The parameters of internal gravity waves in the atmosphere from the amplitude fluctuations of radio occultation signals

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Abstract We develop a method for the reconstruction of the statistical parameters of internal gravity wave (IGW) in the atmosphere from the amplitude fluctuations of signals acquired in radio occultation observations. We discuss the choice of the model of the IGW spatial spectrum, derive the relationships between the IGW spectra and the amplitude fluctuation spectra, develop the reconstruction algorithm and estimate its uncertainties. The IGW spectrum parameters to be retrieved are the outer (dominant) scale and the structure characteristic, the latter determining the power of saturated IGW. The method was tested on the COSMIC (Constellation Observing System for Meteorology, Ionosphere and Climate) observations acquired in the winter and summer seasons 2011. The IGW parameters were reconstructed in the height range from the upper boundary of the tropopause to 28 km. We show the altitude-latitudinal distributions of the IGW parameters and two integral characteristics: the variances of the temperature fluctuations and the IGW potential energy. We perform a detailed comparison of our results with those obtained from radiosonde and radio occultation observations. The developed method can be applied for the global monitoring of IGW parameters and activity in the middle atmosphere.

1. Introduction

Internal gravity waves (IGW) in the atmosphere are the main source of small-scale fluctuations of wind and temperature with characteristic vertical scales from several kilometers to a hundred meters and periods from five minutes to about ten hours. They play the key role in energy exchange and atmospheric global circulation. IGW breaking generates turbulence in the middle atmosphere [1]. In order to include IGW related processes into global atmospheric circulation models, their statistical parameterization based on the global distribution of IGW activity, is necessary. The importance of this problem is confirmed by a large number of publications in this area. The most accurate observations of wind and temperature are acquired by radiosondes [2–5]. However, it is only satellite constellations carrying corresponding instruments that can really provide the global monitoring of IGW. Widespread are studies based on radio occultation (RO) observations performed by low-Earth Orbiters (LEO) receiving the highly stable signals of global navigation satellite systems, such as GPS [6–10]. For example, the COSMIC constellation consisting of 6 LEOs, at its maximum capability provided about 3000 events per day, homogeneously distributed all over the globe. The GPS signals at wavelengths of $\lambda_1=19.03$ cm and...
λ₂=24.42 cm are used for the measurements of the atmospheric excess phase, from which atmospheric temperature profiles including both regular and random components, are retrieved. A detailed analysis of the modern principles of RO sounding of the atmosphere can be found in the monograph [11].

The IGW parameters indicate a high spatiotemporal variability [1]. This fact, along with the importance of the role played by IGW in the atmospheric dynamics, motivates the development of new methods of their global monitoring. In our studies, we use the statistical description of the fluctuations in the amplitude of RO signals probing the atmosphere. In [12], we derived the main relationships connecting the measured spectra of the RO signal amplitude and phase fluctuations with the IGW spatial spectra. In [13], we developed the method for the reconstruction of the main statistical parameters of IGW from the amplitude fluctuation spectra; later the method was tested on COSMIC observations in 2011 [14].

This paper is a brief review of [12–14] complemented with a detailed comparison with other studies. The approximations adopted in our analysis impose weaker restrictions upon the spatial resolution, as compared to the standard geometric optical (GO) retrieval of temperature profiles [7, 8, 11]. From the COSMIC observations in January–February and June–July 2011, we reconstruct the altitude-latitudinal distributions of the main parameters of the IGW spectrum: the structure characteristic and the outer scale; we also obtain the variance of temperature fluctuations and the potential energy per unit mass in the height range from the tropopause to 28 km. We compare our results with other studies based on radiosonde, RO, and stellar occultation observations.

2. The reconstruction method
Fluctuations in the amplitude and phase of signals probing the atmosphere, are primarily caused by turbulence and IGW. In optical occultations, the contributions of these two sources of the light intensity fluctuations are nearly equal [15]. In RO observations, the main role is played by IGW, while the contribution of the Kolmogorov turbulence is 2–3 orders of magnitude smaller [12]. This is explained by the spectral slopes of IGW and turbulence and by a thousandfold difference in the main (Fresnel) scale for optical and radio waves [16]. Therefore, RO observations constitute a convenient tool for the study of IGW.

Our method is based on the statistical description of the random field of the atmospheric inhomogeneities and the RO signal fluctuations caused by them. In the limb observation geometry, a radio wave interacts with a representative ensemble of IGW. To take into account the effects of the atmospheric sphericity in RO observations, we assume that the random field of refractivity inhomogeneities is locally homogeneous in a spherical layer of the atmosphere.

The statistical description of wave propagation in the atmosphere is based on the 3-D spectrum of relative fluctuations of air refractivity, or temperature if humidity is negligible. We employ the 3-D power-law universal spectrum of the temperature fluctuations caused by IGW, developed by A.S. Gurvich [17].

\[
\Phi_{\delta T/T}(\kappa_x, \kappa_y, \kappa_z) = C_W^2 \eta^2 (\kappa^2 + K_W^2)^{-5/2},
\]

\[
\kappa^2 = \kappa_x^2 + \eta^2 \kappa_y^2,
\]

\[
\kappa_z^2 = \kappa_x^2 + \kappa_y^2,
\]

where \(\delta T/T\) are the relative temperature fluctuations, \(T\) is the seasonal-regional mean temperature, \(C_W^2\) is the structure characteristic determining the intensity of temperature fluctuations, \(\eta\) is the anisotropy coefficient characterizing the ratio of the characteristic horizontal to vertical scales (for IGW \(\eta \gg 1\)), \(\kappa_x\) is the vertical wavenumber, \(\kappa_x\) and \(\kappa_y\) are the horizontal wavenumbers, \(x\) axis coincides with the line of sight, \(K_W = 2\pi/L_W\), and \(L_W\) is the vertical outer (dominant) scale.

The single-sided 1-D vertical spectrum equals the integral of \(\Phi_{\delta T/T}\) over \(\kappa_x, \kappa_y\):
The main parameters of spectra (1) and (2), which have to be reconstructed, are the outer scale \( L_W = 2\pi/K_W \) and the structure characteristic \( C_W^2 \) [13, 14]. The outer scale \( L_W \) defines the transition from saturated to unsaturated waves at large scales \( \kappa_z \leq K_W \). The structure characteristic \( C_W^2 \) is the only parameter that defines the power of saturated waves with \( \kappa_z > K_W \). Spectrum \( \Phi_W \) defined by equation (1) is the 3-D generalization of the known universal model of saturated IGW with the vertical spectrum [18, 19]:

\[
V_{\delta T/T}(\kappa_z) = \frac{4\pi}{3} C_W^2 (\kappa_z^2 + K_W^2)^{-\frac{3}{2}}, \quad \kappa_z \geq 0.
\]  

where \( \beta \approx 0.1 \) is the IGW model coefficient that was first introduced theoretically and then confirmed experimentally, \( \omega_{B,V.} \) is the Brunt–Väisälä frequency, and \( g \) is the gravity acceleration. The structure characteristic \( C_W^2 \) is related to the parameters of the universal spectrum (3) as follows:

\[
C_W^2 = \frac{3\beta \omega_{B,V.}^4}{4\pi g^2}.
\]  

We adopt the approximations of phase screen and weak fluctuations, which are traditional for the statistical interpretation of occultation observations. Due to the averaging of the RO amplitude fluctuations over a large Fresnel zone, the condition of weak fluctuations is fulfilled for perigee heights down to several kilometers over the Earth's surface. These approximations greatly simplify the solution of the forward problem and result in a transparent analytical expression of the vertical spectrum of RO amplitude fluctuations though the vertical IGW spectrum in equation (2) [12, 13]:

\[
V_{\delta A/A}(\kappa_z) = \frac{k^2 \bar{\psi}^2}{\sqrt{1 + \kappa_z^2 H_0^2}} V_{\delta T/T}(\kappa_z) \sin^2 \frac{\kappa_z^2}{\kappa_F^2} = \frac{4\pi}{3} \frac{k^2 \bar{\psi}^2}{\sqrt{1 + \kappa_z^2 H_0^2}} C_W^2 (\kappa_z^2 + K_W^2)^{-\frac{3}{2}} \sin^2 \frac{\kappa_z^2}{\kappa_F^2}.
\]  

where \( k = 2\pi/\lambda, \bar{\psi} \) is the regular component of the excess phase [17], \( H_0 = 6–8 \text{ km} \) is the atmospheric vertical scale, \( \kappa_F = 2\pi/\rho_F, \rho_F = \sqrt{\pi q \lambda D} \) is the vertical Fresnel scale, \( q \) is the refractive attenuation, \( D = D_t D_r/(D_t + D_r) \) is the reduced observation distance (\( D_t \approx 3,200 \text{ km} \) and \( D_r \approx 25,800 \) are the distances from LEO and GPS satellites to the ray perigee, respectively). The spectra in equation (5) are written down for wavenumbers \( \kappa_z \geq 0 \) in the phase screen plane. The sine factor describes the diffraction after the phase screen, and the square root term in the denominator takes into account the sphericity of the atmosphere. Due to the strong anisotropy of IGW, practically all occultation events can be considered as vertical ones [12]. The reconstructed parameters were used for the evaluation of the integral characteristics describing the IGW activity: the temperature fluctuation variance \( \sigma_{\delta T}^2 \) and the potential energy per unit mass \( E_p \) [1, 4]:

\[
\sigma_{\delta T}^2 = \int_0^\infty V_{\delta T/T}(\kappa_z) d\kappa z = \frac{1}{3\pi} C_W^2 L_W^2,
\]  

\[
E_p = \frac{g^2}{2\pi^2 \omega_{B,V.}^4} \sigma_{\delta T}^2.
\]  

Our method fits the theoretical spectra of amplitude fluctuation in equation (5) to the experimental ones obtained from COSMIC observations, by varying the two parameters of the IGW model: the outer scale \( L_W \) and the structure characteristic \( C_W^2 \), and minimizing the residual. The residual is defined as the squared difference between the theoretical and experimental spectra integrated over the wavenumber.

The standard method of temperature profile retrieval from RO signals uses the approximations of the geometrical optics and the local spherical symmetry of the atmosphere [7, 8, 20]. The GO resolution is
defined by the Fresnel zone scale $\rho_F$, which is about 1.5 km in the stratosphere [7, 20]. The approximation of the local spherical symmetry limits the horizontal resolution by wavelengths of about 250-300 km [21]. In our method, we take diffraction into account, and the vertical resolution, which is mainly limited by the receiver noise, is about 0.5 km [12]. Moreover, the condition of the spherical symmetry is imposed not upon the whole structure of the atmosphere, but upon the second moments of the random inhomogeneity field only [17]. This result in weaker restrictions upon the IGW anisotropy and, accordingly, upon the horizontal resolution of the method [12, 17]. A higher spatial resolution allows, in particular, separately reconstructing the two parameters, the outer scale and the structure characteristic. A detailed discussion of wave-optical methods of retrieving high-resolution temperature profiles from RO observations can be found in [11].

In order to estimate the uncertainties of the two reconstructed parameters, we used fragments of a large ensemble consisting of about 30,000 events from spring 2011, under the assumption of its statistical homogeneity. Our tests indicated that the reconstruction accuracy is 10–20% for the outer scale and 20–40% for the structure characteristic [13].

3. Altitude-latitudinal distribution of IGW parameters
For the validation of the method, we studied the altitude-latitudinal distribution of IGW parameters in latitude bands $0^\circ–20^\circ$, $20^\circ–40^\circ$, $40^\circ–60^\circ$, $60^\circ–90^\circ$ in the Northern and Southern hemispheres for winter and summer COSMIC observations in 2011. For each season, we used 20,000 events. The IGW parameters were reconstructed in the height range from 4 km above the tropopause up to 28 km. Figure 1 presents the profiles of the outer scale for the local winter (panels A and C) and local summer (panels B and D) in both hemispheres. For the comparison, we present the multi-year measurements of the outer (dominant) scale obtained by other authors from radiosonde temperature observations [2–5, 22], stellar scintillations [23] and RO in the GO approximation [20, 21], the latter being justified by the fact that the outer scale exceeds the Fresnel zone size.

The outer scale in our observations varies from 2 to 4 km. There is a noticeable increase of the outer scale from polar to tropical latitudes, especially in the lower latitude range. Our data are generally in close agreement with the other studies.

Figure 2 presents the profiles of the structure characteristic for the local winter (panels A and C) and local summer (panels B and D) in both hemispheres. Recall that the structure characteristic $C_W$ is the...
only parameter of the saturated IGW spectrum. The structure characteristic varies from $10^{-11} \text{ m}^{-2}$ to $10^{-10} \text{ m}^{-2}$. It indicates a noticeable increase from the poles to the tropics. For a comparison, we also show the results obtained from radiosonde measurements [2, 3, 24] and satellite stellar scintillation observations [23]. RO data, due to their insufficient vertical resolution, do not allow obtaining a reliable estimate of the structure characteristic. On the other hand, radiosonde observations lack the global coverage, which does not leave much data for the comparison. Optical and RO observations only overlap at the upper border of the altitude range used in this study [23].

Figure 2. The structure characteristic profiles. The panel notations are the same as in Figure 1. The vertical bars represent the data from other studies [2, 3, 23, 24].

Figure 3. The temperature fluctuation profiles. The panel notations are the same as in Figure 1. The horizontal bar in panel A shows the variability estimate of the mean-square temperature deviations. The vertical bars represent the data from other studies [8, 21, 24–27].
The structure characteristic estimates indicate a high variability. Typical are the results obtained from the radiosonde measurements performed during 5 years at the same station located on a flat terrain [3]. The complete database was processed using the same algorithm. Still, the spectral amplitudes vary ten and more times for different measurements in close conditions.

Figure 3 presents the profiles of the mean-square temperature fluctuations for the local winter (panels A and C) and local summer (panels B and D) in both hemispheres. The vertical bars of the corresponding colors show the results obtained from radiosonde measurements [24–26] and RO retrieved temperature profiles [8, 21, 27]. The temperature fluctuation variance is determined by large-scale waves; therefore, it can be reliably estimated from both radiosonde data and RO observations processed in the GO approximation. The mean-square temperature deviations in our measurements are in the range from 1 to 3 K. As expected, the temperature fluctuations indicate a significant increase from the poles to the tropics. The mean-square temperature deviations obtained in our and other studies are in close agreement with each other.

Figure 4. The stepwise solid lines show the latitudinal distribution of the integral potential energy of IGW for the altitude intervals of 16–24 km (red) and 24–28 km (black). The dashed lines show the measurement results [8] for 15–25 km (red) and 20–30 km (black). The solid vertical bars show the data of RO observations [8, 10, 21, 27–29, 31], stellar scintillation observations [30], dashed bars show the radiosonde measurement data [4, 24, 29]. The height of the bars is the estimate of the measurement variability, including the uncertainties.

A very important characteristic of the IGW activity is their energy per unit mass. The radiosonde observations allow the determination of both energy components: the kinetic one from the pulsation power of the horizontal wind component and the potential one from the temperature fluctuation intensity. In RO monitoring, the IGW activity indicator is the potential energy per unit mass [1, 4, 21], because the kinetic energy is linked to the potential one by the polarization relationships: in the linear theory their ratio equals the modulus of the exponent of the temporal fluctuation spectrum of temperature [1, 2]. Figure 4 shows the latitudinal distributions of the integral potential energy defined by equation (7), averaged over the two altitude intervals: 16–24 km and 24–28 km, for January–February (left panel) and June–July (right panel). Noticeable are the high values of $E_p$ in the equatorial zone, reaching 10–11 J/kg in the altitude range 16–24 km and 8–9 J/kg in the altitude range 24–28 km. This indicates the leading role of the convective processes for IGW generation in the tropics [1, 8, 20]. An enhanced IGW activity in the equatorial zone may be partially explained by the Kelvin wave contribution into large-scale temperature fluctuations [8, 9, 27]. The IGW activity indicates a fast decay with increasing latitudes, reaching values $E_p$ of 3–4 J/kg in the middle latitudes and 1–2 J/kg in the high latitudes. In
these zones, the potential energy $E_p$ for altitudes 24–28 km is slightly higher and has a flatter distribution as compared to the lower altitude interval. The global potential energy distribution is approximately symmetric for the two hemispheres, however, in Northern latitudes over 60° the winter values of $E_p$ are approximately twice as high as the summer ones.

As noticed above, the temperature fluctuation intensity and IGW potential energy defined by equations (6) and (7), respectively, are determined by large-scale waves close to dominant ones. This justifies the estimates based on RO temperature profiles obtained in the GO approximation, which are intensively used during the two last decades. Below we compare our results with pioneering study [8], which analyzed the global distribution of the IGW potential energy inferred from the GPS/MET RO data 1995–1997. As follows from Figure 4, our results mostly agree with [8], although there are some differences. In [8], an enhanced IGW activity at high latitudes in local winter is noticed; in the Northern hemisphere (left panel) this effect is modest, while in the Southern hemisphere (left panel) for 50°S–80°S latitude band it is noticeable. Besides, in the altitude interval 24–28 km our estimates of $E_p$ are higher. Figure 4 also shows the vertical bars of the corresponding colors representing the results of other studies, obtained both from RO, stellar scintillation observations, and radiosonde measurements. We conclude that our results agree with those from other studies, within the high variability of IGW potential energy.

4. Conclusions
Here we summarize the main statements of our study.

We developed a method of the reconstruction of the main statistical parameters of IGW in the atmosphere: the outer scale and the structure characteristic, from the measurements of amplitude fluctuations in RO experiments.

The approximations of the phase screen and weak fluctuations allowed the derivation of simple relations between the vertical spectra of IGW and RO signal amplitude fluctuations. A higher spatial resolution, as compared to the traditional GO approximation used in temperature profile retrieval from RO observations, allows the separate reconstruction of the IGW parameters.

Our validation indicated that the method can be successfully applied for the global monitoring of the statistical parameters and activity of IGW in the middle atmosphere.

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