Orientation-dependent crossover from retro to specular Andreev reflections in semi-Dirac materials

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Abstract

In the framework of Bogoliubov–de Gennes equation, we theoretically study the transport properties in normal-superconducting junctions based on semi-Dirac materials (SDMs). Owing to the intrinsic anisotropy of SDMs, the configuration of Andreev reflection (AR) and the differential conductance are strongly orientation-dependent. For the transport along the linear dispersion direction, the differential conductance exhibits a clear crossover from retro AR to specular AR with increasing the bias-voltage, and the differential conductance oscillates with the interfacial barrier strength without a decaying profile. Conversely, for the transport along the quadratic dispersion direction, the boundary between the retro AR and specular AR becomes ambiguous when the orientation angle increases, and the differential conductance decays with increasing the momentum mismatch or the interfacial barrier strength. We illustrate the pseudo-spin textures to reveal the underling physics behind the anisotropic coherent transport properties. These results enrich the understanding of the superconducting coherent transport in SDMs.

1. Introduction

In a normal-superconducting (NS) junction with ideal contacts, the transport properties are dominated by the Andreev reflection (AR) in the subgap energy regime of $E \leq \Delta_0$, with $E$ the incident energy and $\Delta_0$ the superconducting gap [1, 2]. In accordance with the origination of the AR-involved electron and hole, the AR can be classified into two typical categories, i.e., the intra-band and inter-band phase-coherent processes. In most conventional-metal-based NS junctions, the chemical potential in the N region satisfies $\mu_N \gg \Delta_0$, thus in the subgap regime the AR is a intra-band phase-coherent scattering process, during which an incident electron from the N region is approximately reto-reflected as a hole [1–4]. While in the NS junctions based on Dirac materials, $\mu_N$ can be continuously tuned to the regime of $\mu_N < E$ [5–7]. Consequently, a conduction-band electron incident from the N region is specularly reflected back as a valence-band hole, leading to a inter-band phase-coherent scattering process known as specular-AR [7–17]. Remarkably, when the N region is weakly doped, a crossover from retro-AR to specular-AR occurs with increasing $E$, manifesting itself as a dip at $E = \mu_N$ in the E-dependent conductance spectrum. This signature has been experimentally observed in bilayer-graphene-based NS junctions [7]. In addition, in Dirac-material-based NS junctions, due to the novel momentum-spin/pseudo-spin textures of Dirac fermions, the subgap differential conductance oscillates with the interfacial barrier strength without a decaying profile [12], as confirmed by recent experimental measurements [18].

Although the scenarios of AR in the systems with pure quadratic [1–4, 7–9] or pure linear [10–23] dispersions have triggered extensive studies, the AR and related subgap differential conductance in NS junctions based on semi-Dirac materials (SDMs) have received no attention to date. Unlike most Dirac materials that possess liner dispersions in all momentum-space directions [5, 6, 24], in SDMs the
low-energy excitations disperse linearly in one direction but quadratically along the other directions [25–29]. The semi-Dirac dispersion has recently been observed in black phosphorus with situ deposition of K [30] or Rb [31] atoms. The unique band structures of SDMs are responsible for a series of novel phenomena [28–33, 33–46], such as the consequences of anisotropic aspect in the superconducting order parameter correlations [47–49]. Recent theoretical efforts have demonstrated that the superconductivity in SDMs can be induced by arbitrarily weak attractions in the present of random chemical potential [47]. Resorting to the mean-field calculation [48] and renormalization group analysis [49], it is revealed that the s-wave superconductivity is more favorable in SDMs. More strikingly, owing to the intrinsic anisotropy, the stiffness of superconducting order parameter and the divergence behavior of correlation length are highly orientation-dependent [48, 49]. These progresses provide foundations for exploring coherent transport properties in SDM-based NS junctions.

Since the low-energy excitations in SDMs host unique dispersions intermediate between the quadratic and linear energy spectra, it is natural to ask how the intrinsic anisotropy manifests itself in the superconducting coherent transport. In this paper we investigate the subgap transport properties in SDM-based NS junctions. The manifestations of the anisotropic dispersion in the subgap transport can be summarized as two points. First, the crossover from the retro–AR to specular–AR is orientation-dependent. When the orientation angle increases, the boundary between the retro-AR and specular-AR becomes ambiguous. Second, the influences of momentum-mismatch and interfacial barrier on the subgap transport are sensitive to the transport direction. For the transport along the quadratic dispersion direction, the subgap differential conductance rapidly decays with increasing the interfacial barrier strength or the momentum-mismatch between the N and S regions. Whereas the transport in the linear dispersion direction is insensitive to the momentum-mismatch, and the subgap differential conductance periodically oscillates with the interfacial barrier strength without a decaying profile. We illustrate the pseudo-spin textures of semi-Dirac fermions to elucidate our findings.

2. Model and method

In the leading order of \( k \), the single-particle effective Hamiltonian of SDM reads \( \hat{h}(k) = \hbar v k_x \hat{\sigma}_x + \eta k_y^2 \hat{\sigma}_y \) [32–34], where \( \hat{\sigma}_x, \hat{\sigma}_y \) label the Pauli matrices operating on the pseudo-spin space, \( v \) is the Fermi velocity, and \( \eta \equiv \hbar^2/(2m) \) with \( m \) the effective mass. The low-energy dispersion of the normal state is \( \tilde{E} = \sqrt{\hbar^2 v^2 k_x^2 + \eta^2 k_y^2} \), which is anisotropic in the momentum space.

To address the effects of the intrinsic anisotropy on the subgap transport properties, we consider a SDM-based NS junction with the normal direction of NS interface defining as \( \mathbf{n} = (\cos \theta, \sin \theta) \), where \( \theta \) indicates the interface orientation angle between directions of the \( x \)-axis and \( \mathbf{n} \), as that carried out in the superconducting hybrids based on monolayer phosphorus [50] and type-II Weyl semimetals [16, 17]. In this way, the NS interface is located at \( x = -y \tan \theta \), as schematically shown in figure 1. For the sake of convenience, we take a coordinate transformation of \((\tilde{x}, \tilde{y})^T = \tilde{R}(x, y)^T\), where the superscript \( T \) denotes the transposition of the row vector and the rotation operator \( \tilde{R} \equiv e^{i\phi \theta} \). In the new coordinate system, as shown in figure 1, the N and S regions are placed within the ranges of \( \tilde{x} < 0 \) and \( \tilde{x} > 0 \), respectively, and the chemical potentials in the N and S regions are assumed to be tuned independently. We consider the transport properties along the \( \tilde{x} \)-direction, and assume that the translational symmetry in the \( \tilde{y} \)-direction is preserved, so that the transverse momentum \( \tilde{k}_y \) can be treated as a good quantum number [8–15]. Moreover, under this assumption the influences of boundary effects on the transport can berationally neglected. In the S region, we take the intra-sublattice/orbit s-wave pairing, as proposed by recent theoretical work [48, 49]. In practice, the superconductivity in the S region can be induced by an s-wave superconductor via the proximity effect, as implemented in similar NS junctions based on graphene [7, 18, 19] and Weyl semimetals [22, 23].

Under these lines, the Bogoliubov–de Gennes (BdG) Hamiltonian is given by [48, 49]

\[
\mathcal{H}_{\text{BdG}} = \begin{pmatrix} \hat{h}(\tilde{k}) - \tilde{\mu}(\tilde{x}) & \tilde{\Delta}(\tilde{x}) \\ \tilde{\Delta}^\dagger(\tilde{x}) & -\hat{h}(\tilde{k}) + \tilde{\mu}(\tilde{x}) \end{pmatrix},
\]

acting on the pseudo-spin \( \otimes \) Nambu space. In the new coordinate system, the single-particle effective Hamiltonian takes the form of \( \hat{h}(\tilde{k}) = \hbar v (\tilde{k}_x \cos \theta - \tilde{k}_y \sin \theta) \hat{\sigma}_x + \eta (\tilde{k}_x \sin \theta + \tilde{k}_y \cos \theta)^2 \hat{\sigma}_y \). The chemical potential \( \tilde{\mu}(\tilde{x}) = \tilde{\mu}_N \Theta(-\tilde{x}) + \tilde{\mu}_S \Theta(\tilde{x}) \), where \( \Theta(\tilde{x}) \) is the Heaviside step function and \( \tilde{\mu}_N \) (\( \tilde{\mu}_S \)) denotes the chemical potential of the S(N) region. In this paper, we assume that the relation of \( \tilde{\mu}_S \gg \tilde{\mu}_N \) is satisfied, so that the leakage of Cooper pairs from the S to N regions can be safely neglected [8–15]. In doing so, the pair potential can be effectively modeled by a step function, i.e., \( \tilde{\Delta}(\tilde{x}) = \tilde{\Delta}_0 \tilde{\sigma}_y e^{i\phi} \Theta(\tilde{x}) \), with \( \phi \) the
superconducting phase and \( \sigma_0 \) a \( 2 \times 2 \) identity matrix operating on the pseudo-spin space. In addition, we assume the relation of \( \tilde{\Delta}_0 \ll \mu_0 \) holds throughout this work, to ensure the validity of mean-field approximation. We note that the BdG Hamiltonian shown in equation (1) can also be derived from the lattice model of a graphene-like system in proximity to an s-wave superconductor, and the details are given in appendix A.

To rescale related parameters as dimensionless ones, it is convenient to introduce three natural scales, i.e., the momentum scale \( p_0 \equiv 2m v \), the energy scale \( E_0 \equiv \hbar p_0^2/(2m) \), and the length scale \( l_0 \equiv h/p_0 \) [34, 35]. In doing so, one can define the dimensionless quantities as \( k \equiv \hbar k_x/p_0, \; q \equiv \hbar k_y/p_0, \; E \equiv \tilde{E}/E_0, \; \mu_{NS} \equiv \tilde{\mu}_{NS}/E_0, \; \Delta_0 \equiv \tilde{\Delta}_0/E_0 \), and \( X(Y) \equiv \tilde{x}(\tilde{y})/l_0 \). Hereafter, we take the proposed dimensionless quantities to express corresponding formulism.

In the N region, by diagonalizing the BdG Hamiltonian given in equation (1), the rescaled dispersion can be written as

\[ E_{\epsilon(h)}^{\pm} = \pm \sqrt{\zeta_{\epsilon(h)}^2 - (\pm)\mu_N}, \tag{2} \]

where \( \zeta_{\epsilon(h)} = (k_{\epsilon(h)} \cos \theta - q \sin \theta)^2 + (k_{\epsilon(h)} \sin \theta + q \cos \theta)^2 \), and the quantity with a subscript \( \epsilon(h) \) denotes the electron-like (hole-like) one. The group velocity in the N region can be parameterized as

\[ v_{\epsilon(h)}^x = v_0 \cos \theta (k_{\epsilon(h)} \cos \theta - q \sin \theta) + 2 \sin \theta (k_{\epsilon(h)} \sin \theta + q \cos \theta)^3/\sqrt{\zeta_{\epsilon(h)}}, \tag{3a} \]

\[ v_{\epsilon(h)}^y = v_0 \sin \theta (q \sin \theta - k_{\epsilon(h)} \cos \theta) + 2 \cos \theta (k_{\epsilon(h)} \sin \theta + q \cos \theta)^3/\sqrt{\zeta_{\epsilon(h)}}. \tag{3b} \]

As can be seen, the group velocity is \( \theta \)-dependent and the two components exhibit distinct behaviors with respect to the momentum, reflecting the anisotropic aspect of SDMs.

According to equation (2), when the interface orientation angle \( \theta \neq 0^\circ \), the low-energy excitations of the N region disperse quadratically in the transport direction. While in the case of \( \theta = 0^\circ \), the low-energy excitations exhibit linear dispersions with respect to the longitudinal momentum \( k_{\epsilon(h)} \). Since the situations of \( \theta \neq 0^\circ \) and \( \theta = 0^\circ \) possess distinct scattering configurations, it is necessary to consider them separately. Without loss of generality, in both situations of \( \theta \neq 0^\circ \) and \( \theta = 0^\circ \) we consider the scattering configurations resulting from electron-like propagating incident modes. The scenarios stemming from hole-like incident modes can be analyzed in the same way.

### 2.1. Scattering configuration with \( \theta \neq 0^\circ \)

In the case of \( \theta \neq 0^\circ \), equation (2) gives four roots of \( k_{\epsilon(h)} \) for a set of fixed \( E, \mu_N \), and \( q \), implying that there are four possible electron-like (hole-like) scattering modes in the N region. We note that to get a propagating incident mode, the condition of \( q \in [q_{L1}^0(\theta), q_{L2}^0(\theta)] \) should be satisfied, where \( q_{L1}^0(\theta) \) and \( q_{L2}^0(\theta) \) are, respectively, the upper and lower critical transverse momenta of an electron-like mode. Under the condition of \( q \in [q_{L1}^0(\theta), q_{L2}^0(\theta)] \), the four electron-like modes can be classified into two propagating modes \( k_{L1}^0 \) with \( v_{L1}^x > 0 \) and \( k_{L2}^0 \) with \( v_{L2}^x < 0 \), and two evanescent ones \( \kappa_{L1}^0 \) with \( \text{Im}(\kappa_{L1}^0) \) \(<\) 0 and \( \kappa_{L2}^0 \) with \( \text{Im}(\kappa_{L2}^0) \) \(>\) 0. The situation for the hole-like mode is more complicated. Defining the range of \( q \) to get a propagating hole-like mode as \( [q_{H1}^0(\theta), q_{H2}^0(\theta)] \), we have \( \{q_{H1}^0(\theta), q_{H2}^0(\theta)\} \subseteq [q_{L1}^0(\theta), q_{L2}^0(\theta)] \) for \( E \geq 0 \) and \( \mu_N \geq 0 \). As a consequence, equation (2) determines two types of \( k_{L1}^0 \) depending on the value of \( q \). In the case of \( q \in [q_{L1}^0(\theta), q_{L2}^0(\theta)] \), there are two propagating hole-like modes \( k_{L1}^0 \) with \( v_{L1}^y > 0 \) and \( k_{L2}^0 \) with \( v_{L2}^y < 0 \), and two complex ones \( \kappa_{L1}^0 \) with \( \text{Im}(\kappa_{L1}^0) \) \(<\) 0 and \( \kappa_{L2}^0 \) with \( \text{Im}(\kappa_{L2}^0) \) \(>\) 0. While the four roots of \( k_{L1}^0 \) are all complex when \( q \in [q_{L1}^0(\theta), q_{L2}^0(\theta)] \) and \( q \notin \{q_{H1}^0(\theta), q_{H2}^0(\theta)\} \), we denote two of which by \( k_{L1} \) and \( \kappa_{L1} \), satisfying \( \text{Im}(k_{L1}) \) \(<\) 0 and \( \text{Im}(\kappa_{L1}) \) \(<\) 0, respectively. In the S region, there are eight evanescent modes in the subgap regime, we choose four of them indicated by \( k_{eq(1,2)} \) and \( \kappa_{eq(1,2)} \), with \( \text{Im}(k_{eq(1,2)}) \geq 0 \) and \( \text{Im}(\kappa_{eq(1,2)}) \) \(<\) 0, respectively. Although the evanescent modes do not contribute to the current, they need to
be included in the wave functions to get correct boundary conditions. With this in mind, the wave function in the N and S regions are, respectively, parameterized as

\[ \Phi_N = \varphi^+_N + r_{re1} \varphi^-_N + r_{re2} \varphi^+_N + r_{re3} \varphi^-_N \]

\[ \Phi_S = t_{eq1} \varphi^+_S + t_{eq2} \varphi^-_S + t_{eq3} \varphi^+_S + t_{eq4} \varphi^-_S, \]

where \( r_{re1,2,3} \) and \( t_{eq1,2} \) are, respectively, the reflection amplitudes of propagating and evanescent modes in the NR(AR) processes, and the coefficients \( t_{eq3,4} \) represent the transmission amplitudes for the propagating and evanescent electron-like (hole-like) quasiparticles, respectively. The details of the related basis scattering states are given in appendix B.1.

In practice, there may exist defects and lattice mismatch at the boundary between the N and S regions, which may profoundly influence the transport properties. To model the effects originating from the interfacial defects and lattice mismatch, we introduce an interfacial barrier modeled by \( \hat{U}(\chi) \equiv \hat{U}_0 \delta(\chi) \), with \( \delta(\chi) \) being the Delta function. In doing so, the boundary conditions can be formulated as

\[ \Phi_S|_{x=0^+} = \Phi_N|_{x=0^-}, \]  

\[ \partial_x \Phi_S|_{x=0^+} = \mathcal{M}(\theta) \Phi_N|_{x=0^-}, \]

where \( \mathcal{M}(\theta) = \partial_x - \frac{i\hbar}{\sin \theta} \sigma \tau_2 \) and \( Z = \frac{\hbar}{q \hbar_0} \) indicating the normalized interfacial barrier strength. The probabilities for the NR and AR processes are, respectively, defined as

\[ R_{re,1(2)} = \left| \frac{\langle \varphi_e^{1(2)} | \mathcal{J}_X(\theta) | \varphi^-_{eV,1(2)} \rangle}{\langle \varphi_e^{1(2)} | \mathcal{J}_X(\theta) | \varphi^+_{eV,1(2)} \rangle} \right|^2, \]

\[ R_{he,1(2)} = \left| \frac{\langle \varphi_h^{1(2)} | \mathcal{J}_X(\theta) | \varphi^-_{eV,1(2)} \rangle}{\langle \varphi_h^{1(2)} | \mathcal{J}_X(\theta) | \varphi^+_{eV,1(2)} \rangle} \right|^2, \]

where the particle current density operator \( \mathcal{J}_X(\theta) \equiv \frac{e}{\hbar} [X, \hat{H}_{\text{BdG}}(k, q)] \) = \( v [\cos \theta \sigma_x + 2 \sin \theta (-i \sin \theta \sigma_x + q \cos \theta) \sigma_y] \tau_2 \), with \( \tau_2 \) the Pauli matrix operating on the Nambu space. We note that the probabilities of evanescent modes \( R_{he,1(2)} \) and \( R_{he,2} \) are vanishing and do not contribute to the current. Therefore, in accordance with the Blonder–Tinkham–Klapwijk (BTK) formula [4], the zero-temperature differential conductance along the X-direction can be written as

\[ G(\theta) = \frac{2e^2}{h} \int_{\Phi^{1(2)}(\theta)} q d\alpha \left[ 1 - R_{re,1}(eV, q, \theta) + R_{he,1}(eV, q, \theta) \right], \]

where \( W \) is the width along the Y-direction of the junction. It is convenient to normalize the conductance by \( G_0(\theta) = \frac{2e^2}{h} [q^{1(2)}(\theta) - q^{1(2)}(\theta)] \), indicating the conductance of a related SDM-based NN junction in the ballistic limit. In the special case of \( \theta = 90^\circ \), the cut-off of transverse momenta for an electron-like mode are given by \( q^{1(2)}(90^\circ) = -q^{1(2)}(90^\circ) = |eV + \mu_N| \), and thus \( G_0(90^\circ) = \frac{2e^2}{\pi} |eV + \mu_N| \).

2.2. Scattering configuration with \( \theta = 0^\circ \)

We now turn to the scattering problem in the NS junction with \( \theta = 0^\circ \), where the concerned transport is along the linear dispersion direction. We note that in the current direction, the low-energy excitations in SDMs are similar as that in Dirac materials, thus the scattering amplitudes can be calculated in term of the standard procedure employed in Dirac-material-based NS junctions [8–13], see appendix B.2 for details. For the scattering problem resulting from an electron-like incident mode, in the limit regime of \( \mu_S \gg \max(\mu_N, E, \Delta_0) \), the reflection amplitudes for the processes of NR and AR are, respectively, given by

\[ r_{re} = \frac{(k^+_S e_\pm - k^+_S e_\mp)}{(k^+_S e_\pm + k^+_S e_\mp)} \cos \beta + i[i(k^+_S k^+_S - e_\pm e_\mp)] \sin \beta, \]

\[ r_{he} = \frac{2k^+_e e_\mp e^{-i\alpha}}{(k^+_S e_\pm + k^+_S e_\mp)} \cos \beta + i[i(k^+_S k^+_S - e_\pm e_\mp)] \sin \beta, \]

where \( e_{\pm} = E \pm \mu_N \), \( k_{e(\theta)} = \text{sgn}[e_{\pm}]\sqrt{e_{\pm}^2 - q^2} \), \( k_{e(\theta)} = k_{e(\theta)} \pm i\eta^2 \), and \( \beta = \cos^{-1}(E/\Delta_0) \Theta(\Delta_0 - E) - i \cosh^{-1}(E/\Delta_0) \Theta(E - \Delta_0) \). As can be seen, both \( r_{re} \) and \( r_{he} \) are independent of \( Z \), implying that in the limit of \( \mu_S \gg \max(\mu_N, E, \Delta_0) \), the transport properties along the linear dispersion direction are insensitive to the interfacial barrier, similar as that in a graphene-based NS junction [12]. However, beyond the limit regime...
of $\mu_s \gg \max(\mu_N, E, \Delta_0)$, the scattering probabilities and hence the differential conductance become sensitive to $Z$, as shown in figure 5(a).

The zero-temperature differential conductance along the linear dispersion direction can be obtained from equation (7), by substituting $R_{\alpha e}$ and $R_{\alpha e,\alpha}$, respectively, with $R_{\alpha e} = \frac{(\psi_{\alpha}^\dagger \phi_{\alpha}^{(0)}(0), \psi_{\alpha}^\dagger \phi_{\alpha}^{(0)}(0))}{|\phi_{\alpha}^{(0)}(0)|^2} |r_{\alpha e}|^2$ and $R_{\alpha e,\alpha} = \frac{(\psi_{\alpha}^\dagger \phi_{\alpha}^{(0)}(0), \psi_{\alpha}^\dagger \phi_{\alpha}^{(0)}(0))}{|\phi_{\alpha}^{(0)}(0)|^2} |r_{\alpha e,\alpha}|^2$. In addition, the cut-off of transverse momenta can be reduced into $\tilde{q}_{\alpha e}^S(0) = -\tilde{q}_{\alpha e}^S(0) = \sqrt{|eV + \mu_N|}$, so that $G(0^+) = \frac{2e^2}{h} \sqrt{|eV + \mu_N|}$.

3. Results and discussion

In this section, we proceed to analyze the numerical results and reveal manifestations of the anisotropic structures of SDMs intermediate between the linear and quadratic spectra. In the case of $\mu_s < \Delta_0$, the subgap differential conductance vanishes at $eV = \mu_N$ due to the absence of AR for any angle of incidence. Since the subgap differential conductance is dominated by the retro-AR and specular-AR, respectively, in the regimes of $eV < \mu_N$ and $eV > \mu_N$, the line of $eV = \mu_N$ serves as a boundary between the retro-AR and specular-AR in the subgap regime [7–15]. In the case of $\theta = 0^\circ$, when $eV \neq \mu_N$ the subgap differential conductance is non-vanishing even in the presence of large momentum-mismatch between the N and S regions (i.e., the large difference between $\mu_N$ and $\mu_S$), as depicted in figure 2(c). Since the subgap differential conductance only vanishes at $eV = \mu_N$, the line of $eV = \mu_N$ serves as an unambiguous boundary between the retro-AR and specular-AR, as shown in figure 2(a). However, for $\theta = 90^\circ$, in the regime of $\mu_N < \Delta_0$ the subgap differential conductance is strongly suppressed by the large momentum-mismatch, so that $G(90^\circ)|_{eV<\Delta_0} \simeq G(90^\circ)|_{eV=\mu_N} = 0$, as illustrated in figure 2(d). Consequently, in the case of $\theta = 90^\circ$, the boundary between the retro-AR and specular-AR disappears, as shown in figure 2(b).

The anisotropic aspect of the subgap transport also manifests itself in the distinct behaviors of the zero-bias differential conductance for $\theta = 0^\circ$ and $\theta = 90^\circ$. As shown in figure 2(c), when $\theta = 0^\circ$ the differential conductance always exhibits a zero-bias peak in the regime of $\mu_N \leq \Delta_0$. Remarkably, in the case of $0 < \mu_N \leq \Delta_0$ $G(0^+) |_{eV=0}$ takes a universal value of $\frac{8}{3}$ (e.g., for $\mu_N/\Delta_0 = 0.1, 0.5, 1$), implying that the zero-bias differential conductance is insensitive to the momentum-mismatch. On the contrary, as can be seen in figure 3(b), the zero-bias differential conductance strongly depends on the momentum-mismatch for $\theta = 90^\circ$ and $G(90^\circ)|_{eV=0}$ almost vanishes in the regime of $\mu_N \leq \Delta_0$. Therefore, the behavior of zero-bias differential conductance is strongly orientation-dependent.

To address the unique anisotropy of the subgap differential conductance in SDM-based NS junctions, we compare our results with those in similar NS junctions based on the Dirac materials with isotropic dispersions. Since the excitations in SDMs disperse linearly for $\theta = 0^\circ$, we compare $G(0^\circ)$ with that in NS junctions based on graphene and topological insulators which possess linear low-energy excitations in normal states [11–13]. As shown in figure 2(c), although the configuration of $G(0^\circ)$ is similar as that in NS junctions based on graphene [11, 12] and topological insulators [13], there is a significant difference on the value of $G(0^\circ)|_{eV=0}$ for $eV = 0$. For $eV = 0$, the reflection probabilities in the regime of $0 < \mu_N \ll \mu_S$ can be analytically expressed as $R_{\alpha e}|_{eV=0} = \frac{d^\dagger}{\mu_S^2}$ and $R_{\alpha e,\alpha}|_{eV=0} = \frac{d^\dagger}{\mu_S^2}$ In terms of the BTK formula [4], we have $G(0^\circ)|_{eV=0} = \sqrt{\frac{\mu_S}{\mu_N}} \int_{0}^{\pi/2} (2 - \frac{d^\dagger}{\mu_S^2})dq = \frac{8}{3}$, differing from the value of $\frac{8}{3}$ in graphene- and topological-insulator-based NS junctions [11–13]. This consequence can be ascribed to the unique band structures of SDMs intermediate between the linear and quadratic spectra. In the case of $\theta = 90^\circ$, the transport is along the quadratic dispersion direction. Because bilayer graphene harbors quadratic low-energy excitations, we compare $G(90^\circ)$ with the differential conductance in bilayer-graphene-based NS junctions. As proposed in references [7–9], in bilayer-graphene-based NS junctions with weakly doped N regions, the zero-bias differential conductance remains finite and the boundary between the retro-AR and specular-AR is unambiguous. In contrast, as charted out in figures 2(b) and (d), in the SDM-based NS junction with $\mu_N \leq \Delta_0$, the zero-bias differential conductance $G(90^\circ)|_{eV=0}$ almost vanishes and the
Figure 2. Panels (a) and (b) depict, respectively, the contour plots of the normalized differential conductance \( \frac{G(0^\circ)}{G_0(0^\circ)} \) and \( \frac{G(90^\circ)}{G_0(90^\circ)} \). Panels (c) and (d) present the bias-voltage-dependent normalized differential conductance for \( \theta = 0^\circ \) and \( \theta = 90^\circ \), respectively. The inset in panel (d) is the zoom-in of the sub-gap conductance for \( \mu_n/\Delta_0 = 0, 0.1, 0.5, \) and 1. In all panels, \( Z = 0 \).

Figure 3. Schematic plots of pseudo-spin texture of an electron-like conduction band in the N region of a SDM-based NS junction, where the arrow denotes the pseudo-spin and the solid curve depicts the iso-energy contour. The symbol \( +(-) \) indicates that the direction of \( \tilde{v}_x \) is parallel (anti-parallel) to the current direction, and the notations IN and NR denote, respectively, the regions of incidence and normal reflection in the momentum space.

crossover behavior from the retro-AR to specular-AR disappears. The novel subgap transport properties in SDM-based NS junctions can be ascribed to the unique pseudo-spin textures of SDMs.

To understand the underlying physics behind the anisotropic subgap transport properties, it is instructive to reveal the pseudo-spin textures of SDMs. To do so, we define the pseudo-spin vector as \( \mathbf{P} \equiv \langle \sigma \otimes \tau_0 \rangle \), which is a unit vector consisting of the expectation values of operator \( \sigma \otimes \tau_0 \) in normalized basis scattering states, where \( \sigma \equiv (\sigma_x, \sigma_y) \) [51–55]. Specifically, the pseudo-spin carried by a propagating
electron-like mode can be formulated as

$$P_e(\theta) = \left( \frac{k_e \cos \theta - q \sin \theta}{E + \mu_N} \right) \left( \frac{E + \mu_N}{E + \mu_N} \right)^2. \tag{9}$$

The pseudo-spin textures of electron-like conduction band for $\theta = 0^\circ$ and $\theta = 90^\circ$ are schematically shown in figure 3. As can be seen, the pseudo-spin configuration is highly anisotropic, in stark contrast to that of Dirac materials [56–58]. In the NS junction with $\theta = 0^\circ$, when $q$ is small enough, the pseudo-spin carried by the electron-like incident mode is nearly opposite to that of the normally reflected one. Consequently, the conservation of pseudo-spin suppresses the NR, giving rise to an enhanced AR even in the presence of strong momentum-mismatch. Moreover, since the conductance is mainly contributed by the modes with small $q$, the differential conductance keeps finite for $eV \neq \mu_N$, leading to a clear boundary between the retro-AR and specular-AR. Whereas for $\theta = 90^\circ$, as depicted in figure 3(b), the incident and normally reflected modes carry the same pseudo-spin, so that the NR is favorable in the presence of large momentum-mismatch. Therefore, in the case of $\theta = 90^\circ$, the AR and subgap differential conductance are strongly suppressed by the large momentum-mismatch, i.e., $G(90^\circ)|_{eV < \Delta_N} \simeq G(90^\circ)|_{eV = \mu_N} = 0$, thus blurring the boundary between the retro-AR and specular-AR.

We now turn to the transport properties in a SDM-based NS junction extending along an arbitrary direction. When the N region is weakly doped satisfying $\mu_N < \Delta_0$, as shown in figure 4(a), for $\theta \lesssim 75^\circ$, the subgap differential conductance exhibits a clear crossover from retro-AR to specular-AR with increasing the bias-voltage. While for $\theta \gtrsim 75^\circ$, the boundary between retro-AR and specular-AR is ambiguous. Therefore, the crossover from the retro-AR to specular-AR is blurred with increasing the orientation angle $\theta$. For the NS junction with a heavily doped N region, we find that the subgap differential conductance non-monotonically varies with $\theta$, as depicted in figure 4(b). This signature is quite different from that in similar NS junctions based on type-II Weyl semimetals [16] and monolayer phosphorus [50], where the differential conductance decays monotonically when the orientation angle increases.

### 3.2. Effects of the interfacial barrier

Owing to the intrinsic anisotropy of SDMs, the effects of the interfacial barrier on the differential conductance are orientation-dependent. For $\theta = 0^\circ$, the pseudo-spin carried by the incident and normally reflected modes are nearly opposite to each other when the incident angle is small enough, resulting in the prohibition of NR and the enhancement of AR. As a consequence, the zero-bias differential conductance periodically oscillates with $Z$ without a decaying profile, as charted out by figure 5(a). In addition, by inspecting figure 5(a) one can find that the oscillation period of $Z$-dependent $G(0^\circ)$ increases by dropping the chemical potential $\mu_N$. In the limit of $\mu_N \gg \max(\mu_N, eV, \Delta_0)$, the oscillation period goes to infinity, so that the differential conductance becomes insensitive to the interfacial barrier. On the contrary, when $\theta \neq 0^\circ$ the differential conductance exponentially decays with increasing the interfacial barrier strength, as shown in figures 5(b)–(d). In this regard, the influences of the interfacial barrier on the subgap transport are orientation-dependent, in stark contrast to that in superconducting hybrids based on type-II Weyl semimetals. In type-II-Weyl-semimetal-based superconducting hybrids, the differential conductance oscillates with the interfacial barrier strength without a decaying profile, although the low-energy excitations in type-II Weyl semimetals are also anisotropic due to the tilting of Weyl cones [17]. The anisotropic aspect of the $Z$-dependent differential conductance in SDM-based NS junctions can be ascribed...
Figure 5. Panels (a) and (b) illustrate the contour plots of the zero-bias differential conductance $G(0^\circ)/G_0(0^\circ)$ and $G(90^\circ)/G_0(90^\circ)$, respectively, where the starting value of $\mu_N$ is chosen as $10^{-3}\Delta_0$. Panels (c) and (d) present the zero-bias differential conductance, with $\mu_N/\Delta_0 = 0.5$ in (c) and $\mu_N/\Delta_0 = 50$ in (d).

to the novel pseudo-spin textures of SDMs as well as the unique band structures of SDMs intermediate between the linear and quadratic spectra.

4. Conclusion

In conclusion, in the framework of BdG equation, we have figured out the manifestations of the intrinsic anisotropy of SDMs in the AR configurations and subgap differential conductance. For the transport along the linear dispersion direction ($\theta = 0^\circ$), the subgap differential conductance exhibits a clear crossover from the retro-AR to specular-AR with increasing the bias-voltage, and the zero-bias differential conductance is insensitive to the momentum mismatch when the N region is weakly doped. Moreover, when the interfacial barrier strength increases, the zero-bias conductance exhibits an oscillating configuration without a decaying profile. However, for the transport along the quadratic dispersion direction ($\theta \neq 0^\circ$), the boundary between the retro-AR and specular-AR fades out with increasing the orientation angle, and the zero-bias differential conductance rapidly drops when the momentum mismatch or the interfacial barrier strength increases. We have elucidated the anisotropic coherent transport properties resorting to the unique pseudo-spin textures of SDMs. These results would provide intriguing insights for the coherent transport in SDMs-based superconducting hybrid structures, and we anticipate more interesting results for the Andreev bound states and supercurrents in SDMs-based Josephson junctions.

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Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

Appendix A. Derivation of the BdG Hamiltonian

We derive the BdG Hamiltonian from the lattice model of a graphene-like system in proximity to an s-wave superconductor. Physically, the semi-Dirac dispersion can be realized in a graphene-like systems with the NN lattice vectors $b_1 = \frac{2}{3} (1, \sqrt{3})$ and $b_2 = \frac{2}{3} (1, -\sqrt{3})$ with $a$ the interatomic distance, and $s = +(-)$ for $\alpha = \uparrow(\downarrow)$. Performing Fourier-transformations according to $a_{k}\alpha = \frac{1}{\sqrt{N}} \sum_{l} a_{l}\alpha e^{-i k l}$ and $b_{k}\alpha = \frac{1}{\sqrt{N}} \sum_{l} b_{l} \alpha e^{-i k l}$, we arrive at

$$H = \sum_{k,\alpha} \left[ t \left( \gamma_k b_{k,\alpha}^{\dagger} a_{k,\alpha} + \gamma_k^{*} a_{k,\alpha}^{\dagger} b_{k,\alpha} \right) - \mu \sum_{k,\alpha} (a_{k,\alpha}^{\dagger} a_{k,\alpha} + b_{k,\alpha}^{\dagger} b_{k,\alpha}) \right] + \sum_{k,\alpha} \left[ s \Delta (a_{k,\alpha}^{\dagger} a_{-k,-\alpha} + b_{k,\alpha}^{\dagger} b_{-k,-\alpha}) + s \Delta^{*} (a_{-k,-\alpha} a_{k,\alpha} + b_{-k,-\alpha} b_{k,\alpha}) \right],$$

where the parameter $\gamma_k$ is given by

$$\gamma_k = e^{-i k \delta_1} + e^{-i k \delta_2} + \frac{\mu}{t} e^{-i k \delta_3} = 2 \cos(\sqrt{3} k a / 2) e^{-i k \delta_3 / 2} + \frac{\mu}{t} e^{i k \delta_3},$$

with the NN lattice vectors being defined as $\delta_1 = \frac{2}{3} (1, \sqrt{3})$, $\delta_2 = \frac{2}{3} (1, -\sqrt{3})$, and $\delta_3 = a(-1,0)$. Accordingly, the dispersion for the normal state is $\epsilon = \pm |\gamma_k|$. For the critical case of $t = 2t$, $\epsilon$ vanishes at $M = (\frac{2\pi}{3 a}, 0)$, and the normal state reaches the semi-Dirac phase. Now we linearize the Hamiltonian $H$ for a small $k$ around the $M$ point by setting $k_x \rightarrow k_x + \frac{2\pi}{3 a}$ and $k_y \rightarrow k_y$, with $k_x a \ll 1$ and $k_y a \ll 1$ being satisfied. Up to the leading order in $k$, we arrive at

$$\gamma_{M+k} \approx \left( 3 i a_k e^{2i \pi / 3} + \frac{3}{4} a^2 k_x^2 e^{2i \pi / 3} \right),$$

$$\gamma_{M+k}^{*} \approx \left( -3 i a_k e^{-2i \pi / 3} + \frac{3}{4} a^2 k_x^2 e^{-2i \pi / 3} \right),$$

$$\gamma_{-M-k} \approx \left. \left( -3 i a_k e^{-2i \pi / 3} + \frac{3}{4} a^2 k_x^2 e^{2i \pi / 3} \right) \right|_{M}$$

Substituting equation (A.4) into equation (A.2), we obtain the Hamiltonian at $q = M + k$ point as

$$H_q = \frac{t}{2} \sum_{q,\alpha} \left[ \left( 3 i a_k + \frac{3 a^2}{4} k_x^2 \right) e^{i \frac{2\pi}{3} q_x} b_{q,\alpha}^{\dagger} a_{q,\alpha} + \left( -3 i a_k + \frac{3 a^2}{4} k_x^2 \right) e^{-i \frac{2\pi}{3} q_x} a_{q,\alpha}^{\dagger} b_{q,\alpha} \right]$$

$$+ \left[ -3 i a_k + \frac{3 a^2}{4} k_x^2 \right] e^{-i \frac{2\pi}{3} q_x} b_{q,\alpha}^{\dagger} a_{-q,-\alpha} + \left( 3 i a_k + \frac{3 a^2}{4} k_x^2 \right) e^{-i \frac{2\pi}{3} q_x} a_{-q,-\alpha}^{\dagger} b_{-q,-\alpha}$$

$$- \mu \sum_{q,\alpha} (a_{q,\alpha} a_{q,\alpha}^{\dagger} + b_{q,\alpha} b_{-q,-\alpha}^{\dagger} + a_{-q,-\alpha} a_{-q,-\alpha} a_{q,\alpha} + b_{-q,-\alpha} b_{q,\alpha} b_{q,\alpha})$$

$$+ \sum_{q,\alpha} \left[ s \Delta (a_{q,\alpha} a_{-q,-\alpha} + b_{q,\alpha} b_{-q,-\alpha}) + s \Delta^{*} (a_{-q,-\alpha} a_{q,\alpha} + b_{-q,-\alpha} b_{q,\alpha}) \right],$$

(A.5)
where the relation of $\sum_{k} = \sum_{-k}$ has been used. Rewrite $\tilde{H}_q$ in the form of $\tilde{H}_q = \sum_{q} \psi_{q}^\dagger \tilde{H}_q \psi_{q}$, with the basis
\[
\psi_{q}^\dagger = [a_{q}^\dagger, b_{q}^\dagger, a_{-q}, b_{-q}, a_{q}^\dagger, b_{q}, a_{-q}, b_{-q}],
\]
and
\[
\tilde{H}_q = \begin{pmatrix} H(k, \Delta) & 0 \\ 0 & \tilde{H}(k, -\Delta) \end{pmatrix}.
\]
By defining $v \equiv \frac{\hbar\Delta}{\mu}$, $\eta \equiv \frac{\hbar v}{\eta_0}$, and $\mu/2 \rightarrow \mu$, the upper block can be formulated as
\[
\tilde{H}(k, \Delta) = \begin{pmatrix} -\mu & -(i\hbar v_x + \eta k_y^2)e^{i\phi} \Delta & 0 \\ (i\hbar v_x + \eta k_y^2)e^{i\phi} \Delta^\dagger & -\mu & 0 \\ 0 & 0 & \mu \end{pmatrix},
\]
where the anticommutation relation has been employed. Taking a unitary transformation $H_{BdG} \equiv U^\dagger \tilde{H} U$ with
\[
U = \begin{pmatrix} e^{i\pi/4} & 0 & 0 \\ 0 & 0 & e^{i\pi/4} \\ 0 & e^{-i\pi/4} & 0 \end{pmatrix},
\]
The BdG Hamiltonian can be compactly written as
\[
H_{BdG} = (\hbar v_x \sigma_x + \eta k_y^2 \sigma_y - \mu \sigma_0)\tilde{\tau}_z + \Delta_0(\cos \phi \tilde{\sigma}_x - \sin \phi \tilde{\sigma}_y),
\]
where we rewrite $\Delta = \Delta_0 e^{i\phi}$ with $\Delta_0$ the amplitude of pairing potential and $\phi$ the superconducting phase, $\tilde{\sigma}_y$ is a $2 \times 2$ identity matrix, and the Pauli matrices $\tilde{\sigma}_{x,y}$ and $\tilde{\tau}_{x,y,z}$ act on the pseudo-spin and Nambu spaces, respectively.

**Appendix B. Calculation of the scattering states in NS junctions based on semi-Dirac materials**

In this appendix we present necessary calculation details regarding the wave functions and related quantities in SDMs-based NS junctions.

**B.1. Scattering states for $\theta \neq 0$**

We assume the translational symmetry in the $Y$-direction is preserved, so that the transverse momentum $q$ can be treated as a good quantum number. By solving the rescaled BdG equation $H_{BdG}(-i\partial_x, q) \varphi = E \varphi$, the basis states in the N region are given by
\[
\varphi_{e}^{(+)} = \begin{pmatrix} (k_{e,1}(2) \cos \theta - q \sin \theta) - i(k_{e,1}(2) \sin \theta + q \cos \theta) \right)^2 E + \mu N \\ 0 \\ 0 \end{pmatrix} e^{i(k_{e,1}(2)X + qY)},
\]
\[
\varphi_{e}^{(-)} = \begin{pmatrix} (k_{e,1} \cos \theta - q \sin \theta) - i(k_{e,1} \sin \theta + q \cos \theta) \right)^2 E + \mu N \\ 0 \\ 0 \end{pmatrix} e^{i(k_{e,1}X + qY)},
\]
\[
\varphi_{h}^{(+)} = \begin{pmatrix} 0 \\ 0 \\ - (k_{h,1}(2) \cos \theta - q \sin \theta) + i(k_{h,1}(2) \sin \theta + q \cos \theta) \right)^2 E - \mu N \end{pmatrix} e^{i(k_{h,1}(2)X + qY)},
\]
\[
\varphi_{h}^{(-)} = \begin{pmatrix} 0 \\ 0 \\ - (k_{h,1} \cos \theta - q \sin \theta) + i(k_{h,1} \sin \theta + q \cos \theta) \right)^2 E - \mu N \end{pmatrix} e^{i(k_{h,1}X + qY)},
\]
where the related quantities $k_{e(h),1(2)}$ and $k_{e(h),1}$ need to be numerically determined in accordance with equation (2) shown in the main text.

In the S region, the rescaled dispersion is given by

$$E_S^{\nu, \lambda} = \nu \sqrt{\left( k_S \cos \theta - q \sin \theta \right)^2 + \left( k_S \sin \theta + q \cos \theta \right)^2 + \Delta_S^2},$$

(B.2)

where $\nu = \pm$ and $\lambda = \pm$, indicating the four different branches. Since the relation of $\Delta_0 \ll \mu_S$ should be satisfied to ensure the validity of mean-field approximation, the branches $E_S^{\nu, +}$ are far away from the subgap regime and only $E_S^{\nu, -}$ bands contribute to the subgap conductance. In terms of the rescaled quantities, the basis states of the S region can be formulated as

$$\varphi_{e(h),1(2)} = \left( \begin{array}{c} k_{e(h),1(2)} \cos \theta - q \sin \theta - i (k_{e(h),1(2)} \sin \theta + q \cos \theta)^2 \\ \sqrt{k_{e(h),1(2)} \cos \theta - q \sin \theta)^2 + (k_{e(h),1(2)} \sin \theta + q \cos \theta)^4} e^{-i \beta + \phi} \\ \sqrt{k_{e(h),1(2)} \cos \theta - q \sin \theta)^2 + (k_{e(h),1(2)} \sin \theta + q \cos \theta)^4} e^{-i \beta - \phi} \end{array} \right),$$

$$\varphi_{h(q),1(2)} = \left( \begin{array}{c} k_{h(q),1(2)} \cos \theta - q \sin \theta - i (k_{h(q),1(2)} \sin \theta + q \cos \theta)^2 \\ \sqrt{k_{h(q),1(2)} \cos \theta - q \sin \theta)^2 + (k_{h(q),1(2)} \sin \theta + q \cos \theta)^4} e^{-i \beta + \phi} \\ \sqrt{k_{h(q),1(2)} \cos \theta - q \sin \theta)^2 + (k_{h(q),1(2)} \sin \theta + q \cos \theta)^4} e^{-i \beta - \phi} \end{array} \right).$$

(B.3)

where $\beta = \cos^{-1}(\varepsilon/\Delta_0) \Theta(\Delta_0 - |\varepsilon|) - i \cosh^{-1}(\varepsilon/\Delta_0) \Theta(|\varepsilon| - \Delta_0)$, and the values of $k_{e(h),1(2)}$ can be numerically calculated from equation (B.2). We note that for $|\varepsilon| > \Delta_0$, $\varphi_{e(h),1(2)}$ denote the electron/hole-like scattering states propagating along the X-direction, while $\varphi_{e(h),1(2)}$ represent the evanescent modes.

B.2. Scattering states for $\theta = 0$

In the S region, the basis scattering states are given by

$$\psi^\pm_{e(h)} = \left( \begin{array}{c} \pm k_{e(h)} - i q^2 \\ \sqrt{k_{e(h)}^2 + q^4} \end{array} \right) e^{i \pm k_{e(h)} X + q Y},$$

(B.4)

$$\psi^\pm_{h(q)} = \left( \begin{array}{c} \pm k_{h(q)} - i q^2 \\ \sqrt{k_{h(q)}^2 + q^4} \end{array} \right) e^{i \pm k_{h(q)} X + q Y},$$

(B.5)

where the related parameter are defined by

$$k_{e(h),1(2)} = \pm(-1) \sqrt{\mu_S + (-1) \sqrt{E^2 - \Delta_0^2}} - q^4.$$  

(B.6)

In the N region, the basis scattering states can be formulated as

$$\psi^\pm_e = \left( \begin{array}{c} \pm k_e - i q^2 \\ E + \mu_N \end{array} \right) e^{i \pm k_e X + q Y},$$

(B.7)

$$\psi^\pm_h = \left( \begin{array}{c} 0 \\ \pm k_h + i q^2 \end{array} \right) e^{i \pm k_h X + q Y},$$

(B.8)
The boundary condition is given by
\[
(t_{\text{eq}}\psi_{\text{eq}}^+ + t_{\text{ha}}\psi_{\text{ha}}^-)^{x=\alpha+} = M(\alpha)(\psi_{\text{eq}}^+ + r_{\text{eq}}\psi_{\text{eq}}^- + r_{\text{ne}}\psi_{\text{ne}}^-)^{x=\alpha-},
\]  
(B.9)
[36] Saha K, Nandkishore R and Parameswaran S A 2017 Valley-selective Landau–Zener oscillations in semi-Dirac $p-n$ junctions *Phys. Rev. B* **96** 045424

[37] Nualpijit P, Sinner A and Ziegler K 2018 Tunable transmittance in anisotropic two-dimensional materials *Phys. Rev. B* **97** 235411

[38] Mawrie A and Muralidharan B 2019 Direction-dependent giant optical conductivity in two-dimensional semi-Dirac materials *Phys. Rev. B* **99** 075415

[39] Adrogue P, Carpentier D, Montambaux G and Orignac E 2016 Diffusion of Dirac fermions across a topological merging transition in two dimensions *Phys. Rev. B* **93** 125113

[40] Islam S F and Saha A 2018 Driven conductance of an irradiated semi-Dirac material *Phys. Rev. B* **98** 235424

[41] Carbotte J P and Nicol E J 2019 Signatures of merging Dirac points in optics and transport *Phys. Rev. B* **100** 035441

[42] Real B et al 2020 Semi-Dirac transport and anisotropic localization in polariton honeycomb lattices *Phys. Rev. Lett.* **125** 186601

[43] Wang J-R, Liu G-Z and Zhang C-J 2017 Excitonic pairing and insulating transition in two-dimensional semi-Dirac semimetals *Phys. Rev. B* **95** 075129

[44] Saha K 2016 Photoinduced Chern insulating states in semi-Dirac materials *Phys. Rev. B* **94** 081103

[45] Dutreix C, Bilteanu L, Jagannathan A and Bena C 2013 Friedel oscillations at the Dirac cone merging point in anisotropic graphene and graphenelike materials *Phys. Rev. B* **87** 245413

[46] Narayan A 2015 Floquet dynamics in two-dimensional semi-Dirac semimetals and three-dimensional Dirac semimetals *Phys. Rev. B* **91** 205445

[47] Wang J-R, Liu G-Z and Zhang C-J 2019 Fate of superconductivity in disordered Dirac and semi-Dirac semimetals *J. Phys. Commun.* **3** 055006

[48] Bruno U and Seo K 2017 Superconducting states for semi-Dirac fermions at zero and finite magnetic fields *Phys. Rev. B* **96** 220503

[49] Uryszek M D, Christou E, Jaefari A, Krüger F and Uchoa B 2019 Quantum criticality of semi-Dirac fermions in 2 + 1 dimensions *Phys. Rev. B* **100** 155101

[50] Linder J and Yokoyama T 2017 Anisotropic Andreev reflection and Josephson effect in ballistic phosphorene *Phys. Rev. B* **95** 144515

[51] San-Jose P, Prada E, McCann E and Schomerus H 2009 Pseudospin valve in bilayer graphene: towards graphene-based pseudospintronic *Phys. Rev. Lett.* **102** 247204

[52] Schomerus H 2010 Helical scattering and valleytronics in bilayer graphene *Phys. Rev. B* **82** 165409

[53] Majidi L and Zareyan M 2011 Pseudospin polarized quantum transport in monolayer graphene *Phys. Rev. B* **83** 115422

[54] Majidi L and Zareyan M 2012 Enhanced Andreev reflection in gapped graphene *Phys. Rev. B* **86** 075443

[55] Habe T 2019 Pseudospin triplet superconductivity in 2H-type transition-metal dichalcogenide monolayers *Phys. Rev. B* **100** 165431

[56] Breunig D, Burset P and Trauzettel B 2018 Creation of spin-triplet cooper pairs in the absence of magnetic ordering *Phys. Rev. Lett.* **120** 037701

[57] Uchida S, Habe T and Asano Y 2014 Andreev reflection in Weyl semimetals *J. Phys. Soc. Japan* **83** 064711

[58] Bai C and Yang Y 2020 Signatures of nontrivial Rashba metal states in a transition metal dichalcogenides Josephson junction *J. Phys.: Condens. Matter* **32** 465302