Dynamical evolution of triplets of galaxies

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Accepted ——-. Received ——-; in original form ——-

ABSTRACT
By means of \( N \)-body simulations we study the global dynamics of triplets of galaxies, considering initial conditions starting from ‘maximum expansion’ and in virial equilibrium. Unlike previous studies we treat galaxies self-consistently, but we restrict ourselves to models with spherical symmetry and do not consider the influence of a primordial common halo of dark matter.

Our results indicate that a low number of triple mergers are expected at the present epoch (\( \approx 10\% \)) for collapsing triplets. Initially virialised conditions yield \( \approx 5 \) percent of triple mergers in \( \approx 10 \) Gyr of evolution; hence, the 3-galaxy problem has stable states. No overmerging problem for these small groups of galaxies is found. Their geometrical properties, as reflected by the Agekyan-Anosova map, do not show an excess of extreme hierarchical triplets. Unlike the 3-body problem no ‘sling-shot’ events are found during triple interactions, both for collapsing and virial initial conditions. The median velocity dispersion of observed compact triplets (\( \sigma \sim 100 \) km s\(^{-1}\)) is not well reproduced in our models at the present epoch: \( \sigma \sim 50 \) km s\(^{-1}\) for collapsing and \( \sigma \sim 80 \) km s\(^{-1}\) for virial. However, about 10 per cent of simulated triplets reaching the present epoch from maximum expansion have dynamical properties very similar to the median of Karachentsev’s compact triplets. Our median values agree, however, very well with new data on triplets. We find that the median of the virial mass estimates do not overestimate, in general, the mass of triplets, but underestimate it by \( \approx 35 \) percent. The median mass estimator appears as a somewhat better mass estimator.

Analysis of the dynamical parameters, as well as information obtained from a pseudo phase-plane constructed using their velocity dispersion and harmonic radius, lead us to conclude that: Karachentsev’s compact triplets probably represent the most advanced stage of gravitational clustering of initially diffuse triplets. To test this thesis we suggest that triplets be studied within a cosmological scenario.

Key words: galaxies: interactions - galaxies: kinematics and dynamics

1 INTRODUCTION

Galaxies in the universe tend to aggregate in different levels of structure, from binaries to large super-clusters. Systems of three galaxies, or triplets, constitute the smallest galaxy groups and are the boundary of \( N \)-body systems that cannot be modeled by analytical methods. On other hand, existing compact triplets of galaxies (Karachentsev 2000) are not very distinct from compact groups of galaxies (Hickson 1997) and their dynamical study may help in our understanding of the latter.

Karachentseva, Karachentsev & Shcherbanovskii (1979) were the first to compile a systematic catalog of triplets and to study some of their observational properties. At the present time, observational surveys in search of triplets and studies to determine their kinematical properties are being conducted by several groups (e.g., Karachentseva & Karachantsev 2000; Infante et al. 2000). On other hand, Heidt et al. (1999) have found that a strongly interacting triplet may be responsible for the production of a BL Lac object, and Iglesias-Páramo & Vilchez (1997) show an example of a probable triple interaction of galaxies in Hickson’s compact group HCG 95. All this observational research points toward the necessity of developing a better understanding of the dynamics of triplets.

Triplets maintain also a close relation with the 3-body problem of celestial mechanics and, hence, their study is also of theoretical interest (Valtonen & Mikkola 1991; Heinämäki et al. 1998; Muzzio, Wachlin & Carpintero 2000). A popular idea in the community (Valtonen & Flynn 2000) is that there are no stable states in a triplet, but as we will show later this is incorrect.

In general, the problem of triplets, or 3-galaxy problem, has not received much attention in contrast to the dynamics of compact groups of galaxies. Several dynamical stud-
ies of triplets have been carried out by different authors in order to explain some of their observational properties, but most of the up-to-date studies have used either a point-particle approach to model galaxy interactions (e.g., Chernin & Mikkola 1991) or included Chandrasekhar’s formula to model dynamical friction (Zheng et al. 1993).

In the 3-body simulations made by Chernin & Mikkola (1991) they found too many hierarchical structures (i.e. a close binary and a far away third body) that does not correspond to what is observed. Chernin et al. (1991) have suggested that dark matter may be the necessary ingredient to help 3-body simulations better reproduce these observations, based up on numerical experiments that show that the probability of binary formation is reduced if dark matter is present in the system (Chernin et al. 1988). On other hand, Zheng et al. (1993) have shown that the excess of hierarchical structures may be reduced somewhat by the inclusion of Chandrasekhar’s dynamical friction formula in simulations of point-like galaxies; it is to note that Zheng et al. start their simulations with five ‘galaxies’.

The use of non self-consistent galaxies in previous studies lead to consider current results on the dynamics of triplets as a first approximation to the 3-galaxy problem. Effects like dynamical friction are not easy to model by an explicit-physics approach, which in any event requires fine tuning using self-consistent galaxies (Velázquez & White 1999). Therefore some doubts are cast on the results obtained when the self-gravity of galaxies is not taken into consideration. The natural following step in the modeling of triplets is to treat galaxies as self-gravitating systems, which will allow them to absorb orbital energy and angular momentum during an encounter.

An important quantity to determine in triplets, as well as in other systems of galaxies, is their gravitational mass. Although large amounts of dark matter are rather well established in groups and clusters of galaxies (Bahcall 1999), the situation in low-multiplicity systems is still a matter of some debate. For example, several authors have suggested that binary galaxies, even with the known uncertainties in their orbital parameters (Binney & Tremaine 1987), do not have much more dark matter than that probably associated with their individual haloes (e.g., Karachentsev 1990; Honma 1999).

On other hand, numerical simulations using point-particles indicate that the mass of a triplet obtained by the virial mass estimator is unreliable, with overestimates up to one or two orders of magnitude (e.g., Kiseleva & Chernin 1988; Chernin & Mikkola 1991). Dolgachev & Chernin (1997), carrying out simulations of wide-triplets (W-triplets, see §2) with free-fall initial conditions, conclude that these systems must hold a considerable amount of dark matter and that their masses are significantly underestimated if their non-stationary state is not taken into consideration. The assessment of typical methods used to determine mass, when using self-consistent galaxies and involving a non-stationary situation, may shed light towards understanding some of these discrepancies. It will be evident later that the self-gravity of galaxies introduces another degree of uncertainty in the determination of mass due to the transfer of orbital to internal energy during interactions.

In this contribution we perform a set of numerical experiments on the dynamics of triplets using a self-gravitating galaxy model. In particular, we will focus on global quantities that are directly comparable with observations in order to test our model. We divide this work as follows. In §2 we present an outline of some of the observational properties of compact and wide triplets, and our adopted set of definitions to estimate the dynamical quantities. In §3 the galaxy model and the initial conditions to be used in the simulations are presented. In §4 our results are presented and discussed. Here we consider the general aspects of the evolution of triplets, merging histories, geometrical properties, and the median values of their observationally related dynamical quantities. Finally, in §5 we summarize the main results of this work.

2 OBSERVATIONAL SETTING

Karachentseva et al. (1979) have presented the first systematic study of the properties of triplets having a compact configuration in the sky (see also Karachentsev 2000; Karachentseva & Karachentsev 2000). According to the statistical criteria of Anosova (1987) to distinguish physical from optical triplets, which has become a standard in selecting physical triplets and is considered to be a strong selection criteria (Zheng et al. 1993), about 45 triplets out of 83 are probably physical; we will refer to the former as K-triplets or compact triplets.

Trofimov & Chernin (1995) have analyzed the triplets in the catalogs of Maia, da Costa & Latham (1989) and Huchra & Geller (1982). For these triplets, they derive a median mean harmonic radius that is about an order of magnitude larger than that of K-triplets, thus calling them wide triplets (W-triplets). In Table 1 the median values of different dynamical quantities, both for K-triplets and W-triplets, are provided for future reference. The quantities written are: the mean harmonic radius, \(R_H\); the one-dimensional velocity dispersion, \(\sigma\); the dimensionless crossing time, \(H_0\tau_c\), and the virial mass estimate \(M_v\). A Hubble constant of \(H_0 = 75\) km s\(^{-1}\) Mpc\(^{-1}\) is assumed throughout this work.

The values for K-triplets in Table 1 were obtained using the information for \(\sigma\) and \(R_H\) provided by Karachentsev et al. (1989), and for W-triplets’ those found in Trofimov & Chernin (1995) and Dolgachev & Chernin (1997). The differences of values in Table 1 with those found in the referred works arise from a difference in the definitions used for the dynamical parameters. The expressions used here, as well as that for the median mass estimator to be used later, are as follows (e.g., Nolthenius & White 1987; Heisler, Tremaine & Bahcall 1987):

\[
\frac{1}{R_H} = \frac{2}{N(N-1)} \sum_{i<j} \frac{1}{R_{ij}},
\]

Table 1. Median dynamical properties of triplets

| Type     | \(R_H\) [kpc] | \(\sigma\) [km/s] | \(H_0\tau_c\) | \(M_v\) [\(M_\odot\)] |
|----------|---------------|------------------|---------------|------------------------|
| K-Triplets| 65.7          | 120.0            | 0.041         | 1.71 \times 10^{12}    |
| W-Triplets| 653.5         | 105.3            | 0.531         | 9.46 \times 10^{12}    |
triplets of galaxies

\[ \sigma^2 = \frac{1}{N-1} \sum_i (v_i - \langle v \rangle)^2, \quad (2) \]
\[ \tau_c = \frac{2 R_{\text{H}}}{\sqrt{3} \sigma}, \quad (3) \]
where \( \langle v \rangle = N^{-1} \sum_i v_i \), \( N = 3 \), and
\[ M_c = \frac{6}{G} R_{\text{H}} \sigma^2, \quad M_{\text{med}} = \frac{6.5}{G} \text{MED}_{ij} \left( (v_i - v_j)^2 R_{ij} \right), \quad (4) \]
with \( v_i \) being the line-of-sight velocity of a galaxy and \( R_{ij} \) the projected pair-wise separation of two galaxies. In these formulae the bulk velocity of galaxies and their position vectors are assumed to be randomly oriented.

The expression for the virial mass yields the standard formula of the virial mass estimator, which leads to accurate results for the total mass of an equilibrium system made of point-like particles (e.g., Aceves & Perea 1999). The median mass estimator is used here due to its insensitivity to extreme values, and also to compare its performance against the virial mass estimator. In the expression for \( M_{\text{med}} \) a Michie’s equilibrium model was used to obtain the constant factor; although this factor is perhaps not true for triplets, we note the less use it as is normal practice and also to assess its accuracy in a triplet of self-consistent galaxies.

In Fig. 1 histograms for the different dynamical quantities are provided, normalized to the total number of triplets assumed physical in each of the above references. We note in passing the existence of overlapping regions between the K-triplets’ and W-triplets’ distributions.

In addition to the previous kinematical quantities, one may consider also the shape of the triangle formed by the galaxies in the sky to characterize a triplet; an idea introduced by Agekyan & Anosova (1968) in terms of an homologous map (AA-map). In this map one normalizes to unity the largest side of the triangle formed by the galaxies in the sky, and the remaining galaxy determines, when a new reference system’s axis is set along the longest side, a new set of coordinates which are to be plotted in such diagram. By symmetry arguments all triangles can be represented inside a particular region; see Fig. 2. In this AA-map hierarchical (H) configurations lie at the lower extreme right, Lagrangian (L) triangles at the upper corner, and alignments (A) or chain-like configurations lie at the bottom of the diagram.

Karachentsev’s triplets are scattered more or less randomly in the AA-map, while W-triplets show an absence of L configurations. An important result found by Chernin et al. (1994) was that projection effects are not responsible for the lack of excess of hierarchical triplets. This indicates that K-triplets are absent of many close pairs. These authors also indicate that W-triplets show only a weak excess of number density in the H-area of the AA-map.

3 MODELS FOR TRIPLET EVOLUTION

3.1 Galaxy model

In this work the focus is on the overall dynamics of triplets. Hence we consider it convenient to use a spherically symmetric N-body system to model a galaxy. In particular, we have adopted a Plummer sphere. We consider that this galaxy model can represent an elliptical or, to a first approximation, the dark halo of a spiral, and can be used to obtain some information about the merging activity of triplets. We have not made any attempt to make an explicit difference between a dark and luminous component.

The mass enclosed within a radius \( r \) and the phase-space density distribution function \( f(\xi) \) for the Plummer model are given, respectively, by:
\[
M(r) = \frac{M(r/R_0)^3}{1 + (r/R_0)^2}^{3/2}, \quad f(E) = \frac{24\sqrt{2}R_0^2}{7\pi^3GM^4} |E|^{7/2},
\]

where \( M \) is the total mass, \( R_0 \) its scale-length, and \( E \) the energy per unit mass (Aarseth, Hénon & Wielen 1974). We constructed each numerical galaxy with \( N = 3000 \) particles, with positions and velocities randomly sampled by an acceptance-rejection technique using the relations in (4).

The \( N \)-body units used here are such that \( G = M = R_0 = 1 \), thus the corresponding unit of time that follows is \( t = (R_0^3/GM)^{1/2} \); the total mass of a triplet is \( M_t = 3 \). We chose the dynamical time for the galaxy model to be \( t_d = 2\sqrt{2}(R_0^3/GM)^{1/2} \); i.e., \( \sqrt{2} \) in our units. The half-mass radius is \( R_h \approx 1.3 \). Given that the mass distribution of a Plummer sphere falls very rapidly, \( \rho \propto r^{-5} \), and about 99 percent of the enclosed mass is contained within \( r \approx 10R_0 \) we take this value as the extent of our galaxy model.

In order to compare the results of the \( N \)-body simulations with those of astronomical systems we proceeded as follows. We denote by a subscript \( \mu \) the astronomical unit of mass to be 1, and the dynamical time for the galaxy model to be \( t_0 \approx 0.07 \) Myr, with \( t_0 \approx 0.2 \).

For both types of IC's, we do not consider the existence of a primordial common envelope of dark matter. This was motivated, on one hand, by the fact that some observations of galaxy clusters and groups tend to suggest that most of their dark matter is associated with the dark haloes of galaxies (e.g., Bahcall, Lubin & Droman 1995; Puche & Carignan 1991). On the other hand, there are indications that in binary galaxies their \( M/L \)-ratios might be rather small suggesting that large quantities of dark matter in a common halo are unnecessary. We make the hypothesis that these conditions prevailed once galaxies were already formed in a triplet, a condition that will have to be relaxed in future work where the effect of a self-gravitating common halo will have to be considered.

Results from simulations of groups of galaxies indicate that the merging rate is smaller when a common extended massive envelope of dark matter is imposed to each galaxy at \( t = 0 \), which, in turn, leads to a particular value of \( t_0 \). This was assumed to be the size of the maximum radius of the dark matter is associated with the dark haloes of galaxies (e.g., Bahcall, Lubin & Droman 1995; Puche & Carignan 1991). On the other hand, there are indications that in binary galaxies their \( M/L \)-ratios might be rather small suggesting that large quantities of dark matter in a common halo are unnecessary. We make the hypothesis that these conditions prevailed once galaxies were already formed in a triplet, a condition that will have to be relaxed in future work where the effect of a self-gravitating common halo will have to be considered.

For IC's, we do not consider the possibility of secondary infall into the region where the triplets will evolve. This is a justifiable assumption in a low density universe similar to the one where we appear to live (e.g., Bahcall 1999; Hradecky et al. 2000), where major infall of matter has ended at an earlier epoch (\( z \sim 1/\Omega_0 \), with \( \Omega_0 \sim 0.2 \)).

3.2 Initial conditions

Theoretically speaking, it will be highly desirable to obtain initial conditions (IC's) from a cosmological scenario for the formation of triplets; however, this is out of the scope of the present work. None the less, we have chosen two typical IC's for our triplets, namely: starting from (1) near 'maximum expansion' in a classical cosmology with \( \Lambda = 0 \), and (2) from virial equilibrium. We assume that galaxies are already formed at the moment of starting the simulations. We do not consider the possibility of secondary infall into the region where the triplets will evolve. This is a justifiable assumption in a low density universe similar the one where we appear to live (e.g., Bahcall 1999; Hradecky et al. 2000), where major infall of matter has ended at an earlier epoch (\( z \sim 1/\Omega_0 \), with \( \Omega_0 \sim 0.2 \)).

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3.2.1 Turnaround

For IC's simulating the ‘turnaround’ moment for triplets, the centre of mass of each galaxy was sampled randomly from a uniform spherical density distribution of radius \( R_{\text{max}} \), which is assumed to be the size of the maximum radius of the density perturbation that led to the formation of a triplet. A small isotropic bulk velocity, derived from a Gaussian with mean zero and one-dimensional velocity dispersion \( \sigma \), was imposed to each galaxy at \( t = 0 \). In a sense, this isotropic \( \sigma \) mimics a state of pre-virialization in our simulations (Davis & Peebles 1977; Lokas et al. 1996).

The numerical values of \( R_{\text{max}} \) and \( \sigma \) were obtained as follows. In the spherical collapse model for the detachment of a density perturbation from the Hubble flow (Gunn & Gott 1972), the maximum expansion radius at a particular epoch \( t \), for a standard cosmology with zero cosmological constant, \( \Lambda = 0 \), is:

\[
R_{\text{max}}(t) = \left( \frac{8GMt^2}{\pi^2} \right)^{1/3},
\]

where \( M \) represents here the mass at a given epoch.

We make the hypothesis here that triplets are on the verge of complete collapse for the first time at the present epoch, \( t_0 \), which, in turn, leads to a particular value of \( R_{\text{max}} \). The present age of the universe \( t_0 \) depends, in the previous cosmology, on the current value of the Hubble constant \( H_0 \) and the dimensionless density parameter \( \Omega_0 \). In particular, for an Einstein-de Sitter universe and a low-density universe we have, respectively (Gott et al. 1974):
triplets of galaxies

Table 2. Turn-around radius in Mpc

| $H_0$ | $\Omega_0 = 0$ | $\Omega_0 = 0.2$ | $\Omega_0 = 1$ |
|-------|----------------|------------------|----------------|
| 50    | 0.84           | 0.74             | 0.64           |
| 75    | 0.64           | 0.57             | 0.49           |
| 100   | 0.55           | 0.47             | 0.40           |

$H_0 t_0 = \begin{cases} 
2/3 & \Omega_0 = 1 \\
1 + \frac{1}{2} \Omega_0 \ln \Omega_0 & \Omega_0 \to 0 
\end{cases}

Therefore for $\Omega_0 \approx 0.2$ we have $H_0 t_0 \approx 0.84$ and, using our choice of the Hubble constant, $t_0 \approx 0.84 (13 \text{ Gyr}) \approx 10 \text{ Gyr}$. In Table 2, the turnaround radius (in Mpc) of a triplet of Galaxy-like objects $(3 \times 5.5 \times 10^{11}$ $M_\odot)$, for different values of $H_0$ and $\Omega_0$, are given; equation (8) was evaluated at a time $t_0/2$ to obtain $R_{\text{max}}$ according to our stated hypothesis. Of the plausible values in Table 2, we have taken $R_{\text{max}} = 500$ kpc as our fiducial value for the simulations under this kind of initial conditions.

Our collapsing triplets are assumed to start with cold initial conditions satisfying a virial ratio of $2T/|W| = 1/4$, as is common in simulations of groups of galaxies (e.g., Barnes 1985). A random velocity, sampled out of a Gaussian distribution with zero mean and 1-D velocity dispersion $\sigma = \frac{V_0}{2\sqrt{3}}$, with $V_0 = \left(\frac{3GM_c}{5R_{\text{max}}}ight)^{1/2}$, was assigned as bulk motion to each galaxy. Using our fiducial values for $R_{\text{max}}$ and the triplet’s mass we obtain a value of $\sigma \approx 30 \text{ km s}^{-1}$. This is consistent with the value found by Lake & Carlberg (1988) for the velocity dispersion of dark matter at the time of turnaround, $\sim 35 \text{ km s}^{-1}$.

3.2.2 Virial

Adopting a realistic virial radius from which to sample the position of galaxies is not straightforward, because we are not considering the process of formation of triplets in a cosmological setting. Nevertheless, early point-like simulations made by Peebles (1970) suggest that by $\approx 3t_c/2$, with $t_c$ being the collapse time scale, a collapsing system has reached an approximate equilibrium state, after having rebounded, with a virial radius $R_c \approx R_{\text{max}}/2$ (Gott & Rees 1975). According to Coles & Lucchin (1995), more recent N-body simulations of structure formation in an expanding universe tend to favour this time-scale.

Therefore, if we make the hypothesis that triplets at the present epoch are virialised, which is suggested by the crossing times of K-triplets, we have that $R_c = R_{\text{max}}(t_0/3)/2$; for our fiducial triplet we take $R_c = 191$ kpc. The bulk velocities of galaxies were chosen initially so as to satisfy the virial ratio $2T/|W| = 1$ exactly.

3.3 Some computational aspects

To obtain the global dynamical properties of triplets, e.g., velocity dispersion and mean harmonic radius, we used the centre of each galaxy. We set the most bounded particle of the galaxy model to have 1 percent of its total mass, set its coordinates to be identically zero in the random realization, and identified this massive particle as the centre of the numerical galaxy. This numerical artifact allowed us to use this massive particle to trace the galaxy’s path and to compute its bulk velocity by time differentiation. A similar approach to identify the centre of a numerical galaxy was used by Aguilar & White (1985).

We performed 30 triplet simulations for each kind of initial conditions using the parameters and method discussed above. Each simulation differed in the initial seed used in the random realization to obtain the positions and velocities of galaxies at $t = 0$. The collapse time for our fiducial triplet, in N-body units, is $t_c = \pi R_{\text{max}}^3/(2GM_c)^{1/2} \approx 290$ (Gott 1975), and its half-mass free-fall time $t_{\text{ff}} = 1/2 \pi R_{\text{ff}}^3/(2GM_c)^{-1/2} \approx 72$ (Mamon 1990); here we have taken $R_{\text{ff}} = R_{\text{max}}/2$. The dynamical time for initially virialised triplets is $t_v = (2R_c)^{3/2}/(3GM_c)^{-1/2} \approx 130$. Each triplet was evolved for $t_v = 321.4$ time units; i.e., $\sim 10$ Gyr. Energy was conserved to better than 0.5 percent throughout this time for all simulations. Each run took $\approx 4$ hrs of CPU time in a Pentium-II 400MHz PC.

4 RESULTS AND DISCUSSION

4.1 Global behaviour

In Figures 3 and 4 we display the XY-projection of the evolution of two particular triplet simulations starting from maximum expansion. These show some of the general qualitative characteristics found in the other simulations, including those starting in virial equilibrium. For example, a ‘binary’ galaxy is in general formed first while the third galaxy ‘orbits’ around it for some period of time. This binary instability is analogous to that found in the 3-body problem (e.g., Valtonen & Mikkola 1991) and, in this respect, supports a generalization of this behaviour to triplets of self-gravitating galaxies.

However, two important qualitative differences between the 3-galaxy and 3-body problem are found: (1) isolated triplets will eventually merge in their centre of mass provided enough time is given, and (2) no ‘sling-shot’ events were found for these initially bound systems. The physical reason for both of these characteristics stems from the fact that galaxies are able to absorb orbital kinetic energy, and angular momentum, and transfer it to their internal degrees of freedom. A clear manifestation of the previous is the large extent that matter in a merger can reach; e.g., in Fig. 3 the final system can occupy a region of $\sim 1$ Mpc in diameter. If we interpret our galaxy models as representing a spiral, this large region would be occupied primarily by dark matter and form a common halo by the present epoch. This has actually been observed in a simulation of a triplet of spirals (Aceves 2000b).

We emphasized above the issue of triplets being isolated because there are some indications that tidal interactions with large-scale structures might disrupt them over a Hubble time (Aceves 2000a). We believe that this may apply more properly to present-day W-triplets due to their large extent.
Figure 3. Projection on the XY-plane of a particular collapsing triplet simulation. Snapshots are at \( \approx 21.4 \) time unit (\( \approx 0.69 \) Gyr) intervals. Time increases from left to right and from top to bottom. The boxes are 80 x 80 units (\( \approx 1 \) Mpc). Here galaxies do not merge during the whole extent of the simulation (\( \sim 10 \) Gyr). The present epoch is approximately in the third frame of the first column from top to bottom.

Figure 4. Similar to Fig. 3, but for virialised triplets. Here, the initial conditions lead to a rapid binary formation and a subsequent triple merger. The initial chain-like structure of the triplet in the sky lasts for \( \sim 1 \) Gyr. Note the large extent of the halo developed during the merging process.

Figure 5. Merging histories of thirty collapsing triplets. Time is in N-body units and measured from the start of the simulations; indicated are the half-mass free-fall time, \( t_{\text{ff}} \), and the present epoch, \( t_0 \), according to our fiducial model. At \( t_0 \) about 10 per cent of triplets have merged. Merging histories have been slightly displaced horizontally and vertically for better appreciation.

4.2 Merging histories

We have adopted a criterion of ‘loss-of-identity’ (Athanassoula et al. 1997) to determine when a pair of galaxies has merged. Although different merging criteria lead to differences in the number of mergers found at a particular time (e.g., García-Gómez & Athanassoula 1993), we consider the adopted criterion well-suited for our purposes, for it appears more closely related to an observational practice.

In particular, we consider that two galaxies have lost their identity when

\[
V_{ij} = |v_j - v_i| < V_{\text{rms}}/2, \quad R_{ij} = |r_j - r_i| < R_0/2; \quad (10)
\]

where \( V_{ij} \) and \( R_{ij} \) are the relative three-dimensional velocity and distance between galaxies \( i \) and \( j \), and \( V_{\text{rms}} \) and \( R_0 \) are the root-mean-squared velocity and scale radius of a Plummer sphere, respectively. In the numerical implementation of criterion (10) a triple merger required a few time-steps to be detected, since Eq. (10) is a pair-wise recipe, but no unreasonable results were obtained.

In Figs. 5 and 6 we show the number of ‘galaxies’ remaining in the triplet at time \( t \), both starting from maximum expansion and in virial equilibrium, respectively. A merged pair was counted as an individual ‘galaxy’. We note that the cosmic time, i.e., the time elapsed since the ‘big bang’, is different for each of the IC’s considered; this is because they were assumed to start a different epochs in the evolution of the universe. For IC’s starting at maximum expansion the cosmic time is \( t_{\text{cos}} = t_0/2 + t \), where \( t \) is the time span of the simulation (e.g., Ishizawa 1986); analogously, for initially virialised triplets we would have \( t_{\text{cos}} = t_0 + t \).

It is interesting to note that by the present epoch,
Figure 6. Similar to Fig 5, but for virialised triplets. The time scale indicated is the dynamical time of the triplet. About 20 percent of triple mergers are found after $\sim 10$ Gyr of evolution.

$t_n \approx 160$, only about 10 percent of triplets starting from maximum expansion have merged and about 60 per cent have not suffered any mergers. At the end of the simulation, i.e. about 5 Gyr after $t_0$, about 30 percent have managed to survive without any merging.

Although there are not many observational studies regarding triplets’ characteristics, the previous results appear qualitatively consistent with the available observations. For instance, a rapid visual inspection of K-triplets in NASA’s database NED does not show strong triple mergers but their luminous components look fairly separated in most of the images, though this may be a consequence of the selection criteria. One example of a possible triple merger in process is in Hickson’s group 95 (Iglesias-Páramo & Vílchez 1997), but since this is a group one cannot even strictly compare such ‘triplet’ with our results, and another is that found by Heidt et al. (1999).

For initially virialised triplets, 4 ($\approx 15\%$) merge completely in within a dynamical time, while 2 ($\approx 7\%$) do not present any merger and 19 ($\approx 60\%$) only present binary mergers in $\sim 10$ Gyr. This behaviour is probably related to the fact that virialised triplets tend to maintain their initial conditions longer, with higher velocity dispersion, hence being less disposed to satisfy the merging criterion (10). Obviously, these results would be altered if a primordial common dark halo had been introduced.

We do not find an overmerging problem for triplets under the kinds of initial conditions considered here, contrary to what is usually found in numerical simulations of compact groups of galaxies (Hickson 1997). The results obtained also show that there are stable states in the 3-galaxy problem, contrary to what is sometimes believed (Valtonen & Flynn 2000). We must recognize, however, that since we are not modeling the actual process of formation of a triplet, the above results need to be taken as a first approximation to the problem of merging activity in triplets.

4.3 Geometrical properties

Several authors have used the AA-map to characterize the geometrical properties of triplets and three-body systems (e.g., Chernin et al. 1994; Heinämäki et al. 1998). Although this approach does not provide much information on the dynamical state of triplets, since a triplet initially in the L-area can migrate toward other regions at later times (e.g., Ivanov, Filistov & Chernin 1995), can none the less provide us with another means to test our models.

In Figs. 7 and 8 we show the AA-map at different times for triplets starting from maximum expansion and virial equilibrium, respectively. Although it is somewhat difficult to assess quantitatively these results and to compare them with the observations of K and W-triplets, in part due to the small number of points for a statistical analysis, it is interesting to note that the AA-map indicates that no strong hierarchical structures are present at the current epoch ($t \approx 6$ Gyr) for collapsing triplets. On the other hand, for initially virialised triplets we obtain a slight excess of triplets in the H-area at $t = 0$ that is still present after a few giga-years of evolution.

We would like to point out also that it is possible that the lack of many observational triplets in the H-area of the AA-map may be due to an observational bias. Real galaxies are able to merge and disappear from the AA-map at some time. Once the haloes of galaxies start to overlap the merging process is rather rapid ($\sim 1$ Gyr), and this will reduce the probability of finding such extreme H-structures in a sky survey. If some W-triplets are truly physical hierarchical triplets one would expect them to disappear from the H-area of the AA-map in the order of a few Gyr.

Figure 7. Evolution of the AA-map for collapsing triplets. Numbers indicate the time elapsed from their initial condition. No significant excess of hierarchical structures are observed at the present epoch ($t \approx 6$ Gyr). Physical hierarchical triplets disappear in this map in a scale of $\sim 1$ Gyr.
The previous results lead us to consider that our models are not inconsistent with the AA-map of observed triplets, both compact and wide. However, these results do not allow us to establish a preference for the current dynamical state of triplets. It appears, on other hand, that a large primordial common halo of dark matter is not required to avoid the existence of hierarchical triplets, as suggested by other authors (e.g., Chernin et al. 1994).

### 4.4 Dynamical quantities

In order to compare our models with observations we have calculated, for both kinds of initial conditions, the one dimensional velocity dispersion, the mean harmonic radius, the dimensionless crossing time, and the virial and mean harmonic radius of the ensemble of simulations. These results are in contrast to others that indicate a factor at most of about 4 higher than the true mass of the system. These results are somewhat as point-like particles and not as self-gravitating systems, part of their mutual orbital kinetic energy is transformed into internal kinetic energy and, thus, no complete virial equilibrium in their translational bulk degrees of freedom can be achieved. A virial equilibrium, but in the complex mixture of the internal and bulk degrees of freedom of galaxies, tends to be established during their interactions.

From Tables 3 and 4, we observe that no overestimates of the median virial mass are obtained at any time, except at the end of the simulations for virialised triplets. Overestimates were found in few of our numerical experiments, but only for brief intervals of time when using the median mass estimator. At most a \( \approx 20 \) per cent overestimate was found for virialised IC’s and \( \approx 10 \) per cent for collapsing triplets. After examining each run, we found that several triplets were a factor at most of about 4 higher than the true mass of the triplet, but this happened only along a particular line-of-sight while along the other two an underestimation always occurred. These overestimates happened before the galaxies overlapped of the galaxies; i.e., when galaxies still behaved somewhat as point-like particles and not as self-gravitating systems. These results are in contrast to others that indicate that overestimates of even a few orders of magnitudes may be obtained in triplets.

The results of Table 3 indicate that triplets arriving virialised triplets tend to evolve much slower than those for the collapsing ones, which is perhaps most noticeable in the small change of the median velocity dispersion (Table 4). On the other hand, a significant evolution toward the establishment of a virial equilibrium among the translational bulk degrees of freedom of galaxies is observed for collapsing triplets. This is noticeable as their virial mass estimate approaches their true mass. However, because galaxies are self-gravitating systems, part of their mutual orbital kinetic energy is transformed into internal kinetic energy and, thus, no complete virial equilibrium in their translational bulk degrees of freedom can be achieved. A virial equilibrium, but in the complex mixture of the internal and bulk degrees of freedom of galaxies, tends to be established during their interactions.

| Table 3. Turnaround Results |
|-----------------------------|
| \( t \) & \( \sigma \) & \( R_H \) & \( H_0\tau_c \) & \( M_V \) & \( M_{med} \) |
| 17.2 & 0.052 & 39.13 & 2.08 & 0.66 & 0.81 |
| 34.5 & 0.067 & 38.73 & 1.71 & 0.94 & 1.30 |
| 64.6 & 0.080 & 33.93 & 1.37 & 1.16 & 1.71 |
| 94.8 & 0.098 & 31.56 & 0.83 & 1.11 & 1.86 |
| 125.0 & 0.090 & 22.20 & 0.65 & 1.04 & 1.51 |
| 159.4 & 0.125 & 21.40 & 0.43 & 1.96 & 2.07 |
| 185.3 & 0.139 & 19.09 & 0.46 & 2.35 & 3.26 |
| 219.7 & 0.116 & 12.65 & 0.33 & 1.28 & 1.58 |
| 249.9 & 0.141 & 9.37 & 0.16 & 1.16 & 1.71 |
| 282.2 & 0.152 & 14.07 & 0.24 & 1.37 & 2.21 |

| Table 4. Virialised Results |
|-------------------------------|
| \( t \) & \( \sigma \) & \( R_H \) & \( H_0\tau_c \) & \( M_V \) & \( M_{med} \) |
| 17.2 & 0.189 & 13.53 & 0.23 & 2.65 & 4.09 |
| 34.5 & 0.152 & 12.73 & 0.18 & 1.62 & 2.23 |
| 64.6 & 0.164 & 14.50 & 0.23 & 2.05 & 2.83 |
| 94.8 & 0.159 & 15.30 & 0.23 & 1.14 & 2.51 |
| 125.0 & 0.135 & 15.96 & 0.29 & 2.22 & 2.78 |
| 159.4 & 0.120 & 14.42 & 0.34 & 1.40 & 2.81 |
| 185.3 & 0.126 & 13.40 & 0.33 & 1.80 & 3.18 |
| 219.7 & 0.176 & 12.71 & 0.17 & 1.87 & 2.29 |
| 249.9 & 0.161 & 8.72 & 0.13 & 1.05 & 2.65 |
| 282.2 & 0.187 & 13.89 & 0.24 & 3.53 & 3.84 |

Figure 8. Similar as in Fig. 1, but for virialised triplets. A slight excess of hierarchical triplets are observed initially, but later they are no longer present. Merging activity depletes the extreme H-area.

\[ 0.5 \text{ Gyr} \quad 2.0 \text{ Gyr} \]

\[ 4.0 \text{ Gyr} \quad 5.9 \text{ Gyr} \]
has the intrinsic propensity to underestimate the total mass of galaxies—even if we had three-dimensional information—in observational practice, that the use of the bulk velocities mentioned transfer of orbital to internal energy. This means, the physical reason behind this underestimate, in these truly bounded systems, is the already-mentioned transfer of orbital to internal energy. This means, in observational practice, that the use of the bulk velocities of galaxies—even if we had three-dimensional information—has the intrinsic propensity to underestimate the total mass of the system. A similar conclusion was reached by Barnes (1985) in his study of small groups of galaxies. For initially virialised triplets (Table 4) the median mass estimator tends to provide a better estimate, though it may overestimate in some cases, by less than ≈ 30 percent.

Overall, we consider the median mass estimator to perform better than the virial mass estimator in these low-multiplicity systems. The accuracy of the mass estimate depends in an important way on the dynamical status of the system. To derive a time-varying expression for the median or virial mass estimator is not the purpose here. See how ever Dolgachev & Chernin (1997) for a study developed to take this into account using point-like particles as a galaxy model.

At the present epoch, $t_a ≈ 160$, our models predict that triplets starting from maximum expansion have their median virial masses underestimated by ≈ 35 percent, with a somewhat better agreement when using the median mass estimator. The physical reason behind this underestimate, in these truly bounded systems, is the already-mentioned transfer of orbital to internal energy. This means, in observational practice, that the use of the bulk velocities of galaxies—even if we had three-dimensional information—has the intrinsic propensity to underestimate the total mass of the system. A similar conclusion was reached by Barnes (1985) in his study of small groups of galaxies. For initially virialised triplets (Table 4) the median mass estimator tends to provide a better estimate, though it may overestimate in some cases, by less than ≈ 30 percent.

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At the present epoch, $t_a ≈ 160$, our models predict that triplets starting from maximum expansion should have a median $R_{\text{HI}} ≈ 290$ kpc and $H_0 \tau_c ≈ 0.43$. When comparing these values with the properties of K-triplets (Table 1), we find them higher by about a factor of 4 and 10, respectively. Therefore, it appears that our models yield results that are difficult to reconcile with the observations. This situation is worse when we note that a median of $\sigma \approx 52$ km s$^{-1}$ is predicted, a value which is about half of the observed one for K-triplets. A maximum of $\sigma ≈ 64$ km s$^{-1}$ is obtained at the end of the simulations. For virialised IC’s, the maximum median is $\sigma ≈ 80$ km s$^{-1}$, occurring at $t = 0$. Afterwards, the median $\sigma$ tends to decrease somewhat due to the transfer of orbital to internal energy during the interactions of galaxies.

The previous results, taken at face value, tend to rule out the models presented for triplets with either kind of initial conditions considered. However, there are some observational matters and numerical aspects that indicate that our models might not be an unreasonable scenario for the global evolution of triplets. In § 3 we will establish which kind of initial conditions we favour the most.

On the other hand, when comparing our results to observations we are assuming implicitly that K-triplets form an homogeneous sample, both in luminosity ($\sim$ mass) and morphological type, but as can be seen from the properties of K-triplets (Karachentsev et al. 1989) this is not the case. In this respect our ensemble of numerical triplets forms, by construction, a homogeneous ‘catalog’. A mass-spectrum, for example, can have effects on the numerical values obtained for $\sigma$ and $R_{\text{HI}}$ that are not easy to estimate. This motivated us to estimate the velocity dispersion of K-triplets weighted by the luminosity of their member galaxies in order to compare it with our numerical results. Weighting by mass appears to be a more uncertain procedure. The median weighted velocity dispersion obtained was $\sigma ≈ 90$ km s$^{-1}$ (about 25 per cent lower the value in Table 1), which in turn is closer to our results.

Different selection criteria can yield significantly different results for the dynamical quantities in galaxy systems. For example, in a recently-announced catalog of small galaxy groups by Makarov & Karachentsev (2000), using physically motivated selection criterion, the following median values, with quartiles, for triplets are reported: (a) velocity dispersion $41^{+16}_{-18}$ km s$^{-1}$, (b) mean harmonic radius $191^{+157}_{-88}$ kpc, and dimensionless crossing time $0.15^{+0.16}_{-0.09}$ (us).
ing $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$). These authors provide a harmonic radius value not corrected for projection effects (i.e. lower by a factor of $\pi/2$ than when using Eq. 3) and use the crossing time definition of Gott, Wrixon & Wamier (1973) (Makarov, private communication). The latter definition is $t_c = 3R_h (5^{1/2}/\sigma)^{-1}$, that, when comparing with Eq. 4 yields $\tau_c \approx 4t_c$. Therefore, comparing our results at $\approx t_0$ for collapsing triplets (Table 3) with the values found by Makarov & Karachentsev a much better agreement is found. Furthermore, we note that while all the previous comparisons were done among the median values, we find that $\approx 10$ per cent of collapsing triplets have $\sigma \sim 100 \text{ km s}^{-1}$ at $\approx t_0$ and small RH's ($\approx 0.1$).

Part of the disagreement of our median results with, for example, the observed velocity dispersion of K-triplets may be somewhat alleviated if we use a larger mass as our fiducial galaxy model, but it does not satisfactorily solve the problem. Zaritsky et al. (1997) after examining the distribution and kinematics of satellites around spiral galaxies conclude that the halo masses of spirals are in the excess of $2 \times 10^{12} \text{ M}_\odot$ within $\approx 200 \text{ kpc}$. Using this type of galaxies, the scaling factors $\{l, \mu\}$ in Eq. 4 are $\{4.4, 39.2\}$. From this we obtain that $v_a = 660v_\sigma \text{ km s}^{-1}$, which leads to a median $\sigma \approx 80 \text{ km s}^{-1}$ at $\approx t_0$ for collapsing triplets. This new scaling provides a much better agreement with the velocity dispersion observed in K-triplets. However, the discrepancy with observations moves now to the $R_h$ median value that results in $\approx 425 \text{ kpc}$, i.e., about an order of magnitude larger than the observed value and marginally consistent with the results of Makarov & Karachentsev (1999). The time scale does not change by much. One may try to force a better agreement by selecting a galaxy with a mass of $\approx 10^{12} \text{ M}_\odot$ within $\approx 100 \text{ kpc}$, but this appears in conflict with the fact that not many galaxies satisfying these conditions, e.g., large ellipticals, exist in K-triplets. To properly address these scaling matters, more realistic simulations including a mass-spectrum would be required.

The models considered here do not properly apply to present day observed W-triplets. A better model for these would be, perhaps, to assume that they are in the early phases of their turnaround. With this assumption $R_{\text{max}} \sim 1 \text{ Mpc}$ and $\sigma \sim 30 \text{ km s}^{-1}$. The latter scenario has been considered briefly elsewhere (Aceves 1999a), where it was found that we will have to wait for about another Hubble time for W-triplets to reach a compact configuration similar to the one of K-triplets. See also Dolgachev & Chernin (1997). However, the current large $\sigma \sim 100 \text{ km s}^{-1}$ for W-triplets does not seem to favour such possibility.

We note that W-triplets appear to be a less homogeneous sample than K-triplets, for they include triplets with a very low harmonic radius similar to those of K-triplets (Fig. 1). It is not the purpose here to make an analysis of the selection criteria used to compile W-triplets, but we point out that some of them would appear more properly classified as K-triplets. The overlapping in the RH-distribution is interesting in itself, however, since some triplets classified as W-triplets may constitute a link between those that are in the early phases of collapse and those that are in the more advanced phase.

Larger samples of triplets and more detail studies of their properties are required to build a more solid observational database for future comparisons with theoretical models. Efforts toward expanding the sample of triplets, for example, in the southern sky are presently being done by several groups (e.g., Karachentseva & Karachentsev 2000).

4.5 Evolutionary Trends

An important question to investigate is how K-triplets have attained their present configuration and dynamical properties. From the previous results, it appears that triplets starting from a diffuse configuration, followed by a collapse, is a probable scenario.

Perhaps one of the best methods to discern the evolution of a dynamical system is to look into its phase-space structure. However this is not possible in galaxy systems since we do not have yet enough information to construct a snapshot of their physical phase-space. None the less, we may construct a pseudo phase-plane for triplets by using their velocity dispersion and mean harmonic radius, compare them with the numerical results, and see if some evolutionary trends are discernible.

In Fig. 11 we show the pseudo phase-plane (hereafter, phase-plane for brevity) for our numerical triplets at different times spanned from $t = 0$, both for collapsing and virialised initial conditions. In Fig. 12 the phase-planes of K and W-triplets are shown. Again it is difficult to assert some conclusions from a comparison of the phase-planes, but it is interesting to note that some tendencies exist. For example, K-triplets show, on one hand, the general trend followed by systems that had suffered a collapse out of a much wider configuration: a decrease in their $\sigma$ as $R_h$ increases. This behaviour is similar to that exhibited by the numerical triplets that have started from maximum expansion in our simulations. On the other hand, W-triplets have a phase-plane that resembles qualitatively systems that are on the verge of suffering a gravitational collapse.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{phase_plane.png}
\caption{Pseudo phase-plane for triplets under both types of initial conditions considered and at different times during the simulations. The present epoch is at $\approx 6 \text{ Gyr}$ for collapsing triplets.}
\end{figure}
(ii) A rather low number of triple mergers ($\approx 10$ per cent) are expected to occur at the present epoch for systems starting from maximum expansion. This appears to be in concordance with observations. Initially virialised systems yield $\approx 5$ per cent of triple mergers in $\sim 10$ Gyr of evolution. Therefore, no overmerging problem is found for triplets. This contrasts to what it is usually found in numerical simulations of small compact groups.

(iii) Under both types of initial conditions considered, we find that the 3-galaxy problem has stable states; i.e., there are certain conditions that do not lead to a triple merger even after $\sim 10$ Gyr of evolution.

(iv) The geometrical properties of numerical triplets, as indicated by their homologous AA-map, are not inconsistent with those of observed triplets, both compact and wide. Physical triplets in the extreme hierarchical region of the AA-map would suffer a rapid dynamical evolution toward a binary merger leading to a depletion of triplets in such region, since they would no longer would be counted as triplets in an observational survey.

(v) Median values of the dynamical parameters for triplets that have started as a diffuse system and are collapsing for the first time at the present epoch are not inconsistent with the data of Karachentsev’s compact triplets. A much better agreement is reached with recent data on triplets. We find that about $\approx 10$ percent of the simulated triplets reproduce well the K-triplets median dynamical quantities at the present epoch.

(vi) The median of the virial mass estimator does not in general overestimate mass, but, on the contrary, it underestimates it. For triplets in an advance stage of collapse at the present epoch an underestimate of $\approx 35$ percent is found. Therefore if K-triplets are truly bound physical systems, with no primordial common dark halo, their $M/L$-ratios are being underestimated by the same factor. Otherwise, the underestimate may be larger. The median mass estimator appears as a somewhat better mass estimator for triplets than the virial mass estimator.

(vii) The plane $\sigma - R_h$ of numerical triplets suggests that K-triplets have acquired their present configuration through a clustering process, and lead us to conclude that they are the triplets with the most extreme values in the literature, i.e., lowest harmonic radius and highest velocity dispersion. Some dynamical properties of K-triplets are also consistent with a virial equilibrium state in their bulk degrees of freedom.

(viii) Our results suggest that the presence of a primordial, large, massive, common dark halo might not be a strong requirement to account for the dynamical evolution of triplets, but they cannot exclude it.

In order to better compare with observations, future models should include, e.g., a mass-spectrum for galaxies, consider different morphological types, and investigate the role of a primordial common dark halo in a cosmological setting.

### ACKNOWLEDGMENTS

The Spanish Ministry of Foreign Affairs (MUTIS program) and México's CONACyT (Proyecto I35546-E) are thanked for financial support at different stages of this work. We...
thank greatly Gary Mamon for fruitful discussions on the issue of initial conditions. Jack Sulentic and Héctor Velázquez are also thanked for comments on this work, and Michael Richer for help with the English writing.

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