Low-frequency peak in the magnetoconductivity of a non-degenerate 2D electron liquid

Frank Kuehnel\textsuperscript{1}, Leonid P. Pryadko\textsuperscript{2}, and M.I. Dykman\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48823
\textsuperscript{2}School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540

We study the frequency-dependent magnetoconductivity of a strongly correlated nondegenerate 2D electron system in a quantizing magnetic field. We first restore the single-electron conductivity from calculated 14 spectral moments. It has a maximum for $\omega \sim \gamma (\hbar \gamma)$ is the disorder-induced width of the Landau level), and scales as a power of $\omega$ for $\omega \to 0$, with a universal exponent. Even for strong coupling to scatterers, the electron-electron interaction modifies the conductivity for low and high frequencies, and gives rise to a nonzero static conductivity. We analyze the full many-electron conductivity, and discuss the experiment.

PACS numbers: 73.23.-b, 73.50.-h, 73.40.Hm

One of the most interesting problems in physics of low-dimensional systems is the effect of the electron-electron interaction (EEI) on electron transport. In many cases the EEI is the major factor, fractional quantum Hall effect (QHE) being an example. At the same time, single-electron picture is often also used for interpreting transport, as in the integer QHE. Another closely related example is magnetotransport of a low-density two-dimensional electron system (2DES) on helium surface \[\omega/\gamma\]. For strong quantizing magnetic fields, experimental data on electron transport in this system are reasonably well described \[\omega/\gamma\] by the single-electron theory based on the self-consistent Born approximation (SCBA) \[\omega/\gamma\]. This theory does not take into account the interference effects that lead to electron localization in the random potential of scatterers. Such a description appears to contradict the phenomenology of the integer QHE, where all but a finite number of single-particle states in the random potential are localized \[\omega/\gamma\]. The static single-electron magnetoconductivity $\sigma_{xx}(0)$ must vanish, as illustrated in Fig. \[\omega/\gamma\], since the statistical weight of the extended states is equal to zero.

In this paper we discuss the case where the EEI is strong and the electrons are correlated, as for 2DES on helium and in fractional QHE. Yet the characteristic force on an electron from the short-range random potential may exceed the force from other electrons. The interrelation between the forces determines the effective strength of the coupling to scatterers. The analysis allows us to understand the strong coupling limit and the crossover to weak coupling, and to resolve the apparent contradiction between localization of single-electron states and the experimental data for electrons on helium.

We show that, for strong coupling to scatterers, the low-frequency magnetoconductivity $\sigma_{xx}(\omega)$ of a nondegenerate 2DES becomes nonmonotonic: it has a maximum at a finite frequency $\omega_{\text{max}} \approx 0.3 \gamma$, where $\gamma$ is the SCBA level broadening [Fig. \[\omega/\gamma\]]. For small but not too small $\omega/\gamma$, the conductivity scales as $\omega^{\mu}$ with a universal exponent $\mu \approx 0.215$. Whereas the onset of the peak is a single-electron effect, the nonzero value of the static conductivity and the form of $\sigma_{xx}(\omega)$ for big $\omega/\gamma$ are determined entirely by the EEI. We obtain an estimate for $\sigma_{xx}(0)$ and analyze the overall shape of $\sigma_{xx}(\omega)$ in the parameter range where $\exp(\hbar \omega_c/k_B T) \gg 1$ and $k_B T \gg \hbar \gamma$ ($\omega_c$ is the cyclotron frequency), the conditions usually met in strong-field experiments on electrons on helium.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Reduced microwave conductivity \[\omega/\gamma\] of a non-interacting 2DES in a short-range disorder potential for $k_B T \gg \hbar \gamma$ (solid line). The electron-electron interaction results in flattening of the conductivity for $\omega \lesssim \omega_c$ \[\omega/\gamma\], and in a much slower decay of $\tilde{\sigma}$ for moderately big $\omega \gg \gamma$, as shown by dashed lines. Inset: convergence of the interpolation factor $G$ \[\omega/\gamma\] with the increasing number of moments $2k$.}
\end{figure}

The single-electron conductivity at low frequencies is determined by the correlation function of the velocity of the guiding center $\mathbf{R}$ of the electron cyclotron orbit in the potential of scatterers. For $\omega \ll k_B T/\hbar$ and $\exp(\hbar \omega_c/k_B T) \gg 1$, it can be written as $\sigma_{xx}(\omega) = (n e^2 l^2 \gamma /8 k_B T) \tilde{\sigma}(\omega)$, where $n$ is the electron density, $l = (\hbar/m \omega_c)^{1/2}$ is the magnetic length, and $\tilde{\sigma}$ is the reduced conductivity,

$$
\tilde{\sigma}(\omega) = \frac{2\hbar \gamma}{m \omega_c} \int_{-\infty}^{\infty} dt e^{i \omega t} \sum_{\mathbf{q}, \mathbf{q}'} \langle \mathbf{q}, \mathbf{q}' \rangle \langle \hat{V}_{\mathbf{q}} \hat{V}_{\mathbf{q}'} \exp[i \mathbf{q} \mathbf{R}(t)] \exp[i \mathbf{q}' \mathbf{R}(0)] \rangle.
$$

\[1\]
Here, $\langle \cdot \rangle$ stands for thermal averaging followed by the averaging over realizations of the random potential of defects $V(\mathbf{r})$, and $\hat{V}_q = (V_q / h \gamma) \exp(-l^2 q^2 / 4)$ are proportional to the Fourier components $V_q$ of $V(\mathbf{r})$. We will assume that $V(\mathbf{r})$ is Gaussian and delta-correlated,

$$\langle V(\mathbf{r}) V(\mathbf{r}') \rangle = e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}')},$$

(2)

in which case $h \gamma = (2/\pi)^{1/2} v l / \ell$. 

Time evolution of the guiding center $\mathbf{R} \equiv (X, Y)$ in Eq. (6) is determined by the dynamics of a 1D quantum particle with the generalized momentum and coordinate $X$ and $Y$, and with the Hamiltonian

$$H = h \gamma \sum_q \hat{V}_q \exp(i \mathbf{q} \mathbf{R}), \quad [X, Y] = -i l^2.$$  

(3)

Because of the Landau level degeneracy in the absence of random potential, the problem of dissipative conductivity is to some extent similar to the problem of the absorption spectra of Jahn-Teller centers in solids [5], which are often analyzed using the method of spectral moments. This method can be applied to the conductivity [1] as well [3]. It allows, at least in principle, to restore $\sigma_{xx}(\omega)$. In addition, the moments

$$M_k = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \frac{d \omega}{\omega^{1/2} \gamma} \hat{\sigma}(\omega)^k$$

(4)

can be directly found from measured $\sigma_{xx}(\omega)$, and therefore are of interest by themselves.

For $\omega, \gamma \ll k_B T / \hbar$, the states within the broadened lowest Landau level are equally populated and the reduced conductivity is symmetric, $\hat{\sigma}(\omega) = \hat{\sigma}(\omega)$. Then odd moments vanish, $M_{2k+1} = 0$. For even moments, we obtain from Eqs. (1), (4)

$$M_{2k} = -2 l^2 \sum (q_1 q_{2k+2}) \langle \hat{V}_{q_1} \cdots \hat{V}_{q_{2k+2}} \rangle$$

$$\times \left[ \cdots [e^{i\mathbf{q}_1 \mathbf{R}}, e^{i\mathbf{q}_2 \mathbf{R}}], \cdots, e^{i\mathbf{q}_{2k+1} \mathbf{R}} \right] e^{i\mathbf{q}_{2k+2} \mathbf{R}},$$

where the sum is taken over all $q_1, \ldots, q_{2k+2}$. The commutators (3) can be evaluated recursively using

$$[e^{i\mathbf{q}_1 \mathbf{R}}, e^{i\mathbf{q}_2 \mathbf{R}}] = 2 i \sin \left( \frac{1}{2} q_1^2 + q_2^2 \right) e^{i(q_1 + q_2) \mathbf{R}}.$$  

(6)

From Eq. (3), $\langle \hat{V}_q \hat{V}_{q'} \rangle = (\pi l^2 / 2 \exp(-l^2 q^2 / 2)) \delta_{q+q'}$, where $S$ is the area. The evaluation of the 2kth moment comes then to choosing pairs $(q_1, -q_1)$ and integrating over $k+1$ independent $q_1$. From Eq. (6), the integrand is a (weighted with $q_1 q_{2k+2}$) exponential of the quadratic form $(l^2 / 2) \sum_{ij} A_{ij} q_i q_j$, where $i, j = 1, \ldots, k + 1$. The matrix elements $A_{ij}$ are themselves $2 \times 2$ matrices, $A_{ij} = -l \delta_{ij} + a_{ij} \hat{\sigma}_y$, where $\hat{\sigma}_y$ is the Pauli matrix, and $a_{ij} = -a_{ji} = 0, \pm 1$. Because of the structure of the matrices $\hat{A}$, the moments $M_{2k}$ are given by rational numbers. For $k = 0, 1, \ldots, 7$ we obtain [10]

$$M_{2k} = 1; \frac{3}{2}; \frac{443}{8}; \frac{25003}{152}; \frac{1360894709}{1152}; \frac{8941363200}{1372}; \frac{447809}{157244}; \frac{637499}{7244}$$

(7)

(we give approximate values of $M_{2k}$ for $k \geq 5$).

To restore the conductivity $\hat{\sigma}(\omega)$ from the calculated finite number of moments, we need its asymptotic form for $\omega \gg \gamma$. It can be found from the method of optimal fluctuation [1], by calculating the thermal average in Eq. (6) on the exact eigenstates $|n\rangle$ of the lowest Landau band of the disordered system. All states $|n\rangle$ are equally populated for $k_B T \gg \hbar \gamma$. Their energies $E_n$ are symmetrically distributed around the band center ($E = 0$), with the density of states $\rho(E) \propto \exp(-4E^2 / h^2 \gamma^2)$ [12]. For large $\omega / \gamma$, the conductivity is formed by transitions between states $|n\rangle, |m\rangle$ with large and opposite sign energies $E_{n,m} (|E_n - E_m| = \hbar \omega)$. The major contribution comes from $E_n = -E_m$. Only those configurations of $V(\mathbf{r})$ are significant, where the states $|n\rangle, |m\rangle$ are spatially close. However, the overlap matrix elements affect only the prefactor in $\hat{\sigma}$ [10], and to logarithmic accuracy,

$$\hat{\sigma}(\omega) \propto |\rho(\hbar \omega / 2)|^2 \propto \exp(-2\omega^2 / \gamma^2).$$

(8)

Since the tail of the conductivity is Gaussian, one is tempted to restore $\hat{\sigma}(\omega)$ from the moments $M_n$ using a standard expansion in Hermite polynomials, $\hat{\sigma}(\gamma x) = \sum_n c_n H_n(\sqrt{2}x) \exp(-2x^2)$. From Eq. (7), the coefficients $c_n$ are recursively related to the moments $M_n$ with $k \leq n$. However, for the moments values (7), such an expansion does not show convergence. This indicates possible nonanalyticity of the conductivity at $\omega = 0$.

For $\omega \rightarrow 0$, the conductivity can be found from scaling arguments [13, 14] by noticing that it is formed by states within a narrow energy band $|E| \ll \hbar \gamma$. The spatial extent of low-energy states is of the order of the localization length $\xi \sim \ell |e|^{-\nu}$, where $e = E / h \gamma$ and $\nu = 2.33 \pm 0.03$ is the localization exponent [3]. The frequency $\omega$, on the other hand, sets a “transport” length $L_\omega \sim \ell (\gamma / \omega)^{1/2}$. It is the distance over which an electron would diffuse in the random field $V(\mathbf{r})$ over time $1 / \omega$, with a characteristic diffusion coefficient $D = \ell^2 \gamma$, if there were no interference effects. For large $\xi$, $L_\omega \gg \ell$, the scaling parameter can be chosen as $g = (L_\omega / \xi)^{1/2} \sim |e| (\gamma / \omega)^{-1/2}$ [14].

The conductivity $\hat{\sigma}(\omega)$ is determined by the states within the energy band where $g \lesssim 1$. For high $T$, all these states contribute nearly equally, and

$$\hat{\sigma}(\omega) \propto \omega^\mu (\omega \rightarrow 0), \quad \mu = (2\nu)^{-1} \approx 0.215.$$  

(9)

With Eqs. (6), (8), the conductivity can be written as

$$\hat{\sigma}(\omega) = x^\mu G(x) \exp(-2x^2), \quad x = |e| / \gamma / \gamma.$$  

(10)

The function $G(x)$ ($x \geq 0$) can be expanded in Laguerre polynomials $L_n^{(\mu-1)/2}(2x^2)$, which are orthogonal for the weighting factor in Eq. (10). We have restored the corresponding expansion coefficients from the moments [7].

2
The resulting conductivity is shown in Fig. 3 with solid line. The expansion for $\sigma$ converges rapidly for $\mu$ between 0.19 and 0.28 (as illustrated in Fig. 1) for $\mu = 0.215$, whereas outside this region the convergence deteriorates.

The electron-electron interaction (EEI) can strongly affect the magnetoconductivity even for low electron densities, where the 2DES is nondegenerate. Of particular interest for both theory and experiment are many-electron effects for densities and temperatures where $\Gamma \equiv e^2(\pi n)^{1/2}/k_BT \gg 1$. The 2DES is then strongly correlated and forms a nondegenerate electron liquid or, for $\Gamma > 130$ 12, a Wigner crystal. The motion of an electron is mostly thermal vibrations about the (quasi)equilibrium position inside the “cell” formed by other electrons. For strong $\mathbf{B}$, the characteristic vibration frequency is $\Omega_p = 2\pi e^2n^{3/2}/m\omega_c$, $\Omega_p \ll \omega_c$ (for a Wigner crystal, $\Omega_p$ is the zone-boundary frequency of the lower phonon branch 11). We will assume that $k_BT \gg \hbar\Omega_p$. Then the vibrations are quasiclassical, with amplitude $\delta \sim (k_BT/e^2n^{3/2})^{1/2} \gg l$.

The restoring force on an electron is determined by the electric field $\mathbf{E}_R$ from other electrons. The distribution of this field is Gaussian, except for far tails, and $E_R = F(\Gamma) n^{3/2}k_BT$, with $F(\Gamma)$ varying only slightly, from 8.9 to 10.5, in the whole range $\Gamma \gtrsim 20$ 11. Since $\delta \gg l$, the field $\mathbf{E}_R$ is uniform over the electron wavelength $l$. The electron motion can be thought of as a semiclassical drift of an electron wave packet in the crossed fields $\mathbf{E}_R$ and $\mathbf{B}$, with velocity $c\mathbf{E}_R/B$.

In the presence of defects, moving electrons will collide with them. If the density of defects is small and their potential $V(r)$ is short-range [cf. Eq. (3)], the duration of a collision is

$$t_c = l(B/c) \langle E_R^{-1} \rangle \sim (\hbar/\epsilon l) n^{-3/4}(k_BT)^{-1/2},$$

and the scattering cross-section is $\propto \gamma^2$. For $\gamma t_c \ll 1$, electron-defect collisions occur independently and successively in time. This corresponds to weak coupling to the defects, and allows one to use a single-electron type transport theory, with the collision rate $\tau^{-1}$ calculated for the electron velocity $c\mathbf{E}_R/B$ determined by the EEI, $\tau^{-1} \sim \gamma^2 t_c$ 11. The many-electron weak-coupling results have been fully confirmed by experiments 11.

For $\gamma t_c \gg 1$, collisions with defects “overlap” in time, which corresponds to the strong coupling limit. In this case, from Eqs. (3), (11), the characteristic force on an electron from the random field of defects $F_R = \hbar\gamma/l \gg c\mathbf{E}_R$. One might expect therefore that the EEI does not affect the conductivity, and the single-electron theory discussed above would apply. It turns out, however, that this is not the case for the low- and high-frequency conductivity.

As a result of the EEI, the energy of an electron in the potential of defects $V(r)$ is no longer conserved. The motion of each electron gives rise to modulation of energies of all other electrons. The overall change of the Coulomb energy of the electron system over a small time interval is given by $\sum_n e \langle \mathbf{E}_n \delta r_n \rangle$, where $\delta r_n$ is the displacement of the $n$th electron due to the potential of defects, and $\mathbf{E}_n$ is the electric field on the $n$th electron from other electrons. Clearly, $\mathbf{E}_n$ and $\delta r_n$ are statistically independent. This allows us to relate the coefficient of energy diffusion of an electron $D_e$ to the characteristic coefficient $D = \gamma l^2$ of spatial diffusion in the potential $V(r)$,

$$D_e = (e^2/2) \langle E_R^2 \rangle D \sim \gamma(\hbar/t_e)^2. \tag{12}$$

Energy diffusion eliminates electron localization which caused vanishing of the single-electron static conductivity. The low-frequency boundary $\omega_l$ of the range of applicability of the single-electron approximation can be estimated from the condition that the diffusion over the energy layer of width $\sim \delta \epsilon_l = (\omega_l/\gamma)^{\mu}$ [which forms the single-electron conductivity (3) at frequency $\omega_l \ll \gamma$] occurred over the time $1/\omega_l$. For $\mu = 1/(2\nu)$, this gives

$$\omega_l/\gamma = C_1(\gamma t_c)^{-2\nu/(\nu+1)}, \quad C_1 \sim 1. \tag{13}$$

All states with energies $|\epsilon| \lesssim \delta \epsilon_l$ contribute to the conductivity for frequencies $\omega < \omega_l$. Therefore the many-electron conductivity may only weakly depend on $\omega$ for $\omega < \omega_l$, as shown in Fig. 1, and the static conductivity

$$\sigma_{xx}(0) \approx \sigma_{xx}(\omega_l) \sim (ne^2/\hbar) \gamma(\gamma t_e)^{-1/(\nu+1)}. \tag{14}$$

We note that there is a similarity between the EEI-induced energy diffusion, which we could quantitatively characterize for a correlated nondegenerate system, and the EEI-induced phase breaking in QHE 21, 22. The cutoff frequency $\omega_l$ can be loosely associated with the reciprocal phase breaking time.

The EEI also changes the high-frequency tail of $\sigma_{xx}(\omega)$ in the range $\omega \ll \omega_c$. In the many-electron system, the tail is formed by processes in which a guiding center of the electron cyclotron orbit shifts in the field $\mathbf{E}_R$ (by $\delta \mathbf{R}$). The energy $\hbar \omega$ goes into the change of the potential energy of the electron system $e\mathbf{E}_R\delta \mathbf{R}$, whereas the recoil momentum $\hbar\delta \mathbf{R}/l^2$ goes to defects. For large $\omega$, it is necessary to find optimal $\delta \mathbf{R}$ and $\mathbf{E}_R$. For weak coupling to defects, $\gamma t_e \ll 1$, the correlator (13) can be evaluated to the lowest order in $\gamma$, which gives

$$\tilde{\sigma}(\omega) = \gamma\omega_l^2 \exp \left[-(2/\pi)^{1/2}/\omega_l t_e\right]. \tag{15}$$

The exponential tail (15) is determined by the characteristic many-electron time (11), and the exponent is just linear in $\omega$. For larger $\omega$, the decay of $\tilde{\sigma}$ slows down to $|\ln \tilde{\sigma}(\omega)| \propto (\omega t_e)^{2/3}(|\ln(\omega/\gamma)|)^{1/3}$, provided $n^{12}2^{12}/\tilde{\delta}^{12}(\omega t_e)^{1/3} \ll 1$ (16). This asymptotics results from anomalous tunneling (22) due to multiple scattering by defects. It also applies for strong coupling to defects, $\gamma t_e \gg 1$, and replaces the much steeper single-electron Gaussian asymptotics (8).
We note that the overall frequency dependence of \( \sigma_{xx}(\omega) \) is qualitatively different for strong and weak coupling to scatterers. In the latter case, \( \sigma_{xx} \) is maximal for \( \omega = 0 \) and decreases monotonously with the increasing \( \omega \), in contrast to the behavior of \( \sigma_{xx}(\omega) \) in the strong-coupling case shown in Fig. 1. Both \( \gamma_t \) and \( t_e \) increase with the magnetic field, and by varying magnetic field, electron density, and temperature one can explore the crossover between the limits of strong and weak coupling.

It is interesting that both the static and the high-frequency conductivities are many-electron even for \( \gamma t_e \gg 1 \), where the coupling to defects is strong. It follows from Eq. (4) (see also Fig. 1) that the many-electron \( \sigma_{xx}(0) \) is of the order of the single-electron SCBA conductivity \( \sigma_{xx}^{SCBA}(0) = (4/3\pi)ne^2\gamma l^2 \) for not extremely large \( \gamma t_e \). This is a consequence of the very steep frequency dependence of the full single-electron conductivity \( \sigma(\omega) \) for \( \omega \to 0 \).

It follows from the above arguments that the random potential of defects does not eliminate self-diffusion in 2DES for \( \Gamma < 130 \), where the electrons form a nondegenerate liquid. For electrons on bulk helium, the results on the static conductivity apply also for \( \Gamma > 130 \), where electrons form a Wigner crystal. In this case the random field comes from thermally excited ripplons or (for \( T \gtrsim 1 \) K) from helium vapor atoms. Ripplons, although they are extremely slow, do not pin the Wigner crystal (we note that, for scattering by ripplons, \( \gamma \propto T^{1/2} \)). Random potential of vapor atoms is time-dependent (and also non-pinning). Vapor atoms stay within the electron layer only for a time \( t_v = a_B/v_T \), where \( a_B \) is the layer thickness and \( v_T \) is the thermal velocity of the atoms. For strong magnetic fields one can have \( \gamma t_v \gg 1 \), and then if \( \gamma t_e \gg 1 \), coupling to the vapor atoms is strong, as observed in Refs. 3, 10-13. The presented strong-coupling theory describes the conductivity for arbitrary \( t_e/t_v \) provided the low-frequency cutoff of the single-electron theory \( \omega(\omega) \) is replaced by \( \min(t_v^{-1},\omega) \).

In conclusion, we have analyzed the magnetoconductivity of a nondegenerate 2D electron liquid in quantizing magnetic field. This is a simple and well-studied experimentally strongly correlated system, where effects of the electron-electron interaction on transport can be characterized qualitatively and quantitatively. It follows from our results that, whereas for weak coupling to short-range scatterers the conductivity \( \sigma_{xx}(\omega) \) monotonically decays with increasing \( \omega \) (\( \omega \ll \omega_c \)), for strong coupling it becomes nonmonotonic. Even for strong coupling, the static conductivity is determined by many-electron effects, through energy diffusion. It is described in terms of the critical exponents known from the scaling theory of the QHE. The frequency dispersion of \( \sigma_{xx} \) disappears for \( \omega \ll \omega_c \propto T^{\nu/(\nu+1)} \), for temperature-independent disorder. In a certain range of magnetic fields and electron densities, the value of \( \sigma_{xx}(0) \) \( \sim \) is reasonably close numerically to the result of the self-consistent Born approximation, which provides an insight into numerous experimental observations for electrons on helium surface.

We are grateful to M. M. Fogler and S. L. Sondhi for useful discussions. Work at MSU was supported in part by the Center for Fundamental Materials Research and by the NSF through Grant no. PHY-972205. L.P. was supported in part by DOE grant DE-FG02-90ER40542.

[1] Two-dimensional electron systems on helium and other cryogenic substrates, ed. by E. Y. Andrei (Kluwer, Boston, 1997).
[2] P. W. Adams and M. A. Paalanen, Phys. Rev. B 37, 3805 (1988).
[3] R. W. van der Heijden et al., Europh. Lett. 6, 75 (1988).
[4] M. J. Lea et al., Phys. Rev. B 55, 16280 (1997) and references therein.
[5] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982).
[6] A. Pruisken, in The Quantum Hall effect, eds. R. Prange and S. M. Girvin (Springer, New York, 1990), p. 117.
[7] B. Huckestein, Rev. Mod. Phys. 67, 357 (1995).
[8] A. M. Stoneham. Theory of defects in solids: electronic structure of defects in insulators and semiconductors (Clarendon Press, Oxford, 1975).
[9] M. I. Dykman, Phys. Stat. Sol. B 88, 463 (1978).
[10] F. Kuehnel et al., in preparation.
[11] L. B. Ioffe and A. I. Larkin, JETP 54, 556 (1981).
[12] F. Wegner, Z. Phys. B 51, 279 (1983).
[13] J. T. Chalker and P. D. Coddington, J. Phys. C 21, 2665 (1988).
[14] Z.-Q. Wang et al., cond-mat/9906454.
[15] S. L. Sondhi, unpublished.
[16] C. C. Grimes and G. Adams, Phys. Rev. Lett. 42, 795 (1979); D. S. Fisher, B. I. Halperin, and P. M. Platzman, Phys. Rev. Lett. 42, 798 (1979).
[17] C. Fang-Yen, M. I. Dykman, and M. J. Lea, Phys. Rev. B 55, 16272 (1997).
[18] M. I. Dykman and L. S. Khazan, JETP 50, 747 (1979).
[19] M. J. Lea and M. I. Dykman, Physica B 251, 628 (1998) and references therein; E. Teske et al., Phys. Rev. Lett. 82, 2772 (1999).
[20] M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Phys. Rev. Lett. 64, 587 (1990).
[21] D. Polyakov and B. Shklovskii, Phys. Rev. Lett. 70, 3796 (1993); ibid. 73, 1150 (1994).
[22] B. I. Shklovskii, JETP Lett. 36, 51 (1982); B. I. Shklovskii and A. L. Efros, JETP 57, 470 (1983). Qin Li and D. J. Thouless, Phys. Rev. B 40, 9738 (1989); T. Martin and S. Feng, ibid. 44, 9084 (1991); J. Haidu, M. E. Raikh, and T. V. Shahbazyan, ibid. 50, 17625 (1994).
[23] M. I. Dykman, JETP 54, 731 (1981).