Quadrupole effects on the motion of extended bodies in Kerr spacetime

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Abstract
The motion of a body endowed with a dipolar as well as a quadrupolar structure is investigated in the Kerr background according to the Dixon model, extending a previous analysis done in the Schwarzschild background. The full set of evolution equations is solved under the simplifying assumptions of constant frame components for both the spin and the quadrupole tensors and that the center of mass moves along an equatorial circular orbit, the total 4-momentum of the body being aligned with it. We find that the motion deviates from the geodesic one due to the internal structure of the body, leading to measurable effects. Corrections to the geodesic value of the orbital period of a close binary system orbiting the galactic center are discussed assuming that the galactic center is a Kerr supermassive black hole.

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1. Introduction

In a recent paper [1], we have investigated the motion of an extended body endowed with a dipolar as well as a quadrupolar structure in the field of a Schwarzschild black hole described by the Dixon model [2–6]. The corresponding evolution equations up to the quadrupole approximation are given by

\[
\frac{DP^{\mu}}{d\tau_U} = -\frac{1}{2} R^{\mu}_{\nu\alpha\beta} U^\nu S^{\alpha\beta} - \frac{1}{6} J^{\rho\gamma\delta}_{\mu} R^{\alpha\beta\gamma\delta}_{\nu} S^{\mu
\text{spin}} + F^{(\text{quad})}_\mu 
\]

\[
\frac{DS^{\mu\nu}}{d\tau_U} = 2 P^{[\mu} U^{\nu]} - \frac{4}{3} J^{\rho\gamma\delta}_{\mu} R^{\nu}_{\alpha\beta\delta} S^{\mu
\text{quad}}
\]
where $P^\mu = m U^\mu_p$ (with $U_p \cdot U_p = -1$) is the total 4-momentum of the particle, and $S^{\mu\nu}$ is a (antisymmetric) spin tensor; $U$ is the timelike unit tangent vector of the ‘center of mass line’ $C_U$ used to make the multipole reduction, parameterized by the proper time $\tau_U$. The tensor $J^{\alpha\beta\gamma\delta}$ is the quadrupole moment of the stress–energy tensor of the body, and has the same algebraic symmetries as the Riemann tensor. We assume here that the only contribution to the complete quadrupole moment stems from the (symmetric) mass quadrupole moment $Q^{\mu\nu}$ [7, 8], implying that

$$J^{\alpha\beta\gamma\delta} = -3 U^{[\alpha}_p Q^{\beta][\gamma}_p U^\delta], \quad Q^{\alpha\beta} U_p = 0. \quad (1.3)$$

Moreover, the following additional conditions [2, 9] should be imposed on the spin tensor:

$$S^{\mu\nu} U_p = 0, \quad (1.4)$$

to ensure the correct definition of the various multipolar terms.

It is convenient to introduce also the spin vector by the spatial (with respect to $U_p$) duality

$$S^\beta = \frac{1}{2} \eta_{\alpha\beta\gamma\delta} U^\alpha S^{\gamma\delta}, \quad (1.5)$$

where $\eta_{\alpha\beta\gamma\delta} = \sqrt{-g} \epsilon_{\alpha\beta\gamma\delta}$ is the unit volume 4-form and $\epsilon_{0123} = 1$ is the Levi-Civita alternating symbol, as well as the scalar invariant

$$s^2 = S^\mu S_\mu = \frac{1}{2} S_{\mu\nu} S^{\mu\nu}, \quad (1.6)$$

which, in general, is not a constant along the trajectory of the body.

The compatibility of the model requires that the mass of the body, its spin as well as its quadrupole moments must all be small enough not to contribute significantly to the background metric. Otherwise, backreaction must be taken into account.

We have solved in [1] the system of equations (1.1)–(1.2) under the simplifying assumptions of constant frame components (with respect to a natural orthonormal frame) of both the spin and the quadrupole tensor, obtaining the kinematical conditions to be imposed on the particle’s structure in order the orbit of the particle itself be circular and confined on the equatorial plane of a Schwarzschild black hole.

We extend here this analysis to the more interesting case of a Kerr background maintaining the same restrictions on the spin and quadrupole tensor components as well as on the circularity of motion in order to completely solve the problem, obtaining an analytical solution to be compared with the corresponding one discussed in [1]. As an example, we calculate the corrections to the geodesic value of the orbital period of a binary pulsar system, with the same parameters of PSR J0737-3039, orbiting close to the galactic center black hole.

Such an analysis can also be extended to other objects of astrophysical interest, e.g. ordinary or neutron stars, orbiting the galactic center supermassive black hole (Sgr A*); in fact, recent measurements of near-infrared periodic flares [10, 11] suggest that Sgr A* is a rapidly rotating Kerr black hole with specific angular momentum in the range $(0.5 \div 1)M$. The interest in studying orbits close to Sgr A* relies on the increasing accuracy in sub-milli-arcsecond astrometry by the near-infrared detectors [12] and on the potentiality of the next-generation radiotelescopes, e.g. the square kilometer array (SKA) [13], to identify some of the $10^4$ compact objects orbiting within 1 parsec around Sgr A* [14].

2. Dynamics of extended bodies in the equatorial plane of a Kerr spacetime

The Kerr metric in standard Boyer–Lindquist coordinates is given by

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr}{\Sigma} \sin^2 \theta \, dt \, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma \, d\theta^2 + \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta \, d\phi^2, \quad (2.1)$$
where $\Delta = r^2 - 2Mr + a^2$ and $\Sigma = r^2 + a^2 \cos^2 \theta$; here $a$ and $M$ are the specific angular momentum and the total mass of the spacetime solution respectively. The event horizons are located at $r_\pm = M \pm \sqrt{M^2 - a^2}$.

Let us introduce the zero angular momentum observer (ZAMO) family of fiducial observers, with 4-velocity

$$n = N^{-1}(\partial_t - N^\theta \partial_\theta); \quad (2.2)$$

where $N = (-g^{tt})^{-1/2}$ and $N^\theta = g_{t\theta}/g_{\theta\theta}$ are the lapse and shift functions respectively. A suitable orthonormal frame adapted to ZAMOs is given by

$$e_l = n, \quad e_r = \frac{1}{\sqrt{g_{rr}}} \partial_r, \quad e_\theta = \frac{1}{\sqrt{g^{\theta\theta}}} \partial_\theta, \quad e_\phi = \frac{1}{\sqrt{g^{\phi\phi}}} \partial_\phi. \quad (2.3)$$

We limit our analysis to the equatorial plane ($\theta = \pi/2$) of the Kerr solution; as a convention, the physical (orthonormal) component along $-\partial_\theta$, perpendicular to the equatorial plane will be referred to as 'along the positive z-axis' and will be indicated by the index $\hat{z}$ when convenient; $e_z = -e_\theta$. Moreover, we will consider our quadrupolar spinning test body as spinning only along the $\hat{z}$-direction and moving along an equatorial circular orbit of the Kerr spacetime, in the sense that both the center-of-mass line $U$ and the momentum $P = mU_p$ (with $U_p \cdot U_p = -1$) have only nonvanishing $t$ and $\phi$ components:

$$U \equiv \gamma [e_t + v e_\phi], \quad \gamma = (1 - v^2)^{-1/2},$$
$$U_p \equiv \gamma_p [e_t + v_p e_\phi], \quad \gamma_p = (1 - v_p^2)^{-1/2}. \quad (2.4)$$

We then proceed introducing observer-adapted frames to both $U_p$ and $U$ as follows. An orthonormal frame adapted to $U_p$ is given by

$$e_0 = U_p, \quad e_1 = e_t, \quad e_2 = \gamma_p [v_p e_t + e_\phi], \quad e_3 = e_\phi, \quad (2.5)$$

and hereafter all frame components of the various fields are meant to be referred to such a frame. The spin vector results then $S = se_3$ and the orthogonality of the quadrupole tensor with respect to $U_p$ implies

$$Q_{00} = Q_{01} = Q_{02} = Q_{03} = 0. \quad (2.6)$$

All the surviving components of the quadrupole tensor are constant along the path, corresponding to the definition of 'quasirigid motion' due to Ehlers and Rudolph [8]. Clearly in a more realistic situation the latter hypothesis should be released.

Being the evolution of the various quantities along $U$ it is also convenient to introduce a Frenet–Serret frame along $U$ [15],

$$E_0 \equiv U = \gamma [n + ve_\phi], \quad E_1 = e_t, \quad E_2 \equiv E_\phi = \gamma [vn + e_\phi], \quad E_3 = e_\phi, \quad (2.7)$$

satisfying the following system of evolution equations:

$$\frac{dE_0}{d\tau_U} = \kappa E_1, \quad \frac{dE_1}{d\tau_U} = \kappa E_0 + \tau_1 E_2,$$
$$\frac{dE_2}{d\tau_U} = -\tau_1 E_1 + \tau_2 E_3, \quad \frac{dE_3}{d\tau_U} = -\tau_2 E_2. \quad (2.8)$$

In this case, with $U$ tangent to an equatorial circular orbit, the second torsion $\tau_2$ is zero while the geodesic curvature $\kappa$ and the first torsion $\tau_1$ are simply related by

$$\tau_1 = -\frac{1}{2} \frac{d \kappa}{2\gamma^2 dv}. \quad (2.9)$$

A direct calculation shows that

$$\kappa = k(\nu e)(v - v^\nu)(v - v^\nu). \quad (2.10)$$
where \( k_{(\text{lie})} \) is the Lie relative curvature of each orbit [16]

\[
    k_{(\text{lie})} = -\partial_t \ln \sqrt{g_{\phi \phi}} = -\frac{(r^3 - a^2 M)\sqrt{\Delta}}{r^2(r^3 + a^2 r + 2a^2 M)},
\]

and

\[
    v^r_\pm = \frac{a^2 \mp 2a \sqrt{Mr + r^2}}{\sqrt{\Delta(a \pm r \sqrt{r/M})}},
\]

are the linear velocities of co/counter-rotating geodesics on the equatorial plane of the Kerr spacetime.

Consider now first the evolution equations (1.2) for the spin tensor. They imply that

\[
    Q_{12} = Q_{13} = Q_{23} = 0,
\]

and introducing the 'structure functions' \( f \) and \( f' \) of the extended body, defined by

\[
    Q_{11} = Q_{33} + f, \quad Q_{22} = Q_{33} + f',
\]

they also give

\[
    0 = \left[-v_p(\tau_1 + \kappa \nu) + v \tau_1 + \kappa \right] s - m(\nu - v_p) - f \frac{\nu_p}{\gamma} \left[H_{\dot{\phi} \dot{\phi}}(1 + v^2_p) - v_p(E_{\phi \phi} - E_{\dot{\phi} \dot{\phi}})\right],
\]

where the electric and magnetic parts of the Weyl tensor with respect to the ZAMO frame (2.3) have been introduced

\[
    E_{\phi \phi} = -E_{\dot{\phi} \dot{\phi}}, \quad E_{\dot{\phi} \dot{phi}} = \frac{M}{r^5} \frac{(r^2 + a^2)^2 + 2a^2 \Delta r^3}{r^3 + a^2 r + 2a^2 M}, \quad E_{\dot{phi} \phi} = \frac{M}{r^5},
\]

\[
    H_{\dot{\phi} \dot{\phi}} = -\frac{3Ma^2 \Delta}{r^5} \frac{r^2 + a^2}{r^3 + a^2 r + 2a^2 M}.
\]

By assuming the tracefree property characterizing the classical (Euclidean) quadrupole moment tensor to hold also in the relativistic case, the components of the quadrupole moment tensor \( Q_{ab} \) in this case are completely determined by \( f \) and \( f' \):

\[
    Q_{11} = \frac{2}{3} f - \frac{1}{3} f', \quad Q_{22} = -\frac{1}{3} f + \frac{2}{3} f', \quad Q_{33} = -\frac{1}{3} (f + f').
\]

Consider then the equations of motion (1.1). They imply that

\[
    0 = \left[H_{\dot{\phi} \dot{\phi}}(1 + \nu v_p) - v_p E_{\phi \phi} + v E_{\dot{\phi} \dot{phi}}\right] s - m \left[-v_p(\tau_1 + \kappa \nu) + v \tau_1 + \kappa \right]
    \quad + \frac{3}{2 \gamma \nu_p} \frac{M \sqrt{\Delta}}{r^5} \left[f' + f \frac{\gamma_p}{\gamma} [c_1 v_p + c_2 (1 + v^2_p) + c_3]\right],
\]

where

\[
    c_1 = \frac{2a}{\sqrt{\Delta}} \frac{(r^2 + a^2)^2 + \Delta(4a^2 + 3r^2)}{r(r^3 + a^2 r + 2a^2 M)},
\]

\[
    c_2 = -\frac{(r^2 + a^2)^2 + 4a^2 (\Delta + Mr)}{r(r^3 + a^2 r + 2a^2 M)},
\]

\[
    c_3 = -\frac{(r^2 + a^2)^2 - a^2 \Delta}{r(r^3 + a^2 r + 2a^2 M)}.
\]

Solving equations (2.15) and (2.18) for \( v \) and \( v_p \) in terms of \( s \) and \( f, f' \) completely determines the motion.

According to [1] it is useful to introduce the following rescaled dimensionless angular and quadrupolar momentum quantities,

\[
    \sigma = \frac{s}{m} \xi_K, \quad F = \frac{f}{m} \xi_K^2, \quad F' = \frac{f'}{m} \xi_K^2,
\]
where $\zeta_K = (M/r)^{1/2}$ is the Keplerian value of the geodesic angular velocity in the absence of background rotation (i.e. $a = 0$), constant along $U$ due to the fact that along a circular orbit $r = \text{const}$.

The quantities $\sigma$, $F$ and $F'$ are necessarily small. Although the quadrupolar terms $f$ and $f'$ are small only for a quasi-spherical body, the further rescaling by $\zeta_K$ indeed makes them small in any case. In fact, the radius of the orbit is assumed to be large enough in comparison with certain natural length scales like $|s|/m$ (also known as the Moller radius [17] of the body), $\langle |f|/m \rangle^{1/2}$, $\langle f'|/m \rangle^{1/2}$ associated with the body itself in order to avoid backreaction effects.

Equations (2.15) and (2.18) then become

$$0 = \zeta_K \left[ -v_p(\tau_1 + \kappa v) + v_\tau + \kappa \right] \sigma - \zeta_K^2 (v - v_p) - F \frac{\gamma_p}{\nu} \left[ H_{\dot{\phi}} (1 + v_p^2) - v_p (E_{\dot{\phi}} - E_{\dot{\theta} \dot{\phi}}) \right],$$

(2.21)

and

$$0 = \zeta_K \left[ H_{\dot{\phi}} (1 + v v_p) - v_p E_{\dot{\phi}} + v E_{\dot{\theta} \dot{\phi}} \right] \sigma - \zeta_K^2 \left[ -v_p (\tau_1 + \kappa v) + v_\tau + \kappa \right]$$

$$+ \frac{3}{2} \frac{M \sqrt{\Delta}}{2 \gamma_p} \frac{\kappa}{r^3} \left[ F' + F \frac{\gamma_p^2}{\nu} c_1 v_p + c_2 (1 + v_p^2) + c_3 \right].$$

(2.22)

The above relations define the kinematical conditions allowing circular motion of the extended body taking into account its spinning and quadrupolar structures.

Let us investigate the case of extended bodies with internal structure (dipolar and quadrupolar) compatible with a nearly geodesic motion. The case of a spinning particle with a vanishing quadrupole moment tensor, i.e. $F = 0$, has been already studied in [18]. In this situation equations (2.21)–(2.22) reduce to

$$0 = \left[ -v_p (\tau_1 + \kappa v) + v_\tau + \kappa \right] \sigma - \zeta_K (v - v_p),$$

(2.23)

$$0 = \left[ H_{\dot{\phi}} (1 + v v_p) - v_p E_{\dot{\phi}} + v E_{\dot{\theta} \dot{\phi}} \right] \sigma - \zeta_K \left[ -v_p (\tau_1 + \kappa v) + v_\tau + \kappa \right].$$

In the limit of small spin $\sigma$ we find

$$v_{\pm} = v_{\pm}^0 + \mathcal{N}_{\pm} \sigma + O(\sigma^2),$$

$$v_{\pm}^{(\pm)} = v_{\pm} + O(\sigma^2),$$

(2.24)

where

$$\mathcal{N}_{\pm} = \mp \frac{1}{2 \zeta_K} \left[ H_{\dot{\phi}} (1 + v_{\pm}^0) - v_{\pm}^0 (E_{\dot{\phi}} - E_{\dot{\theta} \dot{\phi}}) \right],$$

(2.25)

and the signs $\pm$ correspond to co/counter rotating orbits. To first order in $\sigma$ and also neglecting terms like $a\sigma$ the linear velocity (2.24) becomes

$$v_{\pm} \approx \pm v_K - 3 v_K \left[ \zeta_K + \frac{\sigma}{2} \right],$$

(2.26)

where $v_K = [M/(r - 2M)]^{1/2}$ is the Keplerian linear velocity.

If the contribution of quadrupolar terms can be considered negligible with respect to the dipolar ones and comparable with second-order terms in the spin itself, one can consider corrections to equation (2.24) as given by

$$v_{\pm} \approx v_{\pm}^0 + \mathcal{N}_{\pm} \sigma + \mathcal{N}_{\pm} \left\{ \frac{\sigma^2}{v_{\mp}^0 - v_{\pm}^0} - F \left[ 1 - \frac{2 H_{\dot{\phi}} v_{\pm}^0 - E_{\dot{\phi}} + E_{\dot{\theta} \dot{\phi}}}{2 \zeta_K^2} \right] \right\},$$

(2.27)

$$v_{\pm}^{(\pm)} \approx v_{\pm} + 2 \mathcal{N}_{\pm} (F - \sigma^2).$$

The corresponding angular velocity $\zeta_{\pm}$ and its reciprocal are

$$\zeta_{\pm} \approx \zeta_{\pm}^0 + \mathcal{N}_{\pm} \left\{ \gamma_{\phi} \frac{\sigma^2}{v_{\mp}^0 - v_{\pm}^0} - F \left[ 1 - \frac{2 H_{\dot{\phi}} v_{\pm}^0 - E_{\dot{\phi}} + E_{\dot{\theta} \dot{\phi}}}{2 \zeta_K^2} \right] \right\},$$

(2.28)

$$\frac{1}{\zeta_{\pm}} \approx \frac{1}{\zeta_{\pm}^0} (1 + \lambda_\phi^\pm + \lambda_\theta^\pm).$$
where

\[
\lambda_d^{\pm} = -\frac{NN_\pm}{\sqrt{8\phi_0^3}\zeta_\pm^2} \sigma, \quad \lambda_q^{\pm} = -\frac{NN_\pm}{\sqrt{8\phi_0^3}\zeta_\pm^2} \left(1 + N_\pm \left(\frac{1}{v_0^2 - v_0^2} - \frac{N}{\sqrt{8\phi_0^3}\zeta_\pm^2}\right)\right) - F\left(1 - \frac{2H\nu_0^2 - E_{\hat{r}\nu} - E_{\hat{r}\nu}^2}{2\zeta_k^2}\right).\]

The period of revolution around the central source thus turns out to be

\[
T = \frac{2\pi}{|\zeta_\pm|} = T_0^d + \lambda_d^{\pm} + \lambda_q^{\pm}, \quad T_0^q = \frac{2\pi}{|\zeta_\pm^2|},
\]

(2.29)

In the limit of small values of the black hole rotation parameter \(a\) equations (2.27)–(2.28) reduce to

\[
v_\pm \simeq v_K \pm 3v_K \left[a\zeta_K + \frac{\sigma}{2}\right] + \frac{3}{2} \frac{\zeta_K}{v_K} (1 + 4v_K^2) a\sigma + \frac{3}{8} v_K (\sigma^2 + 2F) - \frac{3}{4} \frac{\zeta_K}{v_K} \left[(1 + 10v_K^2)F - (1 + 4v_K^2)\sigma^2\right],
\]

(2.31)

\[
v_p^{(\pm)} \simeq v_\pm \pm 3v_K (F - \sigma^2) - 3\frac{\zeta_K}{v_K} (1 + 4v_K^2) a(F - \sigma^2),
\]

\[
\zeta_\pm \simeq \frac{\zeta_K}{v_K} v_\pm,
\]

\[
\frac{1}{\zeta_\pm} \simeq \frac{1}{\zeta_K} + a + \frac{3}{2\zeta_K} \sigma \mp \frac{3}{2} \frac{r}{M} \sigma \mp \frac{3}{8\zeta_k} (2F - 5\sigma^2)
\]

\[
+ \frac{3}{2} a \left[3(F - \sigma^2) + \frac{r}{2M} (F - 7\sigma^2)\right],
\]

from which the limiting expression for the period of revolution \(T = 2\pi / |\zeta_\pm|\) follows easily.

A direct measurement of \(T\) will then allow us to estimate the quantity \(F\) determining the quadrupolar structure of the body, if its spin is known. Note that the fraction \(\lambda_d^{\pm}\) due to the spin is different depending on whether the body is spinning up or down, whereas the term \(\lambda_q^{\pm}\) due to the quadrupole has a definite sign once the shape of the body is known (\(F\) cannot change its sign).

3. Applications

An interesting opportunity to test the quadrupole effect of an extended body would arise, for instance, from the motion of ordinary or neutron stars around Sgr A*, the supermassive \((M \simeq 10^6 M_\odot)\) rotating \((a \in [0.5, 1] M)\) black hole located at the galactic center \([10, 14, 19]\).

To illustrate the order of magnitude of the effect, we may consider a binary pulsar system similar to PSR J0737-3039 as orbiting Sgr A* at a distance of \(r \simeq 10^8\) km. The PSR J0737-3039 system consists of two close neutron stars (their separation is only \(d_{AB} \sim 8 \times 10^5\) km) of comparable masses \(m_A \simeq 1.4 M_\odot, m_B \simeq 1.2 M_\odot\), but very different intrinsic spin period (23 ms of pulsar A versus 2.8 s of pulsar B) \([20]\). Its orbital period is about 2.4 h, the smallest yet known for such an object. Since the intrinsic rotations are negligible with respect to the orbital period, we can treat the binary system as a single object with reduced mass \(\mu_{AB} \simeq 0.7 M_\odot\) and intrinsic rotation equal to the orbital period. The spin parameter thus turns out to be equal
to \( \sigma \approx 6 \times 10^{-8} \), whereas the quadrupolar parameters are \( F = F' \approx 9.6 \times 10^{-10} \), since we have taken \( f = f' = \mu_{AB}d_{AB}^{\pm} \) as a rough estimate.

Since the rotation parameter of Sgr A* is not small we must use the exact expression (2.30). In the literature one finds different estimates of the spin parameter of the galactic center black hole, ranging from \( a \approx 0.52M \) [21] all the way up to \( a \approx 0.983M \) [22, 23] or even \( a \approx 0.996M \) [24, 25]. We list in table 1 the corresponding values of the geodesic period \( T_g^f \) of the PSR J0737-3039 binary system as well as the corrections \( \lambda_d \) and \( \lambda_q \) due to its dipolar and quadrupolar structures respectively.

Table 1. The estimates of geodesic period \( T_g^f \) as well as the corrections \( \lambda_d \) and \( \lambda_q \) due to both the dipolar and quadrupolar structures of the PSR J0737-3039 binary system are listed for different values of Sgr A* rotational parameter \( a/M \). Note that in order to resolve the dipolar and quadrupolar effects the measured period should be known with very high accuracy.

| \( a/M \) | \( T_g^f \) (cm) | \( \lambda_d \) | \( \lambda_q \) |
|------|-----------|------|------|
| 0.52 | 1.622 36 \times 10^{16} | 8.83 \times 10^{-8} | -7.05 \times 10^{-10} |
| 0.75 | 1.622 38 \times 10^{16} | 8.75 \times 10^{-8} | -6.99 \times 10^{-10} |
| 0.983 | 1.622 40 \times 10^{16} | 8.66 \times 10^{-8} | -6.92 \times 10^{-10} |
| 0.996 | 1.622 41 \times 10^{16} | 8.66 \times 10^{-8} | -6.92 \times 10^{-10} |
| 1 | 1.622 41 \times 10^{16} | 8.66 \times 10^{-8} | -6.92 \times 10^{-10} |

The detection of pulsars in Sgr A* is difficult because of the intense scattering region located in front of Sgr A*. However, these pulsars may be detectable in the next decade by the SKA detector, which promises high frequency sensitivity and large collecting area [13].

Another possible application of our calculations could be the orbital motions of the so-called S-stars [21], i.e. the massive ((30 ÷ 120)M\(_\odot\)), young (<10 Myr) stars within the influence of the supermassive black hole. However, in this case the orbits are no longer circular and the problem of discriminating quadrupole effects would deserve further investigation.

4. Concluding remarks

We have investigated the motion of extended bodies endowed with dipolar as well as quadrupolar structure on a Kerr background following Dixon’s model, generalizing previous results. Under the simplified assumptions of constant frame components (with respect to a natural orthonormal frame) of both the spin and the quadrupole tensor describing the body
we have found the kinematical conditions to be imposed on the particle’s structure in order the orbit of the particle itself be circular and confined on the equatorial plane. The motion turns out to deviate from the geodesic one due to the internal structure of the body, leading to measurable effects. The effect of the quadrupole terms could be important, for instance, when considering the period of revolution of an extended body around the central source. We have then applied such an analysis to the binary pulsar system PSR J0737-3039 as if it were orbiting the galactic center black hole, providing an estimate of the corrections to the geodesic value of the orbital period due to the dipolar as well as quadrupolar terms.

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