Information Missing Puzzle, Where Is Hawking’s Error?

Ding-fang Zeng

Institute of Theoretical Physics, Beijing University of Technology, China, Bejing 100124

Introduction

Steven Hawking walked through his extraordinary life two months ago, leaving us great scientific heritages among which the discovering that black holes can radiate in such a way that information may miss \( \mathcal{E} \) attracts the world’s attention most deeply. Since information missing breaks the basic unitarity principle of quantum mechanics, many physicists consider it a huge conflict between general relativity and quantum mechanics and a strong signal to necessities of the more fundamental theory of quantum gravitations. For this reason, physicists including Hawking himself pay much effort to see if this is indeed the case or not, see refs.\(^1\)\(^,\)\(^2\) for more related works. Basing on general ideas of gauge/gravity duality, most researchers in this area agree that the basic principle of quantum mechanics can not be violated and Hawking must do something wrong somewhere in the calculation or reasonings. However, consensus on where and what he does erroneously that lead to this puzzle is never reached uniformly in the area \( \mathcal{E} \). The purpose of this work is to provide a new but very potential answer to this question which has pictures very similar to the string theory fuzzballs \( \mathcal{E} \) but use ideas only of standard general relativity and canonical quantum mechanics.

From the viewpoint of full quantum gravitation theories, FIG.\(^1\) our answer lies on a level similar to that of quantum mechanics in the working flow of quantum electrodynamics and Hydrogen atom physics. This is a valuable working level rarely explored by previous researchers. Bekenstein and Mukhanov\(^3\)\(^,\)\(^4\) once tried to provide explanations for the black hole entropy and quantisation at the same level but failed. In their tryings, the microscopic degree of freedom of black holes is carried by the horizon area elements directly and the hole has discrete mass levels, which leads to line-shape radiation spectrum, thus contradicting with Hawking’s radiation spectrum explicitly. While in our works, the black hole microscopic degrees of freedom is encoded in the radial eigen motion modes of its matter-energy contents and no discrete mass spectrum is required of the hole itself. Due to the same reason, to get area law entropy in our picture is a highly non-trivial work. We provide persuasive evidences following from concrete calculations in the main text that this is possible and reasonable. Our conclusion is that, using standard general relativity and canonical quantum mechanics, we can provide perfectly consistent answers to most of the mysterious events occurring on both the central point and horizon surface of black holes and the answer is dis/verifiable in the high precision gravitational wave observations.

\[
\begin{align*}
\text{quantum electrodynamics} & \quad \downarrow \\
\text{quantum mechanics} & \quad \downarrow \\
\text{spontaneous radiation of Hydrogen atoms} & \quad \downarrow \text{what we do} \\
\text{Hawking radiation of Black Holes} & \\
\end{align*}
\]

\(\text{FIG. 1: What we do in this work between quantum gravitation and Hawking radiations can be looked as a counterpart of what quantum mechanics do between quantum electrodynamics and hydrogen atom spectrums.}\)

Where is the error? — Simple but straight forward answer

The key idea behind Hawking’s calculation and reasoning is rather simple \( \mathcal{E} \). Surrounding a black hole, both the freely falling observer and fixed position ones have their own mode expansion for quantum fields,

\[
\begin{align*}
\phi &= \sum_\omega \left( a_{\omega}^{\text{fr}} e^{-i \omega u} + a_{\omega}^{\text{fr}} e^{i \omega u} \right), \quad u \sim t - r \\
\phi &= \sum_\omega \left( a_{\omega}^{\text{fx}} e^{-i \omega \bar{u}} + a_{\omega}^{\text{fx}} e^{i \omega \bar{u}} \right), \quad \bar{u} = \bar{u}(u) \quad (1) \\
\end{align*}
\]

\[
\begin{align*}
a_{\omega}^{\text{fx}} &= \sum_k \left( \alpha_{\omega k} a_{k}^{\text{fr}} + \beta_{\omega k}^{*} a_{k}^{\text{fr}} \right), \quad c.c. \quad (2)
\end{align*}
\]

However, after some pure technique derivations, Hawking finds that the vacuum state identified by freely falling observers are not vacuums of the fixed position ones,

\[
\langle \text{vac} | a_{\omega}^{\text{fr}} a_{\omega}^{\text{fr}} | \text{vac} \rangle = 0 \neq \langle \text{vac} | a_{\omega}^{\text{fr}} a_{\omega}^{\text{fr}} | \text{vac} \rangle = \frac{1}{e^{\frac{2\pi}{\hbar}} \pm 1} \quad (3)
\]
That is, the latters see particles radiating from the black hole horizon and the thermal feature of these particles implies that pure states describing the pre-blackhole contents evolve into mixed ones of the radiation products so that the unitarity principle of quantum mechanics is violated during this process.

Directive as it is, Hawking’s train of thought contains a key bug. It focuses on particles being measured outside the horizon but neglects the fact that blackhole has inner structure and micro-state, which is just the goal he tries to show against but provides evidences for in his original work on this question [3]. Just as we pointed out in ref. [2], thermal features of the radiation products can be derived out from averages on the initial- and summations on the final state of blackholes under consideration. Indeed, as long as we look each mass contained in black hole and vacuum state, we will see that the probability it spontaneously radiates a particle of mass \( m \) equals to

\[
P = \frac{e^{4\pi G(m-h\omega)^2} \cdot e^{4\pi Gm^2}}{(e^{4\pi Gm^2} - 1) \cdot (e^{4\pi Gm^2} - 2) \cdots + 1 + 1}
\]

Because both the minimal black hole and vacuum state are once degenerate, we have two +1 terms in the denominator. While due to averages on the possible initial states, all final state becomes equally hard/easy to be radiated into. Considering randomness of the quantum radiation [10], the average energy emitted in one such radiation event reads

\[
\langle E \rangle = \frac{h\omega e^{-h\omega / kT} + 0}{e^{-h\omega / kT} + 1} = \frac{h\omega}{e^{h\omega / kT} + 1}, \text{ for fermions (5)}
\]

\[
\langle E \rangle = \frac{\sum n h\omega e^{-n h\omega / kT}}{\sum n e^{-n h\omega / kT}} = \frac{h\omega}{e^{h\omega / kT} - 1}, \text{ for bosons (6)}
\]

This reproduces the particle spectrum of Hawking’s radiation exactly but leaves no places for information to miss into.

The Schwarzschild singularity’s resolving The reason that S. Hawking neglected inner structures of the black hole has relevance with his another heritage, proofs of a serial of singularity theorems collaborating with R. Penrose [11, 12]. According to these theorem, all matters falling across the horizon of a black hole, including those consisting the hole itself must arrive onto the central point in finite proper time and form singularities there. Correct as it is, in both Hawking himself and the latter followers, this is over interpreted. As can easily imagine and show physically, the more naturally happening when two particles colliding together or a spherical shell collapses is not a static singular point’s formation, but such an point’s instantaneous formation and the closely following up crumbling. That is, the matter contents’ going across each other and moving to the other side. To understand this fact more better, a good starting point is to consider physic pictures of the S-wave positronium, or hydrogen atoms similarly. Classically, such an object consists of a point like electron and its anti-partner moving in one same line and towards each other. It is the uncertainty principle that assigns transverse momentum to the electron and positron so that we get impressions that the S-wave state corresponds to classic circled orbits, while the P-, D- and other wave states correspond to classic ellipse orbits. Properly saying, S-wave corresponds to classic line-type orbits, while other waves correspond to classic ellipse orbits. Obviously, existences of the S-wave positroniums imply that when a classic electron and positron encounter colinearly, what we get is not a stable double massed neutral particle, but such a particle’s instantaneous formation and closely following up splitting into electron and positrons again, with the two exchange positions only.

Now let us consider a freely collapsing zero pressure spherical shell under self gravitations. Spacetimes outside such a shell is Schwarzschildian, while the inside, Minkowskian [13]. Sizes of the shell change the same way as a freely falling test particle in a Shwarzschild background with equal masses. For observers fixed outside far enough and on the central point respectively,

\[
\{ h\dot{t} = \gamma = \dot{x}^0 + \Gamma^0_\mu \dot{x}^\mu \dot{x}^\nu = 0, \{ \dot{t}^2 - \dot{r}^2 = 0, \gamma^2 = 1 \}
\]

\[
\{ h\dot{r}^2 - h^{-1} \dot{t}^2 = 1, \{ \dot{t}^2 - \dot{r}^2 = 1 \}
\]

or

\[
\{ \dot{r}^2 = \gamma^2 - h, \dot{t}^2 = \gamma^2 - h^{-1} = 1, \}
\]

where \( \{t, r\}, \{\tau, \gamma\} \) and dot over them are time, spatial radial coordinates used by the two observers describing the shell and derivatives with respect to proper times of the shell respectively; \( \gamma^2 \) is an integration constant equals to the value of \( h \) on \( r = r_{\text{rel}} \) where the shell is released statically. If the release occurs outside the horizon, \( \gamma \) will be real and less than 1. While if the release occurs inside the horizon, \( \gamma \) will be pure imaginary and \( \gamma^2 < 0 \). Talking about release and motion inside the horizon is meaningful because we can have observers on the central point around which the spacetime is simply Minkowskian before the shell arrives onto, see e.g. [13]. While as the shell arrives onto, its radial speed becomes that of light \( \left( \frac{d\tau}{dt} \right)^2 \left( \frac{t}{\tau} \right) \rightarrow 0 \), so it cannot be stopped there but have to go across that point and oscillate thereafter. This resolves the Schwarzschild singularity classically but avoid contradictions with Hawking and Penrose’s singularity naturally. Singularities indeed form in finite proper times after the collapsing begins. However, it crumbles at the same time it forms, see the following diagram representation.
Quantum mechanically, the resolving of Schwarzschild singularity is also straightforward [2]. We only need to multiply the shell mass on both sides of eqs. (3)’s first line thus translating them into Hamiltonian constraints and quantise canonically, that is, replacing $m \dot{v} \rightarrow i \hbar \frac{\partial}{\partial r}$ and introducing a wave function $\Psi(r)$ to denote the probability amplitude the shell be measured at radius $r$ and let it be function object of the operatorised constraint, we will get

$$[-\hbar^2 \frac{\partial^2}{\partial r^2} - m^2 (\gamma^2 - h)] \Psi = 0, \quad \gamma^2 \equiv h[r_{rel}] \in (-\infty, 0)$$

It can be verified that, the finite at origin, square integrable solution to this equation exists only when

$$\sqrt{-\gamma^2 + 1} = 1, 2, \cdots, n$$

This is very similar to the simple hydrogen atoms [10] and

$$\Psi = \Psi_\beta(\hat{r}) = e^{-\beta^2 \hat{r} L_{q-1}^1(2\beta \hat{r})}, \quad \hat{r} \equiv r \hbar^{-1}$$

$$1 \leq \beta^2 = -\gamma^2 + 1, \quad \frac{2Gmm}{2\beta \hbar} = q = 1, 2, \cdots, q_{\text{max}}$$

where $L_{q-1}^1(x)$ is the first order associated Legendre polynomial. However, two big differences occur here. The first is, because the shell is constrained to be spherical and we are focusing on its radius’ evolution, the question is essentially one-dimensional. As results, a factor of $r^{-1}$ is absent relative to hydrogen atoms from the probability amplitude and the wave function is definitely zero at the origin. This is nothing but the Schwarzschild singularity’s resolving at quantum levels. Classically, this resolving corresponds to the fact that the shell is going across the central point at the speed of light, which is the highest value during whole oscillation process. So the probability the shell be measured on that point takes minimal values relative to those on other places. The second difference is that, since the shell is released statically from inside the horizon thus $\gamma^2 = h(r_{rel}) < 0$, the value of $\beta$ is always larger than 1. This leads to that given masses, the number of allowed quantum states of the shell if finite, symbolically

$$q_{max} = \text{Floor}[Gmm \hbar^{-1}]$$

Even if $0 < \gamma^2 = h(r_{rel})$ thus $\beta < 1$, so that the shell is released from outside the horizon, as long as the release point is finitely further away from the horizon, $q_{max}$ will be finite.

**The origin of Bekenstein-Hawking entropy** The spherical shell is the extremal case of spherical symmetric objects with continuous mass distributions. For more general such objects, we can decompose them into many sub shells with micro states of the whole system denoted by the direct product of all sub-spheres. Superficially looking, this shell decomposition is arbitrary and its number should be non-countable. However, since the micro state of all sub-spheres are quantised similarly as eqs. (9), (10) and (11), the number of distinguishable micro states of the system is countable and finite. Denoting the mass of all sub-spheres in a shell decomposition scheme as $\{m_1, m_2, \cdots, m_k\}$, and the mass of contents inside each shell as $\{M_1, M_2, \cdots, M_k\}$, with $M_1 = m_1, M_2 = m_1 + m_2, \cdots, M_k = \sum^k m_i$, then the micro state of the system and their number will be respectively

$$\Psi = \Psi_\{\beta\} = \Psi_{\beta_1} \otimes \Psi_{\beta_2} \cdots \otimes \Psi_{\beta_k}$$

$$[-\hbar^2 \frac{\partial^2}{\partial r^2} - m^2 (\gamma^2 - h_i)] \Psi_{\beta_i} = 0, \quad \gamma^2_i = 1 - \frac{2GM_i}{r_{rel}} < 0$$

$$\Psi_{\beta_i} = e^{-\beta_i r} \beta_i L_{q_i}^1(2\beta_i r), \quad \hat{r} = r \hbar^{-1}$$

$$1 \leq \beta_i = \sqrt{-\gamma^2_i + 1}, \quad \frac{2Gm_i}{2\beta_i \hbar} = q_i = 1, 2, \cdots, q_{max}^i$$

$$W = \sum_{\{m\}} W_{\{m\}}, \quad W_{\{m\}} = q_{max}^1 \cdot q_{max}^2 \cdots q_{max}^k$$

For small of these objects with horizons, the micro state can be listed out by brute force. We do so in ref.[2] and find that their number falls on the curve $W(M) = e^{k_s A(M)/4G}$ very precisely. This happens to be the area law of Bekenstein-Hawking entropy formulas except for numerical factor $k \approx (8\pi)^{-1}$.

Obviously, if this quantum state counting indeed catches the essence of black hole microscopic degrees of freedom, we will expect that it is dimensionally universal. Just as we pointed out in ref.[2], simply generalising this calculation to 5 dimension encounters the very troublesome eigenvalue problem [14] featured by Calogero potentials. However, if we focus on $3+\epsilon+1$ dimensions and looking 5 as the limit of $\epsilon \rightarrow 1$, then we will get proper
TABLE I: The shell decomposition and micro state quantum number of a freely collapsing dust ball with horizons in 4+1 dimensional space-time. The third column decomposition is non-allowable because its most inner shell of mass $2 - \sqrt{2}$ does not satisfy quantising conditions of eq.(16). The last column should not be counted as a valid decomposition way because it cannot lead to distinguishable quantum state from the second one.

| $\{m_i\}$ | $\{2\}$ | $(2,\frac{7}{2},\frac{2}{2})$ | $(\sqrt{2},\sqrt{2})$ | $\{1,1\}$ |
|----------|---------|----------------|-----------------|-----------|
| $\{M_{i/2}\}$ | $\{2^+\}$ | $(2,\frac{7}{2},\frac{2}{2})$ | $(\sqrt{2},\sqrt{2})$ | $\{1,1\}$ |
| $\{M_{i/2}\}$ | $\{2^+\}$ | $(2,\frac{7}{2},\frac{2}{2})$ | $(\sqrt{2},\sqrt{2})$ | $\{1,1\}$ |
| $\{q_i\}$ | $(1/2)$ | $\{(1/2)\} + 1$ | $\{(1/2)\} + 1$ | $\{1,1\}$ |
| $\{q_{max}\}$ | $\{2\}$ | $(1,1)$ | $(0,1)$ | $(1,1)$ |

The corresponding wave dynamic equations can not be solved exactly. However, using Bohr-Sommerfield estimation, we can get the $\gamma$-quantizing condition

$$ h_i(r) = 1 - \frac{2GM_i}{r} + \gamma_i^2 = h_i(r_{rel}) < 0 \quad (18) $$

The corresponding wave dynamic equations can not be solved exactly. However, using Bohr-Sommerfield estimation, we can get the $\gamma$-quantizing condition

$$ \int_0^{r_{horizon}} \left\{ m_i^2 \left[ -\left( \gamma_i^2 - 1 \right) + \frac{2GM_i}{r_{+\pi}} \right] \right\}^{\frac{1}{2}} dr = (q_i + \nu)\pi \quad (19) $$

$$ \Rightarrow \left( \frac{GM_i}{\beta(\gamma_i)} \right)^{\frac{1}{2}} m_i = (q_i + \nu), \quad 1 \leq \beta_i \sim -\gamma_i^2 + 1 \quad (20) $$

with $q_i$ taking positive integer values and $0 < \nu < 1$. We will take $\nu = 0$ for simplicity. By this quantising condition and completely the same listing method as in 4 dimensions, we can get all possible micro state quantum numbers of a given mass dust ball with horizons in 5 dimension. TABLE I lists all quantum numbers for the micro state of $2M_{pl}$ mass ball as an example. Just the same as 4 dimensional case, quantum mechanically the contents of a 5 dimensional dust ball with horizon can only oscillate in a serial of finite, distinguishable eigen modes. For more higher dimensional cases, although the Bohr-Sommerfield estimation does not apply, it is believable that similar results should be true either.

FIG. 3 displays our quantitative results for the micro-state number of 4 and 5-dimensional dust ball with horizons and oscillating inner contents virus their horizon sizes directly. From the figure, we easily see that, in both 4 and 5 dimensions, the logarithmic values of these object’s micro state number fit with the area law very well. Looking from outside, such dust balls with horizons are not distinguishable from the equal mass Schwarzschild black holes. This implies that, it is totally possible that the micro state of a Schwarzschild black hole follows from the oscillation modes of its consisting matters inside the horizon. Such matter/energy contents and the corresponding layering structure of the pre-hole star are not destroyed on the central point, they are just across-running and going to the other side of the system at all. Two possible questions may arise here. i) why the pressure of contents of the pre-hole star? ii) why our micro state counting did not yield the Bekenstein-Hawking formula $S = \frac{A}{4\pi}$ exactly but a proportional relation? On the first question, our reply is, when all known physic effects cannot resist the collapsing trends of self-gravitation, ignoring pressures of the pre-hole star’s contents is reasonable.

On the second question, we note that when a dust ball is partitioned into several concentric spherical shells, e.g. three layer of mass $\{m_1, m_2, m_3\}$, the matter contents inside each shell have their own horizon sizes, e.g. $\{r_{h1}, r_{h2}, r_{h3}\} \propto \{M_1, M_2, M_3\}$, $M_i \equiv \sum_{j \leq i} m_j$. It is classically allowable that the matter contents of layer 2 moves inside $r_{h3}$ but outside $r_{h2}$, so its integration constant $\beta_2 = \sqrt{-\frac{1}{A} + \frac{1}{A}} < 1$. However, the condition $1 \leq \beta_i$ in eqs. (16) and (20) excludes this possibility, thus underestimates the micro state number unavoidably. A new question arises here, will the remedy of this underestimation breaking the area law following from its being neglected? The answer is, no. Because when this remedy of shell-crossing motion is included, all sub spheres will be equal-footing and the micro state of the system will be symmetric direct product of all sub spheres' and the number of them will be given by approximately

$$ W = c \cdot c \cdot c \cdots = c^{\max \text{ num. layers}} \quad (21) $$

The maximal number of layers such a ball can be divided into is completely determined by the quantisation condition of eqs. (16) and/or (20), with all $\beta_i$ being set to 1. The results is shown in FIG. 4 directly. Comparing with FIG. 4 we easily see that the maximal number of layers and the logarithmic value of micro state number satisfy the same area law, even the proportionality constant. Obviously, as long as we take $c_{4d} = e^{8\pi}$ or
\(c_{5d} = e^{3\pi^2/4}\) similarly, will get the Bekenstein-Hawking formula for black hole entropies exactly.

![Graph](image)

**FIG. 4:** The maximal number of layers a dust ball with horizon can be divided into and their masses in 4- and 5-dimensional space-time. The scattered points is the results maximal number of layers following from the quantisation condition of (10) and/or (20), with all \(\beta_i\) setting to 1 and counting by brute force. While the continuous line is the fitting formula of \(\ell_{\max}\) from the black hole. The \(H_{\text{int}}\) part functions to bring energies from the former to the latter and vice versa. Denoting the quantum state of a mass \(b_v\), binding status \(u \in \{1, 2, \cdots, e^{4k\pi G_5 b_v}\}\), black hole and its environment consisting of Hawking modes as \(|\ell^n, n\ell\rangle\), then

\[
b_v^\dagger b_v \Psi_{\ell^n, n\ell}^* (r) |\ell^n, n\ell\rangle = (|\ell + k\rangle^n, n\ell_k) = (|\ell + k\rangle^n, n\ell_k) (24)
\]

We will constrain ourselves to zero angular momentum radiations, so the transition \(|\ell^n, n\ell\rangle \rightarrow (|\ell + k\rangle^n, n\ell_k)\) is induced by gravitation, i.e. mass monopole interactions only. The proportionality in the definition of \(g_{uv}\) and/or \(g_{vu}\) just reflects the idea that two black holes which have more similar microscopic wave functions could be more rapidly to transit to each other.

Now denoting the initial state of the system as \(|n^{w}, 0\rangle\), and the latter time state as follows

\[
|\psi(t)\rangle = \sum_{\ell=0}^{n} e^{\frac{\pi}{8} b_v^\dagger b_v^\ell c_{\ell^n}(t)} |\ell^n, n\ell\rangle
\]

we will find through Schrödinger equation \(i\hbar \partial_t \psi(t) = H \psi(t)\) that

\[
i\hbar \partial_t c_{\ell^n}(t) = (b_v - b_v^t) c_{\ell^n} + \sum_{j} g_{uv} e^{i(b_j - b)^t} c_{\ell^n}(t) (27)
\]

This will lead to special time-dependent mass or horizon size function for each micro state black hole

\[
m_w(t) = \frac{r_w^2(t)}{2 G_N} = \sum_{\ell=0}^{n} e^{\frac{\pi}{8} b_v^\dagger b_v^\ell c_{\ell^n}^2(t)} (29)
\]

We will call this function evaporation curve of them. For a 2\(M_{\text{pl}}\) mass hole, whose 8 micro states are listed out explicitly in ref. [2], their characteristic evaporation curve, as well as their averages are all displayed in FIG.3 explicitly. The average one corresponds to the variation trends of Hawking radiations. It is because we focus on too small black holes and neglect momentum difference between the radiation particles, thus very few final states, that lead to the non monotone feature of the average curve [17]. To get this figure numerically, we set the proportionality constant in (25) being set to 1.

1. \(b_v, b_u\) are abbreviations for \(b_{uv} = b_v\) and \(b_{(\ell + k)v} = b_{\ell + k}\) respectively.
As long as we know initial status of the black hole, i.e. the \( w \) value in \( n^w \), then we will be able to predict its latter time size variations exactly. On the other hand, if we precisely measure a black hole’s evaporation curve, then we will be able to infer its initial status exactly. So the averaged evaporation curve is not monotone decreasing.

**Comparing with string theory fuzzy ball picture**

Although our micro state picture for black holes is very similar to that of string theory fuzzy balls \([7, 8]\), key differences exist between the two. Most importantly, our picture resolves the Schwarzschild singularity more thoroughly than the string theory fuzzy ball does. In ours, the static, point-like singularity is replaced by a periodically oscillating, continuous radial mass distribution \([2]\). The inner metric of black holes has the form

\[
ds^2 = -(h^{-1} \frac{m}{m^2} + 1) d\tau^2 + h^{-1} dr^2 + r^2 d\Omega_2^2 (30)\]

\[
h = 1 - \frac{2Gm(\tau, r)}{r}, \quad r < r_0 \equiv 2Gm_{\text{total}} (31)\]

where \( m(\tau, r) \) is determined by Einstein equations at the classic level and by Wheeler-de Witt equations at quantum levels. In the latter case, \( m(\tau, r) \) can only take some eigen modes whose total possible number equals to the exponentiated area of the hole. While in string theory fuzzy balls, the point-like central singularity is only replaced by a dynamic string-like singularity. For example, in the NS1-P representation \([8]\), the metric of the string whose compactification in the transverse \( x_i \) space will reduce to the 4 + 1 dimensional fuzzy ball reads

\[
ds^2 = H[-dudv + Kdv^2 + 2A_i dx_i dv] + dx_i^2 + dz_i^2 (32)\]

\[
H = 1 + \frac{Q_1}{|\vec{x} - \vec{F}(t-y)|^2}, \quad K = \frac{Q_1|\dot{\vec{F}}(t-y)|^2}{|\vec{x} - \vec{F}(t-y)|^2} (33)\]

\[
A_i = -\frac{Q_1\dot{F}_i(t-y)}{|\vec{x} - \vec{F}(t-y)|^2}, \quad B_{uv}, B_{vi}, e^{2\phi} = \cdots (34)\]

where \( F_i(t-y) \) are four arbitrary functions featuring the quantum motion of the strings in transverse directions \( x_i, i = 1, 2, 3, 4 \).

Secondly, in our micro state pictures, there are clear horizons at the classic level which divide inner and outside of black holes. At quantum levels, the horizon is blurred by the nonzero (outside horizon) value of the wave function \( |\Psi_{\beta_1}(r) \otimes \Psi_{\beta_2}(r) \cdots \otimes \Psi_{\beta_t}(r)|^2 \) which measures the probability of mass shells consisting the black hole are found at radius \( r \). While in string theory fuzzy ball pictures, the horizon are blurred at both classic and quantum levels. In a sense, string theory fuzzy ball provides only one picture for the black hole. You can say it a pure classic one, you can also say it a pure quantum one.

Thirdly, in our micro state pictures, Schwarzschild black hole is the most simple object whose classic inner metric and quantum wave function can be worked out explicitly. While in string theory fuzzy balls, the picture can only be established for some specially designed black holes which correspond directly or indirectly to the NS1-P system. Schwarzschild black holes, for its lacking of enough symmetries required by string theory method, is the hardest to exploring.

**Conclusion**

To the title question, we show that it is Hawking’s neglecting or implicit averaging of the inner structure or micro state of black holes that leads to the information missing puzzle. While the neglecting of micro state itself follows from an over interpretation of the singularity theorems which say that matters collapsing into their own horizon will fall on to the central point and form singularities there for ever. Both analogue with classic S-wave hydrogen atoms or positronium systems and concrete calculation basing on general relativity reveal that, matters collapsing into their own horizon can go across the central point and oscillate around there. Central singularity is not a static and for ever phenomenon but an instantaneous and dynamic one, although unavoidable.

We provide quantum descriptions for this oscillating motion inside horizons and find that, for a dust ball with horizon and classically continuous radial mass distribution, distinguishable quantum state is countable and finite, with the number happens to be exponentials of the horizon area and with each micro state corresponds to a special oscillating modes of the contents. We verify this point in both 4 and 5 dimensions and argue that...
in extremal cases when all known physic forces cannot resist the trends of self-gravitation collapses, neglecting the pressure of pre-hole contents and looking them as dusts is reasonable. As results, the dust ball with horizon and with radial oscillating contents provides interpretations for the micro state of black holes, both classical and quantum mechanically.

Basing on this micro state interpretation, we construct a hamiltonian thus explicitly unitary formulation for Hawking radiations. Using this formulation, we calculate the characteristic mass variation curve of 8 micro state black holes with equal $2M_{pl}$ mass. Each of them is distinguishable from each other. This forms a concretely answer the question on where the information is going in Hawking radiations.

We also make comparisons between our micro state picture for black holes and that of string theory fuzzy balls and point out their main differences.

**Prospects** The goal of looking for errors in the information missing puzzle is not to find the error itself, but to find the hints of quantum gravitations. Basing on this motivation, physicists have learned many things from this looking for, e.g. the discovering of gauge/gravity duality [18–20], and entanglement/geometry equivalences [21–23]. From this aspects, we may say that no one is welcome to present too simple answers to this puzzle, although such answers are believed [24, 25] to exist in many serious researches, and does not repel works following from more fundamental theory of quantum gravitations, just as we indicated in FIG.1.

However, with the development of gravitational wave phenomenologies [26, 27], the question on inner structure of black holes are no long a pure theoretical one. Gravitational waves following from binary black holes and other compact objects provides us a possible way to measure these objects’ inner contents’ distribution. Obviously, black holes in our micro state pictures have totally different quadrupole moment structure from those of conventional ones featured by the Schwarzschild singularity and the completely empty inner horizon space-time. As results, gravitational waves following from binary holes in our picture are expected to have different features from those in conventional ones [28].

So, no matter we will become the non grata man which encounters the too simple answer to information missing puzzles or not, it is reasonable to expect that studies on the inside horizon structure and micro state of black holes [29–31] will become the subject of gravitational wave era of new decades.

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* Electronic address: dffzeng@bjut.edu.cn

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