Subjective modeling of image shape

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Abstract. The paper presents two approaches to subjective modelling incomplete and uncertain information about possible shape of images. The shape is understood as a set of images of the scene, recorded under all possible conditions (lighting, exposure, etc.). To simulate the likelihood, the measure of likelihood introduced by Yu.P. Pyt'ev is used. This measure is a function defined on a set of statements and ordering them according to plausibility. The obtained subjective models of shapes allow using the theory of optimal strategies for solving morphological problems.

1. Introduction

Methods of morphological analysis of images use their structural and brightness description. The first approaches to the description of the morphology of images appeared in the 1970s. These include mathematical morphology described in the work of J. Serre [1] and methods of morphological image analysis published in the works of Yu.P. Pyt'ev [2-5], while the approaches to the description of the shape of images in these works are fundamentally different.

The methods of morphological image analysis of Yu.P. Pyt’ev were continued by his colleagues under his guidance [6–9]. These methods have been widely and successfully used to solve a number of problems, such as recognizing objects by the shape of their image, distinguishing differences in scenes by their images, estimating the parameters of objects in the depicted scenes, etc. [6, 8, 10]. They are also used to analyze the shape of images and signals when solving a number of applied problems [11–15].

In the methods of morphological analysis of image given by Pyt'ev the image shape is defined as an invariant of image transformations that simulate a change in the conditions of its registration. Thus, the shape of the image does not depend on the lighting, the parameters of the recording equipment, etc. and is determined only by the properties of the objects of the depicted scene. In the classical methods of morphological analysis in terms of the named invariant, the image shape is defined, a comparison of the images by shape operation is introduced, and a set of methods for solving a number of image analysis tasks is proposed.

However, in solving these problems, additional information about the scene or its images may be essential. This information, on the one hand, is difficult to formalize in terms of classical morphological analysis, and on the other hand, its consideration can significantly increase the accuracy and adequacy of solutions. To account for this kind of information in solving mathematical modeling problems in [16] Yu.P. Pyt'ev proposes to use fuzzy plausibility and belief measures on a set
of statements. This paper is devoted to the application and development of this approach in the morphological analysis of images.

2. Image shape

In the classical morphological analysis, a canonical example is the shape of a mosaic image. For its definition, one uses a mathematical model of the image of a certain scene $S$ in the form

$$f(x) = \sum_{i=1}^{n} c_i(x) \chi_i(x) \quad x \in X,$$

(1)

here $X$ is a bounded subset of the two-dimensional plane (field of view), $\chi_i(\cdot)$ is the indicators of the sets $A_i$, $i = 1, \ldots, n$, forming the partition $X = \bigcup_{i=1}^{n} A_i$, $A_i \bigcap A_j = \emptyset$ for $i \neq j$,

$$\chi_i(x) = \begin{cases} 1, & x \in A_i, \\ 0, & x \notin A_i, \end{cases}$$

$c_i(\cdot)$ is a continuous function defined on the set $A_i$ that simulates the brightness (color, if $c_i(\cdot)$ is a vector-valued function) of set $A_i$. In other words, (1) is a mathematical model of a segmented image, the brightness (color) of which continuously changes inside segment $A_i$. Geometric shape of segments $A_i$, $i = 1, \ldots, n$, reflects the optical and geometric properties of scene objects, and brightness $c_i(\cdot)$ of the image of the same scenes can change with variation of registration conditions. Thus, the indicators $\chi_i(\cdot)$ are unchanged in all images of the scene obtained under various registration conditions. In terms of these indicators, a mosaic image shape description of the scene is given.

The mathematical definition of the image shape is as follows: the image shape (1) is the set

$$V_f = \{g(x) = \sum_{i=1}^{n} c'_i(x) \chi'_i(x), c'_i(\cdot) \in F_i, i = 1, \ldots, n\}.$$

(2)

This set is determined by the set of indicators $\chi'_i(\cdot)$, $i = 1, \ldots, n$, and takes into account the restrictions on the possible classes of changes in the brightness (color) of each set $A_i$ with the indicator $\chi'_i(\cdot)$. It is usually assumed that the images $f(\cdot)$ and $g(\cdot)$ are elements of a linear normed space, most often the Euclidean space $R$; then, if $V_f$ is convex and closed in $R$, then the projection operator $P_f : R \rightarrow R$ is one-to-one connected to it, with $g(\cdot) \in V_f$ iff $g(\cdot) \in V_f$. The image $g - P_f g$ represents everything that distinguishes $g(\cdot)$ from the shape $V_f$ of the image $f(\cdot)$, therefore the value

$$\|g - P_f g\|^2$$

is a measure of difference $g(\cdot)$ from shape $V_f$.

To characterize the closeness of the shape of the image of $g(\cdot)$ to $V_f$, in practice, the relation

$$\tau_f = \frac{\|g - P_f g\|^2}{\|P_f g - P_0 g\|^2}$$

is often used. Here, $P_0 : R \rightarrow R$ is a projector on a set of images equal to a constant on $X$, so $P_0 g$ is an image with a constant brightness (color) equal to the average brightness of $g(\cdot)$ in the field of view of $X$. The denominator thus characterizes the magnitude of the image component of $g(\cdot)$ having the shape of $V_f$, which is different from the constant. This denominator determines the value of the "useful signal". The numerator gives the difference between $g(\cdot)$ and $V_f$, which can be interpreted as a hindrance, if we assume that $g(\cdot)$ is a distorted image of the same scene.
as the image $f()$. The ratio $\tau_f = \frac{\|g - Pf g\|^2}{\|P_f g - P_0 g\|^2}$ determines the difference in the shape of the image $g()$ from $f()$. In view of the foregoing, it is possible to give it a meaning of the “noise”/“signal” ratio.

3. **Subjective model of the image shape with fixed segments of mosaic**

The set $V_f$ of all possible images of the scene $S$ in the previous example is a formally “completely deterministic” set: for any element $g() \in R$ there is a definite answer whether $g()$ belongs to the shape $V_f$ or not. However, in practice, the researcher can set a rule that allows for any pair of images, $g_1() \in R$ and $g_2() \in R$, indicate which of them is more similar to the image of scene $S$, or say that they are equally alike. The researcher can model his views by giving a measure of plausibility [16] of the statement that $g()$ is similar in shape to $f()$, and, if necessary, a measure of belief in this statement, describing how far the opposite statement can be mistrusted. Formally, these measures can be defined if, in (1), the coefficients $c_i(), i = 1,...,n$, are considered to be indeterminate elements in terms of [16]. For simplicity, let $c_i()$ in (1) be constants independent of $x \in X$, then the likelihood distribution of an undefined vector $(c_1,...,c_n)$ can be specified as a function $\pi^{c_1,...,c_n}(z_1,...,z_n): R^n \rightarrow [0,1]$. The meaning of this function is as follows: if $\pi^{c_1,...,c_n}(z_1,...,z_n) = \pi^{c_1,...,c_n}(\tilde{z}_1,...,\tilde{z}_n)$, then the image $\tilde{g}(x) = \sum_{i=1}^{n} \tilde{z}_i\chi_i(x)$ is more similar in shape to $f()$ than the image $g(x) = \sum_{i=1}^{n} z_i\chi_i(x)$ according to the subjective view of the researcher.

The proposed technique allows us to subjectively compare mosaic images in shape. However, a $\xi()$ image that is not mosaic may be suggested for analysis. In this case, before applying the subjective comparison, one can calculate the projection $\Pi_f \xi$ of the image $\xi()$ onto the linear subspace $L(\chi_1(),...,\chi_n()) \subset R$ spanned by the indicators $\chi_1(),...,\chi_n()$, and then compare the projection $\Pi_f \xi = \sum_{i=1}^{n} (\xi,\chi_i)\chi_i$ with the subjective shape of the image $f()$, calculating $\pi^{c_1,...,c_n}(z_1,...,z_n)$ at $z_i = (\xi,\chi_i)^2$. However, in this case, we get a two-criterion description, which is not always convenient in practice.

To obtain a single proximity criterion, one should specify the likelihood distribution on the whole set $R$, assuming that the image $\xi(x) = g(x) + \nu(x)$, where $g(x) = \sum_{i=1}^{n} c_i\chi_i(x)$ is a mosaic piecewise constant image, and $\nu()$ is an indefinite element the plausibility distribution of which is a monotonic function of its norm in $R$: $\pi^{\nu}(y) = \mu_0(\|y\|)$. Assuming the indefinite elements $\nu()$ and $(c_1,...,c_n)$ are independent, we obtain the joint distribution of a pair of indefinite elements $\pi^{\nu,c_1,...,c_n}(y,z_1,...,z_n) = \min\{\pi^{\nu}(y),\pi^{c_1,...,c_n}(z_1,...,z_n)\}$. 


Having a plausibility distribution, one can apply the mathematical decision-making technique developed in the theory of subjective modeling described in [16].

4. General subjective model of image shape

In the previous section, a mathematical model of the image shape was constructed, in which the subjective preferences of the researcher concern only the brightness characteristics of the image, and the geometric structure is assumed to be given. However, in practice, it is often necessary to assume that the scene S can generate images with different segmentation. In this paper, it is proposed to choose a subjective model of the image shape based on the visual analysis of the presented image of the scene.

Let the image $\xi(x)$ of the scene S be presented as a mixture of the “ideal” image $f(\cdot)$ and noise $\nu(\cdot)$: $\xi(x) = f(x) + \nu(x)$. The mathematical model of noise is not defined a priori, but it is believed that the researcher has sufficient experience and intuition to distinguish the noise image from the image of a certain scene. As for the shape of the image $f(\cdot)$, it is considered that it is defined as a family of “completely deterministic” sets $V_f(\mathcal{G})$, depending on the parameter $\mathcal{G} \in \Theta$.

The task is to build the distribution of plausibility on the set of shape parameters $\Theta$, and thus on the set of shapes. The basic idea of constructing such a distribution is taken from [17].

Let for each $\mathcal{G} \in \Theta$ shape $V_f(\mathcal{G})$ be a linear subspace of $R$, $P_f(\mathcal{G})$ is the linear operator of orthogonal projection onto $V_f(\mathcal{G})$ in $R$. Then, if the parameter value $\mathcal{G}$ determines the shape of an image that agrees with the researcher’s subjective view of the shape of the image of the scene S, the image $\xi - P_f(\mathcal{G})\xi$ is a noise image, and $P_f(\mathcal{G})\xi$ is the image of the scene S; $P_f(\mathcal{G})\xi$ and $\xi - P_f(\mathcal{G})\xi$ lie in orthogonal subspaces, which allows to consider them independent.

The researcher for each selected value of $\mathcal{G} = t_k \in \Theta$, $k=1,\ldots,K$, by visual analysis of a series of images $P_f(t_k)\xi$ based on experience and intuition indicates his attitude to its similarity with the image of the scene S, setting the plausibility distribution $\pi_f(\mathcal{G})$, $t = t_k$, $k=1,\ldots,K$. Further, based on the visual analysis of a series of images $\xi - P_f(t_k)\xi$, the researcher, based on his experience and intuition, indicates his attitude to the similarity of images $\xi - P_f(t_k)\xi$ with the noise image by constructing a plausibility distribution $\pi_{\nu}(t)$, $t = t_k$, $k=1,\ldots,K$. A combination of these two models, provided that the elements of orthogonal to each other subspaces are independent, gives the result $\pi(\mathcal{G}) = \min\{\pi_f(\mathcal{G}), \pi_{\nu}(t)\}$, $t = t_k$, $k=1,\ldots,K$.

The distribution of plausibility constructed in this way on a set of parameters $\mathcal{G} \in \Theta$ makes it possible to solve a number of image analysis problems. For example, the task of estimating the shape of a scene image from the presented image can be solved by choosing a shape $V_f(t_*)$, where $t_* = \arg \max_{t \in t_1,\ldots,t_K} \pi(\mathcal{G})$ is the most likely value of the parameter $\mathcal{G} \in \{t_1,\ldots,t_K\}$.

5. Conclusion

In this paper, two approaches are proposed for constructing a subjective model of the shape of an image. Both allow taking into account the experience and intuition of the researcher. Models constructed applying these approaches make it possible to use a mathematical apparatus for making optimal decisions, developed in the theory of possibilities, for morphological studies.
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