On Mixed $b$-Fuzzy Topological Spaces

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Abstract
This paper describes the construction of a topological space from two different fuzzy topologies using the fuzzy $b$-$q$-nbd of a fuzzy point with respect to one topology and the fuzzy $b$-closure of a fuzzy set with respect to another topology. Additionally, some relationships are established between the two adopted topologies and the resulting mixed topology. Finally, we define countability on mixed fuzzy topological spaces and investigate the various quasi-type properties of such spaces.

Keywords: Fuzzy $b$-open set, Fuzzy $b$-cluster point, Fuzzy $b$-regularly open sets, Mixed topology, Fuzzy $b$-closure, Fuzzy $b$-$q$-nbd, Fuzzy $b$-closed set

1. Introduction and Preliminaries

The study of mixed topologies expanded based the work of the Polish mathematicians Alexiewicz and Semadini, but it was first invented by Finchtenholz of the Polish School of Mathematics in 1938. Topology mixing is a technique for combining two topologies on the same set to obtain a third topology. Das and Baishya [1] constructed a fuzzy topology called a mixed fuzzy topology from two given topologies on a set $X$ based on the closure of the neighborhoods of one topology with respect to the other topology and then studied various properties of the resulting topology.

Different properties of fuzzy topological spaces have been investigated by Arya and Singal [2, 3], Chang [4], Das and Baishya [1], Ganster et al. [5], Ganguly and Singha [6], Ghanim et al. [7], Tripathy and Debnath [8], etc. Recently, mixed fuzzy topological spaces have been investigated from different perspectives by Das and Baishya [1], Tripathy and Ray [9,10], etc.

Azad [11] defined the concept of fuzzy regular open and fuzzy regular closed sets. Benchalli and Karnel [12] defined the concepts of fuzzy $b$-open sets and fuzzy $b$-continuous mappings in fuzzy topological spaces. In this paper, we introduce and study the concept of mixed $b$-fuzzy sets on mixed fuzzy topological spaces. We investigate this concept in the context of $b$-$q$-neighborhoods, $b$-$q$-coincidence, fuzzy $b$-closures, and fuzzy $b$-interiors. Additionally, some relations are established between the two main topologies considered and their corresponding mixed fuzzy topological spaces. Most of the concepts, notations, and definitions used in this paper are standard, but for the sake of completeness, we recall some basic definitions and results below.

Definition 1.1 ([4]). A fuzzy point in $X$ is a fuzzy set in $X$ and is zero everywhere, except at one point (e.g., $x$), where it takes value such as $r$ with $r \in (0, 1)$ (i.e., $0 < r < 1$). We denote this set as $x_r$, where the point $x$ is its support and $r$ is its value.
Definition 1.2 (4). A fuzzy set $A$ in $\mathcal{X}$ is called $q$-coincident with a fuzzy set $B$ in $\mathcal{X}$, denoted as $AgB$, if and only if $A(x) + B(x) > 1$ for some $x \in \mathcal{X}$. It is clear that if $x_rqA$, then $r + A(x) \leq 1$ for every $x \in \mathcal{X}$ and if $AgB$, then $A(x) + B(x) \leq 1$ for every $x \in \mathcal{X}$.

Let $A$ be a fuzzy subset of a space $\mathcal{X}$. The fuzzy closure of $A$ and fuzzy interior of $A$ are denoted as $Cl(A)$ and $Int(A)$, respectively, and are defined by $Cl(r) = \bigcap\{u : u$ is a closed fuzzy set in $\mathcal{X}$ and $A \subseteq u\}$ and $Int(r) = \bigcap\{v : v$ is an open fuzzy set in $\mathcal{X}$ and $v \subseteq r\}$, respectively. Clearly, $Cl(r)$ ($Int(r)$) is the smallest (largest) closed (open) fuzzy set in $X$ containing (contained in) $r$. If there is more than one topologies on $X$, then the closure and interior of $r$ with respect to a fuzzy topology $\tau$ on $X$ are denoted as $\tau - Cl(r)$ and $\tau - Int(r)$, respectively. A fuzzy subset $r$ of a space $X$ is called fuzzy-pre-open [13] if $r \leq Int(Cl(r))$. The complement of a fuzzy-pre-open set is called a fuzzy-pre-closed set.

Definition 1.3. A fuzzy singleton $x_r$ in $\mathcal{X}$ is said to be quasi-coincident (q-coincident) with a fuzzy set $A$ in $\mathcal{X}$, denoted as $x_rqA$, if and only if $r + A(x) > 1$.

Definition 1.4. A fuzzy set $A$ in a fuzzy time series (FTS) $(\mathcal{X}, T)$ is called a quasi-neighborhood (q-nbd) of $x_r$ if and only if $A \subseteq T$ such that $A \subseteq A$ and $x_rqA_1$. The family of all q-neighborhoods of $x_r$ is called the system of q-neighborhoods of $x_r$. The intersection of two quasi-neighborhoods of $x_r$ is also a quasi-neighborhood. Let $(\mathcal{X}, \mathcal{T})$ and $(\mathcal{X}, \mathcal{T}_1)$ be two fuzzy topological spaces and let $\mathcal{T}_1(\mathcal{T}_2) = \{A \in I^{\mathcal{X}} :$ for every fuzzy set $B$ in $\mathcal{X}$ with $AgB$, there exists a $\mathcal{T}_2$-q-nbd $A_\alpha$ such that $A_\alphaqB$ and we have a $\mathcal{T}_1$-closed $A_\alpha \subseteq B$. Then, $\mathcal{T}_1(\mathcal{T}_2)$ is a fuzzy topology on $\mathcal{X}$ called a mixed fuzzy topology and the space $(\mathcal{X}, \mathcal{T}_1(\mathcal{T}_2))$ is called a mixed fuzzy topological space.

Definition 1.5. A fuzzy set $A$ in a fuzzy topological space $\mathcal{X}$ is called fuzzy $b$-open [2] if $A \leq Int(Cl(A)) \lor Int(Cl(A)) \geq A$. The complement of a fuzzy $b$-open set is called a fuzzy $b$-closed set (i.e., $Int(Cl(A)) \land Cl(Int(A)) \geq A$).

Definition 1.6 (12). A fuzzy set $A$ in a fuzzy topological space $\mathcal{X}$ is called a fuzzy $b$-q-nbd of a fuzzy point $x_r$ in $\mathcal{X}$ if there is a fuzzy $b$-open set $V$ in $X$ such that $x_rqV \leq A$. Additionally, if $A$ is fuzzy $b$-open, then $A$ will be referred to as a fuzzy $b$-open q-nbd of $x_r$.

Lemma 1.7[2]. In a fuzzy topological space $(\mathcal{X}, \mathcal{T})$. Every fuzzy pre-open set is also fuzzy $b$-open.

Definition 1.8 (1[1][4]). Let $A$ be a fuzzy set in an FTS $\mathcal{X}$. Then, $bCl(A) = \land\{u \geq A : u$ is a $fb$-closed set of $\mathcal{X}\}$. Additionally, because $bInt(A) = \lor\{v \leq A, v$ is an $fb$-open set of $\mathcal{X}\}$.

Theorem 1.9 (14). An FTS $\mathcal{X}$ is $fb$-open (fb-closed) if and only if $A = bCl(S)$ ($bInt(S)$).

Remark 1.10 (14). A fuzzy set $A$ in a fuzzy topological space $\mathcal{X}$ is fuzzy $e$-closed if and only if $A = bCl(A)$.

2. Main Results

In this section, we discuss the construction of a mixed fuzzy topology with help from the fuzzy $b$-q-nbd of a fuzzy point.

Definition 2.1. A fuzzy point $x_r$ in a fuzzy topological space $\mathcal{X}$ is called a fuzzy $b$-cluster point of a fuzzy set $A$ in $\mathcal{X}$ if every fuzzy $b$-q-nbd of $x_r$ is $q$-coincident with $A$. The union of all fuzzy $b$-cluster points in $A$ is called the fuzzy $b$-closure of $A$ and is denoted as $bCl(A)$.

Theorem 2.2. Let $(\mathcal{X}, (\mathcal{T}_1(\mathcal{T}_2)))$ be a mixed $b$-topological space. If $U_{aa} = \{U \in I^{\mathcal{X}} :$ for every fuzzy point $x_a$, there exists a fuzzy $\mathcal{T}_2$- $b$-set $V$ in $\mathcal{X}$ such that $x_aqV$ and $\mathcal{T}_1 - bCl(V) \leq U\}$, then there is a fuzzy topology $\mathcal{T}_1(\mathcal{T}_2)$ for which $U_{aa}$ is a fuzzy $b$-q-nbd system of $x_a$ under the condition that the intersection of two $b$-fuzzy sets is also $b$-fuzzy.

Proof. (i) Let $x_a$ be a fuzzy point and $U \in U_{aa}$. We must demonstrate that $x_aqU$. Because $U \in U_{aa}$, there is a fuzzy $\mathcal{T}_2$- $b$-set $V$ in $X$ such that $x_aqV$ and $\mathcal{T}_1 - bCl(V) \leq U$. Now, $x_aqV$ implies $a + V(x) > 1$. Additionally, $V \leq 1 - bCl(V) \leq U$. Therefore, $V(x) \leq U(x)$. Finally, $a + U(x) \geq a + V(x) > 1$, which implies $a + U(x) > 1$, meaning we have $x_aqU$.

(ii) Let $U, V \in U_{aa}$. Then, there are fuzzy $\mathcal{T}_2$-$b$-sets $U_1$ and $V_1$ such that $x_aqU_1$ with $\mathcal{T}_1 - bCl(U_1) \leq U$ and $x_aqV_1$ with $\mathcal{T}_1 - bCl(V_1) \leq V$, respectively. Now, $x_aqU_1$ and $x_aqV_1$

\[\Rightarrow a + U_1(x) > 1 \land a + V_1(x) > 1\]

\[\Rightarrow a + \min\{U_1(x), V_1(x)\} > 1 \Rightarrow a + (U_1 \lor V_1)(x) > 1\]

\[\Rightarrow x_aq(U_1 \lor V_1).

Because $U_1$ and $V_1$ are fuzzy $\mathcal{T}_2$-$b$-sets, according to the given condition, $(U_1 \lor V_1)$ is a fuzzy $\mathcal{T}_2$-$b$-set. Additionally, $\mathcal{T}_1 - bCl(U_1) \lor V_1 \leq \mathcal{T}_1 - bCl(U_1) \lor (\mathcal{T}_1 - bCl(V_1) \leq U \lor V)$. Therefore, $U \lor V \in U_{aa}$.

(iii) Let $U$ and $V$ be two fuzzy sets such that $U \in U_{aa}$ and $U \leq V$. We must demonstrate that $V \in U_{aa}$. Because $U \in U_{aa}$, there is a fuzzy $\mathcal{T}_2$-$b$-set $U_1$ such that $x_aqU_1$ and $\mathcal{T}_1 - bCl(U_1) \leq U \leq V$. Therefore, $V \in U_{aa}$.
Therefore, $T_1(T_2)$ is a fuzzy topology on $\mathcal{X}$ for which $U_{\alpha}$ is a fuzzy $b$-q-nbd system of $x_{\alpha}$.

**Definition 2.3.** Let $(\mathcal{X}, T_1)$ and $(\mathcal{X}, T_2)$ be two fuzzy topological spaces. We define $T_1(T_2) = \{ A \in I \mathcal{X} |$ for every $x_{\alpha} \in A$ and there is a fuzzy $T_2$-b-q-nbd $A_{\alpha}$ of $x_{\alpha}$ such that $T_1\rhd bCl(A_{\alpha}) \subseteq A \}$. Then, $T_1(T_2)$ is a fuzzy topology on $\mathcal{X}$. This fuzzy topology is called a mixed $b$-fuzzy topology and $(\mathcal{X}, T_1(T_2))$ is called a mixed $b$-fuzzy topological space.

**Theorem 2.4.** The intersection of two fuzzy $b$-q-nbds of a fuzzy point $\alpha$ is also a fuzzy $b$-q-nbd of $\alpha$ in the fuzzy topological spaces satisfying the condition that the intersection of two fuzzy $b$-open sets is fuzzy $b$-open.

**Proof.** Let $(\mathcal{X}, T_1)$ be a fuzzy topological space satisfying the condition that the intersection of two fuzzy $b$-open sets is fuzzy $b$-open. Let $A$ and $B$ be two fuzzy $b$-q-nbds of a fuzzy point $\alpha$. Therefore, there are fuzzy $b$-open sets $U_1$ and $U_2$ such that $\alpha \in U_1 \subseteq A$ and $\alpha \in U_2 \subseteq B$. This implies $\alpha \in U_1 \cap U_2 \subseteq A \cap B$. Therefore, $U_1 \cap U_2$ is a fuzzy $b$-open because the intersection of two fuzzy $b$-open sets is also fuzzy $b$-open based on our assumptions. By combining this result with (1) and (2), we can conclude that $U_1 \cap U_2$ is a fuzzy $b$-q-nbd of $\alpha$, which completes the proof.

**Theorem 2.5.** Let $(\mathcal{X}, T_1)$ and $(\mathcal{X}, T_2)$ be two fuzzy topological spaces. Consider the collection of fuzzy sets $T_1(T_2) = \{ A \in I \mathcal{X} |$. For any fuzzy set $B$ in $\mathcal{X}$ with $AqB$, there is a $T_2$-b-q-nbd set $A_\delta$ such that $A_\delta \cap T_1 - bCl(A_\delta) \subseteq A$. Then, this family of fuzzy sets forms a topology on $\mathcal{X}$ and this topology is called a mixed fuzzy topology on $\mathcal{X}$.

**Proof.** Let $(\mathcal{X}, T_1)$ be a fuzzy topology space. If $\alpha + 1 > 1$, for any fuzzy point $x_{\alpha}$, there is $x_{\alpha}\in T_1(T_2)$. Therefore, $T_1 - bCl(A_\alpha) = \mathcal{X}$. Additionally, $T_1 - bCl(A_\alpha) = \mathcal{X}$. Therefore, $x_{\alpha}\in T_1(T_2)$. We must demonstrate that $x_{\alpha}\in T_1(T_2)$. Let $x_{\alpha}$ be a collection of members of $T_1(T_2)$. We must also demonstrate that for every $x_{\alpha}\in T_1(T_2)$, there is a fuzzy $T_2$-b-q-nbd $A_{\alpha}$ of $x_{\alpha}$ such that $T_1 - bCl(A_{\alpha}) = x_{\alpha}$. Consider a fuzzy point $x_{\alpha}$. Now, $x_{\alpha}\in T_1(T_2)$ implies $x_{\alpha}\in T_1(T_2)$. This implies $x_{\alpha}\in T_1(T_2)$.

**Lemma 2.6.** Let $T_1$ and $T_2$ be two fuzzy topologies on a set $\mathcal{X}$. If every fuzzy $T_1$-b-q-nbd of $x_{p}$ is a fuzzy $T_2$-b-q-nbd of $x_{p}$, then $T_1$ is coarser than $T_2$.

**Proof.** Let $A$ be a fuzzy $T_1$-b-q-nbd of $x_{p}$. This indicates that there is a fuzzy $T_1$-b-open set $M$ such that $x_{p}$, $q$, $M$, and $M \subseteq A$. Now, $A$ is fuzzy $T_1$-open, which implies that $A$ is fuzzy $T_1$-pre-open, which also implies that $A$ is fuzzy $T_1$-b-open according to Lemma [17]. Therefore, we can assume $M = A$. Let $A$ be a fuzzy $T_1$-b-q-nbd of $x_{p}$, which is $q$-coincident with $A$. According to our hypothesis, $A$ is a fuzzy $T_2$-b-q-nbd of $x_{p}$. Therefore, there is a fuzzy $T_2$-b-open set $M_1$ such that $x_{p}$, $q$, $M_1$, and $M_1 \subseteq A$. We must prove that $A$ is fuzzy $T_2$-b-q-nbd of $x_{p}$.


\[ T_1 \text{-open. To this end, we prove that } A(x) = \sup M_1(x) \text{ with the supremum being taken over all } M_1 \leq A. \] 

Now, based on the fact that \( A(x) + 1 - A(x) + \varepsilon > 1 \), it follows that 

\[ x_1 + \varepsilon - A(x)/\varepsilon \leq \] 

Therefore, \( x_1 + \varepsilon - A(x)/\varepsilon \) for some \( M_1 \in T_2 \) with \( M_1(x) \leq A(x) \). Consequently, \( 1 + \varepsilon - A(x) + M_1(x) > 1 \) or \( M_1(x) > A(x) - \varepsilon \). Therefore, \( A(x) = \sup M_1(x) \) and each \( M_1 \) is fuzzy \( T_2 \)-open. Therefore, each \( A \) is fuzzy \( T_2 \)-open and \( T_1 \leq T_2 \), which completes the proof.

**Lemma 2.7.** Let \( T_1 \) and \( T_2 \) be two fuzzy topologies on a set \( \mathcal{X} \). Then, the mixed \( b \)-fuzzy topology \( T_1(2) \) is coarser than \( T_2 \), which is denoted as \( T_1(2) \leq T_2 \).

**Proof.** Let \( A \) be a fuzzy \( T_1(2) \)-\( b \)-q-nbd of \( x_p \). We must prove that \( A \) is a fuzzy \( T_2 \)-q-nbd of \( x_p \). \( A \) is a fuzzy \( T_1(2) \)-\( b \)-q-nbd of \( x_p \), which implies that there is a fuzzy \( T_1(2) \)-open set \( M \) in \( \mathcal{X} \) such that \( x_p \in M \) and \( M \leq A \). In other words, for every \( x_p \in M \), there is a fuzzy \( T_2 \)-\( b \)-q-nbd \( M_1 \) of \( x_p \) such that \( T_1 - bC(M_1) \leq M \). This implies that there is a fuzzy \( T_2 \)-open set \( M_1 \) such that \( x_p \in M_1 \leq M \). Therefore, \( A \) is a fuzzy \( T_1(2) \)-\( b \)-q-nbd of \( x_p \), meaning \( T_1(2) \leq T_2 \) according to Lemma 2.6 which completes the proof.

**Definition 2.8.** A fuzzy topological space \((\mathcal{X}, T)\) is said to be fuzzy \( b \)-regular if for each fuzzy point \( x_\alpha \) and each fuzzy \( b \)-closed set \( \mathcal{W} \) with \( x_\alpha \in \mathcal{W} \), there is a fuzzy open set \( A \) and fuzzy \( b \)-open set \( B \) such that \( x_\alpha \in A \), \( A \subseteq B \), and \( A \) is not \( q \)-coincident with \( B \).

**Application 2.9.** Theorem 2.7 can be applied to represent the open subsets of the mixed topology \( T_1(2) \). For example, if \( X = \{a, b\} \), \( T_1 = \{1, 0, (a, A)\} \), then \( T_2 = \{1, 0, (b, 3)\} \) and according to Theorem 2.7 \( T_1(2) \leq T_2 \). To find the exact members of \( T_1(2) \), it is sufficient to determine if \( b \) belongs to \( T_1(2) \). In fact, \( (b, 3) \notin T_1(2) \). Therefore, \( T_1(2) = \{1, 0\} \neq T_2 \). This also implies that \( T_1(2) \) is strictly coarser than \( T_2 \). We now present another example where \( T_1(2) = T_2 \).

**Example 2.10.** Let us consider a non-empty set \( X = \{a, b\} \) and consider the following fuzzy sets in \( X \): \( C = \{(a, 7), (b, 3)\} \), and \( D = \{(a, 3), (b, 7)\} \). Then, the collection of fuzzy sets \( T_1 = \{0', 1', D\} \), and \( T_2 = \{0', 1', C\} \) represent two fuzzy topologies on \( X \). Now, we construct a mixed fuzzy topology on \( X \) from the two fuzzy topologies \( T_1 \) and \( T_2 \). The mixed fuzzy topology is coarser than \( T_2 \), we only need to verify that the fuzzy set \( A \) is in \( T_1(2) \).

Let us consider a fuzzy set \( G \) in \( X \) such that \( CqG \). Now, the \( T_2 \)-open sets are \( 1' \) and \( C \) such that \( CqG \) and \( 1'qG \). This means that the \( T_1 \)-closure of \( C = \wedge \{Q : Q \text{ is } T_1 \text{-closed and } A \subseteq Q\} = 1' \wedge C = C \subseteq C \). Therefore, \( A \subseteq T_1(2) \) and \( T_1(2) = \{0', 1', C\} \). In this mixed fuzzy topology, the fuzzy regular open sets are \( 0', 1' \), but \( C \) is not fuzzy regular open because \( T_1 - \text{Int}(T_2 - \text{Cl}(C)) = 1' \). Now, we demonstrate that \( C \) is fuzzy \( b \)-open. It is sufficient to prove that \( C = D \) is fuzzy \( b \)-closed. Now, \( T_1 - \text{Cl}(T_2 - \text{Int}(C)) = A \).

Therefore, \( D \) is fuzzy \( b \)-closed and \( C \) is fuzzy \( b \)-open. Therefore, we can conclude that a fuzzy \( b \)-open set need not be fuzzy regular open.

We now establish a relationship between \( T_1 \) and \( T_1(2) \) for some specific cases.

**Proposition 2.11.** If \( T_1 \) is fuzzy \( b \)-regular and \( T_1 \leq T_2 \), then \( T_1 \leq T_1(2) \).

**Proof.** Let \( A \) be a fuzzy \( T_1 \)-\( b \)q-nbd of \( x_p \). This implies that there is a fuzzy \( T_1 \)-\( b \)-open set \( M \) in \( X \) such that \( x_p \in M \) and \( M \leq A \). We must prove that \( A \) is a fuzzy \( T_1(2) \)-\( b \)-q-nbd of \( x_p \). Because \( T_1 \) is fuzzy \( b \)-regular, for each \( x_p \) and for each fuzzy \( b \)-open set \( M \) that is \( q \)-coincident with \( x_p \), there is a fuzzy \( T_1 \) open set \( M_1 \) in \( X \) such that \( x_p \in M_1 \leq T_2 - \text{bCl}(M_1) \leq M \). Now, a fuzzy set being fuzzy open implies that the set is fuzzy pre-open, which implies that the set is fuzzy \( b \)-open according to Lemma 2.7. Therefore, \( M_1 \) is fuzzy \( b \)-open. Again, \( T_1 \leq T_2 \) implies that \( M_1 \) is fuzzy \( b \)-open. Therefore, \( M_1 \) is \( T_1 \)-fuzzy open, \( x_p \in M_1 \), and \( \text{bCl}(M_1) \leq M \). Therefore, \( M_1 \) is \( T_1(2) \)-fuzzy open, meaning \( A \) is a fuzzy \( T_1(2) \)-\( b \)-q-nbd of \( x_p \) and \( T_1 \leq T_1(2) \) according to Lemma 2.6 which completes the proof.

3. Conclusion

We introduced mixed \( b \)-fuzzy topological spaces and a fuzzy completely weak \( b \)-irresolute function over an initial universe with a fixed set of parameters. Many results have been established to demonstrate how topological structures are preserved by this \( b \)- irresolute function. We also provided examples in which such properties fail to be preserved. In this paper, we only focused on a few specific ideas. In the future, it will be necessary to conduct more theoretical research to establish a general framework for practical applications.

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