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The Rate of Three-dimensional Hall-MHD Reconnection

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Abstract. There is a discrepancy between reconnection rates observed in a) various plasma environments, b) plasma PIC simulations and c) resistive fluid simulations. Careful observations of solar flares show reconnection rates between $0.001 - 0.1v_A$ (see, e.g., [1]), plasma PIC and Hall-MHD simulations tend to show faster reconnection rate of around $0.1v_A$ and pure MHD resistive simulations tend to converge to $0.015v_A$. The common explanation of the difference between b and c [2] is that plasma reconnection is inherently different from resistive MHD reconnection due to the different mechanisms of how individual field lines break and reconnect. This would imply a universal rate of $0.1v_A$ in electron-proton plasma. In this paper, I report three-dimensional Hall-MHD simulations with resolution up to $2304 \times 4608^2$. Reconnection is mediated by self-excited turbulence in the current layer and is indeed faster than MHD reconnection. However, the reconnection rate goes down as the layer width increases. For the average layer width between two and ten ion skin depths ($d_i$), we recover “classic” value of $0.1v_A$, however, the trend of the rate to decrease may bring it to the fluid value for widths of around $170d_i$, although reaching this width is not possible with current numerical resources. In any case, based on these simulations, now we are certain that the $0.1v_A$ value has been favored before due to the limited range of scales available in numerical simulations.

1. Introduction

Magnetic reconnection is a topological rearrangement of the magnetic field lines, leading to the release of magnetic energy, which is associated with solar X-ray flares [3, 4]. Magnetic field lines are supposed to be frozen into the well-conducting plasma, so quiet laminar reconnection [3, 5] is extremely slow and cannot explain observed phenomena. This prompted research into collisionless reconnection [6, 7, 8]. The stochasticity of magnetic field lines due to ambient turbulence [9, 10, 11] leads to fast reconnection, also the tearing instability of the thin current sheet [12, 13] was proposed as a driver of resistivity-independent reconnection, which was shown to be consistent with two-dimensional simulations [14, 15].

Recent three-dimensional MHD simulations [16] further investigated the development of instability of the current layer and found that unlike 2D case the layer is not dominated by plasmoids but has fully developed turbulence with motion on all scales between the layer width and the dissipation scale. It was confirmed that reconnection is fast (independent on resistivity) and [16] interpreted this as a consequence of turbulence locality. In this picture, the rate was determined not by the critical Lundquist number, but rather by inherent properties of turbulence, such as the ratio of magnetic to kinetic energies and the Kolmogorov constant.

Reconnection rate is an important parameter to understand many space and solar physics phenomena. For example, reconnection releases up to $6 \times 10^{33}$ erg in gigantic X-ray flares on the Sun. The dayside reconnection in front of the Earth magnetosphere allows fast particles from
Run | $n_x \cdot n_y \cdot n_z$ | $d_i$ | $dt$ | $L/d_i$ | $d_i/d$ | $\nu_4$
--- | --- | --- | --- | --- | --- | ---
R0 | $384 \times 768^2$ | 0.32 | 2.25e-5 | 20 | 39 | 9.00e-08
R1 | $768 \times 1536^2$ | 0.16 | 1.06e-5 | 39 | 39 | 9.33e-09
R2 | $1536 \times 3072^2$ | 0.08 | 5.30e-6 | 79 | 39 | 9.26e-10
Q0 | $576 \times 1152^2$ | 0.052 | 5.00e-5 | 121 | 9.5 | 9.50e-09
Q1 | $1152 \times 2304^2$ | 0.026 | 2.75e-5 | 242 | 9.5 | 9.43e-10
Q2 | $2304 \times 4608^2$ | 0.013 | 1.25e-5 | 483 | 9.5 | 9.35e-11

Table 1. Parameters in our Hall-MHD simulations, where grid size $d$, ion skin depth $d_i$, box size $L$, time step $dt$ and hyperdiffusivity $\nu_4$ are in code units, such that $L = 2\pi$, and Alfvén crossing time of $L$ is also $2\pi$. Finally, the code has $\rho = 1$ and the code unit of mass is absent.

The understanding of magnetic reconnection is important not only in the Sun and heliosphere but also in a wide range of astrophysical objects. The push toward larger resolutions and larger numbers of particles in PIC and classical hybrid simulations have resulted in an impressive progress in understanding plasma reconnection on scales up to several hundred ion skin depths [7, 17]. At the same time, MHD fluid simulations also became commonplace [18, 19, 20], moving from 2D to 3D setups [10, 16, 21] and breaking through the critical Lundquist number required for spontaneous reconnection ($10^4$).

Many astrophysical objects, such as the interstellar medium in our Galaxy, feature relatively high level of ambient turbulence and the reconnection is supposed to be fast [9] due to the existing magnetic field stochasticity. In highly magnetized environments, e.g., the solar surface or the pulsar wind nebulae, the level of ambient turbulence is much lower, and the turbulence, spontaneously generated by the current sheet and fueled by the reconnecting field itself is of great interest. Magnetospheric observations which show an enhanced level of turbulence inside current sheets [22, 23], are supportive of this spontaneous reconnection picture.

Despite the success of MHD reconnection, it has been argued that the plasma reconnection rate could be higher and the value around $0.1v_A$ obtained by most authors in PIC and Hall-MHD simulations has been brought as an argument that plasma reconnection is qualitatively different due to the different mechanisms of how individual field lines break and reconnect. In this case, studying MHD limit in reconnection would be of little practical use, because all relevant MHD fluids in space physics and astrophysics are actually plasmas. This will also bring into question each MHD study having unresolved current layers, ostensibly due to an improperly resolved inflow condition at the discontinuity.

The spontaneously evolving turbulent current layer in MHD develops turbulence that pretty much resembles generic MHD turbulence [16]. In this case, we expect turbulence to be local and the large-scale effects to be disconnected from the small-scale plasma effects and the MHD rate...
Figure 1. Left: The all-periodic box reconnection setup with two current layers between the regions of reconnecting field $\pm B_{0y}$. Upper right: the surface of mixing of the electron fluid (the boundary between mixed and unmixed fluid), which is the surface where reconnection occurs, the color of the surface indicates the intensity of mixing. Yellow tubes are the magnetic field lines. Bottom right, we zoom into the surface of the electron fluid mixing and see complex and multi-scale nature of reconnection. The large-scale ridges visible on the surface on the left are characteristic angular outflows of Hall reconnection, while small scales shows a lot of structure pertaining to fast electron MHD turbulence.

of $\sim 0.015$ of Alfvén speed $v_A$ should be universal.

The study of reconnection has split over various pathways, often creating confusion regarding the approximation being made. In the big picture of things, we subdivide reconnection into small-scale or plasma-scale and large-scale cases. (see, e.g., [24] for an overview of various reconnection regimes).

In the first case, the length of the reconnection layer is at most a few tens of ion skin depths and the layer thickness is of order or smaller than ion skin depth. This is the case which is commonly realized in plasma experiments. It can also be applied, with some caution, to the reconnection in the Earth magnetosphere which determines its connection to the solar wind. We believe that this regime has already been extensively studied with PIC codes, see, e.g. [7, 17].

We would like to concentrate on the second case, large-scale reconnection, where the length of the current layer is at least a hundred ion skin depths. Such cases include astrophysical reconnection, where it is important for understanding star formation, flares from pulsars and acceleration in high energy sources, such as blazars or GRBs. The other example is the reconnection in the solar corona, where the typical length of a large reconnection layer is 10
Figure 2. Kinetic and magnetic spectra in simulation R2 and Q2 (see Table 1). On small scales magnetic perturbations are primarily right-circularly-polarized whistlers, which have a steep spectrum of $k^{-7/3}$. While slower velocity perturbations are ion cyclotron waves, the nature of their cascade is still debated.

Mm. In this large-scale case, it has become clear recently that 2D physics is not sufficient to explain the physics of fast reconnection correctly. For example, while 3D results usually show fast reconnection, in the 2D case the three papers [25], [26], [27] all show different results: the first two reported slow reconnection with scalings $(d_i/L)^{3/2}$ and $(d_i/L)^{1/2}$ correspondingly, while the last – fast reconnection with $0.1v_A$. While 2D reconnection show plasmoids, the 3D case often have turbulent flux ropes.

The key role of turbulence in 3D reconnection still has to be studied in great detail. In this project, we decided to concentrate on the spontaneous reconnection that does not have any imposed turbulence in it. This may correspond to solar events, which look very quiet before erupting.

2. Previous MHD findings

Earlier, I simulated spontaneous reconnection in resistive MHD with the main results presented in [16]. The power spectra of turbulent perturbations resembled decaying magnetic turbulence with magnetic contribution dominating over kinetic on large scales and approximate equipartition on small scales. The total energy spectral slope was between -1.5 and -1.7, roughly consistent with Goldreich-Sridhar [28] scaling. Such spectral slopes are indicative of a turbulent cascade, alternatively called local-in-scale turbulence. The concept of such turbulence was introduced by Kolmogorov [29] who showed that the energy transfer between large and small scales is dominated by so-called local interactions, i.e. nonlinear interactions between two scales separated by a factor of two or so. Scale-locality is a key ingredient in theories of turbulent reconnection. Indeed full scale-locality will imply that large-scale quantities, such as reconnection rate and dissipation rate per unit area should be independent on microphysics. Unlike the 2D case where large plasmoids couple large and small scales directly, the 3D turbulence we observe appears to be local-in-scale.

The dynamics of large scales, which determines such properties as the bulk reconnection and dissipation rates, will be disconnected from the dynamics on small scales, plasma parameters, and microscopic dissipation. We were arguing in [16] that this would imply that any large-scale spontaneous reconnection case will produce the same rate, i.e., $0.015v_A$.

3. Hall-MHD Equations

Hall-MHD is a simple framework for electron-proton plasmas. We use one-fluid incompressible Hall-MHD that nevertheless has a defining feature of dispersive plasmas below the ion skip depth $d_i$ – the highly dispersive whistler waves that was argued to result in the higher reconnection
Figure 3. Left: we evolve passive scalars associated with ions (top) and electrons (bottom). Here we show xz slices of the data cubes, i.e. the slices parallel to the mean field $B_z$. Right: Dimensionless reconnection rate as a function of the mixing layer’s width for all simulations in Table 1. The rate shows “classic” value of 0.1 for moderate width and then started to decline. We conjecture that for $w \gg d_i$ the rate will converge to the fluid value of 0.015. Note that the rate is largely insensitive to the size of the datacube. It does depend of the ratio of $d_i/d$ (red vs blue), but only for early times when $w < 3.6 d_i$.

rate because they open an X-point to a constant angle [30, 31].

Although the Hall-MHD is not capturing all elements of plasma dynamics, it had been verified to accelerate reconnection [2]. We hope, therefore, that Hall-MHD will allow us to uncover the puzzle of the discrepancy between MHD rates and plasma rates.

The incompressible Hall-MHD equations with explicit dissipation are

$$
\partial_t \mathbf{v} = \tilde{S}(-\omega \times \mathbf{v} + \mathbf{j} \times \mathbf{b}) - \nu_n \nabla^4 \mathbf{v},
$$

$$
\partial_t \mathbf{b} = \nabla \times ((\mathbf{v} - d_i \mathbf{j}) \times \mathbf{b}) - \nu_n \nabla^4 \mathbf{b},
$$

$$
\partial_t \phi_i = \mathbf{v} \cdot \nabla \phi_i - \nu_n \nabla^4 \phi_i,
$$

$$
\partial_t \phi_e = (\mathbf{v} - d_i \mathbf{j}) \cdot \nabla \phi_e - \nu_n \nabla^4 \phi_e.
$$

We also show evolution equations for passive scalars $\phi_i$, $\phi_e$ for ion and electron fluids respectively. Here we introduced the current density $\mathbf{j} = \nabla \times \mathbf{B}$ and vorticity $\omega = \nabla \times \mathbf{v}$. We renormalized magnetic field to velocity units $\mathbf{b} = \mathbf{B}/\rho_1^{1/2}$ (the absence of $4\pi$ is due to Heaviside units), and used the solenoidal projection operator $\tilde{S} = (1 - \nabla \Delta^{-1} \nabla)$ to get rid of pressure.

The forth-order diffusivity is essential in the Hall-MHD case to terminate the whistler cascade. While natural forth-order diffusive terms exist in plasmas, such as an extra 4th order magnetic diffusivity due to the scalar electron viscosity, we treat this diffusivity as just a tool to terminate the cascade.

4. Simulations and results

We are following our earlier paper on spontaneous reconnection in resistive MHD [16] regarding initial setup, which is the thin planar current layer and tiny initial perturbations (see Fig. 1). Thus we will be able to compare results between MHD and Hall-MHD case directly.
We ran two series of simulations with different ratio of the ion skin depth $d_i$ to the grid size $d$. See Table 1. Cases R0-R2, have “physically motivated” $d_i$, i.e., we assumed that the whistler cascade terminates on electron scales $\sim d_e = d_i \sqrt{m_e/m_i} \approx d_i/42$. This is actually the observed range of the steeper part of the spectrum of the dispersive cascade in the solar wind. This condition is approximately satisfied with $d_iN = 40(2\pi)$.

Cases Q0-Q2 have $d_e - d_i$ range of scales diminished by a factor of 4, e.g. using 4 times smaller $d_i$. Here we are motivated to go to the largest possible box sizes, in units of $d_i$. This also broadly corresponds to PIC simulations with the reduced $m_i/m_e$ ratio.

All simulations were evolved to around $t = 4$ in code units at which point reconnection may be influenced by the finite box size (see Fig. 3).

We introduce $w$, the average width of the mixed scalar, i.e. the volume of the mixed scalar in one layer, divided by the XY cross-section of the datacube, $4\pi^2$ in our case. We determined the reconnection rate by the rate of mixing of the electron passive scalar as

$$v_r = dw/dt.$$  

On Figures 1-3 we show that current layer indeed become fully turbulent. Figure 2 shows energy spectra of the velocity and magnetic perturbation which are broadly consistent with MHD turbulence on large scales and whistler turbulence for B on small scales and ion-cyclotron turbulence for v on small scales. Figure 1, right panel, show mixing surface of the passive scalar (a threshold value of passive scalar of 0.95), which looks like multi scale mixing surface characteristic for turbulence. Figure 3, left panel, indicates that mixing of electron fluid is more intense than of the ion fluid, which is characteristic for plasma reconnection. We found, however, that on large timescales the mixing rates of ion and electron fluids tend to converge, which is indicative of MHD regime. Figure 3, right panel, shows the reconnection rate as a function of average layer width. It is amazing that despite the difference in simulation setups the values for the rate largely converge for $w > 3d_i$. We also confirm that the finite box size is not a problem for $t = 4$, as the simulation in smaller box, say Q1 is reproduced by the simulation in larger box Q2.

The overall conclusion, perhaps greatly surprising that $0.1v_A$ is not a universal number, but rather a constant trend over ranges of scales currently explored by numerics. Starting with $w > 7d_i$ the rate is decreasing. We speculate that given the observed trend, the rate may approach MHD fluid rate at $w \approx 170d_i$, which would require box sizes five times larger than we used in this study, i.e., around $20000^3$, which is currently not possible.

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6. Appendix: Code detail.

ICaru5 is a pseudospectral C++ code which solves hydrodynamic, MHD, Reduced MHD, Hall MHD and Electron MHD equations. Some details may be found in [32, 33]. The pseudospectral code solves above equations as the ordinary differential equation in time for each spacial Fourier harmonic, the “pseudo” coming from the fact that nonlinear term is calculated in real space, and then converted back to Fourier space. Unlike finite difference schemes pseudospectral scheme does not suffer from either dispersion of dissipation errors. The dissipation is explicit and divergence-free condition for velocity and magnetic field are done with simple algebraic
operations in Fourier space. For time integration we use leapfrog scheme which is time-reversible and numerical dissipation is absent, because nonlinear term, calculated in this manner, preserve both energy and cross-helicity.

By testing the code, we verified that the following standard prescriptions for the dissipation and time step are sufficient for the simulation to stay accurate and stable. The fourth-order diffusivity coefficient $\nu_4$ is determined by

$$\left(\frac{\nu_4}{c_d^2}\right)^{1/8} = \frac{3.3}{N}, \quad (6)$$

where $\epsilon$ is the dissipation rate, which in our case was taken 0.08 in code units and $N$ refers to the number of grid points in the larger dimension (which is assumed to have the size $2\pi$ in code units). This condition terminates whistler cascade on grid scale, where 3.3 was found empirically for sufficient numerical accuracy. The time step $dt$ was chosen by CFL number of 0.1 for the whistler mode:

$$dt = 0.1(2\pi/N)^2/v_A d_c. \quad (7)$$

Hall-MHD code with two passive scalars does 18 direct, and 8 inverse Fourier transforms per time step, 26 in total. Since in Hall-MHD velocity evolves on the longer timescale than the magnetic field, it is possible to save some computing time by sub-stepping (see, e.g., [34]). The small-scale evolution of $b$ is faster than $v$ by a factor of around $d_c N/(2\pi)$. We implemented substepping routine when certain $v$ evolution steps are skipped, e.g., in cases Q0-Q3 we will be doing 3 magnetic steps per 1 kinetic, which gives a substepping reduction of SR=4/6=0.67 of the original number of transforms and in the cases R0-R2 we will have 6 magnetic steps per 1 kinetic, SR=7/12=0.58. With substepping improvement, Hall-MHD code is about as fast as hydro code, per time step. Despite massive amounts of FFT transform, each requiring all-to-all communication, 1CARU5 scaled well up to 256k cores on IBM BG/Q platform and reached the peak of 0.3 Pflops on ALCF Mira, mostly because of the highly optimized parallel FFT library P3DFFT [35].

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