Finite Element Analysis of the thermo-Mechanical Behavior of composite Pipe Elbows under Bending and Pressure loading

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ABSTRACT. Elbows under load experience more stress and strain than straight tubes. The strength of the composite tubes plays an important role in their use, the damage under a thermomechanical behavior of a composite tubular structure bent between two linear parts is studied in this work. The HASHIN criterion model is used through finite element analysis. The main objective is to predict the effect of the main parameters by curves of torque, using the calculation code ABAQUS. These evaluative parameters are addressed to the geometrical conditions of the elbow, in loading mode on the pressurized structures and to the danger of the defect, hence the advantage of using Shell elements as a structure. The numerical results obtained illustrate that the parameters studied condition the level and the mode of failure as well as the response of composite elbows.

KEYWORDS. FEM; Damage; Elbows; Hashin criterion; Shell element.

INTRODUCTION

Composite materials used in piping design in industries such as nuclear power plants, oil-gas, energy and environment, are imposed by their operating ranges from -30°C to 120°C and 8 at 15 bars. This also depends on chemical constraints such as corrosion. Their normalization remains difficult given these numerous dependencies. Homogenization models remain the appropriate approach to estimate them from these characteristics. Several important elements in the mechanical analysis of composite materials such as folds that must comply with the surface curvature, dimensioning such as thickness, diameter and radius of curvature have known several international
standards such as ASME16.9 [1-3] and the European standard EN 10253- [4] based on experimental tests and analytical calculations. In view of the geometrical configuration of the elbow, the circulating fluid submits it to bending moments. Several researchers are studying the resistance of composite tubes such as Lee et al [5] and Sivakumar Palanivelu et al [6]. They found that its resistance depends on the fiber in its quality, are orientation and its fraction. Research on the combined influence of internal pressure and bending in composite tubular structures has found its way through the work of Natuski et al [7] on the flexural strength of composite tubes. Experimental tests done by Kitching et al [8] on composite bends subjected to bending with and without pressure. A comparison also made by Kochehseraaii and. Al. [9] which ends with a good agreement between the experimental and the finite element analysis on a composite tubular structure subjected to a combined loading.

The elbow in its geometric configuration will cause a variation of stress along its intrados and the extrados from where the analytical solutions that are possible to be implemented [10]. Several loading conditions are possible giving rise to several types of damage. The study of these tubular composite structures by thermal effect have taken the interest of other researchers as well, Shao [11] evaluated the thermal stresses as well as Kandil et al. [12] by the numerical model. Others such as Bakaiyan et al [13-16] on composite tube responses to combined internal pressure and thermomechanical loading. By their presence, the defects and the temperature, strongly and geometrically destabilize the bends in their resistance, ex; loading mode or misalignment of fibers or orientation anomalies [17]. Matrix cracking and delamination are the main modes that cause the fiber to break [18] and subsequently lead to complete damage.

In numerical calculation and composites, most researchers are based on two energy methods. The virtual crack closure technique (VCCT) such as the work of Wimmer et al. [19, 20] on initiation and propagation of delamination and cohesive zone method (CZM) by Gözdüklü and Coker [21,22] who studied the same phenomenon. However, other models of damage exist. The one used by Garnish et al [23] proposed by Padhi et al [24] where the structure rigidity vanishes entirely at the beginning of the failure. It is identified according to the failure criteria of Tsai-Wu [25]. Given these numerous advantages, the Hashin criterion [26] is widely used in composite damage [27]. This criterion uses six modes of failure between fiber and matrix and the separation between them [28]; it is presented by ultimate constraints. The purpose of the present work using the Hashin criterion is to predict the damage under internal pressure and temperature of composite tubular structures caused by a bending moment and by the presence of defect in its quality and location as parameter dangerousness.

**Hashin criterion and input parameter**

The Hashin criterion is implemented in the standard Abaqus calculation code [29]. The input data given in Tab.(1) are: longitudinal tensile and compressive strengths, transverse tensile and compressive strengths, and longitudinal and transverse shear strengths. All resistance values are assumed to be positive [30]. The damage is continuous in the structure by degradation of rigidity or by removal of the elements. In composites the damage is multimodal, that of fiber and or matrix. In the Hashin criterion, the damage is presented by the following forms:

1. Tensile fiber failure for \( \sigma_{11} \geq 0 \)

\[
\frac{\sigma_{11}}{X_T} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} = \begin{cases} 
\geq 1 & \text{failure} \\
< 1 & \text{no failure}
\end{cases}
\]

(1)

2. Compressive fiber failure for \( \sigma_{11} < 0 \)

\[
\frac{\sigma_{11}}{X_C} = \begin{cases} 
\geq 1 & \text{failure} \\
< 1 & \text{no failure}
\end{cases}
\]

(2)

3. Tensile matrix failure for \( \sigma_{22} + \sigma_{33} > 0 \)

\[
\frac{(\sigma_{22} + \sigma_{33})^2}{Y_T^2} + \frac{\sigma_{23}^2 - \sigma_{23} \sigma_{33}^2}{S_{23}^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} = \begin{cases} 
\geq 1 & \text{failure} \\
< 1 & \text{no failure}
\end{cases}
\]

(3)

4. Compressive matrix failure for \( \sigma_{22} + \sigma_{33} < 0 \)
The simulations made are more reliable through the use of shell elements because there is no interface between the folds or between the fibers and the matrix. The numerical computation parameters have been in agreement with the computational convergence, which allows us more to go towards the evaluation of the quality of defect. These defects are hardly supported by numerical calculations using other criteria. The parameters introduced in the ABAQUS calculation code are:

** MATERIALS **

*Material, name=Material-1
*Damage Initiation, criterion=HASHIN 2050.,1200., 62., 190., 81., 81.
*Damage Evolution, 45., 45., 0.6, 0.6
*Damage Stabilization 0.003,0.003,0.003,0.003
*Elastic, CONSTANTS
170000.,9000.,9000., 0.34, 0.34, 0.34,4800.,4800.4500.,
*Expansion,
0.1E-6, 0.1E-6, 0.1E-6

3. Description of model geometry and material properties

The geometries of the composite structure with these standardized dimensions [31] are illustrated in Fig. (1). These characteristics of elasticity and resistance are reported in Tab.(1): The elbow element is connected to two straight pipes 1100 mm long. This length is sufficient to ensure that no stress interference in the region of the ends of the elbow will occur. It is assumed that no failure occurs at the straight pipe which should only act as a means of uniformly transferring the bending moments to the bends. Numerically the load of bending moment on the elbow was obtained by imposing a rotation about the axis perpendicular to the plane of curvature of the elbow.
Composite tubular structures having the following dimensions: a diameter equal to 50 mm and a thickness equal to 2 mm. The three elbows have the same radius of curvature equal to 20 mm. Only one elliptical defect shape with one dimension is taken for each case. In order to better compare the damage results, these elbows are uniformly subjected to an imposed rotational movement of 60°. In order to cause only the damage, the defect is always in these three locations in the extrados of the elbow Fig. (5).

Figure 2: Schematic representation of flexure moment in (a) in-plane closing moments, (b) in-plane opening moments reverse ovalization, and (c) out-of-plane bending

For all the situations studied, the loading conditions are as follows: the attachment is always the same at the end of the linear part of the tube. This consists in fixing the three displacements following (x, y and z) and in the other end of the linear part of the tube, a rotation according to the studied case in the direction of opening or closing of the elbow. This rotation takes place around the plane of the structure until its damage, except for the case of moment out plans. It is reported that all the structures studied are under pressure with the presence of temperature in the form of a flow passing through the structure. The properties of the carbon / epoxy are chosen from the experimental work of Auwal Muhammad [32], which allowed us to use these stiffness and resistance parameters. These parameters characterize the architecture of a composite with a 45° crossover fabric orientation and with helical advancement along the longitudinal axis of the elbow and the straight portions.

| $E_1$ = 170000MPa | $v_{12}$ = 0.342 | $G_{12}$ = 4800MPa | $X^T$ =2050 | $Y^T$=62 | $\alpha_{11}$=0.1x10⁻⁶K⁻¹ |
|-------------------|-----------------|-----------------|-----------|--------|-----------------|
| $E_2$ = 9000MPa   | $v_{13}$ = 0.342 | $G_{13}$ = 4800MPa | $X^C$=1200 | $Y^C$=190 | $\alpha_{22}$=0.1x10⁻⁶K⁻¹ |
| $E_3$ = 9000MPa   | $v_{23}$ = 0.342 | $G_{23}$ = 4500MPa | $S^L$ =81 | $S^T$=81 | $\alpha_{33}$=0.1x10⁻⁶K⁻¹ |

Table 1: For a volume fraction of 65%

E: Young's modulus, $v$: Poisson’s ratio, G: shear modulus, $\alpha$: orthotropic expansion [33]
$X^T$: Longitudinal tensile strength, $X^C$: Longitudinal compressive strength, $Y^T$: Transverse tensile strength
$Y^C$: Transverse compressive strength, $S^L$: Longitudinal shear strength, $S^T$: Transverse shear strength

Figure 3: Architecture and Schematizations of the local marks for the composite used [31].

The properties resulting from the experimental used by Auwal Muhammad [32], are for a straight tube. However, for the composite architecture at the elbow, it was provided by a local landmark that follows the curvature of the elbow. To evaluate
the behavior of our damaged structure, we use "Assigning Material Orientation". This technique can be done either by "Create Geometric Part" or by "Create Mesh Part" in order to sweep the structure by local marks that indicate the orientation of the fibers, see Fig. (3).

The geometric effect of the elbow on the damage of the structure was limited only on the choice of three angles 30°, 60° and 90° for the same architecture of the composite and each different case (default).

In this analysis, the mesh of the structure is refined around the defect zone in order to better capture the damage zone in a precise manner. The linear behavior of the composite was presented using S4R shell elements with geometrically zero thickness introduced into the properties of the composite. The number of elements used in the structure depends solely on the angle of the elbow. A number of elements of 8484 is taken for the elbow of 30° and 9150 elements for the elbow of 60° and 11270 elements for the elbow of 90°. Fig. (4) shows a detailed example of the mesh used for the calculations.

RESULTS AND ANALYSIS

The typical configuration of bends in tubular systems is widely used. The elbows are much more stressed to the loadings than straight tubes hence the importance of studying their behavior and their damage. By the presence of the defects and under the loading in compression and in bending moment, the damage of these elbows is quickly favored. These defects by their quality and their location in certain configurations, condition the level and the mode of the localized damage. Our problematic in this work is to identify the effect of defect in the various cases of figure.

Our study is to analyze the elbow by three different locations of the defect: in the extrados of the elbow, in the middle or near the end or at the end and by four qualities of defect; the defect according to the whole thickness, half outside or inside and at the end of both sides. Numerically one took several folds with the same orientation of the fibers in order to select the zone of defect to be modified. This modification consists of the cancellation of the continuity of the fibers by a significant decrease of these values of rigidity and resistance. This numerical computation modality has given us the
advantage with the reliability of creating defects in complex geometric and loading situations without a convergence problem.

Figure 6: Damage moment–rotation curve with effect of pressure values in the case of all thickness section type and middle default position in a) 30° b) 60° c) 90° angular elbows composite structure under out-of-plane bending moment.

Effect of the pressure with different angular elbows and loading condition on the failure of structure

The out-of-plane flexural response of a composite elbow is an important research focus in pipe analysis, but with less attention in the literature. Temperature and internal pressure are no longer comparative parameters because they are present
in all calculations. But depending on the conditions in which they occur, they affect the results. On one side the temperature weakened the structure by accumulating the charges and on the other side the pressure opposes the ovalization. Only the presence of fault conditioned by its quality and location, bending mode and elbow angle, are comparative parameters, hence the resistance capacity of the structure depends on it. The response and the evolution of the system to thermomechanical damage under the evaluative parameters are presented hereinafter by the curves of moments-rotation.

Figs. (6 a and b) show that the bend response at bending moment is initially linear and identical for all cases, and take slightly different values just before the maximum moment. This difference is much more caused by the angle of the elbow than by the internal pressure. Figs. (6) also show that during loading, the internal pressure does not delay the damage, in that the bending moment is out of plane. The mode of damage in the out-of-plane bending as shown in Fig. (7), corresponding to a large compression of the matrix on one of the transverse sides of the bend, and tension on the opposite side. The presence of defect has always played the role of amplifying the damage in all the different cases studied.

The out-of-plane bending moment causes twists which later give rise to small, 45° oriented wrinkles. This Hashin damage presentation showed us the zone of initiation of damage in compression and tension for the case of fiber and matrix. The dimensionless value 1.00 corresponds to the total damage. In the Hashin criterion, the damage is done by degradation of the rigidity or by the complete suppression of the elements which satisfy the value 1.00 of the structure. It can be seen in all the structures that localized fiber damage occurs only after extensive damage to the matrix in tension and compression. According to the bending mode, the damage is caused either by only the defect (bending in closing) or by the flattening of the cross section of the bend (bending in opening) or by both at the same time as shown in the Figs. (7) (bending out of planes).

Figure 7: Hashin damage representation in structure with elbows of 60° under out-of-plane bending moment.

Under the damage by the moment of opening and under the same conditions of internal pressure and temperature, the numerical predictions presented by the Fig. (8) show that the level of damage of the structures is conditioned much more by the type of the moment bending only by the effect of internal pressure and the angle of the elbow. And if we compare the response of the structures to the opening moments, they are very different from those which are submitted to the bending moments out of planes. The response to the bending moments applied is the flattening of the cross section of the bend which takes an orientation in the direction of the bending plane. The mode of elbow failure occurs by a local fiber tension at the central cross section of the elbow. At this point, the critical moment for the 60° and 90° elbows is 52% and 28% less than the 30° elbow, respectively, see Fig. (8). There is a very weak pressure effect dominated by the rigidity of the composite that appears weakly for structures with a 90° elbow. This effect on response increases as the moment approaches its critical value, as shown in Fig. (8-c). Load responses have almost the same trends as in Fig. (6).

It is found in Fig. (10) the same responses up to the critical moment with slightly different levels for the three bends of 30°, 60° and 90°. The structure with the 30° elbow allows a large deformation capacity and a very low pressure effect. Excessive ovality is observed in the case where the structure is subjected to closing bending moments. The flattening is perpendicular to the plane of flexion. The response to the loading of structures leads to the mechanism of their rapid damage due to the additive effect of applied temperature. Failure, therefore, occurs when the composite is solicited by its matrix at high voltages at the extrados of the elbow and by these fibers in tension at the finely localized defect. Localized damage caused by defect positioning may not occur, or with the effect of pressure and temperature. This is illustrated in Fig. (11). The presence of internal pressure has a positive effect on the flexural deformation capacity; it allows the deformation in rotation without breaking because it prevents the structure from becoming ovalized, may remain very weak by the fact that the composite behaves linearly until it is damaged.
Figure 8: damage moment–rotation curve with effect of pressure values in the case of all thickness section type and middle default position in a) 30° b) 60° c) 90° angular elbows composite structure under opening moment.

Figure 9: Hashin damage representation in structure with elbows of 60° under opening moment.
Figure 10: Damage moment–rotation curve with effect of pressure values in the case of all thickness section type and middle default position in a) 30° b) 60° c) 90° angular elbows composite structure under closing moment.

Figure 11: Hashin damage representation in structure with elbows of 60° under closing moment.
In order to better compare and group all the effects and their interdependence on the critical bending moment of each situation, this Fig. (12) has been recapitulated under the condition of the same levels of internal pressure and applied temperature. The composite structure does not allow large deformations (ovalization), hence the weak effect of the internal pressure in certain situations. By purely geometrical effect of the elbows and their ovalization in their cross section, the fibers of the composite are subjected locally to the tensions and others to the compressions. The damage to the elbow is more conditioned by the angle than by the bending mode, hence the critical moments observed in the structure with a 30° elbow. Add to this the effect of location of the defect that approaches critical areas of the elbow (tension or compression).

**Figure 12: Recapitulative of damage in**

- a) out-of-plane bending moment,
- b) opening moment,
- c) closing moment with effect of angular elbows composite structure and default position under thermo-mechanical loaded condition.

**Effect of the emplacement and default type with different angular elbows and loading conditions on the failure of structure**

In this part of this study, our work focuses on the response to the damage of various structures by effect of location and quality of defect under the presence of temperature and internal pressure. The presence of defect in itself has a significant effect according to the conditions of our structures in which they are subjected. According to the composite used in its architecture presented by these break parameters that allow several modes of failure, the defect took throughout this study a single form with a single dimension \(a / b = (6/3)\). The defect zone was presented by a low resistance to damage, which has the effect of destabilizing the structure differently in three selected locations; that in extrados of the elbow in the middle (type 1) near the end (type 2) and at its end (type3). The quality of the defect is illustrated in the Fig. (5) which is presented only according to the thickness by: the defect is following the whole thickness, the half outside or inside and with the end of both sides.

The causes of damage in the presence of a defect in the composites of our structure subjected to both the internal pressure and the bending moment are manifold. The effect of defect quality is important under torsion caused by out-of-plane bending moment. The thickness of the elbow is heavily stressed in tension and compression, hence the effect of defect...
quality which results in critical moment values up to 200Nm difference. Fig. (13) shows that in the case where the defect is in the middle of the thickness of the elbow, failure is more favored. This one is much more stressed with the torsion forces which damage the fibers after the total damage of the matrix of the composite than in the case where the defect is in the free surface. Because the latter is absorbed by the angular displacements. The effect of the location of defects on the response and the critical moment in bending out of planes depends on the approach towards the zones of tension or excessive compression from where the disordered one illustrated in the Fig. (13).

Figure 13: Damage moment-rotation curve under hors plans moment in the case of a) middle position with effect of their type and in the case of b) all thickness section default with effect of their position.

Figure 14: Hashin damage representation in structure with elbows of 60° under hors plans moment, type (3).
The defect quality effect is conditioned by its location, in the case of closing bending moment as the opening case, its quality does not have much influence on the level and the structure response up to their damage. This situation is explained by the concentration of efforts on this area. By the fact that the defect is always at these locations in the extrados of the elbow, its dangerousness increase according to the mode of bending, in other words the fibers of the composite are in tension or in excessive compression by the ovalization of the elbow. These fibers will strongly react with the approximation of defects, which is the case for the closing bending moment, particularly the defect in the middle and the one near the end of the elbow or the critical moment's levels. Is weak with a value close to 9000Nm.
As in the previous section, in the case of closing bending moment, the effect of defect quality is not important. The same response is always obtained with the same level of critical moment of flexion in opening. The defect by its position is always stressed in excessive compression, which is why the damage occurs at low values of the critical moment. Fig. (17) shows that the defect location in the composite elbow with the presence of pressure and that of temperature has a significant effect on the structure response and on the critical moment level for damage to the moment case. Bending in opening, with values shifted up to 3000Nm.

Figure 17: Damage moment-rotation curve under opening moment in the case of a) middle position with effect of their type and in the case of b) all thickness section default with effect of their position.

Figure 18: Hashin damage representation in structure with elbows of 60° under opening moment, type (2).
Fig. (19) presents a series of calculations made for different cases of structure, under the same levels of internal pressure and applied temperature. A summary takes place to clarify the effect of both location and quality of defect on the critical moment of each situation. The structure beneath the out-of-plane bending moment shows that the defect quality effect appears slightly in each location, which is not the case for the other two situations of bending moment (in closing and opening), where the quality effect is the same. In all three bending modes, the location has a critical time effect. This effect occurs with significant levels for the fault location near the end, which is where critical areas of tension or fiber compression in the 30° elbow occur.

CONCLUSION

This work has well presented the effectiveness of the damage criterion under complex geometric and loading conditions and in composite structures of a tubular model connected by elbow in the middle. The results obtained allowed us to evaluate and compare various parameters influencing the damage of our structure to study.

• The Hashin criterion technique was used to estimate the critical moment value of the composite pipeline structure under thermomechanical behavior using shell elements.
• Pressurized tubular composite structures can support the same loading with different deformation capabilities.
• The presence of a defect aggravates the damage by its quality and is location and following the mode of bending moment applied. In the tubular structures the internal pressure opposes the ovalization during loading in bending moment.
• The bending mode undergone by the elbow conditions the ovalization planes and its capacity supported by the fibers is dependent on the angle of the elbow.
• The fault approach to areas of ovalization of tension or compression locates and quickly promotes damage to the elbow.
• The deformation in most of the studied structures occurs strongly and rapidly by temperature effect at two locations caused by the flattening of the cross-section.

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