We consider the counter-term describing isoscalar axial two-body currents in the nucleon-nucleon interaction, $L_{1A}$, in the effective field theory approach. We determine this quantity using the solar neutrino data. We investigate the variation of $L_{1A}$ when different sets of data are used.

1. Introduction

Few body reactions play an important role in astrophysics and cosmology. In recent years nuclear effective field theories were developed for few nucleon systems. The question we wish to address here is if astrophysical data (in particular solar neutrino data) can be used to constrain effective field theory description of nuclear reactions.

The goal of effective field theories is to find an appropriate way to integrate over the undesired degrees of freedom. For example one may wish to write an effective theory of photon interactions by integrating over the charged elementary particles in quantum electrodynamics. To represent the photon-photon interaction one may introduce a point interaction of the photons instead of the square box diagram with four external photon lines and charged-particles circulating in the loop integral. However, such a recipe produces a divergent diagram when we go to the next order with one photon loop in the effective theory (the equivalent diagram with two charged particle loops in the original theory, quantum electrodynamics, is normalizable). To circumvent this problem one introduces counter terms in the effective theory to cancel the infinities. Such counter terms should of course be consistent with the symmetries of the original theory.
The effective field theories can be applied to the neutrino-deuteron reactions measured at the Sudbury Neutrino Observatory (SNO)

\begin{align*}
\nu_e + d &\rightarrow e^- + p + p, \\
\nu_x + d &\rightarrow \nu_x + p + n.
\end{align*}

(1)

At low energies below the pion threshold, the \(^3S_1 \rightarrow ^3S_0\) transition dominates these reactions and one only needs the coefficient of the two-body counter term, so called \(L_{1A}\)\(^2,3,4\). This term can be obtained by comparing the cross section \(\sigma(E) = \sigma_0(E) + L_{1A}\sigma_1(E)\) with either cross sections calculated using other, more traditional, nuclear physics approaches \(^5,6,7\) or with direct measurements. Note that once the quantity \(L_{1A}\) is determined one can easily calculate related two-body reactions such as

\begin{equation}
p + p \rightarrow d + \nu_e + e^+.
\end{equation}

(3)

The reaction in Eq. (3) is impossible to measure for the very-low energies in the solar core. For that reason the process of determining \(L_{1A}\) was dubbed “Calibrating the Sun” by Holstein.\(^8\) Note that naive dimensional analysis predicts a value of \(|L_{1A}| \sim 6 \text{ fm}^3\) when the renormalization scale is set to the muon mass. Since the value of \(L_{1A}\) depends on the renormalization scale, this dimensional estimate cannot be used at lower energies.

2. Extraction of \(L_{1A}\)

In our calculations we used the neutrino cross sections given in Refs. 2 and 3. The details of the analysis method and procedures used to calculate the measured neutrino rates and spectra are described in Refs. 9 and 10. To calculate the MSW survival probabilities we used the neutrino spectra and solar electron density profile given by the Standard Solar Model of Bahcall and collaborators.\(^11\) In the calculations presented in this section we took the mixing angle \(\theta_{13}\) to be zero. What we present below is an improved version of our analysis in Ref. 12.

We calculate \(\chi^2\) marginalized over the neutrino parameters \(\delta m^2_{12}\) and \(\theta_{12}\). In Figure 1 we present the quantity \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) calculated as a function of \(L_{1A}\). In this figure \(\Delta \chi^2\) is projected only on one parameter \((L_{1A})\) so that \(n - \sigma\) bounds on it are given by \(\Delta \chi^2 = n^2\). In our global fit we used 93 data points from solar and reactor neutrino experiments; namely the total rate of the chlorine experiment (Homestake\(^13\)), the average rate of the gallium experiments (SAGE\(^14\), GALLEX\(^15\), GNO\(^16\)), 44 data points from the SuperKamiokande (SK) zenith-angle-spectrum,\(^17\) 34 data points from the SNO day-night-spectrum,\(^18\) the neutral current flux measurement at SNO using dissolved salt\(^19\) and 13 data points from the KamLAND spectrum\(^20\). In Figure 1 we show the \(L_{1A}\) values obtained using only the solar neutrino data from SNO as well as various combinations of other experiments. Clearly the \(\chi^2\) minimum is almost the same in all these cases. In all these cases we obtain a best fit value of \(L_{1A}\) around 5 \text{ fm}^3. It should be noted that one and two sigma
 bounds on $L_{1A}$ get significantly reduced when, in addition to SNO, one includes other solar neutrino data as well. The $L_{1A}$ values change between 4 and 8 fm$^3$ with a one-sigma error of 5 fm$^3$.

The dependence of the extracted neutrino parameters on the value of $L_{1A}$ is not very strong. We show how the neutrino parameter space changes with $L_{1A}$ in Figures 2 and 3. The analysis presented in Figure 2 uses only the solar neutrino data as input whereas that presented in Figure 3 uses both the solar neutrino data and results from the KamLAND reactor neutrino measurements.
Fig. 2. The change in the allowed region of the mixing parameter space using only the solar neutrino data as a function of $L_{1A}$. In the calculations leading to this figure the neutrino mixing angle $\theta_{13}$ is taken to be zero. The shaded areas correspond to 90%, 95%, 99%, and 99.73% confidence levels.

Fig. 3. The change in the allowed region of the mixing parameter space using combined solar neutrino data and KamLAND as a function of $L_{1A}$. In the calculations leading to this figure the neutrino mixing angle $\theta_{13}$ is taken to be zero. The shaded areas corresponds to 90%, 95%, 99%, and 99.73% confidence levels.
3. Exploring the effects of $\theta_{13}$

One of the open questions in neutrino physics is understanding the role of mixing between the first and third flavor generations, $\theta_{13}$. Since both of the quantities $\theta_{13}$ and $L_{1A}$ are rather small one can investigate if the uncertainties coming from the lack of knowledge of $\theta_{13}$ and the counter-term $L_{1A}$ are comparable. In the limiting case of small $\cos \theta_{13}$ and $\delta m^2_{31} \gg \delta m^2_{21}$, which seems to be satisfied by the measured neutrino properties, it is possible to incorporate the effects of $\theta_{13}$ rather easily. In this limit the charged-current counting rate at SNO can be linearized in $\cos^4 \theta_{13}$:

$$\text{Count Rate} \sim A + B (1 - \cos^4 \theta_{13}),$$

where the parameters $A$ and $B$ are independent of $\theta_{13}$. The neutral- and charged-current counting rates linearly depend on $L_{1A}$ while elastic scattering rate does not. Conversely the charged-current and elastic scattering rates linearly depend on $\cos^4 \theta_{13}$ while the neutral-current rate does not. We show the allowed $\theta_{13}$ and $L_{1A}$ parameter space in Figure 4 when $\theta_{12}$ and $\delta m^2_{12}$ are taken to give the minimum $\chi^2$ values to fit the data.

![Figure 4](attachment:image.png)

**Fig. 4.** Allowed parameter space for $L_{1A}$ vs $\theta_{13}$ when $\chi^2$ is marginalized over $\theta_{12}$ and $\delta m^2_{12}$. All solar neutrino experiments along with the KamLAND experiment considered. The shaded areas are the 90%, 95%, 99%, and 99.73% confidence levels. Horizontal lines show $\theta_{13}$ bounds from CHOOZ + SK.

Of course there are other experiments that limit the value of $\theta_{13}$. The CHOOZ$^{21}$ and Palo Verde$^{22}$ experiments limit $\sin^2 2\theta_{13}$ to be less than 0.19 at 90% C.L. for $\delta m^2_{\text{atmos}} = 0.002 \text{ eV}^2$. Data from the K2K experiment$^{23}$ provides a limit of $\sin^2 2\theta_{13} < 0.3$. These limits are consistent with SK atmospheric neutrino data.$^{24}$
We also show these bounds in Figure 4. Note that future data from SNO and KamLAND may further limit the value of $\theta_{13}$.\textsuperscript{25}

### 4. Conclusions

Several other authors tried to estimate the value of $L_{1A}$. Using SNO and SK data Chen, Heeger, and Robertson found\textsuperscript{26}

\begin{equation}
L_{1A} = 4.0 \pm 6.3 \text{ fm}^3.
\end{equation}

As we mentioned in the Introduction, the $pp$ fusion cross section also depends on $L_{1A}$. State of the art calculations of this cross section implies a value of\textsuperscript{27}

\begin{equation}
L_{1A} = 4.2 \pm 2.4 \text{ fm}^3
\end{equation}

in third order of power counting. (This is the same order in which the neutrino-deuteron cross-section of Refs. 2 and 3 are calculated).

Helioseismic observation of the pressure-mode oscillations of the Sun can be used to put constraints on various inputs into the Standard Solar Model, in particular the $pp$ fusion cross section. Helioseismology gives a limit of\textsuperscript{28}

\begin{equation}
L_{1A} = 4.8 \pm 6.7 \text{ fm}^3
\end{equation}

in the third order. Finally one can present a constraint on $L_{1,A}$ imposed by existing reactor antineutrino-deuteron breakup data,\textsuperscript{29} which yields

\begin{equation}
L_{1A} = 3.6 \pm 5.5 \text{ fm}^3.
\end{equation}

Various values of $L_{1,A}$ we obtained using different data sets are rather comparable to the values listed above.

The uncertainties in the neutrino-deuteron breakup cross-sections at low energies are dominated by the isovector axial two-body current parametrized by $L_{1A}$. However the contribution of the uncertainty in $L_{1A}$ to the analysis and interpretation of the SNO data is rather small.\textsuperscript{29,12} The effect of this uncertainty is even smaller than effects of a value of $\theta_{13}$ near its currently allowed maximum or than effects of possible solar density fluctuations.\textsuperscript{30}

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