PHENOMENOLOGICAL RELATIONS FOR QUARK AND NEUTRINO MIXING ANGLES

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The recent experimental data on mixing angles for quarks and neutrino have been discussed. Quarks mixing angles are calculated in the phenomenological approach of Fritzsch-Scadron-Delbourgo-Rupp (FSDR approach) using mass values of light and heavy constituent quarks. The neutrino mixing angles have been calculated with the high degree of precision with the help of the hypothesis of quarks and neutrino mixing angles complementarity, the results obtained do not contradict with the present experimental data.

1. Introduction
An elucidation of a character of mixing of quarks and neutrino states is an unsettled problem of Standard Model (SM). Of fundamental importance is the calculation of mixing angles in agreement with experimental data. There are many works devoted to this problem, in which different approaches have been proposed (e.g., see reviews [1-4]), however the answer not conclusively obtained. It is possible that resolution of this issue will appear after generalization of SM, when some new components will be included in for a description of phenomena beyond SM such as neutrino oscillations. An exhaustive resolution of the problem of mixing of quarks and neutrino should appear in a Great Unification Theory (GUT), but the eventual variant of GUT is not available now. A certain intension about a further development of SM can be obtained in studies of various mutual relations between mixing angles of quarks and neutrino. Noted phenomenological regularities in quark and lepton sectors of SM, particularly the hypothesis of quarks and neutrino mixing angles complementarity, tell us about a possible existence of a common physical cause of mixing for these particles. The facts of such kind deduced from data may be useful in elucidation of a way of a SM generalization and a subsequent GUT development. From this point of view it is interesting on the one hand to perform an experimental verification of phenomenological relations for mixing angles with the best accuracy, and on the other to pursue an investigation of a possible interpretation of the relations in the framework of existing models, along with a dependence of mixing angles from other parameters, for instance, masses of fundamental particles.

Inasmuch as the SM is a quantum gauge chiral theory, so in its initial lagrangian mass terms are absent. For example, fermionic masses arise from Yukawa interactions with Higgs fields after a spontaneous breaking of gauge symmetry. In doing so fermionic states of the initial lagrangian \( f'_\alpha \) and fermionic states with definite masses \( f_i \) can be correlated each other with a nontrivial mixing matrix, which assumed to be an unitary one, as a rule. Under condition that \( u \)--type quarks and charged leptons are states with definite masses, the states of \( d \)--type quarks and neutrino entered into the interaction lagrangian are connected with their mass states by \( V_{CKM} \) and \( U_{P M N S} \) matrices, correspondingly:

\[
d'_{\alpha L} = V_{CKM,\alpha i}d_{iL},
\]

\[
\nu'_{\alpha L} = U_{PMNS,\alpha i}\nu_{iL},
\]

where \( \alpha, i = 1, ..., n \). The mixing matrix in the quark sector \( V_{CKM} \) is named as Cabibbo-Kabayashi-Maskawa matrix [5, 6], whereas the mixing matrix in the neutrino sector \( U_{PMNS} \) is Pontecorvo-Maki-Nakagawa-Sakata matrix [7, 8].

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It is common knowledge that the unitary $n \times n$ matrix is determined with $n^2$ real parameters, for instance, $n(n-1)/2$ angles and $n(n+1)/2$ phases. Take into account the SM electroweak
lagrangian structure with currents constructed from quarks, charged leptons and neutrino, when
fermionic fields are Dirac type it is possible to eliminate $2n-1$ phases. For Majorana type neutrino
fields only $n$ phases connected with Dirac type charged leptons can be eliminated [9]. On this basis
$V_{CKM} n \times n-$ matrix which specify quarks mixing in general case is determined with $n(n-1)/2$
angles and $(n-1)(n-2)/2$ phases. However $U_{PMNS} n \times n-$ matrix is determined with the same
number of parameters if neutrino are Dirac type or with $n(n-1)/2$ angles and $n(n-1)/2$ phases
if neutrino are Majorana type.

Conceptually a manner of its own can be used for parameterization of $V_{CKM}$ or $U_{PMNS}$ matrix.
But in order to reveal an association between mixing mechanisms for quarks and neutrino the
parameterization of choice is the same for $V_{CKM}$ and $U_{PMNS}$. For $V_{CKM}$ matrix, which is $3 \times 3$
matrix in SM, a standard parameterization is the following [10]:

\[
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
\]

(3)

where $c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$. If number of lepton generations equal to three and neutrino
are Dirac type, then one can use the similar parameterization for $U_{PMNS}$. When neutrino are
Majorana type, the dependence on two Majorana phases $\chi_1, \chi_2$ should be taken into account for
$U_{PMNS}$. In this case an appropriate form of $U_{PMNS}$ parameterization is as follows:

\[
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\chi_1} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\chi_2} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\chi_2} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\chi_2} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\chi_2} & c_{23}c_{13}
\end{pmatrix},
\]

where $c_{ij} = \cos \eta_{ij}, s_{ij} = \sin \eta_{ij}$. Others methods of $V_{CKM}$ and $U_{PMNS}$ parameterization are
possible, among them it should be noted the Wolfenstein parameterization for $V_{CKM}$ [11], when
the experimental magnitudes of mixing matrix elements are properly accounted for.

\[
s_{12} = \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)
\]

(4)

This parameterization can be used advantageously in all orders of $\lambda$ even if effects of new physics
beyond SM take place [12, 13].

2. Experimental values of quark mixing parameters

Let us consider quark mixing matrix in parameterization (3), in doing so angles $\theta_{ij}$ specify the
mixing between $i-$ and $j-$generations. A qualitative analysis of experimental data alone [1] point
to a number of interesting facts. For instance, small values of angles $\theta_{13}$ and $\theta_{23}$ tell us that third
generation mix weakly with others and at $\theta_{13} = \theta_{23} = 0$ it decouples, and only mixing between
first two generations remains with $\theta_{12} = \theta_C$, where $\theta_C$ is the Cabbibo angle. Although this is a
rough approximation, nevertheless it frequently used at qualitative description of low energy weak
interaction processes. It is known from experimental data, that $\cos \theta_{13}$ differs from unity only in
the sixth place, thus the following equalities can be taken to a quite good approximation: $V_{ud} = c_{12}$
, $V_{us} = s_{12}$, $V_{cb} = s_{23}$, $V_{tb} = c_{23}$. Note that the phase $\delta$, which associated directly with breaking
CP invariance and always appears with the factor $s_{13}$, lies in the range $[0, 2\pi)$ close to the value
$60^\circ$ (more precisely $63^\circ(+15^\circ/ - 12^\circ)$ [1]).

At present a large amount of data is obtained for a determination of matrix element $V_{ud}$. We
have data for beta- decays of a free neutron and a pion on the one hand, on the other data for
beta-decays of nuclei. The most precise value for $V_{ud}$ is found now in super allowed $0^+ \rightarrow 0^+$ beta-
declays of light and intermediate nuclei. In doing so theoretical results used concerning estimations
of matrix elements on the basis of hypothesis of the vector current conservation with Coulomb and
nuclear structure corrections, together with corrections due to exchanges of virtual $\gamma-$quanta and
Z-bosons between nucleons’ quarks and between nucleons’ quarks and final leptons. In Ref. [14] the weighted value of twelve super allowed $0^+ \to 0^+$ beta-decays of nuclei is given, which correlates well with the weighted value for three processes, namely, the super allowed $0^+ \to 0^+$ beta-decays of nuclei, the beta-decay of a free neutron and beta-decay of a pion. This value is equal to

$$|V_{ud}| = 0.9738 \pm 0.0004$$  

(5)

In the recent review [1] the most precise value is given, which obtained only from data for nine super allowed $0^+ \to 0^+$ beta-decays of nuclei:

$$|V_{ud}| = 0.97377 \pm 0.00027$$  

(6)

In regard to the values $|V_{ud}|$, extracted from data for beta-decays of a free neutron and a pion, they should be more free from uncertainties in comparison with nuclear data, as to they relate to elementary processes. However, things are not so simple. The matter is that even in the principal case for $V - A$-variant of weak lepton-hadron interaction in order to extract unambiguously the data there is a need to carry out two independent experiments as a minimum. For the beta-decay of neutron as usual one pick out the experiments devoted to a determination of life time for a neutron and a measurement of a spin-electron correlation in decay of a polarized neutron. In the series of most precise experiments during 90s the technique of ultra cold neutrons (UCN) had been used (e.g., see [15]). As a result the values for life time of a neutron $\tau_n$ and the $A$, which is a correlation parameter, have been obtained ($\tau_n = 885.7 \pm 0.8c, A = -0.1161 \pm 0.0007$). These values yield [14]

$$|V_{ud}| = 0.9741 \pm 0.0020,$$  

(7)

this is in accordance with data for $0^+ \to 0^+$ nuclei beta-decays. But in the early 2000s in the UCN procedure essentially new phenomena have been found, which connected with super small heating and anomalous interaction of UCN with container walls. These facts led to a critical analysis of previous results and a necessity of new methods for setting up experiments. Performed experiments for the neutron life time gave the following value: $\tau_n = 878.5 \pm 0.8c$ [16], which differs from the result of Ref. [17], contributing the major portion in the world averaged value, more than for magnitude of 5$\sigma$. The new value of the neutron life time leads to the value:

$$|V_{ud}| = 0.9781 \pm 0.0020$$  

(8)

This value is not in a good accordance with the unitarity condition for $V_{CKM}$ in view of recent data for the $V_{us}$, it will be discussed below. Thus the question of the neutron life time is still an open question and further experimental investigations are needed.

Some difficulties arose with a determination of a magnitude of matrix element $V_{us}$, because of the value $|V_{us}| = 0.2200 \pm 0.0026$ used previously, which has been obtained from $K_{e3}$ decays, led to the breaking of the unitarity condition on the confidential level of 2$\sigma$ [14, 15]. This problem was discussed repeatedly and the assumption that effects beyond the SM took place in the neutron decay was one of the possible solutions. However, recent experiments for kaon decays give a new value of $V_{us}$ [14]:

$$|V_{us}| = 0.2254 \pm 0.0021,$$  

(9)

which, as it will be seen below, do not violate the $V_{CKM}$ unitarity without regard for the result of Ref. [16].

Really the $V_{ub}$ value, which extract from data for semileptonic decays $b \to u\ell \bar{\nu}$ and charge conjugated processes, now is equal to [1]:

$$|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$$  

(10)

So if one take into account the results (6), (9), (10) the unitarity condition for $V_{CKM}$ matrix is accurate to 1$\sigma$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9991(11)$$  

(11)
Using for $|V_{ud}|$ the value (7) obtained from the neutron life time, one obtain an increase of an uncertainty for the unitarity condition:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9997(40) \tag{12}$$

Whereas using the result of Ref. [16] only we obtain

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0075(40), \tag{13}$$

which lead to the magnitude of right side of Eq. (14) exceeding unity more than $1\sigma$. It is clear, that additional efforts are needed in order to resolve the experimental problem of the neutron life time. At the same time we have the validity of the unitarity condition (11) for the first row of $V_{CKM}$ matrix using the world average values, as it was pointed out above.

Let us go to the next row of the quark mixing matrix, which is connected with processes with charm quarks. The $V_{cd}$ value is extracted from data for production of charm quarks involving neutrino, antineutrino and $d$ quarks. According to the recent data [1] the value is

$$|V_{cd}| = 0.230 \pm 0.011 \tag{14}$$

Using along with experimental data results of evaluations in the frame of the Lattice QCD, the following $|V_{cs}|$ value can be obtained [1]:

$$|V_{cs}| = 0.957 \pm 0.017 \pm 0.093, \tag{15}$$

where the first uncertainty for $|V_{cs}|$ is experimental one, the second one is the uncertainty of theoretical calculations.

The $|V_{cb}|$ value is obtained from data for exclusive decays $B \to D^+ l^+ \nu_l$, $B \to D^+ l^- \bar{\nu}_l$, and for inclusive decays $B \to X l l \bar{l}$ with a determination of a charge lepton characteristics, which subsequently are correlated each other. A difference of $|V_{cb}|$ values derived from exclusive and inclusive processes lies in the range of experimental uncertainties. The $|V_{cb}|$ value averaged is [1]:

$$|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}, \tag{16}$$

Experiments for $W^\pm$ bosons’ decays carried out at LEP-2 make possible to verify the unitarity condition for $V_{CKM}$ matrix for sum $\sum_{a,d,s,b} |V_{ij}|^2$ [1]:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 2.002 \pm 0.027 \tag{17}$$

Note that the unitarity conditions for columns and rows of $V_{CKM}$ matrix can be represented on a complex plane as, so called, "unitary triangles".

Nowadays there are experimental and theoretical difficulties for determinations of matrix elements of the third row. For example, usually absolute values for $|V_{td}|$ and $|V_{ts}|$, which is small enough, ($|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$), are determined at the assumption of unity absolute value for $V_{tb}$. However, at Tevatron only the limitation $|V_{tb}| > 0.78$ has been obtained with the help of comparison of data for $t$ quarks decays into $b$ and $s, d$ quarks [1]. In addition estimations for two-loop contributions for decay width $\Gamma (Z \to b \bar{b})$ give the following value: $V_{tb} = 0.77(+0.18/-0.24)$. It should be noted that at present main uncertainties of some matrix elements are due to theoretical contributions rather than experimental ones. They are caused by using various models for calculations of matrix elements on the basis of existing data. With help of models and data, which give the most precise matrix elements values, the following ranges for $V_{CKM}$ values have been obtained within $1\sigma$ (they do not always symmetric about dominant absolute values) [1-3]:

$$
\begin{pmatrix}
0.97360 \pm 0.97407 & 0.2262 \pm 0.2282 & 0.00387 \pm 0.00405 \\
0.2261 \pm 0.2281 & 0.97272 \pm 0.97320 & 0.04141 \pm 0.04231 \\
0.00750 \pm 0.00846 & 0.04083 \pm 0.04173 & 0.999096 \div 0.999134 
\end{pmatrix}, \tag{18}
$$
Then the angles $\theta_{ij}$, entering in the $V_{CKM}$ matrix in parameterization (3), are equal to:

$$
\theta_{12} = 13.14^\circ \pm 0.06^\circ, \theta_{23} = 2.43^\circ(+0.01^\circ/ -0.05^\circ), \theta_{13} = 0.23^\circ \pm 0.01^\circ
$$

(19)

3. Quark mixing angles $\theta_{ij}$ and masses of constituent quarks

It is known that matrix elements values for $V_{CKM}$ (18) presented above cannot be evaluated from the first SM principles. Nevertheless there are a number of phenomenological approaches, which make it possible to evaluate these values. From our standpoint the most interesting approaches are those, where relations between mixing angles and quark masses can be found. One of the approaches of such kind was developed in the Fritzsch’s works [18], where the connection between mixing angles and current quark masses has been established. The authors of Ref. [19] modified this approach and suggest using masses of constituent quarks instead of current ones. Let us name this approach as the Fritzsch-Scadron-Delbourgo-Rupp approach (FSDR approach). Below quark mixing angles and their uncertainties are evaluated with masses of constituent quarks obtained in the framework of the relativistic model of quasi independent quarks [20, 21]. Mass values for light quarks within uncertainties of the model of Refs. [20, 21] are correspond to values listed in Refs. [19, 22], but mass values for heavy quarks turn out somewhat different.

Relations obtained in the framework of $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory form a basis of FSDR approach, the gauge theory initially is symmetric with respect to left and right currents [18]. The relations between mixing angles and quark masses arise under assumption that a special mechanism of mass generation for quarks proceeds like a cascade. It is suppose that at first mass values for $c, s, d$ and $u$ quarks are zero, while mass values of heavy $b$ and $t$ quarks are initial parameters of the theory. Then the mass generation for light quarks proceeds. First masses of $c, s$ quarks came via the mixing due to the weak interaction. Subsequently masses of $d$ and $u$ quarks emerged in an analogous way. Relations between mixing angles and quark masses obtained by the consideration of such process [18] later on were modified and supplemented in Ref. [19]. In doing this mass values of current quarks, governing mixing angles in the Fritzsch’s model, were substituted with mass values of constituent quarks. Moreover the expression for an auxiliary angle $\varphi_{sd}$ was proposed for a calculation of the Cabbibo angle. The angle $\varphi_{sd}$ can be expressed in terms of $\pi$ and $K$ decay constants $f_\pi, f_K$ and masses of constituent $S$ and $D$ quarks $m_S, m_D$:

$$
\sin^2 \varphi_{sd} = \frac{2\sqrt{2}\pi(f_K - f_\pi)}{\sqrt{3}(m_S - m_D)}
$$

(20)

Other two angles $\varphi_{cu}$ and $\theta_{23}$ can be calculated through formulae, which depend only on quark masses

$$
\sin^2 \varphi_{cu} = \sqrt{\frac{m_S - m_D}{m_C - m_U}}
$$

(21)

$$
\sin^2 \theta_{23} = \sqrt{\frac{m_C - m_U}{m_T - m_C}}
$$

(22)

The angle $\theta_{12}$, which is the Cabbibo angle $\theta_C$, is equal to the difference of angles $\varphi_{sd}$ and $\varphi_{cu}$:

$$
\theta_C = \varphi_{sd} - \varphi_{cu}
$$

In Refs. [19, 22] the constituent quark model for hadrons was employed for a determination of mass values of light constituent quarks with the help of experimental data on mass spectra of mesons and baryons, and magnetic moments of baryons. By this means the following values were obtained:

$$
\begin{align*}
    m_U &= 335.5 MeV, & m_D &= 339.5 MeV, & m_S &= 485.7 MeV
\end{align*}
$$

(23)

Mass values of heavy constituent quarks in Ref. [19] were determined as one-halves mass values for $1^{--}$ quarkonia in their ground states, hence the following values were used:

$$
\begin{align*}
    m_C &= 1550 MeV, & m_B &= 4730 MeV
\end{align*}
$$

(24)
This method of a determination of mass values for heavy quarks may consider as a quite rough estimation. More precise mass values for constituent quarks can be found with the help of solution of problem concerning ground states of quark-antiquark systems considered in hadrons’ potential models, for instance, in the relativistic quasi independent quark model with a linear rising confinement potential and a quasi Coulomb potential [20, 21]. In the framework of this model the hypothesis of the universality of the confinement potential was verified for as heavy as light quarks with the coefficient of a slope of a linear rising potential equal to $\sigma = 0.20 \pm 0.01 GeV^2$. New characteristic constants for a confinement domain, which have dimensions of mass and length, can be associated with the obtained coefficient $\sigma$ ("the string tension"): $\mu_C = 0.45 \pm 0.02 GeV$, $\lambda_C = 0.44 \pm 0.02 Fm$. As this takes place, $\mu_C$ defines typical magnitudes of transversal impulses $<p_T>$ for quarks-partons inside hadrons, while a radius of a perturbative domain surrounded a current quark is equal to $r_C = \lambda_C/2 = 0.22 \pm 0.01 Fm$. The domain $r > r_C$ is most likely to be the domain of a formation for a constituent quark due to nonperturbative interactions. Mass values for light constituent quarks evaluated in the model are in accordance with the values (23), however mass values for heavy quarks differ considerable from the values (24) and are equal to

$$m_C = 1610 MeV, \quad m_B = 4950 MeV$$

(25)

In the framework of the relativistic quasi independent quark model systematic uncertainties can be also estimated for as light as heavy quarks.

$$m_U = 335 \pm 2 MeV, \quad m_D = 339 \pm 2 MeV, \quad m_S = 485 \pm 8 MeV;$$

$$m_C = 1610 \pm 15 MeV, \quad m_B = 4950 \pm 20 MeV.$$  

(26)

Now we use the results (26), for quark mixing angles to be refined. By using formula (21) one can obtain $\sin 2 \varphi_{cu} = 0.338 \pm 0.009$, so the angle magnitude is $\varphi_{cu} = 9.9^\circ \pm 0.3^\circ$. We take the value $20.5 \pm 0.2 MeV$ for $f_K - f_\pi$, which do not contradict to existing data, then with the help of the relation (20) we obtain $\sin 2 \varphi_{sd} = 0.720 \pm 0.041$, that is $\varphi_{sd} = 23.0^\circ \pm 1.7^\circ$. As the result of these calculations we have the Cabbibo angle $\theta_C$:

$$\theta_C = \varphi_{sd} - \varphi_{cu} = 13.1^\circ \pm 1.7^\circ$$

(27)

With account of data [1] the mass of the constituent $T$ quark can be taken as $m_T = 173 \pm 3 GeV$, so with the formula (22) one can obtain the following value for the $\theta_{23}$:

$$\theta_{23} = 2.47^\circ \pm 0.03^\circ$$

(28)

As for the $\theta_{13}$ angle we use a relation, which is immediately apparent from data, namely: $\theta_{13} = \theta_{23}/12$. Thus the $\theta_{13}$ value is equal to

$$\theta_{13} = 0.206^\circ \pm 0.003^\circ$$

(29)

A comparison of the quark mixing angles obtained above with the experimental values for these angles (see equalities (19)) shows that an accordance between these results takes place within $3\sigma$ uncertainties.

4. Hypothesis of complementary for quark and neutrino mixing angles and neutrino mixing angles $\eta_{ij}$

Let us use quark mixing angles evaluated in the previous section for checking and refinement of mixing angles in the neutrino sector with the aid of the hypothesis of a complementary and equality for mixing angles of quarks and neutrinos [23, 24]. In the framework of the hypothesis it is suppose that the angles $\theta_{13}$ are $\eta_{13}$ equal each other, while the angles $\theta_{12}$ and $\eta_{12}$, $\theta_{23}$ and $\eta_{23}$ complement each other within the $\pi/4$ angle, namely:

$$\theta_{13} = \eta_{13}, \quad \theta_{12} + \eta_{12} = \pi/4, \quad \theta_{23} + \eta_{23} = \pi/4$$

(30)
If we insert the quark mixing angles $\theta_{ij}$ evaluated in the previous section, then it is an easy matter to calculate with formulæ (30) neutrino mixing angles $\eta_{ij}$, involved in the $U_{PMNS}$ matrix:

$$\eta_{12} = 31.9^\circ \pm 1.7^\circ, \quad \eta_{23} = 42.53^\circ \pm 0.03^\circ, \quad \eta_{13} = 0.206^\circ \pm 0.003^\circ \quad (31)$$

It should be noted that the predicted value of $\eta_{13}$ angle is rather small, so a experimental determination of this value is a difficult problem. Of course, the uncertainties shown in Eqs. (27), (28), (29) and (31) should be taken as model dependent ones, so should be increased by unknown systematic (theoretical) uncertainties. However if the hypothesis of a complementary and equality for mixing angles is true, the $\eta_{13}$ angle is nonzero because of the $\theta_{13}$ is nonzero (see Eq. (29)). This fact is principal since only in this case a $CP$ breaking can appear in processes involving neutrinos as Dirac particles. For Majorana neutrinos $U_{PMNS}$ matrix contains additionally two phase parameters $\chi_1$ and $\chi_2$, which lead to observable effects in some processes, for instance, in $0\nu2\beta$ decays of nuclei.

The evaluated values for neutrino mixing angles (31) can be compared with angles’ values found experimentally. It is known that recent experiments confirm the theoretical idea about oscillations of electron and muon fluxes in their passage of long distance from a source [7-9] on the one hand, on the other they provide an explanation for the deficit of solar neutrinos and verify once more the Sun standard model [25-27]. Experimental data from KamLAND [28], SNO [29], K2K [30], Super-Kamiokande [31] and CHOOZ [32] give the following restrictions neutrino mixing angles [1]:

$$\theta_{sol} = 34.01^\circ (+1.31^\circ / -1.56^\circ), \quad \theta_{atm} > 36.78^\circ, \quad \theta_{chz} < 12.92^\circ \quad (32)$$

In notations of our paper $\theta_{sol} = \eta_{12}$, $\theta_{atm} = \eta_{23}$, $\theta_{chz} = \eta_{13}$. Besides that an additional restriction for the mixing angle $\theta_{chz}$ can be obtained with the aid of data for neutrino flux from the star SN 1987A [33, 34] under assumption that neutrino masses form a normal hierarchy [35-37]. The correlation of mixing angles (31) and (32) give us a firm evidence for the validity of hypothesis of a complementary for mixing angles within existing experimental uncertainties. As for the equality of the $\theta_{13}$ and $\eta_{13}$ angles then a performance of experiments are required for more precise confirmation. It is worth noting that the calculated uncertainties for $\eta_{12}$, $\eta_{23}$ and $\eta_{13}$ angles are considerably smaller experimental ones. So the hypothesis used is consistent and can be applied for independent estimations neutrino mixing angles on the basis of quark mixing angles since the last ones are measured with a high accuracy.

5. Conclusions and discussion

It is generally recognized that SM is the transition stage of a more perfect and universal theory notwithstanding impressive achievements of the SM. Several problems remain open, such as experimental verification of the existence of Higgs particles, the large number of SM constants, which are pure phenomenological parameters. For instance, there are experimental values of quark and lepton masses, and quark mixing angles. The discovery of neutrino oscillations in fluxes of solar, reactor, atmospheric and accelerator neutrinos only adds to the complexity of the problem discussed, because of shows the necessity of increasing the number of parameters by reason of neutrino nonzero masses and mixing angles. Therefore revealing of relations between SM constants and searching of a more general theory than the SM, for instance, a GUT are urgent problems of present-day investigations. In a GUT framework the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ goes over, as a rule, into a gauge group with more simpler structure. For one of GUT variants [38, 39] the group of this type is the $SO(10)$ group with the following pattern of spontaneous gauge symmetry breaking

$$SO(10) \rightarrow SU(4)_{ec} \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y, \quad (33)$$

where $SU(4)_{ec}$ is the group of extended color [40]. In the $SO(10)$ model the fundamental spinor representation with dimension equal to 16 is filled by one left fermion generation consist of quarks, leptons including a $CP$ partner for a right neutrino. In the framework of this model nonzero
neutrino masses naturally arise. This model does not contradict the existing restrictions associated with the value of the Weinberg angle, the lifetime of the proton and the neutrino masses [1]. Notice that the $SU(2)_L \times SU(2)_R$ gauge symmetry is the basis for the FSDR formulae, which are applied in the present paper. Besides the mass values of constituent quarks and their absolute uncertainties are used, which have been obtained in the framework of the relativistic model for quasi independent quarks with the universal confinement potential. As a consequence the accordance with the data for quark mixing angles has been achieved. Moreover the neutrino mixing angles have been evaluated with a high accuracy by using the hypothesis of a complementary and equality for mixing angles of quarks and neutrinos.

It should be noted, that as distinct from the quark sector the independent description of a generation mass mechanism and mixing do not achieved much success yet. At present it is clear that the neutrino mixing has its peculiar features connected with a possibility for neutrino be a Dirac or Majorana particle. In the last case supplementary circumstances must be taken into account, when one transform flavor neutrino states to mass neutrino states in order to bring a mass matrix into a diagonal form. As analysis performed in the works [41, 42] in the framework of the special two flavor Pauli model shows, if neutrinos are complex states from Dirac and Majorana components then their mixing can be determined by a ratio of Dirac and Majorana contributions in effective neutrino masses. In its turn effective masses govern oscillation lengths of neutrinos. From this point of view, it is possible, that the hypothesis of a complementarity used by us point to a similarity of mixing mechanisms for quarks and neutrinos, and this hypothesis need further consideration with account of peculiarities for quark and neutrino sectors. So tackling the question about a Dirac or Majorana nature of neutrinos has the principal significance. It is known, that the key experiment is a discovery of the neutrinoless mode for double beta nuclear decays. An observation of this mode signals that a neutrino has Majorana properties, while an effective coherent mass

$$m_{\beta\beta} = U_{e1}^2m_1 + U_{e2}^2m_2 + U_{e3}^2m_3,$$

measured in experiments of such kind, depends on Majorana phases entering into $U_{ei}$, $i = 1, 2, 3$. The current data give for the $m_{\beta\beta}$ value the following limitation [43-46]:

$$|m_{\beta\beta}| < 0.55\text{eV}$$

Moreover, direct measurements of neutrino mass in the beta decay of tritium [47, 48] lead to

$$m_\nu < 2.2\text{eV}$$

At the same time oscillation data show the occurrence of the lower limitation for one of neutrino masses [49]:

$$m_\nu > 8.5 \cdot 10^{-3}\text{eV}$$

Hence the question about the character of mixing in the lepton sector is still open.

In conclusion the results of the work indicate that mixing angles for quarks and leptons are not independent SM parameters, supposedly they relate to masses of these particles. Moreover mechanisms of mass generation in the quark and lepton sectors ought to be dependent each other as indicated by correlations between quark and neutrino mixing angles. Undeniably the major importance for the resolution of the mixing problem, primarily in the lepton sector has got and will be got results of experiments on neutrinoless double beta nuclear decays.

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