Soft Binary Processes, NJL Model and Absolute Values of the Amplitudes of Reactions

\[ \pi^{-}p \rightarrow \pi^{0}n \text{ and } \pi^{-}p \rightarrow \eta n \]

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Abstract
Reactions \( \pi^{-}p \rightarrow \pi^{0}n(\eta n) \); \( \pi^{-}p \rightarrow K^{0}\Lambda(\Sigma) \) occur through two phases: 1. Gribov’s diffusion of constituent quarks of each of interacting hadrons in the space of rapidities and impact parameters with production of a flux-tube which has a fast spectator on one edge and a slow reagent on the other. This phase determines a power decrease of the amplitude with energy increase. 2. The charge-exchange of slow reagents \( \bar{u}u \rightarrow \bar{d}d \); \( \bar{u}u \rightarrow s\bar{s} \) determines the value of the residue of the Regge pole. Reaction \( \pi^{-}p \rightarrow \pi^{0}n(\eta n) \) contains: the scalar bilinear \( (\bar{u}u)(\bar{d}d) = (3P_{0})^{2} \) determining the dominant spin-orbital amplitude \( M_{1}(3P_{0}) \) and the pseudoscalar bilinear \( (\bar{u}i\gamma_{5}u)(\bar{d}i\gamma_{5}d) = (1S_{0})^{2} \). In the amplitude \( M_{0}(3P_{0})^{2} \) and \( (1S_{0})^{2} \) interfere destructively and strongly. These facts which follow from the analysis of experimental data agree with the NJL model predictions. Spontaneous chiral symmetry violation leads to that the bilinear weights are independent of the coupling constant of NJL Lagrangian (the ”blackness” condition) and are determined by the transversal dimension \( R^{2} \) in the charge-exchange region \( \bar{u}u \rightarrow \bar{d}d \) entering the physical amplitude expression as a radius of the residue of the Regge pole. \( M_{1}(3P_{0}) \) at small \( q_{T}^{2} \) contains: the colour number \( N_{c} = 3 \); \( R^{2} \) and \( \alpha(0) \). The comparison with the experimental \( M_{1}(exper.) \) gives \( |M_{1}(3P_{0})|/|M_{1}(exper)| = 1.09(1 \pm 0.1); 0.7(1 \pm 0.2) \) for reaction \( \pi^{-}p \rightarrow \pi^{0}n; \eta n \), correspondingly within the interval \( s = 8 - 400 GeV^{2} \) and \( 0.004 \leq q_{T}^{2} \leq 0.1(GeV/c)^{2} \).

In the framework of the formally \( SU(3) \) symmetrical NJL model we discuss the reason of a strong \( SU(3) \) symmetry violation which manifests itself in the observable smallness of \( M_{1} \) in reactions \( \pi^{-}p \rightarrow K^{0}\Lambda(\Sigma) \).

1 Introduction
I will consider the charge-exchange reactions \( 0^{-\frac{1}{2}} \rightarrow 0^{-\frac{1}{2}} \) \( \pi^{-}p \rightarrow \pi^{0}n(1), \pi^{-}p \rightarrow \eta n(2), \pi^{-}p \rightarrow K^{0}\Lambda(\Sigma)(3) \) in the Regge region of \( s \) and \( t \). Figs.1 a,b and 2 show some
data on differential cross sections of these reactions [1,2,3]. The amplitudes $M$ and differential cross sections are expressed via elements $M_0$ (non-flip) and $M_1$ (spin-flip):

$$M = M_0 + i\vec{\sigma}\vec{n}M_1 \frac{q_\perp R}{2}; \quad 64\pi p_c^2 s \frac{d\sigma}{dq_\perp^2} = |M_0|^2 + |M_1|^2 \frac{q_\perp^2 R^2}{4}$$

(1)

where $\vec{\sigma}$ are Pauli matrices, $\vec{n}$ is an orth of normal of the reaction plane, $q_\perp$ is transverse momentum transfer, $p_c$ is momentum in the c.m.s, $R$ is the radius of the residue of Regge poles which dominates in the reactions.

Our choice is due to simple spin structure of these reactions and to presence of exact data on differential cross sections of reactions 1,2 within the interval $s = 8 - 400 GeV^2; \quad q_\perp^2 = 0.004 - 0.3(\frac{GeV}{c})^2 [1,2]$.

The simplest scheme of reactions 1,2,3 is presented by the dual diagram of Fig.3b, which is interpreted by A.B.Kaidalov in [4] as a three-stage process [5] of Fig.3c. In stage I choose configurations of initial hadrons with small rapidities of annihilating constituents. These configurations fuse into intermediate flux-tube (f.t) where the transition $\bar{u}u \rightarrow \bar{d}d$ or $\bar{u}u \rightarrow s\bar{s}$ (stage II) appear, after that f.t. disintegrates into two parts which evolve into finite hadrons (stage III) or continue disintegrating giving rise to a multiple process.

This picture is based on experimental facts:

1. Power decrease of cross-sections with $s$ increasing

$$\frac{d\sigma}{dt} ch.ex. \sim s^{2[\alpha_R(-t)-1]}; \quad \alpha_R(-t) < 1$$

(2)

where $\alpha_R$ – is trajectory of the secondary Regge pole. The power decrease of cross sections agrees with the parton concept providing the rapidities of annihilating constituents to be small and independent of the rapidity of initial hadrons [4,6].

2. Mass relations [4] which follow from factorization of the binary amplitude in $s$ channel are well fulfilled. Factorization in $s$ channel is a good argument in the favour of existence of an intermediate object – f.t.

3. The appearance of f.t. with increase of colour dipole moment of a pair $\bar{q}q$ static test quarks is confirmed in QCD on lattices [7].

The model [4] results in a correct Regge asymptotics of binary processes [8].

In the framework of the dual diagram of Fig.3b the spin structure and the value of residues of physical amplitude poles is determined by the spin structure of soft 4-quark interaction in a subprocess accompanied by a change of flavour $\bar{u}u \rightarrow \bar{d}d, \quad \bar{u}u \rightarrow s\bar{s}$ which holds in f.t. in the II stage and may be represented by five 4-fermionic bilinear amplitudes [6]

$$\langle f.t. | \sum_{n=1}^{5} T_n(\bar{q}_i q_i \rightarrow \bar{q}_f q_f) | f.t. \rangle$$

(3)

where symbol $\langle f.t. | T | f.t. \rangle$ implies the soft pair charge-exchange amplitude averaged over relative moment of $\bar{q}$ and $q$. The pair results (by analogy with [4]) from diffusion of partons of interacting hadrons in the space of rapidities and impact parameters $\vec{b}$ [9]. The weights and signs of bilinears $T_n$ are given by the effective chiral Nambu and Jona-Lasinio Lagrangian (the NJL model [10]), broadened by ’t Hooft [13] (see reviews [11], [12]).
In this paper I will show that such a construction well reproduces the experimental data of reactions 1 and 2.

Since in the NJL model, in the lowest in $1/N_c$ order ($N_c$ is the colour number) there are only bilinears of scalars ($s$) and pseudoscalars ($p.s.$) [12]:

\[
(2p_v\sqrt{s})^{-1} \Im M_b = \langle \mathrm{f.t.}|T(S) - T(P.S)|\mathrm{f.t.}\rangle = \\
\langle \mathrm{f.t.}|(\bar{q}_i q_i) D_s(\bar{q}_f q_f) - (\bar{q}_i i\gamma_5 q_i) D_{p.s.}(\bar{q}_f i\gamma_5 q_f)|\mathrm{f.t.}\rangle
\]

where $D$ are propagators of intermediate states. The sign $-$ in eq.(4) is determined by the sign in the 't Hooft determinant not conserving SU(3) singlet current ($\bar{u}u + \bar{d}d + \bar{s}s$).

The account of bilinear ($\bar{u}u)(\bar{d}d)$ in eqs.(3), (4) makes it possible to explain qualitatively a large spin-orbital effect in reactions 1,2, their planarity and smallness of spin-orbital interaction at the Pomeranchuk pole [6, 14]. Let us remind this explanation.

Fig.3d shows the $^3P_0 = O^+$ scheme of reaction $\pi^- p \rightarrow \pi^0 n$ on a "proton" consisting of a totally polarized along $+z$ axis soft $u$ quark and a leading spectator $ud$ with the impact parameter $+b_u$. The impact parameter of mesonic spectator $-b_d = 0$. The sign $\nearrow$ denotes the transversal spin of quark with $s_z = +\frac{1}{2}$. Z axis is directed along the normal of reaction plane. In $^3P_0$ state the transverse orbital moment $l_z = -1$ denoted as $\Leftarrow$ and the summary transversal spin $\bar{u}u/p$ $S_z = +1$ are antiparallel. That is why the quark charge-exchange, under a nontrivial condition of $S_z$ and $l_z$ conserving in $^3P_0 = 0^+$ state in the $\bar{u}u \rightarrow \bar{d}d$ process follows predominantly at $+b$, since at $-b$ annihilation in $^3P_0$ state is realized with a smaller probability ($l_z$ and $S_z$ are parallel). Thus, $M(+b) - M(-b) = M(-b) - M(+b) \neq 0$, i.e. $\bar{l}\bar{\sigma}$ interaction appears. In the third part of the paper I show that it is $T(S) = T(^3P_0)$ but not $T(^3P_2)$ which determines $M_1$ in reaction 1. At P pole effects due to soft quark annihilation are of no importance and spin-orbital interaction is small.

Large spin-orbital amplitude $M_1$ in reaction 1 is a result of $l_z$ and $S_z$ conservation in $^3P_0 = 0^+$ state in the $\bar{u}u \rightarrow \bar{d}d$ process. We deal with a "hidden orbital moment". Conservation of $l_z$ corresponds to the planary process $\bar{u}u \rightarrow \bar{d}d$ (Fig.4). Thus the presence of a dip in reaction is an experimental evidence for planarity and, consequently, for that dual diagram is appropriate to describe $\pi^- p \rightarrow \pi^0 n$. Some arguments according to which all the mentioned on reaction 1 is also valid for reaction 2, are presented in the third part of the paper.

The next discussion is planned as follows:

In the 2-d part we consider some consequences of extended NJL model for the $\bar{u}u \rightarrow \bar{d}d$ amplitude. One may expect them to reveal in the $M_{0,1}$ amplitudes in reactions 1 and 2. We note

1. Dominance of $S$ and $PS$ bilinears in (3).
2. Their strong destructive interference in $M_0$ provided that the intermediate particle mass in P.S. channel is small.
3. Independence of $\bar{u}u \rightarrow \bar{d}d$ soft amplitudes on the coupling constant of the NJL effective Lagrangian stemming from spontaneous violation of chiral symmetry. This fact is an argument in the favour of existence of the "blackness condition" (12), the physical meaning of which is that the soft $\bar{u}u \rightarrow \bar{d}d$ amplitude is determined by transversal dimension of $R$ region where the pair charge-exchange occurs, but not by the coupling constant. In the 4-th section we see that $R$ enters the expression for $M$ as a radius of residue of the Regge pole.
In the 3-d section we perform the analysis of experimental data based on the Fiertz expansion of the $t$ channel 4-quark amplitude in the dual diagrams of reactions 1 and 2 taking into account their exchange degeneration. This analysis will confirm the 1-st and the 2-d consequence of the NJL model for reactions 1 and 2 (strong destructive interference of $M_0(3P_0)$ and $M_0(1S_0)$ and, consequently, the smallness of the intermediate particle mass in P.S.channel and shows that in the framework of dual diagram $M_1 = M_1(3P_0)$. The experimental $|M_1|/|M_0| = 5; 10$ for reactions 1 and 2 , respectively, are compared with the estimate ($|M_1|/|M_0|)_s < 1$) in reactions $\pi^- p \rightarrow K^0 \Lambda(\Sigma)$ obtained on the basis of the differential cross sections data. It is stated that the dynamics of $\bar{u}u \rightarrow \bar{d}d$ and $\bar{u}u \rightarrow s\bar{s}$ is different. There is a strong SU(3) symmetry violation in $M_1$. Possible reasons for this phenomenon are discussed within the NJL model.

In the 4-th section the relation (3) is written in explicit form (29) basing on the unitarity condition of the f.t. model in $s$ channel. With the account of the ”blackness” condition (12) this enables one to calculate absolute values of the $M_1(3P_0)$ amplitudes which are dominant in reactions 1 and 2. The model expression $M_1(3P_0)$ (39) contains: the colour number $N_c = 3$, the radius of the residue of the Regge pole $R^2$ and $\alpha(-t)$. Since $R^2$ and $\alpha(-t)$ are determined from the energy dependence of the slope of reactions cone, the expressions for the absolute value $M_1(3P_0)$ has no free parameters $M_1(3P_0)$ is compared with the experimental values $M_1(\text{exper.})$ (24). As a result:

$$|M_1(3P_0)|/|M_1(\text{exper.})| = 1.09(1 \pm 0.1); \quad 0.7(1 \pm 0.2)$$

for reactions 1 and 2, correspondingly. This result describes the experimental data within the intervals: $0.004\left(\frac{GeV}{c}\right)^2 \leq q_1^2 < 0.1\left(\frac{GeV}{c}\right)^2$ and $s = 8 - 400GeV^2$.

We have refrained from estimating the $M_0$ amplitudes because of their theoretical ambiguity: $M_0 = M_0(3P_0) - M_0(1S_0)$ is a small difference of large values. The 5-th concluding part deals with some problems following from the results of our analysis.

2 Extended NJL Model and Soft $\bar{u}u \rightarrow \bar{d}d$ and $\bar{u}u \rightarrow s\bar{s}$ Amplitudes.

The minimal $U_L(1) \otimes U_R(1)$ symmetric (chiral) NJL Lagrangian of the $\bar{q}q$ light pair system with one flavour in Dirac vacuum has the form

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma\partial - m)q + G\left[(\bar{q}q)^2 + (\bar{q}\gamma_5q)^2\right]$$

(5)

where $G$ is the positive interaction constant (quarks undergo attraction), $m = 5MeV$ - is the current quark mass, $\gamma$ is the Dirac matrix, $q$ is the quark field operator. The form of this bare Lagrangian is determined by requirement of chiral invariance. The current quark mass is small as compared with the cut-off energy of nonrenormalizable NJL model $\Lambda \simeq 1GeV$. The value $\Lambda = 0.631GeV$ [12] is agreed with the low-energy data array.

The equality of interaction constants with which bilinears of scalar and pseudoscalar currents enter $\mathcal{L}_{NJL}$ is the fact following from requirement of chiral invariance. This equality takes place also for extended NJL model.
When $G$ increases up to a critical value $G_c \geq \pi^2/N_c A^2$ the current quark system becomes energy-unstable. The chiral symmetry spontaneously violates as a result of the instanton transition and phase transition into minimal energy state occurs. Here current quarks of each flavour acquire mass (in the extended NJL model due to scalar virtual $\bar{q}q$ pairs of three flavours) and go over to dynamical (constituent) quarks with the mass $M_c \simeq 0.3 GeV$ which are paired into scalar pairs with $J^P = 0^+$ producing a new ”QCD” vacuum of chiral condensates $< \bar{q}q >$ with the energy density $\epsilon < 0$.

An energetic gap $|\Delta E| \geq |\epsilon| + M_c$ appears between external constituent quarks and the new vacuum. Since the NJL model considers only fermionic degrees of freedom chiral condensate is a condensate of constituent $^3P_0$ pairs (by analogy with super-conductor).

Production of massive $^3P_0$ pairs in which transversal spin of a $\bar{q}q$ is compensated by transversal orbital moment, from light pairs of longitudinal current quarks with opposite moments and equal chiralities (bilinear scalar in (5)) is connected with quark spin flip described by the Bogolyubov-Valatin transformation and should be accompanied by production of pseudoscalar Goldstone bosons (bilinear pseudoscalar in (5)) which are associated with an octet of pseudoscalar mesons.

On the background of new vacuum (chiral condensate) external constituent quarks undergo residual interaction which may be represented, under mean field approximation, by the sum of two terms of the extended NJL model:

$$\mathcal{L}_{NJL}^{\text{res}} = \mathcal{L}_{\text{sym}}^{\text{res}} + \mathcal{L}_{\text{det}}^{\text{res}}$$

$$\mathcal{L}_{\text{det}}^{\text{res}} = G_{\text{IH}} \det \bar{q}_i (1 + \gamma_s) q_f + \det \bar{q}_i (1 - \gamma_s) q_f$$

(6)

The first term has the structure of (5), it does not mix flavours, if small effects due to current mass difference will be excluded. The determinant term constructed by ’t Hooft violates $U_A(1)$ symmetry, does not conserve singlet $SU_3$ current and consequently gives transitions $\bar{u}u \rightarrow \bar{d}d, \bar{u}u \rightarrow s\bar{s}$. In the case of three flavours $\mathcal{L}_{\text{det}}^{\text{res}}$ has six-fermion structure. $\mathcal{L}_{\text{det}}^{\text{res}}$ can be reduced to the effective 4-quark representation [15] which imitates instantonous transition (Fig.5).

$$\mathcal{L}_{\text{IH}}^{\text{res}}(4) = <\bar{q}q > K \sum_{n=0}^{9} \left[ (\bar{q}_i \lambda_n q_i) (\bar{q}_f \lambda_n q_f) - (\bar{q}_i i\gamma_5 \lambda_n q_i) (\bar{q}_f i\gamma_5 \lambda_n q_f) \right]$$

(7)

where $\lambda_0 = -\sqrt{2/3} \cdot I, \lambda_n (n = 1 - 8)$ are Gell-Mann matrices [16], (7) gives the sign $-$ in (4).

For the case of two flavours ($u$ and $d$) and up to $1/N_c^2 \mathcal{L}_{\text{IH}}^{\text{res}}(2)$ has the form [11, 12]

$$\mathcal{L}_{\text{IH}}^{\text{res}}(2) = G^\text{IH} \left\{ [ (\bar{q}_i q_i) (\bar{q}_f q_f) - (\bar{q}_i \tau q_i) (\bar{q}_f \tau q_f) ] - \\
- [ (\bar{q}_i i\gamma_5 q_i) (\bar{q}_f i\gamma_5 q_f) - (\bar{q}_i i\gamma_5 \tau q_i) (\bar{q}_f i\gamma_5 \tau q_f) ] \right\}$$

(8)

where $\tau$ are Pauli matrices.

The transition $\bar{u}u \rightarrow \bar{d}d$ corresponds to $180^0$ rotation of $\bar{u}$ and $u$ around the second axis in the isotopical spin space ($u \rightarrow -d$, $\bar{u} \rightarrow -\bar{d}$) [17]. Since at such rotation bilinears of scalars and pseudoscalars go over into each another we restrict ourselves when analysing reactions 1 and 2, to the term in (8) averaged over isospin

$$\mathcal{L}_{\text{IH}}^{\text{res}}(\bar{u}u \rightarrow \bar{d}d) = G [ (\bar{u}u)(\bar{d}d) - (\bar{u}i\gamma_5 u)(\bar{d}i\gamma_5 d) ]$$

(9)
$\bar{u}u(\bar{d}d)$ are superpositions of states with $I = 0, 1$ that is why in intermediate two-quark states we fix the third component of isospin $I_3 = 0$, but not $I$.

The extended NJL model does not consider the problem on conservation of $l_z, S_z$ in $(\bar{u}u)(\bar{d}d), (\bar{u}u)(ss)$.

In order that to turn from contact amplitudes to amplitudes (4) one should take into account propagators of scalar and pseudoscalar states $D_{S,PS}$. To this end the authors of [11,12] used random phase approximation [18]

$$D_{S,PS} = \frac{2iG}{1 - 2G\Pi_{S,PS}} \quad (10)$$

where $\Pi_{S,PS}$ are loop integrals depending on the total energy of charge-exchanged quarks $k^2 = 4m_q^2 + 4\vec{k}^2$, on the intermediate state mass $m_{S,PS}$ and $\Lambda$.

Note three, important for our following analysis, consequences of the NJL model:

1. It is shown in [11] (eqs.4.14 - 4.32) and furthermore (6.49) that

$$1 - 2G\Pi_{S,PS} = \frac{m}{m_q} + i2G(k^2 - m_{S,PS}^2)I'(k^2; m_q, \Lambda) \quad (11)$$

where $I'$ a is fluent, weakly depending on $k^2$ function. Substituting (11) into (10) we see that after spontaneous violation of chiral symmetry (i.e. at $G \geq G_c$, $\frac{m}{m_q} = \frac{m}{M_c} \simeq 0$, $D_{S,PS}$ stops depending on $G$ provided that $k^2 \neq m_{S,PS}^2$.

G-independence of $D_{S,PS}$ at $G \geq G_c$ suggests that as constituent quarks at the edges of two f.t. appear to be in $^3P_0$ or $^1S_0$ states they annihilate with a unity probability (two f.t. fuse into one with a unity probability):

$$\gamma^2 = \frac{\text{number of annihilations with production of general colour f.t.}}{\text{number of colour } ^3P_0( ^1S_0) \text{ pairs} \cdot c^{-1}} = 1 \quad (12)$$

We call the equality $\gamma^2 = 1$ as "blackness" condition. The "blackness" condition (12) will be used to estimate the absolute value of $M_1$ in the 4-th section. The "blackness" effect is a consequence of spontaneous violation of chiral symmetry. Note that G-independence of amplitudes at $G \geq G_c$ by no means implies that in $T(S)$ and $T(PS)$ (4) the values $G_S$ and $G_{PS}$ are arbitrary. In fact, $G_S = G_{PS} = G$ since (11) follows from (5).

2. In the NJL model $m_S^2 \equiv m_\sigma = 2M_c$ (see experimental confirmation in [25] and references therein. At $G \geq G_c$

$$k^2 = 4M_c^2 + 4\vec{k}^2. \quad (13)$$

Substituting $m_\sigma^2$ into (11) we get (accounting for (13) and (10)) $D_S = (\vec{k} \cdot I')^{-1}$. Thus, $D_S$ in (4) is determined by the 3-moment of charge-exchanged quarks, i.e. by the dimension of the region where charge exchange occurs. In the 4-th section we shall see that the transversal part of this region $R^2 = \langle k_1^2 \rangle^{-1}$ enters the expression for the amplitude $M$ as the radius of the residue of the Regge pole $M$ which dominates in the reaction.

3. Now we can write in (4):

$$T(S) - T(PS) \approx \frac{(\bar{u}u)(\bar{d}d)}{k^2 - m_\sigma^2} - \frac{(\bar{u}\gamma_5u)(\bar{d}\gamma_5d)}{k^2 - m_{PS}^2} = \quad$$

\footnote{The value $\gamma^2$ was formulated by the author together with L.B.Okun in [6].}
\[ \frac{(\bar{u}u)(\bar{d}d)}{4\vec{k}^2} = \frac{(\bar{u}i\gamma_5u)(\bar{d}i\gamma_5d)}{4M_c^2 + 4\vec{k}^2 - m_{PS}^2} \]  

(14)

Since \((\bar{u}u)(\bar{d}d) = 4\vec{k}^2; (\bar{u}i\gamma_5u)(\bar{d}i\gamma_5d) = 4M_c^2 + 4\vec{k}^2 \)

\[ T(S) - T(PS) \approx 1 - \frac{1}{1 - \frac{m_{PS}^2}{4M_c^2 + 4\vec{k}^2}} \]  

(15)

In the minimal NJL model \(m_{PS}^* = m_\pi\). The amplitude \(M_1\) contains only \(M_1^{(3P_0)}\), while \(M_0\) – the combination of \(M_0^{(3P_0)}\) and \(M_0^{(1S_0)}\) (see the next section). Because of (15) the NJL model predicts a strong destructive interference of \(M_0^{(3P_0)}\) and \(M_0^{(1S_0)}\) in \(M_0\).

3 Analysis of experimental data of reactions \(\pi^- p \rightarrow \pi^0 n; \pi^- p \rightarrow \eta n; \pi^- p \rightarrow K^0 \Lambda(\Sigma)\). 

1. \(\pi^- p \rightarrow \pi^0 n, \eta n\).

In papers of the High Energy Physics Institute -CERN [1,2] the world data on differential cross sections of reactions are well parametrized in the region \(s = 8 - 400 GeV^2\) and \(q_{\bot}^2 = 0.004 - 0.3(GeV/c)^2\) as

\[ \frac{d\sigma}{dq_{\bot}^2} = \frac{d\sigma}{dq_{\bot}^2} |_{q_{\bot} = 0}(1 + aq_{\bot}^2)exp[-2q_{\bot}^2(R^2 + \xi\alpha')] \]  

(16)

\[ (d\sigma/dq_{\bot}^2)_{q_{\bot} = 0} = A \exp(-2\beta\xi); \quad \xi = \ln\frac{s}{s_0}; \quad s_0 = 1 GeV^2 \simeq (\alpha')^{-1} \]

The parameters of (16) are listed in Table 1.

We assume that

\[ a = \frac{|M_1|^2}{|M_0|^2} \frac{R^2}{4} \]  

(17)

Whence

\[ |M_1| / |M_0| \equiv \nu = 5.4; 10.3 \]  

(18)

For reactions 1 and 2, correspondingly. The assumption (17) is confirmed by the following data:

1. A direct amplitude analysis of \(\pi N \rightarrow \pi N\) including data on spin flip at \(s \approx 12 GeV^2\) in the region \(q_{\bot}^2 \leq 0.5(GeV/c)^2\) gives for the charge exchange amplitude \(M_1/M_0 = -8\) in accordance with the \(M_{0,1}\) calculations on the basis of dispersion sum rules at finite energy [19,20].

2. The phase analysis \(\pi N \rightarrow \pi N\) at \(P_{lab} = 2 GeV/c\) gives for the charge exchange amplitude \(M_1/M_0 = -4 \div 5\) [21]. \(R^2\) at the points 1 and 2 is taken from the data on \(\pi^- p \rightarrow \pi^0 n\) at \(s = 8 - 400 GeV^2\).

3. Polarization in the region \(q_{\bot}^2 \leq 0.3(GeV/c)^2\) and \(s = 80 GeV^2\) is small [22], so that the relative phase of \(M_0\) and \(M_1\) is \(|\varphi_0 - \varphi_1| \leq 4.10^{-2}\) for both reactions. In what follows we will neglect \(|\varphi_0 - \varphi_1|\) and will consider \(M_0/M_1\) to a real quantity.
Table 1: The list of parameters (16), describing differential cross sections of reactions 1 and 2 within the interval $s = 8 - 400 GeV^2, q^2_\perp = 0.004 - 0.3 (GeV/c)^2$ [1,2 Apel V.D.], $r^* = [1 + \rho^2(0)]^{1/2}$.

|      | $R^2$ (GeV/c)$^{-2}$ | $a$ (GeV/c)$^{-2}$ | $A$ (GeV)$^{-4}$ | $a'$ (GeV/c)$^{-2}$ | $\beta = 1 - \alpha(0)$ | $r^*$   |
|------|---------------------|--------------------|------------------|---------------------|------------------------|--------|
| $\pi^0 n$ | 4.5 (1 ± 0.03) | 33 (1 ± 0.05) | 12 (1 ± 0.05) | 0.8 (1 ± 0.06) | 0.52 (1 ± 0.02) | 1.37   |
| $\eta n$  | 1.16 (1 ± 0.09) | 31 (1 ± 0.2) | 1.49 (1 ± 0.1) | 0.8 (1 ± 0.06) | 0.603 (1 ± 0.03) | 1.70   |

2 Parameters are taken from [1] Apel V.D. et al. Yad.Fiz. V.30, p.373 and [2] Apel V.D. et al. Yad.Fiz. 1979, V.29, p.1519. The parameters $R^2$ and $A$ depend on the value of dimensional factor $s_0$ in $\xi$. In the papers by Apel et al. it was adopted $s_0 = 10 GeV^2$. In this paper $s_0 = 1 GeV^2 \simeq (a')^{-1}$. Parameters $R$ (Apel) and $A$ (Apel) were recalculated taking into account $s_0 = 1 GeV$. When recalculating $R^2$, the absolute values of errors were conserved in $R^2$ (Apel) and in $a'$. When recalculating $A$ relative errors $A$ (Apel) were conserved. In papers by V.D.Apel et al. $(a)$ were parametrized by the functions $a_1 = [12.7(1±0.024) + 1.57(1±0.076) \ln \frac{s}{s_0}]; (2.55(1±0.035) - 0.23(1±0.26) \ln \frac{s}{s_0}]$; $a_2 = [6.0(1±0.03) + 1.6(1±0.06) \ln \frac{s}{s_0}] \times [4.6(1±0.07) - 0.5(1±0.4) \ln \frac{s}{s_0}]$. $a_{1,2}$ weakly depend on $s$ in the interval $s = 8 - 400 GeV^2$.

Table 1 shows means arithmetics $a_{1,2}$ taken at $s = 10, 100, 400 GeV^2$.

4. (16) coincides with the Regge expression for reactions $0^- \rightarrow 0^- \frac{1}{2}$ with one pole in $t$ channel and equal residue radius $R$ in $M_0$ and $M_1$.

5. In the next section eq.(17) will be quantitatively confirmed.

If we restrict ourselves in $s$ channel of the dual diagram of Fig.3b to $S$ and $P$ combinations of charged exchanged quarks (minimum sufficient for self-consistent description of reactions 1 and 2), then the subprocess $\bar{u}u \rightarrow dd$ may involve the following states:

$$S(\text{scalar}) \quad V \quad T \quad A \quad P$$

$^3P_0 \quad ^3S_1, \quad ^1P_1 \quad ^3P_2 \quad ^3P_1 \quad ^1S_0$

Only $^3P_0$ and $^3P_2$ may contribute to $M_1$ since spin-orbital amplitude in the subprocess appears if there is a difference in cross sections in pair annihilation with $l_z = \mp 1$ at $S_z = +1$. Sign inversion in $l_z$ transfers $^3P_0(^3P_2)$ into another state $-^3P_2(^3P_0)$. In the rest cases there are no changes, as either $l_z = 0$ or $S_z = 0$.

$M_0$ can be, in principle, contributed by all 6 states.

In the case of the $^3P_0$ or $^3P_2$ dominance and at $q^2 R^2 \ll 1 \left| \frac{M_2(^3P_0)}{M_0(^3P_0)} \right| = 2$ [6], see also (38). Hence it follows that the experimental values $\nu = 5; 10$ (18) result from destructive interference of $M_0(^3P_0,2)$ with other states [6]:

$$\nu = \left| \frac{M_1}{M_0} \right| = \frac{M_1(^3P_0,2)}{M_0(^3P_0,2)} = \frac{2 \left| M_0(^3P_0,2) \right|}{\left| M_0(^3P_0,2) - M_0(x) \right|},$$

(19)
Table 2: Fiertz expansion of the soft $t$-channel amplitude $\bar{u}d \to \bar{u}d$ over $s$-channel amplitudes $\bar{u}u \to \bar{d}d$. Only bilinear notations are presented.

$$
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{t channel} & \text{s channel} \\
\bar{u}d \to \bar{u}d & \bar{u}u \to \bar{d}d \\
\hline
\rho \text{ exchange } (^3S_1)^2 & (^3P_0)^2 & (^3S_1)^2 & (^3P_1)^2 & (^1S_0)^2 \\
A_2 \text{ exchange } (^3P_2)^2 & (^3P_0)^2 & (^3P_1)^2 & (^1S_0)^2 \\
\hline
\end{array}
$$

Table 3: Relative contributions $|M_0(^3P_0)|$ and $|M_0(^3S_1)|$ in reactions 1 and 2.

$$
\begin{array}{|c|c|c|c|c|c|}
\hline
\nu & |M_1| & |M_0| & \chi(^3P_0) & \chi(^1S_0) & \text{interference} \\
\hline
\pi^- p \to \pi^0 n & 5.4 & 0.6 & 0.4 & \text{destructive} \\
\pi^- p \to \eta n & 10.3 & 0.55 & 0.45 & \text{destructive} \\
\hline
\end{array}
$$

where $x = ^3S_1, ^3P_1, ^3P_2$ and $^1S_0$.

In Table 2 we present the Fiertz expansion of the soft "t"-channel vector ($\rho$-exchange in reaction 1) and tensor ($A_2$ exchange in reaction 2) 4-quark amplitude $\bar{u}d \to \bar{u}d (\bar{q}_1q_3 \to \bar{q}_2q_4$ in Fig.3b) over $s$-channel amplitudes $\bar{u}u \to \bar{d}d$ of interest.

"s"-channel spin structures of the amplitudes $\bar{u}u \to \bar{d}d$ with the opposite "t" channel signature ($\rho$ and $A_2$) differ by their "u" channel singularities. But in "u" channel ($\bar{u}d \to \bar{u}d$) repulsion forces predominate and there are no physical poles; for this reason "u" channel gives no large contribution into "s" channel that is confirmed by exchange degeneration of $\rho$ and $A_2$ trajectories. This argumentation allows one to narrow the set of "s" channel bilinears of Table 2 in reactions 1 and 2 up to two general $(^3P_0)^2 = (\bar{q}q)^2$ and $(^1S_0)^2 = (\bar{q}i\gamma_5q)^2$. Thus, reactions 1 and 2 involve the same bilinear forms as in the minimal NJL model. Now eq.(19) is open:

$$
\nu = \frac{2|M_0(^3P_0)|}{|M_0(^3P_0) - M_0(^1S_0)|}
$$

From (20) we determine the relative contributions

$$
M_0(^3P_0) \quad \text{and} \quad M_0(^1S_0): \quad \chi(^3P_0) = |M_0(^3P_0)| / |M_0(^3P_0) + M_0(^1S_0)|; \quad \chi(^1S_0) = 1 - \chi(^3P_0)
$$

$$
\chi(^3P_0) = \frac{\nu}{2(\nu - 1)}
$$

see Table 3.

The data of Table 3 agree with predictions of the extended NJL model – destructive interference of $S$ and $PS$ bilinears in $M_0$ and their close absolute values (relic of equality $G_S = G_{PS} = G$ in the bare Lagrangian of NJL (5).
II. Reactions with strangeness production of $\pi^- p \rightarrow K^o \Lambda(\Sigma)$.

Besides the differential cross sections data of Fig.3, there are the data [3] (B.Foley et al.) in the region $s = 16 - 30 GeV^2$ and $q_\perp^2 = 0 \div 1\,(GeV/c)^2$ where spin-orbital dip is also invisible, but the errors in $(d\sigma/dq_\perp^2)_{q_\perp = 0}$ are presented and $\alpha'_s$ is estimated. From these data one can estimate the upper limit for the value $\left(\frac{|M_1|}{|M_0|}\right)_s$ according to formula

$$\frac{d\sigma}{dq_\perp^2} = \left(\frac{d\sigma}{dq_\perp^2}\right)_{q_\perp = 0} (1 + \delta) \exp[-2q_\perp^2(R^2 + \alpha'_s \xi)] =$$

$$= \left(\frac{d\sigma}{dq_\perp^2}\right)_{q_\perp = 0} (1 + \left(\frac{|M_1|}{|M_0|}\right)^2 \frac{R^2}{4} \Delta q_\perp^2) \exp[-2q_\perp^2(R^2 + \alpha'_s \xi)]$$

(22)

where $\delta = 0.022$ is an error in $d\sigma/dq_\perp^2$ at the minimal $q_\perp^2$ within the minimal interval $\Delta q_\perp^2 = 0.05\,(GeV/c)^2$, $R^2 = 2.8\,(GeV/c)^2$ and $\alpha'_s = 0.43 \pm 0.36\,(GeV/c)^2$. As a result:

$$\left(\frac{|M_1|}{|M_0|}\right)_s \leq 1.$$  

(23)

$\mathcal{L}^{\prime}_{ij}\,L^{\prime}_{ij}$ (4) (7) is formally $SU(3)$ symmetrical. What is the reason for strong $SU(3)$ violation in $M_1$ at production of strangeness? A possible answer is that in the instanton transition $\bar{u}u \rightarrow s\bar{s}$ [23], the NJL model does not take into account the $l_z$ non conservation in $(\bar{u}u)(s\bar{s})$. In this case $(\bar{u}u)(s\bar{s}) = (3P_0)^2$ does not contribute to the amplitude $M$ in "q" representation (see the next section).

4 Calculation of the absolute value of $M_1$ in reactions 1,2. Comparison with experimental data.

Thus, $M_0 = M_0(3P_0) - M_0(1S_0)$ is a small difference of large quantities and, consequently, is theoretically unstable. The instability is, due, in particular, to a different contribution of Regge cuts $R \otimes P$ in $M_0(3P_0)$ and $M_0(1S_0)$ [14]. While $M_0(3P_0)$ does not practically contain $R \otimes P$ [24], the contribution from $R \otimes P$ into $M_0(1S_0)$ diminishes it by $\approx 20\%$ [14]. That is why we restrict ourselves to calculation of the dominant $M_1$, which within the model equals to $M_1(3P_0)$.

We compare $|M_1(3P_0)|$ with $|M_1(\text{exper.})|$. From (1), (16), (17)

$$|M_1(\text{exper.})| = |M_1(\text{exper.})|_{q_\perp = 0} \cdot \exp[-q_\perp^2(R^2 + \alpha'_s \xi)]$$

(24)

$$|M_1(\text{exper.})|_{q_\perp = 0} = 8\sqrt{\pi} p_c \sqrt{s} \frac{2\sqrt{a}}{R} \cdot A^{1/2} \exp(-\beta_\xi)$$

(25)

(the parameters (24) and (25) in Table 1).

To calculate $|M_1(3P_0)|$ we need the unitarity conditions of f.t. model of binary processes. Multiple processes do not enter the unitarity conditions of f.t. model since they occur after the first disruption of f.t. and causally is almost independent of the binary process. The

3 The part of $M_1$ in the total cross sections of reactions 1 and 2 comprises $\approx 90\%$ at $P_{lab} \approx 10\,GeV/c$
intermediate stage contain only single f.t. which may differ only by transformation properties of the first disruption mode. The unitarity conditions in s channel in ”b” representation have the form [6]

$$\text{Im } M_b(ab \rightarrow cd) = 2p_c \sqrt{s} \sum_{i=1}^{5} M_b(ab \rightarrow \text{f.t.}_i)M_b^*(\text{f.t.}_i \rightarrow cd)$$  \hspace{1cm} (26)$$

where

$$|M_b(ab \rightarrow \text{f.t.})|^2 = W_b(ab \rightarrow \text{f.t.}) = N_c W_b[(ab)_c \rightarrow \text{f.t.}_c]$$  \hspace{1cm} (27)$$

\(W_b[(ab)_c \rightarrow \text{f.t.}_c]\) is the probability of production of a coloured f.t. at annihilation of a coloured \(\bar{q}q\) pair. Since we estimate only \(3P_0\) mode of disruption, index \(i\) in (27) and hereafter is omitted.

Up to exchange degeneration of reactions \(\pi^- p \rightarrow \text{f.t.} \rightarrow \pi^- p\) and \(\pi^0 n \rightarrow \text{f.t.} \rightarrow \pi^0 n\) where only contributions from the \(\rho\) and \(f\) pole, respectively, are taken into account

$$\left(2p_c \sqrt{s}\right)^{-1}|\text{Im } M_b(\pi^- p \rightarrow \pi^0 n)| = \left[|M_\rho(\pi^- p \rightarrow \text{f.t.})|^2|M_f(\pi^0 n \rightarrow \text{f.t.})|^2\right]^{1/2} =$$

$$= |M_\rho(\pi^- p \rightarrow \text{f.t.})|^2$$  \hspace{1cm} (28)$$

From (28) and (27)

$$\left(2p_c \sqrt{s}\right)^{-1}|\text{Im } M_b(\pi^- p \rightarrow \pi^0 n)| = N_c W(b, y_d - y_{ud}) =$$

$$N_c \int d^2(b_u - b_{ud})d^2(b_d - b_u)\omega(-y_d, b_d - b_u)W(b_u - b_u)\omega(y_{ud}, b_{ud} - b_u) \equiv$$

$$\equiv <\text{f.t.}|T(3P_0)|\text{f.t.}>$$  \hspace{1cm} (29)$$

where \(W(b, y_d - y_{ud})\) is a dimensionless probability of production of a coloured f.t. with coloured quark and diquark at the edges with the rapidity difference \(y_d - y_{ud} = \xi\) and with projection \(b = b_d - b_{ud}\) onto the plane of the impact parameters \(y, z\) (Fig.3d);

$$\omega(y_i, b_i - b_k) = \beta(4\pi\alpha' y_i)^{-1}\exp[\beta y_i - (b_i - b_k)^2(4\alpha'y_i)^{-1}]$$  \hspace{1cm} (30)$$

$$\beta = 1 - \alpha_\rho(0)$$

is a normalized [6] density of a probability to find in the impact parameter plane a parton at the point \(b_k\) with rapidity \(\sim 0\) if it comes out from the point \(b_i\) with rapidity \(y_i\) [9].

The dimensionless probability of production of a unite coloured f.t. at annihilation of a coloured pair from \(3P_0\) state:

$$W(b_u - b_{ud}) \approx |\int d^2k_\bot dk_\parallel \exp[-2R^2(k_\parallel^2 + k_\bot^2)]j_\Sigma^\Sigma(k)\exp\left[-ik(b_u - b_u)\right]|^2$$  \hspace{1cm} (31)$$
is the square of the Fourier-image of scalar current with transformation properties $^3P_0 = J^P = O^+$. In the given model the current $J^S_S$ corresponds to annihilation (production) of a constituent pair of converging (diverging) waves with moments $k$ and $-k$ and with unit summary spin and orbital moment with conserved $S_Z = +1$ and $l_Z = -1$, correspondingly:

$$j^S_S = (8\pi)^{1/2} \sum_{m=0,1} C(110; m, -m, 0) \times C(1/2, 1/2, 1; m_1, m_2, m)Y_{1,-m}(k)\chi_{m_1}\chi_{m_2} = 2[i(k_x - ik_y)\chi_{1+1} - ik_z\chi_{10}]$$

where $\chi_{1,m}$ are spin functions of a pair. The parameter $R^2$ determines the scale of annihilating quark momenta. It enters the final expression of the amplitude as the radius of the Regge pole residue. Integration over $k_\parallel = k_x$ is performed within $-\infty < k_\parallel < 0$ (converging wave) Integration over $dk_\parallel$ and $d^2k_\perp$ and squaring results in:

$$W(b_u - b_\alpha) = W(b') = \gamma^2 \cdot 0.464(1 + 1.25|b'|R^{-1}\cos \varphi_{\perp} + 0.39b'^2R^{-2})exp\left(-\frac{b'^2}{4R^2}\right) = \gamma^2 \cdot F(b')$$

where $\varphi_{\perp}$ is counted from the axis $Y$. $F(b')$ - the $b'$-dependence of the pair production probability in $^3P_0$ state is determined such that $0 \leq F \leq 1$. $\gamma^2 = 1$ in accordance with the blackness condition (12).

The scalar current $j^S_{nc}$ with transformation properties $J^P = ^3P_0 = 0^+$ but not conserving $l_z, S_Z$ gives no contribution into amplitude $M(q^2_\perp)$ (1) since in

$$[(q_1q_i)(q_fq_f)]^nc \approx \left(Y_{1-1}\chi_{1+1} + Y_{10}\chi_{10}\right)\left(Y^*_{1+1}\chi_{1-1} + Y_{10}\chi_{10}\right)$$

$Y_{1-1}Y_{1+1}\chi_{1+1}\chi_{1-1} = 0$ by virtue of orthogonality, while $(\chi_{10}\chi_{10})^2$ is out when going over into "$q^\perp_\perp$" representation [14].

After substituting (30) and (33) the convolution (29) is integrated. As a result we obtain the one-pole charge-exchange amplitude where only $T(S)$ in (4) is taken into account:

$$(2p_c)\sqrt{s}^{-1} ImM_b(^3P_0) = \frac{G'}{C'}\left(1 + 1.56\frac{C' - R^2}{C'} + 0.39\frac{R^2b'^2}{C'^2} + 1.25\frac{|b|R}{C'} \cdot \cos \varphi_b\right)exp\left(-\frac{b'^2}{4C'^2}\right)$$

where $P_c|b|\cos \varphi_b = -1\sigma, C' = R^2 + \alpha\xi$; $G' = 0.464N_c\beta^2R^2\exp(-\beta\xi)$; at $\xi = 0$ (35) goes over into (33) up to a coefficient $N_c\beta^2R^2$.

In "$q$" representation ($ImM(q^2_\perp) = \int ImM_b exp(-iqb)d^2b$)

$$ImM(^3P_0) = Im M_0 + i\tilde{\sigma}n \int ImM_1 \frac{q^2_\perp R}{2}$$

\footnote{In [6] (expression 20) the author had obtained the correct value $\gamma^2 \approx 1 (\gamma^2 = \gamma^2$ of this paper). Unfortunately, $\gamma^2 \approx 1$ was obtained as a result of a random compensation of two errors: the numerator in (20) [6] did not take into account the factor $N_c = 3$ in front of $\gamma^2$. The denominator in (20) [6] used $\sqrt{A(Apel)}$ instead of $\sqrt{A}$ from Table 1 of the given work (see footnote 2) and, consequently, factor 3.3 and 4 for reactions 1 and 2, respectively, was not taken into account}
\[ Im \ I \!M_0(\beta P_0) = +G \cdot exp(-C) \]  
\[ Im \ I\!M_1(\beta P_0) = -1.95 \cdot G \cdot exp(-C) \]

where \( G = -2p_c \sqrt{s} \cdot 2\pi R^2 N_c \beta^2 \cdot 0.464 \cdot 5.12 \) at \( q_\perp^2 R^2 \ll 1; \)
\( C = q_\perp^2 (R^2 + \alpha' \xi^2) + \beta \xi. \)

For the choice of signs of \( Im \ I\!M_{01} \) and \( G \) see [6].

Thus:
\[ |M_1(\beta P_0)| = [1 + \rho^2(q_\perp)]^{1/2} |Im \ I\!M_1(\beta P_0)| = [1 + \rho^2(q_\perp)]^{1/2} \cdot 2\pi N_c R^2 \beta^2 \times \]
\[ \times 1.95 \cdot 0.464 \cdot 5.12 \cdot 2p_c \sqrt{s} \cdot exp(-C) \]

where \( \rho(q_\perp) = ctg \frac{\pi}{2} [\alpha_p(0) + \alpha' q_\perp^2]; \ cttg \frac{\pi}{2} [\alpha_A(0) + \alpha' q_\perp^2] \) for reactions 1 and 2, respectively.

We get from (39) and (24):
\[ \frac{|M_1(\beta P_0)|}{|M_1(exper.)|} = \frac{[1 + \rho^2(q_\perp)]^{1/2} \cdot 2\pi N_c R^2 \beta^2 \cdot 4.63 \cdot 2p_c \sqrt{s} \cdot exp(-C)}{8R^{-1} \sqrt{\pi aA} \cdot 2p_c \sqrt{s} \cdot exp(-C)} \]

Substituting into eq.(40) the parameters \( R, a, A, \beta, [1 + \rho^2(0)]^{1/2} \) from Table 1 we have
\[ \frac{|M_1(\beta P_0)|}{|M_1(exper.)|} = 1.09(1 \pm 0.1); \ 0.7(1 \pm 0.2) \]

for reactions \( \pi^−p \rightarrow \pi^0n \) and \( \pi^−p \rightarrow \eta n \), correspondingly. From (41) it follows, that (39) describes, within the errors of existing experiments, \( |M_1(exper.)| \) in reactions 1 and 2 in the interval, \( s = 8 - 400 GeV^2 \) and \( 0.004 \leq q_\perp^2 \leq 0.1(GeV^2)^2. \)

5 Conclusion

Charge-exchange reactions \( \pi^−p \rightarrow \pi^0n, \ \eta n; \ \pi^−p \rightarrow K^0\Lambda(\Sigma) \) occur through two phases.

1. Diffusion of constituent quarks of each interacting hadrons in the space of rapidities and impact parameters involving flux-tube production, one edge of this f.t. having fast spectator and another – slow reagent. Confinement forces (\( \alpha' \) in the diffusion equation (31)) are explicitly taken into account in this phase; it determines the Regge power decrease of the amplitude with energy increase.

2. Charge-exchange of slow reagents: \( \bar{u}u \rightarrow \bar{d}d, \ \bar{d}d \rightarrow s\bar{s}. \) This phase determines the spin structure and the absolute value of the Regge pole residue.

The spin-orbital amplitude \( M_1 \), caused by spin-orbital interaction (SOI) in the process \( \bar{u}u \rightarrow \bar{d}d \), dominates in reactions \( \pi^−p \rightarrow \pi^0n, \ \eta n \) (more than %90 of cross section of the reaction). SOI stems from quark charge-exchange in the state \( J^z = 0^+ = \bar{3} \) \( P_0 \) provided \( l_z \) conservation in this process. Consequently, the process \( \bar{u}u \rightarrow \bar{d}d \) is planary (Fig.4a). This fact is an argument in the favour of that to consider the simplest planary dual diagram of Fig.3b as an experimentally substantiated model of reactions.

The NJL model gives certain predictions on relations between different 4-quark invariants in \( \bar{u}u \rightarrow \bar{d}d \) and, consequently, in charge-exchange physical amplitudes. In the minimal
NJL model only bilinears of scalars \((^3P_0)^2\) and of pseudoscalars \((^1S_0)^2\) are effective. They enter the physical non-flip amplitude \(M_0\) with the opposite signs and with approximately equal weights (strong destructive interference). The weights are independent of the coupling constant of NJL Lagrangian (the "blackness" condition \((12)\) and are determined by the dimensions of the region of charge exchange \(\bar{u}u \rightarrow \bar{d}d\). The transversal radius of this region \(R^2\) enters the finite expression of the physical amplitude as a radius of the Regge pole residue and determines its absolute value. These predictions are direct consequences of dynamical violation of the chiral invariance and of non conservation of the \(SU(3)\) singlet current , underlying the extended NJL model.

Luckily, the most completely experimentally studied reactions \(\pi^-p \rightarrow \pi^0n(\eta n)\) contain the same bilinears \((^3P_0)^2\) and \((^1S_0)^2\) as the minimal NJL model. This conclusion resulted from comparison of the Fiertz expansion of the t-channel soft 4-fermion amplitude in reactions \(\pi^-p \rightarrow \pi^0n(\eta n)\) with the account of their exchange degeneration. The analysis of experimental data had confirmed the consequences of the NJL model - the strong destructive interference \(^3P_0\) and \(^1S_0\) in \(M_0\). We have refrained from estimating the absolute value of \(M_0\) in view of its theoretical instability (a small difference of large values).

The absolute value of \(M_1\) \((^3P_0)\) contain; \(R^2\), the colour number \(N_c = 3\) and \(\beta^2 = (1 - \alpha_R(0))^2\) and, consequently, there are no free parameters. The comparison with the experimental \(M_1(exper.)\) yields:

\[
|M_1(^3P_0)|/|M_1(exper.)| = 1.09(1 \pm 0.1); \quad 0.7(1 \pm 0.2)
\]

for reactions \(\pi^-p \rightarrow \pi^0n(\eta n)\), correspondingly, within the interval \(s = 8 - 400 GeV^2\) and \(0.004 \lesssim q_\perp^2 \lesssim 0.1(\frac{GeV}{c})^2\).

In reactions \(\pi^-p \rightarrow K^0\Lambda(\Sigma)\) SOI are at least by the order of magnitude smaller than in \(\pi^-p \rightarrow \pi^0n(\eta n)\). The transition \(\bar{u}u \rightarrow s\bar{s}\) is usually considered as an instanton one. On the other hand, in the \(SU(3)\) symmetrical model NJL, the process \(\bar{u}u \rightarrow s\bar{s}\) also contains the scalar bilinear. The absence of SOI in \(\pi^-p \rightarrow K^0\Lambda(\Sigma)\) may be, perhaps explained as follows: \(l_z\) is not conserved in the instanton transition \(\bar{u}u \rightarrow s\bar{s}\) in \(^3P_0\) state. In this case \((\bar{u}u)(s\bar{s})\) gives no contribution into physical amplitude \(M\) (see the 4-th section). Thus, a strong \(SU(3)\) violation appears in the formal \(SU(3)\) symmetrical NJL model.

A question arises why in the scalar channel of the soft instanton transition \(\bar{u}u \rightarrow \bar{d}d\) \(l_z\) is conserved, while in \(\bar{u}u \rightarrow s\bar{s}\) it is not?

Reactions \(\pi^-p \rightarrow K\Lambda\nu(\Sigma\nu)\) are very interesting objects for studying transitions \(\bar{u}u \rightarrow s\bar{s}\) since the non conserving parity decays \(\Lambda\nu(\Sigma\nu) \rightarrow \pi^-p\) provide an experimental possibility for the total and exact amplitude analysis of these reactions in the whole region of \(q_\perp\).

In this work we based on: 1. The parton concept of binary processes. 2. Non conservation of the \(SU(3)\) singlet current (gluon anomaly consequence). 3. The "blackness" condition which follows from dynamical violation of chiral symmetry. In view of a general character of these elements, the above mentioned give, as it seems to us, grounds to believe soft binary processes as a good laboratory of binary quark reactions in the nonperturbative region of QCD.

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Fig. 1. a) Differential cross sections of the reaction $\pi^- p \rightarrow \eta n$ at small $q_L^2$. The data [1] V.D. Apel et al. The solid curve (16).
b) Differential cross sections of the reaction $\pi^- p \rightarrow p^0 n$ at small $q_L^2$. The data [1] V.D. Apel et al. The solid curve (16).

Fig. 2. Differential cross sections $\pi^- p \rightarrow K^0 \Lambda^0$ - black circles and $\pi^- p \rightarrow K^0 \sigma^0$ - white circles.

Fig. 3. Space-time scheme of the reaction $\pi^- p \rightarrow p^0 n$ in $^3P_0$ model with flux-tube. The arrow $\nearrow$ corresponds to the spin direction along axis $+Z$.

Fig. 4. $^3P_0$ transition $\bar{u}u \rightarrow \bar{d}d$ for the case of conservation (a), nonconservation (b) of $l_Z, S_Z$. The straight-line arrows label 3-momenta of quarks.

Fig. 5. The "instanton" transition $\bar{u}u \rightarrow s\bar{s}$.
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