INFLATION UNLOADED

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Abstract  I present a brief review of astro-ph/0406099, which argues that there is a limit on the number of efolds of inflation which are observable in a universe which undergoes an eternally accelerated expansion in the future. Such an acceleration can arise from an equation of state \( p = w \rho \), with \( w < -1/3 \), and it implies the existence of event horizons. In some respects the future acceleration acts as a second period of inflation, and “initial perturbations” (including signatures of the first inflationary period) are inflated away or thermalize with the ambient Hawking radiation. Thus the current CMB data may be looking as far back in the history of the universe as will ever be possible even in principle, making our era a most opportune time to study cosmology.

1. INTRODUCTION

Quantum fluctuations produced during inflation are imprinted on the curvature, and are subsequently stretched by inflation to super (Hubble) horizon scales\(^1\). Once there, they “freeze out”, i.e. their amplitude approaches a constant set by the horizon crossing condition, and their wavelength scales with the particle horizon, \( \lambda(t) = \lambda_0 a(t)/a_0 \). What happens next depends on the subsequent evolution of \( a(t) \). When inflation ends, the Hubble horizon begins to grow linearly in time, but the wavelength stretches more slowly, as \( \lambda(t) \sim a(t) \). If the vacuum energy is zero, this situation will persist indefinitely, and the Hubble horizon eventually catches up with the perturbation (see the left panel of Fig.

\(^1\)Because space is tightly constrained in these proceedings, I have included only the most immediately relevant references. Please see [1] for more.
Figure 1.1  Evolution of the wavelengths of some typical inflationary perturbations in a universe without (left panel) and with (right panel) event horizons. In the left panel, all fluctuations eventually reenter the Hubble horizon. In the right panel, in the case a), a fluctuation is stretched outside of the Hubble horizon during inflation, remains there for a time, then reenters during a matter dominated era after inflation, and eventually gets expelled out of the horizon once more during the final stage of acceleration. In b), the fluctuation would have reentered about now, but the late acceleration just prevents that. In c), the late acceleration prevents the fluctuation from ever reentering the Hubble horizon. AH = apparent horizon, RS = reheating surface, PT = photon trajectory, F(P) H = future(past) horizon, $t_0 = \text{now}$.

1.), after which it can collapse and form structure. A patient observer in such a universe can see arbitrarily far back in time by measuring these perturbations: the longer she waits, the farther back during inflation the fluctuations she sees were generated.

However, if at some time the post-inflationary universe begins to accelerate (and continues to do so forever), there will be event horizons [2]. In this case a part of the global spacetime is permanently inaccessible to any given observer, and the evolution of perturbations is very different. Depending on when they were produced, inflationary fluctuations may or may not re-enter the Hubble horizon during matter domination (see the right panel of Fig. 1.). If they do not re-enter by the time the universe begins to accelerate, they will never do so and hence will never be directly observable.

The photons which comprise the CMB originate on the slice (i.e. a sphere) of the last scattering surface which is separated from the observer by null geodesics (labelled PT in Fig. 1.). In a decelerating universe the radius of this last scattering sphere grows without bound, and new information about inflation continues to become available over time.

In a universe which accelerates, the last-scattering sphere asymptotes to the size of the event horizon at the time of last scattering, which is
finite. Therefore, the pattern of temperature anisotropies in the CMB “freezes” after the transition to future acceleration.

Eventually even this remnant will be permanently erased. Spacetimes with event horizons contain Hawking particles, and as the cosmological expansion continues, the CMB redshifts until it is colder than the Hawking radiation. After this time, any remaining information in the CMB will be masked by quantum effects.

2. QUANTIFICATION

The condition that an initial Hubble-scale perturbation (generated at some time $t_i$ during inflation with scale $H_i$) has expanded to fill the observable universe today (subscript 0 refers to now) is:

$$a(t_i)H(t_i) = a_0H_0.$$  

(1.1)

Using $a(t) = a_e \exp(H_e(t - t_e))$ for times during inflation yields

$$N \equiv H_i(t_e - t_b) = \ln \left( \frac{a_eH_i}{a_0H_0} \right),$$  

(1.2)

for some time $t_b$ during inflation. After inflation, the universe grew by a factor of about $a_0/a_e \sim T_e/T_0$, where $T_e$ is the reheating temperature and $T_0 \sim 10^{-3}eV$ the current CMB temperature. Taking this ratio to be about $10^{26} - 10^{28}$ and the scale of inflation to be $H_i \sim 10^{14}GeV$, one finds $N \sim 60$. Hence, to use inflation to solve the horizon and flatness problems requires at least 60 efolds (this is somewhat model dependent).

If the universe accelerates in the future, the comoving Hubble scale $a(t)H(t)$ grows at late times. At a time $t_f$ when the comoving Hubble scale equals its value at reheating, the last perturbation generated during inflation will be larger than the horizon, and afterwards no new structure
will form from inflationary perturbations. The equality
\[ a(t_f)H(t_f) = a(t_e)H_i, \]  
(1.3)
(where \( t_e \) is the reheating time) defines \( t_f \); its value is calculated below.

Spacetimes with event horizons contain (approximately) thermal Hawking particles, with a characteristic temperature \( T_H = H/2\pi \). Being a quantum effect, this spectrum does not redshift in the usual way. If the universe is accelerating, the CMB temperature \( T_{CMB} \) will eventually decrease to a point where it is equal to \( T_H \sim H(t) \). This occurs at a time \( t_T \) when
\[ a(t_T)H(t_T) = a(t_e)T_e. \]  
(1.4)
Taking the ratio \( H_i/T_e \sim 1, t_f \sim t_T \). Using (1.3), eqs. (1.1) and (1.2), and the scaling \( a(t)H(t) \sim a_0H_0(t/t_0)^{-\frac{(1+3w)}{[3(1+w)]}} \) when \(-1 < w < -1/3\):
\[ t_T \sim t_f \sim 10^{78(1+w)/[1+3w]}t_0. \]  
(1.5)
In the limit \( w \to -1/3 \) the time diverges. The limit \( w \to -1 \) yields
\[ t_T \sim \frac{60}{H_0}. \]  
(1.6)

Therefore if the cosmic acceleration never ends, only those inflationary fluctuations produced in the interval between the end of inflation and 60 e-folds before the end will ever be observable. Further, the information which is accessible now will be lost after the time \( t_T \). It is interesting that the future acceleration appears in this sense to shield us from the past incompleteness of inflation, e.g. from the big bang singularity.

3. SUMMARY

Eternal dark energy with \( w < -1/3 \) prevents us from ever detecting inflationary perturbations which originated before the ones currently observable. Further, it slowly degrades the information stored in the currently observable perturbations. This allows us to re-formulate the “Why now?” problem in a novel and interesting way: why are we living in the best time to do cosmology; the time at which we can see back the farthest?

References

[1] N. Kaloper, M. Kleban and L. Sorbo, Phys. Lett. B 600, 7 (2004).
[2] S. Hellerman, N. Kaloper and L. Susskind, JHEP 0106 (2001) 003; W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, JHEP 0107 (2001) 003.