Multiple slip effects on mixed convection of Oldroyd-B fluid towards a stretchy surface with radiation and chemical reaction using Cattaneo-Christov heat flux

K. Loganathanb,#, S. Sivasankaran*a, M. Bhuvaneswari*a, S.Rajanb

aDepartment of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
bDepartment of Mathematics, Erode Arts & Science College,Erode, Tamilnadu, India
#Email: loganathankaruppusamy304@gmail.com, *Email: sd.siva@yahoo.com

Abstract: The steady two-dimensional MHD mixed convection flow and heat transfer characteristics of an Oldroyd-B fluid over a linear stretchy sheet is considered. The flow caused by a linear stretchy sheet with effects of velocity, temperature and concentration slips. The thermal radiation, internal heat generation effects are included in the study. The resulting governing PDEs rebuild into ODE model by applying a suitable similarity transformation and then are solved by homotopy analysis method. The effects of the various constants on the velocity, temperature and concentration fields are discussed. It is found that the velocity, temperature and concentration fields decrease when increasing the corresponding slip constants.

1. Introduction

Mixed convection flow with radiation involves in many experimental positions when heat transfer by only natural convection is not enough. The involvements of mixed convection and non-linear thermal radiation are in electronic cooling, flow movement in the atmosphere, chemical processes etc. Few studies doing with radiation and mixed convection flow of a fluid can be reviewed by Refs. [1]-[3]. The non-Newtonian fluids are not easy to deal when compared with Newtonian fluids because the non-Newtonian fluid shear stress and shear rate are linked non-linearly. Therefore, many researchers are attracted by non-Newtonian models in the literatures [4]-[6]. Convective flow of a fluid towards stretched surface with Cattaneo-Christov heat flux is investigated by Eswaramoorthi et al. [7]. The research of influence of heat and mass transfer with chemical reaction in a flow is of huge practical importance to the engineering industries, few more studies seen in this regionin Refs. [8-10].

In the present study, we investigate the MHD double diffusive mixed convection characteristics over a linearly stretchy sheet with the velocity, temperature and concentration slip effects at the boundary. In addition, we also added the thermal radiation, heat generation and chemical reaction with Cattaneo-Christov heat flux model in the present work.

2. Problem formulation

Consider the steady two-dimensional double diffusive magneto-convection flow of an incompressible Oldroyd-B fluid. The velocity components \((u, v)\) are taken in the \((x, y)\)-directions. The velocity of sheet is assumed as \(u_w = nx\) where \(n > 0\)is the stretching rate. The two temperatures on and apart from the surface are expressed by \(T_w\)and \(T_\infty\),with \(T_w > T_\infty\). The magnetic field of strength \(B_m\)is applied upright to the stretchy surface. It is assumed that there is a first-order chemical reaction inside the fluid. The electric and induced magnetic fields are neglected. The governing equations for the present problem are
\[ u_x + u_y = 0 \]  
\[ uu_t + vv_y + \omega_1 (u^2 u_{xx} + v^2 u_{xx} + 2uv u_{xy}) = \mu u_{yy} - \mu \omega_2 (u u_{xy} + v u_{yy} - u_x u_{yy} - u_y v_{yy}) - \frac{\sigma R_d}{\rho} (u + \omega_1 u y) + g \beta [\beta_r(T - T_w) + \beta_C(C - C_\infty)] \]  
\[ (u T_x + v T_y) = \frac{k}{\rho c_p} T_{yy} + \frac{q}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} q_y \]  
\[ u C_x + v C_y = D_m C_{yy} - k_m (C - C_x) \]  

Then, the boundary conditions are
\[ u = u_w = n x + \epsilon_1 u_y, \ v = 0, T = T_\infty, C = C_w + \epsilon_3 C_y \text{ at } y = 0 \]  
\[ u \rightarrow 0, \ v \rightarrow 0, \ C \rightarrow C_w, \ T \rightarrow T_\infty \text{ as } y \rightarrow \infty \]  

where \((u, v)\) are the velocity components in the \(x\) \& \(y\)-directions, respectively, \(\epsilon_1, \epsilon_2 \& \epsilon_3\) are the velocity, temperature and concentration slip constants, respectively, \(\mu\) the kinematic viscosity, \(\rho\) the density, \(\omega_1\) the relaxation time, \(\omega_2\) the retardation time, \(\sigma\) the electrical conductivity, \(C_p\) specific heat, \(D_m\) the diffusion coefficient and \(k_m\) the first order chemical reaction. Using Cattaneo-Christov heat flux theory, we obtain the following energy equations
\[ u T_x + v T_y + \tau (u^2 T_{xx} + v^2 T_{yy} + (u u_x + v u_y) T_x + 2uv T_{xy}) \]  
\[ + (u u_x + v u_y) = \frac{k}{\rho c_p} T_{yy} + \frac{q}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} q_y \]  

Consider the following similarity transformations
\[ \eta = \frac{n}{\sqrt{\mu}} y, \ u = n x F'(\eta), \ v = -\sqrt{\mu} F(\eta), \ \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_w}, \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \]  

Apply equation (7) in equations (2), (4) and (6) we have
\[ F'' + FF' - F^2 + \alpha (FF'' - F^2 F^\prime) + \beta (F^2 - FF^w) - M (F' - \alpha FF^\prime) + \lambda (\theta + N \phi) = 0 \]  
\[ (1 + \frac{4}{3} Rd) \theta'' + Pr F \theta' - Pr \gamma (F^2 \theta'' + FF' \theta') + H g \theta = 0 \]  
\[ \frac{1}{Sc} \phi'' + F \phi' - C r \phi = 0 \]  

with boundary conditions
\[ F(\eta) = 0, F'(\eta) = 1 + \Gamma_1 F''(0), \ \theta(\eta) = 1 + \Gamma_2 \theta'(0), \ \phi(\eta) = 1 + \Gamma_3 \phi'(0), \text{ at } \eta = 0 \]  
\[ F'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \]  

Here we declare the non-dimensional variables as follows \(\alpha = \omega_1 n\) and \(\beta = \omega_2 n\) are the relaxation time \& retardation time constants respectively, \(\lambda = \frac{G_{r_s}}{R_d}\), the mixed convection constant, \(N = \frac{\beta \lambda (C - C_\infty)}{\beta_T (T - T_m)}\) the buoyancy ratio constant, \(M = \sigma B_m^2 / \rho n\) the Hartmann number, \(Pr = \rho c_p / k\) the Prandtl number, \(Rd = (4 \sigma T_s^2) / (k k^*)\) the radiation constant, \(\gamma = \tau n\) the non-dimensional thermal relaxation time,
\( Cr = \frac{k_m}{n} \) the chemical reaction constant, \( Sc = \mu/D_m \) the Schmidt number, \( \Gamma_1, \Gamma_2, \Gamma_3 \) are the slip constants for velocity, temperature and solute.

The dimensionless forms of skin friction, local heat and mass transfer rates are represented below

\[
Re \frac{1}{2} C_f = \frac{1+\alpha}{1+\beta} F'(0), \quad Re \frac{1}{2} Nu_x = -\left(1 + \frac{4}{3} Rd \right) \theta'(0), \quad Re \frac{1}{2} Sh_x = -\phi'(0)
\]

3. Method of solution

We incorporated the homotopy analysis method (HAM) in order to get the convergent solution of the system of equations. The initial approximations can be put in the form \( F_0 = \frac{1-e^{-\eta}}{1+\Gamma_1}, \quad \theta_0 = \frac{e^{-\eta}}{1+\Gamma_2} \) and \( \phi_0 = \frac{e^{-\eta}}{1+\Gamma_3} \) the auxiliary linear operators \( L_F, L_\theta \) & \( L_\phi \) are derived as \( L_F = F''(\eta) - F'(\eta), \quad L_\theta = \theta''(\eta) - \theta'(\eta), \quad L_\phi = \phi''(\eta) - \phi'(\eta) \) with satisfying the following properties \( L_F [X_1 + X_2 e^\eta + X_3 e^{-\eta}] = 0, \quad L_\theta [X_4 e^\eta + X_5 e^{-\eta}] = 0 \) and \( L_\phi [X_6 e^\eta + X_7 e^{-\eta}] = 0, \) where \( X_j (j = 1 - 7) \) denote the arbitrary conditions. \( F_n(\eta) = F_n^0(\eta) + X_1 + X_2 e^\eta + X_3 e^{-\eta}, \quad \theta_n(\eta) = \theta_n^0(\eta) + X_4 e^\eta + X_5 e^{-\eta} \) and \( \phi_n(\eta) = \phi_n^0(\eta) + X_6 e^\eta + X_7 e^{-\eta} \), where \( F_n^0(\eta) \), \( \theta_n^0(\eta) \) and \( \phi_n^0(\eta) \) are the appropriate solutions.

The auxiliary constants \( h_F, h_\theta \) & \( h_\phi \) play an vital part for convergence series solutions. The \( h \)-fields are drawn at 15\textsuperscript{th} order of similarities to accomplish valid ranges of constants (see Fig. 1). The admissible values of \( h_F, h_\theta \) & \( h_\phi \) are \(-1.5 \leq h_F, h_\theta \leq -0.3, \quad -1.6 \leq h_\phi \leq -0.4 \). More over the solutions converge in the total range of \( \eta \). When \( h_F = h_\theta = h_\phi = -0.7 \). Table.1 displays the order of similarities of HAM solutions.

![Fig.1. h-curves for \( h_F, h_\theta \) and \( h_\phi \)](image)

4. Results and discussion

The calculations are carried out to obtain approximate solutions for various physical constants in velocity, concentration and temperature fields for the fixed values of \( M = 0.5, \quad \beta = 0.2, \quad \alpha = 0.1, \quad \lambda = 0.2, \quad N = 1.0, \quad \gamma = 0.5, \quad Pr = 1.0, \quad Sc = 0.9, \quad Rd = 0.3, \quad Cr = 1.0, \) and \( \Gamma_1 = \Gamma_2 = \Gamma_3 = 1.0 \).

Figs. 2(a-d) show that the velocity for different values of velocity slip (\( \Gamma_1 \)) mixed convection (\( \lambda \)) and buoyancy ratio (\( N \)) constants. Fig. 2a depicts the effect of velocity slip constant on the velocity field. There is a decrement in velocity field as increasing the values of \( \Gamma_1 \). It is observed that slip velocity diminishes the velocity. Fig. 2b illustrates that the velocity enhances when increasing the mixed convection constant \( \lambda \). The buoyancy force boosted up on increasing the mixed convection constant and
then it results the increment in velocity. Fig. 2c reports that the velocity enhances on increasing the values of $N$.

It is observed from the Fig. 3a that temperature field diminishes as thermal slip constant $\Gamma_2$ increases. As $\lambda$ rises, the temperature decreases. Figs. 3c & 3d elucidates the effects of radiation and heat generation constants. It is observed that an increase in thermal boundary layer and temperature of the fluid is observed on increasing these constants. Further scrutinizing these figures, the heat generation effect is more pronounced on temperature distribution.

The concentration field decreases when the solutal slip constant $\Gamma_3$ increases, see Fig. 4a. Fig. 4b presents the effects of chemical reaction constant on concentration field. It shows that concentration reduces on rising the values of chemical reaction constant.

Fig. 5 displays the various combination of physical constants on local skin friction ($C_{fx}$), local heat ($Nu_x$) and mass ($Sh_x$) transfer rates. Fig. 5a indicates local skin friction augments when increasing the value of velocity slip $\Gamma_1$ and mixed convection constant $\lambda$. Figs. 5b & 5c reports local mass and heat transfer rates diminish when increasing the value of slip constants. It is also observed that the chemical reaction enhances the mass transfer rate.

Fig. 2 Velocity field $\left(F'(\eta)\right)$ for various values of $\Gamma_1$, $\lambda$ and $N$. 

(a) $\Gamma_1 = 0.0, 0.1, 0.2, 0.3$

(b) $\lambda = 0.0, 0.3, 0.6, 0.9$

(c) $N = 0.0, 2.0, 4.0, 6.0$
Fig. 3 Temperature field $\theta(\eta)$ for various values of $\Gamma_2$, $\lambda$, $R_d$ and $Hg$.

Fig. 4 Concentration field $\phi(\eta)$ for various values of $\Gamma_3$ and $Cr$. 
Fig. 5: Effects of $\Gamma_1$ vs $\lambda$, $\Gamma_2$ vs $M$ and $\Gamma_3$ vs $Cr$ on local skin friction ($Cf_x$), local Nusselt number ($Nu_x$) and local Sherwood number ($Sh_x$).

Table 1: Order of approximations when $M = 0.5$, $\beta = 0.2$, $\alpha = 0.1$, $\gamma = 0.5$, $Pr = 1.0$, $Sc = 0.9$, $Rd = 0.3$, $Hg = -0.5$, $Cr = 1.0$, $\lambda = 0.2$, $N = 1.0$, $\Gamma_1 = \Gamma_2 = \Gamma_3 = 1.0$ and $h = -0.7$.

| Order of approximation | $-F''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|------------------------|-----------|---------------|--------------|
| 1                      | 0.45807   | 0.43583       | 0.50486      |
| 5                      | 0.44525   | 0.41341       | 0.51082      |
| 11                     | 0.44533   | 0.41339       | 0.51081      |
| 20                     | 0.44533   | 0.41339       | 0.51081      |
| 30                     | 0.44533   | 0.41339       | 0.51081      |
| 40                     | 0.44533   | 0.41339       | 0.51081      |

5. Final remarks
A 2D mixed double diffusive convective flow of an Oldroyd-B fluid towards a linearly stretchy surface is examined. Approximate solutions are presented which depends on mixed convection, radiation, chemical reaction, buoyancy ratio, velocity, thermal and concentration slip constants. The observations of the present study are as follows:

- The slip boundary conditions diminish the velocity, temperature and concentration fields.
- The thermal boundary layer thickness enhances by raising the values of radiation and heat generation constants.
The concentration diminishes on raising the chemical reaction.

The mass and heat transfer rates diminish on raising the slip constant values and the opposite trend is found on skin friction with velocity slip.

6. References

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