Weak G-band stars on the H-R Diagram: Clues to the origin of Li anomaly

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1. Introduction

Defining WGB stars

The weak G-band (hereafter WGB) stars are G and K giants whose spectra show very weak or absent G-bands of the CH $\Delta A - X^2\Pi$ system at 4300Å. These stars were first identified as stellar class by Bidelman [1951], and have been mainly studied in the late seventies, early eighties with a total number of dedicated papers not exceeding 20 (see e.g. for instance Sneden et al. [1978], Rad [1978], Parthasarathy & Rad [1980], Day [1980]). They are rare with less than 30 known to date among the population of G-K giants in the Galaxy.

Chemical composition studies [Sneden et al. 1978, Cottrell & Norris [1978] demonstrated that they are very much underabundant in carbon (typical $[\text{C}/\text{Fe}] \approx -1.4$) and present small overabundances of nitrogen and normal oxygen. Sneden et al. [1978] analyzed the $^{13}\text{CN}$ red-system features in the high resolution spectra of weak G-band stars and found $^{12}\text{C}/^{13}\text{C} \approx 4$. Hartoog et al. [1977] from the photometry of the 2.3μm CO vibration-rotation bands confirmed the underabundances of carbon in a large sample of weak G-band stars. The CH band strengths observed by Rad [1978] also indicate that the weakening of the G-band is due to underabundance of carbon. Sneden & Pilachowski [1984] obtained high resolution spectra of the 2μm first-overtone CO bands in the weak G-band giants and found excellent agreement between the carbon abundances derived from CO data and those determined using features of the CH G-band. They have found $^{12}\text{C}/^{13}\text{C} \lesssim 4$
which is in agreement with the predicted ratio for the CN-cycle in equilibrium. The normal abundance of oxygen and sodium shows that the atmospheres of WGB stars are probably not mixed with ON-cycle processed material (Sneden et al. 1978, Drake & Lambert 1994). Additional chemical constraints are given by lithium and beryllium abundance determinations. Li abundances have been derived for several WGB stars and several stars were found to be lithium rich, e.g. with A(Li) ≥ 1.4 (see Brown et al. 1989 and reaching up to A(Li) = 3 (Lambert & Sawyer 1984, Parthasarathy & Rad 1980). Lambert & Sawyer 1984 also report on the possible presence of 6Li in HR 1299, but the profile of the 6707 Å Li line that is better reproduced with 7Li/6Li ≠ 0, could also be reproduced without having to invoke the presence of 6Li. Be abundances were derived from IUE spectra for 3 Li-rich weak G-band stars by Parthasarathy et al. 1984. Providing a differential analysis between the WGB stars and K giants in the Hyades, Be is found to be similar in both groups, and compatible within presumably large errorbars with the expected post dredge-up Be abundances according to standard stellar evolution models.

Tentative evolutionary status and initial mass estimate

Due to their relative low temperature and gravity, WGB stars are classified as giants and fall amidst the very crowded area of the HR diagram populated with stars on the subgiant branch, the RGB, the core helium burning phase and the AGB. This has made the definition of their evolutionary status and initial mass difficult. They seem however to have been unanimously qualified as core helium burning star. In particular (Sneden et al. 1978) tentatively attribute a mass of = 1M⊙ to HR 6766 (e.g. HD 165634) and exclude it from being more massive than 3 M⊙ because of the magnitude they derive. With such a low-mass, that star should then have undergone the He flash. Let us note that their conjecture is based on a very crude estimation for the luminosity due to the large error bars impairing the parallax of HR 6766. Cottrell & Norris (1978) also identify WGB stars in their sample as being in the mass range 1.5 M⊙ ≤ M ≤ 3 M⊙ past the He flash. Later on, Lambert & Sawyer (1984) proposed that WGB stars could be the progeny of magnetic Ap stars, with initial masses of 2-3 M⊙ that have undergone a dredge-up, but they do not make any clear statement of their evolutionary status. Much recently Parthasarathy & Jasniencic (2000, 2002) suggest that WGB stars should be the descendants of intermediate-mass stars with 2 M⊙ ≤ M ≤ 5 M⊙. In that case, WGB should not go through the He flash, and will undergo a different evolution than that of the lower mass stars. This in turn may have an impact on the possible mechanisms that can be invoked to explain the abundance anomalies.

Comparison with other carbon depleted and Li-rich giants

WGB stars are thus possibly low to intermediate mass evolved stars exhibiting strong depletion of carbon that can be accompanied in several cases by a Li enrichment. One may wonder how these objects compare with the other stars that also present signatures of CN-cycle processed material in their atmospheres and exhibit unexpected Li enrichment. Giant stars with depleted carbon and enhanced nitrogen abundances are found in almost all environments among giants. These are essentially low-mass stars on the RGB. Charbonnel & Do Nascimento (1998) show that 96% of field RGB stars with metallicities from solar to 1/1000 solar and Mv ≤ −0.5 have a carbon isotopic ratio lower than what is predicted by standard theory. In their survey of metal-poor field stars, Gratton et al. (2000) show that the unexpected decrease of the carbon isotopic ratio occurs at the RGB bump, when the outgoing hydrogen burning shell (hereafter HBS) erases the mean molecular weight discontinuity left behind by the retreating convective envelope as a result of the first dredge-up. It is furthermore associated with a decrease of the carbon abundance [C/Fe], an increase of the nitrogen abundance [N/Fe] and a strong decrease of the lithium abundance, all characteristic of CN-cycle processed material. Oxygen does not seem to be affected, as in the case of WGB stars.

Typically, Charbonnel & Do Nascimento (1998) report that around solar metallicity the lower envelope for the 12C/13C ratio in low-mass RGB stars is of about 12. Gratton et al. (2000) report [C/Fe] = −0.58 ± 0.03 as a mean carbon abundance for upper RGB low-metallicity field stars (e.g. log(L/L⊙) > 2 and −2 ≤ [Fe/H] ≤ −1), and 12C/13C between 6-10. These values are higher than the mean obtained for WGB stars.

In addition to that Li-enrichment also exists among K giants. It is a very rare phenomenon found in less than 2% of the K giant stars (Kumar et al. 2011). Charbonnel & Balachandran (2000) have shown that around solar metallicity, giants exhibiting a genuine overabundance of Li (e.g. excluding those giants that are still undergoing the first dredge-up) appear to cluster around the RGB bump and the clump. Very recently however Monaco et al. (2011) have studied Li-rich RGB stars in the Galactic thick disk, and show that at low metallicity Li-rich giants do not cluster at the bump and are found scattered between the bump and the tip of the RGB (see also Lebzelter et al. 2011, Kumar et al. 2011). Charbonnel & Balachandran (2000) confirmed the finding by Charbonnel & Balachandran (2000), with Li-rich stars clustered at the RGB bump, and some new objects that seem to be more evolved, may be past the RGB tip and undergoing central He burning. Let us note that Li-rich K giant stars around the bump region do not present “anomalous” carbon isotopic ratios, e.g. the 12C/13C determined in these stars well agrees with the predictions of post dredge-up dilution (Charbonnel & Balachandran 2000, Kumar et al. 2011).

In these K giants, Li enrichment is thought to have an internal origin (so called Cameron & Fowler [1977], mechanism) and to be a short-lived phase preceding the phase of transport CN-cycle processed material from the HBS to the surface (Charbonnel & Balachandran 2000, Palacios et al. 2001). The

1 With the classical notation A(Li) = log(N(Li)/N(H)) + 12
2 The authors do not indicate any errorbar in the paper and give abundances of Be “rounded to the nearest 0.5 dex.”
extra-mixing mechanism capable of transporting Li in regions where it is preserved from proton captures and connecting the convective envelope to these regions is not clearly identified [Palacios et al. [2001], Palmerini et al. [2011]].

As a summary, in low-mass RGB stars, extra-mixing processes are clearly responsible for bringing up to the stellar surface the products of the CN-cycle occurring in the outer layers of the HBS deeper inside the star, and may be also responsible for strong and short-lived episodes of Li enrichment. The specific location of Li-rich K giants (see, for example [Jasniewicz et al. [1999]]) points towards an extra-mixing process that becomes efficient only when no strong molecular weight barrier shields the CN-cycle processed regions. This occurs only after the RGB bump in low-mass stars ($M \leq 2.3M_\odot$) or on the early AGB for the stars that do not go through the He flash ($M > 2.3M_\odot$). Carbon depletion and nitrogen enhancement at the stellar surface that would result from internal mixing are thus expected to happen during these specific evolutionary stages according to the proposed self-consistent scenarios [Charbonnel & Balachandran [2000], Charbonnel & Lagarde [2010]].

Paper outline

In this paper, we propose a new re-analysis of the data available for WGB stars and investigate the possible origin for their abundance anomalies, giving particular attention to the lithium abundance anomalies. In particular we investigate the possible physical mechanisms that could account for the simultaneous lithium overabundances and strong carbon underabundances. In the following, we describe the homogeneous redetermination of atmospheric parameters and lithium abundances for the WGB stars known to date in §2 and §3. We then place the WGB stars in the HR diagram and make a tentative classification in terms of evolutionary status and initial masses in §4. In §5 and §6 we finally explore possible scenarios to consistently account for the light elements (Li, C, N and O) abundances found in WGB stars based on the results of dedicated stellar evolution models.

2. Atmospheric parameters

The most complete list of weak G-band stars known up to now is presented in Table 1. So far 28 WGB stars have been identified in the literature. This represent a very small fraction (< 1%) of K giants and makes these objects pertain to a very rare class. In order to study their position in the HR diagram we have firstly determined their effective temperature $T_{\text{eff}}$ by using both $(B - V)$ and $(V - I)$ colors from the Hipparcos catalogue [van Leeuwen [2007]] and the improved color-temperature relations for giants from Houdashelt et al. [2000] or Alonso et al. [1999]. Colors of some stars are affected by a little interstellar reddening and we take it into account for the determination of $T_{\text{eff}}$ and the bolometric correction $BC$ by using a least squares approximation in varying accordingly the color excess $E(B - V)$ and $E(V - I)$. The error on $T_{\text{eff}}$ is estimated from our interpolation processing in the Houdashelt et al. [2000] tabular data. The bolometric magnitude $M_{\text{bol}}$ and gravity log($g$) are given by:

$$M_{\text{bol}} = V - 3.1 E(B - V) + BC(V) + 5 \log(\pi) + 5 \quad (1)$$

$$\log(g) = \log(M) + 0.4 M_{\text{bol}} + 4 \log(T_{\text{eff}}) - 12.505 \quad (2)$$

where $\pi$ is the Hipparcos parallax [van Leeuwen [2007]] and $M$ the mass of the star in solar units. The $V$ magnitude is also taken from the release of the Hipparcos catalogue by van Leeuwen [2007]. In the preceding equation, gravity log($g$) is little dependent on the mass of the star in the range 2-4 solar masses. Due to the location of the stars in the HR diagram (see Fig. 1) we assume $M = 3.5 M_\odot$ for the calculation of log($g$) in Table 1. The error on log($g$) comes essentially from the error on $T_{\text{eff}}$. The errors on the luminosities are computed using the quoted uncertainties in the parallaxes and magnitudes. They may be huge for certain WGB stars as they propagate from ill-determined parallaxes. Atmospheric parameters given in Table 1 are in excellent agreement with those calculated recently by other authors [Takeda et al. [2008], Wu et al. [2011]].

3. Lithium Abundance

Lithium abundance for most of the weak G-band stars, derived from the available strengths of Li resonance line at 6707.8 Å, are given in Table 2. The equivalent widths of Li line and metallicities are sourced from the literature [Hartoog [1978], Lambert & Sawyer [1984]]. Solar metallicity is considered for the giants whose metallicity information was not available in the literature. Local thermodynamic equilibrium (LTE) stellar model atmospheres computed by Kurucz [1994] with convection option on, and the revised radiative transfer code MOOG originally written by Sneden [1973] were used for the analysis. The well tested line list [Reddy et al. [2002]] in the region of Li resonance line 6707.8Å was adopted. Hyperfine structure and the $gf$ values of $^7$Li components of Li line at 6707.8Å were taken from Hobbs et al. [1999]. NLTE corrections are applied using the recipe given by Lind et al. [2009].

The information on the lithium abundance, in the Table 2, is given for 19 stars. Lithium studies have not been attempted until now on those nine stars whose Li information is missing in the Table 2. Among the 19 stars with available lithium, 11 giants can be qualified as “Li-rich”. This denomination refers to an unexpected large Li abundance compared to the post dredge-up value of about 1.4 dex expected for giant stars at solar metallicity from standard stellar evolution theory (see this work and Brown et al. [1989]). This represents about 39% of the WGB stars and almost 58% of the WGB for which lithium has been searched for. The remaining eight stars are lithium poor. WGB stars present a large range of Li abundances from -0.2 to 3.0 dex. Contrary to K giants, Li overabundances are much more common in this subclass of stars, amounting to about 58% for those stars where the lithium region around 6707.8 Å has been observed. No weak G-band star is found to have Li abundance more than main sequence or ISM value (A(Li) ~ 3.28). This very large percentage by itself adds to the peculiarity of

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WGB stars among other types of lithium-rich stars, in particular Li-rich giants that are very rare.

We will discuss further the possible origin to these unexpected large lithium abundances in $\S$ 6.

4. HR Diagram

In order to better evaluate the evolutionary status of the weak G-band stars (also with respect to that of the Li-rich K giants), we have computed two grids of dedicated models for stars with solar metallicity (the solar chemical composition adopted as a reference is that of Grevesse & Noels (1993)) in the mass range 3 to 4.5 $M_\odot$. The models were all evolved from the pre main sequence to the early AGB phase. They were computed using the STAREVOL V3.0 code, that has been extensively described in $\S 5$. The models were computed assuming solid-body rotation in the convective envelopes (which should result in a minimal estimate of the amount of shear mixing possibly developing in the radiative region connecting the convective envelope and the nuclear active regions; see Palacios et al. 2006), and rotation and thermohaline mixing were included from the Zero Age Main Sequence up to the early-AGB phase. The formalism used is the same as in Charbonnel & Lagarde (2010) and the reader is referred to this paper for more details. Let us just recall that we assume an efficient thermohaline mixing following Ulrich (1972), which might turn out to be a maximum efficiency approach given the lack of actual good description of this thermodynamic instability in astrophysical regime (Cantiello & Langer 2010). As in Charbonnel & Lagarde (2010), the rotation velocity of the models when they reach the main sequence is assumed to correspond to 45% of their critical velocity, which results in mean equatorial rotation velocities of about 200 km.s$^{-1}$. Mass loss has been included in all models using the classical Reimers law up to the early AGB phase.

Figure 1 displays the luminosity effective gravity of the WGB stars as a function of the effective temperature together with the evolutionary tracks. Only those objects with accurate enough parallaxes (e.g. $\epsilon (\log(L/\odot)) < 0.5$) are shown. The shaded areas in panels a) and c) of Figure 1 do not include mixing outside the convective regions, and the convective boundaries are fixed according to the Schwarzschild criterion for the convective instability. No overshooting is included. The tracks associated with the rotating models are displayed in panels b) and d) of the same figure. They were computed assuming solid-body rotation in the convective envelopes (see text).
Fig. 1. Weak G-band stars with the best parallaxes (error on the log(L/L\(_{\odot}\)) lower than 0.5 dex) are plotted in the Hertzsprung-Russell diagram (upper panels) and in the (log g, T\(_{\text{eff}}\)) plane (lower panels) along with evolutionary tracks for models with initial masses 3, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2 and 4.5 M\(_{\odot}\) as described in § 4 (from bottom to top in the HR diagrams and in the reverse way, from top to bottom, in the (log g, T\(_{\text{eff}}\)) diagrams.). Left panels represent classical models with no rotation nor diffusion processes included, and right panels show models including rotation and thermohaline mixing. The bold line intersecting the racks on the left side of the plots marks the beginning of the first dredge-up. The size of the symbols is proportional to the Li content (the more lithium, the larger the symbol). Empty symbols are those for which no lithium abundance exists to date. The triangle on each of the plots represents star HD 21018 which is a spectroscopic binary. The shaded areas in the HR diagrams indicate the location of the Li-rich K giants, corresponding to the bump area for models with initial masses lower than 2.3 M\(_{\odot}\) and to the early-AGB phase for the models in the mass range presented here.

Overlap exists and Li-rich WGB stars being more luminous appear to clearly detach from the bulk of Li-rich K giants. The thick line intersecting the tracks on the left part of the plots indicates the beginning of the first dredge-up. WGB stars appear clearly to be stars in the mass range 3 to 4.5 M\(_{\odot}\) at the clump. Their position in the HR diagram makes the determination of their evolutionary status ambiguous: they could either be on the red giant branch, undergoing the first DUP, or be in the core helium burning phase. Cottrell & Norris (1978) already suggested that WGB stars could be core helium burning stars in their study of HD 91805, but contrary to what they thought, WGB stars are not passed the helium flash since this event only occurs in stars with a mass lower than approximately 2.3 M\(_{\odot}\) (see e.g. Palmerini et al. 2011). In the work Cottrell & Norris also suggested that meridional circulation induced by rotation in main sequence stars of intermediate mass could be a possible mechanism to explain the carbon strong depletion of WGB stars. We therefore computed models including this transport process along with turbulent shear and thermohaline mixing in order to test this hypoth-
Rotation velocities ($\nu \sin i$) have been measured for a limited number of WGB stars and are reported in Table 2. Figure 2 shows the evolution of the surface equatorial velocity of our rotating models during the RGB and subsequent evolutionary phases. In order to mimic a mean $\nu \sin i$ associated to what can be considered a mean equatorial velocity given by 1-D stellar evolution models, we have multiplied the velocity given by our models by $\frac{\pi}{2}$, following Chandrasekhar & Münch (1950). The observational data seem to suggest rotation rates lower (by a factor of two approximately) than what is predicted by our models. Let us mention here that conservatively adopting a solid-body rotation law for the convective envelope favours larger surface rotation rates. Using a probably more realistic differential rotation in the extended convective envelopes of these stars as they evolve through the giant phases will lead to lower surface rotation velocities (see Brun & Palacios 2009). HD 21018 is on the other hand quite well reproduced considering the large error bars on the parallax.

As can be seen from Fig. 1, the evolutionary tracks are slightly modified by the action of rotation and thermohaline mixing, having less extended and noisier blue loops. The tracks nonetheless match the region occupied by the WGB stars as well as the standard models do.

In the following, we confront the models, standard and with rotation, to the abundance data and discuss the possible origin of the WGB stars in the light of this confrontation.

5. Abundances

The position of WGB stars in Fig. 1 indicates that they are bright stars of intermediate mass. The expected evolution of the surface abundances of lithium, carbon, nitrogen and carbon isotopic ratio for our standard and rotating models is shown in Fig. 3 together with the values reported in Table 2 for the WGB stars. In this figure, the squares in different shades of blue/violet represent correspond to HD 166208 (violet), HD 165634 (brown), HD 91805 (cyan) and HD 18474 (blue).

Lithium

The evolution of the surface abundance of lithium in standard (left) and rotating models (right) is displayed in the first row. The weak G-band stars are either Li-rich or Li depleted. For those stars that have Li abundances lower than 1.4 dex, we clearly see that standard stellar evolution models fail to reproduce them, the level reached after the first and second dredge-up episodes being of about 1.4 dex. On the contrary the models including rotation-induced mixing (e.g. meridional circulation + secular turbulent shear instability) and thermohaline mixing undergo lithium depletion already during the main sequence phase and achieve $A(\text{Li}) \approx 0$ in very good agreement with the NLTE corrected lithium abundance derived for HD 67728, HD 78146 and HD 91805.

The rest of the WGB stars exhibit lithium abundances that would be typical of first dredge-up dilution in standard (non-rotating) models and in models with slow rotation on the ZAMS (50 km s$^{-1}$). This can be seen from the upper panels of Fig. 3 where the grey shaded area corresponds to the location of tracks for slow rotating models. Among those for which lithium lines were actually observed, the proportion of WGB stars with $A(\text{Li}) > 1.4$ dex is of about 58%, which means either that a large proportion of core helium burning WGB stars are Li-rich, or that a large fraction of the WGB stars are young red giants in which the first dredge-up is not completed yet.

We postpone a more detailed discussion to §6

Beryllium

The evolution of the surface Be abundance in our models lead to a mean post-dredge-up value (first and second) of about -0.2 dex for the models computed, while the models including rotation and thermohaline mixing reach $A(\text{Be}) \approx -0.65$ dex. These values are to be compared with the abundances estimated from IUE spectra by Parthasarathy et al. (1984) of about -0.5 dex. Errorbars on observed values are not detailed in the original paper but the value is considered an upper limit so that non-standard models are here again favoured.

Carbon

Actual derivation of carbon abundances exist only for a bunch of stars with known parallaxes, that are represented together with our models predictions in the second row of Fig. 3. While models including rotation-induced and thermohaline mixing
Table 2. Chemical abundances and rotational velocities of weak G-band stars

| HD    | [Fe/H]  | A(Li)   | W_i   | A(Li)   | A(Li)_{WIGE} | [C/H]     | [N/H]     | [O/H]     | ^{12}C/^{13}C | V_{sini} |
|-------|---------|---------|-------|---------|--------------|-----------|-----------|-----------|----------------|---------|
| 18474 | −0.10^a| 1.42^a  | 29^a  | 1.33    | 1.48         | −1.6      | 0.2       | −         | −               | 1.9^i   |
| 18636 | −0.11^b| 1.77^b  | 66^b  | 1.71    | 1.853       | −         | −         | −         | −               | −       |
| 21018^c| 2.89^c  | 245^c   | 3.13  | 3.06    | −           | −         | −         | −         | −               | 20.1^g |
| 26575 | −0.01^b| 2.02^b  | 239^b | 2.16    | 2.34         | −         | −         | −         | −               | −       |
| 28932 | −       | 2.25^a  | 158^a | 2.04    | 2.212       | −         | −         | −         | −               | −       |
| 31274 | −0.27^b| < 0.52^b| < 4^b  | −0.67   | −           | −         | −         | −         | −               | −       |
| 31869 | −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 36552 | −0.17^b| 2.58^b  | 173^b | 2.64    | 2.70         | −         | −         | −         | −               | −       |
| 40402 | −0.45^b| 2.44^b  | 215^b | 2.60    | 2.586       | −         | −         | −         | −               | −       |
| 49960 | −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 56438 | −0.38^b| 1.43^b  | 104^b | 1.59    | 1.774       | −         | −         | −         | −               | −       |
| 67728 | −       | 0.76^a  | 15^a  | −0.22   | 0.06        | −         | −         | −         | −               | −       |
| 78146 | −0.31^b| 1.01^b  | 22^b  | −0.27   | 0.01        | −         | −         | −         | −               | −       |
| 82595 | −10^b   | 1.14^b  | 22^b  | 0.89    | 1.136       | −         | −         | −         | −               | −       |
| 91805 | 0.0^c   | 0.25^b  | 03^b  | 0.06    | 0.202       | −1.4      | 0.6       | 0.1^f     | −               | −       |
| 94956 | −       | 1.07^a  | 15^a  | 0.65    | 0.834       | −         | −         | −         | −               | −       |
| 10285 | −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 119256| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 120213| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 120170| −0.07^a| 3.03^a  | 253^a | 3.14    | 3.02        | −         | −         | −         | −               | −       |
| 124721| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 146116| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 165462| −       | 2.05^a  | 141^a | 1.87    | 2.068       | −         | −         | −         | −               | −       |
| 165634| −0.15^f| 0.75^f  | −     | −       | −1.49 0.23 −0.38^f 4.1^f 1.49^b | −         | −         | −         | −               | −       |
| 166208| +0.2^d  | 1.65^a  | 74^a  | 1.73    | −0.9 0.8    | −9.45^d  2.9^g | 6.3^g | − | − | − | − |
| 188328| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 204046| −       |         |       |         | −           | −         | −         | −         | −               | −       |
| 207774| −       | 1.5^d   | 30^d  | 1.06    | 1.24        | −         | −         | −         | −               | −       |

^a Hartoog (1978) , ^b Lambert & Sawyer (1984) , ^c From our spectra, ^d Day (1980) , ^e Cottrell & Norris (1978) , ^f Sneden et al. (1978) , ^g Glebocki & Gniaciński (2005) , ^h Hekker & Meléndez (2007) , ^1 Binary.

lead to [C/H] ≈ −0.25 dex by the end of the core helium burning phase, about 0.05 dex larger a depletion than predicted for classical stellar evolution models, these values are far from reaching the extreme underabundances derived for HD 166208, HD 165634, HD 91805 and HD 18474 (see also Table 2). The combined action of meridional circulation and turbulent shear and thermohaline mixing that has been proposed as a physical process to explain light-elements (Li, C and N) abundance anomalies in RGB stars (Charbonnel & Lagarde 2010) clearly fails here to account for very large carbon depletion in WGB stars when using the same set of parameters as those adopted for lower mass stars. Let us note that among the WGB stars for which [C/H] is available, HD 91805 is “normal” in terms of lithium (and also nitrogen, see below) abundance when compared to non-standard models including rotation and thermohaline mixing, while the other three stars are Li-rich (A(Li) ≥ 1.4 dex).

The carbon deficiency, the common feature to the WGB stars, thus seems to be decorrelated from the other abundance anomalies, in particular the lithium content, and could have a different origin.

Nitrogen

Nitrogen abundances are available from the literature for the same stars for which the carbon abundance has been derived. The third row (from the top) of Fig. 3 displays the evolution of the [N/H] as predicted by stellar evolution and the abundances derived for HD 166208, HD 165634, HD 91805 and HD 18474. In our non-standard models, nitrogen is enhanced during the RGB phase (up going parts of the tracks) as a result of the continuously ongoing rotation-induced mixing allowing to bridge the gap between the convective envelope and the hydrogen burning shell. Thermohaline mixing only develops near the core, but anyway remains much slower than meridional circulation and shear-induced turbulence, and is not the lead process shaping the surface abundance pattern of our models.

Both standard and non-standard models lead to surface abundances at the core helium burning phase that are in agreement with the observational data within the (large) error bars. We may however distinguish two groups:

1. the more nitrogen rich stars HD 91805 (cyan square at $T_{\text{eff}} = 5350K$) and HD 166208 (purple square at $T_{\text{eff}} = 5050K$) are clearly better fitted by rotating models undergoing core helium burning, in particular HD 91805;
Fig. 3. Abundances of lithium, carbon, nitrogen and carbon isotopic ratio as a function of the effective temperature for the objects listed in Table 2. A conservative error bar of 0.2 dex has been adopted for the NLTE lithium shown in the upper panels. The tracks are for standard (left column) and rotating models (right column) with masses ranging from 3.0 to 4.5 $M_\odot$ (from left to right). The colors of the tracks indicate the evolutionary phase: black for main sequence, green for RGB, blue for central He burning and red for AGB phase. Squares in different shades of blue are used in the upper panel to represent those stars for which $[\text{C}/\text{H}]$ and $[\text{N}/\text{H}]$ is also available from the literature (see text). On the lower panels, the thick black line at $^{12}\text{C}/^{13}\text{C} = 4$ represents the carbon isotopic ratio found in HD 16634 ([Snedden et al. 1978]) and in HD 166208 ([Day 1980]). The grey areas in the left column plots show the region occupied by tracks for rotating models with $v_{ZAMS} = 50$ km s$^{-1}$ (see text).

2. The nitrogen abundances of HD 165634 and HD 18474 fall in between $[\text{N}/\text{H}]_{SB} (\approx 0.05$ dex) and $[\text{N}/\text{H}]_{\text{clump}} (\approx 0.6$ dex) as predicted by both our sets of models. This is quite puzzling and definitely not of any help to pinpoint the evolutionary status of these stars.

6. Origin of the peculiar chemical pattern of WGB stars

From the preceding sections § 4 and § 5 weak G band stars appear to be intermediate mass stars with initial masses in the range 3 to 4.5 $M_\odot$ that are undergoing either the first dredge-up or are in the core helium burning stage. A large fraction of these stars is lithium rich ($A(\text{Li}) > 1.4$ dex), and all of them present large carbon underabundances even when compared to
non-standard stellar evolution models including thermohaline mixing and rotation.

When trying to understand the observations, one needs to be able to account for the following features:

1. All WGB stars are carbon deficient, and for the 4 stars with available measurements for the carbon abundance, [C/Fe] < −1 dex; carbon isotopic ratio is also seemingly very small (around equilibrium value 4). The observed under-abundance are larger by at least 1 dex than the one predicted by standard and non-standard models;
2. A large fraction (about 58%) of the WGB stars for which lithium lines have been observed exhibit A(Li) > 1.4 dex;
3. Lithium and nitrogen surface abundances as well as evolutionary status are well matched by stellar evolution models including rotation and thermohaline mixing for those stars that present a low lithium surface abundance;
4. The rotation velocity of WGB stars with measured $v \sin i$ is marginally reproduced by rotating stellar evolution models with $v_{\text{ZAMS}} \approx 200 \text{ km s}^{-1}$.

6.1. “Li-poor” weak G band stars

As partly discussed in § 5, lithium poor WGB stars are actually normal except for their carbon abundance. HD 91805 is used as a prototype since we have access to its luminosity, temperature, gravity, lithium, carbon and nitrogen abundances. This star is well fitted by models including rotation, rotation-induced mixing (meridional circulation + shear turbulence), and thermohaline mixing with an initial mass of about 4.5 $M_\odot$.

If we consider an internal origin for the carbon underabundance, as proposed by Cottrell & Norris (1978), Tomkin et al. (1984), the evolutionary status of red giants (RGB) stars should be ruled out. Considering their mass, WGB stars experience the end of the first dredge-up while already undergoing core helium burning, and no such extra mixing processes as rotation-mixing or thermohaline mixing are expected (and actually seen in the models) to be efficient enough to create the large carbon depletion. Furthermore, admitting that some extra-mixing acts as an overshooting and makes the dredge-up deeper, the very large extent of the convective envelope at this stage implies a very large dilution factor that will prevent the carbon abundance and carbon isotopic ratio to drop to the observed value within the duration of the RGB phase. Even though the mixing reaches down to the region where CN-cycle occurs, this region is far too shallow compared to the convective envelope for this late region to be completely processed.

Meridional circulation and shear turbulence are actually efficient in earlier evolutionary phases as can be seen in Fig. 5 but they mainly act on the lithium abundance and the associated mixing does not reach the carbon depleted regions that are located deeper in the core. Contrary to what was suggested by Lambert & Sawyer (1984) or Cottrell & Norris (1978), it seems very unlikely that carbon depletion is the result of internal mixing during the early evolution of the progenitors of WGB stars.

During core helium burning, on the other hand, the HBS crosses the composition gradient left by the dredge-up when the stars are located at the “turning point” of the blue loop, and then the regions were CN-cycling occurs are again accessible to extra-mixing processes. However, the physical distance separating the nucleosynthetic regions from the now much shallower convective envelope is very large (more than 22 $R_\odot$) for an extra-mixing process to be efficient; it is then needed that its characteristic timescale would be shorter than the evolutionary timescale of about 20 to 200 Myrs typical for stars in the mass range considered here. In terms of diffusion coefficient, this would imply $D_{\text{extra mix}} \approx 10^8$ – $10^{10} \text{ cm}^2\text{s}^{-1}$. The total diffusion coefficient obtained in our non-standard models ($D_{\text{MC}} + D_{\text{turb}} + D_{\text{Tih}}$) is always smaller by at least one order of magnitude. Such high rates of mixing are difficult to obtain using the available prescriptions for hydrodynamical instabilities.

Altogether these points indicate that the extra-mixing processes included so far in our stellar evolution models (meridional circulation + turbulent shear + thermohaline mixing) cannot produce the huge carbon underabundances but actually help reproduce the lithium and nitrogen abundances measured for the WGB stars if likely rotation velocities are adopted for their main sequence progenitors.

6.2. Li-rich weak G band stars

As already mentioned several times, WGB stars with $A(Li) \geq 1.4$ dex represent a large percentage of these stars. Such lithium abundances will be considered an evidence for lithium over-abundances if the stars are evolved past the first dredge-up, but would be within the expected range for stars that are currently undergoing the first dredge-up.

Let us analyze the available observational data using both interpretations and the consequences of adopting one or the other point of view for the origin of the peculiar abundance patterns of WGB stars.

WGB stars as core helium burning stars

If one assumes from their location at the clump in the HR diagram and the discussion in the previous paragraph concerning the evolutionary status of the Li-poor ones, that WGB stars are core helium burning stars, then lithium enrichment is everything but an exceptional feature among this stellar subclass. The Li-rich stars are rare among other classes of giants: in the case of low-mass stars, they are the result of seemingly very short enrichment episodes at which we manage to catch a glimpse for only 1% - 5% of the RGB stars (Charbonnel & Balachandran 2000; Palacios et al. 2001). For more massive stars, they are produced by Li-enrichment episodes in stars with $M_\star \geq 5M_\odot$ that undergo Hot Bottom Burning during the thermal pulse AGB phase (Palmerini et al. 2011).

Weak G band stars do not appear to be on the TP-AGB phase.
(too warm and not luminous enough). During the core helium burning phase there is no such thing as Hot Bottom Burning, and to explain Li-enrichment by internal mixing, one should invoke Cool Bottom Processing, a general denomination for any unspecified mechanism able to connect the convective envelope with deeper regions where nuclear reactions occur.

Cool Bottom Processing in relation with rotation-induced hydrodynamical instabilities has been proposed to account for Li enrichment at the RGB bump (Palacios et al. 2001) in the lithium flash scenario. In this scenario, the convective envelope of the RGB star is enriched with lithium when the outer regions of the HBS were \(^7\)Li\((p, \alpha)\)\(^4\)He occurs, becomes convectively unstable and connects with the envelope. This episode is very prompt in time, and is rapidly followed by a deepening of the extra-mixing. Carbon and lithium abundances subsequently decrease at the surface.

While one could be tempted to transpose this scenario to the Li-rich WGB stars, a difficulty arises since it excludes a simultaneous increase of Li and depletion of carbon. This could be overcome if we consider that the stars already present a strong carbon depletion prior to the red giant phase. Another difficulty resides in the timescales: short-time enrichment is difficult to reconcile with when almost 40% of the stars in the WGB class being Li-rich, but keeping in mind that WGB stars are very rare per se, this difficulty might not be as difficult to overcome as it may appear at first sight. Lambert & Sawyer (1984) suggested that the abundance anomalies of WGB stars, including Li enrichment and strong carbon depletion could be due to diffusion in the early stages of the evolution of A stars. They speculate that WGB stars could be the progeny of Ap stars which are well known magnetic CP stars. The role they attribute to magnetism is not at all contradictory since it excludes that the Li-rich WGB stars are descendants of A stars and are the progeny of late-B type stars. If we still consider the possibility that WGB stars are descendant of magnetic Bp stars, we can expect in these stars effects similar to those in magnetic Ap stars and a similar evolution. Concerning a diffusion scenario including the combination of gravitational settling and radiative accelerations, any abundance stratification that would be built during the main sequence will be erased by the first dredge-up (O. Richard, private communication). Moreover, for stars with \(T_{\text{eff}} > 12000\) K on the main sequence, the effects of diffusion are expected to be essentially confined to the atmosphere, and non-magnetic evolution models by Turcotte & Richard (2003) show that no specific accumulation or depletion of carbon is expected in the interior of these stars that could appear at their surface as they will evolve up the giant branch.

The evolutionary models for Ap/Bp stars are not yet available since the effects of magnetic fields and how to take them into account is still a matter of debate. Still, Charbonnel & Zahn (2007) suggested that magnetic Ap stars give rise to the very few RGB stars that present light elements abundances (\(^4\)He, \(^7\)Li, C, N) complying to the predictions of standard and rotating models. Strong magnetic fields are actually invoked as a mean to inhibit the thermohaline mixing that is thought to be responsible for the light elements abundance variations seen at the surface of RGB stars that have evolved past the bump. Following this work and considering that for intermediate mass stars, thermohaline mixing is inefficient unless maybe during the TP-AGB phase, that is at a more advanced evolutionary stage than that attributed to WGB stars, magnetic fields are not expected at all to favour carbon depletion of any sort, and the scenario proposed by Lambert & Sawyer (1984) is discarded.

Mass transfer (or mass accretion) appears as one of the very few options left to try and understand the WGB phenomenon. From the HST UV spectra Böhm-Vitense et al. (2000) find evidence for the presence of a white dwarf companion to the weak G-band star HD165634. On the other hand, Tomkin et al. (1984) searched for binarity in a sample of 7 WGB stars of the northern hemisphere, finding a binarity rate of 15%-40% , that is not very different from that expected for normal K giants. This result is however questionable since the number of targets selected was rather small and these observers did not exhibit the persistence that is really needed for an investigation of radial velocity variability because they stopped their observations after only two years (see arguments also given by Griffin 1992).

For the binary scenario to work, the secondary of the system should also be a star from which the accreted matter would be carbon depleted (and possibly sometimes also lithium enriched). According to Forestini & Charbonnel (1997) (see their Table 8), stars in the mass range 5-6 \(M_\odot\) at solar metallicity are expected to produce yields baring such a chemical imprint. In their models, due to the operation of efficient Hot Bottom Burning, they predict that the yields of \(^7\)Li are positive and more interestingly, that the net yields of \(^12\)C are negative with values up to \(-1.21 \times 10^{-2} \, M_\odot\). Forestini & Charbonnel (1997) do not give any expected decrease of C at the surface of the secondary. They find lifetimes of 111 Myrs and 65.8 Myrs for their 5 \(M_\odot\) and 6 \(M_\odot\) respectively at solar metallicity, to be compared with the 473 Myrs and 210 Myrs for their 3\(M_\odot\) and 4 \(M_\odot\) respectively. These numbers imply that the mass accretion from the more massive shorter lived primary onto the secondary would occur during the early main sequence evolution of the latter. As main sequence stars in the mass range 3 to 4.5 \(M_\odot\) have a radiative envelope, it is likely that the material that would be accreted would remain at the surface and possibly slowly diffuse inwards. If WGB stars are intermediate-mass stars undergoing core helium burning, they have already evolved through the first dredge-up, one of the main signatures of which is the decrease of surface abundances of carbon and lithium, and the increase of nitrogen abundance. The dredge-up of carbon depleted, lithium enhanced material could exacerbate the carbon depletion at the end of the dredge-up (but this need to be actually computed and check before drawing any conclusion) but will certainly not affect the lithium abundance decrease since the point is that the base of the convective envelope enters regions where lithium is destroyed by proton captures.

Another uncertainty of the binary scenario is that of the yields of intermediate-mass stars. They indeed vary greatly from one
source to another (see for instance Karakas & Lattanzio 2007), making any conclusion illusive.

**WGB stars as young red giant stars (SGB/RGB)**

Contrary to the Li-poor WGB stars, that are very seemingly rotating core He burning stars with anomalous carbon depletion, it is much more difficult to actually assess the evolutionary status of those that exhibit a lithium abundance larger than that expected after the completion of the first dredge-up. In fact, if no postulate is made concerning a common evolutionary status for all the stars populating the weak G-band subclass, then a strong ambiguity arises. From the sole position in the HR diagram and the comparison with the stellar evolution predictions for lithium abundance (and marginally nitrogen), we can not rule out the possibility that the so-called “Li-rich” WGB stars could turn out to be just “normal” stars as far as lithium is concerned, meaning with lithium abundances consistent with models predictions, according to their evolutionary status and their rotational history. Indeed, from Fig. 3 we see that the stars with large lithium abundance are well fitted by the standard tracks of intermediate mass stars that are undergoing the first dredge-up. According to these tracks, they should thus not be considered as lithium rich since the first dredge-up episode is not completed.

We may argue that the WGB stars should have experienced some rotational mixing being the progeny of B-type stars. The initial rotation spread observed in main sequence B-type stars in the mass range 2 to 4 M☉ as shown by Huang et al. (2010) is actually very large, and it might well be that WGB stars are also the descendants of slow rotating late-B type stars. In order to investigate the effect of a slower rotation velocity of the progenitors of the WGB stars on the ZAMS, we have computed 3 models (with initial masses of 3.0 M☉, 3.5 M☉ and 4.5 M☉) for which we adopted $v_{\text{ZAMS}} = 50$ km s$^{-1}$. The domain covered by these slow rotators is represented by the shaded area in Fig. 3. $v_{\text{ZAMS}} = 50$ km s$^{-1}$ corresponds to a ratio $v_{eq}/v_{\text{crit}} \approx 0.1$. With such a slow rotation, the transport due to meridional circulation and turbulent shear instability is not very efficient during the main sequence evolution so that lithium is little depleted at the surface of our models when they reach the turn-off. The deepening of the convective envelope during the first dredge-up leads to a decrease of the surface lithium and carbon abundances and to an increase of the nitrogen abundance similar to that obtained for standard stellar evolution.

Although it has been commonly assumed that WGB stars shared the same evolutionary status, our models along with the chemical constraints (from lithium and nitrogen) that are available for WGB stars could be telling a different story. This subclass could be populated with the progeny of slow to mild rotating ($v_{eq}/v_{\text{crit}} \leq 0.45$) B-type stars that are either experiencing the first dredge-up or that are past this phase and are core helium burning stars. In this context the large carbon underabundance associated to the G band weakness observed in all these stars, and the narrow mass range become the only common features of WGB stars.

As mentioned earlier a binary scenario is not to be excluded, but would deserve an in depth study before any conclusions are drawn.

### 7. Conclusions

In this study we have made a tentative re-analysis of the data existing for the WGB stars, in particular concerning fundamental parameters and lithium abundances. We have confirmed that WGB stars are within the mass range 3–4.5 M☉. However, the too few abundance determinations of Li, N and C and the fact that the WGB stars are located at the clump where stellar evolution tracks corresponding to the subgiant, red giant and core helium burning phases are intertwined prevent us to assess any sound evolutionary status to the WGB stars. Concerning the chemical peculiarities, we have used dedicated stellar evolution models to test their endogenous origin, and shown that for those WGB stars that are not Li-rich, lithium and nitrogen abundances can be reproduced by rotating models. On the other hand the carbon deficiency of all WGB stars is difficult to reconcile with an internal origin and we propose that it is decorrelated from the other chemical peculiarities. The case of lithium is still under debate as it is not clear, because of the degeneracy of the evolutionary status, whether the WGB stars with $A(\text{Li}) \geq 1.4$ dex are actually Li-rich (core helium burning stars) or normal (subgiant or RGB stars undergoing the first dredge-up).

Carbon depletion could be associated to an early mass transfer from a more massive (5–6 M☉) companion under specific hypothesis, but yields of intermediate mass stars in that mass range are very uncertain and no firm conclusion should be drawn. As the rate of binarity among WGB stars is far from clear, and is at least similar to that in normal giants, a new search for duplicity would be helpful. It is quite clear from this study that more data are needed to shed light on the weak G-band stars puzzle. Considering the work by Miglio (2011) on the use of asteroseismic data to lift the status degeneracy between giants and clump stars, applying such an analysis to the oscillation spectra of WGB stars could be a very important step to clarify the evolutionary status of WGB stars.

Actually deriving proper abundances of light elements for these stars is also needed to start and better understand these overlooked puzzling stars.

### 8. Acknowledgements

The authors thank the referee for very thoughtful comments that have helped improve and clarify the work presented here. MP is thankful to Prof. Shoken Miyama, Prof. Ramanath Cowisk and Prof. Yoichi Takeda for their kind support, encouragement and hospitality. AP thanks Dr. O. Richard and Dr. F. Martins for fruitful discussions on diffusion and modelling.
Appendix A: Notes on Individual Objects

A.0.1. HD 21018
It is a well-known visual binary with an orbital period of 287 days. It has highest Li among all the weak G-band stars. Charbonnel (2000) suggested that it is undergoing the first dredge-up dilution. Due to the duplicity, the position of this object in the HR diagram may be uncertain.

A.0.2. HD 165634
It is reported to have white dwarf binary companion from the IUE spectral studies by Böhm-Vitense et al. (2000). Li information is not available for this star.

A.0.3. HD 188328
It is a well-known visual binary; both components are separated by about 2″ according to Douglass et al. (2000). The position of this object in the HR diagram occupies near the main-sequence turn-off. Due to the visual binarity the position in the HR diagram may be uncertain. Li information is not available.

A.0.4. HD 120213
It may be a binary star. It is classified as a weak G band star in Hartoog et al. (1977). Further studies on this star classified as Ba star of class 0.5 according to Douglass et al. (2000). In general, most of the Ba stars are suggested to be binaries.

A.0.5. HD 166208
HD 166208 is a spectroscopic binary with a period of 5.5 years, low amplitude (3 km/s⁻¹), and moderate eccentricity (0.4) (Griffin, 1992). Griffin (1992) emphasizes the point that the eccentricity of the orbit of HD 166208 and its small mass function make distinctly less likely that in the cases of barium stars that there has been transfer of material between the components during a giant-phase evolution of the companion.

For few stars, like HD 36552, HD 82595, and HD 120170, luminosities are not derived due to the lack of Hipparcos parallaxes. There is no basic data for HD 124721 and HD 31869 available in the literature.

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