A method for calculating the aviation fuel $O_2$ saturation process parameters

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Abstract. This paper presents and substantiates the method of processing the experimental data obtained from the experiments related to air sparging through the jet fuel layer. The experimental data was collected by Henshaw and Lam during the study addressed to oxygen re-absorption characteristics of an oxygen depleted, quiescent Jet A-1 fuel when sparged with air. In this study we accepted the hypothesis that the experimental results should be described primarily based on physical knowledge of the process and common techniques used for the description of individual components of the mass transfer process in sparging. The proposed calculation method of sparging mass transfer is valid for sparging through porous surfaces with pores of 1 $\mu$m or less, with large groups of small bubbles forming in the fuel. Moreover, the proposed method has been well-tested in the parameter range used in the underlying experiments and needs to be thoroughly checked beyond this range.

1. Introduction

Fuel saturation with gas via sparging is often applied for needs of aviation and space-rocket technology. This process has been studied in a large number of experiments. As to the mass transfer in sparging with bubbles forming at the exit of 0.25 mm to 1-2 mm pores, the calculation method for such cases was examined in [1-4]. It should be noted that often the results of experimental data processing and generalization seem to be rather controversial and leave of account several factors that are significant in terms of common understanding of mass transfer process at sparging.

The present paper describes and substantiates a method of experimental data processing related to fuel saturation with oxygen. The proposed method takes into account a number of very important factors that allow making the regeneration time calculations more precise in comparison with the empirical formula obtained by Henshaw and Lam in [1]. The authors studied Jet A-1 fuel saturation by air sparging through porous surfaces. The admixture content on the air in fuel was changed from 0.5 to 0.98, where $C_s$ is the equilibrium value under the given conditions. The pore size, the sparger open part area and volume flow rate of air were measured during the experiments. Using a standard procedure, Henshaw and Lam obtained in [1] in particular that the time of complete saturation of fuel with an oxygen is independent on diffusion coefficient, gravity acceleration, mass transfer surface as it follows from the works of other researchers [3, 4].

The sparging process is similar in many ways to liquid boiling which, is well-known, cannot be described by a closed set of differential equations. This explains the existence of a broad variety of experimental data presentation methods. Therefore, we believe the experimental results should be...
described primarily based on physical knowledge of the process and common techniques used for the description of individual components of the mass transfer process in sparging.

Mass transfer in sparging is intensified by two key factors: first, intensive mixing of the fuel (Jet-A1 in this case) by air bubbles and, second, an extensive mass transfer surface, i.e. bubble surface. This effect is obvious from the solution of the mass balance equation addressed below.

The mixing occurs as a result of the bubbles causing the liquid to rise. The attempt to estimate the velocity of the rising two-phase flow based on the bubble floating velocity failed to produce a valid result, therefore we used another approach which is summarized below.

2. Calculation of the regeneration time

The mathematical description of mass transfer by sparging is based on the solution of the following differential equation [5]:

\[ F_{\text{bub}} \beta (C - C_{\text{surf}}) \cdot d\tau = -V_f dC \]  

Here $C$ is the content of admixture in the fuel volume $V_f$, $C_{\text{surf}}$ is the admixture content on the air (gas) bubble surface, $F_{\text{bub}}$ is the total mass transfer surface area, $\beta$ is the mass transfer coefficient. If the mass transfer surface and mass diffusivity remain constant during the sparging process, the equation can be easily integrated. As a result of integration we can estimate the regeneration time for fuel saturation with oxygen from initial admixture content to the final admixture content which is close to the equilibrium under the given conditions:

\[ \tau_f = \frac{V_f}{F_{\text{bub}} \beta} \ln \frac{C_0 - C_s}{C_f - C_s} \]  

Here $C_0, C_f$ are the admixture content in the fuel volume at the beginning and at the end of the process. The values of $F_{\text{bub}}$ and $\beta$ depend strongly on the method used for gas supply, which produces difficulties in obtaining a general estimation for these characteristics. Nevertheless in some cases it is possible to obtain rather satisfactory estimations. Fairly accurate calculations are possible when air is supplied through porous insert in the pipe. In this case the two-phase flow containing air bubbles and fuel is travelling inside the active volume $V_0$ delimited by a pipe diameter $D$, a perforated part of the pipe, $L_{\text{per}}$, and the distance from the perforated insert to the fuel surface $h$. This volume is delineated by the red dashed lines in figure 1.

![Figure 1](image-url)

**Figure 1.** Flow behavior in the tank with fuel sparged by air through porous insert in the pipe: a) side view, b) top view. I – active volume, II – the total fuel volume (20 liters).

The effective density of the two-phase flow in active volume is less than the liquid density $\rho_f$ in the tank volume. The effective density is described through the gas volume fraction $\alpha$ using the following expression:

\[ \rho_{\text{2ph}} = \rho_f (1 - \alpha) + \rho_{\text{air}} \alpha \]  

The mixing in active volume occurs as a result of the bubbles causing the liquid to rise with a buoyancy velocity

\[ w_{\text{bou}} \approx \sqrt{\frac{g h (\rho_f - \rho_{\text{2ph}})}{\rho_f}} \]  

(4)
but since $\alpha < 1$ and $\rho_{\text{air}} \ll \rho_f$, one can assume with a high degree of accuracy that $\rho_{2ph} = \rho_f (1 - \alpha)$, and the buoyancy velocity can be expressed as

$$w_{\text{bou}} = \sqrt{gh\alpha} \quad (5)$$

The gas volume fraction $\alpha$ can be calculated based on the definition of $\alpha$ as a portion of the two-phase flow occupied by gas: $\alpha = \frac{V_{\text{air}}}{V_0}$. The air volume inside $V_0$ can be calculated from the volume flow rate $\dot{V}_{\text{air}}$ of air coming through the sparger open part area $A$ and the time $\tau_{\text{bou}}$ it takes the bubbles to rise from the sparger surface to the fuel surface $V_{\text{air}} = \dot{V}_{\text{air}}\tau_{\text{bou}}$. Substituting the formulas $\tau_{\text{bou}} = \frac{h}{w_{\text{bou}}}$, $V_0 = Dh$ and $A = \pi DL_{\text{per}}$ into gas volume fraction equation leads to the following expression

$$\alpha = \frac{V_{\text{h}}}{V_0w_{\text{bou}}} = \frac{V_{\text{h}}\pi}{Ah\sqrt{gL_0}\alpha} = \frac{V_{\pi}}{A\sqrt{gh\alpha}}$$

and the final gas volume fraction formula:

$$\alpha = 2.15\left[\frac{V}{A\sqrt{gh}}\right]^2 \quad (6)$$

Mass transfer coefficient $\beta$ can be found from the analogy between the heat and mass transfer [6] using the dependency for the heat transfer between the sphere (bubble) and the flow surrounding it [7]. Replacing the Nusselt number ($Nu = \frac{kD_{\text{bub}}}{\lambda}$) with the Sherwood number ($Sh = \frac{\beta D_{\text{bub}}}{D}$), and the Prandtl number ($Pr = \frac{\nu_f}{\alpha_f}$) with the Schmidt number ($Sc = \frac{\nu_f}{\alpha_f}$), where $\alpha_f$ is thermal diffusivity coefficient, $D$ is the admixture diffusivity (diffusion coefficient) and $\nu_f$ is a fuel kinematic viscosity, allows deriving a relation for the calculation of the mass transfer coefficient

$$Sh = 2 + 0.03Re^{0.54}Sc^{0.33} + 0.35Re^{0.58}Sc^{0.35}, \quad (7)$$

where $\beta = \frac{Sh D_{\text{bub}}}{D_{\text{bub}}^3}$ and $Re = \frac{w_{\text{bou}}D_{\text{bub}}}{\nu_f}$. Thus to calculate the mass transfer coefficient one should know the bubble diameter and floating velocity. The bubble size can be estimated using the Fritz approach [6] to the calculation of the bubble departure diameter in the boiling medium:

$$D_{\text{bub}} = \psi(\theta) \frac{\sigma}{\sqrt{g(\rho_f - \rho_{\text{air}})}} \quad (8)$$

where $\psi(\theta)$ are the functions of the marginal angle $\theta$. Fritz data $\psi(\theta) = 0.02\theta$ [6, 7] can be used but marginal angle value for kerosene is less than that for water and is still under researches. It was assumed in the calculations that $\theta \approx 4.4^\circ$, so to calculate the bubble diameter we used formula (8) with $\psi(\theta) = 0.088$.

The bubbles floating velocity has been extensively studied [2, 5]. As it was already noted, active volume containing the bubbles moves as a single body with fuel therefore the velocity can be calculated by equation (5).

The mass transfer surface can be estimated based on the following considerations. All the bubbles are concentrated in the active volume: $V_0 = \frac{Ah}{\pi}$, the total air volume in it is $V_{\text{air}} = \frac{Ah}{\pi}$. Then the total number of bubbles is $N = \frac{V_{\text{air}}}{V_{\text{bub}}} = \frac{6\alpha Ah D_{\text{bub}}^2}{\pi^2}$. Multiplying $N$ by the sphere surface area $\pi D_{\text{bub}}^2$, we get

$$F_{\text{bub}}^S = \frac{6\alpha Ah}{\pi D_{\text{bub}}} = \frac{6V_{\text{air}}}{D_{\text{bub}}} \quad (9)$$

Equation (9) should be adjusted to account for the fact that the number of bubbles per unit of the sparger surface area depends on the pore size $\phi$. The adjustment can be made by introducing the correction factor $\varepsilon_1 = \left(\frac{\sigma^2}{\phi^2}\right)^{0.0736}$. Its power exponent was defined in [1].
Now we have all the elements required for the calculation of the regeneration time. In Henshaw and Lam study the initial concentration was $C_0 = 0.5$ and the process stopped when it reached $C_f = 0.98 C_s$ and fuel volume was 20 l therefore (see formula 2)

$$
\tau_{\text{calc}} = \frac{0.0643}{(\beta F_{\text{bub}})^{\text{calc}_1}}
$$

where the mass transfer coefficient and mass transfer surface area values are determined by equations (7 - 10). The calculation results obtained using this approach are presented in Table 1. Note that the values calculated taking into account correction factor $\varepsilon_1$ are marked by index "1". Table 1 also features the mass transfer surface and mass transfer coefficient products $(F_{\text{bub}}^E \beta)^{\text{exp}}$ calculated from equation (2) using experimental data $(\tau_{\text{exp}})$. For this purpose equation (2) was resolved with respect to $F_{\text{bub}}^E \beta$.

### Table 1. Regeneration times calculated by formulas.

| $N_{\text{exp}}$ | $A$ | $(F_{\text{bub}}^E \beta)_{\text{calcl}}$ | $(F_{\text{bub}}^E \beta)_{\text{calc2}}$ | $\tau_{\text{calc1}}$ | $\tau_{\text{calc2}}$ | $\tau_{\text{exp}}$ | $\tau_{\text{calc1}}/\tau_{\text{exp}}$ | $\tau_{\text{calc2}}/\tau_{\text{exp}}$ |
|------------------|-----|------------------|-----------------|----------------|----------------|----------------|------------------|------------------|
| -                | -   | -                | -               | -              | -              | -              | -                | -                |
| 1                | 0.0111 | 0.98 | 1.01 | 936 | 914 | 927 | 1.01 | 0.99 |
| 2                | 0.0177 | 1.09 | 0.98 | 1066 | 1182 | 1087 | 0.98 | 1.09 |
| 3                | 0.0111 | 0.98 | 1.01 | 1109 | 1083 | 1065 | 1.04 | 1.02 |
| 4                | 0.0177 | 1.09 | 0.98 | 1269 | 1401 | 1477 | 0.86 | 0.95 |
| 5                | 0.0326 | 1.34 | 1.02 | 238 | 313 | 335 | 0.71 | 0.93 |
| 6                | 0.0517 | 1.48 | 0.99 | 271 | 404 | 405 | 0.67 | 1.00 |
| 7                | 0.0326 | 1.34 | 1.02 | 282 | 371 | 362 | 0.78 | 1.02 |
| 8                | 0.0517 | 1.48 | 0.99 | 321 | 479 | 466 | 0.69 | 1.03 |

A comparison shows that the calculated $F_{\text{bub}}^E \beta$ and experimental results differ by no more than 14.5% at low gas volume fractions ($\alpha < 0.02$). At gas volume fractions $\alpha > 0.02$ the calculations produce results that differ from the experiment by 30 to 33%. Thus, the method described above is suitable for making a tentative estimate of the time but needs to be further adapted for higher gas volume fractions.

We proceed to calculating a more exact mass transfer parameters solving equation (2) using the definition of the product $(F_{\text{bub}}^E \beta)^{\text{exp}}$ obtained on the basis of Henshaw and Lam experimental data, to get the second correction factor for the product $(F_{\text{bub}}^E \beta)_{\text{calc}}$.

The dependence $(F_{\text{bub}}^E \beta)_{\text{calc}}/(F_{\text{bub}}^E \beta)_{\text{exp}} = F(\alpha)$ obtained from these calculations is presented as a graph in figure 2 which also shows the approximating curve (in logarithmic coordinates) which helps to obtain the correction factor in the form of the power function

$$
\varepsilon_2 = (3.776\alpha^{0.276})^{-1}
$$

The resulting $(F_{\text{bub}}^E \beta)_{\text{calc2}}$ calculated by equations (7) and (10) and both correction coefficients and the product $(F_{\text{bub}}^E \beta)_{\text{exp}}$ calculated by using the experimental data [1] are presented in Table 1.

The final form of the equation for the calculation of the mass transfer surface area and mass transfer coefficient product can be presented as follows

$$
(F_{\text{bub}}^E \beta)_{\text{calc2}}^{\text{fin}} = \frac{6\nu_{\text{air}}}{d_{\text{bub}}} \left(\frac{1}{\varphi} \right)^{0.0736} \beta^{1.589} \left(\frac{1}{\varphi} \right)^{0.0736} \alpha^{-0.276}
$$

$$
(F_{\text{bub}}^E \beta)_{\text{calc2}}^{\text{fin}} = \frac{6\nu_{\text{air}}}{d_{\text{bub}}} \left(\frac{1}{\varphi} \right)^{0.0736} = \frac{1.589\nu_{\text{air}}}{d_{\text{bub}}} \left(\frac{1}{\varphi} \right)^{0.0736} \alpha^{-0.276}
$$

$$
(F_{\text{bub}}^E \beta)_{\text{calc2}}^{\text{fin}} = \frac{6\nu_{\text{air}}}{d_{\text{bub}}} \left(\frac{1}{\varphi} \right)^{0.0736} \beta^{1.589} \left(\frac{1}{\varphi} \right)^{0.0736} \alpha^{-0.276}
$$

$$
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$$
Figure 2. The relation \( \frac{F_{\text{calc}}}{F_{\text{exp}}} \) as a function of the gas volume fraction.

It follows from Table 1 that the results of calculations with the use of criterial equation (7) which, strictly speaking, are valid for fuel saturation with oxygen from \( C_0 = 0.5 \) to \( C_1 = 0.98 \), agree quite well (the discrepancy is less than 9% ) with the calculations by the general method (equations (2), (6), (7) and (13)), with allowance for the corrections \( \epsilon_1 \) obtained from [1] and \( \epsilon_2 \) obtained from our work.

Conclusion
The method proposed for generalization of experimental data related to air sparging of jet fuel layer through porous surface was developed on the base of Henshaw and Lam experimental data. Taken into account physical knowledge of the process we developed a calculation chain based on criterial equations that can be used for calculations of mass transfer parameters during a fuel saturation process. The final version of method containing corrections allows calculations of regeneration time with a rather high accuracy.

It should be noted that the above sparging mass transfer calculation method is valid only for sparging through porous surfaces with pores of 1 \( \mu \)m or less, with large groups of small bubbles forming in the fuel. The proposed method has been well-tested in the parameter range used in Henshaw and Lam experiment only [1] and needs to be thoroughly checked beyond this range.

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