The Simulation Evaluation System Fuzzy Reliability Based on Fuzzy Fault Tree

Haidong Du, Junhai Cao, Shen Ying and Lingwei Li

Department of Technical Support Projects of Academy of Armored Forces Engineering, Beijing, 100072, P.R. China

Abstract—For the indeterminism problem existing in dynamic reliability of complex system, the thesis presents the simulation analyze method for reliability based on fuzzy Markov process. Firstly, a reliability model according to the fuzzy theory and fault tree was described; and then, the paper study the transformation method between dynamic fault tree and Markov process, the evaluation model for fuzzy dynamic reliability question was built, and a Monte Carlo simulation for fuzzy reliability membership was given. And last, the scientificity and effectiveness of method was proved by the example of internal combustion engine.

Keywords—fuzzy; markov process; reliability; simulation

I. INTRODUCTION

The failure rate of product is difficult to account because of the mission period which experiences during the task is unstable, the statistics result of system reliability is uncertain. under this situation, the traditional computation based on Markov process according to boolean algebra and fault tree is not suit for the study of system reliability. Briabant V raised a optimization method for the structural system reliability design on the foundation of probability[1], and then, Smith A and Pennetsa used vertex as well as optimization arithmetic for the study of fuzzy reliability analysis[2-3]. Moller B and Adduri PR have study the problem under random and fuzzy condition, but there is a large deviation form the actual statistics[4-5]. Zhang and Weng used Vague function for the question under dynamic state, but the calculation process for the membership is only suitable for the questions under the static state situation[9]. According to the aboving questions, the thesis used fuzzy Markov process to describe the change regulation under the dynamic state, the arithmetic for the membership grade also developed based on optimization simulation.

II. FUZZY FAULT TREE

A. Fundamental principles

In order to solve the fuzzy problem under the multi-phase mission, fuzzy mathematics theory was applied. The basic translation is to assume the range of values for occurrence probability of bottom event is [0,1], which will be replaced by a trigonometric function like triangular(m,α,β) used in fuzzy fault tree and that make the method feasible based on traditional fault tree. The fixed probability of bottom event is convert to a fuzzy variable value, m, α, β are characteristic variables, which determines the top limit and floor limit of the function value.

The computational formula of system reliability from traditional reliability theory is

\[ R(t) = e^{-\int_0^t \lambda(t)dt} = \exp(-\int_0^t \lambda(t)dt) \]

and \( \lambda(t) \) is fault rate, then we could get the reliability of system is

\[ R(t) = e^{-\lambda t} \]

So we could describe the system reliability under the fuzzy situation use \( \tilde{R}(t) = 1 - \tilde{F}(t) \), \( \tilde{F}(t) \) is the probability of system fault.

III. DYNAMIC FUZZY RELIABILITY ASSESSMENT BASED ON SIMULATION

A. Dynamic Fuzzy Reliability

If the total mission time is \( T \), which could be split many discrete periods, the state of bottom event will stay constant during the time \( \Delta T \). If there are \( n \) bottom event existing in fuzzy fault tree of syste, the i-th sample time is \( \xi_i = F_i^{-1}(t) \), \( t_{ji} = t_{ji-1} + \xi_i \), during which \( F_i^{-1}(t) \) is the fuzzy fault probability distribution function of bottom event, \( t_{ji} \) is the sequence for the j-th bottom event, \( \xi_i \) is the sample result for the bottom event. \( T_{max} \) is the longest time among the sampling process, then we could get the sequence as \( 0 = t_0^l \leq t_1^l \leq \cdots \leq t_m^l \leq T_{max} \) sorted by volume in descending order. Then the simulation process could be designed as:

a) Initialization. There are no fault existing in system at the beginning of simulation, \( t_{sum} \) is the total time for the simulation process, \( t_s \) is the operating time without fault, \( A_s \) is the times for the success experiment, \( t_F \) is the fault time for system, \( F_s \) is the fault times. The initial value of them is 0.

b) Sampling from probability distribution function \( F_i^{-1}(t) \) of every bottom event, then the time sequence will be updated;

c) \( \bar{\theta}_{ji}(t) \) is the reliability function of the j-th bottom event from the i-th sample, \( \bar{\theta}_{ji}(t) = 0 \) represents the state of bottom event is operating well, \( \bar{\theta}_{ji}(t) = 1 \) means there are fault exists in system. So, for n-parts system, if \( F_i^{-1}(\bar{\theta}_{i1}(t),\bar{\theta}_{i2}(t),...,\bar{\theta}_{in}(t)) = 0 \), the system operating well, and the operating time of system updates to \( t_s = t_{s-1} + t_{ji-1} \), the last operating time of system is \( t_{s-1} \), \( t_{ji-1} \) is the sampling time, and then \( A_s = 1 \). The fault time of system is \( t_F \), and \( A_s = 0 \) on the contrary.

d) If \( t_s < t_{sum} \), the abving steps will be iterated until \( t_s > t_{sum} \) and the simulation is end.
The result of \( \sum_{i=1}^{m} t_i = \sum_{i=1}^{m} A_i \) could be calculated rely on the statistics data, during which, \( m \) is the iteration times. The point estimation value of system fault probability could be described by the inverse function of \( F^{-1}(t) \), and the result is \( \hat{P}(t) = P(F^{-1}(t) \leq t) \approx \frac{1}{t_{\text{sum}}} \sum_{i=1}^{m} t_i \).

B. Algorithm of fuzzy degree of membership

The reliability function of fuzzy system under random process is \( F(V_R, V_F) \), and \( V_R = (v_{R1}, v_{R2}, ..., v_{Rn}) \) are \( n \) independent variables, and \( V_F = (v_{F1}, v_{F2}, ..., v_{Fm}) \) are \( m \) independent fuzzy variables, and the degree of membership of which is \( D_{\chi}(V_R, V_F) \). The range of current fuzzy variable value under \( \chi \) degree is \( V_R(\chi) \in (V_R^\text{bottom}, V_R^\text{top}) \), and \( V_R^\text{bottom} \) and \( V_R^\text{top} \) are the floor limit and top limit of the fuzzy variable for the range, which determines the reliability of system. The floor limit and top limit for every \( \chi \) could be calculated used the follow expression as:

\[
\begin{align*}
P_{\text{bottom}} &= P(g_{\text{bottom}}(V_R, V_F(\chi)) > 0) \\
P_{\text{top}} &= P(g_{\text{top}}(V_R, V_F(\chi)) > 0)
\end{align*}
\]

The calculation algorithm of fuzzy degree of membership could be designed according to subset iterative simulation method as FIGURE I shows.

![FIGURE I. THE CALCULATION PROCESS OF MEMBERSHIP DEGREE FOR FUZZY RELIABILITY](image)

IV. CASE ANALYSIS

A. Dynamic fuzzy reliability

The fault tree model of the product as FIGURE II shows, the fault probability of the bottom event with trigonometric function just as TABLE I shows.

![FIGURE II. THE FAULT TREE MODEL OF THE SYSTEM](image)

TABLE I. THE FAULT PROBABILITY USING TRIGONOMETRIC FUNCTION

| Bottom event | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( A_4 \) | \( A_5 \) | \( A_6 \) | \( A_7 \) | \( A_8 \) |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|
| \( A_1 \)    | (0.425, 0.500, 0.575) | (1.350, 1.475, 1.600) | (0.612, 0.720, 0.828) | (1.275, 1.500, 1.725) | (0.425, 0.500, 0.575) | (1.350, 1.475, 1.525) | (0.612, 0.720, 0.828) | (0.068, 0.08, 0.092) |
| \( A_8 \)    | (0.722, 0.850, 0.977) | (0.485, 0.570, 0.655) |

The fuzzy reliability of system at time \( t \) is \( R(t) \), then we could get the variation curve of system dynamic reliability as FIGURE III shows during the working time of 500h on the foundation of Anylogic software.

![FIGURE III. THE VARIATION CURVE OF SYSTEM DYNAMIC RELIABILITY](image)

In the aboving figure, the sample result of system reliability fluctuates as dashed shows, and the accurate result using the intermediate value of trigonometric function as the red line shows.

B. Dynamic Degree of Membership

If we could get the dynamic fuzzy reliability of system at every time node of mission phase, then we could build the programming model of bottom event existing in the fault tree according to formula (1), then we could calculate the floor limit and top limit of fuzzy reliability using Opt Quest algorithm. The iteration times of simulation are 500, and the stop time of every simulation is 500h.
According to the aboves steps, we could get the value range of fuzzy reliability of every time node during the mission process, the result as FIGURE IV. shows.

![The variation curve of top limit and floor limit of system fuzzy reliability](image)

Then we could calculate the membership degree of reliability at time 100, 200, 300, 400, 500, and the result as Table.2 shows using trigonometric function. For example, the sample reliability result at time 100 is 0.927, and the membership of which is 0.243, which means the reliability of system is 0.927 with the probability of 24.3% complete the mission.

V. CONCLUSION

It’s difficult for the evaluation of system reliability under multi phase mission with random and fuzzy question, so fuzzy mathematics theory and fault tree was introduced into the solution process, and a simulation calculation method based on Markov process was presented in the thesis. And the algorithm for fuzzy membership degree was also designed, so which make it’s easy to build the reliability model for the system with fuzzy characteristic, and the calculation process is more practical in engineering work.

| Working time $T (h)$ | Sample result $R(t)$ | Top limit and floor limit $[R^+(t), R^-(t)]$ | Membership degree $\mu_{\omega(t)}$ |
|-----------------------|----------------------|---------------------------------------------|----------------------------------|
| 100                   | 0.927               | [0.891, 0.932]                             | 24.3%                            |
| 200                   | 0.843               | [0.779, 0.865]                             | 51.2%                            |
| 300                   | 0.741               | [0.668, 0.798]                             | 87.7%                            |
| 400                   | 0.638               | [0.547, 0.719]                             | 94.2%                            |
| 500                   | 0.509               | [0.422, 0.709]                             | 75.4%                            |

REFERENCES

[1] Briabant V, Oudshoorn, Boyer C, Delcroix F. Nondeterministic possibilistic approaches for structural analysis and optimal design[J]. American Institute of Aeronautics and Astronautics Journal(AIAA Journal), 1999,37(10):1298-1303

[2] Smith S A, Krishnamurthy T, Mason B H. Optimized vertex method and hybrid reliability[C].// Proceedings of 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference. Hilton Head, SC: Aeronautics and Astronautics and aeronautics, 2002:1-12

[3] Penmetsa R C, Grandhi R V. Uncertainty propagation using possibility theory and function approximations[J]. Mech Based Des Struct, 2003,31(2):257-279

[4] Moller B, Graf W, Beer M. Safety assessment of structures in view of fuzzy randomness[J]. Comput Struct, 2003,81(15):1567-1582

[5] Adduri P R, Penmetsa R C. Confidence bounds on component reliability in the presence of mixed uncertain variables[J]. Int J Mech Sci, 2008,50(3):481-489

[6] Du X. Unified uncertainty analysis by the third order reliability method[J]. Journal of Mechanical Design(ASME), 2008,130(9):091401-091410

[7] Zhang Xiangnan, Liu Anxin, Gao Qingzhen, etc. Fuzzy fault tree method for reliability design of complex system in engineering machinery[J]. Journal of Machine Design, 2011,28(1):9-12

[8] Weng Xingguo, Ye Song, Wang Xiaolei, etc. Reliability Analysis of Airborne Unit of NMHEMS Based on Vague Fault Tree[J]. Fire Control & Command Control, 2014, 39 (1):27-31

[9] Yuan Long, Zhao Jiang, Li Meng, etc. Fuzzy fault tree analysis by a high accuracy fast algorithm [J]. Journal of Mechanical Strength, 2013, 35 (6):755-757