An Analytical Approach and Optimization of Curvature Gauge

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Abstract. The ‘curvature gauge’ is sensor for the monitoring of deflection of structures under mechanical loading in applications in which strain gauges have traditionally been used. The sensitive zone of the curvature gauge is precision machined into the plastic optical fibre on grinding or milling machines. The cut-out produced removes a part of the fibre core and introduces a loss of light propagating along. The extent of such loss is related to the bend-radius of the fibre. In this work we present an analytical approach of this sensor. The operation of this curvature gauge is now explained and estimated in order to optimise it and improve its performances. The results are shown that relate the relative light-loss to the fibre curvature for different parameters (its depth, length, number of cuts, bend radius and pitch of cuts). Proposed model allows a quantitative optimization of curvature gauge without the necessity for making of the thousands of sensors with slightly different parameters in order to accomplish a similar objective experimentally.

1. Introduction
In last twenty years, the use of optical fiber sensors has been increasing and fiber optic sensors represent a fast evolving sector in industrial applications. Many ideas have been proposed and various techniques have been developed for different measurands and applications. Optical fiber sensors have advantages such as immunity to electromagnetic interference lightweight, corrosion resistance, small size, high sensitivity, large bandwidth, and ease in signal light transmission. Fiber-optic sensors could be applied to measure many physical quantities, such as voltage, strain, temperature, pleasure, acoustic wave, etc. For example, in the work [1] the use of a low cost, intensity-based plastic optical fibre sensor for curvature and strain measurements in samples subjected to flexural and tensile loading conditions was proposed. Similarly in [2], plastic optical fiber (POF) sensors for crack detection and vertical deflection monitoring in concrete were proposed. The development of fibre gratings has had a significant impact on research and development in telecommunications and fibre optic sensing. Optical fibre long-period grating sensors for applications in sensing strain, temperature, bend radius and external index of refraction is reviewed in [3]. A new interferometer-based method for evaluation of the influence of bending on the propagation constant difference between two adjacent propagated modes in terms of core cladding index difference was presented in [4]. The characteristics of the use of a hollow-core splice scheme for macro-bend measurements are discussed both theoretically and experimentally in [5]. In [6] a new method based on photon counting is utilized to investigate sensing characteristics of POFs using the optical time-domain reflectometry technique. The responses of POF under various disturbances including small-radius bending, clamping, axial strain, etc, are
measured and discussed. In the work [7] an optical fibre sensor for short pulse duration x-ray dosimetry is presented. The sensor is based on luminescence generated in the cladding of a 1 mm core diameter polymer optical fibre which has been doped with a radio luminescent phosphor. In the paper [8] a new hydrogen sensors based on the use of fibre Bragg gratings and long period gratings coated by palladium nanolayers is presented. The sensing principle is based on the palladium–hydrogen interaction. An optical displacement sensor based on bending multi-looped POFs that has multi-structural imperfections on the outer side of its core is presented in [9]. A theoretical model shows that losses of light in such fibres result from the attenuation of light in the structurally imperfect area.

Waveguide bending loss has long been an important topic in both, theory and practice. The successful prediction of radiation loss in curved optical fibers either in analytical or numerical form has great value in the waveguide theory. Sensing applications in particular have created an additional demand to model the waveguide bending loss. Such theoretical models have been proposed by a number of authors [10-13]. Most of these models are based on perturbation analysis of the wave equation. Ray presentation of electromagnetic modes in waveguides is often used to obtain a qualitative analysis of the field solution [14]. Bending loss modeling is of direct relevance to a fibre optic sensor of structural deflection curvature. Such sensor has been proposed as an alternative to strain measurement [15]. Deflection curvature measurement can offer a number of advantages compared to the well-established strain measurement alternative. For example, the thinner the structure, the less strain there is to measure the same structural deflection curvature. The implication is that for any strain sensor as the benchmark, no matter how sophisticated, a break-even thickness exists so that the curvature gauge will provide more sensitivity with structures thinner than that. This break-even thickness is 50 mm, with respect to strain sensors sensitive to 10 micro strain as the benchmark (or 5 mm for 1 micro strain devices, etc). The three-dimensional analysis of light propagation through the optical fibre curvature gauge sensor was reported in [16]. Although the structure of the sensitive zone is very complex, all major cases that may appear when light interacts with the device’s sensitive zone have been considered.

In this work using the principles of geometric optics, curvature gauge was modelled by integration method. According to this model fraction of power refracted on the each plane of the sensitive zone was calculated separately. The model enabled visualization of fundamentals involved in the operation of this fibre optic sensor so that better qualitative understanding could be gained. It is also allowed quantitative optimisation of the gauge without the need for making thousands of sensors with slightly different parameters in order to accomplish a similar objective experimentally.

2. Description of curvature gauge
Curvature gauge is a fiber-optic, intensity-modulated deformation-curvature sensor. While the sensitivity of all optical fibers to large curvature with the arc-radius in the range of only a few centimeters has long been know, this sensitivity is insufficient to detect the deformation of structures in bending. The radii of deformation-curvatures are typically in kilometer range, making them indistinguishable from straight lines under visual inspection. In order to measure deformation curvatures, the optical fibre of curvature gauge is specially sensitized by precision machining of a selected fibre segment by cutting into its core (plastic fibers 0.25 mm thick were used). Such machining consist a much number of closely spaced cuts of equal depth made transversely by a flywheel resulting in the fibre cross section sensitized zone illustrated in Figure 1. The depth of cuts is uniform, and chosen between 20 and 110 μm. The number and spacing of cuts determines whether the sensor is for lumped or distributed measurement. The larger the spacing, the more distributed the sensor measurement will be. The cuts may also be grouped into distinct (separate) zones. Because the machining is done asymmetrically on one side of the fibre only, the convex (“+”) bending of that segment can be differentiated from the concave (“–”) bending. Transmission losses of the fibre increase in the former and decrease in the latter case relative to the straight-line configuration. The length and depth of the sensing zone have a major impact on the gauge sensitivity and their experimental optimization for fixed emitter/detector combination was shown in [17].
The measurement range of curvature gauges is large, making it difficult to design a single experiment that would precisely investigate it wholly. The elastic limit in bending of the sensor’s host structure is one of the constraints to be overcome. Moreover, the theory of bending of beams does not hold for large beam deflections when geometric nonlinearities are large. In the work [17] the graph pertains to an experiment designed to utilize the full 12-bit analog-to-digital converter scale without losing resolution is presented.

3. Calculating of total bound ray-power

A source is specified by the distribution of power among all the ray directions emitted from each differential element $dS$ of its surface area. For example, the source in Figure 2 emits light within a cone of half-angle $\theta_k$.

Light emitted at angle $\theta_0$ to the normal has intensity $I(\theta_0)$ (energy emitted per unit solid angle from the unit area of source). Thus the element of power $dP$ radiated is given by

$$
\begin{align*}
    dP &= \begin{cases} 
        I(\theta_0) d\Omega dS, & \theta \leq \theta_0 \leq \theta_k \\
        0, & \theta_k \leq \theta_0 \leq \pi/2 
    \end{cases}
\end{align*}
$$

(1)

where $d\Omega$ is the element of solid angle as shown. A diffuse or Lambertian source is one where each differential area $dS$ of source area emits light in all directions, i.e. $\theta_k=\pi/2$. This is the most typical source in practice and approximates the output of a light-emitted diode (LED). The intensity distribution of Lambertian source is given by

$$
I(\theta_0) = I_0 \cos \theta_0, \quad 0 \leq \theta_0 \leq \pi/2
$$

(2)

where $I_0$ is a constant. Now, we can calculate the source power carried by bound rays when the fiber is illuminated by the diffuse source. First we calculate the total source power. The element $dP$ radiated into solid angle $d\Omega$ by area $dS$ of the source in medium of refractive index $n_0$ is given by

$$
    dP = I_0 \cos \theta_0 d\Omega dS = I_0 \cos \theta_0 r dr d\varphi \sin \theta_0 d\theta_0 d\theta_\varphi
$$

(3)

where the angles $(\theta_0, \theta_\varphi)$ are spherical polar angles relative to the normal QN and $(r, \varphi)$ are polar coordinates relative to the fiber axis shown in Figure 3.
Figure 3. Angles to describe reflection and refraction of a ray incident at Q on the fiber end face.

The range of source-ray directions satisfies $0 \leq \theta_0 \leq \pi/2$, $0 \leq \theta_p \leq 2\pi$ and the range of positions on the end face satisfies $0 \leq r \leq a$, $0 \leq \varphi \leq 2\pi$. The total source power $P_{tot}$ radiated by the source in all directions over the area of the core cross-section in Figure 3 is obtained by integrating $dP$ of the Eq.3 over the complete range of values of each of the four variables given above. Hence

$$P_{tot} = \int_0^a \int_0^{2\pi} \int_0^{\pi/2} \int_0^{2\pi} I_0 \sin \theta_0 \cos \theta_p d\theta_p d\varphi d\theta_0 dr.$$

When the illumination is uniform, $I_0$ is constant and therefore

$$P_{tot} = \pi^2 a^2 I_0$$

The amount of surface power carried by the bound rays, $P_{br}$, is found by integrating Eq.2 over the complete ranges of values of $\varphi$, $r$ and $\theta_p$ given below Eq.2, and the range of values of $\theta_0$ corresponding to bound rays within the fiber, i.e. $0 \leq \theta_0 \leq \theta_{cl}$, where $\theta_{cl}$ is maximum angle of incidence and satisfied

$$n_0 \sin \theta_{cl} = n_{co} \sin \theta_c.$$ 

Where $\theta_c$ is the complement of the critical angle, defined by

$$\theta_c = \cos^{-1} \left( \frac{n_{cl}}{n_{co}} \right) = \sin^{-1} \left( 1 - \frac{n_{cl}^2}{n_{co}^2} \right)^{1/2}.$$ 

Starting from

$$P_{br} = I_0 \int_0^{2\pi} d\varphi \int_0^a dr \int_0^{\pi/2} d\theta_p \int_0^{\theta_{cl}} \sin \theta_0 \cos \theta_p d\theta_0$$

and by straightforward integration we obtain

$$P_{br} = I_0 \left( \frac{\pi a n_{co}}{n_0} \right)^2 \sin^2 \theta_c.$$ 

Thus $P_{br}$ is proportional to the square of the numerical aperture of the fiber. The index $n_0$ of the source medium usually differs from that of the core, and hence not all of the source power can be transmitted into the core. However, a maximum of about 4 per cent of source power can be reflected from end face in practice. In the case of step optical fiber, the fraction $T$ of power transmitted into the
core is given by the classical Frensel’s coefficient for reflection at a planar dielectric interface, 
\[ T_{00} = 4n_0n_{co}/(n_0+n_{co})^2. \]
It can be accounted for by replacing the intensity \( I(\theta_0) \) in Eq (8) by \( TI(\theta_0) \).
Throughout the rest of this work we rewrite results (8) in the form
\[ P_{br} = T_{00} I_0 \left( \frac{\pi a n_{co}}{n_0} \right)^2 \sin^2 \theta_c. \]  

(10)

4. Gauge modelling and numerical results
In order to make calculation simpler, the following assumptions were introduced: (1) each bound ray propagates nearly parallel to the axis (paraxial approximation); (2) the sensitive zone is much shorter than fibre so that zone was considered as straight part; (3) the critical angle for total reflection in the fibre, \( \theta_c \), is the function of bend radius \( R \) as follows
\[ \theta_c(R) = \theta_c \left(1 - \frac{2a}{R \theta_c^2}\right)^{0.5}; \]  
(18)
(4) bound power, \( P_{br} \), is uniformly distributed across cross section of the fibre; 5) all teeth are regular prisms with height of \( h_t \) and angle \( \delta \); 6) all surfaces of the zone, where the light refracts are smooth.

![Figure 4. The segment of the sensitized fiber.](image)

The fraction of bound ray power transmitted into the sensitized zone at the each plane interface is given by the Fresnel’s transmission coefficient [13, 19]
\[ T = \frac{4n_1n_2 \cos \theta_i \cos \theta_t}{(n_1 \cos \theta_i + n_2 \cos \theta_t)} \]  
(11)
Where \( \theta_i \) and \( \theta_t \) are the angles of incidence and transmission in respect to the normal respectively.
According to the abovementioned assumptions the power distribution of the optical fiber is uniform with density
\[ P_0 = \frac{P_{br}}{S} = \frac{P_{br}}{\pi R^2}. \]  
(12)

Total light power that ‘comes’ to the sensitive zone is \( P_{br} \). One part of this power comes directly to the roughness surface of the zone, and the further analysis is limited to this fraction of the power. A part of light which strike the zone can be lost on it, while the other part can be transmitted further through the fiber. Considering that the angle of total reflection on the surface \( S_0 \) (core/air) is \( \theta_{c,0} = 42^0 \) and the maximal angle of incident rays on this surface is \( \theta_c = 20^0 \), the total reflection would not occur on \( S_0 \), and the power that passes through this surface should be calculated. Calculation for other surfaces of the
zone is the same as it was described in section 2. Let’s note small element $dS_0$ at the surface $S_0$, and $dP_0$ is surface emitted from this segment in space angle $d\Omega$. Then the elementary fraction of power from $dS_0$ is $dP_0=p_0(\theta)T_0dS_0d\Omega$. Total power emitted from $S_0$ was obtained by integration over total range of relevant variables. The refractive power passing through the plane $S_0$ is

$$P_0 = \iint_{S_0} dS_0 \int_0^{2\pi} d\theta_\rho \int_0^{\theta_0} p_0 T_0 \cos \theta_0 \sin \theta_0 d\theta_0,$$  \hspace{1cm} (13)

where $\theta_0$ is the maximum of angle $\theta$, which is related with critical angle for interface core/cladding as: $n_{co} \sin \theta_0(R) = n_0 \sin \theta_{c1}$, and $T_0$ is the coefficient of transmission for core/vacuum interface. Integration over the complete ranges of $S_0$, $\theta_0$ and $\theta_0$ yields

$$P_0 = p_0 S_0 J_0$$  \hspace{1cm} (14)

where are

$$J_0 = 2\pi \int_0^{\theta_0} \frac{4 n_{co} n_0 \cos \theta_0 \cos \theta_0}{(n_{co} \cos \theta_0 + n_0 \cos \theta_0)^2} \cos \theta_0 \sin \theta_0 d\theta_0,$$

and

$$S_0 = \left[ a^2 \tan^{-1}\sqrt{\frac{2a h - h^2}{a - h}} - (a - h) \sqrt{2a h - h^2} \right].$$

Figure 5 presents dependence of $P_0/P_{br}$ as a function of the height of the first tooth for fibre radius of $a=125\mu m$, $\delta=10^6$ and bend radius of $R=500m$. $P_0$ is power transmitted across the first plane of the zone.

![Figure 5](image.png)

**Figure 5.** Relative refractive power though the plane $S_0$.

Figure 6 represents the relative loss of the first plane $(P_{br} - P_0)/P_{br}$ as a function of bend radius for $a=125\mu m$, $\delta=10^6$ and $h=80\mu m$. 

![Figure 6](image.png)

**Figure 6.** Relative refractive power through the plane $S_0$. 

Figure 6 represents the relative loss of the first plane $(P_{br} - P_0)/P_{br}$ as a function of bend radius for $a=125\mu m$, $\delta=10^6$ and $h=80\mu m$. 

![Figure 6](image.png)

**Figure 6.** Relative refractive power through the plane $S_0$. 

Figure 6 represents the relative loss of the first plane $(P_{br} - P_0)/P_{br}$ as a function of bend radius for $a=125\mu m$, $\delta=10^6$ and $h=80\mu m$. 

![Figure 6](image.png)
In similar way we found the power behind the first tooth on the zone as:

$$P_{1}^{\text{out}}(R, h, h_{f}) = p_{2} S_{i}, J_{2} = p_{0} J_{0} J_{1} J_{2},$$

(15)

where:

$$J_{1} = 2\pi \frac{\theta_{2}}{\theta_{1}} \int_{0}^{\theta_{2}} \frac{4 n_{o} n_{0} \cos \theta \cos \theta_{1}}{\left(n_{0} \cos \theta_{1} + n_{o} \cos \theta_{1}\right)^{2}} \cos \theta_{1} \sin \theta_{1} d\theta_{1};$$

(16)

$$J_{2} = 2\pi \frac{\theta_{2}}{\theta_{1}} \int_{0}^{\theta_{2}} \frac{4 n_{o} n_{0} \cos \theta \cos \theta_{1}}{\left(n_{0} \cos \theta_{1} + n_{o} \cos \theta_{1}\right)^{2}} \cos \theta_{1} \sin \theta_{1} d\theta_{1}.$$  

(17)

where: $\theta_{c2} = \arcsin\left(\frac{n_{0}}{n_{o}} \sin(\theta_{c})\right)$, $\theta_{c3} = \arcsin\left(\frac{n_{0}}{n_{o}} \sin(\theta_{c2})\right)$. This procedure was repeated for all N tooth of the zone, and the part of the power that remains in the fiber is after the sensitive zone is

$$P_{in} = p_{0} S_{i} J_{0}^{2} J_{1}^{N} J_{2}^{N}.$$  

(18)

Relative loss due to the zone can be calculated as

$$\delta P = \frac{P_{br} - P_{in}}{P_{br}}.$$  

(19)

An assumption that the angle $\delta$ is small i.e. $\cos\delta \approx 1$ was used in previous derivation. In addition, in evaluation of integrals $J_{1}$ and $J_{2}$ the relationship between $\theta_{1}$ and $\theta$, $n_{o} \sin \theta_{1} = n_{0} \sin \theta$ was used also.

The same procedure was repeated for each tooth of the zone and total relative loss as a function of the number of teeth was determined and presented in Figure 7.
5. Conclusion

Although the structure of the sensitive zone is very complex, all major cases that may occurred when light interacts with the device’s “sensitive zone” have been analytical considered. The model enabled visualization of fundamentals involved in the operation of this fibre optic sensor so that better qualitative understanding could be gained. It also allowed a quantitative optimization of curvature gauge without the necessity for making of thousands of sensors with slightly different parameters in order to accomplish a similar objective experimentally. This model is an improvement over the previous model of the curvature gauge [16] that did not show the optimal number of the cuts on the sensitive zone. From the Figure 4 one can see that the optimal number of teeth is about 50.

Another conclusion is that there is a performance limit for a curvature gauge, at least in this present form: curvature whose radius is shorter than 2 km can be measured. This result is also in good agreement with previous results [16] and with experimental results [20] and with failed attempts in the meantime to improve the sensor performance through experimental optimization of the sensor.

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