Thermoelectric DC conductivities with momentum dissipation from higher derivative gravity

Long Cheng, Xian-Hui Ge

Department of Physics, Shanghai University, Shanghai 200444, P.R. China

physcheng@shu.edu.cn, gexh@shu.edu.cn,

Zu-Yao Sun

College of Arts and Sciences, Shanghai Maritime University, Shanghai 200135, China.

zysun@shmtu.edu.cn

Abstract

We present a mechanism of momentum relaxation in higher derivative gravity by adding linear scalar fields to the Gauss-Bonnet theory. We analytically computed all the thermoelectric conductivities in this theory by adopting the method given by Donos and Gauntlett in [arXiv:1406.4742]. The results show that the DC electric conductivity is not a monotonic function of the effective impurity parameter $\beta$: in the small $\beta$ limit, the DC conductivity is dominated by the coherent phase, while for larger $\beta$, pair creation contribution to the conductivity becomes dominant, signaling an incoherent phase.
1 Introduction

The AdS/CFT correspondence provides a powerful tool in probing many important phenomena of strongly correlated systems in condensed matter physics [1–3]. In the context of AdS/CFT, many charge transport coefficients such as DC conductivity, optical conductivity have been computed by considering the near-equilibrium field theories on the boundary with gravity dual in the bulk. One can perturb the boundary by a time-dependent field with frequency $\omega$ to obtain the optical conductivity [1, 3]. However, under this approach, when getting the DC conductivity with the limit $\omega \to 0$, one will confront the divergence due to the spatial translation invariance of the homogeneous gravitational backgrounds involved. Unfortunately, it is well-known that in the real materials, the spatial translation invariance is not preserved i.e. the momentum are not conserved because of the presence of impurities and lattices.

So to extract the finite DC conductivity holographically, many approaches to breaking of the spatial translation invariance in the bulk have been employed. There are basically two kinds of translational symmetry breaking: One is to introduce the lattices [4–12], part of which relies on the complicated numerical computation technic in solving PDE, or massive term [13–20] or spatial scalar fields [21–24] in the gravitational background by hand. Another way is to spontaneously break the translational invariance by introducing Chern-Simons term [25] or pseudo-scalar [26].

Recently, a new approach to calculation of the DC conductivity have been developed in [27, 28]. This approach does not rely on the zero frequency limit, but rather than a time-independent electric field as perturbation on the boundary. The DC conductivities can be obtained in terms of the horizon data by analysing regularity conditions to the holographic model where the momentum dissipation is due to linear spatial scalar fields. Further discussions on the holographic massive gravity theory and Einstein-Maxwell theory with inhomogeneous, periodic lattices have been studied in [29] and [30] respectively.

In this paper, we generalize the strategy presented in [27, 28] to calculate the DC conductivities in five-dimensional Einstein-Gauss-Bonnet-Maxwell-linear scalar field theory with momentum dissipation. The Gauss-Bonnet (GB) term in string effective action appears as the first curvature stringy correction to Einstein-Hilbert action when considering the semi-
classical effect, then the higher order terms is dual to the finite corrections to the 1/N expansion of field theory on the boundary [31, 32]. So in the framework of AdS/CMT, it is interesting to investigate the holographic conductivity of the quantum field theories with the higher derivative gravity dual before the string theory is fully understood [33–36]. Furthermore, to obtain the finite DC conductivities, we will also introduce spatially dependent massless field leads to the momentum dispassion [21]. Since we will focus on the isotropic bulk metric, we shall include three scalar fields that are linear in all the spatial directions. The anisotropic solutions with one linear axion have been studied in [37–40], and the discussions for condensed matter with the anisotropic black brane dual can be found in [24, 28, 41].

This paper is organised as follows. In section 2, we present the exact solution for Einstein-Gauss-Bonnet-Maxwell gravity with linear scalar fields. Then, following [28], we calculate the DC electrical conductivity \( \sigma \), thermal conductivity \( \bar{\kappa} \) and thermoelectric \( \alpha \) in terms of horizon data in section 3. The conclusions are presented in section 4.

## 2 Black brane solutions in Einstein-Maxwell-Gauss-Bonnet gravity with linear scalar fields

We begin with the following five-dimensional action of Einstein-Maxwell-Gauss-Bonnet gravity with three scalar fields.

\[
S = \frac{1}{2\kappa^2} \int_M d^5x \sqrt{-g} \left( R - 2\Lambda + \tilde{\alpha} \mathcal{L}_{GB} - \frac{1}{2} \sum_{i=1}^{3} (\partial \phi_i)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right),
\]

where \( 2\kappa^2 = 16\pi G_5 \) is the five-dimensional gravitational coupling and \( \Lambda = -6 \) is cosmological constant. \( \tilde{\alpha} \) is Gauss-Bonnet coupling constant with dimension (length)\(^2\) and

\[
\mathcal{L}_{GB} = (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2)
\]

is Gauss-Bonnet term \(^1\). \( \phi_i(x^\mu) (i = 1, 2, 3) \) are 3 massless scalar fields and \( U(1) \) gauge field strength is defined as \( F_{\mu\nu} = (dA)_{\mu\nu} \).

\(^1\)We follow the conventions of curvatures as [42]
The equations of motion are easily obtained as
\[
\nabla_\mu F^{\mu\nu} = 0,
\]
\[
\nabla_\mu \nabla^\mu \phi_i = 0,
\]
\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} \left( R - 12 + \tilde{\alpha}(R^2 - 4R_{\rho\sigma}R^{\rho\sigma} + 4R_{\lambda\rho\sigma\tau}R^{\lambda\rho\sigma\tau}) \right)
+ \tilde{\alpha} \left( 2RR_{\mu\nu} - 4R_{\mu\rho}R_{\nu}^\rho - 4R_{\mu\nu\rho\sigma}R^{\rho\sigma} + 2R_{\mu\rho\sigma\lambda}R_{\nu}^{\rho\sigma\lambda} \right)
- \sum_{i=1}^{3} \left( \frac{1}{2} \partial_\mu \phi_i \partial_\nu \phi_i - \frac{g_{\mu\nu}}{4}(\partial \phi_i)^2 \right)
- \frac{1}{2} \left( F_{\mu\lambda}F^{\nu_\lambda} - \frac{g_{\mu\nu}}{4}F_{\lambda\rho}F^{\lambda\rho} \right) = 0.
\] (3)

We will consider homogeneous and isotropic charged black brane solutions, and then work with the following planar symmetric ansatz:
\[
ds^2 = -f(r)N^2 dt^2 + \frac{1}{f(r)} dr^2 + r^2(dx^2 + dy^2 + dz^2),
\] (4)

where the UV boundary is defined as \( r \to \infty \) and \( N \) is a constant which can be fixed shortly by requiring that the geometry of the spacetime should asymptotically approach to the conformally flat metric at the UV boundary.

To obtain the metric homogeneous, we also assume the scalar fields are linearly dependent on the three spatial coordinates:
\[
\phi_i(x^\mu) = a_i x + b_i y + c_i z
\] (5)

and gauge field as
\[
A = A_t(r) dt.
\] (6)

So the Maxwell equations and Einstein equations can be solved exactly
\[
A_t(r) = \mu \left( 1 - \frac{r_H^2}{r^2} \right),
\]
\[
f(r) = \frac{r^2}{4\tilde{\alpha}} \left( 1 - \sqrt{1 - 8\tilde{\alpha} + \frac{2\beta^2\tilde{\alpha}}{r^2} - \frac{2\beta^2 \alpha r_H^2}{r^4} + \frac{8\alpha r_H^4}{3N^2 r^4} + \frac{8\alpha r_H^4 \mu^2}{3N^2 r^6} + \frac{8\alpha \mu^4}{3N^2 r^6}} \right),
\] (8)

where \( \mu \) is the chemical potential of the dual field theory on the boundary, \( r_H \) is the black brane horizon i.e. \( f(r_H) = 0 \). The positive constant \( \beta^2 = \sum_{i=1}^{3} a_i^2 = \sum_{i=1}^{3} b_i^2 = \sum_{i=1}^{3} c_i^2 \) and the constants \( \{a_i, b_i, c_i\} \) are satisfy \( \sum_{i=1}^{3} a_i b_i = \sum_{i=1}^{3} b_i c_i = \sum_{i=1}^{3} c_i a_i = 0 \).
The temperature can be evaluated directly from the Euclidean continuation of the metric (4), that is
\[ T = \frac{N f'(r_\text{H})}{4\pi} = \frac{6N^2 r_\text{H}^2 - \mu^2}{6\pi N r_\text{H}} - \frac{\beta^2 N}{8\pi r_\text{H}}. \] (9)

Since the entropy of GB black hole satisfies the area formula, from the Bekenstein-Hawking entropy formula, we obtain the entropy density of horizon
\[ s = \frac{r_\text{H}^3}{4G_5}. \] (10)

Finally, we discuss the UV and IR behavior of the solution. First, near the UV boundary \( r \to \infty \), to guarantee that the spacetime is asymptotically conformally flat, we set \( N^2 = \frac{1}{2} (1 + \sqrt{1 - 8\tilde{\alpha}}) \) and assume \( \tilde{\alpha} \leq \frac{1}{8} \). Note that the Einstein limit is obtained by taking the limit \( \tilde{\alpha} \to 0 \), in which the solution (4) reduces to the metric of [21]. To understand the geometry near horizon, we define a new coordinate \( u \),
\[ r - r_\text{H} = \frac{3N r_\text{H}^2}{4(3N^2 r_\text{H}^2 + \mu^2)u}, \] (11)

then at \( T = 0 \), one can readily check that the extremal black brane geometry is topologically equivalent to \( \text{AdS}_2 \times \mathbb{R}^3 \):
\[ ds^2 = \frac{L^2}{u^2} (-dt^2 + du^2) + r_\text{H}^2 (dx^2 + dy^2 + dz^2), \] (12)

where \( L \) is the curvature radius of \( \text{AdS}_2 \):
\[ L \equiv \sqrt{\frac{3\beta^2 N^2 + 4\mu^2}{12N^2(\beta^2 N^2 + 4\mu^2)}}. \] (13)

So we can see that in the absence of \( U(1) \) gauge field, the extremal black brane geometry can still be achieved, and the radius of \( \text{AdS}_2 \) will be bigger with the increase of Gauss-Bonnet coupling constant \( \tilde{\alpha} \).

3 DC conductivities

In this section, we will evaluate the DC electrical conductivity \( \sigma \), thermal conductivity \( \bar{\kappa} \) and thermoelectric conductivity \( \alpha \) in terms of horizon data.
3.1 Electric conductivity

In order to compute the conductivities, we consider the perturbations as follows:

\[
\begin{align*}
g_{tx} &\to \delta g_{tx}(r) \\
g_{rx} &\to r^2 \delta g_{rx}(r) \\
A_x &\rightarrow -Et + \delta A_x(r) \\
\phi_1 &\to a_1 x + \delta \phi(r) + b_1 y + c_1 z. 
\end{align*}
\]

Then linearizing the Maxwell equation, Einstein equations and Klein-Gordon equation, we can obtain four independent equations of perturbations:

\[
\begin{align*}
\delta A''_x + \left( \frac{f'}{f} + \frac{1}{r} \right) \delta A'_x + \frac{2E \mu r^2}{N^2 f r^3} \left( \delta g'_{tx} - \frac{2 \delta g_{tx}}{r} \right) &= 0, \\
\delta \phi' - \frac{\beta^2}{a_1} \delta g_{rx} - \frac{2E \mu r^2}{a_1 N^2 f r^3} &= 0, \\
\delta g''_{tx} + \frac{r^2 + 4\tilde{\alpha}(f - rf')}{r(r^2 - 4\tilde{\alpha} f)} \delta g'_{tx} + \frac{8\tilde{\alpha} f f' - r(4f + \beta^2)}{r f(r^2 - 4\tilde{\alpha} f)} \delta g_{tx} + \frac{2E \mu r^2}{r(r^2 - 4\tilde{\alpha} f)} \delta A'_x &= 0, \\
\delta \phi'' + \frac{3f + rf'}{rf} \delta \phi' - \frac{(3a_1 f + a_1 rf') \delta g_{rx}}{rf} - a_1 \delta g'_{rx} &= 0,
\end{align*}
\]

where the prime denotes derivatives with respect to \( r \). Note that for the special case of \( a_1^2 = b_2^2 = c_3^2 = \beta^2 \), Einstein equation (16) implies the equation of motion for \( \delta \phi \) (18).

From the (15), one can obtain a radially conserved current

\[
J = -\sqrt{-g} F^{rx} = -N r f \delta A'_x - \frac{2E \mu r^2}{N^2 f} \delta g_{tx},
\]

which is a constant. The Einstein equation (16) simply gives

\[
\delta g_{rx} = -\frac{2E \mu r^2}{\beta^2 N^2 f r^3} + \frac{a_1 \delta \phi'}{\beta^2}.
\]

Then it is straightforward to see that the equation of motion for \( \delta \phi \) can be simplified as

\[
\frac{(3f + rf')}{rf} \delta \phi' + \delta \phi'' = 0.
\]

To completely determine the solution of perturbation equations, we also need to impose the boundary condition for fluctuation. First, near the UV boundary \( r \to \infty \), the scaler field
perturbation $\delta \phi'$ should be regular, since $\frac{3f+r f'}{r}$ $\sim r^{-1}$ as $r \to \infty$ which is consistent with (21). $\delta g_{tx}$ behaves as $r^{-1}$ which can be seen from (17).

Now we consider the asymptotic behavior near the horizon $r = r_H$. Since we consider the boundary condition at the future horizon, we will use ingoing Eddington-Finklestein coordinates $(v, r)$ defined as $v = t + \int \frac{dr}{f(r)N}$ here.

First, the gauge field should be regular at the future horizon, which means that $A_x \sim -Ev + ....$ So from the (14), we conclude that $\delta A_x$ should satisfies

$$\delta A_x \sim -E \frac{1}{4\pi T} \log(r - r_H) + O(r - r_H)$$

(22)

near horizon $r = r_H$. On the other hand, it is easy to see that the singular part of the metric (4) can be expressed as

$$2\delta g_{tx} dv dx - \frac{2\delta g_{tx}}{N f(r)} dr dx + 2r^2 \delta h_{rx} dr dx$$

(23)

in the ingoing Eddington-Finklestein coordinates. We can see from (20) that $\delta g_{rx} \sim \frac{1}{r - r_H}$ is divergence as $r \to r_H$, so to get the metric non-singular at the horizon, we should require the metric perturbation behaves as

$$\delta g_{tx} \sim Nr^2 f \delta g_{rx} \bigg|_{r \to r_H}$$

$$= -\frac{2E \mu r_H^2}{\beta^2 N r} \bigg|_{r \to r_H} + O(r - r_H).$$

(24)

Note that we have used the assumption of $\delta \phi$ is regular at the horizon.

Since electric current $J$ is radial conserved, the DC electric conductivity can easily obtained by evaluation of the (19) at the horizon:

$$\sigma = \frac{\partial J}{\partial E}$$

$$= \left( r + \frac{4\mu^2 r_H^4}{\beta^2 N^2 r^3} \right) \bigg|_{r \to r_H}$$

$$= r_H + \frac{4\mu^2 r_H}{\beta^2 N^2}$$

$$= \frac{\pi T}{2N} + \frac{2\pi T \mu^2}{\beta^2 N^3} + \frac{\sqrt{3} \beta^2 N^2 + 6\pi^2 T^2 + 4\mu^2}{2\sqrt{6N}} + \frac{\mu^2 \sqrt{6\beta^2 N^2 + 12\pi^2 T^2 + 8\mu^2}}{\sqrt{3} \beta^2 N^3}.$$  

Note that, in the Einstein limit $\tilde{\alpha} \to 0$, it will reproduce the result of [21].
As a demonstration, we plot the conductivity as a function of temperature in Fig 1, which reflects that the ground state of the model we are studying is not an insulator. It behaves more like semiconductors, since for semiconductors, there are insufficient mobile carriers at low temperatures and resistance is high; but as one heats the material, more and more of the lightly bound carriers escape and become free to conduct. However for normal metals there are plenty of mobile carriers and the motion of the lattice atoms due to thermal energy causes them to interfere with the transport of mobile carriers through the lattice. Thus, the conductivity of metals decreases as temperature goes up. We can see from Fig.1 that what we obtained does not correspond to normal metals.

![Figure 1: σ as a function of temperature T, scalar parameter β, Gauss-Bonnet coupling constant ˜α respectively.](image)

(a) σ as a function of the temperature with β = μ = 0.1, N = 0.9. (b) σ as a function of β with N = 0.9. The lines correspond to μ = β = μ = 0.1, T = 1 (blue); μ = 1, T = 1 (red); μ = 1, T = 0.1 (yellow).

The dependence of conductivity on β are shown in Fig.1 (b), in which we can see that in the small β limit, σ ∝ 1/β^2 means that it is dominated by the coherent phase. But as β becomes larger, σ ∝ β implies that the contribution of the pair production becomes stronger, leading to an incoherent phase [22]. This phenomena strongly signals that there is a competition effect between the Drude conductivity and conductivity due to pair creation at the horizon.

The dependence of conductivity on the Gauss-Bonnet coupling constant ˜α is shown in Fig.1 (c). The upper bound of the Gauss-Bonnet coupling constant and its relation with the causality has been investigated in [43–48]. We can see that the DC conductivity increases as the Gauss-Bonnet constant becomes bigger.
Now let us examine the behaviour of electric conductivity at low temperature. It is easy to see that, in the limit of $T \ll \mu$, the $\sigma$ behaves as

$$\sigma = \frac{(\beta^2 N^2 + 4 \mu^2)\sqrt{3\beta^2 N^2 + 4 \mu^2}}{2\sqrt{6}\beta^2 N^3} + \frac{\pi (\beta^2 N^2 + 4 \mu^2)}{2\beta^2 N^3} T + ...,$$

which means that the electric conductivity $\sigma$ is finite as $T \to 0$, indicating the metallic behaviour. On the other hand, for the case $T \gg \mu$, we have

$$\sigma = \frac{(k^2 N^2 + 4 \mu^2)\pi}{N^3 k^2} T + \frac{3N^4 k^4 + 16N^2 k^2 \mu^2 + 16 \mu^4}{24\pi N^3 k^2} T + ....$$ 

To obtain the transport coefficient $\bar{\alpha}$, we will use a two-form [28]:

$$H^{\rho\nu} = \nabla^\rho K^\nu + \frac{1}{3} K^{[\rho} F^{\nu] \lambda} A_\lambda + \frac{1}{6} (\psi - 3\theta) F^\rho\nu,$$

where $K^\mu$ is the Killing vector. Obviously, the metric (4) possesses the Killing vector $K_\mu = \frac{1}{N} \partial_t$, then it is straightforward to check that $\sqrt{-g} H^{tx}$ is conserved. We can deduce that

$$Q \equiv 2 \sqrt{-g} H^{tx} = rf \delta g'_{tx} - rf' \delta g_{tx} - A_t J.$$ 

As discussed in [28], the quantity $Q$ should be identical to the heat current in the $x$-direction via calculation of holographic stress tensor [51–53]. Note that $Q$ is independent of $r$, so after evaluation at the horizon, one can obtain $Q = \frac{8E\pi Tr_\mu}{kN^2}$, then $\bar{\alpha} = \frac{\partial Q}{\partial E}$ is given by

$$\bar{\alpha} = \frac{8\pi^2 r_\mu}{\beta^2 N^2} = \frac{\pi \mu}{N^2} + \frac{4\pi^3 T^2 \mu}{\beta^2 N^4} + \frac{4\pi \mu^3}{3\beta^2 N^4} + \frac{2\pi^2 T \mu \sqrt{6\beta^2 N^2 + 12\pi^2 T^2 + 8\mu^2}}{\sqrt{3}\beta^2 N^4}.$$ 

### 3.2 Thermal and thermoelectric conductivities

To compute the thermoelectric and thermal conductivities, as in [28], we consider the fluctuations as follows:

$$g_{tx} \to t \delta h(r) + \delta g_{tx}(r)$$
$$g_{rx} \to r^2 \delta g_{rx}(r)$$
$$A_x \to t \delta a(r) + \delta A_x(r)$$
$$\phi_1 \to a_1 x + \delta \phi(r) + b_1 y + c_1 z.$$ 

(31)
Then, similarly, the linearised Maxwell equation involves a conserved current
\[ J = -\frac{2r^2 H_r \mu (\delta g_{tx} + t \delta h) + N^2 r^3 f (t \delta a' + \delta A'_x)}{N r^2}, \] (32)
and the linearised Einstein equation gives
\[ \delta g_{rx} = \frac{2r^2 \mu \delta a}{\beta^2 N^2 r^3 f} + \frac{(r^2 - 4\tilde{\alpha} f)(r \delta h' - 2 \delta h)}{\beta^2 N^2 r^3 f} + \frac{a_1 \delta \phi'}{\beta^2}. \] (33)

As the last section, the heat current is also obtained as
\[ Q = 2 \sqrt{-g} H_{rx} \]
\[ = rf \delta g'_{tx} - r \delta g_{tx} f' + rt f \delta h' - rt \delta h f' - A_t j. \] (34)

To get the transport coefficients \( \alpha \) and \( \bar{\kappa} \), we suppose \( \delta h(r) = -CNf(r) \) and \( \delta a(r) = -E + \frac{C}{N} A_t(r) \) which can cancel the time-dependent terms of the conserved current \( J \) and \( Q \).

To find the behaviours of the perturbations near the horizon, we switch to Kruskal coordinates \((U, V)\) instead, which are defined as \( U = -e^{-f(r) N u/2} \) and \( V = e^{f(r) N v/2} \). Similar to the last section, for the purpose of the metric regularity at the horizon, the perturbation at the horizon should be required as
\[ \delta A_x \sim -\frac{E}{4\pi T} \log(r - r_H) + O(r - r_H), \] (35)
\[ \delta g_{tx} \sim N r^2 f \delta g_{rx} \mid_{r \to r_H} - \frac{CNf}{4\pi T} \log(r - r_H) + O(r - r_H). \] (36)

Note that the positive sign in the first term of (36) is chosen to be satisfied the equation for \( \delta g_{tx} \).

Now the \( \alpha \) and \( \bar{\kappa} \) can be easily obtained. First, because \( J \) and \( Q \) are constants in \( r \) direction, then evaluating the two conserved currents (32) and (34) at the horizon, we get
\[ J = Er_H + \frac{8C \pi T \mu \nu_H^2}{\beta^2 N^2} + \frac{4E \mu^2 r_H}{\beta^2 N^2}, \] (37)
\[ Q = \frac{8\pi ET \mu \nu_H^2}{\beta^2 N^2} + \frac{16C \pi^2 T^2 r_H^3}{\beta^2 N^2}. \] (38)
Consequently, the conductivities $\alpha$ and $\bar{\kappa}$ are given by

$$\alpha = \left. \frac{1}{T} \frac{\partial J}{\partial C} \right|_{\infty} = \frac{8\pi \mu r^2}{\beta^2 N^2}$$

$$= \frac{\pi \mu}{N^2} + \frac{4\pi^3 T^2 \mu}{\beta^2 N^4} + \frac{4\pi^3}{3\beta^2 N^4} + \frac{2\pi^2 T \mu \sqrt{6\beta^2 N^2 + 12\pi^2 T^2 + 8\mu^2}}{\sqrt{3\beta^2 N^4}} ,$$

$$\bar{\kappa} = \left. \frac{1}{T} \frac{\partial Q}{\partial C} \right|_{\infty} = \frac{16\pi^2 T r H^3}{\beta^2 N^2}$$

$$= \frac{3\pi^3 T^2}{N^3} + \frac{8\pi^5 T^4}{\beta^2 N^5} + \frac{4\pi^3 T^2 \mu^2}{\beta^2 N^5} + \frac{\pi^2 T \sqrt{3\beta^2 N^2 + 6\pi^2 T^2 + 4\mu^2}}{\sqrt{5 N^3}}$$

$$+ \frac{2\pi^2 T (6\pi^2 T^2 + \mu^2) \sqrt{2\beta^2 N^2 + 4\pi^2 T^2 + \frac{8}{3} \mu^2}}{3\beta^2 N^5} .$$

At low temperature, these transport coefficients behave as

$$\alpha = \left( \frac{\pi \mu}{N^2} + \frac{4\pi^3}{3 N^4 \beta^2} \right) + \frac{2\pi^2 \mu \sqrt{6 N^2 \beta^2 + 8\mu^2}}{\sqrt{3 N^4 \beta^2}} T + ... ,$$

$$\bar{\kappa} = \frac{\pi^2 (3N^2 \beta^2 + 4\mu^2)^{3/2}}{3 \sqrt{6 N^5 \beta^2}} T + ... ,$$

while at high temperature, the behaviour is

$$\alpha = \frac{8\pi^3 \mu}{\beta^2 N^4} T^2 + \left( \frac{2\pi \mu}{N^2} + \frac{8\pi^3 \mu}{3 N^4 \beta^2} \right) - \frac{\mu (3N^2 \beta^2 + 4\mu^2)^2}{72\pi N^4 \beta^2} \frac{1}{T^2} ... ,$$

$$\bar{\kappa} = \frac{16\pi^5}{N^5 \beta^2} T^4 + \frac{2\pi^3 (3N^2 \beta^2 + 4\mu^2)}{N^5 \beta^2} T^2 + \frac{(3N^2 \beta^2 + 4\mu^2)^3}{864\pi N^5 \beta^2} \frac{1}{T^2} + ... .$$

So we find that thermoelectric conductivity $\alpha$ is finite at $T = 0$, while thermal conductivity $\kappa = 0$, meaning that a heat gradient does not give rise to transport. The presence of Gauss-Bonnet term increases the thermoelectric and thermal conductivities. Our results implies that we can extend [28] to higher dimensions with higher derivative gravity terms.

### 4 Summary

In this paper, we studied holographic conductivities for the higher derivative gravity with momentum relaxation. We presented a exact solution for Gauss-Bonnet-Maxwell theory with scalar fields. Then we derived analytically the DC electric conductivity, thermal and thermoelectric conductivities of the dual conformal filed on the boundary in the Gauss-Bonnet-Maxwell theory with momentum dissipation. We found that when the Gauss-Bonnet
coupling increases, all the conductivities become bigger. The exact form of the conductivities confirmed that the ansatz given in [28] is applicable even in Gauss-Bonnet gravity in AdS space.

Different from the conductivities discussed in [22], the DC electric conductivity derived in this paper is temperature dependent and basically it increases as the temperature goes up. The DC electric conductivity does not vanish even at \( T \to 0 \) limit. In our case, at \( T = 0 \) the black brane approaches \( AdS_2 \times \mathbb{R}^3 \) in the far IR with non-vanishing entropy density. This reflects that the ground states of our system are semiconductors or bad metals. The electric conductivity at zero temperature might be regarded as arising from charged particle-hole pairs evolution.

It is our interests for the future task to work on the viscosity bound and causality problem in this linear scalar fields modified Gauss-Bonnet theory. There are some very recent works on viscosity bound in anisotropic superfluid [49] and backreaction effects [50] in higher derivative gravity. We expect that the presence of the linear scalars may contribute some physics more interesting that would greatly change the causal structure of the boundary theory and the upper and lower bounds of the Gauss-Bonnet coupling constant.

Acknowledgments

We thank R. G. Cai, J. X. Lu, Y. Ling, K.-Y. Kim, S. J. Sin and J. B. Wu for useful discussions at the early stage of this work. This work was partly supported by NSFC, China (No.11375110).

References

[1] S.A.Hartnoll, *Lectures on holographic methods for condensed matter physics*; arXiv:0903.3246[hep-th]

[2] J. McGreevy, *Holographic duality with a view toward many-body physics*, arXiv:0909.0518[hep-th]
[3] C. P. Herzog, *Lectures on Holographic Superfluidity and Superconductivity*, arXiv:0904.1975 [hep-th]

[4] G. T. Horowitz, J. E. Santos and D. Tong, *Optical Conductivity with Holographic Lattices*, JHEP 1207, 168 (2012), [arXiv:1204.0519 [hep-th]].

[5] G. T. Horowitz, J. E. Santos and D. Tong, *Further Evidence for Lattice-Induced Scaling*, JHEP 1211, 102 (2012), [arXiv:1209.1098 [hep-th]].

[6] G. T. Horowitz and J. E. Santos, *General Relativity and the Cuprates*, arXiv:1302.6586 [hep-th].

[7] A. Donos, S. A. Hartnoll, *Metal-insulator transition in holography*, [arXiv:1212.2998 [hep-th]].

[8] J. Erdmenger, X. H. Ge and D. W. Pang, *Striped phases in the holographic insulator/superconductor transition*, JHEP 1311 (2013) 027 [arXiv:1307.4609]

[9] Y. Ling, C. Niu, J. P. Wu, Z. Y. Xian, H. Zhang, *Holographic Lattice in Einstein-Maxwell-Dilaton Gravity*, JHEP 11(2013)006, [arXiv:1309.4580].

[10] Y. Ling, C. Niu, J. Wu, Z. Xian and H. Zhang, “Metal-insulator Transition by Holographic Charge Density Waves,” Phys. Rev. Lett. 113 (2014) 091602 [arXiv:1404.0777 [hep-th]].

[11] A. Donos and J. P. Gauntlett, *Holographic Q-lattices*, JHEP 1404(2014) 040, [arXiv:1311.3292 [hep-th]]

[12] Yi Ling, Peng Liu, Chao Niu, Jian-Pin Wu, Zhuo-Yu Xian, *Holographic Superconductor on Q-lattice*, arXiv:1410.6761 [hep-th].

[13] D. Vegh, *Holography without translational symmetry*, arXiv:1301.0537[hep-th].

[14] B. Goutéraux, *Charge transport in holography with momentum dissipation*, JHEP04(2014)181 [arXiv:1401.5436 [hep-th]].
[15] R. A. Davison, *Momentum relaxation in holographic massive gravity*, Phys.Rev. D88 (2013) 086003, [arXiv:1306.5792 [hep-th]].

[16] M. Blake and D. Tong, *Universal Resistivity from Holographic Massive Gravity*, Phys.Rev D88 (2013) 106004, [arXiv:1308.4970 [hep-th]].

[17] M. Blake, D. Tong, and D. Vegh, *Holographic Lattices Give the Graviton a Mass*, Phys.Rev.Lett. 112(2014) 071602, [arXiv:1310.3832 [hep-th]].

[18] Hua Bi Zeng, Jian-Pin Wu, *Holographic superconductors from the massive gravity*, Phys. Rev. D 90, 046001 (2014), [arXiv:1404.5321 [hep-th]].

[19] R. A. Davison, K. Schalm, and J. Zaanen, *Holographic duality and the resistivity of strange metals*, Phys.Rev. B89(2014) 245116, [arXiv:1311.2451 [hep-th]]

[20] M. Blake and A. Donos, *Quantum Critical Transport and the Hall Angle*, arXiv:1406.1659 [hep-th]

[21] T. Andrade and B. Withers, *A simple holographic model of momentum relaxation*, arXiv:1311.5157 [hep-th].

[22] K. Y. Kim, K. K. Kim, Y. Seo and S.J. Sin, *Coherent/incoherent metal transition in a holographic model*, arXiv:1409.8346[hep-th]

[23] A. Donos, B. Goutraux and E. Kiritsis, *Holographic metals and insulators with helical symmetry*, JHEP 09 (2014) 038 [arXiv:1406.6351]

R.A. Davison, B. Goutraux, *Momentum dissipation and effective theories of coherent and incoherent transport*, arXiv:1411.1062[hep-th]

[24] X. H. Ge, Y. Ling and S. J. Sin, work in progress

[25] S. Nakamura, H. Ooguri, C-S. Park, *Gravity Dual of Spatially Modulated Phase*, Phys.Rev.D81:044018,2010, [arXiv:0911.0679[hep-th]].

[26] A. Donos and J. P. Gauntlett *Holographic striped phases*, JHEP08(2011)140, [arXiv:1106.2004[hep-th]].
[27] A. Donos and J. P. Gauntlett, *Novel metals and insulators from holography*, JHEP06(2014)007, arXiv:1401.5077 [hep-th].

[28] A. Donos, J P. Gauntlett, *Thermoelectric DC conductivities from black hole horizons*, arXiv:1406.4742 [hep-th].

[29] A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, D. Musso, *Analytic DC thermoelectric conductivities in holography with massive gravitons*, arXiv:1406.4742 [hep-th].

[30] A. Donos, J P. Gauntlett, *The thermoelectric properties of inhomogeneous holographic lattices*, arXiv:1409.6875 [hep-th].

[31] R G Cai, *Gauss-Bonnet Black Holes in AdS Spaces*, Phys.Rev.D65:084014,2002, arXiv:hep-th/0109133.

[32] M. Cvetic, S. Nojiri, S.D. Odintsov, *Black Hole Thermodynamics and Negative Entropy in deSitter and Anti-deSitter Einstein-Gauss-Bonnet gravity*, Nucl.Phys.B628:295-330,2002,arXiv:hep-th/0112045

[33] R-G Cai, Z-Y Nie, H-Q Zhang, *Holographic p-wave superconductors from Gauss-Bonnet gravity*, Phys. Rev. D 82, 066007 (2010) [arXiv:1007.3321].

[34] L. Barclay, R. Gregory, S. Kanno, P. Sutcliffe, *Gauss-Bonnet Holographic Superconductors*, JHEP 1012:029,2010 [arXiv:1009.1991].

[35] J. Jing, L. Wang, Q. Pan, S. Chen, *Holographic Superconductors in Gauss-Bonnet gravity with Born-Infeld electrodynamics*, Phys.Rev.D83:066010,2011 [arXiv:1012.0644].

[36] Q. Pan, J.Jing, B. Wang, *Analytical investigation of the phase transition between holographic insulator and superconductor in Gauss-Bonnet gravity*, JHEP 11 (2011) 088 [arXiv:1105.6153].

[37] D. Mateos and D. Trancanelli, *Thermodynamics and instabilities of a strongly coupled anisotropic plasma*, JHEP 1107 (2011) 054 [arXiv:1106.1637[hep-th]].

[38] L. Cheng, X. H. Ge, S.J. Sin, *Anisotropic plasma with a chemical and schemenidenpendent instabilities*, Phys. Lett. B 734 (2014) 116 [arXiv:1404.1994[hep-th]].
[39] L. Cheng, X.H. Ge and S. J. Sin, *Anisotropic plasma at finite U(1) chemical potential*, JHEP 07 (2014) 083 [arXiv:1404.5027[hep-th]]

[40] V. Jahnke, A. S. Misobuchi and D. Trancanelli, *Holographic renormalization and anisotropic black branes in higher curvature gravity*, [arXiv:1411.5964[hep-th]]

[41] L. Q. Fang, X.H. Ge, J. P. Wu and H. Q. Leng, *Anisotropic Fermi surface from holography*, [arXiv:1409.6062[hep-th]]

[42] S. Carroll, *Spacetime and Geometry*, Addision-Wesley Reading, MA, (2004).

[43] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *Viscosity bound violation in higher derivative gravity*, Phys. Rev. D 77 (2008) 126006 [arXiv:0712.0805]

[44] X.-H. Ge and S.-J. Sin, *Shear viscosity, instability and the upper bound of the Gauss-Bonnet coupling constant*, JHEP 05 (2009) 051, [arXiv:0903.2527[hep-th]]

[45] R. G. Cai, Z. Y. Nie and Y. W. Sun, *Shear Viscosity from Effective Couplings of Gravitons*, Phys. Rev. D 78 (2008) 126007 [arXiv:0811.1665[hep-th]].

[46] X.H. Ge, S. Sin, S.Wu, and G.Yang, *Shear viscosity and instability from third order Lovelock gravity*, Phys. Rev. D, 80,104019 (2009), [arXiv:0905.2675[hep-th]]

[47] R. C. Myers, M. F. Paulos and A. Sinha, *Holographic Hydrodynamics with a Chemical Potential*, JHEP 0906, 006 (2009) [arXiv:0903.2834[hep-th]].

[48] X. H. Ge, Y. Matsuo, F.-W. Shu, S.-J. Sin and T. Tsukioka, *Viscosity Bound, Causality Violation and Instability with Stringy Correction and Charge*, JHEP 0810 (2008) 009, [arXiv:0808.2354[hep-th]]

[49] A. Bhattacharyya and D. Roychowdhury, *Viscosity bound for anisotropic superfluids in higher derivative gravity*, [arXiv:1410.3222[hep-th]]

[50] L. K. Joshi and P. Ramadevi, *Backreaction effects due to matter coupled higher derivative gravity*, [arXiv:1409.8019[hep-th]]
[51] V. Balasubramanian, P. Kraus, *A Stress Tensor for Anti-de Sitter Gravity*, Commun.Math.Phys. 208 (1999) [arXiv:hep-th/9902121]

[52] Y. Brihaye, E. Radu, *Black objects in the Einstein-Gauss-Bonnet theory with negative cosmological constant and the boundary counterterm method*, JHEP0809:006, 2008 [arXiv:0806.1396]

[53] J. T. Liu, W. A. Sabra, *Hamilton-Jacobi Counterterms for Einstein-Gauss-Bonnet Gravity*, arXiv:0807.1256[hep-th]