Science with the TianQin Observatory: Preliminary Results on Galactic Double White Dwarf Binaries

Shun-Jia Huang,1 Yi-Ming Hu,1,† Valeriya Korol,2 Peng-Cheng Li,3,4,1 Zheng-Cheng Liang,5,1 Yang Lu,1 Hai-Tian Wang,5,7,1 Shenghua Yu,8,9 and Jianwei Mei1,†

1TianQin Research Center for Gravitational Physics and School of Physics and Astronomy, Sun Yat-sen University (Zhuhai Campus), Zhuhai 519082, P. R. China
2School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom
3Center for High Energy Physics, Peking University, No.5 Yixueyuan Rd, Beijing 100871, P. R. China
4Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, No.5 Yixueyuan Rd, Beijing 100871, P. R. China
5MOE Key Laboratory of Fundamental Physical Quantities Measurements, Hubei Key Laboratory of Gravitation and Quantum Physics, PGMF and School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China
6Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210023, P. R. China
7School of Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, P. R. China
8National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
9The Key Laboratory of Radio Astronomy, Chinese Academy of Sciences (Dated: May 19, 2020)

We explore the prospects of detecting of Galactic double white dwarf (DWD) binaries with the space-based gravitational wave (GW) observatory TianQin. In this work we analyse both a sample of currently known and a realistic synthetic population of DWDs to assess the number of guaranteed detections and the full capacity of the mission. We find that TianQin can detect 12 out of ~100 known DWDs: GW signals of these binaries can be modelled in detail ahead of the mission launch, and therefore they can be used as verification sources. Besides we estimate that TianQin has potential to detect as many as 10^4 DWDs in the Milky Way. TianQin is expected to measure their orbital periods and amplitudes with accuracy of ~10^{-7} and ~0.2 respectively, and to localize on the sky a large fraction (39%) of the detected population to better than 1 deg^2. We conclude that TianQin has the potential to significantly advance our knowledge on Galactic DWDs by increasing the sample up to 2 orders of magnitude, and will allow their multi-messenger studies in combination with electromagnetic telescopes. We also test the possibilities of different configuration of TianQin: 1) the same mission with a different orientation, 2) two perpendicular constellations combined into a network, and 3) the combination of the network with the ESA-lead Laser Interferometer Space Antenna. We find that the network of detectors boosts the accuracy on the measurement of source parameters by 1 – 2 orders of magnitude, with the improvement on sky localization being the most significant.

I. INTRODUCTION

The first direct detection of gravitational waves (GWs) generated from a binary black hole merger (GW150914) was made by the LIGO and Virgo Collaborations in 2015 [1], one hundred years after they were predicted by Albert Einstein [2]. This detection, together with several subsequent ones including a binary neutron star merger (GW170817), started new fields of GW and multi-messenger astronomy [3–6].

The sensitivity band of the currently operational ground-based detectors LIGO and Virgo is limited between 10 Hz and kHz frequencies [7]. However, GW sources span many orders of magnitude in frequency down to fHz. Several experiments aim to cover such a large spectrum: the cosmic micro-wave background polarization experiments [8], pulsar timing array [9, 10] and the space-based laser interferometers sensitive to fHz, nHz and mHz frequencies respectively [11, 12].

The mHz frequency band is populated by a large variety of GW sources: massive black holes binaries (10^8 – 10^7M⊙) formed via galaxy mergers [13–17]; compact stellar objects orbiting massive black holes, called extreme mass ratio inspirals (EMRIs) [18, 19]; ultra-compact stellar mass binaries (and multiples) composed of white dwarfs, neutron stars and stellar-mass black holes in the neighbourhood of the Milky Way [20–23]. Besides individually resolved binaries, stochastic backgrounds of astrophysical and cosmological origin can be detected at mHz frequencies [e.g. 24, 25]. Therefore, this band is expected to provide rich and diverse science ranging from Galactic astronomy to high-redshift cosmology and to fundamental physics [26–30].

Among all kinds of ultra-compact stellar mass binaries, those composed of two white dwarf stars (double white dwarf binaries (DWDs)) comprise the absolute majority (up to 10^8) in the Milky Way. Being abundant and nearby-by, DWDs are expected to be the most numerous GW sources for space-based detectors [20, 31–33].

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* huyiming@mail.sysu.edu.cn
† meijw@sysu.edu.cn
Individual GW detections of DWDs will significantly advance our knowledge on binary formation and white dwarf stars themselves in a number of ways. Firstly, DWD represent the end products of the low-mass binary evolution, and as such they encode information on physical processes such as the highly uncertain mass transfer and common envelope phases [34, 35]. Secondly, DWDs are progenitors to AM CVn systems [36–38]. Thirdly, DWD mergers are thought to originate over a broad range of interesting transient events including type Ia supernovae (SNe Ia) [39–41]. In addition, detached DWDs are particularly suitable for studying the physics of tides. DWDs affected by tides will yield information on the nature and origin of white dwarf viscosity, which is still a missing piece in our understanding of white dwarfs’ interior matter. [42–45]. Finally, by analysing their GW signals one could set constraints on deviations from general relativity [46, 47].

The overall GW signal from DWDs imprints the information on the Galactic stellar population as a whole, and can constrain the structural properties of the Milky Way [33, 48–51]. A significant fraction of the population may present a stellar or sub-stellar tertiary companions, that can be recognised by an extra frequency modulation of the DWD GW signals [23, 30, 52]. GW detectors have the potential to guide the discovery of these populations [53].

TianQin is a space-based GW observatory sensitive to 100 Hz frequencies [12, 54, 55]. Recently, a significant effort has been put into the study and consolidation of the science cases for TianQin [56]. On the astrophysics side, these efforts include studies on the detection prospect of massive black hole binaries [14, 57], EMRIs [19], stellar-mass black hole binaries [58] and stochastic backgrounds [25]; on the fundamental physics side, prospects for testing of the no-hair theorem with GWs from massive black hole binaries [28] and constraints on modified gravity theories [29] have been assessed for TianQin. In this paper we aim to forecast detection of Galactic DWDs with TianQin. Due to low masses, DWD observable horizon in GWs is limited to the Milky Way, possibly reaching nearby satellite galaxies and the Andromeda galaxy [20, 26, 59, 60]. Therefore in this study we focus on the Galactic population only. We concentrate on detached systems only, because they are expected to be orders of magnitude more numerous than other types binaries in the mHz frequency regime [e.g., 61, 62].

The paper is organized as follows. In Section II, we outline the sample of the currently known ultra-compact DWDs and AM CVns, and we present a mock Galactic population. In Section III, we derive analytical expressions for computing the signal-to-noise ratio and uncertainties on binary parameters for TianQin. In Section IV, we present our results on the detectability of the known DWDs and of the mock population. We also present similar results for some mission variations and explore the improvements that could be achieved when a few detectors work as a network. Finally, we summarize our main findings in Section V.

II. GALACTIC DOUBLE WHITE DWARF BINARIES

Currently known electromagnetic (EM) sample amounts to ~100 detached and ~60 interacting (AM CVn) DWD systems with orbital periods ≤1 day [63–65]. Although rapidly expanding with several recent detections [66–69], this sample is still limited and represents only the tip of the iceberg of the overall Galactic population. To quantify the ability of TianQin in detecting DWDs, in this study we consider both the known sample and a synthetic Galactic population. In this section we briefly outline both samples.

A. Candidate verification binaries

Binaries discovered through EM observations are often called verification binaries in the literature [e.g. 70, 71]. This is because we can measure their parameters and therefore accurately model their GW signals; the predicted signal can be used to verify detector’s performance. Here we consider a sample of 81 candidate verification binaries (CVBs) (40 AM CVn type systems and 41 detached DWDs) with orbital periods ≤5 hours. FIG. 1 shows the sky position and the luminosity distance of our CVBs in the ecliptic coordinate system.

We list parameters of verification binaries in Table V in Appendix A. Parameters with poor observational constraints has been inferred from theoretical models. For example, for most verification binaries, trigonometric parallaxes from Gaia Data Release 2 [72] can be used to determine their luminosity distance [71]. Distances to RX J0806.3+1527 (also known as HM Cancri, hereafter J0806 [73]), CR Boo, V803 Cen, SDSS J093506.92+441107.0, SDSS J075552.40+490627.9, SDSS J002207.65–101423.5 and SDSS J110815.50+151246.6, however, are determined using different methods. In particular, J0806 has a largely uncertain distance. Here we use a conservative upper boundary of 5 kpc based on its luminosity observation [74].

In this work we define a DWD system as a verification binary if it: 1) has been detected in the electromagnetic (EM) bands and 2) its expected GW signal-to-noise ratio (SNR) for TianQin is ≥ 5 with a nominal mission lifetime of five years [70, 71]. We adopt a relatively low SNR threshold for the detection of the verification binaries because there is a priori information from the EM observations to fall back on. We also define the potential verification binaries to be the CVBs which have 3 ≤ SNR < 5 [70, 71].
B. Synthetic Galactic population

In this study we employ a synthetic catalog of Galactic DWDs based on models of Toonen et al. [75, 76]. These models are constructed on a statistically significant number of progenitor zero age main sequence systems ($\sim 10^5$) evolved with binary population synthesis code SeBa [77] until both stars became white dwarfs. To construct the progenitor population the mass of the primary star is drawn from the Kroupa initial mass function in the range between 0.95 and 10 $M_\odot$ [78]. Then, the mass of the secondary is drawn from a uniform mass ratio distribution between 0 and 1 [79]. Orbital separations and eccentricities are obtained from a log-flat distribution (considering those binaries that on the zero age main sequence have orbital separations up to $10^6 R_\odot$) and a thermal distribution respectively [79–81]. The binary fraction is set to 50% and the metallicity to Solar. It is important to note that in this paper we use models that employ the $\alpha\gamma$—common envelope evolution model designed and fine-tuned on observed DWDs [36, 82]. We highlight that this model matches well the mass ratio distribution [75] and the number density [76] of the observed DWDs.

Next, we assign the spatial and the age distributions to synthetic binaries. Specifically, we use a smooth Milky Way potential consistent of an exponential stellar disc and a spherical central bulge adopting scale parameters as in [50, see table 1]. The stellar density distribution is normalized according to the star formation history numerically computed by Boissier and Prantzos [83], while the age of the Galaxy is set to 13.5 Gyr. We account for the change in binary orbital periods due to GW radiation from the moment of DWD formation until 13.5 Gyr.

Finally, for each binary we assign an inclination angle $\iota$, drawn randomly from the uniform distribution in $\cos \iota$. The polarization angle and the initial orbital phase (respectively $\psi_S$ and $\phi_0$) are randomized assuming uniform distribution over the interval of $[0, \pi)$ and $[0, 2\pi)$, respectively. The obtained catalog contains the following parameters: orbital period $P$, component masses $m_1$ and $m_2$, the ecliptic latitude $\lambda$ and longitude $\beta$, distance from the Sun $d$ and angles $\iota, \psi_S, \phi_0$. This catalog has been originally employed in the study of DWD detectability with LISA [11]. Therefore, this paper represents a fair comparison with the results in Amaro-Seoane et al. [11].

III. SIGNAL AND NOISE MODELLING

A. Gravitational wave signals from a monochromatic source

The timescale on which DWDs’ orbits shrink via GW radiation is typically $> \text{Myr}$ (at low frequencies). This is significantly greater than the mission lifetime of TianQin of several years. Therefore, at low frequencies these binaries can be safely considered as monochromatic GW sources, meaning that they can be described by a set of...
several parameters: the dimensionless amplitude \( A \), GW frequency \( f = 2/P \), \( \lambda, \beta, \iota, \psi, S \) and \( \phi_0 \). Note, that we do not include eccentricity because DWDs circularize during the common envelope phase.

GWs emitted by a monochromatic source can be computed using the quadrupole approximation \([84, 85]\). In this approximation GW signal can be described as a combination of the two polarizations (+ and ×)

\[
h_+(t) = A (1 + \cos \iota^2) \cos(2\pi ft + \phi_0 + \Phi_D(t)), \quad (1)
\]

\[
h_\times(t) = 2A \cos \iota \sin(2\pi ft + \phi_0 + \Phi_D(t)), \quad (2)
\]

with

\[
A = \frac{2(\gamma \mathcal{M})^{4/3}}{c^4 d}(\pi \gamma)^{2/3}, \quad (3)
\]

where \( \mathcal{M} \equiv (m_1 m_2)^{2/3}/(m_1 + m_2)^{1/3} \) is the chirp mass, \( G \) and \( c \) are the gravitational constant and the speed of light, respectively. Note that the additional term \( \Phi_D(t) \) in the GW phase (Eq. (1)-(2)) is the Doppler phase arising from the periodic motion of TianQin around the Sun:

\[
\Phi_D(t) = 2\pi \frac{R}{c} \sin(\pi/2 - \beta) \cos(2\pi f_m t - \lambda), \quad (4)
\]

where \( R = 1\text{AU} \) is the distance between the Earth and the Sun, and \( f_m = 1/\text{year} \) is the modulation frequency. \( \lambda \) and \( \beta \) are the ecliptic coordinates of the source.

### B. Detector’s response to GW signals

Design of the TianQin mission \([12]\) envisions a constellation of three drag-free satellites orbiting the Earth maintaining the distance between each of \( \sim 10^5 \text{km} \). Satellites will form an equilateral triangle constellation oriented in a way that the normal vector to the detector’s plane is pointing towards J0806\( (\lambda = 120.4^\circ, \beta = -4.7^\circ) \).

In the low frequency limit \( f \ll f_s \) with \( f_s = c/2\pi L \) being the transfer frequency, \( \sim 0.28 \text{Hz} \) for TianQin, signal produced in the detector can be described as a linear combination of the two GW polarizations modulated by the detector’s response \([86]\):

\[
h(t) = h_+(t)F^+(t) + h_\times(t)F^\times(t), \quad (5)
\]

where \( F^+ \times(t) \) are the antenna pattern functions.

For a detector with an equilateral triangle geometry, two orthogonal Michelson signals can be constructed and the antenna pattern functions can be expressed as

\[
F_1^+(t, \theta, \phi_S, \psi) = \frac{\sqrt{3}}{2} \left( \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi_S(t) \cos 2\psi - \cos \theta \sin 2\phi_S(t) \sin 2\psi \right) \quad (6)
\]

\[
F_1^\times(t, \theta, \phi_S, \psi) = \frac{\sqrt{3}}{2} \left( \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi_S(t) \sin 2\psi + \cos \theta \sin 2\phi_S(t) \cos 2\psi \right) \quad (7)
\]

\[
F_2^+(t, \theta, \phi_S, \psi) = F_1^+(t, \theta, \phi_S - \pi/4, \psi) \quad (8)
\]

\[
F_2^\times(t, \theta, \phi_S, \psi) = F_1^\times(t, \theta, \phi_S - \pi/4, \psi) \quad (9)
\]

where \( \sqrt{3}/2 \) represents factor originated from the geometry of the detector and encodes 60° angle between the detector’s arms, \( \theta_S(t) = \phi_S(0) + \omega t \) are the space directional angles of the source in the detector’s coordinate frame, with \( \omega \approx 2 \times 10^{-5} \text{rad/s} \) being the angular frequency of the TianQin satellites. We report the transformation from the ecliptic coordinate \( (\beta, \lambda) \) to the detector coordinate \( (\theta_S, \phi_S) \) can be found in Appendix E. The subscripts 1 and 2 in Eq. (6)-(9) are labels for the two Michelson signals, which are orthogonal to each other, as indicated by the \( \pi/4 \) phase difference between the corresponding antenna pattern functions \([e.g., 86]\).

From Eq. (6)-(9) it follows that TianQin is most sensitive to GWs propagating along the normal direction to the detector’s plane, and least sensitive to GW propagating along the detector plane.

In general, the implementation of the antenna pattern functions is complicated \([e.g., 87]\). In practice, we can introduce the sky-averaged response function \( R(f) \) to simplify the following calculations. The sky-averaged response function \( R(f) \) can be approximated by

\[
R(f) \approx \frac{3}{10} \left( 1 + 0.6(f/f_\ast)^2 \right) \quad (10)
\]

The pre-factor 3/10 \( = 2 \times 3/20 \) is two times (to account for two independent Michelson interferometers) the sky-averaged factor of 3/20, that can be obtained as \( F^{\times,+} \equiv F^{\times,+} \int_0^\pi d\psi \int_0^{2\pi} d\phi \int_0^{2\pi} F^{\times,+} \sin \theta d\theta \).

### C. Detector noise and the scaled sensitivity curve

The huge number of Galactic DWDs can generate a foreground confusion noise that may affect the detection of other types of GW sources. In Sect. IV A we show that...
| Configuration       | TianQin                  |
|---------------------|--------------------------|
| Number of satellites | N=3                      |
| Orientation         | $\lambda = 120.4^\circ$, $\beta = -4.7^\circ$ |
| Observation windows | 2 $\times$ 3 months each year |
| Mission lifetime    | 5 years                  |
| Arm length          | $L = \sqrt{3} \times 10^9$km |
| Displacement measure noise | $S_x = 1 \times 10^{-24}$m$^2$Hz$^{-1}$ |
| Acceleration noise  | $S_a = 1 \times 10^{-30}$m$^2$s$^{-4}$Hz$^{-1}$ |

TABLE I. Key parameters for the TianQin configurations.

FIG. 2. The sensitivity curve of TianQin. The red line corresponds to $\tilde{S}_n(f)$ defined in Eq. (12), while the black line corresponds to the full sky averaged result preserving all the frequency dependence (see Eq. (15)-(16) in [14]).

such foreground can be neglected for TianQin. Therefore, through this paper we only consider the instrumental noise.

The noise spectral density of TianQin can be expressed analytically as

$$S_N(f) = \frac{1}{L^2} \left[ \frac{4S_a}{(2\pi f)^4} \left( 1 + \frac{10^{-4}\text{Hz}}{f} \right) + S_x \right], \quad (11)$$

where $L$, $S_a$, $S_x$ are given in Table I.

From the sky-averaged response function (10) and the detector noise (11), one can construct the sensitivity curve of the detector as

$$\tilde{S}_n(f) = S_N(f) / \tilde{R}(f)$$

$$= \frac{1}{L^2} \left[ \frac{4S_a}{(2\pi f)^4} \left( 1 + \frac{10^{-4}\text{Hz}}{f} \right) + S_x \right]$$

$$\times \left[ 1 + 0.6 \left( \frac{f}{f_*} \right)^2 \right], \quad (12)$$

where

$$\tilde{R}(f) \equiv R(f) \frac{10}{3} = \frac{1}{1 + 0.6(f/f_*)^2}. \quad (13)$$

Note that in this formalism, we assume both the geometry factor $\sqrt{3}/2$ and the antenna pattern to be associated with the signal. The obtained sensitivity curve is represented in FIG. 2.

D. Data analysis

The SNR of a signal is defined as

$$\rho^2 = (h|h), \quad (14)$$

where the inner product $(\cdot|\cdot)$ is defined as [88, 89],

$$(a|b) = 4\Re e \int_0^\infty df \tilde{a}^*(f) \tilde{b}(f)$$

$$\simeq \frac{2}{\tilde{S}_n(f_0)} \int_0^T dt a(t)b(t), \quad (15)$$

where $\tilde{a}(f)$ and $\tilde{b}(f)$ are the Fourier transformation of two generic functions $a(t)$ and $b(t)$, $\tilde{S}_n(f)$ is defined in Eq. (12). The second step is obtained by using Parseval’s theorem and the quasi-monochromatic nature of the signal which acts like a Dirac delta function on the noise power spectral density [86].
For monochromatic GW signal with frequency $f_0$, it is possible to derive an analytical expression of the SNR ($\rho$):

$$\rho^2 = \langle h|\rho| h \rangle \approx \frac{2}{S_n(f_0)} \int_0^T dt h(t)\bar{h}(t) = \frac{2(A^2)T}{S_n(f_0)} , \quad \text{(16)}$$

with

$$\langle A^2 \rangle = \frac{1}{T} \int_0^T h^2(t) dt, \quad \text{(17)}$$

$$\approx \frac{3}{16} A^2 [ (1 + \cos^2 t)^2 \langle F^2 \rangle + 4 \cos^2 t \langle F^2 \rangle ], \quad \text{(18)}$$

$$\langle F^2 \rangle = \frac{1}{4} (1 + \cos^2 \theta_S)^2 \cos^2 2\psi_S + \cos^2 \theta_S \sin^2 2\psi_S, \quad \text{(19)}$$

$$\langle F^2 \rangle = \frac{1}{4} (1 + \cos^2 \theta_S)^2 \sin^2 2\psi_S + \cos^2 \theta_S \cos^2 2\psi_S, \quad \text{(20)}$$

where $T$ is the observation time (which is half the operation time), and we have neglected the $O(T^{-1})$ terms in (18). It is also useful to define the characteristic strain $h_c = AV\sqrt{N}$ with $N = f_0T$ being the number of binary orbital cycles observed during the mission. Analogously, the noise characteristic strain is $h_n(f) = \sqrt{\int_S S_n(f)}$. One can straightforwardly estimate the SNR from the ratio between $h_f$ and $h_n$.

### E. Galactic GW foreground

At frequencies $< 1$ mHz, the number of Galactic sources per frequency is too large to resolve all individual GW signals. These signals can potentially become indistinguishable and form a foreground for the TianQin mission [in analogy with [90]]. We assess the level of such a foreground using synthetic population presented in Section II B.

We follow the method outlined in Littenberg and Cornish [91]. For each binary we construct the signal in the frequency domain, $h(f)$ (cf. Section III A). All signals in each frequency bin are then incoherently added, forming an overall population spectrum. Next, we smooth the spectrum by running a median smoothing function with a set window size and by fitting with cubic spline to it. We define the smoothed Galactic spectrum $S_{\text{DWD}}(f)$ and compute the total noise as the sum of the instrumental noise $S_n(f)$ and $S_{\text{DWD}}(f)$. Using the updated noise

1 Note that our definitions of the SNR and the amplitude differ from those in [22] by a numerical factor, but we both are self-consistent.

### F. Parameter estimation

The uncertainty on the binary parameters can be derived from the Fisher information matrix (FIM) $\Gamma_{ij}$,

$$\Gamma_{ij} = \left( \frac{\partial h}{\partial \xi_i} \frac{\partial h}{\partial \xi_j} \right) , \quad \text{(21)}$$

where the $\xi_i$ stand for the $i$th parameters.

In the high SNR limit ($\rho \gg 1$), the inverse of the FIM equals to the variance-covariance matrix $\Sigma = \Gamma^{-1}$. The diagonal entries $\Sigma_{ii}$ give the variances (or mean square errors) of each parameter, $(\Delta \xi_i)^2$, while the off-diagonal entries describe the covariances. In numerical calculations, we approximate $\partial h/\partial \xi_i$ with numerical differentiation

$$\frac{\partial h}{\partial \xi_i} \approx \frac{h(t, \xi_i + \delta \xi_i) - h(t, \xi_i - \delta \xi_i)}{2\delta \xi_i} . \quad \text{(22)}$$

The differentiation steps $\delta \xi_i$ was chosen to make the numerical calculation stable [92].

Notice that compare with the uncertainty of each coordinates, we are more interested in the sky localisation, which is a combination of uncertainties of both coordinates [86]

$$\Delta \Omega_S = 2\pi |\sin \beta| \left( \Sigma_{\lambda \beta}^2 - \Sigma_{\beta \beta}^2 \right)^{1/2} . \quad \text{(23)}$$

When a network of independent detectors is considered, the total SNR and Fisher information matrix (FIM) of a source can be calculated as

$$\rho_{\text{total}}^2 = \sum_a \rho_a^2 = \sum_a (h_a|h_a) ,$$

The differentiation steps $\delta \xi_i$ was chosen to make the numerical calculation stable [92].

TABLE II. The coefficients for the polynomial fit for the foreground, as $\rho \xi_1 a x^4$, where $x = \log(f/10^{-5})$. Different lines corresponding to increasing operation time $t_O$.

| $t_O$ (yr) | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| 0.5       | -18.73| -1.146| -1.095| 2.0970| -4.931| 7.147 | -4.651|
| 1         | -18.63| -0.1160| 0.0773| 0.840 | -2.784|
| 2         | -18.57| -0.5428| 0.6988| -1.037| -2.115| -0.8659|
| 4         | -18.58| -1.395| -1.1520| 0.0197| 6.041| -7.334| -8.738|
| 5         | -18.74| -1.380| -0.8748| -0.4322| 0.721| 4.536 | -10.14|
\[ \Gamma_{\text{total}} = \sum_a \Gamma_a = \sum_a \left( \frac{\partial h_a}{\partial \xi_i} \right) \left( \frac{\partial h_a}{\partial \xi_j} \right) , \]  

where the subscript \( a \) stands for quantities related to the \( a \)th detector.

IV. RESULTS

In this section we report our results for the TianQin mission. We also consider an alternative version of the mission configuration with the same characteristics (cf. Table I), but oriented perpendicularly to the original TianQin’s configuration (pointing towards Table I), but oriented perpendicularly to the original mission configuration with the same characteristics (cf. mission. We also consider an alternative version of the

where \( x \)

we also explore the possibility of \( \lambda \) TianQin’s configuration as \( \lambda \) and the additional one as \( \lambda \) TQ II. GW observations can be improved if many detectors are working simultaneously in a network (e.g. the LIGO + Virgo network). Therefore, in this work we also explore the possibility of two detectors \( \lambda \) and \( \lambda \) II operating simultaneously, both following the “three month on + three month off” observation scheme in a way to fill the data gaps of each other. We refer to the configuration consisting of the two detectors as \( \lambda \) I+II. In addition, we also explore the possibility of \( \lambda \) and \( \lambda \) I+II operating together with LISA.

A. Galactic foreground

Firstly, we assess the impact of the Galactic confusion foreground for TianQin. In FIG. 3 we show the estimates of the foreground levels corresponding to different operation times (colored lines) obtained according to the procedure described in Section IV A. Each line can be reproduced by using the expression \[ S_{\text{PWF}}(f) = 10 \Sigma_i a_i x^i , \]

where \( x = \log (f/10^{-3}) \) and polynomial coefficients \( a_i \) are reported in Table II for different operation times.

From FIG. 3 it is evident that the foreground hardly exceeds the instrumental noise curve (solid black line) even for operation times \( < 1 \) year. Therefore, we conclude that Galactic foreground can be safely neglect it in the following analysis.

FIG. 3 illustrates that the foreground is dependent on the operation time in two different ways. On one hand, TianQin’s frequency resolution is inversely proportional to the operation time, therefore longer operation times result in better frequency resolution. On the other hand, individual DWD signals also increase as a square root of operation time (cf. Eq.(16)). As the result, at frequencies \( \lesssim 3 \) mHz the foreground decreases because more sources can be resolved with increasing operation time, whereas as frequencies \( \gtrsim 3 \) mHz Galactic foreground decreases because of the improving resolution in frequency.

B. Verification binaries

Out of 81 considered candidates (cf. Table V) we find 12 verification binaries with \( \text{SNR} \geq 5 \): J0806, V407 Vul, ES Cet, AM CVn, SDSS J1908, HP Lib, CR Boo, V803 Cen, ZTF J1539, SDSS J0651, SDSS J0935, SDSS J2322, with J0806 having the highest SNR. In particular, we find that J0806 reaches SNR threshold of 5 already after only two days of observation. We predict that its SNR will reach 36.8 after three months of observation, and will exceed 100 after nominal five years of mission (effectively corresponding to 2.5 years of observation time). In addition, we find 3 potential verification binaries with \( 3 \leq \text{SNR} < 5 \): SDSS J1351, CXOGBS J1751 and PTF J0533. Figure 4 shows the evolution of the SNR with time for all verification binaries in blue for TianQin (TQ).

In Table VI we report dimensionless amplitudes (\( A \)) and SNRs for all 81 CVBs considering mission configurations: TianQin (TQ), TQ II and TQ I+II, assuming five years of mission lifetime and setting \( \phi_0 = \pi \) and \( \psi_S = \pi/2 \) for all binaries. We note that the sky position, orbital inclination and GW frequency of the binary affect SNR by a factor of a few (cf. Eq.(16)-(20)). For example, V803 Cen and SDSS J0651 have comparable GW amplitudes \( (16.0 \times 10^{-23} \text{ and } 16.2 \times 10^{-23}) \) respectively, but their SNRs differ significantly \( (6.2 \text{ and } 26.5 \text{ respectively}) \). This difference arises from the fact that SDSS J0651 is both located in a more favourable position on the sky for TianQin (TQ), and has a higher frequency than V803 Cen. We also note that because TianQin (TQ) is oriented directly towards J0806, its SNR is the largest across the sample, although its amplitude is not the highest. When considering the TQ II configuration with a different orientation, its SNR decreases by a factor of \( \sim 3 \).

We find that TQ II can detect 13 verification binaries with \( \text{SNR} > 5 \) and one potential verification binary with \( 3 < \text{SNR} < 5 \). Being orthogonal to TianQin (TQ), the TQ II configuration is more disadvantageous than for the detection of J0806. However, even with TQ II J0806 be detected with the SNR of 41.6. This is because J0806 has the highest frequency across the sample (cf. Table V). With the frequency of 6.22 mHz, it positions in the amplitude-frequency parameter space where noise level of TianQin is the lowest (see also FIG. 5). Therefore, J0806 is still among the best verification sources for the TQ II configuration.

Similarly, for the network TQ I+II we find 14 verification binaries and one potential verification binary. The SNR evolution for different operation times for these verification binaries is represented in red in FIG. 4. The SNR produced by a source in this case is given by the root sum squared of the SNRs of the two configurations considered independently (see Eq.(24)). Therefore, if TianQin (TQ) and TQ II independently detect a source with a similar SNR, the network TQ I+II would improve the SNR by a factor of \( \sqrt{2} \). However, if the source produces significantly higher SNR in one of the detectors in the network, the improvement is not significant (e.g. J0806 in
Table VI).}

In Table III we fixing all other parameters and only report the estimated uncertainties on the amplitude $A$ and inclination angle $\iota$ by inversing for the 14 verification binaries. These two parameters are typically degenerate (cf. Eq. (1)-(2)). However, for nearly edge-on binaries the degeneracy can be broken by using the asymmetry between two GW polarizations [e.g. 92]. This is reflected in a small correlation coefficient $c_{A, \cos \iota} = 0.157$ of SDSS J0651 with inclination angle of $86.95^\circ$. For decreasing inclination angles the degeneracy increases as can be seen for SDSS J1908 and V803 Cen with inclination angles of $\iota = 15^\circ$ and $\iota = 13.5^\circ$ respectively. These two verification binaries have $\Delta \cos \iota > 1$, meaning that the uncertainty on the inclination angle exceeds the physical range $(0, \pi)$.

For eclipsing binaries the inclination angle can be independently determined from the optical light curves. It can then be used to narrow down the uncertainty on the inclination from GW data by removing the respective row and column of the FIM. In the column denoted “without EM on $\iota$” we report uncertainties on $A$ and $\cos \iota$ derived from inversing the $2 \times 2$ FIM. In the column denoted “with EM on $\iota$” we report uncertainties on $A$ for the case when $\iota$ is known a priori from EM observation.

| Source | without EM on $\iota$ | with EM on $\iota$ |
|--------|----------------------|-------------------|
| J0806  | 0.061 0.055 0.991 | 0.008 7.625 |
| V407 Vul | 0.050 0.039 0.904 | 0.021 2.381 |
| ES Cet  | 0.051 0.039 0.904 | 0.022 2.318 |
| SDSS J1351 | 0.193 0.145 0.905 | 0.082 2.354 |
| AM CVn  | 0.115 0.102 0.984 | 0.020 5.750 |
| SDSS J1908 | 6.102 >1 1.000 | 0.100 >1 |
| HP Lib  | 0.384 0.360 0.997 | 0.030 12.800 |
| CR Boo  | 0.865 0.813 0.997 | 0.066 13.106 |
| V 803 Cen | 4.377 >1 1.000 | 0.062 >1 |
| ZTF J1539 | 0.013 0.012 0.300 | 0.013 1.000 |
| SDSS J0651 | 0.033 0.018 0.157 | 0.032 1.031 |
| SDSS J0935 | 0.073 0.056 0.904 | 0.031 2.355 |
| SDSS J2322 | 1.033 0.979 0.998 | 0.063 16.397 |
| PTF J0533 | 0.219 0.137 0.700 | 0.156 1.404 |

C. Simulated Galactic double white dwarfs binaries

To forecast the total number of binaries detectable by TianQin we employ the simulated population of Galactic DWDs (cf. Sect. II B). Here, we set higher SNR threshold
of 7, assuming that there is no \textit{a priori} information from the EM observations to fall back on.

We estimate the number of resolved DWDs for the three considered configurations (TQ, TQ II and TQ I+II) to be of the order of several thousand for the full mission lifetime of 5 years. In Table IV we summarise our result for increasing operation times. In FIG. 5 we show the dimensionless characteristic strain of DWDs with SNR $>40$ in the mock population compared to 14 verification binaries.

The density of DWDs in the bulge region of the Galaxy is significantly higher than in the disk (see Fig. 3 of Korol \textit{et al.} [50]), therefore the detector’s orientation has a significant impact on the total number of detectable DWDs. The Galactic center (where the density of DWD is the highest) in ecliptic coordinate corresponds to ($\lambda = 266.8^\circ, \beta = -5.6^\circ$). TianQin (TQ) is oriented towards ($\lambda = 120.4^\circ, \beta = -4.7^\circ$), that is about 30$^\circ$ away from the Galactic center; TQ II is oriented towards ($\lambda = 30.4^\circ, \beta = 0^\circ$) is about 60$^\circ$ away from the Galactic center. Consequently, the number of detected DWDs for TianQin (TQ) is about 1.3 – 1.4 times larger that for TQ II (cf. Table IV). When we consider TQ I+II the number of detections increases by $\sim 1.3$ compared to TianQin (TQ) alone. We verify that pointing the detector towards the Galactic center would returns the maximum detections $\sim 1.0 \times 10^4$.

FIG. 6 illustrates the distributions of the SNRs and relative uncertainties on binary parameters $A, P, \cos \iota, \psi_S$ and sky position $\Omega_S$ (Eq. 23). The figure shows that most sources have relatively low SNR ($\lesssim 10$), and that there is a not negligible number of sources with SNR $> 100$ reaching the maximum of $\sim 1000$. These high SNR binaries are also well-localized ones (because $\Delta \Omega_S \propto 1/\rho^2$), therefore they will be good candidates for EM follow-up and multi-messenger studies [93]. We find that for 90% of detections the uncertainty on $\Delta P/P$ ranges between $(0.15 – 4.63) \times 10^{-7}$, on $\Delta A/A$ between $0.04 – 5.02$, on $\Delta \cos \iota$ between $0.02 – 4.95$, on $\Delta \psi_S$ between $0.03 – 4.01$ rad, and on $\Delta \Omega_S$ between $0.02 – 21.36$ deg$^2$.

FIG. 4. The SNR evolution of verification binaries over time. Blue stars represent TianQin (TQ) and red stars represent TQ I+II. The black dashed line corresponds to the SNR threshold of 5.

|                      | 0.5yr | 1yr  | 2yr  | 4yr  | 5yr  |
|----------------------|-------|------|------|------|------|
| TQ                   | 2371  | 3589 | 5292 | 7735 | 8710 |
| TQ II                | 1672  | 2595 | 3943 | 5782 | 6540 |
| TQ I+II              | 3146  | 4716 | 6966 | 10023| 11212|

TABLE IV. The expected detection numbers of resolvable binaries for TianQin (TQ), TQ II and TQ I+II.
FIG. 5. The characteristic strain $h_c$ of the 14 verification binaries (the golden dots and the red star) for TQ I+II and the simulated DWDs with SNR > 40 for TianQin, compared with the noise amplitude $h_n$ of TianQin (red line). J0806 is highlighted with a red star. An operation time of 5 years is assumed.

The median value of uncertainties are: $\Delta P/P = 1.41 \times 10^{-7}$, $\Delta A/A = 0.26$, $\Delta \cos \iota = 0.20$, $\Delta \psi_S = 0.39$ rad and $\Delta \Omega_S = 1.85$ deg$^2$. We highlight that TianQin (TQ) can locate 39% of DWDs to better than 1 deg$^2$, while TQ I+II can locate 54% of detection withing 1 deg$^2$.

Next, we explore the additional cases of TianQin operating in combination with LISA: TQ + LISA and TQ I+II + LISA. For these additional cases the mission lifetime of TQ and TQ I+II are assumed to be 5 years, while that for LISA is taken to be 4 years [11]. We verify that by adding LISA to the network the total number of detected DWD doubles. This is due to the fact that LISA is also sensitive to lower GW frequencies, where number of DWD is larger.

As shown in Eq.(24), an additional detector has an effect of increasing the SNR for a source. This improves also the uncertainties on the source parameters, since roughly all the uncertainties scale inversely with the SNR. We illustrate the improvement by using a network of detectors compared to TianQin alone in FIG. 7.

We also look at the improvement in the parameter uncertainties for the 8710 resolvable binaries for TianQin.

In FIG. 7, we present the histograms of ratio between the uncertainties when measured by TianQin alone, and when measured by a network of detectors. Top left panel of FIG. 7 shows that the improvement on SNR is within a factor of 10. While improvements on the parameters uncertainties are within a factor of a few dozens for $\cos \iota$ and $A$ and are largely comparable for all the three networks. Improvements on SNR, $A$, $P$, $\psi_S$ and $\Omega_S$ are larger for TQ + LISA and TQ I+II + LISA; those on $\psi_S$ and $\Omega_S$ reach up to two-three orders of magnitude.

We remark that: (1) TQ + LISA and TQ I+II + LISA are better than TianQin and TQ I+II in determining DWDs’ periods; (2) TianQin and TQ I+II are slightly better than TQ + LISA and TQ I+II + LISA in determining GW amplitudes and $\cos \iota$; (3) TQ I+II is better than TQ + LISA and TQ I+II + LISA and the latter two are better than TianQin in determining the sky positions; (4) The result for the polarization angle $\psi_S$ is a bit mixed but the three networks of detectors usually perform better than TianQin alone.

D. The estimation of merger rate

In this section we estimate the number of DWD mergers that can be expected for TianQin.

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2 For LISA we adopt the sensitivity curve from [94].
We consider a DWD with equal mass components of $1\,M_\odot$, so that the total mass of the binary is larger than the Chandrasekhar mass limit. We model its chirping signal with the IMRPhenomPv2 waveform [95] and calculate SNR using Eq. (15). Following Wang et al. [14] and assuming a mission lifetime of 5 years for TianQin, we find that the SNR of our example DWD binary is

$$\rho \approx 20 \left( \frac{1 \, \text{Mpc}}{d} \right).$$

This result implies that TianQin can detect SNe Ia explosions within the virial radius of the Local Group.

The SNe Ia rate in the Milky Way is 0.01-0.005/yr [96], and the DWD merger rate is $4.5-7 \times$ the SNe Ia rate (as most DWDs would not exceed the Chandrasekhar limit) [64, 97]. This means that an optimistic estimation of the DWD merger rate is $\sim 0.07/\text{yr}$ in the Galaxy.

To estimate the DWD merger rate in the Local Group, we note the Local Group is consisted of about 60 galaxies, most with masses $< 10^8 \, M_\odot$. Therefore, the total mass of the Local Group galaxies is dominated by the Milky Way and the Andromeda galaxy [98]. The masses of the Milky Way and the Andromeda Galaxy are $0.8 - 1.5 \times 10^{12} \, M_\odot$ and $1 \sim 2 \times 10^{12} \, M_\odot$ [98], respectively. Assuming that the DWD merger rate is proportional to the galaxy mass, we obtain

$$\left( 1 + \frac{2}{0.8} \right) \times 0.07/\text{yr} \approx 0.25/\text{yr}.\quad (26)$$
Therefore, in the optimistic case, TianQin would be able to observe one DWD merger event with its lifetime of 5 years.

V. SUMMARY AND DISCUSSION

In this paper, we carry out the first prediction for the detection of Galactic DWDs with TianQin. For this purpose, we adopt a catalogue of known DWDs discovered with EM observations and a mock Galactic population constructed using binary population synthesis method. We outlined analytical expressions and numerical methods for computing noise curves, SNR and uncertainties on the measured parameters of monochromatic GW sources for the TianQin mission with fixed orientation. By considering different detector orientations, in this work we also address an interesting open question regarding the optimal orientation of mission.

First, we assessed the strength of the foreground arising from unresolved Galactic DWDs. We find that its effect can be largely ignored for the present design sensitivity of the TianQin detector.

When considering the sample of known DWDs, we find that out of 81 CVBs with orbital periods $\lesssim 5$ hour TianQin can detect 12 with SNR $\geq 5$ within 5 years of mission lifetime. In particular, we find that TianQin will be able to detect J0806 (its main verification source) already after two days of observations. We estimate that the expected uncertainty on GW amplitude for verifica-
tion binaries is of a few per cent. For verification binaries with small inclination angles (nearly face-on), this uncertainty can be improved up to a factor of 16, if the binary inclination angle is known a priori.

When analysing a synthetic Galactic population of DWD, we find that the overall number of detections is expected to be $8.7 \times 10^4$ for the full mission duration of 5 years. We find a typical value (median) of $\sim 10^{-7}$ on relative uncertainty of DWDs’ orbital periods, 0.26 on relative uncertainty of GW amplitude, 0.20 uncertainty on $\cos \iota$ and $\sim 1$ deg$^2$ uncertainty on sky positions, respectively. About 39% can be localized to better than 1 deg$^2$.

Finally, we outline a proof-of-principle calculation showing that TianQin is expected to detect one DWD merger event with a supernovae type Ia-like counterpart during its five years of operation time.

In addition to TianQin’s nominal orientation (TQ, pointing towards J0806), we also analysed a variation of the mission oriented perpendicularly (TQ II), and different networks of simultaneously operational GW detectors TQ I+II, TQ + LISA and TQ I+II + LISA. Although TQ II and TQ I+II can detect the same set of 12 verification binaries as TianQin (TQ), the total number of detections increases by $\sim 1.3$ when considering the network TQ I+II. In addition, the total number of binaries localised to better than 1 deg$^2$ also increases to 54% of the total detected sample. We find that the major advantage of combining TianQin and LISA, besides increasing the total number of detections, consists in the improvement on binary parameter uncertainties by $1-2$ orders of magnitude, while the improvement the sky localization can reach up to 3 orders of magnitude.

We are living in the era of large astronomical surveys with the number of known DWDs increasing every year thanks to surveys like the ELM [99] and ZTF [100] surveys. The upcoming LSST [101], GOTO [102] and BlackGem [103] will further enlarge the sample by the time TianQin will fly. We show that the TianQin mission has the potential to push the DWD field in the regime of robust statistical studies by increasing the number of detected DWDs to several thousand. By combining data from GW observatories such as TianQin with those from aforementioned large optical surveys will enable multi-messenger studies and advance our knowledge about these unique binary systems.

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Appendix A: Table of the selected candidate verification binaries

All the selected CVBs are listed in Table V, with the ecliptic coordinates ($\lambda, \beta$), the GW frequency $f = 2/P$, with $P$ being the orbital period of the corresponding binary stars, the luminosity distance $d$, the inclination angle $\iota$ of the source and the heavier and the lighter masses, $M$ and $m$, respectively, of the component stars. In some cases, there is no direct measurement on the masses or the inclination angles, then estimated values are assigned based on the evolutionary stage and the mass ratio of the corresponding system. All such values are given with a square bracket. We make a conservative choice of 5 kpc for the distance to J0806 [74]. The parameters of the listed sources are taken from: i [71], ii [65], iii [22], iv [66], v [69], vi [67], vii [68], viii [99], ix [104] and the references therein.

| Source            | $\lambda$  | $\beta$  | $f$  | $d$  | $M$  | $m$ | $\iota$  | Refs. |
|------------------|------------|----------|-----|-----|-----|-----|----------|-------|
| AM CVn type systems |            |          |     |     |     |     |          |       |
| J0806            | 120.4425   | -4.7040  | 6.22| [5] | 0.55| 0.27| 38       | i     |
| V407 Vul         | 294.9945   | 46.7829  | 3.51| 1.786| 0.177| 60 | i        |
| ES Cet           | 24.6120    | -20.3339 | 3.22| 1.584| 0.161| 60 | i        |
| SDSS J135154.46-064309.0 | 208.3879 | 4.4721 | 2.12 | 1.317 | 0.100 | 60 | i |
| AM CVn           | 170.3858   | 37.4427  | 1.94| 0.299| 0.68 | 0.68| 30       | i     |
| SDSS J190817.67+394036.4 | 298.2172 | 61.4542 | 1.84 | 1.044 | 0.085 | 30 | i |
| HP Lib           | 235.0882   | 4.9597   | 1.81| 0.276| 0.645| 0.068| 30       | i     |

Continued on next page
| Source               | λ [deg] | β [deg] | f [mHz] | d [kpc] | M [M\(_\odot\)] | m [M\(_\odot\)] | t [deg] | Refs. |
|---------------------|---------|---------|---------|---------|----------------|----------------|---------|-------|
| PTF1 J191905.19+481506.2 | 309.0023 | 69.0296 | 1.48    | 1.338   | 0.8            | 0.066         | 60      | i     |
| ASASSN-14cc         | 303.9576 | -42.8640 | 1.48    | 1.019   | 0.0 (1)        | 0.06 (1)      | 60      | ii    |
| CXOGBS J175107.6-294037 | 268.0614 | -6.2526 | 1.45    | 0.971   | 0.8            | 0.064         | 60      | i     |
| CR Boo              | 202.2728 | 17.8791 | 1.36    | 0.337\(^{0}\) | 0.885         | 0.066         | 30      | i     |
| KL Dra              | 334.1334 | 78.3217 | 1.33    | 0.956   | 0.76           | 0.057         | 60      | ii,iii|
| V803 Cen            | 216.1673 | -30.3166 | 1.24    | 0.861   | 0.8            | 0.053         | 60      | i,ii  |
| PTF1 J071912.13+485834.0 | 104.3883 | 26.5213 | 1.24    | 0.857   | 0.85           | 0.032         | 60      | ii,iii|
| SDSS J092638.71+362402.4 | 132.2867 | 20.2342 | 1.18    | 0.577   | 0.8            | 0.053         | 60      | ii,iii|
| CP Eri              | 42.1327  | -26.4276 | 1.36    | 0.337\(^{a}\) | 0.885         | 0.066         | 30      | i     |
| KL Dra              | 334.1334 | 78.3217 | 1.33    | 0.956   | 0.76           | 0.057         | 60      | ii,iii|
| V803 Cen            | 216.1673 | -30.3166 | 1.24    | 0.861   | 0.8            | 0.053         | 60      | i,ii  |

**continued on next page**
### Table V – Continued from previous page

| Source | \(\lambda\) [deg] | \(\beta\) [deg] | \(f\) [mHz] | \(d\) [kpc] | \(M\) [M\(_{\odot}\)] | \(m\) [M\(_{\odot}\)] | \(\iota\) [deg] | Refs. |
|--------|-----------------|----------------|-------------|------------|----------------|--------------|--------------|-------|
| SDSS J12410.36–22082.9 | 188.8196 | 1.1194 | 0.25 | 0.754 | 0.09 | 0.23 | [60] iii |
| SDSS J10554.10+22423.2 | 145.432 | 10.4254 | 0.24 | 0.555 | 0.36 | 0.31 | 88.9 iii |
| SDSS J135219.99+023814.4 | 177.1265 | 1.8106 | 0.23 | 0.718 | 0.47 | 0.41 | 89.2 iii |
| SDSS J105435.78–212155.9 | 173.8923 | -26.0107 | 0.22 | 1.313 | 0.39 | 0.168 | [60] iii |
| SDSS J074511.56+194926.5 | 114.6397 | -1.3939 | 0.20 | 0.875 | 0.16 | 0.156 | [60] iii |
| WD 1242–105 | 194.5586 | -5.5520 | 0.19 | 0.040 | 0.56 | 0.39 | 45.1 i,iii |
| SDSS J110815.50+151246.6 | 162.1662 | 8.9070 | 0.19 | 0.698 | 0.42 | 0.167 | [60] iii |
| WD 1101+364 | 152.2513 | 27.6895 | 0.16 | 0.088 | 0.36 | 0.31 | [60] iii |
| WD 1704+4807BC | 242.3234 | 70.1865 | 0.16 | 0.039 | 0.39 | 0.56 | [60] ix |
| SDSS J011210.25+183503.7 | 23.7268 | 10.1149 | 0.16 | 0.843 | 0.62 | 0.16 | [60] iii |

### Appendix B

**Table VI:** The expected amplitude \(A\) and SNR of 81 candidate verification binaries, assuming a nominal mission lifetime of five years and the three configurations of TianQin. \(A\) is given in units of \(10^{-23}\).

| Source | \(A\) | SNR | TQ | TQ II | TQ I+II |
|--------|------|-----|-----|-------|--------|
| AM CVn type systems |
| J0806  | 6.4  | 116.202 | 41.657 | 123.443 |
| V407 Vul | 11.0 | 41.528 | 21.537 | 46.780 |
| ES Cet | 10.7 | 17.775 | 42.110 | 45.708 |
| SDSS J135154.46–064309.0 | 6.2 | 4.454 | 11.345 | 12.188 |
| AM CVn | 28.3 | 31.245 | 37.499 | 48.810 |
| SDSS J190817.07+394036.4 | 6.1 | 8.622 | 5.077 | 10.006 |
| HP Lib | 15.7 | 16.619 | 29.427 | 33.795 |
| PTF1 J191905.19+481506.2 | 3.2 | 1.526 | 1.122 | 1.894 |
| ASASSN-14cc | 0.5 | 0.338 | 0.188 | 0.387 |
| CXOGBS J175107.6–294037 | 4.2 | 3.022 | 2.172 | 3.722 |
| CR Boo | 12.9 | 5.473 | 14.029 | 15.058 |
| KL Dra | 3.5 | 1.109 | 1.006 | 1.497 |
| V803 Cen | 16.0 | 6.187 | 15.026 | 16.249 |
| PTF1 J071912.13+485334.0 | 3.6 | 1.844 | 0.982 | 2.089 |
| SDSS J092638.71+362402.4 | 3.6 | 1.175 | 0.664 | 1.350 |
| 2QZ J142701.6–012310 | 0.9 | 0.115 | 0.282 | 0.304 |
| SDSS J173047.59+554518.5 | 0.4 | 0.069 | 0.066 | 0.096 |
| 2QZ J142701.6–012310 | 0.9 | 0.115 | 0.282 | 0.304 |
| SDSS J124058.03–015919.2 | 2.8 | 0.436 | 0.842 | 0.948 |
| NSV1440 | 1.0 | 0.140 | 0.130 | 0.191 |
| SDSS J012940.05+384210.4 | 3.1 | 0.389 | 0.882 | 0.964 |
| SDSS J172102.48+273901.2 | 0.4 | 0.069 | 0.063 | 0.093 |
| ASASSN-14mv | 1.5 | 0.388 | 0.174 | 0.425 |
| ASASSN-14ei | 1.4 | 0.133 | 0.192 | 0.233 |

*Continued on next page*
| Source              | A  | SNR   | TQ  | TQ II | TQ I+II |
|---------------------|----|-------|-----|-------|---------|
| SDSS J152509.57+360054.5 | 0.7 | 0.059 | 0.097 | 0.114 |
| SDSS J080449.49+161624.8 | 1.4 | 0.304 | 0.119 | 0.326 |
| SDSS J141118.31+481257.6 | 0.8 | 0.068 | 0.093 | 0.115 |
| GP Com              | 5.0 | 0.400 | 0.882 | 1.099 |
| SDSS J090221.35+381941.9 | 0.7 | 0.121 | 0.053 | 0.132 |
| ASASSN-14cn         | 4.0 | 0.220 | 0.227 | 0.316 |
| SDSS J120841.96+355025.2 | 4.1 | 0.383 | 0.408 | 0.560 |
| SDSS J164228.06+193410.0 | 0.3 | 0.025 | 0.029 | 0.038 |
| SDSS J155252.48+320150.9 | 0.7 | 0.039 | 0.060 | 0.145 |
| V396 Hya/CE 315     | 5.5 | 0.221 | 0.535 | 0.579 |
| SDSS J1319+5915     | 1.3 | 0.063 | 0.062 | 0.089 |

**detached DWD**

| Source              | A  | SNR   | TQ  | TQ II | TQ I+II |
|---------------------|----|-------|-----|-------|---------|
| ZTF J153932.16+502738.8 | 18.4 | 51.351 | 60.184 | 79.114 |
| SDSS J065133.34+284423.4 | 16.2 | 26.535 | 15.700 | 30.831 |
| SDSS J093506.92+441107.0 | 29.9 | 28.797 | 14.245 | 32.128 |
| SDSS J232230.20+050942.06 | 8.7 | 9.973 | 12.371 | 15.891 |
| PTF J053332.05+020911.6 | 7.6 | 4.965 | 4.042 | 6.402 |
| SDSS J010657.39+100003.3 | 8.3 | 0.989 | 1.892 | 2.135 |
| SDSS J163030.58+423305.7 | 11.6 | 1.423 | 1.721 | 2.233 |
| SDSS J082239.54+304857.2 | 10.4 | 1.713 | 0.887 | 1.929 |
| ZTF J190125.42+50929.5 | 6.5 | 0.614 | 0.549 | 0.824 |
| SDSS J104336.27+055149.9 | 3.9 | 0.649 | 0.561 | 0.857 |
| SDSS J105353.89+520031.0 | 9.0 | 0.698 | 0.457 | 0.835 |
| SDSS J065648.23+061141.5 | 9.3 | 0.475 | 0.931 | 1.045 |
| SDSS J105611.02+653631.5 | 8.7 | 0.570 | 0.378 | 0.684 |
| SDSS J092345.59+302805.0 | 26.2 | 2.422 | 1.138 | 2.675 |
| SDSS J143633.28+501026.9 | 6.7 | 0.262 | 0.359 | 0.444 |
| SDSS J085211.90+115236.4 | 3.9 | 0.235 | 0.094 | 0.253 |
| WD 9057–666          | 25.7 | 0.502 | 0.621 | 0.798 |
| SDSS J174140.49+652638.7 | 4.9 | 0.104 | 0.103 | 0.147 |
| SDSS J075552.40+490627.9 | 1.7 | 0.072 | 0.035 | 0.080 |
| SDSS J233821.51–205222.8 | 3.3 | 0.073 | 0.078 | 0.107 |
| SDSS J230919.90+260346.7 | 2.4 | 0.046 | 0.062 | 0.077 |
| SDSS J084910.13+044528.7 | 3.2 | 0.094 | 0.042 | 0.103 |
| SDSS J002207.65+101423.5 | 2.1 | 0.036 | 0.056 | 0.067 |
| SDSS J075141.18–014129.9 | 2.7 | 0.078 | 0.032 | 0.084 |
| SDSS J211921.96–001825.8 | 2.9 | 0.069 | 0.037 | 0.078 |
| SDSS J123410.36–022802.8 | 0.9 | 0.031 | 0.020 | 0.038 |
| SDSS J100559.10+224932.2 | 5.3 | 0.059 | 0.039 | 0.071 |
| SDSS J115219.99+024814.4 | 6.3 | 0.047 | 0.062 | 0.078 |
| SDSS J105435.78–212155.9 | 1.3 | 0.014 | 0.017 | 0.022 |
| SDSS J074511.56–194926.5 | 0.6 | 0.008 | 0.003 | 0.008 |
| WD 1242–105          | 109.2 | 0.755 | 1.656 | 1.820 |
| SDSS J110815.50+151246.6 | 2.4 | 0.022 | 0.020 | 0.030 |
| WD 1101+364          | 25.4 | 0.159 | 0.120 | 0.199 |
| WD 1704+4807BC       | 99.9 | 0.376 | 0.388 | 0.541 |
| SDSS J011210.25+183503.7 | 2.2 | 0.008 | 0.019 | 0.020 |
| SDSS J123316.20+160204.6 | 2.2 | 0.009 | 0.014 | 0.016 |
| SDSS J113017.42+385549.9 | 3.9 | 0.019 | 0.016 | 0.026 |
| SDSS J111215.82+111745.0 | 1.4 | 0.005 | 0.005 | 0.008 |
| SDSS J100554.05+355014.2 | 1.1 | 0.005 | 0.003 | 0.006 |
| SDSS J144342.74+150938.6 | 2.6 | 0.005 | 0.010 | 0.011 |
| SDSS J184037.78+642312.3 | 2.1 | 0.004 | 0.004 | 0.005 |

*Continued on next page*
Appendix C: Re-expression of the responded Gravitational Wave Signal

For the convenience of calculation, we rearrange the expression for the waveform in the detector

$$h(t) = A(t) \cos \Psi(t),$$

where the waveform amplitude $A(t)$ is

$$A(t) = \left[ (A_+ F^+(t))^2 + (A_\times F^\times(t))^2 \right]^{1/2}.$$  

$A_+$ and $A_\times$ are given by

$$A_+ = A(1 + \cos \iota^2), \quad A_\times = 2A \cos \iota.$$  

The phase of the waveform is

$$\Psi(t) = 2\pi f t + \phi_0 + \Phi_D(t) + \Phi_P(t),$$

The polarization phase $\Phi_P(t)$ is given by

$$\Phi_P(t) = \tan^{-1} \left( -\frac{A_\times F^\times(t)}{A_+ F^+(t)} \right).$$

Appendix D: Derivation of the average amplitude

In order to verify our SNR calculation, more specifically the calculation of average amplitude, we can obtain the average amplitude from the antenna beam patterns function given by (13) in [57]

$$F^+(t, \theta, \phi, \psi) = \cos 2\psi \xi^+(t; \theta, \phi) - \sin 2\psi \xi^\times(t; \theta, \phi),$$

$$F^\times(t, \theta, \phi, \psi) = \sin 2\psi \xi^+(t; \theta, \phi) + \cos 2\psi \xi^\times(t; \theta, \phi),$$

and

$$\xi^+(t; \theta, \phi) = \frac{\sqrt{3}}{32} \left( 4 \cos 2(\kappa - \beta')((3 + \cos 2\theta) \sin \theta_\times \sin (\phi - \phi_\times) + 2 \sin (\phi - \phi_\times) \sin 2\theta \cos \theta_\times) 
- \sin 2(\kappa - \beta')(3 + \cos 2\theta)(\phi - \phi_\times)(9 + \cos 2\theta(3 - \cos 2\theta_\times)) + 6 \cos 2\theta_\times \sin^2(\phi - \phi_\times)
- 6 \cos 2\theta \cos^2 \theta_\times + 4 \cos(\phi - \phi_\times) \sin 2\theta \sin 2\theta_\times) \right),$$

$$\xi^\times(t; \theta, \phi) = \frac{\sqrt{3}}{8} \left( -4 \cos 2(\kappa - \beta')(\cos 2\phi - \phi_\times) \cos \theta \sin \theta_\times + \cos(\phi - \phi_\times) \sin \theta \cos \theta_\times
+ \sin 2(\kappa - \beta')(\cos(3 - \cos 2\theta_\times) \sin 2(\phi_\times - \phi - 2 \sin(\phi - \phi_\times) \sin \theta \sin 2\theta_\times) \right).$$

where $\kappa = 2\pi f_{s} t + \lambda', f_{s} \approx 1/(3.656d)$ is the modulation frequency from the rotation of the satellites around the guiding center. $\lambda'$ and $\beta'$ are some of initial phase of constant.

In above expression, $\theta = \pi/2 - \beta$ and $\phi = \lambda$ are source location in the ecliptic coordinate system. $\psi$ is polarization angle. $\theta_\times$ and $\phi_\times$ are the ecliptic coordinate of reference source. For the reference source of TianQin is J0806, there are $\theta_\times = -4.7040^\circ$ and $\phi_\times = 120.4425^\circ$.

By doing the same process as described in Section III D, we get some expressions similar to (18)-(20), given below.

$$\langle A^2 \rangle = A^2 \left[ (1 + \cos^2 \iota)^2 \langle F_+^2 \rangle + 4 \cos^2 \iota \langle F_\times^2 \rangle \right],$$

$$\langle F_\times^2 \rangle = \frac{1}{4} \cos^2 2\psi \langle D_\times^2 \rangle - 4 \sin 2\psi \langle D_+ D_\times \rangle + \sin^2 2\psi \langle D_\times^2 \rangle,$$
where

\[
\langle D_+^2 \rangle = b_1^2 + b_2^2, \\
\langle D_+^2 \rangle = b_3^2 + b_4^2, \\
\langle D_+ D_\times \rangle = -2(b_1b_3 + b_2b_4). 
\]  
(D6)

and

\[
b_1 = \frac{\sqrt{3}}{8}((3 + \cos 2\theta) \sin \theta_s \sin(\phi - \phi_s) + 2 \sin(\phi - \phi_s) \sin \theta \cos \theta_s), \\
b_2 = \frac{\sqrt{3}}{32}(3 + \cos 2\phi)(9 + \cos 2\theta(3 - \cos 2\theta_s)) + 6 \cos 2\theta_s \sin^2(\phi - \phi_s) \\
- 6 \cos 2\theta \cos^2 \theta_s + 4 \cos(\phi - \phi_s) \sin 2\theta \sin 2\theta_s), \\
b_3 = \frac{\sqrt{3}}{2}(\cos 2\phi \cos \theta \sin \theta_s + \cos(\phi - \phi_s) \sin \theta \cos \theta_s), \\
b_4 = \frac{\sqrt{3}}{8}((3 - \cos 2\theta_s) \cos \theta \sin 2(\phi - \phi) + 2 \sin(\phi - \phi) \sin \theta \sin 2\theta_s). 
\]  
(D7)

The average amplitude calculated by (D3)-(D7) is consistent with (18)-(20), with 0.1\% ~ 1\% of relative uncertainty.

Appendix E: Coordinate transformation

The transformation of the source position from the ecliptic coordinate ($\beta, \lambda$) to the detector coordinate ($\theta_S, \phi_S$) and ($\theta_S', \phi_S'$) of the TianQin (TQ) and TQ II is described by the following formula

\[
\begin{pmatrix}
\frac{d \sin \theta_S \cos \phi_S}{d \sin \theta_S \sin \phi_S} \\
\frac{d \sin \theta_S \sin \phi_S}{d \cos \theta_S}
\end{pmatrix} = R_x(\theta = 120^\circ - 90^\circ)R_z(\theta = -4.7^\circ - 90^\circ) \begin{pmatrix}
d \cos \beta \cos \lambda \\
d \cos \beta \sin \lambda \\
d \sin \beta
\end{pmatrix} 
\]  
(E1)

and \( \begin{pmatrix}
\frac{d \sin \theta_S' \cos \phi_S'}{d \sin \theta_S' \sin \phi_S'} \\
\frac{d \sin \theta_S' \sin \phi_S'}{d \cos \theta_S'}
\end{pmatrix} = R_y(\theta = 90^\circ)R_x(\theta = 120^\circ - 90^\circ)R_z(\theta = -4.7^\circ - 90^\circ) \begin{pmatrix}
d \cos \beta \cos \lambda \\
d \cos \beta \sin \lambda \\
d \sin \beta
\end{pmatrix}, \)  
(E2)

where the rotation matrix are

\[
R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}, R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \text{ and } R_z(\theta) = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. 
\]  
(E3)

[1] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016).
[2] A. Einstein, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin , 688 (1916).
[3] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. X 6, 041015 (2016).
[4] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. X 9, 031040 (2019).
[5] The LIGO Scientific Collaboration, the Virgo Collaboration, et al., arXiv e-prints , arXiv:2004.08342 (2020), arXiv:2004.08342 [astro-ph.HE].
[6] B. P. Abbott et al., Astrophysical Journal 892, L3 (2020), arXiv:2001.01761 [astro-ph.HE].
[7] S. T. McWilliams, R. Caldwell, K. Holley-Bockelmann, S. L. Larson, and M. Vallisneri, arXiv e-prints (2019), arXiv:1903.04592 [astro-ph.HE].
[8] M. Kamionkowski and E. D. Kovetz, ARA&A 54, 227 (2016), arXiv:1510.06042.
[9] Z. Arzoumanian et al., Astrophysical Journal 859, 47 (2018), arXiv:1801.02617 [astro-ph.HE].
[10] R. M. Shannon et al., Science 349, 1522 (2015), arXiv:2004.08342 [astro-ph.HE].
