MULTIPLAYER GAMES AND HIV TRANSMISSION VIA CASUAL ENCOUNTERS

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Abstract. Population transmission models have been helpful in studying the spread of HIV. They assess changes made at the population level for different intervention strategies. To further understand how individual changes affect the population as a whole, game-theoretical models are used to quantify the decision-making process. Investigating multiplayer nonlinear games that model HIV transmission represents a unique approach in epidemiological research. We present here 2-player and multiplayer noncooperative games where players are defined by HIV status and age and may engage in casual (sexual) encounters. The games are modelled as generalized Nash games with shared constraints, which is completely novel in the context of our applied problem. Each player’s HIV status is known to potential partners, and players have personal preferences ranked via utility values of unprotected and protected sex outcomes. We model a player’s strategy as their probability of being engaged in a casual unprotected sex encounter (USE), which may lead to HIV transmission; however, we do not incorporate a transmission model here. We study the sensitivity of Nash strategies with respect to varying preference rankings, and the impact of a prophylactic vaccine introduced in players of youngest age groups. We also study the effect of these changes on the overall increase in infection level, as well as the effects that a potential prophylactic treatment may have on age-stratified groups of players. We conclude that the biggest impacts on increasing the infection levels in the overall population are given by the variation in the utilities assigned to individuals for unprotected sex with others of opposite HIV status, while the introduction of a prophylactic vaccine in youngest age group (15-20 yr olds) slows down the increase in HIV infection.

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1. **Introduction.** Since the beginning of the HIV epidemic, it is estimated that 75 million people have been infected with HIV and 36 million have died [40]. HIV spans the globe, affecting every country, although some have fared worse than others. This is especially apparent in sub-Saharan Africa, with an estimated 5% of all adults infected [40]. HIV is not restricted to certain age groups, and, despite prevention programs and awareness campaigns, HIV incidence continues to increase, particularly amongst men who have sex with other men (MSM).

Fortunately, there have been numerous medical advancements since AIDS and HIV were defined in 1982–85. In 1986, the first antiretroviral therapy (ART) was introduced to prolong the life of individuals affected with HIV and reduce its spread. This changed the perspective of HIV to that of a chronic disease. Questions are now raised as to why the incidence rate continues to increase amongst certain population groups. Numerous surveys attempt to explain an individual’s reasoning, but the results have been varied, showing evidence of increased risky behaviour as well as decreased or no change in behaviour. [12, 22, 28, 32]. The imminence of prophylactic HIV vaccines (such as one being developed at The University of Western Ontario [39]) would have an enormous impact worldwide, but also raises some new interesting questions, chief among them being how to assess its impact on individuals contemplating whether to engage in casual sex, specifically in unprotected casual sex.

Population models have been useful in understanding and predicting the spread of HIV [20, 26, 18, 31, 19]. To better understand how one individual can affect change at population level game-theoretical models have been used [30, 63]. These models illuminate a feedback mechanism where individual’s choices may affect the population, which in turn affect the choices an individual is likely to make.

Game theory dates back to late 1940s with the works of von Neumann and Morgenstern [37] and later those of Nash [25] in the early 1950s. A game is a mathematical framework to describe decision-making by individuals engaged in competitive situations, where they can behave noncooperatively or cooperatively. Noncooperative game theory is nowadays widely used in applied areas such as economics, engineering, operations research, evolutionary biology and social sciences (psychology and cognitive sciences) see [4, 15, 16] and many references therein. The question of existence and computation of Nash strategies for a given game can be tackled with various methods, such as the reaction-curves method, optimization techniques, variational inequalities, computational methods (such as genetic algorithms, evolutionary computation), or a replicator-dynamics equilibrium, etc. [4, 10, 21, 8, 11].

Generalized Nash games (GN) with finite dimensional strategy sets were first studied by Arrow and Debreu in [11], followed by [20, 23, 31, 27], with a recent review in [14]. The formulation of the generalized Nash game as a variational inequality problems dates back to Bensoussan [5], while [34] gives first equivalence results for finite-dimensional GN games and quasivariational inequalities.

In this paper, we model casual (sexual) encounters as a noncooperative generalized Nash game between 2, 3, or 4 players, where each player’s HIV status is known to both one’s self, and to the player they choose to interact with. We do not model in this paper a population-level transmission process. All players have personal preferences ranked in utility of unprotected and protected sex outcomes, and they are given expected utilities of the casual encounter, depending on possible outcomes: unprotected sex outcome ($USE$), or protected/no-sex outcome ($PSE$). We model a player’s strategy vector as their probabilities of being engaged in $USE$
with other players. We further introduce multiplayer GN games to analyze casual encounters between players belonging to different age groups, where partner choice is closely tied to a player’s age. This age-stratified game model is introduced here to investigate the possibility of offering a prophylactic HIV vaccine to the youngest age group, perhaps before they become sexually active (similar to the HPV vaccine). Then we study the effects of individual choices on HIV transmission where different age groups interact and have differing levels of access to treatment options.

In our previous work on the topic, we modelled a similar setup of casual encounters with an agent-based model of the population [33, 35], and we analyzed how groups can emerge from coevolution of HIV spread with partner choices and risk perception, where we also assumed that a player’s true HIV status is only known to themselves. In this model, unlike our previous simulation-based work, we construct a theoretical model to identify and analyze Nash equilibria with respect to the decisions players make. This allows us to better understand: a) the impact of personal preferences for unprotected sex; and b) the impact of heterogeneity of players (division in age groups) and initial HIV age composition both in presence and absence of a prophylactic vaccine.

To the best of our knowledge, modelling HIV transmission with age-stratified multiplayer GN games models is absolutely novel in the literature. There are a handful of examples of multiplayer GN games in applied problems (the River Basin problem, electricity markets [25], cap-and-trade agreements [14], voluntary vaccination models [11]), so our work here is unique in pushing the boundaries of modelling using GN games.

The structure of the paper is as follows: In Section 2, we present a 2-player game while in Section 3 we formulate the multiplayer games of casual encounters between players in age groups. Throughout, we investigate the sensitivity of players’ Nash strategies and of HIV transmission when changes in utility rankings, efficacy of prophylactic treatment and group-specific initial HIV age composition are taken into account. We close with a few conclusions and future work.

2. Casual encounter games as generalized Nash games. In general, a multiplayer game involves a finite number of players, denoted here by $N > 0$. A generic player $i \in \{1, \ldots, N\}$ is thought to have a strategy set $S_i \subset \mathbb{R}^{n_i}$, whose strategies are vectors $x_i \in S_i$, and a payoff function $f_i : S_i \to \mathbb{R}$. A Nash equilibrium of a multiplayer game is defined as follows:

**Definition 2.1.** Assume each player is rational and wants to maximize their payoff. Then a Nash equilibrium is a vector $x^* \in K := S_1 \times \cdots \times S_N$ which satisfies the inequalities:

$$\forall x_i \in S_i, f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*), \forall x_i \in S_i$$

where $x_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$.

For several decades, there exists, in the game theoretic literature, the concept of a generalized Nash (GN) game [11] [29], which in brief is a game such as above, where however each player’s strategy set $S_i$ is in fact dependent on the vector of strategies of other players [14]. It is known that in general a GN game has entire sets of Nash equilibria (as defined above), and it is also known that very few solution methods exist to date to compute all these for a given game. In particular, the subclass of GN games with shared constraints (SC) [27] [13] has developed more than its generic GN games counterparts. These are GN games where each player has an individual
constraint set (applied only to their vector of strategies, such as \( S_i := S_i(x_i) \) here), however all players in a game have to also obey a shared constraint, i.e., a set of inequalities involving other players’ strategies. This class of games is detailed in [27, 13, 14] and most recent work by the first author developed a theory and a computational method for finding the entire solutions set of a GNSC [9].

In this paper, we need to use the framework of GN theory, as our player interactions lead to casual sexual encounters in a closed population, thus the encounters have to obey a counting rule from both an HIV+ and and HIV− player viewpoint. This constraint is in fact a shared constraint by all players, which naturally leads us to consider multiplayer GN games.

There are results asserting existence of generalized Nash equilibria for the type of games we model (see for instance [13, 34, 29] and references therein). One such approach is that of finding a small subset of solutions of a GN game by reformulating the GN game into a variational problem as below in Definition 2.2. Once the VI is proven to have solutions, computational methods are employed to find its solution set.

**Definition 2.2.** Given a set \( K \subset \mathbb{R}^n \), closed and convex, and given \( F : K \rightarrow \mathbb{R}^n \) a continuous function, the variational inequality (VI) problem is to find a vector \( x^* \in K \) such that

\[
\langle F(x^*), y - x^* \rangle \geq 0, \quad \forall y \in K.
\]

The existence of a solution to the VI problem in Definition 2.2 can be shown in many mathematical contexts (see [24]) but, specifically for our cases here, we use the results in [13]. Specifically, we solve for generalized Nash strategies of players where \( F(x) := (-\frac{\partial f_1(x)}{\partial x_1}, ..., -\frac{\partial f_N(x)}{\partial x_N}) \) and where \( K = \{S_1 \times ... \times S_N | g(x) \leq 0\} \), where \( g(x) \) is the shared constraint.

To solve the VI problem we compute its solution set as the set of critical points of a projected dynamical system (see [2, 8]) given by

\[
\frac{dx}{d\tau} = P_{T_K(x(\tau))}(F(x(\tau))), \quad x(0) \in K,
\]

where \( F(x) := (\nabla_{x_1} f_1, ..., \nabla_{x_N} f_N) \). The advantages are three-fold: we can assert existence of solutions to the game (given known results of existence of solutions to [1]); we can use computational methods developed for projected systems to analyze and compute Nash equilibria for the game; and we can check if/when uniqueness of Nash strategies can be asserted.

It is known that the system (1) is well-defined if \( F \) is Lipschitz continuous on \( K \), where \( K \) is a closed and convex set. Under these assumptions, solutions to this system exist and are unique through each initial point \( x(0) \in K \). A projection-type algorithm can be used to compute its trajectories and its stationary points such as the ones in [2, 8]. To answer the uniqueness question, we numerically explore the set of initial conditions of system (1) and study how many (and what values of) Nash strategies we uncover.

### 2.1. Two-player casual encounter game

Let us consider now a casual encounter between two individuals from a general population of individuals aged 15 and over. A player can have one of two statuses: HIV negative (HIV−) or HIV positive (HIV+). Let us then denote by \( \epsilon_−, \epsilon_+ \), the proportion of HIV negative, respectively HIV positive individuals so that \( \epsilon_− + \epsilon_+ = 1 \). Now, let us define by \( P_1 \) and \( P_2 \) respectively two players such that their statuses are \( s_1 := HIV+ \)
and $s_2 := HIV−$. Define by $x^i \in [0,1]^2, \ x^i = (x^i, x^i_+), i \in \{1, 2\}$ a player’s probability vector of having unprotected sex, where $x^i_−$ is probability of unprotected sex (USE) with an $HIV−$ player and $x^i_+$ is the probability of USE with an $HIV+$ player. We try to find out, via our game model, what are the probabilities of having unprotected sex as a result of a casual encounter, based on their preference of sex outcomes.

Utility ranking of preferences of each player is given based on the HIV status of the individual they might engage with, which is considered known to them, as well as based on the type of sexual outcome they may find themselves in, namely USE, or PSE (where by PSE we amalgamate preferences of either protected sex or no sex outcomes).

Specifically, we assign as a numerical value for the utility of a sexual encounter the range of $[0, 1]$. We consider throughout the paper that $USE(+) = 1$, $USE(−) = 1$ and that, as in [33], $USE(−, +) < 1$ and $USE(−, −) < 1$. This is to say that players are rational, and prefer unprotected sexual encounters with players of same status over all other outcomes.

Let us define the expected utilities for $P_i, i \in \{1, 2\}$ out of a casual encounter as: $E^i_− :=$ expected utility from interacting with an $HIV−$ player and $E^i_+ :=$ expected utility from interacting with an $HIV+$ player. For $P_1$, who is $HIV+$, these are:

\[ E^1_− = \rho [x^1_− USE(+, −) + (1 − x^1_−)PSE(+) −] \]
\[ E^1_+ = \rho [x^1_+ USE(+, +) + (1 − x^1_+)PSE(+, +)] \]

where $\rho$ is the activity parameter of $P_1$. This represents a multiplicative factor controlling overall sexual activity level. It is outlined further in Section 3.

Then the overall expected utility of the encounter for $P_1$ is:

\[ E^1(x^1_−, x^1_+) = \epsilon_+ E^1_− + (1 − \epsilon_+) E^1_+ . \]

Similarly, for $P_2$, whose status is $HIV−$, we have:

\[ E^2_− = \rho [x^2_− USE(−, −) + (1 − x^2_−)PSE(−, −)], \]
\[ E^2_+ = \rho [x^2_+ USE(−, +) + (1 − x^2_+)PSE(−, +)]. \]

Thus $E^2(x^2_−, x^2_+) = \epsilon_+ E^2_− + (1 − \epsilon_+) E^2_+ .)$

Now let us recall that $\epsilon_+$ depends on the probabilities of players $P_1$ and $P_2$ to have unprotected sex. Consequently, we express $\epsilon_+$ as:

\[ \epsilon_+ = \epsilon_+(0) + [x^2_− \epsilon_+(0) + x^2_+ \epsilon_−(0)] \tau \]  \hspace{1cm} (2)

where $\epsilon_+(0)$ is the initial (before the game) fraction of HIV+ people in the population, and $\tau = 0.02$ is the known transmission probability of HIV [17].

Last but not least, we need to make sure that the number of possible sexual encounters that lead to transmission is the same whether counted from the $P_1$ or $P_2$ perspective. This leads us to the constraint:

\[ \epsilon_− \sum_{i \in HIV−} x^i_− = \epsilon_+ \sum_{i \in HIV+} x^i_+ \Leftrightarrow (1 − \epsilon_+) x^2_+ = \epsilon_+ x^1_− , \]

with $\epsilon_+$ as in [2].

Players $P_1$ and $P_2$ want to maximize their expected utilities $E^1(x^1, x^2)$ and $E^2(x^1, x^2)$ subject to $(x^1, x^2)$ in the set

\[ K := \{S_1 \times S_2 \mid (1 − \epsilon_+) x^2_+ = \epsilon_+ x^1_− \} \]
where
\[ S_i = \{ x^i = (x^-_i, x^+_i) \in [0, 1]^2, 0 \leq x^-_i + x^+_i \leq 1, i = 1, 2 \}. \]

Due to expression (2) we see that these utilities have actual dependencies on the other player’s choices, so our model is a 2-player game with nonlinear payoffs.

2.2. Base case scenarios, parameter values and uniqueness of Nash strategies. At this stage, a discussion of parameter values we define as our “base case scenario” throughout the rest of the manuscript is needed. We outlined above our starting assumptions on the utility ranking values for \( USE(+, +) \) and \( USE(-, -) \). Furthermore, we also assume \( USE(+, -) = 0 \) and \( USE(-, +) = 0 \) and \( PSE(s_1, s_2) = 0 \), for any status values \( s_1 \neq s_2 \in \{+, -\} \) and \( PSE(-, -) = USE(+, +) = 0.25 > 0 \). The latter values mean that a protected sex outcomes or no sex outcome is less preferred than \( USE \) by all players, but a \( PSE \) outcome with a partner of same status has a positive utility for all players.

The base values used in our simulations, unless otherwise noted, are listed in Table 1.

| \( P_1 \) | \( P_2 \) | Utility for \( USE \) | Utility for \( PSE \) | Range |
|---|---|---|---|---|
| HIV+ | HIV+ | \( USE(+, +) = 1 \) | \( PSE(+, +) = 0.25 \) | \([0, 1]\) |
| HIV+ | HIV- | \( USE(+, -) = 0 \) | \( PSE(+, -) = 0 \) | \([0, 1]\) |
| HIV- | HIV+ | \( USE(-, +) = 0 \) | \( PSE(-, +) = 0 \) | \([0, 1]\) |
| HIV- | HIV- | \( USE(-, -) = 1 \) | \( PSE(-, -) = 0.25 \) | \([0, 1]\) |

Table 1. This table outlines the base case preferences for different sexual acts given a players’ status.

In general, it is expected that a Nash game will have multiple equilibria. We took a numerical approach to investigate the type of Nash equilibria we get in the game above. We first set the rest of our game parameters as described in Table 2 below. We then vary the initial conditions of the game reformulated as in equation (1), using experiments with uniformly distributed points from \( K \). We ran 100 simulations, each starting with 50 uniformly distributed initial values \( (x^-_1, x^+_1, x^-_2, x^+_2) \in K \), and the only resulting Nash equilibrium strategies are the ones pictured in Figure 1. This specifically shows a (unique) Nash point \((0, 1, 1, 0)\) regardless of the initial conditions, i.e., players have \( USE \) with others from same status groups only. How-

| Term | Definition | Baseline value | Range |
|---|---|---|---|
| \( \tau \) | Probability of HIV spread from an \( HIV+ \) player to an \( HIV- \) player through \( USE \) | 0.02 | – |
| \( \epsilon_+(0) \) | Initial proportion of \( HIV+ \) individuals in the population. | 0.05 | 5% of population |
| \( \epsilon_-(0) \) | Initial proportion of \( HIV- \) individuals in the population. | 0.95 | 95% of population |

Table 2. Parameter definitions and parameter values for baseline scenario. Here \( \tau \) is a fixed probability of transmission per contact.
ever, this base case, though ideal in the sense of the type of behaviour suggested, is not realistic, since under Nash strategies \((0, 1, 1, 0)\) no one would have unprotected sex with a positive partner, thus HIV would die out in the population.

In the next section, we investigate the sensitivity of these results with respect to changes in utilities for casual sex with players of opposite status.

### 2.2 Analysis of the 2-player encounter game

In this section we are interested in investigating the effects of varying base utilities on the equilibrium strategies of both players to have unprotected sex in a casual encounter, namely on \(x_1^-\) and \(x_2^+\). This question was not investigated in our previous works [33, 35]. We then track the effect of these changes on the overall fraction of infected individuals, \(\epsilon_+\). We use Table 2 to describe the values of our parameters. We compute the general Nash equilibrium points as described in the previous Section 2.1. Figure 2 shows the impact that changing utilities has on \(x_1^*\), \(x_2^*\), as these strategies relate to HIV spread. We plotted 3-dimensional surfaces that observes these changes while varying both \(USE(-, +)\) and \(USE(+, -)\) over the range \([0, 1]\). We also plotted the change in \(\epsilon_+\) according to the same changes in utilities.

We see a change in the equilibrium solution for both \(x_1^*\) and \(x_2^*\) as we vary \(USE(+, -)\) values, and a similar impact on values of \(\epsilon_+\). In contrast, varying \(USE(-, +)\) values has no effect on the equilibrium values of players’ strategies. A new set of Nash strategies, obtained for instance for a value of \(USE(+, -) = 0.5\) is \((x_1^-, x_1^+, x_2^-, x_2^+) = (1, 0.9463, 0.0537)\) as seen in Figure 2, whereas the equilibrium strategies for instance for \(USE(+, -) = 0.08\) are \((x_1^-, x_1^+, x_2^-, x_2^+) = (0.4282, 0.5718, 0.9773, 0.0227)\).

### 3. Multiplayer game

We extend next the 2-player game presented in Section 2.2 to a multiplayer game, to capture interactions between players belonging to different age groups, as a possibly important factor in HIV transmission [7]. We consider here a population with 5 age cohorts, 15–20, 20–30, 30–40, 40–50 and 50+. Age group 1 (\(G_1\)), representing 15–20 year-old individuals, interacts with individuals from their age group plus with individuals of the 20–30 age cohort (\(G_2\)). Age group 2 (\(G_2\)), representing 20–30 year-old individuals, interacts with individuals from their own age cohort, plus with individuals in their adjacent age groups, i.e. \(G_1\) and 30–40 year-olds (\(G_3\)). This continues for the 3rd and 4th age groups, with
the 5th age group (G5) interacting with themselves and the 40–50 (G4) age cohort. Surveys have shown that HIV prevalence varies greatly by age group, with youth accounting for a substantial amount of infections [6]. In general, interactions can be modeled in a variety of ways, subject to differing assumptions. We guided our groups interactions here following our previous model [35].

A game is defined by choosing one HIV+ player (always taken to be Player 1) in one of the groups at a time; the other players in a game will have an HIV− status and will belong to the age cohorts allowed to interact with the age cohort $P_1$ belongs to. Following the age-group interactions allowed above, if $P_1$ belongs to $G_1$ or $G_5$, then the HIV− players will be from the same or adjacent groups; thus we model these interactions as a 3-player game. Whenever $P_1$ is chosen in one of the $G_2$, $G_3$, or $G_4$, then we model their interactions as a 4-player game.

Let us denote by game$_i, i \in \{1, ..., 5\}$ one of the games describes above, such that $P_1$ in game$_i$ belongs to $G_i$.

We assume first that HIV+ individuals are spread among the five groups; thus each age group has a subgroup of HIV+ individuals of size $\epsilon^G_+(0), i \in \{1, ..., 5\}$ so that $\sum_{i=1}^5 \epsilon^G_+(0) = \epsilon_+(0) = 5\%$ as in Section 2. We start first by assuming an even spread of HIV+ individuals among age groups: $\epsilon^G_+(0) = 0.01$.

Finally, each age group has a differing activity parameter $\rho_i, i \in \{1, ..., 5\}$ given by: $\rho_1 = 0.5, \rho_2 = 1, \rho_3 = 0.8, \rho_4 = 0.6, \rho_5 = 0.3$, representing a multiplicative factor controlling overall sexual activity level relative to the second age group, by

\[ \text{Figure 2. The 2-player game showing } x^{1*}, x^{2*} \text{ and } \epsilon_+ \text{ varying } USE(−,+) \text{ and } USE(−,+). \]
age. Each $P_i$ has an expected utility $E^i$ of unprotected sex in a casual encounter, assuming they are aware of their partner status:

$$E^i := \epsilon_+ E^i_+(x) + (1 - \epsilon_+) E^i_-(x), \forall i \in \{1, 2, 3\}.$$ 

### 3.1. 3-player game

We start by setting up the game concerning age cohort 15–20, where we choose $P_1$ to be HIV+ in $G_1$. Then $P_2$ and $P_3$ are HIV– players from age groups $G_1, G_2$ respectively. The vector of strategies of player $P_i$ is $x^i = (x^i_1, x^i_2, x^i_3)$, where $x^i_j$ we denote the probability of $P_i$ to have unprotected sex with an HIV– individual in $G_j$, and by $x^i_j$ the probability of unprotected sex with an HIV+ individual in $G_j$.

We define their expected utilities as:

$$E^i_+ = \rho_i \left[ (x^-_1 + x^-_2) \text{USE}(+, -) + (1 - (x^-_1 + x^-_2)) \text{PSE}(+,-) \right]$$

$$E^i_- = \rho_i \left[ (x^+_1 + x^+_2) \text{USE}(+, +) + (1 - x^+_1) \text{PSE}(+, +) \right]$$

Then $P_1$’s strategies have to satisfy: $0 \leq x^-_1, x^+_1, x^-_2 \leq 1$ and that $x^+_1 + x^-_1 + x^+_2 = 1$.

Similarly for $P_2$ an HIV– individual $\in G_1$, and $P_3$ an HIV– individual $\in G_2$, we get (for $j = \{2, 3\}$):

$$E^j_+ = \rho_{j-1} \left[ (x^j_1 + x^j_2) \text{USE}(-, +) + (1 - (x^j_1 + x^j_2)) \text{PSE}(-, +) \right]$$

$$E^j_- = \rho_{j-1} \left[ (x^j_1) \text{USE}(-, -) + (1 - x^j_1) \text{PSE}(-, +) \right]$$

Each player strategies have to satisfy ($j \in \{2, 3\}$): $0 \leq x^-_1, x^j_1, x^-_2 \leq 1$ and $x^j_1 + x^-_1 + x^j_2 = 1$.

As a consequence of the interaction between players, the fractions of HIV+ individuals in $G_1$ and $G_2$ change now as follows (note that $\epsilon^{G_j}_{+}(\text{game}_1) = \epsilon^{G_j}_{+}(0), j \in \{3, 4, 5\}$):

$$\epsilon^{G_1}_{+}(\text{game}_1) = \epsilon^{G_1}_{+}(0) + \tau \left[ x^1_- \epsilon^{G_1}_{+}(0) + x^2_+ \epsilon^{G_1}_{-}(0) \right]$$

$$\epsilon^{G_2}_{+}(\text{game}_1) = \epsilon^{G_2}_{+}(0) + [x^2_- \epsilon^{G_1}_{+}(0) + x^3_+ \epsilon^{G_2}_{-}(0)] \tau$$

Then we compute

$$\epsilon_{+}(\text{game}_1) := \sum_{i=1}^{5} \epsilon^{G_i}_{+}(\text{game}_1). \quad (3)$$

Last but not least, we need to impose the shared constraint that the number of possible sexual encounters that lead to transmission is the same when counted from each of the + and − players’ perspective. This leads us to the constraint:

$$(1 - \epsilon_{+}(\text{game}_1))(x^2_+ + x^3_+) = \epsilon_{+}(\text{game}_1)(x^1_- + x^2_-),$$

with $\epsilon_{+}(\text{game}_1)$ as in (3).

We investigate uniqueness of solutions for the generalized Nash equilibrium strategies computed as in Section 2.2. Figure 3 shows a non-unique Nash point while varying initial conditions of system (1). We ran 100 simulations each starting with 50 uniformly distributed initial values and the compiled results are always as in Figure 3. As expected, the equilibrium strategies are not unique, however, the equilibrium strategies for players engaging in USE with players of opposite status...
Figure 3. Heat map for 3-player game showing $(x_1^{1*}, x_2^{2*}, x_3^{3*})$ equilibrium values for a uniform spread of initial conditions.

(namely $x_1^{1*}, x_1^{2*}$ for $P_1$ and respectively $x_2^{2*}$ for $P_2$ and $x_3^{3*}$ for $P_3$) are always unique, and all equal to 0.

3.2. Results & discussion: 3-player game. Similar to studying the 2-player game, we study the effects of varying USE(−, +) and USE(+, −) on the choices of players and on the fraction $\epsilon_+$ of the population, with baseline values of Table 2.

Figure 4. Results for 3-player game. The three upper panels show the $(x_1^{1*}, x_1^{2*}, x_1^{3*})$ choices of $P_1$, whereas the lower left panels show the $x_2^{2*}$ choice of $P_2$, $x_3^{3*}$ for $P_3$ dependent on USE(−, +) and USE(+, −). Lower right panel shows the $\epsilon_+(game_1)$ variation.

We know from our previous section that equilibrium strategies found for this game are not unique, thus initial conditions for the computation of equilibrium strategies under varying parameters are important. In order to derive our analysis, we use as initial conditions one of the equilibrium points computed in the subsection above:

$$x^* = \left((0, 1, 0), (0.6369, 0, 0.3631), (0.62, 0, 0.38)\right).$$

(4)

relying on the fact that all equilibrium strategies for players engaging in USE with players of opposite status in the baseline scenario are 0. The new equilibrium strategies pictured above are:

$$x^* = \left((0.5, 0, 0.5), (0.62, 0.02, 0.36), (0.62, 0, 0.38)\right).$$
Figure 4 shows more refined results than those in the 2-player scenario. Choices for \( P_3 \) interacting with the HIV+ player are not shown, as they are not changed from the initial values. Varying \( \text{USE}(+, -) \) affects the strategies of \( P_1 \) interacting with both \( P_2 \) and \( P_3 \) (upper left and upper right panels of Figure 4), while varying \( \text{USE}(-, +) \) affects the strategies of \( P_2 \) interacting with \( P_1 \) (upper middle panel of Figure 4). We see a switch in strategy of \( P_2 \) as \( \text{USE}(-, +) \) increases from 0 to 1. With increase in both utilities of USE with players of opposite status, \( P_1 \)'s choices move to unprotected sex with both \( P_2 \), while \( P_2 \), engages with \( P_1 \), though not exclusively (they maintain nonzero strategies of unprotected sex with players of HIV− status as well). The impact on the infection levels from such a game alone is shown to have an increasing direction. The infected fraction in the population (lower left panel of Figure 4) increases from the baseline value of 0.05% to \( \epsilon_+(\text{game1}) = 0.05028\% \), most affected by values of \( \text{USE}(-, +) \) beyond a threshold of \( \approx 0.5 \).

3.2. 4-player game. We describe next the 4-player game arising from choosing for instance \( P_1 \) as an HIV+ player in \( G_2 \) (the game will be identical for a choice of \( P_1 \) in either \( G_3 \) or \( G_4 \)).

We denote \( P_1 \) as HIV+ from \( G_2 \) and \( P_2 \), \( P_3 \) and \( P_4 \) as HIV− players from \( G_1 \), \( G_2 \) and \( G_3 \) respectively. The vectors of strategies are as follows:

\[
P_1 : \begin{cases} \quad 0 \leq (x_1^1, x_2^2, x_3^3, x_4^4) \leq 1 \text{ s.t. } x_1^1 + x_2^2 + x_3^3 + x_4^4 = 1 \\
\end{cases}
\]

\[
P_2 : \begin{cases} \quad 0 \leq (x_1^1, x_2^2, x_3^3) \leq 1 \text{ s.t. } x_1^1 + x_2^2 + x_3^3 = 1 \\
\end{cases}
\]

\[
P_3 : \begin{cases} \quad 0 \leq (x_1^1, x_2^2) \leq 1 \text{ s.t. } x_1^1 + x_2^2 = 1 \\
\end{cases}
\]

\[
P_4 : \begin{cases} \quad 0 \leq (x_1^1) \leq 1 \text{ s.t. } x_1^1 = 1 \\
\end{cases}
\]

The expected utilities for these individuals are listed below, starting with \( P_1 \) representing HIV+ individuals from \( G_2 \):

\[E_1^1 = \rho_2 (x_1^1 + x_2^2 + x_3^3) \text{USE}(+, -) + (1 - (x_1^1 + x_2^2 + x_3^3)) \text{PSE}(+, -)]\]

\[E_1^1 = \rho_2 [x_2^2 \text{USE}(+, +) + (1 - x_2^2) \text{PSE}(+, +)]\]

Similarly for \( P_2 \) who is an HIV− individual in \( G_1 \) we have:

\[E_2^2 = \rho_1 [(x_1^1 + x_2^2) \text{USE}(-, -) + (1 - (x_1^1 + x_2^2)) \text{PSE}(-, -)]\]

\[E_2^2 = \rho_1 [x_2^2 \text{USE}(-, +) + (1 - x_2^2) \text{PSE}(-, +)]\]

For \( P_3 \) an HIV− individual in \( G_2 \):

\[E_3^3 = \rho_2 [(x_1^1 + x_2^2 + x_3^3) \text{USE}(-, -) + (1 - (x_1^1 + x_2^2 + x_3^3)) \text{PSE}(-, -)]\]

\[E_3^3 = \rho_2 [x_2^2 \text{USE}(-, +) + (1 - x_2^2) \text{PSE}(-, +)]\]

Finally, for \( P_4 \), an HIV− individual in \( G_3 \) we have:

\[E_4^4 = \rho_3 [(x_1^1 + x_2^2 + x_3^3) \text{USE}(-, -) + (1 - (x_1^1 + x_2^2 + x_3^3)) \text{PSE}(-, -)]\]

\[E_4^4 = \rho_3 [x_2^2 \text{USE}(-, +) + (1 - x_2^2) \text{PSE}(-, +)]\]

As a result of interactions allowed in this game, the fraction of the infected individuals changes in groups 1, 2, 3 (note that \( \epsilon_+^G_{j}(\text{game}_2) = \epsilon_+^G_{j}(0), j \in \{4, 5\} \):

\[\epsilon_+^G_{j}(\text{game}_2) = \epsilon_+^G_{j}(0) + \tau [x_1^1 \epsilon_+^{G_2}(0) + x_2^2 \tau \epsilon_+^{G_3}(0)], j \in \{1, 2, 3\}\]
Then we compute
\[ \epsilon_+(\text{game}_2) := \sum_{i=1}^{5} \epsilon_i^G. \] (5)

Last but not least, we need to impose the shared constraint that the number of possible sexual encounters that lead to transmission is the same when counted from each of the + and − players’ perspective. This leads us to the constraint:
\[ (1 - \epsilon_+(\text{game}_2))(x_{2+}^2 + x_{3+}^3 + x_{4+}^4) = \epsilon_+(\text{game}_2)(x_{1-}^1 + x_{2-}^2 + x_{3-}^3), \]
with \( \epsilon_+(\text{game}_2) \) as in (5).

We again investigate uniqueness of solutions as we did in Section 2.2. Figure 5 shows non-unique Nash points while varying initial conditions of (1). We ran 100 simulations each starting with 50 uniformly distributed initial values and the compiled results are always as in Figure 5. As expected, the equilibrium strategies are not unique, however, the equilibrium strategies for players engaging in US with players of opposite status (namely \( x_{1+}^1, x_{1+}^2, x_{1+}^3 \) for \( P_1 \) and respectively \( x_{2+}^2, x_{3+}^3, x_{4+}^4 \) for \( P_2 \), \( x_{2+}^3 \) for \( P_3 \) and \( x_{2+}^4 \) for \( P_4 \)) are always unique, and all equal to 0.

**Figure 5.** Heat map for 4-player game showing \( (x^1, x^2, x^3, x^4) \) equilibrium values for the respective initial conditions.

### 3.4. Results & discussion: 4-player game.

Similar to studying the 2-player and 3-player games, we vary USE(+-) and USE(-+) and study the Nash choices of players, as well as the evolution of the \( \epsilon_+(\text{game}_2) \) fraction. From our previous section, equilibrium strategies found for this game are not unique, thus initial conditions for the computation of equilibrium strategies under varying parameters are important. In order to derive our analysis, we use as initial conditions one of the equilibrium points computed in the subsection above
\[ x^* = \left( (0, 0, 0, 1), (0.2799, 0.7201, 0), (0.3174, 0.3651, 0.3175, 0), (0.5339, 0.4661, 0) \right), \] (6)
relying on the fact that all equilibrium strategies for players engaging in USE with players of opposite status in the baseline scenario are 0.

Figure 6 shows results similar to the 3-player scenario, where HIV− individuals engage with HIV+ individuals as USE(−+) increases. Choices for \( P_1 \) become \( x_{3+}^1 = 0, x_{3-}^1 = x_{3-}^2 \approx 0.334 \), thus \( P_1 \) engages in USE with opposite status HIV players only, beyond a threshold of USE(−+) \( \approx 0.1 \) (note that \( P_1 \) here is in
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$G_2$, which is the most active group). Choices of $P_2$ vary slightly into USE with $P_1$, with $x_{2+}^* ≈ 0.032$, while maintaining positive strategies with $P_3, P_4$. Choices of $P_3$ and $P_4$ for USE with $P_1$ do not change from 0 (not shown).

The impact on the infection levels from such a game alone is shown to have an increasing direction. The infected fraction in the population (lower left panel of Figure 6) increases from the baseline value of 0.05% to $\epsilon + (game 2) = 0.5082\%$, most affected by positive values of $USE(-, +)$.

3.3. Compounded effect of multiplayer games on HIV transmission. Recall that by $game_i, i \in \{1, ..., 5\}$ we denoted the game where $P_1$ who is $HIV+$ belongs to $G_i$. We let $p_i, i \in \{1, ..., 5\}$ be the size of the age group $G_i$ in the population (which was taken to be 20% for all groups in previous sections) so that $\epsilon^0_i(0) := p_i \epsilon^+_i(0) = p_i \cdot 0.01, i \in \{1, ..., 5\}$. We denote by $\gamma_i$ the probability of $game_i$ taking place. Given that each game leads to possible unprotected sex among $HIV+$ and $HIV-$ players, we estimate the infected fraction of individuals in each group, after interactions occur, to be described as follows:

$\epsilon^+_i = \sum_{k=1}^{5} \gamma_k \epsilon^+_{i}(game_k)$ where for each $i, k \in \{1, ..., 5\}$ we have that $\epsilon^+_{i}(game_k)$ are defined as in Section 3. The overall fraction of infected individuals in the population is obtained by adding the fractions above, after compiling the results for

![Figure 6. 3-dimensional results for 4-player game2 showing choices for varying USE(+, −) and USE(−, +) utilities. The upper panels show the change in equilibrium strategies of $P_1$: the upper left panel shows the strategies $x_{1+}^* = 0, x_{1-}^* = x_{2+}^* = x_{3+}^* \approx 0.334$. The likelihoods of $P_2$ to engage in USE with $P_1$ are shown in lower left panel, while $x_{3+}^* = x_{4+}^* = 0$ are not shown. The effect on the infected fraction due to this game is shown in lower right panel.](image-url)
each game: $\epsilon_+ = \left( \sum_{i=1}^{5} \epsilon^G_i \right)$. For two quick examples, we plot $\epsilon_+$ in Figure 7 using $p = (0.086, 0.173, 0.159, 0.170, 0.412)$ the estimated size of the age groups according to U.S. census [40, 38], and $p = (0.21, 0.313, 0.18, 0.12, 0.18)$ using Zimbabwe census data [41], both assuming $\gamma_k = 1, \forall k$.

As we have stated in previous sections, in both analyses of game 1 and game 2, we are dependent on initial meaningful states in order to evolve them while varying $USE(\cdot, \cdot, +)$ and $USE(\cdot, \cdot, -)$. For consistency of all simulations presented here, we use as initial data for the compounded games the points $x^*$ in (4) for the 3player games, and the point $x^*$ in (6) for the 4player games. Figure 7 presents our results, where clearly we see that the level of infection will be higher in the Zimbabwe population, as it contains more numerous younger age groups.

![Figure 7](image)

**Figure 7.** Compounded $\epsilon_+$ using U.S. census data (left) over 5 games vs. the same using Zimbabwe data (right)

### 4. Prophylactic vaccine in youngest group.

In this section we investigate implementing a theoretical prophylactic vaccine within the population, with efficacy $\mu$. Consequently, we adjust some of the baseline utilities as follows: a) the utility for an unvaccinated $HIV^-$ individual interacting with an $HIV^+$ individual, i.e., $USE(-, +)$ is increased from 0 to 0.25 due to the assumption that unvaccinated individuals aware of a vaccination program would place higher utility for $USE$ given an increased level of protection through treatment optimism [12];

b) we define $USE(-, +, vacc) = 0.5$ as the utility for $HIV^-$ vaccinated individuals engaging in $USE$ with $HIV^+$; c) we define $(1 - \mu)\tau$ to be the relative reduction in transmission. This will affect the variation of $\epsilon_+(game1)$ and $\epsilon_+(game2)$, as we assume $P_2$ in game1 and $P_2$ in game2 are now $HIV^-$ and vaccinated.

The population sizes are adjusted to fit $p = (0.086, 0.173, 0.159, 0.170, 0.412)$ with $p_i, i \in \{1, \ldots, 5\}$ the estimated size of the age group according to U.S. census [40, 38] and $p = (0.21, 0.313, 0.18, 0.12, 0.18)$ using Zimbabwe census data [41].

We investigate what impact these changes have on HIV transmission if $HIV^-$ players in youngest age class are vaccinated. We run the compounded games of Section 3 for both U.S. and Zimbabwe populations assuming that $\gamma_k = 1$ for all $k \in \{1, \ldots, 5\}$. We now have:

- game 1: $P_2 \in G_1$ is $HIV^-$ vaccinated, $P_3 \in G_2$ is $HIV^-$ unvaccinated

---

1This is an assumption only; treatment optimism can in fact increase HIV transmission, as we show in [35].
game 2: $P_2 \in G_1$ is HIV$^-$ vaccinated, $P_3, P_4$ are HIV$^-$ unvaccinated. Games 3, 4, 5 do not involve players in $G_1$, by the rules of our interactions, hence there is no vaccinated HIV$^-$ player in these games.

Results presented are dependent on varying the vaccine efficacy $USE(+, -) \in [0, 1]$ and $USE(-, +) \in [0.25, 1]$. Figure 8 illustrates the compounded results for the U.S. and Zimbabwe data. We see that the fraction $\epsilon_+$ show similar behaviour for both populations, with Zimbabwe at higher overall prevalence. It is also notable that if values of $USE(-, +) > 0.4$, the infection level reaches the same value as in the unvaccinated population. Thus, too much confidence in the efficacy of a vaccine leads HIV$^-$ unvaccinated players to raise their preference for $USE$ with HIV$^+$ players, thus leading in fact to a nondecrease in HIV infection.

Figure 8. Compounded $\epsilon_+$ using U.S. (left panel) and Zimbabwe (right panel) census data comparing $USE(+, -) \in [0, 1]$ and $USE(-, +) \in [0.25, 1]$ values, while with $U(USE, -, +, vacc) = 0.5$ and $\mu = 0.75$.

The increase in $\epsilon_+$ remains less than 1% as a result of initial HIV prevalence, transmission probability, as well as implementing a one-off game. In cases where the games are repeated, then every repetition brings a small possible increase in the value of $\epsilon_+$, which over time leads to potentially more significant increases.

5. Conclusions. The previous sections outlined a one-off casual sexual encounters game for 2, 3 and 4 player variations dependent on status and age. Our most interesting conclusions are that preferences of HIV$^-$ players towards unprotected sex have the largest impact on HIV transmission in the population. In games with age group interactions, and assumed treatment introduced for the youngest group, populations with higher proportion of youth see a decrease in HIV prevalence through a possible implementation of a theoretical prophylactic vaccine in these age groups similar to the HPV case.

We showed that generalized Nash equilibria exist and can be computed for these types of games. Moreover, we demonstrated the sensitivity of GN equilibria with respect to varying players’ utilities of unprotected sex with partners of opposite HIV status, giving an appreciation for how changes in individual decisions may contribute to an increase in HIV transmission.

As we expanded from 2-player to 3- and 4-player games, we in fact refined the interactions among the individuals in a population, previously regarded as one-on-one positive–negative outcomes. Given that the sizes of groups and activity parameter
values matter, we saw that the increase in transmission due to a single instance of a 3-player or a 4-player game is smaller than in a 2-player game. However, the biggest contributing factor in changing HIV transmission is found to be due to the $HIV^- \text{ players'}$ ranking of $USE$ with an $HIV^+$ player. Compounding over the age groups and adjusting for HIV prevalence in the population, we showed that a higher increase in transmission arises in populations with a larger youth composition.

We need to stress again here that the value of this modelling framework depends on the initial conditions of a population under observation, especially in the case of multiplayer groups. Moreover, knowing specific equilibrium strategies within the population of players can lead to sensitivity analyses having these specific equilibrium strategies as initial values. Last but not least, the fact that mathematically a generalized Nash game has in fact sets of equilibria gives strength to this modelling paradigm, in the sense that differing initial conditions in a players’ population can give rise to different outcomes, which seems more appropriate to model real life encounter outcomes.

As future work, it would be interesting to include incorporation of ART for $HIV^+$ individuals, which could show how adjusting their utilities may affect transmission. It would also be interesting to conduct a repeated game in order to better see how transmission evolves and how this evolution affects the overall risk assessment and spread of HIV.

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