Self-tuning behavior of a clamped-clamped beam with sliding proof mass for broadband energy harvesting

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Abstract.
Real world systems rarely vibrate at a single resonance frequency and the frequencies drift over time. Tunable devices exist, but generally need additional energy to achieve frequency adaptation. This means that the benefits in power output from this tuning need to be large enough to power the mechanism itself. Passively self-tuning systems go into resonance without requiring active control. This paper focuses on a passively self-tuning system with a proof mass that can slide freely along a clamped-clamped beam. Under external vibration, the slider moves along the beam until the system goes into resonance. A proof-of-concept design is introduced using either a copper or a steel beam and a 3D-printed ABS thermoplastic proof mass. Successful self-tuning is demonstrated in both cases. The frequencies range from 80 – 140 Hz at accelerations as low as 0.007 g rms. Results show the resonance of the beam and the position of the slider along the beam with time. Furthermore, the dynamic magnification and the proof mass position at resonance are discussed, together with the inherent non-linearities of double-clamped beam resonators. The findings support the hypothesis that the effect of the ratio between proof mass and beam mass outweighs the Duffing spring stiffening effects.

1. Introduction
One of the problems faced by resonant vibration energy harvesting is that real world systems exhibit changes in frequency over time. This causes a mismatch between the excitation frequency and the narrow optimal operation frequency of many of these devices [1]. Various tunable designs offer potential solutions [2], but most of the proposed tuning mechanisms require some form of energy to operate. This means that in order for tuning to be viable, the system needs to improve the power output sufficiently to accommodate for the energy that goes into the tuning itself. Another approach is frequency up-conversion, where a random input gets converted into an ideal operation frequency for the transducer [3]. However, this strategy is best for situations where resonant devices are not suitable in the first place. For instance, an advantage of resonant harvesters is the dynamic magnification of small vibration amplitudes, which is beneficial when the internal proof mass displacement limit is large compared to the input excitation.

One method for passive self-tuning, which means that the system naturally goes into resonance without the need to supply additional energy, uses spring-stiffening effects on a...
microelectromechanical device [4]. In contrast, our work expands on a system, first presented in [5], that allows passive self-tuning of the device by incorporating a proof mass that can slide along a double-clamped beam. The principle is similar to work presented in [6], where the resonance of a set-up with a freely sliding bead on a string was investigated. In [7], a spring-suspended slider was used with a simple cantilevered beam and [8] investigated a number of systems, one based on a sphere moving inside a cylinder. The latter two attempts in particular proved the importance of carefully selecting the system parameters. Both showed dissipation of energy, thus achieving self-damping of the system instead of self-tuning. This paper builds on experimental results from [5], in which passive self-tuning is reproducibly shown at various frequencies in three prototype systems, to present new data on the potential role that non-linear spring stiffness plays in the system.

2. Experimental Setup

A schematic of the system setup is shown in figure 1 and a photograph of the prototype is given in figure 2. A clamped-clamped beam carries a freely sliding proof mass. The position of this slider affects the resonance frequency of the beam. Under external vibration the slider will progressively migrate into a position such that the beam matches the driving frequency and goes into resonance. This paper is based around two different types of beams, steel and beryllium copper (60 × 3 × 0.1 mm and 60 × 3 × 0.2 mm respectively); a third configuration is discussed in [5]. It was found that the steel beam can sag through under its own weight during clamping to the U-shaped holder with two bars screwed in on either side. The resulting compressive load causes the beam to buckle and needs to be avoided for the system to work properly. This was much easier to achieve with the thicker copper beams. Furthermore, due to the lower Young’s modulus of copper and the higher specific weight, the operation frequencies stay in a similar range.

The 3D-printed slider carries a screw at the bottom that acts as an additional mass and allows the adjustment of the clearance with the beam. It was marked with a white dot on the side for image processing in Matlab. The position of this white dot can be read from photographs taken at one second intervals, triggered by a Labview program. The results were used to plot the relative position of the slider on the beam over time together with the vibration amplitude at the beam center recorded with a Keyence LK-H087 laser displacement sensor (LDS). The vibration shaker (Labworks ET-126) and accelerometer (Kistler 8315A) were also operated through the same Labview program.
3. Measurement Results
Passive self-tuning was successfully achieved for both steel and copper beams. Figures 3 and 4 show the measurements for the steel beam at the upper and lower frequency limit respectively. The corresponding graphs for the copper beam are shown in figure 5 and figure 6. The mid-point displacement is shown along with the proof mass position, normalized over the length of the beam. It can be observed that the mass slides away from the center until the system hits resonance. The difference between the copper and the steel beam is in the frequency range over which tuning was successfully achieved. In both cases the same sliding mass was used, which means that the proof mass to beam weight ratio is twice as high for the steel beam as for the copper one. A relatively heavier proof mass results in a wider range of frequencies over which tuning will occur. A similar effect of the proof mass weight was observed in [6].

![Figure 3: Self-tuning at 140 Hz on a steel beam at 0.0084 g rms acceleration](image1)

![Figure 4: Self-tuning at 80 Hz on a steel beam at 0.012 g rms acceleration](image2)

![Figure 5: Self-tuning at 135 Hz on a copper beam at 0.007 g rms acceleration](image3)

![Figure 6: Self-tuning at 100 Hz on a copper beam at 0.011 g rms acceleration](image4)

The normalized position of the slider when the system tunes in to resonance for the tested driving frequencies is shown in figure 7. The graph is restricted to the right hand side of the beam. In theory, the proof mass direction of travel should be symmetric, not favouring either side, when the initial position is in the middle of the beam. During the experiments, the slider had a tendency to move to the right. This might be caused by the fact that the initial position
was not exactly in the center because the laser beam of the displacement sensor needed a clear path. However, as expected, there is a trend of the slider to move towards the end of the beam with an increase of frequency. The explanation is simple; if the mass were fixed on the beam, a position in the middle would yield the lowest possible resonance frequency, whereas a position at the end of the beam would yield the highest one. The same trend was observed for a larger aluminum beam prototype in [5].

![Figure 7: Position of the slider on the beam when tuning occurs](image1)

![Figure 8: Magnification of the displacement amplitudes when the system goes into resonance](image2)

Figure 8 depicts the magnification of the beam center displacement in resonance for all the frequencies where tuning was achieved. The results achieved with the copper beam are more consistent and show a slight increase of magnification with an increase of frequency. The reasons behind this are not entirely clear and need further investigation. The scattering of the results for the steel beam could be due to the previously mentioned difficulties in clamping the beams, whereas the increase in magnification for the copper beams is likely due to the higher energy present in the system at higher frequencies.

![Figure 9: Frequency sweep on a steel beam without the proof mass, FFT of the linear displacement sensor output](image3)

![Figure 10: Frequency sweep on a copper beam without the proof mass, FFT of the linear displacement sensor output](image4)

Figures 9 and 10 were obtained by performing a frequency up-sweep and a down-sweep on beams without a proof mass and exhibit the inherent non-linearity of the frequency response of
double-clamped beams. The first finding from these graphs is a good consistency with figure 8; the FFTs show a larger amplitude for the copper beam than for the steel beam. It is assumed that these effects originate from different damping values between the copper and steel set-ups. Those might simply be due to different material damping coefficients or caused by the difficulties in clamping the steel beams. Furthermore, in the FFTs without attached mass, the steel beam exhibits a significantly smaller frequency bandwidth than the copper beam. However, in the case of the self-tuning results previously shown, the opposite was found. What this suggests is that the effect of proof mass to beam mass ratio far outweighs the effects of the non-linearity and the bandwidth of the frequency response of a double clamped beam.

4. Conclusions
This paper presents an experimentally verified self-tuning mechanism based on a sliding proof mass on a beam. It was found that the crucial points are the clamping of the beam to avoid any compression or tensioning of the beam, and the clearance between the slider and the beam. The latter could easily be adjusted during the experiments by tightening or loosening a screw in the base of the slider. However, this process required a fair amount of fine tuning before the self-resonance was successful. If the clearance is too tight, the slider will not move; if it is too large, the slider will start rattling and dissipates energy. The possibility of lubrication could be investigated in a next step.

Further work is required to determine the effects of spring-stiffening and to fully characterize the behaviour of the system. That being said, the results are encouraging, showing self-tuning over a range between 80 – 140 Hz for a steel beam and between 100 – 135 Hz for a copper beam at low acceleration levels. In addition, the comparison between measurements without the proof mass and the results when self-tuning occurred consistently shows the differences in magnification between copper and steel and suggests that the effects of non-linearity are only secondary to the effects of the proof mass to beam mass ratio. The next step for energy harvesting could be the inclusion of piezoelectric patches on the beam to generate power.

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References
[1] Miller L M, Halvorsen E, Dong T and Wright P K 2011 Journal of Micromechanics and Microengineering 21 045029 ISSN 0960-1317
[2] Zhu D, Tudor M J and Beeby S P 2010 Journal of Measurement Science and Technology 21 022001 ISSN 0957-0233
[3] Pillatsch P, Yeatman E M and Holmes A S 2012 Journal of Smart Materials and Structures 21 115018 ISSN 0964-1726
[4] Marzencki M, Defosseux M and Basrour S 2009 Journal of Microelectromechanical Systems 18 1444–1453
[5] Miller L M, Pillatsch P, Halvorsen E, Wright P K, Yeatman E M and Holmes A S 2013 Journal of Sound and Vibration
[6] Boudaoud A, Couder Y and Ben Amar M 1999 The European Physical Journal B 9 159–165 ISSN 1434-6028
[7] Miranda E C and Thomsen J O N J 1998 Journal of Nonlinear Dynamics 167–186
[8] Kozinsky I 2009 PowerMEMS pp 388–391 ISBN 0974361151