Reliability of the past surface temperature reconstruction methods

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Abstract. The analysis of the SVD method for the past surface temperature reconstructions based on the measured borehole temperatures is presented. This method has got wide application in geophysics for interpretation of the wide-world temperature profile data set. We show examples where reconstructions do not reflect the true behavior of the past surface temperatures.

1. Introduction
The proxy climate indicators are used to get information on the past climate. The most reliable data for the past surface temperature reconstructions are the borehole temperatures connected directly with the surface temperature changes while the other data result from empirical correlations and calibrations. The past surface temperature changes penetrate in the Earth depth and disturb the steady-state temperature field. Thus, analysis of deviation of the borehole temperature profile from the steady-state thermal regime allows us to reconstruct the resultant ground surface temperature. Numerous results on the past surface temperature histories based on the ground borehole temperature measurements in different places of the Earth were obtained in the last dozen years. The objective of this paper is to study the reliability of these reconstructions. We show that major past climatic changes derived by the inverse problem solutions can be lost.

2. Mathematical model and method
The underground temperature distribution is mainly determined by two types of processes. The first is the surface temperature changes and the second is the heat flux from the Earth that is subjected to the long-time geological processes [1, 2, 3]. The borehole temperature-depth profile $\chi(z)$ can be expressed by the long-term surface temperature $T_0$, the geothermal gradient $\Gamma_0$ and the temperature perturbation $T(z)$ caused by the ground surface temperature variations [4]:

$$\chi(z) = T_0 + \Gamma_0 z + T(z).$$  

The climatic ground surface signal influences on the Earth’s temperature field and can be described by the one-dimensional thermal diffusivity equation with initial and boundary conditions [5]:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}$$

with the initial condition $T(z, 0) = T_0$ and the boundary conditions $T(0, t) = T_0$ and $T(\infty, t) = T_0$.
\[
\begin{align*}
T_t &= a^2 T_{zz}, & 0 < t < t_f, 0 < z < \infty, \\
T(0, t) &= \mu(t), & 0 \leq t < t_f, \\
T(z, t_f) &= 0, & 0 \leq z < \infty,
\end{align*}
\]

(2)

where \(a^2\) is the thermal diffusivity, and \(\mu(t)\) is the ground surface temperature (GST) anomaly from the steady-state temperature \(T_0\) being at the time \(t_f\).

If the past variations of ground surface temperature \(\mu(t)\) are modeled as a series of \(K\) step changes in temperature, the solution of this problem can be found out analytically:

\[
T(z) = \sum_{k=1}^{K} M_k \left[ \text{erfc} \left( \frac{z}{2\sqrt{a^2 t_k}} \right) - \text{erfc} \left( \frac{z}{2\sqrt{a^2 t_{k-1}}} \right) \right]
\]

(3)

where \(M_k\) is the ground surface temperature at the time interval \((t_k, t_{k-1})\), and \(\text{erfc}\) is the complementary error function.

The inverse problem for the surface temperature reconstruction involves equations (2) and the re-determination condition \(T(z, 0) = \theta(z)\), where \(\theta(z)\) is the measured temperature-depth profile. Then the inverse problem solution can be reduced the solution of the linear system of equations:

\[
\Theta_j = A_{jk} M_k,
\]

(4)

where \(\Theta_j\) is the temperature perturbation at depth \(z_j\), \(M_k\) is the average temperature on the time interval \((t_k, t_{k-1})\), and \(A_{jk}\) is a matrix:

\[
A_{jk} = \text{erfc} \left( \frac{z_j}{2\sqrt{a^2 t_k}} \right) - \text{erfc} \left( \frac{z_j}{2\sqrt{a^2 t_{k-1}}} \right).
\]

(5)

This system is over-determined and can be solved by the singular value decomposition (SVD) [2].

\[
A = U \Lambda V^T,
\]

(6)

where \(\Lambda\) is a \((N \times M)\) diagonal matrix whose elements are the non-zero singular values \(\lambda_r\), \(r = 1,...,R\) (\(R\) is the rank of \(A\)) and zero for the null singular values, \(U\) is an \((N \times N)\) orthonormal matrix of eigenvectors spanning data space and \(V\) is an \((M \times M)\) orthogonal matrix of eigenvectors spanning model space. Therefore, the solution of the equation (4) can be written in the following form:

\[
M = V \Lambda^{-1} U^T \Theta,
\]

(7)

Matrix \(\Lambda^{-1}\) is a \((M \times N)\) diagonal matrix whose elements are \(1/\lambda_r\), \(r = 1,...,R\). Thus, any error in the data will be multiplied by \(1/\lambda_r\) and be amplified for the very small singular values. In order to reduce the impact of noise it is necessary to eliminate the singular values which are smaller than cutoff value.

3. Simulations

We consider influence of the GST on the temperature-depth distribution, and derive what information can be retrieved from this profile. We solve the test problem with parameters similar ones obtained from the borehole measurements. The depth of the borehole is 1000m, step of temperature measurements is 1m with accuracy of 0.01 °C, the thermal diffusivity is
\[ a^2 = 10^{-6} \text{ m}^2/\text{s}. \] For simplicity we assume that the long-term surface temperature \( T_0 \) and the geothermal gradient \( \Gamma_0 \) are exact measured. The GST is given by the step function (Fig. 1 (a)):

\[
\mu(t) = \begin{cases} 
0, & 0 < t < 200, \\
0.8, & 200 < t < 400, \\
0, & 400 < t < 1000.
\end{cases}
\] (8)

**Figure 1.** The test GST (Eq. 8) (a) and five chosen GST (b-f) resulting in the same temperature-depth disturbance in range of 0.01 \(^\circ\text{C}\) error.

This function is close to the Little Ice Age that is the last most significant climatic event at the Earth. The temperature-depth disturbances in the present time \( t = 0 \) are shown in Fig. 2. We also point out some other GST result in the similar temperature-depth profiles with accuracy 0.01 \(^\circ\text{C}\). There are infinite set of such GST. The number of these variants increases with increase of the temperature measurement steps and accuracy. Consider examples of GST in the form of two rectangle signals:
\[ \mu(t) = \begin{cases} 
0, & 0 < t < t_1, \\
\mu_1, & t_1 < t < 200, \\
0, & 200 < t < 400, \\
\mu_2, & 400 < t < t_2, \\
0, & t_2 < t < 1000. 
\end{cases} \] (9)

Figure 2. The temperature-depth disturbances for test GST (Eq. 8).

Figure 3. Deviation of the temperature-depth profiles corresponding to GST (Fig. 1(a) and (b)).

One can see that the chosen (9) and test (8) temperatures differ significantly each other. They have different intervals where each signal is zero (Fig. 1(b-f)). With measurement accuracy it is not possible to determine which GST results in the measured temperature-depth profile: the temperature (8) with climatic signal at $200 < t < 400$ years or the temperature (9) with two climatic signals at $t_1 < t < 200$ and $400 < t < t_2$ years.

In all previous papers based on the step GST, the reconstructed temperatures were interpreted as the average temperatures at the one time interval $[1, 2, 3, 4]$. These reconstructions are not corrected (Fig. 1). This situation can not be fixed due to increase time intervals corresponding to the father past time. It is not possible uniquely to determine average temperature at each interval (Fig. 1(b)) at the existing accuracy Fig. 3. This interval length is used for the past surface reconstructions based on data from the Global Database of Borehole Temperatures and Climate Reconstructions.

The other test problem is based on the GST shown in Fig. 4 (black) and used in several papers [4, 6]. This function was based on typical results obtained from analyses of the geothermal data in Eastern Canada. It was checked to estimate quality of the reconstructions by thye SVD method. The borehole depth is equal to 600m and the thermal diffusivity is $a^2 = 10^{-6} \text{ m}^2/\text{s}$ in this examples. The temperature anomaly in borehole due to GST is shown in Fig. 5. We also consider four other GST accordingly to Fig. 4. These GST result in the same borehole-
depth profile in range of the temperature measurement accuracy $0.01 \, ^\circ C$. Deviations of the four temperature-depth profiles from the test temperature (Fig. 5) are shown in Fig. 6.

![Graph showing temperature change over time](image)

**Figure 4.** The test GST [4] (black) and four GST resulting in the same temperature-depth disturbance in range of $0.01 \, ^\circ C$ error.

![Graphs of temperature anomaly and deviation](image)

**Figure 5.** The temperature-depth disturbances for the test GST (Fig. 4, black).

**Figure 6.** Deviation of the temperature-depth profiles corresponding to GST (Fig. 4).

The solution of the inverse problem for these five temperature anomaly by the piece-wise functions. Each function consists of twenty steps of fifty years duration. In the SVD method
Figure 7. The reconstructed GST based on the SVD method for temperature-depth profiles (Fig. 4).

an eigenvalue cutoff set at 0.025 similar to [4]. The reconstructed temperatures are shown in Fig. 7. Their comparison with GST (Fig. 4) allows us to conclude that important features of the climatic signals at the surface are significantly filtrated (Fig. 7). All reconstructed temperatures are smoothed so that reconstructed temperature at 50 years intervals is not often average surface temperature at the at same interval. It is most noticeable in the range of 200–400 YBP.

4. Conclusions

The Earth thickness filtrates significantly high-frequency components of the surface temperature signals. As a result the surface temperature history information is significantly lost in the measured temperature-depth borehole profiles. We derived examples of the inverse problem for the GST reconstructions where climatic signals can not be found out by the SVD method. Therefore, the numerous paleotemperature reconstructions have to be reconsidered. The correction of the GST reconstructions can be done if there are additional data such as the tree ring width data, isotope data and others [7].

References

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