Rate-Energy Tradeoff for Wireless Simultaneous Information and Power Transfer in Full-Duplex and Half-Duplex Systems

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Abstract: In this paper, we study the rate-energy tradeoff for wireless simultaneous information and power transfer in full-duplex and half-duplex scenarios. To this end, the weighting function of energy efficiency and transmission rate, as rate-energy tradeoff metric is first introduced and the metric optimization problem is formulated. Applying Karush-Kuhn-Tucker (KKT) conditions for Lagrangian optimality and a series of mathematical approximations, the metric optimization problem can be simplified. The closed-form solution of the power ratio is obtained, building direct relationship between power ratio and the rate-energy tradeoff metric. By choosing power ratio, one can make the tradeoff between information rate and harvested power in a straightforward and efficient way. Using the method similar to the half duplex systems, the optimal power ratio can be obtained in the full duplex systems, so as to balance the information transmission rate and energy transmission efficiency. Simulation results validate that the information rate is non-increasing with harvested power in half-duplex systems and the tradeoff of information rate and harvested power can be simply made. In the full duplex systems, the power ratio solution of the rate-energy tradeoff metric optimization problem can be used as the approximate optimal solution of the optimization problem and the approximation error is negligible.

Keywords: Power transfer, full duplex, optimization problem.

1 Introduction

Recently there are increasing research activities towards 5G [Dutkiewicz, Jayawickrama, He et al. (2017); Lee, Wang, Liao et al. (2017); Li, Xu and Zhao (2018)]. Indeed, this trend is a great challenge in future, because The 5G device will need lower delays, smarter, more cost-effective structure than before [He, Xie, Xie et al. (2019); Kader, Shin, Leung et al. (2018); Rahimi, Zibaenejad, Safavi et al. (2018); Xia, Tan, Wang et al. (2019); Yu, Liu, Zhang et al. (2019)]. How to effectively provide energy for sensor nodes for a long period has become a key issue to achieve the energy consumption requirements of 5G.

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Simultaneous wireless information and power transfer (SWIPT) is considered as a convenient, safe and “green” energy collection technology [Assimonis and Fusco (2018); Hannaidh (2018); Ijaz, Zhang, Grau et al. (2016)]. Recently there are increasing research activities towards simultaneous wireless information and power transfer [Liu, Zhang, Chua et al. (2013); Varshney (2008)], which realizes both useful utilizations of RF signals: energy harvest and information transmission. The idea of transmitting information and energy simultaneously is first proposed in Varshney [Varshney (2008)]. Specifically, in view of practical implementations, a general receiver operation scheme is presented in Zhou et al. [Zhou, Zhang, Ho et al. (2012)], namely, dynamic power splitting, by which the signal is dynamically split into two streams with arbitrary power ratio over time. The architecture for the separated information and energy receiver is given in part 2, where $\rho \in (0,1]$ is the power ration. By choosing the value of $\rho$, one can manage the power ratio fed to energy receiver and information receiver. However, the rate-energy relationship is invisible. The question is how to select the value of $\rho$ to make the tradeoff between information rate and energy harvest?

In this paper, we extend the existing work in Zhou et al. [Zhou, Zhang, Ho et al. (2012)], and consider the rate-energy tradeoff for simultaneous wireless information and power transfer. By introducing a rate-energy metric, we build the direct relationship between information rate and energy harvest. Unlike the work in Zhou et al. [Zhou, Zhang, Ho et al. (2012)], one can easily make the rate-energy tradeoff by choosing one preference.

2 System model in half-duplex systems

As shown in Fig. 1, the received signal by the antenna is split into two signal streams, which are then separately fed to the energy receiver and information receiver for harvesting energy and decoding information, respectively. The harvested power is given as

$$E = \rho \eta P,$$

where $P$ is the power of the received signal, $\eta \in (0,1]$ is the conversion efficiency.

![Architecture for the separated information and energy receiver](image-url)
The received signal at the information receiver is given by
\[ y_\text{re} = (1 - \rho)(y_\text{rec} + n_\text{rec}) + n_\text{cov} \] (2)
where \( n_\text{rec} \) and \( n_\text{cov} \) denote the additive white Gaussian antenna noise and processing noise with zero mean and variance \( N_\text{rec} \) and \( N_\text{cov} \), respectively. Then the maximum information rate can be given by
\[ R_\text{rec} = \log_2 \left( 1 + \frac{(1 - \rho)P}{(1 - \rho^2)N_\text{rec} + N_\text{cov}} \right) \] (3)

3 System model in full duplex systems

As shown in Fig. 2, the system is composed of the signal transmitting part, the signal receiving part and the energy collecting part in the scenario of full duplex. The receiving part of the signal and the collecting part of the energy are the same as the half-duplex. There is also much interference between the signal transmitting part and the signal receiving part, the system receives the self-interference through \( h_{\text{rr}} \) channel due to the full-duplex. \( h_{\text{rr}} \) are respectively distributed in complex Gaussian distribution with zero mean and the variance \( \sigma^2_{\text{rr}} \). Then the \( h_{\text{rr}} \) channel gains are represented as \( f_{\text{rr}} = |h_{\text{rr}}|^2 \).

This part of the paper redefines the power ration, which respectively represents the proportion of message received, message transmitted and energy collected in the total energy. By choosing the value of the power ration, system can manage the power ratio fed to energy receiver, information receiver and information transmitter. However, the rate-energy relationship is invisible. As in the half-duplex hypothesis, the similar question is how to select a set of the power ration values \( \rho_1, \rho_2, \rho_3 \) to make the tradeoff among information receive rate, energy harvest and information transmit rate?
In the same way as the half-duplex hypothesis, the harvested power of the full duplex hypothesis can be given by $E_{\text{en-full}} = \rho_\eta P$.

In order to simplify the analysis process, the distribution of transmitted signals can be equivalent to the distribution of received signals, assuming that the receiver and transmitter channels have the same fading.

The signal transmission rate of the system can be solved by the Shannon formula:

$$R_{\text{tr-full}} = \log_2 \left( 1 + \frac{\rho_\eta P}{N_u} \right),$$

where $n_u$ denote the additive white Gaussian antenna noise with zero mean and variance $N_u$.

The received signal at the information receiver is given by

$$y_u = \rho_\eta \left( y_{\text{rec}} + n_{\text{rec}} + h_{\text{rr}} x_u \right) + n_{\text{cov}}.$$  

The self-interfering equivalent power in the receiver is $g_{\text{rr}} = |h_{\text{rr}}|^2 / |h_{\text{com}}|^2$, with $h_{\text{com}}$ is channel fading coefficient of communication link.

Then the maximum information rate of message received can be given by

$$R_{\text{rec-full}} = \log_2 \left( 1 + \frac{\rho_\eta P}{\rho_\eta (N_{\text{rec}} + \rho_\eta g_{\text{rr}}) + N_{\text{cov}}} \right).$$

4 Rate-energy tradeoff in half-duplex systems

Our main goal is to build the visual rate-energy relationship and easy the rate-energy tradeoff processing in half-duplex systems. Denoting $\omega \in [0,1]$ as the preference for information rate, we define a rate-energy tradeoff metric as

$$U_{R-E} = \omega R_{\text{rec}}^{\text{norm}} + (1 - \omega) E_{\text{norm}}$$

where

$$R_{\text{rec}}^{\text{norm}} = \frac{R_{\text{rec}}}{R_{\text{rec}}^{\text{max}}}, \quad E_{\text{norm}} = \frac{E}{E^{\text{max}}}$$

with

$$R_{\text{rec}}^{\text{max}} = \log_2 \left( 1 + \frac{P}{N_{\text{rec}} + N_{\text{cov}}} \right), \quad E^{\text{max}} = \eta P.$$  

Therefore, we can obtain
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\[ U_{R-E} = \omega \log_2 \left( 1 + \frac{(1-\rho)P}{(1-\rho)N_{\text{rec}} + N_{\text{cov}}} \right) \frac{R_{\text{rec}}^\max}{(1-\omega)P} \left[ \frac{\rho \eta P}{E_{\text{max}}} \right]. \] (8)

Now, the question is: How to determine the value of power ratio \( \rho \) for a given information rate preference \( \omega \)? Thus, we obtain a rate-energy optimization problem as

\[ \rho^* = \arg \max_{\rho} U_{R-E} \quad \text{s.t.} \quad \rho \in [0,1]. \] (9)

Applying Karush-Kuhn-Tucker (KKT) conditions [Boyd and Vandenberghe (2004)] for Lagrangian optimality, we can obtain the unique solution after some algebraic manipulations:

\[ \rho^* = \left[ \sqrt{(PR_{\text{rec}}^\max N_{\text{cov}} (1-\omega)\ln(2) + 4\omega N_{\text{rec}} (P + N_{\text{rec}})\ln(2) P N_{\text{cov}}} \right. \\
\left. + \sqrt{(1-\omega)R_{\text{rec}}^\max 2(P + N_{\text{cov}} + N_{\text{rec}})N_{\text{rec}} + P N_{\text{cov}})\ln(2)} \right]^{-1} \times 2 \ln(2) N_{\text{rec}} (P + N_{\text{rec}}) \sqrt{(1-\omega)R_{\text{rec}}^\max}^{-1}. \] (10)

Using this expression, one can easily choosing the value \( \rho \) to make the rate-energy tradeoff. When \( N_{\text{rec}} \gg N_{\text{cov}} \), i.e., the antenna noise is dominant over the processing noise, we have

\[ \rho^* \rightarrow 1. \] (11)

This indicates that \( \rho \) is independent from \( \omega \) and the optimal rate-energy tradeoff is achieved when the vast majority of the received power is fed to the energy receiver, which corresponds to the conclusion given in Zhou et al. [Zhou, Zhang, Ho et al. (2012)]. On the other hand, when, reduces to

\[ \rho^* = \frac{(1-\omega)R_{\text{rec}}^\max (N_{\text{cov}} + P)\ln(2) - \omega P}{\ln(2) P R_{\text{rec}}^\max (1-\omega)}. \] (12)

From (12) we can observe that \( \rho \) is non-increasing with preference \( \omega \).

5 Rate-energy tradeoff in full duplex systems

In full duplex systems, our main goal is to build the visual rate-energy relationship and easy the rate-energy tradeoff processing. Denoting \( \omega_1, \omega_2, \omega_3 \in [0,1] \) as the preference for information rate, we define an information receive rate-energy collect rate-information transmit rate tradeoff metric as
\[ U_{R-E-T} = \omega_1 R_{\text{rec-full}}^{\text{norm}} + \omega_2 E_{\text{full}}^{\text{norm}} + \omega_3 E_{\text{tr-full}}, \quad (13) \]

where
\[ R_{\text{rec-full}}^{\text{norm}} = \frac{R_{\text{rec-full}}}{R_{\text{rec-full}}^{\text{max}}}, \quad E_{\text{full}}^{\text{norm}} = \frac{E_{\text{full}}}{E_{\text{full}}^{\text{max}}}, \quad R_{\text{tr-full}}^{\text{norm}} = \frac{R_{\text{tr-full}}}{R_{\text{tr-full}}^{\text{max}}}, \]

with
\[ R_{\text{rec-full}}^{\text{max}} = \log_2 \left( 1 + \frac{P}{N_{\text{rec}} + N_{\text{cov}}} \right), \quad E_{\text{full}}^{\text{max}} = \eta P, \quad E_{\text{tr-full}}^{\text{max}} = \log_2 \left( 1 + \frac{P}{N_{\text{tr}}} \right). \]

Therefore, we can obtain
\[ U_{R-E-T} = \left[ \omega_1 \log_2 \left( 1 + \frac{\rho_1 P}{\rho_1 N_{\text{rec}} + N_{\text{cov}} + \rho_3 g_{\text{ref}}} \right) \right] + \left[ \omega_2 \log_2 \left( 1 + \frac{\rho_2 P}{N_{\text{tr}}} \right) \right] + \left[ \omega_3 \log_2 \left( 1 + \frac{\rho_3 P}{N_{\text{tr}}} \right) \right], \quad (14) \]

with \( \omega_1 + \omega_2 + \omega_3 = 1, \omega_1, \omega_2, \omega_3 \in [0,1] \).

Thus, we obtain an information receive rate-energy collect rate-information transmit rate optimization problem as
\[ [\rho_1^*, \rho_2^*, \rho_3^*] = \arg \max_{\rho_1, \rho_2, \rho_3} U_{R-E-T} \]
\[ \text{s.t.} \begin{cases} \rho_1, \rho_2, \rho_3 \in [0,1] \smallskip \rho_1 + \rho_2 + \rho_3 = 1 \end{cases} \quad (15) \]

Applying Karush-Kuhn-Tucker (KKT) conditions for Lagrangian optimality, we can obtain the solution after some algebraic manipulations. However, the solution of are too complex to be expressed effectively.

With the development of full duplex technology, the influence of self-interfering signals is small. Therefore, In order to simplify the optimization problem, the approximate optimization problem of the optimization problem Eq. (15) is obtained by taking the upper limit of the variable \( \rho_3 \) in Eq. (6):
\[ [\rho_1^*, \rho_2^*, \rho_3^*] = \arg \max_{\rho_1, \rho_2, \rho_3} U_{R-E-T}' \]
\[ \text{s.t.} \begin{cases} \rho_1^*, \rho_2^*, \rho_3^* \in [0,1] \smallskip \rho_1^* + \rho_2^* + \rho_3^* = 1 \end{cases} \quad (16) \]

with
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\[ U'_{R-E-T} = \frac{\omega_1 \log_2 \left( 1 + \frac{\rho'_1 P}{\rho'_1 N_{rec} + N_{cov} + \rho'_2 g_{rr}} \right)}{R^\text{max}_{\text{rec-full}}} + \omega_2 (1 - \rho'_1 - \rho'_2) + \frac{\omega_1 \log_2 \left( 1 + \frac{\rho'_1 P}{N_{tr}} \right)}{\log_2 \left( 1 + \frac{P}{N_{tr}} \right)} \].

Applying Karush-Kuhn-Tucker (KKT) conditions, the optimization problem Eq. (16) can be transformed into the solution of a system of equations:

\[
\begin{cases}
-\lambda + P N_{cov} \omega_1 = 0 \\
-\lambda + P \omega_2 = 0 \\
-\lambda + R \log[2] \left( N_{cov} + (g_{rr} + N_{rec}) \rho'_1 \right) \left( N_{cov} + (P + g_{rr} + N_{rec}) \rho'_1 \right) = 0 \\
\rho'_1 + \rho'_2 + \rho'_3 = 1
\end{cases}
\]  

(17)

We can obtain the solution after some algebraic manipulations

\[
\begin{align*}
\rho'_1^* &= \left[ 4 g_{rr}^2 \omega_1 + 4 P N_{rec} \omega_1 + 4 N_{rec}^2 \omega_1 \right]^{1/2} \\
&+ \sqrt{P R \ln \left( 2 \right) N_{cov} \omega_2} \\
&- \sqrt{R \omega_2 \ln \left( 2 \right) N_{cov} \left( P + 2 g_{rr} + 2 N_{rec} \right)} \\
&+ 2 \sqrt{R \ln \left( 2 \right) \omega_2 \left( g_{rr} + N_{rec} \right) \left( P + g_{rr} + N_{rec} \right)}
\end{align*}
\]

(18)

\[
\begin{align*}
\rho'_2^* &= 1 - \rho'_1^* - \rho'_3^* \\
\rho'_3^* &= \frac{N_{rec} \omega_2 + P \omega_3}{PR \ln \left( 2 \right) \omega_2}
\end{align*}
\]

where \( R \) is the abbreviation of \( R^\text{max}_{\text{rec-full}} \) in the Eq. (4).

6 Simulation results

This section verifies the above analysis results by computer simulation. Without loss of generality, we assume that \( N_{rec} = N_{cov} = 1 \) and \( P=10 \, dB \).
Figure 3: Normalized information rate, harvested power and tradeoff metric $U_{R-E}$ against preference $\omega$

Normalized information rate $R_{\text{rec}}^{\text{norm}}$, harvested power $E^{\text{norm}}$ and $U_{R-E}$ is presented in Fig. 3. As shown in Fig. 3 the $R_{\text{rec}}^{\text{norm}}$ is non-decreasing with $\omega$ and $E^{\text{norm}}$ is non-increasing with $\omega$, due to the fact that $\omega$ is the preference of information rate. This can provide valuable guideline for the selection of $\omega$ in practical system design, thus facilitating rate-energy tradeoff processing. Furthermore, the three curves do intersect at a unique point, which can be readily clarified by definition of the rate-energy tradeoff metric.

Figure 4: Power ratio $\rho$ against information rate preference $\omega$
For better insights, the power ratio $\rho$ against information rate preference is given in Fig. 4 with $P=0, 5, 10, 15, 20, 25$ dB. The direct relationship between $\rho$ and $\omega$ can be easily figured out and one can simply determine the value of power ratio $\rho$ for different $\omega$. Moreover, rate-energy relationship is given in Fig. 5. Obviously, information rate is non-increasing with harvested power. Through this figure, the tradeoff of information rate and harvested power can be simply made.

The relationship contour of tradeoff metric and power ration is given in Fig. 6. Without losing generality, we assume that $N_{rec}=N_{con}=N_{se}=1$, $P=10$ dB, $\omega_1 = 0.5$, $\omega_2 = 0.2$. 
$\omega_1 = 0.3$ and $P=10 \text{ dB}$. Since power ration $\rho_2$ of energy harvest can be simply calculated by, power ration $\rho_1$ of information receive and power ration $\rho_3$ of information transmission, $\rho_2$ is not taken as a variable in this simulation. The X-axis is the power ration $\rho_1$ of information receive, the Y-axis is the power ration $\rho_3$ of information transmission, and the contour lines represent information receive rate-energy collect rate-information transmit rate tradeoff metric in Fig. 6. The simulation shows that, the tradeoff metric reaches the maximum 0.8932 when $\rho_1 = 0.103, \rho_2 = 0.831, \rho_3 = 0.066$. At the same time, the approximate optimal solution $(\rho_1^*, \rho_2^*, \rho_3^*)=0.8932$ (with $\rho_1^* = 0.1008$, $\rho_2^* = 0.832$ and $\rho_3^* = 0.0672$ ) can be obtained according to the Eq. (18). Simulation results validate the optimal solution of the optimization problem Eq. (16) can be used as the approximate optimal solution of the optimization problem Eq. (15).

Figure 7: Total system power against information rate preference $\omega$

For better insights, the total system power against information rate preference is given in Fig. 7 with $P=5$, 10, 15, 20, 25 dB. Without losing generality, we assume that $N_{\text{rec}} = N_{\text{cov}} = N_{\text{tr}} = 1$, $\omega_1 = 0.2$, $\omega_2 = 0.2$, $\omega_3 = 0.6$. Simulation results validate the approximation error is negligible.

7 Conclusions
This paper investigates the rate-energy tradeoff for wireless simultaneous information and power transfer of 5G embedded sensor nodes. The direct relationship between power ratio and the preference for information rate is built by solving an optimization problem. Therefore, one can make the tradeoff between information rate and harvested power fast
and efficiently by choosing power ratio. Simulation results validate the effectiveness of the proposed power allocation.

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