Electron Self-injection in Multidimensional Relativistic Plasma Wakefields

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We present an analytical model for electron self-injection in nonlinear, multidimensional plasma wave excited by short laser pulse in the bubble regime or by short electron beam in the blowout regime. In this regimes, which are typical for electron acceleration in the last experiments, the laser radiation pressure or the electron beam charge pushes out background plasma electrons forming a plasma cavity - bubble - with a huge ion charge. The plasma electrons can be trapped in the bubble and accelerated by the plasma wakefields up to very high energies. The model predicts the condition for electron trapping and the trapping cross section in terms of the bubble radius and the bubble velocity. The obtained results are in a good agreement with results of 3D PIC simulations.

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Plasma-based charged particle acceleration is now a fast developing area of science. It has attracted much attention due to the recent experimental breakthrough in generation of quasimonoenergetic, dense and short bunches of relativistic electrons with up to GeV energies in the laser-driven acceleration [1, 2, 3, 4] and due to the energy doubling of a 42 GeV electron beam [5]. The high-gradient acceleration is provided by very strong electromagnetic fields in a plasma wake excited by short laser pulse or electron beam. The plasma electrons are radially expelled by ponderomotive force of the laser pulse or by Lorentz force from the space charge of the laser beam that leads to formation of the plasma cavity - bubble with unshielded ions inside. This is the bubble [6] or blowout [7] regime of laser-plasma and beam-plasma interaction. It is important that a small part of the background plasma electrons can be trapped in the bubble providing the electron self-injection. The bubble velocity is close to the speed of light and the trapped relativistic electrons can be continuously accelerated in the plasma fields up to a very high energy.

The electron self-injection is the key phenomenon of the last laser-plasma acceleration experiments [1, 2, 3, 4]. It is a crucial factor for the quality of the accelerated electron beam. Many applications ranging from x-ray free electron lasers to electron-positron colliders require high quality electron beams with a very low emittance and energy spread. The one-dimensional model based on the Hamiltonian formalism has been developed to study electron trapping in relativistic plasma wave [8, 9]. Several optical and plasma techniques have been proposed to enhance the electron self-injection in plasma wakefield like the collision of two counter-propagating laser pulses [10, 11], density transitions [12, 13, 14] etc. In the ultra-high intensity regime the radial force from the driver and from the plasma fields strongly affect electron motion and the electron dynamics become complex and multidimensional. The trajectory of the trapped electrons starts at the bubble front, bends around the cavity and becomes caught at the trailing edge of the bubble. Evidently, such trajectories cannot be described in the framework of a one-dimensional model.

Despite the great interest in plasma-based electron acceleration there is little theory of self-injection. In Ref. [15, 16, 17] a multidimensional model of a relativistic plasma wake including beam loading is proposed, however the electron self-injection was not considered. The electron self-injection has been mainly studied numerically [18, 19, 20]. It was found in Ref. [19] that a sufficiently large bubble, \( R > 4 \), can trap plasma electrons. Here \( R \) is the bubble radius normalized to \( c/\omega_p \), \( \omega_p = \sqrt{4\pi ne^2/m} \) is the (non-relativistic) plasma frequency for the background electron density \( n_0 \), \( e \) and \( m \) are the charge and mass of the electron, respectively.

A different condition for electron self-injection has been proposed in Ref. [20]: \( R > \gamma_0 \), where \( \gamma_0 \) is the bubble gamma-factor. Unfortunately, principal limitations of a numerical approach make the accuracy and the validity range of the obtained conditions unclear and do not provide an insight in the self-injection physics. However, the numerical approach has its natural limitations in accuracy and validity conditions of the obtained results and does not provide insight in the self-injection physics.

In this paper, we present an analytical model for self-injection and check it by direct measurements of model parameters in 3D particle-in-cell (PIC) simulations. For simplicity we assume that the bubble has a spherical shape with large radius \( R \gg 1 \) that is typical for relativistic laser pulses. The numerical simulations [20] and theoretical analysis [17] demonstrate that the bubble shape is close to the spherical one for relativistically intense laser pulse and when the number of the trapped electrons is...
not too large. It can be shown [17, 20] that the space-

time distribution of the electromagnetic field inside the

spherical bubble expressed in cylindrical coordinates is

\[ E_r = \sqrt{\left(1 + \frac{V}{c}\right)^2 - 1} \frac{p_y}{\gamma}, \]  

\[ E_\theta = \frac{p_x}{\gamma}, \]

\[ B_z = \frac{p_y}{\gamma}, \]

where \( E_r \) is the canonical momentum of the electron and

\( \gamma \) is the relativistic \( \gamma \)-factor of the bubble. The square of the distance

\( r > R \). The most critical

instants of time when the electron can leave the bubble

is \( t = t_m > 0 \) when \( dy/dt = p_y = 0 \) and the electron

excursion from the \( \eta \)-axis reaches its maximum. At

this moment we can write \( \gamma - V p_x \approx p_x / \left(2 \gamma_0^2 + 1 / (2 p_x)\right) \),

where \( p_x \gg 1 \) is assumed, \( \gamma_0 = (1 - V^2)^{-1/2} \gg 1 \) is the gamma-factor of the bubble. The square of the distance

of the electron from the bubble center (\( \xi = 0, y = 0 \)) at

\( r^2 \approx 4 + R^2 - \frac{2 p_x(t_m)}{\gamma_0} - \frac{2}{p_x(t_m)} \),

(4)

where we use Eq. (3).

We change the variables \( p_y = R^2 p_y, p_x = R^2 p_x, \xi = X R, y = Y R \) and \( t = R s \). As a result Eqs. (1) and (2) takes a form

\[ \frac{d p_x}{d s} = -X \frac{2}{2} + \frac{P_y}{\sqrt{P_x^2 + P_y^2}} \frac{Y}{4} + O \left(R^{-4}, \gamma_0^{-2}\right), \]

(5)

\[ \frac{d p_y}{d s} = -\left(1 + \frac{P_x}{\sqrt{P_x^2 + P_y^2}}\right) \frac{Y}{4} + O \left(R^{-4}\right), \]

(6)

\[ \frac{d X}{d s} = \frac{P_x}{\sqrt{P_x^2 + P_y^2}} - 1 + O \left(R^{-4}, \gamma_0^{-2}\right), \]

(7)
where \( P_x = P_y = 0 \) and \( dP_y/ds = -1/4, dP_x/ds = 0 \) at \( s = 0 \). In the zeroth order in \( \gamma_0^{-2} \) and \( R^{-2} \) Eqs. (5)-(8) do not depend on any parameters and can be numerically solved to find \( s_m \) and \( t_m = R s_m + O (\gamma_0^{-2}, R^{-2}) \): \( s_1 \simeq 3.2, s_2 \simeq 10.7, s_3 \simeq 23.0, \ldots \). Using Eq. (24) the condition that electron leaves the bubble \( (r_m > R) \) can be written as follows

\[
2 > \frac{R^2 P_x (s_m)}{\gamma_0^2} + O (\gamma_0^{-4}, R^{-2}) .
\]

The electron leaves the bubble if there exists at least one value of \( s_m \) when the condition (9) is satisfied. We can take only \( P_x (s_1) \simeq 1.1 \) since \( P_x (s_1) < P_x (s_2) < P_x (s_3) \ldots \). As a result we come to the condition for electron capture in the bubble

\[
\frac{\gamma_0}{R} \lesssim \frac{1}{\sqrt{2}} ,
\]

which is close to the condition obtained numerically in Ref. [20].

Now we estimate the effect of the impact parameter on the trapping condition. If \( \rho < R \) then electron first moves through the decelerating bubble field where \( \xi > 0 \) in contrast to the electron with \( \rho = R \), which starts motion from position \( \xi = 0 \) with \( P = 0 \). The deceleration leads to reduction of the longitudinal momentum at \( s = s_1 \) for electron with \( \rho < R \) as compared to the electron with \( \rho = R \). We estimate this reduction, \( P_x (s_1, \rho = R) - P_x (s_1, \rho < R) \), as \( \Delta \), that is close to the value calculated by the numerical integration of Eqs. (11). To calculate \( \Delta \) we can use Eq. (3) and assume \( 1/R \ll R - \rho \ll R \). As \( p_y \simeq 0 \) at \( \xi = 0, y \simeq \rho \) then \( H \simeq (1 + R^2 \Delta^2)^{1/2} + R^2 \Delta + \rho^2/4 = 1 + R^2/4 \), so that \( \Delta = \nu (\nu + 2/R^2) / (2 \nu + 2/R^2) \simeq \nu/2 \), where \( \nu = (1 - \rho^2/R^2)^2/4 \). Therefore, \( P_x (s_1, \rho \leq R) \simeq P_x (s_1, \rho = R) - \nu/2 \) and the condition for electron trapping (10) can be rewritten as follows

\[
\rho_0 \lesssim \rho \lesssim R ,
\]

Therefore the trapping cross-section \( \sigma \) near the trapping threshold \( \gamma_0 \simeq 2^{-1/2} R \) takes the form

\[
\frac{\sigma}{\pi R^2} = \int_{\rho_0}^{R} \frac{2 \rho}{R^2} d\rho = 1 - \frac{\rho_0^2}{R^2} \simeq 8 \left( 1 - 2 \frac{\gamma_0^2}{R^2} \right) .
\]

It follows from Eq. (13) that the trapping cross-section decreases as \( \gamma_0/R \) increases and the trapping stops at the threshold \( \gamma_0 \simeq 2^{-1/2} R \).

The developed model abstracts from some important effects observed in numerical simulations: the bubble
shape deformation during laser pulse propagation and the bubble field enhancement at the bubble back due to electron sheet crossing \([19, 21]\). It follows from 3D PIC simulations that the bubble back typically moves slowly than the bubble front \([19, 24]\). The bubble back gamma-factor should be used in Eq. \((\text{19})\) as \(\gamma_0\) since the electrons are trapped at the bubble back. Another effect is that the bubble field can be stronger at the bubble back than it follows from linear approximation for the bubble field in our model. To estimate the effect of field enhancement on the trapping we introduce the field enhancement factor \(k\) so that \(A_x = -\phi = kr^2/8\). It follows from the scalability analysis of Eqs. \((\text{5})-(\text{8})\) that the coefficient at RHS of Eq. \((\text{10})\) becomes \(\sqrt{k}/2\). The considered effects can significantly affect electron self-injection. For parameters of simulations performed in Ref. \([19]\) the minimal radius of the bubble with self-injection is reduced by a factor of 4 when the bubble field enhancement \((\sim 4\) see Fig. 3 in Ref. \([19]\) and the bubble back gamma-factor \((\sim 9\) see Fig. 1 in Ref. \([19]\) \) are taken into account in Eq. \((\text{10})\). The estimated radius is about 1.5 times more than observed in the simulations. Further studies are needed to include the above mentioned effects in our model accurately.

We carry out a numerical simulation of laser-plasma interaction by PIC VPILO3D code. The laser pulse is circularly polarized and has the envelope \(a = a_0 \exp(-r^2/r_0^2) \cos(\pi t/T_L)\), where \(T_L = 30\) fs is the pulse duration, \(r_1 = 9\) \(\mu\)m is the focused spot size and \(a_0 \equiv eA/mc^2 = 1.5\) is the normalized vector potential, which corresponds to the laser intensity \(I = 3 \times 10^{18} \text{W/cm}^2\), \(\lambda = 0.82\) \(\mu\)m is the laser wavelength. The plasma density is \(n_0 = 1.16 \times 10^{19} \text{cm}^{-3}\) so that the parameters are close to that used in experiments \([3]\). The typical distribution of the electron density in the bubble regime is shown in Fig. \(2(a)\) when laser pulse has passed the distance 2000\(\lambda\) in plasma. Because of the large number of trapped electrons the field enhancement effect is suppressed and the bubble shape deviates from the ideal spherical one. We modify the code to track individual particles over the full simulation. To separate trapped electrons, we introduce an energy threshold: we assume that an electron was trapped if its maximum energy exceeds 75 MeV.

The distribution of the number of electrons trapped in the bubble as a function of their initial position \(x_0\) is shown in Fig. \(2(b)\) for three instants of time. Fig. \(2(b)\) demonstrates that the capture process is non-uniform and the number of trapped electrons changes along the laser path. The trapped electrons start motion from position \(x = x_0\) at \(t = t_0\) that corresponds to the electron coordinates \(y = R, \xi = 0\) and the bubble center position \(x = x_0\) according to our model. If the bubble propagates with velocity \(V(t)\) then \(x_0 = \int_{t_0}^{t} V(t')dt'\). For the particle position we have \(x = x_0 + \int_{t_0}^{t} v_x(t_0, t')dt'\), where \(v_x\) is longitudinal component of the electron velocity and \(v_x(t_0, t_0) = 0\). Differentiating this equation with respect to \(t_0\) we get \(dx/dx_0 = 1 + [1/V(t_0)] \int_{t_0}^{t} [dv_x(t_0, t')/dt_0]dt'\), where we use \(dx_0/dt_0 = V(t_0)\). In our model \(v_x(t_0, t)\) is a function of \(t - t_0\) and for a highly relativistic electron \(\gamma \gg \gamma_0 \gg 1\), \(v_x \approx 1\) we can write a \(dx/dx_0 \approx -1/(2\gamma_0^2)\). This derivation does not take into account the bubble shape deformation. However, it follows from our simulations that \(\gamma_0\) calculated from \(dx/dx_0 \approx -1/(2\gamma_0^2)\) largely determines the gamma-factor of the bubble back because the trapped electrons with \(1 < \gamma < \gamma_0\) spent most of time in the bubble back. It is seen from Fig. \(2(b)-(d)\) that the self-injection is strongly correlated with this \(\gamma\)-factor. The number of trapped particles per laser period peaks in the regions \(1100\lambda/c < t < 1400\lambda/c\) and \(t > 1700\lambda/c\), where \(\gamma_0\) is minimal. Vice versa, for \(1400\lambda/c < t < 1700\lambda/c\) when \(\gamma_0 > 2^{-1/2}R\) the number of the trapped particles becomes negligible that agrees with predictions of the model.

In conclusion we present the model for electron self-injection in the relativistic plasma wakefield generated by a short laser pulse or by an electron beam. The density threshold for electron self-injection is predicted by our model that is different from the self-injection condition proposed in Ref. \([19]\). The evidences of such threshold have been observed in experiments \([11]\). The model can also explain electron self-injection scheme based on downward plasma density transition \([14]\). Such transition leads to the bubble elongation because of plasma density decreasing. Further studies are needed and sharply reduces the gamma-factor of the bubble back thereby strongly enhancing self-injection. Recently, a strong impact of the wake phase velocity on electron trapping was also observed experimentally \([22]\) that is consistent with our model. However, the accurate measurements of the bubble velocity and the structure of the electromagnetic field in the bubble are needed for careful verification of the obtained results. Much effort is now mounted in theory and in experiments to control the emittance and the energy spread of the accelerated electron bunch. It follows from the obtained result that the self-injection strongly depends on the wake phase velocity that can be controlled, for example, by plasma density profiling \([22]\). Therefore, the obtained results can be used to optimize the plasma-based acceleration.

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