The role of glassy dynamics in the anomaly of the dielectric function of solid helium

Jung-Jung Su$^{1,2,3,4}$, Matthias J Graf$^1$ and Alexander V Balatsky$^{1,2}$

$^1$ Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
$^2$ Center for Integrated Nanotechnologies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
E-mail: jungjsu@gmail.com

New Journal of Physics 13 (2011) 113024 (9pp)
Received 20 July 2011
Published 18 November 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/11/113024

Abstract. We propose that the acousto-optical (electro-elastic) coupling of the electric field to strain fields localized around defects in disordered $^4$He causes an increase of the dielectric function with decreasing temperature due to the arrested dynamics of defect excitations. A distribution of such low-energy excitations can be described within the framework of a glass susceptibility of a small volume fraction inside solid $^4$He. Upon lowering the temperature the relaxation time $\tau(T)$ of defects increases and an anomaly occurs in the dielectric function $\epsilon(\omega, T)$ when $\omega \tau(T) \sim 1$. Since $\epsilon(\omega, T)$ satisfies the Kramers–Kronig relation, we predict an accompanying peak in the imaginary part of $\epsilon(\omega, T)$ at the same temperature that the largest change in amplitude occurs at a fixed frequency. We also discuss recent measurements of the amplitude of the dynamic dielectric function that indicate a low-temperature anomaly similar to that seen in the resonance frequency of the torsional oscillator and shear modulus experiments.

$^3$ Present address: Edward L Ginzton Laboratory, Stanford University, Stanford, CA 94305, USA.
$^4$ Author to whom any correspondence should be addressed.
1. Introduction

The reported anomalies in resonance frequency and dissipation of torsional oscillators [1–9] and in the dynamic shear modulus [10–12] at low temperatures are the subject of intensive study as they have been suggested to be signatures of supersolidity. It is now generally agreed that disorder plays an essential role in observing these effects. The challenge in unambiguously identifying Bose–Einstein condensation in solid $^4$He arises from the fact that crystal defects are required for supersolidity [13–15]. These defects exhibit their own dynamics and contribute to the observed period of oscillations, shear modulus and specific heat in the same temperature and pressure range where supersolidity is expected. Thus any unambiguous identification of a supersolid state requires a detailed understanding of the behavior of defects.

To study the role of glassy excitations, which make up a small volume fraction of the crystal, we propose a phenomenological framework that captures the dynamics of defects [16–21]. It accounts for observed anomalies in the torsional oscillator, specific heat and shear modulus experiments [16–23] and can be thought of as a distribution of crystal defects forming two-level systems (TLS) [24, 25]. Possible candidates for the TLS are groups of atoms near a crystal defect or pinned segments of dislocation lines. These defects can be removed through annealing or created by applying shear stress, thus drastically changing the mechanical and elastic properties of solid $^4$He [26]. A freezing out of these excitations can account for the anomalies by postulating a relaxation time increasing with decreasing temperature. This is typical of glassy systems overcoming an activation barrier [16–23], which results in visco-elastic behavior, long relaxation times and hysteresis, as well as effective softening of elastic moduli.

Given the growing experimental evidence for the role of disorder in solid $^4$He, we address the question of the effects of a glassy component on the dielectric function at the lowest temperatures. Here, we show that glassy dynamics can affect the strain and position of atoms, which in turn leads to an additional contribution to the polarization. This contribution causes a decrease of the dielectric function at high temperatures. Therefore, we propose that the cooperative motion of displaced (out-of-equilibrium) atoms near defects, in the glasslike regions of a solid, creates local strain and thus reduces the polarization through acousto-optical (electro-elastic) coupling. Note that this effect is not captured by the standard Clausius–Mossotti equation for dielectrics, which attributes the change in dielectric function $\epsilon$ to a change in either polarizability $\alpha$ or mass density $\rho$: $(\epsilon - 1)/(\epsilon + 2) = (4\pi/3)(\alpha\rho/M)$, with molar mass $M$. In fact, the glass model yields an effect opposite in sign, and orders of magnitude larger, than a correction in the polarizability predicted for dipole-induced dipole interactions [27], reported long ago in $^4$He by Chan et al [28].
The glass model describes the anomaly in $\epsilon(\omega, T)$ due to electro-elastic coupling between strain induced by glassy defects and dielectric polarization. It features: (i) a decrease of $\epsilon(\omega, T)$ at high temperatures; and (ii) a dissipation peak in the same temperature range where the amplitude of $\epsilon(\omega, T)$ changes most significantly. The origin for this dynamic behavior is set by the matching condition $\omega \tau(T) \sim 1$ between the frequency $\omega$ and relaxation time $\tau(T)$. The key assumption is that a strain field in the vicinity of crystalline defects induces changes in the polarization, since the magnitude of the local electric field sensed by a helium atom is different from the electric field at the equilibrium position of atoms in a crystal free of internal stress.

Very recent measurements of $\epsilon(\omega, T)$ by Yin et al [29] show that there is an increase of the dielectric function at low temperatures in solid helium, whereas no such change is found in a control experiment with liquid helium. In this paper, we argue that this experiment may be explained by an electro-elastic coupling model of a solid that is out of equilibrium with frozen-in internal stress. We will illustrate that these data cannot be described by the standard Clausius–Mossotti equation through a change in mass density or polarizability, e.g. due to dipole-induced dipole interactions. Neither the measured change of the mean mass density $\delta \rho/\rho \sim 10^{-6}$ nor the predicted correction in polarizability, which actually leads to a decrease of $\epsilon(\omega, T)$ at low temperatures [27, 28], can account for the reported change of dielectric function of order $\delta \epsilon/\epsilon \sim 10^{-5}$ when invoking the Clausius–Mossotti equation.

2. The model

The acousto-optical or electro-elastic coupling (which we will use interchangeably) describes the interaction between the electromagnetic field and the strain field. The minimal model for such coupling is obtained by expanding the dipole moment, $p(r)$, of a helium atom around its equilibrium value:

$$p(r) \approx p(R_i) + (u_i \cdot \nabla) p |_{R_i},$$

where $r_i = R_i + u_i$ is the position of atom $i$, $R_i$ is its equilibrium position and $u_i$ is its displacement. We would like to translate this microscopic description into a macroscopic expression with quantities that can be directly measured. The macroscopic quantity representing dipole moment is polarization, which is obtained by averaging $p$ over a macroscopic volume element $v$, $P(r) = (1/v) \sum_i p(r_i)$, where $r$ is the macroscopic coordinate indicating the center of $v$. The sum can be taken in the continuum limit, $\sum_i p(r_i) \approx (1/v) \int_v \mathrm{d}r' p(r')$, since the macroscopic length scale is much longer than the atomic length. In the presence of a local strain field, one finds from equation (1) in the linear order of the displacement

$$P(r) \approx (1/v^2) \int_v \mathrm{d}R [p(R) + (u \cdot \nabla) p(R)].$$

Integration of the first term gives the macroscopic polarization for zero internal strain (a solid in equilibrium), $P_0$. It is related to the macroscopic electric field $E$ by $P_0 \equiv \chi_0 E$, where $\chi_0$ is the zero-strain susceptibility. The second term in the integration describes $\delta P$, the polarization change due to the atoms’ displacements. The two quantities are related through integration by parts, that is, $\int \mathrm{d}r (u \cdot \nabla) p = \int_S \mathrm{d}A (u \cdot n) p - \int \mathrm{d}r (\nabla \cdot u) p$. We neglect the surface term $\int_S \mathrm{d}A u \cdot n$, because cancellation and screening of other defects localize the strain field.

The second term modifies the polarization by

$$\delta P = - (1/v^2) \int \mathrm{d}R (\nabla \cdot u) p(R) \approx -\varepsilon_{ii} P_0,$$

where

$$\varepsilon_{ii} = \frac{1}{v^2} \int \mathrm{d}R (\nabla \cdot u) p(R).$$

New Journal of Physics 13 (2011) 113024 (http://www.njp.org/)
with \(e_{ii} = (1/v) \int_v d\mathbf{R} (\nabla \cdot \mathbf{u})\) being the macroscopic dilatory strain (we use the notation \(e_{ii} \equiv e_{11} + e_{22} + e_{33}\)). For convenience we will use the strain in the remainder, instead of the atomic displacement \(\mathbf{u}\). Note that in obtaining equation (3), we make the long-wavelength approximation that \(kv^{1/3} \ll 1\). The integration of the product can then be represented as the product of the integration. This long-wavelength approximation holds well for the wavelength of microwaves \((\lambda > 10^{-3} \text{ m})\) and above. The change in local polarization corresponds to a change in dielectric function of

\[\delta\epsilon_{ii}(\omega, T) = -4\pi \chi_0 e_{ii}(\omega, T),\]  

where we have used \(e_{ii} - 1 = 4\pi P_i/E_i\). Note that only the diagonal components of the strain tensor play a role in this leading order expansion of electro-elastic coupling. In principle, the shear strain can couple to the electric field by considering dipole-induced dipole interactions (van der Waals), which is a higher order effect. The same coupling mechanism between photons (electric field) and phonons (strain) gives rise to the acousto-optical effect. Note that our derivation of equation (4) is equivalent up to leading order to the Pockels coefficient for acousto-optical coupling in isotropic or polycrystalline dielectrics, \(\delta\epsilon_{ij} = -\varepsilon^2 [2P_{44} e_{ij} + P_{12} e_{kk}\delta_{ij}]\), where \(P_{kl}\) are the reduced Pockels coefficients (of the order of unity) \([30–32]\).

Next we discuss the origin of the local strain. Depending on the rate of the solidification process of helium, various amounts and kinds of defects can be frozen into the crystal, leaving behind relics of the history of the phase transition. We postulate that defects in the glassy regions of the solid form the TLS. The associated strain fields are localized near the TLS and exhibit glassy dynamics, since nearby atoms reshuffle with an intrinsic time delay. We describe the equation of motion of these atoms within linear response theory by including a back action term \([16–21]\). In an isotropic or polycrystalline medium the elastic stress tensor \(\sigma_{ij} = \lambda_{ijkl} \partial u_k/\partial x_l\) takes a simplified form with the elastic modulus tensor \(\lambda_{ijkl} = \lambda_0 \delta_{ij}\delta_{kl} + \mu_0 (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\). If the electric field couples to local density fluctuations only through dilatory strain, then the important matrix element is the Lamé parameter \(\lambda_0\) of the purely elastic solid. We write the displacement to an out-of-equilibrium internal force in the presence of the back action as

\[\rho \partial_t^2 u_i(t) + \lambda_0 \partial_t^2 u_i(t) = f_{i,\text{INT}}(t) + f_{i,\text{BA}}(t),\]  

where \(u_i\) is the displacement of a volume element in the \(i\)th direction and \(\rho\) is the mass density. \(f_{i,\text{INT}}\) is the out-of-equilibrium internal force density in the \(i\)th direction at the defect. \(f_{i,\text{BA}}\) is the back action force density that describes the time-delayed response of nearby atoms:

\[f_{i,\text{BA}}(t) = \int_{-\infty}^{t} dt' G(t-t'; T) \partial_t^2 u_i(t'),\]  

where \(G\) is the strength of the back action onto bulk \(^4\text{He}\). Integration of equation (5) over the \(i\)th direction and Fourier transformation to frequency domain yields the dynamic strain due to a local dilatory stress \(\sigma_{ii}\):

\[\epsilon_{ii}(\omega, T) = \sigma_{ii}(\lambda_0 - G(\omega, T))^{-1}.\]  

We assumed that the back action can be described by a distribution \(P(t)\) of Debye relaxors, \(G(\omega, T) = \int_{0}^{\infty} dt P(t) [1 - 1/(1 - i\omega \tau(T) t)]\), with relaxation time \(\tau(T)\) of the glassy regions of the solid. The specific form of \(\tau(T)\) can change quantitatively, but not qualitatively, the \(T\) dependence of \(\epsilon_{ii}\). For simplicity, we choose for \(P(t)\) the Cole–Cole distribution resulting in
Figure 1. Experimental data and calculations of the dielectric function versus temperature. The red circles are the experimental data of the dielectric function (data from Yin et al [29]). The black lines are the calculated amplitude and phase lag (dissipation) of $\epsilon(\omega, T)$. We used parameters $\alpha = 1.49$, $e_0 = 8.88 \times 10^{-4}$, $g = 0.21$, $\tau_0 = 10.4$ ns, $\Delta = 1.92$ K and $T_g = -119$ mK.

$G(\omega, T) = g_0\lambda_0/[1 - (i \omega \tau)^\alpha]$. Therefore, the corresponding dilatory strain of a distribution of glassy TLSs becomes

$$e_{ii}(\omega, T) = e_0(1 - g_0/[1 - (i \omega \tau)^\alpha])^{-1}, \quad (8)$$

where $e_0 \equiv \sigma_{ii}/\lambda_0$ at $T = 0$ and $g_0$ is the glass parameter, which depends on the TLS density. From equations (8) and (4), we obtain the change in dielectric function due to local strain fluctuations,

$$\delta\epsilon_{ii} = -4\pi \chi_0 e_0(1 - g_0/[1 - (i \omega \tau)^\alpha])^{-1}. \quad (9)$$

At low temperatures, $\tau \to \infty$ and $e_{ii} \to e_0$; hence, the strain is minimal and the reduction of dielectric function due to strain fluctuations is small. At high temperatures, $\tau \to 0$, and $e_{ii} \to e_0(1 - g_0)^{-1}$ reaches its maximum, resulting in the largest reduction of strain, where solid $^4$He is softest. This is the main result of this study, namely that the dielectric function reflects the arrested dynamics of the glassy components at low temperatures through the acousto-optical (electro-elastic) coupling. Within this approximation, the maximum size of $\delta\epsilon_{ii}$ is independent of frequency and is controlled by $e_0$ and $g_0$.

3. Results and discussion

The results of the electro-elastic model for the dielectric function are shown in figure 1. We obtain excellent agreement with the measurements of Yin et al [29] for an applied alternating voltage at 500 Hz. Additionally, we predict an accompanying dissipation peak in the dielectric function or a phase lag $\phi = \arg(\epsilon)$ between the real and imaginary parts of $\epsilon(\omega, T)$. Experimental confirmation of the dissipation peak will constitute an important test of the glassy nature and the electro-elastic coupling of defects in solid helium. In our calculation, we assumed the Vogel–Fulcher–Tammann form for the glassy relaxation time: $\tau(T) = \tau_0 e^{\Delta/(T - T_g)}$. 

New Journal of Physics 13 (2011) 113024 (http://www.njp.org/)
for \( T > T_g \), and \( \tau(T) = \infty \) for \( T \leq T_g \), which describes non-Arrhenius thermal activation processes for \( T_g \neq 0 \). Here \( \tau_0 \) is the attempt time and \( \Delta \) is the mean activation energy of a TLS. In our fit, we are not biasing the ideal glass transition temperature \( T_g \) to be positive. We find a negative \( T_g \), which means that no real phase transition occurs at finite temperatures, but rather a crossover. This can be due to inherent quantum fluctuations originating from the large zero-point motion of helium atoms, which suppress the onset of a lower phase transition.

A discussion of the glass model in the context of the dielectric function requires a justification for the magnitude of the electro-elastic coupling to strain. The existence of local strain in solid \(^4\)He is inferred from measured pressure gradients. We obtain a rough estimate, for example, from the mass flow measurements in bulk solid \(^4\)He [33], where a pressure difference of \( \Delta P_L \sim 0.1 \text{ bar} \) across a centimeter-sized pressure cell is reported. The estimated local strain, with a bulk modulus \( B = 320 \text{ bar} \), is accordingly \( \Delta P_L / B = 3 \times 10^{-4} \). This is consistent with the value we used for the fit in figure 1, namely \( e_0 = 8.88 \times 10^{-4} \). In fact, the local strain may have been even larger in the experiments of Yin due to the more complicated geometry of the measurement apparatus. Alternatively, one might think of thermal fluctuations as the origin for strain leading to the observed effects. Here we show that the related density fluctuation is too small to account for the observed effect. The relative density change is given by \( \Delta \rho / \rho = \delta \rho / \rho \), where \( \delta \rho \) is the pressure fluctuation due to thermal processes. Since \( \delta P_b \sim \sqrt{k_B T B / V} \) [35], where \( V \sim 10 \text{ mm}^3 \) is the volume between the capacitors, we obtain that even at \( T = 1 \text{ K} \), the change is only \( \Delta \rho / \rho \sim 5 \times 10^{-12} \). This is several orders of magnitude too small to account for the observed effect and we will neglect such density fluctuations.

Since the electro-elastic coupling model depends on the presence of defects, an important question to ask is: what is the quality of the helium crystals? For that reason, we are reanalyzing pressure measurements of Yin et al [29]. The glassy contribution of the TLS to the bulk pressure manifests itself through \( P(T) = P_0 + a_{gl} T^2 + a_{ph} T^4 \), where \( P_0 \) is the crystal pressure of bulk solid at \( T = 0 \), while the coefficients \( a_{gl} \) and \( a_{ph} \) correspond to the contributions of the TLS glass and phonons, respectively. In figure 2, we show the temperature dependence of the deviation \( \Delta P = P(T) - P_0 \). A large glassy contribution can be seen at low temperatures, similar to that in reports by others [36, 37]. The \( P(T) \) curve in Yin’s measurement clearly deviates from a purely Debye lattice behavior at around \( T = 0.4 \text{ K} \), with a large positive intercept corresponding to the order of 100 ppm of TLS [16, 17]. This number is roughly five times larger than the most disordered sample in Lin’s [34] specific heat experiments on ultrapure \(^4\)He with less than 1 ppb of \(^3\)He impurities grown over 4 h using the blocked capillary method [16, 17, 34]. Clearly crystals grown by Yin et al are disordered with sufficiently many TLS defects to support centers of local strain fields.

So far we have assumed that both local and global stress are constant at low temperatures. From figure 2, we can see that the global pressure change between 300 and 40 mK is less than \( \Delta P_T = 0.18 \text{ mbar} \). This is more than three orders of magnitude smaller than the local stress \( \sigma_L = 8.88 \times 10^{-4} \times 320 \text{ bar} \sim 250 \text{ mbar} \) inferred from the dilatory strain \( e_0 \) used in the fit, as well as the static pressure difference \( \Delta P_L = 0.1 \text{ bar} \) measured at two pressure gauges in the mass flow experiment [33]. Taking all these observations together, we find that the change of the dielectric function based on global density changes in the Clausius–Mossotti equation is negligible. The corresponding density change is \( \Delta \rho / \rho = \Delta P_T / B < 10^{-6} \), leading to a change in the dielectric function of \( \delta \varepsilon \approx (\varepsilon - 1) \Delta \rho / \rho = 0.065 \Delta P_T / B < 10^{-7} \), which is more than two orders of magnitude too small to account for the observed effect \( \sim 10^{-5} \).
Figure 2. Low-temperature pressure deviation from lattice contribution of the Debye solid (data from Yin et al [29]). The intercept of $\Delta P/T^2$ versus $T^2$ extrapolated from low-temperature data points is in agreement with a glassy contribution from TLS of the order of 100 ppm. The arrow indicates the onset of deviation from the Debye solid. The large scatter of data points at lowest temperatures is due to the subtraction of a constant pressure $P_0$. Dashed lines are a guide to the eye.

Finally, we point out that the effect described here should only be important for nonpolarized solids. In a polarized material, the change in the dielectric function due to atoms sensing different local fields is even more difficult to detect because of the large polarizability of the material. Among the nonpolarized solids, solid $^4$He favors the observation of this phenomenon, especially because of its softness. Rapid cooling of solid helium allows a large local strain build-up, which is proportional to the size of the effect. For the same reason, we expect that the effect should also be seen in solid $^3$He, hydrogen or xenon. In any case it would be a delicate experiment ($<10^{-5}$ effect) to see this phenomenon.

4. Conclusion

In conclusion, we have shown that arrested glass dynamics cause low-temperature anomalies in strained solid $^4$He through the acousto-optical (electro-elastic) coupling. We argued that the temperature behavior of the dielectric function is coupled to local strain fields near crystal defects. Thus it records the glassy dynamics and concomitant freeze-out of the hypothesized TLS excitations, which also leads to a stiffening of the solid with decreasing temperature. This effect is not captured by the standard Clausius–Mossotti relation, which attributes dielectric function changes to a change in mass density or polarizability of the helium atom. An important consequence of the phenomenological glass susceptibility is the decrease of the dielectric function at high temperatures, accompanied by a broad dissipation peak that can be measured by the imaginary part of the dielectric function. We hypothesize that the cooperative motion of atoms forming the TLS along dislocation segments is the relevant process contributing to
the reported anomaly. In our model, both the change in $\epsilon (\omega, T)$ and the dissipation peak are to leading order independent of the applied frequency. Since the glass parameter $g_0$ of the back action depends on the concentration of defects, we predict that the change in dielectric function will be larger in quench-cooled or shear-stressed samples.

Acknowledgments

We acknowledge fruitful discussions with Z Nussinov, J C Davis and W Harrison. We are grateful to L Yin and N Sullivan for explaining their experiments and sharing their data. This work was supported by the US DOE at Los Alamos National Laboratory under contract no. DE-AC52-06NA25396 through the LDRD program, the Office of Basic Energy Sciences (BES), and the Center for Integrated Nanotechnologies, a BES user facility.

References

[1] Kim E and Chan M H W 2004 Nature 427 225
[2] Rittner A S and Reppy J D 2006 Phys. Rev. Lett. 97 165301
[3] Aoki Y, Graves J C and Kojima H 2007 Phys. Rev. Lett. 99 015301
[4] Kondo M, Takada S, Shibayama Y and Shirahama K 2007 J. Low Temp. Phys. 148 695
[5] Penzev A, Yasuta Y and Kubota M 2007 J. Low Temp. Phys. 148 677
[6] Mukharsky Yu, Penzev A and Varoquaux E Phys. Rev. B 80 140504
[7] Hunt B, Pratt E, Gadagkar V, Yamashita M, Balatsky A V and Davis J C 2009 Science 324 632
[8] Kim D Y, Kwon S, Choi H, Kim H C and Kim E 2010 New J. Phys. 12 033004
[9] Pratt E, Hunt B, Gadagkar V, Yamashita M, Graf M J, Balatsky A V and Davis J C 2011 Science 332 821
[10] Day J and Beamish J 2007 Nature 450 853
[11] Day J, Syschenko O and Beamish J 2009 Phys. Rev. B 79 214524
[12] Syschenko O, Day J and Beamish J 2010 Phys. Rev. Lett. 104 195301
[13] Burovskii E, Kozik E, Kulakov A, Prokof’ev N and Svistunov B 2005 Phys. Rev. Lett. 94 165301
[14] Boninsegni M, Kulakov A B, Pollet L, Prokof’ev N V, Svistunov B V and Troyer M 2007 Phys. Rev. Lett. 99 035301
[15] Pollet L, Boninsegni M, Kulakov A B, Prokof’ev N V, Svistunov B V and Troyer M 2007 Phys. Rev. Lett. 98 135301
[16] Balatsky A V, Graf M J, Nussinov Z and Trugman S A 2007 Phys. Rev. B 75 094201
[17] Su J-J, Graf M J and Balatsky A V 2010 J. Low Temp. Phys. 159 431
[18] Nussinov Z, Balatsky A V, Graf M J and Trugman S A 2007 Phys. Rev. B 76 014530
[19] Graf M J, Balatsky A V, Nussinov Z, Grigorenko I and Trugman S A 2009 J. Phys.: Conf. Ser. 150 032025
[20] Graf M J, Nussinov Z and Balatsky A V 2010 J. Low Temp. Phys. 158 550
[21] Graf M J, Su J-J, Dahal H P, Grigorenko I and Nussinov Z 2011 J. Low Temp. Phys. 162 500
[22] Su J-J, Graf M J and Balatsky A V 2010 Phys. Rev. Lett. 105 045302
[23] Su J-J, Graf M J and Balatsky A V 2011 J. Low Temp. Phys. 162 433
[24] Andreev A F 2009 J. Exp. Theor. Phys. 109 103
[25] Korshunov S E 2009 J. Exp. Theor. Phys. Lett. 90 156

New Journal of Physics 13 (2011) 113024 (http://www.njp.org/)
[26] Reppy J D 2010 Phys. Rev. Lett. 104 255301
[27] Kirkwood J G 1936 J. Chem. Phys. 4 592
[28] Chan M, Ryschkewitsch M and Meyer H 1977 J. Low Temp. Phys. 26 211
[29] Yin L, Xia J S, Huan C, Sullivan N S and Chan M H W 2011 J. Low Temp. Phys. 162 407
[30] Werthamer N R 1969 Phys. Rev. 185 348
[31] Kisliuk A, Kudlik A, Sukmamowski J, Loheider S, Soltwisch M and Quitmann D 1991 J. Phys.: Condens. Matter 3 9831
[32] Bell J S, Kearslev M J, Pitaevskii L P, Landau L D, Lifshitz E M and Sykes J B 1984 Electrodynamics of Continuous Media 2nd edn (Oxford: Pergamon)
[33] Ray M W and Hallock R B 2010 Phys. Rev. Lett. 105 145301
[34] Lin X, Clark A C, Cheng Z G and Chan M H W 2009 Phys. Rev. Lett. 102 125302
[35] Wallace D C 2000 Phys. Rev. E 62 3077
[36] Grigor’ev V N, Maidanov V A, Rubanskii V Yu, Rubets S P, Rudavskii E Ya, Rybalko A S, Syrnikov Ye V and Tikhii V A 2007 Phys. Rev. B 76 224524
[37] Lisunov A A, Maidanov V A, Rubanskii V Yu, Rubets S P, Rudavskii E Ya, Rybalko A S and Tikhiy V A 2011 Phys. Rev. B 83 132201