Numerical study of Free Convective Viscous Dissipative flow along Vertical Cone with Influence of Radiation using Network Simulation method

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Abstract: A two dimensional mathematical model is formulated for the transient laminar free convective flow with heat transfer over an incompressible viscous fluid past a vertical cone with uniform surface heat flux with combined effects of viscous dissipation and radiation. The dimensionless boundary layer equations of the flow which are transient, coupled and nonlinear partial differential equations are solved using the Network Simulation Method (NSM), a powerful numerical technique which demonstrates high efficiency and accuracy by employing the network simulator computer code Pspice. The velocity and temperature profiles have been investigated for various factors, namely viscous dissipation parameter $\varepsilon$, Prandtl number $Pr$ and radiation $R_d$ are analyzed graphically.

1. Introduction

Natural convective flow under the influence of gravitational force has been investigated most extensively because they happen often in nature and also in various fields of science and engineering. When with the fluid is in contact with heated surface the result of temperature variation causes buoyancy force, which causes natural convection. Merk and Prins[1-2] derived the general relations for similar solutions on isothermal axi-symmetric forms and obtained similarity solutions for vertical cone in steady state. Hering and Grosh [3] proved the existence of similarity solutions for free convection flow over non-isothermal vertical cone with variable surface temperature in steady state. Hering [4] extended the problem of Hering and Grosh [3] for low-Prandtl-number fluids and obtained numerical solutions of the transformed boundary-layer equations have been obtained for Prandtl numbers typical of gases and liquid metals as well as for an in viscid fluid. To solve the present problem a well-tested, highly adaptive numerical procedure known as the Network Simulation Method (NSM) has been applied. This method was originally developed by Nagel [5] for semiconductor applications at the University of California at Berkeley. It has been implemented in a wide range of engineering problems subsequently.

The fundamentals of NSM are based on thermoelectric analogy between thermal variables and electrical variables. NSM can be applied for any kind of non-linear problems due to boundary conditions, temperature differences of the thermal properties etc. This technique has been employed quite recently to complex nonlinear thermo fluid dynamic problems. NSM transforms the partial differential equations that represents the mathematical model of the physical process with spatial discretization [6], yields the ordinary differential equations which are the fundamentals for executing the standard electrical network model for an elemental control volume. Time is considered as a continuous factor in the discretized equations. A network model is designed based on these equations.
The whole medium and boundary conditions are implemented by means of special electrical components are connected in series to make the networks. The main advantage is that the network model comprises very few electrical devices connected in series to which the boundary conditions are added to form the whole model of the medium. The design of network model is the combination of resistors, capacitors and current control generators, with minimum programming rules. Once the complete network model is designed, a computer code Pspice [5] is used to simulate it and to provide the numerical solution. The relation between heat transfer and NSM were developed by [7]. Beg et al. [8] presented a mathematical model for the two dimensional, steady, incompressible, laminar free convection flow boundary layer flow over a continuously moving plate immersed in a thermally stratified high porosity non-Darcian porous medium and the governing conservation equations are solved using Network Simulation Method along with the Pspice algorithm. Beg et al. [9] developed a network model to simulate the transient, nonlinear buoyancy-driven double diffusive heat and mass transfer of a viscous, incompressible, grey, absorbing-emitting fluid flowing past an impulsively started moving vertical plate. Beg et al. [10] examined the composite effects of magnetic field, buoyancy, heat generation and also Darcian and Forchheimer drag forces on the boundary layer convection from a sphere embedded in porous regime using the Network Simulation Method. Beg et al. [11] obtained the numerical solution for the viscous, incompressible, Magneto hydrodynamic flow in a rotating channel comprising two infinite parallel plates and containing a Darcian porous medium. Zueco et al. [12] investigate the axisymmetric convective heat and mass transfer boundary layer flow of vertical thin cylinder with uniform heat and mass flux and the resulting boundary layer equations are solved using Network Simulation Method. Zueco and Beg [13] studied the steady-state, magnetohydrodynamic, optically thick, dissipative gas boundary layer flow and heat transfer past a non-isothermal porous wedge embedded in a scattering isotropic Darcy-Forchheimer porous medium are solved using a powerful computational method based on thermoelectric analogy, viz, the Network Simulation Method. Zueco [14] studied transient free convective MHD flow of dissipative fluid past a semi-infinite vertical plate with constant heat flux using NSM. The plates lying under constant pressure gradient using the Network Simulation Method. Zueco et al [15] presented a two dimensional mathematical model for the laminar heat and mass transfer of an electrically conducting, heat generating/absorbing fluid past a perforated horizontal surface in the presence of viscous and ohmic heating and solved using the Network Simulation Method.

The aim of the present study is to investigate unsteady free convective flow from a non-isothermal vertical cone with uniform surface heat flux together with combined effects of viscous dissipation and radiation has not received any attention in literature. Also it has a wide range of applications in the field of solar energy plants, nuclear reactor safety, drying, steam generators and design of space crafts and so forth. Hence, the present work studies and deals with the transient free convective flow from a non-isothermal vertical cone with the above said effects. The solutions of governing boundary layer equations are obtained by using Network Simulation Method. The effects of velocity and temperature for various values of viscous dissipation parameter $\varepsilon$, Prandtl number $Pr$ and radiation $R_{\gamma}$ are studied.
2. Mathematical Formulation

An axisymmetric unsteady, laminar natural convective flow of incompressible viscous dissipative fluid past a vertical cone with uniform surface temperature under the influence of radiation is considered. It is assumed that the effects of pressure gradient along the boundary layer are negligible. It is also assumed that the surrounding fluid which is at rest and are at the surface of the cone are in same temperature $T_\infty$. The temperature of the cone surface is raised to $T'_w$ when time $t' > 0$. The coordinate system is chosen (as shown in Figure 1) such that $x$ measures the distance along surface of the cone from the apex ($x = 0$) and $y$ measures the distance normally outward. The fluid properties are assumed to be constant except the density variations causing a body force in the momentum equation. Applying Boussinesq approximation the governing boundary layer equations of continuity, momentum and energy are as follows:

**Equation of continuity:**

\[
\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0
\]  

(1)

**Equation of Momentum:**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \beta (T' - T'_w) \cos \phi + \nu \frac{\partial^2 u}{\partial y^2}
\]  

(2)

**Equation of energy:**

\[
\frac{\partial T'}{\partial t} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{\kappa}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2
\]  

(3)

Where $u$ and $v$ represents velocity components in the $x$ and $y$ directions, $T'$ temperature in boundary layer, $T'_w$ the temperature away from surface of the cone, $t'$ is time, $g$ acceleration due to gravity, $\beta$ is the volumetric coefficient of thermal expansion, $\nu$ Kinematic viscosity, $\alpha$ Thermal diffusivity, $\rho$ is the density, $c_p$ Specific heat at constant pressure, $\kappa$ thermal conductivity, $\kappa'$ mean absorption coefficient, $\text{R}_d$ radiation parameter. $\mu$ dynamic viscosity.

![Figure 1: Physical Model and co-ordinate system](image-url)
The initial and boundary conditions are

\[
t' \leq 0: \quad u = 0, \quad v = 0, \quad T' = T'_e \quad \text{for all } x \text{ and } y,
\]

\[
t' > 0: \quad u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = \frac{q_w(L)}{k} \quad \text{at } y = 0, \quad (4)
\]

\[
u = 0, \quad T' = T'_e \quad \text{at } x = 0,
\]

\[
u \to 0, \quad T' \to T'_e \quad \text{as } y \to \infty.
\]

\[
q_w = \frac{4\sigma T^4}{M} (T'' + T'' - T''') = 16\sigma T^4 \frac{\partial T}{\partial y}.
\]

Local Skin-friction and local Nusselt number are given by

\[
\tau_x = \mu \left( \frac{\partial u}{\partial y} \right)_{\infty}, \quad Nu_x = -x \left( \frac{\partial T}{\partial y} \right)_{\infty}, \quad (5)
\]

Using the following non-dimensional quantities:

\[
R = \frac{r}{L}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad (Gr_t)^\frac{1}{3} \quad \text{where } r = x \sin \phi,
\]

\[
V = \frac{VL}{U}, \quad U = \frac{UL}{V}, \quad T = \frac{(T' - T'_e)}{q_w/k}, \quad Pr = \frac{\nu}{\alpha}
\]

\[
Gr_t = \frac{g\beta q_w / k' \cos \phi}{\nu^2}, \quad \varepsilon = \frac{g\beta L}{c_r}, \quad R_s = \frac{k' k''}{4\sigma T_e^4}, \quad (6)
\]

Where \( L \) Reference length, \( Gr_t \) Thermal Grashof number, \( Pr \) Prandtl number, \( \sigma \) electrical conductivity, \( q_w \) thermal radiation, \( \phi \) Semi-vertical angle of the cone.

The non-dimensional form of equations (1), (2), (3), (4) and (5) is given by:

\[
\overrightarrow{\partial (U/R)} + \overrightarrow{\partial (VR)} = 0 \quad (7)
\]

\[
\overrightarrow{\partial U} + U \overrightarrow{\partial U} + V \overrightarrow{\partial U} = T^\prime + \overrightarrow{\partial^2 U} \quad (8)
\]

\[
\overrightarrow{\partial T} + U \overrightarrow{\partial T} + V \overrightarrow{\partial T} = \frac{1}{Pr} \left( 1 + \frac{4}{3R_s} \right) \overrightarrow{\partial^2 T} + e \left( \overrightarrow{\partial U} \right)^2 \quad (9)
\]

The corresponding initial and boundary conditions of non-dimensional form are

\[
t \leq 0: U = 0, \quad V = 0, \quad T = 0 \quad \text{for all } X \text{ and } Y
\]

\[
t > 0: U = 0, \quad V = 0, \quad \frac{\partial T}{\partial Y} = -1 \quad \text{at } Y = 0
\]

\[
U = 0, \quad T = 0 \quad \text{at } X = 0
\]

\[
U \to 0, \quad T \to 0 \quad \text{as } Y \to \infty \quad (10)
\]

Local Skin-friction, and local Nusselt number in non-dimensional quantities are
\[ \tau_x = Gr_\text{T} \left( \frac{\partial U}{\partial Y} \right)_{Y=0}, \quad \text{Nu}_x = \frac{X}{T_{\text{r,ad}}} \left( \frac{-\partial T}{\partial Y} \right)_{Y=0} Gr_\text{T} \frac{1}{\text{T}}. \]  

Taking semi-infinite vertical cone slant height as \( L = 1 \), it is taken as region of rectangle with \( X \) varies from 0 to 1 and \( Y \) varies from 0 to \( Y_{\text{max}} = 20 \), where \( X = L \) corresponds to the slant height of the vertical cone and \( Y_{\text{max}} \) is regarded as \( \infty \), where \( Y_{\text{max}} \) lies outside the thermal and momentum boundary layers. The region of integration is considered as a rectangle with mesh sizes \( \Delta X = 0.25 \) and \( \Delta Y = 0.25 \).

### 3. Solution Procedure

Network Simulation Method is applied to solve the transient nonlinear coupled partial differential equations (7)-(9) with the initial and boundary conditions (10). The discretization of the boundary-layer equations is based on the difference-finite formulation, and only discretization of the spatial co-ordinates is necessary, time remaining as a real continuous variable. Based on these equations the design of an electrical network circuit is formulated. Electric analogy is applied in which the variable voltage \((V)\) corresponds to velocities \((U, V)\), temperature \((T)\) and the variable electric current \((J)\) is equivalent to the velocity fluxes and the temperature fluxes. For each dimensionless boundary layer equation, network circuits models are developed. Whole network is converted into a suitable program that is solved by a computer code (electric circuits simulator), Pspice [5]. The Time interval required for the convergence is not a prerequisite since the code Pspice does this work with highly advanced mathematical algorithms developed by Nagel [5]), which are common for most currently used numerical methods.

#### 3.1. Design of the Network Model

The design of the network model is given as follows; Refer Gonza’lez-Ferna’ndez and Alhama [7], Zueco[13] for more information. The finite difference differential equations resulting from dimensionless continuity, momentum balance and energy balance equations are

\[ \frac{d}{dt} \left( U_{i,j} \right) - \frac{1}{\Delta X} \left( \frac{U_{i+1,j} - U_{i-1,j}}{2} \right) = 0 \]  

\[ \frac{d}{dt} \left( V_{i,j} \right) - \frac{1}{\Delta Y} \left( \frac{V_{i,j+1} - V_{i,j-1}}{2} \right) = 0 \]  

\[ \frac{d}{dt} \left( T_{i,j} \right) - \frac{1}{\Delta X} \left( \frac{T_{i+1,j} - T_{i-1,j}}{2} \right) = 0 \]  

\[ \frac{d}{dt} \left( H_{i,j} \right) - \frac{1}{\Delta Y} \left( \frac{H_{i,j+1} - H_{i,j-1}}{2} \right) = 0 \]  

Defining the following currents:

(i) Equation of momentum balance

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

(ii) Equation of buoyancy term

\[ j_{B,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{C,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{D,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{E,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

Where \( j_{U,i,j} \) and \( j_{V,i,j} \) are the currents that leave and enter the cell for the friction term of \( U \). \( j_{B,i,j} \) the current corresponding to buoyancy term, \( j_{C,i,j} \) and \( j_{D,i,j} \) are the currents corresponding to the inertia terms of \( U \) and \( V \), respectively, while \( j_{E,i,j} \) represents the transitory term. The currents

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{B,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{C,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{D,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{E,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{B,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{C,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{D,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{E,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{B,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{C,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{D,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{E,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{B,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{C,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{D,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{E,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]  

\[ j_{U,i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta Y}, \quad j_{V,i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \quad j_{T,i,j} = \frac{T_{i,j} - T_{i,j-1}}{\Delta Y}, \quad j_{H,i,j} = \frac{H_{i,j} - H_{i,j-1}}{\Delta Y} \]
The currents \( j_{x_{i,j},o} \) and \( j_{i_{o},y} \) are executed with two resistors \( R_{x_{i,j},o} \) of values \( \Delta Y / 2 \); while the currents \( j_{x_{i,j},o}, j_{x_{i,j},o}^\Delta, j_{x_{i,j},o}^\Delta \) are executed with voltage control current generators. Thus, the two currents are modeled by the generator \( G_{x_{i,j},o} \) with the control action \( \Delta Y \left(T_{x_{i,j}}\right)^o \), while the other currents are modeled with the generators, \( G_{x_{i,j},o}, G_{x_{i,j},o}^\Delta \) respectively, by means of the voltages \( \Delta Y U_{\Delta x_{i,j}}(U_{x_{i,j}} - U_{x_{i,j}}^\Delta) \) and \( \Delta Y V_{\Delta x_{i,j}}(U_{x_{i,j}} - U_{x_{i,j}}^\Delta) \), respectively; \( U_{x_{i,j}}, U_{x_{i,j}}^\Delta, U_{x_{i,j}}, U_{x_{i,j}}^\Delta \) are the voltages which represent velocities at the nodes \( \left(i + \Delta X, j\right), \left(i - \Delta X, j\right), \left(i, j + \Delta Y\right) \), \( \left(i, j - \Delta Y\right) \) of the cell in the momentum equation, while \( U_{x_{i,j}} \) is the velocity in the centre of this cell \((i, j)\) and \( j_{x_{i,j},o} \) is employed with a capacitor of value \( C_{x_{i,j}} = \Delta Y \), linked to the centre of each cell. (as shown in Figure 2)

\[
\begin{align*}
\text{Figure 2. Network model of the control volume} & \quad \text{Figure 3. Network model of the control volume} \\
\text{Equation of momentum} & \quad \text{Equation of energy}
\end{align*}
\]

\[
(ii) \text{ Equation of energy} \\
\begin{align*}
J_{x_{i,j},o} &= \left(1 + 4/3 R_{x_{i,j},o}\right) \left(T_{x_{i,j}} - T_{x_{i,j}}^o\right) / \left(\Delta Y / 2\right) \\
J_{x_{i,j},o} &= \left(1 + 4/3 R_{x_{i,j},o}\right) \left(T_{x_{i,j}} - T_{x_{i,j}}^o\right) / \left(\Delta Y / 2\right) \\
J_{x_{i,j},o} &= \Pr \left(U_{x_{i,j}} - U_{x_{i,j}}^\Delta\right) / \left(\Delta Y\right) \\
J_{x_{i,j},o} &= \Pr \left(V_{x_{i,j}} - V_{x_{i,j}}^\Delta\right) / \left(\Delta Y\right) \\
J_{x_{i,j},o} &= \Pr \left(T_{x_{i,j}} - T_{x_{i,j}}^\Delta\right) / \left(\Delta Y\right) \\
J_{x_{i,j},o} &= \Pr \left(T_{x_{i,j}}^o\right) / \left(\Delta Y\right)
\end{align*}
\]

where \( j_{x_{i,j},o} \) and \( j_{x_{i,j},o}^\Delta \) are the currents that leave and enter the cell due to the transversal conduction, \( j_{x_{i,j},o} \) the current corresponding to the viscous dissipation, \( j_{x_{i,j},o} \) and \( j_{x_{i,j},o}^\Delta \) are the currents corresponding to the convective terms of \( U \) and \( V \), respectively, while \( j_{x_{i,j},o} \) is the transitory term. The currents \( j_{x_{i,j},o} \) and \( j_{x_{i,j},o} \) are executed with two resistors \( R_{x_{i,j},o} \) of values \( \Delta Y / 2 \); while the currents \( J_{x_{i,j},o}^\Delta, J_{x_{i,j},o}^\Delta, J_{x_{i,j},o}^\Delta \) are executed with the voltage control current generators \( G_{x_{i,j},o}, G_{x_{i,j},o}^\Delta \) respectively; the voltages are \( \Pr \left(U_{x_{i,j}} - U_{x_{i,j}}^\Delta\right) \) and \( \Pr \left(V_{x_{i,j}} - V_{x_{i,j}}^\Delta\right) \) respectively; \( T_{x_{i,j}}, T_{x_{i,j}}^\Delta, T_{x_{i,j}}^\Delta, T_{x_{i,j}}^\Delta \) being the voltages for temperatures at the nodes \( \left(i + \Delta X, j\right), \left(i, j - \Delta Y\right) \), \( \left(i, j + \Delta Y\right) \), \( \left(i, j - \Delta Y\right) \) of the cell of the energy balance, while \( T_{x_{i,j}} \) is the temperature in the centre of this cell \((i, j)\) and \( j_{x_{i,j},o} \) is executed with one capacitor of value \( C_{x_{i,j}} = \Delta Y \Pr \), linked to the centre of each cell. (as shown in Figure 3)
The finite difference formulation corresponding to continuity equation (12) is

$$\nabla V_{ij} = \left\{ U_{i+1,j} - U_{i,j} \right\} \Delta Y / (2 \Delta X) + U_{i,j} \Delta Y / (i \Delta X) - V_{ij} \Delta X$$  \hspace{1cm} (17)

Above Equations \((13) - (14)\) can be written in the form of Kirchhoff’s law as

$$\begin{align*}
\sum_{\text{faces of cell}} j_{ijk} &= 0, \\
\sum_{\text{faces of cell}} j_{ijk} &= 0
\end{align*}$$  \hspace{1cm} (18)

Finally, to implement the velocity and temperature boundary conditions (at \(X = 0\) and \(Y \to \infty\)) ground elements are employed. Constant current and constant voltage at \(Y = 0\) are utilized to simulate the constant heat flux \(\left( \frac{\partial T}{\partial Y} = -1 \right)\). For the initial condition, the voltages \(U = T = 0\) for \(t \leq 0\) are applied to the two capacitors \(C_{U}, C_{T}\).

4. Result and Discussion

The suitable network model has been developed and analyzed free convective viscous dissipative flow along a vertical plate subject to radiation and constant heat flux. The results are presented for velocity and temperature profiles for various parameters \(Pr, \varepsilon\) and \(Rd\).

Figure 4 shows the velocity profile for different values of \(Pr\). The thickness of momentum boundary layer increases with lower values of \(Pr\). The effect of the radiation parameter \(Rd\) on the velocity field is presented in Figure 5. Velocity increases for lower values of \(Rd\). Figure 6 presents the velocity profile for different values of viscous dissipation parameter \(\varepsilon\), velocity increases as \(\varepsilon\) increases. Figure 7 illustrate the effects of \(Pr\) on temperature profile. One can observe that the temperature decreases with increasing values of Prandtl number \(Pr\). Figure 8 depicts the radiation effects on temperature profile, the thickness of the fluid increases for higher values of radiation parameter. The influence of viscous dissipation on temperature is presented in Figure 9; greater viscous dissipative heat causes a rise in the temperature.

![Figure 4](image1.png)

**Figure 4.** Velocity profiles at \(X=1.0\) for various values of \(Pr\)

![Figure 5](image2.png)

**Figure 5.** Velocity profiles at \(X=1.0\) for various values of \(Rd\)
Figure 6. Velocity profiles at X=1.0 for various values of $\varepsilon$

Figure 7. Temperature profiles at X=1.0 for various values of $Pr$

Figure 8. Temperature profiles at X=1.0 for various values of $Ra$

Figure 9. Temperature profiles at X=1.0 for various values of $\varepsilon$
5. Conclusion

A mathematical model has been formulated for the natural convective viscous dissipative flow from a vertical cone with uniform surface heat flux and radiation effect. A parametric study is performed to illustrate the influence of thermo physical parameters on the velocity and temperature profiles. It has been observed that

1. The velocity of the decreases for lower values of ε and higher values of Pr and higher values of Rd
2. The temperature decreases for smaller values of ε and larger values of Pr, Rd

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