Background Analysis around Q_{\beta\beta} for \textsuperscript{76}Ge Double Beta Decay experiments, and Statistics at Low Count Rates

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Abstract

The background in the region of the Q-value for neutrinoless double beta decay of \textsuperscript{76}Ge has been investigated by different methods: Simulation with GEANT 4 of the HEIDELBERG-MOSCOW experiment, analysis of other Ge double beta experiments. Statistical features of the analysis at very low count rates are discussed.
1 Introduction

Recently first experimental evidence has been reported for neutrinoless double beta decay. Analysis of 55 kg y of data, taken by the HEIDELBERG-MOSCOW experiment in the GRAN SASSO over the years 1990 - 2000, has led \[1, 2, 3, 4\] to a half-life

\[ T_{1/2} = (0.8 - 18.3) \times 10^{25} \text{ years} \ (95\% \ C.L.), \]

with best value of \( T_{1/2} = 1.5 \times 10^{25} \text{ y} \), for the decay of the double beta emitter \(^{76}\text{Ge}\)

\[ ^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^- \]  \( (2) \)

Assuming the decay amplitude to be dominated by exchange of a massive Majorana neutrino (see, e.g. \[5\]), this half-life results in a value of the effective neutrino mass

\[ < m > = \sum |U_{ei}^2m_i| = 0.05 - 0.84 \text{ eV} \ (95\% \ C.L.), \]

with best value of 0.39 eV. Here a 50\% uncertainty in the nuclear matrix elements has been taken into account (for details see \[3\]).

This is for the first time that the absolute scale of the neutrino mass spectrum has been fixed, which cannot be achieved by neutrino oscillation experiments. This result restricts possible neutrino mass scenarios to degenerate or (still marginally allowed) inverse hierarchy \[6, 7, 8\]. In the degenerate case it leads to a common neutrino mass eigenvalue of

\[ m_1 = 0.05 - 3.4 \text{ eV} \ (95\% \ C.L.). \]

This result is nicely consistent with later collected or analyzed experimental data, such as results from Large Scale Structure and CMB measurements \[9, 10, 11\], or ultra-high energy cosmic rays \[12\]. The former yield an upper limit of \( \sum m_i = 1.0 \text{ eV} \) (corresponding in the degenerate case to a common mass eigenvalue \( m_0 < 0.33 \text{ eV} \)). The Z-burst scenario for ultra-high energy cosmic rays requires 0.1 - 1.3 eV \[12\]. Tritium single beta decay cuts the upper range in eq.(4) down to 2.2 or 2.8 eV \[13\].

There is further theoretical support for a neutrino mass in the range fixed by the HEIDELBERG-MOSCOW experiment. A model based on an A4 symmetry of the neutrino mass matrix requires the neutrinos to be degenerate
Figure 1: The spectrum taken with $^{76}$Ge detectors Nrs. 1,2,3,4,5 over the period August 1990 - May 2000 (54.98 kg y), in the energy range 2000 - 2100 keV. Simultaneous fit of the $^{214}$Bi-lines and the two high-energy lines yield a probability for a line at 2039 keV of 91% C.L.

and the common mass eigenvalue to be $>0.2$ eV [14].

Starting with the hypothesis that quark and lepton mixing are identical at or near the GUT scale, Mohapatra et al. [15] show that the large solar and atmospheric neutrino mixing angles can be understood purely as result of renormalization group evolution, if neutrino masses are quasi-degenerate (with same CP parity). The common Majorana neutrino mass then must be larger than 0.1 eV.

In this paper we report additional support of the double beta result of [1, 2, 3, 4], by further discussion of the structure of the experimental background, by statistical considerations and by analysis of other double beta experiments investigating the decay of $^{76}$Ge.

Important points in the analysis of the measured spectrum are the structure of the background around $Q_{\beta\beta} (=2039.006(50)$ keV [16]), and the energy range of analysis around $Q_{\beta\beta}$. 

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2 Structure of Background Around $Q_{\beta\beta}$ in Different Ge Double Beta Experiments

Fig. 1 shows the spectrum measured in the range 2000 - 2100 keV in its original binning of 0.36 keV. By the peak search procedure developed on basis of the Bayes and Maximum Likelihood Methods, exploiting as important input parameters the experimental knowledge on the shape and width of lines in the spectrum, weak lines of $^{214}$Bi have been identified at the energies of 2010.78, 2016.7, 2021.6 and 2052.94 keV. Fig. 2 shows the
Figure 4: Analysis of the spectrum measured by D. Caldwell et al. [21], with the Maximum Likelihood Method, in the energy range 2000 - 2060 keV assuming lines at 2010.78, 2016.70, 2021.60, 2052.94, 2039.0 keV. No indication for a signal at 2039 keV is observed in this case.

probability that there is a line of correct width and of Gaussian shape at a given energy, assuming all the rest of the spectrum as flat background (which is a highly conservative assumption), for the HEIDELBERG-MOSCOW experiment.

The intensities of these lines have been shown to be consistent with other, strong Bi lines in the measured spectrum according to the branching ratios given in the Table of Isotopes [17], and to Monte Carlo simulation of the experimental setup [3]. Note that the 2016 keV line, as an E0 transition, can be seen only by coincident summing of the two successive lines $E = 1407.98$ keV and $E = 609.316$ keV. Its observation proves that the $^{238}$U impurity from which it is originating, is located in the Cu cup of the detectors. Recent measurements of the spectrum of a $^{214}$Bi source as function of distance source-detector confirm this interpretation [18].

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These $^{214}$Bi lines occur also in other investigations of double beta decay with Ge - and - even more important - also the additional structures in Fig. 2, which cannot be attributed at present, are seen in these other investigations.

There are three other Ge experiments which have looked for double beta
decay of $^{76}\text{Ge}$. First there is the experiment by Caldwell et al. [21], using natural Germanium detectors (7.8% abundance of $^{76}\text{Ge}$, compared to 86% in the HEIDELBERG-MOSCOW experiment). This was the most sensitive natural Ge experiment. With their background a factor of 9 higher than in the HEIDELBERG-MOSCOW experiment and their measuring time of 22.6 kg years, they had a statistics of the background by a factor of almost four larger than in the HEIDELBERG-MOSCOW experiment. This gives useful information on the composition of the background.

![Figure 5: Result of the peak-search procedure performed for the ITEP/YePI spectrum [22] (left: Maximum Likelihood method, right: Bayes method). On the y axis the probability of having a line at the corresponding energy in the spectrum is shown.](image1)

![Figure 6: Result of the peak-search procedure performed for the IGEX spectrum [23, 24] using the ML approach (left) and the Bayesian statistics (right). On the y axis the probability of having a line at the corresponding energy in the spectrum is shown.](image2)

Applying the same method of peak search as used in Fig. 2 yields indications for peaks essentially at the same energies as in Fig. 2 (see Fig. 3). This
shows that these peaks are not fluctuations. In particular it sees the 2010.78, 2016.7, 2021.6 and 2052.94 keV $^{214}$Bi lines, but also the unattributed lines at higher energies. It finds, however, no line at 2039 keV. This is consistent with the expectation from the rate found in the HEIDELBERG-MOSCOW experiment. About 16 observed events in the latter correspond to 0.6 expected events in the Caldwell experiment, because of the use of non-enriched material and the shorter measuring time. Fit of the Caldwell spectrum allowing for the $^{214}$Bi lines and a 2039 keV line yields 0.4 events for the latter (see Fig. 4).

The first experiment using enriched (but not high-purity) Germanium 76 detectors was that of Kirpichnikov and coworkers [22]. These authors show only the energy range between 2020 and 2064 keV of their measured spectrum. The peak search procedure finds also here indications of lines around 2028 keV and 2052 keV (see Fig. 5, but no indication of a line at 2039 keV. This is consistent with the expectation, because for their low statistics of 2.95 kg y they would expect here (according to HEIDELBERG-MOSCOW) 0.9 counts.

Another experiment (IGEX) used between 6 and 8.8 kg of enriched $^{76}$Ge, but collected since beginning of the experiment in the early nineties till shutdown in 1999 only 8.8 kg years of statistics [23, 24]. The authors of [23, 24] unfortunately show only the range 2020 to 2060 keV of their measured spectrum in detail. Fig. 6 shows the result of our peak scanning of this range. Clear indications are seen for the Bi lines at 2021 and 2052 keV, but also of the unidentified structure around 2030 keV. Because of the conservative assumption on the background treatment in the scanning procedure (see above) there is no chance to see a signal at 2039 keV because of the 'hole' in the background of that spectrum (see Fig. 1 in [23]). With some good will one might see, however, an indication of 3 events here, consistent with the expectation of the HEIDELBERG-MOSCOW experiment of 2.6 counts.

It may be noted that all three experiments with enriched $^{76}$Ge see a line around 2028 keV. It was already mentioned in [25], that it is suspicious that this line which is seen also clearly in the pulse-shape analyzed spectrum of the HEIDELBERG-MOSCOW experiment (see 1, 2, 3) differs in energy from $Q_{\beta\beta}$ just by the K-shell X-ray energies of Ge (Se) of 9.2 (10.50) keV, or the K shell electron atomic binding energies of 11.10 (12.66) keV (see [17]), and that this might point to a partly not understood mechanism of the double beta decay process.
Figure 7: Example of a random-generated spectrum with a Poisson distributed background with 4.0 events per channel and a Gaussian line centered in channel 50 (line-width corresponds to a standard-deviation of $\sigma = 4.0$ channels). The left picture shows a spectrum with a line-intensity of 10 events, the right spectrum a spectrum with a line-intensity of 100 events. The background is shown dark, the events of the line bright.

3 Statistics at Low Count Rates - Peak Search, and Analysis Window

At this point it may be useful to demonstrate the potential of the used peak search procedure. Fig. 7 shows a spectrum with Poisson-generated background of 4 events per channel and a Gaussian line with width (standard deviation) of 4 channels centered at channel 50, with intensity of 10 (left) and 100 (right) events, respectively. Fig. 8 right shows the result of the analysis of spectra of different line intensity with the Bayes method (here Bayes 1-4 correspond to different choice of the prior distribution: (1) $\mu(\eta) = 1$ (flat), (2) $\mu(\eta) = 1/\eta$, (3) $\mu(\eta) = 1/\sqrt{\eta}$, (4) Jeffrey’s prior) and the Maximum Likelihood Method. For each prior 1000 spectra have been generated with equal background and equal line intensity using random number generators available at CERN [27]. The average values of the best values agree (see Fig. 8) very well with the known intensities also for very low count rates (as in Fig. 7 left).

In Fig. 9 we show two simulations of a Gaussian line of 15 events, centered at channel 50, again with width (standard deviation) of 4 channels, on a Poisson-distributed background with 0.5 events/channel. The figure gives an indication of the possible degree of deviation of the energy of the peak maximum from the transition energy, on the level of statistics collected in experiments like the HEIDELBERG-MOSCOW experiment (here one channel
Figure 8: Results of analysis of random-number generated spectra, using Bayes and Maximum Likelihood method (the first one with different prior distributions). For every number of events in the simulated line, shown on the x-axis, 1000 random generated spectra were evaluated with the five given methods. The analysis on the left side was performed with an Poisson distributed background of 0.5 events per channel, the background for the spectra on the right side was 4.0 events per channel. Every vertical line shows the mean value of the calculated best values (thick points) with the $1\sigma$ error area. The mean values are in good agreement with the expected values (black horizontal dashed lines).

corresponds to 0.36 keV). This should be considered when comparing Figs. 2, 3, 5, 6.

The influence of the choice of the energy range of the analysis around $Q_{\beta\beta}$ has been thoroughly discussed in [2, 3]. Since erroneous ideas about this point are still around, a few further comments may be given here. In Fig. 10 we show the analysis of a simulated spectrum consisting of a Gaussian line of width (standard deviation) of 4 channels and intensity of 10 counts on a Poisson-distributed background of 4 events per channel (see fig. 7 left), as function of the width of the range of analysis. It is seen that a reliable result is obtained for a range of analysis of not smaller than 35 channels (i.e. ±18 channels) - one channel corresponding to 0.36 keV in the HEIDELBERG-MOSCOW experiment. This is an important result, since it is of course important to keep the range of analysis as small as possible, to avoid to include lines in the vicinity of the weak signal into the background. This unavoidably occurs when e.g. proceeding as suggested in Aalseth et. al., hep-ex/0202018 and Mod. Phys. Lett. A 17 (2002) 1475, Yu.G. Zdesenko et. al., Phys. Lett. B 546 (2002) 206. The arguments given in those papers are therefore incorrect. Also Kirpichnikov, who states [28] that his analysis...
finds a 2039 keV signal in the HEIDELBERG-MOSCOW spectrum on a 4 sigma confidence level (as we also see it, when using the Feldman-Cousins method [26]), makes this mistake when analyzing the pulse-shape spectrum.

4 Simulation of the Background with GEANT 4

Finally the background around $Q_{\beta\beta}$ will be discussed from the side of simulation. A very careful new simulation of the different components of radioactive background in the HEIDELBERG-MOSCOW experiment has been performed recently by a new Monte Carlo program based on GEANT4 [29, 30]. This simulation uses a new event generator for simulation of radioactive decays basing on ENSDF-data and describes the decay of arbitrary radioactive isotopes including alpha, beta and gamma emission as well as conversion electrons and X-ray emission. Also included in the simulation is the influence of neutrons in the energy range from thermal to high energies up to 100 MeV on the measured spectrum. Elastic and inelastic reactions, and capture have been taken into account, and the corresponding production of radioactive isotopes in the setup. The neutron fluxes and energy distribu-
Figure 10: Result of an analysis as function of the evaluation width. The used spectrum consists of a Poisson distributed background with 4 events per channel, and a line of 10 events (see fig. 7, left part). The dark area corresponds to a 68.3% confidence area with the dark line being the best value. Below an evaluation width of 35 channels the result becomes unreliable, above 35 channels the result is stable.

Measurements were taken from published measurements performed in the Gran Sasso. Also simulated was the cosmic muon flux measured in the Gran Sasso, on the measured spectrum. To give a feeling for the quality of the simulation, Fig. 11 shows the simulated and the measured spectra for a \(^{228}\text{Th}\) source spectrum for 4 of our five detectors. The agreement is excellent.

The simulation of the background of the experiment reproduces all observed lines in the energy range between threshold (around 100 keV) and 2020 keV [29]. Fig. 12 shows the simulated background in the range 2000-2100 keV with all known background components. The black solid line corresponds to the measured data in the period 20.11.1995 - 16.4.2002 (55.57 kg y).

The background around \(Q_{\beta\beta}\) is according to the simulations flat, the only expected lines come from \(^{214}\text{Bi}\) (from the \(^{238}\text{U}\) natural decay chain) at 2010.89, 2016.7, 2021.6, 2052.94, 2085.1 and 2089.7 keV. Lines from cosmogenically produced \(^{56}\text{Co}\) (at 2034.76 keV and 2041.16 keV), half-life 77.3 days, are not expected since the first 200 days of measurement of each detector are not used in the data analysis. Also the potential contribution from decays of \(^{77}\text{Ge}\), \(^{66}\text{Ga}\), or \(^{228}\text{Ac}\), should not lead to signals visible in our measured...
Figure 11: Comparison of the measured data (black line, November 1995 to April 2002) and simulated spectrum (red line) for the detectors Nrs. 1, 2, 3 and 5 for a $^{232}$Th source spectrum. The agreement of simulation and measurement is excellent.

spectrum near the signal at $Q_{\beta\beta}$. For details we refer to [30].

The structures around 2028 keV, 2066 keV and 2075 keV seen - as also the $^{214}$Bi lines - in practically all Ge experiments (see above), cannot be identified at present. The 2028 keV line because of its strong occurrence in the PSA spectrum in the HEIDELBERG-MOSCOW experiment, may play a special role here (see above).

## 5 Conclusion

Concluding, additional support has been given for the evidence of a signal for neutrinoless double beta decay, by showing consistency of the result - for the signal, and for the background - with other double beta decay experiments using non-enriched or enriched Germanium detectors. In particular it has been shown that the lines seen in the vicinity of the signal (including
those which at present cannot be attributed) are seen also in the other experiments. This is important for the correct treatment of the background. Furthermore, the sensitivity of the peak identification procedures has been demonstrated by extensive statistical simulations. It has been further shown by new extensive simulations of the expected background by GEANT4, that the background around $Q_{\beta\beta}$ should be flat, and that no known gamma line is expected at the energy of $Q_{\beta\beta}$. The 2039 keV signal is seen only in the HEIDELBERG-MOSCOW experiment, which has a factor of 10, and much more, statistics than all other double beta experiments.

References

[1] H.V. Klapdor-Kleingrothaus et al. hep-ph/0201231 and *Mod. Phys. Lett. A* **16** (2001) 2409-2420.

[2] H.V. Klapdor-Kleingrothaus, A. Dietz and I.V. Krivosheina, Part. and Nucl. **110** (2002) 57-79

[3] H.V. Klapdor-Kleingrothaus, A. Dietz and I.V. Krivosheina, *Foundations of Physics* **31** (2002) 1181-1223 and Corrigenda, 2003 home-page: [http://www.mpi-hd.mpg.de/nonacc/main_results.html](http://www.mpi-hd.mpg.de/nonacc/main_results.html)

[4] H.V. Klapdor-Kleingrothaus, hep-ph/0303217
[5] H.V. Klapdor-Kleingrothaus, "60 Years of Double Beta Decay - From Nuclear Physics to Beyond the Standard Model", World Scientific, Singapore (2001) 1281 p.

[6] H.V. Klapdor-Kleingrothaus, H. Päś and A.Yu. Smirnov, Phys. Rev. D 63 (2001) 073005 and hep-ph/0003219

[7] H.V. Klapdor-Kleingrothaus, U. Sarkar, Mod. Phys. Lett. A 16 (2001) 2460-2482

[8] H.V. Klapdor-Kleingrothaus, U. Sarkar, hep-ph/0304032

[9] D.N. Spergel et. al., astro-ph/0302209

[10] S. Hannestad, astro-ph/0303076

[11] A. Pierce, H. Murayama, astro-ph/0302131

[12] Z. Fodor et. al., JHEP (2002) 0206046 or hep-ph/0203198 and hep-ph/0210123. D. Fargion et. al., Proc. DARK 2000, Heidelberg, ed. H.V. Klapdor-Kleingrothaus (Springer, Heidelberg, 2001) 455 and Proc. BEYOND02, Oulu Finland, ed. H.V. Klapdor-Kleingrothaus (IOP Bristol, 2003); H. Päś and H. Weiler, Phys. Rev. D 63 (2001) 113015

[13] J. Bonn et. al., Nucl. Phys. B (Proc. Suppl.) 91 (2001) 273

[14] K.S. Babu, E. Ma, J.W.F. Valle, hep-ph/0206292

[15] R. Mohapatra, M. K. Parida, G. Rajasekaran, hep-ph/0301234

[16] G. Douysset et. al., Phys. Rev. Lett. 86 (2001) 4259 - 4262.

[17] R.B. Firestone and V.S. Shirley, Table of Isotopes, 8-th Edition, John Wiley and Sons, Incorp., N.Y. (1998).

[18] H.V. Klapdor-Kleinegrothaus, O. Chkvorets, I.V. Krivosheina, C. Tomei, to be published

[19] H.V. Klapdor-Kleinegrothaus, hep-ph/0205228

[20] G. Gilmore, J. Hemingway, “Practical Gamma-Ray Spectrometry”, Wiley and Sons (1995)

[21] D. Caldwell et. al., J. Phys. G 17 (1991) S137-S144

[22] A.A. Vasenko et. al., Mod. Phys. Lett. A 5 (1990) 1299, and I. Kirpichnikov, Preprint ITEP (1991)

[23] C.E. Aalseth et. al., Yad. Fiz. 63 (2000) 1299
[24] C.E. Aalseth et. al., Phys. Rev. D 65 (2002) 092007

[25] H.V. Klapdor-Kleingrothaus, Proc. BEYOND02, Oulu, Finland 2002, ed. H.V. Klapdor-Kleingrothaus (IOP, Bristol) 2003

[26] A. Dietz, Dissertation, University of Heidelberg, 2003

[27] CERN number generators (see e.g. http://root.cern.ch/root/html/TRandom.html)

[28] I. Kirpichnikov, talk at Conf. on Nucl. Phys Russ. Acad. Sci., Moscow, Dec. 2 (2002), and private communication, Dec. 2002

[29] C. Dörr, diploma thesis, University of Heidelberg, 2002, unpublished

[30] H.V. Klapdor-Kleingrothaus et. al., to be published
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Figure 2: Result of the peak-searching procedure performed on the HEIDELBERG-MOSCOW spectrum using the Maximum Likelihood approach (left) and the Bayesian method (right). On the y axis the probability of having a line at the corresponding energy in the spectrum is shown.

Figure 3: Result of the peak-search procedure performed for the UCBS/LBL spectrum [21] (left: Maximum Likelihood method, right: Bayes method). On the y axis the probability of having a line at the corresponding energy in the spectrum is shown.

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![Figure 5](image1.png)

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![Figure 6](image2.png)

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Applying the same method of peak search as used in Fig. \cite{2} yields indications for peaks essentially at the same energies as in Fig. \cite{2} (see Fig. \cite{3}). This
shows that these peaks are not fluctuations. In particular it sees the 2010.78, 2016.7, 2021.6 and 2052.94 keV $^{214}$Bi lines, but also the unattributed lines at higher energies. It finds, however, no line at 2039 keV. This is consistent with the expectation from the rate found in the HEIDELBERG-MOSCOW experiment. About 16 observed events in the latter correspond to 0.6 expected events in the Caldwell experiment, because of the use of non-enriched material and the shorter measuring time. Fit of the Caldwell spectrum allowing for the $^{214}$Bi lines and a 2039 keV line yields 0.4 events for the latter (see Fig. 4).

The first experiment using enriched (but not high-purity) Germanium 76 detectors was that of Kirpichnikov and coworkers [22]. These authors show only the energy range between 2020 and 2064 keV of their measured spectrum. The peak search procedure finds also here indications of lines around 2028 keV and 2052 keV (see fig. 3), but not any indication of a line at 2039 keV. This is consistent with the expectation, because for their low statistics of 2.95 kg y they would expect here (according to HEIDELBERG-MOSCOW) 0.9 counts.

Another experiment (IGEX) used between 6 and 8.8 kg of enriched $^{76}$Ge, but collected since beginning of the experiment in the early nineties till shutdown in 1999 only 8.8 kg years of statistics [23, 24]. The authors of [23, 24] unfortunately show only the range 2020 to 2060 keV of their measured spectrum in detail. Fig. 6 shows the result of our peak scanning of this range. Clear indications are seen for the Bi lines at 2021 and 2052 keV, but also of the unidentified structure around 2030 keV. Because of the conservative assumption on the background treatment in the scanning procedure (see above) there is no chance to see a signal at 2039 keV because of the 'hole' in the background of that spectrum (see Fig. 1 in [23]). With some good will one might see, however, an indication of 3 events here, consistent with the expectation of the HEIDELBERG-MOSCOW experiment of 2.6 counts.

It may be noted that all three experiments with enriched $^{76}$Ge see a line around 2028 keV. It was already mentioned in [25], that it is suspicious that this line which is seen also clearly in the pulse-shape analyzed spectrum of the HEIDELBERG-MOSCOW experiment (see [11, 21, 3]) differs in energy from $Q_{\beta\beta}$ just by the K-shell X-ray energies of Ge (Se) of 9.2 (10.50) keV, or the K shell electron atomic binding energies of 11.10 (12.66) keV (see [17]), and that this might point to a partly not understood mechanism of the double beta decay process.
Figure 7: Example of a random-generated spectrum with a Poisson distributed background with 4.0 events per channel and a Gaussian line centered in channel 50 (line-width corresponds to a standard-deviation of $\sigma = 4.0$ channels). The left picture shows a spectrum with a line-intensity of 10 events, the right spectrum a spectrum with a line-intensity of 100 events. The background is shown dark, the events of the line bright.

3 Statistics at Low Count Rates - Peak Search, and Analysis Window

At this point it may be useful to demonstrate the potential of the used peak search procedure. Fig. 7 shows a spectrum with Poisson-generated background of 4 events per channel and a Gaussian line with width (standard deviation) of 4 channels centered at channel 50, with intensity of 10 (left) and 100 (right) events, respectively. Fig. 8 right shows the result of the analysis of spectra of different line intensity with the Bayes method (here Bayes 1-4 correspond to different choice of the prior distribution: (1) $\mu(\eta) = 1$ (flat), (2) $\mu(\eta) = 1/\eta$, (3) $\mu(\eta) = 1/\sqrt{\eta}$, (4) Jeffrey’s prior) and the Maximum Likelihood Method. For each prior 1000 spectra have been generated with equal background and equal line intensity using random number generators available at CERN [27]. The average values of the best values agree (see Fig. 8) very well with the known intensities also for very low count rates (as in Fig. 7 left).

In Fig. 9 we show two simulations of a Gaussian line of 15 events, centered at channel 50, again with width (standard deviation) of 4 channels, on a Poisson-distributed background with 0.5 events/channel. The figure gives an indication of the possible degree of deviation of the energy of the peak maximum from the transition energy, on the level of statistics collected in experiments like the HEIDELBERG-MOSCOW experiment (here one channel
Figure 8: Results of analysis of random-number generated spectra, using Bayes and Maximum Likelihood method (the first one with different prior distributions). For every number of events in the simulated line, shown on the x-axis, 1000 random generated spectra were evaluated with the five given methods. The analysis on the left side was performed with an Poisson distributed background of 0.5 events per channel, the background for the spectra on the right side was 4.0 events per channel. Every vertical line shows the mean value of the calculated best values (thick points) with the 1σ error area. The mean values are in good agreement with the expected values (black horizontal dashed lines).

corresponds to 0.36 keV). This should be considered when comparing Figs. 2, 3, 5, 6.

The influence of the choice of the energy range of the analysis around $Q_{\beta\beta}$ has been thoroughly discussed in [2, 3]. Since erroneous ideas about this point are still around, a few further comments may be given here. In Fig. 10 we show the analysis of a simulated spectrum consisting of a Gaussian line of width (standard deviation) of 4 channels and intensity of 10 counts on a Poisson-distributed background of 4 events per channel (see fig. 7 left), as function of the width of the range of analysis. It is seen that a reliable result is obtained for a range of analysis of not smaller than 35 channels (i.e. ±18 channels) - one channel corresponding to 0.36 keV in the HEIDELBERG-MOSCOW experiment. This is an important result, since it is of course important to keep the range of analysis as small as possible, to avoid to include lines in the vicinity of the weak signal into the background. This unavoidably occurs when e.g. proceeding as suggested in Aalseth et. al., hep-ex/0202018 and Mod. Phys. Lett. A 17 (2002) 1475, Yu.G. Zdesenko et. al., Phys. Lett. B 546 (2002) 206. The arguments given in those papers are therefore incorrect. Also Kirpichnikov, who states [28] that his analysis
finds a 2039 keV signal in the HEIDELBERG-MOSCOW spectrum on a 4 sigma confidence level (as we also see it, when using the Feldman-Cousins method [26]), makes this mistake when analyzing the pulse-shape spectrum.

4 Simulation of the Background with GEANT 4

Finally the background around $Q_{\beta\beta}$ will be discussed from the side of simulation. A very careful new simulation of the different components of radioactive background in the HEIDELBERG-MOSCOW experiment has been performed recently by a new Monte Carlo program based on GEANT4 [29, 30]. This simulation uses a new event generator for simulation of radioactive decays basing on ENSDF-data and describes the decay of arbitrary radioactive isotopes including alpha, beta and gamma emission as well as conversion electrons and X-ray emission. Also included in the simulation is the influence of neutrons in the energy range from thermal to high energies up to 100 MeV on the measured spectrum. Elastic and inelastic reactions, and capture have been taken into account, and the corresponding production of radioactive isotopes in the setup. The neutron fluxes and energy distribu-
Figure 10: Result of an analysis as function of the evaluation width. The used spectrum consists of a Poisson distributed background with 4 events per channel, and a line of 10 events (see fig. 7 left part). The dark area corresponds to a 68.3% confidence area with the dark line being the best value. Below an evaluation width of 35 channels the result becomes unreliable, above 35 channels the result is stable.

Simulations were taken from published measurements performed in the Gran Sasso. Also simulated was the cosmic muon flux measured in the Gran Sasso, on the measured spectrum. To give a feeling for the quality of the simulation, Fig. 11 shows the simulated and the measured spectra for a \(^{228}\text{Th}\) source spectrum for 4 of our five detectors. The agreement is excellent.

The simulation of the background of the experiment reproduces all observed lines in the energy range between threshold (around 100 keV) and 2020 keV. Fig. 12 shows the simulated background in the range 2000-2100 keV with all known background components. The black solid line corresponds to the measured data in the period 20.11.1995 - 16.4.2002 (55.57 kg y).

The background around \(Q_{\beta\beta}\) is according to the simulations flat, the only expected lines come from \(^{214}\text{Bi}\) (from the \(^{238}\text{U}\) natural decay chain) at 2010.89, 2016.7, 2021.6, 2052.94, 2085.1 and 2089.7 keV. Lines from cosmogenically produced \(^{56}\text{Co}\) (at 2034.76 keV and 2041.16 keV), half-life 77.3 days, are not expected since the first 200 days of measurement of each detector are not used in the data analysis. Also the potential contribution from decays of \(^{77}\text{Ge}\), \(^{66}\text{Ga}\), or \(^{228}\text{Ac}\), should not lead to signals visible in our measured
Figure 11: Comparison of the measured data (black line, November 1995 to April 2002) and simulated spectrum (red line) for the detectors Nr. 1, 2, 3 and 5 for a $^{232}\text{Th}$ source spectrum. The agreement of simulation and measurement is excellent.

The structures around 2028 keV, 2066 keV and 2075 keV seen - as also the $^{214}\text{Bi}$ lines - in practically all Ge experiments (see above), cannot be identified at present. The 2028 keV line because of its strong occurrence in the PSA spectrum in the HEIDELBERG-MOSCOW experiment, may play a special role here (see above).

5 Conclusion

Concluding, additional support has been given for the evidence of a signal for neutrinoless double beta decay, by showing consistency of the result - for the signal, and for the background - with other double beta decay experiments using non-enriched or enriched Germanium detectors. In particular it has been shown that the lines seen in the vicinity of the signal (including...
those which at present cannot be attributed) are seen also in the other experiments. This is important for the correct treatment of the background. Furthermore, the sensitivity of the peak identification procedures has been demonstrated by extensive statistical simulations. It has been further shown by new extensive simulations of the expected background by GEANT4, that the background around $Q_{\beta\beta}$ should be flat, and that no known gamma line is expected at the energy of $Q_{\beta\beta}$. The 2039 keV signal is only in the HEIDELBERG-MOSCOW experiment, which has a factor of 10, and much more, statistics than all other double beta experiments.

References

[1] H.V. Klapdor-Kleingrothaus et al. hep-ph/0201231 and Mod. Phys. Lett. A 16 (2001) 2409-2420.

[2] H.V. Klapdor-Kleingrothaus, A. Dietz and I.V. Krivosheina, Part. and Nucl. 110 (2002) 57-79

[3] H.V. Klapdor-Kleingrothaus, A. Dietz and I.V. Krivosheina, Foundations of Physics 31 (2002) 1181-1223 and Corrigenda, 2003 home-page: http://www.mpi-hd.mpg.de/non.acc/main_results.html

[4] H.V. Klapdor-Kleingrothaus, hep-ph/0303217
[5] H.V. Klapdor-Kleingrothaus, "60 Years of Double Beta Decay - From Nuclear Physics to Beyond the Standard Model", *World Scientific, Singapore* (2001) 1281 p.

[6] H.V. Klapdor-Kleingrothaus, H. Päss and A.Yu. Smirnov, *Phys. Rev.* D **63** (2001) 073005 and *hep-ph/0003219*

[7] H.V. Klapdor-Kleingrothaus, U. Sarkar, *Mod. Phys. Lett.* A **16** (2001) 2460-2482

[8] H.V. Klapdor-Kleingrothaus, U. Sarkar, *hep-ph/0304032*

[9] D.N. Spergel et. al., *astro-ph/0302209*

[10] S. Hannestad, *astro-ph/0303076*

[11] A. Pierce, H. Murayama, *astro-ph/0302131*

[12] Z. Fodor et. al., JHEP (2002) 0206046 or *hep-ph/0203198* and *hep-ph/0210123*, D. Fargion et. al., Proc. DARK 2000, Heidelberg, ed. H.V. Klapdor-Kleingrothaus (Springer, Heidelberg, 2001) 455 and Proc. BEYOND02, Oulu Finland, ed. H.V. Klapdor-Kleingrothaus (IOP Bristol, 2003); H. Päss and H. Weiler, *Phys. Rev.* D **63** (2001) 113015

[13] J. Bonn et. al., *Nucl. Phys.* B (Proc. Suppl.) **91** (2001) 273

[14] K.S. Babu, E. Ma, J.W.F. Valle, *hep-ph/0206292*

[15] R. Mohapatra, M. K. Parida, G. Rajasekaran, *hep-ph/0301234*

[16] G. Douysset et. al., *Phys. Rev. Lett.* **86** (2001) 4259 - 4262.

[17] R.B. Firestone and V.S. Shirley, Table of Isotopes, 8-th Edition, *John Wiley and Sons*, Incorp., N.Y. (1998).

[18] H.V. Klapdor-Kleingrothaus, O. Chkvorets, I.V. Krivosheina, C. Tomei, to be published

[19] H.V. Klapdor-Kleingrothaus, *hep-ph/0205228*

[20] G. Gilmore, J. Hemingway, “Practical Gamma-Ray Spectrometry”, *Wiley and Sons* (1995)

[21] D. Caldwell et. al., *J. Phys.* G **17** (1991) S137-S144

[22] A.A. Vasenko et. al., *Mod. Phys. Lett.* A **5** (1990) 1299, and I. Kirpichnikov, Preprint ITEP (1991)

[23] C.E. Aalseth et. al., *Yad. Fiz.* **63** (2000) 1299
[24] C.E. Aalseth et. al., Phys. Rev. D 65 (2002) 092007

[25] H.V. Klapdor-Kleingrothaus, Proc. BEYOND02, Oulu, Finland 2002, ed. H.V. Klapdor-Kleingrothaus (IOP, Bristol) 2003

[26] A. Dietz, Dissertation, University of Heidelberg, 2003

[27] CERN number generators (see e.g. http://root.cern.ch/root/html/TRandom.html)

[28] I. Kirpichnikov, talk at Conf. on Nucl. Phys Russ. Acad. Sci., Moscow, Dec. 2 (2002), and private communication, Dec. 2002

[29] C. Dörr, diploma thesis, University of Heidelberg, 2002, unpublished

[30] H.V. Klapdor-Kleingrothaus et. al., to be published