THE ELECTRIC CHARGE ASSIGNMENT
IN $SU(4)_L \otimes U(1)_Y$ GAUGE MODELS

ADRIAN PALCU

Faculty of Exact Sciences - “Aurel Vlaicu” University Arad, Str. Elena Drăgoi 2, Arad - 310330, Romania

Abstract

In this brief report, apart from the usual approach, we discriminate among models in the class of $SU(4)_L \otimes U(1)_Y$ electro-weak gauge models by just setting the versors in the method of the exactly solving gauge models with high symmetries. We prove that the method itself naturally predicts the correct assignment of the electric charge spectrum along with the relation between the gauge couplings of the groups involved therein for each particular model in this class.

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1 Introduction

The general method of exactly solving models with high symmetries - based on the gauge group $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$ that undergoes a spontaneous symmetry breakdown (SSB) in its electro-weak sector - was proposed several years ago by Cotăescu [1] and applied by the author [2] - [7] to the so called 3-3-1 models. This exact algebraical approach is employed here to prove that, in the case of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_Y$ (3-4-1) models [8] - [18], the correct electric charge assignment for all the fermion representations (and thus for all the bosons) can be predicted by just setting the versors $\nu_i$ in the general Weinberg transformation (gWt). The gWt is designed to bring the massive vector fields from the gauge basis to their physical basis through a $SO(n-1)$ rotation in the parameter space. At the same time, the procedure by itself naturally ensures at this stage the correct gauge coupling matching without resorting to any supplemental hypothesis.

The paper is organized as follows: Sec. 2 reviews the main aspects of the general method, Sec. 3 presents its predictions regarding the electric charge spectrum and gauge couplings matching in each particular case of versor setting for the $SU(4)_L \otimes U(1)_Y$ model, while Sec. 4 sketches our conclusions.
2 The general method - a brief review

All the details of constructing a consistent algebraical approach designed to exactly solve gauge models with high symmetries can be found in Ref. [1]. We restrict ourselves here to outline only the main results concerning the electric charge operator and close related topics. The electric charge assignment must ensure such fermion representations that all the anomalies cancel by an interplay between generations.

2.1 Irreducible representations of $SU(n)_L \otimes U(1)_Y$

We focus on the fermion representations of the $SU(n)_L \otimes U(1)_Y$ electro-weak model. The main piece is the group $SU(n)$ and its two fundamental irreducible unitary representations (irreps) $\mathbf{n}$ and $\mathbf{n}^*$ which give different classes of tensors of ranks $(r, s)$ as direct products like $(\otimes \mathbf{n})^r \otimes (\otimes \mathbf{n}^*)^s$. These tensors have $r$ lower and $s$ upper indices for which we reserve the notation, $i, j, k, \cdots = 1, \cdots, n$. As usually, we denote the irrep $\rho$ of $SU(n)$ by indicating its dimension, $\mathbf{n}_\rho$. The $su(n)$ algebra can be parameterized in different ways, but here it is convenient to use the hybrid basis of Ref. [1] consisting of $n-1$ diagonal generators of the Cartan subalgebra, $D_{ij}$, labeled by indices $i, j, \cdots$ ranging from $1$ to $n-1$, and the generators $E_{ij} = H_{ij}/\sqrt{2}, i \neq j$, related to the off-diagonal real generators $H_{ij}$ [19][20]. This way the elements $\xi = D_{ij} \xi^i + E_{ij} \xi^j \in su(n)$ are now parameterized by $n-1$ real parameters, $\xi^i$, and by $n(n-1)/2$ $c$-number ones, $\xi_{ij} = (\xi^i)^*, \, i \neq j$. The advantage of this choice is that the parameters $\xi^i$ can be directly associated to the $c$-number gauge fields due to the factor $1/\sqrt{2}$ which gives their correct normalization. In addition, this basis exhibit good trace orthogonality properties,

$$Tr(D_i D_j) = \frac{1}{2} \delta_{ij}, \quad Tr(D_i E_j^*) = 0, \quad Tr(E_i^* E_j) = \frac{1}{2} \delta_i^j \delta_j^k. \quad (1)$$

When we consider different irreps, $\rho$ of the $su(n)$ algebra we denote $\xi^\rho = \rho(\xi)$ for each $\xi \in su(n)$ such that the corresponding basis-generators of the irrep $\rho$ are $D_i^\rho = \rho(D_i)$ and $E_{ij}^\rho = \rho(E_{ij})$.

2.2 Fermion sector

The $U(1)_Y$ transformations are nothing else but phase factor multiplications. Therefore - since the coupling constants $g$ for $SU(n)_L$ and $g'$ for the $U(1)_Y$ are assigned - the transformation of the fermion tensor $L^\rho$ with respect to the gauge group of the theory reads

$$L^\rho \rightarrow U(\xi^0, \xi)L^\rho = e^{-i(g\xi^0 + g'y_{ch} \xi^0)} L^\rho \quad (2)$$

where $\xi = \in su(n)$ and $y_{ch}$ is the chiral hypercharge defining the irrep of the $U(1)_Y$ group parametrized by $\xi^0$. For simplicity, the general method deals with the character $y = y_{ch} g'/g$ instead of the chiral hypercharge $y_{ch}$, but this mathematical artifice does not affect in any way the results. Therefore, the irreps of the whole gauge group $SU(n)_L \otimes U(1)_Y$ are uniquely determined by indicating the dimension of the $SU(n)$ tensor and its character $y$ as $\rho = (\mathbf{n}_\rho, y_\rho)$. 

2
2.3 Gauge Fields

As in all kinds of gauge theories, the interactions are mediated by gauge fields that are vector fields (massive or massless) that couple different fermion fields in a particular manner, by introducing the so-called covariant derivatives. The gauge fields in our notation \( A^0_\mu = (A^0_\mu)^* \) and \( A_\mu = A^+_\mu \in su(n) \) respectively, while the needed covariant derivatives are defined as \( D_\mu L^\rho = \partial_\mu L^\rho - ig(A^a_\mu T^a_\rho + y_a A^0_\mu L^\rho) \) where \( T^a_\rho \) are generators (let them be diagonal or off-diagonal) of the \( su(n) \) algebra.

2.4 Minimal Higgs Mechanism

The scalar sector, organized as the so-called minimal Higgs mechanism (mHm), is flexible enough to produce the SSB in one step up to the \( U(1)_{em} \) symmetry and, consequently, generate masses for the plethora of particles and bosons in the model. The scalar sector consists of \( n \) Higgs multiplets \( \phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(n)} \) satisfying the orthogonality condition \( \phi^{(i)}+ \phi^{(j)} = \phi^2 \delta_{ij} \) in order to eliminate the unwanted Goldstone bosons that could survive the SSB. \( \phi \) is a gauge-invariant real scalar field while the Higgs multiplets \( \phi^{(i)} \) transform according to the irreps \((n, y^{(i)})\) whose characters \( y^{(i)} \) are arbitrary numbers that can be organized into the diagonal matrix \( Y = \text{diag} \left( y^{(1)}, y^{(2)}, \ldots, y^{(n)} \right) \). In addition, the Higgs sector needs, in our approach, a parameter matrix \( \eta = \text{diag} \left( \eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(n)} \right) \) with the property \( \text{Tr}(\eta^2) = 1 - \eta_0^2 \) in order to supply a non-degenerate boson mass spectrum after SSB took place. The scalar potential is assumed to have an absolute minimum for \( \phi = \langle \phi \rangle \neq 0 \) that is, \( \phi = \langle \phi \rangle + \sigma \) where \( \sigma \) is the unique surviving physical Higgs field. Therefore, one can always define the unitary gauge where the Higgs multiplets, \( \phi^{(i)} \) have the components \( \phi^{(i)}_k = \delta_{ik} \phi = \delta_{ik} (\langle \phi \rangle + \sigma) \).

2.5 Electric and neutral charges

The charge spectrum of the model is close related to the problem of finding the basis of the physical neutral bosons. First of all, the method ensures the separation of the electromagnetic potential \( A^0_\mu \) corresponding to the surviving \( U(1)_{em} \) symmetry. The one-dimensional subspace of the parameters \( \xi^a \) associated to this symmetry assumes a particular direction in the parameter space \( \{\xi^0, \xi^i\} \) of the whole Cartan subalgebra. This is uniquely determined by the \( n-1 \)-dimensional unit vector \( \nu \) and the angle \( \theta \) giving the subspace equations \( \xi^0 = \xi^a \cos \theta \) and \( \xi^i = \nu_i \xi^a \sin \theta \). On the other hand, since the Higgs multiplets in unitary gauge are invariant under \( U(1)_{em} \) transformations, one remains with the condition \( D_i \xi^i + Y \xi^0 = 0 \) which yields \( Y = -D^\nu D^\nu \tan \theta \equiv -(D \cdot \nu) \tan \theta \). In other words, the new parameters \((\nu, \theta)\) determine all the characters \( y^{(i)} \) of the irreps of the Higgs multiplets and hence these will be considered the principal parameters of the model. Therefore we one deal with \( \theta \) and \( \nu \) (which has \( n-2 \) independent components) instead of \( n-1 \) parameters \( y^{(i)} \). Under these circumstances, one can easily compute the mass term of the gauge bosons depending on the parameters \( \theta \) and \( \nu \). Evidently, \( A^0_\mu \) does not appear in the mass term, so it remains massless. The other neutral gauge fields \( A^i_\mu \) have the non-diagonal mass
matrix (Eq. (53) in Ref. [1]). This can be brought in diagonal form with the help of a
$SO(n-1)$ transformation, $A^i_{\mu} = \omega^i_j Z^j_{\mu}$, which leads to the physical neutral bosons
$Z^i_{\mu}$ with well-defined masses. Performing this $SO(n-1)$ transformation the physical neutral bosons are completely determined. The transformation
\[ A^0_{\mu} = A^m_{\mu} \cos \theta - \nu_i \omega^i_j Z^j_{\mu} \sin \theta, \]
\[ A^k_{\mu} = \nu^k A^m_{\mu} \sin \theta + \left( \delta^k_i - \nu^k \nu_i (1 - \cos \theta) \right) \omega^i_j Z^j_{\mu}. \]

which switches from the original diagonal gauge fields, $(A^0_{\mu}, A^i_{\mu})$ to the physical ones,
$(A^m_{\mu}, Z^j_{\mu})$. This is called the generalized Weinberg transformation (gWt).

Now one can identify the charges of the particles with the coupling coefficients of
the currents with respect to the above determined physical bosons. Thus, we find that
the spinor multiplet $L^\rho$ (of the irrep $\rho$) has the following electric charge matrix
\[ Q^\rho = g \left[ (D^\rho \cdot \nu) \sin \theta + y^\rho \cos \theta \right], \]
and the $n-1$ neutral charge matrices
\[ Q^\rho (Z^i) = g \left[ D^\rho_k - \nu_k (D^\rho \cdot \nu) (1 - \cos \theta) - y^\rho \nu_k \sin \theta \right] \omega^k_i. \]

Corresponding to the $n-1$ neutral physical fields, $Z^i_{\mu}$. All the other gauge fields, namely
the charged bosons $A^i_{\mu \nu}$, have the same coupling, $g/\sqrt{2}$, to the fermion multiplets.

### 3 $SU(4)_L \otimes U(1)_Y$ models

The general method must be based on the following assumptions in order to give viable results when it is applied to concrete models:

(I) the spinor sector must be put (at least partially) in pure left form using the charge
conjugation (see for details Appendix B in Ref. [1]):

(II) the minimal Higgs mechanism must be employed with its arbitrary parameters
$(\eta_0, \eta)$ satisfying the condition $\text{Tr}(\eta^2) = 1 - \eta_0^2$ and giving rise to traditional Yukawa
couplings in unitary gauge

(III) the coupling constant, $g$, is the same with the first one of the SM

(IV) at least one $Z$-like boson should satisfy the mass condition $m_Z = m_W / \cos \theta_W$
established in the SM and experimentally confirmed.

Conditions (II) and (IV) lead to a realistic non-degenerate mass spectrum for particular
classes of the 3-4-1 model that will be presented elsewhere [21]. For our purpose
here condition (III) plays a crucial role.

In the following, we will use the standard generators $T_a$ of the $su(4)$ algebra.
Therefore, as the Hermitian diagonal generators of the Cartan subalgebra one deals,
in order, with $D_1 = T_3 = \frac{1}{2} \text{diag}(1, -1, 0, 0)$, $D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2, 0)$, and
$D_3 = T_{15} = \frac{1}{2\sqrt{6}} \text{diag}(1, 1, 1, -3)$ respectively. At the same time, we denote the irreps
of the electroweak model under consideration here by \( \rho = (n, y^\rho_{ch}) \) indicating the genuine chiral hypercharge \( y_{ch} \) instead of \( y \). Therefore, the multiplets - subject to anomaly cancellation - of the 3-4-1 model of interest here will be denoted by \( (n_{color}, n, y^\rho_{ch}) \).

There are three distinct cases leading to a discrimination among models of the 3-4-1 class, according to their electric charge assignment. They are: (i) versors \( \nu_1 = 1, \nu_2 = 0, \nu_3 = 0 \), (ii) versors \( \nu_1 = 0, \nu_2 = 1, \nu_3 = 0 \), and (iii) versors \( \nu_1 = 0, \nu_2 = 0, \nu_3 = -1 \), respectively. At the same time, one assumes the condition \( e = g \sin \theta_W \) established in the SM.

### 3.1 Case 1 (versors \( \nu_1 = 1, \nu_2 = 0, \nu_3 = 0 \))

In this case, the lepton 4-plet obeys the fundamental irrep of the gauge group \( \rho = (4, 0) \). Eq. (4) yields:

\[
Q^{(4,0)} = e T_3^{(4)} \frac{\sin \theta}{\sin \theta_W},
\]

which leads to the lepton 4-plet \( (e^c_\alpha, e_\alpha, \nu_\alpha, N_\alpha)^T_L \sim (4, 0) \) if and only if \( \sin \theta = 2 \sin \theta_W \) holds.

For the two families \( (i = 1, 2) \) of quarks transforming in the same way under the gauge group \( (J_i, u_i, d_i, D_i)^T_L \sim (4^*, -1/3) \) and for the third one that transforms as \( (J_3, d_3, u_3, U_3)^T_L \sim (4, +2/3) \), the electric charge operator will take, respectively, the forms

\[
Q^{(4^*, -1/3)} = e \left[ T_3^{(4^*)} \frac{\sin \theta}{\sin \theta_W} - \frac{1}{3} \left( \frac{g'}{g} \right) \frac{\cos \theta}{\sin \theta_W} \right],
\]

\[
Q^{(4, +2/3)} = e \left[ T_3^{(4)} \frac{\sin \theta}{\sin \theta_W} + \frac{2}{3} \left( \frac{g'}{g} \right) \frac{\cos \theta}{\sin \theta_W} \right],
\]

compatible with the known quark charges if and only if

\[
\frac{g'}{g} = \frac{\sin \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}.
\]

For the sake of completeness we show the fermion representations of this class of models.

**Lepton families**

\[
f_{\alpha L} = \begin{pmatrix} e^c_\alpha \\ e_\alpha \\ \nu_\alpha \\ N_\alpha \end{pmatrix} \sim (1, 4, 0)
\]

**Quark families**

\[
Q_{i L} = \begin{pmatrix} J_i \\ u_i \\ d_i \\ D_i \end{pmatrix} \sim (3, 4^*, -1/3) \quad Q_{3 L} = \begin{pmatrix} J_3 \\ -d_3 \\ u_3 \\ U_3 \end{pmatrix} \sim (3, 4, +2/3)
\]
\[(d_3L)\,^c, \,(d_iL)\,^c, \,(D_3L)\,^c, \sim (3, 1, +1/3) \tag{12}\]

\[(u_3L)\,^c, \,(u_iL)\,^c, \,(U_3L)\,^c \sim (3, 1, -2/3) \tag{13}\]

\[(J_{3L})\,^c \sim (3, 1, -5/3) \quad (J_{iL})\,^c \sim (3, 1, +4/3) \tag{14}\]

with \(\alpha = 1, 2, 3\) and \(i = 1, 2\).

In the representations presented above one can assume, like in majority of the papers in the literature, that the third generation of quarks transforms differently from the other two. This could explain the unusual heavy masses of the third generation of quarks, and especially the uncommon properties of the top quark. The capital letters \(J\) denote the exotic quarks included in each family. They exhibit electric charges \(\pm 4/3\) and \(\pm 5/3\).

This possible choice of the versors \(\nu_1 = 1, \nu_2 = 0, \nu_3 = 0\) has led us to the very class of 3-4-1 models with exotic electric charges [8] - [13] whose phenomenology predicted by our method will be in extenso analysed in Ref. [21].

### 3.2 Case 2 (versors \(\nu_1 = 0, \nu_2 = 1, \nu_3 = 0\))

Due to \(T_8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2, 0)\) there is no room for a plausible electric charge operator assigning only 0 and \(\pm e\) in the lepton 4-plets. Therefore, this case is ruled out as long as one does not allow for exotic electric charges like, for instance \(\pm 2e\), in the lepton sector.

### 3.3 Case 3 (versors \(\nu_1 = 0, \nu_2 = 0, \nu_3 = -1\))

In this case, no 4-plet obeys the fundamental irrep of the gauge group \(\rho = (4, 0)\). Notwithstanding, since for the lepton 4-plet one can assign two different chiral hypercharges \(-\frac{1}{4}\) and \(-\frac{3}{4}\) respectively, we get two sub-cases leading to two different versions of the class of 3-4-1 models without exotic electric charges. The coupling matching, as we will see in the following, assumes the same relation in both sub-cases.

From Eq. (4), it is straightforward that the lepton family exhibits the electric charge operator

\[Q^{(4^*,-\frac{1}{4})} = e \left[ -T^{(4^*)}_{15} \frac{\sin \theta}{\sin \theta W} - \frac{1}{4} \left( \frac{g'}{g} \right) \frac{\cos \theta}{\sin \theta W} \right] , \tag{15}\]

for the first sub-case. This leads to the lepton representation \((\nu_\alpha, \nu_\alpha, N_\alpha, N_\alpha')_L^T \sim (4^*, -\frac{1}{4})\) including two new kinds of neutral leptons \((N_\alpha, N_\alpha')\).

In the second subcase, the electric charge operator will be represented as

\[Q^{(4,-\frac{1}{4})} = e \left[ -T^{(4)}_{15} \frac{\sin \theta}{\sin \theta W} - \frac{3}{4} \left( \frac{g'}{g} \right) \frac{\cos \theta}{\sin \theta W} \right] , \tag{16}\]

leading to the lepton families \((\nu_\alpha, e^-_\alpha, E^-_\alpha, E'^-_\alpha)_L^T \sim (4, -\frac{3}{4})\) that allow for new charged leptons \((E^-_\alpha, E'^-_\alpha)\).
After a little algebra, both Eqs (15) and (16) require - via the compulsory condition $\sin \theta = \sqrt{3}/2 \sin \theta_W$, since the unique allowed electric charges in the lepton sector are 0 and $\pm e$ - the coupling matching:

$$
\frac{g'}{g} = \frac{\sin \theta_W}{\sqrt{1 - \frac{3}{2} \sin^2 \theta_W}}.
$$

Once these assignments are assumed, the quarks will acquire their electric charges from the following operators

For Lepton families

$$
f_{\alpha L} = \begin{pmatrix} e_{\alpha} \\ \nu_{\alpha} \\ N_{\alpha} \\ N'_{\alpha} \end{pmatrix}_{L} \sim (1, 4^*, -1/4)
$$

For Quark families

$$
Q_{4L} = \begin{pmatrix} u_i \\ d_i \\ D_i \\ D'_i \end{pmatrix}_{L} \sim (3, 4, -1/2)
Q_{3L} = \begin{pmatrix} -d_3 \\ u_3 \\ U \\ U' \end{pmatrix}_{L} \sim (3, 4^*, 5/12)
$$

3.3.1 Case 3a

With the first of the above mentioned assumptions, the fermion representations are:

**Lepton families**

$$
f_{\alpha L} = \begin{pmatrix} e_{\alpha} \\ \nu_{\alpha} \\ N_{\alpha} \\ N'_{\alpha} \end{pmatrix}_{L} \sim (1, 4^*, -1/4)
$$

**Quark families**

$$
Q_{4L} = \begin{pmatrix} u_{i} \\ d_{i} \\ D_{i} \\ D'_{i} \end{pmatrix}_{L} \sim (3, 4, -1/2)
Q_{3L} = \begin{pmatrix} -d_{3} \\ u_{3} \\ U \\ U' \end{pmatrix}_{L} \sim (3, 4^*, 5/12)
$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$. We recovered the same fermion content as the one of the model presented in Refs. [14, 18].

3.3.2 Case 3b

With the second of the above mentioned assumptions, the fermion representations are:

**Lepton families**

$$
f_{\alpha L} = \begin{pmatrix} e_{\alpha} \\ \nu_{\alpha} \\ N_{\alpha} \\ N'_{\alpha} \end{pmatrix}_{L} \sim (1, 4^*, -1/4)
$$

**Quark families**

$$
Q_{4L} = \begin{pmatrix} u_{i} \\ d_{i} \\ D_{i} \\ D'_{i} \end{pmatrix}_{L} \sim (3, 4, -1/2)
Q_{3L} = \begin{pmatrix} -d_{3} \\ u_{3} \\ U \\ U' \end{pmatrix}_{L} \sim (3, 4^*, 5/12)
$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$. We recovered the same fermion content as the one of the model presented in Refs. [14, 18].
Quark families

\[ Q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ U_i \\ U_i' \end{pmatrix}_L \sim (3, 4^*, 5/12) \]

\[ Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ D \\ D' \end{pmatrix}_L \sim (3, 4, -1/12) \]

\[ (d_{3L})^c, (d_{iL})^c, (D_{3L})^c, (D'_{iL})^c \sim (3, 1, +1/3) \]  \hspace{1cm} (25)

\[ (u_{3L})^c, (u_{iL})^c, (U_{iL})^c, (U'_{iL})^c \sim (3, 1, -2/3) \]  \hspace{1cm} (26)

with \( \alpha = 1, 2, 3 \) and \( i = 1, 2 \). We recovered the same fermion content as the one of the model presented in Refs. [17, 18].

With this assignment the fermion families (in each of the above displayed cases) cancel the axial anomalies by just an interplay between them, although each family remains anomalous by itself. Thus, the renormalization criteria are fulfilled and the method is validated once more from this point of view. Note that one can add at any time sterile neutrinos - i.e. right-handed neutrinos \( \nu_{\alpha R} \sim (1, 1, 0) \) - that could pair in the neutrino sector of the Ld with left-handed ones in order to eventually generate tiny Dirac or Majorana masses by means of an adequate see-saw mechanism. These sterile neutrinos do not affect anyhow the anomaly cancelation, since all their charges are zero. Moreover, their number is not restricted by the number of flavors in the model.

4 Concluding remarks

In this brief report we have obtained the correct electric charge assignment and matching the gauge couplings for some particular anomaly-free models of the 3-4-1 class, by just using the prescriptions of the general method of exactly solving gauge models with high symmetries. All the results are simply consequences of a proper versor choice in the general Weinberg transformation. This approach represents a complementary way to discriminate among different particular 3-4-1 models, in addition to the well-known classification based on the parameters \( b \) and \( c \). However our approach is a little bit more restrictive, since it leaves out the model investigated in Ref. [15] which can be reproduced by none versor setting in our method.

The complex phenomenology of the above obtained 3-4-1 models - such as boson mass spectrum, neutrino masses, extra-neutral bosons and neutral currents, bileptons etc. - will be investigated in a future work.

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