Cuprate interband model and doping dependence of the coherence length

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Abstract

The free energy expansion of a two-band pair-transfer superconductor is developed. A critical in temperature and a noncritical coherence length appear. The effective in-plane $\xi_{ab}$, $H_{c2}^\xi \sim \xi_{ab}^{-2}$ and the thermodynamic critical magnetic field ($H_{c0}$) on the whole hole doping ($p$) scale are calculated for a "typical" cuprate. A doping-prepared bare spectrum with normal state gaps quenched by doping has been used. The coherence length $\xi_{ab}$ falls with $p$ first rapidly. A moderate enhancement starts when the "cold" defect band overlap with the valence band is reached. At overdoping $\xi_{ab}$ rises markedly. The $H_{c2}^\xi$ curve shows a maximum before $T_{c_{\text{max}}}$. The bell-like $H_{c0}$-curve follows closely $T_c$, superconducting gaps, and superfluid density. The theoretical $\xi_{ab}(p)$ nonmonotonic curve on the whole doping scale agrees with a recent experimental finding.

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Key words: Cuprates; Coherence length; Two-band model

1 Introduction

Two-band superconductivity with the interband pairing channel [1,2] is known for a long time. It is seemingly the most effective mechanism which is able to serve high transition temperatures ($T_c$) in a simple way. The interest for this mechanism and its applications has recently essentially grown in connection with the now recognized two-gap superconductivity, e.g. [3-5], of magnesium diboride. Various two-band approaches with diverse electron spectra have been applied also for cuprates (e.g. introducing reviews [6,7]). Wide experimental data on cuprate energetic characteristics ($T_c$, superconducting-, normal state- and pseudogaps)
and their interrelations have been ordered quite recently. Data characterizing the coherence properties are far from being complete. Cuprate superconductivity mechanism itself remains debatable.

In papers [8-11] a simple, partly postulative model which uses only very general knowledge on cuprates has been proposed. The electron spectrum of a doped cuprate incorporates in this model the valence band and hole-created defect states ("hot" and "cold" subbands [11]) with bare normal state gaps between them. These gaps become quenched on progressive doping. Overlap dynamics of bands appears as a novel source of critical doping concentrations. The leading pairing interaction is supposed to be the pair-transfer between the itinerant and defect subsystem states. The nature of the minimal quasiparticle excitation energy changes on doping. This model is able to describe the observed behaviour of cuprate energetic characteristics on the doping scale. The pseudogap(s) are of "extrinsic" occurrence and transform as precursors on the doping scale (not the energetic scale) to superconducting gaps. Pseudogaps survive as normal state gaps. These latter do not manifest in the superconducting density [12] because of interband nature of the pairing. Superfluid density follows the trends shown by $T_c$ and superconducting gaps with expressed maximum near the optimal doping. The strength of the pairing and the phase coherence develop and vanish simultaneously as recent experimental findings [13-18] stress.

Experimental data on cuprate characteristics reflecting the coherence properties (coherence lengths and critical magnetic fields) are given usually for dispersive dopings and temperatures (e.g. [6,19]). The behaviour for extended dopings have been obtained in few cases, and a common trend has not been formulated. For YBa$_2$Cu$_3$O$_y$ as a general trend the decrease of the in-plane coherence length $\xi_{ab}(0)$ with increased doping has been found until the $T_c$ maximum is reached [20]. It manifests also in $H_c^c$ enhancement with $y$. For La$_{2-x}$Sr$_x$CuO$_4$ the c-axis coherence length amplitude diminishes slowly with $x$ and falls then to a remarkably smaller nearly constant value on overdoping [21]. On the contrary, for the Bi-based systems the opposite behaviour has been found in [22,23]. The pair coherence length rises here with doping in a wide region. The corresponding $H_c^c$ falls off. The authors of [22,23] stress the pseudogap to represent the amplitude of the pairing strength. This is not the case in our model [8-12]. The antiferromagnetic coherence length representing the average distance between the doped holes falls off with enhanced doping as $\sim p^{-1/2}$ [24].

However, in the recent investigation [25] based on the vortex core size measurements, seemingly for the first time, the doping dependence of the cuprate coherence length on the whole doping scale has been obtained from the low temperature data.

In the present communication we apply the model of [11,12] to investigate the in-plane coherence length amplitude at zero temperature $\xi_{ab}(0) = \xi_0$ and the corresponding $H_c^c$, and also the thermodynamic critical field $H_c(0)$, on the full doping scale for a "typical" cuprate. Qualitative agreement with the valley-profile type curve recently found in [25] is obtained.

We start with the derivation of the long-wavelength expansion of the free energy for a two-band superconductor with leading pair-transfer interaction.
2 The free energy

We describe the superconducting system under consideration by the Hamilton operators separation as

\[ H = H_1 + H_2, \quad H_1 = H_0 + H_\lambda, \quad H_2 = -H_\lambda + H_i \]  

with

\[ H_0 = \sum_{\sigma k} \tilde{\epsilon}_\sigma(k)a_{\sigma k\sigma}^+a_{\sigma k\sigma}, \]

\[ H_i = 2W \sum_{\sigma, \sigma'} \sum_{\vec{k}, \vec{k}'} \sum_{\vec{q}} a^+_{\sigma \vec{k}} a^+_{\sigma \vec{-k} + \vec{q}} a_{\sigma' \vec{-k}' + \vec{q}} a_{\sigma' \vec{k}'\uparrow}, \]

\[ H_\lambda = 2W \sum_{\sigma \vec{k} \vec{q}} (\lambda_{\sigma \vec{q}} a^+_{\sigma \vec{k} \uparrow} a^+_{\sigma \vec{-k} \downarrow} + \text{h.c.}). \]

Here \( \tilde{\epsilon}_\sigma = \epsilon_\sigma - \mu \), \( \mu \) is the chemical potential, \( \sigma = 1, 2 \) is the electron subsystem index, \( s \) is the spin index, \( W \) is the interband pair-transfer interaction constant, \( \vec{q} \) stands for the total momentum of an electron pair. Correspondingly the superconducting order parameters \( \lambda_{\sigma \vec{q}} \) can express spatially inhomogeneous fluctuations.

The mean-field free energy of \( H \) reads

\[ F = -k_B T + \ln Z(H_1) + \langle H_2 \rangle_{H_1}, \]

\[ Z(H_1) = Sp \exp \left(-\frac{H_1}{k_B T}\right), \]

and \( \langle \ldots \rangle_{H_1} \) means the average in respect of \( H_1 \). Near the phase transition temperature (5) will be expanded as

\[ F = F_0 + F_2 + F_4, \]

where \( F_0 = -k_B T \ln Z(H_0), F_2 \sim \lambda^2, F_4 \sim \lambda^4 \). For the long-wave fluctuations of \( \lambda, F_4 \) can be taken as in the case of \( \vec{q} = 0 \lbrack 26 \rbrack \). Application of the unitary transformation \( U = \exp(iS) \) with

\[ S = 2Wi \sum_{\sigma \vec{k} \vec{q}} \left[ \frac{a^+_{\sigma \vec{k} \uparrow}a^+_{\sigma \vec{-k} \downarrow}}{\tilde{\epsilon}_\sigma(k) + \tilde{\epsilon}_\sigma(k - \vec{q})} \lambda_{\sigma \vec{q}} + \text{h.c.} \right] \]

enables to calculate the terms in (5) of the necessary order. One ends with the expression
\[ F_2 = |W| \sum_{\vec{q}} \left[ \sum_{\sigma} \eta_{\sigma \vec{q}}(T) |\lambda_{\sigma \vec{q}}|^2 \right. \\
+ \frac{w}{2} \eta_{1\vec{q}}(T) \eta_{2\vec{q}}(T) \left( \lambda_{1\vec{q}}^* \lambda_{1\vec{q}}^* + \lambda_{1\vec{q}} \lambda_{2\vec{q}} \right) \right], \tag{8} \]

where \( w = \text{sgn}(W) \) and

\[
\eta_{\sigma \vec{q}}(T) = 2|W| \sum_{\vec{k}} \left[ \tilde{\epsilon}_{\sigma}(\vec{k}) + \tilde{\epsilon}_{\sigma}(\vec{k} - \vec{q}) \right]^{-1} \\
\times \left[ \text{th} \frac{\tilde{\epsilon}_{\sigma}(\vec{k})}{2k_B T} + \text{th} \frac{\tilde{\epsilon}_{\sigma}(\vec{k} - \vec{q})}{2k_B T} \right]. \tag{9} \]

### 3 Expansions and equilibrium equations

The quantity (9) will be expressed near \( T = T_c (\tau = (T - T_c)T_c^{-1}) \) and \( \vec{q} = 0 \) as

\[
\eta_{\sigma \vec{q}}(T) = \eta_{0\sigma}(\theta_c) - \alpha_{\sigma} \tau - \sum_{i,j} B_{\sigma ij} q_i q_j, \tag{10} \]

where \( (\theta_c = k_B T_c) \)

\[
\alpha_{\sigma} = \frac{|W|}{\theta_c} \sum_{\vec{k}} ch^{-2} \tilde{\epsilon}_{\sigma}(\vec{k}) 2\theta_c \tag{11} \]

and \( B_{\sigma} \) is given by a complicated formula for which an approximate expression will be given later. The linear terms in (10) are absent in connection with the approximation

\[
\epsilon_{\sigma}(\vec{k}) = E_{0\sigma} \pm \sum_i \frac{\hbar^2 k_i^2}{2m_{\sigma i}} \tag{12} \]

for the ”band” energies with \( m_{\sigma i} > 0 \).

The superconducting phase transition temperature \( T_c \) is determined by the equation [7]

\[
\eta_1 \eta_2 = 4 \tag{13} \]

with \( \eta_{0\sigma}(\theta_c) = \eta_{\sigma} \).

The integration in the momentum space is supposed to be performed from \( \Gamma_{0\sigma} \) to \( \Gamma_{c\sigma} \) for an electron band and from \( \Gamma_{c\sigma} \) to \( \Gamma_{0\sigma} \) for a hole band. For constant densities of states \( (\rho_{\alpha}) \)
having in mind CuO\(_2\) planes and supposing that |\(\Gamma - \mu\) > \(\theta_c\), one obtains in the case where \(\mu\) lies inside of the integration limits

\[
\eta_\sigma = 2|W|\rho_\sigma \ln |\Gamma_{0\sigma} - \mu||\Gamma_{c\sigma} - \mu| \left(\frac{2\gamma}{\pi \theta_c}\right)^2 ,
\]

(14)

\[
\alpha_\sigma = 4|W|\rho_\sigma ,
\]

(15)

\[
\sum_{i,j} B_{\sigma ij} q_i q_j = \beta_\sigma \sum_i \frac{\hbar^2 q_i^2}{4m_{\sigma i}} ,
\]

(16)

\[
\beta_\sigma = \frac{7\zeta(3)|W|\rho_\sigma |\mu - E_{0\sigma}|}{(\pi \theta_c)^2}
\]

(17)

with \(\gamma \approx 1.78\) and \(\zeta(3) \approx 1.2\). Note that |\(\mu - E_{0\sigma}\)| determines the Fermi energy in the corresponding band. In the case if \(\mu\) lies out of the integration limits

\[
\eta_\sigma = 2|W|\rho_\sigma \ln \left|\frac{\Gamma_{c\sigma} - \mu}{\Gamma_{0\sigma} - \mu}\right| , \quad \alpha_\sigma = \beta_\alpha = 0 .
\]

(18)

Analogous formula can be found also for \(\mu\) coincidence with one of the integration limits.

After the Fourier transformation to the coordinate space the approximate expression for the second order contribution to the free energy (8) reads

\[
F_2 = \frac{|W|}{V} \int d\vec{r} \left\{ \frac{1}{4} \sum_\sigma \left[ (\eta_\sigma - \alpha_\sigma\tau)|\lambda_\sigma(\vec{r})|^2 
- \beta_\sigma \sum_i \frac{\hbar^2}{4m_{\sigma i}} \left| (\nabla_i - \frac{2ie}{\hbar c} A_i) \lambda_\sigma(\vec{r}) \right|^2 \right] 
+ \frac{w}{2} \left[ 4 - (\eta_1 \alpha_2 + \eta_2 \alpha_1)\tau \right] (\lambda_1(\vec{r})\lambda_2^*(\vec{r}) + \lambda_1^*(\vec{r})\lambda_2(\vec{r})) 
- \sum_i \left( \eta_1 \beta_2 \frac{\hbar}{4m_{2i}} + \eta_2 \beta_1 \frac{\hbar}{4m_{1i}} \right) 
\times \left[ (\nabla_i - \frac{2ie}{\hbar c} A_i) \lambda_1(\vec{r}) \left( (\nabla_i - \frac{2ie}{\hbar c} A_i) \lambda_2(\vec{r}) \right)^* 
+ \left( (\nabla_i - \frac{2ie}{\hbar c} A_i) \lambda_1(\vec{r}) \right)^* (\nabla_i - \frac{2ie}{\hbar c} A_i) \lambda_2(\vec{r}) \right] 
+ \frac{(\text{rot}\vec{A})^2}{8\pi} \right\}
\]

(19)
where the vector potential $\vec{A}$ takes care for the gauge invariance. This $F_2$ expression contains squared gradient terms, however, with opposite signs in comparison with the one band case. Products of gradient terms of different bands appear in (19).

For the fourth order term analogously to [27]

$$F_4 = -\frac{|W|}{V} \int d\vec{r} \left\{ \frac{3}{2} \sum_\sigma \nu_\sigma |\lambda_\sigma(\vec{r})|^4 + \right.$$  
$$+ \frac{w}{2} \left[ \nu_1 \eta_2 |\lambda_1(\vec{r})|^2 + \nu_2 \eta_1 |\lambda_2(\vec{r})|^2 \right]$$  
$$\left[ \lambda_1(\vec{r})\lambda_2(\vec{r})^* + \lambda_1(\vec{r})^*\lambda_2(\vec{r}) \right] \right\} d\vec{r},$$

with

$$\nu_\sigma = \frac{14\zeta(3)|W|^3 \rho_\sigma}{(\pi \theta_c)^2},$$

and $\nu_\sigma = 0$ for $\mu$ out of the band.

### 4 Fluctuative ordering

The expression (19) for $F_2$ can be diagonalized by an orthogonal transformation

$$\lambda_{1q} = \lambda_{r\bar{q}} \cos \varphi_{\bar{q}} + \lambda_{s\bar{q}} \sin \varphi_{\bar{q}}$$
$$\lambda_{2q} = -\lambda_{r\bar{q}} \sin \varphi_{\bar{q}} + \lambda_{s\bar{q}} \cos \varphi_{\bar{q}},$$

$$\tan \varphi_{\bar{q}} = \frac{w \eta_2}{2} \frac{(\eta_1^2 \alpha_2 - \eta_2^2 \alpha_1) \tau + \sum_i \left( \eta_1^2 \beta_2 \frac{m_{2i}}{m_{1i}} - \eta_2^2 \beta_1 \right) \frac{h^2 q_i^2}{4} - 1}{4(\eta_1 + \eta_2)}$$

to the new variables

$$\psi_{s,r}(\vec{r}) = \sqrt{\frac{|W|(\beta_1 + \beta_2)}{2V}} \sum_{\bar{q}} \lambda_{s,r}(\bar{q}) e^{i\vec{q}\cdot\vec{r}}.$$

Then

$$F_2 = \int d\vec{r} \left[ a_s \tau |\psi_s(\vec{r})|^2 + (a_o - a_r \tau) |\psi_r(\vec{r})|^2 \right.$$
\[ + \sum_i \frac{\hbar^2}{4M_{si}} \left| \left( \nabla_i - \frac{2ie}{\hbar c} A_i \right) \psi_s(\vec{r}) \right|^2 \]

\[ - \sum_i \frac{\hbar^2}{4M_{ri}} \left| \left( \nabla_i - \frac{2ie}{\hbar c} A_i \right) \psi_r(\vec{r}) \right|^2 + \frac{(\nabla A)^2}{8\pi} \right]. \]  

(25)

Here

\[ a_0 = \frac{(\eta_1 + \eta_2)^2}{u}, \quad u = \frac{1}{2}(\eta_1 + \eta_2)(\beta_1 + \beta_2), \]  

(26)

\[ a_s = \frac{\eta_1 \alpha_2 + \eta_2 \alpha_1}{u}, \quad a_r = \frac{(2\eta_1 + \eta_2)\alpha_2 + (2\eta_2 + \eta_1)\alpha_1}{u}, \]  

(27)

\[ M_{si}^{-1} = \frac{\eta_1 \beta_2 m_{2i}^{-1} + \eta_2 \beta_1 m_{1i}^{-1}}{u}, \]

\[ M_{ri}^{-1} = \frac{(2\eta_1 + \eta_2)\beta_2 m_{2i}^{-1} + (2\eta_2 + \eta_1)\beta_1 m_{1i}^{-1}}{u}. \]  

(28)

The indices \( s \) and \( r \) correspond here to "soft" and "rigid". The coefficient \( a_s \tau \) changes its sign when temperature passes \( T_c \). On the contrary, the coefficient before \( |\psi_r|^2 \) remains positive. The "soft" variable \( \psi_s \) plays the role of a driver for the phase transition. Indeed, proceeding from the system for the order parameters minimizing the free energy

\[ 2w \bar{\lambda}_1 + \left[ \eta_2 - \sum_i \frac{\beta_2}{4m_{2i}} (-i\hbar \nabla_i - \frac{2e}{c} A_i)^2 \right] - \alpha_2 \tau - \nu_2 |\bar{\lambda}_2|^2 |\bar{\lambda}_2| = 0 \]

\[ 2w \bar{\lambda}_2 + \left[ \eta_1 - \sum_i \frac{\beta_1}{4m_{1i}} (-i\hbar \nabla_i - \frac{2e}{c} A_i)^2 \right] - \alpha_1 \tau - \nu_1 |\bar{\lambda}_1|^2 |\bar{\lambda}_1| = 0, \]  

(29)

for the homogeneous case \((\vec{A} = 0 \text{ and } T < T_c)\), one finds

\[ |\bar{\lambda}_{1,2}|^2 = -\eta_{2,1} \Xi \tau, \]  

(30)

\[ \Xi_2 = \frac{\eta_1 \alpha_2 + \eta_2 \alpha_1}{\eta_2^2 \nu_2 + \eta_1^2 \nu_1}. \]  

(31)

The superconducting gaps \( \Delta_{1,2} = 2W \bar{\lambda}_{1,2} \) vanish simultaneously at \( T_c \). According to (24)

\[ |\bar{\psi}_s|^2 = \frac{|W|}{V} u \Xi \tau, \]  

(32)
\[ |\bar{\psi}_r|^2 = 0, \]
and for \( T > T_c \), \( \bar{\psi}_s = \bar{\psi}_r = 0 \). The superconducting gaps are determined only by the "soft" order parameter.

Minimizing (25) for \( \vec{A} = 0 \) in the normal phase shows that fluctuations of \( \psi_s \) and \( \psi_r \) satisfy the equations

\[ \sum_i \xi_{s,r,i}^2 \nabla_i^2 \psi_{s,r} = \psi_{s,r}, \]

where the corresponding coherence lengths are given by

\[ \xi_{si}^2 = \frac{\hbar^2}{4M_{si}a_s \tau}, \]

\[ \xi_{ri}^2 = \frac{\hbar^2}{4M_{ri}(a_0 - a_r \tau)}, \]

As the result, the fluctuations of band order parameters are governed by two special coherence scales. The \( \xi_s \) acts as an Ornstein-Zernicke type critical coherence length known for one-band systems and diverging at \( T \to T_c \). The other, \( \xi_r \), behaves noncritically (is rigid) and as an imaginary quantity characterizes a periodic spatial coherence wave. In the limit \( \tau \to 0 \), \( \xi_s \to \infty \), however \( \xi_r \) remains finite. Analogously a two-band superconductor with interband pairing possesses a critical and a noncritical relaxation channel [27]. The relaxation times connected with these channels show correspondingly a critical and noncritical dependence on temperature.

The discussion of the nature of the spatial periodic fluctuating structure characterized by the rigid coherence length \( \xi_r \) with weak dependence on temperature remains out of the scope of the present paper. However, it will be tempting to attribute \( \xi_r \) to the recently observed pair density wave in the Bi-compound [28]. Then seemingly \( \xi_r \) characterizes the average distance between the pairs (without phase coherence in the normal state). As a function of doping \( \xi_r \) falls off with leaving the underdoped region and remains nearly constant in the basic region of actual \( T_c \)-s. Then the pair distribution density becomes more homogeneous, cf. [29]. Note that a spatial modulation of gap sizes has also been observed [30].

5 Cuprate superconductivity coherence and critical magnetic fields

In general the cuprate superconductivity two-component scenario supposed in [8-11] supports on the separation of the strongly correlated doped CuO\(_2\) planes in nanoscale structural components. A new superconductivity playground is built up with radical reorganization of the electron spectrum until the creation of states corresponding to the doped-hole bearing
subsystem. This spectrum is doping-variable in band structure and densities. A new pairing channel with interband pair transfer opens between the (mainly) itinerant and defect components.

In what follows the cuprate critical coherence length $\xi_s = \xi_0 \tau^{-1/2}$ will be calculated using the model of [11]. For the valence band $\Gamma_{c\gamma} = -D$, $\Gamma_{0\gamma} = 0$ and for the defect system subbands $\Gamma_{c\alpha} = d_1 - \alpha c$, $\Gamma_{0\alpha} = d_1$ and $\Gamma_{c\beta} = d_2 - \beta c$, $\Gamma_{0\beta} = d_2$ ($d_1 > d_2$, $\alpha > \beta$). The measure of the doped hole concentration $c$ is scaled to the transition temperature maximum at $p = 0.16$ by $p = 0.28c$. The bottoms of the defect subbands (attributed to $(\frac{\pi}{2}, \frac{\pi}{2})$ and $(\pi, 0)$ - type regions of the momentum space) evolve down in energy with doping. The overlap with the itinerant band is reached correspondingly at $c_\beta = d_2/\beta$ and $c_\alpha = d_1/\alpha$. There are the following different arrangements of the bands and $\mu$. At $c < c_\beta$ $\mu_1 = d_2 - \beta c$ remains connected with the ”cold” $\beta$-band. In this very underdoped region the charge carriers concentrate first. For $c > c_\beta$ $\mu_2 = (d_2 - \beta c)\{1 + 2(1 - c)\beta D^{-1}\}^{-1}$ is shifted into the valence band. The overlap of the narrow $\beta$ band with the wide $\gamma$ band leads to the formation of two Fermi surface sheets with a tendency to appearance of a ”flat band” component with lowering $\mu$. For the effective doping near $c_0$, defined by $d_1 - \alpha c_0 = \mu_2$, the role of the ”hot” region is enhanced.

The $\mu_3 = [\alpha d_2 + \beta d_1 - 2\alpha \beta c][\alpha + \beta + (1 - c)2\alpha \beta D^{-1}]^{-1}$ intersects all three overlapping bands and $T_c$ becomes maximized. For the extended overdoping $c > c_1$, $\mu_3 = d_2 - \beta c_1$, the chemical potential falls out of the defect $\beta$-band. In the doping process the mixing of the band components stimulates the Fermi-liquid behaviour of the carriers. The decrease of $T_c$ at overdoping is connected with the deterioration of the interband pairing conditions. This leads to the drop of superfluid density on the background of extending hole concentrations.

One finds in this model ($\xi_s^2 = \xi_0^2 \tau^{-1}$)

$$\xi_0^2 = \frac{\hbar^2 (\eta_\alpha + \eta_\beta) \beta_\gamma m^{-1}_\gamma + \eta_\gamma (\beta_\alpha m^{-1}_\alpha + \beta_\beta m^{-1}_\beta)}{(\eta_\alpha + \eta_\beta)\alpha_\gamma + \eta_\gamma (\alpha_\alpha + \alpha_\beta)}, \tag{36}$$

which must be interpreted as the in-plane coherence length ($\xi_{ab}$).

The effective masses of the bands are connected with the constant densities of states $m^{-1}_i = \frac{V}{2\pi \hbar^2 \rho_i}$, $\rho_\alpha = 1/2\alpha$, $\rho_\beta = 1/2\beta$, $\rho_\gamma = (1 - c)/D$; $V$ is the plaquette area ($a^2$) in the CuO$_2$ plane.

The calculated $\xi_0$ over the whole hole doping region of a ”typical” cuprate together with the $T_c$-curve is given in Fig.1. The same plausible parameter set as in [11] has been taken for the illustration. The order of $\xi_0$ of some tens of Å in the actual region agrees with the values given in the literature for cuprates [6]. This points to some self-consistency of the theoretical scheme concerning the earlier results for energetic characteristics [8-12].

The general trend of $\xi_0(p)$ found starts with the steep decrease until the critical point $c_\beta$ is reached. Then a moderate enhancement until the second critical point $c_0$ with further quick increase follows. The behaviour of $\xi_0$ as projected on the $T_c$ bell-like curve wings seems to be a natural result. At extreme dopings $\xi_0$ diverges.
The theoretical valley-profile like $\xi_0(p)$ curve agrees in its behaviour well with the recent experimental result [25] on the whole doping scale. One can state also some agreement with the different $\xi_{ab}(p)$ behaviour in different fragmental regions given in [20-23], however these data cannot be joined continuously with our curve.

Theoretical doping dependence of the c-axis second critical field ($T = 0$) $H_{c2} = \Phi(2\pi\xi_0^2)^{-1}$ is given in Fig.2. This curve shows an intensive maximum built up from $c_\beta$ until $T_c$ maximum is reached.

In this respect there is also the agreement with the paper [25], however the $H_{c2}$ maximum in [25] lies nearly at the doping level which corresponds to the maximum of the collective pinning energy. Our theoretical $H_{c2}$ peak corresponds to the effective enhancement of the contribution of the cold subsystem with the inside shifted chemical potential. This defect subsystem possesses the smaller superconducting gap and supports larger $\xi_{ab}$. Note that in the present model $\Delta_1 < \Delta_2$ if $\rho_1 > \rho_2$.

The thermodynamic critical field

$$H_c(0) = \left[4\pi[\rho_\gamma \Delta_\gamma^2(0) + (\rho_\alpha + \rho_\beta)\Delta_\alpha^2(0)]\right]^{1/2}$$

(37)

characterizes the condensation energy and is shown vs doping in Fig.3. This is also a curve with a maximum (as observed [31]) which repeats the behaviour of $T_c$, of the superconducting density $n_s$ [11,12] and the superconducting gaps. This means that in the present model the strength of the pairing and the phase coherence develop and vanish simultaneously. Such behaviour of cuprates is stressed as the result of recent experimental investigations [13-18]. The parallel course of the condensation energy and of the superconducting gap has been followed in [32]. It must be mentioned that the present model delivers a sublinear Uemura-type limited segment in the $T_c$ vs $n_s$ plot at underdoping [12] with a followed back-turn along the branch characterizing the overdoped side.

In conclusion, the present simple model, which describes qualitatively correctly the behaviour of the cuprate energetic characteristics, serves the same result for coherence properties in agreement with the recent experimental data on the whole doping scale. The qualitative agreement between the recent experimental data and the present theoretical approach on the nature of the cuprate coherence length doping dependence seems to be of some importance, at least for the reliability of the model.

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Figure captions.

Fig. 1 Cuprate transition temperature (dashed line) and the ab-plane coherence length vs hole doping.

Fig. 2 The c-axis second critical magnetic field vs. doping.

Fig. 3 The thermodynamic critical magnetic field on the doping scale (eV$^{1/2}$).
