Effects of single-particle potentials on the level density parameter

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Abstract

The new definition of the energy dependence for the level density parameter including collective effects depends strongly on the semi-classical approach. For this method, defining an accurate single-particle potential is of great importance. The effect of the single-particle potential terms, which are central, harmonic oscillator, Woods-Saxon and Coulomb potential, on the level density parameter was investigated by examining the local success of the global parameterizations of four different combinations of these terms. Among these combinations, the sum of the central, harmonic oscillator and Coulomb potentials, gives the most accurate predictions compared with experimental data. The local selections of the global parameterizations show that the single-particle models, which are based on Woods-Saxon potential as main term, are more suitable candidates than the models based on harmonic oscillator potential to extrapolate away far from stability. Also it can be concluded that the contribution of the Coulomb interaction, both around the closed and open shells is not neglectable.

Keywords: nuclear level density, semi-classical, single-particle potentials, harmonic oscillator, Woods-Saxon, Coulomb potential, collective effects

1. Introduction

The number of the excited states in an infinitesimal amount of energy around a certain excitation energy is called as the nuclear level density (NLD). The NLD is of vital importance for the theoretical studies of nuclear structure and reactions. The excited levels of the nucleus are very scarce at low excitation energy and can be countable easily, but with the increasing excitation energy, it is not possible to count the levels since the spacing between consecutive levels becomes so narrow. Therefore, a function is needed to describe the distribution of the excited levels. The population of the excited levels is unique for each nuclei, and the knowledge of this distribution is very important for the Hauser-Feshbach calculations of the compound-nucleus cross sections.

To develop a theoretical framework for understanding the unusual properties of the light exotic beams has been of major interest during the last few decades (Tanihata, 1985). Even though very sophisticated nuclear reaction models (Lapoux et al., 2008; Rawitscher, 1974; Sakuragi et al., 1972, 1986; Satchler and Love, 1979; Satchler, 1983; Sinha, 1975; Tamura, 1968; Tobocman and Kalos, 1955; Yamagata et al., 1989), which give accurate predictions in many cases, have been developed, solution of this complex problem cannot be obtained without also considering their structural properties, such as the distribution of their excited levels, arising from both pure single-particle and collective excitations.

A Laplace-like formula for the level density parameter including collective effects has been proposed in a recent paper of ours (Canbula et al., 2014). The new definition of the energy dependence for the level density parameter significantly improved the agreement between predicted and experimental data.

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observed excited energy levels. Furthermore, the asymptotic level density parameter, $\bar{a}$, is redescribed in a more physical and more realistic way. This redesignation takes into account corrections for both the shell and pairing effects in addition to the value obtained from the analytic semi-classical approximation. Using the semi-classical approximation (Canbula and Babacan, 2011) requires a well-defined single-particle potential because it directly determines $\bar{a}$, which is the limit value of the level density parameter for the energies above the neutron separation energy. Therefore, the single-particle potential almost remains the only component to improve the success of the reaction calculations that uses the level density as an ingredient. In other words, the single-particle potential parameterization is still of great importance for the level density.

In our previous work (Canbula et al., 2014), we have performed the global and the local calculations to obtain $\bar{a}$. In global calculation, we used the single-particle potential consists of harmonic oscillator, Coulomb and central potential terms with global potential parameters. In contrast, the local calculation considers the asymptotic level density parameter as a free parameter to be adjusted to the experimental data on the mean resonance spacing and discrete level scheme for each nuclei separately. Although, as expected, locally adjusted values of the asymptotic level density parameter provide much better agreement with the experimental data as compared with the global parameterization, disregarding of the global potential parameters is not permissible, especially, for the nuclei near the driplines. Since there is not enough experimental information on the excited energy levels of these nuclei, this situation makes impossible to adjust the asymptotic level density parameter locally, and therefore, to rely on a global parameterization becomes an obligation rather than a choice.

There are nearly 2000 nuclei, which have at least two experimentally-known excited energy levels (Capote et al., 2009). It seems that the global potential which has the highest predictive power is harmonic oscillator with Coulomb and central terms for all these nuclei over the whole mass range (Canbula et al., 2014). However, various combinations of potential terms can be more suitable in certain mass regions. Moreover, there can be possible correlations between the potential choice and some other properties of nuclei, not only their mass number. In this manner, it is worth to investigate that the behavior of the goodness-of-fit estimators, which results from the global parameterizations of different single-particle potentials, respect to some fundamental properties as well as the mass number of nuclei.

The aim of this paper is to investigate the role of the single-particle potential in the prediction power of the semi-classical level density model for four different combinations of the potential terms as comparing the global parameterizations to each other and also analyzing the local success of them. For this purpose four difference combinations are constructed on the basis of two main potentials, which are harmonic oscillator and Woods-Saxon, with and without Coulomb interaction combined with the central term for all combinations.

The present paper is organized as follows: In Section 2, we briefly discuss the analytical method for obtaining the asymptotic level density parameter $\bar{a}$. Section 3 includes the definition of the goodness-of-fit estimators for phenomenological level density models. Then, in Section 4 we present the single-particle potentials, which we used in this work, and discuss our results for 1136 nuclei. Finally, a summary of our model and some concluding remarks of this paper are given in Section 5.

2. Theory

Many studies of the nuclear level density have been based on the Fermi gas model (Bethe, 1937) in which interactions between nucleons are ignored, therefore, nucleons are assumed to occupy equispaced single-particle states arising from an average nuclear potential. According to this model, one can describe the level density at an excitation energy for a certain total angular momentum $J$ and parity $\Pi$.

$$\rho(U, J, \Pi) = \frac{1}{2} \sqrt{2 \pi \sigma^3} \exp \left[ -\frac{(J + \frac{1}{2})^2}{2\sigma^2} \right] \times \frac{\sqrt{\pi} \exp \left[ 2 \sqrt{aU} \right]}{12 \ a^{1/4} U^{3/4}}$$

(1)

where the factor $\frac{1}{2}$ corresponds equiparity. The remaining ingredients $a$, $U$ and $\sigma^2$ represent the level density parameter, the effective excitation energy and the spin cut-off.
parameter, respectively. For the Fermi gas model, the total level density is described as summing Eq. (1) over all spins

\[ \rho_n(E_s) = \frac{1}{\sqrt{2\pi \sigma}} \sqrt{\pi} \exp\left(\frac{2}{\sqrt{\pi}} U\right). \]  

(2)

The collective effects arising from the collective motion of many nucleons, were not taken into account in the Fermi gas model. However, these effects play an important role in populating the excited states. In the later studies [Hagelund and Jensen, 1977; Ignatyuk, 1983], collective effects have been considered as vibrational and rotational effects separately and included in the model as additional enhancement factors to total level density. In our recent work [Canbula et al., 2014], we have introduced a Laplace-like formula for the energy dependence of the level density parameter which spread the collective effects through the whole level density calculation. The level density parameter \( a_c \) including collective effects is given by

\[ a_c(U) = \tilde{a} \left(1 + A_c S_n \exp\left(-\frac{|U - E_0|}{\sigma_c^2}\right)\right). \]  

(3)

where \( S_n \) is the neutron separation energy, \( A_c \) is the collective amplitude and defined as the shape dependent shell (microscopic) correction energy at a critical temperature \( T_c \), which is the nuclear temperature at \( S_n \). As mentioned above, \( \sigma_c^2 \) is the spin cut-off parameter where \( c \) indicates to its value at \( T_c \), for further details see ref. [Canbula et al., 2014]. \( E_0 \) is the energy of the first phonon level which corresponds to the first excited state caused by vibrational effects [Krane, 1987; Rowe, 1970]. This energy level is also known as the first \( 2^+ \) excitation state for even-even nuclei [Krane, 1987] and its energy can be given with a simple formula 0.2ℏω0 fitted to experimental \( 2^+ \) states [Ring and Schuck, 1980; Siegbahn, 1965]. Finally, having a key role in this study, \( \tilde{a} \) is defined in different forms by many authors. The simplest expression of this parameter is given by [Ericson, 1960; Ignatyuk, 1983]

\[ \tilde{a} = \frac{A}{k} \]  

(4)

or can be taken as liquid drop like formula [Bartel et al., 2006]

\[ \tilde{a} = a_{vol} \left[1 + k_{vol} \left(\frac{N - Z}{A}\right)^2\right] A^+ \]

\[ a_{sur} \left[1 + k_{sur} \left(\frac{N - Z}{A}\right)^2\right] A^{2/3} + a_{Coul} Z^2 A^{-1/3}. \]  

(5)

Another expression for this parameter, fitting to resonance spacings and/or discrete levels, can be written as [Iljinov et al., 1992]

\[ \tilde{a} = a A + \beta A^{2/3} \]  

(6)

where \( \alpha \) and \( \beta \) are adjustable parameters.

Unlike the above expressions, we used a modified expression of the well-known semi-classical formula [Bohr and Mottelson, 1998; Brack and Bhaduri, 1997] for the asymptotic level density parameter in terms of the single-particle level density at Fermi energy of nucleus including the shell and pairing corrections [Canbula et al., 2014]

\[ \tilde{a} = \frac{\pi^2}{6} \left[g_p(E^p_F + S(N, Z) - \Delta) + g_n(E^n_F + S(N, Z) - \Delta)\right]. \]  

(7)

\( S(N, Z) \) denotes the shell correction energy from the liquid drop model [Myers and Swiatecki, 1964]. The pairing correction energy is given by \( \Delta = n \frac{\hbar^2}{4m A} \) with \( n \) is −1 for odd-odd, 1 for even-even, 0 for odd nuclei. Therefore, including the energy shift \( \Delta \) to the Fermi energy substitutes the usage of the expression \( U = E_c - \Delta \) and allows to use effective excitation energy \( U \) instead of pure excitation energy \( E_c \) directly. \( g_p \) and \( g_n \) are proton and neutron single particle level density, respectively, and can be calculated from the semi-classical formula with spin degeneracy [Brack and Bhaduri, 1997; Salasnich, 2004]

\[ g(\varepsilon) = \frac{2}{\pi} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int r^2 \sqrt{\varepsilon - V(r)} \, dr. \]  

(8)

\( m \) is the mass of nucleon and \( V(r) \) is an effective potential. The proton and neutron Fermi energy values \( E^p_F \) can be obtained by inverting the integral, which gives the nucleon number in terms of single-particle level density

\[ N_n = \int_{-\infty}^{E^p_F} g_n(E) dE, \quad N_n = \{N, Z\}. \]  

(9)
Therefore, the crucial role of the single-particle potential in the asymptotic level density parameter motivated us to investigate the effects of the single-particle potential description to predictive power of the semi-classical level density model. For this purpose, we considered four different combinations of various single-particle potential terms and analyzed the results in the view of agreement between their predictions and observations.

3. Goodness-of-fit Estimators

Phenomenological level density models have been needed to agree with two observable, which are average resonance spacings and discrete level schemes. One can test the reliability of the level density models with the aid of these observable. In this study, we have calculated the rms deviation factor of the mean resonance spacings for 289 nuclei, which exist naturally on Earth, and their experimental average resonance spacing data are available. However, the average goodness-of-fit estimator of discrete levels for 1136 nuclei, which have sufficient information on the discrete energy level scheme. The goodness-of-fit estimator $\chi^2$ for average resonance spacings has been minimized to follow as

$$\chi^2_{D,ij} = \left( \frac{D^{\text{theo}}_{0,i} - D^{\text{exp}}_{0,j}}{D^{\text{err}}_{0,j}} \right)^2$$

where the index $i$ indicates the nucleus. $D^{\text{exp}}_{0,i}$ and $D^{\text{err}}_{0,j}$ are respectively experimental data and the uncertainty of the average resonance spacing which its theoretical predictions are obtained from the equation below

$$\frac{1}{D^{\text{theo}}_{0,i}} = \sum_{J=|l-\frac{1}{2}|}^{J=l+\frac{1}{2}} \rho(S_n, J, \Pi).$$

Unlike the average resonance spacings, goodness-of-fit estimator for discrete levels has no experimental error in the cumulative level scheme and is given by

$$\chi^2_{\text{lev},ij} = \sum_{k=N_L}^{N_j} \frac{[N^{\text{cum}}_{\text{lev}}(E_k) - k]^2}{k}.$$  

These estimators allow us to test the agreement between our predictions and experimental data besides making comparisons with the results of the other level density models. The rms deviation factor of mean resonance spacings, which is defined as for all $N$ nuclides reads

$$f_{\text{rms}} = \exp \left[ \frac{1}{N} \sum_{i=1}^{N} \ln \left( \frac{\rho^{\text{theo}}_{0,i}}{D^{\text{exp}}_{0,i}} \right) \right]^{1/2}$$

and the average goodness-of-fit estimator for discrete levels is

$$f_{\text{lev}} = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=N_L}^{N_j} \left( \frac{N^{\text{cum}}_{\text{lev}}(E_k) - k}{k} \right)^2.$$ 

4. Results and Discussion

In our recent work (Canbula et al., 2014), we have calculated the nuclear level density parameter by using the semi-classical approximation with the single-particle potential consists of harmonic oscillator and central potential terms and also Coulomb potential for protons. That study (Canbula et al., 2014) leads us to investigate the single-particle potential’s role of choosing the best agreement for each nuclei in level density calculations. In this study, we used various single-particle potential terms, which are central, harmonic oscillator, Woods-Saxon, and Coulomb potentials. Because of their relatively small effects, spin-orbit and other similar terms are not included into calculations. We have considered four different combinations constructed from these single-particle potential terms:

$$V(r) = V_{\text{central}}(r) + V_{\text{main}}(r) + V_{\text{Coulomb}}(r).$$

The central potential is taken into account for all potential combinations and is given by the equation below

$$V_{\text{central}}(r) = \hbar^2 l(l + 1)/2mr^2$$

where $l$ is the angular momentum. As the main potential, we used harmonic oscillator (HO) or Woods-Saxon (WS)
Table 1: (Color online) Goodness-of-fit estimator values for various potential combinations and comparison with previous results.

| Model | Potential Terms | \( f_{\text{rms}} \) | \( f_{\text{lev}} \) | Reference |
|-------|----------------|----------------|----------------|-----------|
| Model 0 | Local Selections | 1.34 | 1.12 | This work |
| Model 1 | Central + HO | 1.59 | 2.07 | This work |
| Model 2 | Central + HO + Coulomb | 1.51 | 1.37 | This work, (Canbula et al., 2014) |
| Model 3 | Central + WS | 1.99 | 1.84 | This work |
| Model 4 | Central + WS + Coulomb | 2.00 | 2.13 | This work |
|          HO            |              | 1.12 | 43.9 | (Canbula and Babacan, 2011) |
|          HO + Coulomb  |              | 1.16 | 42.6 | (Canbula and Babacan, 2011) |

potential. The HO potential, which has a very convenient form for analytical calculations, is described as follows

\[
V_{\text{HO}}(r) = \frac{1}{2} m \omega^2 r^2 - V_0
\]  

where \( \omega \) is the oscillator frequency, and it has been generally parameterized as \( 41/A^{1/3} \). The depth of the potential well, \( V_0 \), is taken as 50 MeV (Bohr and Mottelson, 1998). These parameters are chosen to represent only the common properties of the nuclei because the other effects like shell and pairing corrections will be applied to Fermi energy explicitly as in Eq. (7). The other option for the main potential is the WS potential, which is a more realistic description compared to HO meanwhile to use WS in calculations causes some difficulties due to the inability to obtain analytical solutions. The WS potential is written as

\[
V_{\text{WS}}(r) = -\frac{V_0}{1 + \exp \left( \frac{r-R}{a} \right)}
\]  

where \( V_0 \), \( R \) and \( a \) are depth, radius and diffuseness parameters of the potential well, respectively. In this study, potential well depth is taken as 50 MeV to be consistent with the HO potential, and radius is defined as \( R = r_0 A^{1/3} \) where \( r_0 \) equals 1.25 fm. Also, diffuseness parameter of the potential well is used as 0.5 fm in calculations.

Furthermore, the potential combination consisting of the central potential term and the selected main potential is complemented with Coulomb potential for protons to investigate Coulomb interaction on the level density parameter. Under the assumption that nucleus is a uniformly charged sphere, the Coulomb potential is given by

\[
V_C(r) = \begin{cases} 
\frac{Ze^2}{2R_C} \left( 3 - \frac{r^2}{R_C^2} \right) & r \leq R_C \\
\frac{Ze^2}{r} & r \geq R_C
\end{cases}
\]  

where \( R_C \) is charge radius and is taken as \( R_C = 1.169A^{0.291} \) from a recent fit (Bayram et al., 2013) to the latest nuclear charge radii data (Angeli and Marinova, 2013).

However, one would note that the first single-particle level has never been in the bottom of the potential well, and this situation should be considered when calculating the Fermi energy level. Therefore, the contribution from the interval between the bottom of the potential well and the first single-particle level with the lowest energy should be zero in the integral. The value of the first single-particle level is well known for the HO potential but to make a similar prediction for WS potential might be difficult. So, considering that the total depth of the well is approximately equal to the sum of the Fermi energy and binding energy (Bohr and Mottelson, 1998), to determine a value for this interval is a reasonable correction which has applied as first 8 MeV for the WS potential.

In Table 1, we define four different single-particle potential combinations with the potential terms which are given above. It also represents the \( f_{\text{rms}} \) and \( f_{\text{lev}} \) values compared with the preceding semi-classical BSFGM calculations (Canbula and Babacan, 2011; Canbula et al., 2014). The results of the models from 1 to 4 are obtained from the global calculations covering all 1136 nuclei, of which model 2 gives the best values of \( f_{\text{rms}} \) and \( f_{\text{lev}} \). In contrast to previous work (Canbula and Babacan, 2011), which examined the effect of the Coulomb potential in the
Table 2: Details of the selections of model 0. The first line represents 289 stable isotopes. The third line represents 1136 isotopes while the selections of the remaining 847 isotopes are given in the second line.

| NoI  | Model 1 | Model 2 | Model 1+2 | Model 3 | Model 4 | Model 3+4 |
|------|---------|---------|-----------|---------|---------|-----------|
|      | NoI     | %       | NoI       | %       | NoI     | %         |
| 289  | 88      | 30.4    | 111       | 38.4    | 199     | 68.8      |
|      | 47      | 16.3    | 43        | 14.9    | 90      | 31.2      |
| 847  | 233     | 27.5    | 102       | 12.0    | 335     | 39.5      |
|      | 342     | 40.4    | 170       | 20.1    | 512     | 60.5      |
| 1136 | 321     | 28.2    | 213       | 18.8    | 534     | 47.0      |
|      | 389     | 34.2    | 213       | 18.8    | 602     | 53.0      |

NoI stands for number of isotopes.

level density parameter, the addition of the Coulomb term
to the HO potential improves the agreement between the
predictions and the observable significantly. The contra-
riety of the results of two studies can be explained by the
difference between the simple description of the charge
radius used in ref. [Canbula and Babacan, 2011], and the
expression deduced from a recent fit (Bayram et al., 2013)
which is employed in this work. Additionally, both mod-
els 1 and 2 based on HO potential give better results com-
pared to models 3 and 4, which are based on WS potential
as the main term, which is rather unexpected because of
the fact that the WS potential is more realistic than the HO
potential. This unforeseen aspect can be explained by the
need for a more comprehensive parameterization of the
WS potential parameters to describe the properties of the
nuclei as much as the HO potential can.

Even though the comparison between these four mod-
els gives a general idea about their success to describe the
common properties of the most of the nuclei, in the case of
the extrapolation to certain mass regions, especially near
the driplines, this point of view becomes deficient. There-
fore, to test the predictions of the global potential param-
terizations locally for each nuclei is the only thing can
help us to conclude that which potential model is suitable
to extrapolate outside the certain mass regions. Consider-
ing this situation, we define model 0, which consists of
the local selections among the global parameterizations of
models 1-4. In model 0, the selections have been made to
give the lowest $\chi^2_i$ contribution to global $\chi^2$ values for the
considered nucleus. Local model selections chosen with
criterion are illustrated in the upper and lower panels of Figure 1 for 289 and 1136 nuclei, respectively.

In Figure 1 the local model selections of 289 stable
isotopes of the total of 1136 isotopes are shown explicitly
in the upper panel because of the interpretations about the
stable isotopes should be completely different from the
stable isotopes.

Figure 1: (Color online) The local selections of the single-particle com-
binations. The upper panel includes 289 stable isotopes while the lower
panel includes 1136 isotopes. Cyan, yellow, magenta and black colored
dots donate models 1 to 4, respectively. The magic numbers are shown
with grid lines.
rest of the isotopes. As already mentioned above in the discussion of Table 1 for stable isotopes, most of the selections are in model 1 and 2, which are HO based single-particle potentials. As seen from Figure 1 model 2 is the almost only option in the heavy mass, $Z > 70$, region, therefore it can be said that the effect of the Coulomb interaction becomes indispensable with the increasing proton number. Also in $Z < 40$ region, model 2 selections are quite intense. However, in the case of 1136 isotopes, a significant increase has been noted in the number of selections of model 3 and 4, which are based on WS potentials. These selections are notably more abundant in the region covering the exterior side of the island shown in the lower panel of Figure 1.

On the other hand, to quantify the total numbers of selections of models can be useful to conclude that which model is better to extrapolate to far from stability. Table 2 shows this quantification. Besides models 1-4, the cumulative number of selections of models based on the same main potential term are shown in Table 2. It can be easily understood that the local selections of 289 stable isotopes, and the remaining 847 isotopes are completely different from each other. For 289 stable isotopes WS potential based models have been selected by only 31.2 percent, however, for remaining 847 isotopes, selection rate increases to 60.5 percent. In contrast to overwhelming superiority of model 2, which is clearly seen from Table 1, WS potential based models, especially model 3, seem to have had considerable success in describing the properties of the isotopes far from stability. Moreover, in the case of 1136 isotopes, the total numbers of selections of models 3 and 4 are slightly more than that of models 1 and 2. When the selections are expanded from 289 isotopes to 1136, model 2, which is the most selected model, becomes equal to model 4, which is the least selected model. Therefore, it can be concluded that HO potential is very suitable for describing the most of the nuclei, but some certain isotopes or mass regions can be described better with WS potential. Nevertheless, WS potential is not suitable for a generalization to the entire mass region, at least with the potential parameters used in this work.

In Figure 2 $f_{\text{rms}}$ and $f_{\text{lev}}$ values are shown as a function of the mass number, in the upper and the lower panels, respectively. In the mass region $A < 40$, $f_{\text{rms}}$ values resulting from models 3 and 4 are better than the values of models 1 and 2, but in the rest, they have tended to be worse. For both $f_{\text{rms}}$ and $f_{\text{lev}}$ values around mass numbers 40 and 80, there is a common peak for all models, which means a failure to describe the properties of isotopes in this mass region. Model 1 gives better $f_{\text{rms}}$ values until the mass number around 190, but after this region, model 2 starts to give better results than all the other models. In a similar way, $f_{\text{lev}}$ values of model 1 also rise dramatically after the mass number around 150 in contrast to smooth behavior of the other models. Also $f_{\text{lev}}$ values of models 3 and 4 are split by a small amount at the mass region around 75 and keep this difference in the further mass region, whereas they are almost equal below the mass num-
Figure 3: (Color online) The asymptotic level density parameter values obtained by using four different single particle potential combinations for 1136 isotopes. Cyan and magenta colored dots denote the HO potential without and with Coulomb potential, and given in the upper-left and the upper-right panels, respectively. Remaining yellow and black colored dots present the WS potential without and with Coulomb potential and are given in the lower-left and the lower-right panels, respectively.

The asymptotic level density parameter values, which are obtained from four different single-particle combinations are illustrated in Figure 3. HO based models are given in upper panels while WS based models are shown in lower panels. The single-particle potential models with Coulomb interaction are given at the right side for both in upper and lower panel. The first impression that emerges from Figure 3 is the existence of the downwards peaks around the closed shells for all models. However, there are also upwards peaks around the open shells only for models 3 and 4, which are WS based on single-particle potentials. In addition, the Coulomb term seems to increase the depth and the height of peaks for models 2 and 4, respectively, compared to models 1 and 3.

In Figure 4 are plotted the asymptotic level density parameter values of model 0, which is the selected values of global parameterizations contributing the least $\chi^2_i$ to global $\chi^2$. The color codes correspond to the single-particle potential models, which the asymptotic level density parameters originated from. Models based on HO potential are selected only for the isotopes which are weakly influenced by the shell effects. This situation is also consistent with the issue which is mentioned above in the dis-
Figure 4: (Color online) The asymptotic level density parameter values obtained by using model 0, which is the local selections of the global parameterizations of models 1-4. Cyan, magenta, yellow and black colored dots indicate the results obtained from model 1 to 4, respectively.

5. Conclusions

Summarizing, the effect of the single-particle potential terms, which are central, harmonic oscillator, Woods-Saxon and Coulomb potential, on the level density parameter was investigated by examining the local success of the global parameterizations of four different combinations of these terms. In the light of above discussions, the following conclusions can be drawn from this study:

(i) Model 2, which is the sum of the central, harmonic oscillator and Coulomb potentials, gives the most accurate predictions compared to experimental data.

(ii) The local selections of the global parameterizations indicate that the single-particle models, which are based on Woods-Saxon potential as the main term, are more suitable candidates than the models based on harmonic oscillator potential to extrapolate away far from stability.

(iii) It is seen from the investigation of the asymptotic
level density parameters obtained from the local selection that the contribution of Coulomb interaction is not ignorable both around the closed and open shells.

(iv) Finally, for the exotic nuclei, which have not any experimental information to adjust the level density parameters, the single-particle potential consists of the central, Woods-Saxon, and Coulomb potential terms is the most reliable potential model to calculate the asymptotic level density parameter.

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