We study a family of equations of state (EoS) for hybrid neutron star (NS) matter. The hybrid EoS are obtained from a Maxwell construction of a first-order phase transition between an instantaneous nonlocal version of NJL model in $SU(2)$ with vector interactions and color superconductivity describing the quark matter (QM) phase and the “DD2” EoS with excluded volume and a crust at low baryon densities for the hadronic phase. The form factor in the nonlocal QM model is fitted to lattice QCD (LQCD) results in the Coulomb gauge. To simultaneously fulfill the constraints from the NICER radius measurement for PSR J0740+6620 and tidal deformability from GW170817 it is necessary to consider a $\mu$-dependent bag constant that mimics confinement. Our results show an asymptotic constant behaviour for the squared sound of sound that reproduces the conformal limit value of $1/3$ from perturbative QCD in the free case and larger, approximately constant, values of $0.4 - 0.6$ in the region of NS core densities when interactions are turned on.

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I. INTRODUCTION

With the advent of multi-messenger observations of neutron stars (NS) due to the detection of the gravitational wave signal from the inspiral phase of the binary NS merger GW170817 [1, 2] stringent new constraints for the equation of state (EoS) of NS matter appeared. The measured low value the tidal deformability $\Lambda = 190^{+390}_{-120}$ of NS at a mass of $1.4 M_\odot$ that was inferred from GW170817 [3] is a challenge for many stiff nuclear matter EoS such as DD2 [4], a standard relativistic density functional (RDF) EoS. On the other hand, soft nuclear EoS like APR [5] or SLy4 [6] which were favorably advocated in the GW170817 discovery paper [2], fail in matching the recent constraint from the NICER-XMM mass-radius measurement on PSR J0740+6620 that requires a large radius $R_{2,0} = 13.7^{+2.6}_{-1.5}$ km [7] ($R_{2,0} = 12.39^{+1.30}_{-0.98}$ km [8] from the Amsterdam team) at the mass $2.08 \pm 0.07 M_\odot$ [9]. Besides this, these nuclear EoS have the caveat that they do not include the appearance of the strangeness degree of freedom due to the onset of hyperons that would occur at masses $\sim 1.5 M_\odot$.

A successful description of the multi-messenger phenomenology of NS including the recent constraints on $R_{1.4}$ and $R_{2.0}$ has been given recently with hybrid stars based on the constant speed of sound (CSS) model for quark matter. A few examples can be found in the references [10–14]. Most of them choose the value of the squared sound speed $c_s^2$ freely, others vary the onset of the deconfinement transition and $c_s^2$ so that a sufficiently high maximum mass is obtained. Allowing values up to the causality limit $c_s^2 = 1$, one reaches maximum masses up to $4 M_\odot$ [10, 11]. Such an ambiguity is not satisfactory. Moreover, a justification for using the CSS model of the quark matter EoS is highly desirable.

This is the main motivation for the present work. Following up on initial indications that the EoS of a nonlocal chiral quark model with an instantaneous, separable interaction potential can be nicely described by a CSS model [15] and that also a covariant formulation of such a nonlocal NJL-type model is nicely fitted by a CSS model [13], we investigate this relationship between modern NS phenomenology and a microphysical approach formulated by a chiral quark model Lagrangian.

In the present work we present a hybrid description in order describe the transition from nuclear to quark
matter (QM). We start from a reliable QM model, with color superconductivity (2SC) and repulsive vector interactions, compatible with Coulomb gauge LQCD, that predicts CSS at high densities, to later enrich the model in order to satisfactory fulfill observational constraints. In Section II we describe the QM model and in Section III the construction of the hybrid EoS with its application to the modern neutron star phenomenological constraints on the mass-radius diagram and the tidal deformability. Then in Section IV we present the summary and conclusions. We added an Appendix with complementary calculations for the QM model, TOV and tidal deformability equations.

II. INSTANTANEOUS NONLOCAL CHIRAL QUARK MODEL WITH VECTOR INTERACTIONS AND 2SC

We describe QM within a nonlocal chiral quark model which includes scalar and vector quark-antiquark interactions and anti-triplet scalar diquark interactions. The corresponding effective Euclidean action in the case of two light flavors is given by

\[ S_E = \int d^4x \left\{ \bar{\psi}(x) (-i\gamma^\mu + m_c) \psi(x) - \frac{G_S}{2} j_S^f(x) j_S^f(x) - \frac{G_D}{2} [j_D^\sigma(x)]^\dagger j_D^\sigma(x) + \frac{G_V}{2} j_V^\mu(x) j_V^\mu(x) \right\} \]  

(1)

Here \( m_c \) is the current quark mass, which is assumed to be equal for \( u \) and \( d \) quarks, whereas the currents \( j_{S,D}(x) \) are given by nonlocal operators based on a separable approximation to the effective one gluon exchange model (OGE) of QCD. In this work we use natural units with \( \hbar = c = k_B = 1 \). The currents read

\[ j_S^f(x) = \int d^4 z \, g(z) \bar{\psi}(x + \frac{z}{2}) \Gamma_f \psi(x - \frac{z}{2}) , \]

\[ j_D^\sigma(x) = \int d^4 z \, g(z) \bar{\psi}_C(x + \frac{z}{2}) \gamma_5 \tau_2 \lambda_a \psi(x - \frac{z}{2}) , \]

\[ j_V^\mu(x) = \bar{\psi}(x) i\gamma^\mu \psi(x) , \]

(2)

where we have defined \( \psi_C(x) = \gamma_2 \gamma_4 \bar{\psi}^T(x) \) and \( \Gamma_f = (1, i\gamma_5 \vec{\tau}) \), while \( \vec{\tau} \) and \( \lambda_a \), with \( a = 2, 5, 7 \), stand for Pauli and Gell-Mann matrices acting on flavor and color spaces, respectively. The functions \( g(z) \) in Eqs. (2) are nonlocal “instantaneous” form factors (3D-FF) characterizing the effective quark interaction \[ 16, 17 \], which in momentum space depend on \( \vec{p} \). Note that the vector current in Eq. (2) is considered to be local. The reason will be given below. The effective action in Eq. (1) might arise via Fierz rearrangement from some underlying more fundamental interactions, and is understood to be used—at the mean field level—in the Hartree approximation. The vector MF plays a special role since it affects the chemical potential and thus the baryon density which is a constraint and not a dynamic degree of freedom. In general, the ratios of coupling constants \( \eta_D = G_D/G_S \), \( \eta_V = G_V/G_S \) would be determined by this microscopic couplings; for example, OGE interactions in the vacuum lead to \( \eta_D = 0.75 \) and \( \eta_V = 0.5 \). However, since the precise derivation of effective couplings from QCD is not known, there is a large theoretical uncertainty in these ratios. Details of the values used in the present work will be given below.

Now we consider the Euclidean action at both finite temperature \( T \) and baryon chemical potential \( \mu_B \). The simplicity of the 3DFF is that the Matsubara summation can be performed analytically. We introduce different chemical potentials \( \mu_{fc} \) for each flavor and color. In principle one has six different quark chemical potentials, corresponding to quark flavors \( u \) and \( d \) and quark colors \( r,g \) and \( b \). However, there is a residual color symmetry (say, between red and green colors) arising from the direction of \( \Delta \) in color space. Moreover, if we require the system to be in chemical equilibrium, it can be seen that chemical potentials are not independent from each other. In general, it is shown that all \( \mu_{fc} \) can be written in terms of three independent quantities: the baryonic chemical potential \( \mu_B \), a quark electric chemical potential \( \mu_{Qc} \) and a color chemical potential \( \mu_R \). The corre-
sponding relations read

\[ \mu_{ur} = \mu_{ug} = \frac{\mu_B}{3} + \frac{2}{3} \mu_{Q_s} + \frac{1}{3} \mu_8 \]
\[ \mu_{dr} = \mu_{dg} = \frac{\mu_B}{3} - \frac{1}{3} \mu_{Q_s} + \frac{1}{3} \mu_8 \]
\[ \mu_{ub} = \mu_{ub} = \frac{\mu_B}{3} + \frac{2}{3} \mu_{Q_s} - \frac{2}{3} \mu_8 \]
\[ \mu_{db} = \frac{\mu_B}{3} - \frac{1}{3} \mu_{Q_s} - \frac{2}{3} \mu_8. \]  

(3)

The chemical potential \( \mu_{Q_s} \), which distinguishes between up and down quarks, as well as the color chemical potential \( \mu_8 \), which has to be introduced to ensure color neutrality, vanish for an isospin symmetric quark matter system.

As we considered here only the vector meson mean field \( \bar{\omega} \), that comes from the term with \( \gamma_0 \) in the vector current in Eqs. (2), the chemical potentials are shifted as

\[ \bar{\mu}_c = \mu_c - \bar{\omega}. \]  

(4)

Since we considered in Eqs. (2) that the vector current is local, the above shown renormalized chemical potentials depends on the mean field \( \bar{\omega} \). If we were considered a non-local vector current, the term driven by \( \bar{\omega} \) would include the form factor, given as a result, a chemical potential dependence on the 3-momentum.

Following Ref. [18], it is convenient to define

\[ \bar{\mu}_c = \frac{\mu_{uc} + \mu_{dc}}{2} \]  

(5)

and

\[ \delta \bar{\mu}_c = \frac{\mu_{uc} - \mu_{dc}}{2}. \]  

(6)

Thus, the corresponding mean field grand canonical thermodynamic potential per unit volume can be written as

\[ \Omega^{MFA} = -\frac{T}{V} \ln Z^{MFA} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\Delta^2}{2G_D} - \bar{\omega}^2 \]
\[ -2 \int \frac{d^3\bar{p}}{(2\pi)^3} \xi(\bar{p}), \]  

(7)

where

\[ \xi(\bar{p}) = \sum_{\kappa,\lambda = \pm} \left\{ \frac{E^\kappa_\lambda}{2} + T \ln \left[ 1 + e^{-\frac{E^\kappa_\lambda + \delta\bar{\mu}_c}{T}} \right] \right\} \]
\[ + \sum_{\kappa,\lambda = \pm} \left\{ \frac{\bar{E}^\kappa_\lambda}{2} + T \ln \left[ 1 + e^{-\frac{\bar{E}^\kappa_\lambda + \delta\bar{\mu}_c}{T}} \right] \right\}. \]  

(8)

In the above expression the \( \pm \) means that one has to consider two terms with each sign. We have defined:

\[ E^\pm_\kappa = E^\pm_\kappa \sqrt{1 + \left[ g(\bar{p})/E^\pm_\kappa \right]^2} \]  

(9)

for the gapped colors, where

\[ E^\pm_\kappa = E \pm \bar{\mu}_r \]  

(10)

and

\[ E^\pm_\delta = E \pm \bar{\mu}_b \]  

(11)

for the ungapped “blue” color of quarks with the dispersion relation

\[ E^2 = \bar{p}^2 + M^2(\bar{p}). \]  

(12)

Here, the momentum-dependent quark mass function is

\[ M(\bar{p}) = m_q + g(\bar{p})\bar{\sigma}. \]  

(13)

The mean field values \( \bar{\sigma}, \Delta \) are obtained from the coupled gap equations together with the constraint equation for \( \bar{\omega} \)

\[ \frac{\partial \Omega^{MFA}}{\partial \bar{\sigma}} = 0, \quad \frac{\partial \Omega^{MFA}}{\partial \Delta} = 0, \quad \frac{\partial \Omega^{MFA}}{\partial \bar{\omega}} = 0. \]  

(14)

Then the corresponding EoS can be obtained by inserting into Eq. (7) the mean field values \( \bar{\sigma}, \bar{\omega}, \Delta \), which are obtained by solving the gap equations of Eqs. (14), explicitly shown in Eqs. (A.2-A.4), and using the regularization prescription of Eq. (A.5). Thus, the pressure of the quark matter is given by

\[ P_q = -\Omega^{MFA}_{\text{reg}}. \]  

(15)

Now, if we want to describe the behavior of quark matter in the core of neutron stars, in addition to quark matter we have to take into account the presence of electrons and muons. Thus, treating leptons as a free relativistic Fermi gas, the total pressure of the quark matter plus leptons is given by

\[ P = P_q + P_{\text{lep}}, \]  

(16)

where \( P_{\text{lep}} \) is given by

\[ P_{\text{lep}} = 2 \frac{T}{\bar{\sigma}} \sum_{\ell = e, \mu, \pm} \int \frac{d^3\bar{p}}{(2\pi)^3} \ln \left[ 1 + e^{\frac{\epsilon_{\ell}^{\ell} + \mu_\ell}{T}} \right], \]  

(17)

with \( \epsilon_{\ell} = \sqrt{\bar{p}^2 + m_\ell^2} \) and the chemical potential \( \mu_e = \mu_\mu \).

In addition, it is necessary to take into account that quark matter has to be in beta equilibrium with electrons and muons through the beta decay reactions

\[ d \to u + l + \bar{\nu}_l, \quad u + l \to d + \nu_l, \]  

(18)
for \( l = e, \mu \). Thus, assuming that (anti)neutrinos escape from the stellar core, we have an additional relation between fermion chemical potentials, namely
\[
\mu_{de} - \mu_{u\ell} = -\mu_{Q_q} = \mu_l \quad (19)
\]
for \( c = r, g, b \), \( \mu_l = \mu_e = \mu_\mu \).

Finally, in the core of neutron stars we also require the system to be electric and color charge neutral, hence the number of independent chemical potentials reduces further. Indeed, \( \mu_l \) and \( \mu_8 \) get fixed by the condition that charge and color densities vanish,
\[
\rho_{Q_{tot}} = \rho_{Q_e} - \sum_{l=e,\mu} \rho_l = 0,
\]
\[
\rho_8 = \frac{1}{3} \sum_{f=u,d} (2 \rho_{fr} + \rho_{fg} - 2 \rho_{f\ell}) = 0, \quad (20)
\]
where the expressions for the different densities can be found in the Appendix 1.

In summary, in the case of neutron star quark matter, for each value of \( T \) and \( \mu_B \) one can find the values of \( \hat{\Delta} \), \( \hat{\sigma} \), \( \hat{\omega} \), \( \mu_l \) and \( \mu_8 \) by solving Eqs. (14), supplemented by Eqs. (19) and (20). This allows to obtain the quark matter EoS in the thermodynamic region we are interested in.

The energy density can be written as
\[
\varepsilon = -P + T \, s + G \quad (21)
\]
where \( s = -\partial G/\partial T \). Here, \( G \) is Gibbs free energy, that depends on conserved charges
\[
G = \sum_{\alpha} \mu_{\alpha} \rho_{\alpha} = \mu_B \rho_B + \mu_Q \rho_Q + \mu_8 \rho_8. \quad (22)
\]
By imposing electric charge and color charge neutrality, the last two terms of the above equation are zero, then, \( G \) can be written as
\[
G = \sum_{f,c} \mu_{f,c} \rho_{f,c} + \sum_{l=e,\mu} \mu_l \rho_l, \quad (23)
\]
where \( \rho_B = (1/3)(\rho_u + \rho_d) \) and \( \rho_f = \sum_c \rho_{f,c} \).

### A. Zero Temperature limit

In the present work we are interested on describing hybrid EoS for cold compact stellar systems. Then, we will take the zero limit for the temperature. The corresponding mean field grand canonical thermodynamic potential per unit volume can be written as in Ref. [18]
\[
\Omega^{MFA}_0 = \frac{\bar{\sigma}^2}{2G_S} + \frac{\hat{\Delta}^2}{2G_D} - \frac{\hat{\omega}^2}{2G_V} - \int \frac{d^3\bar{p}}{(2\pi)^3} \xi_T(\bar{p}) + \Omega^{lep}_T = 0 \quad (24)
\]
where,
\[
\xi_T(\bar{p}) = \sum_{\kappa,s=\pm} \left\{ 2 [\epsilon_{\kappa}^s + s \bar{\mu}_r] - E_0 \right\} + [E_b^\kappa + s \bar{\mu}_b] - E_0 \right\} \quad (25)
\]
For leptons, the thermodynamic potential at vanishing temperature \( \Omega^{lep}_0 \) is given by [16]
\[
\Omega^{lep}_T = - \frac{1}{24\pi^2} \sum_{l=e,\mu} m_l^4 F(\mu_l/m_l), \quad (26)
\]
with
\[
F(x) = 2 x (x^2 - 5/2) \sqrt{x^2 - 1} + 3 \ln (x + \sqrt{x^2 - 1})
\]
valid when \( \mu_l > m_l \).

It should be noticed that, in general, there might be regions for which there is more than one solution for each value of \( \mu_B \). The stable solution correspond to an overall minimum of the thermodynamic potential.

Given the thermodynamic potential, the expressions for all other relevant quantities can be easily derived. The quark and lepton densities are defined as
\[
\rho_{f,c} = - \frac{\partial \Omega^{MFA}_T = \Omega^{lep}_T}{\partial \mu_{f,c}} \quad , \quad \rho_l = - \frac{\partial \Omega^{MFA}_T = \Omega^{lep}_T}{\partial \mu_l}. \quad (27)
\]
For each flavor the quark chiral condensate is defined as
\[
\langle \bar{\psi} \psi \rangle = \frac{\partial \Omega^{MFA}_T = \Omega^{lep}_T}{\partial \mu_{c}}. \quad (28)
\]
Finally, a magnitude which is important to determine the characteristic of the chiral phase transition is the chiral susceptibility \( \chi \), and it can be calculated as
\[
\chi = - \frac{\partial^2 \Omega^{MFA}_T = \Omega^{lep}_T}{\partial m_c^2} = - \frac{\partial \langle \bar{\psi} \psi \rangle}{\partial m_c}. \quad (29)
\]

### B. Form Factors and set of parameters

To fully specify the nonlocal NJL model under consideration, one has to fix the model parameters as well as the instantaneous form factor \( g(\bar{p}) \) that characterize
the nonlocal interactions between quarks in both channels $q\bar{q}$ and $qq$. In this work we consider an exponential momentum dependence for the form factor in momentum space
\[ g(p^2) = \exp[-p^2/\Lambda_0^2] . \]
This form, which is widely used, guarantees a fast ultraviolet convergence of quark loop integrals. Notice that the energy scale $\Lambda_0$ has to be taken as an additional parameter of the model. Other functional forms, e.g., Lorentzian [19–21] or Woods-Saxon [22] form factors, have also been considered in the literature, concluding that the form factor choice does not have in general major impact in the qualitative predictions for the relevant thermodynamic quantities [16–18, 23, 24].

Given the form factor functions, it is possible to set the model parameters to reproduce the observed meson phenomenology. First, we perform a fit to LQCD results (in the Coulomb gauge) from Ref. [25] for the normalized quark effective mass $M(p)/M(0)$. This fit, quoted in Fig. 1, has been carried out considering results up to 3 GeV, obtaining $\Lambda_0 = 885.47$ MeV. 

![FIG. 1: LQCD data from Ref. [25] and our fit for the normalized quark effective mass.](image)

Finally, by requiring that the model reproduce the empirical values of two physical quantities, chosen to be the pion mass $m_\pi = 138$ MeV and the pion weak decay constant $f_\pi = 92.4$ MeV, one can determine the remaining model parameters $m_c = 2.29$ MeV and $G_S = 9.92$ GeV$^{-2}$.

C. Phase Diagram and speed of sound

Let us start by studying the behaviour of the mean field values for representative values of $\eta_D$ and $\eta_V$ as function of the baryonic chemical potential. Our results are shown in Fig. 2. There, as function of $\mu_B$, we quote the VEV’s $\bar{\sigma}$, $\bar{\Delta}$ and $\bar{\omega}$ and the lepton and color chemical potentials, $\mu_l$ and $\mu_8$ for $\eta_D = 1.10$ and $\eta_V = 0.50$. The critical chemical potentials $\mu_B^c$ are denoted with thin black vertical lines. The dotted-one, at $\mu_B = 888$ MeV, indicates the baryonic chemical potential at which the total pressure vanishes. Here, the mean field value of the diquark field vanishes denoting a second order phase transition. On the other hand, at $\mu_B = 931$ MeV, one finds the peak of the chiral susceptibility indicating a crossover phase transition to a region where the chiral symmetry is partially restored. In this narrow region of about 40 MeV, denoted by the grey band in the figure, one has 2SC with a finite and small value of the diquark gap, while the system is still in a chiral symmetry broken phase ($\chi_SB$).

![FIG. 2: $\bar{\sigma}$, $\bar{\Delta}$, $\bar{\omega}$, $\mu_l$ and $\mu_8$ (in MeV) as function of $\mu_B$ (in MeV).](image)

Carrying out the previous analysis in the range allowed by the model for $\eta_D$ and $\eta_V$, namely $0.0 < \eta_V < 1.2$ and $0.9 < \eta_D < 1.2$, we can set up the corresponding phase diagram and analyze the features of the phase transitions in the $\eta_D - \mu_B^c$ plane for different fixed values of $\eta_V$. 

\footnote{The normalization for the LQCD data has been done using data from Fig. (7d) of Ref. [25] together with Eq. (26) evaluated at $k = 0$ with the corresponding value of the LQCD bare quark mass.}
The phase diagram can be sketched by analyzing the numerical results obtained for the relevant order parameters. For the chiral symmetry restoration we take as order parameter the quiral quark condensate, while for the onset of the diquark condensation we take the VEV of the diquark field. The chiral critical chemical potentials are defined by the position of the peaks in the chiral susceptibilities in the region where the transition occurs as a smooth crossover, denoted by dashed lines in Fig. 3. To sufficiently low diquark couplings the chiral restoration takes place as a first order phase transition (solid lines in the figure). However, when \( \eta_D \) is increased, the chiral critical chemical potential gets reduced, and the chiral transition continues to be of first order up to a certain critical end point (CEP). For larger values, the chiral restoration phase transition proceeds as a smooth crossover.

On the other hand, the diquark condensation is always a second order phase transition, quoted by dotted lines in Fig. 3.

The region between both phase transition curves, quoted as a grey band in the figure, is a coexistence phase where the chiral symmetry remains broken with a nonvanishing diquark VEV, the same as in Fig. 2.

When the vector coupling is increased the CEP position is pushed to the left, while the coexistence phase becomes wider.

---

**FIG. 3:** QM phase diagram in the \( \eta_D - \mu_B^{c} \) plane. Solid, dotted and dashed lines correspond to first, second and crossover phase transitions. The shaded band indicates the coexistence region.

To conclude this section we present our numerical results for the speed of sound, whose square \( (c_s^2) \) is defined as the slope of P vs. \( \varepsilon \). This quantity is relevant in the astrophysical applications as it is related with the stiffness of the EoS. Weak coupling (perturbative) QCD suggests that the conformal limit \( (c_s^2 = 1/3) \) is approached from below [26, 27], see also Eq. (5) of [28].

In Fig. 4 we show the squared speed of sound for fixed diquark coupling ratio \( \eta_D = 1.10 \) and for different values of \( \eta_V \), namely 0.10, 0.50 and 1.10 as function of the...
energy density. It is clear that, within the present QM model that includes 2SC, $c_s^2$ is always larger than the conjectured limit 1/3 on QCD, and smaller than 1, preserving causality.

In the range of energy densities which is relevant for the cores of neutron stars, $400 < \varepsilon [\text{MeV/fm}^3] < 2000$, and which is displayed in Fig. 4, the values for $c_s^2$ are approximately constant and lie in the range $\approx c_s^2$ and which is displayed in Fig. 4, the values for $c_s^2$ are approximately constant and lie in the range $\approx 0.4 – 0.6$. In order to understand the role of color superconductivity for $c_s^2$, it is instructive to apply the formula (6) from [29] which quantifies the deviation from the conformal limit by

$$c_s^2 = 1 + \frac{\zeta}{3 + \zeta}, \quad \zeta = \frac{2(3\Delta)^2}{a_4 \mu_B^2},$$  \hspace{1cm} (30)

where $\Delta$ is the pairing gap and $a_4 = 1 – 2\alpha_s/\pi$ is the $O(\alpha_s)$ perturbative correction to the ideal massless quark pressure. With $\Delta = 0.14 \mu_B$, which is a satisfactory approximation to the gap at zero momentum found for $\eta_V = 0.5$ and $\eta_D = 1.1$, we get from Eq. (30) the value $c_s^2 = 0.52$, in rough agreement with the red line in Fig. 4.

In addition, we have checked that, within our model without interactions ($\eta_D = \eta_V = 0$), the asymptotic behaviour of $c_s^2$ corresponds to the conjectured value of 1/3.

III. HYBRID EOS

The aim of the present work is to test the reliability of the presented QM model in astrophysical arena. For that purpose we use a two-phase description to account for the transition from nuclear to QM in the interiors of compact stars.

The nonlocal NJL model is found to provide a basic understanding for the mechanisms governing both the spontaneous breakdown of chiral symmetry and the dynamical generation of massive quasiparticles from almost massless current quarks, in close contact with QCD [30].

However, it does not account for some important features expected from the underlying QCD interactions. In particular, the model predicts the existence of colored quasiparticles in regions of $T$ and $\mu$ where they should be suppressed by confinement. Therefore, to successfully describe the dynamics of QCD, it is necessary to include the effects of color confinement, that could be mimicked throughout a bag pressure term. In the hybrid description we observed that both, hadronic and QM, EoSs have similar behavior in the neighborhood of the phase transition which is a sign of the masquerade effect [16, 31–33]. Then, below, we will show that to satisfactorily fulfill modern astrophysical constraints for the neutron star masses and tidal deformabilities within the present hybrid description, the EoS (total pressure) of the non-local NJL model should be stiffened (modified) near the phase transition (from hadronic to QM phase), in order to avoid the masquerade effect, and softened in the high-density range, to reach the maximum mass prediction for compact stars. An fruitful way to solve both drawbacks at zero temperature is to include a bag constant in the total pressure of the QM description. Therefore, we will start by considering a constant bag scenario in Sect. III D and then, in Sect. III E, we will generalize this approach by a $\mu_B$-dependent bag pressure.

A. Hadronic model

To describe the nuclear matter it has been used the relativistic density-functional approach by Typel [34] which includes meson-exchange interactions within the “DD2” parametrization [4]. Even though this model satisfactorily describes nuclear matter up to saturation density, the higher density region needs improvement, that is achieved considering an excluded volume effect [35]. In the present work it is considered different values for the excluded volume, for instance DD2-p40. Here the label “p40” stands for a positive excluded volume parameter of $v = 4 \text{fm}^3$. This type of nuclear EoS has been extensively used in systematic studies of hybrid star models; see for

FIG. 4: Squared speed of sound for the nonlocal chiral QM model with $\eta_D = 1.10$ and $\eta_V = 0.1, 0.5, 1.1$, respectively.
instance [36–39]. In addition, it has also been considered a neutron stars crust, including the BPS model [40], to fully describe the hadronic EoS at low baryonic densities.

### B. Maxwell construction

In the present work it will be only considered a sharp interphase between QM and hadronic phase. That means that no mixed phase is expected (see Ref.[41] and references therein). Then, the phase transition between the EoSs for nuclear matter and QM, will be described by the Maxwell construction. Both phases satisfy charge-neutrality condition and $\beta$-equilibrium with electrons and muons. Then, both phases are connected by requiring that chemical potentials and pressures of the two phases coincide at the phase transition

$$\mu^H = \mu^{QM} = \mu_c$$

and

$$P^H = P^{QM} = P_c$$

Outside the phase transition, the phase with higher pressure (lower grand canonical potential) is to be chosen as the physical one.

It is worth mentioning that the first result we obtained (not shown here) is that unless additional (de)confining effects are introduced with a bag pressure, the only physical crossings between hadronic and QM EoSs occur at very high $P$ and $\mu_B$ values, producing compact star masses, radii and tidal deformabilities that not accomplish several astrophysical constraints mentioned in the introduction (see Sect. I).

### C. Calculation of astrophysical observables

In order to rate and compare the obtained EoSs we have to calculate possible neutron star properties. In order to generate plausible solutions for neutron star properties, hybrid neutron star EoS has been augmented with the crust EoS by Baym, Pethick and Sutherland (BPS) [40]. In the first step one can calculate the range of possible neutron star radii and masses. These can directly be compared to observations from the combined observations by NICER and XMM Newton of the millisecond pulsar J0740+6620 according to the analysis of Miller et al. [7] To evaluate them it has to be solved the Tolman-Oppenheimer-Volkoff (TOV) equations for a static non-rotating, spherical-symmetric star [42, 43] (see Appendix 2 for details). The tidal deformability $\Lambda$ can be calculated for the considered sequence of neutron star masses [44] and be compared to the constraint obtained from the gravitational wave signal that was observed for the binary neutron star merger GW170818 [3] in the mass range $M \approx 1.4 M_\odot$.

The astrophysical observables were calculated on the basis of a code by Andrea Maselli [45].

### D. Constant B scenario

As a first step towards to astrophysical applications, we consider a constant bag pressure ($B$) effects in the QM EoS, in order to have lower physical crossings between both, hadronic and QM $P$ vs. $\mu_B$ curves.

We renormalize the QM EoS considering that

$$P(\mu_B) \rightarrow P(\mu_B) - B,$$

where a bag pressure shift is included in the QM EoS. Throughout this subsection, we will consider a fixed $B = 10 \text{ MeV/fm}^3$, that lies in the range of 10 - 50 MeV/fm$^3$ used in [46]. With this particular choice we are able to obtained an early onset from hadronic to QM phase as can be see from Fig. 5.

First, in order to explore the effects of setting different parameters values, for example the $\eta_V$ values for QM and the excluded volume parameter for the hadronic phase, we proceed as follows: first we considered a hybrid description for a fixed QM EoS and different hadronic ones. Later we choose one hadronic EoS and different QM EoSs. After that, we will only present some representative hybrid EoSs.

In Fig. 5 (a) we show a particular QM EoS for fixed $\eta_D = 1.1$, $\eta_V = 0.6$ (green dashed line), and different hadronic EoSs: DD2 (orange) or DD2-p10 (black), p40 (light blue) and some values in between, p20 (green) and p30 (purple). These EoSs were considered as input for solving the TOV equations, giving as output the $M$ vs. $R$ shown in Fig. 5 (b). It can be observed earlier onsets for larger excluded volume parameter in the hadronic EoS. In all cases, the maximum mass attained is about 2.3 $M_\odot$ with $R \sim 12.35 \text{ Km}$. 
On the other hand, in Fig. 5 (c) it is shown one hadronic EoS, DD2-p40 (solid line), and different QM EoSs with $\eta_D = 1.1$ and $\eta_V$ from 0.5 to 0.9 (different trace lines). In panel (d), we show the effects of the parameters selections in (c) for the M-R relations. It is observed earlier onsets for smaller $\eta_V$ considered. The maximum mass attained increases with $\eta_V$, being the larger value of about 2.6 $M_\odot$ with $R \sim 13.75$ K$m$, for $\eta_V = 0.9$. Therefore, the larger the vector coupling, the larger both maximum mass and radius.

We summarize in Fig. 6 (a), the EoSs with selected quark-hadron parameter combinations that gives as a result an early phase transition onset (for the $\eta_V = 0.5$-DD2 hybrid EoS, red lines), a high mass phase transition onset (for the $\eta_V = 0.8$-DD2-p40 hybrid EoS, blue lines), and an intermediate onset value (for $\eta_V = 0.6$-DD2-p20 hybrid EoS, green lines). Now, in order to test the obtained hybrid EoS with some well know previous results, we compared with the constraints adjusting a parametric model (piecewise polytrope in this case) to the observations from NICER and X-ray Multi-Mirror (XMM-Newton) observations of PSR J0740+6620 (see ref. [7] for details). This comparison is shown in Fig. 6 (b) where it can be seen a very good agreement for almost all the presented EoS, with the only exception of DD2-p40. Similar result is obtained when one compares with the EoS regions generated by superimposing large numbers of individual EoSs given from the speed-of-sound interpolation method introduced in [47]. In Fig. 6 (c) it is displayed the speed of sound considering the EoSs from the panel.
(a). The asymptotic behavior of $c_s^2$ tends to $0.5 - 0.6$, for $\eta_V = 0.5$ to 0.8 respectively.

M-R plot is presented in panel (d), where we show the hybrid solutions together with the hadronic ones. As in previous very recent works [48–50], it is also included different color regions corresponding to either pulsar measurements or forbidden (striped) regions that serve as constraints for the compact star EoS. The green band region above $2M_\odot$ corresponds to the updated mass measurement of PSR J0740+6620 [9], which was recently upgraded to a mass-radius measurement by the Neutron Star Interior Composition Explorer (NICER), shown by the purple ellipsoidal type region for the result of the Star Interior Composition Explorer (NICER), shown by the wine color region for XMM-Newton Spectroscopy [8]. The orange and red regions around $M=1.4 M_\odot$ correspond to the estimates of the components of the binary system labeled as M1 and M2 of the GW170817 merger [3]. The central big blue region correspond to 95% contour of the joint probability density distribution of PSR J0030+0451 (see Appendix 2 for details) compared to the probability density contours of the GW170817 merger [3]. The central big blue region correspond to 95% contour of the joint probability density distribution of PSR J0030+0451 Mass and Radius from NICER Data [51]. The Green and brown striped bands correspond to excluded regions derived from GW170817 observations by Bauswein et al. [52] and Annala et al. [53]. The light blue band region corresponds to the mass $2.59^{+0.08}_{-0.09} M_\odot$ of the lighter component in the binary merger event GW190814 [54]. Older maximum mass constraints from Shapiro Delay measurement of PSR J0348+0432 and PSR J1614+2230 [55–59] have also been included as additional references.

In Fig. 6 (e) it is shown the tidal deformability as a function of $M/M_\odot$ including its value ($\Lambda = 190^{+390}_{-120}$) from GW170817 [3]. Finally, in panel (f) we present the tidal deformability parameters $\Lambda_1$ and $\Lambda_2$ of the high and low mass components of the binary merger (see Appendix 2 for details) compared to the probability density contours for the analysis of GW170817 signals [1, 60]. The $\Lambda_1$ and $\Lambda_2$ parameters characterize the size of the tidally induced mass deformations of each star.

Note that for tidal deformabilities results, the hybrid EoS are not separated into its QM and hadronic parts, then in pannels (e) and (f) the traced lines represent all the hybrid EoSs and the solid lines are for the pure hadronic EoS as usual, but they result overlapped in all most all the cases.

From the results shown in Figs. 6 (e) and (f) it is evident that the constant B inclusion is not sufficient to fulfill both constraints, maximum mass and tidal deformabilities, simultaneously, therefore a more general approach is necessary.

E. $\mu_B$ dependent B

Now we will consider a $\mu_B$-dependent bag as in Ref. [39], given by the equation

$$B(\mu_B) = B f_<(\mu_B)$$

with

$$f_<(\mu_B) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\mu_B - \mu_\eta}{\Gamma_\mu} \right) \right],$$

where we have set $\mu_\eta = 895$ MeV, $\Gamma_\mu = 180$ MeV and $B = 35$ MeV/fm$^3$ being the optimal values to reproduce the astrophysical observables. Note that $B(\mu_B)$ in the vicinity of the transition from hadronic to QM phase, takes a value around the one considered in Sect. III D.

In Fig. 7 we summarize the results obtained with the proposed hybrid construction but considering now the $B(\mu_B)$ function instead of the constant B. In panel (a) we show P vs. $\mu_B$ for QM and hadronic EoSs. For QM we display the corresponding EoSs considering $\eta_D = 1.1$ and three different values for $\eta_V = 0.5, 0.6, 0.8$. The hadronic EoSs correspond to DD2, DD2-p20 and DD2-p40. It is observed that with the chosen parameter values the crossing of the hadronic and QM EoSs occur at lower P and $\mu_B$ values in comparison with the constant B case. In panel (b), it is included the hybrid EoSs together with NICER constraints where it can be observed a notorious change with respect to the corresponding panel of Fig. 6: now the energy gaps are bigger for lower values of $\eta_V$.

A consequence of the energy gap enhancement could be that the M-R curves obtained in the QM phase have a bigger deviation from the corresponding hadronic lines around the phase transition. In panel (c) it is shown the corresponding curves for $c_s^2$, where it is observed a peak in the region of the transition from hadronic to QM. It can be seen as well a constant behavior, in between 0.5 and 0.6, at high densities.
FIG. 6: Pressure vs. chemical potential for the QM (dashed lines) and the hadronic (solid lines) EoS is shown in panel (a). Pressure vs. energy density is shown in panel (b) and we highlight the fact that NS interiors do not probe the energy densities where pQCD is applicable. In panel (c), the squared sound speed $c_s^2$ is shown for the corresponding hybrid EoS. The M-R relations are given in panel (d). The solid dots indicate the respective phase transition points and vertical bars show the location of the maximum mass stars. The results for the pure hadronic case without phase transition are also shown (in solid lines). For comparison with the NS phenomenology, the mass-radius constraints from NICER observations (see Refs. [7, 51] for details) and from the merger event GW170817 are shown. The tidal deformability $\Lambda$ vs. $M/M_\odot$ for the set of Hybrid EoS (traced lines), hadronic EoS (solid lines, overlapped with QM ones), and comparison to the results of GW170817 is given in panel (e). Panel (f) displays the comparison of $\Lambda_2 - \Lambda_1$ calculated for our EoS in comparison to the analysis of the gravitational wave signal from the merger GW170817 [1, 60].
In panel (d) it is displayed the corresponding Mass-Radius relations, showing both the hybrid solutions and the hadronic ones (solid lines). As in the previous section, it is included different constraints regions from astrophysical observables. It is observed an earlier onset compared with the ones with constant $B$ and that, as in all the previous M-R plots, the lower the vector coupling, the smaller the radius and the mass of the corresponding maximum mass stars. The tidal deformability $\Lambda$ and comparison to the results of GW170817 are shown in panel (e), and finally, in panel (f) it is shown $\Lambda_1 - \Lambda_2$ of the calculated tidal deformabilities analysis with respect to the gravitational wave measurement GW170817.

As can be seen from the $B(\mu_B)$ results, it is possible to
fulfill simultaneously the maximum mass and tidal deformabilities constraints.

IV. SUMMARY AND CONCLUSIONS

In the present work we have investigated the relationship between modern NS phenomenology and a microphysical approach formulated by a chiral quark model Lagrangian. We have used a two-phase description of quark-nuclear matter to obtain a first-order deconfinement phase transition by a Maxwell construction between the DD2 model (with excluded volume and a crust of neutron stars) for nuclear matter and a nonlocal instantaneous NJL type model, including color superconductivity and local vector repulsive interactions for QM. Nonlocality was introduced in the quark currents through a Gaussian form factor in 3D momentum space, except in the vector channel. In order to obtain the input parameters of the QM model we have first performed a fit to the momentum dependence of the quark mass function from LQCD in the Coulomb gauge, to obtain a value for the effective range parameter $\Lambda_0$. Then, the values of the scalar coupling $G_S$ and the current quark mass $m_c$ were fixed by low energy phenomenology. The 3D form factor used in the present work, allows for an analytical Matsubara summation.

One of the remarkable results obtained within the present QM model is the constant asymptotic behavior of the squared sound speed $c_s^2$. First of all, we reproduced the conjectured QCD value of 1/3 when we turn off the interactions in our nonlocal model. Then, turning on the strong diquark coupling and vector interactions, the asymptotic value for $c_s^2$ lies in the range 0.4 - 0.6. Our results show that, to satisfactorily fulfill modern observational constraints, we need to include a bag pressure in the QM EoS. Moreover, a $\mu_B$ dependent bag pressure is essential to simultaneously satisfy both radius constraints, for $R_{2,0}$ for high-mass NS from the NICER observation of PSR J0740+6620 and $R_{1.4}$ for typical-mass NS from the tidal deformability obtained from the inspiral gravitational wave measurement of GW170817. The bag pressure value close to the transition is in the range adopted by some works in the literature.

The reliable hybrid description studied in the present work is the first step of testing the model before applying its finite temperature generalization to the simulation of supernova explosions and NS mergers.

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Appendix:

1. Details of the nonlocal model for quark matter

In this appendix we show some explicit expressions corresponding to the nonlocal chiral quark model considered in Sect. II.

From Eqs. (14) the gap equations for the mean fields $\bar{\sigma}$ and $\bar{\Delta}$ together with the constraint equation for the mean field $\bar{\omega}$ read

$$\bar{\sigma} = 2G_s \int \frac{d^3p}{(2\pi)^3} g(p) \frac{M}{E} \times$$

$$\sum_{\kappa} \left\{ 1 - n_F\left( \frac{E_b^\kappa + \delta \mu_b}{T} \right) - n_F\left( \frac{E_b^\kappa - \delta \mu_b}{T} \right) \right\}, \quad (A.1)$$

$$\bar{\Delta} = 2G_D \bar{\Delta} \int \frac{d^3p}{(2\pi)^3} g^2(p) \times$$

$$\sum_{\kappa=\pm} \left\{ \frac{2}{\epsilon_r^\kappa} \left[ 1 - n_F\left( \frac{\epsilon_r^\kappa + \delta \mu_r}{T} \right) - n_F\left( \frac{\epsilon_r^\kappa - \delta \mu_r}{T} \right) \right] \right\}, \quad (A.2)$$

and

$$\bar{\omega} = 2G_V \int \frac{d^3p}{(2\pi)^3} \times$$

$$\sum_{\kappa=\pm} \left\{ 1 - n_F\left( \frac{E_b^\kappa + \delta \mu_b}{T} \right) - n_F\left( \frac{E_b^\kappa - \delta \mu_b}{T} \right) \right\}, \quad (A.3)$$

$$\sum_{\kappa=\pm} \left\{ \frac{2}{\epsilon_r^\kappa} \left[ 1 - n_F\left( \frac{\epsilon_r^\kappa + \delta \mu_r}{T} \right) - n_F\left( \frac{\epsilon_r^\kappa - \delta \mu_r}{T} \right) \right] \right\}.$$
where \( n_F(x) = (1 + \exp(x))^{-1} \) is the Fermi distribution function. The solutions for (A.2-A.4) in the vacuum, at \( T = \mu = 0 \), are denote with the subscript 0 as \( \bar{\sigma}_0 \) and \( \bar{\omega}_0 \), respectively. In the vacuum \( \bar{\Delta}_0 \) and \( \bar{\omega}_0 \) are zero, so the vacuum thermodynamic potential reads

\[
\Omega_0^{MFA} = \frac{\bar{\sigma}_0^2}{2G_S} - 12 \int \frac{d^3\vec{p}}{(2\pi)^3} E_0/2. \tag{A.4}
\]

Notice that in the above expression \( E_0^2 = \vec{p}_0^2 + M_0^2 \) where \( M_0(\vec{p}) = m_c + g(\vec{p})\bar{\sigma}_0 \). The integral in Eq. (7) turns out to be ultraviolet divergent because of the zero-point energy terms. Since this is exactly the divergence of Eq. (A.4), a successful regularization scheme consists just in the vacuum subtraction

\[
\Omega_{r,g}^{MFA} = \Omega^{MFA} - \Omega_0^{MFA}. \tag{A.5}
\]

Finally, the quarks and lepton densities, \( \rho_{fc} \) and \( \rho_l \), respectively, are given by

\[
\rho_{fc} = -\frac{\partial \Omega^{MFA}}{\partial \mu_{fc}}, \quad \rho_l = -\frac{\partial \Omega_{lep}}{\partial \mu_e}, \tag{A.6}
\]

where \( f = u, d \), \( c = r, g, b \), and \( l = e, \mu \). Thus,

\[
\rho_{fr} = \rho_{fg} = -\int \frac{d^3\vec{p}}{(2\pi)^3} \times \sum_{\kappa = \pm} \left\{ \left[ n_F\left( \frac{E_r^c + \delta \bar{\mu}_r}{T} \right) - n_F\left( \frac{E_l^c - \delta \bar{\mu}_l}{T} \right) \right] \right\}, \tag{A.7}
\]

\[
\rho_{fb} = -\int \frac{d^3\vec{p}}{(2\pi)^3} \times \sum_{\kappa = \pm} \left\{ \left[ n_F\left( \frac{E_b^c + \delta \bar{\mu}_b}{T} \right) - n_F\left( \frac{E_g^c - \delta \bar{\mu}_g}{T} \right) \right] \right\}, \tag{A.8}
\]

\[
\rho_l = -2 \sum_{l=e,\mu} \int \frac{d^3\vec{p}}{(2\pi)^3} \times \left[ n_F\left( \frac{E_l^c + \mu_e}{T} \right) - n_F\left( \frac{E_l^c - \mu_e}{T} \right) \right]. \tag{A.9}
\]

Note that the fermion densities has only the contribution of the first term of the regularized thermodynamic potential (A.5), since the second one has no dependence on the chemical potentials.

The corresponding gap equations and constraint equation are given by

\[
\bar{\sigma} = 2G_S \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{g(\vec{p})M(\vec{p})}{E(\vec{p})} \times \sum_{\kappa,\delta = \pm} \left\{ \frac{E_r^c c_r^c + s \delta \bar{\mu}_r}{|E_r^c c_r^c| + s \delta \bar{\mu}_r} + \frac{1}{2} \frac{E_b^c + s \delta \bar{\mu}_b}{|E_b^c + s \delta \bar{\mu}_b|} \right\}. \tag{A.10}
\]

\[
\bar{\Delta} = 2G_D \bar{\Delta} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{g^2(\vec{p})}{E(\vec{p})} \sum_{\kappa,\delta = \pm} \left\{ \frac{E_r^c c_r^c + s \delta \bar{\mu}_r}{|E_r^c c_r^c| + s \delta \bar{\mu}_r} + \frac{1}{2} \frac{E_b^c + s \delta \bar{\mu}_b}{|E_b^c + s \delta \bar{\mu}_b|} \right\}. \tag{A.11}
\]

The densities read

\[
\rho_{fr} = \rho_{fg} = \int \frac{d^3\vec{p}}{2\pi^2} \times \sum_{\kappa = \pm} \left\{ \frac{E_r^c + \delta \bar{\mu}_r}{|E_r^c + \delta \bar{\mu}_r|} \frac{1}{2} \left[ \frac{E_r^c + \delta \bar{\mu}_r}{|E_r^c + \delta \bar{\mu}_r|} + (\delta_{uf} - \delta_{df}) \right] + \frac{E_r^c - \delta \bar{\mu}_r}{|E_r^c - \delta \bar{\mu}_r|} \frac{1}{2} \left[ \frac{E_r^c - \delta \bar{\mu}_r}{|E_r^c - \delta \bar{\mu}_r|} - (\delta_{uf} - \delta_{df}) \right] \right\}. \tag{A.13}
\]

\[
\rho_{fb} = \int \frac{d^3\vec{p}}{(2\pi)^3} \sum_{\kappa,\delta = \pm} \left\{ \frac{E_b^c + \delta \bar{\mu}_b}{|E_b^c + \delta \bar{\mu}_b|} \frac{1}{2} \left[ \kappa + (\delta_{uf} - \delta_{df}) \right] + \frac{E_b^c - \delta \bar{\mu}_b}{|E_b^c - \delta \bar{\mu}_b|} \frac{1}{2} \left[ \kappa - (\delta_{uf} - \delta_{df}) \right] \right\}. \tag{A.14}
\]

where for leptons

\[
\rho_l = \frac{1}{3\pi^2} (\mu_l^2 - m_l^2)^{3/2}. \tag{A.15}
\]

Finally, for completeness, the chiral quark condensate is defined as

\[
\langle \bar{q}q \rangle = \frac{\partial \Omega^{MFA}}{\partial m_c} = -\int \frac{d^3\vec{p}}{(2\pi)^3} \zeta(\vec{p}) \tag{A.16}
\]

where,

\[
\zeta(\vec{p}) = \sum_{\kappa,\delta = \pm} \left\{ 2 \left[ sgn(c_r^c + s \delta \bar{\mu}_r) \frac{M E_r^c}{|E_r^c|} - \frac{M_0}{E_0} \right] + \left[ sgn(E_b^c + s \delta \bar{\mu}_b) \frac{M E_b^c}{|E_b^c|} - \frac{M_0}{E_0} \right] \right\}. \tag{A.17}
\]
2. Calculation of gravitational mass, radius and tidal deformability of spherical compact stars

In order to compute the internal energy density distribution of compact stars and thus derive the mass-radius relation we utilize the Tolman–Oppenheimer–Volkoff (TOV) equations for a static and spherical star in the framework of general relativity:

\[
\frac{dP(r)}{dr} = G(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r)) \quad r(r - 2GM(r)) \tag{A.18}
\]

\[
\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r), \tag{A.19}
\]

with \(P(r = R) = 0\) and \(P(r = 0) = P_c\) as boundary conditions for a star with mass \(M\) and radius \(R\).

Here we want to briefly describe how to compute the tidal deformability (TD) of a compact star, based on the results of [44, 61–64]. In order to determine the dimensionless tidal deformability parameter \(\Lambda = \lambda/M^5\) that can be computed for small tidal deformabilities in an perturbative way. Here \(\lambda\) is the stellar TD and \(M\) is the stellar gravitational mass. \(\lambda\) is related to the so called love number

\[
k_2 = \frac{3}{2} \lambda R^{-5}. \tag{A.20}
\]

The TD can be thought of a modification of the spacetime metric by a linear \(l = 2\) perturbation of a spherical symmetric star,

\[
ds^2 = -e^{2\Phi(r)} [1 + H(r)Y_{20}(\theta, \varphi)] dt^2 + e^{2\Lambda(r)} [1 - H(r)Y_{20}(\theta, \varphi)] dr^2 + r^2 [1 - K(r)Y_{20}(\theta, \varphi)] (d\theta^2 + \sin^2 \theta d\varphi^2), \tag{A.21}
\]

where \(K'(r) = H'(r) + 2H(r)\Phi'(r)\), primes denoting derivatives with respect to \(r\).

The functions \(H(r), \beta(r) = dH/dr\) obey

\[
\frac{dH}{dr} = \beta \tag{A.22}
\]

\[
\frac{d\beta}{dr} = 2 \left(1 - 2\frac{m(r)}{r}\right)^{-1} H \left\{ -2\pi \left[5\varepsilon(r) + 9P(r) + f(\varepsilon(r) + P(r))\right] + \frac{3}{r^2} + 2\left(1 - 2\frac{m(r)}{r}\right)^{-1} \left(\frac{m(r)}{r^2} + 4\pi rP(r)\right)^2 \right\} \left[ -1 + \frac{m(r)}{r} + 2\pi r^2(\varepsilon(r) - P(r)) \right], \tag{A.23}
\]

where \(f = de/dp\) is the equation of state.

The above equations and the TOV equations have to be solved simultaneously. The system is to be integrated outward starting near the center using the expansions

\[
H(\tau) = a_0 \tau^2 \quad \text{and} \quad \beta(\tau) = 2a_0 \tau \quad \text{as} \quad \tau \rightarrow 0. \quad a_0 \quad \text{is a constant that determines how much the star is deformed and turns out to be arbitrary since it cancels in the expression for the Love number. With the definition of}
\]

\[
y = \frac{R \beta(R)}{H(R)}, \tag{A.24}
\]

the \(l = 2\) Love number is found as

\[
k_2 = \frac{8C^5}{5} (1 - 2C)^2 \left[2 + 2C(y - 1) - y \right] \left\{ 2C^2 \left[6 - 2y + 3C(5y - 8)\right] \right. \]

\[
+ 4C^3 \left[13 - 11y + C(3y - 2) + 2C^2(1 + y)\right] \]

\[
+ 3 \left(1 - C\right)^2 \left[1 - y + 2C(y - 1)\right] \ln 1 - 2C \right\}^{-1} \tag{A.25}
\]

where \(C = M/R\) is the compactness of the star.

Finally, note that the \(\Lambda_1\) and \(\Lambda_2\) parameters for the two components of the NS merger are obtained from Eq. (A.20) with the corresponding M-R values,

\[
\Lambda_{1,2} = \frac{2}{3} k_2 \left(\frac{R_{1,2}}{M_{1,2}}\right)^5. \tag{A.26}
\]

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