DI-QUARKS AND TRI-QUARKS ON THE LATTICE

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The distribution of gluon fields in hadrons is of fundamental interest in QCD. Using lattice QCD we have observed the formation of gluon flux tubes within tri-quark (baryon) systems for a wide variety of spatial distributions of the color sources. In particular we have investigated configurations where two of the quarks are close together and the third quark is some distance away, which approximates a quark plus diquark string. We find that the string tension of the quark - diquark string is the same as that of the quark - antiquark string on the same lattice. We also compare the longitudinal and transverse profiles of the gluon flux tubes for both sets of strings, and find them to be of similar radii and to have similar vacuum suppression.

1. Introduction

Recently there has been renewed interest in studying the distribution of quark and gluon fields in the three-quark static-baryon system. While the earliest studies were inconclusive, improved computing resources and analysis techniques now make it possible to study this system in a quantitative manner. In particular, it is possible to directly compute the gluon field distribution using lattice QCD techniques similar to those pioneered in mesonic static-quark systems. We first investigate the formation of flux tubes, where vacuum gluon field fluctuations are suppressed, in systems where the three quarks are approximately equidistant. In order to compare baryonic and mesonic flux tubes, we have further investigated three-quark static baryons where two of the quarks are close together and the third is some distance away.

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Fig. 1. Gauge-link paths or “staples,” $U_1, U_2$ and $U_3$, forming a three-quark Wilson loop with the quarks located at $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$. $\varepsilon^{abc}$ and $\varepsilon^{a'b'c'}$ denote colour anti-symmetrisation at the source and sink respectively, while $\tau$ indicates evolution of the three-quark system in Euclidean time.

2. Baryon Flux tubes on the Lattice

In order to study flux-tubes on the lattice, we begin with the standard approach of connecting static quark propagators with spatial-link paths in a gauge invariant manner. We use APE-smeared spatial-link paths (with $\alpha = 0.7$ and 30 smearing steps in all cases) to propagate the quarks from a common origin to their spatial positions as illustrated in Fig. 1. The static quark propagators are constructed from time directed link products at fixed spatial coordinate, $\prod_i U_t(\vec{x}, t_i)$, using the untouched ‘thin” links of the gauge configuration. Finally smeared-link spatial paths propagate the quarks back to the common spatial origin. The three-quark Wilson loop is thus defined as:

$$W_{3Q} = \frac{1}{3!} \varepsilon^{abc} \varepsilon^{a'b'c'} U_1^{aa'} U_2^{bb'} U_3^{cc'},$$

where $U_j$ is a staple made of path-ordered link variables as shown in Fig. 1. We consider Y and T shaped paths in the $x-y$ plane, where the three quarks are created at the origin, then are propagated to positions $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$, which approximate an equilateral triangle, before being propagated through time and finally back to a sink at the same spatial location as the source. We investigate seven such triangles, with average inter-quark distance varying from 0.27 fm to 1.7 fm.

The gluon-field fluctuations are characterised by the gauge-invariant action density $S(\vec{y}, t)$. We calculate the action density using the highly-improved $O(a^4)$ three-loop improved lattice field-strength tensor [13] on four-sweep APE-smeared gauge links.

For a Wilson loop of Euclidean time extent $\tau$ we evaluate the correlation function

$$C(\vec{y}; \vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) = \frac{\langle W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) S(\vec{y}, \tau/2) \rangle}{\langle W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) \rangle \langle S(\vec{y}, \tau/2) \rangle},$$

where $\langle \cdots \rangle$ denotes averaging over configurations and lattice symmetries. This correlates the quark positions with the gauge-field action in a gauge invariant manner, and has the advantage of being positive definite, eliminating any sign ambiguity on whether vacuum field fluctuations are enhanced or suppressed in the presence of static quarks. For $\vec{y}$ well away from the quark positions, there are no correlations
Di-quarks and Tri-quarks on the Lattice

Fig. 2. Expulsion of gluon-field fluctuations from the region of static quark sources illustrated by the spheres. An isosurface of \( C(\vec{y}) \) is illustrated by the translucent surfaces. The surface plots (or rubber sheets) describes the values of \( C(\vec{y}) \) for \( \vec{y} \) in the quark plane, \((y_1, y_2, 0)\). Results are for 30-sweep smeared T-shape (left) and Y-shape (right) sources with the largest quark separation considered.

and \( C \to 1 \). We find that \( C \) is generally less than 1, signaling the expulsion of vacuum fluctuations from the interior of heavy-quark hadrons.

We first consider 200 quenched QCD gauge-field configurations created with the \( O(a^2) \)-mean-field improved Luscher-Weisz plaquette plus rectangle gauge action \(^{14}\) on \( 16^3 \times 32 \) lattices at \( \beta = 4.60 \). The long dimension is taken as being the \( x \) direction making the spatial volume \( 16^2 \times 32 \). Using a physical string tension of

\[
\sigma = (0.440 \text{ GeV})^2 = 0.981 \text{ GeV/fm},
\]

the \( QQ \) potential sets the lattice spacing to

\[
a = 0.123(2)\text{fm}.
\]

We use lattice symmetries to improve the signal to noise ratio of our measurements. These include translational invariance (any point on the lattice can be taken as the origin), reflection in the \( x \) plane and 90\(^\circ\) rotational symmetry about the \( x \)-axis. The advantage of this approach is that we do not have to perform any gauge fixing to find a signal in the flux distributions.

In Fig. 2 we show examples of the expulsion of vacuum fluctuations and the formation of flux-tubes for our largest T and Y shaped configurations.

A detailed analysis of these flux tubes is given elsewhere \(^{15,16}\). The major conclusions can be summarised. We do not see any evidence for \( \Delta \) - shaped (empty triangle) flux tubes. At large quark separations we do see Y shaped flux tubes, even from initial T shapes, which relax towards a Y shape. We observe a potential which is a linear function of the minimum length of string needed to connect the quarks to the Fermat point, and the extracted string tension \((\sigma = 0.97(1) \text{ GeV fm}^{-1})\) is in good agreement with the quark - antiquark string tension.

3. Diquarks on the Lattice

There has recently been a renewal of interest in the properties of diquarks in hadronic systems, as they may play an important role in the existence of exotic states, such as the putative \( \Theta^+ \), or in explaining the scarcity of such exotics \(^{15}\).
In QCD, two quarks close together, a diquark, can transform either according to the conjugate representation ($\bar{3}$) or the sextet (6) representation of $SU(3)$. The color hyperfine interaction then leads to attraction in the spin singlet, scalar diquark channel, while the spin triplet, axial vector diquark is disfavoured. Hence diquarks should have positive parity and belong to the color $\bar{3}$ representation, and so have many properties similar to an antiquark. In lattice QCD this should lead to the formation of quark - diquark flux tubes with similar physical characteristics to those of quark - antiquark flux tubes. In particular we would expect the long range linear part of the quark - diquark potential to have the same slope as that of the quark - antiquark potential, corresponding to the flux tubes having the same energy density, and we would expect the flux tubes to have similar transverse size.

The three-quark configurations we use to approximate a quark - diquark string are T-shapes, with the origin at the junction of the T. Two quarks are positioned one lattice step in opposite directions from the origin (approximating the diquark), and the third is placed from 1 to 12 lattice steps in an orthogonal direction. For this part of our work we have used 300 quenched QCD gauge field configurations created with the same action as previously. Two hundred of these configurations were at at $\beta = 4.60$ (as in the previous section) and one hundred at $\beta = 4.80$, to investigate the use of a finer lattice. These values of $\beta$ give lattice spacings $a$ of 0.123 fm and 0.093 fm respectively.

In Fig. 3 we show an example of the expulsion of vacuum fluctuations and the formation of flux-tubes for our quark - diquark configurations.

The effective potential is obtained from the Wilson loops in the standard manner:

$$ aV(\vec{r}, \tau) = \ln \left( \frac{W(\vec{r}, \tau)}{W(\vec{r}, \tau + 1)} \right) . $$

(3)
Di-quarks and Tri-quarks on the Lattice

\[
\sigma \equiv \sigma_{Q\bar{Q}} \label{eq:1}
\]

where \( \sigma \) is the string tension. The three quark potential is

\[
V_{3Q}(r) = \frac{3}{2} V_0 - \frac{1}{2} \sum_{j<k} g^2 C_F \frac{4 \pi r_{jk}}{r} + \sigma_{3Q} L(r), \tag{5}
\]

where \( C_F = 4 / 3 \) and \( L(r) \) is a length linking the quarks. As shown in our earlier work, \( L(r) \) is given by the minimum length of string that connects the three quarks, or the sum of distances from the quarks to the Fermat (or Steiner) point. We expect that the two string tensions \( \sigma_{Q\bar{Q}} \) and \( \sigma_{3Q} \) are equal. In Fig. 4 we plot the extracted effective potentials for the quark - diquark and quark - antiqaurk flux-tubes at each of the values of \( \beta \) for our gauge configurations. The plots show that our expectation is confirmed at both values of \( \beta \). Converting length measurements from lattice units to fermi we obtain the quark - diquark string tension \( \sigma_{3Q} = 0.98 \pm 0.01 \text{ GeV fm}^{-1} \), which is in excellent agreement with the quark - antiquark string tension \( \sigma_{Q\bar{Q}} = 0.97 \text{ GeV fm}^{-1} \).

We can gain further insight into the properties of the flux tubes by examining their profiles close to the quark. We study the values of the correlators \( C_{3Q}(\vec{y}) \) and \( C_{Q\bar{Q}}(\vec{y}) \) where \( (\vec{y}) = (y_1, y_2, 0) \) is constrained to the plane of the color sources, and the origin is at the position of either the antiquark or the join of the T. The quark is then at the position \((\xi, 0, 0)\) where \( \xi \) varies from 2 to 12 lattice steps. First we examine the longitudinal profiles of both quark - diquark and quark - antiquark flux-tubes along the line \( (\vec{y}) = (x, 0, 0) \) in Fig. 5. As expected, the vacuum expulsion close to the diquark is stronger than in the vicinity of the antiquark. However, near the quark the two flux tubes show very similar profiles. Similar results are seen at \( \beta = 4.6 \).

Next we examine the transverse profiles along a line orthogonal to the midpoint of the flux tube, \textit{i.e.} along \((\xi/2, y, 0)\) for \( \xi \) even, or along \(((\xi + 1)/2, y, 0)\) for \( \xi \) odd. In Fig. 6 we show profiles of both quark - diquark and quark - antiquark flux-tubes for \( \xi = 12 \). We find that as long as \( \xi \) is larger than 4 lattice steps, the transverse
Fig. 5. Comparison of longitudinal flux tube profiles for quark - diquark (left) and quark - antiquark (right) flux tubes at $\beta = 4.6$ at all longitudinal separations.

Fig. 6. Transverse profiles for quark - diquark (red lines) and quark - antiquark (black lines) flux tubes at $\beta = 4.6$ (solid lines) and $\beta = 4.8$ (dashed lines).

profiles are close to identical. In earlier work \cite{8} we saw that the profiles saturate for long enough Euclidean time evolution. Using a fit to a Gaussian profile we find that the transverse profiles of quark - diquark and quark - antiquark flux-tubes are statistically identical \cite{16}.

4. Conclusions

We have directly compared gluon flux-tubes for quark plus antiquark and three quark systems. In the three quark systems we kept two quarks close together (two lattice units separation), so that the system approximates a quark - diquark string. We found that the string tension in the quark - diquark string was the same as for the quark - antiquark string. In addition we compared the vacuum expulsion in both sets of flux-tubes. We found that, in the vicinity of the quark, there was no measurable difference between the transverse profiles of the quark - diquark flux-tubes and the quark - antiquark flux-tubes.

These findings confirm the expectation from QCD that a diquark has many properties in common with an antiquark. In particular the long range color interaction between a diquark and a quark is seen to be the same as that between an antiquark and a quark. It would be interesting to repeat this work with dynamical quarks, where the effect of screening by quark - antiquark pairs could be observed.
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References

1. J. Flower, *Nucl. Phys. B* **289** (1987) 484.
2. T. T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suginuma, *Phys. Rev. Lett.* **86** (2001) 18. T. T. Takahashi, H. Suginuma, Y. Nemoto and H. Matsufuru, *Phys. Rev. D* **65** (2002) 114509 [arXiv:hep-lat/0204011].
3. C. Alexandrou, P. De Forcrand and A. Tsapalis, *Phys. Rev. D* **65** (2002) 054503 [arXiv:hep-lat/0107006]; C. Alexandrou, P. de Forcrand and O. Jahn, *Nucl. Phys. Proc. Suppl.* **119** (2003) 667 [arXiv:hep-lat/0209062].
4. Ph. de Forcrand and O. Jahn, *Nucl. Phys. A* **755** (2005) 475 [arXiv:hep-ph/0502039].
5. H. Ichie, V. Bornyakov, T. Streuer and G. Schierholz, *Nucl. Phys. Proc. Suppl.* **119** (2003) 751 [arXiv:hep-lat/0212024]; V. G. Bornyakov et al. [DIK Collaboration], *Phys. Rev. D* **70** (2004) 054506 [arXiv:hep-lat/0401026].
6. F. Okiharu and R. M. Woloshyn, *Nucl. Phys. Proc. Suppl.* **129** (2004) 745 [arXiv:hep-lat/0310007].
7. F. Bissey et al., *Nucl. Phys. Proc. Suppl.* **141** (2005) 22 [arXiv:hep-lat/0501004].
8. F. Bissey et al., *Phys. Rev. D* **76** (2007) 114512 [arXiv:hep-lat/0606016].
9. R. Sommer, *Nucl. Phys. B* **291** (1987) 673.
10. G. S. Bali, K. Schilling and C. Schlichter, *Phys. Rev. D* **51** (1995) 5165 [arXiv:hep-lat/9409005].
11. R. W. Haymaker, V. Singh, Y. C. Peng and J. Wosiek, *Phys. Rev. D* **53** (1996) 389 [arXiv:hep-lat/9406021].
12. M. Falcioni, M. L. Paciello, G. Parisi and B. Taglienti, *Nucl. Phys. B* **251** (1985) 624. M. Albanese et al. [APE Collaboration], *Phys. Lett. B* **192** (1987) 163.
13. S. O. Bilson-Thompson, D. B. Leinweber and A. G. Williams, *Annals Phys.* **304**, (2003) 1 [hep-lat/0203008].
14. M. Luscher and P. Weisz, *Commun. Math. Phys.* **97** (1985) 59; [Erratum-ibid. 98, (1985) 433].
15. R. L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91** (2003) 232003 [arXiv:hep-ph/0307341]; R. L. Jaffe, *Phys. Rept.* **409** (2005) 1 [arXiv:hep-ph/0409065].
16. F. Bissey, A. Signal and D. Leinweber, *in preparation.*