Attitude Regulation for Multiple Spacecrafts Based on the Distributed Fixed-Time Observer

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Abstract. This paper has investigated the distributed attitude regulation problem for multiple spacecrafts on the basis of the leader-follower architecture. In this paper, a control scheme based on the distributed fixed-time observer is proposed. To be specific, with the condition that some following spacecraft can’t obtain the leader’s attitude, a distributed attitude observer is introduced for estimating the leader’s attitude. By using the distributed observer, this issue will be transformed into an attitude tracking control problem for a single spacecraft. It is proved that the proposed distributed observer is in the sense of the fixed-time stability. Finally, simulation results illustrate that the proposed distributed observer is effective.

1. Introduction

Recently, cooperative control for spacecraft formation flight (SFF) has emerged as a concerned research direction because of its wide application in many space missions [1]. In these missions, an expensive and large spacecraft is usually replaced by a group of micro-spacecrafts because of its advantages of higher feasibility, more enhanced fault-tolerance, less expenditure and higher probability of success [2].

Cooperative control is a key technology for distributed multiple spacecrafts. Therefore, various robust and reliable control algorithms were developed for spacecraft cooperative control [3-6]. The leader-follower approach has been one of the most intensively studied control algorithms [7]. Because of some factors, such as the relative distance or direction constraints between spacecrafts, the leader’s attitude information can’t be obtained by some followers. To this end, there are fruitful achievements about distributed attitude control for SFF [8-10]. He [8] designed a nonlinear distributed observer for solving the attitude consensus problem in case that the leader’s states cannot be accessible to each follower. On this basis, He also proposed an attitude consensus algorithm under a bidirectional communication graph [9]. Considering the adverse effect of the communication delays, Guo [10] presented a backstepping-based attitude synchronization control scheme.

It is worth noting that all of the above distributed attitude controllers were asymptotically stable, which means that system states converge to equilibrium as time goes to positive infinity. Compared to the asymptotical controllers, finite-time controllers have better performance in the convergence speed and have been widely used in the distributed control for SFF [11-13]. However, the upper bound of the convergence time for the finite-time controllers are usually depended on the initial system states and it is difficult to give its explicit expressions. Considering this fact, Ref. [14] proposed the concept of the fixed-time stability. For the fixed-time controller, the upper bound of the convergence time is independent of the initial system states. Inspired by the above, Sun et al. [15] proposed a novel fixed-time control scheme for the spacecraft attitude control question. Ref. [16] proposed an adaptive 6-DOF
fixed-time attitude controller for the multiple spacecraft formation problem. In addition, Ref. [17] investigated in the global fixed-time attitude control problem aiming for handling the actuator saturation and fault problem. In conclusion, it is still an open problem for further exploring the distributed fixed-time stability for SFF.

Motivated by the above facts and challenges, this paper mainly focuses on the distributed fixed-time attitude regulation problem for a group of rigid spacecrafts. Inspired by Ref. [13], a distributed observer is adopted for the following spacecrafts to estimating the leader’s attitude, thereby, this issue is transformed to an attitude tracking control problem for a single spacecraft. Additionally, for tracking the estimated attitude of each following spacecraft, a terminal sliding-mode control scheme in the sense of the fixed-time stability is adopted according to Ref. [18].

2. Nonlinear Dynamic Model of Spacecraft
In this paper, the Modified Rodrigues Parameters (MRPs) are used to describe the attitude kinematics of a rigid spacecraft. We define $\sigma_i$ as the MRPs of spacecraft $i$ and the following attitude dynamic equation can be obtained:

$$\begin{align*}
\dot{\sigma}_i &= G(\sigma_i)\omega_i \\
J\dot{\omega}_i &= -\omega_i^T J \omega_i + u_i + d_i
\end{align*}$$

with

$$G(\sigma_i) = \frac{1}{4} (1 - \sigma_i^T \sigma_i) I_{3 \times 3} + 2\sigma_i \sigma_i^T + 2\sigma_i$$

where $\omega_i \in \mathbb{R}^{3 \times 1}$ is the attitude angular velocity of the spacecraft, $J \in \mathbb{R}^{3 \times 3}$ is the inertial matrix, $u_i \in \mathbb{R}^{3 \times 1}$ is the combined control torque from actuators along three mutually perpendicular axes and $d_i \in \mathbb{R}^{3 \times 1}$ is the external disturbance, the notation $\times$ represents the skew symmetric matrix.

3. Preliminaries

3.1. Algebraic Graph Theory
For describing the communication topology of the multi spacecraft system, the graph theory will firstly be introduced. An undirected weighted graph with $n$ nodes can be defined as $G = \{V, E, A\}$, where $V = \{v_1, \ldots, v_n\}$ is a finite set of nodes, $E \subseteq V \times V$ represents a set of edges and $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the weighted adjacency matrix of graph $G$. The elements of matrix $A$ satisfy that

$$a_{ij} = \begin{cases} 
1, & (v_i, v_j) \in E \land i \neq j \\
0, & \text{else}
\end{cases}, \quad i, j \in \{1, 2, \ldots, n\}$$

For an undirected graph, $A$ is a non-negative symmetric matrix, that is, $a_{ij} = a_{ji}$.

3.2. Definitions, Lemmas and Assumptions
Definition 1. Given the following nonlinear system,

$$\dot{\xi}(t) = f(t, \xi(t)), \xi \in \mathbb{R}^n$$

If the system states are globally finite-time stable (GFTS) and the upper bound of the convergence time $T$ can be determined without using the initial system states, that is, there exists constant $0 < T_{\text{max}} < \infty$ with satisfying $T \leq T_{\text{max}}$, then system (3) is fixed-time stable.
Lemma 1 [19]. System (3) is fixed-time stable if there exists a positive definite Lyapunov function

\[ V(\xi) = \sum_{i=1}^{n} \xi_i^2 \]

with satisfying

\[ \sup_{t>0, \xi \neq f(t, \xi)} \xi^T \frac{\partial V(\xi)}{\partial \xi} \leq -aV^p(\xi) - bV^q(\xi) \quad \forall \xi \neq 0 \]  

(4)

where \(a, b > 0\), \(p = 1 + \mu^{-1}\), \(q = 1 - \mu^{-1}\) and \(\mu > 1\). The settling time can be calculated as

\[ T(x) \leq \frac{\pi \mu}{2\sqrt{ab}} \]  

(5)

Lemma 2 [15]. For any positive scalars \(\xi_1, \ldots, \xi_n\), the following inequalities always hold.

\[ \sum_{i=1}^{n} \xi_i^\zeta \geq \begin{cases} \left( \sum_{i=1}^{n} \xi_i \right)^\zeta, & 0 < \zeta < 1 \\ n^{1-\zeta} \left( \sum_{i=1}^{n} \xi_i \right)^\zeta, & \zeta > 1 \end{cases} \]  

(6)

Definition 2. If an undirected edge \((v_i, v_j) \in E\), that is, \(a_{ij} = a_{ji} > 0\), it means spacecraft \(i\) and spacecraft \(j\) can get the attitude information from each other.

Definition 3. For an undirected graph \(G_n\), it is connected if its Laplacian matrix \(L\) has \(n\) nonnegative eigenvalues

\[ \lambda_n \geq \lambda_{n-1} \geq \cdots \geq \lambda_1 \geq 0 \]  

(7)

where the elements of \(L\) are as follows

\[ l_{ij} = \begin{cases} \sum_{j=1}^{n} a_{ij}, i = j \\ -a_{ij}, i \neq j \end{cases} \]  

(8)

Definition 4. For any scalar \(\xi \in \mathbb{R}^n\), the function \(\text{sig}^\alpha(\xi)\) is defined as

\[ \text{sig}^\alpha(\xi) = [\text{sig}^\alpha(\xi_1), \ldots, \text{sig}^\alpha(\xi_n)]^T \]  

(9)

where \(\text{sig}^\alpha(\xi_i) = \text{sign}(\xi_i) |\xi_i|^{\alpha}\) and

\[ \text{sign}(\xi_i) = \begin{cases} 1, & \xi_i > 0 \\ 0, & \xi_i = 0 \\ -1, & \xi_i < 0 \end{cases} \]  

(10)

Assumption 1. The undirected graph \(G_n\) for indicating the communication topology of the multi-spacecraft system is connected and at least one of the following spacecrafts can obtain the leader’s attitude information.

Assumption 2. The leader’s attitude \(\sigma_{\sigma}^d\) is always a constant, that is, \(\dot{\sigma}_{\sigma}^d = 0\).

4. Distributed Controller Design

With all the above, a distributed regulation controller based on fixed-time stability is proposed in this section. The controller is composed of two parts: Firstly, with the incomplete communication topology, not every following spacecraft can obtain the leader’s attitude, a distributed fixed-time observer will be proposed for estimating the leader’s attitude by each following spacecraft. Then, for the single
spacecraft, a tracking control scheme based on the terminal sliding-mode technology is introduced such that each following spacecraft can track the desired attitude.

If the multiple spacecraft system consists of 1 leader and \( n \) followers, with Assumption 1 and Assumption 2, the distributed fixed-time observer for the system (1) is designed as

\[
\dot{\hat{\sigma}}_i = -k_1 \sigma_i^{1.2/\eta} \left[ \sum_{j=1}^{n} \left( a_{ij} \hat{\sigma}_j + b_{ij} \hat{\sigma}_i \right) \right] - k_2 \sigma_i^{1.2/\eta} \left[ \sum_{j=1}^{n} \left( a_{ij} \sigma_j + b_{ij} \sigma_i \right) \right], \quad i = 1, \ldots, n \tag{11}
\]

where \( \hat{\sigma}_j = \hat{\sigma}_j - \hat{\sigma}_j \), \( \sigma_i = \sigma_i - \sigma_j \), \( \sigma_j \) is the leader’s attitude, \( \hat{\sigma}_j \) is the estimate of \( \sigma_j \) for spacecraft \( i \), \( k_1, k_2 > 0 \), \( \eta > 2 \).

Theorem 1. For the system as equation (1), given the distributed observer as equation (11), a desired attitude \( \sigma_j \) can be consistently estimated in a finite time \( T_1 \) which satisfies

\[
T_1 \leq \frac{\pi \eta}{2 \sqrt{\rho_1 \rho_2}} \tag{12}
\]

with \( \rho_1 = (3n)^{-\eta} \times (2 \lambda_1')^{1+\eta} \) \( k_1 \), \( \rho_2 = (2 \lambda_1')^{1+\eta} k_2 \), where \( \lambda_1' \geq 0 \) is the minimum eigenvalue of \( L + B \) with \( B = [b_1, \ldots, b_n]^T \).

The proof for Theorem 1 is given as follows.

Proof. For spacecraft \( i \), define

\[
e_i = \sum_{j=1}^{n} \left( a_{ij} \sigma_j + b_{ij} \sigma_i \right) = \Theta \hat{\sigma}_i
\]

where \( \Theta = (L + B) \otimes I_{3 \times 3} \), \( \otimes \) represents the Kronecker product. With Assumption 2, the first-order derivative of \( \hat{\sigma}_i \) is calculated as:

\[
\dot{\hat{\sigma}}_i = \hat{\sigma}_i - \sigma_i = -k_1 \sigma_i^{1.2/\eta} (e_i) - k_2 \sigma_i^{1.2/\eta} (e_i) \tag{13}
\]

For proving the stability of equation (11), we select the following Lyapunov function:

\[
V_i = \frac{1}{2} \sum_{j=1}^{n} \sigma_i^T \Theta^T \hat{\sigma}_i \tag{14}
\]

Utilizing Lemma 2, \( \dot{V}_i \) is calculated as

\[
\dot{V}_i = \sum_{i=1}^{n} e_i^T \left[ k_1 \sigma_i^{1.2/\eta} (e_i) + k_2 \sigma_i^{1.2/\eta} (e_i) \right] = -k_1 \sum_{i=1}^{n} \sum_{m=1}^{n} e_{i,m}^2 \sigma_i^{2.2/\eta} - k_2 \sum_{i=1}^{n} \sum_{m=1}^{n} e_{i,m}^2 \sigma_i^{1.2/\eta} \leq -(3n)^{-\eta} k_1 \left( \sum_{i=1}^{n} \sum_{m=1}^{n} e_{i,m}^2 \right)^{1+\eta/1} - k_2 \left( \sum_{i=1}^{n} \sum_{m=1}^{n} e_{i,m}^2 \right)^{1+\eta/1} = -(3n)^{-\eta} k_1 \left( \sum_{i=1}^{n} \sigma_i^T \Theta^T \hat{\sigma}_i \right)^{1+\eta/1} - k_2 \left( \sum_{i=1}^{n} \sigma_i^T \Theta^T \hat{\sigma}_i \right)^{1+\eta/1} \tag{15}
\]

From Lemma 1, the observed error \( \hat{\sigma}_i \) will consistently converge to the equilibrium in a finite time \( T_1 \). Consequently, by using Definition 1, the distributed observer as equation (11) is fixed-time stable. Thus, Theorem 1 has been proven.

On the basis of the distributed observer proposed above, define the following state errors:
\[
\begin{align*}
\sigma_{\alpha} &= \frac{(1-\sigma_{\alpha}^T \hat{\sigma}_{\alpha}) \sigma_{\alpha} - (1-\sigma_{\alpha}^T \sigma_{\alpha}) \hat{\sigma}_{\alpha} + 2 \sigma_{\alpha}^T \hat{\sigma}_{\alpha}}{1 + \sigma_{\alpha}^T \sigma_{\alpha}} \sigma_{\alpha}^T \hat{\sigma}_{\alpha} + 2 \sigma_{\alpha}^T \sigma_{\alpha} \\
\omega_{\alpha} &= \omega_{\alpha} - R(\sigma_{\alpha}) \omega_d 
\end{align*}
\]

with
\[
R(\sigma_{\alpha}) = I_{3\times 3} - \frac{4(1-\sigma_{\alpha}^T \sigma_{\alpha})}{(1+\sigma_{\alpha}^T \sigma_{\alpha})^2} \sigma_{\alpha}^T + \frac{8\sigma_{\alpha}^T}{(1+\sigma_{\alpha}^T \sigma_{\alpha})^2} \sigma_{\alpha}^T
\]

where \( \omega_d \) is the desired attitude angular velocity, which can be derived from equation (1) once the desired attitude \( \sigma_{\alpha} \) has been estimated. As a matter of fact, \( \omega_d = 0 \) always holds since the attitude of the leader spacecraft is a constant.

Let \( x_1 = \sigma_{\alpha} \), \( x_2 = \hat{x}_1 \) and to facilitate the controller design, the error dynamic system can be transformed to a Lagrange system as
\[
M_i (x_i) \dot{x}_2 + C_i (x_i, x_2) x_2 + N_i (x_i) = G^{-T} (x_i) u_i + G^{-T} (x_i) d_i 
\]
with
\[
M_i = G^{-T} J_i G^{-1}, \quad N_i = G^{-T} [(R \omega_d)^T J_i (R \omega_d) + J_i R \omega_d]
\]
\[
C_i = -G^{-T} [(J_i G^{-1} x_2)^T G^{-1} - J_i \dot{G}^{-1}] + G^{-T} [J_i (R \omega_d)^T + (R \omega_d)^T J_i] - (J_i R \omega_d)^T G^{-1}
\]

For designing the controller, the following nonsingular terminal sliding-mode surface is firstly introduced
\[
S_i = x_2 + k_3 \text{sig}(x_1)^{1+2\eta_3^2} + k_4 [H(x_{1i}) - H(x_{1})] + k_5 x_1 
\]
with
\[
H(x_{1m}) = \begin{cases} \text{sig}(x_{1m})^{1+2\eta_3^2}, & |x_{1m}| \geq \delta \\ \frac{2\delta}{\pi} \sin \left( \frac{\pi}{2\delta} x_{1m} \right) + (1 - 2/\eta_2) \delta^{-2\eta_2}, & |x_{1m}| < \delta \end{cases} m = 1, 2, 3
\]

where \( k_3 > 0, k_4 > 0.25 \) and \( \eta_2 > 2, \delta = \left( \pi \eta_2 \right)^{0.5\eta_3} \).

For the error dynamic system equation (18), a robust attitude tracking controller is proposed as
\[
u_i = G^T (C_i x_2 + N_i) - G^T M_i \left( 1 + 2/\eta_2 \right) \text{sig}(x_{1m})^{1+2\eta_3^2} x_2 + k_4 [H(x_{1i}) - H(x_{1})] + k_5 x_2 
\]
\[
- G^T M_i \left( k_3 \text{sig}(S_i)^{1+2\eta_3^2} + k_4 \text{sig}(S_i) + D \| M_i^{-1} G^{-T} \| \tanh(3\kappa S_i) \right)
\]

where \( k_6, k_7 > 0 \) and \( \eta_3 > 2 \).

Theorem 2. For the multiple spacecraft system as equation (17), it is fixed-time stable when the feedback control scheme is developed as equations (18)-(20).

Due to limited space, the proof for Theorem 2 can refer to Ref. [18].

5. Simulation Results
For validating the effectiveness of the fixed-time distributed attitude control scheme proposed in this paper, a numerical simulation will be carried in this section. A scenario including 4 following spacecraft and 1 leader will be considered. The undirected communication topology is presented in
The inertia moment matrices of the spacecrafts are given as follows:

\[ J_i = \text{diag } (18, 12, 10), \quad J_3 = \text{diag } (22, 16, 12), \quad J_4 = \text{diag } (17, 14, 12), \quad J_3 = \text{diag } (15, 13, 8) \]

The leader’s attitude and the external disturbances are selected as:

\[ \sigma_i = [-0.2, 0.3, 0.2]^T, \quad \delta_i = 0.01 \times [\sin(t/60), -\cos(t/60), \sin(t/50)]^T \]

The initial states of following spacecrafts are set as:

\[ \sigma_i(0) = [-0.15, 0.3, 0.15]^T, \quad \sigma_i(0) = [-0.2, 0.3, 0.2]^T \]

\[ \sigma_i(0) = [-0.15, 0.15, 0.15]^T, \quad \omega_i(0) = \omega_i(0) = \omega_i(0) = 0_{3 \times 1} \]

The observer gains and the control parameters are chosen as:

\[ k_1 = 1.5, k_2 = 1.5, k_3 = 0.5, k_4 = 1, k_1 = 1, k_2 = 1, k_3 = 2.5, \eta_1 = 2.5, \eta_2 = 2.5, \eta_3 = 2.5 \]

Simulation results of the proposed control scheme are shown in figures 2-6. Firstly, the time response for the estimation errors of the observer is shown in figure 2a, from which we can conclude that the leader’s attitude can be observed (\( |\hat{\sigma}_i| \leq 10^{-7} \)) by each following spacecraft within 3.5 s. It needs to be pointed out that, the curves when \( i = 2, 3, 4 \) are almost coincided, which is determined by their communication topology. In order to introduce the superiority of this observer than other methods, a comparison has been made to the work in Ref. [13]. As shown in figure 2b, with the same conditions, it took at least 15 s to observe the leader’s attitude by using the distributed finite-time observer proposed in Ref. [13]. Hence, we can conclude that the distributed fixed-time observer as equation (11) has a better performance.

(a) Distributed fixed-time observer as equation (11)  \hspace{1cm} (b) Distributed finite-time observer in Ref. [13]
The time response curves of $S_i$, $\sigma_i$, $\omega_i$, and $u_i$ are respectively shown in figures 3-6. According to figure 3, clearly, $S_i$ can consistently converge to a small neighbourhood with containing the origin at about 2 s. The curves of $\sigma_i$ and $\omega_i$ are given in figures 4-5, respectively. It can be seen that that attitude synchronization will be reached within about 5 s and the final attitudes of each following spacecraft will be consistent with the leader’s attitude. It must be pointed out that convergence time of the attitudes is slightly greater than that of the sliding-mode surface. At last, the control torque curves are described by figure 6.

**Figure 3.** The curves of the sliding-mode surfaces.

**Figure 4.** The curves of the attitudes.

**Figure 5.** The curves of the attitude angular velocities.

**Figure 6.** The curves of the control torques.

6. **Conclusions**

In this paper, an attitude control scheme employing distributed fixed-time observer and the terminal sliding-mode technology is proposed for multiple spacecrafts. To achieve the control objective, a fixed-time distributed attitude observer has been developed such that the leader’s attitude will be estimated by each following spacecraft. The effectiveness of the proposed observer has been proved by rigorous mathematical proof and simulation results. Compared to the existing works, the observer in this paper has been proved to have a rapider convergence.

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