Entanglement of the orbital angular momentum states of the photons generated in a hot atomic ensemble

Qun-Feng Chen, Bao-Sen Shi, Yong-Sheng Zhang, and Guang-Can Guo

Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, People’s Republic of China

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Quantum protocols will be more efficient with high-dimensional entangled states. Photons carrying orbital angular momenta can be used to create a high-dimensional entangled state. In this paper we experimentally demonstrate the entanglement of the orbital angular momentum between the Stokes and anti-Stokes photons generated in a hot atomic ensemble using spontaneous four-wave-mixing. This experiment also suggests the existence of the entanglement concerned with spatial degrees of freedom between the hot atomic ensemble and the Stokes photon.

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Entanglement is one of the most fantastic phenomenon of quantum mechanics, and is used as a resource in quantum information field [1]. High-dimensional two-particle entangled states can be used to realize some quantum information protocols more efficiently [2, 3]. Photons carrying orbital angular momenta (OAM) are used to create high-dimensional entangled states, since OAM can be used to define an infinite-dimensional Hilbert space [4]. The first experiment of the entanglement of the OAM states generated via spontaneous parametric down-conversion in a nonlinear crystal was demonstrated in 2001 [5], since then several protocols based on OAM states of photons have been realized experimentally [6, 7, 8]. The transferring of OAM between classical light and cold atoms [9, 10, 11] and hot atoms [12] has also been reported.

In 2001 [5], since then several protocols based on OAM states of the photons generated in a cold atomic system have been clarified by Inoue et al. [13]. The transferring of OAM between classical light and cold atoms [9, 10, 11] and hot atoms [12] has also been reported in the past years. Recently, the entanglement of OAM states of the photons generated in a cold atomic system using Duan-Lukin-Cirac-Zoller (DLCZ) scheme [13] has been clarified by Inoue et al. [14]. So far, there is no experimental discussion about the entanglement of the OAM states of the photons generated in a hot atomic system. In this paper we demonstrate the entanglement of OAM states of the photons generated in a hot atomic ensemble using the spontaneous four-wave-mixing (SFWM) [15, 16]. Our experiment is different from the experiment done by Inoue et al.: In our experiment, SFWM is used to generate a photon pair, in contrast with the experiment of Ref. [14], in which the method based on DLCZ scheme is used. Furthermore, our experiment is based on a hot atomic ensemble, which is more easy to be realized compared with the scheme based on a cold atomic system. In our experiment, we clearly demonstrate the entanglement of the OAM between the Stokes and anti-Stokes photons generated via SFWM in a hot atomic ensemble, the concurrence got in this experiment is about 0.81. This experiment also suggests the existence of the entanglement concerned with spatial degrees of freedom between the hot atomic ensemble and the Stokes photon.

The schematic setup used in this experiment is shown in Fig. 1 [1]. The energy levels and the frequencies of the lasers used are shown in Fig. 1(a). A strong coupling laser, which is resonant with the $|b\rangle \rightarrow |c\rangle$ transition, drives the populations of the atoms into level $|a\rangle$. A weak pump laser, resonant with the $|a\rangle \rightarrow |d\rangle$ transition, is applied to the system. The $|d\rangle \rightarrow |b\rangle$ transition will be induced by the pump laser and the Stokes (S) photons will be generated. When a Stokes photon is emitted, the atomic ensemble collapses into the state $\sum_{j} \alpha_{j} |a_{1}, a_{2}, \ldots, a_{j}, \ldots, a_{N}\rangle$. The strong coupling laser repumps the atomic ensemble back to the state $|a_{1}, a_{2}, \ldots, a_{N}\rangle$, and an anti-Stokes (AS) photon is generated. In this process, the energy, momentum and OAM of the photons will be conserved [12, 17], i.e.,

\[
\omega_{S} + \omega_{AS} = \omega_{P} + \omega_{C},
\]

\[
\vec{k}_{S} + \vec{k}_{AS} = \vec{k}_{P} + \vec{k}_{C},
\]

\[
L_{S} + L_{AS} = L_{P} + L_{C},
\]

(1)

where $\omega_{i}$, $\vec{k}_{i}$ and $L_{i}$ represent the frequency, wave vector and OAM of the corresponding photons respectively. According to Eq. (1), when the pump and coupling lasers carry zero OAM, the Stokes and anti-Stokes photons will be in the entangled state of

\[
|\Psi\rangle = C \sum_{i=-\infty}^{+\infty} \alpha_{i} |i\rangle_{S} |i\rangle_{AS},
\]

(2)

where $C$ is the normalization coefficient, $\alpha_{i}$ are the relative amplitudes of the OAM states. In this work we only investigate the entanglement concerned with $i = 0$ and 1, thus the experimental expected entangled state can be written as:

\[
|\Psi\rangle = C (|0\rangle_{S} |0\rangle_{AS} + \alpha_{1} |1\rangle_{S} |1\rangle_{AS}).
\]

(3)

Although we only discuss the two dimensional case, it is natural to presume that our discussion can be extended into high-dimensional cases over a wide range of OAM [18].

A Gaussian mode beam carrying the well-defined OAM is in Laguerre-Gaussian (LG) mode [18], it can be described by LG_{p,0} mode, where $p + 1$ is the number of the
The power of the pump is about 60 µW, the pump laser, which is counter-propagating with the coupling laser, has an intensity of about 7 mW. The horizontally polarized coupling laser is resonant with |5S_{1/2}, F = 2⟩ → |5P_{1/2}, F = 2⟩ transition of 87Rb. The vertically polarized anti-Stokes photons in the phase matched direction are diffracted by the other CGH (H2). The +1 order diffraction is coupled into SMF2 after being filtered by F2, which increases the OAM of the collected anti-Stokes photons by 1ℏ at 0 displacement. The diffraction efficiency of the CGHs used in this experiment are about 40%. Each of the filters F1 and F2 consists of an optical pumped paraffin-coated 87Rb cell and a ruled diffraction grating. The optical pumped rubidium cell is used to filter out the scattering of the co-propagating laser, and the ruled diffraction grating is used to separate the photons at the D1 and D2 transitions. The collected photons are detected by photon-counting modules (Perkin-Elmer SPCM-AQR-15). The time resolved coincident statistics of the Stokes and anti-Stokes photons are accumulated by a time digitizer (FAST ComTec P7888-1E) with 2 ns bin width and totally 160 bins. In this experiment the Stokes photons are used as the START of the P7888-1E and the anti-Stokes photons after certain delay are used as the STOP of the P7888-1E.

The time resolved coincident counts of the Stokes and...
FIG. 2: (Color online) Time resolved coincidence counting between the Stokes and anti-Stokes photons. The data is accumulated about 1000 seconds and then normalized in time. τ is the relative delay between the Stokes and anti-Stokes photons. The delay between the Stokes photons and anti-Stokes photons is caused by time used to generate anti-Stokes photons, which is mainly determined by the Rabi frequency τ.

In order to evaluate the quantum correlation of the OAM states, we measure the coincident counts with various displacements of the holograms. Figure 3 shows the coincident counts versus the displacement of H2 when the displacement of H1 is fixed at various displacements while the Stokes and anti-Stokes photons are in strongly quantum correlated states. Therefore if Eq. (5) always holds no matter the Stokes photons are collapsed to stationary states or superposition states, the Stokes photon and anti-Stokes photon should be in a quantum correlated state. In Fig. 3(a), the red squares show the results of the coincident counts versus the displacement of H2 when the displacement of H1 is far larger than the waist of the Stokes photons, and the green dots show the results when the displacement of H1 is 0. The red line in Fig. 3(a) is fitted with θ = 0 and the green dashed line is fitted with θ = π/2, which means the Stokes photons are in LG00 and LG01 modes respectively. This figure demonstrates the collapse of the Stokes photon state into the stationary states lead the anti-Stokes photon state collapse into the corresponding stationary states. Therefore this figure indicates clearly the correlation of OAM between the Stokes and anti-Stokes photons. However, such a correlation can be obtained even in the mixture |0⟩S|0⟩AS and |1⟩S|1⟩AS states. To further demonstrate that the Stokes and anti-Stokes photons are in a quantum correlated state, we displace the H1 with a certain amount, which make the collected Stokes photons be in the superposition states 1/√2(|0⟩S|0⟩AS + |1⟩S|1⟩AS), and then sweep H2. The results are shown in Fig. 3(b). The data fit well with the theoretical prediction, which demonstrates that the anti-Stokes photon state collapses into the corresponding superposition states when the Stokes photon state collapses into the superposition states. Therefore the results shown in Fig. 3 demonstrate that the Stokes and anti-Stokes photons are in strongly quantum correlated OAM states.

To further demonstrate the entanglement of the Stokes and anti-Stokes photons, we perform a two-qubit state tomography[21], and get the full state of the Stokes and anti-Stokes photons. The density matrix is reconstructed from the experimentally obtained coincidences.
The data are fitted using the square of Eq. (5) with \( w \). The density matrix is shown in Fig. 4. From the density matrix, the graphical representation of the reconstructed density matrix is retrieved by the coupling laser, which is dependent on the beam waist, superposition of the state \( \text{LG}_{00} \). The speed of the anti-Stokes photon generated is mainly determined by the Rabi frequency of the coupling laser \( \Omega \). Therefore the entanglement of OAM between Stokes photons and the anti-Stokes photons might suggest the existence of the entanglement of OAM between the Stokes photon and the atomic ensemble.

In summary, we have demonstrated the entanglement of OAM states between the Stokes and anti-Stokes photons generated via SFWM in a hot rubidium cell. The entanglement of the Stokes and anti-Stokes photons also suggests that the Stokes photon might entangle with the hot atomic ensemble in spatial degrees of freedom (OAM in this paper).

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