Testing the $k_t$-factorization approach with $P$-wave quarkonia production

S.P. Baranov

P.N. Lebedev Institute of Physics, Leninskii prosp. 53, Moscow 119991, Russia

Received 6 April 2004; received in revised form 14 April 2004; accepted 14 April 2004

Available online 15 June 2004

Editor: P.V. Landshoff

Abstract

A new qualitative test of the $k_t$-factorization approach is proposed. We analyse the production of $P$-wave quarkonium states with different spins $\chi_0$, $\chi_1$, and $\chi_2$ in high energy proton–proton interactions. The predictions of the usual parton model and the $k_t$-factorization approach are compared. We find that the shape of the transverse momentum distributions and, also, the ratio of the production rates $\sigma(\chi_1)/\sigma(\chi_2)$ are qualitatively different in the considered models. The referred processes may serve as an essential test of the validity of the $k_t$-factorization approach.

© 2004 Elsevier B.V. Open access under CC BY license.

PACS: 12.38.Bx; 13.85.Ni; 14.40.Gx

Keywords: $k_t$-factorization; Quarkonia production

1. Motivation

Over the years, the $k_t$-factorization approach [1–4] has demonstrated its ability to describe a large body of experimental data (see [5] and references therein), including the photo- and electroproduction of $D^*$ mesons, $J/\psi$ mesons and forward jets, as well as specific kinematic correlations observed in the associated $D^*$ + jets photoproduction at the DESY $ep$ collider HERA. The semihard approach was also shown to reasonably describe the data on the hadroproduction of open charm and beauty, $J/\psi$, $\chi_c$ and $\Upsilon$ mesons at the Fermilab Tevatron. In many cases, however, the data can also be described within the usual (collinear) parton model, if the relevant next-to-leading order QCD corrections are taken into account or the so-called colour-octet mechanism is included.

In this context, the theoretical predictions on $J/\psi$ spin alignment made in Ref. [6] are of particular interest, as the collinear and $k_t$-factorization approaches show qualitatively different behaviour. Note that the $k_t$-factorization approach provides the only known (up to date) explanation of the $J/\psi$ polarization phenomena observed at the Tevatron [7] and at HERA [8].

It would be interesting and important to find other examples, where the difference between the collinear and noncollinear approaches would manifest in such a clear and unambiguous way. The goal of the present note is to show that such a process is found. We
analyse the production of $P$-wave quarkonium states (namely, the $\chi_c$ and $\chi_b$ mesons) in high energy hadronic collisions and demonstrate the dramatic difference between the different theoretical calculations.

Naively, one could expect the difference from the fact that the production of $\chi_1$ states in the $2 \rightarrow 1$ gluon–gluon fusion process is forbidden if the initial gluons are on shell, but is allowed if the gluons are off shell. However, the real situation is complicated by the necessity to take into account the $2 \rightarrow 2$ processes as well. The results of our analysis are presented in the next section.

2. Numerical results and discussion

We begin our discussion with showing the predictions of the collinear parton model on the production of $P$-wave charmonia at Tevatron conditions. The colour-singlet production scheme refers to the $2 \rightarrow 2$ gluon–gluon fusion subprocess

$$g + g \rightarrow \chi + g,$$

represented by the Feynman diagrams displayed in Fig. 1(a) and (b). (It would be inadequate to rely upon the $2 \rightarrow 1$ subprocess $g + g \rightarrow \chi$ in this case, because the final state particle would then be produced with zero transverse momentum.) The computational technique is explained elsewhere [9,10] in every detail and needs not to be reproduced here again.

For the sake of definiteness, we only present the parameter setting used in our calculations. Throughout the Letter, we use the leading order GRV set [11] of gluon densities in the proton and the value of the $\chi_c$ wave function $\langle R_{\chi_c}^2 (0) \rangle = 0.075 \text{GeV}^5$ taken from the potential model of Ref. [12]. The renormalization scale in the strong coupling constant $\alpha_s(\mu^2 R/\Lambda^2)$ is set to $\mu^2 R = m_{\chi}^2 + p_T^2$ with $\Lambda = 200 \text{MeV}$. The integration over the final state phase space is restricted to the pseudorapidity interval $-0.6 < \eta (\chi_c) < 0.6$, in accord with the experimental cuts used by the CDF Collaboration [13–15].

Since the predictions based on the colour-singlet mechanism alone are known to be inconsistent with the data [13–15], the theory has to be amplified with the so-called colour-octet contribution, as it is commonly assumed in the literature [10]. The relevant parton-level Feynman diagrams are displayed in Fig. 1(a)–(d). Unlike the predictions of the colour-singlet model, the size of the colour-octet matrix elements is not calculable within the theory. Therefore, the corresponding numerical results are always shown with arbitrary normalising factors (just chosen to fit the experimental data when possible).

The numerical predictions of the collinear parton model are summarised in Fig. 2(a). At relatively low transverse momenta, the production of $\chi_c$ states is dominated by the colour singlet mechanism. The differential cross section $d\sigma/dp_T$ diverges when $p_T \rightarrow 0$ for $\chi_2$ states, while it remains finite for $\chi_1$ states. (In fact, it even goes to zero, but this effect is not visible with the histogram binning, which we are using.) The production of $\chi_1$ states at zero $p_T$ is suppressed (in accord with Landau–Yang theorem), because in the limit of very soft final state gluon the $2 \rightarrow 2$ gluon–gluon process degenerates into $2 \rightarrow 1$ process. The shape of the $\chi_0$ spectrum is similar to that of $\chi_2$ (up to an overall normalising factor), and this spectrum is not shown in the figure.

The production of $\chi_c$ mesons at high $p_T$ is dominated by the colour-octet contribution, which mainly comes from the ‘gluon fragmentation’ diagrams shown in Fig. 1(c), (d). Here, the perturbative production of $^3S_1$ colour octet states

$$g + g \rightarrow ^3S_1 + g,$$
Fig. 2. Upper panel (a). Predictions of the collinear parton model at Tevatron conditions. Solid histogram, $\chi_c^1$ production via colour-singlet mechanism; dashed histogram, $\chi_c^2$ production via colour-singlet mechanism; the lower and the upper dotted histograms, $\chi_c^1$ and $\chi_c^2$ production via colour-octet mechanism, respectively. Middle panel (b). Predictions of the $k_t$-factorization approach at Tevatron conditions. Solid histograms, $\chi_c^1$ production; thin and thick dashed histograms, $\chi_c^0$ and $\chi_c^2$ production, respectively. The lower and the upper histograms of each type correspond to the gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases. Lower panel (c). Predictions on the ratio of the differential cross sections $d\sigma(\chi_c^1)/d\sigma(\chi_c^2)$ at Tevatron conditions. Solid histograms, $k_t$-factorization approach with gluon densities of Refs. [1,17]. Only the colour singlet mechanism is assumed in all cases.

is followed by a nonperturbative emission of soft gluons, that results in the formation of physical colour singlet $\chi_c$ mesons:

$$3S^8_0 \to 3P^1_J + ng.$$  \hspace{1cm} (3)

As the co-produced gluons in Eq. (3) are assumed to be soft, the momentum distribution of $\chi_c$ mesons is taken identical to that of the colour-octet $3S^8_1$ state in Eq. (2). The nonperturbative matrix elements responsible for the process (3) are related to the fictitious colour-octet wave functions, which are used in calculations based on Eq. (2) in place of the ordinary colour-singlet wave function: $(0|\chi^8_0|0) = (9/2\pi)|R^8_0(0)|^2$.

It should be noted that the fragmentation of an almost on-shell transversly polarised gluon into $\chi_1$ state via the emission of a single additional gluon $g \to 3S^8_1 \to \chi_1 + g$ is suppressed in accord with Landau–Yang theorem—due to exactly the same reasons, which make the direct on-shell gluon–gluon fusion $g + g \to \chi_1$ or two-gluon decay $\chi_1 \to g + g$ impossible. In terms of the nonrelativistic approximation, it is equivalent to say that the formally leading colour-electric dipole transitions are forbidden. The fragmentation requires the emission of at least two additional gluons (respectively, in the nonrelativistic QCD, one must go to nonleading higher multipoles), and so, the fragmentation probability must be suppressed by some extra powers of $v$, the relative velocity of charmed quarks in the bound state under study. As the degree of this suppression is not calculable within the colour-octet model on its own, we rather arbitrarily set the suppression factor to $1/20$, which corresponds to potential model expectations for the average value of $v^2$. Our numerical results on the colour-singlet contribution (1) agree with the ones presented in Ref. [10]. At the same time, we disagree about the colour-octet contributions (2), (3), because the results presented in [10] are not normalised to the available data [13–15], and, also, the suppression of $\chi_1$ states in the gluon fragmentation channel is not taken into account in [10].

Now, we proceed with showing the results obtained in the $k_t$-factorization approach. In this case, the production of charmonium $\chi_c$ states can be successfully described within the colour-singlet model alone [7], or with only a minor admixture of colour-octet contribution [16]. The consideration is based on the $2 \to 1$
partonic subprocess

\[ g + g \rightarrow \chi, \quad (4) \]

which represents the true leading order in perturbation theory. The nonzero transverse momentum of the final state meson comes from the momenta of the initial gluons. The computational technique, which we are using here is identical to the one described in detail in Ref. [7].\(^1\) In the present calculations, we conserve the parameter setting described in the beginning of this section and accept the Blümlein’s parametrization for the unintegrated gluon distributions [17], where the ordinary LO GRV functions [11] are taken as boundary conditions. This choice is fully consistent with our previous calculations [7].

In order to estimate the degree of theoretical uncertainty connected with the choice of unintegrated gluon density, we also use the prescription proposed in [1]. In this approach, the unintegrated gluon density is derived from the ordinary density \(G(x, q^2)\) by differentiating it with respect to \(q^2\) and setting \(q^2 = k_T^2\). Among the different parametrizations available on the present-day theoretical market, the latter set shows the largest difference with Blümlein’s density [17]. Thus, these two gluon densities can show the theoretical uncertainty band.

The numerical results are exhibited in Fig. 2(b). In contrast with the collinear parton model, the differential cross sections are no longer divergent, even at very low \(p_T\) values. This property emerges from the fact that the relevant \(2 \rightarrow 1\) matrix elements are always finite. One can see that the production of \(\chi_1\) states at low \(p_T\) is strongly suppressed because the initial gluons are almost on-shell. The suppression goes away at higher \(p_T\), as the off-shellness of the initial gluons becomes larger.

In Fig. 2(c) we compare the predictions of the collinear and \(k_T\)-factorization approaches by showing the ratio of the differential cross sections \(d\sigma(\chi_1)/dp_T\) and \(d\sigma(\chi_2)/dp_T\) plotted as a function of \(p_T\). As far as the ratio of the nonperturbative colour-octet matrix elements \([\mathcal{O}^3S_1^8 \rightarrow \chi_1])/[\mathcal{O}^3S_1^8 \rightarrow \chi_2])\) is unknown, the predictions of the collinear parton model are very uncertain. However, in view of the expected suppression of \(\chi_1\) states (as discussed above), the band between the two lowest dotted histograms in Fig. 2(c) may be considered as the most realistic case. The predictions of the collinear and \(k_T\)-factorization approaches clearly differ from each other in their absolute values and show just the opposite trend in the experimentally accessible region \((p_T > 5 \text{ GeV})\). Although the exact shapes of the curves may change depending on the choice of model parameters, the qualitative difference between the models is undoubtful. One can also see from the figure that, even in the asymptotic \(p_T\) regime, the proportion between the production rates \(\sigma(\chi_0): \sigma(\chi_1): \sigma(\chi_2)\) does not follow the naive rule 1:3:5 based on the number of spin degrees of freedom.

We conclude our discussion with showing the predictions on the bottomonium states. The calculations are performed with the parameter setting as above, and with the value of the \(\chi_b\) wave function set equal to \(|\mathcal{R}_{\chi_b}^2(0)|^2 = 1.4 \text{ GeV}^5\) [18]. The integration over the final state phase space is now restricted to the pseudorapidity interval \(-0.4 < \eta(\chi_b) < 0.4\), in accord with the CDF experimental cuts [13–15].

Our numerical results are displayed in Fig. 3. The qualitative features of the differential cross sections are similar to the ones which we have seen in the case of charmonium. It is worth recalling that the production of \(\Upsilon\) mesons has been already measured by CDF Collaboration [13–15] at \(p_T\) values close to zero. Although the \(p_T\) dependence of the direct \((\bar{p} p \rightarrow \Upsilon X)\) and indirect \((\bar{p} p \rightarrow \chi_b X \rightarrow \Upsilon Y \ X)\) contributions has not been studied separately, the net result seems to be at odds with collinear calculations. In fact, the predicted size of indirect contribution coming from the decays of \(\chi_b\) states at \(p_T < 2 \text{ GeV}\) exceeds the total measured \(\Upsilon\) production rate in this region. On the contrary, the measured differential cross section \(d\sigma(\Upsilon)/dp_T\) decreases with decreasing \(p_T\), in perfect agreement with \(k_T\)-factorization predictions [7].

3. Conclusions

We have considered the production of \(P\)-wave charmonium and bottomonium states with different
spins in high energy proton–proton interactions and compared the predictions of the usual parton model (with the colour-octet contributions taken into account) and the $k_t$-factorization approach. We have found that the shapes of the transverse momentum distributions and, also, the ratio of the production rates $\sigma(\chi_1)/\sigma(\chi_2)$ are qualitatively different in the considered models.

One major difference is connected with the behaviour of the differential cross section $d\sigma(\chi_2)/dp_T$ at low transverse momentum. This quantity remains finite in the $k_t$-factorization approach, while it diverges in the collinear parton model when $p_T$ goes to zero. The latter prediction seems to be not supported by the available experimental data on the bottomonium production at the Tevatron.

Another well-pronounced difference refers to the ratio of the production rates $d\sigma(\chi_1)/d\sigma(\chi_2)$. The underlying physics is promptly connected with gluon off-shellness. In the collinear parton model, the relative suppression of $\chi_1$ states becomes stronger with increasing $p_T$ because of the increasing role of the colour-octet contribution: in this approach, the leading-order fragmentation of an on-shell transversely polarised gluon into a vector meson is forbidden. In contrast with that, in the $k_t$-factorization approach, the increase in the final state $p_T$ is only due to the increasing transverse momenta (respectively, virtualities) of the initial gluons, and, consequently, the suppression motivated by Landau–Yang theorem becomes weaker at large $p_T$.

We derive the conclusion that the considered process may be proposed as a direct probe of the gluon virtuality, which can eventually testify for the validity of the noncollinear parton evolution. Our results seem especially promising in view of the fact that the difference between the two theoretical approaches is clearly pronounced at the conditions accessible for direct experimental measurements.

**Acknowledgements**

The author conveys his thanks to Nikolai Zotov for his encouraging interest and useful discussions. The author acknowledges the financial support from Crafoord foundation (Sweden).
References

[1] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rep. 100 (1983) 1.
[2] E.M. Levin, M.G. Ryskin, Phys. Rep. 189 (1990) 267.
[3] S. Catani, M. Ciafaloni, F. Hautmann, Phys. Lett. B 242 (1990) 97;
   S. Catani, M. Ciafaloni, F. Hautmann, Nucl. Phys. B 366 (1991) 135.
[4] J.C. Collins, R.K. Ellis, Nucl. Phys. B 360 (1991) 3.
[5] B. Andersson, et al., Small x Collaboration, Eur. Phys. J. C 25 (2002) 77.
[6] S.P. Baranov, Phys. Lett. B 428 (1998) 377.
[7] S.P. Baranov, Phys. Rev. D 66 (2002) 114003.
[8] A.V. Lipatov, N.P. Zotov, Eur. Phys. J. C 27 (2003) 87.
[9] H. Krasemann, Z. Phys. C 1 (1979) 189;
   G. Guberina, J. Kühn, R. Peccei, R. Rückl, Nucl. Phys. B 174 (1980) 317.
[10] B. Kniehl, et al., Phys. Rev. D 68 (2003) 114002.
[11] M. Glück, E. Reya, A. Vogt, Z. Phys. C 67 (1995) 433.
[12] E.J. Eichten, C. Quigg, Phys. Rev. D 52 (1995) 1726.
[13] F. Abe, et al., CDF Collaboration, Phys. Rev. Lett. 69 (1992) 3704;
   F. Abe, et al., CDF Collaboration, Phys. Rev. Lett. 71 (1993) 2537;
   F. Abe, et al., CDF Collaboration, Phys. Rev. Lett. 75 (1995) 1451;
   F. Abe, et al., CDF Collaboration, Phys. Rev. Lett. 79 (1997) 578.
[14] T. Affolder, et al., CDF Collaboration, Phys. Rev. Lett. 86 (2001) 3963.
[15] T. Affolder, et al., CDF Collaboration, Phys. Rev. Lett. 84 (2000) 2094.
[16] Ph. Hägler, R. Kirschner, A. Shafer, L. Szymanowski, O.V. Teryaev, Phys. Rev. Lett. 86 (2001) 1446.
[17] J. Blümlein, Preprint DESY 95-121, 1995.
[18] K. Hagivara, A.D. Martin, A.W. Peacock, Z. Phys. C 33 (1986) 135.