The $A_4$ flavor symmetry and neutrino phenomenology

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ABSTRACT

It has been shown that tribimaximal mixing can be obtained by some particular breaking pattern of the $A_4$ symmetry, wherein the extra $A_4$ triplet Higgs scalars pick up certain fixed vacuum expectation value (VEV) alignments. We have performed a detailed analysis of the different possible neutrino mass matrices within the framework of the $A_4$ model. We take into account all possible singlet and triplet Higgs scalars which leave the Lagrangian invariant under $A_4$. We break $A_4$ spontaneously, allowing the Higgs to take any VEV in general. We show that the neutrino mixing matrix deviates from tribimaximal, both due to the presence of the extra Higgs singlets, as well as from the deviation of the triplet Higgs VEV from its desired alignment, taken previously. We solve the eigenvalue problem for a variety of these illustrative cases and identify the ones where one obtains exact tribimaximal mixing. All such cases require fine-tuning. We show which neutrino mass matrices would be strongly disfavored by the current neutrino data. Finally, we study in detail the phenomenology of the remaining viable mass matrices and establish the deviation of the neutrino mixing from tribimaximal, both analytically as well as numerically.
1 Introduction

Neutrinos have provided us with a window to physics beyond the Standard Model. A plethora of striking experimental results have propelled us to a juncture where we already know a great deal about the basic structure of the neutrino mixing matrix. Results from KamLAND [1] and solar neutrino experiments [2] can be best explained if $\sin^2 \theta_{12} = 0.32$, while atmospheric neutrino experiments [3] and results from K2K [4] and MINOS [5] pick $\sin^2 \theta_{23} = 0.5$ as the best-fit solution. So far there has been no evidence for $\theta_{13}$ driven oscillations and hence $\theta_{13}$ is currently consistent with zero, with a 3$\sigma$ upper bound of $\sin^2 \theta_{13} < 0.05$ [6, 7]. These results prod us to believe that the mixing matrix in the lepton sector could be of the tribimaximal (TBM) mixing form, first proposed by Harrison, Perkins, and Scott [8, 9],

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (1)$$

It has been well known that for TBM mixing to exist, the neutrino mass matrix should be of the form

$$\mathcal{M}_\nu = \begin{pmatrix} A & B \\ B & \frac{1}{2}(A + B + D) \\ B & \frac{1}{2}(A + B - D) \end{pmatrix}, \quad (2)$$

where $A = \frac{1}{3}(2m_1 + m_2)$, $B = \frac{1}{3}(m_2 - m_1)$ and $D = m_3$, where $m_1$, $m_2$ and $m_3$ are the neutrino masses. The current neutrino data already give very good measurement of mass squared differences, while the best limit on the absolute neutrino mass scale comes from cosmological data. We show in Table 1 the allowed neutrino oscillation parameters within their 3$\sigma$ ranges.

| $\Delta m_{21}^2$ | $\Delta m_{31}^2$ | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{23}$ | $\sin^2 \theta_{13}$ |
|------------------|------------------|------------------|------------------|------------------|
| $(7.1 - 8.3) \times 10^{-5}$ eV$^2$ | $(2.0 - 2.8) \times 10^{-3}$ eV$^2$ | $0.26 - 0.40$ | $0.34 - 0.67$ | $< 0.05$ |

Table 1: The 3$\sigma$ allowed intervals (1 dof) for the three–flavor neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments.

Maximal $\theta_{23}$ and zero $\theta_{13}$ can be easily obtained if $\mathcal{M}_\nu$ possesses $\mu - \tau$ exchange symmetry [10] or the $L_\mu - L_\tau$ symmetry [11]. However, the solar mixing angle $\sin^2 \theta_{12}$ is not so easily predicted to be exactly 1/3, as required for exact TBM mixing. While the current data points towards Eq. (1), better precision on the mixing angles are needed to really test the TBM ansatz and deviation from TBM [12]. The mixing angles $\theta_{13}$ will be probed in the next generation long baseline and reactor...
experiments [13], while deviation of $\theta_{23}$ from maximality could be done in atmospheric neutrino experiments [14]. However, it is extremely crucial that one makes a very accurate measurement of $\theta_{12}$ in the future [15] in order to confirm TBM mixing.

The challenge for model builders lies in explaining all features of the lepton mixing matrix together with the mass pattern of the neutrinos given in Table 1. If indeed the neutrinos have TBM mixing, then one should be able to naturally generate the mass matrix given in Eq. (2). Various symmetry groups explaining the flavor structure of the leptons have been invoked in the literature in order to accommodate the neutrino masses and mixing along with the charged leptons. In particular, the study of the non-Abelian discrete symmetry group $A_4$ has received considerable interest in the recent past [16, 17, 18, 19, 20, 21, 22, 23]. This group has been shown to successfully reproduce the TBM form of the neutrino mixing matrix, in the basis where the charged lepton mass matrix is diagonal [19]. However, the authors of [19] work in a very special framework where only one of the three possible singlet Higgs under $A_4$ is considered and for the $A_4$ triplet Higgs which contributes to the neutrino mass matrix, a particular vacuum alignment is taken\(^1\). In this framework, the mixing matrix emerges as apparently independent of the Yukawa couplings, the VEVs and the scale of $A_4$ breaking. Only the mass eigenstates depend on them.

In this paper, we consider the most general scenario with all possible Higgs scalars that can be accommodated within this $A_4$ model. We give both approximate analytical solutions of the expected phenomenology as well as exact numerical results. We expound the conditions on VEVs and Yukawas needed for obtaining exact TBM mixing and show how the neutrino masses in those cases severely constrain them. We begin by showing that in the model considered in [19], one gets TBM mixing simply through the alignment of the $A_4$ triplet Higgs vev. A concerted effort has been made in the literature to explain naturally this particular vacuum alignment needed for TBM mixing. The mass squared differences can be obtained if one has the vacuum expectation value (VEV) of an additional singlet Higgs. We show that to get the correct $\Delta m^2_{21}$ and $\Delta m^2_{31}$, the product of the VEV and Yukawa of this singlet is determined almost completely by the VEV and Yukawa of the triplet. This emerges as a further undesirable feature of the model, and one would need further explanation for this additional “relative alignment” between the product of VEVs and Yukawas of the triplet and the singlet.

We allow for presence of an additional singlet Higgs, construct the neutrino mass matrices and study their phenomenology. We take one, two and three singlet Higgs at a time and check which combinations produce viable neutrino mass matrices. In particular, we check which ones would produce TBM mixing and under what conditions. We obtain a few combinations in addition to the one considered in [19] which produce TBM mixing. We perturb these mass matrices by slightly shifting the VEVs and/or the Yukawas and study the deviation from TBM and the corresponding effect on the neutrino masses. We note that if just one singlet Higgs is allowed, the model of [19] is the only viable model. We further show that in the simplest version of this model, one necessarily gets the normal mass hierarchy\(^2\). Inverted hierarchy can be possible if we have at least two or all three Higgs scalars with nonzero VEVs. We give predictions for the sum of the absolute neutrino masses ($m_\nu$), the effective mass that will be observed in neutrinoless double beta decay

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\(^1\)We give a brief overview of the $A_4$ model in section 2.

\(^2\)We call this the neutrino mass hierarchy, though what we mean is the ordering of the neutrino mass states.
Table 2: List of fermion and scalar fields used in this model. Two lower rows list additional singlets considered in the present work. In section 4, we also allow for a different VEV alignment for $\phi_S$.

experiments ($\langle m_{ee} \rangle$) and $m_3^2$ for tritium beta decay experiments. Finally, we perturb the VEV alignment for the triplet Higgs and study the deviation from TBM mixing.

In section 2 we give a brief overview of the $A_4$ model considered. In section 3 we begin with detailed phenomenological analysis of the case where there is just one singlet Higgs under $A_4$. We next increase the number of contributing singlet Higgs, give analytical and numerical results. In section 4 we study the impact of the misalignment of the VEVs of the triplet Higgs. We end in section 5 with our conclusions.

2 Overview of the Model

Alternating group $A_n$ is a group of even permutations of $n$ objects. It is a subgroup of the permutation group $S_n$ and has $\frac{n!}{2}$ elements. The non-Abelian group $A_4$ is the first alternating group which is not a direct product of cyclic groups, and is isomorphic to the tetrahedral group $T_d$. The group $A_4$ has 12 elements, which can be written in terms of the generators of the group $S$ and $T$. The generators of $A_4$ satisfy the relation

$$S^2 = (ST)^3 = (T)^3 = 1$$

There are three one-dimensional irreducible representations of the group $A_4$ denoted as

$$1 \quad S = 1 \quad T = 1$$

$$1' \quad S = 1 \quad T = \omega^2$$

$$1'' \quad S = 1 \quad T = \omega$$
It is easy to check that there is no two-dimensional irreducible representation of this group. The three-dimensional unitary representations of $T$ and $S$ are

$$
T = \begin{pmatrix}
1 & 0 & 0 \\
0 & \omega^2 & 0 \\
0 & 0 & \omega
\end{pmatrix}, \quad
S = \frac{1}{3} \begin{pmatrix}
-1 & 2 & 2 \\
2 & -1 & 2 \\
2 & 2 & -1
\end{pmatrix}.
$$

where $T$ has been chosen to be diagonal. We refer the readers to [19] for a quick review of the $A_4$ group. Here we give multiplication rules for the singlet and triplet representations, for the sake of completeness. These multiplication rules correspond to the specific basis of two generators $S,T$ of $A_4$. We have

$$1 \times 1 = 1, \quad 1' \times 1'' = 1 \quad 3 \times 3 = 3 + 3_A + 1 + 1' + 1''.$$  

For two triplets

$$a = (a_1, a_2, a_3), \quad b = (b_1, b_2, b_3)$$

one can write

$$1 \equiv (ab) = (a_1 b_1 + a_2 b_3 + a_3 b_2)$$

$$1' \equiv (ab)' = (a_3 b_1 + a_1 b_2 + a_2 b_3)$$

$$1'' \equiv (ab)'' = (a_2 b_2 + a_3 b_1 + a_1 b_3).$$

Note that while $1$ remains invariant under the exchange of the second and third elements of $a$ and $b$, $1'$ is symmetric under the exchange of the first and second elements while $1''$ is symmetric under the exchange of the first and third elements.

$$3 \equiv (ab)_S = \frac{1}{3} \left(2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, 2a_2 b_2 - a_1 b_3 - a_3 b_1\right)$$

$$3_A \equiv (ab)_A = \frac{1}{3} \left(a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_1 b_3 - a_3 b_1\right).$$

We will be concerned with only 3 and we can see that the first element here has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry, while the third element has 1-3 exchange symmetry.\footnote{There are no $3_A$ terms in the Lagrangian since it is antisymmetric and hence cannot be used for the neutrino mass matrix.}

The full particle content of the model we consider is shown in Table 2. There are five $SU(2) \otimes U_Y(1)$ Higgs singlets, three ($\xi$, $\xi'$ and $\xi''$) of which are singlets under $A_4$ and two ($\phi_T$ and $\phi_S$) of which transform as triplets. The standard model lepton doublets are assigned to the triplet representation of $A_4$, while the right handed charged leptons $e^c$, $\mu^c$ and $\tau^c$ are assumed to belong to the 1, 1'' and 1' representation respectively. The standard Higgs doublets $h_u$ and $h_d$ remain invariant under $A_4$. The form of the $A_4$ invariant Yukawa part of the Lagrangian is\footnote{We assume that $\phi_S$ does not couple to charged leptons and $\phi_T$ does not contribute to the Majorana mass matrix. These two additional features can be obtained from extra-dimensional realization of the model, or from extra abelian symmetries [19].}

$$
\mathcal{L}_Y = y_e e^c (\phi_T l) + y_{\mu} \mu^c (\phi_T l)' + y_{\tau} \tau^c (\phi_T l)'' + x_a \xi (ll) + x_a' \xi' (ll) + x_a'' \xi'' (ll) + x_b (\phi_S ll) + h.c. + ...
$$

(15)
where, following [19] we have used the compact notation, $y_e e^c(\phi_T l) \equiv y_e e^c(\phi_T l)h_d/\Lambda$, $x_a \xi (l l) \equiv x_a \xi (l h_u h_u)/\Lambda^2$ and so on, and $\Lambda$ is the cut-off scale of the theory. After the spontaneous breaking of $A_4$ followed by $SU(2)_L \otimes U(1)_Y$, we get the mass terms for the charged leptons and neutrinos. Assuming the vacuum alignment

\[
\langle \phi_T \rangle = (v_T, 0, 0) ,
\]

the charged lepton mass matrix is given as

\[
\mathcal{M}_l = \frac{v_d v_T}{\Lambda} \begin{pmatrix}
  y_e & 0 & 0 \\
  0 & y_\mu & 0 \\
  0 & 0 & y_\tau \\
\end{pmatrix} ,
\]

(17)

Note that we could also obtain a diagonal charged lepton mass matrix even if we assume that $e^c$, $\mu^c$ and $\tau^c$ transform as $1''$, 1' and 1, and $\langle \phi_T \rangle = (0, v_T, 0)$ with appropriate change in the Yukawa Lagrangian. Similarly, $e^c$, $\mu^c$ and $\tau^c$ transform as 1', 1 and 1'', and $\langle \phi_T \rangle = (0, 0, v_T)$ could give us the same $\mathcal{M}_l$. In what follows, we will assume that the $\mathcal{M}_l$ is of the form given in Eq. (17).

In the most general case, where all three singlet Higgs as well as $\phi_S$ are present and we do not assume any particular vacuum alignment, the neutrino mass matrix looks like

\[
\mathcal{M}_\nu = m_0 \begin{pmatrix}
  a + 2b_1/3 & c - b_3/3 & d - b_2/3 \\
  c - b_3/3 & d + 2b_2/3 & a - b_1/3 \\
  d - b_2/3 & a - b_1/3 & c + 2b_3/3 \\
\end{pmatrix} ,
\]

(18)

where $m_0 = \frac{v_s^2}{\Lambda} b_i = 2x_b \frac{v_{S_i}}{\Lambda}$, $a = 2x_a \frac{u}{\Lambda}$, $c = 2x''_a \frac{u'}{\Lambda}$ and $d = 2x'_a \frac{u}{\Lambda}$ and we have written the VEVs as

\[
\langle \phi_S \rangle = (v_s, v_s, v_s) , \quad \langle \xi \rangle = u , \quad \langle \xi' \rangle = u' , \quad \langle \xi'' \rangle = u'' , \quad \langle h_{u,d} \rangle = v_{u,d} .
\]

(19)

3 Number of Singlet Higgs and their VEVs

In this section we work under the assumption that the triplet Higgs $\phi_S$ has VEVs along the direction

\[
\langle \phi_S \rangle = (v_s, v_s, v_s) .
\]

(20)

This produces the neutrino mass matrix

\[
\mathcal{M}_\nu = m_0 \begin{pmatrix}
  a + 2b/3 & c - b/3 & d - b/3 \\
  c - b/3 & d + 2b/3 & a - b/3 \\
  d - b/3 & a - b/3 & c + 2b/3 \\
\end{pmatrix} ,
\]

(21)

where $b = 2x_b \frac{u}{\Lambda}$. In the following we discuss the phenomenology of the different forms of $\mathcal{M}_\nu$ possible as we change the number of singlet Higgs or put their VEVs to zero. We assume that $\mathcal{M}_\nu$ is real.
3.1 No $A_4$ Singlet Higgs

If there were no singlet Higgs, or if the VEV of all three of them were zero, one would get the neutrino mass matrix

$$M_\nu = m_0 \begin{pmatrix} 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & -b/3 \\ -b/3 & -b/3 & 2b/3 \end{pmatrix}. \quad (22)$$

On diagonalizing this one obtains the eigenvalues

$$m_1 = m_0 b, \quad m_2 = 0, \quad m_3 = m_0 b \quad (23)$$

and the mixing matrix

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (24)$$

Therefore, we can see that the tribimaximal pattern of the mixing matrix is coming directly from the term containing the triplet Higgs $\phi_S$ and does not depend on the terms containing the singlet Higgs scalars$^5$. However, in the absence of the singlet Higgs contributions to $M_\nu$, the ordering of the neutrino masses turn out to be very wrong. In this case, $\Delta m_{21}^2 = -b^2 m_0^2$ and $\Delta m_{31}^2 = 0$, in stark disagreement with the oscillation data.

3.2 Only one $A_4$ Singlet Higgs

If we take only one $A_4$ singlet Higgs at a time then there are three possibilities. The resulting mass matrices are shown in column 2 of Table 3. One could get exactly the same situation with three singlet Higgs and demanding that the VEV of two of them are zero while that of the third is nonzero. The $M_\nu$ given in Table 3 can be exactly diagonalized and the eigenvalues and eigenvectors are shown in column 3 and 4 of the Table, respectively. One can see that only the case where $\xi$ is present gives rise to a viable mixing matrix, which is exactly tribimaximal [19]. One can check that only the first case has the form for $M_\nu$ given in Eq. (2). Note that each of the $M_\nu$ given in Table 3 possesses an $S_2$ symmetry, which reflects the symmetry of Eqs. (10-12)$^6$. While the case with $\xi$ exhibits the $\mu - \tau$ exchange symmetry, the one with $\xi''$ remains invariant under $e - \mu$ permutation and the one with $\xi'$ under $e - \tau$ permutation. This would necessarily demand that while for the first case $\theta_{23}$ would be maximal and $\theta_{13} = 0$, for the second and third cases $\theta_{23}$ would be either 90$^\circ$ or 0 respectively, and $\theta_{13}$ maximal.

Since only the case with $\xi$ reproduces the correct form for the mixing matrix, we do not discuss the remaining two cases any further. This was the model presented by Altarelli and Feruglio in their seminal paper [19]. The mass squared differences in this case are

$$\Delta m_{21}^2 = (-b^2 - 2ab)m_0^2, \quad \Delta m_{31}^2 = -4abm_0^2, \quad (25)$$

$^5$Note that the mixing pattern does not depend explicitly even on the VEV of $\phi_S$.

$^6$Since we have assumed the vacuum alignment $\langle \phi_S \rangle = (v_S, v_S, v_S)$, the $b$ terms of $M_\nu$ are such that it is symmetric under all the three exchange symmetries.
Figure 1: Contour plot of $\Delta m^2_{21}$, $\Delta m^2_{31}$ and sum of absolute neutrino masses $m_t$ in the $a - b$ plane, for three different values of $m_0$ ($m_0=0.016$, $m_0=0.024$ and $m_0=0.032$) for the case with $\xi$ and $\phi_S$. The first row shows contour plots for different values of $\Delta m^2_{21}$ (in $10^{-5}$ eV$^2$), with red dashed lines for 6.5, blue solid lines for 7.1, green dashed lines for 7.7, green solid lines for 8.3, and orange dotted lines for 8.9. The second row shows contour plots for different values of $\Delta m^2_{31}$ (in $10^{-3}$ eV$^2$), with red dashed lines for 1.6, blue solid lines for 2.0, green dashed lines for 2.4, green solid lines for 2.8, and orange dotted lines for 3.2. The third row shows contour plots for $m_t$ (in eV), with red dashed lines for 0.05, blue solid lines for 0.07, green dashed lines for 0.09, green solid lines for 1.1, orange dotted lines for 1.3, turquoise solid lines for 1.4, magenta solid lines for 1.6, and brown solid lines for 1.8.
| Higgs | Neutrino mass matrix | Eigenvalues | Mixing Matrix |
|-------|----------------------|-------------|---------------|
| $\xi$ | $m_0 \begin{pmatrix} a + \frac{2b}{3} & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & 2\frac{b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & 2\frac{b}{3} \end{pmatrix}$ | $(m_0(a + b), m_0a, m_0(b - a))$ | $\begin{pmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ |
| $\xi''$ | $m_0 \begin{pmatrix} \frac{2b}{3} & c - \frac{b}{3} & -\frac{b}{3} \\ c - \frac{b}{3} & 2\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & -\frac{b}{3} & c + 2\frac{b}{3} \end{pmatrix}$ | $(m_0(c + b), m_0c, m_0(b - c))$ | $\begin{pmatrix} \frac{-\sqrt{6}}{3} & 0 & 1 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ |
| $\xi'$ | $m_0 \begin{pmatrix} \frac{2b}{3} & d + \frac{2b}{3} & -\frac{b}{3} \\ d + \frac{2b}{3} & \frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & -\frac{b}{3} & 2\frac{b}{3} \end{pmatrix}$ | $(m_0(d + b), m_0d, m_0(b - d))$ | $\begin{pmatrix} \frac{-\sqrt{6}}{3} & 0 & 1 \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ |

Table 3: The mass matrix taking one singlet at a time, its mass eigenvalues, and its mixing matrix.

Figure 2: Scatter plot showing regions in $a - b$ parameter space for the model considered in [19], which are compatible with the current $3\sigma$ allowed range of values of the mass squared differences. The parameter $m_0$ is allowed to vary freely.
where \( m_0 = v_u^2 / \Lambda \). Since it is now known at more than 6\( \sigma \) C.L. that \( \Delta m_{21}^2 > 0 \) [7], we have the condition that \(-2ab > b^2\). Since \( b^2 \) is a positive definite quantity the above relation implies that \(-2ab > 0\), which can happen if and only if \( sgn(a) \neq sgn(b) \). Inserting this condition into the expression for \( \Delta m_{31}^2 \) gives us \( \Delta m_{31}^2 > 0 \) necessarily in this model. Therefore, inverted neutrino mass hierarchy is impossible to get in the framework of the simplest version of the model proposed by Altarelli and Feruglio [19].

The sum of the absolute neutrino masses, effective mass in neutrinoless double beta decay and prediction for tritium beta decay are given respectively as

\[
m_t = |m_1| + |m_2| + |m_3|, \quad \langle m_{ee} \rangle = m_0(a + 2b/3), \quad m_{\beta}^2 = m_0^2 \left( a^2 + \frac{4ab}{3} + \frac{2b^2}{3} \right). \quad (26)
\]

In Fig. 1 we show the contours for the observables \( \Delta m_{21}^2 \), \( \Delta m_{31}^2 \) and \( m_t \) in the \( a - b \) plane, for three different fixed values of \( m_0 \). The details of the figure and description of the different lines can be found in the caption of the figure.

In Fig. 2 we present a scatter plot showing the points in the \( a - b \) parameter space which are compatible with the current 3\( \sigma \) allowed range of the mass squared differences. We have allowed \( m_0 \) to vary freely and taken a projection of all allowed points in the \( a - b \) plane. Note that while \( a \) is related to the VEV of the singlet \( \xi \), \( b \) is given in terms of the VEVs of the triplet \( \phi_S \). We reiterate the point mentioned before that the TBM form for the mixing matrix comes solely from the vacuum alignment of \( \phi_S \) and \( \xi \) is not needed for that. The singlet \( \xi \) is necessary only for producing the correct values of \( \Delta m_{21}^2 \) and \( \Delta m_{31}^2 \). However, we can note from Fig. 2 that for a given value of \( a \) needed to obtain the right mass squared differences, the value of \( b \) is almost fixed. In fact, we can calculate the relation between \( a \) and \( b \) by looking at the ratio \( \frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{b^2 - 2ab}{4ab} \approx 0.03 \), where 0.03 on the right-hand-side (RHS) is the current experimental value. This gives us the relation \( b \approx -1.88a \). Therefore, not only does one need the alignment \( \langle \phi_S \rangle = (v_S, v_S, v_S) \) to get TBM, one also needs a particular relation between the product of Yukawa couplings and VEVs of \( \phi_S \) and \( \xi \) in order to reproduce the correct phenomenology. This appears to be rather contrived. Even if one includes the 3\( \sigma \) uncertainties on \( \Delta m_{21}^2 \) and \( \Delta m_{31}^2 \), \( |b| \) is fine tuned to \( |a| \) within a factor of about \( 10^{-2} \).

### 3.3 Two \( A_4 \) Singlet Higgs

If we take two singlet Higgs belonging to two different singlet representations and allow for nonzero VEVs for them, then the \( \mathcal{M}_\nu \) obtained for the three possible cases are shown in the first three rows of Table 4. One can again see that of the three possible combinations, only the \( \xi', \xi'' \) combination gives a viable TBM matrix. The other two mass matrices exhibit \( e - \tau \) (\( \xi, \xi'' \)) and \( e - \mu \) (\( \xi, \xi' \)) symmetry respectively and are ruled out. Note that we have chosen \( a = c \) for the \( \xi, \xi'' \) combination, \( a = d \) for the \( \xi, \xi' \) combination and \( c = d \) for the \( \xi', \xi'' \) combination for the results given in Table 4. This is a reasonable assumption to make since the phenomenology of the three cases does not change drastically unless the VEVs of the singlet Higgs vary by a huge amount. In particular, by changing the relative magnitude of the VEVs, we do not expect the structure of the mixing matrix for the first two rows of Table 4 to change so much so that they could be allowed
Table 4: Here we take two singlets at a time, and analytically display eigenvalues and eigenvectors of the neutrino mass matrix.

| Higgs   | Neutrino mass matrix                                                                 | Eigenvalues                                                                 | Eigenvectors                                                                 |
|---------|--------------------------------------------------------------------------------------|----------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\xi,\xi'$ | $m_0 \begin{pmatrix} a + \frac{2b}{3} & c - \frac{b}{3} & -\frac{b}{3} \\ \frac{b}{3} & d - \frac{b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & c + \frac{2b}{3} \end{pmatrix}$ | $a = c$;                                                                 | $\begin{pmatrix} \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \end{pmatrix}$ |
| $\xi,\xi'$ | $m_0 \begin{pmatrix} a + \frac{2b}{3} & -\frac{b}{3} & d - \frac{b}{3} \\ \frac{b}{3} & d + \frac{2b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & c + \frac{2b}{3} \end{pmatrix}$ | $a = d$;                                                                 | $\begin{pmatrix} \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$ |
| $\xi',\xi''$ | $m_0 \begin{pmatrix} \frac{2b}{3} & c - \frac{b}{3} & d - \frac{b}{3} \\ \frac{b}{3} & d + \frac{2b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & c + \frac{2b}{3} \end{pmatrix}$ | $c = d$;                                                                 | $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ |
| $\xi',\xi''$ | $m_0 \begin{pmatrix} \frac{2b}{3} & c - \frac{b}{3} & d - \frac{b}{3} \\ \frac{b}{3} & d + \frac{2b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & c + \frac{2b}{3} \end{pmatrix}$ | $c = d + e$;                                                             | $\begin{pmatrix} \frac{\sqrt{3}}{\sqrt{6}} \\ \frac{\sqrt{3}}{\sqrt{6}} \\ \frac{\sqrt{3}}{\sqrt{6}} \end{pmatrix}$ |

Table 4: Here we take two singlets at a time, and analytically display eigenvalues and eigenvectors of the neutrino mass matrix.
Figure 3: The left panels show $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ vs $\epsilon$ and the right panels show $\Delta m^2_{21}$, $\Delta m^2_{31}$ and $m_t$ vs $\epsilon$ respectively. Here $\xi' \text{ and } \xi''$ acquire VEVs. The other parameters $c$, $b$ and $m_0$ are allowed to vary freely.
Figure 4: Scatter plot showing the $3\sigma$ allowed regions for the $b-c-d$ parameters for the case where $\xi'$ and $\xi''$ acquire VEVs. The top, middle and lower panels show the allowed points projected on the $c-b$, $c-d$ and $d-b$ plane, respectively. The parameter $m_0$ was allowed to take any value. Here we have assumed normal hierarchy.
Figure 5: Same as in Fig. 4 but for inverted hierarchy.
by the current data. In the limit that \( c = d \), it is not hard to appreciate that the resultant matrix with \( \xi' \) and \( \xi'' \) would exhibit \( \mu - \tau \) symmetry, though the \( \xi' \) and \( \xi'' \) terms alone have \( e - \tau \) and \( e - \mu \) symmetry respectively.

Since the \( \xi', \xi'' \) combination is the only one which gives exact TBM mixing in the approximation that \( c = d \), we perform a detailed analysis only for this case. Putting \( c = d \) is again contrived and would also lead to a certain fixed relation between them and \( b \), as in the only \( \xi \) case. This would mean additional fine tuning of the parameters, unless explained by symmetry arguments. Hence, we allow the two VEVs to differ from each other so that \( c = d + \epsilon \). If \( \epsilon \) is small we can solve the eigenvalue problem keeping only the first order terms in \( \epsilon \). The results for this case are shown in the final row of the Table 4. The deviation of the mixing angles from their TBM values can be seen to be

\[
D_{12} \simeq 0, \quad D_{23} \simeq -\frac{\epsilon}{4d}, \quad U_{e3} \simeq -\frac{\epsilon}{2\sqrt{2d}},
\]

where \( D_{12} = \sin^2 \theta_{12} - 1/3 \) and \( D_{23} = \sin^2 \theta_{23} - 1/2 \). We show in the left hand panels of Fig. 3 the mixing angles \( \sin^2 \theta_{12} \) (upper panel), \( \sin^2 \theta_{13} \) (middle panel) and \( \sin^2 \theta_{23} \) (lower panel) as a function of \( \epsilon \). We vary \( \epsilon \) from large negative to large positive values and solve the exact eigenvalue problem numerically allowing the other parameters, \( m_0, b \) and \( d \), to vary freely. For \( \epsilon = 0 \) of course we get TBM mixing as expected. For very small values of \( \epsilon \), the deviation of the mixing angles from their TBM values is reproduced well by the approximate expressions given in Eq. (27). For large \( \epsilon \) of course the approximate expressions fail and we have significant deviation from TBM. The values of \( \sin^2 \theta_{12} \) and \( \sin^2 \theta_{13} \) increase very fast with \( \epsilon \), while \( \sin^2 \theta_{23} \) decreases with it. We have only showed the scatter plots up to the current 3σ allowed ranges for the mixing angles. The mass dependent observables can be calculated up to first order in \( \epsilon \) as

\[
\Delta m^2_{21} \simeq m_0^2 \left( b + d + \frac{\epsilon}{2} (3d - b + \frac{3\epsilon}{2}) \right), \quad \Delta m^2_{31} \simeq (4bd + 2b\epsilon)m_0^2,
\]

\[
\langle m_{ee} \rangle = m_0\frac{2b}{3}, \quad m_t = \sum_i |m_i|, \quad m_\beta \simeq m_0^2\left(\frac{2b^2}{3} + 2d^2 - \frac{4bd}{3} + 2d\epsilon - \frac{2b\epsilon}{3}\right).
\]

Of course, epsilon can be larger and we show numerical results for those cases.

### 3.3.1 Normal Hierarchy

The right panels of Fig. 3 show \( \Delta m^2_{21} \) (upper panel), \( \Delta m^2_{31} \) (middle panel) and \( m_t \) (lower panel) as a function of \( \epsilon \) assuming \( m_1 < m_2 \ll m_3 \). We show \( \Delta m^2_{21} \) and \( \Delta m^2_{31} \) only within their 3σ allowed range. We notice that while \( \Delta m^2_{21} \) and \( \Delta m^2_{31} \) are hardly constrained by \( \epsilon \), there appears to some mild dependence of \( m_t \) on it.

Fig. 4 gives the scatter plots showing the allowed parameter regions for this case. The upper, middle and lower panels show the allowed points projected on the \( b - c, d - c \) and \( b - d \) plane, respectively.
Figure 6: Scatter plot showing the 3σ allowed regions in the model parameter space for the case where $\xi$, $\xi'$ and $\xi''$ all acquire VEVs. The upper panels show allowed regions projected onto the $a-c$, $a-d$, $a-b$ planes. The lower panels show allowed regions projected onto the $c-d$, $c-b$, $d-b$ planes. Normal hierarchy is assumed.

3.3.2 Inverted Hierarchy

For this case it is possible to obtain even inverted hierarchy. We show in Fig. 5 the scatter plots showing the allowed parameter regions for inverted hierarchy. We have allowed $m_0$ to vary freely and show the allowed points projected on the $c-b$, $c-d$ and $d-b$ planes. One can check that only for the points appearing in this plot, $m_3 < m_1 < m_2$. We reiterate that these points also satisfy the 3σ allowed oscillation parameter ranges given in Table 1.

3.4 Three $A_4$ Singlet Higgs

Finally, we let all three singlet Higgs VEVs contribute to $M_\nu$. In this case one has to diagonalize the most general mass matrix given in Eq. (21). This matrix has four independent parameters.
| Higgs    | Neutrino mass matrix                                                                 | Eigenvalues   | Eigenvectors                                                                 |
|----------|--------------------------------------------------------------------------------------|---------------|------------------------------------------------------------------------------|
| ξ, ξ', ξ'' | \[ m_0 \begin{pmatrix} a + \frac{2b}{3} & c - \frac{b}{3} & d - \frac{b}{3} \\ c - \frac{b}{3} & d + \frac{2b}{3} & a - \frac{b}{3} \\ d - \frac{b}{3} & a - \frac{b}{3} & c + \frac{2b}{3} \end{pmatrix} \] | \[ a = c = d; \] | \[ \begin{pmatrix} -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sqrt{3}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}} & \frac{1}{\sqrt{3}} & 0 \end{pmatrix} \] |
|          | \[ (m_0 b, 3m_0 a, m_0 b) \]                                                      | \[ a \neq c = d; \] | \[ \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \] |
|          | \[ (m_0 (a + b - c), m_0 (a + 2c), m_0 (b + c - a)) \]                          | \[ a = c \neq d; \] | \[ \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \] |
|          | \[ (m_0 (b + d - a), m_0 (2a + d), m_0 (a + b - d)) \]                          | \[ a = d \neq c; \] | \[ \begin{pmatrix} -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sqrt{3}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \sqrt{\frac{3}{2}} & \frac{1}{\sqrt{3}} & 0 \end{pmatrix} \] |
|          | \[ (m_0 (b + c - a), m_0 (2a + c), m_0 (a + b - c)) \]                          | \[ a \neq c = d + \epsilon; \] | \[ \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{a_4}{\sqrt{2} - a_5} \\ -\frac{1}{\sqrt{6}} + a_2 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2} - a_6} \\ -\frac{1}{\sqrt{6}} + a_3 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2} - a_6} \end{pmatrix} \] |
|          | \[ (m_0 (a + b - d - \frac{\epsilon}{2}), m_0 (a + 2d + \epsilon), m_0 (b + d - a + \frac{\epsilon}{2})) \] | \[ \text{The correction factors in the last row are: } a_2 = \frac{\sqrt{3} \epsilon}{4\sqrt{2}(a-d)}, \ a_3 = -\frac{\sqrt{3} \epsilon}{4\sqrt{2}(a-d)} \text{ and } a_4 = \frac{\epsilon}{2\sqrt{2}(a-d)}, \ a_5 = \frac{\epsilon}{4\sqrt{2}(a-d)}, \ a_6 = \frac{\epsilon}{4\sqrt{2}(a-d)}. \] |
If we assume that the singlet VEVs are such that $a = c = d$, then the eigenvalues and mixing matrix are given in the first row of Table 5. This gives us a mass matrix whose structure is identical to that given in Eq. (22). Hence, it is not surprising that the corresponding mixing matrix that we obtain has exact TBM mixing and two of the mass eigenstates are degenerate. Therefore, to get the correct mass splitting it is essential that (i) we should have contribution from the singlet VEVs and (ii) the contribution from the the three singlets should be different. If we assume that $a = c \neq d$, then one can easily check that $M_{\nu}$ has $e - \tau$ exchange symmetry, and hence the resulting mass matrix is disallowed. This is because for $a = c$, as discussed before we get $e - \tau$ exchange symmetry and the $\xi'$ term has an in-built $e - \tau$ symmetry. Similarly for $a = d \neq c$, one gets $e - \mu$ symmetry in $M_{\nu}$ and is hence disfavored. Only when we impose the condition $c = d$, we have $\mu - \tau$ symmetry in $M_{\nu}$, since the $\xi$ term and the sum of the $\xi'$ and $\xi''$ terms are now separately $\mu - \tau$ symmetric. Therefore, the case $a \neq c = d$ gives us the TBM matrix and the mass eigenvalues are shown in Table 5.

Since $a \neq c = d$ is the only allowed case for the three singlet Higgs case, we find the eigenvalues
and the mixing matrix for the case where $c$ and $d$ are not equal, but differ by $\epsilon$. We take $c = d + \epsilon$ and for small values of $\epsilon$ give the results in the last row of Table 5, keeping just the first order terms in $\epsilon$. The deviation from TBM is given as follows

$$D_{12} \simeq 0, \quad D_{23} \simeq \frac{\epsilon}{4(a - d)}, \quad U_{e3} \simeq \frac{\epsilon}{2\sqrt{2}(a - d)}.$$  \hspace{1cm} (30)

The mass squared differences are

$$\Delta m_{21}^2 \simeq m_0^2 \left(2a + b + d + \frac{\epsilon}{2}\right)(3d - b + \frac{3\epsilon}{2}), \quad \Delta m_{31}^2 \simeq 2m_0^2 b(2d - 2a + \epsilon).$$  \hspace{1cm} (31)

From the expression of the mass eigenvalues given in the Table, one can calculate the observables $m_t$, $m_\beta^2$ and $\langle m_{ee} \rangle$

$$\langle m_{ee} \rangle = m_0 \left(a + \frac{2b}{3}\right), \quad m_t = \sum_i |m_i|, \quad m_\beta^2 \simeq m_0^2 \left(a^2 + \frac{4ab}{3} + \frac{2b^2}{3} + 2d^2 - \frac{4bd}{3} + 2d\epsilon - \frac{2b\epsilon}{3}\right).$$  \hspace{1cm} (32)

### 3.4.1 Normal Hierarchy

Let us begin by restricting the neutrino masses to obey the condition $m_1 < m_2 \ll m_3$ and allow $a$, $b$, $c$ and $d$ to take any random value and find the regions in the $a$, $b$, $c$ and $d$ space that give $\Delta m_{21}^2$, $\Delta m_{31}^2$ and the mixing angles within their current $3\sigma$ allowed ranges. This is done by numerically diagonalizing $M_\nu$. The results are shown as scatter plots in Fig. 6. To help see the allowed zones better, we have projected the allowed points on the $a - c$, $a - d$ and $a - b$ plane shown in the upper panels, and $c - d$, $c - b$ and $d - b$ plane in the lower panels. There are several things one can note about the VEVs and hence the structure of the resultant $M_\nu$

- $a = 0$ is allowed, since this gives a $M_\nu$ which has contributions from $\xi'$ and $\xi''$, discussed in section 3.3,
- $b = 0$ is never allowed since $b$ is needed for TBM mixing as pointed out before,
- $a = b$, $a = c$ and $a = d$ are never allowed,
- $c = d$ is allowed and we can see from the lower left-hand panel how much deviation of $c$ from $d$ can be tolerated,
- $c = 0$ and $d = 0$ can also be tolerated when $a \neq 0$.

All these features are consistent with the results of Table 5.

### 3.4.2 Inverted Hierarchy

In this case too its possible to get inverted hierarchy. The corresponding values of the parameters of $M_\nu$, which allow this are shown as scatter plots in Fig. 7. Here $m_0$ has been allowed to take any value, and we show the points projected on the $a - c$, $a - b$, $a - d$ plane in the upper panels and $c - b$, $c - d$, $b - d$ plane in the lower panels. Each of these points also satisfy the $3\sigma$ experimental bounds of Table 1. Note that for $a = 0$ we get the same regions in $b$, $c$ and $d$, as in Fig. 5.
4 Vacuum Alignment of the Triplet Higgs

In case we do not confine ourselves to $\langle \phi_S \rangle = (v_S, v_S, v_S)$, we would have the general $M_\nu$ given in Eq. (18). Since we have argued in the previous section that the only viable scenario where one allows for all three Higgs singlet is when $a \neq c \approx d$, we will assume that this condition for the singlet terms holds. We further realize that to reproduce a mixing matrix with $\theta_{13} \sim 0$ and $\theta_{23} \sim 45^\circ$, it might be desirable to keep $\mu - \tau$ symmetry in the mass matrix. Therefore, we show our results for the case $\langle \phi_S \rangle = (v_S, v_S, v_S)$. The mass matrix is then given as

$$
M_\nu = m_0 \begin{pmatrix}
a + 2b_1/3 & d - b/3 & d - b/3 \\
d - b/3 & d + 2b/3 & a - b_1/3 \\
d - b/3 & a - b_1/3 & d + 2b/3
\end{pmatrix}.
$$

Of course for $b_1 = b$ one would recover the case considered in the previous section and TBM mixing would result. The possibility of $b_1 \neq b$ gives rise to deviation from TBM mixing. In order to solve this matrix analytically we assume that $b_1 = b + \eta$ and keep only the first order terms in $\eta$. The mass eigenvalues obtained are

$$
m_1 = m_0 (a + b - d + \frac{\eta}{3}), \quad m_2 = m_0 (a + 2d), \quad m_3 = m_0 (-a + b + d + \frac{\eta}{3}),
$$

Figure 8: Variation of $\sin^2 \theta_{12}$ with $\eta$ for the case where we allow for a misalignment of the triplet Higgs such that $\langle \phi_S \rangle = (v_{S1}, v_S, v_S)$ and $a \neq c = d$. 

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and the mixing matrix is
\[
\begin{pmatrix}
\sqrt{\frac{2}{3}} \left(1 - \frac{2\eta}{3(3d-b)}\right) & \sqrt{\frac{1}{3}} \left(1 + \frac{2\eta}{3(3d-b)}\right) & 0 \\
-\sqrt{\frac{1}{6}} \left(1 + \frac{2\eta}{3(3d-b)}\right) & \sqrt{\frac{1}{3}} \left(1 - \frac{2\eta}{3(3d-b)}\right) & -\sqrt{\frac{1}{2}} \\
-\sqrt{\frac{1}{6}} \left(1 + \frac{2\eta}{3(3d-b)}\right) & \sqrt{\frac{1}{3}} \left(1 - \frac{2\eta}{3(3d-b)}\right) & \sqrt{\frac{1}{2}} \\
\end{pmatrix}.
\]
(35)

Therefore, the only deviation from TBM comes in $\theta_{12}$ and we have
\[
D_{12} \simeq \frac{4\eta}{9(3d-b)}.
\]
(36)

We show in Fig. 8 variation of $\sin^2 \theta_{12}$ with $\eta$. As expected, $\sin^2 \theta_{12}$ is seen to deviate further and further from its TBM value of 1/3 as we increase the difference between $v_S$ and $v_{S1}$. The other two mixing angles are predicted to be exactly at their TBM values due to the presence of $\mu - \tau$ symmetry in $M_\nu$. They would also deviate from TBM once we allow for either $v_{S2} \neq v_{S3}$ or $c \neq d$, and in the most general case, both.

5 Conclusions

Current data seems to be pointing towards the existence of tribimaximal mixing. One needs to invoke some symmetry argument in order to get tribimaximal mixing. The discrete symmetry group $A_4$ has received a lot of attention in recent times as an attractive option for explaining the masses and mixing pattern of the neutrinos along with those of the charged leptons. We made a detailed phenomenological study of the viability of the different mass matrices that can be generated by spontaneous $A_4$ symmetry breaking.

In particular, we considered the model proposed in [19] and studied the phenomenological implications for it. The authors of [19] consider only one $A_4$ singlet and one $A_4$ triplet Higgs contribution to the neutrino mass matrix. Since the singlet transforms as 1 and since they take the vacuum alignment $\langle \phi_S \rangle = (v_S, v_S, v_S)$ for the $A_4$ triplet, the neutrino mass matrix by construction produces tribimaximal mixing. A lot of attention has been paid on justifying the vacuum alignment of the triplet Higgs which is absolutely essential for tribimaximal mixing. In this paper we pointed out that in addition to the vacuum alignment of the triplet, one also needs a certain fixed relation between the product of the VEV and the Yukawa of the singlet and the triplet. In particular, we found that in order to generate the correct ratio of $\Delta m^2_{21}$ to $\Delta m^2_{31}$, one demands that $b \simeq -1.88a$, where $a = 2x_u A$ and $b = 2x_{dS} b_S$. This appears to be extremely contrived and therefore undesirable. Even if one includes the 3σ uncertainties on $\Delta m^2_{21}$ and $\Delta m^2_{31}$, $|b|$ is fine tuned to $|a|$ within a factor of about $10^{-2}$. Even with this fine adjustment of the product of the VEVs and Yukawas, one would be able to generate only normal hierarchy for the neutrino mass spectrum.

We checked if it was possible to generate a viable mass matrix if the singlet Higgs belonged to either the 1$'$ or 1$''$ representation. We found them to be unsuitable due to the in-built wrong $S_2$ symmetry of the neutrino mass matrix for these cases. We studied the possibility of combining two singlet Higgs at a time. We showed that the case where $\xi'$ and $\xi''$ transforming as 1$'$ and 1$''$ are
taken together is the only viable option, since this could lead to an approximate $\mu - \tau$ symmetry for the mass matrix. In the limit that $c = d$, we get exact $\mu - \tau$ symmetry and exact tribimaximal mixing. We allowed the breaking of this $\mu - \tau$ symmetry and showed how the mixing angles deviate from their tribimaximal values. We gave approximate analytical predictions for $\Delta m^2_{21}$, $\Delta m^2_{31}$, $m_t$ and $m^2_2$, when the difference between the model parameters $c$ and $d$ is small. We showed numerically through scatter plots how some of these quantities varied, as the difference between $c$ and $d$ was increased. We identified the regions of the model parameter space which produce values of the neutrino oscillation parameters within their current 3$\sigma$ limits. This model is capable of producing even inverted hierarchy. We showed the regions of the $b - c - d$ parameter space which allow $m_3 < m_1 < m_2$ and at the same time correctly reproduce the neutrino oscillation parameters within their 3$\sigma$ ranges.

We next allowed for all three singlet Higgs contribution to the neutrino mass matrix. The case where $a \neq c = d$ emerged as the only possible case which produced exact tribimaximal mixing. Allowing $c \neq d$ allows for deviation from tribimaximal mixing. We studied this deviation as a function of the difference between $c$ and $d$. This model can give both normal and inverted hierarchy. The regions of the model parameter space which reproduce neutrino oscillations parameters within their current 3$\sigma$ allowed ranges were identified both for the normal as well as for the inverted hierarchy.

Finally, we allowed $\langle \phi_S \rangle$ to deviate from $(v_S, v_S, v_S)$. Changing the vacuum alignment immediately affects the tribimaximal character of the neutrino mixing matrix. We showed the results for $\langle \phi_S \rangle = (v_{S1}, v_S, v_S)$ and $a \neq c = d$ only, just for the sake of illustration. Since the resultant mass matrix by construction still has the residual $\mu - \tau$ symmetry, $\theta_{23}$ and $\theta_{13}$ remain at their tribimaximal values of $45^\circ$ and $0$ respectively. We showed the deviation of $\sin^2 \theta_{12}$ from $1/3$, analytically for small values of $\eta$ and numerically for all values of $\eta$, where $\eta$ quantifies the difference between $v_S$ and $v_{S1}$. The most general case would of course be when $v_{S1} \neq v_{S2} \neq v_{S3}$, where all three mixing angles will deviate from their tribimaximal values. Also, the condition $c = d$ is extremely “unnatural” and any deviation from that would also break tribimaximal mixing.

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