Controller Synthesis for Golog Programs over Finite Domains with Metric Temporal Constraints

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Abstract
Executing a Golog program on an actual robot typically requires additional steps to account for hardware or software details of the robot platform, which can be formulated as constraints on the program. Such constraints are often temporal, refer to metric time, and require modifications to the abstract Golog program. We describe how to formulate such constraints based on a modal variant of the Situation Calculus. These constraints connect the abstract program with the platform models, which we describe using timed automata. We show that for programs over finite domains and with fully known initial state, the problem of synthesizing a controller that satisfies the constraints while preserving the effects of the original program can be reduced to MTL synthesis. We do this by constructing a timed automaton from the abstract program and synthesizing an MTL controller from this automaton, the platform models, and the constraints. We prove that the synthesized controller results in execution traces which are the same as those of the original program, possibly interleaved with platform-dependent actions, that they satisfy all constraints, and that they have the same effects as the traces of the original program. By doing so, we obtain a decidable procedure to synthesize a controller that satisfies the specification while preserving the original program.

1 Introduction
While GOLOG (Levesque et al. 1997), an agent programming language based on the Situation Calculus (McCarthy 1963; Reiter 2001), allows a clear and abstract specification of an agent’s behavior, executing a GOLOG program on a real robot often creates additional issues. Typically, the robot’s platform requires additional constraints that are ignored when designing a GOLOG program. As an example, a robot may need to calibrate its arm before it can use it. One way to deal with such platform constraints is to split the reasoning into two parts (Hofmann et al. 2018): First, an abstract GOLOG program specifies the intended behavior of the robot, without taking the robot platform into account. In a second step, the platform is considered by transforming the abstract program into a program that is executable on the particular platform, given a model of the platform and temporal constraints that connect the platform with the plan.

In this paper, we propose a method for such a transformation: We model the robot platform with a timed automaton (TA) and formulate constraints with \( t\text{-ESG} \) (Hofmann and Lakemeyer 2018), a modal variant of the Situation Calculus extended with temporal operators and metric time. We then synthesize a controller that executes the abstract program, but also inserts additional platform actions to satisfy the platform constraints. To do so, we restrict the GOLOG program to a finite domain, finite traces, and a fully known initial state. This allows us to reduce the controller synthesis problem to the MTL control problem, which has been shown to be decidable (Bouyer, Bozzelli, and Chevalier 2006). Furthermore, for the purpose of this paper, we only use time to formulate temporal constraints on the robot platform and we restrict programs to untimed programs, i.e., in contrast to programs in (Hofmann and Lakemeyer 2018), a program may not refer to time and action preconditions and effects are time-independent. We will revisit these restrictions in the concluding section.

In the following, we first give an overview on the Situation Calculus and GOLOG and related work in Section 2 and summarize \( t\text{-ESG} \) in Section 3. In Section 4, we describe timed automata and Metric Temporal Logic (MTL), before we summarize the MTL synthesis problem. We explain how to transform a GOLOG program over a finite domain with a complete initial state into a TA in Section 5 and how to model a robot platform with a TA and temporal constraints in Section 6. Both TA and the constraints are then used in Section 7 to synthesize a controller that executes the program while satisfying all constraints. We conclude in Section 8.

2 Related Work
The Situation Calculus (McCarthy 1963; Reiter 2001) is a first-order logic for representing and reasoning about actions. Following Reiter, action preconditions and effects as well as information about the initial situation are then encoded as so-called Basic Action Theories (BATS). The action programming language GOLOG (Levesque et al. 1997) and its concurrent variant CONGOLOG (De Giacomo, Lespérance, and Levesque 2000) are based on the Situation Calculus and offer imperative programming constructs such as sequences of actions and iteration as well as non-deterministic branching and non-deterministic choice. The semantics of GOLOG and its
on-line variant INDIGOLOG can be specified in terms of transitions (De Giacomo et al. 2009). The logic $ES$ (Lakemeyer and Levesque 2011) is a modal variant of the Situation Calculus which gets rid of explicit situation terms and uses modal operators instead. The logic $ESG$ (Claßen and Lakemeyer 2008; Claßen 2013) is a temporal extension of $ES$ and used for the verification of GOLOG programs. It specifies program transition semantics similar to the transition semantics of INDIGOLOG and extends $ES$ with the temporal operators $X$ (next) and $U$ (until). The logic $t-ESG$ (Hofmann and Lakemeyer 2018) extends $ESG$ with metric time and timing constraints on the $until$ operator.

MTL (Koymans 1990) is an extension of Linear Time Logic (LTL) with metric time, which allows expressions such as $F_{\leq c}$, meaning eventually within time $c$. In MTL, formulas are interpreted over timed state sequences, where each state specifies which propositions are true, and each state has an associated time value. Depending on the choice of the state and time theory, the satisfiability problem for MTL becomes undecidable (Alur and Henzinger 1993). However, both for finite words and for a pointwise semantics, it has been shown to be decidable (Ouaknine and Worrell 2005; Ouaknine and Worrell 2008).

Similar to the proposed approach, Schiffer, Wortmann, and Lakemeyer (2010) extend GOLOG for self-maintenance by allowing temporal constraints using Allen’s Interval Algebra (Allen 1983). Those constraints are resolved on-line by interleaving the original program with maintenance actions. Closely related is also the work by Finzi and Pirri (2005), who propose a hybrid approach of temporal constraint reasoning and reasoning about actions based on the Situation Calculus. They also allow constraints based on Allen’s Interval Algebra, which are translated into a temporal constraint network. De Giacomo and Vardi describe a synthesis method for LTL and LDL specifications over finite traces (De Giacomo and Vardi 2015). Similar to MTL synthesis, they partition the propositions in controllable and uncontrollable symbols and use games to synthesize a controller. Based on $LTL_f$ synthesis, He et al. describe a synthesis method that controls a robot against uncontrollable environment actions under resource constraints (He et al. 2017). They model the underlying planning problem as a graph, where each vertex describes the state of the world and each edge corresponds to an action, either by the agent or by the environment. In contrast to this work, they do not allow metric temporal constraints.

### 3 Timed $ESG$

In this section, we summarize the syntax and semantics of $t-ESG$ (Hofmann and Lakemeyer 2018), which is based on $ESG$ (Claßen and Lakemeyer 2008) and $ES$ (Lakemeyer and Levesque 2011), modal variants of the Situation Calculus. We refer to (Hofmann and Lakemeyer 2018) for a more complete description.

The language has two sorts: object and action. A special feature inherited from $ES$ is the use of countably infinite sets of standard names for both sorts. Standard object names syntactically look like constants, but are intended to be isomorphic with the set of all objects of the domain. In other words, standard object names can be thought of as constants that satisfy the unique name assumption and domain closure for objects. We assume that object standard names include the rational numbers (including $\infty$) as a subsort. Action standard names are function symbols of any arity whose arguments are standard object names. Examples are $\text{pick}(o)$ and $\text{goto}(t_1, t_2)$ for picking up an object and going from one location to another, respectively. Again, standard action names range over all actions and satisfy the unique name assumption and domain closure for actions. One advantage of using standard names is that quantifiers can be understood substitutionally when defining the semantics. For simplicity, we do not consider function symbols other than actions. Formally the language is defined as follows:

#### 3.1 Syntax

**Definition 1** (Symbols of $t-ESG$). The symbols of the language are from the following vocabulary:

1. object variables $x_1, x_2, x_3, \ldots, y_1, \ldots$.
2. action variables $a, a_1, a_2, a_3, \ldots$.
3. object standard names $N_O = \{ o_1, o_2, o_3, \ldots \}$.
4. action standard names $N_A = \{ p_1, p_2, p_3, \ldots \}$.
5. fluent predicates of arity $k$: $F^k \subseteq \{ F^1_k, F^2_k, \ldots \}$, e.g., $\text{Holding}(o)$; we assume this list contains the distinguished predicate $\text{Poss}$.
6. rigid predicates of arity $k$: $G^k \subseteq \{ G^1_k, G^2_k, \ldots \}$.
7. open, closed, and half-closed intervals, e.g., $[1, 2]$, with rational numbers as interval endpoints.
8. connectives and other symbols: $\land$, $\lor$, $\neg$, $\forall$, $\exists$, $\left[ \right]$, $\llbracket \right]$, $\mathcal{U}$ (with interval $I$).

We denote the set of standard names as $N = N_O \cup N_A$.

**Definition 2** (Terms of $t-ESG$). The set of terms of $t-ESG$ is the least set such that (1) every variable is a term of the corresponding sort, (2) every standard name is a term.

**Definition 3** (Formulas). The formulas of $t-ESG$, consisting of situation formulas and trace formulas, are the least set such that:

1. if $t_1, \ldots, t_k$ are terms and $P$ is a $k$-ary predicate symbol, then $P(t_1, \ldots, t_k)$ is a situation formula.
2. if $t_1$ and $t_2$ are terms, then $(t_1 = t_2)$ is a situation formula.
3. if $\alpha$ and $\beta$ are situation formulas, $x$ is a variable, $\delta$ is a program (defined below), and $\phi$ is a trace formula, then $\alpha \land \beta$, $\neg \alpha$, $\forall x. \alpha$, $\Box \alpha$, $[\delta] \alpha$, and $[\overline{\delta}] \alpha$ are situation formulas.
4. if $\alpha$ is a situation formula, it is also a trace formula.
5. if $\phi$ and $\psi$ are trace formulas, $x$ is a variable, and $I$ is an interval, then $\phi \land \psi$, $\neg \phi$, $\forall x. \phi$, and $\phi \mathcal{U}_I \psi$ are also trace formulas.

A predicate symbol with standard names as arguments is called a primitive formula, and we denote the set of primitive formulas as $P_F$. We read $\Box \alpha$ as “$\alpha$ holds after executing any
sequence of actions”, $[\delta]x$ as “$x$ holds after the execution of program $\delta$”, $[\vec{\delta}]x$ as “$x$ holds during the execution of program $\delta$”, $\phi U_t \psi$ as “$\phi$ holds until $\psi$ holds, and $\psi$ holds within interval $I$”.

A formula is called static if it contains no $[\cdot], \square, \text{ or } [\cdot]$ operators. It is called fluent if it is static and does not mention Poss.

We also write $< c, \leq c, = c, > c,$ and $\geq c$ for the respective intervals $[0, c), [0, c], [c, c], (c, \infty),$ and $[c, \infty)$. We use the short-hand notations $F_1 \phi \overset{def}{=} (\top U_t \phi)$ (future) and $G_1 \phi \overset{def}{=} \neg F_1 \neg \phi$ (globally). For intervals, $c + [s, e]$ denotes the interval $[s + c, e + c]$, similarly for $c + (s, e), c + [s, e], and c + (s, e)$. We also omit the interval $I$ if $I = [0, \infty)$, e.g., $\phi U [0, \infty) \psi$ is short for $\phi U [0, \infty) \psi$.

Finally we define the syntax of GOLOG programs referred to by the operators $[\vec{\delta}]$ and $[\vec{\delta}]$:

**Definition 4 (Programs).**

$$\delta ::= t \mid \alpha ? \mid \delta_1 \delta_2 \mid \delta_1 \parallel x. \delta \mid \delta_1 \parallel \delta_2 \mid \delta^*$$

where $t$ is an action term and $\alpha$ is a static situation formula. A program consists of actions $t$, tests $\alpha ?$, sequences $\delta_1 \delta_2$, nondeterministic branching $\delta_1 \parallel \delta_2$, nondeterministic choice of argument $\parallel x. \delta$, interleaved concurrency $\delta_1 \parallel \delta_2$, and nondeterministic iteration $\delta^*$.

We also use the abbreviation $nil \overset{def}{=} \top ?$ for the empty program that always succeeds. We remark that the above program constructs are a proper subset of the original CONGOLOG (De Giacomo, Lescopérane, and Levesque 2000). We have left out other constructs such as prioritized concurrency for simplicity.

### 3.2 Semantics

**Definition 5 (Timed Traces).** A timed trace is a finite timed sequence of action standard names $w$ with monotonically non-decreasing time. Formally, a trace $\pi$ is a mapping $\pi : N \rightarrow P_A \times Q$, and for any $i, j \in N$ with $\pi(i) = (\sigma_i, t_i), \pi(j) = (\sigma_j, t_j)$, if $i < j$, then $t_i \leq t_j$.

We denote the set of timed traces as $J$. For a timed trace $z = (a_1, t_1) \ldots (a_k, t_k)$, we define $time(z) \overset{def}{=} t_k$ for $k > 0$ and $time(\langle \rangle) \overset{def}{=} 0$, i.e., $time(z)$ is the time value of the last action in $z$. We define the timed trace $z^0$ where all actions occur at time $0$ as $z^0 = (a_1, 0) \ldots (a_n, 0)$.

**Definition 6 (World).** Intuitively, a world $w$ determines the truth of fluent predicates, not just initially, but after any (timed) sequence of actions. Formally, a world $w$ is a mapping $P_F \times Z \rightarrow \{0, 1\}$. If $G$ is a rigid predicate symbol, then for all $z, z' \in Z$, $w[G(n_1, \ldots, n_k), z] = w[G(n_1, \ldots, n_k), z']$.

Similar to $E^*G$ and $E^*G$, the truth of a fluent after any sequence of actions is determined by a world. Different from $E^*G$ and $E^*G$, we require all traces referred to by a world to contain time values for each action. This also means that in the same world, a fluent predicate $F(n)$ may have a different value after the same sequence of actions if the actions were executed at different times, i.e., $w[F(n, \langle a_1, 1 \rangle)]$ may have a different value than $w[F(n, \langle a_1, 2 \rangle)]$. However, for simplicity the actions considered in basic action theories (see Section 3.3) do not make use of this feature.

Next we define the transitions programs may take in a given world $w$. In two places these refer to the satisfaction of situation formulas (see Definition 9 below).

**Definition 7 (Program Transition Semantics).** The transition relation $\Rightarrow$ among configurations, given a world $w$, is the least set satisfying

1. $(\langle z, a \rangle, t) \Rightarrow (\langle z \cdot (p, t), nil \rangle, if \ t \geq time(z), and w, z \models Poss(p)$
2. $(\langle z, \delta_1, \delta_2 \rangle) \Rightarrow (\langle z \cdot (p, \gamma), \delta_2 \rangle, if \ (z, \delta_1) \Rightarrow (\langle z \cdot (p, \gamma), \delta_2 \rangle)
3. $(\langle z, \delta_1, \delta_2 \rangle) \Rightarrow (\langle z \cdot (p, \delta') \rangle, if \ (z, \delta_1) \in F^w$ and $(z, \delta_2) \Rightarrow (\langle z \cdot (p, \delta') \rangle)
4. $(\langle z, \delta_1, \delta_2 \rangle) \Rightarrow (\langle z \cdot (p, \delta') \rangle, if \ (z, \delta_1) \Rightarrow (\langle z \cdot (p, \delta') \rangle$ or $(z, \delta_2) \Rightarrow (\langle z \cdot (p, \delta') \rangle$.

The set of final configurations $F^w$ is the smallest set such that

1. $(\langle z, \alpha ? \rangle \in F^w, if w, z \models \alpha,$
2. $(\langle z, \delta_1 \delta_2 \rangle \in F^w, if (z, \delta_1) \in F^w and (z, \delta_2) \in F^w$
3. $(\langle z, \delta_1 \delta_2 \rangle \in F^w, if (z, \delta_1) \in F^w, or (z, \delta_2) \in F^w$
4. $(\langle z, \parallel x. \delta \rangle \in F^w, if (z, \delta_1) \in F^w for some $i \in N_{x}$
5. $(\langle z, \delta^* \rangle \in F^w$
6. $(\langle z, \delta_1 \delta_2 \rangle \in F^w, if (z, \delta_1) \in F^w and (z, \delta_2) \in F^w$

The program transition semantics is very similar to the semantics of $E^*G$. The only difference is in Rule 1, which has an additional constraint on the time, and which requires the action to be executable.

**Definition 8 (Program Traces).** Given a world $w$ and a finite sequence of action standard names $z$, the set $\parallel \delta \parallel z$ of finite timed traces of a program $\delta$ is

$$\parallel \delta \parallel z = \{z' \in Z \mid (z, \delta) \Rightarrow (z \cdot z', \delta') and (z \cdot z', \delta') \in F^w\}$$

**Definition 9 (Truth of Situation and Trace Formulas).** Given a world $w \in W$ and a situation formula $\alpha$, we define $w \models \alpha$ as $w, \langle \rangle \models \alpha$, where for any $z \in Z$:

1. $w, z \models F(n_1, \ldots, n_k), if \ w[F(n_1, \ldots, n_k), z] = 1$;
2. $w, z \models (n_1 = n_2) if \ n_1$ and $n_2$ are identical;
3. $w, z \models \alpha \land \beta, if \ w, z \models \alpha$ and $w, z \models \beta$;
4. $w, z \models \neg \alpha, if \ w, z \not\models \alpha$;
5. $w, z \models \forall x. \alpha, if \ w, z \models \alpha_n for every standard name of the right sort$;
6. $w, z \models \square \alpha, if \ w, z \cdot z' \models \alpha for all $z' \in Z$;
7. $w, z \models [\delta] \alpha, if \ for all finite $z' \in [\delta]z, w, z \cdot z' \models \alpha$;
8. \( w, z, τ \models [\delta]φ \) if for all \( τ \in [\delta]_{w, z} \).

Intuitively, \( [\delta]α \) means that after every execution of \( δ \), the situation formula \( α \) is true. \( [\delta]φ \) means that during every execution of \( δ \), the trace formula \( φ \) is true.

The truth of trace formulas \( φ \) is defined as follows for \( w ∈ W, z, τ ∈ Z \):

1. \( w, z, τ \models α \) iff \( w, z, τ \models α \) and \( α \) is a situation formula;
2. \( w, z, τ \models φ \wedge ψ \) iff \( w, z, τ \models φ \) and \( w, z, τ \models ψ \);
3. \( w, z, τ \models ¬φ \) iff \( w, z, τ \models φ \) and \( w, z, τ \models ψ \);
4. \( w, z, τ \models ∀x.φ \) iff \( w, z, τ \models φ_n \) for all \( n ∈ N_z \);
5. \( w, z, τ \models φ U_1 psi \) if there is a \( z_1 ⊢ 0 \) such that
   - \( τ = z_1 \cdot τ' \),
   - \( (z_1 + I, 0, τ') \models ψ \),
   - \( w, z \cdot z_1, τ' \models 0 \).

Definition 10 (Validity). A situation formula \( α \) is valid (written \( \models α \)) iff for every world \( w, w \models α \). A trace formula \( φ \) is valid (\( \models φ \)) iff for every world \( w \) and every trace \( τ, w, δ, τ' \models 0 \).

3.3 Basic Action Theories

A basic action theory (BAT) defines the preconditions and effects of all actions of the domain, as well as the initial state:

Definition 11 (Basic action theory). Given a finite set of fluent predicates \( F \), a set \( Σ ⊆ \mathcal{E}_{\mathcal{G}} \) of sentences is called a basic action theory (BAT) over \( F \) iff \( Σ = Σ_0 \cup Σ_{pre} \cup Σ_{post} \), where \( Σ_0 \) mentions only fluents in \( F \) and

1. \( Σ_0 \) is any set of fluent sentences,
2. \( Σ_{pre} \) consists of a single sentence of the form
   \[ \Box Pos(a) \equiv π, \text{where } π \text{ is a fluent formula with free variable } a. \]
3. \( Σ_{post} \) is a set of sentences, one for each fluent predicate \( F' ∈ F \). of the form
   \[ \Box[a] F(x) \equiv γ_{F}. \]

The set \( Σ_0 \) describes the initial state; \( Σ_{pre} \) defines the preconditions of all actions of the domain, and \( Σ_{post} \) defines action effects by specifying for each fluent of the domain whether the fluent is true after doing some action \( a \).

We will also consider BATs restricted to a finite domain of actions and objects:

Definition 12 (Finite-domain BAT). We call a BAT \( Σ \) a finite-domain basic action theory (fd-BAT) iff

1. each \( ∀ \) quantifier in \( Σ \) occurs as \( ∀x.τ_i(x) \equiv φ(x) \), where \( τ_i \) is a rigid predicate, \( i = α \) if \( x \) is of sort object, and \( i = a \) if \( x \) is of sort action;
2. \( Σ_0 \) contains axioms
   - \( τ_n(x) \equiv (x = n_1 \lor x = n_2 \lor \ldots x = n_k) \) and
   - \( τ_a(a) \equiv (a = m_1 \lor a = m_2 \lor \ldots a = m_l) \)
   where the \( n_i \) and \( m_i \) are object and action standard names, respectively. Also each \( m_j \) may only mention object standard names \( n_i \).

We call a formula \( α \) that only mentions symbols and standard names from \( Σ \) restricted to \( Σ \) and we denote the set of primitive formulas restricted to \( Σ \) as \( P_Σ \) and the action standard names mentioned in \( Σ \) as \( A_Σ \). We also write \( \exists x_i.φ \) for \( Σ \cdot τ_i(x) \land φ \) and \( ∀ x_i.φ \) for \( ∀ x_i.τ_i(x) \lor φ \). Since an fd-BAT essentially restricts the domain to be finite, quantifiers of type object can be understood as abbreviations:

\[ \exists x_i.τ_i.φ \equiv \bigwedge_{i=1}^k φ_i, \]

and similarly for quantifiers of type action.

In addition to a finite domain, we also restrict a BAT such that it completely determines the initial situation:

Definition 13 (Determinate BAT). A fd-BAT \( Σ \) is determinate iff every for atomic formula \( φ \) restricted to \( Σ \), either \( Σ_0 \models φ \) or \( Σ_0 \models ¬φ \).

Next, given a world \( w \), we define a world \( w_Σ \) that is consistent with \( Σ \):

Definition 14. For any world \( w \) and basic action theory \( Σ \), we define a world \( w_Σ \), which is like \( w \) except that it satisfies the \( Σ_{pre} \) and \( Σ_{post} \) sentences of \( Σ \).

Lemma 1 ((Lakemeyer and Levesque 2011)). For any \( w, w_Σ \) exists and is uniquely defined.

For a determinate BAT over a set of fluent predicates \( F \), we can show that \( Σ \) fully determines the truth of every fluent \( f ∈ F \), not only initially, but after any sequence of actions:

Lemma 2. Let \( Σ \) be a determinate BAT over \( F, δ \) a program over \( Σ \) and \( w, w' \) two worlds, and \( z ∈ Z \) a finite trace such that \( (⟨⟩, δ) \rightarrow^* (z, δ') \). Then

1. \( (⟨⟩, δ) \rightarrow^* (z, δ') \),
2. for every primitive formula \( F(\vec{f}) \) with \( F \in F; w_Σ[F(\vec{f}), z] = w'_Σ[F(\vec{f}), z] \)

Proof. By induction over the length of \( z \).

1. Let \( z = ⟨⟩ \). By definition of the determinate BAT, we know that \( w_Σ[F(\vec{f}), ⟨⟩] = 1 \iff w'_Σ[F(\vec{f}), ⟨⟩] = 1 \).
2. Let \( z = z' \cdot (p, t) \). By induction, for each atomic formula \( φ \), \( w_Σ[φ], z' \equiv w'_Σ[φ], z' \), and thus, for each fluent situation formula \( γ \), \( w_Σ[γ], z' \equiv γ \iff w'_Σ[γ], z' \equiv γ \). Furthermore, we know from \( (⟨⟩, δ) \rightarrow^* (z, δ') \) that for some \( z'', δ'' \), \( w_Σ[z, δ''], z' \equiv γ \) and \( w'_Σ[z, δ''], z' \equiv γ \). Thus \( w_Σ[γ], z' \equiv γ \). As both \( w_Σ \) and \( w'_Σ \) satisfy \( Σ_{pre} \), it follows that \( w'_Σ[γ], z' \equiv γ \).

We have shown that \( w_Σ[F(\vec{f}), z] = 1 \iff w'_Σ[F(\vec{f}), z] = 1 \).
In fact, we can show that $\Sigma$ fully determines possible traces of $\delta$, as well as the truth of any formula restricted to $\Sigma$:

**Theorem 1.** Let $\Sigma$ be a determinate BAT, $\delta$ a program over $\Sigma$ and $w, w'$ two worlds, and $z \in [\delta][w]$, $\alpha$ a situation formula and $\phi$ a trace formula, both restricted to $\Sigma$. Then:

1. $z \in [\delta][w] \iff [\delta][z] = [\delta][\alpha]$
2. $w \in [\delta][w] \iff [\delta][w'] = [\delta]\alpha$
3. $w \in [\delta][w] \iff [\delta][\phi] = [\delta]\phi$

**Proof.** Follows from Lemma 2.

For the purpose of this paper and in contrast to (Hofmann et al. 2018), we do not have distinguished function symbols now and time that allow referring to time in a situation formula. In particular, this means that we cannot define time-dependent preconditions or effects in a BAT. Thus, time is only relevant for the truth of trace formulas. Also, a program’s traces are not restricted with respect to time:

**Proposition 1.** Given a BAT $\Sigma$, a program $\delta$, and a world $w$. Let $\tau_1, \tau_2$ be two traces with $\tau_1(i) = (a_1, t_i)$, $\tau_2(i) = (a_2, t'_i)$ for every $i$ (i.e., they contain the same action symbols but different time points). Then $\tau_1 \in [\delta][w] \iff \tau_2 \in [\delta][w']$.

A Simple Carrier Bot With the following determinate fd-BAT, we describe a simple carrier bot that is able to move to locations and pick up objects:

$$\square\text{Poss}(a)$$

1. $\exists s: o \exists g: o. a = s_{\text{g}}(s, g) \land \neg\exists a': a. \text{Perf}(a')$ (1)
2. $\forall s: o \exists g: o. a = e_{\text{g}}(s, g) \land \text{Perf}(s, g)$ (2)
3. $\forall s: o, l: o. a = s_{\text{pick}}(o) \land \neg\exists a': a. \text{Perf}(a') \land \text{At}(l) \land \text{At}(o, l)$ (3)
4. $\forall s: o. a = e_{\text{pick}}(o) \land \text{Perf}(\text{pick}(o))$ (4)

The precondition axioms state that it is possible to start the $\text{g}_o$ action if the robot is not performing any action (Equation 1), it can stop the $\text{g}_o$ action if it is currently performing it (Equation 2). Furthermore, it can start picking up an object if it is not performing any other action and it is at the same position as the object (Equation 3). Finally, it can stop picking if it is currently performing a $\text{pick}$ action (Equation 4).

By splitting actions into start and stop actions, we can execute multiple actions concurrently. We will later insert platform actions that are executed in addition and concurrent to the program’s actions. Also, splitting actions into start and stop actions allows us to model that only the start but not the end of an action is under the robot’s control. In Section 7, we will let the environment control all end actions, i.e., the environment will decide when an action ends.

In addition to the precondition axioms, we also define successor state axioms for all fluents of the domain:

1. $\forall s: o. a = e_{\text{g}}(s, l)$ (5)
2. $\forall s: o. a = s_{\text{g}}(s, g')$ (6)
3. $\exists s: o. a = e_{\text{g}}(s, g) \land \text{Perf}(s, g)$ (7)
4. $\exists s: o. a = e_{\text{g}}(s, g) \land \neg\exists s: o. a = s_{\text{g}}(s, g')$ (8)
5. $\forall s: o. a = e_{\text{g}}(s, g) \land \text{Perf}(s, g)$ (9)
6. $\forall s: o. a = e_{\text{g}}(s, g) \land \neg\exists s: o. a = s_{\text{g}}(s, g')$

Listing 1 shows a simple program that picks up one object.

### 4 MTL Synthesis

Timed automata (TA) (Alur and Dill 1994; Alur 1999) are a widely used model for representing real-time systems. Their properties are often described with MTL (Kowman 1990), a temporal logic that extends LTL with metric time. We first summarize timed automata and MTL, and then define the problem of controlling a TA against an MTL specification, following (Bouyer, Bozzelli, and Chevalier 2006; Ouaknine and Worrell 2008).

MTL MTL extends LTL with timing constraints on the Until modality. One commonly used semantics for MTL is a pointwise semantics, in which formulas are interpreted over timed words.

**Definition 15 (Timed Words).** A timed word $\rho$ over a finite set of atomic propositions $P$ is a finite or infinite sequence $(\sigma_0, t_0) \sigma_1, t_1, \ldots$ where $\sigma_i \in P$ and $t_i \in \mathbb{Q}_+$ such that the sequence $(\tau_i)$ is monotonically non-decreasing and non-Zeno. The set of timed words over $P$ is denoted as $TP^*$. For a timed word $\rho = (\sigma_0, t_0) \sigma_1, t_1, \ldots$ and every $k \in \mathbb{N}$ with $k \leq |\rho|$, we also write $\rho_k$ for the prefix $(\sigma_0, t_0) \ldots (\sigma_k, t_k)$.

**Definition 16 (Formulas of MTL).** Given a set $P$ of atomic propositions, the formulas of MTL are built as follows:

$$\phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \phi U \phi$$
We use the same abbreviations as for $t$-$\mathcal{E}SG$, i.e., $\mathbf{F}_I \varphi \overset{\text{def}}{=} (\top \cup \mathbf{U}_I \varphi)$ (future) and $\mathbf{G}_I \varphi \overset{\text{def}}{=} \neg \mathbf{F}_I \neg \varphi$ (globally). As in $t$-$\mathcal{E}SG$, we may omit the interval $I$ if $I = [0, \infty)$. For a given set of atomic propositions $P$, we denote the language of MTL formulas over $P$ as $L_{\text{MTL}}(P)$.

**Definition 17** (Pointwise semantics of MTL). Given a timed word $\rho = (\sigma_0, \tau_0) (\sigma_1, \tau_1) \ldots$ over alphabet $P$ and an MTL formula $\varphi$, $\rho, i \models \varphi$ is defined as follows:

1. $\rho, i \models p$ iff $p \in \sigma_i$.
2. $\rho, i \models \neg \varphi$ iff $\rho, i \not\models \varphi$.
3. $\rho, i \models \varphi_1 \land \varphi_2$ iff $\rho, i \models \varphi_1$ and $\rho, i \models \varphi_2$.
4. $\rho, i \models \varphi_1 \varphi_2$ iff there exists $j$ such that
   (a) $i < j < |\rho|$.
   (b) $\rho, j \models \varphi_2$.
   (c) $\tau_j - \tau_i \in I$.
   (d) and $\rho, k \models \varphi_1$ for all $k$ with $i < k < j$.

For an MTL formula $\varphi$, we also write $\rho \models \varphi$ for $\rho, 0 \models \varphi$ and define the language of $\varphi$ as $L(\varphi) = \{ \rho \mid \rho \models \varphi \}$.

**Alternative definition of MTL** A commonly used alternative definition of MTL, especially in the context of timed automata, requires the symbols in timed words to be from $P$ instead of $2^P$, i.e., for a timed word $\rho = (\sigma_0, \tau_0) (\sigma_1, \tau_1) \ldots$ over $P$, we require $\sigma_i \subseteq P$ (instead of $\sigma_i \subseteq P$). Also, truth of an atomic formula $p$ is defined as:

$I'$. $\rho, i \models p$ iff $\sigma_i = p$.

Intuitively, a timed automaton describes a transition system with actions leading from one state to the other, where formulas describe the occurrence of actions, e.g., $G[a_1 \lor F a_2]$ says that whenever action $a_1$ occurs, $a_2$ will occur afterwards eventually. Here, the set of atomic propositions $P$ is the set of possible actions. At most one action may occur at any point in time. Thus, each $\sigma_i \subseteq P$ defines the action that occurs at time $\tau_i$.

In our context, formulas describe states of the world, e.g., $\text{RAt}(m_1) \land \text{Holding}(a_1)$ says that the robot is at $m_1$ and currently holding $a_1$. Here, the set of atomic propositions is the set of primitive formulas describing possible world states and multiple predicates may be true at the same time. Thus, each $\sigma_i \subseteq P$ describes the primitive formulas that are true at time $\tau_i$.

Let MTL$_e$ and denote MTL with the alternative semantics and $\models_e$ satisfaction in MTL$_e$. We can define mappings between MTL and MTL$_e$. The mapping $\ast : L_{\text{MTL}}(P) \to L_{\text{MTL}_e}(2^P)$ maps a formula of MTL into MTL$_e$, where:

$$\varphi^* = \bigvee_{\{Q \subseteq P \mid p \in Q\}} Q \quad \neg \varphi^* = \neg \varphi^* \quad \varphi \land \psi^* = \varphi^* \land \psi^* \quad \varphi \mathbf{U}_I \psi^* = \varphi^* \mathbf{U}_I \psi^*$$

Note that if $\varphi$ is a formula over $P$, then $\varphi^*$ is a formula over $2^P$, i.e., the atomic propositions in $\varphi^*$ are sub-sets of $P$. As an example, for $P = \{a, b, c\}$:

- $(a \land b)^* = ((a) \lor \{a, b\} \lor \{a, b, c\}) \land (\{a\} \lor \{a, b\} \lor \{b, c\})$.

The mapping $\ast : L_{\text{MTL}_e}(P) \to L_{\text{MTL}}(P)$ maps a formula of MTL$_e$ into MTL by enforcing that each $\sigma_i$ contains exactly one symbol from $P$:

$$\varphi^* = \varphi \land G \bigvee_{p \in P} \left( p \land \bigwedge_{q \in P \setminus \{p\}} \neg q \right)$$

**Theorem 2.** For every $\varphi \in L_{\text{MTL}_e}(P)$ and $\psi \in L_{\text{MTL}_e}(P)$:

$$\models \varphi \iff \models_e \varphi^*$$
$$\models \psi^* \iff \models_e \psi$$

In the following, we will use the semantics from Definition 17. However, related work on MTL synthesis uses the other formalism. In particular, Theorem 4 uses the alternative MTL semantics from above. With Theorem 2, we can apply those results while using the semantics from Definition 17.

**MTL and $t$-$\mathcal{E}SG$** Timed words in MTL are similar to traces in $t$-$\mathcal{E}SG$. In fact, $t$-$\mathcal{E}SG$ subsumes MTL:

**Theorem 3** (Hofmann and Lakemeyer (2018)). Let $\varphi$ be a sentence of MTL. Then $\models t$-$\mathcal{E}SG \varphi$ iff $\models_{\text{MTL}} \varphi$.

**Symbolic transition systems and timed automata** Intuitively, a timed automaton is a finite automaton extended with time. More specifically, a timed automaton has a finite set of clocks; time may pass in the vertices of the graph, which are also called locations. Transitions, also called switches, are the edges of the graph. They are always instantaneous, may have clock constraints, and may reset some clocks to zero. Formally, we first define symbolic transition systems (STSs):

**Definition 18** (Symbolic Transition Systems and Timed Automata (Bouyer, Bozzelli, and Chevalier 2006)). Let $X$ be a finite set of variables (called clocks). The set $G(X)$ of clock constraints over $X$ is defined by the grammar $g ::= g \land g \mid x \geq c$, where $x \in \{<, =, \geq, >\}$, $x \in X$, and $c \in \mathbb{Q}_{\geq 0}$. A valuation $\nu : X \to \mathbb{R}_{\geq 0}$. The set of valuations satisfying a constraint $g$ is denoted as $[g]$. A granularity is defined by a triple $\mu = (X, m, K)$, where $X$ is a finite set of clocks, $m \in \mathbb{N}_{> 0}$, and $K \in \mathbb{N}$. A constraint $g$ is $\mu$-granular if it only uses clocks from $X$ and each constant in $g$ is $\frac{1}{m \cdot \alpha}$ with $\alpha \leq K$ and $\alpha \in \mathbb{N}$.

For alphabet $P$ and clocks $X$, a symbolic alphabet $\Gamma$ is a finite subset of $2^P \times G(X) \times 2^X$, where a symbolic action $(p, g, Y) \in \Gamma$ is interpreted as action $p$ can happen if the constraint $g$ is satisfied, with the clocks in $Y$ being reset after the action. A symbolic word $\gamma = (a_1, g_1, Y_1) (a_2, g_2, Y_2) \ldots$ over $\Gamma$ gives rise to a set of timed words $tw(\gamma)$ over $P$.

A symbolic transition system (STS) over a symbolic alphabet $\Gamma$ based on $(P, X)$ is a tuple $T = (S, s_0, \rightarrow, F)$, where $S$ is a possibly infinite set of states, $s_0 \in S$ is the initial state, $\rightarrow \subseteq S \times \Gamma \times S$ is the transition relation, and
F ⊆ S is a set of accepting states. The timed language accepted by an STS T is denoted as L(T).

A STS is called deterministic if there are no distinct transitions q a(g1Y 0) → q1 and q a(g2Y 2) → q2 with [g1] ∩ [g2] = ∅.

A timed automaton (TA) is an STS with finitely many states.

We also want to compose STSs:

**Definition 19 (STS Compositions).** For two STS T1 = ⟨Q1, q01, →1, F1⟩ over Γ1 based on (P1, X1) and T2 = ⟨Q2, q02, →2, F2⟩ over Γ2 based on (P2, X2), the parallel composition T1 ⊥ T2 and T2 is the STS ⟨Q, q0, →, F⟩ where Q = Q1 × Q2, q0 = (q01, q02), F = F1 × F2 and

\[ p_1, p_2 \stackrel{a}{\rightarrow} (q_1, q_2) \text{ iff } p_1 \stackrel{a(g_1Y 1)}{\rightarrow} q_1 \text{ and } p_2 \stackrel{a(g_2Y 2)}{\rightarrow} q_2 \text{ with } g = g_1 \land g_2 \text{ and } Y = Y_1 \cup Y_2. \]

If P1 ∩ P2 = ∅, then the product STS T1 × T2 is the STS ⟨Q, q0, →, F⟩ where Q = Q1 × Q2, q0 = (q01, q02), F = F1 × F2 and

\[ p_1, p_2 \stackrel{a(gY 1)}{\rightarrow} (q_1, q_2) \text{ iff } p_1 \stackrel{a(g_1Y 1)}{\rightarrow} q_1 \text{ and } p_2 \stackrel{a(g_2Y 2)}{\rightarrow} q_2 \text{ and } a = a_1 \cup a_2, g = g_1 \land g_2, \text{ and } Y = Y_1 \cup Y_2. \]

In the parallel composition T1 ⊥ T2, both T1 and T2 take a transition for the same input simultaneously. The product T1 × T2 takes a transition on a symbol a if a is the union of two input symbols a1 and a2, such that T1 (T2) takes a transition on a1 (a2).

MTL Control Problem Finally, we define the MTL control problem. Intuitively, the goal is to synthesize a controller C that controls a plant P against a specification of desired behaviors Φ such that all resulting traces satisfy the specification Φ without blocking the plant P. In this context, control means that C has control over some actions, while the environment controls the remaining actions. Formally:

**Definition 20 (MTL Control Problem (Bouyer, Bozelli, and Chevalier 2006)).** Let P = P_C ∪ P_E be an alphabet partitioned into a set of controllable actions P_C and a set of environment actions P_E. A plant P over P is a deterministic TA. Let the clocks used in P be X_P and \( m, K \) be a granularity finer than that of the plant. Then, a µ-controller for P is a deterministic STS C over a symbolic alphabet based on (P, X_P ∪ X_C) having granularity µ and satisfying:

1. C does not reset the clocks of the plant: q_C a(gY) → q'_C implies Y ⊆ X_C.
2. C does not restrict environment actions: if σ ∈ L(P ∥ C) and σ(e, t) ∈ L(P) with e ∈ P_E then σ(e, t) ∈ L(P ∥ C)
3. C is non-blocking: if σ ∈ L(P ∥ C) and σ(a, t) ∈ L(P) and σ·(a, t) ∈ L(P), then σ(b, t′) ∈ L*(P ∥ C) for some b ∈ P and t′ ∈ Q.
4. All states of C are accepting.

For a timed language L ⊆ T P^*, we say that a µ-controller C controls P against the specification of desired behaviors Φ iff L(P ∥ C) ⊆ L(Φ). The control problem with fixed resources against desired behaviors is to decide, given a plant P, a set of formulas Φ, and a granularity µ finer than that of P, whether there exists a µ-controller C which controls P against the specification of desired behaviors Φ.

Bouyer, Bozelli, and Chevalier showed that the synthesis problem is decidable, with some restrictions:

**Theorem 4 (Bouyer, Bozelli, and Chevalier (2006)).** The control problem for fixed resources against MTL specifications over finite words representing desired behaviors is decidable. Moreover, if there exists a controller, then one can effectively construct a finite-state one.

We will use this result by constructing a TA PTA(Σ, δ) from a determinate fd-BAT Σ and program δ, modelling the platform as another TA R, and synthesizing a controller C that controls the TA T = PTA(Σ, δ) × R against the platform constraints Φ.

5 Constructing a TA from a Program

We describe how to construct a TA from a program δ over a determinate fd-BAT Σ. We do this by using P = P_E ∪ A_Σ as alphabet for the TA PTA(Σ, δ), i.e., the alphabet P consists of all primitive formulas and action standard names from Σ. In each transition, we encode the occurring action and the resulting situation, such that \( p \stackrel{a}{\rightarrow} q \) for \( a = \{f_1, \ldots, f_k, a\} \) if after doing action a ∈ A_Σ in the corresponding situation, exactly the primitive formulas \( \{f_1, \ldots, f_k\} \subseteq P_Σ \) are true. By doing so, we obtain a correspondence of traces of the program δ with traces in the TA.

We assume that Σ is a determinate finite-domain basic action theory and δ is a program over Σ. We need to restrict Σ to be a determinate BAT as in the resulting timed automaton, each transition encodes which primitive formulas are true in the respective situation. In particular, the transition \( q_0 \rightarrow q_0 \) will encode the primitive formulas that are true in the initial situation. As we cannot encode disjunctions in such a transition, we need Σ_0 to determine the truth for each primitive formula \( f_i \). Also, as each transition can only contain finitely many symbols, Σ needs to be restricted to a finite domain. Furthermore, we assume that δ is terminating, i.e., it only induces finitestate traces, which is necessary to guarantee that the resulting transition system indeed has a finite number of states. We will further discuss those restrictions in Section 8.

**Definition 21 (Program Timed Automata).** Given a program δ over a determinate fd-BAT Σ. We define the timed automaton PTA(Σ, δ) = (S, q0, →, F) as follows:

1. \( q_0 \stackrel{P_0}{\rightarrow} (\emptyset, \delta) \) with P = \( \{f_i \in P_Σ \mid w_Σ[f_i, \emptyset] = 1\} \)
2. \( (z, \delta) \stackrel{P_L(a)}{\rightarrow} (z \cdot a, \delta') \text{ iff } (z_0, \delta) \stackrel{w_Σ}{\rightarrow} (z_0 \cdot a, \delta') \) and P = \( \{f_i \in P_Σ \mid w_Σ[f_i, (z_0 \cdot a)] = 1\} \)
3. \( (z, \delta) \stackrel{P_E}{\rightarrow} (z, \delta) \) with P = \( \{f_i \in P_Σ \mid w_Σ[f_i, z] = 1\} \)
4. \( (z, \delta) \in F \) iff \( (z, \delta) \in F_{Σ,E} \)

A word ρ of the TA PTA(Σ, δ) corresponds to a trace \( \tau \in [\|\|]_{w_Σ} \). We can map ρ to τ:

**Definition 22 (Induced action trace).** Given a word ρ ∈ PTA(Σ, δ), we define the (action) trace \( \mu(\rho) \) induced by ρ inductively:
Proof. Let \( k = 0 \). Thus \( \rho_k = (s_0, t_0) \). By definition of \( PTA(\Sigma, \delta) \), we know that \( \sigma_0 = \Sigma_0 \). For \( z = \langle \rangle \), it follows that \( \mu(\rho_k) = z \) and \( w_{\Sigma, z} : z = \alpha \iff w_{\Sigma} : \alpha \iff \Sigma_0 = \alpha \iff \rho_k = 0 \).

(b) Let \( k = l + 1 \). By induction, there is a \( z' \) such that \( \tau = z' \cdot \tau' \), \( z' = \mu(\rho_k) \), and \( w_{\Sigma, z'} : z' = \alpha \iff \rho_{k+1} = \alpha \).

Now, we have two cases:

i. There is some action symbol \( a \in \sigma_k \). Then, by definition of \( PTA(\Sigma, \delta) \), for \( z = z' \cdot (a, k) \), \( w_{\Sigma, z} : z = \alpha \iff \rho_k = \alpha \).

ii. There is no action symbol in \( \sigma_k \). Then, by definition of \( PTA(\Sigma, \delta) \), \( \sigma_k = \{ f_i \mid w_{\Sigma, z, i} = 1 \} \) and thus, for \( z = z' \), it follows that \( w_{\Sigma, z} : z = \alpha \iff \rho_k = \alpha \).

2. Let \( \tau \in \|\|_{\Sigma, \Sigma} \). By Lemma 3, we know that there is a \( \rho \in L(PTA(\Sigma, \delta)) \). It remains to be shown that for every \( \tau = z \cdot \tau' \), \( \mu(\rho_k) = z \) and \( w_{\Sigma, z} : z = \alpha \iff \rho_k = \alpha \).

By induction over the length of \( z \):

(a) Let \( k = 0 \). Thus \( \rho_k \) is a platform action and \( \mu(\rho_k) = z \) and \( w_{\Sigma, z} : z = \alpha \iff \rho_k = \alpha \).

(b) Let \( k = l + 1 \). By induction, there is a \( z' \) such that \( \tau = z' \cdot \tau' \), \( z' = \mu(\rho_k) \), and \( w_{\Sigma, z'} : z' = \alpha \iff \rho_{k+1} = \alpha \).

Proof. Let \( \rho \in L(PTA(\Sigma, \delta)) \). By Lemma 3, we know that \( \mu(\rho) = \|\|_{\Sigma, \Sigma} \). It remains to be shown that for every \( k \leq |\rho| \), there is a \( z \), \( \tau' \) such that \( \tau = z \cdot \tau' \) and \( \mu(\rho_k) = z \).

We show the existence of \( z \), \( \tau' \) by induction over \( k \):

6 Platform Models

We model the robot platform with timed automata, an example is shown in Figure 2. Similar to PTAs, we expect a platform model to use an alphabet with symbols of the form \( \{ f_1, \ldots, f_k, a \} \), where \( a \in N_{fl} \setminus A \) is a platform action and \( f_i \in P_FL \setminus P_FL \) are exactly those primitive formulas that are true after executing the action. We expect \( f_i \) and \( a \) to be from a different alphabet than the BAT, i.e., the platform does not have any effects on the abstract program and vice versa. Further, to guarantee that the platform model does not block the PTA, we expect it to contain self loops, similar to the self loops of a PTA, and as shown in Figure 2.
Platform Constraints Given a determinate fd-BAT $\Sigma$ and a platform model $R$, we can formulate constraints over $\Sigma$ and $R$:

\begin{align*}
G \neg \text{Calibrated} & \supset -F_{\leq 10} \exists o. \text{Perf}(\text{pick}(o)) \quad (10) \\
G \text{Calibrating} & \supset \exists o. \text{RAI}(l) \land \text{Spacious}(l) \quad (11)
\end{align*}

The first constraint states that if the robot’s arm is not calibrated, it must not perform a pick action in the next 10 seconds, i.e., it must calibrate the arm before doing pick. The second constraint says that if the robot is calibrating its arm, it must be at a location that provides enough space for doing so, i.e., a Spacious location.

7 Synthesizing a Controller

Figure 3: A possible controller that controls the program from Figure 1 and the platform from Figure 2 against the constraints from Equations 10 and 11. The dashed edges are controlled by the environment.

Using the TA $\text{PTA}(\Sigma, \delta)$ that represents the program $\delta$, the TA $R$ for the platform, and constraints $\Phi$, we can use MTL synthesis to synthesize a controller that executes $\delta$ while satisfying the platform constraints. Specifically, we use

1. the plant $P = \text{PTA}(\Sigma, \delta) \times R$,
2. as controllable actions $P_C$ all symbols that contain start actions of the program or the platform model, i.e., $P_C = \{S \mid S \in P; s_a(\vec{t}) \in S \text{ for some } a(\vec{t})\}$,
3. as environment actions $P_E$ all symbols that contain end actions of the program or the platform model, i.e., $P_E = \{E \mid E \in P; e_a(\vec{t}) \in E \text{ for some } a(\vec{t})\}$,
4. a fixed granularity $\mu$, e.g., based on the robot platform’s time resolution
5. the set of MTL formulas $\Phi$ as specification of desired behaviors.

Figure 3 shows a possible controller for our example program from Listing 1, the platform from Figure 2, and the constraints from Section 6.

We can show that (1) the resulting controller indeed satisfies the constraints and (2) each of its traces is equivalent to some trace of the original program, i.e., the resulting controller satisfies the same situation formulas as the original program at any point of the execution:

**Theorem 6.** Let $\Sigma$ be a determinate fd-BAT, $\delta$ a program over $\Sigma$ that only induces finite traces, $R$ a platform model with symbols disjunct with the symbols from $\Sigma$, and let the constraints $\Phi$ be a set of MTL formulas. Let $C$ be the synthesized MTL controller with $L = L((\text{PTA}(\Sigma, \delta) \times R)) \parallel C$.

Then:

1. $L \subseteq L(\Phi)$, i.e., all constraints are satisfied.
2. For every $\rho = \rho' \cdot \rho'' \in L$, $\mu(\rho) \in \|\delta\|_{w_{\Sigma}}$ and for every fluent state formula restricted to $\Sigma$:

$$\rho' \models \alpha \iff w_{\Sigma}, \mu(\rho') \models \alpha$$

**Proof.**

1. Follows directly from Theorem 4.
2. First, note that $L \subseteq L((\text{PTA}(\Sigma, \delta) \times R))$. Second, as $R$ does not contain any action standard name from $\Sigma$, for every $\rho \in L$, there is a $\rho' \in \text{PTA}(\Sigma, \delta)$ such that $\mu(\rho) = \mu(\rho')$. By Theorem 5, for every $\rho' \in \text{PTA}(\Sigma, \delta)$, $\mu(\rho') \in \|\delta\|_{w_{\Sigma}}$ and $\rho' \models \alpha$ iff $w_{\Sigma}, \mu(\rho') \models \alpha$.

Thus, the resulting controller preserves the program’s original effects while satisfying all platform constraints.

8 Conclusion

In this paper, we have described how to synthesize a controller that controls a GOLOG program over a finite domain against a robot platform with metric temporal constraints. We did so by reducing the problem to the MTL synthesis problem, assuming that the initial state is completely known, the original program does not refer to time and only induces finite traces. For this reduction, we generated a timed automaton (TA) from the initial situation $\Sigma_0$, the program $\delta$ and the platform model $R$, where each transition describes all the fluents that are true in the respective situation. We then synthesized an MTL controller that controls the generated TA against a set of MTL constraints $\Phi$. By doing so, we obtain a decidable procedure to control an abstract program against a platform model with metric temporal constraints.

For future work, we plan to implement the proposed synthesis method based on (Bouyer, Bozzelli, and Chevalier 2006).

While the restriction to a finite domain is fundamental for the described synthesis method, in future work, we may want to allow programs that allow infinite traces. This is possible if we restrict the constraints to Safety MTL but requires modifications to the TA representation of the program, as the resulting TA must not have infinitely many states. Furthermore, we may want to allow programs that refer to time, e.g., by defining equivalence classes of traces that may refer to different points in time but imply the same situation formulas. Lastly, it would be interesting to go beyond determinate BATs to allow some form of incompleteness, for example, by considering sets of literals under the open world assumption (Levesque 1998).
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