Comment on “Accumulating Evidence for the Associate Production of a Neutral Scalar with Mass around 151 GeV”

Andrew Fowlie
Department of Physics and Institute of Theoretical Physics,
Nanjing Normal University, Nanjing, Jiangsu 210023, China

A recent paper [1] accumulates evidence for a new fundamental particle by combining several CMS and ATLAS searches for the Standard Model Higgs boson. The putative particle is a neutral scalar, \( S \), with a mass of about 151 GeV. The reported significances are 5.1\( \sigma \) local and 4.8\( \sigma \) global. This nearly reaches the 5\( \sigma \) threshold for a discovery in high-energy physics. In this brief note we cast doubt on the strength of the evidence for a new particle. After taking into account the fact that signals were fitted to six different channels, we find that the significances are only 4.1\( \sigma \) local and 3.5\( \sigma \) global. The code and instructions for reproducing our calculations are available at this github repo.

I. INTRODUCTION

In ref. [1] several ATLAS and CMS resonance searches are combined into six channels. In each channel, the background model predicts a smooth non-resonant spectrum parameterised by four nuisance parameters, collectively denoted by \( \boldsymbol{\theta} \). The signal model, on the other hand, predicts a Crystal-ball shaped resonance in each channel parameterised by seven parameters: a single mass parameter, \( m \), common to each channel, and six positive signal strength parameters \( \mu_i \geq 0 \) for \( i = 1 \ldots 6 \), one for each channel.

Ref. [1] construct a test-statistic based on a profiled likelihood ratio for the background model and background plus signal model,

\[
\lambda \equiv -2 \log \frac{\max_{m,\mu,\theta} \mathcal{L}(m, \mu, \theta)}{\max_{\theta} \mathcal{L}(\mu = 0, \theta)}, \tag{1}
\]

where \( \mathcal{L}(m, \mu, \theta) \) is the likelihood of the observed data for the model with parameters \( m, \mu \) and \( \theta \), \( \mu = 0 \) corresponds to the background model, and the maximisation over the signal strengths is constrained by \( \mu_i \geq 0 \). Ref. [1] find \( \lambda = 26.01 \) [2]; we haven’t verified this result, but assume that it is correct. This test-statistic is used to compute local and global significances. We discuss the interpretation of significances as evidence in ref. [3, 4]; we focus here only on their correct computation. Although the computation is complicated by the look-elsewhere effect (LEE) and the fact that the signal strength parameters must be positive, the issue with ref. [1] is simple: they computed the significances as though they fitted a single signal when in fact they fitted six independent signal strengths.

II. LOCAL SIGNIFICANCE

To compute significances, it is common to resort to asymptotic formulae (see e.g., ref. [5]) based on Wilks’ theorem [6]. Roughly speaking, this theorem states that test-statistics of the form eq. (1) are asymptotically \( \chi^2 \) distributed, where \( k \) is the number of parameters describing the signal. The resonance search, however, violates a required assumption (see e.g., ref. [7]), as the mass parameter isn’t identifiable under the background model, i.e., when \( \mu = 0 \), the model doesn’t depend on \( m \).

By simply fixing the mass parameter to its best-fit value \( \hat{m} \), however, we may compute a local \( p \)-value. In this case, the test-statistic splits into independent contributions for each channel,

\[
\lambda = \sum_{i=1}^{6} -2 \log \frac{\max_{\mu_i, \theta_i} \mathcal{L}_i(\hat{m}, \mu_i, \theta_i)}{\max_{\theta_i} \mathcal{L}_i(\mu_i = 0, \theta_i)}. \tag{2}
\]

There is a further complication, though, as the signal strength parameters must be positive, such that the background model lies at the boundary rather than in the interior of the background plus signal model. This is addressed by Chernoff’s modification to Wilks’ theorem [8] and further explored in ref. [9]. For each channel, if the best-fit signal at a fixed mass fluctuates downwards, that channel’s contribution to the test-statistic in eq. (2) vanishes. If it fluctuates upwards, that channel’s contribution to the test-statistic is described by a chi-squared distribution with one degree of freedom as usual. This is a mixture distribution called a half chi-square distribution, \( \frac{1}{2} \chi^2 \). The probability density function is (see case 5 in ref. [9] or discussion around eq. 52 in ref. [5])

\[
p(\lambda) = \frac{1}{2} \delta(\lambda) + \frac{1}{2} p_{\chi^2}(\lambda). \tag{3}
\]

where \( p_{\chi^2}(\lambda) \) is the probability density function for a \( \chi^2 \) distribution with one degree of freedom. The terms in this mixture correspond to downwards and upwards fluctuations and the coefficients are half because downwards and upwards fluctuations are equally likely.

The test-statistic for \( n \) channels as in eq. (2) for \( n = 6 \) is thus the sum of \( n \) half-chi-squared variates. We denote this as a \( \frac{1}{2} \chi^2_n \) distribution. The density is (see case 9 with
s = 0 in ref. [9])

\[
p(\lambda) = \left(\frac{1}{2}\right)^n \sum_{i=0}^{n} \binom{n}{i} p_{\chi^2}(\lambda),
\]

(4)

where \( p_{\chi^2}(\lambda) = \delta(\lambda) \). The coefficients are just the
binomial coefficients describing the chances of \( i \) upwards fluctuations at the fixed mass in \( n \) channels. When \( i \) of them fluctuate upwards, the test-statistic is a sum of
\( i \) chi-squared variates which follows a \( \chi^2 \) distribution. When \( n = 1 \), this reduces to eq. (3).

In the \( n = 1 \) case, the density in eq. (3) leads to the
simple result for the local significance (eq. 52 in ref. [5]),

\[
Z_{\text{Local}} = \sqrt{\lambda}.
\]

(5)

This does not apply when \( n \neq 1 \) but was used [2] in ref. [1] to calculate
\( Z_{\text{Local}} = 5.1\sigma \). In contrast, using
eq. (4), we obtain \( Z_{\text{Local}} = 4.1\sigma \) when combining \( n = 6 \)
channels as in ref. [1]. We validate this result through
Monte Carlo (MC) simulations described in section III.

Thus to avoid overstating the significance and reaching
potentially faulty conclusions, we must take into account
that we searched six channels.

III. GLOBAL SIGNIFICANCE

We previously considered a local significance based on a
fixed mass. We should compute a global significance by taking into account every test that was performed and
every test that would have been performed were the data
different (see problems 1 and 2 in ref. [10] for a pedagogical if partisan discussion). Unfortunately, we don’t
know the analysis plan of the authors of ref. [1] and thus
there are imponderable look-elsewhere effects associated
with their choices of statistical tests and datasets. Even
if the authors of ref. [1] began with a particular theory of
a new scalar boson in mind, many analysis choices could
have been influenced by the observed data. We are not,
however, suggesting conscious hacking of any sort and
direct readers to the nuanced discussion in ref. [11, 12].

We don’t further discuss these issues and instead compute
the global significance following the scope of the LEE considered in ref. [1] by taking into account the LEE
for the mass range that was searched. In this case, we
consider the test-statistic as a random field over the mass
range and consider its most extreme fluctuation. Ref. [1]
accounted for this LEE by multiplying the local \( p \)-value
by a trial factor of 5 [2]. This was an estimate motivated
by the 140 GeV – 155 GeV range and the 1.5 GeV
and 14 GeV resolutions in the six channels. We instead
compute it.

There are rules of thumb for the trial factor based on
the range, \( \Delta m \), and resolution, \( \sigma_m \), e.g.,

\[
N \approx \frac{1}{3} \frac{\Delta m}{\sigma_m} Z_{\text{Local}}.
\]

(6)

This rule yields a trial factor of about 14, but it was
found and tested only for the case of \( n = 1 \) channels [13]. Thus rather than using a rule of thumb, we
compute it through MC simulations and the Gross-Vitells
method (see e.g., ref. [14–16]).

The Gross-Vitells method is a sophisticated treatment
based on the expected numbers of up-crossings of random fields above specified thresholds. To apply it, we
required the so-called Euler characteristic densities for
the \( \frac{1}{2} \chi^2 \) random field and an estimate of the expected
number of up-crossings at a specified level. From theorem
1 and remark 2 in ref. [17], the Euler characteristic
densities for the mixture \( \frac{1}{2} \chi^2 \) are just the same mixtures
of the known densities for \( \chi^2 \) random fields (see theorem
15.10.1 in ref. [14]). To estimate the expected number
of up-crossings, we perform simulations using a toy treatment
of a resonance problem with five channels with resolution
1.5 GeV and one channel with resolution 14 GeV
in a window 140 GeV – 155 GeV. In this toy treatment,
we fit the mass and signal strengths of a Crystal ball
function on top of a fixed background. The resulting
test-statistic should obey the same asymptotic distribution
as that in the full treatment. We ultimately find a
global significance of about 3.5\( \sigma \), which corresponds to a
trial factor of about 12.

To check the asymptotic results for the local and global
significances, we perform MC simulations of the test-
statistic. We perform 100,000 pseudo-experiments with
the toy treatment of the problem described above, computing
the test-statistics eqs. (1) and (2) in each case. By
simply counting the number of simulations in which the
test-statistics exceeded that observed, we again find 4.1\( \sigma \)
local and 3.5\( \sigma \) global significances, validating our
previous results.\(^{1}\) The distribution of the test-statistic and

\(^{1}\) Note though that this does not validate that the asymptotic limit
was valid in the real problem, as our toy treatment used large
numbers of expected events in all channels.
the test-statistic at a fixed mass of 151 GeV are shown in fig. 1. We see that the simulations in the latter case closely matches a $\frac{1}{2}\chi^2$ distribution, as expected.

**IV. SUMMARY**

Taking into account the fact that in ref. [1] six signals were fitted rather than one and properly computing the LEE, we find the significances for a new neutral scalar are only 4.1$\sigma$ local and 3.5$\sigma$ global. Thus while this anomaly may remain intriguing, it isn’t as significant as first thought.

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