A condition-based maintenance policy for a system deteriorating with age and usage

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Abstract. The rapid development of sensor and maintenance technologies support the implementation of a Condition-Based Maintenance (CBM) policy. The CBM policy is an effective way for reducing or even eliminating failure of a system and hence it will minimize the maintenance cost and increase the system’s availability and productivity. In this study, we propose a CBM policy with periodic inspection to control the system’s deterioration level. The deterioration is modelled by using a Gamma process with its shape parameter is assumed to be affected by the system’s age and usage. The maintenance action is imperfect in the sense that it restores the level of the deterioration to a fixed level that is not as-good-as-new condition. We use a simulation approach to simulate the deterioration process and the impact of maintenance action to the deterioration level, and to find an optimal inspection interval for a given preventive threshold, which minimizes the total maintenance cost.

1. Introduction

A Condition-Based Maintenance (CBM) is defined as a maintenance program that recommends maintenance decisions based on information collected through condition monitoring system [1]. The condition of a system is monitored through its operating condition which can be measured by various monitoring devices [2]. Sensor and maintenance technologies help to monitor the condition of a system and present data or information representing the condition. In the last decade, the technologies have developed very rapidly, and these in turn provide a better support for the CBM implementation, specifically for a system which deteriorates over time [3]. The maintenance action and maintenance schedule required are decided based on these monitored data [4]. The implementation of the CBM policy will minimize or even eliminate system’s failure so that reduce the maintenance costs [3] and increase the system’s availability and productivity. Moreover, as stated in [5], CBM has proved to improve operational safety and reduce the number of in-service system failures [6].

Many researchers have studied the CBM policy for deteriorating systems. Van and Berenguer [7] deal with a CBM model for a production deteriorating system considering both maintenance cost and production capacity aspects. Van and Berenguer [8] extended their previous model by considering a periodic inspection and imperfect maintenance, and the number of repairs allowed for the system. Zhang et al. [9] developed a CBM policy for a leased system by considering the lessees’ satisfaction. The model aims to determine the optimal inspection interval and preventive maintenance threshold to maximize the lessor’s profit. All these papers consider that the rate of the system’s deterioration is constant over time.
In many cases, the system’s deterioration is greatly affected not only by the age but also the usage of the system. As the system is operated with a high usage, then the rate of the deterioration will be accelerated. Hence the system’s failure occurs more frequent and this will increase the downtime of the system or decrease the system’s availability.

In this study, we propose a CBM policy for single-unit system that deteriorates stochastically and gradually. The system’s deterioration is modelled by a Gamma process with its deterioration is assumed to be affected by the age and usage of the system. The system is inspected periodically to control the level of the deterioration. Imperfect Preventive Maintenance (PM) and Corrective Maintenance (CM) actions are done to restore the level of system deterioration to a fixed condition which is not as-good-as-new condition. The imperfect maintenance is a realistic action to be implemented in the real world [10]. As the realization of deterioration process and the impact of maintenance action restoring the system considered in this study is relatively complex, then a simulation approach is used to study the proposed CBM policy. The purpose of this study is to find a best solution of the inspection interval for a given preventive threshold, that minimizes the total maintenance cost.

This paper is organized as follows. The second section describes the system description, includes the assumption and deterioration modelling. The third section presents the CBM model and policy. The fourth section deals with the optimization of maintenance policy and simulation approach. The fifth section presents a simple numerical example with simulation approach. The best solution will be discussed in this section. Finally, the last section gives the conclusion and future research.

2. System description
This study uses following notations:

\[ T \] : Inspection interval

\[ X_t \] : Deterioration level at time \( t \)

\[ X_t' \] : Deterioration level after maintenance action at time \( t \)

\[ D_0 \] : Fixed state of system deterioration after maintenance action

\[ D_p \] : Preventive threshold

\[ D_f \] : Failure threshold

\[ \zeta \] : Time spent on CM after failure occurred that follows Exponential distribution

\[ \pi \] : CM time threshold after failure occurred

\[ \tau \] : Excess time that occurred if \( \zeta \) exceeds \( \pi \)

\[ C_i \] : Cost per inspection

\[ C_p \] : Cost per PM action

\[ C_c \] : Cost per CM action

\[ C_\zeta \] : Penalty cost per unit time

\( n_i \) : Number of inspections

\( P \) : Number of PM actions

\( \Lambda \) : Number of CM actions

\( E(ET) \) : Expected of excess time

\( C_\alpha \) : Total maintenance cost

\( TI \) : Total inspection cost

\( TPM \) : Total PM cost

\( TCM \) : Total CM cost

\( TP \) : Total penalty cost

\( \alpha_g(t) \) : Shape parameter of Gamma distribution

\( \beta_g \) : Scale parameter of Gamma distribution

\[ f(x;\alpha_g(t),\beta_g) \] : Probability Density Function (PDF) followed by increments in the deterioration state per unit time

\( L \) : Maintenance period
The system is inspected periodically;
- The maintenance actions (both PM and CM actions) are imperfect maintenance;
- The system initial state $X_0$ is 0, meaning that the system’s condition is new;
- The system fails if $X_t$ exceeds $D_t$. The $D_t$ level is a failure level that must not be exceeded for economical and safety reasons.

### 2.1. Deterioration modelling

We consider a machine in a manufacturing system, or a heavy equipment in a mining operation system, that plays an essential role in the business process of a company. This system will deteriorate with the age and usage, and ultimately it fails. In other words, failure of such a system follows a gradual process (does not occur instantaneously). It is assumed that the stochastic and gradual deterioration process follows a Gamma process which is the most appropriate to be used as in Van Noortwijk [11]. The increment of the deterioration between two consecutive times $t$, $t-1$ is denoted as $\Delta X_t$. It is a non-negative and independent random variable that follows a Gamma probability density function (PDF) with shape parameter $\alpha_g(t-(t-1))$ and scale parameter $\beta_g$. The Gamma PDF is given by:

$$f(\Delta X_t; \alpha_g(t-(t-1)), \beta_g) = \frac{\beta_g^{\alpha_g(t-(t-1))}}{\Gamma(\alpha_g(t-(t-1)))} \Delta X_t^{\alpha_g(t-(t-1))-1} e^{-\beta_g \Delta X_t}$$

where $\alpha$ is a positive real number.

The shape parameter, which represents the deterioration rate of system, is considered as a function of the system’s age and usage. We propose an Accelerated Failure Time (AFT) formulation to model the shape parameter as a function of the system’s age and usage. With AFT formulation, we can model the system used with a low, medium, and high usage rates, and under a severity degree of the operating condition [12]. The system operating condition, $\gamma$ is equals to 1 ($\gamma=1$) if the system is operated under a normal severity condition. Note that $\gamma < 1$ and $\gamma > 1$ represent a low severity and a high severity condition, respectively. Hence, $\alpha_g(t)$ is given by:

$$\alpha_g(t) = \left(\frac{r}{r_0}\right)^\gamma \alpha_g(t)$$

As a result, the Gamma PDF is given by:

$$f(\Delta X_t; \alpha_g(t-(t-1)), \beta_g) = \frac{\beta_g^{\alpha_g(t-(t-1))}}{\Gamma(\alpha_g(t-(t-1)))} \Delta X_t^{\alpha_g(t-(t-1))-1} e^{-\beta_g \Delta X_t}$$

### 3. Maintenance policy

A system is inspected periodically at interval inspection $kT$, $k=1, 2, \ldots \frac{L}{T}$. Inspection is performed only when system is in the operating condition, if the system is in the failed (shutdown) condition, then the inspection does not need to be performed. The maintenance decisions (i.e., when to do inspection, to perform PM or CM action) are based on the deterioration level. In CBM policy, PM or CM action will be performed according to deterioration level of the system as follows:
• If $X_t < D_p$, no maintenance action is required. The system is on operating condition and the current deterioration level of system remains the same at beginning the next period;

• If $D_p \leq X_t < D_f$, imperfect PM action is performed. Imperfect PM action restores the level of system deterioration ($X_t'$) to a fixed condition $D_0$ that is not as-good-as-new condition. We assume that the system’s reliability $R(t)$ equals 95% after each imperfect maintenance action. By considering the value of $R(t)$, we obtain the value of $D_0$. The time spent to do a PM action is relatively short compared to the inspection period, and therefore, it is ignored.

• If $X_t \geq D_f$, the system fails, and the imperfect CM action must be performed to restore the level of the deterioration ($X_t'$) to a fixed condition $D_0$. The time needed to carry out a CM action ($\zeta$) is assumed to follow Exponential distribution. If $\zeta$ exceeds the threshold value ($\pi$), a production loss (or penalty cost) is incurred and the cost is a function of the excess time ($\tau$) where $\tau = \zeta - \pi$.

Figure 1 shows an illustration of the system’s deterioration evolution and the maintenance policy considered.

![Illustration of the deterioration evolution and maintenance policy](image)

**Figure 1.** Illustration of the deterioration evolution and maintenance policy

4. **Optimization of maintenance policy**

The decision variables of the optimization problem for a CBM studied are the inspection interval ($T$) and the preventive threshold ($D_p$). The optimal values of $D_p$ and $T$ are the solution of the following optimization problem with the objective function being the minimization of the total maintenance cost ($C_a$).

\[
\text{minimize} \quad C_a = TI + TPM + TCM + TP \quad (4)
\]

\[
\text{subject to} \quad T = 1, 2, 3, \ldots, L
\]

\[
0 < D_p < D_f
\]

The components of the total maintenance cost are given by

\[
TI = C_i n_i \quad (5)
\]

\[
TPM = C_p P \quad (6)
\]

\[
TCM = C_r \Lambda \quad (7)
\]

\[
TP = E(ET) * C_\tau \Lambda \quad (8)
\]

\[
E(ET) = \int_0^\infty 1 - F(t) \, dt \quad (9)
\]
\[ F(t) = 1 - e^{2t} \]  

(10)

As the maintenance decisions under the CBM policy studied (described in Section 3) are relatively complex, then a simulation approach is required to find the best solution (instead of the optimal solution) of the inspection interval for a given preventive threshold, which gives the lowest total maintenance cost. In the simulation approach described in the following subsection, TP will be obtained numerically using equations (8), (9), and (10). TP is dependent on the time spent for conducting a CM \((\zeta)\) after a failure occurred, and it is assumed to follow the Exponential distribution with parameter \(\lambda = \frac{1}{\pi}\).

4.1. Simulation approach

A simulation approach will give a better realization of deterioration over time, and the decision of when to do PM and CM actions. Also, it allows to represent precisely the time instance when failure occurs and the improvement of the deterioration level after each PM action or CM action. The simulation procedure involves six steps as follows:

**Step 1:** Initiation - set all parameters used including some values of \(D_p\) and \(T\) that will be evaluated;

**Step 2:** Generate \(\Delta X_t\) based on Gamma density function given in (3) for \(t = 1, 2, \ldots, L\) with shape parameter \(\alpha_g(t)\) and scale parameter \(\beta_g\) that represents the deterioration rate per each time \(t\);

**Step 3:** Update the deterioration level \(X_t\) which is the sum of deterioration level \(X_{t-1}\) and deterioration increment \(\Delta X_t\);

**Step 4:** For \(t = 1, 2, \ldots, L\), if failure occurs, \(X_t \geq D_f\), do CM action and reset \(X_t' = D_o\), then proceed Step 3 with \(t = t + 1\), if not go to **Step 5**;

**Step 5:** For \(t = kT\) with \(k = 1, 2, \ldots \frac{L}{T}\), check the level \(X_t\) for following PM scenarios:

- if \(X_t < D_p\), proceed Step 3 (the simulation is continued with \(t = t + 1\));
- if \(D_p \leq X_t < D_f\), do PM action and reset \(X_t' = D_o\), then update \(t = t + 1\) and proceed Step 3.

**Step 6:** Simulation is terminated at time \(t = L\); Calculate TI, TPM, TCM, TP, and \(C_u\);

We repeat steps 1-6 for five times (or the number of replications equals 5).

5. Numerical example

The parameter values considered to simulate the system’s deterioration are as follows (note that some parameter values taken from [3] and the rest are set according to the assumptions in this study).

\[ \alpha_g = 1.4, \beta_g = 0.3, L = 1, 2, \ldots, 50, r = 300, r_0 = 333.33, \gamma = 1, R(t) = 95\%, D_f = 15, \]

\[ \pi = 1.2, \lambda = 1.2, C_i = 60, \quad C_p = 300, \quad C_r = 800, \quad C_t = 100. \]

The simulation is repeated five times for each combination of \(D_p\) and \(T\), and we calculate its Coefficient of Variation (CV) to evaluate data variability. To find the best value for the PM threshold, we consider \(D_p = 6, 8, 10, 12\), and evaluate the average of the total maintenance cost for each inspection interval in order to obtain the best value. Inspection interval that gives the lowest average of the total maintenance cost will be selected as the best solution.

5.1. Simulation result

The total maintenance cost obtained from the simulation is given in the form of an average value – i.e., the average of the values obtained from five replications.
Figure 2. Simulation result for $D_p = 6$

Figure 3. Simulation result for $D_p = 8$
Figure 4. Simulation result for $D_p = 10$

Figure 5. Simulation result for $D_p = 12$
Figures above show the best values of $T$ for each given $D_p$, which gives the lowest average TMC. The best value of $T$ and its corresponding the average total maintenance cost for each $D_p$ value are shown in Table 1. The best value of $T$ increases as $D_p$ increases. This is as expected since the larger $D_p$ will cause the inspection period ($T$) to increase. As a result, we have $D_p = 8$ and $T = 23$ (as the best values) which result in the lowest average total maintenance cost (i.e. average TMC = 12192.8).

| Preventive Threshold ($D_p$) | Inspection Interval ($T$) | Average Total Maintenance Cost (TMC) |
|-----------------------------|---------------------------|------------------------------------|
| 6                           | 19                        | 12494.4                            |
| 8                           | 23                        | 12192.8                            |
| 10                          | 26                        | 12217.6                            |
| 12                          | 24                        | 122495.2                           |
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Acknowledgements
This work is funded by the Ministry of Education and Culture of the Republic of Indonesia through the scheme of “PUDPT 2020” with contract number SP DIPA-2/E1/KP.PTNBH/2020.