Analysis of the Fusion Hindrance in Mass-symmetric Heavy Ion Reactions

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Abstract

The fusion hindrance, which is also denominated by the term extra-push, is studied on mass-symmetric systems by the use of the liquid drop model with the two-center parameterization. Following the idea that the fusion hindrance exists only if the liquid drop barrier (saddle point) is located at the inner side of the contact point after overcoming the outer Coulomb barrier, the reactions in which two barriers are overlapped with each other are determined. It is shown that there are many systems where the fusion hindrance does not exist for the atomic number of projectile or target nucleus $Z \leq 43$, while for $Z > 43$, all of the mass-symmetric reactions are fusion-hindered.

Keywords: Fusion hindrance; Two center liquid drop model; Saddle point; Contact point
I. INTRODUCTION

Nuclear reactions with heavy-ion beams have been revealing many interesting behaviours of the atomic nucleus. Among them, fusion of two heavy ions into one spherical nucleus is an interesting process which is not yet well understood, although the fusion probability is crucially important for synthesis of the so-called super-heavy elements. It is well known that fusion reactions of light systems and heavy systems have different feature: in light systems, the fusion cross section can be explained by the overcoming of the Coulomb barrier between projectile and target (for instance, see Ref. [1, 2]), while in heavy systems, the experimental fusion cross sections start to be appreciable at higher energies than the Coulomb barrier, that is, are much smaller than those calculated with the same model as that of the light systems (examples at Ref. [3, 4]). It means that the fusion barrier in heavy systems appears to be higher than the Coulomb barrier. The phenomenon in heavy systems is called fusion hindrance, and the corresponding energy difference between the fusion and the Coulomb barriers is called extra-push energy. Theoretical attempts to explain this phenomenon are made with moderate success. In Ref. [5, 6, 7], it was explained by an internal barrier which must be overcome after passing over the usual Coulomb barrier. The internal barrier could be thought as the conditional saddle point in the liquid-drop potential as well as could be attributed to an effective barrier due to the dissipation of the incident kinetic energy [8, 9]. But there was no simple explanation of the mechanism of the hindrance, and therefore no theoretical predictions with quantitative reliability.

Since there are two barriers for the fusion, we have proposed a model where the fusion reaction is divided into two steps: (i) the projectile and target overcome the Coulomb barrier and reach the contact configuration, (ii) the touched projectile and target evolve from di-nucleus to the spherical compound nucleus by passing over the ridge line of the LDM potential [10]. The model explains the extra-push energy and furthermore gives an energy dependent fusion cross sections [11, 12, 13]. According to the compound nucleus theory, production cross sections are given by the fusion and the survival probabilities.

In the present paper, presuming that the internal barrier plays a crucial role in the fusion hindrance, we analyze a relation between the saddle point (more generally a conditional saddle or a ridge line) and the occurrence of the hindrance. The dissipative dynamics of the passing-over of the saddle point has been already studied analytically with the simplification
of the inverted barrier \[14\]. According to the results of Ref. \[14\], the hindrance is given by the saddle point height measured from the energy of the di-nucleus configuration formed by the projectile and the target nuclei of the incident channel. In other words, there is no hindrance for cases with no saddle point height. Therefore, the border between the normal and the hindered fusion is given by the condition that the saddle point height be equal to zero, that is, the di-nucleus configuration be on the top of the saddle point.

In mass-symmetric reactions, the ridge line is simplified into a saddle point, which makes analysis to be simpler. Below, we will find out the region of fusion hindrance for mass-symmetric reactions by using the finite range LDM with two-center parametrization of nuclear shapes \[15, 16\].

Experimentally, it is very difficult to distinguish between fission events coming from the fused compound nuclei and so-called quasi-fission events coming from a di-nuclear system. Therefore, experimental fusion cross sections might not be reliable enough for quantitative comparisons with theoretical calculations. In the present paper, we, thus, focus on the appearance and disappearance of the hindrance that is clearly observed in the symmetric systems.

The present paper is organized as follows: Sec. II recapitulates the parametrization of the di-nuclear system. The determination of the neck parameters which has been recently obtained by the present authors \[17, 18, 19\] is reminded, and a prediction of the fusion hindrance area is shown in Sec. III for mass-symmetric systems. Sec. IV gives a summary.

II. PARAMETRIZATION OF DI-NUCLEAR SYSTEM

There are several ways to parametrize the shape of the amalgamated system. The more accurate description, the more parameters. In this paper we use the two-center parametrization, using three important parameters which are: distance between two centers \(z\), the mass asymmetry parameter \(\alpha\), and the neck parameter \(\varepsilon\), as shown in Fig. 1. The first one is defined as a dimensionless parameter as follows,

\[
z = \frac{R}{R_0},
\]

where \(R\) denotes the distance between two centers of the harmonic potentials, and \(R_0\) the radius of the spherical compound nucleus. The mass-asymmetry parameter is defined as
FIG. 1: Schematic plot of the parametrization of a di-nuclear system. The upper figure shows the harmonic oscillator potential of projectile and target, while the lower one shows the cross section of an equi-potential surface. See text for details.

usual,
\[ \alpha = \frac{A_1 - A_2}{A_1 + A_2}, \]
where \( A_1 \) and \( A_2 \) are mass numbers of the constituent nuclei. The neck parameter \( \varepsilon \) is defined by the ratio of the smoothed height at the connection point of the two harmonic potentials \( (V_1) \) and that of spike potential \( (V_2) \), i.e.,
\[ \varepsilon = \frac{V_1}{V_2}. \]

In this description, nuclear shape is defined by equi-potential surfaces with a constant volume. For example, \( \varepsilon = 1.0 \) means no correction, i.e., complete di-nucleus shape, while \( \varepsilon = 0.0 \) means no spike, i.e., flatly connected potential, which describes highly deformed mono-nucleus. Thus, the neck describes shape evolution of the compound system from di-nucleus to mono-nucleus. The initial parameters for \( z \) and \( \alpha \) are
\[ z_0 = \frac{A_p^{1/3} + A_t^{1/3}}{(A_p + A_t)^{1/3}}, \]
and
\[ \alpha_0 = \frac{A_t - A_p}{A_t + A_p}, \]
respectively. In mass-symmetric case, \( z_0 = \sqrt[3]{4} = 1.5874 \) and \( \alpha = \alpha_0 = 0 \). The initial value of \( \varepsilon \) will be explained in the next section.
III. CALCULATIONS AND ANALYSIS

A. Determination of the neck parameter

With the parametrization of the amalgamated system, the finite range LDM potential can be calculated. In order to study the neck-dependence of the saddle point, the LDM potentials of, as an example, $^{100}\text{Mo} + ^{100}\text{Mo}$ as a function of $z$ for different neck parameters are given in Fig. 2(a). It is clearly shown that the saddle point is very sensitive to the $\varepsilon$: when $\varepsilon$ is smaller (thicker neck), the saddle point is shifted to lower and wider place. Therefore, how the neck changes at contact configuration is a very important problem. To reveal the driving effect of the LDM potential on the neck, the relation between LDM potential at contact configuration and $\varepsilon$ is plotted in Fig. 2(b). The large positive slope of LDM potential with respect to $\varepsilon$ ($dV/d\varepsilon$) drives the neck at contact to be thicker with $\varepsilon$ up to 0. This is natural, considering the strong surface tension of the nuclear matter and a sensitive change of the surface area due to the variation of the $\varepsilon$. Since we also know that the inertia mass for the $\varepsilon$ degree of freedom is small, its momentum is expected to be quickly equilibrated, compared with the other two degrees of freedom: so $\varepsilon$ very quickly reaches the end at $\varepsilon = 0.0$, starting with $\varepsilon = 1.0$. Actually, due to actions of the random force associated to the friction, the $\varepsilon$ reaches the equilibrium quickly, far quicker than the time scale of the radial fusion motion [17, 18, 19]. Thus, when the projectile and target touch with each other, the neck firstly reaches its equilibrium, and then the other two degrees of freedom start to evolve toward the compound stage. The neck parameter at equilibrium can be determined through the average of $\varepsilon$ via

$$\langle \varepsilon \rangle = \int \varepsilon w(\varepsilon) d\varepsilon \int w(\varepsilon) d\varepsilon,$$

where $w(\varepsilon) = e^{-V(\varepsilon)/T}$, and $T$ is the temperature of the system. In most cases, $\langle \varepsilon \rangle$ is close to 0.1. Therefore we take $\varepsilon = 0.1$ in next calculations. This value is also used in the fusion cross section calculations in two-step model [13], which shows a good agreement with experimental data.

From Fig. 2(a), it is obvious that, for $^{100}\text{Mo} + ^{100}\text{Mo}$, the contact point $z_0 (= 1.5874)$ is located inside the saddle point $z_{\text{saddle}} (= 2.40)$ at $\varepsilon = 0.1$, which means that the dinucleus automatically reaches the compound nucleus after overcoming the Coulomb barrier, i.e., no fusion hindrance exists for this case. Following the same way, we can make the
FIG. 2: LDM potential for $^{100}\text{Mo} + ^{100}\text{Mo}$. Left panel: the potential as a function of $z$ with various $\varepsilon$, which gives the $\varepsilon$-dependence of the saddle point. The vertical dashed line represents the contact point. Right panel: the potential at contact configuration with respect to $\varepsilon$, which is used to calculate the average of $\varepsilon$.

same calculation for each mass-symmetric reactions to find out the region where the fusion hindrance disappears, or extra-push energy is zero.

B. Fusion hindrance region

To study the appearance of the hindrance phenomena, one should compare the relative positions of the Coulomb barrier and the conditional saddle point with the neck set at $\varepsilon = 0.1$. The location of the Coulomb barrier depends on the model, but also on the neck parameter [20], but it is always beyond the contact point of the two rigid spheres at contact. Here, with a conservative point of view, we will choose this contact point as a reference.

For a certain mass-symmetric reaction, the conditional saddle point position can be compared with the contact point to determine if the extra-push appears or not. If the contact point is located on the inner side of saddle point, the di-nucleus system will evolve automatically from the touching point to the compound stage by the driving force $dV_{\text{LDM}}/dz$. Otherwise, an additional LDM barrier has to be overcome, which needs an extra-push energy.

To find out the fusion hindrance region of the reaction $^AZ + ^AZ \rightarrow ^{2A}(2Z)$, we fix $Z$ and determine the saddle point for each $A$. Fig. 3 shows an example for $Z = 36$. For
FIG. 3: Determination of the region where the fusion hindrance disappears for mass-symmetric reactions \(^4Z + ^4Z \rightarrow ^{2A}(2Z)\). If the distance \(z\) at saddle point is larger than the \(z_0\) \((A_{\text{low}} \leq A \leq A_{\text{up}})\), fusion hindrance of the reaction does not exist. Otherwise, the extra-push energy is needed to overcome the LDM saddle. The dashed horizontal line represents the contact point.

very small and very large \(A\) the contact point is located outside the saddle point, while for \(A_{\text{low}} \leq A \leq A_{\text{up}}\), the contact point is inside the saddle point, which means that there is no fusion hindrance in this region. Therefore, \(A_{\text{low}}\) and \(A_{\text{up}}\), where the contact point is overlapping with the saddle point, are critical mass numbers corresponding to the \(Z\).

Changing the proton number \(Z\), series of \(A_{\text{low}}\) and \(A_{\text{up}}\) are determined and plotted in Fig. 4. The shadowed area (including the border) represents the reactions of \(A_{\text{low}} \leq A \leq A_{\text{up}}\) and consequently where fusion hindrance does not exist, while the white space represents the contrary. It is interesting that for \(Z < 42\), both \(A_{\text{low}}\) and \(A_{\text{up}}\) increase with increasing \(Z\), but the width of \((A_{\text{up}} - A_{\text{low}})\) becomes narrower and narrower. When \(Z\) is larger than 43, all of the LDM saddle points of the reaction \(^4Z + ^4Z\) are located inside the contact points. Therefore, the extra-push energy should be considered for all mass-symmetric reactions with \(Z > 43\). It is well known that the reactions \(^{90}\text{Zr} + ^{90}\text{Zr}\) and \(^{100}\text{Mo} + ^{100}\text{Mo}\) do not have fusion hindrance while the \(^{110}\text{Pd} + ^{110}\text{Pd}\) reactions does \([21, 22, 23]\). To compare with the above theoretical analysis, the three reactions are also pointed in Fig. 4, in which two dots for \(^{90}\text{Zr}\) and \(^{100}\text{Mo}\) systems are inside the shadowed area, while the dot for \(^{110}\text{Pd}\) system is in the white space. The results show that the theoretical analysis is in a good agreement with experimental data for mass-symmetric reactions. Using \(Z^2\) as a criteria, \(Z^2 > 1849\), which is also in agreement with the empirical rule used to determine the appearance of the
FIG. 4: Critical line for mass-symmetric reactions where the contact point is overlapped with the saddle point. In the shadowed region the contact point is located inside the saddle which means that there is no fusion hindrance in the corresponding reaction. While in white area it is on the contrary and the extra-push energy is needed. Three dots correspond to $^{90}\text{Zr} + ^{90}\text{Zr}$, $^{100}\text{Mo} + ^{100}\text{Mo}$ and $^{110}\text{Pd} + ^{110}\text{Pd}$ reactions. In the calculations, $r_0 = 1.15\text{ fm}$ and $\varepsilon = 0.1$ are adopted.

IV. SUMMARY

In summary, the fusion hindrance of mass-symmetric reactions is studied with the two-center model, in which three parameters (dimensionless distance between two centers $z$, neck parameters $\varepsilon$, mass asymmetry parameter $\alpha$) are employed to describe the di-nuclear system. Because of the very fast evolution of $\varepsilon$ compared to the other two degrees of freedom, $\varepsilon$ is set to its equilibrium value 0.1. In order to find out the reactions where the fusion hindrance does not exist, the position of the saddle point and the contact point for $A^Z + A^Z$ are compared for different $Z$ and $A$. It is found that the mass-symmetric fusion reactions without hindrance are located only in a limited area, and $Z$ should be $\leq 43$. While for systems with $Z$ larger than 43, all of the mass-symmetric reactions are hindered, i.e., the extra-push energy should be required to form the compound nucleus.

Experimental studies around the predicted border are strongly called for. Quantitative comparisons of fusion cross sections or of fusion probabilities should be made between experiments and theoretical results, for which the sticking probability in the two-step model,
i.e., effects of over-coming of the Coulomb barrier have to be taken into account, though there are ambiguities.

Following the same method, the fusion hindrance in mass-asymmetric reactions can also be studied. However, the determination of the saddle in two dimensional LDM potential is more complicate than the symmetric case. Studies along this direction are currently underway and will be addressed in a forthcoming paper.

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[1] Bock R, Chu Y T, Dakowski M, et al. Dynamics of the fusion process. Nucl Phys A, 1982, 388: 334-380
[2] Münzenberg G, Armbruster P, Faust W, et al. TITLE. Actinides in Perspective, Proc. of the Actinides - 1981 Conf., Pacific Grove, California, Ed. N. M. Edelstein, Pergamon Press, 1982: 223-230
[3] Westmeier W, Esterlund R A, Rox A, Patzelt P. Further evidence for extra push. Phys Lett B, 1982, 117: 163-166
[4] Gäggeler H, Sikkeland T, Wirth G, et al. Probing sub-barrier fusion and extra-push by measuring fermium evaporation residues in different heavy ion reactions. Z Phys A, 1984, 316: 291-307
[5] Bjornholm S, Swiatecki W J. Dynamical aspects of nucleus-nucleus collisions. Nucl Phys A, 1982, 391: 471-504
[6] Royer G, Remaud B. Static and dynamic fusion barriers in heavy-ion reactions. Nucl. Phys A, 1985, 444: 477-497
[7] Blocki J P, Feldmeier H, Swiatecki W J. Dynamical hindrance to compound-nucleus formation in heavy-ion reactions. Nucl Phys A, 1986, 459: 145-172

[8] Gross D H E, Kalinowski H. Friction model of heavy-ion collisions. Phys Reports, 1978, 45: 175-210

[9] Fröbrich P. Fusion and capture of heavy ions above the barrier: Analysis of experimental data with the surface friction model. Phys Reports, 1984, 116: 337-400

[10] Abe Y. Reaction dynamics of synthesis of superheavy elements. Eur Phys J A, 2002, 13: 143-148

[11] Shen C W, Kosenko G, Abe Y. Two-step model of fusion for the synthesis of superheavy elements. Phys Rev C, 2002, 66: 061602(1-5)

[12] Abe Y. et al., Fusion Dynamics of Massive Heavy-Ion Systems. Prog. Theor. Phys. Suppl. 2002, no 146 : 104-109

[13] Shen C W, Abe Y, Boilley D, Kosenko G, Zhao E G. Isospin dependence of reactions \( ^{48}\text{Ca} + ^{243-251}\text{Bk} \). Int J of Mod Phys E, 2008, 17 supp: 66-79

[14] Abe Y, Boilley D, Giraud B G, Wada T. Diffusion over a saddle with a Langevin equation. Phys Rev E, 2000, 61: 1125-1133

[15] Suekane S et al. Modified Two-Center Shell Model and Nuclear Fission I. The Modified Two-Center Harmonic Oscillator Shell Model and the Shell Structure. 1974, JAERI-memo 5918

[16] Sato K, Yamaji S, Harada K, Yoshida S. A numerical analysis of the heavy-ion reaction based on the linear response theory. Z Phys A, 1979, 290: 149-156

[17] Abe Y et al. Di-Nucleus Dynamics toward Fusion of Heavy Nuclei. Int. J. Mod. Phys., 2008, E17: 2214-2220

[18] Abe Y et al. From Di-Nucleus to Mono-Nucleus ; Neck Evolution in Fusion of Massive Systems. Int. J. Mod. Phys., 2008, E17: to appear.

[19] Boilley D, Abe Y, Shen C W, et al., in preparation, “Role of the neck in the study of the fusion of heavy ions”.

[20] Iwamoto A, Harada K. Enhancement of the Subbbarrier Fusion Reaction Due to Neck Formation. Z. Phys A, 1987, 326: 201-211

[21] Keller J G, Schmidt K H, Hessberger F, Munzenberg G, Reisdorf W, Clerc H G, Sahm C C. Cold fusion in symmetric \( ^{90}\text{Zr} \)-induced reactions. Nucl Phys A, 1986, 452: 173-204

[22] Schmidt K H, Morawek W. The conditions for the synthesis of heavy nuclei. Rep Prog Phys,
1991, 54: 949-1003

[23] Morawek W, Ackermann D, Brohm T, Clerc H G, Gollerthan U, Hanelt E, Horz M, Schwab W, Voss B, Schmidt K H, Hessberger F P. Breakdown of the compound-nucleus model in the fusion-evaporation process for $^{110}$Pd+$^{110}$Pd. Z Phys A, 1991, 341: 75-78