The role of joint spacing on the stability analysis of wedge failures

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Abstract. Kinematic analyses of wedge failures in rock slopes are usually carried out based on stereographic techniques. Nevertheless, this methodology presents several limitations that could lead to poorly accurate conclusions. In line with this idea, the main objective of this work is to evaluate the effect of joint spacing in the estimation of the factor of safety of rock slopes affected by potential wedge failures. For this purpose, a 3D numerical distinct-element code (3DEC) was selected to carry out a good number of simulations in which the factor of safety a slope, affected by different discontinuity sets, was studied by using the Shear Strength Reduction (SSR) method. Different values of joint spacing, cohesion and friction angles were considered, combined with two angles of the slope face under study. The joint spacing has been found to relevantly affect the values of the factor of safety, which showed variations of up to 40% in comparison with those obtained from limit-equilibrium methods for rock slopes with similar structural features. This work provides an insight into a more realistic interpretation of rock slope analyses against wedge failures, and particularly to more accurate estimations of the factor of safety.

1. Introduction
Excavations in rock masses are common operations in the fields of mining and civil engineering, whether for the development of foundations, tunnels or slopes. All these activities involve an interaction with the natural environment, which presents characteristics that are conditioned by its origin and geological history, which are modified at the time of excavation. A well-known feature of any rock mass is that its mechanical behavior is greatly determined by the presence of discontinuities. According to [1], the discontinuity spacing can be considered as one of the most relevant indicators when determining the so-called ‘quality’ of a rock mass. It seems therefore reasonable that the magnitude of this parameter may influence the general stability of slopes. This feature is illustrated by the photographs shown in Figure 1, in which two rock masses are affected by several discontinuity sets causing wedge failures, but with very different spacings.

A preliminary stability analysis of the slopes shown in Figure 1 would often require plotting the main joint sets by means of a stereographic projection, in such a way that potential instability mechanisms can be identified. This can be carried out by resorting to commercial software like Dips [2] even though other freeware options like Stereographic Projection [3], InnStereo [4] or Stereonet [5] are currently available. After that, the Limit Equilibrium Method (LEM) is often used to estimate the factor of safety of the slope.
Figure 1: (a) Photograph or a rock slope, where a typical wedge failure can be observed, with two persistent joints. Photo taken from [6]; (b) Photograph of a slope excavated in a highly-fractured rock mass (relatively low joint spacing), where several wedge failures occur. Photo taken from [7].

Figure 2: Idealized sketch of two slopes in a rock mass affected by two joint sets with the same dip and dip direction, correspondingly, but different spacings: (a) spacing $= S_a$ and (b) spacing $= S_b$. The corresponding stereographic plots are also included.

In the case of rock slopes potentially affected by wedge failures, their stability can be estimated through deterministic calculations with the help of ad hoc software solutions like SWedge [8] or Rock Stability [9]. Nevertheless, despite their advantages in terms of user-friendliness and graphic output, these programs present some limitations. A relevant drawback of these methodologies
is that they solely consider the dip and dip direction of the involved joint sets so as to generate the wedge geometry to be analyzed. This step is carried out, in the case of SWedge, by applying the principles of the ‘block theory’ [10, 11]. The fact of considering only dip and dip direction for generating the wedge geometry may lead to perform the same stability analysis for rather different slopes, as illustrated in Figure 2. In this line, the existence of smaller or less stable wedges are disregarded and so, the overall safety factor of the slope.

The main goal of our study was to assess the effect of joint spacing on the safety-factor values of rock slopes, highlighting the importance of considering this feature so as to obtain more realistic stability analyses.

2. Methodology

In the present work, stability calculations for a rock slope affected by two joint sets prone to cause wedge failures were carried out by means of two methodologies. On the one hand, a deterministic method, as that implemented in SWedge, was selected to perform analyses of single wedges. On the other hand, once calibrated with FS results obtained from SWedge, the 3D Discrete Element Analysis implemented in 3DEC [12] was used to perform the numerical simulations, by considering different spacings within the selected joint sets.

2.1. Definition of the models

A slope model was defined, according to its general slope angle ($\psi_{fi} = 65^\circ$) [6], height (H) and length (L), as presented in Figure 3. Since the aim of the work was to study the effect of spacing on the stability calculations, six increasing levels of jointing were considered by reducing the spacing of the main joint sets (J1 and J2) that affect the rock-slope model. The orientations of these joint sets were selected so as to cause wedge failures.

As anticipated in Section 1, SWedge does not allow the possibility of assigning any joint spacing value when the deterministic analysis' option is selected, being the analysis restricted to a single wedge. Nevertheless, the joint sets introduced in the numerical models carried out with 3DEC allowed different values of spacing.

The shear-strength behavior of the joint was modeled with the Mohr-Coulomb failure criterion [13], by combining six cohesion values, two friction angles, and six spacing values. This combination generated 84 numerical models. The Mohr-Coulomb failure criterion was selected in order to be able to apply the shear strength reduction method for estimating the FS. The geometrical features and mechanical properties of the rock material and joints are shown in Table 1.
Figure 3: 3DEC model for the studied rock slope with $\psi_f = 65^\circ$.

Table 1: Geometrical characteristics and mechanical properties used in the numerical simulations.

| Material          | Dip dir., $\alpha$ (°) | Dip, $\beta$ (°) | Spacing, S (m) | Cohesion, c (MPa) | Friction angle, $\phi$ (°) | Other properties |
|-------------------|-------------------------|------------------|----------------|-------------------|---------------------------|-----------------|
| Intact rock       | —                       | —                | —              | —                 | —                         | $\rho = 2700$ kg·m$^{-3}$ (modelled as rigid blocks in 3DEC) |
| Slope face        | 180                     | 65; 90           | —              | —                 | —                         | —               |
| Upper face        | 180                     | 0                | —              | —                 | —                         | —               |
| J1 set            | 125                     | 62               | >80; 40; 20; 10; 5; 2.5 | 0; 0.01; 0.05; 0.1; 0.15; 0.20 | 30; 40         | —               |
| J2 set            | 230                     | 55               | >80; 40; 20; 10; 5; 2.5 | 0; 0.01; 0.05; 0.1; 0.15; 0.20 | 30; 40         | $j_{kn} = 1 \cdot 10^{10}$ N·m$^{-2}$; $j_{ks} = 1 \cdot 10^9$ N·m$^{-2}$ |
| Tension crack set | 180                     | 85               | >80; 40; 20; 10; 5; 2.5 | $c_{tc} = c/100$ | $\phi_{tc} = 10$ | —               |
| Basal joint set   | 180                     | 25               | >80; 40; 20; 10; 5; 2.5 | 0; 0.01; 0.05; 0.1; 0.15; 0.20 | 40              | —               |
2.2. Calculation of the factor of safety

The factor of safety was firstly estimated through a deterministic analysis by means of SWedge, for a rock slope affected by two joint sets, as defined in Section 2.1. Different values of cohesion were considered for these calculations, according to Table 1. The factors of safety obtained for each cohesion value with SWedge (FSi), were used to calibrate the corresponding ones obtained with 3DEC, by considering the highest joint spacing (S>80 m), that is, the scenario where a single wedge is formed. The estimation of the factor of safety with 3DEC required the application of the shear strength reduction (SSR) method [14–16], in such a way that an increasing trial factor of safety (f) is used to reduce the shear strength properties of the joints (c and φ, in the current case) in the way shown by Equations 1 and 2, until the slope fails. Failure was considered to be attained when the displacement of the involved blocks of the model reached a value of approximately 10−2 m. At failure, the factor of safety (FS) equals the trial factor of safety (f) [6].

\[ c_{\text{trial}} = \left( \frac{1}{f} \right) c \]  
\[ \phi_{\text{trial}} = \arctan \left[ \left( \frac{1}{f} \right) \tan \phi \right] \]

Figure 4: Calibration process to evaluate the agreement between the factors of safety estimated with SWedge (FSi) and with 3DEC (FS).

The calibration process is presented in Figure 4. After carrying simulations for a single wedge with both programs and considering different cohesion values, the factors of safety were found to be very similar.
3. Results

Once the numerical models for a single wedge were calibrated, it was possible to perform numerical simulations with 3DEC for rock slope models affected by joint sets prone to cause wedge failures. In what follows, the results for different combinations of joint spacings, cohesion and friction angles are presented. Figure 5 shows normalized values of the FS (that is, FS/FSi) as obtained from 36 simulations carried out with 3DEC, represented as a function of normalized (S/H) spacing values.

![Figure 5](image_url)

**Figure 5:** Normalized FS values (FS/FSi) as a function of normalized spacing (S/H), obtained from numerical simulations carried out with 3DEC by considering six different cohesion values and $\phi = 40^\circ$.

As can be seen in Figure 5, in the case of considering joint spacings greater than those of the slope height ($H = 50\, m$), any variation is hardly observed between FS (3DEC) and FSi ($SWedge$). However, a relevant decrease of the FS can be observed from the point where joint spacing is reduced below the order of the slope height ($S/H < 0.8$). This reduction of the FS becomes more relevant as the values of joint cohesion increase, reaching reductions of up to 25% of the values expected to be obtained from a deterministic method, for the same joint sets.
Figure 6: Normalized FS values (FS/FSi) as a function of normalized spacing (S/H), obtained from numerical simulations carried out with 3DEC by considering six different cohesion values and $\phi = 30^\circ$. Results for $\phi = 40^\circ$ are plotted in dashed lines.

In a similar manner as in Figure 5, normalized FS results are plotted in Figure 6 against a normalized spacing, but considering a lower value of the friction angle ($\phi = 30^\circ$). In this case, the cohesion values were selected so as to ensure the initial stability of the wedge, so $c = 0$ was disregarded. Once the results are analyzed, a similar trend like that presented in Figure 5 can be observed. It must be remarked that the reduction of the FS with respect to that obtained from deterministic methods is more relevant for the cases where lower cohesion values are assigned to the joints. Again, a reduction in the range of 10 to 25% of the FS values can be observed when considering different levels of joint spacing.

4. Concluding remarks
In the present work, an analysis of the effect of discontinuity spacing and shear strength parameters on the estimation of the factor of safety has been carried out. For this purpose, multiple numerical simulations were performed in order to compare the wedge stability results, obtained through a LEM —such as that implemented in the software SWedge—with the results yielded by a 3D Discrete Element Method (3DEC).

Once the models analyzed with 3DEC were calibrated, it was possible to study the stability of rock slopes affected by six different levels of jointing, by varying the joint-set spacing values and by assigning different values of cohesion and friction angle. FS were estimated through the application of the shear strength reduction method (SSR), modeling the shear strength behavior of joints with the Mohr-Coulomb criterion.

For all the cases studied, a decreasing trend of the FS was observed when joint spacing was reduced below the magnitude of the slope height, an effect that became more significant for higher cohesion values. Reductions were found in the range of 10 to 25% of those encountered for the same rock slope scenario, but analyzed through deterministic calculations. According to the results obtained in this study, certain limitations have been detected in these methodologies,
like the one implemented in SWedge, since they omit the effect of spacing when performing the stability analysis of wedge failures.

Even though Mohr-Coulomb cannot be considered the most appropriate criterion for modeling the characteristic non-linear behavior of rock joints, it was resorted to for the sake of simplicity when implementing the shear strength reduction method in 3D numerical models. Further research is needed to better analyze the effects of joint spacing on the stability of rock slopes affected by wedge failures, as well as to implement non-linear joint shear-strength constitutive models in 3D numerical simulations, like those carried out in the present study.

This work tries to remark the influence of the selected methodology on the stability assessment of rock slopes, particularly, in those where instabilities are controlled by the structure of the rock mass, such as in the case of wedge failures. The results could also be applied for better back-analyses of this type of instabilities.

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