Constructing Time Machines *

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The existence of time machines, understood as spacetime constructions exhibiting physically realised closed timelike curves (CTCs), would raise fundamental problems with causality and challenge our current understanding of classical and quantum theories of gravity. In this paper, we investigate three proposals for time machines which share some common features: cosmic strings in relative motion, where the conical spacetime appears to allow CTCs; colliding gravitational shock waves, which in Aichelburg-Sexl coordinates imply discontinuous geodesics; and the superluminal propagation of light in gravitational radiation metrics in a modified electrodynamics featuring violations of the strong equivalence principle. While we show that ultimately none of these constructions creates a working time machine, their study illustrates the subtle levels at which causal self-consistency imposes itself, and we consider what intuition can be drawn from these examples for future theories.

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1. Introduction

New insight into fundamental theories may often be achieved by studying their behaviour in extreme or near-paradoxical regimes. Part of the enduring fascination of time machines is that they force us to confront basic questions and assumptions about spacetime inherent in the structure of classical and quantum theories of gravity.

In this article, we review three attempts to construct a time machine, which we understand as a spacetime scenario in which there exist closed signal trajectories which return to the original spacetime point from which they were emitted. Essentially this means showing the existence of physically realisable closed timelike (or lightlike) curves, which are abbreviated as CTCs (CLCs). These three examples share certain common features which we feel makes it interesting to present them together, the paper being a mixture of review, original work and critique. While none of these constructions ultimately leads to a working time machine, their analysis involves many quite subtle issues and reveals how intricate the mechanisms can be by which quantum theory and general relativity respect causality.

Time machines can be studied from at least two points of view. One is to try to formulate general theorems on the occurrence of CTCs in exact solutions of Einstein’s field equations subject to various axioms, physical restrictions on the energy-momentum tensor, or realistic boundary conditions. Such theorems are valuable in making precise the conditions under which CTCs occur, what exactly constitutes a physically realisable CTC, and whether their occurrence can be tolerated in consistent quantum field theories involving gravity. Examples of this approach include Hawking’s chronology protection conjecture [1]. See also refs.[2,3] for a selection of the relevant literature.

A second approach, which we pursue here, is to try to construct relatively simple examples of spacetime scenarios with the potential to exhibit causality-violating trajectories. This relatively low-tech strategy complements the axiomatic, theorem-based approach in important ways. Even simple time machine models can help to sharpen what we should understand by the requirement that CTCs should be ‘physically realisable’ and what should be considered as ‘realistic’ boundary conditions. They can uncover essential ingredients for the axioms of a general theorem, notably in relation to global rather than local properties of spacetime. They can be used as probes to uncover an interesting physical effect – by imposing the absence of CTCs as a self-consistency constraint on the kinematics we may deduce, for example, that an event horizon must form and learn about the conditions under which it does. And finally, of course, they may succeed!

Simple time machine models typically use one of two fundamental mechanisms to generate back-in-time motion. One involves rotation – in some way, the metric term
mixing $t$ and $\phi$ in a rotating spacetime is contrived to make the periodic angle variable into a timelike coordinate. The canonical example of this type of construction is the Gödel universe \[4\]. The second mechanism, which all of our examples exploit, involves propagation that is effectively superluminal. Superluminal motion, as is well-known in special relativity, becomes back-in-time when viewed from a certain class of reference frames, so has at least the potential to lead to causality violation. Here, we consider not only superluminal propagation itself but also two types of spacetime which, like the well-known example of wormholes \[5\], in a sense allow superluminal paths – the conical spacetime around a cosmic string \[6,7\], which can be viewed as flat spacetime with a missing wedge, and a gravitational shock wave spacetime \[8\], which permits discontinuous geodesics jumping back in time.

We begin in section 2 by recalling some very simple elements of superluminal motion in special relativity and establish some principles which we will refer back to at various points in our discussion of the three time machines. Then we consider probably the most well-known example of a time machine construction of this type – the demonstration by Gott \[9\] of the existence of CTCs in the spacetime generated by two cosmic strings in relative motion. This exploits the fact that a trajectory crossing the missing wedge in a single cosmic string spacetime can be effectively superluminal in the sense that it can arrive before a light signal following a direct path. As Gott then showed, by combining two cosmic strings moving with velocities exceeding a certain critical value, CTCs exist which encircle the two strings. The subsequent discussion of whether such CTCs are physical and consistent with realistic boundary conditions is reviewed in section 3. This has proved remarkably fruitful, particularly in terms of emphasising the importance of global conditions in general theorems on the occurrence of physical CTCs.

We then try to repeat the success of this idea in a somewhat similar context. The spacetime generated by a gravitational shock wave is, like the string spacetime, almost everywhere flat and can be thought of as essentially two sections of Minkowski spacetime patched together but with an impact parameter dependent shift \[10\]. Geodesics which cross the spacetime are discontinuous and jump back in time. This gives another mechanism, analogous to the wedge-crossing trajectories in the cosmic string spacetime, for effectively superluminal trajectories. To exploit this to find trajectories which return to the past light cone of the emitter (thereby yielding CTCs) we again need to invoke a second shock wave travelling in the opposite direction to the first. In the approximation of neglecting the interaction between the shock waves, we indeed find that CTCs exist in which a test particle is successively struck by the two shocks. In the Gott cosmic string scenario, the condition for the existence of CTCs was shown to be violated when the ‘interaction’ between the strings (in the sense of the conditions of global self-consistency of the two-
string solution, subject to certain boundary conditions) is included. In the same spirit, our hope in formulating a shock wave time machine was that, while the CTCs would not exist in the presence of interactions, the precise reasons would shed new light on the kinematics of the shock wave collision, in particular the possible emergence of an event horizon in the collision zone. The extent to which this picture is realised is described in section 4.

Our final example is rather different in that it involves genuinely superluminal propagation. As explained in the next section, if we are prepared to relax the strong equivalence principle, then the action for electrodynamics in a gravitational field may contain explicit curvature-dependent photon-gravity interactions. (This SEP-violating effective action is in fact generated by vacuum polarisation in QED in curved spacetime, as first shown by Drummond and Hathrell [11]. See also [12,13].) In sections 2 and 5, we discuss whether superluminal propagation of this (non-tachyonic) type in general relativity necessarily leads to the same inevitable causality violations that accompany superluminal propagation in special relativity. To sharpen this discussion, Dolgov and Novikov [14] have proposed a construction, again involving two gravitational sources in relative motion, in which Drummond-Hathrell superluminal propagation is claimed to allow signals to be sent into the past light cone of the emitter. In section 5, we analyse the Dolgov-Novikov proposal in the context of superluminal propagation in gravitational radiation spacetimes of Bondi-Sachs type [15]. We conclude once more that causality-violating trajectories, although apparently occurring when the two gravitational sources are assumed to act entirely independently, do not arise when the full spacetime involving both sources is considered consistently.

Finally, in section 6 we summarise our conclusions and discuss possible future developments.

2. Superluminal Propagation and Causality in Special and General Relativity

It is generally understood that superluminal propagation in special relativity leads to unacceptable violations of causality. Indeed the absence of tachyons is traditionally employed as a constraint on fundamental theories. In this section, we re-examine some basic features of superluminal propagation in order to sharpen these ideas in preparation for our subsequent discussion of the three time machines, especially the Dolgov-Novikov proposal [14] in section 5.
The first important observation is that given a superluminal signal we can always find a reference frame in which it is travelling backwards in time. This is illustrated in Fig. 1. Suppose we send a signal from O to A at speed \( v > 1 \) (in \( c = 1 \) units) in frame \( S \) with coordinates \((t, x)\). In a frame \( S' \) moving with respect to \( S \) with velocity \( u > \frac{1}{v} \), the signal travels backwards in \( t' \) time, as follows immediately from the Lorentz transformation.

![Fig. 1](image)

Fig. 1  A superluminal \((v > 1)\) signal OA which is forwards in time in frame \( S \) is backwards in time in a frame \( S' \) moving relative to \( S \) with speed \( u > \frac{1}{v} \). However, the return path with the same speed in \( S \) arrives at B in the future light cone of O, independent of the frame.

The important point for our considerations is that this by itself does not necessarily imply a violation of causality. For this, we require that the signal can be returned from A to a point in the past light cone of O. However, if we return the signal from A to B with the same speed in frame \( S \), then of course it arrives at B in the future cone of O. The situation is physically equivalent in the Lorentz boosted frame \( S' \) – the return signal travels forward in \( t' \) time and arrives at B in the future cone of O. This, unlike the assignment of spacetime coordinates, is a frame-independent statement.

The problem with causality arises from the scenario illustrated in Fig. 2.

\[1\] From the Lorentz transformations, we have \( t'_A = \gamma(u)t_A(1 - uv) \) and \( x'_A = \gamma(u)x_A(1 - \frac{u}{v}) \).

For the situation realised in Fig. 1, we require both \( x'_A > 0 \) and \( t'_A < 0 \), that is \( \frac{1}{v} < u < v \), which admits a solution only if \( v > 1 \).
Fig. 2 A superluminal ($v > 1$) signal OC which is backwards in time in frame $S$ is returned at the same speed to point D in the past light cone of O, creating a closed time loop.

Clearly, if a backwards-in-time signal OC is possible in frame $S$, then a return signal sent with the same speed will arrive at D in the past light cone of O creating a closed time loop OCDO. The crucial point is that in special relativity, local Lorentz invariance of the laws of motion implies that if a superluminal signal such as OA is possible, then so is one of type OC, since it is just given by an appropriate Lorentz boost (as in Fig. 1). The existence of global inertial frames then guarantees the existence of the return signal CD (in contrast to the situation in Fig. 1 viewed in the $S'$ frame).

The moral is that both conditions must be met in order to guarantee the occurrence of unacceptable closed time loops – the existence of a superluminal signal and global Lorentz invariance. Of course, since global Lorentz invariance (the existence of global inertial frames) is the essential part of the structure of special relativity, we recover the conventional wisdom that in this theory, superluminal propagation is indeed in conflict with causality.

The reason for labouring this elementary point is to emphasise that the situation is crucially different in general relativity. The weak equivalence principle, which we understand as the statement that a local inertial (or freely-falling) frame exists at each point in spacetime, implies that general relativity is formulated on a Riemannian manifold. However, local Lorentz invariance is not sufficient to establish the link discussed above between superluminal propagation and causality violation. This is usually established by adding to the essential structure of general relativity a second dynamical assumption. The strong equivalence principle (SEP) states that the laws of physics should be identical in the local
frames at different points in spacetime, and that they should reduce to their special relativistic forms at the origin of each local frame. It is the SEP which takes over the role of the existence of global inertial frames in special relativity in establishing the incompatibility of superluminal propagation and causality.

However, unlike the weak equivalence principle which underpins the essential structure of general relativity, the SEP appears to be merely a simplifying assumption about the dynamics of matter coupled to gravitational fields. Mathematically, it requires that matter or electromagnetism is minimally coupled to gravity, i.e. with interactions depending only on the connections but not the local curvature. This ensures that at the origin of a local frame, where the connections may be Lorentz transformed locally to zero, the dynamical equations recover their special relativistic form. In particular, the SEP would be violated by interactions which explicitly involve the curvature, such as terms in the Lagrangian for electromagnetism of the form

$$\Gamma \sim \frac{1}{m^2} \int dx \sqrt{-g} \left[ aRF_{\mu\nu}F^{\mu\nu} + bR_{\mu\nu}F^{\mu\lambda}F^{\nu}_{\lambda} + cR_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho} \right]$$

(2.1)

for some SEP and conformal breaking mass scale $m$. Such interactions are of more than speculative interest, since they have been shown by Drummond and Hathrell [11] to arise automatically due to quantum effects (vacuum polarisation) in quantum electrodynamics. They modify the propagation of photons, which no longer follow null geodesics. Remarkably, such SEP-violating interactions can imply superluminal propagation, with photon speeds depending on the local spacetime curvature [11].

The question of whether this specific realisation of superluminal propagation is in conflict with causality is the subject of section 5. Notice though that by violating the SEP, we have evaded the necessary association of superluminal motion with causality violation that held in special relativity. Referring back to the figures, what is established is the existence of a signal of type OA, which as we saw, does not by itself imply problems with causality even though frames such as $S'$ exist locally with respect to which motion is backwards in time. However, since the SEP is broken, even if a local frame exists in which the signal looks like OC, it does not follow that a return path CD is allowed. The signal propagation is fixed, determined locally by the spacetime curvature.

We return to these questions in section 5, where we discuss attempts to use Drummond-Hathrell superluminal propagation to build a time machine. First, we consider two different proposals for time machines which stay within the conventional formulation of general relativity but invoke spacetimes with special and surprising properties which, in some sense, allow superluminal paths.
3. A Time Machine from Cosmic Strings?

The first of our time machines is the well-known construction due to Gott [9], who showed the existence of CTCs in spacetimes with two cosmic strings in relative motion. This construction exploits the feature that the spacetime around a cosmic string is conical, i.e. flat everywhere except for a missing wedge where points on the opposite edges are identified. This gives rise to the possibility of ‘short-cut’ trajectories which can mimic the properties of superluminal paths.

The exterior metric for an infinitely long cosmic string in the $z$ direction is

$$ds^2 = -dt^2 + dr^2 + (1 - 4\mu)^2 r^2 d\phi^2 + dz^2$$

(3.1)

where $\mu$ is the mass/unit length of the string. (Throughout this paper we use units with $G = c = \hbar = 1$.) To see the conical nature of the spacetime, define $\hat{\phi} = (1 - 4\mu)\phi$ so that the metric becomes

$$ds^2 = -dt^2 + dr^2 + r^2 d\hat{\phi}^2 + dz^2$$

(3.2)

with $0 \leq \phi < (1 - 4\mu)2\pi$. This is clearly Minkowski spacetime apart from a missing wedge with deficit angle $2\alpha = 8\pi\mu$. The idealisation of an infinitely long string reduces the problem to one in $(2 + 1)$ dimensional gravity [16] where there is no dynamics associated with curvature or transverse gravitons and the physics is determined entirely by global topological features of the spacetime.

‘Non-Interacting’ Strings – A Time Machine

Now consider the situation illustrated in Fig. 3.

Fig. 3 Sketch of the $(x, y)$ plane of the cosmic string spacetime where the string runs in the $z$ direction from the vertex of the missing wedge. The deficit angle of the wedge is $2\alpha$. Two trajectories are compared: the direct path AB and the path ACC´B crossing the wedge, where the points C and C´ are identified. The paths intersect the wedge at C and C´ at right angles.
We wish to compare two trajectories. The first is a direct path from A to B; the second also goes from A to B but via the missing wedge. This is shown as AC followed by C′B in the figure, where we identify the points C and C′. The idea is that for some choice of parameters, it should be possible to send a signal along the trajectory ACC′B so that it takes less time than a light signal sent directly from A to B. This would effectively realise superluminal propagation, which we can then try to exploit according to the discussion in the last section to produce back-in-time motion.

From Fig. 3, elementary trigonometry gives the relations

$$h = x \sin \alpha - d \cos \alpha$$
$$x^2 - \ell^2 = h^2 - d^2$$

(3.3)

from which we can show

$$1 - \frac{\ell^2}{x^2} = \sin^2 \alpha \left(1 - 2 \frac{d}{x} \cot \alpha - \frac{d^2}{x^2}\right)$$

(3.4)

The ‘effective speed’ of a light signal sent along ACC′B is $$v = \frac{x}{\ell}$$. It arrives before a direct light signal along AB if $$\ell < x$$, i.e. if the effective speed $$v > 1$$. As in the last section, this implies that the ACC′B signal arrives at B having travelled from A back in time as measured in a frame moving with velocity $$u > \frac{1}{v}$$.

This establishes that back-in-time paths across the wedge exist in a frame moving relative to the cosmic string with velocity $$u > \frac{\ell}{x}$$. From eq.(3.4), this condition is simply

$$u > \cos \alpha$$

(3.5)

The critical speed is therefore directly related to the deficit angle and hence the mass/unit length of the string.

So far, we have realised the analogue of the motion OC in Fig. 2. As explained there, in order to construct a genuine time machine trajectory we also need to show the existence of a further back-in-time path CD into the past light cone of the emitter. The crucial idea in the Gott scenario [9] is that this can indeed be realised if we have two cosmic strings moving in opposite directions, each with velocity $$u$$, as shown in Fig. 4.
Fig. 4 The Gott time machine trajectory. With two cosmic strings with equal and opposite velocities $u > \cos \alpha$, the path ACC’BDD’A can be a CTC.

At this point, we assume that the joining of the two cosmic string spacetimes is trivial (what we may call ‘non-interacting’ strings) and we may freely glue together the two patches of Minkowski spacetime. Later we will need to re-examine carefully the topology of combining the spacetimes in this way. If so, then the same arguments that led to the path ACC’B being effectively superluminal and therefore back-in-time apply equally to a return path via the oppositely moving cosmic string along the wedge path BDD’A. The combined path allows a light signal to be sent from A back into its own past light cone provided the condition (3.5) is realised.

It follows immediately that there exist CTCs looping around the two cosmic strings provided their velocities satisfy $u > \cos \alpha$. This is the Gott time machine [9].

‘Interacting’ Strings and Holonomy

Although locally things seem to be straightforward, patching together single cosmic string solutions (or particles in (2+1) dim gravity) of Einstein’s equations to produce new solutions is subject to subtle and restrictive constraints arising from global considerations. For example, although for small mass parameters the combination rule for static strings is simply additive, if we try to combine two masses $\mu_1$ and $\mu_2$ where the corresponding deficit angles satisfy $\alpha_1 + \alpha_2 > 2\pi$ then we encounter a global constraint. The total mass $M$ may exceed $\frac{1}{2}$ (corresponding to a deficit angle of $2\pi$) but then the 2-space must be closed and $M$ must equal precisely 1, determined by the Euler number of the space [16].
Patching together cosmic string spacetimes when the strings are in motion, even for small masses, also raises interesting global considerations. This can be viewed in terms of holonomy. In a curved spacetime, parallel transport of a vector in the tangent space at given point around a closed curve results in a transformation of that vector due to the non-vanishing curvature tensor. The group of these transformations is the holonomy group, which for a general (pseudo) Riemannian space is just the Lorentz group SO(3,1).

In the (2+1) dim flat spacetime corresponding to an infinite cosmic string, parallel transport around a closed curve encircling the string also produces a non-vanishing SO(2,1) holonomy transformation due to the deficit angle. For a stationary string, this is just a rotation equal to the deficit angle. The group element may be written in the form

\[ T = \exp(8\pi i P^\mu \Sigma_\mu) \]  

where \( \Sigma_\mu \) are the three SO(2,1) generators and \( P^\mu \) is the string energy-momentum vector. For a stationary string, \( P^0 = \mu, P^1 = P^2 = 0 \), \( \Sigma_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \) and \( T \) is the rotation matrix through an angle \( 2\alpha = 8\pi \mu \) in the \( (x,y) \) plane, \( R_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha \\ 0 & -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \). A boost with velocity \( u \) is represented by \( L_u = \begin{pmatrix} \cosh \beta & \sinh \beta & 0 \\ \sinh \beta & \cosh \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \) where the rapidity is defined through \( \tanh \beta = u \).

The element of the holonomy group for a moving cosmic string is thus \( L_u^{-1} R_\alpha L_u \). So for the two cosmic string Gott scenario, a loop encircling both strings is associated with the holonomy \[ T = L_u R_\alpha L_u^{-1} L_u^{-1} R_\alpha L_u \] (3.7) since the strings are moving in opposite directions. If we now try to identify this with the holonomy of a single string with mass/unit length \( M \) and deficit angle \( A = 4\pi M \) with some velocity \( U \), then we would write

\[ T = L_u^{-1} R_A L_U \] (3.8)

Taking the trace and equating these two expressions, we find

\[ \text{tr } R_A = \text{tr } L_u^2 R_\alpha L_u^{-2} R_\alpha \] (3.9)

which gives

\[ \cos A = \cos^2 \alpha - \sin^2 \alpha \cosh 2\beta \] (3.10)
or

$$\sin \frac{A}{2} = \cosh \beta \sin \alpha$$ \hspace{1cm} (3.11)

For non-zero velocities, this gives a non-trivial and non-linear addition rule for masses, viz.

$$\sin 2\pi M = \cosh \beta \sin 4\pi \mu$$

$$= \frac{1}{\sqrt{1-v^2}} \sin 4\pi \mu$$ \hspace{1cm} (3.12)

Now for this identification to work, clearly the r.h.s. of eq. (3.11) must be less than 1. This gives the condition \[17,18,19\]

$$\cosh \beta \sin \alpha < 1 \hspace{1cm} (3.13)$$

or equivalently

$$u < \cos \alpha \hspace{1cm} (3.14)$$

This is exactly the opposite of the criterion for the existence of CTCs in the Gott time machine scenario.

From eq. (3.6), the Gott condition $u > \cos \alpha$ therefore appears to correspond to a holonomy element representing a boost rather than a rotation. Equivalently, the combined two-string system would have a spacelike energy-momentum vector ($P^2 > 0$) corresponding to tachyonic motion. Although both constituent strings have subluminal velocities $u < 1$, the non-linear energy-momentum addition rules following from the holonomies imply that CTCs only arise for velocities for which the combined system is tachyonic and hence unphysical \[17,18,19\].

This appears to be a clean resolution of the cosmic string time machine puzzle, showing neatly that CTCs are necessarily associated with physically unrealisable energy-momentum conditions. However, even closer analysis shows that the situation is not quite so clear-cut. If we consider parallel transport of spinors rather than just vectors around a closed curve, the relevant holonomy group becomes SU(1,1), the double cover of SO(2,1). The group manifold of SU(1,1) reduces to that of SO(2,1) under an identification of points and, as shown explicitly in ref. [20], it is only under this identification that the holonomy element associated with the Gott string pair can be represented as the exponential of a spacelike generator. In the full SU(1,1) group, the Gott holonomy element cannot be represented as the exponential of a generator. (In terms of the geometry of the group manifold, this corresponds to a point which cannot be reached from the identity by following a geodesic path.) In this more general context, it is not true that the Gott condition implies a
spacelike energy-momentum vector. Indeed, the whole idea that the energy-momentum vector in (2+1) dim gravity can be defined by its identification with holonomy elements becomes problematic \[21\].

It remains the case, however, that the Gott spacetime contains CTCs at spacelike infinity \[17\]. In ref.\[17\], this is regarded as an unphysical boundary condition, but this assertion is challenged with some justification in ref.\[21\]. After all, unless we rule out the existence of CTCs as unphysical from the start and use this as a selection criterion as to what matter content and boundary conditions we consider to be ‘physical’ or ‘unphysical’, why should their absence be required as a boundary condition? Presumably, if we were able to make sense of quantum field theory on background spacetimes involving CTCs (see e.g. ref.\[21\] for a selection of references), then we would be happy to regard such spacetimes and boundary conditions as physical.

Finally, an important further result on the Gott scenario is developed in refs.\[19,20,22\]. In the first two of these papers, it is shown that the Gott time machine cannot be created in an open universe with a timelike total energy-momentum. Essentially, in an open universe, there is not enough energy to satisfy the Gott condition. On the other hand, ’t Hooft showed \[22\] that in a closed universe, the spacetime collapses to zero volume, a big crunch, before the Gott CTCs have time to form. So it seems that in a universe with timelike total energy-momentum which is initially free from CTCs, the Gott mechanism does not in fact create a time machine.
4. A Time Machine from Gravitational Shock Waves?

The spacetime describing a gravitational shock wave exhibits the unusual property that, depending on their impact parameter, geodesics which cross the shock may experience a discontinuous jump backwards in time. In this section, we investigate whether this phenomenon can be exploited to construct a time machine.

A gravitational shock wave is described by the Aichelburg-Sexl metric\(^2\)

\[
ds^2 = -dudv + f(r)\delta(u)du^2 + dr^2 + r^2d\phi^2
\]

(4.1)

with light-cone coordinates \(u = t - z\), \(v = t + z\). This describes an axisymmetric, plane-fronted shock wave advancing in the positive \(z\) direction at the speed of light \((u = 0)\) with a profile function \(f(r)\) in the transverse direction. It separates two regions of spacetime which are manifestly Minkowskian in standard coordinates. The Einstein equation

\[
R_{uu} = 8\pi T_{uu}
\]

(4.2)

is satisfied for a matter source \(T_{uu} = \rho(r)\delta(u)\), provided the profile function obeys

\[
\Delta f(r) = -16\pi \rho(r)
\]

(4.3)

where \(\Delta\) is the transverse 2 dim Laplacian. Two matter sources are of particular interest: a particle with \(\rho(r) = \mu\delta(x)\), for which the profile function is \(f(r) = -4\mu \ln \frac{L^2}{r^2}\) for some arbitrary scale \(L\), and a homogeneous beam \(\rho(r) = \rho = \text{const } (r < R)\) for which \(f(r) = -4\pi \rho r^2\) \((r < R)\) and \(-4\pi \rho R^2(1 + \ln \frac{r^2}{R^2})\) \((r > R)\). The geodesics for an infinite homogeneous shell were discussed by Dray and ’t Hooft\(^2\) and generalised to a finite homogeneous beam in arbitrary dimensions by Ferrari, Pendenza and Veneziano\(^3\). (We always consider impact parameters less than the beam size \(R\) in those parts of the following discussion where we specialise to homogeneous beams.)

Geodesics in this spacetime satisfy the following equations (\(\cdot\) denotes differentiation w.r.t. the affine parameter and ‘\(\cdot\)' differentiation w.r.t. \(r\)):

\[
\ddot{u} = 0
\]

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\(^2\) The Aichelburg-Sexl metric can be derived, for example, by taking the Schwarzschild solution for a mass \(M\) and boosting along the \(z\) direction with Lorentz parameter \(\gamma\), taking the infinite boost limit \(\gamma \to \infty\) with \(\gamma M\) fixed. An alternative method\(^4\) involves a technique of ‘cut-and-paste’ along a null hypersurface. For a detailed description of the A-S metric and the collision of two shock waves, see e.g. refs.\(^2\)\(^3\)\(^4\). The idealisation of an infinitely thin shock can of course be smoothed out, in which case the geodesics across the shock simply change very rapidly rather than actually discontinuously.
\[ \ddot{v} - f(r)\delta'(u)\dot{u}^2 - 2f'(r)\delta(u)\dot{u}\dot{r} = 0 \]
\[ \dot{r} - \frac{1}{2}f'(r)\delta(u)\dot{u}^2 + r\dot{\phi}^2 = 0 \]
\[ \ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} = 0 \]

(4.4)

In particular, a null geodesic in the negative \( z \) direction making a head-on collision with impact parameter \( b \) with the shock is described by

\[ v = f(b)\theta(u) + \tan^2 \left( \frac{\phi}{2} \right) u\theta(u) \]
\[ r = b - u\tan\left( \frac{\phi}{2} \right) \]

(4.5)

where the deflection angle \( \phi \) satisfies \( \tan\left( \frac{\phi}{2} \right) = -\frac{1}{2}f'(b) \). Notice that for sufficiently large impact parameters, \( \phi \) can exceed \( \pi/2 \) and the geodesic is reflected back in the positive \( z \) direction.

Fig. 5 Sketches of null geodesics in the Aichelburg-Sexl metric in the \((t, z)\) and \((r, z)\) planes. The incoming geodesic experiences a discontinuous jump on collision with the shock wavefront at \( u = v = 0 \) from \( t = z = 0 \) to \( t_P = z_P = \frac{1}{2}f(b) \) (shown as a dotted line in the figures). For \( b > L \) (particle) or all \( b \) (homogeneous beam), the shift is backwards in time, \( t_P < 0 \). The geodesic is also deflected towards the \( r = 0 \) axis by an angle \( \phi \) where \( \tan\left( \frac{\phi}{2} \right) = -\frac{1}{2}f'(b) \).

The important observation is that the geodesics make a discontinuous jump \( ^3 \) (see Fig. 5) from the collision surface \( u = v = 0 \) to \( u = 0 \), \( v_P = f(b) \), i.e. \( t_P = z_P = \frac{1}{2}f(b) \),

\(^3\) It should immediately be noted that this discontinuity can be removed by a discontinuous
which in the case of a homogeneous beam in particular is always negative for all impact parameters. Collision with the shock front therefore induces a jump *backwards in time*. This is the effect we will use to try and construct a time machine.

Before this, we also need the formulae for null geodesics making a collision with the shock wave at a general angle of incidence, say $\psi$. In this case, the geodesic is described by

\[
\begin{align*}
v &= u\left(\tan \frac{\psi}{2}\right)^2 \theta(-u) + \left[f(b) + u\left(\tan \frac{\phi'}{2}\right)^2\right] \theta(u) \\
r &= \left[b - u \tan \frac{\phi}{2}\right] \theta(-u) + \left[b - u \tan \frac{\phi'}{2}\right] \theta(u)
\end{align*}
\]

(4.6)

The incident ($\psi$) and deflected ($\phi'$) angles satisfy the elegant addition formula

\[
\tan \frac{\phi'}{2} = \tan \frac{\psi}{2} + \tan \frac{\phi}{2}
\]

(4.7)

with the definition $\tan \frac{\phi}{2} = -\frac{1}{2} f'(b)$ as before.

There is a further effect which will be important to us, viz. geodesic focusing. From the formulae above, we see that after collision the geodesics will intersect the $r = 0$ axis at $(t_F, z_F)$ where

\[
\begin{align*}
t_F &= \frac{1}{2} f(b) - \frac{b}{f'(b)} - \frac{1}{4} f'(b) b \\
z_F &= \frac{1}{2} f(b) + \frac{b}{f'(b)} - \frac{1}{4} f'(b) b
\end{align*}
\]

(4.8)

For a particle shock, this results in a caustic, but for the special case of a homogeneous beam, $f(r) = -\alpha r^2$ (where from now on we use the notation $\alpha \equiv 4\pi \rho$ to avoid unnecessary factors of $4\pi$), the geodesics actually *focus* to the single point $u_F = \frac{1}{\alpha}$, $v_F = 0$, i.e.

\[
t_F = -z_F = \frac{1}{2\alpha}
\]

(4.9)

change of coordinates in which the geodesics are continuous (though not differentiable) at the shock, but where the manifest Minkowski nature of the external metric is lost (see, e.g. ref. [23]). For this reason, we prefer to present the discussion of possible time paradoxes in the original Aichelburg-Sexl form, where the use of standard Minkowski coordinates highlights the peculiar effects of the shock wave on geodesic paths.
The greater the energy density in the beam, the closer the focal point is to the collision surface. This is illustrated in the \((t, z)\) and \((r, z)\) plane sketches in Fig. 6 and the 3D plot in Fig. 7. For a finite beam \((\rho(r) = 0 \text{ for } r > R)\) this is simply a focal point \(25\), whereas for an infinite shell it produces a focusing singularity \(24\).

**Fig. 6** Sketches in the \((t, z)\) and \((r, z)\) planes showing the focusing of geodesics with different impact parameters \((b)\) at the point \(u_F = \frac{1}{\alpha}\) following collision with a homogeneous beam with profile \(f(r) = -\alpha r^2\).

**Fig. 7** 3D plot in \((z, r, t)\) coordinates of the paths illustrated in Fig. 5. The transverse \(r\) coordinate is zero at the back of the diagram with larger values of the impact parameter \(r = b\) illustrated towards the front. The discontinuous jump of the geodesics followed by deflection towards \(r = 0\) and focusing is clearly seen.
Non-Interacting Shocks – A Time Machine

These discontinuous jumps backwards in time therefore appear to provide a promising mechanism to construct a time machine. However, as we emphasised in section 2, it is not at all sufficient to find a coordinate system in which motion occurs backwards in ‘time’. A genuine time machine requires that a signal can be sent into the past light cone of the emitter. It is clear that this cannot be achieved by considering the geodesics of only one shock wave.

Inspired by the Gott proposal with cosmic strings, we therefore consider the spacetime formed by two gravitational shock waves moving towards each other along the $z$ direction with zero impact parameter. We shall initially assume that we may neglect the interaction of the shocks themselves, so that in particular the shock fronts remain unchanged in the region $u, v > 0$. (Later, as in the cosmic string example, we will need to examine the effects of the interaction of the two gravitational sources more closely.) For simplicity, we choose the shocks to be generated by finite homogeneous beams, with profile functions $f_1(r) = -\alpha r^2$ and $f_2(r) = -\beta r^2$ (for $r < R$) respectively.

We therefore consider the following scenario, illustrated in Fig. 8.

![Fig. 8 Two gravitational shock waves, comprising homogeneous beams with energy densities given by $4\pi\rho(r) = \alpha r^2$ and $\beta r^2$ respectively, moving in opposite directions along the $z$ axis intersect at $u = v = 0$ with zero impact parameter. They are assumed not to interact so that the shock fronts continue unchanged along the trajectories $u = 0$ or $v = 0$. The ‘time machine’ trajectory SPQR comprises a null geodesic](image-url)
colliding with shock wave 1 at point S, jumping back in time to P and reversing its direction, colliding with shock 2 at Q and again jumping back in time to R. For suitable impact parameters, R lies in the past lightcone of S.

Consider a signal following a null geodesic which is struck head-on by shock 1 at point S with impact parameter $a$. It experiences a discontinuous jump back in time (and back in the longitudinal direction $z$) to P. For a suitable choice of impact parameter, its angle of deflection $\varphi_a$ can be sufficiently large that its direction along the $z$ axis is actually changed and it is reflected back into the path of the oncoming second shock wave. It collides with shock 2 at point Q in an oblique collision with impact parameter $b$ and angle of incidence $\psi$. A further discontinuous jump back in time takes the signal to R, where it continues its further deflected path. We wish to show that for suitable parameter choices, the spacetime interval RS is *timelike*, i.e. R really does, as the figure suggests, lie in the past light cone of S.

The geodesic equations governing this sequence of events are as follows. Approaching S, the initial signal follows the null geodesic

$$
\begin{align*}
  v &= v_0 \\
  r &= a 
\end{align*}
$$

(4.10)

Following the impact with shock 1 and jump to P, the signal along PQ follows the geodesic

$$
egin{align*}
  v &= v_0 + f_1(a) + u\left(\tan\frac{\varphi_a}{2}\right)^2 \\
  r &= a - u\tan\frac{\varphi_a}{2}
\end{align*}
$$

(4.11)

with $\tan\frac{\varphi_a}{2} = -\frac{1}{2}f_1'(a)$. Clearly,

$$
egin{align*}
  v_P &= v_0 + f_1(a) \\
  r_P &= a 
\end{align*}
$$

(4.12)

Under the no-distortion approximation, the signal then makes an oblique impact with the second shock front at Q where $v_Q = 0, u_Q = u_0, r_Q = b$, with $v_0$ and $b$ to be determined, and the angle of incidence $\psi = \pi - \varphi_a$. From the PQ geodesic equations, we deduce

$$
egin{align*}
  v_Q &= u_0 = -(v_0 + f_1(a))\left(\tan\frac{\varphi_a}{2}\right)^{-2} \\
  r_Q &= b = a + (v_0 + f_1(a))\left(\tan\frac{\varphi_a}{2}\right)^{-1}
\end{align*}
$$

(4.13)
After the second jump, the trajectory RS is therefore

\[ u = u_0 + f_2(b) + v \left( \tan \frac{\theta}{2} \right)^2 \]
\[ r = b - v \tan \frac{\theta}{2} \]

where

\[ \tan \frac{\theta}{2} = \tan \frac{\phi_b}{2} + \tan \frac{\psi}{2} \]
\[ = \tan \frac{\phi_b}{2} + \left( \tan \frac{\phi_a}{2} \right)^{-1} \]

In particular, the point R is

\[ u_R = u_0 + f_2(b) \]
\[ r_R = b \]

The spacetime interval \( \Delta s^2_{RS} \) is simply

\[ \Delta s^2_{RS} = -(u_S - u_R)(v_S - v_R) + (r_S - r_R)^2 \]

since the spacetime is flat in region II, and we find (eliminating \( u_0 \) and \( v_0 \) in favour of \( f_1(a) \) and \( f_2(b) \)),

\[ \Delta s^2_{RS} = -f_1(a)f_2(b) - (a - b) \left( f_1(a) \left( \tan \frac{\phi_a}{2} \right)^{-1} + f_1(b) \left( \tan \frac{\phi_a}{2} \right) \right) \]

So far, the discussion has applied to both particles and beams as the source of the shockwaves as the profile functions \( f_1(r) \) and \( f_2(r) \) have not been specified. For definiteness, we now consider homogeneous beams with profiles \( f_1(r) = -\alpha r^2 \) and \( f_2(r) = -\beta r^2 \). In this case, the condition (4.18) becomes

\[ \Delta s^2_{RS} = -(\alpha \beta ab) b^2 + \left( 1 - \frac{b}{a} \right) a^2 \]

The parameter count is as follows. \( \alpha, \beta \) specify the energy densities of the two beams, so are free parameters for us to choose. Likewise, the initial signal trajectory, which we may also choose freely, is specified by the two parameters \( a \) (impact parameter) and \( v_0 \).
(location of the initial collision). Using eq. (4.13), we can trade $v_0$ for the second impact parameter $b$, which may then be considered as the other free parameter. So in eq. (4.19), all the parameters can be chosen freely.

In particular, we can certainly choose the free parameters such that

$$\Delta s^2_{RS} < 0$$

(4.20)

i.e. RS is timelike with R indeed in the past light cone of S. In other words, the trajectory SPQR describes a time machine!

**Interacting Shocks**

As explained in the introduction, the motivation for this construction is not so much the belief that a time machine would actually exist in this spacetime, but that the reason preventing its existence would be interesting and shed new light on the physics of gravitational shock wave collisions. Indeed, there is much current interest in shock wave collisions in connection with the possible production of black holes in transplanckian energy collisions, especially since certain large extra dimension theories can accommodate a lowering of the effective four dimensional Planck energy to the TeV scale with the consequence that transplanckian scattering could actually be realised in TeV colliders such as the LHC (see, e.g. refs. [26,27,28]).

There are three particularly significant effects of the shock wave interaction that we have yet to consider. First, in the future region of the collision surface, the spacetime is curved, gravitational radiation may be emitted, and in general the metric is difficult to determine even perturbatively. (See however ref. [24] for a solution in the case of infinite shells in terms of ‘Robinson’s nullicle’ [29].) Second, the shock wavefronts themselves are distorted and, in the case of homogeneous beams, become concentric spherical shells converging to focal points. Third, for certain impact parameters, a closed trapped surface may form in the collision region.

The most interesting of these from our perspective would be the latter. The identification of a closed trapped surface in the colliding shock wave geometry was first made by Penrose [30] (see also [28]) and also discussed by Yurtsever [31]. Recently, this construction has been generalised to higher dimensions and non-zero impact parameters by Eardley

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4 It is perhaps also interesting to note that if we allow the signal to continue on from R in the negative $z$ direction according to eq. (4.14), it will collide again with shock 1 at a point $T$ with $v_T < v_S$. The null geodesic path therefore continues to cycle round a similar loop to SPQRT, getting closer to the origin $u = v = 0$ with decreasing $r$ in each loop.
and Giddings \[32\], and to the case of beams by Kohlprath and Veneziano \[33\]. Indeed, the original, rather speculative, motivation for pursuing this ‘time machine’ construction was that the impact parameters of the trajectories which would be required to produce a causality-violating loop would be related to the location of a possible trapped surface. In that case, the preservation of causality would be linked to the presence of an event horizon.

Unfortunately, the reason why the time machine fails is rather less exotic and concerns the first two properties of the interaction. At first sight, the future curved zone appears to be simply the wedge IV on the spacetime diagram Fig. 8, i.e. the future region of the initial collision surface \( u = v = 0 \). However, as we see below, the identification of the ‘collision surface’ and thus the curved zone in Aichelburg-Sexl coordinates is slightly more subtle. The other key ingredient is the distortion of the shock fronts themselves.

The shock wavefronts are generated by a congruence of null geodesics, so we identify the position and shape of the shocks using the same geodesic equations described above. In particular, the shock front generators will jump and deflect on collision at \( u = v = 0 \) with the other shock and, for homogeneous beams, will focus at \( v_F = \frac{1}{\beta} \) and \( u_F = \frac{1}{\alpha} \) for shocks 1 and 2 respectively. At first sight, it may seem that our approximation of neglecting the shock front distortion would be good provided the distortion is small, i.e. the signal trajectory lies far from the focusing zone. This would require \( u_Q \ll u_F \) and \( v_S \ll v_F \). In terms of the parameters \( \alpha, \beta, a, b \) we find

\[
\begin{align*}
    u_Q &= u_0 = \frac{1}{\alpha} \left( 1 - \frac{b}{a} \right) \\
    v_S &= v_0 = \alpha ab
\end{align*}
\]

The far-from-focus conditions are therefore:

\[
\begin{align*}
    u_Q \ll u_F & \iff 1 - \frac{b}{a} \ll 1 \\
    v_S \ll v_F & \iff \alpha\beta ab \ll 1
\end{align*}
\]

the second being equivalent to \( \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \ll 1 \).

Now notice the curious fact that these conditions involve precisely the two coefficients in the time machine criterion (4.19). The magnitude of the back-in-time motion \( S \rightarrow R \) is therefore constrained to be small by the far-from-focus condition. Moreover, the fact that the same parameters are controlling both effects strongly suggests we need to look more closely at the assumption that shock front distortion can be neglected.

In the non-interacting idealisation, the shock fronts propagate both before and after the collision as flat discs moving at the speed of light. The collision causes the shock front
generators to jump and deflect, so that for homogeneous beams the wavefront shape of, for example, shock 2 after the collision is as shown in Fig. 9.

![Diagram showing wavefronts](image)

**Fig. 9** The shape of the gravitational wave shockfront for shock 2 after impact at the surface $C_-$ ($z = 0$) with shock 1. The wavefront experiences a discontinuous jump in the negative $z$ direction (and backwards in $t$) to the surface $C_+$. It then focuses to the point $r = 0$, $z_F = \frac{1}{\alpha}$. The equal-time wavefronts are the spherical surfaces $W$. See below for the significance of point P.

![3D plot of equal-time shock wavefronts](image)

**Fig. 10** 3D plot in $(z, x, y)$ coordinates of the equal-time shock wavefronts described in Fig. 8.

The shock wavefronts, defined as equal $t$ surfaces, are spherical shells converging at the speed of light to the focal point $r = 0$, $z = z_F = \frac{1}{\alpha}$. The relevant geodesic equations
for their generators after the jump are

\[
\begin{align*}
v &= -\alpha a^2 (1 - \alpha u) \\
r &= a (1 - \alpha u)
\end{align*}
\]

(4.23)

and it is straightforward to show that the wavefronts are the surfaces

\[
r^2 = -\frac{1}{\alpha} v (1 - \alpha u) \quad \text{(wavefronts)}
\]

(4.24)

The key observation, however, concerns the status of the after-jump surface \( C_+ \). This is easily identified as the surface

\[
z = -\frac{1}{2} \alpha r^2 \quad \text{(collision surface } C_+)\]

(4.25)

This is the ‘collision surface’ regarded as the limit \( u \to 0_+ \) of the shock fronts, which must be distinguished in these coordinates from the ‘collision surface’ \( C_- \) identified as the limit \( u \to 0_- \) of the shock fronts from the pre-collision region. The important point is that the curved spacetime zone is in fact the future region of the collision surface \( C_+ \), not the ‘initial’ \( u = v = 0 \) collision disc \( C_- \).

In particular, this means that any point \( P \) behind the collision surface \( C_+ \) (see Fig. 9) at a later time is in fact in the curved zone\(^5\). It is now reasonably clear to see that the

\(^5\) This is especially evident, indeed trivial, if we use different coordinates. Defining the discontinuous change of coordinates to \( \hat{u}, \hat{v}, \hat{r} \) with

\[
\begin{align*}
\hat{u} &= \hat{u} \\
\hat{v} &= \hat{v} + \theta(\hat{u}) \left( f(\hat{r}) + \frac{1}{4} f'(\hat{r})^2 \hat{u} \right) \\
\hat{r} &= \hat{r} \left( 1 + \theta(\hat{u}) \frac{1}{2} f'(\hat{r}) \frac{1}{\hat{r}} \hat{u} \right)
\end{align*}
\]

then by construction the geodesics expressed in hatted coordinates are continuous across the shock. The shock fronts now remain flat in these coordinates but the spacetime metric is non-trivial everywhere and its Minkowski nature, which allows us some intuition as to the meaning of the ‘time’ coordinate, is hidden. (This is why we chose to analyse the ‘time machine’ trajectory in the original Aichelburg-Sexl coordinates.) The spacetime now looks like the illustration in Fig. 8 in that the shocks \( \hat{u} = 0 \) and \( \hat{v} = 0 \) divide the spacetime into the four regions I, \ldots IV. Now, however, the incoming signal geodesic simply passes straight through the \( \hat{u} = 0 \) shock wave and directly into region IV, which is the curved zone. The collision surfaces \( C_- \) and \( C_+ \), which must be carefully distinguished in Aichelburg-Sexl coordinates, are clearly degenerate in the hatted description.
point P on the time machine trajectory SPQR is in fact in this region. Comparing the point S with impact parameter $a$, corresponding to the collision of the signal and shock 1, with the collision at $u = v = 0$ of the shock 2 generator with $r = a$ and shock 1, we see that both are jumped back in time along the $u = 0$ line by the same amount $f_1(a)$. The point P is therefore necessarily to the future, with greater $z$, than the shock 2 wavefront, as illustrated in Fig. 9. This is perhaps best seen using the 3D plot in Fig. 7, where we can interpret the shaded surface as the wavefront of shock 2; the signal at P would then lie on the flat sloping surface, always behind (greater $t$, $z$ and $v$) the shock front. It is easy to check that this also holds for any angle of incidence of the signal.

The resolution of the ‘time machine’ scenario is therefore simply that the point Q illustrated in Fig. 8 does not in fact represent a collision of the signal with shock 2. Because the shock front has been distorted by the collision, the signal is now behind this curved shell and never catches up. Moreover, because it is in the future of the ‘collision surface’ $C_+$ (though not $C_-$) it is in the curved zone where the full metric is unknown and we cannot describe its path using the geodesic equations above.

Ultimately, therefore, this proposal for a time machine trajectory fails when the interaction between the gravitational shock waves is taken correctly into account. The shock fronts distort in the collision and tend to focus, with the curvature of the shock 2 front sufficient by construction to keep the signal trajectory behind it, in the future curved spacetime zone.

Is there a way to evade this conclusion? We may try, for example, to let the two finite-size shocks pass each other with a bigger impact parameter than their sizes $R_1, R_2$ (regions with non-zero energy density $\rho(r)$). But this does not help. The shock front is defined by the area of transverse space where the profile function $f(r)$ is non-zero and this extends beyond the radii of the shocks. Necessarily the initial signal collision with shock 1 takes place at an impact parameter where the profile of shock 2 is non-zero and, assuming the shock front is governed by the kinematics of the congruence of null geodesics which generate it, necessarily they both experience a back-in-time jump by the same amount. The situation is therefore exactly as above. The only way to recover the time-machine scenario of non-interacting shocks would be if the profile functions vanished outside some radius. But since in particular $f'(r) = -8E(r)/r$ where $E(r)$ is the total beam energy within the radius $r$, $f(r)$ can never vanish. This would imply screening of the gravitational source. Such an ‘anti-gravity’ theory would presumably have even more severe conceptual problems than those provided by CTCs!
In this section, we discuss a proposal by Dolgov and Novikov [14] to create a time machine by exploiting the phenomenon of superluminal photon propagation in gravitational fields originally discovered by Drummond and Hathrell [11]. As in the last two sections, this time machine again involves two gravitating objects in relative motion, but here the essential feature giving rise to backwards-in-time motion is not the spacetime itself but the assumed superluminal propagation of signals in the background gravitational field.

We begin with a very brief review of the Drummond-Hathrell mechanism [11] (see also refs. [34,12]). This exploits the fact that if the strong equivalence principle (SEP) is violated by direct couplings of the electromagnetic field to the curvature, then photons are no longer constrained to follow null geodesics and may, depending on their polarisation and direction, propagate along spacelike curves. The simplest form of such SEP-violating interactions is given by the Drummond-Hathrell effective action

\[ \Gamma = \int dx \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{m^2} \left( a R F_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\lambda} F^{\nu\lambda} + c R_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho} \right) \right] \] (5.1)

This action is induced by vacuum polarisation in QED in curved spacetime (in which case the couplings \( a, b, c \) are \( O(\alpha) \) and \( m \) is the electron mass), where it is valid in a low-frequency approximation where terms involving derivatives of the field strengths or curvature have been neglected. It remains controversial whether or not the characteristics of photon propagation in QED are changed by dispersion. An argument proposed by Khriplovich [35] suggests that there is no dispersion. However, we have recently constructed an extended effective action [13] valid to all orders in the derivative expansion and have used this to deduce the dispersive properties of the Drummond-Hathrell effect [12]. Apart from certain interesting exceptional cases, the photon velocity was shown to be frequency-dependent, with the quantum vacuum in a gravitational field acting as a dispersive medium. Unfortunately, this work was still not sufficiently general to give a definitive result for the high-frequency limit of the phase velocity, which determines the characteristics and thus the causal properties of photon propagation.

For our purposes here we shall set aside the issue of dispersion in QED, and instead simply consider the Drummond-Hathrell action (5.1) (with arbitrary, but small, couplings) as a phenomenological model of SEP violation, which predicts non-dispersive, superluminal signal propagation. Within this model, we want to address the question of whether superluminal propagation necessarily leads to violations of causality and permits the construction of time machines. This is rather different from the previous examples in that here we are concerned with whether there are physical light paths that return to their temporal origin, even if the geometrical structure of the background spacetime obeys standard causality properties.
The effective light cone derived from (5.1) is
\[ k^2 + \frac{(2b+4c)}{m^2}R_{\mu\lambda}k^\mu k^\lambda - \frac{8c}{m^2}C_{\mu\nu\lambda\rho}k^\mu k^\lambda a^\nu a^\rho \equiv G^{\mu\nu}(R,a)k_\mu k_\nu = 0 \] (5.2)

where \( k_\mu \) is the wave-vector and \( a^\mu \) the polarisation (normalised so that \( a^2 = -1 \)). For vacuum spacetimes, only the term depending on the Weyl tensor is present and it is this that gives rise to the polarisation dependence of the speed of light (gravitational birefringence). Since in this model the modified light cone is still quadratic in \( k_\mu \), it can be written as shown in terms of \( G^{\mu\nu}(R,a) \), which depends on the polarisation and local spacetime curvature. The equivalent photon momentum \( p^\mu = \frac{dx^\mu}{ds} \), which is a tangent vector to the light rays \( x^\mu(s) \), is related non-trivially to the wave-vector by
\[ p^\mu = G^{\mu\nu}k_\nu \] (5.3)

With this definition, the photon momentum satisfies
\[ G_{\mu\nu}p^\mu p^\nu = G^{\mu\nu}k_\mu k_\nu = 0 \] (5.4)

where \( G \equiv G^{-1} \) defines a new effective metric which determines the light cones mapped out by the light rays in the geometrical optics construction. Notice that indices are always raised or lowered using the original geometric metric \( g^{\mu\nu} \), so that we need to distinguish explicitly between the effective metric \( G \) and its inverse \( G \).

It is elegant and instructive to re-express this condition using the Newman-Penrose formalism (see, e.g. ref. [37] for a review). We introduce a null tetrad in terms of the null vectors \( \ell^\mu, n^\mu \) and the complex null vectors \( m^\mu \) and \( \bar{m}^\mu \), where \( m^\mu = \frac{1}{\sqrt{2}}(a^\mu + ib^\mu) \) with \( a^\mu \) and \( b^\mu \) spacelike. These are chosen to satisfy the orthogonality and normalisation conditions
\[ \ell.m = \ell.\bar{m} = n.m = n.\bar{m} = 0 \quad \ell.n = 1 \quad m.\bar{m} = -1 \] (5.5)

Components of the Weyl tensor in the corresponding null tetrad basis are denoted by the complex scalars \( \Psi_0, \ldots, \Psi_5 \), where
\[ \Psi_0 = -C_{abcd}\ell^a m^b \ell^c m^d = -C_{1313} \]
\[ \Psi_1 = -C_{abcd}\ell^a n^b \ell^c m^d = -C_{1213} \]
\[ \Psi_2 = -C_{abcd}\ell^a m^b \bar{m}^c n^d = -C_{1342} \]
\[ \Psi_3 = -C_{abcd}\ell^a n^b \bar{m}^c n^d = -C_{1242} \]
\[ \Psi_4 = -C_{abcd}n^a \bar{m}^b n^c \bar{m}^d = -C_{2424} \] (5.6)

Notice that this is not the same relation as for conventional ‘tachyons’, which have an inhomogeneous term in the mass-shell relation, i.e. \( k^2 = -M^2 \) in a metric with signature (+−−−). The speed of such a tachyon is not fixed but depends on its energy.
In this formalism, the modified light cone condition takes a simple form. If we choose the unperturbed photon momentum to be in the direction of the null vector \( \ell^\mu \), i.e. \( k^\mu = \omega \ell^\mu \), and the transverse polarisation vectors as \( a^\mu, b^\mu \), then the light cone condition becomes (for Ricci-flat spacetimes)

\[
k^2 = \pm \frac{4c\omega^2}{m^2}(\Psi_0 + \Psi^*_0) \tag{5.7}
\]

for the two polarisations respectively. It is clear from (5.7) that in Ricci flat spacetimes, if one photon polarisation travels subluminally, the other necessarily has a superluminal velocity. Physical photons no longer follow the geometrical light cones, but instead propagate on the family of effective, polarisation-dependent, light cones defined by eq.(5.7).

Many examples of the Drummond-Hathrell effect in a variety of gravitational wave, black hole and cosmological spacetimes have been studied \[11,38,39,15\] etc. The black hole cases are particularly interesting, and it is found that for photons propagating orbitally, the light cone is generically modified and superluminal propagation occurs. For radial geodesics in Schwarzschild spacetime, however, and the corresponding principal null geodesics in Reissner-Nordstr"om and Kerr, the light cone is unchanged. The reason is simply that if we choose the standard Newman-Penrose tetrad in which \( \ell^\mu \) is the principal null geodesic, the only non-vanishing component of the Weyl tensor is \( \Psi_2 \) since these black hole spacetimes are all Petrov type D, whereas the modification to the light cone condition involves only \( \Psi_0 \).

Our principal interest here is in whether the existence of this type of superluminal propagation in gravitational fields, which after all does imply backwards in time motion in a class of local inertial frames, is necessarily in contradiction with our standard ideas on causality. A precise formulation of the question is whether a spacetime which is stably causal with respect to the original metric remains stably causal with respect to the effective metric defined by the modified light cones (5.7). A precise definition of stable causality can be found in ref.\[3\], proposition 6.4.9. Essentially, this means that the spacetime should still admit a foliation into a set of hypersurfaces which are spacelike according to the effective light cones.

The Drummond-Hathrell effective light cones differ from the geometrical ones by perturbatively small shifts dependent on the local curvature. While it is certainly possible that these shifts are sufficient to destroy stable causality, we have not identified any general argument that says this must be so. Rather, it seems entirely possible that a curved spacetime could remain stably causal with respect to the effective light cones. The issue fundamentally comes down to global, topological properties of spacetime, such as are ad-

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7 See also ref.\[36\] for a closely related discussion in the context of superluminal propagation in flat spacetime between Casimir plates.
dressed in ref. [1], but which lie rather beyond the scope of this paper. Instead, we shall take a more pragmatic approach and simply try, following ref. [14], to identify a time-machine scenario using superluminal propagation.

The Dolgov-Novikov time machine is most simply (though not essentially) described for a spacetime which admits superluminal radial trajectories. Rather than use a black hole spacetime as suggested in ref. [14], we therefore describe the time machine in terms of the asymptotic spacetime surrounding an isolated source of gravitational radiation, which is described by the Bondi-Sachs metric. The Drummond-Hathrell effect in this spacetime has recently been studied in ref. [15].

The Bondi-Sachs metric is

\begin{equation}
\begin{aligned}
ds^2 &= -W du^2 + 2 e^{2\beta} dudr + r^2 h_{ij} (dx^i - U^i du)(dx^j - U^j du)
\end{aligned}
\end{equation}

where

\begin{equation}
\begin{aligned}
h_{ij} dx^i dx^j &= \frac{1}{2} (e^{2\gamma} + e^{2\delta}) d\theta^2 + 2 \sinh(\gamma - \delta) \sin \theta d\theta d\phi + \frac{1}{2} (e^{-2\gamma} + e^{-2\delta}) \sin^2 \theta d\phi^2
\end{aligned}
\end{equation}

The metric is valid in the vicinity of future null infinity \( I^+ \). The family of hypersurfaces \( u = \text{const} \) are null, i.e. \( g^{\mu\nu} \partial_\mu u \partial_\nu u = 0 \) and their normal vector \( \ell_\mu \) satisfies

\begin{equation}
\ell_\mu = \partial_\mu u \quad \Rightarrow \quad \ell^2 = 0, \quad \ell^\mu D_\mu \ell^\nu = 0
\end{equation}

The curves with tangent vector \( \ell^\mu \) are therefore null geodesics; the coordinate \( r \) is a radial parameter along these rays and is identified as the luminosity distance. Outgoing radial null geodesics are associated with \( \ell^\mu \) while incoming null geodesics have tangent vector \( n^\mu \) in the obvious Newman-Penrose basis [41][15]. The metric functions have asymptotic expansions near \( I^+ \) for large luminosity distance \( r \). We just need those for the functions \( \gamma \pm \delta \), which correspond to the two independent gravitational wave polarisations:

\begin{equation}
\begin{aligned}
\frac{1}{2} (\gamma + \delta) &= \frac{c_+}{r} + \frac{q_+}{r^3} + O\left(\frac{1}{r^4}\right) \\
\frac{1}{2} (\gamma - \delta) &= \frac{c_\times}{r} + \frac{q_\times}{r^3} + O\left(\frac{1}{r^4}\right)
\end{aligned}
\end{equation}

where \( c_{+(\times)} \) and \( q_{+(\times)} \) are functions of \((u, \theta, \phi)\). \( \partial_u c_{+(\times)} \) are the Bondi ‘news functions’, while \( \partial_u^2 c_{+(\times)} \) can be identified as the gravitational wave amplitude in a weak-field limit. \( q_{+(\times)} \) denote the ‘quadrupole aspect’ of the gravitational radiation.

As shown in ref. [15], the modifications to the light cone following from eq.(5.2) are as follows. For outgoing photons, with \( k^\mu = \omega \ell^\mu \),

\begin{equation}
\begin{aligned}
k^2 &= \pm \frac{4c^2}{m^2} \left( \Psi_0 + \Psi_0^* \right) = \pm \frac{48c^2}{m^2} \frac{q_+}{r^5}
\end{aligned}
\end{equation}
for the two photon polarisations aligned with the + gravitational wave polarisation. A
similar result holds for alignment with the \( \times \) polarisation. For incoming photons,
\[
k^2 = \pm \frac{4c\omega^2}{m^2} (\Psi_4 + \Psi_4^*) = \pm \frac{8c\omega^2}{m^2} \frac{1}{r} \partial_u c_+ \tag{5.13}
\]
The different dependence on \( 1/r \) is governed by the Peeling Theorem in Bondi-Sachs space-
time [41], according to which the asymptotic form of the Weyl tensor components is \( \Psi_4 \sim \frac{1}{r} \),
\( \Psi_3 \sim \frac{1}{r^2} \), \ldots, \( \Psi_0 \sim \frac{1}{r^5} \).

Depending on the photon (and gravitational wave) polarisations, we therefore have
superluminal propagation for both outgoing and incoming radial photons, albeit only of
\( O\left(\frac{1}{r^5}\right) \) for the photons in the same direction as the gravitational waves. The Bondi-Sachs
spacetime therefore provides a natural realisation of the Dolgov-Novikov (DN) scenario.

The DN proposal is to consider first a gravitating source with a superluminal photon
following a trajectory which we may take to be radial. Along this trajectory, the metric
interval is effectively two-dimensional and DN consider the form
\[
ds^2 = A^2(r)dt^2 - B^2(r)dr^2 \tag{5.14}
\]
(Actually, in order to realise the Bondi-Sachs case, we would need to let the coefficients
\( A \) and \( B \) depend on both \( r \) and \( t \) because of the time-dependence of the gravitational
wave metric. This complication is inessential for the main ideas, so we neglect it and
simply follow the DN calculation based on eq. (5.14).) The photon velocity in the \((t,r)\)
coordinates is taken to be \( v = 1 + \delta v \), so the effective light cones lie perturbatively close
to the geometric ones. The trajectory is forward in time with respect to \( t \).

DN now make a coordinate transformation corresponding to a frame in relative motion
to the gravitating source, rewriting the metric interval along the trajectory as
\[
ds^2 = A^2(t', r')(dt'^2 - dr'^2) \tag{5.15}
\]
The transformation is
\[
t' = \gamma(u)(t - ur - uf(r)) \\
r' = \gamma(u)(r - ut + f(r))
\]
with
\[
f(r) = \int dr \left( \frac{B}{A} - 1 \right) \tag{5.17}
\]
Another simple possibility with superluminal radial trajectories would be the dilaton
gravity model considered in ref. [42].
This is best understood in two steps. First, since any two-dimensional metric is conformally flat, we can always (even if $A$ and $B$ were functions of both $r$ and $t$) find a transformation $(t, r) \rightarrow (\tilde{t}, \tilde{r})$ which takes the metric to the standard form $ds^2 = \Omega^2(d\tilde{t}^2 - d\tilde{r}^2)$. For (5.14), the transformation to standard conformally flat form is achieved by

$$
\tilde{t} = t \\
\tilde{r} = r + f(r)
$$

(5.18)

with $f(r)$ as in (5.17). The second step is just a boost on the flat coordinates $(\tilde{t}, \tilde{r})$, viz.

$$
t' = \gamma(u)(\tilde{t} - u\tilde{r}) \\
r' = \gamma(u)(\tilde{r} - u\tilde{t})
$$

(5.19)

Now, a superluminal signal with velocity

$$
v = 1 + \delta v = \frac{B}{A} \frac{dr}{dt}
$$

(5.20)

emitted at $(t_1, r_1)$ and received at $(t_2, r_2)$ travels forward in $t$ time (for small, positive $\delta v$) with interval

$$
t_2 - t_1 = \int_{r_1}^{r_2} dr \left(1 - \delta v\right) \frac{B}{A}
$$

(5.21)

As DN show, however, this motion is backwards in $t'$ time for sufficiently large $u$, since the equivalent interval is

$$
t'_2 - t'_1 = \gamma(u) \int_{r_1}^{r_2} dr \left(1 - u - \delta v\right) \frac{B}{A}
$$

(5.22)

The required frame velocity is $u > 1 - \delta v$, i.e. since $\delta v$ is small, $u > \frac{1}{v}$.

The situation so far is therefore identical in principle to the discussion of superluminal propagation illustrated in section 2, Fig. 1. In DN coordinates the outward superluminal signal is certainly propagating backwards in time, but a reverse path with the same perturbatively superluminal velocity would necessarily go sufficiently forwards in time to arrive back within the future light cone of the emitter. 

\footnote{Notice that in the Bondi-Sachs case, the return signal could also be arranged to have a superluminal velocity (of $O(\frac{1}{r})$ rather than the outward $O(\frac{1}{r^5})$) assuming the polarisation can be reversed without altering any other features of the kinematics.}
As yet, therefore, we have no time machine but have simply recovered standard features of superluminal motion described in different frames. At this point, DN propose to introduce a second gravitating source moving relative to the first, as illustrated below.

![Diagram](image-url)

**Fig. 11** The Dolgov-Novikov time machine proposal. A superluminal signal from X, described as backwards-in-time in a relevant frame, is sent towards a second gravitating source Y moving relative to X and returned symmetrically.

This is reminiscent of the cosmic string and shock wave proposals in that a single gravitational source provides the essential physical effect but two sources in relative motion are required to construct a genuine time machine scenario.

DN now claim that a superluminal photon emitted with velocity $v(r)$ in the region of X will travel backwards in time (according to the physically relevant coordinate $t'$) to a receiver in the region of Y. A signal is then returned symmetrically to be received at its original position in the vicinity of X, arriving, according to DN, in its past. This would then be analogous to the situation illustrated in Fig. 2.

However, as we emphasised in section 2, we are *not* free to realise the scenario of Fig. 2 in the gravitational case, because the SEP-violating superluminal propagation proposed by Drummond and Hathrell is pre-determined, fixed by the local curvature. The $t'$ frame may describe back-in-time motion for the outward leg, but it does not follow that the return path is similarly back-in-time *in the same frame*. The appropriate special relativistic analogue is the scenario of Fig. 1, not Fig. 2. This critique of the DN time machine proposal has already been made by Konstantinov [43] and further discussion of the related effect in flat spacetime with Casimir plates is given in [36]. The relative motion of the two sources, which at first sight seems to allow the backwards-in-time coordinate $t'$ to be relevant and to be used symmetrically, does not in fact alleviate the problem.

The true situation seems rather more to resemble the following figure.

![Diagram](image-url)

**Fig. 12** A decomposition of the paths in Fig. 11 for well-separated sources.
With the gravitating sources $X$ and $Y$ sufficiently distant that spacetime is separated into regions where it is permissible to neglect one or the other, a signal sent from the vicinity of $X$ towards $Y$ and back would follow the paths shown. But it is clear that this is no more than stitching together an outward plus inward leg near source $X$ with an inward plus outward leg near $Y$. Since both of these are future-directed motions, in the sense of Fig. 1, their combination cannot produce a causality-violating trajectory. If, on the other hand, we consider $X$ and $Y$ to be sufficiently close that this picture breaks down, we lose our ability to analyse the Drummond-Hathrell effect since the necessary exact solution of the Einstein equations describing the future zone of the collision of two Bondi-Sachs sources is unknown.

We therefore conclude that the Dolgov-Novikov time machine, like the others considered in this paper, ultimately does not work. The essential idea of trying to realise the causality-violating special relativistic scenario of Fig. 2 by using two gravitational sources in relative motion does not in the end succeed, precisely because the physical Drummond-Hathrell light cones (5.2) are fixed by the local curvature. It appears, surprising though it may appear at first sight, that in general relativity with SEP-violating electromagnetic interactions, superluminal photon propagation and causality may well be compatible.

6. Conclusions

In this paper, we have analysed three proposals for constructing time machines which, although quite different, share a common basic structure, and have shown how each of them is in fact ultimately consistent with our fundamental idea of causality.

The first, the Gott cosmic string time machine, exploits the conical nature of the cosmic string spacetime to find effectively superluminal paths. Introducing a second cosmic string then leads to the existence of CTCs encircling the two strings, provided the string velocities satisfy the condition $u > \cos \alpha$, where $2\alpha$ is the deficit angle related to the string mass. However, provided appropriate boundary conditions are imposed, global consistency of the two-string solution requires exactly the opposite condition on the velocities, $u < \cos \alpha$. This condition was formulated in the language of holonomy for loops in the two-string spacetime. The intricate question of whether these boundary conditions should be considered physical and the relation of Lorentz boost holonomies to tachyonic motion was also discussed.

The second example mimics the Gott proposal but this time exploits the discontinuous geodesics which arise in gravitational shock wave spacetimes as the mechanism for effec-
tively superluminal paths. In the unrealistic approximation of neglecting the interaction between the shocks, CTCs were also found in which a test particle is struck successively by the two shocks, the first collision reversing its direction of motion as well as inducing a time shift. However, a self-consistent analysis shows that because the shock fronts are themselves generated by a congruence of null geodesics, they are also distorted by the same mechanism responsible for the discontinuities in the test particle geodesics. The shock front distortion is exactly sufficient to invalidate the CTC construction. The motivation for this construction, namely that the reason for the absence of CTCs when the shock wave interactions are included would shed new light on the kinematics of event horizon formation in the collision, was not realised in this case.

The final example exploited the genuinely superluminal propagation arising from a SEP violating effective action to look for causality violations. General arguments why this specific type of superluminal propagation need not be in conflict with stable causality were discussed. A critical analysis of the proposal by Dolgov and Novikov to use this mechanism to build a time machine again involving two gravitational sources in relative motion was then made, with the by now familiar conclusion that while causal violations appear to arise when the two sources are treated independently, a consistent analysis of the combined spacetime shows that closed light trajectories do not occur.

We close with some general reflections on these diverse examples, and their relevance for future theories and theorems. All of the scenarios discussed here rely fundamentally on finding effectively superluminal paths, whether by constructing spacetimes with missing wedges or non-trivially patched regions of Minkowski spacetime as in the cosmic string and shock wave models, or by assuming a modification of electrodynamics in which light itself no longer follows null paths. Both of the first two models fall within the orbit of the chronology protection theorems, even though the Gott spacetime is extremely subtle. The final example is different in that it breaks the assumption made in all conventional work on causality in general relativity that physical signals follow timelike or null geodesics. Now it may well be true (see ref.\[12\] for a careful recent discussion) that despite the Drummond-Hathrell effect in QED, the ‘speed of light’ relevant for causality, i.e. the high-frequency limit of the phase velocity, is indeed \(c\). However, this effect highlights the fact that theoretically consistent generalisations of (quantum) electrodynamics can be constructed in which light signals no longer follow null geodesics and the physical light cones define a new metric which does not coincide with the geometric one. An elegant example of such a bi-metric theory was recently formulated in ref.\[44\]. It may well, therefore, be interesting to revisit the chronology protection theorems in these more general contexts.

Finally, although the models described in this paper have been fundamentally classical, the most interesting questions on the nature of causality arise in quantum theory, where in
particular the weak energy condition may be circumvented. Although the indications from studies of quantum field theory in curved spacetime suggest that the occurrence of CTCs leads to pathologies such as divergent expectation values or violations of unitarity, perhaps these are issues that also deserve renewed attention from the perspective of modern ideas in quantum gravity. There are surely further scenarios to devise which may yet successfully challenge Hawking’s chronology protection conjecture [1]: ‘the laws of physics do not allow the appearance of closed timelike curves’.

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