Numerical study of natural convection nanofluids flow in tilted cavities with a partially thermally heat source

Djamila Benyoucef¹, Samira Noui¹ and Afaf Djaraoui²

Abstract
Numerically, natural convection heat transfer of nanofluids in a two-dimensional tilt square enclosure was investigated, with a partial heat source embedded on the bottom wall subject to a fixed heat flux. The remaining portions of the horizontal bottom wall are assumed to be adiabatic, while the upper horizontal wall and the vertical ones are supposed to be at a relatively low temperature. Using the finite volume method and the SIMPLER algorithm, the governing equations have been discretized and solved. Simulations have been carried out for more than one nanoparticle and base fluid, a range of Rayleigh numbers ($10^3 \leq Ra \leq 10^6$), various values of heat source length and location ($0.2 \leq B \leq 0.8$ and $0.2 \leq D \leq 0.5$, respectively), solid volume fraction ($0 \leq \phi \leq 20\%$) as well as tilt angle ($0^\circ \leq \theta \leq 90^\circ$). The results indicate that the heat transfer performance increases by adding nanoparticles into the base fluid. An optimum solid volume fraction raises and reduces the heat transfer rate and maximum temperature of the surface heat source, respectively. Moreover, the results show a significant impact of the tilt angle on the flow, temperature patterns, and the heat transfer rate with a specific tilt angle depending to the pertinent parameters.

Keywords
Nanofluids, natural convection, CFD, tilted cavities, heat transfer, finite volume method

Date received: 13 August 2021; accepted: 27 October 2021

Handling Editor: Chenhui Liang

Introduction
Free convection fluid flow heat transfer in the confined enclosure has received considerable theoretical, experimental, and numerical attention for its ability to solve many natural phenomena and industrial applications. Through a literature review, we found that natural convection in enclosures in various forms has received great attention until now, following the Benard’s experiments and Rayleigh’s theoretical analysis.¹-³ Changing the thermal properties, boundary conditions in addition to the geometric characteristics experimentally and numerically have been the work of many research extensively in horizontal and inclined enclosures. The transient natural convection heat transfer in inclined rectangular heated from below, the inclination effect for two different aspects and fixed Rayleigh number values on the mean Nusselt number have been experimentally well explained.⁴ Natural convection in a tilted square cavity with two heated adjacent walls is numerically studied,⁵ for different values of the Rayleigh number and tilt angles. The effect of the two parameters on the flow, the temperature distribution, and the Nusselt number behavior across the enclosure are discussed.

¹LPEA Laboratory, Department of Physics, Faculty of Sciences, University of Batna 1, Batna, Algeria
²LMD ST Department, Faculty of Engineering Sciences University of Batna 2, Batna, Algeria

Corresponding author:
Djamila Benyoucef, LPEA Laboratory, Department of Physics, Faculty of Sciences, University of Batna 1, Batna 05000, Algeria.
Email: djamila.phy@gmail.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
The transient natural convection in tilted rectangular cavities experimentally and numerically for a large Rayleigh number has been inspected. The results provided new correlations between Nusselt and Rayleigh numbers. While the effect of the tilt angle on the natural convection in a cavity with a corner heater has been discussed. The four characteristics of the natural convection flow behaviors in cavities distinguished in the steady state regime according to the values of Rayleigh number and aspect ratio have been presented.

Thermal fluids have a major role in many heat industrial applications, but the weak thermal conductivity of those fluids like water and oils prevented from having a good heat transport yield. Improving its performance and compactness is still the aim of much scientific research in engineering applications. Usually known that solid thermal conductivity is greater than the fluid one, for a cite, the copper thermal conductivity compared to water and engine oil is higher great 700 and 3000 times, respectively. Using solid particles in the thermal fluid is the latest newer mechanism to enhance heat transfer. New technical axes grow up recently to solve the daily faced problems, based on nanoparticles. These terms, which have become common in recent years, are still complex. On the other hand, nanoparticles today represent a major technological and economic challenge. They allow very promising innovations in many fundamental areas such as health, energy industry. Enhancing the thermo-physical properties of the fluids involved in the industry (e.g. cooling systems, refrigerators, solar thermal collectors, ...) by adding particles is not new, but to use a nanoparticle have the potential to significantly minimize the corrosion and sedimentation problems encountered with larger size particles. They called them nanofluids.

A nanofluid is a designed colloid by dispersion nanoparticles (typically; the diameter is less than 100 nm) in the base fluid to improve certain properties. This term was introduced for thermal applications and remains commonly used to designate this type of suspension. Since then, considerable research and studies have been done to assess nanofluid thermal conductivity. Results proved that nanofluids thermal conductivity improves the heat transfer compared to the base fluid alone. It found that the effective thermal conductivity and viscosity of the nanofluids are substantially higher than the values of the base fluids by an experimental and theoretical study. Thermal conductivity and viscosity experiments were conducted in temperatures (20°C–60°C) and solid volume fractions (0.3%–1.5%) with Al2O3-nanoparticles dispersed in three different base fluids. Their results prove that thermal conductivity increases with solid volume fraction and temperatures. But the viscosity reflects its behavior with temperature. Similarly, the thermal conductivity, viscosity, and also their potential use in applications have been estimated. The thermal characteristics of (Cu-TiO2) hybrid nanofluid with those of a non-hybrid (Cu and TiO2) nanofluid were scrutinized and compared. Also, various factors that affect thermal conductivity were studied. They found that the thermal conductivity increased according to the solid volume percentage, exhibiting an improvement of 9.8%.

Heat exchangers of thermal systems require high thermal conductivity for thermal fluid, this required fluids with high thermal conductivity. Is expected that dispersing nanoparticles in the base fluid increases their thermophysical properties and thus greatly enhances heat transfer. According to Choi and Eastman, nanofluids have attracted widespread interest in the last two decades, as evidenced by the massive increasing research. A majority of works adopt water as base fluid that was often tested at a reference temperature. Investigating numerically the natural convection using the finite volume method to solve the governing equations in rectangular enclosures filled with nanofluids partially heated. Calculations were performed according to the control parameters \(10^0 \leq Ra \leq 5 \times 10^3\), \(0.1 \leq h \leq 0.75\), \(0.25 \leq \eta_p \leq 0.75\), \(0.5 \leq A \leq 2\), \(0 \leq \phi \leq 0.2\), and (Cu, Al2O3, and TiO2) nanoparticles. An increase was found in the mean Nusselt number, Heat transfer evacuation and also clear that heater location affects the thermal and dynamic flows.

Experimental and numerical investigation of tilt angle effect on natural convective heat transfer of nanofluids in the closed inclined enclosure is so limited. Tilt angle is one of the geometric control parameters which has a wide concentration in studies, for their enormous influence on the dynamic and thermal performance. A numerically the results of natural convection in a tilted enclosure for nanofluid with two walls kept at different temperatures were presented. Results show that the pertinent parameters have improved the heat transfer performances using nanofluid, also the tilt angle has an impact on the performance too. Numerically, the convective heat transfer flow in a two-dimensional tilted square cavity for Cu-water nanofluid was discussed. Its notables that nanofluid enhances the heat transfer, where it is important for low Rayleigh numbers and low tilt angle equal to 90°. The details of the observed numerical results are reported, to envision the heat transfer in an inclined nonuniformly heated cavity. Simulations were performed for a range of Rayleigh numbers, tilt angle, and volume fraction. The natural convection of nanofluid in a square cavity has been studied numerically. The Rayleigh number, tilt angle, and solid volume fraction effect on the behavior were examined. The results found that the tilt angle can be considered as a characteristic control parameter.

Technological progress has led to changing the electronic components size to a smaller scale and lighter weight, which makes cooling it always a big challenge.
The early interest in nanofluids by researchers was the possibility of using these fluids for cooling purposes, primarily from a technological viewpoint. As the high thermal conductivity can be considered an encouraging step of the cooling capabilities of such fluids, it is also important to understand the dynamics flow and the heat transfer theories as well as the convective behavior of nanofluids. The major studies cited considered the problem of natural convection of pure thermal fluids and nanofluids in a two-dimensional tilted cavity, but addressing the problem in an enclosure with a constant heat source has not been sufficiently analyzed. The most objective of the study is to examine numerically the natural convective heat transfer of a nanofluid in a tilted square cavity with under boundary conditions using the mathematical nanofluid model proposed. Four different nanoparticles (Cu, Ag, Al₂O₃, and TiO₂) and two base fluids (water (H₂O) and ethylene glycol (C₂H₆O₂)) are verified the performance of the natural convection. The validity of the obtained results were done with previous results and the effect of the Rayleigh number, solid volume fraction, the length and location of heat source and the tilt angle on flow, and thermal fields are researched.

**Analysis model**

The two-dimensional model considered is represented in Figure 1. We considered a square cavity with dimension L inclined at an angle of \( \phi \) with the horizontal axis, with partial heating in the bottom wall (heat source with fixed flux \( q'' \)) and cooling three walls (sides and top) at fixed low temperature \( T_c \). The remaining unheated lower wall parts are considered adiabatic. The natural convection flow forms inside the cavity due to the temperature gradient and the gravity acceleration. The nanofluid flow analysis is supposed to be a single-phase, Newtonian, incompressible in a laminar regime. The thermal physical properties of the base fluid and the spherical nanoparticles which are considered in thermal equilibrium are written in Table 1.²¹

![Figure 1. Problem description.](image)

### Mathematical modeling

The equations which govern the problem in the stationary case are based on mass, momentum, energy laws, and thermo-physical models of nanofluids. The buoyancy forces are taken into account applying the Boussinesq approximation. According to the assumptions literature, the dimensional equations can be written as²¹:

**Conservation of mass equation**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 
\]

**Conservation of x-momentum equation**

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - g \beta \Delta T \sin \phi
\]

**Conservation of y-momentum equation**

\[
\frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - g \beta \Delta T \cos \phi
\]

**Conservation of energy equation**

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

Nanoparticles are well spread inside the base fluid. Applying this assumption makes the particle concentration uniform across all the cavity hence the thermo-physical properties of nanofluids can be estimated using some known theoretical formulas. The given correlations as follows in Table 2 are to estimate the density,
thermal diffusivity, specific heat, thermal expansion coefficient, viscosity, and thermal conductivity values of the nanofluid.

Density

\[ \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p \]  

thermal diffusivity

\[ \alpha_{nf} = \frac{k_{nf}}{(\rho Cp)_{nf}} \]  

heat capacity

\[ (\rho Cp)_{nf} = (1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_p \]  

Thermal expansion coefficient

\[ (\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_p \]  

Viscosity

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^2} \]  

Thermal conductivity

\[ k_{nf} = k_f \left[ \frac{(k_f + 2k_f) - 2\phi(k_f - k_p)}{(k_f + 2k_f) + \phi(k_f - k_p)} \right] \]

The studied system is subject for the boundary conditions listed below:

\[
\begin{align*}
\text{u} = v = 0 & \text{ and } T = T_C \text{ for } x = 0; x = L \text{ and } 0 \leq y \leq L \quad \text{(11a)} \\
\text{u} = v = 0 & \text{ and } T = T_C \text{ for } y = L \text{ and } 0 \leq x \leq L \quad \text{(11b)} \\
\text{u} = v = 0 & \text{ and } \frac{\partial T}{\partial y} = 0 \text{ for } y = 0, 0 \leq x \leq d - \frac{b}{2} \quad \text{(11c)} \\
\text{u} = v = 0 & \text{ and } \frac{\partial T}{\partial y} = 0 \text{ for } y = 0d + \frac{b}{2} \leq x \leq L \quad \text{(11d)} \\
\text{u} = v = 0 & \text{ and } \frac{\partial T}{\partial y} = q'' \text{ for } y = 0 \text{ and } d - \frac{b}{2} \leq x \leq d + \frac{b}{2} \quad \text{(11e)}
\end{align*}
\]

The following dimensionless parameters used in this presentation work as a control parameter or to plot profiles and field maps are introduced:

\[ X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, \theta = \frac{T - T_C}{\Delta T}, \psi = \frac{\psi}{\rho\alpha_f}, Ra = \frac{\beta y_{nf} A T (\frac{L}{f})^3}{\nu\alpha_f}, \Delta T = \frac{q''L}{k_f}, Pr = \frac{\nu}{\alpha_f} \]  

The ratio of convective heat flux to pure diffusion heat flux defined as the rate of heat transfer and the dimensionless local \( (N_u) \) and average \( (N_{um}) \) Nusselt number described it.

The \( (N_u) \) along the heater source is known:

\[ N_u = \frac{h \times L}{k_f} \]  

\( h \) is the convective heat transfer coefficient and his formula is:

\[ h = \frac{q''}{T_s - T_C} \]  

The integration of the local Nusselt number \( (N_u) \) over the heater source gives us the average Nusselt number \( (N_{um}) \):

\[ N_{um} = \frac{1}{b} \int_{d-b/2}^{d-b/2} N_u dx \]

**Numerical model**

The set system equations above (1–4) together with the initial and boundary conditions (11a–11e) are numerically discretized using the finite-volume method (FVM) formulation presented by Patankar on an uniform structured quadrilateral grid. From the physical viewpoint, the method (FVM) is based on the spatial integration of the conservation equations through control volumes. Converting the governing equations to algebraic equations. The coupling between the continuity and momentum equations is satisfied using the semi-implicit method for the pressure-linked equations (SIMPLE) algorithm. The convective terms of the discretized equations were treated with a Second Order Upwind scheme. Considering the convergence of the numerical results, the under relaxation method is employed and the criterion on one time step is established as \( 10^{-6} \).
The numerical study was proceeded in a CFD commercial code. The grid-independence resolution was examined in order to select the appropriate grid density. An uniform grid sizes were used for all regions in $x$ and $y$ directions. Table 2 displays the grid independence results across from the average Nusselt number and the maximal temperature in the heater source for different mesh combinations are shown. Judging from the uniform grid ($60 \times 60$) it meets the grid independence requests study. The grid independent values are chosen according to the Aminossadati and Ghasemi results (FORTRAN program) with which we compare the obtained results (CFD code). All values show that the difference between the two results is less than (0.2%).

**Validation**

As a whole scientific work, the validate of the employed method and check the code is necessary. Results for natural convection in a nanofluid-filled square cavity were obtained and compared with the Aminossadati and Ghasemi results. The comparison between the two numerical results obtained is shown in Table 3.

To get more precise results, the results obtained are compared and displayed for the four types of the used nanofluids, a considerable range of the Rayleigh number and different solid volume fractions. Figure 2 shows the variation of the average Nusselt number with the volume fraction with different Rayleigh numbers. Very
excellent agreement between the results obtained with the previously published results can be observed.

Results

This section portrays the numerical simulation results of the streamline and isotherm contours of the nano-fluid flow, the local and average heat transfer rate, and the maximum temperature profiles of the heater wall-part for the proposed physical parameters. These parameters are the Rayleigh number \(10^3 \leq Ra \leq 10^6\), solid volume fraction \(0\% \leq \phi \leq 20\%\), tilt angle \(0^\circ \leq \varphi \leq 90^\circ\), heater source length and location \(0.2 \leq B \leq 0.8\) and \(0.2 \leq D \leq 0.5\), respectively, and the nanoparticle (Cu, Ag, Al\(_2\)O\(_3\), TiO\(_2\)). In simulations, pure water (H\(_2\)O) and ethylene glycol (C\(_2\)H\(_6\)O\(_2\)) are considered as base fluids.

Tilt angle effect

In this part of the results, the heater source is located in the middle of the bottom wall \((D = 0.5)\) and length \((B = 0.4)\) in a cavity with Cu–water nanofluid-filled.

To analyze the impact of the tilt angle on the heat transfer. Figure 3 illustrates the impressive effect of the tilt angle \(\varphi = 45^\circ\) on the streamlines and isotherms (upper and lower, respectively) for the nanofluid, \(\phi = 10\%\) (plotted by solid lines) and pure water (plotted by dashed lines) as a function of Rayleigh numbers. The dynamic maps (streamlines) show a single central rotation cell in an anticlockwise direction for all Rayleigh numbers. The length of the rotation cell increases to obtain the egg shape (elliptical pattern) as the Rayleigh number increases (clearly shaped at \(Ra = 10^6\)). Increasing the Rayleigh number values increase the flow strength and become more intense and distinguished, which is caused by the buoyant forces generated due to the applied thermal gradient. The isotherms have various shape form at each Ra, it offers different behaviors as they change. For low Rayleigh numbers \((10^3, 10^4)\): isotherms distributions are almost uniform in an interesting part of the cavity near the heater wall part and tend to be right and parallel which refers to the conduction heat transfer. Higher Rayleigh number leads to an increase in flow intensity, this enhances the convection heat transfer. The isotherm map completely shifts, becomes roughly parallel to the cold wall and wavier in the rest of the enclosure. It is noticeable that the Rayleigh number increases reduced the temperature of the heater part so that at \(Ra = 10^5\) and \(10^6\) the dimensionless heater temperatures are 0.207 and 0.093, respectively.

Figure 4(a) and (b) presents the horizontal and vertical velocities profiles via the middle-axis of the cavity affected by the Rayleigh number at a fixed inclination angle, respectively. Obviously, the magnitude velocity for vertical and horizontal absolute values increases with increasing Rayleigh number, caused by the strong

![Figure 3. Streamlines (a) and isotherms (b) for cavity-filled with nanofluid (\(\phi = 10\%,\) solid lines) and water (dashed lines) at various tilt angle \((D = 0.5, B = 0.4,\) and \(\varphi = 45^\circ\)).](image-url)
buoyancy flow. In both profiles, the values are smaller than near the walls. Moreover, the high velocity is associated with the nanoparticles added into the base fluid.

Figure 5 are prepared to present the impact of the tilt angle. The figure includes streamlines and isotherms (upper and lower, respectively) for the nanofluid, $f = 10\%$ (solid lines) and pure water (dotted lines) for a fixed Rayleigh number ($10^5$). At the tilt angle ($\varphi = 0^\circ$) the fluid moves in a symmetrical flow pattern in the enclosure; a two counter-rotating circulating cells are shown for the streamlines. The fluid rise in the middle and descend on the sides walls, due to buoyant forces generated by the fluid temperature differences. When the tilting angle is applied, a progressive slowing down in the anticlockwise fluid flow is observed. The convective regime always gets across the cavity, a convection cell is formed at the angle ($\varphi = 30^\circ$) with a weak one at the lower right corner, that disappears with high values.
of the tilt angle. The isotherms exhibit the convective characteristics dominated, they are symmetrically distributed in the whole cavity. Applying the tilt angle, the flow becomes more intense and distinct. Isotherms appear to be more curved due to the tilt, more swayed when using a nanofluid (Cu–water), and more confined to the right-hand cold wall.

It seems interesting that at \( u = 0 \), the pattern of thermal and dynamic fields arising within the cavity has been destabilized by the imposed vertical temperature gradient. While, at tilt angle \(( \varphi = 90^\circ )\), the typical thermal and dynamic stratification of the cavity is perfectly presented by the horizontal temperature gradient.

Figure 6(a) and (b) plots the velocity profiles (horizontal (left) and vertical (right), respectively) as a function of the inclination angle at Rayleigh number \( 10^5 \). For pure water as well as the Cu–water nanofluid at the cavity middle-section. A significant variation manifests in velocity plots for the zero angle \(( \varphi = 0^\circ )\) and other angle values. It is regarding the fluid flow according to the inclination angles, where two countercirculation cells arose first with zero tilt angle. The gradual demise of the second circulation cell-associated with the increase of the inclination angles. It is well noted that the \( y \)-velocity and \( x \)-velocity amplitude for pure water is more than the Cu-water nanofluid with smaller values in the middle of the cavity than near the walls.

The effect of the orientation angle of the cavity can be treated on the fluid flow pattern by plotting the Nusselt number (\( \text{Nu} \) and \( \text{Nu}_a \)). Figure 7(a) shows charts of the local Nusselt number via the heater source for some cavity’s tilt angles at \( \text{Ra} = 10^5 \) with three solid volume fractions \(( \phi = 0\% , 10\% , 20\% )\). Symmetric forms are obtained for zero tilt angle, in addition to their lowest local Nusselt number values in the middle of the heater source. It exists a maximum value for the convective parameter (\( \text{Nu} \)) for each tilt angle and which occurs at the inclination point before getting a minimum, as a result of higher buoyancy forces. Moreover, it notices that the local Nusselt number value through the heater source increases with the increase of the solid volume fraction away from the tilt point, which can be explained by the stronger circulation produced by elevated thermal heat transport.

The cavity is tilting upward with each tilt angle and the heater source loses its horizontal shape, this facilitated the convective behavior of the fluid induced by the floating forces. Figure 7(b) displays the \( \text{Nu}_a \) for various values of \( \text{Ra} \) also explains the tilt angle effect \(( \varphi )\). The following can be concluded:

- The \( \text{Nu}_a \) almost rises importantly with solid volume fraction for all values of \( \text{Ra} \).
- For weak Rayleigh numbers \(( 10^3 \) and \( 10^4 \)), the \( \text{Nu}_a \) values remain pretty nearly constant with tilt angles except for a slight variation at the one-thousandth digit after the decimal point. This is well observed at \( \text{Ra} = 10^3 \) whereas at \( \text{Ra} = 10^4 \) the variation appears in the \( \text{Nu}_a \) values with a slight increase especially for the pure water and the first solid volume fraction \(( \phi = 5\% )\). The variation is linear for the other solid volume fraction values \(( \phi = 10\% , 15\% , \text{and} 20\% )\). This can be explained by the stronger viscous forces which make the conduction heat transfer dominate in the cavity. As a result, the tilt angle slightly affects or doesn’t on \( \text{Nu}_a \).
Figure 7. (a) Nu via the heater wall-part at various tilt angles ($Ra = 10^5$, $D = 0.5$, $B = 0.4$, and $\phi = 0\%, 10\%, 20\%$). (b) $Nu_m$ at the heater wall-part for various Rayleigh numbers, solid volume fraction at various inclination angles ($D = 0.5$, $B = 0.4$).
For the first high Rayleigh number ($10^5$), the buoyant forces are intensively influenced by the tilt angle and the convective heat transfer becomes dominant, $N_u_m$ attains its max at a specific tilt angle ($\varphi = 60^\circ$) except for the greater angle ($\varphi = 90^\circ$) where the $N_u_m$ is strongly decreased. As well, the average Nusselt numbers are lower for the pure water-filled cavity compared to those nanofluids-filled (Cu–water) ($5\% \leq \phi \leq 20\%$). It improves the heat transfer performance with nanofluid as a thermal fluid.

For $Ra = 10^6$, an inverse behavior occurs and presents its effects on the $N_u_m$. However, it noted that $N_u_m$ decreases at the beginning of certain tilt angles for the solid volume fraction values. The increase occurs again at the other angles except the last one. The convection controls the fluid flow with strong buoyant forces.

**Heater wall effects**

This section examines the effect of the length of the heater wall part and its position relative to the vertical symmetry axis, for Cu–water nanofluid ($\phi = 10\%$) and pure water ($\phi = 0\%$). The heater source located in the middle of the bottom wall ($D = 0.5$ and $Ra = 10^5$) first with a small length ($B = 0.2$), after the change in the heat source length has been considered. The same thing is produced for the heat source location for ($B = 0.4$ and $Ra = 10^5$).

The $\theta_{max}$ and $Nu_m$ variation for length heater wall part and changing its location at a constant Rayleigh number ($10^5$) against tilt angle when $\phi = 10\%$ are shown in Figure 8(a) and (b), left and right, respectively. The $\theta_{max}$ and $Nu_m$ are varied as seen, due to the increase in the length of the heater wall-part (B), the maximum surface temperature ($\theta_{max}$, Figure 8(a), at the top left) increases. Whereas, it decreases with tilt
angle increases until it reaches the right angle ($\phi = 90^\circ$), as there is a noticeable rise in $\theta_{\text{max}}$. It can be explained by the stronger convective heat transfer with the heater wall-part length increases (increasing energy exchange). Moreover, the evacuation of the surface temperature becomes greater. The same behavior is observed when changing the heat source location with tilt angle ($\theta_{\text{max}}$). Figure 8(b), at the bottom left, it increases with the heat source location change ($D$). The heat source maximum temperature decreases according to the tilt angle increase, except for the first and the last tilt angle ($10^\circ$ and $90^\circ$) where there is an appreciable increase far from the middle location. Figure 8(a) and (b), right up and down, plots the average Nusselt number ($Nu_m$) for the same conditions, it is also noted that $Nu_m$ decreases, increases with a length heater wall-part (or its changing location), and the tilt angle increases, respectively. At this point, the destabilized fluid flow resulting from the low tilt angle ($10^\circ$) implies a decreases $Nu_m$ as a function of the heat source location. This decrease also occurs at the right angle ($90^\circ$), when the heat source changes its location and the cavity becomes subject to a horizontal temperature gradient.

**Effect of nanoparticles type and base fluids**

Nanoparticles type (Cu, Ag, Al$_2$O$_3$, and TiO$_2$) dispersing in base fluids (water ($H_2O$) and ethylene glycol ($C_2H_6O_2$)) are treated. The heater wall-part placed in the middle for length $B = 0.4$ for this section of the study. Figure 9(a) and (b) demonstrates the influence of nanoparticles type and base fluids on Nusselt number ($Nu_m$) for Rayleigh number, tilt angle suggestion ($\phi = 0^\circ, 30^\circ, 60^\circ, 90^\circ$), and solid volume fraction. The main findings in this part are:

- The effect of cavity tilt angle on $Nu_m$ is depicted. A linear effect on the average Nusselt number reflects the growth of the tilt angle. It is also observed that the rise of tilt angle leads the $Nu_m$ along the solid volume fraction axis in the growth direction with nanoparticle type and base fluid. Where the buoyant flows become stronger with the range of Rayleigh number growth.

- It is fully noted that the $Nu_m$ increases approximately monotonically with solid volume fraction for all nanofluids in terms of Rayleigh’s number with the water base fluid. Furthermore, it is more monotonous when using ethylene glycol. Especially for low Rayleigh numbers, as a result of the weak convection in this regime. Notable that the $Nu_m$ curves are more same and more closely related to all the nanoparticles.

- Average Nusselt number values grow up when ethylene glycol is used as a base fluid compared to water in all proposed tilt angles. In both cases of the base fluid, the average Nusselt number reaches their great values at the tilt angle ($\phi = 60^\circ$).

- $Nu_m$ as a function of the solid volume fraction keep the growth behavior in the range of Rayleigh number. For weak Rayleigh number ($Ra = 10^5, 10^6$), $Nu_m$ values at first are identical for the tilt angle ($\phi = 0^\circ$), particularly for water base fluid. Then it starts to diverge at the first solid volume fraction values gradually. Contrary to water, the ethylene glycol diverges starts from zero angle.

- For high Rayleigh numbers ($10^5$ and $10^6$), the growing up of $Nu_m$ values are manifest for all nanoparticle type and both base fluids. At $Ra = 10^5$, $Nu_m$ values increase along the solid volume fraction axis with weakly gradient compared to the low Rayleigh numbers and $Ra = 10^6$. At $Ra = 10^6$, $Nu_m$ weakly increases with the solid volume fraction for Cu and Ag nanoparticles than for Al$_2$O$_3$ and TiO$_2$ in the water base fluid. While this increase is almost disappeared using ethylene glycol.

**Conclusion**

The numerical steady-state of natural convection heat transfer, in tilted cavity partially heated by the bottom wall, tested with more than one nanofluid has been explored. The fluid flow, thermal, and heat transfer rate performance of pertinent parameters such as various tilt angles, range of Rayleigh number, solid volume fraction, heater wall-part length and location, type of nanofluid, and base fluid have been analyzed. The simulation analysis results hint at the following.

Modified thermal physical properties of basic fluids which related to the solid volume nanoparticles adding; strongly effective on the dynamic and thermal behavior such as heat transfer rate (local and mean Nusselt number). Especially for nanoparticles with high thermal conductivity (Cu and Ag), with an increment rate (between 10% and 70%) according to the nanoparticle types used and the Rayleigh numbers. Therefore, the rise in the mean Nusselt number expresses an improvement in heat transfer characteristics.

An optimum tilt angle value that results in the maximum mean Nusselt number is found for two basic fluids (water and ethylene glycol) with all nanoparticles. It is found that the highest heat transfer is formed using ethylene glycol. Especially for low Rayleigh numbers, as a result of the weak convection in this regime. Notable that the $Nu_m$ curves are more same and more closely related to all the nanoparticles.
Figure 9. Nu at the heater wall-part as function of Rayleigh numbers, solid volume fraction, and tilt angles for different nanofluids ($D = 0.5, B = 0.4$). (a) Water and (b) ethylene glycol.
viscosity). Therefore, there is an improvement in the mean heat transfer characteristics.

Two models of heat transfer appeared in the Rayleigh numbers range, the conduction heat transfer where it is dominant for low Rayleigh numbers. The same thermal and dynamic patterns happen for several tilt angles and solid volume fractions. Whereas, for higher Rayleigh numbers, the convective heat transfer is dominating. Considerable changes in the fluid flow and temperature behavior are appeared as in Nusselt number from the zero tilt angle ($\varphi = 0^\circ$) until the right angle ($\varphi = 90^\circ$), with the solid volume fraction increases.

Regardless of the pertinent parameters giving the best heat transfer and improving it using just a pure fluid (such as water), inserting the nanoparticles with a weak volume fraction offers the largest heat transfer gain in an enclosure. It is presupposed that using a basic fluid with high viscosity contrasts with buoyancy forces, thus reduces the heat transfer rate. Nevertheless, it is shown that using ethylene glycol as a base fluid that has high viscosity has a different reflection. Compared to the nanofluid in the fluid (water), the ethylene glycol increases $\text{Nu}_m$ and gives a further rise in reaches to 30%, for all suggestion nanoparticles although their varied thermal conductivity.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Djamila Benyoucef https://orcid.org/0000-0001-9684-0810

References

1. Ostrach S. Natural convection in enclosures. J Heat Transf 1988; 110: 1175–1190.
2. Bejan A. Theory of heat transfer-irreversible power plants—II. The optimal allocation of heat exchange equipment. Int J Heat Mass Transf 1995; 38: 433–444.
3. Khalifa AJN. Natural convective heat transfer coefficient – review: II. Surfaces in two-and three-dimensional enclosures. Energy Convers Manag 2001; 42: 505–517.
4. Elsherbiny SM. Free convection in inclined air layers heated from above. Int J Heat Mass Transf 1996; 39: 3925–3930.
5. Cianfrini C, Corcione M and Dell’Omo PP. Natural convection in tilted square cavities with differentially heated opposite walls. Int J Therm Sci 2005; 44: 441–451.
6. Baïri A, Laraqi N and García de María JM. Numerical and experimental study of natural convection in tilted parallelepiped cavities for large Rayleigh numbers. Exp Therm Fluid Sci 2007; 31: 309–324.
7. Varol Y, Ozt"op HF, Koca A, et al. Natural convection and fluid flow in inclined enclosure with a corner heater. Appl Therm Eng 2009; 29: 340–350.
8. Bejan A. Convection heat transfer. Durham, NC: John Wiley Sons Inc, 2013.
9. Choi SU and Eastman JA. Enhancing thermal conductivity of fluids with nanoparticles. Lemont, IL: Argonne National Lab, 1995.
10. Murshed SM, Leong KC and Yang C. Investigations of thermal conductivity and viscosity of nanofluids. Int J Therm Sci 2008; 47: 560–568.
11. Syam Sundar L, Venkata Ramana E, Singh MK, et al. Thermal conductivity and viscosity of stabilized ethylene glycol and water mixture Al$_2$O$_3$ nanofluids for heat transfer applications: an experimental study. Int Commun Heat Mass Transf 2014; 56: 86–95.
12. Sundar LS, Hortiguela MJ, Singh MK, et al. Thermal conductivity and viscosity of water based nanodiamond (ND) nanofluids: an experimental study. Int Commun Heat Mass Transf 2016; 76: 245–255.
13. Leong KY, Razali I, Ku Ahmad KZ, et al. Thermal conductivity of an ethylene glycol/water-based nanofluid with copper-titanium dioxide nanoparticles: an experimental approach. Int Commun Heat Mass Transf 2018; 90: 23–28.
14. Oztöp HF and Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int J Heat Fluid Flow 2008; 29: 1326–1336.
15. Ghasemi B and Aminossadati SM. Natural convection heat transfer in an inclined enclosure filled with a water-CuO nanofluid. Numer Heat Transf A Appl 2009; 55: 807–823.
16. Abu-Nada E and Oztöp HF. Effects of inclination angle on natural convection in enclosures filled with Cu–water nanofluid. Int J Heat Fluid Flow 2009; 30: 669–678.
17. Oztöp HF, Mobedi M, Abu-Nada E, et al. A heatline analysis of natural convection in a square inclined enclosure filled with a CuO nanofluid under non-uniform wall heating condition. Int J Heat Mass Transf 2012; 55: 5076–5086.
18. Sheremet MA, Pop I and Mahian O. Natural convection in an inclined cavity with time-periodic temperature boundary conditions using nanofluids: application in solar collectors. Int J Heat Mass Transf 2018; 116: 751–761.
19. Abu-Nada E, Masoud Z and Hijazi A. Natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids. Int Commun Heat Mass Transf 2008; 35: 657–665.
20. Öğüt EB and Kahveci K. Mixed convection characteristics of ethylene glycol and water mixture based Al₂O₃ nanofluids. In: Proceedings of the 2nd world congress on mechanical, chemical, and material engineering (MCM’16), Budapest, Hungary.

21. Varol Y and Oztop HF. A comparative numerical study on natural convection in inclined wavy and flat-plate solar collectors. Build Environ 2008; 43: 1535–1544.

22. Pak BC and Cho YI. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. Exp Heat Transf 1998; 11: 151–170.

23. Xuan Y and Roetzel W. Conceptions for heat transfer correlation of nanofluids. Int J Heat Mass Transf 2000; 43: 3701–3707.

24. Brinkman HC. The viscosity of concentrated suspensions and solutions. J Chem Phys 1952; 20: 571–571.

25. Maxwell JC. A treatise on electricity and magnetism. New York: Clarendon Press, 1873. pp.1.

26. Patankar SV. Numerical heat transfer and fluid flow. New York, NY: Hemisphere Publ Corp, 1980. pp.58.

27. Aminossadati SM and Ghasemi B. Natural convection cooling of a localised heat source at the bottom of a nanofluid-filled enclosure. Eur J Mech B Fluids 2009; 28: 630–640.

Appendix

Notations

| Symbol | Description |
|--------|-------------|
| b      | length of heat source, m |
| B      | dimensionless length of heat source (b/L) |
| C_p   | specific heat at constant pressure, kJ/kg/K |
| d      | distance of heat source from the left wall, m |
| D      | dimensionless distance of heat source from the left wall (d/L) |
| g      | gravitational acceleration, m/s² |
| h      | local heat transfer coefficient, W/m²/K |
| k      | thermal conductivity, W/m/K |
| L      | height of the enclosure, m |
| Nu    | local Nusselt number along the heat source surface |
| Nu_m  | average Nusselt number along the heat source surface |
| Pr    | Prandtl number |
| q''    | heat flux, W/m |
| Ra    | Rayleigh number |
| T     | temperature, K |
| u, v  | velocity components in x, y directions, m/s |
| U, V  | dimensionless velocity components (uL/α_f; vL/α_f) |
| x, y  | Cartesian coordinates, m |
| X, Y  | dimensionless coordinates (x/L, y/L) |
| β     | thermal expansion coefficient, K⁻¹ |
| ΔT    | temperature difference, K |
| φ     | solid volume fraction |
| μ     | Dynamic viscosity, N s m⁻² |
| ψ     | dimensional stream function, m²/s |
| ψ*    | dimensionless stream function |
| ϕ     | tilt angle, ° |

Subscripts

| Subscript | Description |
|-----------|-------------|
| C         | cold |
| f         | fluid |
| m         | average |
| max       | maximum |
| nf        | nanofluid |
| p         | particle |
| s         | heat source surface |

Greek symbols

| Symbol | Description |
|--------|-------------|
| α      | fluid thermal diffusivity, m²/s |