The nature of the baryon resonances is one of the important issues in hadron physics [1, 2]. There are many facilities, such as BESIII, LHCb, Belle, et al., that have presented a lot of information about baryon resonances, which provide a good platform to extract the baryon properties. On the other hand, the theoretical work goes parallel, most of the existing states can be well described, and some predictions of the effective theories have been confirmed experimentally.

One of the important predictions is the existence of two poles of the $\Lambda(1405)$ state, which were first reported in Ref. [3], discussed in detail in Ref. [4] and later on confirmed in all theories using the chiral unitary approach [5–17]. The experimental confirmation came from the work of Ref. [18] and the analysis of Ref. [19], but other experiments have come to confirm it too (see Refs. [20, 21] for more details). The low $\Lambda(1405)$ mainly couples to the $\pi\Sigma$ channel, and the high $\Lambda(1405)$ couples to the $K\bar{N}$ channel [20, 22]. In addition, many theoretical works about the $\Lambda(1405)$ production induced by photon [20, 22–26], pion [27], kaon [19, 28–32], neutrino [33], proton-proton collision [24, 34, 35], and heavy meson decay [36–39] were carried out to clarify the molecular nature of the $\Lambda(1405)$ resonance. However, it is problematic to extract the $\Lambda(1405)$ in some conventional reactions involving $K\bar{N}$ or $\pi\Sigma$ final states which may mix $I = 0$ and $I = 1$ contributions.

In order to further understand the molecular structure of the $\Lambda(1405)$ state, we investigate the process $\chi_{c0}(1P) \rightarrow \bar{\Lambda}\Sigma\pi$, by considering the final state interaction of $\pi\Sigma$, which will dynamically generate the two poles of the $\Lambda(1405)$ state in the chiral unitary approach. The $\chi_{c0}(1P)$, with $I^G(J^{PC}) = 0^+(0^{++})$, is a $c\bar{c}$ state, and blind to SU(3), hence behaving like an SU(3) singlet. Since the outgoing particle $\bar{\Lambda}$ has isospin $I = 0$, the $\pi\Sigma$ system must have isospin $I = 0$, to combine to the isospin $I = 0$ of the $\chi_{c0}(1P)$. Thus, this process is a good filter of isospin that guarantees that the $\pi\Sigma$ will be in $I = 0$. Besides, the lower $\Lambda(1405)$ state couples strongly to the $\pi\Sigma$ channel, so the $\pi\Sigma$ final state in this process is an ideal channel to study the molecular structure of the $\Lambda(1405)$ state.

The $\pi$ and $\bar{\Lambda}$ can also undergo final state interaction, and the isospin of $\pi\bar{\Lambda}$ system is $I = 1$, which together with the $\Sigma$ gives rise to the isospin of $\chi_{c0}$. It should be noted that a baryon resonance around the $K\bar{N}$ threshold with $J^P = 1/2^-$, strangeness $S = -1$ and isospin $I = 1$ was predicted in the chiral unitary approach [3, 4] (we label this state as $\Sigma(1380)$ in following), and can couple to the $\pi\Lambda$ channel. It was also suggested to search for this state in the process $\chi_{c0} \rightarrow \Sigma\Sigma\pi$ [40]. In addition, a $\Sigma^-$ state with $J^P = 1/2^-$ and $M \sim 1380$ MeV and $\Gamma \sim 120$ MeV (we label this state as $\Sigma(1380)$ in following), has been predicted in the pentaquark picture [41], which were studied in the processes of $J/\psi$ decay [42, 43], $K^–p \rightarrow \Lambda\pi^+\pi^–$ [44, 45], $\Lambda p \rightarrow \Lambda\pi^0$ [46], and $\Lambda^+_c \rightarrow \eta\pi^+\Lambda$ [47], as so on. Thus, the $\chi_{c0}$ can also decay into a $\Sigma$ and the intermediate resonance $\Sigma(1380)$ in $S$-wave, then the $\Sigma(1380)$ go into the $\pi\Lambda$ states in $S$-wave. As a result, for the $S$-wave final state interaction of $\pi\Lambda$, we will consider the mechanism of the coupled channel in the chiral unitary approach, and the intermediate resonance $\Sigma(1380)$ mechanism1. In this way, the shape of the $\pi\Lambda$ mass distribution of this process can be helpful to test the existence of the $\Sigma(1380)$ and $\Sigma(1380)$ resonances, and to search for the unobserved state $\Sigma^*$.

1 We do not consider the contribution of the intermediate $\Sigma(1385)$ with $J^P = 3/2^-$ in $\pi\Lambda$ system, which will be suppressed, because the $\Sigma(1385)$, as the SU(3) anti-decuplet, together with the SU(3) octet state $\Sigma$, cannot give rise the SU(3) singlet state $\chi_{c0}$.
going to the \( 8 \) the SU(3) isoscalar factors in the PDG. We have the representation for \( 8(\pi) \) in the baryon channel, and stand for the weights of the transition, which will give rise to the dynamically generated \( \pi \) \( \pi \) amplitudes, which will be adopted in this work. Because the thresholds of the channels \( \eta \Lambda \) and \( K \Xi \) lay far above the energies that we consider in this work, and the effect can be effectively reabsorbed in the subtraction constants, as discussed in Refs. [20, 22], we do not consider these two channels in following calculations.

In addition to the \( \pi \Sigma \) final state interaction, the states \( \pi \) and \( \Lambda \) also can undergo the final state interaction in the process of \( \chi_{c0} \rightarrow \Lambda \Sigma \pi \), as depicted in Fig. 2. In this case, we get the weights for different channels from Ref. [40],

\[
\begin{align*}
\hat{h}_{K\bar{N}} &= -\sqrt{\frac{3}{8}} \hat{D} + \sqrt{\frac{1}{6}} \hat{F}, \\
\hat{h}_{\pi \Sigma} &= \sqrt{\frac{2}{3}} \hat{F}, \quad \hat{h}_{\pi \bar{\Lambda}} = \sqrt{\frac{1}{5}} \hat{D}, \quad \hat{h}_{\eta \Sigma} = \sqrt{\frac{1}{5}} \hat{D}, \\
\hat{h}_{K \bar{\Xi}} &= -\sqrt{\frac{3}{8}} \hat{D} + \sqrt{\frac{1}{6}} \hat{F},
\end{align*}
\]

where the parameters \( \hat{D} \) and \( \hat{F} \) are unknown parameters.

The interaction of octet pseudoscalar mesons and the octet 1/2+ baryons, which can dynamically generate the \( \Lambda(1405) \), has been studied with the chiral unitary approach in Refs. [3, 6, 48]. In Ref. [22], a new strategy to extract the position of the two poles of \( \Lambda(1405) \) from \( \pi \Sigma \) photo-production experimental data was done, based on small modifications of the unitary chiral perturbation theory amplitude, which will be adopted in this work. Because the thresholds of the channels \( \eta \Lambda \) and \( K \Xi \) lay far above the energies that we consider in this work, and the effect can be effectively reabsorbed in the subtraction constants, as discussed in Refs. [20, 22], we do not consider these two channels in following calculations.

In addition to the \( \pi \Sigma \) final state interaction, the states \( \pi \) and \( \Lambda \) also can undergo the final state interaction in the process of \( \chi_{c0} \rightarrow \Lambda \Sigma \pi \), as depicted in Fig. 2. In this case, we get the weights for different channels from Ref. [40],

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\begin{align*}
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\hat{h}_{\pi \Sigma} &= \sqrt{\frac{2}{3}} \hat{F}, \quad \hat{h}_{\pi \bar{\Lambda}} = \sqrt{\frac{1}{5}} \hat{D}, \quad \hat{h}_{\eta \Sigma} = \sqrt{\frac{1}{5}} \hat{D}, \\
\hat{h}_{K \bar{\Xi}} &= -\sqrt{\frac{3}{8}} \hat{D} + \sqrt{\frac{1}{6}} \hat{F},
\end{align*}
\]

where the parameters \( \hat{D} \) and \( \hat{F} \) are unknown parameters.

As predicted in Refs. [3, 4], the \( \pi \Lambda \) system with isospin \( I = 1 \) can undergo the S-wave final state interaction, which will dynamically generate a cusp structure around the \( K \bar{N} \) threshold, associated to the \( \Sigma(1430) \) resonance. Taking into the S-wave final state interaction, the total
can also couple to \( \bar{\Sigma}(1380) \) state with \( \bar{K} \bar{\Lambda} \Sigma \) channel in the process \( \chi_{c0} \rightarrow \bar{\Lambda} \Sigma \pi \).

The mechanism of the intermediate resonance \( \Sigma(1380) \) in the \( \pi \bar{\Lambda} \) channel in the process \( \chi_{c0} \rightarrow \bar{\Lambda} \Sigma \pi \) is,

\[
\mathcal{M} = V_p \left[ h_{\pi \Sigma} + \sum_i h_i G_i (M_{\pi \Sigma}) t_i, \pi \Sigma (M_{\pi \Sigma}) \right] + \tilde{V}_p \sum_j \tilde{h}_j G_j (M_{\pi \Lambda}) t_j, \pi \Lambda (M_{\pi \Lambda}) \\
= \mathcal{M}_{\text{tree}} + \mathcal{M}_{\pi \Sigma} + \mathcal{M}_{\pi \Lambda}
\]

where the loop function \( G \) and the transition amplitude \( t \) of the \( \pi \Sigma \) final state interaction are taken from Refs. [22, 40]. In this work, we work with \( R = \tilde{F}/D \), and include the weight \( D \) of Eqs. (1) and (2) in the \( V_p \) and \( \tilde{V}_p \) factors, as done in Ref. [40].

In Eq. (3), the amplitude for the tree level of Fig. 1(a) is,

\[
\mathcal{M}_{\text{tree}} = V_p h_{\pi \Sigma}.
\]

If we consider the transition \( \chi_{c0} \rightarrow \Sigma P \bar{B} \) \((P \bar{B} = K \bar{N}, \pi \Sigma, \bar{\Lambda} \eta, K \bar{K})\), the amplitude for the tree level can also be given as,

\[
\mathcal{M}_{\text{tree}} = \tilde{V}_p \tilde{h}_{\pi \bar{\Lambda}}.
\]

Combining the Eqs. (4) and (5), we can obtain that \( \tilde{V}_p = V_p \times (h_{\pi \Sigma}/h_{\pi \bar{\Lambda}}) = -\sqrt{3}V_p \). Now, the amplitude of Eq. (3) can be rewritten as,

\[
\mathcal{M} = V_p \left[ h_{\pi \Sigma} + \sum_i h_i G_i (M_{\pi \Sigma}) t_i, \pi \Sigma (M_{\pi \Sigma}) \right] - \sqrt{3} \sum_j \tilde{h}_j G_j (M_{\pi \Lambda}) t_j, \pi \Lambda (M_{\pi \Lambda}) \\
= V_p (h_{\pi \Sigma} + T_{\pi \Sigma} + T_{\pi \Lambda}).
\]

**B. Contribution from \( \Sigma(1380) \) with \( J^P = 1/2^- \)**

In addition to the \( S \)-wave interaction, the \( \pi \bar{\Lambda} \) states can also couple to \( \Sigma(1380) \) state with \( J^P = 1/2^- \) in

\[
\frac{d\Gamma}{dM^2_{\pi \Sigma} dM^2_{\pi \Lambda}} = \frac{1}{(2\pi)^3 32 M_{\chi_{c0}}^3} |\mathcal{M}(M_{\pi \Sigma}, M_{\pi \Lambda})|^2.
\]

For a given value of \( M^2_{\pi \Sigma} \), the range of \( M^2_{\pi \Lambda} \) is defined as,

\[
(M^2_{\pi \Lambda})_{\text{max}} = (E_\pi^* + E_\Lambda^*)^2 - (\sqrt{E_\pi^2 - m_\pi^2} - \sqrt{E_\Lambda^2 - m_\Lambda^2})^2,
\]

\[
(M^2_{\pi \Lambda})_{\text{min}} = (E_\pi^* + E_\Lambda^*)^2 - (\sqrt{E_\pi^2 - m_\pi^2} + \sqrt{E_\Lambda^2 - m_\Lambda^2})^2,
\]

here \( E_\pi^* = (M_{\pi \Sigma}^2 - M_{\pi \Lambda}^2 + m_\pi^2)/2M_{\pi \Sigma} \) and \( E_\Lambda^* = (M_{\chi_{c0}}^2 - M_{\pi \Sigma}^2 - M_{\pi \Lambda}^2)/2M_{\pi \Sigma} \) are the energies of \( \pi \) and \( \Lambda \) in the rest frame of \( \pi \) and \( \Sigma \) system.

\footnote{Although the \( \Sigma(1380) \) is predicted in pentaquark picture [41], for simplicity, we use the Breit-Wigner form for its contribution, as used in Refs. [44–47].}
Before presenting the results for the process \( \chi_{c0}(1P) \rightarrow \bar{\Lambda}\Sigma \pi \), we show the module squared of the amplitudes \(|t_{K\Lambda,\pi}\|^2\) and \(|t_{\pi\Sigma,\pi}\|^2\) in \( I = 0 \) in Fig. 4, from which we can see that the peak of \( \pi\Sigma \rightarrow \pi\Sigma \) amplitude mainly comes from the the lower pole, while the one of \( KN \rightarrow \pi\Sigma \) amplitude comes from the higher pole. In Fig. 5, we present the module squared of the transition amplitudes \(|t|^{2}\) in \( I = 1 \). As we can see, a clear cusp structure around the \( KN \) threshold is found, same as the Refs. [22, 40].

As we do not know the exact value of \( R = F/D \), and the production weights of \( KN \) and \( \pi\Sigma \) are expected to be same magnitude, we will vary the \( R \) from \(-2\) to \(2\), as done in Ref. [40]. Finally, we take \( R = 1 \), and \( \alpha = 0.06 \) of the normalization in Eq. (8), which gives rise to a sizeable effect of intermediate \( \Sigma(1380) \). In Fig. 6, up to an arbitrary normalization of \( V_{\rho} \), we show the \( \pi\Sigma \) invariant mass distribution, where the term of \( \pi\Sigma \rightarrow \pi\Sigma \) final state interaction (labeled as 'Tree') gives rise to a peak around 1410 MeV, and the peak moves to low energy because of the interference with the tree level term (labeled as 'Tree'). We also present the \( \pi\Sigma \) invariant mass distribution with different values of \( \alpha \) in Fig. 7, which shows that the contribution from the intermediate \( \Sigma(1380) \) resonance does not significantly affect the peak position of the \( \pi\Sigma \) mass distribution.

The \( \pi\Sigma \) invariant mass distribution with different values of \( R \) from \(-2\) to \(2\) is shown in Fig. 8, where we can see that as the ratio \( R \) decreasing, the peak position of the \( \Lambda(1405) \) in the \( \pi\Sigma \) mass distribution moves to the region of low energies, and a cusp structure around \( KN \) threshold appears. Indeed, the value of \( R \) can be larger than \(2\) or less than \(-2\), and the peak of the \( \pi\Sigma \) invariant mass distribution will vary from 1360 MeV, the position of the peak in \( t_{\pi\Sigma,\pi}\), to 1400 MeV, the one in \( t_{K\Lambda,\pi}\), as shown in Fig. 4.

As discussed in Ref. [49], one of the defining features associated to the molecular states that couple to several hadron-hadron channels is that one finds a strong and unexpected cusp at the threshold of the channels...
corresponding to the main component of the molecular state, and one of the examples is the observations of the cusp, associated to the molecular state $X(4160)$, in the $B^+ \to J/\psi K^+$ decay [50]. The peak and the cusp, observed in Fig. 8, should be the important feature to confirm the existence of the $\Lambda(1405)$ in the decay of $\chi_{c0} \to \bar{\Lambda}\Sigma\pi$. Thus, we strongly encourage to measure the $\pi\Sigma$ invariant mass distribution of the $\chi_{c0} \to \bar{\Lambda}\Sigma\pi$ decay.

In order to check the existence of the $\Sigma(1340)$ and $\Sigma(1380)$, we also present the $\pi\Lambda$ invariant mass distribution in Fig. 9, where we can see that there is a clear bump structure around $1380$ MeV, which is associated to the intermediate resonance $\Sigma(1380)$, and a clear cusp structure around the $K\Sigma$ threshold, which comes from the $\pi\Lambda$ final state interaction, as shown in Eq. (6). By varying the value of the normalization $\alpha$ as depicted in Fig. 10, the bump structure of $\Sigma(1380)$ becomes smoother for a smaller $\alpha$, and more clear for a larger one. It should be stressed that the bump structure of $\Sigma(1380)$, or the cusp structure around the $K\Sigma$ threshold, if confirmed experimentally, should be related to the resonance $\Sigma(1380)$ or $\Sigma(1430)$.

In addition, we show the $\pi\Lambda$ invariant mass distributions by varying the ratio $R$ from -2 to 2 in Fig. 11. As we can see that the peak structure of the $\Sigma(1380)$ and the cusp structure around the $K\Sigma$ threshold are always clear for different values of the ratio $R$.

Finally, it should be noted that the SIDDHARTA measurement of kaonic hydrogen gives an accurate constraint on the $K^-p$ scattering length [51], and the interaction kernel with the next-to-leading order chiral perturbation theory is used with the systematic $\chi^2$ analysis in Refs. [13, 15, 16]. However, as a motivation for measuring this process experimentally, we use the model of the final state interaction of $\pi\Sigma$ of $I = 0$ developed in Ref. [22], which can also produce the main feature of the amplitudes of $\pi\Sigma \to K\Sigma$ and $\pi\Sigma \to \pi\Sigma$ given by the more accurate model.

**IV. CONCLUSIONS**

In this paper, we have studied the process $\chi_{c0}(1P) \to \bar{\Lambda}\Sigma\pi$ by taking into account the final state interactions of $\pi\Sigma$ and $\pi\Lambda$ within the chiral unitary approach. As the isospin $I = 0$ filter in the $\pi\Sigma$ system and the isospin $I = 1$ filter in the $\pi\Lambda$ system, this process can be used to study the molecular structure of $\Lambda(1405)$ state, and to search for the predicted states $\Sigma(1380)$ or $\Sigma(1430)$ with $J^P = 1^{-+}$. We have shown that, there is a peak of $1350 \sim 1400$ MeV with $-2 \leq R \leq 2$, and a cusp around the $K\Sigma$ in the $\pi\Sigma$ mass distribution, which should be the important feature the the molecular state $\Lambda(1405)$.

In summary, the process $\chi_{c0} \to \bar{\Lambda}\Sigma\pi$ can be used to study the molecular structure of the $\Lambda(1405)$ resonance, and also to test the existence of the predicted states $\Sigma(1380)$ and $\Sigma(1430)$.

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