Random Information Sharing over Social Networks

Mert Kayaalp, Virginia Bordignon, and Ali H. Sayed
Adaptive Systems Laboratory, EPFL
{mert.kayaalp, virginia.bordignon, ali.sayed}@epfl.ch

Abstract—This work studies the learning process over social networks under partial and random information sharing. In traditional social learning, agents exchange full information with each other while trying to infer the true state of nature. We study the case where agents share information about only one hypothesis, i.e., the trending topic, which can be randomly changing at every iteration. We show that agents can learn the true hypothesis even if they do not discuss it, at rates comparable to traditional social learning. We also show that using one’s own belief as a prior for estimating the neighbors’ non-transmitted components might create opinion clusters that prevent learning with full confidence. This practice however avoids the complete rejection of the truth.

Index Terms—social learning, distributed inference, distributed hypothesis testing, diffusion strategy, trending topics, partial information sharing

I. INTRODUCTION AND RELATED WORK

Social learning [1]–[4] models opinion formation and decision making by agents connected by a graph topology. In these models, agents observe data and interact with their neighbors in order to infer the true state of nature from a finite set of hypotheses. In a behavioral context, for example, voters may seek to agree on the best political representative among a set of candidates \{A, B, C, D\}, based on their personal biases as well as on their interactions over social networks. Other examples can be found in the context of engineering systems: A sensor network may be cooperating to detect whether the weather is sunny or rainy or to classify imagery captured from a common scene [5].

Non-Bayesian social learning algorithms [6]–[13] are implemented in two steps: \(i\) agents form their local beliefs, based on private observations; \(ii\) agents combine their neighbors’ beliefs using a weighted averaging scheme like consensus [14] or diffusion [15]. An implicit assumption common to these models is that agents are willing to share with neighbors their full belief vector. That is, they share their beliefs about all possible hypotheses. In the context of social networks, this can be an unrealistic assumption. For instance, oftentimes, Twitter users concentrate on particular topics that constitute Twitter Trends. If candidate A gave a recent press release, Twitter users will likely focus on A when exchanging opinions. Furthermore, in the context of engineering systems, transmitting partial beliefs rather than full beliefs can help to design communication-efficient systems under limited resources. Motivated by these examples, we are interested in the case where social agents share partial information.

Partial information was considered in the context of social learning in [16], where each agent shares a single hypothesis of interest, which is fixed over time. This setup does not cover the situation in which the hypothesis of interest might change, which is relevant in real social network dynamics. In our recurrent political example, users can switch the discussion topic from candidate A to candidate B, if suddenly a new event brings relevance to B. To represent this randomness, we choose to model the hypothesis of interest, or trending hypothesis, as a random variable following some underlying probability mass function over the set of hypotheses.

Another related work is [17], where event-triggered information sharing mechanisms are studied. In contrast, we consider a probabilistic choice of the trending hypothesis. In [18], distributing partial hypotheses sets for agents to track is examined, but we do not consider hypotheses assignment problems in this work. Aiming to improve communication efficiency in detection networks, the work [19] proposes communicating with one randomly sampled agent instead of all neighbors at each time instant. Moreover, quantizing the beliefs is possible and studied in [20], [21]. In our work, decreasing the communication burden on the nodes is instead achieved by transmitting partial beliefs.

When agents share partial beliefs, one is then faced with the challenge of estimating the hidden/latent belief components. In [16], the remaining hidden beliefs are assumed to be uniform among non-transmitted components. Under some situations, this strategy is shown to result in being fully confident on a wrong hypothesis. In this work, we propose an algorithm where agents bootstrap. Namely, they utilize their own beliefs for estimating their neighbors’ non-transmitted components. As it is shown in the sequel, this practice helps make the system more robust by precluding learning a wrong hypothesis. However, it might also create a stubbornness effect across all agents which can prevent truth learning. This is different from the stubborn behavior considered in [22]–[24], where the stubborn agents are non-typical agents that do not change their beliefs and possibly supply misinformation to the network.

Contributions.

- We propose a social learning algorithm, where agents share their beliefs on only one randomly chosen hypothesis at each time instant, in Section II-A.
- When a wrong hypothesis is exchanged with positive probability, beliefs evaluated at that hypothesis decay exponentially. The decay rate is the same as the asymptotic learning rate of traditional social learning algorithms almost surely (Theorem II).

This work was supported in part by SNSF grant 205121-184999.
III. PROBLEM FORMULATION

Consider a network of $K$ agents trying to distinguish the true state of nature $\theta^\circ$, among a finite set of $H$ hypotheses $\Theta = \{1, 2, \ldots, H\}$. Belief $\mu_{k,i}(\theta)$ represents the confidence that agent $k$ has that $\theta \in \Theta$ is the true hypothesis at time instant $i$. Each agent $k$ receives partially informative and private data $\xi_{k,i}$ at each time instant $i$. It is distributed according to the marginal distribution $L_k(\cdot|\theta^\circ)$. Note that we denote random variables in bold. Agents know their own likelihood functions $L_k(\cdot|\theta)$ for all $\theta \in \Theta$ but do not need to know the likelihoods for the other agents. We assume that:

$$\text{KL}(L_k(\cdot|\theta)|L_k(\cdot|\theta^\circ)) < \infty$$  \hspace{1cm} (1)

for all agents and hypotheses.

Agents are connected over a strongly-connected network [15]. Namely, there is a path between any agent pair $(\ell, k)$ and there is at least one agent with a self-loop. This implies that the combination matrix $A \triangleq [a_{\ell,k}]$ is a primitive matrix. The entry $a_{\ell,k}$ represents the weight agent $k$ assigns to the information it receives from agent $\ell$. It is non-zero if, and only if, agent $\ell$ is a neighbor of agent $k$, i.e., $\ell \in N_k$. The matrix $A$ is also left-stochastic. Its Perron vector $v$ has positive entries and:

$$1^T A v = 1, \quad A v = v, \quad 1^T A = 1^T$$  \hspace{1cm} (2)

A. Social Learning Strategy

At each iteration, agents first obtain intermediate beliefs via local Bayesian updates based on their personal observations as in standard social learning algorithms [6]–[9], [16]:

$$\psi_{k,i}(\theta) = \frac{L_k(\xi_{k,i}|\theta) \mu_{k,i-1}(\theta)}{\sum_{\theta'} L_k(\xi_{k,i}|\theta') \mu_{k,i-1}(\theta')}$$  \hspace{1cm} (3)

Although each agent computes its intermediate belief $\psi_{k,i}(\theta)$ for every possible $\theta \in \Theta$, the agents will share with their neighbors information about their belief for only one of the hypotheses. Specifically, all agents share information about the same trending hypothesis $\tau_i$. This is a random variable with distribution $\pi$, which means that $\mathbb{P}(\tau_i = \theta) = \pi_\theta$. Since agents receive incomplete belief vectors from their neighbors (actually, they receive only one entry from these vectors), the agents need to complete the missing entries. In this work, we assume agents use their own intermediate local beliefs to fill in for the missing beliefs from their neighbors by using the following construction. Agent $k$ completes the belief vector receiving from its neighbor $\ell$ using:

$$\hat{\psi}_{\ell,i}^{(k)}(\theta) = \begin{cases} \psi_{\ell,i}(\theta), & \theta = \tau_i \\ \frac{1 - \psi_{k,i}(\tau_i) + \psi_{\ell,i}(\tau_i)}{\psi_{k,i}(\theta)}, & \theta \neq \tau_i \end{cases}$$  \hspace{1cm} (4)

Subsequently, the agents combine these approximate intermediate beliefs to update their beliefs as in [8], [9], [12], [13], [16]:

$$\mu_{k,i}(\theta) = \frac{\exp\{\sum_{\ell \in N_k} a_{\ell,k} \log \hat{\psi}_{\ell,i}^{(k)}(\theta)\}}{\sum_{\theta^\prime} \exp\{\sum_{\ell \in N_k} a_{\ell,k} \log \hat{\psi}_{\ell,i}^{(k)}(\theta^\prime)\}}$$  \hspace{1cm} (5)

Notice the differences with standard non-Bayesian social learning [8], [9], and partial information sharing in [16]. In standard social learning, all hypotheses are exchanged and hence:

$$\hat{\psi}_{\ell,i}^{(k)}(\theta) = \psi_{\ell,i}(\theta)$$  \hspace{1cm} (6)

In [16], there is a fixed transmitted hypothesis $\tau_i = \tau$ and non-transmitted hypotheses are assumed to be uniformly likely:

$$\hat{\psi}_{\ell,i}^{(k)}(\theta) = \begin{cases} \psi_{\ell,i}(\tau), & \theta = \tau \\ \frac{1 - \psi_{\ell,i}(\tau)}{H - 1}, & \theta \neq \tau \end{cases}$$  \hspace{1cm} (7)

In contrast, in [4] agents exploit their own beliefs as prior information.

III. MAIN RESULTS

We first formalize the meaning of learning, not-learning and mislearning.

**Definition 1.** For any realization of the system, truth learning occurs when:

$$\mu_{k,i}(\theta^\circ) \to 1$$  \hspace{1cm} (8)

Any other situation is considered as not-learning. Among not-learning cases, we define the mislearning, or alternatively, learning a wrong hypothesis $\theta \in \Theta \setminus \{\theta^\circ\}$ as:

$$\mu_{k,i}(\theta) \to 1$$  \hspace{1cm} (9)

So, in a mislearning situation, agents become fully confident on a wrong hypothesis.

To avoid cases where agents may discard some hypotheses, we introduce the following condition.

**Assumption 1.** All initial beliefs are strictly positive at all hypotheses, i.e., for each agent $k$ and for all hypotheses $\theta \in \Theta$, $\mu_{k,0}(\theta) > 0$. 


A. Truth Learning

We first present results that characterize truth learning under certain conditions.

**Theorem 1 (Asymptotic Learning Rate).** For any wrong hypothesis \( \theta \in \Theta \setminus \{ \theta^o \} \), if the transmission probability is strictly positive, i.e., \( \pi_\theta > 0 \), then the belief will have an asymptotic exponential behavior. Namely, for each agent \( k \):

\[
\lim_{i \to \infty} \log \frac{\mu_{k,i}(\theta)}{\mu_{k,i}(\theta^o)} = \sum_{k=1}^{K} -v_k D_{KL}\left(L_k(\cdot|\theta^o)||L_k(\cdot|\theta)\right) \quad (10)
\]

where \( \xrightarrow{a.s.} \) denotes almost sure convergence.

**Proof.** Omitted due to space limitations. \( \square \)

Observe that this rate is same as the standard social learning rates in [8] and [9], which require transmission of full beliefs. Therefore, a positive probability of transmitting the wrong hypothesis suffices for achieving the same asymptotic performance with probability one. If we also introduce a global identifiability condition, agents can distinguish the true hypothesis from the wrong ones.

**Assumption 2 (Global Identifiability).** For all wrong hypotheses \( \theta \in \Theta \setminus \{ \theta^o \} \), there is at least one agent \( k \) who can distinguish \( \theta \) and \( \theta^o \). Namely, for at least one agent \( k \):

\[
D_{KL}\left(L_k(\cdot|\theta^o)||L_k(\cdot|\theta)\right) > 0 \quad (11)
\]

Combining Theorem 1 and Assumption 2 yields the following result.

**Corollary 1 (Truth Learning).** Under Assumptions 1 and 2, if \( \pi_\theta > 0 \) for all wrong hypotheses \( \theta \in \Theta \setminus \{ \theta^o \} \), then each agent \( k \) learns the truth with probability one, i.e.:

\[
\mu_{k,i}(\theta^o) \xrightarrow{a.s.} 1 \quad (12)
\]

Notice that any asymmetry between entries of \( \pi \) does not affect the learning. In [16], truth learning occurs if and only if the fixed transmitted hypothesis is the true hypothesis. Corollary 1 shows that if agents are bootstrapping as opposed to [16], then learning can occur as long as \( \pi_\theta > 0 \) for all wrong hypotheses \( \theta \). This implies that they can learn the truth even if they do not discuss the true hypothesis, i.e. even if \( \pi_{\theta^o} = 0 \).

B. Truth Sharing

In the previous section, we concluded that exchange of all wrong hypotheses with positive probability is sufficient for learning. What about the exclusive exchange of the true hypothesis? Is it also sufficient for learning? We give a negative answer to this question by providing a toy counterexample where agents do not learn even when \( \pi_{\theta^o} = 1 \).

Consider a fully-connected network of 3 agents (see Fig. 1). The hypotheses set is \( \Theta = \{ 1, 2, 3, 4 \} \) where \( \theta^o = 1 \). Let us denote the locally indistinguishable hypotheses set for each agent \( k \) by \( \Theta_k \). Assume that agent \( k \) cannot distinguish between the true hypothesis \( \theta^o = 1 \) and the hypothesis \( \theta_k \), i.e., \( \Theta_k = \{ \theta^o, \theta_k \} \) and \( D_{KL}(L_k(\cdot|\theta^o)||L_k(\cdot|\theta_k)) = 0 \). Suppose for this example that \( \theta_1 = 2, \theta_2 = 3, \) and \( \theta_3 = 4 \). Since there is no common \( \theta_k \) across all agents, the problem is globally identifiable.

Imagine that at time instant \( i \), \( \mu_{k,i}(\theta^o) = \alpha \) and \( \mu_{k,i}(\theta_k) = 1 - \alpha \) for each agent \( k \). Using expression (3) yields the intermediate beliefs:

\[
\psi_{k,i+1}(\theta^o) = \mu_{k,i}(\theta^o) = \alpha \quad (13)
\]

\[
\psi_{k,i+1}(\theta_k) = \mu_{k,i}(\theta_k) = 1 - \alpha \quad (14)
\]

Applying expression (4), since \( \pi_{\theta^o} = 1, \tau_{i+1} = \theta^o \) we get:

\[
\tilde{\psi}_{k,i+1}^{(k)}(\theta^o) = \psi_{k,i+1}(\theta^o) = \alpha \quad (15)
\]

\[
\tilde{\psi}_{k,i+1}^{(k)}(\theta_k) = \psi_{k,i+1}(\theta_k) = 1 - \alpha \quad (16)
\]

Finally, after the combination step (5):

\[
\mu_{k,i+1}(\theta^o) = \mu_{k,i}(\theta^o) = \alpha \quad (17)
\]

\[
\mu_{k,i+1}(\theta_k) = \mu_{k,i}(\theta_k) = 1 - \alpha \quad (18)
\]

This is an equilibrium (fixed point) for the algorithm. Consequently, if \( \alpha \) is small, agents can get stuck in beliefs where their confidence levels on the wrong hypotheses are higher than confidence on the true hypothesis.

In [16] (see Eq. (7)), when the truth is the fixed transmitted hypothesis, truth learning occurs almost surely. The example described in this section suggests that using one’s own belief for estimating non-transmitted components of neighbors, i.e., bootstrapping, as opposed to using uniform priors may lead to stubbornness in regular agents. It can prevent learning under partial information sharing. In addition to not learning the truth, the network can also fail to reach consensus and opinion clusters might emerge. In Fig. 1, agents having positive beliefs for different hypotheses can have a major effect especially when \( \alpha \) is small. It leads to a strong network disagreement. Network disagreement phenomena were observed in [25] when there are special agents who never change their opinions. Our
result, on the other hand, indicates that even when the network is only composed of regular agents, limited communication can hinder concurrence and truth learning.

C. Impossibility of Mislearning

The previous section demonstrated that bootstrapping might induce network disagreement and poor equilibrium. In this section, we provide a positive result in the opposite direction: Agents will never be fully confident on a wrong hypothesis. Total mislearning cannot occur.

**Theorem 2 (Impossibility of Mislearning).** Under Assumption \([\ref{assumption2}]\) agents will always have positive confidence on the true hypothesis. Namely, for each agent \(k\):

\[
P\left( \lim_{i \to \infty} \mu_{k,i}(\theta^o) = 0 \right) = 0 \tag{19}
\]

or alternatively, for \(\theta \in \Theta \setminus \{\theta^o\}\)

\[
P\left( \lim_{i \to \infty} \sup \mu_{k,i}(\theta) = 1 \right) = 0 \tag{20}
\]

**Proof.** Omitted due to space limitations.

Notice that there is no assumption on the transmission probabilities in Theorem 2. With bootstrapping, agents never learn a wrong hypothesis. In \([\ref{ref16}]\), it was shown that agents might mislearn a wrong hypothesis if the fixed transmitted hypothesis is not the true hypothesis. As a matter of fact, bootstrapping leads to a more robust design in the face of partial communication.

**IV. Numerical Simulations**

Consider a 10-agent strongly-connected network (see topology in the leftmost panel of Fig. 2). The combination matrix is designed using the Metropolis rule \([\ref{ref15}]\), yielding a doubly-stochastic matrix. Agents are trying to detect the true state \(\theta^o\) among a set of five hypotheses, namely \(\Theta \triangleq \{1, 2, 3, 4, 5\}\). Incidentally, we assume that \(\theta^o = 1\). To accomplish this task, agents use the protocol described in \([\ref{assumption2}]\)–\([\ref{assumption3}]\), where the random shared hypothesis \(\tau_i\) is distributed according to the following probability mass function:

\[
P(\tau_i = \theta) = \pi_{\theta} = \begin{cases} 0, & \text{if } \theta = \theta^o \\ 0.25, & \text{otherwise} \end{cases} \tag{21}
\]

Agents consider a family of unit-variance Gaussian densities:

\[
f_n(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{(x - 0.5n)^2}{2} \right\} \tag{22}
\]

for \(n = 1, 2, 3, 4, 5\). The likelihoods of agents are chosen among these Gaussian densities according to the identifiability setup in Table 1. For example, we note that agents 8–10 cannot distinguish hypotheses 1 and 5. Observe that global identifiability condition in Assumption 2 is satisfied.

In the middle panel of Fig. 2 we see the evolution of belief for agent 1, which shows that, although the agents never share information about the true hypothesis, i.e., \(\pi_{\theta^o} = 0\), the agent asymptotically learns the truth, as suggested by Corollary \([\ref{corollary1}]\). A similar behavior happens for the remaining agents.

**TABLE I**

| Agent \(k\) | Likelihood Function: \(f_k(x|\theta)\) |
|------------|-----------------------------------|
| 1 – 2      | \(f_1\) \(f_2\) \(f_3\) \(f_4\) \(f_5\) |
| 3 – 5      | \(f_3\) \(f_2\) \(f_1\) \(f_4\) \(f_5\) |
| 6 – 7      | \(f_3\) \(f_2\) \(f_5\) \(f_1\) \(f_5\) |
| 8 – 10     | \(f_1\) \(f_2\) \(f_3\) \(f_4\) \(f_1\) |

The rightmost panel of Fig. 2 shows that the experimental convergence rates for agent 1, i.e.,

\[
r_{1,i}(\theta) \triangleq \log \frac{\mu_{1,i}(\theta)}{\mu_{1,i}(\theta^o)} \tag{23}
\]

which are shown in colored lines, approach the asymptotic convergence rates of traditional social learning (black dotted lines):

\[
d_{ave}(\theta) \triangleq \sum_{k=1}^{K} -v_k D_{KL}\left( L_k(\cdot | \theta^o) \parallel L_k(\cdot | \theta) \right) \tag{24}
\]

as predicted by Theorem 1. This means that convergence rate-wise, there is no performance loss when only one hypothesis is exchanged at each iteration as long as all wrong hypotheses have positive probability of being transmitted.

In the next simulation, we illustrate that truth sharing is not sufficient for truth learning. For that purpose, we fix the transmitted hypothesis at the true hypothesis \(\tau_i = \theta^o = 1\) for all \(i = 1, 2, \ldots\). The result can be seen in Fig. 3 where we show the evolution of the belief of agent 1 over time.

Despite sharing the true hypothesis, the stubbornness effect described in Section III-B hinders the ability of the agent to learn the truth. We note that agent 1 cannot **decisely** distinguish between hypothesis 1 and 2, which are indistinguishable from its local point of view (see Table 1). This was suggested by the example in Section III-B where agents are caught in an equilibrium where they have non-zero belief values for locally indistinguishable hypotheses. Notice that although truth learning is not observed, there is no mislearning phenomena as well. As suggested by Theorem 2, the confidence in truth is not going to 0.

**V. Conclusion**

In this work, we studied social learning under random and partial information sharing. We first provided a sufficient condition for truth learning and derived the corresponding convergence rate. Then, we demonstrated an example where truth sharing is not enough for truth learning. We discussed the stubbornness effect causing this situation. Then, we presented a result about the impossibility of learning a wrong hypothesis with full confidence, which is a positive result that is important for robustness.
Experimental rates of convergence for agent 1, i.e., $\mu_{1,i}(\theta)$ for different hypotheses (in colored lines), compared with the theoretical asymptotic rate of convergence (in black dotted lines).

Fig. 2. Leftmost panel: Network topology. Middle panel: Evolution of the shared hypothesis $\tau_i$ over time in the upper panel, and belief evolution for agent 1 showing truth learning in the bottom panel. Rightmost panel: Experimental rates of convergence for agent 1, i.e., $\mu_{1,i}(\theta)$ for different hypotheses (in colored lines), compared with the theoretical asymptotic rate of convergence (in black dotted lines).

Fixed shared hypothesis: $\tau_i = 1$

Fig. 3. Belief evolution of agent 1 when the shared hypothesis is fixed over time to be the true state of nature, showing not-learning.

REFERENCES

[1] C. P. Chamley, Rational Herd: Economic Models of Social Learning, Cambridge University Press, 2003.
[2] D. Acemoglu, M. A. Dahleh, I. Lobel, and A. Ozdaglar, “Bayesian learning in social networks,” The Review of Economic Studies, vol. 78, no. 4, pp. 1201–1236, 2011.
[3] P. M. Djurić and Y. Wang, “Distributed Bayesian learning in multiagent systems: Improving our understanding of its capabilities and limitations,” IEEE Signal Processing Magazine, vol. 29, no. 2, pp. 65–76, 2012.
[4] V. Krishnamurthy and H. V. Poor, “Social learning and Bayesian games in multiagent signal processing: how do local and global decision makers interact?,” IEEE Signal Processing Magazine, vol. 30, no. 3, pp. 43–57, 2013.
[5] V. Bordignon, S. Vlaski, V. Matta, and A. H. Sayed, “Learning from heterogeneous data based on social interactions over graphs,” arXiv preprint arXiv:2112.09483, 2021.
[6] A. Jadabaie, P. Molavi, A. Sandroni, and A. Tahbaz-Salehi, “Non-Bayesian social learning,” Games and Economic Behavior, vol. 76, no. 1, pp. 210–225, 2012.
[7] X. Zhao and A. H. Sayed, “Learning over social networks via diffusion adaptation,” in Asilomar Conference on Signals, Systems and Computers, 2012, pp. 709–713.
[8] A. Nedić, A. Olshevsky, and C. A. Uribe, “Fast convergence rates for distributed non-Bayesian learning,” IEEE Transactions on Automatic Control, vol. 62, no. 11, pp. 5538–5553, 2017.
[9] A. Lalitha, T. Javidi, and A. D. Sarwate, “Social learning and distributed hypothesis testing,” IEEE Transactions on Information Theory, vol. 64, no. 9, pp. 6161–6179, 2018.
[10] R. Parassisi, M. Franceschetti, and B. Touri, “Non-Bayesian social learning on random digraphs with aperiodically varying network connectivity,” arXiv preprint arXiv:2010.06695, 2020.
[11] K. Ntemos, V. Bordignon, S. Vlaski, and A. H. Sayed, “Social learning under inferential attacks,” in Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2021, pp. 5479–5483.
[12] V. Bordignon, V. Matta, and A. H. Sayed, “Adaptive social learning,” IEEE Transactions on Information Theory, vol. 67, no. 9, pp. 6053–6081, 2021.
[13] M. Kayaalp, V. Bordignon, S. Vlaski, and A. H. Sayed, “Hidden Markov modeling over graphs,” arXiv preprint arXiv:2111.13626, 2021.
[14] M. H. DeGroot, “Reaching a consensus,” Journal of the American Statistical Association, vol. 69, no. 345, pp. 118–121, 1974.
[15] A. H. Sayed, “Adaptation, learning, and optimization over networks,” Foundations and Trends in Machine Learning, vol. 7, no. 4-5, pp. 311–801, July 2014.
[16] V. Bordignon, V. Matta, and A. H. Sayed, “Social learning with partial information sharing,” arXiv preprint arXiv:2006.13659, 2020.
[17] A. Mitra, S. Bagchi, and S. Sundaram, “Event-triggered distributed inference,” in Proc. IEEE Conference on Decision and Control (CDC), 2020, pp. 6228–6233.
[18] P. Paritosh, N. Atanasov, and S. Martinez, “Hypothesis assignment and partial likelihood averaging for cooperative estimation,” in Proc. IEEE Conference on Decision and Control (CDC), 2019, pp. 7850–7856.
[19] Y. Inan, M. Kayaalp, E. Telatar, and A. H. Sayed, “Social learning under inferential attacks,” in Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2020, pp. 6228–6233.
[20] M. T. Toghani and C. A. Uribe, “Communication-efficient distributed cooperative learning with compressed beliefs,” arXiv preprint arXiv:2102.07767, 2021.
[21] A. Mitra, J. A. Richards, S. Bagchi, and S. Sundaram, “Distributed inference with sparse and quantized communication,” IEEE Transactions on Signal Processing, vol. 69, pp. 3906–3921, 2021.
[22] E. Yildiz, A. Ozdaglar, D. Acemoglu, A. Saberi, and A. Scaglione, “Binary opinion dynamics with stubborn agents,” ACM Trans. Econ. Comput., vol. 1, no. 4, dec 2013.
[23] S. D. Lena, “Non-Bayesian social learning and the spread of misinformation in networks,” Working Papers 2019:09, Department of Economics, University of Venice Ca’ Foscari, 2019.
[24] D. Vial and V. Subramanian, “Local non-Bayesian social learning with stubborn agents,” arXiv preprint arXiv:1904.12767, 2021.
[25] D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar, “Opinion fluctuations and disagreement in social networks,” Mathematics of Operations Research, vol. 38, no. 1, pp. 1–27, 2013.