Rare Decays of $\psi$ and $\Upsilon$

K. K. Sharma and R. C. Verma*

Centre for Advanced Study in Physics, Department of Physics,
Panjab University, Chandigarh -160014, India.

* Department of Physics, Punjabi University, Patiala - 147 002, India.

Abstract

We study two-body weak hadronic decays of $\psi$ and $\Upsilon$ employing the factorization scheme. Branching ratios for $\psi \to PP/PV$ and $\Upsilon \to PP/PV$ decays in the Cabibbo-angle-enhanced and Cabibbo-angle-suppressed modes are predicted.
1 Introduction

A wealth of experimental data on masses, lifetimes and decay rates of charm and bottom mesons has been collected [1]. Models based on factorization scheme [2,3], successfully used to describe weak decays of naked flavor mesons, can also be used to study weak decays of hidden flavor mesons. Low lying states of quarkonia systems usually decay through intermediate photons or gluons produced by the parent $c\bar{c}$ and $b\bar{b}$ quark pair annihilation. For instance, charmonium state $\psi(1S)$ decays predominantly to hadronic states (87.7%), whereas its leptonic modes are observed to be around 6%. The same trend follows for the bottomonium states $\Upsilon(1S)$ also. These OZI violating but flavor conserving decays lead to narrow widths to $\psi$ and $\Upsilon$ states.

In the Standard Model framework, the flavor changing decays of these states are also possible though these are expected to have rather low branching ratios. At present, experiments have crossed over to many million $\psi$-events and the prospectus are good for a 100-fold increase in these events in the high energy electron-positron collider. It is expected that some of its rare decay modes may also become detectable in future. With this possibility in mind, we study two-body weak decays of $\psi$ in Cabibbo-angle-enhanced and Cabibbo-angle-suppressed modes. Employing the factorization scheme, we predict branching ratios for $\psi \rightarrow PP/PV$ decays (where $P$ and $V$ represent pseudoscalar meson and vector meson respectively) and extend this analysis to $\Upsilon \rightarrow PP/PV$ decays involving $b \rightarrow c$ transitions.
2 Two-body Weak Decays of $\psi$

The structure of the general weak current $\otimes$ current Hamiltonian is

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cs}^* \{ a_1 (\bar{s}c)_H (\bar{u}d)_H + a_2 (\bar{s}d)_H (\bar{u}c)_H \} + h.c., \quad (1)$$

for Cabibbo-angle-enhanced mode ($\Delta C = \Delta S = -1$), and

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \{ a_1 \{ (\bar{u}d)_H (\bar{c}d)_H - (\bar{u}s)_H (\bar{s}c)_H \} + \}
\nonumber$$

$$\quad + a_2 \{ (\bar{u}c)_H (\bar{d}d)_H - (\bar{u}c)_H (\bar{s}s)_H \} + h.c., \quad (2)$$

for Cabibbo-angle-suppressed mode ($\Delta C = -1, \Delta S = 0$). $\bar{q}_1 q_2 \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ represents the color singlet V-A current and $V_{ij}$ denote Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements. The subscript H implies that $(\bar{q}_1 q_2)$ is to be treated as a hadron field operator. $a$‘s are the two undetermined coefficients assigned to the effective charge current, $a_1$, and the effective neutral current, $a_2$, parts of the weak Hamiltonian. These parameters can be related to the QCD coefficients $c_{1,2}$ as

$$a_{1,2} = c_{1,2} + \zeta c_{2,1}, \quad (3)$$

where $\zeta$ is usually treated as a free parameter, to be fixed by the experiment. We take

$$a_1 = 1.26, \quad a_2 = -0.51, \quad (4)$$

on the basis of $D \to K\pi$ decays [4].
2.1 $\psi \to PP$ Decays

The decay rate formula for $\psi \to PP$ decays is given by

$$\Gamma(\psi \to PP) = \frac{p_c^3}{24\pi m_\psi}|A(\psi \to PP)|^2,$$

where $p_c$ is the magnitude of the three momentum of final state meson in the rest frame of $\psi$ meson and $m_\psi$ denote its mass. Following the procedure adopted by one of us (RCV) with Kamal and Czarnecki [5] in determination of the weak decay amplitudes of $\psi$ decays, we express the decay amplitude of $\psi \to PP$ as (upto the scale $\frac{G_F}{\sqrt{2}} \times CKM factor \times QCD$ coefficient),

$$A(\psi \to PP) = \langle P|J^\mu|0\rangle \langle P|J_\mu|\psi\rangle,$$

where $J^\mu$ is the weak V-A current. Matrix elements of the weak current are given by

$$\langle P(k)|A_\mu|0\rangle = -i f_P k_\mu,$$

and

$$\langle P|J_\mu|\psi\rangle = \frac{1}{m_\psi + m_P} \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu_\psi (P_\psi + P_P)^\rho V(q^2) - i (m_\psi + m_P) \epsilon^\mu_\psi A_1(q^2)$$

$$- i \frac{\epsilon_\psi \cdot q}{m_\psi + m_P} (P_\psi + P_P)^\mu A_2(q^2) + i \frac{\epsilon_\psi \cdot q}{q^2} (2m_\psi) q^\mu A_3(q^2)$$

$$- i \frac{\epsilon_\psi \cdot q}{q^2} (2m_\psi) q^\mu A_0(q^2),$$

where $f_P$ is the meson decay constant, $\epsilon_\psi$ is the polarisation vector of $\psi$, $P_\psi$ and $P_P$ are the four-momenta of $\psi$ and pseudoscalar meson respectively, and $q^\mu = (P_\psi - P_P)^\mu$. $A_3(q^2)$ is related to $A_1(q^2)$ and $A_2(q^2)$ as

$$A_3(q^2) = \frac{(m_\psi + m_P)}{2m_\psi} A_1(q^2) + \frac{(m_\psi - m_P)}{2m_\psi} A_2(q^2).$$
In the Cabibbo-angle-enhanced mode, $\psi$ can decay to $D_s^+\pi^-$ or $D^0K^0$. To illustrate the procedure, we consider the color enhanced decay $\psi \to D_s^+\pi^-$ whose decay amplitude can be expressed as

$$A(\psi \to D_s^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* a_1 \langle \pi^- | J^\mu | 0 \rangle \langle D_s^+ | J_\mu | \psi \rangle,$$  \hspace{1cm} (10)$$

which gets simplified to

$$A(\psi \to D_s^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* a_1 (2m_\psi) f_\pi (e_\psi \cdot q) A_0^{\psi D_s}(m_\pi^2).$$  \hspace{1cm} (11)$$

Similarly, the factorization amplitude of the decay $\psi \to D^0K^0$ and Cabibbo-angle-suppressed decays of $\psi$ are obtained. We take the following values for the decay constants (in GeV) [6]:

$$f_\pi = 0.132, \quad f_K = 0.161,$$

$$f_\eta = 0.131, \quad f_{\eta'} = 0.118.$$  \hspace{1cm} (12)$$

For the form factors at $q^2 = 0$, we use

$$A_0^{\psi D_s}(0) = 0.61, \quad A_0^{\psi D_s}(0) = 0.66,$$  \hspace{1cm} (13)$$

obtained in the earlier work [5] using the BSW model wavefunctions [7] at $\omega = 0.5$ GeV. The oscillator parameter $\omega$ is the measure of the average transverse momentum of the quark in the meson [7]. We use the following basis for $\eta - \eta'$ mixing:

$$\eta = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \sin \phi_p - (s\bar{s}) \cos \phi_p,$$  \hspace{1cm} (14)$$

$$\eta' = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \cos \phi_p + (s\bar{s}) \sin \phi_p,$$  \hspace{1cm} (15)$$
where $\phi_p = \theta_{\text{ideal}} - \theta_{\text{physical}}$. Using the decay rate formula (5), we compute branching ratios of various $\psi \rightarrow PP$ decays which are listed in Table-1. Among the Cabibbo-angle-enhanced decays, we find that the dominant decay is $\psi \rightarrow D_s^+ \pi^-$, having the branching ratio

$$B(\psi \rightarrow D_s^+ \pi^-) = (0.87 \times 10^{-7})\%,$$  \hspace{1cm} (16)$$

and the next in order is $\psi \rightarrow D^0 K^0$, whose branching ratio is

$$B(\psi \rightarrow D^0 K^0) = (0.28 \times 10^{-7})\%.$$  \hspace{1cm} (17)$$

We hope that these branching ratios would lie in the detectable range.

2.2 $\psi \rightarrow PV$ Decays

Similar to $\psi \rightarrow PP$ decays, the weak decay amplitude for $\psi \rightarrow PV$ decays can be expressed as product of the matrix elements of the weak currents,

$$A(\psi \rightarrow PV) = \langle V|J^\mu|0\rangle \langle P|J_\mu|\psi\rangle,$$ \hspace{1cm} (18)$$

where the vector meson is generated out of the vacuum, and the corresponding matrix element is given by

$$\langle V(k)|V_\mu|0\rangle = \epsilon^*_\mu m_V f_V.$$ \hspace{1cm} (19)$$

Using the matrix elements given in (8), and (9), we obtain

$$A(\psi \rightarrow PV) = \frac{2m_V f_V}{m_\psi + m_P} \epsilon_{\mu\nu\rho\sigma} \epsilon^*_\nu \epsilon^*_\rho P_\psi \epsilon P_\rho V(q^2) + \iota(m_V f_V) \left\{ \epsilon^*_1 \epsilon^*_2 (m_\psi + m_P) A_1(q^2) \right\}$$

6
which yields the following decay rate formula:

\[
\Gamma(\psi \rightarrow PV) = (\text{nonkinematic factor})^2 \frac{p_c}{24\pi m_\psi^2} (m_V f_V)^2 (m_\psi + m_P)^2 \\
\times \{ \alpha |V(q^2)|^2 + \beta |A_1(q^2)|^2 + \gamma |A_2(q^2)|^2 + \delta \text{Re}[A_1^*(q^2) \times A_2(q^2)] \},
\]

(21)

where

\[
\alpha = \frac{8 m_P^2 p_c^2}{(m_P + m_\psi)^2},
\]

(22)

\[
\beta = 2 + \left\{ \frac{m_\psi^2 - m_P^2 - m_V^2}{2 m_\psi m_V} \right\}^2,
\]

(23)

\[
\gamma = \frac{4 m_P^4 p_c^4}{m_\psi^2 m_V^2 (m_P + m_\psi)^2},
\]

(24)

and

\[
\delta = \frac{2 (m_\psi^2 - m_P^2 - m_V^2) m_P^2 p_c^2}{(m_P + m_\psi)^2 m_\psi^2 m_V^2},
\]

(25)

Nonkinematic factor is the product of scale factor \(\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^*\) and the appropriate QCD coefficient \(a_1\) or \(a_2\). The terms corresponding to \(A_1(q^2), V(q^2),\) and \(A_2(q^2)\) represent S, P, and D partial waves in the final state. We have computed the numerical coefficients \((\alpha, \beta, \gamma, \delta)\) for various decays which are given in Table-2. We observe that the numerical coefficient of \(A_1(q^2)\) is the largest, thus the retention of S-wave only would appear to be an excellent approximation. For the sake of simplicity, we have retained only this term in our analysis. Following values of the vector meson decay constants (in GeV) [6] are used in our analysis:

\[
f_\rho = 0.216, \quad f_{K^*} = 0.221, \quad f_\omega = 0.195, \quad f_\phi = 0.237, f_{D^*} = 0.250,
\]

(26)
and the form factors [5]

\[ A_1^{\psi D}(0) = 0.68, \quad A_1^{\psi D^*}(0) = 0.78, \quad (27) \]

at \( \omega = 0.5 \) GeV. Using the decay rate formula (21), we obtain the branching ratios of \( \psi \rightarrow PV \) decays in Cabibbo-angle-enhanced and Cabibbo-angle-suppressed modes which are given in Table-3. For the color enhanced decay of the Cabibbo-angle-enhanced mode, we calculate

\[ B(\psi \rightarrow D_s^+ \rho^-) = (0.36 \times 10^{-6}), \% \quad (28) \]

which is higher than the branching ratio of \( \psi \rightarrow D_s^+ \pi^- \). Our analysis yields

\[ \frac{B(\psi \rightarrow D_s^+ \rho^-)}{B(\psi \rightarrow D_s^+ \pi^-)} = 4.2, \quad (29) \]

and therefore \( \psi \rightarrow D_s^+ \rho^- \) can be expected to be measured soon.

3 Two-body Weak Decays of \( \Upsilon \)

In this section, we extend our analysis to \( \Upsilon \rightarrow PP/PV \) decays.

3.1 \( \Upsilon \rightarrow PP \) Decays

The effective weak Hamiltonian generating the dominant b quark decays involving \( b \rightarrow c \) transition is given by

\[
H_{W}^{\Delta b=1} = \frac{G_F}{\sqrt{2}}\{V_{cb}V_{ud}^*[a_1(\bar{c}b)(\bar{d}u) + a_2(\bar{d}b)(\bar{c}u)] \\
+ V_{cb}V_{cs}^*[a_1(\bar{c}b)(\bar{s}c) + a_2(\bar{s}b)(\bar{c}c)]\} + h.c., \quad (30)
\]
for the CKM favored mode and

\[ H_W^{\Delta b=1} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}V_{us}^*[a_1(\bar{c}b)(\bar{s}u) + a_2(\bar{s}b)(\bar{c}u)] \right. \\
+ V_{cb}V_{cd}^*[a_1(\bar{c}b)(\bar{d}c) + a_2(\bar{d}b)(\bar{c}c)] \left. \right\} + h.c., \]  

(31)

for the CKM singly suppressed mode. In our analysis we use

\[ a_1 = 1.03, \quad a_2 = 0.23, \]  

(32)

as guided by \( B \to PP/PV \) data [8]. Similar to \( \psi \to PP \) decays, the factorization scheme expresses weak decay amplitudes as a product of matrix elements of the weak currents (upto the scale \( \frac{G_F}{\sqrt{2}} \times CKM \text{ factor} \times QCD \text{ factor} \) as:

\[ A(\Upsilon \to PP) = \langle P|J^\mu|0\rangle < P|J^\mu|\Upsilon >. \]  

(33)

For instance the decay amplitude for the color enhanced mode \( \Upsilon \to B_c^+\pi^- \) of the CKM-favored decays is given by

\[ A(\Upsilon \to B_c^+\pi^-) = \frac{G_F}{\sqrt{2}} \times V_{cb}V_{ud}^* \times a_1 f_{\pi}(2m_\Upsilon) A_{0}^{\Upsilon \to B_c} (m_{\pi}^2). \]  

(34)

We take (in GeV)

\[ f_D = 0.240, \quad f_{D_S} = 0.280, \quad f_{\eta_c} = 0.393, \]  

(35)

values [6] in our calculations. Generally the form factors for the weak transitions between heavy mesons are calculated in the quark model framework using meson wave functions. However, in the past few years the discovery of new flavor and spin symmetries has simplified the heavy flavor physics [9], and it has now become possible to calculate these form factors from certain mass
factors involving the Isgur-Wise function. In the framework of heavy quark
effective theory (HQET), these can be expressed as

$$A_0^{\Upsilon \rightarrow B_c}(q^2) = \frac{m_\Upsilon + m_{B_c}}{2\sqrt{(m_{B_c}, m_\Upsilon)}} \xi(\omega),$$  \hspace{1cm} (36)

with

$$\omega = v_\Upsilon \cdot v_{B_c} = \frac{m_\Upsilon^2 + m_{B_c}^2 - m_\pi^2}{2m_\Upsilon m_{B_c}},$$  \hspace{1cm} (37)

where the Isgur-Wise function $\xi$ is normalized to unity at kinetic point $v_\Upsilon \cdot v_{B_c} = 1$. As $\omega \approx 1.08$ for $\Upsilon \rightarrow B_c^+ \pi^-$ decay, we have ignored the $\omega$ dependence of the Isgur-Wise function $\xi$ and calculate

$$A_0^{\Upsilon \rightarrow B_c}(q^2) \approx 0.98.$$  \hspace{1cm} (38)

This in turn yields

$$B(\Upsilon \rightarrow B_c^+ \pi^-) = 0.33 \times 10^{-8} \%.$$  \hspace{1cm} (39)

The decay amplitudes for other CKM-favored and CKM-suppressed decay
modes are obtained similarly. Branching ratios for various $\Upsilon \rightarrow PP$ decays
are given in Table 4. We find that the dominant decays are $\Upsilon \rightarrow B_c^+ D_s^-$ and
$\Upsilon \rightarrow B_c^+ \pi^-$.

### 3.2 $\Upsilon \rightarrow PV$ Decays

The decay rate formula for such decays has been discussed in the section
3. Here also for comparison of the contributions of the various form factors
involved, we have calculated the numerical coefficients ($\alpha, \beta, \gamma, \delta$) for various
\( \Upsilon \to PV \) decays, which are given in Table 5. Like \( \psi \to PV \) decays, here also numerical coefficient \((\alpha)\) of \( A_1(q^2) \) is found to be the largest. Various form factors appearing in \( A(\Upsilon \to PV) \) are mutually related by HQET \([9]\),

\[
\frac{m_\Upsilon + m_{B_c}}{2 \sqrt{(m_{Bc}m_\Upsilon)}} \xi(\omega) = V(q^2) = A_0(q^2) = A_2(q^2)
\]

\[
= \left\{ 1 - \frac{q^2}{(m_\Upsilon + m_{Bc})^2} \right\}^{-1} A_1(q^2),
\]

\( q^2 = m_\Upsilon^2 + m_{Bc}^2 - 2 m_\Upsilon m_{Bc} v_\Upsilon \cdot v_{Bc}. \) (41)

Following the procedure used for \( \psi \to PV \) decays, we determine the decay amplitudes for various \( \Upsilon \to PV \) decays in the CKM-favored and CKM-suppressed modes. In addition to the meson decay constants given in (26), two more decay constants \((\text{in GeV})\) \([6]\)

\[
f_\psi = 0.405, \quad f_{D_s^*} = 0.271,
\]

are used here. Branching ratios for these decays are given in Table 6. Here also, we observe that the dominant mode is \( B(\Upsilon \to B_c^+ D_{s}^{*-}) = (2.57 \times 10^{-8})\% \), which is higher than \( B(\Upsilon \to B_c^+ \pi^-) \) by a factor of 7.9.

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Table 1: Branching Ratios of $\psi \to PP$ decays

| Mode       | Decay         | $Br.(\times 10^{-8}\%)$ |
|------------|---------------|-------------------------|
| $\Delta C = \Delta S = +1$ | $\psi \to D_s^+\pi^-$ | 8.74                    |
|            | $\psi \to D^0K^0$ | 2.80                    |
| $\Delta C = +1, \Delta S = 0$ | $\psi \to D_s^+K^-$ | 0.55                    |
|            | $\psi \to D^+\pi^-$ | 0.55                    |
|            | $\psi \to D^0\eta$ | 0.016                   |
|            | $\psi \to D^0\eta'$ | 0.003                   |
|            | $\psi \to D^0\pi^0$ | 0.055                   |
Table 2: Numerical Coefficients of the Form Factors for $\psi \to PV$ Decays

| Decay                  | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|------------------------|----------|---------|----------|----------|
| $\psi \to D_s^+ \rho^-$| 0.0210   | 3.755   | 0.0032   | 0.150    |
| $\psi \to D^0 K^*$    | 0.0204   | 3.553   | 0.0020   | 0.113    |
| $\psi \to D_s^+ K^{*-}$| 0.0146   | 3.390   | 0.0011   | 0.080    |
| $\psi \to D^+ \rho^-$ | 0.0264   | 3.974   | 0.0047   | 0.192    |
| $\psi \to D^0 \rho^0$ | 0.0267   | 3.984   | 0.0048   | 0.194    |
| $\psi \to D^0 \omega$ | 0.0261   | 3.928   | 0.0044   | 0.184    |
| $\psi \to D^0 \phi$   | 0.0135   | 3.284   | 0.0007   | 0.060    |
Table 3: Branching Ratios of $\psi \to PV$ decays

| Mode | Decay | $Br. (\times 10^{-8} \%)$ |
|------|-------|--------------------------|
| $\Delta C = \Delta S = +1$ | $\psi \to D_s^+ \rho^-$ | 36.30 |
| | $\psi \to D^0 K^{*0}$ | 10.27 |
| $\Delta C = +1, \Delta S = 0$ | $\psi \to D_s^+ K^{*-}$ | 2.12 |
| | $\psi \to D^+ \rho^-$ | 2.20 |
| | $\psi \to D^0 \rho^0$ | 0.22 |
| | $\psi \to D^0 \omega$ | 0.18 |
| | $\psi \to D^0 \phi$ | 0.65 |
| Mode | Decay | $Br. \times 10^{-8} \%$ |
|------|-------|-----------------|
| $\Delta b = 1, \Delta C = 1, \Delta S = 0$ | $\Upsilon \rightarrow B^+_c \pi^-$ | 0.33 |
| | $\Upsilon \rightarrow B^0 D^0$ | 0.10 |
| $\Delta b = 1, \Delta C = 0, \Delta S = -1$ | $\Upsilon \rightarrow B^+_c D_s^-$ | 0.76 |
| | $\Upsilon \rightarrow B^0 \eta_c$ | 0.17 |
| $\Delta b = 1, \Delta C = 1, \Delta S = -1$ | $\Upsilon \rightarrow B^+_c K^-$ | 0.024 |
| | $\Upsilon \rightarrow B^0 D^0$ | 0.005 |
| $\Delta b = 1, \Delta C = 0, \Delta S = 0$ | $\Upsilon \rightarrow B^+_c D^-$ | 0.031 |
| | $\Upsilon \rightarrow B^0 \eta_c$ | 0.010 |
Table 5: Numerical Coefficients of the Form Factors for $\Upsilon \to PV$ Decays

| Decay                  | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|------------------------|----------|---------|----------|----------|
| $\Upsilon \to B_c^+ \rho^-$ | 0.0319   | 13.414  | 0.0759   | 1.862    |
| $\Upsilon \to B^0 D^{*0}$   | 0.0379   | 5.000   | 0.0117   | 0.375    |
| $\Upsilon \to B_c^+ D_s^{*-}$ | 0.0175   | 3.758   | 0.0030   | 0.146    |
| $\Upsilon \to B_s^0 \psi$   | 0.0199   | 3.435   | 0.0014   | 0.090    |
| $\Upsilon \to B_s^0 K^{*-}$  | 0.0311   | 10.550  | 0.0537   | 1.355    |
| $\Upsilon \to B_s^0 D^{*0}$   | 0.0365   | 4.898   | 0.0112   | 0.360    |
| $\Upsilon \to B_c^+ D^{*-}$  | 0.0191   | 3.911   | 0.0040   | 0.174    |
| $\Upsilon \to B^0 \psi$     | 0.0216   | 3.477   | 0.0016   | 0.097    |
Table 6: Branching Ratios of $\Upsilon \to PV$ decays

| Mode | Decay | Br.$(\times 10^{-8}\%)$ |
|------|-------|-------------------------|
| $\Delta b = 1, \Delta C = 1, \Delta S = 0$ | $\Upsilon \to B_c^+ \rho^-$ | 0.88 |
| | | $\Upsilon \to B^0 D^{*0}$ | 0.19 |
| $\Delta b = 1, \Delta C = 0, \Delta S = -1$ | $\Upsilon \to B_c^+ D_{s}^{*-}$ | 2.57 |
| | | $\Upsilon \to B^0_s \psi$ | 0.93 |
| $\Delta b = 1, \Delta C = 1, \Delta S = -1$ | $\Upsilon \to B_c^+ K^{*-}$ | 0.050 |
| | | $\Upsilon \to B^0_s D^{*0}$ | 0.009 |
| $\Delta b = 1, \Delta C = 0, \Delta S = 0$ | $\Upsilon \to B_c^+ D^{*-}$ | 0.11 |
| | | $\Upsilon \to B^0_s \psi$ | 0.053 |