Curvature of Levels and Charge Stiffness of One-Dimensional Spinless Fermions

N. M. R. Peres$^1$, P. D. Sacramento$^2$, D. K. Campbell$^3$, and J. M. P. Carmelo$^1$

$^1$Department of Physics, University of Évora, Apartado 94, P-7001 Évora Codex, Portugal
$^2$Departamento de Física and CFIF, Instituto Superior Técnico, Av. Rovisco Pais, P-1096 Lisboa Codex, Portugal
$^3$Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

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Combining the Bethe Ansatz with a functional deviation expansion and using an asymptotic expansion of the Bethe Ansatz equations, we compute the curvature of levels $D_n$ at any filling for the one-dimensional lattice spinless fermion model. We use these results to study the finite temperature charge stiffness $D(T)$. We find that the curvature of the levels obeys, in general, the relation $D_n = D_0 + \delta D_n$, where $D_0$ is the zero-temperature charge stiffness and $\delta D_n$ arises from excitations. Away from half filling and for the low-energy (gapless) eigenstates, we find that the energy levels are, in general, flux dependent and, therefore, the system behaves as an ideal conductor, with $D(T)$ finite. We show that if gapped excitations are included the low-energy excitations feel an effective flux $\Phi^{eff}$ which is different from what is usually expected. At half filling, we prove, in the strong interacting limit and to order $1/V$ ($V$ is the nearest-neighbor Coulomb interaction), that the energy levels are flux independent. This leads to a zero value for the curvature of levels $D_n$ and, as consequence, to $D(T) = 0$, proving an earlier conjecture of Zotos and Prelovšek.

I. INTRODUCTION

The study of transport and response properties in exactly solvable one-dimensional (1D) correlated fermion systems, e.g. the spinless fermion, Hubbard, and $t-J$ models, has long been an important field of theoretical research. This theoretical interest has been supported by experimental studies that have found both qualitative and quantitative agreement between thermodynamic, spectral, and transport measurements in quasi-1D materials and the corresponding properties calculated in 1D correlated electron models.

Recently, several comparative numerical and analytical studies have explored the differences in the transport properties between integrable and non-integrable 1D models. Most of these studies have dealt with generalizations to finite-temperature of Kohn’s zero-temperature concepts and approach. In reference [1], the concepts of ideal insulator and ideal conductor at finite temperatures were introduced. These concepts refer to the temperature dependence of the real part of the optical conductivity $\sigma_r(\omega, T)$, which is given by

$$\sigma_r(\omega, T) = 2\pi D(T)\delta(\omega) + \sigma_{reg}(\omega, T),$$

where we have taken $\hbar = e^2 = 1$, and $e$ being the electron charge. The quantity $D(T)$ is called the charge stiffness and characterizes the response of the system to a static electric field, within linear response theory. According to Kohn’s zero-temperature criterion, the value of $D(0)$ can be used to distinguish between an ideal insulator $D(0) = 0$ and an ideal conductor $D(0) \neq 0$. The quantity $\sigma_{reg}(\omega, T)$ is called the regular part of the conductivity and describes the absorption of light of finite frequency $\omega$ by the system.

At finite temperatures, the classification of a given system as an ideal conductor or an ideal insulator involves the values of both $D(T)$ and $\sigma_0(T) = \sigma_r(\omega \rightarrow 0, T)$ as follows: (i) if $D(T) > 0$, the system behaves as an ideal conductor; (ii) if $D(T) = 0$ and $\sigma_0(T) = 0$ the system behaves as an ideal insulator; (iii) and, finally, if $D(T) = 0$ and $\sigma_0(T) > 0$ the system behaves as a normal conductor.

Motivated by the results of Refs. [3–5], we compute in this article the curvature of levels $D_n$ and study the effect of the temperature on the charge stiffness of the simplest 1D lattice spinless fermion model, the Hamiltonian for which is

$$\hat{H} = -\frac{J}{2} \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_i (\hat{n}_i - \frac{1}{2})(\hat{n}_{i+1} - \frac{1}{2}),$$

where the spinless fermion operators $c_i^\dagger, c_i$ obey the usual anti-commutation relations, $\langle i, j \rangle$ means summation over nearest neighbors, $\hat{n}_i = c_i^\dagger c_i$ is the usual local number operator, $\frac{J}{2}$ is the hopping integral (normally called $t$ but here called $\frac{J}{2}$ to stress the important exact correspondence between this model and the (anisotropic) Heisenberg $S=1/2$ chain), and $V$ is the nearest-neighbor Coulomb repulsion.

The model (2) is solvable by the Bethe Ansatz (BA) and has a metal-insulator transition at half filling and zero temperature (in this model half filling means a fermion for each two lattice sites, i.e., fermionic density 1/2). The metal-insulator transition occurs when the first-neighbor Coulomb repulsion $V$ is greater than $J$. This is in contrast to the standard Hubbard model, where the Mott-Hubbard transition occurs at half filling for any non-zero value of the on-site Coulomb repulsion (in the Hubbard model half filling means one electron per
lattice site, i.e., electron density $1$).

The optical properties of the model \cite{3} can be determined by studying the current operator $\hat{j}$, which is given by

$$
\hat{j} = i\frac{J}{2} \sum_j (c^\dagger_j c_{j+1} - c^\dagger_{j+1} c_j).
$$

(3)

As in the case of the Hubbard chain, the commutator of $\hat{j}$ with the Hamiltonian \cite{2} is finite (i.e., non-zero) operator (i.e., not a $c$-number). This implies that the real part of the optical conductivity has finite-energy absorption, i.e., $\sigma_{\epsilon}(\omega, T) \neq 0$. The exact calculation of $\sigma_{\epsilon}(\omega, T)$ requires the full computation of the Kubo formula. The changes of the energy eigenvalues $E_n$ in response to an external flux $\phi$ piercing the ring (formed by the 1D chain with periodic boundaries) can be determined by solving Hamiltonian \cite{2} with the hopping integral modified by the usual Peierls phase factor: $t \to te^{i\phi}/N_a$.

The charge stiffness $D(T)$ can be evaluated as a thermodynamic quantity \cite{4} using

$$
D(T) = \frac{1}{N_a} \sum_n p_n D_n = \frac{1}{2N_a} \frac{d^2 F}{d\phi^2} |_{\phi=0} + \frac{1}{2TN_a} \sum_n p_n (j_n)^2,
$$

(4)

where $p_n = Z^{-1} e^{-\beta E_n}$ is the usual Boltzmann weight, $Z$ is the partition function, $2D_n = d^2 E_n / d\phi^2 |_{\phi=0}$ is the curvature of the level $E_n$, $j_n = -d E_n / d\phi |_{\phi=0}$ is the mean value of the current operator in the eigenstate of energy $E_n$, $F$ is the free energy, and $N_a$ is the number of lattice sites. The summation in Eq. (4) is not in general simple to perform, and it must be computed for a given fermionic density $n_f = N_f / N_a$, with $N_f$ the total number of fermions.

Many recent studies have provided crucial insight into various aspects of this general problem. First, using the Mazur inequality, Zotos and collaborators\cite{3-2} derived some analytical inequalities for $D(T)$, valid for $T \to \infty$. Their results imply ideal conductivity away from half filling both for the spinless fermion and Hubbard models but are inconclusive for the half-filled band case. Second, for the Hubbard model, a recent analytical study by Fujimoto and Kawakami\cite{3-3}, using the thermodynamic BA\cite{4}, confirmed the exponentially activated nature of the conductivity $D(T) \sim \sqrt{T}$ \cite{3-4} at half filling, and ballistic transport of charge $D(T) = D_0 - aT^2$ ($a$ is a positive constant), for low temperatures and away from half filling. These results have been confirmed by Kirchner et al. combining both Monte Carlo and BA methods\cite{3-5}. As a result of their study, these authors found numerical evidence that some of the conjectures of Refs. \cite{3-3,3-5} may not be correct. Third, a recent study\cite{3-6} of the “kicked $t-V$” model has focused on the transition from integrability to “ergodicity,” in the thermodynamic limit. This model is a modification of the Hamiltonian \cite{3} in which the interaction term $V$ is time dependent. Reference\cite{3-7} related the infinite number of conservation laws of the model \cite{3} to its nonergodicity, and argued that the deviation from the ergodic behavior led to the anomalous transport properties observed in Refs. \cite{3,3-8}. In particular, Ref.\cite{3-7} pointed out that nonergodicity implies an infinite transport coefficient and, therefore, ideal conductivity away from half filling. In the approach we develop in this paper, the infinite number of conservation laws manifest themselves in the existence of only forward scattering among the exact many-body excitations of the system (called pseudoparticles below) at all energy scales. Fourth, in the regime $-2t < V < 2t$ (so that the interaction in Eq. (2) is parametrized as $V = J \cos \lambda$) the behavior of $D(T)$ has been studied by Narozhny et al.\cite{3-9} using exact diagonalization and finite-size scaling\cite{3-10}. These authors found ideal conductivity behavior arising from a non-zero fraction of degenerate states in the thermodynamic limit.

It is worthwhile to remark that different authors associate ideal conductivity with different physical mechanisms – conservation laws, nonergodicity, degeneracy of the many-body energy states, forward scattering (see below) – but at present the deep relation (if any) among all of them remains unclear.

In this article, we focus on the regime ($V > 2t$) of the spinless fermion model in which the metal-insulator transition can occur and compute the needed mean values of the current operator in any eigenstate for the model \cite{3}. As well as the respective curvature of levels. In the regime of the metal-insulator transition we introduce for later convenience the parametrization $V = J \cosh \lambda$, with $\lambda > 0$.

To carry out part of our study, we use the formalism of the pseudoparticle operator algebra, which has recently been presented in detail for the Hubbard model\cite{4}. In brief, the BA solution (discussed in more detail below) can be shown to refer to an algebra of operators describing “pseudoparticle” excitations. The energy eigenstates are characterized by the pseudoparticle momentum distributions, $N_{c,\gamma}(q)$. Here, $c, \gamma$ with $\gamma = 0, 1, 2, 3, ...$ are the quantum numbers that label the different pseudoparticle branches, with $N_{c,\gamma}(q) = 0$ for $\gamma > 0$ in the ground state. The BA gives the energy and other operator mean values as functionals of the momentum distributions $N_{c,\gamma}(q)$. Hence, as in reference\cite{4}, we can employ a functional deviation expansion (FDE) which refers to the momentum deviations, $\delta N_{c,\gamma}(q) = N_{c,\gamma}(q) - N_{c,\gamma}^{0}(q)$, where $N_{c,\gamma}^{0}(q)$ is the ground-state momentum distribution. This leads to FDE expressions for the energy, current mean values, and other mean values, exactly as in a Fermi liquid. In addition to the central role played by the FDE expansions, there are other similarities between 1D quantum liquids and 3D Fermi liquids\cite{3-10}. These similarities have justified the introduction of the concept of a “Landau liquid”\cite{3-11}, which includes both the 3D Lan-
dau Fermi liquids and the 1D Luttinger liquids as specific cases. There is, however, an important difference between the Fermi liquid quasiparticles and the pseudoparticles in 1D integrable models: namely, the collisions between the quasiparticles are not, in general, of forward scattering type only, whereas in the case of the pseudoparticles in integrable models, there is only forward scattering. Moreover, the quasiparticles are not the true many-body excitations of the quantum liquid. Therefore, it is not to be expected that the transport properties behave in the same way for the two types of quantum liquids.

The remainder of the paper is organized as follows. In Sec. II we treat the BA equations in the thermodynamic limit and compute the low- and high-energy excitation spectra, and the corresponding energy gaps. We show that the elementary excitations (the pseudoparticles) interact via forward scattering interaction (the $f$-function) only, and we give its explicit form. We show that the energy eigenvalues are functionals of the pseudoparticle occupancies and depend on the excitation spectra and on the $f$-functions. Using the BA equations with a flux we obtain general expression for $j_n$ and $D_n$, in the low-energy Hilbert subspace. We develop the FDE, which leads to simpler expressions for $j_n$ and $D_n$, for the low energy eigenstates. In Sec. III we treat the BA equations in a finite-size system and derive the curvature of levels $D_n$ as function of $V$ and $N_n$, in the limits $V = \infty$ and $V \gg t (1/V)$ corrections, for all the eigenstates of the model. We use our results to discuss the ballistic transport of charge at finite temperature in this model. In Sec. IV we discuss our results and present the conclusions.

\begin{equation}
2 \arctan \left( A_\gamma \tan \frac{\Lambda^\gamma_\gamma}{2} \right) = \frac{2 \pi I^\gamma_{c,\gamma}}{N_\gamma} + (\gamma + 1) \frac{\phi}{N_\gamma} + \sum_{\gamma = 0} \sum_j \frac{[\gamma \gamma']}{N_\gamma} 2 \arctan \left( A_p \tan \frac{\Lambda^\gamma_{\gamma'} - \Lambda^\gamma_{\gamma'}}{2} \right),
\end{equation}

where the numbers $\Lambda^\gamma_{\gamma'}$ are called rapidities, the label $c$ is associated with charge, $A_\gamma = \coth((\gamma + 1)\lambda/2)$, $A_p = \coth(p \lambda/2)$, $\phi$ is the twist angle, and the symbol $[\gamma \gamma']$ is given by

\begin{align*}
[\gamma \gamma'] &= 1, \text{ for } p = |\gamma - \gamma'|, \gamma + \gamma' + 2 \\
[\gamma \gamma'] &= 2, \text{ for } p = |\gamma - \gamma'| + 2, |\gamma - \gamma'| + 4, \ldots, \gamma + \gamma', \\
[\gamma \gamma'] &= 0, \text{ otherwise.}
\end{align*}

The energy $E_n$ of a given eigenstate is a function of the chosen set of numbers $\{I^\gamma_{c,\gamma}\}_n$ and can be written as

\begin{equation}
E_n = -J \sinh(\lambda) \sum_\gamma \sinh((\gamma + 1)\lambda) \sum_{\{I^\gamma_{c,\gamma}\}_n} \frac{\cosh((\gamma + 1)\lambda) - \cos(\Lambda^\gamma_{c,\gamma})}{\cosh((\gamma + 1)\lambda)}.
\end{equation}

The exact solution and of the model shows that all its eigenstates, for a given canonical ensemble,

\section*{II. BETHE ANSATZ RESULTS}

Our approach will be to use the general BA solution for the Hamiltonian with a flux through the ring to compute the mean value of the current operator $j_n$ and the level curvature $D_n$ in a given eigenstate of energy $E_n$ using the formalism of Refs. The results will follow directly from the structure of the Hilbert space of the model. In the following, we discuss this structure in a finite-sized system and present a simple physical picture for this problem.

It is well known that the model can be mapped onto the Heisenberg spin-one-half model by the Jordan-Wigner transformation. For a given canonical ensemble there are states with $S_z = S$ and $S_z \neq S$, where $S$ is the total spin and $S_z$ is its projection in the $z$ direction. We shall present the results for those states that are characterized by $S_z = S$, but similar results would be obtained for the states with $S_z \neq S$ (for example, the lowest energy state with $S = 1$ and $S_z = 0$ is degenerate, in the thermodynamic limit, with the ground state characterized by $S = 0$ and $S_z = 0$).

The model has particle-hole symmetry and, therefore, without loss of generality we study the case where the fermion density $n_f$ is less than $1/2$. The general BA solution for the Hamiltonian is obtained by combining the Jordan-Wigner transformation with the BA solution for the Heisenberg spin-one-half model. Our starting point is the set of BA equations with the (Orbach parametrization) twisted boundary conditions, which are

\begin{equation}
N_f = \sum_{\gamma = 0} (\gamma + 1) N_{c,\gamma}.
\end{equation}

The numbers $I^\gamma_{c,\gamma}$ can be positive or negative integers or half-odd integers depending on whether $N_a + N_{c,\gamma}$ are odd or even respectively ($N_a$ is taken to be even). The eigenstates of the model are characterized by the set of numbers $N_{c,\gamma}$ and by the configurations of the occupied quantum numbers $I^\gamma_{c,\gamma}$ among their possible values. The $N_{c,\gamma}$ numbers give the numbers of $c, \gamma$ pseudoparticles and $q = 2\pi I^\gamma_{c,\gamma}/N_a$ is the momentum. For a given canonical ensemble the $I^\gamma_{c,\gamma}$ are distributed in the interval...
and \( M(\gamma, \gamma') = 2\min(\gamma + 1, \gamma' + 1) \) for \( \gamma \neq \gamma' \), and
\( M(\gamma, \gamma') = 2\gamma + 1 \) for \( \gamma = \gamma' \). The \( I_{\gamma,0} \) numbers describe the low energy excitations while the \( I_{\gamma,0} \) numbers are related to the BCS string solutions and describe the gapped excitations.\(^{37}\)

A. Bands, gaps, and \( f \)-functions: \( \phi = 0 \) results

The numbers \( N_{c,\gamma} \) and the momentum \( q \) introduced above have a simple physical interpretation. The index \( \gamma \) labels a set of energy bands which have their (pseudo-) Brillouin-zones and (pseudo-) Fermi points controlled by the relations \(^{3}\) and \(^{4}\). All these bands are separated from each other by energy gaps. These gaps, relative to the lower energy band \( c,0 \), are given by
\[
\Delta_\gamma = \varepsilon_{c,\gamma>0}(0) - (\gamma + 1)\varepsilon_{c,0}(q_{F,c,0}) ,
\]
where \( \varepsilon_{c,\gamma}(q) \) is the energy dispersion of the pseudoparticle and \( q_{F,c,\gamma} = 2\pi I_{c,\gamma}^{max}/N_a \), with \( I_{c,\gamma}^{max} \) being the largest quantum number \( I_{c,\gamma} \), in the compact symmetric distribution of these numbers around zero. For \( n_f = 1/2 \), we have \( q_{F,c,0} = \pi/2 \) and \( q_{F,c,0} = 0 \) in the ground state.

In equation \(^{12}\) \( \Phi_{c,0,c,0}(q,q') \) are pseudoparticle phase-shifts due to forward scattering pseudoparticle-pseudoparticle collisions.\(^{38}\) In the Appendix, we give the integral equations obeyed by the phase shifts \( \Phi_{c,\gamma,c,\gamma'}(q,q') \).

These \( f \)-function control the behavior of the correlation function exponents, as is the case of the conductivity exponents.\(^{34}\) The \( f \)-functions also control the energy eigenvalues \( E_n \), which are written as functionals of \( \delta N_{c,0}(q) \) and are of the form
\[
E_n - E_0 = E^{(1)} + E^{(2)} + \text{(higher order terms)} ,
\]
with
\[
E^{(1)} = \sum_{q,\gamma} \varepsilon_{c,\gamma}(q)\delta N_{c,\gamma}(q) ,
\]
and
\[
E^{(2)} = \frac{1}{N_a} \sum_{q,\gamma} \sum_{q',\gamma'} \frac{1}{2} f_{c,\gamma,c,\gamma'}(q,q')\delta N_{c,\gamma}(q)\delta N_{c,\gamma'}(q') ,
\]
and \( E_0 \) is the ground state energy. Similar results to Eqs. \(^{13}\) and \(^{14}\) have been derived for the Hubbard model,\(^{35}\) and as long as \( \delta N_{c,\gamma}/N_a \)–the number of excited pseudoparticles– is small the higher order terms are negligible.

The relation between the gapless \( c,0 \) excitations and the original fermionic problem is best understood in the Luttinger liquid paradigm.\(^{36}\) These excitations correspond to the density waves obtained by bosonization of Hamiltonian \(^{3}\), which, in turn, are generated by the low energy particle-hole processes around the non-interacting Fermi surface.

A different situation occurs if \( n_f = 1/2 \). For this canonical ensemble, Table \(^{4}\) shows that the number of available \( I_{c,0} \) numbers is equal to the number of \( c,0 \) pseudoparticles occupying those orbitals. As a consequence, no low-energy excitations can occur in the \( c,0 \) band. The lowest energy excitation requires that one \( N_{c,1} \) pseudoparticle is created, which costs a finite energy. In this process two available states appear in the

\[ -\frac{Q_\gamma - 1}{2} \leq I_{c,\gamma} \leq \frac{Q_\gamma - 1}{2} , \quad j = 1, \ldots, N_{c,\gamma} , \]

\[ Q_\gamma = N_a - \sum_{\gamma=0} M(\gamma,\gamma')N_{c,\gamma} , \]

and \( M(\gamma,\gamma') = 2\min(\gamma + 1, \gamma' + 1) \) for \( \gamma \neq \gamma' \), and
\[ M(\gamma,\gamma') = 2\gamma + 1 \) for \( \gamma = \gamma' \). The \( I_{c,0} \) numbers describe the low energy excitations while the \( I_{c,0} \) numbers are related to the BCS string solutions and describe the gapped excitations.\(^{37}\)
c,0 band and therefore the remaining particles can now undergo particle-hole excitations within the c,0 band. For \( V > J \) the c,1 excitation can be seen as an excitation to an “upper Hubbard band” and is related, in the original lattice, to the creation of a pair of two nearest-neighbor occupied sites. In this limit, the gap \( \Delta_1 \) has the asymptotic form \( \Delta_1 \approx V - 2J \). This excitation process is the most important contribution for the zero-temperature behavior of \( \sigma_{reg}(\omega,0) \) at half-filling. In Fig. 1 we plot the c,0 band at half-filling and represent the lowest energy excitation to the c,1 band (represented by a point).

From this description, we see that there is a mapping between an insulator due to correlations in the fermionic picture and a band insulator in the pseudoparticle representation. This mapping holds, in this model, for all \( V > J \) and is not restricted to the “upper Hubbard band” strong-coupling argument given above. The same mapping holds for the Hubbard model.

![Figure 1](image)

**FIG. 1.** Plot of the filled c,0 band (corresponding to half-filling of the spinless fermions) and lowest energy excitation to the c,1 band (creating a pseudoparticle in the c,1 band and leaving two pseudoholes in the c,0 band in accordance with Eq. (1)).

**B. \( D_n \) for \( n_f < 1/2 \) and \( T \ll v_{c,0} q_{c,0} \)**

Due to the presence of the gaps \( \Delta_j \) the behavior of the curvature of levels \( D_n \) at low temperatures and away from half filling is mainly controlled by the c,0 excitations. Therefore in the calculation of \( D_n \) below we ignore the contributions from the states with \( c,\gamma > 1 \) excitations (these will play a major role only at half filling).

We now make these considerations more quantitative. For the calculation of Eq. (4), we need to compute the curvature of levels, \( D_n \). The eigenstates characterized by the \( q = 2\pi I_{c,0}/N_a \) numbers and the corresponding eigenenergies \( E_n \) are expressed as functionals of the distributions \( N_{c,0}(q) \) of these quantum numbers. We write \( N_{c,0}(q) = N_{c,0}^0(q) + \delta N_{c,0}(q) \), where \( N_{c,0}^0(q) \) is a compact distribution of all \( I_{c,0} \) numbers symmetrically distributed around zero. If the deviation \( \delta N_{c,0}(q) \) is small, we can expand the relevant quantities in \( \delta N_{c,0}(q) \). Using the BA equations with twisted boundary conditions, we next derive simple equations for \( j_n \) and \( D_n \) as functions of \( N_{c,0}(q) \).

Following references [13,14,15], the general expression for \( j_n \) reads

\[
\tilde{j}_n = J \sin^2 \lambda \sum_q N_{c,0}(q) \frac{\sin[\Lambda_{c,0}(q)]\Lambda_{c,0}^\phi(q)}{\{\cosh \lambda - \cos[\Lambda_{c,0}(q)]\}^2},
\]

where \( \Lambda_{c,0}(q) \) is defined by taking the limit \( N_a \to \infty \) in Eq. (4) and \( \Lambda_{c,0}^\phi(q) \) is the first derivative of \( \Lambda_{c,0}(q) \) with respect to \( \phi \) such that

\[
\Lambda_{c,0}^\phi(q) = \lim_{N_a \to \infty} \frac{dN_{c,0}^0(2\pi I_{c,0}/N_a)}{d\phi/N_a} \Bigg|_{\phi=0}.
\]

We stress that Eq. (16) is the exact mean value of the current operator (2) of the low energy sector (only \( c,0 \) excitations included). That is, Eq. (16) holds for any \( N_{c,0}(q) \). By methods equivalent to those used previously for the Hubbard model [11] and for small densities of excited c,0 pseudoparticles the FDE gives the following result for the mean value of the current operator (10):

\[
\tilde{j}_n = \sum_q \delta N_{c,0}(q) j_{c,0}(q),
\]

with the spectrum \( j_{c,0}(q) \) given by

\[
j_{c,0}(q) = v_{c,0}(q) Q_{c,0}^\phi(q) + \sum_{j=\pm 1} v_{c,0} \Phi_{c,0}(j,q) Q_{c,0}^\Phi(q) ,
\]

\[
Q_{c,0}^\Phi(q) = 1 + \sum_{j=\pm 1} j \Phi_{c,0}(q,j,qF,0),
\]

\[
v_{c,0}(q) = d\epsilon_{c,0}(q)/dq, \quad v_{c,0} = v_{c,0}(q_{F,c,0}).
\]

Equation (19) has a simple physical meaning, since it is the sum of the velocity plus a dragging term due to the collisions among the pseudoparticles (similar results have been derived for the Hubbard model [11]). This picture is very similar to that occurring in a Fermi liquid [14]. The curvature of the levels \( E_n \) can also be computed. For any low-energy eigenstate, its curvature \( D_n \) is given by
\[ D_n = -J \frac{\sinh^2 \lambda}{2} \sum_q N_{c,0}(q) \left( -\sin[\Lambda_{c,0}(q)]\Lambda_{c,0}^{\phi\phi}(q) + \cos[\Lambda_{c,0}(q)]\Lambda_{c,0}^{\phi}(q) \right)^2 \{ \cosh \lambda - \cos[\Lambda_{c,0}(q)] \}^2 + 2 \sin^2[\Lambda_{c,0}(q)]\Lambda_{c,0}^{\phi}(q) \right)^2 \{ \cosh \lambda - \cos[\Lambda_{c,0}(q)] \}^3 \right), \]  

where \( \Lambda_{c,0}^{\phi\phi}(q) \) is the second derivative of \( \Lambda_{c,0}^\phi \) with respect to \( \phi \) in the sense of Eq. (13).

For small densities of excited \( c,0 \) pseudoparticles using the FDE allows us to express Eq. (20) as

\[ D_n = D_0 + \frac{1}{2} \sum_q \delta N_{c,0}(q) D_{c,0}(q), \]

\[ D_0 = \frac{Na}{4\pi} \sum_{j=\pm 1} v_{c,0} \left[ Q_{c,1}^{\phi,0} (jqF_0) \right]^2 = \frac{Na}{2\pi} v_{c,0}(\xi_{c,0})^2, \]

where \( D_0/Na \) is the zero-temperature charge stiffness and \( \xi_{c,0} \) is the dressed charge. Identical relations have been obtained by other authors for other integrable models. It is simple to show that \( D_0 = j_{c,0}(qF_0)/(2\pi) \), and similar results have been derived for the Hubbard model [14]. The function \( D_{c,0}(q) \) is an even function of \( q \) and is a combination of \( v_{c,0}(q) \), \( \Phi_{c,0}(q,q') \), and their derivatives. We give its general form in the Appendix.

Close to half filling, \( D_{c,0}(q) \to dv_{c,0}(q)/dq \) is the curvature of the energy band and \( D_0 \to 0 \), as computed by Haldane [15]. This latter result is a consequence of the vanishing velocity \( v_{c,0}(q) \) at \( q = \pi/2 \), as can be seen directly from Eq. (12). This signals the zero temperature metal-insulator transition in this model. The spectrum of the curvature, \( D_{c,0}(q) \), has a very simple form at close to half filling, when compared to its general expression given in the Appendix. It bears some resemblance to the result for the independent fermion gas, but here \( dv_{c,0}(q)/dq \) refers to the \( c,0 \) energy excitations of the many-body system.

### III. ASYMPTOTIC BA RESULTS FOR ARBITRARY SYSTEM SIZES

In the previous section we have considered the curvature of levels in the thermodynamic limit and we used the FDE formalism to compute \( j_n \) and \( D_n \) for the low energy states. We now consider the \( N_n \) dependence of \( j_n \) and \( D_n \). The calculation of \( j_n \) and \( D_n \) for all eigenstates for finite-size \( N_a \) is a difficult problem in general, but in the limits \( V = \infty \) and \( V \gg t \), it can be performed explicitly. Our starting point is the \( V = \infty \) solution which we will use to compute the asymptotic corrections of order \( 1/V \).

#### A. \( V = \infty \) solution

The limit \( V \to \infty \) is a well defined limit of the BA equations and therefore is suitable for classical perturba-

tion theory. In the limit \( V = \infty \) the BA equations \( \text{[3]} \) and \( \text{[4]} \) can be written as

\[ N_a\Lambda_{c,0}^{\phi\phi} = 2\pi I_{c,0}^{\phi} + (\gamma + 1)\phi \]

\[ + \sum \sum [\gamma \gamma'] \sum \sum \sum_{N_e\gamma} (\Lambda_{c,0}^{\phi\phi} - \Lambda_{c,0}^{\phi\phi}, j = 1, \ldots, N_e, \gamma) \]

and

\[ E_n = -J \sum_{j=1}^{N_e} \cos \Lambda_{c,0}^{\phi\phi}, \]

respectively. In this limit we see that the set of Eqs. \( \text{[3]} \) transform into a set of coupled linear algebraic systems. This result shows that if \( V = \infty \) only the \( \Lambda_{c,0}^{\phi\phi} \) parameters are of importance, in what concerns the energy eigenvalues, but, nevertheless, Eq. \( \text{[22]} \) is meaningful for all \( \Lambda_{c,0}^{\phi\phi} \). Considering only the \( c,0 \) excitations, Eq. \( \text{[22]} \) can be solved exactly and reads

\[ \Lambda_{c,0}^{\phi\phi} = \frac{\phi}{N_a} + \frac{2\pi}{N_a - N_f} I_{c,0}^{\phi} + \left( \frac{2\pi}{N_a N_f} \right) \sum_{i=1}^{N_f} I_{c,0}^{f} \]

The picture that emerges from this solution is essentialy that of non-interacting particles, in an effective ring of size \( N_a - N_f \). For this case, the curvature of levels is simply given by

\[ D_n = \frac{J}{2N_a} \sum_{\{I_{c,0}^{f}\}_n} \cos \Lambda_{c,0}^{\phi\phi} \]

and the zero temperature charge stiffness can be computed using Eqs. \( \text{[24]} \) in Eq. \( \text{[25]} \) giving

\[ D(0) = \frac{J}{2N_a} \frac{\sin(\pi N_f/(N_a - N_f))}{\sin(\pi/(N_a - N_f))}. \]

Therefore, as long as the system is not at half filling we expect ideal conductivity behavior at finite temperature, in agreement with the picture we developed in the previous section for the thermodynamic limit.

The set of equations \( \text{[22]} \) can be solved for arbitrary occupancy of the numbers \( N_{c,0} \) and this solution can be used as a starting point to an asymptotic calculation of the energy levels \( E_n \) and its corresponding curvature of levels \( E_n \) as function of the system size \( N_a \). The curvature of levels \( D_n \) depends in a crucial way on the flux
dependence of the parameters $\Lambda_{c,\gamma}$. We now determine this dependence by solving Eq. (24).

From Eq. (8) the highest possible occupied band is that characterized by $\gamma_f = N_f - 1$. Let us consider an arbitrary occupancy of the numbers $N_{c,\gamma}$ compatible with Eq. (8). This configuration is always of the form

$$N_{c,0} = \nu_0; N_{c,1} = \nu_1, \ldots; N_{c,\bar{m}} = \nu_{\bar{m}} \quad (27)$$

and

$$N_{c,1+\bar{m}} = 0; \ldots; N_{c,\gamma_f} = 0, \quad (28)$$

with the possibility that $\bar{m} = \gamma_f$. In order to solve our algebraic coupled linear systems we first need to obtain $\Lambda_{c,\bar{m}}^{\infty}$. Incidentally we only need to compute the sum

$$\sum_{j=1}^{\nu_{\bar{m}}} \Lambda_{c,\bar{m}}^{\infty}, \quad (29)$$

which, as can be seen from Eq. (22), is present in all the equations that define $\Lambda_{c,\gamma}^{\infty}$, with $\gamma < \bar{m}$. Using the equation that defines $\Lambda_{c,\bar{m}}^{\infty}$, we obtain

$$(N_a - 2N_f + 2(\bar{m} + 1))\sum_{j=1}^{\nu_{\bar{m}}} \Lambda_{c,\bar{m}}^{\infty} = 2\pi \sum_{j=1}^{\nu_{\bar{m}}} \sum_{\gamma=0}^{\bar{m}-1} (\Lambda_{c,\gamma}^{\infty} - \Lambda_{c,\gamma}^{\infty})$$

Substituting the above sum in Eq. (22) for a given $\gamma < \bar{m}$ we obtain

$$\phi \rightarrow \Phi^{eff} = \frac{(N_a - 2N_f)\phi}{N_a - 2N_f + 2(\bar{m} + 1)} \quad (32)$$

It is simple to prove that if only the c, 0 excitations have non-zero occupancy then Eq. (31) gives Eq. (24). Moreover we see that the flux term, due to the presence of $c, \gamma > 0$ excitations is modified as follows

$$\phi \rightarrow \Phi^{eff} = \frac{(N_a - 2N_f)\phi}{N_a - 2N_f + 2(\bar{m} + 1)} \quad (32)$$

The above equation shows, in the regime $V \gg t$, that if $c, \gamma > 0$ excitations are allowed, the flux felt by the several $c, \gamma$ excitations is $\Phi^{eff}$, with the exception of the $c, \bar{m}$ excitation. Even more important is the fact that the effective flux $\Phi^{eff}$ is zero at half filling. This property turns out to be important in Section III B where we consider the asymptotic solution $V \gg t$.

B. $V \gg t$ solution: $J^2/V$ corrections

Although the set of equations (22) has a solution for $V = \infty$, the energy of the eigenstates with non-zero occupancy of the $c, \gamma > 0$ bands is infinite. In the regime $V \gg t$ the energy of the eigenstates with finite occupancy of the $c, \gamma > 0$ bands is given by

$$E_n = -J \sum_{j=1}^{N_{c,0}} \cos \Lambda_{c,0}^{\infty} + J \cosh \lambda \sum_{\gamma=1}^{\gamma_{\infty}} \gamma N_{c,\gamma} \quad (33)$$

with $\Lambda_{c,0}^{\infty}$ given by Eq. (31). Therefore, up the order where $J^2/V$ corrections are neglected, we have two different situations: (a) away from half filling, the $c, 0$ holes that appear in the $c, 0$ band due to the creation of $c, \gamma > 0$ excitations fell the effective flux $\Phi^{eff}$ but they still behave as independent particles and therefore the system display ideal (infinite) conductivity at finite temperature (this analysis agrees with the thermodynamic limit solution consider in Section IIIB); (b) at half filling $\Phi^{eff} = 0$ and because, up to this order, $\Lambda_{c,0}^{\infty}$ is independent of $\Lambda_{c,\bar{m}}$ all the derivatives of Eq. (33) in order to the flux $\phi$ are identically zero and the system behaves as ideal insulator (or at least does not display infinite conductivity see Eq. (1)).

We now compute the $1/V$ corrections to Eqs. (2) and (6). To keep the calculations as simple as possible we adopt a low temperature scheme by considering a single type of gapped $c, \gamma$ excitations, namely the $c, 1$ excitations, which correspond to the gap $\Delta_1 = V - 2J$ (neglecting $1/V$ corrections). We show below that this approach leads again to an ideal insulator behavior at half filling. We stress that our approach is equivalent to that of Fujimoto and Kawakami for the Hubbard model, using the thermodynamic BA.

Our starting point is the result for $\Lambda_{c,0}^{\infty}$ and $\Lambda_{c,1}^{\infty}$ for $V = \infty$ which we have denoted by $\Lambda_{c,0}^{\infty,1}$. In the limit $V \gg t$ we look for a solution to the parameters $\Lambda_{c,0,1}^{\infty}$ of the form $\Lambda_{c,0,1}^{\infty} = \Lambda_{c,0,1}^{\infty,1} + \Lambda_{c,0,1}^{1/V}$ (with $V = J \cosh \lambda$), where $\Lambda_{c,0,1}^{1/V}$ is the $1/V$ correction to $\Lambda_{c,0,1}^{\infty}$. By straightforward expansion of Eq. (6) we obtain

$$E_n = -J \sum_{j=1}^{N_{c,0}} \left( \cos \Lambda_{c,0}^{\infty} + \frac{J \cos^2 \Lambda_{c,0}^{\infty}}{V} - \Lambda_{c,0}^{1/V} \sin \Lambda_{c,0}^{\infty} \right)$$

$$+$$

$$\sum_{j=1}^{\nu_{\bar{m}}} \Lambda_{c,\bar{m}}^{\infty}, \quad (29)$$
\[-J^2 \sum_{j=1}^{N_{c,1}} \frac{\cos \Lambda_{c,1}^{j,\infty}}{2V} + J^2 (V - V^{-1}) N_{c,1}.\]

The above equation tells us that we need to obtain \(\Lambda_{c,0}^{j,1/V}\) to compute \(E_0\) consistently up to order \(1/V\). From Eq. (b) we obtain

\[\Lambda_{c,0}^{j,1/V} = -\frac{J}{V} \sin \Lambda_{c,0}^{i,\infty} - \frac{J}{N_a} \sum_{j=1}^{\nu_0} \sin (\Lambda_{c,0}^{j,\infty} - \Lambda_{c,0}^{i,\infty}).\]

(35)

From the study of the \(\Lambda_{c,0}^{j,\infty}\) solution of the previous Section it follows that Eqs. (34) and (36) only depend on the flux \(\phi\) through the parameter \(\Lambda_{c,0}^{j,\infty}\), which in the general case corresponds to \(\Lambda_{c,\bar{m}}^{j,\infty}\).

We now prove that the terms \(\sum_{j=1}^{\nu_0} \sin (\Lambda_{c,0}^{j,\infty} - \Lambda_{c,0}^{i,\infty})\) and \(\sum_{j=1}^{\nu_1} \cos (\Lambda_{c,0}^{j,\infty})\) are indeed flux independent at half filling, although \(\Lambda_{c,\bar{m}}^{j,\infty}\) is not. This would mean that any thermodynamic BA calculation of the type developed for the Hubbard model would give a zero finite-temperature charge stiffness. We start by obtaining \(\Lambda_{c,\bar{m}}^{j,\infty}\) at half filling. For the case we are considering this is simple to obtain (we give the general solution for \(\Lambda_{c,\bar{m}}^{j,\infty}\) in the Appendix) and reads

\[\Lambda_{c,\bar{m}}^{j,\infty} = \frac{2\pi I_{c,0}}{\nu_1} - \frac{1}{2\nu_1} \sum_{j=1}^{\nu_0} \Lambda_{c,0}^{j,\infty} + \frac{\phi}{2\nu_1} - \frac{3\pi}{2\nu_1} \sum_{j=1}^{\nu_1} I_{c,1}^j.\]

(36)

Using the above result together with Eq. (b), it is simple to see that both \(\sum_{j=1}^{\nu_2} \sin (\Lambda_{c,1}^{j,\infty} - \Lambda_{c,0}^{i,\infty})\) and \(\sum_{j=1}^{\nu_2} \cos (\Lambda_{c,1}^{j,\infty})\) are zero at half filling.

Hence we see that the curvature of levels \(D_n\) is identically zero and therefore the charge stiffness \(D(T)\) is also zero. We remark here that our result does not depend on fact that only \(c,0\) and \(c,1\) excitations have been considered. The main conclusion is that for arbitrary values of the numbers \(N_{c,\gamma}\), consistent with the Eq. (34), the highest excitation band \(c,\bar{m}\) is always completely filled (see Table II) and, in addition, is the only band that is flux dependent (up to order \(1/V\)). As result we always have \(D_n = 0\) and a zero value for \(D(T)\) at half filling.

IV. DISCUSSION AND CONCLUSIONS

We have proved, up to order \(1/V\), that the charge stiffness of the \(t-V\) model is zero at any temperature for the half-filled band case. This means that the system does not present balistic transport of charge in this regime. Our result proves a conjecture put forward first by Zotos and Prelovšek, based on exact diagonalization of small rings. If we go to the next order \(1/V^2\) there will be contributions to the curvature of levels \(D_n\) that vanishes in the thermodynamic limit. Furthermore, our results show that the Mazur inequality \(-D(T) \geq 0\) derived in Ref. [3] holds as an equality.

Away from half filling, \(D(T)\) is mainly controlled by \(D_0\) for low \(T\) (that is, \(T \ll v_{c,0}^2 \epsilon_{c,0}\)). Since the states with finite \(c, \gamma \geq 0\) pseudoparticle occupancy have energy gaps relative to the ground state, only the \(c,0\) pseudoparticles contribute to the charge transport at finite but small temperatures. This implies that for low \(T\) the model can be thought of as a gas of \(c,0\) pseudoparticles, and therefore \(D(T)\) is finite and the system is an ideal conductor in agreement with Ref. [3]. Of course, the curvature of levels \(D_n\) is not as simple as in a true fermionic gas, due to the forward scattering among the pseudoparticles (these scattering processes are controlled by the phase shifts as in the Hubbard model). Nevertheless, the forward scattering among the pseudoparticles will only renormalize the \(b\) coefficient in \(D(T) = D_0 - bT^2\) away from its non-interacting value. This picture is supported by the analysis of the BA equations (36) in the extreme correlated case of \(V = \infty\), which, for large \(N_a\), can be consider as free fermions in an effective lattice of size \(L_{eff} = N_a - N_f\).

Our results also provide insight into spin transport in the spin-1/2 Heisenberg chain, and similar conclusions to those above hold for the temperature dependence of the spin stiffness. The regime \(V > J\) corresponds to the Ising symmetry of the spin-1/2 anisotropic Heisenberg chain. In this sector, the exchange coupling in the \(z\) direction is larger than the coupling in the \(xy\) plane. The half-filled band (non-half filled band) case in the spinless fermions model corresponds to the zero magnetic field (finite magnetization) case in the Heisenberg chain.

As a final comment, we believe that the anomalous transport properties exhibited by some of the models solvable by the BA arise from the existence of only forward scattering among the pseudoparticles at all energy scales. As discussed in Sec. IV, this is a consequence of the integrability in these models and is not expected to occur in non-integrable many-body systems.

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APPENDIX A: GENERAL EQUATIONS FOR THE PHASE SHIFTS, ENERGY BANDS, AND $D_{c,0}(q)$

In this appendix we give the general equations for the phase shifts $\Phi_{c,\gamma;c'\gamma'}(q,q')$ and for the bands $\varepsilon_{c,\gamma}(q)$. Following standard methods (see, e.g., Ref. 1), we obtain

$$\Phi_{c,\gamma;c'\gamma'}(\Lambda_{c,\gamma}, \Lambda_{c',\gamma'}) = \sum_{p=1}^{\infty} \left\{ \frac{[\gamma \gamma']^p}{\gamma} \arctan \left( A_p \tan \frac{\Lambda_{c,\gamma} - \Lambda_{c',\gamma'}}{2} \right) \right\} - \int_{-\infty}^{\Lambda_{c,\gamma}} \frac{[\gamma p]}{\pi} \frac{A_p \Phi_{c,0;c',\gamma'}(x, \Lambda_{c',\gamma'})}{1 + A_p^2 - (A_p^2 - 1) \cos(x - \Lambda_{c,\gamma})} \right\} \right\} (A1)$$

and

$$\varepsilon_{c,\gamma}(q) = -J \sinh((\gamma + 1)\lambda) \sinh \lambda + J \int_{-\infty}^{\Lambda_{c,\gamma}} \sin^2 \lambda \sin(x) \Phi_{c,0;c',\gamma'}[x, \Lambda_{c,\gamma}^0(q)] \right\} (A2)$$

where $\Phi_{c,\gamma;c'\gamma'}(q,q') = \Phi_{c,\gamma;c'\gamma'}[\Lambda_{c,\gamma}^0(q), \Lambda_{c',\gamma'}^0(q')]$, $x_0 = \Lambda_{c,0}(q)$, and $\Lambda_{c,0}(q)$ is $\Lambda_{c,0}(q)$ in the ground state and can be written in terms of the phase shifts $\Phi_{c,\gamma;c'\gamma'}(q,q')$ as

$$2 \arctan \left( A_c \tan \frac{\Lambda_{c,\gamma}^0(q)}{2} \right) = \frac{q - 2A_c \int_{-\infty}^{\Lambda_{c,\gamma}^0(q)} \Phi_{c,0;c',\gamma'}[x, \Lambda_{c,\gamma}^0(q)]}{1 + A_c^2 - (A_c^2 - 1) \cos x} \right\} (A3)$$

These three equations can be solved in closed form. In general, only a numerical solution is possible. For $n_f = 1/2$ one has $\phi_{c,0}(q) = \pi/2$ and $\Lambda_{c,0}^0(q) = \pi$, and this set of integral equations can be solved by Fourier series. The results for the phase shifts and for the bands are

$$\Phi_{c,0;c',\gamma'}[\Lambda_{c,0}(q), \Lambda_{c',\gamma'}^0(q')] = \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{[\gamma p]}{\pi} \sin(n(\Lambda_{c,0}(q) - \Lambda_{c',\gamma'}^0(q'))) \right\} (A4)$$

and

$$\varepsilon_{c,0}(q) = -J \sinh \lambda - J \sum_{n=1}^{\infty} \cos(n(\Lambda_{c,0}^0(q))) \frac{\sin(n\lambda)}{\sinh(n\lambda)} \right\} (A5)$$

respectively. The relation of these expressions with Eq. (A1) is obtained by using the delta and amplitude functions. The parameter $k$ is a function of $\Lambda_{c,0}^0(q) = 4 \sum_{n=1}^{\infty} \exp(-\lambda(n - 0.5)^2) / [1 + 2 \sum_{n=1}^{\infty} \exp(-\lambda n^2)]^2$. Both the phase shifts and the velocities enter in the general expression for $D_{c,0}(q)$ (see Eq. (A1)). The general form of $D_{c,0}(q)$ reads

$$D_{c,0}(q) = \frac{d}{dq} \left\{ \varepsilon_{c,0}(q)(Q_{c,0}^0(q))^2 \right\} + \varepsilon_{c,0}(q) W_{c,0}(q)$$

+ $\sum_{j=\pm 1} \left\{ \varepsilon_{c,0}(q)(Q_{c,0}^0(q))^2 \right\}$

+ $\varepsilon_{c,0}(q)(Q_{c,0}^0(q)) W_{c,0}(q)$

+ $v_{c,0}(q) \Phi_{c,0,c',\gamma'}(q,q') \right\} (A6)$

The functions $W_{c,0}^{\phi,0}(q)$ and $\Phi_{c,0,c',\gamma'}(q,q')$ are given by

$$W_{c,0}^{\phi,0}(q) = \sum_{j=0}^{\infty} \sum_{j=0}^{\infty} \frac{[\varepsilon_{c,0}(q)(j q F_{c,0})]^2}{d \Phi_{c,0,c',\gamma'}(q,q',q')} \left\{ q' = \pm j q F_{c,0} \right\} \right\} (A7)$$

and

$$\Phi_{c,0,c',\gamma'}(q,q') = \frac{d \Phi_{c,0,c',\gamma'}(q,q',q')}{d q'} \left\{ q' = \pm j q F_{c,0} \right\} \right\} (A8)$$

respectively.

APPENDIX B: GENERAL SOLUTION OF $\Lambda_{c,m}^{j,\infty}$ FOR ALL FILLINGS

The equation that defines $\Lambda_{c,m}^{j,\infty}$ is

$$N_a \Lambda_{c,m}^{j,\infty} = 2 \pi I_{c,m}^{j,\infty} + (m + 1) \phi$$

+ $\sum_{\gamma' = 0}^{\infty} \sum_{\gamma = 1}^{\infty} \Lambda_{c,m}^{\gamma,\infty} - \Lambda_{c,m}^{j,\infty}$

and can be cast in the form

$$\Lambda_{c,m}^{j,\infty} + \Lambda_{c,m}^{j,\infty} + \ldots + \frac{a}{b} \Lambda_{c,m}^{j,\infty}$$

$\ldots + \Lambda_{c,m}^{j,\infty}$

$$= \frac{2 \pi I_{c,m}^{j,\infty}}{b} + B(\phi) \right\} (B1)$$

and

$$a = N_a - 2N_f + \nu_m + 2m + 1, \right\} (B3)$$

$$b = 2m + 1, \right\} (B4)$$

and
This linear system of order $\nu_m$ can be solve, and the solution $\lambda_{c,m}^{j,\infty}$ is given by

$$\lambda_{c,m}^{j,\infty}[a/b(a/b + \nu_m - 2) - \nu_m + 1] = -\frac{2\pi}{b} \sum_{j'=1}^{\nu_m} I_{c,m}^{j'} + \frac{2\pi I_{c,m}^{j_m}}{b} (a/b + \nu_m - 1) - \frac{B(\phi)}{b} (a/b - 1).$$

At half filling we have $a = \nu_m + 2m + 1$ and $\lambda_{c,m}^{j,\infty}$ is equal to

$$\lambda_{c,m}^{j,\infty} = \frac{2\pi I_{c,m}^{j_m}}{\nu_m} - \frac{2m + 1}{m + 1} \frac{\pi}{(\nu_m)^2} \sum_{j'=1}^{\nu_m} I_{c,m}^{j'} + \frac{B(\phi)}{2(\nu_m + 1) \nu_m}.$$  

(B7)

It is easy to see that the above equation gives Eq. (36) for $m = 1$.
insulator transition as a band insulator, although it gives results that are incorrect in an important qualitative manner (it predicts the metal-insulator transition for \( n = 1/2 \) and any non-zero value of \( V \)). See, for example, Shankar, Rev. Mod. Phys. 66, 129 (1994).

45 See, for example, a similar calculation for the super-symmetric \( t-J \) and Hubbard models, Norio Kawakami and Sung-Kil Yang, Phys. Rev. B 44, 7844 (1991).

46 It is interesting to remark that this result, being true for a free and tight-binding electron systems, is somewhat unexpected in a strongly correlated 1D system. Again, this seems to be a result of integrability and fits well with the band picture for the elementary excitations described above.

47 The velocity vanishes linearly with the doping \( \delta = |1/2-n| \). F.D.M. Haldane, Phys. Lett. 81A, 153 (1981).

48 F. V. Kusmarstev, Phys. Lett. A 161, 433 (1992).

49 See Ref. 48 for the differences between the anisotropic Heisenberg chain (“a-cyclic” problem) and the spinless fermion model (“c-cyclic” problem).

50 Equation (33) is equivalent to the diagonalization of Hamiltonian (3) in the Hilbert subspaces of one, two, ..., \( n \) nearest-neighbor occupied sites. In each of these subspaces the energy eigenvalues are of the order of \( E_0, E_0 + V, ..., E_0 + nV \), respectively, where \( E_0 \) is the energy of the ground state.

51 In a previous version of this work we had concluded that \( D_n \) was finite, in the half-filled band case, due to an inappropriate perturbative treatment of the BA equations. That treatment prevented us to detect the elusive cancelation of the flux factors that we show in this paper.

52 Table of Integrals, Series, and Products, Edited by I.S. Gradshteyn and I. Ryzhik, Academic Press, 5. ed., p. 916.

| \( d_{Fc,0} \) | \( N_{c,0} \) | \( N_{c,\gamma>0} \) | \( d_{Fc,\gamma>0} \) | \( N_{h,\gamma>0} \) | \( N_{c,\gamma>0} \) |
|---|---|---|---|---|---|
| G.S. | \( N_a - N_f \) | \( N_a - 2N_f \) | \( N_f \) | \( N_a - 2N_f \) | \( N_a - 2N_f \) | 0 |
| Ex.S. | \( N_a - N_f + (\gamma - 1)N_{c,\gamma} \) | \( N_a - 2N_f + 2\gamma N_{c,\gamma} \) | \( N_f - (\gamma + 1)N_{c,\gamma} \) | \( N_a - 2N_f + N_{c,\gamma} \) | \( N_a - 2N_f \) | \( N_{c,\gamma} \) |

TABLE I. Available and occupied quantum numbers \( I_{c,\gamma} \) for the spinless fermion model. In first column G.S. and Ex.S. stand for the ground state and for the excited states with \( N_{c,\gamma>0} \) pseudoparticles in a single \( \gamma \) band, respectively. In the first row, \( d_{c,\gamma} \) gives the available \( I_{c,\gamma} \) quantum numbers, \( N_{h,\gamma>0} \) gives how many \( I_{c,\gamma} \) are unoccupied, and \( N_{c,\gamma} \) is the number of \( c, \gamma \) pseudoparticles. The numbers \( N_a \) and \( N_f \) are the number of lattice sites and the number of fermions, respectively.