A Set-Matrix Duality Principle
for the Dirac Equation

Rasulkhozha S. Sharafiddinov

Institute of Nuclear Physics, Uzbekistan Academy of Sciences, Ulugbek, Tashkent 100214, Uzbekistan

Abstract

Spontaneous mirror symmetry violation is carried out in nature as the transition between the usual left (right)-handed and the mirror right (left)-handed spaces, in each of which the usual and mirror particles have the different lifetimes. As a consequence, all equations of motion in a unified field theory of elementary particles include the mass, energy and momentum as the matrices expressing the ideas of the left- and right-handed neutrinos are of long- and short-lived objects, respectively. These ideas require in principle to go away from the chiral definitions of the structure of matter fields taking into account that the internally disclosed Dirac matrices are, in the Weyl presentation, reduced to the matrices having an internal undisclosed. We discuss a theory in which a set comes forward at the new level, namely, at the level of set-matrix duality principle as a criterion for matrices. This connection gives the exact mathematical definitions of internally disclosed and undisclosed matrices, allowing to formulate four more definitions, three lemmas and one pair of axioms. Thereby, it involves that there is no single matrix, for which an absolutely empty matrix would not exist. The sets of matrix elements and the matrices of set elements thus found unite all of matrix and set operations necessary for deciding the problems in a unified whole.

Key words: Dirac Space; Regular Matrices; Casual Matrices; An Empty Matrix; A Set-Matrix Duality; Matrix Operations; Set Operations.

Mathematics Subject Classifications: 03A05, 03B60, 03E99, 15A24, 15A99, 15B99, 39B42

1. Introduction

One structural set of the innate properties of matter, which was not internally disclosed before the creation of the first-initial unified field theory, is spontaneous mirror symmetry violation. It is not surprising therefore that in the form as it was accepted, neither of the quantum mechanical equations depending on the mass, energy and momentum is in a state to describe the elementary objects by the mirror symmetry laws.

At the same time, nature itself relates the same left or right spin state of a particle even, in the case of the neutrino ($\nu_l = \nu_e, \nu_\mu, \nu_\tau, ...$), to corresponding component of its antiparticle. It constitutes [1] herewith an individual paraneutrino

$$(\nu_{lL}, \bar{\nu}_{lR}), \quad (\nu_{lR}, \bar{\nu}_{lL}),$$

confirming the availability in it of the transitions between the left and the right.

However, as was accepted in the standard electroweak model [2-4], their existence contradicts one of its postulates that in nature the right-handed neutrinos are absent. Instead it includes the right components of leptons ($l = e, \mu, \tau, ...$) as the usual singlets.
But if we take into account that the mass, energy and momentum of any of elementary particles unite all symmetry laws in a unified whole, then to any type of lepton corresponds in their spectra [5] a kind of neutrino [6]. Thereby, they describe a situation when mirror symmetry violation spontaneously originates in any [1,7] of interconversions

\[ l_L \leftrightarrow l_R, \quad \bar{l}_R \leftrightarrow \bar{l}_L, \]  
\[ \nu_{lL} \leftrightarrow \nu_{lR}, \quad \bar{\nu}_{lR} \leftrightarrow \bar{\nu}_{lL} \]  

by the same mechanism. Such a mechanism can, for example, be simultaneous change of the mass, energy and momentum of a particle at its transition from one spin state into another. It reflects the availability of the usual left (right)-handed and the mirror right (left)-handed Minkowski space-times. Therefore, to understand the nature of elementary particles at the new dynamical level, one must use each interconversion of (1) and (2) as the transition between the usual and the mirror spaces [8], in each of which the usual and mirror particles have the different masses, energies and momenta. This connection expresses, in the case of the C-invariant Dirac neutrino, the idea about that the left-handed neutrino and the right-handed antineutrino are of long-lived leptons of C-invariance, and the right-handed neutrino and the left-handed antineutrino refer to short-lived C-even fermions.

The unidenticality of lifetimes \( \tau_s \) and space-time coordinates \( (t_s, \mathbf{x}_s) \) of left \( (s = L = -1) \) and right \( (s = R = +1) \) types of elementary objects of C-parity establishes in addition the full spin structure of all equations of motion in a unified field theory of particles with a nonzero spin in which the mass, energy and momentum are predicted as the matrices

\[
m_s = \begin{pmatrix} m_V & 0 \\ 0 & m_V \end{pmatrix}, \quad E_s = \begin{pmatrix} E_V & 0 \\ 0 & E_V \end{pmatrix}, \quad \mathbf{p}_s = \begin{pmatrix} \mathbf{p}_V \\ 0 \end{pmatrix},
\]

\[
m_V = \begin{pmatrix} m_L & 0 \\ 0 & m_R \end{pmatrix}, \quad E_V = \begin{pmatrix} E_L & 0 \\ 0 & E_R \end{pmatrix}, \quad \mathbf{p}_V = \begin{pmatrix} \mathbf{p}_L \\ \mathbf{p}_R \end{pmatrix}.
\]

Such a presentation of \( m_s, E_s \) and \( \mathbf{p}_s \) is of course intimately connected with the character of their compound structure depending on a vector \( (V) \) nature [8] of the same space-time, where there exist C-invariant particles.

However, among the sets of C-even objects there are no C-odd particles. Their mass, energy and momentum do not coincide with (3) and (4), since in them appears an axial-vector \( (A) \) nature [9] of the same space-time, where there exist C-noninvariant particles. They can therefore be expressed in the form

\[
m_s = \begin{pmatrix} 0 & m_A \\ m_A & 0 \end{pmatrix}, \quad E_s = \begin{pmatrix} 0 & E_A \\ E_A & 0 \end{pmatrix}, \quad \mathbf{p}_s = \begin{pmatrix} 0 & \mathbf{p}_A \\ \mathbf{p}_A & 0 \end{pmatrix},
\]

\[
m_A = \begin{pmatrix} m_L & 0 \\ 0 & m_R \end{pmatrix}, \quad E_A = \begin{pmatrix} E_L & 0 \\ 0 & E_R \end{pmatrix}, \quad \mathbf{p}_A = \begin{pmatrix} \mathbf{p}_L \\ \mathbf{p}_R \end{pmatrix}.
\]

This difference corresponds in nature to a separation of elementary currents with respect to C-operation, because it admits the existence of C-even and C-odd types of particles of vector \( (V) \) and axial-vector \( (A) \) masses, energies and momenta.

It is also relevant to use [8,9] their sizes as the quantum operators

\[
m_s = -i \frac{\partial}{\partial \tau_s}, \quad E_s = i \frac{\partial}{\partial t_s}, \quad \mathbf{p}_s = -i \frac{\partial}{\partial \mathbf{x}_s}.
\]
Furthermore, if the investigated and the used objects are simultaneously both C-even and C-odd neutrinos, a motion of all types of particles with the spin $1/2$ and the four-component wave function $\psi_s(t, \mathbf{x})$ may in a mirror world [8,9] be described by a latent united equation

$$i \frac{\partial}{\partial t_s} \psi_s = \hat{H}_s \psi_s,$$

(8)

which states that

$$\hat{H}_s = \alpha \cdot \hat{p}_s + \beta m_s.$$

(9)

However, as is now well seen, the sizes of $m_s, E_s$ and $p_s$ are $4 \times 4$ matrices, which are absent in all classical equations of motion for particles, and therefore, there arises a question about the structure of matrices $\alpha = \gamma_5 \sigma$ and $\beta$, about which there is no single unified sight in the mirror behavior dependence of matter fields.

Using a unity $I$ matrix, the Pauli spin $\sigma$ matrices and taking into account the standard presentation of the Dirac [11], for $\alpha$, $\beta$ and $\gamma_5$, we have

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

(10)

At a choice of the above matrices, the solutions of an equation (8) reflect, in the case of both vector [8] and axial-vector [9] types of fermions, the same characteristic features of quantum mechanical helicity operator $\sigma p_s = s|\mathbf{p}_s|$, which indicate to a unified principle that

$$\sigma \mathbf{p}_L = -|\mathbf{p}_L|, \quad \sigma \mathbf{p}_R = |\mathbf{p}_R|.$$

(11)

But for $\alpha$, $\beta$ and $\gamma_5$, the definition (11) is not singular. They can in the chiral presentation of the Weyl [12] have the following form:

$$\alpha = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

(12)

In both presentations (10) and (12), as we can expect from simple reasoning, an equation (8) cannot change his mirror structure, so that there exists so far unobserved relation between the solutions.

Our purpose in a given work is to follow the mathematical logic of an equation (8) in the presence in it of either the matrices (10) or the matrices (12) both from the point of view of vector C-invariant particles and on the basis of C-noninvariance of axial-vector types of neutrinos. This does not exclude of course from the discussion a theory in which a set comes forward as a criterion for matrices.

2. Helicity Operator of Neutrinos of a Vector Nature

A notion about chiral symmetry introduced by Weyl is based factually on the presentation (12), according to which, the matrix $\gamma_5$ becomes chirality operator having the same self-values as the helicity operator. In this case, it is expected that the solutions of an equation (8) including (3) and (4) correspond in the presentations (10) and (12) to the most diverse forms of the same regularity of a C-invariant nature of vector ($V$) types of neutrinos.

To express the idea more clearly, we use a free particle with

$$\psi_s = u_s(\mathbf{p}_s, \sigma)e^{-ip_s \cdot \mathbf{x}_s}, \quad E_s > 0.$$  

(13)
One can define its four-component spinor $u_s$ in the form

$$u_s = u^{(r)} = \begin{bmatrix} \chi^{(r)} \\ u_a^{(r)} \end{bmatrix}$$  \hspace{1cm} (14)$$

in which

$$\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$  \hspace{1cm} (15)$$

and the presence of an index $a$ in one of $u^{(r)}$ and $u_a^{(r)}$ is responsible for their distinction.

So, it is seen that (13) together with (3) and (12) separates (8) into

$$E_V \chi^{(r)} = (\sigma p_V) \chi^{(r)} + m_V u_a^{(r)},$$  \hspace{1cm} (16)$$

$$E_V u_a^{(r)} = - (\sigma p_V) u_a^{(r)} + m_V \chi^{(r)}.$$  \hspace{1cm} (17)$$

Solving a given system concerning $\chi^{(r)}$ and $u_a^{(r)}$, but having in view of (14), it can also be verified that (4) and (15) lead us from

$$u^{(r)} = \sqrt{E_V + (\sigma p_V)} \begin{pmatrix} \chi^{(r)} \\ \frac{m_V}{E_V + (\sigma p_V)} \chi^{(r)} \end{pmatrix}$$  \hspace{1cm} (18)$$

to their explicit form

$$u^{(1)} = \sqrt{E_L + (\sigma p_L)} \begin{pmatrix} 1 \\ 0 \\ \frac{m_L}{E_L + (\sigma p_L)} \\ 0 \end{pmatrix},$$  \hspace{1cm} (19)$$

$$u^{(2)} = \sqrt{E_R + (\sigma p_R)} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{m_R}{E_R + (\sigma p_R)} \end{pmatrix}.$$  \hspace{1cm} (20)$$

At the same choice of a free particle and its four-component wave function, the solutions of an equation (8) depending on (3) and (4) have in the standard presentation (10) the following structure:

$$u^{(1)} = \sqrt{E_L + m_L} \begin{pmatrix} 1 \\ 0 \\ \frac{(\sigma p_L)}{E_L + m_L} \\ 0 \end{pmatrix},$$  \hspace{1cm} (21)$$

$$u^{(2)} = \sqrt{E_R + m_R} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{(\sigma p_R)}{E_R + m_R} \end{pmatrix}.$$  \hspace{1cm} (22)$$

From their point of view, the chiral presentation (12) leading to (19) and (20) replaces the mass of a C-invariant particle by the operator of its helicity and vice versa. In other words, it requires one to make the replacements

$$m_{L,R} \rightarrow \sigma_{PL,R}, \quad \sigma_{PL,R} \rightarrow m_{L,R}.$$  \hspace{1cm} (23)$$
In the same way one can solve the equation (8) for the free antiparticle with
\[ \psi_s = \nu_s(p_s, \sigma)e^{-ip_s \cdot x_s}, \quad E_s < 0. \] (24)

Its four-component spinor \( \nu_s \) must have the form
\[ \nu_s = \nu^{(r)} = \begin{bmatrix} \nu_a^{(r)} \\ \chi^{(r)} \end{bmatrix}. \] (25)

The availability of an index \( a \) in one of \( \nu^{(r)} \) and \( \nu_a^{(r)} \) implies their difference. We see in addition that jointly with (3) and (12), the four-component wave function (24) constitutes from (8) a system of the two other equations
\[ |E_V| \nu_a^{(r)} = -(\sigma p_V)\nu_a^{(r)} - m_V\chi^{(r)}, \] (26)
\[ |E_V|\chi^{(r)} = (\sigma p_V)\chi^{(r)} - m_V\nu_a^{(r)}. \] (27)

Inserting the second of its solutions
\[ \chi^{(r)} = \frac{-m_V}{|E_V| - (\sigma p_V)}\nu_a^{(r)}, \quad \nu_a^{(r)} = \frac{-m_V}{|E_V| + (\sigma p_V)}\chi^{(r)}. \] (28)
in (25) and uniting the finding equality with (4) and (15), it is not difficult to show that
\[ \nu^{(1)} = \sqrt{|E_L| + (\sigma p_L)} \begin{bmatrix} \frac{-m_L}{|E_L| + (\sigma p_L)} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \] (29)
\[ \nu^{(2)} = \sqrt{|E_R| + (\sigma p_R)} \begin{bmatrix} \frac{-m_R}{|E_R| + (\sigma p_R)} \\ 0 \\ 1 \end{bmatrix}. \] (30)

If choose the standard presentation (10), at which the matrix \( \gamma_5 \) is not chirality operator, then for the same case of a free antiparticle when (3), (4) and (24) refer to it, one can establish the compound structure of both types of solutions of an equation (8) in the disclosed form [8] by the following manner:
\[ \nu^{(1)} = \sqrt{|E_L| + m_L} \begin{bmatrix} \frac{-\sigma p_L}{|E_L| + m_L} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \] (31)
\[ \nu^{(2)} = \sqrt{|E_R| + m_R} \begin{bmatrix} \frac{-\sigma p_R}{|E_R| + m_R} \\ 0 \\ 1 \end{bmatrix}. \] (32)

Their comparison with (29) and (30) convinces us in the validity of (23) once more, confirming that the chiral presentation (12) replaces the helicity operator of a C-invariant antiparticle by its mass and vice versa.
3. Helicity Operator of Neutrinos of True Neutrality

Between the vector and the axial-vector spaces \([13]\) there exists a range of fundamental differences, which require the unification of elementary particles with respect to C-operation. However, nature, by itself, does not separate \([8,9]\) each of these forms of Minkowski spaces into left and right spaces, and the transitions between the different spin states are carried out in it spontaneously by a mirror symmetry violation. It chooses here with the mass, energy and momentum matrices so that to the case of C-even \([14]\) or C-odd \([15]\) types of particles corresponds in their unified field theory a kind of equation of motion.

Therefore, from its point of view, it should be expected that an equation \((8)\) including \((5)\) and \((6)\) describe in the presentations \((10)\) and \((12)\) the most diverse forms of the same regularity of a C-noninvariant nature of axial-vector (A) types of neutrinos.

To elucidate these ideas, we use \((13)-(15)\) for the free particles of C-oddity. Then it is possible, for example, \((13)\) in the presence of \((5)\) and \((12)\) transforms \((8)\) into the system

\[
E_A u_a^{(r)} = (\sigma p_A) u_a^{(r)} + m_A \chi^{(r)}, \quad (33)
\]
\[
E_A \chi^{(r)} = -(\sigma p_A) \chi^{(r)} + m_A u_a^{(r)}. \quad (34)
\]

It establishes the corresponding connections

\[
u_a^{(r)} = \frac{m_A}{E_A - (\sigma p_A)} \chi^{(r)}, \quad \chi^{(r)} = \frac{m_A}{E_A + (\sigma p_A)} u_a^{(r)}. \quad (35)
\]

The first of them together with \((15)\) gives the right to define the four-component spinors \(u^{(r)}\) for C-odd types of neutrinos

\[
u^{(1)} = \sqrt{E_L - (\sigma p_L)} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{m_L}{E_L - (\sigma p_L)} \end{bmatrix}, \quad (36)
\]
\[
u^{(2)} = \sqrt{E_R - (\sigma p_R)} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{m_R}{E_R - (\sigma p_R)} \end{bmatrix}. \quad (37)
\]

However, in the C-noninvariant case of a free particle, an equation \((8)\) depending on \((5)\) and \((6)\) can in the standard presentation \((10)\) have \([9]\) the following solutions:

\[
u^{(1)} = \sqrt{E_L - m_L} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{(\sigma p_L)}{E_L - m_L} \end{bmatrix}, \quad (38)
\]
\[
u^{(2)} = \sqrt{E_R - m_R} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{(\sigma p_R)}{E_R - m_R} \end{bmatrix}. \quad (39)
\]

As we see, the chiral presentation \((12)\) establishing \((36)\) and \((37)\) replaces the mass of a C-noninvariant particle by the operator of its helicity and vice versa.
Unification of (8) with (5) and (12) at the discussion of a C-odd antiparticle described by (24) suggests a system
\[ |E_A|\chi^{(r)} = -(\sigma p_A)\chi^{(r)} - m_A\nu^{(r)}_a; \quad (40) \]
\[ |E_A|\nu^{(r)}_a = (\sigma p_A)\nu^{(r)}_a - m_A\chi^{(r)}. \quad (41) \]
Insertion of the first of its solutions \( \nu^{(r)}_a \) and \( \chi^{(r)} \) in (25) allows one to conclude that
\[ \nu^{(r)} = \sqrt{|E_A| - (\sigma p_A)} \left[ \begin{array}{c}
\frac{-m_A}{|E_A| - (\sigma p_A)} \\
\chi^{(r)}
\end{array} \right]. \quad (42) \]
Because of (6) and (15), the latent structure of \( \nu^{(r)} \) is disclosed in the following its sizes:
\[ \nu^{(1)} = \sqrt{|E_L| - (\sigma p_L)} \left[ \begin{array}{c}
\frac{-m_L}{|E_L| - (\sigma p_L)} \\
0 \\
1 \\
0
\end{array} \right], \quad (43) \]
\[ \nu^{(2)} = \sqrt{|E_R| - (\sigma p_R)} \left[ \begin{array}{c}
0 \\
\frac{-m_R}{|E_R| - (\sigma p_R)} \\
1 \\
0
\end{array} \right]. \quad (44) \]
But in the standard presentation (10), the equation (8) for the same C-odd antiparticle with (5), (6) and (24) establishes [9] the two other spinors
\[ \nu^{(1)} = \sqrt{|E_L| - m_L} \left[ \begin{array}{c}
\frac{-\sigma p_L}{|E_L| - m_L} \\
0 \\
1 \\
0
\end{array} \right], \quad (45) \]
\[ \nu^{(2)} = \sqrt{|E_R| - m_R} \left[ \begin{array}{c}
0 \\
\frac{-\sigma p_R}{|E_R| - m_R} \\
1 \\
0
\end{array} \right]. \quad (46) \]
At the action of (23) they coincide with the corresponding values from (43), (44) and that, consequently, the behavior of the chiral presentation (12) is not changed even at a choice of a C-noninvariant antiparticle.

4. Internally Disclosed Matrices of a Dirac Space

Turning again to the structure and the component of the finding wave functions, we remark that the sign in front of a size of \( m_{L,R} \) in \( u^{(1)}_a, u^{(2)}_a, \nu^{(1)}_a \) and \( \nu^{(2)}_a \) for C-even and C-odd particles does not coincide. This, however, does not exclude [8,9] the fact that \( u^{(1)}, \chi^{(1)} \) and \( u^{(1)}_a \) describe the left-handed neutrino, and \( u^{(2)}, \chi^{(2)} \) and \( u^{(2)}_a \) characterize the right-handed neutrino. At the same time, \( \nu^{(1)}, \chi^{(1)} \) and \( \nu^{(1)}_a \) respond to the right-handed antineutrino, and \( \nu^{(2)}, \chi^{(2)} \) and \( \nu^{(2)}_a \) correspond to the left-handed antineutrino.
It is already clear from the foregoing that the neutrino $\nu_{L}$ and the antineutrino $\bar{\nu}_{R}$ refer to the left-polarized fermions, and the neutrino $\nu_{R}$ and the antineutrino $\bar{\nu}_{L}$ are of the right-polarized leptons.

Such a full spin picture corresponding in an equation (8) to the matrices (3)-(6) and (10) can be established by another way starting from (12) if its prediction (23) is carried out in nature.

At first sight, this says in favor of the compatibility of all requirements of a chiral invariance with implications of the helicity operator itself. On the other hand, such a unification of (11) and (23) shows that

$$m_{L} = -|p_{L}|, \quad m_{R} = |p_{R}|,$$

and consequently, (12) is one of those presentations of matrices $\alpha$, $\beta$ and $\gamma_{5}$, in each of which $\nu_{R}$ and $\bar{\nu}_{L}$ come forward as the particles, and $\nu_{L}$ and $\bar{\nu}_{R}$ are predicted as the antiparticles.

The difference in masses, energies and momenta of a particle and an antiparticle violates, in the case of C-even types of leptons, their CPT-symmetry expressing the idea of a Lorentz invariance [16]. At the same time, a C-noninvariant neutrino itself regardless of whether or not an unbroken Lorentz symmetry exists in its nature, is strictly CPT-odd [10]. This does not imply of course that the same neutrino or antineutrino must be either fermion or antifermion.

By following the structure of matrices (3)-(6), (9) and (10), it is easy to see that

$$[\langle \alpha p_{s} + \beta m_{s} \rangle, \sigma p_{s}] = [\sigma p_{s}, \langle \alpha p_{s} + \beta m_{s} \rangle],$$

$$[\langle \alpha p_{s} + \beta m_{s} \rangle, \gamma_{5}] \neq [\gamma_{5}, \langle \alpha p_{s} - \beta m_{s} \rangle],$$

which characterize the behavior of the standard presentation (10) both from the point of view of a C-even and from the point of view of a C-odd particles.

To the same relationships (48) and (49) one can also lead by another way using (3), (4), (9) and (12), but the latter together with (5), (6) and (9) satisfies the inequalities

$$[\langle \alpha p_{s} + \beta m_{s} \rangle, \sigma p_{s}] \neq [\sigma p_{s}, \langle -\alpha p_{s} + \beta m_{s} \rangle],$$

$$[\langle \alpha p_{s} + \beta m_{s} \rangle, \gamma_{5}] \neq [\gamma_{5}, \langle -\alpha p_{s} + \beta m_{s} \rangle].$$

This would seem to say that either unification [10,13] of elementary objects in families of a different C-parity is incompatible with the chiral presentation (12) or $\sigma p_{s}$ is not helicity operator of a C-odd particle. On the other hand, as follows from symmetry laws, any C-invariant or C-noninvariant neutrino cannot simultaneously have both CPT-even vector and CPT-odd axial-vector nature. Such a circumstance becomes more interesting if we take into account that the existence of vector [17] and axial-vector [18,19] mirror Minkowski space-times are by no means excluded [8,9] experimentally.

Thus, it follows that between the spontaneous mirror symmetry violation and the chiral presentation (12) there exists a range of the structural contradictions, which expresses the ideas of the left- and right-handed neutrinos referring to long- and short-lived objects, respectively. These ideas require in principle to go away from the chiral definitions of the structure of matter fields taking into account that $\alpha$, $\beta$ and $\gamma_{5}$ come forward in (12) as the matrices having an internal undisclosure. Therefore, from the point of view of the mass, energy and momentum matrices, each of (47), (50) and (51) must be considered as an indication to the absence in a mirror world of a place for internal undisclosure of internally disclosed matrices.

There exist, however, a range of the structural equations, in each of which appears a part of a kind of system [20,21] of internally disclosed sets [22]. Exactly the same nature itself is not in force to exclude the existence of a kind of set criterion for matrices, for example, for matrices of the Dirac equation. Therefore, to establish the internal disclosure of each matrix
from (10) at the level of a very mirror space, one must follow the mathematical logic of each of
the corresponding types of sets responsible for its distinction among other internally undisclosed
matrices.

5. Set Criterion for Matrices

Our reasoning is such that each of the matrices

\[
A = \begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix},
\]

(52)

namely, of \( m \times n \) matrices corresponds to a kind of set

\[
A = \{a_{11}, a_{12}, \ldots, a_{1n}, a_{21}, a_{22}, \ldots, a_{2n}, \ldots, a_{m1}, a_{m2}, \ldots, a_{mn}\}.
\]

(53)

This is none other than a set-matrix duality principle, which expresses, for a matrix \( \{a_{11}\} \)
consisting only of one element \( a_{11} \), the idea of a one-element set \( \{a_{11}\} \), confirming that we
cannot exclude the existence of an absolutely empty matrix \( \{\} \) corresponding to a kind of
empty set \( \{\} \) in which there is no single element.

The presence of an empty class in all sets [23] implies that in any matrix, an empty matrix not
containing any element is necessarily present. This becomes possible owing to the coexistence
of both sets of a definite cardinality [22] and matrices of a definite dimensionality.

**Lemma 1 (Theorem on the smallest nonempty matrix).** No single fully regular matrix
from the same element exists without an internally disclosed matrix of a higher dimensionality.

Thus, an internally disclosed or undisclosed matrix is mathematically defined as a fully
regular or casual set [22] of its elements.

**Definition 1.** A matrix is called an internally disclosed one if its elements constitute a fully
regular set.

**Definition 2.** A matrix is called an internally undisclosed one if its elements constitute a fully
casual set.

Furthermore, if it turns out that \( m = n = 2 \), we have

\[
A = \begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix},
\]

(54)

and the elements of such a matrix constitute a quadratic set

\[
A = \{a_{11}, a_{12}, a_{21}, a_{22}\}.
\]

(55)

To show this, we must refer to one of the possible groupings of elements \( a_{mn} \), because it
transforms a set \( A \) from (55) into

\[
A = \{A_{1L}, A_{2L}, A_{1T}, A_{2T}, A_{1D}, A_{2D}\}
\]

(56)
in which
\[ A_{1L} = \{a_{11}, a_{12}\}, \quad A_{2L} = \{a_{21}, a_{22}\}, \quad (57) \]
\[ A_{1T} = \{a_{21}, a_{11}\}, \quad A_{2T} = \{a_{12}, a_{22}\}, \quad (58) \]
\[ A_{1D} = \{a_{22}, a_{11}\}, \quad A_{2D} = \{a_{12}, a_{21}\}. \quad (59) \]
Here \( A_{1L} \) and \( A_{2L} \) denote the first and second row or longitudinal (\( L \)) sets of a matrix \( (54) \), \( A_{1T} \) and \( A_{2T} \) describe its first and second column or transversal (\( T \)) sets, \( A_{1D} \) and \( A_{2D} \) characterize in it the first and second diagonal (\( D \)) sets.

A feature of this structure is that we can replace in a set \( (56) \) one longitudinal subset for another subset
\[ A_{1L} \to A_{2L} \iff A_{2L} \to A_{1L} \quad (60) \]
if and only if the very unity of their internal disclosure laws admits a permutation of elements within each of the transversal and diagonal subsets
\[ A_{1T} = \{a_{21}, a_{11}\} \iff A_{2T} = \{a_{22}, a_{12}\}, \quad (61) \]
\[ A_{1D} = \{a_{22}, a_{11}\} \iff A_{2D} = \{a_{21}, a_{12}\}. \quad (62) \]

They constitute for a matrix \( (54) \) one more fully possible form
\[ A = \begin{pmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{pmatrix}. \quad (63) \]

However, we cannot replace one transversal subset for another subset
\[ A_{1T} \to A_{2T} \iff A_{2T} \to A_{1T} \quad (64) \]
until the very unity of set internal disclosure laws is able to permute both the diagonal subsets
\[ A_{1D} \to A_{2D} \iff A_{2D} \to A_{1D} \quad (65) \]
and the elements within each of the longitudinal subsets
\[ A_{1L} = \{a_{12}, a_{11}\} \iff A_{2L} = \{a_{22}, a_{21}\}. \quad (66) \]

They establish for a matrix \( (54) \) one more fully regular form
\[ A = \begin{pmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{pmatrix}. \quad (67) \]

This is exactly the same as when the transitions \( (65) \) are carried out in a set \( (56) \) as an indication in favor of each of \( (64) \) and \( (66) \), namely, in favor of a kind of Theorem.

**Lemma 2 (Theorem on the transitions between diagonals of a matrix).** There is no single transition between diagonals in an internally disclosed matrix without a permutation of its columns.

If we choose \( a_{11}, a_{12}, a_{21} \) and \( a_{22} \) from the matrices \( (10) \), any of \( (63) \) and \( (67) \) expresses in whole the same idea about that
\[ \alpha = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}. \quad (68) \]
is unlike [12] a fully regular presentation of Dirac matrices. Thereby, it describes mathematically the internal disclosure and undisclosure of matrices as one pair of axioms.

**Axiom 1.** An internal disclosure of a matrix is none other than mathematical disclosure of a set of its elements.

**Axiom 2.** An internal undisclosure of a matrix is none other than mathematical undisclosure of a set of its elements.

### 6. Matrix Criterion for Sets

The preceding reasoning says that a matrix makes it possible to introduce the notions of the row, column and diagonal of a set, confirming their availability in the defined order of elements. Thereby, a set-matrix duality principle requires one to follow the mathematical logic, at the new level, of each matrix of (10) from the point of view of his set of an internal disclosure. It chooses herewith the sets $\mathcal{A}, \mathcal{B}$ and $\Gamma_5$ for matrices $\alpha, \beta$ and $\gamma_5$ so that their structure had the form

$$\mathcal{A} = \{0, \sigma, \sigma, 0\}, \quad \mathcal{B} = \{I, 0, 0, -I\}, \quad \Gamma_5 = \{0, I, I, 0\}. \quad (69)$$

To conform with a presentation (56), the first of these sets involves the following longitudinal, transversal and diagonal subsets:

$$\mathcal{A}_{1L} = \{0, \sigma\}, \quad \mathcal{A}_{2L} = \{\sigma, 0\}, \quad (70)$$

$$\mathcal{A}_{1T} = \{0, \sigma\}, \quad \mathcal{A}_{2T} = \{\sigma, 0\}, \quad (71)$$

$$\mathcal{A}_{1D} = \{0, 0\}, \quad \mathcal{A}_{2D} = \{\sigma, \sigma\}. \quad (72)$$

One can also present a set

$$\mathcal{B} = \{\mathcal{B}_{1L}, \mathcal{B}_{2L}, \mathcal{B}_{1T}, \mathcal{B}_{2T}, \mathcal{B}_{1D}, \mathcal{B}_{2D}\} \quad (75)$$

with longitudinal, transversal and diagonal subsets:

$$\mathcal{B}_{1L} = \{I, 0\}, \quad \mathcal{B}_{2L} = \{0, -I\}, \quad (76)$$

$$\mathcal{B}_{1T} = \{I, 0\}, \quad \mathcal{B}_{2T} = \{0, -I\}, \quad (77)$$

$$\mathcal{B}_{1D} = \{I, -I\}, \quad \mathcal{B}_{2D} = \{0, 0\}. \quad (78)$$

As well as in (75), each subset of a set

$$\Gamma_5 = \{\Gamma_5^{1L}, \Gamma_5^{2L}, \Gamma_5^{1T}, \Gamma_5^{2T}, \Gamma_5^{1D}, \Gamma_5^{2D}\} \quad (79)$$

has a self compound structure:

$$\Gamma_5^{1L} = \{0, I\}, \quad \Gamma_5^{2L} = \{I, 0\}, \quad (80)$$

$$\Gamma_5^{1T} = \{0, I\}, \quad \Gamma_5^{2T} = \{I, 0\}, \quad (81)$$

$$\Gamma_5^{1D} = \{0, 0\}, \quad \Gamma_5^{2D} = \{I, I\}. \quad (82)$$
Based on the matrices (85), one can also find that

\[ M_s = \{ m_V, \ 0, \ 0, \ m_V \}, \]  
(83)

\[ E_s = \{ E_V, \ 0, \ 0, \ E_V \}, \]  
(84)

\[ P_s = \{ p_V, \ 0, \ 0, \ p_V \}. \]  
(85)

The structural set \( M_s \) consisting of elements of a matrix \( m_s \) can be presented by the following manner:

\[ M_s = \{ M_s^{1L}, \ M_s^{2L}, \ M_s^{1T}, \ M_s^{2T}, \ M_s^{1D}, \ M_s^{2D} \}. \]  
(86)

Its longitudinal, transversal and diagonal subsets have the form

\[ M_s^{1L} = \{ m_V, \ 0 \}, \ M_s^{2L} = \{ 0, \ m_V \}; \]  
(87)

\[ M_s^{1T} = \{ m_V, \ 0 \}, \ M_s^{2T} = \{ 0, \ m_V \}; \]  
(88)

\[ M_s^{1D} = \{ m_V, \ m_V \}, \ M_s^{2D} = \{ 0, \ 0 \}. \]  
(89)

The elements of a matrix \( E_s \) lead to the substitution of a set \( E_s \) for

\[ E_s = \{ E_s^{1L}, \ E_s^{2L}, \ E_s^{1T}, \ E_s^{2T}, \ E_s^{1D}, \ E_s^{2D} \}, \]  
(90)

which consists of the longitudinal, transversal and diagonal subsets

\[ E_s^{1L} = \{ E_V, \ 0 \}, \ E_s^{2L} = \{ 0, \ E_V \}; \]  
(91)

\[ E_s^{1T} = \{ E_V, \ 0 \}, \ E_s^{2T} = \{ 0, \ E_V \}; \]  
(92)

\[ E_s^{1D} = \{ E_V, \ E_V \}, \ E_s^{2D} = \{ 0, \ 0 \}. \]  
(93)

Finally, for a set

\[ P_s = \{ P_s^{1L}, \ P_s^{2L}, \ P_s^{1T}, \ P_s^{2T}, \ P_s^{1D}, \ P_s^{2D} \} \]  
(94)

including the elements of a matrix \( p_s \), the structural subets of this class can have the following structure:

\[ P_s^{1L} = \{ p_V, \ 0 \}, \ P_s^{2L} = \{ 0, \ p_V \}; \]  
(95)

\[ P_s^{1T} = \{ p_V, \ 0 \}, \ P_s^{2T} = \{ 0, \ p_V \}; \]  
(96)

\[ P_s^{1D} = \{ p_V, \ p_V \}, \ P_s^{2D} = \{ 0, \ 0 \}. \]  
(97)

They together with subets of sets \( A, \ B, \Gamma_s, \ M_s \) and \( E_s \) constitute the eighteen pairs of sets from two elements such that we can formulate one more theorem.

**Lemma 3 (Theorem on the diagonal elements of a set).** There is no single diagonal in an internally disclosed set without a crossing element of its row and column.

So, we must recognize that

\[ A = \{ A_{1L}, \ A_{2L}, \ A_{1T}, \ A_{2T} \}; \]  
(98)

\[ B = \{ B_{1L}, \ B_{2L}, \ B_{1T}, \ B_{2T} \}; \]  
(99)

\[ \Gamma_5 = \{ \Gamma_5^{1L}, \ \Gamma_5^{2L}, \ \Gamma_5^{1T}, \ \Gamma_5^{2T} \}; \]  
(100)

\[ M_s = \{ M_s^{1L}, \ M_s^{2L}, \ M_s^{1T}, \ M_s^{2T} \}; \]  
(101)

\[ E_s = \{ E_s^{1L}, \ E_s^{2L}, \ E_s^{1T}, \ E_s^{2T} \}. \]  
(102)
\[ \mathcal{P}_s = \{ \mathcal{P}_s^{1L}, \mathcal{P}_s^{2L}, \mathcal{P}_s^{1T}, \mathcal{P}_s^{2T} \} \]

are fully regular sets of the Dirac equation

\[ E_s \psi_s = \mathcal{H}_s \psi_s \]

such that in it

\[ \mathcal{H}_s = \mathcal{A}\mathcal{P}_s + \mathcal{B}\mathcal{M}_s, \]

\[ \psi_s = \mathcal{U}_s(p_s, \sigma)e^{-i p_{s} \cdot x_s}, \quad E_s > 0, \]

\[ \mathcal{U}_s = \mathcal{U}^{(r)} = \{ \chi^{(r)}, \ u_{a}^{(r)} \}, \]

\[ \mathcal{U}_s = \{ \mathcal{U}_s^{1L}, \mathcal{U}_s^{2L}, \mathcal{U}_s^{1T} \}, \]

\[ \mathcal{U}_s^{1L} = \{ \chi^{(r)} \}, \quad \mathcal{U}_s^{2L} = \{ u_{a}^{(r)} \}, \quad \mathcal{U}_s^{1T} = \{ \chi^{(r)}, \ u_{a}^{(r)} \}, \]

\[ \psi_s = \mathcal{N}_s(p_s, \sigma)e^{-i p_{s} \cdot x_s}, \quad E_s < 0, \]

\[ \mathcal{N}_s = \mathcal{N}^{(r)} = \{ \nu_{a}^{(r)}, \ \chi^{(r)} \}, \]

\[ \mathcal{N}_s = \{ \mathcal{N}_s^{1L}, \mathcal{N}_s^{2L}, \mathcal{N}_s^{1T} \}, \]

\[ \mathcal{N}_s^{1L} = \{ \nu_{a}^{(r)} \}, \quad \mathcal{N}_s^{2L} = \{ \chi^{(r)} \}, \quad \mathcal{N}_s^{1T} = \{ \nu_{a}^{(r)}, \ \chi^{(r)} \}. \]

Here \( \mathcal{U}_s^{1L} \) and \( \mathcal{U}_s^{2L} \) express the first and second row of a set \( \mathcal{U}_s \) of a matrix \( [14] \), \( \mathcal{U}_s^{1T} \) characterizes in it a single column, \( \mathcal{N}_s^{1L} \) and \( \mathcal{N}_s^{2L} \) describe the first and second row of a set \( \mathcal{N}_s \) of a matrix \( [25] \), \( \mathcal{N}_s^{1T} \) implies a singleness of its column.

If we now use the sets \( [29] \) and \( [103] \), we see that

\[ \mathcal{A}\mathcal{P}_s = \{ A_{1L}, \ A_{2L}, \ A_{1T}, \ A_{2T} \}\{ \mathcal{P}_s^{1L}, \mathcal{P}_s^{2L}, \mathcal{P}_s^{1T}, \mathcal{P}_s^{2T} \} = \]

\[ \{ A_{1L}\mathcal{P}_s^{1T}, \ A_{1L}\mathcal{P}_s^{2T}, \ A_{2L}\mathcal{P}_s^{1T}, \ A_{2L}\mathcal{P}_s^{2T} \} = \]

\[ \{ \{ 0, \ \sigma \}\{ \mathbf{p_V}, \ 0 \}, \quad \{ 0, \ \sigma \}\{ 0, \ \mathbf{p_V} \}, \]

\[ \{ \sigma, \ 0 \}\{ \mathbf{p_V}, \ 0 \}, \quad \{ \sigma, \ 0 \}\{ 0, \ \mathbf{p_V} \} = \]

\[ \{ 0 \cdot \mathbf{p_V} + \sigma \cdot 0, \quad 0 \cdot 0 + \sigma \cdot \mathbf{p_V} , \]

\[ \sigma \cdot \mathbf{p_V} + 0 \cdot 0, \quad \sigma \cdot 0 + 0 \cdot \mathbf{p_V} = \]

\[ \{ 0, \ \sigma \mathbf{p_V}, \ \sigma \mathbf{p_V}, \ 0 \}. \]

Performing the same set operations that led to \( [114] \), but having in mind the sets \( \mathcal{B} \) and \( \mathcal{M}_s \), we found

\[ \mathcal{B}\mathcal{M}_s = \{ \mathcal{B}_{1L}, \mathcal{B}_{2L}, \mathcal{B}_{1T}, \mathcal{B}_{2T} \}\{ \mathcal{M}_s^{1L}, \mathcal{M}_s^{2L}, \mathcal{M}_s^{1T}, \mathcal{M}_s^{2T} \} = \]

\[ \{ \mathcal{B}_{1L}\mathcal{M}_s^{1T}, \mathcal{B}_{1L}\mathcal{M}_s^{2T}, \mathcal{B}_{2L}\mathcal{M}_s^{1T}, \mathcal{B}_{2L}\mathcal{M}_s^{2T} \} = \]

\[ \{ \{ I, \ 0\}\{ m_V, \ 0 \}, \quad \{ I, \ 0\}\{ 0, \ m_V \}, \]

\[ \{ 0, \ -I\}\{ m_V, \ 0 \}, \quad \{ 0, \ -I\}\{ 0, \ m_V \} = \]

\[ \{ I \cdot m_V + 0 \cdot 0, \quad I \cdot 0 + 0 \cdot m_V, \]

\[ 0 \cdot m_V + (-I) \cdot 0, \quad 0 \cdot 0 + (-I) \cdot m_V = \]

\[ \{ m_V, \ 0, \ -m_V \}. \]

At these situations, \( [114] \) and \( [115] \) replace \( [105] \) for

\[ \mathcal{H}_s = \mathcal{A}\mathcal{P}_s + \mathcal{B}\mathcal{M}_s = \]
\{0, \sigma p_V, \sigma p_V, 0\} + \{m_V, 0, 0, -m_V\} = \\
\{0 + m_V, \sigma p_V + 0, \sigma p_V + 0, 0 - m_V\} = \\
\{m_V, \sigma p_V, \sigma p_V, -m_V\} \quad (116)

and thereby confirm that a set
\[ H_s = \{H_s^{LL}, H_s^{TL}, H_s^{IT}, H_s^{TT}\} \quad (117) \]

involves both the row \((L)\) and column \((T)\) subsets:
\[ H_s^{LL} = \{m_V, \sigma p_V\}, \quad H_s^{TL} = \{\sigma p_V, -m_V\}, \quad (118) \]
\[ H_s^{IT} = \{m_V, \sigma p_V\}, \quad H_s^{TT} = \{\sigma p_V, -m_V\}. \quad (119) \]

Thus, it follows that
\[ H_s U_s = \{H_s^{LL}, H_s^{2LL}, H_s^{IT}, H_s^{2IT}\} \]
\[ \{U_s^{LL}, U_s^{2LL}, U_s^{IT}\} = \{H_s^{1L}U_s^{1T}, H_s^{2L}U_s^{2T}\} = \]
\[ \{\{m_V, \sigma p_V\}\{r\}, u_a^{(r)}\}, \{\sigma p_V, -m_V\}\{r\}, u_a^{(r)}\} = \]
\[ \{m_V \cdot \chi^{(r)} + \sigma p_V \cdot u_a^{(r)}, \quad (\sigma p_V)\chi^{(r)} + (-m_V) \cdot u_a^{(r)}\} = \]
\[ \{m_V \chi^{(r)} + (\sigma p_V)u_a^{(r)}, \quad (\sigma p_V)\chi^{(r)} - m_V u_a^{(r)}\}. \quad (120) \]

and consequently, the multiplication \(E_s U_s\) is equal to
\[ E_s U_s = \{E_s^{LL}, E_s^{2LL}, E_s^{IT}, E_s^{2IT}\} \]
\[ \{U_s^{LL}, U_s^{2LL}, U_s^{IT}\} = \{E_s^{1L}U_s^{1T}, E_s^{2L}U_s^{2T}\} = \]
\[ \{\{E_V, 0\}\{r\}, u_a^{(r)}\}, \{0, E_V\}\{r\}, u_a^{(r)}\} = \]
\[ \{E_V \cdot \chi^{(r)} + 0 \cdot u_a^{(r)}, \quad 0 \cdot \chi^{(r)} + E_V \cdot u_a^{(r)}\} = \{E_V \chi^{(r)}, \quad E_V u_a^{(r)}\}. \quad (121) \]

They show [8] that a system
\[ E_V \chi^{(r)} = (\sigma p_V)u_a^{(r)} + m_V \chi^{(r)}, \quad (122) \]
\[ E_V u_a^{(r)} = (\sigma p_V)\chi^{(r)} - m_V u_a^{(r)} \quad (123) \]
is none other than an equality
\[ \{E_V \chi^{(r)}, \quad E_V u_a^{(r)}\} = \{m_V \chi^{(r)} + (\sigma p_V)u_a^{(r)}, \quad (\sigma p_V)\chi^{(r)} - m_V u_a^{(r)}\}. \quad (124) \]

In a similar way, one can define \(E_s N_s, H_s N_s\) and find [8] the equations
\[ |E_V| \nu_a^{(r)} = -(\sigma p_V)\chi^{(r)} - m_V \nu_a^{(r)}; \quad (125) \]
\[ |E_V| \chi^{(r)} = -(\sigma p_V)\nu_a^{(r)} + m_V \chi^{(r)} \quad (126) \]

from an equality
\[ \{|E_V| \nu_a^{(r)}, \quad |E_V| \chi^{(r)}\} = \{- (\sigma p_V)\chi^{(r)} - m_V \nu_a^{(r)}, \quad m_V \chi^{(r)} - (\sigma p_V)\nu_a^{(r)}\}. \quad (127) \]

For completeness, we remark that the matrices [5] suggest
\[ M_s = \{0, m_A, m_A, 0\}, \quad (128) \]
\[ E_s = \{0, E_A, E_A, 0\}. \quad (129) \]
\[ \mathcal{P}_s = \{0, \ p_A, \ p_A, \ 0\}. \]

Therefore, if one starts from (128)-(130) and then performs explicit set operations of the Dirac equation, one can establish that

\[ \{E_Au_a^{(r)}, \ E_A\chi^{(r)}\} = \{(\sigma p_A)\chi^{(r)} + m_Au_a^{(r)}, \ (\sigma p_A)u_a^{(r)} - m_A\chi^{(r)}\} \]

(131)

holds if there exist [9] the united connections

\[ E_Au_a^{(r)} = (\sigma p_A)\chi^{(r)} + m_Au_a^{(r)}, \]

(132)

\[ E_A\chi^{(r)} = (\sigma p_A)u_a^{(r)} - m_A\chi^{(r)}. \]

(133)

As in ratios (125)-(127), the equality

\[ \{|E_V\chi^{(r)}, \ |E_V\nu_a^{(r)}\} = \{-(\sigma p_V)\nu_a^{(r)} - m_V\chi^{(r)}, \ -(\sigma p_A)\chi^{(r)} + m_A\nu_a^{(r)}\}, \]

(134)

found in a given case leads to a system

\[ |E_A|\chi^{(r)} = -(\sigma p_A)\nu_a^{(r)} - m_A\chi^{(r)}; \]

(135)

\[ |E_A|\nu_a^{(r)} = -(\sigma p_A)\chi^{(r)} + m_A\nu_a^{(r)} \]

(136)

as to one more consequence of a set-matrix duality principle.

### 7. Concluding Remarks

Furthermore, if it turns out that there is no single internally disclosed set, for which a fully regular matrix would not exist, we cannot exclude the possibility to strictly change our presentations about set operations. Without such a change, the mathematically united theory construction of sets and matrices still remains not quite in line with logic.

Therefore, to have the right to perform explicit set operations necessary for deciding the problems at the new level, we must at first define the rows, columns and diagonals of each of them if this is not forbidden by quantities of its elements.

For such purposes, it is desirable to use here the following sets:

\[ \mathcal{C} = \{\varepsilon\}, \]

(137)

\[ \mathcal{D} = \{\eta, \ \kappa\}, \]

(138)

\[ \mathcal{E} = \{\lambda, \ \mu, \ \nu\}, \]

(139)

\[ \mathcal{F} = \{\rho, \ \tau, \ \upsilon, \ \omega\}, \]

(140)

\[ \mathcal{H} = \{\phi, \ \psi, \ \varphi, \ \chi\}. \]

(141)

A set \( \mathcal{C} \) consists only of one element. There is neither a row nor a column in a one-element set. The existence of either one row \( \mathcal{C}_L \) or one pair of rows \( \mathcal{C}_{1L} \) and \( \mathcal{C}_{2L} \) of the same column \( \mathcal{C}_T \) for \( \mathcal{C} \) would indicate that it is not one-element. However, according to Lemma 1, a matrix \( (\varepsilon) \) of a set \( \{\varepsilon\} \) exists in a kind of fully regular matrix as one of its nonempty submatrix. We must therefore recall [22] the following Theorem.

**Lemma 4 (Theorem on the smallest nonempty set).** No single fully regular set from the same element exists without an internally disclosed set of a higher cardinality.

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A two-element set \( \mathcal{D} \) must distinguish itself from \( \mathcal{C} \) by either one row or two rows of its one column. Therefore, it should be defined the matrix structure of a set \( \mathcal{D} \) so that its elements have constituted in whole either a set of the row \( \mathcal{D}_L \) such as \([138]\) or two nonempty rows \( \mathcal{D}_{1L} \) and \( \mathcal{D}_{2L} \) of the same nonempty column \( \mathcal{D}_{1T} \) of a set of the column \( \mathcal{D}_T \) in the form

\[
\mathcal{D}_L = \mathcal{D} = \{\eta, \kappa\},
\]

\[
\mathcal{D}_T = \mathcal{D} = \{\mathcal{D}_{1L}, \mathcal{D}_{2L}, \mathcal{D}_{1T}\},
\]

\[
\mathcal{D}_{1L} = \{\eta\}, \mathcal{D}_{2L} = \{\kappa\}, \mathcal{D}_{1T} = \{\eta, \kappa\}.
\]

On this basis, each matrix of \([141]\) and \([25]\) itself has constituted a kind of set of the column from \([108]\) and \([112]\) as a consequence of a set-matrix duality principle.

The possible grouping of elements of a set \( \mathcal{E} \) would seem to say about that it has two rows \( \mathcal{E}_{1L} \) and \( \mathcal{E}_{2L} \) from two elements and one column \( \mathcal{D}_{1T} \) from three elements. This implication, however, does not correspond to reality. The point is that the availability of such subsets in \( \mathcal{E} \) is incompatible with an equality of the number of its rows and the number of elements of its column. In other words, there is no single row with two elements in a set \( \mathcal{E} \) even if it is strictly a set of the row. A set \( \mathcal{E} \) can therefore be presented either as a set of the row \( \mathcal{E}_L \) or as a set of the column \( \mathcal{E}_T \) by the following manner:

\[
\mathcal{E}_L = \mathcal{E} = \{\lambda, \mu, \nu\},
\]

\[
\mathcal{E}_T = \mathcal{E} = \{\mathcal{E}_{1L}, \mathcal{E}_{2L}, \mathcal{E}_{3L}, \mathcal{E}_{1T}\},
\]

\[
\mathcal{E}_{1L} = \{\lambda\}, \mathcal{E}_{2L} = \{\mu\}, \mathcal{E}_{3L} = \{\nu\}, \mathcal{E}_{1T} = \{\lambda, \mu, \nu\}.
\]

One more characteristic moment is an equal number of rows \( \mathcal{X}_L \) and columns \( \mathcal{X}_T \) of a quadratic set \( \mathcal{X}_{LT} \) from

\[
\mathcal{X} = \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{H}, \ldots.
\]

**Definition 4.** An Internally disclosed set is called a quadratic one if it has the same quantity of rows and columns.

Such a correspondence expresses, for each of sets \( \mathcal{F} \) and \( \mathcal{H} \), the idea of a kind of pair of diagonals \( \mathcal{X}_{1D} \) and \( \mathcal{X}_{2D} \), allowing one to define its self matrix structure

\[
\mathcal{F}_{LT} = \mathcal{F} = \{\mathcal{F}_{1L}, \mathcal{F}_{2L}, \mathcal{F}_{1T}, \mathcal{F}_{2T}, \mathcal{F}_{1D}, \mathcal{F}_{2D}\},
\]

\[
\mathcal{F}_{1L} = \{\rho, \tau\}, \mathcal{F}_{2L} = \{v, \omega\},
\]

\[
\mathcal{F}_{1T} = \{\rho, v\}, \mathcal{F}_{2T} = \{\tau, \omega\},
\]

\[
\mathcal{F}_{1D} = \{\rho, \omega\}, \mathcal{F}_{2D} = \{\tau, v\},
\]

\[
\mathcal{H}_{LT} = \mathcal{H} = \{\mathcal{H}_{1L}, \mathcal{H}_{2L}, \mathcal{H}_{1T}, \mathcal{H}_{2T}, \mathcal{H}_{1D}, \mathcal{H}_{2D}\},
\]

\[
\mathcal{H}_{1L} = \{\phi, \psi\}, \mathcal{H}_{2L} = \{\varphi, \chi\},
\]

\[
\mathcal{H}_{1T} = \{\phi, \varphi\}, \mathcal{H}_{2T} = \{\psi, \chi\},
\]

\[
\mathcal{H}_{1D} = \{\phi, \chi\}, \mathcal{H}_{2D} = \{\psi, \varphi\},
\]

which was used above.

An intramatrix feature of the two types of sets, namely, the sets of the row \( \mathcal{X}_L \) and the column \( \mathcal{X}_T \), is their simultaneous absence, their coexistence or both. In the first case, the matrix has a set consisting of one element. The matrices with sets of the same quantity \((m = n)\) of rows
and columns \((nT)\) refer to the second case. An example of the third case is the matrix with a set with elements of either one \(X_L\) row or one \(X_T\) column.

**Definition 5.** An Internally disclosed set is called a set of the row if and only if each element constitutes in whole a kind of empty row of its column.

**Definition 6.** An Internally disclosed set is called a set of the column if and only if each element constitutes in whole a kind of nonempty row of its column.

We will now perform, on the basis of a set-matrix duality principle, the explicit set operations referring to those sets, the matrix structure of which were already established above. This requires one to explain, for example, the equality of sets \(F\) and \(H\) as a consequence of equalities

\[ F_{mL} = H_{mL} \iff F_{nT} = H_{nT} \]  

expressing at \(m = n = 1, 2\) the idea about that

\[ \rho = \phi, \quad \tau = \psi, \quad v = \varphi, \quad \omega = \chi. \]  

Insofar as their inequality is concerned, it is in favor of

\[ F_{mL} \neq H_{mL} \iff F_{nT} \neq H_{nT}, \]  

which at \(m = n = 1, 2\) state that

\[ \rho \neq \phi, \quad \tau \neq \psi, \quad v \neq \varphi, \quad \omega \neq \chi. \]  

Uniting \(F\) with \(H\) having \((148), (149), (152)\) and \((153)\), one can found that

\[ K = F + H = \{F_{1L}, F_{2L}\} + \{H_{1L}, H_{2L}\} = \{\rho + \phi, \tau + \psi, v + \varphi, \omega + \chi\}. \]  

Unification of this type is none other than a set

\[ K_{LT} = K = \{K_{1L}, K_{2L}, K_{1T}, K_{2T}, K_{1D}, K_{2D}\}, \]  

\[ K_{1L} = \{\rho + \phi, \tau + \psi\}, \quad K_{2L} = \{v + \varphi, \omega + \chi\}, \]  

\[ K_{1T} = \{\rho + \phi, v + \varphi\}, \quad K_{2T} = \{\tau + \psi, \omega + \chi\}, \]  

\[ K_{1D} = \{\rho + \phi, \omega + \chi\}, \quad K_{2D} = \{\tau + \psi, v + \varphi\}. \]  

One more consequence of the matrix structure of sets \(F\) and \(H\) is such that their multiplication constitutes in whole another set

\[ M = FH = \{F_{1L}, F_{2L}, F_{1T}, F_{2T}\} \{H_{1L}, H_{2L}, H_{1T}, H_{2T}\} = \{F_{1L}H_{1T}, F_{1L}H_{2T}, F_{2L}H_{1T}, F_{2L}H_{2T}\} = \{\{\rho, \tau\}\{\phi, \varphi\}, \{\rho, \tau\}\{\psi, \chi\}, \{v, \omega\}\{\phi, \varphi\}, \{v, \omega\}\{\psi, \chi\}\} = \{\rho \cdot \phi + \tau \cdot \varphi, \rho \cdot \psi + \tau \cdot \chi, v \cdot \phi + \omega \cdot \varphi, v \cdot \psi + \omega \cdot \chi\} = \]
\[
\{\rho\phi + \tau\varphi, \ \rho\psi + \tau\chi, \ \nu\phi + \omega\varphi, \ \nu\psi + \omega\chi\}
\]  
\tag{165}

and that, consequently, we have

\[
M_{LT} = M = \{M_{1L}, \ M_{2L}, \ M_{1T}, \ M_{2T}, \ M_{1D}, \ M_{2D}\},
\]  
\tag{166}

\[
M_{1L} = \{\rho\phi + \tau\varphi, \ \rho\psi + \tau\chi\}, \quad M_{2L} = \{\nu\phi + \omega\varphi, \ \nu\psi + \omega\chi\},
\]  
\tag{167}

\[
M_{1T} = \{\rho\phi + \tau\varphi, \ \nu\phi + \omega\varphi\}, \quad M_{2T} = \{\rho\psi + \tau\chi, \ \nu\psi + \omega\chi\},
\]  
\tag{168}

\[
M_{1D} = \{\rho\phi + \tau\varphi, \ \nu\psi + \omega\chi\}, \quad M_{2D} = \{\rho\psi + \tau\chi, \ \nu\phi + \omega\varphi\}.
\]  
\tag{169}

If we now suppose that \(\mathcal{K} = M\), the united connections

\[
\mathcal{K}_{mL} = M_{mL} \Leftrightarrow \mathcal{K}_{nT} = M_{nT}
\]  
\tag{170}

suggest at \(m = n = 1, 2\) the same system

\[
\rho + \phi = \rho\phi + \tau\varphi, \quad \tau + \psi = \rho\psi + \tau\chi,
\]  
\tag{171}

\[
\nu + \varphi = \nu\phi + \omega\varphi, \quad \omega + \chi = \nu\psi + \omega\chi.
\]  
\tag{172}

However, the very coexistence of a matrix of set elements and a set of matrix elements indicates the role of the unified system of rows, columns and diagonals in all set operations. A notion about matrix determinant may therefore be connected with a notion about set determinant. Such a nonclassical correspondence, regardless of what is the interaction between the matrix and its set, requires one to solve the question as to what is the value of determinants of the discussed types of quadratic sets. For this we must at first introduce a symbol \(\Delta_X\) for its denotation, allowing us to write at \(X = \mathcal{T}, \mathcal{H}\) the following determinants of the second order:

\[
\Delta_{\mathcal{T}} = \{\mathcal{T}_{1D}, \ \mathcal{T}_{2D}\} = \rho\omega - \tau\nu,
\]  
\tag{173}

\[
\Delta_{\mathcal{H}} = \{\mathcal{H}_{1D}, \ \mathcal{H}_{2D}\} = \phi\chi - \psi\varphi.
\]  
\tag{174}

Finally, insofar as the matrix operations performed at the new level are concerned, all of them together with some aspects of a set-matrix duality principle (not noted here) will be presented in our further works.

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