High Density Strange Star Matter and Observed Parity Doubling of Excited Hadrons

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Abstract

Parity doubling is observed in hadron states around 1.5 GeV - and the estimated energy density is found to be high. When a large excitation energy is available and the pions decouple from the quarks, the QCD interaction is still not perturbative. And signature of such a system can exist in the form of small ratio of the shear viscosity to entropy density. This is true for an equation of state which is applicable to the surface of strange star. We indicate the correspondence of parity doubling with the apparently disconnected model calculation of compactness of some pulsars.

Keywords(pacs) 11.30.Rd Chiral symmetries 11.10.St Bound and unstable states 11.15.Tk Other non-perturbative techniques

1 Introduction

Taking the recently determined experimental hadronic resonances we see that parity doubling occurs at energy densities which are comparable to densities of strange stars. Parity doubling is an indicative of chiral symmetry restoration (CSR), which can be also be found in strange star models, that has successfully explained many inexplicable features of the astrophysical compact objects.

In this model, which we call ReSS, the u, d and s quarks become gradually less massive as the density increases [1, 2]. Pions decouple from quarks and CSR is observed partially in the surface and strongly at the core of the star [3]. Since existence of strange stars cannot be directly proved with the
present day observation technique, the support of CSR from parity doubling in hadronic resonances may play an important role. In an earlier work by Bagchi et al. [3] the density dependent mass of ReSS was included in their paper following the well known model due to Nambu and Jona Lasinio. There the pion coupling to the quarks namely $f_\pi$ was found to decrease signaling decoupling of the pion from the quarks.

The parity doubling in baryon resonances was rediscovered in 2000 by Glozman [4] and there are many follow up papers [5] - including [6]. In the last mentioned paper, the authors think that chiral symmetry realized in the Nambu Goldstone mode does not predict the existence of degenerate multiplets of hadrons of opposite parity. However they assert that their arguments do not preclude the restoration of chiral symmetry at high temperature or high energy density. In the present paper we seek to show that indeed the energy densities involved are comparable to ReSS and are thus expected to lead to CSR. To do this we show the correspondence to a situation where the chemical potential is high in a strange quark matter (SQM) equation of state (EoS). This EoS, along with a density dependent quark mass, has been used to construct strange star models at zero [1] and finite temperature [2].

This strange star model, in contrast to the recent papers seeking to establish parity doubling, consists of a large $N_c$ mean field approximation with a realistic modified Richardson potential between quarks, but is crucial in explaining many observations like (a) super bursts [7], (b) the existence of minimum magnetic field for all observed pulsars [8], and (c) absorption and emission bands along with high redshift [9].

We observe that the list of states from 2 GeV to 2.51 GeV for the mesons, which show the so called clustering observed recently by Afonin [10], have very high energy density. This suggests CSR, which is built-in for our model. With respect to the objections to CSR [6], we agree with Afonin who observes “one should be careful with any statements forbidding CSR which are based on the language of the low - energy field theory like in ... ”. Most of the resonances are recently found and need confirmation according to the Particle Data Group (PDG) and we emphasize that confirmation and determination of new states is important since this is related to CSR.

The possibility that parity doubling is observed in baryon resonances was realized by Dey and Dey in 1993 [11]. This was inspired by the early work of Barut [12] where he had looked at parity doubled states in the conformal O(4,2) model as early as 1965.

There have been many models for indicating why parity doublets appear in QCD. The most recent one is the toy model of Cohen and Glozman [13] in which there are infinite number of pions and $\sigma$'s and which is claimed to mimic large $N_c$ QCD. Before this, there was the question of parity doubling in the baryons treated by Jaffe, Pirjol and Scardicchio [6]. And finally there was the revival of the model of Ademollo, Veneziano and Weinberg [14] by Afonin [10].

In section 2 we deal with the baryonic resonances at high energy. In section 3 we refer to strange stars and low viscosity-entropy ratio which has relevance to the recent relativistic Heavy ion collision (RHIC) experiments. We summarize and conclude in section 4.
2 Resonances at high energy

The large $N_c$ ReSS model employs a potential which has asymptotic freedom with a scale parameter $\Lambda = 100$ MeV and confinement scale of $\Lambda' = 350$. The stars are fitted with a density dependent form for quark masses at zero as well as finite temperature [1, 2] after solving for $\beta$ equilibrium and charge neutrality for u, d and s quarks and electrons. The hydrostatic equilibrium equation (TOV equation) is then solved self consistently to find the properties of the strange stars at all radii, the density varies from about 4.5 to 15 times normal nuclear density for a maximum mass of about 1.5 $M_\odot$.

We must point out that this SQM model fits the ground state baryons and their magnetic moments [15, 16] without the pion degrees of freedom. Also the quarks are found to decouple from the pion [3] as already stated.

Coming to the experimental states, we note that there are 314 even parity mesons and 308 for odd parity in the range $\Delta E = 2$ to 2.51 GeV [17]. Specific examples of parity doubling are striking: for example, $b_1 (I^G(J^{PC})1^+(1^{-+})1960$, $\rho 1^+(1^{--})1965$. Interestingly, mesonic spectrum can be predicted for example from the discrete quark - gluon plasma states of a finite bag as shown by Dey, Tomio and Dey [18]. With a reasonable bag pressure $B$ given by $B = 200$ MeV one can estimate the radii of these mesons in the bag model. We also note that in the 2006 PDG tables there are some states marked X which could soon be cleared up. What is striking is that even and odd parity states come up together. The way the density of states is calculated is standard [18], the entropy is minimized and the Laplace transform of its second derivative leads to the density of states in saddle point approximation. We refer the interested reader to the original paper. The density of states lead to a limiting temperature which is given in a Table in Dey, Tomio and Dey [18] to be 143.7 for a meson mass 2.1 GeV and 142.8 for 2.5 GeV and the radii are calculated in the same Table to be 0.844 $fm$ and 0.895 respectively. We will see that these match with the hadron radii calculated from other simple considerations. Such massive resonances in such small radii lead naturally to very high matter density.

Most of these states are new and we tabulate in Table (1). The excited states of the pions are wrongly marked as iso-singlets in the original Tables [17] but Dr. Eidelman assures us that these will be corrected in the 2007 edition of PDG.

We now estimate the radii of high energy mesons. Let the energy and average radius respectively be $E$, $R_{\text{reson}}$ for a mesonic resonance. Then from the surface energy density of a ReSS of $\epsilon = 627.36$ MeV/fm$^3$ one gets

$$\frac{E}{\epsilon} = \frac{4\pi}{3} R_{\text{reson}}^3. \quad (1)$$

$R_{\text{reson}}$ is 0.913 to 0.977 $fm$ when $E$ is 2 GeV or 2.5 GeV respectively.

The calculations that we present for the strange star are very straightforward. At the surface of the star, where the number density $n_q$ is minimum (about 4.6 times the normal nuclear density), on the average the quarks occupy a sphere of radius $r_n = 0.51$ $fm$ or less. We notice that the surface area of the star at any radius $r$ is $4\pi r^2$ whereas the projection of a quark with an effective volume $V = \frac{4}{3}\pi r^3_n$ is just $\pi (2 r_n)^2$. This gives us the inequality $r_n^2 \leq r^2/n_q$. Since we know the number
density \( n(r) \) at any \( r \) we can get
\[ n_q = 4 \pi r^2 2r_n n(r) \]
for a thin shell of depth \( 2r_n \). We get the above number when the inequality is assumed to be saturated and we put the surface number density appropriate for a star of mass \( \sim 1.5 \, M_\odot \). The smallness of \( r_n \) justifies the thin shell approximation.

Inside the star where the density is about 15 times the normal nuclear density, the number reduces to 0.314 \( fm \) showing that the effective radius of the quarks gets closer to a partonic picture since the chiral symmetry is restored in the model with a density dependent quark mass [1].

The mean inter-particle distance \( r_0 = 0.47 \, fm \) at the surface of a strange star, assuming quarks have an effective volume \( \frac{4}{3} \pi r_0^3 \) and the usual colour, 3-flavour and spin degeneracy. And like \( r_n \), at the centre of the star \( r_0 \) decreases to 0.315 \( fm \).

Thus we see that the average radius of the resonance is close to twice the size of \( r_n \) or \( r_0 \). Indeed the energy density is slightly larger than that at the surface. It also corresponds to a very little depth inside the star from the surface. These radii agree with the estimates of Dey, Tomio and Dey [18] which uses bag model with finite size corrections. But the number of states given by the bag model, which is 6674 between 1700 and 2100 \( MeV \) is too numerous [19] and may be due some bag artifact - or due to many resonances being so far unobserved.

We do a completely different calculation to demonstrate that for baryons also one is dealing with high density system. All models employing group theory seem to give a spectra which looks like a rotational band [20, 11]. We can see that the parity doubling occurs for isobars with a 3/2+ state at \( \sim 1705 \, MeV \) (observed by Manley and Saleski [21] and Li et al. [22] and a 3/2- state at 1700 \( MeV \) [17]. We take the 1700 3/2+ state as a band head and look for rotational states above it. The \( L = 2 \) states are well known as the quartet states 7/2+ (1950), 5/2+ (1905), 3/2+ (1920) and 1/2+ (1910). For \( L = 4 \) there are only three states : 11/2+ (2420), 9/2+ (2300) and 7/2+ (2300), the fourth 5/2 + state is worth looking for. Apart from the \( L=6, 15/2+ \) state there could be a 13/2+ state also at 3230 \( MeV \). The even parity isobar 15/2+ is at 2950 \( MeV \).

Using a simple moment of inertia model with the \( \alpha = h^2/2I \) with \( I = \frac{2}{5} M R_{reson}^2 \) one gets the resonance radius \( R_{reson} = 0.96 \, fm \) and the moment of inertia is \( I = 627 \, MeV \, fm^2 \). The quartet state comes out at 1886, the \( L=4 \) at 2321 and \( L=6 \) states at 3004. For this \( L=6 \) state the energy density comes out to be \( \epsilon = 810 \, MeV/fm^3 \) comparable to an energy density observed well inside a strange star. This is of some interest since the strange star surface marks the onset of a different phase. Taking the \( \Lambda(2350) \) 9/2+ state with \( L=4 \) we get \( I = 518 \, MeV \, fm^2 \), \( R_{reson} = 0.9 \, fm \) and energy density \( \epsilon = 770 \, MeV/fm^3 \).

The resonances have widths so we deal with the centroids and attempts to more accurate fitting would be futile. On the other hand this approximate but simple calculation seems to work.

\section{Strange matter, strange stars and low viscosity-entropy ratio}

With the value of \( r_0 \) at the surface for a ReSS at \( T = 80 \, MeV \), we satisfy the remarkable inequality
\[ 4 \pi \eta/s \geq 1 \] [23]. This was invoked in a much quoted paper by Kovtun, Son and Starinets [24] (KSS). Here \( s \) stands for the entropy density.

The relevant arguments of KSS [24] are very appealing to us since it only invokes general princi-
ples like Heisenberg uncertainty relation for the typical mean free time of a quasi-particle and \( s \) which in turn is proportional to the density of the quasi-particles. From here to our model is just one short step of identifying the quasi-particles to be the dressed quarks of the mean field description for a large colour effective theory.

The matter in the strange star is a so called perfect interacting liquid where bound reaches the fraction \((4\pi)^{-1}\) and thus it may be the same fluid which Lacey marks as RHIC in figure 3 of his paper - which stands for relativistic heavy ion collisions [25]. The point is that in RHIC one obtains a large elliptic flow which demands a very small shear viscosity whereas perturbative QCD yields a value which is almost ten times the bound.

We thus see a possible connection between parity doubling and high energy matter at the surface of a strange star. Then the fact that the strange matter supports such a low shear viscosity constraint relates it to RHIC. Considerable understanding of QCD will result from more definite observational signatures for the existence of strange stars, more experimental study of parity doubling in hadrons and explanation of RHIC data.

4 Summary and conclusion

Recent developments have made the rather perplexing connection to the idea of a limit for low shear viscosity \( \eta \) to entropy density \( s \), \( \eta/s < 1/(4\pi) \) to RHIC phenomenology [24]. The limit is not satisfied for perturbative QCD [26] and the viscosity is some ten times larger. For matter in the centre of a strange star the same conditions apply and the results are consistent. But at the surface of a strange star the limit is satisfied at \( T = 80 \text{ MeV} \), where the free energy has a minimum and the pressure is zero critically ensuring a self bound star [23].

To summarize, parity doubling in excited state spectrum of the hadrons is a high density phenomenon. It matches with ReSS model indicating that quarks decouple from pions. One finds a similarity in simple high density matter calculations rather than in complicated low energy models. The model supports the Kovtun, Son and Starinets bound for the ratio of shear viscosity to entropy density at a temperature of \( 80 \text{ MeV} \) at the surface of a strange star [23]. The elliptic flow observed at RHIC is believed to be due to such low shear viscosity. And one does not get such flow from partonic matter which uses perturbative QCD but rather from interacting QCD which produces dressed quasi-particle like objects.

We conclude suggesting that the high energy density hadronic resonances should be explored more extensively since they support the chiral symmetry restoration in models of strange stars and in RHIC.

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Table 1: List of mesons at high energy which show that including the degeneracy \((2J+1)(2T+1)\) the total number of odd and even parity states match roughly in the range 2 to 2.51 GeV.

| \(I^G(J^P_C)\) | State   | \(I^G(J^P_C)\) | State   |
|----------------|---------|----------------|---------|
| 0\(^+\)(2\(^+\)) | \(f_2(2000)\) | 1\(^+\)(1\(^-\)) | \(\rho(2000)\) |
| 0\(^+\)(0\(^+\)) | \(f_0(2020)\) | 1\(^-\)(2\(^+\)) | \(\pi_2(2005)\) |
| 1\(^-\)(0\(^+\)) | \(a_0(2020)\) | 0\(^+\)(0\(^+\)) | \(\eta(2010)\) |
| 1\(^+\)(3\(^-\)) | \(b_3(2025)\) | 1\(^-\)(1\(^-\)) | \(\pi_1(2015)\) |
| 1\(^-\)(4\(^+\)) | \(a_4(2040)\) | 0\(^-\)(3\(^-\)) | \(h_3(2025)\) |
| 0\(^+\)(3\(^+\)) | \(f_3(2050)\) | 0\(^+\)(2\(^+\)) | \(\eta_2(2030)\) |
| 0\(^+\)(4\(^+\)) | \(f_4(2050)\) | 1\(^-\)(0\(^-\)) | \(\pi(2070)\) |
| 0\(^+\)(0\(^+\)) | \(f_0(2060)\) | | |
| 1\(^-\)(3\(^+\)) | \(a_3(2070)\) | | |
| 1\(^-\)(2\(^+\)) | \(a_2(2080)\) | | |
| 1\(^-\)(1\(^+\)) | \(a_1(2095)\) | | |
| 0\(^+\)(0\(^-\)) | \(f_0(2100)\) | 0\(^+\)(0\(^-\)) | \(\eta(2100)\) |
| 0\(^+\)(2\(^+\)) | \(f_2(2150)\) | 1\(^-\)(2\(^+\)) | \(\pi_2(2100)\) |
| 0\(^-\)(2\(^+\)) | \(a_2(2175)\) | 0\(^-\)(1\(^-\)) | \(\omega(2145)\) |
| 0\(^-\)(1\(^-\)) | | 1\(^+\)(1\(^-\)) | \(\rho(2150)\) |
| 1\(^-\)(1\(^+\)) | | 0\(^+\)(0\(^-\)) | \(\eta(2190)\) |
| 1\(^-\)(2\(^+\)) | | 0\(^-\)(2\(^-\)) | \(\omega(2195)\) |
| 1\(^-\)(3\(^+\)) | | | |
| 1\(^-\)(4\(^+\)) | | | |
| 0\(^+\)(2\(^+\)) | | | |
| 0\(^+\)(3\(^+\)) | | | |
| 0\(^-\)(4\(^+\)) | | | |
| 0\(^+\)(1\(^+\)) | \(f_1(2310)\) | 0\(^+\)(4\(^-\)) | \(\eta_4(2320)\) |
| 1\(^-\)(3\(^+\)) | \(a_3(2310)\) | 0\(^+\)(1\(^-\)) | \(\omega(2330)\) |
| 0\(^+\)(0\(^+\)) | \(f_0(2330)\) | 1\(^+\)(5\(^-\)) | \(\rho_5(2350)\) |
| 1\(^-\)(1\(^+\)) | \(a_4(2340)\) | 1\(^-\)(0\(^+\)) | \(\pi(2360)\) |
| 0\(^-\)(2\(^+\)) | \(f_2(2340)\) | | |
| 1\(^-\)(6\(^+\)) | \(a_6(2450)\) | | |
| 0\(^+\)(6\(^+\)) | \(f_6(2510)\) | | |
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