Fault-Tolerant Time-Varying Formation Tracking Control for Unmanned Aerial Vehicle Swarm Systems with Switching Topologies

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1. Introduction

Formation control of UAV swarm systems has received extensive attention because it can be wildly applied in many fields, such as surveillance [1], targets search [2], cooperative attack [3], coordinated localization [4], and drag reduction [5]. There are many classic control strategies used to solve the formation control problem of UAV swarm systems, such as the leader-follower [6], behaviour-based strategy [7], and virtual structure [8]. However, the study in [9] has pointed out that the abovementioned control strategies have certain weaknesses. For example, the leader-follower control strategy depends on the state of leader, and the leader’s bias can affect the whole formation. It is difficult to accurate quantitative model using the behaviour-based strategy, so it can impact the stability of the UAV swarm systems. Also, the virtual structure control strategy is unable to be fully distributed because it requires each UAV to track its own way points. For large-scale UAV swarm systems, it is difficult to design a centralized formation control protocol because of the limitations of computation and communication. Thus, how to construct a fully distributed controller to solve the formation control problem is a hot issue in current research.

In recent years, great advances have been made for formation control, and many novel control theories have been applied to study the formation control problem. Using consensus theory, a completely distributed formation control protocol has been designed to solve the formation control problem of VTOL unmanned aerial vehicles with communication delays in [10]. The work in [11] has investigated a target-enclosing problem for spacecraft swarm systems by proposing a two-layer affine formation control strategy. A sliding mode control method is proposed for the time-varying formation tracking of a multi-UAV system with a dynamic leader in [12]. The work in [13] has presented an event-triggered method to deal with the discrete-time formation tracking problems of UAV swarm systems. The work in [14, 15] has proposed the consensus control
protocols for the hybrid multiagent systems which are composed of continuous-time and discrete-time dynamic agents by using the matrix theory and system transformation method, respectively. The work in [10–15] have solved the problems of formation control for swarm systems with fixed topology. However, in the actual flight process, the topology structure between UAVs may cause the failure of the communication link and the creation of a new communication link due to the limitation of the communication distance, so that the switching between topologies occurs.

Recently, the formation control problems with switching topologies have been studied in [16–18]. A distributed controller design methodology with fault detection logic for formation flight of the UAV swarm systems has been presented in [16]. The consensus tracking problem for second-order nonlinear multiagent systems with switching topologies and a time-varying reference state has been solved in [17]. The work in [18] has investigated the finite-time formation control of multiagent systems by using the approach of dynamic output feedback. Nevertheless, the research of [16–18] is designed based on time-invariant formation and switching topologies. However, in many practical applications, such as the source seeking and target enclosing, accomplishing the time-varying formation shape is an important task for the tracking control of UAV swarm systems. Considering that, in practical applications, such as the source seeking and target enclosing, the formation will change according to the position of the leader, it is more meaningful to study the time-varying formation tracking problem with switching topologies.

To solve those aforementioned problems, the time-varying formation tracking control has been developed in [19–21]. The work in [19] has designed a time-varying formation control protocol for UAV swarm systems under Markovian switching topologies with partially unknown transition rates. The formation tracking control problem of high-order multiagent systems in the case of unknown leader input has been proposed in [20]. The work in [21] has proposed a novel distributed output feedback formation control protocol with the backstepping method and the dynamic surface control technique. However, note that the actuator fault was not discussed in the abovementioned research results. In practice, there may be actuator faults for each UAV, such as loss of effectiveness fault and bias fault.

At present, fault-tolerant control has been concerned by a lot of researchers, such as in [22–25]. A distributed adaptive fault-tolerant control protocol has been designed to solve the formation control problem of multiple trailing fixed-wing UAV systems in the event of actuator faults in [22]. The work in [23] has studied the time-varying formation tracking problem of second-order multiagent systems in the case of actuator faults and noncooperative targets. The work in [24] has designed an adaptive fault-tolerant control scheme based on the virtual actuator framework to deal with the problem of actuator faults in nonlinear heterogeneous multiagent systems. A distributed adaptive fuzzy fault-tolerant containment control protocol has been proposed to address the problem of heterogeneous nonlinear multiagent systems with actuator faults and external disturbances in [25]. The work in [26] has proposed a distributed finite-time fault-tolerant control method for containment control of multiple unmanned aerial vehicles with actuator faults and input saturation. The work in [27] has designed a new composite adaptive disturbance observer-based decentralized fractional-order fault-tolerant control scheme to deal with the decentralized fractional-order fault-tolerant control problem for unmanned aerial vehicles in the presence of wind disturbances and actuator faults. Nevertheless, The work in [22–27] has considered the formation control problem with the fixed topology. The distributed cooperative controller design problem for multiagent systems in the presence of actuator faults has been investigated in [28], and The work in [29] has proposed a distributed output feedback consensus tracking control scheme for second-order multiagent systems in the presence of the partial loss of actuator effectiveness faults. However, the boundary information of actuator faults is shown in [28, 29].

In this paper, in response to the aforementioned control problem, a novel adaptive time-varying formation tracking control scheme is proposed for the UAV swarm systems with the actuator faults and switching topologies. Compared with the existing research results, the main contributions can be boiled down to the following three aspects. (1) The communication topologies among UAVs can be switching, which improves the practical application of the previous research results with fixed topology. The interaction topologies are restricted to be fixed in [10–15]. Also, the work in [30] has pointed out that formation control problems with switching topologies are more complicated and challenging than the fixed cases. (2) In the case of the UAV swarm systems with actuator faults, the followers can still accomplish the desired time-varying formation and track the state of the leader. The time-varying formation tracking control problem has been solved in [19–21], but these papers have not considered the problem of the actuator faults. (3) The relative information of the neighboring UAVs is used to design a completely distributed fault-tolerant tracking control protocol, and a novel adaptive control algorithm is introduced, which can effectively avoid the occurrence of high gains and make the design of the control protocol not rely on the boundary information of actuator faults. The problem of actuator faults is studied in [28, 29], but the boundary information of actuator faults is known. However, in the actual flight process, the boundary information of actuator fault cannot be obtained. Thus, the adaptive control algorithm designed in this paper is meaningful.

The rest of this paper is organized as follows. Problem description and basic graph theory are given in Section 2. In Section 3, the main results are presented. In Section 4, the simulation results are given. Conclusions are proposed in Section 5.

Notations 1. \( \mathbb{R}^n \) stands for the \( n \)-dimensional real column vector space, and \( \mathbb{R}^{m \times n} \) denotes the set of \( n \times n \) dimensional real matrices. \( I_n \) stands for an identity matrix with dimension \( n \). The Kronecker product is denoted by \( \otimes \). Let
2. Preliminaries and Problem Description

Basic graph theory and problem description are given in this section.

2.1. Basic Graph Theory. A directed graph with \( N \) nodes can be described as \( G = \{V(G), E(G), W(G)\} \), where \( V = \{v_1, v_2, \ldots, v_N\} \) represents the node set, the edge set is denoted by \( E(G) \subseteq \{(v_i, v_j): v_i, v_j \in V(G), i \neq j\} \), and the weighted adjacency matrix is \( W(G) = [w_{ij}] \in \mathbb{R}^{N \times N} \). For \( \forall i, j \in \{1, 2, \ldots, N\}, w_{ij} > 0 \) if and only if \( e_{ij} = (v_i, v_j) \in E \) and \( w_{ij} = 0 \) otherwise. Let \( D(G) = \text{diag}[d_1, d_2, \ldots, d_N] \) with \( d_i = \sum_{j=1, j \neq i}^{N} w_{ij} \) be the in-degree matrix of \( G \).

The Laplacian matrix is \( L = D(G) - W(G) \). Also, \( e_{ij} = (v_i, v_j) \) is denoted as the edge of \( G \), where the vertex \( v_i \) is called the neighbor of the vertex \( v_j \). If a graph \( G \) has at least one root vertex and this vertex has a path with all other vertices, it is called a spanning tree.

In this paper, the interaction topology among UAVs can be switching. Let \( \Sigma \) represent all possible communication topologies of the system, and the topological index set is \( I \subseteq \mathbb{N} \); the set of natural numbers can be represented by \( N \).

Let \( \sigma(t) : \{0, \infty\} \rightarrow I \) be a switching signal, and its value is the index of the topology at \( t \). The graph and Laplacian matrix at \( t \) can be represented by \( G_{\sigma(t)} \) and \( L_{\sigma(t)} \), respectively. The neighboring set of UAV \( i \) at \( \sigma(t) \) is denoted by \( N_{\sigma(i)}^t \). Let \( w_{ij} (i, j \in \{1, 2, \ldots, N\}) \) be the interaction strength related to the edge from nodes \( v_i \) to \( v_j \). In this paper, it is supposed that the admissible switching signal has a dwell time \( T_d > 0 \).

Assumption 1. The topology \( G_{\sigma(t)} \) contains a spanning tree with the leader as the root node. The interactions among the followers are undirected.

If Assumption 1 is satisfied, considering the leader-follower topology, the Laplacian matrix \( L_{\sigma(t)} \) has the following form:

\[
L_{\sigma(t)} = \begin{bmatrix}
L_{\sigma(t)}^{ffi} & 0_{N \times (N-1)} \\
0_{1 \times N} & L_{\sigma(t)}^{ff}
\end{bmatrix},
\]

where \( L_{\sigma(t)}^{ffi} \in \mathbb{R}^{(N-1) \times (N-1)} \) represents the Laplacian matrix between the leader and the follower and \( L_{\sigma(t)}^{ff} \in \mathbb{R}^{N \times (N-1)} \) denotes the Laplacian matrix among the followers.

Remark 1. Time-varying formation tracking control problem for UAV swarm systems with switching interaction topologies is investigated in this paper. The switching topologies are not considered in [10–15]; however, the topological structure among UAVs may change in the actual flight process. In many practical applications, such as the source seeking and target enclosing, due to the change of formation and the influence of the complex terrain environment, the communication link of the UAVs is prone to failure, which will cause the topologies to switch among the UAVs and then change the connectivity of the UAV swarm systems and the interaction relationship between the UAVs.

Hence, it is more necessary to study time-varying formation tracking problems for UAV swarm systems with switching topologies.

2.2. Problem Description. We consider a UAV swarm system consisting of \( N \) UAVs. The formation tracking control for a multi-UAV system can be decoupled into an inner-loop control and an outer-loop control, where the inner-loop controller is used to stabilize the attitude and the outer-loop controller is used to drive the UAV toward the desired position. This paper mainly considers the formation tracking control for the outer-loop of UAV swarm systems.

If a UAV does not have neighbours, it is called a leader, and if it has at least one neighbour, it is called a follower. Suppose that there exist a leader and \( N - 1 \) followers in this system.

The modes of the leader can be written as

\[
\begin{aligned}
\dot{x}_L(t) &= v_L(t), \\
\dot{v}_L(t) &= \alpha_s x_L(t) + \sigma_{v_L}(t),
\end{aligned}
\]

where \( x_L(t) \in \mathbb{R}^n \) and \( v_L(t) \in \mathbb{R}^n \) represent the position and velocity vectors of the leader, respectively. \( \alpha_s \) and \( \sigma_v \) are the known damping coefficients.

This paper assumes that each follower may have actuator faults. The follower’s actuator faults model is

\[
\begin{aligned}
\dot{u}_{i\ell}(t) &= \rho_{i\ell}(t)u_i(t) + b_i(t),
\end{aligned}
\]

where \( u_i(t) \in \mathbb{R}^n \) is the actuator input, \( u_{i\ell}(t) \in \mathbb{R}^n \) is the actuator output with failures, \( \rho_{i\ell}(t) = \text{diag}[\rho_{i1}(t), \rho_{i2}(t), \ldots, \rho_{im}(t)] \) and \( 0 < \rho_{ij}(t) \leq 1 \) for \( j = 1, 2, \ldots, m \) represent the unknown efficiency factor, and the unknown output bias is denoted by \( b_i(t) \in \mathbb{R}^n \).

Considering the impact of actuator faults, the model of follower \( i \), \( i = 2, 3, \ldots, N \) can be represented by

\[
\begin{aligned}
\dot{x}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= \alpha_s x_i(t) + \sigma_{v_i}(t) + \rho_i(t)u_i(t) + b_i(t),
\end{aligned}
\]

where \( x_i(t) \in \mathbb{R}^n \) and \( v_i(t) \in \mathbb{R}^n \) are the position and velocity vectors of the follower \( i \), respectively. Based on the Kronecker product, the results can also be applied in high-dimensional situations.

Remark 2. The dynamics of a UAV can be decoupled into trajectory dynamics and attitude dynamics. Due to the fact that the trajectory dynamics have much larger time constants than the attitude dynamics, the formation tracking control for a multi-UAV system can be classified into an inner-loop control and an outer-loop control; the outer loop drives the UAV toward the desired position, and the inner loop is used to stabilize the attitude. In this paper, the formation tracking control problem for multi-UAV systems only concerns the positions and velocities. Therefore, the dynamics of the leader and follower UAVs in the outer loop can be modeled by (1) and (3). At present, there are many relevant studies which use this method to get the modes of the leader and follower UAVs, such as the work in [31–34].
In this paper, the actuator faults include bias fault and loss of effectiveness fault, and the fault model of follower $i$ can be described as follows:

(i) If $\rho_i(t) = 1$ and $b_i(t) = 0$, the follower does not have actuator fault

(ii) If $0 < \rho_i(t) < 1$ and $b_i(t) = 0$, the follower has loss of effectiveness fault

(iii) If $\rho_i(t) = 1$ and $b_i(t) \neq 0$, the follower has bias fault

(iv) If $0 < \rho_i(t) < 1$ and $b_i(t) \neq 0$, the follower has loss of effectiveness and bias faults

In this paper, the actuator faults are considered to be unknown, time varying, and satisfying the following bounded assumption.

**Assumption 2.** The unknown loss of effectiveness fault $\rho_i(t)$ and bias fault $b_i(t)$ are bounded, and there exist two unknown positive constants $\eta_i$ and $\beta_i$ such that $0 < \eta_i \leq \rho_i(t) \leq 1$ and $\|b_i(t)\| \leq \beta_i$, ($i = 2, 3, \ldots, N$).

**Remark 3.** In this paper, a fully distributed fault-tolerant time-varying formation fault-tolerant tracking controller is designed for the UAV swarm system with actuator faults, which cancels the limitation of knowing the boundary of actuator fault.

Let $\hat{x}_k(t) = [x_k(t), v_k(t)](k = 1, 2, 3, \ldots, N)$. Then, the modes of the leader and followers can be denoted by

$$\begin{align*}
\dot{\hat{x}}_1(t) &= (B_1B_2^T + B_2\alpha)\hat{x}_1(t), \\
\dot{\hat{x}}_i(t) &= (B_iB_2^T + B_2\alpha)\hat{x}_i(t) + B_2\rho_i(t)u_i(t) + B_2b_i(t),
\end{align*}$$

where $B_1 = [1 \ 0]^T$, $B_2 = [1 \ 0]^T$ and $\alpha = [\alpha_x \ \alpha_v]$.

Time-varying formation of followers can be written by $h_i(t) = [h_{ix}(t), h_{vi}(t), \ldots, h_{ix}(t)]^T$, where $h_i(t) = [h_{ix}(t), h_{vi}(t)]^T$, ($i = 2, 3, \ldots, N$) are piecewise continuously differentiable and $h_{ix}(t)$ and $h_{vi}(t)$ represent the position and velocity components of $h_i(t)$, respectively.

**Definition 1.** For any bounded initial states, if

$$\lim_{t \to -\infty} (\hat{x}_i(t) - h_i(t) - \hat{x}_i(t)) = 0, \quad (i = 2, 3, \ldots, N),$$

then the UAV swarm systems (4) can achieve the time-varying formation tracking performance.

The objective of this paper is to design a distributed fault-tolerant control protocol for the UAV swarm systems (4) with unknown actuator faults and switching topologies, such that the time-varying formation tracking performance can be achieved.

3. Main Results

In this section, a fault-tolerant time-varying formation tracking control protocol and the feasibility condition will be proposed. Also, the proof will be given based on Lyapunov stability theory.

For follower $i$ ($i \in \{2, 3, \ldots, N\}$), the local error of time-varying formation tracking is written as

$$S_i(t) = w_{ii}(t) (\hat{\xi}_i(t) - h_i(t) - \hat{\xi}_i(t)) + \sum_{j=1}^{N} w_{ij}(t) \cdot \left( (\hat{\xi}_j(t) - h_j(t)) - (\hat{\xi}_i(t) - h_j(t)) \right).$$

According to the local error of time-varying formation tracking in (6), the fault-tolerant time-varying formation tracking control protocol with switching topologies can be given by

$$u_i(t) = \bar{\alpha}_i(t)K_i(S_i(t)) - \bar{\alpha}_i(t)g_i(S_i(t)) - \bar{\alpha}_i(t)\|\hat{h}_i(t)\|g_i \cdot (S_i(t) - \bar{\alpha}_i(t)\|\hat{h}_i(t)\|g_i(S_i(t))),$$

where $\bar{\alpha}_i(t)$ is adaptive updating laws, $g_i(S_i(t))$ denotes a nonlinear function, constant gain matrix $K = -B_2^TP$, and positive definite matrix $P$ satisfies the following algebraic Riccati inequality:

$$P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^TP - PB_2^TP + mI \leq 0,$$

where $m$ is a positive constant.

The adaptive updating gain of $\bar{\alpha}_i(t)$ is expressed as

$$\dot{\bar{\alpha}}_i(t) = -\sigma_i(\bar{\alpha}_i(t) - \bar{\pi}_i(t)) + \|B_2^TP\| g_i(S_i(t)) + 1 + (\|\hat{h}_i(t)\|),$$

$$\|B_2^TP\| g_i(S_i(t)) + \|\hat{h}_i(t)\|\|B_2^TP\| g_i(S_i(t)),$$

where $k_i$ and $\sigma_i$ are positive constants. The initial values of $\bar{\alpha}_i(t)$ and $\bar{\pi}_i(t)$ satisfy $\bar{\alpha}_i(0) \geq \bar{\pi}_i(0) \geq 0$.

The nonlinear function $g_i(S_i(t))$ is written as

$$g_i(S_i(t)) = \begin{cases} 
\frac{B_2^TPS_i(t)}{\|B_2^TP\|S_i(t)}, & \|B_2^TP\|S_i(t) \neq 0, \\
0, & \|B_2^TP\|S_i(t) = 0.
\end{cases}$$

**Lemma 1** (See [35]). If the topology $G$ contains a spanning tree and takes the leader as the root node, the Laplace matrix $L_G(0)$ has a simple eigenvalue of 0, its corresponding right eigenvector is $1_N$, and the real parts of the remaining $N - 1$ eigenvalues are positive.

Based on the abovementioned analyses, Theorem 1 is given as follows:

**Theorem 1.** For UAV swarm systems (1) and (3), if Assumptions 1 and 2 hold, the time-varying formation control $h_i(t)$ meets the feasibility condition $h_i(t) - \hat{h}_i(t) = 0$, and there is a positive definite matrix $P$ satisfying inequality (8); the time-varying formation tracking control for the UAV swarm system (1) and (3) with actuator faults and switching topologies can be achieved, by using the control protocol (7) and the adaptive
laws (9). Furthermore, the adaptive updating laws $\hat{a}_i(t)$ and $\overline{x}_i(t)$ can converge to the same constant $\varepsilon_i (i \in \{2, 3, \ldots, N\})$.

**Proof.** The time-varying formation tracking error of follower $i (i \in \{2, 3, \ldots, N\})$ can be expressed by

$$\psi_i(t) = \xi_i(t) - \hat{h}_i(t) - \hat{\xi}_i(t) = (B_1B^T_2 + B_2\alpha)(\xi_i(t) - h_i(t) - \xi_1(t))$$

$$+ B_2\rho_i(t)(u_i(t) + B_2b_i(t) - \hat{h}_i(t)) + (B_1B^T_2 + B_2\alpha)h_i(t),$$

(11)

$$= (B_1B^T_2 + B_2\alpha)\psi_i(t) + B_2\rho_i(t)u_i(t) + B_2b_i(t) + B_2ah_i(t) - B_2\hat{h}_i(t).$$

Substituting fault-tolerant control protocol (7) into (11), equation (11) can be written as follows:

$$\dot{\psi}_i(t) = (B_1B^T_2 + B_2\alpha)\psi_i(t) + B_2\rho_i(t)u_i(t) + B_2b_i(t) + B_2ah_i(t) - B_2\hat{h}_i(t),$$

(12)

$$= B_2\rho_i(t)(\tilde{a}_i(t)K\psi_i(t) - \tilde{a}_i(t)\psi_i(t))\|g_i(S_i(t)) - \tilde{a}_i(t)\|\|\hat{h}_i(t)\|g_i(S_i(t))\$$

$$+ (B_1B^T_2 + B_2\alpha)\psi_i(t) + B_2b_i(t) + B_2ah_i(t) - B_2\hat{h}_i(t).$$

Let $\psi = [\psi^T_2, \psi^T_3, \ldots, \psi^T_N]^T$, $S = [S^T_2, S^T_3, \ldots, S^T_N]^T$, $b = [b_2, b_3, \ldots, b_N]^T$, $g = [g^T_2(S_2), g^T_3(S_3), \ldots, g^T_N(S_N)]^T$, and $h_{1\nu} = [h_{2\nu}, h_{3\nu}, \ldots, h_{N\nu}]^T$, and then, equation (12) can be written in the following form:

$$\dot{\psi} = ((I_{N-1} \otimes (B_1B^T_2 + B_2\alpha)) + (rA\mathcal{L}^f_{\alpha} \otimes B_2K))\psi$$

$$- (rA\mathcal{L}^f_{\alpha} \otimes \|\alpha\|)g - (rA \otimes B_2)g - (rA\mathcal{L} \otimes B_2)g$$

$$- (I_{N-1} \otimes B_2)\hat{h}_i + (I_{N-1} \otimes B_2)b + (I_{N-1} \otimes B_2)a)\psi,$$

(13)

$$V = \psi^T(I_{\alpha} \otimes (P(B_1B^T_2 + B_2\alpha) + (B_1B^T_2 + B_2\alpha)^TP))\psi + 2\psi^T(I_{\alpha} \otimes rA\mathcal{L}^f_{\alpha} \otimes PB_2K)\psi - 2\psi^T((I_{\alpha} \otimes rA \otimes PB_2)g$$

$$- 2\psi^T(I_{\alpha} \otimes rA\mathcal{L} \otimes PB_2)g - 2\psi^T(I_{\alpha} \otimes rA\mathcal{L} \otimes \|\alpha\|)g - 2\psi^T(I_{\alpha} \otimes PB_2)\hat{h}_i + 2\psi^T(I_{\alpha} \otimes PB_2)b$$

$$+ 2\psi^T(I_{\alpha} \otimes PB_2)h + 2\eta \sum_{i=2}^{N} \frac{\sigma_i}{k_i}(\tilde{a}_i - d)\tilde{a}_i + 2\eta \sum_{i=2}^{N} (\tilde{a}_i - d)^2\tilde{a}_i.$$

(15)

By derivation of the Lyapunov function with respect to time and substituting (13) into it, the result is as follows:

$$2\psi^T((I_{\alpha} \otimes rA\mathcal{L}^f_{\alpha} \otimes PB_2K)\psi \leq - 2\eta \sum_{i=2}^{N} \frac{\sigma_i}{k_i}(B_2^T P S_i)^2,$$

(16)

$$-2\psi^T(I_{\alpha} \otimes PB_2)\hat{h}_i + 2\psi^T(I_{\alpha} \otimes PB_2)b \leq 2\sum_{i=2}^{N} (B_2^TP S_i)(\beta_i + \|\hat{h}_i\|),$$

(17)
\[
2\psi^T \left( L_{\sigma(t)}^{ff} \otimes PB_2 \alpha \right) h = 2 \sum_{i=2}^{N} S_i^T PB_2 \alpha h_i \leq 2 \sum_{i=2}^{N} \| B_i^T PS_i \alpha \| h_i. \tag{18}
\]

By using (10), it can be obtained that

\[
-2\psi^T \left( L_{\sigma(t)}^{ff} r \Lambda \otimes PB_2 \right) g - 2\psi^T \left( L_{\sigma(t)}^{ff} r \Lambda \Pi \otimes PB_2 \right) g \leq -2\eta \sum_{i=2}^{N} \hat{a}_i \left( 1 + \| h_i \| \right) \| B_i^T PS_i \| , \tag{19}
\]

\[
-2\psi^T \left( L_{\sigma(t)}^{ff} r \Delta Y \otimes PB_2 \| \alpha \| \right) g = -2 \sum_{i=2}^{N} \rho_i(t) \hat{a}_i \| \alpha \| B_i^T PS_i \| h_i \| \leq -2\eta \sum_{i=2}^{N} \hat{a}_i \| \alpha \| h_i \| B_i^T PS_i \|. \tag{20}
\]

It follows from (9) that

\[
2\eta \sum_{i=2}^{N} \frac{d_i}{K_i} (\bar{a}_i - d) \bar{a}_i + 2\eta \sum_{i=2}^{N} \left( \bar{a}_i - d \right) \hat{a}_i = 2\eta \sum_{i=2}^{N} \frac{d_i}{K_i} (\bar{a}_i - d) \left( k_i (\bar{a}_i - \bar{a}_i) \right)
\]

\[
+ 2\eta \sum_{i=2}^{N} \left( \bar{a}_i - d \right) \left( -\sigma_i(\bar{a}_i - \bar{a}_i) + \| B_i^T PS_i \|^2 + \left( 1 + \| h_i \| \right) \| B_i^T PS_i \| + \| \alpha \| h_i \| B_i^T PS_i \| \right).
\]

\[
= 2\eta \sum_{i=2}^{N} \left( \bar{a}_i - d \right) \left( \| B_i^T PS_i \|^2 + \left( 1 + \| h_i \| \right) \| B_i^T PS_i \| + \| \alpha \| h_i \| B_i^T PS_i \| \right) - 2\eta \sum_{i=2}^{N} \sigma_i(\bar{a}_i - \bar{a}_i)^2. \tag{21}
\]

Substituting inequality (16)–(21) into (15), the derivative of the Lyapunov function can be written as follows:

\[
\dot{V} \leq \psi^T \left( L_{\sigma(t)}^{ff} \otimes \left( P \left( B_1 B_2^T + B_2 \alpha \right) + \left( B_1 B_2^T + B_2 \alpha \right)^T P \right) \right) \psi - 2\eta \sum_{i=2}^{N} \hat{a}_i \| B_i^T PS_i \|^2 - 2\eta \sum_{i=2}^{N} \hat{a}_i \| \alpha \| B_i^T PS_i \| h_i \|
\]

\[
+ 2\sum_{i=2}^{N} \| \alpha \| \| B_i^T PS_i \| h_i \| - 2\eta \sum_{i=2}^{N} \hat{a}_i \| B_i^T PS_i \| \left( 1 + \| h_i \| \right) + 2\eta \sum_{i=2}^{N} \left( \bar{a}_i - d \right) \left( \| B_i^T PS_i \|^2 + \left( 1 + \| h_i \| \right) \| B_i^T PS_i \| \right)
\]

\[
+ \| \alpha \| \| h_i \| B_i^T PS_i \| + 2\sum_{i=2}^{N} \| B_i^T PS_i \| \left( \beta_i + \| h_i \| \right) - 2\eta \sum_{i=2}^{N} \sigma_i(\bar{a}_i - \bar{a}_i)^2, \tag{22}
\]

\[
= \psi^T \left( L_{\sigma(t)}^{ff} \otimes \left( P \left( B_1 B_2^T + B_2 \alpha \right) + \left( B_1 B_2^T + B_2 \alpha \right)^T P \right) \right) \psi - 2\eta d \sum_{i=2}^{N} \| B_i^T PS_i \|^2 - 2\sum_{i=2}^{N} (-\beta_i + \eta d) \| B_i^T PS_i \|
\]

\[
- 2 \left( -1 + \eta d \right) \sum_{i=2}^{N} \| B_i^T PS_i \| \| h_i \| - 2\eta \sum_{i=2}^{N} \sigma_i(\bar{a}_i - \bar{a}_i)^2 - 2\| \alpha \| \left( -1 + \eta d \right) \sum_{i=2}^{N} \| B_i^T PS_i \| \| h_i \|. \]

Assuming \( d \) is a sufficiently large positive constant and it satisfies \( d > \max_{i=2,3,...,N}\{ (\bar{\beta}_i/\eta), (1/\eta) \} \), it leads to

\[
\dot{V} \leq \psi^T \left( L_{\sigma(t)}^{ff} \otimes \left( P \left( B_1 B_2^T + B_2 \alpha \right) + \left( B_1 B_2^T + B_2 \alpha \right)^T P \right) \right) \psi - 2\eta d \sum_{i=2}^{N} \| B_i^T PS_i \|^2 - 2\eta \sum_{i=2}^{N} \sigma_i(\bar{a}_i - \bar{a}_i)^2. \tag{23}
\]
Let $\lambda^{(i)}_{\sigma(t)}$ be the eigenvalue of $L^{(i)}_{\sigma(t)}$, and it satisfies $\lambda^{(1)}_{\sigma(t)} \leq \lambda^{(2)}_{\sigma(t)} \leq \ldots \leq \lambda^{(N)}_{\sigma(t)}$. From Lemma 1, $\lambda^{(i)}_{\sigma(t)} = 0$ and $\lambda^{(i)}_{\sigma(t)} > 0$. According to Assumption 1, there exists a matrix $U^{(i)}_{\sigma(t)}$ such that

\[ U^{T(i)}_{\sigma(t)}L^{(i)}_{\sigma(t)}U^{(i)}_{\sigma(t)} = \Lambda^{(i)}_{\sigma(t)} = \text{diag}(\lambda^{2}_{\sigma(t)}, \lambda^{3}_{\sigma(t)}, \ldots, \lambda^{N}_{\sigma(t)}) \], where $\lambda^{(i)}_{\sigma(t)} > 0 (i = 2, \ldots, N)$. Let $\Psi = (U^{(i)}_{\sigma(t)} \otimes I_{2})\psi$; then, it leads to

\[
\dot{\Psi} = \Psi^{T}(A_{\sigma(t)}^{(i)} \otimes (P(B_{1}B_{2}^{T} + B_{2}a) + (B_{1}B_{2}^{T} + B_{2}a)^{T}P)) - \Psi + \eta \sum_{i=2}^{N} \sigma_{i}(\bar{\alpha}_{i} - \bar{\alpha})^{2},
\]

Then, $V$ is bounded. That means $\dot{V} \leq 0$, and $\psi_{i} = 0$ and $\lambda_{\sigma(t)} = 0$, according to LaSalle’s invariance principle, one has $\lim_{t \to \infty} \Psi(t) = 0$ and $\lim_{t \to \infty} (\bar{\alpha}_{i}(t) - \bar{\alpha}(t)) = 0$. Therefore, under the fault-tolerant control protocol (7), the UAV swarm systems (1) and (3) can accomplish time-varying formation tracking.

Next, we prove that the adaptive updating gain of $\bar{\alpha}_{i}(t)$ and $\bar{\alpha}(t)$ converges to the same constant $\epsilon_{i} (i \in \{2, \ldots, N\})$. Let $\bar{\alpha} = \bar{\alpha}_{i}(t) - \bar{\alpha}(t)$, according to the adaptive updating gain (9); then,

\[
\dot{\bar{\alpha}}_{i}(t) = \dot{\bar{\alpha}}_{i}(t) - \bar{\alpha}(t) = -\sigma_{i} + k_{\bar{a}}(t)\bar{\alpha}_{i}(t) + Y(t),
\]

where $Y(t) = \text{diag}(B_{1}^{T}P, S_{i}^{T}B_{2}^{T}P, S_{i}^{T}) + \|\alpha\|h_{i}(t)\|B_{1}^{T}PS_{i}^{T}(t)\|$. Then, it follows from (27) that

\[
\bar{\alpha}_{i}(t) = e^{-\sigma_{i}k_{\bar{a}}(t)}\bar{\alpha}_{i}(0) + \int_{0}^{t} e^{-\sigma_{i}k_{\bar{a}}(t-\tau)}Y(\tau)d\tau,
\]

where $Y(t) = \text{diag}(B_{1}^{T}PS_{i}(t))\|B_{1}^{T}PS_{i}(t)\| + \|\alpha\|h_{i}(t)\|B_{1}^{T}PS_{i}^{T}(t)\|$.

Because $\bar{\alpha}_{i}(0) \geq 0$, it can be inferred that $\bar{\alpha}_{i}(t) \geq 0$, since $\bar{\alpha}_{i}(t) = k_{\bar{a}}(\bar{\alpha}_{i}(t) - \bar{\alpha}(t)) = k_{\bar{a}}(\bar{\alpha}(t) - \bar{\alpha}(t))$ is bounded; therefore, $\bar{\alpha}_{i}(t)$ and $\bar{\alpha}(t)$ converge to the same constant $\epsilon_{i} (i \in \{2, \ldots, N\})$. So, the proof of Theorem 1 is completed.

4. Simulation Results

In this section, the simulation studies illustrate the effectiveness of the controller.
Iffollowerscanachievethedesiredformation $h_i(t)$, then that means the design of the controller is effective. Let the positive constant $m=1$. By solving inequality (8), we can get positive matrix $P = \begin{bmatrix} 0.8553 & 0.2605 \\ 0.2605 & 0.5427 \end{bmatrix}$.

Let the initial values of the adaptive laws $\tilde{\alpha}_{Xi}(0) = \tilde{\alpha}_{Yi}(0) = 1.2$ and $\bar{x}_{Xi}(0) = \bar{x}_{Yi}(0) = 1$, the relevant design parameters can be selected by $k_{Xi} = k_{Yi} = 5$, and $\sigma_{Xi} = \sigma_{Yi} = 5$. We choose the damping coefficients as $\alpha_x = -1$ and $\alpha_v = -1.2$. The initial states of the leader are $x_1(0) = [1, 2]^T$ and $v_1(0) = [0, 1]^T$. Also, the initial value of the followers are set to be $x_2(0) = [1.5, 3]^T$, $v_2(0) = [0, 1]^T$, $x_3(0) = [1.5, 3]^T$, $v_3(0) = [1, 0.5]^T$, $x_4(0) = [1.5, -2]^T$, and $v_4(0) = [0, 0]^T$.

Based on Theorem 1, the time-varying formation tracking error $\psi_i(t)$ of followers described by (5) is given in Figures 3 and 4. Figure 3 shows position error trajectory of each follower in the $X$ and $Y$ directions. The velocity error trajectory of each follower in the $X$ and $Y$ directions is shown in Figure 4. From these two figures and the Definition 1, we can obtain that the time-varying formation tracking can be achieved with the actuator faults. The velocity trajectories in the $X$ and $Y$ directions are drawn in Figure 5. By observing the velocity trajectories, it can be concluded that the velocity of each follower is time-varying periodically, which can achieve the desired velocity trajectory. Also, the velocity of virtual leader can tend to zero, which accords to the modes of the leader (1). The position trajectory within $t = 20$ s of the UAV swarm system is shown in Figure 6; one can see that
three followers can achieve the given time-varying formation when tracking the leader. The position snapshots of the four UAVs are shown in Figure 7, where the UAVs are represented by a circle, plus sign, triangle, and square respectively. Figure 7(a) depicts the process of forming a given formation in 1–7 s. It can be see that the followers approach the leader from all directions, and the leader stabilizes at one point eventually. Figures 7(b) and 7(c) further show the change of UAVs position within one period $2\pi$. From Figures 7(b) and 7(c), it can be obtained that the formation is time-varying and the follower moves around the leader periodically. Therefore, the effectiveness of the time-varying formation...
Figure 7: Position snapshots of the virtual leader and the followers. (a) Position snapshots at $t = 1$ s, $3$ s, $5$ s, and $7$ s for the UAVs. (b) Position snapshots at $t = 9$ s, $10$ s, $11$ s, and $13$ s for the UAVs. (c) Position snapshots at $t = 15.28$ s, $16.28$ s, $17.28$ s, and $19.28$ s for the UAVs.

Figure 8: Position error curves in the $X$ and $Y$ directions. (a) Position error curves in the $X$ direction. (b) Position error curves in the $Y$ direction.
tracking control scheme is validated for the UAV swarm systems (1) and (3) with the actuator faults and switching topologies.

If the control protocol is designed without considering the actuator faults, it can be given as follows:

\[
\begin{align*}
    u_i(t) &= -\alpha \tilde{a}_i(t)B^TP_i(t) - \alpha \tilde{a}_i(t)g_i(S_i(t)) \\
        &\quad - \alpha \tilde{a}_i(t)\|\tilde{h}_i(t)\|g_i(S_i(t)).
\end{align*}
\]

(29)

In order to validate the ability of handling faults of the proposed method, comparison simulations are performed for the abovementioned described quadrotor UAVs system with the proposed control protocol (7) and control protocol (29), respectively. The position error curves and the velocity error curves in the X and Y directions are shown in Figures 8 and 9, respectively. In Figures 8 and 9, the solid lines are obtained by using the proposed control protocol (7) in Theorem 1, and dashed lines are obtained by using the control protocol (29). From the simulation results, it is easy to see that the formation tracking performance is good with the proposed control method, while the formation tracking performance cannot be achieved with the control protocol (29) because it does not have the ability to deal with the actuator faults. Therefore, the effectiveness of the control method in this paper is validated.

5. Conclusions

The fault-tolerant time-varying formation tracking control for the UAV swarm systems with actuator faults and the switching topologies is studied. For each follower, this paper proposed a fully distributed adaptive fault-tolerant formation tracking control protocol to compensate for the effect of actuator faults. The adaptive update law is adopted so that the designed control protocol only depends on the part of the information of neighboring UAVs. Moreover, the limitation of knowing the boundary information of the actuator faults is removed and the high gain is avoided. Based on the Lyapunov stability theory, the stability of the UAV swarm systems and the reliability of the control protocol are proved. Potential future works will bench test the effectiveness with lab rigs in the next stage of study and finally hopefully go to real system demonstration, which has been a commonly adopted procedure from analytical development to final implementation with real systems. Another future direction is studying the fault-tolerant time-varying group formation tracking problem with multiple leaders and prescribed performance.

Data Availability

The data used to support the fundings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] W. He and S. Zhang, “Control design for nonlinear flexible wings of a robotic aircraft,” *IEEE Transactions on Control Systems Technology*, vol. 25, no. 1, pp. 351–357, 2017.

[2] W. He, Z. C. Yan, C. Y. Sun, and Y. N. Chen, “Adaptive neural network control of a flapping wing micro aerial vehicle with disturbance observer,” *IEEE Transactions on Cybernetics*, vol. 47, no. 10, pp. 3452–3465, 2017.

[3] M. Yao and M. Zhao, “Cooperative attack strategy of unmanned aerial vehicles in adversarial environment,” *IASC-Intelligent Automation & Soft Computing*, vol. 19, no. 3, pp. 487–496, 2013.

[4] J. Suh, S. You, S. Choi, and S. Oh, “Vision-based coordinated localization for mobile sensor networks,” *IEEE Transactions on Automation Science and Engineering*, vol. 13, no. 2, pp. 611–620, 2016.

[5] W. R. Williamson, M. F. Abdel-Hafez, I. Rhee et al., “An instrumentation system applied to formation flight,” *IEEE Transactions on Control Systems Technology*, vol. 15, no. 1, pp. 75–85, 2007.

[6] B. Yun, B. M. Chen, K. Y. Lum, and T. H. Lee, “Design and implementation of a leader–follower cooperative control system for unmanned helicopters,” *Journal of Control Theory and Applications*, vol. 8, no. 1, pp. 61–68, 2010.

[7] H. X. Qiu, H. B. Duan, and Y. M. Fan, “Multiple unmanned aerial vehicle autonomous formation based on the behavior mechanism in pigeon flocks,” *International Journal of Control Theory and Applications*, vol. 32, no. 10, pp. 1298–1302, 2015.

[8] N. H. M. Li and H. H. T. Liu, “Formation UAV flight control using virtual structure and motion synchronization,” in *Proceedings of the 2010 American Control Conference*, pp. 1782–1787, Seattle, WA, USA, June 2008.

[9] R. W. Beard, J. Lawton, and F. Y. Hadaegh, “A coordination architecture for spacecraft formation control,” *IEEE Transactions on Control Systems Technology*, vol. 9, no. 6, pp. 777–790, 2001.

[10] A. Abdessameud and A. Tayebi, “Formation control of VTOL unmanned aerial vehicles with communication delays,” *Automatica*, vol. 47, no. 11, pp. 2383–2394, 2011.

[11] Y. Xu, D. Luo, D. Li, Y. You, and H. Duan, “Target-enclosing affine formation control of two-layer networked spacecraft with collision avoidance,” *Chinese Journal of Aeronautics*, vol. 32, no. 12, pp. 2679–2693, 2019.

[12] J. H. Wang, L. Han, X. W. Dong, Q. D. Li, and Z. Ren, “Distributed sliding mode control for time-varying formation tracking of multi-UAV system with a dynamic leader,” *Aerospace Science and Technology*, vol. 111, Article ID 106549, 2021.

[13] W. H. Song, J. N. Wang, S. Y. Zhao, and J. Y. Shan, “Event-triggered cooperative unscented Kalman filtering and its application in multi-UAV systems,” *Automatica*, vol. 105, pp. 264–273, 2019.

[14] Y. S. Zheng, J. Y. Ma, and M. Wang, “Consensus of hybrid multi-agent systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1359–1365, 2018.

[15] Y. S. Zheng, Q. Zhao, and J. Y. Ma, “Second-order consensus of hybrid multi-agent systems,” *Systems & Control Letters*, vol. 125, pp. 51–58, 2019.

[16] J. Seo, Y. Kim, S. Kim, and A. Tsourdos, “Consensus-based reconfigurable controller design for unmanned aerial vehicle formation flight,” *Journal of Aerospace Engineering*, vol. 226, no. 7, pp. 817–829, 2012.

[17] G. G. Wen, Y. G. Yu, Z. X. Peng, and A. Rahmani, “Consensus tracking for second-order nonlinear multi-agent systems with switching topologies and a time-varying reference state,” *International Journal of Control*, vol. 89, no. 10, pp. 2096–2106, 2016.

[18] H. B. Du, S. H. Li, and X. Z. Lin, “Finite-time formation control of multi-agent systems via dynamic output feedback,” *International Journal of Robust and Nonlinear Control*, vol. 23, no. 14, pp. 1609–1628, 2013.

[19] X. W. Dong, Y. Zhou, Z. Ren, and Y. S. Zhong, “Distributed formation control for multiple quadrotor UAVs under Markovian switching topologies with partially unknown transition rates,” *Journal of The Franklin Institute*, vol. 356, no. 11, pp. 5706–5728, 2019.

[20] J. L. Yu, X. W. Dong, Q. D. Li, and Z. Ren, “Time-varying formation tracking for high-order multi-agent systems with switching topologies and a leader of bounded unknown input,” *Journal of The Franklin Institute*, vol. 355, no. 5, pp. 2808–2825, 2018.

[21] Z. Y. Jia, J. Q. Y. Wang, and X. L. Ai, “Distributed adaptive neural networks leader-following formation control for quadrotors with directed switching topologies,” *ISA Transactions*, vol. 93, pp. 93–107, 2019.

[22] Z. Q. Yu, Y. M. Zhang, B. Jiang, X. Yu, and T. Y. Chai, “Distributed adaptive fault-tolerant close formation flight control of multiple trailing fixed-wing UAVs,” *ISA Transactions*, vol. 106, pp. 181–199, 2020.

[23] Z. Y. Hua, X. W. Dong, Q. D. Li, and Z. Ren, “Fault-tolerant time-varying formation tracking for second-order multi-agent systems with actuator faults and a non-cooperative target,” *IFAC-papers OnLine*, vol. 51, no. 24, pp. 68–73, 2018.

[24] M. Yadegar and N. Meskin, “Fault-tolerant Control of nonlinear heterogeneous multi-agent systems,” *Automatica*, vol. 127, 2021.

[25] S. Y. Xiao and J. X. Dong, “Distributed adaptive fuzzy fault-tolerant containment control for heterogeneous nonlinear multi-agent systems,” *IEEE Transactions on Systems, Man, and Cybernetics, Part A*, vol. 49, no. 1, pp. 1–12, 2019.

[26] Z. Q. Yu, X. Z. Liu, Y. M. Zhang, Y. H. Qu, and C. Y. Su, “Distributed finite-time fault-tolerant containment control for multiple unmanned aerial vehicles,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 6, 2020.

[27] Z. Q. Yu, Y. M. Zhang, B. Jiang, J. Fun, Y. Jin, and T. Y. Chai, “Composite adaptive disturbance observer-based decentralized fractional-order fault-tolerant control of networked UAVs,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 99, pp. 1–15, 2020.

[28] G. Chen and Y. D. Song, “Robust fault-tolerant cooperative control of multi-agent systems: a constructive design method,” *Journal of The Franklin Institute*, vol. 352, no. 10, pp. 4045–4066, 2015.

[29] L. Zhao and Y. M. Jia, “Neural network-based adaptive consensus tracking control for multi-agent systems under actuator faults,” *International Journal of Systems Science*, vol. 47, no. 8, pp. 1931–1942, 2016.

[30] W. Ni and D. Z. Cheng, “Leader-following consensus of multi-agent systems under fixed and switching topologies,” *Systems & Control Letters*, vol. 59, no. 3, pp. 655–661, 2010.

[31] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, “Time-varying formation control for unmanned aerial vehicles with
switching interaction topologies,” *Control Engineering Practice*, vol. 46, pp. 26–36, 2016.

[32] X. Dong, B. Yu, Z. Shi, and Y. Zhong, “Time-varying formation control for unmanned aerial vehicles: theories and applications,” *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 340–348, 2015.

[33] Y. Kuriki and T. Namerikawa, “Consensus-based cooperative formation control with collision avoidance for a multi-UAV system,” in *Proceedings of the 2010 American Control Conference*, pp. 2077–2082, Baltimore, MA, USA, June 2014.

[34] X. W. Dong, Y. Z. Hua, Y. Zhou, Z. Ren, and Y. S. Zhong, “Theory and experiment on formation-containment control of multiple multirotor unmanned aerial vehicle systems,” *IEEE Transactions on Automation Science and Engineering*, vol. 16, no. 1, pp. 229–240, 2019.

[35] W. Ren and R. W. Beard, “Consensus seeking in multi-agent systems under dynamically changing interaction topologies,” *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.