Mass spectra and radiative transitions of doubly heavy baryons in a relativized quark model

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We study the mass spectra and radiative decays of doubly heavy baryons within the diquark picture in a relativized quark model. The mass of the $J^P = 1/2^+$ state is predicted to be 3606 MeV, which is consistent with the mass of $\Xi_{cc}^{++}(3621)$ newly observed by the LHCb collaboration. Theoretical predictions of doubly heavy baryons are also estimated by using the realistic wave functions obtained from relativized quark model. The radiative decay widths of $\Xi_{cc}^{++} \to \Xi_{cc}^{++}\gamma$ and $\Xi_{cc}^{++} \to \Xi_{cc}^{+}\gamma$ are predicted to be about 7 and 4 keV, respectively. These predictions of doubly heavy baryons can provide helpful information for future experimental searches.

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I. INTRODUCTION

The doubly heavy baryons, made up of two heavy quarks and one light quark, are particularly interesting because they provide a new platform for studying the heavy quark symmetry and chiral dynamics simultaneously [1–3]. To looking for the doubly heavy baryons, great efforts were made in the past two decades. In 2002, the SELEX collaboration reported some evidence of a doubly charmed baryon $\Xi_{cc}^{++}(3519)$ in the $\Lambda_c^+ K^-\pi^+$ final state, and confirmed it in the $pD^* K^-\pi^+$ decay mode [4, 5]. However, $\Xi_{cc}^{++}(3519)$ cannot be confirmed by the FOCUS, BaBar, Belle and LHCb collaborations [6–9]. Very recently, the LHCb collaboration observed a highly significant structure with a mass of $M \approx 3621$ MeV in the $\Lambda_c^+ K^-\pi^+\pi^-$ mass spectrum in the proton-proton collisions [10]. This structure can be identified as a doubly charmed baryon (denoted by $\Xi_{cc}^{++}(3621)$) from its weakly decaying behaviors. Due to the large mass difference, the newly observed $\Xi_{cc}^{++}(3621)$ by the LHCb collaboration and $\Xi_{cc}^{++}(3519)$ reported by the SELEX collaboration can be hardly categorized as a isodoublet [10–11].

In theory, the mass spectra of the doubly heavy baryons have been predicted with various methods, such as the constituent quark model [12–20], heavy quark symmetry and mass formulas [21–24], Regge behaviors [25–26], QCD sum rule [27–32], lattice QCD [33–35] and so on. For the lowest $\Xi_{cc}$ state with $J^P = 1/2^+$, the theoretical predicted masses lie in a large range 3500 ~ 3700 MeV, thus, both $\Xi_{cc}^{++}(3621)$ and $\Xi_{cc}^{++}(3519)$ can be candidates of the ground $\Xi_{cc}$ state with $J^P = 1/2^+$. To clarify which one should be the lowest $\Xi_{cc}$ state, further investigations are needed. Besides the mass spectra, the weak decays of the doubly heavy baryons are also extensively discussed in the literature [36–43]. However, the studies on the radiative decays are scarce, only several works focusing on the ground doubly heavy baryons are found in the literature [44–49]. A comprehensive review on the doubly heavy baryons can be found in Ref. [1].

Lately, stimulated by the newly observed state $\Xi_{cc}^{++}(3621)$ by the LHCb collaboration, several theoretical works were performed. Using the QCD sum rule, Chen et al. investigated the low-lying doubly charmed baryon spectra, which suggests the newly observed $\Xi_{cc}^{++}(3621)$ may be the ground $1/2^+$ state [50]. The masses and wave functions of doubly charmed baryons were also restudied with the Cornell potential [51]. The radiative transitions between the ground doubly charmed baryons were investigated within the heavy baryon chiral perturbation theory [52] and constituent quark model with simple harmonic oscillator wave functions [53]. Furthermore, the magnetic moments, weak decays, and other related topics are also discussed in the literature [54–63].

In this work, we use a relativized quark model to calculate the mass spectra of doubly heavy baryons. The relativized quark model, proposed by Godfrey, Capstick, and Isgur, has been extensively used to study the properties of the conventional hadrons and gives a unified description of the hadron spectra [64–72]. Therefore, it is suitable to deal with the doubly heavy baryons, where both heavy-heavy and heavy-light systems are included. Moreover, the relativistic effects are also involved in this model, which may be essential for the light quarks. In Ref. [72], Capstick and Isgur adopted this relativized quark model to estimate various baryon spectra, however, they did not extend it to deal with the doubly heavy baryon spectra. Given the similarity between the heavy diquark and heavy antiquark, the doubly heavy baryons looks like heavy-light mesons in the diquark picture [13, 14, 73–81]. Within the diquark picture, we first calculate the masses and wave functions of the $cc$ and $bb$ diquarks. Then, the mass spectra of the doubly heavy baryons and the diquark-quark wave functions can be obtained by solving the Schrödinger-type equation between the diquark and quark. Finally, the total wave function of a doubly heavy baryon can be expressed as the diquark wave function multiplied by the diquark-quark wave function. It should be pointed out that when we treat

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the diquark-quark interactions, the diquark size effects to the potentials are also considered by introducing a form factor as that adopted in Ref. [14]. Our predicted mass for the ground doubly charmed baryon with $J^P = 1/2^+$ is 3606 MeV. It suggests that the newly observed $\Xi_{cc}^{++}(3621)$ state may be assigned as the $J^P = 1/2^+$ ground state.

Using the wave functions obtained from the relativized quark model, we further study the radiative transitions of doubly heavy baryons. In the calculations, we employ an EM transition operator which is extracted in the non-relativistic constituent quark model and has been successfully applied to the study of the radiative decays of $cc$ and $bb$ systems [82,83] and the heavy baryons [53,84]. Due to the absence of hadronic transitions for the low-lying states [53], these electromagnetic decays provide useful information of the internal structures. It is found that the partial decay widths for $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+} \gamma$ and $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{0} \gamma$ are predicted to be about 7 and 4 keV, respectively, which is quite significant and helpful for future experimental searches.

This paper is organized as follows. The relativized quark model is briefly introduced, and then the masses of the heavy quarks are predicted to be about 7 and 4 keV, respectively, which is quite significant and helpful for future experimental searches.

The radiative decays of doubly heavy baryons are estimated for future experimental searches.

The structures of the doubly heavy baryons are illustrated in Fig. [1] where the two heavy quarks are treated as a diquark with a size. Here, the $\rho$-mode excitation corresponds to the internal excitation of the diquark, and the $\lambda$-mode excitation stands for the diquark-quark excitation. When the $cc$ and $bb$ diquarks locate in $1S$ wave, the spin-parities of the diquarks are $J^P = 0^+$ and $J^P = 1^+$, named as the scalar diquarks and axial diquarks, respectively. Constrained by the symmetry, the scalar diquark cannot exist in the $cc$ and $bb$ systems. In the doubly heavy baryons, the first excited state comes from the diquark ($\rho$-mode excitation) rather than the light quark ($\lambda$-mode excitation), so we have to discuss the excited diquark states as well. Here, we only consider the $1^{1}P_{1}$, $2^{3}S_{1}$, and $1^{3}D_{1}$ diquarks in the low lying baryons. We use the Gaussian expansion method to solve the Hamiltonian [11] with $\tilde{V}_{qg}(p, r)$ potential [85]. The obtained masses of these diquarks are presented in Table [1]. The form factors $F(r)$ are introduced to reflect the diquark sizes. The $F(r)/r$ can be obtained from the Fourier transforms of $F(k^2)/k^2$, and $F(k^2)$ can be taken as [14]

$$F(k^2) = \sqrt{\frac{\varepsilon_d M_d}{E_d + M_d}} \left[ \frac{2m_d}{(2\pi)^3} \right] \int \frac{d^3 p}{(2\pi)^3} \sqrt{2} \Psi_d(p+\frac{2m_d}{E_d + M_d}) \Psi_d(p)+(1 \leftrightarrow 2),$$

where $E_d, M_d, k, \Psi_d$ are the energy, mass, total momentum, and wave function of the diquark, respectively. The form factors can be further approximated by the following expression [14]

$$F(r) = 1 - e^{-\xi r - \zeta r^2},$$

where $\xi$ and $\zeta$ are the real numbers. The relevant parameters are calculated by using the obtained diquark wave functions, and also listed in Table [1].

With the diquarks listed in Table [1] one can calculate the masses of the doubly heavy baryons and the wave functions between diquarks and quarks. Then, the total wave function of the doubly heavy baryon can be expressed as the wave function of diquark ($\rho$ mode) multiplied by the wave function of diquark-quark ($\lambda$ mode). The masses of ground states of the doubly heavy baryons together with different calculations are presented in Tab. [11]. The predicted mass of $J^P = 1/2^+$ $\Xi_{cc}$ ground state is 3606 MeV, which is consistent with the newly observed $\Xi_{cc}^{++}(3621)$ by the LHCb collaboration. This assignment also agrees with many other works [14,16,34,50,51]. Our results indicate that the $\Xi_{cc}^{++}(3519)$ reported by SELEX collaboration may not be interpreted as the conventional $\Xi_{cc}^{++}$. The radiative decays of doubly heavy baryons are estimated for future experimental searches.

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FIG. 1: Doubly heavy baryon system with Jacobi coordinates defined by \( r_3 = (1/3, 2/3) \) and \( r_4 = (2/3, 1/3) \). Q and \( Q' \) stand for the heavy quark and light quark, respectively.

TABLE II: Masses of ground states of the doubly heavy baryons compared with different calculations. The units are in MeV.

| State | Our work | ROM | NQ3 | LOAD | T04 |
|-------|----------|-----|-----|------|-----|
| \( \Xi_{cc} S \) | 3610 | 3620 | 3640 | 3650 | 3660 |
| \( \Omega_{cc} S \) | 3675 | 3727 | 3676 | 3716 | 3692 |
| \( \Xi_{bb} D \) | 3610 | 3676 | 3610 | 3740 | 3753 |
| \( \Omega_{bb} D \) | 3692 | 3740 | 3692 | 3778 | 3788 |
| \( \Xi_{cc} P \) | 3610 | 3676 | 3610 | 3740 | 3753 |
| \( \Omega_{cc} P \) | 3692 | 3740 | 3692 | 3778 | 3788 |

III. RADIATIVE TRANSITIONS

Besides the mass spectra, the decay behaviors are also needed to be considered for doubly heavy baryons in experiments. Due to the suppression of phase space of heavy quarkonia in the decays, the radiative transitions should also be considered.

TABLE III: Predicted mass spectra of \( \Xi_{cc} \), \( \Omega_{cc} \), and \( \Xi_{bb} \). The units are in MeV.

| \( \Xi_{cc} \) | \( \Omega_{cc} \) | \( \Xi_{bb} \) |
|-------------|--------------|--------------|
| \( (D1/2)_{1s} \) | 3610 | 3676 |
| \( (S1/2)_{1s} \) | 3675 | 3727 |
| \( (S1/2)_{1p} \) | 3692 | 3740 |
| \( (S1/2)_{1d} \) | 3715 | 3753 |
| \( (S1/2)_{1f} \) | 3778 | 3798 |
| \( (S1/2)_{1g} \) | 3798 | 3808 |

From Table III, we can see that the \( \Omega_{cc} \) mass is lower than the \( \Xi_{cc} \) mass, while the \( \Xi_{bb} \) mass is higher than the \( \Omega_{cc} \) mass.
one-photon radiative decay of a hadron we apply an EM transition operator which has been successfully applied to study the radiative decays of $cc$ and $bb$ systems [82, 83] and $\Omega_c$ baryons [84]. In this model, the quark-photon EM coupling at the tree level is adopted as

$$H_e = -\sum_j e_j \bar{\psi}_j A^\mu(k, r_j) \psi_j,$$

where $\psi_j$ stands for the $j$th quark field with coordinate $r_j$ and $A^\mu$ is the photon field with three-momentum $k$. To match the wave functions obtained by the Schrödinger-type equation, we adopt the quark-photon EM couplings in a nonrelativistic form. In the initial-hadron-rest system, the approximate form can be written as [82, 83, 86–91]

$$h_e \equiv \sum_j \left[ e_j r_j \cdot \epsilon - \frac{e_j}{2m_j} \sigma_j \cdot (\epsilon \times \hat{k}) \right] e^{-ik \cdot r_j},$$

where $e_j$, $m_j$, and $\sigma_j$ stand for the charge, consistent mass, Pauli spin vector for the $j$th quark, respectively. The $\epsilon$ is the polarization vector of the final photon.

One can obtain the standard helicity amplitude $A$ of the radiative process [82, 83]

$$A = -i \sqrt{\frac{\omega_e}{2}} (f| h_e|i).$$

Then, we can estimate the radiative transitions straightforward [82, 83]

$$\Gamma = \frac{|k|^2}{\pi} \frac{2}{2J_i + 1} M_f \sum_{J_f, \ell_e} |A|^2,$$

where $J_i$ is the total angular momentum of the initial baryons, and $J_{f_2}$ and $J_{f_1}$ are the components of the total angular momenta along the $z$ axis of the initial and final baryons, respectively. In present calculation, the masses and wave functions of doubly heavy baryons are adopted from our theoretical predictions.

The radiative transitions between the $J^P = 3/2^+$ and $J^P = 1/2^+$ ground doubly charmed and bottom baryons are predicted, and the results together with different calculations are listed in Tab. IV. For the $J^P = 1/2^+$ ground state, only weak decays can occur. Since the strong decays are forbidden, the electromagnetic transitions dominate for the $J^P = 3/2^+$ ground state. The theoretical radiative decay widths for the $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}\gamma$ and $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+}\gamma$ transitions are

$$\Gamma[\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}\gamma] = 7.21 \text{ keV},$$

$$\Gamma[\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+}\gamma] = 3.90 \text{ keV},$$

respectively, which are roughly compatible with the quark model predictions [44], simple harmonic oscillator [53], the bag model [45, 46], and chiral perturbation theory calculations [52] in magnitude. However, the partial width ratio

$$\frac{\Gamma[\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}\gamma]}{\Gamma[\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+}\gamma]} = 1.85,$$

predicted by us shows special feature compared to other works [44, 46, 52, 53]. More studies of the radiative decay processes $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}\gamma$ and $\Xi_{cc}^{+} \rightarrow \Xi_{cc}^{+}\gamma$ are needed in the future. Since the LHCb collaboration has observed $\Xi_{cc}^{++}$ in the $\Lambda_c^+ K^-\pi^+$ mass spectrum, the large decay rate of $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+}\gamma$ indicates that they may establish $\Xi_{cc}^{++}$ in the $\Lambda_c^+ K^-\pi^+\gamma$ final state via the decay chain of $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}\gamma \rightarrow \Lambda_c^+ K^-\pi^+\gamma$. The radiative partial decay widths of $\Omega_{cc}^+ \rightarrow \Omega_{cc}^+\gamma$, $\Xi_{bb}^{0} \rightarrow \Xi_{bb}^{0}\gamma$, $\Xi_{bb}^{+} \rightarrow \Xi_{bb}^{+}\gamma$, and $\Omega_{bb}^{*} \rightarrow \Omega_{bb}\gamma$ are sensitive to the masses of the initial and final states. In present work, we predict sizeable partial decay widths for these processes. These information should be helpful for searching for the missing $J^P = 3/2^+$ ground states in future experiments.

The radiative decays of the low-lying excited doubly heavy baryons are studied as well. Our results are listed in Tab. IV. It is found that for the $(1P1s) \rightarrow (1S1s)\gamma$, $(1D1s) \rightarrow (1P1s)\gamma$, and $(2S1s) \rightarrow (1P1s)\gamma$ transitions, the partial radiative decay widths of doubly charmed baryons range from several ten to hundred eV, while the decays of doubly bottom baryons approximate vanish. It should be mentioned that the pionic or kaonic strong decays for $(1P1s)$, $(1D1s)$ and $(2S1s)$ containing diquark excitations are forbidden due to the orthogonality of the diquark wave functions. Hence, the $(1P1s)$, $(1D1s)$ and $(2S1s)$ doubly heavy baryons should be very narrow states, and their radiative transitions should play an important role in their decays. These radiative transitions may be useful for looking for the missing $(1P1s)$, $(1D1s)$ and $(2S1s)$ doubly charmed states. Furthermore, it is found that most of the transitions $(1S1p) \rightarrow (1S1s)\gamma$ have large partial decay widths, which can reach up to several hundred keV. However, the radiative decay rates of $(1S1p) \rightarrow (1S1s)\gamma$ may be seriously suppressed by the large decay widths of $(1S1p)$, because the decays of $(1S1p)$ are dominated by the strong decay modes. About the radiative transitions of the excited doubly heavy baryons, few discussions are found in the literature, thus, more studies are expected to be carried out in future.
TABLE IV: Radiative decay widths between the $J^P = 3/2^+$ and $J^P = 1/2^+$ ground doubly charmed and bottom baryons. The units are in keV.

| Transition                | Our work | RQM [44] | SHO [53] | BG [45] | BG [46] | CPT [52] |
|---------------------------|----------|----------|----------|---------|---------|----------|
| $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{+} \gamma$ | 7.21     | 23.46    | 16.7     | 4.35    | 1.43    | 22.0     |
| $\Xi_{cc}^+ \rightarrow \Xi_{cc}^0 \gamma$    | 3.90     | 28.79    | 14.6     | 3.96    | 2.08    | 9.57     |
| $\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 \gamma$ | 0.82     | 2.11     | 6.93     | 1.35    | 0.95    | 9.45     |
| $\Xi_{bb}^{0} \rightarrow \Xi_{bb}^{0} \gamma$ | 0.98     | 0.31     | 1.19     | ...     | ...     | ...      |
| $\Xi_{bb}^{-} \rightarrow \Xi_{bb}^{-} \gamma$ | 0.21     | 0.06     | 0.24     | ...     | ...     | ...      |
| $\Omega_{bb}^+ \rightarrow \Omega_{bb}^0 \gamma$ | 0.04     | 0.02     | 0.08     | ...     | ...     | ...      |

TABLE V: Partial decay widths of the radiative transitions of low-lying excited $\Xi_{cc}^{++}$, $\Xi_{cc}^{+}$, $\Omega_{cc}$, $\Xi_{bb}^{0}$, $\Xi_{bb}^{-}$, and $\Omega_{bb}$ states. The units are in keV.

| Transition                | $\Xi_{cc}^{++}$ | $\Xi_{cc}^{+}$ | $\Omega_{cc}$ | $\Xi_{bb}^{0}$ | $\Xi_{bb}^{-}$ | $\Omega_{bb}$ |
|---------------------------|----------------|---------------|---------------|----------------|----------------|---------------|
| $|(1S1s)1/2^- \rightarrow (1S1s)1/2^+ \gamma$ | 0.85           | 0.85          | 0.90          | ~0             | ~0             | ~0            |
| $|(1S1s)3/2^- \rightarrow (1S1s)1/2^+ \gamma$ | 1.73           | 1.73          | 1.57          | 0.01           | 0.01           | 0.01          |
| $|(1S1s)1/2^- \rightarrow (1S1s)3/2^+ \gamma$ | 0.39           | 0.39          | 0.57          | ~0             | ~0             | ~0            |
| $|(1S1s)3/2^- \rightarrow (1S1s)3/2^+ \gamma$ | 1.03           | 1.03          | 1.16          | 0.01           | 0.01           | 0.01          |
| $|(1D1s)1/2^- \rightarrow (1P1s)1/2^+ \gamma$ | 0.22           | 0.22          | 0.22          | ~0             | ~0             | ~0            |
| $|(1D1s)3/2^- \rightarrow (1P1s)1/2^+ \gamma$ | 0.04           | 0.04          | 0.05          | ~0             | ~0             | ~0            |
| $|(1D1s)1/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.12           | 0.12          | 0.11          | ~0             | ~0             | ~0            |
| $|(1D1s)3/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.20           | 0.20          | 0.22          | ~0             | ~0             | ~0            |
| $|(2S1s)1/2^- \rightarrow (1P1s)1/2^+ \gamma$ | 0.03           | 0.03          | 0.03          | ~0             | ~0             | ~0            |
| $|(2S1s)3/2^- \rightarrow (1P1s)1/2^+ \gamma$ | 0.01           | 0.01          | 0.02          | ~0             | ~0             | ~0            |
| $|(2S1s)1/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.15           | 0.15          | 0.12          | ~0             | ~0             | ~0            |
| $|(2S1s)3/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.17           | 0.17          | 0.14          | ~0             | ~0             | ~0            |
| $|(1S2s)1/2^- \rightarrow (1P1s)1/2^+ \gamma$ | ~0            | ~0            | ~0            | ~0             | ~0             | ~0            |
| $|(1S2s)3/2^- \rightarrow (1P1s)1/2^+ \gamma$ | ~0            | ~0            | ~0            | ~0             | ~0             | ~0            |
| $|(1S2s)1/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.05           | 0.05          | 0.01          | ~0             | ~0             | ~0            |
| $|(1S2s)3/2^- \rightarrow (1P1s)3/2^+ \gamma$ | 0.41           | 0.41          | 0.35          | 0.01           | 0.01           | 0.01          |
| $|(1S1p)1/2^-_{s_{1/2}=1/2} \rightarrow (1S1s)1/2^+ \gamma$ | 171.99         | 142.88        | 192.71        | 296.48         | 59.95           | 56.07        |
| $|(1S1p)3/2^-_{s_{1/2}=1/2} \rightarrow (1S1s)1/2^+ \gamma$ | 430.75         | 233.47        | 237.88        | 531.16         | 114.51          | 79.32        |
| $|(1S1p)1/2^-_{s_{1/2}=3/2} \rightarrow (1S1s)1/2^+ \gamma$ | 27.66          | 6.21          | 0.93          | 20.66          | 5.20            | 1.12         |
| $|(1S1p)3/2^-_{s_{1/2}=3/2} \rightarrow (1S1s)1/2^+ \gamma$ | 126.26         | 28.36         | 4.01          | 78.89          | 19.84           | 4.06         |
| $|(1S1p)5/2^- \rightarrow (1S1s)1/2^+ \gamma$ | 105.45         | 23.69         | 3.26          | 63.15          | 15.88           | 3.10         |
| $|(1S1p)1/2^-_{s_{1/2}=1/2} \rightarrow (1S1s)3/2^+ \gamma$ | 56.29          | 12.62         | 1.97          | 77.15          | 19.41           | 3.96         |
| $|(1S1p)3/2^-_{s_{1/2}=1/2} \rightarrow (1S1s)3/2^+ \gamma$ | 71.96          | 16.14         | 2.67          | 77.86          | 19.58           | 4.20         |
| $|(1S1p)5/2^- \rightarrow (1S1s)3/2^+ \gamma$ | 403.02         | 200.88        | 183.53        | 635.96         | 141.35          | 82.56        |
| $|(1S1p)3/2^-_{s_{1/2}=3/2} \rightarrow (1S1s)3/2^+ \gamma$ | 394.72         | 213.04        | 207.93        | 554.59         | 121.00          | 77.43        |
| $|(1S1p)5/2^- \rightarrow (1S1s)3/2^+ \gamma$ | 31.30          | 31.77         | 65.55         | 61.85          | 11.70           | 18.23        |

IV. SUMMARY

In this work, we study the mass spectra and radiative decays of doubly heavy baryons within the diquark picture in a relativized quark model. The diquark masses and wave functions are calculated from the relativized quark potential, and the doubly heavy baryon spectra are obtained by solving the Schrödinger-type equation between the diquark and
quark. The effects of the diquark sizes are considered by introducing the form factors. Besides the ground doubly heavy baryons, the masses for the low-lying excited doubly charmed and bottom baryons are given. The theoretical mass of the ground state $\Xi_{cc}^+$ ($J^P = 1/2^+$), 3606 MeV, is consistent with the mass of the newly observed $\Xi_{cc}^+(3621)$ by the LHCb collaboration. The predicted mass gap $m(\Xi_{cc}^+) - m(\Xi_{cc}^-)$ is about 69 MeV, which is waited to be tested in future experiments. Furthermore, using the realistic wave functions obtained from the relativized quark model and the quark-photon couplings, we evaluate the radiative transitions between these states. It is interesting to find that the \((1P1)s\), \((1D1)s\) and \((2S1)s\) doubly charmed baryons containing diquark excitations should be very narrow states with a width of several ten to hundred eV, the decays of these states should be dominated by the radiative transitions due to the absence of the strong decay modes. The radiative decay width of $\Xi_{cc}^{++} \to \Xi_{cc}^{++} \gamma$ and $\Xi_{cc}^{++} \to \Xi_{cc}^{++} \gamma$ are quite significant, which are up to about 7 and 4 keV, respectively. We hope these theoretical predictions of doubly heavy baryons should be helpful for future experimental exploration.

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A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Semileptonic decays of doubly heavy baryons, Phys. Lett. B 518, 55 (2001).

A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Semileptonic decays of double heavy baryons in a relativistic constituent three-quark model, Phys. Rev. D 80, 034025 (2009).

C. Albertus, E. Hernandez and J. Nieves, Hyperfine mixing in $b \to c$ semileptonic decay of doubly heavy baryons, Phys. Lett. B 683, 21 (2010).

M. J. White and M. J. Savage, Semileptonic decay of baryons with two heavy quarks, Phys. Lett. B 271, 410 (1991).

R. H. Li, C. D. Liu, W. Wang, F. S. Yu and Z. T. Zou, Doubly-heavy baryon weak decays: $\Xi_{cc}^+ \to pK^-$ and $\Xi_{bc}^- \to \Sigma^{+(2520)}K^-$, Phys. Lett. B 767, 232 (2017).

F. S. Yu, H. Y. Jiang, R. H. Li, C. D. Lü, W. Wang and Z. X. Zhao, Discovery Potentials of Doubly Charmed Baryons, arXiv:1703.09086.

D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Semileptonic decays of doubly heavy baryons in the relativistic quark model, Phys. Rev. D 70, 041008 (2004); Erratum: [Phys. Rev. D 77, 079903 (2008)].

W. Roberts and M. Pervin, Hyperfine Mixing and the Semileptonic Decays of Double-Heavy Baryons in a Quark Model, Int. J. Mod. Phys. A 24, 2401 (2009).

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij and B. Oexl, Radiative decays of double heavy baryons in a relativistic constituent three-quark model including hyperfine mixing, Phys. Rev. D 81, 114036 (2010).

R. H. Hackman, N. G. Deshpande, D. A.Dicus and V. L. Teplitz, M1 Transitions in the MIT Bag Model, Phys. Rev. D 18, 2537 (1978).

A. Bernotas and V. Šimonis, Radiative M1 transitions of heavy baryons in the bag model, Phys. Rev. D 87, 074016 (2013).

W. S. Dai, X. H. Guo, H. Y. Jin and X. Q. Li, Electromagnetic radiation of baryons containing two heavy quarks, Phys. Rev. D 62, 114026 (2000).

C. Albertus, E. Hernandez and J. Nieves, Hyperfine mixing in electromagnetic decay of doubly heavy $bc$ baryons, Phys. Lett. B 690, 265 (2010).

J. Hu and T. Mehen, Chiral Lagrangian with heavy quark-diquark symmetry, Phys. Rev. D 73, 054003 (2006).

H. X. Chen, Q. Mao, W. Chen, X. Liu and S. L. Zhu, Establishing low-lying doubly charmed baryons, arXiv:1707.01779.

B. O. Kerbikov, Doubly charmed baryon: an old prediction facing the LHCb observation, arXiv:1707.04031.

H. S. Li, L. Meng, Z. W. Liu and S. L. Zhu, Radiative decays of the doubly charmed baryons in chiral perturbation theory, arXiv:1708.03620.

L. Y. Xiao, K. L. Wang, Q. F. Lü, X. H. Zhong and S. L. Zhu, Strong and radiative decays of the doubly charmed baryons, arXiv:1708.04384.

H. S. Li, L. Meng, Z. W. Liu and S. L. Zhu, Magnetic moments of the doubly charmed and bottomed baryons, arXiv:1707.02765.

W. Wang, F. S. Yu and Z. X. Zhao, Weak Decays of Doubly Heavy Baryons: the $1/2 \to 1/2$ case, arXiv:1707.02834.

W. Wang, Z. P. Xing and J. Xu, Weak Decays of Doubly Heavy Baryons: SU(3) Analysis, arXiv:1707.06570.

L. Meng, N. Li and S. L. Zhu, Possible hadronic molecules composed of the doubly charmed baryon and nucleon, arXiv:1707.03598.

M. Karliner and J. L. Rosner, Discovery of doubly-charmed $\Xi_{cc}$ baryon implies a stable $bb\bar{u}\bar{d}$ tetraquark, arXiv:1707.07666.

E. J. Eichten and C. Quigg, Heavy-quark symmetry implies stable heavy tetraquark mesons $Q\bar{Q}q\bar{q}$, arXiv:1707.09575.

N. Brambilla, G. Krein, J. Tarrs Castell and A. Vairo, The Born-Oppenheimer approximation in an effective field theory language, arXiv:1707.09647.

T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Decay chain information on the newly discovered double charm baryon state $\Xi_{cc}^+$, arXiv:1708.00703.

M. Karliner and J. L. Rosner, Quark-level analogue of nuclear fusion with doubly-heavy baryons, arXiv:1708.02547.

Z. H. Guo, Prediction of exotic doubly charmed baryons within chiral effective field theory, arXiv:1708.04145.

S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 32, 189 (1985).

S. Godfrey and J. Napolitano, Light meson spectroscopy, Rev. Mod. Phys. 71, 1411 (1999).

S. Godfrey and K. Moats, Bottomonium Mesons and Strategies for their Observation, Phys. Rev. D 92, 054034 (2015).

J. Ferretti, G. Galatá and E. Santopinto, Interpretation of the $X(3872)$ as a charmonium state plus an extra component due to the coupling to the meson-meson continuum, Phys. Rev. C 88, 015207 (2013).

J. Ferretti and E. Santopinto, Higher mass bottomonia, Phys. Rev. D 90, 094022 (2014).

Q. F. Lü and D. M. Li, Understanding the charmed states recently observed by the LHCb and BaBar Collaborations in the quark model, Phys. Rev. D 90, 054024 (2014).

S. Godfrey and K. Moats, Properties of Excited Charm and Charm-Strange Mesons, Phys. Rev. D 93, 034035 (2016).

Q. T. Song, D. Y. Chen, X. Liu and T. Matsuki, Charm-Strange mesons revisited: mass spectra and strong decays, Phys. Rev. D 91, 054031 (2015).

S. Capstick and N. Isgur, Baryons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 34, 2809 (1986).

D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, Phys. Rev. D 76, 114015 (2007).

D. Ebert, R. N. Faustov and V. O. Galkin, Excited heavy tetraquarks with hidden charm, Eur. Phys. J. C 58, 399 (2008).

D. Ebert, R. N. Faustov and V. O. Galkin, Masses of tetraquarks with open charm and bottom, Phys. Lett. B 696, 241 (2011).

M. Momemzadeh, N. Tazimi and P. Sadeghi, Tetraquarks as diquark-antidiquark bound systems, Phys. Lett. B 741, 124 (2015).

M. R. Hadizadeh and A. Khaledi-Nasab, Heavy tetraquarks in the diquark-antidiquark picture, Phys. Lett. B 753, 8 (2016).

J. Ferretti, A. Vassallo and E. Santopinto, Relativistic quark-diquark model of baryons, Phys. Rev. C 83, 065204 (2011).

E. Santopinto and J. Ferretti, Strange and nonstrange baryon spectra in the relativistic interacting quark-diquark model with a Greys and Radicati-inspired exchange interaction, Phys. Rev. C 92, 025202 (2015).

Q. F. Lü and Y. B. Dong, X(4140), X(4274), X(4500), and X(4700) in the relativized quark model, Phys. Rev. D 94, 074007 (2016).

Q. F. Lü and Y. B. Dong, Masses of open charm and bottom tetraquark states in a relativized quark model, Phys. Rev. D 94, 094041 (2016).

W. J. Deng, H. Liu, L. C. Gu and X. H. Zhong, Charmonium spectrum and their electromagnetic transitions with higher multipole contributions, Phys. Rev. D 95, 034026 (2017).

W. J. Deng, H. Liu, L. C. Gu and X. H. Zhong, Spectrum and electromagnetic transitions of bottomonium, Phys. Rev. D 95, 074002 (2017).
[84] K. L. Wang, L. Y. Xiao, X. H. Zhong and Q. Zhao, Understanding the newly observed $\Omega_c$ states through their decays, Phys. Rev. D 95, 116010 (2017).

[85] E. Hiyama, Y. Kino and M. Kamimura, Gaussian expansion method for few-body systems, Prog. Part. Nucl. Phys. 51, 223 (2003).

[86] S. J. Brodsky and J. R. Primack, The Electromagnetic Interactions of Composite Systems, Annals Phys. (N.Y.) 52, 315 (1969).

[87] Z. P. Li, H. X. Ye and M. H. Lu, A unified approach to pseudoscalar meson photoproductions off nucleons in the quark model, Phys. Rev. C 56, 1099 (1997).

[88] Q. Zhao, J. S. Al-Khalili, Z. P. Li and R. L. Workman, Pion photoproduction on the nucleon in the quark model, Phys. Rev. C 65, 065204 (2002).

[89] L. Y. Xiao, X. Cao and X. H. Zhong, Neutral pion photoproduction on the nucleon in a chiral quark model, Phys. Rev. C 92, 035202 (2015).

[90] X. H. Zhong and Q. Zhao, $\eta$ photoproduction on the quasi-free nucleons in the chiral quark model, Phys. Rev. C 84, 045207 (2011).

[91] X. H. Zhong and Q. Zhao, $\eta'$ photoproduction on the nucleons in the quark model, Phys. Rev. C 84, 065204 (2011).