MULTI-AIRCRAFT COOPERATIVE PATH PLANNING FOR MANEUVERING TARGET DETECTION

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(Communicated by Bin Li)

ABSTRACT. Multi-aircraft cooperative path planning is a key problem in modern and future air combat scenario. In this paper, this problem is studied in aspect of airborne radar detection to maintain a continuous tracking of a maneuvering air target. Firstly, the objective function is established in combination with multiple constraints considered, including Doppler blind zone constraint, radar viewing aspect constraint, baseline constraint, and so on. Then, the above optimal control problem is transformed into a nonlinear programming problem with a series of algebraic constraints by hp-adaptive Gauss pseudospectral method (HPAGPM). And it is solved by GPOPS software package based on MATLAB. Simulation results show that the optimized cooperative paths can be got to achieve continuous tracking of maneuvering air target by HPAGPM.

1. Introduction. With the air-to-air missile attack capabilities and the airborne radar detection range improvement, beyond-visual-range (BVR) air combat has become a typical combat mode. First detection, first shot and first hit are the keys to win the BVR air combat. What’s more, air combat is no longer confrontations between individual combat units and operational elements, but rather system to system confrontations in nowadays. Therefore, multi-aircraft cooperative combat [8] has been identified as a typical combat scenario in future. Cooperative path planning, as a key technology of multi-aircraft cooperative confrontation, is a vital requirement for fast detection, stable tracking and precision strike in cooperative air combat.

In the field of multi-aircraft path planning, most results are focused on UAVs and missiles (ground-to-ground, land-to-air, air-to-ground and various cruising). For UAVs trajectory optimization, the research is mainly concerned about the cooperative coverage search [27, 32, 21] and how to avoid multiple threats (such as missiles, radar network and terrain) [20, 22, 25]. With regard to cooperative attack of multiple missiles, generalizing a cooperative path to satisfy the impact time

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and impact angle constraints is the mainly object for the multi-missile trajectory optimization [16, 17, 4]. To sum up, the above mentioned path planning problem can generally be described as the following: given a starting point and a target point, together with several threatening areas, it is required to find a feasible path from the starting point to the ending point. Moreover, the above results are all aimed at ground targets and sea targets which are always fixed or slowly moving. Nevertheless, there are very few reports on path planning in air combat scenario wherein the targets possess large maneuverability and fast moving ability. Thus, the problem of multi-aircraft cooperative path planning in air combat remains open. Note that the air target detection, tracking and midcourse guidance are all based on airborne radar detection information. The pulse Doppler system is most widely adopted in the advanced airborne radar which has an inherent shortcoming of blind zone [19, 7] for low-velocity target detection. In other words, if a target gets into the blind zone of airborne radar via some maneuvers, the target will be lost. However, via cooperative path planning, the multi-aircraft can perform the target detection from multiple perspectives and different times in sequence. This manner can achieve continuous tracking of the maneuverable target.

Up to now, many path planning algorithms have been introduced to identify an optimum trajectory [29]. They are generally divided into dynamic programming, indirect, and direct algorithms. Dynamic programming is a mathematical method to resolve multistage decision process optimization problem. In [3] and [23], the dynamic programming algorithm has been applied to design path planning algorithm. Theoretically, the indirect method could have an optimal solution for path planning problems and possesses the advantage of high accuracy. However, these methods always cannot find a solution in a complex optimization problem. The reason is that the optimal conditions are difficult to be defined and the convergence domain would be very small [2]. The largest drawback of this method is the curse of dimensionality as Bellman himself said. Even for a relatively complex problem, record of the calculated results needs a large amount of memory space. Moreover, as it is pointed by many researchers, it is difficult to estimate the initial value of the conjugate variables and deal with the path constraints [15]. Therefore, the in-direct method is not a popular method in the literature. Particle swarm optimization algorithm [28], genetic algorithm [30] and pseudo spectral [5] algorithms are typical examples of the direct method, which can directly solve the optimal problem without complex and cumbersome derivation process. Particle swarm optimization algorithm and genetic algorithm can be classified as evolutionary methods, which have been widely used in trajectory planning. Nevertheless, a large number of experimental results show that these methods are subject to the defects of premature convergence, low search efficiency, and always falling into local optimal [15]. Furthermore, theories foundation research of evolutionary algorithms is still quite sparse, lacking deep-going theoretical analysis with universal meaning. And the parameter selecting of these methods is still an open problem. Up to now, the parameters are always selected from user’s experience [18]. These deficiencies limit their applications in practice. Another direct method named Gauss pseudospectral method (GPM) [1], as a typical one of pseudo spectral algorithms, has been widely used in the optimization of many projects and its effectiveness has been proven [26]. In the GPM, the problem is discretized by approximating the state with a kind of Lagrange interpolating polynomials. The dynamics are collocated at a set of collocation points called Legendre Gauss points. With high accuracy in both the primal
and dual solutions, the GPM has become a good choice for the path planning problem in this research [12, 11, 14, 33]. The standard GPM is a global pseudospectral method essentially, which could obtain a high accuracy solution with fewer nodes. However, when dealing with a more complex and non-smooth optimal problem, it is ineffective. With regard to this problem, the hp-adaptive GPM (HPAGPM) was proposed by Darby [9, 13, 10]. The terminology ‘hp’ is used because the segment widths (denoted h) and the polynomial degree (denoted p) in each segment are determined simultaneously. It is found that the HPAGPM leads to higher accuracy solutions with less computational effort and memory than is required in a global pseudospectral method [10]. In this paper, the optimization model of multi-aircraft cooperative path planning is established. The objective of the cost function is to realize continuous target tracking with multiple constraints considered, including overload restraint, radar blind zone constraint, baseline constraint and so on. Then, the continuous path planning problem is discretized into a nonlinear programming manner with HPAGPM utilized. At last, the sequential quadratic programming (SQP) algorithm is adopted to solve the nonlinear programming problem [11, 14].

The remainder of this paper is organized as follows. Section 2 describes the problem formulation of radar blind zone and cooperative target tracking. In section 3, a mathematical model of the multi-aircraft cooperative path planning problem is presented. Section 4 gives the process of HPAGPM in solving the optimization problem. Section 5 furnishes the simulation results. Finally, the concluding remarks are given in the last section.

2. Problem formulation. It is known that the frequency detection region of the airborne radar is restricted by pulse Doppler detection threshold. If the Doppler frequency of the echo is less than the threshold, the echo will be neglected. The minimum detectable frequency (MDF) corresponds to the minimum detectable velocity (MDV). It means that if the radial velocity, which is defined as target velocity in the beam direction of the airborne radar, is less than MDV, the Doppler frequency of the echo will be below the MDF. In other words, the target gets into the radar blind zone and it will not be found by the airborne radar. It should be noted that the airborne radar mentioned in this paper is subject to active phased array radar system. In one-to-one air combat scenario, the target always could escape from airborne radar tracking via maneuvering. By means of multi-aircraft cooperative path optimization, the target cannot get into all the blind zones of the aircrafts at one time and it would be tracked by at least one airborne radar.

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![Figure 1](image-url)  
Figure 1. An illustration for multi-aircraft air combat
3. **Description of cooperative path planning model.** (1) Kinematic model of aircraft

In order to facilitate the design without losing accuracy, a three dimension (3D) kinematic model of the aircraft in inertial coordinate with constant velocity hypothesis is described as [12]

\[
\begin{align*}
\dot{x} &= V \cos \vartheta \cos \psi \\
\dot{y} &= V \cos \vartheta \sin \psi \\
\dot{h} &= V \sin \vartheta \\
\dot{\vartheta} &= \frac{a_m}{V} \\
\dot{\psi} &= \frac{a_z}{V \cos \vartheta}
\end{align*}
\]

(1)

where \((x, y, h)\) is position of the aircraft, \(V\) is the constant velocity, \(\vartheta\) and \(\psi\) are the pitch angle and yaw angle, \(a_m\) and \(a_z\) are the accelerations orthogonal to the velocity vector.

(2) Radar detection probability model

The model of airborne radar detection probability is described as

\[
P_d = \exp(\ln(P_f)R^4)K\sigma
\]

(2)

where \(P_f\) is false alarm probability, \(\sigma\) is radar cross section (RCS), and \(K\) is the regularized value corresponding the parameters of radar function. \(R\) is the range between the aircraft and the target.

(3) Constraint of airborne radar blind zone

As above mentioned, if the target radial velocity is less than MDV, the target will get into the radar blind zone. The radial velocity of the target relative to aircraft \(\alpha\) is

\[
\Delta v_\alpha = \frac{(x_T - x, \ y_T - y)}{\sqrt{(x_T - x)^2 + (y_T - y)^2}} \begin{pmatrix} v_{x_T} \\ v_{y_T} \end{pmatrix}
\]

(3)

where \(\alpha = 1, 2, ..., n\), is the number of aircraft in the cooperative formation, \((x_T, y_T)\) is the position of the target, and \(v_{x_T}\) and \(v_{y_T}\) are the velocity in the x-direction and y-direction, respectively. \(\Delta v_{\text{min}}\) denotes the MDV.

In order to maintain continuous tracking of the target, \(\Delta v_\alpha (\alpha = 1, 2, ..., n)\) should meets the constraint

\[
\max\{|\Delta v_1|, |\Delta v_2|, ..., |\Delta v_n|\} \geq \Delta v_{\text{min}}
\]

(4)

In this constraint, it can be guaranteed that at least one echo which over detection threshold could be achieved for the detections of all the aircrafts.

(4) Constraint of radar viewing aspect

The airborne radar viewing aspect is restricted as \([-\theta_o, \theta_o]\) and \([-\varphi_o, \varphi_o]\), where \(\theta_o\) and \(\varphi_o\) are the maximum view aspect angles of the airborne radar in elevation and azimuth, respectively. For example, the azimuth angle \(\bar{\varphi}_\alpha\), denoted as the angle of target position relative to the head direction of the aircraft, should meet the constraint

\[
-\varphi_o \leq \bar{\varphi}_\alpha \leq \varphi_o (\alpha = 1, ..., n)
\]

(5)

Then, the target will be in the viewing aspect of airborne radar of aircraft \(\alpha\).

(5) Baseline constraint
As known that there exists some constraints in baseline between two aircrafts in multi-aircraft cooperative air combat scenario. In this paper, it is described as
\[
D_{\text{min}} \leq D_{\alpha,\beta} \leq D_{\text{max}} \\
(\alpha = 1, \ldots, n; \beta = 1, \ldots, n; \alpha \neq \beta)
\] (6)
where \(D_{\text{min}}\) and \(D_{\text{max}}\) are the minimum and maximize length of baseline, and \(D_{\alpha,\beta}\) is the distance between the aircraft \(\alpha\) and aircraft \(\beta\).

(6) Overload constraint
The command accelerations must satisfy the overload constraint, and the aircraft flies only in the horizontal plane considered in this paper, so the constraint is
\[
|a_z| = 0, |a_m| \leq a_{\text{max}}
\] (7)

(7) Cost function
According to the radar detection theory, a high radial velocity could bring the echo frequency far away from the central frequency of main clutter which means a high detection probability. Therefore, the cost function correspond to the radial velocity is
\[
J_v = \max \sum_{\alpha=1}^{n} \int_{t_0}^{t_f} |\Delta v_\alpha(t)|, \quad \alpha = 1, 2, \ldots, n
\] (8)
where \(t_0\) and \(t_f\) are start time and terminal time, respectively.
Moreover, a small value of \(|\hat{\phi}_\alpha|\) could make the airborne radar to generate a beam with a narrow main-lobe width. It has some advantages for target detections, such as achieving a high-precision measurement and large detection gain. Therefore, the cost function with regard to radar aspect angle in azimuth is
\[
J_a = \min \sum_{\alpha=1}^{n} \int_{t_0}^{t_f} |\phi_\alpha(t)|, \quad \alpha = 1, 2, \ldots, n
\] (9)
Thus, the overall normalized cost function can be summed up as
\[
J = \min \left[ \omega_1 \cdot \frac{1}{\sum_{\alpha=1}^{n} \int_{t_0}^{t_f} |\Delta v_\alpha(t)|} + \omega_2 \cdot \sum_{\alpha=1}^{n} \int_{t_0}^{t_f} \frac{1}{\cos(\phi_\alpha(t))} \right]
\] (10)
where \(\omega_1\) and \(\omega_2\) are weight values.

4. Path planning algorithm.

4.1. Description of Gauss Pseudospectral method. Gauss Pseudospectral method will transform the optimal control problem into a continuous Bolza problem, approximate the dynamics and cost at a particular set of collocation points, and then solve this through a nonlinear program.

The GPM discretizes state and control variables on a set of Legendre-Gauss (LG) points. Lagrange interpolating polynomials is constructed to approximate state and control variables on these points. The derivative of state variables is approximated by differentiating global interpolating polynomials, and differentiation equations are transcribed into algebraic constraints. The integral term in objective function is calculated by Gauss integration [12]. After the transformation, the optimal control trajectory optimization problem is transformed to a parameter optimization problem with a series of algebraic constraints, which is a nonlinear programming problem (NLP).
The derivation of interpolation polynomial (12) with respect to the differential equation constraint of the optimization problem into an algebraic constraint of the continuous Bolza problem, from the time interval \( t = [t_0, t_f] \) to the fixed time interval \( \tau = [-1, 1] \), via the affine transformation

\[
\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0}
\]

Note that this transformation makes it always valid for the free initial time and final time and the time at the midpoint.

(2) State and control approximation

The state and control approximation are obtained by a set of interpolating polynomial at LG points. Assume there is \( N+2 \) points in the fixed time interval as \( \{\tau_0, \tau_1, \ldots, \tau_N, \tau_f\} \). Moreover, \( \{\tau_0, \tau_1, \ldots, \tau_N\} \) are zeros of \( N\)-th order Lagrange polynomial \( P_N(\tau) \) denoted as \( N\)-th order LG points. \( \tau_0 = -1 \) and \( \tau_f = 1 \) are initial time and terminal time, respectively.

The process of state and control approximation by Lagrange polynomial is

\[
X(\tau) \approx \hat{X}(\tau) = \sum_{i=0}^{N} L_i(\tau) X(\tau_i)
\]

\[
U(\tau) \approx \bar{U}(\tau) = \sum_{k=1}^{N} \bar{L}_k(\tau) U(\tau_k)
\]

where \( L_i(\tau) \) and \( \bar{L}_k(\tau) \) are two basic interpolation functions for state and control variables, respectively. The representations are

\[
L_i(\tau) = \prod_{j=0,j\neq i}^{N} \frac{\tau - \tau_j}{\tau_i - \tau_j} \quad (i = 0, 1, \ldots, N)
\]

\[
\bar{L}_k(\tau) = \prod_{j=0,j\neq k}^{N} \frac{\tau - \tau_j}{\tau_k - \tau_j} \quad (k = 1, \ldots, N)
\]

The terminal state is approximated by Gauss quadrature as

\[
X(\tau_f) \approx \hat{X}(\tau_f) = \hat{X}(\tau_0) + \frac{t_f - t_0}{2} \sum_{k=1}^{N} \lambda_k f(\hat{X}(\tau_k), \bar{U}(\tau_k), \tau; t_0, t_f)
\]

where \( f(\cdot) \) is the dynamic function and \( \lambda_k \) is Gauss weight factor expressed as

\[
\lambda_k = \int_{-1}^{1} L_k(\tau) d\tau = \frac{2}{(1 - \tau_k^2)(\bar{P}_N(\tau_k))^2}
\]

(3) Algebraic representation of dynamics

The next step is to convert the differential equation constraint of the optimization problem into an algebraic constraint. The derivation of interpolation polynomial (12) with respect to \( \tau \) is

\[
\hat{X}(\tau_k) \approx X(\tau_k) = \sum_{i=1}^{N} \bar{L}_i(\tau_k) X(\tau_i) = \sum_{i=1}^{N} \bar{D}_{ki} X(\tau_k) \quad k = 1, \ldots, N
\]

The differentiation matrix \( \bar{D}_{ki} \) is given as

\[
\bar{D}_{ki} = \begin{cases} 
\frac{(1+\tau_k)\bar{P}_N(\tau_k)+\bar{P}_N(\tau_k)}{[\tau_k-\tau_i][1+(1+\tau_i)\bar{P}_N(\tau_i)+\bar{P}_N(\tau_i)]} & i \neq k, \\
\frac{(1+\tau_k)\bar{P}_N(\tau_k)+2\bar{P}_N(\tau_k)}{2[(1+\tau_i)\bar{P}_N(\tau_i)+\bar{P}_N(\tau_i)]} & i = k
\end{cases}
\]
Substituting (17) and (18) into the dynamic equation, one can obtain
\[
\sum_{i=0}^{N} D_{ki}(\tau_k) \dot{X}(\tau_k) - \frac{t_f - t_0}{2} f[X(\tau_k), \bar{U}(\tau_k), \tau_k; t_0, t_f] = 0
\]
where \( k = 1, \ldots, N \).

(4) Discretization of constraints The continuous constraints can be discretized at all the LG points as
\[
C_i[\bar{X}(\tau_k), \bar{U}(\tau_k), \tau_k; t_0, t_f] \leq 0
\]
where \( C_i[\cdot] \) is the constraint function.

4.2. hp-adaptive Gauss Pseudospectral method. Hp-adaptive Gauss pseudospectral method (HPAGPM) is an optimal control method combined the standard GPM and hp-adaptive finite element algorithm. The hp adaptive finite element method is allowed to increase \( h \) new nodes. Mesh is divided into \( h+1 \) subintervals and the order of basis functions \( P \) is changed independently, which makes it possible to obtain desired accuracy with less unknown quantities [31]. Compared with the standard GPM, the HPAGPM takes less computation time and memory, but can get more accurate solution.

The HPAGPM divides the time interval into \( N \) sub-intervals at first. Then, the discretization is performed by GPM. The discretized state equation \( \dot{b}^{(k)} \) and error \( e^{(k)} \) in the interval \( k \) are
\[
\dot{b}^{(k)} = C^{(k)}(X^{(k)}, U^{(k)}, \tau^{(k)}, t_k, t_{k-1})
\]
\[
e^{(k)} = \left| \dot{X}^{(k)} - \frac{t_k - t_{k-1}}{2} f[X^{(k)}, \bar{U}^{(k)}, \tau^{(k)}, t_k, t_{k-1}] \right|
\]
If \( \dot{b}^{(k)} \) and error \( e^{(k)} \) both satisfy the maximum allowable error \( \varepsilon_d \), the solved states and control outputs are approximate solutions of the optimal control problem. Otherwise, the interval \( k \) needs to be further refined by the discrete nodes increased, which means the number of subintervals is increased to improve the accuracy.

The curvature function of the \( m \)-th state variable \( X_m^{(k)}(\tau) \) in the interval \( k \) is
\[
\bar{c}^{(k)}(\tau) = \left| \dot{X}_m^{(k)}(\tau) \right| / \left[ \left( 1 + \left( \dot{X}_m^{(k)}(\tau) \right)^2 \right)^{3/2} \right]
\]
If \( j_k \) is smaller than \( j_{m, ax} \) which is a preset threshold, the number of collocation in the interval \( k \) needs to be increased, which means the order of interpolation polynomial increased. Otherwise, divide the interval \( k \) into \( n^{(k)} \) intervals.

The strategy of increasing polynomial interpolation can be implemented as
\[
N_k^+ = N_k^- + \text{ceil}[\log_{10}(e^{(k)}_{\text{max}}) - \log_{10}(\varepsilon_d)] + 1
\]
where \( N_k^+ \) and \( N_k^- \) represent the interpolation polynomial orders of the interval updated and un-updated, respectively. \( e^{(k)}_{\text{max}} \) is the maximum error in the interval \( k \).

The strategy of increasing the interval division is
\[
n^{(k)} = Y \cdot \text{ceil}[\log_{10}(e^{(k)}_{\text{max}}) - \log_{10}(\varepsilon_d)]
\]
where \( n^{(k)} \) is the number of subintervals in the interval \( k \) and \( Y \) is a constant.
After the above transformations, a parameter optimization problem with a series of algebraic constraints is obtained, that is a nonlinear programming problem (NLP). In HPAGPM, the NLP is solved using snopt [24].

5. Simulation results. In this section, the simulation results are presented to show the efficiency of the proposed method. The simulations are performed in two-to-one (two aircrafts to one maneuverable target) and four-to-one (four aircrafts to one maneuverable target) scenarios, respectively. The weights in the cost function are given as $\omega_1 = 0.9$ and $\omega_2 = 0.1$. According to the knowledge about the airborne radar, the parameters of the constrains are always set as $\varphi_o = 60^\circ$, $D_{\text{min}} = 10\text{km}$, and $D_{\text{max}} = 50\text{km}$, respectively. The sampling interval is 1s.

Assume that the aircrafts and the target have a same constant velocity of 300m/s. The maximal normal acceleration is $a_{\text{max}} = 6g$, where $g$ is the gravitational constant. Besides, the differential constraint is given as $(\dot{a})_{\text{max}} = 9g/s$.

(1) Two-to-one scenario

In the two-to-one scenario, the initial positions of the two aircrafts are set at (3km, 20km) and (40km, 3km), and the initial yaw angles are $40^\circ$ and $45^\circ$, respectively. The target initial position is (180km, 180km) and performs s-turns maneuver.

For comparison, the simulations without cooperative path planning are performed firstly. In this condition, the two aircrafts fly straightly to the target with fixed yaw angles set as $40^\circ$ and $55^\circ$, respectively. The results of target radial velocities relative to the two aircrafts are shown in Fig. 2 wherein the target gets into the blind zone of aircraft 1 in [108s-118s] and of aircraft 2 in [115s-125s]. Therefore, both of the two aircrafts will lost the target in the time interval of [115s-118s]. This result illustrates that it cannot be guaranteed for the target to be stably tracked.

Afterwards, we perform the simulations of cooperative path planning by HPAGPM. Fig. 3 presents the results of the cooperative planned flight trajectories which possess the advantages of smoothness. The results of the target radial velocities relative to the two aircrafts are exhibited in Fig. 4, respectively. Accordingly, the airborne radar blind zone of each aircraft is shown in Fig. 5. The target enters the blind zone of aircraft 1 in [81s-91s] and of aircraft 2 in [222s-232s], and in [97s-107s] and [207s-217s] of aircraft 2, respectively. It illustrates that the target can be effectively detected by one aircraft when it enters the airborne radar blind zone of the other one by means of the proposed cooperative path planning strategy. This can guarantee the target would always be stably tracked. For comparison, the simulations with standard GPM adopted were performed and the results were given in Fig. 6. Note that the total time for the target in the blind zones of the two aircrafts is 51s, whereas this time is 40s with HPAGPM as shown in Fig. 5. It is validated that HPAGPM have a better precision. A more comprehensive comparison will be given later.

It should be pointed that we were unable to achieve the target to be tracked by only one aircraft in the whole time. The reason is that when the target is about to enter the blind zone of the airborne radar, we are unable to make a large change right away in flight direction of the aircraft to make the target get out of the blind zone. Moreover, it is difficult to make some prediction of the target trajectory. Consequently, the target will get into the blind zone of one aircraft unavoidably.

The results of azimuth angles of the target relative to the aircraft 1 and aircraft 2 are presented in Fig. 7, respectively. The azimuth angles are almost restricted in [-5deg, 5deg] in the whole time. This performance of small azimuth angle has some
benefits of generating a narrow beam with high-precision measurement ability and large detection gain. The normal accelerations of the aircrafts are depicted in Fig. 8 which satisfies the overload and its differential constraints.

(2) Four-to-one scenario In order to further test the efficiency of the proposed method, the simulations in four-to-one scenario are performed. Initial positions of the four aircrafts are set at (10km, 200km), (3km, 140km), (13km, 10km), and (45km, 45km), and the initial yaw angles are 45°, 3°, 10°, and 8°, respectively. The parameters of the target motion are same as in the two-to-one scenario. In Fig. 9, the flight trajectories for four aircrafts and the target are presented, respectively. It
Figure 5. The airborne radar blind zone in two-to-one scenario with HPAGPM

Figure 6. The airborne radar blind zone in two-to-one scenario with GPM

Figure 7. Azimuth angle of the target relative to the aircraft in two-to-one scenario

Figure 8. The normal accelerations in two-to-one scenario
is also verified that the continuous tracking of the target can be achieved based on the proposed cooperative path planning strategy in Fig. 10. The azimuth angles of the target relative to aircraft 1 to aircraft 4 are shown in Fig. 11 which are almost near 0°. Fig. 12 gives the results of normal accelerations.

![Figure 9. Flight trajectories in four-to-one scenario](image)

![Figure 10. Target radial velocity in four-to-one scenario](image)

![Figure 11. Azimuth angle of the target relative to the aircraft in four-to-one scenario](image)

In order to further test the efficiency of the HPAGPM in solving the cooperative path planning problem, simulations of a comprehensive comparison are performed with HPAGPM, GPM and PSO, respectively. The results are listed in Table 1. Firstly, in the aboved two-to-one scenario, the program of HPAGPM runs in a fastest speed which consumes 109s in contrast with other two algorithms. The blind zone
6. Conclusion. In this paper, the problem of multi-aircraft cooperative path planning with regard to maneuver target detection in air combat was studied based on hp-adaptive Gauss pseudospectral method. Firstly, the optimization model was established, with the expression of objective function, Doppler blind zone constraint, radar viewing aspect constraint, baseline constraint and overload constraint given. Secondly, the optimization model was transformed into a nonlinear programming problem by HPAGPM. And it is solved by GPOPS software package. Contrastive simulations were performed to validate the continuous tracking of manoeuvring air target can be obtained by the proposed cooperative path planning strategy.

However, some important issues still remain open. For instance, the nonlinear optimal problem always has a high complexity of computation and results a shortcoming of low real-time. Moreover, real-time is the first problem to be solved when the cooperative path planning algorithm comes into engineering application. In future, this nonlinear optimal problem can be cut into multi-stage linear optimal problem [14, 6] which requires less computationally effort.

Acknowledgments. We would like to thank you for following the instructions above very closely in advance. It will definitely save us lot of time and expedite the process of your paper’s publication.
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Received July 2020; revised January 2021.

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