A Sinister Extension of the Standard Model
to $SU(3) \times SU(2) \times SU(2) \times U(1)$

Sheldon L. Glashow

Physics Department
Boston University
Boston, MA 02215

This paper describes work done in collaboration with Andy Cohen. In our model, ordinary fermions are accompanied by an equal number ‘terafermions.’ These particles are linked to ordinary quarks and leptons by an unconventional CP’ operation, whose soft breaking in the Higgs mass sector results in their acquiring large masses. The model leads to no detectable strong $CP$ violating effects, produces small Dirac masses for neutrinos, and offers a novel alternative for dark matter as electromagnetically bound systems made of terafermions.
1. Introduction

I am honored to present the closing talk at the XI Workshop on Neutrino Telescopes, but it will not be a summary of our proceedings. Instead, I shall sketch some recent work [1] I have done with Andy Cohen. Our model is based on the gauge group \( SU(3) \times SU(2) \times SU(2)' \times U(1) \) and involves twice as many fermions as the standard model. The familiar quarks and leptons are accompanied by an equal number of much heavier “terafermions.” The model also involves heavy versions of the weak intermediaries: \( W' \) and \( Z' \) bosons. Ordinary fermions form three families, each consisting of 16 left-handed fields transforming as usual under \( SU(3) \times SU(2) \times U(1) \). Terafermions, also forming three families of 16 left-handed fields, transform in exactly the same way but under a different gauge group, \( SU(3) \times SU(2)' \times U(1) \).

We introduce an unconventional \( CP \) operation, hereafter called \( CP' \), which maps ordinary fermions into the conventional \( CP \) conjugates of their tera-equivalents, and \textit{vice versa}. Soft \( CP' \) breaking within a simple Higgs sector (consisting of one \( SU(2) \) doublet and one \( SU(2)' \) doublet) leads to large and empirically acceptable masses for the terafermions, and for \( W' \) and \( Z' \) as well. This ‘sinister’ (\textit{i.e.,} ‘left-left symmetric’) model is somewhat akin to (but more natural than) certain ‘left-right symmetric’ models for which an unconventional space-reflection operation \( P' \), rather than \( CP' \), links ordinary and exotic fermions.

Regardless of the truth of our model, some its consequences may justify its presentation here. The standard model is known to suffer from two serious problems: the mass hierarchy and strong \( CP \). Our resolution to the latter puzzle is subsumed under the mass hierarchy rubric, as contributions to \( \theta \) from quarks are exactly cancelled by opposite contributions from much heavier teraquarks. A natural seesaw mechanism enables observed neutrinos to secure tiny Dirac masses, thereby forbidding neutrinoless double beta decay. Furthermore, we predict the existence of heavy non-standard stable quarks and leptons. If there is a relic abundance of these particles, they will have formed electromagnetically bound states (‘terahelium atoms’) that can serve as novel candidates for the dark matter of the universe.
2. The Model

Each familiar fermion family includes a pair of colored quarks (e.g., $u$ and $d$) with baryon number $B = 1/3$, a lepton pair (e.g., $e^-$ and $\nu_e$) with lepton number $L = 1$ and a neutral left-handed singlet (e.g., $n_e$) to which we assign $L = -1$. Likewise, each terafamily includes a pair of teraquarks (e.g., $U$ and $D$) with terabaryon number $B' = 1/3$, a teralepton pair (e.g., $E^-$ and $\nu'_e$) with teralepton number $L' = 1$ and a neutral singlet singlet (e.g., $n'_e$) with $L' = -1$. For reasons soon to be apparent, we require the conservation of $B - L$ minus its tera-equivalent,

$$F \equiv (B - L) - (B' - L'). \quad (2.1)$$

Our unconventional $CP'$ operation maps each ordinary field (e.g., $u$) to the conventional $CP$ conjugate $\overline{U}$ of the corresponding terafield, and vice versa ($U \rightarrow \overline{U}$). $CP'$ is assumed to be a symmetry of all dimension-4 terms in the Lagrangian, but to be softly and severely broken by the dimension-2 mass terms of scalar mesons. Invariance under the conventional $CP$ operation is not imposed at any level.

The model requires two scalar Higgs multiplets. One ($h$) transforms as an $SU(2)$ doublet but is $SU(2)'$ invariant. It suffers Yukawa couplings to ordinary quarks and leptons with coupling constants denoted by $\lambda$ (which compactly signifies four distinct $3 \times 3$ flavor matrices related to the masses and mixings of $Q = 2/3$ quarks, $Q = -1/3$ quarks, charged leptons and neutral leptons.) The second Higgs multiplet $h'$ transforms as an $SU(2)'$ doublet but is $SU(2)$ invariant. It suffers Yukawa couplings $\lambda'$ to teraquarks and teraleptons. Under $CP'$, $h$ is replaced by the $CP$ conjugate of $h'$ and vice versa.

Conservation of $F$ implies that the Yukawa couplings of $h$ and $h'$ cannot link fermions to terafermions. Furthermore, the $CP'$ invariance of all dimension-4 terms relates the couplings of $h$ and $h'$:

$$\lambda' = \lambda^*, \quad (2.2)$$

where the asterisk denotes complex conjugation. The vacuum expectation value of $h$ has its conventional value $\langle h \rangle \approx 250$ GeV, but soft $CP'$ breaking in the Higgs mass terms results in a much larger vev for $h'$:

$$\langle h' \rangle = S\langle h \rangle \gg 250 \text{ GeV}. \quad (2.3)$$

According to Eq.\,(2.2), the Yukawa couplings of fermions and terafermions are complex conjugates of one another. Thus, each terafermion mass equals that of its lighter
counterpart, but \textit{multiplied by the large factor} $S$. The least massive charged terafermion is the tera-electron $E^-$, a stable particle with mass $S \times 511$ keV. Unsuccessful searches \cite{2} for heavy stable leptons imply that $E^-$ must be heavier than 100 GeV/c$^2$, and therefore:

$$S > 2 \times 10^5.$$  \hfill (2.4)

Our subsequent considerations of terafermions as dark matter will yield a somewhat stronger constraint on $S$.

The other charged teraleptons are unstable. They decay via $W'$ exchange at rates that are a factor $S$ greater than those of their ordinary counterparts. The same is true for the heavier teraquarks, \textit{e.g.}, $\bar{D} \rightarrow \bar{U} + E^- + \nu'_e$, with the release of several TeV of kinetic energy. But the tera-up quark $U$, with mass $\sim S \times 3$ MeV, is stable. With $S = 10^6$, our stable teralepton $E$ lies at $\sim 0.5$ TeV, our stable teraquark $U$ at $\sim 3$ TeV. These particles may be produced and detected at the LHC.

3. Neutrino Masses

Because this Conference focusses on neutrino physics, I shall begin by exploring the implications of our model for that discipline. Conservation of $F$ implies that neutrinos cannot have Majorana masses, and consequently that \textit{neutrinoless double beta decay is absolutely forbidden}. Let us see how neutrinos naturally acquire tiny Dirac masses through a version of the seesaw mechanism.

We have invoked twelve left-handed neutral leptons. The six with $F = -1$ are the $SU(2)$-doublet states $\nu_i$ and the singlets $n'_i$. The six with $F = +1$ are the $SU(2)'$ doublet states $\nu'_i$ and the singlets $n_i$. The Yukawa couplings of $h$ and $h'$ yield the following contributions to neutrino masses:

$$\langle h \rangle (\nu \lambda_\nu n) + S \langle h \rangle (n' \lambda'_\nu \nu') \equiv (\nu m n) + S(n' m^\dagger \nu')$$  \hfill (3.1)

where $\lambda_\nu$ and $m$ are $3 \times 3$ matrices and we have used Eq.(2.2). (Dirac matrices are suppressed and $m^\dagger$ is the hermitean adjoint of $m$.) The eigenvalues of $m$ should be comparable to the charged lepton masses, at least insofar as up and down quark masses are comparable to one another.

This is not the whole story. Only one fermionic operator is compatible with both gauge invariance and $F$ conservation: $(n' M n)$, where $M$ is an arbitrary $3 \times 3$ matrix
(which is hermitean if \( CP' \) is to be a symmetry of dimension-3 terms in the Lagrangian). Its eigenvalues are expected to be large, perhaps comparable to a hypothetical unification scale. Putting this mass operator together with \( (3.4) \), we obtain for the neutral lepton mass terms:

\[
\mathcal{M} = \begin{pmatrix} \nu & n' \end{pmatrix} \begin{pmatrix} 0 & m \\ Sm^\dagger & M \end{pmatrix} \begin{pmatrix} \nu' \\ n \end{pmatrix},
\]

which is explicitly \( \mathcal{F} \) conserving and describes two sets of three Dirac particles. If, as we expect, the eigenvalues of \( M \) are all much larger than those of \( m \sqrt{S} \), the seesaw approximation is applicable. In this limit, the heavy neutral leptons are made up of the states \( n \) and \( n' \), with their masses described by the matrix \( M \). The light neutrinos involve \( \nu \) and \( \nu' \), with their masses described by the \( 3 \times 3 \) matrix:

\[
M_{\nu\nu'} \simeq -S (m M^{-1} m^\dagger).
\]

With \( S \sim 10^6 \), \( M \sim 10^{17} \) Gev and \( m \) plausibly chosen, we obtain suitable neutrino masses and mixings. However, if \( S \) is much larger than \( 10^6 \), the eigenvalues of \( M \) would have to be disconcertingly close to the Planck mass.

Corrections to the seesaw approximation are of order \( |m/M| \) (or \( |S m/M| \)) for active (or sterile) neutrino states. The consequent mixings of \( \nu \) with \( n' \) (and \( \nu' \) with \( n \)) are exceedingly small and lead to no detectable effects. However, the tiny admixtures of active neutrino states within the heavy neutral leptons ensure their decay in the very early universe.

In any model with Dirac neutrino masses, one must examine the contribution of the light sterile states (\( \nu'_i \) and \( \nu'_i \)) to the expansion rate of the universe during Big Bang nucleosynthesis. These states drop out of equilibrium much earlier than ordinary neutrinos:

\[
T_{eq}(\nu') = S^{4/3} T_{eq}(\nu),
\]

or \( T_{eq}(\nu') \sim 300 \) TeV. Many particle species that were in thermal equilibrium at \( T_{eq}(\nu') \) will no longer be present at \( T_{eq}(\nu) \). These include the six ordinary quarks, muons, tau leptons and their antiparticles, as well as \( W \) and \( Z \) bosons and several teraspecies. Their annihilations will reheat the conventional particle species relative to the sterile neutrinos \( \nu' \). As a result, we find \([1]\) the effective number of neutrino species \( N_\nu \) at nucleosynthesis to be about 3.15. This result may be compared to the 2-\( \sigma \) limit \( N_\nu \leq 3.3 \) deduced from astrophysical data with the prior \( N_\nu \geq 3 \). \([3]\)
4. The Strong $CP$ Puzzle

The quark sector of the standard model involves two independent $CP$ violating parameters: the Kobayashi-Maskawa phase $\delta$ and the strong $CP$ parameter $\bar{\theta}$. The puzzle is to explain why $\delta$ is order unity whereas $\bar{\theta} < 10^{-10}$. In our model, the $CP$-violating operator $G\tilde{G}$ is forbidden by our $CP'$ symmetry. The complex Yukawa couplings of $h$ and $h'$ do not contribute to $\bar{\theta}$ because the teraquark mass matrices are proportional to the complex conjugates of the quark mass matrices. Thus, their contributions to $\bar{\theta}$ cancel one another. It follows that $\bar{\theta}$ vanishes in tree approximation.

A finite and small value for $\bar{\theta}$ is generated by radiative corrections. These consist only of multi-loop diagrams such as are present in the standard model: finite six-loop terms and divergent seven-loop terms. In our case, the divergence is replaced by the factor $\ln S$. It follows that strong $CP$-violating effects are entirely negligible in our model, for which electric dipole moments of elementary particles are far too small to be detected.

Our solution to the strong $CP$ puzzle is a particular realization of the Nelson-Barr mechanism [4], with the UV sector (teraquarks) linked to the low-energy sector by a softly broken discrete symmetry. In effect, we have replaced the strong $CP$ puzzle by a novel fermion-terafermion mass hierarchy.

5. Relic Terahelium as Dark Matter?

There are two stable terafermions in our model aside from the right-handed neutrino states $\nu'$. These are the tera-up quark $U$ and the tera-electron $E$. Were there a terabaryon asymmetry in the universe akin to the baryon asymmetry, the alleged dark matter of the universe could consist of relic terafermions. In our discussion of this possibility, we make two technical assumptions: that the net electric charge of relic fermions and of relic terafermions each vanish, and that both the baryon and terabaryon excesses are positive. One of the less attractive features of our model is that each of these relic abundances must be adjusted to yield the known values of the mean densities of baryons $\Omega_b$ and dark matter $\Omega_d$ in the universe [5]:

$$\Omega_b \approx 0.044 \quad \text{and} \quad \Omega_d \approx 0.224 .$$

What happens to relic $U$’s and $E$’s in the early universe? The QCD force among massive $U$’s is both strong and Coulombic. Thus, the binding energy of a color triplet $UU$
diquark is $\sim \alpha_s M$, or hundreds of GeV. That of the color singlet $UUU$ is several times larger. Thus, exothermic processes such as:

$$U + U \rightarrow (UU) + g \quad \text{and} \quad U + (UU) \rightarrow (UUU) + g$$  \hspace{1cm} (5.2)

where $g$ is a gluon, can proceed irreversibly at temperatures far above the quark-hadron transition. However, the cross-sections for these reactions are small, so that not all the $U$’s are aggregated by this means.

Once the temperature becomes low enough for conventional hadrons to form, the unaggregated $U$’s will bind with the more abundant $u$ and $d$ quarks to form super-heavy hadrons such as $Uud$ and $U\overline{d}$. At that point, exothermic exchange reactions such as:

$$Uud + Uud \rightarrow UUU + uudd, \quad U\overline{d} + Uud \rightarrow UUd + ud\overline{d} \quad \text{and} \quad U\overline{d} + U\overline{d} \rightarrow UUU + \overline{ddd}$$  \hspace{1cm} (5.3)

rapidly ensue. Cross-sections for exchange reactions such as (5.3) are much larger than those for reactions (5.2). They are comparable to the cross-sectional areas of the reactants, which are of order tens of millibarns. The aggregation of $U$ quarks will continue via further highly exothermic exchange reactions such as:

$$UUd + Uud \rightarrow UUU + udd, \quad UUd + UUd \rightarrow UUU + Udd \quad \text{and} \quad UUd + UUd \rightarrow UUU + d\overline{d}.$$

\hspace{1cm} (5.4)

Our estimates [1] show that reactions (5.3) and (5.4) will efficiently convert almost all relic teraquarks into bound $UUU$ states prior to nucleosynthesis. These tiny and tightly bound doubly-charged particles, the tera-equivalents of $\Delta^{++}$, have spin $3/2$ and mass $\sim 10$ TeV. Using Eq.(5.1) and $\eta_b \simeq 6 \times 10^{-10}$ for the baryon to photon ratio, we find:

$$\eta_{B'} \approx 3 \times 10^{-13} S_6$$  \hspace{1cm} (5.5)

(with $S_6 = 10^6 S$) for the tera-$\Delta^{++}$ to photon ratio once the exchange reactions have done their job.

What about the tera-electrons, of which there are twice as many as tera-$\Delta$’s? These particles efficiently recombine with protons once the temperature falls below the $Ep$ binding energy of $\sim 25$ keV:

$$E^- + p \rightarrow (E^- p) + \gamma.$$  \hspace{1cm} (5.6)
These $E$-onic atoms thereupon engage in the following exothermic exchange reactions:

$$UUU + Ep \rightarrow (UUUE) + p \quad \text{and} \quad (UUUE) + Ep \rightarrow (UUUEE) + p.$$ (5.7)

The relevant cross-sections for (5.7) are roughly equal to the cross-sectional areas of the $Ep$ atoms, whose radii are $\sim 3 \times 10^{-12}$ cm. Thus they are order of barns! These reactions should proceed to completion because the energy release is far greater than the temperature. As a result, virtually all relic terafermions are expected to form tiny and electrically neutral ‘terahelium atoms,’ with ($UUU)^{++}$ as nuclei and two bound tera-electrons forming a closed shell. These are our candidates for the dark matter of the universe.

To determine whether tera-helium atoms are plausible candidates for dark matter, we have estimated their interaction cross sections with atomic nuclei [1]. There are three distinct mechanisms for terahelium-nuclear scattering:

- via the chromoelectric polarizability of the teranucleus $UUU$,
- via the electric polarizability of the tera-atom, or
- via the magnetic moment of the tera-atom, which is exclusively that of its $UUU$ nucleus because the spins of its two $E$’s cancel.

We find the latter mechanism to dominate: the interaction between the nuclear charge and the magnetic moment of tera-helium. Our rough estimate of the tera-helium–nuclear cross-section implies that $S > 10^6$ (or equivalently, that the terahelium mass must exceed 10 TeV) if our model is to be consistent with the negative results of current dark matter searches [5]. An improvement of the sensitivity of these experiments by one or two orders of magnitude could exclude (or support) the notion of tera-helium as dark matter.

Our proposal for the nature of dark matter faces another potentially serious obstacle. We showed that almost all relic terafermions end up as electrically neutral terahelium, but almost all may not be good enough. Remnant tera-electrons, neutral ($Ep$) atoms or exotic hadrons such as $Uud$, if present on earth or in cosmic rays, may combine with earthly elements to form super-heavy isotopes, the abundance of which is very highly constrained [5]. If our model is to survive, these remnants must have been dealt with by one or more of the following mechanisms: by being more completely aggregated into terahelium during structure formation, or by being gravitationally concentrated within stars because of their great masses, or by being processed during nucleosynthesis into superheavy elements other those for which sensitive searches have been carried out.

**Acknowledgement:** SLG was supported in part by the National Science Foundation under grant number NSF-PHY-0099529 and AGC by the Department of Energy under grant number DE-FG02-91ER-40676.
References

[1] A.G. Cohen and S.L. Glashow, *in preparation.*
[2] P. Achard et al., Phys. Lett. B517 (2001) 75.
[3] V. Barger et al., Phys. Lett. B566 (2003) 8.
[4] A. Nelson, Phys. Lett B136 (1984)387;
   S. Barr, Phys. Rev. Lett 53 (1984) 329.
[5] E.g., C.L. Bennett et al., ApJS 148 (2003) 1.
[6] E.g., A. Benoit et al., Phys. Lett. B545 (2002) 43;
   D.S. Akerib et al., Phys. Rev. D68 (2003) 082002.
[7] E.g., P.F. Smith et al., Nucl. Phys. B206 (1982) 333;
   T.K. Hemmick et al., Phys. Rev. D41 (1990) 2074;
   P. Verkerk et al., Phys. Rev. Lett. 68 (1992) 1116.