I. INTRODUCTION

A single electron occupying two tunnel coupled quantum dots can be operated as a charge qubit [1–8]. Control parameters of this qubit are the interdot tunnel coupling $t$ as well as the detuning $\delta$ defined as the energy difference between the left and right dot electrochemical potentials. Noise protection to first-order in detuning is obtained by operating the qubit at $\delta = 0$ [9]. This operation point is called a first-order "sweet spot" since the first derivative of the qubit energy with respect to the detuning parameter vanishes. At this point, dephasing due to detuning noise is minimal [3,10,11] and charge qubit linewidths below 3 MHz have been reported [9,12].

Additional qubit control parameters are obtained by increasing the number of quantum dots in the linear array by one, i.e., by using a linear triple quantum dot (TQD) [13–16]. For this system, the qubit parameters are the tunnel coupling $t_L$ between the left and the middle and $t_R$ between the right and the middle quantum dots, as well as the left to right dot asymmetry $\delta = \delta_L - \delta_R$ and the middle to outer dot detuning $E_M = \epsilon_M - (\delta_L + \delta_R)/2$. Here, $\epsilon_L$, $\epsilon_M$, and $\epsilon_R$ are the single-particle energies of electrons in the left, middle, and right quantum dot, respectively.

One promising TQD qubit in terms of noise protection is the charge quadrupole qubit [17,18], which we briefly introduce in this paragraph. This single electron qubit utilizes the TQD ground and second excited states with the qubit excitation energy $E_{02}$, whereas the first excited state is a leakage state not connected to the other states by a quadrupole moment. The quadrupole qubit has recently been investigated experimentally [18] by strongly coupling it to a single photon in a superconducting microwave resonator. It has a single sweet spot at $\delta = E_M = 0$ in both detuning parameters, since at this point $\partial E_{02}/\partial \delta = \partial E_{02}/\partial E_M = 0$. Improved coherence was detected operating the qubit on the quadrupolar axis $E_M$ with $\delta = 0$ compared to operating the qubit on the detuning axis $\delta$ with $E_M = 0$.

In this work, we experimentally explore a different TQD qubit that hosts a single electron, which we will call the $CQ_3$-qubit. The device layout is the same as for the quadrupolar qubit [18], but here, the qubit states are chosen to be the ground and first excited state of the TQD system. For symmetric tunnel coupling, the qubit excitation energy $E_{01}$ possesses a third-order sweet spot with respect to the detuning $\delta$ at $\delta = 0$ and the specific value $E_M = E_{01}^{opt}$, meaning that $\partial^3 E_{01}/\partial \delta^3 = \partial^3 E_{01}/\partial E_M^3 = 0$ at this point (see Sec. II for details).

To operate the $CQ_3$-qubit, the resonator is coupled to the left quantum dot [see Fig. 1(a)], leading to a dipolar coupling between the qubit states. This is in contrast to the quadrupole qubit, where the resonator is coupled to the middle quantum dot [18] in order to avoid dipolar coupling. The $CQ_3$ regime has the potential advantage that the two logical qubit states are the two lowest energy levels and there is no intermediate leakage state as for the quadrupolar qubit [18]. However, the $CQ_3$-qubit has no sweet spot in $E_M$. Sacrificing the sweet spot in $E_M$ for a higher-order sweet spot in $\delta$ is useful when the dominant noise originates from charge fluctuations at large distances from the qubit [17]. We find this to be crucial for
In order to have a second-order sweet spot ($\delta^2 E_{01}/\delta^2 \delta |_{\delta=0} = 0$), we set the prefactor of $\delta^2$ in (2) to zero. This leads to

$$E_{M}^{\text{Opt}} = -|t|\sqrt{3\sqrt{5} - 4} \approx -0.493 |t|,$$

where the point ($\delta = 0, E_M = E_{M}^{\text{Opt}}$) defines the $CQ_3$ operation point in the parameter space. By symmetry the third derivative $\delta^3 E_{01}/\delta^3 \delta$ also vanishes, yielding a third-order sweet spot in $\delta$ at this point. This third order sweet spot along the $\delta$ axis helps to protect the qubit against noise originating from long-distance noise sources, which mainly cause noise in the $\delta$ parameter. Noise sources in a short distance to the quantum dot would mostly affect $E_M$. The exact influence on $\delta$ and $E_M$ also depends on the angle of the noise source understanding the properties of charge noise in semiconductor devices.

In this paper, we start by presenting the theory of the $CQ_3$-qubit. We then show measurements of the qubit-resonator system in the dispersive and resonant limits, investigate the qubit linewidth as a function of detuning $\delta$. We also develop a noise model explaining our experimental findings.

II. THEORY

In the following, we explore the Hamiltonian of a single electron confined in a TQD, as schematically shown in Fig. 1(a). We first consider the bare qubit Hamiltonian neglecting coupling to the resonator. Subsequently we calculate the coupling matrix element to the resonator which is capacitively coupled to the left plunger gate.

In the position basis $|L\rangle, |M\rangle, |R\rangle$, referring to an electron residing in either the left, middle, or right dot, the Hamiltonian reads [17]

$$H = \begin{pmatrix} \delta/2 & t_L & 0 \\ t_L^\ast & E_M & t_R \\ 0 & t_R^\ast & -\delta/2 \end{pmatrix},$$

where $t_L$ and $t_R$ describe the tunnel coupling between the middle-left and middle-right quantum dots, respectively. The charge occupation of the TQD as a function of $\delta$ and $E_M$ is shown in Fig. 2(a). In the following we do not consider excited states of the quantum dots as they are several hundred GHz away and can therefore be neglected. In the following, we are interested in the symmetric coupling case, where $|t_L| = |t_R| = |t|$. Assuming that the quantity $|\delta/t|$ is small, we separate the Hamiltonian $H$ into the part $H_0 = H(\delta = 0, E_M)$, which can be diagonalized analytically, and the perturbation $H_1(\delta) = H - H_0$. We then perform second-order perturbation theory to obtain the following approximate expression for the qubit excitation energy at $O(|\delta/t|^2)$:

$$H_1(\delta) \approx \frac{E_{M}^2 + 4|t|^2 + 3E_M\sqrt{E_M^2 + 8|t|^2}}{8|t|^2 \sqrt{E_M^2 + 8|t|^2}} \delta^2.$$

with respect to the quantum dots, being electric fluctuations parallel to the QD array the dominant contributions to noise. From Ref. [17] long-distance noise sources are predicted to be dominant. In this case, sacrificing having a sweet spot in $E_M$ for a higher order sweet spot in $\delta$ is favourable. The possibility to distinguish long- and short-distance noise sources makes this qubit interesting and allows for deeper analysis of noise than qubits with only a single detuning parameter. The energy levels of the qubit are plotted in Fig. 1(b) as a function of $\delta$ at $E_M = E_{M}^{\text{Opt}}$. The energy difference between the $|0\rangle$ and $|1\rangle$ states as well as the $|1\rangle$ and $|2\rangle$ states are plotted in Fig. 1(e). The weights of the wave functions as a function of $\delta$ are shown in Fig. 6, see Appendix B. We see a flat dispersion for $E_{01}$ around $\delta = 0$, consistent with the third-order sweet spot.
We next consider the qubit coupling to a resonator connected to the leftmost quantum dot, as indicated in Fig. 1(a). We describe this coupling in the basis $\{ |L\rangle, |M\rangle, |R\rangle \}$ with the coupling Hamiltonian $H_{\text{int}} = G(a^\dagger + a)$, where $a^\dagger$ and $a$ are the creation and annihilation operators for a photon in the cavity, and $G$ is the coupling matrix

$$G = \hbar \omega \frac{\pi Z}{\hbar/\epsilon^2} \begin{pmatrix} \alpha_{L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{R} \end{pmatrix}.$$ 

Here, $\hbar \omega$ is the energy of a photon in the resonator, $Z$ is the resonator impedance, and $\alpha_{L} = \alpha_{L}^R - \alpha_{L}^M$ and $\alpha_{R} = \alpha_{R}^L - \alpha_{R}^M$ are the differences of the lever arms $\alpha_j^R$ of the left plunger gate on dot $j$ ($j = L, M, R$). The lever arm is defined as $\alpha_j = (e\Delta V_j)/\Delta E_j$, where $\Delta V_j$ is the change in gate voltage applied to plunger gate $j$ and $\Delta E_j$ is the change in electrochemical potential of dot $j$. The qubit resonator coupling is then found by transforming $G$ into its representation $\tilde{G}$ in the qubit basis $|0\rangle, |1\rangle, |2\rangle$. Given the matrix $S$ of eigenvectors of the Hamiltonian (1), which we compute numerically, the transformation is achieved by

$$\tilde{G} = SG^\dagger,$$

and the qubit–cavity coupling strength is

$$g = \langle 0\rangle \tilde{G} |1\rangle.$$

The resulting detuning-dependent coupling strength $g(\delta)$ is shown in Fig. 1(d) for $E_{M} = E_{\text{opt}}^M$. For the plot, we use $G_{LM}/h \approx 203.4$ MHz and $G_{RM}/h \approx 62.1$ MHz as extracted from the experiment (see below). Taking into account that the resonator couples more strongly to transitions between the left and the middle dot, we find that $g(\delta)$ exhibits a pronounced maximum at negative values of $\delta$. It arises at the point where the electrochemical potentials of the left and middle dot are aligned, giving rise to a large dipole moment.

### III. EXPERIMENTAL SETUP

Figure 1(a) shows the schematic of the sample. An scanning electron micrograph can be found in Appendix A. The TQD is defined on a GaAs/AlGaAs heterostructure hosting a two-dimensional electron gas 90 nm below the surface by applying voltages between nano-fabricated aluminum gates on the surface and the electron gas. We electrostatically control the tunnel couplings, as well as the electrochemical potentials of the dots, applying negative voltages to the corresponding gate electrodes. We measure the charge state of the TQD with a nearby quantum point contact (QPC) [19,20]. We can apply a drive tone at frequency $v_d$ to the right plunger gate. The left dot plunger gate is coupled to a lambda/4 SQUID-array resonator [5,6,12,13,18,21–29]. Changing the flux $\Phi$ threading the SQUID loops of the resonator, we can tune the resonator’s bare resonance frequency by several GHz. The resonator impedance is $Z \approx 1.1$ kΩ. This enhances the resonator–qubit coupling strength by a factor of approximately 5 compared to standard 50 Ω resonators. For all the experiments presented in this work the average photon number in the resonator is less than one, see Appendix G for details.
IV. RESULTS

In the following, we experimentally investigate the qubit proposed above. We make use of the charge sensing QPC to tune the TQD into the single electron regime. The relevant charge states in the \(|\{L\}, \{M\}, \{R\}\) basis are \(|1, 0, 0\rangle, |0, 1, 0\rangle, \) and \(|0, 0, 1\rangle\). We plot the qubit excitation energy as a function of the dipolar \(\delta\) and quadrupolar \(E_M\) detuning in Fig. 2(a) for \(|t_L|/h = |t_R|/h = 2.5\) GHz. The thin solid lines indicate contours of constant qubit excitation energy. The dashed black lines indicate the charge transition lines. We schematically depict the quantum dot energy levels of the two charge occupations. Charging energies of quantum dots are indicated with roman numbers. We do not consider higher electronic temperature of 30 mK is 625 MHz [13]. The tunnel coupling \(t\) is determined by two tone spectroscopy of the DQD charge qubit formed between the left-middle (right-middle) dots at \(|\delta| \gg |t|\) and negative \(E_M\), indicated by panels I and II in Fig. 2(a) [30]. These measurements also allow us to relate the qubit detuning parameters \(\delta\) and \(E_M\) to combinations of plunger gate voltages by determining the gate lever arms, see Appendix C. For the measurements presented in the following, we set \(|t_L|/h = |t_R|/h = 2.5\) GHz. The corresponding qubit energy \(E_{01}\) at \(\delta = 0\) and \(E_M = E_{01}^{\text{Opt}}\) is 4.2 GHz.

Next, we map out contour lines of the qubit energy using two tone spectroscopy [30]. We apply a drive tone \(\nu_d\) at 4.2 GHz to the right plunger gate while measuring the reflection \(|S_{11}|\) of the probe tone applied at the bare resonator frequency \(\nu_p = \nu_i = 3.791\) GHz. The resonator and qubit interact off resonance, which leads to a dispersive shift of magnitude \(\approx \pm \delta^2/|\nu_i - \nu_q|\) [30]. The measured reflection as a function of the qubit detuning parameters \(\delta\) and \(E_M\) is shown in Fig. 2(b). On resonance the qubit excited state population increases, which leads to a decrease in the magnitude of the dispersive shift [30], and consequently to a measurable change in \(|S_{11}|\). As expected for the higher-order sweet spot discussed above, we find a flat dispersion along \(\delta\), which is indicated by an arrow. Additionally we observe that the reflected resonator signal is stronger for \(\delta < 0\) as for \(\delta > 0\) (Diagonal yellow line extending from the left side). This can be explained by the resonator qubit coupling strength being larger for \(\delta < 0\), see Fig. 2(d), which leads to a stronger dispersive shift of the resonator.

We compare the result to theory by plotting the calculated qubit energy \(E_{01}/|t|\) for \(|t_L|/h = |t_R|/h = 2.5\) GHz as a function of the qubit detuning parameters \(\delta\) and \(E_M\) in Fig. 2(c). The solid black line corresponds to the energy contour of 4.2 GHz. It shows excellent agreement with the measurement in Fig. 2(b).

Next we operate the qubit at the optimal working point \(E_M = E_{01}^{\text{Opt}} \approx -0.493|t|\) [see Eq. (3)] and probe the qubit on resonance \(\nu_i = E_{01}/|t| = 4.2\) GHz with the resonator. At constant resonator frequency \(\nu_i\) we change the qubit frequency by sweeping \(\delta\) and measure the amplitude of the reflected resonator signal \(\nu_p\), as shown in Fig. 2(d). As a consequence of the coherent qubit-photon hybridization we observe two resonance peaks in \(|S_{11}|\) over a broad range of \(\delta\). On resonance these two hybridized states with equal photon-matter character have an energy splitting of \(2g\). The magnitude of this energy splitting changes as a function of \(\delta\) in agreement with our theoretical model shown in Fig. 1(d). The energy of the two states is approximately equally separated from the bare resonance frequency, indicating that the qubit energy is almost equal to that of the resonator (and therefore constant) for the whole range. This is an indication that the qubit dispersion is flat for a certain range in \(\delta\) as one can see in Fig. 1(c).

We simulate the reflectance spectrum \(|S_{11}|\) of the qubit using an Input-Output model taking into account all relevant energy levels of the system [31–34] in Fig. 2(e). We account for the detuning dependent coupling strength \(g(\delta)\) as well as the detuning dependent decoherence rate discussed below see Fig. 4(c), details are found in Appendices I and E. It is important to note that a simple Jaynes Cummings model considering only the \(|0\rangle\) and \(|1\rangle\) states would not reproduce the observed energy splitting of the two resonances with the same parameters.

We now perform two tone spectroscopy in the dispersive limit. For this purpose, we tune the resonator frequency to \(\nu_i = 5.1\) GHz and keep the tunnel couplings at \(|t|/h = 2.5\) GHz. Sweeping the drive tone frequency \(\nu_d\) applied to the right plunger gate, and stepping \(\delta\), we measure the complex
δ = E

element. We extract a qubit decoherence rate squared vs applied probe power. The blue line is a fit to the measurement.

For δ = 0, the qubit linewidth νq has a local maximum. We observe the minimal linewidth at δ = 0 and δ ≈ 6 GHz. The red dashed line indicates the qubit operation point.

FIG. 4. (a) Measurement of the half width half maximum squared vs applied probe power. The blue line is a fit to the measurement. We extract a qubit decoherence rate γ2/2π = 53 ± 2 MHz.

(b) Measurement of the half width half maximum of the qubit resonance as a function of detuning δ. The blue line shows the fit using a noise model taking charge noise and magnetic noise into account.

(c) Simulated qubit linewidth from 1/ν charge noise of amplitude 1 μeV on either δ or EM. (d) Plot of ν0/h as a function of EM for δ = 0 and δ/h ≈ 6 MHz. The red dashed line indicates the qubit operation point.

The blue line is a fit to the measurement. We extract a qubit decoherence rate γ2/2π = 53 ± 2 MHz.

(b) Measurement of the half width half maximum of the qubit resonance as a function of detuning δ. The blue line shows the fit using a noise model taking charge noise and magnetic noise into account.

(c) Simulated qubit linewidth from 1/ν charge noise of amplitude 1 μeV on either δ or EM. (d) Plot of ν0/h as a function of EM for δ = 0 and δ/h ≈ 6 MHz. The red dashed line indicates the qubit operation point.

The blue line shows the fit using a noise model taking charge noise and magnetic noise into account.

We observe the minimal linewidth at δ = 0 and δ/h ≈ 6 GHz. Additionally, we find that the minimum for negative detuning δ is lower than for positive δ.

The blue line in Fig. 4(b) shows the results from a fit using a noise model taking 1/ν charge-noise and magnetic noise into account [36,37]. In order to understand the evolution of the qubit linewidth as a function of δ shown in Fig. 4(b), we investigate the different contributions building up the noise spectrum of the qubit. The main noise source is charge noise acting either on δ or EM. Figure 4(c) shows the qubit linewidth simulated with a 1/ν charge noise model, considering first δ noise with spectral density Sδ(ω) = Aδ/ω and amplitude Aδ = 1 μeV (blue trace), or considering EM noise with SEM(ω) = ASEM/ω and amplitude ASEM = 1 μeV (orange trace). In case the system is affected by pure dipolar charge noise the simulation shows a flat decoherence rate as a function of detuning, being in agreement with the higher-order sweet spot in δ. The finite linewidth at δ = 0 can mostly be attributed to leakage to the second excited state. If, however, the system is affected only by noise in EM there is a local maximum at δ = 0.
because the proposed qubit operation point has no sweet spot in $E_M$. The two minima of the linewidth in Figs. 4(b) and 4(c) close to $|\delta|/\hbar \approx 6$ GHz are due to sweet spots along the $E_M$ axis.

A plot of the qubit energy $E_Q$ as a function of $E_M$ is shown in Fig. 4(d). The linecut at $\delta = 0$ has a nonvanishing slope, whereas the line cut at $|\delta|/\hbar \approx 6$ GHz shows a local minimum at $E_{Q|\delta = 0}$ indicating the DQD qubit sweet spot. For further details, we refer to Appendix I. We conclude that the maximum around $\delta = 0$ originates from a non-negligible noise contribution along the $E_M$ axis. From this result, we can see that also short distant noise sources have a crucial effect on the noise environment acting on the qubit. As mentioned above the main contribution to the asymmetry of the measured linewidth presented in Fig. 4(b) is due to different power broadening for negative/positive detuning $\delta$.

For completeness, we also consider the effect of magnetic noise due to different Overhauser fields in the dots [37]. A plot for Gaussian-distributed magnetic noise either on the left-middle or the right-middle DQD is shown in Fig. 13 in Appendix I. The amplitude of the magnetic noise due to Overhauser fields is between one and two orders of magnitude smaller than the experimentally measured noise and therefore only plays a minor role. A model considering a combination of charge and magnetic noise, together with a constant detuning shift $\delta'$ (which can arise due to a change of the electrostatic environment of the qubit) shows good agreement with the measurement. From a fit, we find $A_q = 1.949 \pm 0.098 \, \mu$eV, $A_{EM} = 0.935 \pm 0.026 \, \mu$eV, corr $= -0.084 \pm 0.177$, and $\delta' = 0.69 \pm 0.171 \, \mu$eV. The magnitude of $\delta'$ is plausible and within the range of electrostatic jumps observed during the measurements. The magnitude of the values found for $A_q$, $A_{EM}$ and corr are in agreement with our previous work [18]. Although we find that $A_{EM} < A_q$ in this experiment, we have shown that the former is still strong enough to dominate other decoherence mechanisms in the device. This is reasonable because the QD qubit was specifically designed to be protected from noise in the $\delta$ parameter only. These results support a growing body of evidence suggesting that a significant fraction of charge noise in semiconductor qubits originates from sources in the immediate vicinity of the quantum dot [18,38].

V. CONCLUSION

In conclusion, we have proposed and measured a single-electron qubit hosted in a triple quantum dot with a third-order sweet spot in the detuning parameter $\delta$. Using two-tone spectroscopy we mapped the qubit energy contour as a function of the two qubit parameters $\delta$ and $E_M$. We observed a well resolvable vacuum Rabi splitting when bringing the cavity and the qubit into resonance. The reflected cavity signal was calculated using Input-Output theory taking all three qubit levels into account. With two tone spectroscopy we mapped out the qubit dispersion for different values of the middle dot potential $E_M$ and found good agreement with calculations of the energy spectra. The qubit is expected to have reduced sensitivity to charge noise in the dot detuning. At the same time, the energy dispersion as a function of middle dot energy is not flat, making the system susceptible to quadrupolar charge noise. Experimentally we observe that the qubit linewidth around $\delta = 0$ is maximal, proving that quadrupolar noise cannot be neglected in this system. We extract a decoherence rate $\gamma_2/2\pi = 53 \pm 2$ MHz in the limit of zero applied drive power. In comparison to the charge quadrupole qubit [18] where $\gamma_2/2\pi = 32$ MHz was reported, we see that protecting only against long distant noise sources is not beneficial. Other triple quantum dot qubit implementations such as the exchange-only qubit [39] and the resonant exchange qubit [13,24] have shown better coherence. By coupling the charge and spin degrees of freedom, decoherence rates of $\gamma_2/2\pi \approx 10$ MHz have been reported. These qubits are limited by the hyperfine noise from the GaAs host material. For these qubits further improvements can be made by switching to silicon [40]. We are also aware of a GaAs triple quantum dot hybrid qubit [41] which reports decoherence rates of 250 MHz. We further investigate the qubit linewidth as a function of $\delta$. We observe a maximum in linewidth at $\delta = 0$ and two local minima at $|\delta|/\hbar \approx 6$ GHz. We find good agreement to a 1/f charge noise model also considering magnetic noise due to hyperfine interaction. These results indicate that the noise affecting the qubit has a non-negligible contribution coming from short-range noise sources, in contrast to the original hypothesis that noise mostly originates from long-ranged sources.

The data used in this work are made available online [42].

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APPENDIX A: SAMPLE

We present a scanning electron image of the sample in Fig. 5. The three quantum dots reside under the plunger gates and are indicated by the red dashed circles. The plunger gate of the left quantum dot is coupled to the $\lambda/4$ resonator. Since the resonator DC potential is defined by the ground, the DC potential of the left quantum dot is tuned by the gate potential $V_L$ as indicated in Fig. 5. Therefore its potential can not be tuned by applying a gate voltage. To tune the potential of the left dot we use the additional gate coming from the top. To check the occupation of the dots, we use the nearby quantum point contact [16,43]. Gates which are not used are grayed out. They were grounded during the experiment. In principle, this
sample allows to form a fourth quantum dot residing right to
the quantum dots used. Also we have the option of using three
different gates to form a QPC. For this work only the most
right gate is used.

APPENDIX B: QUBIT WAVE FUNCTION

In Fig. 6, we show the calculated wave-function weights of the three states of the Hamiltonian (1) as a function of the
dipolar detuning $\delta$ at $E_M = E_{M,0}^{\text{Opt}}$. At $\delta = 0$, all three eigen-
states have the same center of mass.

![Image](image-url)

FIG. 6. Plot of the wave-function components of the three eigen-
states of the Hamiltonian (1) as a function of $\delta$. The dotted line
represent the $|L\rangle$ components, the dashed lines the $|M\rangle$ components
and the solid lines the $|R\rangle$ components.

APPENDIX C: CALIBRATING THE QUBIT

In the following, we present how we tune the TQD into
the correct regime and calibrate the relevant qubit control
parameters. We start by tuning the TQD into the correct
charge state. In the next step we introduce the voltages $V_{4}$ and
$V_{M,0}$, which tune the potentials of the dots in a symmetric or
antisymmetric fashion, respectively. In the last step we present
how we calibrate the tunnel couplings $t_L(R)$, respectively, and
how we convert the measurement axis into frequency space.

Using the QPC, we tune the QPC into the single electron
regime. In the basis $\{|L\rangle, |M\rangle, |R\rangle\}$ of the left, middle, and
right dots, the relevant charge states in the occupation number
representation are $|1,0,0\rangle$, $|0,1,0\rangle$, and $|0,0,1\rangle$, in other
words, having one electron in one of the three dots. In the first
step, we tune the qubit by directly tuning the plunger gate volt-
gages $V_x$, with $x \in \{L, M, R\}$. In the next step, we parametrize
the plunger gate voltages by $V_{4}$ and $V_{M,0}$, corresponding to
symmetric and antisymmetric voltage changes in the TQD.

The change in $V_{M}$ is determined by measuring two charge
stability diagrams as function of $V_{L}$ and $V_{R}$ where we decrease
$V_{M}$ by 1 mV. We find that the quadruple point of the charge
states $|0,1,0\rangle$, $|1,0,0\rangle$, $|0,0,1\rangle$, $|1,0,1\rangle$, $|1,1,0\rangle$, $|1,1,1\rangle$ shifts by 1.5
and 0.4 mV in $V_L$ and $V_R$, respectively. The lever arms for $V_{4}$
are determined by measuring a charge stability diagram as
function of $V_{L}$ and $V_{R}$ and calculating the tilt of the $|0,1,0\rangle$
and $|1,0,1\rangle$ transition when measured in the $V_{L}$ and $V_{M,0}$ basis.
From these measurements, we find

$$ V_{L,0} = V_{L,0} + 1.5V_{M,0} - V_{4} $$

(C1)

$$ V_{M} = V_{M,0} - V_{E,M} $$

(C2)

$$ V_{R} = V_{R,0} + 0.4V_{E,M} + 0.6V_{4} $$

(C3)

where $V_{4,0}$, with $x \in \{L,M,R\}$ are constants and chosen such
that higher electron occupation numbers of the TQD are not
relevant when operating the qubit. This parametrization is
device-dependent and is determined by the inter dot and gate
capacitances [1]. It is interesting to note however, that the
lever arms for $V_{L}$ are lower than for $V_{R}$ being in agreement
with the fact that the plunger gate is further away from the
dot, see Fig. 5.

In a next step, we want to calibrate the tunnel couplings
$t_L$ and $t_R$ of the qubit and relate voltage changes in $V_{E,M}$
and $V_{4}$ to the physical qubit detuning parameters $\delta$ and $E_M$.
By tuning $V_{E,M}$, more positive we can operate the device like a conven-
tional DQD charge qubit formed between the left-middle
(middle-right) DQD for large negative (positive) detuning
voltage $V_{\delta}$, see Fig. 2(a) energy diagrams I and II. We map
the DQD dispersion along $V_{\delta}$ and $V_{E,M}$ for both the left-middle
and right-middle DQDs. We show the measurement of the
left-middle DQD as a function of $V_{E,M}$ in Fig. 7. The red line
shows a fit to the data using the DQD dispersion [1]

$$ E_{\text{DQD}} = \sqrt{(\alpha_{xy}(V - V_0))^2 + (2|\tau|)^2} $$

(C4)

where $|\tau|$ is the tunnel coupling of the respective DQD, $V_0$
is the voltage at which the dispersion has its minimum, $V$
the corresponding detuning voltage and $\alpha_{xy}$ is the lever arm
that converts the voltage into a frequency. The energy at the
From this we extract the tunnel coupling $t_	heta$ and the lever arm $\alpha_{\text{MR}}^\delta$.

minimum of the dispersion is given as $2\gamma$. Using the gate electrodes we tune $2|\Gamma|/\hbar = 5\text{ GHz}$. From the fit of the frequency response measurement discussed in the main text, we find $|\Gamma_L|/\hbar = 2.47\text{ GHz}$ and $|\Gamma_R|/\hbar = 2.48\text{ GHz}$. This change of approximately 2% is attributed to the change of tunnel couplings when changing the detuning parameters.

In the last step, we want to calibrate the voltage changes $V_\delta$ and $V_{\text{EM}}$ to detuning changes in $\delta$ and $E_{\text{M}}$. From the above mentioned two tone spectroscopy measurements, we extract lever arms $\alpha_{\text{XY}}$. More specifically from two tone spectroscopy on the left-middle DQD, we extract $\alpha_{\text{ML}}^\delta$ from spectroscopy along $V_\delta$ and $\alpha_{\text{ML}}^\text{EM}$ from spectroscopy along $V_{\text{EM}}$. For further calculations, we define the following detuning parameters which we later will relate to the measured lever arms.

$$\delta_{LR} = \varepsilon_L - \varepsilon_R = \delta,$$

$$\delta_{\text{MR}} = \varepsilon_M - \varepsilon_R,$$

$$\delta_{\text{ML}} = \varepsilon_M - \varepsilon_L,$$

$$E_{\text{ML}}^\text{EM} = \varepsilon_M - \varepsilon_L,$$

$$E_{\text{MR}}^\text{EM} = \varepsilon_M - \varepsilon_R.$$

Voltage changes $\Delta V_\delta$ and $\Delta V_{\text{EM}}$ relate to the measured lever arms according to

$$\Delta \delta_{\text{XY}} = \alpha_{\text{XY}}^\delta \Delta V_\delta,$$

$$\Delta E_{\text{XY}}^\text{EM} = \alpha_{\text{XY}}^\text{EM} \Delta V_{\text{EM}}.$$

We obtain the relevant lever arms for the TQD by forming the linear combinations

$$\alpha_{\text{LR}}^\delta = \alpha_{\text{MR}}^\delta - \alpha_{\text{ML}}^\delta,$$

$$\alpha_{\text{LR}}^\text{EM} = \alpha_{\text{MR}}^\text{EM} - \alpha_{\text{ML}}^\text{EM},$$

$$\alpha_{\text{EM}}^\delta = \frac{1}{2}(\alpha_{\text{MR}}^\delta + \alpha_{\text{ML}}^\delta),$$

$$\alpha_{\text{EM}}^\text{EM} = \frac{1}{2}(\alpha_{\text{MR}}^\text{EM} + \alpha_{\text{ML}}^\text{EM}).$$

Introducing normalization parameters $A$, we obtain the following group of linear equations:

$$\begin{pmatrix}
1\text{ GHz} \\
0 \\
1\text{ GHz}
\end{pmatrix} =
\begin{pmatrix}
\alpha_{\text{LR}}^\delta & \alpha_{\text{LR}}^\text{EM} & 0 & 0 \\
0 & 0 & \alpha_{\text{MR}}^\delta & \alpha_{\text{MR}}^\text{EM} \\
0 & 0 & \alpha_{\text{EM}}^\delta & \alpha_{\text{EM}}^\text{EM}
\end{pmatrix}
\begin{pmatrix}
A_{\text{X}} & A_{\text{Y}} \\
A_{\text{X}} & A_{\text{Y}}
\end{pmatrix}.$$

Rewriting this equation in more compact form we find

$$\begin{pmatrix}
1\text{ GHz} & 0 \\
0 & 1\text{ GHz}
\end{pmatrix} =
\begin{pmatrix}
\alpha_{\text{LR}}^\delta & \alpha_{\text{LR}}^\text{EM} \\
\alpha_{\text{EM}}^\delta & \alpha_{\text{EM}}^\text{EM}
\end{pmatrix}
\begin{pmatrix}
A_{\text{X}} & A_{\text{Y}} \\
A_{\text{X}} & A_{\text{Y}}
\end{pmatrix}.$$

By solving this equation we end up with the desired normalization parameters.

**APPENDIX D: CHARGE STABILITY DIAGRAM**

We show a charge stability diagram measured at the qubit operating point in Fig. 8 [19,20]. It shows the measured QPC current as a function of the two qubit detuning parameters $\delta$ and $E_{\text{M}}$.

**APPENDIX E: INPUT-OUTPUT THEORY**

In this section, we discuss the details of the Input-Output model used to simulate the reflected resonator signal. We closely follow the approach presented in Ref. [34]. A schematic of the TQD coupled to a cavity including the relevant loss channels is provided in Fig. 9. We consider the input field $\alpha_{\text{in}}$ of the transmission line at frequency $\nu_0$ coupling at rate $\kappa_{\text{ext}}$ to the $\nu/4$ resonator. We collect all internal losses of the resonator in $\kappa_{\text{int}}$. The resonator and the TQD couple with strength $G$, see Sec. II for derivation. For our qubit, this strength depends on the detuning parameter $\delta$. We summarize the losses of the TQD qubit in $\gamma$, neglecting quantum noise. We start with the system Hamiltonian $H_{\text{sys}} = H_C + H_{\text{TQD}} + H_{\text{int}}$ consisting of the cavity Hamiltonian $H_C$, the TQD Hamiltonian $H_{\text{TQD}}$ and the interaction Hamiltonian.
that diagonalizes further calculations, it is convenient to work in the eigenbasis where the coupling matrix being fed by a input line. The input field \( \sigma_{in} \) of the feed line at frequency \( \gamma_p \) couples at rate \( \kappa_{ext} \) to the \( \lambda/4 \) resonator. The internal losses of the resonator are described by \( \kappa_{int} \). The resonator couples to the TQD with strength \( g \). We summarize the losses of the TQD qubit with the rate \( \gamma \).

\[ H_{int} \]

In a second step we solve the equations of motion for the cavity and qubit operators and derive the Input-Output model. The triple quantum dot Hamiltonian is given by

\[ H_{TQD} = \begin{pmatrix} \delta/2 & t_L & 0 \\ t_L^* & E_M & t_R \\ 0 & t_R^* & -\delta/2 \end{pmatrix} \]  

(E1)

The inter dot tunnel couplings of the left-middle and right-middle quantum dots are given by \( t_L \) and \( t_R \) respectively. The qubit detuning parameters \( \delta \) and \( E_M \) are defined as in the main text and indicated in Fig. 1(a). The resonator at resonance frequency \( \omega_C \) is described by the Hamiltonian

\[ H_C = \omega_C a^\dagger a, \]  

(E2)

where \( a \) is the photon annihilation operator. The coupling between the two quantum systems is described by the interaction Hamiltonian \( H_{int} \)

\[ H_{int} = G(a + a^\dagger), \]  

(E3)

where the coupling matrix \( G \) is defined as in the main text. For further calculations, it is convenient to work in the eigenbasis of \( H_{TQD} \). Like in the main text, let \( S \) be the unitary operator that diagonalizes \( H_{TQD} \). With this, the qubit Hamiltonian reads

\[ \tilde{H}_{TQD} = S H_{TQD} S^\dagger = \sum_{n=0}^{n=2} E_n \sigma_{nn}, \]  

(E4)

where \( E_n \) are the ordered eigenvalues of \( H_{TQD} \). The operator \( \sigma_{nn} \) is defined by \( \sigma_{nn} = |n\rangle \langle n| \), where \( |n\rangle \) is the eigenstate at energy \( E_n \). Under the transformation \( S \) the coupling matrix transforms to

\[ \tilde{G} = SG S^\dagger = \sum_{m,n=0}^{n=2} d_{mn} \sigma_{mn}, \]  

(E5)

where \( d_{mn} = d_{mn}^* \) are the transition matrix elements between the different eigenstates. In a next step we transform into the rotating frame of the probe frequency \( \omega_p = 2\pi \gamma_p \). The unitary transformation is given by

\[ U_R(t) = \exp \left[ -it \left( \omega_p a^\dagger a + \sum_{n=0}^{n=2} \omega_p \sigma_{nn} \right) \right]. \]  

(E6)

The total system Hamiltonian \( \tilde{H}_{sys} \) transforms as

\[ \tilde{H}_{sys} = U_R \tilde{H}_{sys} U_R^\dagger + iU_R \tilde{H}_{int} U_R^\dagger, \]  

(E7)

Applying this transformation, we find

\[ \tilde{H}_{TQD} = \sum_{n=0}^{n=2} (E_n - n\omega_p) \sigma_{nn}, \]  

(E8)

\[ \tilde{H}_{C} = \Delta_0 a^\dagger a, \]  

(E9)

\[ \tilde{H}_{int} = \left( a \sum_{n=0}^{n=2} d_{n+1,n} \sigma_{n+1,n} + \text{H.c.} \right). \]  

(E10)

where \( \Delta_0 = \omega_C - \omega_p \) is the detuning of the cavity frequency from the probe frequency \( \omega_p \). We collect the dissipative losses of the system in the term \( \tilde{H}_{diss} \). It takes the internal losses of the cavity \( \kappa_{int} \) and the qubit \( \gamma \) to the environment into account. In the following, we neglect quantum noise within the TQD. Given the Hamiltonian of the TQD system coupled to a resonator we calculate the cavity response using Input-Output theory. The equations of motion for \( a \) and \( \sigma_{n,n+1} \) read as

\[ \dot{a} = [\tilde{H}_{sys} + \tilde{H}_{diss}, a], \]  

(E11)

\[ \dot{\sigma}_{n,n+1} = [\tilde{H}_{sys} + \tilde{H}_{diss}, \sigma_{n,n+1}], \]  

(E12)

Calculating the commutators from above, we find

\[ \dot{a} = -i \Delta_0 a - i \sum_{n=0}^{n=2} d_{n+1,n} \sigma_{n+1,n+1} \]  

(E13)

\[ + \sqrt{k_{ext} a_{in} - \kappa_{int} + \kappa_{ext}} a, \]  

(E14)

\[ \dot{\sigma}_{n,n+1} = -i (E_{n+1} - E_n - \omega_p) \sigma_{n,n+1} \]  

(E15)

\[ - i d_{n+1,n} (p_n - p_{n+1}) a + \frac{\gamma}{2} \sigma_{n,n+1}, \]  

(E16)

where \( p_n \) is the occupation probability of state \( n \), coming from evaluating terms of the form \( [\sigma_{n+1,n}, \sigma_{n,n+1}] \). In thermal equilibrium, the occupation probability is described by Boltzmann statistics

\[ p_n = \frac{\exp \left( -E_n/\kappa T \right)}{\sum_n \exp \left( -E_n/\kappa T \right)}. \]  

(E17)

Solving Eq. (E14) in the stationary limit, we find an expression for \( a/\sigma_{n,n+1} = \chi_{n,n+1} \)

\[ \chi_{n,n+1} = -\frac{d_{n+1,n} (p_n - p_{n+1})}{E_{n+1} - E_n - \omega_p - i\gamma/2} = \chi_{n,n+1} a. \]  

(E18)

Solving Eq. (E16) and using the above expression, we find an equation for \( a/a_{in} \) \[32\]

\[ \frac{a}{a_{in}} = \frac{\omega_C - \omega_p - i \sqrt{\kappa_{ext} \gamma}}{\sqrt{\sum_{n=0}^{n=2} d_{n+1,n} \chi_{n,n+1}}}. \]  

(E19)

Taking into account that we use a \( \lambda/4 \) cavity, we find the relation

\[ a_{ext} = \sqrt{\kappa_{ext} \gamma} - a_{in}. \]  

(E20)
between the output field and the input field operators. Using this expression, we get the coefficient

\[
A = \frac{a_{\text{out}}}{a_{\text{in}}} = \frac{\omega_p - \omega_c + i\Delta_{\text{in}}}{\omega_c - \omega_p + i\Delta_{\text{in}}} + \sum_{n=0}^{\infty} d_{n,n+1} \chi_{n,n+1}.
\]

This coefficient is related to the measured reflectivity by \(|S_{11}| = |A|^2|.

**APPENDIX F: VACUUM RABI SPLITTING**

Additionally to the measurement presented in Fig. 2(d), we present the vacuum Rabi cut at \(\delta = 0\) in Fig. 10. For this linecut we perform 40 repetitions. The dots represent the average of the 40 line cuts. The red region represents the standard deviation of the average. The solid blue line is taken from the Input-Output model calculations presented in Fig. 2(e) directly, not doing any separate fitting. One can clearly resolve the two peaks in the vacuum Rabi splitting and therefore the strong coupling.

**APPENDIX G: PHOTON NUMBER CALIBRATION**

In the following section, we discuss how we calibrate the photon number in the resonator. The strong coupling of a qubit to a cavity radiation file leads to a dressed qubit state whose energy depends on the occupation number \(n\) of the resonator [30]. The dressed qubit frequency \(\tilde{\nu}_q\) is given as

\[
\tilde{\nu}_q = \nu_q + \frac{2n(g/2\pi)^2}{\Delta_{qr}} + \frac{(g/2\pi)^2}{\Delta_{qr}},
\]

where \(\Delta_{qr} = \nu_q - \nu_c\) is the qubit cavity detuning. In this experiment we use the DQD charge qubit formed between the left-middle DQDs to calibrate the photon number \(n\). Changing the flux \(\Phi\) we tune the resonance frequency of the resonator to \(\nu_c = 4.2\) GHz. The qubit frequency is given by \(2\pi \approx 5\) GHz. Using two tone spectroscopy we measure the qubit frequency \(\tilde{\nu}_q\) while increasing the applied probe tone power to the resonator, see Fig. 11. We find a linear change of \(\tilde{\nu}_q\). We determine the slope \(a\) by linear fit. From this we find

\[
n/P = \frac{a\Delta_{qr}}{2(g/2\pi)^2}.
\]

In the experiment, we apply a resonator power corresponding to \(n \approx 0.05\).

**APPENDIX H: SWEET SPOT CHARACTERIZATION**

In previous studies with charge-quadrupole qubits, the information is encoded in the ground and second excited states, allowing to work in a sweet spot for both dipolar and quadrupolar detunings [17, 18]. In the CQ3 qubit, however, the qubit is encoded in the two lowest states, such that the sensitivity to quadrupolar charge noise is sacrificed in favor of an increased insensitivity to dipolar charge noise. This alternative encoding, gives rise to a different sweet spot landscape which is characterized in this section.

In Fig. 12, the qubit energy is plotted as a function of \(\delta\) and \(E_M\) for \(|n_c|/|n_c| = 2.5\) GHz. The black and red contours show the curves in which there is a dipolar and quadrupolar sweet spot, respectively. It can be seen that, for this encoding, simultaneous sweet spots of dipolar and quadrupolar detunings cannot exist. The optimal working point as

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]

\[
E_{01}/h [\text{GHz}]
\]
referred to in the main text, occurs when the two black curves cross, at $\delta = 0$ and $E_M \approx -0.493 \mu$eV. We note that, the condition $|r| = |r_L| = |r_R|$ is necessary for the second-order sweet spot to occur.

**APPENDIX I: NOISE MODEL**

The qubit decoherence rate $\gamma_2$ is obtained by its relation with the HWHM $\partial v_q$ obtained for a given power $P$:

$$\partial v_q = \sqrt{(\gamma_2/2\pi)^2 + \beta P},$$  \hspace{1cm} (11)

where $\beta$ is a constant that is calibrated as previously explained, see discussion of Fig. 4 in the main text. We model the qubit decoherence rate by simulating Ramsey free induction decay in the CQD qubit. We expect the qubit decoherence rate to be dominated by charge fluctuations in the different detuning parameters, and Overhauser magnetic fluctuations.

1. **Magnetic noise**

   While magnetic noise is not the dominant decoherence mechanism, its impact on the qubit is non-negligible. The magnetic noise fluctuations in GaAs are slow compared to the qubit dynamics, hence, allowing to assume a quasistatic Gaussian noise distribution. Following Ref. [37], we name the difference in Overhauser fields between left-middle dots and middle-right dots as $B_L$ and $B_R$, respectively. We note that, since this is a charge-type of qubit, a global shift in the magnetic field has a negligible influence on the qubit coherence, allowing to characterize the magnetic noise fluctuations in three dots with two parameters.

   The Hamiltonian due to Overhauser fields is

$$H = \frac{g\mu_B}{6} \begin{pmatrix} 2B_L + B_R & 0 & 0 \\ 0 & B_L - B_R & 0 \\ 0 & 0 & -B_L - 2B_R \end{pmatrix}.$$  \hspace{1cm} (12)

Assuming a Gaussian distribution of local magnetic fields with standard deviations $\sigma_L$ and $\sigma_R$, the decoherence rate is [24]

$$\frac{\gamma_2}{2\pi} = g\mu_B \sqrt{\frac{h_L^2\sigma_L^2 + h_R^2\sigma_R^2}{2}},$$  \hspace{1cm} (13)

where $h_{L,R} = d\omega_0i/dB_{L,R}$, being $\omega_0$ the qubit frequency. The result of using the previous formula for $\sigma_L = 4 \mu$eV, $\sigma_R = 0$, and vice versa, is shown in Fig. 13. In GaAs the fluctuations of the nuclear magnetic field are expected to be around 2 to 5 mT. The asymmetry of the results in Fig. 4(b) is not caused by these fluctuations. Moreover, due to the small value of the decoherence rate due to this mechanism compared to the observed values, we assume a typical value of $\sigma_L = \sigma_R = 4 \mu$eV in the following. Realistic deviations from this value would be negligible compared to the numbers in Fig. 4(b).

![FIG. 13. Simulated influence of magnetic noise arising from different Overhauser fields in different dots. We assume Gaussian distributed magnetic noise with a standard deviation of $\sigma = 4 \mu$eV on either the left-middle or middle-right DQD. Note the different axis scales compared to Fig. 4(c).](image)

2. **Charge noise**

   We assume the overall charge fluctuations follow a $1/\delta$ spectral distribution $S_i(\omega) = 2\pi A_i/\omega$, where $i$ indicates parameter $i = \delta$, $E_M$, and $A_i$ is its corresponding noise amplitude. These fluctuations induce variations in the chemical potentials of the different dots, such that $\delta \rightarrow \delta + \Delta\delta(t)$, and $E_M \rightarrow E_M + \Delta E_M(t)$. For fitting the observed decoherence rate, we consider three parameters: the dipolar detuning noise amplitude $A_\delta$, the quadrupolar detuning noise amplitude $A_{\delta\rho}$, and the correlation between each dipolar and quadrupolar fluctuations $\rho$. This coefficient allows the dipolar and quadrupolar fluctuations to be correlated. To account for the asymmetry observed in Fig. 4(b), we assume that the strength of the qubit drive tone experienced by the qubit decays with coupling to the drive gate.

   For given values of the three noise parameters in a certain qubit configuration ($\delta$, $E_M$) the procedure to obtain the value of $\gamma_2$ goes as follows. (1) The qubit is initialized in a coherent superposition $|\psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$. (2) Noise fluctuations $\Delta\delta(t)$ and $\Delta E_M(t)$ are generated following the method described in Refs. [18,36]. (3) The qubit is left to evolve under the noise fluctuations for a time $\tau = 100$ ns. (4) The evolution of the qubit coherence $\rho_{01}(t)$ is saved. (5) The previous steps are repeated 5000 times. (6) The multiple resulting evolutions of the qubit coherence are averaged. (7) The value of $|\rho_{01}(t)|$ follows a decay law $|\rho_{01}(t)| = \exp(-\gamma_2 t/2\pi)^2$. Fitting to such function provides the decay rate $\gamma_2$ and the exponent $\beta$.

   The result of applying this procedure to the simple cases $A_\delta = 1 \mu$eV, $A_{\delta\rho} = 0$, and vice versa, are shown in Fig. 4(c). To fit the result in Fig. 4(b), this procedure is repeated over a grid in $\Delta\delta$, $A_\delta$, $A_{\delta\rho}$, and $\rho$ for $\Delta E_M = E_M^{\text{Opt}}$. This grid is then interpolated into a function to which we add the decoherence rate from magnetic fluctuations. The interpolated function is then used to fit Fig. 4(b). The result of the fit gives $A_\delta = 1.949 \pm 0.098 \mu$eV, $A_{\delta\rho} = 0.935 \pm 0.026 \mu$eV, corr. $= -0.084 \pm 0.177$, with a shift in detuning $\delta' = 0.69 \pm 0.171 \mu$eV. This implies that the dipolar detuning noise is dominating over the quadrupolar noise, in a similar ratio to the one observed in a previous work [18].
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