A magnetohydrodynamic model for multi-wavelength flares from Sagittarius A* (I): model and the near-infrared and X-ray flares

Ya-Ping Li1,2,†, Feng Yuan1,‡, & Q. Daniel Wang3,‡
1Key Laboratory for Research in Galaxies and Cosmology, Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai 200030, China
2Department of Astronomy and Institute of Theoretical Physics and Astrophysics, Xiamen University, Xiamen, Fujian 361005, China
3Department of Astronomy, University of Massachusetts, Amherst, MA 01003, USA

ABSTRACT
Flares from the supermassive black hole in our Galaxy, Sagittarius A* (Sgr A*), are routinely observed over the last decade or so. Despite numerous observational and theoretical efforts, the nature of such flares still remains poorly understood, although a few phenomenological scenarios have been proposed. In this work, we develop the Yuan et al. (2009) scenario into a magnetohydrodynamic (MHD) model for Sgr A* flares. This model is analogous with the theory of solar flares and coronal mass ejection in solar physics. In the model, magnetic field loops emerge from the accretion flow onto Sgr A* and are twisted to form flux ropes because of shear and turbulence. The magnetic energy is also accumulated in this process until a threshold is reached. This then results in a catastrophic evolution of a flux rope with the help of magnetic reconnection in the current sheet. In this catastrophic process, the magnetic energy is partially converted into the energy of non-thermal electrons. We have quantitatively calculated the dynamical evolution of the height, size, and velocity of the flux rope, as well as the magnetic field in the flare regions, and the energy distribution of relativistic electrons in this process. We further calculate the synchrotron radiation from these electrons and compare the obtained light curves with the observed ones. We find that the model can reasonably explain the main observations of near-infrared (NIR) and X-ray flares including their light curves and spectra. It can also potentially explain the frequency-dependent time delay seen in radio flare light curves.

Key words: black hole physics—accretion, accretion discs—Galaxy: centre—magnetic reconnection—(magnetohydrodynamics) MHD—radiation mechanisms: non-thermal.

1 INTRODUCTION
Various observations have confirmed beyond reasonable doubt that our Galaxy hosts a supermassive black hole (SMBH), Sagittarius A* (Sgr A*), with its mass of $M_\bullet \approx 4 \times 10^6 M_\odot$ (where $M_\odot$ is the solar mass; see reviews by Genzel et al. 2010). Multi-wavelength observations of Sgr A* reveal that its bolometric luminosity is $L_{\text{bol}} \sim 10^{-3}L_{\text{Edd}}$ (where $L_{\text{Edd}}$ is the Eddington luminosity), which is five orders of magnitude lower than that predicted by a standard thin disc accretion at the Bondi accretion rate (Baganoff et al. 2003). A number of theoretical efforts have been made accompanied by these observational progresses (see reviews by Genzel et al. 2010 and Yuan & Narayan 2014). We now understand that an advection-dominated accretion flow (ADAF) scenario works for Sgr A*. In this model, the low luminosity is due to both the low radiative efficiency and the strong mass loss via wind of the ADAF (Yuan et al. 2003, 2012a, 2015; Narayan et al. 2012; Li et al. 2013; Wang et al. 2013; Gu 2015; Roberts et al. 2017). Sgr A* is thus an excellent laboratory for studying the accretion and ejection physics in such radiatively inefficient accretion flows (RIAFs), which are ubiquitous in the nearby universe.

Sgr A* is usually in a quiescent state, and occasionally interrupted by rapid flares (on timescales $\sim 1$ hr), most significantly in X-ray (Baganoff et al. 2001) and near-infrared (NIR; Genzel et al. 2003; Ghez et al. 2004). Many such flares have been observed in various wavebands, including those detected by Chandra (Baganoff et al. 2001; Eckart et al. 2004, 2006b; Akhavan et al. 2008; Eckart et al. 2008a; Marrone et al. 2008; Yusef-Zadeh et al. 2008; Eckart et al. 2012; Nowak et al. 2012; Neilsen et al. 2013; Ponti et al. 2015; Yuan & Wang 2016), XMM-Newton (Goldwurm et al. 2003; Porquet et al. 2003, 2005; Béland et al. 2005; Yusef-Zadeh et al. 2006a; Porquet et al. 2008;
Yusef-Zadeh et al. 2009; Trap et al. 2011; Ponti et al. 2015), Swift (Degenaar et al. 2013, 2015), and NuSTAR (Barrière et al. 2014; Dibbi et al.). Many have also been detected in NIR (Genzel et al. 2003; Eckart et al. 2004, 2006b; Ghez et al. 2004; Yusef-Zadeh et al. 2006a; Eckart et al. 2008a; Yusef-Zadeh et al. 2008, 2009; Kunneriath et al. 2010; Trap et al. 2011; Eckart et al. 2012; Haubois et al. 2012; Hora et al. 2014; Shahzamanian et al. 2015), in sub-millimeter (Eckart et al. 2006b; Yusef-Zadeh et al. 2006a, 2009; Kunneriath et al. 2010; Trap et al. 2011; Eckart et al. 2012; Haubois et al. 2012; Dexter et al. 2014; Bower et al. 2015; Brinkerink et al. 2015), and also in radio (Yusef-Zadeh et al. 2006b, 2008, 2009; Bower et al. 2015; Brinkerink et al. 2015).

We now briefly summarize the main properties of these multiwavelength flares (see also Dodds-Eden et al. 2009 for a detailed summary of the general properties of the NIR and X-ray flares from Sgr A*). The flare rate is roughly two per day in X-ray (Neilsen et al. 2013; Ponti et al. 2015; Yuan & Wang 2016) and more frequently in NIR. The NIR and X-ray flares occur simultaneously within 3 mins when both are observed in company (e.g., Eckart et al. 2004, but see Yusef-Zadeh et al. 2012 for a counter-example). The amplitude of the NIR and X-ray flares can be up to ∼ 20 and 160 of the quiescent fluxes, respectively (e.g., Yusef-Zadeh et al. 2009; Nowak et al. 2012). The full width at half maximum (FWHM) of the NIR flare profile is about 60 mins, which is about twice that of the X-ray. There are substructure variations with characteristic timescale of ∼ 20 mins in the NIR light curve occasionally, but not present at the same level in X-ray (Dodds-Eden et al. 2009). The light curves of both X-ray and NIR flares are roughly symmetric, but the brightest flare in Nowak et al. (2012) shows remarkable asymmetry profile with a faster decline than rise. A large fraction of the X-ray flares in the XVP campaign also shows a faster rise and slow decay profile (Yuan & Wang 2016). The flares in the NIR are significantly polarized with the typical polarization degrees of the order of 20 ± 10% (e.g., Eckart et al. 2006a; Shahzamanian et al. 2015).

The X-ray radiation of Sgr A* is believed to have two distinctive states. One is a steady quiescent emission which is dominated by the radiation around the Bondi radius, while the other is point-like flare emission arisen from the innermost region of the accretion flow (e.g., Baganoff et al. 2001, 2003; Wang et al. 2013). Therefore, the X-ray flares look like to be large amplitude, short duration events overlaid on a flat baseline (Neilsen et al. 2013; Li et al. 2015; Yuan & Wang 2016). However, it is less clear that such a conclusion can be applied to the NIR emission. Much of the NIR variation could just represent the red noise of the underlying quiescent emission (Meyer et al. 2008; Do et al. 2009). Flares in sub-millimeter and radio show much shallower and broader profiles than those in NIR and X-ray. There is a general trend that the peak flare emission at a higher radio frequency leads that of a lower one, e.g., 43 GHz leading 22 GHz by 20 ~ 40 mins (Yusef-Zadeh et al. 2006b). There are also some evidence for time lags among radio, sub-millimeter and NIR/X-ray (Marrone et al. 2008; Yusef-Zadeh et al. 2008, 2009; Brinkerink et al. 2015), although some debates on the correlation of variabilities at different bands exist (e.g., Dodds-Eden et al. 2010).

Most theoretical works of the flares focus on their radiation mechanisms. The proposed flare models usually invoke synchrotron and/or inverse Compton radiation processes. The highly polarized NIR emission is the evidence for a synchrotron origin of the NIR flares, produced by a population of non-thermal electrons (e.g., Eckart et al. 2006a; Shahzamanian et al. 2015, see also references therein). The non-thermal electrons are likely accelerated by magnetic reconnection, shock or turbulence in either an accretion flow (Yuan et al. 2004; Dodds-Eden et al. 2009; Li et al. 2015, and references therein) or an assumed jet (Markoff et al. 2001). The suggested radiation mechanisms for the X-ray flares includes synchrotron (Markoff et al. 2001; Yuan et al. 2003, 2004; Dodds-Eden et al. 2009), inverse Compton scattering (Markoff et al. 2001; Yuan et al. 2003; Eckart et al. 2004, 2006b; Liu et al. 2006a; Marrone et al. 2008; Yusef-Zadeh et al. 2012), and/or bremsstrahlung (Liu & Melia 2002). Continuous injections of a population of high-energy electrons are in general required for the synchrotron mechanism in order to balance the fast cooling of the X-ray synchrotron emission. This may be a natural ingredient for the magnetohydrodynamic (MHD) process, e.g., magnetic reconnection considered in this work.

There do exist some theoretical models aimed to explain the physical origin of the flares, including accretion instabilities (Tagger & Melia 2006; Falanga et al. 2008), orbiting hot spots (Broderick & Loeb 2005; Meyer et al. 2006; Tripp et al. 2007; Hamasa et al. 2009), expanding plasma blobs (Yusef-Zadeh et al. 2006b; Eckart et al. 2006b; Yusef-Zadeh et al. 2009; Dodds-Eden et al. 2010; Trap et al. 2011), and tidal disruption of asteroids by the SMBH (Cadez et al. 2008; Kostić et al. 2009; Zubovas et al. 2012). Recently, modeling efforts have been focused on magnetic reconnection within the accretion flow and are usually based on the MHD numerical simulations (Chan et al. 2009; Dexter et al. 2009; Maitra et al. 2009; Dodds-Eden et al. 2010; Chan et al. 2015; Ball et al. 2016). The flares are produced by the radiation of non-thermal electrons accelerated in the reconnection. Note that the ingredient of the invoked particle acceleration process in these modeling efforts (including the present one) have to be phenomenological in the sense that it is taken from other independent works, usually particle-in-cell simulations. One representative work is Dodds-Eden et al. (2010). In this work, they consider the synchrotron radiation from a population of non-thermal electrons transiently accelerated by an episodic magnetic reconnection occurred in the accretion flow. They assume time-dependent profiles for the injection rate and the magnetic field strength in the description of the non-thermal electron distribution evolution, which are responsible for the light curves of the NIR and X-ray flares, as well as the spectral energy distribution (SED). More recently, Ball et al. (2016) find that X-ray variability could result from non-thermal electrons in localized highly magnetized regions, based on their general relativistic MHD (GRMHD) simulations. Most of these works suggest that magnetic reconnection likely plays an important role in producing the NIR, and especially X-ray flares.

In the present work, we propose an alternative model for the flares of Sgr A*. Different from the works mentioned above, we assume that the flares are caused by magnetic reconnection occurred not in the main body of the accretion flow, but instead in the surface or the coronal region of the accretion flow. This work is a development of the MHD model for the formation of episodic jets proposed by Yuan et al. (2009). The model is analogous to the coronal mass ejection (CME) of the Sun (Lin & Forbes 2000). Whereas the work by Yuan et al. (2009) focuses on the dynamics of the ejection of blobs from the black hole accretion flow, the present work moves a step further to model the associated radiation. The modeling is motivated by the following facts: First, based on a statistical

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1 Some other possibilities have also been proposed, such as accretion instabilities (Tagger & Melia 2006; Falanga et al. 2008) and shocks (Dexter & Fragile 2013).
analysis for X-ray flares of Sgr A*, Li et al. (2015) have shown that they are consistent with events from a three-dimensional self-organized criticality (SOC) system, similar to solar flares, which are powered by magnetic reconnection in the corona above the accretion flow. Second, the detailed study to the solar flares over many years has shown that the powerful CMEs are physically associated with the strong solar flares, i.e., these flares and CMEs are likely the different manifestations of the same physical process. In fact, the flare model to be presented in this work is similar to the standard model of solar flares. Third, if the Sgr A* flares seen in NIR and X-ray are physically associated with those seen in radio, then we can naturally speculate that they are all linked to the plasmoid ejection process, as clearly indicated by the observed wavelength-dependent time lag mentioned above (e.g., Yusuf-Zadeh et al. 2006b; Marrone et al. 2008; Yusuf-Zadeh et al. 2008, 2009; Brinkerink et al. 2015). Lastly, extensive MHD numerical simulations of black hole accretion flows have led to the consensus that the hot accretion flow is enveloped by a tenuous corona which is magnetically dominated (see Yuan & Narayan 2014 for a review). The contrast of the density and magnetic-to-gas pressure ratio between the accretion flow and the corona is similar to that between the solar photosphere and solar corona (Aschwanden 2005). These facts provide us with the motivation to make the above analogy between the Sun and the accretion flow.

In fact, several works have been presented along the above line. Without considering the dynamics of the ejection of a blob from the accretion flow, Kusunose & Takahara (2011) calculated the synchrotron radiation from such a blob, assuming some time-dependent profiles for the non-thermal electron injection. The results were then compared with the observed light curve and SED of the NIR and X-ray flares in Sgr A*. More recently, Younsi & Wu (2015) investigated the emissions from the plasmoids with dynamics as described in the CME scenario for episodic jets. They considered special and general relativistic effects (especially the gravitational lensing one), on the light curves, but without paying attention to the spectra. Furthermore, Meng et al. (2014) applied a similar CME model to interpreting the giant flare of three magnetars by assuming the free magnetic energy released in an eruptive process to power the giant flare events.

Here, we present a time-dependent MHD model for the flares of Sgr A* within the framework described by Yuan et al. (2009). We will calculate both the dynamics of the blob ejection and the corresponding radiation, including the light curves and spectrum of NIR and X-ray flares. In a subsequent work (Li et al. in preparation), we will focus on interpreting radio flares based on the same model. As we will see, the model can explain the observations quite well. The paper is organized as follows. We describe our MHD model in Section 2. Numerical results of our model and comparisons with observations are presented in Section 3. We discuss our results in Section 4 and summarize the work in Section 5. Throughout this work, we assume that the SMBH mass is \( M_\odot = 4 \times 10^6 M_\odot \) and its distance is \( d = 8 \) kpc.

## 2 METHODOLOGY

### 2.1 Dynamical Evolutions

In this section we calculate the dynamical evolution of plasmoids ejected from the coronal region of the accretion flow. Our calculation starts with a newly formed plasmoid (also called flux rope), as shown by Figure 1, in which the closed shaded circle represents the core of the plasmoid. The formation of the plasmoid is not addressed here, but should be similar to the formation of the prominence in the Sun. It could be due to the thermal instability of the gas in the corona, or due to the reconnection of a magnetic loop emerged from the accretion flow into the corona (Yuan et al. 2009). The magnetic loops emerged from the accretion flow into the corona has been proposed as one of the main magnetic field configurations by Blandford (2002) and studied by some previous works (e.g., Uzdensky & Goodman 2008; Guan & Gammie 2011).

Initially, the flux rope is enveloped with magnetic loops, with their foot points anchored in the accretion flow, as shown in Figure 1. The subsequent dynamical evolution of the system due to the turbulent motion of the accretion flow consists of two stages connected with a triggering process. While the details can be found in Yuan et al. (2009), here we briefly summarize the whole process. In the first stage, the plasmoid is in an equilibrium when there is a balance among the forces due to the magnetic compression, magnetic tension, and gravity. The foot points of the magnetic field lines are anchored into the accretion flow. The magnetic energy is gradually accumulated in the coronal magnetic field in response to the turbulent motion and differential rotation of the flow. The evolution of the system in this stage is ideal, which means that magnetic reconnection does not take place in the corona. The gradual evolution of the boundary conditions in the accretion flow will inevitably bring the plasmoid into a critical point when the total magnetic energy of the system reaches a threshold, after which further evolution causes the loss of equilibrium of the system initiated by the triggering process. In the second stage, the loss of this equilibrium leads to a catastrophic evolution of the plasmoid, during which most of stored free magnetic field energy is rapidly released due to the magnetic reconnection. It is thus this non-ideal MHD process in this stage that powers the radiative flares and is our focus in the following calculations.

Figure 1 shows an illustration diagram characterizing the disrupting process. The bolded vertical line represents the current sheet, a neutral region separating magnetic field lines of opposite polarity. The motion of the flux rope is mainly subject to the gravity and magnetic forces (Lin et al. 2006). The gas pressure is assumed to be much smaller compared to the magnetic pressure in the coronal region and is thus neglected in this model.

After the loss of the equilibrium, the flux rope is thrust outward and its motion is governed by

\[
my_s \frac{d^2 h}{dt^2} = \frac{1}{c} |\mathbf{B}_{\text{ext}}| - F_g
\]

(1)

to the first order of approximation, where \( m \) is the total mass inside the flux rope per unit length, \( \gamma_c = 1/\sqrt{1-h^2/c^2} \) is its Lorentz factor, \( h \) is the height of the flux rope from the surface of the accretion flow, \( I \) is the total electric current intensity flowing inside the flux rope, \( B_{\text{ext}} \) is the total external magnetic field measured at the centre of the flux rope. The first term on the right-hand side of

\[\text{Using shearing box MHD numerical simulation approach, Guan & Gammie (2011) find the existence of magnetic loops in the coronal region of the disc. Although this simulation is for a thin disc, we believe that the presence of the magnetic loops should remain the same for a thick accretion flow.}\]

\[\text{This is in reference to the radius of the flux rope core } r_{\text{eq}} \text{ being much less than the typical eruption length scale of the system } h, \text{ i.e., } r_{\text{eq}}/h \ll 1 \text{ (Forbes & Isenberg 1991; Isenberg et al. 1993), which is always satisfied in our model.}\]
Equation (1) is the magnetic force and the second term $F_g$ is the gravitational force acting on the mass inside the flux rope, both of which will be given below.

In the zeroth-order approximation, the following equations hold (see Lin & Forbes 2000, for details):

$$j \times B = 0,$$  

$$j = \frac{c}{4\pi} \nabla \times B,$$

where $j$ and $B$ are the electric current density and the magnetic field in the system, respectively.

In the Cartesian coordinate system $(x, y)$, the $x$-axis is parallel to the equatorial plane of the accretion flow and $y$-axis points upward (see also Figure 1). Solving Equations (2) and (3) gives the configuration of the force-free magnetic field of the system (Reeves & Forbes 2005)

$$B(\zeta) = \frac{2iA_0(\lambda^2 + \lambda^2)}{\pi (\lambda^2 - \lambda^2)(\lambda^2 + \lambda^2)} \left( \frac{\lambda^2 + \lambda^2}{\lambda^2 + \lambda^2} \right)^{\frac{1}{2}},$$

where $\lambda = x + iy$, $A_0 = B_0 \sigma_0$, is the source field strength and $B_0 = 2B_0/(cA_0)$ is the normalization of the magnetic field strength on the surface of the accretion flow.

The corresponding vector potential function $A(\zeta)$ is given by

$$A(\zeta) = -\int B(\zeta) d\zeta.$$  

Knowing the magnetic field configuration of the system, we can derive the forces acting on the flux rope. The magnetic force, $F_m$, can be expressed as

$$F_m = \frac{B_0^2 A^2}{8\pi L_{PQ}^2} \left( \frac{H_{PQ}^2}{2h^2} - \frac{(p^2 + A^2)(h^2 - q^2)}{h^2 + A^2} \right),$$

and the gravity, $F_g$, is

$$F_g = \frac{GM_A \rho_m}{(R_0 + h)^2},$$

where $L_{PQ}^2 = (\lambda^2 + p^2)(\lambda^2 + q^2)$, $H_{PQ}^2 = (h^2 - p^2)(h^2 - q^2)$. $\lambda$ is the half-distance between the two field line foot points anchored on the disc surface (see Figure 1 for the illustration of these parameters), the initial enclosed mass per unit length is $m = m_0$ in the flux rope, and is given by $m_0 = \frac{4\pi}{c} \rho_0 \lambda^2$. $h$ is the radius of the flux rope and usually $r_0 = 0.1h$ is adopted according to the experience of the solar flares, $\xi$ the density ratio for the flux rope with respect to the background density $n_0$, $m_0$ is the mass of the Hydrogen atom, and $M_A$ is the mass of the central black hole.

The first term in the square brackets on the right-hand side of Equation (6) denotes the magnetic compression force, while the other two terms in the bracket represent the magnetic tension forces. It is this magnetic compression force that pushes the flux rope outward (upwards), and makes the catastrophic loss of the equilibrium in the system possible. The magnetic tension and gravity term tends to pull the flux rope backwards (downwards). The system is in equilibrium initially as they balance each other. When the compression term dominates over the other, the system loses its equilibrium, and thrusts the flux rope outward in a catastrophic fashion.

The dynamic equation governing the motion of the flux rope can thus be deduced to

$$\frac{\gamma_0^3 d^2 h}{dt^2} = \frac{B_0^2 A^4}{8\pi m L_{PQ}^2} \left( \frac{H_{PQ}^2}{2h^2} - \frac{(p^2 + A^2)(h^2 - q^2)}{h^2 + A^2} \right),$$

From Faraday’s Law, the electric field in the reconnection region is induced by the reconnection process $E(\zeta)$ and is given by

$$E_\zeta(\zeta) = -\frac{1}{c} \frac{\partial A^0}{\partial t} = M_A V_A B_0 (0, y_0) / c,$$

where $A^0 = \frac{A^0}{c}$, is the magnitude of the vector potential along the current sheet, $y_0 = (p + q)/2$ is the height of the current sheet centre, $V_A \equiv B_0 (0, y_0)/\sqrt{\mu_0}$ is the local Alfvén speed, $M_A$ is the Alfvén Mach number of the reconnection inflow and is a measure of the reconnection rate in the current sheet (Lin & Forbes 2000), which is defined to be the reconnection inflow speed $V_{in}$ divided by the local Alfvén speed $V_A$ near the reconnection region (i.e., $M_A \equiv V_{in}/V_A$). In this work, it is taken to be a constant measured at the current sheet centre $y_0$, and the magnetic field $B_0 (0, y_0)$ can be directly obtained from Equation (4) with $x = 0$ and $y = y_0$. The expression of the density profile $\rho(y)$ will be given below.

As the system evolves dynamically and the ejector moves upward at speed $\dot{h}$, the electric field $E_\zeta$ in Equation (9) can be written
as

\[ E_c(t) = -\frac{1}{c} \frac{\partial A^0}{\partial t} = -\frac{1}{c} \frac{\partial A^0}{\partial h} \]

\[ = -\frac{h}{c} \left( \frac{\partial A^0}{\partial p'} + \frac{\partial A^0}{\partial q'} + \frac{\partial A^0}{\partial q} \right) \]

\[ = -\frac{2I_0 h}{c} (A_{0q} p' + A_{0q} q' + A_{0h}), \]

(10)

where \( h = dh/dt \), \( p' = dp/dh \) and \( q' = dq/dh \), \( A_{0q} \), \( A_{0h} \), and \( A_{0b} \) are given in the appendix.

In order to solve Equation (1), another equation is required, namely the frozen magnetic flux condition on the surface of the flux rope. The frozen-flux condition can be expressed as

\[ \frac{2I_0}{c} A_R = A(0, h - r_0) = \text{const.}, \]

(11)

where \( r_0 \) is the radius of the flux rope. Taking the total derivative about \( h \) on the both side of Equation (11) gives

\[ \frac{\partial A_R}{\partial p} p' + \frac{\partial A_R}{\partial q} q' + \frac{\partial A_R}{\partial q} = A_{Rpp} + A_{Rqq} q' + A_{Rh} = 0. \]

(12)

Then \( p' \) and \( q' \) can be obtained from (10) and (12):

\[ p' = \frac{A_{0q} A_{Rq} - A_{Rh} A_{0q}}{A_{Rpp} A_{0q} - A_{Rqq} A_{0q}} \]

(13)

\[ q' = \frac{A_{Rh} A_{0p} - A_{0h} A_{R0}}{A_{Rpp} A_{0q} - A_{Rqq} A_{0q}} \]

(14)

where

\[ \tilde{A}_{0b} = \frac{cE_y}{B_0 dh} + A_{0b} = \frac{M_A V_A B_y(0, y_0)}{B_0 dh} + A_{0b}, \]

(15)

where most of these symbols have its nominal meaning. The other terms used in the above equations are shown in the appendix. Then two equations governing the motions of the current sheet can be expressed as

\[ \frac{dp}{dt} = p'h, \]

(16)

\[ \frac{dq}{dt} = q'h. \]

(17)

Lin et al. (2006) noted that a large amount of plasma in the corona is brought into the flux rope due to magnetic reconnection as the eruption evolves. The evolution of the total mass in the flux rope is governed by:

\[ \frac{dm}{dt} = B_0 M_A \sqrt{\frac{n_e m}{\pi f(y)}} \frac{(q^2 - p^2)(h^2 + \xi^2)}{(h^2 - \xi^2)(q^2 + \xi^2)} \]

\[ \times \sqrt{\frac{f(y_0)(q^2 - p^2)}{(p^2 + \xi^2)(q^2 + \xi^2)}}, \]

(18)

where \( f(y) \) is a dimensionless function of the plasma density distribution against the height \( y \) in the vertical direction of the accretion disc, which is related to the mass-density distribution \( \rho(y) \) as \( \rho(y) = n_e m f(y) \).

There are two choices for the density distribution \( f(y) \). In the previous numerical models (e.g., Yuan et al. 2009; Meng et al. 2015), they adopted the solar model, in which \( f(y) \) followed the empirical S&G atmosphere (e.g., Sittler & Guhathakurta 1999; Lin et al. 2006). In the present work, we adopt a more realistic density distribution in the corona which is taken from a fully general relativistic three-dimensional MHD simulation of hot accretion flows (De Villiers et al. 2005). They found that the number density decreased exponentially with decreasing polar angle (see Figure 3 of that paper). As shown in Figure 2, the two density profiles differ slightly at the large height, but behave similarly in the lower latitude. Our tests show that the final results for the light curve modeling are insensitive to the two different density profiles. Therefore, we will present only the results from adopting the density profile from the numerical simulations (De Villiers et al. 2005).

Now we are ready to investigate the dynamical properties of the system following the catastrophe by solving differential Equations (8, 16, 17, 18) and \( dh/dt = h \). The dynamical properties of the flux rope are described by five physical quantities \( (p, q, h, m, \lambda) \), which are the bottom, top tip of the current sheet, the height and velocity of flux rope, and the mass inside the flux rope per unit length, respectively. For the dynamical evolutions of the system, four free parameters are involved, namely the magnetic field strength \( B_0 \), the electron number density \( n_e \), the density contrast ratio for the flux rope \( \xi \), and the Alfvén Mach number \( M_A \) of the reconnection inflow.

### 2.2 Energies

Based on the Poynting’s theorem, the change rate of the thermal energy is equal to the integral of the Poynting flux along the current sheet \( S(t) \), which is a part of the magnetic energy (Reeves & Forbes 2005). It is this part of magnetic energy that contributes to the observed radiation in the eruption. By calculating the Poynting flux in the current sheet, we obtain the power related to the energy dissipated in the current sheet. Specially, given \( p, q, h, \) and \( \lambda \) as a function of time, we calculate the power associated with the dynamical evolution, which is (Reeves 2006; Meng et al. 2014)

\[ \frac{dW_{EM}}{dt} = S(t) \]

\[ = \frac{c}{2\pi} E_c(t) \int \rho(t) B_y(0, y, t)dy, \]

(19)

where \( E_c(t) \) is the electric field in the reconnection region induced in the reconnection process (see details given by Lin & Forbes).
The magnetic field along the current sheet \( B_0(0, y, t) \) is determined by Equation (4) with \( x = 0 \), \( q \) and \( p \) are the top and bottom tips of the current sheet, respectively, as shown in Figure 1. Here the product of the electric field \( E \) and the magnetic field \( B \) gives the Poynting flux that describes the electromagnetic energy flux entering the current sheet with the reconnection inflow.

Substituting Equations (4) and (9) into Equation (19) and integrating, we have

\[
S(t) = \frac{V_A(0,y_0)}{2\pi} \frac{\epsilon_0^2}{(c^2)} \frac{M_A^2(h_2^2 + \lambda_2^2)}{q(p^2 + \lambda_1^2)(q^2 + \lambda_1^2)} \left( \frac{\sqrt{q^2 - p^2}}{q} \right) \left( \frac{\sqrt{h^2 - y_0^2}}{(h^2 - y_0^2)(y_0^2 + \lambda_1^2)} \right) \left( \frac{p^2 + \lambda_1^2}{h^2 + \lambda_1^2} \right) \left( \frac{q^2 - p^2}{q^2 + \lambda_1^2} \right) \left( \frac{h^2 - p^2}{h^2 + \lambda_1^2} \right) \right),
\]

where \( K \) and \( J \) are the complete elliptic integral of the first kind and second kind, respectively.

Note that \( S(t) \) above is the power released in the reconnection process per unit length. By introducing the third dimensional length scale of the radiative process, i.e., the length of the flux rope, the energy injection rate powering the radiation in the flare region is (refer to the left plot of Figure 1 in Yuan et al. 2009)

\[
\dot{E}(t) = \pi L_0 S(t),
\]

where \( L_0 \) is the distance between two foot points of the flux rope anchored in the accretion flow. The particle inflow rate in the current sheet is related to the local electron number density \( n_e \) and the Alfvén speed \( V_A \), which is expressed as

\[
\dot{N}_{th}(t) = 2\pi M_A V_A (q(t) - p(t)) n_e L_0.
\]

We note that an extra physical parameter \( L_0 \) is added to calculate the power associated with the dynamical evolutions.

### 2.3 Injected Electron Distribution

With the energy and particle injection rates in hand, we now discuss the initial energy spectrum of electrons injected in the flare region at a function of time. We consider a hybrid distribution of electrons, i.e., the electrons are in the mixture of both thermal and power-law distributions (Yuan et al. 2003). The (relativistic) thermal distribution with the total normalized particle number per unit time \( N_{th}(t) \) is

\[
n_{th}(\gamma) = \frac{N_{th}(t) \gamma^2 \beta \exp(-\gamma / \theta_e)}{\theta_e K_2(1/\theta_e)},
\]

where \( \gamma = 1 / \sqrt{1 - \beta^2} \) is the electron Lorentz factor, \( \theta_e = kT_e / m_e c^2 \) is the dimensionless electron temperature, and \( K_2 \) is the modified Bessel function of the second order.

The power-law distribution is described by

\[
n_{pl}(\gamma) = c_{inj}' \gamma^{-p}, \quad \gamma_{min} \leq \gamma \leq \gamma_{max},
\]

where \( \gamma_{min} \) and \( \gamma_{max} \) are the minimum and maximum Lorentz factors of electrons, respectively, \( c_{inj}' \) is the power-law normalization.

We calculate the values of \( \gamma_{min} \) and \( c_{inj}' \) as follows. We assume that the injected energy in non-thermal electrons is equal to a fraction \( \eta \) of the energy in thermal electrons. We further assume that \( \eta \) is small and independent of time. The injected energy flux at a given time \( t \) of thermal electrons at temperature \( \theta_e \) is (Chandrasekhar 1939)

\[
\dot{u}_{th}(t) = a(\theta_e) \dot{N}_{th}(t) m_e c^2 \theta_e,
\]

where the quantity

\[
a(\theta_e) = \left[ \frac{3K_1(1/\theta_e) + K_1(1/\theta_e)}{4K_2(1/\theta_e)} - 1 \right]
\]

varies from 3/2 for nonrelativistic electrons to 3 for fully relativistic electrons, and \( K_n \) are modified Bessel functions of the \( n \)th order. The electron temperature is determined by setting \( u_{th}(t) = \dot{E}(t) \) in Equation (21), which can now be expressed as

\[
\dot{E}(t) = a(\theta_e) \dot{N}_{th}(t) m_e c^2 \theta_e.
\]

The energy density of power-law electrons is

\[
n_{pl} \equiv \frac{c_{inj}'}{p - 2} m_e c^2 \gamma_{max}^{2 - p},
\]

for \( p_e > 2 \). So the normalization of power-law electrons is determined by \( n_{pl} = \eta n_{th} \), which gives

\[
c_{inj}' = (p_e - 2) \gamma_{max} - p_e \gamma_{min} \eta \theta_e \theta_e \dot{N}_{th}(t).
\]

If \( p_e < 2 \), the formula corresponding to Equations (28) and (29) are

\[
n_{pl} \equiv \frac{c_{inj}'}{2 - p_e} m_e c^2 \gamma_{max}^{2 - p},
\]

and

\[
c_{inj}' = (2 - p_e) \gamma_{max} - p_e \gamma_{min} \eta \theta_e \theta_e \dot{N}_{th}.
\]

Another constraint is that the power-law distribution should smoothly match the thermal distribution at \( \gamma_{min} \).

\[
n_{th}(\gamma_{min}) = n_{pl}(\gamma_{min}).
\]

This condition is naturally expected since the non-thermal electrons are presumably accelerated out of the thermal pool. We can then calculate numerically \( c_{inj}' \) and \( \gamma_{min} \) as a function of time by solving Equations (29) and (32) simultaneously.

The value of \( \gamma_{max} \) depends on the details of electron acceleration which are not well understood. We treat it as a constant \( \gamma_{max} = 10^6 \). Note that the exact value of \( \gamma_{max} \) as long as being large enough) is unimportant for the X-ray and NIR flares considered here.

The outflowing particles from the current sheet will flow into two different flare regions, the flux rope and the flare loop as we show in Fig. 1. A reasonable assumption is that half of \( c_{inj}' \) goes into each of them, namely \( c_{inj, rope} = c_{inj, loop} = 1/2 c_{inj}' \). This assumption is equivalent to \( \dot{E}_{loop}(t) = \dot{E}_{rope}(t) = 1/2 \dot{E}(t) \) and \( \dot{N}_{th, rope}(t) = \dot{N}_{th, loop}(t) = 1/2 \dot{N}_{th}(t) \). In the following numerical modeling, we define \( c_{inj}' \) as the injection profile for both the flux rope and the flare loop region to avoid confusions.

What are the values of \( p_e \) and \( \eta \)? Observations of solar flares have revealed a high particle energization efficiency, i.e., 10% - 50% of the magnetic energy ejected into power-law particles (Lin & Hudson 1976). As a fiducial model, we adopt \( \eta = 0.1 \) as a fixed value. The value of \( \eta \) is not very important as it can be partly absorbed by \( L_0 \). The remaining important parameter is \( p_e \). We will show below that many theoretical and simulation works will also constrain this power-law index of the electrons accelerated by magnetic reconnection.
2.4 Light Curve and SED

We follow the method in Dodds-Eden et al. (2010) to calculate the model light curves and spectra. The electron distribution function $N_e(y,t)$ (the number of electrons with Lorentz factor $γ$ at time $t$) evolves according to the following continuity equation (Blumenthal & Gould 1970)

$$\frac{\partial N_e(y,t)}{\partial t} = \frac{\partial}{\partial y} \left( Q_m(y,t) - \frac{\partial y N_e(y,t)}{\partial t} - \frac{N_e(y,t)}{T_{esc}(y,t)} \right).$$

(33)

The escape term $T_{esc}(y,t)$ can be described by the diffusive escape timescale from the system, or by the timescale for catastrophic losses, such as those which occur in the extreme Klein-Nishina limit for electrons, or in secondary nuclear processes for hadronic collisions (Dermer & Menon 2009). Without addressing these microphysics in detail, we simply assume $T_{esc} = 1000$ mins in this work, which is much longer than the typical flare timescales for Sgr A*.

When $T_{esc}(y,t) \to T_{esc}(y)$ and $γ < 0$, Equation (33) has a solution as (Blumenthal & Gould 1970; Dermer & Menon 2009, see their Appendix C)

$$N_e(y,t) = \frac{1}{|y|} \int_1^{∞} dy' Q_m(y',t') \exp \left( -\int_y^{y'} \frac{1}{T_{esc}(y'')} \frac{dy''}{|y''|} \right).$$

(34)

where

$$t' = t - \int_y^{∞} \frac{dy'}{|y'|},$$

(35)

and $Q_m(y,t)$ is the rate at which electrons with the Lorentz factor $γ$ are injected at time $t$ and can be taken as a power law in $γ$: $Q_m(y,t) = c_m(t) \gamma^{γ_p}$.

We consider two cooling processes for $γ$ in Equation (33), synchrotron and adiabatic cooling. For the synchrotron cooling, we have

$$\dot{γ}_{syn} = -\dot{γ}_{t_{syn}}.$$  

(36)

where $t_{syn} = 7.7462 \times 10^8 / (y B^2)$ s for an isotropic pitch angle distribution, and $B$ is in units of Gauss. For the two flare regions, the magnetic field profiles are different, both of which can be determined by Equation (4). We adopt a spatially averaged magnetic field profile in order to simplify the numerical model. For the erupted flux rope region (refer to Fig. 1), it is expressed as

$$B_{loop}(t) = (B(x = 0, q ≤ y ≤ h))$$

(37)

where $q$ means the average over $x = 0, q ≤ y ≤ h$. While for the flare loop below the current sheet (refer to Fig. 1), the magnetic field is averaged over the region $x = 0$ and $y ≤ p$, $B_{loop}(t) = (B(x = 0, 0 ≤ y ≤ p))$.

The adiabatic cooling rate is

$$\dot{γ}_{ad} = -γ d\log R/dt = -γ v_{exp}/R,$$

(39)

where the expansion velocity $v_{exp} = dR/dt$. The adiabatic cooling rate is also different for different flare regions, depending chiefly on the expansion behavior of the radiative blob. For the ejected flux rope enclosing the flux rope region as shown in Figure 1, $R ≈ h - q$, while for the flare loop close to the surface of accretion flow, $R ≈ p$.

The total cooling rate is thus

$$\dot{γ} = \dot{γ}_{syn} + \dot{γ}_{ad}.$$  

(40)

The synchrotron emission is calculated at each time given the instantaneous electron energy distribution using the following formulae (Rybicki & Lightman 1979):

$$j_v = \frac{1}{4\pi} \int_1^{∞} n_e(y)(P_e(y,v,θ)/dy,$$

(41)

where $(P_e(y,v,θ))$ is the pitch angle (LOS and B) averaged spectral power emitted by a single electron and

$$P_e(y,v,θ) = \frac{eB}{\sin θ} \left( \frac{v}{v_{syn}(y,θ)} \right)^3,$$

(42)

with

$$v_{syn}(y,θ) = 3eB\gamma \sin θ/(4Γmc)$$

(43)

and

$$F(τ) = \int_1^{∞} K_{5/3} (τ) dτ.$$  

(44)

The absorption coefficient is

$$α_v = \frac{e^2}{8\pi^2mc^2} \int_1^{∞} n_e(y) \left( \frac{2P_e(y)/γ + dp_e(y)/dy} {dy} \right).$$

(45)

Assuming a homogeneous sphere of radius $R$ the resultant emission is (Gould 1979; Dodds-Eden et al. 2010)

$$v_{L_v} = 4\pi R^2 \frac{v_j}{α_v} \times \left( 1 + \frac{exp(-2α_vR)}{α_vR} - 1 - exp(-2α_vR) \right).$$

(46)

Note that if the emission is optically thin, the luminosity only depends on the total number of accelerated electrons $N_e(y,t) = 4π/3R^3 n_e(y,t)$, so Equation (46) can be reduced to

$$v_{L_v} = 4π R^2 \frac{v_j}{α_v} \left( v \int_1^{∞} N_e(y,t)(P_e(y,v,θ))dy. \right.$$  

(47)

3 NUMERICAL RESULTS

3.1 Fiducial Model

Our model has six parameters, namely $B_0, n_0, ε, M_A, L_0$, and $p_e$. The model can produce light curves and SED of the flares. The characteristic values of several important parameters can be constrained from either the observations of Sgr A* or theoretical works, thus only leave limited room for adjustment.

All spatial lengths of the system are scaled by $r_s = GM_A/c^2$, the gravitational radius of the black hole. The half-distance of two foot points of the magnetic loop is chosen to be $l_0 = 5r_s$ according to Meng et al. (2015). The flux rope radius is hard to estimate and adopted to be $r_0 = 0.1l_0 = 0.5r_s$ following the CME model for our Sun (Lin & Forbes 2000). For the strength of magnetic field in the accretion flow of Sgr A*, many authors suggest that $B ≤ 30$ G (e.g., Yuan et al. 2003; Sharma et al. 2007; Dodds-Eden et al. 2010 and references in the Introduction section). In a localized flare region, however, the field could be much stronger.

We can also reasonably estimate the value of the number density of electrons in the accretion flow, $n_e$, from previous observational and theoretical works. Yuan et al. (2003) found that the number density in the equatorial plane of the innermost region of the accretion flow is $n_e ≈ 10^7$ cm$^{-3}$ by modeling the quiescent
spectrum of Sgr A*. Observationally, we have good constraints on the number density at the Bondi radius by Chandra observation (Baganoff et al. 2003; Wang et al. 2013). Numerical simulations of the hot accretion flow covering four orders of magnitude in the radial dynamical range by Yuan et al. (2012b; see also references therein) has shown that the radial density distribution of the accretion flow can be well described by a power-law form, \( n_e \propto r^{-s} \), with the index \( s \) in the range of 0.5 – 1.0. Combining this result with the observations of Chandra, we can obtain the number density in the inner region of the accretion flow for Sgr A*, which is also close to \( n_e \sim 10^7 \text{ cm}^{-3} \).

The rates of energy release during the rise and decay phases in the process of magnetic reconnection are controlled by two different physical processes. The rise phase is mainly driven by an ideal-MHD process, which is determined by the Alfvén timescale. After the loss of the equilibrium, a current sheet will be formed to halt further evolution of the system, unless reconnection starts. In the decay phase where reconnection begins to dominate in the dynamical evolution, the evolution is determined by the reconnection timescale. Thus, the Alfvén Mach number \( M_A \) can determine the ratio of rise and decline timescales, namely the asymmetry of flare profiles.

In the Sweet-Parker model of the magnetic reconnection, the reconnection speed is only a tiny fraction of Alfvén speed, which means the Alfvén Mach number \( M_A \) is extremely smaller than unity. However, observations in solar flares require the reconnection speed close to the Alfvén speed. One way to speed up the reconnection is to invoke plasma instabilities, for example, the stream instability which makes Ohmic magnetic resistivity anomalously large (Parker 1979). Another way is to consider the presence of turbulence in the current sheet (Lazarian & Vishniac 1999; del Valle et al. 2016). We adopt \( M_A = 0.5 \) as to make the timescale of the rise phase comparable to the decay phase of the flares, which corresponds to the case of quasi-symmetric flares.

The energy spectral index of accelerated electrons is complicated to determine, and depends on the initial and the boundary conditions of the reconnection site (see review by de Gouveia Dal Pino & Kowal 2015). Analytical studies of the first order Fermi process in current sheets predict that the power-law index \( p_e \) is \( 2.5 \) (de Gouveia Dal Pino & Lazarian 2005) or \( 1.0 \) (Druy 2012). However, Kowal et al. (2012) found a hard power-law spectrum with \( p_e = 1.0 \) for particle acceleration in 3D MHD reconnection sites, close to \( p_e \sim 1.5 \) obtained from 2D collisionless PIC simulations considering merging islands (Drake et al. 2010). These uncertainties provide us with some flexibilities to choose the value of \( p_e \).

As for the length of the flux rope, the observational constraint is less certain, and we adopt \( L_0 = 50 r_g \), the same order of magnitude as the height of the flux rope (see Figure 3). As we will see, the impact of \( L_0 \) is simply only to affect the magnitude of the flare luminosity in NIR and X-ray in a same way.

Accordingly, we choose the characteristic values of these parameters mentioned above as summarized in Table 1.

Solving differential Equations (8, 16, 17, 18) and \( dh/dt = \dot{h} \) as shown in Section 2.1 will give the dynamical evolution of the system following the catastrophe. The initial conditions for catastrophe are given by \( h(t = 0), h(t = 0), p(t = 0), q(t = 0) \) and \( m(t = 0) = m_0 \), which control the evolution in the first stage. The determination of \( h(t = 0), h(t = 0), p(t = 0), q(t = 0) \) can be found in Lin \& Forbes (2000), while \( m_0 \) is determined by our model parameters as discussed in Section 2.1. We show the results as the dashed lines in Figure 3. As it is an ideal MHD process without magnetic energy release to power the radiative flare in this stage, we don’t duplicate the calculations here and refer the interested readers to Lin \& Forbes (2000) for details.

The dynamical evolution of the system is shown as the solid lines in Figure 3. It is clear that the flux rope can be accelerated to mid-relativistic speed within several minutes and the height of the flux rope can be as high as several hundred \( r_g \). We can see that the top and bottom ends of the current sheet are very close to each other, as indicated by the blue dot-dashed and black dashed lines in the upper panel of the figure. This closeness mainly arises from a large Alfvén Mach number \( M_A = 0.5 \) that we have adopted, which makes the current sheet reconnect very efficiently. These profiles are quantitatively similar to (although not as dramatic as) the results presented in Yuan et al. (2009). The velocity profile of the flux rope as shown in the bottom panel of Figure 3 is much shallower compared to the dramatic eruption profile in Yuan et al. (2009) due to the fact that the Alfvén timescale inferred there is longer than that in this work. In Figure 4, we present the evolution of the Alfvén velocity.

To obtain the adiabatic cooling term in modeling the time-dependent distribution of injected electrons, we calculate the term \( d\log R/dt \) (refer to Equation (39), as shown in Figure 5). There are two flare regions as discussed in Section 2.3, i.e., the flux rope and flare loop shown in Figure 1. We calculate the corresponding term in these two regions with the method presented in Section 2.3. As shown in Figure 5, the expansion velocity of the size of “flux rope” region is much larger than that of the “flare loop” region. This implies a fast cooling rate in the late stage of the evolution, as shown in the bottom panel of Figure 5. Note that in the beginning of the catastrophe, the cooling rate in the loop region is larger than that of the flux rope. This is partly owing to the much smaller size of the flare loop region in the initial stage following the loss of equilibrium when the bottom end of the current sheet is still close to the accretion flow surface.

Another important factor determining the cooling rate and the
consequent radiation is the distribution of magnetic field in the flare regions. The evolution of the spatially averaged magnetic field in the “flare loop” and “flux rope” regions are shown in Figure 6 based on Equations (38) and (37). It is clearly shown that the magnetic field strength decreases significantly after magnetic reconnection starts. It is surprising that there is a rapid increase initially in the magnetic field in the loop region. We can also see that the maximum field strengths are different in the two different regions. The field strength in the flux rope region can be higher than $B_0$ set in Table 1. This is because the magnetic flux accumulates as the magnetized plasma flowing into the flare regions.

We further discuss the energy release during the eruption process. The power output associated with the dynamical evolution is directly related to $E(t)$ in Equation (21). The shape of $E(t)$ shown in Figure 7 is reminiscent of typical light curves of flares in NIR and X-ray. The quasi-symmetric profile of $E(t)$ is due to the relative large value of $M_A = 0.5$, which results in a steep (or soft) tail in the decline phase of the light curve. Assuming that all the energy dissipated can be converted to radiative flares, Meng et al. (2014) used $E(t)$ to compare with the observed light curve. This simplification could overestimate the efficiency of the radiation. As shown in the upper panel of Figure 7, the energy release rate is about 2 orders of magnitude higher than the peak luminosity of typical flares as shown in Figure 10, indicating a radiative efficiency of only $\sim 1\%$. In the bottom panel of Figure 7, we calculate the electron number flowing into the current sheet. A fraction of which is accelerated into a power-law distribution by the magnetic reconnection. The total particle injection profile $\dot{N}_e(t)$ is slightly narrower than $\dot{E}(t)$, but is also relatively symmetric.

With the time profile of energy injection rate and the particle injection rate, we can now obtain the time evolution of various physical quantities, namely the dimensionless electron temperature.
\( \theta(t) \), the minimum Lorentz factor of power-law electrons \( \gamma_{\text{min}}(t) \) and the injection profile \( c_{\text{inj}}(t) \), as described in Section 2.3. We show the numerical results in Figure 8. Note that \( N(t) \) is the total electron number by integrating over \( \gamma \), while \( c_{\text{inj}} \) represents the normalization of the power-law electrons. It is the injection profile \( c_{\text{inj}} \) that determines the shape of the resulting light curves. As expected, we can see that the shape, including the rise, decay phase and the width of the light curve, resembles the typically observed X-ray light curves.

With the injection profile \( c_{\text{inj}}(t) \) and the minimum Lorentz factor \( \gamma_{\text{min}}(t) \) in hand, we can obtain the injection term \( Q_{\text{inj}}(\gamma, t) \) in Equation (33). The cooling term \( \dot{\gamma} \) can also be modeled with two terms discussed above. The adiabatic term is shown in Figure 5, while the synchrotron term can be determined by the magnetic field profile in the corresponding flare regions. We can then solve the continuity equation in Section 2.4 to obtain the time evolution of the energy spectrum of electrons. The results are shown in Figure 9. A broken power-law feature in the electron spectra at \( \gamma \sim 10^3 \) exists in the rise phase. This is due to the short cooling timescale of electron compared to the injection timescale, which results in a spectral index of \( \rho_\gamma = p_\gamma + 1 \) (Rybicki & Lightman 1979).

The NIR and X-ray emissions considered here are optically thin, which allows us to utilize Equation (47) to directly calculate the emergent light curves and spectra. The results are shown in Figure 10. In the upper panel of the figure, we compare the model light curve (solid line) to the NIR data (black points) taken on April 4, 2007. Since coordinated X-ray flare were observed, we also calculate the model light curve in X-ray based on synchrotron radiation using the same model parameters. We can see that the X-ray light curve almost peaks simultaneously with the NIR, consistent with the observed data. Due to the fact that the cooling timescale of X-ray emitting electrons is much shorter than the injection timescale, the electrons radiate all of the energy supplied via injection. In that case, the shape of the X-ray light curve follows the injection profile very well, independent of \( \gamma_{\text{min}}(t) \) and \( B(t) \).

As shown in Figure 10, the emissions from the two flare regions, i.e., flare loop and flux rope, are comparable for the NIR and X-ray flares. The total contribution at both two bands are thus the sum from these two flare regions. Since the flare loop makes a comparable contribution to the total emission in both NIR and X-ray, observationally these flares appear to be associated with expanding hot spots close to the black hole. Moreover, there are astrometric signatures during strong flares since we find that such flares are associated with the ejection of the magnetized plasmas from the inner region of the accretion flow. Such expanding and/or ejected blobs could be detected by future high resolution instrument, such as Very Large Telescope Interferometer (VLTI) GRAVITY (Eisenhauer et al. 2011). In addition, our preliminary analysis suggests that the emission from the flux rope region dominates over that from the flare loop for radio flares (Li et al. in preparation). Such a rapid expansion velocity for the radio-emitting (flux rope) region is actually consistent with current radio observations (e.g., Brinkerink et al. 2015), and could be falsified by the near future Event Horizon Telescope (EHT) observations, which make the detection of the source size evolution during the sub-millimeter flares possible.

With the magnetic field strength and electron density used in this work, we find that the contribution from synchrotron self-Compton to the X-ray flares can be negligible, as argued in Dodds-Eden et al. (2009). It is also impossible to interpret the X-ray flares under reasonable physical parameters by inverse Compton process with seed photos from sub-millimeter emission. We

**Figure 8.** Time evolution of the total (including flux rope and flare loop regions) injected electron properties. Upper panel: the dimensionless temperature. Middle panel: the minimum Lorentz factor of power-law electrons. Bottom panel: the normalization of the power-law distribution.

**Figure 9.** Time evolution of the electron energy spectrum. The dashed lines correspond to the rise phase, the solid lines are for the decay phase. The spectra are calculated for every 20 min.
thus neglect the Compton process for X-ray flares emission in this work.

With the time-dependent electron energy spectra, we can also calculate the SED of the flares. Here a time-averaged spectrum over the flaring period is calculated. The result is shown in Figure 11. For reference the quiescent model of Yuan et al. (2003) is plotted (dashed gray line) and overlaid with the flare spectrum. The observed SED data are quoted from Dodds-Eden et al. (2009). The dashed and dot-dashed line correspond to the time-averaged (during flare) emission from flare loop and flare loop regions, respectively. The thick solid line is the total SED from our model.

3.2 Parameter Space Exploration

In this subsection, we investigate the effects of the individual model parameters given in Table 1 on the resultant light curves. The general approach is that we only modify one parameter a time (except for the concentration parameter $\xi$) from the above fiducial model in order to explore the effects. Below we explore the effect of the following parameters in turn: $B_0$, $n_e$, $\xi$, $M_A$, $L_0$, and $p_e$.

In Figure 12, we show the results for the effects of the six model parameters on the NIR light curves. It is straightforward to show that the flare luminosity increases with the increasing of the magnetic field strength, $B_0$. To compare the flare durations for different $B_0$ choices, we align their corresponding light curves with their peak times. As the Alfvén timescale $t_A \propto \sqrt{n_e}/B_0$, the rise and decline timescales decrease with increasing $B_0$, which is due to the shorter Alfvén timescale for the stronger magnetic field. One may naturally expect that the duration of a flare would increase with $n_e$ as shown in the upper right panel of Figure 12. The figure indicates that the flare luminosity is, however, anti-correlated with $n_e$. This is because a lighter blob ejected to farther away from the accretion disc leads to a larger volume of the reconnection region.

To investigate the effect of the concentration parameter $\xi$, we make $n_e \xi$ a constant to fix the mass of the flux rope. We find that the NIR light curve is insensitive to $\xi$. As shown in the middle left panel of Figure 12, the NIR luminosity only slightly decreases with increasing $\xi$. This is because an increase of $\xi$ (or decrease of $n_e$) leads to a decrease of the normalization of power-law electrons while the dynamical properties of the flux rope remains the same for a fixed $n_e \xi$. As we have stated previously, the parameter $M_A$ mainly affects the asymmetry magnitude of the light curve. A larger $M_A$ tends to result in a quasi-symmetric profile as shown in the middle right panel of Figure 12. The effect of $L_0$ is quite straightforward to understand, i.e., changing the amplitude of the light curve as a whole, as shown in the bottom left panel of Figure 12. In the bottom right panel of Figure 12, we show the impact of the power-law index $p_e$. It is obvious that a harder electron spectrum leads to a weaker NIR.
Table 1. Parameters for the MHD model of Sgr A* flares

| model | \( \lambda_0 \) | \( r_{00} \) | \( B_0 \) | \( n_e \) | \( \xi \) | \( M_A \) | \( L_0 \) | \( p_e \) | \( \eta \) | \( t_{esc} \) |
|-------|--------------|--------|--------|-------|------|-------|------|-------|------|-------|
| fiducial | 5 | 0.5 | 135 | 1.6\times10^7 | 0 | 50 | 1.95 | 0.1 | 1000 |
| B | 5 | 0.5 | 125 | 1.5\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 130 | 1.5\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 145 | 1.5\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| ne | 5 | 0.5 | 135 | 1\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.8\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 2.2\times10^7 | 0 | 50 | 2.05 | 0.1 | 1000 |
| \( \xi \) | 5 | 0.5 | 135 | 1.8\times10^8 | 1 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 3.6\times10^7 | 5 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.8\times10^7 | 10 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 0.5\times10^7 | 36 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| MA | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.3 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 10 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 10 | 0.6 | 50 | 2.05 | 0.1 | 1000 |
| L0 | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 30 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 40 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 60 | 2.05 | 0.1 | 1000 |
| pe | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 50 | 1.85 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 50 | 1.95 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 50 | 2.05 | 0.1 | 1000 |
| | 5 | 0.5 | 135 | 1.5\times10^7 | 0 | 0.5 | 50 | 2.10 | 0.1 | 1000 |

NOTE: All the length scales are in units of \( r_s = GM_e/c^2 \), \( B_0 \) in units of Gauss, \( n_e \) in units of cm\(^{-3} \), \( L_0 \) the length of the flux rope, \( p_e \) the index of power-law distributed electrons, \( \eta \) the fraction of thermal energy converted into a population of power-law electrons, \( t_{esc} \) in units of minute.

flares suggest that the substructures in NIR flares could be just statistical fluctuations in a red noise spectrum rather than intrinsic flare events (Meyer et al. 2008; Do et al. 2009), it is still interesting to explore the plausible physical mechanism responsible for them. Dodds-Eden et al. (2010) suggest that this puzzling property is because of the different cooling rate responses of the NIR and X-ray synchrotron-emitting electrons to the change of the magnetic field. In the synchrotron radiation model, small magnetic field fluctuations can produce the substructures observed in the NIR. In contrast, a comparatively smooth shape may be expected in X-ray, because the X-ray emission depends primarily on the injection rate but not on the magnetic field.

During the magnetic reconnection, multiple plasmoids can be formed in the current sheet due to tearing of the current-sheet.
structure, as seen in numerical simulations (Samtaney et al. 2009). When these plasmoids are ejected out from the current sheet, the resultant emission will have substructures in the light curves. This picture is also supported by two-dimensional particle-in-cell simulations of magnetic reconnection (Cerutti et al. 2012). These simulations show a strong anisotropy of the particles accelerated by magnetic reconnection and energetic electrons are concentrated into several compact regions inside magnetic islands. The resultant synchrotron light curve of a flare comprises several bright subflares emitted by energetic beams of particles. The concentration of the accelerated electrons, which could take in action here in a similar environment, can thus be an alternative mechanism responsible for the substructures observed in the NIR. The undetected substructures in the X-ray flare at the same level could be explained by the fact that not every ejector can equally contribute to the NIR and X-ray, as we discuss in the Section 3. The substructures are also observed in the light curves of the X-ray flares (Baganoff et al. 2001; Barrière et al. 2014). This indicates variability in the particle injection profile. The resultant X-ray light curve may mimic the combined emission from the multiple ejectors.

Figure 12. The effect of model parameters on the NIR flare light curves. The observed data shown as black points with error bars are superimposed for comparison. For each plot, we only modify one parameter with others fixed. The value for all parameters are listed in Table 1.
4.2 Time Delays

Yusef-Zadeh et al. (2006b) observed that the peak flare emission at 43 GHz leads the 22 GHz peak flare by ~20 – 40 mins. They show that the time delay of the flare emission can be naturally interpreted in terms of the plasmon model of van der Laan (1966) by considering the ejection and adiabatic expansion of a uniform, spherical plasma blob. This is fully consistent with our model, in which flare activities in Sgr A* are associated with blob ejections. In Yusef-Zadeh et al. (2006b) the blob ejection is an assumption. In the present work, we provide a dynamical interpretation for such an ejection. The quantitative calculations based on our dynamical model will be presented in a subsequent work.

For the NIR and X-ray considered in this work, the simultaneity is because the flares emissions at these two bands are always optically thin during the whole evolution of the blob. The resultant emissions would thus not suffer from the synchrotron self-absorption effect, which is the reason for the frequency-dependent time lag observed in radio bands (Yusef-Zadeh et al. 2006b).
4.3 Asymmetry

Although our numerical modellings are focused on the typical light curves which have symmetric profile, there are many events showing asymmetric features. As we have denoted previously, $M_A$ affects the magnitude of the profile asymmetry: The observed fast rise and slow decline light curves can be interpreted with $M_A < 1$, while $M_A > 1$ tends to generate slow rise and fast decline profiles, as seen in the brightest flare in the XVP campaign (Nowak et al. 2012). When $M_A$ is about unity, one may expect a rather symmetric light curve, which may then reduce the discrepancy between the rise phase of the light curves as shown in Figure 10. In addition, here we have not taken into account the general relativistical effect. Due to gravitational lensing and time dilution effect, Yousni & Wu (2015) found that the emitted profile from a ejected plasmoid could be stretched and/or compressed when the plasmoid is close to the central black hole. As a result, asymmetric (fast-rise slow-decay) light curves could lose their original characteristics, and they may even appear to be quasi-symmetric, or as slow-rise fast-decay, as shown in some observations.

4.4 Polarizations

The high degree of polarization in NIR is a consequence of the presence of a relatively ordered field enclosing the ejecta (see Figure 1) and the small optical depth in the substantially inflated plasmoid blobs. The quantitative discussion of this part will again be presented in the subsequent work. A high degree of polarization for X-ray flares should also be expected due to the same nature as the NIR flares in our model. Such a scenario can thus be tested by the near future X-ray polarimetry, such as enhanced X-ray Timing and Polarimetry Mission (eXTP).

5 SUMMARY

We have developed an analytical MHD model for Sgr A* flares. This work is a development of the Yuan et al. (2009) model, which proposed a general scenario for both the formation of episodic ejection of plasmoids from the accretion flow and the associated radiative flares. The model is analogous with the catastrophe model of solar flares and CMEs (Lin & Forbes 2000). Theoretically, the analogy between black hole flares and solar flares is based on the similarity of the structure between the accretion flow and the solar atmosphere (Yuan et al. 2009). The similarity is further supported by the recent statistical study of X-ray flares, which indicates that they are in a self-organized criticality state driven by magnetic reconnection occurred in the surface of the accretion flow (Li et al. 2015).

The basic scenario is briefly summarized as follows. The starting point is a flux rope located in the corona of the accretion flow. The flux rope is anchored to the accretion flow by the magnetic field lines and is in an equilibrium state initially, balanced by gravity, magnetic tension and pressure forces. The magnetic field lines are controlled by the motion of the accretion flow which is differentially rotating and turbulent. Therefore, magnetic energy and helicity are gradually accumulated with time in the system and eventually reach a threshold. Then the equilibrium of the flux rope is broken down and the flux rope is thrust outwards rapidly. Consequently, the magnetic field lines with opposite directions below the flux rope come close enough, leading to reconnection. Magnetic energy is released in this process and converted into the energy of thermal and power-law electrons. These energetic electrons flow into the flux rope and the magnetic loops where they then emit strong synchrotron radiation. This can explain the observed flares. In this scenario, the radiative flares are associated with expanding hot spots close to the black hole, which could be tested by future observations. The schematic figure of the process is shown in Figure 1.

By assuming certain spacial distributions of the magnetic field and density in the coronal region, we have calculated the dynamical evolution of the height of flux rope (Figure 3), the Alfvén speed (Figure 4), the expansion of the flux rope (Figure 5), the magnetic field close to the reconnection region (Figure 6), the released energy in the reconnection (Figure 7), and the minimum Lorentz factor of accelerated electrons (Figure 8). The dynamical evolution of these parameters further allow us to calculate the evolution of the energy distribution of accelerated electrons in the current sheet (Figure 9), and further their radiation. The results of these calculations are then compared with light curves and SED observed on April 4, 2007 (Figures 10 & 11). Our numerical results show that the flux rope ejected from the surface of the accretion flow can be accelerated to mildly relativistic velocity within ~ 1 hr after the loss of equilibrium. With the relatively large Alfvén Mach number $M_A$, the reconnection of the current sheet can be very efficient and thus the bottom and top tips of the current sheet are rather close to each other. (Figure 3). As the reconnection proceeds, large amounts of the energy flux and particles are brought into the current sheet, and then ejected upwards and downwards. One half of the Poynting flux and energetic electrons are flowing into the flux rope and the flare loop, where flaring activities take place. With appropriate choices of the parameters, we find that the total power by the magnetic reconnection can reach $\sim 10^{35}$ erg s$^{-1}$, and the particle injection rate into the flare regions $\sim 10^{45}$ s$^{-1}$ (Figure 7). The radiative efficiency is thus only $\sim 1\%$ for the resultant luminosity of $10^{39}$ erg s$^{-1}$.

Our calculation results can reasonably explain the main characteristics of the observed flares, including their IR and X-ray light curves (Figure 10) and the spectra (Figure 11). The model can explain not only why NIR and X-ray flares occur simultaneously if both of them are observed, but also some of the NIR flares do not have corresponding X-ray counterparts. Moreover, the astrometric signatures during strong flares due to the expansion and/or ejection blob near the black hole could be detected by future high spatial resolution instruments, such as VLTI GRAVITY (Eisenhauer et al. 2011). The scenario of an expanding radio-emitting blob can also naturally explain the observed time lag between the two light curves at two radio frequencies (e.g., Yusuf-Zadeh et al. 2006b; Brinkerink et al. 2015). The quantitative calculation, together with the interpretation of the observed NIR polarization will be presented in a subsequent paper. Our model also predicates a high degree of polarization for X-ray flares, which can be verified by the near future X-ray polarimetry, e.g., eXTP.

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APPENDIX A: SOLVING EQUATION (I)

The other terms involved in Equations (13,14,15) are listed as follows:

\[
A_R = \frac{\lambda H_{PQ}^2}{2hL_{PQ}} \ln \left( \frac{\lambda H_{PQ}^2}{r_{00}L_{PQ}(h^2 - p^2q^2)} \right)
\]

\[
+ \tan^{-1} \left( \frac{4}{\sqrt{h^2 - q^2}} \right) \left[ \sin^{-1} \left( \frac{q}{h} \right) \frac{p}{q} \right]
\]

\[
+ \left( q^2 - p^2 \right) \Pi \left[ \sin^{-1} \left( \frac{q}{h} \right) \frac{p^2 + q^2}{q^2 + h^2} \frac{p}{q} \right]
\]

\[
- \frac{H_{PQ}^2}{\Pi} \left[ \sin^{-1} \left( \frac{q}{h} \right) \frac{p^2 + q^2}{q^2 + h^2} \frac{p}{q} \right]
\]
