Off-shell dimensional reduction of 5D orbifold supergravity

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Abstract

We present a systematic way for deriving a four-dimensional (4D) effective action of the five-dimensional (5D) orbifold supergravity respecting the \( N = 1 \) off-shell structure. As an illustrating example, we derive a 4D effective theory of the 5D gauged supergravity with a universal hypermultiplet and generic gaugings, which includes the 5D heterotic M-theory and the supersymmetric Randall-Sundrum model as special limits of the gauging parameters. We show the vacuum structure of such model, especially the nature of moduli stabilization, introducing perturbative superpotential terms at the orbifold fixed points.

1 Introduction

The five-dimensional (5D) gauged supergravity provides interesting theoretical frameworks for the physics beyond the standard model (SM), e.g., a supersymmetric (SUSY) warped background which can realize hierarchical structures of the SM scale and couplings (SUSY Randall-Sundrum (RS) model \cite{1, 2}), an effective theory of the strongly coupled heterotic string \cite{3} (5D heterotic M-theory \cite{4}), and so on. Therefore, it would be important to study generic features of the 5D gauged supergravity, and then we present a systematic way for deriving a four-dimensional (4D) effective action of such supergravity models. We apply it to a simple but illustrating example which includes both the SUSY RS model and the 5D heterotic M-theory as special limits of the gauging parameters.

2 5D gauged supergravity with universal hypermultiplet

We start from the off-shell formulation on an orbifold \cite{5}. The 5D superconformal multiplets relevant to our study are the Weyl multiplet \( E_W \), the vector multiplets \( V_I \) and the hypermultiplets \( \mathcal{H}^{\hat{a}} \), where \( I = 0, 1, 2, \ldots, n_V \) and \( \hat{a} = 1, 2, \ldots, n_C + n_H \). Here \( n_C, n_H \) are the numbers of compensator and physical hypermultiplets, respectively. These 5D multiplets are decomposed into \( N = 1 \) superconformal multiplets as \( E_W = (E_W, V_E) \), \( V_I = (V^I, \Sigma^I) \) and \( \mathcal{H}^{\hat{a}} = (\Phi^{2\hat{a}-1}, \Phi^{2\hat{a}}) \), where \( E_W \) is the \( N = 1 \) Weyl multiplet, \( V_E \) is the \( N = 1 \) general multiplet whose scalar component is \( e_{\hat{a}}^4 \), \( V^I \) is the \( N = 1 \) vector multiplet, and \( \Sigma^I, \Phi^{2\hat{a}-1}, \Phi^{2\hat{a}} \) are \( N = 1 \) chiral multiplets.

Here we consider the gauged supergravity with a single universal hypermultiplet spanning the manifold \( SU(2,1)/SU(2) \times U(1) \). The scalar manifold has an \( SU(2,1) \) isometry group, which is linearly realized in the off-shell formulation. The situation we consider is realized by taking \( (n_C, n_H, n_V) = (2, 1, 0) \). Then the bulk action has a \( U(2,1) \) symmetry. The most

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general form of the gauging is parameterized by \(igt_{I=0} = \sum_{i=1}^{8} \tilde{\alpha}_i T^i\) acting on \((\Phi^1, \Phi^3, \Phi^5)^t\) or \((\Phi^2, \Phi^4, \Phi^6)^t\), where \(T^i (i = 1, \ldots, 8)\) are \(3 \times 3\) matrix-valued generators of \(SU(2,1)\). In the following, for simplicity, we consider the case that \(\tilde{\alpha}_i\) are parameterized by three parameters \((\alpha, \beta, \gamma)\) as \(\tilde{\alpha}_3 = 2\beta, \tilde{\alpha}_6 = \alpha, \tilde{\alpha}_8 = \alpha + \beta + \gamma, \tilde{\alpha}_i = 0 \quad (i \neq 3, 6, 8)\), with the corresponding generators in our convention
\[
T^3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad T^6 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{pmatrix}, \quad T^8 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}.
\]

In the limit \(\beta, \gamma \to 0\), this model is reduced to the 5D effective theory of heterotic M-theory derived in Ref. \[4\] on-shell. On the other hand, in the limit \(\alpha \to 0\), we obtain SUSY RS model with the AdS curvature scale \(k\) and a kink mass \(m\) for the hypermultiplet, which are related to the gauging parameters as
\[
k = \beta - \gamma/3, \quad m = 3(\beta + \gamma)/2.
\]

3 4D effective theory

In our model the \(Z_2\)-even multiplets are \(\Sigma^0, \Phi^2, \Phi^3\) and \(\Phi^5\). They appear in the action through the combinations of \(\Sigma^0, \Phi^2\Phi^5\) and \(\Phi^5/\Phi^3\) which carry the zero-modes, i.e., the radion multiplet \(T\), 4D chiral compensator \(\phi\) and the universal (chiral) multiplet \(H\), respectively. The 5D Lagrangian can be expressed in the \(N = 1\) superspace \[6\] as
\[
\mathcal{L}_{5D} = -3e^{2\sigma} \int d^4 \theta \left( -\partial_y V^0 + \Sigma^0 + \bar{\Sigma}^0 \right) \left\{ d_a^b \bar{\Phi}^b (e^{-2igt I^I})_a \Phi^c \right\}^{2/3} e^{3\sigma} \left[ \int d^2 \partial \Phi^a d_a^b \rho_{bc} (\partial_y - 2igt I^I) e^{2\sigma} \Phi^d + \text{h.c.} \right] + \sum_{\vartheta = 0, \pi} \delta(y - \vartheta R) e^{3\sigma} \left\{ \int d^2 \partial \Phi^2 \Phi^3 P_\vartheta (Q) + \text{h.c.} \right\} + \cdots,
\]
where \(e^{2\sigma}\) is the warp factor of the background geometry, \(d_a^b = \text{diag}(1_{2nC}, -1_{2nH})\), \(\rho_{ab} = i\sigma_2 \otimes 1_{nC+nH}\) and \(a, b, \ldots = 1, 2, \ldots , 2(nC+nH)\). The ellipsis denotes terms including boundary induced Kähler potential, gauge kinetic function and those of bulk vector multiplets, which are all irrelevant to the following discussions. We introduce polynomial superpotential terms at the fixed points,
\[
P_\vartheta (Q) = \sum_{n \geq 0} w^{(n)}_{\vartheta} Q^n,
\]
where \(w^{(n)}_{\vartheta}\) \((n = 0, 1, 2, \ldots)\) are constants, and study the nature of moduli stabilization in this model.

Following the off-shell dimensional reduction procedure proposed in Refs. \[7, \ 8\] (which is based on the \(N = 1\) description of 5D supergravity \[6\] and subsequent studies \[9\]) we neglect the kinetic terms for \(Z_2\)-odd \(N = 1\) multiplets and integrate them out by which the zero-modes of \(Z_2\)-even \(N = 1\) multiplets are extracted. Then, after integrating the bulk Lagrangian over the extra dimension, we obtain the 4D effective Lagrangian
\[
\mathcal{L}_{4D} = -3 \int d^4 \phi |\phi|^2 e^{-K/3} + \left\{ \int d^2 \theta \phi^3 W + \text{h.c.} \right\},
\]
with the Kähler potential $K$ and the superpotential $W$ given by [10]

$$K = -3 \ln \int_0^\pi d\tilde{t} e^{-2\beta\tilde{t}} \left\{ \cosh(2t\tilde{\gamma}) (1 - |H|^2) + \sinh(2t\tilde{\gamma}) \frac{(\alpha + \gamma)(1 + |H|^2) + \alpha(\sigma + \bar{\sigma})}{\tilde{\gamma}} \right\}^{\frac{1}{2}},$$

$$W = \frac{1}{4} \sum_{\vartheta = 0, \pi} e^{-3\beta\gamma t} \left\{ c_{\vartheta} + (\alpha + \gamma)s_{\vartheta} + \alpha s_{\vartheta}H \right\} P_{\vartheta}(H_{\vartheta}),$$

where

$$H_{\vartheta} = \frac{-\alpha s_{\vartheta} + (c_{\vartheta} - (\alpha + \gamma)s_{\vartheta}) H}{c_{\vartheta} + (\alpha + \gamma)s_{\vartheta} + \alpha s_{\vartheta}H},$$

and $c_{\vartheta} = \cosh(\vartheta T\tilde{\gamma})$, $s_{\vartheta} = \sinh(\vartheta T\tilde{\gamma})/\tilde{\gamma}$, $\tilde{\gamma} = \sqrt{\gamma^2 - 2\alpha \gamma}$.

In the limit $\gamma \to 0$, the $t$-integration in the above Kähler potential is carried out analytically. In this case, we find a SUSY preserving extremum of the scalar potential with constant and tadpole superpotential terms at the fixed points $(w_{\vartheta}^{[n \geq 2]} = 0)$. The gravitino and the lightest modulus mass squares, $m_{3/2}^2$ and $m_{\text{mod}}^2$, at this SUSY point with various values of $\alpha$ is shown in Fig. 1 (a) and (b), respectively. We find that $m_{3/2}^2$ ($m_{\text{mod}}^2$) monotonically decreases (increases) as $\alpha$ decreases.

On the other hand, in the limit $\alpha \to 0$, our model is reduced to SUSY RS model with the AdS curvature $k$ and an independent kink mass $m$ shown before. With the generic superpotential $w_{\vartheta}^{(n)} \neq 0$, the hypermultiplet is stabilized at the origin $H = 0$, where the effective theory of the radius modulus $T$ is described by

$$K = -3 \ln \frac{1 - e^{\pi k(T + T')}}{2k}, \quad W = \frac{1}{4} (w_0^{(0)} + w_{\pi}^{(0)} e^{-3\pi kT}).$$

If the parameters satisfy $-w_0^{(1)}/w_\pi^{(1)} = (-w_0^{(0)}/w_\pi^{(0)})^{m/k + 3/2}$, we easily find a SUSY AdS minimum at $\pi k T = \ln(-w_0^{(0)}/w_\pi^{(0)})$ where the gravitino and moduli mass square are found as $m_{3/2}^2 = k^3 w_0^{(0)} \left\{ 1 - (w_0^{(0)})^2/(w_\pi^{(0)})^2 \right\}^2 /2$ and $(m_{T^+}^2, m_{T^-}^2) = (4m_{3/2}^2, 0)$, respectively.
4 Summary

Based on the off-shell dimensional reduction [7, 8], we studied a 4D effective theory of the 5D gauged supergravity with a universal hypermultiplet and generic gaugings and found SUSY preserving extrema with perturbative superpotential terms at the orbifold fixed points. These results would be useful for further studies of moduli stabilization and the uplifting of AdS minima in this class of models [10, 11].

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