Unravelling the Cosmic Web: An analysis of the SDSS DR14 with the Local Dimension

Suman Sarkar\textsuperscript{1⋆} and Biswajit Pandey\textsuperscript{1†}

\textsuperscript{1} Department of Physics, Visva-Bharati University, Santiniketan, Birbhum, 731235, India

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ABSTRACT

We analyze a volume limited galaxy sample from the SDSS to study the environments of galaxies on different length scales in the local Universe. We measure the local dimension of the SDSS galaxies on different length scales and find that the sheets or sheetlike structures are the most prevalent pattern in the cosmic web throughout the entire length scales. The abundance of sheets peaks at $30 \, h^{-1}$ Mpc and they can extend up to a length scale of $90 \, h^{-1}$ Mpc. Analyzing mock catalogues, we find that the sheets are non-existent beyond $30 \, h^{-1}$ Mpc in the Poisson distributions. We find that the straight filaments in the SDSS galaxy distribution can extend only up to a length scale of $30 \, h^{-1}$ Mpc. Our results indicate that the environment of a galaxy exhibits a gradual transition towards higher local dimension with increasing length scales finally approaching a nearly homogeneous network on large scales. We compare our findings with a semi analytic galaxy catalogue from the Millennium Run simulation which are in fairly good agreement with the observations. We also test the effects of the number density of the sample and the cut-off in the goodness of fit which shows that the results are nearly independent of these factors. Finally we apply the method to a set of simulations of the segment Cox process and find that it can characterize such distributions.

Key words: methods: statistical - data analysis - cosmology: large scale structure of the Universe.

1 INTRODUCTION

Understanding the formation and evolution of the cosmic web remains one of the most fascinating and challenging problems in cosmology. The first observational hint of the existence of the cosmic web came through several early redshift surveys (Chincarini & Rood 1975; Gregory & Thompson 1978; Einasto et al. 1980) which was later confirmed (de Lapparent et al. 1986) by the surveys like CfA (Davis et al. 1982) and LCRS (Shectman et al. 1996). The modern redshift surveys like the 2dFGRS (Colless et al. 2001) and the SDSS (York et al. 2000) have now revealed the cosmic web in its full glory. The cosmic web is a network of galaxies spanning the entire Universe. The network comprises of several distinct morphological components such as clusters, filaments and sheets which are interconnected in a complex manner and are encompassed by voids of numerous sizes. The galaxies form and evolve in different environments inside the cosmic web and the different morphological components provide unique environments for galaxy formation and evolution.

The first theoretical insight into the formation of the cosmic web was provided by the seminal work of Zel’’dovich (1970) which showed how the successive collapse of an overdense region along its longest, medium and shortest axis would produce spatial patterns like sheets, filaments and clusters respectively. Characterizing these spatial patterns in the cosmic web is an important step towards understanding the galaxy formation and evolution in the Universe. A large number of statistical tools have been designed for this purpose. The percolation analysis (Shandarin & Zeldovich 1983; Einasto et al. 2018), the genus (Gott, Mellot & Dickinson 1986; Appleby et al. 2018), the Minkowski functionals (Mecke et al. 1994; Wiegand et al. 2014; Fang et al. 2017), the Shapefinders (Sahni et al. 1998; Bharadwaj et al. 2004; Pandey & Bharadwaj 2005; Bag et al. 2018), the minimal spanning tree (Barrow et al. 1985; Lares et al. 2017), the statistics of maxima and saddle points (Colombi, Pogosyan & Souradeep 2000; Ansari Fard et al. 2018), the multiscale morphology filter based on the Hessian of the density field (Aragón-Calvo et al. 2007, 2010), the
skeleton formalism (Novikov et al. 2006; Sousbie et al. 2008), the local dimension (Sarkar & Bharadwaj 2009; Sarkar et al. 2012) and the Origami approximation (Neyrinck 2012, 2016) are to name a few. Each of these different statistical measures captures some aspects of the cosmic web. But a comprehensive measure of the cosmic web is still awaited. Presently, developing effective tools for the quantification of the cosmic web is an active area of research.

The different structural elements of the cosmic web are characterized by their density and geometry. The galaxy clusters located at the nodes where the filaments intersect, are known to be the densest regions in the cosmic web followed by the filaments and the sheets. The filaments observed in the galaxy distribution from the SDSS has been shown to be statistically significant up to the length scales of $80 h^{-1}$ Mpc (Bharadwaj et al. 2004; Pandey & Bharadwaj 2005). Filaments are elongated structures with a length of tens of Mpc (Colberg 2007) and thickness of $\sim 2-3 h^{-1}$ Mpc (González & Padilla 2010). They can be of different sizes and types (straight, warped, irregular etc.) based on their visual morphology (Pimbblet et al. 2004). The filaments are believed to host $\sim 50\%$ of the baryons in the Universe (Cen & Ostriker 2006) and are expected to play an important role in the formation and evolution of galaxies. The filaments which are one of the most prominent visual features in the galaxy distribution has so far drawn a lot of attention in the literature. Contrary to this, the detection of sheets or the walls in the galaxy distribution has attracted very little or no attention at all.

There are also giant structures like the Sloan Great Wall (Gott et al. 2005) extending over length scales of more than $400$ Mpc. The Saraswati supercluster (Bagchi et al. 2017) which spans at least $200$ Mpc is a massive supercluster recently found in the SDSS. On the other hand, the empty regions or the voids constitute of about $\sim 95\%$ volume of the Universe (Kauffmann & Fairall 1991; El-Ad & Piran 1997; Hoyle & Vogeley 2002; Platen et al. 2007). The voids seen in the galaxy distribution have different sizes such as Bootes void with a radius of $62$ Mpc (Kirshner et al. 1987) and the Eridanus supervoid which extends up to $\sim 300$ Mpc (Szapudi et al. 2015). The existence of these giant structures illustrate the variety and richness of the environments for galaxy formation and evolution in the cosmic web. Galaxy environments are primarily characterized by the local density which is known to play a central role in the galaxy formation and evolution. It has been also argued that besides the density, the morphology of the environment may also play a crucial role in the formation and evolution of galaxies (Pandey & Bharadwaj 2008; Scudder et al. 2012; Darvish et al. 2014; Luparello et al. 2015; Filho et al. 2015; Pandey & Sarkar 2017; Lee 2018). It would be interesting to measure the relative abundance of these structures on different length scales and understand their roles in the galaxy formation and evolution.

The Sloan Digital Sky Survey (SDSS) which is currently the largest redshift survey has mapped the distribution of millions of galaxies in the nearby Universe providing an unprecedented view of the cosmic web in the nearby Universe. This provides an unique opportunity to unravel the cosmic web in greater detail than ever possible. Sarkar & Bharadwaj (2009) propose the local dimension which is a simple measure to characterize the environment in which a galaxy is embedded inside the cosmic web. This has been applied earlier to the SDSS DR7 data by Sarkar et al. (2012) to study the length scale dependence and density dependence of the various morphological components of the cosmic web. The local dimension can be also employed to address several other important issues related to the cosmic web. In this work, we analyze the data from the SDSS DR14 with the local dimension to study how the fraction of galaxies residing in different morphological environments changes with the associated length scales. This allows us to explore the relative abundance of different types of structures at different length scales and identify the length scales which are dominated by any particular type of structures. We also prepare a list of galaxies for which the local dimension can be computed throughout the entire length scale range available for this analysis. This would enable us to track the gradual transition of the environment of a galaxy with the increasing length scales.

We compare our findings with a semi analytic model of galaxy formation by using a semi analytic galaxy catalogue (Henriques et al. 2015) based on the Millennium Run Simulation (MRS) (Springel et al. 2005). Further, some of the filaments and sheets observed in the galaxy distributions are the outcome of random chance alignments. So we also compare our findings against the random mock catalogues from Poisson distributions to quantify the fraction of galaxies identified as part of filaments and sheets which are the products of random chance alignment.

We also test the possible roles of any systematics such as the number density and the cut-off in the goodness of fit in influencing the results of the present analysis. Finally, we test the efficiency of the method by applying it to a set of simulations of the segment Cox process.

We convert redshifts to distances using a $\Lambda$CDM cosmological model with $\Omega_{\text{m}0} = 0.31$, $\Omega_{\Lambda0} = 0.69$ and $h = 1$ throughout the analysis.

A brief outline of our paper is as follows. We describe the method in Section 2 and the data in Section 3. We present our results and conclusions in section 4 and section 5 respectively.

## 2 METHOD OF ANALYSIS

We consider a sphere of radius $R$ centred around each galaxy in the volume limited sample. The centres for which the spheres remain completely inside the survey boundary are identified and we count the number of galaxies $N(<R)$ inside each of these spheres. We repeat these measurements for a number of different radius $R$ within a specified length scale range $R_1 \leq R \leq R_2$. The value of $R_1$ is kept fixed and $R_2$ is gradually increased up to the largest radius accessible within the survey region.

The cosmic web is an interconnected network of sheets, filaments and clusters and each of the galaxies are part of any of these structural elements. We expect the number of galaxies within a sphere of radius $R$ centered around a galaxy to scale as,

$$N(< R) = A R^D$$  \hspace{1cm} (1)$$

where $A$ is a constant and the exponent $D$ is the local di-
mension (Sarkar & Bharadwaj 2009). The local dimension $D$ quantifies the nature of the structural element in which it is embedded. We expect $D = 1$ and $D = 2$ for galaxies residing in the filaments and sheets respectively. $D = 3$ around a galaxy can both indicate a galaxy cluster or any volume filling structures such as the cosmic web on large scales. It may be noted that the intermediate values of the local dimension $D$ are also possible when the counting sphere incorporates multiple structural elements of different types.

We fit the galaxy counts $N(< R)$ around each centre to Equation 1 and measure the $D$ value associated with each galaxy. The local neighbourhood of a galaxy is expected to look different at different length scales. Consequently, the measured $D$ values are expected to change with the increasing length scales and finally approach $D \sim 3$ when the galaxy is surrounded by a homogeneous network. This would occur only beyond the scale of homogeneity.

We consider only those centres for which we have at least 10 neighbouring galaxies within radius $R_2$. The value of $D$ for each galaxy within each length scale range $R_1 \leq R \leq R_2$ is estimated using a least-square fit and a $\chi^2$ value is also calculated for each of these fits. We apply a cut in the Chi-square per degree of freedom $\chi^2 \leq 0.5$ to identify only the good quality fits for our analysis (Figure 1). We classify the galaxies into five classes based on the measured values of their local dimension $D$. Table 1 provides the criteria for this classification. C1 and C2 are the galaxies which are part of a filament or sheet respectively. The C3 galaxies are part of volume filling structures. The I1 and I2 galaxies with intermediate $D$ values may lie near the junction of two different types of structural elements.

For each length scale range $R_1 \leq R \leq R_2$, we find the number and fractions of classified galaxies in each class.

### 3 DATA

#### 3.1 SDSS DR14

We use data from the 14th data release of the Sloan Digital Sky Survey (SDSS) (Abolfathi et al. 2017) which is the second data release of the fourth phase (SDSS IV) of the survey. DR14 has accumulated spectral and imaging data taken from August 2014 to July 2016 by the SDSS 2.5 m telescope and it has the most current and reprocessed data that incorporates the entire coverage of the prior data releases. We use a Structured Query Language (SQL) to get the data from SDSS CasJobs1. We select a contiguous region in the Northern galactic hemisphere using the cuts $0^\circ \leq \delta \leq 60^\circ$ and $135^\circ \leq \alpha \leq 225^\circ$, where $\alpha$ and $\delta$ are the right ascension and declination respectively. We select all the galaxies within redshift $z < 0.3$ and r-band Petrosian magnitude $m_r < 17.77$ in this region. We set the $ZWARNING$ flag to zero to select only the galaxies with good spectrum and reliable redshift. These cuts yield a total 377606 galaxies. We then prepare a volume limited sample from this data by applying a cut $M_r < -20.5$ in the K-corrected and extinction corrected r-band absolute magnitude. The K-corrections are obtained from a polynomial fit provided by Park et al. (2005). The resulting volume limited sample contains 90406 galaxies within redshift $z < 0.1385$ which radially extends upto 406 $h^{-1}$ Mpc. The galaxy sample has a number density of $\sim 2.977 \times 10^{-3} h^3 Mpc^{-3}$ and the mean intergalactic separation of $\sim 6.95 h^{-1}$ Mpc.

#### 3.2 Millennium Run Simulation

The Millennium Run Simulation (MRS) (Springel et al. 2005) is one of the largest high resolution cosmological N-body simulation available today. The Millennium simulation followed the evolution of 2160$^3$ dark matter particles in a comoving box of size $500 h^{-1} Mpc$ from redshift $z = 127$ to $z = 0$. The semi analytic models (SAM) (White & Frenk 1991; Kauffmann, White & Guiderdoni 1993; Cole et al. 1994; Baugh et al. 1998; Somerville & Primack 1999; Benson et al. 2002) provide a powerful and effective tool to study the galaxy formation and evolution. These models parametrise the physics involved in terms of simple models following the dark matter merger trees over time and finally provide the statistical predictions of galaxy properties at any desired epoch. Here we use the data from a semi analytic galaxy catalogue (Henriques et al. 2015) derived from the Millennium run simulation (Springel et al. 2005). Henriques et al. (2015) updated the Munich model of galaxy formation using the values of cosmological parameters from PLANCK first year data. We use a SQL query to extract the data from the Millennium database 2. We map the Millennium galaxies to redshift space by using their peculiar velocities and then construct the mock samples by applying the same absolute magnitude cut as applied to the SDSS data. We ensure that the mock sample has the identical geometry and number density as the actual SDSS sample. We construct 10 such mock SDSS samples from the SAM catalogue from the Millennium Run simulation by placing the observer at different location. These mock samples are not derived from independent regions as we have only one realization of the SAM catalogue.

#### 3.3 Poisson sample

We construct 10 mock SDSS samples from Poisson distributions. These mock random catalogues have exactly the same geometry and number density as the actual SDSS sample used in this analysis.

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1 http://skyserver.sdss.org/casjobs/

2 https://wwwmpa.mpa-garching.mpg.de/millennium/
3.4 Segment Cox process

We simulate a set of segment Cox process (Martinez et al. 1998; Pons-Bordería et al. 1999) inside a cube of sides 250 $h^{-1}$ Mpc to test the efficiency of the method employed in the present work. The segment Cox process is a controlled point process where segments of length $l$ are scattered with random positions and orientations over a given volume. We first generate a random position and then choose a random orientation for a segment. The segment is then populated with points at random locations on it. The process is repeated for the desired number of segments. The segment length, the number of segments per unit volume and the mean number of points per unit length of the segments are the control parameters in the segment Cox process. We generate 10 realizations of the segment Cox process each with segment length 10 $h^{-1}$ Mpc, 30 $h^{-1}$ Mpc, and 50 $h^{-1}$ Mpc. The control parameters of the simulated datasets are described in Table 2.

4 RESULTS

4.1 Scale dependence of the local dimension

We study the scale dependence of the local dimension by identifying the classifiable galaxies at different length scales and estimating the local dimension for each of them. We keep $R_1$ fixed at 5 $h^{-1}$ Mpc and gradually change $R_2$ from 10 $h^{-1}$ Mpc to 100 $h^{-1}$ Mpc in uniform steps of 10 $h^{-1}$ Mpc. The number of classifiable galaxies decreases with increasing length scales. We find that initially 66136 galaxies out of the total 90406 galaxies ($\sim 73\%$) are classifiable at $R_2 = 10$ $h^{-1}$ Mpc which decreases to 2717 ($\sim 3\%$) at $R_2 = 100$ $h^{-1}$ Mpc. We measure the number of galaxies classified in each class (Table 1) and their fractions at each value of $R_2$. The number of SDSS galaxies in each class and their fractions as a function of $R_2$ are shown in the top left and top right panels of Figure 3. At any given length scale $R_2$, the fractions are simply the ratio of the number of galaxies in each class and the total number of classifiable galaxies at that length scale.

The filaments and the sheets are the most striking visible features in the cosmic web. In our analysis, the C1 and C2 types of galaxies are believed to be part of filament and sheet respectively. The right panel of Figure 3 shows the change in the fractions of different types of galaxies with increasing length scales. The figure shows that at $R_2 = 10$ $h^{-1}$ Mpc, $\sim 50\%$ ($\sim 30\%$ in sheets and $\sim 20\%$ in filaments) of all the classifiable galaxies resides in sheets and filaments. The rest $50\%$ galaxies are distributed in C3 type and the intermediate I1 and I2 type environment. The C3 type represent the galaxies inside groups/clusters or volume filling structures such as a homogeneous network of galaxies. The I1 type galaxies are expected to lie in the vicinity where filaments and sheets intersect. Further, this may also include the galaxies which are part of a curved or warped filament. The I2 type galaxies are expected to be a part of the environment where multiple sheets intersect. It is interesting to note that the fraction of C1 type galaxies decreases rapidly from $20\%$ at $10$ $h^{-1}$ Mpc to merely $0.1\%$ at $20$ $h^{-1}$ Mpc. We find very few C1 type galaxies beyond this length scale. It may be noted that only the galaxies residing in the straight filaments would be identified as C1 type. This indicates that

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**Table 2.** This table shows the parameters used to simulate the datasets for the segment Cox process.

| Length of segments ($h^{-1}$ Mpc) | Mean number of points per unit length on the segments ($h$ Mpc$^{-1}$) | Total number of segments |
|----------------------------------|-------------------------------------------------|------------------------|
| 10                               | 1                                               | 3000                   |
| 30                               | 1                                               | 500                    |
| 50                               | 1                                               | 100                    |

**Figure 1.** The left panel shows the best fit lines along with the measured values of $N$ as a function of $R$ for three different galaxies with local dimension 1, 2 and 3 respectively. The fits are carried within the length scale range $5 h^{-1}$ Mpc $\leq R \leq 10 h^{-1}$ Mpc and each of these fits satisfies the criteria $\chi^2/\nu \leq 2$ employed in this work. The right panel shows the same for another three galaxies with $D = 1, 2, 3$ for which the number counts are fitted within length scale range $5 h^{-1}$ Mpc $\leq R \leq 20 h^{-1}$ Mpc.
the straight filaments do not extend beyond a length scale of $30 \, h^{-1} \, \text{Mpc}$.

On the other hand, the fraction of $C2$ type galaxies initially increases with length scales and peaks at $30 \, h^{-1} \, \text{Mpc}$. The fraction of $C2$ type galaxies then decreases steadily with increasing length scales. We note that only $0.5\%$ classifiable galaxies are $C2$ type at $90 \, h^{-1} \, \text{Mpc}$. The presence of a peak at $30 \, h^{-1} \, \text{Mpc}$ for the $C2$ type galaxies indicates that most of the sheets extend up to a length scale $30 \, h^{-1} \, \text{Mpc}$. Sheets of larger sizes also exist in the cosmic web but they become less and less abundant with increasing length scales.

The fraction of $I1$ type galaxies behaves similar to the $C1$ type galaxies but they extend to a larger length scale. The fraction of $I1$ type galaxies changes from $\sim 35\%$ at $10 \, h^{-1} \, \text{Mpc}$ to $0.1\%$ at $60 \, h^{-1} \, \text{Mpc}$ indicating that the size of such environment extend much beyond the size of the straight filaments.

The fraction of $I2$ type galaxies in the SDSS increases from $\sim 15\%$ at $10 \, h^{-1} \, \text{Mpc}$ to $\sim 60\%$ at $100 \, h^{-1} \, \text{Mpc}$. Similarly, the fraction of $C3$ type galaxies grows from $10\%$ at $10 \, h^{-1} \, \text{Mpc}$ to $40\%$ at $100 \, h^{-1} \, \text{Mpc}$. These indicates that more and more galaxies are associated with such environment as the length scales are increased and nearly all the classifiable galaxies are part of either $I2$ or $C3$ type environment on a length scale of $100 \, h^{-1} \, \text{Mpc}$. This trend clearly indicates that a nearly homogeneous network of galaxies emerge on larger length scales.

The two middle panels of Figure 3 show the numbers and fractions of different types of galaxies as a function of length scales for the galaxies from the semi analytic galaxy catalogue from the Millennium simulation. Interestingly, the galaxies in this semi analytic model recovers the observed fractions of different types of galaxies in the SDSS remarkably well. The filaments and sheets extends up to nearly the same length scale in both the SDSS and the semi analytic model. Some small differences in the results can be also noted. For instance at $10 \, h^{-1} \, \text{Mpc}$, a relatively higher fraction of SDSS galaxies reside in sheets as compared those from the semi analytic model and this trend continues till $50 \, h^{-1} \, \text{Mpc}$. Further the fraction of $C2$ type galaxies peaks at $20 \, h^{-1} \, \text{Mpc}$ in the semi analytic model whereas the same peak appears at $30 \, h^{-1} \, \text{Mpc}$ for the SDSS galaxies. This implies that the sheets are relatively less abundant in the semi analytic model as compared to the SDSS. Interestingly, the sheets extend up to nearly $80 – 90 \, h^{-1} \, \text{Mpc}$ in both the distributions. The $I2$ type galaxies which dominates the larger
length scales are also believed to inhabit the regions which are partly sheetlike. These result emphasizes the prevalence of sheets in the cosmic web.

It should be also noted that some of the filaments and sheets identified in the galaxy distribution may be a result of random chance alignment. We would like to examine this by analyzing a set of mock SDSS catalogues from the Poisson random distributions. The results for the Poisson distributions are shown in the bottom two panels of Figure 3. It is interesting to note that a very small number of galaxies (∼5%) are found inside filament at 10 h⁻¹ Mpc in the Poisson distributions. This number is roughly 1/4th of that observed in the SDSS and the semi analytic model. This indicates that although a small number of filaments arise due to ransom chance alignments, the majority of the filaments detected in the SDSS and the semi analytic model are genuine in nature. On the other hand, a significant number of galaxies (∼30% are of C2-type) in the Poisson distribution are found to be part of a sheetlike structures at 10 h⁻¹ Mpc. The fraction of both the C2-type and I1 type galaxies diminish rapidly with increasing length scales and becomes nearly extinct beyond 30 h⁻¹ Mpc in a Poisson distribution. Contrary to this, we observed that the sheetlike structures extend upto 90 h⁻¹ Mpc in the SDSS and the semi analytic model. This suggests that the sheets identified on smaller length scales may be a result of random chance alignment.

Figure 3. The top left panel shows the number of different types of SDSS galaxies classified according to their local dimension (Table 1) at different values of $R_2$. The right panel shows how the fraction of different types of galaxies vary with increasing value of $R_2$. The two middle panels and the two bottom panels show the same but for the mock galaxy samples from a semi analytic galaxy catalogue from the Millennium simulation and the Poisson distributions respectively. The value of $R_1$ is fixed at 5 h⁻¹ Mpc in each case. The error-bars shown for the Poisson distributions and Millennium simulation are obtained from 10 independent realizations. The size of the error-bars are very small for the Poisson distributions. The error-bars are also very small for the Millennium simulation as all the 10 mock samples are derived from the same catalogue.
but the sheetlike structures spanning out to larger length scales in the SDSS and the semi analytic model are significant and genuine.

The fraction of $I_2$ and $C_3$ galaxies rises with increasing length scales in both the SDSS and the semi analytic model. We note that in the Poisson distribution, the fraction of $I_2$ galaxies though initially increases with length scales up to $30\,h^{-1}\text{Mpc}$ but then decreases gradually with increasing length scales. This clearly indicates that both sheetlike ($C_2$ type) and partly sheetlike ($I_2$ type) structures are less likely to emerge on larger length scales in a Poisson distribution due to random chance alignment. This emphasizes the significance of the large sheetlike structures observed in both the SDSS and the semi analytic model. Finally, the fraction of $C_1$ type galaxies steadily increases from 20% at $10\,h^{-1}\text{Mpc}$ to $\sim 90\%$ at $100\,h^{-1}\text{Mpc}$ in the Poisson distribution indicating its homogeneous nature as compared to the galaxy distributions on most length scales.

### 4.2 Transition of the local dimension

We find that the galaxies tend to inhabit regions with higher local dimension when probed on larger length scales. But most of the galaxies which are classified according to their local dimension on different length scales are not available at all scales. The gradual transition of the environment of a galaxy with increasing length scales can be only probed if its local dimension can be calculated at each and every length scale. We identify a subset of the classifiable of galaxies for which the local dimension can be computed throughout the entire length scales probed. We find that there are altogether 2282 galaxies in our SDSS sample for which this can be achieved. We prepare such a sample of galaxies for both the mocks from semi analytic galaxy catalogue and Poisson distribution.

We study the variation in the fraction of different types of galaxies as a function of length scale in each of these samples. The results are shown in Figure 4. The top left and right panel of Figure 4 show the results for the SDSS and the semi analytic model respectively. The results show that the galaxies reside in all sorts of environment when we probe only their immediate neighbourhood. As we in-
The results for the SDSS volume limited sample when the redshifts are converted to distances using the $\Lambda$CDM model and cosmography. The 1-σ errorbars are only shown in the two middle panels which are drawn from 10 different subsamples.

4.3 Systematic effects

We also study the systematics effects which may affect the outcome of the present analysis. While estimating the local dimension, the good quality fits are identified by employing a cut-off in the the Chi-square per degree of freedom $\frac{\chi^2}{\nu} \leq 0.5$. We would like to test if the results of the present analysis which should be kept in mind while analyzing any galaxy distribution to identify various patterns present in them. These results show that sheets can not arise from chance alignment on large scales and the prevalence of sheetlike structures in the SDSS galaxy distribution is an important characteristics of the observed cosmic web.
Figure 6. The top, middle and bottom left panels of the figure show the 3D distributions of the points generated using segment Cox process with segment length and number specified in each panel. The top, middle and bottom right panels show the fraction of points in different classes at different length scales for the datasets shown in the respective left panel.
are sensitive to this criteria. We repeated our analysis for another two cut-off values $\chi^2 \leq 1$ and $\chi^2 \leq 2$. The results of this test on the SDSS data are shown in the top two panels of Figure 5. Comparing these with the top right panel of Figure 3, we find that the fraction of galaxies available in different environment as a function of lengthscale is insensitive to the choice of the cut-off in $\chi^2$. We have checked that the galaxies belonging to a particular class remains in the same class when we change the cut-off in the $\chi^2$. It is only the numbers in each class which gets reduced when more stringent cuts are applied.

Further, we also test if the specific number density in our volume limited sample plays any role in deciding the results of the present analysis. We separately repeated our analysis by randomly discarding 25% and 50% of the galaxies from the SDSS volume limited sample and adopting $\chi^2 \leq 0.5$. The results of this test are shown in the middle two panels of Figure 5. We observe some small differences with the original result when 25% galaxies and 50% galaxies are discarded.

These tests show that the results of the present analysis are robust and nearly independent of the cut-off in $\chi^2$ and the number density of the galaxy sample.

The galaxy distribution analyzed here is restricted within $z < 0.1385$ which probes the local Universe. In this case one may convert redshifts to distances by simply using cosmography without the use of any particular cosmological model. We compare the results from the SDSS using the ΛCDM model and the cosmography in a model independent way in the two bottom panels of Figure 5. We observe that the main findings of the analysis remain nearly model independent.

4.4 Tests with the segment Cox process

We also test the efficiency of the method by simulating a set of segment Cox process and analyzing them with the local dimension. While analyzing the datasets from the segment Cox process, we find that the fraction of points belonging to C1 type or the filaments type gradually decreases with increasing length scales and extends upto a length scale which is somewhat larger than the characteristic segment length in each case. For example, the top right panel of Figure 6 shows that the fraction of filament type points are largest (> 50%) at $10 \, h^{-1}$ Mpc which decays to nearly zero at $30 \, h^{-1}$ Mpc. The datasets with segment length $30 \, h^{-1}$ Mpc shows that more than 65% of the points are filament type at length scales of $10 \, h^{-1}$ Mpc which gradually decays to zero at length scales of $\sim 50 \, h^{-1}$ Mpc. Similarly, we find that the dataset with segment length $50 \, h^{-1}$ Mpc exhibit that $\sim 80\%$ points reside in filaments at $10 \, h^{-1}$ Mpc which diminish to zero at $80 \, h^{-1}$ Mpc. The results suggest the existence of larger number of segments having length smaller compared to the characteristic segment length and smaller number of segments with a length larger than the characteristic segment length. This may arise due to the intersection and chance alignments of multiple segments which could produce both shorter and longer segments as compared to the characteristics segment length. The intersection of multiple segments is more likely to occur as compared to the chance alignment of multiple segments. This explains why we find a larger fraction of shorter filaments than the longer filaments as compared to the characteristic segment length. We also note that the chance alignments of many linear segments from various orientations on large scales can give rise to structures with sheetlike appearance. In all the right panels of Figure 6, we find that a large fraction of points are classified as C2 type or sheet type on increasingly larger scales. These sheetlike structure are the results of pure chance alignments of many linear segments oriented along different directions. Interestingly, we find that the fraction of points belonging to volume filling structures are negligible in each case. So the test suggests that the local dimension is unable to trace the exact size of the linear segments in a segment Cox process but gives the size of longest straight filaments in the distribution which can arise after intersection and alignments of multiple segments are taken into account. It may be noted that this increases with the characteristic segment length. Some spurious sheetlike features are identified on large scales due to the intersection and chance alignments of the linear segments in the segment Cox process. However, the fraction of C2 type points or sheetlike points remain very small on small scales which may be used to distinguish the segment Cox process from other types of distribution. The cosmic web is a much more complex system than a simple superposition of linear segments of uniform length and hence all the findings of this test may not be applicable to the real galaxy distributions. However, the test ascertains that the local dimension method can characterize a distribution which is dominated by linear filamentary structures.

5 CONCLUSIONS

We compute the local dimension of galaxies in a volume limited galaxy sample from the SDSS in the local Universe and study their proportions on different length scales. We find that the galaxies reside in all types of environments when the environment is characterized on smaller length scales. Our results indicate that the filaments in the galaxy distribution extend upto $30 \, h^{-1}$ Mpc whereas the sheets extend upto as large as $90 \, h^{-1}$ Mpc. On large scales, the majority of the galaxies in the SDSS are found to reside in either sheetlike or partly sheetlike environment. We find a very similar trend in the semi analytic galaxy catalogue from the Millennium Run simulation. No filaments and sheets are observed beyond a length scales of $30 \, h^{-1}$ Mpc in the Poisson distribution. The absence of sheetlike structures on large scales in the Poisson distribution show that they can not result from a random chance alignment on those length scales. The present analysis find the prevalence of sheetlike structures in the cosmic web on larger length scales. The filaments are only observed on smaller length scales and are completely absent on larger length scales.

Our analysis indicates that the sheets and the sheetlike structures are the most dominant features on large scales in the galaxy distribution from the SDSS as well as in the semi analytic model. In the Zeldovich scenario, the pancakes are the first non-linear structures formed by gravitational collapse. Doroshkevich (1970) show that the simultaneous collapse along multiple axes is quite unlikely and the filaments and nodes would form later depending on the eigenvalues of the deformation tensor at different Lagrangian co-ordinates.
The pancakes are expected to be the most dominant feature emerging from the first stage of non-linear clustering. So the higher abundance of sheets or sheetlike structures observed on relatively larger scales may be a consequence of the Zeldovich approximation.

Earlier studies find that the filaments are statistically significant up to length scales of $70 \, h^{-1} \, \text{Mpc}$ (Pandey & Bharadwaj 2005) whereas our results indicate that the straight filaments can only extend upto $30 \, h^{-1} \, \text{Mpc}$. The two dimensional sections analyzed by Pandey & Bharadwaj (2005) may also include some filaments which arise due to the projection of sheetlike structures. Further, their study also incorporates the curved or wiggly filaments into consideration.

The observed galaxy distribution shows a tendency towards transition to a homogeneous network on larger length scales. This is consistent with the findings that the Universe is homogeneous around a length scales of $\sim 100 \, h^{-1} \, \text{Mpc}$ (Yadav et al. 2005; Hogg et al. 2005; Sarkar et al. 2009; Scrimgeour et al. 2012; Nadathur 2013; Pandey & Sarkar 2015; Pandey & Sarkar 2016; Avila et al. 2018).

We study the systematics effects of the number density of the sample and the cut-off in the goodness of fit and find that our results are robust against the variation in these parameters. Analyzing simulated datasets of the segment Cox process, we find that the local dimension method can characterize such distributions.

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