BIRKHOFF’S THEOREM AND THE VOID MATTER DENSITY

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ABSTRACT

According to the discussion on Birkhoff’s theorem by Peebles (1993), a void with a negative perturbation $\delta$ may evolve as a separate homogeneous universe with a local expansion parameter $H_V \simeq H_0(1 - \delta/3)$. This slightly low density “universe” will diverge from the mean, producing a void of ever lower density. As a result, the contents of voids will “fall” outward at a velocity equal to the difference between the local, and the mean expansion parameters times the radius of the void. Observational constraints on the outfall velocity can be placed as a result of the fortuitous event that void, and non-void hydrogen Ly$\alpha$ absorbers have distinct characteristics – both in their equivalent width distributions and their Doppler parameter distributions – which are clearly distinguished when cloud environments are measured in terms of the sum of the tidal fields of surrounding galaxies acting on the cloud (Manning 2002). These constraints dictate that the radial outfall velocity from an average void of radius $\sim 15 h^{-1}_{75}$ Mpc must be less than $\sim 100$ km s$^{-1}$. The implications of this are probed with a “negative” top-hat simulation, using a 1-D Lagrangian code, to determine the relationship between the mass deficit in voids and the local expansion parameter in a flat lambda cosmology. The outfall velocity constraint shows that the void matter density must be greater than 75% of the mean matter density. I argue that this implies that $\Omega_m \gtrsim 0.86$. Thus the total amount of matter in voids is of order twice the total mass in the filamentary structures.

Subject headings: intergalactic medium — quasars:absorption lines – dark matter

1. INTRODUCTION

Voids are structures noted for their pronounced lack of galaxies; concentrations of matter are thought to take the form of a luminous “filamentary” structure, so that together they resemble a “foam” of roughly spherical voids, edged by filaments and walls, and sprinkled with richer concentrations of galaxies where the filaments intersect. While galaxies provide an important tracer of mass in the universe, it is possible that there may be significant amounts of undetected matter in the form of highly ionized clouds, or perhaps a significant background of dark and gaseous matter, either within the filamentary structure, or distributed within the voids. Observations of the low-redshift Ly$\alpha$ cloud population have shown that clouds cluster about galaxies (e.g., Tripp et al., 1998; Chen et al., 2001), but there are also many clouds which appear to be isolated.

In Manning (2002) (hereafter Paper 1), a primary cloud catalog (Penton et al., 2000) was separated into complementary void and non-void catalogs through the use of a parameter which assesses cloud environments by weighting the effects of galaxy mass and distance. This parameter, the scalar tidal field, which was summed over galaxies within $7.5 h^{-1}_{75}$ Mpc of a cloud, is optimal for this purpose, as it may easily be calculated from galaxy catalogs. In addition, there is a physical basis for supposing tides affect cloud distributions, since it is the tide which determines the lower limit of cloud density which is stable to dynamical disruption. In Paper 1, a dimensionless form of the tidal field $T$ was evaluated at the locations of each of a catalog of low-redshift H I absorbers (Penton et al., 2000), and used to characterize the clouds as members of a void, or non-void environment, according to whether that field was greater or less than some limiting tide $T_{lim}$. Equivalent width distribution functions (EWDF) are calculated for each sub-catalog (see Eq. 44, Paper 1). These EWDFs are well-approximated as linear in $\log dN/dz$, vs. $\log W$, where $N = n(n_NHI)$, and $W$ is the rest equivalent width (EW) in mA. One may vary $T_{lim}$, calculate the EWDFs of each sub-catalog, and derive the weighted linear fits thereof, based on the number of clouds contributing to the calculation of the EWDF at a given EW. Distributions are fitted to the equation, $\log dN/dz = C + S \log (W/W_0)$ mA. It was found that the trends of fitting parameters for void and non-void catalogs as a function of $T_{lim}$ appear linear at large, or small $T_{lim}$ (see Fig. 1), but undergo a non-linear change within a relatively small range of $T$. As it happens, there is a corresponding change in the distributions of Doppler parameters at a similar range of $T$ for clouds sorted by $T$ (see the differential histogram, Fig. 8 in Paper 1). It appears certain that two distinct populations have been isolated, and that a transition zone lies within the range, $-1.3 \lesssim \log T \lesssim -0.7$.

Apart from the interest in the detailed nature of these clouds, and what they say about the nature of voids (paper in preparation), the very fact that they can be separated says something about the density of voids. For, by Birkhoff’s theorem (Birkhoff, 1923), if voids have a low density, they should behave like a low-density universe, and suffer less deceleration, and have larger local expansion parameters, than the mean. With simulations I calculate the void expansion parameter as a function of the void matter density relative to the average. If the smallness of the range of $T$ over which the transition of void to non-void clouds occurs can be used to constrain the outfall velocity, then this can be used to constrain the the void matter density. This is, in short, my plan.

The cosmology assumed for this paper is a standard flat lambda model with $h = 0.75$. The total matter density of
voids is either referred to as $\Omega_v$ in the intercepts.

There is an apparent strong transition in the slopes in the range $1 < T < 5$ (dotted vertical lines) is plotted against $\log(T_{lim})$. This fraction can be considered to be a fair approximation of the volume filling factor for voids defined by $\mathcal{T} \leq T_{lim}$. Dotted lines refer to the region of transition similarly highlighted in Fig. 1.

voids is either referred to as $\Omega_v = \bar{\rho}_v / \rho_{crit}$, or as $f_0 = \Omega_v / \Omega_m$, where $\Omega_m = 0.3$ is initially assumed. As noted in Paper 1, the large discovered line density of void clouds appears to require that clouds are to a significant degree self-gravitating and discrete. I treat them as such herein.

2. CONSTRAINTS ON CLOUD OUTFALL VELOCITY

Peculiar velocities in galaxy clusters cause a phenomenon known as the “finger of God”. A similar error in attributed position may result when void clouds have large outfall velocities. Large outfall velocities will smear the distinction between void and non-void clouds, as attributed positions are based on the redshift and the mean Hubble constant (see §1). Figure 1 shows the trend of linear fitting parameters for EWDFs defined by various $T_{lim}$. It appears to show a fairly small amount of smearing, suggesting that the transition zone (dotted vertical lines) is $-1.3 \leq \log(T) \leq -0.7$ (two dotted vertical lines), which is also seen in the intercepts.

Fig. 1.— The trend in slopes (panel a) and intercepts (panel b) of equivalent width distribution functions defined by catalogues with tidal field lower (top of each panel), or upper limits (bottom), respectively, for the low-redshift cloud sample (see §6.2.1 Paper 1). There is an apparent strong transition in the slopes in the range $-1.3 \leq \log(T) \leq -0.7$ (two dotted vertical lines), which is also seen in the intercepts.

Fig. 2.— The fraction of redshift space with $\mathcal{T} \leq T_{lim}$, plotted against $\log(T_{lim})$. This fraction can be considered to be a fair approximation of the volume filling factor for voids defined by $\mathcal{T} \leq T_{lim}$. Dotted lines refer to the region of transition similarly highlighted in Fig. 1.

0.1, has a radius of 15 Mpc ($\sim 11.25 h^{-1}$ Mpc) (e.g., Lindner et al., 1995). The range of uncertainty in the position of the transition zone corresponds to radii of $r_{TV} = 15 (f_v / 0.86)^{1/3} = 14.33$, and 15.396 Mpc, for void filling factors $f_v = 0.75$ and 0.91, respectively; an average variation of $\pm 530$ kpc. This uncertainty in the true fiducial radius corresponds to a line of sight (LOS) velocity error of $H_0 \Delta r \simeq 40 {\text{ km s}}^{-1}$, and would roughly reproduce the same range of uncertainty apparent in the transition zone. Admittedly, many clouds will be moving along vectors at large angles to the LOS, and so even with high outfall velocities, they could have low LOS peculiar velocities. But if clouds are randomly distributed in the void relative to our LOS, and radially outfalling, an average of half of the clouds would have to be moving at an angle less than 60 degrees from the LOS, so that the observed LOS outfall velocity at the edge would be greater than half of the radial outfall velocity for half of these clouds. However, if clouds and mass are uniformly distributed in voids, then half of the clouds will be at a radius $r \geq 0.794 (V_{out})$, so that most clouds would have an outfall velocity $v_{out} \geq 80\%$ of the maximum. Therefore, we expect that half of the clouds will have a peculiar outfall velocity along the LOS $\geq 0.4$ times the radial outfall velocity at the void edge. Since the observed range of the LOS positional error corresponds to a velocity error of $\sim 40 {\text{ km s}}^{-1}$, this would seem to limit the true radial outfall velocity at the void edge to be $v_{out} \leq 40/0.4 = 100 {\text{ km s}}^{-1}$. However, since we know that some of the error in the determination of the transition is plausibly due to peculiar velocities of non-void clouds in proximity to galaxies (see §5.2, Paper 1), this is probably an over-estimation of the upper limit on $v_{out}$.

3. THE OUTFALL VELOCITY FROM VOIDS

Voids are thought to grow more rapidly than the scale factor (e.g., Bertschinger, 1985; Piran, 1997). This can be conceptually understood by comparing the evolution of the expansion parameter for a flat (the overall universe), and an open universe (the void). For the former,

$$H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + \Omega_\Lambda},$$  

(1)
and the latter, \cite{Scott:2000}.

\[ H_V(z) = H_V(0) \sqrt{(f_0 \Omega_m z + 1 - \Omega_L)(1+z)^2 + \Omega_L}, \]

where \( H_V \) reflects a Hubble constant in the void, and \( f_0 \) is defined in §1. One might think that as \( z \) approaches infinity, their respective expansion parameters would approach the same value so that the ratio of their current values would simply be,

\[ \frac{R_H}{H_0} = \left( \frac{\Omega_m(1+z)^3 + \Omega_L}{(f_0 \Omega_m z + 1 - \Omega_L)(1+z)^2 + \Omega_L} \right)^{1/2}, \]

so that a “peculiar” outfall velocity could be determined by subtracting off the mean Hubble flow,

\[ v_{out} = (R_H - 1) H_0 V, \]

where \( V \) is the radius of the void. The dashed line in Fig. 3 shows the functional form of Eq. 4 as a function of \( f_0 \).

However, while the theorem says that the lower-density void acts like a separate “universe”, our reconing of it, in terms of the correspondence between lookback time and redshift, is not straightforward, for the lookback time to a given redshift is a function of cosmology, and a flat universe is younger than an open universe with the same \( H_0 \): we cannot so easily derive our answer.

However, we may confidently approach this subject by simulating the void as a “negative” top-hat with perturbation \( \delta \). A 1-dimensional Lagrangian hydro/gravity code was acquired courtesy of one of its authors, (A. A. T.), and adjusted for a flat, \( \Lambda \) cosmology (see §1). A more detailed description of the code and some of its broader uses will be presented in a subsequent paper (Manning, in preparation). Linearized analysis of voids dictate that the appropriate initial velocity perturbation should be \( v = (1 - \delta/3) H_V(z) r \) \cite{Bertschinger:1985, Peebles:1993}, where \( H_V \) is the local, void expansion parameter. Simulations of the dark matter component were performed using this velocity perturbation, with a range of uniform underdensities \( \delta \) embedded in the otherwise flat universe, beginning at \( z = 50 \). A \( \delta = 0 \) case was run to normalize \( H_0 \). The expansion parameters at \( z = 0 \) were derived. The outfall velocity was calculated by subtracting \( H_0 \) from \( H_V \), and multiplying the result by the 15 Mpc of our fiducial void. Figure 3 shows the results (solid line), where \( v_{out} \) is the outfall velocity. For void densities \( f_0 \lesssim 0.2 \), outfall velocities are \( v_{out} \gtrsim 370 \text{ km s}^{-1} \). However, this analysis \cite{Manning:2001} finds that \( v_{out} \lesssim 100 \text{ km s}^{-1} \), and thus, \( f_0 \gtrsim 0.75 \). We may apparently conclude that the Hubble flow in voids is very close to the average Hubble flow, and that therefore the difference in density between voids and the mean is minimal.

4. IMPLICATIONS FOR \( \Omega_m \)

Using the mean filling factors \( f_V = 0.86 \) for voids and \( f_F = 0.14 \) for filaments, we may analyze the distribution of matter implied by the above. The basic equation for this analysis is,

\[ \Omega_m = f_V f_0 \Omega_m + f_F \Omega_F, \]

where \( f_0 \Omega_m \) is substituted for \( \Omega_V \). Solving for \( \Omega_F \), when \( \Omega_m = 0.3 \) and \( f_0 \geq 0.75 \), we find,

\[ \Omega_F \lesssim \frac{\Omega_m(1 - f_V f_0)}{f_F} \approx 0.761, \]

which is absurd, since gravitational clustering could not occur unless \( \Omega_F > 1.0 \). It is apparent that \( \Omega_F \) must be much greater. Introducing commonly accepted values into Eq. 6, for instance, \( \Omega_m = 0.35 \) and \( f_0 = 0.18 \), the mean of the range \( 0.1 \lesssim f_0 \lesssim 0.26 \) \cite{Cen:1997, Il-Ad:1997}, we find \( \Omega_F \approx 2.2 \). If we assume this more reasonable value of \( \Omega_F \), then it turns out that using the above constraint on \( f_0 \), then

\[ \Omega_m = \frac{f_F \Omega_F}{1 - f_V f_0} \gtrsim 0.86. \]

That is, a low outfall velocity from voids mandates a “high” \( \Omega_m \). But is the outfall velocity for large \( \Omega_m \) the same function of \( f_0 \)? When the 1-D code was run for \( \Omega_m = 0.85 \), a graph consistent with that for \( \Omega_m = 0.3 \) resulted (dashed line, open points, Fig. 3), so the same constraints maintain in a high \( \Omega_m \) universe as with \( \Omega_m = 0.3 \).

5. DISCUSSION AND SUMMARY

5.1. Impact on Filaments

First, let us consider the effects of the deposition of energy in voids in the outfall of void clouds onto filaments. If clouds are diffuse, and the kinetic energy of outfall is transformed entirely into random motions, then \( T = \mu m_H V_{out}^2 / 3k \), where \( k \) is the Boltzmann constant, and \( \mu \approx 0.59 \) is the mass of an average particle. A large velocity \( V_{out} = 400 \text{ km s}^{-1} \) would produce a plasma of temperature nearly \( 4 \times 10^8 \text{ K} \), similar to that predicted by \cite{Cen:1999} for filaments, suggesting that absorbers should have Doppler parameters \( b \gtrsim 250 \text{ km s}^{-1} \),
much larger than observed $b$-values of Ly$\alpha$ lines of non-void clouds ($b \approx 60$ km s$^{-1}$, Paper I). For $v_{out} = 100$ km s$^{-1}$; diffuse clouds are converted to a temperature $T = 2.4 \times 10^{5}$ K; $b \approx 62$ km s$^{-1}$. Interestingly, Far Ultraviolet Spectroscopic Explorer observations (Shull et al., 2001) find actual $b$-values may be roughly half that of the observed Ly$\alpha$ line, the excess being attributed to bulk motions. Doppler parameters of order 30 km s$^{-1}$ may be explained if clouds are not diffuse, but held by dark matter halos, and hence centrally condensed. In this case, only gas with a density of order that of the filament will be stripped, leaving the central cloud to proceed relatively unmolested (Murakami & Ikeuchi, 1994), minimizing the thermal broadening of the cloud. Shear-induced vorticity (Manning, 1999), will contribute to the Doppler broadening of the cloud, and ram pressure will heat it, so that these absorbers should have greater Doppler parameters than void clouds, as observed (Paper I, §5.6). According to the scenario in Manning (1999), the induced vorticity of clouds, which are falling into dissipated gaseous envelopes of galaxies and groups of galaxies, introduces an orderly cycle of compression and cooling which results in elevated central H I equivalent widths as the cloud approaches mass concentrations. This inverse correlation between cloud rest equivalent widths and projected galactocentric radii has been observed (Lanzetta et al., 1995; Tripp et al., 1998; Chen et al., 1999; Chen et al., 2001).

5.2. Implications for Cosmology

The conclusion that voids have a high matter density depends heavily on the successful separation of void, from non-void clouds. That the Doppler parameter differential histograms, and trends in EWDF fitting parameters both implicate the same transition zone suggests that this is a robust result. Outside the transition zone $-1.3 \lesssim \log T \lesssim -0.7$, the trends seen in Fig. 1 suggest distinct cloud characteristics with few interlopers. Within this range, however, the trends change in a manner consistent with the effects of peculiar velocities of non-void clouds about galaxies, compounded with that of the outfall of clouds from voids with $v_{out} \lesssim 100$ km s$^{-1}$ ($H_{V} \lesssim 1.1 \ H_{0}$). The simulations have shown that for $v_{out} \lesssim 100$ km s$^{-1}$, the ratio of the void density to the mean is $f_{0} \gtrsim 0.75$, requiring $\Omega_{m}$ to be of order 1. Under the strictures of the flat Lambda model, this requires $\Omega_{\Lambda} \approx 0$, and $\Lambda \approx 0$.

The current standard model postulates two forms of positive energy which, under the analysis of the multipole fluctuations in the CMB, appear to imply that $\Omega_{m} + \Omega_{\Lambda} \approx 1$ (e.g., de Bernardis et al., 2000). Independent assessments of $\Omega_{m}$ are all based on concentrations of matter, usually involving dynamical tests, or biasing estimates (e.g., Fukugita et al., 1995; Verde, 2001), and result in estimates $\Omega_{m} \approx 0.3$. However, since these methods are insensitive to a possible background, or additional quantities of matter in void regions, these estimates are necessarily under-estimates. As we have seen, constraints on the velocity of outfall from voids imply a rather large “background”.

On the other hand, supernova Ia studies strongly imply an exponential expansion of the universe (e.g., Riess et al., 1998; Filippenko, 2001). How is this to be reconciled? What has not been shown by observation or theoretical derivations is that what is created in conjunction with the exponential expansion, and what enables the universe to be flat, is a “dark” energy. The conclusion that $\Omega_{m} \gtrsim 0.85$ and $\Lambda \gg 0$ can only mean that a dark energy is not formed, but in its stead, ordinary matter, dark and baryonic.

It is unclear how one might adjust the parameters of the void evolution simulations to be consistent with these conditions, for if matter is forming in voids, the tendency of the mass density to rapidly decline is reduced. A careful numerical modeling of void cloud absorbers as remnants of sub-galactic perturbations (paper in preparation) may shed light on this problem (Manning, in preparation).

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