Strong–Field Quantum Electrodynamics and Muonic Hydrogen

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We explore the possibility of a breakdown of perturbative quantum electrodynamics in light muonic bound systems, notably, muonic hydrogen. The average electric field seen by a muon orbiting a proton is shown to be comparable to hydrogenlike Uranium and, notably, larger than the electric field achievable using even the most advanced strong-laser facilities. Following Maltman and Isgur who have shown that fundamental forces such as the meson exchange force may undergo a qualitative change in the strong-coupling regime, we investigate a concomitant possible existence of muon-proton and electron-proton contact interactions, of nonperturbative origin, and their influence on transition frequencies in light one-muon ions.

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I. INTRODUCTION

The recent muonic hydrogen experiments [1, 2] have given rise to the most severe discrepancy of the predictions of quantum electrodynamics with experiment recorded over the last few decades. In short, both (electronic, atomic) hydrogen experiments (for an overview see Ref. [3]) as well as recent scattering experiments lead to a proton charge radius of about \( \langle r_p \rangle \approx 0.88 \text{ fm} \), while the muonic hydrogen experiments [1, 2] favor a proton charge radius of about \( \langle r_p \rangle \approx 0.84 \text{ fm} \). The observed difference is consistent with two muonic scattering experiments [4, 5] that were carried out about four decades ago and roughly observe a 4% lower cross section for muons scattering off of protons as opposed to electrons being scattered off the same target. (If one assumes that the shape of the electric Sachs form factor is the same for electron compared to muon scattering, the cross section is proportional to the square of the charge radius.)

All attempts to find a conceivable explanation for the discrepancy based on a calculational error in bound-state quantum electrodynamics [6] or a “subversive” virtual particle [7–9] have failed, mostly because of tight constraints on these terms set by other low-energy tests of quantum electrodynamics [7]. Furthermore, attempts to reconcile the difference based on higher moments of the proton charge distribution (its “higher-order shape”, see Ref. [8]) face difficulty when confronted with scattering experiments which set relatively tight constraints on the higher-order terms.

II. NONPERTURBATIVE LEPTON PAIRS

Recently [10, 11], one of the few remaining theoretical explanations for the discrepancy, namely, the existence of a contact interaction of electron and proton, has been investigated. A contact interaction [10, 11] of nonperturbative origin [10, 11] between the muon and proton, nonuniversal for leptons, can be represented as a simple contact interaction diagram (four-fermion vertex), proportional to \( \delta^3(r) \) in coordinate space (see Fig. 1). The conjecture formulated in Refs. [10, 11] follows Maltman and Isgur who argued [12] that the nuclear force that binds the nucleons together, needs to be modified beyond simple meson exchange for small nuclei with a considerable overlap of the wave functions of two, three, or four hadrons.

One may ask if the Hamiltonian given in Eq. (11) of Ref. [10],

\[
H_{\text{ann}} = \epsilon_p \frac{3\alpha_{\text{QED}}}{2m_e^2} \delta^3(r),
\]

where the subscript “ann” denotes the virtual annihilation channel and \( \epsilon_p \) is a parameter that measures the amount of electron-positron pairs within the proton, has any predictive power. According to Eq. (13) of Ref. [10], a value of \( \epsilon_p = 2.1 \times 10^{-7} \) is sufficient to explain the proton radius puzzle. The Hamiltonian [11] it predicts a specific form of the frequency correction beyond perturbative quantum electrodynamics, namely, that of the Dirac-\( \delta \) potential, which mimics the effect of an apparent change in the square of the proton radius (nuclear size effect).

The parameter \( \epsilon_p \), however, cannot be universal and should depend on the specific details of the proton (p) wave function. A breakdown of perturbative quantum
electrodynamics can be expected in regions with extreme electromagnetic field strengths which can be found inside and in the immediate vicinity of the proton wave function. In a first approximation, the additional presence of a neutron inside the nucleus should not drastically alter the effect, and so, if the contact interaction due to nonperturbative lepton pairs inside the proton exists, then the effect, expressed in frequency units (energy units) for the $2S^1S_0$ transition, should be approximately the same for muonic hydrogen and muonic deuterium (proton and deuteron nuclei), namely, about 0.3 meV. Also, we would expect the discrepancy to point in the same direction, i.e., toward smaller radii being measured in the muonic bound systems.

For transitions in muonic helium, in frequency/energy units, one might conjecture that the parameter $\epsilon_{\alpha}$ which measures the nonperturbative electron-positron pairs inside the helium nucleus, might be enhanced in comparison to the proton, $\epsilon_{\text{He}} > \epsilon_p$. An upper limit of this enhancement (in frequency units) is given by a factor of about 16, because of the presence of two instead of one protons inside the nucleus, and because of the $Z^3$ scaling of the wave function at the origin. However, in order to achieve a more definitive prediction, one should take into account a conceivable electromagnetic polarization of the neutron wave functions near the protons, which might attenuate the electric fields in the nuclear region. Qualitatively, again, we would expect the radius to be smaller when measured in muonic bound systems.

We should re-stress that of course, once a “photon” or another particle “leaves” the proton in a Feynman diagram, then, as pointed out in Ref. [13], the corresponding process is absorbed in the proton polarizability contribution to the Lamb shift. However, additional contact interactions are not excluded by the experiments, and that is the basis for the theoretical suggestion formulated in Refs. [13,14]. From a historical perspective, it is also useful to point out that the contact interaction is exactly of the form proposed in Eq. (2) of Ref. [14] as a conceivable nonlinear correction term in high-field quantum electrodynamics.

III. STRONG FIELDS AND NONPERTURBATIVE EFFECTS

The possibility of a breakdown of the perturbative description of electrodynamic processes in very strong fields [15–18], via the effective action, has been envisaged by Heisenberg, Euler and Schwinger. The question is whether the muonic hydrogen system leaves room for the conceivable occurrence of such effect. One easily estimates that the (fluctuating) electric fields “inside” the proton, given the fact that the three valence quarks cannot be further apart than $0.8$ fermi, are of order

$$E_p \sim 10^{21} \frac{\text{V}}{\text{m}}$$  

and thus exceed the Schwinger critical field strength $E_C$ of about

$$E_C = 1.32 \times 10^{18} \frac{\text{V}}{\text{m}}$$

by three orders of magnitude. The Schwinger critical field strength is the electric field strength where perturbative quantum electrodynamics ceases to be a valid theory [13]. The electric field around the proton reaches the Schwinger critical field already at a distance $0.116$ Bohr radii of the muonic hydrogen system, where the Bohr radius $r_0$ of the muonic bound system is defined as

$$r_0 = \frac{\hbar}{\alpha_{\text{QED}} m_\mu c} = 2.56 \times 10^{-13} \text{ m}.$$  

where $\alpha_{\text{QED}} \approx 1/137.036$ denotes the fine-structure constant. Let us consider bound one-muon ions in the region of low nuclear charge numbers $1 \leq Z \leq 5$. The probability of finding a $1S$ muon inside the region of super-critical field strength, in one-muon ions of nuclear charge number $1 \leq Z \leq 5$, evaluates as follows,

$$p_{\text{cr}}(Z = 1) = 0.17 \%, \quad p_{\text{cr}}(Z = 2) = 1.18 \%, \quad p_{\text{cr}}(Z = 3) = 3.36 \%, \quad p_{\text{cr}}(Z = 4) = 6.73 \%, \quad p_{\text{cr}}(Z = 5) = 11.2 \%.$$  

In Fig. 1 we investigate the electric field strength felt by a bound muon in a “muonic hydrogenlike” system (only one orbiting particle) in the region of low nuclear charge number. We start from the ground-state expectation value of the electric-field operator, which is obtained as the gradient of the Coulomb potential. In SI mksA units, and within the nonrelativistic approximation, the result reads as

$$\langle E \rangle = \left\langle 1S \left| -\frac{\partial}{\partial r} \frac{Ze}{4\pi\epsilon_0 r} \right| 1S \right\rangle = 2Z^3 \frac{m_\mu^2}{m_e^2} \mathcal{E}_0,$$

$$\mathcal{E}_0 = \frac{e\alpha_{\text{QED}}^2 m_e^2 c^2}{4\pi\epsilon_0 \hbar^2} = 5.14 \times 10^{11} \frac{\text{V}}{\text{m}}.$$  

Here, $m_e$ is the reduced mass of the atomic system, $m_e$ is the electron mass, and $\mathcal{E}_0$ denotes the “standard” atomic field strength observed at one Bohr radius in the “standard” hydrogen atom. The prefactor $2$ in Eq. (6a) is a consequence of our taking the quantum mechanical expectation value as opposed to evaluating the classical expression at the (shifted) Bohr radius. We use formulas given on p. 17 of Ref. [19]. For ultra-relativistic systems, Eq. (6a) is replaced by the expectation value of the fully relativistic Dirac–Coulomb wave function [20], but this replacement does not change the order-of-magnitude of the result. The decisive factor in Eq. (6a) is the prefactor $Z^3(m_e/m_\mu)^2$, which is responsible for an enhancement of the field strength by six orders of magnitude in the
range $1 \leq Z \leq 92$ for electronic system, but also for a considerable enhancement in muonic systems, where

$$\left( \frac{m_e}{m_e} \right)^2 \rightarrow \left( \frac{m_\mu m_e}{m_\mu + m_e} \right)^2 \approx 3.45 \times 10^4. \quad (7)$$

For a one-muon ion, the average electric field strength at $Z \geq 3$ surpasses the average electric field strength in hydrogenlike Uranium.

Furthermore, the average field strength felt by a bound 1S electron in one-muon ions with $Z = 4$ and $Z = 5$ is given as

$$\langle E \rangle_{\mu, Z=4} = 1.72 \, E_{cr}, \quad (8a)$$
$$\langle E \rangle_{\mu, Z=5} = 3.36 \, E_{cr}, \quad (8b)$$

thus surpassing (in terms of quantum mechanical average) the Schwinger critical field strength.

The HERCULES laser [21] (still) sets the standard for the highest achievable laser intensities to date, with a peak intensity of about $2 \times 10^{22} \, \text{W cm}^{-2}$. In the future, such facilities are supposed to reach intensities in the range $10^{23} \ldots 10^{24} \, \text{W cm}^{-2}$, an intensity of $10^{24} \, \text{W cm}^{-2}$ corresponds to an electric field strength of

$$E_L = 2.74 \times 10^{15} \, \text{V m}, \quad (9)$$

which is surpassed in the muonic ($1 \leq Z \leq 5$) as well as medium-$Z$ and high-$Z$ bound quantum electrodynamic (QED) systems (with $Z \geq 14$, see Fig. 2). It is thus evident that bound muonic system offer a competing alternative to the exploration of the strong-field QED regime, complementary to strong laser systems [22].

IV. NON–RESONANT EFFECTS AND TRANSITION FREQUENCIES

Discrepancies of Lamb shift experiments and theory have been explored for a long time. For example, a rather well-known accurate Lamb shift experiment in helium [23] has long been in disagreement with theory (see also the theory update in Ref. [24]). Additional experiments on electronic helium ions (as opposed to muonic helium ions) would be able to shed additional light on the “generalized” proton radius puzzle, or “nuclear size effect puzzle” for hydrogenlike ions, because they will enable us to compare the “electronically measured” radius of He$^4$ with the “muonically measured” radius [22]. In particular, one will be able to compare the “anisotropy method” used in Ref. [23] with other, established experimental methods in “electronic” helium ions.

In Sec. III of Ref. [22] [in the text after Eq. (15) ibid.], the authors investigate off-resonant effects differential as opposed to angular-averaged cross sections (so-called “cross-damping” terms). In order to gauge possible concomitant systematic shifts of the accurately measured frequencies, especially those involving highly excited states of hydrogen and deuterium, an improved measurement of the 2S–4P line is currently being pursued in Garching [27], while an improved measurement of the “classic” 2S–2P$_{1/2}$ Lamb shift is pursued at York University [28]. Both of these experiments have the definite potential of clarifying the “electronic hydrogen” side of the proton radius puzzle.

V. CONCLUSIONS

The interaction Hamiltonian $H_{\text{ann}},$ given in Eq. (1), due to nonperturbative lepton pairs, is diagrammatically represented as a four-fermion contact interaction (see Fig. 1). It cannot be excluded [17,11] as an explanation for the proton radius puzzle [1,2] at present. The observed differences in electron-proton versus muon-proton scattering experiments [14,15] support the existence of such an additional interaction in principle. Furthermore, the contact interaction given in Fig. 1 is irrelevant for electron $g - 2$ and muon $g - 2$ experiments. This is of advantage because these experiments otherwise set relatively tight bounds on the parameter ranges available for “subversive” virtual particles that might otherwise “remedy” the proton radius puzzle [1] but would induce a concomitant discrepancy for experiment and theory in $g - 2$ experiments, where the “subversive” particles would also contribute due to virtual loops.
Further clarification will be obtained [29] from ongoing experiments involving muonic helium (understood here as a one-muon ion bound to a helium nucleus, with nuclear charge number $Z = 2$), and from a careful reexamination of the role of nonresonant frequency shifts in angle-resolved spectroscopy (see Sec. III of Ref. [29] and Sec. [IV]). Of prime importance will also be the upcoming MUSE experiment [30] which will allow us to compare electron-proton and muon-proton scattering data, on the basis of intense muon beams. Low-$Z$ muonic ions are among our best probes of strong-field QED to date (see Fig. 2), and the mean field strength seen by muons in ions with nuclear charge numbers $Z = 4$ and $Z = 5$ already exceeds the Schwinger critical field strength for pair production.

These field strengths exceed those achievable in current and projected high-power laser systems. The benefit of the low-$Z$ muonic ions produced in the high-intensity muon beams at the Paul-Scherrer-Institute (PSI) lies in the “clean” environment provided by the one-muon ions, where all other bound electrons have been stripped and the interaction of the muon and the nucleus can be investigated spectroscopically to high accuracy [1, 2].

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