Fat Branes in Infinite-Volume Extra Space

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Abstract

We study branes residing in infinite volume space and of finite extent in the transverse directions. We calculate the graviton propagator in the harmonic gauge both inside and outside the brane and discuss its dependence on the thickness of the brane. Our treatment includes the full tensor structure of the propagator. We obtain two infinite towers of massive modes and tachyonic ghosts. In the thin-brane limit, we recover four-dimensional Einstein gravity. We compare our results to similar recent results by Dubovsky and Rubakov.

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I. INTRODUCTION

The weakness of the gravitational force has been successfully explained by postulating the existence of extra dimensions \([1, 2, 3, 4, 5]\). The effect of the extra dimensions is a high-energy modification of Newton’s Law of gravity due to the tower of Kaluza-Klein modes. When the extra dimensions are of infinite volume, light Kaluza-Klein modes may dominate even at low energies \([6, 7, 8]\). Thus, unlike with finite-volume extra space, Newton’s Law is modified at astronomically large distances \([9, 10, 11, 12, 13, 14, 15, 16, 17]\). Dvali and Gabadadze \([7]\) showed that this is not the case if the infinite space in which the brane lives has dimension \(D > 5\). They studied a three-brane of the \(\delta\)-function type and showed that the graviton propagator has a four-dimensional momentum dependence on the brane even at low energies. This feature is expected to persist if the brane is of finite thickness (“fat”) in the transverse directions for phenomenologically relevant values of the momentum. For extremely low energies, a fat brane should lead to a higher-dimensional behavior of the propagator. This was discussed qualitatively in \([7]\).

Here, we present a quantitative study of a fat brane in infinite volume extra space. By linearizing gravity in the harmonic gauge, we arrive at an explicit expression for the graviton propagator. First, we obtain the propagator for the trace of the metric field over the transverse directions. The trace is a scalar field from the four-dimensional brane point of view. This scalar then contributes to the four-dimensional graviton propagator as a source, in addition to the matter fields. This complicates the tensor structure of the graviton propagator which becomes momentum dependent. We explicitly obtain the solution for the graviton propagator and analyze its momentum dependence and pole structure. We find two infinite towers of massive modes and tachyonic ghosts. In the thin-brane limit we recover four-dimensional Einstein gravity on the brane.

Our discussion is organized as follows. In section \([\text{II}]\) a brane-bulk action is considered which is similar to the action of ref. \([7]\) but generalized to allow for a brane of finite thickness in the bulk. In section \([\text{III}]\) we solve the linearized Einstein field equations in the harmonic gauge and obtain an explicit expression for the graviton propagator. In section \([\text{IV}]\) we analyze the pole structure of the graviton propagator and compare our results to the Dubovsky-Rubakov model \([26]\). In section \([\text{V}]\) we analyze the momentum dependence of the graviton propagator both on the brane
and in the bulk. We finally conclude with a summary of our results in VII.

II. \textbf{D-DIMENSIONAL FAT BRANE MODEL}

We are interested in the dynamics of a 3-brane in a $D$-dimensional infinite space. The 3-brane is allowed to have finite thickness in the bulk-space with extent governed by the density function $\sigma_\Lambda(y)$. The action is similar to the one discussed in ref. \cite{7},

$$S = M^{D-2} \int d^4x d^{D-4}y \sqrt{-g} R^{(D)} + \overline{M}^2 \int d^4x d^{D-4}y \sqrt{-\overline{g}} \sigma_\Lambda(y) \overline{R}^{(4)} + S_{\text{matter}} \quad (1)$$

where $g_{AB}$ is the $D$-dimensional metric which generates the $D$-dimensional Ricci scalar $R^{(D)}$, whereas $\overline{R}^{(4)}$ is generated by the four-dimensional metric $\overline{g}_{\mu\nu}$ which is the induced metric on the slice $\bar{y} = \text{const.}$. Capital Latin indices run over $D$-dimensional space-time ($A, B = 0, 1, 2, \ldots, D$), Greek indices run over the four-dimensional brane worldvolume spanned by coordinates $x^\mu$ ($\mu = 0, 1, 2, 3$) and lowercase Latin indices run over the extra space spanned by $y_m$ ($m = 4, 5, \ldots, D$).

$M$ is the $D$-dimensional Plank mass. $S_{\text{matter}}$ is the (unspecified) matter action giving rise to the fat brane configuration. We set $y = |\bar{y}| = \sqrt{y_1^2 + y_2^2 + \ldots + y_{D-4}^2}$ and $\sigma_\Lambda(y)$ is a smooth function of width $1/\Lambda$ approximating a $\delta$-function. The mass scale $\overline{M}$ is related to the four-dimensional Newton constant. In general, $\overline{M}$ will depend on $M$, but here they will be treated as independent scales.

For explicit calculations, we will choose a step-function form of the density $\sigma_\Lambda$,

$$\sigma_\Lambda(y) = \frac{(D-4)\Lambda^{D-4}}{\omega_{D-4}} \Theta(1/\Lambda - y) \quad (2)$$

where $\omega_n$ is the surface area of the unit $n$-dimensional sphere. The careful reader may wish to smoothen the step-function first and then take the limit in which $\sigma_\Lambda$ becomes discontinuous. Our results are not altered. In the limit $\Lambda \to \infty$, the density $\sigma_\Lambda$ approaches a $\delta$-function: $\sigma_\Lambda(y) \to \delta^{D-4}(\bar{y})$. This continuous distribution of the 3-brane may be thought of as the continuous limit of a discrete set of the four-dimensional hypersurfaces (infinitely thin 3-branes) discussed in \cite{7}.

The Einstein field equations are

$$M^{D-2}G^{(D)}_{AB}(x^\mu, y^m) + \overline{M}^2 \sigma_\Lambda(y) \overline{G}^{(4)}_{AB}(x^\mu, y^m) = T_{AB}(x^\mu, y^m) \quad ,$$

$(3)$
where $G^{(D)}_{AB}$ is the $D$-dimensional Einstein tensor and $\overline{G}^{(4)}_{AB}$ only has brane worldvolume components. Expanding around a flat background,

$$g_{AB} = \eta_{AB} + h_{AB}$$  \hspace{1cm} (4)

the first-order Einstein equations are as follows. The transverse components give

$$2\partial^A \partial^n h_{An} - \partial_n \partial^n h_A^n - \partial_A \partial^A h_n^n = (D - 4) (\partial^C \partial^D h_{CD} - \partial_C \partial^C h^D_D)$$  \hspace{1cm} (5)

The mixed components give

$$\partial_A \partial^A h_{\alpha n} = \partial_\alpha \partial^A h_{An} + \partial_n \partial^A h_{A\alpha} - \partial_\alpha \partial_n h_A^A$$  \hspace{1cm} (6)

and the brane worldvolume components imply

$$M^{D-2} (\partial_\alpha \partial^A h_{\beta A} + \partial_\beta \partial^A h_{\alpha A} - \partial_A \partial^A h_{\alpha \beta} - \partial_\alpha \partial_B h^B_A) - \eta_{\alpha \beta} (\partial^A \partial^B h_{AB} - \partial_B \partial^B h^A_A)$$

$$+ \overline{\sigma}_A(y) (\partial_\alpha \partial^\nu h_{\beta \nu} + \partial_\beta \partial^\nu h_{\alpha \nu} - \partial_\nu \partial^\nu h_{\beta A} - \partial_A \partial_\beta h_{\nu \nu} - \eta_{\alpha \beta} (\partial^\mu \partial^\nu h_{\mu \nu} - \mu \partial^\mu h_{\nu \nu}))$$

$$= T_{\alpha \beta}(x^\mu, \vec{y})$$  \hspace{1cm} (7)

where we have chosen a matter source described by the stress-energy tensor $T_{\mu \nu}$ whose transverse components vanish ($T_{mn} = T_{\mu n} = 0$). Indices are raised and lowered by the flat metric tensor $\eta_{AB}$.

To solve the field equations, we shall choose the harmonic gauge,

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h^A_A$$  \hspace{1cm} (8)

We obtain from eqs. (5) and (6), respectively,

$$(6 - D) \partial_A \partial^A h^n_n = (D - 4) \partial_A \partial^A h^\mu_\mu$$  \hspace{1cm} (9)

$$\partial_A \partial^A h_{m\alpha} = 0$$  \hspace{1cm} (10)

so we may set

$$h_{m\alpha} = 0$$  \hspace{1cm} (11)

$$(D - 6) h^n_n + (D - 4) h^\mu_\mu = 0$$  \hspace{1cm} (12)
Then the brane worldvolume components of the Einstein equations can be written in the following form:

\[- M^{D-2} \partial_A \partial^A (h_{\alpha \beta} - \frac{1}{2} \eta_{\alpha \beta} h_B^B) + \underline{\mathcal{M}}^2 \sigma_\Lambda \left( - \partial_\nu \partial^\nu h_{\alpha \beta} + \partial_\alpha \partial_\beta h_n^n - \frac{1}{2} \eta_{\alpha \beta} \partial_\mu \partial_\mu (h_n^n - h_\nu^\nu) \right) \]

\[ = \mathcal{T}_{\alpha \beta} (x^\mu, \tilde{y}) \]  

(13)

Performing a Fourier transform in the brane worldvolume coordinates \( x^\mu \) and multiplying by an arbitrary conserved stress-energy tensor \( \mathcal{T}'_{\alpha \beta} \), which for simplicity is assumed to have no \( \tilde{y} \)-dependence, we obtain

\[ \left( M^{D-2} (p^2 + \partial_\mu \partial^\mu) + \underline{\mathcal{M}}^2 \sigma_\Lambda p^2 \right) \tilde{h}_{\alpha \beta} = \tilde{T}_{\alpha \beta} (p^\mu, \tilde{y}) \tilde{T}'_{\alpha \beta} 
\]

\[ + \frac{1}{2} \tilde{T}_{\mu \nu} \left( \underline{\mathcal{M}}^2 \sigma_\Lambda p^2 \left( \tilde{h}_\nu^\nu - \tilde{h}_n^n \right) + M^{D-2} (p^2 + \partial_\nu \partial^\nu) \tilde{h}_n^n \right) \]

where the Fourier transformed, \( D \)-dimensional d’Alembertian is \( \partial_A \partial^A = - \partial_\nu \partial^\nu - p^2 \) with \( p^2 = p_0^2 - \tilde{p}^2 \) the worldvolume Minkowski four-momentum. In the next section, we shall solve this equation for the graviton propagator.

**III. GRAVITON PROPAGATOR**

In general, the spread functions of the brane and the matter source are different. However, it was argued by Dvali, et al. \cite{25} that the two spreads coincide at lowest order with correction terms suppressed by factors \( o(M/\mathcal{M}) \). We shall therefore adopt a source stress-energy tensor of the form

\[ T_{\alpha \beta} (x^\mu, \tilde{y}) = T_{\alpha \beta} (x^\mu) \sigma_\Lambda (y) \]  

(15)

in the explicit calculation of the tensor structure and momentum dependence of the graviton propagator. Taking the trace of eq. (13), we obtain

\[ - \frac{(D-2)}{(D-4)} M^{D-2} (p^2 + \partial_\nu \partial^\nu) \tilde{h}_n^n + \frac{2(D-5)}{(D-4)} \underline{\mathcal{M}}^2 \sigma_\Lambda p^2 \tilde{h}_n^n = \tilde{T}_{\alpha} \sigma_\Lambda (y) \]  

(16)

where we used eq. (12) to express \( \tilde{h}_{\mu}^\mu \) in terms of \( \tilde{h}_n^n \). This is an equation for the field \( \tilde{h}_n^n \) (trace over transverse directions of the metric field), which is a scalar from a four-dimensional point of view. The solution is obtained on the brane and in the bulk in terms of the Green function to the wave equation,

\[ \left( M^{D-2} (p^2 + \partial_\nu \partial^\nu) - (\lambda - 1) \underline{\mathcal{M}}^2 p^2 \sigma_\Lambda \right) \mathcal{G}_\Lambda (p, y) = \sigma_\Lambda (y) \]  

(17)
as
\[ \tilde{h}_n^\alpha(p, y) = -\frac{(D - 4)}{(D - 2)} \tilde{T}_n^\alpha G_\lambda(p, y), \quad \lambda = \frac{3(D - 4)}{(D - 2)} \] (18)

After some algebra, we obtain a spherically symmetric solution expressed in terms of Bessel functions as
\[ G_\lambda(p, y) = -\frac{1}{M^2 p^2(\lambda - 1)} \left( 1 - \frac{1}{B_\lambda} \left( \frac{1}{y\Lambda} \right)^{(D-6)/2} H_{(D-4)/2}^{(1)}(p/\Lambda) J_{(D-6)/2}(k_\lambda y) \right) \] (19)
inside the brane \((y \leq 1/\Lambda)\), and
\[ G_\lambda(p, y) = -\frac{1}{M^2 p^2(\lambda - 1)} \frac{k_\lambda}{B_\lambda} \left( \frac{1}{y\Lambda} \right)^{(D-6)/2} I_{(D-4)/2}(k_\lambda p/\Lambda) H_{(D-6)/2}^{(1)}(py) \] (20)
in the bulk \((y > 1/\Lambda)\), where
\[ B_\lambda = k_\lambda I_{(D-4)/2}(k_\lambda p/\Lambda) H_{(D-6)/2}^{(1)}(p/\Lambda) + I_{(D-6)/2}(k_\lambda p/\Lambda) H_{(D-4)/2}^{(1)}(p/\Lambda) \] (21)

and we have introduced the constant \(k_\lambda\) given by
\[ k_\lambda^2 = (\lambda - 1) \frac{(D - 4)\Lambda^{D-4}}{\omega_{D-4} M^{D-2}} \approx (\lambda - 1) \frac{(D - 4)\Lambda^{D-4}}{\omega_{D-4} M^{D-2}} \] (22)

To obtain the graviton propagator, we will also need the Green function which is the solution to eq. (17) when \(\lambda = 0\). Notice that when \(\lambda = 0\), eq. (17) turns into the wave equation for a scalar field in the thin-brane limit. It is derivable from a scalar field action. Explicitly,
\[ G_0(p, y) = \frac{1}{M^2 p^2} \left( 1 + \frac{1}{B} \left( \frac{1}{y\Lambda} \right)^{(D-6)/2} H_{(D-4)/2}^{(1)}(p/\Lambda) J_{(D-6)/2}(\kappa y) \right) \] (23)
inside the brane \((y \leq 1/\Lambda)\), and
\[ G_0(p, y) = -\frac{1}{M^2 p^2} \frac{\kappa}{B} \left( \frac{1}{y\Lambda} \right)^{(D-6)/2} J_{(D-4)/2}(\kappa p/\Lambda) H_{(D-6)/2}^{(1)}(py) \] (24)
in the bulk \((y > 1/\Lambda)\), where
\[ B = \kappa H_{(D-6)/2}^{(1)}(p/\Lambda) J_{(D-4)/2}(\kappa p/\Lambda) - H_{(D-4)/2}^{(1)}(p/\Lambda) J_{(D-6)/2}(\kappa p/\Lambda) \] (25)
and (see eq. (22))
\[ \kappa^2 = -k_0^2 = 1 + \frac{(D - 4)\Lambda^{D-4}}{\omega_{D-4} M^{D-2}} \approx \frac{(D - 4)\Lambda^{D-4}}{\omega_{D-4} M^{D-2}} \] (26)
Notice that inside the brane, $G_0(p, y)$ oscillates rapidly over the transverse width of the brane.

We are now ready to deduce the full graviton propagator. To this end, let us massage eq. (14) into the form

$$\left( M^{D-2}(p^2 + \partial_n \partial^n) + \overline{M}^2 p^2 \sigma \right) \left\{ \tilde{h}_{\alpha \beta}(p, y) \tilde{T}^{\alpha \beta} + \frac{(D-5)}{3(D-4)} \tilde{h}^n(p, y) \tilde{T}_n^{\nu} \right\} = \left\{ \tilde{T}_{\alpha \beta} \tilde{T}^{\alpha \beta} - \frac{1}{3} \tilde{T}_{\mu}^{\nu} \tilde{T}^{\nu}_{\mu} \right\} \sigma (y)$$

(27)

The solution for the graviton propagator is readily obtained in terms of the scalar propagators,

$$\tilde{h}_{\alpha \beta}(p, y) \tilde{T}^{\alpha \beta} = \left\{ \tilde{T}_{\alpha \beta} \tilde{T}^{\alpha \beta} - \frac{1}{3} \tilde{T}_{\mu}^{\nu} \tilde{T}^{\nu}_{\mu} \right\} G_0(p, y) + \frac{(D-5)}{3(D-2)} \tilde{T}_{\mu}^{\nu} \tilde{T}^{\nu}_{\mu} G_{\lambda}(p, y)$$

(28)

where we used eqs. (17) and (18).

IV. POLES OF THE GRAVITON PROPAGATOR

Next, we analyze the pole structure of the graviton propagator. We then compare the results of our model with that of Dubovsky and Rubakov [26].

A. Our model

Using the expressions (23) for $G_0(p, y)$ and (19) for $G_{\lambda}(p, y)$, the graviton propagator (28) inside the brane ($y \leq 1/\Lambda$) can be written in the form

$$\tilde{h}_{\alpha \beta}(p, y) \tilde{T}^{\alpha \beta} = \left\{ \tilde{T}_{\alpha \beta} \tilde{T}^{\alpha \beta} - \frac{1}{2} \tilde{T}_{\alpha}^{\hat{\alpha}} \tilde{T}_{\beta}^{\hat{\beta}} \right\} \frac{1}{M^2 p^2} + \left( \frac{1}{y \Lambda} \right)^{(D-6)/2} H_{(D-4)/2}(p/\Lambda)$$

$$\times \left\{ \tilde{T}_{\alpha \beta} \tilde{T}^{\alpha \beta} - \frac{1}{3} \tilde{T}_{\mu}^{\nu} \tilde{T}^{\nu}_{\mu} \right\} \frac{1}{B} J_{(D-6)/2}(kpy) + \frac{1}{6} \tilde{T}_{\alpha}^{\hat{\alpha}} \tilde{T}_{\beta}^{\hat{\beta}} \frac{1}{B_{\Lambda}} I_{(D-6)/2}(k_\lambda py)$$

(29)

For convenience, we have separated the term that corresponds to the tensor structure and momentum dependence of the four-dimensional graviton propagator. To study the pole structure, we shall introduce the average value of the graviton propagator over the transverse directions of the brane (see [24] for problems associated with the definition of observables on the brane) defined by

$$\tilde{h}_{\alpha \beta}^{\text{Brane}}(p) = \int d^{D-4}y \sigma \left( y \right) \tilde{h}_{\alpha \beta}(p, y)$$

(30)
Integrating (29), we obtain

\[
\tilde{h}^\text{Brane}_{\alpha\beta}(p) \tilde{T}^{\mu\alpha\beta} = \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\mu\alpha\beta} - \frac{1}{2} \tilde{T}_\alpha^\gamma \tilde{T}_\gamma^\mu \right\} \frac{1}{M^2 p^2} + \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\mu\alpha\beta} - \frac{1}{3} \tilde{T}_\alpha^\gamma \tilde{T}_\gamma^\mu \right\} \frac{(D-6)}{(\tilde{M} \kappa p^2 / \Lambda)^2 [1 - \mu_0(\kappa p / \Lambda)]}
\]

from which we may easily deduce the pole structure of the graviton propagator inside the brane.

The above expression is valid for \( D > 6 \) (for \( D = 6 \), we obtain logarithmic corrections, but the results are similar and will not be explicitly discussed here). The functions that appear in the denominators in (31) are

\[
\mu_0(z) = \frac{D-6}{z} \frac{J_{(D-6)/2}(z)}{J_{(D-4)/2}(z)}, \quad \mu_\lambda(z) = \frac{D-6}{z} \frac{I_{(D-6)/2}(z)}{I_{(D-4)/2}(z)}
\]

for \( D > 6 \). The poles of the propagator are solutions to the equations

\[
\mu_0(\kappa p / \Lambda) = 1, \quad \mu_\lambda(k \lambda p / \Lambda) = -1
\]

Using (32) and the Bessel function identity

\[
z J_{\nu-1}(z) + z J_{\nu+1}(z) = 2\nu J_\nu(z)
\]

for \( \nu = (D-6)/2 \), it is easily shown that the solutions to \( \mu_0(z) = 1 \) are the roots of \( J_{\nu-1} = J_{(D-8)/2} \). As is well-known, there are infinitely many zeros for \( \nu > 0 \), i.e., \( D > 6 \), which is the case we are considering here. We shall denote them by \( z_j \),

\[
J_{(D-8)/2}(z_j) = 0, \quad j = 1, 2, \ldots
\]

We therefore obtain an infinite tower of massive poles with masses given by

\[
m^2_j = z_j^2 \frac{\Lambda^2}{\kappa^2}
\]

Similarly, the condition \( \mu_\lambda(z) = -1 \), together with the Bessel function identity

\[
z I_{\nu-1}(z) - z I_{\nu+1}(z) = 2\nu I_\nu(z)
\]

and the relation \( I_\nu(z) = e^{-\pi i \nu/2} J_\nu(iz) \), lead to a tower of tachyonic poles with masses given by

\[
m^2_{\nu j} = -z_j^2 \frac{\Lambda^2}{k^2_\lambda}
\]
To obtain the behavior of the propagator near a massive pole, observe that

\[ 1 - \mu_0(z) = -\frac{J_{(D-8)/2}(z)}{J_{(D-4)/2}(z_j)} = -\frac{J'_{(D-8)/2}(z_j)}{J_{(D-4)/2}(z_j)} (z - z_j) + o((z - z_j)^2) \]  

(39)

Using the Bessel function identity

\[ z J' - (\nu - 1) J - z J = -J' - J \]  

(40)

together with (34), we deduce

\[ 1 - \mu_0(z) = \frac{1}{2(D - 6)} (z^2 - z_j^2) + \ldots \]  

(41)

near \( z = z_j \). It follows that the graviton propagator on the brane (31) behaves as

\[ \tilde{h}_{\alpha\beta}^{\text{Brane}}(p) \tilde{T}^{\alpha\beta} \sim \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\alpha\beta} - \frac{1}{3} \tilde{T}_{\alpha} \tilde{T}^{\alpha} \right\} \frac{2(D - 6)^2 / z_j^4}{M^2 (p^2 - m_j^2)} \]  

(42)

near the massive pole \( p^2 = m_j^2 \). Similarly, near the tachyonic pole \( p^2 = m_{*j}^2 \), we obtain

\[ \tilde{h}_{\alpha\beta}^{\text{Brane}}(p) \tilde{T}^{\alpha\beta} \sim -\frac{1}{6} \tilde{T}_{\alpha} \tilde{T}^{\alpha} \frac{2(D - 6)^2 / z_j^4}{M^2 (p^2 - m_{*j}^2)} \]  

(43)

The minus sign of the residue of the tachyon implies that the tachyon is a ghost.

Notice that both the massive modes (36) and the tachyons (38) are expressed in terms of the same mass scale parameter \( p_c \), where

\[ p_c^2 \sim \frac{\Lambda^2}{k^2} \sim \frac{\Lambda^2}{k_{\perp}^2} \sim \frac{M^{D-2}}{M^2 \Lambda^{D-6}} \]  

(44)

In the thin-brane limit (\( \Lambda \to \infty \)), we have \( p_c \to 0 \) and the infinite towers of massive modes and tachyons turns into continuous spectra. The form of the propagator in this limit is easily deduced from eq. (31). For momenta away from the critical scale (\( |p| \gg p_c \)), the two terms in (31) that give rise to the massive and tachyonic poles become vanishingly small and we are left with

\[ \tilde{h}_{\alpha\beta}^{\text{Brane}}(p) \tilde{T}^{\alpha\beta} \sim \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\alpha\beta} - \frac{1}{2} \tilde{T}_{\alpha} \tilde{T}^{\alpha} \right\} \frac{1}{M^2 p^2} \]  

(45)

recovering four-dimensional Einstein gravity.
B. The Dubovsky-Rubakov model

It is interesting to note that similar results have been obtained by Dubovsky and Rubakov [26] using a slightly different model. In order to directly compare our results with theirs, we shall assume that the spread function (denoted by \( f^2(y) \) in [26]) is given by eq. (2). Then the Einstein field equations proposed in [26] can be written as

\[
F(\Box)G_{AB}^{(D)}(x^\mu, \bar{y}) + \overline{M}^2 \sigma_\Lambda(y) \int d^{D-4}y' \sigma_\Lambda(y') G_{AB}^{(4)}(x^\mu, \bar{y}') = T_{AB}(x^\mu, \bar{y}) \tag{46}
\]

to be compared with the Einstein eq. (3) in our model. In eq. (46), the four-dimensional Einstein tensor only has brane worldvolume components (i.e., \( G_{aB}^{(4)} = 0 \)) and the form-factor \( F \approx M^2 \) at low energies. Also, the matter source on the brane will be assumed to have only space-time components \( T_{\mu\nu} \) and a spread function same as that of the brane,

\[
T_{\mu\nu}(x, y) = T_{\mu\nu}(x) \sigma_\Lambda(y) \tag{47}
\]

where \( T_{\mu\nu}(x) \) is conserved in the four-dimensional sense (cf. eq. (15) in our model). The inverse width \( \Lambda \) of the spread function is assumed to be \( \Lambda \sim M \) in [26] to be contrasted with our model in which \( \Lambda \sim \overline{M} \), since it coincides with the inverse width of the brane [7].

Working as in section II, we linearize the Einstein equations and obtain the graviton propagator in the form

\[
\tilde{h}_{\mu\nu}(p, y) T'_{\mu\nu} = \frac{2}{C} \left\{ T_{\mu\nu} T'_{\mu\nu} - \frac{1}{3} T_{\mu\nu} T'_{\mu\lambda} \right\} G_1(p, y) - \frac{1}{3C_*} T'_{\mu\nu} T'_{\mu\lambda} G_1(p, y) \tag{48}
\]

where we multiplied by the arbitrary stress-energy tensor \( T'_{\mu\nu} \) to absorb the longitudinal part which is not gauge-invariant. It is given in terms of the Green function which satisfies eq. (17) for \( \lambda = 1 \),

\[
M^{D-2} (p^2 + \partial_s \partial^s) G_1(p, y) = \sigma_\Lambda(y) \tag{49}
\]

(denoted by \( Df \) in [26]). The denominators are

\[
C = 1 + \overline{M}^2 p^2 \overline{G}_1, \quad C_* = 1 - \overline{M}^2 p^2 \overline{G}_1 \tag{50}
\]

where \( \overline{G}_1 \) is the average of \( G_1 \) over the spread function (defined as in eq. 30 and denoted by \( Df \) in [26]). Explicitly,

\[
\overline{G}_1(p) = -\frac{\kappa^2}{M^2 \Lambda^2} f(p/\Lambda), \quad f(z) = \frac{1}{z^2} \left[ 1 - \frac{i\pi}{2} (D-4) H^{(1)}_{(D-4)/2}(z) J_{(D-4)/2}(z) \right] \tag{51}
\]
where we introduced the function \( f(z) \) for convenience and the scale \( \kappa \), which coincides with our earlier definition \( \Lambda \) in the large \( \Lambda \) limit,

\[
\kappa^2 = \frac{(D - 4)\Lambda^{D-4}M^2}{\omega_{D-4}M^{D-2}}
\]

(52)

The poles of the propagator \( G \) are the zeros of \( C \) and \( C^\ast \). They can easily be seen to correspond to small \( z \), therefore we may approximate \( C \approx 1 - \kappa^2 f(0)p^2/\Lambda^2 \), whose root is

\[
m^2 \approx \frac{\Lambda^2}{\kappa^2 f(0)} \sim \frac{M^{D-2}}{M^2 \Lambda^{D-6}}
\]

(53)

which is a massive pole. Similarly, the root of \( C^\ast \) is a tachyonic pole

\[
m^\ast_2 \approx -m^2 \sim -\frac{M^{D-2}}{M^2 \Lambda^{D-6}}
\]

(54)

Notice that the mass scale is similar to the mass scale of the poles in our model \( \Lambda \), although in this model only one pair of poles is obtained instead of the infinite tower we found in our model. This scale matches the one found in \( \Lambda \) if we set \( \Lambda \sim M \), in which case \( m \sim M^2/\Lambda \).

V. MOMENTUM DEPENDENCE OF THE GRAVITON PROPAGATOR

Having understood the large \( \Lambda \) limit, we now turn to a study of the momentum dependence of the graviton propagator keeping \( \Lambda \) finite. By introducing the width \( 1/\Lambda \), we have added a scale to the theory in addition to the mass scales \( M \) and \( \Lambda \). It follows from the explicit form of the propagator that the relevant scales are \( \Lambda \) and \( \Lambda/k \), where \( k \sim k_\Lambda \sim \kappa \) is a dimensionless parameter given by \( \Lambda \) or \( \kappa \). Phenomenologically, one expects \( \Lambda \sim \Lambda \) and \( M \ll \Lambda \). So we shall restrict attention to momenta that are well below the scale \( \Lambda (p \ll \Lambda) \). This range is divided by the scale given by eq. \( \Lambda \) into a small momentum \( (p \ll p_c) \) and a large momentum \( (p \gg p_c) \) regime. Qualitatively, one expects four-dimensional behavior of the graviton propagator for large momenta and \( D \)-dimensional behavior for small momenta. We wish to study this behavior quantitatively.

For small momentum, \( p \ll p_c \), we have

\[
G_0(p, y) \approx G_\Lambda(p, y)
\]

(55)
as can easily be verified from eqs. (20) and (24) in the bulk and eqs. (19) and (23) on the brane. The resulting tensor structure of the graviton propagator (28) is
\[
\tilde{h}_{\alpha\beta}(p; y) \bar{T}^{\alpha\beta} \approx \left\{ \tilde{T}_{\alpha\beta} \bar{T}^{\alpha\beta} - \frac{1}{(D - 2)} \tilde{T}_{\alpha} \tilde{T}_{\beta} \right\} G_0(p, y) \tag{56}
\]
In the bulk, we deduce from (24),
\[
G_0(p; y) \sim i \left( \frac{1}{py} \right)^{(D-6)/2} H^{(1)}_{(D-6)/2}(py) \tag{57}
\]
which is the propagator for a D-dimensional scalar field. Therefore, the graviton behaves as a D-dimensional field in both its momentum dependence and its tensor structure in the bulk.

On the brane, after averaging over its transverse width, eq. (56) yields in the regime \( p \ll p_c \)
\[
\tilde{h}^{\text{Brane}}_{\alpha\beta}(p) \bar{T}^{\alpha\beta} \sim \frac{1}{M^2 p^2} \left\{ \tilde{T}_{\alpha\beta} \bar{T}^{\alpha\beta} - \frac{1}{(D - 2)} \tilde{T}_{\alpha} \tilde{T}_{\beta} \right\} \times \left( 1 + \frac{1}{B \Gamma \left( \frac{D-4}{2} \right)} \frac{(\kappa p)}{2\Lambda} \right)^{(D-6)/2} H^{(1)}_{(D-4)/2}(p/\Lambda) \tag{58}
\]
where we used eqs. (23) and (30). It is easy to see that the \( 1/p^2 \) pole vanishes. The first non-analytic term can be found from the expansion for small argument
\[
H_\nu(z) = -\frac{i}{\pi} \Gamma(\nu) \left( \frac{z}{2} \right)^{-\nu} (1 + \ldots) + \frac{2i}{\pi \Gamma(\nu + 1)} \left( \frac{z}{2} \right)^{\nu} \ln \left( \frac{z}{2} \right) + \ldots \tag{59}
\]
for integer \( \nu \), where the dots represent higher-order and analytic terms. Applying this to eq. (58), we obtain
\[
\tilde{h}^{\text{Brane}}_{\alpha\beta}(p) \bar{T}^{\alpha\beta} \sim \left\{ \tilde{T}_{\alpha\beta} \bar{T}^{\alpha\beta} - \frac{1}{(D - 2)} \tilde{T}_{\alpha} \tilde{T}_{\beta} \right\} \left( \frac{p}{\Lambda} \right)^{D-6} \ln(p/\Lambda) \tag{60}
\]
exhibiting D-dimensional behavior. Similar conclusions may be drawn for the trace \( \tilde{h}_n^n \) in the small momentum regime \( p \ll p_c \).

In the large momentum regime \( (\Lambda \gg p \gg p_c) \), the results are similar to those in the large \( \Lambda \) limit, which we discussed in the previous section. In this regime, the scalar Green functions are related by
\[
G_0(p, y) \approx (\lambda - 1) G_\lambda(p, y) \tag{61}
\]
to be contrasted with the relation (63) in the regime \( p \ll p_c \). Thus the tensor structure of the graviton propagator (28) becomes
\[
\tilde{h}_{\alpha\beta}(p; y) \bar{T}^{\alpha\beta} \approx \left\{ \tilde{T}_{\alpha\beta} \bar{T}^{\alpha\beta} - \frac{1}{2} \tilde{T}_{\alpha} \tilde{T}_{\beta} \right\} G_0(p, y) \tag{62}
\]
exhibiting four-dimensional behavior. Inside the brane, we deduce from (23)
\[
\hat{h}^{\text{Brane}}_{\alpha\beta}(p)\tilde{T}^{\alpha\beta} = \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\alpha\beta} - \frac{1}{2} \tilde{T}^{\alpha}_{\alpha} \tilde{T}^{\beta}_{\beta} \right\} \frac{1}{M^2 p^2} + o \left( \left( \frac{p_c}{p} \right)^2 \right)
\]  
(63)

exhibiting the distance dependence of Newtonian gravity with the tensor structure of four-dimensional Einstein gravity. This is in agreement with our earlier conclusion (45) in the large \( \Lambda \) limit. In the bulk (\( y > 1/\Lambda \)), we deduce from (24)
\[
\hat{h}_{\alpha\beta}(p, y)\tilde{T}^{\alpha\beta} \approx \frac{-i\pi}{\Gamma\left(\frac{D-6}{2}\right)} \frac{1}{M^2 p^2} \left( \frac{p^2}{2\Lambda^2} \right)^{(D-6)/2} \left\{ \tilde{T}_{\alpha\beta} \tilde{T}^{\alpha\beta} - \frac{1}{2} \tilde{T}^{\alpha}_{\alpha} \tilde{T}^{\beta}_{\beta} \right\} \left( \frac{1}{py} \right)^{(D-6)/2} H^{(1)}_{(D-6)/2}(py)
\]  
(64)

Therefore, the propagator vanishes in the thin brane limit (\( \Lambda \to \infty \)). These results for the momentum dependence of the tensor structure of the graviton propagator are in agreement with the qualitative suggestions of [7].

VI. CONCLUSION

The main objective of the present work was to analyze quantitatively the gravitational effects on a brane of finite extent in the transverse directions (“fat”). We obtained an equation for the graviton propagator which we then proceeded to solve in two steps. First, we obtained the propagator for the trace \( h^n \) over the transverse directions of the metric field, which is a scalar from the four-dimensional point of view. The trace \( h^n \) acted as an effective source term for the graviton propagator in addition to the contribution from the matter source. This complicated the tensor structure of the graviton propagator which became momentum dependent. We found a solution for the graviton propagator which explicitly revealed its pole structure. We obtained infinite towers of massive gravitons and tachyonic ghosts with a discrete mass spectrum. We found the contributions from the massive gravitons and tachyonic ghosts in the thin-brane limit and showed that the tensor structure and distance dependence of four-dimensional Einstein gravity is recovered in this limit. We then analyzed the tensor structure of the momentum dependent graviton propagator for a brane of finite thickness. In the small momentum regime, the graviton propagator exhibited a \( D \)-dimensional behavior, which was in contrast to the large momentum regime (above the critical scale \( p_c \) (eq. (44)) but well below the inverse brane width \( \Lambda \)), where the contributions from the massive gravitons and tachyonic ghosts conspired to produce
a propagator on the brane whose tensor structure and distance dependence was that of four-dimensional Einstein gravity.

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