Spin Hall effect and circular birefringence in polymers

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Spin-Orbit Interactions (SOI) of light refer to the coupling of different internal degrees of freedom of the radiation field, such as polarization and spatial degrees of freedom, as a result of propagation of light in different media. SOIs of light have recently attracted attention in a number of fast growing fields, ranging from photonics, plasmonics, nano-optics and quantum optics to meta-materials [1]. Most SOI effects originate from space- or wavevector-variant geometric phases and result in spin-dependent redistribution of light intensity [1]. When the system has cylindrical symmetry with respect to the z-axis, SOIs produce spin-to-orbital angular momentum conversion, i.e., generation of a spin-dependent vortex in the z-propagating light [1–14]. If the cylindrical symmetry is broken, SOIs bring about the spin-Hall effect of light, i.e., a spin-dependent transverse y-shift of light intensity [12–20]. An example of the latter effect is the so-called transverse Imbert–Fedorov (IF) beam shift or spin Hall effect of light, which occurs when a paraxial optical beam is reflected or refracted at a plane interface [18–20].

Up to present, spin-Hall effect has been observed for various material and geometrical settings, such as gratings [3, 9], liquid crystals [4], dielectric crystals [10], metal [21] and magnetic [22] films, uniaxial birefringent crystals [23], graphene [24]. Furthermore, recently plasmonic [25] and dielectric [26] meta-surfaces [27], as well as by hyperbolic metamaterials [28], have been intensively studied due to their ability to manipulate polarization states by the design of artificial surface structures in subwavelength scale. Photonic spin-Hall effect offers device applications ranging from spin-dependent beam splitters [27] to surface sensors [29].

In this letter, we demonstrate experimentally that the spin-Hall effect of light and the transverse spin dependent beam shift appears in the light transmission through a birefringent polymer with a tilted anisotropy axis as illustrated in Fig. 1. This new type of spin-Hall effect is quite surprising for traditional optics, as it has recently been demonstrated for a Quartz crystal plate [23] because it implies weak circular birefringence of a uniaxial crystal plate. To the best of our knowledge, this is the first time this effect is reported in a polymeric material. Polymers are widely used as optical materials from transmission media to light sources with the advantages of low processing cost, mechanical flexibility, ease of large area fabrication and so on [30]. Moreover, birefringent polymers are typically produced by electronic modulation, such as liquid crystals [31] or by stress induced mechanical effects [32]. Therefore, this provides an external means of control for the birefringence induced in the polymer and could in turn be used as an optical switch in the nanometer range, as opposed to natural crystals or metal where the birefringence is fixed by the parameters of the media.

The complete theory for light transverse shifts in uniaxial crystals was described in [23]. For an input state described by a Jones vector |ψ⟩ and a transmitted state described by Jones vector |ψ′⟩, the anisotropic transverse shift or spin-Hall effect of light is given by the expectation value of the position operator:

\begin{equation}
\langle Y \rangle = \langle \psi' | \hat{Y} | \psi' \rangle = \frac{\cot(\theta)}{k} [-\sigma(1 - \cos(\phi_0)) + \chi \sin(\phi_0)],
\end{equation}

where \(\phi_0\) represent the phase difference between ordinary and extraordinary wave propagating through the birefringent medium.

The spin-Hall effect can be measured either directly, via subwavelength shift [Eq. (1)] of the beam centroid [18, 19, 33], or via various other methods including quantum weak measurements [20, 34–39]. The latter method allows for significant amplification of the shift using almost crossed polarizers at the input and

![Fig. 1. 3D geometry illustrating transmission of a paraxial beam through a tilted transparent polymer film. The beam experiences nano-meter scale transverse shift \(\langle Y \rangle\) due to spin Hall effect in a polymer. The paraxial angles \((\Theta_y, \Theta_z)\) determine the direction of propagation of the wave vectors \(k\) in the incident beam.](image-url)

\[\text{Image 333x378 to 558x546}\]
output of the system, respectively. As before, the input polarizer corresponds to a pre-selected state $|\psi\rangle = (a, \beta)^T$ (where $T$ stands for transpose operation), while the output polarizer corresponds to another post-selected polarization state $|\psi\rangle = (\alpha', \beta')^T$. The resulting beam shifts are determined by the weak value $\langle Y \rangle_{\text{weak}}$ instead of expectation values $\langle Y \rangle$, which can exhibit quantum weak amplification effect and lay outside the bounds of the spectrum of the operator. We analyze quantum weak amplification of the spin-Hall effect shift, considering an initial beam with polarization $|\psi\rangle = (1, 0)^T$, while the post-selection polarizer is nearly orthogonal $|\psi\rangle = (\epsilon, 1)^T, |\epsilon| << 1$. The weak value yields:

$$\langle Y \rangle_{\text{weak}} = \frac{1}{\epsilon k} \sin(\phi_0) \cot(\theta) + \frac{z}{\epsilon k} (1 - \cot(\phi_0)) \cot(\theta), \tag{2}$$

where $z_R$ is the Rayleigh length. The second (angular) term, becomes dominant in the far field zone, and presents weak amplification due to two reasons. Firstly, because $|\epsilon| << 1$, and secondly because $z >> z_R$. Note that the maximal achievable weak amplification at $|\epsilon| \approx (k v_0)^{-1}$ is of the order of the beam waist $\omega_0 z / z_R$.

To determine the classical beam shift we calculate the expectation value of the centroid displacement for a birefringent polymer, modeled as a tilted Quartz plate of thickness $d = 50 \mu m$. The phase difference can be expressed as:

$$\phi_0(\theta) = k [n_o d_o(\theta) - n_e d_e(\theta)]. \tag{3}$$

Here, $n_o = 1.54$ is the refractive index for the ordinary wave, $n_e = n_o / \sqrt{n_o \cos(\theta)} + n_o \sin(\theta)$ is the refractive index for the extraordinary wave propagating at the angle $\theta$ to the optical axis, $n_o = n_e (\pi / 2) = 1.60$, and the distances of propagation of the ordinary and extraordinary rays in the tilted plate are:

$$d_o(\theta) = \frac{n_o d}{\sqrt{n_o^2 - \cos(\theta)^2}}, \quad d_e(\theta) = \frac{n_e d}{\sqrt{n_e^2 - \cos(\theta)^2}}. \tag{4}$$

Using Eqs. (1), (2) and (3), in Fig. 2 (a) and (b) we plot the phase difference and spin-Hall shifts (blue curves) as functions of the tilt angle $\theta$. One can see that the transverse shift $\langle Y \rangle$ due to the spin-Hall effect reaches wavelength-order magnitude, typical for other spin-Hall systems in optics [20–24]. In contrast to the IF shift in the reflection/refraction problems, here the transverse shift as a function of $v$ displays two-scale behavior. Namely, the fast oscillations in Fig. 2(b) originate from the term $1 - \cos(\phi_0)$, whereas the slow envelope corresponds to the universal cot(\theta) factor in SOI terms.

To verify the above theoretical predictions, we performed a series of experimental measurements using the setups shown in Fig. 3. We use a sample of free-standing birefringent polymer foil, similar to the type Newport 05RP32-1064. As a source of incident Gaussian beam, we employed a He-Ne laser (Melles Griot Griot 05-LHR-111) of wavelength $\lambda = 633$ nm. The laser radiation was collimated using a microscope objective lens. We measure the anisotropic phase difference $\phi_0$ versus the angle of the tilt $\nu$ via Stokes polarimetry methods [40]. For this purpose we used the setup shown in Fig. 1(a). The double Glan–Laser polarizer (Thorlabs GL10) (P1) selected the desired linear-polarization state in the incident beam. In the first experiment, this was $45^\circ$ polarization, i.e., $a = \beta = 1 / \sqrt{2}$. The beam then propagates through the polymer, and the Stokes parameters are measured using a Quarter Wave-Plate (QWP) at a retardation angle $\delta$, and a second polarizer P2, with angle $\gamma$, as indicated in Fig. 3(a). The phase difference can be obtained via the Stokes parameters using the expression:

$$\phi_0 = \tan^{-1} \left( \frac{S_3}{S_2} \right), \tag{5}$$

where $S_1 = I(90^\circ, 45^\circ) - I(90^\circ, 135^\circ)$ is the normalized Stokes parameter for circular polarization, and $S_2 = I(0^\circ, 45^\circ) - I(0^\circ, 135^\circ)$ is the normalized Stokes parameter in the diagonal basis, where the normalization factor $S_0$ is given by the total intensity of the beam. The measured phase using Eq. (5) is wrapped in the range $(-\pi, \pi)$. In order to determine the unwrapped phase difference we use an unwrapping algorithm [23], with a tolerance set to 0.001 radians. The measured unwrapped phase difference is displayed in Fig. 2(a) (purple dots). The experimental spin Hall effect using Stokes polarimetry is shown in Fig. 2(b) (purple dots). The agreement between experiment and theory is apparent. We note that we observe a spin-Hall effect via Stokes polarimetry ($k < Y >$) using a $50 \mu m$ polymer film which is 10 times larger than the shift observed in Ref. [23], for a $1000 \mu m$ Quartz sample. We ascribe this increase to the larger effective birefringence in the polymers [32].

Next, we performed the weak measurement of spin Hall shift, and observed quantum weak amplification effect using the quantum weak measurement setup in Fig. 3(b). The beam...
We reported experimental observations of this nano-meter scale amplification factor becomes pre-selected and post-selected states, with polarization states polarimetric and (b) quantum weak measurements. P1 and PDE 2017, Argentina; Villum Fonden DarkSILD project, Den- ica, PICT2015-0710 Startup, UBACyT PDE 2016 and UBACYT Funding. Agencia Nacional de Promocion Cientifica y Tecnolog- ival circular birefringence of a tilted birefringent polymer, the approximately $\Delta = \pi k z / Y = 1000$; the amplification factor due to crossed polarizers results $1/e \approx 1.83 \times 10^{-2}$ For $k = \frac{2\pi}{\lambda}$, the overall weak amplification factor becomes $A = \frac{2\pi}{\lambda} \approx \frac{2\pi}{\lambda}$, this is confirmed in the experiment where a displacement between centroids of $\Delta Y = 1000\mu m$ between post-selection polarizers oriented at $\epsilon = -1/1.83 \times 10^{-2}$ (Fig. 4(a)), and $\epsilon = +1/1.83 \times 10^{-2}$ (Fig. 4(c)) is measured at a tilt angle $\theta = 20^\circ$, thus amplifying the spin Hall effect by a factor $A = 200$. For crossed polarizers ($\epsilon = 0$), the input Gaussian beam is split into a Hermite-Gaussian distribution (Fig. 4(b)), and the separation of the two peaks is approximately $\Delta Y = 1000\mu m$ in $< Y >$.

In conclusion, we demonstrated experimentally the fine lateral circular birefringence of a tilted birefringent polymer, the first example of the spin-Hall effect of light in a polymer material. We reported experimental observations of this nano-meter scale effect using Stokes polarimetry techniques and quantum-weak-measurement techniques, reporting a quantum weak amplification factor of 200. The birefringence in the polymer can be tuned using voltage in the case liquid crystals or using mechanical stress in the case of stress-induced birefringence, therefore this lateral shift could be used as an optical switch in the nano-meter scale, opening the doors to a myriad of novel applications in photonics, nano-optics, quantum optics, and metamaterials.

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