Polarization fluctuations in vertical cavity surface emitting lasers: a key to the mechanism behind polarization stability

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We investigate the effects of the electron-hole spin dynamics on the polarization fluctuations in the light emitted from a vertical cavity surface emitting laser (VCSEL). The Langevin equations are derived based on a rate equation model including birefringence, dichroism and two carrier density pools separately coupled to right and left circular polarization. The results show that the carrier dynamics phase lock the polarization fluctuations to the laser mode. This is clearly seen in the difference between the fluctuations in ellipticity and the fluctuations in polarization direction. Separate measurements of the polarization fluctuations in ellipticity and in polarization direction can therefore provide quantitative information on the non-linear contribution of the carrier dynamics to polarization stability in VCSELs.

I. INTRODUCTION

Although vertical cavity surface emitting lasers (VCSELS) are often highly symmetric around the axis of laser emission, practical devices usually emit linearly polarized light. The investigation of the weak anisotropies responsible for this polarization stability has been the object of a number of recent experimental and theoretical studies [1–6]. One of the questions raised in this search for a better understanding of the polarization properties of VCSELs is whether it is sufficient to consider the effects of the optical anisotropies separately from the carrier dynamics or whether the highly anisotropic saturation of the gain in quantum well VCSELs must be taken into account as well. This question is particularly complicated because semiconductor lasers are type B lasers and usually it is not realistic to adiabatically eliminate the carrier dynamics from the laser equations. The correct description of gain saturation effects therefore requires the inclusion of carrier densities as dynamical variables. A rate equation model for the polarization dynamics of quantum well VCSELs including the spin dynamics of the carriers was introduced in 1995 by San Miguel and coworkers [7,8]. This model shows that the effects of the linear anisotropies can be greatly modified as a result of the carrier dynamics [8,9]. In the case of relaxation oscillations frequencies greater than the spin relaxation rate the carrier dynamics can be observed as relaxation oscillations in the fluctuations of ellipticity and of polarization direction [9]. Recent experimental results, however, suggest that the spin relaxation rate is likely to be greater than the relaxation oscillation frequency [8–10]. In one study, the contributions of the carrier dynamics have been neglected altogether since the observed polarization stability could be interpreted in terms of a type A laser model including only linear optical anisotropies [9]. A direct experimental determination of the carrier dynamics contribution to polarization stability would thus be useful to test the validity of the different models.

In this paper we therefore investigate the different contributions of the linear anisotropies to the polarization fluctuations in the case of fast spin relaxation. In section I the rate equations are introduced and the Langevin equations at the stable point are formulated. In section II the Langevin equation is solved and the resulting polarization fluctuations are presented. In section III the difference between the contributions of birefringence and dichroism and the contributions of the relaxation oscillation dynamics of the carriers is discussed and experimental possibilities of identifying the contributions are proposed. In section IV the conclusions are presented.

II. POLARIZATION DYNAMICS OF THE SPLIT DENSITY MODEL

A. The rate equations

In the model introduced by San Miguel [7], the carrier density is subdivided into two carrier density pools interacting only with right or left circular polarized light, respectively. The physical justification for this assumption is the conservation of angular momentum around the axis of symmetry. In the following equations, we will use the parameters D for the total carrier density above transparency and d for the difference between the carrier density interacting with right circular polarized light and the carrier density interacting with left circular polarized light. n is the total number of photons in the cavity. The polarization is described using the normalized Stokes parameters. In terms of the complex amplitudes of the circular polarized light field modes $E_+$ and $E_-$, these are

$$P_1 = \frac{E_+^* E_- + E_-^* E_+}{E_+^* E_+ + E_-^* E_-} \quad (1a)$$

$$P_2 = -i \frac{E_+^* E_- - E_-^* E_+}{E_+^* E_+ + E_-^* E_-} \quad (1b)$$

$$P_3 = \frac{E_+^* E_- - E_-^* E_+}{E_+^* E_+ + E_-^* E_-} \quad (1c)$$
The relevant timescales of the laser process are given by the rate of spontaneous emission into the laser mode, $2w$ (usually around $10^6 - 10^7 s^{-1}$), the rate of emission into non-laser modes $\gamma$ (usually around $10^9 - 10^{10} s^{-1}$), and the rate of emission from the cavity, $2\kappa$ (usually around $10^{12} - 10^{13} s^{-1}$). In addition, the spin flip scattering rate $\gamma_s$ is an important timescale for the polarization sensitive interaction between the light field and the carrier densities. It is expected to be around $10^{10} - 10^{12} s^{-1}$. In light of the experimental results presented in [3,4,9] we will in the following consider $\gamma_s$ to be much faster than the relaxation oscillation and adiabatically eliminate the spin dynamics. Note that the opposite case, i.e. where $\gamma_s$ is much smaller than the relaxation oscillation frequency, has been described in [8].

Similar to the definition of the Stokes vector, the anisotropies can be defined as vectors. The orientation of the vector indicates the polarization for which the respective physical quantities are at a maximum. There are three types of anisotropies:

1. the relative gain anisotropy given by $g$, such that the rate of spontaneous emission into the laser mode is $2w(1 + \mathbf{P} \cdot g)$
2. the loss anisotropy $l$, such that the rate of photon emission from the cavity is given by $2\kappa(1 + \mathbf{P} \cdot l)$
3. the frequency anisotropy $\Omega$, such that the length of $\Omega$ is equal to the frequency difference between the modes of orthogonal polarization.

Since we only consider small anisotropies, we will neglect the effects of gain and loss anisotropies on the total intensity of the laser process by assuming that $1 + \mathbf{g} \cdot \mathbf{P} \approx 1$ and $1 + \mathbf{g} \cdot \mathbf{P} \approx 1$. The rate equations are then given by

\[
\frac{d}{dt}D = -wDn - \gamma D - wdnP_3 + \mu \tag{2a}
\]

\[
\frac{d}{dt}n = wDn - 2\kappa n + wdnP_3 \tag{2b}
\]

\[
\frac{d}{dt}d = -wDn - (\gamma + \gamma_s)d - wDnP_3 \tag{2c}
\]

\[
\frac{d}{dt} \mathbf{P} = [(wDg + dw \hat{e}_3) - 2\kappa l] \times \mathbf{P} \times \mathbf{P}
+ (\Omega + w\alpha d\hat{e}_3) \times \mathbf{P} \tag{2d}
\]

$\hat{e}_3$ indicates the unit vector in the direction of the 3rd component of the stokes vector.

$\mu$ is the injection current above transparency and $\alpha$ is the linewidth enhancement factor which describes a shift in frequency due to the electron-hole density in the quantum well.

### B. Adiabatic elimination of the carrier density difference $d$

If we assume that the spin flip scattering rate $\gamma_s$ is much faster than all the other timescales involved in the laser process, we can adiabatically eliminate the carrier density difference $d$ by using

\[
d = \frac{wDnP_3}{\gamma_s} \tag{3}
\]

We can also assume that $d$ will always be much smaller than the total carrier density $D$, such that $wDn + wdnP_3 \approx wDn$. The rate equations of the total carrier density $D$ and the total photon number $n$ are then independent of the polarization dynamics.

\[
\frac{d}{dt}D = -wDn - \gamma D + \mu \tag{4a}
\]

\[
\frac{d}{dt}n = wDn - 2\kappa n \tag{4b}
\]

\[
\frac{d}{dt} \mathbf{P} = [(wDg + w\frac{\kappa nP_3}{\gamma_s} \hat{e}_3) - 2\kappa l] \times \mathbf{P} \times \mathbf{P}
+ (\Omega + wD\frac{\kappa nP_3}{\gamma_s} \hat{e}_3) \times \mathbf{P} \tag{4c}
\]

In order to describe the polarization dynamics at constant laser intensity the stationary solutions of the total carrier density $D = 2\kappa/w$ and the stationary photon number $n$ may be applied to the polarization dynamics. The equation for the Stokes parameter dynamics then reads

\[
\frac{d}{dt} \mathbf{P} = [(s + 2\kappa w n P_3 \hat{e}_3) \times \mathbf{P} \times \mathbf{P}
+ (\Omega + \alpha \frac{2\kappa w n P_3 \hat{e}_3}{\gamma_s} \hat{e}_3) \times \mathbf{P} \tag{5}
\]

with $s = 2\kappa(g - 1)$ as the total dichroism. Note that for $2\kappa w n / \gamma_s = 0$ this equation is essentially the Stokes parameter version of the linear equation used in [3]. The interpretation of the polarization stability in terms of a type A laser model is possible because $d$ can be adiabatically eliminated. It should be noted, however, that the type A model does not correctly describe the dynamics of the field intensity. The use of normalized Stokes parameters is therefore more appropriate. In particular, the possibility of relaxation oscillations in the laser intensity is not included in the equations used in [3].

The effects of the split density model are given by the terms proportional to $2\kappa w n / \gamma_s$. $2\kappa w n$ is approximately equal to the square of the relaxation oscillation frequency. To estimate the importance of the split density model contributions to the laser dynamics it is therefore useful to compare the relaxation oscillation frequency with the spin flip scattering rate.

### C. Langevin equation

In the case of stable linear polarization with parallel birefringence $\Omega = \Omega \hat{e}_3$ and dichroism $s = s \hat{e}_1$ the stationary Stokes vector is $\mathbf{P} = \hat{e}_1$. Fluctuations of $\mathbf{P}$ are
given by the components $P_2$ for fluctuations in polarization direction and $P_3$ for fluctuations in ellipticity. The linearized Langevin equation for $P_2$ and $P_3$ derived from equation (6) is

$$
\frac{d}{dt} \begin{pmatrix} P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} -s & -\Omega - \alpha \chi_n \\ \Omega & -s - \chi_n \end{pmatrix} \begin{pmatrix} P_2 \\ P_3 \end{pmatrix} + f(t)
$$

(6)

where $\chi = 2kw/\gamma_0$ is the coefficient of the laser intensity dependent contribution of the carrier dynamics in the split density model.

The contributions of dichroism $s$, birefringence $\Omega$ and carrier dynamics $\chi_n$ can be recognized clearly in this matrix equation. The noise term $f(t)$ is a consequence of the vacuum fluctuations in the electromagnetic field entering the cavity and in the dipole density of the gain medium. The magnitude of the noise terms for the Langevin equations may be derived by considering that photonic shot noise must be present both in the circular polarized modes and in the linear polarized modes at 45 degrees to the polarization of the laser light.

$$
\langle f_{p_2}(t)f_{p_2}(t+\tau) \rangle = \frac{4\kappa}{n} \delta(\tau)
$$

(7a)

$$
\langle f_{p_3}(t)f_{p_3}(t+\tau) \rangle = \frac{4\kappa}{n} \delta(\tau).
$$

(7b)

The factor of $4\kappa/n$ is the rate $4\kappa n$ at which photons enter and leave the cavity, divided by the squared normalization of the Stokes parameter. This is the minimum noise term necessary to satisfy the quantum mechanical uncertainty relations. Additional noise may arise from reabsorption of photons into the laser medium due to incomplete inversion. This effect will be extremely strong in ultra low threshold lasers. For typical VCSELs however, the minimal noise terms should be the main contribution to $f(t)$.

III. SOLUTION OF THE LANGEVIN EQUATION

A. Linear response near the stationary point

The eigenvalues $\lambda_{\pm}$ and the left and right eigenvectors $a_{\pm}$ and $b_{\pm}$ of the $2 \times 2$ matrix describing the relaxation dynamics of polarization fluctuations in equation (6) can be analytically determined to obtain the linear response of the laser to the polarization fluctuations.

$$
\lambda_{\pm} = -s - \frac{\chi_n}{2} \pm i\Omega \sqrt{1 + \frac{\alpha \chi_n}{\Omega} - \left(\frac{\chi_n}{2\Omega}\right)^2}
$$

(8a)

$$
a_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp i\sqrt{1 + \frac{\alpha \chi_n}{\Omega} - \left(\frac{\chi_n}{2\Omega}\right)^2} \end{pmatrix}
$$

(8b)

$$
b_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp i\sqrt{\frac{\Omega}{\Omega + \alpha \chi_n} \left(\frac{\chi_n}{2\Omega} \pm i\sqrt{1 + \frac{\alpha \chi_n}{\Omega} - \left(\frac{\chi_n}{2\Omega}\right)^2}\right)} \end{pmatrix}
$$

(8c)

The eigenvalues already indicate an intensity dependent contribution to both the frequency and the relaxation rate of polarization fluctuations. The linear response to the polarization fluctuations is then given by the Greens function

$$
G(\tau) = e^{\lambda^+ \tau} b_+ \otimes a_+ + e^{\lambda^- \tau} b_- \otimes a_-.
$$

(9)

The fluctuations in the polarization of the laser light can be determined by applying this Greens function to the noise term $f(t)$ in the Langevin equation.

B. Polarization fluctuations

The fluctuations of polarization direction $P_2$ and ellipticity $P_3$, as well as their correlations are given by the following correlation functions:

$$
\langle P_2(t)P_2(t+\tau) \rangle = \frac{4\kappa}{(2s+\chi_n)n}(1 + \frac{\alpha \chi_n}{\Omega})e^{-\left(s+\chi_n/2\right)\tau} \cos(\omega_0\tau)
$$

(10a)

$$
\langle P_3(t)P_3(t+\tau) \rangle = \frac{4\kappa}{(2s+\chi_n)n}(1 + \frac{\alpha \chi_n}{\Omega})e^{-\left(s+\chi_n/2\right)\tau} \frac{\Omega}{\Omega + \alpha \chi_n} \cos(\omega_0\tau)
$$

(10b)

$$
\langle P_2(t)P_3(t+\tau) \rangle = \frac{4\kappa}{(2s+\chi_n)n}(1 + \frac{\alpha \chi_n}{\Omega})e^{-\left(s+\chi_n/2\right)\tau} \left(\frac{\chi_n/2}{\Omega + \alpha \chi_n} \cos(\omega_0\tau) + \frac{\omega_0}{\Omega + \alpha \chi_n} \sin(\omega_0\tau)\right)
$$

(10c)

$$
\langle P_3(t)P_2(t+\tau) \rangle = \frac{4\kappa}{(2s+\chi_n)n}(1 + \frac{\alpha \chi_n}{\Omega})e^{-\left(s+\chi_n/2\right)\tau} \left(\frac{\chi_n/2}{\Omega + \alpha \chi_n} \cos(\omega_0\tau) - \frac{\omega_0}{\Omega + \alpha \chi_n} \sin(\omega_0\tau)\right).
$$

(10d)

The frequency $\omega_0$ is given by the imaginary part of the eigenvalues,

$$
\omega_0 = \Omega \sqrt{1 + \frac{\alpha \chi_n}{\Omega} - \left(\frac{\chi_n}{2\Omega}\right)^2}.
$$

(11)

Figure 3 shows the two-time correlations as a function of the delay time $\tau$ for a typical choice of parameters.

In addition to the quantitative changes in the relaxation rate and the oscillation frequency, the non-linear contribution of the carrier dynamics introduce clear qualitative modifications to the polarization fluctuations. This is a consequence of the split density model in which the ellipticity is stabilized by the carrier dynamics in addition to the dichroism. Therefore the fluctuations in ellipticity are smaller than the fluctuations in polarization direction. Also, the phase shift between the oscillations of ellipticity and of polarization direction is not $\pi/2$ as one would expect for a purely linear birefringence. These differences between a linear optical model and the split density model may provide possibilities for experimental investigations of the split carrier density contributions to polarization stability.
C. Polarization noise in the emission spectrum

Because the laser field amplitude \( E_\parallel \) is large compared to the noise, the two-time correlation function of the field amplitude polarized orthogonally to the laser mode \( E_\perp \) may be determined from the two-time correlations of the normalized Stokes parameters. In fact, the measurement of the Stokes parameters corresponds to a heterodyne detection of the field dynamics of \( E_\perp \). Consequently, the two-time correlations of \( E_\perp \) may be obtained using

\[
E_\perp = \frac{P_2 + iP_3}{2} E_\parallel.
\]

The measurement of the fluctuations in ellipticity and in polarization direction is therefore equivalent to a phase sensitive measurement of the fluctuations in the laser cavity mode of orthogonal polarization to the lasing mode. The difference between the in-phase fluctuations corresponding to the polarization direction and the out-of-phase fluctuations corresponding to ellipticity corresponds to a phase locking effect between the emissions into the non-lasing polarization mode and the lasing mode.

The two-time correlation function of \( E_\perp \) also shows some features of the phase locking. However, the information is somewhat hidden by the summation over the separate contributions. Neglecting the fluctuations in \( E_\parallel \), the two-time correlation function of \( E_\perp \) is

\[
\langle E_\perp^*(t) E_\perp(t + \tau) \rangle = \frac{n}{4} \left( \langle P_2(t) P_2(t + \tau) \rangle
\right.
\]

\[
+ \langle P_3(t) P_3(t + \tau) \rangle
\]

\[
+ i\langle P_2(t) P_3(t + \tau) \rangle
\]

\[
- i\langle P_3(t) P_2(t + \tau) \rangle
\]

\[
= \frac{2\chi(\Omega + \alpha \chi n/2)}{2s + \chi n(\Omega + \alpha \chi n)} \left( (1 + \frac{\alpha \chi n}{2\Omega}) \cos(\omega_0 \tau) + \frac{i\alpha \chi n}{2\Omega} \sin(\omega_0 \tau) \right) e^{-(s + \chi n/2) \tau}
\]

\[
= \frac{2\chi(\Omega + \alpha \chi n/2)}{2s + \chi n(\Omega + \alpha \chi n)} \left( \frac{\omega_0 + \Omega + \alpha \chi n/2}{2\Omega} e^{i\omega_0 \tau} - \frac{\omega_0 - \Omega - \alpha \chi n/2}{2\Omega} e^{-i\omega_0 \tau} \right) e^{-(s + \chi n/2) \tau}.
\]

The intensity spectrum \( I_\perp(\omega) = \langle E_\perp^*(\omega) E_\perp(\omega) \rangle \) is the Fourier transform of the two-time correlation function. It is given by two Lorentzians,

\[
I_\perp(\omega) = \frac{2\chi(\Omega + \alpha \chi n/2)}{2s + \chi n(\Omega + \alpha \chi n)} \left( \frac{\omega_0 + \Omega + \alpha \chi n/2}{2\Omega} \right) \frac{s + \chi n/2}{\pi((s + \chi n/2)^2 + (\omega + \omega_0)^2)}
\]

\[
- \frac{\omega_0 - \Omega - \alpha \chi n/2}{2\Omega} \frac{s + \chi n/2}{\pi((s + \chi n/2)^2 + (\omega - \omega_0)^2)}.
\]

An example of such a spectrum is given in figure \( 1 \) using the same parameters as for the two-time correlations shown in figure \( 4 \). Note that the intensity is given in units of photon number inside the cavity. The intensity emitted is given by \( 2s \) times this value. The minimum noise assumption of equations (12) and (13) may be tested by comparing the total intensity in the orthogonally polarized mode with the predictions of equation (14).

For \( \chi = 0 \) the theory predicts not only the peak at \(-\omega_0 \) from the laser line but also a much smaller peak at \(+\omega_0 \). To clarify the quantitative relation between the two peaks it is useful to treat \( \chi n / \Omega \) as a small perturbation.

\[
I_\perp(\omega) = \frac{2s}{2s + \chi n} \frac{s + \chi n/2}{\pi((s + \chi n/2)^2 + (\omega + \omega_0)^2)} + (\frac{\chi n}{2s})^2 \left( \alpha^2 + 1 \right) \frac{s + \chi n/2}{\pi((s + \chi n/2)^2 + (\omega - \omega_0)^2)}.
\]

If \( \chi n / \Omega \) is large enough the small noise peak at the opposite side of the laser line should be sufficient for a determination of \( \chi \). The fact that no such peak was observed in \( 3,4,9 \) indicates that \( \chi n \) is indeed small compared to \( \Omega \) in the devices studied.

IV. EXPERIMENTAL POSSIBILITIES

A. Spectrum of light polarized orthogonally to the lasing mode

If the carrier density dynamics is negligible for polarization stability the birefringence \( \Omega \) is given by the frequency difference \( \omega_0 \) between the laser line and the emission line of the orthogonally polarized mode and the dichroism \( s \) is given by one half of the linewidth at half maximum \( \Delta \omega_{FWHM} \). This is the assumption used in \( 9 \). That the linewidths reported in that paper seem to be larger than \( 2s \) may be a consequence of the carrier density dynamics. In particular, the linewidth with carrier density dynamics is given by

\[
\Delta \omega_{FWHM} = 2s + \chi n.
\]

The frequency shift \( \omega_0 \) is much harder to identify. According to the theory the frequency should increase with intensity until \( \chi n = 2\alpha \Omega \) and then decrease again until overdamping occurs at \( \chi n = 2\Omega(\alpha + \sqrt{\alpha^2 + 1}) \). The linear increase of frequency is

\[
\omega_0 \approx \Omega + \alpha \chi n/2.
\]

This frequency shift has first been derived by van der Lem and Lenstra \( 11 \) who argue that the shift is an alpha-enhanced saturation effect.

The most important feature predicted by the split density model is the peak on the opposite side of the laser
The ratio of the peak intensity at \( \omega = \omega_0 \) and at \( \omega = -\omega_0 \) is approximately given by

\[
\frac{I_{\perp}(\omega_0)}{I_{\perp}(-\omega_0)} \approx \frac{\chi_n}{4\Omega}^2(\alpha^2 + 1). \tag{18}
\]

If the frequency shift \( \omega_0 \), the linewidth \( \Delta \omega_{FWHM} \) and the intensity ratio \( I_{\perp}(\omega_0)/I_{\perp}(-\omega_0) \) are measured it is possible to determine the contribution of the carrier density dynamics \( \chi_n \) as well as the dichroism \( s \) and the birefringence \( \Omega \) at a fixed laser intensity. Note that to obtain correct quantitative results the linewidth enhancement factor \( \alpha \) must be known. The peak at \( \omega = +\omega_0 \) may be very small because of the second order dependence on \( \chi_n/\Omega \). For \( \chi_n/\Omega = 0.1 \) the intensity in that peak would be 100 to 1000 times lower than that in the main peak at \( \omega = -\omega_0 \). This may be the reason why it has not been observed in previous experiments. If there really is no peak at \( \omega = +\omega_0 \) however, it must be concluded that the split density model is not a valid description of the polarization dynamics in the device under consideration.

### B. Separate measurements of the fluctuations in ellipticity and in polarization direction

While it is possible to observe the effects of phase locking between the \( E_\perp \) mode and the lasing mode \( E_\parallel \) in the spectrum by measuring the small peak at the opposite side of the laser peak separate measurements of the fluctuations in ellipticity \( \chi_n \) and in polarization direction \( \chi_n \) are more sensitive to the effects of \( \chi_n \) and reveal more details of the phase locking. This is most clearly seen in figures 1 and 2. At \( \chi_n/\Omega = 0.5 \) the intensity at \( \omega = +\omega_0 \) is still very small while the features of the polarization fluctuations clearly reveal strong effects of the carrier dynamics.

The most important indicator is the ratio of fluctuations in polarization direction and in ellipticity. This ratio can even be measured at low time resolutions since it is a constant over all times and frequencies.

\[
\frac{\langle P_2(t)P_2(t + \tau) \rangle}{\langle P_3(t)P_3(t + \tau) \rangle} = 1 + \frac{\alpha \chi_n}{\Omega}. \tag{19}
\]

Note that the \( \alpha \) factor is responsible for the different magnitude of the fluctuations. Without the effects of the \( \alpha \) factor the phase locking effects would only appear in the correlations between ellipticity and polarization direction \( \langle P_2(t)P_3(t + \tau) \rangle \) and \( \langle P_3(t)P_2(t + \tau) \rangle \). If these correlations are measured it is interesting to determine the phase shift \( \delta \phi \) with

\[
\delta \phi = \arctan\left(\frac{\langle P_2(t)P_3(t + \tau) \rangle + \langle P_3(t)P_2(t + \tau) \rangle}{\langle P_2(t)P_3(t + \tau) \rangle - \langle P_3(t)P_2(t + \tau) \rangle}\right)
= \arctan\left(\frac{\chi_n}{\sqrt{4\Omega^2 + 4\alpha \chi_n \Omega - (\chi_n)^2}}\right)
\approx \frac{\chi_n}{2\Omega}. \tag{20}
\]

This phase shift shows how the additional damping of the ellipticity by the carrier density dynamics makes the ellipticity fluctuate less than \( \pi/2 \) out of phase with the polarization direction. The frequency \( \omega_0 \), the relaxation rate \( s + \chi_n/2 \), the fluctuation ratio \( \langle P_2(t)P_2(t + \tau) \rangle/\langle P_3(t)P_3(t + \tau) \rangle \) and the correlation phase \( \delta \phi \) provide all the information needed to determine the dichroism \( s \), the birefringence \( \Omega \), the contribution of the carrier density dynamics \( \chi_n \) and the linewidth enhancement factor \( \alpha \). Note that the time resolution necessary to measure the dynamics of polarization fluctuations in this regime is given by the birefringence \( \Omega \). This suggests that a time resolution of several nanoseconds may actually be sufficient.

### V. CONCLUSIONS

The calculations presented here clearly demonstrate how the carrier dynamics modify the polarization fluctuations both in the spectrum of \( E_\perp \) and in the Stokes parameters of ellipticity \( P_3 \) and of polarization direction \( P_2 \). Even if the contribution of the carrier density dynamics \( \chi_n \) is very small compared to the birefringence \( \Omega \) a careful analysis of the experimental data on polarization fluctuations should reveal these modifications. Such an analysis would show whether an interpretation of polarization stability in terms of birefringence and dichroism only is valid or not. If the carrier dynamics of the split density model are relevant to the polarization stability of a given device, the measurement of polarization fluctuations provides information on the optical anisotropies \( \Omega \) and \( s \), on the spin relaxation rate \( \gamma_s \) and on the linewidth enhancement factor \( \alpha \). If the non-linear contribution \( \chi \) turns out to be zero, it seems likely that valence bands with an angular momentum other than 3/2 around the axis perpendicular to the quantum well also contribute to the laser process in vertical cavity surface emitting lasers.

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FIG. 1. Two-time correlations of the polarization fluctuations \( \langle P_i(t)P_j(t+\tau) \rangle \) for \( \alpha = 2 \), \( s = 0.5 \) GHz, \( \Omega = 4 \) GHz and \( \chi_n = 2 \) GHz. (a) shows the fluctuations in ellipticity \( \langle P_3(t)P_3(t+\tau) \rangle \) and the fluctuations in polarization direction \( \langle P_2(t)P_3(t+\tau) \rangle \). (b) shows the correlations between ellipticity and polarization direction, \( \langle P_2(t)P_2(t+\tau) \rangle \) and \( \langle P_3(t)P_3(t+\tau) \rangle \).

FIG. 2. Spectrum \( I_{\perp}(\omega) \) of the emission polarized orthogonally to the lasing mode for \( \alpha = 2 \), \( s = 0.5 \) GHz, \( \Omega = 4 \) GHz and \( \chi_n = 2 \) GHz. Even though \( \chi_n/\Omega = 0.5 \) the peak near +5 GHz is still very small.