New Physics Effect on the Higgs Self-Coupling

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Abstract

One-loop corrections to the self-coupling constant of the lightest CP-even Higgs boson are studied in the two Higgs doublet model. After renormalization is performed, quartic contributions of heavy particle’s mass can appear in the effective coupling. We find that these non-decoupling effects can yield $\mathcal{O}(100)$ % deviations from the Standard Model prediction, even when all the other couplings of the lightest Higgs boson to gauge bosons and fermions are in good agreement with the Standard Model.
I. INTRODUCTION

The cause of the electroweak symmetry breaking (that generates masses for the weak gauge bosons $W^\pm$ and $Z$) and the origin of the flavor symmetry breaking (that generates mass spectrum for quarks and leptons) are great mysteries in the elementary particle physics. In the Standard Model (SM), both symmetry breaking mechanisms are explained by introducing a scalar iso-doublet field which includes a physical scalar state, i.e., the Higgs boson ($h$). The $W$ and $Z$ bosons obtain their masses through the Higgs mechanism, and the fermions gain their masses via the Yukawa interaction.

The present precision data have shown an excellent agreement with the prediction of the SM \cite{1,2}. It also suggests the Higgs boson to be light, with a central value $m_H = 81$ GeV (below the LEP2 direct search bound which requires $m_H > 114.4$ GeV \cite{3}) and a 95% C.L. upper limit on Higgs boson mass to be 193 GeV \cite{4}. However, a more close examination reveals that the present data only strongly constrains the couplings of the gauge bosons with the fermions (except top quark) as well as those of the triple gauge boson vertices, but says little about the interaction of Higgs boson with gauge bosons and fermions. With the data coming from the current Run-II of the Fermilab Tevatron collider, the CERN Large Hadron Collider (LHC), and future Linear Colliders (LC’s), more precise tests of these sectors of the SM become possible. For example, at the LC, the Higgs boson can be produced via the processes $e^+e^- \rightarrow Z^* \rightarrow Zh$ and $e^+e^- \rightarrow W^+\nu W^-\bar{\nu} \rightarrow h\bar{\nu}\nu$ \cite{5}, and the $hZZ$ and $hW^+W^-$ couplings can be determined to a few percent from the precise measurement of the above production cross sections \cite{6}. Furthermore, the Yukawa couplings of the Higgs boson to fermions can be determined from measuring the decay branching ratios of the Higgs boson.

To test the electroweak symmetry breaking (EWSB) sector of the SM, not only should the couplings of the Higgs boson to gauge bosons and fermions be measured, but also the self-coupling of the Higgs boson which originates from the Higgs potential. Unfortunately, at the LHC, it would be extremely difficult to measure the SM Higgs boson self-couplings, either the trilinear coupling $hhh$ or the quartic coupling $hhhh$ \cite{7}. At the LC, the trilinear coupling $\lambda_{hhh}$ can be measured via the Higgs boson pair production in $e^+e^- \rightarrow Z^* \rightarrow Zhh$ and $e^+e^- \rightarrow W^+\nu W^-\bar{\nu} \rightarrow hh\bar{\nu}\nu$, if the Higgs boson is not too heavy \cite{8,9,10,11}. It is expected that at a 500 GeV (3 TeV) $e^+e^-$ collider with an integrated luminosity of 1 ab$^{-1}$ (5
ab\(^{-1}\)), \(\lambda_{hhh}\) can be measured to about 20% (7%) accuracy for the Higgs boson mass around 120 GeV \[^{10}\]. The expectation for such a precise measurement motivates to study radiative correction to the Higgs self-couplings. In the SM, the one-loop contribution of the heavy quarks is substantial due to their non-decoupling property, especially its leading effect grows as the quartic power of the top-quark mass in the large mass limit. The precise measurement of the self-coupling at the LC also makes it possible to test extended Higgs models, which have different structures of the Higgs potential from the SM \[^{9,12}\].

In this Letter, we discuss quantum corrections to the self-coupling of the lightest CP-even Higgs boson in the two-Higgs-doublet model (THDM) \[^{13}\]. The THDM is the simplest model of extended Higgs sectors, and its Higgs potential has a rich structure for various physics motivations. The Higgs sector of the Minimal Supersymmetric Standard Model (MSSM) is a special case of the *weakly coupled* THDM \[^{14}\]. Some models of dynamical electroweak symmetry breaking also yield the THDM as their low-energy effective theory \[^{15}\], in which the Higgs self-couplings are relatively strong. The tree-level \(hhh\) coupling in the THDM generally differs from that predicted by the SM, depending on the other parameters of the model \[^{9,12}\]. In Ref. \[^{16}\], the one-loop effects on the Higgs self-couplings in the MSSM have been calculated, and their decoupling property has also been studied in detail. As to be shown below, in contrast to the MSSM, it can happen that the heavy mass effects in a general THDM do not decouple. To illustrate this point, we calculate the Higgs boson self-coupling by the diagrammatic approach as well as the effective potential method. After rewriting the one-loop effect in terms of the renormalized mass of the lightest Higgs boson, quartic dependence on the mass of the heavier Higgs bosons contributing in the loop diagrams appears in the effective \(hhh\) coupling. We find that even when the Higgs couplings to gauge bosons and fermions are almost SM-like, the deviation in the \(hhh\) self-coupling from the SM prediction can be at the order of 100% due to the non-decoupling effects of the additional heavier Higgs bosons in loops.

II. LEADING SM CORRECTIONS TO THE \(hhh\) COUPLING

Before proceeding to the discussion of the results for the THDM, it would be instructive to see how the leading one-loop effects appear in the \(hhh\) coupling in the SM. The tree-level trilinear Higgs coupling is expressed in terms of the Higgs boson mass \((m_h)\) and the vacuum
expectation value \( v \) by \( \lambda_{hhh}^{tree}(SM) = \frac{3m_h^2}{v} \). The leading one-loop contribution of the top quarks to the effective coupling \( \lambda_{hhh}^{eff}(SM) \) is derived as

\[
\lambda_{hhh}^{eff}(SM) = \frac{3m_h^2}{v} \left[ 1 - \frac{N_c}{3\pi^2} \frac{m_t^4}{m_h^2} \left\{ 1 + \mathcal{O}\left(\frac{m_h^2}{m_t^2}, \frac{p_i^2}{m_t^2}\right) \right\} \right],
\]

where \( m_h \) and \( m_t \) are the physical masses of the Higgs boson and the top-quark, respectively, and \( p_i \) \((i = 1-3)\) represent the momenta of external Higgs lines. As an interesting feature, the non-vanishing one-loop effect of the top-quarks appears as \( \mathcal{O}(m_t^4) \).

The easiest way to understand the appearance of the quartic mass term in the effective \( hhh \) coupling is to study the one-loop effective potential; \( V_{eff}[\varphi] = V_{tree}[\varphi] + \Delta V[\varphi] \). The one-loop contribution \( \Delta V[\varphi] \) is given by

\[
\Delta V[\varphi] = \frac{1}{64\pi^2} \sum_f N_{c_f} N_{s_f} (-1)^{2s_f} (M_f[\varphi])^4 \left\{ \ln\left(\frac{M_f[\varphi]^2}{Q^2}\right) - \frac{3}{2} \right\},
\]

where \( \varphi = \langle \phi \rangle = v + \langle h \rangle \), \( N_{c_f} \) is the color number, \( s_f \) \((N_{s_f})\) is the spin (degree of freedom) of the field \( f \) in the loop, \( M_f[\varphi] \) is the field dependent mass of \( f \), and \( Q \) is an arbitrary energy scale. The effective coupling of \( hhh \) can be expressed in terms of the physical mass of the Higgs boson and \( \Delta V[\varphi] \) by

\[
\lambda_{hhh}^{eff} = \frac{\partial^3 V_{eff}}{\partial \varphi^3} \bigg|_v = \frac{3m_h^2}{v} + \left( \frac{3}{v^2} \frac{\partial}{\partial \varphi} - \frac{3}{v} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^3}{\partial \varphi^3} \right) \Delta V[\varphi] \bigg|_v,
\]

up to the wave function renormalization contributions. We note that \( m_h^2 \) is obtained as the second derivative of \( V_{eff}[\varphi] \). The leading top-quark loop effect in Eq. (3) is easily obtained from Eq. (3) with the field dependent mass \( M_t[\varphi] = y_t \frac{\varphi}{\sqrt{2}} \), where \( y_t \) is the top-Yukawa coupling constant in the SM. Although each individual term inside the parenthesis (present in the right-hand side) of Eq. (3) can contribute a large logarithmic term \( m_t^4 \ln(m_t^2/Q^2) \), they all cancel with each other in the sum so that the remaining leading contribution to \( \lambda_{hhh}^{eff} \) is the constant \( m_t^4 \) term.

The appearance of this non-vanishing \( m_t^4 \) term is a striking feature of the one-loop correction to the self-coupling constant. In contrast, the one-loop effective couplings of \( hVV \) \((VV = ZZ, W^+W^-) \) \((g_{hVV})\) have non-decoupling power-like contributions of at most \( \mathcal{O}(m_t^2) \). Therefore, if a new heavy particle has the similar non-decoupling property to the top quark in some extension of the SM, its loop effect on the \( hhh \) coupling can become important, because the quartic mass contribution is expected to make the correction large. In the following, we examine this point in the context of the THDM.
III. ONE LOOP EFFECT ON THE $hhh$ COUPLING IN THE THDM

The Higgs potential of the CP-conserving THDM is given by

$$V_{\text{THDM}} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 \left( \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4$$

$$+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 \left| \Phi_1^\dagger \Phi_2 \right|^2 + \frac{\lambda_5}{2} \left\{ \left( \Phi_1^\dagger \Phi_2 \right)^2 + \left( \Phi_2^\dagger \Phi_1 \right)^2 \right\},$$

(4)

where we imposed a softly-broken discrete symmetry under the transformation of $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$. We assume all the coupling constants and mass parameters to be real, so that there are eight real parameters in the potential (4). The discrete symmetry allows two types of the Yukawa interaction, so called Model I and Model II [14]. The discrete symmetry ensures natural suppression of flavor changing neutral current processes.

Diagonalizing the mass matrices, we have the five physical scalar states; i.e., two CP-even ($h$, $H$), one CP-odd ($A$), and a pair of charged ($H^\pm$) Higgs bosons. Notice that masses of the heavier Higgs bosons ($H$, $H^\pm$ and $A$) schematically take the form as

$$m_\Phi^2 \simeq M^2 + \lambda_i v^2,$$

(5)

where $\Phi$ represents $H$, $H^\pm$ or $A$, $M$ is the soft-breaking scale of the discrete symmetry defined by $M = m_3/\sqrt{\sin \beta \cos \beta}$, and $\lambda_i$ is some linear combination of $\lambda_1$-$\lambda_5$. As indicated in Eq. (5), there are two origins of the masses; one is the soft-breaking scale $M$, and another is the vacuum expectation value of the electroweak symmetry breaking $v$. The origin of the mass determines the decoupling property of the heavy Higgs bosons [17]. When $M^2 \gg \lambda_i v^2$, $m_\Phi$ is determined by $M$, and is independent of $\lambda_i$. Consequently, the loop effects of $\Phi$ vanish in the large mass limit (i.e., $m_\Phi \to \infty$) because of the decoupling theorem\(^1\). On the other hand, when $M^2 \lesssim \lambda_i v^2$ the large value of $m_\Phi$ is realized by large coupling constants $\lambda_i$. In this case, the decoupling theorem cannot be applied. Hence, we expect positive power (or logarithmic) contributions of $m_\Phi$ in the radiative correction [18, 19]. We refer such power-like contribution as the non-decoupling effect. In these scenarios, theoretical consistencies and present experimental data generally provide strong constraints to the model parameters. For instance, too large values of $\lambda_i$ clearly break validity of perturbation calculation.

\(^1\) The MSSM Higgs sector corresponds to this case, in which coupling constants $\lambda_1$-$\lambda_5$ are given to be $O(g_i^2)$ due to supersymmetry, where $g_i$ represent the electroweak gauge coupling constants.
Now we discuss the $hhh$ coupling in the THDM. At the tree level, the $hhh$ coupling is expressed in terms of the input parameters of the Higgs sector by

$$\lambda^{\text{tree}}_{hhh}(\text{THDM}) = -\frac{3}{4v\cos\beta\sin\beta} \left[ 4M^2\cos^2(\alpha - \beta)\cos(\alpha + \beta) \right.$$  
$$- \left\{ \cos(3\alpha - \beta) + 3\cos(\alpha + \beta) \right\} m_h^2 \right].$$

For general values of $\alpha - \beta$, the coupling depends on the mixing angles and the soft-breaking scale of the discrete symmetry, and thus its value can be completely different from the SM prediction. In general, when $\lambda^{\text{tree}}_{hhh}(\text{THDM})$ is significantly different from $\lambda^{\text{tree}}_{hhh}(\text{SM})$, the new physics effect will also manifest in the coupling of Higgs bosons to gauge bosons and fermions, which can be detected experimentally at future high energy colliders.

We have calculated the one-loop correction to the effective $hhh$ vertex function by the diagrammatic approach in the on-shell scheme. In this Letter, we present the results of our calculations, and details of the calculation will be shown elsewhere [20]. To simplify our discussion on the one-loop radiative corrections to the Higgs self-coupling in the THDM, we shall assume the following scenario: (i) Only one Higgs boson ($h$) is found and its mass ($m_h$) is measured to be light ($<\sim 200\ \text{GeV}$). (ii) Experimental data on the $hVV$ couplings ($g_{hVV}$) and the Higgs decay branching ratios agree with their SM prediction in good accuracy. This implies $\sin^2(\alpha - \beta) \simeq 1$ (or $\alpha \simeq \beta - \pi/2$) in the context of the THDM [19]. In this case, the tree $hhh$ coupling given in Eq. (6) takes the same form as the SM prediction; i.e.,

$$\lambda^{\text{tree}}_{hhh}(\text{THDM}) = \frac{3m_h^2}{v}.$$

Using the Feynman diagrammatic method, we calculate the leading contributions originated from the heavy Higgs boson loops and the top quark loops. We find that at the one loop level, the effective $hhh$ coupling can be written as

$$\lambda^{\text{eff}}_{hhh}(\text{THDM}) = \frac{3m_h^2}{v} \left\{ 1 + \frac{m_H^4}{12\pi^2m_h^2v^2} \left( 1 - \frac{M^2}{m_H^2} \right)^3 + \frac{m_A^4}{12\pi^2m_h^2v^2} \left( 1 - \frac{M^2}{m_A^2} \right)^3 \right.$$  
$$+ \frac{m_{H^\pm}^4}{6\pi^2m_h^2v^2} \left( 1 - \frac{M^2}{m_{H^\pm}^2} \right)^3 - \frac{N_c m_t^4}{3\pi^2m_h^2v^2} + \mathcal{O}\left( \frac{p_i^2m_H^2}{m_h^2v^2}, \frac{m_H^2}{m_h^2v^2}, \frac{p_i^2m_A^2}{m_h^2v^2}, \frac{m_t^2}{m_h^2v^2} \right) \right\},$$

where $m_q$ and $p_i$ represent the mass of $H$, $A$ or $H^\pm$ and the momenta of external Higgs lines, respectively. We note that in Eq. (7) $m_h$ is the renormalized physical mass of the lightest CP-even Higgs boson $h$. The leading contribution of the above result can also be obtained by using the effective potential method. As expected, the contribution from the top quark loop is the same as the SM prediction because the tree level coupling of the top quark to $h$ is
identical to that in the SM when $\alpha = \beta - \pi/2$. Similar to the top quark loop, the contribution from the Higgs boson loops also grows as $m_\Phi^4$ but with a suppression factor $(1 - M^2/m_\Phi^2)^3$, where $\Phi$ represents $H$, $A$ or $H^\pm$. Furthermore, the Higgs boson loop contributes an opposite sign to the top quark loop because the former is a boson loop and the latter is a fermion loop. Because of the suppression factor $(1 - M^2/m_\Phi^2)^3$, the maximum non-decoupling effect is realized in the limit of $M^2 \to 0$, and the Higgs boson loop contribution is enhanced by $m_\Phi^4$. On the other hand, when the Higgs boson mass $m_\Phi$ is at the same order as the soft mass scale $M$, the Higgs boson loop contribution becomes diminished and decoupled from $\lambda_{hhh}^{eff}(THDM)$.

To examine the numerical effect of the one loop radiative corrections to the trilinear coupling $\lambda_{hhh}$ in the THDM, we have to take into account various theoretical and experimental constraints. Some of them are discussed below. The choice of the THDM parameters should satisfy the requirement of perturbative unitarity for the $S$-wave amplitudes of the $2 \to 2$ scattering processes of the Higgs bosons and longitudinally polarized weak bosons [21]. For example, the unitarity condition requires that when $m_h = 120$ GeV, $m_A = m_H = m_{H^\pm} (\equiv m_\Phi)$ and $M = 0$, the upper bound on the masses of the heavier Higgs bosons is about 550 GeV to 600 GeV [22]. This upper bound is generally weakened as $M$ increases, so a heavier Higgs boson is allowed. We also include the constraints imposed from the low-energy precision data on the THDM [23], especially, the $\rho$ parameter constraint ($\Delta \rho (\equiv \rho - 1) \sim 10^{-3}$). To satisfy this constraint, the THDM has to have an approximate custodial ($SU(2)_V$) symmetry [24]. In the Higgs sector of the THDM, there are typically two options of the parameter choice in which $SU(2)_V$ is conserved according to the assignment of the $SU(2)_V$ charge; (1) $m_{H^\pm} \simeq m_A$, and (2) $m_{H^\pm} \simeq m_H$ with $\sin^2(\alpha - \beta) \simeq 1$ or $m_{H^\pm} \simeq m_h$ with $\cos^2(\alpha - \beta) \simeq 1$ [24, 25]. In our numerical analysis, we choose the parameters that satisfy these conditions. In addition, in Model II of the THDM, it is known that the $b \to s\gamma$ branching ratio excludes the small mass of the charged Higgs boson [26].

In Fig. 1, we show $\Delta \lambda_{hhh}^{THDM}/\lambda_{hhh}^{eff}(SM)$ as a function of $m_\Phi (= m_H = m_A = m_{H^\pm})$ in the SM-like scenario ($\sin^2(\alpha - \beta) = 1$) for $m_h = 100, 120$ and 160 GeV, where $\Delta \lambda_{hhh}^{THDM} \equiv \lambda_{hhh}^{eff}(THDM) - \lambda_{hhh}^{eff}(SM)$. The solid curves are the results from a full calculation of the

\[^2\text{In terms of the coupling constants, these conditions are expressed by (1) } \lambda_4 = \lambda_5, \text{ and (2) } \lambda_1 = \lambda_2 = \lambda_3 \text{ with } m_1^2 = m_2^2.\]
FIG. 1: The $m_Φ^4$ behavior in $∆λ_{hhh}^{THDM}$, where $∆λ_{hhh}^{THDM} ≡ λ_{hhh}^{eff}(THDM)−λ_{hhh}^{eff}(SM)$. The results of the full one-loop calculation are shown as solid curves, while the quartic mass contributions given in Eq. (7) are plotted as dotted curves.

Higgs boson loop contributions, and the dotted curves are the leading contributions in Eq. (7). Here, we have chosen $M = 0$ to explore the maximal non-decoupling effect. Because of the quartic power dependence of $m_Φ$, the non-decoupling effect becomes greater for larger values of $m_Φ$ with smaller $m_h$. However, the allowed value of $m_Φ$ is bounded from above by the perturbative unitarity bound ($m_Φ ≲ 550$-600 GeV in this case). As shown in Fig. 1, the deviation from the SM prediction is about 30% (100%) for $m_Φ = 300$ (400) GeV, in the maximal non-decoupling scenario\(^3\). We note that the large, of $\mathcal{O}(1)$, one-loop radiative correction to $λ_{hhh}$ in the THDM does not imply the breakdown of the perturbative expansion, for the large contribution originates from new types of couplings, e.g., $λ_{hΦΦ}$ and $λ_{hhΦΦ}$, that enter in loop calculations. Needless to say that we do not expect such kind of large correction to occur beyond the one-loop order.

As shown in Eq. (7), the non-decoupling effect of the heavier Higgs bosons is suppressed by the factor of $(1 − M^2/m_Φ^2)^3$ for a non-vanishing $M$. In the case of $M^2 ≫ λ_i v^2$, cf. Eq (6), this

\(^3\) Although the expression in Eq. (7) does not depend on $tan β$, the allowed value of $tan β$ is constrained to be $\mathcal{O}(1)$ due to the requirement of the perturbative unitarity when large values of $m_Φ$ are taken with $M = 0$. Hence, the large deviation from the SM prediction occurs at $tan β = \mathcal{O}(1)$. We note that the parameter set $m_H = m_A = m_{H±}$, $M = 0$, $α = β − π/2$ and $tan β = 1$ corresponds to $λ_1 = λ_2 = λ_3 = (m_h^2 + m_H^2)/v^2$ and $λ_4 = λ_5 = −m_H^2/v^2$. 

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FIG. 2: The decoupling behavior of $\Delta \lambda_{hhh}^{T H D M}$, where $\Delta \lambda_{hhh}^{T H D M}$ is defined by $\lambda_{hhh}^{eff}(T H D M) - \lambda_{hhh}^{eff}(S M)$ calculated in the one-loop diagrammatic approach. The mass of the heavy Higgs bosons $m_\Phi(\equiv m_H = m_A = m_{H^\pm})$ is given by $m_\Phi^2 = \lambda v^2 + M^2$.

The factor behaves as

$$f = \frac{1}{16\pi^2 v^2 m_h^2} \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \longrightarrow \frac{\lambda^2}{16\pi^2} \frac{v^2 v^2}{m_h^2 m_\Phi^2},$$

and thus decouples in the limit of $m_\Phi^2(\approx M^2) \rightarrow \infty$. In Fig. 2, we show the decoupling behavior of the heavier Higgs contribution as a function of $M$ with fixed $\sqrt{\lambda v^2} = 200 - 450$ GeV, in the case of $\sin^2(\alpha - \beta) = 1$ and $m_h = 120$ GeV, where the mass of the heavier Higgs bosons $m_\Phi (= m_A = m_H = m_{H^\pm})$ is given by $m_\Phi^2 = \lambda v^2 + M^2$. (We note that $\lambda$ corresponds to $\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta - m_h^2/v^2 = \lambda_3 - m_h^2/v^2 = -\lambda_4 = -\lambda_5$ in this case.) It is evident that the heavier Higgs boson contributions reduce rapidly for a larger value of $M$. Nevertheless, a few tens of percent of the correction remains at $M = 1000$ GeV. Since the Higgs sector of the MSSM is a special case of the Type-II THDM with $\lambda_i v^2 \simeq \mathcal{O}(m_W^2)$, as required by supersymmetry, it belongs to the class of models in which the heavier Higgs bosons decouple. Hence, the effect of the Higgs boson loops to $\lambda_{hhh}^{eff}(MSSM)$ is expected to be small. A detailed study on this decoupling behavior of the one-loop corrected $hhh$ coupling in the MSSM can be found in Ref. [16]. We confirmed that our results for large values of $M$ are consistent with those in Ref. [16].

New physics model can strongly modify the trilinear coupling of the Higgs boson, which can be measured from studying the scattering processes $e^+e^- \rightarrow Z^* \rightarrow Zh^* \rightarrow Zhh$ and $e^+e^- \rightarrow \bar{\nu}W^+W^- \rightarrow \bar{\nu}h^* \rightarrow \bar{\nu}hh$ in the $e^+e^-$ collision, and $\gamma\gamma \rightarrow h^* \rightarrow hh$ at the
FIG. 3: The momentum dependence of $\Delta \lambda_{hhh}^{THDM}(q^2) \equiv \lambda_{hhh}^{eff}(THDM) - \lambda_{hhh}^{eff}(SM)$ calculated in the diagrammatic approach, where $\sqrt{q^2}$ is the invariant mass of $h^*$ in $h^* \rightarrow hh$ for each value of $m_\Phi (\equiv m_H = m_A = m_{H^\pm})$, when $m_h = 120$ GeV, $\sin(\alpha - \beta) = 1$ and $M = 0$.

$\gamma \gamma$ option of the LC. In Fig. 3, we show the momentum dependent effective self-coupling of the Higgs boson, $\lambda_{hhh}^{eff}(q^2)$, as a function of the invariant mass ($\sqrt{q^2}$) of the virtual $h$ boson, for various values of $m_\Phi (= m_A = m_H = m_{H^\pm})$ with $\sin^2(\alpha - \beta) = 1$ and $m_h = 120$ GeV. Again, to show the maximal non-decoupling effect, we have set $M$ to be zero. The Higgs boson one-loop contribution is always positive. Below the peak of the threshold of the heavy Higgs pair production, $\lambda_{hhh}^{eff}(q^2)$ is insensitive to $\sqrt{q^2}$. We note that the low $\sqrt{q^2}$ (but $\sqrt{q^2} \gtrsim 2m_h$) is the most important region in the extraction of the $hhh$ coupling from the data of the double Higgs production mechanism, because the $h^*$ propagator $1/(q^2 - m_h^2)$ in the signal process becomes larger. On the contrary, the fermionic (top-quark) loop effect strongly depends on $\sqrt{q^2}$, because the threshold enhancement at $\sqrt{q^2} = 2m_t$ contributes an opposite sign to the quartic mass term contribution. Including both the scalar and the fermion loop contributions, the one-loop radiative correction to the $hhh$ coupling changes sign when $\sqrt{q^2}$ is somewhere between $2m_h$ and $2m_t$.

Finally, we comment on the cases in which the assumption $\sin^2(\alpha - \beta) = 1$ is slightly relaxed; i.e., the coupling for $hVV$ ($VV = ZZ$ and $W^+W^-$) deviates from the SM prediction by a factor of $\sin(\beta - \alpha)$ at the tree level. When $g_{hVV}$ are measured to be nearly (not exactly) SM-like with a few percent deviation, the tree-level $hhh$ coupling can also be different from the SM prediction for the given $m_h$, depending on the soft-breaking scale $M$ for the discrete
symmetry as well as \( \tan \beta \), as shown in Eq. (6). By scanning the parameters \( m_h \) and \( \sin(\alpha - \beta) \) under the available phenomenological constraints and the theoretical bounds, such as the perturbative unitarity and vacuum stability, one can obtain the range of the allowed deviation in the tree-level \( hhh \) coupling from the SM prediction. When the measured \( hhh \) coupling is found to be out of the allowed range of the tree-level deviation for each set of the parameters \( m_h \) and \( \sin(\alpha - \beta) \), the one-loop effect to the \( hhh \) coupling can be readily identified. Especially, in the case of \( M^2 \lesssim \lambda v^2 \), we found that the tree-level \( hhh \) coupling can at most differ from that in the SM by about 10\%, assuming \( g_{hVV} \) only deviates from the SM prediction by a few percent. Hence, a large positive deviation arising from the one-loop \( \mathcal{O}(m_A^4) \) contribution can be much larger than the tree-level deviation from the SM \( hhh \) coupling, as \( \sin^2(\alpha - \beta) \) slightly deviates from 1. A more detailed discussion on this point will be presented in Ref. [20].

IV. CONCLUSION

We have examined the one-loop correction to the Higgs self-coupling \( hhh \) in the THDM. There can be non-decoupling quartic mass contributions of the heavier Higgs bosons in the \( hhh \) coupling. Because of these effects, deviation from the SM prediction can be \( \mathcal{O}(100)\% \), even when all the measured Higgs couplings with gauge bosons and fermions are consistent with the SM values. At LC’s, such a large difference in the \( hhh \) coupling from the SM prediction may be detected. In the weakly coupled THDM, the one-loop effect is small and decouples in the large mass limit for the heavier Higgs bosons. The quartic mass effect on the effective \( hhh \) coupling is a general characteristic in any new physics model which has the non-decoupling property.

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