A model of grassroots changes in linguistic systems
Janet B. Pierrehumbert\textsuperscript{1,2,3*}, Forrest Stonedahl\textsuperscript{4}, Robert Daland\textsuperscript{5}
1 Linguistics Department, Northwestern University, Evanston, IL, USA
2 Northwestern Institute on Complex Systems, Evanston, IL USA
3 New Zealand Institute of Language, Brain, and Behaviour, Univ. of Canterbury, Christchurch, NZ
4 Computer Science, Augustana College, Rock Island, IL, USA
5 Linguistics Department, Univ. California Los Angeles, Los Angeles, CA, USA
* E-mail: jbp@northwestern.edu

Abstract

Linguistic norms emerge in human communities because people imitate each other. A shared linguistic system provides people with the benefits of shared knowledge and coordinated planning. Once norms are in place, why would they ever change? This question, echoing broad questions in the theory of social dynamics, has particular force in relation to language. By definition, an innovator is in the minority when the innovation first occurs. In some areas of social dynamics, important minorities can strongly influence the majority through their power, fame, or use of broadcast media. But most linguistic changes are grassroots developments that originate with ordinary people. Here, we develop a novel model of communicative behavior in communities, and identify a mechanism for arbitrary innovations by ordinary people to have a good chance of being widely adopted.

To imitate each other, people must form a mental representation of what other people do. Each time they speak, they must also decide which form to produce themselves. We introduce a new decision function that enables us to smoothly explore the space between two types of behavior: probability matching (matching the probabilities of incoming experience) and regularization (producing some forms disproportionately often). Using Monte Carlo methods, we explore the interactions amongst the degree of regularization, the distribution of biases in a network, and the network position of the innovator. We identify two regimes for the widespread adoption of arbitrary innovations, viewed as informational cascades in the network. With moderate regularization of experienced input, average people (not well-connected people) are the most likely source of successful innovations. Our results shed light on a major outstanding puzzle in the theory of language change. The framework also holds promise for understanding the dynamics of other social norms.
Introduction

Many human needs and goals are shared by people in different cultures. But the social norms for addressing them can be highly arbitrary in their details. This is especially true for norms that promote communication and social cohesion. The value of such norms may arise more from the existence of a consensus than from its specifics. A fax machine has value only insofar as other people also have compatible fax machines. Facebook supports social contact only insofar as one’s friends and relations are also on Facebook. This situation is epitomized in the most complex and highly evolved communication system of all, namely human language. Relationships between word forms and word meanings are very idiosyncratic across languages, and hence must be learned for any individual language [1, 2]. The usefulness of language comes about because people come to agree on ways to refer to entities, events, and abstractions. Agreement enables them to pool individual knowledge and benefit from cooperative activities over great expanses of time and space.

Once a linguistic consensus is in place in a community, why and how can it ever change? This question, the central focus of our paper, is a major challenge for models of language dynamics. It is widely agreed that linguistic systems emerge through innate propensities for people to imitate others in their communities [3–9]. Frequent forms, and forms produced by many different speakers, are at an advantage in being learned, remembered, and used [10–13]. In contrast, infrequent forms are vulnerable to loss over time [12, 14]. These basic findings all mitigate against linguistic innovations, which by definition are rare and poorly disseminated when they first occur. Nonetheless, natural languages are never perfectly stable. They change at a wide range of time scales, and the fastest changes (such as the adoption of new slang expressions and jargon) are exhibited on time-scales of a few months or less within the speech of individual adults. In this regard, linguistic systems resemble cultural norms in other domains such as fashion, dancing, politics, and religion. In these domains as well, people are strongly influenced by the consensus of their neighbors, but norms still evolve over time. Complex patterns of variation across groups emerge when distinct groups follow different trajectories of cultural evolution.

Here, we develop a new model of how linguistic innovations can become widely adopted in a community. We focus on a type of changes that are particularly challenging to model: changes that we characterize as fast, arbitrary, and egalitarian. By fast changes, we mean ones that occur through learning and adaptation by individual speakers in the course of repeated interaction with their peers, without any generational replacement. Ref [15] characterizes within-generation changes in social norms as cases of horizontal transmission. The slow changes that occur during cross-generational transmission of language fall outside of the scope of this paper; modeling them requires distinct mechanisms [6, 16].

By arbitrary changes, we mean changes that involve no global functional advantage. They may be neutral, offering no identifiable functional advantage at all. Or, if there is any
advantage to some people from adopting them, it is balanced by some equal disadvantage to others. We do not deny that some changes in language may be globally advantageous. For example, human languages are shaped by a persistent bias to encode more predictable information using shorter expressions, which improves the efficiency of communication [17]. However, prior work in language evolution, drawing on evolutionary biology, already reveals how positive utility can cause an innovation to spread. In contrast, for many linguistic changes, no global advantage is apparent. Explaining how arbitrary changes can take hold is essential to explain the vast profusion of apparently arbitrary detail that is observed synchronically [1].

We use the term *egalitarian* to highlight our interest in changes that originate with ordinary people and spread without regard to social status. Sociolinguistic field work has shown that people at the forefront of linguistic change typically belong to the lower middle class or upper working class. Although imitation of upper-class people may play a role in some conscious choices, such as product purchases, it cannot be the fundamental mechanism for linguistic changes. Typically, upper-class speakers eventually adopt patterns set by lower classes, rather than the other way around [18]. This pervasive pattern could not arise if imitating the few people at the top of the social hierarchy (perhaps in an effort to increase social capital) is the engine of linguistic change. However, the empirical findings do not necessarily indicate that people preferentially imitate lower-class people. Ordinary people are far more numerous than upper-class people, and this factor alone might explain the results. We thus take as a starting point the null hypothesis by which listeners weight input from all speakers equally, without assigning more value or prestige to some speakers than to others. Unlike Refs [3,4], we do not assume that people with many social connections have more status than others, or are preferentially taken as linguistic models. This null hypothesis takes us surprisingly far. We identify a confluence of social and cognitive factors that enables fast, arbitrary, and egalitarian innovations to propagate with considerable probability. The key cognitive factor is the decision rule relating the perception of linguistic norms to the choice of what form to produce on any particular occasion. The key social factor is the distribution of biases over the members of the community. If some people are conservative, and others love novelty, the linguistic system is more likely to change than if nobody is biased in either direction. These findings challenge previous claims [3,4,19] that differences in social prestige are a necessary component in a theory of how linguistic innovations can spread.

Our model of how innovations propagate integrates technical ingredients from work in several disciplines. Following recent work in network theory, we consider the linguistic community to be a social network in which nodes represent people and links represent social relationships. Specifically, a link between two nodes means that the two communicate with each other and are disposed to imitate each other in their choices of expression. Social ties that do not involve this level of mutual influence exist of course, but these ties are omitted from the model.

Next, we draw a careful distinction between the mental state of a node and the signals
that the node emits. We assume that signals are categorical. For example, on each individual occasion when some speaker wishes to quote someone else, she either uses the conservative form (e.g. *he said*) or an innovative form (e.g. *he was like*). Mental states, in contrast, are numbers on a continuous scale of probabilities, representing the speaker’s current estimate of the linguistic norm. They take values from 0 (“I don’t believe anybody uses that expression”) to 1 (“I believe everybody uses that expression”). The value 0 will be used here for the conservative norm, and 1 for the innovation, so that in this example, a mental state of 0.6 would represent the speaker’s belief that people in general use *like* to express quotation with \( P = 0.6 \). The model is applicable to any linguistic innovation that can be described as a new category competing with a pre-existing one. Examples include novel slang, such as *lol* in competition with *ha-ha*; novel affixes such as *uber-* in competition with *super-*, as well as new constructions, such as *gonna* in competition with *will* for expressing the future. It does not encompass gradient changes in the signals themselves, such as the gradual shifts in vowel quality explored in Ref [20].

Signals and mental states are linked by a decision function, whose input is the mental state and whose output is the probability of emitting different signals. This decision process is unconscious, but it is not trivial. To characterize variability in the decision function, we adopt the concepts of temperature and bias from the field of reinforcement learning. The bias captures the extent to which an individual generally favors conservative norms versus innovations. Temperature characterizes the extent to which the individual chooses the optimal production based on input received so far, versus exploring other options. A higher temperature means that individuals make more random choices, an analogy to the behavior of gas molecules, which exhibit more random motion under higher temperatures. In this application, we construe the optimal signal to be the one that others are most likely to accept as the norm. A central innovation of our model is a novel mathematical treatment of the decision function. In standard treatments of temperature, the decision function is the SoftMax function, which reduces to the logistic in the case of binary choices. The logistic is a sigmoidal function that maps inputs on the interval \((-\infty, \infty)\) to values on the interval \((0, 1)\). Varying the free temperature parameter \( \tau \) causes its form to vary between a step function as \( \tau \to 0 \) and a constant function as \( \tau \to \infty \).

We analyze the behavior of the logistic function in the context of the emergence of linguistic norms, and identify serious problems with its behavior and interpretability. Linguistic norms resemble other application areas for reinforcement learning, because people learn them incrementally from experiences over time. However, a critical difference is that the signals produced by each individual provide the input for the neighbors, so that repeated signaling gives rise to positive feedback loops in the community. The logistic equation does not display sensible behavior under such iteration. For any temperature \( \tau > 0 \), it cannot characterize the situation in which an absolute agreement in the linguistic community is stable. It is also unable to capture probability-matching behavior, in which people produce linguistic variants with the same relative frequencies that they experience them. Since probability-matching behavior and stable agreement are two of the principle
phenomena we need to characterize, we develop a new sigmoidal decision function that incorporates the broad insights behind the logistic, while also avoiding these problems. This is the clog ('cognitive logistic') function. Like the logistic, the clog is a sigmoidal function that reduces to a step function as $\tau \rightarrow 0$. Unlike the logistic, it is designed to map probabilities onto probabilities. It converges to the line $y = x$, and not a constant function, as $\tau \rightarrow \infty$. Variation in $\tau$ is usefully viewed as variation in the categoriality of the decision process. As $\tau \rightarrow 0$, the output approaches two categories, $y = 1$ (or Boolean TRUE) and $y = 0$ (or Boolean FALSE). The variation in the input is completely regularized in the output. As $\tau \rightarrow \infty$, the categorization vanishes as all experienced probabilities become equally available to the production system. The output faithfully reflects the input, without any regularization at all.

Finally, these ingredients come together in our treatment of the diffusion of innovations as informational cascades [21]. Informational cascades occur in a social system when a local change propagates through a large group as a result of people’s repeated interactions and tendencies to conform to each other. Initially developed in the area of opinion dynamics and product adoption [21–23], the concept is applicable to language dynamics under the assumption that linguistic norms are collective opinions about how to express concepts [4]. Figure 1, the result of one run of our model, illustrates a partial informational cascade in a social network in which people vary in their biases. At the beginning of the run, the innovator (at the left edge) is the only person in the network who has adopted the novel expression. Although the innovator is directly connected to only three other people, the expression is adopted by many others in the network to varying degrees after a large number of communicative interactions have occurred.

To explore the behavior of the clog in relation to likelihood of informational cascades in social networks, we report simulations on networks of 256 nodes. This size corresponds to a village, social club, or adolescent peer group that might adopt a new opinion, fashion, or linguistic expression in a short time. The social network is idealized as a scale-free network generated by a preferential attachment rule [24].

We have already said that bias heterogeneity is a key factor in the successful propagation of linguistic innovations. Variation in individual biases is generated by sampling from a uniform distribution. However, values are sampled in pairs to ensure the overall functional neutrality of the innovative variant: each individual who is biased toward the innovation is balanced by a different individual in the same run with an equal but opposite bias. In the field of opinion dynamics, the adoption of an innovation is typically treated as absolute and irreversible (c.f. [16,25]). Under these assumptions, bias heterogeneity has already proven to be an important factor in the propagation of innovations [26]. In contrast to these models, we treat individual changes as reversible, in the interests of empirical realism in the language domain. Reversibility does not affect which end states can be reached by the system, but it does impact the likelihoods of the different end states, which is our primary interest. The previous observations about the importance of bias heterogeneity generalize to this new model structure.
Figure 1. A partial cascade of a novel form of expression originating from a low-degree innovator. For each individual $i$, the degree of preference $m$ for the novel opinion is represented as a probability from blue ($m_i = 0$) to red ($m_i = 1$). The network layout is governed by the distance from the innovator. Heterogeneous individual biases are assigned on the basis of this distance as described in the text for the nearby scenario. The decision rule is absolutely categorical (model parameter $\tau = 0$).

Our Monte Carlo simulation, implemented as an agent-based model, systematically explores the interaction between categoriality, innovator degree, and the distribution of biases in the population. We contrast two baseline scenarios, for which we expect rather stable norms, to three novel scenarios in which change is more likely. In one baseline scenario, people simply reproduce the frequencies they encounter. This scenario is comparable to neutral evolution in evolutionary biology, and is already known to make incorrect predictions about rates of language change [27]. In a second baseline, people categorize the frequencies they encounter (to greater or lesser extents), but no one has any bias for or against the change; the only heterogeneity in the population is in the number of social connections in the network. This setup is known to be highly stable if the decision function is a step function [22][26], and we will extend this result to less categorical decision functions. Of greatest interest are the three other scenarios, in which people in the network both categorize frequencies, and vary in their biases. We designate these as the random, hubs, and nearby scenarios. In the random scenario, bias values are distributed randomly.
in the community. In the hubs scenario, innovation-favoring bias is preferentially allocated to high degree nodes (hubs), and bias against the innovation is allocated to low degree nodes. In the nearby scenario, innovation-favoring bias is preferentially allocated to the individuals topologically nearest to the innovator (regardless of the innovator’s degree), and bias against the innovation is allocated to the individuals that are furthest from the innovation.

Cascading is rare in the random scenario, though this scenario does support some partial cascades beyond those that are observed in a neutral evolution model. Two different regimes in which change is far more likely are identified. In the hubs scenario, the probability of a cascade increases monotonically with categoriality and innovator degree; total cascades become certain for sufficiently high categoriality and innovator degree. This regime may be relevant for social phenomena in which a few highly connected individuals are able to trigger sudden changes in collective opinion through their ability to broadcast their preferences to a large number of people at once. However, it is less relevant to language change than the nearby scenario. In the nearby scenario, cascades are most likely at a moderate level of categoriality, and are most likely to be initiated by individuals with low to middle degrees of connectivity. This novel result captures the phenomenon of changes that are fast, arbitrary and egalitarian. It represents a previously unsuspected regime for informational cascades. The discovery of this regime is due to our model’s ability to formalize the concept of moderate categoriality and explore the interaction of categoriality with heterogeneity in the population.

1 Model definition and justification

1.1 Social network

We consider $N$ individuals in a community whose network structure is represented by a graph with adjacency matrix $A$. The matrix entry $a_{ij} = 1$ if individual $i$ is a neighbor of individual $j$ (e.g., individuals $i$ and $j$ are connected) and 0 otherwise. As explained above, the links are interpreted to mean that the two individuals communicate with each other and are disposed to imitate each other; social connections that lack this force are omitted. The number of neighbors of individual $i$, $n_i = \sum_j a_{ij}$, defines the individual’s degree in the network. Individuals interact with their neighbors in the network, and these interactions are assumed to be bidirectional and uniformly weighted. Adjacency matrices are generated using preferential attachment [24], with an average degree of 4. This represents an intermediate degree of network connectivity that falls within the range for global cascades to occur [22]. Results for average degree 3 are similar, and are not reported here.
1.2 Social norms and signals

Competing forms of expression are represented within the minds of individuals. With each communicative action, an individual elects to use one of them. For example, to express the past tense of *slink*, a speaker of English has a choice between *slinked* and *slunk*, but in each specific utterance, only one of these is used. The choice of signal is treated as a binary (Boolean) variable that assumes values of 0 (FALSE) or 1 (TRUE). In our simulations, 1 will always denote the innovation that may or may not cascade, and 0 will always denote the pre-existing consensus. This binary treatment of the available choices entails little loss of generality. If a new expression, such as *snowpocalypse*, competes against a variety of pre-existing expressions, such as *blizzard*, *large snowstorm* and *massive snowfall*, 0 denotes the use of any of the earlier forms.

We define the observable signal from individual $i$ as a function of time as $s_i(t)$. The signal is communicated to all of the neighbors in the network at the same time. This means that individuals with many neighbors (represented by nodes with high degrees) are heard by many more people than individuals with few neighbors; however, by the egalitarian assumptions of the model, these better-connected people also listen to more people. The mental state $m_i(t)$ of an individual $i$ represents his or her private beliefs about norms of language, as these beliefs develop over time. The model is initialized with $m_i(0) = 0$ for all individuals except a single innovator $v$, for whom $m_v(0) = 1$. The innovator could deviate from the group consensus because his beliefs were formed before joining the network, or through a mutation process whose details fall outside of the scope of this paper. As individuals acquire experience over time, their mental states change because they are influenced by their impressions of their neighbors’ norms. We next describe the learning rule that determines $m_i(t)$. We return later to the characterization of the relationship between $m_i(t)$ and the signal output at time $t + 1$.

1.3 Social assimilation

In general, social learning is driven by general inclinations to assimilate one’s opinions and behaviors to those of others in the community [16,23,28,29]. Applying these ideas to language, individuals estimate the community norm from the relative frequency of each signal type in their aggregate input. Taking $N_i$ to be the set of neighbors of individual $i$ (e.g. $\{j | a_{ij} = 1\}$) and $n_i$ to be number of neighbors, the input to that individual at each timepoint is thus the mean of the signals $s_j$ emanating from the members of $N_i$.

\[
\text{input}_i(t) = \frac{1}{n_i} \sum_{j \in N_i} s_j(t)
\]

However, individuals do not instantaneously copy the current input as their new mental state. People’s beliefs generally reflect a combination of prior experience and current input. Standard learning models capture this by using a weighted average of the prior
mental state and the current input as the update rule, with a \textit{learning rate} parameter \(0 \leq \alpha \leq 1\). \cite{15,30}

\[ m_i(t) = \alpha \cdot \text{input}_i(t) + (1 - \alpha) \cdot m_i(t - 1) \]  

The memory term \((1 - \alpha) \cdot m_i(t)\) holds individual \(i\) back from slavishly following the neighbors. \(\alpha\) has no fixed interpretation in relation to the time scale of learning, but rather reflects the temporal granularity of a batch-processed approximation to social reality. We will report model runs for \(\alpha = 0.1\), selected to give insightful results using the available computational resources. With this value of \(\alpha\), individuals give 9 times more weight to their existing knowledge than to the immediate input from their neighbors.

### 1.4 Unbiased decision rules

The mental state \(m_i\) and the learning rule Eq. 2 capture the way in which the individual’s beliefs about the ambient social norm evolve as input arrives. Now we turn to the question of how individuals act on the basis of this knowledge, where action means making a choice of which signals to express. We first consider the case in which individuals act without any kind of preference or bias.

#### 1.4.1 Criterion or threshold decision rules

The criterion choice rule (also known as a threshold choice rule) maximizes the expected utility in the case of perfect knowledge \cite{31,32}. As a consequence of its mathematic definition, utility can assume values from \(-\infty\) (an infinite loss) to \(\infty\) (an infinite gain). Considering a binary choice between mutually exclusive alternatives \(a\) and \(\neg a\), we define \(Q(a)\) as the utility of \(a\), and \(Q(\neg a)\) as the utility of \(\neg a\). The criterion choice rule then takes the form:

\[ P(a) = \begin{cases} 
0 & \text{if } Q(a) < Q(\neg a) \\
0.5 & \text{if } Q(a) = Q(\neg a) \\
1 & \text{if } Q(a) > Q(\neg a) 
\end{cases} \]  

An individual should choose \(a\) if it offers a net utility benefit over \(\neg a\). The same equation can be also applied to perceptual decisions based on an infinite scale of evidence; an individual should reach conclusion \(a\) if the total evidence for \(a\) is greater than the evidence for \(\neg a\).

For language, let’s imagine that the individual wants to produce the signal that is most likely to be accepted by any random member of the community. This means producing the signal that most other people use, on the assumption that what people use is what they will accept. This goal is similar to the goals of maximizing economic utility or reaching the best-supported conclusion, except that the input to the decision is not a utility or scale of evidence, but rather the \(m_i\), the mental estimate of community norms. The criterion choice rule can be put into following form.
The individual $i$ should choose $s = 1$ if, according to his beliefs, it is more likely than $s = 0$ to be the general norm. Eq. 3 and Eq. 4 are the same except for the range of the input variable. $Q(a)$ and $Q(\neg a)$ can take values anywhere in the interval $(-\infty, \infty)$, whereas $m$ can only take values in the interval $[0, 1]$. This difference in range has little importance if the decision function is a step function. However, for other decision functions the difference in range has important consequences.

Eq. 4 is an absolutely categorical decision rule. Even if the input to the individual has been variable, the individual treats one of the competing forms as normative, and the other input as deviant, invariably producing the normative form as the output. This is not what people do. If linguistic norms vary amongst individuals in a group (as happens during periods of language change), then many individual speakers also vacillate in what form they choose to express [33, 34]. In experiments where some regularization of the input is found, it is never absolute [35, 37]. To model real linguistic behavior, a less categorical decision rule is needed.

### 1.4.2 The SoftMax and logistic functions

In economics, a threshold decision rule is only guaranteed to be optimal in the case of perfect knowledge. It may not be optimal in the case of imperfect knowledge, because the threshold must be set on the basis of information already acquired, and it is possible that some information about the real optimum has not yet come to light. Speakers in a linguistic community also have imperfect knowledge of what other speakers do, because they only receive input from communications they are involved in. The statistical properties of communications that take place in more distant parts of their social networks are invisible to them, as are the communications that will take place in the future. These observations lead naturally to the conjecture that the cognitive system processes the frequencies of linguistic inputs in a manner that resembles reasoning under uncertainty in other areas of cognition.

The theory of reinforcement learning provides a general framework for understanding how learning proceeds incrementally as more and more evidence becomes available from experience. The SoftMax equation contains a free temperature parameter $\tau$ that captures the extent to which the learner makes the optimal choice based on the evidence so far, or makes a random choice which may yield more information in the future [31]. If there are only two choices, the SoftMax reduces to the logistic function Eq. 5, which is a generalization of Eq. 3 and is expressed here in a form that emphasizes the parallelism.

$$P(a) = \frac{e^{Q(a)/\tau}}{e^{Q(a)/\tau} + e^{Q(\neg a)/\tau}}$$
The parameter $\tau$ captures the categoriality of the decision. As $\tau \to 0$, the learning system becomes completely categorical, rigidly applying a threshold based on past evidence. This is the low temperature situation because the decision function can be viewed intuitively as a frozen reflex of the past. As $\tau \to \infty$, the decision becomes random without regard to past evidence.

The logistic function is a sigmoidal function that is widely used in analyzing cognitive, social, and biological processes that involve a competition between two states. Its many applications include not only decisions under uncertainty in economics, but also analysis of the time course of linguistic changes [38][39], the categorization of perceptual inputs [40][41], and binary choices as a function of a continuous and unbounded scale of evidence [32].

The crux of the problem is whether it makes sense to add $\tau$ in just the same manner to a decision function that maps mental probabilities onto signaling actions, as shown in Eq. 6.

$$P(s = 1) = \frac{e^{m/\tau}}{e^{m/\tau} + e^{(1-m)/\tau}}$$

(6)

The answer to this question rests on the behavior of Eq. 6 as it is iterated through repeated social interactions. Figure 2 graphs Eq. 6 as the temperature $\tau$ is varied over its full range. To understand the behavior of the function, it will be convenient to have an equation for the angle $\phi$ at the inflection point.

$$\phi = \arctan \frac{1}{2 \cdot \tau}$$

(7)

Varying $\tau$ is accomplished by varying $\phi$ from 90° to 0°. $\phi = 90°$ in the limit as $\tau \to 0$ and $\phi = 0°$ in the limit as $\tau \to \infty$. The points of most interest are the fixed points, defined as the points for which the output and input are the same (eg. the points where Eq. 6 crosses the identity function $P(s_i = 1) = m_i$). The stable fixed points act as attractors; that is, slight deviations in the vicinity of a stable fixed point push the result back onto the same point. The unstable fixed points act as repellers; slight deviations send the result away from the fixed point. Thus, the stable fixed points represent predictions about the possible long-term states for the system.

The location and stability of the fixed points for Eq. 6 vary as $\phi$ is varied. For $45° < \phi < 90°$, there is an unstable fixed point at $(0.5, 0.5)$, which means that for moderate temperatures, free variation between two competing forms is unstable towards a system in which one variant is preferred. This behavior is somewhat realistic, but not entirely so, because the location of the stable fixed points is problematic. No value of $\phi$ other than 90° has a stable fixed point at $(0, 0)$ or $(1, 1)$. For $45° < \phi < 90°$, there are indeed two stable fixed points. However, one is always located at $0 < m_i < 0.5$ and the other is always located at $0.5 < m_i < 1$. 
Figure 2. Behavior of the logistic function as $\phi$ is varied. $\phi$ determines the location and nature of the fixed points. A: Except for $\phi = 90^\circ$, the function lacks fixed points at $(0, 0)$ and $(1, 1)$. For $\phi \leq 45^\circ$, the unstable fixed point at $(0.5, 0.5)$ becomes a stable fixed point. B: Detailed view of behavior in the lower-left quadrant. $\phi$ varied in 5° steps. For $\phi > 45^\circ$, there is an unstable fixed point at $(0.5, 0.5)$. For $\phi \leq 45^\circ$, the fixed point at $(0.5, 0.5)$ is stable.

This situation has the unfortunate consequence that inputs of $m_i = 0$ and $m_i = 1$ yield outputs that are shifted towards the center. It means that all binary competitions in language continue to the end of time; no form or expression can ever truly go extinct. To make a comparison to norms of attire, some 600 years ago, garments called chausses were in competition with garments called breeches; breeches eventually won out. If neither I nor anyone in my community has any knowledge or experience of chausses, then we have all adopted the innovative form and a model of our speech community has $m_i = 1$ for all individuals $i$. A model using a logistic choice function would nonetheless predict that the archaic form chausses has a predictable and persistent tendency to reappear as a competitor to breeches, as we continue to interact with each other in the future. In fact in language, just as in biology, something that is extinct is gone forever. New competitions arise, but these are due to fresh innovations. The invention of trousers is what created later competition for breeches, and not the persistent return of chausses from the graveyard of fashion history.

A further problem is revealed for $0^\circ \leq \phi \leq 45^\circ$. In this case, the fixed point at $(0.5, 0.5)$ becomes stable. At high temperatures, the system is predicted to converge towards free
variation. For language systems, this asymptotic behavior is highly questionable. This would mean that if people weakly categorize the input they receive about two competing words or constructions, their output persistently reverts towards completely free variation. This would behavior would count as a tendency towards true synonymy (in which two forms completely share their denotation and their range of use). But, in the psycholinguistic literature, avoidance of true synonymy is documented in people of all ages, and it is argued to play a key role in the emergence of meaning categories in language [42–44]. The formalization of the concept of temperature in the context of language and collective opinions needs to be reconsidered.

Note finally that the logistic never falls on the line \( y = x \). For \( \phi = 45^\circ \), it tracks \( y = x \) in the middle of the range, but curves away from this line for more extreme values. This means that it cannot be used to model probability-matching behavior, which is widely assumed to be relevant for language [45–48].

1.5 Introducing the \textit{clog} function: categoriality and bias

The problems with the applicability of Eq 6 arise from the fact that it converges to \( P(s = 1) = 0.5 \) as \( \tau \to \infty \). Although Eq. 3 and Eq. 4 have similar behavior as formulations of a criterion (or expectation maximization) decision rule, this similarity disappears as soon as nonzero temperatures are introduced. This problem is solved if we assume that the sigmoidal function should instead approach \( P(s_i) = m_i \) as \( \tau \to \infty \). We now develop the mathematical apparatus to explore this assumption.

We define a new relative of the logistic, the \textit{clog} (‘cognitive logistic’) function that converges to the identity function as \( \tau \to \infty \):

\[
\text{clog}_\tau(m) = \frac{m \cdot e^{m/\tau}}{m \cdot e^{m/\tau} + (1 - m) \cdot e^{(1-m)/\tau}}
\]

Just as for the \textit{logistic}, it will be convenient to characterize the degree of nonlinearity in the \textit{clog} function in terms of the slope \( \phi \) at the inflection point, which we will denote as \( m^* \). It is:

\[
\phi = \arctan \left( 1 + \frac{1}{2 \cdot \tau} \right)
\]

In the limit of \( \phi \to 45^\circ \) (i.e. \( \tau \to \infty \)), \textit{clog} reduces to the identity function; the production probabilities are exactly the same as the current mental state. As \( \phi \to 90^\circ \) (i.e. \( \tau \to 0 \)), the function approaches a step function with the discontinuity at \( m^* \). For all \( \tau < \infty \) (\( \phi > 45^\circ \)), Eq. 8 has attracting fixed points at \((0, 0)\) and \((1, 1)\). This means that uncertain opinions tend to become polarized in the course of repeated interactions towards more absolute opinions. The stability of these fixed points provides for the ultimate extinction of losing competitors. The inflection point \( m^* \) in between \( m = 0 \) and \( m = 1 \) (located at \( m = 0.5 \)) is a repelling fixed point. Because \( m^* \) is always unstable, there is never a regime in which the system is attracted towards free variation.
The limiting case represented by $\phi = 45^\circ$ is studied in the literature under the name of a *probability-matching* rule, a *relative goodness* rule or *Luce’s Choice Rule* [32].

Thus far, we have considered only unbiased decisions, which rest entirely on the balance of utility, evidence, or likelihood of the two alternatives. In practice, biased decision-making processes are frequently encountered. Such biases are normally accommodated in Eq. 5 for the *logistic* by introducing a second free parameter $\beta$.

$$P(a) = \frac{e^{(Q(a)-\beta)/\tau}}{e^{(Q(a)-\beta)/\tau} + e^{(Q(\neg a)+\beta)/\tau}}$$

(10)

Varying $\beta$ has the effect of shifting the entire function. If $\beta < 0$, the individual is more likely to pick the innovative variant than would be expected from its utility. If $\beta > 0$, he is less likely to do so.

Figure 3 illustrates the results of attempting to recast Eq. 10 as a function from mental probabilities to signaling actions, as shown in Eq. 11.

$$P(s = 1) = \frac{e^{(m-\beta)/\tau}}{e^{(m-\beta)/\tau} + e^{(1-m+\beta)/\tau}}$$

(11)

**Figure 3.** Behavior of the *logistic function* for $\phi = 60^\circ$ as $\beta$ is varied. The unstable fixed point at (0.5, 0.5) is lost for small biases $\beta = 0.2$ and $\beta = -0.2$. Each of these $\beta$ values results in a system with a single stable fixed point.
The shift in the function shifts the location of the inflection point occurring in the unbiased case at \( m = 0.5 \). The number and stability of the fixed points is greatly affected. A small positive bias gives rise to a single stable fixed point representing a high level of acceptance of the innovation, and a small negative bias yields a stable fixed point that represents a low level of acceptance of the innovation.

In contrast to the logistic, the \( \text{clog} \) can incorporate a bias parameter without restructuring the fixed points. Including a bias parameter \( \beta \) into the \( \text{clog} \) function yields the following fully elaborated form.

\[
clog_{r,\beta}(m) = \frac{m \cdot e^{(m-\beta)/\tau}}{m \cdot e^{(m-\beta)/\tau} + (1 - m) \cdot e^{(1-m+\beta)/\tau}}
\]  

(12)

For the \( \text{clog} \), \( \beta \) controls the location of the unstable fixed point \( m^* \), which is always located at \( m = 0.5 + \beta \). If \( \beta_i < 0 \), \( m_i^* < 0.5 \), and individual \( i \) has a greater propensity to produce 1 than the input from the neighbors would dictate. Similarly, if \( \beta_i > 0 \), \( m_i^* > 0.5 \), and the individual has a disproportionate tendency to produce 0. The stable fixed points are unaffected by the value of \( \beta \). The introduction of a function that smoothly interpolates between a criterion choice rule and a probability-matching choice rule, under under any choice of bias, while always having fixed points at \((0, 0)\) and \((1, 1)\), is a novel contribution of this paper.

Heterogeneity amongst community members in \( \beta \) is the primary type of heterogeneity that we explore. The different scenarios for informational cascades depend on positive and negative values of \( \beta \) being distributed in different ways across the individuals in the network.

Figure 4 illustrates the behavior of \( \text{clog} \) as \( \phi \) and \( \beta \) are varied. In this figure, note particularly that varying \( \beta \) has no effect on the fixed points at \((0, 0)\) and \((1, 1)\) for \( \phi > 45^\circ \). This entails that the \( \text{clog} \) (unlike the logistic) cannot represent a uniform distribution on the closed interval \([0, 1]\). Note also that the effects of \( \beta \) are neutralized if \( \phi = 45^\circ \); the concept of bias is only meaningful if the choice rule is at least somewhat categorical.

### 1.6 Implementation

The model is implemented in the NetLogo agent-based modeling environment [19]. It is initialized at time \( t = 0 \) with \( m_i(0) = 0 \) for all individuals \( i \) in \( A \), except for a single innovator \( v \), for whom \( m_v(0) = 1 \). During each model cycle \( t + 1 \), every individual first randomly produces an expression of opinion, 0 or 1, with probability:

\[
P(s_i(t) = 1) = clog_{r,\beta}(m_i(t))
\]  

(13)

Then they update their mental state from \( m_i(t) \) to \( m_i(t + 1) \) on the basis of the set of signals emitted by their neighbors \( N_i \) (e.g. \( \{s_j(t) | j \in N_i\} \)), using Eq. 2. Each model run is terminated when the population reaches a consensus (defined as \( m_i < 10^{-8} \) or \( m_i > 1 - 10^{-8} \) for all \( i \)), or at the 10,000th iteration if no convergence has occurred.
Figure 4. Behavior of the clog function as $\beta$ and $\phi$ are varied. Blue: No bias. Magenta: Bias against the pre-existing consensus, equivalent to bias toward the innovation. Green: Bias toward the pre-existing consensus, equivalent to bias against the innovation. When $\phi = 45^\circ$, the clog reduces to the dashed line $P(s_i=1) = m_i$ for all values of $\beta$.

The probabilities of cascades as a function of innovator degree are estimated by a Monte Carlo method, in which innovator degrees from 2 to 55 are sampled equally in the form of 500 runs for each degree. Innovators with degrees of 1 and > 55 are not included because the preferential attachment rule for network construction does not generate nodes with such degrees frequently enough. A fresh network is generated for each run, and all individuals share a value of $\phi$. $\phi$ was smoothly varied from $\phi = 45^\circ$ (linear) to $\phi = 90^\circ$ (categorical) by increments of 1°. In both baseline scenarios, all individuals are unbiased. (In the first baseline, this is because they follow a probability-matching decision rule. In the second baseline, the decision rule is more categorical and $\beta_i = 0$). In all other scenarios, heterogenous values of $\beta_i$ are assigned, as described in the text.

1.7 Summarizing outcomes

In our Monte Carlo simulations, we vary the categoriality, the degree of the innovator and the way that bias values are distributed across the community. Each model configuration yields a distribution of cascade sizes; the size of a cascade is distinct from the likelihood
of a cascade. In order to summarize the results, however, we conflate these two factors. We use the following classifications of the average mental state $\bar{m}$ of all the individuals in the network when the model run ends at time $t_{final}$.

- **Survival**: The innovation avoids extinction, defined as $\bar{m}(t_{final}) > 10^{-4}$.

- **Dominance**: The innovation becomes dominant by surpassing the established variant, i.e. $\bar{m}(t_{final}) \geq 0.5$.

- **Completion**: The innovation drives the previous consensus opinion to extinction i.e. $\bar{m}(t_{final}) \geq 1 - 10^{-4}$.

## 2 Results in different scenarios

We first present two baseline cases in which all individuals are unbiased. Then, we compare three scenarios in which individuals have heterogeneous biases.

### 2.1 Baseline scenarios

If $\phi = 45^\circ$, each individual expresses each opinion in direct proportion to their experience. The model instantiates the case of neutral evolution (also known as random drift [27]). For a fully continuous system, the population would converge to the average of the initial states of the individuals; however, because emitting a signal is a discrete probabilistic event, the mean state instead exhibits a random walk on the frequencies of the alternative signals 0 and 1 [27]. The probability of a complete cascade is low, and is directly proportional to the innovator’s degree, as shown in Figure 5.

If $\phi > 45^\circ$ (e.g. individuals have at least some tendency toward categorical behavior), and all individuals are unbiased ($\beta_i = 0$ for all $i$), then the innovation always becomes extinct; no cascades of any size are found. This very strong result on the impossibility of change without bias heterogeneity occurs because memory is included in the model of learning. Eq. [2] entails that a single signal $s_i = 1$ from an innovator $i$ can never cause the mental state of a neighbor $j$ to increase from its initial value $m_j = 0$ to a value greater than the repelling fixed point at $m_j^* = 0.5$. The innovator cannot convert anyone else, and is eventually reconverted to the original consensus.

### 2.2 Scenarios with heterogeneous biases

To explore the role of heterogeneous biases while maintaining overall functional neutrality, bias values are generated through random sampling on a uniform distribution over the interval $[0, 0.5]$. Neutrality is enforced by taking both the $+\beta$ and $-\beta$ for every $|\beta|$ that is selected. The resulting set of values is distributed over the individuals in the social network according to one of three different methods. In the hubs scenario, the
bias against the pre-existing consensus is preferentially allocated as a function of degree centrality. The highest degree individuals are most disposed to change, and the lowest degree individuals are least disposed to change. In the nearby scenario, the bias against the pre-existing consensus is allocated as a function of the distance from the innovator; individuals nearest to the innovator are the most disposed to change, and those farthest from the innovator are least disposed to change. In the random scenario, bias values are randomly distributed.

The hubs and nearby scenarios idealize skewed patterns in the distribution of biases in the network. To motivate these idealizations, let us consider what the bias $\beta$ means. The case of negative bias generalizes the concept of early adopters, defined in [22, 23] as individuals who will adopt an innovation after encountering a single example of it. Negative bias means that an individual is inclined to abandon the old norm, even if most of the neighbors do not. Positive bias is a generalization of the concept of late adopters. It means that an individual is inclined to continue using the old expression, even if the neighbors use its more innovative competitor.

Such individual biases could arise as a reflex of personality traits such as levels of adventurousness and nonconformity, which are factors in the standard theory of personality [50] and are partially innate in people and other animals [51]. The differences could also arise from knowledge or habits that provide a latent potential or impediment to the adoption of the innovation. As a simple example, the use of text messaging provides a platform for the use of slang acronyms like lol (laughing out loud) and btw (by the way), and it would not be surprising if heavy users of text messaging were among the first to adopt these expressions upon encountering them. In modern linguistic theories, the adoption of one construction may also provide a foundation for further change through

Figure 5. Relationship of innovator degree to cascade probability in the neutral evolution model. For all three summary cases, the relationship is linear. The probability of a complete cascade is extremely small.
its indirect impact on the encoding system\textsuperscript{34}. It is possible to develop a connection between such individual biases and the concept of utility. We already provided an interpretation of $\tau$ in relation to the global value of addressing any member of the community in a cooperative manner, using a form they are likely to accept. $\beta$ encapsulates any sort of subjective personal utility that is independent from the utility to the community as a whole of having shared norms.

The distribution of biases in the hubs scenario thus can be understood as one in which individuals with many social connections share any type of knowledge, practice, or experience that makes it easy for a specific linguistic innovation to take root. The nearby scenario is even more plausible, because people with shared traits tend to form social connections to each other\textsuperscript{52}. Field studies of Belfast English indicate that feelings of group solidarity affect people’s choice of modes of expression\textsuperscript{53}. The rate of adoption of linguistic innovations peaks in adolescence\textsuperscript{39}. In choices of language, music, and attire, adolescents tend to be most influenced by the central members of their immediate peer group (rather than by the highest status people in their whole world of experience)\textsuperscript{54-56}. A group of young people who share rebellious attitudes and a love of novelty is indeed the very prototype of a group with innovative language.

Results for the three scenarios are shown in Figure 6.
Figure 6. Interaction of bias distribution with innovator degree and categoriality in determining the likelihood of cascades.
Non-zero likelihoods of a cascade in each of the three categories survival, dominance, completion are color-coded as shown. Black indicates the absence of any cascades in the simulation results. A: Bias values are assigned to individuals as a function of degree centrality under the hubs scenario. B: Bias values are assigned to individuals as a function of proximity to the innovator under the nearby scenario. C: Bias values are assigned randomly to individuals.
2.2.1 Results for the hubs scenario

In the hubs scenario (Figure 6A), cascading is frequent and consistent for high-degree innovators and high $\phi$ values. As the innovator degree decreases, the likelihood of success also decreases. Furthermore, cascading is basically an all-or-nothing phenomenon; if a cascade avoids extinction, it is very likely to go all the way to completion. Intuitively, if an innovation survives the initial stage in this scenario, it has done so by spreading to a backbone of high-degree adopters. These in turn can dominate the overall signal to the rest of the network, because each of their expressions of opinion is sent to many neighbors.

Interestingly, when the input-output relation is not strongly categorical (i.e., $\phi \lesssim 75^\circ$), assigning the favorable bias values to the hubs is not effective for causing cascades (regardless of innovator degree). As seen in Figure 6A, a phase transition occurs in the high-$\phi$ regime ($\phi > 75^\circ$). As categoriality increases, the outcome passes from invariable cascade extinction, through degree-sensitive all-or-nothing cascades, to complete cascades (apart from very low-degree innovators).

Just visible in blue at the bottom of the graph is a sporadic set of cascades arising from neutral evolution ($\phi = 45^\circ$), corresponding to those displayed in Figure 5.

2.2.2 Results for the nearby scenario

We identify three regimes in the nearby scenario (Figure 6B). For $45^\circ \leq \phi \lesssim 55^\circ$, no cascades occur, apart from the few previously discussed for the $\phi = 45^\circ$ neutral evolution model. For $55^\circ \lesssim \phi \lesssim 65^\circ$, cascades are more frequent, and often go to completion. For $65^\circ \lesssim \phi \leq 90^\circ$, cascades become increasingly likely, but do not go to completion even if they succeed in dominating the network. In particular, in the fully categorical limit, where $\phi = 90^\circ$, the cascade proceeds only a little further than halfway: $\bar{m}(t_{\text{final}}) = 0.56$ (with the standard deviation $\sigma = 0.069$). The network already displayed in Figure 1 is a final configuration (at the 10,000th time step) for this partial cascade situation. It shows how the novel opinion diffuses through some (but not all) of the network, leaving an entrenched opposition elsewhere. This means that for innovations occurring in a social network with a nearby bias distribution, a highly categorical decision rule tends to split the social network into subgroups with different norms.

In-depth examination of the case $\phi = 60^\circ$ (Figure 7) reveals another striking point. The likelihood of success in triggering a cascade peaks in the vicinity of innovator degrees $7 \leq n_i \leq 15$ for this level of categoriality. The individuals with degrees in this range are more likely to succeed than very low-degree individuals, an effect also visible for other values of $\phi$ at the left edge of the panels in Figure 6B. Surprisingly, they are also somewhat more likely to succeed than high-degree individuals. With this combination of categoriality and bias, it is clearly not appropriate to describe the best-connected nodes as influentials; their influence is actually constrained by the variability of their many neighbors. Taking the network’s degree distribution into account, we can also calculate the probability of the innovator degree, given that a cascade occurred. As Figure 7 shows,
this probability peaks at innovator degree \( n_i = 4 \), which is by construction the average degree of the network. Thus in this scenario, a successful innovation is most likely to have originated from Joe or Jane Average.

![Figure 7. Relationship of innovator degree to complete cascades for the nearby scenario with \( \phi=60^\circ \). Red: The probability of a complete cascade, given the degree of the innovator. Blue: The probability of the innovator degree, given that a complete cascade occurred. Each data point is estimated from the 500 runs per degree in the Monte Carlos simulation. Results are highly similar for cascade survival and dominance.](image)

### 2.2.3 Results for the random scenario

The hubs and nearby scenarios are highly idealized, and yield exorbitantly high success rates for innovation. In reality, the bias distribution is probably less stringently tied to network position, and very many innovations fail. A perspective on this reality is provided by results on the random scenario in Figure 6C. Survival of an innovation is fairly common when \( \phi \gtrsim 70^\circ \). The rate of survival is notably higher than in the neutral evolution model as shown in Figure 5, also visible in this figure panel as a blue line at \( \phi = 45^\circ \). Dominating cascades are more rare. In addition to the few dominating cascades found for \( \phi = 45^\circ \), a small cluster of dominating cascades is found for degree \( n_i \gtrsim 10 \) and \( \phi \gtrsim 80^\circ \). Thus, moderate categoriality in combination with heterogeneity permits a moderate to high degree innovator to have some chance of seeing an innovation widely adopted. This likelihood becomes attenuated as \( \phi \) approaches 90°. Qualitatively, the pattern suggests a mixture of cases skewed towards the nearby scenario and cases skewed towards the hubs scenario. Complete cascades were not observed at all above \( \phi = 45^\circ \). Of course, a complete cascade could occur if the random bias distribution happened to distribute the bias in exactly the same way as in one of the more favorable scenarios in
Figure 6AB. The fact that this outcome is too rare to occur at all in our simulations suggests that bias heterogeneity alone cannot fully explain how neutral innovations may come to be adopted by an entire community. Instead, systematic patterns are needed in the distribution of decision biases across individuals.

Discussion

We have presented a framework for exploring the interaction of categoriality, bias, and innovator degree in permitting arbitrary linguistic innovations to succeed in a social network. As anticipated from prior related models [23, 26, 27], model configurations in which individuals are homogeneous and unbiased exhibit negligible success rates for innovations. We identified two regimes involving heterogeneous biases that are conducive to change. Under the hubs scenario, a highly categorical decision rule supports a mechanism for the widespread adoption of innovations initiated by highly connected people. Under the nearby scenario, a moderately categorical decision rule leads to a mechanism for the grassroots changes to succeed. This second regime, which was previously unsuspected in research on informational cascades, is highly relevant to linguistic change because linguistic changes typically originate with ordinary people.

Previous models of language dynamics have explored the ways that functional factors can have major impacts on the linguistic structures through their iterated effects in language acquisition, perception, and production [46, 57]. Here we have shown that such functional factors are not necessary for global changes to occur. Our results connect the understanding of language dynamics with classic observations about the arbitrariness of many words and expressions in human languages [1, 2].

Statistical variability in language is pervasive and often persists through language learning from one generation to the next [45]. Many researchers have assumed that the cognitive system for language uses probability-matching decision rules [17, 48]. However, tendencies toward regularization of observed frequencies have been found in the area of phonotactics [38, 59], and in some experiments involving learning of artificial languages [35, 36]. It has been argued that people discount ambiguous evidence in the acquisition of morphology and syntax [60, 61]. Ambiguity and vacillation may be cognitively costly for linguistic processing. Categorical processing is argued to support fast, accurate encoding and decoding, as well as the ability to create new words and sentences by recombining basic elements [7].

Our results reconcile these disparate threads in the literature by developing a formal apparatus for analyzing regularization as the categorization of experienced frequencies. We then suggest that the cognitive processing of frequencies in language is moderately categorical. The regime in which we observe grassroots changes has $\phi \approx 60^\circ$. As shown in Figure 4, this is a level of categoriality that deviates only moderately from a probability-matching decision rule. Although this moderate nonlinearity is sufficient to completely
change the prospects for an innovation to succeed, it is probably too weak to be detected in a typical psycholinguistics experiment. Very large scale data collection over the entire range of input probabilities would also be needed to assess the heterogeneity in individual biases that also plays a key role in our model. In short, the statistical power needed to rigorously compare a probability-matching (or neutral evolution) model with the regime for complete cascades in the nearby scenario is much higher than has been available in many classic experimental studies.

Why isn’t language processing strongly categorical? A probability-matching decision rule can be optimal at the group level when coordinated actions are needed to divide up resources. Language presents a related case, because coordinated actions are needed for a group to maintain a mutually intelligible language, which benefits the group as a whole by permitting knowledge to be transmitted from any person to any other. In the nearby scenario, a completely categorical decision rule jeopardizes mutual intelligibility. For $65^\circ \leq \phi \leq 90^\circ$, arbitrary innovations dominate the system but do not go to completion. This outcome means that the social group has split into subgroups with distinct norms. In contrast, the regime $55^\circ \leq \phi \leq 65^\circ$ yields a greater likelihood that mutual intelligibility will be maintained through periods of change. Assuming that the distribution of biases in the nearby scenario is empirically well-founded, language processing can be understood as striking a balance between fast, accurate processing and the maintenance of a shared linguistic code.

In research on opinion dynamics, an early and prominent hypothesis held that a small number of individuals with high degree in a social network had disproportionate importance in triggering global cascades. That is, hubs in the network were thought to have special importance as so-called *influentials*. Some previous models of language change achieve realistic cascade rates by assuming that people weight positively the input from highly-connected people, who are also assumed to be high-status. In the extreme, these assumptions lead to the conjecture that broadcast media should be a major factor in language change, since the media transmit the linguistic patterns of political leaders and celebrities to such large numbers of people. This conjecture is not well supported. For example, establishing any effect whatsoever of television on local dialects has been an elusive goal, and only recently has such an effect been found. In a detailed field study of Glaswegian English, some TV watchers were found to be influenced by the East End dialect of London, but only if they had personal ties to the East End or felt strong psychological engagement with characters in a popular TV soap drama that is set there.

In our model of egalitarian changes (the nearby scenario), informational cascades occur without any positive weighting of input from highly connected individuals. These individuals do play an important role, in that successful innovations succeed in part by being adopted by one or more highly connected individuals, who then broadcast them to many people. On the average, however, the highest degree individuals are somewhat conservative compared to less connected individuals, because their linguistic patterns are stabilized by the large number of incoming signals. Individuals who are far from the
innovator, and biased against the change, eventually adopt it with some probability simply because a core of positively biased individuals near the innovator allows the change to gain traction and in some cases eventually dominate the signal to the whole community. Some previous work has argued that the concept of *covert prestige* (understood as an implicit value system that countervails overt norms) is necessary to explain why high status people often adopt stigmatized expressions originating in lower status circles [63]. However efforts to substantiate this notion in terms of perceived lower-class attributes such as toughness or friendliness have enjoyed mixed success [18]. Our model can explain why high degree or high status people eventually adopt grassroots linguistic innovations without recourse to a covert value system. Overall, our results support claims that social solidarity, and not differences in status, is the dominant factor in the adoption of linguistic innovations [53, 55].

We saw that categorical decisions in the hubs scenario permit a well-connected group of people to effect a rapid and complete change that it favors. Less well-connected people may be initially biased against the change, but are swept along. This scenario does not appear to be relevant for linguistic changes. Is the model appropriate for some other type of change in social norms or collective opinions? A categorical decision function maximizes an individual’s statistical expectations [32] in the case of perfect knowledge. This low-temperature decision rule is normally used in economic models [32], a reflex of economists’ assumption that individuals have complete information and make independent and rational decisions to maximize their gains. A recent example of a shift in public opinion in the economic domain was the 1999 repeal of Glass-Steagall Act provisions that separated investment banking from commercial banking. Our model shows how a well-connected network of investment bankers who were biased in favor of opportunities for investment banking could cause this change to be widely accepted (even if other people held the opposite bias). More generally, a categorical decision rule is adaptive if the utility of the competing options is well-defined and generally known, or if the cost of waiting for more information is great. In times of revolution or war, it may be clear that people who delay or vacillate in their allegiances are less safe than people who take a side. Insofar as political behavior reflects this perception of the risks, our model predicts people would apply a highly categorical decision process, be strongly influenced by a slight advantage of one option over the other, and that as a result public opinion in a time of crisis could be effectively controlled by a well-connected minority with shared biases. Insofar as political behavior strikes a balance between taking a stand and waiting for more information, our model suggests that grassroots political changes could take place without any definitive evidence about their ultimate utility.
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