Localized Fermions and Anomaly Inflow
via Deconstruction

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Abstract

We study fermion localization in gauge theory space. We consider four dimensional product gauge groups in which light chiral fermions transform under different gauge factors of the product group. This construction provides a suppression of higher dimensional operators. For example, it can be used to suppress dangerous proton decay operators. The anomalies associated with the light chiral fermions are compensated by Wess-Zumino terms, which in the continuum limit reproduce the five dimensional Chern-Simons term.
I. INTRODUCTION

Extra-dimensional scenarios provide a novel mechanism of generating small coefficients [1]. The overlap of spatial wave-functions can be small for fields localized at different positions. Operators involving those fields can thus be suppressed. In an effective four-dimensional description such small coefficients appear to be accidental. There are several important applications of this idea, for example explaining proton stability in theories with a low fundamental scale, or modeling small Yukawa couplings of the Standard Model fermions.

In this article we study fermion localization in four dimensions. Localization in extra dimensions is replaced by localization in “theory space” using the recent notion of deconstruction [2,3]. Deconstruction trades extra dimensions for lattices of four dimensional gauge theories. The sites of such lattices are gauge groups, so extra dimensions are replaced by products of gauge groups. The product gauge group is spontaneously broken to a single gauge group, which is the lowest Kaluza-Klein (KK) mode of the extra dimensional gauge group. The massive gauge bosons corresponding to the broken generators play the role of the massive KK tower.

Deconstruction is interesting for several reasons. First, it provides a UV completion of higher dimensional theories. Second, it motivates interesting model building. It is certainly not surprising that a space-time dimension can be successfully discretized. When the lattice spacing is small, lattice theories should provide a sufficiently accurate description of their continuum counterparts. What is interesting for model building is that some extra-dimensional features are preserved by very coarse lattices. When the number of lattice sites is small the spectrum does not resemble higher dimensional theories at all. Several models motivated by deconstruction illustrate this point. Examples include novel extensions of the Standard Model [4,5], mechanisms for communicating supersymmetry breaking [6,7], low scale unification [8], and models for breaking the unified gauge symmetries [9]. In some cases, there exist higher dimensional theories which guide four-dimensional model building, in other cases four dimensional models cannot be extended into higher dimensions.

Deconstruction has yielded some formal developments as well. These include a description of little string theories in terms of gauge theory [10], a single gauge group description of extra dimensions in the large number of colors limit [11], and models of noncommutative geometry [12,13]. Other works investigated warped background geometry [14], topological objects in field theory [15], Seiberg-Witten curves [16], and even ventured into gravity [17]. One of our goals is to investigate anomalies associated with chiral fermions localized in theory space and to derive the Chern-Simons term in the continuum limit.

The article is organized as follows. In the next section we describe how to localize fermions in a product group theory. Our mechanism relies on decoupling massive fermions and it is not a lattice description of fermionic zero modes trapped on topological defects. In Section II, we also explain how small coefficients are generated and why high-energy scattering is suppressed [18] in our framework. In Section III, we consider the continuum limit of a theory with localized fields. Since chiral fermions transform under different gauge groups, the theory appears to be anomalous. The anomalies are canceled by the Wess-Zumino (WZ) terms. In the continuum limit, the WZ terms associated with different lattice sites combine to reproduce the bulk Chern-Simons term. In Section IV, we present supersymmetric models that realize our idea, paying particular attention to proton decay operators. The models are
extensions of the minimal supersymmetric Standard Model (MSSM), in which light quarks and leptons transform under different gauge groups.

II. LOCALIZED FERMIONS

In this section we introduce the construction of localized fermions, which is central to the rest of the article. We then explore a few consequences of such localization. We will not summarize the inner workings of deconstruction and the emergence of extra dimensions since it has been described extensively in the literature and instead we refer the reader to Refs. [2, 3].

A useful notation for deconstructed theories is the moose notation [19], in which individual gauge groups are denoted as circles. Fermionic (scalar) fields are denoted by solid (dashed) oriented lines. A line incoming to a circle indicates a field transforming in the fundamental representation, outgoing one in the antifundamental. Consequently, lines connecting two circles represent fields transforming under two gauge groups in the “bifundamental” representation, that is fundamental under one of the gauge groups, antifundamental under the other.

Consider an $SU(k)^n$ product gauge group with a pair of Weyl fermions, $\psi_i$ and $\bar{\psi}_i$, for each group, as illustrated in Fig. 1. In addition to the fermion fields, there are bifundamental scalar “link” fields, $U_i$. The link fields obtain vevs, $v$, proportional to the identity, which break $SU(k)^n$ to the diagonal subgroup $SU(k)$. We do not define the dynamics that provides the vevs for the link fields. It could either be arranged by a choice of potential for the link fields or emerge dynamically if the link fields are condensates of more fundamental objects.

In addition to gauge interactions, we add the following mass terms and Yukawa couplings.

$$\mathcal{L} = \lambda \sum_{i=1}^{n-1} \bar{\psi}_i U_i \psi_{i+1} + \mu \sum_{i=2}^{n} \bar{\psi}_i \psi_i,$$

where $\lambda$ is a common Yukawa coupling and $\mu$ a common mass term. Note that we have omitted a mass term for $\bar{\psi}_1$ and $\psi_1$. When the link fields obtain vevs, the Yukawa interactions become mass terms with mass $M = \lambda v$. In what follows it will be crucial that $\mu < M$. Our objective for now is to explain how localization works and we postpone to Section IV the obvious questions on how to obtain such a pattern of couplings, which includes the hierarchy of masses, and the absence of Yukawa couplings $\bar{\psi}_{i+1} U_i^\dagger \psi_i$.

From Eq. (1) the mass matrix for fermions is
It is apparent from the construction that $\psi_1$ is massless irrespectively of the values of $\mu$ and $M$ since it does not couple to any other fermion. There must also be another massless fermion, $\bar{\psi}$, because at most $n - 1$ $\psi$'s are massive. When $\mu = 0$, $\bar{\psi}_n$ is massless, while all other fermions have mass $M$. So in this simple case massless fermions transform under the first and the last $SU(k)$ factors of the moose.

When $\mu \neq 0$, the zero modes are $\psi_1$ and

$$\bar{\Psi} = \left(\frac{\mu}{M}\right)^{n-1} \bar{\psi}_1 - \left(\frac{\mu}{M}\right)^{n-2} \bar{\psi}_2 + \ldots + (-1)^{n-1} \bar{\psi}_n \sqrt{1 + \left(\frac{\mu}{M}\right)^2 + \ldots + \left(\frac{\mu}{M}\right)^2(n-1)}.$$  \hspace{1cm} (3)

We assume that $\left(\frac{\mu}{M}\right)^{n-1} \ll 1$, so that $\bar{\psi}_1$ appears with a small coefficient in the linear combination comprising the $\bar{\Psi}$ zero mode. Thus, operators involving both zero modes will generally be suppressed as their overlap is small. The coupling suppression is the result of a symmetry present for special values of parameters. When $\mu = 0$, there is a $U(1)$ global symmetry that rotates $\bar{\psi}_n$ only and does not act on any other field. For $\mu \neq 0$ this symmetry is approximate.

When unknown or poorly understood dynamics becomes relevant at a high energy scale, $\Lambda$, one needs to regard the theory as an effective theory with a cutoff $\Lambda$. At $\Lambda$, all operators allowed by gauge symmetries can be generated, presumably with order one coefficients. We are interested in such operators if they involve the zero modes since these are the operators that are accessible to experiment. High energy dynamics can generate, for example, the following gauge invariant combinations in our moose theory

$$\bar{\psi}_1 \psi_1, \quad \frac{1}{\Lambda^{n-1}} \bar{\psi}_n U_{n-1}^\dagger \ldots U_1^\dagger \psi_1.$$  \hspace{1cm} (4)

Expressing these operators in terms of the zero modes and replacing the link fields by their vevs reveals the suppressions associated with them. The first operator is small due to the $\bar{\Psi}$ zero mode wave-function, the second one is suppressed if $v < \Lambda$. Therefore, these gauge invariants always bring in small coefficients

$$\bar{\psi}_1 \psi_1 \rightarrow \left(\frac{\mu}{M}\right)^{n-1} \bar{\Psi} \psi_1,$$

$$\frac{1}{\Lambda^{n-1}} \bar{\psi}_n U_{n-1}^\dagger \ldots U_1^\dagger \psi_1 \rightarrow \left(\frac{\nu}{\Lambda}\right)^{n-1} \bar{\Psi} \psi_1.$$  \hspace{1cm} (5)

We will apply these results to models of quark-lepton separation in Section IV.

Another interesting consequence of localized fermions is the suppression of high-energy scattering cross-sections. The $t$ and $u$ channel scattering amplitudes turn out to be subdued compared to the low-energy results when their natural scaling with $1/t$ or $1/u$ is factored out. In extra dimensions the suppression appears for momentum transfers comparable to the inverse of the separation distance and becomes an exponential suppression in
the large momentum limit. The moose theory resembles a five dimensional theory only at distances shorter than the compactification scale, but longer than the lattice spacing. At distances shorter than the lattice spacing, the theory exhibits four dimensional behavior. Therefore, in the deconstructed theory the suppression will reach its maximum near the energies corresponding to the inverse lattice spacing and will not increase after that.

To illustrate this point we study the scattering

$$\psi_1 + \bar{\Psi} \rightarrow \psi_1 + \bar{\Psi}$$

(6)

in the tree-level approximation. When the link fields get vevs the spectrum of the gauge bosons includes a massless multiplet of the diagonal $SU(k)$ and $n-1$ massive adjoints of $SU(k)$. Let us assume that all the gauge couplings are equal to $g$. Then, in terms of the original gauge bosons, $A_i$, the mass eigenstates $a_j$ are [3]:

$$a_0 = \sqrt{\frac{1}{n}}(A_1 + A_2 + \ldots + A_n),$$

$$a_j = \sqrt{\frac{2}{n}} \sum_{i=1}^{n} \cos \left( \frac{(2i-1)j\pi}{2n} \right) A_i,$$

(7)

where $j = 1, \ldots, n-1$. The masses of the vector bosons are $m_j^2 = 2g^2v^2\sin^2\left(\frac{j\pi}{2n}\right)$. We express the gauge couplings of $\psi_1$ and $\bar{\Psi}$ in terms of the mass eigenstates. The mass eigenstates defined in Eq. (7) can be summarized as

$$A_i = \sum_{j=0}^{n-1} \gamma_{ij}a_j, \quad \gamma_{ij} = \sqrt{\frac{2}{n(1+\delta_{i,0})}} \cos \left( \frac{(2i-1)j\pi}{2n} \right).$$

(8)

The amplitude for the scattering can be computed by summing up the exchanges of all mass eigenstates

$$A(\psi_1 + \bar{\Psi} \rightarrow \psi_1 + \bar{\Psi}) \propto \sum_{j=0}^{n-1} \sum_{i=1}^{n} \frac{(\mu/M)^{2(n-i)}}{1 + (\mu/M)^2 + \ldots + (\mu/M)^{2(n-1)}} \frac{1}{t - m_j^2}$$

$$\rightarrow \left( \frac{\mu}{M} \right)^{2(n-1)} \frac{1}{t}, \quad \text{when} \quad t \gg m_j^2,$$

(9)

where the last identity follows from the orthogonality of $\gamma_{ij}$. We have assumed that $\mu \ll M$ and kept only the lowest order term in $\frac{\mu}{M}$.

This result for the suppression of the amplitude could have been anticipated. At high energy it is more convenient to work with the original gauge bosons, $A_i$, rather than the mass eigenstates $a_j$. Since $\psi_1$ couples only to $A_1$, the sum over all mass eigenstates can be replaced by the exchange of $A_1$ at high energies, and the amplitude suppression arises simply from suppression of the coupling of $A_1$ to $\bar{\Psi}$. The amplitude for the inclusive scattering $\psi_1 + \bar{\Psi} \rightarrow \psi_1 + X$ is down by only a factor of $(\mu/M)^{n-1}$ since at sufficiently high energies massive fermions can be produced. The massive $\bar{\psi}$’s couple more strongly to $A_1$ than $\bar{\Psi}$ couples to $A_1$, so the suppression comes only from the initial state. If the fermions are heavier than the gauge bosons ($\lambda > g$) there could be an interesting intermediate regime, in which the inclusive cross-section is highly suppressed and it is less suppressed at both lower and higher energies.
III. CONTINUUM LIMIT AND ANOMALY INFLOW

In this section we study the issue of anomaly cancellation in an effective theory obtained by decoupling heavy fermions. For simplicity, we consider the non-supersymmetric theory described in the previous section, where we set $\mu = 0$ in Eq. (1). We choose the Yukawa couplings to be larger than the gauge couplings, so we can study the theory after the fermions are integrated out, while all gauge bosons are still present in the effective theory.

Fermions that obtain masses via the Yukawa couplings leave well-known non-decoupling remnants in the effective theory. The classic examples are the Goldstone-Wilczek current \[21\] and the WZ term \[21\]. The link fields in Fig. 1 transform as $(\bar{k}, k)$ under the neighboring gauge groups. When the link fields obtain vevs the setup is identical to that considered in Refs. \[21, 22\]. In the moose theory we obtain a gauged WZ term for each link field.

It is important that we consider the appropriate energy regime for this derivation. At the highest energy, the theory is non-anomalous since there is a vector-like pair of fermions for each gauge group. After integrating out fermions we need to include WZ terms involving the link fields and the gauge bosons. If we integrated out all massive gauge bosons there would be no WZ term left. The massless theory, described by the diagonal subgroup of $SU(k)^n$, is not anomalous. It contains two massless Weyl fermions with opposite gauge charges.

The gauged parts of the WZ terms \[23, 24\] summed over all link fields are

$$
\mathcal{L}_{\text{eff}} = \frac{1}{48\pi^2} \sum_{i=1}^{n-1} \left\{ \text{tr} \left[ (A_i dA_i + dA_i A_i)(U_i A_{i+1} U_i + U_i dU_i) + (dU_i U_i dA_i A_i A_{i+1} U_i) - \text{p.c.} \right] \right. \\
+ \left. \left\{ \text{tr} \left[ A_i^2 (U_i A_{i+1} U_i^+ + U_i dU_i^+) - (A_{i+1} U_i^+ dU_i A_i + A_i U_i) - \text{p.c.} \right] \right\} \\
- \left\{ \frac{1}{2} \text{tr} \left[ A_{i+1} U_i^+ A_i U_i A_{i+1} U_i^+ A_i U_i \right] \right\} \\
+ \left\{ \text{tr} \left[ A_i (dU_i U_i)^3 + \frac{1}{2} A_i dU_i U_i^+ A_i dU_i U_i^+ + U_i A_{i+1} U_i^+ dU_i U_i^+ dU_i U_i^+ - \text{p.c.} \right] \right\} .
$$

In the above equation p.c. denotes the parity conjugate under which $A_i \leftrightarrow A_{i+1}$ and $U_i \leftrightarrow U_i^+$. The parity conjugate applies to all the terms inside each square bracket. Meanwhile, $d$ denotes the four dimensional derivative and all terms are contracted with the four dimensional $\epsilon$ tensor even though we omitted Lorentz indices.

After integrating out $\psi_i$ and $\bar{\psi}_{i+1}$ the Lagrangian is not gauge invariant, and the addition of the $i$-th WZ term from Eq. (11) compensates for that. Under gauge transformations $\omega_i$, $\omega_{i+1}$ the variation of the $i$-th WZ term is

$$
\delta \mathcal{L}_{\text{eff}}^i = -\frac{1}{24\pi^2} \text{tr} \left[ \omega_i (dA_i dA_i + \frac{1}{2} d(A_i^2)) - (i \rightarrow i + 1) \right] .
$$

It is clear that the gauge variations of the neighboring WZ terms cancel for their common gauge group. That is, the sum of $i$ and $i + 1$ WZ terms is invariant under the gauge transformation $\omega_{i+1}$. In the full effective Lagrangian the cancellation occurs for all gauge groups, except for the endpoints. This is, of course, what the sum of the WZ terms is supposed to do in the effective theory. Only at the endpoints are there light chiral fermions, which are not gauge invariant on their own.

We now consider the continuum limit, in which the lattice spacing $a \to 0$. We evaluate only terms of order $a$ and neglect terms $\mathcal{O}(a^2)$. We can then identify $A(y) = A \equiv A_i$, where
$y$ is the fifth coordinate. Therefore, $A_{i+1} = A + a\partial_5 A$ and $U_i = 1 + aA_5$. The lowest order terms are linear in $a$, so we can pull $a$ out and convert the sum over all lattice sites into an integral $a\sum_i \to \int dy$.

The terms in Eq. (11) are grouped such that the terms inside the first curly bracket contribute only to $AdA A A$ and $A^3 dA$, inside the second one to $A^3 dA$ and $A^5$, while inside the third one only to $A^5$. The terms inside the last curly bracket are all $O(a^2)$ and do not contribute.

From now on $A$ denotes the five dimensional (5D) vector boson, while $d$ denotes 5D derivatives. Terms with five vector indices are all contracted with the 5D $\epsilon$ tensor. After a bit of algebra we obtain

$$
\mathcal{L}_{\text{eff}} = -\frac{1}{24\pi^2} \int d^4xdy \left[ AdA A A + \frac{3}{2} A^3 dA + \frac{3}{5} A^5 \right] + \int d^4xdy (\text{total 4D derivatives})
$$

The first line in the above equation is precisely the 5D Chern-Simons term [24]. The total derivative terms act on either $A^4$ or $A^2 dA$. After integrating by parts these terms do not contribute since at large distances they decay at least as fast as $1/r^4$.

The Chern-Simons term is what compensates for the anomaly with localized fermions in the 5D theory [23–24]. The effect of gauge transformations on fermions localized at the endpoints of the 5D space is canceled by the anomaly inflow from the bulk of five dimensions. It is an interesting check of the deconstruction procedure that the continuum limit of the moose theory reproduces properly the Chern-Simons term. If the moose theory did not reproduce the Chern-Simons term it would be very puzzling.

**IV. MODELS**

Here we use the localization mechanism introduced in Section II to construct extensions of the MSSM that feature effective quark-lepton separation. In these models dangerous dimension-five proton decay operators are suppressed by gauge invariance.

How is proton stability maintained in the ordinary MSSM? Unlike the situation in the Standard Model, baryon number is not an accidental symmetry of the renormalizable interactions. For example, $QDL$ is an allowed operator in the superpotential. Such unwanted interactions are usually prohibited by imposing R-parity. Although a global symmetry, R-parity is likely to be gauged in the underlying theory; in the models considered here R-parity will arise as an unbroken discrete subgroup of $U(1)_{B-L}$ gauge symmetry.

However, it is well known that R-parity alone is not sufficient for stabilizing the proton, because there exist R-parity conserving non-renormalizable operators that violate baryon and lepton number. For example, proton decay bounds require the effective suppression scale $\Lambda$ in the superpotential operator, $\frac{1}{4}QQQL$ to be at least $\sim 10^{26}$ GeV, whereas naively, without invoking small couplings, one would expect this operator to have at most $\sim M_{\text{Planck}}^{-1}$ suppression. Similarly, the operator $UUDE$ must be suppressed by at least $\sim 10^{22}$ GeV. Thus, we see a motivation for considering effective quark-lepton separation in supersymmetric theories even if the cutoff is quite high.
There were three crucial ingredients required in the localization framework described in Section II. First, there were no mass terms for fermions transforming under the first gauge group. Second, Yukawa couplings acted only in one direction, for example coupling fermions from left to right but not the other way around. Third, there was a hierarchy between $\mu$ and $M = \lambda v$ such that $(\frac{\mu}{M})^{n-1}$ was small. We will address these issues in turn.

Explicit mass terms are absent in any chiral theory. Assuming that one of the endpoints of the moose diagram has the field content of the Standard Model chirality takes care of the first requirement.

The second requirement is trivially satisfied in models with two lattice sites. In models with more lattice sites, supersymmetry can address this requirement. Because the superpotential terms are holomorphic, hermitian conjugates of the link fields cannot appear in the superpotential.

This brings us to the hierarchy of masses. In supersymmetric theories, the choice of a small value for the parameter $\mu$ is guaranteed to be radiatively stable. In the models we construct, $\mu$ will appear in front of operators like $L_L$ and $E_N$ and will be on similar footing to the $\mu$ parameter appearing in front of $H_U$ and $H_D$ in the superpotential.

![Moose diagram for the $MSSM \times U(1)_{(B-L)} \times U(1)_{(B-L)'}$ model.](image)

Our simplest model contains just two lattice sites. Models with additional lattice sites allow the successful gauge coupling unification of the MSSM to be preserved, or the cutoff scale to be lowered far beneath the Planck scale.

The model with two lattice sites is depicted in Fig. 2. The gauge group is taken to be the product of the Standard Model gauge group with two additional $U(1)$’s, which we call $U(1)_{(B-L)}$ and $U(1)_{(B-L)'}$. The idea is that the light quarks and leptons will be separated in “$B–L$ space,” rather than in a space having to do with the Standard Model gauge group. Three generations of MSSM matter fields transform under $U(1)_{(B-L)}$ alone with the usual $B–L$ charges. Thus we have $Q(1/3, 0), U(-1/3, 0), D(-1/3, 0), L(-1, 0), E(1, 0),$ and $N(1, 0)$, where we list charges under $U(1)_{(B-L)} \times U(1)_{(B-L)'}$ in parenthesis. Meanwhile, transforming under $U(1)_{(B-L)'}$ alone are three generations of vector-like pairs $L'(0, -1) + \overline{L'}(0, 0), E'(0, 1) + \overline{E'}(0, -1),$ and $N'(0, 1) + \overline{N'}(0, -1)$, again with ordinary $B–L$ charge assignments, and with the usual transformation properties under the Standard Model gauge group. The Higgs doublet superfields $H_U$ and $H_D$ are neutral under $U(1)_{(B-L)} \times U(1)_{(B-L)'}$, so they are spread out over $B–L$ space, and Yukawa couplings may be written down at both lattice sites.

We assume that $U(1)_{(B-L)} \times U(1)_{(B-L)'}$ is broken to its diagonal subgroup by vevs of link fields $\phi(1, -1)$ and $\overline{\phi}(-1, 1)$, which are singlets under the Standard Model gauge group. This breaking can be arranged through the superpotential term $S(\phi \overline{\phi} - v^2)$, where $S$ is some singlet chiral superfield. The Yukawa interactions.
\[ \lambda_L L \phi L' + \lambda_E E \phi E' + \lambda_N N \phi N' \]  (13)

and the \( \mu \) terms

\[ \mu_L L L' + \mu_E E E' + \mu_N N N' \]  (14)

give the leading contributions to the masses for the lepton superfields. In the absence of electroweak symmetry breaking, there is a massless zero mode

\[ l = \frac{L' - \frac{\mu_L}{M_L} L}{\sqrt{1 + \left(\frac{\mu_L}{M_L}\right)^2}} \]  (15)

where \( M_L = \lambda_L v \). Similarly, the zero modes \( e \) and \( n \) are linear combinations of \( E \) and \( E' \), and \( N \) and \( N' \), respectively.

To have effective quark lepton separation, there are two requirements we must impose. The first is that \( v \ll \Lambda \), where \( \Lambda \) is the cutoff scale suppressing non-renormalizable operators. This ensures, for instance, that the operator

\[ \frac{1}{\Lambda} QQQ \frac{\phi}{\Lambda} L' \]  (16)

yields a \( QQQL' \) operator with strong extra suppression \( v/\Lambda \). The second requirement is \( \mu \ll M \) for \( L \) and \( E \). This ensures that the zero modes \( l \) and \( e \) will essentially be equal to \( L' \) and \( E' \). Then, for example, the operator

\[ \frac{1}{\Lambda} QQQL \]  (17)

contains a \( QQQL \) coupling suppressed by the additional small factor \( \mu_L/M_L \).

Taking \( \Lambda \sim M_{\text{Planck}} \), we require \( v/\Lambda \) and \( \mu/M \) both to be less than roughly \( 10^{-8} \) to evade proton decay bounds. For example, one could have \( v \sim 10^{11} \) GeV and set the \( \mu \)'s to be \( \sim \) TeV, roughly equal to Higgs \( \mu \) parameter of the MSSM.

As things stand, the model still has an unbroken \( U(1)_{B-L} \), which can be broken relying on standard methods. For instance, we can introduce \( X \) and \( \overline{X} \) superfields, with charges \((0,2)\) and \((0,-2)\) under \( U(1)_{(B-L)} \times U(1)_{(B-L)'} \), and neutral under the Standard Model gauge group, and add the superpotential couplings

\[ T(X\overline{X} - v_{B-L}^2) + XN'N', \]  (18)

where \( T \) is a singlet chiral superfield. The vevs of \( X \) and \( \overline{X} \) then break the diagonal \( U(1)_{B-L} \) to a discrete subgroup that contains R-parity. Taking \( v_{B-L} \sim 10^{15} \) GeV gives roughly the right size seesaw neutrino masses to account for atmospheric neutrino oscillations.

How low can the cutoff be taken in this model? Taking \( \mu_L \) and \( \mu_E \) to zero leaves only the contributions to \( QQQL \) and \( UU'De \) coming from the coupling through link fields, as in Eq. \((16)\). Requiring \( v > \text{TeV} \) then leads to the bound \( \Lambda > 10^{14} \) GeV. The cutoff scale can be reduced further below the Planck scale in models with additional sites. In fact, there is a completely different motivation for considering models with larger numbers of sites, coming from gauge coupling unification. In the model with two sites, the scale \( v \) is forced to be
below $\sim 10^{11}$ GeV, leaving us with extra matter at this scale that spoils the successful gauge coupling unification in the MSSM. By taking more sites, the upper bound on $v$ coming from proton decay is raised, and with enough sites $v$ can be raised to the GUT scale.

Consider the following extension of the two-site model, shown in Fig. 3: the gauge group is the product of the Standard Model group with $2n$ additional $U(1)$ factors, called $U(1)^{(B-L)}_i$ for $i$ running from 1 to $2N$. Link fields $\phi_i$ have charges $(1, -1)$ under $U(1)^{(B-L)}_i \times U(1)^{(B-L)}_i$ (with the index $2n + 1$ identified with 1), and acquire vevs that break the product of $U(1)$’s to their diagonal subgroup. To cancel mixed $U(1)^{(B-L)}_i \times U(1)^{(B-L)}_i$ anomalies we introduce $\phi_i(-1, 1)$ fields that do not acquire vevs. Alternatively, one could have eight copies of link fields $\phi_i(1, -1)$ and one copy of $\Phi_i(-2, 2)$. Quark-lepton separation is preserved even if all link fields acquire vevs since Yukawa couplings with $\Phi_i$ are forbidden. At site 1 three generations of MSSM matter superfields reside as before. For sites 2 through $n + 1$ we have vector-like pairs $L + \bar{L}$ at each site, while for sites $n + 1$ through $2n$ we have both $E + \bar{E}$ and $N + \bar{N}$ pairs at each site. Suppose for simplicity that we have universal $\mu$ terms for $L, N,$ and $E$ at each site where they appear, and universal Yukawa couplings to the link fields, $\lambda$. Then, as before, we require $\mu \ll M = \lambda v$ and $v \ll \Lambda$. The zero modes $l, n,$ and $e$ are then dominantly $L_{n+1}, N_{n+1},$ and $E_{n+1}$, respectively. The higher-dimensional operator

$$\frac{1}{\Lambda^n}Q_1Q_1Q_1\phi_1\phi_2...\phi_nL_{n+1}$$  \hspace{1cm} (19)$$

leads to a $QQQl$ coupling suppressed by an extra factor $(v/\Lambda)^n$. For example, taking $n = 3$ and $\Lambda = M_{\text{Planck}}$ allows us to take $v$ near the GUT scale. Alternatively, for fixed, small $v$, $\Lambda$ can be lowered by taking $n$ large. Meanwhile, the contribution to $QQQl$ coming from

$$\frac{1}{\Lambda}Q_1Q_1Q_1L_1$$  \hspace{1cm} (20)$$

is suppressed by an extra factor $(\mu/M)^n$ (identical suppressions arise for $UUDe$). Thus,

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1We thank Erich Poppitz for bringing these to our attention.
even a mild hierarchy between $\mu$ and $M$ can lead to a sufficient suppression of proton decay if one makes $n$ large enough.

V. CONCLUSIONS

In theories with extra dimensions fermion localization can be used to suppress certain operators. One of the most striking results obtained using this mechanism is that the cutoff scale can be taken quite low without violating proton decay bounds, by spatially separating quark fields and lepton fields. The anomalies associated with the separated chiral fermions are canceled in this framework by the anomaly inflow associated with the five-dimensional Chern-Simons term.

In this paper we have deconstructed this picture by building four-dimensional models with fermions localized in gauge theory space. The localization results from particular patterns of couplings and is not due to topological features. Our low-energy theory contains light chiral fermions transforming under different factors of a product gauge group. The anomaly associated with these fermions is canceled by the Wess-Zumino terms generated by integrating out heavy fermions. We checked that in the continuum limit these Wess-Zumino terms combine to form the Chern-Simons term.

We used deconstruction to obtain four-dimensional models with effective quark-lepton separation. In the models we constructed, the product gauge group is comprised of factors of $U(1)_{(B-L)}$. The degree of separation (and thus the level of suppression of dangerous proton decay operators) depends on the number of lattice sites, and on the ratios of $v/\Lambda$ and $\mu/(\lambda v)$. Here, $v$ is the scale of spontaneous breaking to the diagonal $U(1)_{(B-L)}$, $\Lambda$ is the UV cutoff, $\mu$ is the mass between vector-like pairs at a given site, and $\lambda$ is the Yukawa coupling involving the link fields and fermions at adjacent sites. Models with very few sites require large hierarchies among these mass scales; models with only mild hierarchies require taking a larger number of sites.

One model building challenge is localizing the matter fields without localizing the Higgs doublets, so that both $QDH_D$ and $LEH_D$ are present. This is accomplished trivially in our models because the Higgs fields are neutral under the “moose” $B-L$ gauge groups. Neutral fields are naturally spread out over the whole theory space since they can couple at any site without suppression. In this setup, the subject of flavor is essentially set aside. One could construct alternative models in which different generations of quarks and leptons have different “profiles” in the gauge space (see extra-dimensional models of Ref. [27]). Moreover, one could localize the Higgs fields, e.g. in theories where the Standard Model gauge group is discretized, so that certain Yukawa couplings are suppressed by factors of $v/\Lambda$ required to link the Higgs to the relevant fermions. In this way, a connection could be drawn between suppressed proton decay and small Yukawa couplings in this framework.

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REFERENCES

[1] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000), hep-ph/9903417.
[2] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001), hep-th/0104003.
[3] C. T. Hill, S. Pokorski and J. Wang, hep-th/0104035.
[4] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001), hep-ph/0105239.
[5] H. Cheng, C. T. Hill, S. Pokorski and J. Wang, hep-ph/0104179; H. Cheng, C. T. Hill and J. Wang, hep-ph/0105323.
[6] C. Csaki, J. Erlich, C. Grojean and G. D. Kribs, hep-ph/0106043.
[7] H. C. Cheng, D. E. Kaplan, M. Schmaltz and W. Skiba, Phys. Lett. B 515, 395 (2001), hep-ph/0106098.
[8] N. Arkani-Hamed, A. Cohen and H. Georgi, hep-th/0108089; P. H. Chankowski, A. Falkowski and S. Pokorski, hep-ph/0109272.
[9] C. Csaki, G. D. Kribs and J. Terning, hep-ph/0107266; H. Cheng, K. T. Matchev and J. Wang, hep-ph/0107268.
[10] N. Arkani-Hamed, A. G. Cohen, D. B. Kaplan, A. Karch and L. Motl, hep-th/0110146.
[11] I. Rothstein and W. Skiba, hep-th/0109175.
[12] M. Alishahiha, Phys. Lett. B 517, 406 (2001); hep-th/0105153.
[13] A. Adams and M. Fabinger, hep-th/0111079.
[14] K. Sfetsos, Nucl. Phys. B 612, 191 (2001) hep-th/0106126.
[15] C. T. Hill, hep-th/0109068.
[16] C. Csaki, J. Erlich, V. V. Khoze, E. Poppitz, Y. Shadmi and Y. Shirman, hep-th/0110188.
[17] M. Bander, Phys. Rev. D 64, 105021 (2001), hep-th/0107130.
[18] N. Arkani-Hamed, Y. Grossman and M. Schmaltz, Phys. Rev. D 61, 115004 (2000), hep-ph/9909411.
[19] H. Georgi, Nucl. Phys. B 266, 274 (1986).
[20] J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981).
[21] J. Wess and B. Zumino, Phys. Lett. B 37, 95 (1971); E. Witten, Nucl. Phys. B 223, 422 (1983).
[22] E. D’Hoker and E. Farhi, Nucl. Phys. B 248, 59 (1984); Nucl. Phys. B 248, 77 (1984).
[23] A. Manohar and G. W. Moore, Nucl. Phys. B 243, 55 (1984).
[24] L. Alvarez-Gaume and P. Ginsparg, Nucl. Phys. B 243, 449 (1984); Annals Phys. 161, 423 (1985) [Erratum-ibid. 171, 233 (1985)].
[25] C. G. Callan and J. A. Harvey, Nucl. Phys. B 250, 427 (1985).
[26] S. G. Naculich, Nucl. Phys. B 296, 837 (1988).
[27] E. A. Mirabelli and M. Schmaltz, Phys. Rev. D 61, 113011 (2000), hep-ph/9912265; D. E. Kaplan and T. M. Tait, JHEP 0111, 051 (2001), hep-ph/0110126.