Construction of Lorentz-Invariant Amplitudes from Rest-Frame Wave Functions in HQET
– Application to the Isgur-Wise Function –

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Succeeding in predicting 0++ and 1++ states of D and Ds heavy mesons with our semi-relativistic quark potential model, we examine a method for constructing Lorentz-invariant scattering amplitudes and/or decay widths and develop a formulation to calculate Lorentz-boosted ones given the rest-frame wave functions in our model.

To illustrate the effectiveness of our method, we apply the formulation to the problem of calculating the semileptonic weak form factors from the rest-frame wave functions of heavy mesons and numerically calculate the dynamical 1/mQ corrections to those for the process B → D(*)ℓν based on our model of heavy mesons. It is shown that nonvanishing expressions for ρ1(ω) = ρ2(ω) and ρ3(ω) = ρ4(ω) = 0 are obtained in a special Lorentz frame, where ρi(ω) are the parameters used in the Neubert-Rieckert decomposition of form factors. Various values of form factors are estimated. These values are compatible with recent experimental data as well as other theoretical calculations.

§1. Introduction

The discovery of the narrow meson states Ds0(2317) by BaBar and Ds1(2460) by CLEO and the following confirmation by Belle has motivated many theorists to explain these states, as previous studies of these states (Refs. 6 and 7) using quark potential model apparently fail to reproduce these mass values. More recent experiments have found many other heavy-light mesons: broad Ds0(2308) and Ds1(2427) mesons found by the Belle collaboration, which are identified as c̅q (q = u/d) excited (ℓ = 1) bound states and have the same quantum numbers, jP = 0+ and 1+, as DsJ, respectively (The masses of these mesons have been reported with quite different values in the CLEO and FOCUS experiments, and hence are not yet determined with definiteness;6,7); narrow B and B̄ states of ℓ = 1, B1(5720), B1(5745), and B2(5839) found by CDF and D0 whose decay widths are also narrow because of their decay through the D-waves; and seemingly radial excitations (n = 2) of the 0+ Ds0(2860) state found by BaBar and the 1− Ds*(2715) state found by Belle. Furthermore several c̅c quarkonium-like states have been discovered: X(3872), X(3940), Y(3940), Z(3930), and Y(4260).

The discovery of these mesons triggered a series of studies of the spectroscopy of heavy-light and/or heavy-heavy hadrons using many kinds of ideas, including our semirelativistic quark potential model. Because we do not discuss these ideas in this

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paper, we refer the reader to the review articles\cite{12,13,14}. In previous papers\cite{15} we formulated a method for calculating the spectrum of heavy-light mesons in order to construct the Schrödinger equation for a bound state. In that method, the mass is expressed as an eigenvalue of the Hamiltonian of the heavy-light system, in which a bound state consists of a heavy quark and a light antiquark, and negative energy states of a heavy quark appear in the intermediate states when calculating an energy eigenvalue. In order to make our formulation more reliable, we attempt to reproduce narrow $B$ and $B_s$ states of $\ell = 1$, $B_1(5720)$, $B_2^*(5745)$ and $B_{s2}^*(5839)$, together with higher states of $J^P$ for $D$, $D_s$, $B$ and $B_s$\cite{16} and also to reproduce radial excitations of the $0^+$ state of $D_{s0}(2860)$ and the $1^-$ state of $Ds^*(2715)$\cite{17}. These calculations show our model’s reliability, as these results fit well with experiments. What we need to do next is to show that our approach can also produce a method for calculating scattering amplitudes and decay widths using the rest-frame wave functions. Here, we concentrate on the construction of such a formulation, and therefore we refer readers the review paper Ref. \cite{18} for explanations of other methods and their results, because there are many papers that give estimates of widths of many decay modes of heavy mesons.

The heavy quark effective theory (HQET) is a very convenient tool for studying heavy mesons/baryons, including at least one heavy quark\cite{19,20}. In particular, there are many studies on the Isgur-Wise function\cite{21–32} but most of the calculations in those works use a simple-minded Gaussian form for the meson wave function or a solution to a single-particle Dirac equation in a potential. Some other people have used the Bethe-Salpeter solution for a meson wave function, but it seems there has been no systematically developed way to calculate the higher-order terms in $1/m_Q$\cite{25,30}, although some papers\cite{31} claim to have done so. Other people\cite{32} used the light-front formalism to calculate the scattering amplitudes, so that in the heavy quark limit they obtained the Isgur-Wise function, in which they adopted the Gaussian-type wave function for the heavy mesons. There is a recent paper\cite{33} in which the authors use a "relativistic" formulation to calculate the masses of heavy mesons and apply it to calculate the Isgur-Wise functions for semileptonic $B$ decays. Although that paper adopts an approach that is different from ours, they present a method for calculating higher-order corrections in $1/m_Q$ to form factors, which is one of the purposes of this paper.

Most of the other calculations of form factors used the so-called trace formalism developed by Falk et al.\cite{19} which assumes the Lorentz covariance of the final form and fits it to the experimental data, or the QCD sum rule\cite{34} was used to obtain the relation with the quark/gluon condensates. In previous papers\cite{15} we developed a semi-relativistic formulation for calculating the meson masses and wave functions for the heavy-light $Q\bar{q}$ system with one heavy quark, $Q$, and one light antiquark, $\bar{q}$, introducing phenomenological dynamics. That is, assuming a Coulomb-like vector potential as well as a confining scalar potential to $Q\bar{q}$ bound states, we expanded the effective Hamiltonian and then perturbatively solved the Schrödinger equation as an expansion in $1/m_Q$. The meson wave functions thereby obtained and expanded in $1/m_Q$ can be used in principle to calculate ordinary form factors or Isgur-Wise functions and their corrections higher-order in $1/m_Q$ for semileptonic weak or other decay
processes. However, what we have obtained are wave functions in the rest-frame, and therefore we need to develop a method to obtain Lorentz-invariant amplitudes or Lorentz-boosted wave functions. In this paper we would like to address the problem of how accurately we can calculate scattering amplitudes and/or decay widths of heavy-light systems using the rest-frame wave functions. In the following, explaining the results obtained in Ref. [15], we show how to calculate the Lorentz-invariant amplitudes by taking the Isgur-Wise function as an example. This calculation is general enough to apply our formulation to other scattering/decay amplitudes.

Following a previous study, in which we derived the zeroth-order form of the Isgur-Wise function expanded in $1/m_Q$, we develop a formulation for calculating higher-order corrections to the semi-leptonic weak form factors in $1/m_Q$ from our semi-relativistic wave functions for heavy mesons in the rest-frame obtained in Ref. [15]. The problem involved in constructing a Lorentz-boosted wave function is that there is ambiguity in determining the space-time coordinates of two composite particles from information regarding one bound state, in this case a heavy meson. We study four cases for the reference frames of composite particles and then give a prescription for calculating matrix elements of currents using the rest-frame wave functions in §2. In §3 we give the form factors at zeroth order for four reference frames, show that they agree with each other in the HQET limit, and then obtain them at first-order for one special reference frame. The calculation to lowest order in $1/m_Q$ gives the numerical value of the slope for the Isgur-Wise function at the origin, and the semi-leptonic weak form factors are calculated up to first-order in $1/m_Q$ in §4. It is found that there are no dynamical contributions to the form factors; i.e., the first-order corrections to the wave functions do not contribute to the form factors. In §4 studying $\bar{B} \rightarrow D\ell\nu$ and $\bar{B} \rightarrow D^*\ell\nu$ processes, physical quantities related to the CKM matrix elements are obtained. Finally, §5 is devoted to summary and discussions of the results obtained in this paper. All the details associated with calculations of the form factors are given in the appendices.

§2. Formulation

2.1. Schrödinger equation

In Ref. [15], we calculated the mass spectrum of heavy mesons in a so-called Cornell potential model, which includes both scalar and vector potentials. Although we needed only a stationary system, or rest-frame wave function there, we must now treat a moving frame of a heavy meson to calculate weak form factors. Although we cannot work in a fully relativistic system as far as the Hamiltonian formulation is concerned, we adopt the notation of the Nambu-Bethe-Salpeter equation as much as possible. First, we define a heavy meson ($X$) wave function composed of a light anti-quark, $q^c(x)$, and a heavy quark, $Q(y)$, as

$$
\langle 0| q^c_\alpha(t, \vec{x}) Q_\beta(t, \vec{y}) |X; P_X \{\ell\} \rangle = \psi_\ell^{X,\alpha\beta} ((0, \vec{x} - \vec{y}); P_X) e^{-iP_X \cdot \vec{y}}, \tag{1}
$$

where $\alpha$ and $\beta$ are Dirac indices of four-spinors, $P_{X\mu}$ is the heavy meson four-momentum, $\{\ell\}$ is a set of quantum numbers, and $P_X \cdot y = P_{X\mu} y^\mu$. Note that this
definition of the wave function, $\psi_X^\ell$, equals to that used in Ref. [15] multiplied by -1. The heavy meson state is an eigenstate of the four-momentum operator $P_\mu$ given by

$$P_\mu |X; P_X \{\ell\} = P_{X\mu} |X; P_X \{\ell\}, \quad P_{X0} = \sqrt{M_X^2 + \vec{P}_X^2},$$

(2)

with $M_X$ a heavy meson mass. In this case, the Schrödinger equation in the $X$ moving frame is given by

$$H \psi_X^\ell ((0, \vec{x} - \vec{y}); P_X) = \sqrt{M_X^2 + \vec{P}_X^2} \psi_X^\ell ((0, \vec{x} - \vec{y}); P_X),$$

(3)

where the Hamiltonian is given by

$$H = (\vec{\alpha}_q \cdot \vec{p} + \beta_q m_q) + \left[ \vec{\alpha}_Q \cdot (\vec{P}_X - \vec{p}) + \beta Q m_Q \right] + \sum_{i,j} \beta_q O_{qi} V_{i,j} (\vec{x} - \vec{y}) \beta Q O_{Qj}. \quad (4)$$

Here, $\vec{p} = -i\nabla_x$ and $\vec{P}_X$ is the heavy meson momentum. In our case, Eq. (4) gives $O_{qi} = O_{Qj} = 1$ for the scalar potential, $V_{i,j}(\vec{x} - \vec{y}) = S(r)$ and $O_{qi} = \gamma_{q\mu}$ and $O_{Qj} = \gamma_Q^\mu$ for the vector potential, $V_{i,j}(\vec{x} - \vec{y}) = V(r)$ with $r = |\vec{x} - \vec{y}|$. Actually, we have included the transverse part of the vector potential and the total potential in Eq. (4) is given by

$$\beta_q \beta Q S(r) + \left\{ 1 - \frac{1}{2} \left[ \vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n}) \right] \right\} V(r),$$

(5)

with

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4 \alpha_s}{3 r}, \quad \vec{n} = \frac{\vec{r}}{r}. \quad (6)$$

In Ref. [15], we solved the Schrödinger equation given by Eq. (3) with $\vec{P}_X = \vec{0}$ order by order in $1/m_Q$ by expanding $H$, $\psi_X^\ell$, and $M_X$ in $1/m_Q$, in order to numerically calculate the mass spectrum, i.e., an eigenvalue, $M_X$, in each order of perturbation, and then to relate the results to those of HQET.

2.2. Lorentz boost

In order to calculate the decay rate, we need to Lorentz boost the rest-frame wave function that we obtained in previous papers. To explain the idea, we consider the $B \to D^{(*)}\ell\nu$ decay process as an example and express the wave function in the moving frame in terms of that in the rest-frame.

Assume that the heavy meson is moving in the $+z$ direction with a velocity $\beta$. Below, we consider two cases of composite particles, i.e., $q^c$ and $Q$. In the following, for the initial state of a heavy meson, we adopt the notation $q^c(x^0, \vec{x})$ and $Q(y^0, \vec{y})$ and for the final state, $q^c(x'^0, \vec{x}')$ and $Q(y'^0, \vec{y}')$. The relation between the constituent particles in the rest and moving frames is given by

$$G q_\alpha^c (x^0, \vec{x}) G^{-1} = G_{\alpha\beta}^{-1} q^c_\beta (x'^0, \vec{x}'), \quad (7)$$

$$G Q_\alpha (y^0, \vec{y}) G^{-1} = G_{\alpha\beta}^{-1} Q_\beta (y'^0, \vec{y}'), \quad (8)$$
where $G$ is the Lorentz-boost operator, $G$ is its spinor representation, and their definitions are given by

\begin{align}
G \left| X; \left( M_X, \vec{0} \right) \{ \ell \} \right> &= | X; \left( P_X \{ \ell \} \right> \\
G &= \cosh \frac{\varphi}{2} + \alpha^3 \sinh \frac{\varphi}{2}, \quad \beta = \tanh \varphi, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \cosh \varphi.
\end{align}

(9)

(10)

We have assumed that the heavy meson is boosted in the $+z$ direction. Here, we have $\vec{\alpha} = \gamma^0 \vec{\gamma}$, and $\beta$ and $\gamma$ are the velocity and Lorentz factor of the heavy meson, respectively, which can be expressed in terms of one parameter, $\varphi$.

Even if two constituent quarks have equal times in one frame, they have different times in another frame. In order to pull back the time of the heavy quark to that of the light quark, we make an approximation that the heavy quark propagates freely over a short time interval, assuming

\[ Q_\beta \left( t + \delta t, \vec{y} \right) \simeq \exp \left( -i m_Q \gamma \beta x^3 \right) Q_\beta \left( t, \vec{y} \right). \]

(11)

Below we consider two cases. In the first case, the two constituent quarks have equal time in the rest-frame of the meson, and we apply the above approximation to the heavy quark in the moving frame. In the second case, the two constituents have equal times in the moving frame of the meson, and we apply the approximation to the heavy quark in the rest-frame. In these cases, different relations are derived between the wave function of the meson with finite momentum and that of the meson at rest. These are as follows:

i) $t = x^0 = y^0$ and $x^0 \neq y^0$ (equal time in the rest-frame)

\begin{align}
& x^3 = x^3 \cosh \varphi + t \sinh \varphi, \quad x^0 = t \cosh \varphi + x^3 \sinh \varphi, \\
& y^3 = y^3 \cosh \varphi + t \sinh \varphi, \quad y^0 = t \cosh \varphi + y^3 \sinh \varphi.
\end{align}

(12)

In this case, an approximate relation between the rest and moving frame wave functions is given by

\[ \psi^f_{X\alpha\beta} \left( (0, \vec{x}) \; P_X \right) \simeq G_{\alpha \gamma} G_{\beta \delta} \psi^f_{X\gamma\delta} \left( (0, \vec{x}_\perp, \gamma^{-1} x^3) \; \left( M_X, \vec{0} \right) \right) e^{i(M_X - m_Q) \gamma \beta x^3}. \]

(13)

ii) $t' = x^0 = y^0$ and $x^0 \neq y^0$ (equal time in the moving frame)

\begin{align}
& x^3 = x^3 \cosh \varphi - t' \sinh \varphi, \quad x^0 = t' \cosh \varphi - x^3 \sinh \varphi, \\
& y^3 = y^3 \cosh \varphi - t' \sinh \varphi, \quad y^0 = t' \cosh \varphi - y^3 \sinh \varphi.
\end{align}

(14)

In this case, an approximate relation between the rest and moving frame wave functions is given by

\[ \psi^f_{X\alpha\beta} \left( (0, \vec{x}) \; P_X \right) \simeq G_{\alpha \gamma} G_{\beta \delta} \psi^f_{X\gamma\delta} \left( (0, \vec{x}_\perp, \gamma x^3) \; \left( M_X, \vec{0} \right) \right) e^{i(M_X - m_Q) \gamma \beta x^3}. \]

(15)

The derivation of the Lorentz boosted wave functions given by Eqs. (13) and (15) is given in Appendix B.
2.3. Wave function

The explicit form of the wave function we obtained in the previous paper has the form

\[
U_X(p)\psi_X^f(\vec{x}) = \psi_{X \text{FWT}}^f(\vec{x}), \quad (16a)
\]

\[
U_X(p) \equiv U_c U_{\text{FWT}}(p), \quad \psi_X^f(\vec{x}) \equiv \psi_X^f\left((0, \vec{x}); (M_X, \vec{0})\right), \quad (16b)
\]

\[
\psi_{X \text{FWT}}^f(\vec{x}) = \psi_{X0}^f(\vec{x}) + \psi_{X1}^f(\vec{x}) + \ldots, \quad (16c)
\]

\[
\psi_{X0}^f(\vec{x}) = \Psi_+^f(\vec{x}) \equiv \sqrt{M_X} \begin{pmatrix} 0 & \Psi_{j+}^k \end{pmatrix}. \quad (16d)
\]

We have solved the Schrödinger equation in terms of the wave function \(\psi_{X \text{FWT}}^f\), not \(\psi_X^f\). Here, the \(4 \times 4\) wave function, \(\psi_X^f\), is transformed just once with the Foldy-Wouthuysen-Tani transformation, \(U_{\text{FWT}}\), acting on the heavy quark and with the charge conjugation operator, \(U_c\), into \(\psi_X^f\), and its \(i\)-th order term in \(1/m_Q\) is given by \(\psi_{X,i}^f\). Higher-order corrections depend on the positive and negative components of the heavy quark, \(\Psi_{+}^f\), which are given by

\[
\Psi_{+}^f(\vec{x}) \equiv \sqrt{M_X} \begin{pmatrix} 0 & \Psi_{j+}^k \end{pmatrix}, \quad \Psi_{-}^f(\vec{x}) \equiv \sqrt{M_X} \begin{pmatrix} \Psi_{j-}^k & 0 \end{pmatrix}. \quad (17)
\]

Here, \(\vec{p} = \vec{p}_Q\) appearing in the argument of \(U_X(p)\) above is the initial momentum of the heavy quark, and henceforth, the color index \((N_c = 3)\) is omitted, because the form of the wave function is the same for all colors, \(\ell\) stands for the set of quantum numbers, \(j, m, k\), and \(\ell\), and the positive/negative component wave functions, \(\Psi_{\pm}^f\), are given in terms of a \(4 \times 2\) wave function, \(\Psi_{j \pm}^f(\vec{x})\), as

\[
\Psi_{j \pm}^k(\vec{x}) = \frac{1}{r} \begin{pmatrix} u_k(r) \\ -i v_k(r) (\vec{n} \cdot \vec{\sigma}) \end{pmatrix} y_{j \pm}^k(\Omega), \quad (18)
\]

where \(r = |\vec{x}|\), \(\vec{n} = \vec{x}/r\), \(j\) is the total angular momentum of the meson, \(m\) is its \(z\) component, \(k\) is a quantum number that takes only the values \(k = \pm j\), \(\pm (j + 1)\) and \(\neq 0\). (More details are given in Appendix C)

We showed in the previous paper that \(\Psi_{+}^f(\vec{x})\) is an eigenstate of the operator \(K \equiv -\beta_q (\vec{\Sigma}_q \cdot \vec{\ell} + 1)\), with eigenvalue \(k\), and we classified the spectra in terms of \(k\). People normally classify the spectra in terms of \(s_{\ell}P\), which is defined to be an eigenvalue \(s_{\ell}(s_{\ell} + 1)\) of the operator \((\vec{s}_{\ell})^2\) \(^{30}\) together with parity, \(P\). We can show the following direct relation among \(k\), \(s_{\ell}\), and \(P\); i.e., \(s_{\ell}\) and \(P\) can be explicitly written only in terms of \(k\) as \(^{37}\)

\[
k = \pm \left(s_{\ell} + \frac{1}{2}\right) \quad \text{or} \quad s_{\ell} = |k| - \frac{1}{2}, \quad (19a)
\]

\[
P = \frac{k}{|k|} (-1)^{|k|+1}, \quad (19b)
\]

where we have definitions

\[
\vec{s}_{\ell} \equiv \vec{\ell} + \frac{1}{2} \vec{\Sigma}_q, \quad (\vec{s}_{\ell})^2 = \vec{\ell}^2 + \vec{\ell} \cdot \vec{\Sigma}_q + \frac{3}{4}, \quad \vec{\ell} = -i \vec{r} \times \vec{\nabla}, \quad \vec{\Sigma}_q = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}. \quad (20)
\]
The corresponding physical quantities between $k$ and $s_{\ell}$ and the first few states are listed in Tables I and II respectively.

### Table I. Corresponding values for $k$, $s_{\ell}$, and $P$.

| $P$   | $(-)^{j+1}$ | $(-)^{j+1}$ | $(-)^{j+1}$ | $(-)^{j+1}$ |
|-------|-------------|-------------|-------------|-------------|
| $k$   | $(-j+1)$    | $j+1$      | $-j$        | $j$         |
| $s_{\ell}$ | $j+\frac{1}{2}$ | $j+\frac{1}{2}$ | $j-\frac{1}{2}$ | $j-\frac{1}{2}$ |
| $\Psi_{j}^{+}$ | $\Psi_{j}^{-(j+1)}$ | $\Psi_{j}^{j+1}$ | $\Psi_{j}^{-j}$ | $\Psi_{j}^{j}$ |

### Table II. States classified with respect to various quantum numbers.

| $j$   | $0^-$ | $1^-$ | $0^+$ | $1^+$ | $1^-$ | $2^+$ | $2^-$ | $2^+$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $k$   | 0     | 1     | 0     | 1     | 1     | 0     | 1     | 1     |
| $s_{\ell}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $\Psi_{j}^{0}$ | $\Psi_{j}^{1}$ | $\Psi_{j}^{-1}$ | $\Psi_{j}^{2}$ | $\Psi_{j}^{-2}$ | $\Psi_{j}^{3}$ | $\Psi_{j}^{-3}$ | $\Psi_{j}^{4}$ | $\Psi_{j}^{-4}$ |

From Tables I and II, it is seen that there are both advantages and disadvantages of using these quantities to classify the heavy mesons. The quantity $s_{\ell}^{P}$ has an intuitive meaning that it represents the total angular momentum of the light degrees of freedom, while $k$ does not. However, $k$ includes information concerning parity, while $s_{\ell}$ does not. Hence we need to add $P$, writing it as $s_{\ell}^{P}$. Certainly either one can be used to classify the heavy meson spectra equally well; i.e., they are equivalent.

### 2.4. Normalization

Normalization of the state $|X; P_{X} \{\ell\}|$ is formally given by

$$\langle X; P_{X} \{\ell\} | X; P_{X} \{\ell\} \rangle = (2\pi)^{3} 2P_{X0} \delta^{3} \left( \vec{P}_{X} - \vec{P}_{X}' \right) \delta_{\ell' \ell}$$

or

$$\int d^{3}z \text{tr} \left[ \psi_{X}^{\ell'\dagger} \left( 0, \vec{z} ; P_{X} \right) \psi_{X}^{\ell} \left( 0, \vec{z} ; P_{X} \right) \right] \simeq 2P_{X0} \delta_{\ell' \ell},$$

whose derivation is given in Appendix B. Actually, the normalization given by Eq. (22) does not hold, because of approximations adopted in the previous subsection, and hence, instead of Eq. (22), we define the normalization in the rest-frame as follows:

$$\int d^{3}z \text{tr} \left[ \psi_{X}^{\ell'\dagger} \left( 0, \vec{z} ; \left( M_{X}, \vec{0} \right) \right) \psi_{X}^{\ell} \left( 0, \vec{z} ; \left( M_{X}, \vec{0} \right) \right) \right] = 2M_{X} \delta_{\ell' \ell}.$$  \hspace{1cm} (23)

In order to calculate the normalization in a moving frame, we have to specify the quantum numbers in which we are interested. Because we would like to compute the Isgur-Wise function and its higher orders, the cases of the pseudoscalar state $0^-$ and vector state $1^-$ are calculated in the following.

The normalization condition given by Eq. (23) can be rewritten in terms of $\psi_{X \text{FWT}}(\vec{x})$ as

$$\int d^{3}x \text{tr} \left[ \psi_{X \text{FWT}}^{\ell'\dagger} (\vec{x}) \psi_{X \text{FWT}}^{\ell} (\vec{x}) \right] = 2M_{X} \delta_{\ell' \ell}.$$  \hspace{1cm} (24)
All the details concerning the wave functions are given in Appendix C.

In order to calculate the normalization of the wave function in a moving frame and/or the matrix elements of the semi-leptonic decay processes, we need to develop a formulation of their calculations in terms of the rest-frame wave functions. Let us assume that a physical quantity is already given in terms of the rest-frame wave functions and rewrite them in terms of the FWT-transformed ones as

\[
\int d^3 x \left( O_q \right)_{\alpha \alpha'} (O_Q)^{\beta \beta'} A^{\ast \ast} p_{\beta \delta} \psi_{X' \gamma \delta} A_{\alpha' \gamma'} B_{\beta \delta'} \psi_{X \gamma \delta} = \int d^3 x \text{tr} \left( \psi_{X' \F W T}^\dagger A^\dagger \psi_{X \F W T} B^* \right)
\]

\[
= \int d^3 x \text{tr} \left[ \psi_{X' \F W T}^\dagger A^\dagger \psi_{X \F W T} U_X^{-1} p_{Q} B^* U_{X'}(p_{Q}') \right], \tag{25}
\]

where \((O_q)^{\alpha \alpha'}\), \(A^\ast\), and \(B_{\beta \delta}\) act on light quarks, while \((O_Q)^{\beta \beta'}\), \(B_{\beta \delta}'\) and \(B_{\beta \delta'}\) act on heavy quarks, and use has been made of

\[
\psi_{X} = U_{X}^{-1}(p_{Q}) \otimes \psi_{X \F W T}(p_{Q}) = \psi_{X \F W T} U_{X}^{-1}(p_{Q}), \quad \psi_{X'} = \psi_{X' \F W T} U_{X'}^{-1}(p_{Q}'),
\]

\[
U_X^{-1}(p_{Q}) B^T Q_{O} B^* U_{X'}(p_{Q}') = U_{X \F W T}(p_{Q}) U_{c \F W T}^{-1}(p_{Q}) B^T Q_{O} B^* U_{c \F W T} U_{X \F W T}(p_{Q}'). \tag{26}
\]

In the course of obtaining relativistic results from the rest-frame wave functions, we calculate the normalization of the wave function in a moving frame in two extreme cases, i) \(t = x^0 = y^0\) and ii) \(t' = x^0 = y^0\). The FWT-transformed rest-frame wave function is given by, to first order in \(1/m_Q\),

\[
\psi_{X \F W T}^\dagger (0^-) = \Psi_{-1}^+ + c_{-1}^{-1,1} \Psi_{1}^- + O \left( 1/m_Q^2 \right), \tag{27a}
\]

\[
\psi_{X \F W T}^\dagger (1^-) = \Psi_{-1}^+ + c_{1}^{-1,2} \Psi_{2}^+ + c_{-1}^{-1,1} \Psi_{1}^- + c_{-1}^{-1,-2} \Psi_{-2}^- + O \left( 1/m_Q^2 \right), \tag{27b}
\]

where only the value of the \(k\) quantum number is written as a subscript of \(\Psi_{\pm}^j\). The total angular momentum, \(j\), though not given explicitly, should be the same on both sides of the equations and the coefficients, \(c_{1 \pm}^{k \pm}\), are given in \([15]\). Their explicit expressions are not necessary here, since we show that the higher-order corrections do not contribute to the physical quantities at this order, \(O(1/m_Q)\). Actually, Eq. (27) can be derived from the conservation of total angular momentum and parity alone, without explicit interaction terms specified, since we know the complete set of eigenfunctions with \(j\), \(m\), and \(k\) quantum numbers. (Details are given in Appendix C.)

We calculate the normalization up to first-order in \(1/m_Q\) below.

i) \(t = x^0 = y^0\) and \(x^0 \neq y^0\) (equal time in the rest-frame)

Using Eqs. (13), (24) and (25), we obtain, to first-order in \(1/m_Q\) and both for \(0^-\) and \(1^-\) states given by Eq. (27),

\[
\int d^3 x \text{tr} \left[ \psi_{X}^\dagger ((0, \bar{x}); P_{X}) \psi_{X} ((0, \bar{x}); P_{X}) \right] = 2 M_X \gamma^3 + O \left( 1/m_Q^2 \right). \tag{28}
\]

ii) \(t' = x^0 = y^0\) and \(x^0 \neq y^0\) (equal time in the moving frame)
Similarly using Eq. (13), (24) and (25), we obtain
\[
\int d^3 x \; \text{tr} \left[ \psi_X^\dagger ((0, \vec{x}) ; P_X) \; \psi_X^\dagger ((0, \vec{x}) ; P_X) \right] = 2 M_X \gamma + O \left( 1/m_Q^2 \right).
\]  
(29)

Only this case agrees with the relativistic normalization given by Eq. (22).

2.5. Matrix elements

The light anti-quark is treated as a spectator; that is, it does not interact with other particles, except for gluons represented by potentials in our model. The heavy quark has a weak vertex, and its current is in general given by
\[
j(t, \vec{x}) = Q^\dagger_{X', \alpha}(t, \vec{x}) \; O_{\alpha \beta} \; Q_{X \beta}(t, \vec{x}),
\]  
(30)

where \( O \) is a \( 4 \times 4 \) matrix. By inserting the number operator of the anti-quark and by assuming vacuum dominance among intermediate states, we obtain the formula for the matrix element of the above current, Eq. (30), between heavy mesons:
\[
\langle X'; P_{X'}\{\ell'\} | j(t, \vec{x}) | X; P_X \{\ell\} \rangle 
\simeq \int d^3 y \; \text{tr} \left[ \psi_{X'}^\dagger ((0, \vec{y} - \vec{x}) ; P_{X'}) \left( O \otimes \psi_X^\dagger ((0, \vec{y} - \vec{x}) ; P_X) \right) \right] \\
\times e^{-i(P_X - P_{X'})x}
\]  
(31)

or
\[
\langle X'; P_{X'}\{\ell'\} \left| j(0, \vec{0}) \right| X; P_X \{\ell\} \rangle 
\simeq \int d^3 x \; \text{tr} \left[ \psi_{X'}^\dagger ((0, \vec{x}) ; P_{X'}) \; \psi_X^\dagger ((0, \vec{x}) ; P_X) \; O^\dagger \right].
\]  
(32)

Now we would like to evaluate the matrix element of the process \( \bar{B} \to D^{(*)} \ell \bar{\nu}_\ell \) in two frames, the \( \bar{B} \) rest-frame and the Breit frame. (The velocities of \( \bar{B} \) and \( D \) have the same magnitude and have the opposite directions.) In each frame, we apply two approximate relations, Eqs. (12) and (14). As mentioned in the previous subsection, the normalization condition Eq. (22) of the wave function holds in the approximation ii) (equal time in the moving frame) but not in the approximation i) (equal time in the rest-frame). In the approximation i), we need to renormalize the matrix elements by normalizing the wave function, and we redefine the matrix elements of the current as follows:
\[
\langle D^{(*)} | j(0, \vec{0}) | \bar{B} \rangle 
= \frac{\sqrt{2 P_{B_{0}} \cdot 2 P_{D^{(*)}_0}}} {\sqrt{\int d^3 x \; \text{tr} \left[ \psi_{D^{(*)}}^\dagger ((0, \vec{x}) ; P_{D^{(*)}}) \; \psi_{\bar{B}}^\dagger ((0, \vec{x}) ; P_{\bar{B}}) O^\dagger \right]}} \\
\times \frac{\int d^3 x \; \text{tr} \left[ \psi_{D^{(*)}}^\dagger ((0, \vec{x}) ; P_{D^{(*)}}) \right]^2} {\int d^3 x \; \text{tr} \left[ \psi_{\bar{B}}^\dagger ((0, \vec{x}) ; P_{\bar{B}}) \right]^2}.
\]  
(33)

The numerator here is calculated below using the results of §2.2 while the denominator is calculated in §2.4 and given by Eqs. (28) and (29).
In this case, the matrix element is given by, with our approximations,

\[
\int d^3 x \tr \left[ \psi^\dagger_{D^{(*)}}((0, \vec{x}); P_{D^{(*)}}) \psi^F_B((0, \vec{x}); P_B) \mathcal{O}^T \right] \\
= \int d^3 x G^*_{\gamma \beta} G^{\gamma} \delta^\alpha \delta^\epsilon \psi^F_{D^{(*)}}(0, \vec{x}, \gamma^{-1} x^3; (M_{D^{(*)}}, 0)) \mathcal{O} \psi^F_B(\vec{x}) \\
\times e^{-i(M_{D^{(*)}} - m_e) \gamma \beta x^3}. 
\]

(34)

1-i) \( t = x^0 = y^0 \) for the \( D^{(*)} \) meson

In this case, the matrix element is given by

\[
\int d^3 x \tr \left[ \psi^\dagger_{D^{(*)}}((0, \vec{x}); P_{D^{(*)}}) \psi^F_B((0, \vec{x}); P_B) \mathcal{O}^T \right] \\
= \int d^3 x G^*_{\gamma \beta} G^{\gamma} \delta^\alpha \delta^\epsilon \psi^F_{D^{(*)}}(0, \vec{x}, \gamma^{-1} x^3; (M_{D^{(*)}}, 0)) \mathcal{O} \psi^F_B(\vec{x}) \\
\times e^{-i(M_{D^{(*)}} - m_e) \gamma \beta x^3}. 
\]

(35)

1-ii) \( t' = x^0 = y^0 \) for the \( D^{(*)} \) meson

In this case, the matrix element is given by

\[
\int d^3 x \tr \left[ \psi^\dagger_{D^{(*)}}((0, \vec{x}); P_{D^{(*)}}) \psi^F_B((0, \vec{x}); P_B) \mathcal{O}^T \right] \\
= \gamma \int d^3 x \psi^F_{D^{(*)}}(\vec{x}) \left( G^T \mathcal{O}^{-1} \right)_{\alpha \beta} \psi^F_B(\vec{x}) \\
\times \exp \left[ -\frac{i}{2} (M_{D^{(*)}} - m_e + M_B - m_b) \gamma^2 \beta x^3 \right]. 
\]

(36)

2) Breit frame

2-i) \( t = x^0 = y^0 \) for the \( \bar{B} \) and \( D^{(*)} \) mesons

In this case, the matrix element is given by

\[
\int d^3 x \tr \left[ \psi^\dagger_{D^{(*)}}((0, \vec{x}); P_{D^{(*)}}) \psi^F_B((0, \vec{x}); P_B) \mathcal{O}^T \right] \\
= \gamma^{-1} \int d^3 x \psi^F_{D^{(*)}}(\vec{x}) \left( G^T \mathcal{O}^{-1} \right)_{\alpha \beta} \psi^F_B(\vec{x}) \\
\times \exp \left[ -\frac{i}{2} (M_{D^{(*)}} - m_e + M_B - m_b) \beta x^3 \right]. 
\]

(37)

\section{Semi-Leptonic weak form factor}

Using the results derived in the previous section, we can now calculate semi-leptonic weak form factors including the Isgur-Wise function. In this case the currents are given by

\[
\begin{align*}
j_\mu &= \bar{c} \gamma^\mu b(t, \vec{x}), \\
j_5^\mu &= \bar{c} \gamma^\mu b(t, \vec{x}). \quad \text{(38a)}
\end{align*}
\]
Define the six independent semi-leptonic weak form factors as:

\[ \langle D | j_{\mu}(0,\vec{0}) | \bar{B} \rangle = \sqrt{M_B M_D} (\xi_+ (\omega) (v_B + v_D)_\mu + \xi_-(\omega) (v_B - v_D)_\mu), \]  

(39a)

\[ \langle D^* | j_{\mu}(0,\vec{0}) | \bar{B} \rangle = i \sqrt{M_B M_D} \xi_V (\omega) \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu v_D^\rho v_B^\sigma, \]  

(39b)

\[ \langle D^* | j_{\mu}(0,\vec{0}) | \bar{B} \rangle = \sqrt{M_B M_D} ((1 + \omega) \xi_{A_1}(\omega) \epsilon_\mu - \xi_{A_2}(\omega) (\epsilon^* \cdot v_B) v_B^\mu - \xi_{A_3}(\omega) (\epsilon^* \cdot v_B) v_D^\mu), \]  

(39c)

with

\[ v_X^\mu = P_X^\mu / M_X, \quad \text{for } X = \bar{B}, \quad (s), \quad \omega = v_B \cdot v_D. \]  

(40)

3.1. Zeroth order (Isgur-Wise function)

After some calculations, all the above form factors are found to be proportional to the Isgur-Wise function, \( \xi(\omega) \), or vanish and are given by

\[ \xi_+ (\omega) = \xi_V (\omega) = \xi_{A_1}(\omega) = \xi_{A_3}(\omega) = \xi(\omega), \]  

(41a)

\[ \xi_-(\omega) = \xi_{A_2}(\omega) = 0. \]  

(41b)

Below we only briefly show how to calculate \( \xi_+(\omega) = \xi(\omega) \) from Eq. (39a), since the other functions are obtained similarly. All the details are given in Appendix E. We define the Isgur-Wise function as

\[ \langle D | j_0(0,\vec{0}) | \bar{B} \rangle = \frac{\sqrt{2 P_{B_0}^2 P_{D_0}^2} \int d^3 x \text{ tr} \left[ \psi_D^{\ell \dagger} ((0,\vec{x}) ; P_D) \psi_{\bar{B}0}^{\ell} ((0,\vec{x}) ; P_{\bar{B}}) \right]}{\sqrt{\int d^3 x \text{ tr} \left| \psi_D^{\ell} ((0,\vec{x}) ; P_D) \right|^2 \int d^3 x \text{ tr} \left| \psi_{\bar{B}0}^{\ell} ((0,\vec{x}) ; P_{\bar{B}}) \right|^2}}, \]  

(42)

The expression Eq. (42) can be further reduced in two cases of the \( \bar{B} \) frame as follows.

1) \( \bar{B} \) rest-frame

In this frame, \( \bar{B} \) is at rest, \( D \) is moving in the +z direction with velocity \( \beta \) and \( \xi(\omega) \) is given by

\[ \xi(\omega) = \frac{\sqrt{2 \omega} \int d^3 x \text{ tr} \left[ \psi_D^{\ell \dagger} ((0,\vec{x}) ; P_D) \psi_{\bar{B}0}^{\ell} ((0,\vec{x}) ; P_{\bar{B}}) \right]}{\sqrt{M_B (1 + \omega) \int d^3 x \text{ tr} \left| \psi_D^{\ell} ((0,\vec{x}) ; P_D) \right|^2}}, \]  

(43)

with the relations

\[ P_{B_0} = M_B, \quad P_{D_0} = M_D v_{D0} = M_D \omega, \quad v_{B_0} = 1, \]

\[ v_{D0} = v_B \cdot v_D = \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \omega, \quad \beta = \frac{\sqrt{\omega^2 - 1}}{\omega}. \]  

(44)

2) Breit frame

In this frame, \( \bar{B} \) is moving in the +z direction, \( D \) in the −z with the same velocity \( \beta/2 \), and \( \xi(\omega) \) is given by

\[ \xi(\omega) = \frac{\int d^3 x \text{ tr} \left[ \psi_D^{\ell \dagger} ((0,\vec{x}) ; P_D) \psi_{\bar{B}0}^{\ell} ((0,\vec{x}) ; P_{\bar{B}}) \right]}{\sqrt{\int d^3 x \text{ tr} \left| \psi_D^{\ell} ((0,\vec{x}) ; P_D) \right|^2 \int d^3 x \text{ tr} \left| \psi_{\bar{B}0}^{\ell} ((0,\vec{x}) ; P_{\bar{B}}) \right|^2}}, \]  

(45)
with the relations
\[ P_{B0} = M_B \gamma, \quad P_{D0} = M_D \gamma, \quad v_{B0} = v_{D0} = \gamma = \frac{1}{\sqrt{1 - (\beta/2)^2}}, \]
\[ v_B \cdot v_D = \gamma^2 \left(1 + \frac{\beta^2}{4}\right) = \omega, \quad \gamma = \sqrt{\frac{\omega + 1}{2}}, \quad \frac{\beta}{2} = \sqrt{\frac{\omega - 1}{\omega + 1}}. \] (46)

We note that here and below we have used the same notation for the velocity and Lorentz factor both in the rest-frame and the Breit frame, although they are different quantities, as one can see from Eqs. \([14]\) and \([16]\).

We calculate this Isgur-Wise function in four cases below. The details are given in Appendix B.1.
1-i) $\bar{B}$ rest-frame and $t = x^0 = y^0$. In this case, we have
\[ \xi(\omega) = \frac{1}{\omega} - \frac{1}{6} \beta^2 \omega \bar{E}_D^2 \langle r^2 \rangle + \frac{\beta^2}{4} + O(\beta^4), \] (47)
where
\[ \langle r^2 \rangle = \int d^3x \text{tr} \left[ \psi_{X0}^{\dagger}(\vec{x}) r^2 \psi_{X0}(\vec{x}) \right], \quad \text{for } X = D, \text{ or } \bar{B}, \] (48)
both of which give the same result, since this is the lowest-order calculation.

1-ii) $\bar{B}$ rest-frame and $t' = x^0 = y^0$:
\[ \xi(\omega) = 1 - \frac{1}{6} \beta^2 \omega \bar{E}_D^2 \langle r^2 \rangle - \frac{\beta^2}{4} + O(\beta^4). \] (49)

2-i) Breit frame and $t = x^0 = y^0$:
\[ \xi(\omega) = \gamma^{-2} - \frac{1}{6} \left(\frac{\beta}{2}\right)^2 \gamma^2 (\bar{E}_B + \bar{E}_D)^2 \langle r^2 \rangle + O(\beta^4). \] (50)

2-ii) Breit frame and $t' = x^0 = y^0$:
\[ \xi(\omega) = \gamma^{-2} - \frac{1}{6} \left(\frac{\beta}{2}\right)^2 \gamma^{-2} (\bar{E}_B + \bar{E}_D)^2 \langle r^2 \rangle + O(\beta^4). \] (51)

In the above we have used
\[ \bar{E}_D \equiv M_D - m_c, \quad \bar{E}_B \equiv M_B - m_b. \] (52)

The rhs of Eq. (52) can be expanded in $1/m_Q$ as
\[ \bar{E}_X = M_X - m_Q = \left(m_Q + \sum_{i=0} C_{X}^{i} \left(\frac{m_Q}{m_Q}\right)^i\right) - m_Q = \sum_{i=0} C_{X}^{i} \left(\frac{m_Q}{m_Q}\right)^i, \] (53)
where the $i$-th order in $1/m_Q$ of $M_X$ is given by $C_{X}^{i}/(m_Q)^i$, and it is shown in Ref. \[15\] that $C_{X}^{i}$ depends only on the light quark mass $m_q$, and hence $C_{D}^{i} = C_{B}^{i} = C^{i}(m_q)$ in our case, since light quarks are either $u$ or $d$, and we have set $m_u = m_d$. Equation (53) is for the $0^-$ state. When it is written for the $1^-$ state, the coefficients are given by $C_{X}^{1}$. All the expressions obtained above have the same form, up to first-order in $\omega$ in the vicinity of $\omega = 1$, and up to the zeroth-order in $1/m_Q$, as given by
\[ \xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3} \tilde{A} \langle r^2 \rangle\right) (\omega - 1), \] (54)
\[ \tilde{A} = \lim_{m_Q \to \infty} \bar{E}_D = \lim_{m_Q \to \infty} \bar{E}_B = C^0, \] (55)
that is,
\[ \xi(1) = 1, \quad \xi'(1) = -\frac{1}{2} - \frac{1}{3} \bar{\Lambda}^2 \langle r^2 \rangle. \] (56)

The form for \( \xi(\omega) \) given by Eq. (54) is derived from our semi-relativistic formulation and coincides with those derived from other considerations, but we believe that our derivation is the most general and does not depend on any specific model. This form gives the lower bound \(-\frac{1}{2}\) for \( \xi'(1) \) or the upper bound \( \frac{1}{2} \) for \( -\xi'(1) \), as shown in Refs. [24], [27] and [29]. All the other form factors, \( \xi_-(\omega) \), \( \xi_V(\omega) \), and \( \xi_{Ai}(\omega) \) for \( i = 1 - 3 \), are similarly calculated and given by Eq. (41) up to this order, \( (1/mQ)^0 \).

Using the values for the parameters obtained in Ref. [15] to calculate the mass spectra of heavy mesons, we can calculate \( \xi'(1) \) to first and second orders in \( 1/mQ \) as
\[ \xi'(1) = -1.44, \] (57)
where we have used the values \( \bar{\Lambda} = 0.752 \) GeV and \( \langle r^2 \rangle = 5.009 \) GeV\(^{-2} \), taken from Ref. [16]. The values of \( \xi'(1) \) given in references are listed in Ref. [27].

3.2. First order

There is only one kind of the first-order correction, i.e., that from the wave functions. The first-order corrections to the form factors defined by Eq. (39) from the wave functions are straightforwardly calculated, although the calculation is cumbersome. Hence, here we do not repeat calculations similar to §3.1 and give only the final results below.

We have calculated all four cases, 1-i) - 2-ii). The results show that there are no contributions from the first-order corrections of the wave functions to the form factors. Although we attempted to obtain relativistically invariant results from knowledge of the rest frame by applying four different Lorentz-boost frames, the first-order corrections in all cases are not invariant, except one, the case 2-ii). The Breit frame with \( t' = x^0 = y^0 \) gives relativistic results that coincide with those of Ref. [21]. Hence, the following results could be a conjecture from our model, but we believe that our results are very plausible, since they agree with relativistic results which come from a semi-relativistic potential model. Brief derivations of these form factors are given in Appendix E.2.

As stressed in the Introduction, our approach does not use fields, and hence there is no other kind of first order, i.e., the currents cannot be expanded in \( 1/mQ \) in terms of the effective velocity-dependent fields, as in the HQET given in many papers as
\[ j_\mu = c^\dagger_v \gamma^0 \gamma_\mu b_v + c^\dagger_v \left( \frac{-i}{2m_c} \gamma^0 \gamma_\mu \vec{P} + \frac{i}{2m_b} \gamma^0 \gamma_\mu \vec{D} \right) b_v. \]

Six form factors including zeroth-order terms for completeness with the decomposition of Neubert and Rieckert [24] are given by
\[ \xi_+(\omega) = \left[ 1 + \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \rho_1(\omega) \right] \xi(\omega), \] (58a)
\( \xi_-(\omega) = \left[ -\frac{A}{2} + \rho_4(\omega) \right] \left( \frac{1}{m_c} - \frac{1}{m_b} \right) \xi(\omega), \) (58b)

\( \xi_V(\omega) = \left[ 1 + \frac{A}{2} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \rho_2(\omega) + \frac{1}{m_b} (\rho_1(\omega) - \rho_4(\omega)) \right] \xi(\omega), \) (58c)

\[ \xi_{A_1}(\omega) = \left[ 1 + \frac{A}{2} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \rho_2(\omega) + \frac{1}{m_b} \left( \rho_1(\omega) - \frac{\omega - 1}{\omega + 1} \rho_4(\omega) \right) \right] \xi(\omega), \] (58d)

\[ \xi_{A_2}(\omega) = \frac{1}{\omega + 1} \left[ -\frac{A}{\omega + 1} + \rho_3(\omega) - \rho_4(\omega) \right] \xi(\omega), \] (58e)

\[ \xi_{A_3}(\omega) = \left[ 1 + \frac{A}{2} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \rho_2(\omega) + \frac{1}{m_b} \left( \rho_1(\omega) - \rho_3(\omega) - \frac{1}{\omega + 1} \rho_4(\omega) \right) + \frac{1}{m_b} (\rho_1(\omega) - \rho_4(\omega)) \right] \xi(\omega), \] (58f)

where \( \xi(\omega) \) is given by Eq. (54) and

\[ \rho_1(\omega) = \rho_2(\omega) = -\frac{1}{3} C^1 \left( \langle r^2 \rangle \right) (\omega - 1), \] (59a)

\[ \rho_3(\omega) = \rho_4(\omega) = 0. \] (59b)

Thus, there are \( 1/m_Q \) corrections to \( \rho_1 \) and \( \rho_2 \) coming from phase factors of the wave functions, together with kinetic terms. (See Appendix E.2)

§4. CKM matrix element

4.1. \( \bar{B} \to D \ell \bar{\nu} \)

With the results given in §§3.1 and 3.2, we now calculate the CKM matrix element, \(|V_{cb}|\). We first evaluate the differential decay rate of the process \( \bar{B} \to D \ell \bar{\nu} \) to extract the value \(|V_{cb}|\). Note that since we have neglected the isospin invariance in our formulation, there is no distinction between \( \bar{B}_0 \) and \( B^- \) in the following. Similarly, \( D^{(*)} \) means either \( D \) or \( D^* \), which is equal to \( D^0 \) and \( D^+ \) or \( D^{*0} \) and \( D^{*+} \), respectively, in our formulation. The expression for this differential decay rate is given by

\[ \frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 M_D^3 (M_B + M_D)^2 \omega^2 (\omega^2 - 1)^{3/2} F_D(\omega)^2, \] (60)

where we have defined

\[ F_D(\omega) = \xi_+(\omega) - \frac{1}{1 + r} \xi_-(\omega), \quad r = \frac{M_D}{M_B}. \] (61)

Using the above expressions and the form factors obtained in the previous subsections, we can evaluate values of the form factor \( F_D(\omega) \) at zero recoil and its first derivative.
i) Zeroth-order in $1/m_Q$

In this case, since $\xi_+(\omega) = \xi_V(\omega) = \xi_{A_1}(\omega) = \xi_{A_3}(\omega) = \xi(\omega)$ and $\xi_-(\omega) = \xi_{A_2}(\omega) = 0$, given by Eq. (41), we have

$$\mathcal{F}_D(1) = \xi(1) = 1, \quad \mathcal{F}_D'(1) = \xi'(1) = -1.44.$$  \hspace{1cm} (62)

ii) First-order in $1/m_Q$

In this case, Eq. (58), together with Eq. (59) gives

$$\mathcal{F}_D(1) = \xi_+(1) - \frac{1-r}{1+r} \xi_-(1) = 1.07, \quad \mathcal{F}_D'(1) = -0.875.$$  \hspace{1cm} (63)

The combined recent experimental data of CLEO\textsuperscript{38} and Belle\textsuperscript{39} lead to the following value of the product of the form factor and the CKM matrix element:

$$\mathcal{F}_D(1) |V_{cb}| = 0.0414 \pm 0.0064.$$  \hspace{1cm} (64)

Using our prediction Eq. (63), we find the value of CKM matrix element to be

$$|V_{cb}| = 0.0387 \pm 0.0060.$$  \hspace{1cm} (64)

4.2. $\bar{B} \to D^* \ell \bar{\nu}$

As noted by Neubert\textsuperscript{40} the differential decay rate of the process $\bar{B} \to D^* \ell \bar{\nu}$ is the best quantity to extract the value $|V_{cb}|$. The expression for this differential decay rate is given by

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 M_{D^*}^2 (M_B - M_{D^*})^2 \sqrt{\omega^2 - 1(\omega + 1)^2} \times \left[ 1 + \frac{4\omega}{\omega + 1} \frac{1 - 2\omega r^* + r^{*2}}{(1 - r^*)^2} \right] \mathcal{F}_{D^*}(\omega)^2,$$  \hspace{1cm} (65)

where we have defined

$$(1 - r^*)^2 \left[ 1 + \frac{4\omega}{\omega + 1} \frac{1 - 2\omega r^* + r^{*2}}{(1 - r^*)^2} \right] \mathcal{F}_{D^*}(\omega)^2$$

$$= 2(1 - 2\omega r^* + r^{*2}) \left( \xi_{A_1}(\omega)^2 + \frac{\omega - 1}{\omega + 1} \xi_V(\omega)^2 \right) + \left\{ (\omega - r^*) \xi_{A_1}(\omega) - (\omega - 1) \xi_{A_3}(\omega) \right\}^2$$

$$= \left\{ 2(1 - 2\omega r^* + r^{*2}) \left( 1 + \frac{\omega - 1}{\omega + 1} R_1(\omega)^2 \right) + [(\omega - r^*) - (\omega - 1) R_2(\omega)]^2 \right\} \times \xi_{A_1}(\omega)^2,$$  \hspace{1cm} (66)

and

$$r^* = \frac{M_{D^*}}{M_B}, \quad R_1(\omega) = \frac{\xi_V(\omega)}{\xi_{A_1}(\omega)}, \quad R_2(\omega) = \frac{\xi_{A_1}(\omega) + r^* \xi_{A_3}(\omega)}{\xi_{A_1}(\omega)}.$$  \hspace{1cm} (67)

Similarly to (44), one can evaluate values of the form factor $\mathcal{F}_{D^*}(\omega)$ at zero recoil and its first derivative.
Table III. Comparison of our model predictions for the values of the product \( F_D^* |V_{cb}| \) with theoretical predictions and experimental data\(^{33}\)

| param. | our \( F_D^* |V_{cb}| \) | Ref. \( 33^\)   |
|--------|-----------------------------|----------------|
| \( F_D^* |V_{cb}| \) | 0.0380(21)               | 0.0343(12) |

| params. | CLEO\(^{41}\) | BaBar\(^{42}\) | Belle\(^{43}\) | DELPH\(^{44}\) |
|---------|----------------|---------------|---------------|---------------|
| \( F_D^* |V_{cb}| \) | 0.0376(3)(16)a  | 0.0343(13)(18)a | 0.0354(19)(18)a | 0.0377(11)(19)a |
|         | 0.0328(5)b     | 0.0360(20)b   | 0.0315(12)b   |               |

\(^{a}\) fit using the form factor \( \xi_{A_1} \) parameterization of the model\(^{19}\)

\(^{b}\) fit using the form factor predictions of the model\(^{33}\)

i) Zeroth-order in \( 1/m_Q \)

In this case, we have

\[
F_D^* (1) = \xi (1) = 1, \quad F_D^* ' (1) = \xi ' (1) = -1.44. \quad (68)
\]

ii) First-order in \( 1/m_Q \)

In this case, we have

\[
F_D^* (1) = \xi (1) = 1, \quad F_D^* ' (1) = -1.09, \quad (69a)
R_1 (1) = 1.45, \quad R_1 ' (1) = -0.222, \quad R_2 (1) = 0.942, \quad R_2 ' (1) = 0.0286, \quad (69b)
\]

where the first equation is consistent with the so-called Luke theorem\(^{21}\) and we have used \( m_c = 1.032 \text{ GeV} \), \( m_b = 4.639 \text{ GeV} \), and \( r^* = M_D^*/M_B = 0.3804 \) used in Ref. \(^{17}\) and \( C_1 = 0.19022 \) in Eq. \(^{69a}\). The estimated values given by Eqs. \(^{63}\) and \(^{69}\) in our paper are consistent with other theoretical values, except for \( R_2 ' (1) \), whose values are given in Ref. \(^{33}\). The value of \( R_2 ' (1) \) is one order of magnitude smaller than the other theoretical values, and there is yet no experimental data for it.

The combined fit of the experimental data from CLEO\(^{41}\) Belle\(^{43}\) and BaBar\(^{42}\) gives

\[
F_D^* (1) |V_{cb}| = 0.0380 \pm 0.0021.
\]

Using our value of \( F_D^* (1) = 1 \), we have

\[
|V_{cb}| = 0.0380 \pm 0.0021. \quad (70)
\]

\section*{§5. Summary and discussion}

In this paper, we have formulated a method for constructing Lorentz-invariant amplitudes by examining four kinds of Lorentz-boosted systems after normalizing their amplitudes. Although all four are not always consistent for higher orders in \( 1/m_Q \), they may be corrected by comparing with other methods, for instance, relativistic and kinematic results. Despite of this defect, we believe that this formulation certainly provides a method to calculate reliable amplitudes from wave functions in the rest frame.
Table IV. Comparison of theoretical predictions of for the ratios $R_1(1)$ and $R_2(1)$ and their derivatives, $R_1'(1)$ and $R_2'(1)$.

| Ref. | $R_1(1)$ | $R_2(1)$ | $R_1'(1)$ | $R_2'(1)$ |
|------|----------|----------|-----------|-----------|
| our  | 1.45     | -0.22    | 0.94      | 0.029     |
| [39] | 1.39     | -0.23    | 0.92      | 0.12      |
| [40] | 1.25     | -0.10    | 0.81      | 0.11      |
| [47] | 1.27     | -0.12    | 0.80      | 0.11      |
| [20] | 1.35     | -0.22    | 0.79      | 0.15      |
| [49] | 1.15     |          | 0.94      |           |
| [50] | 1.01(2)  | 1.04(1)  |           |           |

As an example of this formulation, we have calculated the semi-leptonic weak form factors up to first-order in $1/m_Q$ following a previous study and using the results obtained in that paper. In this paper, we have found the following results.

1. The Isgur-Wise function has the following form, up to first-order in $1/m_Q$ and in $(\omega - 1)$:

$$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3} \bar{\Lambda}^2 \langle r^2 \rangle \right) (\omega - 1), \quad \bar{\Lambda} = \bar{\Lambda}_u = \lim_{m_Q \to \infty} \bar{E}_D = \lim_{m_Q \to \infty} \bar{E}_B.$$ 

Hence, we have

$$\xi(1) = 1, \quad \xi'(1) = -\frac{1}{2} - \frac{1}{3} \bar{\Lambda}^2 \langle r^2 \rangle .$$

Here, since $\bar{\Lambda}$ depends only on the light quark mass and we treat only the heavy mesons $D, D^*$, and $B$, which include only $u$ and $d$ light quarks with $m_u = m_d$, the subscript of $\bar{\Lambda}_u$ expresses this fact.

2. We find that there is no contribution to the six form factors from correction terms at first-order in $m_Q$ for the rest-frame wave functions. That is, no terms, except for the first in Eq. (27), which include negative and positive energy states of a heavy quark contribute to the physical quantities.

3. The first-order corrections are derived from phase factors of the wave functions and also given by kinetic terms, and there are no contributions from the first-order corrections to the wave functions. They are explicitly given by Eqs. and . That is, in the terminology of Neubert and Rieckert in Ref. 21), we have

$$\rho_1(\omega) = \rho_2(\omega) = -\frac{1}{3} C^1 \bar{\Lambda} \langle r^2 \rangle (\omega - 1), \quad \rho_3(\omega) = \rho_4(\omega) = 0.$$ 

4. We have calculated the values for the form factor $F(\omega)$ at zero recoil and/or their first derivatives up to first-order in $1/m_Q$ as

$$F_D(1) = 1.07, \quad F_D'(1) = -0.875,$$

$$F_{D^*}(1) = \xi(1) = 1, \quad F_{D^*}'(1) = -1.09,$$

$$R_1(1) = 1.45, \quad R_1'(1) = -0.222, \quad R_2(1) = 0.942, \quad R_2'(1) = 0.0286.$$ 

The first equations here were obtained by analyzing the $\bar{B} \to D \ell \bar{\nu}$ process, and the second by analyzing $\bar{B} \to D^* \ell \bar{\nu}$. These values are consistent with
experimental data as well as other theoretical estimates listed in Tables I and II of Ref. [33].

5. The above values can be used to estimate the CKM matrix element $|V_{cb}|$, which we have obtained as

$$|V_{cb}| = 0.0387 \pm 0.0060$$

for the $\bar{B} \to D\ell\nu$ process, and

$$|V_{cb}| = 0.0380 \pm 0.0021$$

for the $\bar{B} \to D^*\ell\nu$ process. These values are consistent with the value in PDG,[51]

$$|V_{cb}| = 0.0409 \pm 0.0018 \quad \text{(exclusive)}.$$  

Here to obtain theoretical predictions for $|V_{cb}|$, we have ignored theoretical uncertainties, although experimental errors are taken into account.

We have developed a method to obtain relativistically invariant results using rest-frame wave functions, and to do this, four different Lorentz-boosted frames were adopted to check the validity of our results. The same form is obtained for the Isgur-Wise function in all four cases up to zeroth-order in $1/m$, and first-order in $(\omega - 1)$. However, the first-order corrections in $1/m$ are not the same in all four cases. Only the case denoted by 2-ii) in the main text, i.e., in the Breit frame with $t' = x^0 = y^0$ both for $\bar{B}$ and $D^*$, gives results consistent with the relativistic ones given in Ref. [21].

---

**Appendix A**

**Schrödinger Equation**

The Schrödinger equation given by Eq. (3) is derived by calculating

$$\langle X'; P_X \{\ell'\} | H | X; P_X \{\ell\} \rangle = \langle X; P_X \{\ell\} | P^0 | X; P_X \{\ell\} \rangle,$$  

(A.1)

with $P^0$ being the 0-th component of the four-momentum $P^\mu$, and by varying with respect to $\psi^\ell_{X'}$. Here the Hamiltonian density is given by

$$\mathcal{H} = \int d^3x \left\{ q^c(\bar{\alpha} \cdot \bar{p} + \beta_q m_q)q^c(\bar{\alpha} \cdot \bar{p} + \beta_q m_q)Q(\bar{x}) \right\}$$

$$+ \int d^3xd^3y q^c(\bar{\alpha} \cdot \bar{p} + \beta_q m_q)Q(\bar{x} - \bar{y})Q(\bar{y}) \beta Q_Q Q(\bar{y}).$$  

(A.2)

The lhs of Eq. (A.1) can be approximated as

$$\langle X'; P_X \{\ell'\} | H | X; P_X \{\ell\} \rangle$$

$$= \left\langle X; P_X \{\ell\} \left| \int d^3x \int d^3y Q^c_\alpha(t, \bar{x}) q^c_\beta(t, \bar{x}) Q^c_\beta(t, \bar{y}) Q_Q(t, \bar{y}) \mathcal{H} \right| X; P_X \{\ell\} \right\rangle$$

$$\simeq \int d^3x \int d^3y \left\langle X'; P_X \{\ell'\} \left| Q^c_\alpha(t, \bar{x}) q^c_\beta(t, \bar{x}) \right| 0 \right\rangle$$

$$\times \left\langle 0 \left| q^c_\gamma(t, \bar{x}) Q_\gamma(t, \bar{y}) Q_Q(t, \bar{y}) \mathcal{H} Q_Q^c(t, \bar{y}) q^c_\gamma(t, \bar{x}) Q_\gamma(t, \bar{y}) \right| X; P_X \{\ell\} \right\rangle$$
\[ \simeq \int d^3x \int d^3y \psi^\ell_{X', \alpha \beta} ((0, \bar{x} - \bar{y}); P_{X'}) \ H_{\alpha \gamma, \beta \delta} \psi^\ell_{X, \gamma \delta} ((0, \bar{x} - \bar{y}); P_X) \]
\[ \times e^{-i(P_X - P_{X'}) \cdot y}, \quad (A.3) \]

where the number operators for \( q^c \) and \( Q \) have been inserted, and the vacuum dominance has been used among the intermediate states. The rhs of Eq. (A.3) is given by

\[ \langle X'; P_{X'} \{ \ell' \} | P^0 | X; P_X \{ \ell \} \rangle = \sqrt{M_X^2 + \vec{P}_X^2} \langle X'; P_{X'} \{ \ell' \} | X; P_X \{ \ell \} \rangle, \quad (A.4) \]

and thus we obtain the Schrödinger equation, Eq. (3).

**Appendix B**

**Lorentz Boost and Normalization**

The derivations of Eqs. (13), (15) and (22) are similar to those given in Eq. (A.3), we obtain

\[ \langle X; P_{X'} \{ \ell' \} | X; P_X \{ \ell \} \rangle \]
\[ \simeq \int d^3x \int d^3y \psi^\ell_{X', \alpha \beta} ((0, \bar{x} - \bar{y}); P_{X'}) \psi^\ell_{X, \alpha \beta} ((0, \bar{x} - \bar{y}); P_X) \ e^{-i(P_X - P_{X'}) \cdot y} \]
\[ = (2\pi)^3 \delta^3 \left( \vec{P}_{X'} - \vec{P}_X \right) \int d^3z \ tr \left[ \psi^\ell_{X}((0, \bar{z}); P_X) \psi_{X'}^\dagger((0, \bar{z}); P_X) \right]. \quad (B.1) \]

The lhs is given by

\[ \langle X; P_{X} \{ \ell \} | X; P_X \{ \ell \} \rangle = (2\pi)^3 2P_{X_0} \delta^3 \left( \vec{P}_X - \vec{P}'_X \right) \delta_{\ell' \ell}, \quad (B.2) \]

and hence

\[ \int d^3z \ tr \left[ \psi^\ell_{X}((0, \bar{z}); P_X) \psi_{X'}^\dagger((0, \bar{z}); P_X) \right] \simeq 2P_{X_0} \delta_{\ell' \ell} \quad (B.3) \]

holds.

We have to calculate the normalization of the wave function in the moving frame in two cases, i) \( t = x^0 = y^0 \) and ii) \( t' = x'^0 = y'^0 \). To do so, we need to have a relation between the rest-frame (RF) and Lorentz-boosted (LB) wave functions. i) \( t = x^0 = y^0 \) and \( x^0 \neq y^0 \) (equal time in the rest frame).

The relation between RF and LB is derived as follows. By definition, we have

\[ \langle 0 \mid q^c_\alpha (t, \vec{x}) Q_\beta (t, \vec{y}) \mid X; \left( M_X, \vec{0} \right) \{ \ell \} \rangle \]
\[ = G^{-1}_\alpha G^{-1}_\beta \langle 0 \mid q^c_\gamma (x'^0, \vec{x}') Q_\delta (y'^0, \vec{y}') \mid X; P_X \{ \ell \} \rangle. \quad (B.4) \]

Hence, we obtain

\[ \langle 0 \mid q^c_\alpha (x'^0, \vec{x}') Q_\beta (y'^0, \vec{y}') \mid X; P_X \{ \ell \} \]
\[ \simeq \psi^\ell_{X, \alpha \beta} ((0, \bar{x}' - \bar{y}'); P_X) \ \exp \left( i[-M_X t + (M_X - m_Q) \gamma^2 \beta (x^3 - y^3)] \right) \]
\[ = G_\alpha G_\beta \langle 0 \mid q^c_\gamma (t, \vec{x}) Q_\delta (t, \vec{y}) \mid X; \left( M_X, \vec{0} \right) \{ \ell \} \rangle \]
\[ = G_\alpha G_\beta \psi^\ell_{X, \gamma \delta} (\bar{x} - \bar{y}) \ e^{-iM_X t}, \quad (B.5) \]
where use has been made of the approximation
\[ Q_\beta \left( y^0, \vec{y}' \right) \simeq \exp \left[ -i m \gamma \left( y^0 - x^0 \right) \right] Q_\beta \left( x^0, \vec{y}' \right). \] (B.6)

Rewriting Eq. (B.5), we obtain the relation
\[ \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) \simeq G_{\alpha \gamma} G_{\beta \delta} \psi_{X \gamma \delta}^{\ell} \left( (0, \vec{x}_\perp, \gamma^{-1} x^3); (M, 0) \right) e^{i (M - m q) \gamma \beta x^3}. \] (B.7)

Using this equation, we find
\[ \int d^3 x \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) = \gamma \int d^3 x \psi_{X \alpha \beta}^{\ell} (\vec{x}) G^2 \psi_{X \alpha \beta}^{\ell} (\vec{x}) G^2 T = 2 M \gamma. \] (B.8)

ii) \( t' = x^0 = y^0 \) and \( x^0 \neq y^0 \) (equal time in the moving frame)
Similarly to the case i), the relation between RF and LB is, in this case, given by
\[ \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) \simeq G_{\alpha \gamma} G_{\beta \delta} \psi_{X \gamma \delta}^{\ell} \left( (0, \vec{x}_\perp, \gamma x^3); (M, 0) \right) e^{i (M - m q) \gamma \beta x^3}. \] (B.9)

Then, using Eq. (B.9) we obtain
\[ \int d^3 x \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) \psi_{X \alpha \beta}^{\ell} ((0, \vec{x}); P_X) = \gamma^{-1} \int d^3 x \psi_{X \alpha \beta}^{\ell} (\vec{x}) G^2 \psi_{X \alpha \beta}^{\ell} (\vec{x}) G^2 T = 2 M \gamma. \] (B.10)

Here use has been made of the following:
\[ \frac{1}{2} \text{tr} \left[ \begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i (\vec{\sigma} \cdot \vec{n}) v_{-1}(r) \end{pmatrix} \right]^\dagger G^2 \left[ \begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i (\vec{\sigma} \cdot \vec{n}) v_{-1}(r) \end{pmatrix} \right] G^2 T \]
\[ = \frac{\gamma^2}{2} \text{tr} \left[ \begin{pmatrix} 0 & 0 \\ u_{-1}(r) & i (\vec{\sigma} \cdot \vec{n}) v_{-1}(r) \end{pmatrix} \right] \left[ \begin{pmatrix} 1 & \sigma^3 \beta \\ \sigma^3 \beta & 1 \end{pmatrix} \right] \left[ \begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i (\vec{\sigma} \cdot \vec{n}) v_{-1}(r) \end{pmatrix} \right] \]
\[ \times \left[ \begin{pmatrix} 1 & \sigma^3 \beta \\ \sigma^3 \beta & 1 \end{pmatrix} \right] \]
\[ = \gamma^2 \left( u_{-1}^2 + v_{-1}^2 \right). \] (B.11)

**Appendix C**

**Wave Functions**

The wave function is generally defined as
\[ \langle 0 | q_\alpha^{\ell}(t, \vec{x}) Q_\beta(t, \vec{y}) | X; P_X \{ \ell \} \rangle \]
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\[ = \langle 0 | q_0(0, \eta (\vec{x} - \vec{y}) ) Q_\beta (0, \zeta (\vec{y} - \vec{x}) ) | X; P_X \{ \ell \} \rangle e^{-iP_0^0 t} \]
\[ \equiv \psi^\ell_{X \alpha \beta} ((0, \vec{x} - \vec{y}); P_X ) e^{-iP_X \cdot r}, \quad (C.1) \]

with

\[ r^\mu = \zeta x^\mu + \eta y^\mu, \quad \zeta + \eta = 1. \quad (C.2) \]

Here, without loss of generality, we adopt \( \zeta = 0, \ \eta = 1. \)

The Foldy-Wouthuysen-Tani transformation and charge conjugation are defined as

\[ U_{\text{FWT}}(p) = \exp \left( W(p) \vec{\gamma} \cdot \vec{p} \right) = \cos W + \vec{\gamma}_Q \cdot \vec{p} \sin W, \quad (C.3a) \]
\[ \vec{\rho} = \frac{\vec{p}}{p}, \quad \tan W(p) = \frac{p}{m_Q + E}, \quad E = \sqrt{\vec{p}^2 + m_Q^2}, \quad (C.3b) \]
\[ U_c = i \gamma^0 \gamma_2 Q = -U_{c}^{-1}. \quad (C.3c) \]

Here, \( \vec{p} = \vec{p}_Q \) is the initial momentum of the heavy quark, and \( U_{\text{FWT}}(p) \) operates on heavy quarks. Henceforth the color, \( N_c = 3, \) is ignored since the form of the wave function is the same for all colors. The quantity \( \ell \) stands for a set of quantum numbers, \( j, m, \) and \( k, \) and

\[ \Psi^{k j m}(\vec{x}) = \frac{1}{r} \left( \begin{array}{c} u_k(r) \\ -i v_k(r) \left( \vec{n} \cdot \vec{\sigma} \right) \end{array} \right) y^{k j m}_j(\Omega), \quad (C.4) \]

where \( r = |\vec{x}|, \ \vec{n} = \vec{x}/r, \ j \) is the total angular momentum of the meson, \( m \) is its \( z \) component, \( k \) is a quantum number which takes only values, \( k = \pm j, \ \pm (j+1) \) and \( \neq 0, \) whose operator form is given by \( \hat{k} = -\beta_q \left( \vec{\Sigma}_q \cdot \vec{\ell} + 1 \right). \) The scalar functions \( u_k(r) \) and \( v_k(r) \) are polynomials of the radial variable \( r \) and satisfy

\[ \int dr \left( u^2_k(r) + v^2_k(r) \right) = 1. \quad (C.5) \]

Here, \( y^{k j m}_j(\Omega) \) are functions of angles and spinors of the total angular momentum, \( \vec{j} = \vec{\ell} + \vec{s}_q + \vec{s}_Q, \) with \( \vec{\ell} = -i \vec{r} \times \nabla, \) whose first few explicit forms are given by

\[ y^{-1}_{00} = \frac{1}{\sqrt{4\pi}}, \quad y^{-1}_{1m} = \frac{1}{\sqrt{4\pi}} \sigma^m, \]
\[ y^2_{1m} = -\frac{1}{\sqrt{4\pi}} \frac{3}{\sqrt{6}} \left( n^i n^m - \frac{1}{3} \delta^i_m \right) \sigma^i. \quad (C.6) \]

These functions satisfy the following relation and normalization condition:

\[ y^{-k}_{j m} = - (\vec{n} \cdot \vec{\sigma}) y^{k}_{j m}, \quad (C.7) \]
\[ \frac{1}{2} \text{tr} \left( \int d\Omega \ y^{k' j' m'}(\Omega) y^{k}_{j m}(\Omega) \right) = \delta^{k' k} \delta_{j j'} \delta_{m m'}. \quad (C.8) \]
The relative phases among \( y_{jm}^k \) given by Eq. (C.6) are determined so that they give the correct relative phases among the form factors, which are determined by Eqs. (39a)-(39c). From this point, whenever the trace of \( y_{jm}^k \) appears, it is understood to be in the sense of Eq. (C.8).

The leading-order pseudoscalar state \((0^-)\) corresponds to \( \ell = (k = -1, \ j = m = 0) \), and hence we have the wave function

\[
\psi_{X \ 0}^\ell (\vec{x}) = \sqrt{M_X} \begin{pmatrix} 0 & \psi_{00}^{-1}(\vec{x}) \end{pmatrix} = \sqrt{M_X} \begin{pmatrix} 1 & u_{-1}(r) \\
4\pi & -i(n \cdot \vec{\sigma})v_{-1}(r) \end{pmatrix},
\]

with \( X = D \) or \( B \). On the other hand, the leading-order vector state \((1^-)\) has the set of quantum numbers \( \ell = (k = -1, \ j = 1) \) and is given by

\[
\psi_{X \ 0}^\ell (\vec{x}) = \sqrt{M_X} \begin{pmatrix} 0 & \psi_{1m}^{-1}(\vec{x}) e^m \end{pmatrix} = \sqrt{M_X} \begin{pmatrix} 1 & u_{-1}(r) \\
4\pi & -i(n \cdot \vec{\sigma})v_{-1}(r) \end{pmatrix} (\vec{e} \cdot \vec{\sigma}),
\]

with \( e^m \) being a polarization vector \((\vec{e}^2 = -1)\) and \( X = D^* \) or \( B^* \).

**Appendix D**

**Matrix Elements**

The light anti-quark is treated as a spectator. That is, it does not interact with other particles, except for gluons represented by potentials in our model. The heavy quark has a weak vertex and its current is in general given by

\[
j(\vec{x}, t) = Q_{X' \alpha}^\dagger (t, \vec{x}) \mathcal{O}_{\alpha \beta} Q_X(t, \vec{x}),
\]

where \( \mathcal{O} \) is a \( 4 \times 4 \) matrix. By inserting the number operator of the anti-quark and by assuming vacuum dominance among intermediate states, we obtain the following formula for the matrix element of the above current, i.e., Eq. (D.1), between heavy mesons:

\[
\langle X'; P_{X'} \{ \ell' \} | j(\vec{x}, t) | X; P_X \{ \ell \} \rangle
\]

\[
= \mathcal{O}_{\alpha \beta} \langle X'; P_{X'} \{ \ell' \} | Q_{X' \alpha}^\dagger (t, \vec{x}) \int d^3y \ q_{\ell'}^c(t, \vec{y}) q_{\ell}^c(t, \vec{y}) Q_X(t, \vec{x}) \rangle X; P_X \{ \ell \} \rangle
\]

\[
\approx \int d^3y \ \mathcal{O}_{\alpha \beta} \langle X'; P_{X'} \{ \ell' \} | Q_{X' \alpha}^\dagger (t, \vec{x}) q_{\ell}^c(t, \vec{y}) \rangle \langle 0 \rangle \times \langle 0 | q_{\ell'}^c(t, \vec{y}) Q_X(t, \vec{x}) \rangle X; P_X \{ \ell \} \rangle
\]

\[
= \int d^3y \ \text{tr} \left[ \psi_{X'}^{\ell'} (\vec{y} - \vec{x}; P_{X'}) \psi_{X}^\ell (\vec{y} - \vec{x}; P_X) \mathcal{O}^T \right] e^{-i(P_X - P_{X'}) \cdot x},
\]

or

\[
\langle X'; P_{X'} \{ \ell' \} | j(0, 0) | X; P_X \{ \ell \} \rangle = \int d^3x \ \text{tr} \left[ \psi_{X'}^{\ell'} (\vec{x}; P_{X'}) \psi_{X}^\ell (\vec{x}; P_X) \mathcal{O}^T \right],
\]

where in Eq. (D.2) we have assumed vacuum dominance among intermediate states in order to make some approximations.
Appendix E

Semileptonic Weak Form Factor

In this appendix, we use the Foldy-Wouthuysen-Tani transformed wave functions, $\psi^X_{FWT}$, instead of $\psi^X$, although the subscript FWT is omitted for simplicity.

E.1. Zeroth order (Isgur-Wise function)

Here we present the derivations of the Isgur-Wise function given in the main text in four cases.

1-i) $\bar{B}$ rest-frame and $t = x^0 = y^0$:

$$
\xi(\omega) = \frac{2\sqrt{\omega}}{(1 + \omega)\sqrt{2M_B}} \int d^3x \left| \left[ \psi^{\ell+}_{D0}((0, x^0); P_D) \psi^\ell_{B0}((0, x^0); P) \right] \right| \sqrt{\int d^3x \left| \psi^{\ell+}_{D0}((0, x^0); P_D) \right|^2}
$$

$$
= \frac{2\sqrt{\omega}}{(1 + \omega)\sqrt{2M_B}2M_D\gamma^2} \int d^3x \left[ x^3 G^*_{\alpha\gamma} G^*_{\beta\delta} \psi^{\ell+}_{D0} \left( (0, x^0, \gamma x^3); (M_D, 0) \right) \right] \times e^{-i(M_D - m_e)\gamma\gamma^3} \psi_{B0\alpha\beta}(x)
$$

$$
= \frac{2\sqrt{\omega}}{2M(1 + \omega)\gamma^3/2} \int d^3x \left\{ x^3 \left[ \psi^{\ell+}_{D0\alpha\beta}(x) + (\gamma^3 - 1)x^3 \frac{\partial}{\partial x^3} \psi^{\ell+}_{D0\alpha\beta}(x) \right] \right\}
$$

$$
\times G^*_{\alpha\gamma} G^*_{\beta\delta} \psi_{B0\alpha\beta}(x) e^{-i(M_D - m_e)\gamma\gamma^3} + O(\beta^4)
$$

$$
= \frac{1}{\omega} - \frac{1}{6} \beta^2 \omega \bar{E}_D^2 \langle r^2 \rangle \gamma^3/2 - \frac{\beta^2}{2} \frac{1}{2M} \int d^3x x^3 \left( \frac{\partial}{\partial x^3} \psi^{\ell+}_{D0\alpha\beta}(x) \right) \psi_{B0\alpha\beta}(x) + O(\beta^4)
$$

$$
= \frac{1}{\omega} - \frac{1}{6} \beta^2 \omega \bar{E}_D^2 \langle r^2 \rangle + \frac{\beta^2}{4} + O(\beta^4),
$$

where we have used

$$
\int d^3x x^3 \left( \sum_2 \phi^{\ell+}_{D0\alpha\beta}(x) \right) \psi_{B0\alpha\beta}(x)
$$

$$
= \frac{2M}{4\pi} \int d^3x x^3 \left[ \frac{\partial}{\partial x^3} \left\{ \frac{1}{r} \left( \begin{array}{c}
0 \\
u_{-1}
\end{array} \right) \right\} \right] \int d^3x \left( \begin{array}{c}
u_{-1}
\end{array} \right)
$$

$$
= \frac{2M}{4\pi} \int d^3x x^3 \left[ \frac{\partial}{\partial x^3} \left( \begin{array}{c}
u_{-1}
\end{array} \right) \right] \int d^3x \left( \begin{array}{c}
u_{-1}
\end{array} \right)
$$

$$
= \frac{2M}{4\pi} \int d^3x x^3 \left( \sum_2 \psi^{\ell+}_{D0\alpha\beta}(x) \right) \psi_{B0\alpha\beta}(x)
$$

$$
= \frac{2M}{4\pi} \int d^3x x^3 \left( \sum_2 \psi^{\ell+}_{D0\alpha\beta}(x) \right) \psi_{B0\alpha\beta}(x)
$$

with $M = \sqrt{M_B M_D}$.

1-ii) $\bar{B}$ rest-frame and $t' = x'^0 = y'^0$:

$$
\xi(\omega) = \frac{2\sqrt{\omega}}{(1 + \omega)\sqrt{2M_B}2M_D\gamma} \int d^3x \left[ x^3 G^*_{\alpha\gamma} G^*_{\beta\delta} \psi^{\ell+}_{D0\alpha\beta}(x) \right] e^{-i(M_D - m_e)\gamma\gamma^3}
$$
2-i) Breit frame and $t = x^0 = y^0$:

$$
\xi(\omega) = \frac{\int d^3 x \text{ tr} \left[ \psi_D^\dagger (0, \vec{x}) ; P_D \right] \psi_B (0, \vec{x})}{\sqrt{\int d^3 x \text{ tr} \left[ \psi_D^\dagger (0, \vec{x}) ; P_D \right]^2} \int d^3 x \text{ tr} \left[ \psi_B (0, \vec{x}) ; P_B \right]^2}
= \frac{1}{\sqrt{2M_B \gamma^3 2M_D \gamma^3}} \int d^3 x \psi^\dagger \psi_B(0, \vec{x}) \psi_B(0, \vec{x}) \psi_B(0, \vec{x}) e^{-i(M_D - m_c) \gamma^2 \frac{\beta}{2} x^3}
= \gamma^2 - \frac{1}{6} \left( \frac{\beta}{2} \right)^2 \gamma^2 (\tilde{E}_B + \tilde{E}_D)^2 \langle r^2 \rangle + O(\beta^4).
\tag{E.3}
$$

2-ii) Breit frame and $t' = x^0 = y^0$:

$$
\xi(\omega) = \frac{1}{\sqrt{2M_B \gamma^3 2M_D \gamma^3}} \int d^3 x \frac{G^*_{\alpha \gamma} G_{\beta \delta} \psi_D^\dagger (0, \vec{x})}{(0, \vec{x}_\perp, \gamma^{-1} x^3) ; (M_D, 0)} \psi_B(0, \vec{x}) \psi_B(0, \vec{x}) \psi_B(0, \vec{x}) e^{-i(E_B + E_D) \gamma^2 \frac{\beta}{2} x^3}
= \gamma^{-1} \left( \frac{\beta}{2} \right)^2 \gamma^{-2} (\tilde{E}_B + \tilde{E}_D)^2 \langle r^2 \rangle + O(\beta^4).
\tag{E.4}
$$

E.2. First order

In this appendix, we show how to calculate the first-order corrections to the form factors by using Eqs. (25)–(27) and (29) and the equations in Appendix C. Only the case 2-ii) gives six form factors that are consistent with those of Neubert and Rieckert among the four different Lorentz frames considered in the previous subsection:

$$
\xi_+(\omega) = \frac{1}{2M^2} \int d^3 z \text{ tr} \left[ \psi_D^\dagger (z) \psi_B(0, \vec{x}) U_B^{-1} G^{-1} G^* U_D \right] e^{-2i(\frac{\beta}{2} z^3)}
$$

among the four different Lorentz frames considered in the previous subsection:
Comparing the result with Eq. (58a), we obtain

\[
\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3} A^2 \langle r^2 \rangle\right) (\omega - 1),
\]

(E.8)

where \( U_X \) for \( X = \bar{B} \) and/or \( D \) are defined by Eqs. (16) and (C.3) and \( M = \sqrt{M_B M_D} \). Here, \( A \) is defined by

\[
A = \frac{\bar{E}_D + \bar{E}_\beta}{2} = \frac{(E_D - m_c) + (E_\beta - m_b)}{2} = C^0 + \frac{C^1}{2} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + O \left( (m_Q)^2 \right),
\]

(E.7)

which comes from the expansion of the phase factor \( \exp \left( -2iA^2 \bar{z}^3 \right) \). Here, we have \( C^0 = \bar{A} = \bar{A}_u \) and \( C^1 \) is defined in Eq. (53) and depends only on \( m_q = m_u = m_d \). Comparing the result with Eq. (58a), we obtain

\[
\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3} A^2 \right) \langle r^2 \rangle (\omega - 1),
\]

(E.8)

\[
\xi_+(\omega) = \left[ 1 + \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \rho_1(\omega) \right] \xi(\omega), \quad \rho_1(\omega) = \frac{1}{3} A C^1 \langle r^2 \rangle (\omega - 1). \quad (E.9)
\]

Similarly, the five other form factors and \( \rho_i(\omega) \) are given by the following:

\[
\xi_-(\omega) = \frac{1}{2M^2 \gamma^2 \frac{d}{2}} \int d^3z \text{tr} \left( \psi^f_D (\bar{z}) \psi_B^f (z) U_B^{-1} G^{-1} \alpha^3 T G^* U_D \right) e^{-2iA^2 \bar{z}^3}
\]

\[
= \frac{1}{2M^2 \gamma^2 \frac{d}{2}} \int d^3z \text{tr} \left( \psi^f_{D0} (\bar{z}) \psi_B^f (z) \left( \alpha^2 + \frac{1}{2} \left[ \frac{1}{m_c} p_c^3 + \frac{1}{m_b} p_b^3 \right] \gamma^0 \right) \right) e^{-2iA^2 \bar{z}^3}
\]

\[
= \left( \frac{A}{2} + \rho_4(\omega) \right) \left( \frac{1}{m_c} - \frac{1}{m_b} \right) \xi(\omega),
\]

(E.10)

\[
\rho_4(\omega) = C^1 \left( \frac{1}{4} + \frac{1}{6} A^2 \right) \langle r^2 \rangle (\omega - 1) \left( \frac{1}{m_c} + \frac{1}{m_b} \right) = O(1/mQ), \quad (E.11)
\]

\[
\xi_\nu(\omega) = \frac{1}{2M^2 \gamma^2 \frac{d}{2}} \int d^3z \text{tr} \left( \psi^f_D (\bar{z}) \psi_B^f (z) U_B^{-1} G^{-1} \alpha^1 T G^* U_D^* \right) e^{-2iA^2 \bar{z}^3}
\]

\[
= \frac{1}{2M^2 \gamma^2 \frac{d}{2}} \int d^3z \text{tr} \left( \psi^f_{D0} (\bar{z}) \psi_B^f (z) \right)
\]
Here, we define 

\[
\rho_2(\omega) = \rho_1(\omega), \quad \rho_4(\omega) = 0, 
\]

where \(\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}\), \(\vec{\sigma}\) is a static polarization vector for \(D^*\), and \(\overline{M}^* = \sqrt{M_B M_D^*}\).

Here, we define

\[
N' = \frac{\tilde{E}_{D^*} + \tilde{E}_B}{2} = \frac{(E_{D^*} - m_c) + (E_B - m_b)}{2} = C^0 + \frac{C^{1'}}{2m_c} + \frac{C^1}{2m_b} + O\left(1/(mQ)^2\right),
\]

with \(C^{0'} = C^0 = \overline{A} = \overline{A}_u\),

\[
\xi_{A_1}(\omega) = \frac{1}{2M' \gamma^2 \epsilon_{1^*}} \int d^3z \, \text{tr} \left( \psi_{D^*}^\dagger(\vec{z}) \psi_B^\dagger(\vec{z}) U_B^{-1} G^{-1} \gamma_5^T \alpha^1 G^* U_D \right) e^{-\frac{2iN' \omega}{2} z^3} 
\]

\[
= \frac{1}{2M' \gamma^2 \epsilon_{1^*}} \int d^3z \, \text{tr} \left( \psi_{D^*}^\dagger_{0} (\vec{z}) \psi_B^\dagger_{0}(\vec{z}) \right. 
\]

\[
\times \left( \alpha^1 + \frac{i \beta}{2} \Sigma^2 + \frac{1}{2} \left[ \frac{1}{m_c} p_c^3 - \frac{1}{m_b} p_b^3 \right] \gamma_0 \Sigma^1 \right) \left. \right) e^{-\frac{2iN' \omega}{2} z^3} 
\]

\[
= \left[ 1 + \frac{\tilde{A}'}{2} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \right] \xi(\omega) 
\]

\[
= \left[ 1 + \frac{\tilde{A} \omega - 1}{2 \omega + 1} \left( \frac{1}{m_c} + \frac{1}{m_b} \right) + \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \rho_2(\omega) \right] \xi(\omega), 
\]

\[
\rho_2(\omega) = \rho_1(\omega), \quad \rho_4(\omega) = 0, 
\]

\[
-\xi_{A_1}(\omega) + \xi_{A_2}(\omega) + \xi_{A_3}(\omega)
\]

\[
= \frac{1}{2M' \gamma^4 \epsilon_{3^*}} \int d^3z \, \text{tr} \left( \psi_{D^*}^\dagger(\vec{z}) \psi_B^\dagger(\vec{z}) U_B^{-1} G^{-1} \gamma_5^T G^* U_D \right) e^{-\frac{2iN' \omega}{2} z^3} 
\]

\[
= -\frac{1}{2M' \gamma^4 \epsilon_{3^*}} \int d^3z \, \text{tr} \left( \psi_{D^*}^\dagger_{0} (\vec{z}) \psi_B^\dagger_{0}(\vec{z}) \gamma_5 \left( 1 + \frac{1}{2} \left[ \frac{1}{m_c} p_c^3 + \frac{1}{m_b} p_b^3 \right] \right) \right) e^{-\frac{2iN' \omega}{2} z^3} 
\]

\[
= \frac{\tilde{A}'}{2} \left( \frac{1}{m_c} - \frac{1}{m_b} \right) \xi(\omega), 
\]

\[
\xi_{A_1}(\omega) + \frac{\omega - 1}{\omega + 1} \left( \xi_{A_2}(\omega) - \xi_{A_3}(\omega) \right)
\]

\[
= \frac{1}{2M' \gamma^4 \epsilon_{3^*}} \int d^3z \, \text{tr} \left( \psi_{D^*}^\dagger(\vec{z}) \psi_B^\dagger(\vec{z}) U_B^{-1} G^{-1} \gamma_5^T \alpha^3 G^* U_D \right) e^{-\frac{2iN' \omega}{2} z^3} 
\]
\[ = -\frac{1}{2M^2} \int d^3z \text{tr} \left( \psi_D^\dagger \psi_D^0 \left( \Sigma^3 - \frac{1}{2} \left[ \frac{1}{m_c^3} - \frac{1}{m_b^3} \right] \gamma_5 \gamma_0 \right) \right) e^{-2i\Lambda' \frac{3}{2} z^3} \]
\[ = \left( 1 - \frac{\omega - 1}{\omega + 1} \right) \xi(\omega). \]  
(E.18)

From these, we derive

\[ \xi_{A_2}(\omega) = -\frac{\Lambda}{\omega + 1} \frac{1}{m_c} \xi(\omega), \quad \rho_3(\omega) = \rho_1(\omega) = 0, \]  
(E.19)

\[ \xi_{A_3}(\omega) = \left[ 1 + \frac{A}{2} \left( \frac{\omega - 1}{\omega + 1} \frac{1}{m_c} + \frac{1}{m_b} \right) + \left( \frac{1}{m_c} + \frac{1}{m_b} \right) \rho_1(\omega) \right] \xi_+(\omega), \]  
(E.20)

\[ \rho_2(\omega) = \rho_1(\omega), \quad \rho_3(\omega) = \rho_4(\omega) = 0. \]  
(E.21)

Therefore, all the expressions for \( \rho_i(\omega) \) included in \( \xi_i(\omega) \) are consistent up to first order in \( 1/m_Q \). Even though the first-order corrections to the wave functions are included as given by Eq. (27), their contributions vanish, because of the matrix structure after taking the trace over indices. From the above equations, we can easily reproduce our results given by Eq. (58), together with Eq. (59).

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