Heavy quark potential at finite imaginary chemical potential

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We investigate chemical-potential ($\mu$) dependence of static-quark free energies in both the real and imaginary $\mu$ regions, using the clover-improved two-flavor Wilson fermion action and the renormalization-group improved Iwasaki gauge action. Static-quark potentials are evaluated from the Polyakov-loop correlator in the deconfinement phase and the imaginary $\mu = i\mu_I$ region and extrapolated to the real $\mu$ region with analytic continuation. As the analytic continuation, the potential calculated at imaginary $\mu = i\mu_I$ is expanded into a Taylor-expansion series of $i\mu_I/T$ up to 4th order and the pure imaginary variable $i\mu_I/T$ is replaced by the real one $\mu_R/T$. At real $\mu$, the 4th-order term weakens $\mu$ dependence of the potential sizably. Also, the color-Debye screening mass is extracted from the color-singlet potential at imaginary $\mu$, and the mass is extrapolated to real $\mu$ by analytic continuation. The screening mass thus obtained has stronger $\mu$ dependence than the prediction of the leading-order thermal perturbation theory at both real and imaginary $\mu$.

This talk is based on [1].

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1. Introduction

The free energies between two static quarks are fundamental quantities to understand inter-quark interactions. Particularly above $T_{pc}$, the static-quark potentials determined from the free energies characterize quark-gluon dynamics in QGP; for example, the inverse of the range of the color-singlet potential is the color-Debye screening mass. The potential largely affects the behavior of heavy-quark bound states such as $J/\Psi$ and $\Upsilon$ in QGP created at the center of heavy-ion collisions \[2\]. In lattice QCD (LQCD) simulations, the static-quark potential is determined from the Polyakov-loop correlation function. For zero chemical potential, $T$ dependence of the static-quark potential was investigated by quenched QCD \[3, 4, 5\] and full QCD with staggered-type \[6\] and Wilson-type quark actions \[7, 8, 9\]. For small $\mu/T$, it was analyzed by the Taylor-expansion method with staggered-type \[10\] and Wilson-type quark actions \[11\]. In the analysis \[11\], the expansion coefficients are taken up to 2nd order of $\mu/T$.

In this report, we present $\mu$ dependence of the static-quark free energies and the color-Debye screening mass in both the imaginary and real $\mu$ regions, performing LQCD simulations at imaginary $\mu$ and extrapolating the result to the real $\mu = \mu_R$ region with analytic continuation. We consider two temperatures above $T_{pc}$, i.e., $T/T_{pc} = 1.20$ and 1.35. Following the previous LQCD simulation \[11\] at small $\mu/T$, we compute static-quark free energies along the line of constant physics at $m_{PS}/m_V = 0.80$. As the analytic continuation, the static-quark potential at imaginary $\mu = i\mu_I$ is expanded into a Taylor-expansion series of $i\mu_I/T$ and pure imaginary variable $i\mu_I/T$ is replaced by real one $\mu_R/T$. In the present work the Taylor-expansion coefficients of the static-quark potential are evaluated up to 4th order, whereas the coefficients were computed up to 2nd order in Ref. \[11\]. It is found that the 4th-order term yields non-negligible contributions to $\mu$ dependence of the static-quark potentials at real $\mu$. At long distance, all of the color singlet and non-singlet potentials tend to twice the single-quark free energy, indicating that the interactions between heavy quarks are fully color-screened. Although this property is known for finite $T$ and zero $\mu$ \[8\], the present work shows that the property persists also for imaginary $\mu$. For imaginary $\mu$, the color-singlet $q\bar{q}$ and the color-antitriplet $qq$ interaction are attractive, whereas the color-octet $q\bar{q}$ and the color-sextet $qq$ interaction are repulsive. The color-Debye screening mass at imaginary $\mu$ is extracted from the color-singlet potential there. The mass at real $\mu$ is extrapolated from the mass at imaginary $\mu$ by analytic continuation, i.e., by expanding the mass at imaginary $\mu$ into a power series of $i\mu_I/T$ up to 2nd order and replacing $i\mu_I$ by $\mu_R$. The $(\mu/T)$ dependence of the screening mass is found to be stronger than the prediction of the leading-order thermal perturbation theory.

2. Static-quark free energies

The Polyakov loop is defined as

$$L(x) = \prod_{t=1}^{N_t} U_\mu(x, t)$$  \hspace{1cm} (2.1)$$

with link variables $U_\mu \in SU(3)$. At imaginary $\mu$, the ensemble average of the Polyakov loop becomes a complex number, $\langle TrL(0) \rangle \equiv \Phi e^{i\theta}$. The modulus is related to the single-quark free
energy $F_q$ as $\Phi = \exp[-F_q/T]$. After an appropriate gauge fixing, one can derive the static-quark free energies (potentials)$V_M$ of color channel $M$ from the Polyakov-loop correlator [12, 13]:

$$e^{-V_1(r,T,\mu)/T} = \frac{1}{3} (\langle \text{Tr} L^+(x)L(y) \rangle),$$  \hspace{2cm} (2.2)

$$e^{-V_8(r,T,\mu)/T} = \frac{1}{8} (\langle \text{Tr} L^+(x)\text{Tr}L(y) \rangle - \frac{1}{24} (\text{Tr}L^+(x)L(y)),$$  \hspace{2cm} (2.3)

$$e^{-V_{35}(r,T,\mu)/T} = \frac{1}{6} (\langle \text{Tr}L(x)\text{Tr}L(y) \rangle - \frac{1}{6} (\text{Tr}L(x)L(y)),$$  \hspace{2cm} (2.4)

$$e^{-V_6(r,T,\mu)/T} = \frac{1}{12} (\langle \text{Tr}L(x)\text{Tr}L(y) \rangle + \frac{1}{12} (\text{Tr}L(x)L(y)),$$  \hspace{2cm} (2.5)

where $r = |x - y|$ and the subscripts $M = (1, 8, 3^*, 6)$ mean the color-singlet, -octet, -antitriplet and -sextet channels, respectively. We adopt the Coulomb gauge fixing.

In general, the $V_M$ ($M = 1, 8, 3^*, 6$) are complex at finite imaginary $\mu$. The real part of $V_M$ is $\mathcal{C}$-even and the imaginary part is $\mathcal{C}$-odd. This can be easily understood by expanding $V_M$ into a power series of $i\mu/T$:

$$\frac{V_M(r,T,\mu)}{T} = v_0(r) + iv_1(r) \left( \frac{\mu_1}{T} \right) + v_2(r) \left( \frac{\mu_1}{T} \right)^2 + iv_3(r) \left( \frac{\mu_1}{T} \right)^3 + v_4(r) \left( \frac{\mu_1}{T} \right)^4,$$  \hspace{2cm} (2.6)

where we consider terms up to 4th order. The potential $V_M$ at real $\mu$ is obtained from that at imaginary $\mu$ by analytic continuation, i.e., by replacing $i\mu_1/T$ by $\mu_R/T$:

$$\frac{V_M(r,T,\mu_R)}{T} = v_0(r) + v_1(r) \left( \frac{\mu_R}{T} \right) - v_2(r) \left( \frac{\mu_R}{T} \right)^2 - v_3(r) \left( \frac{\mu_R}{T} \right)^3 + v_4(r) \left( \frac{\mu_R}{T} \right)^4.$$  \hspace{2cm} (2.7)

The WHOT-QCD Collaboration calculated the Taylor-expansion coefficients of $V_M$ up to 2nd order by using the Taylor-expansion method and the reweighting technique with the Gaussian approximation for the distribution of the complex phase of the quark determinant [11]. In this work, meanwhile, we obtain the coefficients up to 4th order from $V_M$ at imaginary $\mu$ by expanding it as in (2.6).

3. Results of the lattice simulations and the analytic continuation

We employ the clover-improved two-flavor Wilson fermion action and the renormalization-group improved Iwasaki gauge action. Finite temperature simulations are performed on $16^3 \times 4$ lattices along the line of constant physics with $m_{PS}/m_{V} = 0.80$. We consider two temperatures $T/T_{pc} = 1.20$ and 1.35. We generated 16,000 trajectories and removed the first 1,000 trajectories as thermalization for all the parameter set. We measured the static-quark potential at every 100 trajectories.

The coefficients $v_2(r)$ and $v_4(r)$ of $V_1(r)$ are shown in Fig. 1. The ratio $v_4(r)/v_2(r)$ is about 3/4 for $T/T_{pc} = 1.20$ and about 1/4 for $T/T_{pc} = 1.35$. Thus the contribution of $v_4(r)$ to $V_1(r)$ is significant near $T_{pc}$ such as $T/T_{pc} = 1.20$. Even at higher $T$ such as $T/T_{pc} = 1.35$, the contribution is not negligible.

Figure 2 shows the color-singlet potential at imaginary and real $\mu$ for (a) $T/T_{pc} = 1.20$ and (b) $T/T_{pc} = 1.35$. The chemical potential is varied from $(\mu/T)^2 = -1.0$ to 1.0. The potential $V_1$ is $\mathcal{C}$-even, so that $v_1(r) = v_3(r) = 0$. Furthermore, if $v_4(r) = 0$, the potential $V_1/T$ will linearly
Figure 1: Taylor-expansion coefficients, $v_2(r)$ and $v_4(r)$, of $V_1(r)$ at (a) $T/T_{pc} = 1.20$ and (b) $T/T_{pc} = 1.35$.

The interactions between heavy quarks are thus color screened also for imaginary $\mu$. Following the previous works [8, 4, 5, 9, 11], we then subtract $2F_q$ from $V_M(r)$. The subtracted static-quark potentials are shown in Fig. 3(a) for the color-singlet and -octet channels and in Fig. 3(b) for the color-antitriplet and -sextet channels. Needless to say, the physical interpretation of gauge dependent quantities is not straightforward; this is the case also for the potentials. See ref. [14]. Our results show distinctively different behaviors for the singlet/antitriplet channel and the octet/sextet channel; the former is "attractive" and the latter is "repulsive". The attractive interactions have strong $\mu_1/T$ dependence, but the repulsive interactions have weak $\mu_1/T$ dependence.
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Figure 3: \( \mu_I/T \) dependence of the subtracted \( q\bar{q} \) in (a) the color-singlet and -octet channels and \( qq \) potentials in (b) the color-antitriplet and -sextet channels at \( T/T_{pc} = 1.20 \).

4. Color-Debye screening mass

In order to analyze the color screening effect, we fit the static-quark potential to the screened Coulomb form

\[
V_M(r, T, \mu) = C_M \frac{\alpha_{\text{eff}}(T, \mu)}{r} e^{-m_D(T, \mu)r},
\]

where \( C_M \equiv \langle \sum_{a=1}^{8} t_1^a \cdot t_2^a \rangle_M \), \( \alpha_{\text{eff}} \) and \( m_D(T, \mu) \) are the Casimir factor, the effective running coupling and the color-Debye screening mass, respectively. Here, we focus our discussion on the color-singlet channel that is most important in the real world, and the Casimir factor in the singlet channel is \( C_1 = -4/3 \). Since \( V_1 = 0 \) in the limit of large \( r \) in (4.1), we extract the screening mass from the subtracted static-quark potential. Following the previous work [9], we choose a fit range of \( \sqrt{11} \leq r/a \leq 6.0 \).

In the leading-order (LO) hard thermal loop (HTL) perturbation theory, the color-Debye screening mass is obtained [15] by

\[
\frac{m_D(T, \mu)}{T} = g_{21}(\nu) \sqrt{\left(1 + \frac{N_f}{6}\right) + \frac{N_f}{2\pi^2} \left(\frac{\mu}{T}\right)^2},
\]

with the 2-loop running coupling \( g_{21} \) given by

\[
g_{21}^{-2}(\nu) = \beta_0 \ln \left(\frac{\nu}{\Lambda}\right)^2 + \frac{\beta_1}{\beta_0} \ln \ln \left(\frac{\nu}{\Lambda}\right)^2,
\]

where the argument in the logarithms is rewritten into \( \nu/\Lambda = (\nu/T)(T/T_{pc})(T_{pc}/\Lambda) \) with \( \Lambda = \Lambda_{MS}^{N_f=2} \approx 261 \text{ MeV} \) [16] and \( T_{pc} \approx 171 \text{ MeV} \) [17], and the renormalization point \( \nu \) is assumed to be \( \nu = \sqrt{(\pi T)^2 + \mu^2} \) [18].

Figure 4 shows the \( (\mu/T)^2 \) dependence of the color-Debye screening mass for (a) \( T/T_{pc} = 1.20 \) and (b) \( T/T_{pc} = 1.35 \). The lattice-simulation results are plotted by the cross symbols. The screening mass is then expanded up to 2nd order of \( \mu/T \):

\[
\frac{m_D}{T} = a_0(T) + a_2(T) \left(\frac{\mu}{T}\right)^2,
\]
where note that $m_D$ is $\mathcal{C}$-even and hence it has no linear term of $\mu/T$. The screening mass at real $\mu$ is extrapolated from that at imaginary $\mu$ by using (4.4).

The results of the extrapolation, represented by the hatching area, are consistent with the previous LQCD result, denoted by a circle symbol, at $\mu = 0$ [8] for both cases of $T/T_{pc} = 1.20$ and 1.35. Comparing the hatching area (the result of the extrapolation) with the solid line (the prediction of the leading-order thermal perturbation theory), one can see that the present LQCD results show stronger $\mu/T$ dependence than the prediction of the perturbation theory.

\begin{figure}[h]
\centering
\includegraphics[scale=0.5]{fig4.png}
\caption{(\mu/T)^2 dependence of the color-Debye screening mass for (a) $T/T_{pc} = 1.20$ and (b) 1.35. The screening mass is determined from the singlet potential. Crosses with error bars denote results of the present lattice simulations at imaginary $\mu$, while a circle with an error bar is a result of the previous lattice simulations at $\mu = 0$ [8].}
\end{figure}

5. Summary

We have investigated $\mu$ dependence of the static-quark potential and the color-Debye screening mass in both the imaginary and real $\mu$ regions, performing LQCD simulations at imaginary $\mu$ and extrapolating the result to the real $\mu$ region with analytic continuation. LQCD calculations are done on a $16^3 \times 4$ lattice with the clover-improved two-flavor Wilson fermion action and the renormalization-group improved Iwasaki gauge action. We took an intermediate quark mass and considered two cases of $T/T_{pc} = 1.20$ and 1.35.

The static-quark potential at real $\mu$ was obtained by expanding the potential at imaginary $\mu$ into a Taylor-expansion series of $i\mu_I/T$ up to 4th order and replacing $i\mu_I$ to $\mu_R$. Since the expansion series was taken only up to 2nd order in the previous analysis [11], this is the first analysis that investigates contributions of the 4th-order term to the potential. We found that at real $\mu$ the 4th-order term weakens $\mu$ dependence of the potential sizably. This effect becomes more significant as $T$ decreases toward $T_{pc}$. We have also investigated color-channel dependence of the static-quark potentials. At large distance, all the potentials tend to twice the single-quark free energy, indicating that the interactions are fully color screened. Although this property is known for finite $T$ and zero $\mu$ [8], the present analysis shows that the property persists also for imaginary $\mu$. For imaginary $\mu$, the color-singlet $q\bar{q}$ and the color-antitriplet $qq$ interaction are attractive, whereas the color-octet $q\bar{q}$ and the color-sextet $qq$ interaction are repulsive.
The color-Debye screening mass is evaluated from the color-singlet potential at imaginary $\mu$. The screening mass thus obtained at imaginary $\mu$ is extrapolated to real $\mu$ by expanding the mass at imaginary $\mu$ into a power series of $i\mu I/T$ up to 2nd order and replacing $i\mu I/T$ by $\mu I/T$. The resulting mass has stronger $\mu$ dependence at both imaginary and real $\mu$ than the prediction of the leading-order thermal perturbation theory.

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