TOPOLOGY OF LARGE-SCALE STRUCTURE BY GALAXY TYPE:
HYDRODYNAMIC SIMULATIONS

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ABSTRACT

The topology of large scale structure is studied as a function of galaxy type using the genus statistic. In hydrodynamical cosmological CDM simulations, galaxies form on caustic surfaces (Zeldovich pancakes) then slowly drain onto filaments and clusters. The earliest forming galaxies in the simulations (defined as “ellipticals”) are thus seen at the present epoch preferentially in clusters (tending toward a meatball topology), while the latest forming galaxies (defined as “spirals”) are seen currently in a spongelike topology. The topology is measured by the genus (= number of “donut” holes - number of isolated regions) of the smoothed density-contour surfaces. The measured genus curve for all galaxies as a function of density obeys approximately the theoretical curve expected for random-phase initial conditions, but the early forming elliptical galaxies show a shift toward a meatball topology relative to the late forming spirals. Simulations using standard biasing schemes fail to show such an effect. Large observational samples separated by galaxy type could be used to test for this effect.

Cosmology: large-scale structure of Universe – galaxies: clustering – galaxies: formation – hydrodynamics – stars: formation
1. INTRODUCTION

The formation of large-scale structure in the universe remains one of the most interesting problems in cosmology. Much progress in understanding this question has been made in the last quarter century. The standard picture for structure formation has been the gravitational instability picture: small random-phase initial fluctuations growing under the action of gravity (for discussions, see Gunn & Gott 1972; Doroshkevich, Zeldovich & Sunyaev 1976; Peebles 1980). This picture predicted an essentially undistorted thermal spectrum for the cosmic microwave background (confirmed by the COBE satellite, cf. Mather et al. 1990), and small angular fluctuations in the cosmic microwave temperature (Sachs & Wolfe 1967), also confirmed by the COBE satellite (Smoot et al. 1992). Models where the mass in the universe is dominated by Cold Dark Matter (CDM) (Peebles 1982) (with $\Omega$ not necessarily unity) have been particularly successful in explaining a number of observations (cf. White et al. 1987; Park 1990; Park & Gott 1991; Weinberg & Gunn 1990), in particular, the order of magnitude of the amplitude of the COBE fluctuations and the observed pattern of great walls and voids seen in the data (de Lapparent, Geller & Huchra 1986; Geller & Huchra 1989). A review of the virtues and defects of the CDM models is presented in Ostriker (1993). While the standard ($\Omega_{CDM} = 1$) model is almost certainly not correct, the best fitting models now under discussion are variants of standard CDM, which retain many of the essential features of the original model. In CDM models, caustics form (Zeldovich Pancakes, cf. Doroshkevich, Zeldovich, & Sunyaev 1976; Einasto, Joeveer & Saar 1980) and thus Great Walls are produced naturally (cf. Park 1990; Weinberg & Gunn 1990; Park & Gott 1991). As such models evolve in time, there is a draining of matter by gravity from minor walls onto filaments which produces an even larger network of voids as minor walls become so thinly populated as to effectively disappear (Weinberg 1990). This leaves a distinctive pattern of major and minor walls. The observational sample shows such features as well. In the original de Lapparent et al. 6°-wide slice, the large elliptical void on the left is nearly bisected by a very sparse and very thin wall of galaxies (Park et al. 1992a) which was noticed in a topology study and which looks just like the
minor walls seen in the simulations. Deep pencil beams to redshift $z = 0.5$ show great walls with a median separation of $133h^{-1}\text{Mpc}$ in the CDM simulations (Park & Gott 1991) and $128h^{-1}\text{Mpc}$ in the observations (Broadhurst et al. 1990). Broadhurst et al. reported a regular periodicity for these walls as measured by a power spectrum test, but 1 of the 12 simulated pencil beams showed an even greater periodicity by the same test (Park & Gott 1991), so the Broadhurst et al. results were certainly consistent with the standard model at the $2\sigma$ level. The inflationary CDM spectrum (Bardeen et al. 1986), gives an excellent fit to the large scale power seen in a variety of samples (Maddox et al. 1990; Saunders et al. 1991 [IRAS]; Sheetman et al. 1995 [deep slices]; Park, Gott & da Costa 1992 [Southern Sky Survey]) providing that $\Omega h \sim 0.3$ where $h = (H_o/100\text{km}s^{-1}\text{Mpc}^{-1})$.

Given current estimates of the Hubble constant of $h \sim 0.8$ (Freedman et al. 1994; Pierce et al. 1994), this would seem to favor either low-density ($\Omega < 1$), single-bubble inflationary models (cf. Gott 1982; Gott & Statler 1984; Gott 1986; Bucher, Goldhaber & Turok 1995; Yamamoto, Sasaki, & Tanaka 1995; Ratra & Peebles 1994; Kamionkowski & Spergel 1994) or $k = 0, \Omega_{CDM} \sim 0.4, \Omega_{\Lambda} \sim 0.6$ inflationary model (cf. Bahcall & Cen 1992; Efstathiou, Bond, & White 1992; Kofman, Gnedin, & Bahcall 1993; Cen, Gnedin, and Ostriker 1993; Cen & Ostriker 1994). Alternatively $\Omega_{CDM} = 1, h = 0.3$ models (Bartlett, Blanchard, Silk, & Turner 1995), Mixed (70% cold + 30% hot) Dark Matter Models (Davis, Summers & Schegel 1992; Taylor & Rowan- Robinson 1992; Klypin et al. 1993; Cen & Ostriker 1994; Ma & Bertschinger 1994), or tilted inflationary power spectra (Cen et al. 1993) have also been proposed. While standard biased CDM simulations (which do not match the COBE normalization) produce excellent fits to the observations in many ways, they simply designate proto-galaxies using the expected number density of galaxy-sized peaks in the CDM distribution found in the initial conditions and ignore the important gas dynamical and atomic radiative effects that must surely be important at small scales. Thus Cen & Ostriker (1992a) have embarked on a series of hydrodynamical simulations to model these effects allowing for as much of the relevant physics as possible.

These hydrodynamical simulations have given promising results in a number of ways.
As the density fluctuations grow, the gas dynamics are followed including shock heating from convergent flows, subsequent cooling etc. If there is a collapsing flow and the local cooling time is shorter than the local collapse time, one expects that the gas can cool to permit star formation and a proto-galaxy is formed. Proto-galaxies are then followed via collisionless dynamics and they are grouped into galaxies (via a friend-of-friend algorithm) at each epoch only to examine their properties (we do not group them during the evolution). Galaxies form on caustics (Zeldovich pancakes = walls) and filaments - then drain via gravity onto clusters. If the first 25% of galaxies formed are identified as “ellipticals”, the second 25% as “S0” galaxies, and the last 50% as “spirals”, a convenient and simple if crude representation of the observed morphology-mean age relation, then the morphology-density relation observed by Dressler (1980) and Bhavsar (1981) is reproduced well (Cen & Ostriker 1993a, Figure 6a), since ellipticals have migrated toward clusters while spirals are still on a filamentary net (see Fig. 1 panels a and c). Also, the elliptical galaxies produced have a higher covariance function among themselves than spiral galaxies have among themselves (Cen & Ostriker 1993a, Figure 7b), also in agreement with the observations (cf. Davis & Geller 1976). The distribution of hot and cold intergalactic gas produced in the simulations is good, giving approximately the correct number of X-ray clusters (Kang et al. 1994) and approximately the correct spectrum for the X-ray background (Cen et al. 1995). The distribution of Lyman alpha clouds is surprisingly realistic (Cen et al. 1994).

Now the morphology-density relation of Dressler (1980) and Bhavsar (1981) may be explained by a variety of reasons. Elliptical galaxies may simply be those with the shortest collapse times as suggested by Gott & Thuan (1977). They then have the highest mass densities at turn-around and the highest ratios of collapse times to cooling times and star formation times, so that they are also the galaxies that complete their star formation before their collapse (cf. Ostriker & Rees 1977). S0 and spiral galaxies do not complete their star formation before their collapse, so in addition to a spheroidal component of stars they have some left-over gas which infalls after the spheroid is formed and dissipates into a disk. Further star formation can then occur in the disk, either reaching completion (as
in the S0’s) or not (as in the spirals). Spirals subsequently infalling into clusters filled with hot virialized intergalactic gas can be stripped by ram pressure stripping (Gott & Gunn 1971; Gunn & Gott 1972) and turned into S0’s. This would explain the complete absence of spirals from the cluster cores of great (regular) virialized clusters like Coma (which are also X-ray clusters). Since spirals and S0’s have small spheroidal components, they would look like small ellipticals if their infall gas did not exist. Thus, an elliptical galaxy with a total luminosity of L* might have a larger total mass (including its CDM heavy halo), than an L* luminosity spiral. Now the 3-point function for galaxy clustering (Groth & Peebles 1977) [with Q ∼ 1 as observed] shows that the covariance function of a tight binary galaxy with all galaxies should be just twice the amplitude of the galaxy-galaxy covariance function (Gott 1980). This is reasonable since an object that is twice as massive should have twice the infall toward it, and a tight binary is dynamically equivalent to a single object of twice the total mass. Indeed, Gott, Turner, & Aarseth (1979) did a simulation with point particles of mass 2M* representing elliptical and S0 galaxies (with their lack of young bright stars) and particles of mass M* representing spiral galaxies and found \( \xi_{E,S0-gal} \sim 2\xi_{gal-gal} \) just as in the observations (Davis & Geller 1976). In the quoted hydrodynamical simulations, the elliptical galaxies formed, for example, have an average mass that is about 3 times as large as the average mass in the spirals. Also, if elliptical galaxies are those which collapse first, they represent higher initial amplitude peaks in the initial conditions, and according to standard biased galaxy formation schemes, they would represent peaks with a higher required threshold than other galaxies, and therefore would constitute a more biased initial distribution with a higher amplitude covariance function in the initial conditions (Bardeen et al. 1986).

Thus, there are a number of effects that would independently or together tend to act to produce a morphology-density relation similar to that observed. The hydrodynamical simulations naturally produce a distribution of galaxies that is biased relative to the CDM mass distribution: \( (\delta \rho_{gal}/\bar{\rho}_{gal}) \sim b(\delta \rho/\bar{\rho}) \) where \( b \sim 1.3 \) on a scale of \( 8h^{-1}\text{Mpc} \). This occurs by natural processes occurring as the gas turns into galaxies and explains why a
standard biasing scheme of picking initial peaks in the initial conditions might work fairly well in practice. The hydrodynamic simulations that use real physics thus offer us a good reason why we might expect the galaxy distribution to be somewhat biased relative to the mass distribution. In this paper we are looking for some additional testable telltale signs that the observed distribution of galaxies is in fact being produced by the detailed mechanisms actually occurring in the hydrodynamic simulations rather than just those generic to any model possessing a biased galaxy distribution. Topology seems to possess this property, for if the spirals are truly formed and found on a filamentary net (which would have a spongelike topology) and the ellipticals are preferentially found in clusters (a meatball topology) although perhaps also occasionally found on filaments (cf. Cen & Ostriker 1993a), then using the genus curve might permit differentiation between the two distributions in a large observational sample. This could provide a test of the detailed processes that are occurring in the hydrodynamical simulations. In this paper we show how such a topological analysis can be done and how its results differ when applied to the hydrodynamical simulations and simple but naive biasing models. Also we indicate how these techniques can be applied to large observational samples in the future.

2. Hydrodynamic Simulations

The numerical methods and treatment of detailed atomic physics are described in Cen (1992) and Cen & Ostriker (1992). To summarize briefly, we use a hydrocode based on Jameson’s (1989) aerospace code, modified extensively for cosmological applications. Poisson’s equation is solved on a periodic mesh with a FFT routine. All the principal line and continuum atomic processes are computed for each cell and each time step assuming a plasma of standard primordial composition. The radiation field from 1 eV to 100 keV is treated in detail with allowance for sources, sinks and cosmological effects, but only in a spatially averaged fashion.

We model galaxy formation in a heuristic but hopefully plausible way. The details of how we identify galaxy formation, follow the motions of formed galaxies and treat feedback processes (UV and supernovae energy input into the IGM from young massive stars) have
been presented (Cen & Ostriker 1993a). First we check cells for baryonic overdensity and only examine further those with \((\delta \rho / \bar{\rho}) > 4.5\) as candidates for regions within which galaxy formation will occur. Then, of these cells, we tag those that satisfy the following criteria:

\[
\nabla \cdot \vec{v} < 0 \quad \Rightarrow \text{contraction} \quad (1)
\]

\[
t_{\text{cool}} < t_{\text{dyn}} \equiv \sqrt{\frac{3\pi}{32G\rho_{\text{tot}}}} \quad \Rightarrow \text{cooling rapidly} \quad (2)
\]

\[
m_B > m_J \equiv G^{-3/2} \rho_b^{-1/2} C^3 [1 + \left( \frac{\delta \rho_d}{\delta \rho_b} \right) \left( \frac{\bar{\rho}_d}{\bar{\rho}_b} \right)]^{-3/2}
\]

\[
\Rightarrow \text{gravitationally unstable} \quad (3)
\]

Here \((\rho_b, \rho_d)\) are baryonic and dark matter densities, \((m_b, m_J)\) baryonic mass in the cell and Jeans mass of the cell, \(C\) isothermal sound speed and other symbols have their usual meanings. If all of these criteria are satisfied, it seems impossible to prevent collapse of the gas towards the center of the cell with subsequent condensation into a stellar system. This, of course, we cannot follow with our code. Instead we adopt a model inspired by the classic work of Eggen, Lynden-Bell & Sandage (1962, ELS) and assume that the dynamical free-fall and galaxy formation timescales are simply related. We remove from the gas in the cell in question the mass that would collapse in \(\Delta t\) and create a collisionless particle:

\[
m_* = +m_b \Delta t / t_{\text{dyn}} \quad \text{and} \quad \Delta m_b = -m_b \Delta t / t_{\text{dyn}} \quad (4)
\]

at the center of the cell, giving it the same proper velocity as the gas in the cell. These collisionless particles are given three labels at birth: their mass, \(m_*\), the epoch of creation, \(z_*\), and \(t_{\text{dyn}}\), the free-fall time in the birth cell. After creation, these new particles are treated dynamically the same as dark matter particles. The three components (gas, galaxies and dark matter) interact through gravity. Feedback input into the IGM from young stars, through the processes such as UV and supernovae, is allowed for through adoption of some fairly conservative values (see below) of “efficiencies” utilized to parameterize these processes.
We adopt conventional CDM parameters $h = 0.5$, $\Omega = 1$, $\Omega_b = 0.06$, $\sigma_8 = 0.77$, so the amplitude is somewhat (35%) below the COBE normalization but within 2.5$\sigma$ of the COBE results. For the scales of interest the power spectrum adopted is close to that in the COBE normalized variants of CDM, namely the tilted, mixed, and low $\Omega$ models all of which are adjusted to give $\sigma_8 \sim 0.8$ to match observational constraint. The baryon density is taken from standard light element nucleosynthesis (Walker et al. 1991). Our box size is $80h^{-1}$Mpc, so, with $200^3$ cells and dark matter particles, our nominal resolution is $400h^{-1}$kpc, but actual resolution, as determined by extensive tests (Cen 1992), is approximately of a factor of 2.5 worse than this ($\sim 1h^{-1}$Mpc). In addition, we have made a higher resolution run with box size $8h^{-1}$Mpc, the same number of cells and nominal resolution $40h^{-1}$kpc. In the small box galaxy formation occurs earlier (because there are more nonlinear waves in this box at earlier times) and more vigorously. There is about 40% less galaxy formation in the $L = 80h^{-1}$Mpc than in the $L = 8h^{-1}$Mpc, partly due to the missing waves with wavelengths larger than $L = 8h^{-1}$Mpc which, if present as they should be, would have heated the temperature to a higher level to reduce galaxy formation. We ran the $L = 8h^{-1}$Mpc box first, then we added the UV/X-ray emissivities from sources in the smaller box to the larger box when the $L = 80h^{-1}$Mpc simulation was run. After the galaxy subunits are created, they release energy in two forms: UV from young stars and thermal energy from supernovae shocks (see Cen & Ostriker, 1993b, for details).

For comparison with the hydrodynamical simulations, we have run in parallel a conventionally biased CDM N-body simulation with also $\Omega = 1$, $h = 0.5$ and $b = 1.3$. The CDM evolves under the influence of gravity. Some CDM particles are tagged in the initial conditions (at $z=20$) as being peaks in the initial conditions above a certain threshold where the cell size of $0.4h^{-1}$Mpc /$(1 + z_{\text{initial}})$ is used with no smoothing. In order to achieve the desired bias of $b = 1.3$, the lower threshold for peaks to become galaxies is $\rho > 0.92\bar{\rho}$ where $\bar{\rho}$ is the mean density at $z = 20$. The biased particles are divided into groups. The top 25% (elliptical galaxies) are those peaks with $\rho > 1.735\bar{\rho}$, the next 25% (S0 galaxies) are those peaks with $1.735\bar{\rho} > \rho > 1.653\bar{\rho}$, and the bottom 50% (spiral galax-
ies) are those peaks with $1.653\bar{\rho} > \rho > 0.92\bar{\rho}$ where density $\rho$ and mean density $\bar{\rho}$ in the universe are both evaluated at $z = 20$. Now, for a perturbation that is bound, its radius $a_p(t)$ is given parametrically by (cf. Gunn & Gott 1972)

$$a_p(\eta) = \frac{1}{2}a_{\text{max}}(1 - \cos \eta)$$

$$t_p(\eta) = \frac{1}{2\pi}T_c(\eta - \sin \eta)$$

where $T_c$ is the collapse time and $a_{\text{max}}$ is the maximum radius, achieved when $t = T_c/2$. Meanwhile the mean expansion of the $\Omega = 1$ universe is given by

$$a(t) = \frac{a_{\text{max}}}{4}(\frac{12\pi t}{T_c})^{2/3}$$

which in the limit of early times agrees with that of the perturbation. Now

$$\frac{\rho_p}{\bar{\rho}} = \frac{a^3(t)}{a_p^3(t)} = \frac{9}{2} \frac{(\eta - \sin \eta)^2}{(1 - \cos \eta)^3}$$

Thus at maximum expansion at $t = T_c/2$, $\eta = \pi$, $a_p = a_{\text{max}}$, $\rho_p/\bar{\rho} = 5.55$ (cf. Gunn & Gott 1972), and $\rho_p/\bar{\rho} = 1.735$ at $t = 0.1458T_c$, and $\rho_p/\bar{\rho} = 1.653$ at $t = 0.1297T_c$, where $t = t_o(1/21)^{3/2} = 0.135$ billion years at $z = 20$ and $t_o = 13$ billion years is the current age of the universe in this $\Omega = 1$, $h = 0.5$ model. Thus, ellipticals are simply those galaxies with collapse times $T_c < 0.93$ billion years which form at $z_c > 4.8$ (i.e., those peaks with $\rho/\bar{\rho} > 1.735$ at $z = 20$). This is as in the Gott & Thuan (1976) proposal that ellipticals are just those galaxies that have the shortest collapse times. Then S0’s are those galaxies that have collapse times 0.93 billion years $< T_c < 1.04$ billion years which form at $4.38 < z_c < 4.8$ (i.e. those peaks with $1.653 < \rho/\bar{\rho} < 1.735$ at $z = 20$). Finally, spirals are those galaxies that have collapse times that are longer than 1.04 billion years and form at $z_c < 4.38$. By way of comparison, the (25%) ellipticals formed in the more realistic hydrodynamic simulation formed in the interval $7.5 > z_c > 3.6$, the (25%) S0’s formed in the interval $3.6 > z_c > 2.7$, and the (50%) spirals formed since $z_c = 2.7$. Thus the simple biased N-body model with the biased particles divided into quartiles provides
an ad hoc way of producing ellipticals, S0’s and spirals which might naturally produce a reasonable morphology-density relation for these galaxies today but without following the detailed physics supplied by the hydrodynamic simulations.

We will measure the topology of the large-scale structure outlined by elliptical, S0, and spiral galaxies, both as produced by the hydrodynamical simulations and by the simpler, more naive, biased N-body simulations. This will show whether the detailed physics of galaxy formation on caustics and then galaxies draining onto clusters by gravity which occurs in the hydrodynamic simulations leaves a trace on the topology which is noticeably different from that produced by a simple N-body biasing scheme in which galaxy formation is dictated by collapse-time arguments alone.

3. TOPOLOGICAL TECHNIQUES

Important clues as to the origin of large-scale structure lie in its topology. Gott, Melott, & Dickinson (1986), Hamilton, Gott, & Weinberg (1986), and Gott et al. (1989) have shown how topology can be measured using the genus statistic. First, the galaxy distribution is smoothed with a Gaussian window function

\[ W(r) = \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{r^2}{2\lambda^2}\right), \]

where smoothing length \( \lambda \) is chosen so that \( 2\lambda^2 = \lambda''^2 > l^2 \) where the maximum mean separation between galaxies in the sample is defined as \( l = n_{ex}^{-1/3} \) where \( n_{ex} \) is the minimum expected volume density of galaxies anywhere in the sample (usually at the outer edge) calculated from the selection function. For volume-limited samples with \( N \) galaxies brighter than some limiting absolute magnitude within a volume \( V, l = (V/N)^{1/3} \). Note: we are using a new definition of the smoothing length \( \lambda \) as defined in equation (9), our old smoothing length was \( \lambda''^2 = 2\lambda^2 \), so in previous papers such as Gott et al. 1989, when we have used \( \lambda' = 6h^{-1}\text{Mpc} \), this would correspond to \( \lambda = 4.24h^{-1}\text{Mpc} \) with the new definition. Since in most magnitude- or volume-limited samples \( l = 5h^{-1}\text{Mpc} \), we expect to use smoothing lengths \( \lambda > 3.5h^{-1}\text{Mpc} \). We use smoothing because we wish to study large-scale structure and are not interested in the fact that the galaxies are isolated points.
Once the smoothing has been done, density-contour surfaces in the smoothed density field are computed and are identified by a variable $\nu$ which is a measure of the volume fraction $f_\nu$ contained on the low-density side of the density contour surface:

$$f_\nu = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\nu} \exp(-t^2/2)dt.$$  \hspace{1cm} (10)

Thus $\nu = 0$ is the median density contour which contains 50\% of the volume. [This definition is such that if the density field were gaussian random phase then $\rho_{\text{surface}} - \bar{\rho} = \nu(\delta \rho)_{\text{rms}}$.] The ($\nu = -2$, $\nu = -1$, $\nu = 0$, $\nu = 1$ and $\nu = 2$) surfaces contain respectively (2.5\%, 16\%, 50\%, 84\%, and 97.5\%) of the volume on their low density sides. Alternatively, we may define the contours by the fraction of mass $f_M$ of the smoothed-galaxy mass distribution contained on their low-density sides.

The topology of each density-contour surface is measured by the genus statistic.

$$G = \text{Number of “donut” holes} - \text{Number of isolated regions}.$$  \hspace{1cm} (11)

By this definition, a sphere with $N$ handles would have a genus of $G = N - 1$ because it is one isolated region and has $N$ holes. Also by this same definition, 3 spheres have a genus of $G = -3$ because they have no holes and are 3 isolated regions. As shown by Gott, Melott, & Dickinson (1986) using the Gauss-Bonnet Theorem

$$G = -\frac{1}{4\pi} \int K dA,$$  \hspace{1cm} (12)

where $K = 1/r_1 r_2$ is the Gaussian curvature, and $r_1$ and $r_2$ are the two principle radii of curvature, and the integral is carried out over the entire surface area of the contour surface. Thus 1 sphere has a genus $G = -1$ because $K = 1/r^2$, $\int dA = 4\pi r^2$, so by eq. 12 $G = -1$. The above formula allows us to develop a computer program which goes pixel by pixel in a smoothed density array and computes the genus for each density-contour surface (see Gott, Melott, & Dickinson 1986, and Weinberg 1988 for a copy of the program called CONTOUR).
What if we did such a topology study in the initial conditions in a standard inflationary model where the fluctuations are produced by quantum noise and are gaussian random phase? Then we expect that the average genus per unit volume $g(\nu)$ is given by

$$g(\nu) = A(1 - \nu^2) \exp(-\nu^2/2),$$

(13)

where

$$A = \frac{1}{4\pi^2} \left( \frac{1}{3} \int k^2 P'(k) d^3k \right)^{3/2},$$

(14)

and where $P'(k) = P(k) \exp(-k^2\lambda^2)$ is the smoothed power spectrum and $P(k)$ is the primordial power spectrum (Hamilton, Gott, & Weinberg 1986). We note that while the amplitude of the genus curve $A$ depends on the power spectrum [for $P(k) \propto k^n$, $A = ([3 + n]/6)^{3/2}/4\pi^2\lambda^3$] - with a smaller amplitude for models with more power at large scales - the form of the genus curve as a function of $\nu$ is independent of the power spectrum $g(\nu) \propto (1 - \nu^2) \exp(-\nu^2/2)$. It is positive for $-1 < \nu < 1$, indicating holes and a spongelike topology, it is negative for $\nu > 1$ indicating isolated clusters, and negative for $\nu < 1$ indicating isolated voids. Now, while the fluctuations are in the linear regime, they simply grow in place increasing in amplitude ($\delta\rho/\rho \propto a$) so that the topology does not change at all. If $r_o$ is the correlation length at the present epoch, then $\xi(r_o) = 1$, and if we choose $\sqrt{2}\lambda > r_o$, we can guarantee that the fluctuations we are looking at are still approximately in the linear regime. To lower statistical noise the curve is smoothed using boxcar averaging $g(\nu) = 1/3[g(\nu - 0.1) + g(\nu) + g(\nu + 0.1)]$ (cf. Vogeley et al. 1994).

Since making the prediction that the topology of large-scale structure should be spongelike and approximately random phase (Gott, Melott, & Dickinson 1986; Hamilton, Gott, & Weinberg 1986; Gott, Weinberg, & Melott 1987), this prediction has been confirmed in every observational sample studied: [CfA, Giovanelli & Haynes, Tully, Thuan & Schneider, and Abell cluster samples] Gott et al. (1989); [IRAS] Moore et al. (1992); [Abell clusters] Rhoads, Gott, & Postman (1994); [CfA 1+2] Vogeley et al. (1994). In all cases, $g(\nu = 0) > 0$, indicating a spongelike topology for the median density contour.
We examine the galaxies produced in the hydrodynamical simulations as they appear at the present epoch. This simulation has $\Omega_{CDM} = 1$, $h = 0.5$ and amplitude $\sigma_8 = 1/b = 0.77$ (i.e., $(\delta \rho/\rho)_{CDM} = 1/1.3$ for spheres of radius $8h^{-1}\text{Mpc}$). The galaxies that have formed and evolved in the hydrodynamical simulations are observed to have $(\delta \rho/\rho)_{gal} = 1$ for spheres of radius $8h^{-1}\text{Mpc}$, giving a natural bias factor of 1.3. We regard this particular model, not as necessarily the best or only one, but following the discussion in the introduction, to simply be representative of the physics obtained in the broad class of standard inflationary CDM models (with $\Omega$ not necessarily 1) in which the structure originates from random quantum noise in the early universe. Starting with a magnitude-limited survey with an optimally chosen outer boundary, one can typically produce a volume-limited survey with a mean separation of bright galaxies of $l = 5h^{-1}\text{Mpc}$. Thus our computational volume is simulating a volume-limited observational sample of 4096 galaxies (which could be drawn from a magnitude-limited survey of 16,000 galaxies) and is just somewhat larger in volume than the CfA 1+2 sample. We divide this sample, as noted, into quartiles by epoch of formation: the earliest forming 25% we will call ellipticals, the second 25%, S0’s, and the final 50%, spirals. Figure 1 shows the distribution of these subsamples of galaxies in a slice of size $80 \times 80 \times 15h^{-3}\text{Mpc}^3$. Now for each quartile the mean separation between galaxies within that quartile is $l = (4)^{1/3}5h^{-1}\text{Mpc} = 7.9h^{-1}\text{Mpc}$. To avoid discreteness effects we therefore should adopt $\sqrt{2}\lambda > l$, $\lambda > 5.6h^{-1}\text{Mpc}$. To be on the cautious side we will adopt $\lambda = 8h^{-1}\text{Mpc}$. This is large enough that we may ignore peculiar velocities and, since $\lambda > r_o$, we are also assured that we are looking at scales where the fluctuations are still approximately in the linear regime and we may expect the random-phase topology formula to approximately hold true except for small deviations caused by non-linear effects (which are precisely those we are investigating).

4. RESULTS

Figure 2 (solid line) shows the genus curve for the CDM in the sample smoothed at $\lambda = 8h^{-1}\text{Mpc} = 800\text{km/s}^{-1}$. It is approximately random-phase - the best fitting theoretical random phase genus curve $[g(\nu) \propto (1 - \nu^2) \exp(-\nu^2/2)]$ is shown for comparison (dashed
Figure 3 shows the genus curve (and best fit theoretical random phase curve) for all galaxies in the hydrodynamic simulations. In these genus curves and all that follow, the total genus for the \((80h^{-1}\text{Mpc})^3\) volume is recorded. The error bars are computed by dividing the cube into 8 subcubes, each of \((40h^{-1}\text{Mpc})^3\) volume, and computing the standard deviation of the mean genus per unit volume for the entire cube from these 8 independent estimates. The smoothing length is \(\lambda = 8h^{-1}\text{Mpc} = 800\text{km}\text{s}^{-1}\) in all cases. Both the CDM and the galaxies approximately follow the random-phase curve. It is significant that the galaxies still approximate the random-phase curve, for it shows that the non-linear hydrodynamic effects in the simulations do not wipe out the influence of the random phase initial conditions. This means that we can still use the genus curve as a test of whether or not the initial conditions were random phase. If larger smoothing lengths were used, the agreement with the theoretical curve would be better and better, provided that large enough survey volumes were used.

Figures 4a,b,c show the genus curves of each group of galaxies classified by epoch of formation: 4a, oldest 25% = ellipticals; 4b, second 25% = S0’s; and 4c, last 50% = spirals. The spirals are approximately random phase in reasonable agreement with the expected spongelike topology.

The ellipticals show a significant systematic shift toward the left called a "meatball shift" (cf. Weinberg, Gott & Melott 1987; Gott et al. 1989). If galaxies are placed down in a meatball distribution these isolated clusters will retain their identity (and provide negative genus contour surfaces, indicating isolated high-density regions) down to a lower threshold value of \(\nu\) than would otherwise be the case. In a random-phase distribution with \(g(\nu) \propto (1 - \nu^2) \exp(-\nu^2/2)\) we expect to find \(g(\nu) < 0\) indicating isolated clusters for \(\nu > 1\). For a meatball topology we expect \(g(\nu) < 0\) for \(\nu > \nu_+\) where \(\nu_+ < 1\) (cf. Weinberg, Gott, & Melott 1987). This has the effect of pushing the entire genus curve to the left, in particular, moving the peak of the genus-curve to \(\nu < 0\) and compressing the region of negative genus isolated voids \(g(\nu) < 0\), for \(\nu < \nu_-\) where \(\nu_- < -1\).
shows that the older galaxies designated as “ellipticals” in the hydrodynamic simulations are preferentially located in clusters relative to the younger galaxies designated as “spirals” which have a more nearly random-phase spongelike topology characteristic of a filamentary net. The “S0’s” are intermediate.

We can quantify this shift by use of the shift statistic $\Delta \nu$ (cf. Vogeley et al, 1994) tabulated in Table 1.

$$
\Delta \nu = \int_{\nu=-1}^{\nu=+1} \nu g(\nu) d\nu / \int_{\nu=-1}^{\nu=+1} g_{fit}(\nu) d\nu .
$$

(15)

This gives the mean shift in the genus curve. A negative value of $\Delta \nu$ indicates a shift to the left or a meatball shift, a positive value of $\Delta \nu$ indicates a shift to right or a “swiss cheese” shift (i.e., toward a swiss cheese topology of isolated voids - see Gott, Weinberg, & Melott 1987). For ellipticals $\Delta \nu = -0.150$, while for spirals $\Delta \nu = -0.0864$, showing that the ellipticals have a larger meatball shift than the spirals, apparent from a visual inspection of the genus curves in fig. 4a and 4c.

To further illustrate, in fig. 5a and b we show the $\nu = -1$ density contour surfaces for the ellipticals and the spirals. This contour contains 16% of the volume on its low-density side and in a random-phase distribution would mark the transition from isolated voids to a spongelike topology. Because of the meatball shift of the genus curve of the ellipticals relative to the spirals, this particular contour, for the ellipticals, is more multiply connected (has more “donut” holes) than the surface for spiral galaxies. This is shown by the tube connecting the two voids at the bottom and on the right of the elliptical cube (which adds +1 to the genus) which is absent in the spiral cube, where those two voids are separate.

Other statistics in table 1 include the best fit amplitude of the random phase curve as a fraction of the amplitude expected from the initial power spectrum (i.e., the amplitude that would have been measured for the initial conditions themselves). A value of $R_G < 1$ indicates there has been some non-linear merging of structures, a little of which is to be expected (Melott, Weinberg, & Gott 1988). Finally, there is the width of the genus curve $W_\nu = \nu_2 - \nu_1$ which measures the width of the region that has positive genus or spongelike
topology. For a random-phase distribution \( g(\nu) > 0 \) for \(-1 < \nu < 1\) so \( W_\nu = 2.0 \). If \( W_\nu > 2.0 \), as is true for these distributions, it indicates a filamentary net which retains its spongelike topology over a larger range of \( \nu \) than would be true in a random phase distribution. The CfA 1 + 2 sample for all galaxies also shows a \( W_\nu \) value somewhat larger than 2 (Vogeley et al. 1994).

Another way to illustrate the difference in the distribution between ellipticals and spirals in the hydrodynamical simulations is to note that each density-contour surface can be labeled either by the volume fraction \( f_V \) contained on its low-density side or by the mass fraction \( f_M \) contained on its low-density side. Each density contour surface thus appears as a point on the \( f_V, f_M \) plane and the entire family of density contours for (ellipticals, say, or spirals) can be plotted as a curve in the \( f_V, f_M \) plane (see fig. 6). In the initial conditions \( f_V \approx f_M \), so departures from this line are entirely due to non-linear effects. The curves for dark matter, all galaxies, ellipticals, and spirals are shown. The curve for ellipticals deviates more from the \( f_V = f_M \) line than any of the others, showing them to be more strongly clumped. This is in line with the morphology-density relation shown by the hydrodynamic simulation which is in agreement with that observed.

The difference in topology in the hydrodynamical simulations between the ellipticals and the spirals can be displayed most graphically by plotting genus versus \( f_M \) for each as is done in figures 7a and b. The ellipticals show a positive genus (spongelike topology) for \( 0.03 < f_M < 0.58 \); the genus is negative for \( f_M > 0.58 \). This means that 42\% of the smoothed mass distribution in ellipticals is contained in isolated clusters. For spirals, the genus is positive for \( 0.06 < f_M < 0.70 \); the genus is negative for \( f_M > 0.70 \). This means that only 30\% of the smoothed mass distribution in spirals is contained in isolated clusters. This quantifies the propensity for the ellipticals to be found in clusters.

The genus curves for ellipticals and spirals in the biased N-body simulations are shown in figures 8a and c. There is not any dramatic meatball shift of the ellipticals relative to spirals. This is quantified in Table 1: \( \Delta \nu = -0.0455 \) for ellipticals and \( \Delta \nu = -0.0671 \) for spirals. By comparison, in the hydrodynamic simulations, \( \Delta \nu = -0.150 \) for ellipticals and
\[ \Delta \nu = -0.0864 \] for spirals, giving a much larger differential difference.

Genus versus \( f_M \) curves drawn from the biased N-body simulation (shown in figs. 9a
and b) are more nearly similar to each other and do not show such a large difference as seen
in the hydrodynamic simulations. For these ellipticals, the genus is negative for \( f_M > 0.68 \),
and for these spirals, the genus is negative for \( f_M > 0.71 \). Thus 32\% of the smoothed
density distribution of ellipticals is in isolated clusters versus 29\% for spirals - a difference
of only 3\% as opposed to the 12\% difference predicted by the hydrodynamic simulations.
Thus we could test future observational samples for this effect by plotting the genus as a
function of \( f_M \) for both ellipticals and spirals, and then comparing them.

Topology can of course be studied as a function of smoothing length \( \lambda \) (or \( \lambda' =
(2)^{1/2}\lambda \)). For \( \lambda > 800\text{km}s^{-1} \) the elliptical and spiral curves should both become more
nearly random phase and therefore more nearly alike. As studies we have done by plotting
\( f_M \) versus \( f_v \) for various smoothing lengths indicate, for \( \lambda < 800\text{km}s^{-1} \) the elliptical and
spiral curves become more and more differentiated. The smallest smoothing length one
would want to use for a topology study by galaxy type would be \( \lambda' = l_e = (4)^{1/3}5h^{-1}\text{Mpc} \),
or \( \lambda = 560\text{km}s^{-1} \), where \( l_e \) is the minimum mean distance between ellipticals in a typical
volume or magnitude limited sample. Smaller than this we would begin to encounter
discreteness effects. At a smoothing length of \( \lambda = 560\text{km}s^{-1} \), some care would also have
to be taken to ensure that the simulation had a peculiar velocity dispersion for ellipticals
that was in agreement with the observed sample, since the line of sight rms peculiar
velocity dispersion for ellipticals in the simulations (or observations) might be as large as
560km\(^{-1} \). Also, operating at the edge where discreteness effects might become important
at \( \lambda = 560\text{km}s^{-1} \) would require more accurate simulation of the luminosity function.

As a further check against discreteness effects, we could divide the spirals into 2 halves
and check that the ellipticals show the same meatball shift relative to each half of the
spirals and that there is no difference between the average of the genus curves for the
spiral halves, and that for all the spirals taken together. (This is certainly the case at
\( \lambda = 800\text{km}s^{-1} \).) Against these cautions would be the benefits of having a larger mean
difference expected between the elliptical and spiral curves and smaller error bars. Since the errors are primarily counting errors and since for a power spectrum $P(k) \sim k^{-1}$ (for CDM at these scales) we expect $A \propto V/\lambda^3$, and so $\delta g/g \propto V^{1/2}\lambda^{-3/2}$. So going from $\lambda = 800\text{km}s^{-1}$ to $\lambda = 560\text{km}s^{-1}$ could lower the relative statistical errors by a factor of 1.7. Figures (10a,b) show the equivalent of Figures (7a,b) but with $\lambda = 560\text{km}s^{-1}$.

As expected, the elliptical and spiral genus curves are even more differentiated from each other. At this smoothing length 52% of the smoothed mass distribution in the ellipticals is in isolated clusters whereas only 37% of the smoothed mass distribution in the spirals is in isolated clusters (a difference of 15%). Alternatively, as we shall discuss below, the simplest way to lower the relative statistical errors is simply to study a larger survey volume - which should be possible in the near future.

5. CONCLUSIONS

Topology can provide many important clues as to the formation of large-scale structure in the universe. Hydrodynamic simulations with CDM and baryons, following known physics, give rise to structures quite promisingly like those seen in the observations. Starting with Gaussian random-phase initial conditions, fluctuations initially grow entirely by gravitational instability, caustics and shocks form when the fluctuations go non-linear. Galaxies are formed in these shocks and so are initially located along walls and filaments. This picture developed in Cen & Ostriker (1993a) and supported by the present work is the following. The first-forming galaxies are identified with ellipticals. Later, S0’s and finally spirals form. This designation in the simulation produces samples which qualitatively reproduce the properties of real ellipticals, S0’s and spirals with regard to the density-morphology relation, gas-to-total mass ratio, dark matter-to-stellar ratio and other properties. After formation, galaxies, under the action of gravity, drain off walls onto filaments and then into clusters. Ellipticals, which get a head start on this process, are therefore seen at the present epoch in the simulations preferentially in clusters relative to spirals which are still primarily in a filamentary net. The topology can be measured quantitatively with the genus statistic. Random-phase initial conditions produce a characteristic spongelike
topology if the density field is smoothed on a scale larger than the mean intergalactic distance and larger than the correlation length. That is because in the gravitational instability picture, fluctuations grow in place (increasing in amplitude) as long as they are still in the linear regime and they retain the random-phase topology they inherited from the initial conditions. If genus is measured as a function of density, we expect to find isolated voids at very low density, isolated clusters at very high density, and we expect the median density-contour surface to be spongelike. This produces a symmetric genus curve \( g(\nu) \propto (1 - \nu^2) \exp(-\nu^2/2) \). The hydrodynamic simulations show for all galaxies a genus curve that approximately follows this random phase law if a smoothing length of \( \lambda = 8h^{-1}\text{Mpc} \) is adopted. This shows that the hydrodynamic effects do not mask the topology inherited from the initial conditions and that measuring the genus curve can be used as a test for whether the initial conditions were random phase or not. This is quite important since inflationary models in which structure arises from random quantum functions in the early universe do have random-phase initial conditions whereas certain other models (e.g., textures, cosmic strings, domain walls) do not. The random-phase nature of the initial fluctuations can be checked independently by using a 2D genus statistic (hot spots - cold spots) (Gott et al. 1990) on the fluctuations seen in the cosmic microwave background. Three groups (cf. Torres 1994; Smoot et al 1994; Park & Gott 1994) have independently shown that the year-1 COBE microwave background sky maps give an excellent fit to the theoretical random-phase curve (Melott et al. 1989) predicted for this statistic \( g_{2D}(\nu) = \nu \exp(-\nu^2/2) \).

As expected, however, the simulations do show some small but measurable deviations from the random-phase theoretical genus curve due to non-linear effects. The genus curve for “ellipticals” is shifted to the left \( (\Delta \nu = -0.150) \) more than that for the “spirals” \( (\Delta \nu = -0.086) \). This shows the greater propensity for the ellipticals to be found in clusters. The difference between the topology of the ellipticals and the spirals shows up most dramatically when we measure the genus of contour surfaces as a function of the mass fraction in the smoothed distribution. This treatment shows that 42% of the smoothed
mass distribution of ellipticals is in isolated clusters as opposed to only 30% of spirals. These topological properties are signs of the physical origin of the “bias” found in the hydrodynamic simulations. Effects this large do not show up in standard biased N-body simulations based on a peaks approach.

Studies of the topology of large-scale structure by galaxy type in future observational samples should easily be able to test for this effect. If, as expected, real ellipticals show a meatball shift similar to the “elliptical” subset in the simulation, and dissimilar to the “elliptical” set picked out by standard peak biasing technique, this would provide evidence for the physical plausibility of the hydrodynamic simulations and for the designation of ellipticals as a statistically old population of galaxies. The current CfA 1+2 sample is almost as large in volume as these simulations and the Sloan Digital Sky Survey (SDSS) will be 62 times larger in volume. Automated typing algorithms will be implemented to type the galaxies in the SDSS from the imaging phase of the survey. This should allow us to implement topology studies by galaxy type as described in this paper on a grand scale. Likewise, over the next few years computer hydrodynamic simulations are likely to also improve by a large factor from $(200)^3$ grids to $\sim (1600)^3$ grids allowing more resolution $(200h^{-1}\text{kpc})$ while giving survey volumes 64 times larger than at present. New effects may also be added such as ram pressure stripping of spirals falling into clusters (cf. Gunn & Gott 1972), converting them to S0’s.

Currently observational samples show approximately random-phase genus curves (Gott et al. 1989; Moore et al. 1992; Vogeley et al. 1994). This supports the claim that the structures we see today originated from small-amplitude Gaussian random initial conditions. Although all observational samples are approximately random phase (all have a spongelike median-density-contour surface, for example), some small differences can be noted. In general, samples including all galaxies are either random phase or show a small shift in the direction of a meatball topology ($\Delta \nu < 0$) like that seen in the ellipticals and spirals in the hydrodynamic simulations. The largest sample, the CfA 1+2, has a genus curve with a width $W_\nu$ - somewhat wider than that for a random-phase distribution, an
effect also seen in these hydrodynamic simulations. Since the elliptical curve is shifted relative to the spiral curve, when both populations are combined, an overall wider genus curve should be produced. The amplitude of the genus curve for the CfA 1+2 is best fit by a CDM model with $\Omega h = 0.3$. Such models, with more power at large scales than the hydrodynamic $\Omega h = 0.5$ CDM model, are expected to show for all galaxies a genus curve that is less meatball shifted overall (cf. Vogeley et al. 1994). We would expect the same difference between ellipticals and spirals, but for an $\Omega h = 0.3$ model both populations would show somewhat less propensity to be found in isolated clusters. So if the galaxies were typed, we might expect that, with $\lambda = 8 h^{-1}\text{Mpc}$ smoothing, somewhat less than 42% of the smoothed mass fraction in ellipticals would be in isolated clusters and somewhat less than 30% of the smoothed mass fraction in spirals would be in isolated clusters, but an overall difference of 12% in the fraction of each population in isolated clusters might still be expected. In the future, comparison of newer, larger, even more realistic hydrodynamic simulations with much larger observational samples using these topological techniques – each surveying a volume $\sim 64$ times as large as that depicted here (and with error bars on all graphs 8 times as small relative to the curves) offers exciting possibilities for testing and illuminating the details of the galaxy formation process.

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REFERENCES

Bahcall, N.A., & Cen, R. 1992, ApJ, 398, L81

Bardeen, J.M., Bond, J.R., Kaiser, N., & Szalay, A.S. 1986, ApJ, 304, 15.

Bardeen, J.M., Steinhardt, P.J., & Turner, M.S. 1983, Phys. Rev. D., 28, 679.

Bartlett, J.G., Blanchard, A., Silk, J., & Turner, M. 1995, Science, 267, 980.

Bhavsar, S.P. 1981, ApJ, 246, L5.

Broadhurst, T.J., Ellis, R.S., Koo, D.C., & Szalay, A.S. 1990, Nature, 343, 726.

Bucher, M., Goldhaber, A.S., Turok, N., submitted to Physical Rev. D.

Cen, R. 1992, ApJS, 78, 341.

Cen, R., & Ostriker, J.P. 1992a, ApJ 393, 22.

Cen, R., & Ostriker, J.P. 1992b, ApJ, 399, L113.

Cen, R., & Ostriker, J.P. 1993a, ApJ, 417, 415.

Cen, R., & Ostriker, J.P. 1993b, ApJ, 417, 404.

Cen, R., & Ostriker, J.P. 1994, ApJ, 431, 451.

Cen, R., Gnedin, N., & Ostriker, J.P. 1993 ApJ, 417, 387.

Cen, R., Miralda-Escude, J., Ostriker, J.P, & Rauch, M. 1994, ApJ, 437, L2.

Cen, R., Kang, H., Ostriker, J.P, & Ryu, D. 1995, ApJ, submitted

Davis, M., & Geller, M.J. 1976, ApJ, 208, 13.

de Lapparent, V., Geller, M.J., & Huchra, J.P. 1986, ApJ, 302, L1.

Doroshkevich, A.G., Zeldovich, Ya.B., & Sunyaev, R.A. 1976, Formation and Evolution of
   Galaxies and Stars, Moscow.

Dressler, A. 1980 , ApJ, 236, 351.
Eggen, O.J., Lynden-Bell, D., & Sandage A.R. 1962, ApJ, 136, 748.

Efstathiou, G., Bond, J.R., & White, S.D.M. 1992, MNRAS, 258, 1p

Einasto, J., Joeveer, M., & Saar, E. 1980, MNRAS, 193, 353.

Freedman, W.L. et al. 1994, Nature, 371, 757.

Geller, M.J., & Huchra, J.P. 1989, Science, 246, 897.

Gott, J.R. 1977, Ann. Rev. As. Astrophysics, 15, 235.

Gott, J.R. 1980 Physical Cosmology, Les Houches, eds. J. Audouze, R. Balain, & D.N. Schramm (North Holland Publishing Company, Amsterdam) 561.

Gott, J.R. 1982 Nature, 295, 304.

Gott, J.R., & Gunn, J.E. 1971ApJ, 169, L13.

Gott, J.R., Turner, E.L., & Aarseth, S.J. 1979 ApJ, 234, 13.

Gott, J.R., Weinberg, D.H., & Melott, A. 1987 ApJ, 319, 1.

Gott, J.R., & Statler, T.S. 1984 Physics Letters, 136B, 157.

Gott, J.R., Miller, J., Thuan, T.X., Schneider, D.E., Weinberg, D.H., Gammie, C., Polk, K., Vogeley, M., Jeffrey, S., Bhavsar, S., Melott, A.L., Giovanelli, R., Haynes, M.P., Tully, R.B., & Hamilton, A.J.S. 1989 ApJ, 340, 625.

Gott, J.R., Park, C., Juskiewicz, R., Bies, W.E., Bennett, D.P., Bouchet, F.R., & Stebbins, A. 1990, ApJ, 352, 1.

Gott, J.R. 1986 Inner Space/Outer Space, The Interface Between Cosmology and Particle Physics, eds. E.W. Kolb, M.S. Turner, D. Lindley, K. Olive, & D. Seckel (Chicago and London, University of Chicago Press), 362.

Gott, J.R., Melott, A., & Dickinson, M. 1986, ApJ, 306, 341.

Gott, J.R., & Thuan, T.X. 1976, ApJ, 204, 649.
Groth, E.J., & Peebles, P.J.E. 1977, ApJ, 217, 385.

Gunn, J.E., & Gott, J.R 1972, ApJ, 176, 1.

Hamilton, A.J.S., Gott, J.R., & Weinberg, D. 1986, ApJ, 309, 1.

Jameson, A. 1989, Science, 245, 361.

Kamionkowski, M. & Spergel, D. 1994, ApJ, 432, 1.

Kang, H., Ostriker, J.P., Cen, R., Ryu, D., Hernquist, L., Evard, A.E., Bryan, G.L., & Norman, M.L. 1994, ApJ, 430, 83.

Klypin, A. Holtzman, J., Primack, J., & Regos, E. 1993, ApJ, 416, 1.

Kofman, L., Gnedin, N.Y., & Bahcall, N.A. 1993, ApJ, 413, 1.

Ma, C.P., & Bertschinger, E. 1994, ApJ, 435, L5

Maddox, S.J., Efstathiou, G., Sutherland, W.J., & Loveday, J. 1990, MNRAS, 242, 43p.

Mather, J.C. et al. 1990, ApJ, 354, L37.

Melott, A.L., Weinberg, D.H., & Gott, J.R. 1988, ApJ, 328, 50.

Melott, A.L., Cohen, A.P., Hamilton, A.J.S., Gott, J.R., & Weinberg, D.H. 1989, ApJ, 345, 618.

Moore, B., Frenk, C.S., Weinberg, D.H., Saunders, W., Lawrence, A., Ellis, R.S., Kaiser, N., Efstathiou, G., & Rowan-Robinson, M. 1992, MNRAS, 256, 477.

Ostriker, J.P. 1993, ARAA, 31, 689

Park, C. 1990, MNRAS, 242, 59P.

Park, C., & Gott, J.R 1991, MNRAS, 249, 288.

Park, C., Gott, J.R., Melott, A., & Karachentsev, I.D. 1992a, ApJ, 387, 1.

Park, C., Gott, J.R., da Costa, L.N. 1992b, ApJ, 392, L51.

Park, C., & Gott, J.R. (unpublished).
Peebles, P.J.E. 1980, The Large-Scale Structure of the Universe, Princeton University Press, Princeton, NJ.

Peebles, P.J.E. 1982, ApJ, 263, L1.

Peebles, P.J.E. 1984, ApJ, 284, 439.

Pierce, M.J., Welch, D.L., McClure, R.D., van den Bergh, S., Racine, R., & Stetson, P.B. 1994, Nature, 371, 385.

Ratra, B., & Peebles, P.J.E. 1994, ApJ, 432, L5.

Rees, M.J., & Ostriker 1977, MNRAS, 179, 451.

Rhoads, J., Gott, J.R., & Postman, M. 1994, ApJ, 421, 1.

Sachs, R.K., & Wolfe, A.M. 1967, ApJ, 147, 73.

Saunders, W., Frenk, C., Rowan-Robinson, M., Efstathiou, G., Lawrence, A., Kaiser, N., Ellis, R., Crawford, J., Xia, X-Y, & Parry, I. 1991, Nature, 349, 32.

Shectman, S.A., Landy, S.D., Oemler, A.A., Tucker, D., Kirshner, R.P., Lin, H., & Schechter, P.L. 1995, Harvard-Smithsonian Center for Astrophysics Preprint, to appear in the 35th Herstmonceaux Conference, Wide Field Spectroscopy and the Distant Universe.

Smoot, G.F. et al. 1992, ApJ, 396, L1.

Smoot, G.F., Tenorio, L., Banday, A.J., Kogut, A., Wright, E.L., Hinshaw, G., Bennett, C.L. 1994, ApJ, 437, 1.

Taylor, A.N., & Rowan-Robinson, M. 1992, Nature, 359, 396.

Torres, S. 1994, ApJ, 423, L9.

Vogeley, M.S., Park, C., Geller, M.J., Huchra, J.P., & Gott, J.R. 1994, ApJ, 420, 525.

Walker, T.P., Steigman, G., Schramm, D.N., Olive, K.A., & Kang, H.S. 1991, ApJ, 376, 51.
Weinberg, D. 1990, PhD Thesis, Princeton University.

Weinberg, D.H., & Gunn, J.E. 1990, ApJ, 352, L25.

Weinberg, D.H. 1988, P.A.S.P., 100, 1373.

Weinberg, D.H., Gott, J.R., & Melott, A.L. 1987, ApJ, 321, 2.

White, S.D.M., Frenk, C.S., Davis, M., & Efstathiou, G. 1987, ApJ, 313, 505.

Yamamoto, K., Sasaki, M., & Tanaka, T. 1995, preprint.
### Table 1. Genus Curve Statistics

|                      | Amplitude | Width | Shift |
|----------------------|-----------|-------|-------|
|                      | $R_G$     | $W_\nu$ | $\Delta_\nu$ |
| I. Hydrodynamic Simulation |           |       |       |
| A. Cold Dark Matter  | 0.844     | 2.06  | -0.0529 |
| B. All Galaxies      | 0.877     | 2.03  | -0.119  |
| C. “Ellipticals” (oldest 25%) | 1.00    | 2.03  | -0.150  |
| D. “Spirals” (youngest 50%) | 0.861 | 1.978 | -0.0864 |
| II. Biased N-Body Simulation |          |       |       |
| All Initial Bias Particles (IBP) | 1.189 | 2.27  | -0.0482 |
| 25% Highest IBP = “Ellipticals” | 1.126 | 2.25  | -0.0455 |
| 50% Lowest IBP = “Spirals” | 1.193 | 2.26  | -0.0671 |
FIGURE CAPTION

Fig. 1– A slice through the hydrodynamic simulations with co-moving dimensions $80h^{-1}\text{Mpc} \times 80h^{-1}\text{Mpc} \times 15h^{-1}\text{Mpc}$. (a): sample of 1/2 of the spirals (youngest 50%) at $z=0$, (b): S0’s (intermediate 25%) at $z=0$, (c): ellipticals (oldest 25%) at $z=0$, (d): ellipticals at $z=3.6$. At the present epoch, spirals are seen on a network of walls and filaments, while ellipticals congregate more in clusters. The picture at $z=3.6$ shows that these ellipticals originally formed on a network of walls and filaments just like the spirals and then later drained gravitationally onto the clusters. The S0’s form an intermediate population.

Fig. 2– Genus curve for the dark matter ($\text{CDM} \ \Omega = 1, \ h = 0.5$) in the hydrodynamical simulations at the present epoch. In this diagram and in those that follow, the total genus for the simulation cube $(80h^{-1}\text{Mpc})^3$ is shown, which is equal to the number of “donut” holes minus the number of isolated regions in the smoothed density-contour surfaces. The volume fraction $f_V$ on the low-density side of the contour surface is indicated by the parameter $\nu$: $f_V = (2.5\%, 16\%, 50\%, 84\%, 97.5\%)$ for $\nu = (-2, -1, 0, 1, 2)$ respectively. The data is shown as a solid curve. The 1 $\sigma$ error bars shown are calculated using the standard deviation of the mean genus for the entire simulation cube derived from the genus values from the 8 independent subcubes of $(40h^{-1}\text{Mpc})^3$ which together make up the entire simulation cube of $(80h^{-1}\text{Mpc})^3$. The best fit random-phase curve $g(\nu) \propto (1 - \nu^2) \exp(-\nu^2/2)$ is shown as a dashed line. The overall genus curve for the dark matter is approximately random phase, reflecting the topology of the initial conditions. The median density contour is sponge-like [$g(\nu = 0) > 0$]. The smoothing length is $\lambda = 8h^{-1}\text{Mpc} = 800\text{km}s^{-1}$.

Fig. 3– The genus curve for all galaxies in the hydrodynamic simulations at the present epoch, plotted as in fig. 2. Again the genus curve is consistent
with the predicted random-phase curve inherited from the initial conditions. There is a slight shift to the left (toward a meatball topology) characteristic of that seen in many biased CDM models (cf. Gott et al., 1989).

Fig. 4– Figure 4a shows the genus curve for ellipticals (the first 25% of galaxies formed) from the hydrodynamic simulations at the present epoch. Figure 4b shows the genus curve for S0 galaxies (the second 25% of galaxies formed) from the hydrodynamic simulations at the present epoch. This population is intermediate between ellipticals and spirals. Similarly, Figure 4c shows the genus curve for spirals (the 50% of galaxies most recently formed). The ellipticals (Fig. 4a) show a genus curve that is shifted to the left (toward the meatball topology) relative to that of the spirals (Fig. 4c).

Fig. 5– Figure 5a shows the $\nu = -1$ density-contour surface is shown for ellipticals in the simulation cube. This surface encloses the 16% of the volume which is lowest density. For comparison, Figure 5b shows the same $\nu = -1$ density contour surface is shown for the spirals. Because of the meatball shift to the left for the ellipticals, this particular contour has a more spongelike topology than for spirals. Fig. 4a,c shows that $g(-1) \approx 3$ for ellipticals while $g(-1) \approx 2$ for spirals. In the figure this difference of +1 in the elliptical contour surface is caused by the tube shown in 5a connecting the two voids at the bottom and on the right, which is absent in the spiral contour surface in 5b. One more connection (one more “donut” hole) means a difference in genus of +1. Figures (5c,d) show the $\nu = 0$ median density contour surfaces for ellipticals and spirals are shown in figures 5c and 5d respectively; both are spongelike.

Fig. 6– Density-contour surfaces can be labeled by either the fraction of the volume contained on their low-density side $f_V$ or by the mass fraction in
the smoothed mass distribution $f_M$ contained on their low-density side. This graph shows the relation between $f_V$ and $f_M$ for dark matter, all galaxies, ellipticals, and spirals.

Fig. 7— Figure 7a shows the genus curve for elliptical galaxies plotted as a function of $f_M$. Figure 7b shows that for spirals. The genus curve for ellipticals becomes negative (signifying isolated regions) for $f_M > 0.58$ while for spirals the genus becomes negative for $f_M > 0.70$. This means that 42% of the smoothed mass distribution of ellipticals is in isolated clusters, while only 30% of the smoothed mass distribution of spirals is in isolated clusters.

Fig. 8— Results at the present epoch from a standard biased N-body simulation for comparison. Figure 8a shows genus curve for the 25% highest biased particles drawn from the initial conditions, these will be the galaxies that form first - the ellipticals. Figure 8b shows genus curve for the 25% next highest biased particles drawn from the initial conditions, these will form next - the S0’s. Figure 8c shows genus curve for the 50% lowest biased particles drawn from the initial conditions, these will form last - the spirals. There is no systematic shift of the elliptical curve to the left relative to the spirals as was seen in the hydrodynamical simulations.

Fig. 9— Genus curves for ellipticals (9a) and spirals (9b), respectively, plotted as a function of $f_M$ for the biased N-body simulation (to compare with figs. 7a and b for the hydrodynamical simulations). In 9a, the genus becomes negative at $f_M = 0.68$, while in 9b the genus becomes negative at $f_M = 0.71$. Thus, the fraction of the smoothed mass distribution in ellipticals which is in isolated clusters (32%) differs from the fraction of the smoothed mass distribution in spirals which is in isolated clusters (29%) by only a small amount (3%).

Fig. 10— Figure 10a shows the genus curve for elliptical galaxies plotted as a func-
tion of $f_M$ with a smoothing length of 560kms$^{-1}$. Figure 10b shows that for spirals. As expected, the elliptical and spiral genus curves are even more differentiated from each other. At this smoothing length 52% of the smoothed mass distribution in the ellipticals is in isolated clusters whereas only 37% of the smoothed mass distribution in the spirals is in isolated clusters (a difference of 15%).