Implementation of quantum state manipulation in a dissipative cavity

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Scientists have explored techniques to perform quantum state manipulation in a cavity. This work shows that atomic spontaneous emission and cavity decay can be used to drive many atoms into a many-body steady-state entanglement. The protocol offered here dramatically improves fidelity as noise strength increases. Additionally, the dephasing noise is effectively suppressed, allowing high-fidelity target states to be obtained in a noisy environment.

Controlling quantum state superposition is a fundamental goal in quantum information science. Quantum coherence is often destroyed by environmental interactions, motivating much research into manipulating quantum states in noisy environments. Dissipative dynamics have been studied in two-qubit systems, where some initial states can be driven to steady states with nonzero entanglement. Although the entanglement generated by noise is small, these approaches open up new perspectives for engineering quantum states. Many methods have been developed to protect entanglement against decoherence.

The approaches for dissipative preparation satisfaction with the requirement of a nearly perfect fidelity of quantum state and robustness against various types of noise. Controlling this coherently is vital for realizing useful quantum processes. Here, we investigate quantum state manipulation in a cavity via engineering dissipative processes. By suitably choosing the intensities and detunings of fields, we build the effective decay channels through which the system is driven to the desired steady state. It is well known that the fidelity will be reduced with increasing noise strength. Using our method, high-fidelity quantum state can be obtained even when the level of noise increases. Moreover, we are particularly interested in reducing the negative effect of dephasing noise.

We consider that many identical three-level atoms interact with a cavity. Each atom has one excited state (|E⟩) and two stable ground states (|L⟩ and |R⟩). The transitions |L⟩ ↔ |E⟩ and |R⟩ ↔ |E⟩ are driven by a classical field and a quantized cavity field, respectively. The parameter ∆ is the detuning of driving field frequency from atomic transition frequency. Another cavity mode drives the transition |R⟩ ↔ |L⟩ resonantly. The Hamiltonian for the whole system is written as...
\[ H = H_d + H_c, \]

where

\[ H_d = \sum_{j=1}^{n} \left( \Omega e^{-i \Delta t} [E]_j \{E\} + ga [R]_j \{E\} + H.c. \right), \]

\[ H_c = \sum_{j=1}^{n} \left( \Omega, b^\dagger [E]_j \{R\} + H.c. \right), \]

where subscript \( j \) corresponds to the \( j \)th identical atom. \( \Omega \) is the Rabi frequency of classical pulse. \( a \) and \( b \) are the annihilation operators for cavity modes. \( g \) and \( \Omega \) are the atom-cavity coupling constants. In the system, the source of decoherence originates from cavity photon decay and atomic spontaneous emission. With considering the dissipation within Markovian approximation, the time evolution operator is given by a master equation with a Lindblad form

\[ \dot{\rho} = -i [H, \rho] - \sum_{k=1}^{n} \frac{\tau_{jk}}{2} \left( \sigma_{jk}^\dagger \sigma_{jk} \rho - \sigma_{jk} \rho \sigma_{jk}^\dagger \right) \]

\[ - \frac{\kappa}{2} \left( (a^\dagger a - 2a^\dagger a + \rho a^\dagger a) - \frac{\kappa'}{2} \left( (b^\dagger b - 2b^\dagger b + \rho b^\dagger b) \right) \right), \]

where \( \sigma_{jk}^\dagger = [E]_j \{L\} \) and \( \sigma_{jk}^\dagger = [E]_j \{R\} \). \( \tau_{j1}, \tau_{j2} \) are the spontaneous emission rates which are related to the decay channels \( [E] \rightarrow [L] \) and \( [E] \rightarrow [R] \), respectively. \( \kappa \) and \( \kappa' \) are the photon decay rates (for the sake of simplicity, we set that \( \kappa = \kappa' \)). The central idea of our work can be understood by considering three atoms in an open cavity. When the ground state \([L]_1 [L]_2 [L]_3\) is initially populated and the strength of classical field \( \Omega \) is sufficiently weak, there is only one single excitation in the whole system. The transition from \( |\psi\rangle \{0\}_a \{0\}_b \) to \( |\psi\rangle \{1\}_a \{0\}_b \) occurs with the coupling constant \( g \). Here \( |\psi\rangle_1 = 1/\sqrt{3} \left( |E\rangle_1 |L\rangle_2 |L\rangle_3 + |L\rangle_1 |E\rangle_2 |L\rangle_3 + |L\rangle_1 |L\rangle_2 |E\rangle_3 \right) \)
and \( |\psi\rangle_2 = 1/\sqrt{3} \left( |R\rangle_1 |L\rangle_2 |L\rangle_3 + |L\rangle_1 |R\rangle_2 |L\rangle_3 + |L\rangle_1 |L\rangle_2 |R\rangle_3 \right) \). The subscripts \( a \) and \( b \) denote the cavity modes \( a \) and \( b \), respectively. Then the excited manifold may show energy splitting. Under the condition that \( \Delta = g \), the state \([L]_1 [L]_2 [L]_3\) is resonantly coupled to the state \( \chi \left( |\psi\rangle_1 \{0\}_a \{0\}_b + |\psi\rangle_2 \{1\}_a \{0\}_b \right) \) (\( \chi \) is a normalization parameter). The atoms decay through the cavity mode \( a \) from the ground state to the state \( |\psi\rangle_2 \) which is our wanted state. With considering the homogeneous collective spontaneous emission of atoms, the net transfer is only possible from the ground state to state \( |\psi\rangle_2 \). A similar behavior has been reported for the system of an optical cavity containing many atoms in the presence of collective spontaneous emission. When the other atomic decay channels are included, the final state will be a mixed state instead of the steady state \( |\psi\rangle_2 \). Similarly, for many-atom case, the coupling constants \( \Omega_1, \Omega_2, \Omega_3 \) are chosen as \( \Omega_1 e^{i \pi/3}, \Omega_2 e^{i \pi/3}, \Omega_3 \) and \( \Omega_1 e^{i \pi/3}, \Omega_2 e^{i \pi/3}, \Omega_3 \) sequentially. If the atoms are not in state \( |\psi\rangle_2 \), they will be transferred to the ground state. The classical field drives the atoms from ground state to excited state again. Coherent driving corresponding to \( H_d \) and \( H_c \) are performed repeatedly. Consequently, in the presence of symmetry breaking, the competition between the coherent and dissipative dynamics can drive the system to the steady state \( |\psi\rangle_2 \). Similarly, for many-atom case, the coupling constants \( \Omega_1, \Omega_2, \ldots, \Omega_n \) are set to be \( \Omega e^{i \pi/3}e^{i \pi/3}, \ldots, \Omega e^{i \pi/3}e^{i \pi/3} \) sequentially. Here \( k = \{1, 2, \ldots, n - 1 \} \). We can use this method to drive many atoms into the state with single excitation.

In Fig. 1(a), we consider the case that three atoms in a cavity are initially in ground state. By a direct numerical simulation of master equation with the Hamiltonian in Eq. (1), the temporal evolution of density matrix is calculated numerically. As expected, the numerical results show that the fidelity of state \( |\psi\rangle_1 \) can reach 0.92. In addition, the fidelity of four-qubit state is about 0.9 if \( \tau_{j1} = 0.02g \) and the other parameters are chosen the same as those in three-qubit case. Without loss of generality, in the following, we will use three atoms as an example. With the increasing of \( \tau_{j1} \) and \( \kappa \), the Lindblad operators corresponding to noise terms will drive the transition from \( |\psi\rangle_2 \) to the other states, which makes the stationary state be away from \( |\psi\rangle_2 \), so the fidelity is reduced greatly. Similar problem arises in Refs. [15-18]. Is there any way to improve the fidelity under strong dissipation? In Fig. 1(b), one can observe that the density matrix can be expressed as \( \rho = \alpha |L\rangle_1 |L\rangle_2 |L\rangle_3 \{L\} + \beta |\psi\rangle \{\psi\} \) approximately because the other matrix elements have negligible amplitudes. The stronger the noise strengths are, the larger the parameter \( \alpha \) is. To improve the fidelity, the single qubit operations \( \sigma_{jk}^\dagger \) will be performed on the atom \( j \) \( \left( \sigma_{jk}^\dagger = [L]_j \{F\} + [F]_j \{L\} \right) \). Here \( [F] \) is an auxiliary ground state which is not coupled to the cavity mode. Then we set the parameters \( \Omega_j \) to be equal to \( \Omega' \). Through the driving \( H_c \), the atoms in state \( [R] \) will evolve
to state $|L\rangle$. The photon in mode $b$ need be detected by a detector. The process is described by the following master equation

$$\dot{\rho} = -i [H_d, \rho] - \frac{\kappa_d}{2} \left\{ (db^d b \rho - 2d^b \rho d^b + \rho d^b d^b b) \right\},$$

where $d$ denotes the annihilation operator of a detector mode. The master equation describes an irreversible detection process. By detecting the photon at time $t$ ($t > t_d$), the system is projected to the subspace where the detector clicks. Correspondingly, the density matrix is expressed by

$$\rho_m = \frac{\hat{P} \rho \hat{P}^{-1}}{\text{Tr}(\hat{P} \rho \hat{P}^{-1})}, \quad \hat{P} = I_1 \otimes I_2 \otimes I_3 \otimes I_4 \otimes I_5 \otimes |1\rangle_d \langle 1|,$$

where $I_1$, $I_2$, and $I_3$ are the identity operations of atoms. $I_4$ and $I_5$ are the identity operations of cavity modes $a$ and $b$, respectively. The state $|1\rangle_d \langle 1|$ corresponds to the Fock state of detector mode. Then the target state is $|\psi\rangle_2 = 1/\sqrt{3} [|L_1\rangle |F_2\rangle |F_3\rangle + |F_1\rangle |L_2\rangle |F_3\rangle + |F_1\rangle |F_2\rangle |L_3\rangle]$. In Fig. 2, the fidelity of three-qubit is improved from 0.8 to 0.98 when the cooperativity $g^2/\kappa \tau$ is about 20.

It is very important to discuss the variation of interaction time to reach the stationary state when the number of atoms increases. For many-atom system, we cannot simulate the dynamics evolution directly because of the increased complexity. In order to get additional insight, we use Monte Carlo wave function method to calculate the time evolution of four- and five-atom states. The simulations are performed...
under 200 quantum trajectories for each time point. Because the time needed to complete the prepa-
ration process is longer than its corresponding detection time, we will consider the time evolution in the
preparation process. In Fig. 3(a), one observes that the fidelity of four- or five-atom state is about 0.9.
The equilibration time corresponding to four- and five-atom systems is slightly different. However, with
increasing of atomic number, the interaction time should increase. In Figs. 3(b,c), the density matrix
elements are shown. The steady-state density matrix of the atoms can still be written as
\[
\rho = \alpha|x(0)\rangle\langle x(0)| + \beta|x(1)\rangle\langle x(2)| + \cdots + |x(n)\rangle\langle x(n)|)
\]
approximately. Here \(|x(0)\rangle = |L_1\rangle|L_2\rangle\cdots|L_n\rangle\rangle,
\(|x(1)\rangle = |L_1\rangle|L_2\rangle\cdots|L_n\rangle\rangle,\cdots, |x(n)\rangle = |L_1\rangle|L_2\rangle\cdots|L_n\rangle\rangle (j = 1, 2, \cdots, n)\rangle.
As the interaction time decreases, the ratio \(\alpha/\beta\) becomes large. Then high-fidelity state can still be
obtained in the detection process. For example, when the interaction time is chosen as 3000/g and the
other parameters are the same as those in Fig. 3, the final fidelities of four- and five-atom states are 0.99
and 0.97, respectively.

It is necessary to consider the influence of inhomogeneous dephasing on the creation of quantum
state. The super-operator for dephasing noise is given as follows
\[
\mathbb{L}_\kappa \rho = -i \sum_{k=1}^{3} \varepsilon_k \sigma_k^z \rho \sigma_k^z
\]
where \(\sigma_k^z = |L_k\rangle\langle L_k| - |R_k\rangle\langle R_k|\) and \(\varepsilon_k = \gamma(7 - 2k)/5\). The fidelity is calculated by solving the master
equation including the dephasing noise term. Unfortunately, the fidelity is significantly decreased when
the dephasing noise is taken into account, i.e., the fidelity is only 0.1 (0.8) before (after) detecting the
photon, if we chose \(\gamma = 0.01g\) in Fig. 5(a). How do we reduce the undesired effect of dephasing noise
in the whole process? Based on our method, a simple modification can be made to resist the influence of
inhomogeneous dephasing noise. In the beginning, the state of atoms is driven by \(H_d\) within a time
interval \(\delta t\). Then the \(\sigma_x\) operations drive the atomic transition between the states \(|L_k\rangle\rangle\) and \(|R_k\rangle\rangle\). In the
following, we apply \(H = 0\) for the same time interval \(\delta t\). Another \(\sigma_x\) operation is performed on each atom
again. That is to say, after each coherent driving over a time interval \(\delta t\), the operations \(\sigma_x, e^{-iHt}\), and \(\sigma_z\)
are done on the system sequentially. The basic unit of the whole process can be described by open system
dynamical maps shown in Fig. (4). From Fig. 5(b), we show that the final fidelity is about 0.97 in the
presence of dephasing noise. The physical mechanism can be understood as follows: the dephasing noise
will induce an unwanted phase fluctuation that destroys the coherence of our target state. The unwanted
phase can be cancelled by applying the operations \(\sigma_x, e^{-iHt}\), and \(\sigma_z\) in the noisy environment. In addition,
the dephasing noise results in a larger or smaller energy shift which does not change the dynamics
evolution dramatically within a short time interval. We also must point out that, with the increasing of
time interval, the influence of phase noise on the dynamics will no longer be neglected. However, the
fidelity is more than 0.9 if the time interval is less than 50/g.

Next we will discuss the influence of atomic spontaneous emission and photon decay on the manip-
ulation of quantum state. Because the noise is not easy to control, the strength of noise parameter may
vary within a wide range. In our work, the dissipation term for the channel \(|E\rangle\rangle \rightarrow |L\rangle\rangle\) plays a neglected
role due to the fact that the atoms are driven to ground state along the channel. On the other hand,
the varied spontaneous emission rate for the channel \(|E\rangle\rangle \rightarrow |R\rangle\rangle\) incoherently changes the distribution of
populations. Thus we will fix the parameter \(\tau_{j1}\) and consider the effect of the variation of sponta-
neous emission rate \(\tau_{j2}\) corresponding to the channel \(|E\rangle\rangle \rightarrow |R\rangle\rangle\). In Fig. 6(a), one finds that the final

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**Figure 3.** (a) Time evolution of fidelity versus \(gt\) with different atom numbers \(n\). The real parts of density
matrix elements of (b) four-atom state and (c) five-atom state. The spontaneous emission rates are
\(\tau_{j,k} = 0.03g\) and the other parameters chosen are the same as in Fig. 2.
**Figure 5.** Time evolution of fidelity for $g_0$ in (a) the preparation process and (b) detection process. The red/blue line corresponds to the original/modified method. Parameters set: $t = 25\,\gamma$ and $\tau_{j,k} = 0.05\,\gamma$. The other parameters are the same as those in Fig. 2.

**Figure 4.** Quantum circuit for the realization of dissipative maps. The processes of implementing dissipative maps are consisted of (a) the elementary preparation process and (b) the detection process. The atoms are represented as black circles and the operations as rectangles. The yellow dots denote the operations $\sigma_x$.

**Figure 6.** Fidelity versus time for various values of decay rate. The parameters are (a) $\kappa = 0.34g$; (b) $D = 0.9$. We have taken $\tau_{1,2} = D\tau$, $t = 25\,\gamma$, $\tau_{2,2} = 2D\tau$, $\tau_{3,3} = 1.5D\tau$, and $\tau = 0.05\,\gamma$. The other parameters are the same as those in Fig. 2.
fidelity is about 0.974 when the noise parameters vary in a wide range. In Fig. 6(b), when photon decay rate $\kappa$ is changed from 0.1g to 0.3g, we observe that the fidelity is 0.975. The results show that our method can be robust against the variation of noise parameters.

Now we would like to give a brief analysis of the experimental implementation. The configuration of atom may be realized with existing atom-cavity system in experiment\textsuperscript{25,26}. In our proposal, we chose the parameters as $g = 100\text{MHz}$, $\Omega = 1\text{MHz}$, $\Omega' = 5\text{MHz}$, $\gamma = 1\text{MHz}$, $\tau_{jk} = 10\text{MHz}$, $\kappa_d = 60\text{MHz}$ and $\kappa = 10\text{MHz}$, then the target quantum state is obtained with a fidelity of 0.98. The total interaction time is about $73\ \mu\text{s}$, which is smaller than the lifetime of metastable state. In the detection process, the conventional detector is only required to distinguish the vacuum and non-vacuum Fock number states because the total excitation number is less than or equal to 1 under the condition of weak driving. In addition, does the imperfect efficiency of detector influence the implementation? To evaluate the effect of detection efficiency on the fidelity, the dissipative term is arranged into the Lindblad form as

$$\eta_{d, g}/2[(d^\dagger b b d b - d b^\dagger d b - \rho d^\dagger b d b) + (1 - \eta)\kappa'/2[(2 b b^\dagger - b^\dagger b - \rho b^\dagger b)].$$

If the efficiency is low, the photon might leak into the environment. Then the detector will not be clicked. As a result, the success probability will be decreased. However, the fidelity is almost not affected, i.e., when the efficiency of detector $\eta$ is 0.8, $\kappa' = 0.34\kappa$, and the other parameters are the same as in Fig. 2, the fidelity and success probability for three-qubit state are about 0.98 and 0.81, respectively. Therefore, the method might be used to obtain a high-fidelity quantum state in an open system.

In conclusion, we have studied the dissipative dynamics of many atoms in a cavity. We showed that the target quantum state can be obtained by engineering the source of noise. Both cavity decay and atomic spontaneous emission have been changed from a detrimental source to a useful resource. The fidelity inevitably drops with the increasing of the noise strengths. However, the fidelity of steady state can be further improved by detecting the photon in cavity. Furthermore, a slight modification of our method allows the creation of target quantum state in the presence of dephasing noise. Thus our protocol might open up a promising perspective for manipulating quantum state in a noisy environment.

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Acknowledgements
This work was supported by the MOST of China (973 Project No. 2013CB328702), the National Natural Science Foundation of China under Grants No. 11205037, No. 51176041, No. 11105030, Doctoral Fund of Ministry of Education of China under Grant No. 20122302120003, PIRS OF HIT A201412, the Fundamental Research Funds for the Central Universities under Grant No. AUGA5710056414, and Natural Science Foundation of Heilongjiang Province of China Grants No. A201303.

Author Contributions
J.S. and J.Y.D. came up with the initial idea for the work and performed the simulations for the model. Y.X. performed the calculations for the model. J.Y.D. and Y.Y.J. performed all the data analysis and the initial draft of the manuscript. All authors participated in the writing and revising of the text.

Additional Information
Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Song, J. et al. Implementation of quantum state manipulation in a dissipative cavity. Sci. Rep. 5, 10656; doi: 10.1038/srep10656 (2015).

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