Numerical Simulation and Hopf Bifurcation of Flutter-Type Oscillation of Two-Dimensional Blade

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Abstract. A numerical method is proposed to simulate the Flutter-type oscillation of the two-dimensional blades in a flow with low speed. The finite element method is used to solve numerically the Laplace equation, and then the aerodynamic forces can be obtained using the unsteady Bernoulli equation. A two-degree-of-freedom dynamic model is introduced to describe the blade oscillation, and Runge-Kutta method is applied to solve the equation of motion. The coupled fields can be solved alternately, and the oscillation orbit of the two-dimensional blade can be obtained. Furthermore, the results are presented in phase plane and studied based on Hopf bifurcation. The influence of the flow velocity on the blade flutter is studied, and it can be concluded that the appearance of flutter-type oscillation is the result of the occurrence of Hopf bifurcation, as the flow velocity increases.

1. Introduction

It can be seen from many reports that the fracture of blades of wind power generator and large capacity turbo-machinery occur even in flow with low speed. One main reason of such phenomenon is the flutter-type oscillation. Flutter-type oscillation is a kind of flow-induced vibration, which is caused by the coupling interaction of aerodynamic, inertial and elastic forces. Aeroelastic experiment is an effective way of studying such self-excited vibration, and the results can be used directly in the design procedure of real blade. However, the experiment is usually costly, and it is infeasible for conditions with high pressure as well as high temperature. Numerical simulation is another available way for the study of flutter-type oscillation. Compared with experiment method, it has lower cost, less time-consuming and has the advantage of good repeatability.

Recently, much work has been carried out for the numerical simulation of flutter-type oscillation [1-5]. In these papers, for the sake of simplification, a number of assumptions are introduced. For example, the flow is usually assumed to be two-dimensional, sometimes inviscid and incompressible, and the blade is straight, the vibration of which is restricted in pitch and plunge directions. Euler/Navier-Stokes equations are used to obtain the unsteady aerodynamic forces, and a high accuracy can be obtained, using high accuracy algorithms and modern grids generation technologies. For flutter-type oscillation in flow with low speed, the existing methods are available theoretically. However, for low velocity, further simplification can be introduced to the flow model. So it will be somewhat bothered to use Euler or Navier-Stokes equations.

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The purpose of this paper is to propose an efficient numerical method to simulate the flow-induced vibration of the straight blade in flow with low speed. In this paper, the influence of viscosity and compressibility are neglected. Hence, Laplace equation, in term of velocity potential, and unsteady Bernoulli equation are available and feasible. Then, based on unstructured triangular grids, finite element method is applied to compute the numerical solution of aerodynamic forces. To depict the vibration of a straight blade, an existing two-degree-of-freedom structural dynamic model is used, and the equation of motion is solved by Runge-Kutta method. Calculating flow and structure equations alternately, the simulation of the blade flutter is carried out.

2. Dynamic models
For flutter-type oscillation of a blade in flow with low speed, some assumptions are introduced as follows,

(1) Since the flow velocity is low, namely, Ma<0.3, the incompressibility hypothesis is satisfied.
(2) For the sake of simplification, the flow is assumed to be inviscid.
(3) Two-dimensional blades are selected, and their motion is restricted in plunge and pitch directions.

2.1. Model of flow field
With simplifications given before, the flow field around an oscillating blade can be described by a model with two-dimensional flow around an airfoil, as shown in Fig. 1. In this model, domain boundaries $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ should be placed far enough from the airfoil in order to satisfy the undisturbed flow conditions. Furthermore, the no-penetration boundary condition is applied on the surface of the airfoil $\Gamma_4$. Since the internal boundary $\Gamma_4$ is an airfoil with lift, a circulation $\Gamma$ around the airfoil exists, which means that the velocity potential is multi-valued in the flow field. In order to obtain a unique solution, a pair of infinite close boundaries, e-f and g-h, are introduced behind the airfoil tip, as shown in Fig.1. There is a jump of velocity potential across this splitting boundary, and the value is $\Gamma$[6].

![Fig. 1 Solution domain for flow around an airfoil](image)

Taking velocity potential $\phi$ as the primary variable, the Laplace equation for incompressible, irrotational flow with two-dimension is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$  \hspace{1cm} (1)

The boundary conditions are

$$\frac{\partial \phi}{\partial n}|_{\Gamma_1} = -u, \quad \frac{\partial \phi}{\partial n}|_{\Gamma_2} = u, \quad \frac{\partial \phi}{\partial n}|_{\Gamma_3} = 0, \quad \frac{\partial \phi}{\partial n}|_{\Gamma_4} = v_n, \quad \phi|_{\Gamma} - \phi|_{\phi} = \Gamma.$$  \hspace{1cm} (2)
where \( u \) is the velocity of free stream, and \( v_n \) the normal velocity of airfoil surface\[7\]. The reference point of the velocity potential is point \( a \), the velocity potential of which is supposed to be zero.

The flow is unsteady because of the vibration of airfoil. Hence, the unsteady Bernoulli equation is applied to calculate the pressure distribution \[8\]. That is

\[
\frac{\partial \phi}{\partial t} + \frac{dp}{\rho} + \frac{1}{2} \nabla \phi \cdot \nabla \phi - G = F(t),
\]

where \( F(t) \) is the unsteady Bernoulli constant, which is constant in the entire flow field at any instant. For the sake of simplification, the flow density is assumed to be constant, and the influence of the gravity is neglected. Then, Eq.(3) can be reduced to a simpler form

\[
\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2} \nabla \phi \cdot \nabla \phi = F(t).
\]

### 2.2. Model of the structural dynamics

Figure 2 presents a simplified structural dynamic model of a straight blade \[9\]. In this model, the vibration is restricted in plunge and pitch directions. Springs \((k_y, k_\theta)\) and dampers \((D_y, D_\theta)\) in two directions are attached to the rotating centre, which is at quarter of the blade chord \((c)\).

![Model of the structural dynamics](image)

Fig.2 Model of the structural dynamics

The two-degree-of-freedom equations of motion of this structural dynamic model \[10\] are given as

\[
\ddot{y} - \left( \frac{S_y}{m} \right) \dot{\theta} + 2 \zeta_y \omega_y \dot{y} + \omega_y^2 y = \frac{L(t)}{m},
\]

\[
\ddot{\theta} - \left( \frac{S_\theta}{I_\theta} \right) \dot{y} + 2 \zeta_\theta \omega_\theta \dot{\theta} + \omega_\theta^2 \theta = \frac{M(t)}{I_\theta},
\]

where \( m \) is the total mass of the straight blade, \( I_\theta \) the mass moment of inertia about the rotating center, \( S_y/m \) the static unbalance distance between gravity center and rotating center, \( L \) and \( M \) the aerodynamic lift and moment. Furthermore, \( \zeta \) and \( \omega \) stand for damping ratio and natural frequency, which are defined by

\[
\omega_y = \sqrt{\frac{k_y}{m}}, \quad \omega_\theta = \sqrt{\frac{k_\theta}{I_\theta}}, \quad \zeta_y = \frac{D_y}{2m\omega_y}, \quad \zeta_\theta = \frac{D_\theta}{2I_\theta\omega_\theta}.
\]
3. Numerical simulation

3.1. Finite element method for flow field
Finite element method presented by E. Baskharone and A. Hamed [6] for steady flow is extended to unsteady conditions, namely, the boundary-value problems described by Eqs.(1) and (2). The solution domain is discretized by Delaunay method, and the linear shape function is applied based on unstructured triangular grids. Galerkin method of weighted residuals is used to solve the Laplace equation, and the final linear algebraic equations are solved by Gauss method.

Based on solution of velocity potential from finite element method, the unsteady Bernoulli equation can be used to calculate the pressure distribution. At each grid node, the discretized form of Eq.(3) is

$$\frac{\phi_{n+1} - \phi_n}{\Delta t} + \frac{P_{n+1}}{\rho} + \frac{1}{2} \nabla \phi_{n+1} \cdot \nabla \phi_{n+1} = F_{n+1},$$

(5)

where superscripts $n$ and $n+1$ refer to successive instant $t$ and $t+\delta t$ respectively. $F_{n+1}$ is the unsteady Bernoulli constant at time $t+\delta t$ and can be presented based on flow parameters at reference point $a$. Then, the aerodynamic lift $L$ and moment $M$ at time $t+\delta t$ can be obtained by integrating flow pressure along the airfoil surface.

A program is developed to generate the unstructured grids, based on Delaunay triangulation method [11]. Importing coordinates of boundary nodes to this subprogram, the domain will be discretized automatically. Since $\Gamma_4$ is a moving boundary, the grids will be regenerated at each calculating time.

3.2. Numerical simulation of the structure dynamic model
Let $y_i = \dot{y}, \theta_1 = \dot{\theta}$. Eq. (4) can be rewritten as

$$\begin{align*}
\dot{y} &= y_1, \\
\dot{\theta} &= \theta_1, \\
y_1 &= (S_t M(t) + I_{\rho} L(t) - 2m_l \omega^2 y_1 - 2I_0 \omega \omega \theta_1 - m_l y \omega^2) \left( m_l - S_{\theta}^2 \right), \\
\dot{\theta}_1 &= (m M(t) + S_t L(t) - 2m S_{\theta} \omega \omega \theta_1 - m S_{\theta} \omega^2 y - m \omega^2 \theta_1) \left( m_l - S_{\theta}^2 \right),
\end{align*}$$

(6)

which is a group of first-order ordinary differential equations. We assume that $L$ and $M$ are invariable in the sufficient small time step $\delta t$. Then, Runge-Kutta method is available to approach Eq.(6), and vibration variables $y, y_1, \theta$ and $\theta_1$ at time $t+\delta t$ can be obtained.

Fig. 3 Unstructured triangular grids of the solution domain
4. Results and discussion

Flow-induced vibration of a straight blade is simulated. The blade is 1m long in span direction, and 0.0824m in chord. Fig.3 presents the unstructured triangular mesh of the flow field, which has 447 grid nodes and 745 triangular elements.

The time step of the numerical simulation is defined by \( \delta t = T/40 \), where \( 1/T \) is the natural frequency of the model of structural dynamic. The velocity of the free stream is increased from 10m/s to 100m/s. At each computation velocity, 4000 time steps are calculated. More parameters are shown in Table 1.

| \( \rho_\infty \) (kgm\(^{-3}\)) | \( S_0/m \) (m) | \( I_\theta \) (kgm\(^2\)) | \( M \) (kg) | \( c \) (m) | \( \omega_y \) | \( \zeta_y \) | \( \omega_\theta \) | \( \zeta_\theta \) |
|---|---|---|---|---|---|---|---|---|
| 1.186 | 0.0151 | 2.782E-3 | 4.534 | 0.0824 | 100 | 0.05 | 100 | 0.05 |

Figure 4 shows the phase portraits of flow-induced vibration of the straight blade at some flow velocities. In Fig.4(a), for flow with lower velocity, there exists stable focus in phase plane, which indicates that the dynamic system will be asymptotically stabilized to an equilibrium position after a transient process.

However, with the increase of the velocity of free stream, the topology of the equilibrium position changes significantly. Stable focus becomes unstable, and limit cycle appears in phase plane, as shown in Fig.4(b). This is a Hopf bifurcation if the flow velocity is taken as the bifurcation parameter.

Increase the flow velocity further. The limit cycle oscillation loses its stability. A chaotic like motion happens, which can be seen from Fig.4(c). Although the phase portraits in plunge and pitch directions are disorder, the oscillation is restricted in a limited area.

However, in Fig.4(d), as flow velocity exceeds 50m/s, the vibration of the blade is out of control due to its large amplitude. In this case, the vibration amplitude will increase continuously along, which means violent oscillations occur in plunge and pitch directions.

![Fig.4 (a) Phase portraits in plunge and pitch directions when \( u=20 \) m/s](image1)

![Fig.4 (b) Phase portraits in plunge and pitch directions when \( u=37.5 \) m/s](image2)
Fig.4 (c) Phase portraits in plunge and pitch directions when $u=40$ m/s

Fig.4 (d) Phase portraits in plunge and pitch directions when $u=50$ m/s

5. Conclusions
A numerical method has been proposed to simulate the flow-induced vibration of the straight blade in flow with low speed. The flutter-typed oscillation of a straight blade is simulated, and the influence of free stream velocity is investigated. It has been found that the dynamic system has different equilibrium states for different flow velocities. For flow with lower velocity, the vibration is asymptotically stabilized to an equilibrium position, and there are stable focuses in the phase plane. When flow speed is increased, the equilibrium position becomes unstable, and limit cycle oscillation happens, which is a Hopf bifurcation. With further increase of the velocity, the stability of the limit cycle oscillation is lost. Chaotic, violent oscillation appears. Hence, the flutter-typed oscillation is actually the result of the Hopf bifurcation, as the velocity of the flow is increasing.

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