Modeling of rheological properties of cloudy apple juice using master curve

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ABSTRACT
The effect of temperature and concentration on the rheological behavior of pasteurized apple juice has been studied using a rotational rheometer at shear rates ranging from 0.5 to 12 s\textsuperscript{-1}. The rheological properties were measured at different conditions changing from 10°C to 60°C and from 15°Brix to 40°Brix. On the basis of the obtained results, flow curves were prepared and consistency coefficients and flow indexes were determined. The superposition technique was applied to model behavior of apple juice. The obtained master curve is very important for the industry because it shows the apple juice in the whole range of temperature and soluble solid content. The influence of temperature on consistency coefficient was well described by the Arrhenius equation with activation energy values between 23.3 and 33.6 kJ\textcdot mol\textsuperscript{-1}. A single equation was proposed for describing the combined effect of temperature, concentration, and shear rate on shear stress at constant flow behavior index. The model can be useful for the easy comparison of the rheological properties of different fluid food.

Modelado de las propiedades reológicas del jugo de manzana turbio usando la curva maestra

RESUMEN
El presente estudio investigó el efecto de la temperatura y la concentración sobre el comportamiento reológico del jugo de manzana pasteurizado, utilizando para ello un reómetro de rotación a velocidades de corte variables entre 0,5 y 12 s\textsuperscript{-1}. Las propiedades reológicas se midieron en diferentes condiciones, cambiando de 10 a 60°C y de 15 a 40 °Brix. A partir de los resultados obtenidos se elaboraron curvas de flujo, determinándose los coeficientes de consistencia y los índices de flujo. Se aplicó la técnica de superposición al modelo de comportamiento del jugo de manzana. La curva maestra obtenida es muy importante para la industria, porque muestra al jugo de manzana en todo el rango de temperaturas y contenido de sólidos solubles. La influencia de la temperatura en el coeficiente de consistencia pudo ser bien descrita por la ecuación de Arrhenius, con valores de energía de activación variables entre 23,3 y 33,6 kJ\textcdot mol\textsuperscript{-1}. Para describir el efecto combinado de la temperatura, la concentración y la velocidad de corte en la tensión de corte a un índice de comportamiento de flujo constante se propuso una única ecuación. El modelo puede ser útil para comparar fácilmente las propiedades reológicas de diferentes alimentos líquidos.

1. Introduction
Poland is one of the biggest apple producers in the European Union. A large part of this production is processing for juices and nectars. Apple juices are an excellent source of many valuable compounds such as phenolics, vitamins (A, B, C, D, and PP), organic acids, and microelements. The presence of the compounds contributes to health promotion and disease prevention (Boyer & Liu, 2004). In recent years, the interest in cloudy apple juices has increased. This kind of juice requires mild methods of preservation to prolong its shelf life. The most widely applied methods for preservation are thermal methods such as evaporation and pasteurization. During these processes, juices change its temperature and concentration. Both factors influence flow properties and behavior of fluids. The rheological properties of juice products are important in calculating the power requirements for pumping and mixing, sizing of pipes (Nindo, Tang, Powers, & Takhar, 2007; Sestak, Zitny, & Houska, 1983), estimation of holding time in heat exchanger (Chin, Chan, Yusof, Chuah, & Talib, 2009), understanding changes in food structure during processing (Guerrero & Alzamora, 1998; Holdsworth, 1993), and sensory evaluation (Bozdogan, 2017). Due to their importance of fluid properties for further processing, rheological models are frequently constructed to describe the obtained data. Many rheological models are used to represent the flow behavior of liquid food. The most popular are Newtonian, Power law, Bingham, Casson, and Herschel–Bulkley.

The rheological properties of apple juices were measured by several authors (Constenla, Lozano, & Crapiste, 1989; Ibarz, Vicente, & Graell, 1987; Ibrahim et al., 2011; Rao, Cooley, Ortloff, Chang, & Wijts, 1993; Saravacos, 1970; Torres et al., 2011).
The rheological character of apple juice depends on many parameters such as clarity, soluble solid content, and method of extraction and preservation. Clarified apple juices behave like Newtonian fluids at both positive (Bayindirli, 1992; Ibarz et al., 1987) and subzero temperatures (Falguera, Vicente, Garvin, & Ibarz, 2013). Cloudy apple juices change their rheological behavior with the concentration. Saravassos (1970) reported that cloudy apple juice exhibits Newtonian behavior at soluble solid content below 50°Brix, while up to 50°Brix behaves like non-Newtonian fluids. Newtonian behavior of cloudy apple juice with soluble solid content in the range from 10°Brix to 50°Brix was also described by Genovese and Lozano (2000) and by Brugnoni, Pezzutti, and Gonzalez (2013) for juice at 47°Brix. Torres et al. (2011) founded that unpasteurized fresh apple juice behaved as a non-Newtonian fluid.

The most liquid food does not exhibit Newtonian behavior. It means that their viscosity is dependent on the shear rate. Due to this, there are some problems in comparing the viscosity curves of juices produced from different raw materials. Temperature and soluble solid content are other factors which modify the behavior of fruit juices (Kobus, Nadulski, Guz, Mazur, & Panasiewicz, 2015). The high variability of the rheological properties with temperature and concentration sometimes makes measurements impossible for all shear rates because of exceeding the working range of viscometer.

The effect of temperature and concentration on the behavior of liquid can be modeled using the time–temperature superposition principle (Steffe, 1996). This technique equates the influence of time and temperature on rheological properties. Superposition of these parameters yields a master curve that covers a wide range of rheological properties that are unavailable due to the experimental limitations. This method was first described by William, Landel, and Ferry. The principle of the equivalency of time and temperature is basically described by time–temperature superposition and is given by (Fischer & Rehage, 1997):

$$\log \sigma_T = \frac{b_1}{b_2 + T - T_0}$$

The time–temperature superposition has proven to be a valuable method in studying aequous cetylpyridinium chloride and sodium salicylate solutions (Fischer & Rehage, 1997), carbon black suspended in base stock oil (Trappe & Weitz, 2000) and thickeners (guaranate, alginate, and etherified starch) (Ziad, Wang, Viallier & Dupuis, 2010).

This technique was also successfully used to model the effect of temperature and concentration on the rheological behavior of pomelo juice (Chin et al., 2009) and sourosop juice (Quek, Chin, & Yusof, 2013). In this case, the idea of the method is based on the use of a shift factor defined as the shear rate at one temperature to the shear rate at the reference temperature. A similar technique is used in the shear rate–concentration superposition.

Until now, there were no studies focused on the application of the superposition technique for modeling of apple juice behavior. Meanwhile, this method allows for quick comparison of rheological properties in a wide range of concentrations and temperatures. The variability of the consistency coefficient is often described by a single equation with the combined effect of concentration and temperature.

The paper attempts to extend this type of model with the third parameter, which is the shear rate. The objective of the paper was to compare rheological data of apple juice using master curve technique and to work out a new model describing the combined effect of shear rate, temperature, and concentration on shear stress.

2. Materials and methods

2.1. Preparation of apple juice

Apples of variety Eliza were purchased from a local ultra-low oxygen atmosphere cold storage. Fruits were washed with water and disintegrated. Juice was extracted by pressing on a basket press. The press consisted of a perforated cylinder with holes of 3 mm diameter, piston, construction frame, and hydraulic system (UHJG 20/C/2, Hydrotech, Lublin, Poland) which allows obtaining the pressure of 4.5 MPa. Next, juice was pasteurized in sterilized jars at 85°C for a time period of 10 min (Lopez-Sanz, Montilla, Moreno, & Villamiel, 2015) by putting them in a water bath (Brookfield TC-502, volume 6 l).

After thermal treatment, the juice was concentrated to 50°Brix using rotary glass vacuum evaporator (Rotavapor R205, Buchi, Switzerland). The concentrated juice was later diluted to six concentrations 15°Brix, 20°Brix, 25°Brix, 30°Brix, 35°Brix, and 40°Brix with distilled water. The obtained juice was sampled into 250 ml glass bottles and stored aseptically in a refrigerator at 4°C. The soluble solid content of each juice was determined by hand refractometer at 20°C (Atago, Pal-3, Japan).

2.2. Rheological measurements

The rheological properties of juice were determined using a rotational type Brookfield Viscometer (Model LDV-II Pro +, Brookfield Engineering Laboratories, USA). A juice sample of 16 ml was placed into a cylindrical chamber (ultra low viscosity adapter, ULA) and was allowed to equilibrate at the desired temperature using a circulation water bath (Brookfield TC-502, volume 6 l). The rheological parameters of concentrated apple juice were measured at six levels of temperatures 10°C, 20°C, 30°C, 40°C, 50°C, and 60°C at shear rate from 0.5 to 12 s⁻¹ in accordance with viscometer working range (Kobus et al., 2015).

2.3. Modeling of rheological properties of apple juice

The rheological data from the experiments were fitted to existing models such as Newtonian and Ostwald-de-Waele. The above-mentioned models are represented by the following equations:

$$\text{Newtonian } \sigma = \eta \gamma$$

$$\text{Ostwald–de–Waele } \sigma = K(\gamma)^n$$

The effect of temperature on the consistency coefficient was described by Arrhenius relationships:

$$K = A_0 \exp \left( \frac{E_a}{RT} \right)$$

The technique of the master curve was applied to compare the rheological data obtained at different temperatures and
concentrations. A total number of 36 flow curves were combined by using the superposition technique to construct one master curve. The shear rates were horizontally shifted at shear stress equals 0.02 Pa. The dimensionless temperature shift factors \( \alpha_T \) were defined as the ratio of shear stress at other temperatures to the shear rate at the reference temperature (30°C):

\[
\alpha_T = \frac{\gamma_i}{\gamma_{ref}}
\]

In the next step, the six concentration master curves were combined to construct a single temperature–concentration master curve. In this case, the concentration of 30°Brix was selected as the reference concentration. The concentration shift factor \( \alpha_C \) was defined as the ratio of shear rate at other concentrations to the shear rate at reference concentration:

\[
\alpha_C = \frac{\gamma_i}{\gamma_{RC}}
\]

2.4. Statistical analysis

The experiments were carried out in triplicates. The data were evaluated by analysis of variance, and differences among means were determined by the Tukey’s multiple-comparison post-hoc test with \( p < 0.05 \). The normality of the distribution was verified by means of the Shapiro–Wilk test. To verify the suitability of models, the determination coefficient \( R^2 \) and the deviation modulus \( D \) were calculated. The mean relative percentage deviation modulus \( \bar{D} \) was calculated from the following equation (Bozdogan, 2015):

\[
\bar{D} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{A_{\exp} - A_{\text{pred}}}{A_{\exp}} \right| 100\%
\]

All analyses were performed with Statistica software (Statistica 12, StatSoft Inc., Tulsa, OK, USA). The graphs show mean values and whiskers representing standard deviations.

3. Results and discussion

3.1. Rheological behavior of apple juice concentrates

The rheograms of shear stress versus shear rate are shown in Figure 1. The relationship between the shear stress and shear rate was fitted to two models (Newtonian and Ostwald-de-Waele). Both models showed high values of the goodness of fitting; however, the Ostwald-de-Waele model was perfectly fitted to experimental data with a coefficient of determination \( R^2 = 0.991 \). The determination coefficient for the Newtonian model was 0.958.

Due to the best fitting, Ostwald-de-Waele model was used for further analysis. This approach allowed to determine the values of consistency coefficient and flow behavior index (Table 1).

As shown in Table 1, all the values of the flow behavior index of apple juice are below unity for all measured temperatures and concentrations. It indicates that apple juice exhibits non-Newtonian, shear-thinning behavior. A close observation on each of the \( n \) values shows that there is no effect of concentration and temperature on the flow behavior index.

3.2. Modeling of rheological properties of apple juice using master curve

The values of temperature shift factors for each concentration obtained at different temperatures are shown in Table 2.

The temperature shift factor is equal to unity at the reference temperature. The first reduced shear rate \( (\gamma_1) \) was calculated as the original shear rate \( (\gamma) \) divided by the dimensionless shift factors \( (\alpha_T) \). The master curves were plotted as the shear stress \( (\sigma) \) versus first reduced shear rate. In this way, the six flow curves for each temperature were combined in one master curve. The plot of six concentration master curves is presented in Figure 2.

The obtained concentration master curves were fitted to the power law model expressed by the following equation (Chin et al., 2009):

\[
\sigma = K_T \left( \frac{\gamma}{\alpha_T} \right)^n_C
\]

The results are listed in Table 3.

The dependency between consistency coefficient and concentration was expressed by the exponential model (Equation (9)):

\[
K_T = 7.73210^{-4}\exp(0.1358C) \quad R^2 = 0.998
\]

\[
D = 2.16
\]

In the next step, the six concentration master curves were combined to construct a single temperature–concentration master curve. The values of concentration shift factor are demonstrated in Table 4.

It is worth to notice that concentration shift factor at reference concentration equals also to unity.

Finally, the single master curve was constructed as shear stress versus the second reduced shear rate \( (\gamma_2) \) (Figure 3).

The obtained master curve was again fitted to power law model according to equation (Chin et al., 2009):

\[
\sigma = K_T \left( \frac{\gamma}{\alpha_T \alpha_C} \right)^n_C
\]

The relationship between shear rate and the second reduced shear stress was expressed by the following equation:

\[
\sigma = 0.0575 \left( \frac{\gamma}{\alpha_T \alpha_C} \right)^{0.8621} \quad R^2 = 0.9943 \quad D = 3.26
\]

Master curve gives an overall picture of the flow behavior of apple juice irrespective of its temperature and concentration. The multitude of rheological parameters and dependence (of some) on the temperature and concentration impedes comparison of the rheological properties of juices originating from different sources (fruit species, variety, and extraction methods). For instance, the number of apparent viscosities in this study is equal to the number of combinations of the shear rate, temperature, and concentration, e.g., it is 360 (10 shear rates × 6 temperatures × 6 concentrations). The use of a single master curve enables easy and quick comparison of rheological properties of two different juices.

Quek et al. (2013) used this technique to model the rheological behavior of soursop juice concentrates. The authors obtained the following values of rheological properties: the consistency coefficient \( K' = 0.308 \) and flow behavior
index $n = 0.506$, respectively. A quick comparison allows to notice that the soursop juice has a several times greater consistency coefficient and is characterized by a much higher degree of pseudoplasticity compared to apple juice.

### 3.3. The effect of temperature on rheological properties of apple juice

The influence of temperature on consistency coefficient and flow behavior index is presented in Figure 4a,b, respectively. The consistency coefficient decreases with the increase in temperature. The results showed significant differences ($p < 0.05$) between the consistency coefficient for different temperatures (Table 1). Similar results were found by Constenla et al. (1989) and Ibarz et al. (1987), respectively. They reported a decrease in consistency coefficient in clarified apple juice with increasing temperature.

Movement of particles depends on temperature. The general rule is that the higher the temperature, the lower the resistance of the fluid. This inverse relationship has been likened to the incidence of a freer molecule-to-molecule interaction at elevated temperatures. Since viscosity and consistency coefficient are indications of the resistance to flow, such a freer interaction is expected to minimize the resistance (Maskan, 1999).

The effect of temperature on the consistency coefficient can be described by Arrhenius relationships. The Arrhenius parameters ($A_0$ and $E_0$) together with determination coefficient are listed in Table 5.

Activation energy reflects the sensitivity of viscosity or consistency coefficient to temperature changes. Higher...
Table 1. The rheological parameters of pasteurized apple juices obtained from the power law model.

| Concentration (°Brix) | 15  | 20  | 25  | 30  | 35  | 40  |
|-----------------------|-----|-----|-----|-----|-----|-----|
| Temperature (°C)      | 15  | 20  | 25  | 30  | 35  | 40  |
| Consistency coefficient, K | 0.0121* | 0.0265* | 0.0590* | 0.1369* | 0.2456* | 0.4819* |
| 20                   | 0.0083* | 0.0172* | 0.0322* | 0.0829* | 0.1359* | 0.2722* |
| 30                   | 0.0058* | 0.0116* | 0.0207* | 0.0493* | 0.0903*4 | 0.1706* |
| 40                   | 0.0046* | 0.0086* | 0.0144* | 0.0339* | 0.05698* | 0.1057* |
| 50                   | 0.0035* | 0.0064* | 0.0115* | 0.0267* | 0.03938* | 0.0790* |
| 60                   | 0.0026 | 0.0050 | 0.0087 | 0.0222 | 0.02942* | 0.0602 |

Flow behavior index, n

| Concentration (°Brix) | 10  | 20  | 30  | 40  |
|-----------------------|-----|-----|-----|-----|
| Temperature (°C)      | 10  | 20  | 30  | 40  |
| Shift factor a*       | 0.965* | 0.951* | 0.940* | 0.931* | 0.940* |
| 20                   | 0.947* | 0.942* | 0.933* | 0.924* | 0.941* | 0.949* |
| 30                   | 0.947* | 0.955* | 0.957* | 0.949* | 0.936* | 0.947* |
| 40                   | 0.954* | 0.949* | 0.967* | 0.937* | 0.951* | 0.935* |
| 50                   | 0.950* | 0.962* | 0.939* | 0.939* | 0.960* | 0.944* |
| 60                   | 0.963* | 0.970* | 0.952* | 0.953* | 0.961* | 0.941* |

Table 2. Shift factors of apple juice (a*) for different concentrations and temperatures.

Changes in flow behavior index as a function of temperature and concentration are shown in Figure 4b. There were no statistical differences in the values of the flow behavior index. That means that the pseudoplastic character of apple juice was independent of temperature and concentration.

The combined effect of shear rate and temperature on shear stress can be expressed as follows (Steffe, 1996):

\[
\sigma = f(T, \gamma) = A_0 \exp \left( \frac{E_a}{RT} \right) \cdot (\gamma)^n
\]  

(13)

The models described combined effect of shear rate and absolute temperature on shear stress for each concentration are demonstrated in Table 6.

3.4. Effect of concentration on rheological properties

The consistency coefficient increases with the concentration of apple juice. In general, a higher concentration of juice requires more energy to flow due to its relatively thick texture, which is related to the solid content (Abdullah, Chin, Yusof, & Talib, 2018). This phenomenon is consistent with many other studies, e.g. clarified apple juice reported

![Figure 2. Master curves for different concentrations of apple juice.](image1)

![Figure 2. Curvas maestras para diferentes concentraciones de jugo de manzana.](image2)
by Constenla et al. (1989) or cloudy apple juice described by Genovese and Lozano (2000). Also, Kimball (1986) reported that high pulp levels and soluble components contribute to the increase in consistency.

The effect of concentration on consistency coefficient was modeled using power type relationship (Equation (14)) and exponential type relationship (Equation (15)):

$$ K = K_1 \cdot C^b $$  \hspace{1cm} (14)

$$ K = K_2 \cdot \exp(b_2 C) $$  \hspace{1cm} (15)

The values of consistency coefficient and flow behavior index for both evaluated models obtained at different temperatures are shown in Table 7.

As presented in Table 7, the exponential model has slightly higher determination coefficients than the power law one. For this reason, to describe the combined effect of shear rate and concentration on shear stress, the exponential equation was selected. The values of the exponential model describing the simultaneous effect of shear stress and soluble solid content on shear stress are listed in Table 8.
The variation of the consistency coefficient $K_2$ and flow behavior index $b_2$ can be described with the help of the following Equations (16) and (17):

$$K_2 = K_1 \cdot \exp(d_k \cdot t)$$  \hspace{1cm} (16)

$$b_2 = b_1 \cdot \exp(d_b \cdot t)$$  \hspace{1cm} (17)

Thus, the combined effect of temperature and concentration can be expressed as:

$$K = K_1 \cdot \exp(d_k \cdot t) \cdot \exp(b_1 \cdot \exp(d_b \cdot t) \cdot C)$$  \hspace{1cm} (18)

Then, after simplification, we obtain:

$$K = K_1 \cdot \exp(d_k \cdot t + b_1 \cdot C \cdot \exp(d_b \cdot t))$$  \hspace{1cm} (19)

And after value substitution, we receive:

$$K = 16.14 \cdot 10^{-4} \cdot \exp(-0.0224t + 0.1522C \cdot \exp(-0.0037t))$$  \hspace{1cm} (20)

Because the expression “$\exp(-0.0037t)$” equals approximately to unity, then Equation (19) can be written as:

$$K = K_1 \cdot \exp(d_k \cdot t + b_1 \cdot C)$$  \hspace{1cm} (21)

In the study case, the flow behavior index is independent of concentration and temperature. Therefore, we can take an average value of the flow behavior index ($\bar{n}$). Thus, the equation for the combined effect of shear rate, concentration, and temperature can be written as follows:

$$\sigma = K_1 \cdot \exp(d_k \cdot t + b_1 \cdot C) \cdot (\gamma)^n$$  \hspace{1cm} (22)

After value substitution, we receive:

$$\sigma = 3.13910^{-3} \cdot \exp(-0.04314t + 0.1366C) \cdot (\gamma)^{0.948}$$  \hspace{1cm} (23)

The equation has very high determination coefficient, which indicates the goodness of fit of the model to experimental data. The derived equation allows to estimate the behavior of apple juice in the whole tested range of shear rate, temperatures, and concentrations. The numerical values of this equation indicate that the concentration has a greater effect on changes in rheological properties of apple juice than temperature. The constant value of the flow behavior index ($n$) indicates the lack of influence of concentration and temperature on the rheological characteristic of the juice, and its value (below 1) shows the pseudoplastic character of the apple juice.

4. Conclusions

Pasteurized cloudy apple juice behaved like non-Newtonian fluid in the range of studied soluble solid content (15–40°Brix) and temperatures (10–60°C). There was no influence of temperature and concentration on the flow behavior index. The dependency between temperature and concentration coefficient was well described by Arrhenius equation with activation energy ranging from 23.3 to 33.6 kJ/mol. Modeling of rheological properties using master curve has shown that the consistency coefficient increases exponentially with the concentration. The combined effect of shear rate, temperature, and soluble solid content was described by a new semi-empirical model. This model is the first one that describes the simultaneous influence of concentration, temperature, and shear rate on the rheological properties of apple juice. It is more useful than the models used so far, which only took into account the effect of temperature and concentration. The additional parameter in the form of a shear rate shows how strong the impact of flow velocity on changes in rheological properties of apple juice is. Derived model can be very useful for the optimization of processes involving apple juice production, transport, and evaporation.

### Nomenclature

- $\sigma$ shear stress (Pa),
- $\sigma_0$ yield stress (Pa),
- $\gamma$ shear rate ($s^{-1}$),
- $\gamma_{1s}, \gamma_{2s}$ first and second reduced shear rate ($s^{-1}$),
- $\gamma_{RT}$ shear rate at the reference temperature ($s^{-1}$),
- $\gamma_{RC}$ shear rate at the reference concentration ($s^{-1}$),
- $K_1, K_2, K_{15}, K_5$ consistency coefficient (Pas$^2$),
- $n_1, n_2, n_3$ shear flow index,
- $n_m$ arithmetic mean of flow behaviour index,
- $K_1, n_1$ constant in power equation,
- $K_1, n_2$ constant in exponential equation,
- $K_1, d_k, b_1, d_b, K_1, d_k, b_1$ constant in power law model,
- $n_c$ number of experimental data,
- $A_0$ frequency factor (Pas$^2$),
- $A_{exp}$ experimental value,
- $\bar{A}_{pred}$ predicted value,
- $\Delta$ mean relative percentage deviation modulus (%),
- $E_a$ activation energy (kJ/mol$^{-1}$),
- $R$ universal gas constant (J·mol$^{-1}$·K$^{-1}$),
- $T$ absolute temperature (K),
- $T_0$ reference temperature (K),
- $t$ temperature (°C),

### Table 7. Effect of concentration on rheological properties of apple juice.

| Temperature (°C) | $K_1$ | $b_1$ | $R^2$ | $K_2$ | $b_2$ | $R^2$ | $D$ |
|------------------|-------|-------|-------|-------|-------|-------|-----|
| 10               | 3.412.10^{-2} | 3.79 ± 0.014 | 0.9825 | 13.280.10^{-4} | 0.149 ± 0.0007 | 0.995 | 1.92 |
| 20               | 4.32.10^{-2} | 3.56 ± 0.021 | 0.9815 | 10.355.10^{-4} | 0.140 ± 0.0006 | 0.996 | 1.63 |
| 30               | 4.11.10^{-2} | 3.45 ± 0.021 | 0.9775 | 7.561.10^{-4} | 0.136 ± 0.0010 | 0.997 | 1.58 |
| 40               | 5.99.10^{-2} | 3.23 ± 0.022 | 0.9755 | 6.746.10^{-4} | 0.128 ± 0.0022 | 0.995 | 1.90 |
| 50               | 5.66.10^{-2} | 3.16 ± 0.042 | 0.9760 | 5.461.10^{-4} | 0.124 ± 0.0021 | 0.997 | 1.57 |
| 60               | 4.05.10^{-2} | 3.17 ± 0.022 | 0.9760 | 4.146.10^{-4} | 0.125 ± 0.0015 | 0.991 | 2.12 |

### Table 8. The parameters of models describing the combined effect of shear rate and concentration on shear stress for apple juice.

| Temperature (°C) | Equation | $R^2$ | $D$ |
|------------------|----------|-------|-----|
| 10               | $\sigma = 13.280 \cdot 10^{-4} \exp(0.149t) \cdot (C)^{0.946}$ | 0.854 | 15.91 |
| 20               | $\sigma = 10.355 \cdot 10^{-4} \exp(0.140t) \cdot (C)^{0.940}$ | 0.957 | 5.56 |
| 30               | $\sigma = 7.561 \cdot 10^{-4} \exp(0.136t) \cdot (C)^{0.934}$ | 0.990 | 2.38 |
| 40               | $\sigma = 6.746 \cdot 10^{-4} \exp(0.128t) \cdot (C)^{0.949}$ | 0.995 | 1.94 |
| 50               | $\sigma = 5.461 \cdot 10^{-4} \exp(0.124t) \cdot (C)^{0.949}$ | 0.992 | 2.06 |
| 60               | $\sigma = 4.146 \cdot 10^{-4} \exp(0.125t) \cdot (C)^{0.937}$ | 0.988 | 3.07 |
R², determination coefficient, temperature shift factor, consistency shift factor, empirical constants.

Disclosure statement
No potential conflict of interest was reported by the authors.

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