Supersymmetry in Slow Motion

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ABSTRACT: We construct new theories of electroweak symmetry breaking that employ a combination of supersymmetry and discrete symmetries to stabilize the weak scale up to and beyond the energies probed by the LHC. These models exhibit conventional supersymmetric spectra but the fermion-sfermion-gaugino vertices are absent. This closes many conventional decay channels, thereby allowing several superpartners to be stable on collider time scales. This opens the door to the possibility of directly observing R-hadrons and three flavors of sleptons inside the LHC detectors.

KEYWORDS: Beyond Standard Model, Supersymmetric Standard Model, Phenomenology of Field Theories in Higher Dimensions
1. Introduction

The stability of the weak scale against radiative corrections from higher scales is a mystery. While the puzzle itself is conceptual, virtually all proposed solutions involve new particles and interactions at the weak scale, providing rich collider phenomenology to be explored at the Large Hadron Collider (LHC). Since the reach of the LHC is limited to about 5 TeV, it is important to explore theories of new physics which can stabilize the weak scale up to these energies, and lead to interesting collider signals. Several interesting mechanisms have recently been proposed, including little Higgs theories [1], [2], twin Higgs theories [3], [4], and folded supersymmetry [5].

In this paper we propose a new class of models that stabilize the weak scale up to and beyond the energies probed by the LHC, and which give rise to exotic collider signatures. These theories are based on the following observation. In conventional supersymmetric theories such as the Minimal Supersymmetric Standard Model (MSSM), the quadratically divergent
contributions to the squared Higgs mass in the Standard Model (SM) are cancelled by the superpartners. However, while this cancellation relies on the couplings of the superpartners to the Higgs, conventional supersymmetric collider phenomenology actually depends more critically on the fermion-sfermion-gaugino couplings, through which most superpartners decay. Since the latter interactions do not play a role in stabilizing the Higgs mass at one loop, the relationship between supersymmetric naturalness and supersymmetric phenomenology at LHC energies is somewhat indirect.

This observation then begs the following question. Do there exist consistent effective field theories that exhibit supersymmetric spectra and stabilize the weak scale up to the energies probed by the LHC, but where the fermion-sfermion-gaugino vertices are absent? Such theories could in general give rise to collider signatures that are completely different from those of conventional supersymmetric models!

In this paper we construct one realization of such a scenario where supersymmetry, in combination with a set of discrete symmetries, stabilizes the weak scale even in the absence of the fermion-sfermion-gaugino vertices. These discrete symmetries lead to robust phenomenological consequences. In particular, several superpartners become stable or quasi-stable on collider time scales, their decays effectively happening in slow-motion. This opens the door to the exciting possibility of directly observing several of the superpartners in the LHC detectors.

This particular realization borrows from the ideas of folded supersymmetry [5], but the phenomenology of that case [6] (see also [7]) is very different. For every quark or lepton superfield in the MSSM, consider adding to the theory an additional chiral superfield with exactly the same gauge quantum numbers. We give the MSSM fields a subscript A to distinguish them from these new fields to which we give a subscript B. We impose the following three $Z_2$ symmetries: $Z_2^{AB}$ which interchanges the $A$ and $B$ fields, $Z_2^A$ and $Z_2^B$ which flip the sign of the $A$ and $B$ fields, respectively. Then the up-type Yukawa couplings in this theory take the form

$$
\int d^2 \theta y_u (H_u Q_A U_A + H_u Q_B U_B)
$$

where $H_u$ denotes the up-type Higgs, $Q$ the SU(2) doublet quarks and $U$ the SU(2) singlet anti-quarks. The discrete symmetries have ensured the equality of the $A$ and $B$ Yukawa couplings and the absence of mixing between $A$ and $B$. The one-loop quadratic divergences to the squared Higgs mass from the fermions are cancelled by loops involving the corresponding superpartners.

This theory also has a $Z_2^F$ symmetry under which all fermions are odd and all bosons even. We now construct a new theory (the “daughter” theory) from this theory by projecting out all states that are odd under the combined $Z_2^A \times Z_2^F$ symmetry. Then the daughter theory only contains the SM fermions from $A$, the scalars from $B$, and the gauge bosons. The scalars from $A$, the fermions from $B$ and all the gauginos have been projected out. We refer to the scalars from $B$ as the pseudo-sfermions of the SM fermions from $A$. The pseudo-sfermions have the exactly same quantum numbers as the true sfermions, but do not
form supermultiplets with the SM fermions. In particular, the pseudo-sfermions do not have a vertex with the SM fermion and the gaugino. Therefore, the daughter theory is fundamentally non-supersymmetric. Furthermore, \( Z_2^{AB} \) is also broken. However, provided that the form of (1.1) can be justified, the one-loop quadratic divergences to the squared Higgs mass from the Yukawa couplings are still cancelled, but now between the SM fermions in \( Q_A \) and \( U_A \) and the pseudo-sfermions in \( Q_B \) and \( U_B \).

Of course the daughter theory by itself does not possess any symmetry that can ensure the equality of the \( A \) and \( B \) Yukawa couplings necessary for these cancellations to go through. Therefore it must emerge as the low energy limit of some other theory where the equality of these couplings is a consequence of a symmetry. Furthermore, since gauginos are required to cancel quadratic divergences from gauge loops, they must be reintroduced at some level, but without reintroducing fermion-pseudo-sfermion-gaugino vertices. In the next section we shall give an explicit example of such a construction.

Remarkably, some phenomenological aspects of this scenario are already clear. The theory possesses a \( Z_2^B \) parity symmetry under which all the pseudo-sfermions are odd. Therefore the lightest pseudo-sfermion is stable. Furthermore, notice that the \( B \) sector has its own conserved baryon number, and also three conserved lepton numbers (neglecting the neutrino masses). Therefore, the lightest pseudo-sfermion with any of these quantum numbers is necessarily stable. In order to avoid conflict with the observational bounds on stable charged particles [8], [9], [10], these conservation laws cannot be exact. Since it is technically natural for the breaking to be small, these pseudo-sfermions can be long-lived or stable on collider time-scales. Therefore this scenario can give rise to spectacular signatures involving displaced vertices and stable exotics at the LHC.

2. A Model

Here we present a concrete model which realizes the features presented above, and constitutes an existence proof of the scenario. The model provides a complete self-consistent description up to and beyond LHC energies, and serves as a useful benchmark for the study of collider phenomenology.

2.1 Fields and Symmetries in the Bulk

In this subsection, we focus on the physics in the bulk. The fields and symmetries at the boundaries will be discussed in section 2.2. We take the bulk to be a 5D Minkowski space with the 5th coordinate \( x^5 \equiv y \) restricted to an interval \( 0 \leq y \leq \pi R \), where \( R^{-1} \) is taken to be in the 5-10 TeV range. We also assume that the bulk has 5D \( \mathcal{N} = 1 \) supersymmetry (SUSY).

All SM fields except the Higgs live in the bulk. (The Higgs will be located on the boundary at \( y = 0 \).) The 5D gauge supermultiplets are denoted by \((A_{aM}, \lambda_a, \lambda^c_a, \sigma_a)\), where \( a = 1, 2, 3 \) refer to \( U(1)_Y, \) \( SU(2)_L \), and \( SU(3)_C \) respectively. Here, \( A_{aM} \) corresponds to the SM gauge field, \( \lambda \) and \( \lambda^c \) to the gauginos, and \( \sigma \) to the adjoint scalar which completes the 5D gauge
supermultiplet. In accordance with the scenario outlined above, the matter fields are doubled and we label them as $q_{ip}$, $u_{ip}$, $d_{ip}$, $\ell_{ip}$ and $\epsilon_{ip}$ (all being left-handed Weyl spinors), where $p$ runs over $A$ and $B$, and $i = 1, 2, 3$ refer to the three generations. They form supersymmetric hypermultiplets with the fermions $q^c_{ip}$, $u^c_{ip}$, $\cdots$ in the corresponding conjugate representations and the scalar partners $\bar{q}_{ip}$, $\bar{u}_{ip}$, $\cdots$, $\bar{\epsilon}_{ip}$, $\cdots$. These fields are collectively referred to as $\psi_{ip}$, $\bar{\psi}_{ip}$, $\phi_{ip}$, $\phi^c_{ip}$, respectively. By definition the SM fermions are the zero modes of $\psi_{1A}$.

The bulk $\mathcal{N} = 1$ SUSY possesses an SU(2)$_R$ symmetry under which $(\phi_{ip}, \phi^c_{ip})$ and $(\lambda_a, \lambda^c_a)$ transform as doublets. For our purposes, it is important to distinguish two different ways of embedding 4D $\mathcal{N} = 1$ multiplets into a 5D $\mathcal{N} = 1$ multiplet. One embedding—the “unprimed SUSY”—is $\Phi_{ip} = (\phi_{ip}, \psi_{ip})$ and $\Phi^c_{ip} = (\phi^c_{ip}, \psi^c_{ip})$, while the other—the “primed SUSY”—is $\Phi'_{ip} = (\phi'^{ip}_c, \psi_{ip})$ and $\Phi'^{c}_{ip} = (-\phi'^{ip}_c, \psi^c_{ip})$. Similarly, we can have $V_a = (A_{a5}, \lambda_a)$ and $\Sigma_a = (\sigma_a + i A_{a5}, \lambda^c_a)$, or $V'_a = (A_{a5}, \lambda^c_a)$ and $\Sigma'_a = (\sigma_a + i A_{a5}, -\lambda_a)$. The SU(2)$_R$ symmetry renders irrelevant whether we use the unprimed or primed basis in the bulk, but the distinction will be important at the boundaries.

We require the bulk Lagrangian to possess the following $Z_2$ symmetries.

$$
Z_2^A : \Phi_{iA} \rightarrow -\Phi_{iA}, \quad \Phi^c_{iA} \rightarrow -\Phi^c_{iA}.
$$

$$
Z_2^B : \Phi_{iB} \rightarrow -\Phi_{iB}, \quad \Phi^c_{iB} \rightarrow -\Phi^c_{iB}.
$$

$$
Z_2^{AB} : \Phi_{iA} \leftrightarrow \Phi_{iB}, \quad \Phi^c_{iA} \leftrightarrow \Phi^c_{iB}.
$$

$$
Z_2^{AB} : \begin{cases} 
\Phi'_{iA} \leftrightarrow \Phi'^{c}_{iB}, \\
\Phi'_{iB} \leftrightarrow -\Phi'^{c}_{iA}, \\
V'_a \rightarrow -(V'_a)^T, \quad \Sigma'_a \rightarrow -(\Sigma'_a)^T.
\end{cases}
$$

Note that $Z_2^{AB}$ and $Z_2^{AB}$ together forbid bulk masses for the matter fields.

### 2.2 Fields and Symmetries at the Boundaries

Having described the bulk fields and symmetries, we now introduce the boundaries. The boundaries do not preserve all the symmetries of the bulk; in particular, the bulk SUSY is broken by the following boundary conditions. For the gauge fields, we choose

$$
A_a : (+, +), \quad \lambda_a : (+, -),
$$

$$
\sigma_a : (-, -), \quad \lambda^c_a : (-, +),
$$

where the first (second) $\pm$ refers to the boundary condition at $y = 0$ ($y = \pi R$). “+” means the field is allowed to freely fluctuate at the boundary, while “−” means the field is constrained, i.e., not an independent degree of freedom at the boundary. These boundary conditions only preserve the unprimed (primed) SUSY at $y = 0$ ($y = \pi R$). This is an example of SUSY breaking by the Scher-Schwarz mechanism [11], [12], [13]. Furthermore, notice that only the

[1] If we ignore boundary-localized terms, “+” and “−” would reduce to the usual Neumann and Dirichlet boundary conditions, respectively, or “even” and “odd” in the orbifold language. But boundary terms are important as we will see later.

\[2\]
SM gauge fields have zero modes, precisely in accord with the low-energy spectrum of the scenario described in Sec. 1.

For the $A$-type matter fields, we choose
\[ \phi_{iA} : (+, -), \quad \psi_{iA} : (+, +), \]
\[ \phi_{iA}^c : (-, +), \quad \psi_{iA}^c : (-, -), \quad (2.6) \]

while for the $B$ sector, we choose
\[ \phi_{iB} : (+, +), \quad \psi_{iB} : (+, -), \]
\[ \phi_{iB}^c : (-, -), \quad \psi_{iB}^c : (-, +). \quad (2.7) \]

Note that only $\psi_{iA}$ (the SM fermions) and $\phi_{iB}$ (the pseudo-sfermions) have zero modes. The $\phi_{iB}$ will acquire mass, but only radiatively. This again exactly realizes the low-energy spectrum of our scenario.

Where should the Higgs be located? Note that, of the four bulk $Z_2$ symmetries (2.1)-(2.4), $Z_2^{A\bar{B}}$ is broken at $y = 0$ while $Z_2^{AB}$ is broken at $y = \pi R$. In order to ensure the desired form of the Yukawa couplings (1.1), it is crucial to sequester the Higgs from $Z_2^{AB}$ breaking. Therefore, we must place the Higgs and the Yukawa couplings at $y = 0$. The most general supersymmetric Yukawa couplings consistent with the unbroken $Z_2^A$, $Z_2^B$ and $Z_2^{A\bar{B}}$ are given by
\[ W_{y=0} = \sum_{i,j,p} (N_{ij}^{(u)} y_{ij}^{(u)} H_u Q_{ip} U_{jp} + N_{ij}^{(d)} y_{ij}^{(d)} H_d Q_{ip} D_{jp} + N_{ij}^{(\ell)} y_{ij}^{(\ell)} H_d L_{ip} E_{jp}), \quad (2.8) \]
where $y_{ij}^{(u,d,\ell)}$ are the SM Yukawa couplings, and $N_{ij}^{(u,d,\ell)}$ account for the normalizations of the zero-mode wavefunctions:
\[ N_{ij}^{(u)} \equiv \frac{1}{\zeta_{uA}^{(0)}(y) \xi_{uA}^{(0)}(0)} \quad \text{etc.}, \quad (2.9) \]

where $\xi_{uA}^{(n)}(y)$ is the normalized $n$-th KK mode of $q_{uA}$, etc. (We have also implicitly imposed an R parity, which has the same charge assignments as in the MSSM for each of the fields in $A$ and $B$.)

The symmetries at $y = 0$ also allow brane-localized kinetic terms at this point. The $Z_2^{A\bar{B}}$ symmetry ensures that the kinetic terms for the $A$ and $B$ fields have the same coefficient, and therefore the cancellation of the quadratic divergences arising from Yukawa couplings is maintained. On the other hand, at the $y = \pi R$ boundary, the only quadratic terms allowed by $Z_2^A$, $Z_2^B$ and $Z_2^{A\bar{B}}$ are the brane-localized kinetic terms for $\Phi_{iA}^c$ and $\Phi_{iB}^c$. $Z_2^{A\bar{B}}$ ensures that these two kinetic terms have the same coefficients, which again guarantees the cancellation.

In summary, our choice of boundary conditions ensures that the only zero modes arising from the bulk fields are the SM fermions from the $A$-sector, the pseudo-sfermions from the $B$-sector, and the SM gauge bosons. The discrete symmetries relating the $A$ and $B$ fields...
ensure that one loop quadratic divergences to the Higgs mass arising from loops involving the SM fermions are cancelled by the pseudo-sfermions. What about loops involving the gauge fields? The lightest gauginos have masses of order $1/R$. The fact that the above boundary conditions break supersymmetry only non-locally ensures that contributions to the Higgs mass from gauge loops are finite and cutoff at this scale. Provided that the compactification scale $1/R$ is less than about 5-10 TeV, radiative corrections to the Higgs mass from gauge loops are under control even though there is no fermion-pseudo-sfermion-gaugino vertex. Therefore this model is a concrete realization of the scenario described in Sec.

2.3 The Long-lived Pseudo-sfermions

Now, as anticipated in the introduction, this model as it stands conserves $B$-baryon and $B$-lepton numbers, implying that the lightest $B$ scalars are necessarily stable. This is incompatible with observation. To resolve this problem, we allow for small violations of $Z_A^2$ and $Z_B^2$ by adding to (2.8) the following terms:

$$
\Delta W_{y=0} = \sum_{i,j} \left( N^{(u)}_{ij} y^{(u)}_{ij} H_u Q_i A_j B + N^{(d)}_{ij} y^{(d)}_{ij} H_d Q_i A_j B + N^{(\ell)}_{ij} y^{(\ell)}_{ij} E_i A_j B \right) + (A \leftrightarrow B),
$$

(2.10)

where $\epsilon^{(u,d,\ell)}_{ij}$ are dimensionless parameters, and $N^{(u,d,\ell)}_{ij}$ are given in (2.9). Note that this still preserves $Z_A^{AB}$, so the cancellation of the quadratic divergences in the squared Higgs mass is not spoiled.

One technical remark is in order. Note that while there are no terms with a lower dimension than those in (2.10) which can mix $A$ and $B$ while preserving $Z_A^{AB}$, brane-localized kinetic mixing terms, such as $Q_{iA}^\dagger e^{2V} Q_{jB}$, have the same dimension. However, unlike the mixed Yukawa couplings above, such terms do not affect the zero modes and therefore do not give rise to decays of the light scalars. If we begin with such kinetic $A$-$B$ mixing but without the mixed Yukawas (2.10), then the mixed Yukawas would never be induced, due to the non-renormalization theorem of superpotential. On the other hand, if we begin with mixed Yukawas but without kinetic $A$-$B$ mixing, it would be induced radiatively. Since the only phenomenological relevance of the $A$-$B$ mixings is to cause $q_i B$ to decay, we neglect the brane-localized kinetic $A$-$B$ mixings, which can self-consistently be assumed to be suppressed by a loop factor with respect to the mixed Yukawa couplings.

2.4 The Cutoff of the Theory

Here we estimate the scales suppressing higher dimensional operators in our Lagrangian, which we have neglected in our analysis up till now. In principle, locality allows three separate scales, i.e. the cutoffs at $y = 0$, in the bulk, and at $y = \pi R$, which we denote by $\Lambda_0$, $\Lambda_b$, and $\Lambda_{\pi R}$, respectively. We take the true cutoff of the model to be the lowest of the three.

The most severely constrained is $\Lambda_0$, because the brane-localized top Yukawa interactions can rapidly become strong above $1/R$ due to the $O(1)$ top Yukawa coupling, color multiplicity,
and the fact that the Higgs couples to both $A$ and $B$ fields. Since there is no large or small number involved here, one might naively expect that $\Lambda_0 \sim 1/R$, i.e. no significant gap between the compactification scale and the cutoff. Then the effects of higher dimensional operators can potentially be large, invalidating the 5D effective field theory.

However, in estimating $\Lambda_0$, it is crucial not to neglect the brane-localized kinetic terms, because their effects are larger for the heavier KK modes which are important when analyzing the UV behavior. In terms of the dimensionless coefficient $Z$ defined via

$$L_{\text{brane kinetic}} = \int d^4 \theta Z \pi R \Phi^+ e^{2V} \Phi \bigg|_{y=0}, \quad \text{(2.11)}$$

a rough estimate of the cutoff $\Lambda$ ($= \Lambda_0$) as a function of $Z$ (taken to be equal for $Q_3$ and $U_3$ for simplicity) is plotted in Fig. 1. As one can see, the cutoff $\Lambda_0$ can easily exceed $\sim O(10)/R$ for $Z \gtrsim 1/10$. This then becomes comparable to the bound on $\Lambda_b$ from gauge loops in the bulk, and there is no advantage to further raising $Z$.

The final check is to make sure that such values of $Z$ do not significantly lower the masses of the lightest KK fermions in the $B$ sector, which would jeopardize our scenario. The lightest KK $B$-fermion mass can be computed from (B.8) in Appendix B, and one finds that when varying $Z$ from 0 to 0.25 the mass only decreases from $0.5/R$ to $0.4/R$. Therefore, we conclude that our 5D model with $R^{-1}$ in the 5-10 TeV range provides a good effective field theory realization of our scenario up to and beyond energies accessible to the LHC.

One might wonder if the overall cutoff of the theory could be raised simply by warping the 5D spacetime. This possibility is however difficult to reconcile with the Scherk-Schwarz mechanism for supersymmetry breaking [14], which plays a crucial role in our construction.
Figure 2: The functions $K_g(Z)/(1 + Z)$ (solid curve) and $(1 + Z)K_Y(Z)$ (dashed curve).

2.5 The Pseudo-sfermion Spectrum

The one-loop gauge contributions to the squared soft mass of the scalar $i$ are given by

$$
\delta m_{i,\text{gauge}}^2 = \frac{1}{4\pi^4 R^2} \frac{K_g(Z_i)}{1 + Z_i} \sum_G g_G^2 C_2^{(G)}(i),
$$

(2.12)

where $G = U(1)_Y, SU(2)_L, SU(3)_C$, and $g_G$ and $C_2^{(G)}$ are respectively the gauge coupling and the quadratic Casimir for the group $G$. $K_g(Z)$ is an $O(1)$ dimensionless integral:

$$
K_g(Z) \equiv \int_0^\infty dx \frac{x^2 [2 - Z + \frac{Z^3 x^2}{2} + Z(1 + 2Z)x \coth x]}{(4 + Z^2 x^2) \sinh x + 4Z x \cosh x}.
$$

(2.14)

The combination $K_g(Z)/(1 + Z)$ is represented in Fig. 2 by the solid curve.

For the third generation squarks, the one-loop contributions from the large top Yukawa coupling are also important.

$$
\delta m_{\tilde{q}_3}^2 = \frac{g_t^2}{8\pi^4 R^2} (1 + Z_{U_3}) K_Y(Z_{U_3}),
$$

$$
\delta m_{\tilde{u}_3}^2 = \frac{g_t^2}{4\pi^4 R^2} (1 + Z_{Q_3}) K_Y(Z_{Q_3}),
$$

(2.13)

where

$$
K_Y(Z) = \int_0^\infty dx \frac{2x^2}{(4 + Z^2 x^2) \sinh x + 4Z x \cosh x}.
$$

(2.14)

The combination $(1 + Z) K_Y(Z)$ is represented in Fig. 2 by the dashed curve.
2.6 The Higgs Sector and Electroweak Symmetry Breaking

There are a few points regarding the Higgs sector and electroweak symmetry breaking which must be discussed. First, recall that the bulk possesses an $SU(2)_R$ symmetry, which contains a $U(1)_R$ subgroup generated by rotations about the $\sigma^3$ direction. In the convention where the gauginos $\lambda$ and $\lambda^c$ have $U(1)_R$ charges $+1$ and $-1$, $\Phi_{ip}$ and $\Phi_{c_{ip}}$ have $U(1)_R$ charges $+1$ and $-1$, respectively, while $V_a$ and $\Sigma_a$ have no $U(1)_R$ charge. The bulk also possesses a global $U(1)$ symmetry under which $\Phi_{ip}$ and $\Phi_{c_{ip}}$ have charge $-1/2$ and $+1/2$ respectively. The sum of this $U(1)$ and the above $U(1)_R$ is also an $R$-symmetry, which we label $U(1)_{R'}$. Then, the superpotential (2.8) is $U(1)_{R'}$ invariant if $H_u$ and $H_d$ are assigned unit $U(1)_{R'}$ charge. The $\mu$-term $\delta(y) \int d^2\theta \mu H_u H_d$ also respects $U(1)_{R'}$. Therefore, as things stand, the theory is $U(1)_{R'}$ invariant, and a $B\mu$-term $B \mu \tilde{H}_u \tilde{H}_d$ will not be generated.

Therefore, in order to have realistic electroweak symmetry breaking, we must introduce explicit $U(1)_{R'}$ violation. It is also desirable to have additional contributions to the Higgs quartic couplings to get a Higgs heavier than the LEP bound without too much tuning. As shown in [5], both objectives can be simultaneously realized by introducing additional SM singlets in the Higgs sector. Below, we summarize this analysis.

We extend the Higgs sector by adding to the theory an extra singlet $S$ which is localized to the brane at $y = 0$ and replaces the $\mu$ term $\delta(y) \int d^2\theta \mu H_u H_d$ by

$$\delta(y) \int d^2\theta \left[ \alpha S + \lambda S H_u H_d + \kappa S^3 \right].$$
(2.15)

Now $U(1)_{R'}$ is explicitly broken. The Higgs sector has no continuous global symmetries, which ensures the absence of an unwanted Goldstone boson. The Higgsino mass, or the $\mu$ term, will be supplied by the VEV of the scalar $S$. (The origin of the negative squared mass for the scalar $S$ will be discussed below.) The above superpotential also provides an additional tree-level Higgs quartic coupling. For example, for $\tan \beta \sim O(1)$ and $\lambda \gtrsim 0.7$, the tree-level Higgs masses will be greater than the experimental lower bound. Such "large" values of $\lambda$ are allowed since the cutoff of the theory is low. We choose the value of $\alpha$ to be of order weak scale size to obtain consistent electroweak breaking. This choice is technically natural. We leave the problem of naturally generating $\alpha$ of this size for future work.

Now we are ready to compute the radiatively generated soft mass of the Higgs. First, there is a one-loop contribution from gauge loops, given by the formula (2.12):

$$\delta m_H^2|_{\text{gauge}} = \frac{2.1}{4\pi^4 R^2} \left( \frac{3g_2^2}{4} + \frac{g_1^2}{4} \right) \simeq \frac{0.075}{4\pi^2} R^{-2},$$
(2.16)

where we have ignored $Z_H$. (The values corresponding to nonzero $Z_H$ can be read off from Fig. 4.) There is a two-loop contribution from top and stop loops where the stop masses are generated at one loop given by the formulae (2.13) and (2.14), giving rise to

$$\delta m_H^2|_{\text{top}} \simeq -\frac{3y_t^2}{4\pi^2} m_t^2 \log \frac{R^{-1}}{m_t},$$
(2.17)
where $\bar{m}_t^2$ is the average of the left and right stop mass-squareds. Taking $Z_{Q_3} = Z_{U_3} \equiv Z_t$ for simplicity, one finds that $\bar{m}_t^2$ varies from $0.017 R^{-2}$ to $0.012 R^{-2}$ as $Z_t$ is varied from 0 to 2. In this range the total $\delta m_H^2$ is negative, thereby triggering electroweak symmetry breaking.

Finally, let us discuss the origin of a negative squared mass for the scalar in $S$. A simple way to generate this is to introduce into the bulk two SM singlet hypermultiplets $\hat{P}_A$ and $\hat{P}_B$. The boundary conditions on these fields are such as to allow only a fermion zero mode for each of $\hat{P}_A$ and $\hat{P}_B$. The bulk $Z_{AB}$ symmetry interchanges $\hat{P}_A$ and $\hat{P}_B$. In addition, under the $Z_{AB}'$ symmetry, $\hat{P}_A$ and $\hat{P}_B$ are also interchanged. Then on the brane at $y = 0$ we can write the interaction

$$\delta (y) \int d^2 \theta \left[ \lambda_{P} S P_A P_B + \mu_{P} \left( P_A^2 + P_B^2 \right) \right]$$

(2.18)

The effect of the coupling $\lambda_{P}$ is to generate a negative mass squared for the scalar in $S$ at one loop. Note that the theory possesses a $Z_2$ symmetry under which $P_A \to -P_A$ and $P_B \to -P_B$ while all other fields are invariant. Then, the lightest fermion in $P_{A,B}$ is stable and therefore can be a viable dark matter candidate [13].

The extended Higgs sector has no impact on the essential aspects of the theory such as the mechanism for cancellation of the one-loop quadratic divergences, or the lifetimes of the long-lived pseudo-sfermions. Further, the new couplings introduced above do not affect our estimate of the cutoff of the theory in section 2.4. The reason is that this estimate is dominated by the behavior of the top Yukawa coupling, which rapidly grows strong above $1/R$, whereas the new couplings in the Higgs sector need not run rapidly.

3. Collider Phenomenology

Let us first contrast the phenomenology of this scenario with that of the MSSM (and extensions of the MSSM that include additional singlets) with a similar spectrum. We therefore consider a spectrum where the gauginos are heavy, at a few TeV, while the sfermions and Higgsinos are at a few to several hundred GeV. We further specialize the case where the Higgsino is the LSP. In the MSSM with such a spectrum, gauginos are not directly accessible to the LHC. Therefore, sfermions predominantly decay to the corresponding SM fermions and a Higgsino. These decays are prompt.

In stark contrast, in our scenario, the lightest pseudo-sfermions (i.e. the lightest pseudo-squark and the three lightest $e$, $\mu$, $\tau$-type pseudo-sleptons) can decay only via the couplings (2.10). Since these couplings break symmetries, it is technically natural for them to be small. Therefore, these four lightest sfermions can be naturally long-lived or even collider-stable!

In our specific extra-dimensional construction, the form of the soft masses (2.12) implies that SU(2) doublet pseudo-sfermions are heavier than the SU(2) singlet ones with the same baryon, $e$, $\mu$, or $\tau$-number. Thus, the doublets will promptly decay to the corresponding singlet scalars. Neglecting the masses of the decay products, the SU(2) singlet pseudo-sfermions’
decay rates are given by

$$\Gamma^{-1} = \left(\frac{y^2 \varepsilon^2}{8\pi \tilde{m}}\right)^{-1} \simeq 50 \mu m \frac{100 \text{ GeV}}{\tilde{m}} \left(\frac{10^{-6}}{y \varepsilon}\right)^2,$$

(3.1)

where $\tilde{m}$ is the pseudo-sfermion mass, while $y \varepsilon$ represents the relevant combination of the Yukawa coupling and the $\varepsilon$ factors as in (2.10). For example, the pseudo-selectron could have a displaced vertex for $\varepsilon \sim O(0.1)$ and $\tan \beta \sim O(1)$. It is also possible that two pseudo-sleptons of different generations both decay inside the detector. For example, for $\varepsilon \sim O(10^{-3})$ and $\tan \beta \sim O(1)$, a pseudo-smuon will have a displaced vertex of about $100 \mu m$, while a pseudo-selectron decays after travelling a meter or so.

Due to their large masses, these long-lived charged pseudo-sleptons hardly lose any energy while coasting through the detector material [16]. If the produced pair of the pseudo-sleptons are collider-stable, we expect to see two highly-ionizing tracks. On the other hand, if each decays into a SM fermion and a Higgsino in the detector, there will be two tracks with a kink. In this regard, our scenario shares some similarity with the slepton co-NLSP scenario [17] in gauge mediation.

The long-lived pseudo-squarks, on the other hand, will hadronize into $R$-hadrons, which may be neutral or charged. If charged, they will again appear as a highly-ionizing tracks, although a neutral $R$-hadron can sometimes be converted to a charged one by interacting with nucleons in the detector. Slow enough $R$-hadrons can be stopped [18], as in the case of the long-lived gluino in Split Supersymmetry [19].

Note that these signals are quite robust expectations of our low-energy scenario described in Sec. 1, independent of the details of any particular UV completion, and should make this scenario straightforward to distinguish at the LHC.

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**A. Boundary Conditions with Boundary-Localized Kinetic Terms**

We analyze a massless bulk fermion with a brane-localized kinetic term at the $y = 0$ boundary. First, consider the contribution to the 4D action from the 5D bulk kinetic term:

$$\mathcal{L}_{4D} \equiv \int_{0}^{\pi R} dy \mathcal{L}_{\text{bulk kin.}} \quad \text{(Naive)}, \quad (A.1)$$

where

$$\mathcal{L}_{\text{bulk kin.}} = \bar{\psi} p \cdot \bar{\sigma} \psi + \bar{\psi}^c p \cdot \sigma \bar{\psi}^c + \frac{1}{2} \left[ \bar{\psi}^c \partial_y \psi - (\partial_y \bar{\psi}^c) \psi + \text{h.c.} \right]. \quad (A.2)$$
We have carefully distributed $\partial_y$ such that $L_{\text{bulk kin.}}$ is real without integration by parts.)

The reason that (A.1) is “naive” is the following. From the equations of motion in the bulk

$$
\begin{align*}
p \cdot \bar{\sigma} \psi - \partial_y \bar{\psi}^c &= 0, \\
p \cdot \sigma \psi + \partial_y \psi &= 0,
\end{align*}
$$

(A.3)

we expect that $L_{4D}$ can depend on only two of the four variables $\psi(0)$, $\psi^c(0)$, $\psi(\pi R)$, and $\psi^c(\pi R)$. For example, when $\psi$ and $\psi^c$ have the $A$-type boundary conditions ($(+, +)$ and $(-, -)$ respectively), $L_{4D}$ by definition should only depend on $\psi(0)$ and $\psi(\pi R)$. Thus, $\delta L_{4D}$ should only contain $\delta \psi(0)$ and $\delta \psi(\pi R)$. Similarly, when $\psi$ and $\psi^c$ have the $B$-type boundary conditions ($(+, -)$ and $(-, +)$ respectively), $\delta L_{4D}$ should only depend on $\delta \psi(0)$ and $\delta \psi^c(\pi R)$. However, notice that the variation of (A.1) is

$$
\delta L_{4D} = \frac{1}{2} \left[ \psi^c \delta \psi - \delta \psi^c \psi + \text{h.c.} \right]_{y=0}^{y=\pi R}
$$

(A.4)

which depends on all of $\delta \psi(0)$, $\delta \psi^c(0)$, $\delta \psi(\pi R)$, and $\delta \psi^c(\pi R)$.

We usually fix this problem by hand by imposing the boundary condition $\psi^c(0) = \psi^c(\pi R) = 0$ for the $A$-type, or $\psi^c(0) = \psi(\pi R) = 0$ for the $B$-type. This is analogous to imposing constraints by hand when solving a constrained mechanical system. Alternatively, we can let mathematics take care of the constraints by adding Lagrange multipliers. In our case, a suitable mathematical trick is to add to (A.1) the following terms: for the $A$-type

$$
L_{4D}^{(A)} \equiv \int_0^{\pi R} dy \, L_{\text{bulk kin.}} + \frac{1}{2} \left[ -\psi^c \psi(0) + \psi^c \psi(\pi R) + \text{h.c.} \right]
$$

(Correct),

while for the $B$-type

$$
L_{4D}^{(B)} \equiv \int_0^{\pi R} dy \, L_{\text{bulk kin.}} + \frac{1}{2} \left[ -\psi^c \psi(0) - \psi^c \psi(\pi R) + \text{h.c.} \right]
$$

(Correct)

Then, instead of (A.4), we now obtain

$$
\delta L_{4D}^{(A)} = -\psi^c \delta \psi(0) + \psi^c \delta \psi(\pi R) + \text{h.c.},
$$

(A.5)

and

$$
\delta L_{4D}^{(B)} = -\psi^c \delta \psi(0) - \delta \psi^c \psi(\pi R) + \text{h.c.}.
$$

(A.6)

Note that (A.5) shows that $L_{4D}^{(A)}$ is a function of only $\psi(0)$ and $\psi(\pi R)$ as it should be. In the absence of other terms at the boundaries, demanding that $\delta L_{4D}^{(A)}$ vanish for arbitrary variations gives us the usual boundary conditions $\psi^c(0) = \psi^c(\pi R) = 0$, which, together with the equations of motion (A.3), further implies that $\partial_y \psi(0) = \partial_y \psi(\pi R) = 0$. This is what we would have got in the orbifold language by assigning $(+, +)$ and $(-, -)$ parities to $\psi$ and $\psi^c$ respectively. Similarly, demanding that the variation (A.6) vanish is equivalent to assigning $(+, -)$ and $(-, +)$ parities to $\psi$ and $\psi^c$. 

– 12 –
The advantage of using $L^{(A)}_{4\text{D}}$ and $L^{(B)}_{4\text{D}}$ becomes clear once there are extra terms at the boundaries. For example, consider the $A$-type case and let us add a boundary-localized kinetic term at $y = 0$:

$$L^{(A)}_Z = L^{(A)}_{4\text{D}} + Z\pi R \overline{\psi} p \cdot \bar{\sigma} \psi(0) .$$  \hfill (A.7)

Then, extremizing $L^{(A)}_Z$ readily give us

$$\overline{\psi}'(0) = Z\pi R p \cdot \bar{\sigma} \psi(0) ,$$

$$\overline{\psi}'(\pi R) = 0 .$$  \hfill (A.8)

Combining these with the equations of motion (A.3), we obtain

$$\partial_y \psi(0) = -Z\pi R p^2 \psi(0) ,$$

$$\partial_y \psi(\pi R) = 0 ,$$  \hfill (A.9)

and

$$Z\pi R \partial_y \psi^c(0) = \psi^c(0) ,$$

$$\psi^c(\pi R) = 0 .$$  \hfill (A.10)

These are the correct boundary conditions for the $A$-type fermion with a brane-localized kinetic term at $y = 0$.

Similarly, for the $B$-type fermion, extremizing

$$L^{(B)}_Z = L^{(B)}_{4\text{D}} + Z\pi R \overline{\psi} p \cdot \bar{\sigma} \psi(0)$$  \hfill (A.11)

gives

$$\overline{\psi}'(0) = Z\pi R p \cdot \bar{\sigma} \psi(0) ,$$

$$\overline{\psi}'(\pi R) = 0 ,$$  \hfill (A.12)

which, combined with (A.3), implies

$$\partial_y \psi(0) = -Z\pi R p^2 \psi(0) ,$$

$$\psi(\pi R) = 0 ,$$  \hfill (A.13)

and

$$Z\pi R \partial_y \psi^c(0) = \psi^c(0) ,$$

$$\partial_y \psi^c(\pi R) = 0 .$$  \hfill (A.14)

These are the correct boundary conditions for the $B$ type fermion with a brane-localized kinetic term at $y = 0$. 

"-- 13 --"
Of course, one could re-derive all of the above results in the orbifold language, but must be careful in doing so, because, at the \( y = 0 \) brane, "odd" fields jump while even fields have a kink. So, the usual advantage of the orbifold language, namely the simple relation between the parity of a field and its boundary condition, is lost.

Although the above analysis was done for fermions, it should be obvious that, by supersymmetry, bulk scalars with \((+,+), (-,-), (+,-),\) and \((-,+)\) parities obey the same boundary conditions, \((\ref{a9}), (\ref{a10}), (\ref{a13}),\) and \((\ref{a14}),\) respectively.

**B. The KK Modes**

We expand an \( A \)-type 5D fermion field as

\[
\psi(p,y) = \sum_n \xi_n^{++}(y)\psi_n(p),
\]

\[
\psi^c(p,y) = \sum_n \xi_n^{--}(y)\psi_n^c(p),
\]

where \( \Psi_n = (\psi_n, \psi_n^c) \) satisfies the 4D Dirac equation \( \not{\! p}\Psi_n = m_n \Psi_n \). Then, the 5D Dirac equation \((\ref{a3})\) implies that both \( \xi(y) \)'s satisfy the bulk equation of motion \( (m_n^2 + \partial_y^2)\xi_n(y) = 0 \) in the interval \( 0 < y < \pi R \). The boundary conditions for \( \xi_n^{++} \) and \( \xi_n^{--} \) are given by \((\ref{a3})\) and \((\ref{a10}),\) respectively. It is trivial to repeat the exercise for the \( B \)-type fermion and also for scalars.

The solutions for the \((+,+)\) and \((-,-)\) modes are then given by

\[
\begin{cases}
\xi_0^{++}(y) = 1/\sqrt{\pi R(1 + Z)} \\
\xi_n^{++}(y) = N(m_n^{\pm\pm}) \cos[m_n^{\pm\pm}(y - \pi R)]
\end{cases}
\]

\( (B.2) \)

\[
\xi_n^{--}(y) = N(m_n^{\pm\pm}) \sin[m_n^{\pm\pm}(y - \pi R)],
\]

\( (B.3) \)

where \( n = 1, 2, 3, \ldots \), while for the \((+,-)\) and \((-,+)\) modes

\[
\begin{align*}
\xi_n^{+-}(y) &= N(m_n^{\pm\mp}) \sin[m_n^{\pm\mp}(y - \pi R)] \\
\xi_n^{-+}(y) &= -N(m_n^{\pm\mp}) \cos[m_n^{\pm\mp}(y - \pi R)]
\end{align*}
\]

\( (B.4) \)

\( (B.5) \)

where again \( n = 1, 2, 3, \ldots \). The corresponding KK masses and normalization factors are given by

\[
m_0^{++} = 0,
\]

\( (B.6) \)

\[
Z\pi R m_n^{\pm\pm} = -\tan(m_n^{\pm\pm}\pi R),
\]

\( (B.7) \)

\[
Z\pi R m_n^{\pm\mp} = \cot(m_n^{\pm\mp}\pi R),
\]

\( (B.8) \)

and

\[
N(m) = \sqrt{\frac{2}{\pi R}} \left( 1 + \frac{Z}{1 + (Z\pi R m)^2} \right)^{-1/2}.
\]

\( (B.9) \)
As they should be, all these modes are orthonormal. Note that, because of the brane-localized kinetic term, the appropriate inner-products are

\[
\langle f | g \rangle = Z \pi R f^*(0) g(0) + \int_0^{\pi R} f^*(z) g(z) \, dz ,
\]

for the “(+, +)” and “(+, -)” fields, while

\[
\langle f | g \rangle = \int_0^{\pi R} f^*(z) g(z) \, dz ,
\]

for the “(-, +)” and “(-, -)” types.

C. The Propagators

First, consider four massless bulk scalars \( \phi_{\alpha\alpha'} \) with \( \alpha, \alpha' = \pm, \pm \). First, all four of them satisfy the bulk equation of motion \((p^2 + \partial_y^2)\phi_{\alpha\alpha'}(p, y) = 0\) in the interval \(0 < y < \pi R\), so all the propagators satisfy

\[
(p^2 + \partial_y^2 + i\epsilon)G_{\alpha\alpha'}(y, y'; p) = i\delta(y - y') \quad \text{(C.1)}
\]

in this interval. Then, viewing \(G_{\alpha\alpha'}(y, y'; p)\) as a function of \(y\), it satisfies \((A.9), (A.11), (A.13),\) and \((A.14)\) for \((\alpha, \alpha') = (+, +), (-, -), (+, -),\) and \((-, +)\), respectively. For example,

\[
\begin{aligned}
\partial_y G_{++}(y, y'; p) |_{y=0} &= -Z \pi R p^2 G_{++}(y, y'; p) |_{y=0} , \\
\partial_y G_{++}(y, y'; p) |_{y=-\pi R} &= 0 .
\end{aligned}
\]

Solving these, we obtain

\[
G_{++}(y, y', p; Z) = -\frac{i \cosh[p_E(y_\geq - \pi R)] (\cosh[p_E y_\leq] + Z \pi R p_E \sinh[p_E y_\leq])}{p_E (\sinh[p_E \pi R] + Z \pi R p_E \cosh[p_E \pi R])} , \quad \text{(C.3)}
\]

where \(p_E \equiv (-p^2 - i\epsilon)^{1/2}\), and \(y_\geq\) and \(y_\leq\) are respectively the larger and the smaller of \(y\) and \(y'\). Similarly, we have

\[
\begin{aligned}
G_{--}(y, y', p; Z) &= \frac{i \sinh[p_E(y_\geq - \pi R)] (\sinh[p_E y_\leq] + Z \pi R p_E \cosh[p_E y_\leq])}{p_E (\sinh[p_E \pi R] + Z \pi R p_E \cosh[p_E \pi R])} , \\
G_{+-}(y, y', p; Z) &= \frac{i \sinh[p_E(y_\geq - \pi R)] (\cosh[p_E y_\leq] + Z \pi R p_E \sinh[p_E y_\leq])}{p_E (\cosh[p_E \pi R] + Z \pi R p_E \sinh[p_E \pi R])} , \\
G_{-+}(y, y', p; Z) &= -\frac{i \sinh[p_E(y_\geq - \pi R)] (\sinh[p_E y_\leq] + Z \pi R p_E \cosh[p_E y_\leq])}{p_E (\cosh[p_E \pi R] + Z \pi R p_E \sinh[p_E \pi R])} .
\end{aligned}
\]

Using these scalar propagators, we can also write down the propagators for fermions. For the \(A\)-type fermion, we have chirality-preserving propagators

\[
\begin{aligned}
\langle \psi_\alpha(y) \bar{\psi}_\beta(y') \rangle(p) &= p \cdot \sigma_{\alpha\beta} G_{++}(y, y', p; Z) , \\
\langle \bar{\psi}^\alpha(y) \psi^{\beta}(y') \rangle(p) &= p \cdot \dot{\sigma}^{\dot{\alpha}\dot{\beta}} G_{--}(y, y', p; Z) , \quad \text{(C.7)}
\end{aligned}
\]

- 15 -
and chirality-flipping propagators

\[ \langle \psi_\alpha(y) \psi^\beta(y') \rangle(p) = \delta_\alpha^\beta \partial_y G_--(y, y', p; Z), \]
\[ \langle \bar{\psi}^\alpha(y) \bar{\psi}_\beta(y') \rangle(p) = -\delta_\beta^\alpha \partial_y G_{++}(y, y', p; Z), \]  

where \( \langle \cdots \rangle(p) \) denotes the time-ordered correlation function in the mixed momentum-position representation. Similarly, for the \( B \)-type fermion, we have

\[ \langle \psi_\alpha(y) \bar{\psi}_\beta(y') \rangle(p) = p \cdot \sigma_{\alpha \beta} G_--(y, y', p; Z), \]
\[ \langle \bar{\psi}^\alpha(y) \psi^\beta(y') \rangle(p) = p \cdot \sigma^{\alpha \beta} G_{++}(y, y', p; Z), \]  

and

\[ \langle \psi_\alpha(y) \psi^\beta(y') \rangle(p) = \delta_\alpha^\beta \partial_y G_--(y, y', p; Z), \]
\[ \langle \bar{\psi}^\alpha(y) \bar{\psi}_\beta(y') \rangle(p) = -\delta_\beta^\alpha \partial_y G_{++}(y, y', p; Z) \]  

\[ \text{(C.8)} \]

\[ \text{(C.9)} \]

\[ \text{(C.10)} \]

D. Computation of the Soft Masses

D.1 Gauge Contributions

We ignore the brane-localized kinetic terms for the gauge fields for simplicity, as they have little relevance to the phenomenology we are concerned with in this paper. The relevant bulk gauge interactions involving the zero mode of the \( B \) scalar are

\[ \mathcal{L}_{\text{bulk}} = -\sqrt{2}g_{5D}(\phi_B^* \lambda \psi_B - \phi_B \lambda^c \psi_B^c) \]
\[ \geq -\frac{\sqrt{2}g}{\sqrt{1 + Z}} (\phi_B^{(0)*} \lambda \psi_B^{(0)} - \phi_B^{(0)} \lambda^c \psi_B^{(0)c}), \]  

where the relation \( g = g_{5D}/\sqrt{\kappa R} \) was used in the second line.

Then, the one-loop contribution to the squared soft mass of \( \phi_B^{(0)} \) from bulk fermion loops is then given by

\[ -\frac{2ig^2 C_2}{1 + Z} \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 y}{(2\pi)^4} \int \frac{d^4 y'}{(2\pi)^4} \]
\[ \left( -\text{tr}[p \cdot \sigma \cdot p' \cdot \bar{\sigma}] \left[ G_{++}(y, y', p; Z) G_{++}(y, y', p; 0) + G_{--}(y, y', p; Z) G_{--}(y, y', p; 0) \right] \right. \]
\[ + \text{tr}[1] \left[ \partial_y G_{++}(y, y', p; Z) \partial_y G_{++}(y, y', p; Z) + \partial_y G_{--}(y, y', p; Z) \partial_y G_{--}(y, y', p; Z) \right] \]
\[ = \frac{4g^2 C_2}{1 + Z} \int \frac{d^4 p_E}{(2\pi)^4} \int \frac{d^4 y}{(2\pi)^4} \int \frac{d^4 y'}{(2\pi)^4} \]
\[ \left( p_E^2 \left[ G_{++}(y, y', p; Z) G_{++}(y, y', p; 0) + G_{--}(y, y', p; Z) G_{--}(y, y', p; 0) \right] \right. \]
\[ + \partial_y G_{++}(y, y', p; Z) \partial_y G_{++}(y, y', p; Z) + \partial_y G_{--}(y, y', p; Z) \partial_y G_{--}(y, y', p; Z) \right) , \]  

\[ \text{(D.1)} \]
where $p_E \equiv (-p^2 - i\varepsilon)^{1/2}$. The first term is from the loop of $\psi$ and $\lambda$, while the second from $\bar{\psi}^c$ and $\bar{\lambda}^c$. These do not contain chirality flips, and therefore the propagators (C.9) have been used. The third and forth terms come from the diagrams containing chirality flips, thus the propagators (C.10) have been used.

The bosonic contribution can be calculated by the following trick. Imagine changing the boundary conditions for the fermions such that supersymmetry is preserved. In this situation, we know that the bosonic and fermionic contributions cancel with each other. Since we did not change the boundary conditions for the bosons when switching from non-supersymmetric to supersymmetric case, the bosonic contribution stays the same. Thus, the bosonic contribution in the case of our interest is just the negative of the fermionic contribution in the supersymmetric case, i.e.,

$$-(D.2) \text{ with } ^{\pm \pm \pm \pm} + \text{ and } ^{\pm \pm \pm \pm} - \text{.}$$ (D.3)

Adding (D.2) and (D.3) gives the squared soft mass from the bulk gauge interactions.

We also have contributions from the boundary-localized gauge interactions

$$\mathcal{L}_{\text{boundary}} = -Z\pi R \sqrt{2} g_{5D} \phi^+_B \lambda \psi_B |_{y=0}
\quad \supset -\frac{\sqrt{2} g Z\pi R}{\sqrt{1 + Z}} \phi^{(0)\dagger}_B \lambda \psi_B |_{y=0}. \quad \text{ (D.4)}$$

Using the above trick to obtain the bosonic part, this gives the following contribution to the squared soft mass:

$$\frac{4g^2(Z\pi R)^2 C_2}{1 + Z} \int \frac{d^4 p_E}{(2\pi)^4} p^2_E \left[ G_{+-}(0,0,p;Z) G_{+-}(0,0,p;0) - G_{++}(0,0,p;Z) G_{++}(0,0,p;0) \right], \quad \text{(D.5)}$$

Adding up (D.2), (D.3) and (D.5), we obtain the formula (2.12).

### D.2 Yukawa Contributions

The relevant Yukawa couplings are

$$\mathcal{L}_{\text{Yukawa}} = -y_{5D} \tilde{h}_u (q_{3B} \bar{u}_{3B} + \bar{q}_{3B} u_{3B})
\quad \supset -y_t \sqrt{\pi R(1 + Z_{Q_3})} \tilde{h}_u q_{3B} \bar{u}^{(0)}_{3B} - y_t \sqrt{\pi R(1 + Z_{U_3})} \bar{h}_u q^{(0)}_{3B} u_{3B} \quad \text{(D.6)}$$

where

$$y_{5D} = y_t \sqrt{\pi R(1 + Z_{Q_3})} \sqrt{\pi R(1 + Z_{U_3})}. \quad \text{(D.7)}$$

Then, again using the above trick to obtain the bosonic contribution, we get

$$\delta m^2_{\tilde{q}_3} = y_t^2 \pi R(1 + Z_{U_3}) \int \frac{d^4 p}{(2\pi)^4} \left( -\frac{\text{tr}[p \cdot \sigma p \cdot \bar{\sigma}]}{p^2 - \mu^2} \left[ G_{+-}(0,0,p;Z_{U_3}) - G_{++}(0,0,p;Z_{U_3}) \right] \right)
\simeq 2y_t^2 \pi R(1 + Z_{U_3}) \int \frac{d^4 p_E}{(2\pi)^4} \left[ -iG_{+-}(0,0,p;Z_{U_3}) + iG_{++}(0,0,p;Z_{U_3}) \right], \quad \text{(D.8)}$$
where $\mu$ was neglected in the second step. This is the first equation of (2.13). Similarly, we have

$$\delta m_{\alpha_3}^2 \simeq 4g_i^2\pi R(1 + Z_{Q_3})\int d^4p_E\left[-iG_+(0,0,0; Z_{Q_3}) + iG_{++}(0,0,0; Z_{Q_3})\right],$$

which is the second equation of (2.13).

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