Conductance oscillations with magnetic field of a two-dimensional electron gas-superconductor junction

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We find the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Typically in experiments $L/2R_c \gtrsim 3$, where $L$ is the surface length and $R_c$ is the cyclotron radius. The conductance oscillates in experiments usually as $G = g_0 + g_1 \cos(2\nu \pi + \delta_1)$; higher harmonics, $g_2 \cos(4\nu \pi + \delta_2), \ldots$, are hardly seen. Theories based on the assumption of the interface perfection can hardly describe qualitatively the visibility of the conductance oscillations and the amplitudes of the harmonics: they predict $g_1 \sim g_2 \sim g_3, \ldots$. Our approach with the surface roughness qualitatively agrees with experiments. It is shown how a disorder at a 2DEG-S interface suppresses the conductance oscillations with $\nu$.

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I. INTRODUCTION

The study of hybrid systems consisting of superconductors (S) in contact with clean 2D normal metals (2DEG) in magnetic field has attracted considerable interest in recent years.\textsuperscript{1,2,3} The quantum transport in this type of structures can be investigated in the framework of Andreev reflection.\textsuperscript{4} When an electron quasiparticle in a normal metal (N) reflects from the interface of the superconductor (S) into a hole, Cooper pair transfers into the superconductor. A number of very interesting phenomena based on Andreev reflection had been studied in the past. For example, if the normal metal is surrounded by superconductors, so we have a SNS junction, a number of Andreev reflections appear at the NS interfaces. In equilibrium this leads to Andreev quasiparticle levels in the normal metal that carry considerable part of the Josephson current; out of the equilibrium, when superconductors are voltage biased, quasiparticles Andreev reflect about $2\Delta/eV$ times transferring large quanta of charge from one superconductor to the other. This effect is called Multiple Andreev Reflection (MAR).\textsuperscript{5}

Effect similar to MAR appears at a long enough NS interface in magnetic field when the magnetic field bends quasiparticle trajectories and makes quasiparticles reflect many times from the superconductor. If phase coherence is maintained interference between electrons and holes can result in periodic, Aharonov–Bohm-like oscillations in the magnetoconductance. The conductance $G$ of a S–2DEG interface in magnetic field was measured in experiments.\textsuperscript{6,7,8,9,10} It showed highly nonmonotonic dependence with the magnetic field $B$ [large filling factors were considered]; the most interesting effect was the oscillations of $G$ with the filling factor $\nu$ in a somewhat similar manner as in Shubnikov-de Gaas effect.

A phenomenological analytical theory of these phenomena based on an “analogy” with the Aaranov-Bohm effect was suggested in Ref.\textsuperscript{11}. Numerical simulation was made in Ref.\textsuperscript{12}. It was theoretically shown that the transport along the infinitely long S/2DEG interface can be described in the framework of electron and hole edge states.\textsuperscript{13} 2DEG-S interfaces investigated in the experiments were not infinitely long, but with the length, $L$, of the order of few cyclotron orbits, $R_c$, of an electron in 2DEG at the Fermi energy. Quasiclassical theory of the charge transport through 2DEG-S interface at large filling factors, arbitrary length of the 2DEG-S interface was suggested in Ref.\textsuperscript{14}. Most mentioned above theoretical papers considered the ideal 2DEG-S interface: no roughness. It was shown\textsuperscript{15} that when $L \sim 2R_c$, $G(\nu)$ oscillates nearly harmonically with $\nu$, as $\cos(2\nu)$. When $L \sim 4R_c$, harmonics $\cos(4\nu)$ becomes visible and so on... In experiments $L \gtrsim 6R_c$, so one would expect good visibility of $\cos(n\pi \nu)$-harmonics in the conductance, where $n = 1, 2, \ldots$. But if we try to compare theoretically predicted $G(\nu)$ with the experimentally measured one then we find that 1) at $L \gtrsim 6R_c$, only the lowest harmonic $\cos 2\nu$ is seen in the conductance and higher harmonics are absent; 2) the visibility [amplitude] of the conductance oscillations is much smaller than theories predict. The reason of this disagreement is probably the roughness of the 2DEG-S interface in experiments and ideal flatness of this interface in theory.

We try to find in this paper the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Our approach with the surface roughness possibly helps to make a step towards explanation of the the experimental results. It is shown that the a disorder at a 2DEG-S interface suppresses high harmonics of the the conductance oscillations with $\nu$.

We consider a junction consisting of a superconductor, 2DEG and a normal conductor segments (see Fig.1). Magnetic field $B$ is applied along $z$ direction, perpendicular to the plane of 2DEG. It is supposed that quasiparticle transport is ballistic (the mean free path of an electron $l_r \gg L$, where $L$ is the length of the 2DEG-S boundary). The current $I$ is supposed to flow between normal (N) and superconducting (S) terminals (the voltage $V$ is applied between them).

Following Ref.\textsuperscript{15,16,17,18} we shall describe the transport properties of the junction in terms of electron
and hole quasiparticle scattering states, which satisfy Bogoliubov-de Gennes (BdG) equations. Then the current through the 2DEG-S surface is

\[ I(V) = \frac{e}{h} \int_0^\infty dE \{ f_e \text{Tr}[\hat{1} - R_{ee} + R_{eh}] - f_h \text{Tr}[\hat{1} - R_{eh} + R_{eh}] \}, \tag{1} \]

where

\[ f_e(h) = \frac{1}{eE + eV + 1}, \]

\[ V \] is the voltage of the normal terminal, the energy \( E \) is counted from \( \mu \) of the superconductor, \( R_{ee}(E, n_e, n_h) \) is the probability of the (normal) reflection of an electron with the energy \( E \) incident on the superconductor in the edge channel with quantum number \( n \), to an electron going from the superconductor in the channel \( n \); the trace is taken in the channel space. Spin degrees of freedom are included into the channel definition.

So the main task of our work is to find the probabilities \( R_{ee}, \text{ etc.} \). Then we’ll be able to evaluate the current, the conductance, noise and so on. We’ll focus on the situation when \( R_c \ll L \). Then quasiparticles reflected from the superconductor (S) due to normal and Andreev reflection of the electron return to S again due to bending of the trajectories by magnetic field.

From the first glance it may seem that the reflection probabilities \( R_{ab}, a, b = e(h) \) could be found using the “standard” approach: by matching the incident and outgoing quasiparticle wave functions at \( y = 0 \) and \( y = L \) with the linear combinations of the quasiparticle wave functions at the 2DEG-S boundary corresponding to Andreev edge states. However this procedure does not look efficient at large filling factors [experimental parameter range] and especially for a disordered 2DEG-S interface. Next it is difficult to do the matching in practice because the Andreev bound states wave functions and the wave functions of 2DEG edge states are localized in different domains in \( x \) direction. The Andreev bound states wave functions penetrate inside the superconductor on the length scale of the order of \( \xi \), but 2DEG edge states wave functions of the incident and outgoing electrons do not penetrate inside the 2DEG edges so deep as \( \xi \).

At large filling factors the quasiclassical approximation is applicable. We show in this paper that within the quasiclassical approximation the matching problem can be solved and \( R_{ab}, a, b = e(h) \) explicitly evaluated.

An electron (hole) quasiparticle in 2DEG can be viewed in semiclassics as a beam of rays (in a similar way propagation of light is described in optics within eikonal approximation in terms of ray beams). Trajectories of the quasiparticle rays can be found from the equations of classical mechanics. In terms of the wave functions this description means that we somehow make wave packets from edge states wave functions. Reflection of an electron from the superconductor is schematically shown in Figs. 1 and 2.

II. IDEALLY FLAT 2DEG-S INTERFACE

The transport properties of the ideally flat 2DEG-S interface can be most simply described. The edge channels do not mix at such interface. It means in quasiclassical
language that electrons (holes) skip along the 2DEG edge along the same arc-trajectories, like it is shown in Fig.2. Then $n_a = n_i$ and, e.g., $R_{he}(y_0; n_a, n_i) \propto \delta_{n_a, n_i}$. 

The probability of Andreev reflection can be found as follows for the trajectories shown in Fig.2-3:

$$R_{he}(y_0; n_a, n_i) = 
\delta_{n_a, n_i} \left| e^{i(S_e - \pi/2)} \left\{ r_{he} e^{i\phi} r_{ee} e^{i(3iS_e - i\pi/2 - i\phi(y))} + r_{hh} r_{he} e^{i2iS_e - i\pi/2 - i\phi(y)} + \ldots \right\} \right|^2,$$

(2)

where $r_{ba}$ is the amplitude of reflection of a quasiparticle (ray) $a$ into a quasiparticle (ray) $b$ from the superconductor; $S_{e(h)}$ is the quasiclassical action of an electron (hole) along the part of the trajectory connecting the adjacent points of reflection: $\pm \pi/2$ is the Maslov index of the electron trajectory. The phase $\phi(y)$ arises due to the screening supercurrents. We assume that the superconductor satisfies the description within the London theory [usual in experiments] then $\phi(y) = \phi(0) + \hbar^{-1} \int_0^y d\gamma 2m v_s(\gamma) \xi$, where $v_s$ is the superfluid velocity evaluated at $x = 0$ and $m$ is electron mass in the superconductor. We used here the property of London superconductors that the spatial dependence of the vector potential and $v_s$ are small in the perpendicular direction to the superconductor edge on the length scale $\xi$ [on which the wave functions of the scattering electron and hole quasiparticles penetrate in the superconductor].

We introduce the matrix

$$M(y) = 
\left( \begin{array}{cc} r_{ee} e^{i(S_e - \pi/2)} & r_{eh} e^{i(S_h + \pi/2) + i\phi(y)} \\ r_{he} e^{i(S_e - \pi/2) - i\phi(y)} & r_{hh} e^{i(S_h + \pi/2)} \end{array} \right),$$

(3)

that contains the amplitudes of Andreev ($r_{he}, r_{eh}$) and normal ($r_{ee}, r_{hh}$) quasiparticle reflection from the superconductor at the point $y$. Then the matrix product

$$S^{(3)} = M(y_0) M(y_1) M(y_0),$$

(4)

describes the Andreev and normal scattering amplitudes for the case shown in Fig.2-3. So,

$$R_{he}(E; y_0; n_a, n_i) = \left| S^{(3)}_{21} \right|^2,$$

(5)

$$R_{ee}(E; y_0; n_a, n_i) = \left| S^{(3)}_{11} \right|^2.$$

(6)

If there are $n$ reflections from the 2DEG-S interface then $n \rightarrow n - 1$.

Below we show that the matrix $S^{(n)}$ can be calculated analytically for any integer $n$ when the superfluid velocity $v_s(x = 0, y)$ is constant so the phase $\phi(y)$ is a linear function of $y$. Then the difference, $\phi(y_n) - \phi(y_{n-1}) = \delta \phi$, does not depend on $n$ because the 2DEG-S interface is flat and $y_n - y_{n-1} = \ldots = y_1 - y_0$. The matrix $M(y_n)$ can be written as

$$M(y_n) = \Phi^\dagger(n) M(y_0) \Phi(n),$$

(7)

where

$$\Phi(n) \equiv \left( \begin{array}{cc} \exp\left\{-\frac{i}{2} n \delta \phi \right\} & 0 \\ 0 & \exp\left\{\frac{i}{2} n \delta \phi \right\} \end{array} \right).$$

(8)

Thus

$$S^{(n)} = \Phi^\dagger(n) (M \Phi)^{n+1} \Phi^\dagger,$$

(9)

where $M = M(y_0)$, $\Phi = \Phi(1)$.

Then [see Appendix 1]

$$(M \Phi)^{n+1} = \det(n+1)/2,$$

$$(m_{ee} U_n(a) - U_{n-1}(a)) e^{i(S_e + S_h)},$$

$$m_{eh} U_n(a) e^{i(S_e + S_h)},$$

(10)

where $\det = \{r_{ee} r_{hh} - r_{he} r_{ee}\} \exp[i(S_e + S_h)]$, $a = r_{ee} e^{i(S_e - \pi/2) + \phi_y} + r_{hh} e^{i(S_h + \pi/2) + \phi_y}$,

(11)

$$m_{ee} = \frac{r_{ee} e^{i(S_e - \pi/2)}}{\sqrt{\det}},$$

(12)

$$m_{eh} = \frac{r_{eh} e^{i(S_e + S_h) + \phi_y}}{\sqrt{\det}},$$

(13)

$$m_{he} = \frac{r_{he} e^{i(S_e - \pi/2)}}{\sqrt{\det}},$$

(14)

$$m_{hh} = \frac{r_{hh} e^{i(S_h + \pi/2)}}{\sqrt{\det}}.$$

(15)

Eqs. (9)-(15) give an opportunity to find the probabilities $R_{eh}$, if given the amplitudes of local reflection, $r_{ab}$. The probabilities $R_{ab}$ depend on the position of the first reflection from the 2DEG-S interface, $y_0$, that varies in the range $(0, d)$, see Fig.2. The number of reflections, $n$, depends on the choice of $y_0$. So the solution strategy is to calculate the current using Eq.(11) with the probabilities $R_{ab}$ defined in Eqs.(9)-(15) and average the result over $y_0$. The natural choice for the distribution of $y_0$ is the uniform distribution: $P_{\delta}(y_0) = \theta(d(n_0) - y_0)/2R_c$. So

$$I(V) = \frac{e}{\hbar} \int_0^\infty dE \sum_{n_i} \int_{y_0}^{y_0 + \delta} dy_0 P_{\delta}(y_0) \left\{ [1 - R_{ee} - R_{he}] \frac{e}{\hbar} c - \left( 1 - R_{hh} + R_{eh} \right) \right\} f_h.$$

(16)

The action $S_e = \int \vec{k}_e \cdot d\vec{l}$, where $\vec{k}_e$ is the generalized momentum and the integral is over the quasiparticle trajectory that connects the adjacent points of reflection from the superconductor-2DEG interface.

$$S_e = k_e |e| \int \mathbf{A} \cdot d\mathbf{l} = k_e |e| e - \frac{|e|}{\hbar} \Phi_e,$$

(17)

$$S_h = -k_h l_h + \frac{|e|}{\hbar} \Phi_h,$$

(18)

where $k_e(\pm) = \sqrt{2m/2 \pi e \pm (E + g u_B \sigma B)/|e|^2}$, $\sigma = \pm 1$, $l$ is the trajectory length and $\Phi_e(\pm)$ is the absolute value of the magnetic field flux through the area bounded by
where the quasiparticle trajectory arc and the 2DEG-S interface. The actions can be explicitly written in terms of the filling factor \( \nu \) and the y-component of the quasiparticle velocity, \( v_y^{e(h)} \), at the 2DEG-S interface when \( E, g \mu_B \sigma B \ll \mu_{2DEG}/\nu \):

\[
S_{e(h)} = s_{e(h)} \pm \pi(\nu + 1/2),
\]

\[
s_{e(h)} = 2\left(\nu + \frac{1}{2}\right)\left(\arcsin \frac{\nu}{n} - \frac{2}{\sqrt{1 - \nu^2}}\right),
\]

where \( \nu = v_y^{e(h)}/v^{e(h)} \).

Often the Andreev approximation can be used. Then the Andreev-reflected hole velocity direction may be considered opposite to the velocity direction of the incident at the superconductor electron [see Fig.2]. The conditions are:

\[
v_s \ll v_p^{(2DEG)}, \quad \max(|eV|, T, g \mu_B B) < \Delta < E_F^{(2DEG)}.
\]

Then \( s_e = s_h \) and

\[
S_e - S_h = \frac{|e|}{2\hbar c} \Phi = 2\pi \left(\nu + \frac{1}{2}\right),
\]

where \( \Phi \) is the flux through the Larmor ring-trajectory of an electron in magnetic field \( B \) at the Fermi shell.

The problem how to evaluate the \( r_{ab} \) amplitudes also simplifies within the parameter range, Eq. (21). The conditions mean that the magnetic field could be neglected in the Bogoliubov-de Gennes (BdG) equations \([B \text{ is already taken into account by the phase } \phi]\). Then \( r_{ab} \) can be evaluated according to the BTK theory.\(^{15}\)

When the conditions, Eq. (21), are violated our quasiclassical transport picture in terms of quasiparticle rays can be still applied, see Fig.3. Then the amplitudes \( r_{ab} \) are the solutions of the scattering problem for Bogoliubov–de Gennes equations:

\[
(E - g \mu_B B)u = \left(\frac{|p + n v_s|}{2m} - \mu\right)u + \Delta v,
\]

\[
(E - g \mu_B B)v = -\left(\frac{|p - m v_s|}{2m} - \mu\right)v + \Delta u.
\]

Here \( m, g \) and \( \mu \) should be considered different in S and 2DEG. The spatial distribution of the superfluid velocity is fixed by the London equation, rot \( mv_s = -B e/c \). Usually there is a barrier at the 2DEG-S interface, we did not write its contribution to BdG explicitly.

It follows from Eq. (19) that at zero temperature and voltage the conductance is:

\[
G = \frac{2e^2}{h} \sum_n \sum_s |r_{e(h)}|^2 \sin^2[s \arccos(\sqrt{|r_{e(h)}|^2 \cos(\Omega)})]/\left(1 - |r_{e(h)}|^2 \cos^2(\Omega)\right),
\]

where \( s \) is defined as above. The function \( P_s \) is the probability that the orbit describes \( s \) reflections from the surface of the superconductor; this function originates from the averaging over \( \nu_0 \) discussed above. \( P_s \) can be expressed though the maximum number of jumps, \( g_m = \lfloor L/d \rfloor \), over the S-2DEG surface with the length \( L \), where \( \lfloor \ldots \rfloor \) denotes the integer part:

\[
P_s = \begin{cases} \frac{L - g_m d}{d} & \text{if } s = g_m + 1, \\ 1 - \frac{L - g_m d}{d} & \text{if } s = g_m, \\ 0 & \text{otherwise.} \end{cases}
\]

The conductance, Eq. (25), as well as the current, Eq. (19), is an oscillating function of \( \nu \):
where \( g_n \) are the Fourier coefficients and \( \delta_n \) – the “phase shifts”. When the length of the interface, \( L \lesssim 2R_c \), then the leading contribution to the conductance (current) gives the zero harmonic; while \( 2R_c \lesssim L \lesssim 4R_c \) then \( G \approx g_0 + g_1 \cos(2\pi\nu + \delta_1) \); when \( 4R_c \lesssim L \lesssim 6R_c \) the second harmonics becomes relevant, and so on...

How the conductance changes with \( \nu \) is illustrated in Fig.4. Typically in experiments \( L/2R_c \gtrsim 3 \) [thin black curve]. But the conductance behaves in experiments as if \( G = g_0 + g_1 \cos(2\pi\nu + \delta_1) \); higher harmonics, \( g_{31}, \ldots \), are not seen. But our theory based on the assumption of the interface flatness predicts \( g_1 \sim g_2 \sim g_3 \) while \( L/2R_c \gtrsim 3 \), see Fig[1]. The reason of the discrepancy between our theory and the experiment is the assumption that the 2DEG-S interface is ideally flat. Disorder at the 2DEG-S interface makes \( g_0 > g_1 > g_2, \ldots \) Below we demonstrate it.

### III. DISORDERED 2DEG-S INTERFACE

Usually 2DEG-S interface is not ideally flat. The disorder at the interface can be divided at two classes long range and short range with the respect to the characteristic wavelength, \( \lambda_{26} \), in 2DEG. Presence of the long range disorder implies that the 2DEG-S interface position fluctuates around the line \( x = 0 \) at length scales much larger than the \( \lambda_{26} \sim 10^{-6} \mu m \). Photographs of the experimental setups do not allow to think that 2DEG-S interface bends strongly from the line \( x = 0 \). So this kind of the disorder is likely not very important.

The short-range disorder includes the fluctuations of the surface at length scales smaller than \( \lambda_{26} \), impurities, clusters of atoms at the surface due to defects of the lithography and so on... When, for example an electron ray falls on the disordered 2DEG-S surface the reflected electron rays go off the surface not at a fixed angle but they may go at any angle with certain disorder induced probability distribution. The phases that carry the reflected electron rays going off the surface at different angles may be considered random, so the reflected electron rays can be considered as incoherent. But to any reflected electron ray an Andreev reflected hole ray is attached that is coherent with the electron. So the interference of rays [that produces the conductance oscillations] may be not killed completely by the short range disorder. Below it will be clarified.

“Weak” short range disorder at 2DEG-S interface does not destroy the Andreev edge states but it induces transitions between the edge states, see Fig[5b]. Andreev edge states in quasiclassics fix electron-hole orbit-arcs with the same beginning and end. The quasiclassical picture of the disorder-induced transitions is shown in Fig[3].

It is shown below how to describe the transport properties of the weakly short-range disordered 2DEG-S interface in magnetic field. We assume that the Andreev approximation conditions, Eq.[21], are fulfilled [general case brings to qualitatively similar results for the current, it requires same idea of calculations but it is more cumbersome] then \( s_c = s_h \) and the fluctuating quantity is \( \delta \). Similarly as before we introduce the matrix

\[
M(y_n) = \Phi^\dagger(n) M \Phi(n) e^{i \sum_{s=0}^{n} s_i},
\]

where \( s \) is defined in Eq.[20], and

\[
\Phi(n) \equiv \left( \exp\left( -\frac{i}{2} \sum_{0}^{n} \delta \phi_i \right) \right), \quad \exp\left( \frac{i}{2} \sum_{i=0}^{n} \delta \phi_i \right),
\]

and

\[
M = \begin{pmatrix}
r_{ee} e^{i \pi \nu} & r_{eh} e^{-i \pi \nu} \\
r_{he} e^{i \pi \nu} & r_{hh} e^{-i \pi \nu}
\end{pmatrix}.
\]

As before, for the situation sketched in Fig[6a], \( S^{(3)} = M(y_3)M(y_2)M(y_1)M(y_0) e^{i \sum_{s=0}^{3} s_i} \), and, e.g., \( R_{he} = \left| S^{(3)} \right|^2 \). It is clear that \( e^{i \sum_{s=0}^{3} s_i} \) does not influence on the probabilities \( R_{ab} \) so we’ll omit this term below. In general case:

\[
S^{(n)} = \Phi^\dagger(n)[M \Phi_1 \ldots M \Phi_n] \Phi_0^\dagger,
\]

here

\[
\Phi_n \equiv \left( \begin{array}{c}
\exp\left( -\frac{i}{2} \delta \phi_n \right) \\
0
\end{array}
\right), \quad \exp(\frac{i}{2} \delta \phi_n),
\]

where \( \delta \phi_n = \phi_n - \phi_{n-1} \) and \( \delta \phi_0 = 0 \).
The probabilities can be found in the following way:

\[ R_{ee} - R_{he} = \frac{1}{2} \text{Tr} \left\{ \sigma_z M \Phi_n M \Phi_{n-1} \ldots \right. \]
\[ \left. \times \Phi_2 M \Phi_1 M \sigma_z M \Phi_1 M^\dagger \Phi_2^\dagger \ldots \Phi_{n-1} M^\dagger \Phi_n^\dagger M^\dagger \right\}. \quad (33) \]

Here we neglected the term \( \Phi_1 \) that enters Eq. (31) because it does not contribute to the probabilities \( R_{ab} \). If one wants to find \( R_{ee} \) then \( \sigma_z \) should be substituted by \( (\sigma_0 + \sigma_z)/2 \) in the last equation. If \( R_{he} \) is wanted then the first \( \sigma_z \) should be substituted by \( (\sigma_0 + \sigma_z)/2 \), the second – by \( (\sigma_0 - \sigma_z)/2 \).

As in the previous section we should average the current over \( y_0 \). This operation is closely related to the disorder averaging because shifting \( y_0 \) we’ll make the trajectories go through different disorder realizations. The phase jumps \( \delta \phi_n \) fluctuate due to the disorder.

\[ \langle \delta \phi_n \rangle = \delta \overline{\phi}, \quad (36) \]
\[ \langle \delta \phi_n \delta \phi_m \rangle = 2 \eta \delta_{nm}, \quad (37) \]

where \( \langle \ldots \rangle \) means the irreducible average. Then

\[ e^{i \delta \phi_n} = e^{i \overline{\delta \phi}} e^{-\eta}. \quad (38) \]

While \( \eta \ll 1 \), weak fluctuations, the results of the previous section are valid. The opposite case we discuss below.

Let \( \eta \gg 1 \). In the zero order over \( \Lambda \), for \( n = 3 \), the average probabilities

\[ R_{ee} - R_{he} = \text{Tr} \left\{ (\sigma_z)_{i_2 i_3} |M_{i_2 i_3}|^2 |M_{i_3 i_4}|^2 |M_{i_4 i_5}|^2 |M_{i_5 i_6}|^2 (\sigma_z)_{i_6 i_7} \right\}. \quad (39) \]

Thus we see that in the “completely incoherent case” [zero order over \( \Lambda \)] the average probabilities can be found as the corresponding elements of the matrix \( \tilde{S}^n \):

\[ \tilde{S}^n = \left( \frac{r_{ee}}{r_{he}} \right)^2 \left( \frac{r_{ch}}{r_{hh}} \right)^n. \quad (40) \]
for example \( R_{ee} = [\tilde{S}^{n}]_{11} \). Explicit form of \( \tilde{S}^{n} \) can be easily found using the theorem mentioned in the Appendix [A]. More interesting is the first order expansion of the probabilities \( R_{ab} \) over \( \Lambda \). After calculations diagrammatically illustrated in Fig. 7 (see Appendix [B]) we find for the average probabilities

\[
A_{n} = R_{ee} - R_{he} = 
R^{n} \left[ 1 + e^{-\eta} \frac{4(n-1)|r_{ee}|^{2}|r_{eh}|^{2}}{R^{2}} \cos(2\Omega) \right] 
\]  
(41)

where \( \Omega = \pi \nu + \theta_{ee} - \delta\phi/2 \), \( R = |r_{ee}|^{2} - |r_{eh}|^{2} \) and \( n \) is the number of reflections from the 2DEG-S interface. All trajectory-dependent quantities that enter Eq.(11) should be evaluated for the trajectory with equal electron and hole arcs \( |T_{e(h)}| = 0 \).

So, finally the zero-bias conductance is

\[
G = \frac{4e^{2}}{h} \nu \sum_{n} P_{n} [1 - A_{n}], 
\]  
(42)

where \( P_{n} \) is defined in Eq.(20), but with \( d \rightarrow 2R_{c} \). The conductance oscillations according to Eq.(42) are illustrated in Fig. 8.

The result similar to Eqs. (11), (12) would be obtain calculating the influence of the disorder at the edges of 2DEG-S interface, Fig. 5, on the magneto-conductance.

**IV. CONCLUSIONS**

We found in this paper the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Our approach with the surface roughness possibly removes the contradiction between the theory and the experiment. It is shown that the a disorder at a 2DEG-S interface suppresses the conductance oscillations with \( \nu \). The measurement of the magneto-conductance of a 2DEG-S boundary is the test for the degree of the boundary roughness.

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**APPENDIX A: APPENDIX 1**

For the sake of a reader’s convenience, we present in this appendix the following theorem. Let us \( Q \) be a \( 2 \times 2 \) (complex) matrix with the determinant equal to unity then

\[
Q^{n+1} = \begin{pmatrix} 
Q_{11}U_{n}(a) - U_{n-1}(a) & Q_{12}U_{n}(a) \\
Q_{21}U_{n}(a) & Q_{22}U_{n}(a) - U_{n-1}(a) 
\end{pmatrix}, \]

(A1)

where \( a = \text{Tr}Q/2 \) and \( U_{n}(a) \) is the Chebyshev polynomial of the second kind

\[
U_{n}(a) = \sin[(n + 1) \arccos(a)]/\sqrt{1-a^2}. 
\]
APPENDIX B: DISORDER AVERAGING

In this appendix we present the details of derivation for Eqs. (11). Let us consider the matrix

\[
\begin{pmatrix}
A_{n+1} & B_{n+1} \\
C_{n+1} & D_{n+1}
\end{pmatrix} = \begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix} \Phi_{n+1} M \Phi_{n+1}^+ \tag{B1}
\]

with

\[
\begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} = M \sigma_2 M^\dagger. \tag{B2}
\]

Then the quantity \( R_{ee} - R_{he} \) in Eq. (33) is equal to \( A_{n-1} \). It is convenient to introduce a vector \( \Psi_n = (A_n, D_n, B_n, C_n)^T \) such that

\[
\Psi_{n+1} = T(-\eta) \Psi_n, \quad T(x) = \begin{pmatrix} \tilde{S} & -x U \\ W & -x V \end{pmatrix} \tag{B3}
\]

where the elements \( U, V \) and \( W \) of the transfer-matrix \( T \) are the following 2 × 2 matrices

\[
W = \begin{pmatrix} r_{ee} r_{ee}^* & r_{eh} r_{he}^* \\ r_{he} r_{ee}^* & r_{he} r_{he}^* \end{pmatrix}, \tag{B4}
\]

\[
U = \begin{pmatrix} r_{ee} r_{eh} e^{-i 2 \pi \nu - i \phi} & r_{eh} r_{ee} e^{-i 2 \pi \nu + i \phi} \\ r_{he} r_{he} e^{-i 2 \pi \nu - i \phi} & r_{he} r_{he} e^{-i 2 \pi \nu + i \phi} \end{pmatrix}, \tag{B5}
\]

\[
V = \begin{pmatrix} r_{ee} r_{eh} e^{-i 2 \pi \nu - i \phi} & r_{eh} r_{ee} e^{-i 2 \pi \nu + i \phi} \\ r_{he} r_{eh} e^{-i 2 \pi \nu - i \phi} & r_{he} r_{he} e^{-i 2 \pi \nu + i \phi} \end{pmatrix}. \tag{B6}
\]

By using that \( \Psi_0 = T(0) \Psi_{-1} \) where \( \Psi_{-1} = (1, -1, 0, 0)^T \), we find

\[
\Psi_n = T^n(x) T(0) \Psi_{-1}. \tag{B7}
\]

To the lowest order in \( \exp(-\eta) \) we find from Eq. (B7)

\[
A_n = \lim_{x \to 0} \text{Tr} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[ \tilde{S}^{n+1} - 2 e^{-\eta} \text{Re} e^{i (2 \pi \nu - \phi)} \right] \times \frac{\partial}{\partial x} (\tilde{S} + x Z)^n \tag{B8}
\]

where

\[
Z = \begin{pmatrix} r_{ee}^2 r_{ee} r_{eh}^* \\ r_{he} r_{ee}^2 r_{he} \\ r_{ee}^2 r_{he} r_{eh} \end{pmatrix}. \tag{B9}
\]

1. Zero temperature \( T = 0 \)

At vanishing temperature \( T = 0 \) the matrices \( \tilde{S} \) and \( Z \) can be simplified drastically

\[
\tilde{S} = \begin{pmatrix} |r_{ee}|^2 & |r_{eh}|^2 \\ |r_{eh}|^2 & |r_{ee}|^2 \end{pmatrix}, \quad Z = |r_{ee}|^2 |r_{eh}|^2 e^{2i \theta} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{B10}
\]

With the help of the following identity

\[
\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \frac{\sigma_x - \sigma_z}{\sqrt{2}} \begin{pmatrix} a - b \sigma_z \sigma_x - \sigma_z \end{pmatrix} \tag{B11}
\]

we obtain Eq. (11) as

\[
A_{n-1} = R^n \left[ 1 + 4(n-1) e^{-\eta} |r_{ee}|^2 |r_{eh}|^2 R^{-2} \cos 2\theta \right]. \tag{B12}
\]

2. Arbitrary temperature

At arbitrary temperature (energy) where is no special relations between \( r_{ab} \). Then with the help of Eq. (A1) we find

\[
A_n = A^{(0)}_n - 2 e^{-\eta} \text{Re} e^{i(2 \pi \nu - \phi)} A^{(1)}_n \tag{B13}
\]

where

\[
A^{(0)}_n = \text{det}_0^{n/2} \left[ |r_{ee}|^2 - |r_{he}|^2 \right] \frac{U_{n-1}(a_0) - U_{n-2}(a_0)}{\text{det}_0}, \tag{B14}
\]

and

\[
A^{(1)}_n = \text{det}_0^{n-1} \left[ U_{n-1}(a_0)(r_{ee}^* r_{ee} r_{eh}^* - |r_{eh}|^2 r_{ee} r_{hh}) - \text{det}_0 \left[ \alpha a_0 U_{n-2}(a_0) + \frac{n}{2} \beta U_{n-2}(a_0) \right] \right]
\]

\[
+ \text{det}_0^{-1} \left[ \frac{n-1}{2} U_{n-1}(a_0) - \alpha a_0 U_{n-2}(a_0) \right] (|r_{ee}|^2 - |r_{eh}|^2) \tag{B15}
\]

Here we have introduced the following notations

\[
\text{det}_0 = |r_{ee}|^2 |r_{hh}|^2 - |r_{eh}|^2 |r_{he}|^2, \tag{B16}
\]

\[
a_0 = \frac{|r_{ee}|^2 + |r_{hh}|^2}{2 \sqrt{\text{det}_0}}. \tag{B17}
\]

and

\[
\beta = \text{det}_0^{-1} \left[ |r_{hh}|^2 r_{ee}^* r_{eh}^* r_{he} + |r_{ee}|^2 r_{hh}^* r_{ee} r_{hh} r_{rh} - 2 |r_{eh}|^2 |r_{he}|^2 r_{ee} r_{hh}^* r_{he} \right], \tag{B18}
\]

\[
\alpha = |r_{ee}^* r_{eh}^* r_{he} + r_{eh}^* r_{he} r_{he}^* r_{ee} r_{hh}^2 - \frac{\beta}{2}. \tag{B19}
\]
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