Stochastic Processes Occurring during the Transition of Technical State of the Structure

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Abstract. The possibility of applying the theory of stochastic processes for evaluating the dynamic pattern of different states is studied for critical-duty structures. Heterogeneous Markovian processes of technical state transition for metallurgical overhead crane structure, Markovian theorem and Kolmogorov-Chapman equation are analyzed. Markovian chain is reviewed at \( t \to \infty \), i.e. under marginal steady-state (stabilized) condition. Real values of limit probabilities are obtained for the structure of the metallurgical overhead crane under review. The proposed approach redefines and elaborates the existing methods and procedures for evaluating the technical state of structures and reduces the level of ambiguity associated with such kind of problems.

1 Introduction

Stochastic processes explore the dynamic evolution of random phenomena [1]. This particular approach is necessary for addressing various physical, technological and other types of problems. For certain reasons, the evaluation of the existing technical state of any structures is essential these days. This particularly applies to the heavy structures operating under different conditions such as cranes of different applications, bridges, pipelines, spaceship masts, etc. [2-6]. For the majority of these structures, the existing technical state is far from perfect [2-5]. The dynamic pattern of their technical state transition can be described by the theory of stochastic processes [1, 6-11]. Representatives of different science departments from the Nosov Magnitogorsk State Technical University have successfully applied the principles of structural-matrix approach, which allows to describe various states and transitions occurring with process or production facilities in their entirety [4, 5].

We shall review the structure of a metallurgical crane that is operating under heavy and super-heavy conditions [2-5].

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2 Research Materials and Methods, Discussion

Average operating life of a metallurgical crane is 25-30 years although its guaranteed service life is 15 years. The load-carrying capability of such cranes is verified and calculated according to specific procedures and based on the first and the second marginal conditions [12]. Application of the stochastic process theory [1] and probabilistic risk analysis [2, 3, 13] will allow to elaborate these procedures and apply a more well-reasoned approach to the operation of such structures.

We shall review the heterogeneous Markovian chain, for which the transition probabilities $p_{ij}$ will vary with each step. We shall define the probability of transition between structural states $S_i$ and $S_j$ at the k-th step as $p_{ij}^k$ and assume that we know the matrices $P_k$ of transition probabilities at each step to obtain the following Kolmogorov-Chapman equation:

$$p(k) = p(k - 1)P_k.$$  \hfill (1)

The equation (1) represents an example of recurrent relationship, which allows to calculate the probabilities of different stochastic Markovian process states at any step when information on the preceding states is available.

$$p(k) = P_1 * P_2 * ... * P_k.$$  \hfill (2)

The equation (2) will determine the probability of transition from the initial step over k-number of steps. It can be noticed that the elements in the columns become equivalent for some matrices $P$ when the k-values are high, which means that the limit probabilities of states are independent of the initial state. This indicates that the probability of the system state $S_j$ is essentially independent on what occurred in the distant past. In other words, the Markovian theorem applies in this case.

Thus, we can proceed to solving the practical problem of applying the theory of stochastic processes to the evaluation of the existing technical state of the structure.

During long-time operation, any structure can be regarded as a physical system existing in one of the following states based on the results of evaluation:

$S_1$ means that the structure (system) is in good state;

$S_2$ means that the structure (system) has slight defects but can still perform the assigned operational tasks;

$S_3$ means that the structure (system) has significant defects but can still perform a limited number of operational tasks;

$S_4$ means that the structure (system) broke down, got destroyed or failed.

We shall regard the process occurring in this system as a heterogeneous Markovian chain (one step corresponds to one scheduled inspection of the crane structure after 15 years of operation) with respective transition matrices [2, 3, 6]:

$$p(t) = p_0 \exp \left[ - \int_0^t \lambda(t) dt \right].$$  \hfill (3)

$$\lambda(t) = 0.005 + 0.003t.$$  \hfill (4)

$p(t)$ is the probability of failure-free operation after the expiry of guaranteed service life; $p_0$ is the initial value when the structure is in good state ($S_1$); $\lambda(t)$ is the failure density function.

Based on the conducted analysis [3], the expected life of a metallurgical crane with given parameters after the expiry of guaranteed service life was estimated to be 5 years.
This was followed by performing expert examination of the crane and making the decision regarding its further operation.

Let us find the limit probabilities under steady-state (stabilized) condition, i.e. at $t \to \infty$, by using the following theorem: when the number of system states is finite and the transition between any such states is possible over a finite number of steps, then the limit probabilities can exist [1, 8].

The structure will remain in S1 (good) state for 48.4% of time, in S2 (slightly defective) state for 30.3% of time, in S3 (significantly defective) state for 20.3% of time and in S4 (emergency) state for 1.0% of time (figure 1).

![Fig. 1. Limit probabilities of structural states](image)

Speaking about the adequacy of such application of the stochastic process theory to a realistic process task, it can be justified that this theory is applicable since it fits in with the realistic operating lifetime period of metallurgical cranes and the obtained limit probabilities match with the actual data.

The use of the above approach definitely elaborates the existing methods and procedures [6, 12] by reducing the entropy of information ambiguity associated with such kind of problems.

### 3 Conclusions

The theory of stochastic processes can definitely apply to address the evaluation of the existing technical state of various critical-duty structures.

The obtained limit probabilities of different structural states provide the opportunity to take appropriate technical and engineering decisions regarding the further operation of structures.

The proposed approach will make the assessment of risk involved with critical-duty structures more accurate.
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