Actin network architecture and elasticity in lamellipodia of melanoma cells

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Abstract. Cell migration is an essential element in the immune response on the one hand and in cancer metastasis on the other hand. The architecture of the actin network in lamellipodia determines the elasticity of the leading edge and contributes to the regulation of migration. We have implemented a new method for the analysis of actin network morphology in the lamellipodia of B16F1 mouse melanoma cells. This method is based on fitting multi-layer geometrical models to electron microscopy images of lamellipodial actin networks. The chosen model and F-actin concentrations are thereby deterministic parameters. Using this approach, we identified distinct structural features of actin networks in lamellipodia. The mesh size which defines the elasticity of the lamellipodium was determined as 34 and 78 nm for a two-layer network.

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at a total actin concentration of 9.6 mg ml$^{-1}$. These data lead to estimates of the low frequency elastic shear moduli which differ by more than a magnitude between the two layers. These findings indicate an anisotropic shear modulus of the lamellipodium with the stiffer layer being the dominant structure against deformations in the lamellipodial plane and the softer layer contributing significantly at lower indentations perpendicular to the lamellipodial plane. This combination creates a material that is optimal for pushing forward as well as squeezing through narrow spaces.

1. Introduction

Cell migration proceeds within the context of tissue structures. Thus, motile cells have to overcome the mechanical resistance of the surrounding cells and extracellular matrix by adapting their shape and physical properties (Wolf et al 2003). The actin filament network can regulate the elasticity of the cell and, consequently, influences cell migration (Beil et al 2003; Park et al 2005). The movement of cells starts with a locally confined reorganization of subcortical cytoplasmic compartments which enables the directed motility of cells (Danuser 2005; Vallotton et al 2004; Yamazaki et al 2003). Formation and protrusion of the leading edge are regulated by the local remodelling of the actin network which consists of at least two morphologically distinct structures: lamellipodia and filopodia with the latter arising from the dendritic actin network of lamellipodia (Pollard and Borisy 2003; Svitkina et al 2003). Although simulation models for the formation of actin networks could explain the propulsive activity of lamellipodia (Maly and Borisy 2001; Mogilner and Oster 1996), global morphological properties of real actin networks in cells have not yet been characterized. This is mainly due to the fact that most microscopic techniques employed for the imaging of individual actin filaments are two-dimensional (2D), i.e. the 3D architecture of actin networks is projected into 2D images (Svitkina and Borisy 1998). Although lamellipodia represent a very thin cytoplasmic compartment (Atilgan et al 2005), they can contain several superimposed layers of flat actin networks (Small et al 1995). This topological problem has to be dealt with when
extracting structural data and estimating biophysical features from 2D images (MacKintosh et al 1995; Ananthakrishnan et al 2006a, b).

The architecture of the cytoskeleton is determined by deterministic as well as stochastic processes which modulate the build-up and remodelling of filament networks. Taking these characteristics into account, we have developed a new approach to the morphological analysis of cytoskeletal filament networks (Beil et al 2006). This method is based on the fitting of statistical geometrical models to cytoskeletal networks with the choice of the specific model and the density of structural components representing deterministic elements. The set of models comprises Poisson-line tessellations (PLT), Poisson–Voronoï tessellations (PVT) and Poisson–Delaunay tessellations (PDT). From a biological point of view, the PLT model represents a set of randomly positioned filaments whose shape is not influenced by interconnections (figure 1(a)). In contrast, both PDT and PVT models describe a network of branched filaments (figures 1(b) and (c)). The PVT model reflects a functionally isotropic pattern of interconnected filaments, whereas the PDT model appears to represent the build-up of major branching points inducing filament arrangements in a few preferred directions. Although these models represent complex topological scenarios, they can be described by a single parameter $\gamma$, i.e. the density of filaments or branching points (Gloaguen et al 2006). Thus, this approach provides the opportunity to reduce the morphological complexity of the cytoskeleton to a small number of parameters. The fitting of these models to images of filament networks is performed by comparing the morphological characteristics of both theoretical and real networks in an $m$-dimensional space (Gloaguen et al 2006).

In our previous work, single layer models were used to study intermediate filament networks. In this study, we extended this approach by superimposing a finite number of these 2D models to match the characteristics of actin networks in the lamellipodia of mouse melanoma cells.

A proper architecture of the actin network is a prerequisite for the protrusion of the cytoplasm at the leading edge (Desmarais et al 2004). Based on the specific morphological properties of the leading edge, i.e. flat structure (Atilgan et al 2005), the geometrical models used in this study assume that the architecture of lamellipodia can be modelled by different layers of flat actin networks. The structural properties of these layers are then used to estimate the elasticity of lamellipodia.
2. Materials and methods

2.1. Specimen preparation and image processing

The mouse melanoma cell line B16F1 was provided by Dr C Ballestrem (Weizmann Institute of Science, Rehovot, Israel). Cell culture, specimen preparation and electron microscopy imaging were performed as described by Svitkina et al (2003). Figure 2 depicts the typical architecture of lamellipodia in these cells.

The superposition of actin filament networks can be regarded as a network of line segments with all endpoints being connected. Thus, a watershed segmentation (Soille 2003) could be applied for greyscale skeletonization. In the resulting dam structure, each pixel was classified into crosspoints and linepoints. In the next step, this dam structure was compared to the original image. For each dam pixel the greyscale values at the corresponding coordinates are regarded along lines. If the minimum of these values is bigger than a given threshold \( t \) the line is kept; otherwise it is discarded. This procedure removes lines which result from topological disturbances and do not correspond to actin filaments. Assuming that the image of an actin filament approximates a straight line due to a sufficiently large persistence length, dams are then replaced by straight lines between the two endpoints of the dam. Finally, topological disturbances of the primary segmentation are corrected by merging neighbouring branching points having a distance below a given threshold. This merging is done such that pairs of branching points with a smaller distance to each other are merged first. The final result is a graph structure consisting of nodes and connecting line segments (figure 3).
2.2. Fitting of tessellation models

To evaluate global geometric properties of the actin networks, we compared the features of real networks with those of theoretical models. The result is the selection of a tessellation model that best represents the pivotal structural properties of lamellipodia. With regard to the input data, we consider the characteristics \( \lambda_1, \ldots, \lambda_4 \) that correspond to the mean number of vertices, the mean number of edges, the mean number of meshes and the mean total length of edges, respectively, always measured with respect to the unit area. The entries of the vector

\[
\lambda = (\lambda_1, \ldots, \lambda_4)^\top
\]  

(2.1)

are estimated from the graph structure that arises from the original electron microscopy images. To compare a vector of estimated characteristics from the input data with a vector of theoretical mean values, a distance measure is needed. In this paper, we consider the relative Euclidean distance measure given by

\[
d_e(y, z) = \left( \sum_{i=1}^{m} \left( \frac{y_i - z_i}{y_i} \right)^2 \right)^{1/2},
\]

(2.2)

where \( y = (y_1, \ldots, y_m)^\top \) and \( z = (z_1, \ldots, z_m)^\top \) denote two vectors with \( m \) (in our case 4) entries. Notice that in this context the distance measure does not have a geometrical interpretation, but refers to the distance of two vectors with respect to this measure and could be interpreted as a qualitative matching likelihood. We also assessed other choices of distance measures such as the relative absolute distance and the relative maximum distance (Gloaguen et al 2006). Due to the fact that the results were similar with respect to the optimal model choice and optimal parameters they are omitted in the following discussion. Such similarities are an indication of the robustness of the method since the results of the fitting must not depend on the specific distance measure used, if the distance measure is reasonably chosen.
Table 1. Values of $\lambda_1$ (mean number of vertices), $\lambda_2$ (mean number of segments), $\lambda_3$ (mean number of meshes) and $\lambda_4$ (mean total length of segments; all measured per unit area) for a given tessellation with respect to $\lambda_4$.

| Tessellation | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|-------------|
| PLT         | $\frac{1}{\pi}\lambda_4^2$ | $\frac{2}{\pi}\lambda_4^2$ | $\frac{1}{\pi}\lambda_4^2$ | $\lambda_4$ |
| PVT         | $\frac{1}{2}\lambda_4^2$ | $\frac{3}{4}\lambda_4^2$ | $\frac{1}{4}\lambda_4^2$ | $\lambda_4$ |
| PDT         | $\frac{9\pi^2}{1024}\lambda_4^2$ | $\frac{27\pi^2}{1024}\lambda_4^2$ | $\frac{18\pi^2}{1024}\lambda_4^2$ | $\lambda_4$ |

In figure 1, realizations of the three basic tessellation models are displayed. For these three realizations, the model parameters were chosen such that the mean total length of the edges per unit area of the three different models is equal. The relationships between the four different characteristics $\lambda_1, \ldots, \lambda_4$, introduced in subsection 2.1, and the parameter $\gamma$ of a basic tessellation are known for these types of tessellations (Stoyan et al 1995; table 1). Note that for each type of basic tessellation the parameter $\gamma$ has a different meaning that is explained below. The PLT is induced by a random Poisson line process in the plane. This Poisson line process can alternatively be regarded as a Poisson point process on $\mathbb{R} \times [0, \pi]$ due to the fact that each line is uniquely determined by the signed perpendicular distance to the origin and by the angle in anti-clockwise direction between the orientation vector of the line and the $x$-axis. The parameter $\gamma_{\text{PLT}}$ represents the mean total length of edges per unit area. The PVT is based on a Poisson point process $X = \{X_n\}$ in the plane with intensity $\gamma_{\text{PVT}}$ representing the mean number of points per unit area. Afterwards Voronoi meshes are constructed with respect to the nuclei $\{X_n\}$ by using the nearest neighbour principle. Finally, the PDT can be generated by a PVT. If a generating PVT is considered and nuclei of these meshes that share a common edge are connected, then a PDT is formed. Hence, we obtain that the vertices of a PDT are given by the nuclei of the generating PVT and that an edge of the PDT always perpendicularly crosses an edge of the generating PVT. Since in the case of Poisson processes almost surely three different points do not lie on one and the same line, the meshes of the PDT are triangles with probability one. The intensity $\gamma_{\text{PDT}}$ denotes the mean number of vertices of the PDT per unit area. These basic tessellations can be used to construct tessellation models that are more sophisticated by a superposition of $n$ ($n \geq 2$) independent layers (figure 4). Such a superposed tessellation can of course be the initial tessellation for a further tessellation step. If the superposition is performed once and if we consider PLTs, PVTs and PDTs as possible model classes for the two tessellation layers, we end up with at most nine different possible model classes. Notice that a symmetry occurs in the sense that, for example, a PLT/PVT superposition with parameters $\gamma_0$ and $\gamma_1$ has the same distribution as a PVT/PLT superposition with parameters reversed. Hence, the nine possible model classes are reduced to six. Notice furthermore that, after rescaling, a PLT/PLT superposition is equal in distribution to a basic PLT tessellation, whereas this property is not true for PVT/PVT and PDT/PDT with respect to a PVT and a PDT tessellation, respectively. Mean value formulae for these six different possible superposition models are given in table 2 (Maier and Schmidt 2003). Notice that also for superpositions consisting of three or more layers mean value formulae can be derived in a similar fashion. For a three-layer superposition with PLT,
Figure 4. Realizations of superposed tessellation models with layers given in blue and red, respectively. (a) PLT/PLT, (b) PLT/PVT, (c) PLT/PDT, (d) PVT/PVT, (e) PVT/PDT and (f) PDT/PDT.

Table 2. Mean-value formulae for one-fold superpositions.

|      | PLT/PLT | PLT/PVT | PLT/PDT |
|------|---------|---------|---------|
| $\lambda_1$ | $\frac{1}{\pi} (\gamma_0 + \gamma_1)^2$ | $\frac{1}{\pi} \gamma_0^2 + 2 \gamma_1 + \frac{4}{\pi} \gamma_0 \sqrt{\gamma_1}$ | $\frac{1}{\pi} \gamma_0^2 + \gamma_1 + \frac{64}{3\pi^2} \gamma_0 \sqrt{\gamma_1}$ |
| $\lambda_2$ | $\frac{2}{\pi} (\gamma_0 + \gamma_1)^2$ | $\frac{2}{\pi} \gamma_0^2 + 3 \gamma_1 + \frac{8}{\pi} \gamma_0 \sqrt{\gamma_1}$ | $\frac{2}{\pi} \gamma_0^2 + 3 \gamma_1 + \frac{128}{3\pi^2} \gamma_0 \sqrt{\gamma_1}$ |
| $\lambda_3$ | $\frac{1}{\pi} (\gamma_0 + \gamma_1)^2$ | $\frac{1}{\pi} \gamma_0^2 + \gamma_1 + \frac{4}{\pi} \gamma_0 \sqrt{\gamma_1}$ | $\frac{1}{\pi} \gamma_0^2 + 2 \gamma_1 + \frac{64}{3\pi^2} \gamma_0 \sqrt{\gamma_1}$ |
| $\lambda_4$ | $\gamma_0 + \gamma_1$ | $\gamma_0 + 2 \sqrt{\gamma_1}$ | $\gamma_0 + \frac{32}{3\pi} \sqrt{\gamma_1}$ |

|      | PVT/PVT | PDT/PVT |
|------|---------|---------|
| $\lambda_1$ | $2\gamma_0 + 2\gamma_1 + \frac{8}{\pi} \sqrt{\gamma_0 \gamma_1}$ | $2\gamma_0 + \gamma_1 + \frac{128}{3\pi^2} \sqrt{\gamma_0 \gamma_1}$ | $\gamma_0 + \gamma_1 + \frac{2048}{9\pi^3} \sqrt{\gamma_0 \gamma_1}$ |
| $\lambda_2$ | $3\gamma_0 + 3\gamma_1 + \frac{16}{\pi} \sqrt{\gamma_0 \gamma_1}$ | $3\gamma_0 + 3\gamma_1 + \frac{256}{3\pi^2} \sqrt{\gamma_0 \gamma_1}$ | $3\gamma_0 + 3\gamma_1 + \frac{4096}{9\pi^3} \sqrt{\gamma_0 \gamma_1}$ |
| $\lambda_3$ | $\gamma_0 + \gamma_1 + \frac{8}{\pi} \sqrt{\gamma_0 \gamma_1}$ | $\gamma_0 + 2\gamma_1 + \frac{128}{3\pi^2} \sqrt{\gamma_0 \gamma_1}$ | $\gamma_0 + 2\gamma_1 + \frac{2048}{9\pi^3} \sqrt{\gamma_0 \gamma_1}$ |
| $\lambda_4$ | $2\sqrt{\gamma_0} + 2\sqrt{\gamma_1}$ | $2\sqrt{\gamma_0} + \frac{32}{3\pi} \sqrt{\gamma_1}$ | $\frac{32}{3\pi} \sqrt{\gamma_0} + \frac{32}{3\pi} \sqrt{\gamma_1}$ |
PVT and PDT as basic models, we obtain seven new possible model classes due to the fact that the 27 original possibilities are reduced by similar effects as for the two-layer superpositions.

2.3. Algorithm summary

The model choice algorithm applied can be summarized as follows. Given an input image, the vector of characteristics $\lambda = (\lambda_1, \ldots, \lambda_4)$ is estimated, leading to estimates $\hat{\lambda} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_4)$ (see Beil et al (2006) for details with respect to estimators). By using the relative Euclidean distance measure introduced in section 2.2 for each of the six possible superposed tessellation models separately a relative distance function

$$f_{\text{model}}(\gamma_0, \gamma_1) = \left( \sum_{i=1}^{4} \left( \frac{\hat{\lambda}_i - \lambda_{\text{model}}}{\hat{\lambda}_i} \right)^2 \right)^{1/2},$$

(2.3)

can be constructed, where $\gamma_0$ and $\gamma_1$ denote the parameters for the two layers of the superposed tessellation. The values $\lambda_{\text{model}}$ are then the theoretical model characteristics depending on the choice of the superposed tessellation model and of $\gamma_0$ and $\gamma_1$. Then for each model separately an optimal parameter vector is determined; this means a vector $(\gamma_0, \gamma_1)$ that minimizes $f_{\text{model}}(\gamma_0, \gamma_1)$ for the given model. A model is considered optimal among all possible models if the optimal value $f_{\text{model}}(\gamma_0, \gamma_1)$ is minimal with respect to all models. Optimal in this context means that the (relative Euclidean) distance of the measured vector of characteristics is minimal to the vector of theoretical characteristics compared to the distances measured for other models. In the given data set instead of investigating single data samples, it is reasonable to investigate mean samples since samples of the same group can be considered to be independent and identically distributed. Here, the averaging is performed in an arithmetical sense, meaning that a vector of characteristics $(\bar{\hat{\lambda}}_1, \bar{\hat{\lambda}}_2, \bar{\hat{\lambda}}_3, \bar{\hat{\lambda}}_4)$ is estimated from the $n$ samples by

$$\bar{\hat{\lambda}}_i = \frac{1}{n} \sum_{j=1}^{n} \hat{\lambda}_{ij},$$

(2.4)

where $\hat{\lambda}_{ij}$ is the estimated $i$th characteristic for the $j$th sample. Notice that such an averaging is comparable to the data being measured in a single but much larger sampling window. All software that was developed and used for the image analysis as well as for the model fitting procedure is included in the GeoStoch library which is a Java-based library system (Mayer et al 2004; http://www.geostoch.de).

3. Results

Seven sample regions from different melanoma cells were analysed (figure 2). Assuming that the samples are independently drawn from one and the same distribution, the analysis of the vector of mean characteristics is comparable to an analysis of a single but larger sampling window. Hence it is important to note that it is not the number of images analysed that is the most important factor for the accuracy of the analysis but the total area that is considered. Here, the total area analysed was 10.91 µm². The total run-time for the computations (including image segmentation) was less than 1 h.
Table 3. Distance values and model parameters for actin networks. Recall that parameters \( \gamma \) may have different meanings depending on the tessellation type (for PLT \( \gamma \) is given in nm nm\(^{-2} \), for PVT and PDT the unit for \( \gamma \) is nm\(^{-2} \)).

| Model          | Distance | \( \gamma_0 \)     | \( \gamma_1 \) | \( \gamma_1 \) |
|----------------|----------|---------------------|----------------|----------------|
| Basic tessellations |          |                     |                |                |
| PLT            | 0.2496   | 6.981 × 10\(^{-2} \)| –              | –              |
| PVT            | 0.2534   | 1.086 × 10\(^{-3} \)| –              | –              |
| PDT            | 0.7369   | 7.905 × 10\(^{-4} \)| –              | –              |
| One-fold superpositions |        |                     |                |                |
| PLT/PLT        | 0.2496   | 3.491 × 10\(^{-2} \)| 3.491 × 10\(^{-2} \)| –              |
| PLT/PVT        | 0.0562   | 2.167 × 10\(^{-2} \)| 5.488 × 10\(^{-4} \)| –              |
| PLT/PDT        | 0.2467   | 6.850 × 10\(^{-2} \)| 1.168 × 10\(^{-7} \)| –              |
| PVT/PVT        | 0.0584   | 2.920 × 10\(^{-4} \)| 2.920 × 10\(^{-4} \)| –              |
| PVT/PDT        | 0.0322   | 5.802 × 10\(^{-4} \)| 4.116 × 10\(^{-5} \)| –              |
| PDT/PDT        | 0.3923   | 1.254 × 10\(^{-4} \)| 1.254 × 10\(^{-4} \)| –              |
| Two-fold superpositions |        |                     |                |                |
| PLT/PVT/PVT    | 0.0562   | 1.680 × 10\(^{-2} \)| 4.141 × 10\(^{-6} \)| 5.447 × 10\(^{-4} \)|
| PLT/PVT/PDT    | 0.0322   | 7.105 × 10\(^{-14} \)| 5.802 × 10\(^{-4} \)| 4.116 × 10\(^{-5} \)|
| PLT/PDT/PDT    | 0.2467   | 6.850 × 10\(^{-2} \)| 1.168 × 10\(^{-7} \)| 5.317 × 10\(^{-15} \)|
| PVT/PVT/PVT    | 0.0584   | 5.448 × 10\(^{-5} \)| 2.789 × 10\(^{-5} \)| 4.664 × 10\(^{-4} \)|
| PVT/PVT/PDT    | 0.0322   | 5.802 × 10\(^{-4} \)| 4.253 × 10\(^{-27} \)| 4.116 × 10\(^{-5} \)|
| PVT/PDT/PDT    | 0.0322   | 5.802 × 10\(^{-4} \)| 5.351 × 10\(^{-30} \)| 4.116 × 10\(^{-5} \)|
| PDT/PDT/PDT    | 0.3277   | 5.633 × 10\(^{-5} \)| 5.633 × 10\(^{-5} \)| 5.633 × 10\(^{-5} \)|

3.1. Concentration of filamentous actin in the lamellipodium

The total amount of filamentous actin was calculated by measuring the total length of skeleton lines in the images. Note that the accuracy of this method is hardly affected by an overlay of actin filaments from different layers since the skeleton lines have a negligible width and they are mainly orientated orthogonally to the vertical axis. Therefore, assuming that an actin filament with a length of 1 \( \mu \)m consists of 370 actin molecules with a molecular weight of 43 k Da (Tang and Janmey 1996) and that the lamellipodium has a thickness of 200 nm (Atilgan et al 2005), the concentration of filamentous actin was calculated as 9.6 mg ml\(^{-1} \) for a measured total length of the skeleton lines of \( 7.24 \times 10^{-2} \) nm nm\(^{-2} \).

3.2. Actin network morphology

To obtain objective features of global network architecture, multi-layer tessellation models were fitted to the graphs representing actin filament networks. Table 3 contains the results of the model fitting procedure. The relative Euclidean distance measure was applied here, but the results do not vary much if other distance measures are applied. The data of the fitting procedure show that the optimal superposed model is given by a PVT/PDT superposition (table 3). The fitting of three-layer models resulted in virtually the same model choices as the fitting of two-layer models, i.e. there was one layer with a corresponding \( \gamma \) value of approximately 0.

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Figure 5. Comparison between segmented sample image and realization of the optimal PVT/PDT superposed model fitted to actin networks.

Note that the detected distance value is only a technical characteristic in order to determine the optimal tessellation model with respect to a minimal distance of the estimated vector of mean characteristics from images to the vector of theoretical mean characteristics. In particular, it is not possible to transfer this distance to a linear scale, i.e. the distance has only a qualitative not a quantitative meaning. From the result that models consisting of three layers show no improvement in the optimal distance one can deduce that an additional layer does not provide a better fit (of course it has to be at least as good as a fit with a two layer model consisting of two of the three basic tessellation models involved).

In order to check the significance of our model decision, we also regarded a model fitting to actin networks of the individual sample images. In this case as well, the PVT/PDT model was found to be the optimal model for all seven individual images (as explained below) having similar optimal parameters compared to the result for the mean sample (data not shown). The images are independent and they represent similar (i.e. identically distributed) random structures. Due to this fact, we obtain, under the null hypothesis of the data representing a model different from a PVT/PDT model, that the number of images for which the PVT/PDT model is falsely considered optimal is a binomial distributed random variable with parameters \( n = 7 \) and \( p_{\text{bin}} \). Here, \( p_{\text{bin}} \) represents the probability that the decision is in favour of a PVT/PDT model. Under the (natural) assumption that \( p_{\text{bin}} \) is less than 0.5, the significant decision in favour of the PVT/PDT model for all individual sample images strongly enhances our decision on the vector of averaged characteristics. In other words, the hypothesis that our data represents a specific model different from the PVT/PDT model is clearly rejected (\( p < 0.01 \)).

Figure 5 depicts a segmented sample image of actin networks and a realization of the corresponding optimal PVT/PDT superposed model. Notice that, in general, the similarity between the real images of actin networks and the realizations of the optimal model is only given in a structural sense, i.e. with respect to geometric network characteristics, and not imagewise.

With respect to the total segment lengths, the parameters of the optimal model (PVT/PDT) yield a proportion of 69% for the PVT layer and 31% for the PDT layer. Assuming an equal division of the cytoplasmic volume between the two actin network layers, the actin concentrations for the PDT and PVT layers were determined as 13.25 and 5.95 mg ml\(^{-1}\), respectively.
3.3. Estimation of actin network elasticity

The elastic plateau shear modulus $G$ of an isotropic crosslinked actin filament network can be calculated as (MacKintosh et al 1995):

$$G = \frac{6 k_b T l_p^2}{(l_e^2 \xi^2)},$$  \hspace{1cm} (3.1)

with $l_p$ being the persistence length of actin filaments (17 $\mu$m), $l_e$, the entanglement length and $\xi$ the mean mesh size. Note that (3.1) refers to isotropic 3D networks. Although the actin networks in lamellipodia represents flat structures, this formula can provide a good estimate for $G$. The entanglement length $l_e$ is defined as the average distance between points along an actin filament that are effectively constrained; for a fully crosslinked network, this is determined by the distance between crosslinks. The entanglement length is 2.2 $\mu$m for a 1 mg ml$^{-1}$ actin network and scales as concentration $\rho^{-0.4}$ (Käs et al 1996). The mean mesh size $\xi$ represents the average distance between neighbouring actin filaments. Due to the superposition of filament layers in 2D images of actin networks, $\xi$ cannot be determined directly from these images. Instead, the value of $\xi$ was estimated by the average maximum diameter of circles inscribed into the meshes of simulated networks for each of the two layers (PVT and PDT). The mean mesh size $\xi$ was found to be 34 nm for the PVT layer and 78 nm for the PDT layer. The mean mesh size of an actin network depends on its actin concentration $\rho$ and this relation can be described by (Morse 1998):

$$\rho = \frac{1}{\xi^2}.$$  \hspace{1cm} (3.2)

Actin concentrations of 13.25 and 5.95 mg ml$^{-1}$ as determined for the two network layers would result (using 3.2) in a mean mesh size of 44 nm for the PVT layer and 66 nm for the PDT layer. The relative differences between these mesh size values and those obtained above from the simulations are 23 and 18%.

Based on 3.1 and mesh sizes of 34 and 78 nm, the elastic shear modulus $G$ can be calculated as 23.4 kPa for the PVT layer and 0.6 kPa for the PDT layer. This calculation depends on $l_e$. Due to slack and projection errors, however, it is difficult to determine $l_e$ directly from the images. In view of the mesh sizes of the fitted tessellation models, the values of $l_e$ determined according to Käs et al (1996) appear to be over-estimates. Thus, the values of $G$ as determined above are probably under-estimates.

The elasticity $G$ as determined above represents the maximum elasticity of a fully crosslinked actin network. However, since actin crosslinks in vivo are only transient, the effect of transient crosslinking on elasticity ($G$ value) needs to be considered. To account for dynamic crosslinking, we have applied a model for transiently or partially crosslinked actin networks (Ananthakrishnan et al 2006b) that determines the elastic shear modulus depending on the concentration of actin and actin crosslinkers, such as $\alpha$-actinin. This simplified system represents the minimal set of components necessary to establish an elastic network. Adding other proteins such as filamin or Arp2/3 to the model would further approximate the real intracellular scenario but would significantly increase the complexity of the mathematical model. Figure 6 depicts the relationship between the concentration of a representative crosslinker, i.e. $\alpha$-actinin, and the elastic shear modulus $G$ for the two actin network layers. Note that at low $\alpha$-actinin concentrations (<10 $\mu$M) as detected, for e.g. in Acanthamoeba (Pollard et al 2000), the two graphs demonstrate a different shape resulting in a different tuneability of the network elasticity. Independent of the strong tuneability of the mechanical strength

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Figure 6. Estimations of elasticity for actin networks with a mean mesh size of 34 nm (left panel) and 78 nm (right panel) and $\alpha$-actinin concentrations between 5 and 25 $\mu$M.

of lamellipodial actin networks by the amount of crosslinker present, it becomes clear that the significantly different mesh sizes in the two detected layers result in a very soft and a distinctively stiffer actin layer.

4. Discussion

Directed migration of cells is regulated by remodelling actin networks in lamellipodia (Abraham et al 1999). Thus, the analysis of actin network morphology provides essential information for assessing the functional characteristics of the leading edge. Due to the small diameter of actin filaments, electron microscopic techniques have to be applied for the visualization of individual filaments (Svitkina and Borisy 1998; Shao et al 2006) which is necessary to obtain the structural information required for the analysis of filament network architecture. With the exception of cryoelectron tomography (Medalia et al 2002), electron microscopy produces 2D images which are projections of 3D cellular structures. Analysing projections of originally 3D filament networks by conventional image analysis techniques would result in a biased estimation of several important network features, e.g. mesh size. The new method presented here is based on the observation that lamellipodia are composed of flat actin networks (Shao et al 2006). Thus, the models for filament networks applied in this study were confined to superpositions of 2D geometrical models. The three mathematical models (PLT, PVT, PDT) chosen in this study represent a wide variety of different scenarios with respect to the relationship between the density of filaments, branching points and meshes (table 1). Although these stochastic models cannot be fitted to strictly deterministic topologies, e.g. Arp2/3-mediated branching patterns, they represent actin network features which are pivotal for network elasticity. Moreover, the three models are still mathematically tractable with respect to explicit mean value formulae for geometric characteristics and can easily be simulated. The different ratios between the model characteristics $\lambda_n$ can be related to biochemical processes regulating actin network synthesis and remodelling which can prefer filament elongation or branching. Based on the same amount of polymerized actin, a network represented by the PVT model contains almost 50% more meshes and more than 500% of the branching points in comparison to a network modelled by PDT. In contrast to a PLT network, where one could imagine that the long filaments could be related
mostly to formin-mediated processes (Harris et al 2006), the PVT and PDT models describe networks of branched filaments. Realizations of PVT with $\gamma$ values shown in table 3, which are dominated by short interconnected filaments, appear to represent an actin network architecture that is preferentially regulated by Arp2/3. The PDT model suggests a different balance between elongation and branching processes permitting the occurrence of local anisotropies within the network (figure 1), which might occur, for example, in the early phases of filopodia formation (Svitkina et al 2003). Of course this is not an indication of a strict separation of the network formation processes caused by Arp2/3 and formin with respect to the different layers of actin networks.

The small number of actin filament layers in lamellipodia allowed the detection of almost all individual filaments, although the geometry of filament branching patterns or crosslinks might not have been detected with perfect accuracy. This was compensated for by fitting statistical tessellation models which reflect major structural characteristics of the filament network while taking stochastic variations into account. The comparison of the tessellation models with images of lamellipodia from B16F1 mouse melanoma cells resulted in the detection of the PVT/PDT superposition model as the optimal fit for these actin networks. The consistent selection of this model for all randomly selected sample images emphasizes the existence of a uniform network morphology for this cytoskeletal compartment. A further increase of the number of superposed layers in the model did not yield different model characteristics which strongly suggests that lamellipodia can be modelled best by two different flat actin networks indicating axial asymmetry, i.e. anisotropy. This fits well with previous observations by Ponti et al (2004), who have shown that the lamellipodium is composed of two distinct actin networks. Recent work by Giannone et al (2007) also supports this finding. Furthermore, Atilgan et al (2005) demonstrated that the observed thickness of lamellipodia can only be achieved by filament growth proceeding in 2D. Theoretical studies on actin networks also suggest that actin filaments in lamellipodia are, in fact, organized in 2D layers (Maly and Borisy 2001; Kruse et al 2006). However, other network configurations, e.g. with a more or less continuous transition between different network compartments might exist. Nevertheless, a discrete model consisting of two layers could be a useful tool to assess the axial anisotropy in lamellipodia.

In summary, we introduced a new method of fitting multi-layer geometrical random tessellation models to actin network structures. The fitting is based on the comparison of several geometrical mean characteristics such as the mean number of nodes or the mean total length per unit area. The optimal model is chosen from a pool of given random tessellation models which reflect a large variety of possible network architectures. Note that the results of our fitting procedure do not seem to depend on the choice of the distance measure since results for other reasonable distance measures were very similar. Our results seem to be robust considering the following facts (i) we obtain the same optimal model for all individual images and (ii) we do not observe smaller optimal distances for models with more than two basic tessellation layers, indicating that two layers are already optimal to describe the architecture of lamellipodia.

The total concentration of filamentous actin in the lamellipodium was found to be 9.6 mg ml$^{-1}$ which is within the range of previously measured data (Zigmond 1993; Pollard et al 2000). This measurement was based on determining the length of actin filaments in 2D images. Since actin networks in lamellipodia appear to be spatially restricted in 2D (Shao et al 2006), the calculation of the total filament length by measuring the projection of filaments provides a good estimate of the actin concentration. The two-layer model provided
an opportunity to analyse the structural properties of different actin network compartments. This analysis revealed that lamellipodia are anisotropic in the axial direction with respect to morphology and actin concentration. This anisotropy might be caused by the attachment of cells to the substratum which imposes specific demands on the lower compartment of the actin network. Alternatively, the axial anisotropy may have been generated through the activity of two different actin filament nucleators, Arp2/3 complex and formins, which nucleate branched and linear filaments, respectively. The tessellation model selected as best fit for the architecture of lamellipodia consists of a PVT and a PDT layer. The parameters of the PVT model indicate a more than 10-fold increase of the branching point density in comparison to the PDT model. This results in longer filaments at the PDT level, thus, providing a good match with the original image. However, there cannot be a perfect match between individual images and simulated networks, since the fitted models reflect the inherent structure of these networks through their global characteristics. Hence, random realisations of these statistical models may cause variations of the position and properties of local filament arrangements. For the purpose of obtaining approximations for individual samples based on these model classes, Bayesian image analysis techniques based on Markov-chain-Monte-Carlo methods might prove to be useful (Blackwell and Møller 2002; Nicholls 1998).

The low frequency elastic shear modulus of lamellipodia depends on the mesh size of the actin network (MacKintosh et al 1995). The mesh size of multi-layer networks cannot be determined directly from 2D images. The two-layer model provided the opportunity to estimate mesh sizes of the layers independently. The mesh sizes obtained from model simulations differed only by 18 and 23% from those calculated from actin concentrations (Morse 1998). This variation may be due to an unequal division of the cytoplasmic space between the two layers of the actin network in our model affecting concentration values. The small differences emphasize the fact that the properties of the fitted model indeed reflect the structural characteristics of multi-layer actin networks. Note that the determined mesh sizes of 34 and 78 nm for the two different layers fit quite well with mesh sizes determined by Plastino et al (2004) of 53 nm for a Act-A produced actin gel/network and 16 nm for a PRO produced actin gel. Furthermore, Luby-Phelps et al (1987) reported actin meshwidths of approximately 26 nm that compare reasonably well with the results of our study.

Based on the determined mesh sizes, the PVT and PDT layers of the actin network were found to have an elastic shear modulus of 23.4 and 0.6 kPa, respectively. These estimates describe the elasticity of a fully crosslinked actin network and are within the range of previously measured data for cell elasticity (Ananthakrishnan et al 2006a). As examples, Brunner et al (2006) recently determined that the lamellipodial elasticity (shear modulus G) of keratocytes is 0.32 kPa, while another work measured an elastic shear modulus of approximately 0.4 kPa in the thin regions near the leading edge of fibroblasts (Mahaffy et al 2004). These results are comparable to the lower limits of shear modulus we obtain, especially when we account for transient crosslinking. However, the accuracy of the calculations for the elastic shear modulus performed in our work here is restricted by various factors the modelling could not account for, e.g. slack in the actin network and interconnections between adjacent actin filament layers. Nevertheless, the differences of the two layers indicates an anisotropy of the elastic properties of the lamellipodium with the stiffer compartment determining the leading edge’s ability to push the cell forward and, thus, supporting the leading edge to penetrate soft tissue compartments. The response to small forces perpendicular to the lamellipodium, however, might be determined by the softer compartment modelled by the PDT layer. This provides the
cell with enough softness to move through small tissue slits. Thus, the structural differences between different compartments of lamellipodia suggest that cells can shift production of actin networks in order to tune elasticity for agility or resistance.

Another mechanism to regulate the elasticity of the leading edge is the occurrence of transient actin crosslinks, e.g. by $\alpha$-actinin. This provides lamellipodia with a wide range of elasticity to respond to the biophysical demands imposed on them by the mechanical properties of the surrounding tissue compartments. Importantly, the shape of the tuning curve varies between the two layers of the actin network. Thus, the establishment of an asymmetrical actin network architecture in lamellipodia as revealed in this study may enable the lamellipodium to further fine tune its local structural properties without changing actin crosslinker concentrations.

The lamellipodium represents an interface where a variety of signals is translated into movements of cytoplasmic compartments. This goal is achieved by a dynamic remodelling of the actin filament network which is mainly controlled by actin-associated regulatory proteins. The major mechanisms involved are actin polymerization and filament branching (Bear et al 2002; Mejillano et al 2004; Pollard and Borisy 2003). Although these processes appear to be well understood (Mogilner and Oster 1996; Maly and Borisy 2001), the interactions of signaling pathways resulting in the formation of specific network patterns remain to be determined. The method presented in this study could be a useful tool to analyse these complex processes.

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