The Polarization Sensitive FDA-MIMO Radar: Performance Analysis of Beamforming Based on Oblique Projection

Binbin Li*, Weixiong Bai, Qin Zhang, and Guimei Zheng
Air and Missile Defense College, Air Force Engineering University, 710051 Xi’an, China
Email: binbinli_1025@163.com; wxb369@126.com; kinzh@263.com; zheng-gm@163.com

* Corresponding Author: Binbin Li; email:binbinli_1025@163.com; phone: +8618192394878; fax: +8602984789478

Abstract: In this paper, a beamforming algorithm based on oblique projection technique is proposed for a hybrid radar of polarization sensitive array (PSA), frequency diversity array (FDA) and multiple-input multiple-out (MIMO) radar, termed as PSFDA-MIMO radar. Firstly, the signal model of the proposed PSFDA-MIMO radar is established. Secondly, oblique projection technique is proposed to solve the beamforming problem of the novel radar. Finally, the simulation results show that the overall performance of beamforming based on the proposed radar is better than that of other combined radar systems.

1. Introduction
Polarization sensitive array (PSA), multiple-input multiple-out (MIMO) radar and Frequency diverse array (FDA) are three species of interests due to their own distinctive characteristics. PSA [1] can sense the additional polarization information compared with phased-array radar. MIMO radar [2] provides a platform for the use of spatial diversity and waveform diversity, and has great potential in improving resolution and suppression of interference. FDA has a range-angle-dependent beampattern [3], which has great potential in suppressing mainlobe jamming. The distinctive performance of these new system radars is not available in traditional phased-array radar. To obtain better performance of radar, many scholars combine them to construct a new combined radar system with better performance. Reference [4] solves the problem of the parameter estimation of the radar system combined with PSA radar and MIMO radar (PSA-MIMO). References [5, 6] study the beam forming problem of the radar system with the combination of PSA radar and FDA radar (PSFDA). References [7, 8] investigate the beamforming problem of the radar with the combination of FDA radar and MIMO radar (FDA-MIMO).

However, there is no work to research the radar system combined with three radar systems. Therefore, this paper proposes to integrate PSA radar, MIMO radar and FDA radar into a new radar system, termed as PSFDA-MIMO radar, which provides a platform for the use of diversity technologies, such as spatial diversity, waveform diversity, polarization diversity and frequency diversity. Firstly, the signal model of the proposed PSFDA-MIMO radar is established. Secondly, oblique projection technique is proposed to solve the beamforming problem of the novel radar. Last, by comparing the output signal-to-interference-plus-noise ratio (SINR) after oblique projection filtering, simulations demonstrate...
that the overall performance of the proposed PSFDA-MIMO radar is better than that of other combined radar systems, such as FDA-MIMO radar, PSA-MIMO radar and PSFDA radar.

The remainder of this paper is organized as follows. Section II gives the signal model of the proposed PSFDA-MIMO radar. The beamforming algorithm based on oblique projection technique is proposed in Section III. Section IV gives the simulation results and analyzes the performance of the proposed radar. Section V concludes the paper.

2. Signal model of proposed PSFDA-MIMO radar

The signal model of the proposed PSFDA-MIMO radar will be derived in this section. Just as the name implies, the proposed PSFDA-MIMO radar is the combination of PSA radar, FDA radar and MIMO radar. As shown in Figure 1, the array element of the transmitting array is composed of $M$ ordinary elements. The spacing of the element is marked with $d_T$. The carrier frequency of the $m$-th element is expressed as

$$f_m = f_0 + (m-1)\Delta f, \quad m = 1, 2, \ldots, M$$

where $f_0$ represents the reference carrier frequency and $\Delta f$ denotes the frequency increment. The receiving array is composed of $N$ crossed-dipoles and the spacing of the element is marked with $d_R$.

The proposed radar is assumed to be a colocated MIMO radar. So the direction of departure (DOD) and direction of arrival (DOA) can be assumed as $\theta$ when the target signal is assumed to be a far-field narrow-band signal. The transmitting steering vector can be expressed as

$$a(r, \theta) = r(r) \odot d(\theta) \in \mathbb{C}^{M \times 1}$$

$$r(r) = [1, e^{jk \lambda r/c}, \ldots, e^{jk (M-1) \lambda r/c}]^T \in \mathbb{C}^{M \times 1}$$

$$d(\theta) = [1, e^{j2\pi d_RE \sin \theta/c}, \ldots, e^{j2\pi d_RE (N-1) \sin \theta/c}]^T \in \mathbb{C}^{N \times 1}$$

where $r$ represents the range of the target. $c$ represents the propagation velocity of electromagnetic wave, $\odot$ represents Hadamard product, $(\cdot)^T$ represents the transpose. The receiving steering vector can be expressed as

$$b(\theta) = [1, e^{i2\pi d_RE \sin \theta/c}, \ldots, e^{i2\pi d_RE (N-1) \sin \theta/c}]^T \in \mathbb{C}^{N \times 1}$$

The response of the crossed-dipole can be expressed as

$$c(\theta, \xi, \eta) = \begin{bmatrix} 0 & -1 \sin \xi e^{iq} \\ \cos \theta & 0 \cos \xi \\ \cos \theta \sin \xi e^{iq} \end{bmatrix}$$

where $\xi$ and $\eta$ denote the auxiliary polarization angle and polarization phase difference, respectively. Therefore, the joint steering vector of the proposed radar can be expressed as

$$\vec{a}(\theta, r, \xi, \eta) = c(\theta, \xi, \eta) \odot a(r, \theta) \odot b(\theta) \in \mathbb{C}^{2M \times 1}$$

It is assumed that one target signal and $J$ jamming signals impinge upon the proposed array. The received data can be expressed as

$$z(t) = \sum_{i=1}^{J} \vec{a}(\theta_s, r_s, \xi, \eta_s) s_i(t) + n(t)$$
where \( \{ \theta_i, r_i, \zeta_i, \eta_i \} \) and \( \{ \theta_j, r_j, \zeta_j, \eta_j \} \) denote the parameters of target and jamming, respectively. \( s_i(t) \) and \( s_j(t) \) are the waveform of target and jamming, respectively, which are assumed as a zero-mean, complex Gaussian, random process. \( n(t) \) denotes a \( 2MN \times 1 \) complex Gaussian white noise vector with zero mean and covariance matrix \( \sigma^2 I_{2MN} \). \( \sigma^2 \) represents the noise variance. \( I_{2MN} \) is a \( 2MN \times 2MN \) identity matrix. The matrix \( B = [ \bar{\alpha}(\theta_1, r_1, \zeta_1, \eta_1), \cdots, \bar{\alpha}(\theta_J, r_J, \zeta_J, \eta_J) ] \) denotes the subspace of jamming.

3. Beamforming based on oblique projection

In this section, the oblique projection technique is employed to produce a spatial fine focusing beam-pattern. In the following, \( \bar{\alpha}(\theta, r, \zeta, \eta) \) is abbreviated to \( \bar{\alpha} \). According to the oblique projection theory [6], the oblique projection operator can be expressed as

\[
E_{ab} = \bar{\alpha} (\bar{\alpha}^H P_a^+ \bar{\alpha})^{-1} \bar{\alpha}^H P_a^+
\]

(9)

\[
P_a^+ = I - P_a
\]

(10)

\[
P_a = BB^+
\]

(11)

\[
B^+ = (B^H B)^{-1/2}
\]

(12)

where \( (\cdot)^H \) represents the conjugate transpose. The oblique projection operator \( E_{ab} \) has the following properties

\[
E_{ab} \bar{\alpha} = \bar{\alpha}
\]

(13)

\[
E_{ab} B = 0
\]

(14)

The output of the oblique projection filtering can be expressed as

\[
y(t) = E_{ab} z(t)
\]

\[
= E_{ab} \bar{\alpha} s_i(t) + E_{ab} B s(t) + E_{ab} n(t)
\]

\[
= \bar{\alpha} s_i(t) + E_{ab} n(t)
\]

(15)

Then the desired signal can be recovered by the spatial matched filtering

\[
\hat{s}_i(t) = \bar{\alpha}^H y(t) = s_i(t) + \bar{\alpha}^H E_{ab} n(t)
\]

(16)

4. Simulation results

In order to prove the superior performance of the proposed radar, FDA-MIMO radar, PSA-MIMO radar and PSFDA radar are chosen for comparison. Next, the advantages of the proposed radar are demonstrated from two aspects: a) comparing their beampatterns; b) comparing their output SINR.

4.1 Beampatterns for four classes of radar systems

It is supposed that the parameter of the desired signal is \( \{ \theta, r, \zeta, \eta \} = \{33^\circ, 35.1 \text{km}, 60^\circ, 30^\circ \} \), the parameters of two jamming signals are \( \{ \theta_i, r_i, \zeta_i, \eta_i \} = \{45^\circ, 40.2 \text{km}, 75^\circ, 45^\circ \} \), \( \{ \theta_j, r_j, \zeta_j, \eta_j \} = \{60^\circ, 65.1 \text{km}, 65^\circ, 60^\circ \} \). SNR=10dB, SINR=30dB. The number of array elements for both transmitting and receiving arrays is set to be 10. The frequency increment is set to be \( 10^3 \text{HZ} \). Figure 2 shows the beampatterns of the PSFDA-MIMO radar, FDA-MIMO radar, PSA-MIMO radar and PSFDA radar, respectively. As can be seen from Figure 2, the beam of the PSFDA-MIMO radar and FDA-MIMO radar can be gathered at the desired angle and range point. But the beam of the other two classed of radars can not.
(a) Beampattern of PSFDA-MIMO radar

(b) Beampattern of FDA-MIMO radar

(c) Beampattern of PSA-MIMO radar
According to the results of the first simulation, we know that the beam of both PSFDA-MIMO radar and FDA-MIMO radar can be gathered at the desired angle and range point. But it is impossible to judge which radar has a better performance. Therefore, the second simulation experiment is implemented to compare the output SINR of PSFDA-MIMO radar and FDA-MIMO radar. SNR varies from 0dB to 40dB with 5dB interval. As shown in Figure 3, as the input SNR increases, the output SINR increases too. In the same condition, the output SINR of the PSA-MIMO radar is the largest, but whose beam can only be gathered at the desired angle region, but can not gather at the desired range region. As can be seen from Figure 3, the output SINR of the proposed PSFDA-MIMO radar is higher than that of the FDA-MIMO radar.

In summary, the overall performance of PSFDA-MIMO radar is the best in all combined radar systems.

5. Conclusions
In this paper, the beamforming of a novel radar system has been investigated. The beam of proposed PSFDA-MIMO radar can be gathered at the desired angle and range point, which is beneficial to suppress the mainlobe jamming. The overall performance of the proposed PSFDA-MIMO radar is better than that of existing combined radar systems. However, the corresponding research of the proposed radar system is still in its infancy. Next, the parameter estimation and anti-interference of the proposed radar system will further be researched.
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