GD 99: re-investigation of an old ZZ Ceti friend

Zs. Bognár$^{1,2,3,\star}$, Á. Sódor$^{1,2,3}$, and Gy. Mező$^{1,3}$

1 Konkoly Observatory, Eötvös Loránd Research Network (ELKH), Research Centre for Astronomy and Earth Sciences, Konkoly Thege Miklós út 15-17, H–1121, Budapest, Hungary
2 MTA CSFK Lendület Near-Field Cosmology Research Group
3 MTA Centre of Excellence

ABSTRACT

Context. Thanks to the photometric space missions, we know more and more about the properties of white dwarf stars, especially the pulsating ones. In the case of pulsators, we have the opportunity to get an insight into their otherwise hidden interiors by the means of asteroseismology. Besides the space-based observations, we also have the opportunity to study the pulsations of white dwarf stars from the ground, either as observations complementary to the space-based measurements, or individual, long time-base observing runs on selected targets, respectively.

Aims. We aim at the long-term, single-site observations of the bright and not well-studied ZZ Ceti star, GD 99. Our main goals were to determine as many eigenmodes for asteroseismology as it is possible, and then perform the seismic analysis of this target.

Methods. We performed Fourier analysis of the light curves obtained in different epochs. After finding the normal modes of the pulses, we run the 2018 version of the White Dwarf Evolution Code to build model grids for the period fits. We compared the seismic distance of the best-fit model with the geometric value provided by Gaia measurements.

Results. We find that GD 99 is rich in pulsation modes in the $\sim 200 - 1100$ s period range, as we detected seven new periods. Together with literature data, we were able to use 11 modes for the asteroseismic fits. We accepted an asteroseismic model solution with $T_\text{eff} = 12 600$ K and $M_\odot = 0.85 M_\odot$ as a best fit, however, this suggests a more massive star than we expected based on the spectroscopic values. The difference between the seismic distance derived by this model solution and the Gaia geometric distance is about 4 pc. We also estimated the rotational rate of the star based on TESS observations to be 12.1 h.

Key words. techniques: photometric – stars: individual: GD 99 – stars: interiors – stars: oscillations – white dwarfs

1. Introduction

The star GD 99 is one of the first-discovered ZZ Ceti variables. Its light variations were observed by [McGraw & Robinson 1976] for the very first time. Pulsation periods were published by [Mukadam et al. 2006]. Apart from this, no additional photometric observations were published so far, despite the fact that GD 99 is one of the brightest known ZZ Ceti stars. This is why we selected this target for further, long-term, ground-based photometric observations, as one of the targets in our observing programme, resulted in the measurements and in many cases the asteroseismological investigations of our target objects, see e.g. [Bognár et al. 2009; Paparó et al. 2013; Bognár et al. 2014, 2016, 2018, 2019, 2021; Kalup et al. 2021].

ZZ Ceti, or DAV stars represent the most populous group amongst the white dwarf (WD) variables. Most of the stars (about 97%) will finally end their evolution as WDs, and occupy a wide range of effective temperatures from the extremely hot (about 200 000 K) to the now cool objects around 4000 K. They also can have a wide variety of masses from 0.15 to 1.3 solar mass, although most of them are between 0.5 – 0.7 $M_\odot$. As these stellar remnants are about earth-sized, their surface gravities are relatively high, $\log g \sim 8$, and because of the gravitational settling, we find the heavier constituting elements deeper than the lighter ones. Considering a representative DA spectral type white dwarf, it has a degenerate carbon–oxygen (C–O) core, surrounded by a non-degenerate thin helium (He) and an even thinner hydrogen (H) layers.

$\star$ e-mail: bognar.zsofia@csfk.org

As these stars cool and reach the so-called ZZ Ceti instability strip around $10 500 - 13 000$ K, they become pulsationally unstable, and show low-amplitude (∼ mmag), short-period (100 – 1500 s) g-mode pulsations, which we can detect as brightness variations of the stars due to the surface temperature changes. The excitation is provided by the classical $\kappa - \gamma$ mechanism [Dolez & Vauclair 1983, Winget et al. 1982], in combination with the convective driving [Brickhill 1991, Goldreich & Well 1999]. One would have thought that the ZZ Ceti stars behave similarly through the entire DAV instability strip, but observations contradict this presumption. As e.g. [Hermes et al. 2017] clearly described, the pulsational behaviour of the WDs depend on their situation in the instability strip: close to the hot (blue) edge, the stars show the lowest-amplitude and shortest-period light variations; in the middle of the strip we observe much larger amplitudes, and as we are approaching the cool (red) edge of the instability region, the light-variation amplitudes start to decrease with longer and longer periods. We also observe changing stability of the pulsational behaviour with changing temperature. Cooler variables are more likely to show short-term (from days to weeks) amplitude and frequency variations, while the pulsation properties are more stable of the hotter objects, close to the blue edge of the instability strip. Space observations revealed that some of the cool ZZ Ceti stars show irregularly recurring outburst events, when the stellar flux can increase up to 15% [Bell et al. 2017]. For comprehensive reviews on the characteristics of pulsating white dwarf stars, see the papers of [Winget & Kepler 2008], [Fontaine & Brassard 2008], [Althaus et al. 2010], [Côrsico et al. 2019], and [Côrsico 2020].
Pulsating white dwarfs are otherwise regular white dwarf stars. These are ideal space laboratories to study the behaviour of matter under extreme physical conditions. However, the only way we can investigate the interior of these stars is through the study of the global oscillations excited in them, by the apparatus of asteroseismology. Our primary goal is to determine as many as possible pulsational frequencies in oscillating stars, to impose stronger constraints for asteroseismic modelling.

We present our ground-based observations from different years, and the measurements obtained by the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) in Sect. 2, followed by the light curve analyses of the data sets in Sects. 3 and 4. We summarise our period findings in Sect. 5. The asteroseismic analysis is presented in Sect. 6, and the summary and discussion of our results can be found in Sect. 7.

### 2. Observations and data reduction

We observed GD 99 ($G = 14.56$ mag, $\varpi_2000 = 09^h01^m49^s$, $\delta_2000 = +36^\circ07'00''$) on 30 nights from the ground, while we also obtained data with the SI camera used in the TESS space telescope.

#### 2.1. Ground-based observations

We performed observations between 2002 and 2022 with the 1-m Ritchey–Chrétien–Coudé telescope at the Piszkéstető mountain station of Konkoly Observatory, Hungary. The detector was a Photometrics Inc. Thomson TH7896M CCD chip, and we applied 40 s exposure times besides 20 s readout times in 2002. The following observations were performed in 2018 with an FLI Proline 16803 CCD camera and with a read-out time of ∼ 3 s. We continued the observations in the 2020/2021 observing season, when the telescope was equipped with a Spectral Instruments (SI) 1100S CCD. The readout time of this camera is ∼ 22 s. In the 2021/2022 observing season we utilised two cameras: the SI 1100S CCD and an Andor iXon+888 Electron-multiplying CCD (EMCCD). In this later case the readout time was ∼ 4 s.

In all cases but in 2002 we observed in white light to maximise the photon counts, while in 2002 we observed in V band. The observing log is presented in Table 1 with the corresponding dates, exposure times, numbers of data points, and observing lengths.

We performed the reduction of the raw data frames the standard way utilising IRAF tasks. After bias, dark and flat corrections, we carried out the aperture photometry of the field stars. We fitted second- or third-order polynomials to the resulting light curves, correcting for long-period atmospheric and instrumental trends. We converted the observational times of every data point to barycentric Julian dates in barycentric dynamical time (BJD$_{TDB}$) using the applet of Eastman et al. (2010).

The panels of Figs. 1 and 2 show the plots of all ground-based data obtained in chronological order.

#### 2.2. TESS observations

TESS observed GD 99 for 27.4 days in sector 21 with the 120-second short-cadence mode between 21st January and 18th February, 2020. We downloaded the light curves from the Mikulski Archive for Space Telescopes (MAST), and extracted the

### Table 1. Log of ground-based observations of GD 99. ‘Exp’ is the integration time, $N$ is the number of data points, and $\delta \Omega$ is the length of the data sets including gaps. Weekly observations in the case of the SI camera are denoted by ‘a–g’ letters in parentheses. We also listed the cameras used for the corresponding observations.

| Run | UT Date | Start time (BJD-2450000) | Exp. (s) | $N$ | $\delta \Omega$ (h) |
|-----|---------|--------------------------|---------|-----|-----------------|
| Photometrics Inc. CCD | 01 | 2002 Feb 04 | 2310.274 | 40 | 519 | 9.46 |
| 02 | 2002 Feb 05 | 2311.276 | 40 | 542 | 9.53 |
| Total: | | | | | 1061 | 18.99 |
| FLI Proline CCD | 03 | 2018 Oct 12 | 8403.540 | 30 | 309 | 2.87 |
| 04 | 2018 Oct 14 | 8405.548 | 30 | 284 | 2.64 |
| 05 | 2018 Oct 15 | 8406.546 | 30 | 284 | 2.69 |
| Total: | | | | | 877 | 8.20 |
| SI CCD | 06(a) | 2021 Feb 15 | 9261.248 | 30 | 426 | 6.27 |
| 07(b) | 2021 Mar 15 | 9289.320 | 30 | 409 | 6.01 |
| 08(b) | 2021 Mar 16 | 9290.308 | 30 | 450 | 6.80 |
| Total: | | | | | 1285 | 19.08 |
| EMCCD | 09 | 2021 Nov 05 | 9523.505 | 30 | 170 | 1.61 |
| 10 | 2021 Nov 05 | 9524.448 | 30 | 601 | 5.68 |
| 11 | 2021 Nov 08 | 9527.478 | 30 | 500 | 4.73 |
| 12 | 2021 Nov 09 | 9528.463 | 30 | 524 | 4.97 |
| Total: | | | | | 1795 | 16.99 |
| SI CCD | 13(c) | 2021 Dec 07 | 9556.400 | 30 | 467 | 7.55 |
| 14(d) | 2022 Jan 06 | 9586.308 | 30 | 664 | 9.97 |
| 15(d) | 2022 Jan 07 | 9587.320 | 30 | 221 | 3.20 |
| 16(d) | 2022 Jan 10 | 9589.527 | 30 | 329 | 4.91 |
| 17(d) | 2022 Jan 10 | 9590.437 | 30 | 446 | 6.80 |
| 18(d) | 2022 Jan 11 | 9591.311 | 30 | 658 | 9.71 |
| 19(d) | 2022 Jan 12 | 9592.317 | 30 | 589 | 8.64 |
| 20(e) | 2022 Feb 11 | 9622.430 | 30 | 402 | 6.15 |
| 21(e) | 2022 Feb 12 | 9623.229 | 30 | 711 | 11.06 |
| 22(f) | 2022 Feb 24 | 9635.239 | 30 | 262 | 4.93 |
| 23(f) | 2022 Feb 27 | 9638.252 | 30 | 247 | 3.58 |
| 24(f) | 2022 Feb 28 | 9639.247 | 30 | 452 | 7.21 |
| 25(f) | 2022 Mar 01 | 9640.236 | 30 | 455 | 7.72 |
| 26(g) | 2022 Mar 24 | 9663.267 | 30 | 261 | 3.81 |
| 27(g) | 2022 Mar 25 | 9664.326 | 30 | 308 | 4.93 |
| 28(g) | 2022 Mar 26 | 9665.303 | 30 | 448 | 6.76 |
| 29(g) | 2022 Mar 27 | 9666.262 | 30 | 398 | 5.85 |
| 30(g) | 2022 Mar 28 | 9667.261 | 30 | 395 | 6.54 |
| Total: | | | | | 7713 | 119.34 |

PDCSAP fluxes provided by the Pre-search Data Conditioning Pipeline (Jenkins et al. 2016). We omitted the obvious outliers. The resulting light curve consists of 17 005 data points (with a gap). The cleaned light curve can be seen in Fig. 3.

### 3. Light curve analyses of the ground-based observations

For the standard Fourier analysis of the data sets, we utilised the photometry modules of the Frequency Analysis and Mode Identification for Asteroseismology (fiamas) software package (Zima 2008). Considering our experience we gained on the light curve analyses of several other white dwarf variables, we accepted a frequency peak as significant if its amplitude reached the five signal-to-noise ratio ($S/N$) instead of the widely used 4 $S/N$ limit. We calculated the noise level by the average Fourier amplitude in a ∼ 1700 µHz radius vicinity (150 $d^{-1}$) on the peak in question (see e.g. in Bognár et al. 2019). We chose the 5 $S/N$ significance level, as below this, we can find several peaks with similarly low amplitudes in many cases, which may often be the results of the amplitude-frequency-phase variations frequently detected in white dwarf variables.
Fig. 1. Normalised differential light curves of GD 99 – part one.
We analysed the data sets obtained with different CCDs separately. In the case of the Photometrics Inc., the FLI Proline, and the Andor EMCCD, we observed GD 99 on one observing week, respectively, while we observed the star on seven weeks altogether (weeks a–g in Table 1) with the SI CCD camera. Besides the simple analysis of the weekly data, we tested our frequency solutions analysing combinations of different consecutive weekly observations. These subsets were selected from the SI-observations, and consist of the data of weeks (a+b+c), (b+c+d), (c+d+e), (d+e+f), and (e+f+g), see the markings in Table 1.

We identified closely spaced peaks in the 935 – 1045 µHz frequency range, while we see more or less well separated peaks at around 1150, 1685, 2890, 3340, 3500, 4390, and 4525 µHz. We reviewed the frequency analysis results of the weekly and combined weekly data, and searched for similar independent frequencies at the different frequency domains, which we can accept as real pulsation frequencies. We summarised our findings in Table 2.

We identified 11 frequencies, listed in Table 2, however the presence of further frequencies is possible at 1030, 1175, and 2840 µHz, but we do not list them in Table 2. At the ~1030 µHz frequency peak there are 1 d−1 alias ambiguities, and in the other two cases we can find a significant peak only in one data set, while we were looking for repeatedly appearing frequencies. The only exception we see in Table 2, is the peak at 1147.49 µHz. We accepted this frequency as it was the dominant peak in the 2018 (FLI) data set. Notice that week(b), which consists of two nights only, is not listed in Table 2 because the only significant peak found in this data is at 4374 µHz, which is about ~1 d−1 distance from the 4386 µHz peak exists in almost all of the other data sets. Note also that in many cases we can detect a peak at 4374 µHz or at the 4374 – 1 d−1 µHz frequency. This suggests that there is another, closely spaced frequency to the 4386 µHz peak. We resolved this ambiguity by analysing the TESS data (see Sect. 4).

We also analysed the complete data set of the most extensively observed season, SI 2021/2022 (18 nights). These results are also listed in Table 2.
4. Light curve analysis of the TESS observations

We also performed frequency-analysis on the TESS data. Surprisingly, the only two frequencies above the 5 S/N significance level are at 3948.04, and 3959.86 µHz, with ~ 1 d\(^{-1}\) separation. We cannot see peaks at these frequencies in the ground-based data, so one can assume that we found additional frequencies to the ones listed in Table 2. However, as we investigate the Fourier transform (FT) of the TESS data set above the Nyquist limit (4167 µHz), we can find the pairs of these frequencies at the already known 4385.38 and 4373.58 µHz. That is, the 3948.04, and 3959.86 µHz peaks are Nyquist aliases of the real frequencies at 4385 and 4374 µHz. The TESS data revealed that there is a doublet at this frequency domain, also suggested by the ground-based data. The separation of the frequency components is very close to 1 d\(^{-1}\), which explains why we could not unambiguously determine the lower-amplitude 4374 µHz component in the ground-based data.

5. Periods for asteroseismology

Our main goal with these several weeks of observations is to determine periods as inputs for asteroseismology. We decided to utilise the SI 2021/2022 data set consist of 18 nights as a reference for seismology with ten observed periods, and complement this list with the dominant frequency of the FLI (2018) data set. This means 11 periods for the asteroseismic investigations. Table 3 lists the corresponding frequencies, periods, and amplitudes. As we see, the star shows both long- and short-period light variations. However, we must keep in mind that observations performed at different epochs can reveal additional pulsation modes for a given star. For this reason, we complemented our list of pulsational modes with the periods presented by Mukadam et al. (2006) for our asteroseismic investigations. We compare our observational results with those presented in Mukadam et al. (2006) in Table 3. In the case of closely spaced periods, which can be treated as common frequencies, we calculated the amplitude weighted mean periods for asteroseismology. The final list of periods we used for asteroseismic fits can be found in the last column of Table 3. As we see, GD 99 is rich in pulsation frequencies, with 18 periods listed.

Figures 4 and 5 show the FTs of the SI 2021/2022 and FLI (2018) data sets, respectively. We marked the accepted frequencies listed in Table 3 in these plots. The FT of the TESS data can be seen in Fig. 6. The doublet at 3948.04 and 3959.86 µHz appears clearly, together with their super-Nyquist pairs at 4385.38 and 4373.58 µHz.

6. Asteroseismology

A new version of the White Dwarf Evolution Code (wdec) was presented in 2018 by Bischoff-Kim & Montgomery (2018), which now uses the Modules for Experiments In Stellar Astrophysics (MESA) (Paxton et al., 2011; version r8118) opacity routines and equation of states. We utilised this wdec version to build model grids for the asteroseismic investigations of GD 99. We start every model with a ~ 100 000 K polytrope, which is then evolved down to the requested temperature. Finally, we obtain a thermally relaxed solution to the stellar structure equations. The program treats the convection operates in the white dwarf envelopes within the framework of the mixing length theory (Bohm & Cassinelli 1971). The code permits to define how to treat the α parameterization. We chose to take α according to the results of Tremblay et al. (2015).

We determined the ℓ = 1 and 2 pulsation modes for each model according to the adiabatic equations of non-radial stellar oscillations (Unno et al. 1989). For comparison of the observed (P\(_{o\ell}\)) and calculated (P\(_{c\ell}\)) eigenmodes, we applied the fitper
Table 4. Comparison of the periods presented this work with the solution listed in \textcite{Mukadam et al. (2006)}. The last column shows the periods utilised for the asteroseismic fits.

\begin{tabular}{lllll}
\hline
 & \textbf{This work} & \textbf{Mukadam et al. (2006)} & \textbf{For seismology} \\
\hline
$P$ & $\text{Ampl.}$ & $P$ & $\text{Ampl.}$ & $P$ \\
[s] & [mmag] & [s] & [mma] & [s] \\
\hline
$-06$ & 220.91 & 1.63 & - & - & 105.2 \\
$-01$ & 227.99 & 7.40 & 228.9 & 4.5 & 228.3 \\
$-04$ & 285.97 & 2.10 & - & - & 286.0 \\
$-07$ & 299.62 & 1.41 & - & - & 299.6 \\
$-10$ & 346.00 & 1.18 & - & - & 346.0 \\
$-11$ & 593.26 & 1.16 & - & - & 593.3 \\
$-02$ & 871.47 & 6.38 & - & - & 871.5 \\
$-09$ & 957.85 & 1.23 & - & - & 957.9 \\
$-08$ & 1007.00 & 1.36 & 1007.0 & 6.5 & 1007.0 \\
$-05$ & 1060.31 & 2.15 & 1058.0 & 8.3 & 1058.4 \\
$-03$ & 1069.88 & 2.75 & - & - & 1069.9 \\
$-01$ & 1088.0 & 4.3 & - & - & 1088.0 \\
$-08$ & 1151.0 & 1.9 & - & - & 1151.0 \\
\hline
\end{tabular}

Fig. 4. Fourier transform of the SI 2021/2022 data set. We also marked the frequencies listed in Table 3 for completeness.

Fig. 5. Fourier transform of the FLI (2018) data set. We marked the $f_{02}$ frequency listed in Table 3.

The tool of \textcite{Kim (2007)}, which calculates the root mean square ($\sigma_{\text{rms}}$) for every model as follows:

$$\sigma_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^{N} (P_{\text{calc}} - P_{\text{obs}})^2}{N}}$$

where $N$ is the number of observed periods. That is, $\sigma_{\text{rms}}$ characterises the goodness of the fits.
First, we built a coarse (master) model grid covering a wide parameter space in the main physical properties: stellar mass and effective temperature. We scanned the parameter space as follows: $T_{\text{eff}}, M_*, M_{\text{env}}$ (the mass of the envelope, determined by the location of the base of the mixed helium and carbon layer), $M_{\text{He}}$, $X_{\text{He}}$ (the helium abundance in the C/He/H region), and $X_0$ (the central oxygen abundance). While we scanned several parameters, we fixed the mass of the helium layer ($M_{\text{He}}$) at $10^{-2} M_*$. This is the theoretical maximum for $M_{\text{He}}$, and we did not vary it, considering the results of Romero et al. (2012). They found that $M_{\text{He}}$ can be as much as a factor of 3–4 lower than the values according to evolutionary calculations, but not orders of magnitudes lower, therefore, it does not affect the periods substantially.

Table 5 lists the parameter space we covered by the master grid, and the corresponding step sizes. Note that we calculated the possible pulsation periods of the models in the 70–1500 s period range.

Investigating this coarse grid, we considered six different cases with different restrictions on the degree of the pulsation modes: (a) We assumed that all 18 periods are $\ell = 1$. (b) We treated the dominant periods of both detected by this work and listed in Mukadam et al. (2006) as $\ell = 1$ modes, while we let the other modes to be either $\ell = 1$ or $\ell = 2$. That is, in this case, both the 228.3 and 1058.4 s modes were required to be $\ell = 1$ periods during the fitting process. (c) We assumed that the 228.3 s mode is an $\ell = 1$, while we did not fix the $\ell$ values of the other modes. (d) Similarly to the previous case, we assumed that at least one mode should be $\ell = 1$, but this time this mode was the 1058.4 s period. (e) We selected models with at least ten $\ell = 1$ modes, that is, we assumed that more than half of the modes are $\ell = 1$. (f) Finally, we assumed only that all observed periods are either $\ell = 1$ or $\ell = 2$ modes, but we did not apply any further restriction on the degree of the modes.

Our results show the followings: (a) We did not find any solution with all detected periods being $\ell = 1$ degree modes with better than $\sigma_{\text{rms}} = 15$ s. (b) With the two or more $\ell = 1$ modes, the best-fit model has the physical parameters of $T_{\text{eff}} = 12,250$ K and $M_* = 0.90 M_\odot$ ($\sigma_{\text{rms}} = 2.69$ s). (c) Assuming the 228.3 s period mode to be $\ell = 1$ degree, the best fit model was found at $T_{\text{eff}} = 13,500$ K and $M_* = 0.80 M_\odot$ ($\sigma_{\text{rms}} = 2.52$ s). (d) Assuming the 1058.4 s period mode to be $\ell = 1$ degree, the best fit model was found at $T_{\text{eff}} = 14,000$ K and $M_* = 0.85 M_\odot$ ($\sigma_{\text{rms}} = 2.58$ s). (e) The best fit model that has at least 10 $\ell = 1$ degree modes is $T_{\text{eff}} = 13,750$ K and $M_* = 0.90 M_\odot$ ($\sigma_{\text{rms}} = 2.70$ s). (f) Without any restriction on the $\ell$ values, the best fit model was found to be the same as in case (c).

In sum, our best fit solutions are situated at relatively high effective temperatures, in the range of $12,000$–$14,000$ K, and also show high stellar masses between 0.8 and 0.9 $M_\odot$. That is, our fit results suggest GD 99 being a hot and massive star, hotter and more massive than it was found by previous spectroscopic measurements. The spectroscopic solutions obtained previously on GD 99 are listed in Table 6 while Fig. 7 demonstrates the first stage of our search for the best fit model utilising a master grid.

Considering these results, we built a second, higher-resolution subgrid to further investigate the possible asteroseismic solutions for GD 99. Table 7 lists the parameter space we covered by this subgrid, and the corresponding step sizes.

Utilising the higher-resolution subgrid, we found solutions for all six cases. We list the physical parameters of the best fit models both for the master grid and the subgrid in Table 8.

We can check the reliability of our best-fit solutions listed in Table 8 considering the known geometric distance of the star provided by the Gaia space telescope. For our target, this geometric distance is $d_{\text{Gaia}} = 33,939^{+9027}_{-6030}$ pc (Bailer-Jones et al. 2021), which is based on the Gaia early third release EDR3 (Gaia Collaboration et al. 2021). The method we followed to determine the seismic distances of the selected models utilising the luminosity of the given model and the apparent visual magnitude of the star is detailed e.g. in Bell et al. (2019) and Bognár et al. (2021). For the apparent visual magnitude of

Table 5. Physical parameters varied while building the master grid. The step sizes applied are in parentheses.

| Parameter   | $10,000 – 14,000$ | 250 |
|-------------|------------------|-----|
| $T_{\text{eff}}$ [K] | $10,000 - 14,000$ | 250 |
| $M_*$ [M$_\odot$] | $0.35 – 0.90$ | 0.5 |
| $-\log(M_{\text{env}}/M_*)$ | $1.5 – 1.9$ | 0.1 |
| $-\log(M_{\text{He}}/M_*)$ | $2$ | fixed |
| $-\log(M_0/M_*)$ | $4 – 6$ | 1.0 |
| $X_{\text{He}}$ | $0.5 – 0.9$ | 0.1 |
| $X_0$ | $0.5 – 0.9$ | 0.1 |

Table 6. Spectroscopic values for GD 99. The source of this list is the Montreal White Dwarf Database (Dufour et al. 2017).

| $T_{\text{eff}}$ [K] | $M_*$ [M$_\odot$] | Ref. |
|------------------|------------------|-----|
| $11,830 ± 174$ | $0.66 ± 0.03$ | Liebert et al. (2002) |
| $12,380 ± 190$ | $0.75 ± 0.03$ | Gianninas et al. (2011) |
| $12,080 ± 179$ | $0.70 ± 0.03$ | Limores et al. (2013) |
| $11,681 ± 72$ | $0.68 ± 0.01$ | McCleery et al. (2020) |
| $11,575 ± 38$ | $0.663 ± 0.004$ | Kille et al. (2020) |

![Fig. 7. Models on the $T_{\text{eff}} – M_*$ plane of the coarse grid. We show the goodness of the fit of each grid-point with no restrictions on the degree of the observed pulsation modes. The model with the lowest $\sigma_{\text{rms}}$ is denoted with a white open circle, while the spectroscopic solutions are marked with black dots. The parameter space further investigated by a higher-resolution subgrid is indicated by a black square.](image-url)
GD 99, the fourth US Naval Observatory CCD Astrograph Catalog (Zacharias et al. 2012) gives $m_V = 14.584 \pm 0.06$ mag. We found that the best matching model to the Gaia geometric distance is the $T_{\text{eff}} = 13 \, 500 \, \text{K}$ and $M_* = 0.80 \, M_\odot$, with the seismic distance of 34.19 $\pm$ 0.94 pc. There is another model with $T_{\text{eff}} = 14 \, 000 \, \text{K}$, $M_* = 0.85 \, M_\odot$, and the seismic distance of 33.25 $\pm$ 0.92 pc, in which case the seismic and Gaia geometric distances are practically the same within the errors. In the other cases listed in Table 8 the distances suggest a star closer to us than we found by the Gaia data.

Being somewhat closer in effective temperature and stellar mass to the spectroscopic values, we selected the $T_{\text{eff}} = 13 \, 500 \, \text{K}$ and $M_* = 0.80 \, M_\odot$ model for further investigation. Table 9 summarises the calculated and observed periods for this model.

We plot the chemical composition profiles and also the Brunt–Väisälä frequency profile of our accepted model in Fig. 8. As Table 8 shows, we obtained 60 per cent oxygen abundance for the core. Comparing it with the expected ~ 65 per cent value by the evolutionary calculations of Romero et al. (2012), our solution is close to this probable core oxygen abundance.

Note that considering our model solutions, we obtain models in which the chemical profiles show a pure carbon buffer, in spite of the fact, that fully evolutionary models are not in agreement with this characteristics. Instead, the pure carbon buffer can disappear by chemical diffusion (e.g. De Gerónimo et al. 2019).

7. Summary and discussion

The long known GD 99 is one of the brightest ZZ Ceti type pulsating white dwarf, yet, not a detailed investigation both on its pulsational behaviour and asteroseismology have been published thus far. That is the reason why we selected GD 99 for a long-time ground-based follow-up measurement, which resulted in 30 nights of observations, spanning more than 119 observing hours. Our extended observations enabled the detection of 11 eigenmodes for seismology, and we also revealed that the peaks detected in the TESS light curve are Nyquist aliases of the frequencies derived by the ground-based observations. This highlighted the usefulness of ground-based observations, even when space-based measurements are also available. Moreover, with the modes detected by our ground-based measurements, and as we can complete our set of modes with the ones presented by Mukadam et al. (2006), GD 99 joined the small group of known white dwarf pulsators rich in oscillation modes.

We performed a preliminary asteroseismic analysis of GD 99 by comparing the observed and calculated pulsating periods. Our preferred model is the one with $T_{\text{eff}} = 13 \, 500 \, \text{K}$ and $M_* = 0.80 \, M_\odot$, that is, according to our seismic fittings, GD 99 may be at the blue edge of the empirical ZZ Ceti instability strip, see e.g. fig. 3 in Hermes et al. (2017).

Considering the frequency separation $(\delta f)$ of the well-resolved doublet found by the TESS light curve, we can estimate the rotation period of the star, as, in the case of slow rotation, the frequency differences of the $m = -1, 0, 1$ rotationally split components can be approximated to first order by the following relation:

$$\delta f_{l,m} = \delta m (1 - C_{l,i}) \Omega,$$

where the coefficient $C_{l,i} \approx 1/(l+1)$ for high-overtone $(k \gg l)$ g-modes and $\Omega$ is the (uniform) rotation frequency. The frequency separation of the TESS doublet of the GD 99 data is 11.81 $\mu$Hz, and the corresponding $C_{l,i}$ value is 0.44, as these are low radial-order frequencies. Finally, we obtain a $P = 0.55$ d = 13.17 h rotation period for GD 99. Thanks to the TESS observations, this is the first time we could derive the rotational rate of this star.

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Fig. 8. Chemical composition profiles in fractional abundances, and the corresponding logarithm of the squared Brunt–Väisälä frequency ($\log \nu$) for our selected model by the refined grid ($T_{\text{eff}} = 13 \, 500 \, \text{K}$, $M_* = 0.80 \, M_\odot$, $-\log \nu_{\text{lin}} = 1.6$, $-\log \nu_{\text{osc}} = 2.0$, $-\log \nu_{\text{rot}} = 4.0$, $X_{\text{He}} = 0.8$, $X_O = 0.6$).
Table 8. Physical parameters and seismic distances ($d$) of the best-fit models. Short summary on the different cases: case (a) – all modes are $\ell = 1$, case (b) – the 228.3 and 1058.4 s modes are $\ell = 1$, case (c) – the 228.3 s mode is $\ell = 1$, case (d) – the 1058.4 s mode is $\ell = 1$, case (e) – at least ten $\ell = 1$ modes, case (f) – no restrictions on the modes’ $\ell$ values.

| $T_{\text{eff}}$ [K] | $M_\ast$ [$M_\odot$] | -log($M_{\text{env}}/M_\ast$) | -log($M_{\text{He}}/M_\ast$) | -log($M_{\text{H}}/M_\ast$) | $X_{\text{He}}$ | $X_{\text{O}}$ | $\sigma_{\text{rms}}$ [s]) | $d$ [pc] |
|----------------------|---------------------|------------------|------------------|------------------|----------------|----------------|------------------|----------|
| master grid – case (b): | 12 250 | 0.90 | 2.0 | 4.0 | 0.9 | 0.8 | 2.69 | 27.96 ± 0.77 |
| master grid – case (c) – same as case (f): | 13 500 | 0.80 | 1.6 | 2.0 | 4.0 | 0.8 | 0.6 | 2.52 | 34.19 ± 0.94 |
| master grid – case (d): | 14 000 | 0.85 | 1.6 | 2.0 | 4.0 | 0.8 | 0.6 | 2.58 | 33.25 ± 0.92 |
| master grid – case (e): | 13 750 | 0.90 | 1.5 | 2.0 | 4.0 | 0.5 | 0.8 | 2.70 | 30.72 ± 0.85 |
| subgrid – case (a): | 14 000 | 0.94 | 1.6 | 2.0 | 4.5 | 0.9 | 0.5 | 5.76 | 29.45 ± 0.81 |
| subgrid – case (b): | 13 000 | 0.94 | 1.6 | 2.0 | 4.0 | 0.7 | 0.9 | 2.04 | 27.81 ± 0.77 |
| subgrid – case (c): | 12 200 | 0.91 | 1.9 | 2.0 | 4.0 | 0.9 | 0.6 | 2.03 | 27.52 ± 0.76 |
| subgrid – case (d) – same as case (f): | 12 400 | 0.93 | 1.7 | 2.0 | 4.0 | 0.8 | 0.6 | 2.13 | 27.21 ± 0.75 |
| subgrid – case (e): | 12 400 | 0.93 | 1.7 | 2.0 | 4.0 | 0.9 | 0.6 | 2.13 | 27.21 ± 0.75 |

Gaia: 33.939 ± 0.027

Table 9. Observed and calculated periods of the selected model (see master grid – case (c) in Table 8). In the last column we also list the period differences.

| Obs. period [s] | Calc. period [s] | $\ell$ | Difference [s] |
|-----------------|-----------------|------|----------------|
| 105.2           | 109.0           | 2    | 3.8            |
| 222.6           | 218.9           | 2    | 3.7            |
| 228.3           | 227.9           | 1    | 0.4            |
| 286.0           | 287.5           | 2    | 1.5            |
| 299.6           | 301.7           | 1    | 2.1            |
| 346.0           | 342.4           | 1    | 3.6            |
| 593.3           | 594.3           | 2    | 1.1            |
| 633.1           | 632.8           | 2    | 0.3            |
| 853.2           | 856.3           | 2    | 3.1            |
| 871.5           | 873.7           | 2    | 2.3            |
| 924.7           | 925.9           | 2    | 1.2            |
| 957.9           | 954.5           | 1    | 3.4            |
| 976.0           | 974.1           | 2    | 1.9            |
| 1007.0          | 1007.2          | 2    | 0.2            |
| 1058.4          | 1060.3          | 2    | 1.9            |
| 1069.9          | 1066.3          | 1    | 3.6            |
| 1088.0          | 1086.8          | 2    | 1.2            |
| 1151.0          | 1146.9          | 2    | 4.1            |

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