The effect of empirical log yield observations on carbon storage economics

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Abstract

An empirical model for log yield from trees is established and applied in microeconomics of carbon storage in a boreal spruce estate. The transition from pulpwood to sawlogs is a smoother function of stem diameter in the empirical data, in comparison to literature values. Correspondingly, the value transition of trees along with increasing size is gentler. Due to price premium of sawlogs from clearcuttings, all economically feasible treatment schedules terminate in clearcutting. Best capital return rates are gained with two heavy thinnings from above before clearcutting. Present carbon emission prices allow moderate carbon storage increment if the increment is compensated by proportional carbon rent. Doubling the present carbon prices would allow strong carbon storage increments if compensated by carbon rent. Application of nonproportional carbon rent is proposed.

Keywords: capitalization; capital return rate deficiency; expected value; carbon storage; timber stock; carbon rent

1. Introduction

Boreal forests constitute a potentially significant carbon sink. A particular benefit of boreal regions is significant carbon storage in the soil. It has been approximated that the amount of soil carbon may exceed the carbon storage in living biomass [1–4]. However, living biomass produces the litter resulting in soil carbon accumulation. The rate of carbon storage depends on the rate of biomass production on the site. The biomass production rate, in turn, is related to the amount of living biomass [2,5–7].

In the occurrence of clearcutting and soil preparation, the net release of carbon from the soil to the atmosphere begins [8–11]. Correspondingly, maintenance of canopy cover possibly is essential in carbon sequestration.
Human activity related to the sequestration and release of carbon in the atmosphere forms a complex system of industries and disciplines [12–16]. Instead of trying to propose some kind of holistic, interdisciplinary optimum, we here discuss microeconomics of carbon sequestration in forestry, focusing on the estate level.

Within the business of forestry, a straightforward policy might appear as a carbon trade system, possibly administered by public agents [17]. An unbiased carbon trade, however, would incorporate a high initial expense [18]. Fortunately, it has been recently shown that a carbon rental system is equivalent to the carbon trade system, without much of an initial expense [18]. In general, the amount of productive biomass is not constant within any forest estate. The canopy cover is also not constant, but rather subject to change over time. A natural reason for the variation in time is that any estate has some variety of stand ages and stand biomass densities. The development of the state of any particular estate can be designed in terms of some sort of dynamic programming [19].

From the viewpoint of generic instructions, or policy actions, it might be beneficial to reduce the variety of initial estate states by adopting some kind of unifying boundary conditions. A tempting candidate is the normal forest principle [20]. This principle simply refers to postulating that stand ages are evenly distributed and stand characteristics are uniquely determined by stand age. Such a postulation, even if often departing from reality, simplifies many treatments, producing idealized systems that are stationary in time.

Within the normal forest principle, quantities such as rotation age and average stand trunk volume per area unit become well defined, not only for a single stand but also on the estate level. Correspondingly, microeconomic discussion regarding such quantities becomes simplified. One can even state that the normal forest principle allows for determination of microeconomically optimal rotation age, as well as an expected value of stand volume.

Microeconomically optimal rotation age and expected value of stand volume are not necessarily optimal from the viewpoint of national economics, nor the viewpoint of carbon sequestration. It has been recently shown that microeconomics often favors solutions with relatively low capitalization [19,21,22,23]. Low capitalization has adverse effects on volumetric growth rate, as well as litter accumulation rate [21,24,23].
A rather recent paper clarified microeconomics of carbon sequestration within a boreal spruce forest estate [23]. It was found that the best capital return rate is achieved by multiple repeated thinnings from above, to the transition diameter where sawlogs are gained instead of pulpwood only [21,22,23]. Increased carbon storage is most economically achieved by increasing the harvesting limit diameter [23]. However, carbon rent derivable from present carbon market prices appeared not high enough to compensate for the induced capital return deficiency [23].

There are some possibly important sources of uncertainty in the recent treatment [23]. Firstly, the growth model applied in this study discussed stem sizes at 50 mm intervals [25]. The main transition of assortments from pulpwood to sawlogs happens between diameter classes centered at 175 mm and 225 mm [26,27]. Correspondingly, computations implemented in the paper assumed cutting limit diameters at diameter-class boundaries 200, 250, and 300 mm.

Secondly, literature values [26,27] regarding the yield of sawlogs and pulpwood from trees of different sizes were missing experimental verification. Even if such values have been used in a variety of investigations [26,21,22,23], the origin of the applied data has not been rigorously reported.

This paper intends to clarify the uncertainties mentioned above. An empirical log yield data is collected and used in the valuation of trees of different sizes. The resolution of the applied growth model is refined and used in a more accurate determination of financially appropriate harvesting schedules, as well as the consequences of intentionally increased carbon storage.

First, experimental materials are presented, as well as related computational methods. Then, microeconomic methods, as well as carbon rent formulae are introduced. Third, results are reported, for six different kinds of treatment schedules. Finally, the outcome is discussed, and a carbon sequestration subsidy system is proposed.

2. Materials and Methods

2.1. Empirical Observations
Eleven circular plots of area 314 square meters were taken from typical spots of 11 spruce-dominated forest stands in November 2018 at Vihtari, Eastern Finland. Seven of the stands had experienced only young stand cleaning, whereas four of the stands were previously thinned commercially. Breast-height diameters were recorded, as well as tree species, and a quality class was visually determined for any measured tree.

On measured plots on sites without any previous commercial thinning, the basal area of tree trunks at breast height varied from 32 to 48 m$^2$/ha, and stem count from 1655 to 2451 per hectare. On measured plots on stands previously thinned commercially, the basal area varied from 29 to 49 m$^2$/ha and the stem count from 891 to 955 per hectare. Further details of the experimental stands are reported in earlier papers [21].

The particular estate is characterized by measurements from the 11 sample plots. However, such a sample does not necessarily represent the entire estate accurately. More importantly, the sampling does not conform to the normal forest principle, with assumptions of constant age distribution and stand characteristics uniquely determined by stand age. Here, we utilize the normal forest principle in terms of establishing a “normal stand” based on the observations and then approximating the development of this “normal stand” as a function of stand age.

The normal stand is established on the basis of two never-thinned sample plots of medium site fertility and younger range of stand age, among the empirical observations. The two sample plots were combined into one experimental plot of area 628 square meters. Within the representative sample plot, the basal area of acceptable-quality trees was 35.3 m$^2$/ha, and the corresponding stem count 1401 per hectare. The age of the normal stand was 35 years, and the dominating tree height 15 m.

Instead of using literature values [26,21,22,23] as an estimate of the yield of sawlogs and pulpwood from any tree of particular breast-height diameter, a dataset of 6123 spruce trees were collected. The data was collected by four different single-grip harvesters, operated by six individuals at four harvesting sites. Log cutting instructions provided by three different sawmilling companies were applied. One thinning site was located at Vihtari, two thinning sites and one clearcutting site at Ilomantsi, all in Eastern Finland. The proportion of sawlogs of the total harvester-measured volume, as a function of breast height diameter, is shown in Fig. 1. Fig. 1 also shows the literature values used in [26,21,22,23]. It is found that the empirical
sawlog yield function is smoother than the function taken from literature. Some sawlogs are gained from trees of diameter 175 mm, unlike in the literature data. On the other hand, the sawlog proportion of trees thicker than 200 mm is smaller in the empirical function. The latter probably is due to harvester operators being trained not to produce sawlogs that would be rejected at the mill.

![Figure 1](image_url)

**Fig. 1.** Literature values, as well as empirical values of sawlog proportion within the commercial section of spruce stems of different sizes.

### 2.2. Growth Model

For prognostication of further development of the normal stand, some kind of a growth model is needed. The growth model of Bollandsås et al. [21,22,25] is adopted, discussing not only growth but also mortality and recruitment. The original growth model [21,22,25] discussed 50 mm breast-height diameter classes within a temporal resolution of five years. Any diameter class was represented by its central tree, and the process of growth was described in terms of the probability of any tree to transfer to the next diameter class [21,22,25]. The underlying idea of the description of growth is that any tree either remains in the same diameter class or transfers to the next diameter class within the five-year time interval. This underlying idea naturally greatly simplifies computation.
It is found from Fig. 1 that according to the literature values of sawlog yield, there is a sharp transition between diameter classes centered at 175 and 225 mm. Such a sharp transition would induce a huge value increment. The empirical yield function being smoother, there is a need for a greater resolution in tree size description. It is not too complicated to modify the growth model from the size resolution of 50 mm to 25 mm. However, to retain the underlying principle of the growth model, this requires a corresponding change in the temporal resolution, from 5-year time steps to 30 months.

The simultaneous change in the time and size resolutions retains the probability of any tree to transfer to the next diameter class. The same is not true in the case of recruitment and mortality. The latter two quantities are affected by the time resolution only. Correspondingly, the recruitment and mortality values become scaled along with the time step by a factor of $\frac{1}{2}$. 

For any 25-mm diameter class of trees, the volumetric amount of two assortments, pulpwood and sawlogs, are taken from the empirical observations as the expected values for any diameter class.

Description of stand development until the time of observation in 2018 requires another kind of approach. An exponential growth function was fitted between approximated worthless initial volume of 15 m$^3$/ha and the 2018 commercial volume estimate of 274 m$^3$/ha.

Possibly the simplest way to approximate financial history is to determine an internal rate of operative return for the period from stand establishment to 2018. In other words, we require

$$V(\tau_3)e^{-\tau_3} - R(\tau_1)e^{-\tau_1} - C(\tau_2)e^{-\tau_2} = 0$$  \hspace{1cm} (1)$$

where $R$ is regeneration expense at regeneration time $\tau_1$, $C$ is young stand cleaning expense at cleaning time $\tau_2$, and $V$ is stumpage value of trees at observation time $\tau_3$. In this study, the observation time $\tau_3$ corresponds to November 2018, regeneration time is clarified according to known stand age, and young stand cleaning is assumed to have occurred ten years after regeneration. It is assumed that prices and expenses do not evolve in real terms, and thus presently valid expenses can be used in Equation (1). The regeneration expense is taken as 1250 Eur/ha, and young stand cleaning 625 Eur/ha.
It is worth noting that the operative internal rate of return \( s \) in Equation (1) does not correspond to the capital return rate in the entire activity, as the latter depends on nonoperative capitalization such as bare land value.

2.3. Financial Considerations

To determine a momentary capital return rate, we need to discuss the amount of financial resources occupied \([21,24,28,23]\). This is done in terms of a financial potential function, defined in terms of capitalization per unit area \( K \). The momentary capital return rate becomes

\[
r(t) = \frac{dK}{K(t)dt}
\]

(2)

where \( K \) in the numerator considers value growth, operative expenses, interests and amortizations, but neglects investments and withdrawals. In other words, it is the change of capitalization on an economic profit/loss basis. \( K \) in the denominator gives capitalization on a balance sheet basis, being directly affected by any investment or withdrawal. It is worth noting that timber sales do not enter the numerator of Equation (2): selling trees at market price levels does not change the amount of wealth, it only converts wealth from trees into the form of cash. However, harvesting naturally changes capitalization appearing in the denominator of Equation (2). Harvesting also likely changes the change rate of capitalization occurring after the harvest. Equation (2) gives a momentary capital return rate, not necessarily sufficient for management considerations. By definition, the expected value of capitalization per unit area is

\[
\langle K \rangle = \int_{-\infty}^{\infty} p(K)KdK
\]

(3)

where \( p(K) \) is the probability density function of capitalization \( K \). By change of variables we get

\[
\langle K \rangle = \int_{0}^{\tau} p(K)K \frac{dK}{d\tau}d\tau = \int_{0}^{\tau} p(\tau)K(\tau)d\tau
\]

(4)

where \( \tau \) is the rotation age. The expected value of the change rate of capitalization is
\[ \left\langle \frac{d\kappa}{dt} \right\rangle = \int_0^r p(a) \frac{d\kappa(a)}{dt} da \]  \hspace{1cm} (5)

Correspondingly, the expected momentary rate of relative capital return is

\[ \left\langle r(t) \right\rangle = \frac{\left\langle \frac{d\kappa}{dt} \right\rangle}{\left\langle K \right\rangle} = \frac{\int_0^r p(a) \frac{d\kappa(a,t)}{dt} da}{\int_0^r p(a) K(a,t) da} = \frac{\int_0^r p(a) K(a,t) r(a,t) da}{\int_0^r p(a) K(a,t) da} \]  \hspace{1cm} (6)

We find from Equation (6) that the expected value of capital return rate within an estate generally evolves in time as the probability density of stand ages evolves. However, Equation (6) can be simplified to be independent of time by adopting the normal forest principle, where stand age probability density is constant [20]. Besides, the constancy of the expected value of capital return rate in time requires that prices and expenses do not evolve in real terms. Then, the expected value of the capital return rate becomes

\[ \left\langle r \right\rangle = \frac{\int_0^r \frac{d\kappa(a)}{dt} da}{\int_0^r K(a) da} = \frac{\int_0^r K(a) r(a) da}{\int_0^r K(a) da} \]  \hspace{1cm} (7)

It has been recently shown that Equation (7) corresponds to the ratio of the partition functions of the change rate of capitalization and capitalization itself [29]. It also has been recently shown that the maximization of the net present value of future revenues may result in financially devastating consequences [30]. Momentary capital return rate as given in Equation (2) was introduced in 1860 [28]; an expected value was mentioned in 1967 [31,32], however applications have been introduced only recently [21,22,29,30,23].

High capital return rates are gained by an improvement harvesting including diameter-limit cutting to the transition diameter between pulpwood and sawlogs [21,23]. It may be possible to retain a state of high capital return for decades, implementing further diameter-limit cuttings frequently, provided there is an abundant supply of pulpwood-size stems of an at least semi-shade-tolerant tree species [21,23]. In this investigation, however, the focus is on procedures inducing an increment of biomass density on the one hand, and extension of semiclosed canopy cover on the other hand. The former can simply be achieved by applying cutting diameter limits.
greater than the transition diameter between pulpwood and sawlogs. It is however worth noting
that the application of the empirical yield data may change any of the results achieved in
previous investigations.

In the context of the improvement harvesting, 20% of the stem count of good-quality trees is
removed in all diameter classes due to the establishment of striproads.

The stumpage value is determined in terms of roadside price, deduced by harvesting expense.
We here use the roadside prices recently applied by Parkatti et al. [33], 34.04 Eur/m3 for spruce
pulpwood, and 58.44 Eur/m3 for sawlogs. We further use the same harvest-expense function
as Parkatti et al. [33], stated to be based on a productivity study of Nurminen et al. [34],
however with one correction. Model parameter C5 value is taken as 2, instead of 1. With this
correction, the expense function corresponds to present local circumstances, including the
transfer expense of machinery. In addition to the expense function, we include a fixed
harvesting entry expense per hectare. Justification of the latter is that some sites require at least
partial pre-harvest cleaning. The entry expense is approximated as 200 Eur/ha. Again, is
assumed that prices and expenses do not evolve in time. In other words, the capital return rate
is discussed in real terms.

The original version of the growth model operates in five-year time steps [21,22,25,23], as
discussed above. Consequently, an eventual harvesting entry may take place every five years.
According to recent investigations, it often is favorable to harvest every five years [21,23].
However, the fixed harvesting entry cost of 200 Eur/ha restricts low-yield harvesting entries.
Numerical investigations indicated that it is not reasonable to harvest if the yield would be less
than 20–30 m²/ha. Such an entry limit is in concert with practices applied in the area.

The feasibility of harvesting is investigated in five-year intervals, always considering two
options: thinning or clearcut. Clearcutting expenses are lower than thinning harvesting costs,
according to Parkatti et al. [33], stated to be based on a productivity study of Nurminen et al.
[34]. Besides, a 15% clearcutting premium for the roadside price of sawlogs is applied,
following with local tradition. The premium, as well as the harvesting cost reduction, are
applied within the last 30-month period before eventual clearcutting.
Thinning procedures are iteratively designed to maximize the expected value of capital return rate, up to the rotation age providing the maximum expected value. After the maximum expected value, any thinning procedure is designed to maximize the capital return rate within the next five years.

In thinnings from above, applied cutting diameter limit is used as a restriction. The applied restrictions are the upper limits of 25-mm diameter classes centered at 175, 200, 225, and 275 mm. Also, a case of heavy thinning is investigated. This case corresponds to removing 50% of good-quality trees in all diameter classes in the context of the improvement harvesting.

In addition to high thinnings intended to maximize capital return rate [21,22,29,30,23], the consequences of following semi-official silvicultural guidance commonly applied in the area [35] are discussed. Thinnings are predominantly applied from below, and any rotation is terminated in clearcutting. The first thinning from below, combined with quality thinning, as well as opening strip roads, is conducted to basal area 21 m²/ha. The second thinning is implemented when the basal area exceeds 32 m²/ha, and it is conducted to basal area 25 m²/ha. No further thinnings from below are applied as they do not tend to increase the capital return rate.

Application of Equation (7) results as an expected value of capital return rate for any treatment schedule investigated. Treatment schedules not corresponding to the most economical one induce some amount of capital return rate deficiency, in terms of percentage per annum. Such deficiency is related either to the amount of timber stock deviating from the optimal timber stock, or the rotation age deviating from the optimal rotation age, or both. In the former case, the deficiency often is due to an excess standing volume, which allows for expressing the deficiency per excess volume unit.

2.4. Carbon Rent Considerations

The last issue in this section of methods regards carbon trade and carbon renting. It is worth noting that carbon prices or rents do not enter the analysis described above in any way. Carbon prices and rents are discussed to enable a comparison of the capital return deficiency per excess volume with any hypothetical carbon rent.
It has been recently shown that policies based on carbon rent are equivalent to policies based on carbon sequestration subsidies and taxies [18]. Unbiased carbon sequestration trade would require a huge initial investment; correspondingly mostly carbon rent procedures are practically feasible [18]. We will here present a brief derivation of the equivalency of the two principles of subsidies, however adopting boundary conditions possibly less restrictive than those of Lintunen et al. [18].

Let us establish a carbon sequestration subsidy system at a particular time $\tau_1$. Within a time range up to time $\tau_2$, the total carbon trade compensation is

$$p_\tau C_\tau + \int_{\tau_1}^{\tau_2} d(pC) dt = p_\tau C_\tau + \int_{\tau_1}^{\tau_2} \left[p \frac{dC}{dt} + C \frac{dp}{dt} \right] dt \tag{8}$$

where $p_\tau$ is carbon price at time $t$, and $C_\tau$ is carbon inventory at time $t$. On the other hand, the revenue from carbon rentals is

$$\int_{\tau_1}^{\tau_2} uC dt \tag{9}$$

where $u$ is the rent rate per carbon unit. Now, to establish equivalency between the carbon storage trade and rent, Equations (8) and (9) must become equal. Equality naturally should apply in any possible circumstance. One of the circumstances is that the time change rate of prices, as well as inventories, is zero. In such a case, the latter term of Equation (8) vanishes. Consequently, the long-term flow of carbon rents should equal a one-time initial storage purchase payment. If the duration of the rent payments extends towards infinity, the only possibility is that the present value of rent payments forms a contracting series. One possibility of such a contracting series is

$$u_\tau C_\tau \int_{0}^{\infty} e^{-qt} dt = p_\tau C_\tau \tag{10}$$

where $q$ is a discount rate. The corresponding solution for the carbon rent rate is

$$u_\tau = qp_\tau \tag{11}$$

One can readily show that Equation (11) applies not only to a steady-state of Equations (8) and (9) but also to any incremental carbon price and inventory.
3. Results

Figure 2 shows the capital return rate according to Equation (2) for any 30-month period after the improvement harvesting for four different cutting limit diameters, the heavy thinning-case, as well as with thinning from below. In any 30-month annualized capital return rate, there is no integration over the site lifespan, neither is any clearcutting price premium or clearcutting harvesting pricing considered. The latter issues are neglected in Fig. 2 since they would indicate clearcutting and thus specify rotation age.

The lower is the cutting limit diameter, the higher is the capital return rate in Fig. 2. The case of the heavy thinning falls to the midrange, thinning from below being the worst performer. Nonsmoothness of the curves is due to irregular harvesting. Typically, there is a depression in the capital return rate during 30-month periods including thinning, for two reasons. Firstly, there often is greater capitalization before the thinning is implemented. Secondly, thinning-induced harvesting entry expense reduces net value increment.

It is worth noting that in Fig. 2, the leftmost data point within any curve corresponds to capital return rate appearing during the first 30-month period after observations in November 2018.

![Graph showing capital return rate for different treatments after 30-month periods.](image)

Fig. 2. Capital return rate according to Eq. (2) for six different treatments applied on a normal stand representing the example estate. The capital return rate is computed for any 30-month
period, without integration over the site lifespan, and without considering any clearcutting price premium or clearcutting harvesting pricing.

The expected value of capital return rate, integrated over the stand lifespan according to Eq. (7) is shown in Fig. 3. Any feasible treatment cycle terminates in clearcutting, and thus utilizes the premium in clearcutting price, as well as the harvesting expense reduction. The achievable expected value of capital return rate is the greater the smaller the cutting diameter limit. However, the difference between 188 and 213 mm limits is small, the latter mostly renders a younger rotation age. Heavy thinning induces a somewhat smaller expected value of capital return rate than the 238 mm cutting diameter limit. 288 mm cutting diameter limit induces a monotonically decreasing capital return rate, indicating that the suitable rotation age would be 40 years. In that case, the 288 mm limit would actually not be used, since there would be only one thinning, referring to improvement harvesting without any thinning from above, corresponding to opening striproads and removing bad-quality trees. Also thinning from below shows a monotonically decreasing capital return rate, along with the worst overall performance. In the last case, the second thinning is not to be implemented if the capital return rate is maximized.

![Fig. 3. The expected value of capital return rate for six different treatment schedules applied on a normal stand representing the example estate. The capital return rate is integrated according to Eq. (7) over stand age until any rotation age. The clearcutting price premium is applied within the terminal 30 months, as well as clearcutting harvesting pricing.](image-url)
Figure 4 shows representative commercial stand volume for any 30-month period after the improvement harvesting for four different cutting limit diameters, the heavy thinning-case, as well as with thinning from below. The higher is the cutting limit diameter, the higher is the total volume of trees. Thinning from below results in a higher range of stand volumes, however, showing a relative decline along with time. The case of heavy thinning results in a low stand volume, however increasing in its relative position since there is a large delay before a such stand can be thinned again.

The integrated (average) stand volume increases with rotation age in all cases (Fig. 5). The greatest volume increment is gained by thinning above with the 288 mm cutting diameter limit, followed by thinnings from below. Initial heavy thinning leads to average stand volumes of the lower range.

![Stand volume, m³/ha](image)

Fig. 4. Commercial stand volume for six different treatments applied on a normal stand representing the example estate. A representative stand volume is computed for any 30-month period, without integration over the site lifespan.
Fig. 5. The expected value of commercial stand volume for six different treatment schedules applied on a normal stand representing the example estate. The volume is integrated over stand age until any rotation age.

Representative capitalization per hectare for any 30-month period (Fig. 6) differs from the stand volume (Fig. 4) most in the case of thinning from below. This due to the removal of the least valuable trees in thinning. The heavy thinning case also shows greater capitalization, in relation to stand volume, partially for the same reason. Again, the capitalization within the 30-month periods does not consider any premium in clearcutting sawlog price, neither the harvesting expense reduction, since such measures would indicate clearcutting and thus specify rotation age.
Fig. 6. Capitalization per hectare for six different treatments applied on a normal stand representing the example estate. The capitalization is computed for any 30-month period, without integration over the site lifespan, and without considering any clearcutting price premium or clearcutting harvesting pricing.

In integrated (average) capitalization increases monotonically along with rotation age, except for of the two lowest cutting limit diameters (Fig. 7). The average capitalization does consider the premium in clearcutting sawlog price, as well as the harvesting expense reduction, as it is computed as a function of rotation age. The average capitalization of thinning from above with the 288 mm limit almost unifies with thinning from below (Fig. 7), unlike the case of average volume (Fig. 5). For the second half of rotation ages, the heavy thinning case almost unifies with the 238 mm thinning (Fig. 7), unlike in the case of average volume (Fig. 5).

Fig. 7. The expected value of capitalization for six different treatment schedules applied on a normal stand representing the example estate. The capitalization is integrated over stand age until any rotation age.

Figure 8 shows the annual growth rate per hectare for any 30-month period after the improvement harvesting for four different cutting limit diameters, the heavy thinning-case, as well as with thinning from below. The higher is the cutting limit diameter, the higher is the growth rate. The heavy thinning case is the worst performer after first thinning but improves along with time. Thinning from below results in a strongly declining growth along with time.
Fig. 8. Net annual growth rate per hectare for six different treatments applied on a normal stand representing the example estate. The growth rate is computed for any 30-month period, without integration over the site lifespan.

The integrated (average) growth rate declines monotonically in three cases and shows a maximum in Fig. 9 in the remaining three cases. The expected value of growth rate over the rotation age is the greater the larger is the harvesting limit diameter. At young rotation ages, the heavy thinning performs worst. At older rotation ages, thinning from below is the worst performer.

Fig. 9. The expected value of net annual growth rate for six different treatment schedules applied on a normal stand representing the example estate. The growth rate is integrated over stand age until any rotation age.
As Figure 3 shows the maximal expected value of capital return rate is found with thinnings from above with 188 mm cutting diameter limit and rotation age 60 years, other treatments are principally deficient. The deficiencies are plotted in Fig. 10. At the youngest rotation ages, the most severe thinnings show the greatest deficiencies. At greater rotation ages, thinning from below shows superior deficiencies, followed by thinning above with 288 mm cutting diameter limit, and the heavy thinning. 238, 213 and 188 diameter limits do not differ drastically, but in all cases increasing rotation age induces a significant capital return rate deficiency.

![Graph of capital return rate deficiency](image)

Fig. 10. The deficiency of the expected value of capital return rate, in comparison to maximum value available according to Eq. (7) and Fig 3. The deficiency is plotted for six different treatment schedules and any rotation age.

The capital return rate deficiency in terms of percentages (Fig. 10) can be converted to Euros per hectare and year by multiplying by the capitalization appearing in Fig. 7. The result is shown in Fig. 11. It is found that the deficiency may be hundreds of Euros per hectare and year. Doubling the expected value of stand volume (Fig. 5) would induce a capital return deficiency in the order of 150 Euros per hectare and year (Figs. 5 and 11).
Fig. 11. The deficiency of the expected value of capital return rate in terms of Euros per hectare and year. The deficiency is plotted for six different treatment schedules and any rotation age.

From the viewpoint of any carbon rent policy, or any other carbon sequestration policy, it is of interest to consider what is the capital return rate deficiency due to any excess stand volume. First, the excess commercial stand volume in cubic meters per hectare is plotted in Fig. 12. This plot actually is a more detailed analysis of the data appearing in Fig. 5. The expected value of stand volume (Fig. 5) corresponding to the maximum value of the expected capital return rate (Fig. 3) is 104 m³/(ha•a). Figure 12 reveals that the excess stand volume achievable may be more than that.
Fig. 12. The expected value of stand volume over the stand volume providing the greatest expected value of capital return rate according to Eq. (7) and Fig. 3. The excess volume is plotted for six different treatment schedules and any rotation age.

The annual capital return rate deficiency per excess stand volume is shown in Figure 13. Figure 13 simply shows the deficiency per hectare shown in Fig. 11 divided by the excess volume per hectare shown in Fig. 12. It is found that thinning from below always shows a deficiency of at least 2 Eur/(excess m$^3$/a), heavy thinning performing somewhat better at greater rotation ages.

![Capital return rate deficiency, Eur/(Excess m$^3$/a)](image)

Fig. 13. The deficiency of the expected value of capital return rate in terms of Euros per excess volume and year. The deficiency is plotted for six different treatment schedules and any rotation age. Negative values are omitted from the Figure.

To establish strategies for financially effective carbon storage, it is beneficial to plot the annual capital return rate deficiency per excess stand volume as a function of excess volume, instead of stand age (Fig. 14). We find that the financially most effective way of increasing timber volume is to increase the cutting diameter limit. An excess volume up to 30 m$^3$/ha is best gained by increasing the cutting diameter limit to 213 mm, then up to 50 m$^3$/ha by increasing the cutting diameter limit to 238 mm. An excess volume greater than 60 m$^3$/ha requires a cutting diameter limit greater than 238 mm (Fig. 14).
Fig. 14. The deficiency of the expected value of capital return rate in terms of Euros per excess volume and year. The deficiency is plotted for six different treatment schedules as a function of excess volume. Negative values are omitted from the Figure.

4. Discussion

Issues related to increment of biomass density on the one hand, and extension of semiclosed canopy cover, on the other hand, have been discussed at the estate level, with application to fertile boreal spruce estates. The results of this investigation can be readily compared with a recent paper that utilized similar methodology but different data for the yield of logs from trees, as well as a coarser version of the growth model [23].

The most striking difference of the present results to the recent ones [23] is that all financially feasible treatment cycles terminate in clearcutting, instead of multiple thinnings from above slowly leading to diminishing growth and yield. A simple reason for this difference is the gradient in the sawlog proportion shown in Fig. 1. In the recent paper, the sharp value increment from 175 mm to 225 mm of diameter favored multiple thinnings from above, utilizing the value transition [23]. The smoother value transition in the present paper redirects the focus from the utilization of a single value step towards simultaneous utilization of the clearcutting premium and control of capitalization through thinnings from above.

In the recent paper [23] the capital return rate was not sensitive to rotation age. In the present results, it is sensitive to rotation age (Fig. 3). Also this difference is due to the yield functions.
Sharp value transition made it possible to utilize the value step even with low stem count, within a wide temporal tolerance [23]. Slow recruitment, along with a smooth value transition applied in this paper makes the capital return rate sensitive to rotation age, the optimal rotation age depending on the applied thinning practices (Fig. 3).

In the recent paper [23] the capital return rate was sensitive to cutting limit diameter. In the present results, such dependency is much less (Fig. 3). Again, the difference is due to the yield functions. Sharp value transition between 175 mm and 225 mm diameter induced low value increment rates for trees of at least 250 mm of diameter [23]. In the present paper, the smooth value transition allows a range of cutting limit diameters where the capital return rate does not differ much, the thinning pattern mostly affecting the suitable rotation age (Fig. 3).

Manipulation of biomass density is possible even if the rotation age would not be manipulated. The possibilities for this are particularly pronounced if thinnings are done from above (Fig. 5). It is possible to double the density of living biomass, in comparison to the financially optimal treatment schedule (Fig. 5), and consequently, the growth rate is increased (Fig. 9). However, another consequence is a deficiency in the capital return rate (Figs. 3 and 10).

It is possible to allocate the capital return rate deficiency to the excess value of commercial stand volume. With prices applicable at the time of writing, the deficiency per excess volume is in the order of 1.5 Euros per excess standing cubicmeter, if the biomass density is doubled (Fig. 14). With smaller excess volume, the specific deficiency is less (Fig. 14).

An applicable carbon rent can be derived from carbon storage market price according to Equation (11). At the time of writing, the market price of carbon dioxide emissions is in the order of 25 Euros per ton. This inserted to Equation (11) together with a 3% discount rate, often applied in Forestry [36–38], results as an annual carbon rent of 0.75 Euros per ton of carbon dioxide.

In coarse terms, a cubicmeter of commercial trunk volume in boreal forest stores a ton of carbon dioxide (in living biomass, litter, and soil) [1,2,4,7,39,40]. Correspondingly, considering the prices valid at the time of writing, one might reasonably expect a carbon rent of 0.75 Euros per standing cubicmeter. In comparison to Fig. 14, that is enough only up to an excess volume of 30 m³/ha. For greater levels of carbon storage, the deficiency of capital return rate per standing
cubic meter is greater than the appropriate rent. The situation will naturally change if the market price of carbon dioxide emissions changes.

It has recently been shown that a carbon sequestration trade policy is equivalent to a carbon rent policy [18]. The same treatment, abbreviated and with somewhat less restrictive boundary conditions, is given in Equations (8) to (11) of this paper. There are reasons why any carbon sequestration trade compensation must be proportional to the carbon storage on the estate level [18]. If it would not be, agents with large carbon inventories would initially suffer heavy and unjustified release taxes.

Unlike the carbon trade, there is no particular reason why the carbon rent should be proportional. If the carbon rent is made nonproportional, the marginal rent may reasonably reach or exceed the capital return rate deficiency of Fig. 14, also on the right on the Figure.

All the quantitative results in this paper are estate-specific. It is of interest to consider how they would change if the properties of the estate would change. Changing soil fertility, as well as changing temperature sum, would change the values of all time derivatives. Correspondingly, capital return rates, as well as annual capital return deficiencies would change. Greater fertility would result in greater time change rates, and vice versa. The effect of adopting other dominating tree species but Norway spruce on the estate is somewhat more difficult to estimate.

It has recently been proposed that on a national level, sites of low fertility and regions of cool climate are most suitable for carbon sequestration, while regions with high production capacity are best suitable for wood raw material supply [17]. Consequently, one might think that carbon rent derived from present emission prices might well suffice to compensate for capital return deficiency on forests at lower fertility or temperature sum. However, maximizing capital return rate results in low capitalization on productive sites, and correspondingly as reduced growth (Figs. 3, 5, 7, and 9), which in turn reduces timber supply. A functional carbon sequestration subsidy would increase capitalization, growth, and timber supply from productive forests.

Many of the quantitative results in this paper depend on the market prices of roundwood assortments, bare land, silvicultural and harvesting expenses, carbon emission, as well as the discount rate applied in Equations (10) and (11). Considering eventual changes in the latter two is straightforward in terms of Equations (10) and (11). Changes in the former quantities
contribute trivially if they are proportional. In the case of significant nonproportional changes in prices, most of the Figures of this paper will have to be redrawn.

It might be of interest to try to relate the present results with previous approaches with a focus on large-scale systems. Some investigations claim the benefits of intensive forestry with significant harvesting, considering the displacement of nonforest products with forest products [12,13]. Other investigators claim displacement factors are not large enough, and consequently minimized harvesting, resulting in large carbon stocks in forests, should be favored [15,16]. The present results happen to be in concert with both views. Maximization of capital return rate results in small carbon inventory, as well as a smallish rate of wood production. Carbon stocking, as well as growth rate, can be increased at the expense of capital return deficiency (Figs. 5, 9, and 10).

5. Conclusions

Extending rotations induces a large capital return rate deficiency if thinnings from below are applied. With thinnings from above, the capital return rate also is sensitive to rotation age. It is less sensitive to cutting limit diameter than indicated by previous studies. All feasible growing cycles terminate in clearcutting, provided there is a price premium in clearcutting sawlogs. Manipulation of biomass density is most feasible by manipulating the intensity of thinnings from above.

Maximization of capital return rate results in small carbon inventory, as well as a smallish rate of wood production. Carbon stocking, as well as growth rate, can be increased at the expense of a capital return rate deficiency.

It is possible to allocate the capital return rate deficiency to an excess value of commercial stand volume. Present carbon prices allow only moderate excess volumes, if carbon rent is proportional to carbon storage. However, unlike carbon trade, carbon rent can be made nonproportional.

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