The Analysis of Student Error in Solve the Problem of Spherical Trigonometry Application

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Abstract. Students tend to memorize the concept of trigonometry given by lecturers or written in textbooks without understanding the purpose and content in solving the problem trigonometry, so that students often experience errors in solving the problem trigonometry. Error student in doing trigonometry problem is one of the clues to detect student difficulties. The failure is a form of deviation from the right, the prescribed procedure, or the deviation from the expected. This research type is descriptive qualitative, which aims to get information about mistakes of students of mathematics education semester 2 of trigonometry of Academic Year 2017/2018 Universitas Al Asyariah Mandar, in solving the problem of spherical trigonometric applications consisting of 2 questions, (1) determining the direction qibla; and (2) distance between 2 countries. The types of errors in solving the problems referred to in the study grouped according to the classification of Newman's faults namely, (1) the failure of process skills; (2) carelessness/inaccuracies; (3) misunderstanding of problems; (4) errors in the use of notation; and (5) misconceptions of concepts. The instrument used to reveal the type of error is a diagnostic test that contains two questions concerning the application of spherical trigonometry and interviews. After triangulation, the most common mistake made by the student is the misconception of the concept. Student made a mistake because of lack of understanding the correct concept in this case that is the analogy of Napier in solving the matter of calculating the distance between two countries and the spherical triangle formula to solve the problem calculate the direction of qibla.

1. Introduction

One of the subjects in the course of trigonometry is spherical trigonometry application, which discuss the implementation of the formula of spherical trigonometry among others: (1) Terrestrial Sphere; (2) Terrestrial Triangle; (3) The Celestial Sphere; (4) Calculating Direction of qibla. Studying the spherical trigonometry application, the students tend to memorize the formula given by the lecturer or written in the textbook without understanding the intent and
content, so that students often make mistakes in solving the problem. The students ability not only at the memorization stage, but the students already must be at the level of metacognitive knowledge, namely the development of thinking analyze (C4) to the creation (C6). Based on [1] the development of thinking The Anderson’s Bloom’s Taxonomy dimension (cognitive process dimension) is categorized into two levels: (1) Lower Order Thinking Skills (LOTS), ie remembering (C1) the knowledge form is fact and understand (C2) 2) Higher order thinking Skills (HOTS), the ability to analyze (C4), evaluate (C5) and creativity (C6). The field shows the students they are still make mistakes in solving the problem.

Related to the spherical trigonometry application, it is essential to minimize the mistakes of conceptualization, conceptual to understanding and problem-solving skills, in line [2] in his research that the students' performance in solving mathematical problems needs to be analyzed to know which errors or at which stage the student made a mistake. In the application of spherical trigonometry one of the subjects raised is calculate the direction of qibla and calculate the distance between the two countries.

The earth, in reality, is not a perfect sphere. To simplify calculations which involve measurement of long distances on the surface of the earth, astronomers, surveyors, and navigators treat the earth as a sphere of radius approximately 3,959 statute miles (1 statute mile = 5,280 ft.). A sphere of such magnitude is called a terrestrial sphere [3].

The earth rotates about a diameter called its axis. The ends of this diameter are called the north pole \( P_n \) and the south pole \( P_s \) as shown in fig. 1.1. The great circle having \( P_n \) and \( P_s \) as poles is called the equator.

Any half great circle which connects the north and south poles is called a meridian. Every meridian intersects the equator at right angles. If \( A \) is point distinct from \( P_n \) and \( P_s \), the meridian of \( A \) is the half great circle that extends from \( P_n \) to \( P_s \) and passes through \( A \). The meridian which passes through the astronomical laboratory at Greenwich, England is called the prime meridian. In fig. 1., \( P_n \) \( GP_s \) is the prime meridian with \( G \) representing Greenwich and \( P_nAP_s \) is the meridian of \( A \).

The latitude \( L \) of a point on the terrestrial sphere is the angular distance of the point from the equator. It measured along the meridian of the point from \( 0^\circ \) to \( 90^\circ \), north or south of the equator. In fig. 2., the latitude of \( A \) is measured by the arc \( AA' \) or the central angle \( AOA' \) subtended by the arc. If angle \( AOA' = 40^\circ \), then the point \( A \) is said to have a latitude of \( 40^\circ \)N. Note that \( A \) is north of the equator as shown in the given figure. Similarly, a point \( 35^\circ \) south of the equator is designated \( 35^\circ \)S.

![Figure 1. Prime Meridian](image-url)
The angular distance of a point from $P_n$ is called the *colatitude*. It is denoted by the symbol \( co-L \). In fig. 2., the colatitude of $A$ is the arc $AP_n$. Since any arc from the north pole to the equator has an arc length of $90^\circ$, then we can easily determine the colatitude of any point $A$ on the arc. Thus if $A$ has latitude $L$, then its colatitude is

\[
\begin{align*}
E(1.1) & \quad co-L = 90^\circ - L \quad & \text{if } A \text{ is north of the equator} \\
E(1.2) & \quad co-L = 90^\circ + L \quad & \text{if } A \text{ is south of the equator}
\end{align*}
\]

For example, since Manila has latitude $14^\circ36'\ N$, then its colatitude is $co-L = 90^\circ - 14^\circ36' = 75^\circ24'$. Sidney with latitude $33^\circ52'\ S$ has colatitude equal to $co-L = 90^\circ + 33^\circ52' = 123^\circ52'$. 

Small circle cut by planes perpendicular to the axis $(P_nP_s)$ of the terrestrial sphere are called *parallels of latitude*. All points on a parallel of latitude have the same latitude. In fig. 2., the points $A$ and $B$ have the same latitude.

The difference in latitude $DL$ between two points is the degree measure of the arc of a meridian that connects the parallels of latitude of the points. Consider two points $A$ and $B$ having latitude $L_1$ and $L_2$ respectively. The difference in latitude of these points are given by the following:

1. If $A$ and $B$ are both north or both south of the equator, then
   \[
   E(1.3) \quad DL = L_1 - L_2 \quad \text{if } L_1 > L_2 \\
   DL = L_2 - L_1 \quad \text{if } L_1 < L_2
   \]
2. If $A$ and $B$ are on opposite sides of the equator, then
   \[
   E(1.4) \quad DL = L_1 + L_2
   \]

For example, since Manila has latitude $14^\circ36'\ N$ and Sydney has latitude $33^\circ52'\ S$, then the difference in latitude between them by E(1.4) is $DL = 14^\circ36' + 33^\circ52' = 47^\circ88'$. 

**Figure 2.** Colatitude of $A$ is the arc $AP_n$. 

**Figure 3.** The Longitude of $A$ is Measured by the Arc $G' \hat{A}$ or the Central Angle $A'OG'$. 

The longitude $\lambda$ of a point on the terrestrial sphere is the angular distance between the prime meridian and the meridian of the point. It is measure from $0^\circ$ to $180^\circ$, east or west of the prime meridian. For example, since Manila is $121^\circ5'\text{E}$ of the prime meridian, its longitude is $121^\circ5'\text{E}$. If a point is $24^\circ$ west of the prime meridian, its longitude is designated $24^\circ\text{W}$.

The difference in longitude $D\lambda$ between two points on the terrestrial sphere is the degree measure of the shorter arc that connects the meridians of the points. If $A$ and $B$ have longitude $\lambda_1$ and $\lambda_2$ respectively, then the difference in longitude between them may be found by the following:

1) If $A$ and $B$ are both east or both west of the prime meridian, then
   \[ D\lambda = \lambda_1 - \lambda_2 \text{ if } \lambda_1 > \lambda_2 \]
   \[ D\lambda = \lambda_2 - \lambda_1 \text{ if } \lambda_1 < \lambda_2 \]

2) If $A$ and $B$ are on opposite sides of the prime meridian, then
   \[ D\lambda = \lambda_1 + \lambda_2 \text{ if } \lambda_1 + \lambda_2 < 180^\circ \]
   \[ D\lambda = 360^\circ - (\lambda_1 + \lambda_2) \text{ if } 180^\circ < \lambda_1 + \lambda_2 < 360^\circ \]

In fig. 3, $A$ and $B$ are both west of the prime meridian. Hence the difference in longitude between them is $D\lambda = B'G' - A'G'$.

We may consider the prime meridian and the equator as a pair of coordinate axes on the earth’s surface. Then concerning these axes, the latitude $L$ and the longitude $\lambda$ will serve as the coordinates of points on the terrestrial sphere. If the coordinates of each point are considered as latitude first and longitude second, then any point on the sphere is an ordered pair $(L, \lambda)$. Thus to indicate that Manila has latitude $14^\circ36'\text{N}$ and longitude $121^\circ5'\text{E}$, we simply write $(14^\circ36'\text{N}, 121^\circ5'\text{E})$.

Two ways can be done to calculate the direction of qibla, by using spherical trigonometry and shadow gnomon [4] and [5]. One of the expected skills possessed by students is the skill of calculating the direction of qibla by using spherical trigonometry formula. This skill is essential to be mastered by the students because as Muslim Ummah has been ordered by Al-Qur'an Surah Al-Baqarah verse 142-145. Here is the Surah Al-Baqarah verse 144

\[
\text{قد رأى نقلت وجُهك في السماء فلنَلْتِكما وَتُرْضَاهَا فَولِيَتَكْ}
\]
\[
\text{وَجَهَتْ نَظَرَ المسْجِدَ الْحَرَامَ وَتَحْيَبَ مَا كُنْتُمْ فُوْلُوا وَجِوْهُكَمُ نَظُرْهُندَ وَإِذَا}
\]
\[
	ext{الَّذِينَ أُوتُوا الْكِتَابَ لِيُعْلَمُوا أَنَّهُمْ أَلْحَقُّونَ أَنَّهُمْ أَلْحَقُّونَ وَمَا إِلهَٰنَا غَيْرُ إِلهٍ عَمَّا يُصَلِّمُونَ}
\]

The meaning: We have certainly seen the turning of your face, (O Muhammad), toward the heaven, and We will surely turn you to a qibla with which you will be pleased. So turn your face toward al-Masjid al-Haram. And wherever you (believers) are, turn your faces toward it (in prayer). Indeed, those who have been given
the Scripture well know that it is the truth from their Lord. And Allah is not unaware of what they do.

To find out if the student has skills, it is necessary to test and analyze the test results. The test results activities examine to see what kind of mistakes made by students in solving the problem. The purpose of these activities undertaken by the lecturers so that the lecturers know the difficulties of the students in learning to solving the problem spherical trigonometry and make improvements in future learning. The type of error resolving the question in the study grouped according to Newman's classification error, can be seen in table 1.

| Number | Error Type                        | Procedure    |
|--------|-----------------------------------|--------------|
| 1.     | Mistake to Understanding Problem  | Decoding     |
| 2.     | Careless / Inadequate Mistakes    | Comprehension|
| 3.     | Error In Notation Usage           | Transformation|
| 4.     | Error of Process Skill            | Process Skill|
| 5.     | Error in Understanding Concept    | Encoding     |

Based on the description above, the purpose of this study is to description of student error in solving the problem of spherical trigonometry application. This research is expected to give benefit that is as one references related to the type of student error in solving the problem (1) determining the direction of qibla; and (2) the distance between the two countries in the trigonometric course.

2. Research Methods
The research type is qualitative research. [6] Qualitative research is used to reveal a phenomenon experienced by research subjects in the form of behavior, perception, motivation, action and others holistically and by way of description in the kind of words and language, in a particular context that is natural and by utilizing various scientific methods.

The subjects of this study were the students of the A-grade mathematics education program which amounts to 22 people in academic 2017/2018 year. The instruments were used to identify the type of error, they were (1) diagnostic test in the form of a description that contains two questions concerning the spherical trigonometry application that is determining the direction of qibla and the distance between two countries; and (2) interviews. To measure the validity of data used triangulation. Triangulation done through tests and interview results. The tests used previously has been validated by the expert, while in the interview process focused to reveal five types of errors namely (1) the failure of process skills; (2) carelessness / inaccuracies; (3) misunderstanding of problems; (4) errors in the use of notation; and (5) misconceptions of concepts.

The data of the student’s test result then analyzed based on Flow model [7] which includes (1) data reduction; (2) data presentation and (3) conclusion/verification. Data reduction consists of the selection process, focusing on simplification, abstraction, and rough data transformation. This reduction activity resulted in the classification of Newman’s faults. Furthermore, the data presented in the form of narrative and table. Results clarification, data presentation and conclusion/verification is a complete configuration and then sought the meaning of research results.
3. Results And Discussion
This research includes quantitative and qualitative data. The result of performance analyze test consist of two test were (1) determination of direction of qibla; and (2) the distance between the two countries in the trigonometric course. Can be seen in table 2. and 3.

Table 2. Percentage of Student Error in Performing Performance Test

| Number. | Errors type                      | Procedure | Percentage (%) |
|---------|----------------------------------|-----------|----------------|
|         |                                  |           | Test of qibla Direction calculation | Test of direction two countries calculation |
| 1.      | Mistake to Understanding Problem | Decoding  | 13.64          | 22.73          |
| 2.      | Careless / Inadequate Mistakes   | Comprehension | 22.73 | 40.91          |
| 3.      | Error In Notation Usage          | Transformation | 31.82 | 50.00          |
| 4.      | Error of Process Skill           | Process Skill | 50.00 | 59.09          |
| 5.      | Error in Understanding Concept   | Encoding  | 59.09          | 68.18          |

Based on the table 2., it known that the fifth mistake type is the most common misconception of the concept that is 59.09 (test calculate the direction of qibla) and 68.18 (test calculate the distance between 2 countries). While the types of errors that are not done by the students are the first type of error that is the process skill error of 13.64 (test calculates the direction of qibla) and 22.73 (the test calculate the distance of 2 countries). For the description of the type of error and stage of student error procedure in doing the test calculate the distance between two countries and calculate the direction of qibla shown in table 3.

Table 3. Description of Student Error in Performing the Performance Test

| Number. | Error Type                        | Procedure | The Description of Students Error |
|---------|----------------------------------|-----------|----------------------------------|
| 1.      | Mistake to Understanding Problem | Decoding  | 1. The test determines the direction of qibla: (a) Errors that occur because students can not remember the position of latitude and longitude of mecca (in the question not given); (b) can not distinguish between the symbols for the latitude and longitude of the location to be measured in the direction of the qibla. |
|         |                                  |           | 2. The test calculates the distance between two |

| Number. | Error Type                  | Procedure                 | The Description of Students Error |
|---------|----------------------------|---------------------------|-----------------------------------|
| 2.      | Careless / Inadequate Mistakes | Comprehension              | The test to count the calculation of Qibla direction and the distance between the two countries has the same error, that is the error occurs because the student does not understand the term, phrase or do not know the question comprehensively. |
| 3.      | Error In Notation Usage       | Transformation             | 1. Test to calculates the direction of Qibla: Students are not able to change the information in question to mathematical symbols, operations and mathematical sentences correctly.  
2. Test to calculate the distance between two states: a) Errors that occur because students are not able to determine the colatitude at the point for both countries (whether using the formula co-L = 90° - L atau co-L = 90° + L ) is dikarena not able to determine the position of whether the point it is just north of the equator or south of the equator. (b) is unable to determine this longitude difference (D\lambda) due to the inability to distinguish between the two countries |
| Number | Error Type               | Procedure | The Description of Students Error |
|--------|--------------------------|-----------|----------------------------------|
| 4.     | Error of Process Skill   | Process Skill | on the East or both on the west of the main meridian or if the two countries are on the opposite side of the prime meridian. |
|        |                          |           | 1. Test to calculates the direction of qibla. Errors that occur due to the inability of students in applying the calculation steps correctly when using the procedure/algorithm despite having successfully wrote the spherical triangles formula. |
|        |                          |           | 2. The test to calculates the distance between the two countries. Errors that occur due to the inability of the student in applying the calculation steps correctly when using the procedure of logarithmic rules despite having successfully wrote the Napier's analogy formula. |
| 5.     | Error in Understanding Concept | Encoding | Both of the tests calculate the direction of qibla and calculate the distance between the two countries, the student has the same mistake, that is not able to write the correct or correct answer in the form of numbers, symbol or verbal. |

Based on the data from table 3., found that the fifth kind of mistake that most students do. Students can not write correct or correct answers in numerical, symbolic or verbal form. After interviewing the students who made the most mistakes in solving the problem, the average student's answer says that the error done because the students do not understand the correct concept. In this case that is the analogy of Napier’s in solving the matter of calculating the distance between two countries and the formula spherical triangles to solve the problem of calculating the direction of qibla.
4. Conclusion
After triangulation the data, the most common mistake made by the students of the mathematics education program of Al Asyariah Mandar University is the misconception of the concept. Students make mistakes because of lack of understanding of the correct concept in this case that is the analogy of Napier’s in solving the matter of calculating the distance between two countries and spherical angles formula to solve the problem of computing the direction of qibla.

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