Learning Active Constraints to Efficiently Solve Linear Bilevel Problems

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Abstract—Bilevel programming can be used to formulate many engineering and economics problems. However, solving such problems is hard, which impedes their implementation in real-life. In this paper, we propose to address this tractability challenge using machine learning classification techniques to learn the active constraints of the lower-level problem, in order to reduce it to those constraints only. Unlike in the commonly used reformulation of bilevel programs with the Karush–Kuhn–Tucker conditions as a mixed-integer linear problem, our approach avoids introducing binaries and big-M constants. The application of machine learning reduces the online solving time, and is particularly necessary when the same problem has to be solved multiple times. In particular, it is very adapted to power systems problems, and especially to market applications in which the same problem is solved many times for different loads. Three methods are developed and applied to the problem of a strategic generator, with a DC Optimal Power Flow in the lower-level. We show that for networks of varying sizes, the computational burden is significantly reduced with a good probability of retrieving the optimal solution. We manage to find solutions for problems that were previously intractable.

Index Terms—Bilevel programming, Stackelberg games, classifier, active set, mixed-integer linear programming (MILP)

I. INTRODUCTION

Bilevel problems were formulated for the first time in 1934 by H.v. Stackelberg [1]. Since then, they have been widely used in economics and game theory, in particular to model strategic behaviors. One issue is that these problems are NP-hard to solve [1], [2]. Linear bilevel problems can easily be reformulated as one-level problems, but the introduction of binary variables render these reformulations intractable for large systems [3]. In power systems, bilevel problems can be used to model the behavior of a price-maker in electricity markets, to evaluate investment in production facilities, to model the best transmission network investments [4], to evaluate the vulnerability of power systems to deliberate [5] or unintentional [6] outages, and more recently for demand response management by tariff design in a smart grid setup [7]. Due to the size of the networks, the tractability of bilevel problems is critical for these applications.

The approach used here derives from the active-set strategy [8]. The lower-level problem is replaced with its active constraints in a one-level reformulation in order to obtain a more tractable version of the bilevel problem. However, there can be multiple possible sets of active constraints to consider, depending on the value of the variables of the upper-level problem. To avoid having to identify the possible active sets at every run of a model, machine learning techniques can be used. This is particularly interesting for power systems applications in which similar calculations may have to be carried out very often with only a few parameters changing, especially when the market is involved.

In this paper, the application example studied is the problem of a strategic generator optimizing its bids on the market; with the market modelled with a DC Optimal Power Flow (DCOPF). Machine learning has already been used to learn the active constraints of a DCOPF problem with promising results [9]–[11]. In [12], a similar approach is considered, except that the active constraints are not learned directly. Instead, they are derived from the gradient of the cost with respect to the loads, which is itself the output of a Neural Network classifier. The combination of active constraints and machine learning is the chosen approach here, with the additional challenge that the decision variables of the upper-level problem should not intervene in the classification process, although they are parameters in the DCOPF. To our knowledge, this application has not been considered yet.

Looking at the literature for approaches regarding a more efficient solving of linear bilevel problems, several directions have been explored, such as genetic algorithms [13] and evolutionary algorithms [14], [15]. In [16], regularization approaches are combined with mixed-integer reformulation, by first finding a local optimal solution to provide initial values of the binary variables, which reduces the computational burden. In these regards, the application of machine learning techniques is, to the best of our knowledge, a new approach, and a promising one.

The objectives of this paper are to show that linear bilevel problems can be solved more efficiently by learning active sets and complex bilevel problems can become tractable. This paper introduces three methods based on machine learning to achieve that. They allow replacing the lower-level problem with its active constraints, avoiding the use of binaries and big-M constants, reducing the solving time and even solving problems that were intractable before.

The rest of this paper is organized as follows: Section II introduces bilevel problems as well as the example considered in the rest of the paper with its reformulation as a mixed-integer linear problem (MILP). Section III describes the methods proposed. The application of the methods to test systems is given in Section IV and Section V concludes the paper.

II. SOLVING BILEVEL PROBLEMS

A. Formulation of KKTs and linearization

Bilevel problems are optimization problems in which constraints are in part defined by another optimization problem. One common example in the fields of economics and game theory is Stackelberg games, in which one player, the leader, anticipates the decision of the other agents, or followers, and
and can generally be solved. However, when the number of problems that were intractable before.

The variables under consideration are

\[
\begin{align*}
\min_{x,y} & \quad F(x,y) \\
\text{s.t.} & \quad H(x,y) = 0 \\
& \quad G(x,y) \leq 0 \\
& \quad \min_y f(x,y) \\
& \quad h(x,y) = 0 \\
& \quad g(x,y) \leq 0
\end{align*}
\]

where \( x \in \mathbb{R}^n \) and \( y \in \mathbb{R}^m \). Equations (1a) to (1f) describe the embedded problem, referred to as lower-level or follower problem. The lower-level objective function is \( f(x,y) \) and \( h(x,y) \) and \( g(x,y) \) are the lower-level constraints. The dual variables associated with these equality and inequality constraints respectively are \( \lambda \) and \( \mu \). \( F(x,y) \) is the objective function of the global problem, called upper-level or leader problem. \( H(x,y) \) and \( G(x,y) \) are the upper-level constraints. The Variables under \( x \) are decision variables to the upper-level problem and parameters in the lower-level problem. On the other hand, \( y \), stands for the decision variables of the lower-level problem.

The problem is non-linear and intractable, due to equation (1d). In order to be solved, it can be reformulated as a one-level problem. The most common approach, under the condition that the lower-level problem is convex and regular, is to replace the problem with its Karush-Kuhn-Tucker conditions (KKTs). The following problem is obtained:

\[
\begin{align*}
\min_{x,y} & \quad F(x,y) \\
\text{s.t.} & \quad H(x,y) = 0 \\
& \quad G(x,y) \leq 0 \\
& \quad \nabla_y L(x,y,\lambda,\mu) = \nabla_y f(x,y) + \lambda^T \nabla_y h(x,y) + \mu^T \nabla_y g(x,y) = 0 \\
& \quad h(x,y) = 0 \\
& \quad 0 \leq \mu \perp -g(x,y) \geq 0
\end{align*}
\]

where \( \nabla_y L(x,y,\lambda,\mu) \) represents the Lagrangian derivatives with regards to the components of the vector \( y \).

The problem is thus to replace these binary variables in order to significantly decrease the solving time and to enable solving problems that were intractable before.

**B. Strategic generator as a MILP**

In the rest of this paper, the methods proposed will be applied to one particular instance of bilevel problem which formulates the decision-making of a strategic producer wanting to determine its bids on the day-ahead market, in order to maximize its profit. The lower-level problem is the market clearing as a DCOPF:

\[
\begin{align*}
\min_{P_{g,i},\theta} & \quad c_s P_{g,1} + \sum_{i \neq 1} c_i P_{g,i} \\
\text{s.t.} & \quad P_{g,i} - P_{d,i} - \sum_{l \in I} B_l \Delta \theta_l = 0, \quad \forall i \quad (a) \\
& \quad P_{g,i}^{\text{min}} \leq P_{g,i} \leq P_{g,i}^{\text{max}}, \quad \forall i \quad (b) \\
& \quad -f_{l,i}^{\text{max}} \leq B_l \Delta \theta_i \leq f_{l,i}^{\text{max}}, \quad \forall l, \forall i \quad (c) \\
& \quad \theta_{\text{ref}} = 0 \quad (d)
\end{align*}
\]

where \( i \in I \), represents the bus of the system studied and \( l \in L \), the lines connecting the bus of this system. The decision variables of the DCOPF are the power output of all the generators in the system \( P_{g,i} \) \((i \in I)\) and the voltage angles at the bus \( \theta_i \) \((i \in I)\). \( \Delta \theta_l \) represents the voltage angle difference between the from bus and the to bus of line \( l \). The strategic generator is placed in bus 1. The slack bus is denoted with the subscript \( \text{ref} \). It might or might not be the bus where the strategic generator is located. The price bid of the strategic generator \( c_s \) is a parameter to the DCOPF.

The objective is to minimize the total cost of the system, \( c_s \) gives the real cost of generator \( i \). Equation (4a) is the power balance at bus \( i \), \( B_l \) being the susceptance of line \( l \). Equation (4b) gives the limits of the generator at bus \( i \) in terms of minimum \( P_{g,i}^{\text{min}} \) and maximum \( P_{g,i}^{\text{max}} \). Equation (4c) gives the limits of the power flow in line \( l \), bounded by the line constraint \( f_{l,i}^{\text{max}} \). Finally, the angle of the slack bus \( \theta_{\text{ref}} \) is set to 0. The dual variables of Equations (4b) to (4c) are given in parentheses next to each equation, and will be used to formulate the KKTs of the lower level problem.

The bilevel problem can be formulated as:

\[
\begin{align*}
\min_{c_s, P_{g,i}, \theta} & \quad c_s P_{g,1} - \alpha_1 P_{g,1} \\
\text{s.t.} & \quad c_s \leq c_i \leq c_s^{\text{max}} \\
& \quad c_s - c_i \geq 0 \quad (e) - (f)
\end{align*}
\]

The upper-level problem objective (Equation (5a)) is the maximization of profit for the strategic generator, as the difference between its operating cost \( c_s \) and the price received, given by the dual variable of the power balance in bus 1, \( \alpha_1 \). The profit is here expressed in the maximization form (standard form). Equation (5b) belongs to the upper-level problem and sets some limits to the cost of the strategic generator to ensure that the problem is bounded. The minimum is equal to the real cost of production \( c_s \) and the maximum is \( c_s^{\text{max}} \).

The objective function in Equation (5a) is not linear because of the term \( \alpha_1 P_{g,1} \). However it is possible to linearize it, using the KKTs of the lower level problem. The proof is not given
here but is available in [4]. This together with the KKTs and Fortuny-Amat McCarl linearization gives the following MILP:

\[
\begin{align*}
\min_{c_S, P_{g,i}, \theta, u, y, \alpha, \rho, \gamma} & \sum_{i \in I} c_i P_{g,i} + \phi_i^{\text{max}} P_{g,i}^{\text{max}} - \phi_i^{\text{min}} P_{g,i}^{\text{min}} - \alpha_i P_{d,i} \\
\quad & + c_S P_{d,i} + \sum_{l \in I} f_l^{\text{max}} (\rho_l^{\text{min}} + \rho_l^{\text{max}}) \\
\text{s.t.} & c_S \leq c_S \leq c_S^{\text{min}} \\
& P_{g,i} - P_{d,i} - \sum_{l \in I} B_l \Delta \theta_l = 0, \quad \forall i \\
& \theta_{l,ref} = 0 \\
& c_S - \phi_i^{\text{min}} + \phi_i^{\text{max}} = 0 \\
& \alpha_i - \phi_i^{\text{min}} + \phi_i^{\text{max}} = 0 \quad \forall i \neq 1 \\
& \sum_{l, i = \text{from}} B_l (\alpha_l - \alpha_{l,ref} - \rho_l^{\text{min}} + \rho_l^{\text{max}}) \\
& + \sum_{l, i = \text{to}} B_l (\alpha_l - \alpha_{l,ref} - \rho_l^{\text{min}} + \rho_l^{\text{max}}) = 0 \quad \forall i \neq ref \\
& \sum_{l, ref = \text{from}} B_l (\alpha_{l,ref} - \alpha_{l,ref} - \rho_l^{\text{min}} + \rho_l^{\text{max}}) \\
& + \sum_{l, ref = \text{to}} B_l (\alpha_{l,ref} - \alpha_{l,ref} - \rho_l^{\text{min}} + \rho_l^{\text{max}}) + \gamma = 0
\end{align*}
\]

where \( u_i^{\text{min}}, u_i^{\text{max}}, y_i^{\text{min}}, y_i^{\text{max}} \) \((i \in I, l \in L)\) are the binary variables introduced by Fortuny-Amat McCarl linearization, and \( M \) is a large enough constant. Equations (6c) and (6d) are the equalities of the lower-level problem. Equations (6e) to (6h) are obtained by setting to zero the derivatives of the Lagrangian of the lower-level problem with regard to all the variables. Equations (6e) to (6g) are the linearized complementarity constraints. They contain and replace the inequality constraints in Equations (4c) and (4d).

Solving Equations (6a) to (6d) directly will be used as a baseline for the case studies in Section IV.

III. METHODS

The reformulation proposed here is based on the model given in [3], but aims at only keeping the constraints that are active at the optimal point. In the case of a linear problem, those are sufficient to describe the system at optimality. Three methods have been established to achieve this. The process for each of these methods is illustrated in Figure 1. They follow the same general structure. First, as part of an offline process, a database is built, mapping the variables of the lower-level problem with the corresponding active constraints. This is described in Section III-A.2. This database is used to train a decision tree (DT), as explained in Section III-A.3. For a given value of the parameters, this DT allows to retrieve sets of active constraints, in order to build and solve a reduced bilevel problem. This process is detailed in Section III-B.

A. Database generation and learning

The general idea is to reduce the lower-level problem to its active constraints, which will eliminate the binaries introduced by the linearization of the complementarity constraints. The optimal solution will be different for different value of the input parameters (such as the load in the case of the strategic generator problem) and so will the active constraints. As a consequence, the identification of these active constraints must be carried out for each new value of the input parameters. It can be tedious as the variables of the upper-level problem are also parameters to the lower-level problem. So in order to consider all possible reductions of the lower-level problem, multiple setups would have to be tested, even when the parameters are known. In the context of power systems, this is particularly critical as decisions have to be made very often and the parameters vary and are uncertain, especially demand and renewable energy generation. To avoid a long decision process, the idea is to move the selection of the active constraints to an offline process, using machine learning classification techniques. In this paper, DTs are used to perform this classification. The challenge is to remove the decision variables of the upper-level problem from the classifier, unless the DT is included in the optimization problem as in [17]. This option has been considered but discarded as it would introduce unnecessary binary variables and additional constraints, while preventing the removal of constraints. The three methods detailed in the following offer three different ways to exclude the decision variables of the upper-level problem from the classification. Solving the problem for each possible active set could be a way of proceeding. But while this works well for a small number of active sets, it becomes inefficient when there are many. On the other hand, for a system that has few active sets, it will be more suitable to directly solve the problem for each of the sets than to use the DT approach.

1) Identification of the active constraints: The focus here is on the lower-level optimization problem. In optimization problems, the optimal solution always lies at the boundaries of the feasible space. The corresponding constraints are considered as binding. The other constraints are inactive. The active set that corresponds to the optimal solution regroups the constraints that are satisfied with equality at the optimal point. If the problem is reformulated following this set, by replacing the active inequality constraints with equalities and removing the other constraints, it will recover the optimal solution. This reduced problem is easier to solve since some constraints are dropped.

The active constraints of a linear problem (LP) can be identified by looking at the value of the dual variables associated with the inequalities of the problem, at the optimal point (equality constraints are always binding). All dual variables that are non-zero indicate an active constraint.

In the example of a strategic generator, the DCOPF problem is solved for a given load and bidding cost of the strategic producer. The value of the dual variables associated with constraints (4c) to (4d) is then analyzed in order to retrieve the set of active constraints.

2) Database building process: In order to train the DT classifier, a database of points has to be built. The way this database is generated is different in each method we propose but the general idea is the same: the DT should take the load as input and return one or several active sets. For all methods, the algorithm DiscoverMass, as presented in [10] is applied.
As a stopping criterion for building the database. The idea is to keep generating points from a given distribution, until a sufficient share of the possible active sets have been recovered; that is, until the probability mass of the discovered sets reaches a chosen threshold. A safety limit to the number of steps of the algorithm is also defined, in case it would not converge fast enough. The interested reader can refer to [10] for details on the algorithms and the theorems and proofs associated, in particular regarding termination.

a) Method VarLower: In this method, the generated point consists of the varying parameters of the lower-level and the variables of the upper-level problem that are parameters to the lower-level problem. In the selected example, that would be the load along with the cost bid by the strategic generator. For the randomly generated point, the lower-level problem (DCOPF) is run and the set of active constraints at the optimal point is retrieved.

b) Method AllSets: As illustrated in Figure 1b, the database in this method associates for a given load all observed sets of active constraints obtained by varying the cost bid by the strategic generator. For each randomly created load, multiple instances of the DCOPF are solved for a range of $c_S$ values. For each value of $c_S$, an active set is retrieved. All these active sets are gathered in a set of active sets to be associated with this load in the database.

c) Method BestSet: This works similarly to the previous method: for a given load, the DCOPF is solved for a range of $c_S$ values. In this case, however, only the active set corresponding to the value of $c_S$ that returns the best value of the upper-level objective function is kept to be part of the database. The intention here is to keep only the active sets corresponding to optimal points of the bilevel problem.

3) Decision Tree training: Once the database is created, a DT is trained in order to later predict, for any given value of the parameters of the lower-level problem, the set of active constraints to apply. A decision tree is a classifier that keeps splitting the data according to one of the features of the input until reaching a separation per class. It consists of nodes, which represent decisions made based on a given feature, branches, and leaves, which are the final nodes, in which the class is selected. Figure 2 shows representations of such decision trees.

a) Method VarLower: In this method, a first DT is generated from the database, that is from samples consisting
of load and $c_S$ values. However, $c_S$ should not be an input parameter to the final DT, which will be applied outside and before the bilevel problem is solved. As a consequence, all the nodes in which the feature is $c_S$ are retrieved and the corresponding critical values of $c_S$ are extracted, as shown in Figure 2. The database is then split following the identified intervals of $c_S$. Then, for each of these intervals, a DT is built, taking as input the load and returning an active set. Online, each of these sub-decision trees will be applied, thus returning as many active sets as there are sub-decision trees. This whole process is illustrated in Figure 2.

b) Method AllSets: For the method AllSets, the database is already built for the load only, so a single DT is built, taking as input the load and returning a set of active sets.

c) Method BestSet: Similarly to the previous method, one DT is built with the load as input, but this time only one active set is returned, the one corresponding to the optimal solution of the bilevel problem.

B. Reduced bilevel

Once one (or more for method VarLower) DT(s) has/have been trained offline, it can be applied online to predict the active set corresponding to a given load. The lower-level problem is then reduced to keep the active constraints only, replacing them with equalities. This reformulation is applied to the inequalities (3a) to (3b) in the MILP formulation. For each inequality of the original lower-level problem:

- If it is considered as active, it is replaced with equality and the corresponding complementarity constraint is discarded, as well as the constraints on the dual variables, since they are inactive. In the general formulation of the bilevel program, for $g$ active, we would replace Equations (3a) to (3b) with:

$$g(x, y) = 0$$

(7a)

- If it is considered as inactive, the inequality is dropped and the corresponding dual variable is set to be equal to 0. In the general formulation of the bilevel program, for $g$ inactive, we would replace Equations (3a) to (3b) with:

$$\mu = 0$$

(8a)

As a consequence, the binary variables are completely removed from the formulation and the bilevel problem is now an LP. Inequality constraints are also removed. Big-$M$ constants are not necessary anymore, which is also a considerable advantage of our approach over the traditional one. This will be discussed in Section IV.

Note that we still have the equalities (6c) to (6h) associated with the Lagrangian derivatives in the new formulation. In general, for a nondegenerative LP it would be possible to simply replace the lower-level problem with its active constraints, without having to formulate any Lagrangian, as in [12]. However, as the duals of the lower-level problem intervene in the objective function (6a), this cannot be applied to our problem.

In the cases where several active sets are returned by the DT(s), several LPs are solved and compared in terms of value of the objective function, to keep the best one only.

a) Method VarLower: In the method VarLower, there are $n_V$ decision trees to be applied, one per interval of $c_S$ as illustrated in the online part of Figure 1a. As a result, $n_V$ sets of active constraints are returned. The bilevel problem, reformulated as an LP with the corresponding active constraints only, is solved $n_V$ times.

b) Method AllSets: With AllSets, one DT is applied, and the output is a set of $n_A$ active sets. $n_A$ different versions of the reduced bilevel are solved and the best result is identified. This is shown in the online part of Figure 1b.

c) Method BestSet: In the method BestSet, one DT is applied, returning one active set. Only one reduced bilevel problem is solved in this case.

IV. CASE STUDIES

In this section, the three methods will be applied to 5 different test cases and compared to the baseline. The systems are based on Matpower cases [18]. The details of the cases are given in Table I. The costs of the generators have been modified in order to increase the number of possible outcomes for the strategic generator. The files are available online [19]. All simulations were carried out in Python using Gurobi to solve the optimization problems and Python library scikit-learn to build and apply the decision trees. The corresponding code is also available online [20].

| Test case | Characteristics of the test cases chosen |
|-----------|----------------------------------------|
| 5-bus     | Load interval around default value      |
| 9-bus     | VarLower                               |
| 2869-bus  | AllSets                                |
| 1354-bus  | Best Set                               |

Note that the table contains the number of sets for each method for each test case.
A. Baseline method: Choice of big-M

As mentioned in Section 2, the baseline method used for comparison is obtained by directly solving the MILP in Equations (6a) to (6c). The choice of $M$ is in this case an important matter to take into consideration [21], [22]. Too small, it can interfere with the physics of the model, too big it can lead to numerical ill-conditioning. Here we take advantage of the database generation to set $M$. First, two constants are defined: $M_p$ for the primal constraints and $M_d$ for the dual variables. The choice of $M_p$ is straightforward as it is linked with the existing bounds on the primal variables. However, the choice of $M_d$ is not straightforward as it is complicated to evaluate and bound the dual variables. When solving the DCOPF in the database creation process, the maximum values of the dual variables are retrieved and stored to be later assigned to $M_d$ in the solving process. In order to have some safety margin, we used 10 times these maximum values both for $M_p$ and $M_d$.

It should be highlighted here again, that, in contrast with the baseline method, the method we propose in this paper have the additional advantage of avoiding the problem of big-M selection, as these parameters are removed from the reformulations.

B. Modeling parameters

a) Databases generation: The parameters used in DiscoverMass algorithm for the database generation are the ones suggested in [10]. The resulting number of points in the databases is given in Table I. As the number of active sets varies depending on the method (one with BestSet, $n_A$ with AllSets and $n_V$ with VarLower in Figure 1), so does the number of points for a given system. The loads applied are selected randomly with equal probability, and independently for each bus, in an interval $[(1-x_m)P_{d,i},(1+x_p)P_{d,i}]$, where $P_{d,i}$ is the default load (from the test case data) and $x_m$ and $x_p$ are the percentages of sample range under and above the default load, respectively. All the points which are infeasible for the DCOPF are not included in the database. The maximum value of $c_S$, as defined in Section II is chosen as:

$$c_{S}^{max} = 10 \times \max\{c_i, i \in I\}$$

(9)

where $\max\{c_i, i \in I\}$ is the cost of the most expensive generator in the system. This way we avoid the situation where $c_S$ is unbounded and the chosen bound will not impact the resulting dispatch. For methods AllSets and BestSet, each sample of load was tested with 10 values of $c_S$, between $c_1$ and $c_{S}^{max}$.

b) Decision Trees building: When building a classifier, the features of the data used for the training are of great importance. Here, the main features are the load at each bus and the cost bid by the strategic generator (for Method VarLower only). It was found that adding the total load of the system as a feature is a very valuable information. Figure 3 illustrates this. The DTs performance improves significantly with this addition as shown in Table III. It has, however, almost no effect on the big 2869-bus system. To build and evaluate the DTs, the database is randomly separated into training and testing sets, with 70% of the samples kept for training. The performance is evaluated by the achieved DT accuracy on the test set. The accuracy measures the percentage of the test samples that are correctly classified. A randomized search is performed in order to select the hyperparameters that maximize the DT accuracy while avoiding overfitting.

![Fig. 3. Importance of the different features in the resulting DT. Example with 39-bus system and VarLower general DT.](image)

TABLE II

| Test case | Without total load as a feature | Including total load as a feature |
|-----------|---------------------------------|----------------------------------|
|          | VarLower | AllSets | BestSet | VarLower | AllSets | BestSet |
| 39-bus    | 95.0%    | 97.5%   | 97.0%   | 96.8%    | 96.0%   | 92.6%   |
| 89-bus    | 86.0%    | 70.3%   | 82.0%   | 91.6%    | 86.4%   | 91.0%   |
| 1354-bus  | 66.0%    | 72.7%   | 78.1%   | 75.2%    | 25.6%   | 29.3%   |
| 2869-bus  | 52.6%    | 43.5%   | 79.0%   | 52.6%    | 43.7%   | 79.2%   |

c) Models evaluation: In order to assess the performance of our methods, 20,000 scenarios of feasible load are randomly generated, with a uniform distribution between the values used for the database generation. The reduced LPs are run in parallel. In order to ensure feasibility of the solution, we check that the solution of each reduced LP does not violate any constraint of the initial problem. The solution maximizing the profit of the strategic generator is chosen among all feasible solutions. For all optimization problems, a running time limit of 5 minutes is set.

C. Results

The results for all the test cases are gathered in Table III. The running time with our methods is compared to the duration of the baseline in parenthesis (mean duration with the method divided by the mean duration of the baseline). The number of binaries is shown for the baseline in the column #Bin, and the number of reduced LPs solved by our methods in the column #LPs. The proportion of scenarios for which the optimal solution in terms of the objective function value of the bilevel problem is recovered is given in %Opt (the optimal solution is considered to be the one obtained with the baseline). The percentage of test scenarios returning infeasible points with the proposed methods (feasible in the baseline) is available in %Inf. The difference %Opt−%Inf indicates the percentage of sub-optimal results. This corresponds to cases for which the wrong set of active constraints is returned by the DTs but the LPs obtained are still feasible. As a result, the bilevel problem finds a feasible solution but not the optimal one. Except for the smaller 9-bus system, for which the baseline method already solves fast, we achieve a time reduction of 80% to 95% compared to the baseline. This means that our proposed methods achieve a computation speedup of 6-20 times faster than the conventional method. The binaries are completely removed and the number of constraints #Cstr is also reduced in half with all the test cases. Very slow outliers
TABLE III
RESULTS OF THE THREE METHODS COMPARED TO THE BASELINE

|          | Baseline | VarLower | AllSets | BestSet |
|----------|----------|----------|---------|---------|
|          | Duration (s) | #Bin | #Cstr | Duration (s) | #LPs | #Cstr | %Opt | %Inf | Duration (s) | #LPs | #Cstr | %Opt | %Inf |
| Case 9   | 0.011    | 24    | 94    | 0.007 (66%) | 2.8  | 46    | 99.4% | 0%    | 0.007 (66%) | 1.78 | 46    | 99.5% | 0%    |
| Case 39  | 0.012    | 112   | 425   | 0.021 (17%) | 2.7  | 201   | 95.9% | 0%    | 0.020 (16%) | 2.12 | 201   | 95.9% | 0%    |
| Case 1354| 29.22    | 3384  | 13121 | 1.290 (4%)  | 6.1  | 653   | 94.8% | 5.9%  | 1.445 (5%)  | 17.9 | 6353  | 75.0% | 0.4%  |
| Case 2869| 111.01*  | 6506  | 25767 | 5.379 (5%*) | 8.0  | 12755 | 66.3%*| 4.8%  | 6.412 (6%*) | 42.6 | 12755 | 96.1%*| 1.2%  |

* : The mean duration and optimality recovery rate are calculated for the tractable scenarios only. Refer to Section IV-D for results regarding intractability.

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TABLE IV
RESULTS OF THE THREE METHODS COMPARED TO THE BASELINE FOR THE MORE COMPLEX CASES, EXTRACTING MORE CLASSES FROM THE DTs

|          | Baseline | VarLower | AllSets | BestSet |
|----------|----------|----------|---------|---------|
|          | Duration (s) | #Bin | #Cstr | Duration (s) | #LPs | #Cstr | %Opt | %Inf | Duration (s) | #LPs | #Cstr | %Opt | %Inf |
| Case 1354| 111.01*  | 6506  | 25767 | 5.275 (5%*) | 16.8 | 12755 | 73.3%* | 5.4%  | 6.336 (6%*) | 42.6 | 12755 | 96.1%* | 1.2%  |
| Case 2869| 4.936 (4%*) | 36   | 12755 | 97.6%* | 0.4%  |

* : The mean duration and optimality recovery rate are calculated for the tractable scenarios only. Refer to Section IV-D for results regarding intractability.

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For the simple systems (9-bus, 39-bus, and 89-bus), the accuracy of the decision trees is above 85%, and as a consequence, the methods manage to recover the optimal solution for more than 90% of the test scenarios. With BestSet, as only one set is applied, the accuracy of the method corresponds to the accuracy of the decision tree. For the two other methods, as more versions of the reduced bilevel are generated, the accuracy is greater than in the decision tree. However, they are more costly in terms of computing time. Method VarLower is a good option for this problem, especially because there is only one variable of the upper-level problem that is applied.

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as a parameter in the lower-level problem. If there were more, the number of sub decision trees could increase significantly, which would result in more LPs to solve.

The performance of our methods has been evaluated on the ability to recover the value of the profit for the strategic generator (the value of the objective function of the upper-level problem), as in this kind of problems there is usually a range of values of the decision variables which lead to the same solution.

D. Results for complex systems

a) Improved accuracy: The complex systems studied (1354-bus and 2869-bus) do not demonstrate an equally good performance in terms of optimality (%Opt) and frequency of infeasible cases (%Inf), as we also see in Table [III]. This comes from the fact that the DTs have a low accuracy. Figure [5] gives t-SNE plots for the databases with the method VarLower. The t-SNE technique allows to represent a high-dimensional set of data in 2 dimensions. It shows points that are related by clustering them. Ideally, there is a cluster for each class of the data represented. Further information on t-SNE can be found in [23]. Here, each class corresponds to a set of active constraints. However, we cannot identify proper clusters, which means that the classes of our problem are very similar. It is then difficult for the classifier to distinguish how to best separate the data, while avoiding over-fitting. But similar classes will be classified in the same area of the created. This idea is applied to extract extra information from the DT and significantly increase the accuracy for these systems. When applying the DT to a given load, the corresponding leaf is extracted and its parent is identified. Then, the classes of all samples of the training data of the DT that would be classified by the parent node of this leaf are retrieved. As a result, more LPs are formulated but a good performance is recovered, as shown in Table [V].

b) Solve intractable problems: By applying this, we manage to calculate solutions for some cases that are intractable with the baseline method, with the 2869-bus system. For 71% of the scenarios, the baseline method does not solve within 5 minutes while our methods manage to find a feasible solution for respectively 93.6%, 98.9% and 99.5% of these intractable scenarios.

Note that for this 2869-bus system, the difference %Opt−%Inf does not give valuable information, since %Opt is calculated on the cases that are tractable for the baseline method only, while %Inf gives the percentage of infeasibility with our methods over all the scenarios tested. However, 100−%Inf gives the percentage of scenarios for which our methods manage to obtain a feasible solution.

V. CONCLUSION

In this paper, a new efficient approach to solve linear bilevel problems was introduced. Three different methods using decision trees to learn the active sets for different values of the parameters of the lower-level problem were presented. They were applied to the power system problem of a strategic generator optimizing its bidding in the market, represented by a DCOPF in the lower-level of the bilevel problem. To evaluate the performance of our methods, they were applied to systems of different sizes. For smaller systems, we showed that we were able to recover the optimal solution for more than 90% of the load scenarios, with a consequent time reduction of up to 85%, or a speedup of 7 times compared to solving the MILP obtained with the KKTs and Fortuny-Amat McCarl reformulation. For more complex systems, the structure of the decision trees has been used to extract more information. As a result, more than 80% of the optimal solutions were recovered for a time reduction of 94%, or a speedup of 18 times at least. For the 2869-bus system, our methods were able to recover a solution for 93.6% to 99.5% of the scenarios while the baseline method was intractable for 71% of the scenarios. In general, we saw that there was a trade-off between the prediction accuracy and the computational complexity of the problem (the number of LP problems solved). By avoiding the use of binary variables, the proposed methods also do not need a big-M constant to be defined, which solves a major problem of bilevel models.

REFERENCES

[1] S. Dempe, Foundations of Bilevel Programming, ser. Nonconvex Optimization and Its Applications. Springer US, 2006.
[2] B. Colson, P. Marcotte, and G. Savard, “Bilevel programming: A survey,” 4OR, vol. 3, pp. 87–107, 2005.
[3] D. Pozo, E. Sauma, and J. Contreras, “Basic theoretical foundations and insights on bilevel models and their applications to power systems,” Annals of Operations Research, vol. 254, 07 2017.
[4] S. Gabriel, A. Conejo, J. Fuller, B. Hobbs, and C. Ruiz, Complementarity Modeling in Energy Markets, ser. International Series in Operations Research & Management Science. Springer New York, 2012.
[5] J. M. Arroyo and F. D. Galiana, “On the solution of the bilevel programming formulation of the terrorist threat problem,” IEEE Transactions on Power Systems, vol. 20, no. 2, pp. 789–797, 2005.
[6] J. M. Arroyo, “Bilevel programming applied to power system vulnerability analysis under multiple contingencies,” IET Generation, Transmission & Distribution, vol. 4, no. 2, pp. 178–190, 2010.
[7] A. Kovács, “Bilevel programming approach to demand response management with day-ahead tariff,” Journal of Modern Power Systems and Clean Energy, vol. 7, no. 6, pp. 1632–1643, 2019.
[8] M. Fukushima and P. Tseng, “An implementable active-set algorithm for computing a b-stationary point of a mathematical program with linear complementarity constraints,” SIAM Journal on Optimization, vol. 12, no. 3, pp. 724–739, 2002. [Online]. Available: https://doi.org/10.1137/S1052623499363232
[9] Y. Ng, S. Misra, L. A. Roald, and S. Backhaus, “Statistical learning for dc optimal power flow,” 2018.
[10] S. Misra, L. Roald, and Y. Ng, “Learning for constrained optimization: Identifying optimal active constraint sets,” 2018.
[11] D. Deka and S. Misra, “Learning for dc-opp: Classifying active sets using neural nets,” 2019.
[12] Y. Chen and B. Zhang, “Learning to solve network flow problems via neural decoding,” 2020.
[13] H. I. Calvete, C. Galí, and P. M. Mateo, “A new approach for solving linear bilevel problems using genetic algorithms,” European Journal of Operational Research, vol. 188, no. 1, pp. 14 – 28, 2008. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0377221707003773.
[14] H. Li and L. Fang, “An evolutionary algorithm for solving bilevel programming problems using duality conditions,” Mathematical Problems in Engineering, vol. 2012, 01 2012.
[15] A. Sinha, P. Malo, and K. Deb, “Efficient evolutionary algorithm for single-objective bilevel optimization,” 2013.
[16] S. Pineda, H. Bylling, and J. Morales, “Efficiently solving linear bilevel programming problems using off-the-shelf optimization software,” Optimization and Engineering, vol. 19, no. 1, pp. 187–211, 2018.
[17] L. Hallbäck, F. Thams, A. Venzke, S. Chatzivasileiadis, and P. Pinson, “Data-driven security-constrained ac-opp for operations and markets,” in Proceedings of 20th Power Systems Computation Conference. United States: IEEE, 2018, 20th Power Systems Computation Conference, PSCE 2018 - Conference date: 11-06-2018 Through 15-06-2018. [Online]. Available: http://www.pscce2018.net/index.html.
[18] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, “Mat-power: State-of-the-art hardware platforms, options and analysis tools for power systems research and education,” IEEE Transactions on Power Systems, vol. 26, no. 1, pp. 12–19, Feb 2011.
[19] “Online appendix (datasets),” https://zenodo.org/record/4081513#.X4SfvdAzzNl.
[20] “Online appendix (code),” https://github.com/eleaprat/Bilevel---Active-Constraints

[21] T. Kleinert, M. Labbé, F. Plein, and M. Schmidt, “There’s No Free Lunch: On the Hardness of Choosing a Correct Big-M in Bilevel Optimization,” Jun. 2019, working paper or preprint. [Online]. Available: https://hal.inria.fr/hal-02106642

[22] S. Pineda and J. M. Morales, “Solving linear bilevel problems using big-mds: Not all that glitters is gold,” IEEE Transactions on Power Systems, vol. 34, no. 3, pp. 2469–2471, 2019.

[23] L. van der Maaten and G. Hinton, “Visualizing data using t-sne,” Journal of Machine Learning Research, vol. 9, pp. 2579–2605, 11 2008.