Rare $t$-quark decays $t \to c \ell_j^+ \ell_k^-$, $t \to c \tilde{\nu}_j \nu_k$

in the minimal four color symmetry model

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Abstract

The rare $t$-quark decays $t \to c \ell_j^+ \ell_k^-$, $t \to c \tilde{\nu}_j \nu_k$ via the scalar leptoquark doublets are investigated in the minimal four color symmetry model with the Higgs mechanism of the quark-lepton mass splitting. The partial widths of these decays are calculated and the total width of the charged lepton mode $\Gamma(t \to c \ell^+ \ell^-) = \sum_{j,k} \Gamma(t \to c \ell_j^+ \ell_k^-)$ and the total width of the neutrino mode $\Gamma(t \to c \tilde{\nu}_j \nu_k) = \sum_{j,k} \Gamma(t \to c \tilde{\nu}_j \nu_k)$ are found. The corresponding branching ratios are shown to be

$$Br(t \to c \ell^+ \ell^-) \approx (3.5 - 0.4) \cdot 10^{-5},$$
$$Br(t \to c \tilde{\nu} \nu) \approx (7.1 - 0.8) \cdot 10^{-5}$$

for the scalar leptoquark masses $m_S = 180 - 250$ GeV and for the appropriate values ($\sin \beta \approx 0.2$) of the mixing angle of the model. The search for such decays at LHC may be of interest.

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The search for the possible signs of a new physics beyond the Standard Model (SM) will be one of the goals of the experiments at LHC and at the other future colliders. Putting LHC into operation will essentially enlarge the possibilities for the experiments with t-quark, including the search for the non-SM effects in the top physics [1, 2]. In particular, the LHC with its very large top samples (about $10^7 - 10^8$ top quark pairs per year [2]) will allow the experimental investigation of the rare t-quark decays which are forbidden or have unobservable small widths in the SM but can be essentially enhanced in some extensions of the SM. The detection of such decays at LHC would be an evident signal of new physics beyond the SM.

The most investigated rare decays of t-quark are the FCNC decays $t \rightarrow cX$, $X = \gamma, Z, g, H$. These decays are very suppressed in the SM ($Br_{SM}(t \rightarrow cX) \sim 10^{-13}$ and $\sim 10^{-11}$ for $X = \gamma, Z, H$ and for $X = g$ [3–5]) but they can be essentially enhanced in some extensions of the SM. For example, in the minimal supersymmetric standard model (MSSM) the branching ratios of these decays can amount to the values $Br_{MSSM}(t \rightarrow cX) \sim 10^{-8}, \sim 10^{-6}, \sim 10^{-4}$ for $X = \gamma, Z, X = g, X = h^0$ [6–12], in the two Higgs doublet model they can be enhanced up to $Br_{2HDM}(t \rightarrow cX) \sim 10^{-7}, \sim 10^{-8}, \sim 10^{-5}, \sim 10^{-4}$ for $X = \gamma, Z, g, h$ [3, 13, 14], some enhancement of these decays can also take place in the model with additional quark singlets [15].

One of the possible variants of new physics beyond the SM can be the variant induced by the four color symmetry between quarks and leptons of Pati-Salam type [16]. The immediate consequence of this symmetry is the prediction of the gauge leptoquarks which, however, occur to be relatively heavy. For example, the most stringent lower mass limit for the vector leptoquarks (resulted from the unobservation of the $K^0_L \rightarrow \mu^\pm e^\mp$ decays) is of order of $10^3$ TeV. Such heavy leptoquarks can affect the physics at energies of order of or below 1 TeV very weakly.

It should be noted, however, that in addition to the gauge leptoquarks the four color symmetry can predict also the new particles in the scalar sector. Thus, in the case of the Higgs mechanism of splitting the masses of quarks and leptons the four color symmetry in its minimal realization on the gauge group $G = SU_V(4) \times SU_L(2) \times U_R(1)$ (MQLS-model [17, 18]) predicts the existence of the two scalar leptoquark doublets $S^{(\pm)}$ belonging to the $(15, 2, 1)$ - multiplet of the group $G$. These scalar leptoquark doublets together with the other components of the $(15, 2, 1)$ - multiplet and with the $(1, 2, 1)$ - doublet are necessary [19] for splitting the masses of quarks from those of leptons by the Higgs mechanism and for generating the
quark - lepton mass splittings including the so large mass splittings as the $b \rightarrow \tau$ and $t \rightarrow \nu_\tau$ ones. Unlike the vector leptoquarks the scalar leptoquark doublets $S^{(\pm)}$ can be relatively light, with masses of order of 400 GeV or less, without any contradictions with the $K^0_L \rightarrow \mu^\pm e^\mp$ data or with the radiative correction limits [20,21]. Because of their Higgs origin the coupling constants of these scalar leptoquark doublets are proportional to the ratios $m_f/\eta$ of the fermion masses $m_f$ to the SM VEV $\eta$. The effects of these scalar leptoquarks in the processes with the ordinary u-, d-, s- quarks are small because of the smallness of the corresponding coupling constants ($m_u/\eta \sim m_d/\eta \sim 10^{-5}, m_s/\eta \sim 10^{-3}$), whereas these effects can be significant in c-, b- and, especially, in top-physics ($m_c/\eta \sim m_b/\eta \sim 10^{-2}, m_t/\eta \sim 0.7$).

Ones of the possible new effects which can be induced by the scalar leptoquark doublets $S^{(\pm)}$ are the specific decays of $t$-quark
\begin{equation}
    t \rightarrow c \ l_j^+ \ l_k^-, \quad (1)
\end{equation}
\begin{equation}
    t \rightarrow c \ \bar{\nu}_j \ \nu_k \quad (2)
\end{equation}
with the production of c-quark with the pairs $l_{j_k}^+ l_{j_k}^-$, $j, k = 1, 2, 3$ of charged leptons $l_{j_k}^- = e^-, \mu^-, \tau^-$ and antileptons $l_{j_k}^+ = e^+, \mu^+, \tau^+$, in general of the different generation, or with the neutrino-antineutrino pairs $\bar{\nu}_j \nu_k$, $\nu_k$ are the mass eigenstates of neutrinos, $\bar{\nu}_j$ are the antineutrinos. The decays (1) in general are different from the generation diagonal decays $t \rightarrow c X \rightarrow c l_j^+ l_j^-$ predicted in models mentioned above [3-15] and the detection of the decays (1), (2) would be the signal of the new physics, possibly, induced by the four color symmetry between quarks and leptons.

In the present Letter we calculate the contributions of the scalar leptoquark doublets into decays (1), (2) in frame of the minimal four color quark-lepton symmetry model with the Higgs mechanism of splitting the masses of quarks and leptons(MQLS-model [17, 18]) and evaluate and discuss the widths and the branching ratios of these decays in dependence on the scalar leptoquark masses and on the mixing parameters of the model.

In MQLS model the basic left (L) and right (R) quarks $Q'_{iaa}^{LR}$ and leptons $l'_{ia}^{LR}$ form the fundamental quartets of $SU_V(4)$ color group and can be written, in general, as superpositions
\begin{equation}
    Q'_{iaa}^{LR} = \sum_j (A_{Q_{iaa}^{LR}})_{ij} Q_{jaa}^{LR}, \quad l'_{ia}^{LR} = \sum_j (A_{l_{ia}^{LR}})_{ij} l_{jaa}^{LR} \quad (3)
\end{equation}
of the quark and lepton mass eigenstates $Q_{iaa}^{LR}, l_{ia}^{LR}$, where $i, j = 1, 2, 3$ are the generation indexes, $a = 1, 2$ and $\alpha = 1, 2, 3$ are the $SU_L(2)$ and $SU_c(3)$
indexes, \( Q_{11} \equiv u_i = (u, c, t) \), \( Q_{12} \equiv d_i = (d, s, b) \) are the up and down quarks, \( l_{j1} \equiv \nu_j \) are the mass eigenstates of neutrinos and \( l_{j2} \equiv l_j = (e^{-}, \mu^{-}, \tau^{-}) \) are the charged leptons. The unitary matrixes \( A_{Q_a}^{L,R} \) and \( A_{l_a}^{L,R} \) describe the fermion mixing and diagonalize the mass matrixes of quarks and leptons.

The scalar leptoquark doublets \( S^{(\pm)} \) have the SM hypercharge \( Y_{\pm}^{SM} = 1 \pm 4/3 \) and can be written in the form

\[
S_{aa}^{(\pm)} = \left( \begin{array}{c} S_{1a}^{(\pm)} \\ S_{2a}^{(\pm)} \end{array} \right),
\]

(4)

where the up \((a = 1)\) leptoquarks \( S_{1a}^{(\pm)} \) have electric charge 5/3 and 1/3 and the down \((a = 2)\) leptoquarks \( S_{2a}^{(\pm)} \) have the charge \pm 2/3. In general case the scalar leptoquarks \( S_{2a}^{(+)} \) and \( S_{2a}^{(-)} \) with electric charge 2/3 are mixed and can be written as superpositions

\[
S_{2a}^{(+)} = \sum_{m=0}^{3} c_{m}^{(+)} S_m, \quad S_{2a}^{(-)} = \sum_{m=0}^{3} c_{m}^{(-)} S_m
\]

(5)

of three physical scalar leptoquarks \( S_1, S_2, S_3 \) with electric charge 2/3 and a small admixture of the Goldstone mode \( S_0 \). Here \( c_{m}^{(\pm)} \), \( m = 0, 1, 2, 3 \) are the elements of the unitary scalar leptoquark mixing matrix, \( |c_{0}^{(\pm)}|^2 = \frac{1}{3} g_4^2 \eta_3^2 / m_V^2 \ll 1 \), \( g_4 \) is the \( SU_V(4) \) gauge coupling constant, \( \eta_3 \) is the VEV of the \((15,2,1)\)-multiplet and \( m_V \) is the vector leptoquark mass.

With account of the fermion mixing (3) the interaction of the scalar leptoquark doublets (4) with the fermions can be described by the lagrangian

\[
\mathcal{L}_{S\bar{Q}L} = ( \overline{Q}_{i\alpha a}^{L} (h_{\nu}^{(+)}))_{ij} \, \nu_{ji}^{R} \) \( S_{aa}^{(+)} \) + ( \overline{h}_{i\alpha a}^{L} (h_{d}^{(-)}))_{ij} \, d_{ji}^{R} \) \( S_{aa}^{(-)} \) + \( ( \overline{Q}_{i\alpha a}^{L} (h_{\nu}^{(-)}))_{ij} \, \nu_{ji}^{R} \) \( \tilde{S}_{aa}^{(-)} \) + ( \overline{h}_{i\alpha a}^{L} (h_{u}^{(+)}))_{ij} \, u_{ji}^{R} \) \( \tilde{S}_{aa}^{(+)} \) + h.c.,
\]

(6)

where \( \tilde{S}_{a}^{(\pm)} = \varepsilon_{ab} S_{b}^{(\pm)}, \varepsilon_{12} = -\varepsilon_{21} = 1, \varepsilon_{11} = \varepsilon_{22} = 0 \) and

\[
Q_{i\alpha a}^{L} = \left( \begin{array}{c} u_{i\alpha a}^{L} \\ (C_{Q})_{ik} d_{ka}^{L} \end{array} \right), \quad \nu_{i\alpha a}^{L} = \left( \begin{array}{c} (C_{l})_{ik} \nu_{k}^{L} \\ l_{i}^{L} \end{array} \right)
\]

(7)

are the \( SU_L(2) \) doublets of the left quarks and leptons, \( C_{Q} = (A_{Q_1}^{L})^{\dagger} A_{Q_2}^{L} \) is the CKM-matrix, \( C_{l} = (A_{l_1}^{L})^{\dagger} A_{l_2}^{L} \) is the analogous matrix in the lepton sector
and \((h_{f}^{(\pm)})_{ij}\) are the coupling constant matrices, the index \(f\) denotes the right fermions \(f\) entering into \((3)\).

In particular case of zero fermion mixing \((C_{Q} = C_{l} = I, (h_{f}^{(\pm)})_{ij} = h_{f_{i}}^{(\pm)}\delta_{ij})\)
the lagrangian \((3)\) upon substitutions \(S_{a}^{(+)} = \varepsilon_{ab}R_{2b}, S_{a}^{(-)} = (\bar{R}_{2a})^{*}, h_{e}^{(+)} = h_{2R}, h_{d}^{(-)} = (\bar{h}_{2L})^{*}, h_{u}^{(+)}, h_{u}^{(-)} = 0\) reproduces the lagrangian of ref. \([22]\), \(R_{2}, \bar{R}_{2}\) and \(h_{2L}, h_{2R}\) are the scalar leptoquarks and the phenomenological coupling constants of ref. \([22]\).

As a result of the Higgs splitting of the masses of quarks and leptons the general form \([19]\) of Yukawa interactions of the scalar doublet \(\Phi(2)\) in MQLS-model \([17, 18]\) gives for the coupling constants of the lagrangian \((3)\) the expressions

\[
(h_{l}^{(+)})_{ij} = -(C_{Q}h_{2}^{R})_{ij}, \quad (h_{d}^{(-)})_{ij} = -(h_{2}^{L})_{ij},
\]

\[
(h_{u}^{(-)})_{ij} = -(h_{1}^{R})_{ij}, \quad (h_{u}^{(+)})_{ij} = -(\frac{C_{l}}{h_{1}^{L}})_{ij}
\]

with

\[
(h_{a}^{L,R})_{ij} = \sqrt{3} \frac{1}{2\eta \sin \beta} (M_{Q_{a}}K_{a}^{L,R} - K_{a}^{R,L}M_{a})_{ij},
\]

where \((M_{f_{a}})_{ij} = m_{f_{a}}\delta_{ij}\) are the diagonal mass matrices of quarks and leptons, \(f_{a} = Q_{ia}, l_{ia}, K_{a}^{L,R} = (A_{Q_{a}}^{L,R})^{\dagger}A_{L,R}^{L,R}\) are the mixing matrices specific for the model with the four color quark-lepton symmetry, \(\beta\) is the angle of the mixing of the \((1,2,1)\)-doublet \(\Phi(2)\) with the 15th scalar doublet \(\Phi(3)\) of the \((15, 2, 1)\)-multiplet \(\Phi(3)\), \(\tan \beta = \eta_{3}/\eta_{2}\), \(\eta_{2}\) and \(\eta_{3}\) are the VEVs of the \(\Phi(2)\) and \(\Phi(3)\) multiplets, \(\eta = \sqrt{\eta_{2}^{2} + \eta_{3}^{2}}\) is the SM VEV.

From the formulas \((4) - (9)\) we obtain the responsible for the t-quark decays \((1)\), \((2)\) interactions of scalar leptoquarks \((4), (5)\) with quarks and leptons in the form

\[
L_{S_{1}^{(+)}Q_{i1a}} = \bar{Q}_{i1a} \left[ h_{ij}^{L}P_{L} + h_{ij}^{R}P_{R} \right] l_{2S_{1a}^{(+)}a} + \text{h.c.,}
\]

\[
L_{S_{m}Q_{i1a}} = \bar{Q}_{iaa} \left[ (h_{am}^{L})_{ij}P_{L} + (h_{am}^{R})_{ij}P_{R} \right] l_{jaS_{ma}} + \text{h.c.,}
\]

with

\[
(h_{ij}^{L})_{ij} = (h_{1}^{L}C_{l})_{ij}, \quad (h_{ij}^{R})_{ij} = -(C_{Q}h_{2}^{R})_{ij}, \quad (h_{am}^{L,R})_{ij} = -(h_{a}^{L,R})_{ij}c_{am}^{L,R}.
\]
where the matrices \( h_{a}^{L,R} \) are given by eq. (11), \( c_{1m}^{L,R} = c_{m}^{(\pm)} \), \( c_{2m}^{L,R} = c_{m}^{(\mp)} \), \( c_{m}^{(\pm)} \) are the elements of the scalar leptoquark mixing matrix in (5) and \( P_{L,R} = (1 \pm \gamma_{5})/2 \) are the left and right projection operators.

The interactions (11), (12) induce in the tree approximation the \( t \)-quark decays

\[
t \to u_{i} \ell_{j}^{+} l_{k}^{-},
\]

\[
t \to u_{i} \tilde{\nu}_{j} \nu_{k}
\]

with the production of the up quarks \( u_{i} = (u, c) \), \( i = 1, 2 \) according to the diagrams on Fig.1 We have calculated the widths of the decays (14), (15) with neglect of the final fermion masses

\[
m_{u_{i}}, m_{\ell_{j}}, m_{\nu_{k}} \ll m_{t}, m_{S_{1}^{(+)}}/m_{S_{m}}
\]

and assuming that

\[
m_{S_{1}^{(+)}}/m_{S_{m}} > m_{t}.
\]

The widths of the decays (14) are described by the diagram of Fig.1(a) and can be written in the form

\[
\Gamma(t \to u_{i} \ell_{j}^{+} l_{k}^{-}) = m_{t} H_{ij} H_{3k} \cdot f_{1}(\mu_{S_{1}^{(+)}})/128(2\pi)^{3},
\]

where

\[
H_{ij} = |h_{L_{ij}}^{L}|^{2} + |h_{R_{ij}}^{R}|^{2}, \quad i', j = 1, 2, 3,
\]

\[
f_{1}(\mu) = 6\mu^{2} - 5 - 2(\mu^{2} - 1)(3\mu^{2} - 1) \ln \frac{\mu^{2}}{\mu^{2} - 1}
\]

and \( \mu_{S_{1}^{(+)}} = m_{S_{1}^{(+)}}/m_{t} \). One can see from (13), (10) that the widths (18), (19) are the largest ones for the heaviest final quark \( u_{2} = c \) and the corresponding largest coupling constants entering the equation (18), (19) can be approximately written as

\[
h_{L_{ij}}^{L} \approx \sqrt{\frac{3}{2}} \frac{m_{u_{i'}}}{\sin^{2}\beta} (K_{L}^{L} C_{l})_{i' j}, \quad i' = 2, 3, \quad m_{u_{i'}} = m_{c}, m_{t}
\]

(the coupling constants \( h_{ij}^{R} \) are suppressed by the non-diagonal matrix element of CKM matrix \( C_{Q} \). As a result the widths (18), (19) for the decays (1) are simplified and take the form

\[
\Gamma(t \to c \ell_{j}^{+} l_{k}^{-}) = m_{c} \frac{\gamma_{ec}}{\sin^{4}\beta} k_{2j} k_{3k} f_{1}(\mu_{S_{1}^{(+)}}),
\]
\[ \gamma_{tc} = \frac{9}{512(2\pi)^3} \cdot \frac{m_t^2 m_c^2}{\eta^4}, \]  (23)

\[ k_{i'j} = |(K^L_1 C_l)_{i'j}|^2, i', j = 1, 2, 3. \]  (24)

The eq. (22) predicts the \( t \to c l^+_j l^-_k \) decays with the production of the generation diagonal charged lepton pairs \( e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^- \) as well as of the non-diagonal ones such as \( e^+ \mu^-, \mu^+ e^-, \mu^+ \tau^-, \ldots \) in dependence on the fermion mixing parameters \( k_{2j}, k_{3k} \). Summarizing the partial widths (22) over the generation indexes and using the unitarity of the matrixes \( K^L_1, C_l \) we obtain the total width of the charged lepton mode

\[ \Gamma(t \to c l^+_j l^-_k) = \sum_{j,k} \Gamma(t \to c l^+_j l^-_k) = m_t \frac{\gamma_{tc}}{\sin^4 \beta} f_1(\mu_{s_1}^{(+)}) \]  (25)

which is fermion mixing independent and includes all the decays with the production of the every possible charged lepton pairs both the diagonal and non-diagonal ones. In particular case of the zero fermion mixing \( (K^L_1 C_l = I) \) the total width (25) is saturated by the non-diagonal decay \( t \to c \mu^+ \tau^- \) which is in this case the only allowed decay of type (1).

The widths of the decays (15) described by the diagram of the Fig. 1(b) are calculated with account of (12) and can be written in the form

\[ \Gamma(t \to u_i \tilde{\nu}_j \nu_k) = m_t \sum_{m,n} H^m_{ij} H^m_{kn} \cdot f_2(\mu_{s_m}, \mu_{s_n}) / 128(2\pi)^3, \]  (26)

where

\[ H^m_{ij} = (h^L_{1m})_{i'j}(h^L_{1m})^{*}_{i'j} + (h^R_{1m})_{i'j}(h^R_{1m})^{*}_{i'j}, i', j = 1, 2, 3, \]  (27)

\[ f_2(\mu_1, \mu_2) = 2(\mu_1^2 + \mu_2^2) - 3 - 2\frac{\mu_1^2(\mu_1^2 - 1)^2}{\mu_1^2 - \mu_2^2} \ln \frac{\mu_2^2}{\mu_1^2 - 1} - 2\frac{\mu_2^2(\mu_2^2 - 1)^2}{\mu_2^2 - \mu_1^2} \ln \frac{\mu_1^2}{\mu_2^2 - 1} \]  (28)

and \( \mu_{s_m} = m_{s_m} / m_t \). Keeping in (13), (10) only the largest terms proportional to \( m_t \) and \( m_c \)

\[ (h^L_{1m})_{i'j} \approx -\sqrt{\frac{3}{2}} \frac{m_{u_{i'}}}{\eta \sin \beta} (K^L_1)_{i'j} c_m^{(+)}, i' = 2, 3, m_{u_{i'}} = m_c, m_t \]  (29)
we obtain from (26), (27) the widths of the decays (2) in the form
\[ \Gamma(t \to c \tilde{\nu}_j \nu_k) = m_t \frac{\gamma_{tc}}{\sin^4 \beta} \sum_{m,n=1}^3 k_{2j}^{mn} k_{3k}^{mn} f_2(\mu s_m, \mu s_n), \]

(30)

where
\[ k_{ij}^{mn} = |(K^L_i)_{ij}^m c_n^{(*)} + (K^R_i)_{ij}^m c_n^{(*)}|^2, \]

(31)

and \( \gamma_{tc} \) is defined by the eq. (23).

Summarizing the partial widths (30) over the generation indexes and accounting the unitarity of the matrixes \( K^L,R \) we obtain the total width of the neutrino mode
\[ \Gamma(t \to c \tilde{\nu} \nu) = \sum_{j,k} \Gamma(t \to c \tilde{\nu}_j \nu_k) = m_t \frac{\gamma_{tc}}{\sin^4 \beta} \sum_{m,n=1}^3 k_{mn} f_2(\mu s_m, \mu s_n) \]

(32)

which is fermion mixing independent and contains the parameters \( k_{mn} = (c_m^{(*)} c_n^{(*)} + c_m^{(*)} c_n^{(*)})^2 \) depending on the scalar leptoquark mixing (5). The expression (32) can be simplified in the particular case of the scalar leptoquark mixing (5) with \( c_3^{(+)} = 0 \) when \( S_2^{(+)} \) and \( S_2^{(-)} \) are approximately the superpositions of two physical scalar leptoquark \( S_1 \) and \( S_2 \) (the small admixture of the Goldstone mode can be neglected because of the smallness of \( c_0^{(+)} \)). In this case \( k_{mn} \approx \delta_{mn} \) for \( m, n = 1, 2 \) and the width (32) takes the form
\[ \Gamma(t \to c \tilde{\nu} \nu) = m_t \frac{\gamma_{tc}}{\sin^4 \beta} [f_1(\mu s_1) + f_1(\mu s_2)], \]

(33)

here the relation \( f_2(\mu, \mu) = f_1(\mu) \) has been taken also into account.

The widths (25), (33) depend on the masses \( m_{S_1^{(+)}}, m_{S_1}, m_{S_2} \) of the scalar leptoquarks and on the \( \Phi^{(2)} - \Phi^{(3)} \) mixing angle \( \beta \).

The current data on the direct search for the leptoquarks set the lower mass limits [23]
\[ m_{LQ} > 242 \text{ GeV}, 202 \text{ GeV}, 148 \text{ GeV} \]

(34)

for the scalar leptoquarks of the first, of the second and of the third generation respectively. As mentioned above the scalar leptoquarks \( m_{S_1^{(+)}}, m_{S_1}, \)
$m_{S_2}$ couple most intensively with t-quark. In the case they will decay predominantly into $t\bar{t}_{j\alpha}$ pairs and should be regarded as the third generation ones. In this case the condition (17) is consistent with lower experimental limit 148 GeV in (34) and we use (17) when choosing the lower scale leptoquark masses in (25), (33). With account of the total width of t-quark $\Gamma_t^{tot} \approx \Gamma(t \rightarrow bW) \approx 1.56$ GeV we obtain from (25), (33) that

$$Br(t \rightarrow c l^+ l^-) = (5.7 \cdot 10^{-8} - 0.6 \cdot 10^{-8} - 0.7 \cdot 10^{-9})/\sin^4 \beta,$$  \hspace{1cm} (35)

$$Br(t \rightarrow c \bar{\nu}^\prime \nu) = (11.3 \cdot 10^{-8} - 1.2 \cdot 10^{-8} - 1.4 \cdot 10^{-9})/\sin^4 \beta$$ \hspace{1cm} (36)

for $m_{S_1^{(+)}}, m_{S_1}, m_{S_2} = 180 - 250 - 400$ GeV.

The mixing angle $\beta$ enters into the coupling constants (13), (10) and we restrict it by the smallness of the perturbation theory parameter

$$(h^{S,P})^2/4\pi < 1,$$ \hspace{1cm} (37)

where $h^{S,P} = (h^L \pm h^R)/2$ are the scalar and pseudoscalar coupling constants and $h^{L,R}$ are the chiral coupling constants (13), (10). With account of the t-quark chiral coupling constants in (21), (29) as of the largest ones the condition (37) gives that $\sin \beta > 0.12$.

The Fig.2 shows the branching ratio $Br(t \rightarrow c l^+ l^-)$ of the charged lepton mode as the function of the scalar leptoquark mass $m_{S_1^{(+)}} = 180 - 300$ GeV for a) $\sin \beta = 0.15$, b) $\sin \beta = 0.2$ and c) $\sin \beta = 0.25$. The corresponding branching ratios $Br(t \rightarrow c \bar{\nu}^\prime \nu)$ of the neutrino mode for $m_{S_1} = m_{S_2} = 180 - 300$ GeV are twice as large as those of the charged lepton mode.

As is seen from the Fig.2 in all three cases a), b) and c) there is the mass region with $Br(t \rightarrow c l^+ l^-) \sim 10^{-5}$. For example for $\sin \beta = 0.2$ from Fig.2-b and from (35), (36) we obtain

$$Br(t \rightarrow c l^+ l^-) = (3.5 - 0.4) \cdot 10^{-5},$$ \hspace{1cm} (38)

$$Br(t \rightarrow c \bar{\nu}^\prime \nu) = (7.1 - 0.8) \cdot 10^{-5}$$ \hspace{1cm} (39)

for $m_{S_1^{(+)}}, m_{S_1}, m_{S_2} = 180 - 250$ GeV. As is seen the branching ratios of the decays under consideration occur to be of the same order as the sensitivity of LHC to the $t \rightarrow c X$ decays $Br(t \rightarrow c X) > 5 \cdot 10^{-5}$ [1,2,12,24].

It would be of interest to estimate the sensitivity of LHC to the decays (11). It can be performed in an order-of-magnitude manner by using the studies [25,26] of LHC sensitivity to the decays

$$t \rightarrow c Z \rightarrow c l^+ l^-,$$ \hspace{1cm} (40)
where \( l^+l^- \) are both \( e^+e^- \) and \( \mu^+\mu^- \) pairs.

The decays (1) together with the decays

\[
\tilde{t} \rightarrow \tilde{c} l_j^+ l_k^-
\]

(the widths of the decays (1) differ from (22) by the transmutations \( j \leftrightarrow k \) of the indices in the mixing parameters) can manifest themselves at LHC through the processes

\[
p + p \rightarrow t\tilde{t} \rightarrow \{ (c l_j^+ l_k^-)(W^-\tilde{b}) \rightarrow (c l_j^+ l_k^-)(j\tilde{j}\tilde{b}) \} = c'l_j^+ l_k^- jjb',
\]

where one of the quarks (\( t \) or \( \tilde{t} \)) decays according to (1) or (41) whereas the other one (\( \tilde{t} \) or \( t \)) decays in the standard way followed by the \( W^\pm \rightarrow jj \) decays into two hadron jets. Although the case with leptonic decay modes \( W \rightarrow l\nu_l \) is more sensitive [25, 26] to the decays (40) we use here for the analysis of the \( t \rightarrow c l_j^+ l_k^- \) decays the case with hadronic decays \( W^\pm \rightarrow jj \) as the more simple one. In this case the final states are \( c'l_j^+ l_k^- jjb' \) with \( c' = c, \tilde{c} \) and \( b' = \tilde{b}, b \) and the experimental signature includes, therefore, two charged leptons (in general, of different generations) and four energetic jets.

For the generation diagonal signal process

\[
p + p \rightarrow t\tilde{t} \rightarrow c'l^+l^- Wb' \rightarrow c'l^+l^- jjb' \rightarrow l^+l^- + 4\text{jets}
\]

with \( l^+l^- = e^+e^-, \mu^+\mu^- \) the dominant backgrounds are the same as for the process (40).

\[
p + p \rightarrow Z + \text{jets} \rightarrow l^+l^- + \text{jets},
p + p \rightarrow ZW \rightarrow l^+l^- + \text{jets},
p + p \rightarrow t\tilde{t} \rightarrow W^+bW^-\tilde{b} \rightarrow l^+\nu_l\bar{b}\nu\bar{\tilde{b}} \rightarrow l^+l^- + \text{jets}.
\]

For the expected number of the signal events \( N_{l^+l^-}^s = N_{e^+e^-}^s + N_{\mu^+\mu^-}^s \) and for the background ones we obtain the estimations

\[
N_{l^+l^-}^s = 1.1 \cdot 10^7 \cdot Br(t \rightarrow c l^+l^-),
\]

\[
N_{l^+l^-}(Z + \text{jets}) = 5.7 \cdot 10^6, \quad N_{l^+l^-}(ZW) = 1.3 \cdot 10^4, \quad N_{l^+l^-}(t\tilde{t}) = 1.9 \cdot 10^5
\]

for the expected LHC cross sections \( \sigma_{l\tilde{t}} = 833pb, \sigma_{Zj} = 8478pb, \sigma_{ZW} = 28pb \) [26] and for the integrated luminosity \( L = 10fb^{-1} \), here and below \( Br(t \rightarrow c l^+l^-) = Br(t \rightarrow c e^+e^-) + Br(t \rightarrow c \mu^+\mu^-) \).
To reduce the backgrounds one can use, in part, the cuts which have been used for estimations of the sensitivity of LHC to the decay \([40]\). In the Table 1 we show the cuts which have been used in ref. \([26]\) in analysys of the events \(t\bar{t} \rightarrow cZWb \rightarrow l^+l^- + 4jets\) for the reconstruction of \(t \rightarrow cZ\) decays. We also show the relative efficiencies \(\tilde{\varepsilon}^s\), \(\tilde{\varepsilon}^b\) of each cut for signal and backgrounds, which we have evaluated by using the results of ref. \([26]\) where the detailed descriptions of the cuts can be also found.

| cut                          | relative efficiencies \(\varepsilon\), % | usefulness of the cuts |
|------------------------------|----------------------------------------|------------------------|
|                             | signal | backgrounds | \(l^+l^-\) | \(\epsilon\) |
| 1 \(l^+l^- j\) satisfying \(p_T, \eta\) cuts | 26.5   | 0.54       | 1.4        | 6.0       | +          | +          |
| 2 \(m_Z\) cut               | 80.0   | 80.2       | 81.8       | 9.4       | -          | -          |
| 3 \((\geq 4)\)jets, \(p_T \geq 30\) GeV | 57.5   | 7.8        | 76.3       | 85.6      | +          | -          |
| 4 \# b-tag=1                | 41.8   | 26.7       | 0.9        | 7.2       | +          | +          |
| 5 \(m_W\) cut              | 64.7   | 32.4       | 25.4       | +         | +          |
| 6 \(m_{t\rightarrow Wb}\) cut | 81.8   | 39.5       | 52.3       | +         | +          |
| 7 \(p_T(\text{jets}) \geq 50\) GeV | 14.8   |            |            | 11.1      | +          | -          |

Table 1: Cuts used for reducing the backgrounds to the signal events a) \(t\bar{t} \rightarrow cZWb \rightarrow l^+l^- + 4jets\) \([26]\), b) \(t\bar{t} \rightarrow c(l^+l^-Wb) \rightarrow l^+l^- + 4jets\) and c) \(t\bar{t} \rightarrow c\mu\mu Wb \rightarrow \epsilon\mu + 4jets\). \(l^+l^-\) includes both \(e^+e^-\) and \(\mu^+\mu^-\) pairs, \(\epsilon\mu\) includes both \(e^-\mu^+\) and \(e^+\mu^-\) ones. The relative efficiencies \(\tilde{\varepsilon}\) are evaluated with using the results of ref. \([26]\). The last two column mark the cuts used in the cases b) and c).

All the cuts are applicable to the analysis of the events \(t\bar{t} \rightarrow c(l^+l^-Wb) \rightarrow l^+l^- + 4jets\) except the \(m_Z\) cut. This cut requires the invariant mass of lepton pair to be near \(m_Z\) (within 4 GeV of \(m_Z\)) and it is not suitable for the decays \(t \rightarrow c l^+l^-\). Assuming that after excluding the \(m_Z\) cut the relative efficiencies of the remaining six cuts are unchanged and multiplying them we obtain the resulted efficiency \(\varepsilon_{t+l^-} = 5.0 \cdot 10^{-3}\) for the signal events and \(\varepsilon_{t+l^-} = \varepsilon_{t+l^-}(\tilde{t}) = 5.4 \cdot 10^{-5}\) for the background ones. As a result of these cuts we can expect \(n_{t+l^-}^s = \varepsilon_{t+l^-} N_{t+l^-}^s = 5.6 \cdot 10^4 \cdot Br(t \rightarrow c l^+l^-)\) signal events and \(n_{t+l^-}^b = n_{t+l^-}^b(\tilde{t}) = \varepsilon_{t+l^-}(\tilde{t}) N_{t+l^-}^b(\tilde{t}) \approx 10\) background ones. Taking for the signal significance \(s_{t+l^-} = n_{t+l^-}^s / \sqrt{n_{t+l^-}^b}\) the value \(s_{t+l^-} = 5\) we obtain...
the expected sensitivity of LHC to the $t \rightarrow c l^+ l^-$ decays

$$Br(t \rightarrow c l^+ l^-) > 3 \cdot 10^{-4}$$

at $5\sigma$ level for $L = 10 fb^{-1}$.

For comparison, in the case of hadronic $W$ decay modes $W^\pm \rightarrow jj$ the sensitivity of LHC to the decay $t \rightarrow c Z$ is $Br(t \rightarrow c Z) > 1.7 \cdot 10^{-3}$ [26] for $L = 10 fb^{-1}$, which corresponds to the sensitivity $Br(t \rightarrow c Z \rightarrow c l^+ l^-) > 1.1 \cdot 10^{-4}$ to the decays (43). As is seen, the sensitivity (43) is approximately by a factor three worse than that for the decay (40).

For the nondiagonal signal process

$$p + p \rightarrow t\bar{t} \rightarrow c'e\mu Wb' \rightarrow c'e\mu jjb' \rightarrow e\mu + 4\text{jets}$$

where $e\mu$ are both $e^-\mu^+$ and $e^+\mu^-$ pairs the dominant background is given only by the process

$$p + p \rightarrow t\bar{t} \rightarrow W^+bW^-\bar{b} \rightarrow (e^+\nu_b)(\mu^-\nu_b) + (\mu^+\nu_b)(e^-\nu_b) \rightarrow e\mu + \text{jets}.$$  

For numbers $N_{e\mu}^s = N_{e^-\mu^+}^s + N_{e^+\mu^-}^s$ and $N_{e\mu}^b = N_{e\mu}(t\bar{t})$ of the signal and background events we have the estimations

$$N_{e\mu}^s = 1.1 \cdot 10^7 \cdot Br(t \rightarrow c e\mu), \quad N_{e\mu}^b = 1.9 \cdot 10^5,$$

where $Br(t \rightarrow c e\mu) = Br(t \rightarrow c e^-\mu^+) + Br(t \rightarrow c e^+\mu^-)$.

In the case of $e\mu$ pairs the $m_Z$ cut should be excluded, in addition, the third cut and the seventh one (see the Table 1) can be also excluded because these two cuts are suitable for reducing the $Zj$ background which is absent in the case of $e\mu$ events. Assuming the relative efficiencies of the remaining four cuts to be unchanged and multiplying them we obtain the resulted efficiencies $\varepsilon_{e\mu}^s = 5.9 \cdot 10^{-2}, \varepsilon_{e\mu}^b = \varepsilon_{e\mu}(t\bar{t}) = 5.7 \cdot 10^{-4}$ and the corresponding numbers

$$n_{e\mu}^s = \varepsilon_{e\mu}^s N_{e\mu}^s = 6.6 \cdot 10^5 \cdot Br(t \rightarrow c e\mu), \quad n_{e\mu}^b = \varepsilon_{e\mu}^b N_{e\mu}^b \approx 108 \text{ of the signal and background events.}$$

As a result for the signal significance $s_{e\mu} = 5$ we can expect the LHC sensitivity to the $t \rightarrow c e\mu$ decay

$$Br(t \rightarrow c e\mu) > 8 \cdot 10^{-5}$$

at $5\sigma$ level for $L = 10 fb^{-1}$.

The approximate estimations (43), (44) relate to the case of hadronic $W$ decay modes $W^\pm \rightarrow jj$ and correspond to an integrated luminosity
$L = 10 fb^{-1}$. These estimations can be improved approximately by order of magnitude

$$Br(t \to cl^+l^-) \gtrsim 10^{-5}, \quad Br(t \to ce\mu) \gtrsim 10^{-6}$$

for $L = 100 fb^{-1}$ and, possibly, the further improvement can be achieved by accounting the leptonic $W$ decay modes $W \to ll\eta$.

As is seen the branching ratios (38), (39) are of the same order as the expected sensitivities (45) of LHC to the decays $t \to cl^+l^-$ and $t \to ce\mu$ type having the more clean signature may be of interest. The detection of such decays would be the clear sign of the new physics, possibly, induced by the four color symmetry between quarks and leptons.

It should be noted that for the masses 180 - 300 GeV the leptoquarks can be directly produced at LHC and by the additional studies of their decay modes, branching ratios etc. it will be possible to to clean up the origin of the observed leptoquarks. In this situation the detection of the $t \to cl^+_j l^-_k$ decays could be an additional argument in favour of the scalar leptoquarks discussed in this paper. It is worth noting that the non-diagonal decays of $t \to cl^+_j l^-_k$ type are also predicted in the general two Higgs doublet model but with the essentially less branching ratios, for example with $Br(t \to c\tau^-\mu^+) \sim 10^{-8}$ as the largest one [27].

In conclusion we resume the results of the work. The rare $t$-quark decays $t \to cl^+_j l^-_k, t \to c\tilde{\nu}_j \nu_k$ induced by the scalar leptoquark doublets are investigated in the minimal four color symmetry model with the Higgs mechanism of the quark-lepton mass splitting. The partial widths $\Gamma(t \to cl^+_j l^-_k)$ and $\Gamma(t \to c\tilde{\nu}_j \nu_k)$ of these decays are calculated in tree approximation and the total width of the charged lepton mode $\Gamma(t \to cl^+l^-) = \sum_{j,k} \Gamma(t \to cl^+_j l^-_k)$ and the neutrino one $\Gamma(t \to c\tilde{\nu}'\nu) = \sum_{j,k} \Gamma(t \to c\tilde{\nu}_j \nu_k)$ are found in the fermion mixing independent form. The corresponding branching ratios $Br(t \to cl^+l^-), Br(t \to c\tilde{\nu}'\nu)$ are shown to be of order of $10^{-5}$ for the scalar leptoquark masses $m_{S_1^{(2)}}, m_{S_1}, m_{S_2} = 180 - 250$ GeV and for $\sin \beta \approx 0.2$, $\beta$ is the $\Phi^{(2)} - \Phi^{(3)}$ mixing angle of the model. These estimations are close to the possible sensitivity of LHC to these decays and the search for the decays $t \to cl^+_j l^-_k, t \to c\tilde{\nu}_j \nu_k$ at LHC may be of interest.

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Figure captions

Fig. 1. The diagrams of the rare $t$-quark decays a) $t \rightarrow c l^+_j l^-_k$ and b) $t \rightarrow c \tilde{\nu}_j \nu_k$ via scalar leptoquarks $S_1^{(+)}$ and $S_m$, $m = 1, 2, 3$ of the MQLS-model.

Fig. 2. The branching ratio $Br(t \rightarrow c l^+ l^-) = \sum_{j,k} Br(t \rightarrow c l^+_j l^-_k)$ of the charged lepton mode as a function of the scalar leptoquark mass $m_{S_1^{(+)}}$ for a) $\sin \beta = 0.15$, b) $\sin \beta = 0.20$, c) $\sin \beta = 0.25$. 
Fig. 1
Fig. 2