Thermohaline mixing in low-mass giants: RGB and beyond

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Abstract. Thermohaline mixing has recently been proposed to occur in low mass red giants, with large consequence for the chemical yields of low mass stars. We investigate the role of thermohaline mixing during the evolution of stars between $1M_\odot$ and $3M_\odot$, in comparison to other mixing processes acting in these stars. We use a stellar evolution code which includes rotational mixing and internal magnetic fields. We confirm that thermohaline mixing has the potential to destroy most of the $^3$He which is produced earlier on the main sequence during the red giant stage, in stars below $1.5M_\odot$. We find this process to continue during core helium burning and beyond. We find rotational and magnetic mixing to be negligible compared to the thermohaline mixing in the relevant layers, even if the interaction of thermohaline motions with the differential rotation may be essential to establish the time scale of thermohaline mixing in red giants.

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INTRODUCTION

Thermohaline mixing is usually not considered as an important mixing process in single stars, since the ashes of thermonuclear fusion consists of heavier nuclei than its fuel, and stars usually burn from the inside out. The condition for thermohaline mixing, however, is that the mean molecular weight ($\mu$) decreases inward. Recently Charbonnel & Zahn (2007, CZ07) identified thermohaline mixing as an important mixing process which significantly modifies the surface composition of red giants after the first dredge-up. The work by CZ07 was triggered by the paper of Eggleton et al. (2006, EDL06), who found a mean molecular weight ($\mu$) inversion — i.e., $(\frac{d\log\mu}{d\log P}) < 0$ — below the red giant convective envelope in a 1D-stellar evolution calculation.

EDL06 found a $\mu$-inversion in their $1M_\odot$ stellar evolution model, occurring after the so-called luminosity bump on the red giant branch, which is produced after the first dredge-up, when the hydrogen-burning shell source enters the chemically homogeneous part of the envelope. The $\mu$-inversion is produced by the reaction $^3$He($^3$He,2p)$^4$He (as predicted by Ulrich (1972)). It does not show up earlier, since the magnitude of the $\mu$-inversion is small, and negligible if compared to a stabilizing $\mu$-stratification.

The mixing process below the convective envelope in models of low mass stars turns out to be essential for the prediction of the chemical yield of $^3$He (EDL06); this process is also essential to understand the surface abundances of red giants, in particular the $^{12}$C/$^{13}$C ratio, the $^7$Li and the carbon and nitrogen abundances (CZ07). We investigate the evolution of solar metallicity stars between $1M_\odot$ and $3M_\odot$ from the ZAMS up to the
thermally-pulsing AGB stage, based on models computed during the last years. We show for which initial mass range, and during which evolutionary phase thermohaline mixing occurs, and with which consequences. Besides thermohaline mixing, our models include convection, rotation-induced mixing, and internal magnetic fields, and we compare the significance of these processes in relation to the thermohaline mixing.

**METHOD**

We compute evolutionary models of 1.0, 1.5, 2.0 and 3.0 M⊙ with solar metallicity (Z=0.02). We use a hydrodynamic stellar evolution code which includes the effect of rotation and magnetic fields (e.g. [Heger et al., 2000, Yoon & Langer, 2005]). Mixing of chemical species is treated as a diffusive process. The condition for the occurrence of thermohaline mixing is

$$\frac{\varphi}{\delta} \nabla \mu \leq \nabla - \nabla_{ad} \leq 0$$

i.e. the instability operates in regions that are stable against convection (according to the Ledoux criterion) and where an inversion in the mean molecular weight is present. Numerically, we treat thermohaline mixing through a diffusion scheme ([Braun, 1997, Wellstein et al., 2001]). The corresponding diffusion coefficient is based on the work of [Stern (1960), Ulrich (1972), and Kippenhahn et al. (1980)]; it reads

$$D_{th} = -\alpha_{th} \frac{3K}{2c_p \rho} \frac{\varphi}{\delta} \nabla \mu \nabla_{ad} - \nabla$$

where $K = 4acT^3/(3\kappa T)$, $\varphi = (\partial \ln \rho / \partial \ln \mu)T$, $\delta = -(\partial \ln \rho / \partial \ln T)\mu T$, $\nabla \mu = d \ln \mu / d \ln P$, $\nabla_{ad} = (\partial \ln T / \partial \ln P)_{ad}$, and $\nabla = d \ln T / d \ln P$. The quantity $\alpha_{th}$ is a efficiency parameter for the thermohaline mixing. The value of this parameter depends on the geometry of the fingers arising from the instability and is still a matter of debate ([Ulrich, 1972, Kippenhahn et al., 1980, Charbonnel & Zahn, 2007]). We use a value of $\alpha_{th}$ corresponding to the prescription of [Kippenhahn et al., 1980].

**RGB AND BEYOND**

The surface composition of low mass stars is substantially changed during the first dredge-up: lithium and carbon abundances as well as the carbon isotopic ratio decline, $^3$He and nitrogen abundances increase. After the first dredge-up the hydrogen-burning shell is advancing while the convective envelope retreats; the shell source then enters the chemically homogeneous part of the envelope. EDL06 and CZ07 have shown how in this situation an inversion in the molecular weight is created by the reaction $^3$He($^3$He,2p)$^4$He in the outer wing of the hydrogen-burning shell in models of 1.0 and 0.9 M⊙. This inversion is responsible for thermohaline mixing to develop.

We compute stellar models of 1.0, 1.5, 2.0 and 3.0 M⊙ with solar metallicity including the effects of rotation and magnetic fields. We confirm the presence of an inversion in the mean molecular weight, in the outer wing of the H-burning shell, after the luminosity bump on the red giant branch. According to inequality (1) this inversion gives
rise to thermohaline mixing in the radiative buffer layer, the radiative region between the H-burning shell and the convective envelope.

In our 1M_☉ model, thermohaline mixing develops at the luminosity bump and transports chemical species in the radiative layer between the H-burning shell and the convective envelope. This results in a change of the stellar surface abundances. The left panel of Fig. 2 shows the evolution of 3He surface abundance and of the ratio 1/213 at surface as a function of time, confirming the result of EDL06 and CZ07, namely that thermohaline mixing is efficient in depleting 3He and lowering the ratio 1/213 on the giant branch.

While CZ07 and EDL07 investigate thermohaline mixing only during the RGB phase, we followed the evolution of our models until the TP-AGB phase. Indeed a μ-inversion is always created if a H-burning shell is active in a chemically homogeneous layer; this happens not only during the RGB phase, but also during the HB and AGB phases. The size of the μ-inversion is depending on the local amount of 3He, that comes from the incomplete PP chain, as well as from the chemically homogeneous layer.

After core He-flash, helium is burned in the core, while a H-burning shell is still active below the convective envelope. We found that during this phase thermohaline mixing is present and can spread through the whole radiative buffer layer in our 1 M_☉ model (left panel in Fig. 1). In this model the surface abundances change also during this phase because the H-burning shell and the envelope are connected. This is shown in Fig. 2 left panel, where surface abundances change also after the luminosity peak corresponding to the He-flash. We stress that using the prescription of Kippenhahn et al. (1980) for thermohaline mixing allows our model to reach this phase without completely burning the 3He; models of CZ07 almost completely deplete 3He in the envelope already during the RGB phase because of their higher diffusion coefficient. In this case thermohaline mixing would be much less efficient, during the subsequent evolutionary phases, due to the lower abundance of 3He.

The subsequent evolutionary phase of a low mass star is referred as Asymptotic Giant Branch (AGB), and is characterized by the presence of two burning shells and
FIGURE 2. Left panel: evolution of the surface abundance of the $^{12}$C/$^{13}$C ratio (dotted red line) and $^3$He (dashed green line), and of the luminosity (solid blue line) from the onset of thermohaline mixing up to the AGB phase for a 1.0$M_\odot$ star. Right panel: diffusion coefficients in the region between the H burning shell and the convective envelope for the 1.0$M_\odot$ model during the RGB phase ($t = 1.267 \times 10^{10}$ years). The black, continuous line shows convective and thermohaline mixing diffusion coefficients, the green, dashed line is the sum of the diffusion coefficients due to rotational instabilities while the blue, dot-dashed line shows the magnitude of magnetic diffusion coefficient.

a degenerate core. The star burns H in a shell and the ashes of this process feed a underlying He shell. During the most luminous part of the AGB the He shell periodically experiences thermal pulses (TPs); in stars more massive than 2$M_\odot$ these thermal pulses are associated with a deep penetration of the convective envelope, the so-called third dredge-up (3DUP). We find thermohaline mixing to be present also in the TP-AGB phase. Depending on the mass of the model the diffusion process is able to connect the H-burning shell with the convective envelope during the whole interpulse phase. In a 1$M_\odot$ model thermohaline mixing connects the H-burning shell to the convective envelope (Fig. 3), confirming that this mixing process is more efficient at lower masses.

**ROTATION AND MAGNETIC FIELDS**

In our models we found that in the relevant layers thermohaline mixing has generally higher diffusion coefficients than rotational instabilities and magnetic diffusion. The right panel of Fig. 2 clearly shows that rotational and magnetic mixing are negligible compared to the thermohaline mixing in our 1.0$M_\odot$ model. The only rotational instability acting on a shorter timescale is the dynamical shear instability, visible in the right panel of Fig. 2 as a spike present at the lower boundary of the convective envelope. This instability works on the dynamical timescale in regions of a star where a high degree of differential rotation is present; it sets in if the energy that can be gained from the shear flow becomes comparable to the work which has to be done against the potential for an adiabatic turn-over of a mass element ("eddy") (Heger, 1998). However, if present, this instability acts only in a very small region (in mass coordinate) at the bottom of the convective envelope. As a result thermohaline mixing is still setting the timescale for the diffusion of chemical species from the convective envelope to the Hydrogen-burning
FIGURE 3. Evolution of the region between the H burning shell source and the convective envelope during a thermal pulse in a $1 \, M_\odot$ star. Green hatched regions indicate convection and red crossed regions indicate thermohaline mixing. Blue shading shows regions of nuclear energy generation.

We will discuss qualitatively the interaction of thermohaline motions with magneto-rotational instabilities in a forthcoming paper (Cantiello et al., 2007).

DISCUSSION

We confirm the results of EDL06 and CL07: thermohaline mixing in low mass giants is capable of destroying large quantities of $^3$He, as well as decreasing the ratio $\Delta 1213$. Thermohaline mixing indeed starts when the hydrogen burning shell source moves into the chemically homogeneous layers established by the first dredge-up. Our models show further that thermohaline mixing remains important during core helium burning, and can still be relevant during the AGB phase — including the termally-pulsing AGB stage. This results in important changes in the surface abundances of low mass stars. The quantitative discussion is complicated by the fact that thermohaline mixing is strongly dependent on the mass of the star and on the efficiency of thermohaline mixing, which is still a matter of debate.

Moreover, our calculations show that in the relevant layers thermohaline mixing has generally a higher diffusion coefficient than rotational instabilities and magnetic diffusion.

We will discuss qualitatively the interaction of thermohaline mixing with magneto-rotational instabilities in a forthcoming paper, where we will also explore the effect of using different prescriptions for thermohaline mixing diffusion coefficient (Cantiello et al., 2007).
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