Thermal description of hadron production in $e^+e^-$ collisions revisited

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Abstract

We present a comprehensive analysis of hadron production in $e^+e^-$ collisions at different center-of-mass energies in the framework of the statistical model of the hadron resonance gas. The model is formulated in the canonical ensemble with exact conservation of all relevant quantum numbers. The parameters of the underlying model were determined using a fit to the average multiplicities of the latest measurements at $\sqrt{s} = 10, 29-35, 91$ and $130-200$ GeV. The results demonstrate that, within the accuracy of the experiments, none of the data sets is satisfactorily described with this approach, calling into question the notion that particle production in $e^+e^-$ collisions is thermal in origin.

1 Introduction

The analysis of hadron yields measured in central heavy ion collisions from AGS up to RHIC energies has shown \cite{12341507891011} that hadron multiplicities can be described very well with a hadro-chemical equilibrium approach which is governed by the chemical freeze-out temperature $T$, baryo-chemical potential $\mu_b$, and the fireball volume $V$; for a recent review see \cite{12}. The natural question arising here is whether this statistical behavior is a unique feature of high energy nucleus-nucleus collisions or whether it is also applicable in elementary collisions like, e.g., $e^+e^-$. Previous publications indicated that indeed hadron production in $e^+e^-$ collisions at 14-43 GeV \cite{131415} and 91 GeV \cite{1316} can be well described within a thermal model provided that local quantum number conservation is properly implemented. The main result of these investigations was that the temperature values deduced are almost constant near $T = 160$ MeV and that the volume increases with energy, while strangeness is undersaturated. These results were taken, together with the results for nucleus-nucleus collisions where a similar temperature is reached at high energies, as evidence for the interpretation that the thermodynamical
state is not reached by dynamical equilibration among constituents but rather is a generic fingerprint of hadronization [17,18] or a feature of the excited QCD vacuum [19]. Alternatively, it was argued in [20] that the quark-hadron phase transition drives the equilibration dynamically for nucleus-nucleus collisions. Equilibration in $e^+e^-$ collisions is not easy to explain in this latter approach.

In our new analysis we are using the latest multiplicity measurements summarized and published by the Particle Data Group (PDG) [21]. Since the aim is a precision calculation for a small system we employ a fully canonical form of the statistical model [12,22] conserving baryon number $N$, charge $Q$, strangeness $S$, charmness $C$, and bottomness $B$. In the present analysis charmed and bottom hadrons are relevant only via their feed-down contributions to the yields of the lighter hadron species (for more details see below). To reach within the model a precision comparable to that of the data from the LEP collider (a few percent), we have performed computations including quantum statistics (see below).

2 The model

The canonical statistical model we will base our investigations on is described in [12,13,14,16,22]. Here we present a short summary with emphasis on the way the quantum number conservation is implemented. Most hadronic events in high energy $e^+e^-$ annihilations are two-jet events, originating from quark-antiquark pairs of the five lightest flavors. Since we would like to address the issue of overall equilibration in these systems, we are using a scheme in which each jet is treated as a fireball with vanishing quantum numbers as fixed by the entrance channel; we call this the “no flavor” scheme. It is clear at this point that hadrons from jets with heavy quarks ($c$ and $b$) will be greatly underestimated by the model because of the large Boltzmann suppression factors. In this approach the issue of equilibration is effectively addressed only for hadrons with light quarks ($u, d, s$).

It is important to recognize that the measured yields of these hadrons contain the contribution from the $e^+e^-$ annihilation events into $c\bar{c}$ and $b\bar{b}$. Heavy-quark production is indeed significant and is very precisely measured, in particular at the $Z^0$ mass ($\sqrt{s}=91.2$ GeV), where the measurements are very well described by the standard model [21]. Hence, heavy-quark production is manifestly non-thermal in origin. We therefore consider three scenarios: i) we fit the data as measured. ii) we subtract from the yields of hadrons carrying light quarks the contribution originating from charm and bottom decays based on available data for charmed and bottom hadron production (and their branching ratios) at 91 GeV. iii) we perform a fit to the data at 91 GeV in a 5-flavor ($u, d, s, c, b$) approach. In this case $e^+e^- \rightarrow q\bar{q}$ events are treated in as 2-jet initial state where each jet carries the relevant quantum numbers. The fractions of the quark flavors in hadronic events [21] are thus external input values, unrelated with the thermal model (see also Table II in ref. [23]). We will treat the heavy quark sector in detail in a forthcoming publication [24].

Scenarios i) and ii) differ from that used e.g. in ref. [23], where a 5-flavor scheme is used throughout. A significant difference between our approach and that of [23] is in the treatment of the volume entering the statistical model calculations, as explained below. In our case the volume is the hadronization volume of each jet, meaning that each jet hadronizes separately, as in deep inelastic scattering. The yields calculated for each jet
are then added to compare with data. In ref. [23] the volume is that of both jets together. Because of different canonical corrections (see below) this leads to significant differences in the final results. While we consider our jet hadronization picture as more suited to describe two-jet events within a statistical framework, we acknowledge that the approach of hadronization in one volume [23] cannot be excluded at present.

The appropriate tool to deal in a statistical mechanics framework with a system where all quantum numbers are zero is the canonical partition function with exact conservation of the N, Q, S, C and B quantum numbers [25]:

\[
Z(\vec{X}) = \frac{1}{(2\pi)^5} \int d^5\vec{\phi} \exp\{\sum_{j=1}^{N_B} \sum_k \ln(1 - e^{-\beta \epsilon_{j,k} - i\vec{\epsilon}_{j}\vec{\phi}})^{-1} \\
+ \sum_{j=1}^{N_F} \sum_k \ln(1 + e^{-\beta \epsilon_{j,k} - i\vec{\epsilon}_{j}\vec{\phi}})\}
\]  

where \(\vec{\epsilon}_j\) is a five component vector \(\vec{\epsilon}_j = (N_j, Q_j, S_j, C_j, B_j)\) containing the quantum numbers of hadron \(j\), and \(\phi = (\phi_N, \phi_Q, \phi_S, \phi_C, \phi_B)\) contains the parameters of the symmetry group \([U(1)]^5\). The sum over \(k\) is for the phase space cells for each hadron, \(\beta = 1/T\), \(\epsilon_{j,k} = \sqrt{p_k^2 + m_j^2}\). In this expression, each \(\phi_X\) corresponds to the conservation of the corresponding quantum number \(X\); \(N_B\) and \(N_F\) count the total number of boson and fermion states, respectively.

The usual procedure is to solve Eq. 1 in the Boltzmann approximation, i.e. \(\ln(1 \pm x) \approx x\) to perform the calculation with full quantum statistics we employ the series expansion:

\[
\ln(1 - x)^{-1} = \sum_{k=1}^{\infty} \frac{x^k}{k}
\]  

in Eq. 1 leading to

\[
Z(\vec{X}) = \frac{1}{(2\pi)^5} \int d^5\vec{\phi} e^{i\vec{X}\vec{\phi}} \exp\{\sum_{j} z_j^1 e^{-i\vec{\epsilon}_j\vec{\phi}} + \sum_{k=2}^{\infty} z_b^k e^{-i k \vec{\epsilon}_{b}\vec{\phi}}\}
\]  

with

\[
z_j^k = g_j V \frac{m_j^2}{k(2\pi)^3} \int d^3p \ e^{-\frac{\sqrt{p^2 + m_j^2}}{k}}
\]  

where \(m_j\) is the particle mass and \(g_j\) is the spin-isospin degeneracy factor. The index \(j\) runs over all particles species in the hadronic gas and \(b\) runs only over bosons. Here and in the following we use units with \(\hbar = c = 1\). The complete derivation of the partition function (for quantum statistics) will be the subject of a separate publication [26]. The differences between calculations with Boltzmann and quantum statistics are presented in Table 1.

The set of hadron species used here was updated compared to [11] with the most recent information available from the PDG [21] and includes all resonances listed there. We

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1. Three and more jet events are neglected here as they are in other approaches too.
2. We perform calculations with quantum statistics for bosons only. Due to the higher masses, for fermions the Boltzmann approximation is sufficiently accurate.
3. We have included the \(\sigma\) meson, as in ref. [27].
have also, as in ref. [11], included the resonance width explicitly.

The integral representation of the partition function in Eq. (3) is not convenient for numerical analysis as the integrand is a strongly oscillating function. Thus we have applied an expansion into series of Bessel functions [28,29,30] to obtain a result that is free of oscillations. We obtain the multiplicity \( \langle n_j \rangle \) per jet for particle species \( j \) by introducing a fugacity parameter \( \lambda_j \) which multiplies the particle partition function \( z_j^k \) and by differentiating

\[
\langle n_j \rangle = \frac{\partial \ln Z}{\partial \lambda_j} \bigg|_{\lambda_j=1}.
\]  

(5)

One has to take into account that \( \langle n_j \rangle \) is the yield resulting from one jet. The multiplicity for the whole event is then the sum over the two jets.

Table 1

Comparison of particle yields obtained with our code and with the THERMUS code [31] for both initial production (prior to strong decays) and for the final values (after strong and electromagnetic decays). We show yields (sum of particles and antiparticle yields and for the 2 jets) calculated with our code with quantum statistics (QS) and with Boltzmann statistics (BS) for the parameter set \( T=158 \text{ MeV}, V=30 \text{ fm}^3 \) and \( \gamma_s=0.80 \).

| particle | this work | THERMUS |
|----------|-----------|----------|
|          | final     | initial  | final |
|          | QS        | BS       | QS    | BS    |
| \( \pi^+ \) | 18.54     | 18.07    | 5.12  | 4.74  | 4.72   | 14.28  |
| \( \pi^0 \) | 11.08     | 10.73    | 2.97  | 2.68  | 2.68   | 8.29   |
| \( K^+ \)  | 2.028     | 2.016    | 0.954 | 0.945 | 0.940  | 1.89   |
| \( K^0 \)  | 1.952     | 1.939    | 0.938 | 0.928 | 0.924  | 1.84   |
| \( \eta \)  | 1.090     | 1.087    | 0.472 | 0.468 | 0.472  | 0.890  |
| \( \rho^0 (770) \) | 1.12     | 1.12     | 0.756 | 0.753 | 0.756  | 1.044  |
| \( K^*(892) \) | 0.597    | 0.595    | 0.447 | 0.445 | 0.442  | 0.570  |
| \( \phi (1020) \) | 0.974    | 0.966    | 0.232 | 0.230 | 0.229  | 0.668  |
| \( \Lambda \) | 0.364    | 0.361    | 0.0706| 0.0703| 0.0677 | 0.239  |
| \( \Sigma^+(1385) \) | 0.0393   | 0.0389   | 0.0313| 0.0310| 0.0300 | 0.0316 |
| \( \Xi^- \)  | 0.0202    | 0.0200   | 0.0115| 0.0114| 0.0108 | 0.0190 |
| \( \Xi^0 (1530) \) | 0.00768  | 0.00760  | 0.00738| 0.00731| 0.00679| 0.00679|
| \( \Omega \)   | 0.00125  | 0.00123  | 0.00125| 0.00123| 0.00117| 0.00117|

Before engaging in the present data analysis we have performed a comparison of results from the model described above with those obtained using the THERMUS code [31], which is available publicly. The results for a particular set of parameters are presented in Table 1.

4 We have not used the method applied in [13,14,15,16] to include the uncertainties in hadron mass, width and branching ratios as additional systematic errors.
Here and in the following the quoted yields include the corresponding antiparticle states (i.e. the yield labelled $\pi^+$ is actually the sum of the yields of $\pi^+$ and $\pi^-$) and are the sum over the two jets, as summarized and published by the PDG [21]. The agreement between the particle yields obtained with both codes is generally very good, lending strong support also to the numerical implementation of our methods. Note that, in our calculations, we employ quantum statistics, whereas in the THERMUS code the Boltzmann approximation is used for the canonical ensemble. Neglecting quantum statistics causes an error of about 3% for the final pion yield, i.e. larger than the uncertainty in the data. As expected, the effect on all other hadron yields is smaller. If we employ the Boltzmann approximation in our code we get, prior to strong decays, values for mesons and non-strange baryons which are in agreement with THERMUS results at the percent level, while for strange baryons our yields are systematically higher by about 2% compared to those obtained with THERMUS. Inspecting the yields after strong decays, one notices a discrepancy between results from our code and from THERMUS for most of the hadrons. Our larger yields are due to a more complete set of hadron species used in the calculations. Concerning the decays, we have ensured that, in our code, the decay tables are symmetrical for particles and antiparticles and the decay widths always add up to the total width, even if particular channels are not measured. As the branching ratios are not well known for some of the high-mass states, we have used the known BR of the nearest state with the same quantum numbers.

3 The fit procedure

The multiplicity calculation proceeds in two steps. First, a primary hadron yield, $N_{h}^{th}$, is calculated using (1) and (5). A crucial assumption of the model is that the final yields of all particles are fixed at a common temperature, the chemical decoupling point. As a second step all resonances in the gas which are unstable against strong and electromagnetic decays are allowed to decay into lighter stable hadrons, using appropriate branching ratios ($B$) and multiplicities ($M$) for the decay $j \rightarrow h$ published by the PDG [21]. The abundances in the final state are thus determined by

$$N_h = N_{h}^{th} + \sum_j N_j \cdot B(j \rightarrow h)M(j \rightarrow h)$$

(6)

where the sum runs over all hadron species.

In the resonance gas model the results are determined by the basic thermal parameters, temperature $T$ and volume $V$ (the volume corresponding to one jet). Following the approach of ref. [13], we also introduce an additional parameter $\gamma_s$ into the partition function to account for a possible deviation of strange particle yields from their chemical equilibrium values. If a hadron contains $n_s$ strange valence quarks, its production is reduced by a factor $\gamma_s^{n_s}$. This parameter is also applied to neutral mesons such as $\eta, \eta', \phi, \omega, f_2(1270)$ and $f'_2(1525)$ according to the fraction of $s\bar{s}$ content in the meson itself. The relevant fraction is determined using mixing formulae quoted in [21].

For the fit procedure we use the complete set of all measured yields of hadrons carrying light quarks. The second scenario, namely subtracting the contribution originating from charm and bottom decays, is investigated only at $\sqrt{s}=91$ GeV as a case study, since the
measurements needed to allow the subtraction are complete only at this energy. The $\chi^2$ fit is performed by minimizing

$$\chi^2 = \sum_h \frac{(N_{h}^{exp} - N_{h})^2}{\sigma_h^2}$$ (7)

as a function of the three parameters $T$, $V$ and $\gamma_s$, taking account of the experimental uncertainties $\sigma_h$.

4 Results

The resulting best fit to the data at the energy $\sqrt{s}=91$ GeV is shown in Fig. 1. Shown are the three cases of the fit, the non-flavor approach without and with the subtraction of the contribution from heavy quarks (see below for the magnitude of this contribution) and the 5-flavor approach. We first note the overall behavior of the data, namely an approximately exponential decrease of particle yield with increasing particle mass. Such a behavior is expected in the hadron resonance gas model due to the Boltzmann factors, thus indicating the presence of statistical features of hadron production in elementary collisions.

The quantitative description of the data with the statistical model is, however, rather poor and certainly no improvement is visible for the case of subtracting charm and bottom contributions. The poor fit quality which is already visible in Fig. 1 becomes striking when, as in Fig. 2 we show for the four energies the difference $\Delta$ (in units of the experimental error) between the experimental data and the statistical model calculations for the best fit values.

A summary of the fit parameters obtained for the various data sets is presented in Table 2. The errors of the thermal parameters listed in Table 2 are the statistical errors as extracted from a minimization done with MINUIT, interfaced with our code. For the discussion of the systematic errors see below.

Table 2

| $\sqrt{s}$[GeV] | $T$[MeV] | $V$[fm$^3$] | $\gamma_s$ | $\chi^2$/dof |
|-----------------|----------|-------------|-----------|-------------|
| 10              | 166±1.7  | 10±1.5      | 0.80±0.02 | 314/20      |
| 29-35           | 162±1.7  | 16±1.4      | 0.96±0.03 | 108/17      |
| 91 i)           | 160±0.5  | 26±1        | 0.82±0.007| 595/28      |
| 91 ii)          | 168±0.5  | 18±1        | 0.66±0.01 | 994/28      |
| 91 iii)         | 170±0.5  | 16±1        | 0.66±0.01 | 499/28      |
| 130-200         | 154±2.8  | 40±4.3      | 0.82±0.03 | 12/2        |

Typical $\chi^2$ values per degree of freedom lie between 5 and 20 and discrepancies between single data points and fit values larger than 5 standard deviations are not rare. Furthermore, there is no clear pattern observed: the fits are comparably poor for baryons and mesons, as well as for non-strange and strange hadrons. In particular, for all energies the
Figure 1. Comparison between the best fit thermal model calculations and experimental hadron multiplicities (sum of particles and antiparticle yields and for the 2 jets) for $e^+ e^-$ collisions at $\sqrt{s}=91$ GeV. The upper panel shows the fit in the non-flavor scheme of data including the feed-down contribution from heavy quarks, the middle panel is after subtraction of this contribution, the lower panel is for a 5-flavor scheme fit where the flavor abundances are extra input parameters from data (see text). The best fit parameters are listed for each case.

yields of $\phi$ mesons and of hyperons are poorly reproduced. Large deviations are seen also for kaons.

The case of the fit after subtraction of the contribution of the decays of charmed and bottom hadrons (scenario ii), explored at $\sqrt{s}=91$ GeV, is characterized by a larger $\chi^2$ value compared to the overall fit (scenario i). The extracted fit parameters differ somewhat in the two cases. In particular, $T$ is higher and $V$ is smaller for scenario ii) compared to scenario i), following the $(T,V)$ correlation discussed below. Scenario iii) gives the lowest $\chi^2$ value, although the value is still far from that of a good fit.
Figure 2. Difference (in units of experimental error) between experimental data and thermal model fits at four energies. For $\sqrt{s}=91$ GeV with open squares we show the results of the fit to data after subtraction of the heavy quark contribution (scenario ii), while the triangles are for the fit for the 5-flavor scheme (scenario iii).

It should be mentioned that, while the general agreement among the four LEP experiments is excellent, the measured yields of $\Sigma^*$ hyperons differ by more than 70%. Excluding the $\Sigma^*$ hyperons would cause an increase of $\gamma_s$ at $\sqrt{s}=91$ GeV, because the $\Sigma^*$ yields calculated in the model are overestimated (see Fig. 1). This would slightly improve the situation for the $\Xi$ and $\Lambda$ multiplicities which are higher than those predicted by the model, but with only a marginal improvement of the $\chi^2$ values, as is discussed below.

In Table 3 we show for selected hadron species the calculations in the no flavor scheme and the experimental data after the subtraction of the charm and beauty contribution. The relative magnitude of this contribution is also listed in Table 3.

A difficulty for the determination of fit parameters is visible if one inspects the $\chi^2$ contour lines as shown in Fig. 3 in $(T,V)$ plane for fits at 91 GeV. One notices in this figure a strong anticorrelation between the fit parameters which is also present in the $(T,\gamma_s)$ space (not shown). Closer inspection reveals, in addition, a series of local minima which indicates the difficulty in the determination of the fit parameters. Such local minima are typical for poor fits and imply that the true uncertainties in the fit parameters are likely much larger than the values obtained from the standard fit procedure [32] employed here.

Despite these caveats about fit quality and uncertainties it is noteworthy that the tem-
Table 3
Calculated yields (corresponding to the fit of the subtracted data at 91 GeV, \( T=168 \text{ MeV}, V=18 \text{ fm}^3 \) and \( \gamma_s=0.66 \)) for selected hadron species for the non-flavor scheme and for the 5-flavor one assuming vanishing or fractional baryon and charge quantum numbers. In the third column we show the percentage of the yields arising from the contribution of charm and bottom events, as calculated from the experimental data (complemented by calculations using scenario iii) for \( \Xi_c, \Omega_c \) and \( \Omega_b \) hadrons.

| particle Calculations | Data without contribution | c,b contribution | from c,b (in %) |
|------------------------|--------------------------|-----------------|----------------|
| \( \pi^+ \) | 15.80 | 14.97 | 12.0 |
| \( \pi^0 \) | 9.45 | 8.50 | 9.8 |
| \( K^+ \) | 1.43 | 1.69 | 24.2 |
| \( \rho^0(770) \) | 1.042 | 1.209 | 1.8 |
| \( K^*(892)^0 \) | 0.436 | 0.630 | 14.7 |
| \( p \) | 0.965 | 0.992 | 5.5 |
| \( \phi(1020) \) | 0.090 | 0.0.054 | 44.2 |
| \( \Lambda \) | 0.300 | 0.336 | 14.3 |
| \( \Sigma^+(1385) \) | 0.0337 | 0.0232 | 3.0 |
| \( \Xi^- \) | 0.0123 | 0.0231 | 10.6 |
| \( \Omega \) | 0.00056 | 0.00140 | 12.8 |

The temperature parameters obtained from most data sets are close to 160 MeV and nearly independent of energy, similar to results of previous investigations. In contrast, the volume increases with the center of mass energy. The values obtained for the strangeness undersaturation parameter \( \gamma_s \) range between 0.96 and 0.70 and exhibit no clear trend with energy. For the 5-flavor case, at 91 GeV we obtain a volume of 18 fm\(^3\). This is smaller than the results reported in [23] by about a factor of 2 because, in our case, the 2 jets hadronize separately.

In the following we will take a closer look at the 91 GeV fit, as the data set at that energy contains the largest number of measured hadron yields. We consider scenario i), that is, the fit to data without subtracting the charm and bottom decay contributions to calculations in the non-flavor scheme. To check whether the high \( \chi^2/\text{dof} \) values are caused by discrepancies for a few particular particles, we excluded 3 hadron species (the \( \Sigma^* \)'s and \( \phi \)) and repeated the fit. Naturally, the fit is better, but we still found a high \( \chi^2/\text{dof}=208/25 \) (for \( T=158 \text{ MeV}, V=28 \text{ fm}^3, \gamma_s=0.90 \)).

The data from LEP comprise very many particle species and their yields typically are measured with accuracies of a few percent. Clearly the precision of the experimental data set provides a stringent test of the statistical model. To provide a quantitative estimate at which accuracy level the statistical model breaks down and to allow comparison with the situation encountered for RHIC data we performed a fit at \( \sqrt{s} = 91 \text{ GeV} \) to those hadron yields which were used for the fit of central nucleus-nucleus collision data at \( \sqrt{s_{NN}}=200 \text{ GeV} \) [11], namely \( \pi^+, \pi^0, K^+, K^0, K^{*+}, K^{*0}, p, \Lambda, \Xi^-, \Omega^-, \) and \( \phi \). The resulting thermal parameters are: \( T=162\pm2 \text{ MeV} \), \( V=24\pm2 \text{ fm}^3 \), and \( \gamma_s=0.82\pm0.02 \), with a still poor \( \chi^2/\text{dof} \)
Figure 3. \( \chi^2 \) contour lines in temperature and volume space for the overall fit (left panel) and after subtraction of charm and bottom decays (right panel) for \( \sqrt{s} = 91 \text{ GeV} \) for the no flavor scheme. The contours correspond to \( \chi^2_{\text{min}} + 10\%, +20\%, +50\%, +100\% \). The best fit values are indicated by the crosses.

of 278/8. Excluding the \( \phi \)-meson from the fit yields \( T = 160 \pm 2 \text{ MeV}, \ V = 24 \pm 2 \text{ fm}^3 \) and \( \gamma_s = 0.96 \pm 0.02 \), with \( \chi^2/\text{dof} = 62/7 \). A more direct comparison between thermal fits for heavy ion and \( e^+e^- \) data can be obtained by arbitrarily assigning the uncertainties of the RHIC data to be corresponding LEP yields (namely uncertainties of the order of 10%). Constraining, as for the RHIC data, the fit parameters to \( T \) and \( V \), i.e. setting \( \gamma_s = 1 \) yields then \( T = 158 \pm 2 \text{ MeV} \) and \( V = 24 \pm 2 \text{ fm}^3 \), with \( \chi^2/\text{dof} = 49/9 \). A reasonable fit can only be obtained by also letting \( \gamma_s \) vary freely, with resulting parameters \( T = 168 \pm 2 \text{ MeV}, \ V = 18 \pm 2 \text{ fm}^3 \) and \( \gamma_s = 0.80 \pm 0.02 \), with \( \chi^2/\text{dof} = 18/8 \). These exercises demonstrate that a statistical model description of \( e^+e^- \) data fails badly without the introduction of the non-equilibrium parameter \( \gamma_s \). Even using \( \gamma_s \) the statistical description breaks down completely at an accuracy level for the data better than 10%.

Another noteworthy difference between fireballs in \( e^+e^- \) and nucleus-nucleus collisions is their energy content. For its determination we have computed the energy density \( \epsilon \) in the hadronic gas to yield the thermal energy content \( E_j = \epsilon V \) of the jet at chemical decoupling. For the \( e^+e^- \) case and the parameters reported in Table 2, this procedure leads to \( E_j = 3.23, 5.45, 8.6, 10.2 \text{ GeV} \) at \( \sqrt{s} = 10, 29-35, 91, 130-200 \text{ GeV} \), respectively. Hence, the thermal energy within each jet is only a small fraction of \( \sqrt{s}/2 \), (e.g. about 19% at 91 GeV). Apparently, in the thermal interpretation of \( e^+e^- \) collisions, most of the c.m. energy is not available for particle production. This is in strong contrast with results for nucleus-nucleus collisions. We have analyzed for this purpose central collision events for 20 and 40 GeV/nucleon Pb-Pb collisions [11], where it makes sense to consider data integrated over the full phase space. In these cases we find that the energy content in the fireball amounts to 61% and 63% of the total c.m. energy at 20 and 40 AGeV, respectively, implying that most of the total c.m. energy in a nucleus-nucleus collision is thermal, with the remaining non-thermal fraction likely to be due to collective flow. We note that these differences are not consistent with the finding that particle production in \( e^+e^- \), pp and nucleus-nucleus collisions is universally governed by the available c.m. energy [33,34,35].
5 Summary and conclusions

We analyzed the comprehensive set of measured yields of hadrons with light quarks ($u$, $d$, $s$) in $e^+e^-$ collisions in a range from 10 GeV up to 200 GeV within an equilibrium thermodynamics picture. The calculations were performed in the framework of canonical partition functions to conserve explicitly baryon number, strangeness, charge, charmness and bottomness and made use of the hadron resonance gas description. Our results corroborate previous findings [13,14,15,16] that statistical features are present in hadron production in $e^+e^-$ collisions. At $\sqrt{s}=91$ GeV, three scenarios were considered, namely fitting data without and with subtraction of the decay products of charmed and bottom hadrons in our no flavor approach (vanishing jet quantum numbers) and employing a 5-flavor scheme where the $q$ and $\bar{q}$ jets (with the relative abundancies of the $q\bar{q}$ events as extra parameters taken from measurements at LEP) are carrying the flavor quantum numbers. The two thermodynamical parameters temperature $T$ and volume $V$, and a strangeness undersaturation factor $\gamma_s$ were obtained from a $\chi^2$ minimization procedure. While we find, as in previous investigations [13,14,15,16], that the resulting temperature value is close to 160 MeV, independent of energy, the overall description of the high-precision LEP data is rather poor, independent of whether heavy quark contributions are subtracted or not. The $\chi^2$/dof values larger than 5 for all fits call into question the validity of the thermodynamical approach for these data. This conclusion still holds even if the analysis is restricted to the same set of hadrons which were analyzed in the context of thermal model fits to Au-Au collision data from the RHIC accelerator.

The apparent statistical fingerprint visible in the LEP data and first observed in [13] breaks down at an accuracy level of about 10%. Even at that level the $e^+e^-$ data cannot at all be described without the explicit assumption of strangeness undersaturation, implying that hadron production in $e^+e^-$ originates from a state which is quite far from true thermodynamic equilibrium. This conclusion is further supported by the observation that the corresponding fireball volume contains only a small fraction of the overall c.m. energy, implying that most of the c.m. energy is not available for particle production. This is in strong contrast to the situation in nucleus-nucleus collisions.

A striking feature observed for nuclear fireballs is the complete absence of strangeness suppression. In this context it would be very interesting to probe the equilibrium features observed in central nuclear collisions with increasing accuracy. Will non-equilibrium features there also be first revealed in the strangeness sector? To clarify these issues will be a challenge for future measurements.

Acknowledgments

We acknowledge illuminating discussions with Francesco Becattini. We acknowledge the support of the Alliance Program of the Helmholtz Association HA216/EMMI. K.R. acknowledges partial support from the Polish Ministry of Science (MENiSW) and the

\[^{5}\text{In contrast, analysis of nuclear fireballs [11] yields temperature values which decrease with decreasing energy.}\]
Deutsche Forschungsgemeinschaft (DFG) under the Mercator Programme. F.B acknowledges helpful discussions with Kai Schweda.

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