Conditional optimization of geometric parameters of aircraft hydraulic system elements

A I Demidov, I O Bobarika, I N Gusev
Irkutsk National Research Technical University, 83, Lermontov St., Irkutsk, 664074, Russia

E-mail: demidov-ai@yandex.ru

Abstract. The article describes the method of selecting the optimal geometric parameters of the hydraulic system of the aircraft. The minimum weight of the system and the minimum power consumption of the power supply are chosen as the optimal criteria. There is a mathematical model providing effective flow distribution in any branched hydraulic system. Target functions and all equations are obtained in general form.

Key words. Hydraulic system; optimization; flow distribution; weight; power; pressure; diameter.

1. Introduction

Nowadays, the domestic aviation equipment is being actively modernized and updated. A number of civil aircrafts have been upgraded and completely new units of aircraft have been put into production. The Irkutsk Aviation Plant is actively launching serial production of the MC-21, a promising short- and mid-range aircraft developed jointly by the Yakovlev Design Bureau and PJSC Irkut Corporation. The launch of the latest aircraft, such as the MC-21, is associated with a large number of innovative solutions, development of the latest technologies, which have not been previously used in the domestic civil aviation industry. Power on-board systems, in particular hydraulic systems, are used in MC-21. Permalswitch system was introduced to connect pipelines, which allows to reduce weight due to pipeline fittings and improve the quality of installation of hydraulic systems. Reduction of the hydraulic system weight can be obtained by providing a qualitative study of the hydraulic system at the initial stage of design [1-3]. Mathematical modeling methods are used for this purpose, methods of selection of the best design solutions are developed, and methods of selection of optimal parameters are developed [4-6].

The Training and Research Laboratory for Aerodynamics, Structure and Strength Modeling of Aircraft at IRNITU deals with advanced methods of aircraft design and, in particular, with the improvement of hydraulic systems. Employees of the laboratory are engaged in the complex engineering analysis of hydraulic systems, beginning from the optimum circuit design, a choice of optimum geometrical parameters of pipelines and consumers at the set output characteristics of hydraulic systems from the point of view of the mechanism (efforts, moments, speeds etc.) Taking into account the dynamics of the working bodies before the "point" consideration and improvement of individual components, joints, fixing points, on the basis of hydrogas-dynamic analysis of the fluid flow, analysis of the conditions of cavitation [7], strength analysis in static and dynamic statements, taking into account the high vibration load of hydraulic system elements [8]. One of the current scientific works: Development of methods for selecting the optimal geometric parameters of hydraulic system elements.
2. Hydraulic system optimization

Reducing the weight of the hydraulic system will increase the payload or fuel reserve. The power to drive hydraulic system pumps is selected from the aircraft engine, then reducing the power required for hydraulic system operation will reduce energy consumption and reduce fuel consumption. On the basis of the above, we can highlight the criteria for optimizing the hydraulic system of the aircraft: minimum weight, minimum power consumption.

Based on the physical meaning of the design of the hydraulic system, we will accept as input data the direct purpose of the hydraulic drive in the mechanism: at a certain point in time to make a progressive movement with the required force and speed by means of a hydraulic cylinder or a rotational movement with the required torque and angular speed by means of a hydraulic motor, etc. The kinematic system of the driven mechanism is known, then the necessary movements of hydraulic cylinder rods are known [9]. The system is defined in space, i.e. there are known distances from the power supply source to the driven mechanisms and the place of branching of pipelines.

The process of finding and extreme values of the target function of the object is called optimization [10,11]. The criteria for optimizing an aircraft's hydraulic system are the minimum weight and minimum power requirement of the power supply, which are the output parameters. The optimization task is to find such values of input variables at which the output variables take the minimum value [12]. Then the input variables will be optimal.

The purpose of the work is to determine the optimal characteristic sizes of pipelines and consumers. Then the vector of input variable parameters

\[ \mathbf{b} = (d_{fr_i}, d_{hz_j}) \]  

(1)

where \( d_{hz_j} \) – inner diameter of the consumption's working cavity \( j \); \( d_{fr_i} \) – branch pipeline inner diameter \( i \).

In general, the hydraulic system has an arbitrary number of consumers and branches, and then the vector (1) has an arbitrary length. At such statement of a problem it is impossible to define the unique decision as the same value of a minimum of function can be received by various combinations of values in a series of diameters. To ensure the uniqueness of the solution, it is necessary to impose additional conditions and proceed to conditional optimization [13]. Let us introduce the requirement that the vector of input parameters must satisfy the effective flow distribution.

Efficiency by definition is a property of the system to fulfill the set goal in the given conditions of use and with certain quality. before proceeding to the receipt of the target functions of optimization of the hydraulic system according to the accepted criteria, it is necessary to pay attention to the issue of efficiency of the hydraulic system, namely, the effective flow distribution. In order to consider and minimize any parameter for the hydraulic system as a whole, it is necessary to ensure energy efficiency within the system - in each functional subsystem simultaneously. Efficient flow distribution - the state of the system, in which the design operation of the actuators is ensured: the pump capacity is provided with the necessary and sufficient amount of working fluid in each consumer for, for example, the output of rods for the design time of all hydraulic cylinders, while the size of pipelines and other local resistances are provided with such pressure losses, so that the combination of pressure in the consumer and its size led to the design effort. There is no sense to bring to the consumer a working liquid with the pressure exceeding necessary for creation of settlement effort (settlement effort here the required characteristic of the executive mechanism which has received taking into account a stock or without that - from the point of view of a research problem does not matter), as well as has no sense to count pipelines on the expense of a liquid which will not be claimed. The conclusion is obvious, but not simple from the point of view of mathematics in the case of an arbitrary branched hydraulic system with unknown parameters. There are mathematical models of flow distribution, representing a system of nonlinear equations, which can be solved only as a result of additional iterative calculations [14-16].

In the course of work on this topic we have managed to obtain a system of equations linking the physical parameters of the hydraulic system, such as the power supply, pressure in consumers, costs in all functional subsystems with the geometric parameters of all pipelines and all consumers of the system.
The obtained system of equations provides an effective flow distribution at any initial data, while excluding the iterative process in the solution.

For any branch of an arbitrary branched hydraulic system it is possible to write the Bernoulli equation

\[
\frac{\rho v^2}{2} + \rho g h_i + P_i = \frac{\rho v'^2}{2} + \rho g h_{i'} + P_{i'} + \Delta P_l
\]

(2)

where \(\rho\) – working fluid density; \(v\) – velocity of the working fluid; \(i\) - number of the branch in question; \(i'\) – branch number before the one under consideration; \(g\) – freefall acceleration; \(h\) – height; \(P\) – pressure; \(\Delta P\) – pressure losses.

The contribution of the summand taking into account the height difference is not significant within the framework of consideration of the aircraft hydraulic system, as the operating pressure in the system, as a rule, exceeds by several orders of magnitude the possible increase in pressure from the height difference.

A branch is a section of the network consisting of one or more constant-diameter pipes, including fittings and other local resistances [17]. Local resistances in the form of switchgear and fittings are taken into account by adding to the actual length of the pipelines an additional length, the resistance of which is equivalent to the local resistance, the so-called equivalent length.

The number of consumers in the system is denoted as \(n\), and the number of branches - \(m\). In the course of mathematical transformations with application of Kirchhoff’s laws for hydraulic chains, equation of material balance, Darcy-Weisbach equation[18,19], Reynolds number expressions and taking into account liquid flow regime by analytical method the system of equations was obtained at \(i=1…m\) and \(j=1…n\)

\[
Q_i = \frac{\sum_{j=1}^{n} C_j \sum_{i=1}^{m} B_i r_i}{p_j},
\]

\[
P_i = P_{i'} - \frac{B_i r_i}{Q_i}, \text{as } df
\]

(3)

where \(Q_i\) – branch flow \(i\); \(P_{i'}\) – pressure at the beginning of branch \(i\) or at the end of the previous branch \(i'\); \(n_i\) – node number at the beginning of branch \(i\), which coincides with the number of the previous branch; \(m_i\) – many of the contour numbers that the branch belongs to \(i\); \(B_i\) – customer parameter \(j\). For hydraulic cylinder \(C = F \cdot v_n m_i\) – a lot of branch numbers, the working fluid to which passes through the branch \(i\); \(B_i\) – a parameter that takes into account the fluid flow mode. For laminar flow \(B = 8 \pi r v_p \rho^2\).

For any arbitrary hydraulic system a system of equations (3) can be obtained from \(2m\cdot k\) equations for \(2m\) unknown, where \(k\) is the number of branches with the initial node in the power supply.

The resulting equation system clearly defines the effective flow distribution for all source data, without the need for additional nodal linking of flow rates or pressures, where the consumers involved operate at the right force and speed, and the most efficient mode of operation of the power supply is clearly defined. It is important to note that an iterative process is not required to solve the equation system.

Target input parameters (internal diameters of pipelines, diameters of working cavity of consumers) are not specified in the system of equations in an explicit form, but their values are unambiguously determined as a result of the system solution. If we leave the vector of input parameters in the initial form (1), then at each optimization step we will need to check the conditions of effective flow distribution. In addition, when an effective flow distribution condition is introduced, the input parameters become dependent on other variable input parameters, which contradicts with the requirements of the optimization task. In this case, it is decided to replace the vector of input variable geometric parameters with the vector of physical parameters, then the optimization of geometric parameters occurs in the space of physical parameters of the hydraulic system.

It is determined that if you set some value of the velocity of the working fluid and set the boundary conditions at the inlet to the system in the form of power supply pressure, the system of equations (3) will be solved with an equal number of equations and unknown. According to the condition that the hydraulic system design problem is formulated, these parameters are unknown. They should be chosen as the variable input physical parameters, then the vector will be as
\[ \vec{b} = (v, P_0), \]  
\[ (4) \]
where \( v \) – speed of the working fluid, average pipeline cross section; \( P_0 \) – rated pressure of the power supply.

**A. To solve the optimization task by several criteria, it is necessary to consider the function of each criterion separately**

*Minimum power consumption of the power supply* \( N \to \min \)

The power to drive the hydraulic system is selected from the aircraft powertrain, so the reduction in hydraulic power consumption will increase the share of power consumed by the main powertrain function.

Let us consider the first equation in the system (3) for the branch originating directly in the power supply. The flow rate \( Q_i \) and pressure \( P_i' \) for such a branch are the pump capacity and nominal pressure, respectively. Then the equation is converted to an equation that uniquely determines the power requirement of the power supply

\[ N = \sum_{j=1}^{n} C_j \sum_{i=1}^{m} B_i L_{fr_i}, \]  
\[ (5) \]

where \( N \) – power consumption of the power supply source; \( n \) - number of consumers in the system; \( m \) - number of branches in system; \( C_j \) – customer parameter \( j \). For hydraulic cylinder \( C = F \cdot \frac{u}{\rho} \); \( B_i \) – a parameter that takes into account the fluid flow mode. For laminar flow \( B = 8\pi\nu\rho u^2 L_{fr_i} \) – branch pipeline length \( i \).

Targeted minimum power optimization function

\[ N = f (P_0, v) \to \min. \]  
\[ (6) \]

**B. Minimum weight* \( M \to \min \)

Weight is one of the most significant characteristics of aeronautical equipment. Its overall efficiency depends on the mass characteristics of the aircraft. The minimum mass is most often accepted as the main criterion in optimizing the glider design, selection of aggregates, optimization of systems and equipment [shmyrev]. Hydraulic system mass consists of masses of pipelines, connecting fittings, equipment, units and other equipment.

In order to take into account changes in the mass of other elements within the framework of the research task, it is reasonable to use the following coefficients:

- coefficient \( k_1 \), taking into account the weight of the equipment and connecting fittings, depending on the size of the conditional passage, i.e. the diameter of the pipelines;
- coefficient \( k_2 \), taking into account the increase of the system weight due to the fixing points with the increase of the pipeline diameter;
- coefficient \( k_3 \), that takes into account the weight of the fluid in the tank and the units;
- coefficient \( k_4 \), taking into account changes in the weight of the accumulator with changes in the flow rate and pressure in the system;
- coefficient \( k_5 \), taking into account the change in the weight of the hydraulic pump depending on the required power;
- coefficient \( k_6 \), taking into account the weight of the heat exchanger depending on the pressure loss.

\[ M = k_1 k_2 k_3 k_4 k_5 k_6 \left( k_1 k_2 \sum_{j=1}^{n} M_{fr_i} + \sum_{j=1}^{n} M_{hz_j} + k_3 M_i \right), \]  
\[ (7) \]

where \( k_1, k_2, k_3, k_4, k_5, k_6 \) – coefficients; \( M_{fr_i} \) – branch pipe weight \( i \); \( M_{hz_j} \) – consumption weight \( j \); \( M_i \) – weight of working fluid in pipelines and consumers

From the resulting system of equations (3) obtained as a result of the solution of geometric characteristics of pipelines and consumers, it will not be difficult to obtain a mass of these elements in terms of strength on the basis of the characteristics of the received material.
Taking into account the solution of the equation system (3) by varying the speed of the working fluid and the pressure of the power supply, the target function of optimization by mass of the hydraulic system

\[ M = f(P_0, v) \rightarrow \text{min}. \]  

(8)

It should be noted here that the paper deals with the solution of the research task of designing a hydraulic system, so the methods of design and calculation of specific units are not considered, their mass is taken into account by means of coefficients. In order to solve the practical problem of hydraulic system optimization, it is better to switch from accounting for the mass of aggregates by means of coefficients to specific aggregates samples selected from the corresponding catalogues.

3. Optimization method

Target optimization functions are defined only algorithmically; direct search methods are usually used to find the minimum of such functions. Direct search methods do not use information about derivative functions, and also do not require analytical representation of functions [8]. In addition to direct search methods, the random search method is also available here. Such methods are also called zero order methods.

In zero order methods, the derivatives of the target function do not need to be calculated to determine the direction of the descent. In this case, the direction of minimization is fully determined by consecutive calculations of the function values.

To solve the optimization task, several criteria of the function of each criterion are considered separately. Then, to find a common solution, the front of the solution by Paretto is determined.

As a result of the solution, the optimal values of the input parameters that make up the vector (4) are determined, as well as the physical parameters corresponding to the optimal effective distribution. For each branch the pressures in the initial and final nodes, as well as the flow rates are known. The required geometric parameters are unambiguously defined on the basis of these data and the data of the solution vector (4)

\[ d_{fr_i} = \sqrt{\frac{4Q_i}{u\pi}}, \]  

(9)

\[ d_{pj} = \sqrt{\frac{4P_j}{F_j\pi}}, \]  

(10)

where \( d_{fr_i} \) – branch pipeline diameter \( i \); \( Q_i \) – branch flow \( i \); \( u \) – Average velocity of the bath fluid over the pipe section; \( d_{pj} \) – the diameter of the consumer's cavity \( j \); \( P_j \) – pressure of consumer \( j \); \( F_j \) – customer effort required \( j \) (hydraulic cylinder or single ram of the hydraulic motor).

4. Conclusion

The article describes the method of selecting the optimal geometric parameters of the hydraulic system of the aircraft. As criteria of optimality the minimum weight of the system and the minimum required power of a power supply source are chosen. The mathematical model providing effective flow distribution in any branched hydraulic system is presented. Target functions and all equations are obtained in general form. The technique allows to set the output characteristics of the hydraulic system (force, speed and movement on the actuators) and its spatial position, to determine the optimal characteristic dimensions of pipelines and consumers in providing the calculated operation of the mechanisms.

The resulting system of equations connects the physical and geometric parameters of the hydraulic system. Optimization of geometric parameters is carried out within physical parameters of the hydraulic system.

Practical significance of the obtained results lies in the possibility of obtaining a design solution with indication of pump capacity, pipeline diameters, executive cavity diameters of consumers, based on the required output characteristics of the hydraulic system.

The basic methodical results and algorithms are invariant concerning what formulas are included in mathematical models.

The obtained system of equations is a mathematical model of the hydraulic system with effective flow distribution [20]. It is convenient to use such a model as a basic one and by means of time
differentiation to pass, for example, to account for the presence of regulating devices in the hydraulic system, to account for changes in the characteristics of the working fluid during the flow (heating, viscosity change, etc.).

The developed technique is very convenient for mathematical modeling. Currently, work on the implementation of the presented mathematical model and methods of optimization of hydraulic systems in the form of a program code based on Matlab is actively conducted. Such a program can be used as part of the expert system for the design of hydraulic systems of aircraft.

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