Sinusoidal Fuzzy Soft Set and its Application in Mcdm

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Abstract. This paper deals with the study of extension of classical soft sets into a Sinusoidal fuzzy soft set based on Sinusoidal fuzzy number and the basic operations on a sinusoidal fuzzy soft set are defined. Then a new algorithmic approach for Multi Criteria decision making based on Sinusoidal fuzzy soft set has been furnished using the ranking techniques, finally it has been analyzed in detail and illustrated with a numerical example.

Key words: Sinusoidal Fuzzy number, SFSS, MCDM, linguistic variables, Choice value.

1. Introduction:

In this uncertain environment fuzzy numbers plays an important role in various data of real life. Some of the complicated problems in Engineering, Medical Sciences and many other fields involves uncertainty. These problems face lots of practical issues in solving using the traditional classical types of sets. Some of the well-known theories are theory of fuzzy sets [25], Theory of intuitionistic fuzzy sets [2,16], theory of vague sets [5] theory of interval mathematics [7], theory of rough sets [16] and other mathematical tools. But these theories have their own difficulties as discussed by Molodtsov [14]. The concept of Soft sets which is one of the new mathematical tools was initiated by Molodtsov in 1999[13] to overcome these difficulties in uncertainties. This was called as soft set theory and it seems to be free from difficulties affecting the existing methods and can be used for approximate description of objects without any restriction. In recent years, its applications have been big advantage and are extended to data analysis [8,26], medical diagnosis [12], decision making evaluation process [19], forecasting [21] and so on. Based on the issue of combination between fuzzy sets and soft sets, some of the new concepts has been proposed by the researchers. Later, some of the new operations and complement of soft set has been introduced by Ali et al. [1]. Park et. al; [15] has given few properties of equivalence soft set relations.

Furthermore, in connection with fuzzy sets and soft sets, a study on hybrid structures has been initiated by Maji et al. [11]. The notion of fuzzy soft sets was introduced which was a fuzzy generalization of soft sets. Majumdar and Samanta [12] modified the definition of fuzzy soft sets and the generalized fuzzy soft set theory based on [15]. By combining interval valued fuzzy sets [3,6] and soft set models, Yang.et.al [23] presented the concept of interval-valued fuzzy soft sets. He also presented the concept of the multi fuzzy soft set and provided its application in decision making process [24]. Though there exist various fuzzy numbers like trapezoidal fuzzy number and triangular fuzzy number their membership function is piecewise linear and trapezoidal, which can express vagueness information caused by linguistic assessments into numerical variables. By combining these fuzzy numbers along with soft set models, Xiao et.al; [20] presented a new concept of trapezoidal fuzzy soft sets with certain linguistic assessments. Multi criteria Decision making problem is one of the basic classical tools in which we assume the ratings of alternatives and the weights given to each criterion as a crisp number, but it is impossible in real-life situations. In solving MCDM problems
various types of membership functions which express some of the vagueness factors are applied such as linear membership function [18], S-curve membership function [4,17], etc.

Sinusoidal fuzzy number as a vital concept of fuzzy set is often applied in various Engineering, medical and economical fields. The membership function of a Sinusoidal fuzzy number is piecewise linear and sinusoidal [9], which can express vagueness information caused by linguistic assessments through transforming them into numerical values. In this paper we have combined the sinusoidal fuzzy number and the soft set. Some of the previous research about soft sets focus on the application of soft sets in some areas, includes data analysis in incomplete evaluation system [26]. The aim of this paper is to introduce a sinusoidal fuzzy soft set, which can deal with the linguistic assessments. Some of the operations on a Sinusoidal fuzzy soft set are defined, such as “AND”, “OR” and Complement operations. Then an algorithmic approach for the decision-making problem based on Sinusoidal fuzzy soft set has been introduced with the several concepts of normalized weighted Sinusoidal fuzzy soft set and the ranking function. Finally, a MCDM problem has been illustrated with a numerical example. Finally, the conclusions have been given.

2. Preliminaries:

Definition 2.1: [13]
Suppose that U is an initial Universe set and A is a set of parameters. Let P(U) be the set of all the subsets of U, a pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

Clearly a soft set is a mapping from parameters to P(U) and it is not a set, but a parameterized family of subsets of the universe U for e ∈ A, F(e) may be considered as the set of e-approximate elements of the soft sets (F, A)

Definition 2.2: [6]
Let P(U) be the set of all fuzzy subsets of U, a pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A → P(U).

2.3 Sinusoidal Fuzzy numbers (SFN): [9]
A fuzzy number $\mathcal{S}$ is a Sinusoidal fuzzy number denoted by $\mathcal{S} = (s_1, s_2, s_3, s_4, s_5)$ where $s_1, s_2, s_3, s_4, s_5$ are real numbers and its membership function $\mu_{\mathcal{S}}(x)$ is given below

$$
\mu_{\mathcal{S}}(x) =
\begin{cases}
0 & \text{for } x < s_1 \\
\frac{(x - s_1)}{(s_2 - s_1)} & \text{for } s_1 \leq x \leq s_2 \\
\frac{(s_3 - x)}{(s_3 - s_2)} & \text{for } s_2 \leq x \leq s_3 \\
\frac{(s_4 - s_2)}{(s_4 - s_3)} & \text{for } s_3 \leq x \leq s_4 \\
\frac{(s_5 - s_4)}{(s_5 - s_4)} & \text{for } s_4 \leq x \leq s_5 \\
0 & \text{for } x > s_5
\end{cases}
$$

The membership function of a Sinusoidal fuzzy number captures the vagueness of the linguistic assessment as shown in the figure 3.1

3. Sinusoidal Fuzzy Soft Sets (Sfss):
A set which is consisted by Sinusoidal fuzzy number is called Sinusoidal Fuzzy set. This Section introduces Sinusoidal fuzzy soft sets and some of its relevant operations.

Definition 3.1
Let $P(U)$ be the set of all Sinusoidal fuzzy subsets of $U$, a pair $(\mathcal{S}, A)$ is called a Sinusoidal fuzzy soft set over $U$, where $\mathcal{S}$ is a mapping given by $\mathcal{S}: A \rightarrow P(U)$.

A sinusoidal fuzzy soft set is a mapping from parameter $A$ to $P(U)$. It is a parameterized family of Sinusoidal fuzzy subsets of $U$. The universe of a SFSS is the set of all Sinusoidal fuzzy subsets of $U$. A SFSS is a special case of a soft because it is still a mapping from parameters to the universe $P(U)$.

**Example : 1**

Consider a problem in which Mr. John describes the optional of 5 different jewellery shops under various attributes with linguistic variables intuitively as in table 3.1

| $U$     | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop |
|---------|--------------|-----------|-------------|-------------|------------|
| $J_1$   | Poor         | good      | Medium      | Very Good   | Very Good  |
| $J_2$   | Good         | Medium    | Medium      | Fair        | Medium     |
| $J_3$   | Fair         | Very Good | Fair        | Good        | Fair       |
| $J_4$   | Very Good    | poor      | Good        | Medium      | Medium     |
| $J_5$   | Poor         | Fair      | Good        | Good        | Medium     |

**Table-3.1 Linguistic variables**

Then we have a corresponding Sinusoidal fuzzy soft set $(\mathcal{S}, A)$ over the universe $U$ through the rule of conversion between linguistic variables and numeric variables showed in figure 3.1 in which $U = \{J_1, J_2, J_3, J_4, J_5\}$ and $A = \{e_1, e_2, e_3, e_4, e_5\}$ represent the set of jewel shops and its attributes, respectively.

$\mathcal{S}(e_1) = \{(J_1/(0.0,0.1,0.2,0.25,0.3), (J_2/(0.6,0.7,0.8,0.85,0.9), (J_3/(0.2,0,3,0.4,0.45,0.5), \\
(J_4/(0.7,0,8,0.9,0.95,1), (J_5/0.05,0.1,0.16,0.2)\}

$\mathcal{S}(e_2)=\{(J_1/(0.64,0.7,0.8,0.86,0.9),(J_2/(0.15,0.2,0.3,0.35,0.44),(J_3/(0.7,0.8,0.9,0.95,1),(J_4/(0.1,0.15,0.2, \\
0.25,0.35) (J_5 / (0.2,0.3,0,4,0.5,0.6))\}

$\mathcal{S}(e_3) =\{(J_1/(0.1,0.2,0.3,0.35,0.4), (J_2/(0.1,0.2,0.3,0.4,0.45), (J_3/(0.3,0.4,0.5,0.6,0.65), \\
(J_4/(0.5,0.6,0.7,0.8,0.9), (J_5/0.6,0.7,0.8,0.85,0.9)\}

$\mathcal{S}(e_4) = \{(J_1/(0.7,0.8,0.9,0.9,0.98), (J_2/(0.7,0.8,0.9,0.9), (J_3/(0.3,0.4,0.5,0.6,0.65), \\
(J_4/(0.5,0.5,0.6,0.7,0.8), (J_5/0.5,0.6,0.7,0.8,0.85,0.9))\}

$\mathcal{S}(e_5)=\{(J_1/(0.7,0.8,0.9,0.95,1),(J_2/(0.01,0.2,0.3,0.4,0.5),(J_3/(0.2,0.3,0.4,0.5,0.6),(J_4/(0.1,0.2,0.3,0.4,0.5), \\
(J_5/0.1,0.2,0.3,0.4)\}


Definition 3.2:
Suppose that \((\mathcal{S}, A)\) and \((\mathcal{T}, B)\) are two Sinusoidal fuzzy soft sets over the universe \(U\) and \(E\) be a set of parameters where \(A, B \subseteq E\), then \((\mathcal{S}, A)\) is said to be a Sinusoidal fuzzy soft set subset of \((\mathcal{T}, B)\) iff \(A \subseteq B\) and \(\tilde{s}_{ia} \leq \tilde{t}_{ia}\) in \(\mathcal{S}(e)\) and \(\mathcal{T}(e)\) \(\forall e \in E, i \in E\), which is denoted as \((\mathcal{S}, A) \subseteq (\mathcal{T}, B)\). Here \(\tilde{s}_{ia}\) and \(\tilde{t}_{ia}\) are SFNs respectively in \(\mathcal{S}(e)\) and \(\mathcal{T}(e)\).

Definition 3.3:
Assume that \((\mathcal{S}, A)\) and \((\mathcal{T}, B)\) are two SFSSs, we can say that \((\mathcal{S}, A)\) and \((\mathcal{T}, B)\) are equal if and only if \((\mathcal{S}, A) \subseteq (\mathcal{T}, B)\) and \((\mathcal{T}, B) \subseteq (\mathcal{S}, A)\). This is denoted by \((\mathcal{S}, A) = (\mathcal{T}, B)\).

3.1 Operations on SFSS:
Definition 3.4:
The Complement of a SFSS, \((\mathcal{S}, A)\) is denoted and defined by \((\mathcal{S}, A)^c = (\mathcal{S}^c, A)\), where \(\mathcal{S}^c : A \rightarrow \mathcal{P}(U)\) is a mapping given by \(\mathcal{S}^c(\alpha)\) for all \(\alpha \in A\), \(\mathcal{S}^c(\alpha) = \{J_i \mid \tilde{t}_{ic} \subseteq \tilde{s}_{ic}\}\). Here \(\tilde{s}_{ic}\) and \(\tilde{t}_{ic}\) is the sinusoidal fuzzy number of the \(i\) object under the \(\alpha\) attribute in the sinusoidal fuzzy soft set \((\mathcal{S}, A)\).

Clearly, \((\mathcal{S}, A)^c = U - (\mathcal{S}, A)\) and \(((\mathcal{S}, A)^c)^c = (\mathcal{S}, A)\).

Definition 3.5:
The “AND” operations on two Sinusoidal fuzzy soft sets \((\mathcal{S}, A)\) and \((\mathcal{T}, B)\) which is denoted by \((\mathcal{S}, A) \wedge (\mathcal{T}, B)\) and defined by \((\mathcal{S}, A) \wedge (\mathcal{T}, B) = (\mathcal{W}, A \times B)\). Where \(\mathcal{W}(\alpha, \beta) = \mathcal{S}(\alpha) \cap \mathcal{T}(\beta)\) = \(\{J_i/\tilde{t}_{ia} \cap \tilde{s}_{ib}\}\) \(\forall (\alpha, \beta) \in A \times B\), \(\tilde{t}_{ia}\) denotes the sinusoidal fuzzy number of the \(i\) object under the \(\alpha\) attribute.
attribute in the sinusoidal fuzzy number $(\tilde{S}, A)$, $\tilde{s}_{i\beta}$ denotes the sinusoidal fuzzy number of the $i$ object under the $\beta$ attribute in the Sinusoidal fuzzy soft set $(\tilde{T}, B)$.

**Example: 2**

If $(\tilde{S}, A) = (0.0.05,0.1,0.15,0.2)$ and $(\tilde{T}, B) = (0.0.1,0.2,0.25,0.3)$

Then $(\tilde{S}, A) \land (\tilde{T}, B) = (0.0.05,0.1,0.15,0.2]$

**Definition 3.6:**

The “OR” operations between two Sinusoidal fuzzy soft sets $(\tilde{S}, A)$ and $(\tilde{T}, B)$ which is denoted by $(\tilde{S}, A) \lor (\tilde{T}, B)$ and defined by $(\tilde{S}, A) \lor (\tilde{T}, B) = (\tilde{Y}, A \cup B)$ Where $\tilde{Y}(a, \beta) = \tilde{S}(a) \lor \tilde{T}(\beta) = \{j_i/\max(\tilde{s}_{i\alpha}, \tilde{s}_{i\beta})\} \forall (a, \beta) \in AXB$ or $= \{j_i/\tilde{t}_{i\alpha} \lor \tilde{s}_{i\beta}\}$ $\tilde{t}_{i\alpha}$ denotes the sinusoidal fuzzy number of the $i$ object under the $\alpha$ attribute in the sinusoidal fuzzy number $(\tilde{S}, A)$, $\tilde{s}_{i\beta}$ denotes the sinusoidal fuzzy number of the $i$ object under the $\beta$ attribute in the Sinusoidal fuzzy soft set $(\tilde{T}, B)$.

**Example: 3**

If $(\tilde{S}, A) = (0.0.05,0.1,0.15,0.2)$ and $(\tilde{T}, B) = (0.0.1,0.2,0.25,0.3)$

Then $(\tilde{S}, A) \lor (\tilde{T}, B) = (0.0.1,0.2,0.25,0.3]$

**4. Application of Sinusoidal Fuzzy Soft Set:**

Molodtsov[13] has presented various applications of soft set theory in many branches like probability theory of measurement, operations research, game theory etc. In this section an application of Sinusoidal fuzzy soft set theory in a Multi-criteria Decision making (MCDM) problem has been furnished. Here the linguistic variables and the importance of various choice value is also considered and hence the decision-making problem has been expressed.

In case if a decision maker have to choose one of the best alternatives such as $J_1, J_2, \ldots J_n$ based on $m$ parameters: $e_1, e_2, \ldots \ldots e_m$ and suppose $w_j$ is the relative weight of attribute which is given for each set of parameters where $\sum_{j=1}^{m} w_j = 1$, denote a weight vector by $w= (w_1, w_2, \ldots \ldots w_m)^T$ and $(w_j \geq 0)$.

In some cases, it is impossible to get more accurate values for the decision maker and there exists some vagueness information and piecewise sinusoidal membership functions are good enough to capture the vagueness of those linguistic assessments. Hence, the sinusoidal fuzzy soft set is effective to express these decision-making problems under this type of circumstances which are too complex to be defined.

In this way, the ranking of $J_i$ should be a sinusoidal fuzzy number: $J_i = (J_{i1}, J_{i2}, J_{i3}, J_{i4}, J_{i5})$ and the weights has been assumed for each parameter as $w_j$. Then a MCDM problem can be expressed by sinusoidal fuzzy soft set $(\tilde{S}, E)$ as follows:

$$\tilde{S}(e_1) = \{(J_{11}/J_{12}/J_{13}/J_{14}/J_{15}), (J_{21}/J_{22}/J_{23}/J_{24}/J_{25}), \ldots \ldots (J_{m1}/J_{m2}/J_{m3}/J_{m4}/J_{m5})\}$$

$$\tilde{S}(e_2) = \{(J_{11}/J_{12}/J_{13}/J_{14}/J_{15}), (J_{21}/J_{22}/J_{23}/J_{24}/J_{25}), \ldots \ldots (J_{m1}/J_{m2}/J_{m3}/J_{m4}/J_{m5})\}$$
As it is difficult to calculate each Sinusoidal fuzzy number in sinusoidal fuzzy soft sets, we use ranking of sinusoidal fuzzy number to normalize the sinusoidal fuzzy soft sets and the definition is as follows:

**Definition: 4.1** [10]

If \( S, E = (J_1, J_2, J_3, J_4, J_5) \) is a sinusoidal fuzzy number in the Sinusoidal fuzzy soft then the normalization of the Sinusoidal fuzzy soft is given by the ranking of a sinusoidal fuzzy number to get the crisp value is obtained using the \( \alpha \) cut operations interval for which \( \alpha \in [-1,1] \) is as follows.

\[
S'(\alpha) = \frac{1}{2} \int_{-1}^{1} (\frac{1+\alpha}{2} + \alpha (\frac{2-1-\alpha}{2} - L_2 + \frac{1+\alpha}{2} - L_4)) \, d\alpha + \frac{1}{2} \int_{-1}^{1} (\frac{1+\alpha}{2} + \alpha (\frac{1+\alpha-2L_4}{2}))) \, d\alpha
\]

Where \( S'(\alpha) = f_{ij} \) is the rank of the alternative in the Sinusoidal fuzzy soft set \((S', E)\).

**Definition: 4.2** [22]

\((S'', E)'\) is the weighted normalized Sinusoidal fuzzy soft set of the Sinusoidal Fuzzy soft sets \((S', E)\) and \(J_{ij}'' = J_{ij} \times w_i\), where \( J_{ij}'' \) is the score of alternatives in the weighted normalized sinusoidal fuzzy soft set \((S', E)\).

**4.1 Algorithm:**

Algorithm for Multi-Criteria Decision making (MCDM) problem based on Sinusoidal fuzzy soft sets is as follows:

1. **Step-1:** Input the fuzzy soft set
2. **Step-2:** Identify the parameters/attributes for which \( e \in A \) is taken as Input and the number of alternative \( u \in U \).
3. **Step-3:** Using the rules of linguistic variables determine the corresponding Sinusoidal fuzzy soft set \((S, E)\).
4. **Step-4:** Normalize the Sinusoidal fuzzy soft set using the ranking of sinusoidal fuzzy number and determine the \((S', E)\).
5. **Step-5:** Determine the weighted normalized table of the Sinusoidal fuzzy soft set \((S'', E)\).
6. **Step-6:** Determine the total weighted choice value of an object
7. **Step-7:** Choose the optimal object(s) \( J_i, i=1,2,...,n \), whose choice value are maximum.

**4.2 Numerical Example:**

Considering Example 1, suppose if Mr. John decides to buy a jewel from a jewellery shop, considering 5 attributes: \( e_1 \) (less wastage), \( e_2 \) (beautiful/variety of unique designs), \( e_3 \) (Gold purity) \( e_4 \) (the famous certified shop) and \( e_5 \) (the lucky shop). He has taken a survey with the customers and the people located in his area and listed the top 5 jewel shops as \( J_1, J_2, J_3, J_4, J_5 \). His aim is to choose the best shop among the five-jewellery shops. To solve this problem, we undergo three different cases as given below to arrive at the best alternative.
Case: 1:

In the first case, we use the classical fuzzy soft sets to compare the effectiveness of the alternatives. Thus, a fuzzy soft set \((\mathcal{F}, A)\) is obtained from the linguistic variables, which has been rated by the customers as shown in Table 3, then its choice value is calculated by:

\[ c_i = \sum_{j=1}^{m} J_{ij} \]

Where \(J_{ij}\) is the fuzzy number of objects \(J_i\) on attribute \(e_j\) in the fuzzy soft set \((\mathcal{S}, A)\), \(1 \leq i \leq n, 1 \leq j \leq m\). The fuzzy soft set \((\mathcal{S}, A)\) and choice values \(c_i\) is shown in Table 4.2.1.

| \(U\) | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop | Choice values |
|-------|--------------|-----------|-------------|-------------|------------|--------------|
| \(J_1\) | 0.2 | 0.8 | 0.3 | 0.9 | 0.9 | 3.1 |
| \(J_2\) | 0.8 | 0.3 | 0.3 | 0.4 | 0.2 | 2 |
| \(J_3\) | 0.4 | 0.9 | 0.5 | 0.7 | 0.4 | 2.9 |
| \(J_4\) | 0.9 | 0.2 | 0.7 | 0.3 | 0.3 | 2.4 |
| \(J_5\) | 0.1 | 0.4 | 0.8 | 0.8 | 0.3 | 2.4 |

Table 4.2.1 Classical Fuzzy soft set

From the table, it has been observed that \(J_1\) could be selected as the optimal alternative. Here the order of the alternatives is as follows: \(J_1 > J_2 > J_4 = J_5 > J_3\).

Case-2:

In this case, we solve the same problem based on the algorithm in which the Sinusoidal fuzzy soft sets can be applied into the same MCDM problem. Hence by the steps 1 and 2, fuzzy soft set and its attributes with respect to the customers rating for each criterion is determined.

Then the linguistic variables have been transformed into Sinusoidal fuzzy numbers to construct the Sinusoidal fuzzy soft set \((\mathcal{S}, \mathcal{E})\). It is represented as shown in Table 4.2.2.

| \(U\) | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop |
|-------|--------------|-----------|-------------|-------------|------------|
| \(J_1\) | (0.0,0.1,0.2,0.25,0.3) | (0.64,0.7,0.8,0.86,0.9) | (0.1,0.2,0.3,0.35,0.4) | (0.7,0.8,0.9,0.9,0.98) | (0.7,0.8,0.9,0.95,1) |
| \(J_2\) | (0.6,0.7,0.8,0.85,0.9) | (0.15,0.2,0.3,0.35,0.44) | (0.1,0.2,0.3,0.4,0.45) | (0.2,0.3,0.4,0.5,0.6) | (0.01,0.2,0.3,0.4) |
| \(J_3\) | (0.2,0.3,0.4,0.45,0.5) | (0.7,0.8,0.9,0.95,1) | (0.3,0.4,0.5,0.6,0.65) | (0.5,0.6,0.7,0.8,0.9) | (0.2,0.3,0.4,0.5,0.6) |
| \(J_4\) | (0.7,0.8,0.9,0.95,1) | (0.1,0.15,0.2,0.25,0.35) | (0.5,0.6,0.7,0.8,0.9) | (0.1,0.2,0.3,0.4,0.5) | (0.1,0.2,0.3,0.4,0.5) |
| \(J_5\) | (0,0.05,0.1,0.16,0.2) | (0.2,0.3,0.4,0.5,0.6) | (0.6,0.7,0.8,0.85,0.9) | (0.6,0.7,0.8,0.85,0.9) | (0.01,0.1,0.2,0.3,0.4) |
| \(W_j\) | 0.7 | 0.6 | 0.8 | 0.4 | 0.3 |

Table 4.2.2 (Weighted Sinusoidal Fuzzy soft set)

Finally, the decision making has been done based on the selection of the maximum value of the optimal alternatives by step 7, i.e., we can obtain \(J_2 > J_4 > J_1 > J_5 > J_3\). Thus from this, we can conclude that Mr. John can choose the Jewel shop \(J_3\) as the best choice which has the maximum choice value.
**Case-3:**

In the previous case we have taken the weights as a crisp value, suppose if the weights taken in the normalization of Sinusoidal fuzzy soft is again a Sinusoidal fuzzy soft then let us determine the best alternative. This is done by considering the two different operations of Sinusoidal fuzzy soft sets.

By considering the same problem as shown in the above case 2, we take two sets of normalized weights assigned by the two different sets of customers for each of the parameter as a sinusoidal fuzzy number are shown below,

Let

\[
\begin{align*}
  w_j &= \left( \frac{w_1}{0.0, 0.05, 0.1, 0.16, 0.2}, \frac{w_2}{0.2, 0.3, 0.4, 0.5, 0.6}, \frac{w_3}{0.6, 0.7, 0.8, 0.85, 0.9}, \frac{w_4}{0.7, 0.8, 0.9, 0.95, 1}, \frac{w_5}{0.7, 0.8, 0.9, 0.95, 1} \right) \\
  w'_j &= \left( \frac{w'_1}{0.4, 0.45, 0.5, 0.55, 0.6}, \frac{w'_2}{0.1, 0.2, 0.3, 0.4, 0.5}, \frac{w'_3}{0.7, 0.8, 0.9, 0.95, 1}, \frac{w'_4}{0.4, 0.45, 0.5, 0.55, 0.6}, \frac{w'_5}{0.2, 0.3, 0.4, 0.45, 0.5} \right)
\end{align*}
\]

be the two set of weights assigned by different set of customers for each parameter.

Here by the definitions 3.5 and 3.6, we make use of the operations of Sinusoidal fuzzy soft set to get the most approximate weight as shown in the table 4.2.4 and 4.2.5. It has been transformed into a sinusoidal fuzzy soft crisp weight by the definition 4.1 and the problem has been solved by 4.2. Then by the step 6 and 7 the maximum choice value is obtained, and the best alternative results has been discussed as shown in the tables 4.2.6 and 4.2.7.

| J_1 | 0.175 | 0.732 | 0.275 | 0.86 | 0.875 | 1.3882 |
| J_2 | 0.775 | 0.286 | 0.28375 | 0.4 | 0.2 | 1.1611 |
| J_3 | 0.375 | 0.875 | 0.49375 | 0.7 | 0.4 | 1.5825 |
| J_4 | 0.875 | 0.20625 | 0.7 | 0.3 | 0.3 | 1.50625 |
| J_5 | 0.1025 | 0.4 | 0.775 | 0.775 | 0.2 | 1.30175 |
| w_j | 0.7 | 0.6 | 0.8 | 0.4 | 0.3 | 

**Table 4.2.3 (Optimal solution)**

| U | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop |
|---|---|---|---|---|---|
| J_1 | (0.0, 0.1, 0.2, 0.25, 0.3) | (0.64, 0.7, 0.8, 0.86, 0.9) | (0.1, 0.2, 0.3, 0.35, 0.4) | (0.7, 0.8, 0.9, 0.9, 0.98) | (0.7, 0.8, 0.9, 0.9) |
| J_2 | (0.6, 0.7, 0.8, 0.85, 0.9) | (0.15, 0.2, 0.3, 0.35, 0.44) | (0.1, 0.2, 0.3, 0.4, 0.45) | (0.2, 0.3, 0.4, 0.5, 0.6) | (0.1, 0.1, 0.2, 0.3, 0.4) |
| J_3 | (0.2, 0.3, 0.4, 0.45, 0.5) | (0.7, 0.8, 0.9, 0.95, 1) | (0.3, 0.4, 0.5, 0.6, 0.65) | (0.5, 0.6, 0.7, 0.8, 0.9) | (0.2, 0.3, 0.4, 0.5, 0.6) |
| J_4 | (0.7, 0.8, 0.9, 0.95, 1) | (0.1, 0.15, 0.2, 0.25, 0.35) | (0.5, 0.6, 0.7, 0.8, 0.9) | (0.1, 0.2, 0.3, 0.4, 0.5) | (0.1, 0.2, 0.3, 0.4, 0.5) |
| J_5 | (0.05, 0.1, 0.16, 0.2) | (0.2, 0.3, 0.4, 0.5, 0.6) | (0.6, 0.7, 0.8, 0.85, 0.9) | (0.6, 0.7, 0.8, 0.85, 0.9) | (0.1, 0.2, 0.3, 0.4, 0.5) |
| w_j v w'_j | (0.6, 0.7, 0.8, 0.85, 0.9) | (0.4, 0.5, 0.6, 0.7, 0.8) | (0.7, 0.8, 0.85, 0.95, 1) | (0.4, 0.45, 0.5, 0.55, 0.6) | (0.2, 0.3, 0.4, 0.4) |

**Table 4.2.4: (OR operation on SFSS)**
### Table 4.2.5: (AND operation on SFSS)

| U    | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop | Optimal Max Value |
|------|--------------|-----------|-------------|-------------|------------|-------------------|
| J₁   | 0.175        | 0.732     | 0.275       | 0.86        | 0.875      | 1.58016           |
| J₂   | 0.775        | 0.286     | 0.28375     | 0.4         | 0.2        | 1.2994            |
| J₃   | 0.375        | 0.875     | 0.49375     | 0.7         | 0.4        | 1.7543            |
| J₄   | 0.875        | 0.20625   | 0.7         | 0.3         | 0.3        | 1.6751            |
| J₅   | 0.1025       | 0.4       | 0.775       | 0.775       | 0.2        | 1.4558            |
| wᵢ  ∨ wⱼ | 0.78       | 0.6125    | 0.8625      | 0.5         | 0.375      |                   |

### Table 4.2.6: Optimal value (OR operation on SFSS)

| U    | Less wastage | Beautiful | Gold Purity | Famous Shop | Lucky shop | Optimal Max Value |
|------|--------------|-----------|-------------|-------------|------------|-------------------|
| J₁   | 0.17         | 0.732     | 0.275       | 0.86        | 0.875      | 1.0742            |
| J₂   | 0.775        | 0.286     | 0.28375     | 0.4         | 0.2        | 0.88485           |
| J₃   | 0.375        | 0.875     | 0.49375     | 0.7         | 0.4        | 1.267188          |
| J₄   | 0.875        | 0.20625   | 0.7         | 0.3         | 0.3        | 1.17711           |
| J₅   | 0.1025       | 0.4       | 0.775       | 0.775       | 0.2        | 1.0526            |
| wᵢ  ∨ wⱼ | 0.5         | 0.5375    | 0.7         | 0.2625      | 0.2        |                   |

### Table 4.2.7: Optimal value (AND operation on SFSS)

It has been found that the maximum choice value obtained by the maximum operator and the minimum operator of the SFSS is at the J₃. Hence the decision maker Mr. John gets the closest approximation to choose the best alternative which is J₃.

### 5. Discussion:

In the above section fuzzy soft sets are taken in three different cases, in case -1 the numerical problem has been solved by classical fuzzy soft set method and has found that the best alternative as J₁ but in the ranking order of the alternatives it was found that two of the objects are equal such as J₄ = J₅, which seems to be there is some ambiguity in the earlier method. So, to get the more approximated closeness results we have converted the linguistic variables into a sinusoidal fuzzy soft sets into the same MCDM problem and the results in case 2 shows that the two objects are not equal and J₄ is better than J₅, according to the decision-making algorithm based on Sinusoidal fuzzy soft sets. Moreover, the best alternative has been changed to J₁, which is comparatively maximum. This result is different from the results obtained in traditional fuzzy soft set method. To improvise the results in case 3 some operations have been introduced in weights and the decision-making algorithm has been applied in the same numerical problem. Later it was found that there no change in the results which we obtained from case 2, i.e., J₁ holds to be maximum where J₃ > J₄ > J₁ > J₅ > J₂. This result shows that the decision making based on sinusoidal fuzzy soft sets gives preferable to react with reality. Hence this method is extremely helpful to deal with MCDM problems.

### 6. Conclusions:

In this uncertain environment Linguistic assessments plays a vital role in many practical problems. In this paper a new concept has been introduced by combining the fuzzy soft sets and membership function of the Sinusoidal fuzzy number as a Sinusoidal fuzzy soft set which can deal with certain linguistic assessments. Some of the operations of Sinusoidal fuzzy soft sets are also defined with an example in this paper to deal with some vague linguistic variables effectively. Then a new algorithmic approach of transforming the linguistic variables of Sinusoidal fuzzy soft sets into a MCDM problem has been proposed and a case study with a numerical example has been briefly explained. As the
results shown above gives more closest approximation of the attributes for the decision makers. Hence this method helps the decision makers preferably to solve various real life practical problems like Signal processing, Image processing etc and choose the best out of it in future.

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