The Public and Private Marginal Product of Capital

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Why does capital not flow to developing countries as predicted by the neoclassical model? What are the direction and degree of capital misallocation across nations? We revisit these questions by removing public capital from total capital to achieve a more accurate estimate of the marginal productivity of private capital. We calculate marginal product of capital schedules in a large sample of advanced and developing countries. Our main result is that, in terms of the Lucas Paradox, private capital is allocated remarkably efficiently across nations. Tentative estimates of the marginal productivity of public capital suggest that the deadweight loss from public capital misallocation across countries can be much larger than that from private capital.

INTRODUCTION

If in poor countries capital–labour ratios are low and returns are high, as the standard one-sector growth model predicts, why does more capital not flow from rich to poor countries? This fundamental question, known as the Lucas Paradox, a name coined after Lucas’ seminal paper (Lucas 1990), is a focal point for many key areas of economic development, such as the efficacy of aid, the extent of international capital market frictions, or the importance of institutions and complementary factors. The paradox presupposes a downward-sloping financial return to investment in the cross-section of nations. Lucas himself posited that the explanation could be that of failing to account for complementary factors to physical capital, such as human capital, resulting in an overstating of the marginal product of capital (MPK). Lucas placed little credence on the argument of capital market frictions.1

The aggregate MPK is the most common measure employed to approximate the return to investment. Unfortunately, estimating the MPK is no easy task (e.g. see Banerjee and Duflo (2005) for a review). In a persuasive, yet provocative, contribution to the literature, Caselli and Feyrer (2007) (CF hereafter) propose a measurement approach based on development accounting. More specifically, they present the case for direct MPK estimation using easily accessible macroeconomic data, namely, the income share of capital, GDP, and the value of the capital stock.2 Their approach assumes competitive markets and imposes no restrictions on production functions other than that of constant returns to scale.

CF’s main contribution is that they derive an MPK measure that is more suitable for the purpose of international credit flows. They modify the standard MPK derived from the one-sector growth model to remove natural resource rents from the income share of capital and correct for higher relative costs of capital in poor countries. By making these two reasonable adjustments, so the Lucas Paradox is resolved, CF find that the cross-country MPK is roughly flat, and the overall efficiency loss due to capital misallocation is only 0.1% of global GDP.3 Yet CF’s measure is based on a capital stock that includes both public and private components, whereas the relevant MPK for investors is only the return to private capital. Consequently, in this paper, we attempt to go beyond CF by stripping out public capital from total capital to achieve a more accurate estimate of the

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marginal productivity of private capital. An additional goal from our analysis is to shed light on the allocation of public capital across countries—although, as will become clear later, our estimates of the marginal productivity of public capital are more tentative and based on *ad hoc* and harder-to-verify assumptions.

More specifically, we look at the difference between the total and the private MPKs, and at the public MPK. We study the shape of their estimates with respect to income per capita in the cross-section of countries and the percentage loss in global GDP due to the possible misallocation of both types of capital. The private MPK is calculated as a straightforward extension of the methodology proposed by CF. The calculation of the public MPK is, on the other hand, more problematic because national accounts do not provide any net income from public capital since the government sector performs a non-market activity. Therefore, in order to measure the share of public capital in output, we follow two approaches. First, as in Cooley and Prescott (1995), we assume that the net rate of return between private and public investment is the same; we see this case as the most conservative that provides a possible minimum value of the deadweight loss due to public capital misallocation. Second, we employ regression estimates of the output elasticity of public capital.

Our work follows Pritchett (2000) and Caselli (2005), among others, who argue for the future separation of public and private investment in the context of development accounting. These authors thought that data issues would make it near impossible to convincingly estimate the private and public MPK. One main contribution of this paper is to break the impasse and carry out this exercise for the first time. For that purpose, we employ improved data on the sectoral share of investment from the IMF’s World Economic Outlook (WEO).

The distinction between the two types of capital is important for at least two reasons. The first stems from the observed variation of public investment across nations. As Table 1 and Figure 1 show, the public sector plays a disproportionately large role in investment in developing countries compared to advanced economies—the relationship looks more flat when public investment as a percentage of GDP is considered because investment as a proportion of GDP rises in income. Therefore eliminating the public capital component of the overall MPK to obtain the private MPK can have important implications for the slope of the MPK.

The second reason is that the theory behind MPK determination is likely to differ significantly between the two sectors. There is much literature elsewhere with results that hinge on the contrasting behavioural idiosyncrasies of public and private agents (e.g. Becker 1957; Fama 1980; Pritchett 2000; Besley and Burgess 2002; Robinson and Torvik 2005). Governments probably follow objective functions that not only take into account efficiency considerations but also rent capture and redistributive and other policies. If the

| Variable | Low-income countries | Middle-income countries | Advanced countries |
|----------|----------------------|-------------------------|--------------------|
| Mean public share in total investment (2010) | 42.6% | 29.4% | 16.6% |
| Mean public investment as % of GDP (2007) | 6.0% | 6.9% | 3.9% |

*Source:* World Economic Outlook, Penn World Tables.
The public sector tends to invest in production infrastructure such as roads, ports, airports, railways and energy-transportation grids—goods where markets fail. As a consequence, public and private MPK should be considered imperfect substitutes in a country’s production function (CF implicitly considered them to be perfect substitutes). The fact that no net income from public capital is reflected in national accounts and the non-rival nature of public goods lead us to incorporate private inputs into the production function as being characterized by constant returns, and public capital as a complementary factor that induces increasing returns.

The analysis is carried out in a broad sample composed of 26 advanced economies and 42 developing countries that is taken from Monge-Naranjo et al. (2016) (MSS hereafter). The reason for adopting the MSS sample is that they construct estimates of the income share of reproducible capital by directly employing natural-resource rents hereafter). The reason for adopting the MSS sample is that they construct estimates of the income share of natural resources that represent an improvement of the measure of the income share of natural resources used by CF. Using these new share estimates, MSS revise the CF misallocation results but focusing only on overall capital.

Most of our results are driven by a strongly positively sloped and highly dispersed ratio of output to public capital. In particular, we find that the overall MPK is flat and the private one slightly downward sloping—although following MSS, when we split the sample using an openness indicator, economies classified as closed depict upward frictions in the flow of private capital. The analysis also shows that, whereas the cross-country dispersion of the overall MPK did not change much from 1990 to 2005, that of the private MPK has decreased rapidly (mainly in developing nations). This has contributed to ameliorating the global output loss due to private capital misallocation, which in our sample for the year 2005 is only about 1.9%, a number that is larger than that found by CF for 1996 but still relatively small. Hence in terms of the Lucas Paradox, our results suggest that private capital is allocated remarkably efficiently.
The analysis yields another interesting result—albeit more tentative due to the strong assumptions employed to compute the public MPK. In all nations, but especially in developing countries, the marginal productivity of public capital varies much more than its private counterpart, which implies a potentially much greater misallocation in public than in private capital. This implication is supported by our calculations. In the counterfactual exercises conducted, the cost of capital misallocation is about 4.5 times larger for the public component than for the private one. All in all, our findings point to public sector frictions as a key constraint to enhancing the MPK and accelerating international capital inflows.

We proceed as follows. Section I takes a close look at the primary sources of the data used to disaggregate total capital into its public and private components, and discusses the steps followed to calculate the public and private MPK. Section II presents the new trends on the private MPK unravelled from the data disaggregation, and compares them to CF’s overall MPK measure. Section III introduces the results for the public MPK and discusses its implications. Section IV calculates the worldwide deadweight loss that can be attributed to private and public capital misallocation. Robustness checks that split the countries into open and closed are performed in Section V. Section VI concludes.

I. DATA

In this section we show in detail the steps followed to construct the private and public MPK. Following CF, suppose that there are \( J \) final goods in the economy, including capital and consumption products. In any one of these sectors, let us say sector \( i \), production occurs using a set of complementary inputs that include private capital \( (K_{pi}) \), public capital \( (K_{gi}) \), and other factors \( (X_i) \) according to

\[
Y_i = K_{gi}^c F(K_{pi}, X_i).
\]

The role of public capital in production could be indirect, if the public sector provides a flow of services to the private economy, instead of a stock of capital. In this case, the government’s output would represent an intermediate input, and we could think of the above production function as a reduced form.

Suppose as well that \( F \) displays constant returns to scale over \( K_{pi} \) and \( X_i \), \( \gamma > 0 \), and there is perfect competition in all markets. These assumptions are made for the sake of simplicity, and their only purpose is to guarantee that private capital exclusively obtains the value of its marginal product in return for its service to the production activity. Nevertheless, we notice that perfect competition might not constitute a good representation of market structure, especially in low-income countries, and that the constant returns assumption implies, as a byproduct, that public capital enters the production function as a total factor productivity enhancing variable.

If the available amount of capital is allocated efficiently among sectors, then it must hold that \( P_{j} MPK_{Pi} = P_{j} MPK_{Pi} \) and \( P_{j} MPKG_{j} = P_{j} MPKG_{j} \), for all \( i \) and \( j \), where \( P_{i} \) is the price of good \( i \), and \( MPK_{Pi} \) and \( MPKG_{j} \) represent the marginal products of private and public capital in sector \( i \), respectively. \(^6\) Under this premise, the return to one unit of income invested in public and private capital employed in a given sector is the same across all final-good industries. Let us denote by \( MPK \) and \( MPKG \) these common private and public capital returns, respectively. Focusing on sector \( i \) and abstracting from capital gains, we can write

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\(^6\) This assumption simplifies the analysis and allows for comparability across sectors and countries.
where $P_{kp}$ and $P_{kg}$ give the prices of private and public capital, respectively.

One additional step is needed to obtain the private MPK. Notice that income that should be attributed to $K_p$ equals $\sum_{j=1}^{J} P_j MPKP_j K_p$. Hence we can write capital income derived from its private component as $P_j MPKP_j K_p$, where $K_p = \sum_{j=1}^{J} K_p$ is the stock of private capital. In addition, if $\beta$ denotes the share of private capital in aggregate output, then we obtain that

$$\beta = \frac{P_j MPKP_j K_p}{P_y Y},$$

where $P_y Y$ is the value of GDP. Combining the $MPKP$ equality in (2) with expression (3), it is straightforward to get an expression for this common return as

$$MPKP = \beta \frac{P_y Y}{P_{kp} K_p}.$$ 

The derivation of the public MPK, in turn, uses the Cobb–Douglas form assumed for public capital in the production function. In particular, using expression (2), we can write that $MPKG * P_{kg} \sum_{j=1}^{J} K_{gj} = \gamma \sum_{j=1}^{J} P_j Y_j$, which implies that

$$MPKG = \gamma \frac{P_y Y}{P_{kg} K_g},$$

where $K_g$ is the stock of public capital in the economy. Notice that even though the Cobb–Douglas form given to the public capital input is instrumental in obtaining (5), it is completely irrelevant for expression (4).

A key issue is how to calculate $\beta$ and $\gamma$. The parameter $\beta$ represents the share of private capital in GDP and can be easily obtained from national accounts data. The output elasticity of public capital $\gamma$, on the other hand, cannot be derived in that way, as will be explained shortly, and we resort to counterfactual and regression estimates. Next, we focus on the calculation of $\beta$ and the description of the data. The discussion on possible estimates of $\gamma$ is postponed to Section III.

To get estimates of $\beta$ we can directly employ the share of reproducible capital in GDP from the national income accounts. This approach implicitly assumes that all income attributed to capital in national accounts comes from private capital. Although not completely correct, this is a fairly accurate approximation. To see this, notice that national accounts collect data on private sector output (sales plus the change in inventory levels) and costs (intermediate inputs and labour costs) to calculate capital income in the private sector. The return on capital or operating surplus is then calculated as the difference between output and the costs. In contrast, government accounts are based on solely the costs incurred because the public sector performs a non-market activity, hence does not have sales. As a result, the only capital income that is attributed to public capital in the national accounts is the consumption of fixed...
capital (i.e. depreciation). Put differently, given that firms do not pay for the services provided by public capital, all capital income generated by reproducible capital except depreciation of public capital is, by construction, paid to the owners of private capital.\footnote{Our country sample employed to estimate the MPKs is taken from MSS. It is comprised of 68 countries with private and overall MPK data in 2005—the latest year for which the MSS shares of capital are available. We also look at time series data from 1990 to 2005, with the number of nations beginning at 66, increasing to 67 from 1993, and to 68 from 2000. We measure the cross-country MPK using current-price local currency data from World Development Indicators (WDI), rather than real data from Penn World Tables (PWT) adjusted for relative price differences as in CF. The use of current-price local currency data is preferred here since it sidesteps any reliance on purchasing power parity (PPP) adjustments and extrapolated International Comparison Program data shown to be quite unstable for non-OECD countries (see Johnson et al. 2013). In addition, it has been argued elsewhere (e.g. Knowles 2001) that investment shares are more accurately measured using local price data, rather than data from PWT. In any case, for our analysis the two approaches yield essentially the same results. The data that we require are income shares of private capital and government operating surplus \((a_k, a_g)\), GDP in current-price local currency \((P_yY)\), private capital \((P_{ kp}K_{ p})\), and public capital \((P_{ kg}K_{ g})\).

Current-price local currency data on GDP and investment are taken from WDI. In principle, capital series could be obtained by using the perpetual inventory method on current-price historic investment data, deflating the past capital stock each year by a sector-specific investment deflator (see expression (6)). This deflator should differ between the two sectors because government investment is based largely on structures and equipment, while items like software, whose price shows a steeper trend than other investment items, have more weight on the private sector. Unfortunately, in practice only a common investment deflator exists for both private and public figures, which is the one that we apply to the investment series.\footnote{With current- and constant-price investment numbers, the next step is to split these investment flows into their private and public sector constituents. This split is crucial as it drives the resulting differences in the private, public and total MPK. To do this disaggregation, we use private and public investment share data from the World Economic Outlook (WEO). For the sample of 50 countries in 2006, the mean number of time series observations of the private investment share is 33 (ranging from a minimum of 12 to a maximum of 49). Before total investment is disaggregated, the first available observation of the investment share is extrapolated back to the first year of investment data. In the absence of any investment data at all prior to 1960, it is necessary to set initial conditions for both the public and private capital stocks. As is common practice (given the notion of a steady-state capital stock), we set the initial condition, \(K_{0,t} = I_{0,t}/(g+\delta_{0})\), for countries with available data.}

\[100 \times \frac{{\text{current-price local currency gross fixed capital formation}}}{{\text{constant-price local currency gross fixed capital formation}}}\]

Missing constant investment data are set equal to the product of constant-price GDP and gross fixed capital formation as a proportion of GDP, for countries with available data.
where private and public sectors are indexed by \( j = p, g \). The term \( I_{j0} \) is current-price investment in the first year available, \( g_j \) is the country- and sector-specific average growth rate of constant price investment over the first 20 years of available data, \( \delta_j \) is the relevant depreciation rate for the first year of available investment data. Caselli (2005) shows that sufficiently recent capital measures tend to be insensitive to the exact assumptions made on these initial conditions.

In principle, time- and country-specific depreciation rates would be preferred. However, as far as we know, those rates for a sufficiently large number of countries in our sample are not available. Instead, we follow Kamps (2006) and Gupta et al. (2014) to choose the depreciation rates employed in the construction of the capital series. Using US Bureau of Economic Analysis data, Kamps estimates a time-varying depreciation rate that increases gradually from 2.5% in 1960 to 4% in 2001 for government assets, and from 4.25% to 8.5% for private non-residential assets. In turn, Gupta et al. take into account the different composition of capital in different set of nations and adapt those estimates for the period 1960–2008 as follows: for public capital, the depreciation rate equals 2.5% in low-income countries, 2.5% rising to 3.4% in middle-income countries, and 2.5% rising to 4.3% in advanced countries; for private capital, 4.25% in low-income countries, 4.25% rising to 7.6% in middle-income countries, and 4.25% rising to 9.6% in advanced countries. These last income- and sector-specific depreciation rates are the ones that we use in our calculations. For subsequent years, we extrapolated the 2008 figures. Alternatives to these baseline assumptions, including a constant rate of 0.5 for the whole period and different time-varying profiles suggested by other papers, change the capital stocks only marginally.

Armed with disaggregated investment and deflator data, assumptions on initial conditions and a pattern of depreciation rates, we apply the perpetual inventory method to construct current-price capital series for each country as follows:

\[
P_{kt}K_{jt} = \left(1 - \delta_j\right)\left(\frac{P_{kt}}{P_{kt-1}}\right)P_{kt-1}K_{jt-1} + I_{jt-1},
\]

so

\[
P_{kt}K_{jt} = \left(1 - \delta_j\right)\left(\frac{P_{kt}}{P_{kt0}}\right)I_{j0} + \sum_{i=1}^{t} \left(1 - \delta_j\right)^{t-i} \left(\frac{P_{kt}}{P_{kt0}}\right)I_{jt-1}.
\]

The total capital stock \( (P_K) \) is then simply set equal to the sum of the private and public stocks. The capital measures become less sensitive to the initial conditions and investment share extrapolation as \( t \) becomes closer to the present. This suggests that time series results have to be interpreted with greater care than the cross-section results in 2005.

Having constructed public and private capital stocks, the remaining specification choice is that for income share \( \beta \). We choose to adopt the cross-country estimates constructed by MSS. These share data initially derive from Bernanke and Gurkaynak (2001), adjusted, as proposed by CF, to account for natural capital using wealth data from the World Bank (2006). MSS then improve the measurement of the income share of natural resources used by CF by directly employing natural resource rents. Once income from natural capital is removed from capital income, the result is data on the share of reproducible private capital \( \beta \).
II. PRIVATE MPK CALCULATION

With the necessary data at hand, we turn to calculating each country’s private and overall MPK. The private MPK is given by expression (4), whereas the overall MPK—CF’s preferred measure—equals

$$MPK = \beta \frac{P_y Y}{P_k K}.$$ 

As explained in Section I, our approach is to measure the marginal productivity using current-price data on income and capital along with income share data, whereas CF employed PPP numbers. In the Appendix, we present Figure A1, which plots $MPK$ obtained using current-price data and the CF capital shares against real GDP per capita for the exact same cross-section of countries as CF for the years 1996 and 2005, the former being the year on which CF focus. The figure shows that the current-price approach yields essentially the same slightly upward-sloping overall MPK as CF. Moreover, little has changed over the 9-year period 1996–2005. The advanced economies remain bunched closely around a financial rate of return of 10%, while the developing nations have a similar mean but greater variation, with rates of return from below 1% to 16%.

Coming back to our main calculations, which employ the MSS benchmark sample and reproducible-capital shares, Table 2 presents baseline summary statistics for 2005, unless stated otherwise. The sample originally had 79 nations. However, due to either lack of private and public investment data or not fulfilling our capital-stock quality restriction (see above), our MSS sample composes only 68 countries.11

The charts contained in Figure 2 show the price-corrected private MPK ($MPKP$), and the gap between the private and the overall MPKs ($MPKP - MPK$) for 1996 (left-hand column) and 2005 (right-hand column). Table 3 shows respective summary statistics for 2005. Four observations on Figure 2 are particularly notable. First and most important, with the MSS shares, the overall MPK is no longer upward-sloping—it is flat. In particular, the fitted lines for the $MPK$ measure (top row) are insignificant. Moreover, when we eliminate the public capital component, we obtain a downward-sloping private MPK with a statistically significant fitted line. Second, there are no qualitative differences between the two time periods; nevertheless, the private MPK looks more dispersed in 1996 than in 2005.

Third, there is an interesting pattern among the advanced economies: the private MPKs are relatively similar. According to this, the graphs potentially imply that private capital is allocated efficiently in advanced economies, but inefficiently in poor countries. Last but not least, public investment is less efficiently allocated in developing nations than in advanced economies. We can deduce this from the last row of Figure 3, which shows that the dispersion of the gap between $MPKP$ and $MPK$ is substantially larger in the lower half of the income distribution. This gap is due to public capital investment. Therefore the larger dispersion suggests that the public capital stock can be subject to a larger degree of misallocation in middle- and lower-income countries—notice that under MPK equalization, two economies with the same income level should have the same public capital stock if they share the same production technology and input prices.

Figure 3 contains the temporal evolution from 1990 to 2005 of the mean values of the overall MPK and $MPKP$ (left-hand column), and their standard deviations (right-hand column). These results should be interpreted with care given the greater sensitivity to
### Table 2
Core Sample Summary Statistics

| Country               | ISO  | \( y \) | \( \text{Sh\#} \) | \( MPK \) (1996) | \( MPK \) (2005) | \( MPKP \) (2005) |
|-----------------------|------|---------|-------------------|----------------|----------------|-----------------|
| Argentina*            | ARG  | 21,155.6| 18                | 0.28           | 0.19           | 0.22            |
| Australia*            | AUS  | 74,000.0| 23                | 0.12           | 0.14           | 0.19            |
| Austria*              | AUT  | 72,685.3| 40                | 0.11           | 0.13           | 0.16            |
| Bahrain               | BHR  | 47,787.7| 24                | 0.13           | 0.13           | 0.25            |
| Barbados              | BRB  | 40,181.5| 33                | 0.07           | 0.07           | 0.16            |
| Belgium**             | BEL  | 75,843.0| 16                | 0.13           | 0.15           | 0.18            |
| Bolivia               | BOL  | 7,757.0 | 30                | 0.16           | 0.02           | 0.03            |
| Brazil                | BRA  | 16,440.1| 34                | 0.34           | 0.22           | 0.26            |
| Bulgaria              | BGR  | 21,089.6| 28                | 0.08           | 0.18           | 0.26            |
| Burkina Faso          | BFA  | 1,905.2 | 39                | 0.10           | 0.11           | 0.20            |
| Cameroon              | CMR  | 4,617.7 | 36                | 0.09           | 0.10           | 0.12            |
| Canada**              | CAN  | 67,539.8| 20                | 0.15           | 0.16           | 0.20            |
| Chile                 | CHL  | 27,005.8| 26                | 0.26           | 0.22           | 0.27            |
| China                 | CHN  | 8,117.2 | 18                | 0.09           | 0.12           | 0.29            |
| Colombia              | COL  | 16,566.3| 42                | 0.22           | 0.19           | 0.34            |
| Costa Rica            | CRI  | 22,325.6| 20                | 0.15           | 0.17           | 0.22            |
| Cote d’Ivoire*       | CIV  | 3,399.4 | 41                | 0.12           | 0.10           | 0.22            |
| Cyprus*               | CYP  | 37,323.9| 23                | 0.19           | 0.21           | 0.28            |
| Denmark**             | DNK  | 65,345.9| 31                | 0.14           | 0.14           | 0.16            |
| Dominican Rep.*       | DOM  | 18,243.2| 42                | 0.30           | 0.29           | 0.62            |
| Ecuador               | ECU  | 14,006.1| 32                | 0.17           | 0.14           | 0.27            |
| Finland**             | FIN  | 63,092.2| 15                | 0.12           | 0.12           | 0.19            |
| France*               | FRA  | 67,382.4| 15                | 0.12           | 0.14           | 0.19            |
| Germany**             | DEU  | 63,691.6| 20                | 0.10           | 0.14           | 0.17            |
| Greece*               | GRC  | 58,907.8| 40                | 0.15           | 0.18           | 0.30            |
| Guatemala             | GTM  | 15,210.9| 42                | 0.23           | 0.19           | 0.33            |
| Honduras              | HND  | 8,884.4 | 42                | 0.09           | 0.09           | 0.16            |
| Hong Kong*            | HKG  | 63,557.9| 32                | 0.20           | 0.19           | 0.25            |
| India                 | IND  | 6,609.7 | 42                | 0.11           | 0.16           | 0.28            |
| Indonesia             | IDN  | 7,057.4 | 12                | 0.21           | 0.16           | 0.24            |
| Iran                  | IRN  | 23,472.3| 42                | 0.14           | 0.27           | 0.56            |
| Ireland*              | IRL  | 75,179.2| 49                | 0.23           | 0.22           | 0.30            |
| Israel*               | ISR  | 58,576.1| 27                | 0.22           | 0.18           | 0.23            |
| Italy*                | ITA  | 69,842.6| 23                | 0.15           | 0.16           | 0.20            |
| Japan*                | JPN  | 62,944.7| 31                | 0.12           | 0.13           | 0.20            |
| Jordan                | JOR  | 14,307.6| 48                | 0.10           | 0.15           | 0.28            |
| Kenya                 | KEN  | 2,506.2 | 42                | 0.12           | 0.08           | 0.13            |
| Korea, Rep.*          | KOR  | 46,243.0| 41                | 0.10           | 0.16           | 0.20            |
| Kuwait                | KWT  | 92,999.7| 31                | 0.17           | 0.07           | 0.19            |
| Malaysia              | MYS  | 25,016.9| 42                | 0.26           | 0.10           | 0.19            |
| Mexico                | MEX  | 29,665.0| 38                | 0.16           | 0.24           | 0.33            |
| Morocco               | MAR  | 8,350.4 | 41                | 0.10           | 0.15           | 0.19            |
| Mozambique            | MOZ  | 1,315.4 | 31                | 0.25           | 0.22           | 0.54            |
| Netherlands*          | NLD  | 73,072.6| 31                | 0.07           | 0.14           | 0.18            |
| New Zealand*          | NZL  | 52,145.2| 26                | 0.10           | 0.18           | 0.26            |
| Niger                 | NER  | 1,636.2 | 41                | 0.14           | 0.06           | 0.09            |
| Norway*               | NOR  | 93,836.1| 21                | 0.11           | 0.13           | 0.18            |
initial conditions as we go back in time. The standard deviation of each of the MPKs reflects efficiency in the distribution of resources across countries.

Focusing on the left-hand column of Figure 3 shows that the annual means have remained more or less constant since 1990 in the full sample (first row). This pattern, however, does not hold when we split the sample into advanced and developing nations. In advanced economies (last row), the means have monotonically increased since 1990, whereas for the developing group (middle row), MPK is fairly flat and MPKP has declined. The trend in the developed-world private MPK is most likely due to technical change. The patterns observed in developing nations are, on the other hand, more difficult to explain, and we leave this to future research. The left-hand column also shows that the distance between MPK and MPKP during 1990–2005 has changed (declining) only for the developing group.

The annual standard deviation of the MPK, depicted in the right-hand column of Figure 3, is more closely related to the concept of capital misallocation. In particular, a falling variation suggests more efficient allocation of capital worldwide. We see that the overall MPK shows some variation (a slight decline) of its standard deviation only in advanced economies. The private MPK, on the other side, experiences a significant decrease in both groups—but especially in developing nations—contributing to diminish the gap between the two MPK measures. Therefore, on average, private capital has become more efficiently allocated across nations since 1990.

### Table 2

| Country    | ISO | \(y\) | Sh# | MPK (1996) | MPK (2005) | MPKP (2005) |
|------------|-----|-------|-----|------------|------------|-------------|
| Panama     | PAN | 17,354.6 | 42  | 0.15       | 0.28       | 0.40        |
| Paraguay   | PRY | 7,889.9  | 42  | 0.10       | 0.12       | 0.20        |
| Peru       | PER | 12,880.6 | 42  | 0.24       | 0.28       | 0.38        |
| Philippines| PHL | 5,924.0  | 25  | 0.34       | 0.20       | 0.27        |
| Portugal*  | PRT | 38,234.9 | 16  | 0.19       | 0.10       | 0.12        |
| Qatar      | QAT | 129,535.6| 21  | 0.19       | 0.17       | 0.21        |
| Saudi Arabia| SAU | 54,796.6 | 31  | 0.09       | 0.04       | 0.08        |
| Senegal    | SEN | 3,462.4  | 42  | 0.17       | 0.14       | 0.22        |
| Singapore* | SGP | 79,579.4 | 32  | 0.21       | 0.22       | 0.31        |
| South Africa | ZAF | 19,042.5 | 40  | 0.15       | 0.21       | 0.29        |
| Spain*     | ESP | 58,147.3 | 40  | 0.12       | 0.12       | 0.15        |
| Sri Lanka  | LKA | 7,985.8  | 34  | 0.14       | 0.12       | 0.16        |
| Sweden*    | SWE | 68,093.9 | 49  | 0.13       | 0.16       | 0.21        |
| Switzerland* | CHE | 63,664.9 | 31  | 0.08       | 0.09       | 0.11        |
| Tanzania   | TZA | 1,916.1  | 31  | 0.12       | 0.16       | 0.21        |
| Thailand   | THA | 12,569.5 | 42  | 0.19       | 0.14       | 0.21        |
| Tunisia    | TUN | 15,168.7 | 36  | 0.14       | 0.15       | 0.19        |
| Turkey     | TUR | 26,600.1 | 42  | 0.35       | 0.39       | 0.58        |
| United Kingdom* | GBR | 67,544.3 | 32  | 0.14       | 0.16       | 0.22        |
| United States* | USA | 83,541.8 | 32  | 0.16       | 0.16       | 0.21        |
| Uruguay    | URY | 18,550.0 | 42  | 0.26       | 0.17       | 0.34        |

**Notes**

ISO is the country isocode; \(y\) indicates PPP real GDP per worker from PWT 7; Sh# is the number of time series observations of sectoral investment shares.

* denotes an advanced economy.
Our next task consists in digging deeper into the large cross-country public capital variation found by our analysis of the gap between the total and private MPKs.
This is an important issue because it may indicate that the most significant loss in world GDP is due to the misallocation of public capital, not private capital. This challenge requires the use of the public MPK measure $MPKG$ contained in expression (5).

As we already know, the problem with calculating $MPKG$ is that it is not possible to estimate $\gamma$ using national accounts because the public sector performs a non-market activity. To circumvent this issue, we carry out three exercises. First, we abstract completely from income shares and focus instead exclusively on the ratio of output to capital ($P_y Y / P_y K_y$ and $P_y Y / P_y K_p$), a main driving force of the value and dispersion of the MPK measure. Second, as in Cooley and Prescott (1995), we assume that the net rate of return between private and public investment is the same, which allows us to get country-specific values of the share of public capital in output. This second exercise provides a conservative assessment of the gap between the private and public MPKs. Third, we employ regression estimates of the output elasticity of public capital to proxy the parameter $\gamma$. These regression estimates, except in one case, do not allow for cross-country variation in the shares, and should tend to overestimate the private–public MPK difference. Albeit each of them in isolation is an imperfect measure, we believe that taken together they provide a possible interval of variation for the public MPK.
Counterfactual shares

Results from this first approach are displayed in Figure 4. It uses the current-price measures of capital and output as in Figure 2, though with a much enlarged sample size of 130 nations given that we no longer need income share numbers. To ensure the quality of the capital stock data, only countries with at least 10 investment share observations and 20 observations of the investment deflator are included.

The first row of charts in Figure 4 shows the public (left-hand) and private (right-hand) components of the output–capital ratios. The results are striking. For private capital, the familiar relatively constant and flat shape is obtained. For public capital, however, it is a quite different story: the cloud of points displays a much larger dispersion than the private one (as expected), and is significantly upward-sloping.

**Figure 4.** Counterfactual and actual shares. [Colour figure can be viewed at wileyonlinelibrary.com]
Following the same method, and in order to further look into the extent of capital misallocation, we can ask the question: given the observed output–capital ratios, what pattern of public/private output elasticities would be needed to rationalize the data if the world is one of perfect capital markets (i.e. with equalized MPKs)? The reader can then think about whether the pattern and magnitudes seem reasonable given whatever prior on output elasticities is held. Specifically, we assume a counterfactual in which the returns are equalized across countries, and across sectors, opting for \( r + \delta = 0.13 \) —that is, an arbitrary number within the interval of \( MPKP \) in Table 3. By taking a stance on this hypothetical equalized return to capital, we can then back out the output elasticities as \( \beta = 0.13/(P_y Y/P_y K_y) \) and \( \gamma = 0.13/(P_y Y/P_y K_y) \), ensuring that for each country in 2005 we have \( MPKP = MPKG = 0.13. \)

The middle row of Figure 4 gives the pattern of counterfactual shares consistent with equalized MPKs. A key aggregate pattern of the public capital share (left-hand chart) is that the hypothetical output elasticity is low compared to the private one (right-hand chart), especially for high-income economies (above $35,000 real GDP per worker); in this country group, the average \( \hat{\gamma} \) is 0.07, that is, much smaller than the mean value of \( \hat{\beta} \), which equals 0.19.

It is also interesting that the counterfactual elasticities for private capital are relatively similar across country groups, showing the same average of 0.19 for both high- and low-income economies. However, the ones for public capital are very different, with a mean value of 0.07 for the former group, 0.15 for the latter, and a strongly significant downward-sloping relationship with income levels. In both cases, the heterogeneity among developing countries is clearly larger, with a standard deviation that more than doubles that of high-income countries. A summary observation is then that for equalized MPKs we require, on average, higher public capital output elasticities in developing than advanced economies, and similar elasticities for private capital.

At this point, it is important to test whether output elasticities can be considered essentially constant in the cross-section. If this is the case, then output–capital ratios would be informative about MPK differences across nations. In order to do this test, the final row of Figure 4 turns away from agnosticism and shows national accounts data for the reproducible-capital share (\( \beta \)) in our sample of 68 countries (right-hand column) and for the government gross operating surplus share in a subsample of 49 nations (left-hand column). The latter share captures the depreciation of public capital and therefore provides information about one component of \( \gamma \). Both charts may suggest a slightly negative relationship of the shares with income; however, this relationship is not statistically significant.

Taking all these results together, they suggest a common aggregate production function across countries, and therefore hint at a large misallocation of public capital alongside the relatively efficient allocation of private capital. These patterns are consistent with profit motives bringing private output–capital ratios in line across countries and political motives keeping the public output–capital ratios out of step.

A Cooley–Prescott approach

So far, we have not tried to give actual values to the output elasticity of public capital. However, this is important to get closer to the actual degree of public capital misallocation. In order to advance in that direction, we first take a very conservative view and, following Cooley and Prescott (1995), assume that the net return to investment is
equalized across sectors. This can be justified when governments compete with private enterprises in the loanable-funds market to borrow from private agents and finance budget deficits. A desirable feature of the method is that it delivers country-specific values for $\gamma$.

Suppose that if output included the whole return to public capital ($GDP$ in national accounts includes only its depreciation), then both the public and the private MPK would differ only due to the depreciation rates of their respective capital stocks. Under this assumption, we can compute the share of public capital $\gamma$ in adjusted income ($Y_{adj}$) as

$$
\gamma = (r + \delta_g) \frac{P_{k_g} K_g}{P_y Y_{adj}},
$$

where

$$
P_y Y_{adj} = P_y Y + rP_{k_g} K_g,
$$

and $r$ represents the net return to capital investment. Then expressions (5), (7) and (8) obtain the marginal product of public capital (that we denote $CPMPKG$ for convenience) as follows:

$$
CPMPKG = (r + \delta_g) \left(1 + r \frac{P_{k_g} K_g}{P_y Y}\right)^{-1}.
$$

In principle, $CPMPKG$ can be bigger or smaller than $MPKP$—which equals $r+\delta_p$ by assumption—depending on the difference between the two depreciation rates ($\delta_p$ and $\delta_g$) and the value of the ratio of public capital to GDP. A higher $\delta_g$ and a smaller ratio will tend to make the public MPK larger compare to its private counterpart.

We still need to get values for the variable $r$. From (4), under the cross-sector net-return-equalization assumption, we can use the CF estimates to recover the net return through the expression

$$
r = \alpha_F \frac{P_y Y}{P_{k_p} K_p} - \delta_p.
$$

Notice that $Y_{adj}$ does not belong to expression (9) because the share of non-reproducible capital derived by CF is a fraction of GDP, that is, $P_y Y$.

Figure 5 presents the results for the core sample. The left-hand chart gives the estimated shares, and the right-hand chart gives the estimated public MPK. The cloud of the shares provides a negative relationship, with a fitted line whose slope is significant at the 5% level. The public MPK cloud, on the other hand, still shows a large dispersion, but its fitted line depicts a significant negative slope—that is, the estimated shares more than offset the upward-sloping ratio of GDP to public capital. The main lesson from this second exercise is that the assumption that the net returns of investment in public and public capital are the same does not leave much room for an increasing public MPK. An additional result, although not shown in Figure 5, is that $MPKP$ is above $CPMPKG$ in all countries, because the depreciation rate in the private sector is sufficiently larger.
These depreciation rate differences are therefore a potential source of overinvestment in public capital in all nations.

Regression estimates

The Cooley–Prescott exercise imposes a possible lower bound on the difference between the marginal returns to private and public capital. In order to have a wider view, we next resort to production function estimates of the output elasticity of public capital. This is certainly an imperfect approach, for two reasons: first, there are concerns about proper identification of the elasticity; second, the method does not offer sufficient heterogeneity in relative shares across countries. On the first issue, while concerns about identification are warranted, we offer results for a wide range of economically plausible output elasticities that leave the main results unchanged. On the second issue, the equalization of shares across countries does not allow for the offsetting effect displayed by the downward trend in the left-hand chart of Figure 5, and therefore, as previously mentioned, this exercise should tend to overestimate public MPK differences.

A useful reference point to achieve our goal is given by Bom and Ligthart (2014), who perform a meta-analysis on a sample of 578 estimates from 68 studies carried out from 1983 to 2008, estimating the private output elasticity of public capital. Even given much variation across the studies, they find the average true output elasticity of public capital to be positive and significant—giving support for the implicit assumption throughout this paper that public capital is productive and should appear in the production function.

To be precise, after correcting for linear publication bias, the unconditional average output elasticity of public capital is found to be 0.106. However, the output elasticity is quite heterogeneous. In the short run, \( \gamma \) is only 0.083 for public capital installed at the central level of government. This value increases to 0.193 when long-run estimates of core public capital such as roads and railways installed by regional and local governments are considered.

Though many of the studies in the Bom and Ligthart (2014) sample are for the USA or other advanced economies, thus can be thought of as not completely applicable to our sample, the one study that focuses on low-income countries (Dessus and Herrera 2000) yields a similar output elasticity of 0.13. In addition, a more recent contribution by Gupta et al. (2014) incorporates a large number of low-income countries to their sample.
Their approach is to estimate system-GMM panel regressions assuming a Cobb–Douglas production function with skill-adjusted labour, private capital and public capital as its arguments. As shown in columns (2) and (3) in Table 6 of Gupta et al. (2014), they estimate a value of $\gamma$ equal to 0.253 for low-income countries and 0.167 for middle-income and advanced economies.

Taking these estimates on board, Figure 6 shows results for the sample of 68 countries in 2005 using values of $\gamma$ equal to the minimum ($\gamma = 0.083$, top row) and average ($\gamma = 0.106$, middle row) reported by Bom and Ligthart (2014), and the values estimated by Gupta et al. (2014) ($\gamma = 0.253$ for low-income countries and $\gamma = 0.167$ for the rest, bottom row) that provide a $\gamma$ for low-income countries larger than the values reported by Bom and Ligthart (2014). The left-hand column of charts gives

**FIGURE 6.** $\text{MPKG}$ applying regression estimates. [Colour figure can be viewed at wileyonlinelibrary.com]
values for $MPKG$, whereas the right-hand column provides the ratio of $MPKG$ to $MPKP$. This last ratio is interesting because we can look at efficiency as requiring that marginal returns are equalized not only across countries, but also across sectors.

The $MPKG$ plots reproduce the results obtained in Figure 4 for the output–capital ratios, but this time with a smaller sample. The public MPK increases, on average, with income per worker, and the slope is significant at the 5% level. Results suggest that if there are barriers to the flow of capital across countries, then these obey, on average, upward rigidities in the flow of investment that finances public capital.

The ratio of the public return to the private return, on the other hand, gives information about how countries deviate from cross-sector equalization. The natural interpretation (given a benchmark that the private sector behaves optimally) is that a ratio below 1 reflects a government that overinvests in public capital, whereas a number above 1 suggests underinvestment. Regardless of the value of $\gamma$, few nations show values around 1—the degree of dispersion is high. Another pattern that arises independently of the value of $\gamma$ is the case for public capital underinvestment in advanced economies. Only Japan and Singapore seem to be close to an efficient stock of public capital when the value of $\gamma$ is around its average estimate.

The case of underinvestment in also dominant for emerging countries. When the output elasticity of public capital is 0.083, only 3 out of 42 developing countries in 2005 provide a ratio below 1, making the case for overinvestment in public capital. This number falls to 2 for $\gamma$ equal to 0.106, and becomes zero when $\gamma$ is 0.253 in low-income countries. In all cases, emerging nations that show overinvestment are always middle-income. Hence we conclude that using regression estimates, most developing countries in the sample suffer from underinvestment in public capital.

IV. DEADWEIGHT LOSS CALCULATIONS

A direct measure for the efficiency loss from capital misallocation is the deadweight loss (DWL), which we define here, as in CF, as

$$\frac{\sum_{n=1}^{N} (Y_n^* - Y_n)}{\sum Y_n},$$

where $Y_n^*$ is counterfactual GDP with capital (public, private or overall) efficiently allocated in nation $n$, and $N$ is the number of countries in the sample. The greatest asset

| Sample | Shares | No. of nations | MPK | MPKP | MPKG ($\gamma = 0.106$) | CPMPKG |
|--------|--------|----------------|-----|------|-------------------------|--------|
| CF     | CF     | 50             | 0.40| 0.30 | 8.68                    | 3.48   |
| MSS    | MSS    | 68             | 1.40| 1.90 | 9.32                    | 9.09   |
| Open MSS | MSS    | 46             | 1.14| 1.31 | 8.56                    | 6.24   |
| Closed MSS | MSS    | 18             | 1.26| 2.14 | 8.93                    | 15.64  |

Table 4
DEADWEIGHT LOSS IN 2015, PERCENTAGE INCREASE OVER WORLD GDP
of this measure here is that we can start to quantify the relative loss from public versus private capital misallocation. The calculations extend the approach of CF to account for complementarity of public and private capital in the production function.

Assuming that all industries in the country use the same Cobb–Douglas production technology, we transform expression (1) into the following aggregate production function for country \( n \):

\[
Y_n = K_{gn}^{\gamma_n} K_{pn}^{\beta_n} X_n^{1-\beta_n},
\]

where all variables and parameters are now country-specific.

Profit-maximization and price-taking ensure that the following conditions hold for every \( n \):

\[
\frac{P_n}{P_{K_p}} \beta_n K_{gn}^{\gamma_n} K_{pn}^{\beta_n-1} X_i^{1-\beta_n} = MPKP_n,
\]

(10)

\[
\frac{P_n}{P_{K_g}} \gamma_n K_{gn}^{\gamma_n-1} K_{pn}^{\beta_n} X_n^{1-\beta_n} = MPKG_n.
\]

(11)

In the counterfactual case where the returns to private and public capital (separately) are equalized across countries, expressions (10) and (11) imply that

\[
\frac{P_n}{P_{K_p}} \beta_n K_{gn}^{\gamma_n} (K_{pn}^*)^{\beta_n-1} X_i^{1-\beta_n} = MPKP^*,
\]

(12)

\[
\frac{P_n}{P_{K_g}} \gamma_n (K_{gn}^*)^{\gamma_n-1} K_{pn}^{\beta_n} X_n^{1-\beta_n} = MPKG^*.
\]

These conditions can be manipulated to show that the counterfactual capital stocks can be calculated as

\[
K_{pn}^* = \left( \frac{MPKP_n}{MPKP^*} \right)^{1/(1-\beta_n)} K_{pn},
\]

(13)

\[
K_{gn}^* = \left( \frac{MPKG_n}{MPKG^*} \right)^{1/(1-\gamma_n)} K_{gn}.
\]

However, \( MPKP^* \) and \( MPKG^* \) are unknown. To solve for these, we require an additional resource constraint—we impose that the aggregate counterfactual private/public capital stock is equal to the existing aggregate stocks:

\[
\sum_{n=1}^{N} K_{pn}^* = \sum_{n=1}^{N} K_{pn} = \sum_{n=1}^{N} \left( \frac{MPKP_n}{MPKP^*} \right)^{1/(1-\beta_n)} K_{pn},
\]

(14)
We solve for $MPKP^*$ and $MPKG^*$ in equalities (14) and (15) to an accuracy of two decimal places. Once we know the counterfactual equalized MPKs, it is straightforward to find counterfactual capital stocks country by country. Counterfactual income with private capital efficiently allocated is then simply

$$Y_n^* = Y_n \left( \frac{K_{pn}}{K_{pn}} \right)^{\beta_n},$$

or with efficient allocation of public capital it is

$$Y_n^* = Y_n \left( \frac{K_{gn}}{K_{gn}} \right)^{\gamma_n}.$$

The DWL measure is then calculated as the overall percentage increase in income from capital reallocation. Since the calculations in this section require comparable capital measures across countries, we calculate real capital measures using PWT 7.0 data, rather than the current-price local currency measures used for our preferred measures of the MPK.

Recalling that CF calculate the DWL to be 0.1% of income in 1996 using PWT 6.1 data, we find a comparable result with PWT 6.1 data employing the CF 50-country sample and reproducible-capital shares along with our approach to capital stock construction (which differs slightly to CF in its initial conditions and depreciation rates assumed)—in particular, we find the DWL to be 0.054% of GDP. Using the latest PWT 7.0 data on the same CF sample however, we calculate the DWL for the same year to be 0.41% of income; that is, the update to the data itself yields an update to the DWL.

Our interest is more in finding the DWL by sector, for two reasons. First, the figure of 0.1% (or 0.41%) could understate the actual DWL if public and private capital are complements in the production function. The simplest intuition is that a completely flat overall MPK schedule (DWL of zero) could conceal an upward-sloping public MPK offset by a downward-sloping private MPK (positive DWL in each sector). Second, we are interested in quantifying the difference in efficiency losses between the sectors.

The calculations are presented in Table 4. The first two rows confirm our priors. In 2005, using PWT 7.0 data, the CF sample and shares—which are used just on this occasion for comparison—deliver an overall DWL of 0.40%, very close to the 1996 value of 0.41%. Once capital is disaggregated, the DWL in the private sector (assuming that the allocation of public capital is unchanged) is only 0.30%, while the loss from public capital misallocation goes from 3.48% to 8.68%, depending on whether we use the Cooley–Prescott assumption or the Bom and Ligthart average (0.106) for $\gamma$, respectively. These numbers represent a substantial gain from public capital reallocations.
Returning to our 68-country sample and the MSS shares, it is interesting to compare our results to the ones in the MSS paper. MSS study misallocation only due to total non-reproducible capital. We get a DWL of 1.40% for overall capital in 2005 (second row of Table 4), whereas MSS find 2% for the same year. The different values obtained can be a consequence of the reduction from 79 to 68 in the number of countries that compose the benchmark sample, or from the different version of the PWT employed—they use PWT 8.0. The second row of Table 4 also implies that, compared to the CF case, the losses from overall capital and public capital misallocation (CPMPKG case) almost triple: they rise from 0.40% and 3.48% to 1.40% and 9.09%, respectively. For the private MPK, the impact is more striking: it experiences a sixfold increase, from 0.30% to 1.90%. Perhaps more importantly, the DWL from public capital misallocation remains much larger than from private capital: the former is at least 4.8 times larger.

V. ROBUSTNESS CHECK

In this section, we carry out an experiment to see how the results about the direction and degree of capital misallocation change. We study whether policy distortions can be behind the discrepancy in the MPK across countries. In particular, following MSS, we employ the openness indicator originally developed by Sachs and Warner (1995) and later extended by Wacziarg and Welch (2008). The indicator takes the value 1 if a country is classified as open, and 0 otherwise. As argued by Rodriguez and Rodrik (2001), among many others, the beauty of this indicator is that it reflects a range of policies and institutional differences that go further beyond the degree of international trade liberalization.

Figure 7 and the last two rows of Table 4 provide results when the sample is split into open and closed economies through the adjusted openness indicator constructed by Wacziarg and Welch (2008).16 We see that for closed economies (right-hand column of Figure 7), the fitted lines show significant positive slopes and call for upward frictions in the international flow of capital.17 It is the set of open economies (left-hand column of Figure 7) that is responsible for the patterns obtained previously. As in previous sections, the first chart shows that the private MPK is downward-sloping. So is the public MPK obtained with the Cooley–Prescott approach; it is, however, upward-sloping using regression estimates.

In terms of DWL (see Table 4), the message does not change either: the loss from capital misallocation is much larger for public capital than for private capital. Nevertheless, we find some differences between countries classified as open (third row of Table 4) and closed (last row). Closed economies show a higher DWL, and the difference between the DWL induced by private and public capital is also larger. More specifically, the DWL is 1.31% and 2.14% for private capital (fourth column) in open and closed nations, respectively; those numbers become 6.24% and 15.64% when the CPMPKG values (final column) are equalized across countries, which represent 4.8 and 7.3 multiples compared to their private capital counterparts.

VI. CONCLUSION

Caselli and Feyrer (2007) deliver an intriguing result: after appropriately adjusting the share and relative price of capital, the overall MPK is shown to be broadly the same across a large group of advanced and developing economies, casting doubt on the international capital frictions explanation of the Lucas Paradox, and leaving not much
room for international physical capital misallocation. Motivated by the extensively documented and remarkable differences between public and private sector incentives, especially in developing countries, we have attempted in this paper to unpack the overall MPK into its public and private components.

Given the difficulties associated with the calculation of the output share of public capital and its importance for the public MPK measure, we have followed two approaches that together generate possible upper and lower bounds for the difference between the public and private MPKs in each nation. One approach supposes that net returns are equalized between the public and private sector in each economy (the lower bound). The other approach adopts regression-based estimates under the assumption that the output elasticity of public capital $\gamma$ is relatively stable in the cross-section of countries (the upper bound).
Our results are as follows. First, using data from WDI, WEO and PWT 7.0, and the MSS capital shares, we have shown that the cross-country schedule of the total MPK is flat with respect to income-per-worker levels. Second—and this is our key result—we have shown that even though the private MPK turns out to be negatively-sloped, the DWL from private capital misallocation is relatively small, about 1.9% of global GDP in 2005. Therefore, in terms of the Lucas Paradox, our results suggest that private capital is allocated remarkably efficiently. Third, we have found that the DWL from public capital misallocation can be substantial, much larger than that from cross-country differences in the return to private capital. In particular, public MPK differences across countries produce a loss in global GDP of 9% in our most conservative scenario, where we assume that the net returns to investment in public and private capital are equalized.

Searching for the roots of the differences in the public MPK across countries, we have split the sample into open and closed economies. This exercise has revealed that for closed economies, the cost of public capital misallocation can be substantially larger than for open ones. Another important result from this exercise is that previous findings hold, except for closed economies, where the case for upward frictions in the international flow of private capital is supported.

Our approach also suggests a refinement of the outlook on aid presented in Caselli and Feyrer (2007), namely a sceptical view on aid, concluding that greater flows of aid would be displaced only by capital outflows, given the flat MPK. Our disaggregation brings an alternative view. Given imperfect substitutability between private and public capital in the production function, investment in public capital may lead not to capital outflows, but to inflows of private capital, since the greater stock of public capital raises the returns to private capital.

This alternative view may fit too closely with a story of Tanzania’s ability to attract foreign capital. Taking a walk in the busy streets of Dar es Salaam, the capital city, one is impressed by the vibrant private economic activity, entrepreneurship and many bank branches (local and multinational) scattered across town. One gets the favourable impression that, although at an embryonic stage, the private sector operates under close proximity to ‘market’ conditions. A look at public goods (e.g. railroads and ports) and the provision of public services (e.g. power generation) reveals clear deficiencies. Experts correctly insist on the major progress, including in the public sector, that Tanzania has been through over the last two decades, as captured by the country’s 7% average GDP growth. But by all accounts this progress is not sustainable unless capital starts to flow inwards from abroad. This paper points to public sector frictions, such as public-infrastructure investment mismanagement, rather than financial frictions or complementarities to low human capital or total factor productivity as the key constraint to enhancing the MPK, and with it, accelerating international capital inflows.

Still, our public MPK results should be considered tentative due to the strong assumptions made. There is still much work to be done to improve measurement of the return to public capital investment. In this paper, we have just started to scratch the surface to understand public capital allocations across countries. Nonetheless, we have clearly shown that aggregate estimates can provide a very good start in this line of research, and that existing aggregate datasets are adequate for taking on the task. But such aggregate estimates should be compared against micro-evidence that is just as crucially important in understanding the pattern of capital flows.
**APPENDIX**

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**NOTES**

1. In a recent contribution, Gourinchas and Jeanne (2013) argue that international capital flows move towards developing countries with lower (not higher) productivity growth. While the Lucas Paradox is about the small magnitude of capital inflows to developing countries, the ‘allocation puzzle’, as named by Gourinchas and Jeanne, is about the allocation of the already small size of capital flows across developing countries. Nevertheless, Alfaro et al. (2014) suggest that the allocation puzzle is entirely driven by sovereign-to-sovereign transactions.

2. Taylor (1998) measures the MPK similarly for Argentina, and Bai et al. (2006) use a similar approach to measure the return to capital (both in aggregate and by sector/region) in China, though they use current price data to measure \( P_T Y / P_T K \) rather than real data followed by a price adjustment as in CF.

3. Having said that, other adjustments can be suggested that in principle have the ability to overturn the result of a flat MPK. Chirinko and Mallick (2008) draw attention to the role played by adjustment costs, finding that a large MPK differential re-emerges once adjustment costs are accounted for.

4. We thank a referee for suggesting this exercise.

5. Robinson and Torvik (2005), for example, aim to explain why governments do not act like profit maximizers when it comes to investing. In particular, the model explains the political motivation behind the construction of white elephants. Politicians construct these inefficient projects when they find it difficult to make credible promises to political supporters. The general point of this and other political economy models is that governments are driven more by an electoral motive than by a profit motive.

6. As previously mentioned, when the government provides services, it may not be equalizing returns among industrial activities or across regions (e.g. through ethnic favouritism). The equalization of returns across different sectors may not be fully appropriate for the private sector either. For instance, if capital is heterogeneous, then different sectors might use a different composition of capital goods, hence the rates of return on two different composite capital stocks would not need to be equalized. Nevertheless, the equalization of returns should help to shed light on the allocation of capital across countries.

**Figure A1.** CF results with current-price data. [Colour figure can be viewed at wileyonlinelibrary.com]
7. An implication of this paragraph is that in order to get the private $MPK$ using expression (4), we should subtract (i) the depreciation of public capital from GDP in the numerator, and (ii) the share of $K_d$ depreciation from the share of reproducible capital to obtain $\beta$. These modifications, however, do not significantly change the results (available from the authors on request). The reason is that the depreciation of public capital—that is, the gross operating surplus of the general government in national accounts—amounts to only 2.36% of GDP on average in our sample, with a relatively low standard deviation.

8. It is possible, though, to find price indices for different types of capital. PWT, for example, offers different price indices for four categories: residential and non-residential structures; machinery and (non-transport) equipment; transport equipment; and other assets. However, it is not clear how to go from these categories into private and public capital.

9. For seven countries (Austria, Denmark, Greece, Ireland, Jamaica, Spain and Sweden), investment share data were missing from the latest WEO. We opted to take the share data from WEO 2003, using forecasted shares for the years 2004–8.

10. Since a negative $RGj$ could result in implausibly large or impossibly negative initial conditions, the measure was bounded at zero. For the core sample of 50 countries, this bounding affected only the public capital initial condition for Zambia.

11. In particular, from the original MSS benchmark sample, we lose Hungary, Iceland, Jamaica, Luxembourg, Malta, Nigeria, Oman, Poland, Taiwan, Trinidad & Tobago and Zimbabwe. The 68 nations that remain are the ones listed in Table 2.

12. The distinction between advanced and developing economies follows Table 2.

13. Note that other things being equal, the choice of $r + \delta$ affects only the scale of the cloud of points, not its shape. In particular, a larger choice will lead to larger elasticities, and vice versa.

14. Actually, the CF core sample is composed of 52 nations. We lose Jamaica and Trinidad & Tobago due to a lack of recent investment data covering these countries.

15. Comparing the middle-right chart in Figure 2 ($MPK$) to the right-hand chart in Figure 5 ($CPMPKG$), it is not evident why the DWL from private capital is substantially smaller than from public capital. Notice that, for example, the cloud of points for $CPMPKG$ seems to be more compressed than the one for the private $MPK$. The key to understanding the result is that the $CPMPKG$ average is also smaller, which makes $CPMPKG$ have a coefficient of variation 30% higher than the one for $MPK$.

16. A country is classified as closed if it displayed at least one of the following five characteristics: (1) average tariff rates of 20% or more; (2) non-tariff barriers covering 20% or more of trade; (3) a black market exchange rate at least 10% lower than the official exchange rate; (4) a state monopoly on major exports; (5) a socialist economic system (as defined by Kornai (1992)). In the original Sachs and Warner (1995) criteria, the thresholds for (1), (2) and (3) were 40%, 40% and 20%, respectively. The reason for choosing the adjusted instead of the original openness indicator is that using the latter, the number of countries classified as closed in our sample was too small.

17. The 18 economies classified as closed are Brazil, Burkina Faso, China, Cote d’Ivoire, Cyprus, Dominican Republic, India, Indonesia, Iran, Kenya, Republic of Korea, Sri Lanka, Morocco, Paraguay, Senegal, Tanzania, Thailand and Tunisia. The rest of the countries in the sample are considered open.

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