State estimation accuracy enhancement for optimal power system steady state modes

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Abstract. State estimation is an important tool for system operators, which determine the state of the system and accordingly act for control and operate the system. Ill conditioned system is a problem facing the state estimation algorithms, which results in numerical instability and inaccurate results of power system state estimation. This paper proposes a method to overcoming the ill-conditioned problem based on their linear correlations between the output parameters for obtaining optimal power systems steady state mode.

1. Introduction

The Energy management system (EMS) in electrical power system is a vital function for the reliable delivering energy to customers and efficient operation of the system. Power system state estimation (SE) is one of the most important operations of (EMS), where providing a database of the real time state of the system for using in other (EMS) functions [1]. Therefore, an efficient and accurate state estimation is an obligatory for an efficient operation of the power system.

The most frequently used method for power system state has so far been the weighted least squares (WLS) method which is solved often based on mathematical algorithms [2,3]. There are sundry solution techniques, e.g., Cholesky decomposition [4] and orthogonal factorization [5], which are varying in computational efficiency and numerical stability, but using of very high weights for modeling highly accurate virtual measurements, lead up to ill-conditioning system. One of the properties of ill-conditioned system is that, a small error in inputs results in significant error in outputs of the system, that means, the algorithm of the state estimation may diverge and producing imprecise state [6,7].

Some efforts are introduced to overcoming ill-conditioned power system state estimation problems, as in [8] offers an alternative solution to the problem based on peters and Wilkinson method, equality constraints for modeling zero injection buses are introduced in the WLS formulation, instead of artificially assigning high weights to avoiding ill conditioning problem [4] and regularization techniques using singular value decomposition [9,10]. Linear relations between output elements of the ill-conditioned system is an important property for that system, helping in the solution of that system [11]. In the view of that concept, this paper introduces a method for accuracy improving of the power system state estimation for optimization purposes for power system steady-state mode.

2. Methodology

For an electrical power system network, the nodal voltage at each bus can be calculated from the following equation:
Where, $Y$ – is the bus admittance matrix of the network, $V$ – is the vector of nodal voltages, $I$ – is the vector of the injected currents at nodes. Adding adjustment device with controlled reactance to the network for controlling the bus voltages and power system state optimization purposes, as FACTs devices. Nodal voltages can be expressed as [12]:

$$v_i = \frac{A_i + B_i y_v}{1 + C_y_v},$$

(2)

Where $v_i$ – is the nodal voltage at bus ‘$i$’, $A$, $B$, $C$ – are constants, $A$ and $B$ depends on bus number ‘$i$’ and $c$ depends on position of the adjustment device $y_v$ is the controlled admittance of the added device. Equation (2) shows that, the nodal voltages are fractionally polynomial dependences on the controlled admittance of the adjustment device. To determine the values of these constants in equation (2), the nodal voltages of the network should be measured (calculated) at three various values of $y_v$. Under that condition’s equation (2) will be:

$$\begin{bmatrix}
1 & y_{v,1} & -v_{i,1}c \\
1 & y_{v,2} & -v_{i,2}y_{v,2} \\
1 & y_{v,3} & -v_{i,3}y_{v,3}
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
= 
\begin{bmatrix}
v_{i,1} \\
v_{i,2} \\
v_{i,3}
\end{bmatrix}
\Leftrightarrow SX = T
$$

(3)

Where $y_{v,1}, y_{v,2}, y_{v,3}$ – are three arbitrary values of the admittance of the adjustment device, $y_{v,1}, y_{v,2}$ and $y_{v,3}$ – are three measured (calculated) values of the nodal voltage of bus ‘$i$’ at that values of $y_v$. The known elements in the system of equations (3) are measured quantities, therefore, any changes in these quantities due to measurement errors, will lead to changes in the values of constants $A$, $B$ and $C$, consequently affects the final estimated value of the nodal voltages. These amount of changes of the estimated nodal voltages depends on the value of the condition number of the system of equations (3), that occasional give us the ill-conditioned system. Based on linear relations between the solution elements ($A$, $B$ and $C$ in our system) of ill-conditioned system [13,6], The constants values will be determined as follows:

Pre-multiplying both sides of system of equations (3) by the coefficient matrix powered by ($n$-1), where ‘$n$’ is an integer number, system of equations (3) will be as:

$$\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
= 
\begin{bmatrix}
d_1 \\
d_2 \\
d_3
\end{bmatrix}
\Leftrightarrow S^n X_1 = S^{n-1} T_1
$$

(4)

First equation of system (4) will be as:

$$a_{11}A + a_{12}B + a_{13}C = d_1$$

(5)

I. Determination of linear correlations between the constants $a$, $b$ and $c$ from equation (5) as:

$$B = \frac{d_1}{a_{11}} - \frac{a_{12}}{a_{11}} A - \frac{a_{13}}{a_{11}} C = \beta - \alpha A - \theta C$$

(6)

substituting equation (6) into equation (1) yields:

$$v_i = \frac{a_i + (\beta - \alpha A - \theta C) y_v}{1 + C y_v}$$

(7)
II. Carrying out two measurements (calculations) of the nodal voltages at two different values of controlled admittance of the added device, $y_{v,4}$ and $y_{v,5}$. Equation (7) under that conditions will be:

$$
\begin{bmatrix}
\theta v_4 + v_4 y_{v,4} \\
\theta v_5 + v_5 y_{v,5}
\end{bmatrix}
\begin{bmatrix}
\alpha v_4 - 1 \\
\alpha v_5 - 1
\end{bmatrix}
= \begin{bmatrix}
\beta v_4 - v_4 \\
\beta v_5 - v_5
\end{bmatrix}
$$

(8)

If the system (8) is well-conditioned so from solving (8) and equation (6) the value of constants will be determined, but if the system still ill-conditioned:

III. Repeating step (I) for the system (8) it will be as:

$$
\begin{bmatrix}
f_{11} & f_{12} \\
f_{12} & f_{22}
\end{bmatrix}
\begin{bmatrix}
k_1 \\
k_2
\end{bmatrix}
= A
$$

\( S_2^n X_2 = S_2^n T_2 \)

(9)

First equation of system (9) will be as:

$$
A = \frac{k_1}{f_{12}} - \frac{f_{11}}{f_{12}} C = \psi - \alpha C
$$

(10)

IV. Measuring (calculating) the nodal voltages of the network at one value of $y_v$ different from other mentioned values. Substituting equations (6), (10) into equation (1) yields:

$$
v_i = \psi - \alpha C + (\beta - \alpha A - \theta C) y_v
$$

(11)

Solving equation (11) for $c$:

$$
C = \frac{\beta v_6 - \psi - \psi y_{v,6} - v_6}{y_{v,6}(v_6 + \theta - \alpha) + \psi}
$$

(12)

Values of constants $A$, $B$ and $C$ can be determined from equations (11),(5) and (12) respectively, from these values the nodal voltages can be estimated using equation (2) with low relative error percentage as will be shown in the next section.

3. Simulation and results

The proposed method is applied on IEEE 14 and 30 bus systems as tested system. Error level in measurements is introduced in calculation as a random quantity uniformly distributed in the interval [10^{-4}, 0.01].
As shown in Figures 1 and 2, the relative voltage error percentage of the estimated nodal voltage changes with the error level of measurements, where varies from 0.18% and 0.2% at measurement error level \(10^{-4}\) to 35% and 32% at measurement error level 0.01 for IEEE 14 and 30 bus systems respectively. Highly increasing in the error of estimated value of the nodal voltage with small change in measurement errors can be interpreted due to ill-condition system in the stage of constant determination. The condition number was large as shown in Figures 3 and 4.

After linear correlation process and measurements of the nodal voltages at different two values of \(y_v\), to calculate the system constants, which reduced to only two constants \(A\) and \(C\), the condition number of the modified systems is shown in figures 5 and 6, which is lower than the first one, but the system still ill-conditioned, so another linear correlation process performed again and executed
another measurement for the nodal voltages at one value of $y_p$ to calculate the last constant of the system.

**Figure 5.** Condition number of the constant determination system after second measurement stage for IEEE 14 bus system.

**Figure 6.** Condition number of the constant determination system after second measurement stage for IEEE 30 bus system.

The percentage of the estimated relative voltage error is shown in figures 7,8. The higher relative percentage error of the estimated value of the nodal voltage reaches to 4.5% for IEEE 14 bus system and 4% for IEEE 30 bus system, which is allowable in the accuracy of the circuit parameter standards, while this value was 35% and 32% for IEEE 14 and 30 bus system respectively without applying the proposed method.

4. Conclusions

Nodal voltages of the power system network are estimated through an iterative process based on the fractional polynomial relationship between these nodal voltages and the controlled parameter of adjustment device for optimization of the state of the power system. This relation contains three unknown constants are determined through ill-conditioned system. This paper introduces a method for the solution of that ill-conditioned problem to improve the nodal voltages estimation by using the fractional polynomial relationships. The proposed method applied on IEEE 14 and 30 bus system, resulted in reducing the error of the estimated voltage to 4.5% and 4% compared to 35% and 32% for IEEE 14 and 30 bus system respectively without using the proposed method.
5. References

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Figure 7. Histogram of relative estimated voltage error for IEEE 14 bus system – with applying the proposed method - at different measurement error levels.

Figure 8. Histogram of relative estimated voltage error for IEEE 30 bus system - with applying the proposed method - at different measurement error levels.
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