Signature of tilted Dirac cones in Weiss oscillations of $8 - Pmnm$ borophene

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Polymorph of $8 - Pmnm$ borophene exhibits anisotropic tilted Dirac cones. In this work, we explore the consequences of the tilted Dirac cones in magnetotransport properties of a periodically modulated borophene. We evaluate modulation induced diffusive conductivity by using linear response theory in low temperature regime. The application of weak modulation (electric/magnetic or both) gives rise to the magnetic field dependent non-zero oscillatory drift velocity which causes Weiss oscillation in the longitudinal conductivity at low magnetic field. The Weiss oscillation is studied in presence of an weak spatial electric, magnetic and both modulations individually. The tilting of the Dirac cones gives rise to additional contribution to the Weiss oscillation in longitudinal conductivity. Moreover, it also enhances the frequency of the Weiss oscillation and modifies its amplitude too. Most remarkably, It is found that the presence of out-of phase both i.e., electric and magnetic modulations can cause a sizable valley polarization in diffusive conductivity. The origin of valley polarization lies in the opposite tilting of the two Dirac cones at two valleys.

I. INTRODUCTION

In recent times, Dirac materials have attracted intense research interests after the most celebrated discovery of atomically thin two dimensional (2D) hexagonal carbon allotrope-graphene$^{1,2}$, owing to their peculiar band structure and applications in next generation of nanoelectronics. The polymorph of borophene with tilted anisotropic Dirac cones (named as $8 - Pmnm$ borophene)$^3$ is the latest 2D material to the family of Dirac systems. Very recently, experimental confirmation of such material has been reported$^4$, followed by a detailed analysis of its ab-initio properties$^5$. Similar to the strained graphene$^6$, a pseudo magnetic field has been recently predicted in $8 - Pmnm$ borophene by using tight-binding model$^7$. An effective low energy Hamiltonian in the vicinity of Dirac points has been proposed based on symmetry consideration$^7$, which has recently been used to investigate collective excitations (plasmons)$^8$ and optical properties$^9$ theoretically.

Magnetotransport properties have always been appreciated for providing a powerful and experimentally reliable tool to probe a 2D fermionic system. The presence of magnetic field, normal to the plane of the 2D sheet of electronic system, discretizes the energy spectrum by forming Landau levels (LLs) which manifests itself via oscillatory longitudinal conductivity in inverse magnetic field-known as Shubnikov-de Hass (SdH) oscillation$^{10,11}$. In addition to the SdH oscillation, another type of quantum oscillations appears in low magnetic field regime when the 2D fermionic system is subjected to an weak spatial electric/magnetic modulation. This oscillation is known as Weiss oscillation which was first observed in magnoeto-resistance measurements in the electrically modulated usual two dimensional electron gas (2DEG)$^{12-14}$. The Weiss oscillation is also known as Commensurability oscillation as it is caused by the commensurability of the two length scales i.e., cyclotron orbit’s radius near the Fermi energy and the modulation period$^{15-17}$. An alternative explanation was also given by Beenakker$^{18}$ by using the concept of guiding-center-drift resonance between the periodic cyclotron orbit motion and the oscillating drift of the orbit center induced by the potential grating.

Apart from the electric modulation case, magnetic modulation has also been considered theoretically$^{19-25}$ as well as experimentally$^{26-28}$. Weiss oscillation has been studied in Rashba spin-orbit coupled electrically/magnetically modulated 2DEG and beating pattern was predicted$^{29,30}$. The higher Fermi velocity associated with the linear band dispersion significantly enhances the Weiss oscillation in an electrically modulated graphene$^{31,32}$. Concurrently, same has been studied in a magnetically modulated graphene too and enhancement of the amplitude and opposite phase in comparison to the case of electrically modulated graphene was observed$^{33}$. Similar investigations have been carried out in electrically modulated bilayer graphene$^{34}$, silicene$^{35,36}$, $\alpha - T_3$-lattice$^{37}$ and phosphorene$^{38}$. However, magnetotransport properties of modulated borophene are yet to be explored.

In this work, we investigate the modulation induced longitudinal conductivity of borophene in low temperature regime by using the linear response theory. First, we obtain exact LLs and corresponding density of states (DOS) in $8 - Pmnm$ borophene. Numerically, we notice that the tilting of the Dirac cones lowers the Fermi level. We observe modulation induced Weiss oscillation in the longitudinal conductivity in low magnetic field regime. Interestingly, we find that the opposite tilting of the Dirac cones at two valleys can cause sizable valley polarization in the longitudinal conductivity at low magnetic field regime under the combined effects of out-of phase electric and magnetic modulation, which is in contrast to the non-tilted isotropic Dirac cones-graphene. Moreover, the tilting of the Dirac cones also enhances the frequency of Weiss oscillation.

The paper is organized as follows. In Sec. II, we introduce the low energy effective Hamiltonian and derive LLs. The effect of tilting of Dirac cones on the Fermi level and DOS are also included in this section. The Sec. III
devoted to calculate the modulation induced Weiss oscillation in the longitudinal conductivity. Finally, we summarize and conclude in Sec. IV.

II. MODEL HAMILTONIAN AND LANDAU LEVEL FORMATION

We start with the single particle low energy effective model Hamiltonian of the tilted anisotropic Dirac cones as \(^7,8\)

\[
H = \xi(v_{px}p_x\sigma_x + v_{py}p_y\sigma_y + v_t p_y\sigma_0),
\]

where the first two terms corresponds to the kinetic energy term and the last term describes the tilted nature of Dirac cones. The two Dirac points are at \(k = \pm \mathbf{k}_D\), described by the valley index \(\xi = \pm\). Hereafter, we shall denote two valleys as \(K\) and \(K'\), corresponding to \(\xi = +\) and \(\xi = -\), respectively. Three velocities are given by \(\{v_x, v_y, v_t\} = \{0.86, 0.69, 0.32\}\) in units of \(v_0 = 10^6\) m/s. The velocity \(v_t\) arises due to the tilting of the Dirac cones. Also, \((\sigma_x, \sigma_y)\) are the Pauli matrices and \(\sigma_0\) is identity matrix. The energy dispersion of the above Hamiltonian can be readily obtained as

\[
E_{\xi, k} = \xi hv_tk_y + \lambda \sqrt{v_x^2k_x^2 + v_y^2k_y^2},
\]

where \(\lambda = \pm\) is the band index and the 2D momentum vector is given by \(k = \{k_x, k_y\}\). The energy dispersion for \(K\)-valley is shown in Fig. (1), which is tilted along \(k_y\) due to the presence of \(v_t\) term. In \(K'\)-valley, dispersion will be identical except the tilting is in opposite direction. In addition to this, Dirac cones are anisotropic which is in contrast to graphene. Note that because of the tilted feature of the Dirac cones, particle-hole symmetry is broken in borophene.

1. Inclusion of magnetic field

The perpendicular magnetic field \((B = B\hat{z})\) is incorporated via the Landau-Peierls substitution as: \(\mathbf{p} \rightarrow \mathbf{p} + eA\) in low energy single electron effective Hamiltonian of borophene, lying in the \(x-y\) plane, as

\[
\mathcal{H} = \xi[v_{px}p_x\sigma_x + v_{py}(p_y + eBx)\sigma_y + v_t(p_y + eBx)\sigma_0],
\]

under the Landau gauge \(A = (0, xB, 0)\). Here, \(A\) is the magnetic vector potential. The commutator relation \([\mathcal{H}, p_y] = 0\) guarantees the free particle nature of electron along the \(y\)-direction. Using this fact, the above Hamiltonian reduces to

\[
\mathcal{H} = \xi \left\{ \frac{\hbar v_t}{l_c} X\sigma_0 + \frac{\hbar v_c}{l_c} \left[ \sqrt{\frac{v_x}{v_y}} \sigma_x P + \sqrt{\frac{v_y}{v_x}} \sigma_y X \right] \right\},
\]

where \(l_c = \sqrt{\hbar/eB}\) is the magnetic length, \(P = -i\partial/\partial(x/l_c)\), \(v_c = \sqrt{v_x^2/v_y}\) and \(X = (x + x_0)/l_c\) with the center of cyclotron orbit is at \(x = -x_0 = -k_0l_c^2\). The above Hamiltonian is now similar to the case of monolayer graphene under crossed electric and magnetic field\(^{39}\) except the velocity anisotropy inside the third bracket. The first term is analogous to a pseudo in-plane effective field \(E_{eff} = \xi hv_t/\sqrt{(e\hat{L})^2}\). The typical values of the pseudo-electric field are \((320 \times B)\) kV. Now Eq. (4) can be re-written as

\[
\mathcal{H} = eE_{eff}(x + x_0)\sigma_0 + \xi \hbar \omega_c \left[ \begin{array}{cc} 0 & -ia \\ ia^\dagger & 0 \end{array} \right],
\]

where \(\omega_c(v_c/l_c)\) is the cyclotron frequency and ladder operators are defined as: \(a = (\hat{X} + i\hat{P})/\sqrt{2}\) and \(a^\dagger = (\hat{X} - i\hat{P})/\sqrt{2}\). Here, \(\hat{X} = \sqrt{v_x/v_y} X\) and \(\hat{P} = \sqrt{v_x/v_y} P\), which satisfy the commutator relation \([\hat{X}, \hat{P}] = i\). In absence of \(E_{eff}\), the above Hamiltonian can be diagonalized to obtain graphene-like LLs

\[
E_{\zeta} = \lambda \hbar \omega_c \sqrt{2n}
\]

and eigenfunctions as

\[
\psi_{\zeta}(\mathbf{r}) = \frac{e^{ik_y y}}{\sqrt{2L_y}} \left[ \begin{array}{c} \lambda \phi_n(X) \\ -i\xi \phi_{n-1}(X) \end{array} \right],
\]

where \(\zeta = \{n, k_y\}\) and \(\phi_n(X)\) is the well known simple harmonic oscillator wave functions. In presence of \(E_{eff}\), direct diagonalization of the above Hamiltonian is difficult. However, there is a standard way, given by V. Lukose et al. in Ref.\(^{39}\), to solve this problem exactly. An alternative approach of solving this problem in graphene was also given by NMR Peres et al.\(^{40}\). Following Ref.\(^{39}\), we transform the above Hamiltonian into a moving frame along the \(y\)-direction with velocity \(V = E_{eff}/B = v_t\) where the transformed electric field vanishes and magnetic field reduces to \(B' = B\sqrt{1 - \beta_B^2}\). Here, \(\beta_B = v_t/\sqrt{v_x v_y} (= 0.4154)\) is termed

![FIG. 1. (Color online) The energy band dispersion in k-space representing Eq. (1). The momentum vectors are normalized by \(k_0 = 10^6/m\).](image-url)
as “tilt parameter”. Note that the role of velocity of light is played by $v_F$ in borophene whereas in graphene it is $v_F$. In the moving frame, LLs can be obtained as $\tilde{E}_n, \tilde{\beta}_g = \hbar \omega_c \sqrt{2n(1 - \beta_g^2)}^{3/4}$. However, required LLs and eigen states in the rest frame can be obtained by Lorentz boost back transformation as $^{39, 41}$:
$$E'_n = \lambda \hbar \omega_c \sqrt{2n(1 - \beta_g^2)}^{3/4},$$  
(8)
where the argument of the wave functions becomes
$$X' = \frac{(1 - \beta_g^2)^{1/4}}{\lambda_c} \left[ x + k_y l_c^2 + \lambda \sqrt{2n \lambda c \beta_g} (1 - \beta_g^2)^{1/4} \right],$$  
(9)
after using the Lorentz back transformation of momentum, giving the wave function in rest frame as
$$\Psi_n(x) = \frac{e^{ik_y y}}{\sqrt{2L_y \gamma}} \left( \begin{array}{c} \cosh(\theta/2) \\ -i \sinh(\theta/2) \end{array} \right) \lambda \phi_n(X') - i \xi \left[ \begin{array}{c} i \sinh(\theta/2) \\ \cosh(\theta/2) \end{array} \right] \phi_{n-1}(X').$$  
(10)
with $\tanh \theta = \beta_g$ and $\cosh \theta = \gamma$. Here, we have used the form of hyperbolic rotation matrix as
$$e^{-\left(\frac{x-y}{z}\right)s_g} = \left[ \begin{array}{cc} \cosh(\theta/2) & i \sinh(\theta/2) \\ i \sinh(\theta/2) & \cosh(\theta/2) \end{array} \right].$$  
(11)

On the other hand, the LLs of graphene under the in-plane real electric field ($E_x$) is given by $^{39}$
$$E'_x = \lambda \hbar \omega_c \sqrt{2n(1 - \beta_g^2)}^{3/4} - \hbar k_y \frac{E_x}{B},$$  
(12)
where $\beta_g = \frac{E_x}{B}$ with $v_F$ is the Fermi velocity. Note that in cyclotron frequency, $\omega_c$, should be replaced by $v_F$ in graphene. In Eq. (8), $\beta_g$ is a constant and acting like a system parameter, whereas $\beta_g$ is tunable and governed by the strength of the in-plane electric field in graphene. As the tilt parameter ($\beta_g$) is constant, the LLs are protected from being collapsed in borophene which is in contrast to graphene where LLs may get collapsed under the suitable strength of the electric field (i.e., when $\beta_g$ becomes 1). Note that the LLs in boronene, derived in Eq. (8), exhibit $k_y$ degeneracy whereas in graphene [see Eq. (12)] this degeneracy is removed under the influence of an in-plane electric field. This is because the in-plane electric field in graphene gives rise to the potential energy as $eE_x x$ whereas in boronene for pseudo electric field it is $eE_x f(x + x_0)$. The idea of relativistic Lorentz boost transformation was also used in an organic compound $\alpha - (BEDT - TTF)_2 I_3$ $^{42}$, exhibiting quite similar band structure.

The LLs, derived in Eq. (8), show that the tilt parameter ($\beta_g$) renormalizes each LLs, which should be reflected in the longitudinal conductivity oscillations. Before going into conductivity, we now discuss how Fermi energy and DOS are affected by the tilting of the Dirac cones.

2. Fermi energy and density of states

In this subsection, we compute the Fermi energy ($E_F$) and DOS in terms of the tilt parameter and the magnetic field. In presence of the magnetic field, the Fermi energy can be obtained by solving the following equation self consistently
$$n_e = \int_{-\infty}^{\infty} D(E) f(E) dE,$$  
(13)
where
$$D(E) = \frac{g_s g_v}{\Omega} \sum_n \delta(E - E_n),$$  
(14)
is the DOS per unit energy and per unit area. Here, $g_s$ and $g_v$ are the spin and valley degeneracy, respectively. Carrier density and the area of the system are denoted by $n_e$ and $\Omega(= L_x L_y)$, respectively. The Fermi distribution function is given by $f(E) = (1 + \exp[(E - E_F)/k_B T])^{-1}$. The summation over $k_y$ can be computed by using the fact that the center of cyclotron orbit is always restricted by the system dimension i.e., $0 \leq |x_0 + G_n| \leq L_x$ or $0 \leq k_y \leq L_x/L_y^2$. Then we can replace $\sum_{k_y} \rightarrow \frac{L_x}{2\pi} \int_{L_y^2}^{0} dk_y = \Omega/(2\pi L_y^2)$ known as $k_y$-degeneracy. The factor $L_y/(2\pi)$ preserves periodic boundary condition. Using these, finally Eq. (13) simplifies to
$$\pi n_e L_y^2 = 2 \sum_n f(E_n),$$  
(15)
which is solved numerically to plot the Fermi energy as a function of the magnetic field in Fig. (2). Here we have also substituted spin and valley degeneracy as $g_s = 2$ and $g_v = 2$, respectively. In the same plot, first twenty LLs

![FIG. 2. (Color online) The behavior of the Fermi energy and the LLs with magnetic field. For the plot of $E_F$ versus B, we use $T = 1$ K and carrier density $n_e = 10^{15} m^{-2}$. Green and grey solid lines denote first 20 LLs with and without $v_t$, respectively.](image-url)
are also shown. The Fermi level is found to be fluctuating between two successive LLs with the variation of the magnetic field. The amplitude of fluctuation increases with the increase of the magnetic field, because of the increasing LLs spacing. To understand how the tilting feature of the Dirac cones affects the Fermi energy, we consider the two cases i.e., when $v_t = 0$ and $v_t = 0.32$ unit. It can be seen that for the same carrier density, tilt factor ($v_t$) actually lowers the Fermi level. On the other hand, it causes a shift in the LLs as can be seen from Eq. (8). Note that the position of the jumping of Fermi level between two successive Landau levels remains unchanged in both cases.

Now we will examine the effects of the tilting of the Dirac cones on the behavior of the DOS in borophene. To plot the behavior of DOS, we assume impurity induced Gaussian broadening of the LLs and hence Eq. (14) reduces to

$$D(E) = D_0 \sum_n \exp \left[ \frac{(E - E_n)^2}{2\Gamma_0^2} \right]$$

where

$$D_0 = \frac{g_s g_v}{2\pi l_c^2} \frac{1}{\Gamma_0 \sqrt{2\pi}}.$$  

The DOS is plotted in Fig. (3) by using Eq. (16). It is an established fact that the impurity induced LLs broadening in 2D Dirac material is directly proportional to $\sqrt{B}$. To plot dimensionless DOS, we consider LLs broadening width $\Gamma_0 = 0.1\hbar\omega_c$. To explore the effects of the tilted Dirac cone, we consider the both cases i.e., with and with out $v_t$. The DOS shows oscillation with the magnetic field-known as SdH oscillation. The presence of $v_t$ is causing a significant impact on the frequency of the SdH oscillations. It is also observed that below a certain magnetic field, the SdH oscillation vanishes because of the reduction in the LLs spacing and overlapping of the LLs to each other due to the impurity induced broadening.

### III. MAGNETOCONDUCTIVITY

In this section, we evaluate magnetoconductivity in the presence of a periodic electric/magnetic modulation at low magnetic field regime. In the presence of a spatial electric/magnetic modulation along the $x$-direction, electron gains a finite drift velocity along the $y$-direction for which an additional contribution to the $y$ component of the longitudinal conductivity appears, known as diffusive conductivity, i.e., $\sigma_{yy} = \sigma_{yy}^{\text{diff}} + \sigma_{yy}^{\text{col}}$. Here, $\sigma_{yy}^{\text{col}}$ is the collisional conductivity which arises due to LL induced oscillatory DOS without any external modulation whereas $\sigma_{yy}^{\text{diff}}$ is the diffusive conductivity arises because of the modulation. On the other hand, the longitudinal conductivity along the $x$-direction is $\sigma_{xx}^{\text{col}}$ because $\sigma_{xx}^{\text{diff}} = 0$. However, in this work our major focus will be modulation induced diffusive conductivity which can be evaluated by

$$\sigma_{\mu\nu}^{\text{diff}} = \frac{\beta e^2}{\Omega} \sum_{\xi} f_\xi (1 - f_\xi) \tau(E_\xi) V_\mu V_\nu$$

provided the scattering processes are elastic or quasi elastic. Here, $f_\xi = [1 + \exp(-\beta(E_\xi - E_F))]^{-1}$ is the Fermi-Dirac distribution function with $\beta = (k_B T)^{-1}$ where $k_B$ is the Boltzmann constant. In the above formula, $\tau(E_\xi)$ denotes the energy dependent collision time and the group velocity $V_\mu(v) = (1/\hbar)\partial E_\xi/\partial k_\mu(v)$. In general, electron does not possess any non-zero drift velocity inside the bulk i.e., $V_x = V_y = 0$. However, the application of a spatial electric/magnetic modulation can induce a non-zero finite drift velocity and concurrently gives rise to the diffusive conductivity as discussed below.

#### A. electric modulation

The application of a weak electric modulation to the borophene sheet is described by the total Hamiltonian $\mathcal{H}_F = \mathcal{H} + V_e \sin(\Delta x)$, where $V_e$ is the modulation strength and $\Delta = 2\pi/a$ with $a$ is the period. Using perturbation theory, we evaluate the first order energy correction as

$$\Delta E_\xi^e = \int_0^{L_x} dy \int_{-\infty}^{\infty} \Psi_\xi^\dagger(r)V_e \sin(\Delta x)\Psi_\xi(r)dx$$

$$= V_e \left[ \xi \beta_0 R_n(u) \cos(\Delta x_0) - F_n(u) \sin(\Delta x_0) \right].$$

Here $x_0 = x_0 + G_n$ with $G_n = \sqrt{2\pi \gamma_c \beta_k}$. Also,

$$F_n(u) = e^{-u^2/2}[L_{n-1}(u) + L_n(u)]$$

where $L_n(u)$ are the Laguerre polynomials.
The presence of modulation broadens the LLs width by

\[ E_u \approx \frac{\Gamma_0}{\gamma} \left( 1 - \tilde{\beta}_b \right)^{1/2} \]

with the inverse magnetic field, as the Laguerre polyno-
mial exhibits oscillatory feature. Note that the first term
in Eq. (19) is purely due to the tilting of the Dirac cones
with the inverse magnetic field. The Laguerre poly-
nomial exhibits oscillatory feature. Now, the drift velocity
\[ V_y = \frac{V_e u}{h \Lambda} \left[ \beta_b R_n(u) \sin(\Delta x_0) + F_n(u) \cos(\Delta x_0) \right] \]

and \( V_x = 0 \) which suggests that the diffusive conductivity
arises along the transverse direction to the applied mod-
ulation. Now, after inserting \( V_y \) into Eq.(18), we obtain
diffusive conductivity as

\[ \sigma_{yy} = \frac{e^2 V_e^2 u}{h 4 \Gamma_0} \gamma \sum_n \left[ \frac{\partial f_n}{\partial E} \right] \left\{ [F_n(u)]^2 + [\beta_b R_n(u)]^2 \right\} \]

where \( \Gamma_0 \) is the impurity-induced broadening. Here, we
assume that the collisional time \( \tau(E_c) \) is very insensitive
to the energy i.e., \( \tau(E_c) \approx \tau_0 \) which is a good approx-
imation under low magnetic field regime. We have also
substituted \( \Gamma_0 \approx h/\tau_0 \). The major effects of modula-
tion arise via non-zero drift velocity. On the other hand,
modulation effect on Fermi distribution function is very
small, and hence we ignore it. The diffusive conductivity
in the above Eq. (23) is oscillatory with magnetic field
because of the oscillatory nature of the bandwidth (\( \Delta_c \)).
This oscillation is known as Weiss oscillation.

For the numerical plots, we use the following physical parameters: modulation period \( a = 350 \) nm, charge
density \( n_e = 10^{13} m^{-2} \), and temperature \( T = 3 \) K. The
diffusive conductivity for the electric modulation is pla-
ted with inverse magnetic field in Fig. (4)a. To explore
the effects of the tilted Dirac cones, we consider both sit-
uations i.e., \( \nu_t = 0 \) and \( \nu_t = 0.32 \) unit. The diffusive con-
ductivity exhibits the Weiss oscillation at low magnetic
field regime with the inverse magnetic field. However,
with the increase of the magnetic field, SDH oscillations
start to superimpose over the Weiss oscillation. The SDH
oscillations appear as small oscillation over the envelope
of the Weiss oscillation. The tilted Dirac cones cause a
significant changes in the frequency of the Weiss oscilla-
tion too. To understand the effect of tilting Dirac cones
more explicitly, we shall obtain an approximate analytical
expression of the diffusive conductivity. Following the Refs.[31 and 33], the diffusive conductivity for electric
modulation can be simplified to an analytical form by
using the higher Landau level approximation

\[ e^{-u/2} L_n(u) \rightarrow \frac{1}{\sqrt{\pi \sqrt{nu}}} \cos \left( 2\sqrt{nu} - \frac{\pi}{4} \right) \]

as

\[ \sigma_{yy} \approx \frac{e^2}{h 8 \pi^2 \gamma \Gamma_0} \left\{ U_0^c - U_1^c R_0^W \left( \frac{T}{T_W} \right) + 2 R_1^W U_1^c \left( \frac{T}{T_W} \right) \right\} \]

\[ \cos^2 \left[ 2\pi \left( \frac{f}{B} - \frac{1}{8} \right) \right] \}

Here, \( U_0^c = (V_e)^2 (1 + 2\beta_b^2) \) and \( U_1^c = V_e^2 (2\beta_b^2 - 1) \). The amplitude of the conductivity is governed by \( U_0^c \) which
indicates that it enhances with the tilting of Dirac cones.

On the other hand, the Weiss oscillation amplitude is
due to magnetic modulation. Let the perpendicular monolayer graphene (as mentioned in the Sec. I). Along field and low temperature regime, extensive theoretical bation which can be written as
\[ H = \left(A \right) \sinh(\Lambda) \]. Similar to the case of electrical modulation, the terms involv-
\[ \Delta_e = \frac{\text{e}^2}{\hbar \gamma} \Lambda \sum_n \left[ \left\{ V_{ct} F_n(u) \right\}^2 + \left\{ V_{tc} R_n(u) \right\}^2 \right]. \] (31)
with \( U_0 = (V_{ct}^c)^2 + (\sqrt{2}V_{tc}^c)^2 \) and \( U_1 = (\sqrt{2}V_{tc}^c)^2 - (V_{ct}^c)^2 \). The amplitude of the Weiss oscillations is now the factor \( U_1 \) and it is suppressed in presence of the tilt induced term \( V_{ct}^c \). The tilt induced additional contribution to the diffusive conductivity does not appear in non-tilted Dirac material graphene.

The diffusive conductivity for magnetic modulation is plotted numerically in Fig. (4)b, where Weiss oscillation is found to be weakly suppressed. The origin of this suppression can be understood from the analytical expression of diffusive conductivity given in Eq. (31). The presence of the tilt induced term \( V_{ct}^c \) reduces the amplitude of the oscillation, governed by \( U_1 \). Note that we have taken the strength of magnetic modulation \( B_m = 0.028 \text{ T} \), such that \( V_c = V_{ct}^c = 0.5 \text{ meV} \) and \( V_{tc}^c = 1.2 \text{ meV} \). Note that, in comparison to the case of electrical modulation, the amplitude of Weiss oscillation is enhanced.

C. presence of both modulation

Now we consider the situation when both types of modulation i.e., electric and magnetic are present together. The presence of both modulations may give rise to some new features to the Weiss oscillation. In usual 2DEC, it was found that Weiss oscillation can be pronounced or suppressed depending on whether both modulations are in phase or out of phase. Recently, we have observed that the presence of both modulations can break particle-hole symmetry in Dirac materials like graphene and \( \alpha - T_3 \) lattice. First we consider that the electric and mag-
of electric/magnetic modulation, we obtain the valley de-

\[ \sigma_{yy} \approx \frac{e^2}{\hbar} \frac{\beta_{W} u}{8 \gamma_1 \Gamma_0} \sum_n \left[ \frac{\partial f_n}{\partial E} \right] \left\{ [W_{1n}^{\xi} R_n(u)]^2 + [W_{2n}^{\xi} F_n(u)]^2 \right\}. \]  

(34)

The most remarkable point here is that now the diffusive conductivity is very sensitive to the valley index and can cause sizable valley polarization, which is attributed to the presence of the tilt induced term \( V^t_n \) and \( \beta_\gamma \). In non-tilded Dirac cones like graphene, valley polarization does not appear even in presence of the both modulations because of the absence of the term \( V^t_m \) and \( \beta_\gamma \). Using the similar approach as in the electric/magnetic modulation, analytical form of the diffusive conductivity can be obtained as

\[ \sigma_{yy}^\xi = \frac{e^2}{\hbar} \frac{\beta_{W} U_{0}^\xi}{16 \pi^2 \gamma_1 \Gamma_0} \left\{ 1 - \left( \frac{U_{1}^\xi}{U_{0}^\xi} \right)^2 R^W \left( \frac{T}{T_W} \right) \right\} \]

\[ + 2 \left( \frac{U_{1}^\xi}{U_{0}^\xi} \right)^2 R^W \left( \frac{T}{T_W} \right) \sin^2 \left[ 2\pi \left( 1 - \frac{1}{8} \right) \right], \]

(35)

where \( U_{0}^\xi = (W_{1}^\xi)^2 + (W_{2}^\xi)^2 \) and \( U_{1}^\xi = (W_{1}^\xi)^2 - (W_{2}^\xi)^2 \). The amplitude of the Weiss oscillation is determined by a valley dependent factor \( U_{1}^\xi \). In \( K \)-valley (\( \xi = + \)), the Weiss oscillation amplitude is much suppressed than \( K' \)-valley (\( \xi = - \)), which is shown in the lower panel of Fig. (5). The upper panel of this figure shows that when electric and magnetic modulations are applied individually, the Weiss oscillation in one valley (\( K \) or \( K' \)) exhibits opposite phase with an amplitude mismatch. The origin of the opposite phase is well addressed in Ref.[33]. However, when both modulations are applied together then two valleys respond differently. The Weiss oscillation amplitudes are enhanced in both valleys but the enhancement in \( K' \)-valley is much higher than \( K \)-valley, as shown in the lower panel of Fig. (5). These features can be understood from analytical expression of diffusive conductivity in Eq. (35). In \( K \)-valley, the amplitude of Weiss oscillation is determined by \( U_{1}^+ \) which is smaller than its counterpart in \( K' \)-valley i.e., \( U_{1}^- \).

On the other hand, if we consider that the both modulations are in the same phase i.e., electric modulation is \( V_r \cos(\Delta x) \) and the magnetic modulation is of the form of \( B_m \cos(\Delta x) \), then diffusive conductivity will be

\[ \sigma_{yy}^m = \frac{e^2}{\hbar} \frac{u}{4 \gamma_1 \Gamma_0} \sum_n \left[ - \frac{\partial f_n}{\partial E} \right] \left\{ [W_{3}(n, u)]^2 + [W_{4}(n, u)]^2 \right\}, \]

(36)

where \( W_3(n, u) = [V_r \beta_b R_n(u) + V_m^c F_n(u)] \) and \( W_4(n, u) = [V_m^c R_n(u) - V_c F_n(u)] \). In this case, diffusive conductivity does not depend on the valley index. A similar analytical expression can also be found by following the same approach. The Weiss oscillation for the presence of in-phase and out-of-phase both modulations are presented together in Fig. (6), which shows that the Weiss oscillations in both cases are in opposite phase with

FIG. 5. Diffusive conductivity (in units of \( e^2/\hbar \)) versus inverse magnetic field for electric and magnetic modulation (upper panel) and in presence of both out-of-phase modulations (lower panel) in each valley. The tilt velocity \( v_t = 0.32v_0 \) and Fermi energy is 0.035 eV. All other parameters are taken same as in Fig. (4).

FIG. 6. Diffusive conductivity (in units of \( e^2/\hbar \)) versus inverse magnetic field in presence of the both modulations. All other parameters are kept same as in the Fig. (5).
amplitude mismatch. This features are the direct consequences of the total effective energy correction in both cases. As we have seen that the diffusive conductivity may be sensitive to valley index depending on the phase relationship between electric and magnetic modulations, it is interesting to examine the valley polarization, for which we plot it versus inverse magnetic field in Fig. (7). Because of the unequal suppression of Weiss oscillations in two valleys in presence of the out-of-phase both modulations, a sizable valley polarization arises in the diffusive conductivity. To plot valley polarization, we define it as

$$P_v = \frac{\sigma_{yy}^+ - \sigma_{yy}^-}{\sigma_{yy}^+ + \sigma_{yy}^-}.$$  (37)

The valley polarization oscillates with the inverse magnetic field with the frequency of Weiss oscillation as shown in Fig. (7)a. Appearance of valley polarization strongly depends on the phase relationship between both modulations. Valley polarization appears in Weiss oscillation only when electric and magnetic modulations are in out of phase. It is also interesting to examine the evolution of valley polarization with $v_t$. In Fig. (7)b, we also show evolution of valley polarization in diffusive conductivity with the smooth variation of tilt velocity $v_t$. It shows that valley polarization oscillates with $v_t$, which can be easily understood from the fact that Weiss oscillation frequency has a strong dependency on $v_t$ too. The valley polarization shows emergence of regular peaks with increasing height towards 1 with the increase of $v_t$. We mention here that we have taken Fermi energy (0.035 eV) as constant while plotting the Fig. (7)b although a weak dependency of Fermi energy on $v_t$ exists as shown in Fig. (2). The rise of valley polarized Weiss oscillation in diffusive conductivity is one of our main results which differs from graphene. Here, we mention that the valley polarized Weiss oscillation was predicted in electrically modulated silicene too. However, in that case a gate voltage between two planes of sub-lattices is necessary in addition to the strong spin-orbit interaction.

Finally, we discuss if tilt parameter can be extracted from the Weiss oscillation experiment. The frequency of the Weiss oscillation can be easily obtained from magnetoresistance measurement of borophene, which can be directly used to extract the tilt parameter once we know the Fermi level and Fermi velocity. The direct method of obtaining Fermi level and velocity was recently reported in Ref.[47].

IV. SUMMARY AND CONCLUSIONS

In this work, we have studied magnetotransport properties of a 2D sheet of the polymorph of a periodically modulated $8$-$Pmmn$ borophene which exhibits tilted anisotropic Dirac cones. We have evaluated the modulation induced diffusive conductivity by using the linear response theory. The diffusive conductivity exhibits the Weiss oscillation with the inverse magnetic field, the frequency of which is enhanced by the tilted feature of the Dirac cones. The amplitude of Weiss oscillation is also enhanced or suppressed depending on the types of modulation. Most remarkably, we have found that the presence of out-of-phase electric and magnetic modulations can cause very high valley polarization in Weiss oscillation at low magnetic field. The appearance of the valley polarization in the Weiss oscillation is the direct manifestation of the tilted Dirac cones in borophene. It is in complete contrast to the non-tilted isotropic Dirac material-graphene where such valley polarization does not appear.

As far as the practical realization of this material is concerned, a borophene structure can be formed on the surface of Ag(111) as reported recently in Ref.[4]. On the other hand, periodic modulation can be imparted to the system by several methods. For example, an array of biased metallic strips on the surface of a 2D electronic system has been used by Winkler et al. to achieve electric modulation. Magnetic modulation can be generated by placing a few patterned ferromagnets or a superconductor on the surface of the 2D material26-28.

ACKNOWLEDGMENTS

SFI thanks to Tarun K. Ghosh and Tutul Biswas for useful comments. Authors also thank the anonymous referee for pointing out an error in the calculation. One of us AMJ thanks DST, India for J.C. Bose National Fellowship.

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FIG. 7. Valley polarization versus (a) inverse magnetic field and (b) tilt velocity ($v_t$). Fermi energy is kept fixed at $E_F = 0.035eV$. All parameters are kept same as Fig. (4).

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