*K*D*N* molecular state as a “uuuds\(\bar{c}\) pentaquark” in a three-body calculation

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We predict a new three-body hadronic molecule composed of antikaon \(\bar{K}\), anticharm meson \(\bar{D}\), and nucleon \(N\) with spin/parity \(J^P = 1/2^+\) and isospin \(I = 1/2\). This state behaves like an explicit pentaquark state because its minimal quark configuration is uuuds\(\bar{c}\) or uuuds\(\bar{c}\). Owing to the attraction between every pair of two hadrons, in particular the \(\bar{K}D\) attraction which dynamically generates \(D_s(2317)^-\) and \(KN\) attraction which dynamically generates \(\Lambda(1405)\), the \(K\bar{D}N\) system is bound, and its eigenenergy is calculated as 3244 – 17i MeV in a nonrelativistic three-body potential model. We discuss properties of this \(K\bar{D}N\) quasi-bound state which emerge uniquely in three-body dynamics.

I. INTRODUCTION

Studying strong interactions between hadrons is one of the most important issues in hadron physics. The best known strong interaction is the nuclear force between nucleons (Ns), which generates a large number of atomic nuclei composed of protons and neutrons. In addition to the nuclear force, recent interest in the strong interactions between hadrons is to explore bound states of mesons and baryons governed by strong interactions between them, which are so-called hadronic molecules. A classic example of hadronic molecule candidates is the \(\Lambda(1405)\) resonance, which may be an \(S\)-wave quasibound state of antikaon \(\bar{K}\) and nucleon \(N\) \([1]\). Recently an analysis of the lattice QCD energy levels with an effective-field-theory model showed that the \(\Lambda(1405)\) is dominated by the bound \(KN\) component with isospin \(I = 0\) \([2]\). The \(KN\) molecular picture for the \(\Lambda(1405)\) was supported also in Refs. 3, 4 in terms of the compositeness, which is defined as the norm of a two-body wave function for hadronic resonances 5, 6. Furthermore, recent experiments in high-energy colliders such as Belle, BaBar, BESIII, and LHCb have revealed fruitful physics in the charm- and bottom-quark sectors. Besides the \(X, Y,\) and \(Z\) resonances as exotic candidates, the \(D_{s0}(2317)^-\) resonance 8, 9 is of interest from the viewpoint of hadronic molecules. Because its mass is located just below the \(K\bar{D}\) threshold, it is natural to think that the \(D_{s0}(2317)^-\) is an \(S\)-wave \(K\bar{D}\) bound state with isospin \(I = 0\). A dominant \(K\bar{D}\) component for the \(D_{s0}(2317)^-\) was implied by theoretical calculations 10, 11, and was supported also by theoretical analyses of lattice QCD data 12, 13 and of experimental data 14.

We can extend discussions on hadronic molecules of two-body systems to those of three-body systems. In this respect, three-body systems are not only a key to understand interactions between hadrons but also a good ground to investigate three-body dynamics. For example, properties of two-body bound states may disappear in three-body bound states, or, conversely, some properties may emerge uniquely in the three-body bound states. Three-body forces may become significant, and in general its form may differ from the three-nucleon force. Furthermore, in case that a two-body interaction depends on the energy owing to implicit channels which do not appear as explicit degrees of freedom, it is not trivial how to treat the energy dependence of the two-body interaction in the three-body calculations. We can discuss these three-body dynamics from properties of three-body hadronic molecules by applying and extending approaches to solve few-body problems developed for usual atomic nuclei.

An important progress for three-body hadronic molecules takes place recently on the \(KNN\) quasi-bound state. The \(KNN\) quasi-bound state was predicted based on the strong attraction in the \(KN\) system in Ref. 16, which was followed by more sophisticated theoretical calculations 17, 30. Eventually, the J-PARC E15 experiment very recently observed a peak structure which can be a signal of the \(KNN\) quasi-bound state 31, 32. The study of the \(KNN\) quasi-bound state also triggered theoretical studies of similar three-body hadronic molecules: for instance, \(KKN\) 33, 34, \(KK\) 36, 40, \(KKK\) 39, 41, and \(KDN\) 42.

In this study we propose a new candidate of three-body hadronic molecules, the \(K\bar{D}N\) three-body system with spin/parity \(J^P = 1/2^+\) and isospin \(I = 1/2\). This system has two kinds of attraction which could be essential to make the \(K\bar{D}N\) bound state. One is the \(KN(I = 0)\) interaction and the other is the \(K\bar{D}(I = 0)\) interaction, which dynamically generate the \(\Lambda(1405)\) and \(D_{s0}(2317)\), respectively. On the other hand, the \(\bar{D}N\) interaction is moderate, but some models implied that the coupling to the \(D^*N\) channel brings attraction to the \(DN\) interaction and generates an \(S\)-wave \(\bar{D}N(I = 0)\) bound state with binding energy \(\sim\) 1 MeV 33, 44. To clarify whether the \(K\bar{D}N\) three-body system is bound or not, in this manuscript we will solve a nonrelativistic three-body potential model for the \(K\bar{D}N\) system and search for the bound state. Throughout this study, we assume isospin

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symmetry for the hadron masses and interactions, and concentrate on the zero-charge $K^0N$ system, which exists in the $K^-D^0p-K^0D^-p-K^0D^0n$ coupled channels.

We here mention that the $K^0N$ three-body system with $I = 1/2$ has the minimal quark configuration of $uuds\bar{c}$ or $udsd\bar{c}$, so this bound state, if exists, is explicitly a “pentaquark” state. This is in contrast of $uud\bar{s}$ or $udsd\bar{s}$, so this bound state, if exists, is explicitly a “pentaquark” state. This is in contrast of the charmonium-pentaquark $P_c(4450)$ \cite{43}, in which the charm and anticharm quarks are hidden inside the $P_c(4450)$ and the minimal quark configuration is $uud$.

This paper is organized as follows. In Sec. II we construct the two-body local potentials for the subsystems of $K^0N$, $K^0D$, and $DN$ for the subsystems of $K^0N$. By using this two-body local potentials, we formulate the $K^0N$ three-body problem in Sec. III. In Sec. IV we show our numerical results and discuss properties of the $K^0N$ system. Section V is devoted to the summary and concluding remarks of this study.

II. TWO-BODY SYSTEMS

A. How to construct two-body local potentials

First of all, we explain how to construct the two-body interactions for the $K^0N$, $K^0D$, and $DN$ systems. Because we are interested in the two-body interactions in $S$ wave, we extract the $S$-wave projected interaction and then construct the local and orbital-angular-momentum independent potentials for the two-body systems which reproduce the two-body phenomena in $S$ wave.

In general, the $K^0N$, $K^0D$, and $DN$ channels couple to inelastic channels, but in this study we integrate out them so that only the $K^0N$, $K^0D$, and $DN$ channels are explicit degrees of freedom, according to the method in Ref. \cite{46}. For instance, in the $K^0D_\pi D_\pi\eta D_\eta$ coupled channels for the $K^0D$ interaction, the $\pi D_\pi$ and $\eta D_\eta$ channels are taken as inelastic and are integrated out. We refer to the two-body interactions in which inelastic channels are integrated out as effective interactions.

We start with a full coupled-channels interaction in isospin basis $V_{jk}$ with the channel indices $j$ and $k$, which is calculated in a certain model. We project the interaction to the $S$ wave and take the so-called on-shell factorization \cite{47}, so $V_{jk}$ depends only on the two-body center-of-mass energy $\epsilon$. This interaction generates the full coupled-channels scattering amplitude $T_{jk}(\epsilon)$ as

\[ T_{jk}(\epsilon) = V_{jk}(\epsilon) + \sum_i V_{ji}(\epsilon) G_i(\epsilon) T_{ik}(\epsilon) = \left\{ \left[ 1 - V(\epsilon)G(\epsilon) \right]^{-1} V(\epsilon) \right\}_{jk}. \]

Here, $G_j$ is the hadron–hadron loop function

\[ G_j(\epsilon) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_j^2} \frac{1}{(P - k)^2 - M_j^2}. \]

with $m_j$ and $M_j$ are masses of particles in channel $j$ and $P^\mu = (\epsilon, \mathbf{0})$. We calculate the loop function with the dimensional regularization, which brings a subtraction constant corresponding to the cutoff for the loop.

Now suppose that we explicitly treat only channel $j = 1$ and integrate out inelastic channels $j > 1$. In this condition, we can calculate the effective interaction $V_{\text{eff}}$ as

\[ V_{\text{eff}}(\epsilon) = \left\{ \left[ 1 - V(\epsilon)\tilde{G}(\epsilon) \right]^{-1} V(\epsilon) \right\}_{j=1,k=1}, \]

with

\[ \tilde{G}_j(\epsilon) = \begin{cases} 0 & (j = 1), \\ G_j(\epsilon) & (j > 1). \end{cases} \]

Physically, $V_{\text{eff}}$ is the sum of the bare interaction $V_{11}$ and terms which include resummation of loop contributions from the inelastic channels $j > 1$ to all orders \cite{46}. Then, the effective interaction $V_{\text{eff}}$ is translated into the local two-body potential $U(r)$ with the relative distance $r$ in the nonrelativistic reduction:

\[ U(r; \epsilon) = \frac{g(r)}{4\omega_1(\epsilon)\Omega_1(\epsilon)} V_{\text{eff}}(\epsilon). \]

Here, $g(r)$ is a form factor defined as

\[ g(r) = \frac{1}{\pi^{3/2}b^3} e^{-r^2/b^2}, \]

and

\[ \omega_1(\epsilon) = \frac{\epsilon^2 + M^2 - m_1^2}{2\epsilon}, \quad \Omega_1(\epsilon) = \frac{\epsilon^2 + M^2 - m_1^2}{2\epsilon}. \]

The range parameter $b$ can be fixed independently in three systems: $K^0N$, $K^0D$, and $DN$. Note that the local potential $U$ depends on the energy of the two-body system $\epsilon$ according to the integration of the implicit channels as well as intrinsic energy dependence of the full interaction $V_{jk}$.

The above potential $U(r; \epsilon)$ is described in isospin basis. The translation into the potential in particle basis is straightforward.

B. $K^0N$ system

In the $K^0N$ subsystem in $K^0D$, we consider three channels: $K^-p$, $K^0n$, and $K^0p$. The former two channels couple to each other.

We employ the Kyoto $K^0N$ effective potential developed in the above manner in Ref. \cite{48}, which reproduces experimental results on the $K^-p$ scattering phenomena based on chiral SU(3) coupled-channels dynamics \cite{49,50}. The range parameter is $b = 0.38$ fm. The Kyoto $K^0N$ potential in its original form is written in
isospin basis as $U_{\bar{K}N(I=0)}(r;\epsilon)$ and $U_{\bar{K}N(I=1)}(r;\epsilon)$. The expression in particle basis is

$$U_{K^-p\rightarrow K^-p} = U_{\bar{K}^0n\rightarrow \bar{K}^0n} = \frac{U_{\bar{K}N(I=0)} + U_{\bar{K}N(I=1)}}{2},$$

(8)

$$U_{K^-p\rightarrow \bar{K}^0n} = U_{\bar{K}^0n\rightarrow K^-p} = \frac{U_{\bar{K}N(I=0)} - U_{\bar{K}N(I=1)}}{2},$$

(9)

$$U_{\bar{K}^0p\rightarrow \bar{K}^0p} = U_{\bar{K}N(I=1)},$$

(10)

where we omitted parameters $(r;\epsilon)$ for $U$.

The $KN(I=0)$ effective potential $U_{\bar{K}N(I=0)}(r;\epsilon)$ generates two $\Lambda(1405)$ poles at $\epsilon_{\text{pole}} = 1424 - 26i$ MeV and 1381 - 81i MeV as $S$-wave bound-state solutions of the Schrödinger equation

$$m_K + m_N - \frac{\nabla^2}{2\mu_{KN}} + U_{\bar{K}N(I=0)}(r;\epsilon_{\text{pole}}) \psi(r) = \epsilon_{\text{pole}} \psi(r),$$

(11)

where $m_K$ and $m_N$ are kaon and nucleon masses, respectively, and $\mu_{KN} \equiv m_Km_N/(m_K + m_N)$ is the reduced mass of the $KN$ system. Among the two $\Lambda(1405)$ poles, the higher pole at $\epsilon_{\text{pole}} = 1424 - 26i$ MeV corresponds to the $KN$ quasi-bound state in chiral dynamics [3, 4]. Properties of the $KN$ quasi-bound state in Kyoto $KN$ potential was discussed in Ref. [13]; here we quote that the average of the $KK$ distance is\(\sqrt{(r^2)} = 1.06 - 0.57i\) fm with the Gamow-vector normalization method [14].

**C. $KD$ system**

In the $\bar{K}D$ subsystem in $\bar{K}DN$, we consider three channels: $K^-D^0$, $\bar{K}^0D^-$ and $\bar{K}^0\bar{D}^0$. The former two channels couple to each other.

We employ a phenomenological Lagrangian constructed in Ref. [10]. An important point is that in this model the $D_{s0}(2317)$ state is dynamically generated by the strong attraction of the elastic $\bar{K}D$($I=0$) interaction. In isospin $I=0$, we have $KD-\pi D_s$ coupled channels with the interaction

$$V_{\bar{K}D\rightarrow KD(I=0)}(\epsilon) = -\frac{1}{3f_D} \left[ \gamma(\bar{t}-\bar{u}) + \epsilon^2 - \bar{u} + m_D^2 + m_K^2 \right],$$

(12)

$$V_{\bar{K}D\rightarrow \eta D_s}(\epsilon) = V_{\eta D_s\rightarrow \bar{K}D}(\epsilon) = -\frac{1}{6\sqrt{3}f_D} \left[ \gamma(\bar{u} - \bar{t}) - (3 + \gamma)(\epsilon^2 - \bar{u}) - m_D^2 - 3m_K^2 + 2m_s^2 \right],$$

(13)

where $f_D$ and $m_D$ are decay constant and mass of pion ($D$ meson), respectively, and $\gamma$ is the squared ratio of the masses of the light to heavy vector mesons. In the above expressions, $\bar{t}$ and $\bar{u}$ are the Mandelstam variables projected to the $S$ wave in the on-shell factorization. Owing to the $S$-wave projection, $\bar{t}$ and $\bar{u}$ are functions only of $\epsilon$:

$$\bar{t} = -\frac{1}{2\epsilon^2} \left[ \epsilon^4 - \epsilon^2(m_a^2 + m_b^2 + m_c^2 + m_d^2) + (m_a^2 - m_b^2)(m_c^2 - m_d^2) \right],$$

(15)

$$\bar{u} = -\frac{1}{2\epsilon^2} \left[ \epsilon^4 - \epsilon^2(m_a^2 + m_b^2 + m_c^2 + m_d^2) - (m_a^2 - m_b^2)(m_c^2 - m_d^2) \right],$$

(16)

$$V_{\bar{K}D\rightarrow KD(I=1)}(\epsilon) = 0,$$

(17)

$$V_{\bar{K}D\rightarrow \pi D_s}(\epsilon) = V_{\eta D_s\rightarrow \bar{K}D}(\epsilon) = -\frac{1}{6\sqrt{3}f_D} \left[ \gamma(\epsilon^2 - \bar{t}) + \epsilon^2 - \bar{u} - m_D^2 - m_K^2 \right],$$

(18)

$$V_{\eta D_s\rightarrow \pi D_s}(\epsilon) = 0.$$  

(19)

Note that in our model the $KD$($I=1$) interacts only through the $\pi D_s$ intermediate channel.

Then, the inelastic channels $\pi D_s$ and $\eta D_s$ are integrated out and $KD$ effective potentials in isospin basis, $U_{\bar{K}D(I=0)}(r;\epsilon)$ and $U_{\bar{K}D(I=1)}(r;\epsilon)$, are calculated in the method in Sec. [14A]. These potentials are translated into those in particle basis as:

$$U_{K^-D^0\rightarrow K^-D^0} = U_{\bar{K}D(I=0)} + U_{\bar{K}D(I=1)},$$

(20)

$$U_{K^-D^0\rightarrow \bar{K}^0D^-} = U_{\bar{K}D(I=0)} - U_{\bar{K}D(I=1)},$$

(21)

and

$$U_{\bar{K}D^0\rightarrow \bar{K}^0\bar{D}^0} = U_{\bar{K}D(I=1)}. $$

(22)

In this study, we use parameters fixed in Ref. [14A]: $\gamma = (800 \text{ MeV}/2050 \text{ MeV})^2 \approx 0.152$, $f_D = 93$ MeV, and $f_D = 165$ MeV. Furthermore, we fix the range of the
effective local potential as $b = 0.36$ fm. With these parameters, we can generate the scalar meson $D_{s0}(2317)^-$ with its mass $\epsilon_{\text{pole}} = 2317$ MeV in the $\bar{K}D(I = 0)$ channel as an $S$-wave bound-state solution of the Schrödinger equation with the effective potential $\tilde{U}_{\bar{K}D(I=0)}(r; \epsilon)$

\[
\frac{m_K + m_D - \nabla^2}{2\mu_{\bar{K}D}} + U_{\bar{K}D(I=0)}(r; \epsilon_{\text{pole}}) \psi(r) = \epsilon_{\text{pole}} \psi(r), \tag{23}
\]

with the $\bar{K}D$ reduced mass $\mu_{\bar{K}D} \equiv m_K m_D / (m_K + m_D)$. In Fig. 1 (solid line) we plot the density distribution for the $\bar{K}D$ system in the $D_{s0}(2317)$

\[
\rho_{\bar{K}D}(r) \equiv r^2 |\psi(r)|^2, \tag{24}
\]

calculated from the wave function of the $\bar{K}D$ bound state $\psi(r)$ with the normalization that integral of $\rho_{\bar{K}D}$ with respect to $r$ in the range $[0, \infty)$ is unity. The average of the $\bar{K}D$ distance is calculated as $\sqrt{\langle r^2 \rangle_{\bar{K}D}} = 0.93$ fm, where $\langle r^2 \rangle_{\bar{K}D}$ is defined as

\[
\langle r^2 \rangle_{\bar{K}D} \equiv \int_0^\infty dr r^2 \rho_{\bar{K}D}(r). \tag{25}
\]

In Fig. 2 to compare the strength of the potentials, we show the real parts of the effective local potentials for the $\bar{K}D(I = 0)$ and the $\bar{K}D(I = 1)$ systems as thick and thin dashed-dotted lines, respectively. The energy for the potentials is fixed as the threshold, $\epsilon = m_K + m_D$. As one can see, the $\bar{K}D(I = 0)$ potential has very strong attraction, while the $\bar{K}D(I = 1)$ potential is very tiny and negligible.

We here mention that in terms of heavy quark symmetry we may have to introduce the $D^*$ and $D^*_s$ vector mesons, which exist $\sim 140$ MeV above the ground $D$ and $D_s$ mesons, respectively. However, in this study we do not take into account them in the $\bar{K}D$ system because the contributions from the $\bar{K}D^*$, $\pi D^*$, and $\eta D^*$ channels are expected to be negligible compared to the $\bar{K}D$ dynamics around its threshold.

\section{DN system}

In the $\bar{D}N$ subsystem in $\bar{K}D\bar{N}$, we consider three channels: $D^*p$, $D^0n$, and $\bar{D}^0\bar{p}$. The former two channels couple to each other.

For the $\bar{D}N$ interaction, we take the approach discussed in Ref. [44]. We introduce the $S$-wave channels $\bar{D}N$ (specified by the channel $j = 1$) and $\bar{D}^*N$ ($j = 2$) both in isospin $I = 0$ and $1$.\footnote{We neglect the $D^*\Delta$ channel in $I = 1$, which was included in Ref. [44] but was not important.} We calculate the interaction with a Lagrangian invariant under SU(8) rotations which treats heavy pseudoscalar and vector mesons on an equal footing as required by heavy quark symmetry. The $S$-wave interaction can be expressed as \cite{44}

\[
V_jk(\epsilon) = \frac{\xi_{jk}}{2f_D} (\epsilon - m_N) \sqrt{\Omega_j(\epsilon) + m_N [\Omega_k(\epsilon) + m_N]} \tag{26}
\]

with the on-shell nucleon energy in $j$th channel $\Omega_j(\epsilon)$. The coefficient $\xi_{jk}$ comes from the SU(8) group structure of the couplings, whose expression is

\[
\xi_{j(0)} = \left( \begin{array}{cc} 0 & -\sqrt{72} \\ -\sqrt{72} & 4 \end{array} \right), \quad \xi_{j(1)} = \left( \begin{array}{cc} 2 & 4/\sqrt{3} \\ 4/\sqrt{3} & -2/3 \end{array} \right), \tag{27}
\]

Parameters are taken from Ref. [44]: $f_D = 157.4$ MeV. An interesting feature is that, although the elastic $\bar{D}N$
interaction is zero in \( I = 0 \), dynamics with the \( D^*N \) coupled channel generates a \( DN \) bound state in \( I = 0 \) with binding energy \( \sim 1 \text{ MeV} \).\(^2\) With such an attractive \( DN \) interaction, it is possible to study the formation of \( D \) mesic nuclei in, e.g., Ref. \([51–53]\).

Then we integrate out the \( D^*N \) channel and obtain \( DN \) effective potentials in isospin basis, \( U_{DN(I=0)}(r; \epsilon) \) and \( U_{DN(I=1)}(r; \epsilon) \), with which we calculate the potentials in particle basis as

\[
U_{D^p\to D^p} = U_{D^0n\to D^0n} = \frac{U_{DN(I=0)} + U_{DN(I=1)}}{2},
\]

\[
U_{D^p\to D^0n} = U_{D^0n\to D^p} = -\frac{U_{DN(I=0)} - U_{DN(I=1)}}{2},
\]

\[
U_{D^0p\to D^0p} = U_{DN(I=1)}.
\]

As for the range parameter \( b \), we fix it so as to reproduce an \( S \)-wave \( DN(I = 0) \) bound state with 1 MeV binding (\( \epsilon_{\text{pole}} = 2805 \text{ MeV} \)) as a solution of the Schrödinger equation with the effective potential \( U_{DN(I=0)}(r; \epsilon) \)

\[
\left[ m_D + m_N - \frac{\nabla^2}{2 \mu_{DN}} + U_{DN(I=0)}(r; \epsilon_{\text{pole}}) \right] \psi(r) = \epsilon \psi(r),
\]

with the \( DN \) reduced mass \( \mu_{DN} = m_D m_N / (m_D + m_N) \). The result of the range parameter is \( b = 0.26 \text{ fm} \). In Fig. 1 (dashed line) we plot the density distribution for \( DN \) potentials for the \( I = 0 \) and \( I = 1 \) potentials in isospin basis, as thick and thin dotted lines, respectively. The energy for the potentials is fixed as the threshold, \( \epsilon = m_D + m_N \). As one can see, the \( DN(I = 0) \) potential is the smallest among the \( KN, KD, \) and \( DN \) potentials with \( I = 0 \). The \( DN(I = 1) \) potential is much smaller than the \( DN(I = 0) \) potential.

**III. \( K\bar{D}N \) THREE-BODY PROBLEM**

Next let us formulate the \( K\bar{D}N \) three-body problem. For this purpose, we set the coordinates of the \( K, D, \)

\(^2\) Importance of the \( D^*N \) channel in the \( D\bar{N} \) dynamics was pointed out also in Ref. [53], where a \( D\bar{N} \) bound state with binding energy \( \sim 1 \text{ MeV} \) was predicted as well.

![Fig. 3: Three types of Jacobi coordinates.](image-url)
between $\hat{H}$ and $|\Psi\rangle$ in the left-hand side, we obtain
\[ \sum_{k=1}^{3} \left[ \delta_{jk} \hat{H}_0 + V_{jk}(\lambda_1, \rho_1; E) \right] \Psi_k(\lambda_1, \rho_1) = E \Psi_j(\lambda_1, \rho_1). \] (40)

The kinetic term of the three-body Hamiltonian $\hat{H}_0$ is
\[ \hat{H}_0 = M_{KDN}^{-1} \frac{1}{2\mu'_1} \left( \frac{\partial}{\partial \lambda_1} \right)^2 - \frac{1}{2\mu_1} \left( \frac{\partial}{\partial \rho_1} \right)^2 \] (41)
where $\mu'_1$ and $\mu_1$ are the reduced masses
\[ \mu'_1 = \frac{(m_D + m_N)m_K}{M_{KDN}}, \quad \mu_1 = \frac{m_D m_N}{m_D + m_N}. \] (42)

As for the potential term $V_{jk}$, we employ the orbital-angular-momentum independent potentials developed in the previous section. The diagonal parts $V_{jj}$ consist of all the three combinations of two particles among $K, D,$ and $N$ in each channel:
\[ V_{11} = U_{D^0p-pD^0p}(\rho_1; \epsilon_{DN}) + U_{K^-p-K^-p}(\rho_2; \epsilon_{KN}) + U_{K^-D^-D^-0}(\rho_3; \epsilon_{KD}), \] (43)
\[ V_{22} = U_{D^-p-D^-p}(\rho_1; \epsilon_{DN}) + U_{K^0p-K^0p}(\rho_2; \epsilon_{KN}) + U_{K^0D^-D^-0}(\rho_3; \epsilon_{KD}), \] (44)
\[ V_{33} = U_{D^0n-nD^0n}(\rho_1; \epsilon_{DN}) + U_{K^0n-nK^0n}(\rho_2; \epsilon_{KN}) + U_{K^0D^-D^-0}(\rho_3; \epsilon_{KD}), \] (45)
where $\epsilon_{DN}, \epsilon_{KN},$ and $\epsilon_{KD}$ are energies of the subsystems $KN, DN,$ and $KD,$ respectively, fixed later. The nondiagonal components of the potential consist of the charge transition of two particles among $K, D,$ and $N$:
\[ V_{12} = V_{21} = U_{K^-D^-0-K^-0}(\rho_3; \epsilon_{KD}), \] (46)
\[ V_{13} = V_{31} = U_{K^-p-K^0n}(\rho_2; \epsilon_{KN}), \] (47)
\[ V_{23} = V_{32} = U_{D^-p-D^0n}(\rho_1; \epsilon_{DN}). \] (48)

Because the potentials have imaginary parts according to the implementation of the open channels, the Hamiltonian $\hat{H}$ is not Hermitian. Therefore, the Hamiltonian can have an eigenstate with a complex eigenvalue, called a quasibound state. We do not consider three-body forces in this study.

As we have constructed in Sec. III, the two-body potential in the subsystem depends on its energy. There is ambiguity to fix energy of a two-body subsystem in a three system, but we here simply divide the total energy $E$ among three particles according to the ratio of masses, i.e.,
\[ \epsilon_{DN} = \frac{m_D + m_N}{M_{KDN}} E, \] (49)
\[ \epsilon_{KN} = \frac{m_K + m_N}{M_{KDN}} E, \] (50)
\[ \epsilon_{KD} = \frac{m_K + m_D}{M_{KDN}} E. \] (51)

Note that the subsystem energy is complex when the total energy $E$ is complex.

In this study we concentrate on the ground state of the $KDN$ system with spin/parity $J^P = 1/2^-$, so we limit the basis function for the three-body wave function in the channel to having zero orbital angular momenta both for the $\lambda_c$ and $\rho_c$ modes: $l_{\lambda_c} = l_{\rho_c} = 0$. We then employ the Gaussian expansion method \[54\] and take the sum of all the three rearrangements of the Jacobi coordinates, which results in
\[ \Psi_j = \sum_{c=1}^{3} \sum_{n,n'=1}^{N} C^c_{j,n,n'} \exp \left( -\frac{\lambda^2}{r_a^2} - \frac{\rho^2}{r_{am}^2} \right), \] (52)
with number of the expansion $N$, coefficients $C^c_{j,n,n'}$, and different ranges $r_n$ in a geometric progression
\[ r_n = r_{\min} \times \left( \frac{r_{\max}}{r_{\min}} \right)^{(n-1)/(N-1)}. \] (53)

The minimal and maximal ranges, $r_{\min}$ and $r_{\max}$, respectively, are fixed according to the physical condition of interactions. We comment that, although each $c$ channel in Eq. (52) have zero orbital angular momentum, the sum of all the three rearrangements allows us to take into account components with nonzero orbital angular momenta of two-body subsystems.

By using the wave function (52), the Schrödinger equation (10) becomes
\[
\sum_{c=1}^{3} \sum_{n,n'=1}^{N} \sum_{k=1}^{3} \left\{ \delta_{jk} \frac{1}{\mu'_c} \left( \frac{d}{dr_c} \right)^2 \left( 3 - 2 \frac{\lambda^2}{r^2_a} \right) + \frac{1}{\mu'_c} \frac{d}{dr_c} \left( 3 - 2 \frac{\rho^2}{r^2_{am}} \right) + M_{KDN} - E \right\} \times C^c_{j,k,n,n'} \exp \left( -\frac{\lambda^2}{r^2_a} - \frac{\rho^2}{r^2_{am}} \right) = 0, \]
where we introduced the reduced masses
\[ \mu'_2 = \frac{(m_K + m_N)m_D}{M_{KDN}}, \quad \mu_2 = \frac{m_K m_N}{m_K + m_N}, \] (55)
\[ \mu'_3 = \frac{(m_K + m_D)m_N}{M_{KDN}}, \quad \mu_3 = \frac{m_K m_D}{m_K + m_D}. \] (56)

Then we multiply $\exp(-\lambda^2/r^2_a - \rho^2/r^2_{am})$ to the Schrödinger equation (53) and integrate it with respect
to $\lambda_1$ and $\rho_1$, which results in
\[
\sum_{\beta} \left[ T_{\alpha\beta} + V_{\alpha\beta}(E) + (M_{\bar{K}DN} - E)N_{\alpha\beta} \right] C_{\beta,n,n'}^c = 0,
\]  
(57)
where we introduced sets of indices $\alpha = \{j, m, n', a\}$ and $\beta = \{k, n, n', c\}$, and define $T_{\alpha\beta}$, $V_{\alpha\beta}$, and $N_{\alpha\beta}$ as
\[
T_{\alpha\beta} \equiv \delta_{jk} \int d^3\lambda_1 d^3\rho_1 \exp \left( -\frac{\lambda^2}{r^2_m} - \frac{\rho^2}{r^2_m'} - \frac{\lambda^2}{r^2_n} - \frac{\rho^2}{r^2_n'} \right)
\times \left[ \frac{1}{\mu cr^2_n} \left( 3 - \frac{2\lambda^2}{r^2_n} \right) + \frac{1}{\mu cr^2_n'} \left( 3 - \frac{2\rho^2}{r^2_n'} \right) \right],
\]
(58)
\[
V_{\alpha\beta}(E) \equiv \int d^3\lambda_1 d^3\rho_1 \exp \left( -\frac{\lambda^2}{r^2_m} - \frac{\rho^2}{r^2_m'} - \frac{\lambda^2}{r^2_n} - \frac{\rho^2}{r^2_n'} \right)
\times V_{jk}(E),
\]
(59)
\[
N_{\alpha\beta} \equiv \delta_{jk} \int d^3\lambda_1 d^3\rho_1 \exp \left( -\frac{\lambda^2}{r^2_m} - \frac{\rho^2}{r^2_m'} - \frac{\lambda^2}{r^2_n} - \frac{\rho^2}{r^2_n'} \right),
\]
(60)
respectively. We can regard Eq. (57) as a generalized eigenvalue problem of linear algebra. We numerically solve this to evaluate the eigenvalue $E = E_{\text{pole}}$ and eigenvector $C_{j,n,n'}^c$.

IV. $\bar{K}DN$ MOLECULAR STATE

A. Eigenenergy

Now we solve the Schrödinger equation (57) in the $K^- D^0 p - K^0 D^- p - K^0 \bar{D}^0 n$ coupled channels and search for the $\bar{K}DN$ bound state. For the study of the $\bar{K}DN$ system, we fix $r_{\text{min}} = 0.1 \text{ fm}$ and $r_{\text{max}} = 20.0 \text{ fm}$. Taking the number of the expansion $N = 10$, we find a solution of Eq. (57) with its eigenenergy $E_{\text{pole}} = 3244 - 17i \text{ MeV}$. The convergence of the expansion can be checked by the trace of the eigenenergy $E_{\text{pole}}$ from $N = 4$ to $N = 10$, which is plotted in the complex energy plane of Fig. 4. As one can see, we achieve the convergence of the expansion with $N \geq 8$.

The real part of the eigenenergy $E_{\text{pole}}$ is below the $D_{s0}(2317) + N$ threshold (3256 MeV) as well as below the $\Lambda(1405) + \bar{D}$ and $\bar{K}DN$ thresholds (3291–261 MeV and 3302 MeV, respectively). This means that this state is indeed a $\bar{K}DN$ quasibound state which cannot decay into $D_{s0}(2317) + N$, $\Lambda(1405) + \bar{D}$, nor $\bar{K}DN$. The binding energy of the quasibound state is 58 MeV measured from the $\bar{K}DN$ threshold, 48 MeV from the $\Lambda(1405)\bar{D}$ threshold, and 13 MeV from the $D_{s0}(2317)N$ threshold.

The imaginary part of the eigenenergy indicates decay of the quasibound state, as we introduced complex-valued potentials reflecting implicit decay channels such as $\pi\Sigma$ in $\bar{K}N$. We emphasize that the imaginary part of the eigenenergy is obtained in a full calculation rather than in a perturbative one. From the eigenenergy, we find that the decay width of the quasibound state is $-2 \times \text{Im} E_{\text{pole}} = 34 \text{ MeV}$. The decay of the quasibound state will be discussed in the next subsection.

B. Decay

Because $\Lambda(1405)$ decays into $\pi\Sigma$ and $D_{s0}(2317)^{-}$ into $\pi D_{s}$, we expect that the main decay channels of the $\bar{K}DN$ quasibound state may be $\pi\Sigma + \bar{D}$ and $\pi D_{s} + N$. To check this, we perform the same three-body calculation but without the imaginary parts of the two-body potentials.

When we neglect the imaginary part of the $\bar{K}N$ potential, we obtain the bound-state eigenenergy at 3240 – 0i MeV. On the other hand, when we neglect the imaginary part of the $\bar{K}D$ ($\bar{D}N$) potential, we obtain the bound-state eigenenergy at 3244 – 18i MeV (3245 – 20i MeV). These results indicate that the decay width of the $\bar{K}DN$ quasibound state originates from the $\bar{K}N$ interaction.

Hence, one could conclude that the decay is dominated by the $\pi\Sigma + \bar{D}$ channel, but it is not the all of the main decay modes. As we will see in the next subsection, the $\bar{K}DN$ quasibound state has a significant $\bar{K}N(I = 1)$ component. Furthermore, the $\bar{K}N$ effective potential has similar values of the imaginary parts for the $I = 0$ and $I = 1$ channels (see Figs. 7 and 8 of Ref. [48]). These indicate that the bound-state decay originating from the $\bar{K}N$ interaction includes not only the $\pi\Sigma(I = 0) + \bar{D}$ mode but also the $\pi\Lambda + \bar{D}$ and $\pi\Sigma(I = 1) + \bar{D}$ modes. As a consequence, the main decay modes of the $\bar{K}DN$ quasibound state are $\pi\Lambda + \bar{D}$ and $\pi\Sigma + \bar{D}$ channels.

FIG. 4: Eigenenergy $E_{\text{pole}}$ as a function of the number of the Gaussian expansion $N$ (open circles) from $N = 4$ to $N = 10$. We also plot the threshold points for the $\Lambda(1405) + D$, $D_{s0}(2317) + N$, and $\bar{K} + D + N$ as filled symbols.
We here note that the decay width of the $\bar{K}DN$ quasibound state, 34 MeV, is smaller than that of the $\Lambda(1405)$ as the $\bar{K}N$ quasibound state, $\sim$ 50 MeV. If the $\bar{K}N$ subsystem in the $\bar{K}DN$ quasibound state behaved like the $\Lambda(1405)$, the decay width of the $\bar{K}DN$ quasibound state would be similar to the width of the $\Lambda(1405)$. Indeed, this reduction of the decay width is caused by the three-body dynamics, in particular the slight extension of the $\bar{K}N$ distance in the $\bar{K}DN$ quasibound state compared to that in the $\Lambda(1405)$, as we will see in the next subsection. From this result we can say that, in general, the decay width of a three-body quasibound state, because the three constituents should meet at a point for the contact interaction is strongly suppressed for the $\bar{K}N$ quasibound state behavior like the two-body quasibound state. This is not the sum of the decay widths of the two-body quasibound states $\bar{K}N$, $\bar{K}D$, and $\bar{K}N$, as which are not included in our formulation. For these two-hadron decay modes, we can use the same argument as in Ref. \cite{34} (see also Fig. 4 therein). First, the transition to two-hadron states via a contact interaction is strongly suppressed for a three-body quasibound state, because the three constituents should meet at a point for the contact interaction to take place. Second, the transition to two-hadron states via virtual meson exchanges is also suppressed due to the dilute nature of the three-body quasibound state. Such a virtual meson exchange process is expected to take place in the nonmesonic decay of $\bar{K}$-nucleus systems \cite{52,58}. Therefore, in analogy to the nonmesonic decay of $\bar{K}$-nucleus systems, we can estimate that the branching ratio of the two-hadron decays of the $\bar{K}DN$ quasibound state will be $\sim$ 20\%, about as large as the empirical values of the branching ratio of the nonmesonic decay of $\bar{K}$-nucleus systems.

C. Structure

Then, we investigate the internal structure of the $\bar{K}DN$ quasibound state by using the wave function $\psi_j(\lambda_1, \rho_1)$ which is normalized as

$$\sum_{j=1}^{3} \int d^3\lambda_1 d^3\rho_1 |\psi_j(\lambda_1, \rho_1)|^2 = 1.$$  \hspace{1cm} (61)

We emphasize that, because the $\bar{K}DN$ quasibound state is a resonance, we calculate the complex value squared of the wave function rather than the absolute value squared to normalize the resonance wave function $\psi_j$ as a Gamow vector.

We first perform the isospin decomposition. To this end, we construct the projection operator to the $\bar{K}N(I = 0)$ state as

$$\mathcal{P}_{\bar{K}N(I = 0)} = \frac{1}{2} |K^- p + \bar{K}^0 n\rangle \langle K^- p + \bar{K}^0 n|.$$  \hspace{1cm} (62)

By using this projection operator, we can calculate the fraction of the $\bar{K}N(I = 0)$ component in the $\bar{K}DN$ quasibound state as

$$X_{\bar{K}N(I = 0)} = \langle \psi | \mathcal{P}_{\bar{K}N(I = 0)} | \psi \rangle = \frac{\langle \psi_1^2 + 2\psi_1\psi_2 + \psi_3^2 \rangle}{2},$$  \hspace{1cm} (63)

where

$$\langle \psi_j \psi_k \rangle = \int d^3\lambda_1 d^3\rho_1 \psi_j(\lambda_1, \rho_1) \psi_k(\lambda_1, \rho_1).$$  \hspace{1cm} (64)

While the $\bar{K}N(I = 1)$ component is

$$X_{\bar{K}N(I = 1)} = 1 - X_{\bar{K}N(I = 0)}.$$  \hspace{1cm} (65)

Similarly, we can express the projection operators and fractions of the components for other states as

$$\mathcal{P}_{\bar{K}D(I = 0)} = \frac{1}{2} |K^- \bar{D}^0 + \bar{K}^0 D^-\rangle \langle K^- \bar{D}^0 + \bar{K}^0 D^-|,$$  \hspace{1cm} (66)

$$X_{\bar{K}D(I = 0)} = \langle \psi | \mathcal{P}_{\bar{K}D(I = 0)} | \psi \rangle = \frac{\langle \psi_1^2 + 2\psi_1\psi_2 + \psi_3^2 \rangle}{2},$$  \hspace{1cm} (67)

$$X_{\bar{K}D(I = 1)} = 1 - X_{\bar{K}D(I = 0)},$$  \hspace{1cm} (68)

$$\mathcal{P}_{D(N)(I = 0)} = \frac{1}{2} |D^0 n - D^- p\rangle \langle D^0 n - D^- p|,$$  \hspace{1cm} (69)

$$X_{D(N)(I = 0)} = \langle \psi | \mathcal{P}_{D(N)(I = 0)} | \psi \rangle = \frac{\langle \psi_2^2 - 2\psi_2\psi_3 + \psi_1^2 \rangle}{2},$$  \hspace{1cm} (70)

$$X_{D(N)(I = 1)} = 1 - X_{D(N)(I = 0)},$$  \hspace{1cm} (71)

respectively. The results of the fractions $X$ are listed in Table \ref{tab:isospin_fractions}. All the fractions are complex because the quasibound state is a resonance. Nevertheless, the $\bar{K}D$ component in isospin $I = 0$ is very close to unity with small imaginary part, which implies the dominant $\bar{K}D(I = 0)$ component inside the $\bar{K}DN$ quasibound state. This is

| $X_{\bar{K}N(I = 0)}$ | $X_{\bar{K}N(I = 1)}$ | $d_{\bar{K}N}$ |
|---------------------|---------------------|--------------|
| 0.24 $\pm$ 0.02i   | 0.76 $- 0.02i$      | 1.13 $- 0.39i$ fm |
| $X_{\bar{K}D(I = 0)}$ | $X_{\bar{K}D(I = 1)}$ | $d_{\bar{K}D}$ |
| 0.98 $- 0.01i$    | 0.02 $+ 0.01i$       | 0.79 $- 0.05i$ fm |
| $X_{DN(I = 0)}$ | $X_{DN(I = 1)}$ | $d_{DN}$ |
| 0.27 $- 0.02i$ | 0.73 $+ 0.02i$ | 1.05 $- 0.35i$ fm |
a consequence of the three-body dynamics to maximize the attraction among three constituents. Namely, as one can see from Fig. 2, the $\bar{K}D(I = 0)$ interaction is most attractive among the pairs of the two constituents in the present formulation, and $\bar{K}N(I = 0)$ comes next. Besides, in contrast to the moderate $\bar{K}N(I = 1)$ attraction, the $\bar{K}D(I = 1)$ interaction is negligible, as the $\bar{K}D(I = 1)$ interacts only through the $\pi D_s$ intermediate channel [see Eq. (77)]. Therefore, the three-body dynamics increase the $\bar{K}D(I = 0)$ fraction as much as possible so as to maximize the attraction in the $\bar{K}D$ quasibound state. In this sense, the $\bar{K}D$ interaction, both the $I = 0$ and $I = 1$ components, is most essential for the internal structure of the $\bar{K}D$ quasibound state. Furthermore, we have checked that the following relations hold:

$$\langle \Psi_1 \rangle \approx \langle \Psi_1 \Psi_2 \rangle \approx \langle \Psi_2 \rangle \approx \frac{1}{2}, \quad (72)$$

$$\langle \Psi_1 \Psi_3 \rangle \approx \langle \Psi_2 \Psi_3 \rangle \approx \langle \Psi_3 \rangle \approx 0. \quad (73)$$

These relations indicate that the quasibound state is indeed described by the $[\bar{K}D(I = 0)]p$ configuration. The negligible contribution from the $K^0D^0n$ channel also explains the results that the $\bar{K}N(I = 0)$ and $DN(I = 0)$ components are close to 1/4.

Next we investigate how the two-hadron subsystems behave in the $\bar{K}DN$ quasibound state by calculating the density distribution for each pair of two constituents, which is defined as

$$P_{\bar{K}N}(\rho_2) \equiv \rho_2^3 \int d\Omega_\rho d^3 \rho_2 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (74)$$

$$P_{\bar{K}D}(\rho_3) \equiv \rho_3^3 \int d\Omega_\rho d^3 \rho_3 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (75)$$

$$P_{DN}(\rho_1) \equiv \rho_1^3 \int d\Omega_\rho d^3 \rho_1 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (76)$$

where $\Omega_\rho$ is the solid angle of the vector $\rho$. The integration of $P_{\bar{K}N}$, $P_{\bar{K}D}$, and $P_{DN}$, with respect to $\rho_2$, $\rho_3$, and $\rho_1$, respectively, in the range $[0, \infty)$ is unity according to the normalization (71). The resulting density distributions for the pairs of two constituents are plotted in Fig. 5. From the figure, the $\bar{K}N$ and $DN$ distributions are similar to each other and extend typical hadronic scale 1 fm. However, the $\bar{K}D$ distribution in the $\bar{K}DN$ quasibound state is significant only below 1 fm, which is similar to the $\bar{K}D$ distribution in the $D_{s0}(2317)$ in Fig. 1. This result supports the $[\bar{K}D(I = 0)]p$ configuration for the $\bar{K}DN$ quasibound state. Furthermore, because the $\bar{K}D(I = 0)$ subsystem is compact, the distributions of the $\bar{K}N$ and $DN$ in the $\bar{K}DN$ quasibound state are similar to each other.

From the density distributions, we can calculate the averages of the distances between two constituents:

$$d_{\bar{K}N} \equiv \sqrt{\int_0^\infty d\rho_2 \rho_2^3 P_{\bar{K}N}(\rho_2)}, \quad (77)$$

$$d_{\bar{K}D} \equiv \sqrt{\int_0^\infty d\rho_3 \rho_3^3 P_{\bar{K}D}(\rho_3)}, \quad (78)$$

$$d_{DN} \equiv \sqrt{\int_0^\infty d\rho_1 \rho_1^3 P_{DN}(\rho_1)}. \quad (79)$$

The results are listed in Table 1. As one can see, the averages of the distances have small imaginary parts due to the resonance nature but their real parts are dominant. Therefore, below we focus on the real parts of the distances. Among the three distances, the distance between $\bar{K}D$ is the smallest, which indicates the compact $\bar{K}D(I = 0)$ subsystem. The distance between $\bar{K}D$ in the $\bar{K}DN$ quasibound state is smaller than that in the $D_{s0}(2317)$ as the $\bar{K}D$ two-body bound state, 0.93 fm. This is because the $N$ assists the $\bar{K}D$ attraction in the $\bar{K}DN$ quasibound state via the $\bar{K}N$ and $DN$ interactions. The distance between $DN$ in the $\bar{K}DN$ quasibound state becomes much smaller than that of the $DN$ two-body bound state, 3.66 fm, owing to the compact $\bar{K}D$ subsystem and the strong attraction between $\bar{K}N$. We also note that, although the distance between $\bar{K}N$ in the $\bar{K}DN$ quasibound state is similar to that in the $\Lambda(1405)$, the $\Lambda(1405)$ is not effective degrees of freedom in the $\bar{K}DN$ quasibound state because the isospin component $X_{\bar{K}N}(I = 0)$ is only about 1/4.

In terms of the structure, we can understand the decrease of the decay width of the $\bar{K}DN$ quasibound state compared to the $\Lambda(1405)$. This is caused by the fact that the $\bar{K}N$ distance in the $\bar{K}DN$ quasibound state is slightly larger than that in the $\Lambda(1405)$, which is crucial to the decay width of the $\bar{K}DN$ quasibound state.

$$P_{\bar{K}N}(\rho_2) \equiv \rho_2^3 \int d\Omega_\rho d^3 \rho_2 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (74)$$

$$P_{\bar{K}D}(\rho_3) \equiv \rho_3^3 \int d\Omega_\rho d^3 \rho_3 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (75)$$

$$P_{DN}(\rho_1) \equiv \rho_1^3 \int d\Omega_\rho d^3 \rho_1 \sum_{j=1}^{3} |\langle \Psi_j(\lambda_1, \rho_1) \rangle|^2, \quad (76)$$

$^3$The measures of the Jacobi coordinates satisfy $d^3 \lambda_1 d^3 \rho_1 = d^3 \lambda_2 d^3 \rho_2 = d^3 \lambda_3 d^3 \rho_3$. 

**Fig. 5:** Density distributions for the $\bar{K}N$, $\bar{K}D$, and $DN$ subsystems in the $\bar{K}DN$ quasibound state.
originating from the $\bar{K}N$ interaction. Actually, because the $\bar{K}N$ effective potential has a finite range, which is $b = 0.38$ fm in our study, the increase of the $\bar{K}N$ distance directly reduces the probability of overlapping $\bar{K}N$ for the decay. As a consequence, we obtain the smaller decay width of the $\bar{K}DN$ quasibound state than that of the $\Lambda(1405)$. We note that the isospin structure of the $\bar{K}DN$ quasibound state, i.e., dominant $K\bar{N}(I = 1)$ component, is irrelevant to the decrease of the decay width, because the $\bar{K}N$ effective potential takes similar values of the imaginary parts for the $I = 0$ and $I = 1$ channels.

D. Theoretical ambiguities

Finally, we discuss theoretical ambiguities in our scenario of the generation of the $\bar{K}DN$ quasibound state.

As we have seen, the $\bar{K}DN$ quasibound state is generated by the two kinds of strong attraction, the $\bar{K}N(I = 0)$ interaction and $\bar{K}D(I = 0)$ interaction, and moderate $\bar{D}N$ attraction. Among them, the $\bar{D}N$ interaction is not well determined due to poor experimental data. To check the influence of the $\bar{D}N$ interaction, we switch off the $\bar{D}N$ interaction and perform the three-body calculations. As a result, we obtain the $\bar{K}DN$ quasibound state with eigenenergy $3248 - 21i$ MeV, whose value is similar to the full-calculation value $3244 - 17i$ MeV. Therefore, we can say that ambiguity of the $\bar{D}N$ interaction is irrelevant.

Besides, although the $\bar{K}D$ interaction is fixed to reproduce the $D_{s0}(2317)$ as the $\bar{K}D$ bound state, it is not clear how much the $D_{s0}(2317)$ contains a “bare” $\bar{s}c$ component rather than the $\bar{K}D$ molecular component. Such an $\bar{s}c$ component will weaken the attraction of the $\bar{K}D$ effective potential and may affect our scenario. However, the $D_{s0}(2317)$ generated in the present formulation already contains some missing-channel contribution rather than the $\bar{K}D$-$\bar{D}_s$ channels via the intrinsic energy dependence of the interaction [12]. Actually, the intrinsic energy dependence of the $\bar{K}D$-$\bar{D}_s$ interaction introduces missing-channel fraction $= 22\%$ to the $D_{s0}(2317)$, as seen in Ref. [11], which can be interpreted as a bare $\bar{s}c$ component. Therefore, our scenario allows the $D_{s0}(2317)$ to have $\sim 20\%$ fraction of the bare $\bar{s}c$ component.

Our scenario would be affected by the treatment of the energy dependence of the two-body interaction in the three-body dynamics [see Eqs. (49), (50), and (51)]. To check this, we firstly fix two-body energy of only one of the three pairs in the three-body system to its threshold energy while we keep the energy dependence for other two pairs. When we keep the energy dependence for the $\bar{K}D$ and $\bar{D}N$ potentials but fix the $\bar{K}N$ energy as $\epsilon_{\bar{K}N} = m_K + m_N$, we obtain the eigenenergy $3242 - 21i$ MeV. Similarly, when we fix the $\bar{K}D$ ($\bar{D}N$) energy as $\epsilon_{\bar{K}D} = m_K + m_D$ ($\epsilon_{\bar{D}N} = m_D + m_N$), we obtain the eigenenergy $3238 - 18i$ MeV ($3231 - 25i$ MeV). The shifts of the eigenenergy in these cases are not significant because the energy dependence of the two-body effective potentials is not essential in the energy region of interest, i.e., around their thresholds. Secondly, if we fix all the two-body energies to the two-body threshold energies, $\epsilon_{\bar{D}N} = m_D + m_N$, $\epsilon_{\bar{K}N} = m_K + m_N$, and $\epsilon_{\bar{K}D} = m_K + m_D$, the eigenenergy becomes $3219 - 34i$ MeV. Thirdly, when we fix the two-body energies to the pole positions of the two-body bound states, i.e., $\epsilon_{\bar{K}N} = 1424 - 26i$ MeV, $\epsilon_{\bar{K}D} = 2317$ MeV, and $\epsilon_{\bar{D}N} = 2805$ MeV, the eigenenergy becomes $3226 - 28i$ MeV. These treatments bring more binding energy and width to the $\bar{K}DN$ state, but the $\bar{K}DN$ quasibound state exists in any case. From the above discussions, we conclude that the $\bar{K}DN$ quasibound state will exist even if we take into account theoretical ambiguities.

V. SUMMARY AND CONCLUDING REMARKS

In this study we investigated the $\bar{K}DN$ quasibound state with spin/parity $J^P = 1/2^-$ and isospin $I = 1/2$ in a nonrelativistic three-body potential model. Following the approach in Refs. [46, 48] for the $K\bar{N}$ effective local potential, we constructed the $\bar{K}D$ and $\bar{D}N$ effective local potentials based on phenomenological models, with which we obtained the $D_{s0}(2317)$ as a $\bar{K}D$ bound state and $\bar{D}N$ bound state, respectively. These two-body effective potentials implicitly contain inelastic channels. In particular, the inclusion of the open channels is essential to describe the $\bar{K}DN$ system as a decaying state. By solving the three-body Schrödinger equation in the $K^-\bar{D}^0p - K\bar{D}^-p - K\bar{D}^0\bar{D}^0n$ coupled channels with the constructed two-body effective local potentials, we obtained the $\bar{K}DN$ quasibound state with eigenenergy $3244 - 17i$ MeV. The real part of the eigenenergy is below the $D_{s0}(2317) + N$ and $\Lambda(1405) + \bar{D}$ thresholds as well as the $\bar{K}DN$ threshold, so the $\bar{K}DN$ quasibound state cannot decay into $D_{s0}(2317) + N$, $\Lambda(1405) + \bar{D}$, nor $\bar{K}DN$. From the imaginary part of the eigenenergy, we calculated the decay width of the $\bar{K}DN$ quasibound state into $p\Delta$, $\pi\Sigma$ to be 34 MeV. In addition to the three-hadron decay modes, the $\bar{K}DN$ quasibound state will have two-hadron decay modes $K\bar{D}N \to \bar{D}\Lambda$, $\bar{D}\Sigma$, and $\bar{D}_sN$, and the branching ratio of the two-hadron decays was estimated to be $\sim 20\%$.

As for the internal structure, the $\bar{K}DN$ quasibound state takes $[\bar{K}D(I = 0)]p$ configuration with compact $\bar{K}D$ subsystem because this configuration can maximize the attraction among three constituents by utilizing the strong $\bar{K}D(I = 0)$ attraction fully. We found that the three-body dynamics may increase distance between two constituents and hence may reduce decay width of the three-body quasibound state compared to that of the two-body quasibound state of the constituents, as is the relation between the $\bar{K}DN$ quasibound state and $K\bar{N}$ quasibound state [$\Lambda(1405)$]. We also discussed theoretical ambiguities, and conclude that the $\bar{K}DN$ quasibound state will exist even if we take into account theoretical ambiguities.
Finally, we remark the possibility of the experimental search for the $K\bar{D}N$ quasibound state. Because the $K\bar{D}N$ quasibound state has both strangeness $S = -1$ and charm $C = -1$, practical candidate is the production in relativistic heavy-ion collisions [17, 50]. One can search for the $K\bar{D}N$ quasibound state in, e.g., the $\pi\bar{D}A$ and/or $\bar{D}A$ invariant-mass spectra of relativistic heavy-ion collisions. The $B$ meson decays in $B$ factories are feasible as well. In this case, for instance, the $B_s(s\bar{b}) \rightarrow \pi\bar{D}A + \bar{p}$, $\bar{D}A + \bar{p}$ processes are suitable.

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