Probability entropy approach to the calculation of time before failure of seaport electrical systems

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Abstract. The paper substantiates the method of calculating the operating time to failure of the seaport electrical equipment under the influence of a set of destabilizing factors on the basis of the principle of the maximum entropy production. The interpretation of the results of applying the principles including the concept of entropy to the mathematical models of wear that are used in the theory of reliability is given. The paper considers the probability characteristics of the electrical contacts of mooring columns exposed to the influence of the marine airborne environment. A method for calculating the operating time to failure of marine equipment based on the principle of the maximum entropy production (Ziegler’s principle) was developed. The proposed method is based on the physical principle, which includes the fundamental physical law – the second principle of thermodynamics. In this paper, a new value of the working time corresponding to the maximum observed rate of entropy change is determined. The presented results on finding the operating time to failure of electrical contacts, when exposed to a marine multifactorial environment (as a system with a system-forming connection, in the form of entropy), confirm that the use of the maximum entropy production principle as a criterion for operating time corresponds to the real values of the operating time of devices, and the principle itself gives a physical representation of occurring processes. The paper uses an exponential representation of the probability function of the operational state with an approximated, non-stationary failure rate.

1. Introduction

The studies of failures of technical systems in marine environment suggest that the totality of destabilizing environmental factors may be combined into a system with a system-forming connection defined as entropy. With such an external systemic impact of the environment on technical facilities, irreversible changes occur due to the intra-system destabilization caused by the operation process itself. The illustration of the sharp increase in the entropy of the technical system may be the accelerated destruction of the electric machine caused by rotor vibrations during operation. The emerging phenomena of intersystem and intrasystem interactions in a complex technical system require not only an adequate mathematical description, but also a physical interpretation. Indeed, the mathematical apparatus of reliability theory is being improved, but the structural and functional complexity of technical systems and the variety of its interactions with the environment is also growing. In this case, in order to
fully understand the processes occurring in complex technical systems, including the above interactions, it is necessary to develop an analytical apparatus with a wider conceptual and physical basis. The paper draws special attention to the role of entropy when a technical object loses its operable state, as well as to ideas and principles of non-equilibrium thermodynamics. The purpose of this study is not only to obtain information for the validity of conclusions, but also to move towards expanding the theoretical basis of reliability to obtain new results.

The failures in the reliability theory are associated with the transition of the technical system to an inoperable state and are due to the law of entropy increase. Long-term operation of devices leads to negative factors contributing to failures (i.e. entropy is increasing), and the main task of the reliability theory is to predict them, and make recommendations for their prevention. From this point of view, the current task is to interpret the development of devices in time in terms of the characteristics of the entropy change, at which the speed of its production is maximum. For this purpose, the mathematical model of reliability must be replenished with mathematical models related to the concept of entropy, which would give a physical assessment of the degree of proximity of a technical object to a failure state.

The term “entropy” introduced in 1862 by R. Clausius reflects a quantitative characteristic indicating the direction of the process. This concept underlies one of the basic laws of nature, the second law of thermodynamics, according to which entropy may only increase if an irreversible process occurs in the system, or remains unchanged if the process is reversible. Consequently, the most significant characteristic of changes occurring in any system is irreversibility, which is expressed by the direction of the observed changes and, in fact, involves the time factor, which is one of the main factors in the reliability theory.

It is known that the reliability theory explores the processes associated with the entropy increase, although this is not explicitly formulated in the classical interpretation. For example, the occurrence of sudden and gradual failures is an example of processes in which entropy increases, and there is no such process in the reliability theory that leads, with its spontaneous development, to the entropy decrease. The importance of introducing the concept of entropy into the reliability problems forces us addressing the history of the quantitative representation of this concept.

There were attempts in the history of science to theoretically design a process in which the second law of thermodynamics was violated, all such attempts were unsuccessful, but some of them contributed to new scientific achievements. One example of that kind is the Maxwell demon, with the help of which D. Maxwell considered the theoretical possibility of a physical process with the violation of the second law of thermodynamics. In 1928, analyzing the functioning of the Maxwell demon in the work of L. Szilard, Z. I. Physik, 53, 810 (1928) L. Szilard anticipated the theory of information by C. Shannon for 20 years. In this work, the definition of entropy was given, which became one of the fundamental in the future theory of information. In order to further understand the meaning of the term entropy, it should be noted that the quantitative representation of entropy by L. Szilard arose from the solution of the physical problem, and had a physical meaning.

When developing his information theory C. Shannon used the expression \( H = -\sum_{i=1}^{n} P_i \log P_i \) in signal theory, thus determining the increase in the amount of information as the value of eliminated uncertainty [2, 3]. Thus, in the signal theory it became possible to determine the amount of information and measure its eliminated entropy.

The difference between physical and information entropy is shown in the work of Yu. P. Petrov (p. 46) [1]. It should be noted that this difference exists, but mathematical expressions for representing entropy, which are used in the information theory, were used by L. Boltzmann in his statistical theory and proof of the existence of the \( H \) function known as the H-theorem [2]. The main point of proof of the H-theorem, as P. Chambadal notes in his monograph, is the expression \( H = -\sum_{i=1}^{n} P_i \log P_i \). L. Szilard used the expression \( H_{\text{Szilard}} = -k \sum_{i=1}^{n} P_i \log P_i \) in a heat machine with one heat source, in which the
working body is reduced to one single molecule. L. Szilard’s work uses the expression of the form
\[ H = -\sum_{i=1}^{n} P_{i} \cdot \log P_{i} \]
that he multiplied by the Boltzmann’s constant \( k = 1.28 \cdot 10^{-13} \), i.e. the expression had
the form \( H_{\text{Szilard}} = -k \sum_{i=1}^{n} P_{i} \cdot \log P_{i} \).

The interpretation of entropy by L. Boltzmann made it possible to establish the connection of en-
tropy with probability and opened new scientific theories and applications of the concept of entropy in
other sciences, in particular, the information theory [6, 7, 8]. This study considers the processes asso-
ciated with wear and tear of electric radio devices and the increase of entropy when exposed to desta-
bilizing factors, the appearance of new factors and their role from the standpoint of physical interpreta-
tion of the change in entropy of the studied process.

2. Calculation of operating time before failure of marine equipment
Let us consider the probabilistic characteristics of electrical contacts of mooring columns affected by
marine airborne environment. The experimental studies of the reliability of electrical contacts at the
port moorings made it possible to collect data and build the probability function of the failure of con-
tacts depending on time. As a defining parameter for constructing the probability function, contact re-
sistance was adopted, which randomly changed over time and had a growing mathematical expecta-
tion. According to statistical data, the probability function of failure-free operation is obtained as  fol-
lows:

\[
P(t) = 0.5 + F \left\{ \frac{R_{0} - R_{0\text{ch}} - \gamma_{R_{0}}(t) \cdot t}{\sqrt{\sigma_{R_{0\text{ch}}}^2 + \sigma_{\gamma_{R_{0}}(t)^2} \cdot t^2}} \right\},
\]

where the approximated expression for the resistance speed change \( \gamma_{R_{0}} \) is written as:

\[
\gamma_{R_{0}}(t) = 5.183 \cdot 10^{-2} \cdot \exp(6.802 \cdot 10^{-5} \cdot t),
\]

and the approximated function of the standard deviation of the contact resistance speed change is rep-
resented by the expression:

\[
\sigma_{\gamma_{R_{0}}(t)}(t) = 0.037 \cdot \left[ 1 - \exp(-2.393 \cdot 10^{-3} \cdot t^{0.52}) \right].
\]

The expression for the probability of failure-free operation of the three-phase contact depending on
time in case of three contacts \( P(t) \cdot P(t) \cdot P(t) = P(t)^3 \) was written as:

\[
P(t)_{\text{ch}} = \left\{ 0.5 + F \left\{ \frac{R_{0\text{max}} - R_{0\text{ch}} - \gamma_{R_{0}}(t) \cdot t}{\sqrt{\sigma_{R_{0\text{ch}}}^2 + \sigma_{\gamma_{R_{0}}(t)^2} \cdot t^2}} \right\} \right\}^3.
\]

![Figure 1](image)

Figure 1. Graphical representation of the probability function of failure-free operation of electric-
ental contacts of columns depending on time.
In order to be able to use electrical concepts, we need to present the probability function in the exponential form with a transient failure rate:

$$\lambda a(t) = -\frac{P_{\text{resub}}(t)}{P_{\text{resub}}(t)} = -\frac{f(t)}{P_{\text{resub}}(t)},$$

where the approximated failure rate (approximated by the least squares method) is written as:

$$\lambda a(t) = 1.56 \cdot 10^{-6} + 1.55 \cdot 10^{-13.2} \cdot t^2 + 22.10^{-22.2} \cdot t^5,$$

Figure 2. Graphical representation of the approximating function of failure rate of electrical contacts of columns a) solid line – true failure rate, b) dotted line – approximated failure rate

The standard scheme for determining the operating time $T$ of a system or a device before failure is to calculate the average operating time before failure $\overline{T} = P_{\text{resub}}(t) \cdot dt$. For the above failure probability function, this value makes $T = 1.626 \cdot 10^4$ hours. The obtained value represents a mathematical expectation of random operating time of the device. The study considers another method to determine the operating time of equipment before failure associated with physical processes occurring in the device. Moreover, we will rely on non-equilibrium, irreversible physics characteristic of degradation processes, which are considered in the reliability theory.

In the second half of the 20th century, the Swiss scientist G. Ziegler, studying non-equilibrium thermodynamic systems, formulated the principle of the maximum production of entropy [4]. A similar principle, called the Mises principle, was used in the theory of plasticity, and was called the principle of the maximum dissipation rate of mechanical energy [4, 5]. The core of the principle was that the dissipation rate of mechanical energy in a unit volume during plastic deformation has the maximum value for the actual stress state among all stress states allowed by this plasticity condition [7]. G. Ziegler generalized this principle to non-equilibrium thermodynamics [8]. The idea of G. Ziegler’s principle was to assert that nature is arranged so that with given thermodynamic forces and the law of dissipation the entropy production can be maximized. The main result is that he managed to obtain all known results and, most importantly, the law to increase entropy.

As mentioned above, the study utilizes the exponential representation of the operable state probability function with approximated, non-stationary failure rate:

$$\lambda a(u) = 1.56 \cdot 10^{-6} + 1.55 \cdot 10^{-13.2} \cdot u^2 + 22.10^{-22.2} \cdot u^5,$$

$$P(t) = \exp \left[ -\int_0^t \lambda a(u) \cdot du \right] = \exp \left[ -\int_0^t (1.56 \cdot 10^{-6} + 1.55 \cdot 10^{-13.2} \cdot u^2 + 22.10^{-22.2} \cdot u^5) \cdot du \right].$$

With a given probability of the operable state, the entropy of the state will be described by the function:
\[ H(t) = - \int_0^t P(u) \cdot \log(P(u)) \, du. \]

The integration over long periods of time takes a lot of machine time, so numerical integration in Mathcad is required. We will use the trapezoid method for integration.

The corresponding expressions for numerical integration are given below.

The numerical calculation of integrals by the method of trapezoids in Mathcad is written as:

\[
I(f,a,b,n) = \sum_{i=1}^{n} \frac{b-a}{n} \left[ \frac{f\left(a + \frac{b-a}{n} \cdot i\right) + f\left(a + \frac{b-a}{n} \cdot (i-1)\right)}{2} \right]
\]

The probability of failure-free operation \( p(t) \) is represented by the expression that is calculated in the range from 0 to \( t \):

\[
p(t) = \exp\left(-I(\lambda t, 0, 50)\right).
\]

**Figure 3.** Graphical representation of the probability of failure-free operation depending on time

Entropy \( H(t) \) is calculated within the range from 0 to \( t \):

\[
f(t) = -\frac{P(t) \cdot \log(P(t))}{\log(2)}, \quad H(t) = I(f, 0, 50),
\]

**Figure 4.** Graphical representation of entropy depending on time

The derivative \( D(t) \) of the entropy \( H(t) \) within the range from 0 to \( t \) is given by the following expression:

\[
D(t) = \frac{I(f, 0, t + 10^{-3}, 50) - I(f, 0, t, 50)}{10^{-3} \cdot t},
\]
Computer-graphical solution of operating time before failure at the maximum change of entropy $t = \max D(t)$ is performed graphically in Figure 6.

As a result of the computer-graphical solution, which consists in sequential, numerical integration of the above functions and their graphical display (Figure 5), there is a point at which the rate of entropy change is maximum $\max D(t)$. We call this point on the time axis a critical irreversibility point $T_{KTN}$. When the point $T_{KTN}$ is reached, qualitative changes occur in the object (system) under study. The graphs on Figure 5 and 6 show that the right half of the entropy change rate function $D(t)$ quickly drops to 0, and entropy $H(t) = I(f,0,t,50) \to 1$. From this it follows that the object quickly approaches the deterministic state, in this case, the failure state. Figure 6 shows that the maximum operating time at the point of no return is $T_{KTN} = 1.797 \cdot 10^4$ hours. In classical theory, when using the expression $T_s = \int_0^\infty P_{\text{result}}(t) \cdot dt$, the average time between failures is $T_s = 1.626 \cdot 10^4$ hours.

3. Conclusion
A method for calculating the operating time to failure of marine equipment based on the principle of the maximum entropy production (Ziegler’s principle) was developed. The proposed method is based on the physical principle, which includes the fundamental physical law – the second principle of thermodynamics. The new value of the operating time – $T_{KTN} = 1.797 \cdot 10^4$ hours – corresponding to the
maximum observed rate of entropy change, which differs from the average operating time to failure – $T_r = 1.626 \times 10^4$ hours, is determined.

The stated results on finding the operating time before failure of electrical contacts when exposed to a marine multifactorial environment (as a system with a system-forming connection, in the form of entropy), confirm that the use of the maximum entropy production principle as a criterion for operating time corresponds to the real values of the operating time of devices, and the principle itself gives a physical representation of occurring processes.

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