Coincident-Frequency Entangled Photons in a Homogenous Gravitational Field - A Thought Experiment

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Assuming that the Principle of energy conservation holds for coincident-frequency entangled photons propagating in a homogeneous gravitational field. It is argued that in this physical context, either Quantum entanglement or the weak equivalence principle are broken by the photons.

**Introduction** — After reviewing the relativistic effect of the gravitational redshift for a photon propagating in a homogeneous gravitational field, a similar experiment is idealized for the case of two coincident-frequency entangled photons propagating in a homogeneous gravitational field. The simultaneous validity of the principle of energy conservation, the weak equivalence principle, and the entanglement properties are investigated for the entangled system. Ultimately one concludes that quantum coherent systems and classical systems cannot simultaneously comply with the principle of equivalence and possess entanglement properties, if the principle of energy conservation is applicable on average to both sets of physical systems.

**Gravitational Frequency Shift of Light in a Homogeneous Gravitational Field** — The law of the gravitational redshift can be derived directly from the principle of energy conservation applied to a photon moving against a homogeneous gravitational field \( \mathbf{g} \), cf. Fig.1. A photon source located at point \( A \) where the gravitational potential energy is by convention set to zero, emits a photon with frequency \( \nu \) and total electromagnetic energy \( \epsilon = h \nu \). The photon propagates against the gravitational field \( \mathbf{g} \). As it moves away (along \( z \) direction) from the source its gravitational potential energy, \( \epsilon_g = m_g g z \), increases; and its electromagnetic energy, \( \epsilon'_e = h \nu' \) must decrease accordingly in order to maintain the total energy of the photon equal to its initial value. This should be verified until the photon reaches the spectrometer located at point \( B \) at a distance \( z = L \) above \( A \).

\[
h \nu = h \nu' + m_g g L
\]  
(1)

Where \( g \) is the module of the gravitational field \( \mathbf{g} \), and \( m_g \) is the photon gravitational mass, which is equal to its inertial mass \( m_i \) on the bases of the weak equivalence principle.

\[
m_g = m_i = \frac{h \nu'}{c^2}
\]  
(2)

After substituting Eq.(2) in Eq.(1) one can calculate the relative frequency shift of the photon.

\[
\frac{\nu - \nu'}{\nu'} = \frac{g L}{c^2}
\]  
(3)

Eq.(3) indicates that the photon’s frequency decreases (redshift) as it moves against the gravitational field \( g \).

Of course a blueshift appears for photons path collinear with respect to \( g \) (from \( B \) to \( A \)).

In the theory of general relativity the photon gravitational redshift is attributed to a slowing down of clock’s frequency with a reduction of the gravitational field. This is usually derived from the assumption that the interval \( ds^2 = c^2 dt^2 \) connected with the period of oscillation of an atom \( dt \) (considering that the space coordinates of the atom are fixed), remains unchanged if the atom is put into a gravitational field with Schwarzschild metric:

\[
ds^2 = \left(1 - \frac{2GM}{c^2 R}\right) c^2 dt^2
\]  
(4)

The relativistic gravitational redshift has been experimentally observed by Pound and Rebka in 1960 in the Earth laboratory, by measuring the gravitational frequency shift on gamma rays propagating in a 22.5 meters height tower [1].

**Coincident-Frequency Entangled Photons in a Homogeneous Gravitational Field** — Let us modify the traditional gravitational frequency shift experiment, described schematically in Fig.1 by considering two coincident-frequency entangled photons emitted

![FIG. 1: The frequency of a photon propagating against a homogeneous gravitational field \( g \) decreases, in order to comply with the law of energy conservation.](image-url)
simultaneously at point \( A \) with frequencies \( \nu_0 \), instead of simply one single photon. In this new version of the experiment Photon 1 is propagating along the vertical direction \( z \) against a homogeneous gravitational field \( \vec{g} \), photon 2 is propagating in the horizontal direction \( x \), orthogonal to \( \vec{g} \), cf. Fig. 2.

Two coincident-frequency entangled photons,

\[
|\psi\rangle = \int d\nu \phi(\nu) |\nu'_1 + \nu\rangle_1 |\nu'_2 + \nu\rangle_2
\]  

(5)

consists of a pair of entangled photons with identical frequencies \( \nu'_1 = \nu'_2 \): the two photons are positively correlated in frequency, and hence anti-correlated in time\(^2\).

Photon 1 is subject to the relativistic gravitational redshift. Its frequency decreases according to Equ. 3.

Since Photon 1 and 2 have entangled frequencies the frequency of Photon 2 should also decrease as Photon 1 is approaching point B. Although the gravitational potential energy of photon 2 is kept constant (equal to zero, since \( AC \) defines the level of zero gravitational potential energy), its electromagnetic energy would be decreasing. Therefore, if the frequency entanglement between photon 1 and 2 is preserved while photon 1 propagates against the homogeneous gravitational field \( \vec{g} \), then photon 2 should violate the principle of energy conservation.

Imposing that the principle of energy conservation should not be violated by the entangled photons, then the energy of photon 1 and 2 at point A should remain constant at any time posterior to the photons emission until their detection at points B and C.

\[
2h\nu_0 = h\nu'_1 + h\nu'_2 + m_{g1} gL
\]  

(6)

Since it is assumed that the weak equivalence principle is holding for the photons, one has for photon 1:

\[
m_{g1} = m_{i1} = \frac{h\nu'_1}{c^2}
\]  

(7)

Substituting in Equ. (6) and solving with respect to \( \nu'_2 \) one obtains:

\[
\nu'_2 = \nu_0
\]  

(8)

This shows that the constraint imposed by the quantum entanglement on the frequencies, \( \nu'_1 = \nu'_2 \), must be relaxed if one assumes that the principle of energy conservation and the weak equivalence principle are both holding.

If one now assumes that the principle of energy conservation and that entanglement must be preserved, then the constraint of having \( \nu'_1 = \nu'_2 \) in Equ. (6) leads to the following equation for energy conservation.

\[
2h\nu_0 = 2h\nu'_1 + m_{g1} gL
\]  

(9)

The adjustable variable in Equ. (9) is now photon 1 gravitational mass. Solving the equation with respect to this variable.

\[
m_{g1} = \frac{2h(\nu_0 - \nu'_1)}{gL}
\]  

(10)

FIG. 2: Two coincident-frequency entangled photons propagating in different gravitational potentials either lose entanglement, break the weak equivalence principle or violate the law of energy conservation.

**PHOTON 1 INERTIAL MASS, \( m_{i1} \), REMAINS:**

\[
m_{g1} = \frac{h\nu'_1}{c^2}
\]  

(11)

Dividing Equ. (10) by Equ. (11) one estimates any possible violation of the weak equivalence principle for photon 1 as being proportional to the gravitational redshift:

\[
\frac{m_{g1}}{m_{i1}} = 2 \frac{c^2}{gL} \left( \frac{\nu_0 - \nu'_1}{\nu'_1} \right)
\]  

(12)

If the classical gravitational redshift is observed in this system, then substituting Equ. (3) in Equ. (12) one deduces that the gravitational mass of photon 1 is the double of its inertial mass, between its emission and its reception\(^7\).

\[
\frac{m_{g1}}{m_{i1}} = 2
\]  

(13)

**DISCUSSION AND CONCLUSIONS** — In the previous section one has demonstrated that two coincident-frequency entangled photons propagating in a homogeneous gravitational field along non-collinear optical paths, cannot comply simultaneously with the principle of energy conservation and the weak equivalence principle. Thus assuming that the principle of energy conservation is preserved on average by entangled systems, then either the entanglement between the photons is lost\(^3\) or the weak equivalence principle is violated at least for one of the photons according to Equ. (12).

Assuming the universality of the principle of energy conservation, one concludes that quantum entanglement and the weak equivalence principle cannot hold simultaneously in a physical system. Thus we are left with the
physical possibilities outlined in table 1, which divides the physical possible systems in two major sets: On the one side, **coherent systems**, like for example: superconductors, superfluids, Bose Einstein Condensates, Entangled photons, Entangled quantum bits, which would exhibit macroscopic and / or microscopic entanglement, and would violate the weak equivalence principle. On the other side **classical systems**, made out of macroscopic material bodies which do not possess any form of quantum entanglement between their different building blocks, but do comply with the weak equivalence principle.

|                     | Entanglement | Equivalence Principle |
|---------------------|--------------|----------------------|
| Coherent systems    | Yes          | No                   |
| Classical systems   | No           | Yes                  |

Table 1: Discriminating between classical and quantum coherent physical systems using entanglement properties and the weak equivalence principle.

[1] R. V. Pound, G. A. Rebka, "Apparent Weight of Photons", Phys. Rev. Lett. 4, 337-341 (1960)
[2] O. Kuzucu, M. Fiorentino, M. A. Albota, F. N. C. Wong, F. X. Krtner, "Two-Photon Coincident-Frequency Entanglement via Extended Phase Matching", Phys. Rev. Lett. 94, 083601 (2005)
[3] T. C. Ralph, G. J. Milburn, T. Downes, "Quantum connectivity of space-time and gravitationally induced decorrelation of entanglement", Phys. Rev. A 79, 022121 (2009)
[4] C. J. de Matos, "Physical Vacuum in Superconductors" J. of Superconductivity and Novel Magnetism DOI: 10.1007/s10948-010-0793-x (2010), also available from: arXive: 0908.4495
[5] C. J. de Matos, "Are Vortices in Rotating Superfluids Breaking the Weak Equivalence Principle?", J. of Superconductivity and Novel Magnetism, in press, (2010), also available from: arXiv:0909.2819
[6] C. J. de Matos, "Are Superfluid Vortices in Pulsars Breaking the Weak Equivalence Principle?", arXiv:1005.2360 (2010)
[7] This result is coherent with the experimental results on the deflection of light by the Sun, which indicate a gravitational photon mass equal to the double of its classical inertial value. Could this mean that every photon in the universe is part of a correlated photon pair?
This figure "Entangled.jpg" is available in "jpg" format from:

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This figure "redshift.jpg" is available in "jpg" format from:

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