Research on Quasi-geoid Fitting Method Based on EGM2008 Model and Quadratic Polynomial Fitting Function

Jiaxin Huo, Xin Yang*
College of Chemical Engineering, Harbin Institute of Petroleum, Harbin, Heilongjiang, 150000, China
*Corresponding author’s e-mail: 3600282@163.com

Abstract. Obtaining the elevation of ground points is a basic work for studying the shape of the earth and engineering construction, and can be used in civil construction, water conservancy engineering, mineral engineering, etc. Aiming at the problem of low leveling accuracy in complex mountainous area, this paper proposes a "remove-restore" leveling method based on EGM2008 model and quadratic polynomial fitting. Taking an example of a complex mountainous area, the results show that the external accuracy of this method can reach 4.3 cm.

1. Introduction
Level fitting is a hot research direction in the field of engineering surveying and mapping. The emergence of satellite positioning technology has had a huge impact on traditional geodesy, overcoming the time and space limitations of traditional geodetic technology, and provides new technologies and means for geodetic surveying, with the release of the 2008 Earth Gravity Field Model by the National Space Intelligence Agency, the accuracy and reliability of leveling fitting using the Earth’s gravity model combined with GPS data have been greatly improved [1-3]. Using the EGM2008 Earth's gravity field model combined with the "remove-restore" method is becoming a fast method for obtaining high-precision leveling fittings. This paper proposes a method that uses the EGM2008 gravity field model, combined with the "remove-restore method", and uses a quadratic polynomial fitting method to achieve GPS leveling fitting.

2. Principle Method

2.1 Quadratic polynomial fitting method
Quadratic polynomial fitting method is often used in leveling fitting [4,5]. The expression between elevation anomaly and plane coordinates is as follows:

\[ \zeta = \alpha_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i \] (1)

In the formula, \( \alpha_0, a_1, a_2, a_3, a_4, a_5 \) are the parameters to be determined.

Therefore, this area requires at least 6 known points. When there are more than 6 known points, the corresponding error equations can be listed:

\[ V_i = \alpha_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i - \zeta_i \] (2)
The matrix form is: \( \mathbf{V} = \mathbf{X}\mathbf{A} - \mathbf{\zeta} \). The solution of the height anomaly of unknown point is to determine the coefficient of polynomial. Under the condition of \( \sum V^2 = \min \), \( \mathbf{A}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V} \) can be obtained according to the principle of least squares method, and then the height anomaly of any point can be obtained.

2.2 EGM2008 Model

According to Bruns formula, the height anomaly at any point on the ground can be obtained\[6,7\],

\[
\zeta_{\text{EGM}} = \frac{GM}{\rho \gamma} \sum_{n=2}^{190} \sum_{m=0}^{n} \left( \overline{C}_{mn} \cos(m\lambda) + \overline{S}_{mn} \sin(m\lambda) \right) \overline{P}_{mn}(\sin \phi) \tag{4}
\]

In the formula, \( GM \) is the gravitational constant; \( \rho, \gamma, \lambda \) and \( \phi \) are the geocentric diameter, normal gravity value; longitude and latitude of the points; \( a \) is the ellipsoid radius; \( \overline{C}_{mn} \) and \( \overline{S}_{mn} \) are the fully normalized potential coefficients; and \( \overline{P}_{mn}(\sin \phi) \) is the fully normalized associated function\[8\].

The EGM2008 earth field gravity model is a high-precision global gravity field model launched by the NGA at the European Geoscience Union conference in 2008. The EGM2008 model is based on PGM2007 as a reference frame, combined with gravity and altimetry data measured by GRACE satellites and TOPEX satellites. The high-precision earth gravitational field model constructed by high-precision DEM data has a maximum order of 2190 and a spatial resolution of 5'. EGM2008 provides global 5'×5', 2.5'×2.5' geoid grid data, and the final results of the global 5'×5' grid vertical deviation, as shown in Table 1.

| Area                | Number of GPS leveling points | Standard deviation |
|---------------------|------------------------------|-------------------|
| Global              | 12353                        | 13.0              |
| Continental U.S.    | 4201                         | 7.1               |
| Australia           | 534                          | 26.6              |

2.3 Remove-restore Principle

According to the theory of physical geodesy, elevation anomaly can be expressed as\[9\]:

\[
\zeta = \zeta^G + \zeta^{AG} + \zeta^T \tag{5}
\]

In the formula, \( \zeta^G \) is the long wave term calculated by gravity field model; \( \zeta^{AG} \) is the medium wave part, which can be obtained by solving the boundary value problem of gravity anomaly; \( \zeta^T \) is the short wave part, which can be obtained by solving topographic correction. In this paper, part \( \zeta^{AG} \) and part \( \zeta^T \) are combined and fitted by mathematical model\[6\].

\[
\zeta = \zeta^{\text{EGM}} + \zeta^{\text{res}} \tag{6}
\]
The idea of "remove-restore" is to remove $\zeta^{EGM}$ from the elevation anomaly and get $\zeta^{res}$, and to fit it with known level points. In this paper, quadratic polynomials and polyhedral functions will be used for fitting.

The principle of the "remove-restore" method in GPS leveling fitting is that the known points are first removed from the model elevation anomaly obtained through the earth's gravity field model to obtain the remaining elevation anomaly, and then the remaining elevation anomaly part is performed through the fitting model. Fit, get the GPS level fitting function, and finally get the elevation anomaly of the point.

3. Analysis of Level Fitting Experiments

3.1 Experimental area

A complex mountainous area is selected as the survey area, which is located at $40^\circ01'N$~$41^\circ31'N$, $113^\circ55'E$~$115^\circ24'E$, among which 15 third-class GPS leveling joint measurement points are selected as experiments. According to the data, the highest point is 1513.438m, the lowest point is 615.470m, the maximum height difference is about 897.968m, the height difference is large, and the terrain is undulating. The point distribution is shown in Figure 1.

![Figure 1. GPS points](image)

3.2 Experimental scheme

- Scheme 1: Directly use the quadratic polynomial method for level fitting.
- Scheme 2: EGM2008 model and the "remove and restore" method of quadratic polynomial method. The EGM2008 model is used to solve the residual elevation model, the residual elevation anomaly is solved according to the known point elevation, and the quadratic polynomial method is used to fit the residual elevation.

The fitting residuals are shown in the table 2 below:
Table 2. Fitting Residual Table

| Number | Scheme 1/m | Scheme 2/m |
|--------|-----------|-----------|
| 1      | 11.5      | -0.5      |
| 2      | -43.6     | -2.1      |
| 3      | 16.5      | 0.5       |
| 4      | 26.2      | 3.3       |
| 5      | -31.4     | 0.0       |
| 6      | -3.7      | -0.6      |
| 7      | 24.6      | -1.8      |
| 8      | -18.4     | 0.6       |
| 9      | 14.0      | -1.9      |
| 10     | 4.4       | 2.4       |
| 11     | 27.0      | -1.5      |
| 12     | -14.9     | 4.8       |
| 13     | 65.7      | 0.5       |
| 14     | 10.7      | -6.7      |
| 15     | 40.7      | -1.6      |

3.3 Analysis of Level Fitting Accuracy

The fitting accuracy of various level fitting methods is evaluated by internal and external coincidence accuracy, so as to analyze and compare the advantages and disadvantages of various level fitting methods.

Table 3. Fitting Accuracy Table

| Programmes | Programme 1 | Programme 2 |
|------------|-------------|-------------|
| Internal accuracy /cm | 23.9 | 1.8 |
| External accuracy /cm  | 42.0 | 4.3 |

(1) The internal coincidence accuracy of scheme 1 is 23.9 cm, and scheme 2 is 1.8 cm. In the second scheme, the coincidence accuracy is increased by about 90%, and the accuracy is greatly improved.
(2) The external accuracy of scheme 1 is 42.0 cm, and scheme 2 is 4.3 cm. The accuracy is relatively improved by about 90%.

4. Conclusion

Through the experiments, it can be concluded that in the level fitting of mountain area, the accuracy of scheme 2 is significantly higher than scheme 1, and the accuracy of internal and external compliance with scheme 2 has been greatly improved, indicating that this method has high fitting accuracy in the experiment and can be better the ground fitting locally resembles a geoid. The fitting accuracy of EGM2008 combined with quadratic polynomial fitting method is higher.

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