Semiclassical Strings in Supergravity PFT

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ABSTRACT: Puff Field Theory (PFT) is an example of a non-local field theory which arises from a novel embedding of D-branes in Melvin universe. We study several rotating and pulsating string solutions of the F-string equations of motion in the supergravity dual of the PFT. Further, we find a PP-wave geometry from this nonlocal spacetime by applying a Penrose limit and comment on its similarity with the maximally supersymmetric PP-wave background.

KEYWORDS: AdS-CFT correspondence, Bosonic Strings.
1. Introduction and Summary

It is not uncommon to find examples of Quantum Field Theories (QFT) which violate Lorentz invariance in high energy limit. These theories might play a crucial role in understanding physics beyond the Standard Model of particle physics. In the context of string theory, for example, a few Lorentz violating theories are constructed from the local deformation of the $N = 4$ Super Yang-Mills (SYM) theory. The UV-completeness of such theories are recovered by constraining the conformal dimensions of such deformation operators, although at IR limit, action for these theories can approach to that of $N = 4$ SYM theory. Example of such a theory includes $N = 4$, $U(N)$ SYM on a space of non-commutative $\mathbb{R}^4$, which in the IR limit looks like N=4 SYM deformed by an operator of conformal dimension $\Delta = 6$, breaking the Lorentz group $SO(3,1)$ to $SO(2) \times SO(1,1)$. The non-commutativity introduces a fundamental linear non-locality into the construction of such a theory. It is worth mentioning that in many of these theories the fundamental particles can become extended non-local objects, making them intriguing for string theorists. It is therefore, interesting to explore such possible extensions of field theories that incorporate the violation of Lorentz invariance at some typical mass scales.

Puff Field Theory (PFT) is such an example of a Lorentz violating non-local field theory. The idea follows the construction of Non-Commutative SYM (NCSYM) by Douglas and Hull. In NCSYM we consider $n$-coincident $D0$ branes in type IIA string theory compactified on a small $T^2$. This theory is T-dual to type IIA on a large $T^2$ with $n$ D2 branes. But this T-duality does not simply map the small $T^2$ to a large one if a NS NS 2-form flux $B_{\mu\nu}$ is turned on along $T^2$ as an obstruction. It was argued by Douglas and Hull...
that the D2-branes in this setting will be described by non local interactions in the NCSYM. The construction of PFT is a variant of such a small/large volume duality. Now consider a Kaluza-Klein particle with \(n\) units of momentum in type IIA string theory compactified on a \(T^3\). An appropriate U-duality transformation transforms this setting into \(n\) D3-branes on type IIB theory compactified on large \(T^3\). Instead of \(B_{\mu\nu}\) flux as in the previous case, we give a geometrical twist that will prevent U-duality from producing type IIB on a large \(T^3\). It has been argued in [2] that in the low energy limit the Kaluza-Klein particle is described by a decoupled non-local field theory that breaks Lorentz symmetry \(SO(3,1)\) but preserves rotational invariant group in three dimensions, \(SO(3)\). This conjectured field theory, where the particle carrying a R-charge now expands to occupy a D3 brane worldvolume proportional to the R-charge and the dimensionful deformation parameter, is termed as Puff Field Theory (PFT). Nothing is known about the explicit lagrangian form of PFT, but the supergravity description of PFT can be obtained from the non-trivial embedding of D-brane geometry in a Melvin universe, as done in [4]. The result is a type IIB supergravity background supported by a 4-form RR flux and a constant dilaton. While constructing the supergravity dual background of PFT it has been demanded that the setting should preserve a few of the supersymmetries to avoid instability altogether. It has also been argued that the supersymmetry preservation for this field theory will depend on the nature of symmetry of the deformation parameter. This can in turn be fixed by choosing the geometrical twist accordingly.

Now we can see that the background dual to PFT looks incredibly complex. But, in this work we find that the near horizon geometry of the background, under Penrose limit, reduces to the PP-wave of \(AdS_5 \times S^5\). This result prompts us to look for solutions of the F-string equations of motion in this background in the semiclassical limit. In the context of AdS/CFT duality, string solutions in the semiclassical limit have proved to be of key importance in exploring various aspects of the correspondence. According to AdS/CFT correspondence [5], [6], [7] quantum closed string states in bulk should be dual to local operators on the boundary. This state-operator matching can be tractable only in large angular momentum limit, on both sides of the duality [8], [9], [10], [11], [12], [13], [14], [15], [16], as both the string theory and the gauge theory are integrable in the semiclassical limit, see for example [17], [18], [19], [20], [21]. In this connection a large number of rotating and pulsating string solutions have been studied in various string theory backgrounds, see for example, [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50]. Here, we try to extract some simple solutions following results from these works.

In the case of our background, we expand it in the near horizon limit keeping only \(AdS_5 \times S^5\) plus the leading order deformation terms, containing the mixing of coordinates from both \(AdS\) and sphere part. It it already shown in [2] that this leading order term, in the dual gauge theory corresponds to a deformation operator of conformal dimension \(\Delta = 7\) to N=4 SYM. That is, in the low energy limit the total Lagrangian can be written as

\[
L = L_{N=4} + \eta O^{(7)} + \ldots
\]  

(1.1)
Where $\eta$ is the dimensionful deformation parameter. Thus, we choose to ignore the higher order deformation terms in our metric and study a general class of rotating string solutions in some approximation. We find that the dispersion relation among various conserved quantities differ slightly from that of general $AdS_n \times S^n$. Next we study a class of solutions both rotating and pulsating in this background. Such kind of string states are expected to be dual to highly excited sigma model operators. As oscillation number is a quantum adiabatic invariant, the series relation of energy in terms of oscillation number and other conserved quantities is presented as the solution to characterize the dynamics of these string states.

The rest of the paper is organized as follows. In section-2 we note down the supergravity description of PFT and take the appropriate near horizon limit for studying the rotating string solutions. In section-3, we study penrose limit of the supergravity dual background of PFT. Section-4 is devoted to the study of rigidly rotating strings in this background. We present the regularized dispersion relations among various conserved charges corresponding to the string motion. We also present solutions for strings which are both rotating and pulsating in the above background. Finally, in section-5 we conclude with some comments.

2. Supergravity description of PFT

Following \cite{4} we know the supergravity dual background of PFT is given by the following metric and the 4-form field as,

$$\frac{ds^2}{\alpha'} = K^{\frac{1}{2}} \left( -H^{-1} dt^2 + dU^2 + U^2 ds_2^2 + \sum_{i=8}^{9} dY_i^2 \right)$$

$$+ K^{-\frac{1}{2}} \left( \sum_{i=1}^{3} dx_i^2 + HU^2(d\phi + A + \Delta^{3} H^{-1} dt)^2 \right),$$

$$\frac{A}{\alpha'^2} = K^{-1}(-dt + U^2 \Delta^3 (d\phi + A)) \wedge dx_1 \wedge dx_2 \wedge dx_3,$$

$$e^\phi = g_{IIB} = 2\pi \gamma_{YM}, \quad (2.1)$$

where the harmonic functions $H$ and $K$ are,

$$H = \frac{4\pi g_{IIB}N}{(U^2 + ||Y||^2)^2}, \quad K = \frac{4\pi g_{IIB}N}{(U^2 + ||Y||^2)^2} + \Delta^6 U^2, \quad (2.2)$$

also $ds_2^2 = \frac{1}{4}(d\theta^2 + \sin^2 \theta d\varphi^2)$ is the “Fubini- Study” metric and $A = -\frac{1}{2}(1 - \cos \theta) d\varphi$ is the connection of Hopf fibration. Note that to obtain this background one needs to take the decoupling limit $\alpha' \rightarrow 0$. However, in this limit the value of $\Delta^3 = \eta \alpha'^2$ is held fixed for large value of deformation parameter $\eta$.

Now, considering $U = V \cos \zeta$ and $||Y|| = V \sin \zeta$ i.e. $Y_8 = V \sin \zeta \cos \psi$ and $Y_9 = V \sin \zeta \sin \psi$ we can rewrite the metric and 4-form as follows \cite{4},

$$\frac{ds^2}{\alpha'} = K^{\frac{1}{2}} \left( -K^{-1} dt^2 + dV^2 + V^2 d\zeta^2 + V^2 \sin^2 \zeta d\psi^2 \right) + \frac{1}{4} K^{\frac{1}{2}} V^2 \cos^2 \zeta d\theta^2$$
\[ + \frac{1}{4} K^{-\frac{1}{2}} V^2 \cos^2 \zeta (K \sin^2 \theta + H (1 - \cos \theta)^2) d\varphi^2 + K^{-\frac{1}{2}} H V^2 \cos^2 \zeta d\phi^2 \]
\[ + K^{-\frac{1}{2}} \sum_{i=1}^{3} dx_i^2 + 2 K^{-\frac{3}{2}} V^2 \cos^2 \zeta \Delta^3 dtd\phi \]
\[ - K^{-\frac{1}{2}} H V^2 \cos^2 \zeta (1 - \cos \theta) d\phi d\varphi - K^{-\frac{1}{2}} H V^2 \cos^2 \zeta \Delta^3 (1 - \cos \theta) dtd\varphi , \]
\[ A_{\alpha'^2} = K^{-1}(-dt + \Delta^3 V^2 \cos^2 \zeta (d\phi - \frac{1}{2} (1 - \cos \theta) d\varphi)) \wedge dx_1 \wedge dx_2 \wedge dx_3 , \]
\[ e^\phi = 2 \pi g_M^2 , \quad (2.3) \]

with \( K = H + \Delta^6 V^2 \cos^2 \zeta, H = \frac{8 \pi^2 g_M^2 N}{V^4} \). Now we want to take near horizon limit on this full generalised metric. Note that in near horizon limit (i.e. \( V \to 0 \)), \( H = \frac{C^2}{V} \approx K \), where \( C^2 = 8 \pi^2 g_M^2 N \), and we have kept terms upto \( V^4 \). The resulting metric and the four form field is,
\[ \frac{ds^2}{\alpha'^2} = \frac{V^2}{C} (-dt^2 + \sum_{i=1}^{3} dx_i^2) + C dV^2 \frac{V^2}{V^2} + \frac{2 \Delta^3 V^4}{C} \cos^2 \zeta dt (d\phi - \sin^2 (\frac{\theta}{2}) d\varphi) \]
\[ + C[d\zeta^2 + \sin^2 \zeta d\psi^2 + \cos^2 \zeta \{(\frac{d\theta}{2})^2 + d\phi^2 + \sin^2 (\frac{\theta}{2}) d\varphi^2 - 2 \sin^2 (\frac{\theta}{2}) d\phi d\varphi}] , \]
\[ A_{\alpha'^2} = -\frac{V^4}{C^2} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 . \quad (2.4) \]

Now making the following change of variables,
\[ \theta = 2 \theta, \quad \varphi = \phi_1 - \phi_2, \quad \phi = \phi_1, \quad \zeta = \zeta - \frac{\pi}{2} , \]
we get,
\[ \frac{ds^2}{\alpha'^2} = \frac{V^2}{C} (-dt^2 + \sum_{i=1}^{3} dx_i^2) + C dV^2 \frac{V^2}{V^2} + \frac{2 \Delta^3 V^4}{C} \sin^2 \theta d\phi \sin^2 (\frac{\theta}{2}) d\phi d\varphi \]
\[ + C[d\zeta^2 + \cos^2 \zeta d\psi^2 + \sin^2 \zeta (d\theta^2 + \cos^2 \theta d\phi_1^2 + \sin^2 \theta d\phi_2^2)] , \]
\[ A_{\alpha'^2} = -\frac{V^4}{C^2} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 . \quad (2.5) \]

This is the metric we are interested in taking a Penrose limit.

### 3. Penrose limit

In this section we would like to find out a PP-wave metric by applying Penrose limit on the background \((2.5)\). To take Penose limit on \((2.5)\), we start with a null geodesic in \((t, V, \psi)\) plane following \[51\]. Keeping other coordinates fixed, the metric becomes
\[ \frac{ds^2}{\alpha'^2} = C[-V^2 dt^2 + \frac{dV^2}{V^2} + d\psi^2] . \quad (3.1) \]
To change the coordinates from \( (t, V, \psi) \) to \( (u, v, y) \), which are more suitable to adapt the null geodesic, we use the following transformation

\[
\begin{align*}
    dV &= \sqrt{1 - l^2V^2} du , \\
    dt &= \frac{du}{V^2} + ldy - dv , \\
    d\psi &= ldu + dy ,
\end{align*}
\]

(3.2)

where \( l = \frac{J}{E} \), \( J \) and \( E \) respectively are angular momentum and energy along the geodesic (3.1). Substituting (3.2) in (2.5), and making the change of coordinates,

\[
    u = u, \quad v = \frac{v}{C}, \quad y = \frac{y}{\sqrt{C}}, \quad x_i = \frac{x_i}{\sqrt{C}}, \quad \zeta = \frac{z}{\sqrt{C}}, \quad \Omega_3 = \Omega_3,
\]

followed by a large \( C \) limit, the metric and the field strength reduces to,

\[
\begin{align*}
    ds^2 &= 2dudv - z^2l^2du^2 + (1 - l^2V^2)dy^2 + V^2 \sum_{i=1}^{3} dx_i^2 + d\bar{z}^2 + z^2d\Omega_3^2 , \\
    F &= dA = -4V^3 l \sqrt{1 - l^2V^2} du \wedge dy \wedge dx_1 \wedge dx_2 \wedge dx_3 .
\end{align*}
\]

(3.3)

Again rescaling \( u \rightarrow \mu u \) and \( v \rightarrow \frac{v}{\mu} \), we get,

\[
\begin{align*}
    ds^2 &= 2dudv - \mu^2z^2l^2du^2 + (1 - l^2V^2)dy^2 + V^2 \sum_{i=1}^{3} dx_i^2 + d\bar{z}^2 , \\
    F_{uyx_1x_2x_3} &= -4\mu V^3 l \sqrt{1 - l^2V^2} .
\end{align*}
\]

(3.4)

where \( d\bar{z}^2 = d\bar{z}^2 + z^2d\Omega_3^2 \). This is the Rosen form of the PP wave. To convert this into Brinkman form we make the following substitution,

\[
\begin{align*}
    u &= u, \quad v = \frac{v}{\sqrt{1 - l^2V^2}}, \quad x_i = \frac{x_i}{V}, \quad \bar{z} = \bar{z} , \\
    v &= v + \frac{1}{4}\left[\frac{\partial_u(1 - l^2V^2)}{1 - l^2V^2}y^2 + \frac{\partial_u(V^2)}{V^2}\sum_{i=1}^{3} x_i^2\right] ,
\end{align*}
\]

(3.5)

Substituting these we get the Brinkman form of the PP-wave as,

\[
\begin{align*}
    ds^2 &= 2dudv + (F_1y^2 + F_2x_i^2 - \mu^2z^2l^2)du^2 + dy^2 + \sum_{i=1}^{3} dx_i^2 + d\bar{z}^2 , \\
    F_{uyx_1x_2x_3} &= -4\mu l ,
\end{align*}
\]

(3.6)

where

\[
\begin{align*}
    F_1 &= \frac{1}{2}\left[\partial_u\left(\frac{1 - l^2V^2}{1 - l^2V^2}\right)\right] + \frac{1}{2}\left(\frac{\partial_u(1 - l^2V^2)}{1 - l^2V^2}\right)^2 , \\
    F_2 &= \frac{1}{2}\left[\partial_u\left(\frac{V^2}{V^2}\right)\right] + \frac{1}{2}\left(\frac{\partial_u(V^2)}{V^2}\right)^2 .
\end{align*}
\]

(3.7)
This form is similar to the form that is obtained by taking a Penrose limit on the geometry of a stack of N D3-branes in the near horizon limit. String propagation in this background has been studied in detail [8]. The main output of this section is that the very complicated metric (2.5) reduces to a well known form in Penrose limit. That signifies that when we consider the deformation term to be small, the local geometry will behave like $AdS_5 \times S^5$ to a local observer on the geodesic mentioned in this section. In the next section we will be interested in finding solutions of string equation of motion in semiclassical limit in the background (2.3).

4. Semiclassical String solutions

If we neglect $V^4$ term in (2.5), then the metric takes the form of $AdS_5 \times S^5$, for which the rigidly rotating string solutions are well studied. It would be interesting if we can find more general solutions by keeping $V^4$ term. By rescaling, $t \rightarrow \Delta C^2 t$, $x_i \rightarrow \Delta C^2 x_i$ and substituting $V = \frac{1}{\Delta} W C^2$, we get,

$$ds^2 = C \left[ W^2 (-dt^2 + \sum_{i=1}^{3} dx_i^2) + \frac{dW^2}{W^2} + 2W^4 \sin^2 \zeta dt (\cos^2 \theta d\phi_1 + \sin^2 \theta d\phi_2) + d\zeta^2 + \cos^2 \zeta d\psi^2 + \sin^2 \zeta (d\theta^2 + \cos^2 \theta d\phi_1^2 + \sin^2 \theta d\phi_2^2) \right],$$

$$\frac{A}{\alpha'^2} = -C^2 W^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 . \quad (4.1)$$

Clearly finding out equations of motion for the fundamental string in the above background (4.1) is very hard. However, we can simplify considering the subspace of (4.1) with $W = W_0$ and $\theta = \theta_0$. For these values, the metric (4.1) becomes,

$$ds^2 = C \left[ W_0^2 (-dt^2 + \sum_{i=1}^{3} dx_i^2) + 2W_0^4 \sin^2 \zeta dt (\theta_0 d\phi_1 + \sin^2 \theta_0 d\phi_2) + d\zeta^2 + \cos^2 \zeta d\psi^2 + \sin^2 \zeta (\theta_0^2 d\phi_1^2 + \sin^2 \theta_0 d\phi_2^2) \right],$$

where $W_0$ and $\theta_0$ are constants. In the following analysis we will keep the terms upto $O(W_0^4)$ only.

4.1 Rigidly Rotating Strings

We start our analysis by writing down the Polyakov action of the F-string in the background (4.2),

$$S = -\frac{1}{4\pi \alpha'} \int d\sigma d\tau [\sqrt{-\gamma} \gamma^\alpha_\beta g_{MN} \partial_\alpha X^M \partial_\beta X^N] , \quad (4.3)$$

where $\gamma^{\alpha\beta}$ is the world-sheet metric. Under conformal gauge (i.e. $\sqrt{-\gamma} \gamma^{\alpha\beta} = \eta^{\alpha\beta}$) with $\eta^{\tau \tau} = -1$, $\eta^{\sigma \sigma} = 1$ and $\eta^{\tau \sigma} = 0$, the Polyakov action in the above background takes
the form,
\[ S = -\frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau \left[ W_0^2 \{ -(t'^2 - t^2) + x_i'^2 - x_i^2 \} + \zeta'^2 - \zeta^2 \\
+ \cos^2 \zeta (\psi'^2 - \psi^2) + \sin^2 \zeta \{ \cos^2 \theta_0 (\phi_1'^2 - \phi_1^2) + \sin^2 \theta_0 (\phi_2'^2 - \phi_2^2) \} \\
+ 2W_0^4 \sin^2 \zeta \{ \cos^2 \theta_0 (t'^2 \phi_1' - i\phi_1) + \sin^2 \theta_0 (t'^2 \phi_2' - i\phi_2) \} \right], \tag{4.4} \]
where ‘dots’ and ‘primes’ denote the derivative with respect to \( \tau \) and \( \sigma \) respectively, also \( \text{'t Hooft coupling} \sqrt{\lambda} = C \). For studying the rigidly rotating strings in this background we choose the following ansatz,
\[ t = \tau + h_0(y), \quad x_i = \nu_i(\tau + h_i(y)), \quad i = 1, 2, 3, \quad \zeta = \zeta(y), \]
\[ \phi_1 = \omega_1(\tau + g_1(y)), \quad \phi_2 = \omega_2(\tau + g_2(y)), \quad \psi = \omega_3(\tau + g_3(y)), \tag{4.5} \]
where \( y = \sigma - \nu \tau \). Variation of the action with respect to \( X^M \) gives us the following equation of motion
\[ 2\partial_\alpha (\eta^{\alpha \beta} \partial_\beta X^N g_{KN}) - \eta^{\alpha \beta} \partial_\alpha X^M \partial_\beta X^N \partial_K g_{MN} = 0, \tag{4.6} \]
and variation with respect to the metric gives the two Virasoro constraints,
\[ g_{MN}(\partial_\tau X^M \partial_\tau X^N + \partial_\nu X^M \partial_\nu X^N) = 0, \]
\[ g_{MN}(\partial_\nu X^M \partial_\nu X^N) = 0. \tag{4.7} \]
Next we have to solve these equations by the ansatz we have proposed above in eqn. \[4.5\]. Solving for \( t, \phi_1 \) and \( \phi_2 \) we get,
\[ -\frac{\partial h_0}{\partial y} + \omega_1 W_0^2 \cos^2 \theta_0 \sin^2 \zeta \frac{\partial g_1}{\partial y} + \omega_2 W_0^2 \sin^2 \theta_0 \sin^2 \zeta \frac{\partial g_2}{\partial y} = \frac{1}{1 - v^2} \left[ c_4 - v W_0^2 \sin^2 \zeta \{ \omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0 \} \right], \]
\[ W_0^4 \sin^2 \zeta \frac{\partial h_0}{\partial y} + \omega_1 \sin^2 \zeta \frac{\partial g_1}{\partial y} = \frac{1}{1 - v^2} \left[ c_5 - v \sin^2 \zeta(\omega_1 + W_0^4) \right], \]
\[ W_0^4 \sin^2 \zeta \frac{\partial h_0}{\partial y} + \omega_2 \sin^2 \zeta \frac{\partial g_2}{\partial y} = \frac{1}{1 - v^2} \left[ c_6 - v \sin^2 \zeta(\omega_2 + W_0^4) \right], \tag{4.8} \]
where \( c_4, c_5 \) and \( c_6 \) are integration constants. Solving \[4.8\], we get
\[ \frac{\partial h_0}{\partial y} = \frac{1}{1 - v^2} \left[ W_0^2 (c_5 \cos^2 \theta_0 + c_6 \sin^2 \theta_0) - c_4 \right], \]
\[ \frac{\partial g_1}{\partial y} = \frac{1}{1 - v^2} \left[ \frac{1}{\omega_1} \left\{ \frac{c_5}{\sin^2 \zeta} - W_0^4 (v - c_4) \right\} - v \right], \]
\[ \frac{\partial g_2}{\partial y} = \frac{1}{1 - v^2} \left[ \frac{1}{\omega_2} \left\{ \frac{c_6}{\sin^2 \zeta} - W_0^4 (v - c_4) \right\} - v \right]. \tag{4.9} \]
Solving for \( \psi \) and \( x_i \), respectively we get,
\[ \frac{\partial g_3}{\partial y} = \frac{1}{1 - v^2} \left[ \frac{c_7}{\cos^2 \zeta} - v \right], \quad \frac{\partial h_i}{\partial y} = c_i, \tag{4.10} \]
where \(c_7\) and \(c_i, (i = 1, 2, 3)\) are integration constants. Solving for \(\zeta\) we get,

\[
(1 - v^2)^2 \frac{\partial^2 \zeta}{\partial y^2} = \sin \zeta \cos \zeta \left[ \frac{d^2}{\sin^4 \zeta} - \frac{\omega_2^2 c_7^2}{\cos^4 \zeta} - \omega^2 \right], \tag{4.11}
\]

where \(\omega^2 = \omega_1^2 \cos^2 \theta_0 + \omega_2^2 \sin^2 \theta_0 - \omega_3^2 + 2 \omega_1 W_0^\prime \cos^2 \theta_0 + 2 \omega_2 W_0^\prime \sin^2 \theta_0\) and \(d^2 = c_5^2 \cos^2 \theta_0 + c_6^2 \sin^2 \theta_0\). Integrating (4.11), we get,

\[
(1 - v^2)^2 \left( \frac{\partial \zeta}{\partial y} \right)^2 = -\frac{d^2}{\sin^2 \zeta} - \frac{\omega_2^2 c_7^2}{\cos^2 \zeta} - \omega^2 \sin^2 \zeta + c_8, \tag{4.12}
\]

where \(c_8\) is integration constant. It can be easily checked that the Virasoro constraints are consistent with equation (4.12). Considering the limit, \(\frac{\partial \zeta}{\partial y} \rightarrow 0\) as \(\zeta \rightarrow \frac{\pi}{2}\), in (4.12) implies \(c_7 = 0\) and \(c_8 = d^2 + \omega^2\). Substituting this in the above equation we get,

\[
\frac{\partial \zeta}{\partial y} = \frac{\sqrt{\omega_2 \cot \zeta}}{1 - v^2} \sqrt{\sin^2 \zeta - \sin^2 \zeta_0}, \tag{4.13}
\]

where \(\sin \zeta_0 = \frac{d}{\omega}\). Looking at the symmetry of the background (4.2), a number of conserved charges can be constructed as follows,

\[
E = -\int \frac{\partial L}{\partial t} d\sigma = \frac{\sqrt{\lambda}}{2\pi} W_0^2 \int [(1 - v^2 - vc_4) - W_0^2 (\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0)] d\sigma, \\
P_i = \int \frac{\partial L}{\partial x_i} d\sigma = \frac{\sqrt{\lambda}}{2\pi} v_i W_0^2 (1 - vc_i) \int d\sigma, \\
J_\psi = \int \frac{\partial L}{\partial \psi} d\sigma = \frac{\sqrt{\lambda}}{2\pi} \omega_3 \int \cos^2 \zeta d\sigma, \\
J_{\phi_1} = \int \frac{\partial L}{\partial \phi_1} d\sigma = \frac{\sqrt{\lambda}}{2\pi} \omega_3 \cos^2 \theta_0 \int [(\omega_1 + W_0^\prime) \sin^2 \zeta - vc_5] d\sigma, \\
J_{\phi_2} = \int \frac{\partial L}{\partial \phi_2} d\sigma = \frac{\sqrt{\lambda}}{2\pi} \sin^2 \theta_0 \int [(\omega_2 + W_0^\prime) \sin^2 \zeta - vc_6] d\sigma. \tag{4.14}
\]

Also the deficit angles are given by,

\[
\Delta \phi_1 = \omega_1 \int \frac{\partial g_1}{\partial y} d\sigma = \frac{1}{1 - v^2} \int \left[ \frac{c_5}{\sin^2 \zeta} - W_0^\prime (v - c_4) - v \omega_1 \right] d\sigma, \\
\Delta \phi_2 = \omega_2 \int \frac{\partial g_2}{\partial y} d\sigma = \frac{1}{1 - v^2} \int \left[ \frac{c_6}{\sin^2 \zeta} - W_0^\prime (v - c_4) - v \omega_2 \right] d\sigma, \\
\Delta \phi = \Delta \phi_1 - \Delta \phi_2 = \frac{1}{1 - v^2} \int \left[ \frac{c_5 - c_6}{\sin^2 \zeta} - v (\omega_1 - \omega_2) \right] d\sigma. \tag{4.15}
\]

In what follows, we will find relations among various charges in different limiting cases.

### 4.1.1 Case I

For this case, we choose \(c_5 - c_6 = v (\omega_1 - \omega_2)\), and the angle deficit becomes,

\[
\Delta \phi = \frac{2v (\omega_1 - \omega_2)}{\omega} \int_{\zeta_0}^{\pi/2} \frac{\cos \zeta d\zeta}{\sin \zeta \sqrt{\sin^2 \zeta - \sin^2 \zeta_0}} = \frac{2v (\omega_1 - \omega_2)}{d} \arccos (\sin \zeta_0), \tag{4.16}
\]
which implies \( \sin \zeta_0 = \cos \left( \frac{\Delta \phi}{2} \right) \), where \( \Delta \phi = \frac{\Delta \phi_0}{v(\omega_1 - \omega_2)} \). In this condition the expression of energy can be written as,

\[
E = \frac{\sqrt{\lambda}}{\pi} \frac{W_0^2}{\omega} \left[ 1 - v^2 + vc_4 - W_0^2 (\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0) \right] \int_{\zeta_0}^{\pi} \frac{\sin \zeta d\zeta}{\cos \zeta \sqrt{\sin^2 \zeta - \sin^2 \zeta_0}} + \frac{\sqrt{\lambda}}{\pi} \frac{W_0^4}{\omega} \left[ \omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0 \right] \int_{\zeta_0}^{\pi} \frac{\sin \zeta \cos \zeta d\zeta}{\sqrt{\sin^2 \zeta - \sin^2 \zeta_0}}, \tag{4.17}
\]

which diverges because of the first integration. \( P_1 \) also diverges as,

\[
P_1 = \frac{\sqrt{\lambda} \nu I W_0^2 (1 - vc_4)(1 - v^2)}{\pi} \int_{\zeta_0}^{\pi} \frac{\sin \zeta d\zeta}{\cos \zeta \sqrt{\sin^2 \zeta - \sin^2 \zeta_0}}. \tag{4.18}
\]

But,

\[
J_\psi = \frac{\sqrt{\lambda} \omega_3}{\pi} \cos \zeta_0, \tag{4.19}
\]

is finite. And the combined angular momentum,

\[
J_\phi = J_{\phi_1} + J_{\phi_2} = \frac{\sqrt{\lambda} \omega_2 + W_0^4 - vc_6 + (1 - v^2)(\omega_1 - \omega_2) \cos^2 \theta_0}{\pi} \int_{\zeta_0}^{\pi} \frac{\sin \zeta \cos \zeta d\zeta}{\sqrt{\sin^2 \zeta - \sin^2 \zeta_0}} - \frac{\sqrt{\lambda} \omega_1 \cos^2 \theta_0 \omega_2 \sin^2 \theta_0 + W_0^4}{\pi} \int_{\zeta_0}^{\pi} \frac{\sin \zeta \cos \zeta d\zeta}{\sqrt{\sin^2 \zeta - \sin^2 \zeta_0}}, \tag{4.20}
\]

diverges. On regularization,

\[
E_{\text{reg}} = E - \left( 1 - v^2 + vc_4 - W_0^2 (\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0) \right) P_1, \tag{4.21}
\]

and

\[
(J_\phi)_{\text{reg}} = J_\phi - \left( \omega_2 + W_0^4 - vc_6 + (1 - v^2)(\omega_1 - \omega_2) \cos^2 \theta_0 \right) P_1, \tag{4.22}
\]

Defining,

\[
J = (J_\phi)_{\text{reg}} + \frac{W_0^4}{\omega_3} J_\psi = -\frac{\sqrt{\lambda} \omega_1 \cos^2 \theta_0 \omega_2 \sin^2 \theta_0}{\pi} \cos \zeta_0, \tag{4.23}
\]

and they are found to satisfy the dispersion relation

\[
E_{\text{reg}} = \sqrt{J^2 - f(\lambda) \sin^2 \left( \frac{\Delta \phi}{2} \right)}, \tag{4.24}
\]

where \( f(\lambda) = \frac{\lambda}{\pi^2} \left[ \omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0 \right]^2 \). The above dispersion relation is very similar to what we normally observe in case of rotating strings in \( AdS \times S \).  

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4.1.2 Case II

For this case, choosing $c_5 - c_6 = \frac{1}{5}(\omega_1 - \omega_2)$, we see that deficit angle $\Delta \phi$ diverges, but on regularization,

$$(\Delta \phi)_{reg} = \Delta \phi - \frac{2\pi(\omega_1 - \omega_2)}{\sqrt{\lambda} \nu_1 v W_0^2 (1 - v c_i)} P_i = -\frac{2(\omega_1 - \omega_2)}{vd} \arccos(\sin \zeta_0) ,$$

(4.25)

which implies $\sin \zeta_0 = \cos \left( \frac{(\Delta \phi)_{reg}}{2} \right)$, where $(\Delta \phi)_{reg} = \frac{(\Delta \phi)_{reg} vd}{(\omega_1 - \omega_2)}$. In this condition $P_i$ is also found to be diverging as,

$$P_i = \sqrt{\frac{\lambda}{\pi}} \nu_i W_0^2 (1 - v c_i)(1 - v^2) \int_{\frac{\pi}{2}}^{\zeta_0} \frac{\sin \zeta d\zeta}{\cos \zeta \sqrt{\sin^2 \zeta - \sin^2 \zeta_0}} .$$

(4.26)

Though the energy $E$ diverges, on regularization,

$$E_{reg} = E - \frac{[1 - v^2 + v c_4 - W_0^2 (\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0)]}{v_i (1 - v c_i)(1 - v^2)} P_i ,$$

$$= -\sqrt{\frac{\lambda}{\pi}} W_0^4 [\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0] \cos \zeta_0 .$$

(4.27)

Angular momenta $J_\psi$,

$$J_\psi = -\frac{\sqrt{\lambda}}{\pi} \frac{\omega_3}{\omega} \cos \zeta_0 ,$$

(4.28)

is finite. But the combined angular momenta $J_\phi$ diverges and on regularization,

$$(J_\phi)_{reg} = J_\phi - \frac{[\omega_2 + W_0^4 - v c_6]}{\nu_i W_0^2 (1 - v c_i)(1 - v^2)} P_i ,$$

$$= \sqrt{\frac{\lambda}{\pi}} \frac{[\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0 + W_0^4]}{\omega} \cos \zeta_0 .$$

(4.29)

Again defining,

$$J = (J_\phi)_{reg} + \frac{W_0^4}{\omega_3} J_\psi = \sqrt{\frac{\lambda}{\pi}} \frac{[\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0]}{\omega} \cos \zeta_0 ,$$

(4.30)

we see that the constants of motion satisfy the following dispersion relation

$$J = \sqrt{E_{reg}^2 + f(\lambda) \sin^2 \left( \frac{(\Delta \phi)_{reg}}{2} \right)} ,$$

(4.31)

where $f(\lambda) = \frac{\lambda \pi^2}{2} \frac{[\omega_1 \cos^2 \theta_0 + \omega_2 \sin^2 \theta_0]^2}{\omega^2}$. Here, we can see that the deficit angle had to be regularized unlike the previous case, where it was already finite.

4.2 Rotating and Pulsating Strings with two equal spins

In this section we will focus on a class of long semiclassical strings which are both pulsating and rotating in the background (2.5). Here we follow the simple procedure as done in, for
example [52] for our analysis. 1 We put $W = W_0$ and $\theta = \frac{\pi}{4}$ for simplicity in the metric. Then we get,

$$\frac{ds^2}{\alpha^2} = C[W_0^2(-dt^2 + \Sigma(dx_i)^2) + d\zeta^2 + \cos^2 \zeta d\psi^2 + \frac{1}{2}\sin^2 \zeta(\dot{\phi}_1^2 + \dot{\phi}_2^2) + W_0^4 \sin^2 \zeta dt(\dot{\phi}_1 + \dot{\phi}_2)] .$$

(4.32)

We shall look for circular string propagation in this background using the following ansatz,

$$t = t(\tau), \ x_i = x_i(\tau), \ \psi = \psi(\tau), \ \zeta = \zeta(\tau), \ \phi_1 = \phi_1(\tau) + m_1 \sigma, \ \phi_2 = \phi_2(\tau) + m_2 \sigma.$$  

(4.33)

In this ansatz, we can evaluate the constants of motion from the action as,

$$E = \sqrt{\lambda}E = \sqrt{\lambda}[W_0^2 \dot{t} - \frac{1}{2}W_0^4 \sin^2 \zeta(\dot{\phi}_1 + \dot{\phi}_2)] ,$$

$$P_i = \sqrt{\lambda}P_i = \sqrt{\lambda}W_0^2 \dot{x}_i ,$$

$$J_{\phi_1} = \sqrt{\lambda}J_{\phi_1} = \sqrt{\lambda}\sin^2 \zeta[\dot{\phi}_1 + W_0^2 \dot{t}] ,$$

$$J_{\psi} = \sqrt{\lambda}J_{\psi} = \sqrt{\lambda}\sin^2 \zeta \dot{\psi} .$$

(4.34)

Now, we can see that the second Virasoro constraint in this case implies that

$$m_1 J_{\phi_1} + m_2 J_{\phi_2} = 0 .$$

(4.35)

In this calculation we will be interested in the subset of solutions which have two equal spins i.e.

$$J_{\phi_1} = J_{\phi_2}, \quad m_1 = -m_2 = m .$$

(4.36)

Here, the first Virasoro constraint gives the equation of motion for $\zeta$

$$\dot{\zeta}^2 = W_0^2(\dot{t}^2 - \dot{x}_i^2) - \cos^2 \zeta \dot{\psi}^2 - \frac{1}{2}\sin^2 \zeta[\dot{\phi}_1^2 + \dot{\phi}_2^2 + 2W_0^4(\dot{\phi}_1 + \dot{\phi}_2)\dot{t} + 2m^2] .$$

(4.37)

To find the equation of motion in terms of the conserved quantities, we ignore the terms higher than $O(W_0^4)$ keeping in with our approximation in the previous sections.

Putting in the values, we get

$$\dot{\zeta}^2 = \frac{\tilde{E}^2}{W_0^2} - \frac{J_{\psi}^2}{\cos^2 \zeta} - \frac{J_\theta^2}{\sin^2 \zeta} - m^2 \sin^2 \zeta ,$$

(4.38)

where $\tilde{E}^2 = E^2 - \sum P_i^2 + 2W_0^4(J_{\phi_1} + J_{\phi_2})$ and $J_\theta^2 = 2(J_{\phi_1}^2 + J_{\phi_2}^2)$, so that $J$ is a real quantity. Now the equation of motion for $\zeta$ looks like the classical equation for a particle

1Recently more generalized rotating and pulsating strings have been studied in [53].
moving in a potential. Notice that the potential here grows to infinity at both $\zeta = 0$ as well as $\zeta = \frac{\pi}{2}$. So the functional form suggests an infinite potential well with a minimum in between the extrema. The $\zeta$ coordinate must then oscillate in this well between a maximum and minimum value. We define the Oscillation number for the system as

$$N = \frac{1}{2\pi} \int d\zeta \frac{\partial}{\partial m} = \frac{1}{\pi} \int_{\zeta_{\text{min}}}^{\zeta_{\text{max}}} d\zeta \sqrt{\frac{E^2}{W_0^2} - \frac{\mathcal{J}_\psi^2}{\cos^2 \zeta} - \frac{\mathcal{J}^2}{\sin^2 \zeta} - m^2 \sin^2 \zeta}, \quad (4.39)$$

with $N = \frac{N}{\sqrt{\lambda}}$ being an Adiabatic invariant, which should have integer values in the usual quantum theory. Putting $\sin \zeta = x$ into the integral for oscillation number, we get

$$N = \frac{1}{\pi} \int_{\sqrt{R_1}}^{\sqrt{R_2}} dx \frac{x^3}{1 - x^2 \sqrt{\frac{E^2}{W_0^2} (1 - x^2) - \mathcal{J}_\psi^2 - \frac{\mathcal{J}^2}{x^2} (1 - x^2) - m^2 x^2 (1 - x^2)}}, \quad (4.40)$$

where $R_1$ and $R_2$ are two positive appropriate roots of the polynomial

$$g(z) = m^2 z^3 + (-\frac{E^2}{W_0^2} - m^2) z^2 + (\frac{E^2}{W_0^2} + \mathcal{J}_\psi^2 - \mathcal{J}^2) z - \mathcal{J}^2, \quad z = x^2. \quad (4.41)$$

Naturally, we will be interested in the region of parameter space where the roots to the above polynomial are real. Now taking the partial derivative of $N$ w.r.t $m$ we get

$$\frac{\partial N}{\partial m} = -\frac{m}{\pi} \int_{\sqrt{R_1}}^{\sqrt{R_2}} dx \frac{x^3}{\sqrt{\frac{E^2}{W_0^2} (1 - x^2) - \mathcal{J}_\psi^2 - \frac{\mathcal{J}^2}{x^2} (1 - x^2) - m^2 x^2 (1 - x^2)}} \quad (4.42)$$

Now to find the roots of the polynomial $g(z)$ we do an approximate analysis. In the large $\bar{E}$ but small $\mathcal{J}$ and $\mathcal{J}_\psi$ limit, we can find the three distinct roots as,

$$\alpha_1 = \frac{\bar{E}^2}{mW_0^2} + \frac{W_0^2 \mathcal{J}_\psi^2 - \mathcal{J}^2}{E^2} + O[W_0^4 \bar{E}^{-4}],$$

$$\alpha_2 = \frac{W_0^2 \mathcal{J}^2}{E^2} + O[W_0^4 \bar{E}^{-4}],$$

$$\alpha_3 = 1 - \frac{W_0^2 \mathcal{J}_\psi^2}{E^2} + O[W_0^4 \bar{E}^{-4}]. \quad (4.43)$$

Clearly we can see, $0 \leq x^2 \leq 1$, so in the large $\bar{E}$ limit, we choose the appropriate upper and lower limit to the integral accordingly. Putting $x^2 = z$ we write the integral as

$$\frac{\partial N}{\partial m} = -\frac{m}{2\pi} \int_{\alpha_2}^{\alpha_3} dz \frac{z}{\sqrt{m^2 z^3 + (-\frac{E^2}{W_0^2} - m^2) z^2 + (\frac{E^2}{W_0^2} + \mathcal{J}_\psi^2 - \mathcal{J}^2) z - \mathcal{J}^2}} \quad (4.44)$$

Using standard integral tables we can transform this into a combination of the usual Elliptic integrals of first and second kind as,

$$\frac{\partial N}{\partial m} = -\frac{m}{\pi} \frac{1}{\sqrt{\alpha_1 - \alpha_2}} \left[ \alpha_1 K \left( \frac{\alpha_3 - \alpha_2}{\alpha_1 - \alpha_2} \right) - (\alpha_1 - \alpha_2) E \left( \frac{\alpha_3 - \alpha_2}{\alpha_1 - \alpha_2} \right) \right]. \quad (4.45)$$
We expand the equation again in the large $\tilde{E}$ but small $J$ and $\mathcal{J}_\psi$ limit to get,

$$\frac{1}{W_0} \frac{\partial \mathcal{N}}{\partial m} = c_1 m^2 \tilde{E}^{-1} + c_2 m^4 \tilde{E}^{-3} \left[ c_3 + \frac{\mathcal{J}^2 - \mathcal{J}_\psi^2}{m^2} \right] + \mathcal{O}[W_0^5 \tilde{E}^{-5}] , \quad (4.46)$$

where the numerical constants are given by $c_1 = c_2 = -0.25$ and $c_3 = 0.375$. Integrating this equation we get a series for $\mathcal{N}$,

$$\mathcal{N} = \mathcal{N}_0 + \frac{c_1}{3} m^2 W_0 \tilde{E}^{-1} + \frac{c_2}{5} m^5 W_0 \tilde{E}^{-3} \left[ c_3 + \frac{5 \mathcal{J}^2 - \mathcal{J}_\psi^2}{3 m^2} \right] + \mathcal{O}[W_0^5 \tilde{E}^{-5}] . \quad (4.47)$$

The integration constant $\mathcal{N}_0$ can be evaluated by considering the integral for $m = 0$, i.e.

$$\mathcal{N}_0 = \frac{1}{\pi} \int_{\beta_1}^{\beta_2} \frac{dx}{1-x^2} \sqrt{\frac{\tilde{E}^2}{W_0^2} (1-x^2) + \mathcal{J}^2 (1 - \frac{1}{x^2}) - \mathcal{J}_\psi^2} , \quad (4.48)$$

where the limits are given by

$$\beta^2 = \beta_{2,1}^2 = \frac{-\left( \frac{\tilde{E}^2}{W_0^2} + \mathcal{J}^2 - \mathcal{J}_\psi^2 \right) \pm \sqrt{\left( \frac{\tilde{E}^2}{W_0^2} + \mathcal{J}^2 - \mathcal{J}_\psi^2 \right)^2 - 4 \frac{\tilde{E}^2}{W_0^2} \mathcal{J}^2}}{-2 \frac{\tilde{E}^2}{W_0^2}} . \quad (4.49)$$

Now using $\frac{\tilde{E}^2}{W_0^2} + \mathcal{J}^2 - \mathcal{J}_\psi^2 = \frac{\tilde{E}^2}{W_0^2} \beta^2 + \frac{\mathcal{J}^2}{\beta^2}$ and changing the variable, we transform the integral to

$$\mathcal{N}_0 = \frac{\beta_1}{\pi} \int_{\beta_1}^{\tilde{E}} \frac{dx}{1 - \beta_1^2 x^2} \sqrt{\frac{\tilde{E}^2}{W_0^2} \beta_1^2 (1-x^2) + \frac{\mathcal{J}^2}{\beta_1^2} (1 - \frac{1}{x^2})} = \frac{1}{2} \left( \frac{\tilde{E}}{W_0} - \mathcal{J} + \mathcal{J}_\psi \right) . \quad (4.50)$$

We put back this value into and then, by reverting the series we get

$$\frac{\tilde{E}}{W_0} = 2 \mathcal{N} + (\mathcal{J} - \mathcal{J}_\psi) + a_1 m^3 \mathcal{N}^{-1} - a_2 m^3 \mathcal{N}^{-2} (\mathcal{J} - \mathcal{J}_\psi)$$

$$+ a_3 m^6 \mathcal{N}^{-3} A(m, \mathcal{J}, \mathcal{J}_\psi) - a_4 m^6 \mathcal{N}^{-4} (\mathcal{J} - \mathcal{J}_\psi) B(m, \mathcal{J}, \mathcal{J}_\psi) + \mathcal{O}[\mathcal{N}^{-5}] , \quad (4.51)$$

which reduces to the usual linear scaling relation of energy with spins and oscillation number in the large $\mathcal{N}$ limit. Here $a_1 \simeq 0.08334$, $a_2 \simeq 0.04167$, $a_3 \simeq 0.00347$, $a_4 \simeq 0.00521$, and

$$A(m, \mathcal{J}, \mathcal{J}_\psi) = -1 + \frac{d_1}{m} + \frac{d_2 (\mathcal{J} - \mathcal{J}_\psi)}{m^3} ,$$

$$B(m, \mathcal{J}, \mathcal{J}_\psi) = -1 + \frac{d_1}{m} + \frac{d_2 (2\mathcal{J}^2 - \mathcal{J} \mathcal{J}_\psi - \mathcal{J}_\psi^2)}{3m^3} , \quad (4.52)$$

with $d_1 \simeq 1.35$, $d_2 \simeq 12$. We can see that no higher powers of $W_0$ appears in the series, so we can claim that our approximation on $W_0$ does not bring any divergences in the
spectrum of $\tilde{E}$. Also we recall that $\tilde{E}^2 = \mathcal{E}^2 - \sum P_i^2 + 2W_0^2(J_{\phi_1} + J_{\phi_2})$, and for the sake of completeness we compute the expansion for $\tilde{E} = \sqrt{\mathcal{E}^2 - \sum P_i^2}$. It is easy to find that the solution can be written as,

$$\tilde{E} = 2NW_0 + (\sqrt{2(J_{\phi_1}^2 + J_{\phi_2}^2) - J_{\psi}})W_0 + \sum_{n=1}^{\infty} \left( \frac{1}{N} \right)^n \mathcal{G}(n)(m, J_{\phi_1}, J_{\phi_2}, J_{\psi}), \quad (4.53)$$

where

$$\mathcal{G}(n) = [f_1(m, J, J_{\psi})W_0 + (J_{\phi_1} + J_{\phi_2})f_2(m, J, J_{\psi})W_0^3 + (J_{\phi_1} + J_{\phi_2})^2 f_3(m, J, J_{\psi})W_0^5 + (J_{\phi_1} + J_{\phi_2})^3 f_4(m, J, J_{\psi})W_0^7 + ...]. \quad (4.54)$$

Here the functions $f_k(m, J, J_{\psi})$ are of rather complicated form and we do not present them here explicitly. But again, it seems clear that even without terms higher than $O(W_0^4)$ the series does not show any divergences, hinting at a well behaved energy spectrum.

5. Conclusion

In this paper we have studied few examples of semiclassical strings in the near horizon geometry of PFT. We have found the most general solutions of the equations of motion of the probe fundamental strings in this background and found out dispersion relations among various conserved quantities using some regularization technique. However, while studying semiclassical strings in PFT background we have used some simplification and kept terms up to $O(V^4)$, where $V$ is the radial coordinate. This approximation is justified by following [4], which would correspond to the leading order deformation to $N = 4$ SYM. We can try to study string propagation in the background with full generality. It will also be highly challenging to study the boundary theory operators corresponding to these states as the dual gauge theory is almost unknown beyond the leading order. Hence, the semiclassical analysis of the string states might give us hints about the possible nature of dual gauge theory operators next to leading order. Furthermore it will be interesting to study the Wilson loops in this background to have a better understanding of this. We hope to come back to some of these issues in future.

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