Interacting generalized ghost dark energy in a non-flat universe

Esmaeil Ebrahimi1,3 *, Ahmad Sheykhi2,3 † and Hamzeh Alavirad 4‡

1 Department of Physics, Shahid Bahonar University, PO Box 76175, Kerman, Iran
2 Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
3 Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P. O. Box 55134-441, Maragha, Iran
4 Institute for Theoretical Physics, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

We investigate the generalized Quantum Chromodynamics (QCD) ghost model of dark energy in the framework of Einstein gravity. First, we study the non-interacting generalized ghost dark energy in a flat Friedmann-Robertson-Walker (FRW) background. We obtain the equation of state parameter, \( w_D = p/\rho \), the deceleration parameter, and the evolution equation of the generalized ghost dark energy. We find that, in this case, \( w_D \) cannot cross the phantom line \( (w_D > -1) \) and eventually the universe approaches a de-Sitter phase of expansion \( (w_D \to -1) \). Then, we extend the study to the interacting ghost dark energy in both a flat and non-flat FRW universe. We find that the equation of state parameter of the interacting generalized ghost dark energy can cross the phantom line \( (w_D < -1) \) provided the parameters of the model are chosen suitably. Finally, we constrain the model parameters by using the Markov Chain Monte Carlo (MCMC) method and a combined dataset of SNIa, CMB, BAO and X-ray gas mass fraction.

Keywords: ghost; dark energy; acceleration; observational constraints.

I. INTRODUCTION

The cosmological data from type Ia Supernova, Large Scale Structure (LSS) and Cosmic Microwave Background (CMB) indicate that our universe is currently accelerating [1]. To explain such an acceleration in the framework of standard cosmology, one is required to introduce a new type of energy with a negative pressure usually called “dark energy” (DE) in the literature. A great variety of DE scenarios have been proposed to explain the acceleration of the universe’s expansion. One can refer to [2, 3] for a review of DE models. On the other hand, many people believe in a modification of gravity, seeking an explanation for the late time acceleration. According to this idea the acceleration will be a part of the universe’s expansion and does not need to invoke any kind of DE component. As examples of this approach one can look at Refs. [4–8]. It is important to note that the detection of gravitational waves should be the ultimate test for general relativity or alternatively the definitive endorsement for extended theories [9].

In most scenarios for DE, people usually need to consider a new degree of freedom or a new parameter, in order to explain the acceleration of the cosmic expansion (see e.g. [10] and references therein). However, it would be nice to resolve the DE puzzle without presenting any new degree of freedom or any new parameter in the theory. One of the successful and beautiful theories of modern physics is QCD which describes the strong interaction in nature. However, resolution of one of its mysteries, the U(1) problem, has remained somewhat unsatisfying. Veneziano ghost field explained the U(1) problem in QCD [11]. Vacuum energy of the ghost field can be used to explain the time-varying cosmological constant in a spacetime with nontrivial topology, since the ghost field has no contribution to the vacuum field explained the U(1) problem in QCD [11].

The energy density of the vacuum ghost field is proportional to \( \Lambda_{QCD}^3 H \), where \( \Lambda_{QCD} \) is the QCD mass scale and \( H \) is the Hubble parameter [13]. It is well-known that the cosmological constant model of DE suffers the coincidence and the fine tuning problems. However, with correct choice of \( \Lambda_{QCD} \), the ghost dark energy (GDE) model does not encounter the fine tuning problem anymore [12, 13]. Phenomenological implications of the GDE model were discussed in [14]. In [15] GDE in a non-flat universe in the presence of interaction between DE and dark matter was explored. The instability of the GDE model against perturbations was studied in [16]. It was argued that the perfect fluid for GDE is classically unstable against perturbations. Other features of the GDE model have been investigated in Refs. [17–24].

In all the above references [14–24] the GDE was assumed to have the energy density of the form \( \rho_D = \alpha H \), while, in general, the vacuum energy of the Veneziano ghost field in QCD is of the form \( H + O(H^2) \) [25]. This indicates that in the previous works on the GDE model, only the leading term \( H \) has been considered. Motivated by the argument given in [26], one may expect that the subleading term \( H^2 \) in the GDE model might play a crucial role in the early evolution of the universe, acting as the early DE. It was shown [27] that taking the second term into account can give better agreement with observational data compared to the usual GDE. Hereafter we call this model the generalized
ghost dark energy (GGDE) and our main task in this paper is to investigate the main properties of this model. In this model the energy density is written in the form \( \rho_D = \alpha H + \beta H^2 \), where \( \beta \) is a constant.

In addition to the DE component, there is also another unknown component of energy in our universe called "dark matter" (DM). Since the nature of these two dark components are still a mystery and they seem to have different gravitational behaviour, people usually consider them separately and take their evolution independent of each other. However, there exist observational evidence of signatures of interaction between the two dark components [28, 29].

On the other hand, based on the cosmological principle the universe has three distinct geometries, namely open, flat and closed geometry corresponding to \( k = -1, 0, +1 \), respectively. For a long time it was a general belief that the universe has a flat \( (k = 0) \) geometry, mainly based on the inflation theory [30]. With the development of observational techniques people found deviations from the flat geometry [31]. For example, CMB experiments [32], supernova measurements [33], and WMAP data [34] indicate that our universe has positive curvature.

All the above reasons indicate that although people believe in a flat geometry for the universe, astronomical observations leave enough room for considering a nonflat geometry. Also about the interaction between DM and DE there are several signals from nature which guides us to let the models explain such behaviour. Based on these motivations we would like here to extend the studies on GGDE, to a non-flat FRW spacetime in the presence of an interaction term. Our work differs from [15, 19] in that we consider the GGDE model while in [15] and [19], the original GDE model in Einstein and Brans-Dicke theory were studied, respectively. To check the viability of our model, we also perform the cosmological constraints on the interacting GGDE in a non-flat universe by using the Markov Chain Monte Carlo (MCMC) method. We use the following observational datasets: Cosmic Microwave Background Radiation (CMB) data from WMAP7 [35], 557 Union2 dataset of type Ia supernova [36], baryon acoustic oscillation (BAO) data from SDSS DR7 [37], and the cluster X-ray gas mass fraction data from the Chandra X-ray observations [38]. To put the constraints, we modify the public available CosmoMC [39].

The outline of this paper is as follows. In section III, we study the cosmological implications of the GGDE scenario in the absence of interaction between DE and DM. In section III, we consider interacting GGDE in a flat geometry. In section IV we generalize the study to the universe with spatial curvature in the presence of interaction between DM and DE. In section V cosmological constraints on the parameters of the model are performed by using the Markov Chain Monte Carlo (MCMC) method. We summarize our results in section VI.

II. GGDE MODEL IN A FLAT UNIVERSE

Consider a flat homogeneous and isotropic FRW universe, the corresponding Friedmann equation is

\[ H^2 = \frac{8\pi G}{3} (\rho_m + \rho_D), \]

where \( \rho_m \) and \( \rho_D \) are, the energy densities of pressureless DM and DE, respectively. The generalized ghost energy density may be written as [27]

\[ \rho_D = \alpha H + \beta H^2, \]

where \( \alpha \) is a constant of order \( \Lambda_{QCD}^3 \) and \( \Lambda_{QCD} \) is QCD mass scale, and \( \beta \) is also a constant. In the original GDE \( (\beta = 0) \) with \( \Lambda_{QCD} \sim 100 MeV \) and \( H \sim 10^{-33}eV \) , \( \Lambda_{QCD}^3 H \) gives the right order of magnitude \( \sim (3 \times 10^{-3}eV)^4 \) for the observed DE density [13]. In the GGDE, \( \beta \) is a free parameter and can be adjusted for better agreement with observations.

As usual we introduce the fractional energy density parameters as

\[ \Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G \rho_m}{3H^2}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{8\pi G(\alpha + \beta H)}{3H}, \]

where \( \rho_{cr} = 3H^2/(8\pi G) \). Thus, we can rewrite the first Friedmann equation as

\[ \Omega_m + \Omega_D = 1. \]

Through this section we consider GGDE in the absence of the interaction term, thus DE and DM evolves independent of each other and hence they satisfy the following conservation equations

\[ \dot{\rho}_m + 3H \rho_m = 0, \]

\[ \dot{\rho}_D + 3H \rho_D(1 + w_D) = 0. \]
FIG. 1: These figures show the evolutions of $w_D$ and $q$ against $\Omega_D$ in a flat GGDE and GDE models. Solid lines correspond to GGDE when $\xi = 0.1$ and the dashed lines belong to GDE model.

If we take the derivative of relations (1) and (2) with respect to the cosmic time, we arrive at

$$\dot{H} = -4\pi G \rho_D (1 + u + w_D),$$

(7)

$$\dot{\rho}_D = \dot{H}(\alpha + 2\beta H).$$

(8)

where $u = \rho_m/\rho_D$. Combining relations (7) and (8) with continuity equation (6), we get

$$(1 + w_D)[3H - 4\pi G(\alpha + 2\beta H)] = 4\pi G(\alpha + 2\beta H).$$

(9)

Solving the above equation for $w_D$ and noticing that $u = \Omega_m/\Omega_D$, and

$$\frac{4\pi G}{3H}(\alpha + 2\beta H) = \frac{\Omega_D}{2} + \frac{4\pi G\beta}{3},$$

(10)

we obtain

$$w_D = \frac{\xi - \Omega_D}{\Omega_D(2 - \Omega_D - \xi)}.$$  

(11)

where $\xi = \frac{8\pi G\beta}{3}$. It is clear that this relation reduces to its respective one in the GDE when $\xi = 0$ [13]. In Fig. 1a we have plotted the evolution of $w_D$ versus $\Omega_D$. It is easy to see that at the late time where $\Omega_D \to 1$, we have $w_D \to -1$, which implies that the GGDE model mimics a cosmological constant behaviour. One should notice that this behaviour is the same as for the original GDE model. This is expected since the subleading term $H^2$ in the late time can be ignored due to the smallness of $H$ and the difference between these two models appears only at the early epochs of the universe. From figure (1a) we see that $w_D$ of the GGDE model cannot cross the phantom divide and the universe has a de Sitter phase at the late time. It is important to note that the universe is filled with two dark components namely, DM and GGDE. Thus to discuss the acceleration of the universe we should define the effective EoS parameter, $w_{\text{eff}}$, as

$$w_{\text{eff}} = \frac{p_t}{\rho_t} = \frac{p_D}{\rho_D + \rho_m},$$

(12)

where $\rho_t$ and $p_t$ are, respectively, the total energy density and the total pressure of the universe. As usual, we have assumed the DM is in the form of pressureless fluid ($p_m = 0$). Using relation (4) for the spatially flat universe, one can find

$$w_{\text{eff}} = \Omega_D w_D = \frac{\xi - \Omega_D}{2 - \Omega_D - \xi}.$$  

(13)

Let us now turn to the deceleration parameter which is defined as

$$q = -\frac{a \ddot{a}}{a^2} = -1 - \frac{\dot{H}}{H^2},$$

(14)
where $a$ is the scale factor. Using Eq. (7) and definition $\Omega_D$ in (3) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \Omega_D (1 + u + w_D).$$

Replacing this relation into (13), and using (11) we find

$$q = \frac{1}{2} - \frac{3}{2} \frac{\xi - \Omega_D}{\xi - \Omega_D - 2},$$

which indicates that for $\xi < 2$ the universe is at the deceleration phase at early times while for $\xi > 2$, the universe could experience an acceleration phase, the former is consistent with the definition $\xi = \frac{8\pi G}{\beta}$. On the other side, we find that at the late time where the DE dominates ($\Omega_D \to 1$), independent of the value of the $\xi$, we have $q = -1$. We have plotted the behaviour of $q$ in Fig. 1b. Besides, taking $\Omega_D = 0.72$ and adjusting $\xi = 0.01$ we obtain $q_0 \approx -0.34$, in agreement with observations [40]. Choosing the same set of parameters leads to $w_{D0} \approx -0.78$ and $w_{\text{eff}0} \approx -0.56$.

One can easily check that the deceleration parameter in GDE is retrieved for $\xi = 0$ [15]. We can also take a look at the early and the late time behaviour of the deceleration parameter. At the early stage of the universe where $\Omega_D \to 0$, the deceleration parameter becomes

$$q = \frac{1}{2} - \frac{3}{2} \frac{\xi}{\xi - 2}.$$  

which indicates that for $\xi < 2$ the universe is at the deceleration phase at early times while for $\xi > 2$, the universe could experience an acceleration phase, the former is consistent with the definition $\xi = \frac{8\pi G}{\beta}$. On the other side, we find that at the late time where the DE dominates ($\Omega_D \to 1$), independent of the value of the $\xi$, we have $q = -1$. We have plotted the behaviour of $q$ in Fig. 1b. Besides, taking $\Omega_D = 0.72$ and adjusting $\xi = 0.01$ we obtain $q_0 \approx -0.34$, in agreement with observations [40]. Choosing the same set of parameters leads to $w_{D0} \approx -0.78$ and $w_{\text{eff}0} \approx -0.56$.

At the end of this section we present the evolution equation of the DE density parameter $\Omega_D$. To this goal we take the time derivative of Eq. (3), after using relation $\dot{\Omega}_D = H \frac{d\Omega_D}{d\ln a}$ as well as Eq. (14) we reach

$$\frac{d\Omega_D}{d\ln a} = -3 \Omega_D (1 - \Omega_D) w_D.$$  

Using Eq. (11) we get

$$\frac{d\Omega_D}{d\ln a} = -3 \frac{(1 - \Omega_D)(\xi - \Omega_D)}{2 - \Omega_D - \xi}.$$  

Once again for the limiting case $\xi = 0$, the above relation reduces to its respective evolution equation for the original GDE presented in [15].

III. INTERACTING GGDE IN A FLAT UNIVERSE

In the previous section, the evolution of the DE and DM components were discussed separately. Here we would like to extend the study to the interacting case, seeking new features of GGDE. In the first look investigating interacting models of DE are valuable from two perspective. The first is the theoretical one, which states that we have no reason against interaction between DE and DM components. For example, in the unified models of field theory DM and DE can be explained by a single scalar field, thus they will be allowed to interact minimally. Besides, one can get rid of the coincidence problem by taking into account the interaction term between DM and DE. One can refer to [41–45] for detailed discussion. The other feature which motivates us to consider interacting models of DE and DM comes from observations which indicate the interaction between two dark components of our universe [28]. Thus, there exist enough motivations to consider the GGDE in the presence of an interaction term. To this end, we start with the energy balance equations for DE and DM, namely

$$\dot{\rho}_m + 3H \rho_m = Q,$$  

$$\dot{\rho}_D + 3H \rho_D (1 + w_D) = -Q,$$

where $Q > 0$ represents the interaction term which allows the transition of energy from DE to DM. The form of $Q$ is a matter of choice and can be taken as [15]

$$Q = 3b^2 H (\rho_m + \rho_D) = 3b^2 H \rho_D (1 + u).$$  

with $b^2$ being a coupling constant. Inserting Eqs. (8) and (22) in Eq. (21) and taking into account $u = \Omega_D/\Omega_D^\text{eff}$, we find

$$w_D = -\frac{1}{2} - \frac{1}{2} - \frac{\xi}{2} \left(1 + \frac{2b^2}{\Omega_D} - \frac{\xi}{\Omega_D}\right).$$

At first look one can find that setting $b = 0$, $w_D$ reduces to the respective relation in the absence of interaction obtained in Eq. (11). When $\xi = 0$ the result recovers those in [15] for original GDE. The first interesting point about the EoS parameter of the GGDE is that in the interacting case independent of the interaction parameter, $b$, for $0 < \xi < 1$, $w_D$ can cross the phantom line in the future where $\Omega_D \to 1$. At the present time, by choosing $\xi = 0.03$, $b = 0.15$ and $\Omega_{D0} = 0.72$, we find that $w_{D0} = -0.82$ and $w_{D0}^\text{eff} = -0.59$ which the latter favored by observations. One can easily check that for a same coupling constant these values for the original GDE are $w_{D0} = -0.83$ and $w_{D0}^\text{eff} = -0.60$ which clearly show that the square term in the energy density of the GGDE slow down the evolution of the universe compared to the original GDE model. For a better insight we have plotted $w_D$ against $\Omega_D$ in Fig. 2a. This value for coupling constant, $b$, in the figure is consistent with recent observations [46]. It is worth mentioning that at the late time where $\Omega_D \to 1$ the effective EoS parameter approaches less than $-1$, i.e. $w_{D0}^\text{eff} < -1$, which reminds a super acceleration for the universe in the future. Next we take a look at the deceleration parameter in the presence of an interaction term. Substituting (15) in (14) and using (23) yields

$$q = \frac{1}{2} - \frac{3}{2} \frac{\Omega_D}{\Omega_D - \xi} \left(1 + \frac{2b^2}{\Omega_D} - \frac{\xi}{\Omega_D}\right).$$

Once again it is clear that setting $b = 0$, the respective relation in the previous section is retrieved. When $\xi = 0$ the result of [15] is recovered. For the set of parameters ($\xi = 0.03, b = 0.15, \Omega_{D0} = 0.72$), we find that according to the GGDE the universe enters the acceleration phase at $\Omega = 0.48$ while this transition happens earlier for the GDE model. This point is clear from Fig. 2a. The present value of the deceleration parameter for the interacting GGDE model is $q_0 = -0.38$ which is consistent with observations [46].

Finally, we would like to obtain the evolution equation of DE in the presence of interaction. First we take the time derivative of (3) and obtain

$$\dot{\Omega} = \Omega \left[\frac{\dot{\rho}}{\rho} - 2 \frac{\dot{H}}{H}\right].$$

Using relation (21) as well as (16), it is a matter of calculation to show

$$\frac{d\Omega_D}{d\ln a} = 3\Omega_D \left[\frac{1 - \Omega_D}{2 - \Omega_D - \xi} \left(1 + \frac{2b^2}{\Omega_D} - \frac{\xi}{\Omega_D}\right) - \frac{b^2}{\Omega_D}\right].$$

In the limiting case $\xi = 0$ the equation of motion of interacting GDE is recovered [15].

**IV. INTERACTING GGDE IN A NON-FLAT UNIVERSE**

The flatness problem in standard cosmology was resolved by considering an inflation phase in the evolution history of the universe. Following this theory it became a general belief that our universe is spatially flat. However, later
FIG. 3: These figures show the evolutions of \( w_D \) and \( q \) against \( \Omega_D \) for an interacting GGDE and GDE models in a non-flat universe. Solid lines correspond to the GGDE when \( \xi = 0.1 \) and the dashed lines belong to the GDE model. For both cases \( b = 0.15 \).

It was shown that exact flatness is not a necessary consequence of inflation if the number of e-foldings is not very large \([47]\). So it is still possible that there exists a contribution to the Friedmann equation from the spatial curvature, though much smaller than other energy components according to observations. Thus, theoretically the possibility of a curved FRW background is not rejected. In addition, recent observations support the possibility of a non-flat universe and detect a small deviation from \( k = 0 \) \([48–51]\). Furthermore, the parameter \( \Omega_k \) represents the contribution to the total energy density from the spatial curvature and it is constrained as \(-0.0175 < \Omega_k < 0.0085\) with 95% confidence level by current observations \([52]\). Our aim in this section is to study the dynamic evolution of the GGDE in a universe with spatial curvature. The first Friedmann equation in a non-flat universe is written as

\[
H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_m + \rho_D),
\]

where \( k \) is the curvature parameter with \( k = -1, 0, 1 \) corresponding to open, flat, and closed universes, respectively. Taking the energy density parameters \([3]\) into account and defining the energy density parameter for the curvature term as \( \Omega_k = k/(a^2H^2) \), the Friedmann equation can be rewritten in the following form

\[
1 + \Omega_k = \Omega_m + \Omega_D.
\]

Using the above equation the energy density ratio becomes

\[
u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = \frac{1 + \Omega_k - \Omega_D}{\Omega_D}.
\]

The second Friedmann equation reads

\[
\dot{H} = -4\pi G (p + \rho) + \frac{k}{a^2},
\]

while the time derivative of GGDE density is

\[
\dot{\rho}_D = \dot{H} (\alpha + 2\beta H).
\]

Inserting Eq. \( (30) \) into \( (31) \) and combining the resulting relation with the conservation equation for DE component \( (21) \), after using \( (22) \) and \( (29) \), we find the EoS parameter of interacting GGDE in non-flat universe

\[
w_D = -\frac{1}{2 - \Omega_D - \xi} \left( 2 - \left( 1 + \frac{\xi}{\Omega_D} \right) \left( 1 + \frac{\Omega_k}{3} \right) + \frac{2b^2}{\Omega_D} (1 + \Omega_k) \right).
\]

From the second Friedmann equation, \( (30) \), one can easily obtain

\[
\frac{\dot{H}}{H^2} = -\Omega_k + \frac{3}{2} \Omega_D [1 + u + w_D],
\]
and therefore the deceleration parameter in a non-flat background is obtained as

\[ q = -1 - \frac{\dot{H}}{H^2} = -1 - \Omega_k + \frac{3}{2} \Omega_D[1 + u + w_D]. \]  

(34)

Substituting Eqs. (29) and (32) in (33) we obtain

\[ q = \frac{1}{2} (1 + \Omega_k) - \frac{3 \Omega_D}{2(2 - \Omega_D - \xi)} \left[ 2 - \left( 1 + \frac{\xi}{\Omega_D} \right) \left( 1 + \frac{\Omega_k}{3} \right) + \frac{2b^2}{\Omega_D}(1 + \Omega_k) \right]. \]  

In a non-flat FRW universe, the equation of motion of interacting GGDE is obtained following the method of the previous section. The result is

\[ \frac{d\Omega_D}{d\ln a} = 3\Omega_D \left[ \frac{\Omega_k}{3} + \frac{1 - \Omega_D}{2 - \Omega_D - \xi} \left( 2 - \left( 1 + \frac{\xi}{\Omega_D} \right) \left( 1 + \frac{\Omega_k}{3} \right) + \frac{2b^2}{\Omega_D}(1 + \Omega_k) \right) - \frac{b^2}{\Omega_D}(1 + \Omega_k) \right]. \]  

(36)

In the limiting case \( \Omega_k = 0 \), the results of this section restore their respective equations in a flat FRW universe derived in the previous sections, while for \( \xi = 0 \) the respective relations in [13] are retrieved. The evolutions of \( u_D \) and \( q \) against \( \Omega_D \) for a non-flat interacting GGDE and GDE models are plotted in Fig.4. Let us explore different features of GGDE in non-flat universe by a numerical study. First of all we study the EoS parameter of the GGDE in the future where \( \Omega_D \rightarrow 1 \). In this case, taking \( \xi = 0.1 \), \( b = 0.15 \) and \( \Omega_k = 0.01 \) leads to \( u_D = -1.05 \) which indicates that the GGDE is capable to cross the phantom line in the future. The present stage of the universe can be achieved by the same set of parameters but \( \Omega_D = 0.72 \). In such a case we see that \( u_{D0} = -0.78 \) while the effective EoS parameter becomes \( w_{e0} = -0.6 \) which is consistent with observations. The deceleration parameter of the model can also be obtained which is in agreement with observational evidences. For example, for the above choice of parameters one finds \( q_0 = -0.34 \) [40]. Transition from deceleration to the acceleration phase, in the interacting non-flat case, take place at \( \Omega_D = 0.52 \).

V. COSMOLOGICAL CONSTRAINTS

In order to constrain our model parameters space and check its viability, we apply the Marcov Chain Monte Carlo (MCMC) method. Observational constraints on the original GDE with and without bulk viscosity, was already performed [24]. Our work differs from [24] in that we consider the GGDE with energy density \( \rho_D = \alpha H + \beta H^2 \), while the authors of [24] studied the original GDE with energy density \( \rho_D = \alpha H \). Besides, we have extended here the study to the universe with any spacial curvature. To make a fitting on the cosmological parameters the public available CosmoMC package [39] has been modified.

A. Method

We want to get the best value of the parameters with 1σ error at least. Thus, following [24], we employ the maximum likelihood method where the total likelihood function \( L = e^{-x^2/2} \) is the product of the separate likelihood functions

\[ \chi^2_{tot} = \chi^2_{SN} + \chi^2_{CM} + \chi^2_{BAO} + \chi^2_{gas}. \]  

(37)

Here SNIa stands for type Ia supernova, BAO for baryon acoustic oscillation and gas stands for X-ray gas mass fraction data. The best fitting values of parameters are obtained by minimizing \( \chi_{tot}^2 \). In the next subsection, every dataset will be discussed separately.

We employ the following datasets. CMB data from WMAP7 [35], 557 Union2 dataset of type Ia supernova [36], baryon acoustic oscillation (BAO) data from SDSS DR7 [37], and the cluster X-ray gas mass fraction data from the Chandra X-ray observations datasets [38].

1. Cosmic Microwave Background

For the CMB data, we use the WMAP7 dataset [37]. The shift parameter R, which parametrize the changes in the amplitude of the acoustic peaks is given by [53]

\[ R = \sqrt{\frac{\Omega_{m0}}{c}} \int_0^{z_*} \frac{dz'}{E(z')}, \]  

(38)
where \( z_* \) is the redshift of decoupling. In addition, the acoustic scale \( l_A \), which characterizes the changes of the peaks of CMB via the angular diameter distance out to the decoupling is defined as well in [53] by

\[
l_A = \frac{\pi r(z_*)}{r_s(z_*)},
\]

The comoving distance \( r(z) \) is defined

\[
r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')},
\]

and the comoving sound horizon at the recombination \( r_s(z_*) \) is written

\[
r_s(z_*) = \int_0^{a(z_*)} \frac{c_s(a)}{a^2 H(a)} da,
\]

and the sound speed \( c_s(a) \) is defined by

\[
c_s(a) = \left[ 3(1 + 3\Omega_0 h^2) \right]^{-1/2},
\]

where the seven-year WMAP observations gives \( \Omega_{r0} = 2.469 \times 10^{-5} h^{-2} \) [33].

The redshift \( z_* \) is obtained by using the fitting function proposed by Hu and Sugiyama [54]

\[
z_* = 1048[1 + 0.00124(\Omega_{b0} h^2)^{-0.738}[1 + g_1(\Omega_{m0} h^2)^{g_2}],
\]

where

\[
g_1 = \frac{0.0783(\Omega_{b0} h^2)^{-0.238}}{1 + 39.5(\Omega_{b0} h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_{b0} h^2)^{1.81}},
\]

Then one can define \( \chi^2_{CMB} \) as \( \chi^2_{CMB} = X^T C_{CMB}^{-1} X \), with [24, 33]

\[
X = \begin{pmatrix} l_A - 302.09 \\ R - 1.725 \\ z_* - 1091.3 \end{pmatrix},
\]

\[
C_{CMB}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 293689 & 6825.270 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix},
\]

where \( C_{CMB}^{-1} \) is the inverse covariant matrix.

2. Type Ia Supernovae Data

We shall use the SNIa Union2 dataset [36] which includes 577 SNIa. The Hubble parameter \( H(z) \) determines the history of the universe. However, \( H(z) \) is specified by the underlying theory of gravity. To test this model, we can use the observational data for some predictable cosmological parameter such as luminosity distance \( d_L \). One may note that the Hubble parameter \( H(z; \alpha_1, ..., \alpha_n) \) can describe the universe, where parameters \( (\alpha_1, ..., \alpha_n) \) are predicted by the cosmological model. For such a cosmological model we can define the theoretical 'Hubble-constant free' luminosity distance as

\[
D_L^{th} = H_0 \frac{d_L}{c} = (1 + z) \int_0^z \frac{dz'}{E(z'; \alpha_1, ..., \alpha_n)} = H_0 \frac{1 + z}{\sqrt{|\Omega_k|}} \sin \left[ \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z'; \alpha_1, ..., \alpha_n)} \right],
\]

where \( E = \frac{H}{H_0}, z \) is the redshift parameter, and

\[
sin \left[ \sqrt{|\Omega_k|} x \right] = \begin{cases} 
\sin(\sqrt{|\Omega_k|} x) & \text{for } \Omega_k < 0 \\
\sqrt{|\Omega_k|} x & \text{for } \Omega_k = 0 \\
\sinh(\sqrt{|\Omega_k|} x) & \text{for } \Omega_k > 0.
\end{cases}
\]
Then one can write the theoretical modulus distance
\begin{equation}
\mu_{th}(z) = 5 \log_{10}[D_L(z)] + \mu_0 ,
\end{equation}
where \( \mu_0 = 5 \log_{10}(cH_0^{-1}/\text{Mpc}) + 25 \). On the other hand, the observational modulus distance of the SNIa, \( \mu_{obs}(z_i) \), at redshift \( z_i \) is given by
\begin{equation}
\mu_{obs}(z_i) = m_{obs}(z_i) - M ,
\end{equation}
where \( m \) and \( M \) are apparent and absolute magnitudes of SNIa respectively. Then the parameters of the theoretical model, \( \alpha_i \)'s, can be determined by a likelihood analysis by defining \( \chi^2_{SNIa}(\alpha_i, M') \) in Eq. (37) as
\begin{equation}
\chi^2_{SNIa}(\alpha_i, M') = \sum_j \left( \frac{\mu_{obs}(z_j) - \mu_{th}(\alpha_i, z_j)}{\sigma_j^2} \right)^2 = \sum_j \left( \frac{5 \log_{10}[D_L(\alpha_i, z_j)] - m_{obs}(z_j) + M'}{\sigma_j^2} \right)^2 ,
\end{equation}
where the nuisance parameter, \( M' = \mu_0 + M \), can be marginalized over as
\begin{equation}
\bar{\chi}^2_{SNIa}(\alpha_i) = -2 \ln \int_{-\infty}^{+\infty} \exp\left[ -\frac{1}{2} \chi^2_{SNIa}(\alpha_i, M') \right] dM' .
\end{equation}

3. Baryon Acoustic Oscillation

The baryon acoustic oscillations data from the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) [37] is used here for constraining the model parameters. The data constrains \( d_z \equiv r_s(z_d)/D_V(z) \), where \( r_s(z_d) \) is the comoving sound horizon at the drag epoch (where baryons were released from photons) and \( D_V \) is given by [55]
\begin{equation}
D_V(z) \equiv \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \frac{cz}{H(z)} \right)^{1/3} ,
\end{equation}
The drag redshift is given by the fitting formula [56]
\begin{equation}
z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}] ,
\end{equation}
where
\begin{equation}
b_1 = 0.313(\Omega_m h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.607}] , \quad b_2 = 0.238(\Omega_m h^2)^{0.223} .
\end{equation}
Then we can obtain \( \chi^2_{BAO} \) by \( \chi^2_{BAO} = Y^T C^{-1}_{BAO} Y \), where
\begin{equation}
Y = \begin{pmatrix}
d_{0.2} - 0.1905 \\
d_{0.35} - 0.1097
\end{pmatrix} ,
\end{equation}
and its covariance matrix is given by [37]
\begin{equation}
C^{-1}_{BAO} = \begin{pmatrix}
30124 & -17227 \\
-17227 & 86977
\end{pmatrix} .
\end{equation}
These results are similar to those obtained in [24] for original GDE in flat universe.
X-Ray Gas Mass Fraction

The ratio of the X-ray gas mass to the total mass of a cluster is defined as X-ray gas mass fraction \( \frac{f_{gas}}{f_{total}} \). The \( \Lambda \)CDM model proposed \[38\]

\[
f_{gas}^{\Lambda CDM}(z) = \frac{KA\gamma b(z)}{1 + s(z)} \left( \frac{\Omega_b}{\Omega_{0m}} \right) \left( \frac{D_{\Lambda}^{\Lambda CDM}(z)}{D_{A}(z)} \right)^{1.5}. \tag{56}
\]

The elements in Eq. \[56\] are defined as follows: \( D_{\Lambda}^{\Lambda CDM}(z) \) and \( D_{A}(z) \) are the proper angular diameter distance in the \( \Lambda \)CDM and the interested model respectively. Angular correction factor \( A \)

\[
A = \left( \frac{\theta_{2500}^{\Lambda CDM}}{\theta_{2500}} \right) = \left( \frac{H(z)D_{A}(z)}{[H(z)D_{A}(z)]^{\Lambda CDM}} \right)^{\eta}, \tag{57}
\]

is caused by the change in angle for the model proposed \[38\] in comparison with \( \theta_{2500}^{\Lambda CDM} \), where \( \eta = 0.214 \pm 0.022 \) \[38\]. The factor \( K \) describes the combined effects of the residual uncertainties, such as the instrumental calibration, and a Gaussian prior for the 'calibration' factor is considered by \( K = 1.0 \pm 0.1 \) \[38\].

Then, \( \chi^2_{gas} \) is defined as \[38\]

\[
\chi^2_{gas} = \sum_{i} \left[ \frac{f_{gas}^{\Lambda CDM}(z_i) - f_{gas}(z_i)}{\sigma_{f_{gas}}^2(z_i)} \right]^2 + \frac{(s_0 - 0.16)^2}{0.0016^2} + \frac{(K - 1.0)^2}{0.01^2} + \frac{(\eta - 0.214)^2}{0.022^2} \; ; \tag{59}
\]

with the statistical uncertainties \( \sigma_{f_{gas}}(z_i) \).

### B. Results

Finally, the maximum likelihood method is applied for the interacting GGDE in a non-flat universe by using the CosmoMc code \[38\]. Figure \[5\] shows 2-D contours with 1\(\sigma\) and 2\(\sigma\) confidence levels where 1-D distribution of the model parameters are shown as well. Best fit parameter values are shown in Table \[1\] with 1\(\sigma\) and 2\(\sigma\) confidence levels. From Table \[1\] we can see that the best fit results are given as: \( \Omega_{0DE} = 0.7145^{+0.0427}_{-0.0264} \pm 0.0484 \), \( \Omega_m = 0.2854^{+0.0204}_{-0.0427} \pm 0.0452 \), \( \Omega_k = 0.0285^{+0.0014}_{-0.0274} \). In addition for the model parameters the best fit values are obtained as: \( \xi = 0.2300^{+0.4769}_{-0.0129} \), \( b = 0.0592^{+0.1407}_{-0.0492} \). The age of the universe in this model is given by \( 13.7385^{+0.3302}_{-0.2907} \pm 0.3796^{+0.0492}_{-0.3313} \) Gyr. We have also plotted the evolution of \( \omega_D \), \( \Omega_D \) and \( q \) against the scale factor \( a \) for the interacting GGDE in a nonflat universe by using the best fit values of the model parameters.

### VI. SUMMARY AND DISCUSSION

In order to resolve the DE puzzle, people usually prefer to handle the problem by using existing degree’s of freedom. GGDE is a prototype of these models which discusses the acceleration of the universe and originates from vacuum energy of the Veneziano ghost field in QCD. This model can address the fine tuning problem \[15\]. An extended version of this model called GGDE was recently proposed by Cai et. al., \[27\], seeking a better agreement with observations.

In this paper we explored some features of GGDE in both flat and non-flat FRW universe in the presence of an interaction term between the two dark components of the universe. In section II, we discussed the GGDE in a flat FRW background. We found that the EoS parameter approaches \(-1\) which is the same as the cosmological constant. The next section was devoted to the interacting GGDE in a flat geometry. An interesting feature which we found was the capability of crossing the phantom line in this case. This behaviour is also seen in the last section for interacting GGDE in a universe with spatial curvature.
Parameter | Best Fit value | Mean Value |
---|---|---|
$\Omega_b h^2$ | $0.0226^{+0.0016}_{-0.0016} +0.0015$ | $0.02257^{+0.0009}_{-0.0010}$ |
$\Omega_{DM} h^2$ | $0.1153^{+0.0061}_{-0.0060} +0.0059$ | $0.1132^{+0.0030}_{-0.0030} +0.0063$ |
$\Omega_{DM}$ | $0.2854^{+0.0244}_{-0.0137} +0.0142$ | $0.2769^{+0.0129}_{-0.0131} +0.0294$ |
$\Omega_k$ | $0.0285^{+0.0001}_{-0.0001}$ | $0.0187^{+0.0012}_{-0.0017}$ |
$\Omega_{DE}$ | $0.7145^{+0.0427}_{-0.0264} +0.0484$ | $0.7230^{+0.0131}_{-0.0129} +0.0242$ |
$b$ | $0.0593^{+0.1407}_{-0.0492}$ | $0.1082^{+0.0917}_{-0.0982}$ |
$\xi$ | $0.2300^{+0.0479}_{-0.0129}$ | $0.2228^{+0.2128}_{-0.2174}$ |
$H_0$ | $69.5401^{+3.5998}_{-2.6037} +4.2626$ | $70.0610^{+1.1566}_{-1.1338} +2.7773$ |
Age (Gyr) | $13.7389^{+0.3022}_{-0.2907} +0.3796$ | $13.7596^{+0.1072}_{-0.1065} +0.2173$ |

**TABLE I:** The best fit and mean values of the model parameter with $1\sigma$ and $2\sigma$ regions from MCMC calculation by using CMB, SN1a Union2, X-gas and BAO datasets. The Hubble parameter is in the unit of $kms^{-1}Mpc^{-1}$.

**FIG. 4:** These figures show the evolutions of $w_D$, $\Omega_D$ and $q$ against the scale factor $a$ for the interacting GGDE models in a nonflat universe, where $\xi = 0.23$, $b = 0.05$ and $\Omega_k = 0.028$ which are chosen from the best fit values of Table I.

Then, we applied the Markov Chain Monte Carlo method together with the latest observational data to constrain the model parameters. The results are presented in Table I and Fig. 5. The main result found through this paper is that in the GGDE model, there is a delay in different epochs of the cosmic evolution in comparison with original GDE model. This result was also pointed out in [27] due to the negative contribution of the square term in the energy density of GGDE.

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FIG. 5: 1-D constraints on parameters and their 2-D contours with 1σ and 2σ regions. To obtain these plots, Union2+CMB+BAO+X-gas with BBN constraints are used. In 1-D plots, the solid lines are mean likelihoods of samples and dotted lines are marginalized probabilities for each parameter.

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