Research on Harmonic Resonance Mitigation Based on Improved Modal Sensitivity Analysis

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Abstract. Among the methods used for mitigation of harmonic resonance, modal analysis theory and modal sensitivity can be used to illuminate the nature and properties of this phenomenon, and are thus widely applied. In this paper, an improved modal sensitivity-based method is proposed to reliably suppress the harmonic resonance. First, we show that the current resonance mitigation schemes that are guided by traditional modal sensitivity are not always robust in some cases. Then, through a theoretical analysis, the roots of the shortcomings of the traditional modal sensitivity are determined. The key factor is that the modal sensitivity of a network component is not always constant but tends to vary with adjustment of the component parameters. Finally, to quantify this sensitivity variation and improve the robustness of the analysis results, a second-order modal sensitivity and a critical index are proposed. The results of the test system indicate that the proposed indexes can accurately show the effects of each network component on the resonance, thus are useful in the design of the corresponding mitigation scheme.

1. Introduction

Recently, power electronic converters have been widely used in various industrial fields including HVDC systems, renewable energy sources, and electric railway systems [1][2]. With increasing application of these devices, the harmonic pollution of power systems has been exacerbated significantly, leading to problems such as increased energy losses, line overheating, and reduction of the available capacity of power equipment [3]. This problem becomes even more serious and complex by the occurrence of harmonic resonance [4][5]. Furthermore, power cables with specific capacitive behavior are widely used in city grids and may potentially add the possibilities of both resonance and harmonic amplification [6][7]. To alleviate these harmonic problems, it is crucial to evaluate the nature and the properties of the resonance phenomenon.

The resonance problem can be analyzed by frequency scan analysis or by the modal analysis method [8][9][10]. Frequency scan analysis, as one of the most widely used methods in resonance studies, can identify the existence of resonance and determine the resonance frequency. However, this method cannot offer additional information to mitigate resonance effectively [11]. In comparison, the modal analysis method can assess the effects of various network components on a particular resonance mode, and is thus more useful in the design of a mitigation scheme for the resonance [12].

Notably, modal sensitivity, which can identify the network components most relevant to the resonance, plays an important role in modal analysis theory [13][14][15]. For example, in [14] and [15], based on analysis of the modal sensitivity, resonance alleviation schemes were proposed for railway...
electrification systems that could shift the resonance frequencies to a safe area. Besides, the studies in \cite{16} and \cite{17} presented a modal frequency sensitivity index that can determine each resonance mode and investigate the frequency sensitivity with respect to the network component parameters. In addition, based on this frequency sensitivity, the study in \cite{18} investigated the harmonic problems of a wind park by considering the sensitivities of the various parameters to the frequency and amplitude of the resonance. Furthermore, in \cite{19}, based on the modal sensitivity, a solution for compensating the harmonics in an offshore wind power plant is proposed by using hybrid filters. These researches all indicated the wide applications of modal sensitivity in resonance mitigation analysis.

However, despite the fact that modal sensitivity is effective in guiding resonance suppression, we find in this study that in some situations, the analysis results obtained from this index are not robust. Specifically, sometimes the component with the highest modal sensitivity is not always the most sensitive component, while a component with lower modal sensitivity could be more relevant to the resonance. The main cause of this phenomenon is that the modal sensitivity of a component is not always constant and can change with variation of the component parameters. Remarkably, this flaw in the traditional modal sensitivity can potentially lead to inaccurate evaluation results in the resonance mitigation analysis. However, no existing researches are currently focusing on this question.

Therefore, to improve the robustness of traditional modal sensitivity in resonance analyses, two indexes, called the second-order modal sensitivity and the critical index for variation of the component parameters, are proposed in this paper. The second-order modal sensitivity index can quantify the changes of the traditional modal sensitivity with the variation of the component parameter. In addition, in combination with the proposed critical index, the second-order modal sensitivity can guide resonance mitigation schemes more reliably compared with the traditional index. A three-bus test system is used to validate the proposed indexes.

2. Basic Theory of Modal analysis and modal sensitivity

2.1. Eigenvalue decomposition

When resonance occurs, some elements of the voltage vector given in Eq. (1) have large values.

$$V_f = Y_f I_f,$$

where $V_f$ and $I_f$ are the nodal voltage vector and the nodal current injection vector, respectively. $Y_f$ is the network admittance matrix. $V_f$, $I_f$, and $Y_f$ are discussed at the frequency $f$, which is omitted in the rest of this paper for simplicity.

Resonance usually occurs when $Y$ approaches singularity. To study the properties of this phenomenon, eigenvalue decomposition, which forms the basis of modal analysis, is used to decompose $Y$ as follows:

$$Y = L \Lambda T,$$

where $A = \text{diag}(\lambda_1, \ldots, \lambda_m, \ldots)$ is the diagonal eigenvalue matrix. $L = [l_1, \ldots, l_m, \ldots]$ and $T = [t_1, \ldots, t_m, \ldots]^T$ (which satisfy $L = T^{-1}$) are the left and right eigenvector matrices, respectively. The subscript $m$ represents the $m$-th mode.

By defining $U = TV$ and $J = T I$ and based on Eq. (1) and (2), we obtain

$$U = \Lambda^{-1} J,$$

where $U$ and $J$ are defined as the modal voltage and current vector, respectively. $\Lambda^{-1}$ is defined as the modal impedance.

Eq. (3) reflects that harmonic resonance is more likely to occur when $\lambda_m$ is very small. Therefore, the factors that influence $\lambda_m$ are the focus of this research.

2.2. Modal sensitivity

In modal analysis, the participation factor \cite{8}, defined as $PF_{bm} = l_{bm} t_{mb}$, is a satisfactory index for assessment of the excitability and observability of bus $b$ with respect to mode $m$ resonance. However, this index can only locate to nodes but not network components. To analyze the effect of each component on a specific resonance, another index known as the modal sensitivity \cite{12}\cite{13} was developed.
Based on the previous studies in [13], we define

\[
\begin{align*}
S_m &= l_m t_m \\
S_{mb} &= S_r + jS_i \\
\lambda_m &= \lambda_r + j\lambda_i
\end{align*}
\]  

(4)

where \( S_m \) is the sensitivity matrix at mode \( m \). \( S_{mb} \) represents the element of matrix \( S_m \) at row \( b \) and column \( b \).

The sensitivity indexes of \( |\lambda_m| \) with respect to a parallel component \( (v_{sh} = G + jB) \) located on bus \( b \) are as follows [13]:

\[
\begin{align*}
\frac{\partial |\lambda_m|}{\partial B} &= \frac{S_r \lambda_r - S_i \lambda_i}{\sqrt{\lambda_r^2 + \lambda_i^2}} \\
\frac{\partial |\lambda_m|}{\partial G} &= \frac{S_r \lambda_r + S_i \lambda_i}{\sqrt{\lambda_r^2 + \lambda_i^2}}
\end{align*}
\]  

(5)

For a series component \( (z_{se} = R + jX) \) located between buses \( i \) and \( j \), the sensitivity indexes of \( |\lambda_m| \) are given as [13]:

\[
\begin{align*}
\frac{\partial |\lambda_m|}{\partial X} &= \frac{-2\mu RX + v(X^2 - R^2)}{(R^2 + X^2)^2} \\
\frac{\partial |\lambda_m|}{\partial R} &= \frac{\mu(X^2 - R^2) + 2vRX}{(R^2 + X^2)^2}
\end{align*}
\]  

(6)

We can then calculate \( \mu = \partial |\lambda_m|/\partial G \) and \( v = \partial |\lambda_m|/\partial B \) using Eq. (5), where \( S_r \) and \( S_i \) for the series components satisfy

\[
S_r + jS_i = S_{m,ii} + S_{m,ij} - S_{m,ji} - S_{m,ji}.
\]  

(7)

Modal sensitivity can reflect the effects of a component parameter on \( |\lambda_m| \). Based on this approach, a specific resonance can be mitigated efficiently by adjusting the parameters of the components with high modal sensitivity. To make the modal sensitivity of each component comparable, this index is normalized as follows [13]:

\[
\sigma_\alpha = \left\| \frac{\partial |\lambda_m|}{\partial \alpha} \right\|_{norm} = \frac{\partial |\lambda_m|}{\partial \alpha} \frac{\partial \alpha}{\partial \alpha} = \frac{\partial |\lambda_m|}{\partial \alpha} \frac{\partial |\lambda_m|}{\partial \lambda_m},
\]  

(8)

where \( \alpha \) represents a network component (e.g., a susceptance).

3. Robustness of Traditional Modal Sensitivity and the idea of improvements

3.1. Robustness analysis

Modal sensitivity is widely used as an index to guide resonance mitigation. However, our studies have found that this index is not always robust in applications. Specifically, in some cases, the component with the highest \( \sigma_\alpha \) is not actually the most sensitive component. Additionally, the components with smaller \( \sigma_\alpha \) can sometimes be even more sensitive. This abnormal phenomenon is analyzed using the test system shown in Fig. 1 (a).
There are three critical modes in this circuit, as illustrated in Fig. 1 (b). For mode 2, a sensitivity of $|\lambda_2|$ to each component, i.e., $\sigma_\alpha$, is shown in Fig. 1 (c) (for ease of comparison, the results are presented in absolute values form). To test the validity of $\sigma_\alpha$, we produce $\Delta \alpha\%$ changes for these components (if the $\sigma_\alpha$ of a component is positive, the change is +2%; otherwise, the change is -2%). The corresponding decline of $\lambda_2^{-1}$ is also presented in Fig. 1 (c).

To simplify the notation, we define the symbol $X(X_1)$ as representing the reactance of component $X_1$, while $B(B_2)$ represents the susceptance of component $B_2$. Fig. 1 (c) shows that, for mode 2, although $|\sigma_{X_1}|$ has the largest value among all $|\sigma_\alpha|$, while $|\sigma_{B_2}|$ has the smallest value, it is $B(B_2)$ but not $X(X_1)$ that has the best effect in relieving the resonance. Obviously, this phenomenon is not consistent with the results expected from the modal sensitivity analysis.

To explore the cause of this unusual phenomenon, we analyze how $|\lambda_2^{-1}|$ varies with $\Delta \alpha$ for each component (Fig. 2). For ease of comparison, $\Delta \alpha$ is presented in its absolute value form (if $\sigma_\alpha$ of a component is positive, $\Delta \alpha$ is positive; otherwise, $\Delta \alpha$ is negative). Fig. 2 shows for mode 2 that when $|\Delta \alpha|$ is low, the resonance mitigation effect of $B(B_2)$ is indeed the worst. However, as $|\Delta \alpha|$ increases, the effect of $B(B_2)$ gradually surpasses that of the other components at each of the critical points. This phenomenon is attributed to the fact that $\sigma_{B_2}$ is not constant but changes with the variation of $B(B_2)$. Eventually, when changes of $\pm 2\%$ occur to each component, the most sensitive component is replaced with $B(B_2)$.
3.2. Directions for Improvement of the Traditional Modal Sensitivity

As shown in the analysis above, when the parameter of a component varies, its corresponding modal sensitivity may also show a specific change when compared with the initial value. If this change is obvious, then the previously calculated sensitivity becomes invalid.

Because the change in $\alpha$ causes its instability, quantification of this change is essential to improve the reliability of $\alpha$. Mathematically, the variation of $\alpha$ can be evaluated using its derivative; therefore, in this paper, we propose the more robust index $22\left(\frac{\partial}{\partial \alpha}\right)$ and combine it with $\alpha$ to analyze the resonance mitigation measures.

4. Simulation Verification

To verify the validity of the proposed indexes, the three-bus test system mentioned earlier is analyzed in this section. Fig. 4 shows $\alpha$ and $\alpha'$ for each component in mode 2. For ease of comparison, these parameters are presented in absolute values form.

Fig. 4 shows that for $B(B_2)$, while $\left|\alpha_{B(B_2)}\right|$ has the smallest value among all $\left|\alpha\right|$, $\left|\alpha'_{B(B_2)}\right|$ has the largest value among all $\left|\alpha'\right|$. In contrast, for $X(X_1)$, $\left|\alpha'_{X(X_1)}\right|$ has the largest value among all $\left|\alpha\right|$, but $\alpha'_{X(X_1)}$ is relatively small. Therefore, with the variations of component parameters, $\left|\alpha'_{B(B_2)}\right|$ will increase rapidly and will eventually have the largest value, as shown in Fig. 4 (if $\alpha$ of a component is positive, $\Delta\alpha$ is positive; otherwise, $\Delta\alpha$ is negative).
Theoretically, there is a critical value $\sigma_{\text{critical}}$ for components $B(B_2)$ and $X(X_1)$. When $|\Delta \alpha|$ is lower than $\sigma_{\text{critical}}$, adjustment of $X(X_1)$ is more effective in mitigating the harmonic resonance. Conversely, if $|\Delta \alpha|$ is larger than $\sigma_{\text{critical}}$, $B(B_2)$ will become the more sensitive parameter. Additionally, Fig. 4 also shows that for each component, $|\sigma_{\alpha}|$ varies linearly with the variation of the corresponding component parameter. Therefore, $\sigma_{\alpha}'$ for each component remains constant within a specific range.

5. Conclusions
This paper proposes a second-order modal sensitivity and a critical index for the variation of the component parameters to guide resonance mitigation processes. The main contributions of this study are described as follows.

(1) It is proved that because the modal sensitivity changes with variation in the corresponding component parameter, the analysis results from the traditional modal sensitivity approach are not always reliable in certain cases.

(2) The proposed second-order modal sensitivity can quantify the changes in the modal sensitivity with the variation in the component parameters, and thus can improve the reliability of the traditional modal sensitivity approach.

(3) By combining it with the proposed critical index, the second-order modal sensitivity can identify the most sensitive network component in a particular resonance mode, and thus can guide the resonance mitigation strategy more reliably.

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