Quasiparticle Nernst effect in the cuprate superconductors from the $d$-density wave theory of the pseudogap phase

Chuanwei Zhang,$^1$ Sumanta Tewari,$^2$ and Sudip Chakravarty$^3$

$^1$Department of Physics and Astronomy, Washington State University, Pullman, WA 99164
$^2$Department of Physics and Astronomy, Clemson University, Clemson, SC 29634
$^3$Department of Physics and Astronomy, University of California, Los Angeles, CA 90095

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We consider the Nernst effect in the underdoped regime of the cuprate high temperature superconductors within the $d$-density wave (DDW) model of the pseudogap phase. By a combination of analytical and numerical arguments, we show that there is a robust low-temperature positive peak (i.e., maximum) in the temperature dependence of the Nernst coefficient when the DDW state is ambipolar, i.e., when the broken symmetry supports the coexistence of both electron- and hole-like quasiparticles in the excitation spectrum, and the electron pocket dominates at the low temperatures. In contrast, the Nernst coefficient is negative and there is no such positive peak if the underlying state is non-ambipolar, i.e., when it supports only one type of quasiparticles. More generally, in the ambipolar state, the sign of the Nernst coefficient can be positive or negative depending on the dominance of the electron or hole pockets, respectively, in the low temperature thermoelectric transport. By modeling the pseudogap phase by a doping-dependent DDW order parameter with a Fermi surface topology that supports both hole and electron pockets, and assuming energy-independent transport scattering times, we analyze the evolution of the Nernst effect with doping concentration at low temperatures in the cuprate phase diagram. Even though the chosen ambipolar DDW state with a specific Fermi surface topology is not the only possible explanation of either the recent quantum oscillation experiments or the recent observation of a negative Hall coefficient at low temperatures in the underdoped cuprates, it is at least one possible state qualitatively consistent with both of these experiments. As such, the calculations in this paper present at least one possible scenario for the observed enhanced Nernst signals in the underdoped cuprates.

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I. INTRODUCTION

Even after two decades of intensive efforts, the normal state properties of the cuprate superconductors in the intermediate range of hole doping, called underdoping, are still poorly understood. At low doping, close to the undoped antiferromagnetic phase the behavior of the system is influenced by the parent Mott insulator. At doping level above that corresponding to the maximum superconducting transition temperature ($T_c$), the mobile holes in the normal state constitute a Fermi liquid. However, at the doping range intermediate between these two limits, the system evinces a gap in the spectrum of unidentified origin (pseudogap) below a temperature scale $T^* > T_c$. Many properties of the system in this phase, called the pseudogap phase, are strongly influenced by the gap, which is, similar to the superconducting gap below $T_c$, anisotropic in the momentum space. An understanding of the pseudogap, and the associated loss of the spectral weight from the Fermi surface, is widely believed to hold the key to the high transition temperature in the cuprates. The existence of the gap, even in the absence of super-conduction above $T_c$, have led many theorists to propose exotic non-Fermi-liquid states to be responsible for the pseudogap in the cuprates. However, recent quantum oscillation experiments have found evidence of Fermi pockets even in the enigmatic pseudogap phase. This has rekindled the encouraging prospect of describing this phase in terms of a state with a broken symmetry and a reconstructed Fermi surface treating its hole- and electron-like low energy quasiparticles within a well-defined Fermi-liquid-like description. Note that the Fermi arc picture, as observed in the angle resolved photoemission (ARPES) experiments and the Fermi pocket picture inferred from quantum oscillation are at odds with each other, constituting a major puzzle in the field. There have been many density wave scenarios in which the coherence factors involved in ARPES, but not in quantum oscillation calculations, destroy half of the pockets, giving the appearance of a Fermi arc. On the other hand, ARPES has also revealed the existence of pockets in some recent experiments.

One of the important unsettled questions about the pseudogap phase concerns the low temperature Nernst effect. The Nernst effect experiments measure the transverse electric field response of a system to a combination set-up of an externally-imposed temperature gradient and an orthogonal magnetic field. Early experiments on the Nernst effect in the cuprates revealed a very large signal (compared to that of a Fermi liquid) near $T_c$, which is expected because of the presence of the large number of mobile vortices at these temperatures. The large signal, however, appeared to onset at a temperature far above $T_c$, leading to speculations that there are well-defined, vortex-type, excitations even at such high temperatures. More recent experiments have claimed
to find two peaks in the temperature dependence of the Nernst coefficient, one arising from the onset of a density wave order in the pseudogap phase, and the other due to the onset of the superconducting phase. There is also recent evidence of finding a weak peak in the Nernst signal in the pseudogap phase whose sign is opposite to that expected from the vortex-like excitations. These recent developments, therefore, point to the importance of the quasiparticle Nernst effect associated with an underlying density wave state in the pseudogap regime of the cuprates. While the experimental scenario still needs to be settled, in this paper we deduce the full temperature and doping dependence of the quasiparticle Nernst effect associated with the $d$-density wave state, which has otherwise shown encouraging consistency with the anomalous phenomenology of the underdoped cuprates.

Since the pseudogap has a $d$-wave symmetry, one natural density wave state which could explain it is the $d_{x^2−y^2}$-density wave state. Indeed, much of the phenomenology of the cuprates in the underdoped regime can be unified by making a single assumption that the ordered DDW state is responsible for the pseudogap. The development of the DDW order below optimal doping can lead to a consistent explanation of numerous experimental observations including the abrupt suppression of the superfluid density, and Hall numbers below optimal doping as well as the more recent quantum oscillation experiments. Theoretically speaking, any appropriate Hamiltonian that leads to $d$-wave superconductivity in the underdoped regime of the cuprates will almost certainly favor DDW order as well. The DDW order might also have been directly observed in two polarized neutron scattering experiments, even though some other experiments failed to observe it. The ordered ambipolar DDW state and its associated Fermi surface topology (Fig. 1) are also qualitatively consistent with the quantum oscillation experiments in the pseudogap regime. The quantum oscillation experiments indicate that the Fermi surface in the underdoped cuprates is made up of small reconstructed Fermi pockets, giving rise to both hole and electron-like charge carriers (quasiparticle ambipolarity) in the excitation spectrum. Such a feature is quite robust for the DDW state, in which, for generic values of the band structure and gap parameters, the low energy spectrum consists of both electron and hole-like quasiparticles (Fig. 1).

We will derive the implications of the above important new ingredient in the cuprate physics on the quasiparticle Nernst coefficient of the DDW state. Using quasiclassical Boltzmann theory of transport, we will show that the reconstructed Fermi surface in the DDW state and its low energy quasiparticle ambipolarity can successfully explain the enhanced Nernst signals as found in the experiments at temperatures much above $T^*$.

Even though strong electronic interactions present in the host material are crucial for the formation of the DDW state, deep in the ordered state the quasiparticles can be assumed to be non-interacting (or weakly-interacting).

Therefore, we assume that the Boltzmann theory is still applicable to calculate the transport properties of the quasiparticles in the presence of a well-developed DDW order parameter.

By a combination of analytical and numerical arguments, we show that a low-temperature peak in the Nernst coefficient is very robust in the ambipolar $d$-density wave state. In fact, the existence of the peak is solely due to the dominance of the two types of quasiparticles (electron and hole) at different regimes of temperatures, and is insensitive to the microscopic details. Therefore, quasiparticle ambipolarity of the underlying state, as indicated in the quantum oscillation experiments, is also crucial for the low temperature peak in the Nernst coefficient. We also find that the sign of the peak of the Nernst effect can be positive or negative depending on the dominance of the electron or hole pockets, respectively, in the low temperature thermoelectric transport. By modeling the pseudogap by a suitable, doping-dependent, $d$-density wave order parameter, we analyze the doping dependent evolution of the Nernst effect at a fixed low temperature in a range of hole-concentrations in the underdoped regime of the cuprate phase diagram. The quasiparticle Nernst effect has also been recently studied within the stripe order model of the underdoped cuprates.

The paper is organized as follows: Section II introduces the commensurate DDW state and the corresponding Hartree-Fock Hamiltonian. Section III gives a brief description of the Nernst coefficient. Section IV is devoted to the temperature dependence of the Nernst coefficient. We find, both numerically and analytically, that there is a positive peak of the Nernst signal for the ambipolar DDW state when the electron pocket dominates the transport at low temperatures. In contrast, there is no such peak of the Nernst effect if the underlying state is non-ambipolar. In fact, the Nernst signal from individual electron or hole pockets are both negative, while the combination of them can lead to a positive peak. In Section V, we discuss the doping dependence of the Nernst coefficient. A positive peak in the Nernst signal as a function of hole doping is also found. Finally, we summarize and conclude in Section VI.

The main assumptions (to be explained in more detail below) we use to derive the results of this paper are: 1) Boltzmann theory is applicable to calculate the transport properties of the DDW quasiparticles, 2) The transport scattering lifetimes $\tau_e, \tau_h$ (Eqs. 8, 9) in the underdoped regime are constant over the Fermi surface and are also taken to be independent of energy in a small (temperature-dependent) interval around the Fermi energy, 3) The underlying band structure consists of both electron and hole pockets. The third assumption, that of quasiparticle ambipolarity, is the most crucial one for the qualitative robustness of the temperature and doping-dependent peaks in the DDW Nernst coefficient. Even though the existence of both types of quasiparticles as in the ambipolar DDW state is likely not the only explana-
The Hamiltonian density in Eq. (2) operates on the two-component spinor \( \hat{\Psi} \) described by \( k \) field DDW state is given by, and gives rise to spontaneous currents along the bonds of the square lattice.

A part of the results (numerical calculation for the temperature dependence of the Nernst coefficient) contained in this paper were published earlier.\(^\text{23}\) In addition to giving a more complete discussion of these previously published results, the present paper contains the following new results: 1) An analytical explanation for the peak in the temperature dependence of the Nernst signal, 2) A fact that even though both electron and hole pockets can give negative Nernst effects individually, the combination of them can yield a positive peak as a function of temperature and 3) The full doping dependence of the DDW Nernst coefficient in the underdoped regime of the cuprates. Specifically, we show that there is a well-defined positive peak in the DDW Nernst coefficient as a function of hole doping, which is consistent with the underdoped regime in the cuprate phase diagram.

**II. COMMENSURATE DDW STATE**

The commensurate DDW state\(^\text{28}\) is described by an order parameter which is a particle-hole singlet in spin space,

\[
\left\langle \hat{c}_k^{\dagger} \hat{c}_{k+Q,\alpha} \right\rangle \propto iW_k \delta_{\alpha\beta}, \quad W_k = \frac{W_0}{2}(\cos k_x - \cos k_y). \tag{1}
\]

Here \( \hat{c}^{\dagger} \) and \( \hat{c} \) are the electron creation and annihilation operators on the square lattice of the copper atoms, \( \mathbf{k} = (k_x, k_y) \) is the two-dimensional momentum, \( Q = (\pi, \pi) \) is the wave vector of the density wave, and \( \alpha \) and \( \beta \) are the spin indices. For simplicity, we have taken \( h = 1 \) and the lattice constant \( a = 1 \). In Eq. (1), \( iW_k \) is the DDW order parameter with the \( i = \sqrt{-1} \) symmetry in the momentum space. For \( Q = (\pi, \pi) \), it is purely imaginary\(^\text{28}\) and gives rise to spontaneous currents along the bonds of the square lattice.

The Hartree-Fock Hamiltonian describing the mean-field DDW state is given by,

\[
\hat{H} = \sum_{\mathbf{k}\in\text{RBZ}} \left( \varepsilon_{\mathbf{k}} - \mu \right) \hat{c}_k^{\dagger} \hat{c}_{\mathbf{k}+\mathbf{Q}}, \tag{2}
\]

\[
\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) + 4t'(\cos k_x \cos k_y) - \mu, \tag{3}
\]

where \( \varepsilon_{\mathbf{k}} \) is the band dispersion of the electrons, and \( \mu \) is the chemical potential. The Hamiltonian density in Eq. (2) operates on the two-component spinor \( \hat{\Psi}_{\mathbf{k}} = (\hat{c}_{\mathbf{k}}, \hat{c}_{\mathbf{k}+\mathbf{Q}}) \) defined on the reduced Brillouin zone (RBZ) described by \( k_x \pm k_y = \pm \pi, \) and can be expanded over the Pauli matrices \( \mathbf{\tau} \) and the unity matrix \( \mathbf{I} \),

\[
\hat{H}_k = w_0(k) \mathbf{\hat{I}} + w(k) \cdot \mathbf{\tau}, \quad w_0 = \frac{\varepsilon_k + \varepsilon_{k+\mathbf{Q}}}{2} - \mu, \tag{4}
\]

where, \( w_1 = 0, \quad w_2 = -W_k, \quad w_3 = \frac{\varepsilon_k - \varepsilon_{k+\mathbf{Q}}}{2} \). The spectrum of the Hamiltonian consists of two branches with the eigenenergies given by,

\[
E_{\pm}(k) = w_0(k) \pm w(k), \tag{5}
\]

where, \( w(k) = |w(k)| \). For a generic set of band structure parameters, we use \( t = 0.3 \text{ eV}, \quad t' = 0.3t^\text{41} \) and \( \mu \) corresponding to a non-zero hole doping, \( x \), appropriate for the underdoped regime of the cuprates, the reconstructed Fermi surface consists of two hole pockets near the \( (\pi/2, \pm \pi/2) \) points and one electron pocket near the \( (\pi, 0) \) point in the reduced Brillouin zone. The hole and the electron pockets of the DDW state are shown in Fig.\(^\text{1}\) for two different values of the chemical potential corresponding to different values of the hole doping. The existence of both hole and electron-like excitations in the quasiparticle spectrum generically makes this state an ambipolar state.

**III. NERNST COEFFICIENT**

In Nernst experiments\(^\text{21–24,43}\) a temperature gradient, \(-\nabla T\), is applied on the sample along the \( \hat{x} \) direction. For such a temperature gradient, and with a magnetic field \( \mathbf{B} \) along the \( \hat{z} \) direction, the charge current due to quasiparticles along \( \hat{x} \) driven by \(-\nabla T\) produces a balancing electric field \( \mathbf{E} \). The total charge current in the presence of \( \mathbf{E} \) and \(-\nabla T\) is thus given by,

\[
J_i = \sigma_{ij}E_j + \alpha_{ij}(-\partial_j T), \tag{6}
\]

where \( \sigma_{ij} \) and \( \alpha_{ij} \) are the electric and the thermoelectric conductivity tensors, respectively. In the experiments, \( J \)
is set to zero and the Nernst coefficient can be written as,
\[ \nu_N = \frac{E_y}{-\nabla T \cdot B} = \frac{\alpha_{xy} \sigma_{xx} - \alpha_{xx} \sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \] (7)
where \( \sigma_{ij} \) and \( \alpha_{ij} \) are the electric and the thermoelectric conductivity tensors, respectively.

For the direction of the temperature gradient as above (\( T \) decreases in the positive \( \hat{z} \) direction), and \( B \) in the positive \( \hat{z} \) direction, the vortices of a superconductor produce a Nernst signal in the positive \( \hat{y} \) direction. This is because, due to entropic reasons, the vortices flow towards the cooler end. Due to the Josephson effect, the mobile vortices then produce a transverse electric field, \( E = B \times \mathbf{v} \), which is in the positive \( \hat{y} \) direction. Note that quasiparticles in the same set up, depending on their effective charge, would produce a transverse electric field in positive or negative \( \hat{y} \) direction. Because of the uniqueness of the direction of the vortex Nernst signal, a transverse electric field in the positive \( \hat{y} \) direction is taken as the positive Nernst signal. According to this sign convention, the Nernst coefficient of quasiparticles is positive if it is calculated to be so according to Eq. (7), where \( \nu_N \) is defined in terms of \( E_y \). This modern sign convention is opposite to the older convention sometimes also used in the literature. In the Nernst experiments on the high-\( T \) cuprates, the modern sign convention is universally used so that the vortex signal is positive by definition.

We calculate the off-diagonal element of the conductivity tensor, \( \sigma_{xy} \), by using the solution of the semi-classical Boltzmann equation,
\[ \sigma_{xy}(\mu) = e^2 B \int \frac{d^2 k}{(2\pi)^2} \frac{\partial E_+(k)}{\partial k_x} \frac{\partial E_+(k)}{\partial k_y} \frac{\partial^2 E_+(k)}{\partial k_x \partial k_y} \frac{\partial^2 E_+(k)}{\partial k_x \partial k_y} \left( \frac{\partial f(E_+(k) - \mu)}{\partial E_+} \right) \]
\[ + (E_+ \to E_-; \tau_e \to \tau_h). \] (8)
Here, the momentum integrals are over the reduced Brillouin zone. In the DDW band-structure, the electron pocket near \( (\pi, 0) \) is associated with the upper band, \( E_+(k) \). The first integral in Eq. (8), therefore, embodies the contribution to \( \sigma_{xy} \) due to the electron-like quasiparticles. We have denoted the corresponding transport scattering time as \( \tau_e \), which, for simplicity, is taken to be independent of the location on the electron Fermi line. The second integral in Eq. (8), where \( \tau_e \) is replaced by the scattering time for the hole-like careers, \( \tau_h \), calculates the contribution to \( \sigma_{xy} \) from the hole pockets. \( \tau_h \) is also taken to be constant everywhere on the hole Fermi lines. Even though both the scattering times can be energy-dependent since the Fermi surface integrals in Eq. (8) and Eq. (10) (see below) extend only over a small (temperature-dependent) interval around the Fermi energy, we assume \( \tau_e \) and \( \tau_h \) to be energy independent in our calculations.

In general, there is no obvious reason to expect \( \tau_e = \tau_h \). For a consistent interpretation of the Hall effect experiments, it has been recently argued that the scattering times, which are directly proportional to the career mobilities, may in fact be different for the electron and the hole-like charge carriers. Since at low temperatures the Hall coefficient is negative, Ref. [3] argues that, at least at low \( T \), \( \tau_e > \tau_h \). With the above definition of the parameters, the diagonal element of the conductivity tensor is given by,
\[ \sigma_{xx}(\mu) = e^2 \tau_e \int \frac{d^2 k}{(2\pi)^2} \left( \frac{\partial E_+(k)}{\partial k_x} \right)^2 \left( \frac{\partial f(E_+(k) - \mu)}{\partial E_+} \right) \]
\[ + (E_+ \to E_-; \tau_e \to \tau_h). \] (9)

From the solution of the Boltzmann equation at low \( T \), the thermoelectric tensor \( \alpha_{ij} \) is related to the conductivity tensor \( \sigma_{ij} \) by the Mott relation,
\[ \alpha_{ij} = -\frac{\pi^2}{3} k_B^2 T \frac{\partial \sigma_{ij}}{\partial \mu}. \] (10)
Here \( e > 0 \) is the absolute magnitude of the charge of an electron. Using Eq. (7), the formula for the Nernst coefficient reduces to,
\[ \nu_N = -\frac{\pi^2}{3} k_B^2 T \frac{\partial \Theta_H}{\partial \mu} \] (11)
Here,
\[ \Theta_H = \tan^{-1} \left( \frac{\sigma_{xx}}{\sigma_{xy}} \right). \] (12)

Using Eqs. (11) and with reasonable phenomenological assumptions about the temperature dependence of the scattering times and the DDW order parameter, we can now calculate \( \nu_N \) as a function of \( T \) in the ambipolar DDW state. Using a phenomenological ansatz for the doping dependence of the DDW order parameter, and computing the chemical potential self-consistently, we can also use the same equations to calculate the doping dependence of \( \nu_N \). This way we can evaluate the evolution of the Nernst coefficient in the cuprate phase diagram within the ambipolar DDW model.

It is important to emphasize that the negative Hall coefficient at low \( T \), as seen in the presence of strong magnetic fields in Ref. [3], does not automatically imply the presence of the electron pockets at low enough magnetic fields. However, conversely, the existence of the electron pockets in the band-structure is at least one possible scenario consistent with the negative Hall coefficient. In addition, the existence of the electron pockets is also qualitatively consistent with the observed frequencies in the recent quantum oscillation experiments. Our calculated enhanced Nernst coefficients are for the ambipolar DDW state which has both electron- and hole-like quasiparticles in the excitation spectrum. As such, the results of this paper give at least one possible explanation of the observed enhanced Nernst signals in the underdoped cuprates.
IV. TEMPERATURE DEPENDENCE OF THE NERNST COEFFICIENT

A. Phenomenological temperature dependence of the parameters

In the first step in the evaluation of the temperature dependence of the Nernst coefficient, we have to make suitable assumptions for the behavior of the scattering times $\tau_e$ and $\tau_h$ with temperature. An important hint regarding this can be obtained from the recent Hall effect experiments in Ref. [3]. In these experiments, the normal state Hall coefficient, 

$$R_H = \frac{\sigma_{xy}}{B(d_{xx})^2},$$

has been measured as a function of $T$ in three different samples of underdoped YBCO. In all three samples, $R_H$ is large and positive above $T^*$, which is consistent with the systems being moderately hole doped. $R_H$, however, shows a sharp decline below $T^*$, and subsequently changes its sign from positive (hole-dominated) to negative (electron-dominated) at a crossover temperature $T_0 < T^*$. This anomalous $T$-dependence of the Hall coefficient can be understood naturally if the state in question below $T^*$ is hole occasional ambipolar, and the mobilities of the oppositely charged quasiparticles are assumed to be unequal and changing with temperature.

With Eqs. (8,9), we can calculate the contributions of the electron and hole pockets of the DDW state to the overall $R_H$, which leaves some residual freedom in choosing the values of these constants. However, we have checked that our conclusions for the behavior of $\nu$ with $T$ are robust to any reasonable variation of the $T$-dependence of $\tau_e, \tau_h$ and $W_0(T)$ as long as they satisfy the experimental constraints set by the temperature dependence of $R_H$.

We now use Eqs. (8,9,11) to calculate $\nu_N$ as a function of $T$ for a specific value of the hole doping $x = 10\%$. Using a mean field $T$-dependence of $W_0(T)$ and the phenomenological form of $\tau_e(T)$ and $\tau_h(T)$ above, we plot in Fig. (2) the calculated $\nu_N$ in the ambipolar DDW state as a function of $T$ for $x = 10\%$. It is clear from Fig. (2) that the Nernst coefficient has a pronounced low temperature peak which, as we argue below, is a direct manifestation of the quasiparticle ambipolarity of the DDW state.

B. Sign and temperature dependence of Nernst signal from individual hole and electron pockets

To elucidate the importance of the quasiparticle ambipolarity in the temperature dependence of $\nu_N$, let us
first consider the Nernst effect due to the quasiparticles associated with a hole pocket. In the presence of only hole pockets in the excitation spectrum, we can write the Nernst coefficient as

$$\nu_N = -C\tau_h T \frac{\partial}{\partial \mu} \bar{\Theta}^h, \quad (17)$$

where we take

$$\bar{\Theta}^h \approx \bar{\sigma}_{xy}^h / \sigma_{xx}^h, \quad (\bar{\sigma}_{xy}^h \ll \bar{\sigma}_{xx}^h) \quad (18)$$

$$\bar{\sigma}_{xy}^h = \sigma_{xy}^h / \tau_h, \quad \bar{\sigma}_{xx}^h = \sigma_{xx}^h / \tau_h \quad (19)$$

and $C$ is a numerical constant, $C = \frac{\pi^2 k_B^2}{12 \hbar^2}$, $\bar{\sigma}_{xy}^h$ and $\bar{\sigma}_{xx}^h$ (superscript $h$ indicates the contribution from the hole-like quasiparticles) depend only on the hole Fermi surface integrals in Eq. (8) and Eq. (9), respectively. We have rewritten the expression for $\nu_N$ (Eq. (11)) in Eq. (17) so that the manipulation of the explicit $T$-dependence of the scattering time becomes easier. Similarly, when there are only electron-like quasiparticles in the system, the Nernst coefficient can be written as,

$$\nu^e_N = -C\tau_e T \frac{\partial}{\partial \mu} \bar{\Theta}^e, \quad (20)$$

where $\bar{\Theta}^e \approx \bar{\sigma}_{xx}^e / \sigma_{xx}^e, \bar{\sigma}_{xy}^e = \sigma_{xy}^e / \tau_e, \bar{\sigma}_{xx}^e = \sigma_{xx}^e / \tau_e$, and the superscript $e$ indicates electron-like quasiparticles.
right hand side is the result of the explicit T-dependence of \( \nu_N \) via the scattering time and the explicit factor of \( T \). Since \( \frac{\partial \bar{\Theta}^h}{\partial \mu} \) is positive, this term is strictly negative. The second term in Eq. (22) depends on the implicit T-dependence of \( \nu_N^h \) via the T-dependence of \( W_0 \). To calculate this term, we write,

\[
\frac{\partial}{\partial T} \frac{\partial \bar{\Theta}^h}{\partial \mu} = \frac{\partial}{\partial \mu} \left( \frac{d\bar{\Theta}^h}{dW_0} - \frac{1}{2} \frac{\partial \bar{\Theta}^h}{\partial W_0} \right) \frac{1}{T^* - T}^{1/2},
\]

where we have used the mean field ansatz for the T-dependence of the amplitude of the DDW order parameter. To calculate the right hand side, noting that \( T \) is independent of \( \mu \), we only need to calculate,

\[
\frac{\partial}{\partial \mu} \frac{d\bar{\Theta}^h}{dW_0} = \frac{\partial}{\partial \mu} \left( \frac{d\bar{\Theta}^h}{dW_0} - \bar{\Theta}^h \frac{d\bar{\Theta}^h}{dW_0} \right) \]

\[
\approx - \frac{1}{(\bar{\Theta}^h)^2} \frac{\partial \bar{\Theta}^h}{\partial \mu} \left( \frac{d\bar{\Theta}^h}{dW_0} - \bar{\Theta}^h \frac{d\bar{\Theta}^h}{dW_0} \right)
\]

\[
- \frac{1}{\bar{\Theta}^h} \frac{\partial \bar{\Theta}^h}{\partial W_0} \frac{d\bar{\Theta}^h}{dW_0}.
\]

In the derivation of the above equation, we neglect the small terms \( \frac{\partial d\bar{\Theta}^h}{\partial \mu} \) and \( \frac{\partial d\bar{\Theta}^h}{\partial W_0} \) (see Eq. (25) below for justification). It is clear that we first need the leading \( W_0 \)-dependence of the Fermi surface integrals \( \bar{\Theta}^h \) and \( \bar{\Theta}^e \). We find that the leading \( W_0 \) dependence of these integrals is linear. This linear dependence arises from the integration region around the so-called "hot spots," points on the Fermi surface which also fall on the surface of the RBZ. On the surface of the RBZ \( (k_x \pm k_y = \pm \pi) \), the dominant part of the band structure, \( -2t(\cos k_x + \cos k_y) = 0 \). The linear \( W_0 \) dependence of \( \bar{\Theta}^h \) and \( \bar{\Theta}^e \) comes from the region around the hot spots which satisfies

\[
t | \cos k_x + \cos k_y | < \frac{W_0}{4} | \cos k_x - \cos k_y |,
\]

which has a size \( \sim tW_0 \). Since the relevant integration region is itself \( O(W_0) \), by expanding the integrand in Eq. (8) to zeroth order in \( W_0 \), we get the leading \( W_0 \) dependence of \( \bar{\Theta}^h \), as,

\[
\bar{\Theta}^h \sim 64tW_0 t^3 \sin^2 k_x \cos k_x \cos k_y.
\]

The above is negative since \( \cos k_x = -\cos k_y \) on the surface of RBZ. Therefore, it follows that, \( \frac{\partial \bar{\Theta}^h}{\partial W_0} < 0 \). From similar manipulations, it is straightforward to show that \( \frac{\partial \bar{\Theta}^h}{\partial W_0} > 0 \). We also note that, in Eq. (24), \( \bar{\Theta}^h \) itself is positive for the hole pockets (\( \bar{\Theta}^h \) has the same sign as the Fermi surface integral for the Hall conductivity, \( \bar{\Theta} \)), which is, of course, positive for the hole pockets. Furthermore, noting that \( \frac{\partial \bar{\Theta}^h}{\partial \mu} < 0 \) and \( \frac{\partial \bar{\Theta}^h}{\partial W_0} > 0 \) for the hole pocket, we infer from Eq. (24) that \( \frac{\partial \bar{\Theta}^h}{\partial W_0} < 0 \). Therefore, from Eq. (23),

\[
\frac{\partial}{\partial T} \frac{\partial \bar{\Theta}^h}{\partial \mu} = \frac{\partial}{\partial \mu} \frac{\partial \bar{\Theta}^h}{\partial T} > 0
\]

for \( T < T^* \). Finally, using Eq. (22), we conclude that \( \frac{\partial \bar{\Theta}^h}{\partial T} > 0 \) is negative definite for the hole pockets. This implies that, for only hole-type quasiparticles in the DDW state, the temperature-derivative of the Nernst coefficient can never be zero: there is no low temperature peak of \( \nu_N^h(T) \). This analytical result has been confirmed by the numerical results, as seen in Fig. 4.

In contrast to \( \nu_N^h \), the sign of \( \nu_N^e \) is hard to determine analytically. The reason is that, in

\[
\frac{\partial \bar{\Theta}^e}{\partial \mu} = \frac{1}{(\bar{\Theta}^e)^2} \left( \frac{d\bar{\Theta}^e}{dW_0} \frac{d\bar{\Theta}^e}{d\mu} - \frac{d\bar{\Theta}^e}{d\mu} \frac{d\bar{\Theta}^e}{dW_0} \right),
\]

the terms \( \frac{\partial \Theta^e}{\partial \mu} \bar{\Theta}^e (\bar{\Theta}^e) (\bar{\Theta}^e) < 0 \) and \( \frac{\partial \bar{\Theta}^e}{\partial W_0} \bar{\Theta}^e (\bar{\Theta}^e) (\bar{\Theta}^e) > 0 \) have opposite signs, and the sign of \( \nu_N^e \) should be determined by the relative magnitudes of these two terms. For the band structure parameters used here, we find that \( \nu_N^e \) is negative, as seen in Fig. 4. This implies that, for the electron pocket, the second term in \( \frac{\partial \bar{\Theta}^e}{\partial \mu} \) dominates over the first one, leading to a positive \( \frac{\partial \bar{\Theta}^e}{\partial \mu} \).

C. Temperature dependence of the Nernst coefficient from ambipolar DDW state

In view of the above analysis, a natural question is then why two individually negative contributions from the electron and the hole pockets ‘add up’ to a positive Nernst signal at low temperatures when the two types of pockets coexist. The underlying reason can be most

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FIG. 4. (a) Plot of the Nernst coefficient, \( \nu_N \), versus temperature for the electron (solid line) and hole (dashed line) pockets separately. The parameters used in the plot are same as those in Fig. 2. Nernst coefficient in \( V \) \( K^{-1} \) can be derived by multiplying the dimensionless \( \nu_N \) in the figure with the factor \( 2\pi^2 k_B^2 e \) \( 3h \approx 138 \) nV/KT. The temperature in the horizontal axis is in \( K \). The hole doping \( x = 10\% \). (b) Plots of \( \bar{\Theta} \) versus temperature for the electron and hole pockets separately. The plots in (a) are obtained by multiplying these values by \( -k_B T \) in eV.
clearly expressed by writing down the formula for $\frac{\partial \Theta^t}{\partial \mu}$, where the superscript ‘t’ now represents the total Nernst effect as given by multiplying $\frac{\partial \Theta}{\partial \mu}$ by $-CT$,

$$\frac{\partial \Theta^t}{\partial \mu} = \frac{1}{(\sigma^e)^2} \left( \frac{d \sigma^t_{xy}}{d \mu} \sigma^e_{xx} - \frac{d \sigma^e_{xy}}{d \mu} \sigma^t_{xx} \right)$$

(27)

Here, $\sigma^e_{xx} = \sigma^h_{xx} + \sigma^c_{xx}$, and $\sigma^t_{xy} = \sigma^h_{xy} + \sigma^c_{xy}$. Because of the ambipolar spectrum, the second term in Eq. (27) is much smaller than the first one (since the total Hall conductivity, $\sigma^e_{xy}$, is small), and the sign of $\frac{\partial \Theta^t}{\partial \mu}$ is entirely determined by the first term. It follows that if the contribution from the electron pocket dominates over that from the hole pocket, then $\frac{\partial \Theta^t}{\partial \mu}$ is negative, since the first term in Eq. (27) is negative for the electron pocket. On the other hand, if the contribution from the hole pocket is greater than that from the electron pocket, then $\frac{\partial \Theta^t}{\partial \mu}$ is positive because $\frac{\partial \Theta^h}{\partial \mu}$ is positive for the hole pocket, see Fig. 3. In our calculations, the former is the situation at low $T$, and $\nu_N$ is positive at low temperatures. At high temperatures, the contribution from the hole pockets dominates transport because of their larger size and the first term in Eq. (27), and consequently $\frac{\partial \Theta^t}{\partial \mu}$ becomes positive, leading to a negative Nernst signal. Therefore, $\nu_N$, which is zero at $T = 0$, first increases at low temperatures and then decreases at high temperatures, yielding a positive peak if there are both electron- and hole-type quasiparticles present in the spectrum at the same time.

In the context of the cuprates, at low temperatures, the electron pocket dominates via $\tau_e \gg \tau_h$, and $\frac{\partial \Theta^e}{\partial \mu} > 0$. On the other hand, at high temperatures, the hole pockets dominate when $\tau_h \sim \tau_e$. In this case, we have $\frac{\partial \Theta^e}{\partial \mu} < 0$. In practice, determining an analytical expression for the peak temperature by solving the implicit equation, $\frac{\partial \Theta^t}{\partial \mu} = 0$, is not very illuminating, since it depends on many parameters. However, we have conclusively shown here that the existence of a low temperature positive peak in the Nernst coefficient is a robust consequence of quasiparticle ambipolarity, and, therefore, is independent of any assumptions about the microscopic parameters. Furthermore, the sign of the peak value of $\nu_N(T)$ depends on the relative dominance of the electron and the hole pockets at different regimes of $T$. For example, if the hole (electron) pockets were dominant at low (high) temperatures, the peak value of $\nu_N(T)$ would be negative. However, in the experimentally relevant case where the electron pocket is more dominant at low $T$ (so that the zero temperature Hall coefficient is negative), the peak is on the positive side.

V. DOPING DEPENDENCE OF THE NERNST COEFFICIENT

To calculate the Nernst coefficient as a function of doping in the underdoped regime, we have to start by assuming a phenomenological doping dependence of the DDW order parameter $W_0$. Using the values of the doping-dependent amplitude of the DDW order parameter and the set of parameters $t, t', x$, we can calculate the chemical potential $\mu$ as a function of $x$. This way, the Nernst coefficient can be calculated as a function of the doping in the pseudogap phase. In the absence of a concrete theoretical result for the doping dependence of the DDW order parameter, we assume the mean-field doping dependence,

$$W_0(x, T) = W_0(x_0, T) \left\{ \begin{array}{ll} \sqrt{\frac{x_{\max} - x}{x_{\max} - x_0}} & \text{if } x \geq x_0 \\ \sqrt{\frac{x - x_{\min}}{x_0 - x_{\min}}} & \text{if } x < x_0 \end{array} \right. $$

(28)

where $x_0 = 10\%$ is the doping percentage that yields the maximum DDW order. This kind of a doping dependence is physically motivated, since we expect the DDW order to gradually weaken for both high and low values of $x$. We choose $x_{\min} = 4\%$ and $x_{\max} = 17\%$ as the minimum and the maximum doping where the DDW order may exist. We use the value of $x$ and the set of parameters $t, t', W_0(x, T = 0)$ to self-consistently calculate the value of $\mu$, which determines the size and curvature of the hole and the electron pockets.

To illustrate the behavior of $\nu_N$ with underdoping, we plot in Fig. 5 the temperature dependence of $\nu_N$ for $x = 7\%$. It is clear that the Nernst signal remains negative in a wider range of temperatures at this value of hole-doping than that at $x = 10\%$. However, there is still a small low temperature peak in the positive side because of the larger mobility of the electron pocket at low temperatures. Since the Nernst effect at temperatures close to and below the superconducting $T_c$ is almost entirely dominated by the vortex Nernst signal, the low temperature positive peak due to the DDW quasiparticles may not be visible in the experiments. In this case, the DDW quasiparticle Nernst effect may appear negative above the superconducting $T_c$. 

FIG. 5. The Nernst coefficient as a function of temperature for $x = 7\%$. Nernst coefficient in $\text{V K}^{-1}\text{T}^{-1}$ can be derived by multiplying $\nu_N$ with the factor $2\pi^2k_Ba^2/3h \approx 138 \text{ nV/KT}$. The temperature is in $K$. The corresponding $\mu = -0.238 \text{ eV}$, $T^* = 80 \text{ K}$. We see that the Nernst signal is negative in a broader temperature regime than that for $x = 10\%$. 
It is clear that there is a peak of the Nernst signal at a hole doping $x = 10\%$, where the DDW order parameter is assumed to be the largest. The sign of $\nu_N$ near its peak is positive.

In Fig. 6, we plot the Nernst coefficient at a fixed low temperature as a function of hole doping $x$. To construct this plot, we have taken the transition temperature of the ordered DDW state to scale with the value of the zero temperature order parameter, as would be expected from the mean field theory. This implies that $T^*(x)$ has been determined via

$$T^*(x) = \frac{W_0(x, T = 0)}{W_0(x_0, T = 0)} T^*(x_0).$$

It is clear that there is a peak of the Nernst signal at $x = 10\%$ on the positive side, in agreement with the Nernst effect experiments\textsuperscript{14,23} in the cuprates. The signal weakens on either side of $x = 10\%$ because the magnitude of the DDW order parameter weakens with $x$ on either side of this value of doping.

VI. CONCLUSION

In conclusion, we show that the Nernst signal from an ambipolar DDW state has a robust low temperature peak which occurs below its mean field transition temperature. The onset of the Nernst signal, however, occurs at the transition temperature itself, which may account for the sizable Nernst effect found in the experiments in the pseudogap phase of the high temperature cuprate superconductors. The sign of the peak value of the Nernst coefficient can be positive or negative depending on whether electron- or hole-pockets, respectively, dominate the low temperature thermoelectric transport. For the experimental situation in some cuprates, where the low temperature Hall coefficient is found to be negative indicating the dominance of the electron pocket in transport, we find that the peak in the temperature dependent Nernst coefficient is on the positive side. In contrast, there is no such peak when the DDW state is non-ambipolar. In this case, with only one type of pockets in the excitation spectrum, the Nernst signal is negative for both electron and hole pockets. However, quite surprisingly, we find that these two individual negative contributions ‘add up’ to produce a net positive Nernst effect in the ambipolar DDW state. We prove these results both by numerical calculations as well as analytical arguments, which establishes the robustness of the existence of the low temperature peak, making it insensitive to any reasonable variations of the microscopic parameters.

With a reasonable assumption about the doping dependence of the DDW order parameter, which is physically motivated and stipulates weakening of the order parameter with hole doping on either side of the underdoped regime, we find that the low temperature Nernst coefficient also has a pronounced peak as a function of hole concentration. The peak of the Nernst coefficient coincides with the value of doping where the DDW order parameter is assumed to be the strongest, and the signal weakens on either side of this value of the hole concentration. At low value of the hole doping, we find that the Nernst coefficient remains negative over a wider range of temperature than at moderate underdoping, $x = 10\%$.

To derive these results, we model the pseudogap phase by a doping-dependent DDW order parameter and assume transport scattering times which are constant throughout the Fermi surface and also independent of energy in a small ($T$-dependent) energy interval around the Fermi energy. Even though our chosen ambipolar DDW state (with its specific Fermi surface topology (Fig. (1)) is not the only possible state consistent with either the recent quantum oscillation experiments\textsuperscript{4,14,23} or the observed negative Hall coefficient\textsuperscript{20} in the underdoped cuprates, it is at least one possible state qualitatively consistent with both. As such, the calculations for the Nernst coefficient in the ambipolar DDW state given in this paper present at least one possible scenario for the observed enhanced Nernst signals in the underdoped cuprate superconductors.

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