Super resolved imaging via variable pinholes array and time multiplexed object's coding

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Abstract. In this paper, we describe a novel super resolution method for pinhole optics with variable pinholes array. The imaging system is based on super resolved time multiplexing method using variable or moving pinhole array. The improved resolution and signal to noise ratio are achieved with improved light intensity in the same exposure time, compared as in a single pinhole system. This new configuration preserves the advantages of pinhole optics while solving the resolution limitation problem and the long exposure time of such systems. The system can also be used as an addition to several existing optical systems with visible and invisible light as well as to x rays systems.

1. Introduction
The advantages of lenses in optics are the main reasons to their popularity in optical imaging systems. Although pinhole optics is probably one of the earlier configurations used in optical imaging systems, over the years, lenses were preferred over pinholes in most optical applications. Low resolution, dim image and long exposure time for photon amount accumulation are the disadvantages in usage of pinholes use. Nevertheless, pinhole optics is still used in many applications today. The disadvantages of using lenses, such as depth of focus problems, aberrations, resolution and limited acceptance angle with micro lenses due to the small numerical aperture (NA), are the main motivation to use pinholes. Pinhole optics is also useful in x-rays and gamma rays imaging or particle streams, where no lens materials are available. On the other hand, the advantages of pinhole optics, beside simplicity, are complete freedom from linear distortion, virtually infinite depth of focus and a very wide angular field that can be made to exceed 90°. In pinhole optics there is also a trade off between large and small pinholes, between resolution and light intensity. A large pinhole can pass large amount of photons in a discrete time exposure but will produce a low resolution image. However, a small pinhole can pass small amount of photons but produces a high resolution image. In pinhole optics, for large pinholes the resolution is characterized by the geometric limit and for very small pinholes by the diffraction limit while the situation between the two limits is more complicated. The optimum pinhole size will be a compromise between the large spot image produced by a large pinhole and the one produced by Fraunhofer diffraction coming from a small pinhole. The optimal image thus occurs roughly where the geometrical optics and Fraunhofer diffraction approximations give the same result. Several methods where suggested to calculate the optimum size for pinhole [1-4].
In order to preserve the advantages of pinhole optics together with solving the problems of light intensity and low signal to noise ratio (SNR), many methods have been proposed over the years. Such methods include the use of high sensitivity detectors and camera films. Another method is based on the concept of using coded aperture imaging (CAI). The basic step in coded aperture imaging systems is the use of multiple pinhole apertures to form numerous images of the object. The resulting recoded image is decoded using a digital or optical method. There are many techniques and applications for multiple pinholes coded aperture systems [5-9]. Over the years, a wide range of pinholes designs have been proposed: random array, mosaic pattern array, cyclic array, uniformly redundant array, some with separate multiple images and some with multiplexed overlapped images [10-23].

In this paper we present a novel technique for lensless imaging system. The system preserves the advantages of pinhole optics together with solving the problems of light intensity and low SNR by using a variable and time multiplexed pinholes array. The system provides the advantages of simplicity, freedom from linear distortion, virtually infinite depth of focus and a very wide angular field. It is done while providing a super resolved image with improved SNR, better light intensity and all in the overall the same exposure time as compared to a single pinhole system or a lenses system.

2. Operation principle

The optical setup is composed of a variable pinholes array consisting from several time varying pinholes arrays (a set of $K$ arrays) set one after the other and a computational reconstruction step (Fig. 1). During the overall capture part, each array in the set is used to capture an image consisting of replications of the object according to the pinholes number and position in the array. The final image is reconstructed after time multiplexing. The time parameter for each image capture is variable and depends on the pinholes arrays design. A single pinhole causes loss of information due to two zero points in its spectral plane. A multi pinholes array also causes loss of information and will badly affect the final reconstruction. The reason for this is that multi pinholes arrays also have zero points in their spectral plane. Therefore, the outcome involves loss of the object’s information and the lack of capability to fully reconstruct the image. Using the proposed concept involving a time variable array can solve the above mentioned problem. Although each array in the set has zero points, the overall array maintains all spatial frequencies and it is able to achieve super resolved image (i.e. an image that does not contain partial losses of spectral information). The usage of a set of multi pinholes arrays instead of a single multi pinhole array with the same number of holes is essential in order to cover the spectral information loss that each array by itself has (i.e. in the proposed time multiplexing concept each one of the arrays in the set has zeros in different spectral locations and thus the combination of all arrays avoids the above mentioned spectral loss). The variable array creates the complementary filter with minimum information loss. In order to preserve all the spatial frequencies of the original object image we must control the pinholes arrays design. There are four free parameters in the configuration: the number of pinholes in each array ($N_k$), the spatial order (location) of the pinholes in each array ($L_{x,y}^{(n,k)}$), the size (diameter) of each pinhole in each array ($D_{n,k}$) and the capture time intervals for each pinholes array ($t_k$). The use of such kind of variable array does not extend the overall exposure time and enables to perform the reconstruction after a single capture as in a single array.
The variable array can be referred as a priori known, unique and changeable filter $F(u,v)$ applied in the spatial frequency domain multiplied with the spectrum of the imaged object. This can be clearly seen by computing the spectrum of the spatial distribution of a single time interval capture:

$$F_k(u,v) = \sum_{n=1}^{N} e^{-2\pi i (u x_n + v y_n)}$$

(1)

The spectrum of the spatial distribution is given by the Fourier transform and so the final captured image in the frequency domain is:

$$\mathcal{F}\{ I(x,y) \} = \sum_{k=1}^{K} \mathcal{F}\{ i_k(x,y) \} \cdot t_k = \hat{O}(u,v) \cdot \sum_{k=1}^{K} F_k(u,v) \cdot t_k$$

(2)

where $I$ is the captured image, $i_k$ is the captured image distribution from each array position, $O$ is the original object and $\mathcal{F}\{ \ldots \}$ is the Fourier transform operation. Thus, by extracting the spectral distribution, one can reconstruct the original image. The reconstruction is performed in the spatial Fourier plane by, first, applying inversion operation on the filters' sum function, then, multiplying the inverse of the filters' sum function with the final image captured through the variable pinholes array. The final reconstruction is done by applying Wiener filtering that filters out the distortion function, $F(u,v)$:

$$\hat{I}(u,v) \cdot F(u,v)^{-1} = \hat{O}(u,v) \cdot \sum_{k=1}^{K} F_k(u,v) \cdot t_k \cdot F(u,v)^{-1} = \hat{O}(u,v)$$

(3)

In the far field case ($Z >> F$ in Fig. 1), the difference between the replication distances in the image plane and the pinholes spatial order in the array will be negligible. However in the near field situation the distances between the object’s replications in the image plane are not the same as the distances of the pinholes in the array. The ratio between the two set of distances is related to the realized magnification:

$$L_{s,y}^{(n,k)} = (1 + M) \cdot L_{s,y}^{(n,k)}$$

(4)
where the magnification factor is \( M = \frac{F}{Z} \). As a result, in the near field case, the image reconstruction is performed by using the inverse of function of \( F'(u,v) \) that corresponds to the scaled set of distances and ratios as they appear in Eq. 4 (it is the same equation as Eq. 1 for \( F(u,v) \) but while using the distances of Eq. 4).

### 3. Simulations and experimental validation results of the super resolved system

In order to preserve all the spatial frequencies of the original object image we control the variable pinholes array design. We choose the four free parameters (\( N_k, L_{xy}^{(n,k)}, D_{n,k}, t_k \)) in our configuration such that there will be no frequency loss in the filters’ sum \( F(u,v) \) as compared to the single pinhole system. Our considerations for choosing the number of pinholes and positions were first to prevent zero points in the filters' sum \( F(u,v) \) spectral plane. The second consideration was to make sure that the filters’ sum \( F(u,v) \) transmission value is higher than the reference single pinhole filter in the region of interest. The last consideration was related to the desired relative light intensity improvement (LII) of the multi pinholes variable array system as compared to a single pinhole system. The relative LII is determined by the number, size and time exposure of each hole in the variable array:

\[
\text{Relative LII} = \frac{\sum_{k=1}^{K} \sum_{n=1}^{N} \pi R_{k,n}^2 \cdot t_k}{T \cdot \pi R_{ref}^2}
\]

where \( R \) is the radius of the pinhole.

The next \( F(u,v) \) simulation's results are related to the pinholes arrays design. We have used three different configurations in order to show the influence of the free parameters on the spatial frequencies of the \( F(u,v) \) function. The single pinhole reference system had pinhole size which is referred as 1 radius unit. All three variable arrays had three different positions (\( K=3 \)). The first two variable arrays had the same hole size (one \( R \) unit) for all pinholes in the array (\( D_{n,k} \) constant). One variable array had a total holes number of 17 and the other had 38 total pinholes. The third variable array had total holes number of 36 but with different holes size (3, 1, 2/3, 1/2, 1/3, 1/4.5). The relative LII of the three variable arrays were 5.67, 12.67, 13.21 respectively for constant integration time \( t_k \) in all the arrays. The Three variable arrays are presented in Fig. 2.

![Figure 2. The three variable arrays (K=3): (a). The three positions of the variable array consist of](image-url)
total 17 pinholes with the same holes size; (b). The three positions of the variable array consist of total 38 pinholes with the same holes size; (c). The three positions of the variable array consist of total 38 pinholes with different holes size (0.5:1:2 ratio).

The number and location of the pinholes in the array and the variable array positions enable to prevent zero points in the spatial frequencies of $F(u,v)$. The difference between the three designs and the influence on the $F(u,v)$ function can be seen in Fig. 3. The time parameter $t_k$ for each array position is variable and depends on the pinholes arrays design. In the figure below (Fig. 3), P.T.F.T. stands for P.T.F. x 1 time unit – that is the Pinholes’ Transfer Function computed in relative units. One P.T.F.T. unit is the transmission equals the one of a single pinhole, multiplied by the photons integration time $t_k$. The time parameter helps to achieve the second consideration of the design.

Figure 3. The filters’ sum $F(u,v)$ comparison (top red line) as compared to one single pinhole (thick blue line) for the three above mention variable arrays ($K=3$). The time interval is equal for each array position (and it equals to 60 sec). The straight green line is the maximum level of the single pinhole reference level (since the designs are equal to the x-y axes for simplicity the simulation was performed in 1D only).

The variable pinholes arrays were designed by simulations done according to the conditions that were mentioned above. The pinholes focal length was 45 mm and the magnification was 1. The light source was a LUXEON Rebel ES neutral white light source with a diffuser and the camera was DCC1545M 1280x1024 pixels, 5.2 µm x 5.2 µm pixel size. The object was a spoke resolution target (Siemens star). Fig. 4 shows the experimental setup that was constructed. The single pinhole reference system was with a hole diameter of 250 µm. To the variable array of total 17 holes (250 µm each) we added another reference system of single pinhole having hole diameter of 350 µm with high light intensity for the same time exposure.

Figure 4. The variable pinholes array experimental setup.
The experimental results are shown in Fig. 5. The results show that in the variable array system compared to the single pinhole reference system with the same hole size (250 µm), we get better resolution with better light intensity (an improvement factor of 5.67) and better SNR. The second reference system with the large hole size (350 µm) achieved high light intensity in the same time exposure but low resolution according to the holes area size ratio.

![Image: Comparison of systems](image)

**Figure 5.** The comparison between systems (the same exposure time): (a). One pinhole reference system (250µm) – low light intensity; (b). Super resolved reconstructed image: the reconstructed image of the variable pinholes array system (250µm) with high light intensity (c). One pinhole reference system (350µm) – high light intensity but with bad resolution.

4. Conclusions
In this paper we have presented a method for super resolved imaging for pinholes optics that can improve light intensity and SNR in the same exposure time as compared to a single pinhole system. These improvements are accomplished by using known designs of variable multiple pinholes array coded aperture. The design considerations and the influence of the various parameters on the obtainable results were presented and discussed. The proposed concept was validated numerically and experimentally. The method can be applied for near and far field systems and for many applications achieving the advantages that pinholes optics can offer.

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