Analysis of students critical thinking ability in solving trigonometric problems

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Abstract. In mathematics learning, students’ critical thinking ability is required to understand and solve mathematics problems that require reasoning, analysis, evaluation, and interpretation. Therefore, to describe students’ critical thinking ability, research was conducted on 30 students (15/16 years old) in West Java, Indonesia. This research is qualitative descriptive research and the data collection technique is written test with 3 description questions about trigonometric problems. The test results were analyzed using three indicators of critical thinking ability, (1) composing questions with reasons, (2) identifying relevant and irrelevant data of a mathematical problem, and (3) solving mathematical problems with reasons. The results showed that: (1) almost one-third of the number of students can compile the questions with the reasons, (2) more than one-third of students can identify the relevant and irrelevant data of a mathematical problem, and (3) more than one-third of students can solve the mathematical problems with reasons. The average percentage of students in trigonometric problems is 31.86%. Based on the research result, it is seen that most students do not have good critical thinking ability in solving trigonometric problems.

1. Introduction
In didactic teaching, the content is inadvertently treated with static, as something to be replicated and repeated to be exterminated [1]. And because the content is taught in ways that make it impossible to think about it, students’ minds retreat to memory and do not try to understand it logically. Things like this will make their critical thinking ability do not develop optimally. [2] in solving the problem, students only imitate what the teacher does and assume that it is done quite like an example. This makes the students do not develop the ability to solve the problem with other alternatives and students are less free to take the opportunity to express themselves.

Some scientists identify critical thinking with specific skill mastery and provide a scheme or taxonomy to reveal their interconnectedness. [3], critical thinking is the activity of analyzing ideas in a more specific direction, distinguishing sharply, choosing, identifying, studying and developing them more fully. Furthermore, critical thinking is a process that involves mental activity such as problem-solving, decision making, and science inquiry. Glaser [4] defines mathematical critical thinking as the ability and disposition that combine early knowledge, reasoning, mathematical, and cognitive strategies to generalize, prove, and evaluate reflective mathematical situations. So, critical thinking is a process of solving a problem that involves various aspects, such as prior knowledge or problem-solving strategies.
Ennis [5], describes the indicators of critical thinking ability in detail that focus on questions; analyze and clarify questions, answers, and arguments; select trusted references; observe and analyze deduction; induce and analyse induction; formulate the explanation, conclusions, and hypotheses; throw conclusions, determine action; and interaction. However, [6] describes indicators of critical thinking ability as the skills to (1) examine the truth of the argument, statement, and solution process, (2) construct questions with reasons, (3) compile answers/solve math problems with reasons, (4) identify relevant and irrelevant data of mathematical problem, and (5) identify the underlying assumptions of problem-solving.

Dunn et al. [1], at every level and all subjects, students need to learn how to: ask questions; define context and goals appropriately; find out relevant information; analyze key concepts; get conclusions; throw good reasons; recognize axioms and postulates; track important implications, and; think different. The indicators of critical mathematical thinking ability used to measure students' critical thinking ability in this research are: (1) Compile questions with reasons, (2) Indicate relevant and irrelevant data of a mathematical problem, and (3) Compile answers / solve mathematical problems.

Many innovations have been made in education, including methods and instructional media. But the facts prove that the learning that has been done still using conventional methods. Surely this does not encourage the development of students' critical mathematical thinking ability. This is evidence from the national examination report in 2014 [4] issued by the Center for Educational Assessment that shows the achievement of trigonometric competency decreased by 27.55% from 2012 to 2013 (from 78.63% to 51.08%) and increased to 58.41% in 2014. The data shows that students have difficulty in solving trigonometric problems. Therefore, this study aims to describe how students' critical thinking ability in solving problems on trigonometric.

2. Methods

This research is qualitative research with descriptive problem because this research purpose is to describe or explore a symptom, event, incident thoroughly, and deeply [7]. The data was collected by using the test instrument that was tested on 30 students of class XI SMA N in West Java. The result of this research was obtained through deep research on students' mathematical critical thinking ability on trigonometric material. In this article, only subjects with the highest and lowest scores of test results were described. The written test is given in Table 1.

| No | Mathematics problems |
|----|----------------------|
| A  | **Indicators: Compile questions with reasons**<br>Make a question using the picture beside then answer it completely (you can give additional information to your question). |
| B  | Indicator: Identify relevant and irrelevant data of a mathematical problem<br>ABC is an equilateral triangle with side length of 1 unit. Through point B is made a perpendicular line BC where the line intersects the extension of the CA line at point D, what is the length of the BD? Include the underlying formulas (you can create images based on known information).  |
| C  | **Indicators: Compile answers / solve math problems**<br>To measure the height of a mountain, an observer uses the scheme as in the picture beside. First, the observer is at A and observes the mountain at an elevation angle of 600, then walks away from the mountain and stops at B, 600 m from A. At B he observes a mountain with an elevation angle of 300. |
The mountain’s foot is 1,800 m above sea level. Is the data enough to determine the height of the mountain above sea level? If it’s enough, finish it! If not, what data do you think is missing?

Complete and finish it.

3. Result and Discussion
The discussion in this section will be about the general findings first, then followed by the discussion of students’ critical thinking in solving the problems. There are 30 students participated in the test. Table 2 represents the numerical data on students' critical thinking ability in solving trigonometric problems.

| Indicator of critical thinking ability | Details of answers | %     |
|----------------------------------------|--------------------|-------|
| Compile questions with reasons          | Identify known data and issues to be asked from the information provided | 27.33% |
|                                        | Establish the depth / complexity of the questions to be asked |       |
|                                        | Compile questions relevant to the information provided along with the reasons |       |
| Identify relevant and irrelevant data of a mathematical problem | Identify known and asked data | 31.25% |
|                                        | Identify math problems |       |
|                                        | Identify terms for solving math problems |       |
|                                        | Examine the conformity of known data with terms for a mathematical solution |       |
|                                        | Identify relevant / irrelevant data with reasons |       |
|                                        | Check the correct solution of the main problem |       |
| Compile answers / solve math problems   | Identify the process / mathematical concepts of the given situation / problem, asked, and check the adequacy of the elements | 37%   |
|                                        | Develop a mathematical model |       |
|                                        | Identify the calculation steps along with the reason for explanation of the mathematical process / concept / rule used |       |
|                                        | Complete the mathematical model of the problem with the reasons or include the process / concepts / rules of mathematics used |       |
|                                        | Establish relevant solutions |       |

Based on Table 2, the average percentage of student work result is 31.86%.

3.1 Indicator: compile questions with reasons
In this indicator, students are expected to identify known data and problems to be asked of the information provided; set the depth / complexity of questions to be asked; and compile questions relevant to the information provided along with the reasons. Leicester and Taylor in [8], students learn to gradually critical thinking through diligent habits in formulating problems and answering questions that require explanation. This indicator is contained in question 1 in Table 1. Most students make questions based on the information of the picture, but not complex / depth. For example, one student made a question: “Berapa bukit dan lembah dalam gambar di atas?" means what is the wavelength of the picture above? This question is related to physics, not mathematics. The other example is shown in Figure 2. The student’s question is “Sebutkan koordinat titik puncak!” means what is the coordinates of the turning point and the answer of that question is “($\frac{\pi}{2}$, 2). Figure 2 appears that student made question but only use partial information and did not use all of the information available on the picture and this is done by most students. The best answer is shown in Figure 1. The student’s question is “Sebutkan nama grafik di atas?” means what kind of graph is this? And the answer to that question is “$y = 2 \sin x$”. The answer to the question made is not correct. Nevertheless, she tried to prove her answer using the maximum point and symbol $\pi$ on the picture.
Figure 1. Example of student’s answer in making a question based on picture. The student’s question is “Sebutkan nama grafik di atas?”, while the answer is “$y = 2 \sin x$”

Figure 2. Example question of the other student. The question is “Sebutkan koordinat titik puncak!” and the answer is $\left(\frac{-\pi}{2}, 2\right)$ dan $\left(\frac{\pi}{2}, 2\right)$

The average percentage of students’ critical thinking ability on this indicator is still low at 27.33%.

3.2 Indicator: compile answers / solve math problems

Students are expected to solve the problem using the concepts appropriately. Most of the students solve the problem with pictures. Interesting findings were found on this indicator because of pictures that they made. Some students made picture to illustrate the problem, but the illustrate was not correct as shown in Figure 4. If the students’ answers are analyzed based on Polya’s steps, then for the students’ work in Figure 3 already includes the four steps of solving Polya namely understanding the problem, making a problem-solving plan, executing the plan, and checking back the solution that has been obtained. While the work of students in Figure 4, based on Polya’s step, students have not understood the problem, so it can not represent the problem into a picture. Zheng [9], students’ failure in the examination is language difficulties, not the failure to grasp the relevant algorithms. Result of Zhe’s research, the root cause affecting students’ appropriate mathematical representation is their comprehension, conversion and representation abilities of mathematical language [9]. In addition, [9], teachers believe that in problem-solving, the factors affecting mathematical representation are students’ knowledge structure, mastery of mathematical language, usage of mathematical metacognitive-strategy, logical reasoning ability, written representation ability of mathematical language, time and environment of mathematical problem solving, subject material contents, and so on. The average percentage of students’ critical thinking ability on this indicator is still low at 37%.

Figure 3. Example of student’s answer based on item B
3.3 Indicator: identify relevant and irrelevant data of a mathematical problem

In this indicator, students are expected to identify the information known, the requirements needed to solve a problem, and identify the adequacy of data. Although a student’s answers in Figure 5 did not identify the information known, the answer was correct. Judging from Polya's problem solving step, she has covered all four steps of Polya’s solution. She prefers to find the numerical solution directly from the first information she read rather than to construct the illustrate. This is consistent with the research of [10] that students prefer to find the numerical solution directly from the first information they read rather than to construct its algebraic expression. This is also evidenced by their strong attention to numbers and symbols rather than text and images.

While from Figure 6, it appears that student did not analyze the problem given. He answers that the problem could not be solved, but he was unable to give the reason/the missing data, so the problem could not be solved. Judging from Polya’s problem solving step, he did not understand the problem well. However, he tried to understand the problem. This is evidenced by the writing on the available picture.

Students' critical thinking ability should be more advanced in high school. This is supported by [11], there are ways to stimulate the critical thinking ability at every level of education and in every
management learning. Students should be able to make in-depth questions from available information, identify data or information to solve problems, even make an illustration of the problem given. However, based on this research show that students did not have good critical thinking in solving the trigonometric problem. [10], this is evidenced by their strong attention on numbers and symbols rather than to texts and pictures. These cases are supported by previous research which found that students tend to directly jump into calculation, without reading or observing what is given in the problem [10].

At every level and in all subjects, students need to learn how to: appropriately ask questions, determine contexts and purposes, pursue relevant information, analyze key concepts, derive sound inferences, generate good reasons, recognize axioms and postulates, trace important implications, and think in different perspective [11]. For all educational reforms, critical thinking can be a key organizing concept. Based on experts thinking, students should be given more opportunities to practice their critical thinking ability [12].

4. Conclusion
Based on the research results and discussion, it can be concluded that the average percentage of critical students' critical thinking skills of the three indicators of critical thinking is low, ie 31.86%. Students' critical thinking skills are influenced by many things. One of them is the teacher's habit of teaching which makes students not practice their thinking ability. Students are not taught to make questions from available data, to identify data relevance in solving problems, and to illustrate problems. If this is allowed continuously, students' critical thinking ability will not develop. Whereas in this millennial era, everyone is required to have critical thinking ability in the face of the times. In the future, the investigation of critical thinking ability may be expanded by investigating a strategy to practice students' critical thinking ability in solving mathematics problems (in other topics). In the end, because critical thinking ability is very important for students, teachers need to innovate the learning; both media and learning methods.

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