A binary-coded symbols signal design method for ground-based wireless navigation

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Abstract
Global navigation satellite system (GNSS) like signal has been widely used in ground-based navigation as an augment of global navigation satellite system system. The binary-coded symbols (BCS) spreading modulation has been successfully used in global navigation satellite system systems to pursue the frequency reuses, and it is proved to have better ranging performance if the potential ambiguity risk is avoided. In the paper, the BCS coding is designed for ground-based transmitter with the comprehensive consideration of potential tracking accuracy, anti-multipath performance and ambiguous acquisition probability. The binary-coded symbol codes are obtained by optimising the corresponding performance with the condition of an acceptable false-lock probability. In this way, a moderate performance tracking performance can be achieved when the designed binary-coded symbol codes are adopted with the global navigation satellite system-like signal. The performance of the proposed signal is analysed in theory and assessed by computer simulation, and the unambiguous two-dimensional processing is given as an alternative for the proposed binary-coded symbol signals. The result indicates the proposed binary-coded symbol codes obtain obvious advantages over the traditional binary phase shift keying (BPSK) modulation and the existing binary-coded symbol coding schemes when applied with ground-based occasions.

1 | INTRODUCTION

Ground-based navigation system, which adopts global navigation satellite system (GNSS) like signal, is usually called the pseudolites. It is designed to provide augmented navigation information for the users in addition to the GNSS signals. For the GNSS system, the signal structure is very important for the positioning quality and interference resistance capacity, thus the signal design has been an important research issue for the wireless navigation system. Due to the code division multiply access (CDMA) technique, the GNSS systems obtain a reliable multi-path mitigation capacity and tracking accuracy [1]. The CDMA-based navigation signal can be further modified to adapt to different occasions and pursue a better ranging performance. When it comes to ground-based occasions, the first issue for the signal structure design is how to mitigate the near-far effect [2]. The ground-based transmitters are usually deployed in a specific area, thus they are not far away from each other. Fortunately, the combined TDMA + CDMA structure has been successfully proposed to solve the near-far problem [3]. However, there are few existing outcomes about how to improve the inherent tracking performance of the ground-based signal.

In recent two decades, many innovative CDMA modulation schemes have been proposed to give the signal a certain frequency feature. The most dramatic evolution is the proposal of the binary offset carrier (BOC) signal for Galileo system [4], which provides a split power spectral density (PSD) function and allows frequency reuse with less overlapping PSD. The BOC signal is generated by multiplying the binary offset sub-carrier with the pseudo-random noise (PRN) code in the time domain, which is very easy to be implemented. The auto-correlation function (ACF) is changed to be multi-peaked and the main peak becomes sharper compared to that of the BPSK signal [5]. Since then, people got to realise the sub-carrier would greatly affect the ACF shape and the inherent ranging capability. In general, the CDMA modulations which adopt binary offset sub-carrier can be classified into a wider category, the binary-coded symbol (BCS) modulation [6,7]. Different from the BOC sub-carrier, the sub-carrier of the BCS is no longer restricted to be square sine or cosine waveform. The sub-carrier is only...
required to be binary and periodic over a code period. Moreover, when the sub-carrier has multiply levels instead of only two levels, the multiply coded symbol (MCS) modulation \[8,9\] is obtained for an alternative of the wireless navigation signal. The AltBOC modulation \[10,11\], which is adopted in Galileo E5 band can be seen as a special implementation of MSK modulation. The BCS and MCS modulations have many advantages over the traditional BPSK modulation. However, the ambiguous problem is also introduced by sub-carrier which could add the false-lock risk of the signal acquisition \[12,13\]. The unambiguous signal processing becomes another issue for BCS signal processing. To solve this problem, many unambiguous signal processing algorithms are proposed such as the bump-jumping correlator \[14\], the double estimator (DE) \[15,16\] tracking loop and double-phase estimator (DPE) \[17\] tracking loop.

When it comes to ground-based occasions, the BCS signal design is completely different from that of the satellite-based navigation, as the high spectral side lobes are not necessary for the pseudolite signal. As a supplementary system of the GNSS system, the frequency reuse is not needed, however the inherent anti-multipath and tracking accuracy are more important. The most efficient way for this is to increase the code rating of the CDMA signal. For example, the P code rate in GPS system is 10 times that of the C/A code and can suppress multipath with less than 10 times delay of the multipath, and the inherent anti-noise tracking accuracy is enhanced in the same way. However, it is unreasonable to increase the code rate infinitely. Hence, it is meaningful to design a specific BCS for ground-based navigation signal \[18,19\]. The pseudolite navigation signal design diagram is demonstrated in Figure 1.

With no restriction of a split PSD, we propose to analyse the relationship between the inherent performance and the BCS coding. In this way, the desired BCS can be obtained by optimising the comprehensive performance weighting function. Firstly, the inherent tracking accuracy is proved to be lower bounded to the Cramér-Rao lower bound (CRLB) decided by the Gabor bandwidth in the frequency domain \[20,21\]. Thus, the concept of Gabor bandwidth is first investigated in this paper. Secondly, the multipath sensitivity is another essential index for ground-based navigation signal. There could be multipaths mixed with the line-of-sight (LOS) component causing tens of meters of tracking error. Therefore, a good BCS should provide satisfactory multipath rejection capacity along with a good inherent tracking accuracy. The concept of multipath error envelope (MEE) is usually adopted to assess the anti-multipath capacity. The key point to obtain a better multipath rejection is to make the sub-peaks of the ACF as small as possible and keep the main peak as narrow as we can. In this way, the MEE area can be significantly reduced. Thirdly, the ambiguity problem should be completely avoided. The tracking loop could lock on the ACF’s sub-peaks falsely, which is a non-negligible problem for BCS signal. The solution is to adjust the BCS coding to reduce the amplitude of the most obvious sub-peak. In this way, the false lock is less likely to happen under the parallel acquisition algorithm, even though the sub-peaks are not eliminated thoroughly \[21\].

Furthermore, more flexible signal processing approaches like DE tracking loop could also be applied to BCS signals to achieve an absolutely no ambiguity signal processing, which is originally applied for BOC signal \[22,23\]. This paper investigates the two-dimensional (2D) correlation function for each proposed BCS signal and demonstrates the potential of unambiguous tracking. Overall, the BCS signal can be processed in a traditional way with controllable false-lock probability, or alternatively, with the unambiguous method. This is another obvious advantage over some ground-based signals, such as the signal with the linear sweep sub-carrier which is proposed in \[24,25\], which adopts complicated sub-carrier and raises the receiver complexity significantly and has no possibility to get an unambiguous signal processing. Overall, the proposed BCS modulation schemes are easier to be implemented and have many advantages to be used in ground-based occasions.

The remainder of the paper is organised as follows. In Section 2, the general structure of the TDMA + CDMA signal structure is introduced. In Section 3, the concept of the BCS signal as well as the time and frequency features is introduced. In Section 4, the inherent anti-multipath, tracking accuracy and false-lock probability of BCS signal are analysed. In Section 5, the BCS sub-carrier is optimised based on the proposed scheme, and the most basic unambiguous processing method, the DE tracking loop, is introduced for the proposed BCS signal. In Section 6, the proposed method is performed with simulation. The conclusion and discussion are given in the final section.

2 | GROUND-BASED NAVIGATION SYSTEM SIGNAL

The ground-based wireless navigation system introduced here transmits GNSS-like signal, which is usually called the pseudolites. It is usually working collectively or independently with the

![FIGURE 1 Pseudolite navigation signal design diagram](image-url)
GNSS system, however, the time synchronisation and initialisation of the pseudolites must refer to the GNSS information. A brief schematic of radio navigation transmitter system including GNSS and ground-based transmitters is given in Figure 2. The GNSS-like signal for pseudolites must have strong penetration and ability to resist the near-far effect because the transmitters are usually near the user. The best way to solve this near-far problem is to adopt TDMA structure based on the GNSS-like CDMA signal. The TDMA + CDMA signal structure is demonstrated in Figure 3.

The ground-based navigation system includes a master station and several slavery stations. The working process of the ground-based navigation system can be concluded as follows:

1. The transmitters are installed in the workplace and the system is initialised. The space–time information is coordinated with the information from the GNSS signal.
2. The master station obtains high-precision space–time information and it broadcasts it to the slavery stations. The transmitters also communicate with each other by transmitting and receiving the wireless signal to refine the system synchronisation result.
3. When the space–time synchronisation is finalised, the transmitters broadcast the TDMA + CDMA signal to the workplace with its own spatio-temporal information.
4. The user receives the TDMA + CDMA signals from each ground-based transmitter and the CDMA signals from the GNSS, and the receiver begins to process each signal with different channel. The user’s positioning solution is obtained with the collective spatio-temporal information provided by each channel.

The main challenge of the ground-based transmitter is the near-far effect and multipath distortion. The former problem has been well solved by the TDMA + CDMA scheme, whereas it is still difficult to mitigate all the multipath effect on the receiver. When the transmitters are deployed in urban or mountainous areas, the multipath interference would be severe, therefore it is necessary to improve the anti-multipath capacity of the system.

3 | BCS SIGNAL STRUCTURE AND CHARACTERISTICS

3.1 | Signal structure

The concept of the BCS signal originates from the BOC signal, and is composed of the PRN code and the binary sub-carrier, which can be expressed as

\[ s(t) = \sum_{k} c_k q(t - kT_s), \]  

where \( c_k \) denotes the PRN code with period \( T_c \), and \( q(t - kT_s) \) denotes the sub-carrier with period \( T_s \). Both PRN code and sub-carrier are binary sinusoidal square waveforms for BCS signal, and the concept of ‘segment’ corresponds to each symbol’s
duration, which is defined as

$$\psi_k(t) = \begin{cases} 1, & t \in [kT_c, (k+1)T_c] \\ 0, & \text{others} \end{cases}.$$  (2)

Hence, the sub-carrier sequence $q(t)$ during one PRN chip can be expressed as the combination of different symbols:

$$q(t) = \sum_{k=1}^{n} \psi_k(t)x_k,$$  (3)

where $n = T_c/T_s$ denotes the number of the sub-carrier segments in one PRN chip, and $x_k = \pm 1$ is the corresponding binary symbol during each segment. The structure of the BCS signal is demonstrated in Figure 4.

### 3.2 Auto-correlation function

The ACF is the basis of signal acquisition and tracking, and the general cross-correlation function (CCF) between two BCS signals, $s(t)$ and $s'(t)$, can be expressed as

$$R_{ss}(\tau) = \frac{1}{T} \int_0^T s(t)s'(t+\tau)dt$$

$$= \frac{1}{T} \sum_{r=1}^{N-1} \sum_{f=0}^{n-1} \sum_{m=0}^{n-1} (-1)^{\gamma+f+m} x_r x_r^*$$

$$\int_0^T \psi_k(t-inT_c)\psi_{k'}(t-jnT_c+\tau)dt.$$  (4)

Only when the segments of both signals, $\psi_k(t-inT_c)$ and $\psi_{k'}(t-jnT_c+\tau)$, overlap or partially overlap with each other, the above function could be a non-zero value. The non-zero condition is thus obtained as

$$| (i-j)T_c + (k-q)T_s + \tau | < T_s.$$  (5)

The delay can be divided into the unit of $T_s$, and the remaining part can be further divided into the unit of $T_c$. Let $\tau = aT_s + bT_c + \epsilon$ [21], where $a$ is the integer delay in the PRN code scale and $b = 0, 1, ..., n-1$ is the integer delay in the sub-carrier segment scale. $\epsilon \in [0, T_c)$ is the remaining decimal part of the first two terms. After some straightforward calculations, the CCF can be expressed as

$$R_{ss}(\tau) = \frac{1}{T} \left[ r_a + r_{a+1} \right]$$

$$+ \left( \frac{\tau}{T_c} \left[ r_a + r_{a+1} \right] \right),$$  (6)

where

$$r_a = \frac{1}{N} \sum_{r=0}^{N-1} (-1)^{\gamma+f+m}, r_b = \frac{1}{n} \sum_{k=0}^{n-1} x_r x_r^*.$$  (7)

According to the PRN characteristic, only when the two PRN codes of the CCF are identical and the $a$ is zero, $r_b$ can be a non-zero value. In this way, the ACF of a BCS signal can be obtained:

$$R_{ss}(\tau) = \begin{cases} \left( \frac{\tau}{T_c} - b \right) \left( r_{b+1} - r_b \right) + r_b, & \tau \in [bT_s, (b+1)T_s] \\ \left( \frac{\tau}{T_c} - b + n \right) \left( r_{b+1} - r_b \right), & \tau \in [(b-n)T_s, (b+1)T_s] \end{cases}.$$  (8)

It can be seen that the ACF of the BCS signal is wholly determined by the shape factors of the sub-carrier, $x_k = \pm 1, k = 1, 2, ..., n$. Therefore, the optimisation of a BCS signal is equal to design the BCS sub-carrier vector:

$$x = \{x_1, x_2, x_3, ..., x_n \}, x_k \in \{ \pm 1 \}, k = 1, 2, ..., n.$$  (9)

To exemplify the ACF function, the ACFs with respect to BOC signals and some selected BCS signals are demonstrated in Figure 4. Let $n = 12$, the BCS sub-carrier vectors with respect to BOC(1,1), BOC(1.5,1), BOC(2,1) and three BCS signals are given in Table 1.

| Signal  | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| BOC(1,1)| +1    | +1    | +1    | +1    | +1    | -1    | -1    | -1    | -1    | +1       | +1       | +1       |
| BOC(1.5,1)| +1    | +1    | +1    | +1    | -1    | -1    | -1    | -1    | +1    | +1       | +1       | +1       |
| BOC(2,1)| +1    | +1    | +1    | -1    | -1    | -1    | -1    | +1    | +1    | +1       | +1       | +1       |
| BCS1   | -1    | +1    | +1    | -1    | -1    | -1    | -1    | +1    | +1    | +1       | +1       | +1       |
| BCS2   | -1    | -1    | +1    | -1    | -1    | -1    | -1    | -1    | +1    | +1       | +1       | +1       |
| BCS3   | +1    | +1    | +1    | -1    | -1    | -1     | 1     | 1     | 1     | 1        | 1        | 1        |
As can be seen from Figure 5, the ACF of BCS signals obtain multiple peaks in addition to the main peak. The ACF for BOC(2,1) has six sub-peaks totally, whereas this value is 4 for that of the BOC(1.5,1). The ACF of BOC(1,1) has only two sub-peaks, whereas the main peak is much more wider than its components. BCS1, BCS2 and BCS3 share the same sub-peak number, whereas the shapes of the ACFs are obviously different because the length and position of the negative ‘−1’ symbols varies. Overall, the time and frequency characteristics are determined by BCS sub-carrier.

3.3 | Power spectral density function

When it comes to the frequency domain, the PSD of the BCS signal is determined by both PRN code and the sub-carrier, and the PSD of a CDMA signal can be obtained via its ACF function:

$$G_{BCS}(f) = f_c \left| S_{BCS}(f) \right|^2,$$

where $S_{BCS}(f)$ refers to the chip waveform spectrum, which can be obtained by the Fourier transform of a generic BCS signal as

$$S_{BCS}(f) = e^{-\frac{\pi f}{n f_c}} \sum_{k=1}^{n} \xi_k e^{\frac{-j 2\pi k f_m}{n f_c}}.$$

A general expression for the Fourier transform of a BCS signal is obtained by means of Euler transformations and combinations, which can be expressed as

$$G_{BCS}(f) = f_c \left( \frac{\sin \left( \frac{\pi f}{n f_c} \right)}{\pi f} \right) \left| \sum_{k=1}^{n} \xi_k e^{\frac{-j 2\pi k f_m}{n f_c}} \right|^2.$$

$$= f_c \left( \frac{\sin \left( \frac{\pi f}{n f_c} \right)}{\pi f} \right) \left\{ \sum_{l=1}^{n} \xi_l^2 + 2 \sum_{l=1}^{n} \xi_l \sum_{l'=l+1}^{n} \xi_{l'} \cos \left[ \left( f_f - \frac{2\pi}{n f_c} \right) \right] \right\}.$$

Like the ACF, the PSD of a BCS signal is determined by its sub-carrier vector. Observing the PSD plotted in Figure 6, the high-order BOC signal has a larger spectrum separation coefficient compared to those of the low-order BOC signal. The BCS signals have incompletely split PSD, which is different from those of the BOC signals. The inherent tracking performance is closely linked with the time and frequency characteristics of the BCS signal, which is analysed in the next section.

4 | INHERENT SIGNAL CAPACITY

4.1 | Tracking accuracy

The most important performance characteristics the ground-based signal should have are the inherent tracking and anti-multipath capacities. The CRLB of the code tracking error is the generic theorem to reflect the lower bound of code tracking accuracy in a specific occasion. Assuming that the front-end bandwidth is $\beta_r$ in hertz, the loop bandwidth $B_L$ and the early-minus-late (EML) space $\Delta$ are small enough, the CRLB of the code-tracking error versus the carrier-to-noise density $C/N_0$ is expressed, in meters, as

$$\sigma_{BCS} = c \times \sqrt{\frac{B_L}{\beta_r}} \frac{1}{\sqrt{2\pi C/N_0}},$$

where $c$ is the speed of light, $\beta_{Gabor}$ is the Gabor bandwidth [20], which is the root-mean-square bandwidth of the received signal in hertz:

$$\beta_{Gabor} = \sqrt{\frac{\beta_r}{2}} \sqrt{\int G_{BCS}(f) df},$$

It is obvious to see that the minimisation of the inherent tracking error is equivalent to getting the maximum value of $\beta_{Gabor}$ under bandwidth limited condition. Once the front-end bandwidth is given, the only effective way to increase $\beta_{Gabor}$ is...
to make more high-frequency components of the PSD included by the bandwidth filter.

### 4.2 Multipath insensitivity

Multipath insensitivity is another essential design driver for BCS signals. The multipath error is caused by signals reflected from diverse transmission paths. The carrier-demodulated signal mixed with several multipath components is expressed as

$$r(t) = \alpha_0 s(t - \tau_0) e^{j\varphi_0} + \sum_{i=1}^{M} \alpha_i s(t - \tau_0 - \tau_i) e^{j(\varphi_i + \theta_i)},$$  \quad (15)

where $\tau_0$ denotes the LOS code phase and $\varphi_0 = \varphi_0 - \hat{\varphi}_0$ is the carrier phase demodulation error. $\tau_i$ and $\theta_i$ denote the multipath code and carrier phase code differences relative to the LOS signal. The MEE [27] is usually computed to assess the multipath insensitivity of a given signal. Assuming that the received signal is composed by the direct signal and only one reflected signal with the relative attenuation $\alpha$ and code phase delay $\tau$, respectively, the relative carrier phase delay is set to 0 and $\pi$ to simulate the extreme cases, and the MEE with finite front-end bandwidth is given by

$$\gamma(\tau) = \pm \alpha \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} G_{BCS}(f) \sin(\pi f \tau) \sin(2\pi f \tau) df,$$

$$\gamma(\tau) = \pm \frac{\alpha}{2\pi} \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} G_{BCS}(f) \sin(\pi f \tau) \left[ 1 \mp \alpha \cos(2\pi f \tau) \right] df.$$

(16)

Furthermore, the running-averaged MEE is calculated based on the MEE by averaging all the possible multipath-induced tracking error by sweeping the code phase delay $0$ to the value $\tau$:

$$\bar{\gamma}(\tau) = \frac{1}{\tau} \int_{0}^{\tau} \left[ |\gamma(\tau)| + |\gamma(\tau)| \right] d\tau.$$  \quad (17)

Compared to MEE, the running-averaged MEE shows that the averaged multipath introduced error against the multipath delay $\tau$, hence it is more efficient and visible to reflect the overall multipath mitigation capacity of a navigation signal by running-averaged MEE.

### 4.3 Acquisition ambiguity

Acquisition ambiguity should be considered along with the inherent tracking capabilities because a false acquisition might cause an immense error to the tracking loop. For example, high-order BOC signal obtains better tracking accuracy, however, the amplitude of the sub-peaks is significantly larger than those of the low-order BOC signal, which makes it risky to process the high-order BOC signal with the traditional discriminator. Generally, the parallel strategy, instead of the serial strategy, should be adopted for BCS signal acquisition. Assuming that the carrier demodulation is perfect, the signal power would be concentrated on the in-phase branch. Thus, the following function is considered approximately the false-lock probability:

$$L(\tau) = \frac{1}{T_{coh}} \int_{0}^{T_{coh}} \left[ \sqrt{C s(t) + n(t)} s(t + \tau) \right] dt,$$

$$L(\tau) = \sqrt{C R(\tau) + \eta(\tau)},$$  \quad (18)

where $C$ is the signal power and $\eta(\tau)$ defines the noise term after the correlation operation, which is Gaussian distribution with the variance $\frac{N_0}{2}$. There could be several pairs of sub-peaks with the ACF, but only the sub-peak with the largest size is considered because it has the most potential to introduce false-lock phenomenon. Therefore, the false-lock probability is calculated by comparing the amplitudes of the main peak and the sub-peak in term of Gaussian noise interference

$$P'_{false} = P \left\{ L(0) - L(\tau) < 0 \right\},$$  \quad (19)
The correlation coefficient between \( L(0) \) and \( L(\tau_{\text{sub}}) \) is 
\[
\rho = \frac{L(\tau_{\text{sub}}) \cdot L(0)}{\sqrt{\text{Var}(L(\tau_{\text{sub}})) \cdot \text{Var}(L(0))}}
\]
therefore the distribution of \( L(0) - L(\tau_{\text{sub}}) \) can be obtained after some straightforward calculations:

\[
L(0) - L(\tau_{\text{sub}}) \sim N \left\{ \sqrt{C} \left\{ R(0) - R(\tau_{\text{sub}}) \right\}, \frac{N_0}{T} (1 - \rho) \right\}
\]

\[
\cong N \left\{ \sqrt{C} (1 - R(\tau_{\text{sub}})), \frac{N_0}{T} (1 - R(\tau_{\text{sub}})) \right\}
\]

Therefore, the false-lock probability can be calculated in terms of Marcum-Q function [26]:

\[
P_{\text{false}} = Q \left\{ \frac{\sqrt{C} (1 - R(\tau_{\text{sub}}))}{\sqrt{\frac{N_0}{T} (1 - R(\tau_{\text{sub}}))}} \right\}
\]

\[
= Q \left\{ \sqrt{C/N_0} T (1 - R(\tau_{\text{sub}})) \right\}
\]

(20)

The sub-peaks must appear in pair because of the symmetry of the ACF, therefore, the actual false-lock probability should take into consideration the sub-peak on the both sides:

\[
P_{\text{false}} = 1 - P \left\{ L(0) - L(\tau_{\text{sub}}) < 0 \right\} P \left\{ L(0) - L(-\tau_{\text{sub}}) < 0 \right\}
\]

\[
= 1 - \left( 1 - P'_{\text{false}} \right)^2
\]

(22)

Overall, the amplitudes of the sub-peaks should be small enough to control the false-lock probability. Thus, the BCS signal can be processed with the traditional discriminator when \( C/N_0 \) is acceptable.

5 BCS WAVEFORM SELECTION

The ground-based BCS signal should have strong tracking capacity and multipath insensitivity with acceptable false-lock probability. Moreover, the sub-carrier vector should not be too complicated, otherwise the main lobe of the signal could get larger, and correspondingly the minimum signal receiving bandwidth could be increased. Another reason for a simply sub-carrier is to increase the possibility of flexible signal processing. A complicated sub-carrier might correspond with an unpredictable CCF and it might have been impossible to use unambiguous processing methods to modify it. Based on the above analysis, the following form of sub-carrier with only two transients during one PRN code chip is chosen as the candidate:

\[
\kappa = \left\{ 1, \ldots, 1, -1, \ldots, -1, \right\}
\]

(23)

As analysed in the last section, the Gabor bandwidth is expected to be as large as it is under a band-limited condition. The front-end bandwidth is set to \( \beta_s = mB \), where \( B \) is the single-side bandwidth of the main lobe of the PRN code which is equal to the PRN code rate. The code rate is assumed to be \( f_c = \frac{1}{T_c} = 1.023 \text{MHz} \). The first BCS signal is obtained by maximising the Gabor bandwidth under the condition of an acceptable false-lock probability:

\[
\arg \max_{\kappa_1} \beta_{\text{Gabor}} = \frac{1}{T} \int \frac{\beta_s}{2} \sqrt{G_{\text{BCS}}(f)} df
\]

s.t. \( P_{\text{false}} < 0.01 \)

Here, the segments number is set to \( n = 60 \) and the single-side front-end bandwidth is \( \beta_s = 8.184 \text{MHz} \). In terms of the false-lock probability, \( C/N_0 \) is set to 35 dB-Hz with the coherent integration period \( T = 5 \text{ms} \). Thus, sub-carrier \( \kappa_1 \) can be calculated by comparing the Gabor bandwidth of all possible BCS combinations. Similarly, another BCS sub-carrier \( \kappa_2 \) can be obtained by optimising the running-averaged MEE, to achieve a strongest anti-multipath capacity:

\[
\arg \min_{\kappa_2} \gamma = \frac{1}{T} \int \left\{ |y^+(\tau)| + |y^-(\tau)| \right\} d\tau
\]

s.t. \( P_{\text{false}} < 0.01 \)

In addition to the above two BCS sub-carrier, the third BCS sub-carrier \( \kappa_3 \) can be obtained by optimising a weighted function accounting for both Gabor bandwidth and running-averaged MEE:

\[
\arg \max_{\kappa_3} \beta_{\text{Gabor}} + \frac{\mu}{\gamma}
\]

s.t. \( P_{\text{false}} < 0.01 \)

where \( \mu \) defines a weighted coefficient to balance the two factors with different units numerically. Here, \( \mu \) is set to 0.5 with \( \beta_{\text{Gabor}} \) in megahertz and \( \gamma \) in meters. Thus, this BCS is expected to have harmonious capacity compared to those of the BCS1 and BCS2.

The inherent capacity with respect to different \( \ell_1 \) and \( \ell_2 \) are demonstrated in Figure 7. The area that satisfies the condition \( P_{\text{false}} < 0.01 \) is shown in green. The desired BCS sub-carrier vectors are selected according to (27)–(29) from the area which are \( \ell_1 = 4, \ell_2 = 22 \) for \( \kappa_1 \), \( \ell_1 = 5, \ell_2 = 18 \) for \( \kappa_2 \) and \( \ell_1 = 4, \ell_2 = 20 \) for \( \kappa_3 \). The sub-carriers are demonstrated in Figures 8, and the ACF and normalised PSD are demonstrated in Figures 9 and 10, respectively. Overall, the selected BCS signals obtain sharp main peaks but the sub-peak amplitudes are tolerable. The BOC signals could have obvious ambiguous risks when they are processed in a traditional way, whereas the false-lock probabilities of the proposed BCS signals are much slighter. The PSD is not absolutely spite compared to that of the BOC signal, and the minimum single-side frond-end bandwidth required for
In some situations, the potential ambiguous risk is not allowed even though the risk is low. The code discrimination must be unambiguous and some flexible processing methods can be designed referring to those of the BOC signals. The DE tracking loop, based on the 2D CCF, is the most fundamental and efficient unambiguous signal processing method, which estimates the PRN code and sub-carrier two individually. The proposed BCS signals can be also processed with their 2D CCFs, which are plotted in Figure 9. The 2-D CCF can be expressed as

$$
\chi(\tau_c, \tau_sc) = \frac{1}{T} \int_0^T s(t) s(t + \tau_c, t + \tau_sc) dt
$$

$$
= \frac{1}{T} \int_0^T s(t) s(t + \tau_c + \tau_sc) dt, \quad (27)
$$

where \( \tau_c \) and \( \tau_sc \) define the PRN code delay and sub-carrier delay respectively. The 2D ACF with respect to \( \kappa^1 \), \( \kappa^2 \) and \( \kappa^3 \) are slightly different in their shapes, whereas it is possible to obtain an unambiguous tracking via 2D code processing (Figure 11).

The DE tracking loop works in two steps: The rough stage and precision stage. The rough estimation denotes the code phase estimation by EML discriminator to find the initial code phase estimation:

$$
\psi_c = \chi \left( \tau_c - \frac{\Delta c}{2}, \tau_sc \right) - \chi \left( \tau_c + \frac{\Delta c}{2}, \tau_sc \right).
$$

(28)

The precision estimation means the sub-carrier phase estimation, which is implemented with an additional sub-carrier
Two-dimensional cross-correlation functions: (a) BCS1, (b) BCS2 and (c) BCS3 discriminator and tracking loop:

\[ \psi_n = \chi \left( \tau_c, \tau_n - \frac{\Delta}{2} \right) - \chi \left( \tau_c, \tau_n + \frac{\Delta}{2} \right). \]  \hspace{2cm} (29)

The sub-carrier estimation result is used to determine the accurate code phase based on the rough estimation result. The final result for the code estimation is obtained by combining both estimations:

\[ \hat{\tau} = \hat{\tau}_n - \text{round} \left( \frac{\hat{\tau}_n - \bar{\tau}}{\bar{T}_n} \right) \bar{T}_n. \] \hspace{2cm} (30)

It can be seen that the rough estimation is unambiguous because the 2D CCF is single peaked in the PRN dimension.

With one more discriminator and tracking loop employed, the hardware complexity of the DE tracking loop increases. However, this method is able to provide another alternative for BCS signal processing.

6 \hspace{1cm} \text{PERFORMANCE ANALYSIS}

The tracking performance of the proposed BCS signals is analysed with computer simulation in this section. The simulation diagram is given in Figure 12. The BCS signal is generated, and noise and multipath are added into the signal. The signal is processed with a software-defined receiver.

6.1 \hspace{1cm} \text{Acquisition performance}

The false-lock probability with respect to BOC(1,1), BOC(1.5,1), BOC(2,1), BCS1 and BCS2 and BCS3 signals are plotted in Figures 13 and 14. The linear sweep sub-carrier proposed in [24] is also analysed as a counterpart. The linear sweep technique is able to improve the inherent performance of the ground-based navigation signal, however, it is much more complex as the sub-carrier rate is not constant. It can be seen in the following simulation, it has no obvious advantages over the proposed BCS scheme. Here, the sweep bandwidth is set to 2.046 MHz and the signal is denoted as SpPN(2,1). The \( C/N_0 \)
is set to sweep from 25 to 45 dB-Hz and the integration periods are set to 5 and 10 ms, and the false-lock probability is estimated via 1000 time runs from the computer simulation. The solid line representing the simulation results are slightly higher than the theoretical results plotted in dashed lines, and the BCS1, BCS2 and BCS3 obtain a similar false-lock probability in all conditions, and the values are significantly lower than those of the BOC(1,1), BOC(1.5,1) and BOC(2,1) signals. When the integration time is 5 ms, the minimum $\frac{C}{N_0}$ required to satisfy $P_{\text{false}} < 0.01$ is 37 dB-Hz for the proposed BCS signals, whereas this value is about 39 dB-Hz for the SpPN(2,1) and 40 dB-Hz for BOC(1,1) signal. The same conclusion can also be found when the integration time is 10 ms, which indicates that the proposed signals withstand less risk when processed with the traditional approach compared to the BOC signals.

6.2 | Multopath performance

The MEE and running-averaged MEE are plotted in Figures 15 and 16 to illustrate the anti-multipath advantages of the proposed BCS signals with the EML space $\Delta = 0.1$ chip. It can be seen that BCS2 and SpPN(2,1) obtain the smallest MEE followed by BCS3 and BCS1, respectively. The running-averaged MEE of the BCS signals are about 2 and 3 meters smaller than those of the BOC signals. Thus, the overall multipath mitigation performance advantage of the proposed BCS signals is verified.

6.3 | Tracking performance

The steady-state tracking error versus $\frac{C}{N_0}$ is plotted in Figure 17. The loop bandwidth $B_L$ is set to 1 Hz. The carrier-to-noise ratio $\frac{C}{N_0}$ sweeps from 30 to 50 dB-Hz with coherent integration time 1 ms. The steady-state tracking error is calculated and averaged with 1000 times simulation runs. It can be seen that the overall trends is the BOC(2,1) has the smallest tracking error followed by BCS1, BCS3 and BCS2, but this
7 CONCLUSION

The BCS sub-carriers are optimised for ground-based transmitter on the basis of different concerns including the inherent tracking accuracy, multipath insensitivity, unambiguous processing probability and implementation complexity. The BCS sub-carriers are selected by maximising the Gabor bandwidth or minimising the running-averaged MEE with an acceptable false-lock probability. It is also possible to process the proposed BCS signals with unambiguous approaches, such as the DE tracking loop. The desired BCS signals would be different if the goal functions or parameters were changed, and the goal function could be modified according to the requirement. Overall, the proposed BCS signal design scheme is more suitable for ground-based navigation system.

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