THE WHITE DWARF DEFICIT IN OPEN CLUSTERS: DYNAMICAL PROCESSES
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ABSTRACT
In Galactic open clusters, there is an apparent paucity of white dwarfs compared with the number expected assuming a reasonable initial mass function and assuming that main-sequence stars with an initial mass of less than \( \sim 8 M_\odot \) become white dwarfs. We suggest that this lack of white dwarfs is due at least in part to dynamical processes. Nonspherically symmetric mass loss during the post–main-sequence evolution would lead to an isotropic recoil speed of a few kilometers per second for the white dwarf remnant. This recoil speed can cause a substantial fraction of the white dwarfs formed in a cluster to leave the system. We investigate this dynamical process by carrying out high-precision \( N \)-body simulations of intermediate-mass open clusters, where we apply an isotropic recoil speed to the white dwarf remnants. Our models suggest that almost all white dwarfs would be lost from the cluster if the average recoil speed were to exceed twice the velocity dispersion of the cluster.

Subject headings: methods: \( N \)-body simulations — open clusters and associations: general — white dwarfs

1. INTRODUCTION
Weidemann (1977) first argued that the number of white dwarf (WD) members of the Hyades was unexpectedly low. Although membership information was quite incomplete in 1977, reasonable estimates for the predicted number of initial massive stars that could have evolved into WDs suggested that approximately half the WDs were missing. The cooling time for the faintest Hyades WD was also less than half the cluster age, again pointing to missing WDs, specifically the oldest ones. Adopting an initial mass function (IMF) and assuming that stars with an initial main-sequence mass of less than \( 6 M_\odot \) evolve to become WDs, Weidemann et al. (1992) quantified the Hyades WD "deficit." With these assumptions, 28 WDs are predicted to have been formed in the cluster. After three decades of searching, only seven WD members are known. This deficiency of WDs in young open clusters has been seen in the few other open clusters for which extensive searches for WDs have been made (e.g., Kalirai et al. 2001).

As has been recognized by all of the groups working in this area through the years, there are three obvious explanations for the missing WDs in intermediate-age clusters. First, it is possible that, in significant conflict with stellar evolution models, the critical initial mass above which stars explode as core-collapse supernovae is much less than the canonical value of 6–8 \( M_\odot \). The presence of the lone Pleiades WD member, LB 1497, with a progenitor mass of at least \( 7 M_\odot \) (Claver et al. 2001) is not consistent with this explanation. Second, because of their low luminosity, WDs are difficult to detect as members of binary systems. As demonstrated most recently by Williams (2003), with reasonable assumptions for the binary fraction and mass ratio distribution, a significant fraction (20%–50%) of WDs is likely to be hidden in binary systems. This is at least a part of the explanation. Finally, most open clusters are steadily losing stars because of dynamical evaporation. The possibility that WDs have been preferentially lost from clusters has been investigated in various ways starting with Aarseth & Woolf (1972), Wielen (1974), and Pels, Oort, & Pels-Kluyver (1975) and is still a topic of active investigation today (e.g., Hurley & Shara 2003). Although there is no unanimous consensus among the studies, it is generally agreed that preferential evaporation of WDs from clusters is not a significant contributor to the observed WD deficit.

Here we consider an alternative scenario, as already proposed by Weidemann (1977), in which the paucity of WDs is due to their escape from the cluster potential as a consequence of the recoil speed attained during the nonspherically symmetric loss of their red giant envelope. The typical velocity dispersion of the Galactic open clusters is less than 2 km s\(^{-1}\) (e.g., the Hyades one-dimensional velocity dispersion is 0.3 km s\(^{-1}\); de Bruijne, Hoogerwerf, & de Zeeuw 2001). The mass of the main-sequence turnoff stars is \( \sim 5 M_\odot \) for a 100 Myr old cluster and 2 \( M_\odot \) for a 1 Gyr old cluster. The typical mass of the first WD remnants is \( \sim 0.8 M_\odot \) (Weidemann 2000; Claver et al. 2001). The first cluster stars to become WDs therefore lose 80% or more of their initial mass through a combination of stellar winds and planetary nebula ejections. Typical stellar wind velocities for the giant branch phases of stars that become the first WDs in a cluster are on the order of 10 km s\(^{-1}\) (Kudritzki & Reimers 1978), and even the "slow," high-density planetary nebula wind is typically expanding away from the central star at 20 km s\(^{-1}\) (e.g., Kaler & Aller 1974). Thus, even a 1% deviation from spherical symmetry in the integrated mass-loss history would lead to a recoil speed of a few kilometers per second for the first WDs formed in clusters. Spruit (1998) argues that an asymmetric mass-loss fraction of the order of 10\(^{-3}\) during the asymptotic giant branch phase could explain the rotation period distribution of WDs. A larger asymmetric mass loss would also induce a nonnegligible recoil speed. Furthermore, the author points out that such asymmetries can in principle be observed by proper-motion studies of the clumps in interferometric images of SiO maser emission. A similar problem has been considered for neutron stars (Spruit & Phinney 1998). The recoil speed of many pulsars is observed to be greater than 100 km s\(^{-1}\) (Hansen & Phinney 1997). These large speeds are probably induced by a nonspherically symmetric supernova explosion. Monte Carlo simulations of supernova explosions in primordial binaries show that a large recoil speed leads to a substantial loss of the neutron star remnants even in rich and strongly bound globular clusters (Pfahl, Rappaport, & Podsiadlowski 2002).
We consider a much less volatile situation in which the WDs are formed in an open cluster environment with a few kilometers per second recoil speed. In § 2, we briefly describe our numerical method and the formulation of the problem. The results of our numerical computation are presented in § 3, and we discuss their implications in § 4.

2. NUMERICAL SCHEME AND MODEL PARAMETERS

We perform $N$-body simulations using the direct $N$-body code NBODY6 (Aarseth 1999). This numerical scheme enables us to follow the orbits of the stars in an open cluster with high accuracy. It is a direct summation code with block time steps; i.e., the time steps are quantized to powers of 2 (Makino 1991). It has an Ahmad-Cohen neighbor scheme, which splits the force polynomial of a particle into an irregular part because of the neighboring particles and a regular part of the more distant particles (Ahmad & Cohen 1973). It has a Hermite integrator that is a fourth-order predictor-corrector scheme with a coordinate truncation error proportional to $(M a_{\text{K}})_{\text{h}}$. It is a direct summation code with block time steps; i.e., the time steps are quantized to powers of 2 (Makino 1991).

We perform simulations for a certain parameter set. The last column gives the number of random realizations performed with this parameter setting.

### TABLE 1

**PROPERTIES OF THE OPEN CLUSTER MODELS**

| Parameter                      | $N = 2000$ | $N = 10,000$ |
|--------------------------------|------------|--------------|
| Total mass ($M_\odot$)         | 1317.1     | 6668.1       |
| Crossing time (Myr)            | 6.2        | 2.7          |
| Relaxation time (Myr)          | 180.7      | 109.1        |
| Tidal radius (pc)              | 15.6       | 26.7         |
| Half-mass radius (pc)          | 2.5        | 2.4          |
| Core radius (pc)               | 0.9        | 1.0          |
| Velocity dispersion (km s$^{-1}$) | 0.8    | 1.8          |

### TABLE 2

**RESULTS OF OUR SIMULATIONS AFTER 100 MYR OF EVOLUTION**

| $N$   | $f_b$ | $v_{\text{kick}}$ | WD$_{\text{tot}}$ | WD$_{\text{fN}}$ | $f_{\text{WD}}$ | $f_b$ | Run |
|-------|-------|------------------|------------------|------------------|-----------------|-------|-----|
| 2000  | 0.0   | 0                | 8                | 0.01             | 0.99            | 0.0   | 1   |
| 0.0   | 1     | 8                | 8                | 1.00             | 0.99            | 1     |     |
| 0.0   | 2     | 8                | 4                | 0.50             | 0.98            | 1     |     |
| 0.0   | 5     | 8                | 1                | 0.13             | 0.98            | 1     |     |
| 0.0   | 2     | 6                | 5                | 0.83             | 0.98            | 3     |     |
| 0.2   | 5     | 6                | 1                | 0.17             | 0.98            | 2     |     |
| 0.4   | 5     | 5                | 1                | 0.09             | 0.98            | 2     |     |
| 0.4   | 2     | 5                | 4                | 0.80             | 0.98            | 3     |     |
| 0.4   | 5     | 5                | 1                | 0.20             | 0.98            | 1     |     |
| 0.8   | 1     | 3                | 3                | 1.00             | 0.98            | 1     |     |
| 0.8   | 2     | 3                | 3                | 1.00             | 0.99            | 1     |     |
| 0.8   | 5     | 1                | 0                | 0.00             | 0.97            | 1     |     |
| 10,000| 0.0   | 1                | 43               | 1.00             | 1.00            | 2     |     |
| 0.0   | 2     | 43               | 42               | 0.98             | 1.00            | 1     |     |
| 0.0   | 5     | 43               | 38               | 0.88             | 0.99            | 2     |     |
| 0.2   | 2     | 38               | 39               | 0.97             | 0.99            | 1     |     |
| 0.2   | 5     | 39               | 37               | 0.95             | 0.99            | 1     |     |
| 0.4   | 2     | 27               | 27               | 1.00             | 0.99            | 1     |     |

Note.—The columns are, from left to right, the number of stars initially, the initial binary fraction, the mean velocity of the kick in kilometers per second, the number of WDs formed in the cluster, the number of WDs remaining in the cluster, the fraction of WDs remaining, and the fraction of total stars remaining in the cluster. Because of the small numbers in the $N = 2000$ simulations, we performed several runs with the same parameters but with different random realizations. The absolute numbers given in the table represent the first run of each set; the fractions are the statistical mean out of all simulations for a certain parameter set. The last column gives the number of random realizations performed with this parameter setting.

(Kustaanheimo & Stiefel 1965). Close encounters between single stars and binaries, binaries and binaries, or even a higher multiplicity of close particles are studied by a special method, known as chain regularization (Mikkola & Aarseth 1993; Mikkola 1997). The code is also able to treat primordial binaries and has a scheme to implement the stellar evolution of the stars in the cluster (Hurley, Pols, & Tout 2000).

For our open cluster initial models, we chose a multimass King model (King 1966; Michie & Bodenheimer 1963) with the concentration parameter $W_o = 5$. We perform simulations with $N_{\text{tot}} = 2000$ and 10,000 particles. The tidal field is adjusted to a Galactic central distance of 10 kpc. We perform simulations with different initial binary fractions ($f_b = 0$, 0.2, 0.4, and 0.8). The IMF is taken from Kroupa et al. (1993). The binary population contains only hard binaries. Most soft binaries would be disrupted by the intracluster forces before the first WD is formed. For primordial binaries, the total mass of the binary was chosen from the Kroupa et al. (1993) IMF, which was not corrected for the effect of binaries, and the component masses were then assigned according to a uniform mass ratio distribution. The orbital separation was taken from the lognormal distribution of Eggleton, Fitchett, & Tout (1989) with a maximum of 100 AU, and the orbital eccentricity was taken from a thermal distribution (Heggie 1975). The properties of the two open cluster models can be found in Table 1.

Our low-mass model resembles an open cluster like the Hy-
However, when we include a kick acquired during evolution to the WD phase, some WDs gain enough velocity to leave the system. In the low-mass systems ($N = 2000$), a mean kick velocity of $2 \, \text{km} \, \text{s}^{-1}$ depletes the number of WDs significantly. In the case of high-mass systems ($N = 10,000$), a higher mean kick velocity of about $5 \, \text{km} \, \text{s}^{-1}$ is needed to deplete the cluster of a significant fraction of its WDs.

The results show that WDs are depleted significantly if the mean kick velocity exceeds twice the velocity dispersion of the open cluster. This may be understood in terms of the escape velocity that is also approximately twice the velocity dispersion in these systems. Hence, if the kick velocity is roughly equal to the internal velocity dispersion, the clusters lose a noticeable amount of WDs.

The binary fraction plays only a secondary role. With the caveat that our sample of simulations is small (we would need many random realizations of one set of parameters to reduce the error bars), there is no significant difference between the simulations with and without initial binaries. In principle, hard binaries with high orbital velocities together with low kick velocities could inhibit WDs from leaving the cluster, but mostly the binaries leave the system as a whole. If we take into account that the progenitor of the WD is more massive than its companion that remains on the main sequence, the recoil has a comparable impact on the binary system as a whole. Again, the escape velocity plays an important role here. The dividing line between hard and soft binaries is defined as the orbital velocity of the binary being equal to the escape velocity from the cluster. Therefore, only kick velocities higher than the escape velocity of the system are able to break up such binaries.

Table 2 shows the results of our simulations taken at 100 Myr. Some WDs have formed, and this is also approximately 1 relaxation time of the clusters. To show the time evolution of these results, we followed certain calculations to 500 Myr and some up to 1 Gyr. This time evolution is shown in Figure 1 for the $N = 2000$ cases and in Figure 2 for the $N = 10,000$ cases. In each plot, two lines are shown. The dotted line is the evolution of the total number of stars for the cases. In principle, hard and soft binaries remain in the system. The solid line shows how many of the produced WDs are still in the system. If there is no kick velocity, the line of the WD fraction is above the line of all stars, as expected. Including the kick velocity and increasing it will move the line of the WDs downward, and as soon as the mean kick velocity exceeds the escape velocity of the cluster, the fraction of WDs is significantly lower than the fraction of stars remaining in the system. This means there is a significant WD deficit in these systems.

The remaining WDs are mainly in binaries and preferentially close to the center of the cluster in the simulations with low kick velocities, and they more likely to be found as single stars in calculations with high kick velocity.

4. SUMMARY AND DISCUSSION

With our direct $N$-body simulations, we have shown that if the rapid mass loss involved in the formation of a WD progenitor is asymmetrical at a level of 1% or a few percent, the newly formed WD suffers a recoil kick that is able to deplete an open cluster of almost all of its WDs. From energy arguments, this recoil velocity is at least of the order of a few kilometers per second. But this is already enough to exceed the escape velocity, i.e., to deplete a low-mass open cluster of almost all its WDs. More massive open clusters are able to
retain WDs within the cluster but still show a significant de-
pletion of WDs.

The best studied intermediate-age open clusters in the Milky
Way exhibit a deficit of WDs, even after correcting for WDs
hidden in binaries. Our results are in agreement with the data
compilation of von Hippel (1998). The open clusters studied
by von Hippel exhibit a steep IMF slope of $-2.35$ to $-3$. We
suggest that this deficiency can be explained via the combined
processes of small kicks being imparted during the mass-loss
history and their subsequent evaporation from the cluster. These
processes act preferentially to deplete clusters of the first WDs
to form and could mimic a steep IMF slope. These stars suffer
the largest amount of mass loss and statistically have the largest
kicks and have spent the longest amount of time (among the
WD population) as relatively low mass stars. This preferential
loss of the first-formed WDs may need to be accounted for
when using WD cooling times to estimate cluster ages or when
tracing cluster WDs back to the main sequence and attempting
to determine the critical main-sequence mass at which WDs
first begin to form.

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REFERENCES

Aarseth, S. J. 1999, PASP, 111, 1333
Aarseth, S. J., & Woolf, N. J. 1972, Astrophys. Lett., 12, 159
Ahmad, A., & Cohen, L. 1973, J. Comput. Phys., 12, 389
Claver, C. F., Liebert, J., Bergeron, P., & Koester, D. 2001, ApJ, 563, 987
de Bruijne, J. H. J., Hoogerwerf, R., & de Zeeuw, P. T. 2001, A&A, 367, 111
Eggleton, P. P., Fitchett, M., & Tout, C. A. 1989, ApJ, 347, 998
Hansen, B. M. S., & Phinney, E. S. 1997, MNRAS, 291, 569
Heggie, D. C. 1975, MNRAS, 173, 729
Hurley, J. R., Pols, O. R., & Tout, C. A. 2000, MNRAS, 315, 543
Hurley, J. R., & Shara, M. M. 2003, ApJ, 589, 179
Kalirai, J. S., Ventura, P., Richer, H. B., Fahlman, G. G., Durrell, P. R.,
D’Antona, F., & Marconi, G. 2001, AJ, 122, 3239
King, I. 1966, AJ, 71, 64
Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS, 262, 545
Kudritzki, R. P., & Reimers, D. 1978, A&A, 70, 227
Kustaanheimo, P., & Stiefel, E. 1965, J. Reine Angew. Math., 218, 204
Makino, J. 1991, PASJ, 43, 859
Makino, J., & Hut, P. 1988, ApJS, 68, 833
Michie, R. W., & Bodenheimer, P. H. 1963, MNRAS, 126, 269
Mikkola, S. 1997, in Visual Double Stars: Formation, Dynamics and Evolu-
tionary Tracks, ed. J. A. Docobo Durantez, A. Elipe, & H. A. McAlister
(Dordrecht: Kluwer), 269
Mikkola, S., & Aarseth, S. J. 1993, Celest. Mech. Dyn. Astron., 57, 439
Perryman, M. A. C., et al. 1998, A&A, 331, 81
Pfahl, E., Rappaport, S., & Podsiadlowski, P. 2002, ApJ, 573, 283
Spruit, H. C. 1998, A&A, 333, 603
Spruit, H. C., & Phinney, E. S. 1998, Nature, 393, 139
von Hippel, T. 1998, AJ, 115, 1536
Weidemann, V. 1977, A&A, 59, 411
———. 2000, A&A, 363, 647
Weidemann, V., Jordan, S., Iben, I., Jr., & Casertano, S. 1992, AJ, 104, 1876
Wielen, R. 1974, in Stars and the Milky Way System, ed. L. N. Mavridis
(Berlin: Springer), 326
Williams, K. A. 2003, Ph.D. thesis, Univ. California, Santa Cruz