A Momentous Arrow of Time*

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Abstract

Quantum cosmology offers a unique stage to address questions of time related to its underlying (and perhaps truly quantum dynamical) meaning as well as its origin. Some of these issues can be analyzed with a general scheme of quantum cosmology, others are best seen in loop quantum cosmology. The latter's status is still incomplete, and so no full scenario has yet emerged. Nevertheless, using properties that have a potential of pervading more complicated and realistic models, a vague picture shall be sketched here. It suggests the possibility of deriving a beginning within a beginningless theory, by applying cosmic forgetfulness to an early history of the universe.

1 Introduction

Time in quantum theory is something of a black sheep. In non-relativistic quantum mechanics it remains a classical parameter labelling the evolution of states, but is not allowed to fluctuate as position does. Even in relativistic systems and quantum field theory, time often appears as a disciplined parameter trained to order events, much as it is used in classical physics. Crucial for particle physics is the direction time provides for interaction events scattering initial states into final ones. But any directedness is simply put into the formalism. At the level of elementary reactions, time knows no order: if a reversal of time were allowed, events would still occur in any way, nearly unchanged. It is only our choice of initial and final states which determines a scattering amplitude.

All this is different on the macroscopic level and especially in cosmology. Here, structures change with a trend. One often thinks of a simple initial state evolving into complexity, a puzzle to be explained by an arrow of time. If this is to be derived rather than postulated, a theory of initial states is required.

The main part of this contribution will be an exploration of the possibility that true quantum degrees of freedom, those such as fluctuations which completely lack a classical analog, could play a role of or for time. In this way, we will take seriously quantum space-time, not (just) as a new and possibly discrete structure but as a fully dynamical

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The laws are, of course, not completely time reflection symmetric, which might be exploited in the context discussed here.†
quantum entity. More specifically, cosmological models will lead us to an analysis of quantum correlations as quantities changing with a trend. If consistently realized, such a perspective is very different from the traditional ones regarding time: time would be inherently quantum; it would not exist in a classical world. In semiclassical physics, it remains only as a shadow of the quantum physics that lies beneath.

We will take advantage of a useful description of quantum dynamics (sketched in the Appendix) based on the evolution of characteristic quantum variables, rather than whole but partially redundant wave functions. The same kind of description can be used to explore the nature of non-singular big bangs. Such events, while still playing the role of the moment of commencement for the part of the universe accessible to us, can no longer be viewed as entire initial states of the universe: with the singularity being resolved, there is a universe before the big bang. But specific realizations of such scenarios do have derived features of special initial states as they may be posed at the big bang. In this way, dynamical properties give insights into the question of initial states and the directed evolution that ensues. Especially the phenomenon of cosmic forgetfulness shows that much of the state before the big bang remains hidden after the big bang. Without remembrance, the arrow of time might well be considered blunt — or do we just see the blunt end of a reversed arrow?

By its nature, our analysis will be incomplete and preliminary. No clear scenario emerges yet; just several indications exist. But they may show that the topics touched here are still worth pursuing.

2 The problem and the arrow of time

Many questions are to be addressed in the context of time. The most important one is, of course, the aptly named problem of time [2]. It arises mainly in canonical formulations of gravity and attempted quantizations, but its nature reaches farther. Independently of technical aspects, it is about the question whether there is an unambiguous degree of freedom in generally relativistic theories which can play the role of time, or of a parameter whose values arrange causally related events and thus separate the past, present and future.

This is to be distinguished from the question of the arrow of time [3], which irreversibly orders events already separated into past, present and future by the time variable. Such an arrow is often related to thermodynamical questions via entropy, or to the selection of special initial states in quantum cosmology. The question of the arrow of time builds on an existing time variable and is thus to be separated from the problem of time, which is more basic. In this contribution, we start with a discussion of the problem of time.

The problem initially arose in canonical quantizations of gravity, with a dynamics governed only by a Hamiltonian constraint, not by a true Hamiltonian. Thus, quantum states do not seem to evolve, quite obviously in conflict with the perception of change. Without

\footnote{It is interesting to note that the problem of time and motion becomes pressing when quantum gravity is considered. Quantum gravity is often tied to another expectation, that of discreteness or an atomic nature of space-time. Maybe solving the problem of time would lead us to establishing an atomic nature of
any notion of space-time coordinates in canonical quantizations of gravity, which provide operators for geometrical quantities derived from the space-time metric but nothing for coordinates, the usual way out by coordinatizing time is blocked. One is forced to identify an appropriate time degree of freedom from the physical variables, such as geometrical ones or matter fields. The problem is that none of them seems to be a globally valid choice for time as an unambiguous labelling of events.

While this problem becomes technical and pressing in canonical quantum gravity, it is more general as well as deeper than might be indicated at first sight. If we were able to identify a global time variable from the physical degrees of freedom, we would be led to attributing a new physical meaning to time. Time would cease to be a conventional description of observed change and become a physical quantity on par with all others. It would be subject to physical laws, and would fluctuate in quantum theories. In that case, one might as well look for a global time variable among the true quantum degrees of freedom of a relativistic system, a degree of freedom such as quantum fluctuations or correlations without a classical analog. From a dynamical systems perspective, these are degrees of freedom in their own right. (Such variables do play a special role from the perspective of quantum observables since they are not obtained through expectation values of one linear operator. But quantum fluctuations, for instance, are certainly measurable in the same statistical sense as expectation values.) The fundamental notion of time would crucially be tied to quantum physics, while classical physics would have to resort to time coordinates, a poor substitute for a truly deep notion.

Several indications exist for the squeezing of quantum matter [4, 5, 6, 7] or gravitational waves [8] to play the role of time. Here, following suggestions in [9, 10] we describe results indicating an emergent concept of time in a quantum description of universe models. If these models and ideas are correct, the quantity ultimately playing the role of time is not the one put in initially to set up the evolution equations, and it is not the one used in a classical description. This quantity, the true nature of time in the picture proposed, does not at all exist in the classical theory. So far, these considerations are inconclusive concerning the problem of time. But the methods will set the stage for a discussion of the arrow of time.

3 Classical Dynamics

We start with a cosmological system where the problem of time is solved trivially: a model sourced by a free, massless scalar field $\phi$. Thanks to the absence of any non-trivial potential, the value of the scalar is monotonic in any time coordinate and thus can itself be used as

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3A different perspective on the importance of gravitational degrees of freedom is discussed in [11].
time. While these models are rather simple, some exactly solvable versions provide a basis for a much more general analysis.

With a free, massless scalar as the sole matter content of an isotropic universe with cosmological constant $\Lambda$, the expansion history, for the different choices of spatial curvature via $k = 0$ or $k = \pm 1$, is determined by the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{4\pi G p_{\phi}^2}{3} - \frac{1}{a^6} + \Lambda$$

for the scale factor $a$. The dot denotes a derivative by proper time, leading to the Hubble parameter $\dot{a}/a$. The coupling to matter is quantified by the gravitational constant $G$, multiplying the energy density of matter. Here, for a free, massless scalar, only kinetic energy is contributed via the momentum $p_{\phi} = a^3 \dot{\phi}$. In what follows, we will use $k = 0$ and $\Lambda < 0$ to be specific, though not realistic. (The case of a positive cosmological constant is very similar to the negative sign as far as classical dynamics is concerned, but is much more subtle at the quantum level. One can find hints of this subtlety in the existence of different self-adjoint extensions of the quantum Hamiltonian [12] or in the dynamical behavior of quantum states.)

To employ canonical quantization later on, we now introduce the classical canonical formulation. Choosing the (rescaled) volume $V = a^3 / 4\pi G$ as configuration variable, it follows from the Einstein–Hilbert action that its momentum is $P = \dot{a}/a$: we have the Poisson bracket $\{V, P\} = 1$. In these variables, the Friedmann equation takes the form

$$C := (P^2 + |\Lambda|)V^2 - \frac{1}{12\pi G} p_{\phi}^2 = 0 \quad (1)$$

of a constraint rather than an equation of motion. A Wheeler–DeWitt quantization [13] would turn this expression into an operator $\hat{C}$ — for instance in the volume representation where wave functions are of the form $\psi(V, \phi)$ and $\hat{P}$ acts as $-i\hbar \partial / \partial V$ while $p_{\phi}$ acts as $-i\hbar \partial / \partial \phi$, with Planck’s constant $\hbar$ — and solve the state equation $\hat{C}\psi = 0$. Compared to the Schrödinger equation, time is absent and change would have to be recovered indirectly from the solution space.

What must be absent is time coordinates since they have no role in a quantum theory of gravity, not based on classical space-time manifolds. But other, more physical time parameters may well and should indeed exist. Realizing this is facilitated by eliminating time coordinates already at the classical level, and finding an alternative formulation of classical evolution. To do so, we write equations of motion for $V$ and $P$ with respect to the scalar $\phi$. Such equations can be obtained by dividing equations of motion with respect to coordinate time, such as $dV / d\phi = \dot{V} / \dot{\phi}$. But any reference to coordinate times can be avoided altogether if we solve the Friedmann equation for the momentum

$$p_{\phi} = \pm 2\sqrt{3\pi G V} \sqrt{P^2 + |\Lambda|} =: H(V, P) \quad (2)$$

and take $H(V, P)$ as the Hamiltonian for evolution in $\phi$. (We will choose the $+$-sign in what follows, letting $\phi$ run along with coordinate time.) The Hamiltonian equation of
motion \( \frac{dO}{d\phi} = \{O, H\} = \frac{\partial O}{\partial V} \cdot \frac{\partial H}{\partial P} - \frac{\partial O}{\partial P} \cdot \frac{\partial H}{\partial V} \) for any function \( O \) of \( V \) and \( P \) then equals what we would obtain from dividing coordinate equations of motion.

The case \( \Lambda = 0 \) is particularly simple. It provides a quadratic Hamiltonian \( H \propto |VP| \) and thus constitutes an example of harmonic cosmology \([14, 15]\). Just as the harmonic oscillator in mechanics, it leads to an exactly solvable quantum system — not just in the sense that solutions can be found in closed form, but with the much stronger property that no quantum back-reaction occurs. The evolution of expectation values is entirely unaffected by changing shapes of a state. In the next section we will see what that entails for dynamics, and how perturbation theory can be used to step from the exactly solvable model to more realistic cases as they are obtained for \( \Lambda \neq 0 \) or with a non-trivial matter potential.

4 Quantum Dynamics

We now turn to the quantum dynamics of our systems. A quantum system is characterized by the presence of additional, non-classical degrees of freedom which can change independently of the classical variables, given by \( V \) and \( P \) above. While the latter can be brought in correspondence with expectation values \( \langle \hat{V} \rangle \) and \( \langle \hat{P} \rangle \) in a quantum state, a whole wave function (or density matrix) contains much more information. Indeed, while classical phase space functions \( f(V, P) \) are merely combinations of the canonical coordinates and are fully determined if only a phase space point is specified, products of operators in a quantum system provide independent kinds of information. In general, for instance, \( \langle \hat{V}^2 \rangle \) can take values irrespective of what the value of \( \langle \hat{V} \rangle^2 \) is. The difference \( (\Delta V)^2 = \langle (\hat{V} - \langle \hat{V} \rangle)^2 \rangle \) is a measure for quantum fluctuations, an important quantity in a quantum system. Similarly, all moments

\[
G_{v \ldots v}^{p \ldots p} = \langle (\hat{V} - \langle \hat{V} \rangle)^a (\hat{P} - \langle \hat{P} \rangle)^b \rangle_{\text{Weyl}}
\]

(3)
defined for \( a + b \geq 2 \) are independent parameters of a (density) state. (The subscript “Weyl” indicates that operator products are ordered totally symmetrically before taking the expectation value.) As discussed in the Appendix, these moments are all dynamical, forming an infinite-dimensional coupled system. Solutions tell us how expectation values of a state behave, but also how the state and its moments evolve. Fig. 1 shows the example of a quantum cosmological state during a recollapse, with the spreads changing characteristically.

From these moments we will attempt to form an arrow of time.

4.1 Monotonicity

When a quantum state changes, its moments change. Just like the expectation values, the moments must satisfy equations of motion which, as derived in the Appendix, follow from the Hamiltonian. Having equations of motion for the second order moments, we can check if any one of them would serve as a good time coordinate \([10]\). Since quantum states
Figure 1: Dispersing wave at the recollapse of a Friedmann–Robertson–Walker model with $k = 0$ and $\Lambda < 0$. The top and bottom curves indicate changing spreads $\Delta V$ around the central expectation value trajectory of volume $V$ as a function of the scalar $\phi$. The latter serves as a measure for time in this free, massless case. For the solid lines, the state is unsqueezed, without quantum correlations, at the recollapse point, and fluctuations symmetric around the recollapse result. Non-vanishing correlations (dashed lines), on the other hand, lead to non-symmetric fluctuations.
tend to spread out, one may expect fluctuations to have an interpretation of internal time. For $G^{PP}$, however, this is clearly not the case since its change depends on the sign of curvature $\langle \hat{P}_i \rangle$. It would decrease in an expanding universe but increase in a collapsing one. Around a recollapse or a bounce, this behavior cannot be monotonic. Similarly, the sign of the rate of change of volume fluctuations $G^{VV}$ is not unique from since neither $G^{VP}$ nor $\langle \hat{P}_i \rangle$ is required to have a definite sign throughout the history of a universe.

Of more interest for our purposes is the covariance, subject to

$$
d \frac{G^{VP}}{d \phi} = \frac{3}{2} \frac{|\Lambda|}{(|P|^2 + |\Lambda|)^{3/2}} G^{PP}
$$

from (17). With $G^{PP} = (\Delta P)^2$ required to be positive, the covariance can only grow. Thus, the positivity of fluctuations, or the uncertainty relation in even stronger form, implies a fixed tendency for correlations.

The monotonicity of $G^{VP}$ hints at a possible role in the context of time. In the model considered so far, it certainly does not improve the problem of time since it can anyway be solved trivially by using the scalar (with respect to which $G^{VP}$ now is monotonic). But if we have a look at models with a non-trivial scalar potential $W(\phi) \neq 0$, where $\phi$ would no longer serve as global time, one can see that $G^{VP}$ is better behaved than just $\phi$. In such a case, with a time-dependent potential in the formulation where $\phi$ plays the role of time, equations can be derived as before provided that the potential is not too large [16, 17].

The classical constraint (still for $\Lambda < 0$) now is

$$
\left( P^2 + |\Lambda| - \frac{8\pi G}{3} W(\phi) \right) V^2 - \frac{1}{12\pi G} p_{\phi}^2 = 0
$$

and effective equation of motion for the covariance changes to

$$
d \frac{G^{VP}}{d \phi} = \frac{3}{2} \frac{\langle \hat{V} \rangle |\Lambda| - 8\pi GW(\phi)/3}{(|\hat{P}|^2 + |\Lambda| - 8\pi GW(\phi)/3)^{3/2}} G^{PP}.
$$

(4)

For sufficiently small potentials, $G^{VP}$ is still monotonic for wide ranges of evolution. Also here, this refers to monotonicity with respect to $\phi$, which now is a good time variable only for finite stretches between turning points in the potential. If we approach a turning point of $\phi$, however, the behavior changes. At a turning point, $p_{\phi} = 0$ and thus $\langle \hat{P} \rangle^2 = -|\Lambda| + 8\pi GW(\phi)/3$ from the constraint. Near a turning point, $|\Lambda| - 8\pi GW(\phi)/3$, appearing in the numerator of (4), thus becomes negative. Even before the turning point of $\phi$ is reached, $G^{VP}$ according to (4) turns around.

Near a turning point the potential is important and there may be extra terms in the quantum equations of motion. Our analysis at this stage remains incomplete, but it suggests a situation as follows. As a global time variable through periods of oscillation of $\phi$, $G^{VP}$ appears no better than the scalar. But it is monotonic in a range around the turning point and can thus be used as local internal time, to which we may transform from $\phi$ (or other time choices) when a turning point is approached. Thus, it would extend its role of time into a wider region. As a quantum variable without classical analog, this at least suggests that time in a fully relativistic situation can be assisted by quantum aspects.
4.2 Before the big bang

So far, we have discussed only low-curvature regimes where \( P \ll 1 \). At larger curvature, new effects from quantum gravity and quantum geometry are expected to take over which are not included in the Wheeler–DeWitt quantization \([13]\) understood up to now. Loop quantum cosmology \([18]\) is one such candidate for an extension, and one of its effects is to provide higher order terms to the Friedmann equation. Its new form then is

\[
\frac{\sin^2(\mu P)}{\mu^2} = \frac{1}{12\pi G} \frac{p_\phi^2}{V^2}
\]

where \( \mu \) is a length scale (see, e.g., \([19]\)).

Such higher order terms of \( P \) or \( \dot{a} \) are expected in quantum gravity if we realize the Friedmann equation as the time-time component of Einstein’s tensorial equation. Higher curvature corrections change the action, and thus the Einstein tensor. Correspondingly, the Friedmann equation is amended by higher order terms. (The same reasoning would suggest higher derivative terms, too, which generically are also present. We will, however, be dealing with a solvable model of a free, massless scalar where they are absent \([14, 15]\).) With this analogy, the expansion parameter \( \mu \) is the same as the one multiplying higher curvature terms, and thus should indeed be dimensionfull. One may think of it as being near the Planck length, which is in fact often assumed. But in loop quantum gravity, it has a dynamical origin related to the discreteness of an underlying quantum gravity state \([20, 21]\). Generically, \( \mu \) changes as the universe expands or contracts and cannot always be close to the Planck length. In fact, if it were, other corrections (from inverse scale factor terms \([22]\), based on \([23]\)) would have to be considered as independent quantum corrections, which we avoid here.

The form of the higher order terms, obtained by expanding \( \sin(\mu P) \) by powers of \( P \) when curvature is small, as well as the length scale \( \mu \) determining when quantum corrections become important, is not fixed. It may be constrained further by relating such a Hamiltonian to one that is formulated in the full theory, without any symmetry assumptions. But this has currently not been achieved, and so the precise form remains subject to quantization ambiguities. What we discuss in what follows only involves generic qualitative features which depend on some crucial aspects of the loop quantization but not on the specific form. As an important effect we include lattice refinement, leading to a possible \( V \)-dependence of the parameter \( \mu \): the characteristic length scale where discreteness effects happen might depend on the volume and change dynamically \([20]\). Conceptual \([21]\) as well as phenomenological constraints \([25, 26]\) on the dependence exist, and it is clear that \( \mu \) cannot be \( V \)-independent in all models \([27]\); but in no case has it been fixed uniquely. A power-law dependence of \( \mu \propto V^\kappa \) on \( V \), which can realistically describe at least bounded ranges of evolution, can be taken into account by a canonical transformation \( P \mapsto V^\kappa P, \quad V \mapsto V^{1-\kappa}/(1-\kappa) \) which will not change the following results.

Now, the Hamiltonian for \( \phi \)-evolution is not quadratic in \( V \) and \( P \) even for \( k = 0 = \Lambda \), suggesting non-perturbative effects at strong curvature \( P \gg 1/\mu \). Fortunately, the system is “resummable” \([14]\): it is solvable and free of quantum back-reaction if we use the variables
$V, J = V \exp(i\mu P)$ instead of canonical ones. These variables satisfy a linear Poisson algebra
\[
\{V, J\} = i\mu J, \quad \{V, \bar{J}\} = i\mu \bar{J}, \quad \{J, \bar{J}\} = -2i\mu V
\]
and they provide the basis for solvability even at the dynamical level. In fact, the Hamiltonian for $\phi$-evolution, solving (5), is
\[
\dot{\langle V \rangle} = \alpha \cosh(2\sqrt{3\pi G} \phi) + \beta \sinh(2\sqrt{3\pi G} \phi)
\]
with constants of integration $\alpha$ and $\beta$ to be fixed by initial values. Using (7), we then obtain
\[
\text{Re} \langle \dot{J} \rangle(\phi) = \frac{1}{2\sqrt{3\pi G}} \frac{dV}{d\phi} = \alpha \sinh(2\sqrt{3\pi G} \phi) + \beta \cosh(2\sqrt{3\pi G} \phi).
\]
The imaginary part of $\langle \dot{J} \rangle$ is fixed to be
\[
\text{Im} \langle \dot{J} \rangle(\phi) = \langle V \sin(\mu P) \rangle = \frac{\mu}{2\sqrt{3\pi G}} p_\phi
\]
by the preserved $\phi$-Hamiltonian, using (5).

The constants of integration $\alpha$ and $\beta$ determine whether or not $\langle \dot{V} \rangle$ can reach zero, where a singularity would occur. Due to reality conditions, these constants are not arbitrary: classically we have $|J|^2 - V^2 = 0$, which is to be imposed as an operator equation $\hat{J}\hat{J}^\dagger - \hat{V}^2 = 0$ after quantization. (Otherwise the curvature parameter $P$ would not become self-adjoint and physical states obtained by solving the evolution equations would not be correctly normalized.) Since the reality condition is quadratic, it implies
\[
0 = \langle \dot{J}\hat{J}^\dagger - \hat{V}^2 \rangle = \langle \dot{J}\rangle\langle \hat{J}^\dagger \rangle - \langle \dot{V}^2 \rangle + G^{JJ} - G^{VV} + \mu \langle \hat{V} \rangle
\]
with extra terms from fluctuations. (The last term arises from ordering $\hat{J}\hat{J}^\dagger$ symmetrically.) A state which is semiclassical at a given time has fluctuations of the order $O(\hbar)$, such that the reality condition takes the classical form up to small terms of order $\hbar$. Then, our dynamical solutions must satisfy
\[
(\text{Re} \langle \dot{J} \rangle)^2 + (\text{Im} \langle \dot{J} \rangle)^2 - \langle \dot{V} \rangle^2 = -\alpha^2 + \beta^2 + \frac{\mu^2}{12\pi G} p_\phi^2 = O(\hbar)
\]
Figure 2: Dispersing through a bounce. Here, the volume $V(\phi)$ as a function of the scalar, again indicating time, is shown for a bounce rather than a recollapse as in Fig. 1. As before, the top and bottom curves indicate fluctuations $\Delta V$ around the expectation value $V$ — solid curves for a state uncorrelated at the bounce, dashed curves for a correlated one. Fluctuations “before” the big bang may have been quite different from what they are “afterwards” — see also Eq. (11) — to a degree that can be considered forgetful.

which determines $\beta$ in terms of $\alpha$. With $V_{\text{min}} := \mu p/12\pi G$ and the identity $B \cosh(x + \cosh^{-1}(A/B)) = A \cosh(x) + \sqrt{A^2 - B^2} \sinh(x)$ for arbitrary $A$ and $B$, the volume is

$$\langle \hat{V} \rangle(\phi) = V_{\text{min}} \cosh(2\sqrt{3\pi G\phi + \delta})$$

with $\delta = \cosh^{-1}(\alpha/V_{\text{min}})$. This function never becomes zero, proving that the model has a bounce but no singularity. At the bounce point, the density of the scalar field takes the value

$$\rho_{\text{crit}} = \frac{p^2}{2a^8} = \frac{p^2}{32\pi^2 G^2 V_{\text{min}}^2} = \frac{3}{8\pi G \mu^2}$$

which depends on the scale $\mu$ but is independent of any initial condition. (The same behavior initially arose from numerical studies [28].)

To evaluate the reality condition, we have used semiclassicality. One might worry that this invalidates conclusions about the bounce, typically expected to occur in a highly quantum regime. However, we had to make assumptions about semiclassicality only at one time, which can be arbitrarily far away from the bounce. We only need $G^{JJ} - G^{VV} = O(\hbar)$ throughout, which is at first ensured by an initial condition at large volume. As the state evolves, it may become more quantum. But from equations of motion for the moments it
follows that $G^{JJ} - G^{VV}$ is a constant of motion \[15\], even if the state spreads, making $G^{VV}$ change. Thus, if this combination is of the order $\hbar$ once, it will remain so. In this solvable model the bounce is realized even for states which may not be semiclassical at the bounce.

The high control in this solvable model persists at the state level. Dispersions as well as squeezing can be followed for general states, as well as specifically for the moments of dynamical coherent states; see Fig. 2 for examples. In principle, we could thus test how covariances evolve and whether they remain monotonic. However, moments with easy access are now those of $V$ and $J$, not $P$. Volume fluctuations thus can easily be studied, but the covariance $G^{VP}$ of our earlier interest would, with $P$ related non-linearly to $J$, be a complicated expression in terms of all the moments involving $V$ and $J$. Nevertheless, we can find approximate information about the behavior of covariance. Near the bounce, we have $\mu P \sim \pi/2$ for $\sin(\mu P)$ and thus the scalar density to be close to its maximum. This allows us to use the approximation

$$\text{Re}(\langle \hat{J} \rangle) = \frac{1}{2} \langle \hat{V} e^{i\mu P} + e^{-i\mu P} \hat{V} \rangle \approx \frac{1}{2} \langle e^{i\pi/2} \hat{V} i(\mu \hat{P} - \pi/2) - e^{-i\pi/2} i(\mu \hat{P} - \pi/2) \hat{V} \rangle$$

by Taylor expansion around $\mu P \sim \pi/2$. Noting that

$$\text{Re}(\langle \hat{J} \rangle) = V_{\text{min}} \sinh(2\sqrt{3\pi G\phi} + \delta)$$

from (7) and (9) is monotonic in $\phi$, we are led to suggest that also the covariance on the right hand side is monotonic. Combining all conclusions, it will thus be a good measure for time through several cosmological phases, including recollapses and bounces.

5 Beyond Exactitude

So far, we have considered a free, exactly solvable model to shine some light on the universe at small volume. Such models rarely give the full picture of a physical situation they may be applied to. There are several additional ingredients to be required for a physically reliable analysis of a whole universe through and before the big bang, most importantly inhomogeneous configurations. No general description of inhomogeneities is available around bounce regimes in loop quantum cosmology, not even in perturbative form.

A crucial issue is that of the consistency of higher order terms, such as those appearing in (5), in a context which is no longer homogeneous. Then, the full anomaly issue strikes and modifications to the classical constraints are highly restricted: it is not easy to implement quantum corrections while still maintaining the same level of general covariance as it is realized classically. If covariance transformations are broken, the theory will be anomalous and inconsistent; such transformations could only be deformed by quantum corrections but must remain present in the same number. (Effective actions starting from quantum corrected isotropic equations have been determined \[29\] \[30\] \[31\]. But trying to embed a finite-dimensional model in a fully inhomogenous system is highly ambiguous, and so
quantum corrections for inhomogeneities remain unknown in the presence of higher order corrections such as \( (5) \); examples do, however, exist for special modes \([32, 33]\) or other effects of loop quantum gravity \([34, 35, 36, 37, 38]\).)

Consistency issues arise due to general covariance, which implies that one is dealing with a system of constraints, or an overdetermined set of equations. While there is only one, trivially consistent constraint \( (5) \) in isotropic models, several independent ones exist when geometries become inhomogeneous. Their algebra under Poisson brackets obeys certain conditions for the system to be well-defined, which must also be realized for the quantum representation. A possibility to sidestep the quantization of constraints is reduced phase space quantization, where one tries to find the classical solution space to all constraints and quantizes it. The usual problems are that (i) constraints may be difficult to solve completely and (ii) the solution space may be of complicated structure, for instance in its topological properties, and thus be difficult to quantize in its own right.

In the context of perturbative inhomogeneities, the first problem does not arise at least at the linear level since all gauge-invariant perturbations can easily be written down; see e.g. \([39, 40, 41, 42, 43, 44, 45]\). For linear perturbations, moreover, topological properties of solution spaces mostly disappear such that a reduced quantization here may be viable \([46, 47]\). Alas, it cannot present a full theory if it is simply added on to the bouncing background as treated so far, which was by the Dirac rather than the reduced phase space procedure. One may deal with the background dynamics also by reduced phase space techniques \([48, 49]\), but that would work easily only for a free, massless scalar trivializing the problem of time. By the Dirac procedure, on the other hand, the theory can be formulated for general interacting scalars \([50]\), even though it may be solved easily only in free scalar cases or perturbations around those.

In a reduced phase space quantization of perturbative inhomogeneities, no fully defined theory would be available. This may be acceptable if it can be seen as a valid approximation to some other full theory, but this is not the case. In fact, in systems not involving the bounce, where consistent quantizations of perturbative inhomogeneities in loop quantum gravity are available \([34]\), one can see that a reduced phase space quantization would overlook crucial effects. As shown in \([35]\), quantum corrections can induce effective anisotropic stress terms even in systems which classically have no anisotropic stress. A reduced phase space formulation based on the classical identities between gauge-invariant quantities could not see this new quantum effect, and thus must remain incomplete. Similarly, gauge-fixed treatments (as used e.g. in \([51, 52]\) for recent examples) often hide crucial quantum properties. (Also the more general reduced phase space formulation of \([44, 45]\) is subject to these remarks. Moreover, even in this reduced context, consistency conditions remain which are yet to be implemented in a possible quantization. While valuable, these formulations so far do not suffice to see how an inhomogeneous universe may evolve through a bounce.)

Such a situation makes the task of developing cosmological scenarios based on bounces difficult. But some indications can nonetheless be derived from models if they are understood for the local behavior of a patch of space-time near the moment of its smallest size. How different patches connect may be impossible to say in the absence of a fully inhomogeneous description, but the evolution of a single patch may still carry some surprises.
Concrete properties, such as the density when a patch bounces, may easily change or go away when a sufficiently general situation is considered. But in addition to such positive, affirmative properties there are negative ones which tell us about limitations of what can be said for early stages of the patch. Negative statements of this form are much more reliable, for if knowledge of something is constrained in a simple model, it is unlikely to become better known in a general situation.

There is such a negative property which, rather surprisingly, shows up even in the exactly solvable model [53]. It is not about classical variables, or the expectation values, but rather about quantum fluctuations or other moments. As before, we can derive equations of motion for all the moments, say of second order, forming a closed set of equations. There are several independent second order moments and their equations, with correspondingly many initial values to be chosen for a state. One can cut down the number of parameters by selecting dynamical coherent states: those that saturate the uncertainty relations at all times. The calculations are somewhat lengthy but can be completed [54], with the result that volume fluctuations at early and late times are related by

$$\Delta := \left| \lim_{\phi \to -\infty} G_{VV}^V \langle \hat{V} \rangle - \lim_{\phi \to \infty} G_{VV}^V \langle \hat{V} \rangle \right|$$

$$= 4 \frac{H}{V_{\text{min}}} \sqrt{\left(1 - \frac{H^2}{V_{\text{min}}^2} + \frac{1}{4} \frac{\hbar^2}{V_{\text{min}}^2}\right)^2 \frac{(\Delta H)^2}{V_{\text{min}}^2} - \frac{1}{4} \frac{\hbar^2}{V_{\text{min}}^2} + \left( \frac{H^2}{V_{\text{min}}^2} - 1 \right) \frac{(\Delta H)^4}{V_{\text{min}}^4}}$$

where $H$ is the expectation value of the $\phi$-Hamiltonian and $\Delta H$ its fluctuation. This parameterizes the behavior for all dynamical coherent states.

Of particular interest is the behavior when $H$ is large, which means that one would use the model for a patch containing a large amount of matter. As shown in Fig. 3, the asymmetry in such a case depends very sensitively on the parameters, for instance the ratio $V_{\text{min}}/H$. Moreover, its value can differ significantly from one; fluctuations of the state by no means have to remain unchanged when phases before and after the bounce are considered. There is a degree of cosmic forgetfulness [53]: due to the high sensitivity it is practically impossible to recover the full state before the bounce from its properties after the bounce.

6 An arrow of moments

In our discussion of the covariance, the big bang, resolved to a bounce, did not appear special in any way regarding the direction of time. It did not suggest a turn-around in the rate of change of the covariance. Had it done so, it would have led us to conclude that $G_{VP}$ cannot serve as a good time in that phase, rather than suggesting a flip of the arrow of time.

The role of moments concerning the arrow of time is more subtle. We will first reformulate the usual context to see how it may be related to evolving quantum states. One often says that what distinguishes the past from the future is that we remember the former and
Figure 3: Sensitivity: The asymmetry $\Delta$ of volume fluctuations from (11), depending on the ratio $V_{\text{min}}/H$ with $H$ the value of the scalar momentum as a measure for the amount of matter. Different curves correspond to various values of $H$, growing to the left. Thus, the steep leftmost curves are obtained for a universe with a large amount of matter, the more realistic scenario within the simple solvable models used here. The asymmetry (11) depends very sensitively on the initial values that determine $V_{\text{min}}/H$; unrealistically sensitive measurements would be required at one side of the bounce to determine the volume fluctuations at the other side. It is practically impossible to recover the complete state due to this cosmic forgetfulness [54].
try to predict the latter. It may be more honest to define the past as what we can (and typically do) forget. For human behavior, one of the most important and most annoying consequences of the arrow of time is indeed forgetfulness. In a more general sense, this is true also for thermodynamical systems, although it may not be so clear whether this is really annoying. A thermodynamical system evolving toward equilibrium forgets any sense of being special as it might have been encoded in its initial configuration. In quantum cosmology, even the whole universe has a case of cosmic forgetfulness which one may relate to the arrow of time.

To illustrate this, we return to the resummed solvable model of loop quantum gravity. Now considering its own moments for $V$ and $J$, we can look for all choices giving rise to dynamical coherent states: evolving states which saturate the uncertainty relations at all times. Such states provide the best control one may have on a quantum system, and thus highlight when anything becomes inaccessible — for instance by being forgotten. As already described, this is exactly what happens. Although one could not easily use the solvable model to draw strong conclusions about the universe before the big bang, what it tells us about limitations has to be taken seriously.

The kind of cosmic forgetfulness realized in this model provides an orientation of time, telling us not only which of the properties before the big bang can be forgotten, but also what direction "before the big bang" is. An observer after the bounce would be unable to reconstruct the full state before the bounce, but could easily predict the future development toward larger volume. This arrow agrees with the standard notion.

Now asking how an observer before the big bang would experience the same situation, the answer is also clear: such an observer would be unable to determine the precise state at larger values of $\phi$ beyond the bounce, but could easily extrapolate the state to smaller values of $\phi$. The state at smaller values of $\phi$ can be predicted, while the state at large values of $\phi$ is forgotten once the bounce is penetrated. Since one cannot forget the future, such an observer must be attributed a reversed arrow of time, pointing toward smaller $\phi$. At the bounce, two arrows would emerge pointing in opposite directions as far as $\phi$ is concerned. In this sense, the model resembles [58, 59, 60, 61].

While degrees of freedom propagating in a bouncing universe still have to be understood much better, indications do exist that what appears as a simple bounce in homogeneous models may have to be interpreted rather differently when degrees of freedom other than the purely classical homogeneous ones are considered. Here, this has been discussed for homogeneous quantum degrees of freedom; inhomogeneities will be the next crucial and

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4 In thermodynamics, coarse-graining plays an important role. Cosmic forgetfulness may be interpreted as forcing us to coarse-grain over many of the quantum variables. One should also note that cosmic forgetfulness is much stronger than decoherence (see e.g. [3]) since it appears even in exactly solvable models. It takes into account the specific dynamics of loop quantum cosmology, rather than being a generic property of quantum systems with many degrees of freedom.

5 Cosmic forgetfulness has been perceived as a challenge, heroically taken up in [55] by deriving bounds alternative to (11) for semiclassical states. However, those bounds are much weaker, allowing changes in the fluctuations by several orders of magnitude [56]. (Also this renewed challenge has been taken up in [57], though less heroically so.)
7 Conclusions

“If he had smiled why would he have smiled? To reflect that each one who enters imagines himself to be the first to enter whereas he is always the last term of a preceding series even if the first term of a succeeding one, each imagining himself to be first, last, only and alone, whereas he is neither first nor last nor only nor alone in a series originating in and repeated to infinity.” This describes the thoughts of Leonard Bloom after a long eventful day. Will we be led to similar thoughts after a long eventful journey in quantum gravity?

If we cannot reconstruct the entire past, we may as well forget about it. The part of the universe we see would appear to have originated with its big bang, even though a theoretical formulation, but only the theoretical formulation, may contain a pre-history. Two questions should immediately be asked: Would this be testable? And why would we not apply Occam’s razor to the pre-history? We could clearly not directly test whether there is a part of the history of the universe that is inaccessible. But we may attempt to access it and, if we succeed, falsify the claim; this makes it scientifically viable as a hypothesis. More importantly, the underlying scenario would have further implications for the structures we see after the big bang. Then, we would have an option to test such a model indirectly.

Why do we then consider the pre-history as part of the mathematical modelling? Also this has its justification. Describing a true physical beginning of the universe, where nothing would turn into something, has proved to be challenging. Pretending that there was something before the big bang and describing it by deterministic but forgetful equations may be the best solution to deal with a beginningless beginning, even though we may not be able to use those equations to fully reconstruct the past.

Taking the simplest models of loop quantum cosmology at face value is often seen as suggesting the big bang transition to be viewed as a smooth bounce, as one further element not just in a long history of the universe itself but also in a long history of bouncing cosmological models. Some indications, however, suggest otherwise. The bloomy scenario of loop quantum cosmology may well be this: a universe whose time-reversed pre-history we cannot access but which we grasp in the form of initial conditions it provides for our accessible part; a pseudo-beginning; an orphan universe, shown the rear-end by whatever preceded (and possibly created) it.

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**Appendix: A momentous formulation of quantum mechanics**

Quantum dynamics can usefully be described in terms of the moments of a state. Taken together, they form an infinite-dimensional phase space which can be used to describe the quantum system. At order \( a + b = 2 \) we have the fluctuations \( G_{V}^{2,0} = (\Delta V)^{2} \) and \( G_{P}^{0,2} = (\Delta P)^{2} \) as well as the covariance \( G_{V}^{1,1} = \frac{1}{2}(\bar{V}\bar{P} + \bar{P}\bar{V}) - \langle \bar{V} \rangle \langle \bar{P} \rangle \). While independent variables, the moments cannot be chosen arbitrarily. They are subject to constraints, most importantly the uncertainty relation

\[
G_{V}V - (G_{P}P)^{2} \geq \frac{\hbar^{2}}{4}.
\]

Poisson brackets between the moments can be computed using the general identity

\[
\{\langle \hat{A} \rangle, \langle \hat{B} \rangle \} = \frac{\langle [\hat{A}, \hat{B}] \rangle}{i\hbar}
\]

as well as linearity and the Leibniz rule. This immediately gives \( \{\langle \hat{V} \rangle, \langle \hat{P} \rangle \} = 1 \) and, e.g., \( G_{V}^{V}G_{P}^{P} = 4G_{V}P \). (See [66][67] for further details.)

The moments allow a convenient description of quantum evolution without having to take the usual detour of solving for states first, followed by computing expectation values. Instead, expectation values obey the general evolution law

\[
\frac{d\langle \hat{O} \rangle}{d\phi} = \frac{\langle [\hat{O}, \hat{H}] \rangle}{i\hbar}
\]

which can be used to derive a coupled set of equations of motion for expectation values together with the moments. For non-polynomial \( \hat{H} \), it may be difficult to compute the commutator, followed by taking an expectation value. Semiclassical equations can more easily be obtained in an expansion by moments, which is formally analogous to background-field expansion around expectation values. We write [66]

\[
\langle H(\hat{V}, \hat{P}) \rangle = \langle H(\langle \hat{V} \rangle + (\hat{V} - \langle \hat{V} \rangle), \langle \hat{P} \rangle + (\hat{P} - \langle \hat{P} \rangle)) \rangle
\]

\[
= H(\langle \hat{V} \rangle, \langle \hat{P} \rangle) + \sum_{a,b:a+b\geq 2} \frac{1}{a!b!} \frac{\partial^{a+b}H(\langle \hat{V} \rangle, \langle \hat{P} \rangle)}{\partial \langle \hat{V} \rangle^{a} \partial \langle \hat{P} \rangle^{b}} G_{a,b}
\]

and use this in

\[
\frac{\langle [\hat{O}, \hat{H}] \rangle}{i\hbar} = \{\langle \hat{O} \rangle, \langle H(\hat{V}, \hat{P}) \rangle \}.
\]
Poisson relations between the moments then provide all equations of motion order by order in the moments.

For the cosmological systems with Hamiltonian \( \mathcal{H} \) introduced before, we have

\[
\frac{d\langle \hat{V} \rangle}{d\phi} = \frac{3}{2} \frac{\langle \hat{V} \rangle \langle \hat{P} \rangle}{\sqrt{\langle \hat{P} \rangle^2 + |\Lambda|}} - \frac{9}{4} |\Lambda| \langle \hat{V} \rangle \langle \hat{P} \rangle \frac{G^{VP}}{\langle \hat{P} \rangle^2 + |\Lambda|}^{3/2} + \frac{3}{2} |\Lambda| \frac{G^{VP}}{\langle \hat{P} \rangle^2 + |\Lambda|}^{3/2} + \cdots
\]

\[
\frac{d\langle \hat{P} \rangle}{d\phi} = -\frac{3}{2} \sqrt{\langle \hat{P} \rangle^2 + |\Lambda|} - \frac{3}{4} |\Lambda| \frac{G^{PP}}{\langle \hat{P} \rangle^2 + |\Lambda|}^{3/2} + \cdots
\]

expanded by the moments (kept here to second order only). This is accompanied by the evolution of moments

\[
\frac{dG^{PP}}{d\phi} = -3 \frac{\langle \hat{P} \rangle}{\sqrt{\langle \hat{P} \rangle^2 + |\Lambda|}} G^{PP} + \cdots
\]

(16)

\[
\frac{dG^{VP}}{d\phi} = \frac{3}{2} |\Lambda| \frac{\langle \hat{V} \rangle}{\langle \hat{P} \rangle^2 + |\Lambda|^{3/2}} G^{PP} + \cdots
\]

(17)

\[
\frac{dG^{VV}}{d\phi} = 3 |\Lambda| \frac{\langle \hat{V} \rangle}{\langle \hat{P} \rangle^2 + |\Lambda|^{3/2}} G^{VP} + 3 \frac{\langle \hat{P} \rangle}{\sqrt{\langle \hat{P} \rangle^2 + |\Lambda|}} G^{VV} + \cdots
\]

(18)

Solving or analyzing this coupled set of equations would tell us how the state changes its shape by the evolving moments, and how this back-reacts on the motion of expectation values. In some regimes it is possible to summarize the effect of all moments in an effective potential for expectation values depending only on the classical variables. But in general, higher-dimensional effective systems including the moments as independent variables are required.

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