values of sample sizes and sample intervals are given for the SVSSI-np control chart.

Table 1 is around here
In this research, the optimal values of AATS and ATS for different states are obtained by searching among different values of sample sizes and sampling intervals in the space of the desired charts. In other words, the initial parameters for which the given criterion has the lowest value have been obtained. Then, at these points and for the obtained initial parameters, the optimal values of the given criterion are calculated in each step so that comparing the different sampling methods could be possible. The results of the optimal values of AATS and ATS in all the calculations presented are based on the level parameter (λ) of 0.05 and the coefficient (r) of 1, 2, and 3, which are considered respectively for $K_i$, $W_i^2$ and $W_i^3$. Therefore, the optimal AATS and ATS values of different sampling methods along with the optimal parameters of the SVSSI sampling method are presented in Tables 2-4 and Tables 5-7, respectively. More information regarding the computation of AATS and ATS can be found in Appendix B.

Tables 2-7 are around here

As shown, comprehensive investigations of different sampling methods were provided based on AATS and ATS. According to the results in Tables 2-7, using VSS, VSI, and VSSI sampling schemes have improved the performance of the np control chart in detecting changes, among which the VSSI sampling scheme shows a better performance. Changing the fixed control limits of the VSSI scheme to the variable control limits of the FA scheme has significantly reduced the AATS and ATS values. This is compatible with the results previously stated by Katabi and Moghaddam [13]. Although, it brings more complexity when $K_1 \neq K_2$ changes in the FA scheme, the obtained results of AATS and ATS show a decrease in both.

Furthermore, it is obvious that SVSSI sampling method, compared to the FA scheme, detects the shifts in the process in a shorter time. Generally, comparing the SVSSI sampling method with other sampling methods indicates its superiority in most cases. Therefore, using three sample sizes and two sampling intervals in the SVSSI scheme along with variable warning and control limits has more impact on improving the detection speed of shifts in the np control chart than other schemes.

For more investigation, Monte Carlo simulation method is used to evaluate the results of Markov method of SVSSI sampling scheme. The simulation method is applied to approximately calculate ATS values by generating random data from Binomial distribution in 10,000 iterations. In this evaluation, the parameters are set according to Tables 7-9. As depicted in Figures 2-4, the results of simulation are almost the same as those obtained in Tables 7-9 using the Markov chain method.

Figures 2-4 are around here
The performance of each control chart is highly dependent on the variations of sample size and sampling interval. Therefore, the proper design of the control chart is of great importance. For this reason, another assumption is considered and another method called \textit{VSSI}_n is examined. In this method, the next sampling is performed after the interval \( h_1 \) whenever the statistic is placed in the state of \( W_1 \leq np < W_2 \). Therefore, only \( p_{2j} \) values change as follows (other \( p_{ij} \) values are calculated in the same way as \textit{SVSSI} method):

\[
\begin{align*}
p_{21} &= F\left(n_2, W_2^1, p_0\right) \times \exp(-\lambda h_1), \\
p_{22} &= \left(F\left(n_2, W_2^2, p_0\right) - F\left(n_2, W_2^1, p_0\right)\right) \times \exp(-\lambda h_1), \\
p_{23} &= \left(F\left(n_2, K_2, p_0\right) - F\left(n_2, W_2^2, p_0\right)\right) \times \exp(-\lambda h_1), \\
p_{24} &= (1 - F\left(n_2, K_2, p_0\right)) \times \exp(-\lambda h_1), \\
p_{25} &= F\left(n_2, W_2^1, p_1\right) \times (1 - \exp(-\lambda h_1)), \\
p_{26} &= \left(F\left(n_2, W_2^2, p_1\right) - F\left(n_2, W_2^1, p_1\right)\right) \times (1 - \exp(-\lambda h_1)), \\
p_{27} &= \left(F\left(n_2, K_2, p_1\right) - F\left(n_2, W_2^2, p_1\right)\right) \times (1 - \exp(-\lambda h_1)), \\
p_{28} &= (1 - F\left(n_2, K_2, p_1\right)) \times (1 - \exp(-\lambda h_1)).
\end{align*}
\]

Besides, the \textit{AATS} and \textit{ATS} values are equal to:

\[
\begin{align*}
\text{AATS} &= B \times (1 - Q)^{-1} \times (h_1, h_1, h_2, h_2, h_3, h_3, h_4)^\prime - \frac{1}{\lambda^2}, \\
\text{ATS} &= B^d \times (I^d - Q^d)^{-1} \times (h_1, h_1, h_2)^\prime.
\end{align*}
\] (4) (5)

Similar to the previous investigations, the \textit{VSSI}_n sampling method is evaluated based on \textit{AATS} and \textit{ATS}. The results of comparing this method and the \textit{SVSSI} sampling method are presented in Tables 8-10.

Tables 8-10 are around here

In each Table, the values of \textit{AATS} and \textit{ATS} metrics for \textit{VSSI}_n and \textit{SVSSI} sampling methods are compared. Moreover, the percentage difference between the optimal values of each criterion is calculated. It is clear that a small change in the sampling method has led to weaker results and reduced the performance of the \( np \) control chart in most cases. Therefore, choosing the correct sampling interval and sample size in each state of the process is very important to achieve better results.

5. Concluding remarks and future research

By introducing variable sampling methods, significant improvement in the performance of control charts has been achieved in terms of faster identification of shifts in the processes. In this research, after reviewing \textit{VSS}, \textit{VSI}, \textit{VSSI}, and \textit{FA} sampling methods, we
proposed a new \textit{SVSSI} sampling method for the \textit{np} control chart in which three sample sizes and two sampling intervals are used to design. After introducing, designing of this method based on the concept of the Markov chain was considered. To evaluate the \textit{SVSSI} method and compare it to other sampling methods, the minimum adjusted average time to signal (\textit{minAATS}) and the minimum average time to signal (\textit{minATS}) were considered as criteria. Moreover, comparisons were made based on different values of \(p_0, d, n_0, \text{ and } h_0\) to evaluate the sampling methods from different aspects. Optimal values of the criteria and parameters of the control chart were obtained by grid search and exact method.

The results of different numerical comparisons indicated the superiority of \textit{SVSSI} sampling method. Besides, by making a change in the design of the method, we introduced \textit{VSSI} method. The results of comparing to \textit{VSSI}, the superiority of \textit{SVSSI} was proved again. Therefore, by choosing the proper sampling method and parameters, the duration of the out-of-control state can be reduced and the changes that occurred in the process mean can be detected fast. As a result, generating more defective items than those from other methods can be avoided. Thus, from a theoretical point of view and taking into account practical considerations, it can be admitted that using three sample sizes and two sampling intervals, as shown in Figure 1, is a superior and more efficient method than other methods in detecting the changes.

The \textit{np} control chart is used to decide whether the observations match the technical specifications and pre-specified requirements or not. Thus, several qualitative characteristics can be examined simultaneously in this control chart. Based on such features, the np chart control is widely applied in practice. This chart is a useful tool to control and evaluate the behavior of a process, from low-cost operations to mass production, over time. Thus, practitioners can identify and stop problems before experiencing great losses. The main purpose of this paper was to investigate this issue by evaluating \textit{AATS} and \textit{ATS} criteria for different sampling schemes in order to present the best design in terms of reducing the time of signal. Therefore, the practitioners can use the \textit{SVSSI} scheme as an efficient scheme to reduce the time of signal that results in a reduction in the production of defective items. For this purpose, after considering the parameters of the control chart and evaluating its performance, they can decide whether to reduce or increase the sample size and sampling interval.

For future research, one can consider cases in which the distribution of the failure or the duration of in-control time is not exponential. Moreover, the \textit{SVSSI-np} control chart can be evaluated for further studies from different economic and economic-statistical points of view (see e.g. [47]). The improvement of the \textit{np} control chart by designing other sampling methods necessitates more investigations. For the time being, we are trying to extend this study for an expanded fraction defective chart in [48]. The occurrence of an assignable cause in the process was one of the main assumptions of this study. Assuming the occurrences of multiple assignable causes is another suggestion for future research to pursue.
Acknowledgements

The authors would like to thank the anonymous reviewers for their useful comments and suggestions.

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Appendices

Appendix A: Abbreviations and symbols used in designing SVSSI Sampling Scheme

ATS: Average time to signal
AATS: Adjusted average time to signal
SVSSI: Three sample sizes and two sampling intervals scheme
ARL: Average run length
VSS: Variable sample size
VSI: Variable sampling interval
VSSI: Variable sample size and sampling interval
FA: Full adaptive
DWL: Double warning lines
n: Sample size (i=1, 2, 3)
h: Sampling interval (i=1, 2)
W1i and W2i: Lower and higher warning limits (i=1, 2, 3)
K1: Higher control limit (i=1, 2, 3)
p0: The in-control defective ratio
p1: The out-of-control defective ratio
r: The coefficient of control and warning limits
pij: The transition probability between the states i and j
d: The level of the shift in process
F(.): The binomial cumulative distribution function
λ: The exponential distribution parameter
B: The initial probabilities vector
I: The identity matrix
Q: The identity matrix
VSSIln: Three sample sizes and two different sampling intervals scheme (with different method and assumption with SVSSI scheme)

Appendix B: Procedure of computing AATS and ATS for SVSSI sampling scheme

1. Input p0, n0, h0, and d
   1.1. Calculate p1 using Equation 1
2. Set values of AATS=+∞ or ATS=+∞
3. Loop n1i from {1, 2, ..., n0-1},
   3.1 Calculate W1i*, W2i*, K1*.
4. Loop $n_3^*$ from $\{n_0+1, ..., 50\}$,
   
4.1. Calculate $W_{3,1}^{*}, W_{3,2}^{*}, K_{3}^{*}$.

5. Loop $n_2^*$ from $\{n_1+1, ..., n_3-1\}$,
   
5.1. Calculate $W_{2,1}^{*}, W_{2,2}^{*}, K_{2}^{*}$.

6. And similarly loop $h_2^*$ and $h_1^*$ from $\{0.1, 0.2, ..., h_0-0.1\}$ and $\{h_0, ..., 8\}$, respectively.

7. Calculate $AATS^{*}$ or $ATS^{*}$ using Equation 2 and Equation 3.

7.1. If $AATS^{*} < AATS$ or $ATS^{*} < ATS$ then;

7.1.1. $AATS = AATS^{*}$ or $ATS = ATS^{*}$,

7.1.2. $n_1 = n_1^{*}, n_3 = n_3^{*}, n_2 = n_2^{*}, h_2 = h_2^{*}, h_1 = h_1^{*}$,

7.1.3. $W_{1,1}^{*} = W_{1,1}^{*}, W_{1,2}^{*} = W_{1,2}^{*}, K_{1} = K_{1}^{*}$.

7.1.4. $W_{3,1}^{*} = W_{3,1}^{*}, W_{3,2}^{*} = W_{3,2}^{*}, K_{3} = K_{3}^{*}$, and

7.1.5. $W_{2,1}^{*} = W_{2,1}^{*}, W_{2,2}^{*} = W_{2,2}^{*}, K_{2} = K_{2}^{*}$.
Figures

Figure 1. SVSSI-np control chart

Figure 2. Comparing the optimal results of ATS obtained by Markov and Simulation methods for SVSSI-np control chart when $n_0=4$ and $h_0=1$
Figure 3. Comparing the optimal results of ATS obtained by Markov and Simulation methods for SVSSI-$np$ control chart when $n_0=6$ and $h_0=1.5$
Figure 4. Comparing the optimal results of ATS obtained by Markov and Simulation methods for $SVSSI-np$ control chart when $n_0=8$ and $h_0=2$
Tables

Table 1. The ranges of sample size and sampling interval for SVSSI-np control chart

|       | Min   | Max   | Step |
|-------|-------|-------|------|
| $n_1$ | 1     | $n_0$-1 | 1    |
| $n_2$ | $n_1$+1 | $n_3$-1 | 1    |
| $n_3$ | $n_0$+1 | 50    | 1    |
| $h_1$ | $h_0$  | 8     | 0.1  |
| $h_2$ | 0.1    | $h_0$.1 | 0.1  |

Table 2. Comparing optimal AATS values of np control charts when $n_0=4$ and $h_0=1$

| $p_0$ | $r$ | $n_1$ | $n_2$ | $n_3$ | $h_1$ | $h_2$ | SVSSI | FA  | VSSI | VSS  | VSI  | Shewhart |
|-------|-----|-------|-------|-------|-------|-------|-------|------|------|-------|-------|--------|
| 0.03  | 0.05| 3     | 9     | 10    | 1     | 0.1   | 8.5   | 8.5  | 12.69| 12.69 | 103.17| 117.76 |
| 0.1   |     | 3     | 10    | 33    | 1     | 0.2   | 6.93  | 6.93 | 10.33| 10.34 | 67.94 | 79.71  |
| 0.3   |     | 3     | 47    | 48    | 1     | 0.8   | 3.83  | 3.87 | 5.88 | 5.88  | 21.35 | 27.76  |
| 0.5   |     | 3     | 47    | 48    | 1     | 0.9   | 2.54  | 2.59 | 3.75 | 4.06  | 9.97  | 14.2   |
| 0.7   |     | 3     | 47    | 48    | 1     | 0.9   | 1.91  | 1.94 | 2.36 | 3.08  | 5.65  | 8.7    |
| 0.9   |     | 3     | 47    | 48    | 1     | 0.9   | 1.54  | 1.56 | 1.81 | 2.46  | 3.6   | 5.91   |
| 0.05  | 0.05| 1     | 5     | 6     | 1     | 0.2   | 15.92 | 15.92| 55.06| 62.7  | 48.31 | 48.32  |
| 0.1   |     | 1     | 6     | 48    | 1     | 0.4   | 13.42 | 13.43| 37.85| 43.87 | 35.15 | 35.16  |
| 0.3   |     | 3     | 47    | 48    | 1     | 0.1   | 4.56  | 6.86 | 8.34 | 10.71 | 14.18 | 14.18  |
| 0.5   |     | 3     | 47    | 48    | 1     | 0.1   | 2.18  | 2.66 | 2.85 | 3.84  | 7.73  | 7.74   |
| 0.7   |     | 3     | 47    | 48    | 1     | 0.1   | 1.6   | 1.68 | 1.73 | 2.46  | 4.91  | 4.92   |
| 0.9   |     | 3     | 47    | 48    | 1     | 0.1   | 1.3   | 1.32 | 1.35 | 2.02  | 3.41  | 3.42   |
| 0.08  | 0.05| 1     | 4     | 8     | 1     | 0.1   | 10.19 | 10.19| 40.06| 40.05 | 21.14 | 21.15  |
| 0.1   |     | 1     | 4     | 50    | 1     | 0.1   | 8.84  | 8.84 | 30.73| 30.72 | 16.32 | 16.34  |
| 0.3   |     | 3     | 49    | 50    | 1     | 0.1   | 3.35  | 5.58 | 8.33 | 11    | 7.51  | 7.52   |
| 0.5   |     | 3     | 49    | 50    | 1     | 0.1   | 1.58  | 2.62 | 2.71 | 3.66  | 4.36  | 4.38   |
| 0.7   |     | 3     | 49    | 50    | 1     | 0.1   | 1.19  | 1.75 | 1.78 | 2.43  | 2.88  | 2.89   |
| 0.9   |     | 3     | 49    | 50    | 1     | 0.1   | 0.99  | 1.4  | 1.42 | 2.04  | 2.05  | 2.06   |
| 0.12  | 0.05| 2     | 5     | 8     | 1     | 0.1   | 42.24 | 53.36| 285.66| 297.61| 101.36| 109.6  |
| 0.1   |     | 2     | 40    | 45    | 1     | 0.1   | 27.59 | 42.47| 176.07| 201.8 | 71.92 | 79.1   |
| 0.3   |     | 3     | 45    | 50    | 1     | 0.1   | 3.17  | 11.94| 11.94| 14.52 | 23.99 | 28.54  |
| 0.5   |     | 3     | 47    | 50    | 1     | 0.1   | 1.26  | 3.78 | 3.78 | 4.61  | 10.45 | 13.58  |
| 0.7   |     | 3     | 47    | 50    | 1     | 0.1   | 0.96  | 2.5  | 2.5  | 3.08  | 5.32  | 7.56   |
| 0.9   |     | 3     | 47    | 50    | 1     | 0.1   | 0.82  | 2   | 2   | 2.58  | 3.02  | 4.66   |
| 0.18  | 0.05| 2     | 6     | 31    | 1     | 0.1   | 22.29 | 24.7 | 128.41| 150.23| 620.01| 634.49 |
| 0.1   |     | 2     | 49    | 50    | 1     | 0.1   | 16.62 | 20.47| 97.47 | 115.55| 426   | 438.88 |
| 0.3   |     | 2     | 49    | 50    | 1     | 0.1   | 2.78  | 9.57 | 9.57 | 12.84 | 122.49| 131.09 |
| 0.5   |     | 2     | 49    | 50    | 1     | 0.1   | 1.26  | 2.68 | 2.68 | 3.59  | 45.59 | 51.67  |
| 0.7   |     | 2     | 49    | 50    | 1     | 0.1   | 0.98  | 1.82 | 1.82 | 2.4   | 19.7  | 24.12  |
| 0.9   |     | 2     | 46    | 50    | 1     | 0.1   | 0.82  | 1.42 | 1.44 | 2.04  | 9.34  | 12.59  |
Table 3. Comparing optimal AATS values of np control charts when $n_0=6$ and $h_0=1.5$

| $p_0$ | $r$ | $n_1$ | $n_2$ | $n_3$ | $h_1$ | $h_2$ | SVSSI | FA | VSSI | VSS | VSI | Shewhart |
|-------|-----|-------|-------|-------|-------|-------|-------|-----|------|-----|-----|----------|
| 0.03  | 0.05| 3     | 9     | 10    | 1.5   | 0.1   | 12.75 | 12.75| 19.01| 19.02| 73.96| 73.97    |
| 0.1   | 3    | 10    | 33    | 1.5   | 0.2   |       | 10.39 | 10.4 | 15.49| 15.49| 50.5 | 50.52    |
| 0.3   | 3    | 47    | 48    | 1.5   | 1.2   |       | 5.71  | 5.79 | 8.8  | 8.8  | 18.15| 18.17    |
| 0.5   | 5    | 47    | 48    | 1.5   | 0.1   |       | 2.88  | 3.26 | 3.54 | 5.08 | 9.55 | 9.56     |
| 0.7   | 5    | 47    | 48    | 1.5   | 0.1   |       | 2.05  | 2.14 | 2.23 | 3.37 | 6    | 6.01     |
| 0.9   | 5    | 47    | 48    | 1.5   | 0.1   |       | 1.66  | 1.68 | 1.73 | 2.78 | 4.17 | 4.19     |
|       |      | 1     | 6     | 20    | 1.5   | 0.3   | 23.88 | 23.88| 44.97| 44.97| 31.02| 31.04    |
| 0.05  | 0.05| 1     | 6     | 48    | 1.5   | 0.5   | 20.12 | 20.14| 32.88| 32.88| 22.8 | 22.82    |
| 0.1   | 1    | 6     | 48    | 1.5   | 0.1   |       | 4.06  | 10.2 | 12.38| 13.49| 9.53 | 9.55     |
| 0.3   | 5    | 48    | 49    | 1.5   | 0.1   |       | 2.02  | 3.95 | 4.22 | 5.7  | 5.37 | 5.39     |
| 0.5   | 5    | 48    | 49    | 1.5   | 0.1   |       | 1.52  | 2.49 | 2.55 | 3.64 | 3.51 | 3.53     |
| 0.7   | 5    | 48    | 49    | 1.5   | 0.1   |       | 1.27  | 1.95 | 1.98 | 2.98 | 2.52 | 2.54     |
| 0.9   | 5    | 48    | 49    | 1.5   | 0.1   |       | 15.29 | 15.29| 60.06| 60.05| 103.61| 113.03   |
| 0.08  | 0.05| 1     | 4     | 8     | 1.5   | 0.1   | 13.26 | 13.26| 46.06| 46.06| 69.56| 77.57    |
| 0.1   | 1    | 4     | 50    | 1.5   | 0.1   |       | 3.67  | 8.33 | 12.34| 16.41| 20.71| 25.43    |
| 0.3   | 4    | 49    | 50    | 1.5   | 0.1   |       | 1.81  | 3.88 | 4.01 | 5.42 | 8.75 | 11.86    |
| 0.5   | 4    | 49    | 50    | 1.5   | 0.1   |       | 1.39  | 2.6  | 2.63 | 3.59 | 4.52 | 6.68     |
| 0.7   | 4    | 49    | 50    | 1.5   | 0.1   |       | 1.17  | 2.08 | 2.1  | 3.02 | 2.7  | 4.23     |
| 0.9   | 4    | 49    | 50    | 1.5   | 0.1   |       | 62.81 | 72.76| 139.17| 149.78| 353.44| 364.56   |
| 0.12  | 0.05| 2     | 5     | 8     | 1.5   | 0.1   | 40.84 | 53.09| 97.59| 106.55| 229.29| 238.93   |
| 0.3   | 3    | 45    | 50    | 1.5   | 0.1   |       | 4.6   | 14.41| 17.76| 21.64| 58.62| 64.65    |
| 0.5   | 3    | 47    | 50    | 1.5   | 0.1   |       | 1.84  | 4.7  | 4.95 | 6.38 | 21.32| 25.38    |
| 0.7   | 3    | 47    | 50    | 1.5   | 0.1   |       | 1.4   | 2.82 | 2.87 | 3.94 | 9.49 | 12.34    |
| 0.9   | 3    | 47    | 50    | 1.5   | 0.1   |       | 1.2   | 2.05 | 2.09 | 3.15 | 4.85 | 6.88     |
| 0.18  | 0.05| 2     | 6     | 31    | 1.5   | 0.1   | 33.04 | 37.06| 191.41| 208.87| 82.17| 88.98    |
| 0.1   | 2    | 49    | 50    | 1.5   | 0.1   |       | 24.51 | 30.7 | 135.77| 144.52| 57.67| 63.65    |
| 0.3   | 2    | 49    | 50    | 1.5   | 0.1   |       | 4.04  | 14.16| 14.16| 19.17| 17.94| 21.73    |
| 0.5   | 2    | 49    | 50    | 1.5   | 0.1   |       | 1.85  | 3.97 | 3.97 | 5.32 | 7.25 | 9.76     |
| 0.7   | 5    | 49    | 50    | 1.5   | 0.1   |       | 1.34  | 2.69 | 2.69 | 3.54 | 3.5  | 5.18     |
| 0.9   | 5    | 49    | 50    | 1.5   | 0.1   |       | 1.09  | 2.04 | 2.08 | 3.02 | 1.96 | 3.07     |
### Table 4. Comparing optimal AATS values of \( np \) control charts when \( n_0 = 8 \) and \( h_0 = 2 \)

| \( p_0 \) | \( r \) | \( n_1 \) | \( n_2 \) | \( n_3 \) | \( h_1 \) | \( h_2 \) | \( SVSSI \) | \( FA \) | \( VSSI \) | \( VSS \) | \( VSI \) | \( Shewhart \) |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.03  | 0.05| 3   | 9   | 10  | 2   | 0.1 | 17  | 17  | 25.34| 25.34| 55.16| 55.19|
| 0.1   | 3   | 10  | 33  | 2   | 0.3 |     | 13.86| 13.87| 20.64| 20.64| 37.96| 37.99|
| 0.3   | 7   | 47  | 48  | 2   | 0.1 |     | 5.81 | 7.69 | 11.72| 11.72| 14.03| 14.06|
| 0.5   | 7   | 48  | 50  | 2   | 0.1 |     | 2.81 | 4.32 | 4.68 | 6.69 | 7.55 | 7.59 |
| 0.7   | 7   | 48  | 50  | 2   | 0.1 |     | 2.04 | 2.83 | 2.94 | 4.44 | 4.85 | 4.88 |
| 0.9   | 7   | 48  | 50  | 2   | 0.1 |     | 1.68 | 2.22 | 2.28 | 3.65 | 3.46 | 3.49 |
| 0.05  | 0.05| 1   | 6   | 20  | 2   | 0.4 | 31.84| 31.85| 59.94| 59.94| 184.99| 198.49|
| 0.1   | 6   | 48  | 49  | 2   | 0.1 |     | 21.91| 26.85| 43.82| 43.83| 114.87| 126.02|
| 0.3   | 6   | 48  | 49  | 2   | 0.1 |     | 4.46 | 12.4 | 16.42| 17.98| 29.17 | 35.35 |
| 0.5   | 7   | 47  | 48  | 2   | 0.1 |     | 2.12 | 5.24 | 5.59 | 7.51 | 11.63 | 15.58 |
| 0.7   | 7   | 47  | 48  | 2   | 0.1 |     | 1.61 | 3.3  | 3.38 | 4.78 | 5.9  | 8.6  |
| 0.9   | 7   | 47  | 48  | 2   | 0.1 |     | 1.38 | 2.58 | 2.62 | 3.91 | 3.52 | 5.43 |
| 0.08  | 0.05| 1   | 4   | 12  | 2   | 0.1 | 20.39| 20.39| 79.2 | 80.04| 61.48 | 61.51 |
| 0.1   | 1   | 4   | 50  | 2   | 0.2 |     | 17.68| 17.69| 53.27| 60.15| 42.92 | 42.95 |
| 0.3   | 4   | 49  | 50  | 2   | 0.1 |     | 4.83 | 10.39| 15.37| 19.9 | 14.97 | 15   |
| 0.5   | 4   | 49  | 50  | 2   | 0.1 |     | 2.39 | 3.85 | 4.25 | 6.4  | 7.37  | 7.4  |
| 0.7   | 4   | 49  | 50  | 2   | 0.1 |     | 1.84 | 2.41 | 2.5  | 4.04 | 4.37  | 4.4  |
| 0.9   | 4   | 49  | 50  | 2   | 0.1 |     | 1.56 | 1.83 | 1.89 | 3.34 | 2.91  | 2.95 |
| 0.12  | 0.05| 2   | 5   | 40  | 2   | 0.1 | 83.38| 97.01| 185.31| 199.66| 121.58| 130.07|
| 0.1   | 2   | 40  | 45  | 2   | 0.1 |     | 54.09| 70.77| 129.91| 142.03| 80.18 | 87.48 |
| 0.3   | 3   | 45  | 50  | 2   | 0.1 |     | 6.03 | 19.15| 23.58 | 26.89 | 21.77 | 26.14 |
| 0.5   | 7   | 47  | 50  | 2   | 0.1 |     | 2.39 | 6.18 | 6.56 | 8.39 | 8.45  | 11.25 |
| 0.7   | 7   | 47  | 50  | 2   | 0.1 |     | 1.7  | 3.43 | 3.51 | 5.01 | 4.12  | 5.96 |
| 0.9   | 7   | 40  | 47  | 2   | 0.1 |     | 1.39 | 2.35 | 2.4  | 3.9  | 2.4   | 3.61 |
| 0.18  | 0.05| 2   | 6   | 31  | 2   | 0.1 | 43.8 | 49.42| 254.4 | 278.43| 186.16| 194.59|
| 0.1   | 2   | 49  | 50  | 2   | 0.1 |     | 32.39| 40.94| 180.82| 192.63| 122.33| 129.72|
| 0.3   | 2   | 49  | 50  | 2   | 0.1 |     | 5.3  | 14.61| 18.76 | 25.45 | 31.22 | 35.81 |
| 0.5   | 2   | 49  | 50  | 2   | 0.1 |     | 2.45 | 4.54 | 4.89 | 7    | 11.02 | 14   |
| 0.7   | 5   | 49  | 50  | 2   | 0.1 |     | 1.77 | 2.67 | 2.73 | 4.22 | 4.82  | 6.76 |
| 0.9   | 5   | 46  | 50  | 2   | 0.1 |     | 1.44 | 1.89 | 1.95 | 3.42 | 2.54  | 3.77 |
Table 5. Comparing optimal ATS values of np control charts when \( n_0 = 4 \) and \( h_0 = 1 \)

| \( p_0 \) | \( r \) | \( n_1 \) | \( n_2 \) | \( n_3 \) | \( h_1 \) | \( h_2 \) | SVSSI | FA | VSSI | VSS | VSI | Shewhart |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.03 | 0.05 | 3 | 9 | 10 | 1 | 0.1 | 8.5 | 8.6 | 11.86 | 12.76 | 102.78 | 118.26 |
| 0.05 | 0.05 | 1 | 5 | 6 | 1 | 0.1 | 15.59 | 15.75 | 53.07 | 61.53 | 47.92 | 48.82 |
| 0.08 | 0.05 | 2 | 5 | 40 | 1 | 0.1 | 41.94 | 52.87 | 280.47 | 293.45 | 100.96 | 110.09 |
| 0.12 | 0.05 | 2 | 6 | 31 | 1 | 0.1 | 22.52 | 24.85 | 123.99 | 146.67 | 619.61 | 634.98 |
| 0.18 | 0.05 | 2 | 45 | 49 | 1 | 0.1 | 16.7 | 20.46 | 93.49 | 112.35 | 425.6 | 439.38 |
| 0.5 | 3 | 45 | 49 | 1 | 0.1 | 1.65 | 6.74 | 6.74 | 10.6 | 122.09 | 131.58 |
| 0.7 | 2 | 48 | 49 | 1 | 0.1 | 0.2 | 0.54 | 0.54 | 1.92 | 45.19 | 52.17 |

SVSSI design parameters
| $p_0$ | $r$ | $n_1$ | $n_2$ | $n_3$ | $h_1$ | $h_2$ | SVSSI | FA | VSSI | VSS | VSI | Shewhart |
|-----|----|------|------|------|------|------|-------|----|------|------|-----|---------|
| 0.03| 0.05| 3    | 9    | 10   | 1.5  | 0.1  | 12.67 | 12.86| 17.74| 19.14| 73.31| 74.71   |
| 0.1 | 3    | 10   | 33   | 1.5  | 0.1  | 10.05| 10.32| 14.22| 15.62| 49.86| 51.26   |
| 0.3 | 3    | 47   | 48   | 1.5  | 0.1  | 2.93 | 3.87 | 5.83 | 8.97 | 17.51| 18.91   |
| 0.5 | 5    | 47   | 48   | 1.5  | 0.1  | 0.51 | 0.98 | 1.05 | 3.16 | 8.9  | 10.3    |
| 0.7 | 5    | 47   | 48   | 1.5  | 0.1  | 0.17 | 0.29 | 0.29 | 1.92 | 5.35 | 6.75    |
| 0.9 | 5    | 47   | 48   | 1.5  | 0.1  | 0.11 | 0.14 | 0.14 | 1.61 | 3.53 | 4.93    |
| 0.05| 0.05| 1    | 6    | 20   | 1.5  | 0.1  | 23.56 | 23.57| 43.71| 45.11| 30.38| 31.78   |
| 0.1 | 1    | 6    | 48   | 1.5  | 0.1  | 19.16| 19.66| 30.34| 33.04| 22.16| 23.56   |
| 0.3 | 5    | 47   | 48   | 1.5  | 0.1  | 2.04 | 6.94 | 8.31 | 12.75| 8.89 | 10.29   |
| 0.5 | 5    | 47   | 48   | 1.5  | 0.1  | 0.31 | 1.23 | 1.3  | 3.42 | 4.73 | 6.13    |
| 0.7 | 5    | 47   | 48   | 1.5  | 0.1  | 0.13 | 0.28 | 0.28 | 1.9  | 2.87 | 4.28    |
| 0.9 | 5    | 47   | 48   | 1.5  | 0.1  | 0.11 | 0.13 | 0.13 | 1.58 | 1.88 | 3.28    |
| 0.08| 0.05| 1    | 4    | 8    | 1.5  | 0.1  | 15.5  | 15.39| 58.2 | 59.6 | 102.96| 113.77  |
| 0.1 | 1    | 4    | 50   | 1.5  | 0.1  | 13.17| 13.27| 44.23| 45.63| 68.91| 78.31   |
| 0.3 | 4    | 49   | 50   | 1.5  | 0.1  | 1.9  | 5.76 | 8.09 | 13.08| 20.05| 26.17   |
| 0.5 | 4    | 49   | 50   | 1.5  | 0.1  | 0.25 | 0.88 | 0.9 | 3.02 | 8.1  | 12.6    |
| 0.7 | 4    | 49   | 50   | 1.5  | 0.1  | 0.12 | 0.19 | 0.19 | 1.74 | 3.87 | 7.42    |
| 0.9 | 4    | 49   | 50   | 1.5  | 0.1  | 0.1  | 0.11 | 0.11 | 1.53 | 2.05 | 4.97    |
| 0.12| 0.05| 2    | 5    | 40   | 1.5  | 0.1  | 62.3  | 72.27| 137.18| 149.15| 352.79| 365.3   |
| 0.1 | 2    | 40   | 45   | 1.5  | 0.1  | 39.6 | 51.95| 95.71| 106  | 228.62| 239.68  |
| 0.3 | 3    | 45   | 50   | 1.5  | 0.1  | 2.54 | 9.58 | 11.68| 16.66| 57.97| 65.39   |
| 0.5 | 3    | 47   | 50   | 1.5  | 0.1  | 0.26 | 0.97 | 1.01 | 3.04 | 20.66| 26.12   |
| 0.7 | 3    | 49   | 50   | 1.5  | 0.1  | 0.11 | 0.17 | 0.17 | 1.68 | 8.83 | 13.08   |
| 0.9 | 3    | 49   | 50   | 1.5  | 0.1  | 0.1  | 0.1  | 0.1  | 1.52 | 4.19 | 7.62    |
| 0.18| 0.05| 2    | 6    | 31   | 1.5  | 0.1  | 33.34 | 37.22| 184.73| 207.68| 81.52 | 88.72   |
| 0.1 | 2    | 45   | 49   | 1.5  | 0.1  | 24.54| 30.64| 133.12| 143.4 | 57.01| 64.39   |
| 0.3 | 2    | 45   | 49   | 1.5  | 0.1  | 2.3  | 9.9  | 9.9  | 15.89| 17.29| 22.47   |
| 0.5 | 5    | 45   | 49   | 1.5  | 0.1  | 0.22 | 0.74 | 0.74 | 2.88 | 6.6  | 10.5    |
| 0.7 | 5    | 48   | 49   | 1.5  | 0.1  | 0.11 | 0.14 | 0.14 | 1.64 | 2.84 | 5.92    |
| 0.9 | 5    | 48   | 49   | 1.5  | 0.1  | 0.1  | 0.1  | 0.1  | 1.51 | 1.31 | 3.81    |
Table 7. Comparing optimal ATS values of np control charts when $n_0=8$ and $h_0=2$

| $p_0$ | $r$ | $n_1$ | $n_2$ | $n_3$ | $h_1$ | $h_2$ | SVSSI | FA | VSSI | VSS | VSI | Shewhart |
|-------|-----|-------|-------|-------|-------|-------|-------|-----|-----|-----|-----|--------|
| 0.03  | 0.05| 3     | 9     | 10    | 2     | 0.1   | 16.85 | 17.11| 23.62| 25.52| 54.27| 56.17  |
| 0.1   | 3   | 10    | 33    | 2     | 0.1   | 13.35 | 13.72 | 18.93| 20.83| 37.08| 38.98|
| 0.3   | 7   | 47    | 48    | 2     | 0.1   | 3     | 5.12  | 7.68 | 11.96| 13.14| 15.04|
| 0.5   | 7   | 47    | 48    | 2     | 0.1   | 0.53  | 1.26  | 1.34 | 4.21 | 6.67 | 8.57  |
| 0.7   | 7   | 47    | 48    | 2     | 0.1   | 0.18  | 0.34  | 0.35 | 2.56 | 3.97 | 5.87  |
| 0.9   | 7   | 47    | 48    | 2     | 0.1   | 0.12  | 0.15  | 0.15 | 2.15 | 2.57 | 4.47  |
| 0.05  | 0.05| 1     | 6     | 20    | 2     | 0.1   | 31.36 | 31.39| 58.25| 60.15| 184.08| 199.48 |
| 0.1   | 6   | 20    | 48    | 2     | 0.1   | 21.36 | 26.18 | 42.15| 44.05| 113.97| 127   |
| 0.3   | 7   | 47    | 48    | 2     | 0.1   | 2.28  | 8.64  | 10.97| 17   | 28.27| 36.33 |
| 0.5   | 7   | 47    | 48    | 2     | 0.1   | 0.33  | 1.6   | 1.68 | 4.57 | 10.72| 16.56 |
| 0.7   | 7   | 47    | 48    | 2     | 0.1   | 0.13  | 0.33  | 0.34 | 2.54 | 4.99 | 9.58  |
| 0.9   | 7   | 47    | 48    | 2     | 0.1   | 0.11  | 0.14  | 0.14 | 2.11 | 2.62 | 6.41  |
| 0.08  | 0.05| 1     | 4     | 12    | 2     | 0.1   | 20.72 | 24.49| 77.57| 79.47| 60.59| 62.49 |
| 0.1   | 1   | 4     | 50    | 2     | 0.1   | 17.51 | 17.66 | 51.9 | 60.49| 42.03| 43.93 |
| 0.3   | 4   | 49    | 50    | 2     | 0.1   | 2.43  | 7.04  | 10.06| 16.83| 14.09| 15.99 |
| 0.5   | 4   | 49    | 50    | 2     | 0.1   | 0.29  | 0.91  | 0.98 | 3.85 | 6.49 | 8.39  |
| 0.7   | 4   | 49    | 50    | 2     | 0.1   | 0.12  | 0.19  | 0.19 | 2.3  | 3.48 | 5.38  |
| 0.9   | 4   | 49    | 50    | 2     | 0.1   | 0.1   | 0.11  | 0.11 | 2.04 | 2.03 | 3.93  |
| 0.12  | 0.05| 2     | 5     | 40    | 2     | 0.1   | 82.66 | 96.32| 182.62| 198.87| 120.67| 131.05|
| 0.1   | 2   | 40    | 45    | 2     | 0.1   | 52.4  | 69.22 | 127.37| 141.33| 79.27 | 88.47 |
| 0.3   | 3   | 45    | 50    | 2     | 0.1   | 3.27  | 12.68 | 15.46| 22.21| 20.87| 27.12 |
| 0.5   | 7   | 47    | 50    | 2     | 0.1   | 0.3   | 1.23  | 1.29 | 4.05 | 7.54 | 12.23 |
| 0.7   | 7   | 49    | 50    | 2     | 0.1   | 0.12  | 0.19  | 0.19 | 2.23 | 3.21 | 6.94  |
| 0.9   | 7   | 49    | 50    | 2     | 0.1   | 0.1   | 0.11  | 0.11 | 2.02 | 1.5  | 4.6   |
| 0.18  | 0.05| 2     | 6     | 31    | 2     | 0.1   | 44.15 | 49.59| 245.47| 276.91| 185.25| 195.58|
| 0.1   | 2   | 45    | 49    | 2     | 0.1   | 32.39 | 40.82 | 177.24| 191.21| 121.42| 130.7 |
| 0.3   | 2   | 45    | 49    | 2     | 0.1   | 2.95  | 10.32 | 13.05| 21.19| 30.31| 36.79 |
| 0.5   | 5   | 49    | 50    | 2     | 0.1   | 0.24  | 0.84  | 0.88 | 3.79 | 10.11| 14.98 |
| 0.7   | 5   | 48    | 49    | 2     | 0.1   | 0.11  | 0.14  | 0.14 | 2.17 | 3.91 | 7.74  |
| 0.9   | 5   | 48    | 49    | 2     | 0.1   | 0.1   | 0.1   | 0.1 | 2.01 | 1.63 | 4.75  |
Table 8. Comparing optimal \(AATS\) and \(ATS\) values of \(VSSI_n\) and \(SVSSI\) sampling methods when \(n_0=4\) and \(h_0=1\)

| \(p_0\) | \(r\) | \(AATS\) | \(ATS\) |
| --- | --- | --- | --- |
|   | \(VSSI_n\) | \(SVSSI\) | \(\Delta_{AATS}\) (%) | \(VSSI_n\) | \(SVSSI\) | \(\Delta_{ATS}\) (%) |
| 0.03 | 0.05 | 8.5013 | 8.4971 | 0.05 | 8.7896 | 8.4952 | 3.35 |
|   | 0.1  | 6.9334 | 6.9272 | 0.09 | 7.0473 | 6.754  | 4.16 |
|   | 0.3  | 3.8312 | 3.8266 | 0.12 | 2.3251 | 2.0114 | 13.49 |
|   | 0.5  | 2.5413 | 2.5409 | 0.02 | 0.5978 | 0.4659 | 22.06 |
|   | 0.7  | 1.911  | 1.9117 | -0.04 | 0.2096 | 0.1689 | 19.42 |
|   | 0.9  | 1.5413 | 1.542  | -0.05 | 0.1243 | 0.1137 | 8.53 |
| 0.05 | 0.05 | 15.9262| 15.919 | 0.05 | 15.9077| 15.5929| 1.98 |
|   | 0.1  | 13.4314| 13.4159| 0.12 | 13.2502| 12.8262| 3.2  |
|   | 0.3  | 5.3062 | 4.557  | 14.12 | 3.0394 | 2.1169 | 30.35 |
|   | 0.5  | 2.32  | 2.1823 | 5.94  | 0.5317 | 0.3076 | 42.15 |
|   | 0.7  | 1.6022| 1.5982 | 0.25  | 0.1815 | 0.1152 | 28.37 |
|   | 0.9  | 1.2799| 1.3006 | -1.62 | 0.1152 | 0.1049 | 8.94 |
| 0.08 | 0.05 | 10.1952| 10.1935| 0.02  | 10.4754| 10.3715| 0.99 |
|   | 0.1  | 8.8402 | 8.8374 | 0.03  | 8.9986 | 8.8367 | 1.8  |
|   | 0.3  | 4.847 | 3.3472 | 30.94 | 2.923 | 1.6902 | 42.18 |
|   | 0.5  | 2.1268| 1.5804 | 25.69 | 0.3978 | 0.2322 | 41.63 |
|   | 0.7  | 1.599 | 1.1882 | 25.69 | 0.1379 | 0.1162 | 15.74 |
|   | 0.9  | 1.3343| 0.9856 | 26.13 | 0.1044 | 0.1021 | 2.2  |
| 0.12 | 0.05 | 52.2464| 42.2385| 19.16 | 51.8923| 41.9391| 19.18 |
|   | 0.1  | 36.4141| 27.5882| 24.24 | 35.5929| 26.8033| 24.69 |
|   | 0.3  | 4.8657 | 3.1654 | 34.94 | 3.1696 | 1.8134 | 42.79 |
|   | 0.5  | 1.7806| 1.2646 | 28.98 | 0.3785 | 0.2238 | 40.87 |
|   | 0.7  | 1.3086| 0.9575 | 26.83 | 0.1278 | 0.1118 | 12.52 |
|   | 0.9  | 1.1198| 0.8173 | 27.01 | 0.1021 | 0.1010 | 1.08 |
| 0.18 | 0.05 | 28.8104| 22.2859| 22.65 | 29.0045| 22.521 | 22.35 |
|   | 0.1  | 23.4137| 16.6226| 29   | 23.3607| 16.6961| 28.53 |
|   | 0.3  | 4.5333 | 2.7774 | 38.73 | 3.1359 | 1.6543 | 47.25 |
|   | 0.5  | 1.7627| 1.2556 | 28.77 | 0.3427 | 0.195  | 43.1 |
|   | 0.7  | 1.3426| 0.976 | 27.31 | 0.1177 | 0.1074 | 8.75 |
|   | 0.9  | 1.1494| 0.8241| 28.3  | 0.1009 | 0.1004 | 0.5  |
Table 9. Comparing optimal AATS and ATS values of VSSI<sub>n</sub> and SVSSI sampling methods when \( n_0 = 6 \) and \( h_0 = 1.5 \)

| \( p_0 \) | \( r \) | AATS | VSSI<sub>n</sub> | SVSSI | \( \Delta_{\text{AATS}}(\%) \) | ATS | VSSI<sub>n</sub> | SVSSI | \( \Delta_{\text{ATS}}(\%) \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.03 | 0.05 | 12.7536 | 12.7483 | 0.04 | 13.1341 | 12.6747 | 3.5 |
| 0.1 | 10.4022 | 10.3922 | 0.1 | 10.5133 | 10.0515 | 4.39 |
| 0.3 | 5.7177 | 5.7095 | 0.14 | 3.4204 | 2.9325 | 14.26 |
| 0.5 | 3.0163 | 2.8848 | 4.36 | 0.7305 | 0.508 | 30.46 |
| 0.7 | 2.0646 | 2.0529 | 0.57 | 0.2366 | 0.1738 | 26.54 |
| 0.9 | 1.6457 | 1.6597 | -0.85 | 0.1299 | 0.1146 | 11.78 |
| 0.05 | 0.05 | 23.8918 | 23.8822 | 0.04 | 23.9837 | 23.5588 | 1.77 |
| 0.1 | 20.1496 | 20.1205 | 0.14 | 19.8199 | 19.1602 | 3.33 |
| 0.3 | 6.2904 | 4.0606 | 35.45 | 3.9829 | 2.0388 | 48.81 |
| 0.5 | 2.8966 | 2.0213 | 30.22 | 0.7203 | 0.3133 | 56.5 |
| 0.7 | 2.0862 | 1.5169 | 27.29 | 0.2183 | 0.1313 | 39.85 |
| 0.9 | 1.7101 | 1.2669 | 25.92 | 0.2108 | 0.1051 | 50.14 |
| 0.08 | 0.05 | 15.2945 | 15.2924 | 0.01 | 15.6583 | 15.4978 | 1.03 |
| 0.1 | 13.2626 | 13.2589 | 0.03 | 13.4414 | 13.1742 | 1.99 |
| 0.3 | 5.759 | 3.6676 | 36.32 | 3.6801 | 1.9008 | 48.35 |
| 0.5 | 2.5532 | 1.8064 | 29.25 | 0.5024 | 0.2513 | 49.98 |
| 0.7 | 1.8969 | 1.3874 | 26.86 | 0.1514 | 0.1177 | 22.26 |
| 0.9 | 1.7101 | 1.2669 | 25.92 | 0.1058 | 0.1022 | 3.4 |
| 0.12 | 0.05 | 73.7149 | 62.809 | 14.79 | 73.3425 | 62.2976 | 15.06 |
| 0.1 | 50.1509 | 40.8394 | 18.57 | 48.9955 | 39.5995 | 19.18 |
| 0.3 | 7.2481 | 4.5974 | 36.57 | 4.6493 | 2.5397 | 45.37 |
| 0.5 | 2.6563 | 1.8447 | 30.55 | 0.5033 | 0.2619 | 47.96 |
| 0.7 | 1.9551 | 1.4046 | 28.16 | 0.1388 | 0.1139 | 17.94 |
| 0.9 | 1.6721 | 1.198 | 28.35 | 0.1028 | 0.1011 | 1.65 |
| 0.18 | 0.05 | 43.1859 | 33.0408 | 23.49 | 43.4258 | 33.3357 | 23.24 |
| 0.1 | 35.0879 | 24.5061 | 30.16 | 34.9532 | 24.5435 | 29.78 |
| 0.3 | 6.7612 | 4.0397 | 40.25 | 4.6031 | 2.3004 | 50.02 |
| 0.5 | 2.5605 | 1.8513 | 27.7 | 0.4331 | 0.2199 | 49.23 |
| 0.7 | 1.7901 | 1.3406 | 25.11 | 0.1225 | 0.108 | 11.84 |
| 0.9 | 1.4402 | 1.0884 | 24.43 | 0.101 | 0.1004 | 0.59 |
Table 10. Comparing optimal AATS and ATS values of VSSI\(_n\) and SVSSI sampling methods when \(n_0=8\) and \(h_0=2\)

| \(p_0\) | \(r\) | AATS VSSI | \(\Delta_{AATS}\) (%) | ATS VSSI | \(\Delta_{ATS}\) (%) |
|-------|------|---------|----------------|---------|----------------|
| 0.03  | 0.05 | 17.0079 17.0017 0.04 | 17.4782 16.8542 3.57 |
| 0.1   | 0.5  | 13.8732 13.8584 0.11 | 13.9788 13.3489 4.51 |
| 0.3   | 0.7  | 7.5865  5.8079  23.44 | 4.5157  2.9951  33.67 |
| 0.5   | 0.7  | 3.9793  2.8055  29.5   | 0.9301  0.5265  43.39 |
| 0.7   | 0.1  | 2.7303  2.0385  25.34 | 0.2744  0.1788  34.84 |
| 0.9   | 0.1  | 2.1779  1.6849  22.64 | 0.1372  0.1158  15.6  |
| 0.05  | 0.05 | 31.8596 31.8447 0.05 | 31.945  31.3639 1.82 |
| 0.1   | 0.5  | 26.8694 21.9112 18.45 | 26.3896 21.3569 19.07 |
| 0.3   | 0.7  | 7.2704  4.4564  38.7   | 4.7945  2.2782  52.48 |
| 0.5   | 0.7  | 3.2422  2.1247  34.47 | 0.877   0.3334  61.98 |
| 0.7   | 0.1  | 2.2409  1.6059  28.34 | 0.2499  0.134   46.38 |
| 0.9   | 0.1  | 1.8509  1.3823  25.32 | 0.1249  0.1055  15.53 |
| 0.08  | 0.05 | 20.396  20.3942 0.01 | 20.8412 20.7221 0.57 |
| 0.1   | 0.5  | 17.6871 17.682  0.03  | 17.8843 17.5118 2.08 |
| 0.3   | 0.7  | 7.6622  4.8259  37.02 | 4.8429  2.4282  49.86 |
| 0.5   | 0.7  | 3.4061  2.3904  29.79 | 0.6148  0.2851  53.63 |
| 0.7   | 0.1  | 2.3458  1.8421  21.47 | 0.1637  0.12   26.7  |
| 0.9   | 0.1  | 1.795   1.5611  13.03 | 0.1071  0.1023  4.48  |
| 0.12  | 0.05 | 98.2828 83.3819 15.16 | 97.746  82.656  15.44 |
| 0.1   | 0.5  | 66.8477 54.093  19.08 | 65.2661 52.3957 19.72 |
| 0.3   | 0.7  | 9.3683  6.0316  35.62 | 5.8789  3.2659  44.45 |
| 0.5   | 0.7  | 3.3331  2.3928  28.21 | 0.6056  0.2977  50.84 |
| 0.7   | 0.1  | 2.2666  1.7003  24.98 | 0.1479  0.1154  21.97 |
| 0.9   | 0.1  | 1.8029  1.3909  22.85 | 0.1034  0.1011  2.22  |
| 0.18  | 0.05 | 57.5619 43.798  23.91 | 57.847  44.1504 23.68 |
| 0.1   | 0.5  | 46.7633 32.392  30.73 | 46.5457 32.3909 30.41 |
| 0.3   | 0.7  | 8.9931  5.3045  41.02 | 6.0703  2.9465  51.46 |
| 0.5   | 0.7  | 3.3994  2.4492  27.95 | 0.5307  0.2447  53.89 |
| 0.7   | 0.1  | 2.3808  1.7697  25.67 | 0.1278  0.1086  15.02 |
| 0.9   | 0.1  | 1.8612  1.4368  22.8  | 0.1012  0.1004  0.79  |