Effect of Rayleigh Number and Density Stability Ratio on characteristics of Double-Diffusive Salt Fingers

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Abstract. In this paper, we investigate the finger evolution pattern and physics behind the layer formation in a two layer model using numerical simulations. A three-dimensional (3D) numerical code has been developed to solve Navier-Stokes equations along with energy and concentration conservation equations. Simulations were carried out to study the effect of Rayleigh number and density ratio on the characters of salt finger. Instabilities in form of rolls grow from side walls in all simulated cases. Square planform is found to be stable only at low $\text{Ra}_T$. Individual components flux is found to be larger in 3D case compared to 2D, but the flux ratio is found to be less in 3D.

1. Introduction
Salt finger convection is a buoyancy driven flow in which the density of fluid depends upon two scalar quantities with different diffusion rate and distributed such that fluid is initially gravitationally stable. The presence of faster diffusing component makes the fluid lighter and slower diffusing increases the heaviness. The increase in density due to presence of slower diffusing component in top layer is balanced by faster diffusing component. Initially, for system to be stable the higher diffusing component in top layer has to be higher than the slower diffusing component. Salt fingers are found in many natural and engineering environments, a few examples are oceanography, water bodies [1], metallurgy, geology, and stellar dynamics [2–5]. The difference in diffusion rate of components make the system gravitationally unstable, locally, near the interface, which makes it conducive for perturbations to develop and grow. The perturbations develop in forms of cells which grows slowly inside the density inverted zone and at much faster rate after crossing this zone. While doing so, they exchange more of higher diffusing components horizontally which makes the system run for longer time.

Since last decade, researchers have started reporting results from 3D simulations, which are more close to reality and provide and extra dimensions giving useful insights into the system. With advancements made in computer and electronics the size of computational problem doesn’t pose a big challenge any more. If in 2D system model, fingers evolve from line interface as alternating rising and falling structures, in 3D they rise from a plane and the pattern they form, on the interface plane, is still an unresolved topic.

The linear stability analysis of governing equations gives degenerated solutions for wave number selection and the suitable combination of variables chosen by system is attributed to the non-linear interactions between the fingers.

\[ k^2 + l^2 = k_{\text{max}}^2 \]
Table 1: Basic Notations

| Symbol | Description                                      | Symbol | Description                                      |
|--------|--------------------------------------------------|--------|--------------------------------------------------|
| H      | Height of domain                                  | α      | coefficient of thermal expansion                 |
|        | u,v,w components of velocity vector              | k      | coefficient of thermal diffusivity               |
|        |                                                  | β      | coefficient of salinity expansion                |
|        |                                                  | k_s    | coefficient of salt diffusivity                  |
| ΔT     | Initial temp difference between two layers       | ν      | coefficient of salt diffusivity                  |
| ΔS     | Initial salinity difference between two layers   | RaT   | Thermal Rayleigh no.                             |
| Ra_s   |                                                  | R_p   |                                                  |
|        | $\frac{g\beta_s \Delta T H_3}{\nu k_s}$ (Thermal Rayleigh no.) |        | $\frac{R_{\rho} \Delta T H_3}{R_a S}$ |

where, $k_{max}^2$ corresponds to the maximum growth rate wave number. $k$ and $l$ are the in-plane (horizontal) wave numbers. Strauss [6] made first attempt to resolve this mystery, by proposing that rolls are more stable but this was contradicted by experiment conducted by Shirtcliffe and Turner [7] in which they showed square patterns in a triangular tank. Procter and Holyer [8] showed mathematically using weak non-linear analysis that equilibrated fingers prefer rolls over square cells. Simulations were run on a three dimensional in-house code to solve N-S equations, and parallelized it using OpenMP. The importance of Rayleigh number on salt finger has been shown by [9–12] The system at different values of $R_{\rho}$ and Rayleigh numbers, $R_a S$ and $R_a T$ is explored by us. In this paper, observations are reported from a limited range of dimensionless parameters for a thermohaline system, which are, Rayleigh number ($R_a T$) = $10^4$, $10^5$, $10^8$ for density stability ratio ($R_{\rho}$) = 1, 1.1 and 1.5.

2. Mathematical Formulation

Mathematical model of double diffusive convection involves incompressible continuity equation, Navier-Stokes equations for momentum and energy, and equation for concentration transport. The boundary conditions are slip, and adiabatic for mass and heat. The mathematical expressions are as follows:

**Continuity:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

(2)

**X-momentum:**

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

(3)

**Y-momentum:**

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

(4)

**Z-momentum:**

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + Pr \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Ra_s Pr_s + Ra_T Pr_T$$

(5)

**Energy:**

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

(6)

**Concentration:**

$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} + w \frac{\partial S}{\partial z} = \frac{Pr}{Sc} \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \right)$$

(7)
Figure 1: Comparison of 2D and 3D concentration field at $R_\rho = 1.5$, $\tau = 100$ (a) 2D, $Ra_T = 7E4$, time 2 hours, (b) 3D, $Ra_T = 7E4$, at time = 1.5 hours, (c) 2D, $Ra_T = 7E8$ at time= 25 sec, and (d) 3D, $Ra_T = 7E8$ at time = 20 sec

3. Planforms in 3-Dimensions

As shown in figure 1a and 1b, the wave number is a strong function of $Ra_T$. From figure 2 One thing which is common to all results is that the perturbation starts with rolls from each side wall and crosses where they meet, because they may be the result of T-points (where boundary and two layers of fluid meets, which is a highly unstable region). The widening of these rolls as they propagate towards centre can be ascribed to the changes brought by near wall fluxes. The roll which are thin and high in energy due to density inversion is adjacent to the wall is always a trough, due to the density inversion, they are thin and high in energy. Each trough undergoes constructive interference with the perpendicular roll which produces high velocity jets. The second roll, which is a crest, interferes constructively with second perpendicular roll, but it takes time to invert the flow of previously generated thin high energy roll. This roll eventually produces patterns of up and down-going cells developed near the corners.

At fixed $R_\rho = 1.5$, for $Ra_T = 7 \times 10^8$ when these rolls meet at the centre of interface, a round spot in vertical velocity field was observed. Outer rolls wither away to give a symmetric pattern which lasts only for 1~2 minutes but, the inner roll is still there due to its large width. At centre the round spot converts slowly to cross, as the rolls emerge from bottom and at one time they completely disrupts the previous symmetry to divide the whole plane into four symmetric quadrants. The convection at these parameters is violent at the interface and after 15 minutes all the symmetry is gone from the plane and patterns observed was similar to what Radko and Stern [13] showed in their paper. Merging, splitting and generation of new fingers might be the cause for randomness which appear in plane-form.

Vertical velocities in rolls looks like to be dependent upon their width; thinner near the walls having large velocity, and wider towards the centre with less velocity. In the corners large vertical velocities are found due to constructive interference of thin rolls. At $Ra_T = 7 \times 10^8$ with $R_\rho = 1.1$ (figure 2h, i), the effect of high $Ra_S$ is disruptive on fingers which makes system highly unstable and a proper interference of rolls at the centre was not observed, however a very random breakup was present at central zone. At $Ra_T = 7 \times 10^4$ and $R_\rho = 1.5$, the most stable system simulated, adopts widely reported structure of squares, which continues till the fingers reaches mid height of the domain. For $R_\rho = 1.5$ and $Ra_T = 7 \times 10^8$ at coarse grids of $97^2 \times 241$, with aspect ratio of 0.4 : 0.4 : 1.0, worm like patterns on mid-section are observed as shown in figure 2g. They stays like that with very subtle changes over large time period.
Figure 2: (a) to (h) shows the changes in planform with time at 13, 20, 32, 60, and 107 seconds, for $R_\rho = 1.5$, $Ra_t = 7 \times 10^8$. Figure (h) shows almost square planform at same $R_\rho$ but at $Ra_t = 7 \times 10^4$. Figure (g) for $R_\rho = 1.1$ and $Ra_t = 7 \times 10^8$ at equilibrium state. Figure (h) and (i) for $R_\rho = 1.1$ and $Ra_t = 7 \times 10^8$ and at time = 20 and 25 seconds, respectively.

4. Staircases formation
After the system has passed through its strong convection period and left with small kinetic energy, and steps in concentration field (as shown in figure 4) sandwiching small magnitude vertical fingers were observed. Based upon the observed rolls, the reason might be side heating phenomenon. In this phenomenon, fluid is stably stratified with concentration of salt and heat decreasing from top to bottom and is heated from side vertical wall. A fluid parcel adjacent to the side wall rises due to heat, owing to buoyancy. During its rise salt diffuses very slowly out of it and its vertical motion is halted when its density becomes equal to that of surrounding. This parcel is forced to take slightly tilted path from horizontal towards the opposite wall and turns downward due to heat lost in this journey. The trailing fluid comes from its originating region. This completes the whole circulation and stacks of them has been observed in experiments.

Just before staircases starts to happen, full domain divides into two horizontal halves based on concentration of heat and salt in the fluid. Figure 5 shows the slice of concentration and temperature when staircase begin to form. More heat and salt concentrates in one half and less in the other half maintaining zero horizontal gradient in density, and clearly two separate zones are observed all through the height of system. Heat diffuses from high concentration zone and sets in many layers of horizontally downward tilted flow towards the other half, which is seen from the velocity components in the horizontal plane. What remains unexplained by analagising it with side wall heating phenomena is the existence of vertical cells within two layers.

5. Flux Ratio
Convective fluxes of individual components are higher in 3D system as compared to its counterpart in 2D. The reason owes to high kinetic energy which gives very less time for
Figure 3: (a) Shows the vertical velocity field and (b) the horizontal velocity field. These figures are the evidence of vertical fingers between parallel flows. Slices are taken at $x = 0.1, 0.2, 0.3; y = 0.2$ and $z = 0.5$, the magnitude of vertical an horizontal flow is almost equal.

Figure 4: Slice at $y = 0.2$ (a) is concentration field, (b) horizontal mean of concentration and (c) shows the vertical velocity field.

Figure 5: (a) Temperature field and, (b) Concentration field at value of $Z = 0.21, 0.42, 0.63$ and 0.84.
horizontal diffusion to happen. This small time is too less for salt to diffuse and is a cause for large difference in individual components flux. Due to this large difference in individual component’s flux, flux ratio is lower in 3D as compared to 2D fingers as shown in figure 6. Also flux ratio in 3D have less fluctuations compared to its counterpart in 2D.

6. Conclusion
In this paper we addressed the salt fingering in 2D and 3D. We shows density ratio alone is not sufficient and any one Rayleigh number is necessary to describe the system uniquely. The new observation of staircase in 3D is advantage over 2D model. From numerical simulation we observed that perturbations in 3D starts with rolls, however it is only observable at low values of Rayleigh number ($Ra_T$) at fixed density ratio that square planform is stable. We further notice that individual fluxes of heat and salt remains high in 3D in comparision to 2D, however flux ratio have less values.

7. References
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