INFLATION INDUCED SUSY BREAKING
AND FLAT VACUUM DIRECTIONS

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Abstract

We discuss how the inflation induced supersymmetry breaking affects the flat directions of SUSY vacua. We show that under general assumptions all gauge nonsinglet fields, parameterizing flat directions (and in particular squarks and sleptons), get large radiative masses which are related to the value of the Hubble constant ($H$) and to the expectation value of the inflaton field. This mass (typically $\sim H$) is of “non-gravitational” origin and does not vanish in the global SUSY limit. Large radiative corrections are induced by $F$-term (or $D$-term) density which dominates the inflationary universe and strongly breaks supersymmetry. In such theories it is difficult to treat squarks and sleptons as a light fields in the inflationary period. In the generic supergravity theories all flat directions, including moduli, are getting curvature of order $H$. However, for the gauge-nonsinglet flat directions radiative contribution to the curvature (induced by renormalizable gauge interactions) may be dominant.

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1 Introduction

Characteristic feature of many supersymmetric theories is the existence of the noncompact flat directions in the vacuum. Such directions are parameterized by the vacuum expectation values (VEVs) of the scalar fields, whose masses are induced only by the supersymmetry breaking, which provides (typically small) curvature for the flat vacua. Commonly accepted scenarios [1] assume that supersymmetry breaking takes place in certain isolated “hidden sector” and then gets transferred to other sectors by some universal messenger interaction. Resulting masses are of the order

\[ m^2 \sim \frac{|F|^2}{M^2} \]  

(1)

where \( F \) is a VEV of the \( F \)-term that breaks supersymmetry in the hidden sector (\( D \)-type breaking is also possible) and \( M \) is a scale of messenger interaction. In the generic supergravity theories the messenger is assumed to be gravity and thus \( M = \frac{M_{\text{Planck}}}{\sqrt{8\pi}} \) (reduced Planck mass). Clearly, in this situation one has to arrange the theory in such a way that \( |F|^2 \sim M_W M \) (in the vacuum with zero cosmological constant), in order to generate soft masses \( \sim M_W \) (weak scale) in the matter sector (compatible with the solution of the hierarchy problem).

There is a large class of the flat directions, which are parameterized by the fields having only nonrenormalizable \( (M \) suppressed) interactions. Such fields are usually called moduli. Due to the universal nature of the gravity transfer supersymmetry breaking (1), the resulting masses of moduli are \( \sim M_W \), very much like the squarks and sleptons. This fact may lead to the grave cosmological difficulties [2] due to the very late decay of moduli Bose
condensate. The problem (partially) results from the assumption that moduli masses during inflation are $\sim M_W$ and, therefore, it becomes very difficult to dilute such a condensate, unless the Hubble constant ($H$) is $\sim M_W$. However, as we have shown recently [3], in generic supergravity theories (independently from the details of the inflation) the moduli masses are of order $H$ and as a matter of fact in wide class of theories are larger than $H$. This has to do with the fact that any inflation breaks supersymmetry since it provides a large cosmological constant and thus large $F$-term (or $D$-term) density. Such inflation induced SUSY breaking stabilizes flat directions giving mass $\sim H$ to the moduli. This fact can have an important consequences for the cosmological moduli problem, since now the moduli condensate can be eliminated by any inflation, provided there is no large displacement of the moduli minimum (from its present value) [3]. As we have shown, in particular this can be the case if moduli has no couplings (other than canonical term) in the Kahler potential.

Besides, in the SUSY theories there are many other flat directions whose zero modes carry ordinary gauge quantum numbers and/or have renormalizable interactions with the other fields. Simplest example is provided by some components of squarks and sleptons in minimal SUSY standard model or GUTs[4]. Such flat directions are not problematic cosmologically, but can be of certain importance for the baryogenesis[5]. Very much like moduli fields, gauge nonsinglet flat directions are getting curvature of order $H$ by generic supergravity inflation. However, unlike the moduli their masses get contribution from the other (nongravitational) sources as well. In the present paper we show that such contributions in general are not less impor-
tant and in many cases can be even dominant. The generic reason is that for
the gauge nonsinglet flat directions there are several candidate forces which
can transfer the message about SUSY breaking. In particular such are the
gauge interactions and precisely this was the basic idea of the “old” globally
supersymmetric models with “geometric hierarchy”[6]. In this models the
message about the SUSY breaking from the hidden sector (say GUT Higgses) was transferred to the standard model fields radiatively, by the gauge
interaction. Clearly, this is not a case in the conventional supergravity the-
ories in which the hidden sector is assumed to be trivial under visible gauge
symmetries [1]. The crucial point however is that, even if such mechanism
is not operative in the present vacuum (with zero cosmological constant), in
general it had to be effective in the early universe, since the hidden sector
$F$-term, which provides SUSY breaking “today”, in general is not the one
that was dominating energy density in the inflationary universe. As it is
known [7], during inflation universe has to be in the state with large cosmo-
logical constant, which may or may not be a local minimum of the theory.
However, the essential requirement is that classical expectation values of the
fields change slowly. Below we will assume that this change is slow enough, so
that it makes sense to perform the quantum expansion in perturbation the-
ory about the points of inflationary trajectory (of course, if the inflationary
state is a local minimum, this assumption is automatically valid).

Under above assumption, we show that strong corrections had to be pre-
ented in the large class of scenarios, in which the inflaton sector is not
isolated (at least) from some gauge nonsinglet fields. They give universal
(up to a gauge quantum numbers) contribution $\sim H$ to the curvature of

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all gauge-nonsinglet flat directions, which in some cases can dominate over the similar contribution induced by supergravity transmitted SUSY breaking. This corrections are induced by the large SUSY breaking in the early universe and vanish (or become negligible) once the system settles in to the present minimum with “weakly” broken supersymmetry.

2 Globally supersymmetric inflation

In this section we will consider the behavior of the SUSY vacua flat directions during inflation in the globally supersymmetric theory. The effect is of the special importance due to the fact that it persists also in the locally supersymmetric generalizations, but since its origin is “non-gravitational”, it is more convenient to study it first in the global SUSY case. So let us consider the system of \( n \) chiral superfields \( S_i \) (where \( i = 1, 2, \ldots, n \)). The scalar component of each superfield we will denote by the same symbol, whereas the fermionic and auxiliary \( (F) \) components we will denote as \( \Psi_i \) and \( F_i \) respectively. We will assume that our theory is invariant under a simple gauge group \( G \) (in reality such factors can be several) and that some superfield form its irreducible representations. Now, in the case of global supersymmetry the scalar potential is given by [8]:

\[
V = |F_i|^2 + (D - terms)
\]

Let us assume that, in some way, the above system appears in the inflationary state. The basic idea of any inflationary scenario is that at some time the universe is dominated by the large vacuum energy density \( V \), from which
it follows that any inflation in the SUSY framework implies large $F$-term or $D$-term density. Below we will assume that $D$-terms vanish and thus $F$-terms are dominating inflation. Let’s take such to be an $F_s$-term of $S$ superfield. Thus, in our system the supersymmetry is broken in the inflationary period by the amount that is measured by the expectation value of $F_s$. Clearly, in the state with such a strongly broken supersymmetry, the flat directions of the SUSY vacua can easily be shut down (or destabilized) and corresponding zero modes in general can get large masses, provided the SUSY breaking is transmitted to the respective sector by some interaction. However, in the case of global SUSY there is no universal messenger interaction that can transmit supersymmetry breaking from one sector to another. Therefore, the effects under consideration can only take place if some general conditions are satisfied. In particular, such condition is:

(*) The superfield $S$, whose $F_s$-term is dominating inflation, is coupled to some of the gauge nonsinglet fields in the superpotential.

Let $\phi$ be a gauge nonsinglet field in the real (say adjoint) representation of $G$ (alternatively one can consider two fields $\phi, \bar{\phi}$ in conjugate representations), which is coupled to $S$ in the superpotential

$$\Delta W = g S \phi^2$$

(obviously $G$-invariant contraction of the indexes is assumed). In such a theory, the all $G$-nonsinglet flat directions will obtain a radiative two loop masses of the order

$$m^2_{\text{rad}} \sim \left( \frac{\alpha}{4\pi} \right)^2 g^2 \frac{|F_s|^2}{M_\phi^2}$$

(4)
where $M_\phi$ is a supersymmetric mass term of $\phi$ and $\alpha$ is a gauge coupling of $G$. If (3) is the only contribution to the mass, then

$$M_\phi = gS$$

where $S$ is the expectation value of the scalar component during inflation. Notice, that $S$ need not necessarily be the inflaton field. Experts will easily recognize in (4) the well known expression for the radiative two loop scalar mass in the “old” globally supersymmetric models with “geometric hierarchy”[6]. In this models such corrections where arranged to take place in the phenomenological minimum (present vacuum) and they were the major source for transfer of SUSY breaking to the visible sector trough the gauge interactions. Naturally, in these schemes the $F$-terms were restricted to be $\sim M_W M_\phi$, resulting in the radiatively induced mass $m_{rad} \sim M_W$.

Our observation here is that, independently whether such corrections are zero in the present vacuum, they had to be presented during inflation (provided (*) is valid). Once again, this is consequence of the large $F_S$ density in the early universe. Of course, in reality the global minimum is “slightly” nonsupersymmetric, but this does not affects significantly present discussion. Above correction induces large mass to all scalar fields parameterizing $G$-nonsinglet flat directions (squarks sleptons) and in general to all $G$-nonsinglet light fields as well. However, for the fields which do not correspond to flat vacua, there can be a larger tree level contribution from the other sources.

Using relation between vacuum energy and the Hubble constant (in the slow roll approximation) [7]

$$H^2 = \frac{V}{3M^2} = \frac{|F_S|^2}{M^2}$$

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we can rewrite (4) in the following form

\[ m_{rad}^2 \sim H^2 \left( \frac{\alpha}{4\pi} \right)^2 \left( \frac{M}{|S|} \right)^2 \] (7)

This form shows that, in general, \( m_{rad} \) can be even larger than the Hubble constant, if \( |S| \) is somewhat below \( M \). Usually, \( |S| \) changes (slowly) during inflation and so does the ratio \( m_{rad}/H \).

**Example**

Now we wish to demonstrate above effect on the particular example of ‘hybrid’ inflation which originally was considered by Linde [9] in nonsupersymmetric framework and later was studied in the supersymmetric context in [10,11]. This scenario automatically satisfies condition (*), since it implies that inflaton couples to gauge nonsinglet Higgs field. The simplest superpotential which leads to the hybrid inflation is

\[ W = \frac{1}{2} g S \phi^2 - S \mu^2 \] (8)

where \( \mu \) is some large mass scale and if \( G \) is grand unification symmetry, then one has to assume \( \mu \sqrt{\frac{2}{g}} = M_{GUT} \). In order to study inflationary dynamics, let us assume for a moment that \( \phi \) also is a gauge singlet field. Then, the scalar potential is given by

\[ V = |F_S|^2 + |F_\phi|^2 = \frac{1}{2} g \phi^2 - \mu^2 |\phi|^2 + g^2 |S|^2 |\phi|^2 \] (9)

This theory has a unique supersymmetric vacuum with \( \phi^2 = \frac{2}{g} \mu^2 \) and \( S = 0 \) in which all \( F \)-terms vanish. However, if we minimize \( V \) with respect
to $\phi$ for the fixed values of $S$, we can easily find that for $S > S_C = \frac{\mu}{\sqrt{g}}$, the minimum is at $\phi = 0$, potential is flat in $S$ direction and has $S$-dependent curvature ($\sim g|S|$) in $\phi$ direction. Following [11], let us consider the chaotic initial conditions with $S >> S_C$. Since the curvature in the $\phi$ direction is very large, we expect that $\phi$ will rapidly settle in its “minimum” with $\phi = 0$. Contrastly, the curvature in $S$ direction is zero and, therefore, the system can stay at $S >> S_C, \phi = 0$ quit long. This state is dominated by large $|F_s| = \mu^2$ term density and inflation results. Notice, that classically there is no force that can drive $S$ to the global minimum. Such a force can be provided by the positive SUSY-violating soft mass term $m^2|S|^2$ [10]. However, as it was shown in [11], independently from the existence of the soft terms, nonzero curvature of the $S$ slope in any case is provided by one loop corrections to the effective potential[12]

$$\Delta V = \frac{(-1)^F}{64\pi^2 Tr M^4 ln \frac{M^2}{\Lambda}}$$

where the summation is over all helicity states, factor $(-1)$ stands for the fermions and $\Lambda$ is a renormalization mass. Crucial point is that in the region $S > S_C$ this corrections are nonzero, even if the global vacuum of the theory is supersymmetric. One may think that the existence of the nonzero radiative corrections to the effective potential, in the theory with supersymmetric ground state, may be somewhat surprising and contradict to the “nonrenormalization” theorem [13]. Notice however, that expansion is performed about the (classically flat) points $S > S_C$ and $\phi = 0$ for which supersymmetry is broken by the large $F_S = \mu^2$ density and the Fermi Bose masses are not degenerated. In the other words, in the early universe the system is far away
from its own global minimum and does not “knows” whether this minimum is supersymmetric or not. So, in this epoch there are nonzero corrections. In fact, the nonexistence of these corrections would be more surprising. Let us neglect gravity and inflation for a moment and imagine that our system is at some point $S >> S_C$ and $\phi = 0$. Since classically there is no driving force, the system can stay there long enough, so that the hypothetic observer can measure the particle spectrum. He would find: (1) one massless scalar $S$ and one massless fermionic partner $\psi_S$ (goldstino); (2) one fermion $\psi_\phi$ with mass $M_\phi = gS$ and two real scalars $\phi + \phi^*$ and $i(\phi - \phi^*)$ with mass $M_\phi^2 = g^2|S|^2 - g\mu^2$ and $M_\phi^2 = g^2|S|^2 + g\mu^2$ respectively. Of course, with such a spectrum our observer can never conclude that his universe is supersymmetric.

One loop corrected effective potential (for $S >> S_C$) is given by\cite{11}

$$V = \mu^4\left(1 + \frac{g^2}{32\pi^2}\left[2\ln\frac{g\mu^2}{\Lambda^2} + (x - 1)^2\ln(1 - x^{-1}) + (x + 1)^2\ln(1 + x^{-1})\right]\right) \quad (11)$$

where $x = \frac{g|S|^2}{\mu^2}$. In order to feel more comfortable with this expression, we can obtain the same result by taking the exact SUSY limit of the softly broken theory, for which the point $S >> S_C, \phi = 0$ is a local minimum. For this, let us add to the potential (9) the soft SUSY breaking terms.

$$V_{soft} = \epsilon^2|S - |S_0||^2 \quad (12)$$

where $|S_0| >> S_C$. Now the theory has a well defined local minimum at $S = S_0$ and $\phi = 0$ where curvature in all directions is positive. The tree level spectrum in this minimum is precisely the same as in the case $\epsilon = 0$.
with only modification that now inflaton $S$ gets a soft positive $[mass]^2 = \epsilon^2$. The one loop effective potential now is given by (11) plus an additional term proportional to

$$V_\epsilon = \epsilon^2 \ln \frac{\epsilon^2}{\Lambda^2} \tag{13}$$

which vanishes in the limit $\epsilon \to 0$ and we are left with (11). Above discussion is not altered significantly for the gauge nonsinglet $\phi$, since for $S >> S_C$ the gauge symmetry is unbroken. The phase transition with gauge symmetry breaking takes place only after the $S$ field drops to its critical value $S_C$. Below this point all the fields rapidly adjust their VEVs in supersymmetric minimum. Inflation ends when the slow roll condition breaks down and this happens when $S$ approaches $\sim S_C$ (from above). We will not provide here all the details of this scenario for which reader is referred to [11]. For us the most important thing about this scenario is that the $F_S$-term, which dominates the inflationary universe, is coupled to the $G$-nonsinglet superfields and splits the masses of its Fermi-Bose components. This results in to the large radiative masses of all $G$-nonsinglet flat directions (and in particular quarks and leptons) given by (7). In above model inflation ends when $S \sim S_C$ and thus the squark masses can be in general larger then the Hubble constant (provided $\mu$ is small enough).

The considered universal two loop radiative corrections to $[mass]^2$ are positive and the flat directions are stabilized at the origin. However, in some cases the dominating negative one loop corrections can also appear. This may happen if the nonzero $F_S$-term is coupled to the pair of fields $\phi, \bar{\phi}$ in the non-selfconjugate representations, whose supersymmetric masses are splitted
due to the mixing with other (non-selfconjugate) representations. In general, such a situation may lead to the nonzero one loop contribution to the $\text{mass}^2$, which are proportional to the Abelian generators of $G$ (e.g. hypercharge in the GUT case) and therefore, can have either sign. These corrections can destabilize flat directions. Their presence in the present vacuum would be a disaster, but in the early universe they can play very important role for the baryogenesis via Affleck-Dine mechanism [5], since this mechanism requires large expectation values of squarks and sleptons along the flat vacua.

3 Supergravity

The effects considered in the previous section will be operative in the supergravity scenarios as well if the large $F$-term couples to some of the gauge nonsinglet fields. However, as we have shown in [3], in the supergravity framework flat directions (and in particular moduli) get curvature $\sim H$ through the gravity transferred supersymmetry breaking. Since the gravity is universal messenger, flat directions of the both type (gauge-nonsinglets and moduli) are in general disturbed by the equal strength and resulting curvature is of the order

$$m^2 \sim \frac{|F_s|^2}{M^2}$$

(14)

But, as we know, G-nonsinglet zero modes can get extra contribution (4),(7) from the gauge interactions which can be equally important. Let us consider generic supergravity scalar potential [14]:

$$V = \exp\left(\frac{K}{M^2}\right)[K_{ij}^{-1}F_i^*F^j - 3\frac{|W|^2}{M^2}]$$

(15)
where $K(S_i, S^*_i)$ is a Kahler potential, $W(S_i)$ is superpotential and $S_i$ are chiral superfields. $F_i$ -terms are given by $F_i = W_i + \frac{W}{M^2} K_i$ where upper (lower) index denotes derivative with respect to $S_i$ ($S^*_i$) respectively. (Again, we neglect possible $D$-terms and assume that they vanish during inflation).

For simplicity we assume that flat direction mode (which we denote by $Z$) enters in the Kahler potential only through the canonical term:

$$K = |Z|^2 + K'(S_i, S^*_i) \quad W = W_s + W'(S_i, Z)$$

(16)

Where $K'$ and $W_s$ are arbitrary functions independent of $Z$. Note that if $Z$ is moduli, then only Planck scale suppressed couplings are allowed in $W'$. G-nonsinglet zero modes are allowed to have renormalizable couplings in $W'$.

By convention, let us put the present minimum of the flat direction at $Z = 0$ (at least for the squarks and sleptons this is the necessary requirement). In the class of models in which the minimum is not displaced during inflation, the mass of $Z$-mode is given by:

$$m_z^2 = 3H^2 + e^{\frac{K'}{M^2}} [\frac{|W|^2}{M^2} + |W'|^2 + K''_{ij} F^{ijz} F_{z}^{*z} - \frac{|W'_{z}|^2}{M^2}]$$

(17)

where lower (upper) index $z$ denotes derivative with respect to $z(z^*)$. The value of the second term in brackets is in general model dependent. For the moduli (having only nonrenormalizable couplings in $W$) it can be shown that, under some general conditions, this quantity is positive[3] and thus in such cases moduli masses are larger than the Hubble constant. In this situation inflation can dilute moduli condensate and solve the problem. For squark flat directions this contribution will again be positive if, for example, there
is some symmetry (e.g. matter parity) for which \( W'_z \) automatically vanishes in the minimum with zero matter VEVs. The generic message, however, is that supergravitational induced curvature of the flat directions is \( \sim H \). One may compare this contribution with the one that can be transferred by gauge interaction (7). We see that in general this two sources are comparable and gauge contribution may be even dominant if inflation ends (slow roll condition breaks down) when \( S \) is somewhat below \( M \).

4 conclusions

In conclusion, we have shown that under general conditions all gauge non-singlet flat directions get large \( \sim H \) radiative contribution to the curvature in the inflationary epoch. These corrections result from the strong breaking of the supersymmetry, induced by the inflation, and disappear after system adjusts to its present vacuum. In generic supergravity theories all flat directions (including moduli) are getting curvature \( \sim H \) by the universal gravity transferred SUSY breaking. For gauge non-singlet flat directions this two contributions can be comparable and in some cases the gauge contribution may be the dominant one.

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