Gaugings at angles from orientifold reductions

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Abstract
We consider orientifold reductions to $\mathcal{N} = 4$ gauged supergravity in four dimensions. A special feature of this theory is that different factors of the gauge group can have relative angles with respect to the electro-magnetic $SL(2)$ symmetry. These are crucial for moduli stabilization and de Sitter vacua. We show how such gaugings at angles generically arise in orientifold reductions.

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1. Introduction

An important issue in string theory is the stabilization of moduli. Compactifications to four dimensions generally lead to an abundance of scalar fields, which need to be stabilized at some point in moduli space. Flux compactifications are an attractive route to such a scenario$^1$. In addition, one would like to accommodate for a positive value of the scalar potential in this vacuum. Although at first this seemed hard to realize, there are now a number of possible models for de Sitter spacetimes within string theory$^2$.

Parallel to the ‘top-down’ approach of string compactifications, one can also take a ‘bottom-up’ perspective. There have been systematic investigations of the possibilities for moduli stabilization and de Sitter vacua in four-dimensional gauged supergravity, irrespective of any higher dimensional origin. For $\mathcal{N} \geq 4$ extended supergravity, the de Sitter vacua found so far are unstable and have a value for the slow-roll parameter of order 1 [5–7]. For $\mathcal{N} = 2$, on the other hand, there are examples with stable de Sitter vacua [8, 9]. The higher dimensional origin and relation to string theory of these cases is unknown.

In this paper, we focus on $\mathcal{N} = 4$ supergravity, since the relevant aspects are very clear in this case. Both moduli stabilization and de Sitter vacua crucially depend on a specific property of the gauging. First, the gauge group needs to be a product of factors. In addition,

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1 This has been reviewed in [1, 2].
2 Early references are [3, 4].
these gauge factors need to have different angles with respect to the electro-magnetic $SL(2)$ symmetry that rotates vectors into their electro-magnetic duals [10]. As will be discussed in more detail later, without such a structure the scalar potential $V$ has an overall exponential dependence on the dilaton, making it impossible to stabilize the dilaton at a finite value of $V$. Therefore, it is crucial to have a product of gauge factors with relative $SL(2)$ angles, i.e. **gaugings at angles**.

Despite many results on the relation between $\mathcal{N} = 4$ gaugings and their higher dimensional ancestors, see e.g. [11–17], the higher dimensional origin of non-trivial $SL(2)$ angles has never been clearly pointed out. In this paper, we work out in detail a simple orientifold reduction and identify the resulting $\mathcal{N} = 4$ supergravity. The latter turns out to have gaugings at angles, thus providing a higher dimensional origin for this feature. In particular, we show how moduli stabilization is achieved by combining contributions to the scalar potential that originate from the bulk action and from the local source terms due to the orientifold. By clarifying the relation between gaugings at angles and orientifold reductions, we aim to close the gap between the ‘top-down’ and ‘bottom-up’ approaches.

The organization of this paper is as follows. In section 2, we review a number of general aspects of $\mathcal{N} = 4$ supergravity, after which we focus on a particular(ly useful) truncation. The structure of the gauging and scalar potential is emphasized. Section 3 discusses the orientifold reduction of IIA. Again we restrict ourselves to the simple truncation and show the equivalence to a specific $\mathcal{N} = 4$ theory. Finally, section 4 contains our conclusions and a number of remarks on possible extensions and the relation to other work.

2. $\mathcal{N} = 4$ gauged supergravity

In this section we discuss the structure of the $\mathcal{N} = 4$ theory and its gaugings. We will briefly summarize the general case and focus on a simple truncation which, while technically almost trivial, nevertheless retains the special feature of gaugings at angles that we want to highlight.

In the following section, this will be related to a simple orientifold reduction of IIA. For the general $\mathcal{N} = 4$ discussion we follow the conventions of [16], where more details and further references can be found.

The scalars of $D = 4, \mathcal{N} = 4$ supergravity parametrize a scalar coset of the form

$$\frac{SL(2)}{SO(2)} \times \frac{SO(6, 6 + n)}{SO(6) \times SO(6 + n)}. \quad (2.1)$$

The first factor contains the scalars of the supergravity multiplet. It is denoted by $M_{\alpha\beta}$, for which we use the following explicit parametrization:

$$M_{\alpha\beta} = e^\phi \left( \begin{array}{cc} \chi^2 + e^{-2\phi} & -\chi \\ -\chi & 1 \end{array} \right), \quad \alpha = (+, -). \quad (2.2)$$

The $SL(2)$ indices are raised and lowered with $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta}$, where $\epsilon^{+-} = -\epsilon^{-+} = 1$. The second factor in (2.1) is spanned by the matter multiplets. We focus on the case of six matter multiplets, corresponding to $n = 0$. In this case, it is convenient to use light-cone coordinates for the $SO(6, 6)$ group. The invariant metric is of the form

$$\eta_{MN} = \eta^{MN}_{\bar{I}\bar{J}} = \left( \begin{array}{c} I_6 \\ \bar{I}_6 \end{array} \right), \quad M = (1, \ldots, 6, \bar{1}, \ldots, \bar{6}). \quad (2.3)$$

It was anticipated in [18] that orientifold reductions involving the Romans’ mass parameter and NS–NS flux would lead to non-trivial $SL(2)$ angles. However, no orientifold contributions and tadpole conditions were included (this was done subsequently for $\mathcal{N} = 1$ in [19]). More recently, the connection between orientifold reductions of massive IIA and non-trivial $SL(2)$ angles has been conjectured in [20].
The corresponding $SO(6, 6)$ element that parametrizes the scalar coset is denoted by $M_{MN}$. We will introduce an explicit parametrization later. Together with the Einstein–Hilbert term for the metric, the scalars have the following kinetic terms:

$$L_{\text{kin}} = \sqrt{-g} \left[ R + \frac{1}{8} \partial_{\mu} M_{\alpha \beta} \partial^{\mu} M^{\alpha \beta} + \frac{1}{8} \partial_{\mu} M_{MN} \partial^{\mu} M^{MN} \right].$$ (2.4)

In addition, the theory contains $12 + n$ vectors, transforming into the fundamental representation of $SO(6, 6 + n)$. A noteworthy feature is that under the compact part of the $SL(2)$ symmetry these transform into their electro-magnetic dual. This symmetry is therefore only realized on-shell. This is a particular feature of four-dimensional theories and leads to the following intricate structure of gaugings.

The possible gaugings of this theory have been classified within the framework of the embedding tensor [21, 22]. It turns out that one can introduce two $SO(6, 6)$ representations of gauge parameters: an anti-symmetric 3-form $f_{\alpha MNP}$ and a fundamental $\xi_{\alpha M}$, both of which transform as a doublet under $SL(2)$. Consistency of such gaugings requires a number of quadratic constraints on the embedding tensor, which can be seen as generalized Jacobi identities. For later purposes, we will give the constraints for the case with $\xi_{aM} = 0$, for which one finds

$$f_{\alpha R[MNP} f_{\beta PQR]} = 0, \quad \epsilon^{\alpha \beta} f_{\alpha MNP} f_{\beta PQ} = 0. \quad (2.5)$$

The combination of supersymmetry and gaugings induces the following scalar potential:

$$L_{\text{pot}} = -\sqrt{-g} V, \quad V = \frac{1}{8} f_{\alpha MNP} f_{\beta QRS} M^{\alpha \beta} \left[ \frac{1}{4} M^{MO} M^{NR} M^{PS} + \left( \frac{1}{2} M^{MO} - M^{MO} \right) \eta^{NR} \eta^{PS} \right] - \frac{1}{16} f_{\alpha MNP} f_{\beta QRS} e^{\alpha \beta} M^{MNQR} + \frac{3}{8} \xi_{a} f^{a MNP}, \quad (2.6)$$

where the definition of $M^{MNPQRS}$ in terms of $M^{MN}$ can be found in [16].

As mentioned before, an important aspect of this four-dimensional supergravity is that vectors are transformed into their electro-magnetic dual under the $SO(2) \subset SL(2)$ symmetry. This on-shell symmetry is responsible for the $SL(2)$ doublet structure of the gauge parameters. Depending on the $SL(2)$ orientation, the embedding tensor picks out a vector or its dual (or a linear combination) to gauge a part of the global symmetry of the theory. Moreover, when the gauge group is a product of different factors, it is possible to choose a different $SL(2)$ orientation for the different factors. In terms of the embedding tensor, this corresponds to

$$f_{\alpha MNP} = \sum_{i} \xi_{a} f_{i MNP}^{(i)}, \quad \xi_{aM} = \sum_{i} \delta_{a}^{(i)} \xi_{M}^{(i)}. \quad (2.7)$$

where $f_{i MNP}^{(i)}$ and $\xi_{M}^{(i)}$ specify a factor of the gauge group and $\delta_{a}^{(i)}$ do not necessarily point in the same $SL(2)$ directions. This possibility is referred to as different $SL(2)$ (or $SU(1, 1)$) or de Roo–Wagemans angles [10]. If all the $SL(2)$ factors are identical, one can always rotate these to the $a = +$ direction, corresponding to a zero angle. It follows from (2.6) that in such cases the scalar potential has an overall dependence of $e^{\phi}$ and hence a runaway direction. Therefore gaugings at angles play a crucial role in moduli stabilization [6, 7].

Instead of the full $\mathcal{N} = 4$ supergravity we consider the following truncation. The $SO(6, 6)$ symmetry can be decomposed into

$$SL(3) \times SL(3) \times \mathbb{R}^{+} \times \mathbb{R}^{+} \subset SL(6) \times \mathbb{R}^{+} \subset SO(6, 6). \quad (2.8)$$

We focus on the subsector of the theory that is invariant under both $SL(3)$ factors. The group-theoretic nature of this truncation guarantees its consistency.

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4 We have multiplied the total action of [16] with a factor of 2.
A drastic consequence is that all vectors are projected out. This follows from the decomposition of the fundamental representation of $SO(6, 6)$ into $SL(3) \times SL(3)$ (omitting the $\mathbb{R}^+$ weights):

$$12 \rightarrow (3, 1) \oplus (3', 1) \oplus (1, 3) \oplus (1, 3'),$$

(2.9)

where no singlets appear.

In the scalar sector, the $SL(2)$ scalars are unaffected by this truncation. In contrast, from the decomposition of the adjoint representation one learns that many of the $SO(6, 6)$ scalars are projected out:

$$66 \rightarrow (1, 1) \oplus (1 \oplus 3 \oplus 3', 1 \oplus 3 \oplus 3') \oplus (1, 8) \oplus (8, 1).$$

(2.10)

Since there are only two singlets, the truncation preserves two dilatonic scalars. One can take the following parametrization of the $SO(6, 6)$ element $M_{MN}$ in terms of these scalars $\varphi_1$ and $\varphi_2$:

$$M_{MN} = \begin{pmatrix} e^{-\sqrt{2}\varphi_1} & e^{-\sqrt{2}\varphi_2} \\ e^{\sqrt{2}\varphi_1} & e^{\sqrt{2}\varphi_2} \end{pmatrix} \otimes 1_3.$$  

(2.11)

Inserting this in (2.4) gives rise to the following kinetic terms for the four scalars that survive the truncation:

$$L_{\text{kin}} = \sqrt{-g} \left[ R - \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} e^{2\varphi} (\partial \chi)^2 - \frac{1}{2} (\partial \varphi_1)^2 - \frac{1}{2} (\partial \varphi_2)^2 \right].$$  

(2.12)

where we have also included the Einstein–Hilbert term for the metric.

We now come to effect of the truncation of the embedding tensor. As the components $\xi_{aM}$ also transform into the fundamental representation of $SO(6, 6)$, these suffer the same fate as the vectors, and are all projected out. The other components $f_{aMNP}$ give rise to a number of $SL(3) \times SL(3)$ representations, including four singlets (omitting the $SL(2)$ doublet structure):

$$220 \rightarrow 4 \cdot (1, 1) \oplus \text{non-singlet representations}.$$  

(2.13)

So there are four $SO(6, 6)$ components that survive the truncation. In our light-cone basis these correspond to $f_{a123}$, $f_{a456}$, $f_{a125}$ and $f_{a356}$. Moreover, the quadratic constraints (2.5) result in the simple conditions

$$f_{a123} f_{b123} = 0, \quad f_{a456} f_{b356} = 0.$$  

(2.14)

Hence there are four possibilities of gauge parameters in this truncated theory, taking either nonzero ‘unbarred’ or ‘barred’ components in the (123) and independently in the (456) directions.

These four models can in fact be related to each other by particular elements of the global symmetry, which interchange the two types of light-cone directions. For example, an $SO(6, 6)$ transformation of the form (2.3) interchanges the six ‘unbarred’ and ‘barred’ directions. For an odd number of interchanged directions, this transformation needs to be accompanied by a sign flip of the $SL(2)$ axion. Therefore the four models are physically equivalent, and in the following we will only consider the case with gauge parameters $f_{a123}$ and $f_{b456}$.

Note that the model has a product of gauge factors\(^5\): one in the (123) directions and one in the (456) directions. These gaugings are specified by four real parameters: two can be seen as gauge coupling constants while the other two correspond to the $SL(2)$ angles of the two

\(^5\) This is a slight abuse of notation, as the truncated model does not have any vectors. However, by including the fields that have been truncated out, this model can be restored to a unique $N = 4$ supergravity with a gauging defined by these parameters.
gauge factors. One of the angles can be set equal to zero, i.e. point in the $\alpha = +$ direction, by an $SO(2) \subset SL(2)$ transformation. We will use this to rotate away $f_{-123}$. Moreover, if $f_{-456}$ does not vanish, one can perform an $SL(2)$ transformation that shifts the axion to set the second angle to $90^\circ$. This corresponds to setting $f_{+456} = 0$. In the case of two different angles, one can therefore always take these orthogonal. We will not use this and keep the second angle arbitrary, however.

Let us analyse the form of the scalar potential and its extrema for the truncated model. By writing out the general scalar potential (2.6) and using $f_{-123} = 0$, one finds

$$V = \frac{1}{4} (f_{123} e^{Φ/2\sqrt{2}v_2} - f_{-456} e^{-Φ/2\sqrt{2}v_2})^2 + \frac{1}{2} (f_{+456} + Φf_{-456})^2 e^{Φ+\sqrt{2}v_2}. \quad (2.15)$$

Strikingly, the potential combines into the sum of two squares and is positive definite. This relies crucially on the different $SL(2)$ angles: the crossterm in the first square is independent of the $SL(2)$ scalars and comes from the last line of (2.6). Only in the presence of such terms one can have moduli stabilization. In the extremum with respect to $Φ$, the term on the second line vanishes. The remaining square has an extremum if and only if $f_{+123}$ and $f_{-456}$ have equal signs, while the dilatonic scalars are such that the first square in the potential vanishes as well. In the extremum both squares that make up the scalar potential vanish, and we have a Minkowski solution.

Next, we investigate the issue of stability. The axion can be seen to decouple from this issue as $δΦ V$ is positive while $δΦ δV$ vanishes, where $Φ$ represents the three dilatonic scalars and $δV$ etc. The matrix $δΦ δV$ turns out to have one positive and two vanishing eigenvalues. The Minkowski solution is therefore a minimum of the scalar potential—at least in the truncation to $SL(3) \times SL(3)$ invariant scalars that we consider.

An interesting question is which gaugings are induced by gauge parameters of the form above. The answer can be found in [23], where so-called $CSO(p,q,r)$-gaugings are considered. These groups can be seen as group contractions of $SO(p'+q')$ with $p' + q' = p + q + r$. It turns out that each component of the structure constants that we consider induces a $CSO(1,0,3)$ gauging inside $SO(6,6)$. Our gauge group therefore consists of a product of two such factors. The total dimension of these gauge groups is 12, in accordance with the number of vectors. Reference [23] also performed a stability analysis with respect to all scalars and found a number of unstable directions. The Minkowski solution is therefore a saddlepoint of the full $\mathcal{N} = 4$ theory.

3. Orientifold reduction of IIA

In this section, we will consider a simple orientifold reduction of the IIA theory, which will be related to the previous $\mathcal{N} = 4$ truncation. Further details on different aspects and more complicated cases can be found in, e.g., [11–15, 18, 19, 24, 25].

Consider the toroidal reduction of massive IIA to four dimensions. Introducing an $O6$-plane corresponds to modding out by $(-)^F$, $I_{7,8,9}$. Here $(-)^F$ and $Ω$ correspond to the left-moving fermion number and world-sheet parity, respectively, whose combined action on the IIA bosonic fields is

$$\{g_{μν}, φ, ̂C_3, ̂C_7\} \rightarrow +\{g_{μν}, φ, ̂C_3, ̂C_7\},$$

$$\{B, ̂C_1, ̂C_5, ̂C_9\} \rightarrow -\{B, ̂C_1, ̂C_5, ̂C_9\}. \quad (3.1)$$

6 Modulo two typo’s in these expressions, the $CSO(1,0,3)$ structure constants given in appendix B of [23] in a Cartesian basis correspond to $f_{123}$ in our light-cone basis.
In addition, the spacetime parity operation \( I_{7,8,9} \) reverses the sign of three of the coordinates on the torus:
\[
\{x^7, x^8, x^9\} \rightarrow -\{x^7, x^8, x^9\}. \tag{3.2}
\]
The indices in (3.1) are taken inside the O6-plane, i.e. in the directions \((0,\ldots, 6)\). Other components with indices transverse to the O6-plane will acquire additional signs due to (3.2).

Furthermore, the Romans’ mass parameter \( \hat{G}_0 \) of IIA is invariant under the above involution.

Instead of the general orientifold reduction we will focus on the following truncation. Consider the two \( T^3 \)'s in the directions \( \{x^4, x^5, x^6\} \) and \( \{x^7, x^8, x^9\} \). Diffeomorphisms leaving the two factors separately invariant generate an \( SL(3) \times SL(3) \) symmetry in the four-dimensional description. Completely analogous to the truncation of the \( N = 4 \) theory of the previous section, we will retain only singlets with respect to both factors.

The most general Ansatz for the ten-dimensional metric that is consistent with \( SL(3) \times SL(3) \) invariance is of the form
\[
\hat{g}_{\mu
u} = \begin{pmatrix}
\sqrt{3/2 \sigma_1} g_{\mu \nu} & e^{-\sigma_2/2} \sqrt{3} \sigma_1 / \sqrt{3} \sigma_3 & e^{-\sigma_2/2} \sqrt{3} \sigma_3 / \sqrt{3} \sigma_3 \\
e^{-\sigma_2/2} \sqrt{3} \sigma_1 / \sqrt{3} \sigma_3 & \sigma_2 & \sqrt{2/3} \sigma_1 \\
e^{-\sigma_2/2} \sqrt{3} \sigma_3 / \sqrt{3} \sigma_3 & \sqrt{2/3} \sigma_3 & \sigma_2
\end{pmatrix}, \tag{3.3}
\]
consisting of the four-dimensional metric \( g_{\mu \nu} \) and two scalars \( \sigma_2 \) and \( \sigma_3 \). Both the Kaluza–Klein vectors and other scalars, parametrizing deformations of the internal torus, are projected out by the truncation to \( SL(3) \times SL(3) \) invariant fields. Furthermore, the normalization of \( \sigma \)'s is chosen to ensure canonical normalization. The ten-dimensional bulk action is\(^7\)
\[
\hat{S} = 2\pi \int d^{10}x (\hat{\mathcal{L}}_1 + \hat{\mathcal{L}}_2), \tag{3.4}
\]
where the first term contains the Einstein–Hilbert term and the dilaton kinetic term, while the second term is concerned with the gauge potentials. For the first part, after reduction to four dimensions we find
\[
\hat{\mathcal{L}}_1 = \sqrt{-g} \left[ \hat{R} - \frac{1}{2} (\hat{\phi})^2 \right] \rightarrow \mathcal{L}_1 = \sqrt{-g} \left[ R - \frac{1}{2} \sum_{i=1,2,3} (\partial \sigma_i)^2 \right], \tag{3.5}
\]
where we have set \( \hat{\phi} = \sigma_1 \). Note that we use the Einstein frame both in ten and in four dimensions.

Next, we turn to the gauge potentials. The NS–NS 2-form potential is odd under (3.1) and hence has to wrap an odd cycle in the torus. However, there is only one such form that is invariant under \( SL(3) \times SL(3) \): a 3-form. The field strength of this gauge potential therefore only gives a constant parameter \( h_3 \):
\[
\hat{H} = d\hat{B} = h_3 \, dx^7 \wedge dx^8 \wedge dx^9. \tag{3.6}
\]
The R–R gauge potentials are either even or odd. First of all, the Romans’ mass parameter, which can be seen as a 0-form R–R field strength, is even and also gives rise to a constant parameter in four dimensions: \( \hat{G}_0 = g_0 \). The R–R vector is odd under the orientifold involution. Its field strength necessarily vanishes,
\[
\hat{G}_2 = d\hat{C}_1 + \hat{G}_0 \hat{B} = 0, \tag{3.7}
\]
as there are no odd \( SL(3) \times SL(3) \) invariant zero-, one- or two-cycles on the torus. Finally, the R–R 3-form is even. Its magnetic part will be proportional to the even \( SL(3) \times SL(3) \) invariant 3-form and give rise to a scalar \( \chi \),
\[
\hat{G}_4^{(m)} = d\hat{C}_3 - \hat{H} \wedge \hat{C}_1 + \frac{1}{2} \hat{B} \wedge B = d\chi \wedge dx^4 \wedge dx^5 \wedge dx^6. \tag{3.8}
\]
\(^7\) Our IIA conventions agree with, e.g., [26]. To avoid cluttering our formulae we have set \( 4\pi^2 a' = 1 \).
It can also have an electric part. This will be more conveniently described in terms of the dual field strength, which is related by \( \hat{G}_6^{(m)} = e^{\phi/2} \hat{G}_4^{(e)} \). The dual 5-form gauge potential is odd under (3.1) and can wrap the total six-torus:
\[
\hat{G}_6^{(m)} = d\hat{C}_5 - \hat{H} \wedge \hat{C}_3 + \frac{1}{6} \hat{B} \wedge B = (g_6 + h_3 \chi) \, dx^4 \wedge \cdots \wedge dx^9.
\] (3.9)
Quantization of these parameters requires \( g_6, h_3 \) and \( g_6 \) all to be integer. Moreover, we will assume \( g_0 h_3 \) to be positive, for reasons that will become clear later.

With the Ansätze above, the kinetic terms for the IIA gauge potentials reduce to a kinetic term for \( \chi \) and potential terms for the three constants \( h_3, g_0 \) and \( g_6 \):
\[
\hat{L}_2 = \sqrt{-g} \left[ -\frac{1}{2} e^{-\phi/2} \hat{H} \cdot \hat{H} - \frac{1}{4} e^{\phi/2} \hat{G}_6^2 - \frac{1}{2} e^{\phi/2} \hat{G}_4^2 \cdot \hat{G}_4^{(m)} - \frac{1}{2} e^{-\phi/2} \hat{G}_6^{(m)} \cdot \hat{G}_6^{(m)} \right] \rightarrow
\hat{L}_2 = \sqrt{-g} \left[ -\frac{1}{2} e^{\phi/2} \hat{G}_6^2 \cdot \hat{G}_4 - \frac{1}{2} e^{-\phi/2} \hat{G}_6^{(m)} \cdot \hat{G}_6^{(m)} \right] - \frac{1}{2} g_0^2 e^{\phi/2} \sqrt{g} \sigma_1 - \frac{1}{2} (g_6 + h_3 \chi)^2 e^{-\phi/2} \sqrt{g} \sigma_3.
\] (3.10)

Note that there are no topological Chern–Simons terms in the democratic formulation of IIA [27]; the kinetic terms for the different R–R potentials suffice. These are therefore all the contributions from the ten-dimensional bulk action.

In addition to the bulk, one must also include the orientifold planes induced by (3.1) and (3.2). We further allow for a number of D6-branes with the same orientation (ignoring the world-volume excitations). These give rise to the following contributions to the scalar potential:
\[
\hat{S}_{O6/D6} = 2\pi N \int d^7x [e^{3/4\phi} \sqrt{-g}] \rightarrow \hat{L}_3 = \sqrt{-g} [N e^{3/4\phi_1 + 3/4\phi_2 + 3/4\phi_3}].
\] (3.11)
where \( N = 2N_{O6} - N_{D6} \). An orientifolded three-torus has \( 2^3 \) fixed points under (3.2) and would lead to \( N_{O6} = 8 \). The unusual dilaton coupling stems from the fact that we are using the Einstein frame. Furthermore, we have not included the Wess–Zumino term, as this will not contribute to the four-dimensional action. The total resulting action consists of three pieces (3.5), (3.10) and (3.11).

The Bianchi identities for the different field strengths read
\[
d\hat{H} = 0, \quad d\hat{G}_{2n+2} = \hat{H} \wedge \hat{G}_{2n}.
\] (3.12)
These are satisfied by the Ansätze above modulo the following two points. The first is that, due to the Wess–Zumino term of the O6-planes and D6-branes that involves \( \hat{C}_7 \), the Bianchi identity of \( \hat{G}_7 \) is modified:
\[
d\hat{G}_2 = \hat{G}_6 \hat{H} - N \, dx^7 \wedge dx^8 \wedge dx^9, \quad \Rightarrow \quad g_0 h_3 = N,
\] (3.13)
leading to a tadpole condition that will be essential. Furthermore, the reader might worry about the Bianchi identity for the electric part of \( \hat{G}_6 \), which does not vanish identically. However, this will be proportional to the four-dimensional field equation for \( \chi \) and vanishes on-shell.

Turning to the three pieces of which the action consists, we can now appreciate the beauty of the orientifold reduction and the underlying supersymmetry. The contribution due to the orientifold is such that the scalar potential terms (3.10) and (3.11) involving \( g_0 \) and \( h_3 \) can be combined into a square. This crucially relies on the tadpole condition (3.13). The scalar potential is now a positive definite sum of two squares.

The orientifold breaks half of supersymmetry and the resulting four-dimensional description is an \( \mathcal{N} = 4 \) supergravity. Since our truncation to \( SL(3) \times SL(3) \) singlets
coincides with that of the previous section, there must be a relation to the model discussed there. Indeed the two can be related by the following field redefinition for \( \sigma ' s:\)

\[
\begin{pmatrix}
  \phi \\
  \varphi_1 \\
  \varphi_2
\end{pmatrix} = \frac{1}{4\sqrt{2}} \begin{pmatrix}
  \sqrt{2} & 3\sqrt{3} & -\sqrt{3} \\
  \sqrt{6} & 1 & 5 \\
  -2\sqrt{6} & 2 & 2
\end{pmatrix}
\begin{pmatrix}
  \sigma_1 \\
  \sigma_2 \\
  \sigma_3
\end{pmatrix},
\]

(3.14)

in terms of the \( SL(2) \) dilaton \( \phi \) and the \( SO(6, 6) \) dilatons \( \varphi_1 \) and \( \varphi_2 \) of the previous section. Furthermore, one must identify the gauge parameters of both models as

\[
(f_{+123}, f_{-456}, f_{+456}) = \sqrt{2}(g_0, h_3, g_6).
\]

(3.15)

These redefinitions turn the Lagrangian consisting of (2.12) and (2.15) into that consisting of (3.5), (3.10) and (3.11). Moreover, since the \( SL(3) \times SL(3) \) invariant model defines a unique \( \mathcal{N} = 4 \) gauged supergravity, this connection extends to the full theory: an orientifold reduction that retains all fields and includes these three fluxes will lead to an \( \mathcal{N} = 4 \) supergravity with gauge parameters (3.15).

Our simple orientifold reduction therefore leads to an \( \mathcal{N} = 4 \) supergravity with \( CSO(1, 0, 3) \times CSO(1, 0, 3) \) gauge group, where the two gauge factors have a non-vanishing relative \( SL(2) \) angle. Note that the tadpole condition implies a relation on the gauge parameters: they have to be of the form (3.15) with \( g_0, h_3, g_6 \) integer and subject to \( g_0 h_3 = N \). Furthermore, the condition on the signs of \( f_{+123} \) and \( f_{-456} \) of the previous section justifies our assumption that \( g_0 h_3 \) is positive.

4. Discussion and outlook

In the previous sections, we have seen that the simple IIA orientifold reduction with fluxes \( (g_0, h_3, g_6) \) leads to the \( \mathcal{N} = 4 \) supergravity with \( CSO(1, 0, 3) \times CSO(1, 0, 3) \) gauge group of [23]. The two gauge factors generically are at a non-vanishing \( SL(2) \) angle with respect to each other, leading to moduli stabilization. From the orientifold side, this important feature was achieved by a collaboration of contributions to the scalar potential from the IIA bulk action (3.10) and the local source terms (3.11) due to the orientifolding. In order to avoid this, one must tune the \( O6/D6 \) content such that \( N = 0 \), in which case the two gauge groups have the same angle or one of the them disappears. Thus we have clarified the higher dimensional origin of the important \( \mathcal{N} = 4 \) phenomenon of \( SL(2) \) angles. Our simple model demonstrates that such gaugings at angles will be a generic outcome of IIA orientifold reductions.

Due to \( T \)-duality our results can be related to other orientifold cases. For instance, consider the case where we \( T \)-dualize in the three toroidal directions \( (x^4, x^5, x^6) \) of the \( O6 \)-plane worldvolume. The resulting IIB reduction involves an \( O3 \)-plane and has been studied at length in, e.g., [11, 12, 28, 29]. Our results have a clear counterpart in this IIB case. The parameters \( g_0, h_3 \) and \( g_6 \) now come from the IIB 3-form components \( \hat{G}_{456}, \hat{H}_{780} \) and \( \hat{G}_{789} \), respectively. The tadpole condition relates \( D3 \)-branes and \( O3 \)-planes to a contribution due to the complex 3-form flux, and the resulting action also contains a sum of squares. In this case, the vanishing of the squares corresponds to the well-known imaginary self-duality condition on the 3-form flux. Again the non-trivial \( SL(2) \) angles play an important role in the stabilization of moduli.

On the other hand, one could consider \( T \)-duality in any of the directions \( (x^7, x^8, x^9) \) transverse to the \( O6 \)-plane. In contrast to the previous case, \( T \)-duality in these directions does not leave the 3-form flux invariant. Instead it has been argued that this will be transformed into geometric or even non-geometric flux [30]. Therefore \( T \)-duality in the transverse directions,
giving rise to O7-, O8- or O9-planes, does not lead to the simple reductions we considered with only gauge fluxes.

Coming back to the O6-plane, the reduction to four dimensions can in fact be split up into two steps. The first consists of the reduction over the transverse space of the orientifold, while the second reduces over its three toroidal world-volume directions. One could stop after the first step and thus obtain a seven-dimensional half-maximal supergravity theory with two parameters $g_0$ and $h_3$ (the remaining $g_6$ only shows up after the second step). The gaugings of this theory are encoded in representations $\xi_m$ and $f_{mnp}$ of $SO(3,3)$, while there is a single topological mass parameter $m$ [31]. Restricting to $SL(3,\mathbb{R})$ invariant components leads to $f_{123}$ and $f_{\bar{1}\bar{2}\bar{3}}$ in addition to the mass parameter. The product of the two gauge parameters vanishes due to the Jacobi identity. Therefore, the orientifold parameters $g_0$ and $h_3$ are to be identified with $m$ and, e.g., $f_{123}$. A subsequent normal toroidal reduction to four dimensions leads to the theory that we have considered in this paper.

Two lessons can be drawn from this discussion. First, the $D=7$ topological mass parameter $m$ has a higher dimensional origin from orientifold reductions. Second, gaugings at angles in four dimensions are induced by a toroidal reduction of the seven-dimensional massive theory. To our knowledge, this is the first time that a higher dimensional origin of gaugings at angles from $4 < D \leq 7$ half-maximal supergravity has been put forward. It would be interesting to investigate this connection in more detail. Due to the above discussion involving $Op$-planes with $p > 6$ we do not expect such an origin from dimensions higher than 7. This ties in nicely with the absence of mass parameters in these theories [31].

In this paper, we have restricted ourselves to a very simple truncation to $SL(3) \times SL(3)$ singlets. Needless to say this can be relaxed to allow for many more possibilities [11–15]: different components of gauge fluxes can be turned on and one could reduce over twisted tori with non-vanishing geometric fluxes $\omega$. This would lead to additional structure constants, inducing different gaugings of the four-dimensional $\mathcal{N} = 4$ theory. For instance, including $\omega$ and $G_2$ fluxes in a specific way could lead to $C SO(3,0,1) \times C SO(3,0,1)$ gaugings [18]. It would be interesting to investigate a possible relation to the $SU(2) \times SU(2)$ reduction of [32]. Furthermore, such reductions might give a higher dimensional origin to the unstable de Sitter vacua of [6,7].

Finally, one can consider orientifold reductions that break more supersymmetry. It would be of great interest if one could find, e.g., a string-theoretic origin for the stable de Sitter vacua in $\mathcal{N} = 2$ supergravity [8,9], for which non-trivial $SL(2)$ angles are a necessary ingredient.

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References

[1] Grana M 2006 Flux compactifications in string theory: a comprehensive review Phys. Rep. 423 91–158 (arXiv:hep-th/0509003)
[2] Douglas M R and Kachru S 2007 Flux compactification Rev. Mod. Phys. 79 733–96 (arXiv:hep-th/0610102)
[3] Kachru S, Kallosh R, Linde A and Trivedi S P 2003 De Sitter vacua in string theory Phys. Rev. D 68 046005 (arXiv:hep-th/0301240)
[4] Burgess C P, Kallosh R and Quevedo F 2003 De Sitter string vacua from supersymmetric D-terms J. High Energy Phys. JHEP10(2003)056 (arXiv:hep-th/0309187)
