Study on Tensile-Shear and Compressive-Shear Coupling Strength of Saturated Clay

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Abstract. Traditional soil strength research mainly focused on compressive shear or uniaxial tensile stress state, and rarely considered the tensile-shear coupling strength characteristics in soil. Based on the tensile-shear and compressive-shear coupling strength model established earlier, which consider the tensile-shear coupling strength comprehensively, relevant tests are carried out to verify the strength model. Different types of saturated clay are selected for the uniaxial tensile test, direct shear test and triaxial shear test. The strength model is verified based on experimental data and compared with the existing joint strength model, and the coupling strength model shows good applicability and accuracy. The result shows the strength model can accurately describe the nonlinear strength characteristics of soil under tensile-shear coupling stress.

Keywords. Saturated clay, tensile-shear, compressive-shear, uniaxial tensile test.

1. Introduction

Due to the mutual transformation of tensile and compressive stress states in the natural state of soil failure, the failure of soil is affected by the interaction of tensile-shear coupling stress and compressive-shear coupling stress. However, the tensile shear strength of soil is low and it is unstable. Generally, it is not allowed to use its tensile shear strength in engineering construction. Therefore, there are relatively few studies on the tensile and shear strength of soil, and a relatively complete theory of the tensile and shear strength of soil has not yet been formed. The tensile and shear strength of soil deserves more in-depth research and attention.

The existing soil joint strength theory is mainly established for clay [1-4], which can be roughly divided into two types: one is to use an empirical curve to represent the strength line and establish the strength equation based on the shear test and tensile test data under low confining pressure; the other is to derive and establish the strength equation based on the shear strength index and tensile strength under certain assumptions. Scholars who used the first method to set up the corresponding joint strength theory and equation include R-Haefeli [5], Vesga [6] and Shen Zhunjiang [7]; Bishop [8], Guan Debin [9] and other scholars used the second method to obtain the research results. However, the existing joint strength studies still have the following defects: the three-dimensional strength characteristics of soil are not considered, only the strength study under plane condition is studied; some studies do not distinguish the three-dimensional tensile strength from the uniaxial tensile strength, and there is conceptual confusion; Most of the strength equations are established on the basis of compressive shear strength index.

Based on the analysis of the mechanism of tensile-shear stress and compressive-shear stress in soil, the author established the tensile-shear and compressive-shear coupling strength model of saturated...
clay [10] which considered the tensile-shear coupling strength and the compressive-shear strength comprehensively. In this paper, different types of saturated clay are selected for the uniaxial tensile test, direct shear test and triaxial shear test. Based on the test results, the strength model is verified.

2. Tensile-Shear and Compressive-Shear Coupling Strength Model

In the paper [10], the author analyzes the mechanism of tensile-shear coupling strength in soil, and gives a definition of closed stress point that is suitable for soil. By comprehensively considering the strength properties of soil under the action of tensile-shear coupling stress and compressive-shear stress, the tensile-shear and compressive-shear coupling strength model of saturated clay under plane stress are established, and the specific strength failure envelope and strength line is derived. The following is a brief review of the tensile-shear and compressive shear coupling strength model and the specific strength equations.

2.1. The Strength Failure Envelope

With reference to the concept of closed stress point, the failure point of the unconfined compression test is taken as the boundary point between the tension-shear coupling zone and the compression-shear zone, and the specific physical meaning of the closed stress point is given. The strength failure envelope is showed in figure 1, the intercept of the coupling failure envelope is unconfined shear strength $\tau_0$, and it is smoothly connected to M-C failure envelope at $\sigma_b$. $\sigma_b$ is the uniaxial compression damage strength.

![Figure 1. Failure envelope on saturated clay of T-S and C-S coupling strength model.](image)

In the $\sigma' - \tau$ plane, Four characteristic parameters $\sigma_t$, $\tau_0$, $(\sigma_b, \tau_b)$ and $\varphi'$ can be used to describe the strength failure envelope of the tension-shear coupling zone. We use the quadratic curve $\sigma' = a\tau^2 + b\tau + c$ as the mathematical expression. To derive the strength equation of the model, $\sigma_t$, $(\sigma_b, \tau_b)$ and $\varphi'$ are adopted of the derivation, and we use $\tau_0$ to verify the accuracy of the equation. According to the geometric relationship, the following equations can be obtained.

\[
\begin{align*}
    a\tau_b^2 + b\tau_b + c &= \sigma_b \\
    a\tau_t'^2 + b\tau_t' + c &= \sigma_t' \\
    2a\tau_b + b &= 1/\tan \varphi' \\
    c &= \sigma_b - \tau_b / \tan \varphi' - \tau_b^2 m
\end{align*}
\]

(1)

Where, $\sigma_b = q_u(1-\sin \varphi')/2$, $\tau_b = q_u\cos \varphi'/2$, $\sigma_t' = \sigma_t(1 + \sin \alpha)/2$, $\tau_t' = -\sigma_t\cos \alpha/2$, $m = \left[\sigma_b - \sigma_t' - 1/\tan \varphi' (\tau_b - \tau_t')\right]/\left(\tau_b - \tau_t'\right)^2$, $\alpha = \chi\varphi' (1 < \chi \leq 1.5)$. 


is the effective internal friction angle, \( \sigma' \) is the uniaxial tensile strength, \( q_u \) is the unconfined compressive strength and \( \chi \) is a physical parameter. Substituting the above equation into equation (1), we can obtain the expression of correlation coefficient and tensile-shear coupling strength.

Based on the simultaneous equations of compression-shear strength and tension-shear coupling strength, the complete equation of tension-shear coupling strength can be expressed as follows.

\[
\begin{align*}
\alpha \tau^2 + b \tau &= \sigma' - c & \sigma' < q_u (1 - \sin \varphi') / 2 \\
\tau &= c' + \sigma' \tan \varphi' & \sigma' \geq q_u (1 - \sin \varphi') / 2
\end{align*}
\] (2)

### 2.2. Strength Line (Kf line)

In order to more intuitively and completely reflect the strength model and strength characteristics of the proposed tension-shear coupling stress state, the equation of strength line is established within the plane of \( \bar{\sigma}(p) - \bar{\tau}(q) \), as shown in figure 2. Compared with the previous expression, the coupling strength can be expressed as \( \bar{\sigma} = A \bar{\tau}^2 + B \bar{\tau} + C \).

![Figure 2. Strength line on saturated clay of T-S and C-S coupling strength model.](image)

Figure 2. Strength line on saturated clay of T-S and C-S coupling strength model.

\[
\begin{align*}
A \bar{\tau}^2 + B \bar{\tau} + C &= \bar{\sigma}_b \\
A(-\sigma_i/2)^2 + B(-\sigma_i/2) + C &= \sigma_i/2 \\
2A \bar{\tau}_b + B &= 1 / \tan \phi
\end{align*}
\] (3)

Where, \( \bar{\tau} = (\sigma_i - \sigma_s)/2 \), \( \bar{\sigma} = (\sigma_i + \sigma_s)/2 \), \( \bar{\sigma}_b = q_u/2 \), \( \bar{\tau}_b = q_u/2 \), \( \phi = \arctan \sin \varphi' \),

\[
\begin{align*}
A &= -2(q_u \sin \varphi' - \sigma_i \sin \varphi' - \sigma_i - q_u)/(\sigma_i + q_u)^2 \sin \varphi' \\
B &= (q_u - \sigma_i)(2q_u \sin \varphi' - \sigma_i - q_u)/(\sigma_i + q_u)^2 \sin \varphi' \\
C &= q_u \left( \sigma_i^2 \sin \varphi' + 3\sigma_i q_u \sin \varphi' - \sigma_i q_u - \sigma_i^2 \right) / (2(\sigma_i + q_u)^2) \sin \varphi'
\end{align*}
\] (4)

So the complete equation of strength line is:

\[
\begin{align*}
A \bar{\tau}^2 + B \bar{\tau} &= \bar{\sigma} - C & \bar{\sigma} < q_u/2 \\
\bar{\tau} &= c' \cos \varphi' + \bar{\sigma} \sin \varphi' & \bar{\sigma} \geq q_u/2
\end{align*}
\] (5)
3. Tests of Different Saturated Clays

Laboratory tests were conducted for two different saturated clay samples, and the prediction results of tensile-shear and compressive-shear coupling strength model of saturated clay were analyzed and verified based on the test data. Uniaxial tensile test, direct shear test and triaxial shear test were carried out for saturated samples of two types of clay, and the soil samples were prepared according to the test standards in regulation [11]. The basic soil parameters and sample preparation standards of clay applied in the test are listed in table 1.

| Type of soil | Specific gravity | Plastic limit /% | Plasticity index | Dry density /(kN/m³) | Water content /% |
|--------------|------------------|------------------|------------------|----------------------|-----------------|
| Clay I       | 2.74             | 20.4             | 23.1             | 16.1                 | 19.7            |
| Clay II      | 2.72             | 17.5             | 16.8             | 16.8                 | 16.9            |

3.1. Uniaxial Tensile Test

In the uniaxial tensile test, a new type of soil tensile strength test apparatus [12] is selected, and the saturated clay sample is loaded into the tensile mold of the uniaxial tensile test device. Firstly, the displacement grating and the "s" type force sensor are adjusted, and the test parameters are set. The sample is stretched at a constant rate of 0.1 mm / min. the tensile force and displacement during the tensile process will be transmitted through the displacement grating and force sensor to the computer. When the observed tensile force value drops sharply and significant cracks are observed in the tensile part of the sample, the soil sample can be considered as damaged and the test is ended.

Through the above tensile tests of the two types of clay, the tensile force-displacement curve and stress-strain curve are obtained, as shown in figure 3 and figure 4. After the two types of clay samples reach the peak strength, the tensile force decreases rapidly, and the decrease speed gradually slows down when the tensile force reaches a lower value. Taking the peak value of stress-strain curve as the tensile strength value of soil sample, so the uniaxial tensile strength of two kinds of saturated clay is 53.1 kPa and 41.7 kPa respectively.

3.2. Direct Shear Test

The conventional direct shear equipment of strain controlled is used for the direct shear test. Load the saturated clay sample into the direct shear tester. After installation, the samples were sheared at a shear rate of 0.8mm/min without applying vertical pressure to the samples. We carry on the test until the
peak value of dynamometer reading appears, continue to shear until the shear deformation is 4mm, then stop the machine and record the failure value.

According to the direct shear test results of the two types of clay, the relationship curve between shear stress and shear displacement is drawn, as shown in figure 5. The shear strength is selected as the peak value of the shear stress curve, and the unconfined shear strength of two types of saturated clay is 40.4 kPa and 33.5 kPa, respectively.

![Figure 5. Shear stress-displacement curve of two types of clay.](image)

3.3. Triaxial Shear Test

In the triaxial shear test, the conventional strain controlled triaxial shear apparatus is used. After the saturated clay sample is installed, the back pressure saturation process is continued for two or three times to ensure that the sample reaches the saturation state. The same group of saturated clay samples are subjected to confining pressure of 0 kPa, 100 kPa, 200 kPa and 400 kPa respectively. The shear mode is consolidation drainage shear with fixed confining pressure and increased axial pressure. The shear rate is 0.1 mm/min. The test procedure is carried out according to the procedure of consolidated drained shear test.

The deviatoric stress axial strain curves of the two types of clays in conventional triaxial shear tests are shown in figure 6 and figure 7. According to the test data, it can be found that the initial deformation modulus of the stress-strain curve of the two kinds of clays under different confining pressures are basically the same, which may be due to the over consolidation of the soil caused by the larger compaction work in the process of preparing soil samples. At the same time, the stress-strain curves of the two kinds of clays under different confining pressures show strain hardening phenomenon, and along of the increase of confining pressure, the strain hardening phenomenon tends to be significant.

According to the triaxial shear test, the failure stress circle and strength failure envelope of the two types of clay are obtained, as shown in and figure 8 and figure 9, and the corresponding shear strength indexes are given. The effective cohesion of the two types of saturated clay are 45.3 kpa and 37.8 kpa, and the effective internal friction angle is 16.4 ° and 13.1 ° respectively.
4. Verification of the Strength Model

In order to verify the tensile-shear and compressive-shear coupling strength model, different types of saturated clay are selected for the uniaxial tensile test, direct shear test and triaxial shear test. Table 2 shows the basic indexes of clay got in the test. Substituting the physical and mechanical parameters of two saturated clays into equations (2) and (5) can obtain the specific equations of the coupling strength, and according to different verification needs, the strength envelope equation is given (6), (7). Where, the model parameter $\chi$ is selected as 1.5 and 1.45.

| Type of soil | $\sigma_t$(kPa) | $\tau_0$(kPa) | $q_u$(kPa) | $c'$(kPa) | $\phi'$(°) |
|--------------|----------------|--------------|------------|-----------|------------|
| Clay I       | 53.1           | 40.4         | 111.7      | 45.3      | 16.4       |
| Clay II      | 41.7           | 33.5         | 96.8       | 37.8      | 13.1       |

\[
\begin{align*}
0.043\sigma^2 - 1.210\tau & = \sigma + 18.524 & \text{if } \sigma < 40.1 \\
\tau & = 45.3 + 0.294\sigma & \text{if } \sigma \geq 40.1 
\end{align*}
\]

(6)
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\[
\begin{align*}
0.062\tau^2 - 1.548\tau &= \sigma + 27.37 & \sigma < 37.43 \\
\tau &= 37.8 + 0.233\sigma & \sigma \geq 37.43
\end{align*}
\]  

(7)

Figure 10 and figure 11 show the tensile-shear and compressive-shear coupling strength of clay I and clay II, which are compared with different strength models.

Figure 10. Comparison between experimental data (clay I) from the prediction data.  
Figure 11. Comparison between experimental data (clay II) from the prediction data.

As shown in figure 10 and figure 11, M-C strength criterion significantly overestimates the strength of tensile-shear coupling area; Griffith strength criterion, which is suitable for brittle materials such as rocks, is not suitable for the analysis of clay; Griffith-Mohr strength model [9] and the hyperbolic strength model [7] have poor prediction effect on the strength of the tensile-shear coupling region, which obviously underestimates the strength of the tensile-shear coupling zone; The main reason is that none of the strength models can accurately describe the mechanism of tensile-shear and compressive-shear coupling strength, and there are problems such as deviations in the selection of closed stress point. The tensile-shear and compressive-shear coupling strength envelope and strength line are well fitted by test results, and the model can accurately describe the nonlinear strength characteristics of soil compared with different strength criterions.

5. Conclusions
In this paper, we review the tensile-shear and compressive-shear coupling strength model for saturated clay which is obtained earlier. Based on the uniaxial tensile test, direct shear test and triaxial shear test of different saturated clay, the coupling strength model is analyzed and verified. The results show that the strength model can accurately describe the characteristics of tensile-shear strength and compressive-shear strength of saturated clay. Compared with the current joint strength model, the tensile-shear and compressive-shear coupling strength models can accurately predict the nonlinear strength of clay under the tensile-shear coupling stress. The tensile-shear and compressive-shear coupling strength model of unsaturated soil involves many factors, such as water contents and matrix suction, will be studied later.

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