Wavelet Solution for Biosensor Response at Mixed Enzyme Kinetics

S.G.Venkatesh, K. Balasubramanian, S. Raja Balachandar

Abstract: This paper presents the approximate solution of the reaction diffusion equation based on the hybridization of classical polynomials and Legendre wavelets. The systems of equations are generated first for the differential equations by the properties of Legendre wavelets. Theoretical analysis for the proposed scheme is discussed and computed solutions are also compared with other numerical solutions available in the literature.

Index Terms: Legendre wavelets; Convergence analysis; Reaction-diffusion system.

I. INTRODUCTION

In recent days, differential equations are used to analyze the behaviour of Semiconductors gas sensors and biosensors. One such equation is the steady-state solution of the substrate reaction-diffusion process given by

$$\frac{\partial^2 u}{\partial x^2} - \frac{Ku}{1 + au + \beta u^2} = 0$$

(1)

Subject to the boundary conditions $u(1)=1$ and $\frac{\partial u(0)}{\partial x} = 0$.

The gas sensors [1-7] and biosensors [8-16] models have been studied by differential equations in the literature. To improve the sustainability of our society, we use the gas sensors to detect, monitor and control the presence of dangerous gases in the atmosphere. Many metal oxide semiconductor-based gas sensors are used to detect the different kinds of gas molecules namely H₂, CO, NH₃, H₂S, SO₂, odours, other applications are reported in the field such as the detection of explosive gases, fire detection, and air quality[1-7]. This semiconductor-based gas sensor consists of sintered block, thick film and thin film. The manufacturing and production of this gas sensor considered towards those higher sensitivities, quicker selectivity and response. They also concentrate on easier fabrication and portability.

The analytical devices made up of biological entity. Usually enzyme are termed as biosensors[8-16]. These sensors recognize a particular analyte and translates the changes in the bio-molecules into an electric signal. Due to high sensitivity, simplicity and low cost these sensors are widely used in many industries. The mathematical models have discussed the analytical behavior of bio sensors. In particular, authors [16] studied the enzymatic reaction in the enzyme membrane and mass transport from both sides, which is used to study the biochemical behavior in dimensionlessform. This model is categorized as a nonlinear reaction-diffusion equations in which the nonlinear term related to non-Michaelis-Menton kinetics of the enzymatic reaction, its steady-state version is given in (1), and the authors have studied the analytical and numerical solution for (1) by using the variational iteration method (VIM) and Homotopy perturbation method (HPM) rigorously. They have also compared the simulation results with average percentage deviation of 0.516 and 0.4676 for $\alpha =10$ and $K=0.1$ for VIM and HAM. The error percentage (0.5656, 1.8451) and (7.1357, 1.5644) subject to ($\alpha=1, K=1$) and ($\alpha=0.1, K=5$) for VIM and HAM respectively. To reduce this error percentage and accuracy of concentration, we try Legendre wavelet-based technique to solve (1) in this paper. Generally solving a nonlinear equation is a tedious task. To solve these kinds of equations, we have approximate, numerical and semi-analytical techniques. At the same time, each method has its own merits and demerits based on their applicability. Not all nonlinear problems have been solved by a single approach. These methods have deviated on their nature of the nonlinearity, initial and boundary conditions. Symmetries and transformations methods are also used to find the exact solutions for the same without considering their initial conditions. So the researchers keep on trying the new methods to overcome the limitations stated in the nonlinear differential equations. Many researchers have investigated the substrate reaction-diffusion process through differential equations in nonlinear form, and they have also discussed the various methods to solve, and it can be found in [1-16]. The dynamic behaviour of such a process is encountered in various physical and chemical phenomena and very helpful in modeling certain parameters like reaction-diffusion equations. Chemical kinetics, Populationdynamics, neurophysiology are the areas where this equation plays a pivotal role. In this paper, we solve this partial differential equation through Legendre wavelet-based hybrid method. Wavelet-based methods are usually applied in image processing, restoration, compression etc. The orthogonal properties of the various wavelets are effectively used to solve the differential equations in nonlinear nature. The remaining sections organized as In section 2, we describe the Legendre wavelets and function approximation. Section 3 is devoted to the proposed approach for solving Eq. (1). In Section 4, the convergence and error analysis of the same is given. Numerical examples and concluding remarks are presented in section 5 and 6.
Wavelet solution for nonlinear reaction-diffusion equation

\[ \frac{\partial^2 u}{\partial x^2} - \frac{Ku}{1 + \alpha u + \beta u^2} = 0 \]
\[ \int_0^1 u_{m} \, dx + \alpha \int_0^1 u_{m} \, dx + \beta \int_0^1 u_{m} \, dx - \int_0^1 K \, dx = 0. \]
\[ + \beta \int_0^1 u^2 \, dx - \int_0^1 K \, dx = 0. \]

By using (4), we have
\[ \int_0^1 C^T \Psi'(x) \, dx \quad + \quad \int_0^1 C^T \Psi'(x)C^T \Psi'(x) \, dx \]
\[ + \beta \int_0^1 (C^T \Psi'(x))^2 \quad + \quad \int_0^1 K \Psi(x) \, dx = 0 \]
\[ \int_0^1 C^T \Psi'(x) \, dx \quad + \quad \int_0^1 C^T \Psi(x)C^T \Psi(x) \, dx \]
\[ + \beta \int_0^1 (C^T \Psi'(x))^2 \quad + \quad \int_0^1 K \Psi(x) \, dx = 0 \]

We now collocate the above equation at $2^{k-1}$ points at $x_i$
\[ \int_0^1 C^T \Psi(x) \, dx \quad + \quad \int_0^1 C^T \Psi(x)C^T \Psi(x) \, dx \]
\[ + \beta \int_0^1 (C^T \Psi'(x))^2 \quad + \quad \int_0^1 K \Psi(x) \, dx = 0 \]

The system of equations in $2^{k-1}$ unknowns are constructed by assigning the values to $k$ and $M$.

**IV. CONVERGENCE ANALYSIS**

In this section, we discuss the theoretical background for the function approximation defined in the previous section through the process of convergence and error estimation. We prove the idea of convergence by restricting the given connection coefficients by some constants, and thereby, it converges to the original solution uniformly. For uniform convergence, the $\delta$ value depends only on $\varepsilon$ but not on $x$. For error estimate, we go with the difference of the original approximation with the truncated series in the approximation. Hence the estimate or the bound is attained for those terms that remain after truncation. The bound value that is obtained is validated through some examples, and the error value sticks to the upper bound. This error bound expression falls to zero uniformly when the value of $M$ gets increased.

**Theorem 1 (Convergence Theorem)**

The function $u_{m}(x)$ converges towards the exact solution $u(x)$ where $u(x)$ is continuous and bounded in the second derivative.

Proof:
\[ \leq \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} |c_{nm}| \| \psi_{nm}(x) \| \leq \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} |c_{nm}| \]

**III. THE SOLUTION OF THE BOUNDARY VALUE PROBLEM USING LEGENDRE WAVELETS METHOD**

Consider the Eq. (1)
We can compute the coefficients \( c_{nm} \) through \( 2^{k-1}M \) system of equations.

**Theorem 2 (Error Analysis)**

The error between exact and approximate solution is given by

\[
\varepsilon_{mn} \leq 2^{-(k-1)} \lambda \sqrt{2^n} \sum_{j=0}^{n} \frac{1}{\sqrt{2j+1}} \frac{1}{\sqrt{2(j-2)+2u-3}}
\]

where

\[
\frac{\partial^2 u}{\partial x^2} \leq \lambda \text{ on } [0,1)
\]

and \( \varepsilon_{mn} = \left\| u(x) - u_{mn}(x) \right\|_{L^2} \).

**Numerical Application**

We apply the Legendre wavelets method to the steady-state solution of Eq.(1) for \( k=2 \) and \( M=7 \) is given by

\[
\frac{1}{\cosh \sqrt{k}} \left( 1 + \frac{k x^2}{2!} + \frac{(k x^2)^2}{4!} + \frac{(k x^2)^3}{6!} \right)
\]

and for larger values of \( K \), the solution converges to \( u(x) = \frac{\cosh(\sqrt{k}x)}{\cosh(\sqrt{k})} \) coincides with the solution given in [16].

The values of \( u(x) \) are reported in Tables 1, and 2 for various values \( \alpha, \beta \) and \( K \). Table 1 exhibits the values of \( u(x) \) for \( K = 0.1;1;5 \) at \( \alpha=\beta=0 \) and it also confirms that the LWM solution converges to exact solution when \( x \) approaches 1. The case is shown in Table 2 as the values of \( u(x) \), \( 0<x<1 \) at different values \( \alpha, \beta \) and \( K \). Comparison of LWM solution with other methods VIM and HPM has been reported in the same Table. From both Tables 1 and 2, we conclude that this LWM solution coincides with exact and accurate in less computational effort than the other methods reported in the literature. Figures 1(a) to 1(d) are also confirmed that this LWM converges to the exact solution as \( x \) tends to 1. The analytical conclusion about this model is the concentration decreases when \( K \) increases for all values of \( \alpha \) and \( \beta \). The values of \( u(x) \) are almost equal to 1 for \( K \leq 1 \). The simulation results [16] of this biosensor study coincide with LWM solution with zero percentage error so that the simulation results have not been shown explicitly in the table.

**Table 1:** Steady-state concentration comparison when \( \alpha=0 \) and \( \beta=0 \).

| \( x \) | \( K = 0.1 \) | \( K = 1 \) | \( K = 5 \) |
|-------|---------|---------|--------|
| \( x \) | VIM | HPM | LWM | VIM | HPM | LWM | VIM | HPM | LWM |
| 0.0 | 0.9520 | 0.9520 | 0.9522 | 0.6471 | 0.6481 | 0.6580 | 0.1892 | 0.2113 | 0.2114 |
| 0.25 | 0.9550 | 0.9550 | 0.9550 | 0.9550 | 0.9550 | 0.9550 | 0.2201 | 0.2452 | 0.2459 |
| 0.50 | 0.9639 | 0.9639 | 0.9634 | 0.7298 | 0.7308 | 0.7306 | 0.3286 | 0.3578 | 0.3589 |
| 0.75 | 0.9789 | 0.9789 | 0.9786 | 0.8384 | 0.8386 | 0.8386 | 0.5622 | 0.5851 | 0.5849 |
| 1 | 1 | 1 | 1 | 1.0001 | 1 | 0.9999 | 1 | 1 | 1 |

**Figure. 1.** \( du(x) \) vs \( dx \) for different values of \( K \) when \( \alpha=0 \) and \( \beta=0 \).

Accuracy wise the wavelet-based solution supports exact and experimental solutions. Both VIM and HPM also provide the closest solutions for this problem, but the amount of computational work involved in these methods are very high, time-consuming, and tediousness to evaluate non-linear integration. HPM requires the calculation of coefficients of the infinite series wherein VIM needs more function integrals which are a complicated task to get the good solution. LWM requires only coefficients which can be identified through a solving system of algebraic equations, can be identified by using collocation methods and solved by traditional numerical techniques. With regard to computational effort and solution accuracy, we conclude that LWM is superior to the other methods for this study.

**V. CONCLUSION**

In this paper, the solution of the partial differential equation in the action of a biosensor at mixed enzyme kinetics has been discussed using wavelets. The theoretical analysis, such as the convergence analysis and the error estimation for the proposed technique has been discussed. The solution obtained through the proposed solution has been compared with Homotopy perturbation method and Variational Iteration methods. The quality of the solution has also been investigated through tables and figures. The numerical results are in good agreement with the solution obtained through other traditional methods.
Wavelet solution for nonlinear reaction-diffusion equation

REFERENCES

1. S. Seto., et al. Mathematical Model of Semiconductor Gas Sensor. Sensors and Materials, 18(1) (2006) 001-016.
2. A. Mahmoudi., et al. Analysis of Simulated Output Characteristics of Gas Sensor Based on Graphene Nanoribbon. Journal of Nanomaterials. Vol 2016 (2016) Article ID 9835340, 8 pages.
3. S. Ahlers., et al. A rate equation approach to the gas sensitivity of thin-film metal oxide materials. Sensors and Actuators. 107 (2006) 587-599.
4. K. Selvaraj., et al. Analytical expression for concentration and sensitivity of a thin film semiconductor gas sensor. Ain Shams Engineering Journal. 5 (2014) 885-893.
5. E. Akgari Analytical modeling and simulation of I-V characteristics in carbon nanotube-based gas sensors using ANN and SVR methods. Chemometrics and Intelligent Laboratory Systems. 137 (2014) 173-180.
6. Z. Meng., et al. Legendre wavelets method for solving fractional integro-differential equations. International Journal of Computer Mathematics. 92(6) (2015) 1275-1291.
7. J.D.Winefordner Principles of Chemical and Biological Sensors, (1994) John Wiley & Sons, New York.
8. Scheller F, Schubert F, Biosensors 7, Elsevier, (1988) Amsterdam.
9. Wollenberger U, Lisdat F, Scheller FW Enzymatic Substrate Recycling Electrodes. Frontiers in Biosensors. B and II, Practical Applications.Birkhauser Verlag, Basel, (1997) 45-70.
10. Schulmeister T Mathematical modelling of the dynamic behavior of amperometric enzyme electrodes. Selective Electrode, Rev. 12 (1990) 203-260.
11. Baumann A, Jones C, Wong CY, Price A A generic sandwich-type biosensor with nmol detection limits. Anal. Bioanal. Chem. 378 (2004) 1587-1593.
12. Baronas R, Ivanauskas F, Kulys J The influence of the enzyme membrane thickness on the response of amperometric biosensors. Sensors 3 (2003) 248-262.
13. Aris R The Mathematical Theory of Diffusion and Reaction in Permeable Catalysts. The Theory of the Steady State, Clarendon Press Oxford. 1975.
14. Biemiasz LK, Britz D Recent developments in digital simulation of electro analytical experiments. Pol. J. Chem. 78 (2004) 1195-1219.
15. Rahamathunissa G, Rajendran L Application of He's Variational Iteration Method in nonlinear boundary value problems in enzyme substrate reaction diffusion processes. Part1. The steady-state amperometric response. J. Math. Chem. 44 (2008) 849-861.
16. Manimozhi P, Subbiah A, Rajendran L Solution of steady-state substrate concentration in the action of biosensorresponse at nked enzyme kinetics. Sensors and Actuators B, 147(2010) 290-297.
17. B. Yuttanan, M. Razzagi Legendre wavelets approach for numerical solutions of distributed order fractional diferential equations. Applied Mathematical Modelling. 70 (2019) 350-364.
18. M.H. heydar, Z. Avazzadeh Legendre wavelets optimization method for variable-order fractional Poisson equation. Chaos, Solitons & Fractals, 112: (2018) 180-190.
19. J.Chen, et al.,A fast multiscale Galerkin method for solving second order linear Fredholm integro-differential equation with Dirichlet boundary conditions, Journal of Computational and Applied Mathematics, 364 (2020) 112352, 1-11.
20. Y.Wang, et al.,A new algorithm for the solution of nonlinear two-dimensional Volterra integro-differential equations of high-order. Journal of Computational and Applied Mathematics, 364 (2020) 112301, 1-16.
21. D. Čermá, et al.,Galerkin method with new quadratic spline wavelets for integral and integro-differential equations. Journal of Computational and Applied Mathematics, 363(2020) 426-443.

Authors Profile

S.G. Venkatesh is working as an Assistant Professor in the Department of Mathematics, SASTRA Deemed University, Thanjavur, India. His main research interests include Differential equations and Numerical analysis. To his credit he has published more than 25 papers in international journals.

K. Balasubramanian is working as an Assistant Professor in the Department of Mathematics, Srinivasa Ramanujan Centre, SASTRA Deemed University. He received his Ph.D degree from SASTRA Deemed University in functional analysis and other areas of research are Differential equations and Fixed Point Theorems.

S. Raja Balachandar is also working as an Assistant Professor in the Department of Mathematics, SASTRA Deemed University. His interested research topics are Differential equations and Combinatorial Optimization. He has published more than 50 papers in Journals, International and National Conference Proceedings.

Received Number F88710886192019©BEIESP
DOI: 10.35940/ijeat.F8871.088619
Published By: Blue Eyes Intelligence Engineering & Sciences Publication