Efficient Parametric Optimisation of Support Loss in MEMS beam resonators via an enhanced Rayleigh-Ritz method

H.T.D. Grigg¹, B.J. Gallacher²

¹Ph.D. Candidate, School of Mechanical and Systems Engineering, Newcastle University-Stephenson Building, Claremont Road, Newcastle upon Tyne NE1 7RU, UK
E-mail: harry.grigg@newcastle.ac.uk

²Senior Lecturer, School of Mechanical and Systems Engineering, Newcastle University-Stephenson Building, Claremont Road, Newcastle upon Tyne NE1 7RU, UK
E-mail: barry.gallacher@newcastle.ac.uk

Abstract: MEMS resonators offer attractive prospects in several application areas, including high-performance, low cost sensors, among several others. The performance of many resonant MEMS depends critically on the Q factor, and an important, poorly quantified contribution to the overall Q is the support loss. Additionally, the parameter space for the geometry can be of moderately high dimension, making FEA based parametric optimisation computationally inefficient. Thus motivated, a numerical method based on the Rayleigh-Ritz substructure synthesis using quasicomparison functions is developed, applicable to a wide and important class of beam resonators. It is shown to be highly efficient by comparison with classical FEA methods, facilitating a detailed examination of the support Q as a function of position in parameter space. Selected results are presented and briefly discussed, with particular attention given to convergence, computational efficiency and design optimisation. General design principles for multiply-supported framelike beam resonators are considered in the light of the results, and extensions to the modelling are briefly covered.

KEYWORDS: MEMS resonators, Rayleigh-Ritz, numerical vibration modelling, optimisation, Xylophone Bar Resonator, Support Loss, Q factor.

1. Introduction.

The last decade has seen considerable interest devoted to MEMS devices based on resonators, including RF MEMS[1], resonant accelerometers and gyroscopes[2], magnetometers[3], energy harvesters, biosensors, and other applications. The performance and behaviour of many of these devices depends critically on the quality (Q) factor of the underlying resonator.

A common model for the Q factor of a MEMS resonator can be stated as:
Little study has been dedicated to the subject of support loss([4], [5], etc). The phenomenon consists of energy transport from the resonator to the environment via elastic waves excited in the substrate by the stresses at the interface between the resonator supports and the substrate.

1.1. Xylophone Bar Resonator

The method was developed in the context of performance optimisation of a Xylophone Bar Resonantor (XBR) based magnetometer. The resonator consists of a main sense beam element supported transversely by ancillary support beams, of comparatively small flexural stiffness, forming an H-shaped frame.

A periodic voltage is applied to the device such that any out-of-plane component of magnetic flux excites the mode of interest at its natural frequency, with a steady-state amplitude proportional to the flux density and to the device Q factor.

2. Resonator Model.

To model the resonator dynamics, a Rayleigh-Ritz substructuring model[6] based on the decomposition of the frame structure into 1D elements intersecting at nodes and imposing geometric conditions of compatibility there and at the substrate interfaces. The field Lagrangian was chosen as

\[
L = T^* - V = \left\{ \int_0^L \rho A \left[ \frac{\partial^2 U}{\partial t^2} + \frac{\partial V}{\partial t} \right] \, dx \right\} - \left\{ \int_0^L E I \left[ \frac{\partial^2 U}{\partial x^2} \right]^2 \, dx + \int_0^L E A \left[ \frac{\partial V}{\partial x} \right]^2 \, dx \right\} \quad (2)
\]

Where \( U \) is a transverse displacement and \( V \) an axial displacement. Polynomial trial functions were employed for the axial field, while quasicomparison functions were developed and used based on beam modes under appropriate constraints for the transverse field.
3. Substrate Model.

The problem of elastic wave radiation in a semi-infinite half space has been studied previously. Miller and Pursey’s 1953 paper[7] arrives at closed-form expressions for the energy radiated by localised stress sources on the surface of an infinite linear elastic half-space.

It can be shown that[4]:

\[
W = \int_{0}^{2\pi} Re(F_T \bar{v}_T + F_N \bar{v}_N) \, dt = \frac{\pi b K}{\rho S^2} \{ |\sigma_N|^2 Im(Y_N) + |\sigma_T|^2 Im(Y_T) \} \tag{3}
\]

Where

\[
Im(Y_N) = 0.2215, \quad Im(Y_S) = 0.3350
\]

This formula is an explicit relation between the distal forces of constraint acting at the interface between the resonator and the substrate, and the corresponding support loss, in a compact form suited to evaluating the Q factor.

4. Results and discussion.

![Figure 2](image-url)  

**Figure 2.** Natural frequencies obtained by simulation using the commercial FEA package COMSOL; Rayleigh-Ritz models using 1, 2, and 4 Pinned-Pinned mode shapes per substructure as trial functions; and Rayleigh-Ritz-Meirovitch models using 1 and 4 quasi-comparison trial functions. Agreement between the 3D FEA and the RRM models is superb.
Figure 3. Shear and normal forces at the support-substrate interface, obtained by simulation. Models are as in Fig.1. Note that the PP models do not converge to the QCF/FEA solutions, even in the regime where the natural frequencies have converged.

Excellent convergence and agreement are observed. It is worth noting that over 10000 DOF were needed for 3D FEA and 2000 DOF for 2D FEA to converge to this result, while the 4QCF model contained 71 DOF. From an optimisation point of view, then, the RRM approach is at least two orders of magnitude more efficient under the conditions considered.

Figure 3, taken together with the form of Equation 1, gives an insight into the relationship between resonator geometry and support Q factor. The zero crossing of the normal force that occurs between NR=0.22 and 0.23 for the converged results. This matches to the third significant figure the result for the node points of a free-free beam. This corresponds to the nulling of the component of support Q due to axial forcing of the support, and is a corollary of the impedance tuning of the resonator.

Figure 4. Support quality factor vs. parameters \( NR \) and \( L_{sup} \). The parameters at the peak are \( L_{sup} = 4.3 \text{mm} \); \( NR = 0.2242 \). The support Q is \( 7.541 \times 10^5 \).
Figure 3 illustrates a local maximum of the Q factor in the NR - L_{sup} parameter plane as a surface plot. There are $10^4$ parametric combinations on display in a 100x100 grid, and the solve time was of the order of 1 minute on a desktop system. This compares favourably with the commercial benchmarks. The half-maximum dimensions of the local Q maximum are 0.094% in NR and 0.0026 m in L_{sup}, quantifying the sensitivity of the support Q to mistuning in these parameters.

Figure 5. Support quality factor of the device as a function of H_{sup}, plotted for several values of L_{sup}.

The unconstrained global support Q maximum for the XBR system is unphysical, and cannot be approached closely, requiring an infinitely thin, infinitely short support. Of more interest for practical problems is the constrained maximum obtained by applying manufacturing considerations to the geometry. Figure 5 is of interest in this context. If an arbitrary 100 micron limit is imposed from below on the parameter H_{sup}, then the constrained optimal value for L_{sup} is not a limiting value, but is close to 500 microns. Using the RRM approach allows for detailed examination of the response in the parameter space and optimisation of the dynamics for support loss.

5. Conclusions and further work.

Dynamical modelling of XBR geometry using an RRM approach was shown to be a valid and efficient route to examine and optimise the resonator for support loss. The results imply the efficacy of impedance tuning the resonator-support geometry as a means to mitigate support loss.

Further work would include extending the RRM analysis to include nonlinear axial effects, and developing and integrating more complete and representative substrate models. Integration of
a constrained optimisation method such as simulated annealing, PSO or genetic algorithms as appropriate should be a straightforward extension of utility for more complex frame topologies, for which a qualitative understanding of the impedance tuning conditions is not accessible or satisfactory.

6. References.

[1] J. Basu and T. K. Bhattacharyya, "Microelectromechanical resonators for radio frequency communication applications," *Microsystem Technologies*, vol. 17, no. 10–11, pp. 1557-1580, Aug. 2011.

[2] K. M. Harish, B. J. Gallacher, J. S. Burdess, and J. a Neasham, "Experimental investigation of parametric and externally forced motion in resonant MEMS sensors," *Journal of Micromechanics and Microengineering*, vol. 19, no. 1, p. 015021, Jan. 2009.

[3] A. L. Herrera-May et al., "Mechanical design and characterization of a resonant magnetic field microsensor with linear response and high resolution," *Sensors and Actuators A: Physical*, vol. 165, no. 2, pp. 399-409, Feb. 2011.

[4] Z. Hao, "An analytical model for support loss in micromachined beam resonators with in-plane flexural vibrations," *Sensors and Actuators A: Physical*, vol. 109, no. 1–2, pp. 156-164, Dec. 2003.

[5] B. Chouvion and I. Lyon, "Vibration Transmission and Support Loss in MEMS Sensors," University of Nottingham, 2010.

[6] A. Hale and L. Meirovitch, "A general substructure synthesis method for the dynamic simulation of complex structures," *Journal of Sound and Vibration*, vol. 69, no. 2, pp. 309-326, Mar. 1980.

[7] G. F. Miller and H. Pursey, "The Field and Radiation Impedance of Mechanical Radiators on the Free Surface of a Semi-Infinite Isotropic Solid," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 223, no. 1155, pp. 521-541, May 1954.