Proximal Policy Gradient: PPO with Policy Gradient

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Abstract

In this paper, we propose a new algorithm PPG (Proximal Policy Gradient), which is close to both VPG (vanilla policy gradient) and PPO (proximal policy optimization). The PPG objective is a partial variation of the VPG objective and the gradient of the PPG objective is exactly same as the gradient of the VPG objective. To increase the number of policy update iterations, we introduce the advantage-policy plane and design a new clipping strategy. We perform experiments in OpenAI Gym and Bullet robotics environments for ten random seeds. The performance of PPG is comparable to PPO, and the entropy decays slower than PPG. Thus we show that performance similar to PPO can be obtained by using the gradient formula from the original policy gradient theorem.

Introduction

Policy gradient algorithms introduced in (Williams 1992; Sutton et al. 2000a,b) compute the gradient of the expected return with respect to policy parameters. An action-independent term, typically the state value function, can be subtracted from this gradient. Then, the policy gradient is in the form of gradient of log probability multiplied by the advantage function. The resulting policy gradient algorithm respectively boosts (reduces), the probability of advantageous (disadvantageous) actions. This policy gradient algorithm is too slow to be used as a modern day baseline. Therefore, a reasonable choice of recent advances are applied to the classic policy gradient theorem based methods. In (Achiam 2018), the generalized advantage estimate (GAE) (Schulman et al. 2015b) and the advantage normalization are applied. The resulting algorithm is called the Vanilla Policy Gradient (VPG) (Achiam 2018; Dhariwal et al. 2017).

Recently, the trust region policy optimization (TRPO) (Schulman et al. 2015a) and the proximal policy optimization (PPO) (Schulman et al. 2017) present significant performance improvement over VPG by using the proposed surrogate objective function. In particular, PPO is very popular due to the learning stability and implementation simplicity. PPO provides a novel solution to the problem of increasing the number of policy updates by introducing the clipping idea. In practice, the number of iterations cannot be indefinitely large. Instead, the iteration continues until a target KL divergence is reached. Thus a much larger number of policy update iterations per each roll out is possible, greatly increasing the sample efficiency while maintaining the steady improvement on the average return. This is the baseline PPO algorithm we use to perform experiments. Note that there are variety of variations of PPO (Schulman et al. 2017) for various enhancements.

We explore a slight modification to both VPG and PPO, devise an objective that appears to be in between, and study the performance. The objective without clipping has the gradient exactly the same as VPG’s gradient. This objective can be considered as a partial variation of the VPG objective, i.e., PPG optimizes the changes of the VPG objective. In contrast, PPO optimizes the expected importance-weighted advantage, and hence, the gradient of the PPO objective is different from VPG, which is based on the policy gradient theorem (Sutton et al. 2000a). To achieve performance similar to PPO, we introduce a new clipping algorithm designed for the PPG objective. The clipping is applied to samples that have their probabilities adjusted enough, so that these samples are excluded from the training batch.

We perform experiments in the popular continuous control environments in OpenAI Gym (Brockman et al. 2016) with the MuJoCo simulator: Ant, HalfCheetah, Hopper, and Walker2d, and also in the corresponding Bullet environments: AntBulletEnv, HalfCheetahBulletEnv, HopperBulletEnv, and Walker2DBulletEnv. We found that PPG and PPO are very similar with a few cases where PPG appears to have marginally higher average returns. The entropy tends to decay slower in PPG than in PPO. Therefore, PPG solutions appear to be more stochastic than PPO solutions. Thus, we show that policy gradient can be used for the PPO algorithm.

Related Work

Model free reinforcement learning algorithms have evolved from the classic dynamic programming, policy and value iterations (Sutton and Barto 2018) to Q-learning (Watkins 1989) and direct policy search methods using neural networks for policy and value functions (Mnih et al. 2013, 2016; Andrychowicz et al. 2018; Lillicrap et al. 2015; Fujimoto et al. 2018; Haarnoja et al. 2018).

On-policy algorithms are popular for their stability, i.e., applicability to a wide set of problems. One of the early on-policy policy-gradient based methods is REINFORCE (Williams 1992) that directly used Monte-Carlo returns. Instead, value estimate was used in the Actor-Critic method.
surrogate objective, and still performance similar to PPO can be improved by a variety of secondary optimization techniques. For example, value function clipping and reward scaling have been studied in (Engstrom et al. 2020), where these secondary optimizations are shown to be crucial for PPO’s performance. Since the success of model-free algorithms depends on its exploration policy, and PPO tends to not sufficiently explore, Wang et al. (2019) proposed to adjust the clipping range within the trust region to encourage the exploration. The performance of PPO can be improved by a variety of secondary optimization techniques. For example, value function clipping and reward scaling have been studied in (Engstrom et al. 2020), where these secondary optimizations are shown to be crucial for PPO’s performance. Since the success of model-free algorithms depends on its exploration policy, and PPO tends to not sufficiently explore, Wang et al. (2019) proposed to adjust the clipping range within the trust region to encourage the exploration. The performance of PPO is also enormously influenced by its entropy. Ahmed et al. (2019) and Liu et al. (2019) present higher entropy can be utilized as a regularizer that produces smoother training results.

In this paper, we choose a PPO implementation with clipping and KL divergence break, and then study whether the original policy gradient theorem can be used instead of the surrogate objective. TRPO uses natural policy gradient with KL divergence constraints while PPO uses a simple clipping mechanism. Despite the simplicity, PPO often outperforms TRPO.

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Policy Optimization Objectives

In this section, we first define the objectives of VPG, PPO, and PPG, and provide the detailed discussions in the later sections.

VPG: Vanilla Policy Gradient

Policy Gradient (PG) methods, originally developed in (Williams 1992; Sutton et al. 2000b), are to update a parameterized policy by ascending along the stochastic gradient of the expected return. The gradient of PG has the form

$$\nabla_\theta J(\pi_\theta) = \mathbb{E}_\pi \left[ \nabla_\theta \log\pi_\theta(a_t|s_t)A^\pi(s_t,a_t) \right],$$

(1)

where $\pi_\theta$ is a stochastic policy with the parameter $\theta$, $A^\pi$ is commonly referred to as the advantage function $A^\pi(s_t,a_t) = Q^\pi(s_t,a_t) - V^\pi(s_t)$ where $Q^\pi(s_t,a_t)$ is a state-action value function and $V^\pi(s_t)$ is a state value function. Since this advantage $A^\pi$ is noisy, the generalized advantage estimation (Schulman et al. 2015b) is applied to reduce the variance. We denote the resulting advantage as $A^\pi_{\text{GAE}}$. Finally, we normalize $A^\pi_{\text{GAE}}$ and compute the normalized advantage $A_t$, such that the mean is zero and the standard deviation is 1. We exploit this $A_t$ to define the objective of VPG, denoted as $L_{\text{VPG}}$.

$$L_{\text{VPG}} = \frac{1}{N} \sum_{i,t=1}^{N,T} \log \pi_\theta(a_{i,t}|s_{i,t})A_t.$$  

(2)

where $N$ is the number of episodes in the rollout, $T_i$ is the time steps in episode $i$, and $N = \sum T_i$ is the size of the rollout.

PPO: Proximal Policy Optimization

Proximal Policy Optimization (PPO) (Schulman et al. 2017) and TRPO (Wang et al. 2019) use the surrogate objective (Zhang, Wu, and Pineau 2018; Schulman et al. 2015a).

$$\max_{\theta} \mathbb{E}_t \left[ \pi_\theta(a_{i,t}|s_{i,t}) A_t \right]$$

subject to $\mathbb{E}_t \left[ D_{KL}(\pi_\theta(\cdot|s_t)||\pi_{\text{old}}(\cdot|s_t)) \right] \leq \delta$.

(3)

To avoid the high computational cost and implementation complexity of TRPO, PPO proposes simple techniques: clipped surrogate objective or adaptive KL penalty (Schulman et al. 2017).

Among many variations of PPO, we select an OpenAI’s implementation found in (Achiam 2018). The loss objective of PPO using the clipped surrogate objective method is $L_{\text{PPO}}$ in (4). We also provide the objective without clipping $L_{\text{NCLIP}}$.

$$L_{\text{NCLIP}} = \frac{1}{N} \sum_{i,t=1}^{N,T} r_{i,t}(\theta)A_t$$

$$L_{\text{PPO}} = \frac{1}{N} \sum_{i,t=1}^{N,T} \min(r_{i,t}(\theta), \text{clip}(r_{i,t}(\theta), 1 \pm \epsilon))A_t$$

(4)

where $r_{i,t}(\theta) = \pi_\theta(a_{i,t}|s_{i,t}) / \pi_{\text{old}}(a_{i,t}|s_{i,t})$

PPG: Proximal Policy Gradient

PPG, which we propose in this paper, is a simple variant of VPG, or a simple variant of PPO. We design the objective so that its gradient is the same as the gradient of $L_{\text{VPG}}$, while a clipping method similar to PPO is applicable. The resulting form has the log probability difference between the current policy and the old policy, $d_{i,t}(\theta) = \log \pi_\theta(a_{i,t}|s_{i,t}) - \log \pi_{\text{old}}(a_{i,t}|s_{i,t})$. This can also be interpreted as the application of the logarithmic operation to the surrogate objective as $d_{i,t} = \log r_{i,t}$. The objectives without clipping are
clipping $L_{NCLIP}^{PPG}$ and with clipping $L_{NCLIP}^{NCLIP}$ are

$$L_{NCLIP}^{PPG} = \frac{1}{N} \sum_{i,t=1}^{N,T} d_{i,t}(\theta) \hat{A}_t,$$

$$L_{NCLIP}^{PPG} = \frac{1}{N} \sum_{i,t=1}^{N,T} \delta_{i,t}(\theta),$$

$$\delta_{i,t}(\theta) = \begin{cases} \min(d_{i,t}(\theta), b), & \hat{A}_t \geq 0 \\ \max(d_{i,t}(\theta), b), & \hat{A}_t < 0 \end{cases},$$

$$d_{i,t}(\theta) = \log \pi_\theta(a_{i,t}|s_{i,t}) - \log \pi_{\text{old}}(a_{i,t}|s_{i,t}),$$

where $b$ is the clipping upper bound constant, and $l_b$ is the clipping lower bound constant.

Note that this clipping depends on the sign of the advantage $\hat{A}_t$. For the design details of $L_{NCLIP}^{PPG}$, we defer the discussion to the next section.

Since $\pi_{\text{old}}$ is a constant, the derivative of the $L_{NCLIP}^{PPG}$ is exactly equal to $L_{NCLIP}^{PPG}$, which is based on the gradient from the policy gradient theorem. Also note that the gradient of $L_{NCLIP}^{PPG}$ is different from $L_{NCLIP}^{PPG}$.

$$\nabla_\theta L_{NCLIP}^{PPG} = \nabla_\theta L_{NCLIP}^{PPG} = \frac{1}{N} \sum_{i,t=1}^{N,T} \nabla_\theta \log \pi_\theta(a_{i,t}|s_{i,t}) \hat{A}_t$$

$$\neq \nabla_\theta L_{NCLIP}^{PPG} = \frac{1}{N} \sum_{i,t=1}^{N,T} \nabla_\theta \pi_\theta(a_{i,t}|s_{i,t}) \hat{A}_t$$

The samples clipped by $L_{NCLIP}^{PPG}$ do not contribute to the gradient of $L_{NCLIP}^{PPG}$, and hence, for unclipped samples, $\nabla_\theta L_{NCLIP}^{PPG} = \nabla_\theta L_{NCLIP}^{PPG} = \nabla_\theta L_{NCLIP}^{PPG}$.

**PPG Clipping in Advantage Policy Plane**

When training a policy using VPG, it is difficult to match the performance of PPO simply by increasing the number of iteration as shown in Fig. 2. In order to achieve similar performance of PPO, a clipping mechanism similar to PPO is needed. In this section, we develop this clipping formula $L_{NCLIP}^{PPG}$ in [5], which we call PPG clipping.

In a policy update iteration, if $\hat{A}_t > 0$, the policy parameters should be updated to increase the probability $\pi_\theta(a_t|s_t)$, and if $\hat{A}_t < 0$, the policy parameters should be updated in the direction of decreasing $\pi_\theta(a_t|s_t)$. As illustrated in Fig. 4, the gradient of $L_{NCLIP}^{PPG}$ should have a downward $y$-component in the 2nd and the 3rd quadrants, and a upward $y$-component in the 1st and the 4th quadrants.

Suppose a sample is in the 2nd quadrant. This means that the probability has been increased even though the action is disadvantageous. To penalize this undesirable update, we do not set a clipping upper bound. Then all samples in the 2nd quadrant are used in the gradient ascent, and their probabilities are likely to be decreased for the next policy update. Now, suppose that a sample is in the 4th quadrant. This represents that the probability of that advantageous action is
Figure 3: KL divergences for PPG (light blue) and PPO (light orange). The graphs show the KL divergences of PPG tend to stay lower than that of PPO throughout the training.

Figure 4: Advantage-Policy plane. In left half plane, since $\hat{A}_t < 0$, probability should decrease and on the right half plane, since $\hat{A}_t > 0$, probability should increase. We apply a clipping such that gradient is zero in the blue shaded regions since log probabilities have updated more than the clipping thresholds $u_b$ and $l_b$. See Fig. 1 for an example.

As a policy update iteration continues, $\pi_\theta$ becomes different from $\pi_{\theta,old}$ as demonstrated in Fig. 1. The KL divergence, or relative entropy, measures the difference between $\pi_\theta$ and $\pi_{\theta,old}$ as

$$D_{KL}(\pi_\theta || \pi_{\theta,old}) = \int \pi_\theta(a|s) \log \frac{\pi_\theta(a|s)}{\pi_{\theta,old}(a|s)} da \quad (6)$$

$D_{KL}$ can be approximated by the following $D_{MC}$, i.e., $D_{KL} \rightarrow D_{MC}$ as $n \rightarrow \infty$ (Hershey and Olsen 2007).

Discussions on KL Divergence

As policy update iteration continues, $\pi_\theta$ becomes different from $\pi_{\theta,old}$ as demonstrated in Fig. 1. The KL divergence, or relative entropy, measures the difference between $\pi_\theta$ and $\pi_{\theta,old}$ as

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where $D_{KL}$ can be approximated by the following $D_{MC}$, i.e., $D_{KL} \rightarrow D_{MC}$ as $n \rightarrow \infty$ (Hershey and Olsen 2007).

$$D_{MC}(\pi_\theta || \pi_{\theta,old}) = \frac{1}{N} \sum_{i,t=1}^{N,T_i} (\log \pi_\theta(a|s) - \log \pi_{\theta,old}(a|s))$$

$$= \frac{1}{N} \sum_{i,t=1}^{N,T_i} d_{i,t}$$
Therefore, the average y coordinates of all samples in the advantage-policy plane is just the approximated KL divergence $D_{MC}$. The objective $L_{\text{PPG NCLIP}} = \frac{1}{N} \sum_{i,t=1}^{N,T} d_{i,t}(\theta) \hat{A}_t$ has some relation to $D_{MC}$. Although $d_{i,t}(\theta)$ and $\hat{A}_t$ are correlated, one can easily see some trivial relations. For example, $L_{\text{PPG NCLIP}} \leq \max(\hat{A}_t) D_{MC}$ in the right half plane where $\hat{A}_t > 0$. Since $\hat{A}_t$ is normalized to have the standard deviation of 1, $\max(\hat{A}_t)$ is not large, typically less than 10.

When the iteration starts, all samples are on the x-axis. As the current policy is updated, the samples gradually spread over to all quadrants, while moving to the first and third quadrants are encouraged by the gradient. In the left half plane, since samples are driven to have decreased probabilities by the optimization, the contribution to the approximate KL divergence, $\sum \hat{A}_{t} < 0 d_{i,t}$ is likely to be negative. In the right half plane, $\sum \hat{A}_{t} > 0 d_{i,t}$ is likely to be positive. Since the policy network outputs log probabilities, the log probability changes in both of the half planes are often similar in magnitude. Therefore, the two KL contributions tend to cancel, yielding a small approximate KL divergence as shown in Fig. 5.

Finally, we note that we have performed preliminary tests using a number of $D_{KL}$ constraint ideas such as $D_{KL}$ penalty added to the loss $L_{\text{PPG}}$ with or without various dead zones designs, for example, penalize only if $d_{i,t} > c$, where $c$ is a small constant. We also penalized using the average of exact KL divergence at each sample. In general, we found $D_{KL}$ is easy to control in PPG iterations, and the results were often positive, yet a principle that guides the choice among these methods and a study on general applicability on various environments need more thorough investigations, and hence, are left as future works.
Experiments

Algorithms and Settings

We present Alg. 1 based on Achiam [2018] and Dhariwal et al. [2017]. The policy \( \pi_\theta \) and the value network \( V_\phi \), parameterized by \( \phi \), are the fully connected networks with two layers of size 64. We used hyperbolic tangent activation functions for all layers. For each epoch, after extracting trajectory samples, \( A_1^{SAFE} \) is computed and normalized. For VPG, the policy iteration is done once, and for PPG and PPO, the iteration is continued up to 80.

PPG and PPO perform the clipping step to update the policy within the trust region. This clipping functions have the clip range \( [\epsilon, 1] \) (In our experiments, \( u_b = 0.2, \ l_b = -0.2, \ \epsilon = 0.2 \)). The lines 9 to 11 in Alg. 1 implement a secondary optimization technique: if the approximated KL divergence reaches to the target \( D_{MC}^{Target} \), which we choose 0.015, the policy update iteration is halted. Without this policy learning became unstable in most of the environments. After the policy iteration, \( V_\phi \) is updated by minimizing the mean square error \( ||\hat{\Pi}_t - V_\phi||^2 \).

Algorithm 1 VPG, PPO, PPG

1: Input: an initialized policy \( \pi_\theta \) and value function \( V_\phi \)
2: for \( k \) = 1 to epoch
3: \( \pi_{\theta, old} \leftarrow \pi_\theta \)
4: Collect trajectories \( \{\tau_1, \tau_2, \tau_3, \ldots\} \) from \( \pi_{\theta, old} \)
5: Compute return \( \hat{R}_t \)
6: Compute each \( A_1^{SAFE} \) using \( V_\phi \)
7: for \( i = 1 \) to \( n \) (\( n=1 \) when VPG)
8: Compute loss \( (2), (4), \) or \( (5) \)
9: \( mKL \leftarrow \) the mean of the approximated \( D_{MC} \)
10: if \( mKL > D_{MC}^{Target} \) and not VPG
11: break
12: end if
13: Update \( \pi_\theta \) by maximizing loss \( (2), (4), \) or \( (5) \)
14: end for
15: Update \( V_\phi \) by minimizing \( ||\hat{R}_t - V_\phi||^2 \)
16: end for

Evaluation on Benchmark Tasks

We evaluated PPG on the OpenAI Gym robotics tasks (Brockman et al. 2016), and the corresponding the Bullet tasks (Ant-v2, HalfCheetah-v2, Hopper-v2, Walker2d-v2, AntBulletEnv-v0, HalfCheetahBulletEnv-v0, HopperBulletEnv-v0, Walker2DBulletEnv-v0). For each environment, we trained 10 models using ten seeds (10000,10001,10002,...,10009). In the OpenAI Gym tasks, the models are trained for 20 million interactions with the each environments, and 4 million interactions for Bullet tasks. Training results are shown in Fig. 5 and trained model evaluations are provided in Table 1. For this evaluation, we simulated 100 episodes for each model to measure the performance. We can observe that the entropy decays slower in PPG, while the average return is similar to PPO. In addition, in Fig. 3 PPG has smaller the KL divergence than PPO, under the exactly same condition in Alg. 1.

Table 1: Trained model evaluations. Each entry has ten different trained models for 10 seeds. The average rewards are computed from 1000 episodes (100 episodes for each of 10 seeds). The entropy is the average of 10 models.

| Environment                  | Reward       | (Entropy)    | VPG       |
|------------------------------|--------------|--------------|-----------|
| Ant                          | 5527±947.5   | 5583±652.8   | 2360±900.6|
| HalfCheetah                  | 4783±1708    | 4310±1537    | 3463±1480 |
| Hopper                      | 1865±769.7   | 2292±970.2   | 1756±817.9|
| Walker2d                    | 3943±1400    | 3957±1330    | 3209±1307 |
| Ant                          | -1.08±0.04   | -1.27±0.02   | 0.02±0.01 |
| HalfCheetah                  | -1.28±0.15   | -1.39±0.14   | 0.03±0.07 |
| Hopper                      | 0.07±0.14    | -1.16±0.11   | 0.61±0.02 |
| Walker2d                    | -0.07±0.09   | -0.68±0.08   | 0.71±0.01 |

Conclusions

We have shown that the surrogate objective can be replaced by a partial variation of the original policy gradient objective that has exactly the same gradient with the original policy gradient theorem based algorithm, VPG. Similar to PPO’s clipping mechanism that updates a policy within the trust region, PPG formulates the clipping in the log space. PPG shows the average returns comparable to PPO, and the smaller KL divergence changes than PPO.

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