Glassy Ratchets For Collectively Interacting Particles

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We show that ratchet effects can occur in a glassy media of interacting particles where there is no quenched substrate. We consider simulations of a disordered binary assembly of colloids in which only one species responds to a drive. We apply an asymmetric ac drive that would produce no net dc drift of an isolated overdamped particle. When interacting particles are present, the asymmetric ac drive produces a ratchet effect. A simple model captures many of the results from simulations, including flux reversals as a function of density and temperature.

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In ratcheting systems, a dc current or flux arises upon application of a periodic drive, such as when the underlying potential substrate is flashed, or, for a fixed substrate, when alternating forces are applied [1]. Ratchet effects, which can be deterministic or stochastic in nature, have been studied in a wide variety of systems, including biological motors [2], flux motion in superconductors [3], granular transport [4], and colloidal transport. In many of these systems, an underlying asymmetric potential acts to break the left-right spatial symmetry. Alternative ratchet systems that have recently been investigated [5] have symmetric substrates, and rely on ac drives to provide symmetry breaking. In each of these cases, the particles move through some form of fixed substrate.

In this work we study a ratchet effect that arises due to the collective interactions between driven and non-driven particles in a system that does not contain a fixed substrate. The particular system we consider is a binary mixture of interacting repulsive particles which form a disordered or glassy media. These particles, referred to as the active species, interact only with other particles, and do not respond to an applied ac drive. We introduce a small number of a separate species of particles, termed the passive species, which respond both to the other particles and to the ac drive. Using numerical simulations we demonstrate that a ratchet effect can be achieved for the active particles in this system when the coupling between the active and passive particles is strong enough and a symmetry breaking is introduced by means of the ac drive. This is in contrast to a recent theoretical proposal of an asymmetric drive ratchet which transports undriven superconducting vortices [6]. In addition, for specific ac drives, we find current reversals as a function of density and temperature. We also show that the magnitude of the ratchet effect goes through a maximum as a function of the number of active particles, due to the collective string-like motions of the particles.

We explicitly demonstrate a ratchet effect for mixtures of colloids. Our system is similar to recent studies of binary colloidal models driven with ac fields where one species of the colloids moves in the direction of the applied drive while the other species moves in the opposite direction [7]. These studies have found interesting collective effects in which the moving particles organize into lanes. In extensive studies of driven diffusive models [8], where the two species of oppositely moving particles are placed on a lattice, a rich variety of nonequilibrium phases were observed. Continuum models, with different portions of particles moving in opposite directions, have also been studied in the context of pedestrian flows [9]. A remarkable phenomena termed “freezing by heating,” where the system can jam at higher temperatures, has recently been observed in these types of systems [10]. Since individual colloids or groups of colloids can be manipulated easily with optical traps [11], colloidal systems are ideal for experimentally realizing a system in which only a fraction of the particles respond to the applied drive. The active species could be created using charged or magnetic colloids, while the passive particles would be charge-neutral or non-magnetic. For example, in recent experiments, magnetic colloids were driven with ac [12] and dc drives [13] through a glassy assembly of other non-magnetic colloids. It should also be possible to drive a fraction of the colloids using optical or magnetic traps.

We consider a binary two-dimensional (2D) assembly of colloids using the same simulation method as in our previous works [14,15]. The overdamped equation of motion for colloid $i$ is

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{f}_{ij} + \mathbf{f}_{AC} + \mathbf{f}_T ,$$

where $\mathbf{f}_{ij} = -\sum_{j \neq i}^N \nabla_i V(r_{ij})$ is the interaction force from the other colloids, $\mathbf{f}_{AC}$ is the force due to an applied driving field, and $\mathbf{f}_T$ is a thermal force from Langevin kicks. The colloids interact via a Yukawa or screened Coulomb interaction potential, $V(r_{ij}) = (q_i q_j / |\mathbf{r}_i - \mathbf{r}_j|) \exp(-\kappa |\mathbf{r}_i - \mathbf{r}_j|)$. Here $q_i$ is the charge on colloid $i$, $1/\kappa$ is the screening length, and $\mathbf{r}_{i(j)}$ is the position of particle $i(j)$. The system size is measured in units of the lattice constant $a$, which is set to $a = a_0$ for most of this work, giving a particle density $\rho = 1/a_0^2$. We take the screening length $\kappa = 3/a_0$. Our system is periodic in both the $x$ and $y$ directions. The total number of particles is $N = N_A + N_B + N_D$, with $N_A$ colloids of passive species $A$ and charge $q_A$, and $N_B$ colloids of passive species $B$ and charge $q_B$. We add $N_D$ active colloids...
with charge $q_D$ that respond to an applied ac drive. For the results presented here, we fix $q_A/q_B = 1/2$. We have also considered other ratios, including the crystalline case when $q_A/q_B = 1$. To initialize the system, we start at a high temperature and anneal either to $T = 0$ or to a fixed $T > 0$. The binary particles of species A and B form a disordered assembly, with the active colloids scattered randomly throughout. After annealing, we apply a two-step ac drive during a single cycle of period $\tau$. The first stage is a fast drive where a constant force of $f_1$ is applied for a time $\tau_1$ in the positive $x$-direction. The second stage is a slow drive where a force $f_2 < f_1$ is applied for a time $\tau_2 > \tau_1$ in the negative $x$-direction. The ac drive has the properties $f_1 \tau_1 = f_2 \tau_2$, with $\tau = \tau_1 + \tau_2$, so that the particle experiences zero average force during a single cycle. The average drift velocity $v_d$ can be obtained after many cycles. We conduct a series of simulations for different particle densities, system sizes, temperatures, $q_D$, ac amplitudes, frequencies, and $N_D$.

We first consider a simple model that predicts that a ratchet effect can arise in such a system. Recently it was shown theoretically and in simulations that, for a single colloid driven with a dc drive through a glassy assembly of other colloids [14], the colloid velocity follows a power law $V \propto f^\alpha$ with $\alpha = 1.5$ for certain parameters. The power law appears in regimes where the driven colloid produces collective disturbances of the surrounding colloids on length scales of the order of $a$, the lattice constant. When the density of the surrounding colloids is low, these collective distortions are lost and the velocity of the driven colloid is linear with the drive. Linear behavior also occurs when the charge $q_D$ of the driven particle is much smaller than that of the surrounding particles. Recent experiments on driven magnetic colloids have also found power law velocity-force relations for the driven colloid with exponents $\alpha = 1.5$ or higher in systems with high density [12]. In low density systems, the velocity is linear with drive.

We find that for a group of driven particles in a glassy media, the velocity-force curve is also of a power law form. In a system with a power law velocity-force relationship, ratcheting can occur under the application of an asymmetric ac drive. During a single cycle the colloids move a distance $\delta x = f_1^\beta \tau_1 - f_2^\beta \tau_2$. The net drift velocity is then $v_d = \delta x/(\tau_1 + \tau_2)$. Using the condition $f_1 \tau_1 = f_2 \tau_2$, the drift velocity can be expressed as

$$v_d = \frac{f_1^\beta f_2 - f_2^\beta f_1}{f_2 + f_1}$$

(2)

If $\beta = 1$, as in the case of a single overdamped particle, then $v_d = 0$, as expected. For any value of $\beta > 1$, however, the drift velocity will be positive, $v_d > 0$. Ratcheting can still occur if $f_1$ is close to $f_2$ as long as $\beta > 1$. This model predicts that the ratchet effect becomes more efficient for larger values of $\beta$. In Fig. 1(a) we plot $v_d$ vs $\beta$ from Eq. (2), with $f_1 = 1$ and ratios of $f_2/f_1 = 0.5$ to 1. For $f_2/f_1 = 1$, $v_d = 0$ for all $\beta$, as expected. For $f_2 < f_1$ and $\beta > 1$, ratcheting in the positive direction ($v_d > 0$) occurs. $v_d$ increases with larger $\beta$ and larger $f_2/f_1$. The velocity reverses for $\beta < 1$. This is an unphysical limit for the colloid system, where the lowest possible value is $\beta = 1$. However, some physical systems such as non-Newtonian fluids could have $\beta < 1$.

Since the velocity becomes linear with the drive at low densities or high drives, it should be possible to select ac drive parameters that reverse the ratchet effect and give current reversals for $\beta > 1$. This can be achieved by setting $\tau_1 < \tau_2$, where the inequality is not too large. In the highly nonlinear regime the particle will still ratchet in the positive direction; however, upon approaching the linear regime, the particle will ratchet in the negative direction. In Fig. 1(b) we show $v$ vs $\beta$ from Eqn. (2) for $f_2/f_1 = 0.5$ and increasing $\tau_2/\tau_1$, where a crossover from a positive to negative ratchet effect occurs at $\beta_c$. Here $\beta_c$ increases with increasing $\tau_2/\tau_1$.

We next consider simulation results. In Fig. 2 we show a real space image of the simulated system with $N_D = 11$...
active particles (black circles) and \( N_A + N_B = 678 \) passive particles (open circles) after many cycles of the ac drive. The active particles, which were originally scattered throughout the system, have clustered into a lane, and they also drift over time in the positive \( x \)-direction.

In Fig. 3 we plot the velocity vs time of a single overdamped colloid in the absence of any other particles (dashed curve), showing short large positive pulses and long small negative pulses of velocity. Here we consider the case of \( f_2/f_1 = 0.25 \) with \( f_1 = 0.56 \). The average velocity of the isolated colloid is zero in a single period. The solid line shows the velocity of a driven colloid in the interacting particle system with \( N = 370 \) and \( N_D = 11 \), in a regime where the dc velocity-force curves have a power-law form. Here, the magnitude of the particle velocity \( v \) during both the positive and the negative portions of the ac drive cycle is smaller than that of the single particle, and shows considerable fluctuations. The integrated velocity during the positive portion of the ac cycle is on average larger in magnitude than the integrated velocity during the negative portion of the cycle. To show this more clearly, in the inset of Fig. 3 we plot the time-averaged drift velocity \( v_d \) for each system over many ac cycles. For the single particle case (lower dashed curve), \( v_d = 0 \), while for the interacting particle system, \( v_d > 0 \), with an long time average drift velocity of \( <v_d> \approx 0.03 \), indicating that a positive ratchet effect is occurring.

We next consider the dependence of the ratchet effect on various system parameters. In Fig. 4(a) we plot the drift velocity \( v_d \) vs the fraction of driven particles, \( N_D/N \), for fixed ac drive parameters \( f_1 = 0.56, f_2/f_1 = 0.25 \), and \( \tau_1/\tau_2 = 0.25 \), with \( a/a_0 = 1 \). \( v_d \) goes through a maximum near \( N_D/N = 0.03 \), and then slowly falls off for higher \( N_D/N \) before reaching \( v_d = 0 \) just below \( N_D/N = 1.0 \). We have also considered different system sizes by varying \( a/a_0 \). We obtain the same behaviors, indicating that the ratcheting is not a finite size effect. It is simple to understand why \( v_d \to 0 \) in the limit of \( N_D/N \to 1 \), since here all the particles move in unison with the ac drive, and the plasticity responsible for the nonlinear velocity-force response is lost. For a single driven particle \( N_D = 1 \), we found a dc velocity-force scaling exponent of \( \beta = 1.5 \). As the number of driven particles \( N_D \) increases, \( \beta \) increases to \( \beta = 2 \) near \( N_D/N = 0.03 \), and then decreases back toward \( \beta = 1 \) as \( N_D/N \) increases further. This result is consistent with the predictions of Eq. 2, where a larger ratcheting effect is expected for larger exponents \( \beta \).

In Fig. 4(b) we consider a system with fixed \( N_D/N = 0.03 \) and ac drive parameters, and plot \( v_d \) vs the system density \( \rho \) measured in units of \( 1/a_0^2 \). For low densities \( \rho < 0.5 \), the active particles interact only weakly with the passive particle background, and the dc velocity-force
curves are close to linear ($\beta \approx 1$) so little ratcheting occurs. For large densities $\rho > 1.3$, the ratcheting drift velocity $v_d$ decreases since it becomes more difficult for the active particles to pass the passive particles. We have also fixed the system density and varied the charge $q_D$ of the active particles. The behavior is similar to Fig. 4(b): the ratchet effect is lost at small $q_D$ when the velocity-force curve becomes linear. In Fig. 4(c) we show the effect of a nonzero temperature on the ratchet effect. We plot $v_d$ vs $T/T_m$, where $T_m$ is defined as the temperature at which a completely ordered single-species colloid assembly with charge $q_D$ and density $\rho = 1/a_0^2$ melts. For high $T$, the velocity increases linearly with force, so the ratchet effect is lost.

Equation 2 predicts that a reverse ratchet effect should occur for $\beta < 1$, as shown in Fig. 1(a), or for $f_1\tau_1 \neq f_2\tau_2$. The reverse ratchet effect can occur in systems where the media responds easily to slow moving particles but becomes stiffer under faster perturbations. For example, in granular media the grains respond in a fluid manner to a slow moving object, while for a fast moving object the granular material jams and acts like a solid. Many polymer systems show a similar response. In order for a ratchet effect to occur in a substrate-free system, there must be a combination of the nonlinear behavior of the particle motion and the left-right symmetry breaking by the ac drive. In general the ratchet effect we observe should occur for any media that has a nonlinear viscosity-frequency response.

In Fig. 4(d,e) we consider two cases where flux reversals occur with $f_1\tau_1/f_2\tau_2 = 0.9$. For the parameters chosen here, the drift velocity of an isolated particle would be $v_d = -0.016$. In Fig. 4(d) we plot $v_d$ vs density $\rho$ for $N_D/N = 0.03$. For low density $\rho < 0.3$, the velocity-force response curve is linear ($\beta = 1$) and the particle moves in the negative direction as expected for this ac drive. As the density increases, the velocity-force response curve becomes nonlinear, and $v_d$ crosses over to a positive value $v_d > 0$ as the ratcheting effect reaches a maximum near $\rho = 1/a_0^2$. The drift velocity drops at high density just as in Fig. 4(b). In Fig. 4(e) we show $V$ vs $T/T_m$ for the same system with a fixed density of $\rho = 1.26/a_0^2$. For low temperatures, the velocity-force response curve is nonlinear, and a ratchet effect with positive velocity occurs. As the temperature increases, the drift velocity $v_d$ drops and crosses back to a negative value as the system enters the linear response regime. $v_d$ saturates to $v_d \approx -0.01$, slightly smaller in magnitude than the value expected for a single isolated particle. We have also considered the effect of changing $q_D$ at fixed density and $T = 0$, and observe behavior similar to that shown in Fig. 4(d).

In conclusion we have investigated a “glassy ratchet” effect, which occurs in a glassy system of interacting particles without a quenched substrate. We considered the specific case of a disordered assembly of colloids with overdamped dynamics where only one colloid species couples to an ac drive. The ac drive is asymmetric, with a large positive drive applied for a short time followed by a small negative drive applied for a longer time, and obeys the constraint that the net applied force on the driven colloids is zero in a single cycle. A ratchet effect occurs when the driven and the non-driven colloids are coupled, producing a nonlinear velocity-force response. We find that the dc drift velocity has a maximum value as a function of the fraction of driven particles, temperature, and particle density. A simple model for the ratchet effect agrees well with our simulation results, including the occurrence of ratchet reversal regimes. The system considered here can be realized experimentally for magnetic or charged colloids driven through non-magnetic or uncharged colloids. Our results may also have applications for new types of electrophoresis devices.

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