Research Article

Dynamic Propagation Characteristics of a Mode-III Interfacial Crack in Piezoelectric Bimaterials

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Received 14 October 2021; Revised 3 December 2021; Accepted 18 January 2022; Published 27 March 2022

1. Introduction

1.1. Background. Curie brothers have presented the piezoelectric effect. Due to their coupling behaviors, piezoelectric materials have been widely used in modern smart devices and structures such as actuators, sensors, and ultrasonic generators. At present, piezoelectric materials widely used in engineering practice are mainly piezoelectric ceramics and piezoelectric composites made of piezoelectric ceramics and polymers. Piezoelectric ceramics are brittle materials and have the disadvantage of low fracture toughness. However, defects often cause the structural failure of piezoelectric materials. Cracks as their main disadvantages in piezoelectric media have been widely investigated [1–9]. One of the challenging problems is the fracture behavior of interfacial crack in piezoelectric bimaterials. A few fundamental problems of the crack in homogeneous and inhomogeneous piezoelectric bimaterials have been discussed [10–13]. The fracture analyses of these piezoelectric materials have provided much knowledge for improving the performance of piezoelectric devices. However, most results have focused on static propagation.

Dynamic fracture can be divided into three types according to the time dependence of load and crack state: the mechanical-electric load is time-independent and the crack propagates rapidly [14]; not only is the electromechanical load time-dependent but also the crack propagates rapidly [15]; the crack is stable and the electromechanical load varies with time [16].

A great deal of research has been done on the first and the second kind of fracture conditions [17, 18]. For the third type of dynamic fracture, the dynamic propagation of electroacoustic wave (elastic wave) [19] and impact is included. The crack propagation becomes dynamic behavior under the action of explosion or impact load. The propagation behavior of the crack was analyzed under a concentrated point load [20, 21]. The solutions are derived in an explicit way. When the “electrode” Bleustein–Gulyaev wave velocity is...
above the crack propagation speed, the stress displacement and electric displacement are exhibited $r^{-1/2}$, which is similar to that of static fracture behavior. Narita [22] researched the crack in a piezoelectric layer. The interface was normal to the interface. The results show it is feasible and practical for the design of piezoelectric devices.

The interfacial crack propagation in piezoelectric bimaterials is the most complicated problem in the dynamic fracture of piezoelectric media. Ray investigated the dynamic crack propagation behaviors in piezoelectric layers [23]. The transient response of a Griffith crack between two piezoelectric layers under finite widths was investigated [24]. The problem was formulated using integral transforms, and the path-independent integral $G$ was evaluated at the crack tip to obtain the dynamic energy release rate. The dynamic fracture toughness of a mode-III interfacial crack in two piezoelectric half-spaces was analyzed [25, 26]. Shen et al. provided a unified method to analyze a interfacial crack propagating in piezoelectric bimaterials [27]. The problem of an antiplane Griffith crack moving along with the interface of dissimilar piezoelectric materials was performed using the integral transform technique [28]. A semi-infinite crack in piezoelectric and piezomagnetic bimaterials was researched through the Wiener–Hopf techniques together with the integral transform method [29]. Nourazar and Ayatollahi analyzed the multiple moving cracks along with the integral transform technique [28]. A semi-infinite crack in piezoelectric and piezomagnetic bimaterials was researched through the Wiener–Hopf techniques together with the integral transform method [29]. Nourazar and Ayatollahi analyzed the multiple moving cracks along with the interface of two piezoelectric layers under electrically impermeable or permeable loading, respectively [30].

At the same time, numerical simulations were used for studying the crack. Fracture in piezoelectric materials was analyzed by using a cell-based smoothed X-FEM [31]. Numerical methods were required to deal with the dynamic fracture behavior under loading conditions and general geometries. The dynamic problems of piezoelectric materials were researched by using the X-FEM [32], and it is effective in solving the dynamic fracture problems of isotropic materials. The SGBEM was raised about the dynamic crack in the piezoelectric composite materials [33].

However, piezoelectric bimaterials are widely used in modern life. The crack propagation caused by impact load is often hard to observe, and the crack runs through the interface of bimaterials, which is a complex transient process. The fracture mechanism and process of the material cannot be observed. Therefore, the macroscopic process of crack propagation is carried out to research questions about the materials.

The interfacial fracture of piezoelectric bimaterials is caused by its mismatching characteristics under working loading conditions. There are few kinds of research on the mechanical properties of piezoelectric bimaterials and a variation of the crack as time changes. Therefore, the analysis of dynamic crack along with the interface in piezoelectric bimaterials is essential to the practical engineering design.

1.2. Outline. The presentation of the study is structured as follows. In Section 2, the mixed initial boundary conditions and constitutive equations are presented in detail. In Section 3, the solution method of dynamic crack propagation in the transformed domain is presented. The dynamic stress intensity factor and electric displacement intensity factor of the crack propagation behavior in the interface are given in Section 4. Section 5 presents the numerical results. In Section 6, we give some immediate conclusions.

2. Governing Equations

According to [34] and the variational principle, the energy conservation laws can be written as

$$\frac{d}{dt}(\xi + \Xi) = \int_S \tau_{ij} n_j dS + \int_V E_i D_i dV,$$

where $\xi = 1/2 \rho \ddot{w}$, $\Xi = \int_U dS, S, r, p, E, D$, and $v$ are, respectively, the kinetic energy, internal energy (including electrical and mechanical energies), a surface in the volume domain $V$, outward normal vector, the stress tensor, density, electric field, electric displacement, and velocity vector.

The basic equations for linear piezoelectric medium can be written as follows.

Kinematic equations:

$$\tau_{ij,j} = \rho \ddot{w}_i,$$

where $w$ is the mechanical displacement.

Electrostatic charge conservation:

$$D_{j,j} = 0.$$

Gradient equations:

$$\varepsilon_{ij} = \frac{w_{ij,j} + w_{ji,j}}{2},$$

$$E_i = -\varphi_{j,j},$$

where $w$ is the mechanical displacement, $\varepsilon_{ij}$ is the strain tensor, and $\varphi$ is the electric potential.

From equations (2) and (3), we will obtain the local form of the energy conservation laws:

$$dU = \tau_{ij} d\varepsilon_{ij} + E_i dD_i.$$

We introduce electric enthalpy density $H(\varepsilon_{ij}, E_i)$,

$$H = U - E_i D_i,$$

$$dH = \frac{\partial H}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial H}{\partial E_i} dE_i.$$

The electric enthalpy density $H(\varepsilon_{ij}, E_i)$ is expanded in Taylor series near the natural state ($\tau_{ij} = 0, E_i = 0$):

$$H = \frac{1}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} - c_{klj} \varepsilon_{ij} E_k + \frac{1}{2} \varepsilon_{ij} E_k E_j,$$

where $c_{ijkl}$ are the elastic stiffness constants, $c_{klj}$ are the piezoelectric constants, and $\varepsilon_{ij}$ are the dielectric constants.
By substituting equation (7) to (8), the change of H can be written as follows:

$$dH = \tau_{ij}d\epsilon_{ij} - D_idE_i.$$  \hspace{1cm} (11)

It is noted that

$$\tau_{ij} = \frac{\partial H}{\partial \epsilon_{ij}}, \quad D_i = \frac{\partial H}{\partial E_i}. \hspace{1cm} (12)$$

According to equations (10) and (12), the nontrivial constitutive equations can be obtained:

$$\tau_{ij} = c_{ijkl}\epsilon_{kl} - \epsilon_{kij}E_k, \hspace{1cm} (13)$$
$$D_i = \epsilon_{ikl}\epsilon_{kl} + \epsilon_{ik}E_k. \hspace{1cm} (14)$$

Using the symmetry of strain tensor and stress tensor, the symmetry relation of material constants can be obtained:

$$c_{ijkl} = c_{jikl} = c_{klij} = c_{iklj} = \epsilon_{kij} = \epsilon_{ij}. \hspace{1cm} (15)$$

From equations (3)-(4) and (13)-(15), we have the following basic governing equations of piezoelectric theory:

$$c_{ijkl}\omega_{kj} + \epsilon_{kij}\phi_{kj} = \rho\omega_i, \hspace{1cm} (16)$$

$$\epsilon_{ikl}\omega_{kh} - \epsilon_{i}k\phi_{kl} = 0.$$

For the transversely isotropic piezoelectric medium, the constitutive equations [35] are

$$\tau_{xz} = c_{11}\epsilon_{xx} + c_{12}\epsilon_{yy} + c_{13}\epsilon_{zz} - \epsilon_{31}E_z,$$
$$\tau_{yz} = c_{13}\epsilon_{xx} + c_{11}\epsilon_{yy} + c_{13}\epsilon_{zz} - \epsilon_{31}E_z,$$
$$\tau_{zz} = c_{13}\epsilon_{xx} + c_{13}\epsilon_{yy} + c_{33}\epsilon_{zz} - \epsilon_{33}E_z,$$
$$\tau_{xy} = 2\epsilon_{44}\epsilon_{yz} - \epsilon_{15}E_y,$$
$$\tau_{zx} = 2\epsilon_{44}\epsilon_{zx} - \epsilon_{15}E_x,$$
$$\tau_{xy} = (c_{11} - c_{12})\epsilon_{xy},$$
$$D_x = 2\epsilon_{15}\epsilon_{xz} + \epsilon_{11}E_x,$$
$$D_y = 2\epsilon_{15}\epsilon_{yz} + \epsilon_{11}E_y,$$
$$D_z = \epsilon_{33}\epsilon_{xx} + \epsilon_{33}\epsilon_{yy} + \epsilon_{33}\epsilon_{zz} + \epsilon_{33}E_z. \hspace{1cm} (17)$$

If only the in-plane electric fields and the out-of-plane displacement are considered, the dynamic antiplane governing equations for the transversely isotropic piezoelectric material can be described by

$$c_{44}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + c_{15}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = \rho\frac{\partial^2 \omega}{\partial t^2},$$
$$c_{15}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) - \epsilon_{11}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = 0. \hspace{1cm} (18)$$

The constitutive equations become

$$\tau_{xz} = 2\epsilon_{44}\epsilon_{xz} - \epsilon_{15}E_x = \epsilon_{44}\frac{\partial \omega}{\partial x} + \epsilon_{15}\frac{\partial \phi}{\partial x},$$
$$D_x = 2\epsilon_{15}\epsilon_{xz} + \epsilon_{11}E_x = \epsilon_{15}\frac{\partial \omega}{\partial x} - \epsilon_{11}\frac{\partial \phi}{\partial x},$$
$$\tau_{yz} = 2\epsilon_{44}\epsilon_{yz} - \epsilon_{15}E_y = \epsilon_{44}\frac{\partial \omega}{\partial y} + \epsilon_{15}\frac{\partial \phi}{\partial y},$$
$$D_y = 2\epsilon_{15}\epsilon_{yz} + \epsilon_{15}E_y = \epsilon_{15}\frac{\partial \omega}{\partial y} - \epsilon_{11}\frac{\partial \phi}{\partial y}. \hspace{1cm} (19)$$

In Figure 1, the piezoelectric material one and piezoelectric material two occupy the domains $\Omega_1$ and $\Omega_2$, respectively. A semi-infinite crack along with the interface in the transversely isotropic piezoelectric bimaterials is considered. It is located at $Y = 0, X < 0$. The crack at any time $t < 0$ is in a static equilibrium state in Figure 1. The crack tip begins to move when $t > 0$. The position is $x = vt$ when $t > 0$.

Only if the in-plane electric fields and the out-of-plane displacement are considered, the dynamic antiplane governing equations for the transversely isotropic piezoelectric bimaterials can be described by

$$\epsilon_{44}\left(\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y}\right) + \epsilon_{15}\left(\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right) = \rho \frac{\partial \omega}{\partial t},$$
$$\epsilon_{15}\left(\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y}\right) - \epsilon_{11}\left(\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right) = 0. \hspace{1cm} (20)$$

For the abovementioned equations, $w = w(x, y, t)$ is the mechanical displacements, and $\psi(x, y, t)$ is the electric potential. $\rho$ is the mass density, $\epsilon_{44}^{(k)}$ are the elastic stiffness constants, $\epsilon_{15}^{(k)}$ are the piezoelectric constants, and $\epsilon_{11}^{(k)}$ are the dielectric constants, where $k = 1, 2$ signify piezoelectric material one and piezoelectric material two, respectively.

The shear stress components ($\tau_{xz}, \tau_{yz}$) and electric displacements ($D_x, D_y$) are related to the mechanical displacements and electric potential by the nontrivial constitutive equations:

$$\tau_{xz} = \epsilon_{44}^{(k)}\frac{\partial \omega}{\partial x} + \epsilon_{15}^{(k)}\frac{\partial \phi}{\partial x},$$
$$D_x = \epsilon_{15}^{(k)}\frac{\partial \omega}{\partial x} - \epsilon_{11}^{(k)}\frac{\partial \phi}{\partial x},$$
$$\tau_{yz} = \epsilon_{44}^{(k)}\frac{\partial \omega}{\partial y} + \epsilon_{15}^{(k)}\frac{\partial \phi}{\partial y},$$
$$D_y = \epsilon_{15}^{(k)}\frac{\partial \omega}{\partial y} - \epsilon_{11}^{(k)}\frac{\partial \phi}{\partial y}. \hspace{1cm} (25)$$

For uniform crack propagation, a moving coordinate system is commonly used:

$$x = X - vt, \ y = Y, \ z = Z. \hspace{1cm} (26)$$

By using transformations, equation (21) may be converted to
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![Diagram of a semi-infinite mode-III interfacial propagating crack in piezoelectric bimaterials.](image)

\[ \begin{equation}
\left[ 1 - \frac{\nu}{\varepsilon_k} \right]^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{e_{15}^{(k)}}{\varepsilon_{11}^{(k)}} \left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) = \frac{1}{\varepsilon_k} \left( \frac{\partial^2 w}{\partial t^2} - 2 \nu \frac{\partial^2 w}{\partial x \partial t} \right), \tag{27} \end{equation} \]

where \( \varepsilon_k = [\varepsilon_{44}^{(k)}/\rho]^{1/2} \).

The boundary conditions of mixed mechanical are

\[ \begin{align*}
\tau_{yz}(x, 0^+, t) &= \tau_{yz}(x, 0^-, t) = -\tau_0 H(t), -\infty < x < 0, \\
D_y(x, 0^+, t) &= D_y(x, 0^-, t), -\infty < x < 0, \\
w(x, 0^+, t) &= w(x, 0^-, t) = 0, 0 < x < +\infty, \\
\varphi(x, 0^+, t) &= \varphi(x, 0^-, t) = 0, 0 < x < +\infty, \\
\tau_{yz}(x, 0^+, t) &= \tau_{yz}(x, 0^-, t), 0 < x < +\infty, \\
D_y(x, 0^+, t) &= D_y(x, 0^-, t), 0 < x < +\infty, \\
\end{align*} \tag{29-34} \]

where \( H(t) \) is the Heaviside step function of the time \( t \).

The mechanical displacements and electric potential will disappear when \( y \to +\infty \):

\[ \begin{align*}
\tau_{yz}(x, y, t) &= 0, w(x, y, t) = 0, D_y(x, y, t) = 0, \varphi(x, y, t) = 0, y \to +\infty. \\
\end{align*} \tag{35} \]

The static initial conditions are

\[ \begin{align*}
w(x, y, 0) &= 0, \frac{\partial w(x, y, 0)}{\partial t} = 0, -\infty < x, y < +\infty, \\
\varphi(x, y, 0) &= 0, \frac{\partial \varphi(x, y, 0)}{\partial t} = 0, -\infty < x, y < +\infty. \tag{36-37} \end{align*} \]

3. Solution to the Problem

Following [20, 21], a function is introduced as

\[ \varphi = \varphi - \frac{e_{15}^{(k)}}{\varepsilon_{11}^{(k)}} w. \tag{38} \]

The equations are converted into the canonical form by substituting equations (38) into (27)-(28).

\[ \begin{equation}
\left[ 1 - \frac{\nu}{\varepsilon_k} \right]^2 \frac{\partial^2 w}{\partial x^2} + \frac{e_{15}^{(k)}}{\varepsilon_{11}^{(k)}} \left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) = \frac{1}{\varepsilon_k} \left( \frac{\partial^2 w}{\partial t^2} + 2 \nu \frac{\partial^2 w}{\partial x \partial t} \right), \tag{39} \end{equation} \]

\[ \begin{equation}
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0. \tag{40} \end{equation} \]

To obtain the solution of equations (22)-(25), the following are the Fourier transform concerning \( x \) and Laplace transform concerning the time \( t \):

\[ \begin{equation}
f^*(x, y, p) = \int_{0}^{\infty} f(x, y, t) e^{-pt} dt, \quad f(x, y, t) \tag{41} \end{equation} \]

where \( \xi = \xi_1 + i\xi_2 \) is complex, and \( B_r \) denotes the Bromwich path of integration.

From equations (41) and (42), the governing equations (27), (28), and (38) in the transformed domain can be obtained as follows:
By substituting equations (43) and (45) to equations (39) and (40), respectively, using zero initial conditions in equations (36) and (37), the transformed governing equations become

\[
\begin{bmatrix}
\partial^2 w^* / \partial y^2 \\
\partial^2 \phi^* / \partial y^2
\end{bmatrix}
\begin{bmatrix}
w^* \\
\phi^*
\end{bmatrix}
= 0,
\]

where

\[
w^*(\zeta, y, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x, y, t)e^{-pt}e^{i\zeta x}dtdx,
\]

\[
\phi^*(\zeta, y, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y, t)e^{-pt}e^{i\zeta x}dtdx,
\]

\[
\phi^*(\zeta, y, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y, t)e^{-pt}e^{i\zeta x}dtdx
\]

Applying the conditions given in equations (35), (36), and (37) at infinity, the solutions of equation (46) are

\[
a_k(\zeta) = \frac{\left[f_k - (v/c_k)^2\right]^{1/2} + p^2/c_k^2 + 2vip/c_k^2}{f_k}
\]

\[
= s_k\sqrt{\zeta - i\pi/(c_k + v)\sqrt{\zeta + i\pi/(c_k - v)}
\]

\[
= a_k(\zeta)a_k(-\zeta), s_k = \sqrt{1 - (v/c_k)^2}, f_k\]

\[
= 1 + \frac{e^{i\zeta k}}{c_k} \frac{e^{i\zeta k}}{c_k} b_k(\zeta) = \lim_{\lambda \to 0} \sqrt{\lambda^2 - \zeta^2}
\]

where \(c_k = \sqrt{f_k e^{-2\zeta}}\) denotes the shear bulk wave velocity.

We have the following form with the inverse Fourier transform to equation (50):

\[
w^*(x, y, p) = \frac{1}{2\pi} \int_{i\kappa_{\alpha} = -\infty}^{i\kappa_{\alpha} = \infty} A_{ik} e^{i(3-2k)\alpha y} e^{-i\zeta x} d\zeta,
\]

\[
\phi^*(x, y, p) = \frac{1}{2\pi} \int_{i\kappa_{\alpha} = -\infty}^{i\kappa_{\alpha} = \infty} B_{ik} e^{i(3-2k)\beta y} e^{-i\zeta x} d\zeta.
\]

From equations (52a) and (52b), we have the following form:

\[
\phi^*(x, y, p) = \frac{e^{i\zeta k}}{c_k} \phi^*(x, y, p) + \phi^*(x, y, p)
\]

\[
\phi^*(x, y, p) = \frac{e^{i\zeta k}}{c_k} \frac{1}{2\pi} \int_{i\kappa_{\alpha} = -\infty}^{i\kappa_{\alpha} = \infty} A_{ik} e^{i(3-2k)\alpha y} e^{-i\zeta x} d\zeta
\]

\[
+ \frac{1}{2\pi} \int_{i\kappa_{\alpha} = -\infty}^{i\kappa_{\alpha} = \infty} B_{ik} e^{i(3-2k)\beta y} e^{-i\zeta x} d\zeta
\]

in which

\[
\frac{p}{c_k - v} < \kappa_{\alpha} < \frac{p}{c_k + v}
\]

\[-\zeta < \kappa_{\alpha} < \zeta.
\]

By using equations (52a), (52b) and (53), the transformed results of the stress and electric displacement are
$$
abla \tau_{yz}(x, y, p) = \left( \epsilon_{11}^{(k)} + \frac{\epsilon_{15}^{(k)} \epsilon_{15}^{(k)}}{\epsilon_{11}^{(k)}} \right) (3 - 2k) \left( \frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} a_{ik} e^{i(3-2k)\kappa_{h}^{\text{co}}y} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta \right) + \epsilon_{15}^{(k)} (3 - 2k) \left( \frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{B}_{ik} b_{ik} e^{i(3-2k)\kappa_{h}^{\text{co}}y} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta \right)
$$

(55)

$$D_{y}^{*}(x, y, p) = -\epsilon_{11}^{(k)} (3 - 2k) \left( \frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} a_{ik} e^{i(3-2k)\kappa_{h}^{\text{co}}y} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta \right) \quad \text{(56)}$$

To further improve the speed for solving problems, the boundary conditions equations (29)–(34) expressed in the Laplace transform domain are

$$\tau_{yz}^{*}(x, 0^+, p) = \tau_{yz}^{*}(x, 0^-, p) = \frac{-\tau_{0}}{p}, \quad x < 0 \quad \text{(57)}$$

$$D_{y}^{*}(x, 0^+, p) = D_{y}^{*}(x, 0^-, p), \quad x < 0 \quad \text{(58)}$$

$$w^{*}(x, 0^+, p) = w^{*}(x, 0^-, p) = 0, \quad x > 0 \quad \text{(59)}$$

$$\varphi^{*}(x, 0^+, p) = \varphi^{*}(x, 0^-, p) = 0, \quad x > 0 \quad \text{(60)}$$

$$\tau_{yz}^{*}(x, 0^+, p) = \tau_{yz}^{*}(x, 0^-, p), \quad x > 0 \quad \text{(61)}$$

$$D_{y}^{*}(x, 0^+, p) = D_{y}^{*}(x, 0^-, p), \quad x > 0 \quad \text{(62)}$$

By substituting equations (59) and (60) into equations (52a) and (53), respectively, we have

$$\frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = 0, \quad x > 0 \quad \text{(63)}$$

$$\frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{B}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = 0, \quad x > 0 \quad \text{(64)}$$

Substitution equations (57) and (58) into equations (55) and (56), we will have

$$\frac{3 - 2k}{2\pi} \left[ \left( \epsilon_{14}^{(k)} + \frac{\epsilon_{15}^{(k)} \epsilon_{15}^{(k)}}{\epsilon_{11}^{(k)}} \right) \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta \right] + \epsilon_{15}^{(k)} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{B}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = \frac{-\tau_{0}}{p}, \quad x < 0 \quad \text{(65)}$$

From equation (64), we have

$$\frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} a_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = \frac{\epsilon_{11}^{(k)}}{\epsilon_{14}^{(k)} + \frac{\epsilon_{15}^{(k)} \epsilon_{15}^{(k)}}{\epsilon_{11}^{(k)}}} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta$$

Then, rewriting \( a_{ik} \), we have

$$a_{k} = \sqrt{(p - i\kappa_{h}^{(k)})(p - i\kappa_{h}^{(k)})}, \quad \text{(66)}$$

in which

$$\kappa_{h} = \nu + \sqrt{f_{e}^{-2}}, \quad h_{k} = \nu + \sqrt{f_{e}^{-2}}. \quad \text{(67)}$$

When \( \nu = 0 \), \( g_{k} \) and \( h_{k} \) become

$$g_{k} = \sqrt{f_{e}^{-2}} = h_{k} = c_{k}. \quad \text{(68)}$$

It can be verified that this result is consistent with reference [36] in the case \( \nu = 0 \). Now, let us calculate the case \( \nu \neq 0 \). Given the four boundary conditions equations (59)–(62), the displacement and stress boundary conditions must extend to the entire range of the \( x \)-axis. Then, we introduce two unknown functions:

$$w^{*}(x, 0, p) = \begin{cases} 0 & x \geq 0 \\ w_{*}(x, 0, p) & x < 0 \end{cases} \quad \text{(69)}$$

$$\tau_{yz}^{*}(x, 0, p) = \begin{cases} \tau_{*}(x, 0, p) & x \geq 0 \\ \frac{-\tau_{0}}{p} & x < 0 \end{cases} \quad \text{(70)}$$

$$\varphi^{*}(x, 0, p) = \begin{cases} 0 & x \geq 0 \\ \varphi_{*}(x, 0, p) & x < 0 \end{cases} \quad \text{(71)}$$

Equations (69), (70), and (71) can be written as

$$w^{*}(x, 0, p) = w_{*}(x, 0, p), \quad \text{(72)}$$

$$\tau_{yz}^{*}(x, 0, p) = \tau_{*}(x, 0, p) - \frac{\tau_{0}}{p} f_{e}^{-2} H(-x), \quad \text{(73)}$$

$$\varphi^{*}(x, 0, p) = \varphi_{*}(x, 0, p). \quad \text{(74)}$$

Substituting equations (52b) and (56) into equation (74), respectively, we can obtain

$$\frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{B}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = \varphi_{*}(x, 0, p). \quad \text{(75)}$$

Substituting equations (52a), (52b), (55), and (64) into equation (73), we can obtain

$$\frac{1}{2\pi} \int_{\kappa_{h}^{\text{co}}}^{\kappa_{h}^{\text{co}}} \text{A}_{ik} e^{-i\kappa_{h}^{\text{co}}\kappa_{h}^{\text{co}}} d\zeta = \frac{-\tau_{0}}{p} f_{e}^{-2} H(-x) \left( \epsilon_{44}^{(k)} + \frac{\epsilon_{15}^{(k)} \epsilon_{15}^{(k)}}{\epsilon_{11}^{(k)}} ight) \left( 3 - 2k \right) \quad \text{(76)}$$

Equations (76) and (77) are a set of dual integral equations. The inverse Fourier transform is applied to equations (76)-(77):
\[ W_{\kappa}^{*} (\zeta, 0, p) = A_{\kappa} = \int_{-\infty}^{0} u_{\kappa}^{*} (x, 0, p) e^{ik_{\kappa}x} \, dx, \quad (78) \]

Equation (79) can be rewritten as
\[ W_{\kappa}^{*} M_{\kappa} = R_{\kappa}^{*} - \frac{\tau_{0}}{ip\zeta} + q_{k}D_{\kappa}^{*}, \quad (80) \]

where
\[ M_{\kappa} = p_{k} e_{k} \sqrt{\zeta - ip/(d_{k} + v)} \sqrt{\zeta + ip/(d_{k} - v)} \]
\[ = \left( c_{44}^{(k)} + \frac{e_{15}^{(k)}}{\varepsilon_{11}^{(k)}} \right) (3 - 2k), q_{k} = c_{15}^{(k)} R_{k}^{*} = \int_{0}^{\infty} r_{\kappa}^{*} (x, 0, p) e^{ik_{\kappa}x} \, dx, \quad D_{\kappa}^{*} \]
\[ = \left( \frac{c_{44}^{(k)} c_{44}^{(k)}}{\varepsilon_{11}^{(k)} e_{11}^{(k)}}, \frac{e_{15}^{(k)}}{\varepsilon_{11}^{(k)} e_{11}^{(k)}} + 2c_{15}^{(k)} e_{11}^{(k)} e_{15}^{(k)} \right) + \frac{\left( c_{44}^{(k)} c_{44}^{(k)} + e_{15}^{(k)} e_{15}^{(k)} \right) p}{\left( \varepsilon_{11}^{(k)} e_{11}^{(k)} + e_{15}^{(k)} e_{15}^{(k)} \right) p} \]

\[ d_{k} \text{ are the speeds of the Bleustein–Gulyaev wave. If } a_{1} (\zeta - k_{a}^{2} b_{1} (\zeta) + a_{2} (\zeta - k_{a}^{2} b_{2} (\zeta) = 0 \text{ exists a real root } \nu, \text{ then the real root } \nu \text{ is called the Maerfeld–Tournois wave speed. If the Maerfeld wave exists, we define } c = c_{a}; \text{ otherwise, } c = c_{i}. \text{ The functions } H (\zeta) \text{ (Appendix) can be defined as follows:} \]
\[ H (\zeta) = \frac{\left[ z (c + v) - ip \right] \left[ z (c - v) + ip \right]}{c \sqrt{\varepsilon_{1}}} \frac{\sqrt{z (c_{1} - v) + ip}}{\sqrt{z (c_{1} + v) - ip}} \frac{\left( a_{k} (v) - k_{a}^{2} b_{k} (v) \right)}{s - ic_{k} / p} \]
\[ = H_{k}^{*} (\zeta) H_{k}^{*} (\zeta) r_{k} \]

where
\[ k_{a} = \sqrt{\frac{\varepsilon_{44}^{(k)} \varepsilon_{11}^{(k)}}{\varepsilon_{15}^{(k)} e_{11}^{(k)} + e_{15}^{(k)} e_{15}^{(k)}}} \]
\[ S_{a}^{*} = \exp \left\{ -\frac{1}{\pi} \int_{\varepsilon_{\text{min}} v}^{\varepsilon_{\text{max}} v} \arctan \left( \frac{\text{Im} (H (ip))}{\text{Re} (H (ip))} \right) \frac{ds}{s - ic_{k} / p} \right\}, \quad (83) \]
\[ O^{*} = \frac{d_{1} d_{2}}{(\zeta (d_{1} + v) - ip)(\zeta (d_{2} + v) - ip) (\zeta (d_{1} - v) + ip)(\zeta (d_{2} - v) + ip)} = O_{a}^{*} O_{a}^{*}, \]

(Maerfeld–Tournois wave function) depends on the MT wave, \( M_{k} (\zeta) \) are defined and coefficiented as follows:
\[ M_k = \frac{\sqrt{(d_k - v)(c_k + v)} \sqrt{p - i\xi (d_k + v)} \sqrt{p + i\xi (c_k - v)} H_x^* H_y^* O^* O^*}{\sqrt{(d_k + v)(c_k - v)} \sqrt{p + i\xi (d_k - v)} \sqrt{p - i\xi (c_k + v)} S^*_x S^*_y S^*_y^{-1} r_k} \]

\[ = F^*_k F^*_k \left( L^*_k L^*_k r_k \right) \]

The product decomposition of equation (80) is given by [20, 21, 37]

\[ p_k r_k W^*_k F^*_k F^*_k L^*_k L^*_k = R^*_k - \frac{\tau_0}{i p\xi} + q_k D^*_k, \]

where

\[ F^*_k = \frac{\sqrt{p (d_k + v) - i\xi (1 + v/c) - i p/c)} \sqrt{\bar{\xi} (1 + v/c) - i p/c}}{\sqrt{p (c_k + v) - i\xi (1 + v/d_1) - i p/d_1)} (\bar{\xi} (1 + v/d_2) - i p/d_2)} \]

\[ F^*_k = \frac{\sqrt{p (c_k - v) + i\xi (\xi (c - v) + i p)} \sqrt{\bar{\xi} (c_k - v) + i p)} \sqrt{p (d_k - v) + i\xi (\xi (d_1 - v) + i p)} \sqrt{\bar{\xi} (d_k - v) + i p})}{\sqrt{p (c_k + v) - i\xi (1 + v/d_1) - i p/d_1)} (\bar{\xi} (1 + v/d_2) - i p/d_2)} \]

\[ L^*_k = \frac{C^*_k, C^*_k, C^*_k}{S^*_k}, \quad L^*_k = \frac{C^*_k, C^*_k}{S^*_k} \]

Moreover, equation (85) is expressed as follows:

\[ p_k r_k W^*_k F^*_k F^*_k L^*_k L^*_k = R^*_k - \frac{\tau_0}{i p\xi} + q_k D^*_k \]

\[ \lim_{x \to 0^{+}} \tau^*_y (x, y, p) x^{1/2} \sim \lim_{|\bar{\xi}| \to \infty} R^*_x (\zeta, 0, p)|\xi|^{1+\theta}, \]

\[ \lim_{x \to 0^{+}} W^*_x (x, 0, p)|\xi|^{-\theta} \sim \lim_{|\bar{\xi}| \to \infty} W^*_x (\zeta, 0, p)|\xi|^{-1}. \]

Using the Able theorem [38], we can obtain

\[ \left| p_k r_k W^*_k F^*_k F^*_k L^*_k L^*_k \right| \sim o \left( |\xi|^{1-1/2-\theta} \right), \]

\[ \sim o \left( |\xi|^{-1} \right). \]

According to the extended Liouville’s theorem [37], the polynomial must be less than the maximum value \([-1/2 - \theta, -1/2] \). It can only be equal to zero in the case that

\[ W^*_x (\zeta, 0, p) = \frac{1/(d_1 - v) - i\xi/p \sqrt{1/(d_2 - v) - i\xi/p \sqrt{p (c_k - v) + i\xi \tau_0} \sqrt{1/(c_k - v) - i\xi/p(1/c - v) - \tan(\xi q/p) \sqrt{p (d_k - v) + i\xi \tau_0}}}}{(1 - v/d_1)(1 - v/d_2) \sqrt{1/(c_k)^{1/2}(1 - v/c)^{1/2} i p k r_k L^*_k}} \]
The functions $A_k (\zeta)$ are obtained by substituting equation (92) into (78):

$$A_{k}(\zeta) = \frac{1}{\sqrt{2d_1 - v}} - i\zeta/p\sqrt{1/2d_2 - v} - i\zeta/p\sqrt{p(c_k - v) + i\zeta}
\frac{1}{\sqrt{1/c_1 - v}} - i\zeta/p(1/c - v) - i(1/p)\sqrt{p(d_k - v) + i\zeta}
\frac{(1 - v/d_k)(1 - v/d_1)}{(1 - v/c_1)^{1/2}(1 - v/c)\sqrt{ipK}} \tau_0. $$

(93)

According to the procedure by Li and Mataga [20, 21], the functions $B_k (\zeta)$ can be obtained:

$$B_{k}(\zeta) = \frac{1}{\sqrt{2d_1 - v}} - i\zeta/p\sqrt{1/2d_2 - v} - i\zeta/p\sqrt{p(c_k - v) + i\zeta}
\frac{1}{\sqrt{1/c_1 - v}} - i\zeta/p(1/c - v) - i(1/p)\sqrt{p(d_k - v) + i\zeta}
\frac{(1 - v/d_k)(1 - v/d_1)q_k}{(1 - v/c_1)^{1/2}(1 - v/c)\sqrt{ipK}} \tau_0. $$

(94)

4. Intensity Factors

Substituting equations (93) and (94) into equations (55) and (56), respectively, and applying the inverse Fourier transform, $D^* (x, p)$ and $\tau_{xy}^* (x, p)$ can be obtained. According to Abel’s theorem [37], we can obtain the stress intensity factor and electric displacement intensity factor as follows:

$$K_{III}(p) = \lim_{x \to 0^+} \sqrt{2\pi x} \tau_{xy}^* (x, p)
= \sqrt{2} \left(1 - v/d_1 \right) \left(1 - v/d_2 \right) \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}} p^{3/2}.$$  

(95)

$$K_{D1}(p) = \lim_{x \to 0^+} \sqrt{2\pi x} D^* (x, p)
= \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}} p^{3/2}.$$  

(96)

$$K_{D2}(p) = \lim_{x \to 0^+} \sqrt{2\pi x} D^* (x, p)
= \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}} p^{3/2}.$$  

(97)

Using the inverse Laplace transform to equations (95)–(97) leads to

$$K_{III}(t, v) = \frac{2\sqrt{2}}{L^- (0)} \sqrt{\pi \tau_0} \sqrt{t},$$

$$K_{D1}(t, v) = \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}} \sqrt{\pi \tau_0} \sqrt{t},$$

$$K_{D2}(t, v) = \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}} \sqrt{\pi \tau_0} \sqrt{t}. $$

(98)

According to the procedure in [38] for the problem of the static semi-infinite crack propagation, we introduce a normalization.

The stress intensity factor for the static semi-infinite crack is

$$K_{III}(t, 0) = \frac{2\sqrt{2}}{L^- (0)} \sqrt{\pi \tau_0} \sqrt{t}. $$

(99)

The electric displacement intensity factor for a stationary semi-infinite crack is

$$K_{D1}(t, 0) = \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (0)}} \sqrt{\pi \tau_0} \sqrt{t},$$

$$K_{D2}(t, 0) = \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (0)}} \sqrt{\pi \tau_0} \sqrt{t}. $$

(100)

As a result, the dynamic stress intensity factor and the dynamic electric displacement intensity factors can be represented as

$$K_{III}(t, v) = f(v)K_{III}(t, 0),$$

$$K_{D1}(t, v) = g_1 (v)K_{D1}(t, 0),$$

$$K_{D2}(t, v) = g_2 (v)K_{D1}(t, 0), $$

(101)

where

$$f(v) = \frac{\tau_0}{\sqrt{1 - v/c_1 (1 - v/c)L^- (v)}},$$

$$g_1 (v) = \frac{1 - k_{1e}^2}{(s_1 - k_{1e}^2)} f(v),$$

$$g_2 (v) = \frac{1 - k_{2e}^2}{(s_2 - k_{2e}^2)} f(v), $$

(102)

are the nondimensional dynamic intensity factors.
5. Numerical Examples

The material properties and the mass density [20, 36], elastic stiffness constant, piezoelectric constant, dielectric constant, electromechanical coupling coefficient, and shear wave speed are given in Table 1. Incidentally, the values $c$ and $d$ for PZT65/35 and ZnO, respectively, in [20] are incorrect, and the correct values are given in Table 1.

There are three parts in this section. First, when the research is degenerated into the simple case, some numerical applications are provided to verify the conclusions obtained. Second, some analyses and comparisons are made between the results obtained by us and the existing references [25, 26] (the methods used are different from those we used). Third, the influence of material constants, velocity of the crack propagation, and time of impact are discussed. The researches have led to some new conclusions.

5.1. Study of Degenerate into the Simple Case. When bimaterials degenerate into a single material, it follows that

$$
K_{III}(t, v) = f(v)K_{III}(t, 0),
$$

$$
K_D(t, v) = g(v)K_D(t, 0),
$$

where $f(v) = (1 - v/d)/\sqrt{(1 - v/c) L^*(v)}$, $g(v) = (1 - k^2_0)/(s - k^2_0)f(v)$.

It is noted that the conclusions agree with [20, 21]. We choose the single material PZT-4 and PZT-5H, respectively, for research. The corresponding analysis diagram is shown in Figure 2. They show the variations $f(v)$ and $g(v)$ with the normalized speed $v/d$ of the dynamic crack propagation. As shown in Figure 2(a), $f(v)$ decreases with increasing velocity $v$ of the dynamic crack propagation. The variations $f(v)$ vanish when $v$ reaches the Bleustein–Gulyaev $d$. Furthermore, the nondimensional function of the stress $f(v)$ for an antiplane crack propagating in piezoelectric solids as considered is plotted for comparison [20, 21]. Their changes are consistent. However, as shown in Figure 2(a), when the speed of the dynamic crack propagation reaches Bleustein–Gulyaev $d$, the variations $g(v)$ do not have to go to zero. It has an initial decrease as the speed $v$ of the dynamic crack propagation increases from zero. The variations $g(v)$ increase again as the speed $v$ of the dynamic crack propagation reaches Bleustein–Gulyaev $d$. Furthermore, the variations $g(v)$ for the crack propagating in piezoelectric solids [20, 21] as considered are plotted for comparison. Their changes are consistent.

5.2. Analysis and Comparison with the Existing References. Some analyses and comparisons are made between the results obtained by us and the existing references [25, 26] (the methods used are different from those we used). We choose PZT-4&PZT65/35 and PZT-2&ZnO, respectively, for research. The corresponding analysis diagram is shown in Figure 3. In Figures 3(a) and 3(c), it represents $f(v)$, $g_1(v)$, and $g_2(v)$ of PZT-4&PZT65/35 versus the normalized speed $v/d$ and $v/c$ of the dynamic crack for two cases. In Figures 3(b) and 3(d), it represents $f(v)$, $g_1(v)$, and $g_2(v)$ of PZT-2&ZnO versus the normalized speed $v/d$ and $v/c$ of the dynamic crack for two cases.

It is obvious that the nondimension functions $f(v)$ and $g_1(v)$ decrease with the velocity $v$ of the crack propagation increasing, as shown in Figure 3. The nondimension functions $f(v)$ and $g_1(v)$ reach zero, with the velocity $v$ of the dynamic crack propagation reaching the slower velocity $d$. It is consistent with [25, 26]. However, the nondimension function $g_2(v)$ does not necessarily decrease to zero and is relatively complex, with the dynamic crack speed reaching the Bleustein–Gulyaev wave speed $d$. It has an initial decrease as the speed $v$ of the dynamic crack propagation increases from zero. The variations increase again as the speed $v$ of the dynamic crack propagation reaches Bleustein–Gulyaev $d$. The whole process of changes are consistent with [25, 26].

5.3. The Influence of Material Constants, Velocity of the Crack Propagation, and Time of Impact. Figure 4(a) shows the change of the nondimension functions $f(v)$ with the electromechanical coefficient $k$, with each plotted against the nondimension velocity $v/c$ for five different piezoelectric bimaterials. Suppose the values of $k$ are larger, the shape of the curve flats. Figure 4(b) shows the change of the nondimension functions $f(v)$ with the electromechanical coefficient $k$, with each plotted against the nondimensional velocity $v/d$ for five different piezoelectric bimaterials. As shown, the nondimension functions $f(v)$ are controlled by the electromechanical coefficient $k$, that is, if the values of $k$ are larger, the nondimension functions $f(v)$ are lower.

Figures 5(a) and 5(b) show the change of the nondimensional functions $g(v)$ with the electromechanical coefficient $k$, with each plotted against the nondimension velocity $v/c$ for five different piezoelectric bimaterials. In Figure 5, the change of each function $g(v)$ remains almost constant as $k$ increases. The change of each $g(v)$ drops sharply as $k$ approaches unity. The Bleustein–Gulyaev wave speed $d$ and the shear wave speed $c$ both increase as $k$ increases. The change of $g(v)$ decreases as $c$ increases, and the change of $g(v)$ increases as $c$ increases. However, the effect is not an obvious effect when $k$ is at low values. The variation of $g(v)$ is complex, and the nondimension functions $g(v)$ attains a finite nonzero value with $v$ approaches $c$ or $d$. The change of $g(v)$ is affected by the Bleustein–Gulyaev wave term $1 - k^2_0$. Since $1 - k^2_0$ go to zero with $v$ approaching the velocity $d$ of Bleustein–Gulyaev wave, the functions $g(v)$ do not have to go to zero at large $v = d$.

It is obvious that the nondimension functions $f(v)$ and $g_1(v)$ decrease with the velocity $v$ of the crack propagation increasing, as shown in Figures 6(a) and 6(b). The nondimension functions $f(v)$ and $g_1(v)$ reach zero, with the velocity $v$ of the dynamic crack propagation reaching the slower velocity $d$. It is similar to the nondimension function $K_{III}(v)$ of purely elastic solids. However, the nondimension function $g_2(v)$ does not necessarily decrease to zero and is relatively complex, with the dynamic crack speed reaching the Bleustein–Gulyaev wave speed $d$. In Figures 6(c) and 6(d), it represents $f(v)$, $g_1(v)$, and $g_2(v)$ versus the
### Table 1: Material properties and electroacoustic constants.

| Constant | Material          | PZT-4 | PZT-5H | ZnO | BaTiO₃ | PZT-65/35 | PZT-2 |
|----------|------------------|-------|--------|-----|--------|-----------|-------|
| ρ (10³ kg/m³) |                  | 7.5   | 7.5    | 5.68| 5.7    | 7.825     | 7.6   |
| ε₁ (10⁻⁹ F/m) |                  | 6.4634| 15.052 | 0.0757| 9.8722| 5.66      | 4.4624|
| c₄₄ (10¹⁰ N/m²) |                | 2.56  | 2.3    | 4.247| 4.4    | 3.890     | 2.22  |
| ε₁₅ (C/m) |                  | 6.4634| 15.052 | 0.0757| 9.8722| 5.66      | 4.4624|
| k = √(ε₁₁¹₁₁ + ε₁₃¹₃) |             | 0.7026| 0.6745 | 0.2586| 0.4799| 0.4921    | 0.7016|
| c = √(c₄₄ + ε₁₅/ε₁₁) / ρ (10³ m/s) |         | 2.5963| 2.3721 | 2.8307| 3.1668| 2.5611    | 2.3985|
| d = c√1 - k² (10³ m/s) |            | 2.2579| 2.1123 | 2.8244| 3.0817| 2.4849    | 2.0878|

**Figure 2:** Variations of $f(v)$ and $g(v)$ versus the nondimensional velocity $v/d$ for a single material PZT-4 or PZT-5H.

**Figure 3:** Continued.
normalized speed \( v/d \) of the dynamic crack for two cases. It is clear that the function \( f(v) \) decreases as the velocity of the dynamic crack propagation increases. Because the material is consistent, the speed of the Bleustein–Gulyaev wave and the speed of the Maerfeld–Tournois wave are the same. The non-dimensional functions \( g_1(v) \) and \( g_2(v) \) are identical in the condition. It has an initial decrease as the speed \( v \) of the dynamic crack propagation increases from zero. The variations increase again as the speed \( v \) of the dynamic crack propagation reaches the Bleustein–Gulyaev \( d \).

To verify the accuracy of the proposed theoretical calculation, a central crack of the piezoelectric bimaterials is under a mechanical impact loading. The impact is caused by mechanical loading \( \tau(t) = \tau_0 H(t) \), where \( \tau_0 \) is the loading
Figure 5: Variations of $g(v)$ versus the nondimensional velocity of the dynamic crack propagation for different piezoelectric bimaterials. (a) $v/c$. (b) $v/d$.

Figure 6: Continued.
Figure 6: Example: the nondimension functions, $f(v), g_1(v)$, and $g_2(v)$, for the following pairs of half-spaces: (a) PZT-4 & PZT-5H. (b) PZT-5H & ZnO. (c) BaTiO$_3$ & BaTiO$_3$ ($g_1(v) = g_2(v) = g(v)$). (d) PZT-5H & PZT-5H ($g_1(v) = g_2(v) = g(v)$).

Figure 7: Continued.
Figure 7: The time of six different piezoelectric bimaterials in $M$ when $a$ takes to attain 1 cm: (a) $M = 0.3$. (b) $M = 0.5$. (c) $M = 0.8$.

Figure 8: Variation of $f(v)$ in $M = 0.3$, $M = 0.5$, and $M = 0.8$ with time when $a$ takes to attain 1 cm: (a) PZT-4 & PZT-4. (b) ZnO & ZnO. (c) PZT65/35 & PZT65/35.
amplitude. The $M$ is a parameter of $v/c$, and $a$ is a parameter to measure the strength of the dynamic crack propagation. In this section, we examine the performance of $f(v)$ through six different piezoelectric bimaterials for which the experimental results are available. Figures 7(a) – 7(c) show that the time $t$ of six different piezoelectric bimaterials take to attain $a = 1$ cm in $M = 0.3$, $M = 0.5$, and $M = 0.8$, respectively. The following characteristics are obvious: (a) the dependence of $f(v)$ on the electromechanical coefficient $k$ is significant for a given $M$; (b) the values of $f(v)$ decrease with increasing the electromechanical coefficient $k$ for a given time and a given $M$; (c) it is found that for a given strength of the crack, the length of time is independent of the electromechanical coefficient $k$. This characteristic has also not been reported in the piezoelectric bimaterials.

In order to analyze $f(v)$ with $M = 0.3$, $M = 0.5$, and $M = 0.8$ of the proposed method, we examined PZT-4&PZT-4, ZnO&ZnO, and PZT65/35&PZT65/35, respectively. The experimental setup is shown in Figures 8(a)–8(c). The following characteristics are obvious: (a) the dependence of $f(v)$ on $M$ is significant for given piezoelectric bimaterials; (b) the values of $f(v)$ decrease with increasing $M$; (c) it is found that for a given strength of the crack, the larger $M$ is, the smaller the time is, under the same loading.
This characteristic has also not been reported in the piezoelectric bimaterials.

In Figures 9(a)–9(c), the effects of the electromechanical coupling coefficient $k$ on $K_{III}(t,v)/\tau_0$ are investigated numerically by calculating $K_{III}(t,v)/\tau_0$ of six different piezoelectric bimaterials, respectively. In this study, we specify $M = 0.5$, $M = 0.6$, and $M = 0.7$, respectively, and the corresponding results of $K_{III}(t,v)/\tau_0$ are shown in Figures 9(a)–9(c). The calculated results $K_{III}(t,v)/\tau_0$ of six different curves are shown. The following characteristics are obvious: (a) the dependence of $K_{III}(t,v)/\tau_0$ on the electromechanical coefficient $k$ is significant; (b) it can also be observed that $K_{III}(t,v)/\tau_0$ are equal to zero at $t = 0$. The results show that the electromechanical coupling coefficient $k$ of the piezoelectric half-space can be increased from zero to unity. In contrast, the material properties of the elastic half-space are held constant; (c) the values of $K_{III}(t,v)/\tau_0$ decrease with increasing electromechanical coefficient $k$ for a given time and a given $v$. This characteristic has also not been reported in the piezoelectric bimaterials.

In Figures 10(a)–10(c), the calculated results $K_{III}(t,v)/\tau_0$ of four different curves are shown. The following characteristics are obvious: (a) the dependence of $K_{III}(t,v)/\tau_0$ and $M$ is significant; (b) the values of the $K_{III}(t,v)/\tau_0$ decrease with increasing electromechanical coefficient $k$; (c) the values of $K_{III}(t,v)/\tau_0$ increase with $M$.
6. Conclusions

The dynamic behavior of an interfacial mode-III crack under mechanical impact loading is considered. The dynamic stress intensity factor, the dynamic electric displacement intensity factor, and their nondimension functions are derived with the analytical expressions. According to the solution established in the study, we have some conclusions: (1) the behavior of the dynamic fracture in piezoelectric bimaterials is more to do with the electromechanical coefficient $k$; (2) the dynamic stress intensity factor always vanishes with the velocity of dynamic crack propagation arriving at the speed of the generalized Raleigh wave; (3) it is verified that the nondimensional stress intensity factors are controlled by the electromechanical coefficient $k$, that is, the bigger the electromechanical coefficient $k$, the smaller the nondimensional stress intensity factor; (4) the existence of the Maerfeld–Tournois wave will increase the value of the dynamic intensity factors; (5) the values of $K_{III}(t, v)/\tau_0$ increase with $M$ decreasing; (6) consequently, the effects of the electromechanical coefficient $k$ on $K_{III}(t, v)/\tau_0$ in piezoelectric bimaterials are significant; (7) it is found that for a given strength of the crack, the larger varying $M$ is, the smaller the time is, under the same loading. In a word, it analyzes the relationship between the dynamic crack propagation in the crack plane, and the velocity of the crack. This study shows that the research idea is consist of analyzing fracture, selecting the type of crack, establishing the model, defining the boundary conditions, and confirming fracture criterion. In this study, the existing crack is mode-III crack. Dynamic propagation characteristics of the crack under impact loading are presented. It is considered under the in-plane electric field and out-of-plane displacement. The mode-I, mode-II, and composite cracks will be analyzed and discussed in the future research. At the same time, nanoscale piezoelectric bimaterials are widely used in daily life. Our analysis in this section does not apply to thin nanomaterials. In the next research, we can consider discussing the interface cracks of nanoscale piezoelectric bimaterials in not only analyzing the semi-infinite III cracks but also thinking about deeply the influence of nanomaterial thickness on material design.

Appendix

This decomposition provides the necessary conditions for the solution of equation (80):

$$ M_k = p_k s_k \left( \zeta - i p/(d_k + v) \right) \left( \zeta + i p/(d_k - v) \right) $$

$$ \cdot \sqrt{\zeta - i p/(c_k + v)} \sqrt{\zeta + i p/(c_k - v)} $$

$$ = p_k N^*_k O^*_k, $$

where

$$ N_k = \frac{1}{c_k d_k^2} (\zeta(d_k + v) - ip)(\zeta(d_k - v) + ip) $$

$$ \cdot \sqrt{(c_k + v) - i p} \sqrt{(c_k - v) + ip} = N^*_k N^*_k, $$

$$ O^*_k = \frac{d_k^2}{(\zeta(d_k + v) - ip)(\zeta(d_k - v) + ip)} $$

$$ = \frac{d_k}{\zeta(d_k + v) - ip} \frac{d_k}{\zeta(d_k - v) + ip} = O^*_k O^*_k. $$

It is convenient to express this in the following form [20, 21]:

$$ \ln N_k = \ln(N^*_k) + \ln(N^*_k) $$

Moreover, $N^*_k$ and $N^*_k$ can be expressed as

$$ \ln N^*_k = -\frac{1}{2ni} \int_{\tilde{\eta}_k - \infty}^{\tilde{\eta}_k + \infty} \ln N^*_k dz, \ -\varepsilon < \tilde{\eta}_k < \Im(\zeta) $$

$$ \ln N^*_k = -\frac{1}{2ni} \int_{\tilde{\eta}_k - \infty}^{\tilde{\eta}_k + \infty} \ln N^*_k dz, \ \Im(\zeta) < \tilde{\eta}_k, < \varepsilon, $$

where $z$ is a complex variable.

The argument of function $N^*_k$ along branch cuts are as follows:

$$ \arg N^*_k(\zeta) = \begin{cases} \arctan \frac{\Im[H_k(z)]}{\Re[H_k(z)]} & -\frac{1}{c_k - v} < \Im(z) < -\varepsilon \\
\frac{\pi}{2} & -\frac{1}{d_k - v} < \Im(z) < -\frac{1}{c_k - v} \\
0 & -\infty < \Im(z) < -\frac{1}{d_k - v} \end{cases} $$

$$ \cdot \sqrt{(c_k + v) - i p} \sqrt{(c_k - v) + ip} $$

$$ = p_k N^*_k O^*_k, $$

where

$$ N_k = \frac{1}{c_k d_k^2} (\zeta(d_k + v) - ip)(\zeta(d_k - v) + ip) $$

$$ \cdot \sqrt{(c_k + v) - i p} \sqrt{(c_k - v) + ip} = N^*_k N^*_k, $$

$$ O^*_k = \frac{d_k^2}{(\zeta(d_k + v) - ip)(\zeta(d_k - v) + ip)} $$

$$ = \frac{d_k}{\zeta(d_k + v) - ip} \frac{d_k}{\zeta(d_k - v) + ip} = O^*_k \cdot O^*_k. $$

It is convenient to express this in the following form [20, 21]:

$$ \ln N_k = \ln(N^*_k) + \ln(N^*_k) $$

Moreover, $N^*_k$ and $N^*_k$ can be expressed as

$$ \ln N^*_k = -\frac{1}{2ni} \int_{\tilde{\eta}_k - \infty}^{\tilde{\eta}_k + \infty} \ln N^*_k dz, \ -\varepsilon < \tilde{\eta}_k < \Im(\zeta) $$

$$ \ln N^*_k = -\frac{1}{2ni} \int_{\tilde{\eta}_k - \infty}^{\tilde{\eta}_k + \infty} \ln N^*_k dz, \ \Im(\zeta) < \tilde{\eta}_k, < \varepsilon, $$

where $z$ is a complex variable.

The argument of function $N^*_k$ along branch cuts are as follows:

$$ \arg N^*_k(\zeta) = \begin{cases} \arctan \frac{\Im[H_k(z)]}{\Re[H_k(z)]} & -\frac{1}{c_k - v} < \Im(z) < -\varepsilon \\
\frac{\pi}{2} & -\frac{1}{d_k - v} < \Im(z) < -\frac{1}{c_k - v} \\
0 & -\infty < \Im(z) < -\frac{1}{d_k - v} \end{cases} $$
where

$$H_k(z) = \frac{\sqrt{z(c_k - v) + ip\sqrt{z(c_k + v) - ip}}}{c_k}$$  \hfill (A.8)

$$\arg N_k^*(\zeta) = \begin{cases} \pm \arctan \frac{\Im [H_k(z)]}{\Re [H_k(z)]} & 0 < \Im (z) < \frac{1}{c_k + v} \\ \frac{\pi}{2} & \frac{1}{c_k + v} < \Im (z) < \frac{1}{d_k + v} \\ 0 & \frac{1}{d_k + v} < \Im (z) < +\infty \end{cases}$$  \hfill (A.9)

Then,

$$\ln N_k^* = \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{s - ic/p} ds - \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Im (H_k(ip))}{\Re (H_k(ip))} ds$$  \hfill (A.10)

$$= \ln \left[ \frac{1/(c_k - v) + ic/p}{1/(d_k - v) + ic/p} \right] \exp \left \{ -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Im (H_k(ip))}{\Re (H_k(ip))} ds \right \}$$  \hfill (A.11)

Similarly,

$$\ln N_k^* = \frac{1}{2} \int_{\epsilon}^{1/(c_k + v) - ic/p} \frac{1}{s - ic/p} ds - \frac{1}{\pi} \int_{\epsilon}^{1/(c_k + v) - ic/p} \frac{\Im (H_k(ip))}{\Re (H_k(ip))} ds$$  \hfill (A.12)

Then,

$$N_k^* = \frac{\sqrt{1/(d_k + v) - ic/p \sqrt{1/(c_k + v) - ic/p \exp}}}{1/(c_k + v) - ic/p}$$  \hfill (A.13)

Finally,

$$N_k^* = \sqrt{\frac{1/(d_k + v) - ic/p}{1/(c_k + v) - ic/p}} \frac{1/(c_k + v) - ic/p}{1/(d_k + v) - ic/p} C_k^* C_k^*$$  \hfill (A.14)

**Data Availability**

All the numerical calculated data used to support the findings of this study can be obtained by calculating the equations in the study, and piezoelectric material parameters are taken from reference [20, 36]. The codes used in this
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