Non-perturbative Lee-Wick gauge theory:
BRST, confinement & RGE with strong couplings

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Abstract

We consider non-Abelian non-local Lee-Wick gauge theory and prove Becchi-Rouet-Stora-Tyutin (BRST) invariance. It contains fourth-order derivative as extensions of the kinetic term, leading to massive ghosts in the theory upon quantization. We particularly focus on the confinement conditions in strongly-coupled regimes, using the Kugo-Ojima approach, and obtain the exact $\beta$—functions in the non-perturbative regimes. This is achieved using a set of exact solutions of the corresponding local theory in terms of Jacobi elliptical functions. We obtain an identical $\beta$—function just as for the local Yang-Mills theory but the main difference is that now, the cut-off arises naturally from the Lee-Wick heavy mass scales ($M$). We show that the fate of the ghosts are fixed in these regimes: they are no more the propagating degrees of freedom in the infrared (IR)-limit. In the limit $M \rightarrow \infty$, one recovers the standard results for the local non-Abelian Yang-Mills theory.

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I. INTRODUCTION

Despite the discovery of the 125 GeV Higgs boson at the Large Hadron Collider (LHC) in 2012 completing the missing block of the Standard Model (SM) puzzle it also immediately received lots of motivations for extending beyond the minimal SM, one of the most prolific of them being the question of the hierarchy problem: why extreme fine-tuning is needed to keep the Higgs mass small compared to the Planck scale (at $10^{19}$ GeV). Solutions to this problem often involving protections against dangerous radiative corrections from the cut-off or the Planck Scale via SUSY, or by postulating extra dimensions or by making the Higgs scalar as composite particle. The Lee–Wick approach as described in Refs. 1, 2 with higher-order derivatives augments the SM via TeV-scale Lee–Wick partner particles with negative-sign kinetic and mass terms. This evidently leads to a cancellation of quadratic divergences in scalar self-energies, with the predictions of the precise particle spectrum of Lee–Wick resonances at the laboratory experiments to determine. The theory is unitary, 3 (see also 4), are causal at the macroscopic level 5, have subsequently been generalized to include additional partner particles 6, 7, and have received much attention in the BSM particle physics phenomenology literature as well 8. Although no Lee–Wick partner particles have been found hitherto, thereby putting a lower bound from $\sqrt{s} \sim O(10)$ TeV 9, on the Lee-Wick partner masses, thereby pushing them to higher energies.

The last decades have also seen the development of non-local field theories, in general, in particular ghost-free ones 12, 24, 26, 45, 127, 27, 29 and in context to to p-adic string theory and non-commutative geometry 30, 33, 1 and recently in this context higher-derivative approaches to a UV-completion of QFT have become popular 10, 15, and particularly, the infinite higher-derivative approach was motivated starting from string field theory 16, 23, 32, 33, 44, 48 where attempts were made to address the divergence problem by generalizing the kinetic energy operators of the Standard Model (SM) to an infinite series of higher order derivatives suppressed by the scale of non-locality ($M$) at which the higher order derivatives come into the picture 49, 50, also to readily cure the vacuum instability problem in the SM 51; theory is ghost-free 52, predicts conformal invariance in the ultra-violate (UV), trans-

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1 For cosmology of these theories, see Ref. 34, nonlocality in string theory 35, 44, regularization of the gravitational field via nonlocality 36, 38 on one hand, and on the other hand, the role of the Wick rotation vis-à-vis unitarity and causality 39, 43, 108.
planckian scale transmutation and dark matter phenomenology \cite{53, 54} and free of Landau poles thus a candidate theory for UV-completion of 4D QFT, valid and perturbative up to infinite energy scales \cite{51, 53}. Strong coupling regimes of the theory were studied in Refs. \cite{56, 57, 147}, where it was shown that the mass gap obtained gets diluted in the UV due to non-local effects restoring conformal invariance in the UV. Interestingly, it was shown that Lee Wick theory (higgs, abelian and non-Abelian gauge theories) with $N$ propagator poles, having $(N - 1)$ Lee-Wick partners can be understood as flowing to infinite-derivatives in the $N \rightarrow \infty$ asymptotic limit \cite{60–62}.

Renormalization group equations (RGEs) helps us to understand the relevance of UV fixed points for quantum field theories \cite{63, 64}, for example, in quantum chromodynamics, the property of the asymptotic freedom manifests the reliability of the theory by the use of the standard perturbation theory \cite{65–67}. Now when the fixed point corresponds to an interacting theory, we dub this as asymptotic safety \cite{68}. It may be desirable for UV-completion theories, that are neither asymptotically free nor renormalizable, to have such a UV fixed point behaviour \cite{60}. This idea was recently developed for quantum gravity \cite{69–75}. Applications to the Standard Model also showed that such a possibility \cite{76–78}. The authors studied the RGEs of non-local infinite-derivative theories in Ref. \cite{51, 55, 147}, but however the story remains unclear for Lee-Wick theories particularly in the non-perturbative regimes which we investigate in this paper.

Besides the RGE approach in QFT, deeper understanding of the confinement of quarks in the Standard Model (QCD sector) (see \cite{79} and references there-in) was proposed by Kugo and Ojima first proposed a confinement condition from the BRST invariance based on charge annihilation and then later on extended to color confinement by others \cite{80–89}.\footnote{Studies by Gribov \cite{90} and Zwanziger \cite{91} suggested confinement in QCD with the gluon propagator running to zero as momenta go to zero and an enhanced ghost propagator running to infinity more rapidly than the free case in the same limit of momenta. This scenario was not confirmed by studies of the gluon and ghost propagators on the lattice \cite{92–94}. Indeed, the existence of a mass gap was proven unequivocally in lattice computations for the spectrum of Yang-Mills theory without fermions \cite{95, 96}. These results found theoretical basis in Refs. \cite{97–102} in terms of a closed form formula for the gluon propagator (see Ref. \cite{103} for a review).}

Confinement, in its simplest form, can be understood as the combined effect of a potential obtained from the Wilson loop of a Yang–Mills theory without fermions and the running coupling yielding a linearly increasing potential, in agreement with lattice data \cite{104}.
Here in this work, we will apply the condition derived in [82–86] (via reducing it to the case of the Kugo–Ojima criterion [81]) to non-local non-Abelian Lee-Wick gauge theories without including fermions, following similar approach as studied in the case of ghost-free infinite-derivative non-local Yang-Mills [147]. This uses the form of exact solutions of the background using elliptical functions and the technique has been successfully applied to calculate non-perturbative local theory phenomenology: hadronic contribution of muon \((g-2)_\mu\) [58] and tunneling of the false vacuum [59]. We will prove that non-Abelian Lee-Wick gauge theory with no fermions is confining in 4-D, besides having a mass gap coming from the derived correlation functions. This occurs purely because of the fact that confinement arises due to the BRST invariance of the theory and the asymptotic freedom of the theory, as well as the existence of a mass gap. We show that the Lee-Wick higher-derivative operators defined in the UV yields finite contributions also in the IR-limit and provide a proof of confinement, granted by the absence of the Landau pole. Finally we show that the ghosts do not propagate in theory and does not appear as physical states in the mass spectrum of the theory.

The paper is organized as follows: in section II, we review Lee-Wick gauge theory followed by studying the equations of motion in section III. In section IV., we introduce BRST transformation in Lee-Wick gauge theory, and proof the theory is BRST-invariant. Following this, in section V, we prove confinement of Lee-Wick gauge field propagators in the strongly-coupled regimes and derive the exact RGE in that context. Next in section VII., we discuss the fate of Lee-Wick ghosts and we end by discussing the salient conclusions drawn from our study in the last section.

II. LEE-WICK GAUGE THEORIES

We know the action for the SU(N) pure Yang-Mills theory, in the local case, takes the form

\[ L_g = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}. \]  

(1)

where the repeated indexes imply summation both for space-time and group indexes and the field strength tensor \( F^{a\mu\nu} \) is given by

\[ F^{a\mu\nu} = \partial_{[\mu} A^{a}_{\nu]} - g f^{abc} A^{b}_{\mu} A^{c}_{\nu}, \]  

(2)
with the group structure constants is denoted by $f^{abc}$ and the dimensionless gauge coupling is $g$. We extend the theory to the non-local case by following the approach given in Refs. [51, 55]. When involving higher-derivative extensions of field theory, the free part of the theory can be written as [49–51, 53, 55]:

$$L_f = -\frac{1}{4} F^a_{\mu\nu} U(D^2) F^{a\mu\nu}. \quad (3)$$

where $U(D^2)$ is some function of $D^2$, where $D^a_{\mu} = \partial_\mu \delta^{ab} - ig A^c_\mu (T^c)^{ab}$ is the covariant derivative in the adjoint representation. We have introduced a mass scale $M$ for the scale of new physics, here in this case, is where the higher-order derivatives come into play$^3$:

Particularly, in Lee-Wick extensions model, the proper choice of $U(D^2)$ is granted by

$$U(D^2) = I + \frac{D^2}{M^2}. \quad (5)$$

Total Lagrangian is then given by

$$L = L_g + L_{g-f} + L_{ghost}. \quad (6)$$

where

$$L_{g-f} = \frac{\xi}{2} B^2 + B^a \partial_{\mu} A^a_{\mu}, \quad (7)$$

with $B^a$ an auxiliary field, and

$$L_{ghost} = \bar{c}^a (-\partial^\mu U_1(\square) D^a_{\mu}) c^c. \quad (8)$$

For Lee-Wick gauge theory extensions, let us choose the function, involving higher-order derivatives,

$$U_1(\square) = 1 - \frac{\square}{2M^2}. \quad (9)$$

Please note that both the functions $U$ and $U_1$ are chosen in such a way that, in the limit $M \to \infty$ the local theory is properly recovered$^4$.

$^3$ A different take is $2$.

$$L_f = -\frac{1}{2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{M^2} D^{ab\mu\nu} F_{\mu\nu}^b D^{ac\rho\sigma} F_{\rho\sigma}^c. \quad (4)$$

We will consider both in our analysis. Anyway, it important to emphasize that both choices are gauge invariant and therefore, also BRST invariant.

$^4$ Throughout the paper we will work in the Euclidean metric as done in other work in the literature (see references ipf Lee-Wick in the introduction).
III. CLASSICAL EQUATIONS OF MOTION

In the Lagrangian (4), the higher-order derivative can be removed by adding an auxiliary field. We can write

\[ L_f = -\frac{1}{2} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} M^2 \dddot{A}_\mu^a \dddot{A}^{a\mu} + 2 F^{a\mu\nu} D_{\mu}^a A_{\nu}^b. \]  

(10)

Then, we make the change of variable \( A_\mu^a \rightarrow A_\mu^a + \dddot{A}_\mu^a \) and we obtain

\[ L_f = -\frac{1}{2} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} (D_{\mu}^a \dddot{A}_\nu^b - D_{\nu}^a \dddot{A}_\mu^b)(D^{ac\mu} \dddot{A}^{cv} - D^{acv} \dddot{A}^{c\mu}) + 4 g f^{abc} \dddot{A}_\mu^a \dddot{A}_\nu^b D_{\nu}^d \dddot{A}_{\mu}^d - \frac{1}{2} g^2 f^{abc} f^{cde} \dddot{A}_\mu^a \dddot{A}_\nu^b \dddot{A}_{\mu}^d \dddot{A}_{\nu}^e + \frac{1}{2} M^2 \dddot{A}_\mu^a \dddot{A}^{a\mu}. \]  

(11)

This Lagrangian yields the following classical equations of motion

\[ D_{\mu}^a F^{b\mu\nu} - g f^{abc} \dddot{A}_\mu^b (D^{cd\mu} \dddot{A}_{\nu}^d - D^{cd\nu} \dddot{A}_{\mu}^d) - 2 g f^{abc} f^{cde} \dddot{A}_\mu^a \dddot{A}_{\mu}^d \dddot{A}_{\nu}^e - g f^{abc} D_{\mu}^d \dddot{A}_{\mu}^a \dddot{A}_{\nu}^d = 0 \]
\[ -D_{\mu}^a (D^{bc\nu} \dddot{A}^{c\mu} - D^{bc\mu} \dddot{A}^{c\nu}) - 2 g f^{abc} \dddot{A}_\mu^a (D^{cd\nu} \dddot{A}_{\mu}^d - D^{cd\mu} \dddot{A}_{\nu}^d) - 3 g^2 f^{abc} f^{cde} \dddot{A}_\mu^a \dddot{A}_{\mu}^d \dddot{A}_{\nu}^e - g f^{abc} \dddot{A}_\mu^a \dddot{A}_\nu^b F^{c\mu\nu} - M^2 \dddot{A}_{\mu}^a = 0. \]  

(12)

These equations can be solved by introducing two scalar fields \( \phi \) and \( \dddot{\phi} \), by directly applying the mapping theorem, proved in Refs., and as

\[ A_\mu^a = \eta_\mu^a \phi, \quad \dddot{A}_\mu^a = \eta_\mu^a \dddot{\phi} \]  

(13)

provided the constants \( \eta_\mu^a \) have the properties presented in and are obtained by taking for SU(2)

\[ \eta_\mu^a = ((0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)), \]  

(14)

that yields

\[ \eta^1_\mu = (0, 1, 0, 0), \quad \eta^2_\mu = (0, 0, 1, 0), \quad \eta^3_\mu = (0, 0, 0, 1), \]  

(15)

that implies \( \eta_\mu^a \eta^{a\mu} = 3 \). This easily generalizes to SU(N) as

\[ \eta_\mu^a \eta^{a\mu} = N^2 - 1. \]  

(16)

[^5]: This mapping theorem has been widely utilised in Refs. 56, 58, 147.
Similarly, by generalizing the SU(2) case,
\[ \eta^a_\mu \eta^{b\mu} = \delta_{ab}, \]  
(17)
and
\[ \eta^a_\mu \eta^b_\nu = \frac{1}{2} (g_{\mu\nu} - \delta_{\mu\nu}), \]  
(18)
being \( g_{\mu\nu} \) the Minkowski metric and \( \delta_{\mu\nu} \) the identity tensor. This mapping will yield the field equations in terms of scalar fields as:
\[ \partial^2 \phi + Ng^2 \phi^3 = 2Ng^2 \tilde{\phi}^3, \]
\[ \partial^2 \tilde{\phi} + Ng^2 \phi^2 \tilde{\phi} + M^2 \tilde{\phi} = 3Ng^2 \tilde{\phi}^3. \]  
(19)
Therefore, if we use the exact solution for the \( \phi \) field,
\[ \phi(x) = \mu \left( \frac{2}{Ng^2} \right)^{\frac{1}{4}} \text{sn}(p \cdot x + \theta, i), \]  
(20)
where \( \mu \) and \( \theta \) are arbitrary integration constants and sn is a Jacobi elliptical function, that holds provided
\[ p^2 = \mu^2 \sqrt{Ng^2/2}, \]  
(21)
we will get the trivial solution \( \tilde{\phi} = 0 \), the auxiliary field decouples and the spectrum of the quantum theory is left untouched. So, even though the propagator of the ghost could be not trivial, the field cannot propagate being 0. This conclusion should be supported by a computation of the Dyson-Schwinger set of equations as done in [102] for a local non-Abelian gauge theory.

IV. CONFINEMENT & BRST INVARIANCE IN YANG-MILLS

In the following sections, we present the confinement condition as devised in [82–86], reducing it to the Kugo-Ojima criterion [81], projecting all the discussion on the exact solutions obtained in [56, 57, 102].

A. Local Theory

The formalism we present here for the local case is the same as used in [115].
The Yang-Mills Lagrangian can be subdivided in three parts as follows

\[ L = L_f + L_{gf} + L_{FP}. \]  

We have \( L_{inv} \) for the classical gauge-invariant part, \( L_{gf} \) for the gauge-fixing terms and \( L_{FP} \) for the Faddeev–Popov (FP) ghost term as

\[ L_f = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu}, \]
\[ L_{gf} = \partial_\mu B \cdot A^\mu + \frac{1}{2} \xi B \cdot B, \]
\[ L_{FP} = i\partial_\mu \bar{c} \cdot D^\mu c, \]

where \( \xi \) denotes the gauge parameter and \( D_\mu \) is the covariant derivative given by

\[ D_\mu \psi = (\partial_\mu - igT \cdot A_\mu)\psi, \]
\[ D_\mu c^a = \partial_\mu c^a + gf^{abc} A_\mu c^c. \]

BRST transformations for a generic field \( \chi \) can be expressed by BRST charges \( Q_B \) and \( \bar{Q}_B \) given by

\[ \delta \chi = i[Q_B, \chi]_\mp, \quad \bar{\delta} \chi = i[\bar{Q}_B, \chi]_\mp, \]
\[ Q_B^2 = \bar{Q}_B^2 = Q_B \bar{Q}_B + \bar{Q}_B Q_B = 0. \]

We will take the \((-\)\) sign in (26) when \( \chi \) is even (odd) in the ghost fields \( c \) and \( \bar{c} \). We recognize them as anti-commuting scalar fields.

The BRST transformations can be generally defined in the following way

\[ \delta A_\mu = D_\mu c, \]
\[ \bar{\delta} A_\mu = D_\mu \bar{c}, \]

By imposing for the auxiliary fields \( B, c \) and \( \bar{c} \)

\[ \delta L = \bar{\delta} L = 0, \]

one gets

\[ \delta B = 0, \quad \delta \bar{c} = iB, \quad \delta c^a = -\frac{1}{7} g f^{abc} (c^b \bar{c}^c), \]
\[ \bar{\delta} B = 0, \quad \bar{\delta} c = i\bar{B}, \quad \bar{\delta} \bar{c}^a = -\frac{1}{2} g f^{abc} (\bar{c}^a \bar{c}^c), \]
with $B$ defined by the following equation

$$B^a + \bar{B}^a - igf^{abc}(\bar{c}^b c^c) = 0.$$  \hspace{1cm} (30)

By a direct application of the Noether’s theorem, one has a conserved current given by

$$j_\mu = \sum_{\{\Phi\}} \frac{\partial L}{\partial (\partial_\mu \Phi)} \delta \Phi = B^a(D_\mu c)^a - \partial_\mu B^a c^a + i\frac{1}{2}g f^{abc}\partial_\mu \bar{c}^a c^b c^c,$$  \hspace{1cm} (31)

with $\{\Phi\}$ being the set of all fields present in the Lagrangian. Therefore, the corresponding charge $Q_B$ is given by

$$Q_B = \int d^3x \left(B^a(D_0 c)^a - \bar{B}^a c^a + i\frac{1}{2}g f^{abc}\bar{c}^a c^b c^c\right).$$  \hspace{1cm} (32)

Therefore, we have

$$\delta(L_{gf} + L_{FP}) = \delta(-i\partial_\mu \bar{c} \cdot A_\mu - i\frac{1}{2}\xi \bar{c} \cdot B),$$  \hspace{1cm} (33)

confirming that

$$\delta L_f = 0.$$  \hspace{1cm} (34)

Given this Lagrangian, the equations of motion are given by

$$D^{\mu ab} F_{\mu \nu}^b + j^a_\nu = i\delta A^a_\nu.$$  \hspace{1cm} (35)

The contributions of the auxiliary fields are on the right-hand side. These represents massless particles at tree level. It is also easy to see that the $B$ field is not propagating. The consequences of this is that such fields will not give any contribution to the physical spectrum of the theory. In order to evaluate such a contribution, we have to compute

$$\langle i\delta \bar{A}^a_\mu(x), A^b_\nu(y) \rangle.$$  \hspace{1cm} (36)

By the Kugo-Ojima technique, one has

$$\delta \bar{A}^a_\mu = -\{Q_B, \{Q_B, A^a_\mu\}\}.$$  \hspace{1cm} (37)

Then, because of $\langle 0|Q_B = Q_B|0 \rangle = \bar{Q}B|0 \rangle = \langle 0|\bar{Q}B = 0$, one has

$$\langle i\delta \bar{A}^a_\mu(x), A^b_\nu(y) \rangle = \langle i\bar{A}^a_\mu(x), \delta A^b_\nu(y) \rangle = i\langle D_\mu \bar{c}^a(x), D_\nu \bar{c}^b(y) \rangle.$$  \hspace{1cm} (38)
But it is
\[ \int d^d x e^{ipx} \langle D_\mu c^a(x), D_\nu c^b(y) \rangle = \delta^{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - i\epsilon} \right) u(p^2) - \delta^{ab} \frac{p_\mu p_\nu}{p^2 - i\epsilon}, \]
and the no-pole condition yields here
\[ 1 + u(p^2 = 0) = 0, \]
which is the Kugo–Ojima condition for confinement granting that no massless pole appears in the spectrum of the theory. Indeed, this condition removes the massless term from Eqn.(39).

The form of the \( u(p^2) \) function has been given explicitly in [115] proving confinement for such theories. The \( \beta \)-function was also obtained. Below, we will extend this method for the Lee-Wick theory.

### B. Lee-Wick Gauge Theory

The complete BRST-invariant infinite-derivative gauge theory in the quantized action is of the form [55]:
\[ L_{inv} = L_f + \frac{\xi}{2}(B^a)^2 + B^a \partial^\mu A^a_\mu + \bar{c}^a(-\partial^\mu U_1(\Box) D_{\mu} c^c), \]
where \( \xi \) is the gauge fixing parameter, \( B \) is the auxiliary field, and \( c \) and \( \bar{c} \) are the ghost and anti-ghost fields, respectively. The BRST transformations for non-Abelian gauge theories express a residual symmetry of the effective action which remains after the original gauge invariance has been broken by the addition of the gauge-fixing and ghost action terms. Our BRST transformations are modified in the following way:

\[ \delta A_\mu = D_\mu c, \]
\[ \bar{\delta} A_\mu = D_\mu \bar{c}, \]
and
\[ \delta B = 0, \quad \delta \bar{c} = iU_1^{-1}(\Box) B, \quad \delta c^a = -\frac{1}{2}gf^{abc}(\bar{c}^b c^c), \]
\[ \bar{\delta} \bar{B} = 0, \quad \bar{\delta} c = iU_1^{-1}(\Box) \bar{B}, \quad \bar{\delta} \bar{c}^a = -\frac{1}{2}gf^{abc}(\bar{c}^b \bar{c}^c). \]
We show the BRST-invariance of $S_{inv}$ by noting that the BRST transformation of the gauge field is just a gauge transformation of $A_\mu$ generated by $c_a$ or $\bar{c}_a$. Therefore, any gauge-invariant functionals of $F_{\mu\nu}$, like the first term in Eqn. (41) gives $\delta L_f = 0$. The second term in Eqn. (41) gives $\delta \left( \frac{\varepsilon}{2} (B^a)^2 \right) = 0$ from Eqn. (43). For the third term in Eqn. (41), the transformation of $A_\mu^a$ cancels the transformation of $\bar{c}$ in the last term due to Eqs. (42), leaving us with

$$\delta (D_\mu^a c^c) = D_\mu^{ac} \delta c^c + gf^{abc} \delta A_\mu^b c^c,$$  \hfill (44)

which is is equal to 0, using the properties of the covariant derivative and the Jacobi identity (see Ref. [116]). The transformation of $c^\sigma$ is nilpotent,

$$\delta (\delta_\mu c^a c^b) = 0,$$  \hfill (45)

while the transformation of $A^\mu$ is also nilpotent,

$$\delta ((D_b^a)^a c^b) = 0.$$  \hfill (46)

Hence, the action in Eqn. (41) is BRST-invariant. Noting the fact that the only part of the ghost action which varies under the BRST transformations is that of the anti-ghost ($\bar{c}_a$), the central idea behind our proof of BRST-invariance is that we have chosen the BRST variation of the anti-ghost ($\bar{c}_a$) (see Eqs. (43)) to cancel the variation of the gauge-fixing term.

It is not difficult to see that, in the limit of the non-local mass $M \to \infty$, the BRST transformations given in Eqn. (42)-(43) become identical to those of the local case. Formally, the confinement condition of Eqn. (40) remains untouched as the effects of the non-locality, if present, are kept into the $u$ function.

V. CONDITION OF CONFINEMENT IN LEE-WICK THEORY

In this section, we derive the confinement for the non-local theory, following Ref. [115]. See Appendix C for a brief review of this technique.

From the action (41), we derive the equations of motion,

$$U(D^2) D^\mu F^{a}_{\mu \nu} + j^a_{\nu} = i \delta \bar{c} A^a_{\nu}.$$  \hfill (47)

The RHS can be evaluated as already done for the local case, and we write down

$$\int d^4 x e^{ipx} \langle D_\mu \bar{c}^a(x), D_\nu c^b(0) \rangle = \delta_{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u(p^2) - \delta_{ab} \frac{p_\mu p_\nu}{p^2} U_1(-p^2).$$ \hfill (48)
Indeed, this is the most general form for the given correlation function but, for the massless contribution, we have also to take into account the contribution of the non-locality. The interesting part here is that all the non-local contributions enters into the definition of the function $u$. These non-localities arise from the two-point functions of the non-local theory but also that fluctuations from UV can yield a significant contribution to confinement as they are summed up in the integral where they cannot be neglected. Then, the confinement condition is again

$$1 + u(p^2 = 0) = 0. \quad (49)$$

A. Confinement in Lee-Wick gauge theory

In our preceding works, we obtained the 2P-functions for infinite-derivative Yang-Mills theory [57]. We have for the 2P-function for the gluon field in the Landau gauge

$$D^{ab}_{\mu\nu}(p) = \left( \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) G_2(p), \quad (50)$$

being

$$G_2(p) = \frac{e^{\frac{1}{2} f(-p^2)}}{p^2 + \Delta m^2 e^{\frac{1}{2} f(-p^2)}} \frac{1}{1 - \Pi(p)}. \quad (51)$$

In the local limit, $M \to \infty$, Eqn. (51) reduces to a Yukawa form that yields a fair approximation to the exact local propagator obtained in [102]. In the non-local case, one has the mass gap

$$\Delta m^2 = \mu^2 \left( 18 N g^2 \right)^{\frac{1}{2}} \frac{4 \pi^2}{K^2(i)} \frac{e^{-\pi}}{1 + e^{-\pi}} e^{f\left(-\frac{\mu^2}{4K^2(i)p^2}\right)} + \delta m^2. \quad (52)$$

This must be completed by the gap equation

$$\delta m^2 = 2Ng^2G_2(0) = 2Ng^2 \int \frac{d^4p}{(2\pi)^4} G_2(p). \quad (53)$$

The function $\Pi(p)$ can be neglected as also the shift $\delta m^2$ as a first approximation. Similarly, for the ghost one has

$$K_2(p) = -\frac{1}{p^2} e^{\frac{1}{2} f(-p^2)}. \quad (54)$$

Therefore, one has the integral, in the infinite-derivative case [147]

$$\int d^4x e^{ipx} \langle D_\mu \bar{c}^a(x), D_\nu c^b(0) \rangle = -\frac{\delta^{ab} P_\mu P_\nu}{k^2} \quad (55)$$

$$+ \frac{(N^2 - 1)^2}{2N} g^2 \delta^{ab} \left( \delta_{\mu\nu} - \frac{P_\mu P_\nu}{p^2} \right) \int \frac{d^4p'}{(2\pi)^4} K_2(p - p') G_2(p').$$
This will yield for the confinement condition

\[ u(0) = -\frac{(N^2 - 1)^2}{2N} g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + \Delta m^2 e^{f(-p^2)}} \, \text{e}^{f(-p^2)} + \frac{\Delta m^2}{p^2} \cdot \text{e}^{f(-p^2)} \]  

In order to recover the Lee-Wick limit, it is enough to substitute

\[ e^{f(-p^2)} \to 1 + \frac{p^2}{M^2}. \]

This will yield the integral

\[ u(0) = -\frac{(N^2 - 1)^2}{2N} g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{e^{f(-p^2)} + 1}{p^2 e^{-\frac{1}{2}f(-p^2)} + \Delta m^2} = \]

\[ \frac{(N^2 - 1)^2}{2N} g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 e^{-\frac{1}{2}f(-p^2)} + \Delta m^2}. \]

This integral is finite, both in IR and UV, if we fix a UV cut-off at the non-local scale \( M \). We will have

\[ u(0) = \frac{(N^2 - 1)^2}{4\pi N} \alpha_s \left( \frac{1}{2} \arctanh \frac{1}{\sqrt{2z + 1}} + \frac{1}{4} \ln \left( 1 + \frac{1}{2z} \right) \right), \]

being \( \alpha_s = g^2/4\pi \). Here it is \( z = \Delta m^2/M^2 \). It is easy to see that, for \( z \to 0 \), we get a finite result,

\[ u(0) = \ln 2 \frac{(N^2 - 1)^2}{8\pi N} \alpha_s, \]

that does not grant confinement being greater than 0. The mass gap is given by

\[ \Delta m^2 \approx \mu^2 \alpha_s^\frac{1}{4} \eta_0 \left( 1 - \eta_1 \mu^2 \alpha_s^\frac{1}{2}/M^2 \right), \]

with the numerical constants given by

\[ \eta_0 = (72\pi)\frac{1}{2} \frac{4\pi^2}{K^2(i)} \frac{e^{-\pi}}{(1 + e^{-\pi})} \]

and

\[ \eta_1 = \frac{\pi^2}{4K^2(i)} (2\pi)^2. \]

**VI. RENORMALIZATION GROUP EQUATION**

We can consider \( \mu \) as a running mass and obtain the beta function in the limit \( z \to 0 \). Then, our results can be trusted only in the IR limit. We just note that

\[ z = \frac{\Delta m^2}{M^2} = \mu^2 \alpha_s^\frac{1}{4} \eta_0/M^2 + O(\mu^4/M^4) \]

\[ \text{\textsuperscript{13}} \]
so that, the leading order suffices. Therefore, the confinement condition becomes
\[-\frac{(N^2 - 1)^2}{8\pi N} \alpha_s \ln z = -\frac{(N^2 - 1)^2}{8\pi N} \alpha_s \ln \left( \frac{\mu^2 \alpha_s^2 \eta_0}{M^2} \right) \approx -1.\] (65)
In Fig. 1, we plot the running coupling obtained from eq. (65).

![Figure 1: Running coupling obtained from eq. (65) given as a function of \(\ln(\mu^2/M^2)\). Confinement arises from the coupling going to infinity in the IR limit without manifesting a Landau ghost.](image)

This yields the beta function
\[
\frac{d\alpha}{dl} = -\frac{\beta_0 \alpha^2}{1 - \frac{\beta_0}{2} \alpha},
\] (66)
being \(\beta_0 = (N^2 - 1)^2/8\pi N\). We can conclude that the theory is IR confining and has asymptotic freedom. It is interesting to point out that, in this approximation, we have recovered the beta function of the local Yang-Mills theory [115]. This is due to the fact that, in this case, the computation is practically the same but we have a natural cut-off in the theory arising from the Lee-Wick term.

**VII. FATE OF THE LEE-WICK GHOST IN THE INFRARED**

We consider again the propagator given in eq. (51). This gives
\[G_2(p) = \frac{1}{p^2 \left( 1 - \frac{p^2}{M^2} \right) + \Delta m^2 \frac{1}{1 - \Pi(p)}},\] (67)
This decomposes into
\[ G_2(p) = \left( -\frac{M^2}{m_1^2 - m_2^2 p^2 - m_1^2} + \frac{M^2}{m_1^2 - m_2^2 p^2 + |m_2|^2} \right) \frac{1}{1 - \Pi(p)}, \] (68)
but \( m_1^2 > 0 \) and \( m_2^2 < 0 \) and so, the unphysical Lee-Wick ghost is not propagating (no poles on the imaginary axis).

This means that the effect of the interaction is to remove the ghost from the spectrum of the theory, in a similar manner to what happens to the Faddeev-Popov ghost that decouples in the infrared instead. The physical state represents a genuine gluonic-like degree of freedom mediating the interaction at low-energies. The behavior of the Lee-Wick ghost is that expected from a confining theory.

As the Lee-Wick theory also shows asymptotic freedom in the UV-limit, we should expect that, at these fixed points, the theory is physically well defined.

VIII. CONCLUSION AND DISCUSSION

We investigated strongly coupled higher-derivative Lee-Wick gauge theory in the 4-D in context of the confinement aspects of the theory. We compared the results with that of the standard Yang-Mills theory and discussed the infra-red behaviour. We presented the \( \beta \)-function in the strongly-coupled regime. Below, we summarize the main findings of our paper:

- For the first time as per our knowledge we showed BRST invariance of Lee Wick Yang Mills theory (see eqns. 42,43,44,45, and 46).
- We derived the confinement conditions in the Lee-Wick gauge theories and showed that confinement is determined by the scale \( M \) where the ghosts of the theory come into play (see eqn.(65) and (66)).
- We derived the Renormalization Group Equations in the strongly-coupled regimes of the theory and showed that the coupling runs to infinity in the low energy limit, without encountering the problem of Landau ghosts. (see eq. (65)) The interesting point is that the beta function is identical to the local case but we do not need to introduce any UV cut-off as the theory has a natural one from the non-local mass. (see Fig. (1)).
• The result we obtained is trustworthy both for IR and UV. We recover asymptotic freedom at higher energies and so, the ghost does not propagate in the IR-limit and decouples in the UV-limit.

• We found that the fate of Lee-Wick ghosts are sealed: they stop propagating and instead get confined and consequently do not appear as physical degrees of freedom in the strong coupling regimes (as shown in eq. (68)).

We envisage our results will shed light on more detailed understanding of confinement and β-function analysis in the framework of re-normalizable Quadratic gravity theories, for which Lee-Wick theory acts as a prototype. Ghosts in Quadratic Gravity have already been speculated to be confined and do not appear as physical degrees of freedom in the mass spectrum, in analogy to quarks and gluons in QCD-like theories, in Refs. [148, 149]. Using our methodology we provided an understanding of the fate of ghosts in Lee-Wick QFT and subsequently we look to prove this in a mathematical manner in Quadratic Gravity as well involving the masses of the ghost particles but this is currently beyond the scope of the present draft and will be taken up in another publication [118].

IX. ACKNOWLEDGEMENT

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6 See Ref. [146] for recent conference on this topic.
Appendix A: Dyson-Schwinger Equations & Bender-Milton-Savage Technique

In this Appendix, we briefly present the Bender-Milton-Savage technique \[117\]. We start from the following partition function for a scalar field

\[ Z[j] = \int [D\phi] e^{iS(\phi)+i \int d^4x j(x)\phi(x)}. \]  

(69)

The equation for the 1P-function is given by

\[ \langle \frac{\delta S}{\delta \phi(x)} \rangle = j(x) \]  

(70)

where

\[ \langle \ldots \rangle = \frac{\int [D\phi] \ldots e^{iS(\phi)+i \int d^4x j(x)\phi(x)}}{\int [D\phi] e^{iS(\phi)+i \int d^4x j(x)\phi(x)}}. \]  

(71)

One has

\[ \langle \phi(x_1)\phi(x_2)\ldots \phi(x_n) \rangle = \frac{\delta^n \ln(Z[j])}{\delta j(x_1)\delta j(x_2)\ldots \delta j(x_n)}, \]  

(72)

and

\[ \frac{\delta G_k(\ldots)}{\delta j(x)} = G_{k+1}(\ldots, x). \]  

(73)

We apply all this to a \( \phi^4 \) theory and get

\[ S = \int d^4x \left[ \frac{1}{2}(\partial \phi)^2 - \frac{\lambda}{4} \phi^4 \right], \]  

(74)

so that,

\[ \partial^2 \langle \phi \rangle + \lambda \langle \phi^3(x) \rangle = j(x). \]  

(75)

For the 1P-function is

\[ Z[j]\partial^2 G_1^{(j)}(x) + \lambda \langle \phi^3(x) \rangle = j(x), \]  

(76)

and by definition

\[ Z[j]G_1^{(j)}(x) = \langle \phi(x) \rangle. \]  

(77)

After derivation with respect to \( j(x) \), we get

\[ Z[j][G_1^{(j)}(x)]^2 + Z[j]G_2^{(j)}(x, x) = \langle \phi^2(x) \rangle, \]  

(78)

and after another derivation step it one has:

\[ Z[j][G_1^{(j)}(x)]^3 + 3Z[j]G_1^{(j)}(x)G_2(x, x) + Z[j]G_3^{(j)}(x, x, x) = \langle \phi^3(x) \rangle. \]  

(79)
We insert this into Eqn. (75) to obtain
\[ \partial^2 G_1^{(j)}(x) + \lambda[G_1^{(j)}(x)]^3 + 3\lambda G_2^{(j)}(0)G_1^{(j)}(x) + G_3^{(j)}(0, 0) = Z^{-1}[j]j(x) \] (80)
Setting \( j = 0 \), we get the first Dyson-Schwinger equation
\[ \partial^2 G_1(x) + \lambda[G_1(x)]^3 + 3\lambda G_2(0)G_1(x) + G_3(0, 0) = 0, \] (81)
where we realize that quantum corrections induced a mass term.

Our next step is to derive Eqn. (80) again with respect to \( j(y) \). This gives the result
\[ \partial^2 G_2^{(j)}(x, y) + 3\lambda[G_1^{(j)}(x)]^2G_2^{(j)}(x, y) + 3\lambda G_3^{(j)}(x, y)G_1^{(j)}(x) + 3\lambda G_2^{(j)}(x, y)G_1^{(j)}(x) + G_4^{(j)}(x, x, y) = Z^{-1}[j]\delta^4(x-y) + j(x)\frac{\delta}{\delta j(y)}(Z^{-1}[j]). \]
By setting \( j = 0 \), one has for the 2P-function
\[ \partial^2 G_2(x, y) + 3\lambda[G_1(x)]^2G_2(x, y) + 3\lambda G_3(0, y)G_1(x) + 3\lambda G_2(0)G_2(x, y) + G_4(0, 0, y) = \delta^4(x-y). \] (82)
In principle, one can iterate such a procedure to whatever desired order yielding the full hierarchy of Dyson-Schwinger equations into PDE form.

**Appendix B: Dyson-Schwinger equations for Yang-Mills theory**

Following similar steps as in Appendix A, for the 1P-function one gets
\[ \Box G_{1\mu}^{(j)a} + gf^{abc}e^{-\frac{j}{2}f^{(\Box)}} \left\langle \partial^\mu \left[ e^{\frac{j}{2}f^{(\Box)}}A_\mu^b \right] e^{\frac{j}{2}f^{(\Box)}} \right\rangle + gf^{abc}e^{-\frac{j}{2}f^{(\Box)}} \left\langle \left[ e^{\frac{j}{2}f^{(\Box)}}A^{\mu b} \right] e^{\frac{j}{2}f^{(\Box)}} (\partial^\mu A^c_\mu - \partial^\mu A^c_\nu) \right\rangle + g^2f^{abc}f^{cde}e^{-\frac{j}{2}f^{(\Box)}} \left\langle \left[ e^{\frac{j}{2}f^{(\Box)}}A^{\mu b} e^{\frac{j}{2}f^{(\Box)}} \right] A^d_\nu e^{\frac{j}{2}f^{(\Box)}} A^e_\mu \right\rangle + gf^{abc}e^{\frac{j}{2}f^{(\Box)}} \left\langle \bar{c}^{\mu} \partial^\mu c^\nu \right\rangle = e^{\frac{j}{2}f^{(\Box)}} j^{(j)a}_\mu, \] (83)
similarly, for the ghost field is
\[ -\Box P_1^{(a)b} + gf^{abc} \left\langle \left( e^{\frac{j}{2}f^{(\Box)}} A^a_\mu \right) \partial^\mu c^b \right\rangle = e^{\frac{j}{2}f^{(\Box)}} \eta^a. \] (84)
For our needs, we consider the following functions
\[ G_{1\mu}^{(j)a}(x) = Z^{-1}(A^a_\mu(x)) \]
\[ P_1^{(a)b}(x) = Z^{-1}(c^a(x)), \] (85)
A new 2P-function has been introduced defined as

\[ Z[\eta, \bar{\eta}] P_1^{(\eta) a}(x) = \langle e^{\frac{j}{Z}} A_{\mu}^a(x) \rangle. \]  

(86)

We derive one time again with respect to \( j(x) \) on the first equation yielding

\[ Z e^{\frac{j}{Z}} G_2^{(\eta)ab}(x, x) + Z e^{\frac{j}{Z}} G_1^{(\eta)ab}(x) e^{\frac{j}{Z}} G_1^{x,x}(x) = \langle e^{\frac{j}{Z}} A_{\mu}^a(x) e^{\frac{j}{Z}} A_{\nu}^a(x) \rangle, \]  

(87)

and apply the derivative \( \partial^{\nu} \)

\[ Z e^{\frac{j}{Z}} \partial^{\nu} G_2^{(\eta)ab}(x, x) + Z e^{\frac{j}{Z}} \partial^{\nu} G_1^{(\eta)ab}(x) e^{\frac{j}{Z}} G_1^{x,x}(x) = \langle e^{\frac{j}{Z}} \partial^{\nu} A_{\mu}^a(x) e^{\frac{j}{Z}} A_{\nu}^a(x) \rangle. \]  

(88)

Then, a further derivation of Eqn. (87) with respect to \( j^{\mu\nu} \) yields

\[ Z e^{\frac{j}{Z}} \partial^{\mu} G_2^{(\eta)ab}(x, x) + Z e^{\frac{j}{Z}} \partial^{\mu} G_1^{(\eta)ab}(x) e^{\frac{j}{Z}} G_1^{x,x}(x) + Z e^{\frac{j}{Z}} \partial^{\mu} G_3^{(\eta)abc}(x, x, x) + Z e^{\frac{j}{Z}} \partial^{\mu} G_2^{(\eta)abc}(x, x, x) + Z e^{\frac{j}{Z}} \partial^{\mu} G_2^{(\eta)abc}(x, x, x) \]

\[ = \langle e^{\frac{j}{Z}} A_{\mu}^a(x) e^{\frac{j}{Z}} A_{\nu}^a(x) \rangle, \]  

(89)

and the same to be done on the ghost field. Eqn. (86) yields

\[ Z[\eta, \bar{\eta}] P_1^{(\eta) a}(x) = \langle c^a(x) \rangle. \]  

(90)

The following equation is obtained after deriving with respect to \( \partial_{\mu} \) and then with respect to \( \bar{\eta} \)

\[ Z \bar{P}_1^{(\eta)b}(x) e^{\frac{j}{Z}} \partial^{\mu} P_1^{(\eta)a}(x) + Z \partial^{\mu} K_2^{(\eta)ab}(x, x) = \langle \bar{c}^{b} \partial^{\mu} c^{a}(x) \rangle. \]  

(91)

A new 2P-function has been introduced defined as

\[ K_2^{(\eta)ab}(x, y) = \frac{1}{Z} \frac{\delta P_1^{(\eta)a}(x)}{\delta \bar{\eta}^b(y)}, \]  

(92)

and the other 2P-functions

\[ J_{2\mu}^{(\eta)ab}(x, y) = \frac{1}{Z} \frac{\delta P_1^{(\eta)a}(x)}{\delta j^{b\mu}(y)}. \]  

(93)

Then, we derive Eqn. (90) with respect to \( j^{\mu\nu}(x) \) and obtain

\[ Z e^{\frac{j}{Z}} G_1^{(\eta)b}(x) \partial^{\mu} P_1^{(\eta)a}(x) + Z \partial^{\mu} J_2^{(\eta)ab}(x, x) = \langle A_{\mu}^b(x) \partial^{\mu} c^{a}(x) \rangle. \]  

(94)
Gathering everything together yields

\[ \square G^{(j)a}_{1\mu} + g f^{abc} e^{-\frac{1}{2} f^{(\Box)}} \partial^\nu \left[ e^{\frac{1}{2} f^{(\Box)}} G^{(j)bc}_{2\mu\nu}(x, x) + e^{\frac{1}{2} f^{(\Box)}} G^{(j)b}_{1\mu}(x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)c}_{1\nu}(x) \right] - \\
g f^{abc} e^{-\frac{1}{2} f^{(\Box)}} \left[ e^{\frac{1}{2} f^{(\Box)}} \partial^\nu G^{(j)bc}_{2\mu\nu}(x, x) + e^{\frac{1}{2} f^{(\Box)}} \partial^\nu G^{(j)b}_{1\mu}(x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)c}_{1\nu}(x) \right] - \\
g f^{abc} e^{-\frac{1}{2} f^{(\Box)}} \left[ e^{\frac{1}{2} f^{(\Box)}} \partial_\mu G^{(j)bc}_{2\nu}(x, x) + e^{\frac{1}{2} f^{(\Box)}} \partial_\mu G^{(j)b}_{1\nu}(x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)c}_{1\mu}(x) \right] + \\
g^2 f^{abc} f^{cde} e^{-\frac{1}{2} f^{(\Box)}} \left[ e^{\frac{1}{2} f^{(\Box)}} G^{(j)bd}_{2\mu\nu}(x, x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)ce}_{1\mu}(x) + e^{\frac{1}{2} f^{(\Box)}} \partial^\nu G^{(j)bde\nu}_{3\mu\nu}(x, x, x) + \\
e^{\frac{1}{2} f^{(\Box)}} G^{(j)b}_{1\mu}(x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)c}_{1\nu}(x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)e}_{1\mu}(x) + \\
e^{\frac{1}{2} f^{(\Box)}} G^{(j)de\nu}_{2\mu\nu}(x, x) e^{\frac{1}{2} f^{(\Box)}} G^{(j)b}_{1\nu}(x) \right] - \\
g f^{abc} e^{\frac{1}{2} f^{(\Box)}} \left\{ P^{(n)b}_{1}(x) e^{\frac{1}{2} f^{(\Box)}} \left[ \partial_\mu P^{(n)e}_{1}(x) \right] + \partial_\mu \left[ K^{(n)bc}_{2}(x, x) \right] \right\} = \\
e^{\frac{1}{2} f^{(\Box)}} j^{a}_{\mu}, \quad (95) \]

It is possible to recover the local case discussed in [102] by setting the non-locality factor to 1, or just taking the limit \( M \to \infty \). For the ghost field, it is

\[ -\square P^{(n)c}_{1} - g f^{abc} e^{\frac{1}{2} f^{(\Box)}} G^{(j)a}_{1\mu}(x) \partial^\mu P^{(n)b}_{1}(x) - g f^{abc} \partial^\mu J^{(n, i)ab}_{2\mu}(x, x) = e^{\frac{1}{2} f^{(\Box)}} \eta^{c}. \quad (96) \]

The final step is given by setting all the currents to zero.
From Eqn. (95), we derive it with respect to \( j^h(y) \) and get

\[
\begin{align*}
\&\Box G_{2\mu \lambda}^{(j)ah}(x, y) + gf^{abc} e^{-\frac{1}{2}f^{(\square)} \partial^\nu \left[ e\frac{1}{2}f^{(\square)} G_{3\mu \nu \lambda}^{(j)bch}(x, y) + e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)b}(x, y) \times \right. \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)c}(x) + + e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)c}(x) \right] } \\
&gf^{abc} e^{-\frac{1}{2}f^{(\square)} \left[ e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)bch}(x, y) + e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)bh}(x, y) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)c}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)c}(x) \right] } \\
&gf^{abc} e^{-\frac{1}{2}f^{(\square)} \left[ e\frac{1}{2}f^{(\square)} \partial_\mu G_{3\nu \lambda}^{(j)bchu}(x, y) + e\frac{1}{2}f^{(\square)} \partial_\mu G_{2\nu \lambda}^{(j)bh}(x, y) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)c}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} \partial_\mu G_{1\mu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)c}(x) \right] } \\
&g^2 f^{abc} f^{cde} e^{-\frac{1}{2}f^{(\square)} \left[ e\frac{1}{2}f^{(\square)} G_{3\mu \nu \lambda}^{(j)bchu}(x, y) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)ve}(x) + \\
&e\frac{1}{2}f^{(\square)} G_{2\mu \nu}^{(j)bd}(x, x) e\frac{1}{2}f^{(\square)} G_{2\nu}^{(j)ve}(x, y) + e\frac{1}{2}f^{(\square)} \partial^\mu G_{4\mu \lambda}^{(j)bchu}(x, x, y) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)d}(x, y) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)ve}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)d}(x, y) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)ve}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)d}(x, y) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)ve}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)b}(x) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)d}(x, y) e\frac{1}{2}f^{(\square)} G_{1\nu}^{(j)ve}(x) + \\
&\left. e\frac{1}{2}f^{(\square)} G_{3\mu \lambda}^{(j)bchu}(x, x, y) e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)b}(x) + e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)dev}(x, x, y) e\frac{1}{2}f^{(\square)} G_{2\nu \lambda}^{(j)b}(x, y) \right] \\
&gf^{abc} e\frac{1}{2}f^{(\square)} \left\{ \frac{f^{(\square)} f^{(\square)} G_{2\mu \lambda}^{(j)b}(x, y) e\frac{1}{2}f^{(\square)} \left[ \partial_\mu P_1^{(\eta)b}(x) \right] } + \right. \\
&\left. P_1^{(\eta)b}(x) e\frac{1}{2}f^{(\square)} \left[ \partial_\mu P_2^{(\eta)c}(x, y) \right] + \partial_\mu \left[ W_{3\lambda}^{(\eta)bchu}(x, y, z) \right] \right\} = \\
&\frac{e^2 f^{(\square)} G_{1\mu}^{(j)b}(x, y) e\frac{1}{2}f^{(\square)} \left[ \partial_\mu P_1^{(\eta)c}(x) \right] }.
\end{align*}
\]

after the introduction of the 3P-function

\[
W_{3\lambda}^{(\eta)abc}(x, y, z) = Z^{-1} \frac{\delta K_{2\alpha}^{(\eta)ab}(x, y)}{\delta j^{\lambda c}(z)}.
\]

Taking the 1P-function for the ghost, we derive it with respect to \( \eta^h(y) \) and obtain

\[
-\Box K_{2\alpha}^{(\eta)ch}(x, y) - igf^{abc} f^{(\square)} L_{2\mu}^{(\eta)j} ah(x, y) \partial^\mu P_1^{(\eta)b}(x) \]

\[
-igf^{abc} e\frac{1}{2}f^{(\square)} G_{1\mu}^{(j)a}(x) \partial^\mu K_{2\nu}^{(\eta)bh}(x, y) - igf^{abc} \partial^\mu W_{3\mu}^{(\eta)abh}(x, x, y) \]

\[
= e\frac{1}{2}f^{(\square)} \delta^h \delta^4(x - y).
\]

We have introduced the 2P-function

\[
L_{2\mu}^{(\eta)j} ab(x, y) = \frac{\delta G_{1\mu}^{(j)a}(x)}{\delta \eta^p(y)}.
\]
We derive with respect to \( j^{h\nu}(y) \) and this yields the equation for \( J_2 \)

\[
-\Box J_2^{(\eta)h\nu}(x, y) - ig f^{abc} e^{\frac{i}{2}f(\Box)} G_{2h\mu}^{(j)ab}(x, y) \partial^\mu P_1^{(\eta)h}(x) \\
-ig f^{abc} e^{\frac{i}{2}f(\Box)} G_{1h\mu}^{(j)a}(x) \partial^\mu J_2^{(\eta)h\nu}(x, y) \\
-ig f^{abc} \partial^\mu J_3^{(\eta)h\nu}(x, x, y) = 0,
\]

(101)

with the introduction of the 3P-function

\[
J_{3h\mu}^{(\eta,j)abc}(x, y, z) = \frac{\delta J_{2h\mu}^{(\eta,j)ab}(x, y)}{\delta j^\mu(z)}.
\]

(102)

Again, we repeat the final step setting all the currents to zero. This produces the equations given in the main text.

**Appendix C: Confinement in local Yang-Mills theory**

The argument followed in the main text is just straightforwardly obtained by the approach devised in Ref. [115]. One observes that

\[
\int d^4xe^{ipx} \langle D_\mu \bar{c}^a(x), D_\nu c^b(0) \rangle = -\delta^{ab} \frac{p_\mu p_\nu}{k^2} \\
+ \frac{(N^2 - 1)^2}{2N} g^2 \delta^{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \int \frac{d^4p'}{(2\pi)^4} K_2(p - p') G_2(p').
\]

(103)

We know the form of the propagators and these are

\[
K_2(p) = -\frac{1}{p^2 + i\epsilon}
\]

(104)

for the ghost field and

\[
G_2(p) = \frac{\pi^3}{4K^3(i)} \sum_{n=0}^{\infty} \frac{e^{-(n+\frac{1}{2})\pi}}{1 + e^{-(2n+1)\pi}} \frac{1}{(2n + 1)^2} \frac{1}{p^2 - m_n^2 + i\epsilon}
\]

(105)

for the gauge field, provided the mass spectrum

\[
m_n = (2n + 1) \frac{\pi}{2K(i)} \left( \frac{Ng_2}{2} \right)^\frac{\mu}{4},
\]

(106)

where \( K(i) \) is the complete elliptical integral of the first kind and \( \mu \) is one of the integration constants in the theory. This is a fine approximation to the full propagator as we have
omitted the mass shift induced by quantum corrections. Then, the Kugo-Ojima confinement condition takes the form

\[ u(0) = -\frac{(N^2 - 1)^2}{2N}g^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + i\epsilon} \frac{\pi^3}{4K^3(-1)} \sum_{n=0}^{\infty} \frac{e^{-(n+\frac{1}{2})\pi}}{1 + e^{-(2n+1)\pi}} \frac{1}{(2n+1)^2} = -1. \]  

(107)

This integral can be evaluated by known techniques and one obtains the \( \beta \)-function in closed form

\[ \beta_{YM} = -\beta_0 \frac{\alpha_s^2}{1 - \frac{1}{2}\beta_0 \alpha_s}, \]

(108)

with \( \beta_0 = (N^2 - 1)^2/8\pi N \). This beta function grants confinement with the coupling running to infinity lowering momenta with no Landau pole. In the UV we recover the asymptotic freedom as we expected.

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