A minimal Little Higgs model

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Abstract

We discuss a Little Higgs scenario that introduces below the TeV scale just the two minimal ingredients of these models, a vectorlike T quark and a singlet component (implying anomalous couplings) in the Higgs field, together with a pseudoscalar singlet η. In the model, which is a variation of Schmaltz’s simplest Little Higgs model, all the extra vector bosons are much heavier than the T quark. In the Yukawa sector the global symmetry is approximate, implying a single large coupling per flavour, whereas in the scalar sector it is only broken at the loop level. We obtain the one-loop effective potential and show that it provides acceptable masses for the Higgs h and for the singlet η with no need for an extra µ term. We find that m_η can be larger than m_h/2, which would forbid the (otherwise dominant) decay mode h → ηη.
1 Introduction

The Large Hadron Collider (LHC) at CERN should reveal in the next few years the value and the nature of the Higgs boson mass. The confirmation of the standard model (SM) Higgs sector, with no signs of new physics, would certainly be a very interesting possibility \cite{1, 2}. Most of the community, however, expects (or hopes) that there is a dynamical explanation to the hierarchy problem, and that this explanation will become apparent at the LHC.

One of the possibilities that recently has attracted most attention is the Little Higgs (LH) scenario \cite{3, 4, 5}. Its main motivation has been experimental: the absence of any deviations to the SM predictions in all precision data. Supersymmetry (SUSY), technicolor or the presence of extra dimensions could rise the natural cutoff of the SM and define a theory that is consistent up to the fundamental scale. To do the work, however, this new physics should appear below the TeV, whereas the experiments seem to bound it to be above 5–10 TeV \cite{6}. LH models would release this tension by providing an explanation for the gap between the electroweak (EW) scale and the scale of the new physics. It is not that LH does not imply physics beyond the SM (it does), but being its objective and its structure more simple it tends to be more consistent with the data than these other fundamental mechanisms. The LH idea of the Higgs as a pseudo-Goldstone boson (pseudo-GB) of a broken symmetry could be incorporated into a SUSY \cite{7, 8, 9, 10} or a strongly interacting theory \cite{11, 12} to explain the little hierarchy between the Higgs vacuum expectation value (VEV) and the SUSY breaking scale or the mass of the composite states. The most important consequence is then that it would describe all the new physics to be explored at the LHC.

In this paper we consider a variation of Schmaltz’s model \cite{13} based on a \( SU(3) \times SU(3) \) global symmetry, the so called simplest LH model (see \cite{14, 15} for a review). This model includes two VEVs, \( f_1, f_2 \), that break the global symmetry giving mass to a vectorlike \( T \) quark and to several nonstandard gauge bosons. The modifications that we propose are the following. First, we separate by a sizeable factor the two VEVs, \( f_1 \approx 0.1 f_2 \). This region of the parameter space, identified by other authors \cite{16, 17, 18} as the most promising from a phenomenological point of view, implies a \( T \) quark that can be light (and cancels quadratic corrections) while the extra vector bosons are heavier (and consistent with precision EW data). Second, we also change the usual structure of the couplings in the top-quark sector. Instead of two similar Yukawa couplings that break collectively the global symmetry, we propose that the symmetry is approximate, \( i.e., \) there is one unsuppressed (symmetric) coupling and the rest of them break the symmetry but are smaller by a factor of (at least) \( \approx 0.1 \). This has important implications in the one-loop Higgs potential. In particular, the model is consistent without the need of an extra \( \mu \)-term.
The model includes a lighter scale \( f_1 \) that can be close to the electroweak (EW) scale, \( v/\sqrt{2} \). As a consequence, the non-linear expansion of the Higgs field requires a treatment beyond the usual one at first order in \( v/f_1 \). Here we sum the whole series and express the result as a function of the sine of the ratio \( v/\sqrt{2}f_1 \). The model is minimal in the sense that, in addition to the \( T \) quark, only a neutral scalar field \( r_1 \) gets its mass at \( f_1 \): the rest of scalars, the extra gauge bosons and the nonstandard fermions (up-type quarks and right-handed neutrinos) get masses of the order of the larger scale \( f_2 \). Below \( f_1 \) one is left in the scalar sector with the pseudo-GBs of the global symmetry: the SM Higgs fields plus the extra CP-odd singlet \( \eta \). Therefore, all the exotic physics that the LHC would face below 3 TeV would be a Higgs boson with anomalous gauge and Yukawa couplings (see next section), the neutral scalars \( \eta \) and (possibly) \( r_1 \), and the extra \( T \) quark. We find the one-loop effective potential and show that this setting naturally provides an acceptable EW symmetry breaking with large masses for the Higgs and \( \eta \). We also find that the singlet \( \eta \) may be heavy enough to close the interesting decay channel \( h \to \eta \eta \).

2 Little Higgs or extra singlet model?

Let us start reviewing the model in some detail \cite{13}. The scalar sector contains two triplets, \( \phi_1 \) and \( \phi_2 \), of a global \( SU(3)_1 \times SU(3)_2 \) symmetry:

\[
\phi_1 \to e^{i\theta_1 T^a} \phi_1 \ , \quad \phi_2 \to e^{i\theta_2 T^a} \phi_2 ,
\]

where \( T^a \) are the generators of \( SU(3) \). It is then assumed that these triplets get VEVs \( f_{1,2} \) and break the global symmetry to \( SU(2)_1 \times SU(2)_2 \). The spectrum of scalar fields at this scale will consist of 10 massless modes (the GBs of the broken symmetry) plus two massive fields (with masses of order \( f_1 \) and \( f_2 \)). If one combination of the two global \( SU(3) \) is made local, some of the GBs will be eaten by massive gauge bosons and the rest will define the EW scalar sector.

In particular, if the two VEVs are

\[
\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} , \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} ,
\]

and the diagonal combination of \( SU(3)_1 \times SU(3)_2 \) is local,

\[
\phi_{1(2)} \to e^{i\theta_a^a T^a} \phi_{1(2)} ,
\]
then the VEVs will break the local $SU(3) \times U(1)_Y$ to the standard $SU(2)_L \times U(1)_Y$, a process that takes 5 GBs. The other 5 GBs (the complex doublet $(h^0 h^-)$ and a CP-odd singlet $\eta$) can be parametrized non-linearly \[25]\:

$$
\phi_1 = e^{+ i \frac{f_1}{f_1}} \begin{pmatrix}
0 \\
0 \\
f_1 + \frac{r_1}{\sqrt{2}}
\end{pmatrix}, \quad \phi_2 = e^{- i \frac{f_2}{f_2}} \begin{pmatrix}
0 \\
0 \\
f_2 + \frac{r_2}{\sqrt{2}}
\end{pmatrix},
$$

(4)

where

$$
\Theta = \frac{1}{f} \begin{pmatrix}
\eta/\sqrt{2} & 0 & h^0 \\
0 & \eta/\sqrt{2} & h^- \\
\theta^0 & h^+ & \eta/\sqrt{2}
\end{pmatrix},
$$

(5)

$$
f = \sqrt{f_1^2 + f_2^2}, \quad \text{and} \quad r_1(2) \text{ is a scalar with mass of order } f_{1(2)}.
$$

If the extra vector bosons present in these models were not heavy enough (we deduce their masses in Section 4), they would introduce unacceptable mixing with the EW gauge bosons and four fermion operators upsetting LEP2 data and atomic parity experiments. This forces that $f$ must be large, above 3 TeV \[13, 16, 17, 18\]. However, this can be achieved with just one large VEV, $f_2 \geq 3$ TeV, leaving $f_1$ unconstrained. We will then assume that $f_2$ is large and consider values of $f_1$ between $v/\sqrt{2} = 174$ GeV and 1 TeV.

The global symmetry is not exact (see below), so the Higgs boson will get a one-loop potential and a VEV,

$$
\langle h^0 \rangle = u/\sqrt{2}.
$$

(6)

Such VEV implies the triplet VEVs

$$
\langle \phi_1 \rangle = \begin{pmatrix} if_1 s_1 \\
0 \\
f_1 c_1
\end{pmatrix}, \quad \langle \phi_2 \rangle = \begin{pmatrix} -i f_2 s_2 \\
0 \\
f_2 c_2
\end{pmatrix},
$$

(7)

where

$$
s_1 \equiv \sin \frac{u f_2}{\sqrt{2} f_1}, \quad s_2 \equiv \sin \frac{u f_1}{\sqrt{2} f_2}.
$$

(8)

Since the two upper components in the triplets transform as an $SU(2)_L$ doublet, it is clear that to obtain the observed $W$ and $Z$ masses one needs

$$
\sqrt{f_1^2 s_1^2 + f_2^2 s_2^2} = \frac{v}{\sqrt{2}} \approx 174 \text{ GeV}.
$$

(9)

In the limit with $f \approx f_2 \gg f_1$ that we are considering $s_1$ may be large, as $s_1 \approx v/(\sqrt{2} f_1)$.

\[^1\]These terms would be absent in models with a $T$ parity \[20\] or a smaller gauge group \[27\].
An important observation here is the following. Once $h^0$ gets the VEV $u/\sqrt{2}$ and we expand it (in the unitary gauge) as $h^0 = (u + h)/\sqrt{2}$, we obtain that the physical Higgs $h$ has both doublet and singlet components (in the first and third entries of the triplet, respectively):

$$
\begin{pmatrix}
  i f_1 (s_1 \cos \frac{h}{\sqrt{2} f_1} + c_1 \sin \frac{h}{\sqrt{2} f_1}) \\
  0 \\
  f_1 (c_1 \cos \frac{h}{\sqrt{2} f_1} - s_1 \sin \frac{h}{\sqrt{2} f_1})
\end{pmatrix}
= \frac{1}{\sqrt{2}}
\begin{pmatrix}
  ic_1 h \\
  0 \\
  -s_1 h
\end{pmatrix}
+ ...
$$

(10)

If $f_1$ is much larger than the EW scale, $s_1$ is small and $h$ is predominantly a doublet. However, as $f_1$ approaches $v/\sqrt{2}$ the singlet component $s_1$ grows. Where did the doublet component go? It is easy to see that it went to the scalar $r_1$ that gets massive at $f_1$.

This is a generic feature in any LH model. The scale $f$ of global symmetry breaking is always defined by the VEV of an $SU(2)_L$ singlet, that gets a mass of order $f$. Then the EW symmetry breaking mixes the Higgs $h$ (the pseudo-GB of the global symmetry) with this massive singlet. Since the singlet component of $h$ does not couple, both its gauge couplings $g$ and $g'$ and its Yukawa couplings $\sqrt{2}m_f/v$ will appear suppressed by a factor of $c_1$. These anomalous LH couplings have nothing to do with the non-linear realization of the pseudo-GBs, they just reflect the mixing with the scalar singlet massive at the scale of global symmetry breaking. In our case, since $f_1$ can be close to $v/\sqrt{2}$ while consistent with all precision data, the effect may be large and observable at the LHC [23].

Notice also that the Little Higgs $h$, not being a pure doublet, only unitarizes partially the SM cross sections involving massive vector bosons. In particular, the cutoff at $\approx 1.7$ TeV set by $WW$ elastic scattering would be moved up to $(1.7/s_1)$ TeV. Below that scale the massive scalar $r_1$ (or other field) should complete the unitarization.

A final comment concerns the limit $f_1 \to v/\sqrt{2}$. The pseudo-GB $h$ becomes there a pure $SU(2)_L$ singlet, and the (unprotected) field $r_1$, massive at the scale $f_1$, becomes a doublet and is the real Higgs that breaks the EW symmetry. In this limit the naturality cutoff would be the same as in the SM, whereas in the general case with $f_1 > v/\sqrt{2}$ it is at $\approx 4\pi f_1$.

### 3 Fermion masses

Let us start discussing the top quark Yukawa sector. Since at the scale $f$ the local symmetry is $SU(3) \times U(1)_X$, we must include the doublet $Q^T = (t \ b)$ in a triplet $\Psi_Q^T = (t \ b \ T)$, together
with two singlets, \( t_1^c \) and \( t_2^c \). The Lagrangian may then contain the four couplings

\[
- \mathcal{L}_t = \lambda_1 \phi_1^\dagger \Psi_Q t_1^c + \lambda_2 \phi_2^\dagger \Psi_Q t_2^c + \\
\quad \lambda_1' \phi_1^\dagger \Psi_Q t_2^c + \lambda_2' \phi_2^\dagger \Psi_Q t_1^c + \text{h.c.},
\]

where all the fermion fields are two-component spinors. We will assume that only \( \lambda_1 \) is of order one and that the rest of the couplings are one order of magnitude smaller. This could be justified if the global \( SU(3)_1 \times SU(3)_2 \) symmetry is approximate in this sector. If \( \Psi_Q \) is a triplet under \( SU(3)_1 \), then the terms \( \lambda_1 \) and \( \lambda_1' \) will be unsuppressed (symmetric), whereas \( \lambda_2 \) and \( \lambda_2' \) break the symmetry and will be smaller. We can then redefine the fields \( t_{1,2}^c \) so that \( \lambda_1 \to \sqrt{\lambda_1^2 + \lambda_1'^2} \) and \( \lambda_1' \to 0 \), i.e., with all generality we can take \( \lambda_1' = 0 \) and \( \lambda_2 \) and \( \lambda_2' \) small.

In the original simplest LH model \( \lambda_1' = 0 \) and \( \lambda_1, \lambda_2 \) break collectively the symmetry (i.e., only in diagrams that contain simultaneously both couplings)\(^2\). Here the diagrams that only involve the large top-quark Yukawa coupling (\( \approx \lambda_1 \)) do not break the symmetry, and this is enough to rise the natural cutoff of the SM above LHC energies.

Keeping the exact dependence on \( f_{1,2} \), on the Higgs VEV \( u \), and on the possible \( CP \)-odd singlet VEV \( \langle \eta \rangle = y \), and performing appropriate phase redefinitions of the fermion fields, we obtain the mass matrix

\[
- \mathcal{L}_t \supset \begin{pmatrix} t & T \end{pmatrix} \begin{pmatrix} \lambda_1 f_1 s_1 - e^{i \theta} \lambda_2 f_2 s_2 & -\lambda_2 f_2 s_2 + e^{-i \theta} \lambda_1' f_1 s_1 \\ \lambda_1 f_1 c_1 + e^{i \theta} \lambda_2 f_2 c_2 & \lambda_2 f_2 c_2 + e^{-i \theta} \lambda_1' f_1 c_1 \end{pmatrix} \begin{pmatrix} t_1^c \\ t_2^c \end{pmatrix},
\]

(12)

where

\[
\theta = \frac{y f}{\sqrt{2} f_1 f_2}
\]

(13)

and \( \lambda_{1,2} \) are both real. Several comments are here in order. First, the mass of the extra \( T \) quark is just

\[
\begin{align*}
\lambda_1^2 + \lambda_1'^2 \end{align*}
\]

\[
(\lambda_1^2 + \lambda_1'^2) f_1^2 + (\lambda_2^2 + \lambda_2'^2) f_2^2 + 2(\lambda_1 \lambda_2' + \lambda_2 \lambda_1') f_1 f_2 c_{12} c_\theta - m_T^2,
\]

(14)

with

\[
c_{12} \equiv \cos \frac{u f}{\sqrt{2} f_1 f_2}.
\]

(15)

Since all the couplings except for \( \lambda_1 \) are small, and this coupling only contributes to \( m_T \) multiplied by the lower VEV \( f_1 \), the extra \( T \) quark will have a mass of order \( f_1 \). Second, notice that if \( \lambda_1' = 0 \) (the collective breaking case) then the fermion masses do not depend on the value \( y \) of the singlet \( \eta \). As a consequence, \( \eta \) will not get an effective potential and

\(^2\) If \( \lambda_2' = 0 \) two-loop diagrams generate corrections of order \( \lambda_1^3 \lambda_2^3/(16\pi^2)^2 \).
will remain massless at that order. We show in the next section that in this case the Higgs mass is always below present bounds from LEP [24].

The smaller up and charm quark masses could appear if the assignments for the quark triplets under the approximate symmetry are different: triplets under the second $SU(3)_2$, singlets under the first one. In particular, the only large Yukawas (one per family) should couple these triplets with $\phi_2$. That would make the extra up-type quarks very heavy ($m_{C,U} \approx f_2$), whereas the up and the charm fields would couple to the Higgs with suppressed Yukawa couplings.

Down-type quarks (and also charged leptons) may get their mass through dimension 5 operators [13] like

$$-L_b \approx \frac{y_b}{f} \phi_1 \phi_2 \Psi_Q b^c + \text{h.c.} ,$$

but they do not require extra fields not large couplings.

Finally, here the lepton doublets become triplets that include a $SU(2)_L$ singlet: $\psi^T_L = (\nu e N)$. This forces the addition of a fermion singlet $n_c$ per family and the Yukawa couplings

$$-\mathcal{L}_\nu = \lambda_1^{\nu} \phi_1^\dagger \Psi_L n^c + \lambda_2^{\nu} \phi_2^\dagger \Psi_L n^c + \text{h.c.} .$$

The approximate symmetry should imply then $\lambda_2^{\nu} \approx 1 \gg \lambda_1^{\nu}$, and the two extra fermions ($N$ and $n^c$) would combine into a Dirac field of mass $\approx f_2$. For the light neutrinos, in these models there is an alternative to the usual see-saw mechanism. In [28] it is shown that a small lepton number violating mass term

$$-\mathcal{L}_\nu \supset \frac{1}{2} m_n n^c n^c + \text{h.c.}$$

of order 0.1 keV would generate a one-loop neutrino mass $m_\nu \approx 0.1$ eV.

In summary, in this model all the extra right-handed neutrinos and up-type quarks except for the one cancelling top-quark quadratic corrections can be very heavy, with masses around $f_2 \approx 3$ TeV. As for the extra $T$ quark, all precision bounds are respected if its mixing $V_{Tb}$ with the standard top quark is smaller than $\approx 0.2$ [29] [23].

4 Gauge boson masses

The $SU(3) \times U(1)_X$ gauge boson masses come from terms $(D^\mu \Phi_i)^\dagger (D_\mu \Phi_i)$ in the Lagrangian. In the charged sector we have

$$D_\mu \phi_1 \supset -ig \sum_{i=1,2,6,7} A_\mu^i T^i \phi_1 = \frac{gf_1}{\sqrt{2}} \begin{pmatrix} 0 \\ s_1 W_\mu - c_1 W'_\mu \\ 0 \end{pmatrix} ,$$

\[ 
\begin{pmatrix} 0 \\ s_1 W_\mu - c_1 W'_\mu \\ 0 \end{pmatrix} ,
\]
where we have defined
\[ W = \frac{1}{\sqrt{2}} \left( A_1^1 + i A_2^1 \right) ; \quad W' = \frac{1}{\sqrt{2}} \left( A_1^7 + i A_2^6 \right), \]
(20)
with an analogous expression for \( D_\mu \phi_2 \). In this basis the mass matrix reads
\[
\frac{g^2}{2} \begin{pmatrix}
- g f_1^2 s_1^2 + g f_2^2 s_2^2 & g f_2^2 s_2 c_2 - g f_1^2 s_1 c_1 \\
- g f_2^2 s_2 c_2 - g f_1^2 s_1 c_1 & g f_1^2 c_1^2 + g f_2^2 c_2^2
\end{pmatrix},
\]
(21)
and has the eigenvalues
\[
M_{W_1(W_2)}^2 = \frac{g^2 f_2^2}{4} \left( 1 - (\pm) \sqrt{1 - s_{2\beta}^2 s_{12}^2} \right),
\]
(22)
where
\[
s_{2\beta} \equiv 2 f_1 f_2 / f_2. \tag{23}
\]
Notice that the two masses add to a constant independent of the Higgs VEV (in \( s_{1,2} \)), which will imply no quadratic divergencies in the potential at the one-loop level.

In the neutral sector we find
\[
D_\mu \phi_1 \supset \left( -ig \sum_{i=3,4,5,8} A_i^i T^i \phi_1 + \frac{igX}{3} A^X_\mu \right)
\]
\[
= \frac{gf_1}{2} \begin{pmatrix}
- s_1 \sqrt{1 + t^2} Z_\mu + s_1 \frac{1 - t^2}{\sqrt{3 - t^2}} Z'_\mu - c_1 A^5_\mu - i c_1 A^4_\mu \\
0
\end{pmatrix},
\]
(24)
where
\[
t = \frac{g'}{g} = \sqrt{3} \frac{gX}{\sqrt{3 g^2 + g_X^2}}\tag{25}
\]
and
\[
Z'_\mu = \sqrt{1 - \frac{t^2}{3}} A^8_\mu + \frac{t}{\sqrt{3}} A^X_\mu,
\]
\[
Z_\mu = \frac{1}{\sqrt{1 + t^2}} \left( A^3_\mu + \frac{t^2}{\sqrt{3}} A^8_\mu - t \sqrt{1 - \frac{t^2}{3}} A^X_\mu \right). \tag{26}
\]
The mass matrix of \((Z_\mu, Z'_\mu, A^4_\mu, A^5_\mu)\) is then
\[
\frac{g^2}{2} \begin{pmatrix}
(1 + t^2)(f_1^2 s_1^2 + f_2^2 s_2^2) & \frac{(1-t^2)\sqrt{1+t^2}}{\sqrt{3-t^2}} (f_1^2 s_1^2 + f_2^2 s_2^2) & 0 & -\sqrt{1+t^2}(f_1^2 s_1 c_1 - f_2^2 s_2 c_2) \\
\frac{(1+t^2)\sqrt{1+t^2}}{\sqrt{3-t^2}} (f_1^2 s_1^2 + f_2^2 s_2^2) & \frac{(1-t^2)\sqrt{1+t^2}}{3-t^2} (f_1^2 s_1^2 + f_2^2 s_2^2) & 0 & \frac{1+t^2}{\sqrt{3-t^2}}(f_1^2 s_1 c_1 - f_2^2 s_2 c_2) \\
0 & 0 & f^2 & 0 \\
-\sqrt{1+t^2}(f_1^2 s_1 c_1 - f_2^2 s_2 c_2) & \frac{1-t^2}{\sqrt{3-t^2}}(f_1^2 s_1 c_1 - f_2^2 s_2 c_2) & 0 & f^2
\end{pmatrix}.
\]

It is easy to see that, again, the trace of this matrix does not depend on the Higgs VEV:

\[\text{Tr} [M^2] = \frac{g^2}{2} \left( \frac{4}{3 - t^2} + 2 \right) f^2.\] (28)

The mixing terms of $A_5^\mu$ with $Z_\mu$ and $Z_\mu'$ were overlooked in [13]. Although they cancel at the lowest order in $v/(\sqrt{2} f)$, we show in the next section that they are essential to obtain the right ultraviolet (UV) dependence of the effective potential.

### 5 One-loop potential from collective breaking

Gauge and Yukawa couplings break in this model the global symmetries and introduce a one-loop potential for the pseudo-GBs. To be realistic, we need that the potential implies the right Higgs VEV and an acceptable mass for $h$. The potential can be given in terms of the fermion and boson masses expressed as a function of the Higgs. We will consider here the contributions from the top-quark (the rest of Yukawas do not introduce any new effects) and the gauge sectors.

From the top-quark sector we have

\[V_{\text{top}} = -\frac{3}{16\pi^2} \Lambda^2 \text{Tr} [m^\dagger m] + \frac{3}{16\pi^2} \text{Tr} [(m^\dagger m)^2 \log \left( \frac{\Lambda^2}{m^2} \right)].\] (29)

Let us first discuss the collective breaking case, with $\lambda'_{1,2} = 0$ in Eq. (11). The two mass eigenvalues are just

\[m^2_{i(T)}(h) = \frac{M^2}{2} \left( 1 - (+) \sqrt{1 - s_{2\alpha}^2 s_{12}^2(h)} \right),\] (30)

with

\[s_{12}(h) = \sin \frac{hf}{\sqrt{2 f_1 f_2}};\]
\[
\begin{align*}
M^2 &= \lambda_1^2 f_1^2 + \lambda_2^2 f_2^2; \\
\alpha_2 &= \frac{2\lambda_1 \lambda_2 f_1 f_2}{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}.
\end{align*}
\]

\(V_{\text{top}}\) presents in this case several important features. First, \(m_t^2 + m_T^2\) is a constant (does not depend on \(h\)), so the quadratic divergence (first term in (29)) is zero. Second, we can write (up to a constant)

\[
V_{\text{top}} = \frac{3}{16\pi^2} m_t^4 \log \left( \frac{m_T^2}{m_t^2} \right) + \frac{3}{16\pi^2} (m_t^4 + m_T^4) \log \left( \frac{\Lambda^2}{m_T^2} \right).
\]

As noticed in [15], this potential can be understood as the usual quartic up-quark correction below \(m_T\), plus an \(SU(3)\)-symmetric correction proportional to

\[
m_t^4 + m_T^4 = \frac{M_4^4}{2} (2 - \alpha_2^2) \ s_{12}^2(h)
\]

above that scale. This second contribution is logarithmically divergent, and it would redefine (renormalize) the quartic

\[
V_{UV} = a \left( \phi_1^+ \phi_2 \right) \left( \phi_2^+ \phi_1 \right) \supset a \ f_1^2 f_2^2 \left( 1 - \alpha_2^2 \ s_{12}^2(h) \right).
\]

The sensitivity of the potential to the physics in the UV can be accounted by taking \(a\) as a free parameter or, equivalently, setting \(a = 0\) and varying freely the cutoff. We will take this second approach, defining in this sector an arbitrary cutoff \(\Lambda_t\) that may be different from the one in the gauge sector, \(\Lambda_g\). Notice also that any UV contribution to the (adimensional) parameter \(a\) should be small, as this parameter breaks the global symmetry.

Let us analyze now the gauge sector. The one-loop contribution to the effective potential is

\[
V_{\text{gauge}} = \frac{3}{64\pi^2} \Lambda_g^2 \text{Tr} \left[ M^2 \right] + \frac{3}{64\pi^2} \text{Tr} \left[ M^4 \log \left( \frac{\Lambda_g^2}{M^2} \right) \right]
\]

Again, (i) the quadratic divergence vanishes, (ii) below the scale \(\approx gf\) of the massive vector bosons one has the usual \(W^\pm, Z\) corrections, and (iii) above that scale there is an \(SU(3)\)-symmetric logarithmic divergence proportional to the sum of all vector bosons masses to the fourth power. In particular, in the charged sector \(W_i\) carries particle plus antiparticle

\[
M_{W_1}^4 + M_{W_2}^4 = \frac{g^4 f^4}{4} \left( 1 - \frac{1}{2} s_{2\theta}^2 \ s_{12}^2(h) \right)
\]

whereas the four neutral vectors give

\[
\sum_{i=1}^{4} M_{Z_i}^4 = \frac{g^4 f^4}{2} \left( 1 + \frac{8}{(3 - t^2)^2} - \frac{1 + t^2}{3 - t^2} s_{2\theta}^2 \ s_{12}^2(h) \right).
\]
Figure 1: Higgs mass in the collective breaking case for \( f_2 = 3 \) TeV and different values of \( f_1 \).

These divergent terms will renormalize a combination of the operator in (34) and

\[
V'_{\text{UV}} = b \left( \phi_1^4 \phi_2 \right)_8 \left( \phi_2^4 \phi_1 \right)_8 \supset \frac{2b}{3} f_1^2 f_2^2 \left( 2 + s_{12}^2(h) \right). \tag{38}
\]

We find that the UV physics (quartic terms proportional to \( a \) and \( b \) or the cutoffs \( \Lambda_t, \Lambda_g \) in the top-quark and gauge sectors) can only define the coefficient of a term proportional to \( s_{12}^2(h) \). This single arbitrary parameter from the UV completion will not be enough (see below) to obtain an acceptable Higgs mass.

Let us fix \( f_2 \) at 3 TeV and vary \( f_1 \) between 200 GeV and 1 TeV. In the model with collective breaking (i.e., \( \lambda'_{1,2} = 0 \) in the top-quark sector) the effective Higgs potential will change with the values of \( \lambda_{1,2} \) and the cutoff \( \Lambda_t \) (as explained before, the potential is only sensitive to a combination of \( \Lambda_t \) and \( \Lambda_g \), so we fix \( \Lambda_g \) at 5 TeV). These three parameters must produce \( M_Z = 91 \) GeV (i.e., the right Higgs VEV) and \( m_t = 171 \) GeV. We will require that the extra \( T \) quark has a mass below 2 TeV (in order to cancel naturally top-quark quadratic corrections), and that its mixing \( V_{Tb} \) with the top is smaller than 0.25. In Fig. 1 we plot the maximum value of the Higgs mass for different values of \( f_1 \) and any consistent value of the other parameters. All these values of \( m_h \) are far below the LEP bound of 121 GeV \cite{24}, a fact that does not change increasing \( f_2 \).
6 Effective potential in the minimal model

As described in Section 2, we propose a model with $f_1 \approx 0.1 f_2$, $\lambda_1 \approx 1$, and the rest of the couplings in the top quark sector at least one order of magnitude smaller. The model allows heavy extra gauge bosons while the $T$ quark that cancels top corrections can be below 1 TeV. Notice that, since the global symmetry in the top sector is approximate, the cancelation of one-loop quadratic corrections will also be approximate. This suffices to increase the natural cutoff of the model from 1 TeV up to 5–10 TeV. On the other hand, in the one-loop effective potential there will be new operators breaking the global symmetry that do not appear in the collective breaking case. As a consequence, the Higgs will get a mass above LEP bounds, and also the scalar singlet $\eta$ will acquire an acceptable mass.

The suppressed couplings $\lambda'_{1,2}$ in the top-quark Yukawa sector imply that $\Phi_Q t^c$ couple both to $\Phi_1$ and $\Phi_2$, introducing the one-loop quadratic divergence

$$\Delta V_{\text{top}} = - \frac{3}{16\pi^2} \Lambda_t^2 \left( f_1^2 \left( \lambda_1^2 + \lambda_1'^2 \right) + f_2^2 \left( \lambda_2^2 + \lambda_2'^2 \right) + 2 f_1 f_2 \left( \lambda_1 \lambda_2 + \lambda_2 \lambda'_1 \right) \cos \frac{h f}{\sqrt{2} f_1 f_2} \cos \frac{\eta f}{\sqrt{2} f_1 f_2} \right).$$

(39)

This term is determinant in order to obtain an acceptable potential because it is proportional to $c_{12}(h)$, while before all the UV-dependent contributions (both from the top-quark and the gauge sectors) were proportional to $s_{12}^2(h)$ (i.e., $c_{12}(h)$). To illustrate that, we consider a particular set of parameters for $f_1 = 400$ GeV and $f_2 = 3$ TeV. We take $\lambda_1 = 1.19$, $\lambda_2 = -0.25$, $\lambda_2' = 0.03$ and $\lambda_1' = 0$ (see Section 3). We fix the UV cutoff in the top-quark sector to $\Lambda_t = \Lambda_g = 5$ TeV, but we include the UV dependent coupling $a$ in Eq. (34) with the value $a = 1/(16\pi^2)$. In Fig. 2 we separate the contributions from the top and the gauge sectors (we have added $V_{\text{UV}}$ to $V_{\text{top}}$). We plot the potential expressed as a function of $s_1(h)$. In the minimum, $s_1 = 0.43$ (i.e., $u = 259$ GeV), which reproduces the values $M_Z = 91$ GeV and $m_t = 171$ GeV, with an extra $T$ quark of 920 GeV. This potential implies $m_h = 156$ GeV and $m_\eta = 107$ GeV. Increasing the value of $f_1$, changing the value of $\lambda_2'$, and varying the parameter $a$ we obtain Higgs masses above 200 GeV.

In order to see if the solutions that we find involve any amount of fine tuning, we have varied in a $\pm 5\%$ the VEV $f_1$, the large Yukawa coupling $\lambda_1$, and the UV-dependent coupling $a$ from the values given above. In Fig. 3 we plot the changes in the Higgs potential caused by the variation of each one of these parameters. We obtain that the EW scale $v/\sqrt{2}$ changes between $+20\%$ and $-25\%$ respect to the central values, whereas the Higgs mass moves between 126 and 178 GeV. This result shows that the Higgs sector of the model does not involve any severe degree of fine tuning (just $1 - 5/20 = 75\%$ cancellations).
Figure 2: One-loop Higgs potential as a function of \( s_1(h) \) for the choice of parameters given in the text.

Figure 3: Variation of the Higgs potential for a ±5% variation of \( f_1 \) (short dashes) \( \lambda_1 \) (long dashes) and \( a \) (dots) versus the central values given in the text. The EW scale \( v/\sqrt{2} \) changes in up to a +20% or a −25%, whereas \( m_h \) varies between 126 and 178 GeV.
7 Summary and discussion

The Higgs doublet may be the pseudo-GB of a global symmetry broken spontaneously at a higher scale $f$. The EW symmetry breaking will then define a physical Higgs $h$ that has $SU(2)_L$ doublet and singlet components, while a scalar singlet of mass $\approx f$ acquires a doublet component. LH models include an extra $T$ quark that cancels top-quark quadratic corrections to the Higgs bilinear. This cancellation is effective (making the model natural) if the mass of the $T$ quark is below 1 TeV. On the other hand, the extra gauge bosons present in these models also get a mass of order $f$, which may conflict precision EW data if $f < 3$ TeV.

We have shown that Schmaltz’s simplest LH model can naturally accommodate heavy gauge bosons, a lighter $T$ quark, and an acceptable Higgs mass if (i) $f_1 \approx 0.1 f_2$ and (ii) in the top-quark sector there is only one large Yukawa coupling, $\lambda_2, \lambda'_{1,2} \lesssim 0.1 \lambda_1$. The second condition makes the global symmetry approximate, in contrast with the usual scenario with $\lambda_1 \approx \lambda_2$ and $\lambda'_{1,2} = 0$.

We have studied in detail the Coleman-Weinberg potential and have shown that under these two conditions the model gives naturally acceptable EW minima with a Higgs mass above present bounds. It is essential to work at all order in $v^2/f^2$ (we express the results in terms of the sine of this ratio), as the usual first order expansion fails in the cases with low $f_1$ considered here. In particular, we have found that in the collective breaking case with $\lambda_{1,2} = 0$ it is impossible to obtain a Higgs mass above LEP bounds. The basic reason is that both top-quark and gauge corrections give a logarithmic divergent term proportional to $s_{12}(h)$. This fact is not apparent in the calculation of the potential at first order in $h^2/f^2$ given in [13], as the author overlooks the mixings of $A_\mu^5 (W_0^\prime, 0)$ there with the other neutral gauge bosons. We have shown that in the framework with $f_1 \ll f_2$ it is natural to have an approximate global symmetry in the Yukawa sector, since just one large coupling per flavour is enough to generate the top-quark mass and masses above the TeV for all the extra fermions but $T$. This also provides an acceptable Higgs mass with no need for a $\mu$ term $\phi_1^\dagger \phi_2$ put by hand in the scalar potential. In addition, we showed that the mass of the pseudoscalar $\eta$ may be here larger than in the usual scenario with collective breaking and an extra $\mu$ term, closing (or reducing) kinematically the Higgs decay channel $h \rightarrow \eta \eta$. This decay mode could provide very interesting signals in a hadron collider when the Higgs is produced together with a $W$ or a $Z$ gauge boson [22].

The framework discussed here is a minimal departure from the SM with features that

\footnote{The authors in [21] find acceptable cases working at first order in $v^2/f^2$}
should be observed at the LHC for the preferred values of $f_1$ below 500 GeV. The Higgs appears mixed with a singlet that may be as heavy as $4\pi f_1$ (when the LH is completed with strongly interacting physics [12]). The observation of its anomalous gauge or fermion couplings, and/or the observation of a vectorlike $T$ quark, would then reveal the new scale $f_1$. The resulting model, with a natural cutoff higher than the one in the SM but right above LHC energies, would certainly be an invitation to plan for a bigger collider.

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