Noncommutative Tachyon from
Background Independent Open String Field Theory

Kazumi Okuyama

*Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan*

kazumi@post.kek.jp

We analyze the tachyon field in the bosonic open string theory in a constant $B$-field background using the background independent open string field theory. We show that in the large noncommutativity limit the action of tachyon field is given exactly by the potential term which has the same form as in the case without $B$-field but the product of tachyon field is taken to be the star product.

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1. Introduction

The problem of describing the process of tachyon condensation has attracted many people. By turning on a constant $B$-field, one can handle the behavior of tachyon by using the so-called noncommutative tachyon [4,5] and it enables us to construct the lower dimensional brane easily as a topological defect on an unstable brane containing tachyonic modes. To understand the fate of tachyon more thoroughly we need the information of the form of tachyon potential. Very recently, the problem of the tachyon potential was studied in [6] using the background independent open string field theory [7].

In this paper, combining these two ideas we consider tachyon field in the bosonic open string theory, or tachyon on the $D25$-brane, in a constant $B$-field background and calculate the action of tachyon field using the background independent open string field theory. We show that in the large noncommutativity limit the potential of the form $e^{-T}(T+1)$ with the product of fields taken by the star product gives the exact action of tachyon.

This paper is organized as follows: In section 2, we calculate the action of tachyon field in a $B$-field background following [8]. In section 3, we study this action in the derivative expansion and in the large noncommutativity limit. Section 4 is devoted to discussions.

2. Open String Field Theory in a Constant $B$-Field Background

In this section, we calculate the action of tachyon field in the presence of a background $B$-field using the background independent open string field theory following the procedure in [8]. As we will see, most of the calculation are parallel to those in [8] without $B$-field.

2.1. Green’s Function

The worldsheet action in the bulk of the disc $\Sigma$ in the background of the metric $g_{\mu\nu}$ and the $B$-field $B_{\mu\nu}$ is given by

$$S_\Sigma = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\xi \sqrt{h^{ab}g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2} \int_{\Sigma} B_{\mu\nu} dX^\mu \wedge dX^\nu.}$$

We take $\Sigma$ to be a unit disc $\{|z| \leq 1\}$ and the worldsheet metric to be flat:

$$h_{ab}d\xi^a d\xi^b = dzd\overline{z} = dr^2 + r^2 d\sigma^2$$
where \( z = re^{i\sigma} \). In the formalism of \([7]\), the space of string field is identified as the space of boundary deformation. For the tachyon field \( T(X) \), the boundary action is given by

\[
S_{\partial\Sigma} = \int_0^{2\pi} \frac{d\sigma}{2\pi} T(X(\sigma)).
\]  

(2.3)

Following \([8]\), we consider the tachyon configuration which is quadratic in \( X^\mu \):

\[
T(X) = a + \frac{1}{2\alpha'} u_{\mu\nu} X^\mu X^\nu
\]  

(2.4)

with \( u_{\mu\nu} = u_{\nu\mu} \). Then the worldsheet theory is free and exactly solvable. From \( S_\Sigma \) and \( S_{\partial\Sigma} \), the boundary condition of \( X^\mu \) becomes

\[
\left( g_{\mu\nu} \partial_\tau X^\nu - i2\pi\alpha' B_{\mu\nu} \partial_\sigma X^\nu + u_{\mu\nu} X^\nu \right)_{|_{\tau=1}} = 0.
\]  

(2.5)

To calculate the action of tachyon field, the Green’s function

\[
M^{\mu\nu}(z, w) = \left\langle X^\mu(z) X^\nu(w) \right\rangle
\]  

(2.6)

plays a crucial role. \( M(z, w) \) should satisfy the Laplace equation

\[
-\frac{1}{2\pi\alpha'} g_{\mu\nu} \Delta_z M^{\nu\rho}(z, w) = \delta^2(z - w) \delta^\rho_\mu
\]  

(2.7)

and the boundary condition (2.5). By a straightforward calculation, we find the Green’s function to be

\[
\begin{align*}
\frac{2}{\alpha'} M(z, w) &= G^{-1} F_N(z, w) - \Theta G \Theta F_D(z, w) + 2\pi i \Theta F_\epsilon(z, w) \\
&\quad + 2u^{-1} - 2 \sum_{k=1}^{\infty} \frac{1}{k} \left( \frac{E_+ u}{k + E_+ u} E_+(z\bar{w})^k + E_- \frac{uE_-}{k + uE_-} (\bar{z}w)^k \right)
\end{align*}
\]  

(2.8)

where

\[
\begin{align*}
F_N(z, w) &= -\log |z - w|^2 - \log |1 - z\bar{w}|^2, \\
F_D(z, w) &= -\log |z - w|^2 + \log |1 - z\bar{w}|^2, \\
F_\epsilon(z, w) &= \frac{1}{\pi i} \log \left( \frac{1 - \bar{z}w}{1 - z\bar{w}} \right),
\end{align*}
\]  

(2.9)

and \( G, \Theta \) are the open string parameters defined by

\[
\frac{1}{g + 2\pi\alpha' B} = G^{-1} + \Theta.
\]  

(2.10)

\footnote{In the case \( g_{\mu\nu} = \delta_{\mu\nu} \), the Green’s function in \( B \)-field background was obtained in \([10]\).}
\( E_{\pm} \) in (2.8) denotes the combination

\[
E_{\pm} = G^{-1} \pm \Theta. \tag{2.11}
\]

In the component notation, \( G = (G_{\mu\nu}) \) and \( G^{-1} = (G^{\mu\nu}) \).

The Green's function at the boundary of \( \Sigma \) becomes

\[
\frac{2}{\alpha'} M(\sigma, \sigma') = 2u^{-1} + 2\sum_{k=1}^{\infty} \left[ \frac{1}{k + E_+} e^{ik(\sigma - \sigma')} + E_- \frac{1}{k + uE_-} e^{-ik(\sigma - \sigma')} \right]. \tag{2.12}
\]

Note that \( F_\epsilon \) at the boundary of \( \Sigma \) is reduced to the sign function \( \epsilon(\sigma) \)

\[
F_\epsilon(e^{i\sigma_1}, e^{i\sigma_2}) \to \epsilon(\sigma_1 - \sigma_2) \tag{2.13}
\]

in the limit \( \sigma_1 - \sigma_2 \to 0 \). From this relation, the boundary coordinates become noncommutative \([11,12,13,14]\)

\[
[X^\mu(\sigma), X^{\nu}(\sigma)] = i\theta^{\mu\nu} \tag{2.14}
\]

where \( \theta \) is related to \( \Theta \) by

\[
\Theta^{\mu\nu} = \frac{\theta^{\mu\nu}}{2\pi \alpha'}. \tag{2.15}
\]

### 2.2. Partition Sum

The second step to calculate the action is to calculate the partition sum \( Z \) on the disc which is defined by

\[
Z(a, u) = \int D X e^{-S_\Sigma - S_{\partial \Sigma}} = e^{-a} Z(u). \tag{2.16}
\]

Here we factored out the dependence of the zero mode \( a \) of tachyon field. To calculate \( Z \), we first consider the derivative of it:

\[
\frac{d}{du_{\mu\nu}} \log Z(u) = -\frac{1}{4\pi \alpha'} \int_0^{2\pi} d\sigma \left< X^\mu(\sigma) X^{\nu}(\sigma) \right>. \tag{2.17}
\]

We define the Green’s function at the same point by subtracting the divergent part

\[
\left< X^\mu(\sigma) X^{\nu}(\sigma) \right> = \lim_{\delta \to 0} \left[ \left< X^\mu(\sigma + \delta) X^{\nu}(\sigma) \right> - \frac{\alpha'}{2} C^{\mu\nu}(\delta) \right] \tag{2.18}
\]

where

\[
C(\delta) = G^{-1} F_N(\sigma + \delta, \sigma) + 2\pi i \Theta F_\epsilon(\sigma + \delta, \sigma) = -2E_+ \log(1 - e^{i\delta}) - 2E_- \log(1 - e^{-i\delta}). \tag{2.19}
\]
With this regularization we find

\[
\frac{d}{du} \log Z(u) = -\frac{1}{2} u^{-1} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{k} \left( \frac{E_+ u}{k + E_+ u} E_+ + E_- \frac{u E_-}{k + u E_-} \right). \tag{2.20}
\]

Using the identity of Gamma function

\[
\frac{d}{dx} \log \Gamma(x) = -\frac{1}{x} - \gamma + \sum_{k=1}^{\infty} \frac{x}{k(k+x)}, \tag{2.21}
\]

(2.20) becomes

\[
\frac{d}{du} \log Z(u) = \frac{1}{2} u^{-1} + \frac{1}{2} \frac{d}{du} \left( \text{tr} \log \Gamma(E_+ u) + \text{tr} \log \Gamma(u E_-) \right) + \frac{1}{2} \gamma(E_+ + E_-). \tag{2.22}
\]

Here \(\gamma\) is the Euler’s constant. We use the notation ‘\text{tr}’ and ‘\text{det}’ for the trace and the determinant over the spacetime indices \(\mu, \nu\). By integrating this relation, we find

\[
Z(a, u) = e^{-a + \gamma \text{tr}(G^{-1} u)} \det \frac{1}{2} \left( \Gamma(E_+ u) \Gamma(u E_-) u E_- \right) \\
= e^{-a + \gamma \text{tr}(G^{-1} u)} \det \frac{1}{2} \left( \Gamma(E_+ u) \Gamma(1 + u E_-) \right). \tag{2.23}
\]

Here we include the factor \(\det \frac{1}{2} (E_-)\) which cannot be determined from (2.22). We will see in section 3 that this factor is needed to reproduce the Born-Infeld action. In principle we can calculate the normalization of \(Z\) as reviewed in [15], but we do not discuss it here.

Using the identity for an arbitrary function \(f(x)\) and finite size matrices \(A, B\)

\[
\det \left( f(B A B^{-1}) \right) = \det \left( f(A) \right), \quad \det \left( f(A) \right) = \det \left( f(A^T) \right), \tag{2.24}
\]

\(Z\) can be written as

\[
Z(a, u) = e^{-a + \gamma \text{tr}(G^{-1} u)} \det \left( (E_+ u) \frac{1}{2} \Gamma(E_+ u) \right). \tag{2.25}
\]

Note that \(Z(a, u)\) depends on the \(B\)-field only through the combination \(E_\pm\).

2.3. Evaluation of Action

As the final step, we construct the action of tachyon field in a \(B\)-field background. The action \(S\) of open string field theory is given by [7]

\[
dS = \frac{1}{2} \int_{0}^{2\pi} \frac{d\sigma d\sigma'}{(2\pi)^2} \langle d\mathcal{O}(\sigma) \{ Q_B, \mathcal{O} \}(\sigma') \rangle. \tag{2.26}
\]
For the generic boundary deformation

\[ \int_0^{2\pi} \frac{d\sigma}{2\pi} \mathcal{V}, \quad (2.27) \]

\( \mathcal{O} \) and \( \mathcal{V} \) are related by \( \mathcal{O} = c \mathcal{V} \). The exterior derivative \( d \) is taken on the couplings in the boundary interaction \( \mathcal{V} \), or \( a \) and \( u \) in this case. The BRST transformation of \( \mathcal{O} \) corresponding to the tachyon is given by

\[ \{ Q_B, cT(X) \} = c \partial_\sigma c(1 - \Delta_T) T(X) \quad (2.28) \]

where \( \Delta_T \) is the dimension of \( T \). As a differential operator acting on \( T(X) \), \( \Delta_T \) can be written as

\[ \Delta_T = -\alpha' G^{\mu\nu} \frac{\partial^2}{\partial X^\mu \partial X^\nu}. \quad (2.29) \]

Therefore

\[ \{ Q_B, cT(X) \} = c \partial_\sigma c \left( \text{tr}(G^{-1}u) + a + \frac{1}{2\alpha'} u_{\mu\nu} X^\mu X^\nu \right). \quad (2.30) \]

Using the ghost correlation function

\[ \langle c(\sigma)c(\sigma')\partial_\sigma c(\sigma') \rangle = 2 \left[ \cos(\sigma - \sigma') - 1 \right], \quad (2.31) \]

\((2.26)\) becomes

\[ dS = \int_0^{2\pi} \frac{d\sigma d\sigma'}{(2\pi)^2} \left( \cos(\sigma - \sigma') - 1 \right) \]

\[ \times \left\langle \left( da + \frac{1}{2\alpha'} d\mu_{\mu\nu} X^\mu X^\nu(\sigma) \right) \left( a + \text{tr}(G^{-1}u) + \frac{1}{2\alpha'} u_{\rho\tau} X^\rho X^\tau(\sigma') \right) \right\rangle. \quad (2.32) \]

To show that the right-hand-side of \((2.32)\) is an exact form, we need an identity corresponding to eq.(2.22) in [8]:

\[ \int_0^{2\pi} \frac{d\sigma d\sigma'}{(4\pi\alpha')^2} \cos(\sigma - \sigma') d\mu_{\mu\nu} u_{\rho\tau} \left\langle X^\mu X^\nu(\sigma) X^\rho X^\tau(\sigma') \right\rangle = \text{tr}(G^{-1}u). \quad (2.33) \]

Using the following relations

\[ \int_0^{2\pi} \frac{d\sigma d\sigma'}{(2\pi)^2} \left( \cos(\sigma - \sigma') - 1 \right) \frac{1}{2\alpha'} \left\langle X^\mu X^\nu(\sigma) \right\rangle = \frac{\partial}{\partial u_{\mu\nu}} Z, \quad (2.34) \]

\[ \int_0^{2\pi} \frac{d\sigma d\sigma'}{(4\pi\alpha')^2} \left\langle X^\mu X^\nu(\sigma) X^\rho X^\tau(\sigma') \right\rangle = \frac{\partial^2}{\partial u_{\mu\nu} \partial u_{\rho\tau}} Z, \quad (2.35) \]
and (2.33), we finally find that $S(a, u)$ is related to $Z(a, u)$ by

$$S(a, u) = \left[ \text{tr}(G^{-1}u) - a \frac{\partial}{\partial a} - \text{tr} \left( u \frac{\partial}{\partial u} \right) + 1 \right] Z(a, u). \tag{2.35}$$

### 3. Noncommutative Tachyon

In this section, we examine the action of tachyon field given by (2.35). As shown in [16], a $D$-brane in a $B$-field background can be described by either commutative or noncommutative language. We will show that the derivative expansion of the action (2.35) leads to the commutative description. On the other hand, by taking the large noncommutativity limit $S(a, u)$ reproduce the action of the noncommutative tachyon. In the following discussion, we are not careful about the overall normalization of the action. See [6,5] for recent discussions on the normalization of action. See also [17] for the early discussion on the action of tachyon in a constant $B$-field background.

#### 3.1. Commutative Description of Tachyon

Let us consider the case of nearly constant tachyon, i.e., $u \sim 0$. Then the expansion of $S(a, u)$ with respect to $u$ corresponds to the derivative expansion of tachyon field. When $u$ is small, partition sum $Z(a, u)$ becomes

$$Z(a, u) = e^{-a} \det^{-\frac{1}{2}}(E_+ u) + \cdots = T_{D25} \int d^{26}x \mathcal{L}_{B1}(B) e^{-T(x)} + \cdots \tag{3.1}$$

with $\mathcal{L}_{B1}(B) = \sqrt{\det(g + 2\pi\alpha'B)}$ and $T_{D25} = (2\pi\alpha')^{-13}$ in this normalization of $Z$. However, remember that the overall normalization of $Z$ cannot be determined within this framework. What we can say at most is that $T_{D25}$ is proportional to $(\alpha')^{-13}$ from the dimensional analysis. The dots in (3.1) denote the higher order terms in $u$, or the higher derivative terms of $T(x)$. (See [4] for the structure of the higher derivative terms.)

From this form of partition sum, we can calculate the action of tachyon field $S(a, u)$. The first term in (2.35), which originates from $-\Delta_T$, gives the kinetic term for the tachyon and the other terms correspond to the potential. First we calculate the kinetic term. Using the relation

$$\text{tr}(G^{-1}u) = -\Delta_T T = \alpha' G^{\mu\nu} \partial_\mu \partial_\nu T, \tag{3.2}$$

we find that the first term in (2.35) is related to the kinetic term of $T(x)$ by

$$\text{tr}(G^{-1}u) \int d^{26}x e^{-T} = \int d^{26}x e^{-T} \alpha' G^{\mu\nu} \partial_\mu \partial_\nu T = \int d^{26}x e^{-T} \alpha' G^{\mu\nu} \partial_\mu T \partial_\nu T. \tag{3.3}$$
The differential operator in the second and the third term in (2.35) generates the scale transformation $T(x) \rightarrow \lambda T(x)$. Therefore, we find

$$\left[-a \frac{\partial}{\partial a} \text{tr} \left( u \frac{\partial}{\partial u} \right)\right] \int d^{26} x \, e^{-T(x)} = - \left. \frac{d}{d\lambda} \right|_{\lambda=1} \int d^{26} x \, e^{-\lambda T(x)} = \int d^{26} x \, T(x) e^{-T(x)}. \quad (3.4)$$

Combining (3.3) and (3.4), the action is found to be

$$S = T_{D25} \int d^{26} x \, L_{BI}(B) \, e^{-T} \left( \alpha' G^{\mu\nu} \partial_{\mu} T \partial_{\nu} T + T + 1 \right) + \cdots. \quad (3.5)$$

Since the $B$-dependence in the above action is given by the Born-Infeld form, this expansion corresponds to the commutative description of $D25$-brane.

3.2. Large Noncommutativity Limit

As was pointed out in [1], in the large noncommutativity limit, the problem of the tachyon condensation is drastically simplified since the kinetic term of tachyon disappears in this limit. We show that this phenomenon also occurs in $S(a, u)$ and the structure of star product emerges, which cannot be seen in the derivative expansion.

By the large noncommutativity limit, we mean the situation

$$G^{-1} \ll \Theta \quad (3.6)$$

or equivalently $G^{-1}$ is set to zero while $\Theta$ is kept finite. In this limit, the partition sum $Z(a, u)$ becomes

$$\lim_{G \Theta \rightarrow \infty} Z(a, u) = e^{-a} \det \frac{1}{2} \left( \Gamma(\Theta u) \Gamma(1 - \Theta u) \right) = e^{-a} \det \frac{1}{2} \left( \frac{\pi}{\sin \pi \Theta u} \right). \quad (3.7)$$

To show that this form of $Z$ leads to the noncommutative description of tachyon, let us introduce the quantity $\Xi(a, u)$ by

$$\Xi(a, u) = \int \frac{d^{26} x}{\text{Pf}(2\pi \theta)} \exp_{\star}(-T(x)) \quad (3.8)$$

where $\exp_{\star}$ means that the product of $T(x)$ is taken by the star product

$$f \star g = f \exp \left( \frac{i}{2} \partial_{\mu} \theta^{\mu\nu} \partial_{\nu} \right) g. \quad (3.9)$$
In the operator language, $\Xi$ is written as

$$\Xi(a, u) = \text{Tr}_\mathcal{H} \exp \left( - T(\hat{x}) \right)$$

(3.10)

where the trace is taken over the Hilbert space $\mathcal{H}$ on which the operators $\hat{x}^\mu$ satisfy the relation

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta_{\mu\nu}.$$  

(3.11)

Eq. (3.10) shows that $\Xi$ can be interpreted as a thermal partition function on the phase space $\{x^\mu\}$ with Hamiltonian $T(x)/2\pi$ and inverse temperature $\beta = 2\pi$. Therefore, $\Xi$ can be written as a path integral on $S^1$:

$$\Xi(a, u) = \int \mathcal{D}x \exp \left( \int_0^{2\pi} d\sigma i \frac{1}{2} x^\mu(\sigma)(\theta^{-1})_{\mu\nu} \partial_\sigma x^\nu(\sigma) - \frac{T(x(\sigma))}{2\pi} \right).$$

(3.12)

Up to a proportionality constant, $\Xi$ can be evaluated as

$$\Xi(a, u) = e^{-a} \int \mathcal{D}x \exp \left( \int_0^{2\pi} d\sigma \frac{i}{2} x^\mu(\sigma)(\theta^{-1})_{\mu\nu} \partial_\sigma x^\nu(\sigma) - \frac{u_{\mu\nu} x^\mu(\sigma) x^\nu(\sigma)}{4\pi\alpha'} \right)$$

$$= e^{-a} \text{Det}^{-\frac{1}{2}} \left( -i\theta^{-1} \partial_\sigma + \frac{u}{2\pi\alpha'} \right)$$

$$\approx e^{-a} \text{Det}^{-\frac{1}{2}} \left( -i\partial_\sigma + \Theta u \right)$$

$$= e^{-a} \det^{-\frac{1}{2}} \prod_{n \in \mathbb{Z}} (n + \Theta u)$$

$$\approx \frac{e^{-a}}{\det \frac{1}{4} \left( \sin \pi \Theta u \right)}.$$

(3.13)

The proportionality constant can be fixed from the behavior of the right-hand-side of (3.8) in the limit $\theta \to 0$. Then we find that $\Xi$ is given by

$$\Xi(a, u) = \frac{e^{-a}}{\det \frac{1}{4} \left( 2 \sin \pi \Theta u \right)}.$$  

(3.15)

\footnote{This form can also be deduced from the partition function of a harmonic oscillator}

$$Z_{\text{osci}} = \text{Tr} e^{-\beta H} = \frac{1}{2 \sinh \left( \frac{1}{2} \beta \hbar \omega \right)}$$

(3.14)

with $H = \frac{1}{2} (p^2 + \omega^2 x^2)$ and $[x, p] = i\hbar$. The difference between $\sin$ and $\sinh$ in $\Xi$ and $Z_{\text{osci}}$ comes from the fact that the eigenvalues of $\Theta$ correspond to $\pm i\hbar$.  

From (3.7) and (3.15), we conclude that in the large noncommutativity limit $Z(a,u)$ is equal to $\Xi(a,u)$ up to a normalization factor:

$$\lim_{G\Theta \to \infty} Z(a,u) = \text{Tr}_H \exp(-T(\hat{x})) = \int \frac{d^{26}x}{\text{Pf}(2\pi\theta)} \exp_*(-T(x)).$$ (3.16)

Since $G^{-1} \ll \Theta$ is equivalent to $g \ll 2\pi\alpha' B$, the large noncommutativity limit can be rephrased as the large $B$ limit. In this limit, the bulk worldsheet action (2.1) is given by the $B$-field term alone, which is a total derivative. Therefore, we expect that the partition sum $Z$ is reduced to the quantum mechanics on the boundary of $\Sigma$, which is nothing but (3.12) because $\theta = B^{-1}$ in this limit. Our result (3.16) strongly suggests that the regularization (2.18) used in the calculation of $Z$ is the correct choice, since it leads to the expected result in the large $B$ limit.

From the relation (3.16), we can calculate the action of $T$. Since we have set $G^{-1} = 0$, the first term in (2.35) is zero. Using the relation

$$\left[-a \frac{\partial}{\partial a} - \text{tr} \left(u \frac{\partial}{\partial u}\right)\right] \text{Tr}_H \exp(-T(\hat{x})) = \text{Tr}_H T(\hat{x}) \exp(-T(\hat{x}))$$ (3.17)

the action of $T(x)$ in the large noncommutativity limit is found to be

$$S = \text{Tr}_H \left((T(\hat{x}) + 1) \exp(-T(\hat{x}))\right) = \int \frac{d^{26}x}{\text{Pf}(2\pi\theta)} (T(x) + 1) \exp_*(-T(x)).$$ (3.18)

As is clear from (3.7), this action contains terms of all order in $u$ and is exact in this limit. Note that the higher derivative terms represented by dots in (3.5) appear only through the star product. In [16], it was shown that the S-matrix depends on $B$-field only through the Moyal phase in the noncommutative description. Our result can be thought of as an off-shell extension of the argument in [16].

In this subsection, we assumed that $\Theta$ has the maximal rank. In the lower rank case, the derivatives of tachyon vanish along the directions of non-zero $\Theta$ in the large $\Theta$ limit.

4. Discussion

In this paper, we constructed the action of tachyon field in a $B$-field background using the background independent open string field theory. In the large noncommutativity limit, we found that the action of tachyon is given by the potential term which has the same form as in the case of vanishing $B$-field but the product is taken by the star product.
It will be important to compare our result to the cubic open string field theory in a constant $B$-field background studied in [18]. It may be also interesting to study the relation to the discussion in [21] which says that in the large noncommutativity limit a string field factorizes into a oscillator part and an element of the algebra representing the noncommutative geometry. Another interesting problem is the tachyon condensation in the $D\bar{D}$ system. But due to the non-Abelian nature of Chan-Paton factors it seems that the calculation is not so straightforward. In a $B$-field background, a $D$-brane can be described as a collection of infinitely many lower dimensional branes [22,23,24,25]. The relation between this Matrix Theory picture and the open string field theory deserves to be studied further.

We comment on the kinetic term of tachyon field in the case of non-zero $G^{-1}$. As is well known, the noncommutative gauge theory naturally appears in Matrix Theory when it is expanded around the noncommutative classical solution. In this picture, the derivative of a field is given by the commutator of the matrix coordinate and that field, and it becomes naturally the noncommutative covariant derivative. Therefore, it may be important to include the gauge field into the analysis of tachyon field and study the background independent description using the matrix variables along the line of [26].

**Note added:** After this work was completed, we received a paper [27] which has some overlaps with ours.

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