Bounding Noncommutative QCD

Carl E. Carlson\textsuperscript{a}, Christopher D. Carone\textsuperscript{a,†}, and Richard F. Lebed\textsuperscript{b,‡}

\textsuperscript{a}Nuclear and Particle Theory Group, Department of Physics, College of William and Mary, Williamsburg, VA 23187-8795

\textsuperscript{b}Department of Physics and Astronomy, Arizona State University, Tempe, AZ 85287-1504

(July, 2001)

Abstract

Jurčo, Möller, Schraml, Schupp, and Wess have shown how to construct noncommutative SU(N) gauge theories from a consistency relation. Within this framework, we present the Feynman rules for noncommutative QCD and compute explicitly the most dangerous Lorentz-violating operator generated through radiative corrections. We find that interesting effects appear at the one-loop level, in contrast to conventional noncommutative U(N) gauge theories, leading to a stringent bound. Our results are consistent with others appearing recently in the literature that suggest collider limits are not competitive with low-energy tests of Lorentz violation for bounding the scale of spacetime noncommutativity.

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I. INTRODUCTION

Recent interest in the possibility of extra spatial dimensions with large radii of compactification [1] has led a number of groups to consider other modifications of spacetime structure that may have observable consequences. One such possibility is that the usual four spacetime dimensions become noncommutative at some scale $\Lambda_{NC}$ higher than currently accessible energies, viz.,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu},$$

(1.1)

where a hat indicates a noncommuting coordinate, and where $\theta$ is a real, constant matrix with elements of order $(\Lambda_{NC})^{-2}$ [2, 7]. Such noncommutative geometries have been shown to arise in the low-energy limit of some string theories [8], and lead to quantum field theories that are interesting in their own right [9–13]. Experimental signals of noncommutativity have been discussed from the point of view of collider physics [2, 3] as well as low-energy non-accelerator experiments [4–7]. Two widely disparate sets of bounds on $\Lambda_{NC}$ can be found in the literature: bounds of order a TeV from colliders [2, 3], and bounds of order $10^{11}$ GeV [4] or higher [5] from low-energy tests of Lorentz violation. Bounds of the latter type will be of interest to us in this letter.

The Lorentz violation in noncommutative field theories originates from the constants $\theta^{\mu\nu}$ appearing in Eq. (1.1). Noting that $\theta^{\mu\nu}$ is antisymmetric, one can regard $\theta^{0i}$ and $\theta^{ij}$ as constant three-vectors indicating preferred directions in a given Lorentz frame. How $\theta$ appears in the low-energy effective theory determines the ways in which Lorentz violation may be manifested in experiment. As pointed out by Mocioiu et al. [5], the existence of an operator of the form

$$\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q,$$

(1.2)

where $q$ is a quark field, leads to a shift in nuclear magnetic moments and an observable sidereal variation in the magnitude of hyperfine splitting in atoms [14]. From here, they
obtain the bound $\Lambda_{NC} > 5 \times 10^{14}$ GeV. Anisimov, Banks, Dine, and Graesser [4] have shown that an operator of this form is generated via quantum corrections in a number of toy models. Both Refs. [4] and [5] are speculative in the sense that the relevant calculation was not done in a well-defined noncommutative generalization of QCD. This stems from long-standing difficulties in formulating noncommutative SU(N) gauge theories [10,11]. (Similar difficulties arise in constructing noncommutative U(1) theories with fractionally-charged matter fields [9].) Recently, Jurčo et al. [12] have demonstrated how properly to construct a noncommutative SU(N) gauge theory by implementing consistency conditions order by order in the noncommutativity parameter $\theta^{\mu\nu}$. We review this formulation below. The purpose of our work is two-fold: we first present the Feynman rules derived from this consistent formulation of noncommutative QCD, which differ from those that have appeared previously in the literature [3]. Using these results, we show that an operator of the same form as Eq. (1.2) is indeed generated. From here we can immediately place a bound on the scale of spacetime noncommutativity.

In the conventional formulation of noncommutative gauge theory, one works with an equivalent theory of quantum fields that are functions of commuting spacetime coordinates by promoting ordinary multiplication to a Moyal star product,

$$f(\hat{x})g(\hat{x}) \rightarrow f(x) \star g(x) \quad (1.3)$$

where

$$f(x) \star g(x) = e^{\frac{i}{2} \frac{\partial}{\partial x^\mu} \theta^{\mu\nu} \frac{\partial}{\partial y^\nu} f(x) g(y)|_{y \to x}. \quad (1.4)$$

For a non-Abelian gauge group, with generators $T^a$ and a Lie algebra-valued gauge parameter $\alpha \equiv \alpha^a T^a$, one expects the gauge field $\hat{A}^\mu \equiv A^{\mu a}(\hat{x}) T^a$ to transform infinitesimally as

$$\delta \hat{A}^\mu = \partial^\mu \alpha + i[\alpha, \hat{A}^\mu]. \quad (1.5)$$

In particular, the commutator can be expanded in terms of the group generators,

$$[\alpha, \hat{A}] = \frac{1}{2} (\alpha_r \hat{A}_s + \hat{A}_s \alpha_r) [T^r, T^s] + \frac{1}{2} (\alpha_r \hat{A}_s - \hat{A}_s \alpha_r) \{T^r, T^s\}. \quad (1.6)$$
The problem in formulating an SU(N) gauge theory is now manifest: while the first term lives within an SU(N) representation, the second term does not. For example, in the case of SU(2), the second term is proportional to the identity, and tracelessness of the gauge generators is not maintained.

This problem has led others to focus on U(N) groups to approximate the phenomenology that might be relevant in a noncommutative SU(N) theory [3,4]. We will instead focus on a consistent formulation of noncommutative SU(N), and study the associated phenomenology directly.

II. NONCOMMUTATIVE SU(N)

The stumbling block presented by Eq. (1.6) is that it implies $\delta A$ lives in the enveloping algebra of the gauge group of interest. Jurčo et al. show that it is nonetheless possible to define gauge transformations in this larger algebra that depend only on the gauge parameter and fields of the desired theory. For concreteness, consider a noncommutative SU(N) gauge theory in which the fields transform infinitesimally as

$$\delta \alpha_\psi = i\Lambda\alpha \star \psi , \quad \delta \alpha A_\mu = \partial_\mu \Lambda\alpha + i[A\alpha \star A_\mu] .$$

(2.1)

Here, $\Lambda\alpha$ is a U(N) matrix function that we wish to associate with an element of SU(N) corresponding to the gauge parameter $\alpha$. The appropriate consistency condition is

$$(\delta \alpha \delta \beta - \delta \beta \delta \alpha)\psi(x) = \delta_{\alpha \times \beta} \psi(x) ,$$

(2.2)

where $\alpha \times \beta$ represents $\alpha_\beta f^{abc} T^c$, with $f^{abc}$ and $T^a$ the structure constants and generators of SU(N), respectively. With some algebra, one may show that this constraint is satisfied if

$$\Lambda\alpha[A^0] = \alpha + \frac{1}{4} \theta^{\mu\nu} \{\partial_\mu \alpha , A^0_\nu\} + O(\theta^2) ,$$

(2.3)

where $A^0_\nu \equiv A^0_{\nu a} T^a$ is the usual SU(N) gauge field. The formulation of Jurčo et al. requires the gauge parameter $\Lambda\alpha$ to be a nontrivial function of the gauge field $A^0$. The same holds
true for the fields $\psi$ and $A$: the requirement that the matter and gauge fields transform as in Eq. (2.4) implies that

$$A^\mu = A^0\mu - \frac{1}{4} \theta_{\rho\nu} \{ A^0\rho, \partial^\nu A^0\mu + F_{0\nu\mu} \} \quad (2.4)$$

and

$$\psi = \psi^0 - \frac{1}{2} \theta^{\mu\nu} A^0_\mu \partial_\nu \psi^0 + \frac{i}{4} \theta^{\mu\nu} A^0_\mu A^0_\nu \psi^0 \quad (2.5)$$

to linear order in $\theta$. While $A^0$ and $\psi^0$ have the usual transformation properties of fields in an SU(N) gauge theory, the Lagrangian expressed in terms of these fields is different. Starting with the action

$$S = \int d^4x \left[ \bar{\psi} \left( i \slashed{\nabla} - m \right) \psi - \frac{1}{2g^2} \text{Tr} F_{\mu\nu} \star F^{\mu\nu} \right], \quad (2.6)$$
in which

$$\slashed{D}_\mu \psi \equiv \partial_\mu \psi - i A^0_\mu \star \psi, \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu], \quad (2.7)$$

one may expand in terms of $\psi^0$, $A^0_\mu$, and $\theta$:

$$S = \int d^4x \left[ \bar{\psi}^0 \left( i \slashed{\nabla} - m \right) \psi^0 - \frac{1}{4} \theta^{\mu\nu} \bar{\psi}^0 F^0_{\mu\nu} \left( i \slashed{\nabla} - m \right) \psi^0 - \frac{1}{2} \theta^{\mu\nu} \bar{\psi}^0 \gamma^\rho F^0_{\rho\mu} i D_\nu \psi^0 \right.$$

$$- \frac{1}{2g^2} \text{Tr} F^0_{\mu\nu} F^0_{0\mu\nu} + \frac{1}{4g^2} \theta^{\mu\nu} \text{Tr} F^0_{\mu\sigma} F^0_{\rho\sigma} F^0_{0\rho\sigma} - \frac{1}{g^2} \theta^{\mu\nu} \text{Tr} F^0_{\mu\sigma} F^0_{\nu\sigma} F^0_{0\sigma} \right]. \quad (2.8)$$

Here we have corrected trivial typographical errors that appear in Ref. [12] and adopted Bjorken and Drell conventions [13] for the gamma matrices. Note that derivatives and field strengths now have their ordinary meaning,

$$\slashed{D}_\mu \psi^0 \equiv \partial_\mu \psi^0 - i A^0_\mu \psi^0, \quad F^0_{\mu\nu} \equiv \partial_\mu A^0_\nu - \partial_\nu A^0_\mu - i [A^0_\mu, A^0_\nu] \quad (2.9)$$

Equation (2.8) is, to order $\theta$, a Seiberg-Witten map [8] for noncommutative SU(N), i.e., a physically equivalent theory written in terms of fields that are functions of commuting spacetime coordinates.
Feynman rules may be extracted from Eq. (2.8) in the usual way. The complete $O(\theta^1)$ Lagrangian is seen to contain a variety of interaction vertices, namely, those with two fermions and 1, 2, and 3 gauge bosons, and pure gauge vertices with 3, 4, 5, and 6 bosons. Here we present the Feynman rules for vertices up to $O(g^2\theta^1)$; this includes the new quark-quark-gluon vertex that will be relevant to the calculation presented in the next section. Our conventions are as follows: The structure constants for SU(N) are defined by

$$[T^a, T^b] = i f^{abc} T^c \quad {\text{and}} \quad \{T^a, T^b\} = d^{abc} T^c + \frac{1}{N} \delta^{ab} \mathbb{1}.$$

In addition, we define the following abbreviations for tensor contractions: $\theta^\mu \cdot p \equiv \theta^\mu_{\nu} p^\nu$, and $p \cdot \theta \cdot q \equiv \theta^{\mu\nu} p_\mu q_\nu$, for any four-vectors $p, q$. Finally, it is convenient to introduce the totally antisymmetric three-index symbol

$$\theta^{\mu\nu\rho} \equiv \theta^{\mu\nu}_{\gamma} + \theta^{\nu\rho}_{\mu} + \theta^{\rho\mu}_{\nu}.$$

The Feynman rules for the $O(\theta^1)$ contributions to the vertices shown in Fig. 1 are

**qqg** vertex (i):

$$\frac{g}{2} T^a [\theta^\mu \cdot p (p' - m) - \theta^\mu \cdot p' (p' + m) - p' \cdot \theta \cdot p \gamma^\mu].$$

**qqgg** vertex (ii):

$$\frac{g^2}{2} \{ T^a T^b [m \theta^{\mu\nu} + \theta^{\mu\rho} (p + q)_\rho] - T^b T^a [m \theta^{\mu\nu} + \theta^{\mu\rho} (p + r)_\rho] \}.$$

**ggg** vertex (iii):

$$-\frac{1}{2} g d^{abc} \{ r \cdot \theta \cdot q [(q - r)^\mu g^{\nu\rho} + (p - q)^\rho g^{\mu\nu} + (r - p)^\nu g^{\mu\rho}] + (q^2 g^{\rho\nu} - q^\rho q^\nu) \theta^\mu \cdot r + (r^2 g^{\beta\nu} - r^\beta r^\nu) \theta^\mu \cdot q + (q^2 g^{\mu\nu} - q^\mu q^\nu) \theta^\rho \cdot p + (p^2 g^{\mu\nu} - p^\mu p^\nu) \theta^\nu \cdot r + (q \cdot p r^\nu - r \cdot q p^\nu) \theta^{\mu\rho} + (r \cdot q p^\rho - p \cdot r q^\rho) \theta^{\mu\nu} + (p \cdot r q^\mu - q \cdot p r^\mu) \theta^{\rho\nu} \}.$$

**gggg** vertex (iv):

$$\frac{g^2}{2} \{ T^a T^b [m \theta^{\mu\nu} + \theta^{\mu\rho} (p + q)_\rho] - T^b T^a [m \theta^{\mu\nu} + \theta^{\mu\rho} (p + r)_\rho] \}.$$
There are 24 permutations of the four gluon lines in the gggg vertex; four of these yield the explicit part of the result shown above, while the rest correspond to the indicated permutations. Note that the first, third and fourth vertices above provide $O(\theta)$ corrections to the corresponding standard model results, while the second has no standard model counterpart. These Feynman rules differ significantly from those quoted in Ref. [3] for noncommutative QCD. For example, only part of the Feynman rule for the quark-quark-gluon vertex in the construction described here can be interpreted as a simple phase factor expanded to order $\theta$. This has phenomenological consequences that we explore in the next section.

III. LORENTZ VIOLATION

Noncommutative theories violate Lorentz invariance. Experimental searches for Lorentz violation [14,16] allow one to place limits on the scale of noncommutativity through the effects of operators like the one in Eq. (1.2). In this section we isolate the most significant Lorentz-violating operators generated radiatively in noncommutative QCD, and then study their phenomenological implications.

Bounds from Lorentz violation have been studied previously [4–7]. In particular, operators like the one given above have been considered by Anisimov et al. [4] and shown to arise via quantum effects at the two-loop level in a theory with noncommutative Yukawa and U(1) gauge interactions. Mocioiu et al. [5] have suggested that stronger bounds could
arise from noncommutative effects in QCD, but did not show how to obtain this result from a consistent noncommutative, non-Abelian theory. With the formulation of noncommutative QCD suggested by the last section, we can explicitly calculate effective operators that violate Lorentz symmetry and find that they arise already at one loop.

Figure 2 shows the one-loop diagram that generates an effective $q\theta^{\mu\nu}\sigma_{\mu\nu}q$ operator at lowest order in perturbation theory. (The diagram with a single $qqgg$ vertex vanishes since this vertex is antisymmetric in its Lorentz indices, while the gluon propagator is symmetric.) For noncommutative QED with one charge \[9\], as well as noncommutative U(N) \[3\], the diagram in Fig. 2 is the same as in the commutative case, because the relevant Feynman rules give only phases at each vertex, which precisely cancel in the amplitude. For noncommutative SU(N) we have

$$i\mathcal{M} = -g^2T^aT^a \int \frac{(dq)}{q^2 - \lambda^2} \times \left\{ \gamma^\mu(1 - \frac{i}{2}p \cdot \theta \cdot q) + \frac{i}{2} \theta^{\mu\nu}[(p + q)\nu(p - m) - p\nu(p + q - m)] \right\}$$

$$\times \frac{1}{\bar{p} + \bar{q} - M} \times \left\{ \gamma^\rho(1 - \frac{i}{2}q \cdot \theta \cdot p) + \frac{i}{2} \theta^{\rho\nu}p^\tau(p + q - m) - (p + q)^\tau(p - m) \right\}.$$ \hspace{1cm} (3.1)

where $(dq) \equiv d^4q/(2\pi)^4$. To evaluate the diagram as shown, we set $\lambda = 0$ and $M = m$; the more general notation is useful since we will employ Pauli-Villars regularization to handle the divergences.

Keeping just the $O(\theta)$ terms,

$$i\mathcal{M}(\lambda^2, M^2) = \frac{2}{3}g^2\{(\bar{p} - m), \sigma_{\mu\alpha}\} \times \int \frac{(dq)}{(q^2 - \lambda^2)((p + q)^2 - M^2)}q^\alpha\theta^{\mu\nu}(p + q)\nu,$$ \hspace{1cm} (3.2)

for SU(3). This result is gauge invariant; it would be obtained in any generalized Lorentz gauge. The Pauli-Villars regulated amplitude is $\mathcal{M} \to \mathcal{M}(0, m^2) - \mathcal{M}(\Lambda^2, m^2) - \mathcal{M}(0, \Lambda^2) + \mathcal{M}(\Lambda^2, \Lambda^2)$, where $\Lambda$ is a large mass scale. The result is

$$\mathcal{M} = \frac{g^2}{96\pi^2}\left\{ (m - \bar{p}), \Lambda^2\theta^{\mu\nu}\sigma_{\mu\nu} \right\} - \frac{2}{3}\left\{ (m - \bar{p}), p\mu\theta^{\mu\nu}\sigma_{\nu\tau}p^\tau\ln\Lambda^2 \right\},$$ \hspace{1cm} (3.3)

for the term leading in $\Lambda$ for each Dirac structure. The same result follows if one just cuts off the integral in $\mathcal{M}(0, m^2)$ at a large (Euclidean) $q^2 = \Lambda^2$, the approach that was adopted in \[4\].
We conclude that at lowest order in perturbation theory, the formulation of noncommutative QCD that we have described leads to the set of Lorentz violating operators

\[ m\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q, \quad \theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}\not{D}q, \quad \theta^{\mu\nu}D_{\mu}\bar{q}\sigma_{\nu\rho}D_{\rho}q, \quad (3.4) \]

with coefficients as given by Eq. (3.3). Note that an operator with three derivatives can be eliminated using \( \{\not{p}, \sigma_{\nu\tau}\not{p}\} = 0 \).

Phenomenological bounds follow because the noncommutativity matrix \( \theta^{\mu\nu} \) has a fixed orientation. The most significant phenomenological bound comes from the first term in Eq. (3.3), which corresponds to the leading divergence. Part of \( \theta^{\mu\nu}\sigma_{\mu\nu} \) acts like a \( \vec{\sigma}\cdot\vec{B} \) interaction with a fixed \( \vec{B} \), which leads to sidereal variations in, for example, hyperfine energy splittings. Such variations in the differences between Cs and Hg atomic clocks, which have different sensitivities to an external \( \vec{\sigma}\cdot\vec{B} \)-like interaction, are bounded at the \( 10^{-7} \) Hz, or few \( \times 10^{-31} \) GeV, level [14].

We estimate our operator in a nucleon or nuclear environment. The up and down quarks are very light, but are off shell by about the mass of a “constituent quark,” and we take \( p^0 \approx m_{\text{cons}} \approx 300 \) MeV and \( \not{p} \approx 0 \). Then,

\[ \frac{\alpha_s}{12\pi}\Lambda^2\theta m_{\text{cons}} < \Delta E, \quad (3.5) \]

where \( \theta \) is a typical scale for elements of the matrix \( \theta^{\mu\nu} \) and \( \Delta E \) is a bound on the sidereal variation of an energy difference. For the atomic clock bound cited above, and using \( \alpha_s = 1 \), one finds

\[ \theta\Lambda^2 \lesssim 10^{-29} . \quad (3.6) \]

**IV. DISCUSSION**

We have derived the Feynman rules for a consistent formulation of noncommutative QCD and have used them to compute the most dangerous, Lorentz-violating operator that is generated through radiative corrections. While this operator vanishes when quarks are on shell,
those in a nucleon are typically off shell by an amount comparable to a constituent quark mass, and thus the matrix element between nucleon states should remain nonvanishing. From here we obtained the approximate bound

$$\theta \Lambda^2 \lesssim 10^{-29},$$

(4.1)

where $\Lambda$ is an ultraviolet regularization scale. This result follows from tests of Lorentz invariance in clock-comparison experiments, sensitive to the sidereal variation in the relative Larmor precession frequencies of Hg and Cs [14]. The calculational uncertainty in our analysis lies in the extrapolation of our perturbative result to the nonperturbative regime in which nucleon magnetic moments and hadronic matrix elements are evaluated. While we do not address this issue directly here, it is clearly reasonable to assume that our nonvanishing result changes smoothly as the QCD coupling is increased, and that the estimate of Eq. (3.5) gives a reasonable indication of the magnitude of the effect. This has been the approach adopted almost uniformly in the literature [4,5], pending a more detailed strong interaction calculation. We have also shown that our result persists in two different regularization schemes (and no doubt will appear in others). Even allowing for uncertainty in how the regularization scale is best defined, it is clear that if $\Lambda \sim 1$ TeV (as one would expect if the Planck scale is low) then Eq. (3.1) requires $\Lambda_{NC} \gtrsim 10^{17}$ GeV, which is far above the cutoff. This observation, or equivalently the appearance of an extremely small dimensionless number in the theory, Eq. (1.1), is an indication of the unnaturalness of spacetime noncommutativity, as it is presently defined.

**Acknowledgments**

C.E.C. and C.D.C. thank the National Science Foundation for support under Grant No. PHY-9900657. In addition, C.D.C. thanks the Jeffress Memorial Trust for support under Grant No. J-532. R.F.L. thanks the Department of Energy for support under Grant No. DE-AC05-84ER40150. We thank P. Amore, A. Aranda, A. Armoni, A. Kostelecký, and N. Zobin for useful communications.
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FIG. 1. Feynman vertices up to $O(g^2\theta^1)$.

FIG. 2. A diagram contributing to $\bar{q}\theta^{\mu\nu}\sigma_{\mu\nu}q$. 