Effects of non-magnetic impurities on spin-fluctuations induced superconductivity

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We study the effects of non-magnetic impurities on the phase diagram of a system of interacting electrons with a flat Fermi surface. The one-loop Wilsonian renormalization group flow of the angle dependent diffusion function \( D(\theta_1, \theta_2, \theta_3) \) and interaction \( U(\theta_1, \theta_2, \theta_3) \) determines the critical temperature and the nature of the low temperature state. As the imperfect nesting increases the critical temperature decreases and the low temperature phase changes from the spin-density wave (SDW) to the d-wave superconductivity (dSC) and finally, for bad nesting, to the random antiferromagnetic state (RAF). Both SDW and dSC phases are affected by disorder. The pair breaking depends on the imperfect nesting and is the most efficient when the critical temperature for superconductivity is maximal.

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Interesting and new physics appears when density wave (DW) and superconducting (SC) correlations interfere with each other and with the disorder. One finds realizations of such interferences in correlated electronic systems whose Fermi surface (FS) has at least imperfect nesting properties. The Bechgaard salts (TMTSF)\(_2\)X and the Fabre salts (TMTTF)\(_2\)X have two open quasi-one-dimensional Fermi sheets responsible for DW correlations and, very probably, for superconductivity.\(^1\) In these compounds the disorder strength can be tuned by the cooling speed or by x-ray irradiation, while the imperfect nesting is controlled either by chemical structure or by pressure. Another example of correlated system with important disorder effects are the two-dimensional (2D) organic compounds from the BEDT-TTF (ET) family.\(^1\)\(^2\) All ETs have a metal-insulator and/or a DW transition in the phase diagram. They have two FSs with several more or less perfect inter- and intra-band nesting wave vectors. Their superconductivity is induced by the DW fluctuations. In the high-\(T_c\) superconductors the pseudogap and the superconducting states are both related to the antiferromagnetic (commensurate SDW) insulating phase. The effects of disorder on the high-\(T_c\) superconductivity has been widely studied experimentally and theoretically.\(^2\)\(^3\)\(^4\) It has been shown by a number of methods, and in particular by the angle resolved renormalization group (RG) that this interplay between the DW, the pseudogap and the SC exists in models with at least flat segments of the FS or with nested van Hove singularities.\(^3\)\(^4\)\(^5\) However, no attempt has been made to implement both disorder and correlations in the N-patch RG, which would provide a perturbative, but non-biased and controlled way for constructing the phase diagram.

In this letter we report a perturbative angle-resolved (N-patch) RG theory for the model consisting of two flat FS segments in regard. The imperfect nesting parameter is treated approximately, by introducing a cut-off \( \epsilon \) in the Peierls channel. For the pure case this problem was solved in the Parquet formulation by Zheleznyak et al.\(^2\) and later closely reexamined in the field-theoretical RG language.\(^2\) While previous theories in more than one dimension (1D) were based on the assumption of one dominant order parameter,\(^1\)\(^10\)\(^11\) we take into account in a non-prejudiced way both Cooper and Peierls correlations together with the diffusion processes, generalizing to 2D the 1D theory of Giamarchi and Schulz.\(^12\)

We first consider a non-interacting isotropic disordered electronic system with bare dispersion \( \xi(k) \). We constrain the momenta to lie within the shell of \( \pm \Lambda \) around the Fermi surface and follow the Wilsonian one-loop flow starting from \( \Lambda_0 \). At the scale \( l = \ln(\Lambda_0/\Lambda) \) the propagator has the form

\[
G_l^{-1}(k, \omega) = i\omega f_l(\omega) - \xi(k),
\]

where \( f_l \) is a flowing function of \( \omega \). Considering only the self-energy correction due to the disorder, see Fig.\(^11\)\(^a\) and discarding for simplicity one-loop graphs renormalizing the diffusion \( D \) (Figs.\(^11\)\(^b\) and \( c \)), we find

\[
\frac{df_l(\omega)}{d\Lambda} = -\frac{\nu_F}{\pi\tau} \frac{f_l(\omega)}{(\nu_F\Lambda)^2 + [\omega f_l(\omega)]^2}, \quad \text{with } f_{l=0}(\omega) = 1,
\]

where \( \tau = \nu_F/2D \) is the elastic scattering mean free time. The Born approximation \( f^{(B)} = 1 + 1/(2|\omega|\tau) \) is recovered in the limit \( \Lambda \to 0 \), when setting \( f_l(\omega) \) to its bare value \( f_1 = 1 \), in the RHS of Eq.\(^11\). In Fig.\(^12\) we see how the \( \omega \)-dependence of \( f \) is constructed successively by mode elimination. For any finite \( \Lambda \), the one-particle propagator is a regular function of frequency. The characteristic momentum scale for disorder is \( \Lambda_d = 2D/(\pi\nu_F^2) \). In the Born approximation, and for \( \Lambda/\Lambda_d \ll 1 \), the slope...
of $\omega f_{l}^{(b)}(\omega)$ at the origin is $f_{l}^{(b)}(0) \simeq \Lambda_{l}/\Lambda$. The exact solution of Eq. (2) is much steeper since its slope is

$$f_{l}(0) \simeq \exp(\Lambda_{l}/\Lambda).$$

We now consider the disordered and interacting system whose FS consists of two flat segments in regard. We denote the marginal part of the interaction. We do not take the one-particle propagator at scale $l$ is $G_{l}(k,\omega,\theta) = i\omega f_{l}(\omega,\theta) - \nu_{l}k$. A complete description of the disordered phases would require a fully functional RG, taking the whole frequency dependence of the propagator into account. Here we consider the problem from the weakly coupled Fermi liquid side. We want to study how the model is driven away from its Fermi liquid fixed point by the combined effects of weak interactions and disorder. For this purpose the zero-order scaling arguments allow us to keep only terms linear in energy in the non-interacting part of the replicated (see e.g. [11]) effective action, which then reads

$$S_{0} = -\int^{\Lambda} dX \frac{2\pi}{2\pi} [i\omega Z_{l}^{-1}(\theta) - \nu_{l}k] \bar{\psi}(X)\psi(X).$$

(3)

$X = (k, \omega, \theta, \sigma, \alpha)$ is a multiple index. The integral over $X$ is a shorthand notation for integrals over $k$, $\omega$ and $k_{\parallel}$ and discrete summations over the replica index $\alpha$, spin index $\sigma$ and the left-right index. In Eq. (3) the quantity $Z_{l}^{-1}(\theta) \equiv f_{l}(\omega = 0, \theta)$. It should not be confused with the one-particle weight, discussed in [14]. In fact, the effective action (3) can be interpreted as the marginal part of the diffusion pole as “seen” by the electrons at the RG scale $l$. The interaction and the disorder parts of the replicated effective action read

$$S_{\text{int}} = 2\pi\nu_{l} \int^{\Lambda} \frac{dX_{1} \cdots dX_{4}}{(2\pi)^{4}} [2\pi\delta(4 + 3 - 2 - 1)]\delta_{\alpha_{1},\alpha_{2}}\delta_{\alpha_{2},\alpha_{3}}\delta_{\alpha_{1},\alpha_{4}}U_{l}(\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}) \bar{\psi}_{1}\bar{\psi}_{2}\psi_{1},$$

(4)

$$S_{\text{dis}} = 2\pi\nu_{l} \int^{\Lambda} \frac{dX_{1} \cdots dX_{4}}{(2\pi)^{4}} [2\pi\delta(4 + 3 - 2 - 1)]\delta_{\alpha_{1},\alpha_{2}}\delta_{\alpha_{2},\alpha_{3}}[2\pi\delta(\omega_{1} - \omega_{4})] D_{l}(\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}) \bar{\psi}_{4}\bar{\psi}_{3}\psi_{2}\psi_{1}.$$  

(5)

Schematically, the angle resolved flow equations read

$$\partial_{l}\tilde{U}_{l}(\theta_{1}, \theta_{2}, \theta_{3}) = \beta_{U} \tilde{U}_{l},$$

$$\partial_{l}\tilde{D}_{l}(\theta_{1}, \theta_{2}, \theta_{3}) = \tilde{D}_{l} + \beta_{D} \tilde{D}_{l}.$$  

(6)

with $\tilde{U}_{l}(\theta_{1}, \theta_{2}, \theta_{3}) = Z_{l}^{1/2} U_{l}(\theta_{1}, \theta_{2}, \theta_{3})$ and $\tilde{D}_{l}(\theta_{1}, \theta_{2}, \theta_{3}) = Z_{l}^{1/2} D_{l}(\theta_{1}, \theta_{2}, \theta_{3})$, where $Z_{l} = Z_{l}(\theta_{l})$. Note that the equation for the disorder contains a linear term in $D$, explicitly showing the relevance of the disorder. The $\beta$-functions are homogeneous quadratic functions in $\tilde{U}$ and $\tilde{D}$. The disorder is not renormalized by a term quadratic in $\tilde{U}$ and the interactions are not renormalized by a term quadratic in $\tilde{D}$. Indeed, starting from a pure situation, interactions alone cannot induce disorder and starting from a non-interacting disordered situation, elastic couplings (the disorder) cannot induce inelastic couplings (the interactions). We checked our RG equation in two limits. In the 1D limit, neglecting the contributions quadratic in the disorder, we recover the (weak coupling) equations of Giamarchi and Schulz. We The effect of disorder on a...
conventional s-wave superconductor is obtained from our equations when dropping the particle-hole contributions and using initial conditions of the attractive Hubbard type. In this case we recover Anderson’s theorem.

After discretization of angular variables, we have integrated numerically the equations. The robustness of our results upon increasing number of patches is satisfactory above \( N = 16 \) patches per side of the FS. The dependence of the critical scale on the imperfect nesting gives the phase diagram. For the pure case, i.e. the pair-breaking parameter \( \alpha = 1/\tau = 0 \), we reproduce the result of Zheleznyak et al, shown in dotted line on Fig. 1. As one can conclude from the flow of dominant eigenvalues of the coupling function \( U \), shown in thin lines on Fig. 1, the part of the phase diagram with \( T_c > \epsilon \) shows a transition to the SDW state with enhanced dSC fluctuations, while the \( T_c < \epsilon \) part corresponds to the onset of \( d \)-wave superconductivity. We now introduce a finite disorder via the pair breaking parameter \( \alpha = 2D_0/\nu_F = 0.39T_c\alpha, D_0 \) being the bare diffusion constant. The SDW and dSC phases are both affected by disorder. For \( \epsilon \) greater than a disorder dependent critical imperfect nesting \( \epsilon^* \), the dSC phase is completely suppressed and a new low temperature state appears. The signatures of all three phases are visible on Fig. 1. At perfect and good nesting, i.e. as long as \( \epsilon < T_c \), the flow near the divergence is almost the same as for the pure case: the SDW correlations are dominant while the dSC ones are also divergent but weaker. The disorder remains finite at the critical scale. For moderate imperfect nesting, \( T_c < \epsilon < \epsilon^* \), the dSC becomes the dominant instability because the particle-hole logarithms are cut. The disorder remains finite, but is stronger than in the perfect nesting case. Finally, at very bad nesting, \( \epsilon > \epsilon^* \) the most diverging “coupling” is the disorder. The dSC is irrelevant, while the SDW amplitude shows the intriguing re-appearance at the very last moment before divergence. This divergence is induced by the particle-particle \( U-D \) terms in the renormalization of the interaction \( U \). Its very small residue reflects the fact that the SDW interactions which are driven by disorder are only these with total momentum equal zero. Consequently only a small subset of all SDW amplitudes diverges. Altogether the flow in Fig. 1(c) allows us to characterise rather precisely the corresponding low temperature state: it is a charge-localized state with weakly enhanced SDW correlations. We expect that the properties of this phase are similar to the 1D random antiferromagnetic (RAF) phase, discussed in Ref.

We have calculated the critical pair breaking \( \alpha^* \) as a function of the imperfect nesting parameter \( \epsilon \). The result is shown in the inset of Fig. 2. As long as the flow is entirely in the parquet regime the pair breaking efficiency increases with imperfect nesting, i.e. \( \alpha^*/T_c\alpha = 0 \) decreases. At the point \( \epsilon = T_c \), where the flow starts to be affected by the imperfect nesting, \( \alpha^*/T_c\alpha = 0 \) reaches its minimum. This is the point of the most efficient pair breaking. It corresponds to the maximal value of the dSC critical temperature. As \( \epsilon \) increases further the pair breaking gets progressively less efficient. The dependence of the pair breaking on the imperfect nesting is a very consequence of the interplay between Peiers, Cooper and localization tendencies through the angular dependence of corresponding amplitudes. In the Abrikosov-Gor'kov theory, where particle-hole logarithms are neglected, and a dSC order parameter is phenomenologically assumed, the ratio \( \alpha^*/T_c\alpha = 0 \approx 0.88 \) is independent of nesting.

In conclusion, we have used the one-loop N-patch renormalization group to compute the non-magnetic disorder effects on a system with spin-density wave fluctuations induced d-wave superconductivity. The pair breaking is the most efficient when the \( T_c \) for superconductivity is maximal.

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[1] T. Ishiguro, K. Yamaji, and G. Saito, *Organic Superconductors IIe* (Springer-Verlag, Berlin, 1998).
[2] M. Lang and J. Mueller, in *The Physics of Superconductors - Vol. 2*, edited by K.-H. Bennemann and J. B. Ketterson (Springer-Verlag, 2003).
[3] A. T. Zheleznyak, V. M. Yakovenko, and I. E. Dzyaloshinskii, Phys. Rev. B **55**, 3200 (1997).
[4] D. Zanchi and H. J. Schulz, Phys. Rev. B **61**, 13609 (2000).
[5] N. Furukawa, T. M. Rice, and M. Salmhofer, Phys. Rev. Lett **81**, 3195 (1998).
[6] K. Maki, in *Lectures in the Physics of Highly Correlated Electron Systems*, AIP Conference Proceedings **438**, edited by F. Mancini (Woodbury, New-York, 1998).
[7] N. E. Hussey, Adv. Phys. **51**, 1685 (2002).
[8] S. Dusuel, F. Vistulo de Abreu, and B. Douçot, Phys. Rev. B **65**, 94505 (2002).
[9] A. A. Abrikosov, L. P. Gor’kov, and I. Y. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice Hall, New York, 1963).
[10] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys **57**, 287 (1985).
[11] D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).
[12] T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988).
[13] R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
[14] D. Zanchi, Europhys. Lett. **55**, 376 (2001).
[15] P. W. Anderson, J. Phys. Chem. Solids **11**, 26 (1959).
[16] Y. Sun and K. Maki, Phys. Rev. B **51**, 6059 (1995).
[17] A. A. Abrikosov and L. P. Gor’kov, Soviet Phys. JETP **12**, 1243 (1961).