STRONG LIMITS ON THE DFSZ AXION MASS WITH G117-B15A

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Received 2007 August 13; accepted 2007 November 2

ABSTRACT

We compute rates of period change (\( \dot{P} \)) for the 215 s mode in G117-B15A and the 213 s mode in R548, first for models without axions, and then for models with axions of increasing mass. We use the asteroseismological models for G117-B15A and R548 we derived in an earlier publication. For G117-B15A, we consider two families of solutions, one with relatively thick hydrogen layers and one with thin hydrogen layers. Given the region of parameter space occupied by our models, we estimate error bars on the calculated \( \dot{P} \) values using Monte Carlo simulations. Together with the observed \( \dot{P} \) for G117-B15A, our analysis yields strong limits on the DFSZ axion mass. Our thin hydrogen solutions place an upper limit of 13.5 meV on the axion, while our thick hydrogen solutions relaxes that limit to 26.5 meV.

Subject headings: dense matter — elementary particles — stars: oscillations — stars: variables: other — white dwarfs

Online material: color figures

1. ASTROPHYSICAL CONTEXT

G117-B15A and R548 are pulsating white dwarfs with atmospheres dominated by hydrogen. These stars are called DAVs or ZZ Ceti stars. Their effective temperatures are close to 12,000 K. They are nonradial, g-mode pulsators (the restoring force is buoyancy). G117-B15A is the most stable optical clock known; the rate of change in period (\( \dot{P} = dP/dt \)) of its most stable mode (215 s) is 3.57 ± 0.82 × 10^{-15} s^{-1} (Kepler et al. 2005). That is, G117-B15A loses one “tick” every 1.7 billion years. We can use this well-measured \( \dot{P} \) to constrain, among other things, the mass of the axion, the best motivated dark matter candidate and in general, the emission rate of weakly interacting particles. While measuring a \( \dot{P} \) for R548’s most stable mode (near 213 s) is less trivial because it is a doublet, the current upper limit, 5.5 ± 1.9 × 10^{-15} s^{-1} (Mukadam et al. 2003), also indicates that in the near future R548 may become useful to constrain the axion mass as well.

A \( \dot{P} \) of order 10^{-15} for DAVs is consistent with a slow increase in period due to the cooling of the interior. It is an evolutionary timescale—a measure of how fast the star is cooling. Knowing \( \dot{P} \), we can determine how much energy a star like G117-B15A is losing every second. This energy loss includes of course the radiation of photons (Mestel cooling; Mestel 1952), but in principle could also include the emission of weakly interacting particles. While neutrinos immediately come to mind, the Standard Model of particle physics predicts that neutrino emission should be negligible in G117-B15A and R548 (Winget et al. 2004; Kim 2007). One possibility includes axions, and we apply our method to the latter in the present study.

Axions arise from an elegant solution to a problem with the Standard Model, the strong charge parity problem (e.g., Turner 1990; Kim 2007). Along with supersymmetric particles, axions are currently favored candidates for dark matter. But they have not been discovered (neither have supersymmetric particles), and the theory of axions fails to place any constraint on their mass. The possible contribution of axions to dark matter depends on their mass. The mass of axions determines how strongly they interact with the matter we know, with more massive axions interacting more strongly. In turn, the stronger the interaction of axions with matter or light, the larger their emission rate. With pulsating white dwarfs, we can constrain the axion emission rates and therefore their mass.

We determine limits on the emission of weakly interacting particles by comparing their cooling effect on a white dwarf model with the observed cooling rates of that star (given by \( \dot{P} \)). The strength of those limits depends on the uncertainties involved in the measurement of the \( \dot{P} \) and in the modeling. This method has been used before by Isern et al. (1992) and Córósico et al. (2001) to derive an upper mass limit for axions. While the limits on the axion obtained by those authors were stronger than those obtained through other methods (see Kim 2007 for a review across fields of study), we now have the tools to greatly strengthen them. We present here the first study that takes into account the modeling uncertainties in a systematic way, allowing us to derive a strong upper mass limit for the axion. We present our results in a way that facilitates their application to other weakly interacting particles. We also publish for the first time the \( \dot{P} \) for R548 as a function of the emission rate of weakly interacting particles. Together with a future measurement of \( \dot{P} \) for R548, these results can be used to reinforce the limits given by G117-B15A.

In Bischoff-Kim et al. (2008, hereafter Paper I), we used a systematic, fine grid search to find best fit models to G117-B15A and R548. We were able to define the regions of parameter space the fits occupied. In this work, we use Monte Carlo simulations to calculate \( \dot{P} \) values that include theoretical uncertainties, for varying axion mass. We begin with a brief introduction to axions in § 2. In § 3, we give an overview of the work done by Isern et al. (1992) and Córósico et al. (2001). We detail our method in § 4, discuss our results in § 5, and conclude in § 6.

2. AN INTRODUCTION TO AXIONS

The Standard Model of particle physics does not explain why charge parity (CP) should be violated in weak interactions but not in strong interactions. One would therefore expect the neutron to have an electric dipole moment. Experiments have placed an upper limit on the neutron’s electric dipole of 10^{-25} e·cm (electron charge × length of dipole), 10 orders of magnitude below the predicted value (Turner 1990).
In more technical terms, in QCD, the neutron electric dipole moment arises from a CP (as well as C and P) violating term in the Lagrangian (Turner1990). This electric dipole moment is predicted to be of order $5 \times 10^{-10}$ e-cm, where $\theta$ is a free parameter in the theory. The experimental limit of $10^{-25}$ e-cm means that $\theta$ is less than $10^{-10}$. Theoretical physicists do not understand why $\theta$ has to be so small. This is called the strong CP problem and can be solved elegantly by the introduction of a new symmetry.

This symmetry, added to the Lagrangian of the fundamental interactions, is called the Peccei-Quinn symmetry (Peccei & Quinn 1977). The spontaneous breaking of this symmetry gives rise to axions. The theory does not place any limit on their mass. Axions can couple to photons, electrons (leptons), and nucleons (baryons). The relative strength of each coupling depends on the Peccei-Quinn charges of the (baryons). The relative strength of each coupling depends on one or more of the Peccei-Quinn charges of the $u$ and $d$ quarks and the electron (denoted $X_u/X_d$ and $X_e$, respectively.) Those charges can be between 0 and a number of order 1 and are not constrained by the theory. This gives rises to a continuum of axion models. Two simple models include the KVSZ model (Kim1979; Shifman et al. 1980), where $X_e = 0$ (no coupling to electrons), and the DFSZ model (Dine et al. 1981; Zhitnitsky 1980), where $X_u \sim X_d \sim X_e \sim 1$.

In this paper, we focus on DFSZ axions, those that interact with electrons. For a more complete review of the different axion models and their interaction with light and matter, see Kim (2007). In white dwarf interiors, the dominant axion emission mechanism would be electron bremsstrahlung, where an electron emits an axion as it gets accelerated in the neighborhood of an ion. The axion emission rate (per unit mass) for that process may be expressed as (Nakagawa et al. 1988)

$$\epsilon_a = 1.08 \times 10^{23} \text{ergs g}^{-1} \text{s}^{-1} \alpha a^2 T^4 F(T, \rho),$$  \hspace{1cm} (1)

where $T_7 = T/10^7$ K, $\alpha = g^2_{a\gamma} / 4\pi$, and $g_{a\gamma\gamma}$ is the strength of the axion-electron coupling, is given by equation (2). The term $F(T, \rho)$ is a numerical fit provided by Nakagawa et al. (1988); it is of order 1 throughout most of the interior of a typical white dwarf model (e.g., Fig. 1).

The strength of the axion-electron coupling is

$$g_{a\gamma} = (2.8 \times 10^{-11}) \frac{m_a \cos^2 \beta}{1 \text{ eV}},$$  \hspace{1cm} (2)

where $m_a$ and $\beta$ are free parameters. For DFSZ axions, the coupling to electrons is $10^8$ orders of magnitude greater than the coupling to photons (assuming $\cos^2 \beta \sim 1$). Following in the footsteps of Isern et al. (1992), we shall not make any assumptions about the value of $\beta$, and simply state that the quantity we are constraining is $m_a \cos^2 \beta$. We shall, however, refer to it simply as the “axion mass.”

3. EARLY WORK

In a pioneering work, Isern et al. (1992) used G117-B15A’s $\dot{P}$ to obtain a limit on the axion mass. At the time, the $\dot{P}$ measured ($12.0 \pm 3.5 \times 10^{-15}$ s$^{-1}$; Kepler et al. 1991) for that star was uncertain, and much higher than the one expected from simple Mestel cooling. Using models available at the time (Wood1990; D’Antona & Mazzitelli 1989) and a simple semianalytical treatment, Isern et al. found an average axion mass of 8 meV. Individual values, depending on the model chosen and value of observed $\dot{P}$ considered ($\dot{P} - \Delta \dot{P}$, $\dot{P}$, and $\dot{P} + \Delta \dot{P}$) allowed a range between 0 and 20 meV for the axion mass.

Côrsico et al. (2001) revisited the problem with a new measured value of $\dot{P}$, which had since decreased to what was expected from simple Mestel cooling (Mestel 1952); ($2.3 \pm 1.4) \times 10^{-15}$ s$^{-1}$ (Kepler et al. 2000). In their work, Côrsico et al. performed an asteroseismological study of G117-B15A to find its mass, helium layer mass, and hydrogen layer mass. To reduce the number of parameters to fit, they fixed the internal composition to that found from stellar evolution calculations by Salaris et al. (1997) and the effective temperature to the latest spectroscopic estimate at the
time, 11,620 K (Bergeron et al. 1995). Their best-fit model had a mass of 0.55 \(M_e\), a helium layer mass \(M_{\text{He}} = 10^{-2}M_e\), and a hydrogen layer mass \(M_{\text{H}} = 10^{-4}M_e\). On average, the periods of that model are 5 s away from the observed periods. In contrast, the models we use in the present study fit to better than 1 s.

Côrsico et al. (2001) considered only small uncertainties in effective temperature (200 K) and found that they led to a 4% uncertainty in the calculated effective temperature. A mass uncertainty of 0.2 \(M_e\) (3%) led to a 6% uncertainty in \(P\). And considering a full range of core composition (0% carbon to pure carbon) changed the \(P\) only by 5%. They concluded that uncertainties other than the one in the measured \(P\) were insignificant and could be ignored altogether. They found a tight constraint on the axion mass: 4 meV. If we adopt Corsico et al.’s best-fit parameters for G117-B15A, and follow the same method using our models, we obtain a similar limit on the axion mass.

To place things in perspective, the best DFSZ axion mass limit aside from that of Côrsico et al. (2001) and the ones presented here come from the observation of red giants and horizontal branch stellar populations in clusters. If axions were massive enough, they would cool the core of stars and have an observable effect on the morphology of those populations. The best limit found using this method is 9 meV (Raffelt & Weiss 1995). Such limits depend on the assumed core mass in the models, a rather uncertain quantity. The pulsating white dwarf method does not suffer from such large uncertainties.

Since the work done by Côrsico et al. (2001), a newer value of \(P = 3.57 \pm 0.82 \times 10^{-15} \text{ s} \cdot \text{s}^{-1}\) has been obtained (Kepler et al. 2005). The new value has a smaller measurement error, and could help constrain the axion mass better. More importantly, we take into account additional modeling uncertainties that have been ignored in previous analyses. The better we account for those uncertainties, the stronger our limit on the axion mass will be. We will see that this leads us to different conclusions from the authors mentioned above.

4. METHOD

4.1. Axion Emission in Our Models

For a mixture of carbon and oxygen, as in the interior of our white dwarf models, the axion emission rate is given by

\[
e_a = X_C e_C + X_O e_O,
\]

where \(X_C\) and \(X_O\) are the carbon and oxygen abundance, and \(e_C\) and \(e_O\) are the axion emission rates in pure carbon and pure oxygen, respectively. Combining equations (1) and (3), we have

\[
e_a = 1.08 \times 10^{23} \alpha T_{\text{eff}}^3 [3X_C F_C(\rho, T) + 4X_O F_O(\rho, T)].
\]

We distinguish \(F(T, \rho)\) in pure carbon and pure oxygen by using subscripts.

Integrating over the mass of the model, we obtain the axion luminosity. While we carry the exact integration numerically in our models, it is useful at this point to write down an approximate expression for the axion luminosity. We use the internal properties of a fiducial model, chosen because it matches G117-B15A’s periods and \(P\) for the 215 s mode very closely. This fiducial model has \(T_{\text{eff}} = 12656 \text{ K}, M_e = 0.602 M_e\), a helium layer mass of \(3.55 \times 10^{-3}M_e\), and a hydrogen layer mass of \(4.79 \times 10^{-4}M_e\). We display the internal properties of the fiducial model in Figure 1.

Following the example set forth in Isern et al. (1992), we make the following approximations: (1) we set \(T = T_{\text{center}} = 1.2 \times 10^7 \text{ K throughout the model, and (2) we also set } F_C \text{ and } F_O \text{ constant } (F_C \approx 1,1 \text{ and } F_O \approx 1,2). \) The axion luminosity is then

\[
L_a = 1.08 \times 10^{23} \alpha T_{\text{eff}}^3 M_e \nonumber \\
\times \left( 3F_C \int_0^1 X_C dm + 4F_O \int_0^1 X_O dm \right),
\]

where \(m = M_i/M_e\). For the fiducial model’s core composition, \(\int_0^1 X_C dm \approx 0.4\) and \(\int_0^1 X_O dm \approx 0.6\). We obtain

\[
L_a = 1.13 \times 10^{57} \alpha \text{ ergs s}^{-1}.
\]

This approximate expression is 7% accurate for our fiducial model.

4.2. Asteroseismological Fits

Before we include axions in our models, we need to make sure we have reliable models of the stars under study, G117-B15A and R548. We performed our own asteroseismological analysis of those stars in Paper I. We varied four parameters, including the effective temperature and the helium layer mass, \(M_{\text{He}}\) (in addition to the stellar mass and the hydrogen layer mass, \(M_{\text{H}}\)). In Paper I, we explored systematically a number of sources of uncertainties in our models and concluded that we could adequately account for them by treating equally all models that matched the observed periods to better than 1 s. With that cutoff, we defined the regions of parameter space our models occupied. We found that our best-fit models were hot and/or massive compared to the spectroscopy, although they were not completely inconsistent with the latter. Work is in progress to resolve this apparent discrepancy. Our calculations are self-consistent, and we proceed to derive meaningful results.

4.3. Monte Carlo Simulations

Having narrowed down the regions of parameter space occupied by best-fit models for G117-B15A and R548, the next step is to derive rates of period change for those models along with the associated uncertainties. This is a classic Monte Carlo problem. For G117-B15A, we chose \(T_{\text{eff}}, \Delta M\) (instead of \(M_i\), \(M_{\text{He}}, \) and \(M_{\text{H}}\) as our four independent variables, where \(\Delta M\) is defined by

\[
\Delta M = b - aT_{\text{eff}} + \Delta M,
\]

and \(a\) and \(b\) are defined so that the simulated populations occupy the region of parameter space in the \(M_e-T_{\text{eff}}\) plane defined by the results from our asteroseismological fits. For each family of solutions we found in Paper I, without including axions just yet, we generated a random population \((N = 500)\). We display the distribution of the models of each of the populations in Figure 2.

We have two possible families of fits for G117-B15A. One has a thicker hydrogen layer \((M_{\text{H}} = 6.3 \times 10^{-6}M_e)\) and occupies the high-mass, low effective temperature region of the \(M_e-T_{\text{eff}}\) plane. The other has a thinner hydrogen layer \((M_{\text{H}} < 6.3 \times 10^{-6}M_e)\) and occupies the higher effective temperature (lower mass) region of the \(M_e-T_{\text{eff}}\) plane. The thin hydrogen fits to G117-B15A are very similar to the fits to R548. While this makes the thin hydrogen fits more attractive than the thick hydrogen fits (by “thick” we mean “not as thin”), we cannot discard the latter as a viable possibility on that basis alone. We shall consider both families when constraining the axion mass.

For each model in each population, we proceeded with calculating a \(P\). We checked that the period and \(P\) distribution for
each of the populations were close to Gaussian and found that they were. For each population (family of fits) we calculated the mean period and $\dot{P}$. We list the results in Table 1 with $1\sigma$ error bars. Remember that these results assume that axions do not exist and that the cooling is due entirely to photons. For G117-B15A, the thick hydrogen fit $\dot{P}$ is $2\sigma$ below the observed value. However, we cannot discard this solution based on a low $\dot{P}$ alone, as we can always increase the cooling rate and raise the value of $\dot{P}$ by adding axion emission to the models.

Next we included axion production in our models and repeated the procedure for each different axion mass. We confirm an important fact Córtsico et al. (2001) first pointed out: the periods themselves are insensitive to the axion mass (for $m_a$ up to 30 meV at least). The reason for this small dependence is that axion emission does not introduce any “bumps” into the Brunt-Väisälä frequency. It merely slightly lowers it throughout the degenerate core. This allows us to meaningfully compare the calculated $\dot{P}$ with the observed value.

5. RESULTS

In Figure 3 we show $\dot{P}$ as a function of the axion luminosity for G117-B15A and R548. While we included axions specifically in...
our models, the $\dot{P}$ values depend only on the total luminosity, not on the exact nature of the weakly interacting particles responsible for the “unseen” loss of energy. Our results may therefore directly be generalized to weakly interacting particles other than axions.

The thin hydrogen solution for G117-B15A places an upper limit of $1.1 \times 10^{-31}$ erg s$^{-1}$ for the energy-loss rate due to weakly interacting particles. For axions, this translates to an upper limit of $3.78 \times 10^{-13}$ on the axion-electron coupling constant $g_{ae}$ (13.5 meV on the axion mass). The thick hydrogen family of fits (taken alone) constrains the “unseen” luminosity to be between $2.7 \times 10^{-31}$ and $4.4 \times 10^{-31}$ erg s$^{-1}$. These limits translate to $2.9 \times 10^{-13} \leq g_{ae} \leq 7.4 \times 10^{-13}$ (axions between 10.4 and 26.5 meV in mass).

Combining the two families together, we can place a conservative upper limit of $4.4 \times 10^{-31}$ erg s$^{-1}$ on the luminosity of weakly interacting particles that would be emitted in the interior of G117-B15A, and a 26.5 meV upper limit on the axion mass. For R548, we are still awaiting a better determined $\dot{P}$ to allow us to obtain a potentially tighter constraint on the emission of weakly interacting particles in white dwarfs.

6. CONCLUSIONS

We computed $\dot{P}$ for the 215 s mode in G117-B15A and the 213 s mode in R548 first for models without axions, and then for models with axions of increasing mass. The thin hydrogen, zero axion models for G117-B15A have a $\dot{P}$ of $2.98 \pm 0.17 \times 10^{-15}$ s$^{-1}$, consistent with the observed value of $3.57 \pm 0.82 \times 10^{-15}$ s$^{-1}$. The zero axion, thick hydrogen models have a $\dot{P}$ of $1.92 \pm 0.26 \times 10^{-15}$ s$^{-1}$, 2$\sigma$ below the observed $\dot{P}$, and require additional cooling through axions to become consistent with that observed value. The zero axion models for R548 have a $\dot{P}$ similar to the thin hydrogen $\dot{P}$ for G117-B15A ($\sim 3 \times 10^{-15}$ s$^{-1}$), consistent with the best current limit on $\dot{P}$ for R548’s 213 s mode.

If we believe that G117-B15A must be similar to R548, then we find an upper limit on the axion luminosity of $1.1 \times 10^{-31}$ erg s$^{-1}$, corresponding to an axion-electron coupling constant $g_{ae} = 3.78 \times 10^{-13}$, or an axion mass of 13.5 meV. A second family of fits with thicker hydrogen layers relaxes that limit to $L_a \leq 4.4 \times 10^{-31}$ erg s$^{-1}$, $g_{ae} \leq 7.4 \times 10^{-13}$, or $m_a \leq 26.5$ meV. What makes this limit unique is not the fact that it is lower than previous limits. What makes it unique is its strength. It is based on an unbiased exploration of the parameter space of the models. Compared to typical axion searches (see Kim 2007 for a review), the present analysis involved well-known physics and relatively small, well-identified uncertainties, considered systematically. Improved asteroseismological models will help us choose between the two families of best fits for G117-B15A and improve this limit.

An even longer data baseline for G117-B15A will reduce the size of the error bars on the measured $\dot{P}$ and further narrow the allowed range for the axion mass. We can soon expect a useful $\dot{P}$ for R548 as well, which will reinforce the results of this study. As a part of his planet search around pulsating white dwarfs, Mulally has determined upper limits of $10^{-12}$ s$^{-1}$ for an additional 9 white dwarfs (Mulally et al. 2008) and has identified more blue-edge DAVs with stable modes. With Argos (Mukadam & Nather 2005), the high-speed photometry data collected at McDonald

### Table 1

| Parameter       | G117-B15A | R548  |
|-----------------|-----------|-------|
| $M_{\text{BH}}$ | $6.3 \times 10^{-7}$ | $4 \times 10^{-8}$ |
| $P$ (observed)  | 215.197389 | 213   |
| $P$ (calculated) | $214.9 \pm 2.5$ | $215.7 \pm 2.0$ |
| $P$ (observed) (10$^{-15}$ s s$^{-1}$) | $3.57 \pm 0.82$ | $< 5.5 \pm 1.9$ |
| $P$ (calculated) (10$^{-15}$ s s$^{-1}$) | $1.92 \pm 0.26$ | $2.98 \pm 0.17$ |

![Graph](image_url)
observatory as a part of the ongoing observation of hot DAVs should begin to yield measurements of $P$ for those stars within the next 3 to 5 years. It is merely a question of time until we can place an even tighter constraint on the emission of weakly interacting particles in these stars.

In the present study, we assumed that any unseen source of energy loss was due to axions. The results we present can readily be used to constrain the emission of other weakly interacting particles possibly produced inside white dwarfs, such as supersymmetric particles.

The authors would like to thank E. L. Robinson for key suggestions that helped us improve this work, and the referee for ideas to augment its impact. This research was supported by NSF grant AST-0507639.

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