Phase transition of finite size quark droplets with isospin chemical potential in the Nanbu–Jona-Lasinio model

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Abstract

Making use of the NJL model and the multiple reflection expansion approximation, we study the phase transition of the finite size droplet with $u$ and $d$ quarks. We find that the dynamical masses of $u$, $d$ quarks are different, and the chiral symmetry can be restored at different critical radii for $u$, $d$ quark. It provides a clue to understand the effective nucleon mass splitting in nuclear matter. Meanwhile, it shows that the maximal isospin chemical potential at zero temperature is much smaller than the mass of pion in free space.

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The stability of quark matter has been investigated recently with various approaches, such as lattice QCD \[1, 2\], MIT bag model \[3, 4, 5, 6, 7\], NJL model \[8, 9, 10, 11, 12\]. The strange quark matter containing \(u\), \(d\) and \(s\) quarks are considered more stable than those only containing \(u\), \(d\) quarks \[4, 5\]. In the MIT bag model, hadrons consist of free (or only weakly interacting) quarks which are confined to finite region of space (in the “bag”). The confinement is not a dynamical result of the theory, but an input parameter with appropriate boundary conditions in the model. However, an important feature of MIT bag model is the absence of the dynamical chiral symmetry breaking. When the effect of the dynamical chiral symmetry breaking is taken into account, the strange quark matter is not absolutely stable \[9, 13, 14\]. Meanwhile, the transition from \(u\), \(d\) quark to \(u\), \(d\), \(s\) quark by weak process is not favored, because when the chiral symmetry of \(u\), \(d\) quark is restored, the symmetry of \(s\) quark is still broken \[11\]. It is then necessary to study the property of the system consisting of \(u\) and \(d\) quarks.

So far all the foregoing analysis are either restricted to the quark matter that \(u\), \(d\) quarks have the same chemical potential and the isospin is symmetric in the Lagrangian, or that of infinite volume. This means that, for the infinite quark matter, all the quantities related to \(u\), \(d\) quarks in the matter are the same, in particular the dynamical quark mass and the quark condensates are equal for both flavors. However, there are many situations in the real world that the \(u\), \(d\) quark numbers are not equal. According to the principle of statistical physics, the chemical potential should be different for \(u\), \(d\) quark, respectively. For example, the neutron stars must be electrically neutral to a very high degree (even though there are electrons in neutron stars, the ratio is very small) \[12\]. Therefore if the neutron star consists of deconfined \(u\), \(d\) quarks, the number of the \(d\) quark must be about two times of that of \(u\) quarks to ensure the electrical neutrality. On the other hand, the finite size effect including the contribution of the surface tension and the curvature, which has been shown to be very important to the properties of strangelets \[6, 7\], and of great interest in the study of nucleus, neutron stars, and heavy ion collisions \[15\], has not yet been taken into account for the isospin asymmetric quark matter \[2, 16, 17, 18, 19, 20, 21\]. It is thus imperative to investigate the variation behavior of the isospin asymmetry and the chiral symmetry breaking in finite size quark matter (or quark droplet). Since the Nanbu–Jona-Lasinio (NJL) model \[8\] at quark level \[9, 12\] involves the information of chiral symmetry breaking and restoration, and of which the interaction among quarks is easy to deal with, in this paper we take the NJL model and the multiple reflection expansion (MRE) approximation \[6, 7, 10, 11, 22\] to study the finite size effect on the
isospin asymmetric quark matter. For simplicity, we take only the $u, d$ quark matter into account.

In the two-flavor NJL model, the Lagrangian is written as

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_0 + \mu \gamma_0)q + \mathcal{L}_{int},$$  \hspace{1cm} (1)

where $m_0$ is the current quark mass. For the flavors $u$ and $d$, one usually takes approximation $m_u = m_d = m_0$. The interaction part is

$$\mathcal{L}_{int} = G[(\bar{q}q)^2 + (\bar{q}i\gamma_5 \gamma q)^2].$$  \hspace{1cm} (2)

In order to describe the isospin symmetry breaking, we introduce different chemical potential for $u, d$ quark, respectively,

$$\mu = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix}. \hspace{1cm} (3)$$

It is obvious that this breaks the isospin symmetry, and the breaking can be manifested by the isospin chemical potential $\mu_I = \mu_u - \mu_d$, so different quark condensates $\phi_u = \langle \bar{u}u \rangle$ and $\phi_d = \langle \bar{d}d \rangle$ should be considered. The quark condensate is generally given by

$$\langle \bar{q}q \rangle = -i \int \frac{d^4p}{(2\pi)^4} tr S(p),$$  \hspace{1cm} (4)

where $S(p)$ is the dressed quark propagator and the trace is on the Dirac and color spaces. After some derivations, the thermodynamic potential at finite temperature $T$ and chemical potential $\mu_f$ ($f = u, d$) can be written as

$$\Omega(T, \mu_u, \mu_d, \phi_u, \phi_d) = 2G(\phi_u^2 + \phi_d^2) + \sum_{f=u,d} \Omega_{M_f}(T, \mu_f),$$  \hspace{1cm} (5)

with

$$\Omega_{M_f}(T, \mu_f) = -2N_c \int \frac{d^3k}{(2\pi)^3} \left\{ E_{k,f} + T \ln[1 + \exp(-\frac{1}{T}(E_{k,f} - \mu_f))] \\ + T \ln[1 + \exp(-\frac{1}{T}(E_{k,f} + \mu_f))] \right\},$$

where $E_{k,f} = \sqrt{M_f^2 + k^2}$, $M_f$ is the constituent quark mass and can be given at the mean field level as

$$M_f = m_0 - 2G\phi_f.$$

$$\hspace{1cm} (6)$$
For the $u, d$ quark system in a spherical bubble with radius $R$ (or referred as a quark droplet), the chemical potential is dependent on $R$, so we can write the Lagrangian as

$$L = \bar{q}(i\gamma^\mu \partial_\mu - m_0 + \mu(R)\gamma_0)q + \mathcal{L}_{\text{int}},$$  \hspace{1cm} (7)

where $\mu(R) = \mu_\rho$, the concrete form of $\rho$ will be given later. Because the droplet (or bubble) has a finite surface and the curvature provides a pressure difference between the inner and the outer parts of the droplet, not only the quark matter in the droplet, but also the surface and the curvature contribute to the thermodynamical potential of the system \[23\]. To incorporate all these effects, an approach denoted as multiple reflection expansion approximation has been developed \[6, 7, 10, 11, 22\]. In the multiple reflection expansion approximation, for a droplet composing of quarks with flavor $i$, the thermodynamical quantities can be derived from a density of states in the form

$$\frac{dN_i}{dk} = 6 \left[ \frac{k^2 V}{2\pi^2} + f_S\left(\frac{k}{m_i}\right) kS + f_C\left(\frac{k}{m_i}\right) C + \ldots \right],$$  \hspace{1cm} (8)

where $V$ is the volume of the droplet, $S = 4\pi R^2$ and $C = 8\pi R$ are the area and the extrinsic curvature of the surface of the droplet. $f_S\left(\frac{k}{m}\right), f_C\left(\frac{k}{m}\right)$ are the contributions to the density of the states from the surface, the curvature of the droplet, respectively, and can be given explicitly as \[6, 7\]

$$f_S\left(\frac{k}{m}\right) = -\frac{1}{8\pi} \left( 1 - \frac{2}{\pi} \arctan \left(\frac{k}{m}\right) \right),$$  \hspace{1cm} (9)

$$f_C\left(\frac{k}{m}\right) = \frac{1}{12\pi^2} \left[ 1 - \frac{3k}{2m} \left( \frac{\pi}{2} - \arctan \left(\frac{k}{m}\right) \right) \right].$$  \hspace{1cm} (10)

Then the density of states of the $i$ flavor quarks can be given in the multiple reflection expansion approximation as

$$\rho_i = \rho_{\text{MRE}}(k, m, R) = 1 + \frac{6\pi^2}{kR} f_S\left(\frac{k}{m}\right) + \frac{12\pi^2}{(kR)^2} f_C\left(\frac{k}{m}\right).$$  \hspace{1cm} (11)

From this expression we know that, if we do not consider the contributions from the surface and the curvature, the Lagrangian in Eq. \[7\] has the general form used in studying infinite quark matter in the NJL model.

By using the multiple reflection expansion approximation, we obtain then the density of states as $(k^2 \rho_{\text{MRE}})/2\pi^2$ and the thermodynamic potential can be given as
\[ \Omega(T, \mu_u, \mu_d, \phi_u, \phi_d) = \sum_{f=u,d} \frac{(M_f - m_0)^2}{2G} - 2N_c \sum_{f=u,d} \int \frac{\rho_{MRE} k^2 dk}{2\pi^2} \left\{ E_{k,f} + T \ln[1+\exp(-\frac{1}{T}(E_{k,f} - \mu_f))] + T \ln[1+\exp(-\frac{1}{T}(E_{k,f} + \mu_f))] \right\}, \]

In the zero-temperature limit \( T \to 0 \)

\[ \Omega(T, \mu_u, \mu_d, \phi_u, \phi_d) = \sum_{f=u,d} \frac{(M_f - m_0)^2}{2G} - 2N_c \int_{k_F^u}^{\Lambda} \frac{E_{k,u}\rho_{MRE} k^2 dk}{2\pi^2} - 2N_c \mu_u \int_{k_F^u}^{\Lambda} \frac{\rho_{MRE} k^2 dk}{2\pi^2} \]

\[ -2N_c \int_{k_F^d}^{\Lambda} \frac{E_{k,d}\rho_{MRE} k^2 dk}{2\pi^2} - 2N_c \mu_d \int_{k_F^d}^{\Lambda} \frac{\rho_{MRE} k^2 dk}{2\pi^2}, \quad (13) \]

where \( \mu_u = \sqrt{(M_u)^2 + (k_F^u)^2} \), \( \mu_d = \sqrt{(M_d)^2 + (k_F^d)^2} \); \( k_F^u, k_F^d \) are the Fermi momentums of \( u, d \) quark, respectively.

Consequently we can obtain the \( u, d \) quark numbers

\[ N_u = V n_u = 2N_c V \int_{0}^{k_F^u} \frac{\rho_{MRE} k^2 dk}{2\pi^2}, \quad (14) \]

\[ N_d = V n_d = 2N_c V \int_{0}^{k_F^d} \frac{\rho_{MRE} k^2 dk}{2\pi^2}, \quad (15) \]

where \( V = 4\pi R^3/3 \) is the volume of the droplet. \( n_f = -\frac{\partial \Omega}{\partial \mu_f} \) is the quark number density. For given \( N_u \) and \( N_d \), the Fermi momentum of the \( u, d \) quark can be determined, respectively.

If the system (droplet) is stable, the thermodynamic potential must satisfy the stationary conditions

\[ \frac{\partial \Omega}{\partial M_u} = 0, \quad \frac{\partial \Omega}{\partial M_d} = 0. \quad (16) \]

After some derivation we obtain the constituent quark mass

\[ M_u = m_0 + 2G N_c \partial_m \int_{k_F^u}^{\Lambda} \frac{\rho_{MRE} k^2 E_{k,u} dk}{2\pi^2} + 2G N_c \mu_u \partial_m \int_{0}^{k_F^u} \frac{\rho_{MRE} k^2 dk}{2\pi^2}, \quad (17) \]

\[ M_d = m_0 + 2G N_c \partial_m \int_{k_F^d}^{\Lambda} \frac{\rho_{MRE} k^2 E_{k,d} dk}{2\pi^2} + 2G N_c \mu_d \partial_m \int_{0}^{k_F^d} \frac{\rho_{MRE} k^2 dk}{2\pi^2}, \quad (18) \]

Then we can fix the chemical potentials

\[ \mu_u = \sqrt{(M_u)^2 + (k_F^u)^2}, \quad \mu_d = \sqrt{(M_d)^2 + (k_F^d)^2}, \quad (19) \]
and the isospin chemical potential

\[ \mu_I = \mu_u - \mu_d. \]  

(20)

By varying the radius \( R \) of the droplet, we can investigate the finite size effect on the isospin symmetry breaking and chiral symmetry restoration.

As a numerical example we take a charge neutral droplet with total quark number 9000, and the ratio of \( u, d \) quark number 1:2. For the current quark mass, we take \( m_0 = 6 \) MeV. For the cutoff and the coupling constant, we take \( \Lambda = 590 \) MeV and \( G\Lambda^2 = 4.7 \), with which the pion decay constant and pion mass \( (f_\pi = 93 \) MeV, \( m_\pi = 140 \) MeV) are reproduced well [11].

Figure 1: The dynamical quark masses of \( u, d \) quarks as a function of the droplet radius \( R \).

Since the dynamical quark mass can usually be taken to identify the chiral symmetry breaking and isospin asymmetry, we investigate the variation of the masses of \( u \) and \( d \) quarks at first. By solving Eqs. (14)-(18), we obtain the dynamical quark mass \( M_f \) \((f = u, d)\) as a function of radius \( R \) of the droplet. The result is illustrated in Fig. 1. From Fig. 1 we can notice that the constituent quark mass \( M_u \) is larger than \( M_d \) at the same radius if the radius of the droplet is neither very small nor extremely large. It means that
the isospin symmetry is broken. Whereas with the increase of \( R \), the difference between \( M_u \) and \( M_d \) becomes small. It indicates that, when the radius \( R \) of the droplet is large, the quark number density decreases rapidly, and hence the isospin asymmetry becomes small. Meanwhile, as the radius decreases (i.e., the quark number density increases since the quark number is fixed), the dynamical mass of the quarks can descend to current quark mass suddenly. It shows that the chiral symmetry is restored except for the explicit breaking. The figure also shows that the critical radius for the chiral symmetry of \( u \) quark to be restored \((R = 17 \text{ fm})\) is smaller than that of \( d \) quark \((R = 20.5 \text{ fm})\). When \( R \) is smaller than 17 fm, the dynamical masses of both \( u \) quark and \( d \) quark descend to current quark masses. It manifests that the isospin symmetry can be restored. In order to understand the phase transition in a more clear way, we also study the quark condensates as a function of baryon number density. The numerical result is illustrated in Fig. 2. Fig. 2 shows evidently that the critical density of the chiral symmetry restoration of \( u \) quark is larger than that of \( d \) quark. More concretely, the critical density of \( d \) quark is smaller than the normal nuclear matter density, whereas, that of \( u \) quark is near the normal nuclear matter density. All the results show that the contributions of the surface and the curvature play important roles for the droplet with a small radius.

\[ \langle \bar{q}q \rangle_{VAC} \]

**Figure 2:** Quark condensate as a function of baryon number density. \( \langle \bar{q}q \rangle_{VAC} \) refers to the quark condensate with \( R \to \infty \).
Then, we discuss the isospin chemical potential $\mu_I$ in the droplet. By solving Eqs. (14)-(20), we obtain the function $\mu_I$ versus the droplet radius. The result is shown in Fig. 3. It clearly shows that when $R < 17$ fm, the isospin chemical potential $\mu_I$ has the minimal value 0, and it increases to the maximal value when $R = 20.5$ fm, then decreases with the increase of $R$. When the radius $R$ is larger than 500 fm, the difference between $\mu_u$ and $\mu_d$ is less than 1 percentage of the current quark mass. This means that the isospin symmetry can be restored when the quark number density is small. The zero isospin chemical potential at very small radius shows that the isospin symmetry can also be restored if the quark number density is quite large. Another thing important we recognize is that, if the droplet takes a size so that only the chiral symmetry of $u$ quark could be restored, the isospin chemical potential takes the maximal value about 29.7 MeV in the present example of the droplet. It is obvious that such a value is much smaller than the critical isospin chemical potential for pion condensate ($\mu^*_I = m_\pi$ [18, 24]) to occur. It indicates that it may be difficult for pion condensate to happen in the droplet at zero temperature.

The quark droplets may possibly be formed in the QCD phase transition in the early universe or in astro-objects, for example, the main ingredient in the inner part of pulsars is quark matter composed of $u$, $d$ quarks due to the high pressure [25]. Furthermore, some works once proposed that the new phase of quark matter may emerge as droplets.

Figure 3: The isospin chemical potential as a function of the droplet radius $R$. 

![Figure 3: The isospin chemical potential as a function of the droplet radius $R$.](image)
in nuclear matter at low density \[26\], even appears as \( u-d \) quark stars (P-stars) \[27, 28\]. Our present results indicate that, as the density of the \( u-d \) quark matter in the droplet is not very small, the chiral symmetry can be restored, \( i.e. \), the chiral phase transition takes place. Since the phase transition influences the equation of state of the matter drastically, the pressure, the moment of inertia and other characteristics of the matter may change suddenly. As shown in Figs. 1 and 2, when the density of the matter in our present example is about 0.85\( n_0 \) (the radius of the droplet is about 17 fm), the dynamical mass of \( u \) quark changes from 127 MeV to 6 MeV. Such a sudden change may induce a quake and a glitch for the astro-objects. And the light emission strength may also be changed abruptly. When the density \( n \in (0.48n_0, 0.85n_0) \) (corresponding to the present example with radius 17 fm\(< R < 20.5 \) fm), only the dynamical mass of \( u \) quark changes. With a further decrease of the baryon number density, the dynamical mass of \( d \) quark also changes suddenly. Then another glitch may appear in pulsars. These phenomena may be taken to identify the effect of chiral phase transition in the droplet of \( u \) and \( d \) quarks \[29, 28\]. The brightest giant flare from soft gamma-ray repeaters and anomalous X-ray pulsars and other observations may also be taken as the observable evidences to signal the chiral phase transition effect. It is certain that further studies are necessary to carry out a practical analysis. On the other hand, the Figs. 1 and 2 manifest that, in a quitelarge region of nuclear matter density, the isospin symmetry is broken, and hence the dynamical mass of \( u \) quark is larger than that of \( d \) quark. In the view point of bag models and soliton models of a hadron, the effective mass of a proton (composing of 2 \( u \) quarks and 1 \( d \) quark) in nuclear matter \( m^*_{p} \) is larger than that of a neutron (consisting of 1 \( u \) quark and 2 \( d \) quarks) \( m^*_{n} \). Together with the result given in QCD sum rules \[30\], we can recognize that the result \( m^*_{p} > m^*_{n} \) given in relativistic approaches \[31, 32, 33\] has solid microscopic foundation. Our present result of the isospin asymmetry provides then a clue to solve the controversial problem of the nucleon effective mass splitting in nuclear matter \[31, 32, 33, 34, 35, 36\] on the theoretical side. It is now rather important since experiments have not yet given any conclusion \[36\].

In summary, by taking the two-flavor NJL model and the multiple reflection expansion approximation, we have studied the phase transition of a finite size droplet with \( u \) and \( d \) quarks. We find that, the isospin symmetry is preserved and the chiral symmetry is broken if the radius of the droplet is extremely large (or the matter density is very small). With the radius changes from infinite to about several hundred fms, both the chiral symmetry and the isospin symmetry are broken. In such a case, the dynamical quark
mass $M_u$ is larger than $M_d$ at the same radius $R$. It provides a clue that the effective mass of proton in nuclear matter may be larger than that of neutron. If the radius of the droplet is small enough (less than 20.5 fm for $N = 9000$), the chiral symmetry can be restored. And the critical radius for $u$-quark is smaller than that for $d$-quark. Meanwhile, possible observations to identify the chiral phase transition are proposed in some astronomical phenomena. In addition, the maximal isospin chemical potential in the case of zero temperature is not large enough so as to be comparable with the critical isospin chemical potential for pion condensate to emerge. However in our present study, we have not taken the flavor mixing \[12\] into account, where the dynamical quark mass $M_f$ ($f = u, d$) depends not only on the condensate $\phi_f$ but also on $\phi_f'$. Meanwhile the finite temperature effect has not yet been included either. Moreover the experimental or astronomical observables identifying the phase transition need also detailed investigations. The related studies are under progress.

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References

[1] J.B. Kogut, D.K. Sinclair. Phys. Rev. D 66, 034505 (2002).

[2] Y. Nishida, Phys. Rev. D 69, 094501 (2004).

[3] A. Chodos, R.L. Jaffe, K. Johnson, and C.B. Thorn, Phys. Rev. D 9, 3472 (1974).

[4] E. Witten, Phys. Rev. D 30, 272 (1984).

[5] E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).

[6] M.S. Berger and R.L. Jaffe, Phys. Rev. C 35, 213 (1987); C 44, 566(E) (1991).

[7] J. Madsen, Phys. Rev. Lett. 70, 391 (1993); Phys. Rev. D 47, 5156 (1993); Phys. Rev. D 50, 3328 (1994); Phys. Rev. Lett. 85, 4687 (2000).
[8] Y. Nanbu, and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); ibid, 124, 246 (1961).

[9] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 91 (1991); S.R. Klevansky, Rev. Mod. Phys. 64, 649 (1992); T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 241 (1994).

[10] O. Kiriyama, A. Hosaka. Phys. Rev. D 67, 085010 (2003)

[11] S. Yasui, A. Hosaka and H. Toki, Phys. Rev. D 71, 074009 (2005).

[12] M. Buballa, Phy. Rep. 407, 205 (2005)

[13] M. Buballa and M. Oertel, Nucl. Phys. A 642, 39 (1998); ibid, Phys. Lett. B 457, 261 (1999).

[14] D. K. Kim, Y. D. Han and I. G. Koh, Phys. Rev. D 49, 6943 (1994).

[15] A.R. Bodmer, Phys. Rev. D 4, 1601 (1971).

[16] D.T. Son, and M.A. Stephanov, Phys. Rev. Lett. 86, 592 (2001).

[17] J. B. Kogut, and D. Toublan, Phys. Rev. D 64, 034007 (2001); D. Toublan, and J.B. Gogut, Phys. Lett. B 564, 212 (2003).

[18] A. Barducci, R. Casalbuoni, G. Pettini, L. Ravagli, Phys. Lett. B 564, 217 (2003); ibid, Phys. Rev. D 69, 096004 (2004).

[19] B. Klein, D. Toublan, and J.J.M. Verbaarschot, Phys. Rev. D 68, 014009 (2003).

[20] M. Frank, M. Buballa, and M. Oertel, Phys. Lett. B 562, 221 (2003).

[21] M. Loewe, and C. Villavicencio. Phys. Rev. D 67, 074034 (2003); ibid, D 70, 074005 (2004).

[22] R. Balian, and C. Bloch, Ann. Phys. (N.Y.) 60, 401 (1970).

[23] M. L. Olesen, and J. Madsen, Phys. Rev. D 47, 2313 (1993).

[24] Lianyi He, Pengfei Zhuang, Phys. Lett. B 615, 93 (2005).

[25] M. L. Olesen, and J. Madsen, Phys. Rev. D 49, 2698 (1994).

[26] H. Heiselberg, C. J. Pethick, and E. F. Staubo, Phys. Rev. Lett. 70, 1355 (1993)
[27] P. Cea, J. High Energy Phys. **02**, 031 (2003); Int. J. Mod. Phys. **D 13**, 1917 (2004).

[28] P. Cea, Astron. AstroPhys. in press (arxiv: astro-ph/0511787).

[29] J. Granot, E. Ramirez-Ruiz, G. B. Taylor, etc. astro-ph/0503251

[30] E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova, Phys. Rev. **C 70**, 065205 (2004).

[31] B. Liu, V. Greco, V. Baran, M. Colonna, and M. Di Toro, Phys. Rev. **C 65**, 045201 (2002).

[32] Q. F. Li, Z. X. Li, and E. G. Zhao, Phys. Rev. **C 69**, 017601 (2004).

[33] E. N. E. van Dalen, C. Fuchs, and A. Faessler, Phys. Rev. Lett. **95**, 022302 (2005); Phys. Rev. **C 72**, 065803 (2005).

[34] B. A. Li, Phys. Rev. **C 69**, 064602 (2004).

[35] Z. Y. Ma, J. Rong, B. Q. Chen, Z. Y. Zhu, and H. Q. Song, Phys. Lett. **B 604**, 170 (2004).

[36] J. Rizzo, M. Colonna, M. Di Toro, and V. Greco, Nucl. Phys. **A 732**, 202 (2004); J. Rizzo, M. Colonna, and M. Di Toro, Phys. Rev. **C 72**, 064609 (2005).