Spontaneous compactification of bimetric theory

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Abstract

We propose a model of bimetric gravity in which the mixing of metrics naturally provides mass to a graviton by compactification with the flux of two gauge fields in extra dimensions. We assume that each metric in the solution for the background geometry describes four-dimensional Minkowski spacetime with an $S^2$ extra space, though the two radii of $S^2$ for two metrics take on different values in general. The solution is derived by the effective potential method assuming the existence of magnetic fluxes on the extra spheres. We find that a massive graviton is governed by the Fierz–Pauli Lagrangian in the weak field limit and one massless graviton emerges in four-dimensions.

Keywords: modified theories of gravity, Kaluza–Klein theories, Einstein–Maxwell spacetimes

1. Introduction

In the last ten years or so, there have been a number of attempts to understand dark matter and dark energy. One of the possibilities to understand dark matter is to consider the modifications (for reviews, [1, 2]) of Einstein gravity, which has been the theoretical basis for analyzing the universe as a whole. The nature of gravitational interaction at large distances and very small distances has not been understood thus far. This has led to attempts at modifications of general relativity at large and small distances. Recently, many authors have begun to study massive graviton theory [3, 4] (for reviews, [5, 6]) with considerable interest to better understand cosmology and other aspects of gravity. Another approach to modify general relativity is the bigravity theory or bimetric theory of gravity, which was proposed in the 1970s. The early works [7, 8] suffer from ghost problems, while the ghost-free bimetric theory has been established in recent years [9, 10]. The bimetric theory describes massless and massive gravitons in general. In the massive theories, the mass of the graviton is given as...
a free parameter (by hand). The lack of a theoretical explanation of the origin of the mass scale is a shortcoming of the generic massive gravity models.

The mixing term of the two metrics used in the theory gives mass to a graviton. In the present study, we consider a model in which the mixing originates from the fluxes of gauge fields in extra dimensions.

A model of multiple gauge fields with kinetic mixing was first proposed three decades ago [11], and in this backdrop, several studies on hidden sectors in similar models have been published in recent times [12–14]. A possible stringy origin of such models has also been investigated [15, 16]. In these models, the Lagrangian for two gauge fields is written in the form:

\[
\mathcal{L} = -\frac{1}{4} \left( F_1 \mu \nu F_1^\mu \nu + F_2 \mu \nu F_2^\mu \nu + 2\alpha F_1 \mu \nu F_2^\mu \nu \right),
\]

where \(\alpha\) denotes a dimensionless constant. The models suggest the existence of exotic particles that can be possible candidates for dark matter. This line of approach partially motivated the idea of using two gauge fields as well as two metrics in the theory. The mixing term of two gauge kinetic components can represent the mixing of two metrics simultaneously.

We consider the simplest compactification in a six-dimensional model with \(U(1)\) gauge field strengths in extra dimensions. This yields the non-derivative interaction of two metrics in the vierbein formalism [17–21]. In the present study we analyze our model at the on-shell level; the quantum aspects of the model are not discussed here.

In the next section, we define our model. In section 3, after our assumptions for the metrics with compactification and magnetic fluxes in the extra space are declared, compactification with four-dimensional Minkowski spacetime is investigated by means of the effective potential. In section 4, the effective four-dimensional Lagrangian for gravitons in the weak field limit is derived. Finally, we summarize our work and remark on the general significance of our study in section 5.

2. Model

Our model has two metrics, \(g_{MN}\) and \(f_{MN}\), and two \(U(1)\) gauge fields, \(A_g^M\) and \(A_f^M\), where \(M, N\) vary over the range 0, 1, 2, 3, 5, and 6. The field strengths are defined in general as

\[
F_{g_{MN}} = \partial_M A_g^N - \partial_N A_g^M \quad \text{and} \quad F_{f_{MN}} = \partial_M A_f^N - \partial_N A_f^M.
\]

We consider the following action for six-dimensional spacetime expressed as

\[
S = S_g \left[ g, F_g \right] + S_f \left[ f, F_f \right] + S_{\text{int}} \left[ g, f, F_g, F_f \right],
\]

where

\[
S_g = \int d^6x \sqrt{-g} \left[ \frac{1}{2\kappa_g^2} R_g - \frac{1}{4} g^{MK} g^{NL} F_{g_{MN}} F_{g_{KL}} - \Lambda_g \right]
\]

and

\[
S_f = \int d^6x \sqrt{-f} \left[ \frac{1}{2\kappa_f^2} R_f - \frac{1}{4} f^{MK} f^{NL} F_{f_{MN}} F_{f_{KL}} - \Lambda_f \right].
\]

Here, \(R_g\) and \(R_f\) denote Ricci scalars constructed from the metrics \(g\) and \(f\), respectively. The quantities denoted by \(\kappa_g, \kappa_f, \Lambda_g, \text{and} \Lambda_f\) are constants.
To analyze the preferred action $S_{\text{int}}$ including the mixing term, we introduce two sechbeins, $e_S$ and $e_f$, that satisfy the following relations:

$$e^A_S \eta_{AB} e^B_S = g_{MN}, \quad e^A_f \eta_{AB} e^B_f = f_{MN},$$

where $\eta_{AB} = \eta^{AB} = \text{diag.}(-1, 1, 1, 1, 1, 1)$. Next, we adopt the following action $S_{\text{int}}$ written using antisymmetric symbols:

$$S_{\text{int}} = -\frac{\alpha}{96} \int d^6x e^{MNRLST} e^A_M e^B_N e^C_R e^D_E e^F_S e^G_T F_{JK} e_g e_f,$$

where $\eta_{AB} e^{A(f)_M} e^{B(f)_N} = g_{MN}$. The dimensionless coupling constant $\alpha$ satisfies $|\alpha| < 1$. We note that this term has two independent reflection symmetries, $e_S \leftrightarrow -e_S$ and $e_f \leftrightarrow -e_f$, and an exchange symmetry $e_S \leftrightarrow e_f$. Though this action seems to have a complicated form, the other terms in $S_{\text{int}}$ can also be rewritten using sechbeins, and consequently, we have

$$\sqrt{-g} F_g^2 = \frac{1}{48} e^{MNRLST} e^A_M e^B_N e^C_R e^D_E e^F_S e^G_T F_{JK} e_g e_f,$$

(7)

$$\sqrt{-g} = \text{det } e_S = \frac{1}{720} e^{MNRLST} e^A_M e^B_N e^C_R e^D_E e^F_S e^G_T e_g e_f,$$

(8)

and

$$\sqrt{-g} R_g = \frac{1}{48} e^{MNRLST} e^A_M e^B_N e^C_R e^D_E e^F_S e^G_T e_g e_f,$$

(9)

where $R_g^{EF}_{TL}$ is identical to the coefficient in the curvature two-form as $\Theta^{EF} = \frac{1}{2} R^{EF}_{TL} dx^T \wedge dx^L$. We note that if the two metrics are identical, $S_{\text{int}}$ becomes $-\frac{\alpha}{2} \int d^6x \sqrt{-g} F_{MN} F_{f}^{MN}$.

3. Compaction of background geometry

Compaction with flux in the Einstein–Maxwell theory was investigated by Randjbar-Daemi, Salam, and Strathdee (RSS) more than three decades ago [22]. Since our model is based on principles similar to those of the RSS model, we expect the existence of a similar solution.

Next, we assume that each metric describes a direct product of four-dimensional flat spacetime and an extra two-sphere, $S^2$. We assume that the two metrics have different scales as described by

$$g_{mn} dx^m dx^n = a^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad f_{mn} dx^m dx^n = b^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),$$

(10)

where $\theta$ and $\phi$ denote the standard coordinates on $S^2$, $m, n = 5, 6$, and $a$ and $b$ denote the radii of the spheres. Consequently, the Ricci tensors associated with two metrics are independently given by

$$R_{mn} = \frac{1}{a^2} g_{mn}, \quad R_{f mn} = \frac{1}{b^2} f_{mn},$$

(11)

where $R_{mn}$ denotes the Ricci tensor.
Next, we suppose that a constant magnetic flux penetrates the extra sphere, similar to the RSS model \[22, 23\]. Namely, we set

\[ F_\theta = dA_\theta = -\frac{n_g}{2ea^2} d\theta \wedge a \sin \theta d\phi \] (12)

and

\[ F_\phi = dA_\phi = -\frac{n_f}{2eb^2} b d\theta \wedge b \sin \theta d\phi. \] (13)

Here, the electric charge $e$ is assumed to have a common value for simplicity.

We attempt to obtain the background solution with the four-dimensional flat spacetime. As per the work of Wetterich [24], we use the method of the effective potential for a static solution instead of solving the equation of motion derived from the action directly. Next, we define a potential corresponding to the action and ansatz as

\[
V(a, b) = a^2 \left(-\frac{1}{\kappa_g^2 a^2} + \frac{n_g^2}{8e^2 a^4} + A_g \right) + b^2 \left(-\frac{1}{\kappa_f^2 b^2} + \frac{n_f^2}{8e^2 b^4} + A_f \right) + 2aabb \left(\frac{n_g n_f}{8e^2 a^2 b^2}\right).
\] (14)

If we set $y \equiv ab$ and $x \equiv b/a$, the potential takes the form:

\[
V(x, y) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{1}{8e^2 y} \left(n_g^2 x + \frac{n_f^2}{x} + 2an_f n_f \right) + y \left(\frac{A_g}{x} + A_f x \right). \] (15)

Consequently, the equations of motion are satisfied if \[24\]

\[
\frac{\partial V}{\partial x} \bigg|_{x=x_0, y=y_0} = \frac{\partial V}{\partial y} \bigg|_{x=x_0, y=y_0} = 0,
\] (16)

which are the conditions for the ground state governed by the classical field equations, and

\[ V(x_0, y_0) = 0, \] (17)

which is the condition for the vanishing effective four-dimensional cosmological constant. To simultaneously satisfy equations (16) and (17), we need to tune $\kappa_g$ and $\kappa_f$ to specific values.

Since the last two terms in (15) are positive, the minimum value of $V$ when the value of $y$ moves turns out to be

\[
V(x, y_0) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{1}{\sqrt{2} e y} \left(n_g^2 x + \frac{n_f^2}{x} + 2an_f n_f \right) \left(\frac{A_g}{x} + A_f x \right),
\] (18)

with

\[
y_0 = \frac{1}{2\sqrt{2} e y} \left(n_g^2 x + \frac{n_f^2}{x} + 2an_f n_f \right) \left(\frac{A_g}{x} + A_f x \right)^{-1}. \] (19)
For the special case of $\Lambda_g = n_g^{-2} \lambda$ and $\Lambda_f = n_f^{-2} \lambda$, the minimum of $V(x, y_0)$ corresponds to $x_0 = \left| \frac{n_y}{n_x} \right|$, and it takes the value

$$V(x_0, y_0) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{\sqrt{2\lambda}}{e} \sqrt{1 + \alpha \text{sign}(n_y/n_x)},$$

(20)

and consequently, $y_0 = \left| \frac{n_y}{2e} \right| \sqrt{1 + \alpha \text{sign}(n_y/n_x)}$. Finally, we tune the constants as

$$\frac{1}{\kappa_g^2} + \frac{1}{\kappa_f^2} = \frac{\sqrt{2\lambda}}{e} \sqrt{1 + \alpha \text{sign}(n_y/n_x)}.$$  

(21)

In the present naive approach, the geometry of the background is determined while the individual values for $\kappa_g$ and $\kappa_f$ cannot be specified. The stability condition for flat four-dimensional spacetime is needed for this determination. We examine this condition in the subsequent section.

With further simplification upon setting $\Lambda_g = \Lambda_f \equiv \Lambda$, $\kappa_g = \kappa_f \equiv \kappa$, and $n_g = n_f \equiv n$, we determine $x_0 = 1$ and $y_0 = \frac{n}{2e} \sqrt{1 + \alpha}$, or equivalently, $a^2 = b^2 = \frac{n}{2e} \sqrt{1 + \alpha}$.

4. Masses of gravitons

In this section, we consider the dynamical graviton modes of the lowest excitation on the background. Here we do not discuss the Kaluza–Klein excited modes. Provided that the background geometry is the one obtained in the previous section, the four-dimensional action for gravitons can be obtained by the Kaluza–Klein reduction as

$$S^{(4)} = 4\pi \int d^4x \left\{ -\sqrt{-g^{(4)}} \left[ a g^{(4)}_{\mu\nu} R^{(4)}_{\mu\nu} + \frac{1}{\kappa_g^2} - \frac{n_y^2}{8e^2} - \frac{\Lambda_g}{x_0} \right] + \frac{\sqrt{f^{(4)}}}{2\kappa_f^2} \left[ b g^{(4)}_{\mu\nu} R^{(4)}_{\mu\nu} + \frac{1}{\kappa_f^2} - \frac{n_f^2}{8e^2} - \frac{\Lambda_f}{y_0} \right] - \frac{\alpha}{12} \varepsilon^{\mu\nu\rho\sigma} e^{a}_{\mu} e^{b}_{\nu} e^{c}_{\rho} e^{d}_{\sigma} h_{g}^{(4)} n_{g} n_{f} \right\},$$

(22)

where $\mu, \nu = 0, 1, 2, 3$, $a, b = 0, 1, 2, 3$, and the superscript ‘(4)’ indicates the four-dimensional quantities constructed from four-dimensional metrics. The background values $x_0$ and $y_0$ are considered as determined in the previous section. Hereafter, $e^{a}_{\mu}$ denotes vierbeins. We note that the model seems to be a ghost-free theory in four dimensions, which was examined in [21].

In the weak field limit [17–19], i.e., $e_{g} = \eta + \frac{1}{2} h_{g}$, $e_{f} = \eta + \frac{1}{2} h_{f}$, we find that

$$\sqrt{-g^{(4)}} = \det e_{g} = 1 + \frac{1}{2} \left[ h_{g} \right] + \frac{1}{8} \left[ h_{g} \right]^2 - \frac{1}{8} \left[ h_{g} \right]^2 + O \left( h^3 \right).$$

(23)
with a similar expression for $\sqrt{-f^{(4)}}$, and

$$\frac{1}{24} \epsilon^{\mu\rho\sigma} e_{\alpha \beta} e_{\gamma \delta} e_{\eta \phi} e_{\zeta \theta} e_{\tau \sigma} e_{\nu \mu} e_{\xi \rho} e_{\omega \sigma} e_{\kappa \lambda} e_{\mu \nu} e_{\rho \sigma} e_{\theta \omega} e_{\zeta \eta} e_{\tau \xi} e_{\kappa \lambda} e_{\mu \nu} e_{\rho \sigma} e_{\theta \omega} e_{\zeta \eta} e_{\tau \xi} e_{\kappa \lambda} = 1 + \frac{1}{4} \left( [h^g] + [h_f^g] \right) + \frac{1}{48} \left( [h^g_2]^2 + 4[h^g h^f] + [h^f_2]^2 \right) \right)$$

$$- \frac{1}{8} \left( [h^g_2]^2 + 4[h^g h^f] + [h^f_2]^2 \right) + O(h^3). \quad (24)$$

Here, $\eta$ denotes the four-dimensional flat metric, and $[A] \equiv \text{tr} \ A$ for notational simplicity. It is known that the asymmetric part of $h$ can be omitted [19].

Next, we can write down the non-derivative terms in the four-dimensional action. From equation (17), we note that the constant term reduces to zero for the background metrics obtained in the previous section. The appearance of the linear term in $[h^g]$, $[h^f_1]$ leads to instability of flat four-dimensional spacetime. The vanishing of the coefficients of linear terms in the four-dimensional action yields

$$\frac{1}{\kappa^2_g} = \frac{n^2}{8e^2\gamma_0} + \frac{\alpha n_f}{8e^2\gamma_0} + A_g \frac{\gamma_0}{\gamma_0}, \quad (25)$$

$$\frac{1}{\kappa^2_f} = \frac{n^2}{8e^2\gamma_0\gamma_0} + \frac{\alpha n_f}{8e^2\gamma_0} + A_f \gamma_0 \gamma_0. \quad (26)$$

The summation of equations (25) and (26) is equal to equation (17), and the values of $\kappa_g$ and $\kappa_f$ can be obtained consistently. Incidentally, for the case of the RSS model, the single condition of the vanishing cosmological constant in four dimensions (17) implies stability of flat four-dimensional spacetime because there is only one gravitational field. Actually, equations (25) and (26) turn out to be the constraints that originate from the variations of the two lapse functions $e_{\nu_0}^g$ and $e_{\nu_0}^f$.

When all the equations on the metrics including equations (25) and (26) are satisfied, the non-derivative term in the four-dimensional action is obtained simply as

$$\frac{\alpha n_g n_f}{96e^2\gamma_0^2} \left( \left[ h^g \right] - \left[ h^f \right] \right) - \left[ h^g - h^f \right]^2 \right). \quad (27)$$

On the other hand, the kinetic terms for graviton fields are obtained as

$$\int d^4x \sqrt{-g^{(4)}} R^{(4)} = \int d^4x \left[ -\frac{1}{4} \partial^\mu h_{\nu \rho} \partial^\nu h_{\rho \mu} + \frac{1}{4} \partial^\mu h_{\rho \nu} \partial^\rho \partial^\mu h_{\nu \rho} \right] - \frac{1}{2} \partial^\mu h_{\nu \rho} \partial^\nu h_{\rho \mu} + \frac{1}{4} \partial^\mu h_{\rho \nu} \partial^\rho h_{\nu \mu} + O(h^3) \right] \quad (28)$$

with a similar expression for $f$, where $h \equiv [h]$ for simplicity.
Therefore, the Lagrangian for linearized graviton fields is written as
\[
\frac{\gamma_0}{2\kappa_g^2 x_0} \left[ -\frac{1}{4} \partial_\mu h^\mu_{\rho\sigma} \partial^\rho h^\mu_{\sigma\nu} + \frac{1}{2} \partial_\rho h^\rho_{\mu\nu} \partial^\mu h^\nu_{\rho\sigma} \right.
\]
\[
+ \frac{x_0 \gamma_0}{2\kappa_f^2} \left[ -\frac{1}{4} \partial_\mu h^\mu_{\rho\sigma} \partial^\rho h^\mu_{\sigma\nu} + \frac{1}{2} \partial_\rho h^\rho_{\mu\nu} \partial^\mu h^\nu_{\rho\sigma} \right]
\]
\[
\left. + \frac{\alpha_n n_f}{96e^2 \gamma_0} \left( h^2 - h_f \right)^2 - \left( h_{\rho\mu} - h_{f\mu} \right)^2 \right] = 0.
\]

where
\[
H_0 \equiv \sqrt{\frac{\kappa_g^2 x_0 + \kappa_f^2}{x_0}}, \quad H_1 \equiv \sqrt{\frac{\kappa_g^2 x_0 + \kappa_f^2}{x_0}}.
\]

The quadratic term of \( H_1 \) corresponds to the Fierz–Pauli mass term [25]. Therefore, we conclude that the present model with the previously obtained background geometry contains one massless graviton field \( H_0 \) and one massive graviton field \( H_1 \). The mass \( m \) of \( H_1 \) is given by the equation
\[
m^2 = \frac{\alpha_n n_f}{12e^2 \gamma_0^2} \left( \kappa_g^2 x_0 + \kappa_f^2 \right)
\]
if \( \alpha_n n_f \) is positive.

Now we examine the simple cases that have already been discussed in the last part of the previous section. In the special case that \( \Lambda_g = n_e^2 \lambda \) and \( \Lambda_f = n_f^2 \lambda \), we determined that \( x_0 = \frac{n_f}{n_g} \) and \( y_0 = \frac{1}{2e} \sqrt{\frac{1 + \left| \alpha \right|}{2}} \). The gravitational constants should be chosen to satisfy equations (25) and (26) as
\[
\frac{1}{\kappa_g^2} = \frac{1}{\kappa_f^2} = \frac{\sqrt{2}}{2} \sqrt{1 + \left| \alpha \right|}.
\]
In this case, we find that the mass of the massive graviton is

\[ m^2 = \frac{2\sqrt{2}\Lambda ae}{3(1 + |\alpha|)^{3/2}} \frac{n_x^2 + n_y^2}{n_x^2 n_y^2}. \]  

(33)

In the further simple case that \( \Lambda_x = \Lambda_y \equiv \Lambda, \) \( \kappa_x = \kappa_y \equiv \kappa, \) and \( n_x = n_y \equiv n, \) we determined that \( \chi_0 = 1 \) and \( \gamma_0 = \frac{n}{2\pi} \sqrt{\frac{1 + \alpha}{2\pi}} \cdot \frac{1}{2n} = \frac{n\sqrt{2\Lambda}}{e} \sqrt{1 + \alpha}, \) and \( 0 < \alpha < 1. \) The mass of the massive graviton is given by the equation

\[ m^2 = \frac{4\sqrt{2}\Lambda ae}{3n(1 + \alpha)^{3/2}}. \]  

(34)

It is to be noted that since the ratio \( (m^2/\Omega^2) = \frac{n}{3(1 + \alpha)} \) is always smaller than unity, the massive graviton is expected to be lighter than the first Kaluza–Klein excited mode of the massless graviton.

5. Summary and outlook

In this study we presented a model of bimetric theory in six-dimensions. We showed that compactification with fluxes in the extra space leads to four-dimensional massive and massless gravity. The relation of parameters that allows the compactification was obtained. We found the mass of the massive graviton in terms of the parameters in the model.

We studied the flat four-dimensional geometry in this paper. Although it is important to investigate the nonlinear structure of bimetric interaction, it is difficult to solve the nonlinear dynamics of two metrics. Solutions for generic non-flat metrics will be studied in future.

It is of significant interest that the two radii of the extra spheres in two metrics can have different values in general. This fact gains further significance when we consider the Kaluza–Klein excitation of gravitons as well as matter fields. The possible variety of mass spectra is worth studying in certain phenomenological and cosmological contexts. It is interesting to study the model as a quantum field theory because of the complexity of interactions among the infinite Kaluza–Klein excited modes of gravitons, gauge fields, and matter fields to be added in the case of ‘asymmetric compactification’ for \( a \neq b. \) It is also interesting to introduce dilaton fields into bimetric models, as in the case of the six-dimensional supergravity model [26, 27]. We speculate that similar compactifications can be considered in such models. However, we suspect that there is the possibility of removing the tuning conditions among couplings and even elimination of the asymmetry of two spheres in models with a dilatonic field.

The cosmological application of our model offers considerable potential as massive gravity is expected to solve the riddle of cosmic acceleration. The compactification to the four-dimensional de Sitter spacetime should be studied in this context. Our simple model is also suitable to study classical and quantum cosmology since cosmological aspects of the RSS model have previously been studied by Okada [28, 29], Halliwell [30, 31], and other authors.

Finally, we remark that it is straightforward to generalize the present model to the model of multigravity [20, 32]. In such a theory, one may expect to find a new hierarchical spectrum of gravitons and other fields in the theory. It is interesting to study such a model because the excitation modes may lead to interesting quantum effects as well as a novel cosmological evolution.
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