PROPERTIES OF NUCLEI IN THE NEUTRON STAR CRUST

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Abstract. In the present study we investigate the static properties of nuclei in the inner crust of neutron stars. Using the Hartree-Fock method in coordinate space, together with the semiclassical approximation, we examine the patterns of phase transitions.

The properties of the outer parts of neutron stars are important for the understanding of several observational issues. Despite the fact that the crust is typically less than 10\% of the radius of the star one expects that such phenomena as: thermal X-ray emission from the stellar surface, X-ray burst sources, or the sudden speed-ups in the rotation rate of some neutron stars may be related to the structure of the crust.

In the inner crust of the star, the increasing pressure and density force nuclei to lose some of their neutrons. Hence deeper in the star, the nuclei are immersed in a neutron gas. Since nuclei in such an environment cannot be explored experimentally, our understanding of matter under these conditions is based on modelling. An agreement has been reached in the literature concerning the existence of the following chain of phase transitions as the density increases: nuclei $\rightarrow$ rods $\rightarrow$ plates $\rightarrow$ tubes $\rightarrow$ bubbles $\rightarrow$ uniform nuclear matter \cite{1}. The appearance of different phases is attributed to the interplay between Coulomb and surface energies. Most published works are based on the minimization of some density functional in a single Wigner–Seitz cell and neglect the shell effects. The Hartree–Fock calculations performed so far are limited to Wigner–Seitz cells with geometries reflecting the nuclear shape and not the lattice structure \cite{2, 3}. Hence the
shell effects associated with the scattering of unbound neutrons on nuclear inhomogeneities were not properly taken into account.

There is no well established terminology for the energy corrections we are considering here, even though the problem has been addressed to some extent by other authors. In the case of finite systems, the energy difference between the true binding energy and the liquid drop energy of a given system is typically referred to as shell correction energy. In field theory a somewhat similar energy appears, due to various fluctuation induced effects and it is generically referred to as the Casimir energy [4]:

\[ E_{\text{Casimir}} = \frac{1}{2} \int_{-\infty}^{\infty} d\varepsilon \varepsilon [g(\varepsilon, l) - g_0(\varepsilon)], \]

(1)

where \( g_0(\varepsilon) \) is the density of states per unit volume for the fields in the absence of any objects, \( g(\varepsilon, l) \) is the density of states per unit volume in the presence of some “foreign” objects, such as plates, spheres, etc., and \( l \) is an ensemble of geometrical parameters describing these objects and their relative geometrical arrangement. A similar formula can be written for neutron matter energy [5, 6]

\[ E_{\text{nm}} = \int_{-\infty}^{\mu} d\varepsilon g(\varepsilon, l) - \int_{-\infty}^{\mu_0} d\varepsilon g_0(\varepsilon, l), \]

(2)

with the notable difference in the upper integration limit. In the above equation \( g_0(\varepsilon, l) \) stands for the Thomas–Fermi or liquid drop density of states of the inhomogeneous phase and \( g(\varepsilon, l) \) is the true quantum density of states in the presence of inhomogeneities. The parameters: \( \mu \) and \( \mu_0 \) are determined from the requirement that the system has a given average density:

\[ \rho = \int_{-\infty}^{\mu} d\varepsilon g(\varepsilon, l) = \int_{-\infty}^{\mu_0} d\varepsilon g_0(\varepsilon, l). \]

(3)

Since in infinite matter the presence of various inhomogeneities does not lead to the formation of discrete levels, the effects we shall consider here arise from the scattering states, which is in complete analogy with the procedure for computing the Casimir energy.

We shall only consider here spherical bubble–like, rod–like and plate–like phases. One can distinguish two types of “bubbles”: i) nuclei–like structures embedded in a neutron gas and ii) void–like structures. By voids we mean the regions in which the nuclear density is significantly lower than in the surrounding space. In the first case i), the single particle wave functions can be roughly separated into two classes, those localized mostly inside the nuclei–like structures and those which are completely delocalized. A fermion in a delocalized state will spend some time inside the “nuclei” too, but since
the potential experienced by a nucleon is deeper there, the local momentum is larger and thus the relative time and relative probability to find a nucleon in this region is smaller [7]. One can approximately replace then the “nuclei” by an effective repulsive potential of roughly the same shape. In the case of a “bubble”, when the probability to find a nucleon inside a “bubble” is reduced, again such an approximation appears reasonable. The same holds true for rod–like or plate–like structures. There are of course a number of “resonant” delocalized states, whose amplitudes behave in an opposite manner. However, the number of such “resonant” states is small and brings only small corrections. In all these phases the shell effects depend on the structure and stability of periodic orbits in the system [8].

In the semiclassical approximation it can be shown that the leading contribution to the shell energy (which may be thought of as the interaction energy), for two obstacles being either spherical \((i = 0)\), rod–like \((i = 1)\) or slab–like \((i = 2)\) nuclei located at a distance \(d\) is given by [9]:

\[
E_{\text{shell}} \approx \frac{\hbar^2 L^i R^{2-i}}{8m_n} \left( \frac{3}{\pi} \right)^{(2+i)/4} \left( \frac{\rho_{\text{out}}}{d^{2+i}/4} \right) \cos \left( 2k_F^i d - \frac{i\pi}{4} \right),
\]

where \(m_n\) is the neutron mass, \(k_F^n\) and \(\rho_{\text{out}}\) denote the Fermi momentum and density of unbound neutrons, respectively. We have assumed that rods and slabs are parallel to each other. In this equation, \(L\) defines the length of the obstacle, and \(R\) its radius (in the case of a slab it is defined as half of its width). It turns out that the semiclassical estimations yield energy corrections of the same order of magnitude as the liquid drop energy differences between the various phases of the inhomogeneous nuclear matter [5, 10]. It has lead to the hypothesis that the inner crust may have a quite complicated structure, probably even completely disordered [5].

Unfortunately the semiclassical approach has some drawbacks. First, it assumes that the obstacles are impenetrable scatterers which may overestimate the amplitude of the shell energy. Second, the method lacks of the mutual interplay between the shell energy and the liquid drop energy. Such a coupling is quite important since it allows a part of the shell energy to be “absorbed” into deformation. Hence a microscopic treatment of the problem is needed where both effects, i.e. the one coming from the liquid drop energy (where mainly the surface and Coulomb terms are involved) and the shell energy term are treated on the same footing. Therefore we have applied the Hartree-Fock (HF) approach in the coordinate space with a Skyrme effective interaction (see [9, 11] for details). The analysis of coexistence and stability of different nuclear phases requires construction of

\(^1\)In the Refs.[5, 10] the factor “1/\(\pi\)” has been missed in the expression for the shell energy of rod–like phase. However it does not change the conclusions.
an adiabatic path between different configurations. It has lead us to abandon the Wigner-Seitz approximation for the Coulomb interaction, solving instead the Poisson equation for the electric potential within the box with periodic boundary conditions, the total charge of the box being zero. In our approach the electrons are assumed to form a uniform relativistic gas where screening effects are neglected.

Summarizing, we minimize an energy functional for the neutron-proton-electron matter within the box. The energy functional describes both the liquid drop part of the energy and the shell effects. Namely:

\[ E_{\text{tot}}(\rho_p, \rho_n, \rho_e) = E_{TF}(\rho_p, \rho_n, \rho_e) + E_{\text{shell}}^{\text{in}}(\rho_p^\text{in}, \rho_n^\text{in}) + E_{\text{shell}}^{\text{out}}(\rho_n^\text{out}, \rho_p^\text{out}), \]  

where \( \rho_n, \rho_p, \rho_e \) denote the density of neutrons, protons and electrons, respectively. The charge neutrality condition implies \( \rho_e = \rho_p \). The total nuclear density \( \rho = \rho_n + \rho_p \). In the above expression \( E_{TF} \) represents the Thomas-Fermi energy of the system obtained by constructing the energy functional for a Skyrme interaction. We have used the SLy4 force [12] which give a correct description of neutron matter at low densities [3].

The shell energy consists of two parts. The one denoted by \( E_{\text{shell}}^{\text{in}} \) is determined by the density of neutrons \( \rho_n^\text{in} \) and protons \( \rho_p^\text{in} \) inside the nucleus, whereas \( E_{\text{shell}}^{\text{out}} \) is a function of unbound neutrons \( \rho_n^\text{out} \) and protons \( \rho_p^\text{out} \). Since \( \rho_p^\text{out} = 0 \) for \( \rho < 0.08 \text{ fm}^{-3} \) [3], one can neglect the contribution to the energy due to the unbound protons.

The HF calculations confirm the importance of the shell effects associated with unbound neutrons [9]. In particular, they show that the shell effects associated with unbound neutrons may reverse the phase transition order predicted by the liquid drop based approaches. Moreover, the number of phase transitions increases since the same phase may appear for various density ranges. Our results suggest also that the number of different phases present in the crust is larger since one has to take into account various lattice geometries which may compete. This important fact was disregarded in earlier investigations since in liquid drop based approaches the system favours only one lattice type for a given nuclear shape. However it is no longer the case when one considers the neutron shell energy which is very sensitive to the spatial order in the system.

In Fig.1, we show several nuclear configurations that can be formed at subnuclear densities. One notices the presence of different types of crystal lattices for the same nuclear shapes. The integrated densities are defined by:

\[ \rho_i^z(x, y) = \int_0^d \rho_i(x, y, z)dz, \]  

\[ \rho_i^y(x, z) = \int_0^d \rho_i(x, y, z)dy, \]  

(6)

(7)
where \(i = n, p\) and \(d\) denotes the size of the box.

Once a phase is formed, there is a positional order maintained by the Coulomb repulsion between spherical nuclei, rods or slabs. The Coulomb energy is a smooth function of the void displacement but not the shell energy. Since several orbits contribute to the shell effects (except for the slab–like phase), the displacement of a single bubble–like or rod–like void from its equilibrium position in the lattice will give rise to interference effects [5].

The main conclusion of the present work is that the amplitude of the shell effects is comparable to the energy differences between the various phases determined in simpler liquid drop type models. Our results suggest that the inhomogeneous phase has a complicated structure, maybe completely disordered, with several types of coexisting shapes. The few results highlighted here shows the richness of these systems where both static and dynamic properties are challenging to describe.

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Figure 1. The integrated densities (see text for definition) for various ground state or isomeric nuclear configurations at constant density and $Z/A$ ratio. The size of the box was equal to $d = 26\, fm$ - subfigures a) and b), $d = 23.4\, fm$ - subfigures c) and d), $d = 20.8\, fm$ - subfigures e) and f). The spherical nuclear phase is shown in the subfigures a) (scc) and d) (bcc), rod-like phase - subfigures b) and c), slab-like phase - subfigure e), bubble-like phase: subfigure f). Note different lattice types for the same nuclear shape.
