Superconformal symmetry and higher-derivative Lagrangians

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Frascati,
Breaking of supersymmetry and Ultraviolet Divergences in extended Supergravity
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Interest in higher-derivative terms:

- appear as $\alpha'$ terms in effective action of string theory
- corrections to black hole entropy
- higher order to AdS/CFT correspondence
- counterterms for UV divergences of quantum loops
Plan

1. What we know about general sugra/susy theories
2. The superconformal method (and in which SUGRAs can we use it)
3. Higher derivative sugra actions and sugra loop results
4. Dirac-Born-Infeld– Volkov-Akulov and deformation of supersymmetry (example of an all order higher-derivative susy action)
5. Conclusions
1. What we know about general sugra/susy theories

- There are many good books that explain the basics, but ...
‘Ordinary’ susy/sugra

- Bosonic terms in the action have at most two spacetime derivatives, and fermionic terms at most one.
- E.g. $D=4$: fields of spin $2, 1, 0, 3/2, 1/2$

$$e^{-1} \mathcal{L} = \frac{1}{2} R +$$
$$+ \frac{1}{4} \text{Im} \ N_{IIJ} \mathcal{F}_{\mu \nu}^I \mathcal{F}_{\mu \nu}^J$$
$$- \frac{i}{8} \text{Re} \ N_{IIJ} \epsilon_{\mu \nu \rho \sigma} \mathcal{F}_{\mu \nu}^I \mathcal{F}_{\rho \sigma}^J$$
$$- \frac{1}{2} g_{uv} D_{\mu} \phi^u \ D_{\nu} \phi^v - V$$
$$\{- \bar{\psi}_{\mu i} \gamma^{\mu \nu \rho} D_{\nu} \psi_{\rho i}$$
$$- \frac{1}{2} g_A B \bar{\lambda}^A \phi \lambda_B + \text{h.c.}\} + \ldots$$
Possibilities for susy depend on the properties of irreducible spinors in each dimension

- Dependent on signature. Here: Minkowski
- **M**: Majorana
  **MW**: Majorana-Weyl
- **S**: Symplectic
  **SW**: Symplectic-Weyl

| Dim | Spinor | min.# comp |
|-----|--------|------------|
| 2   | MW     | 1          |
| 3   | M      | 2          |
| 4   | M      | 4          |
| 5   | S      | 8          |
| 6   | SW     | 8          |
| 7   | S      | 16         |
| 8   | M      | 16         |
| 9   | M      | 16         |
| 10  | MW     | 16         |
| 11  | M      | 32         |
Maximal susy / sugra

- From representations in 4 dimensions:
  maximal $N=8 \rightarrow 32$ susys for supergravity
  maximal $N=4 \rightarrow 16$ susys for supersymmetry

- based on particle states and susy operator
  - transforming a boson state in a fermion state,
  - and squaring to translations
Particle representations of $\mathcal{N}$–extended supersymmetry

$\#$ bosonic d.o.f. = $\#$ fermionic d.o.f., based on $\{Q,Q\} = P$ (invertible)

$$\sum_{s,h} \langle p^\mu, s, h | \left( Q_{i\alpha} Q^{\dagger j\alpha} + Q^{\dagger j\alpha} Q_{i\alpha} \right) e^{-2\pi i J_3} | p^\mu, s, h \rangle$$

$$= \delta^j_i \sum_{s,h} \langle p^\mu, s, h | P^0 e^{-2\pi i J_3} | p^\mu, s, h \rangle$$
Maximal susy / sugra

- From representations in 4 dimensions:
  - maximal N=8 → 32 susys for supergravity
  - maximal N=4 → 16 susys for supersymmetry

- based on particle states and susy operator
  - transforming a boson state in a fermion state,
  - and squaring to translations

- maximal spin 2 for gravity theories → N≤ 8 or 32 susys
- maximal spin 1 without gravity → N≤ 4 or 16 susys

- any higher dimensional theory can be dimensionally reduced on tori to D=4.
  This keeps the same number of susy generators
The map: dimensions and # of supersymmetries

| D | susy | 32 | 24 | 20 | 16 | 12 | 8 | 4 |
|---|------|----|----|----|----|----|---|---|
| 11 | M   | M  | M  | M  | M  | M  | M  | M  |
| 10 | MW  | IIA| IIB| IIB| IIB| IIB| IIB| IIB|
| 9  | M   | N=2| N=2| N=2| N=2| N=2| N=2| N=2|
| 8  | M   | N=2| N=2| N=2| N=2| N=2| N=2| N=2|
| 7  | S   | N=4| N=4| N=4| N=4| N=4| N=4| N=4|
| 6  | SW  | (2,2)|(2,1)|(2,1)|(2,1)|(2,1)|(2,1)|(2,1)|
| 5  | S   | N=8| N=6| N=6| N=6| N=6| N=6| N=6|
| 4  | M   | N=8| N=6| N=6| N=6| N=6| N=6| N=6|

SUGRA
SUGRA/SUSY
SUGRA
SUGRA/SUSY

- ![vector multiplets](green)
- ![tensor multiplet](blue)
- ![multiplets up to spin 1/2](olive)

Strathdee, 1987
Basic supergravities and deformations

- **Basic supergravities:**
  - have only gauged supersymmetry and general coordinate transformations (and U(1)’s of vector fields).
  - No potential for the scalars.
  - Only Minkowski vacua.

- In any entry of the table there are ‘**deformations**’: without changing the kinetic terms of the fields, the couplings are changed.
  - Many deformations are ‘gauged supergravities’: gauging of a YM group, introducing a potential.
  - Produced by fluxes on branes
  - There are also other deformations (e.g. massive deformations, superpotential)
Embedding tensor formalism

The gauge group is a subgroup of the isometry group $G$, defined by an embedding tensor. 

\[
(\partial_\mu - A_\mu^M \Theta_M^\alpha \delta_\alpha) \phi
\]

determines which symmetries are gauged, and how: e.g. also the coupling constants. 

There are several constraints on the tensor.

Structure to get a complete picture of supergravities with at most two spacetime derivatives in Lagrangian 

(although: to get all the explicit solutions of constraints still needs more work)

Nicolai, Samtleben, 0010076

de Wit, Samtleben and Trigiante, 0507289

Cordaro, Frè, Gualtieri, Termonia and Trigiante, 9804056
Higher-derivative actions: no systematic knowledge

- Various constructions of higher derivative terms
  - e.g. susy Dirac-Born-Infeld: Cecotti, Ferrara, 1987; Tseytlin; Bagger, Galperin; Roček; Kuzenko, Theisen; Ivanov, Krivonos, Ketov; Bellucci,

- but no systematic construction, or classification of what are the possibilities; (certainly not in supergravity)
Constructions of actions

Possible constructions:

- order by order Noether transformations: the only possibility for the maximal theories (Q>16)
- superspace:
  - very useful for rigid N=1: shows structure of multiplets.
  - very difficult for supergravity. Needs many fields and many gauge transformations
- (super)group manifold:
  - Optimal use of the symmetries using constraints on the curvatures
- superconformal tensor calculus:
  - keeps the structure of multiplets as in superspace but avoids its immense number of unphysical degrees of freedom
  - extra symmetry gives insight in the structure
2. The superconformal method

- Superconformal symmetry is the maximal extension of spacetime symmetries according to Coleman-Mandula theorem

- Here: not about Weyl supergravity: \[ \int d^4x \left[ R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3}R^2 \right] \]

- Tool for construction of actions
  - allows to use multiplet calculus similar to superspace
  - makes hidden symmetries explicit
Gravity as a conformal gauge theory

The strategy

- scalar field (compensator)

### Conformal Gravity

\[ \mathcal{L} = -\frac{1}{2} \sqrt{g} \phi \Box_C \phi = -\frac{1}{2} \sqrt{g} \phi \Box \phi + \frac{1}{12} \sqrt{g} R \phi^2 \]

### Dilatational Gauge Fixing

\[ \phi = \sqrt{6}/\kappa \]

\[ \mathcal{L} = \frac{1}{2\kappa^2} \sqrt{g} R \]

- First action is conformal invariant,
- gauge-fixed one is Poincaré invariant.
- Scalar field had scale transformation \( \delta \phi (x) = \lambda_D (x) \phi (x) \)
Schematic: Conformal construction of gravity

- Conformal scalar action (contains Weyl fields)
- Gauge fix dilatations and special conformal transformations
- Poincaré gravity action

Local conformal symmetry
Local $\square$ symmetry
Superconformal construction
The idea of superconformal methods

- Difference susy- sugra: the concept of multiplets is clear in susy, they are mixed in supergravity
- Superfields are an easy conceptual tool for rigid susy

- (Super)gravity can be obtained by starting with (super)conformal symmetry and gauge fixing.
- With matter:
  Before gauge fixing: everything looks like in rigid supersymmetry + covariantizations
Superconformal algebra

In general

\[
\begin{pmatrix}
\text{conformal algebra} \\
\text{Q, S} \\
\text{R-symmetry}
\end{pmatrix}
\]

according to dilatational weight: \( \text{e.g. N=1} \)

\[
\begin{align*}
1 & : P_\mu \\
\frac{1}{2} & : Q \\
0 & : D, M_{ab}, U(1) \\
-\frac{1}{2} & : S \\
-1 & : K_\mu
\end{align*}
\]

\[
[D, Q] = \frac{1}{2} Q, \quad [D, S] = -\frac{1}{2} S
\]

\[
\begin{align*}
\{ Q_\alpha, Q^\beta \} &= -\frac{1}{2} (\gamma^a)_{\alpha}^{\beta} P_a, \\
\{ S_\alpha, S^\beta \} &= -\frac{1}{2} (\gamma^a)_{\alpha}^{\beta} K_a, \\
\{ Q_\alpha, S^\beta \} &= -\frac{1}{2} \delta_{\alpha}^{\beta} D - \frac{1}{4} (\gamma^{ab})_{\alpha}^{\beta} M_{ab} + \frac{1}{2} i (\gamma_*)_{\alpha}^{\beta} U(1)
\end{align*}
\]
The strategy: superconformal construction of \( N=1 \) supergravity

- chiral multiplet + Weyl multiplet
- superconformal action

Gauge fix dilatations,
- special conformal transformations,
- local R-symmetry and
- special supersymmetry

Poincaré supergravity action
Superconformal construction of N=4 supergravity

De Roo, 1985

Weyl multiplet +

6 gauge compensating multiplets (on-shell)

superconformal action

gauge-fixing

Weyl symmetry, local SU(4), local U(1), S-supersymmetry and K-conformal boosts

pure N=4 Cremmer-Scherk-Ferrara supergravity

\[ \frac{1}{4} R - \frac{1}{8} \frac{\partial \tau \partial \bar{\tau}}{(\text{Im} \tau)^2} + \frac{i}{4} \tau F^{+I}_\mu \delta_{IJ} F^{+J}_\mu + \text{h.c.} \]
On-shell and off-shell multiplets

- Action should be invariant
- Algebra can be closed only modulo field equations
- Problem: No flexibility in field equations
- Examples:
  - hypermultiplets $N=2$;
  - $N=4$ gauge multiplets (are compensator multiplets);
In which sugras can we use superconformal methods?

- We should have a superconformal algebra
- We should have compensating multiplets
Superconformal groups

conformal algebra is $\text{so}(D,2)$

| $D$ | supergroup       | conf                  | $R$                  | ferm. |
|-----|------------------|-----------------------|----------------------|-------|
| 3   | $\text{OSp}(N|4)$ | $\text{SO}(3,2) = \text{Sp}(4)$ | $\text{SO}(N)$       | $4N$  |
| 4   | $\text{SU}(2,2|N)$ | $\text{SO}(4,2) = \text{SU}(2,2)$ | $\text{U}(N)$ for $N \neq 4$ | $8N$  |
|     |                  |                       | $\text{SU}(4)$ for $N = 4$ |       |
| 5   | $F^2(4)$         | $\text{SO}(5,2)$     | $\text{SU}(2)$       | $16$  |
| 6   | $\text{OSp}(8^*|2N)$ | $\text{SO}(6,2) = \text{SO}^*(8)$ | $\text{USp}(2N)$   | $16N$ |

covering group always compact

Other superalgebras have been considered, where the conformal algebra is not a factor, but a subalgebra of the bosonic part symmetry e.g. $\text{SO}(11,2) \subset \text{Sp}(64) \subset \text{OSp}(1|64)$

But not sucessfully applied for constructing actions

JW van Holten, AVP, 1982
D’Auria, S. Ferrara, M. Lledò, V. Varadarajan, 2000
# The map: dimensions and # of supersymmetries

| D | susy | 32 | 24 | 20 | 16 | 12 | 8 | 4 |
|---|------|----|----|----|----|----|---|---|
| 11 | M |     |     |     |     |     |    |    |
| 10 | MW | IIA | IIB |     |     |     |    |    |
| 9  | M  | N=2 |     |     |     |     | N=1|    |
| 8  | M  | N=2 |     |     |     |     | N=1|    |
| 7  | S  | N=4 |     |     |     |     | N=2|    |
| 6  | SW | (2,2)| (2,1)|   | (1,1)| (2,0)|   | (1,0)|
| 5  | S  | N=8 | N=6 |     |     |     | N=4| N=2|    |
| 4  | M  | N=8 | N=6 | N=5 | N=4 | N=3 | N=2| N=1|    |

- **SUGRA**
- **SUGRA/SUSY**
- **SUGRA**

**have SC algebra**
**can be used for SC methods**
vector multiplets +
multiplets up to spin 1/2

**vector multiplets**
**tensor multiplet**

3. Higher derivative supergravity actions and supergravity loop results
## The Null Results

| Miracle #1 | 2007 | N= 8, D=4 is UV finite up to 3-loops |
|------------|------|-----------------------------------|
|            |      | Bern, Carrasco, Dixon, Johansson, Kosower, Roiban |

| Miracle #2 | 2009 | N= 8, D=5 is UV finite up to 4-loops |
|------------|------|-----------------------------------|
|            |      | Bern, Carrasco, Dixon, Johansson, Roiban |

| Miracle #3 | 2012 | N= 4, D=4 is UV finite up to 3-loops |
|------------|------|-----------------------------------|

| Miracle #4 | 2012 | N= 4, D=5 is UV finite up to 2-loops |

- Bern, Davies, Dennen, Huang: 3-loop D=4 computation in pure supergravity
- September: 2-loop D=5 UV finite

- If there are divergences: 
  - supersymmetric counterterms should exist
  - (or supersymmetry anomalies)
- We do not know enough to be sure whether invariants do exist.
Higher derivative sugra actions

- examples with superconformal tensor calculus
  - S. Cecotti and S. Ferrara, `Supersymmetric Born-Infeld actions’, 1986
  - N=2 constructions:
    B. de Wit, S. Katmadas, M. van Zalk, arXiv:1010.2150 ;
    W. Chemissany, S. Ferrara, R. Kallosh, C. S. Shahbazi, 1208.4801
  - Higher derivative extension in D=6 (1,0)
    E. Bergshoeff, F. Coomans, E. Sezgin, AVP 1203.2975

- other methods
  starting with S. Deser, J.H. Kay and K.S. Stelle, 1977, ...
  more recent: G. Bossard, P. Howe, K. Stelle, P. Vanhove;
  M. Koehn, J-L Lehners, B. Ovrut ... (using superspace)
N=2 D=4 construction

- Based on tensor calculus as in superspace:
  - chiral multiplets: \( S = \{X, \Omega_i, \ldots, C\} \)
  - also Weyl multiplet: \( W^2 = \{T_{ab} T^{ab}, \ldots\} \)
  - kinetic multiplet: \( \mathcal{T}(\bar{S}) = \{\bar{C}, \ldots\} \)

- made superconformal invariant
  - restriction on possible actions (homogeneity)

- Everything off-shell

- many possibilities, e.g. invariants contributing to entropy and central charges of black holes

\(^{*}\) arbitrary power is still chiral

de Wit, Katmadas, van Zalk, 2011
N=2 higher derivative terms with auxiliary fields

- The term quartic in the auxiliary field from the Weyl multiplet is a partner of the term quartic in the Weyl curvature.

\[ \lambda(C, \ldots)^4 \quad \lambda(\partial \mathcal{T})^4 \]

- Deformed EOM for the Weyl multiplet auxiliary

\[ T^+_{ab} = \frac{2}{X} F^+_{ab} + \lambda (\partial^4 T^3)^+_{ab} + \ldots \]

- Solve recursively: infinite number of higher derivative terms with higher and higher powers of the graviphoton, N=2 supergravity vector

\[ \mathcal{T}^{def} = \mathcal{F} + \lambda [\partial^4 \mathcal{F}^3] + \lambda^2 [\partial^4 \mathcal{F}^2][\partial^4 \mathcal{F}^3] + \ldots \]

- The action with auxiliary field eliminated: Born-Infeld with higher derivatives

\[ S^{def} = -\frac{1}{4} \mathcal{F}^2 + \lambda ([\partial \mathcal{F}]^4 + \lambda^2 [\partial^8 \mathcal{F}^6] + \ldots \]

Chemissany, Ferrara, Kallosh, Shahbazi, 1208.4801
Also transformation laws deform

\[ \delta \psi_\mu^i = D_\mu \epsilon^i - \frac{1}{16} \gamma^{ab} T_{ab} \epsilon^{ij} \gamma_\mu \epsilon_j - \gamma_\mu \eta^i \]

Deformation of the supergravity local N=2 supersymmetry after S-supersymmetry gauge-fixing and expanding near the lowest order solution for auxiliary fields

Order by order

\[ \phi_{aux} = \phi_{aux}^0 + \Delta \phi_{aux} \]

\[ \Delta \phi_{aux} = \sum_{n=1} \lambda^n \phi^{(n)}_{aux}, \]

The deformation of the gravitino supersymmetry due to higher derivative term is

\[ \Delta \psi_\mu = -4\lambda [\partial^4 F^3]_{\mu}^{\nu} \gamma_\nu \epsilon^i + \ldots \]
Are all valid counterterms (broken) superconformal actions?

\(N=0\) : Locally conformal \(\mathbb{R}^4\) ; gauge fixed: is 3-loop counterterm

\[
\int d^4x \sqrt{-g} \phi^{-4} C_{\alpha\beta\gamma\delta} C_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} C^{\alpha\beta\gamma\delta} C^{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}
\]

\(N=2\) superconformal \(\mathbb{R}^4\)
chiral kinetic action with inverse powers of the compensator superfield \(S\)

\[
\int d^4\theta \frac{W^2}{S^2} T \left( \frac{W^2}{S^2} \right)
\]

\(N=4\) superconformal \(\mathbb{R}^4\)

???
N=4 has no tensor calculus

- Since compensating multiplets cannot be multiplied, ... we cannot make constructions as those for N=2.
- Algebra only valid on shell: modified actions imply modified field equations:

  \[ \Rightarrow \text{transformations (or superfields) have to be deformed.} \]
How for N=4 ?

- No tensor calculus; no auxiliary fields
- How to establish the existence/non-existence of the consistent order by order deformation of N=4 on shell superspace?
- **Conjecture**: if it does not exist: explanation of finiteness (if Bern et al do not find N=4, D=4 is divergent at higher loops)
- Until invariant counterterms are constructed (conformal?) we have no reason to expect UV divergences

**Two points of view**

1. Legitimate counterterms are not available yet
2. Legitimate counterterms are not available, period

S. Ferrara, R. Kallosh, AVP, 1209.0418
N=4 conjecture

If the UV finiteness will persist in higher loops, one would like to view this as an opportunity to test some new ideas about gravity. E.g. : is superconformal symmetry more fundamental?

Repeat: Classical N=4 is obtained from gauge fixing a superconformal invariant action:
The mass $M_{Pl}$ appears in the gauge-fixing procedure

Analogy:
• Mass parameters $M_W$ and $M_Z$ of the massive vector mesons are not present in the gauge invariant action of the standard model.
• Show up when the gauge symmetry is spontaneously broken.
• In unitary gauge they give an impression of being fundamental.
• In renormalizable gauge (where UV properties analyzed) : absent

S. Ferrara, R. Kallosh, AVP, 1209.0418
The non-existence of (broken) superconformal-invariant counterterms and anomalies in $N=4$, $D=4$ could explain ‘miraculous’ vanishing results.

- **simplest** possible explanation of the 3-loop finiteness and predicts perturbative UV finiteness in higher loops
- the same conjecture applies to higher derivative superconformal invariants and to a consistent superconformal anomaly
- the conjecture is **economical**, sparing in the use of resources: either the local $N=4$ superconformal symmetry is a good symmetry, or it is not.
- **Falsifiable** by $N=4$ $L=4$ computations
  (which are already underway)

If the conjecture survives these computations (if UV finite): hint that the models with superconformal symmetry serve as a basis for constructing a consistent quantum theory where $M_{Pl}$ appears in the process of gauge-fixing superconformal symmetry.

Also **falsifiable** by our own calculations: if we find a way to construct (non-perturbative) superconformal invariants
The non-existence of (broken) superconformal invariants and anomalies in $N=4$, $D=4$ could explain 'miraculous' vanishing results.

- **simplest** possible explanation of the 3-loop finiteness and predicts perturbative UV finiteness in higher loops
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- **Falsifiable** by $N=4$ $L=4$ computations (which are already underway)

If the conjecture survives these computations (if UV finite): hint that the models with superconformal symmetry serve as a basis for constructing a consistent quantum theory where $M_{pl}$ appears in the process of gauge-fixing superconformal symmetry.

Also **falsifiable** by our own calculations: if we find a way to construct (non-perturbative) superconformal invariants

“We are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress.” (Feynman)
4. Dirac-Born-Infeld–Volkov-Akulov and deformation of supersymmetry

A super new paper (arXiv:1303.5662)

on the search of deformations of N=4 theories, we find all-order invariant actions in rigid susy with extra supersymmetries (Volkov-Akulov (VA) – type)
Down-up approach: start deformations

\[ S = \int d^D x \left\{ -\frac{1}{4}(F_{\mu\nu})^2 + \bar{\chi} \phi \chi \right\} \]

gauge field \((D-2)\) on-shell dof;
fermion = \#spinor comp / 2

| Dim | Spinor | min.# comp |
|-----|--------|------------|
| 2   | MW     | 1          |
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| 4   | M      | 4          |
| 5   | S      | 8          |
| 6   | SW     | 8          |
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| D | susy  | 32 | 24 | 20 | 16 | 12 | 8 | 4 |
|---|-------|----|----|----|----|----|---|---|
| 11 | M     | M  |    |    |    |    |    |    |
| 10 | MW    | IIA| IIB|    | I  |    |    |    |
| 9  | M     | N=2|    |    |    |    |    |    |
| 8  | M     | N=2|    |    |    |    |    |    |
| 7  | S     | N=4|    |    |    |    |    |    |
| 6  | SW    | (2,2)| (2,1)| (1,1)| (2,0)| (1,0) | | |
| 5  | S     | N=8| N=6|    |    |    |    |    |
| 4  | M     | N=8| N=6| N=5|    |    |    |    |

**SUGRA** | **SUGRA/SUSY** | **SUGRA** | **SUGRA/SUSY**

- **vector multiplets**
- **tensor multiplet**
- **multiplets up to spin 1/2**
Down-up approach: start deformations

\[ S = \int d^D x \left\{ -\frac{1}{4} (F_{\mu\nu})^2 + \bar{\lambda} \phi \lambda \right\} \]

gauge field (D-2) on-shell dof;
fermion = \#spinor comp / 2

\[ \delta_\epsilon A_\mu = \bar{\epsilon} \Gamma_\mu \lambda, \quad \delta_\epsilon \lambda = \frac{1}{4} \Gamma^{\mu\nu} F_{\mu\nu} \epsilon \]

extra (trivial) fermionic shift symmetry

\[ \delta_\eta A_\mu = 0, \quad \delta_\eta \lambda = -\frac{1}{2\alpha} \eta \]

D=10: MW;
D=6 SW;
D=4 M;
D=3 M;
D=2 MW

normalization for later use
Bottom-up deformation

\[ S = \int d^D x \{ -\frac{1}{4} F^2 + \bar{\lambda} \phi \lambda \} - 2\alpha c_4 F^{\mu \nu} \bar{\lambda} \Gamma_\mu \partial_\nu \lambda \]

\[ + \frac{1}{8} \alpha^2 [ \text{Tr} F^4 - \frac{1}{4} (F^2)^2 + 4(1 + 4c_4^2)(F^2)^\mu\nu \bar{\lambda} \Gamma_\mu \partial_\nu \lambda \]

\[ + (1 - 4c_4^2) F_\mu \lambda (\partial_\lambda F_{\nu\rho}) \bar{\lambda} \Gamma^{\mu\nu\rho} \lambda + \frac{1}{2} (c_1 + 8c_4^2) F^2 \bar{\lambda} \phi \lambda \]

\[ - \frac{1}{2} C_2 F_{\mu\nu} (\partial_\lambda F^\lambda_{\rho}) \bar{\lambda} \Gamma^{\mu\nu\rho} \lambda - \frac{1}{2} (c_3 + 4c_4^2) F_{\mu\nu} F_{\rho\sigma} \bar{\lambda} \Gamma^{\mu\nu\rho\sigma} \phi \lambda \]

\[ + \mathcal{O}(\alpha^2 \lambda^4) + \mathcal{O}(\alpha^3) \]

free coefficients \( c_i \), but these are related to field redefinitions

\[ A_\mu(0) = A_\mu - \frac{1}{16} \alpha^2 C_2 F^{\nu\rho} \bar{\lambda} \Gamma_{\mu\nu\rho} \lambda, \]

\[ \lambda(0) = \lambda + \frac{1}{2} \alpha c_4 F_{\mu\nu} \Gamma^{\mu\nu} \lambda + \frac{1}{32} \alpha^2 c_1 F^2 \lambda - \frac{1}{32} \alpha^2 c_3 F_{\mu\nu} F_{\rho\sigma} \Gamma^{\mu\nu\rho\sigma} \lambda, \]

\[ \Rightarrow \text{answer unique;} \]

also transformation rules deformed.

As well \( \epsilon \) as \( \eta \) parameter transformations can be defined continues from E. Bergshoeff, M. Rakowski and E. Sezgin, 1987
bottom-up deformation

e.g.

\[ \delta \eta A^\mu = \frac{\alpha}{4} \overline{\eta} F^\nu{}^\mu \Gamma_{\nu} \lambda + \frac{\alpha}{8} \overline{\eta} \Gamma^{\mu\nu\rho} F_{\nu\rho} \lambda - \frac{1}{16} \alpha c_2 F_{\nu\rho} \overline{\eta} \Gamma^{\mu\nu\rho} \lambda + O(\alpha \eta \lambda^3) + O(\alpha^2), \]

\[ \delta \eta \lambda = -\frac{1}{2\alpha} \eta + \alpha \left[ -\frac{1}{32} F^2 - \frac{1}{64} \Gamma^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right] \eta \]

\[ + \frac{1}{4} c_4 F_{\mu\nu} (c) \Gamma^{\mu\nu} \left[ \eta - \frac{1}{2} \alpha c_4 F_{\rho\sigma} (c) \Gamma^{\rho\sigma} \eta \right] \]

\[ + \frac{1}{64} \alpha c_1 F^2 \eta - \frac{1}{64} \alpha c_3 F_{\mu\nu} F_{\rho\sigma} \Gamma^{\mu\nu\rho\sigma} \eta + O(\alpha \eta \lambda^2) + O(\alpha^2) \]

already complicated; but only use of 
- Majorana flip relations

\[ \bar{\lambda}_1 \Gamma^\mu \lambda_2 = -\bar{\lambda}_2 \Gamma^\mu \lambda_1 \]

- cyclic (Fierz) identity

\[ \Gamma_\mu \lambda_1 \bar{\lambda}_2 \Gamma^\mu \lambda_3 + \Gamma_\mu \lambda_2 \bar{\lambda}_3 \Gamma^\mu \lambda_1 + \Gamma_\mu \lambda_3 \bar{\lambda}_1 \Gamma^\mu \lambda_2 = 0. \]

which are valid in D=10,6,4,3,2

looks hopeless to continue to all orders
Dp-brane action

Start with $\kappa$–symmetric Dp brane action

\[
S_{\text{DBI} + S_{\text{WZ}}} = -\frac{1}{\alpha'^2} \int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} + \frac{1}{\alpha'^2} \int \Omega_{p+1}
\]

\[
G_{\mu\nu} = \eta_{mn} \prod_{\mu}^{m} \prod_{\nu}^{n}, \quad \prod_{\mu}^{m} = \partial_{\mu} X^{m} - \bar{\theta} \Gamma^{m} \partial_{\mu} \theta
\]

\[
F_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha^{-1}\bar{\theta} \sigma_{3} \Gamma_{m} \partial_{[\mu} \theta \left( \partial_{\nu]} X^{m} - \frac{1}{2} \bar{\theta} \Gamma^{m} \partial_{\nu] \theta} \right)
\]

Dp brane: IIB theory $m=0,..., 9$ and $\mu=0,..., p=2n+1$

space-time coordinates $X^{m}$; $\theta$ is doublet of MW spinors;

$F_{\mu\nu}$ Abelian field strength

Symmetries:

rigid susy doublet $\epsilon^{1}; \epsilon^{2}$

local $\kappa$ symmetry doublet (effectively only half (reducible symmetry))

world volume gct

$\delta_{\kappa} \theta = (1 + \Gamma) \kappa$
Dp-brane action

Start with $\kappa$–symmetric Dp brane action

$$S_{\text{DBI}+\text{SWZ}} = -\frac{1}{\alpha'^2} \int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} + \frac{1}{\alpha'^2} \int \Omega_{p+1}$$

$$G_{\mu\nu} = \eta_{mn} \prod_{\mu}^m \prod_{\nu}^n, \quad \prod_{\mu}^m = \partial_\mu X^m - \bar{\theta} \Gamma^m \partial_\mu \theta$$

$$F_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha^{-1} \bar{\theta} \sigma_3 \Gamma_m \partial_{[\mu} \theta (\partial_{\nu]} X^m - \frac{1}{2} \bar{\theta} \Gamma^m \partial_\nu \theta)$$

Dp brane: IIB theory $m=0,...,9$ and $\mu=0,...,p=2n+1$

space-time coördinates $X^m$; $\theta$ is doublet of MW spinors; $F_{\mu\nu}$ Abelian field strength

Symmetries:

rigid susy doublet $\mathcal{S}$

local $\kappa$ symmetry doublet $(\mathcal{S}, \mathcal{S'})$ (effectively only half (reducible symmetry)

world volume get

Same applies for $D=6$ (2,0) (also called iib):

$m=0,...,5$

brane interpretation: see talk Eric Bergshoeff

Also $D=4$ N=2, $m=0,...,3$ (BH solutions)
Gauge fixing

\( X^m = \{ \delta^m_\mu \sigma^\mu, \phi^I \} \), \quad m' = 0, 1, \ldots, p, \quad I = 1, \ldots, 9 - p \\
\theta = (\theta^1 = 0, \theta^2 \equiv \alpha \lambda) \\

worldvolume gct \( \xi^m \) and \( \kappa \) symmetry gauge-fixed \\
to stay in the gauge (‘decomposition laws’): parameters become function of parameters of other symmetries \\
\( \Rightarrow \) the two deformed \( \epsilon^1 \) and \( \epsilon^2 \) supersymmetries preserved \\
suitable combinations are called \( \epsilon \) and \( \zeta \)
Complete DBI-VA model
for the p=9 case (no scalars $\phi^I$)

$$S = -\frac{1}{\alpha^2} \int d^{10}x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} - 1 \right\}$$

$$G_{\mu\nu} = \eta_{mn} \Pi^m_{\mu} \Pi^n_{\nu}, \quad \Pi^m_{\mu} = \delta^m_{\mu} - \alpha^2 \bar{\lambda}\Gamma^m \partial_{\mu} \lambda,$$

$$F_{\mu\nu} \equiv F_{\mu\nu} + 2\alpha \bar{\lambda}\Gamma_{[\nu} \partial_{\mu]} \lambda, \quad \mu = 0, 1, \ldots, 9, \quad m = 0, 1, \ldots, 9,$$

16 $\epsilon$ transformations, deformation of the Maxwell supermultiplet supersymmetries

$$\delta_\epsilon \lambda = -\frac{1}{2\alpha} (1 - \beta) \epsilon - \frac{1}{2} \alpha \partial_\mu \lambda \bar{\lambda}\Gamma^\mu (1 + \beta) \epsilon,$$

$$\delta_\epsilon A_\mu = -\frac{1}{2} \bar{\lambda}\Gamma_{\mu} (1 + \beta) \epsilon + \frac{1}{2} \alpha^2 \bar{\lambda}\Gamma_m (\frac{1}{3} 1 + \beta) \epsilon \bar{\lambda} \Gamma^m \partial_\mu \lambda - \frac{1}{2} \alpha \bar{\lambda}\Gamma^\rho (1 + \beta) \epsilon F_{\rho\mu}$$

16 VA-type $\zeta$ transformations

$$\delta_\zeta \lambda = \alpha^{-1} \zeta + \alpha \partial_\mu \lambda \bar{\lambda}\Gamma^\mu \zeta,$$

$$\delta_\zeta A_\mu = \bar{\lambda}\Gamma_\mu \zeta + \alpha \bar{\lambda}\Gamma^\rho \zeta F_{\rho\mu} - \frac{1}{3} \alpha^2 \bar{\lambda}\Gamma_m \zeta \bar{\lambda}\Gamma^m \partial_\mu \lambda$$

$$\beta = [\det (\delta_\mu \nu + \alpha F_{\mu\rho} G_{\rho\nu})]^{-1/2} \sum_{k=0}^{5} \frac{\alpha^k}{2^k k!} \bar{\Gamma}_1 \nu_1 \cdots \nu_k \nu_k F_{\mu_1 \nu_1} \cdots F_{\mu_k \nu_k} = 1 + O(\alpha)$$

Note: $\Lambda$ susys: do not transform fermion to boson states; are not the regular susys
Complete DBI-VA model for the p=9 case (no scalars $\phi^I$)

$$S = -\frac{1}{\alpha^2} \int d^{10}x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} - 1 \right\}$$

$$G_{\mu\nu} = \eta_{mn} \Pi^m_{\mu} \Pi^n_{\nu} , \quad \Pi^m_{\mu} = \delta^m_{\mu} - \alpha^2 \bar{\lambda} \Gamma^m \partial_{\mu} \lambda ,$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + 2\alpha \bar{\lambda} \Gamma_{[\nu} \partial_{\mu]} \lambda , \quad \mu = 0, 1, ..., 9 , \quad m = 0, 1, ..., 9 ,$$

16 $\epsilon$ transformations, deformation of the Maxwell supermultiplet supersymmetries

$$\delta_{\epsilon} \lambda = -\frac{1}{2\alpha} (1 - \beta) \epsilon + \frac{1}{2} \alpha \partial_{\mu} \bar{\lambda} \Gamma^\mu (1 + \beta) \epsilon ,$$

$$\delta_{\epsilon} A_{\mu} = -\frac{1}{2} \bar{\lambda} \Gamma_{\mu} (1 + \beta) \epsilon + \frac{1}{2} \alpha^2 \bar{\lambda} \Gamma_m (1 + \beta) \epsilon \bar{\lambda} \Gamma^m \partial_{\mu} \lambda - \frac{1}{2} \alpha \bar{\lambda} \Gamma^\rho (1 + \beta) \epsilon F_{\rho\mu}$$

16 VA-type $\zeta$ transformations

$$\delta_{\zeta} \lambda = \alpha^{-1} \zeta + \alpha \partial_{\mu} \bar{\lambda} \Gamma^\mu \zeta ,$$

$$\delta_{\zeta} A_{\mu} = \bar{\lambda} \Gamma_{\mu} \zeta + \alpha \bar{\lambda} \Gamma^\rho \zeta F_{\rho\mu} - \frac{1}{3} \alpha^2 \bar{\lambda} \Gamma_m \zeta \bar{\lambda} \Gamma^m \partial_{\mu} \lambda$$

Note: VA susys: do not transform fermion to boson states; are not the regular susys
Comparing bottom-up with top-down

\[ S = \int d^Dx \left\{ -\frac{1}{4}F^2 + \bar{\lambda}\phi\lambda \right\} - 2\alpha c_4 F^{\mu\nu} \bar{\lambda}\Gamma_{\mu}\partial_\nu\lambda \]

+ \frac{1}{8}\alpha^2 \left[ \text{Tr} F^4 - \frac{1}{4} \left(F^2\right)^2 + 4(1 + 4c_4^2)(F^2)^{\mu\nu}\bar{\lambda}\Gamma_{\mu}\partial_\nu\lambda \right]

+ (1 - 4c_4^2)F_\mu^{\lambda}(\partial_\lambda F_{\nu\rho})\bar{\lambda}\Gamma^{\mu\nu\rho}\lambda + \frac{1}{2}(c_1 + 8c_4^2)F^2\bar{\lambda}\phi\lambda

- \frac{1}{2}c_2 F_{\mu\nu}(\partial_\lambda F_\rho^{\lambda})\bar{\lambda}\Gamma^{\mu\nu\rho}\lambda - \frac{1}{2}(c_3 + 4c_4^2)F_{\mu\nu}F_{\rho\sigma}\bar{\lambda}\Gamma^{\mu\nu\rho\sigma}\phi\lambda + O(\alpha^2\lambda^4) + O(\alpha^3)

- Expanding the all-order result, one re-obtains indeed the result that was obtained in the bottom-up calculation to order \( \alpha^2 \) using a particular field definition (choice of coefficients \( c_i \) : such that no \( \partial F \) terms: \( c_1 = 2, c_2 = 0, c_3 = -1, c_4 = -1/2 \)).

- For the transformation laws: agree modulo a ‘zilch symmetry’: (on-line trivial symmetry) (\( \zeta \) is linear combination of \( \epsilon \) and \( \eta \))

- This proves that our all-order result is indeed the full deformation that we were looking for !
The map: dimensions and # of supersymmetry

| D | susy | 32 | 24 | 20 | 16 | 12 | 8 | 4 |
|---|------|----|----|----|----|----|---|---|
| 11 | M   | M  |     |    |    |    |   |   |
| 10 | MW  | IIA| IIB|    |    | D9| I |   |
| 9  | M   | N=2|    |    |    | N=1|   |   |
| 8  | M   | N=2|    |    |    | D7|   |   |
| 7  | S   | N=4|    |    |    | N=2|   |   |
| 6  | SW  | (2,2)|(2,1)| D5| (1,1)|(2,0)|   |   |
| 5  | S   | N=8| N=6|    | N=4|    | N=2|   |
| 4  | M   | N=8| N=6| N=5| N=4| N=3| N=2| N=1|

SUGRA

SUGRA/SUSY

vector multiplets +
tensor multiplet
multiplets up to spin 1/2
Complete DBI-VA model for the $p=3$ case

$$S = -\frac{1}{\alpha^2} \int d^4 x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} - 1 \right\}, \quad \mu = 0, 1, 2, 3$$

$$G_{\mu\nu} = \eta_{mn} \Pi^m_{\mu} \Pi^n_{\nu} = \eta_{m'n'} \Pi^m_{\mu} \Pi^{n'}_{\nu} + \delta_{IJ} \Pi^I_{\mu} \Pi^J_{\nu}, \quad m' = 0, 1, 2, 3, \quad I = 1, \ldots, 6$$

$$\Pi^m_{\mu} = \delta^m_{\mu} - \alpha^2 \bar{\lambda} \Gamma^m \partial_{\mu} \lambda, \quad \Pi^I_{\mu} = \partial_{\mu} \phi^I - \alpha^2 \bar{\lambda} \Gamma^I \partial_{\mu} \lambda$$

$$F_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha \bar{\lambda} \Gamma_{[\mu} \partial_{\nu]} \lambda - 2\alpha \bar{\lambda} \Gamma_I \partial_{[\mu} \lambda \partial_{\nu]} \phi^I$$

16 $\epsilon$ and 16 $\zeta$ symmetries and shift symmetry of scalars

Can be compared with $N = 4, D=4$
\[ N=4, \ D=4 \]

\[ S_{\text{Maxw}} = \int d^4x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2 \bar{\psi}_i \phi \psi^i - \frac{1}{8} \partial_\mu \varphi_{ij} \partial^\mu \varphi^{ij}) \]

one vector; 4 Majorana (or Weyl) spinors and 6 scalars) recognized as \( \alpha=0 \) part of all order action and transformations when

\[ \alpha \varphi_{ij} = \phi_a \beta_{ij}^a \rightleftarrows i \phi_a + 3 \alpha_{ij}^a, \quad a = 1, 2, 3 \]

Gliozzi, Scherk, Olive \( 4 \times 4 \) matrices rewriting the \( D=10 \) Majorana-Weyl fermion as

\[ \lambda = \begin{pmatrix} \psi^i \\ \psi_i \end{pmatrix} \]

\[ \Gamma^\mu = \gamma^\mu \otimes 1_8, \quad \Gamma^a = \gamma_\ast \otimes \begin{pmatrix} 0 & \beta^a \\ -\beta^a & 0 \end{pmatrix}, \quad \Gamma^{a+3} = \gamma_\ast \otimes \begin{pmatrix} 0 & i\alpha^a \\ i\alpha^a & 0 \end{pmatrix}, \]

\[ C_{10} = C_{4} \otimes \begin{pmatrix} 0 & 1_4 \\ 1_4 & 0 \end{pmatrix}, \quad \Gamma_\ast = \gamma_\ast \otimes \begin{pmatrix} 1_4 & 0 \\ 0 & -1_4 \end{pmatrix} \]
All-order deformations of N=4, D=4

- Since the action is invariant at all orders under 16+16 supersymmetries, this is the full result!
- The action has both type of supersymmetries: ordinary SUSY and VA-type
- The 10-dimensional formulation is much simpler.

\[ S = -\frac{1}{\alpha^2} \int d^4x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha F_{\mu\nu})} - 1 \right\}, \quad \mu = 0, 1, 2, 3 \]

\[ G_{\mu\nu} = \eta_{mn} \Pi^m_{\mu} \Pi^n_{\nu} = \eta_{m'n'} \Pi^m_{\mu} \Pi^{n'}_{\nu} + \delta_{IJ} \Pi^I_{\mu} \Pi^J_{\nu}, \quad m' = 0, 1, 2, 3 \]

\[ \Pi^m_{\mu} = \delta^m_{\mu} - \alpha^2 \lambda \Gamma^m \partial_{\mu} \lambda, \quad \Pi^I_{\mu} = \partial_{\mu} \phi^I - \alpha^2 \lambda \Gamma^I \partial_{\mu} \lambda, \quad I = 1, \ldots, 6 \]

\[ F_{\mu\nu} \equiv F_{\mu\nu} - 2\alpha \lambda \Gamma_{[\mu} \partial_{\nu]} \lambda - 2\alpha \lambda \Gamma^I \partial_{[\mu} \lambda \partial_{\nu]} \phi^I \]
World volume theory in AdS background

\[
\begin{align*}
S_{cl} &= S_{\text{DBI}} + S_{\text{WZ}} \\
S_{\text{DBI}} &= -\int d^{p+1}\sigma \sqrt{-\det (g_{\mu\nu}^{\text{ind}} + F_{\mu\nu})} \\
g_{\mu\nu}^{\text{ind}} &= \partial_\mu X^M \partial_\nu X^N G_{MN}
\end{align*}
\]

rigid symmetries inherited from solution:
• AdS superisometries (incl. susys)
• isometries of sphere
local
• GCT in \((p+1)\)-dimensional worldvolume
• \(\kappa\) symmetry

P. Claus, R. Kallosh and AVP, hep-th/9711161 and 9812066
P. Claus, R. Kallosh, J. Kumar, P. Townsend and AVP, hep-th/9711161
Gauge fixing of GCT on brane and of $\kappa$ - symmetry

- After gauge fixing: remaining symmetries (from rigid super-AdS and gauge-fixed GCT and $\kappa$) appear as conformal rigid symmetries on the brane.
- Fermionic ones are $\epsilon$ and $\eta$.
  (similar to $\epsilon$ and $\zeta$ in this new work).
- Hope to get all-order result with conformal symmetry in the cases where these superalgebras exist (as for D3)
V-branes: DBI-VA actions with 8+8 supersymmetries

- Our results apply also when we start with D=6, and then can take either p=5 or p=3: these are related to branes called V5 and V3 (see talk Eric Bergshoeff)

- Same formulas as for D=10 lead to actions in 6 and 4 dimensions with 8+8 supersymmetries
The map: dimensions and # of supersymmetries

| D | susy | 32 | 24 | 20 | 16 | 12 | 8 | 4 |
|---|------|----|----|----|----|----|---|---|
| 11 | M    | M  |     |    |    |    |    |   |
| 10 | MW   | IIA| IIB|    |    |    |    |   |
| 9  | M    | N=2|    |    |    |    |    |   |
| 8  | M    | N=2|    |    |    |    |    |   |
| 7  | S    | N=4|    |    |    |    |    |   |
| 6  | SW   | (2,2)|    |    |    |    |    |   |
| 5  | S    | N=8| N=6|    |    |    |    |   |
| 4  | M    | N=8| N=6| N=5|    |    |    |   |

- **D9**: N=1
- **D7**: N=1
- **D5**: (2,0)
- **V5**: (1,0)

- **V3**: N=2
- **N=1**
- **N=2**
- **N=3**
- **N=4**
- **N=5**
- **N=6**
- **N=8**

**Legend**:
- Green:_vector multiplets
- Blue: tensor multiplet
- Gold: multiplets up to spin 1/2

**Notes**:
- **SUGRA**
- **SUGRA/SUSY**
- **SUGRA/SUSY** vector multiplets +
5. Conclusions 1

- Superconformal symmetry has been used as a tool for constructing classical actions of supergravity. - also higher-derivative terms can be constructed
- Quantum calculations show that there are unknown relevant properties of supergravity theories.
- Can (broken) superconformal symmetry be such an extra quantum symmetry?
- The non-existence of (broken) superconformal-invariant counterterms and anomalies in N=4, D=4 could in that case explain ‘miraculous’ vanishing results.
Conclusions 2

- We do not have a systematic knowledge of higher-derivative supergravity action.
- **Perturbative approach**: construct actions and transformation laws order by order in $\alpha$.
- Starting from Dp brane actions in $D=10$ we can construct DBI-VA actions with $16+16$ susys ($p=9,7,5,3,...$). For $p=3$ this is the deformation of $N=4, D=4$ with higher order derivatives.
- Same can be done from $D=6$: DBI-VA actions (V-branes) with $8+8$ susys, e.g. containing the deformation of $D=4, N=2$.
- Hope that this can lead also to supergravity actions using the superconformal methods.