Testing Tensor-Vector-Scalar Theory with latest cosmological observations

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Tensor-Vector-Scalar (TeVeS) is considered as a viable theory of gravity. It produces the Milgrom’s modified Newtonian dynamics in the nonrelativistic weak field limit and is free from ghosts. We perform the test of TeVeS theory by confronting to cosmological observations. We find that by including a sterile neutrino, it can mimic ΛCDM model and is compatible with Planck data. But there are tensions between Planck and supernova measurements or recent observations of the kinetic Sunyaev-Zel’dovich effect. We observe that the growth rate in TeVeS theory is scale dependent, which can serve as a potential probe in distinguishing it from ΛCDM model given future precise observations.

I. INTRODUCTION

The convincing observational evidences from the scale of galaxies to the scale of the Cosmic Microwave Background (CMB) radiation accumulated over the past few decades raised the missing mass problem: there is a mismatch between the dynamics and distribution of visible matter [1–10]. To explain this problem, one usually postulate the existence of a new form of matter in nature, called dark matter (DM). DM is considered non-baryonic and does not emit light or interact with electromagnetic field. The only way to detect DM is through its gravitational effect. Traditionally, DM can be classified into “hot dark matter”, which is composed of relativistic particles such as massive neutrinos; “cold dark matter” (CDM), which is composed of very massive slowly moving and weakly interacting particles; and in between the possible “warm dark matter”, which is also sometimes considered. One attributes the observed extra gravitational force to DM component whose abundance greatly exceeds the visible matter. In the standard ΛCDM model, DM contributes about 25% to the total energy budget in the universe.

The discrepancy between the dynamics and distribution of visible matter happens on galactic to cosmological scales. Decades after the proposal of DM, it is discovered that the expansion of our universe is accelerating, which calls for another new substance, dark energy (DE) to contribute the mysterious missing energy at cosmological scales. The Einstein’s General Relativity (GR) has been vigorously tested in the solar system, but on galaxy or larger scales its validity has not been completely proved. Considering that the law of gravity plays a fundamental role at every instance where discrepancies have been observed, it is possible that the phenomena attributed to DM and DE are just a different theory of gravity in disguise. The research relates to modifications of gravity is not extensive. In literatures, modified gravity theories usually contain a Newtonian limit for low velocity, weak potential case. Considering that mass discrepancy problem appears on extragalactic scales where Newtonian gravity is expected to be a good approximation, these theories cannot solve the problem without the help of invisible matter component. This has been resolved in the Milgrom’s Modified Newtonian Dynamics(MOND) proposal[11–13], which assumes that Newtonian gravity fails in low acceleration cases. Instead, the acceleration squared is proportional to the induced by the gravitational force was proposed as $\tilde{\mu}(a/a_0)a = -\nabla\Phi_N$, where $a_0$ is a characteristic acceleration scale, $\Phi_N$ is the usual Newtonian potential. $\tilde{\mu}(x) \approx x$ for $x \ll 1$ and $\tilde{\mu}(x) \to 1$ for $x \gg 1$. In laboratory and solar system experiments, $a \gg a_0$, MOND returns to the Newtonian dynamics; while in extragalactic regime where $a \ll a_0$, the acceleration squared is proportional to the gravitational force. MOND is extremely successful in explaining galactic rotation curve [14–20] and the Tully-Fisher law [21, 22]. Some other predictions of MOND can be found in [23–28].

To be able to make predictions for cosmological observations, a relativistic theory of MOND is required. After some early attempts [29–33], Bekenstein succeeded in constructing Tensor-Vector-Scalar (TeVeS) theory [34], which is a relativistic theory of gravity and produces MOND in the nonrelativistic weak field limit. The name comes from that the theory contains a scalar and a vector field in addition to the metric (a tensor field). TeVeS theory has been proved successful in explaining astrophysical data at scales larger than that of the solar system without the need of an excessive amount of invisible matter [35–42]. Moreover TeVeS theory is proved free of ghosts [43], which makes TeVeS, including a sterile neutrino, it can mimic ΛCDM model and is compatible with Planck data. But there are tensions between Planck and supernova measurements or recent observations of the kinetic Sunyaev-Zel’dovich effect. We observe that the growth rate in TeVeS theory is scale dependent, which can serve as a potential probe in distinguishing it from ΛCDM model given future precise observations.

In order to predict large scale structure in TeVeS cosmology, we need the linear cosmological perturbation theory in TeVeS, which was constructed in a pioneer work [44]. Based on the perturbation theory, the large scale structure in TeVeS cosmology was first discussed in [45], where it was argued that perturbations to the scalar field may induce enhanced growth in the matter perturbations. Analytic explanation of the growth of structure was subsequently given in [46], where it was claimed that the perturbations to the vector field are key to the enhanced growth. It was further clarified in [47] that even if the contribution of the TeVeS fields to the background FLRW equations is negligible, one can still get a growing mode which drives structure formation. This explains analytically the numerical results in [48].
It is of great interest to examine whether TeVeS theory can give predictions for large scale structure similar to ΛCDM model and whether it is compatible with cosmological observations. In [45] the CMB angular power spectrum for TeVeS was first calculated numerically by solving the linear Boltzmann equations in the case of TeVeS theory. By using the initial conditions close to adiabatic, it was found that the power spectrum provides poor fit to observations compared to ΛCDM model. It was observed that if a cosmological constant and/or three massive neutrinos are incorporated into the matter budget, the first peak of the CMB angular power spectrum could locate at the right position. It was argued that by including a fourth sterile neutrino, TeVeS theory can have good fits to the CMB angular power spectrum [47]. However in their research, they assumed that there were no MOND effects before the recombination, so that the MOND effects do not influence the CMB. Thus it would be fair to say that their fitting result has nothing to do with TeVeS features. In this work, we will concentrate on the TeVeS features and their corresponding influences on the CMB. We will examine whether we can get a good fit to current CMB observations by including the cosmological constant and the fourth neutrino.

The mechanism of structure growth in TeVeS theory is different from that in ΛCDM model. In ΛCDM, after decoupling from photons, baryons fall into the gravitational wells induced by CDM. While in TeVeS, the growth of perturbations is driven by the vector field whose perturbation grows rapidly after recombination. This may lead to difference in the growth of baryon density perturbation and the amplitude of the matter peculiar velocity. The change on the matter peculiar velocity can further induce temperature fluctuations on the CMB map at small scales via the conventional kinetic Sunyaev Zel’dovich (kSZ) effect. The kSZ effect is generated through CMB photons scattering off free electrons in the diffuse intergalactic medium and the unresolved cluster population. The study of the kSZ effect is appealing, since it can be observed with the new generation CMB experiments. Recently, the kSZ effect has been found as a potential probe of reionization, the radial inhomogeneities in the Lemaitre-Tolman-Bondi cosmology [48], the missing baryon problem [49], the dark flow [50] and the interaction between the dark sectors [51]. Here we will further investigate the kSZ effect in the frame of TeVeS theory, and disclose whether it can be used to constrain the TeVeS parameters.

In addition to the signatures in the CMB power spectrum, we will also consider the growth rate of baryon density perturbation in TeVeS theory. The growth rate is generally a function of the cosmic scale factor $a$ and the comoving wavenumber $k$, defined as $f(k; a) = d\ln \delta(k; a)/d\ln a$. Although the temporal dependence of the growth rate has been readily measured by galaxy surveys using redshift-space distortion measurements [52–54], the spatial dependence is currently only weakly constrained [55–57]. However the theoretical study of the latter has an undoubted importance, for it is a critical test of theories of gravity. A characteristic prediction of ΛCDM is a scale-independent growth rate, while modified gravity models commonly induce a scale dependence in the growth rate. Thus the measurement of the growth rate, especially its spatial dependence can distinguish modified gravity theories from the standard ΛCDM model, even if they produce the same expansion history of the universe. In this work we will examine the scale dependence of growth rate in TeVeS and see whether TeVeS can be distinguished from ΛCDM model using current observations.

The paper is organized as follows. In Sec. II we will go over the TeVeS model and its application in cosmology. In Sec. III we will concentrate on its influence on the CMB angular power spectrum in the presence of the sterile neutrino and we will fit the TeVeS model to Planck data to constrain the model parameters. In the following section, we will examine the evolution of the density perturbation (Sec. IV A) and the peculiar velocity (Sec. IV B) of baryon in the TeVeS theory. In Sec. IV C we will show that the kSZ effect is a potential probe to constrain the TeVeS parameters. In Sec. IV D we focus on the scale dependence of growth rate in TeVeS model and compare with that of ΛCDM model by confronting to observational data. Finally we draw the conclusions in Sec. V.

II. FUNDAMENTALS OF TEVES THEORY

There are two metrics in Bekenstein’s TeVeS theory [34]. In addition to the Einstein frame metric $\tilde{g}_{\mu\nu}$ whose dynamics is governed by the standard Einstein-Hilbert action, it also has the matter frame metric $g_{\mu\nu}$. These two metrics are related through[34]

$$g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu} - 2\sinh(2\phi)A_\mu A_\nu,$$

where $\phi$ is a scalar field and $A_\mu$ is a vector field. The vector field is required to be unit timelike in the Einstein frame, $g^{\mu\nu}A_\mu A_\nu = -1$. The dynamics of the scalar and vector fields is given by the action $S_\phi$ and $S_A$:

$$S_\phi = -\frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}}(\tilde{g}^{\mu\nu} - A_\mu A_\nu)\nabla_\mu \phi \nabla_\nu \phi + V(\mu),$$

$$S_A = -\frac{1}{32\pi G} \int d^4x \sqrt{-g}[K_\mu F_{\mu\nu}F^{\mu\nu} - 2\lambda(A_\mu A_\nu + 1)],$$

where $K_\mu$ is a vector field and $F_{\mu\nu}$ is the field strength of $A_\mu$. The dynamics of the vector field is determined by the action $S_A$.
where \( \mu \) is a non-dynamical dimensionless scalar field, \( F_{\mu \nu} = 2\tilde{\nabla}_{[\mu} A_{\nu]} \), \( F_{\mu \nu} = \tilde{g}^{\rho \alpha} \tilde{g}^{\beta \gamma} F_{\alpha \beta} \), \( A^\mu = \tilde{g}^{\mu \nu} A_\nu \), \( \lambda \) is a Lagrange multiplier ensuring the unit timelike constraint on \( A_\mu \) and \( K_\mu \) is a dimensionless constant. \( G \) is the bare gravitational constant, whose value does not equal to the measured Newton’s constant. The relation between the gravitational constant and Newton’s constant depends on the quasistatic, spherically symmetric solution to the TeVeS field equations and the free function \( V(\mu) \). \( V(\mu) \) typically depends on a scale \( l_B \). In Bekenstein’s original work, he proposed \( \tilde{\nabla} \)

\[
\frac{dV}{d\mu} = -\frac{3}{2\pi l_B^2 \mu_0^2} \mu^2 (\mu - 2 \mu_0)^2, \tag{4}
\]

where \( \mu_0 \) is a dimensionless constant. A generalization to this function was proposed in \( [59, 61] \) and Angus et al. \( [62] \) suggested alternative functions that also lead to MOND.

The action for matter fields is usually written in the matter frame, where it takes the same form as in GR. Hence the matter frame metric is sometimes called the physical metric. Generically denoted the matter fields by \( \chi^A \), we have

\[
S_m = \int d^4 x \sqrt{-g} \mathcal{L}[g, \chi^A, \partial \chi^A]. \tag{5}
\]

### A. Background dynamics in TeVeS cosmology

The solutions for the homogeneous and isotropic universe in TeVeS theory have been studied in \( [34, 59, 63–66] \). Assuming that the spacetime is flat, the physical metric takes the form

\[
ds^2 = a^2 (-dt^2 + dr^2), \tag{6}
\]

and the Einstein metric has the similar form

\[
ds^2 = b^2 (-e^{-4\phi} dt^2 + dr^2). \tag{7}
\]

\( a \) and \( b \) are the scale factors in the matter and Einstein frames. They are related through \( a = be^{-\phi} \). In the Einstein frame, the Friedmann equation reads \( [44] \):

\[
3 \dot{b}^2 = a^2 \left[ \frac{1}{2} e^{-2\phi}(\mu V' + V) + 8\pi G e^{-4\phi} \rho \right], \tag{8}
\]

where \( \rho \) is the matter energy density which does not include the scalar field. The vector field is not dynamical in FLRW cosmology. It always points to the time direction, and does not contribute to the total energy density. The background dynamics is completely described if we have the equation of motion for \( \phi \),

\[
\ddot{\phi} = \dot{\phi} \left( \frac{\dot{a}}{a} - \dot{a} \right) - \frac{1}{U} \left[ 3\mu_b \dot{b}^2 \phi + 4\pi G a^2 e^{-4\phi}(\rho + 3P) \right], \tag{9}
\]

where \( U \equiv \mu + 2V'/V'' \) and \( P \) denotes the pressure which does not include the pressure of the scalar field.

In the matter frame, the Hubble parameter is defined as \( H \equiv \frac{\dot{a}}{a} \), where the dot denotes the derivative w.r.t the conformal time in the matter frame. The effective Friedmann equation then reads \( [44] \):

\[
3H^2 = 8\pi G e_{\text{eff}}(\rho + \rho_\phi), \tag{10}
\]

where the effective gravitational constant is \( G_{\text{eff}} = G \frac{e^{-4\phi}}{(1 + 3\mu_0^2)^2} \). The effective energy density of the scalar field is

\[
\rho_\phi = \frac{1}{16\pi G} e^{2\phi}(\mu V' + V). \tag{11}
\]

If the free function \( V \) takes the form of \( [1] \), the scalar field energy density will track the matter energy density \( [26, 45, 48] \). Defining the effective density fraction as \( \Omega_\phi = \frac{\rho_\phi}{\rho_{\text{eff}}} \), the tracker is \( \Omega_\phi = \frac{1}{(1+w)^2} \), where \( w \) is the equation of state of the background matter field. The typical value of \( \mu_0 \) has the order of \( 10^2 \), so the scalar field is always subdominant in the history of our universe.

It is free to add an arbitrary integration constant to \( V \). This will only change the Lagrangian of the scalar field by a constant, thus has no influence on the field equations and the evolution of the gravitational fields. Adding a constant in \( V \) is equivalent to include a cosmological constant in the effective Friedmann equation \( [10] \). This leads to the desired accelerated expansion of our universe.
B. Linear perturbation theory in TeVeS cosmology

In this subsection we will go over the linear perturbation theory on the background described above. This will allow us to link TeVeS theory with observations of structure formation on large scale as well as the CMB anisotropies.

The linear perturbation theory for TeVeS cosmology was first constructed in \[44\]. We will employ the formalism in \[44\] and consider only scalar perturbations.

We work under the conformal synchronous gauge, for which \(\delta g_{\mu \nu} = 0\) and \(\delta g_{ij} = 2H\delta_{ij} + (\partial_i \partial_j - \frac{1}{3} \delta_{ij} \Delta)H_T\). It is conventional to write in Fourier space \(H_L = h/6\) and \(H_T = -(h+6\eta)/k^2\). The evolution equations for the matter density contrast and velocity take the same forms as GR in the matter frame

\[
\dot{\delta} = -3\frac{\dot{a}}{a} (C_s^2 - w)\delta - (1 + w) \left( k^2 \theta + \frac{1}{2} \dot{h} \right), \tag{12}
\]

\[
\dot{\theta} = -\frac{\dot{a}}{a} (1 - 3w)\theta + \frac{C_s^2}{1 + w} \delta - \frac{w}{1 + w} \theta - \frac{2}{3} k^2 \Sigma. \tag{13}
\]

We denote the perturbation to the scalar field by \(\phi\), so that \(\delta = \phi + \varphi\). The vector field perturbation is defined as \(A_\mu \equiv \dot{A}_\mu + a e^{-\tilde{\phi}} \alpha_\mu\). Its scalar mode is \(\Delta = a e^{-\tilde{\phi}}\). The evolution equations for the scalar field are given by

\[
\dot{\varphi} = -\frac{1}{2U} a e^{-\tilde{\phi}} \gamma - \dot{\varphi}, \tag{14}
\]

\[
\dot{\gamma} = -\frac{b}{a} \gamma + \frac{b}{a} e^{-3\tilde{\phi}} k^2 (\varphi + \dot{\varphi} + \frac{3}{a} \mu \tilde{\phi} h + 6\varphi + 2k^2 (1 - e^{4\tilde{\phi}}) \alpha) + 8\pi G a e^{-3\tilde{\phi}} [\delta \rho + 3\delta P - 3(\bar{\rho} + 3\bar{P}) \varphi]. \tag{15}
\]

The equations for the perturbed vector field obey

\[
\dot{\alpha} = E - \varphi + \left( \frac{\dot{\varphi}}{\dot{\alpha}} - \frac{\alpha}{a} \right) \alpha, \tag{16}
\]

\[
K_B \left( \bar{E} + \frac{\dot{b}}{b} E \right) = -\bar{\mu} \varphi (\varphi - \varphi) + 8\pi G a^2 (1 - e^{-4\tilde{\phi}})(\bar{\rho} + \bar{P}) (\theta - \alpha). \tag{17}
\]

The perturbed modified Einstein equations yield

\[
2k^2 (\varphi - \eta) + e^{4\tilde{\phi}} \frac{b}{a} \left( \frac{\dot{h}}{h} + 2k^2 (1 - e^{-4\tilde{\phi}}) \alpha + 6\frac{\dot{a}}{a} \varphi \right) + a e^{3\tilde{\phi}} \left( \frac{\dot{\varphi}}{\dot{\alpha}} - \frac{3}{U} \frac{\dot{b}}{b} \right) \gamma \tag{18}
\]

\[
- K_B k^2 E = 8\pi G a^2 (\bar{\rho} - 2\varphi),
\]

\[
2k^2 \dot{\eta} - 2k^2 \left( \frac{\dot{a}}{a} + \bar{\mu} \dot{\varphi} \right) \varphi + \frac{k^2}{U} a e^{-\tilde{\phi}} \gamma = 8\pi G a^2 e^{-4\tilde{\phi}} (\bar{\rho} + \bar{P}) k^2 \theta. \tag{19}
\]

To solve these perturbation equations, we need to specify the initial conditions. In \[47\] the adiabatic initial conditions of scalar mode perturbations during radiation era were proposed. In our numerical computations in the following discussions we will adopt those initial conditions for the selected special potential \[4\].

III. COSMIC MICROWAVE BACKGROUND RADIATION IN TEVES THEORY

In \[45\], the first numerical calculation of CMB angular power spectrum in TeVeS theory was done by using the original Bekenstein’s potential \[4\]. They found that a flat universe composed of about 5% baryon and 95% cosmological constant today matches the observations poorly. The angular distance relation is modified as compared to the standard adiabatic ΛCDM universe. The positions of the peaks in the CMB angular power spectrum are shifted to higher \(l\) which leads to a severe mismatch with the observational data. This problem was argued to be cured if the three neutrinos have a mass of \(m_\nu \approx 2eV\) \[45\]. In \[47\] it was argued that if include a sterile neutrino with \(\Omega_\nu \approx 0.23\) in addition to the three massless neutrinos, the peaks of CMB power spectrum will locate at the right positions which match the observational data. Furthermore by fitting a MOND-like model to the WMAP five year data, it is concluded that the model with the sterile neutrino is compatible with the observation. But in \[47\], it is assumed that there was no MOND effects before the recombination, therefore the MOND effects have no influence on the CMB power spectrum. It was commented that the fitting result in \[47\] has nothing to do with TeVeS features\[20\].
Here we will focus on TeVeS model. We numerically calculated the CMB power spectrum in TeVeS theory in the presence of a sterile neutrino. Our results are demonstrated in Fig. 1. The black line is for the fiducial ΛCDM model. We take the cosmological parameters: $\Omega_b h^2 = 0.022$, $\Omega_c h^2 = 0.12$, $h = 0.68$, $\tau = 0.09$, $n_s = 0.96$ and $\ln(10^{10} A_s) = 3.1$, where the Hubble constant $H_0 = 100 h \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, $\tau$ is the optical depth to the last scattering surface, $n_s$ and $A_s$ are the spectral index and amplitude of the primordial power spectrum. We will use these parameters for the fiducial ΛCDM model throughout this paper. The red lines are for TeVeS models with various $\Omega_\nu h^2$. To illustrate the qualitative influence of the abundance of the sterile neutrino, we fix the parameters in the TeVeS models by taking $K_B = 0.1$, $l_B = 100$ and $\mu_0 = 300$. The other parameters are the same as in the ΛCDM model except that we have no CDM in TeVeS and $\ln(10^{10} A_s)$ is adjusted such that the first peaks of the power spectra have the same height. In the y-axis, $D_l \equiv l(l+1)C_l/2\pi$. The data points and error bars are from the Planck 2013 results[68]. It is clear in Fig. 1 that including the fourth neutrino can move the locations of the acoustic peaks to larger angular scales. Moreover, it can also enhance the third acoustic peak to almost as high as the second peak, which is usually considered the signature of CDM in the universe. With the increase of the abundance of the fourth neutrino, there is clearly a competition between the shift of the peak positions and the enhancement of the third peak.

In [45], they found that changing the TeVeS parameters will modify the CMB power spectrum. It was observed that sufficiently small TeVeS parameters, $K_B$, $l_B$ and $\mu_0$, can cause the excess of the CMB power at large scales. Their conclusion was obtained in the absence of the sterile neutrino. We can see the similar property in Fig. 2 where the fourth neutrino has an abundance of $\Omega_\nu h^2 = 0.15$. Smaller TeVeS parameters consistently enhance the large scale power in CMB. The CMB power spectrum at small $l$s is more sensitive to the parameter $K_B$ than the other two parameters. Considering that $K_B$ regulates the dynamics of the vector field, our observation here supports the argument in [46] that the vector field perturbation plays an important role in the growth of structure in TeVeS. Furthermore we display in Fig. 2 that the influence of TeVeS parameters on the CMB power spectrum at small scales is totally overshadowed by that of the abundance of the fourth neutrino. The change of the positions and amplitudes of acoustic peaks is mainly caused by the change of $\Omega_\nu h^2$.

**TABLE I:** The priors and fitting results of the cosmological parameters.

| Parameter | Best-fit | 68% limits       | Prior           |
|-----------|----------|------------------|-----------------|
| $K_B$     | 0.0535   | $< 0.0701$       | [0.05, 0.5]     |
| $l_B$     | 278      | $> 229$          | [10, 300]       |
| $\mu_0$  | 326      | $329^{+0.41}_{-0.37}$ | [10, 400]     |
| $\Omega_b h^2$ | 0.157 | $0.156^{+0.003}_{-0.002}$ | [0.01, 0.5] |
| $\Omega_c h^2$ | 0.0209 | $0.0209 \pm 0.0002$ | [0.01, 0.03] |
| $h$       | 0.504    | $< 0.508$        | [0.5, 0.85]     |
| $\tau$    | 0.00390  | $< 0.031$        | [0, 0.3]        |
| $n_s$     | 0.898    | $0.900^{+0.005}_{-0.007}$ | [0.8, 1.4] |
| $\ln(10^{10} A_s)$ | 2.89 | $2.93^{+0.02}_{-0.06}$ | [2.3, 3.5] |

**FIG. 1:** The CMB temperature angular power spectra for the fiducial ΛCDM model (solid black curve) and TeVeS models (red curves) having various amount of sterile neutrino. The data points with error bars are from Planck 2013 results.
FIG. 2: The CMB temperature angular power spectra for TeVeS models with different parameters. The black curve is for the fiducial ΛCDM model. The TeVeS parameters are \( K_B = 0.05 \), \( l_B = 300 \) and \( \mu_0 = 300 \) if not specified. The fourth neutrino abundance is \( \Omega_\nu h^2 = 0.15 \)

In order to test the viability of TeVeS theory in explaining the observed CMB power spectrum and constrain the TeVeS parameters as well as the amount of the sterile neutrino, we confront TeVeS model to the Planck 2013 results\cite{68}. We performed the numerical fitting using Markov chain Monte Carlo (MCMC) method. In the fitting, we allow 9 parameters to vary, which are \( K_B, l_B, \mu_0, \Omega_\nu h^2, \Omega_b h^2, h, \tau, n_s, \) and \( \ln(10^{10} A_s) \). The priors of these parameters are listed in the last column in Table I. We modify the public code CMBEASY\cite{69} to compute the CMB power spectra and generate the Markov chains.

The TeVeS parameters have similar influence on CMB. The CMB power spectrum for large \( l \)s hardly depends on \( K_B, l_B \) or \( \mu_0 \), while the low-\( l \) power is suppressed when one of the TeVeS parameters increases. So one expects degeneracy among them when fitting to the CMB observations. Nevertheless, we can get moderate constraints for them by using Planck data alone, as indicated in Table I.

It is interesting that the constrained \( \Omega_\nu h^2 \) is larger than the CDM abundance\((\simeq 0.12)\) got in the concordance ΛCDM model\cite{70}. Meanwhile the obtained \( h \) is much smaller than the prediction in \cite{17}. Recall that the locations of the acoustic peaks shift towards smaller \( l \) when \( \Omega_\nu h^2 \) increases, see Fig I. This explains why our constraint on \( H_0 \) is so small. When Hubble’s constant decreases, the angular diameter distance to the last scattering surface is increased. Thus the angular scales of the acoustic peaks are reduced, which compensates the effect of excessive \( \Omega_\nu h^2 \).

Comparing with ΛCDM model, the constrained optical depth to the last scattering surface in TeVeS model is significantly smaller. Inferring from Planck data, the 68% C.L. limit for the optical depth is \( \tau = 0.09 \pm 0.038 \) for ΛCDM model\cite{70}. Assuming instantaneous reionization, the best-fit value for TeVeS model, \( \tau = 0.0039 \), implies that reionization completed at \( z = 1.2 \). This is certainly ruled out by astronomical observations which suggests the end of reionization was at \( z \simeq 6 \) or earlier. But if we take the 68% C.L. limit \( \tau = 0.31 \), the situation becomes better. The end of reionization was at \( z = 6.2 \). Thus TeVeS cosmology is still marginally allowed by current constraints of
IV. LARGE SCALE STRUCTURE IN TEVES THEORY

A. The growth of the baryon density fluctuation

The growth of structure in TeVeS theory was first discussed in [45]. It was reported that with the decrease of the TeVeS parameters, the small scale power spectrum of the baryon density fluctuations can be boosted to mimic that in the adiabatic ΛCDM model. In [46] it was pointed out that the growth of structure in TeVeS is mainly due to the vector field. In [26], it was further clarified that even if the contribution of the TeVeS fields to total energy budget in the background FLRW universe is negligible, we can still have a growing mode of density fluctuations which drives structure formation.

Although the matter power spectrum in TeVeS theory can mimic that in ΛCDM cosmology, the mechanism of structure growth in two models are different. In ΛCDM model, after decoupled from photons, baryons fall into the gravitational wells induced by CDM. In TeVeS theory, the growth of perturbations is mainly driven by the vector field which grows rapidly after recombination. Thus it may be possible to distinguish them by studying the evolution history of the perturbations.

In the left column of Fig.3, we demonstrate the evolutions of baryon density perturbation in synchronous gauge, bδ, in TeVeS models. The density perturbations are evaluated for k = 0.1Mpc⁻¹. For comparison, we also plot the evolution of δb in the fiducial ΛCDM model. As we expected, the growth rate of δb in TeVeS theory differ from that in ΛCDM model. In most cases, the perturbations grow faster in TeVeS theory than in ΛCDM model at low redshifts. And the smaller the TeVeS parameters are, the more the growth rate deviates from ΛCDM model. The growth rate is specially sensitive to K_B when it is small. Thus observing the structure growth can also help in constraining the TeVeS parameters.

Besides the evolution of the growth rate, its spatial dependence rate is more attractive in distinguishing TeVeS theory from ΛCDM. We will discuss this topic in the last subsection below.

B. The peculiar velocity

The peculiar velocity is related to the time derivative of the density perturbation in the linear perturbation theory. In Newtonian gauge, we have the relation

\[ \langle \nu_b^2 \rangle_b = \frac{\langle \delta_b(N) \rangle_b}{k} = -aH f^{(N)}(N) \delta_b(N), \]

where f(N) = \frac{\text{dlna}(N)}{\text{dlna}} is the linear growth factor and 'N' means the quantity is evaluated in Newtonian gauge. For conciseness, we will omit 'N' in \( \nu_b^{(N)} \) in the following.

To estimate the magnitude of \( \nu_b \), we first solve equation (12)-(19) and derive the peculiar velocity of baryon in Newtonian gauge. Then we compute the root mean square (rms) dispersion of \( \nu_b \) within a sphere of radius \( r \) by

\[ \langle \nu_b^2 \rangle = \int d^3k W_r^2(k) P_v(k), \]

where \( W_r(k) \) is a top hat window function of radius \( r \) and \( P_v(k) \) is the power spectrum of \( \nu_b \). The magnitude \( \langle \nu_b^2 \rangle^{1/2} \) represents the mean velocity of baryons within a sphere of radius \( r \) with respect to the mean matter distribution. For comparison, we also compute the same magnitude for the fiducial ΛCDM model.

We present the calculated \( \langle \nu_b^2 \rangle^{1/2} \) at \( z = 0.1 \) in the right column of Fig.3. We see that the velocity in TeVeS model is larger than that in ΛCDM at the scale of 10Mpc, which is consistent with the fast growth rate displayed in the left column. Depending on the parameters, \( \nu_b \) in TeVeS can be as large as twice of the ΛCDM value. With the increase of radius \( r \), the velocity dispersion in TeVeS model decays faster than in ΛCDM model. When \( r \) reaches 100Mpc, the velocity of the TeVeS models with small values of \( l_B \) and \( \mu_0 \) can become lower than that of ΛCDM. In general smaller TeVeS parameters lead to lower velocity. This may be anti-intuitive since \( \delta_b^{(N)} \simeq \delta_b \) grows faster for smaller TeVeS parameters. This is because \( \nu_b \) is proportional to the density perturbation \( \delta_b^{(N)} \) as well as the growth factor. With the decrease of TeVeS parameters, \( \delta_b^{(N)} \) is getting smaller. At \( z = 0.1 \), the influence of low density fluctuation overwhelms high growth rate of baryons and the net effect is the decrease of \( \nu_b \).
FIG. 3: The figures on the left column are the evolutions of $\delta_b$ for $k = 0.1\text{Mpc}^{-1}$, normalized to its present value; on the right column are the rms dispersion of baryon peculiar velocities. The black curves correspond to the fiducial $\Lambda$CDM model. The colored curves are for TeVeS models with different parameters. The curves in figures on the same row follow the same convention. The TeVeS parameters are $K_B = 0.05$, $l_B = 300$ and $\mu_0 = 300$ if not specified.

Observationally it is difficult to measure the peculiar velocity on scales above $50h^{-1}\text{Mpc}$ using galaxies. The kSZ effect provides an alternative method of great promise to measure peculiar velocity at cosmological distances, without resorting to distance indicators. High resolution and low noise CMB experiments have the potential to measure various statistical average of cluster velocity such as the bulk flow (e.g. [71, 72]), the mean pairwise momentum (e.g. [73]) and the momentum power spectrum (e.g. [74]). Advanced CMB experiments even have the capability of measuring peculiar velocity of individual galaxy clusters (e.g. [75–80]). In [73] Hand et. al. reported the measurement of the mean pairwise momentum of clusters using the CMB sky map made by the Atacama Cosmology Telescope(ACT). Planck found the radial peculiar velocity rms to be below three times the $\Lambda$CDM prediction at $z = 0.15$ [81]. While the results from ACT and Planck seem to be consistent with the $\Lambda$CDM model, given their large uncertainties they are also compatible with the TeVeS cosmology. To conclude, while at present the data does not have the statistical
power to constrain the TeVeS parameters, the peculiar velocity field could become an important test of TeVeS theory with future data sets of higher resolution and lower noise.

C. The kinetic Sunyaev-Zel’dovich effect

The Sunyaev-Zel’dovich(SZ) effect[82] is generated through the scattering of CMB photons by free electrons while the photons travel through ionized gas after reionization. The SZ effect is commonly classified into two sorts: the thermal SZ (tSZ) effect, which is characterized by the thermal motion of free electrons, and the kinetic Sunyaev-Zel’dovich (kSZ) effect, which is characterized by their bulk motion. Because free electrons produced after reionization of the intergalactic medium share the same motion as the plasma, it is expected that the kSZ effect can serve as a probe of baryon peculiar velocity field.

The kSZ effect induces distortions on the CMB temperature map. The kSZ temperature anisotropy is given by

\[
\frac{\Delta T(\hat{n})}{T_{\text{CMB}}} = -\int_{t_0}^{t} n_e \sigma_T e^{-\kappa} (\boldsymbol{v}_e^{(N)} \cdot \hat{n}) \, dt,
\]

where \(n_e\) is the electron density, \(\sigma_T\) is the Thomson cross section and \(\kappa\) is the Thomson optical depth, \(\boldsymbol{v}_e^{(N)}\) is the peculiar velocity of free electrons; the integral is along the line of sight (l.o.s.) out to the reionization epoch and \(\hat{n}\) is the unit vector along the l.o.s. The contribution of the kSZ effect to the CMB temperature angular power spectrum is

\[
C_l^{kSZ} = \frac{16\pi^2}{(2\pi)^3} (\bar{n}_e(0)\sigma_T)^2 \int_0^{z_{re}} (1 + z)^3 \chi_e^2 \frac{2}{\kappa} \Delta B^2(k, z)|_{k=\ell/\pi} e^{-2\kappa} x(z) \frac{dz(z)}{dz},
\]

where \(x\) is the comoving distance, \(\bar{n}_e(0)\) is the mean electron number density at present, \(\chi_e\) is the ionization fraction and \(\Delta B^2(k, z) = \frac{k^3}{2\pi^2} P_B(k, z)\). \(P_B\) is the power spectrum of the curl part of \(p \equiv (1 + \delta_e^{(N)})\boldsymbol{v}_e^{(N)}\). In the linear regime, \(\boldsymbol{v}_e^{(N)}\) is curl-free and only the combination \(\delta_e^{(N)}\) contributes to \(P_B\). Given \(\delta_e^{(N)} = \delta_b^{(N)}\), \(\boldsymbol{v}_e^{(N)} = \boldsymbol{v}_b\) and \(\boldsymbol{v}_b\), \(P_B\) can be written as

\[
P_B(k, z) = \frac{1}{2} \int \frac{d^3k'}{(2\pi)^3} \left( \frac{D(z)}{D(z')} \right)^2 \delta_b^{(N)}(k') \delta_b^{(N)}(k) \delta_b^{(N)}(k - k') \beta^2(k, k'),
\]

where \(D(z) = \delta_b^{(N)}(z)/\delta_b^{(N)}(0)\) is the growth function of baryon, \(P(k)\) is the baryon power spectrum in Newtonian gauge, \(W_g(k)\) is the transfer function which takes into account the suppression of baryon density fluctuations at small scales due to physical processes[83, 84] and \(\beta(k, k') = (k^2 - k(k \cdot k')/k^2)^2\). For simplicity, we have set \(W_g(k)\) to unity in our numerical calculations.

The non-linear evolution of density perturbations enhances the power spectrum at small scales. To account for this effect, we rewrite (23) into (24)

\[
P_B(k, z) = \frac{1}{2} \int \frac{d^3k'}{(2\pi)^3} \left( \frac{D(z)}{D(z')} \right)^2 \delta_b^{(N)}(k') \delta_b^{(N)}(k) \delta_b^{(N)}(k - k') \beta^2(k, k'),
\]

where \(W_g(k)\) and \(T_{NL}(k)\) are the non-linear power spectrum and the transfer function respectively. The non-linear corrections affect the density perturbation only and the velocity field is still linear[80, 85]. We found that the other non-linear power spectrum should also be replaced by the nonlinear one to better describe the simulated \(\Delta B^2\). This is likely caused by the extra contribution from the curl velocity component generated by shell crossing. To include the non-linear correction we need to specify \(T_{NL}(k)\) for TeVeS model, which is usually done by using adequate fits to N-body simulations. However, such simulation has not been carried out in TeVeS theory. It is then difficult to give a reliable description of the non-linear corrections. As a first guess, we borrow the halofit fitting formula[86, 87] for \(\Lambda CDM\) model to evaluate the non-linear power spectrum. We have to emphasize that this is only a rough estimation because TeVeS theory is significantly different from GR at cluster scales where the kSZ effect becomes important in the CMB anisotropies.

In Fig[4] we present the theoretical predictions of both linear and non-linear kSZ power spectrum in TeVeS model and the fiducial \(\Lambda CDM\) model. For consistency, we have assumed \(\tau = 0.09\) for all models. The solid lines represent the
FIG. 4: The kSZ anisotropy power spectra for TeVeS model with different parameters. The black curve is for the fiducial ΛCDM model. The solid lines represent the linear kSZ power spectra and the dashed lines are for the kSZ power spectra taking into account the non-linear corrections. The TeVeS parameters are the best-fit values taken from Table I if not specified. The only exception is that we adopt $\tau = 0.09$ instead of the best-fit value.

The power spectra for TeVeS are always smaller than that of ΛCDM model. Increasing the TeVeS parameters will further suppress the kSZ effect. Taking into account the non-linear effect, the power spectra for TeVeS are enhanced and become comparable with ΛCDM model. In contrast to the linear kSZ effect, increasing the TeVeS parameters enhances the power spectrum. The difference may be the consequence of the scale-dependant evolution of perturbations in TeVeS. $T_{NL}(k)$ varies with $k$, which means that the main contributions to the linear and non-linear kSZ power spectrum come from different scales. And the linear matter power spectrum $P(k)$ at different scales changes differently when the parameters vary. Therefore the linear and non-linear power spectra respond differently to the changing of the parameters. Again we emphasize that this phenomenon depends heavily on the estimation of $P_{NL}(k)$, and it is premature to make solid conclusion before we can have accurate non-linear matter power spectrum in TeVeS theory.

We include two data points for the kSZ power spectrum in Fig. 4. The rectangle indicates the upper limit of $D_l$ at $l = 3000$ with 95% C.L. derived from ACT data, $D_{kSZ}^{3000} < 8.6 \mu K^2$ [92]. The circle with error bar indicates the measurement of the SPT-SZ survey using data from the South Pole Telescope(SPT), $D_{kSZ}^{3000} = 2.9 \pm 1.3 \mu K^2$ with 68% C.L. [93]. These measurements heavily rely on modeling of cosmic infrared background and tSZ contributions, and therefore suffer from significant systematic uncertainty. Meanwhile our theoretical predictions have considerable uncertainties. Besides the non-linear effect, we have assumed a simple instantaneous reionization model with $\tau = 0.09$ while the kSZ effect from the patchy reionization is expected to be important. Hence the kSZ power spectrum may be underestimated. On the other hand, we did not include the smoothing in the gas density caused by the gas pressure in our calculation, which could potentially reduce the amplitude of the kSZ power spectrum. Despite these uncertainties, our computations indicate that the linear kSZ power spectrum in TeVeS model is consistent with the upper limits of the observations. The fact that it is smaller than the lower limit of SPT measurement does not rule out TeVeS since the linear kSZ power spectrum is essentially a lower limit to the realistic one. But if we look at the nonlinear kSZ power spectra, they are certainly ruled out by the SPT observation. The nonlinear kSZ power spectrum obtained by
using the best-fit TeVeS parameters in Table I also lies outside the upper limit from ACT, though it is very similar to ΛCDM model. This implies a friction in TeVeS model between the CMB and kSZ effect observations.

D. The scale dependence of growth rate

One of the characteristic features of ΛCDM model is the scale-independent growth rate in the subhorizon approximation. This property was found violated if the gravity goes beyond GR, if DE clustering cannot be neglected, or if DE couples to DM. Now we will investigate the scale dependence of the growth rate in TeVeS theory and see whether it can serve to distinguish TeVeS from ΛCDM model.

Since observations are in fact sensitive to the normalized growth rate \( f_8(k; z = 0) \) instead of \( f(k; z) \), in Fig. 5 we display \( f_8(k) \) in synchronous gauge for baryon w.r.t. \( k/h \) at redshift \( z = 0 \). The growth rate in TeVeS is systematically higher than that in ΛCDM model. The black curve for the fiducial ΛCDM model is almost a horizontal line, reflecting the scale-independent growth of density perturbations. In contrast to ΛCDM model, \( f_8(k) \) in TeVeS theory clearly varies with scale. We see that the growth rate is bigger at small scales than large scales. Increasing the TeVeS parameters, \( f_8(k) \) at given \( k \) becomes smaller, which is consistent with the behavior seen in Fig. 3. Furthermore, \( f_8(k) \) converges for different parameters when \( k \to 0 \), if \( \sigma_8 \) is equally normalized.

We compare the theoretical prediction of \( f_8(k; z = 0) \) with the measurement using the observations of peculiar motions of galaxies of 6-degree Field (6dF) Galaxy Survey velocity sample together with a newly-compiled sample of low-redshift type Ia supernovae. The measurement was done in 5 \( k \) bins: \( k_1 = [0.005, 0.02], k_2 = [0.02, 0.05], k_3 = [0.05, 0.08], k_4 = [0.08, 0.12] \) and \( k_5 = [0.12, 0.15] \). The data points in different color refers to results deriving by different data sets and methodologies. The measurement does not show strong evidence for a scale dependence in the growth rate. But we see that the TeVeS prediction matches the measured \( f_8 \) for a wide range of parameters.

FIG. 5: \( f_8(k; z = 0) \) for TeVeS model with different parameters. The black curve is for the fiducial ΛCDM model. The data points are from the 6dF Galaxy Survey. The TeVeS parameters are the best-fit values taken from Table I if not specified. For all curves, \( \sigma_8(0) = 0.834 \).
We hope that more precise data on the spatial dependence of growth rate in the future can help to distinguish TeVeS theory from ΛCDM and constrain the TeVeS parameters.

V. CONCLUSIONS

In this paper we have tested the TeVeS theory with several cosmological observations. We found that a TeVeS universe containing a sterile neutrino can produce the CMB power spectrum similar to ΛCDM model and is compatible with the CMB measurements. Fitting to the Planck 2013 data, we have constrained the parameters for TeVeS cosmology. The constraints for the abundance of the sterile neutrino and the Hubble’s constant read \( \Omega_{\nu}h^2 = 0.156_{-0.002}^{+0.003} \) and \( H_0 < 50.8 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \) at 68% C.L. The obtained Hubble parameter is much lower than the observed value obtained from supernovae measurement[104], which indicates a tension between CMB and supernovae observations in the TeVeS theory.

From the fitting of the TeVeS model to the Planck 2013 data, we have also noticed that although the constrained optical depth at the border of the 68% C.L. limit can give the end of the reionization marginally allowed by the constraints on the reionization history of our universe, the best-fit value of the optical depth is extremely small, which indicates that the end of reionization happened at \( z = 1.2 \). This is certainly not allowed and clearly indicates the problem of the TeVeS model in cosmology.

We have investigated the magnitude of the baryon peculiar velocity in TeVeS cosmology. The velocity dispersion at \( r < 100 \text{Mpc} \) in TeVeS is usually larger than that in ΛCDM model and \( \langle v_b^2 \rangle \) decays faster in TeVeS when the scale increases. We have computed the linear and non-linear kSZ anisotropy power spectrum in TeVeS theory, assuming \( \tau = 0.09 \). The linear kSZ power spectra are within upper limits measured by SPT and ACT, although they are much lower than that of ΛCDM model. The non-linear kSZ power spectra in TeVeS can be comparable with that of ΛCDM model, especially when we take the Planck best-fit value for the cosmological parameters. But the kSZ power spectrum for the Planck best-fit model is ruled out by SPT measurement and have a tension with the ACT measurement. Despite the uncertainties to the non-linear kSZ power spectra, there is a friction in TeVeS model between CMB observations and the kSZ effect measurements.

We have extended our discussions to the scale dependence of the evolution of large scale structure. In TeVeS cosmology, we have shown that the normalized growth rate \( f\sigma_8(k) \) rises with the increase of \( k \) at \( z = 0 \). This is clearly in contrast to the scale-independent growth at subhorizon scales in ΛCDM model. Although the predicted \( f\sigma_8 \) in TeVeS theory is consistent with the current measurement using 6dF data, we expect that the distinct scale dependence of the growth rate in TeVeS model can potentially serve as a powerful probe in distinguishing TeVeS from GR in future observations.

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