Electromagnetic induction in the icy satellites of Uranus

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Abstract

The discovery of subsurface oceans in the outer solar system has transformed our perspective of ice worlds and has led to consideration of their potential habitability. The detection and detailed characterisation of induced magnetic fields due to these subsurface oceans provides a unique ability to passively sound the conducting interior of such planetary bodies. In this paper we consider the potential detectability of subsurface oceans via induced magnetic fields at the main satellites of Uranus. We construct a simple model for Uranus’ magnetospheric magnetic field and use it to generate synthetic time series which are analysed to determine the significant amplitudes and periods of the inducing field. The spectra not only contain main driving periods at the synodic and orbital periods of the satellites, but also a rich spectrum from the mixing of signals due to asymmetries in the uranian planetary system. We use an induction model to determine the amplitude of the response from subsurface oceans and find weak but potentially-detectable ocean responses at Miranda, Oberon and Titania, but did not explore this in detail for Ariel and Umbriel. Detection of an ocean at Oberon is complicated by intervals that Oberon will spend outside the magnetosphere at equinox but we find that flybys of Titania with a closest approach altitude of 200 km would enable the detection of subsurface oceans. We comment on the implications for future mission and instrument design.

1. Introduction

1.1 Ocean worlds

Evidence of subsurface liquid oceans beneath the frozen ice crusts of the icy satellites of Jupiter and Saturn has changed our perspective of small icy worlds in the outer solar system; these worlds are not inert. Furthermore, the possibility of extant microbial life in their oceans has forced us to reconsider the meaning of planetary ‘habitability’ throughout the solar system and beyond (e.g., Lazcano and Hand, 2012). Measurements of Europa’s gravity field pointed towards a three-shell model consisting of an \text{H}_2\text{O} layer overlying a silicate mantle and metallic core. Although the state of the \text{H}_2\text{O} layer was unknown, surface geomorphology, including cryovolcanic features, chaos terrain, topography and global tectonics, argued for a liquid subsurface ocean (Anderson et al., 1998; Pappalardo et al., 1999).

One of the principal pieces of evidence for the existence of a liquid subsurface ocean was the detection of magnetic field perturbations, in the vicinity of the satellite, that were consistent with an induced field from the interior. As Jupiter rotates, the tilted dipole causes diurnal oscillations in the magnetospheric magnetic field at the satellites, providing an alternating ‘primary’ or ‘inducing field’. Ions dissolved in the subsurface ocean respond to this inducing field and generate eddy currents which produce a secondary or induced field that acts to exclude the varying field from
the conducting interior. Colburn and Reynolds (1985) first considered the consequences for electromagnetic induction in the icy mantle of Europa, focusing on magnitudes of the eddy currents and their ability to heat the interior through ohmic effects (see also Chyba, Hand and Thomas (2021) for a recent perspective). Kargel and Consolmagno (1996) considered the induced field and concluded it could be detectable, later verified in Galileo measurements (Khurana et al., 1998; Kivelson et al., 2000; Zimmer, Khurana and Kivelson, 2000; Khurana et al., 2009).

The strength of the induced field relative to the inducing field is a diagnostic of the extent and conductivity of the subsurface ocean. Although the signature at Europa is modified by the plasma interaction of Europa’s weak ionosphere with Jupiter’s magnetosphere, the induced field is sufficiently strong that it can be resolved against the field of the plasma currents surrounding the satellite (e.g., Khurana et al., 1998; Zimmer, Khurana and Kivelson, 2000; Schilling, Neubauer and Saur, 2007).

1.2 Uranus’ main icy satellites and their interiors

In comparison to the satellites of Jupiter (and Saturn), we know very little about the main icy satellites at Uranus: Miranda, Ariel, Umbriel, Titania and Oberon. These satellites show evidence of cryovolcanic resurfacing (e.g., Croft and Soderblom, 1991) and tectonic activity in differing proportions; for example, Oberon is heavily cratered and less faulted than Titania.

Hussmann, Sohl and Spohn (2006) constructed detailed models of the thermal and mechanical equilibrium of a range of icy bodies in the outer solar system, including those at Uranus. They considered two- and three-layer models consisting of a rocky core surrounded by two water layers and calculated the heat flux, the resulting density, pressure and temperature profile, and hence an indication of whether the subsurface water layer could be in a liquid state. All model solutions were constrained to satisfy the observational constraints of radius, mean density and moment of inertia. Two factors that increased the likelihood of a subsurface ocean were a) a larger core size, which gives a larger radiogenic heating rate and potentially higher subsurface temperature; and b) a larger fraction of NH$_3$ that lowers the freezing point of H$_2$O. Their models assumed that the satellites were differentiated but this does not appear to be an unrealistic assumption since the satellite surfaces are predominantly icy, yet have mean densities higher than water indicating at least some differentiation of heavier material to the interior. As the authors only included radiogenic heating, and not tidal heating, for example, they considered that their solutions were a lower bound on the presence of subsurface oceans.

Among the uranian satellites, solutions with subsurface oceans were found only for Titania and Oberon. Three solutions were found for Titania (R=519.8 km), with ocean depths of 16, 39.4 and 51.5 km and ice (I) shell thicknesses of 253.1, 229.7, and 217.7 km respectively. Two solutions were found for Oberon (R=481.0 km) with oceans between 16 and 39.3 km thick, overlaid by an ice (I) shell of thickness between 264.4 and 241.1 km respectively (Hussmann, Sohl and Spohn, 2006). Figure 1 shows a schematic of these models.
It is important to highlight that this does not preclude the presence of subsurface oceans at the other uranian icy satellites since the models do not predict a subsurface ocean at Enceladus, contradicting observational evidence from Cassini (e.g., Thomas et al., 2016). However, these models for Titania and Oberon provide our motivation to explore the induced response from ions dissolved in the ocean, and hence the capability of passive magnetic sounding to search for the presence of an ocean in a future mission.

1.3 Induction

Magnetic flux diffuses through the conducting regions of a satellite according to the Helmholtz equation (1) and can be derived from Faraday’s law, Ampère’s law, $\nabla \cdot \mathbf{B} = 0$ and Ohm’s law under the assumption of spatially-uniform conductivity, sufficiently low frequencies such that displacement currents can be neglected, and an absence of advection (e.g., Khurana et al., 2009; Saur et al., 2010),

$$\nabla^2 \mathbf{B} = \sigma \mu_0 \frac{\partial \mathbf{B}}{\partial t}$$

where $\sigma$ is the conductivity, $\mathbf{B}$ is the magnetic field, $t$ is time, and $\mu_0$ is the permeability of free space. In the insulating (non-conducting) regions, e.g., an icy outer mantle, equation (1) reduces to Laplace’s equation. The magnetic field can be written as a linear superposition of harmonics at frequencies $\omega_i$, with complex amplitudes $B_m$, plus a constant background field, $\mathbf{B}_c$ (2).

$$\mathbf{B}(t) = \mathbf{B}_c + \sum_m B_m e^{-i\omega_m t}$$

In this work we restrict ourselves to a three-layer model consisting of an insulating core, conducting liquid water layer, and insulating mantle. Assuming that the conductivity is finite and spherically-symmetrical and the primary field
harmonic is spatially uniform, solution of (1) gives an expression for the secondary field as an induced dipole with a
dipole moment that is a phase lagged function of the primary field,

\[
B_{\text{sec}}(r, t) = -A \exp[-i(\omega t - \phi)] \left[3(r \cdot B_{\text{pri}})r - r^2 B_{\text{pri}}\right] \frac{R^3}{2r^3}
\]

(3)

where \(r\) is the position, \(t\) is time, \(B_{\text{pri}}\) is the primary driving field at frequency \(\omega\) and represents one of the harmonic
terms in equation (2) \(B e^{-i\omega t}\), and \(R\) is the radius of the moon. \(Ae^{i\phi}\) is a complex amplitude which describes the
amplitude, \(A\), of the secondary field relative to the primary, and its phase lag, \(\phi\). With the simplifying assumptions
above, the complex amplitude can be obtained from,

\[
A e^{i\phi} = \left(\frac{r_0}{R}\right)^3 \frac{\xi_{J_5/2}(r_0 k) - J_{-5/2}(r_0 k)}{\xi_{J_5/2}(r_0 k) - J_{-1/2}(r_0 k)}
\]

(4)

\[
\xi = \frac{r_5 k_{J_{-5/2}}(r_1 k)}{J_{J_{5/2}}(r_1 k) - J_{-1/2}(r_1 k)}
\]

(5)

, where \(r_1\) and \(r_0\) are the radius of the inner and outer edges of the conducting layer respectively (schematically
illustrated in Figure 1), \(k\) is the complex wave vector \(k = (1 - i)\sqrt{\mu_0 \sigma \omega / 2}\), and \(J_n\) are Bessel functions of the first kind
(e.g., Zimmer, Khurana and Kivelson, 2000).

1.4 The uranian magnetosphere: factors affecting the primary field

There are three key differences between the magnetospheres of Jupiter and Uranus that we must consider in
understanding the primary field at the satellites, and which guide us in how we may need to treat these systems
differently. Figure 2 shows a schematic of Uranus’ planetary system at two different seasons and two different rotation
phases (separated by half a planetary rotation, “8.5 hours).

1. Large dipole tilt: Uranus’ dipole axis is tilted by an angle of 60° relative to its rotation axis, and furthermore
has an offset that may be important for the closer satellites such as Miranda, and possibly Ariel.

2. Proximity of the magnetopause: Even Callisto, the most distant Galilean satellite, is only around half the
distance from Jupiter to the magnetopause, whereas Titania and Oberon have semi-major axes of 17.1 R\(_U\) and
22.8 R\(_U\) respectively compared with the subsolar magnetopause distance of around 19 R\(_U\) (see section 2.1),
where 1 R\(_U\) = 25 559 km. Hence, Oberon is likely to orbit outside the magnetopause at certain orbital
phases, seasons and solar wind conditions, with Titania less likely to spend time outside the magnetosphere.

3. Large obliquity: Uranus’ ~97° obliquity implies strong seasonal behaviour that we must consider. For
example, near solstice the satellite orbital planes lie close to the terminator plane, whereas near equinox they
are approximately in the noon-midnight plane of the magnetosphere. At solstice Oberon rarely leaves the
magnetosphere whereas at equinox it periodically enters the magnetosheath. This implies that we must consider the primary field at different seasons.

The configuration of the magnetosphere at a given moment can be coarsely parameterised by the solar wind attack angle (e.g., Lepping, 1994) which is the angle, $\alpha$, that the dipole axis makes to the solar wind (where $\alpha=0$ represents the dipole pointing into the solar wind). Earth’s magnetosphere has an attack angle of $\approx 90^\circ \pm 30^\circ$ which means that the dipole is more-or-less perpendicular to the incoming solar wind flow and varies only slightly with Earth’s rotation. The combination of large dipole tilt and obliquity at Uranus means that near solstice the attack angle is relatively constant (the magnetosphere essentially rotates around the planet-Sun line) but at equinox the dipole can, at times, be pole-on to the solar wind (Figure 2 bottom right) and so the solar wind attack angle can vary considerably over one Uranus rotation. These significant diurnal and seasonal variabilities are a unique feature of the uranian magnetosphere (e.g., Lepping, 1994; Arridge and Paty, in press).

Figure 3 shows the solar wind attack angle for one Uranus year (approximately 84 years). The variation can be thought of a slowly varying mean solar wind attack angle representing seasonal variations with a high frequency diurnal variation forming an ‘envelope’ around the mean. The figure also shows the angle between the spin axis and the solar wind that informs us about the orientation of the satellite orbital plane relative to the Sun (e.g., the changing orientation in Figure 2). These two angles expose the different characteristics of the dipole-solar wind geometry and satellite orbital plane-solar wind geometry over time. We split this time series into ‘zones’ to reflect different
characteristics at different epochs. The solar wind attack angle is quantised into 45° segments, for example where $0 \leq \alpha < 22.5^\circ$ would be represented by 0°, $22.5^\circ \leq \alpha < 67.5^\circ$ by $45^\circ$, and $67.5^\circ \leq \alpha < 112.5^\circ$ by $90^\circ$. Thus we coarsely represent the attack angle as quasi-pole-on, intermediate, and quasi-perpendicular, etc. The spin axis angle is similarly quantised into 30° segments. Each unique combination of quantised attack angle and spin axis angle forms a unique ‘zone’ that represents some quantised representation of the geometry of Uranus’ magnetosphere and its seasonal and diurnal variation.

For example, near the Voyager 2 flyby in 1986, the solar wind attack angle was quasi-perpendicular to the solar wind and the spin axis was approximately parallel to the solar wind implying that the satellites only experienced the magnetospheric flanks and polar regions over an orbit, never encountering the dayside and nightside. Whereas during 2015, the magnetosphere transitioned between pole-on and perpendicular configurations over a diurnal rotation and the spin axis was perpendicular to the solar wind implying that the satellites experienced the nightside and dayside of the magnetosphere every orbit.

Figure 3: Angles between (a) the solar wind and dipole (attack angle) and (b) the solar wind and spin axis for a full Uranus orbit. Insets show diurnal variation of the solar wind attack angle at different epochs. Non-sinusoidal behaviour in the solar wind attack angle is particularly evident near 2015. Banding indicates the division of the time series into particular ‘zones’ that have similar characteristics. There is no specific meaning to the colouration.

Apart from being a useful intuitive tool, the variations in the solar wind attack angle have implications for the primary field at the satellites. The distribution and magnitude of currents on the magnetopause change with solar wind attack angles and so affect the primary field by changing the contribution from the magnetopause currents. Furthermore, as can be seen in the insets, the solar wind attack angle variation is also not purely sinusoidal so introduces additional harmonics to the diurnal variation of the magnetopause field. Although the solar wind attack angle is a useful and intuitive tool, the real configuration can be quite difficult to fathom as the field is truly three-dimensional and simple 2D cuts do not fully capture the configuration (Arridge and Paty, in press); 3D tools are essential in understanding the geometry at a given epoch (Arridge and Wiggs, 2019).
1.5 Motivation

In this paper we explore the induced response from sub-surface oceans that have been suggested to exist within Titania and Oberon. Hence, we explore the capability of passive magnetic sounding to provide empirical evidence for the presence of an ocean, as carried out at Europa (Khurana et al., 2009), and examine whether such oceans might be detectable by potential future missions. Furthermore, because Uranus’ magnetosphere is so diurnally-variable, and this variability changes with season, there may be seasons where an induced response from the oceans is richer and which might provide additional opportunities to learn more about these elusive worlds through multi-frequency passive sounding. Although we focus primarily on Titania, we have learned from the Cassini mission that small icy bodies can be more active than previously thought and so we also include some examination of the inducing field at Uranus’ other satellites. We first construct a model for the magnetospheric magnetic field at Uranus and use it to calculate the primary field at each of Uranus’ main satellites over a Uranus orbit (section 2). The amplitude of the primary field harmonics are evaluated in section 3 and applied to compute the induced response using the internal structure models of Hussmann, Sohl and Spohn (2006). Finally, we estimate the field at Titania during potential satellite flybys and conclude that the calculated subsurface oceans from Hussmann, Sohl and Spohn (2006) would result in a detectable induced signature from a flyby of this satellite.

2. Methods

2.1 Magnetospheric magnetic field model

To calculate the primary (inducing) field at the satellites we require a model for the magnetospheric magnetic field. In the terrestrial magnetosphere, the principal contributions to this field are the internal (planetary) magnetic field, the field of the Chapman-Ferraro (magnetopause) currents, the field of the ring current, and the field of the cross-tail current sheet (e.g., Tsyganenko, 2002). At Jupiter and Saturn these must also be augmented by the field of a magnetodisc current sheet which has a dramatic effect on the near-equatorial field (e.g., Smith et al., 1974; Arridge and Martin, 2018). At Uranus we only include the internal planetary field and the Chapman-Ferraro field. Voyager 2 observations demonstrated the presence of a well-developed helical tail current sheet at solstice (Behannon et al., 1987), however, its geometry for other solar wind attack angles, e.g., at equinox with periods in both pole-on and perpendicular orientations, is much more complex and so we chose not to include a tail current sheet. The implications of this will be considered shortly. In contrast to the tail current sheet, Voyager 2 observations suggested a very weak ring current, relative to the internal field, around an order of magnitude smaller than that at Earth (Connerney et al., 1987) and so we chose not to include this in our model. We use the most recent internal field model, which uses magnetometer data and ultraviolet auroral data from Voyager 2 to constrain a 4th degree (hexapole) spherical harmonic model ‘AH5’ (Herbert, 2009).

The final ingredient is a geometrical model for the magnetopause. We use the functional form introduced by Shue et al. (1997) where a position \((r, \theta)\) on the magnetopause (with \(\theta\) measured from the planet-Sun line) is given by \(r = r_0[2(1 + \cos \theta)^{-1}]^\xi\) where \(r_0\) is a constant representing the distance to the subsolar point on the magnetopause and \(\xi\) is a constant that describes the shape of the magnetopause, with \(\xi > \frac{1}{2}\) representing a magnetopause that flares out.
with distance downtail. These constants were derived by approximately matching the shape of the magnetopause in
the MHD simulations of Tóth et al. (2004, their Figure 5) yielding constants \( r_0 = 19 R_U \) and \( \xi = 0.72 \). This geometry
was fixed for all of our calculations and was not varied with solar wind dynamic pressure (discussed further below), or
varied to reflect different seasonal/diurnal configurations. This would be expected to introduce additional variations in
the magnetic field at the appropriate periods, but since we have little information to calculate such changing geometry
we chose to exclude such effects.

The effect of the Chapman-Ferraro currents is to ‘confine’ or ‘shield’ the magnetic field inside the magnetopause such
that no magnetic flux crosses the boundary. This corresponds to a condition where the magnetic field normal to the
magnetopause is zero everywhere on the boundary. In our model we only consider the field of the planet, \( \mathbf{B}_{\text{int}} \), and so
we can state this problem as a zero-valued integral over the boundary. For the practical purpose of finding a Chapman-
Ferraro field, \( \mathbf{B}_{\text{mp}} \) (and its parameters, \( \mathbf{\theta} \)), that confines the planetary field within the magnetopause, it is sufficient to
consider the minimisation of a sum over a finite number, \( N \), of points, \( \mathbf{r}_i \), on the magnetopause with unit normal
vectors \( \mathbf{n}(\mathbf{r}_i) \) (Schulz and McNab, 1996):

\[
\min \left[ \frac{1}{N} \sum_{i=0}^{N-1} \left| \left( \mathbf{B}_{\text{int}}(\mathbf{r}_i) + \mathbf{B}_{\text{mp}}(\mathbf{r}_i, \mathbf{\theta}) \right) \cdot \mathbf{n}(\mathbf{r}_i) \right|^2 \right] \quad (6)
\]

As the Chapman-Ferraro currents flow entirely on the surface of the magnetopause, their field is curl-free inside the
magnetosphere and can be represented by the gradient of a scalar magnetic potential, \( \mathbf{B}_{\text{mp}} = -\nabla U \), and so \( U \) can be
solved from Laplace’s equation using suitable eigenfunctions as appropriate for the geometry of the magnetopause.

We used a modified form of Cartesian ‘box’ harmonics (Tsyganenko, 2002, equations 30-32) with separate harmonics to
confine the part of the internal field parallel, \( U_{\|} \), and perpendicular, \( U_{\perp} \), to the planet-Sun line: \( \mathbf{B}_{\text{mp}} = -\nabla U_{\perp} - \nabla U_{\|} \):

\[
U_{\|}(x, y, z) = \sum_{j=1}^{4} \sum_{l=1}^{4} \left( c_{jl} \sin \Psi + d_{jl} \sin 2\Psi \right) \exp \left[ x \left( \frac{1}{q_j} + \frac{1}{s_l} \right) \cos \frac{y}{q_j} \cos \frac{z}{s_l} \right] \quad (7)
\]

\[
U_{\perp}(x, y, z) = \sum_{k=1}^{3} \sum_{l=1}^{3} \left( a_{ik} \cos \Psi + b_{ik} \cos 2\Psi \right) \exp \left[ x \left( \frac{1}{p_{ik}} + \frac{1}{r_{ik}} \right) \cos \frac{y_{ik}}{p_{ik}} \sin \frac{z_{ik}}{r_{ik}} \right] \quad (8)
\]

where \( a, b, c \) and \( d \) are amplitude coefficients, \( q, s, p \) and \( r \) are scale parameters, \( \Psi = \frac{\pi}{2} - \alpha \) is the dipole tilt angle,
and \( x, y, \) and \( z \) are coordinates in the right-handed Uranocentric Solar Magnetospheric (USM) system, where \( \hat{e}_x \) is the
unit vector from Uranus to the Sun, and the X-Z plane contains the dipole vector, \( \mathbf{M} \), such that \( \hat{e}_y = \mathbf{M} \times \hat{e}_x \). Equations
(7,8) include 25 harmonics with 64 free parameters to be found through the minimisation of (6).

As the quadrupole and higher degree terms are weak at the magnetopause, only the dipole component was included in the
minimisation of (6) with a dipole moment of 23000 nT (Connerney et al., 1987). For simplicity the dipole offset was
not included in this computation, but only causes a scalar error of less than \( dB = 3B \frac{dr}{r} \approx 0.06 \) nT in the field strength
at the magnetopause, smaller than the quadrupole field at the magnetopause. The minimisation was performed using
the Downhill Simplex algorithm (Nelder and Mead, 1965) as implemented in the Python package SciPy (Virtanen et al., in press). The minimiser was iteratively called, each time increasing the number of points that the field was evaluated on, until the root-mean-square normal field and maximum absolute normal field were less than 0.005 nT and 0.01 nT respectively, or until the optimiser had terminated successfully four times in a row. The optimisation of (6) was completed with 37440 points with a root-mean-square normal field of 0.00199 nT and a maximum normal field of 0.0104 nT. Figure 4 shows field lines traced for four different dipole tilt angles showing the quality of the magnetopause shielding and the coefficients required for equations (7) and (8) are included in the appendix.

Figure 4: Shielded dipole field lines for dipole tilt angles of 0, 30, 60, and 90 degrees in the USM X-Z plane. Field lines are drawn from seed points on a circular slice of the planet at 2 degree intervals.

The entire model was built in Python using NumPy (Oliphant, 2006), SciPy and the SPICE toolkit (Acton, 1996) via SpiceyPy (Annex et al., 2020) packages, and a custom magnetic field modelling package. As the AH₅ model is defined in the Uranus Longitude System (ULS) (Connerney et al., 1986) (also known as the U1 coordinate system) which has its pole opposite to that of the IAU defined pole of Uranus, the model was constructed in these coordinates with conversions to/from USM as necessary. USM coordinates can become degenerate for the case where the dipole is exactly parallel or anti-parallel to the solar wind, where the orientation of the X-Z plane around the X axis becomes undefined. This was not encountered during the period of study. Figure 5 shows the diurnal structure in the field for the Voyager 2 epoch, where the configuration remained largely static with the field rotating around the planet-Sun line, and January 2000 where considerable changes in configuration are found over a diurnal cycle. Figure 6 shows a
comparison of the residual field, with the $\text{AH}_5$ internal field model subtracted, and the model at the Voyager 2 flyby.

The model comparison is generally very good on the dayside, with a significant discrepancy near closest approach (possibly reflecting additional unconstrained structure in the internal field) but is worse on the nightside where our neglect of the tail field can be seen.

Figure 5: Field lines traced every $1/7^{th}$ of a planetary rotation for the Voyager 2 epoch (top) and January 2000 (bottom). Each panel contains the dipole tilt angle at that time.
The model has the capability to be varied with the upstream solar wind dynamic pressure through changes in the size of the magnetosphere. We do not include this effect for two reasons: a) to focus on the seasonal and geometrical effects, and b) due to the difficulty in constructing a physically-meaningful timeseries of solar wind pressure over a period of 84 years, from a short time series around the Voyager 2 mission. We leave this for future work. However, experiments show that for near solstice conditions using Voyager 2 data the amplitude of such variations amount to ~0.1 nT at both Titania and Oberon. As might be expected, there are also harmonics of the solar rotation period ~500 hours and frequency mixing with orbital periods that amount to a ~0.01 nT amplitude.

2.2 Construction of the time series data set
As we were particularly interested in the seasonal effects within the system and were concerned to include effects such as nodal precession (e.g., Jacobson, 2014a), we used SPICE to compute the positions of the satellites, rather than using mean orbital elements as in other studies in the jovian system analyses (e.g., Seufert, Saur and Neubauer, 2011). The positions of Miranda, Ariel, Umbriel, Titania and Oberon were calculated between 01 January 1966 and 01 July 2050, bounding a full Uranus year. The time series was constructed at a cadence of 300 s which produces around 400 data points during one orbit of Miranda and approximately 2500 for Titania. The magnetic field model was used to compute
the field at each satellite and the individual sources (dipole, quadrupole, octapole, hexapole, magnetopause) were saved along with the total. Data was stored in HDF5 files for further analysis and are publicly available at doi:10.17635/lancaster/researchdata/411. During analysis a discrepancy was noted between the orbital period of the satellites (from mean elements) and the data from SPICE and so we also computed the orbital period from SPICE for consistency with the analysis. Orbital elements were obtained from state vectors using SPICE using a GM of 5.793951322279009 × 10^6 km^3 s^-2 from ephemeris URA112 (Jacobson, 2014b).

3. Analysis of the inducing field

3.1 General overview

Figure 7 shows hodograms of the magnetic field in the $B_\phi - B_r$ plane for the five satellites near solstice and near equinox. In this plane the hodograms have an elliptical polarisation however in the $B_\theta - B_r$ plane they are almost linearly polarised, in common with similar analyses in the jovian system (Seufert, Saur and Neubauer, 2011). The finite thickness in $B_r$ extrema for Miranda is due to the eccentricity of Miranda’s orbit. As expected, the hodograms show increasing departures from the dipole model (indicated in red) with increasing orbital distance, and increasing differences between seasons, just visible at Umbriel to significant at Titania. Interestingly, the hodogram is relatively constrained near solstice, possibly due to the narrow range of solar wind attack angles at that time and that the orbits are almost in the terminator plane. We do not show the hodogram for Oberon at equinox due to the regular excursions outside the magnetosphere when the satellite is on the dayside. In the jovian system between Ganymede and Callisto, the field becomes almost linearly polarised which is a consequence of the magnetodisc current sheet (Seufert, Saur and Neubauer, 2011). This is absent at in our model results, partly due to the lack of such a current sheet at Uranus, but also due to our neglect of a tail current sheet which would introduce some linear polarisation at particular orbital phases. This may affect Titania but would most strongly affect Oberon.

Figure 7: Hodograms for the variation of the field in the $B_\phi - B_r$ space for the five satellites in two epochs (near solstice (a-e) and near equinox (f-i). The near equinox hodogram for Oberon is not shown due to periods spent outside of the
magnetosphere. Grey backgrounds indicate the envelope of the whole hodogram over an entire Uranus orbit covering
the most highly-visited part of the hodogram space; the red envelope is just the hodogram from the dipole component
of Uranus' field at the satellite; and the blue envelope shows the total field.

To explore the seasonal variation in more detail, Figure 8 shows hodograms for Umbriel and Titania over half a Uranus
orbit. As expected due to its smaller orbital distance, the hodograms for Umbriel display relatively little variation
compared to Titania, however, the hodograms for Umbriel are more elliptically-polarised near solstice, and more
circular near equinox. The hodograms for Titania show a large degree of variation with season. Near solstice, in the
centre two rows of Figure 8, the hodogram has a relatively narrow elliptical polarisation and occupies a relatively small
fraction of the overall envelope of variation over a Uranus year. Near equinox it is broader and almost circular, but it
must be highlighted that these intervals are over a wider period of time which may also contribute to the breadth of
the hodogram. These changes in the hodograms around equinox are not completely correlated with changes in the
range of solar wind attack angles, but more associated with the changing orientation of the equatorial plane relative to
the Sun. Near solstice, the equatorial plane is almost perpendicular to the planet-Sun line and so the satellite orbits
roughly in the terminator plane, whereas near equinox the planet-Sun line is roughly in the plane of the equator and
the orbits take satellites between the dayside and nightside every orbit. Seasonal variations in the inducing (primary)
field should also produce seasonal variations in the amplitude of different driving periods, and hence seasonal
variations in the induced (secondary) field. This has clear consequences for the exploration of the icy satellites at
different epochs; particular epochs may present a richer scenario for the detection and study of subsurface oceans.
Figure 8: Hodograms for Umbriel (left) and Titania (right) for different seasons. The sketches on the right show the configuration of the system; at the top of each panel field lines at a fixed L-shell show extrema (orange and purple) of the diurnal motion of the dipole with respect to the Sun, and the bottom of each panel shows the orientation of the satellite system, spin axis in blue, with the orbital velocity vector in red. Both sketches have the Sun off to the left-hand
side, the top panel is projected into the plane of the dipole magnetic field and the planet-Sun line, the bottom panel is viewed from above the orbital plane. The curved surface represents the magnetopause. For clarity, the axis scales have been suppressed but they have the same ranges and grid as those in Figure 7. Representative centre-times for each row are approximately 1970, 1976, 1982, 1986, 1991, 1996.

3.2 Spectral analysis overview

From these time series we estimated the power spectral density using `scipy.signal` both in the form of line spectra (Welch, 1967) and spectrograms. In both cases a Hamming window was used and was chosen to provide a balance between with the width of main lobe and relative amplitude of the side lobes. The number of samples used for spectral estimation was determined algorithmically to give good resolution near relevant periods. Specifically we required $\Delta t/T$ of 5% at 17 hours (approximate rotation period of Uranus) and 10% at 200 hours (approximate orbital period of Titania) with the constraint that the number of samples was a power of two. This required $2^{16}=65536$ samples and $\Delta t/T$ of 0.311% and 3.662% for 17 and 200 hours respectively. The maximum period that could be examined was 5461 hours (4 s.f.). Time series were linearly detrended before spectral estimation, although the effects of detrending on the power spectral density were examined and did not find any significant differences between the choices of detrending (constant Vs. linear). Spectrograms were not calculated with overlaps between samples but line spectra used the `scipy.signal.welch` default of half the number of samples per windowed interval (32768 in this case).

Figure 9 shows spectrograms of the $B_r$ component of the total magnetic field for each main icy satellite as a function of time. As noted earlier, and schematically illustrated in Figure 2, Oberon has an orbit that takes it outside the magnetosphere, indicated in Figure 9 by bars at the top of the figure. Such excursions are not found near solstice when the spin axis is approximately parallel to the solar wind, and where the satellite orbits are in a plane perpendicular to the solar wind. In our synthetic time series these excursions outside the magnetosphere give a null field (by construction). Hence, we exclude these intervals from the spectral analysis. In each panel we indicate the orbital and synodic period, the beat period between the orbital and Uranus rotation period, $T_{syn}=\left[\frac{1}{T_{ura}} - \frac{1}{T_{orb}}\right]^{-1}$, and their second harmonics.

At each satellite the amplitude at the synodic period is approximately constant in time, however, the amplitude at the orbital period exhibits variation over an order of magnitude between different epochs. Near solstice, where the angle between the solar wind and the spin axis is near 0° or 180°, the time series shows relatively low amplitudes at the orbital period and its harmonics and the spectrum is relatively simple and uncluttered. Near equinox, where the angle between the solar wind and the spin axis is approximately 90 degrees, there is significant power at the orbital period and its harmonics and also a plethora of other spectral peaks, particularly at periods not consistent with either periods or their harmonics. These additional spectral peaks are more important for Titania, less important for Umbriel and Ariel, and essentially absent (relative to the main peaks) at Miranda. As expected, the dipole field component appears at the synodic period, and occasionally at the orbital period if there are effects due to orbital eccentricity of the
satellite. The quadrupole field appears at the synodic period in $B_\phi$ and the 2nd harmonic in $B_r$ and $B_\phi$ due to the
symmetry of the quadrupole. The magnetopause field always appears at the orbital period of the satellites due to the
changing position of the satellite relative to the magnetopause over an orbit. There is also power from the
magnetopause field at the synodic period of the satellites due to the variation in dipole tilt over a planetary rotation.

Figure 9: Amplitude spectra for the radial component of the magnetic field at each of the main icy satellites as a
function of time. Intervals where Oberon spends periods is outside the magnetosphere (indicated by the red bars at
the top of the figure) have not been analysed leaving blank spaces in panel (f). The orbital and synodic periods are
indicated by red and black/white arrows respectively, with the fundamental frequency as a solid line and the higher harmonics as dashed lines. Panel (a) also shows a summary of the solar wind attack angle and spin axis-solar wind angle from Figure 3.

The fine spectral structure we see in Figure 9 was determined to be due to a set of heterodynes, caused by mixing of different frequency components: $T_{het \pm} = \frac{1}{T_1} \pm \frac{1}{T_2}$. This was found to be a persistent feature in the magnetopause field due to a mixing of the synodic period with the orbital period and its harmonics, interpreted as the motion of the satellite through an asymmetrical magnetopause cavity thus experiencing a magnetopause field that varies over an orbit. Weaker heterodynes, from mixing of the orbital period with higher harmonics of the synodic period, are interpreted as a consequence of the non-sinusoidal behaviour of the solar wind attack angle over a rotation period. For example, at solstice Titania experiences relatively symmetrical motion relative to the magnetopause as the orbit is roughly in the terminator plane and so the satellite remains near the flanks of the magnetopause. Near equinox, Titania experiences both the dayside and nightside magnetosphere and so experiences an asymmetrical magnetopause field over an orbit. As expected, the heterodyne spectrum is much richer near equinox. There are some hints of such behaviour in similar power spectra for Ganymede presented by Seufert et al. (2011), although the resolution of their spectra is much lower and the amplitudes are smaller, as expected for satellites that are much farther from the magnetopause.

### 3.3 Line spectra: Titania and Miranda

Figure 10 shows the amplitude spectrum for each field component at Titania near equinox. This spectrum is similar to that near solstice but with larger amplitude heterodynes at equinox. The solstice spectrum for Titania is also similar to Oberon near solstice. The plot highlights the fundamental periods, their harmonics, and heterodynes. Generally the fundamental period and the second harmonics are the strongest lines. For clarity we do not plot heterodynes amongst higher harmonics of both the orbital and synodic periods but note that heterodynes between harmonics generally have a much lower amplitude. As expected for the orientation of the internal field, $B_r$ and $B_i$ have the larger amplitudes amongst the internal field terms. The magnetopause field is somewhat different since the orientation of our spherical coordinate system with respect to the solar wind direction strongly varies with season due to the large obliquity. The magnetopause field presents a great deal of fine structure in the form of heterodynes near the synodic period.

Some additional structure associated with the internal field can be seen near the orbital period, for example signals at around 300 and 600 hours, and are not predicted by sets of heterodynes between the orbital and synodic periods. With the assumption that these are additional heterodynes, they require a modulating signal with a period of around 600 hours to produce heterodynes between the orbital period and the fundamental and 2nd harmonic of this modulating period. Without an obvious source within the system, and its restriction to the internal field, made us suspect this was perturbations in the satellite orbits. We computed power spectra of the orbital elements and found a range of spectral peaks in the semi-major axis (computed using SPICE) of Titania matching the periods of low-amplitude peaks in our field data. Similar spectral peaks were also found in the orbit of Oberon and which match similar spectral peaks in the spectrum of Oberon. This also provides an interpretation for the signals near 1000 hours seen in Figure 9.
This was sufficient to for us to conclude a non-magnetospheric effect and conclude that this was to do with the orbital evolution of the satellites and we made no further investigation.

Figure 10: Amplitude spectrum for Titania near equinox. The spectrum is similar to that near solstice but with larger amplitude heterodynes at equinox. The four terms in the internal field (dipole, quadrupole, octupole, hexapole) and the magnetopause field (dipole shielding) are shown with the total in black. Orbital and synodic periods are indicated by solid vertical grey lines and their higher harmonics by heavy grey dashed lines. Dotted lines indicate a range of the most significant heterodynes.

For contrast, Figure 11 shows the amplitude spectrum at Miranda. As the orbital and synodic periods are very close together we show the spectrum over a smaller range of periods. Similar to the spectrum at Titania, the heterodynes produce a great deal of fine structure but, in contrast, this fine structure is greatly compressed due to the proximity of the orbital and synodic periods and is compressed into packets near the harmonics of the orbital and synodic periods. Many of the peaks are under-resolved or compressed together and any further work on the inducing signals at Miranda should use a larger sampling frequency to enable the resolution of this fine structure. One significant difference between Miranda and Titania is the importance of the higher degree structure in Uranus’ internal field. The quadrupole, octupole and hexapole all present significant contributions to the inducing field and the magnetopause field is around two orders of magnitude smaller than these contributions.
3.4 Primary field amplitudes

To extract the primary field amplitudes we fit the linear decomposition in equation (2) to the time series rather than determining the amplitudes from the spectra. For each harmonic, $m$, we represent the primary field as $\mathbf{B}_m = \mathbf{B}_q \cos \omega_m t + \mathbf{B}_s \sin \omega_m t$, where $\mathbf{B}_q$ and $\mathbf{B}_s$ are constant real-valued vectors that encode the amplitude, phase and polarisation of the primary field (e.g., the hodogram structure in Figures 7 and 8) as $\mathbf{B}_q = (d_x q \cos \phi_x q, d_y q \cos \phi_y q, d_z q \cos \phi_z q)$ and $\mathbf{B}_s = (d_x s \cos \phi_x s, d_y s \cos \phi_y s, d_z s \cos \phi_z s)$. These harmonics, plus a constant term, were fitted to the synthetic time series using a linear least squares matrix inversion using scipy.optimize.least_squares without bounds. For $n$ samples and $p$ harmonics, the matrix problem for the $B_x$ component can be written as:

$$
\begin{bmatrix}
B_x(t_0) \\
B_x(t_1) \\
\vdots \\
B_x(t_{n-1})
\end{bmatrix} =
\begin{bmatrix}
1 & \cos \omega_1 t_0 & \sin \omega_1 t_0 & \cdots & \cos \omega_p t_0 & \sin \omega_m t_0 \\
1 & \cos \omega_1 t_1 & \sin \omega_1 t_1 & \cdots & \cos \omega_p t_1 & \sin \omega_m t_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
1 & \cos \omega_1 t_{n-1} & \sin \omega_1 t_{n-1} & \cdots & \cos \omega_p t_{n-1} & \sin \omega_m t_{n-1}
\end{bmatrix}
\begin{bmatrix}
C \\
B_{xq,1} \cos \phi_{xq,1} \\
B_{xs,1} \sin \phi_{xs,1} \\
B_{xq,p} \cos \phi_{xq,p} \\
B_{xs,p} \sin \phi_{xs,p}
\end{bmatrix}
$$

(9)
The selection of harmonics was performed algorithmically. Prior to the fit, a list of trial periods was constructed from the orbital, synodic and Uranus rotation periods, their higher harmonics, and heterodynes up to a maximum harmonic degree of 8. At each step of the fitting a period was removed from this list and added to a list of periods. The matrix (9) is constructed for each field component and the matrix inverted to provide a list of coefficients. The residuals are calculated and from these the root-mean-square error, the maximum relative (to the peak in that component) error, and the reduced $\chi^2$. The added period is only retained if the quality of the fit is improved, defined by the root-mean-square, reduced $\chi^2$, or maximum relative error decreasing by a factor of $1 \times 10^{-7}$, for all three field components.

Table 1 contains the amplitudes and driving periods for Titania, at solstice and equinox, and Oberon at solstice. Driving periods were only included that were within a factor of $10^{-3}$ of the maximum amplitude for that field component. The identified source of the driving harmonic is listed in the right-hand column. For some harmonics there are multiple physical sources that contribute to that harmonic and we list all the contributors up to 90% of the amplitude of that harmonic; for example, if the dipole can explain 89% of a harmonic then we also list the next most important contribution to get to at least 90%. 


|            | Titania (Equinox) | Titania (Solstice) | Oberon (Solstice) | Source                                      | Period                                      |
|------------|------------------|-------------------|-------------------|---------------------------------------------|---------------------------------------------|
|            | A [nT]           | P [h]             | A [nT]            | P [h]                                       |                                             |
| B<sub>r</sub> | 0.137            | 9.395             | 0.137             | 9.395                                       | Quadrupole Synodic (2nd)                     |
| B<sub>r</sub> | 0.288            | 15.93             | 0.084             | 15.93                                       | MP H+ Orbital (2nd) / Synodic               |
| B<sub>r</sub> | 0.371            | 17.24             | 0.115             | 17.24                                       | MP Uranus                                   |
| B<sub>r</sub> | 6.15             | 18.79             | 6.63              | 18.79                                       | Dipole / MP Synodic                         |
| B<sub>r</sub> | 1.15             | 20.65             | 0.485             | 20.65                                       | MP H- Orbital / Synodic                     |
| B<sub>r</sub> | 0.419            | 22.91             | 0.122             | 22.91                                       | MP H- Orbital (2nd) / Synodic               |
| B<sub>r</sub> | 0.117            | 25.73             | 0.0413            | 21.92                                       | MP H- Orbital (3rd) / Synodic               |
| B<sub>r</sub> | 0.106            | 209               | 0.077             | 209                                         | MP (+dipole at Titania)                     |
| B<sub>b</sub> | 0.00672          | 9.395             | 0.077             | 209                                         | Octupole Synodic (2nd)                      |
| B<sub>b</sub> | 0.0119           | 14.8              | 0.046             | 15.93                                       | MP H+ Orbital (3rd) / Synodic               |
| B<sub>b</sub> | 0.087            | 15.93             | 0.046             | 15.93                                       | MP H+ Orbital (2nd) / Synodic               |
| B<sub>b</sub> | 0.448            | 17.24             | 0.409             | 17.24                                       | MP Uranus                                   |
| B<sub>b</sub> | 0.481            | 18.79             | 0.795             | 18.79                                       | MP (+quadrupole at Titania)                 |
| B<sub>b</sub> | 0.345            | 20.65             | 0.304             | 20.65                                       | MP H- Orbital / Synodic                     |
| B<sub>b</sub> | 0.106            | 22.91             | 0.0555            | 22.91                                       | MP H- Orbital (2nd) / Synodic               |
| B<sub>b</sub> | 0.025            | 25.73             | 0.0223            | 21.92                                       | MP H- Orbital (3rd) / Synodic               |
| B<sub>b</sub> | 0.0328           | 69.66             | 0.0422            | 104.5                                       | MP Orbital (3rd)                            |
| B<sub>b</sub> | 0.159            | 104.5             | 0.0422            | 104.5                                       | MP Orbital (2nd)                            |
| B<sub>b</sub> | 0.626            | 209               | 0.328             | 209                                         | MP Orbital                                  |
| B<sub>b</sub> | 0.0912           | 9.395             | 0.0913            | 9.395                                       | Quadrupole Synodic (2nd)                     |
| B<sub>b</sub> | 0.235            | 15.93             | 0.0741            | 15.93                                       | MP H+ Orbital (2nd) / Synodic               |
| B<sub>b</sub> | 5.53             | 18.79             | 5.27              | 18.79                                       | Dipole / MP Synodic                         |
| B<sub>b</sub> | 1.09             | 20.65             | 0.598             | 20.65                                       | MP H- Orbital / Synodic                     |
| B<sub>b</sub> | 0.425            | 22.91             | 0.129             | 22.91                                       | MP H- Orbital (2nd) / Synodic               |
| B<sub>b</sub> | 0.118            | 25.73             | 0.046             | 21.92                                       | MP H- Orbital (3rd) / Synodic               |
Table 1: Amplitudes and periods of the inducing field at Titania near equinox and solstice, and Oberon near solstice. Amplitudes are given to 3 s.f. and periods to 4 s.f.. The right hand columns indicate the source of the signal and the nature of the inducing period where Uranus is the rotation period of Uranus, Orbital indicates the orbital period of the satellite, and Synodic indicates the synodic period of the satellite. H- and H+ indicate a heterodyne between the two listed periods, e.g., H+ Orbital (2nd) / Synodic indicates a positive heterodyne between the 2nd harmonic of the orbital period and the synodic period, and MP indicates the field of the magnetopause currents.
4. Detectability of an induced field: Implications for future missions

4.1 Amplitude and phase

Figure 12 shows the amplitude and phase, from equations (4) and (5), of the induced response at Titania, Oberon and Miranda for representative driving periods and as a function of ocean conductivity. The conductivity spans four orders of magnitude to explore the parameter space and to highlight the solutions, although it is important to highlight that conductivity at the upper end of the range is unrealistic as salts reach saturation at ~6 S/m for MgSO$_4$ and ~18 S/m for NaCl (Hand and Chyba, 2007). Although the 9.395 hour second harmonic of the synodic period at Titania is not a high amplitude signal, this is included to provide contrast with the signals near the synodic period. The amplitude of the induced response is typically below 0.2 and is very different to the scenario at Europa with an amplitude of 0.97±0.02 (Schilling et al., 2004). This much smaller amplitude is principally due to the thick ice shell overlying the ocean; thinner ice shells of around 50 km in depth produce larger amplitudes near 0.8. Although the Hussman, Sohl and Spohn (2006) models do not include solutions with such thin ice shells, they also fail to do so for Enceladus and it has been shown via Cassini observations of Enceladus’ gravity field that there is a relatively thin ice shell overlying an ocean (Čadek et al., 2016). The models for Europa also contain much thicker ice shells than constraints from observations (e.g., Hand and Chyba, 2007). Thus, thin ice shells may also be common at the uranian icy satellites but we restrict our focus to the model of Hussman, Sohl and Spohn (2006) as physically motivated ice shell thicknesses. In the following examination we explore two conductivity limits, a) 2.75 S/m, consistent with the conductivity of Earth’s oceans and requiring 96.8 g MgSO$_4$ per kg of H$_2$O in the ocean (Hand and Chyba, 2007), and b) 0.275 S/m, requiring only 4 g MgSO$_4$ per kg of H$_2$O.

At Titania, the amplitude of the induced field at 18.79 hours ranges from 0.43 to 1.2 nT for the different ocean configurations, and 0.014 to 0.026 nT at 9.395 hours (“3% of the response at the synodic period). The heterodynes near the synodic period provide relatively strong responses around 10% of those at the synodic period, however their phases are lagged by more than 15° from the response at the synodic period. At a conductivity of 0.275 S/m the response at 18.79 hours ranges between 10 and 17% of the values at the higher conductivity for the three ocean configurations. However, crucially, the phase is further lagged between 20° and 46° suggesting the possibility to distinguish between ocean depth and conductivity from future observations.

By way of contrast, at Miranda the amplitudes (Figure 12) are lower but the primary field is much higher indicating the possibility for detecting an ocean. The clear separation of synodic periods provides more driving periods. At the synodic period of 35.04 hours the amplitudes are between 2.0 and 8.5 nT for 10 to 30 km oceans, and at the second harmonic of 17.52 hours the amplitude is 0.27 to 1.1 nT. The difference in phase lag varies by almost 20° between the first and third harmonics of the synodic period for the 30 km ocean, but only 6° for the 10 km thick ocean. At the lower conductivity the responses are between 0.2 to 0.86 nT, although the 10 km ocean cannot be accurately modelled by an induced dipole at the lower conductivity (explored further in the discussion and conclusions).
Figure 12: The amplitude and phase lag of the induced response at Titania (top), Oberon (middle) and Miranda (bottom) for two representative driving periods. Each panel shows three separate ocean depths and ice shell thicknesses broadly following the solutions of Hussman et al. (2006), except for Miranda which is more speculative. The vertical dashed lines indicate solutions for conductivities of 0.275 and 2.75 S/m.

4.2 Synthetic flyby

For a more practical assessment of the visibility of an ocean we examine the magnetic field perturbation from a flyby of Titania. To estimate the effect of random and systematic errors on the measurements we constructed a simple forward...
model of the Voyager magnetometer. We incorporate the transformation from the sensor to spacecraft frame, \( \mathbf{M}_{\text{sensor-sc}} \), spacecraft to geophysical frame, \( \mathbf{M}_{\text{sc-geo}} \), and incorporated scale factors, \( \mathbf{k} \), and offsets, \( \mathbf{z} \), according to equation (10) (after Acuña, 2002) to convert from engineering (measured) units, \( \mathbf{V} \), to field strength, \( \mathbf{B} \).

\[
\mathbf{B} = \mathbf{k}(\mathbf{V} - \mathbf{z})\mathbf{M}_{\text{sensor-sc}}\mathbf{M}_{\text{sc-geo}} \tag{10}
\]

In a real set of spacecraft observations the transformations \( \mathbf{M}_{\text{sensor-sc}} \) and \( \mathbf{M}_{\text{sc-geo}} \) contain errors due to twisting and bending motions of the boom away from some calibrated alignment and a finite knowledge and control of the spacecraft attitude. The scale factors and offsets are also subject to uncertainty. These can all be controlled to some degree through calibration but systematic and random errors persist. The data in engineering units are also quantised into a finite number of bits thus generating some quantisation noise. In the case of Voyager the data are quantised into 12 bits, although some of these bits are used for ‘guard bands’ at the upper and lower extrema of each sensitivity range (Behannon et al., 1977; Berdichevsky, 2009) reducing the available bits by five for Voyager. To assess the impact of these uncertainties and quantisation our forward model takes the modelled magnetic field during a flyby, transforms the modelled data into quantised engineering units via equation (10), and then reinverts the data to produce a synthetic timeseries. This is schematically illustrated in Figure 13.

The attitude of the spacecraft was specified with some constant axial tilt and a time-varying roll rate to give some constant changing attitude with respect to the ambient field. Both angles were perturbed with normally-distributed angles with a standard deviation of 0.035° to give a maximum RMS error of around 0.05° thus simulating finite knowledge/control of the spacecraft attitude. For simplicity, the error on the alignment of the boom was simulated by twisting the boom around its long axis and we did not consider bending of the boom. This was effected by generating a set of random boom twist angles, equally-spaced in time, that were converted into a continuous boom twist angle via cubic interpolation. The random twist angles were generated from a normal distribution with a standard deviation of 0.25° to give a twist amplitude less than 1° (Miller, 1979). Small errors in the offsets and scale factors were introduced by perturbing the offsets by \( \pm 6 \) counts (Berdichevsky, 2009) and the scale factors were scaled by a normally distributed factor \( \sim \mathcal{N}(1, 0.01) \) to simulate an error of up to around 4%, e.g., instead of 0.005 nT/count, for example, the scale might be \( \sim 0.0048 \) or \( \sim 0.0052 \) nT/count. No attempt is made to specifically emulate the Voyager magnetometer in great detail – just as a template for a reasonable magnetometer that might measure the fields at Uranus.
Each primary harmonic was calculated and used to determine the secondary field for a conductivity of 2.75 S/m. The total field was calculated and the constant terms added to generate the ideal field which was then subjected to the transformations described above. We computed the field along a flyby trajectory with a closest approach altitude of 200 km at a 13° inclination and a flyby speed of 4.5 km/s consistent with the orbital tour presented in the Ice Giants Pre-Decadal Survey Mission Study Report (Hofstdater, Simon et al., 2017). Figure 14 shows the results of our synthetic time series. The small amplitude fluctuations in the $B_z$ component are due to the simulated attitude and boom twisting uncertainties. Quantisation noise is generally small and unimportant on this scale and in this magnetometer range.

Ocean thicknesses of 40 and 52 km should be readily detectable from the bipolar signature in $B_x$ and negative peak in $B_y$. The perturbation in $B_z$ is generally masked by orientation errors with magnitudes of around 0.1 nT. Ocean thicknesses of 16 km with a conductivity of 2.75 S/m present a more marginal case for detection from a 200 km altitude flyby, especially if there is a significant plasma interaction that may mask the induction signature. However, it is important to stress that we consider this as a conservative examination of the errors and careful calibration work could mitigate the errors we have considered. It also provides input for constraints required on a future magnetometer instrument and spacecraft platform.
557 Figure 14: Synthetic time series for a flyby of Titania in satellite-centred interaction coordinates: y is orientated 558 towards Uranus from the satellite, x along the orbit of the satellite, and z perpendicular to the orbit plane. The inset 559 shows a period ±15 minutes from closest approach on a smaller scale. The induced perturbations due to the 39 and 52 560 km ocean models are relatively clearly identified by the perturbation from the 16 km ocean is relatively small.

5. Discussion and conclusions

In this paper we have explored the possibility of detecting subsurface oceans at the uranian icy satellites, focusing on 563 the outer two satellites, Titania and Oberon, as thermal and structural models have identified these as candidates for 564 hosting subsurface oceans. An analytical model for the uranian magnetospheric magnetic field was constructed and 566 used to generate magnetic field time series at the orbits of the five main satellites. These time series were subjected to 567 a spectral analysis to identify the periods of driving signals and their amplitudes were determined via fitting a model 568 harmonic time series. The amplitude of the induced field was calculated at the identified periods to examine the 569 strength of a possible induced response. We found significant periodic signals near the synodic and orbital periods, and 570 their higher harmonics, alongside a rich spectrum of heterodynes particularly associated with the magnetopause field.

The heterodynes were found to be a persistent feature in the magnetopause field due to a mixing of the synodic period 572 with the orbital period and its harmonics. This was interpreted as the product of two effects:
Orbital period: A satellite would experience a changing magnetopause field as it orbited within an asymmetrical magnetospheric cavity.

Diurnal period (+harmonics): As Uranus rotates the solar wind attack angle varies in a (generally) non-sinusoidal fashion (e.g., Figure 3) and therefore the magnetopause field has a diurnal periodicity plus higher order harmonics due to the non-sinusoidal variation in the attack angle.

We found that the identified induced field amplitudes at Titania can vary by a factor of around three between equinox and solstice, due to the variation of both satellite orbit geometry relative to the magnetopause and the solar wind attack angle with season, although this variation in amplitude was mostly restricted to the rich spectrum of heterodynes. The spectrum was found to be generally richer at equinox but contained many closely spaced spectral lines around the synodic period. It remains to be seen if these could be separated and used to constrain a subsurface ocean. It is worthwhile commenting that there is some evidence for similar fine structure at Ganymede, e.g. via an inspection of Figure 4 in Seufert et al., (2011) although the amplitudes are smaller as expected for a satellite much deeper within the magnetosphere (up to around 50% of the magnetopause subsolar distance) than Titania and Oberon.

The major seasonal effect is the proximity of Oberon to the magnetopause. For a period of around ±7 years centred on 2030 Oberon should remain inside the magnetosphere and the results from Table 1 apply. After that time, e.g., for missions arriving later in the 2030s or in the 2040s, it may be possible to detect an ocean, but only from signals near the synodic period and where the satellite has been inside the magnetosphere for a significant period while the eddy currents establish themselves. This places clear constraints on flyby locations and would require a flyby to be timed for after Oberon had left the vicinity of the dayside magnetosphere (moving towards the nightside) and preferably just before it re-emerges into the dayside from the nightside magnetosphere.

These driving periods were combined with a model for the induction response and we showed that induced signatures should be detectable and this was confirmed with a magnetometer forward model and synthetic time series from a Titania flyby. We found that ocean thicknesses of 40 and 52 km should be readily detectable from a flyby with a 200 km altitude closest approach, although a 16 km thick ocean was at the limit of detectability. This analysis demonstrated that a 200 km altitude flyby would be acceptable, but would limit ocean depth/conductivity/ice shell ranges and so lower altitude flybys are strongly recommended. Given the weakness of some signals, this also demonstrates that maintaining an AC spacecraft magnetic field below 0.1 nT (preferably below 0.01 nT) at the magnetometer would be advantageous in our ability to resolve less conducting, thinner, and or deeper oceans; although more accurate constraints require further study. It is important to highlight that this work has been guided by the work of Hussmann et al. (2006) and thinner overlying ice shells would result in stronger induced fields and thus would be more readily detectable than the somewhat thicker ice shells predicted by Hussmann.

We used an induced dipole model to represent the response of a subsurface ocean. This model is valid when the ocean depth is much greater than the electromagnetic skin depth, \( \delta = \sqrt{2/\sigma \mu_0 \omega} \) (e.g., Khurana et al., 1998). For Titania this is almost always satisfied for the 39 and 52 km thick oceans, over a wide range of conductivities above 0.1 S/m. The exception to this is that the 209 hour orbital period signal can only be analysed for conductivities above 1 S/m with this model. The thinner 16 km ocean requires higher conductivities >1 S/m for the main driving periods under this model.
A similar set of restrictions also apply to Oberon. At Miranda the model is only valid above around 3 S/m for the 10 km ocean case, but is less stringent for the 20 and 30 km ocean cases.

Attempting to model fields due to a plasma interaction was deemed beyond the scope of this work and requires further analysis. From a stellar occultation in 2001, Widemann et al. (2009) placed an upper limit of 9-17 nbar on the surface pressure of a CO atmosphere with temperatures of 60-80 K respectively, far below the surface pressure at Triton, Pluto and Europa. Hence, due to Uranus’ large heliocentric distance and relatively low densities of sufficiently high temperature electrons to drive electron impact ionisation (e.g., Sittler et al., 1987), there may be a negligible interaction between the plasma in the magnetosphere and only a tenuous ionosphere. Therefore the interaction may consist of a downstream plasma wake with Alfvén wings. However, the surface pressure may well be seasonally dependent and one shouldn’t rule out plume-like activity such as that found at Enceladus. Regardless, although the plasma beta is generally low, ~ 1 near the plasma sheet (Behannon et al., 1987), the field strength is also lower and so plasma interaction currents and therefore their field, may be higher in order to balance the forces associated with the plasma interaction. The location of Titania and Oberon close to the magnetotail may introduce additional influences due to magnetospheric dynamics.

We specifically excluded solar wind variability in order to focus on the asymmetrical and seasonal drivers, but future work should incorporate this source of variability. We have begun to explore this effect and have identified drivers at the solar periodicity with amplitudes of order 0.1 nT at Titania and Oberon.

In summary:

1. From our simple model of Uranus’ magnetospheric magnetic field there are a rich spectrum of magnetic field periodicities at the natural satellites of Uranus. The more distant satellites show considerable seasonally-dependent fine structure associated with the geometry of the solar wind-magnetosphere interaction and orientation of the satellite orbital plane, whereas the inner satellites show stronger signals from the asymmetrical internal field.

2. From models of expected subsurface ocean structure (Hussmann, Sohl and Spohn, 2006), an induction model, and our model of the inducing field, the amplitude of the modelled induced response is typically below 0.2 and is very different to Europa with an amplitude of 0.97±0.02 (Schilling et al., 2004). This much smaller amplitude is principally due to the thick ice shell overlying the ocean and thinner ice shells produce larger responses of order 0.8. It is worth remarking that the models of Hussmann, Sohl and Spohn (2006) also fail to reproduce the thin shell at Enceladus as inferred from Cassini observations (Čadek et al. (2016).

3. Thick ocean models at Titania are detectable from a 200 km altitude flyby, but a 16 km thick ocean is at the limit of detectability. Detection of oceans at Oberon is complicated by the proximity of Oberon to the magnetopause and so flybys should be designed to encounter Oberon on the nightside of Uranus near equinox.

4. Further work should explore i) additional periodicities driven by solar wind variations; ii) the relative importance of plasma interaction currents; and iii) the inclusion of tail current systems.
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Appendix: Coefficients for dipole shielding field

Units of $a_{ij}, b_{ij}, c_{ij},$ and $d_{ij}$ are nT R$_U$ and $p_i, r_k, q_j, s_i$ are in units of R$_U$.

| $a_{00}$ | -0.5287296425170684 | $b_{21}$ | 0.1722800052080468 |
| $a_{01}$ | 15.077691459563148 | $b_{22}$ | 0.010840039677544875 |
| $a_{02}$ | 12.542608435251966 | $c_{00}$ | 0.529043977018929 |
| $a_{10}$ | 0.6790677915590768 | $c_{01}$ | 0.5531924779259334 |
| $a_{11}$ | 0.52839329955896 | $c_{02}$ | 2.795636187534518 |
| $a_{12}$ | 5.187345418230727 | $c_{03}$ | 4.165392238057216 |
| $a_{20}$ | 7.588002567699361 | $c_{10}$ | 9.47249250384742 |
| $a_{21}$ | -0.038955334291481634 | $c_{11}$ | 3.1398706541002945 |
| $a_{22}$ | -0.4309246786275432 | $c_{12}$ | 0.5607158782730544 |
| $b_{00}$ | -0.067659794456119 | $c_{13}$ | 15.577086936808046 |
| $b_{01}$ | -0.265084622023287 | $c_{20}$ | 0.6801897898848649 |
| $b_{02}$ | 0.039048819175832444 | $c_{21}$ | 8.167236925544106 |
| $b_{10}$ | 0.003658445181747277 | $c_{22}$ | 8.544135882214677 |
| $b_{11}$ | 0.380059786028047 | $c_{23}$ | -7.368520962970561 |
| $b_{12}$ | -0.32854084061998434 | $c_{30}$ | -0.000593516601684975 |
| $b_{20}$ | 0.0793912960739725 | $c_{31}$ | 4.51881410634166 |

| $c_{32}$ | -2.629656639894243 | $d_{00}$ | -0.0158599591234329 |
| $c_{33}$ | 0.07375174811174984 |
| $d_{01}$ | 0.4876191637729948 |
| $d_{02}$ | -0.8042519008224641 |
| $d_{03}$ | 0.31507763470267774 |
| $d_{10}$ | -0.27099432374589136 |
| $d_{11}$ | -1.3517952498602952 |
| $d_{12}$ | 1.9871775394184894 |
| $d_{13}$ | 0.519371982872029 |
| $d_{20}$ | 0.4998831443486746 |
| $d_{21}$ | 0.0010817323978511068 |
| $d_{22}$ | 4.23796706092375 |
| $d_{23}$ | -0.08589911614621044 |
| $d_{30}$ | -0.2790143410517384 |
| $d_{31}$ | 1.9206493946310075 |
| Symbol | Value |
|--------|-------|
| $d_{32}$ | -3.329992459682197 |
| $d_{33}$ | -3.114276641196077 |
| $p_0$ | 54.20597580715924 |
| $p_1$ | 18.161591584063434 |
| $p_2$ | 57.09281695782331 |
| $r_0$ | 14.88166750097028 |
| $r_1$ | 32.31074443198433 |
| $r_2$ | 27.699339701283346 |
| $q_0$ | 15.054040480643655 |
| $q_1$ | 33.8559135376921 |
| $q_2$ | 143.5427552567043 |
| $q_3$ | 72.07887351621275 |
| $s_0$ | 15.970961048424488 |
| $s_1$ | 33.513749485292834 |
| $s_2$ | 120.97202513997192 |
| $s_3$ | 53.377584869814584 |
