Are M-atrix theory and Maldacena’s conjecture related?

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Abstract

We give arguments in the support of a relation between M-atrix theory and Maldacena’s conjecture. M-atrix theory conjecture implies the equivalence of 11-D light-cone supergravity and strongly-coupled (0+1)-D SYM. Maldacena’s SUGRA/SYM duality conjecture implies, in the one dimensional SYM case, the equivalence between strongly-coupled (0+1)-D SYM and 11-D supergravity compactified on a spatial circle in the formal Seiberg-Sen limit. Using the classical equivalence between 11-D supergravity on a light-like circle and on a spatial circle in the formal Seiberg-Sen limit, we argue that in the (0+1)-D SYM case, the large-N M-atrix theory in the supergravity regime is equivalent to SUGRA/SYM duality.

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1 Introduction

M-atrix theory [1, 2] and Maldacena’s conjecture [3] are similar in the sense that both are about the equivalence between M/string theories (i.e. theories in the bulk of space-time) and brane world-volume theories. But does this similarity go beyond a mere analogy? Maldacena’s conjecture relates M/string theory on a curved background to SYM in the flat space, whereas M-atrix theory relates M-theory on a transverse torus to SYM on a dual torus. For example, consider finite-N M-atrix theory on a 3-torus. If $V_3$ is the volume of M-theory transverse 3-torus and $R$ the radius of light-like circle, then the corresponding D=4 SYM is defined on a dual 3-torus of volume $\tilde{V}_3 = (RM^3)^3V_3^{-1}$. Dual 3-torus decompactifies in the limit $V_3 \to 0$, but $g^2_{YM} = (M_3^3V_3)^{-1}$ also blows up in this limit. In the large-N limit one has to take $R \to \infty$. Thus we have $g^2_{YM} \gg 1$ and $N \gg 1$, and this limit is different from the one involved in AdS$_5$/CFT$_4$ correspondence.

![Figure 1: M-atrix and Maldacena maps](image)

These formal arguments do not, however, imply that the two conjectures are in conflict with each other, but rather suggest the following possibility. Imagine two moduli spaces: $\mathcal{M}_1$ of M-theory and $\mathcal{M}_2$ of SYM (coordinates of these moduli spaces are: the dimension $d$ of SYM, the rank $N$ of gauge group, the type of gauge theory base space, the type of space on which M-theory is compactified and so on). One can think of the two conjectures as two one-to-one maps $\mathcal{F}_{\text{M-atrix}}$ and $\mathcal{F}_{\text{Maldacena}}$ between certain regions of $\mathcal{M}_1$ and $\mathcal{M}_2$. It is clear that if these mappings map two non-overlapping regions in $\mathcal{M}_1$ into two non-overlapping regions in $\mathcal{M}_2$, it does not mean that they contradict each other (see Figure 1). A natural question to ask is whether the domains of these two mappings intersect, and if so, are the two mappings identical in the overlap? The motivation for writing this paper was to answer this question.

In this paper we will consider (0+1)-D SYM case. One dimensional SYM is simplest

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1. In what follows we will consider M-atrix theory in flat backgrounds. M-atrix theory in curved backgrounds is still very poorly understood.
2. For some other discussions of M-atrix/Maldacena relation, see refs. [5–10].
3. Supersymmetric matrix quantum mechanics was first studied in ref. [11].
in the sense that no compactification is involved. The purpose of this paper is to establish a relation between large-$N$, uncompactified M-atrix theory in the supergravity regime and SUGRA/SYM duality (large-$N$ version of Maldacena’s conjecture) in this one dimensional case.

The paper is organized as follows. In Sec.2 we give a brief review M-atrix theory. In Sec.3 we discuss some aspects of SUGRA/SYM duality and propose a new interpretation of SUGRA/(0+1)-D SYM duality. In Sec.4 we argue that M-atrix theory and SUGRA/(0+1)-D SYM duality are related by a classical equivalence between 11-D supergravities on a light-like and a spatial circles.

2 M-atrix theory

The finite-$N$ version of M-atrix theory conjecture [2], as clarified by Sen and Seiberg [12, 13], states that discrete light-cone quantization (DLCQ) of M-theory (with the Planck scale $M_P$ and a transverse length scale $x$) compactified on a light-like circle ($x^− \equiv x^− + 2\pi R$) in the sector with the total longitudinal momentum $p_− = \frac{N}{R}$ is equivalent to M-theory (with the Planck scale $M_P$ and a transverse length scale $\tilde{x}$) compactified on a spatial circle ($x_{11} \equiv x_{11} + 2\pi \tilde{R}$) in the sector with the total Kaluza-Klein momentum along the $x_{11}$ direction $p_{11} = \frac{N}{\tilde{R}}$ in the limit $\tilde{R} \to 0 \ , \tilde{M}_P \to \infty \ , \tilde{R}M^2_P = R M^2_P \ , \quad \tilde{x}M_P = xM_P \ . \quad (2.1)$

The large-$N$ version of M-atrix theory conjecture [1] is a related statement about the uncompactified 11-D light-cone M-theory. It is assumed that the large-$N$ version of M-atrix theory is the $R, N \to \infty \ (\text{with } N/R = \text{fixed})$ limit of finite-$N$ theory.

Compelling arguments for the validity of finite-$N$ M-atrix theory conjecture were given by Seiberg and Sen [13, 12]. Let $T$ be a field theory (with a mass scale and a set of dimensionless parameters $\{y_i\}$) formulated on a flat background (or on a background of the form $R^{1,1} \times M$ which has $SO(1,1)$ isometry). Let $\mathcal{H}_{N}^{DLCQ}(M, R, \{y_i\})$ be the $N$-body DLCQ quantum mechanical hamiltonian describing the dynamics of the theory $T$ compactified on a light-like circle ($x^- \equiv x^- + 2\pi \tilde{R}$) in the sector with the total longitudinal momentum $P_− = \frac{N}{\tilde{R}}$. The dynamics of the theory $T$ compactified on a small spatial circle ($x \equiv x + 2\pi \tilde{R}$) in the sector with $N$ units of total momentum along the $x$ direction is also expected to be described by a non-relativistic $N$-body hamiltonian. We denote it by $\mathcal{H}_{N}^{KK}(\tilde{M}, \tilde{R}, \{y_i\})$. Now, thinking of the light-like circle as an almost light-like circle and boosting it, one can show that

$$\mathcal{H}_{N}^{DLCQ}(M, R, \{y_i\}) = \lim_{R \to 0} \mathcal{H}_{N}^{KK}(\tilde{M} = M\sqrt{\frac{R}{\tilde{R}}}, \tilde{R}, \{y_i\}) \ . \quad (2.2)$$

4 We follow a very nice review of these arguments given in Sen’s review article [14].

5 The argument goes as follows: $p_{11} = \frac{N}{\tilde{R}}$, $H = \sqrt{p^2_{11} + p^2_\perp} = p_{11} + \frac{p^2_{12}}{2p_{11}} + \cdots$ and in an appropriate limit when $\tilde{R} \to 0$ one is left with a non-relativistic hamiltonian.
Eq. (2.2) follows from the following kinematical relation between two coordinate frames related by a boost $\beta$:

$$\frac{\partial}{\partial x'^+} = e^\beta \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right),$$  

(2.3)

and the assumption that the quantum operators which generate $i(\partial/\partial x'^+)$, $i(\partial/\partial t)$ and $i(\partial/\partial x)$ are $\mathcal{H}_N^{DLCQ}$, $\mathcal{H}_N^{KK} + N/\tilde{R}$ and $-N/\tilde{R}$, respectively.

3 SUGRA/SYM duality

The essence of SUGRA/SYM duality is the following: surround a D-brane source in supergravity by a sphere of radius $\sqrt{\alpha'}$. Supergravity is a low energy limit of string theory and hence, a priori, it is valid only outside of the sphere. It is claimed that if the curvature and effective string coupling of the supergravity solution are small, supergravity is valid also inside of the sphere. It should be noted that this idea does not give a precise recipe for the SUGRA/SYM correspondence. The rules of the game should be specified for a particular supergravity background, etc. For example, in the AdS/CFT case, such a recipe was given in refs.\[15, 16\]. In this case, the correspondence is between the fluctuations of supergravity fields on the curved AdS background and SYM operators.

We are interested in SUGRA/(0+1)-D SYM duality which was discussed in ref.\[17\]. In the latter paper it was argued that the motion of a D0-brane probe carrying one unit of RR-charge in the curved near-horizon geometry of a source D0-brane of charge $N \gg 1$ is dual to a process in (0+1)-D SYM with the gauge group $U(N+1)$ broken to $U(N) \times U(1)$ by a Higgs vev. Motivated by AdS/CFT correspondence, one may argue that (0+1)-D SYM is dual to supergravity on a curved near-horizon D0-brane background. It seems that this is a widely accepted point of view\[16\]. Let A be a proponent of this point of view and B—a proponent of the following interpretation (which is our interpretation): SUGRA/(0+1)-D SYM duality is a relation between the scattering of 11-D supergravity states in a flat background and the corresponding SYM processes. The meaning of our interpretation is best illustrated by the following discussion between A and B.

A: Taking Maldacena’s low energy limit in the D0-brane metric, one obtains the metric of curved near-horizon geometry of D0-brane. Thus, (0+1)-D SYM is dual to supergravity on the curved space. For example, consider the scattering of three D0-branes. Let the first two of these D0-branes carry one unit of RR-charge each and the third one carries $N \gg 1$ units of RR-charge. One can think of the scattering of the first two D0-branes as taking place in the curved near-horizon geometry of the third D0-brane. One should for instance use a curved-space propagator to compute scattering amplitudes. This curved-space supergravity picture is dual to (0+1)-D SYM with the gauge group $U(N+2) \to U(N) \times U(1) \times U(1)$.

\[6\] For a discussion of AdS$_2$/SYM$_{0+1}$ duality see ref.\[18\]. We will argue that AdS$_2$/SYM$_{0+1}$ duality conjecture of ref.\[18\] can be correct only approximately.
**B:** I agree that in the case of $U(N+2) \rightarrow U(N) \times U(1) \times U(1)$ your curved-space picture works. But I have problem with the other cases. Suppose that the gauge group $U(N)$ is broken to $U(N_1) \times U(N_2) \times \cdots \times U(N_k)$ with $N_i \sim N$. In this case I cannot talk about scattering as taking place in a definite background since the back-reaction effects are not negligible.

**A:** Well, I think SUGRA/(0+1)-D SYM duality in this case should be interpreted as a duality between the excitations of supergravity fields on the 'near-horizon' multi-center D0-brane static background and SYM processes. In ref. [19] the centers of multi-center static solution were promoted to dynamical variables. By the way, an analogous point of view, in the context of AdS$_5$/CFT$_4$ correspondence, was first advocated in Maldacena’s paper [3].

**B:** You forgot to mention one important point about AdS$_5$/CFT$_4$ correspondence on the Coulomb branch. SYM$_4$ has moduli space of supersymmetric vacua which is identical to the parameter space of the general multi-center D3-brane solution. But we know that (0+1)-D SYM does not have moduli space of vacua because symmetry breaking scalar expectation values are not frozen variables [20]. Therefore, one cannot associate the parameter space of multi-center D0-brane solution with the moduli space of vacua of (0+1)-D SYM.

**A:** But may be instead of associating this parameter space with the moduli space of vacua (which does not exist in SUSY matrix quantum mechanics), one should associate it with the space of slow variables in a Born-Oppenheimer approximation. It was argued in ref. [20] that Born-Oppenheimer approximation is justified in the large-N limit.

**B:** I have little to say about it. Instead of trying to resolve problems inherent to the curved-space picture, I want to propose the following simple flat-space picture. Let $A_{n}^{SUGRA}(\tilde{R}, \cdots)$ be a $n$-graviton scattering amplitude in the flat 11-D supergravity compactified on a spatial circle of radius $\tilde{R}$. Let $A_{n}^{SQM}$ be a corresponding SUSY quantum mechanical amplitude. A simple recipe for the SUGRA/(0+1)-D SYM reads

$$A_{n}^{SQM} = \lim_{\tilde{R} \to 0} A_{n}^{SUGRA}(\tilde{R}, \cdots). \quad (3.1)$$

The curved-space picture is approximately correct and useful in some particular cases like the case of broken $U(N+2) \rightarrow U(N) \times U(1) \times U(1)$, but the flat-space picture is more general and contains the curved-space picture as a special case. Note that by the flat space picture, I actually mean the procedure of calculating scattering amplitudes in the supergravity on a flat background and taking Seiberg-Sen limit in the resulting amplitudes. In the flat-space picture context, the curved-space picture emerges effectively in some cases as a result of taking $\tilde{R} \to 0$ limit. The supergravity description that follows from Eq. (3.1) should be tested for its validity in each particular scattering problem. The analysis of ref. [17] shows that supergravity description breaks down at long distances $r > N^{1/3} l_p$. I think the latter inequality should be replaced by a more general one in the case of $U(N) \rightarrow U(N_1) \times \cdots \times U(N_k)$.

**A:** Did not Maldacena use the same arguments in his analysis [3]? Namely, he started with an asymptotically flat D-brane metric, took a low energy limit $\alpha' \to 0$ in it and obtained
an asymptotically curved metric.

B: The relation Eq. (3.1) is a generalization of Maldacena’s idea of taking low energy limit in the D-brane supergravity solution to the cases when there is no well-defined background in which scattering takes place. Moreover, I am emphasizing flatness of supergravity because the justification for taking $\tilde{R} \to 0$ limit in the supergravity amplitudes involves a classical equivalence between supergravities on a light-like and a space-like circles, and the latter equivalence requires the formulation of supergravity on a flat background.

4 M-atrix theory–SUGRA/SYM duality relation

We see that in the M-atrix theory context (0+1)-D SYM in the large-N limit is equivalent to 11-D light-cone supergravity, and in the context of SUGRA/SYM duality it is equivalent to 11-D supergravity on a spatial circle in the formal Seiberg-Sen limit. Assuming that M-atrix theory and SUGRA/SYM duality are related, this suggests that there should exist a relation between light-cone 11-D supergravity and 11-D supergravity on a spatial circle in the formal Seiberg-Sen limit (see Figure 2). Indeed, it was noticed in refs.[21, 22] that the scattering potentials calculated in these two supergravities agree. It is also known that infinitely boosting the D0-brane supergravity solution is equivalent to taking a near-horizon limit (Seiberg-Sen limit) [21]. The authors of refs.[6, 8] used this fact to argue that SUGRA/SYM duality and M-atrix theory are equivalent. Their arguments can be summarized as follows. “Motivated by the finite-N version of M-atrix theory, let us up-lift the D0-brane SUGRA metric to 11-D and then compactify it on $x^−$. As a result, we find that the harmonic function $H_0$ is replaced by $H_0 \rightarrow 1$. Now rewrite this metric in terms of new symbols ($\tilde{r}, \tilde{g}_s$, etc.) that are related to the original ones ($r, g_s$, etc.) by Seiberg-Sen’s relations Eq. (2.1). Let us appropriately rescale (using Seiberg-Sen’s relations) the resulting metric written in terms of new symbols. We find that this rescaled metric is precisely that of the D0-brane in the the near-horizon limit! So, we see the validity of Seiberg-Sen’s prescription when applied to the background geometry of supergravity associated with the DLCQ of M-theory. We also see that (0+1)-D SYM is equivalent to fluctuations of SUGRA fields on the curved near-horizon D0-brane background. Since we know that the latter statement is precisely that of SUGRA/SYM duality, we seem to have shown that SUGRA/SYM duality follows from M-atrix theory.”

There are two objections against their line of reasoning. First, the meaning of taking Seiberg-Sen limit in the expression for the metric is not clear. In Seiberg-Sen’s derivation of M-atrix theory, this limit was taken in the full M-theory and not just in supergravity. Second, the discussion of the authors of refs.[6, 8] seems to imply that two M-theories formulated on the curved backgrounds are related via Seiberg’s boost, but

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7The discussion of this justification will be given below.

8We think these authors implicitly assumed this when they talked about the background geometry etc.
the backgrounds they considered do not have SO(1,1) isometry. But is not it the case that in the Seiberg-Sen’s derivation of Matrix theory one breaks Lorentz invariance by compactification on a circle? It is so, but the underlying theory is supposed to be an SO(1,1) invariant theory formulated on a background of the form \( R^{1,1} \times \mathcal{M} \). The backgrounds considered in refs.\[6, 8\] are not SO(1,1) isometric before the compactification.

We resolve the first problem by clarifying the meaning of taking Seiberg-Sen limit in supergravity expressions. Its meaning is the classical equivalence of 11-D supergravities on light-like and spatial circles. The second problem is resolved by interpreting SUGRA/(0+1)-D SYM duality in a way different from what they did. We stated earlier our interpretation of this duality in the one-dimensional case: ...a relation between the scattering of supergravity states in a flat background and the corresponding SYM processes.

In order to show that Matrix theory and SUGRA/SYM duality are related one has to use the “rectangle of relations” (see Figure 2) and the classical equivalence of two supergravities. Let us prove the classical equivalence of two supergravities. Let us denote by \( H_{N}^{l.c.}(M, R, \{y_{i}\}) \) and \( H_{N}^{s.l.}(\tilde{M}, \tilde{R}, \{\tilde{y}_{i}\}) \) classical hamiltonians of a theory compactified on light-like and spatial circles in the sectors with \( N \) units of longitudinal momentum. In the hamiltonian formulation we associate them with the vector fields \( (\partial/\partial x^{+}) \) and \( (\partial/\partial t) \).

The analog of Eq. (2.2) in this classical case reads

\[
H_{N}^{l.c.}(M, R, \{y_{i}\}) = \lim_{\tilde{R} \to 0} H_{N}^{s.l.}(\tilde{M}, \tilde{R}, \{\tilde{y}_{i}\}) = M \left( \frac{\tilde{R}}{R}, \tilde{R}, \{\tilde{y}_{i}\} \right) .
\]

Note that the statements expressed by Eq. (2.2) and Eq. (4.1) are very different. The latter states the equivalence of two classical field theory hamiltonians and the former states the equivalence of two quantum mechanical \( N \)-body hamiltonians. It is the exploitation of this difference that enables us to relate the two conjectures. Applying Eq. (4.1) to 11-D supergravity formulated in a flat background, one establishes the classical equivalence...
between the two supergravities.

It should be noted that our analysis is different from that of [5]. The arguments of the authors of ref. [5] go as follows. “From the expression for the 11D supergravity plane wave solution \( ds^2 = dx^+dx^- + (Q/r^7)(dx^-)^2 + dx_i^2 \), one sees that \( x^- \), the putative compact “null” direction, is in fact spacelike for \( r < \infty \). Thus it seems that the gravitational effects automatically provide with the almost light-likeness of the circle which is crucial in Seiberg’s derivation of M-atrix theory. It is the reason why supergravity solution is valid at short distances.” As we understand, the discussion of refs. [10] seems to imply that the DLCQ quantization of M-theory in such a plane wave background cannot yield a theory of pure D0-branes (an ordinary matrix SUSY quantum mechanics) because the circle is only asymptotically null and negative momentum modes do not decouple. In our analysis the validity of supergravity solution at short distances follows from the matrix theory conjecture and the classical equivalence of 11-D supergravities on a light-like and a spatial circles, and therefore, our analysis is different from that of ref. [5].

Let us give some examples [21, 22] which illustrate Eq. (4.1). The potential for graviton-graviton scattering with no longitudinal momentum transfer in 11-D supergravity compactified on a light-like circle reads [21]

\[
H_{l.c.} = -\frac{N(2)}{R} \frac{\sqrt{1 - \frac{h \nu^2}{h}} - 1}{h}, \quad h = \frac{15N(1)}{2R^2M_P^9r^7},
\]

(4.2)

where \( N(1)/R \) and \( N(2)/R \) are the longitudinal momenta \( p_- \) of the source and probe gravitons, \( \nu \)–their relative transverse velocity and \( r \)–their transverse separation. The corresponding expression for the graviton-graviton scattering potential in 11-D supergravity compactified on a spatial circle reads

\[
H_{s.l.} = -\frac{N(2)}{R} \frac{\sqrt{1 - \frac{(1 + \tilde{h})\tilde{\nu}^2}{1 + \tilde{h}}} - 1}{1 + \tilde{h}}, \quad \tilde{h} = \frac{15N(1)}{2R^2M_P^9\tilde{r}^7}.
\]

(4.3)

Using the relations

\[
\tilde{\nu}M_P = \nu M_P, \quad \tilde{r}M_P = r M_P,
\]

(4.4)

which follow from Eq. (2.1), it is easy to show that \( H_{t.c.} = \lim_{R \to 0} H_{s.l.} \).

The next example is a non-trivial one. Let us consider membrane – membrane interaction in \( D = 11 \) supergravity with the compact light-like direction. Let \( p_1 = N(1)/R, p_2 = N(2)/R \) and \( m_1, m_2 \) be the longitudinal momenta and masses of the source and probe membranes, respectively. In the case of zero longitudinal momentum transfer the interaction potential of two membranes moving with the relative transverse velocity \( v \) and
separated by a distance \( r \) reads \[22\]

\[
H_{t.c.} = \left( \frac{m_{(1)}}{p_{(1)}^{(1)}} \right)^2 p_{(2)}^{(2)} W^{-1} \left( 1 - \sqrt{\left( 1 - W \left( \frac{v p_{(1)}}{m_{(1)}} \right)^2 \right) \left[ 1 + \left( 1 - \frac{m_{(2)} p_{(1)}}{m_{(1)} p_{(2)}^{(1)}} \right)^2 W \right]} \right)
+ \frac{p_{(2)}^{(2)}}{2} \left( \frac{m_{(1)}}{p_{(1)}^{(1)}} \right)^2 - \frac{m_{(1)} m_{(2)}}{p_{(1)}^{(1)}} , \quad W = \frac{3 m_{(1)}}{2 p_{(1)}^{(1)} R M_p r^5} .
\]

The corresponding potential in the spatially compactified \( D = 11 \) supergravity reads

\[
H_{s.l.} = p_{11}^{(2)} - \bar{m}_{(2)} \bar{K}^{-1} \sqrt{\bar{K} \left( \frac{1}{W \sinh^2 \beta + \bar{K}} - \bar{v}^2 \right) \left[ 1 + \frac{(\bar{K} p_{11}^{(1)}/\bar{m}_{(2)})^2}{W \sinh^2 \beta + \bar{K}} \right]}
+ \bar{m}_{(2)} (\bar{K}^{-1} - 1) \cosh \beta - \frac{\bar{W} p_{11}^{(1)}}{W \sinh^2 \beta + \bar{K}} \sinh \beta ,
\]
where

\[
p_{11}^{(1)} = \frac{N_{(1)}}{R} = \bar{m}_{(1)} \sinh \beta , \quad p_{11}^{(2)} = \frac{N_{(2)}}{R} , \quad \bar{W} = \frac{3}{2 R M_p^6 \cosh \beta r^5} ,
\]

\[
\bar{K} = 1 + \bar{W} , \quad p_{11}' = p_{11}^{(2)} + \bar{m}_{(2)} (\bar{K}^{-1} - 1) \sinh \beta .
\]

Using Eq. (2.4), Eq. (4.4) and Eq. (4.7), it can be shown that

\[
\lim_{R \rightarrow 0} H_{s.t.} = \lim_{R \rightarrow 0} p_{11}^{(2)} (\bar{W} \sinh^2 \beta)^{-1} \left( 1 - \sqrt{(1 - \bar{W} \sinh^2 \beta \bar{v}^2)[1 + \bar{W} \sinh^2 \beta \left( \frac{\bar{m}_{(2)}}{p_{11}^{(2)}} - \frac{1}{\sinh \beta} \right)^2]} \right)
+ \frac{p_{11}^{(2)}}{2 \sinh^2 \beta} - \frac{\bar{m}_{(2)}}{\sinh \beta}
= H_{t.c.} .
\]

It would be interesting to extend these scattering potential calculations to the cases involving longitudinal momentum exchange, recoil effects, etc. In view of relation Eq. (4.4), one expects agreement in these cases. Note that the equivalence between two supergravities holds for arbitrary \( N \), but in relating M-atrix theory and SUGRA/SYM duality we use only the equivalence for large \( N \).

Let us conclude by summarizing our arguments. M-atrix theory conjecture implies the equivalence of strongly coupled one dimensional SYM and 11-D light-cone supergravity.

\[\text{In ref.}[22] \text{ an equivalent potential was obtained from ten dimensional IIA supergravity. It is easier to obtain it directly from 11-D supergravity, starting with the membrane probe action } S = \int L dt = -T_2 \int d^3x (\sqrt{-\det g_{mn}} - C_{ixjx2} - \dot{x}_{11}C_{ixjx1x2}) \text{ and performing Legendre transformation } L \rightarrow L' = L(\dot{x}_{11}(p_{11})) - \dot{x}_{11} p_{11} .\]

\[10\text{11-D supergravity on a light-like circle at finite-N is not necessarily the same as the low energy limit of DLCQ M-theory [23] and therefore the equivalence of two supergravities at finite-N is useless for our purposes.}\]
On the other hand, SUGRA/SYM duality implies the equivalence of one dimensional SYM and 11-D supergravity on a spatial circle in Sen-Seiberg’s limit. We showed that the hamiltonian of 11-D supergravity on a light-like circle is equal to a limit of the hamiltonian of 11-D supergravity on a spatial circle. In this way we argued that the large-N M-atrix theory in the supergravity regime and SUGRA/(0+1)-D SYM duality are equivalent.

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