Universal quantum Controlled-NOT gate

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An investigation of an optimal universal unitary Controlled-NOT gate that performs a specific operation on two unknown states of qubits taken from a great circle of the Bloch sphere is presented. The deep analogy between the optimal universal C-NOT gate and the ‘equatorial’ quantum cloning machine (QCM) is shown. In addition, possible applications of the universal C-NOT gate are briefly discussed.

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I. INTRODUCTION

Manipulation of information encoded in states of quantum systems has remarkable advantages compared to classical information processing [1]. Unlike classical information, however, there is a fundamental limitation on the basic operations that one can perform on quantum systems. This limitation is known as the non-cloning theorem [2] and has a manifest impossibility of conducting several exact operations in quantum information theory, such as ‘cloning’ [2], ‘inversion’ [3] and ‘entangling’ [4] of unknown quantum states. If one does not demand these operations to be perfect, it is possible to construct quantum devices that provide the required operations approximately. Many examples of approximate operations on states of quantum systems have been shown in the last decades: universal symmetric [5] and asymmetric [6] quantum cloning machines (QCM’s), universal NOT gate [8] as well as universal symmetric [1] and asymmetric [7] entanglers.

In this work we pay attention to a quantum operation, Controlled-NOT gate, that plays an important role in quantum information theory and especially in quantum computing [1]. Although this gate is usually associated with a computational basis, for a deeper understanding of the fundamental principles of operating with quantum systems, it is essential to investigate a universal operation that is basis independent. While the impossibility of constructing an exact basis independent C-NOT gate has already been shown [3], we shall present a universal C-NOT gate that provides an approximate transformation on two (input) qubits in unknown quantum states. At first, however, we shall discuss the universal NOT gate that is an essential part of universal C-NOT gate. Although an exact universal NOT gate for a qubit in an unknown input state does not exist [3], we show that there is an exact universal NOT gate for an unknown qubit state chosen from a great circle of the Bloch sphere. With the help of this exact NOT gate, we then construct an optimal universal C-NOT gate for two unknown states of qubits chosen from the great circle of the Bloch sphere. This optimal C-NOT gate has similar structure to the ‘equatorial’ QCM [9] while the fidelity between the ideal and the actual output states of the universal C-NOT gate equals $F = 1/2 + \sqrt{1/8}$ for both qubits.

This paper is organized as follows. In the next section we discuss universal NOT operation [3] and construct an exact universal NOT gate for an unknown qubit state taken from a great circle of the Bloch sphere. Having the exact NOT gate, we then suggest an ‘idealized’ universal C-NOT operation which, however, can not be achieved due to the non-cloning principle. In Section II we present an optimal (approximate) universal C-NOT gate for unknown input states of qubits taken from a great circle of the Bloch sphere. Finally, several concluding remarks are drawn in Section IV.

II. UNIVERSAL QUANTUM GATES

At the beginning of discussion of the universal quantum gates we note that we always use the Bloch sphere representation of a qubit state. A pure state of a qubit can be written as $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\varphi} |1\rangle$, and with $|0\rangle$ and $|1\rangle$ being computational basis states. In this representation, the parameters $\theta$ and $\varphi$ take values in the range $0 \leq \theta \leq \pi$ and $0 \leq \varphi < 2\pi$, respectively, and we shall often use this Bloch sphere in order to visualize the states of interest.

A. NOT gate

Let us start our discussion of the universal quantum gates by recalling the properties of the universal quantum NOT operation [3]. According to the definition, for a
given input qubit state $|\psi\rangle$, this gate generates the orthogonal state $|\psi^\perp\rangle$ at the output, i.e.

$$\text{NOT} |\psi\rangle = |\psi^\perp\rangle, \quad (1)$$

so that $\langle \psi | \psi^\perp \rangle \equiv 0$. An exact unitary transformation (1) for an arbitrary input qubit state $|\psi\rangle$ does not exist. To provide this transformation approximately, Bužek et al. considered an ensemble of $N$ input qubits that are prepared in the state $|\psi\rangle$. It was shown that the transformation (1) can be performed approximately on the ensemble with fidelity $F = \langle \psi^\perp | \rho | \psi^\perp \rangle = (N+1)/(N+2)$ between the approximate output $\rho$ of the transformation and the ideal output $|\psi^\perp\rangle$. If the input ensemble consists of a single state $|\psi\rangle$, the universal unitary NOT transformation has the structure (2).

$$|\psi\rangle |X\rangle \rightarrow \sqrt{\frac{2}{3}} |\psi^\perp\rangle |A\rangle + \sqrt{\frac{1}{3}} |\psi\rangle |B\rangle, \quad (2)$$

where $|X\rangle$, $|A\rangle$ and $|B\rangle$ are the state vectors of the device (with an auxilliary system). The fidelity between the approximate output of the transformation (2) and the ideal output $|\psi^\perp\rangle$ equals $F_{\text{NOT}} = 2/3$.

The universal NOT transformation (2) for an arbitrary input qubit state has low fidelity. To improve the fidelity of the universal NOT transformation one may consider a restricted set of input states, for example, a one-dimensional subspace of the two-dimensional Hilbert state space of a qubit. Using the Bloch sphere representation of the qubit state, a one-dimensional subspace can be visualized as an intersection of the Bloch sphere with a plane. Let us consider the one-dimensional subspace, the main circle, that is formed by the intersection of the Bloch sphere with the $x$-$z$ plane. An arbitrary state of a qubit in this circle can be parameterized as

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle \pm \sin \frac{\theta}{2} |1\rangle. \quad (3)$$

We found that for an arbitrary input state (3) the operator

$$\text{NOT} = -i \sigma_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (4)$$

provides an exact NOT transformation, i.e. $\langle \psi | \text{NOT} | \psi \rangle \equiv 0$. This operator introduces an exact universal NOT gate for the input state (3). Moreover, from the symmetry of the Bloch sphere it follows that a universal NOT gate can be constructed for any restricted one-dimensional set of input states.

Knowing the expressions for universal NOT transformations for an arbitrary input state (2) and for an input state from the main circle (3), one may use one of these transformations in order to construct a universal C-NOT gate. Since the universal gate (2) has low fidelity $F_{\text{NOT}} = 2/3 \approx 0.67$ we shall not discuss a universal C-NOT gate based on this approximate transformation (2). Instead, we shall focus on the universal C-NOT gate that is based on an exact universal NOT gate (1). Of course, such a C-NOT gate is restricted by the input states (3) of qubits taken from the main circle of the Bloch sphere.

### B. C-NOT gate

A quantum Controlled-NOT gate provides a unitary transformation on two qubits, one of which is called control and the other - target. According to the definition (1), the gate leaves the meaning of the target qubit unchanged, if the control qubit is given in the state $|0\rangle$. If the control qubit is in the state $|1\rangle$, the gate performs a NOT operation on the target qubit. If the control qubit is given in a superposed state, the quantum C-NOT gate can be defined on a computational basis as

$$U = |0\rangle \langle 0| \otimes I_c + |1\rangle \langle 1| \otimes (\sigma_x)_t, \quad (5)$$

where $\sigma_x = |0\rangle \langle 1| + |1\rangle \langle 0|$. The gate (5) leaves the states of the control and the target qubits separable, if the control qubit is in one of the basis states $|0\rangle$ or $|1\rangle$, and creates entanglement between the control and the target qubits otherwise.

In contrast to the definition (5), the universal (basis independent) C-NOT gate should always leave the states of the control and the target qubits separable. Indeed, a superposed state of the control qubit in a given basis can be always transformed in one of the basis states into a new basis by means of a basis transformation. In this new basis the states of the control and the target qubits are separable according to the definition given above. On the other hand, the basis independent C-NOT gate should be invariant with regard to a basis transformation. Therefore, for a given superposed input state of the control qubit, the output states of the control and the target qubits are separable.

As mentioned in the previous section, we are going to construct a universal C-NOT transformation for the states of the control and the target qubits taken from the main circle of the Bloch sphere. Let us introduce the following notations for these qubits

$$|\psi_\pm\rangle_c = \cos \frac{\phi}{2} |0\rangle_c \pm \sin \frac{\phi}{2} |1\rangle_c, \quad (6)$$

$$|\chi_\pm\rangle_t = \cos \frac{\phi}{2} |0\rangle_t \pm \sin \frac{\phi}{2} |1\rangle_t, \quad (7)$$

where $|\psi_\pm\rangle_c$ and $|\chi_\pm\rangle_t$ denote the states of the control and the target qubits respectively. Although the state $|\psi_\pm\rangle_c$ of the control qubit is given in a superposition of the two basis states $|0\rangle_c$ and $|1\rangle_c$, it is of course sufficient to know the C-NOT transformation of just the basis in order to obtain a proper transformation for states (6)- (7). For the input states $|0\rangle_c$ and $|1\rangle_c$ of the control qubit, the universal unitary C-NOT should perform the transformation

$$|0\rangle_c |\chi_\pm\rangle_t |Q_d \rightarrow |0\rangle_c |\chi_\pm\rangle_t |Q_0\rangle_d, \quad (8)$$

$$|1\rangle_c |\chi_\pm\rangle_t |Q_d \rightarrow |1\rangle_c |\chi_\pm\rangle_t |Q_1\rangle_d, \quad (9)$$
as required by the definition. The state vectors $|Q\rangle_d$, $|Q_0\rangle_d$ and $|Q_1\rangle_d$ denote the initial and the final states of the device that provides this transformation. The output state $|\chi_{c,t}\rangle_t$ of the target qubit is orthogonal to the input target qubit state $|\chi_{d,t}\rangle_t$ and is obtained by applying the \textsc{not} gate $\text{\textsc{not}}$ to the input state $|\chi_{d,t}\rangle_t$, i.e. $|\chi_{\pm,t}\rangle_t = \text{\textsc{not}}|\chi_{\pm,t}\rangle_t$. If the state of the control qubit is given in the superposed state $|\psi\rangle_c$, the universal unitary $\text{\textsc{not}}$ applies the transformation $|\psi_{t}\rangle_c = U(\psi)|\chi_{t}\rangle_t$. On the other hand, making a superposition of Eqns. (8-9) we obtain

$$
\left(\frac{\cos\theta}{2} |0\rangle_c + \frac{\sin\theta}{2} |1\rangle_c\right) |\chi_{\pm,t}\rangle_t |Q\rangle_d \rightarrow (11)
$$

$$
\cos\frac{\theta}{2} |0\rangle_c |\chi_{\pm,t}\rangle_t |Q\rangle_d + \sin\frac{\theta}{2} |1\rangle_c |\chi_{\pm,t}\rangle_t |Q\rangle_d .
$$

Let us analyze this transformation (11) in order to specify the function $f(\psi, \chi)$ in the transformation (10). Suppose one has two qubits prepared in the states $|\psi_0\rangle_c = \cos\frac{\phi_0}{2} |0\rangle_c + \sin\frac{\phi_0}{2} |1\rangle_c$ and $|\chi_0\rangle_t$ respectively. If one performs the transformation (11) on them, so that the qubit $|\psi_0\rangle_c$ is the control and the qubit $|\chi_0\rangle_t$ is the target, the two-qubit state

$$
\cos\frac{\theta_0}{2} |0\rangle_c |\chi_0\rangle_t + \sin\frac{\theta_0}{2} |1\rangle_c |\chi_0\rangle_t ,
$$

is obtained at the output, as it follows from Eqns. (3-9) and (11). Making a projective measurement on the target qubit in the $\{|\chi_0\rangle_t, |\chi_{\pm,t}\rangle_t\}$ basis one obtains the outcomes $|\chi_0\rangle_t$ and $|\chi_{\pm,t}\rangle_t$ with probabilities $\cos^2\frac{\theta_0}{2}$ and $\sin^2\frac{\theta_0}{2}$ respectively. For the universal \textsc{not} gate (10), we suggest this transformation take the following structure

$$
|\psi_{+}\rangle_c (|\psi_{+}\rangle_c = \cos\frac{\theta}{2} |\chi_{\pm,t}\rangle_t + \sin\frac{\theta}{2} |\chi_{\pm,t}\rangle_t) |Q\rangle_d \rightarrow (13)
$$

$$
|\psi_{+}\rangle_c \left(\frac{\cos\theta}{2} |\chi_{\pm,t}\rangle_t + \sin\frac{\theta}{2} |\chi_{\pm,t}\rangle_t\right) |Q\rangle_d .
$$

On the right hand side of this transformation (13), the control qubit is left without changes as is required by Eqn. (10) while the unitary transformation

$$
U(\psi) = \left(\begin{array}{cc}
\cos\frac{\theta}{2} - \sin\frac{\theta}{2} \\
\sin\frac{\theta}{2} & \cos\frac{\theta}{2}
\end{array}\right)
$$

is to be performed on the target qubit $|\chi_{t}\rangle_t$. After simple algebraic manipulation we find that the universal \textsc{not} operation can be written as

$$
|\psi_{+}\rangle_c \left(\cos\frac{\phi}{2} |0\rangle_t \pm \sin\frac{\phi}{2} |1\rangle_t\right) |Q\rangle_d \rightarrow (15)
$$

$$
|\psi_{+}\rangle_c \left(\cos\frac{\phi - \theta}{2} |0\rangle_t \pm \sin\frac{\phi - \theta}{2} |1\rangle_t\right) |Q\rangle_d ,
$$

where we have shown the states of the target qubit before and after the transformation explicitly. The transformation (15) leaves the control qubit without changes and rotates the target qubit on the angle $\theta$ clockwise. We note that if the state of the control qubit is given in the state $|\psi_{-}\rangle_c$, the transformation (15) rotates the target qubit counterclockwise to the angle $\theta$. It is also remarkable that the state of the output target qubit depends only on the difference $\phi - \theta$ and does not depend on a particular basis (as it should be for a basis independent transformation).

The transformation (15) introduces the ‘idealized’ universal \textsc{not} gate that can not be performed exactly due to the non-cloning principle (8). Indeed, to perform the rotation of the target qubit, the device needs to obtain some information about the input state $|\psi\rangle_c$ of the control qubit. The non-cloning principle implies that any information can not be obtained from the unknown state $|\psi\rangle_c$ without changing the state. Nevertheless, in the next section we shall construct an optimal universal \textsc{not} gate that provides the transformation (15) approximately with constant fidelity for both the control and the target qubit states taken from the main circle.

### III. Explicit Form of the Universal \textsc{not} Gate

To obtain an explicit form of the (approximate) universal \textsc{not} transformation (15), let us consider the most general quantum transformation for two-qubit (control + target) state which can be cast in the form

$$
|0\rangle_c |\chi_{\pm,t}\rangle_t |Q\rangle_d \rightarrow \sum_{m,n=0}^{1} |m\rangle_c |n\rangle_t |Q_{mn}\rangle_d ,
$$

$$
|1\rangle_c |\chi_{\pm,t}\rangle_t |Q\rangle_d \rightarrow \sum_{m,n=0}^{1} |m\rangle_c |n\rangle_t |Q'_{mn}\rangle_d ,
$$

where $|Q\rangle_d$ denotes again the initial state of the device. Once, the transformation has been performed, $|m\rangle_c$ and $|n\rangle_t$ denote the output basis states of the control and target qubits, while $|Q_{mn}\rangle_d$ and $|Q'_{mn}\rangle_d$ are the corresponding states of the apparatus. In order to ensure that the transformation (16) is unitary,

$$
\sum_{i} c_i |i\rangle_{cd} |Q\rangle_d \rightarrow \sum_{i,\lambda} c_i U_{i\lambda} |\lambda\rangle_{ctd} ,
$$

for all possible input states, i.e. for $|i\rangle = \{|0\rangle_c |\chi_{\pm,t}\rangle_t , |1\rangle_c |\chi_{\pm,t}\rangle_t\}$, the three-partite basis $\{|\lambda\rangle_{cd}\}$ refers to a complete and orthonormal basis for the overall system (qubits c, t + device). Thus, the requested unitarity $UU^\dagger = 1$ of the transformation (15) implies the condi-
tary transformation \((16)\) realizes the universal transformation \((16)\). Let us require that the unitary transformation with regard to some specific conditions. For any explicit construction of transformation \((16)\), we must therefore ‘determine’ the final states \(|Q_{mn}\rangle_d\) and \(|Q'_{mn}\rangle_d\) of the device in line with the conditions \((15)\) and an additional optimality condition that specify a particular transformation \((16)\). Let us require that the unitary transformation \((16)\) realizes the universal C-NOT gate \((15)\) with maximal average fidelity between the input and the output states of the control as well as the target qubits. The average fidelity is defined as an integral of a fidelity function over a set of states and is given by \([11, 12]\)

\[
\mathcal{F} = \int_\Omega \frac{d\phi}{A} F_c(\phi) = \int_\Omega \frac{d\theta}{A} F_t(\theta),
\]

where \(A\) is a normalization factor. The fidelity functions \(F_c(\phi)\) and \(F_t(\theta)\) are defined as \(F_c(\phi) = c\langle \psi^{id} | \rho^{out}_{c} | \psi^{id}\rangle_c\) and \(F_t(\theta) = t\langle \chi^{id} | \rho^{out}_{t} | \chi^{id}\rangle_t\), where \(|\psi^{id}\rangle_c\) and \(|\chi^{id}\rangle_t\) denote the ideal output states of the control and the target qubits while \(\rho^{out}_c\) and \(\rho^{out}_t\) are the actual (approximate) output states of the control and the target qubits from the transformation \((16)\) respectively. In the expression \((19)\) the integration of the fidelity functions is to be done over all the states \(\Omega\) from the main circle of the Bloch sphere.

To find the maximum of the average fidelity \((19)\) we used the general method of semidefinite programming \([11, 12]\) which allows one to find the optimal unitary transformation with regard to some specific conditions. Using this method we found the optimal universal unitary C-NOT gate for the input states of the control and the target qubits taken from the main circle of the Bloch sphere. This gate can be given in a chosen basis by the transformation

\[
\begin{align*}
|0\rangle_c |x\rangle_t |Q\rangle_d &\longrightarrow \left( \frac{1}{2} + \sqrt{\frac{1}{8}} \right) |0\rangle_c |x\rangle_t |0\rangle_d \\
&+ \sqrt{\frac{1}{8}} \left( |0\rangle_c |\bar{x}\rangle_t + |1\rangle_c |\bar{x}\rangle_t \right) |1\rangle_d \\
&+ \left( \frac{1}{2} - \sqrt{\frac{1}{8}} \right) |1\rangle_c |\bar{x}\rangle_t |0\rangle_d ,
\end{align*}
\]

\((20)\)

\[
\begin{align*}
|1\rangle_c |x\rangle_t |Q\rangle_d &\longrightarrow \left( \frac{1}{2} + \sqrt{\frac{1}{8}} \right) |1\rangle_c |x\rangle_t |1\rangle_d \\
&+ \sqrt{\frac{1}{8}} \left( |0\rangle_c |\bar{x}\rangle_t + |1\rangle_c |\bar{x}\rangle_t \right) |0\rangle_d \\
&+ \left( \frac{1}{2} - \sqrt{\frac{1}{8}} \right) |0\rangle_c |\bar{x}\rangle_t |1\rangle_d .
\end{align*}
\]

This transformation is invariant with regard to a basis transformation by construction. For the transformation \((20)-(21)\) the fidelity between the ideal output and the actual output for the states of the control as well as the target qubits equals \(F = 1/2 + \sqrt{1}/8\) and is constant for arbitrary input states of the control and target qubits taken from the main circle of the Bloch sphere.

The transformation \((20)-(21)\) has similar structure to the ‘equatorial’ QCM \([9]\). This similarity has an important implication. The ‘idealized’ universal CNOT transformation \((15)\) can be formally treated as a two-step transformation. The first stage of the device provides the cloning transformation on the input control qubit, the second stage rotates the state vector of the copy in the main circle over the angle \(\phi\) which describes the state of the target qubit. While the first stage (cloning) transformation is strongly restricted by the non-cloning principle, there are no limitations on the second stage transformation. Thereby the problem to find an optimal C-NOT transformation for the input states of the qubits taken from the main circle reduces to a search for the optimal cloning transformation for such input states. Since the ‘equatorial’ QCM is the optimal cloning transformation for the input states from the main circle \([9]\), it is not surprising that the universal C-NOT transformation \((20)-(21)\) has a structure similar to the ‘equatorial’ QCM.

IV. CONCLUDING REMARKS

Unlike the well-known basis dependent C-NOT gate \([5]\), we have presented an optimal universal C-NOT gate that performs the transformation \((10)\) approximately on two unknown input qubits taken form the main circle of the Bloch sphere. The obtained universal C-NOT gate provides the transformation \((20)-(21)\) on the input states of the qubits with constant fidelity \(F = 1/2 + \sqrt{1}/8\) between the ideal output and the actual output for both the control and the target qubits. Moreover, we have shown the analogy between universal C-NOT gate and QCM which makes possible the construction of C-NOT gates with various properties. For example, one may construct a universal asymmetric C-NOT gate that provides the transformation \((10)\) with different fidelities for the control and the target qubits \(F_c \neq F_t\). This universal ‘asymmetric’ C-NOT gate represents an analog of the universal asymmetric QCM \([8]\). Another possibility is to construct a universal C-NOT gate for the input states (of control and target qubits) from a small circle on the Bloch sphere that is formed by a plane that crosses the sphere away from its center (similar QCM was considered by Fiurášek \([12]\)). Finally, one may consider the possibility to construct a universal probabilistic C-NOT gate that allows one to perform the transformation \((10)\) exactly with a distinct probability \([13, 14]\).

Besides the pure theoretical interest, the universal C-NOT gate may find its applications in quantum commun-
nication and quantum computing, since it has some advantages compared to the basis dependent C-NOT gate \[5\]. Apart from the fact that the universal C-NOT gate operates with unknown input states of qubits, it may efficiently operate with mixed input states since the optimal cloning transformation for mixed input states has already been developed \[15\]. It has recently been shown that quantum computing with mixed quantum states has advantages over the best possible classical computation \[10\] and may, in particular, provide the computational speed up of Deutsch-Jozsa and Simon problems in comparison to the best known classical algorithms \[17\]. The universal C-NOT gate introduces a basic element for possible schemes of quantum computation based on mixed states.

We also hope that the universal C-NOT gate can be realized experimentally with good accuracy, since efficient experimental realizations of QCM have already been demonstrated \[15\]. For example, an optical implementation of the universal QCM \[5\] based on parametric down-conversion has been achieved with fidelity 0.810 ± 0.008 \[18\] which is in a good agreement with the theoretical prediction $5/6 = 0.833$.

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