On the Performance of the Energy Detector Subject to Impulsive Noise in $\kappa - \mu$, $\alpha - \mu$, and $\eta - \mu$ Fading Channels

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Abstract: This paper presents a unified theoretical analysis of the energy detection of Gaussian and $M$-PSK signals in $\kappa - \mu$, $\alpha - \mu$, and $\eta - \mu$ fading channels at the output of an energy detector subject to impulsive noise (Bernoulli-Gaussian model). As a result, novel, simple, and accurately approximated expressions for the probability of detection are derived. More precisely, the generalized Gauss-Laguerre quadrature is applied to approximate the probability of detection as a simple finite sum. Monte Carlo simulations corroborate the accuracy and precision of the derived approximations. The results are further extended to cooperative energy detection with hard decision combining information.

Key words: spectrum sensing; energy detector; fading channel; impulsive noise

1 Introduction

Spectrum sensing is the primary technique to enable dynamic spectrum sharing among a Primary User (PU) and a Secondary User (SU), such that an SU might access the spectrum opportunistically, without causing interference on PU’s transmissions. To do so, an SU must be able to distinguish (decide) whether or not the spectrum is occupied. Among several techniques for spectrum sensing, such as cyclostationary spectrum sensing[1] and matched filter[2], the energy detector has attracted widely interests, primarily because of its low-complexity and reasonable performance even in severe fading channels[3, 4].

This paper presents novel approximated expressions for the probability of detection of Gaussian and $M$-PSK signals in generalized fading channels, namely, $\kappa - \mu$[5], $\eta - \mu$[5], and $\alpha - \mu$[8], at the output of the energy detector subject to impulsive noise. The assumption for Gaussianity of the transmitted signal occurs in several practical scenarios, such as when the receiver has no prior information about the transmitted signal and in Orthogonal Frequency Division Multiplexing (OFDM) in which the envelope of the signal weakly converges to a Gaussian process when the number of subcarriers is sufficiently large[7].

The impulsive noise is modeled as a Bernoulli-Gaussian (BG) channel[8, 9]. This channel has attracted attention due to its practical importance especially in multi-carrier transmission systems based on OFDM[10]. Furthermore, Vu et al.[11] presented estimators for the ergodic Shannon and constrained capacities of the BG impulsive noise channel in Rayleigh fading.

The probability of detection is approximated by using the generalized Gauss-Laguerre quadrature, which approximates a class of integrals to a finite sum[12]. Additionally, the approximation error quickly goes to zero even for a small number of terms in the sum. Furthermore, this approximation presents an attractive alternative due to its low computational cost and high accuracy. In contrast to the assumption of the Central Limit Theorem, which has been commonly used to approximate the expressions for...
the probabilities of detection and false alarm \[13\], the proposed approximation does not require a large number of samples in order to become accurate, and therefore it does not compromise the sensing time. In fact, it works even for a small number of samples and for a wide range of the parameters involved.

The remainder of the paper is organized as follows. Section 2 states the physical assumptions of the spectrum sensing system and describes its underlying mathematical setting. Section 3 presents the generalized fading distributions considered in the proposed analysis. Numerical and analytical results are presented and discussed in Section 4. Section 5 presents conclusions and remarks.

2 Energy Detector

Consider the following hypotheses,

\[ \mathcal{H}_0 : X_n = W_n + C \cdot U_n \]

\[ \mathcal{H}_1 : X_n = H \cdot S_n + W_n + C \cdot U_n \]

for \( n = 1, 2, \ldots, N \), in which \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) stand for the hypotheses that the PU’s signal is absent and present, respectively. It is assumed that the channel gain \( H \) has probability density function (pdf), \( p_H \), the transmitted signal is Gaussian distributed, i.e., \( S_n \sim \mathcal{N}(0, \sigma^2_S) \) i.i.d., and so is the noise process \( W_n \sim \mathcal{N}(0, \sigma^2_W) \) i.i.d..

The impulsive noise component is modeled according the BG model, which has the following probability mass function \( \text{Pr}[C = 1] = 1 - \text{Pr}[C = 0] = p, p \in [0, 1] \), and \( U_n \sim \mathcal{N}(0, \sigma^2_U) \) i.i.d. Furthermore, it is assumed that \( C \) and \( H \) are constant during the sensing time, i.e., the time for collecting a set of \( N \) samples. It is also assumed that \( H, S_n, W_n, C, \) and \( U_n \) are mutually independent.

Finally, define \( \rho_W \triangleq \frac{\sigma^2_S}{\sigma^2_W} \) and \( \rho_U \triangleq \frac{\sigma^2_S}{\sigma^2_U} \) as the Signal-to-Noise Ratio (SNR) and the Signal-to-Impulsive-noise Ratio (SIR), respectively, and, without loss of generality, assume that \( \sigma^2_S = 1 \) throughout.

The well-known energy detection rule, used to decide between the two aforementioned hypotheses, is defined as follows:

\[ d_X(Y_N) = \begin{cases} 1, & Y_N \geq \lambda; \\ 0, & Y_N < \lambda \end{cases} \]

in which \( Y_N \triangleq \sum_{n=1}^{N} |X_n|^2 \), the threshold \( \lambda \) is a strictly positive real number, and \( d_X(Y_N) = j, j \in \{0, 1\} \) means that the detector has decided in favor of the hypothesis \( \mathcal{H}_j \).

2.1 Hypothesis \( \mathcal{H}_0 \)

Under \( \mathcal{H}_0 \), the distribution function of \( Y_N \) may be written as

\[ P_{Y_N}(y) = (1 - p) \gamma \left( \frac{N}{2}, \frac{y \rho_W}{2} \right) + p \gamma \left( \frac{N}{2}, \frac{y \rho_W \rho U}{2\rho_W + \rho U} \right) \]

for \( y \geq 0 \), in which \( \gamma(\cdot, \cdot) \) is the lower incomplete gamma function defined as \( \gamma(a, z) \triangleq \frac{1}{\Gamma(a)} \int_0^z t^{a-1} e^{-t} dt \).

Considering that the noise processes are complex Gaussian, the distribution function of \( Y_N \) may be written as

\[ P_{Y_N}(y) = (1 - p) \gamma \left( \frac{N}{2}, \frac{y \rho_W}{2} \right) + p \gamma \left( \frac{N}{2}, \frac{y \rho_W \rho U}{2\rho_W + \rho U} \right) \]

2.2 Hypothesis \( \mathcal{H}_1 \)

Given \( H = h, h > 0 \), the distribution function of \( Y_N \) conditioned on \( H \) may be written as

\[ P_{Y_N|H}(y|h) = (1 - p) \gamma \left( \frac{N}{2}, \frac{y \rho_W}{2(h^2 \rho_W + 1)} \right) + p \gamma \left( \frac{N}{2}, \frac{y \rho_W \rho U}{2(h^2 \rho_W \rho U + \rho_W + \rho U)} \right) \]

for \( y > 0 \) and \( h > 0 \).

For the case \( S_n \) is an \( M \)-PSK signal such that every symbol has the same probability of occurrence, i.e., \( \text{Pr}(S_n = s_n) = \frac{1}{M} \), the conditional distribution is given as

\[ P_{Y_N|R}(y|h) = 1 - \left( 1 - p \right) Q_N \left( \sqrt{\frac{2N h^2 E_S}{\sigma^2_W}}, \sqrt{\frac{2y}{\sigma^2_W}} \right) + p Q_N \left( \sqrt{\frac{2N h^2 E_S}{\sigma^2_W + \sigma^2_U}}, \sqrt{\frac{2y}{\sigma^2_W + \sigma^2_U}} \right) \]

in which \( Q_N(\cdot, \cdot) \) is the Marcum-\( Q \) function and \( E_S \) is the energy of an \( M \)-PSK symbol. Henceforth, for the case \( S_n \) is an \( M \)-PSK signal, define the SNR as \( E_S / \sigma^2_W \) and the SIR as \( E_S / \sigma^2_U \). In addition, it is assumed, without loss of generality, that the energy of the PSK constellation is equal to unity, so that \( E_S = \frac{1}{M} \), allowing a fair comparison of the performance of the spectrum sensing system for different sizes of PSK constellations.
In both cases, given an arbitrarily chosen threshold \( \lambda > 0, \lambda \in \mathbb{R} \), the probability of false alarm \( P_F \) and the probability of detection \( P_D \) may be written as

\[
P_F = \Pr \{ d_Y(Y_N) = 1 | H_0 \} = 1 - P_{Y_N}(\lambda)
\]

\[
P_D = \Pr \{ d_Y(Y_N) = 1 | H_1 \} = 1 - E_H \left[ P_{Y_N}(\lambda | H) \right]
\]

(8) (9)

in which \( E_H \) stands for the expected value with respect to the random variable \( H \).

The threshold \( \lambda \) is selected based on the Neyman-Pearson criterion, i.e., \( \lambda \) is the solution of Eq. (8) for a given probability of false alarm. Although an analytical solution seems unfeasible, it may be easily solved numerically, since Eq. (8) is a strictly decreasing function in \( \lambda \).

3 Generalized Fading Channels

3.1 \( \eta - \mu \) distribution

The \( \eta - \mu \) distribution is suitable to model small-scale fading variations in non-line-of-sight conditions. The \( \eta - \mu \) distribution includes the following ones as special cases: Hoyt (Nakagami-\( q \)), Nakagami-\( m \), Rayleigh, and one-sided Gaussian. Let \( H \) be \( \eta - \mu \) distributed. Therefore, its pdf, in its normalized version, may be written as

\[
p_H(h) = \frac{4\sqrt{\pi} \mu^{\frac{\eta^2}{2}}}{\eta^{\frac{\mu-1}{2}}} \exp \left( \frac{-2\mu h^2}{1 - \eta^2} \right) \times I_{\mu-1/2} \left( 2\eta \mu h^2 \right)
\]

(10)

for \( h > 0, \mu > 0, \) and \( -1 < \eta < 1 \), in which \( I_{\alpha}(z) \) is the modified Bessel function of first kind and order \( \alpha \).

3.2 \( \kappa - \mu \) distribution

The \( \kappa - \mu \) distribution is a general fading distribution appropriate to model the small-scale fading variations in line-of-sight applications. The \( \kappa - \mu \) distribution includes Rice (Nakagami-\( n \)), Nakagami-\( m \), Rayleigh, and one-sided Gaussian as special cases. Assuming that \( H \) is \( \kappa - \mu \) distributed, its density, in its normalized form, is

\[
p_H(h) = \frac{2\mu(1 + \kappa)^{\frac{\mu-1}{2}}}{\kappa^{\frac{-1}{2}} \exp(\kappa\mu)} h^\mu \exp \left( -\mu(1 + \kappa)h^2 \right) \times I_{\mu-1} \left( 2\mu \sqrt{\kappa(1 + \kappa)}h \right)
\]

(11)

for \( h > 0, \mu > 0, \) and \( \kappa > 0 \).

3.3 \( \alpha - \mu \) distribution

The \( \alpha - \mu \) fading arises from nonlineaities caused by the propagation medium to the transmitted signal, due to multipath propagation in a non-homogeneous environment. The \( \alpha - \mu \) distribution includes several others, namely, Gamma, Erlang, Nakagami-\( m \), Chi, exponential, Weibull, one-sided Gaussian, and Rayleigh as special cases. Let \( H \) be \( \alpha - \mu \) distributed, then its density, in its normalized version, may be written as

\[
p_H(h) = \frac{\alpha \mu^\alpha h^{\alpha-1}}{\Gamma(\alpha)} \exp \left( -\mu h^\alpha \right)
\]

(12)

for \( h > 0, \alpha > 0, \) and \( \mu > 0 \).

4 Theoretical Results and Numerical Analysis

In order to evaluate the performance of the energy detector, it is necessary to solve the integral in Eq. (9). As far as the authors are concerned, in case \( p_H \) is any of the pdfs considered in this paper (even in simple special cases, e.g., Nakagami-\( m \)), an analytic solution for the probability of detection Eq. (9) does not exist. Therefore, the generalized Gauss-Laguerre quadrature (shown in Appendix) is utilized to approximate the probability of detection Eq. (9).

For both assumptions on the transmitted, i.e., either Gaussian distributed or \( M \)-PSK, Eq. (9) may be approximated as Eqs. (13)–(15), for \( \eta - \mu, \kappa - \mu \), and \( \alpha - \mu \) fading channels, respectively. The parameter \( K \) represents both the order of the Laguerre polynomial and the number of terms in the sum.

\[
P_D^{\eta,\mu} \approx 1 - \frac{\sqrt{\pi} \Gamma(K + \mu + \frac{1}{2})}{(2\eta)^{\frac{\mu-1}{2}} \Gamma(\mu) \Gamma(K + 1)^2} \sum_{k=1}^{K} \frac{\eta^k I_{\mu-1/2}(\eta v_k)}{L_{K+1}^{\mu-1/2}(v_k)} P_{Y_N|R}(\lambda, \frac{(1 - \eta^2)v_k}{2\mu})
\]

(13)

\[
P_D^{\kappa,\mu} \approx 1 - \frac{(\kappa \mu)^{\frac{1}{2}} - \mu}{\kappa ! (K + 1)^2} \exp(\kappa \mu) \times \sum_{k=1}^{K} \frac{\eta^k I_{\mu-1/2}(\sqrt{\kappa \mu} v_k)}{L_{K+1}^{\mu-1/2}(v_k)} P_{Y_N|R}(\lambda, \sqrt{\frac{v_k}{\mu(1 + \kappa)}})
\]

(14)

\[
P_D^{\alpha,\mu} \approx 1 - \frac{\mu^K}{\Gamma(K + 1)^2} \sum_{k=1}^{K} \frac{v_k P_{Y_N|R}(\lambda, (\frac{v_k}{\mu})^{1/\alpha})}{L_{K+1}^{\mu-1/2}(v_k)^2}
\]

(15)
In the following figures, lines designate theoretical results (approximated using $M = 30$), while markers denote Monte Carlo simulation results with $10^6$ realizations. For all figures, it is noted a substantially agreement between the Monte Carlo simulation results and the proposed approximations. Additionally, the number of terms in the sum, which was selected to keep the approximation error below $10^{-4}$, is reasonably small.

Figures 1–3 show the performance of the energy detector for $\kappa - \mu$, $\eta - \mu$, and $\alpha - \mu$ fading channels, respectively. More precisely, Fig. 1 shows the effect of increasing the signal power (thereafter increasing the SIR) on the probability of detection. Figure 2 illustrates the effectiveness of increasing the number of samples $N$ in overcoming fading, low SIR, and moderate $p$. Figure 3 compares the performance of the energy detector by either increasing the number of samples or the SIR. Figure 4 depicts the influence of $p$ on the probability of detection for Nakagami-$m$ and Hoyt fading channels for several values of SNR and SIR. Figure 5 presents the complementary Receiver Operating Characteristic (ROC) for the discussed generalized fading scenarios, when the transmitted signal is Gaussian distributed. It may be noted that the performance of the energy detector may be significantly increased if the probability of false alarm is greater than the probability of occurrence of impulsive noise.

Additionally, consider the complementary ROC depicted in Figs. 6–9. In these figures, the transmitted
Fig. 5 Probability of miss \((1 - P_D)\) versus \(P_{\text{FA}}\) in generalized fading for different values of the probability of occurrence of impulsive noise \((p)\). Markers represent Monte Carlo simulations with \(10^6\) realizations, while lines represent Eqs. (13) - (15) for \(K = 30\).

Fig. 7 Complementary ROC for \(M\)-PSK signals. Markers represent Monte Carlo simulation with \(10^6\) realizations, while lines represent results from Eq. (13).

Fig. 8 Complementary ROC for \(M\)-PSK signals. Markers represent Monte Carlo simulation with \(10^6\) realizations, while lines represent results from Eq. (13).

signal comes from an \(M\)-PSK constellation. It can be noted that, for \(P_F < p\), the pair of curves \((p = 0\) and \(p \neq 0\)) decay roughly with the same rate as \(P_F\) increases, and the difference between them remains approximately constant. This fact is due to the influence of the impulsive noise on the receiver. But, it may also be observed that, for \(P_F > p\), the performance of the energy detector is significantly improved and the effect of the impulsive noise is strongly reduced as \(P_F\) increases. Furthermore, note that the SNR and SIR were chosen such that the variances of the noises, \(\sigma_W^2\) and \(\sigma_U^2\), remained the same for each scenario.

Therefore, while the \(\eta - \mu\) fading impairs the spectrum sensing performance of the energy detector as a whole, i.e, for all values of \(P_F\), the impulsive noise effect turns out to be negligible for \(P_F > p\).

The aforementioned observations are not particular of those examples, in fact, they were verified in several other simulated scenarios of fading, impulsive noise, and signal to noise ratios.

The performance of the energy detector may be undoubtedly improved when there exist \(L\) secondary users that share their local (individual) decisions, and thereby jointly decide on the occupation of the channel. In a hard decision cooperative scheme, in which the global decisions are based on a \(J\)-out-of-\(L\) rule, the detector decides for \(H_1\) if and only if at least \(J\) secondary users have decided so.
When \( J = 1 \), \( J = L \), or \( J = \left\lceil \frac{L}{2} \right\rceil \), the rules are called OR, AND, and Majority, respectively. It has been shown that the OR rule usually outperforms AND and Majority rules in many scenarios of practical interest\textsuperscript{14}. Therefore, only the OR rule shall be considered henceforth. In particular, when \( K = 1 \), the probabilities of false alarm and detection are given as

\[
Q_D(\lambda) = 1 - (1 - P_D(\lambda))^L \quad (16)
\]
\[
Q_T(\lambda) = 1 - (1 - P_T(\lambda))^L \quad (17)
\]

Figure 10 depicts the performance of the cooperative energy detector for several values of \( L \) under Weibull \((\mu = 1)\) and \( \kappa - \mu \) fading channels. The performance of the detector improves significantly as the number of cooperating users increases. Hence, the cooperative detection may effectively overcome the impairments caused by both low SIR and SNR, and severe fading. In fact, for \( \text{SNR} = 10 \text{ dB} \), the probability of detection increases from 0.2 to roughly 0.9, for the case of no cooperation \((L = 1)\) and for a cooperative sensing with ten users \((L = 10)\), respectively.

5 Conclusion

This paper has proposed novel, simple, and accurate approximations for the performance evaluation of the energy detector employed to detect either Gaussian signals or \( M \)-PSK signals, subject to impulsive noise, in generalized fading channels, namely, \( \kappa - \mu, \eta - \mu, \) and \( \alpha - \mu \). The approximations are based on the generalized Gauss-Laguerre quadrature, which approximates the integrals that are required to compute the probability of detection to finite sums, reducing the complexity and ensuring accuracy.

The effects of the probability of occurrence of impulsive noise and the signal-to-impulsive-noise ratio on the performance of the energy detector were described as well. It was postulated that the detector presents a significant improvement in performance when \( P_f \) is slightly greater than \( p \). Additionally, the impulsive noise effect may be negligible when \( P_f \gg p \).

Furthermore, as has been shown, hard decision combining cooperative schemes based on the OR rule effectively overcome the effects of low SIR and severe fading.

The simple form of the proposed expressions provide a tool for design engineers to determine parameters such as the minimum number of samples and the minimum number of cooperating users to achieve a given performance under a variety of fading and impulsive noise scenarios.

Appendix A

The generalized Gauss-Laguerre quadrature\textsuperscript{12} states that, for any real number \( \beta > -1 \),

\[
\int_{0}^{\infty} t^\beta e^{-t} f(t) \, dt \approx \sum_{k=1}^{K} v_k f(r_k) \quad (18)
\]

in which \( r_k \) is the \( n \)-th root of the generalized Laguerre polynomial of order \( M \), i.e., \( L^\beta_K \), and the weight \( v_n \) is given as

\[
v_k = \frac{r_k \Gamma(K + \beta + 1)}{K!(K + 1)^2[L^\beta_{K+1}(r_k)]^2} \quad (19)
\]

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