BITTWISITOR FORMULATION OF MASSIVE SPINNING PARTICLE

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Twistor formulation of massive arbitrary spin particle has been constructed. Twistor space of such particle is formed two twistors and two complex scalars which form together ‘bosonic supertwistor’. The formulation is deduced from space–time one for spinning particle by means of introducing auxiliary harmonic variables and consequent partial fixing of gauges. It is carried out the canonical quantization of twistor massive particle with nonzero spin. It is found the eigenvalues of Casimir operators on particle states and harmonic expansion of wave function in spectrum.

**KEYWORDS:** twistors; spinning particle; harmonic variables.

At present the twistor formulations of point-like and extended objects are acquired grate role in modern particle physics [1-16]. Up to now massless (super)particles was considered in (super)twistor approach in the main. In this article we construct twistor formulation of massive spinning particle.

At construction of twistor formulation of massive spinning particle it is necessary to solve the question of describing of massive spinning states in form of twistors. The natural method in resolving this task is introducing greater than one twistors [1-4], [16] i. e. in some terminology it is obtained the description of massive state in form of two or more massless states [8]. But the problem of spin description and in fact the right chooses of variables in twistor formulation, the constraints and Lagrangian are remained not solved finally.

The constructive way to finding of twistor formulation of spinning particle implies using the appropriate space–time formulation. For this aim from all space–time formulations the more appropriate formulation are those in which the spin degrees of freedom are described by means of commuting variables. Also from such formulations there are appropriate ones in which spin variables are spinors (for obtaining arbitrary spins including half–integer ones) and describing of arbitrary spins is realized in uniform way. The formulation relativistic spinning particle with index spinor [17-20] is more appropriate formulation for these aims.

**SPINNING PARTICLE IN INDEX SPINOR FORMULATION**

From all space–time formulations of spinning particle the formulations with spinning variables of bosonic type are appropriate ones to construction of twistor formulation of it. For this end we prefer formulation of spinning particle with index spinor. Its advantages are confined in use spinor variables for description of particle spin that improve transition to twistor formulation. Also spinning particle with index spinor has a some analogy with usual superparticle which use Grassmannian spinor variable. Therefore for our aim we can exploit the some elements of transition from space–time formulation of superparticle to twistor one.

In index spinor formalism spinning particle is described with space–time vector \(x^\mu\) and commuting Weyl spinor \(\zeta^\alpha\). In first order formalism its Lagrangian has the form [17-20]

\[
L = p\Pi - V(p^2 + m^2) - \Lambda(\zeta\bar{\zeta} - j),
\]

where the bosonic ‘superform’ is

\[
\Pi = \Pi d\tau = dx - id\zeta\sigma\bar{\zeta} + i\zeta\sigma d\bar{\zeta}.
\]

Here \(p_\mu\) is momentum vector of particle with mass \(m\). Real scalars \(V\) and \(\Lambda\) are Lagrange multipliers. In spinor notation

\[
L = -\frac{1}{2}p_{\alpha\dot{\alpha}}\Pi^{\dot{\alpha}\alpha} + \frac{1}{2}V(p_{\alpha\dot{\alpha}}p^{\dot{\alpha}\alpha} - 2m^2) - \Lambda(\zeta^{\alpha}p_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} - j),
\]

where the bosonic ‘superform’ is

\[
\Pi^{\dot{\alpha}\alpha} = \Pi^{\dot{\alpha}\alpha} d\tau = dx^{\dot{\alpha}\alpha} + i\bar{\zeta}^{\dot{\alpha}} d\zeta^{\alpha} - id\bar{\zeta}^{\dot{\alpha}} \zeta^{\alpha}.
\]
We use spinor notations which coincide with [21]. In particular, $p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}}$, $x^{\alpha\dot{\alpha}} = x^\mu \sigma^\mu_{\alpha\dot{\alpha}}$, where matrices $\sigma_\mu$ satisfy $\sigma^{\dot{\alpha}\alpha}_\mu \sigma^\mu_{\alpha\dot{\alpha}} = -\eta_{\mu\nu} \delta^\alpha_{\dot{\alpha}}$ with $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$.

Apart from the constraints inserted into the action explicitly, i.e. the mass constraint

$$ T \equiv \frac{1}{2} (p^2 + m^2) \approx 0, \tag{3} $$

and the spin one

$$ \zeta^\alpha p_{\alpha\dot{\alpha}} \zeta_{\dot{\alpha}} - j \approx 0 \tag{4} $$

the Hamiltonization [8] of the theory reveals also the spinor Bose–constraints

$$ d_{\zeta} \equiv ip_{\zeta} + p_{\alpha\dot{\alpha}} \zeta_{\dot{\alpha}} \approx 0, \quad \bar{d}_{\zeta} \equiv -ip_{\bar{\zeta}} + \zeta^\alpha p_{\alpha\dot{\alpha}} \approx 0. \tag{5} $$

On the constraints surface the spin constraint is equivalent to the constraint

$$ S \equiv S - j \equiv \frac{i}{2} (\zeta^\alpha p_{\alpha\dot{\alpha}} - \bar{p}_{\bar{\alpha}} \zeta_{\dot{\alpha}}) - j \approx 0, \tag{6} $$

because $S \equiv \frac{1}{2} (d_{\zeta} - \bar{d}_{\bar{\zeta}}) + (\zeta \bar{p} - j$).

Immediately it is found the constraint algebra, whose nontrivial brackets are

$$ \{d_\zeta, d_{\bar{\zeta}}\} = 2i \hat{p}, \quad \{S, d_\zeta\} = \frac{i}{2} d_\zeta, \quad \{S, d_{\bar{\zeta}}\} = -\frac{i}{2} \bar{d}_{\bar{\zeta}}. $$

So, the constraints $T$ and $S$ belong to the first class whereas the spinor constraints $d_{\zeta}$ and $\bar{d}_{\bar{\zeta}}$ relate to the second class for particle with nonzero mass, i.e. $\hat{p} \bar{p} = m^2 > 0$. Certainly in the procedure of Hamiltonization the spinor constraints are primary whereas the mass constraints and the spin one are constraints of the second step of the procedure. As a consequence of reparametrization invariance the total Hamiltonian is a linear combination of the first class constraints $T \approx 0$ (3) and $S \approx 0$ (6).

After quantization the wave function of the spinning particle is expressed by (anti)holomorphic polynomial on spinor $\zeta$ [17]. Therefore spinor $\zeta$ was been called index spinor. The information about Lorentz properties of wave function is encoded in a polynomial structure on index spinor. The spin of particle after quantization (only one value of spin!) is equal constant $j$ in Lagrangian renormalized by ordering constants.

**TWISTOR FORMULATION FROM LORENTZ HARMONIC APPROACH**

Let us obtain the twistor formulation from space–time formulation (2).

We introduce two spinors

$$ v^i_\alpha, \quad \bar{v}^{\dot{i}}_{\dot{\alpha}} = \overline{(v^i_\alpha)}, \quad i = 1, 2 \tag{7} $$

which considered as Lorentz harmonics [22-24]. Its formed $2 \times 2$ complex matrix with unit determinant. The harmonic spinors $v^i_\alpha$ are subjected to the conditions

$$ h = v^{\alpha i} v_{\alpha i} + 2 \approx 0, \quad \bar{h} = \bar{v}^{\dot{i}}_{\dot{\alpha}} \bar{v}_{\dot{\alpha} \dot{i}} + 2 \approx 0. \tag{8} $$

where $v_{\alpha i} = \epsilon_{ij} v^j_\alpha$, $\bar{v}^{\dot{i}}_{\dot{\alpha}} = \epsilon^{\dot{i}j} \bar{v}_{\dot{\alpha} j}$ and components of skew–symmetric tensor $\epsilon_{ij}$ are equal matrix elements of matrix $i\sigma_2$, $\epsilon^{ij}\epsilon_{jk} = \delta^i_k$. The conditions (8) can be inscribed in the equivalent form

$$ v^{\alpha i} v^\alpha_i - \epsilon^{ij} \approx 0, \quad \bar{v}_{\dot{\alpha} i} \bar{v}_{\dot{\alpha} j} - \epsilon_{ij} \approx 0 $$

or also in form

$$ v^{\alpha i} \bar{v}^\alpha_i - \epsilon^{\alpha \beta} \approx 0, \quad \bar{v}_{\dot{\alpha} i} \bar{v}^{\dot{i} \dot{\alpha}} - \epsilon^{\dot{\alpha} \dot{\beta}} \approx 0. $$

Let us complement the system (2) by pure gauge sector of Lorentz harmonics. Namely we add to Lagrangian (2) standard kinetic terms for harmonics $v^i_\alpha, \bar{v}^{\dot{i}}_{\dot{\alpha}}$ and canonically conjugate variables $p_{v^i_\alpha}, \bar{p}^{\dot{i}}_{\bar{v}^{\dot{i}}_{\dot{\alpha}}}$, $\{v^i_\alpha, p_{v^j_\alpha}\} = \delta^i_j \delta^\alpha_\beta$, $\{\bar{v}^{\dot{i}}_{\dot{\alpha}}, \bar{p}^{\dot{i}}_{\bar{v}^{\dot{j}}_{\dot{\alpha}}}\} = \delta^{\dot{j}}_{\dot{i}} \delta^\dot{\alpha}_{\dot{\beta}}$ and linear combination of the full set of the constraints, the coefficients of which are Lagrange multipliers. The number of constraints must be sufficient to exclude all harmonic variables. In addition to kinematic constraints (8) we impose the following natural set of constraints on harmonic variables

$$ p_{v^i_\alpha} \approx 0, \quad \bar{p}^{\dot{i}}_{\bar{v}^{\dot{i}}_{\dot{\alpha}}} \approx 0. \tag{9} $$
i.e. all conjugate variables for harmonics $v^i_\alpha$, $\bar{v}_{\alpha i}$ are zero in weak sense. Of course, the constraints (9) mean that the variables $v^i_\alpha$, $\bar{v}_{\alpha i}$ are constant on equations of motion, $\dot{v}^i_\alpha = 0$, $\dot{\bar{v}}_{\alpha i} = 0$.

The system of constraints (8) and (9) contain two pairs of second class constraints and six of first class constraints. The separation of constraints (9) on classes is realized by projection of them on spinors $v^i_\alpha$, $\bar{v}_{\alpha i}$. Because of nonsingularity of harmonic matrix $x^i_\alpha$ (8), the set of constraints (9) and Lorentz–invariant constraints

$$p_{v^i_\alpha} v^j_\alpha \approx 0, \quad \bar{v}_{\alpha i} \bar{p}^j_\alpha \approx 0$$

are equivalent.

The trace parts of constraints (10)

$$p_{v^i_\alpha} v^j_\alpha \approx 0, \quad \bar{v}_{\alpha i} \bar{p}^j_\alpha \approx 0$$

are conjugate ones for kinematic constraints (8). In real quantities the constraints

$$i(h - \bar{h}) = i(v^\alpha v_{\alpha i} - \bar{v}_{\alpha i} \bar{v}^\alpha) \approx 0, \quad h + \bar{h} = v^\alpha v_{\alpha i} + \bar{v}_{\alpha i} \bar{v}^\alpha + 4 \approx 0$$

and

$$D_0 = i(p_{v^i_\alpha} v^j_\alpha - \bar{v}_{\alpha i} \bar{p}^j_\alpha) \approx 0, \quad B_0 = p_{v^i_\alpha} v^j_\alpha + \bar{v}_{\alpha i} \bar{p}^j_\alpha \approx 0$$

form pairs $(i(h - \bar{h}), D_0)$ and $(h + \bar{h}, B_0)$ of conjugate each other second class constraints.

The traceless parts of constraints (10), which commute with constraints (12) and (13), are first class constraints. It is convenient to represent these constraints in form real Lorentz invariant 3–vectors

$$D_r = \frac{i}{2}(\sigma_r)_{ij}(p_{v^i_\alpha} v^j_\alpha - \bar{v}_{\alpha i} \bar{p}^j_\alpha) \approx 0, \quad B_r = \frac{1}{2}(\sigma_r)_{ij}(p_{v^i_\alpha} v^j_\alpha + \bar{v}_{\alpha i} \bar{p}^j_\alpha) \approx 0$$

where matrices $\sigma_r$, $r = 1, 2, 3$ are usual Hermitian Pauli matrices.

Thus spinning particle, added pure gauge sector of Lorentz harmonics, is described phase space variables $x^\mu$, $p_\mu$, $\xi$, $\bar{\xi}$, $\bar{\zeta}$, $\bar{p}_\alpha$, $\bar{p}^\alpha$, $v^i_\alpha$, $\bar{v}_{\alpha i}$, $p_{v^i_\alpha}$, $p_{\bar{v}_{\alpha i}}$, $p_{v^i_\alpha} \bar{p}^j_\alpha$ and constraints (3), (5), (6), (8), (13), (14). Let us exclude with using of part of the constraints the variables $x, p, \zeta, \bar{\zeta}$ and c.c. which are used in space–time formulation.

For that it is convenient to transform by Lorentz harmonics the initial variables $x, p, \zeta, \bar{\zeta}$ and c.c. to Lorentz–invariant quantities

$$x^{(0)} = \frac{1}{2} \bar{v}_{\alpha k} x^{\alpha k} v_\alpha, \quad x^{(r)} = \frac{1}{2} \bar{v}_{\alpha j} x^{\alpha j} v^j_\alpha(\sigma_r)^{ij}, \quad (15)$$

$$p^{(0)} = \frac{1}{2} \bar{v}_{\alpha k} P_{\alpha\alpha} \bar{v}^{\alpha k}, \quad p^{(r)} = \frac{1}{2} \bar{v}_{\alpha j} P_{\alpha\alpha} \bar{v}^{\alpha j}(\sigma_r)^{ij}, \quad (16)$$

$$\xi^i = m^{1/2} \xi^i v_\alpha, \quad \bar{\bar{\xi}}^i = m^{1/2} \bar{\bar{\xi}}^i \bar{v}_{\alpha i}, \quad (17)$$

$$p_{\xi i} = m^{-1/2} \xi^i p_\alpha, \quad \bar{p}_{\xi i} = m^{-1/2} \bar{\bar{\xi}}^i \bar{p}^\alpha_{\alpha i}. \quad (18)$$

At this transition harmonic variables $v^i_\alpha$, $\bar{v}_{\alpha i}$, $p_{v^i_\alpha}$, $\bar{p}^\alpha_{\alpha i}$ transform to variables

$$\lambda^i_\alpha = m^{1/2} v^i_\alpha, \quad \bar{\lambda}_{\alpha i} = m^{1/2} \bar{v}_{\alpha i}. \quad (19)$$

Momenta $\bar{\omega}^\alpha_i$, $\bar{\omega}^{\alpha i}$ for $\bar{\lambda}^i_\alpha$, $\bar{\lambda}_{\alpha i}$ are defined by generating function lower.

The transformation (15)–(19) is canonical transformation. The generating function of the canonical transformation from system with phase variables $x^\mu$, $p_\mu$, $\zeta$, $\bar{\zeta}$, $\bar{p}_{\alpha i}$, $\bar{p}^\alpha$, $v^i_\alpha$, $\bar{v}_{\alpha i}$, $p_{v^i_\alpha}$, $p_{\bar{v}_{\alpha i}}$ to the system with phase variables $x^{(0)}$, $x^{(r)}$, $p^{(0)}$, $p^{(r)}$, $\xi^i$, $\bar{\bar{\xi}}^i$, $\xi_{\alpha i}$, $\bar{\bar{\xi}}_i^\alpha$, $\lambda^i_\alpha$, $\bar{\lambda}_{\alpha i}$, $\bar{\omega}^\alpha_i$, $\bar{\omega}^{\alpha i}$ has the form

$$F = p^{(0)} x^{(0)}(x, v, \bar{v}) + p^{(r)} x^{(r)}(x, v, \bar{v}) + p_{\xi i} \xi^i(\zeta, v, \bar{v}) + \bar{p}_{\xi i} \bar{\xi}^i(\bar{\zeta}, \bar{v})$$

$$+ \bar{\omega}^\alpha_i \lambda^i_\alpha(v) + \bar{\lambda}_{\alpha i}(v) \bar{\omega}^{\alpha i}. \quad (20)$$

Here the expressions for new variables $x^{(0)}(x, v, \bar{v})$, $x^{(r)}(x, v, \bar{v})$, $\xi^i(\zeta, v, \bar{v})$, $\bar{\bar{\xi}}^i(\bar{\zeta}, \bar{v})$, $\lambda^i_\alpha(v)$, $\bar{\lambda}_{\alpha i}(v)$ in term of old variables from the right hand side of the equations (15), (17), (19) have been used. That construction of the generating function reproduces the expressions (15)–(19), whereas the expressions of harmonic momenta $p_{v^i_\alpha}$, $\bar{p}^\alpha_{\alpha i}$ are

$$p_{v^i_\alpha} = m^{1/2} \left[ v^i_\alpha + \frac{1}{2m} p^{(0)} \bar{\lambda}_{\alpha i} x^{\alpha i} + \frac{1}{2m} p^{(r)} \bar{\lambda}_{\alpha i} x^{\alpha i} (\sigma_r)^{ij} + p_{\xi i} \zeta^i \right]. \quad (21)$$
\[ \hat{p}_{v}^{\alpha i} = m^{1/2} \left[ \omega^{\alpha i} + \frac{1}{2m} p^{0} x^{\alpha} \lambda_{\alpha}^{i} + \frac{1}{2m} p^{(r)} x^{\alpha} \lambda_{\alpha}^{i} (\sigma_{r})^{i} + \bar{p}_{\zeta}^{i} \bar{\zeta}^{i} \right]. \]  

Therefore in new variables the constraints (13), (14) acquire the additional terms

\[ D_{0} = D_{0} - i(\bar{\xi}_{j} \bar{\xi}^{j} - p_{\xi} \xi^{i}) \approx 0 , \]  

\[ B_{0} = B_{0} + 2x^{(0)} p^{(0)} + 2x^{(r)} p^{(r)} + (\xi_{i} \bar{p}_{\zeta}^{i} + p_{\xi} \xi^{i}) \approx 0 , \]  

\[ D_{r} = D_{r} - \frac{1}{2} \epsilon_{r} s p^{(s)} x^{(p)} - \frac{1}{2} (\sigma_{r})^{i} (\xi_{j} \bar{p}_{\zeta}^{i} - p_{\xi} \xi^{i}) \approx 0 , \]  

\[ B_{r} = B_{r} + x^{(0)} p^{(0)} + x^{(r)} p^{(r)} + \frac{1}{2} (\sigma_{r})^{i} (\xi_{j} \bar{p}_{\zeta}^{i} + p_{\xi} \xi^{i}) \approx 0 , \]

where in new constraints \( D_{0}, B_{0}, D_{r}, B_{r} \) the expressions for \( D_{0}, B_{0}, D_{r}, B_{r} \) as in (13), (14) with new \( \lambda_{\alpha}^{i}, \bar{\lambda}_{\alpha i}, \) \( \bar{\omega}^{\alpha i}, \omega^{\alpha i} \) in place of \( v_{\alpha}^{i}, \bar{v}_{\alpha i}, p_{\xi}^{\alpha i}, \bar{p}_{\xi}^{\alpha i} \)

\[ D_{0} \equiv i(\bar{\omega}^{\alpha i} \lambda_{\alpha}^{i} - \bar{\lambda}_{\alpha i} \omega^{\alpha i}) \approx 0 , \quad B_{0} \equiv \bar{\omega}^{\alpha i} \lambda_{\alpha}^{i} + \bar{\lambda}_{\alpha i} \omega^{\alpha i} \approx 0 , \]  

\[ D_{r} \equiv \frac{1}{2} (\sigma_{r})^{i} (\bar{\omega}^{\alpha i} \lambda_{\alpha}^{i} - \bar{\lambda}_{\alpha i} \omega^{\alpha i}) \approx 0 , \quad B_{r} \equiv \frac{1}{2} (\sigma_{r})^{i} (\bar{\omega}^{\alpha i} \lambda_{\alpha}^{i} + \bar{\lambda}_{\alpha i} \omega^{\alpha i}) \approx 0 . \]

The constraints (3)–(5) in new variables are

\[ -(p^{0})^{2} + p^{(r)} p^{(r)} + m^{2} \approx 0 , \]  

\[ -\frac{1}{m} \xi^{i} p^{j} \bar{\xi}^{j} - j \approx 0 , \]  

\[ v_{\alpha}^{i} d_{\alpha} = m^{-1/2} (ip_{\xi} + \frac{1}{m} p^{j} \xi^{j}) \approx 0 , \]  

\[ \bar{d}_{\alpha} \bar{v}^{\alpha} = m^{-1/2} (ip_{\zeta} - \frac{1}{m} \xi^{i} p^{j}) \approx 0 , \]

where \( p^{i}_{j} \equiv p^{(0)} \delta^{i}_{j} + p^{(r)} (\sigma_{r})^{i} \) (in term of initial space–time momentum \( p^{i}_{j} = v_{\alpha}^{i} p_{\alpha} \bar{v}_{\alpha}^{j} = \frac{1}{m} \lambda_{\alpha}^{i} p_{\alpha} \bar{\lambda}_{\alpha}^{j} \)).

The transformations generated the constraints (26) are Wigner transformations. By these transformations we can transform four–momentum (in harmonic basis) to standard form with

\[ p^{(r)} \approx 0 , \quad r = 1, 2, 3 . \]

This conditions are gauge fixing conditions for constraints \( B_{r} \approx 0 \) from which we obtain the expressions for \( x^{(r)} \). Because of resolved form of the gauge fixing conditions (32) the Poisson brackets for another variables do not exchanged. Now the mass–shell condition (29) takes the form

\[ p^{(0)} \pm m \approx 0 , \]  

fixed by means condition

\[ x^{(0)} \approx 0 , \]  

which has resolved form also.

The constraints (30), (31) take the form

\[ \pm \xi_{j} \xi^{i} - j \approx 0 , \]  

\[ \psi_{i} \equiv ip_{\xi_{i}} \mp \bar{\xi} \approx 0 , \]  

\[ \bar{\psi}^{i} \equiv ip_{\zeta} \pm \xi^{i} \approx 0 . \]

The last constraints (36) \( \psi_{i} \approx 0 , \bar{\psi}^{i} \approx 0 \) are the pairs of second class constraints, \( \{ \psi_{i}, \bar{\psi}^{j} \} = \mp 2i \delta^{i}_{j} \). After introducing Dirac brackets for them

\[ \{ A, B \}^{*} = \{ A, B \} \mp \frac{1}{2} \left( \{ A, \psi_{i} \} \{ \bar{\psi}^{i}, B \} - \{ A, \bar{\psi}^{i} \} \{ \psi_{i}, B \} \right) \]

the variables \( p_{\xi}, \bar{p}_{\zeta} \) are excluded whereas remaining variables \( \xi^{i}, \bar{\xi} \) have nonzero Dirac brackets \( \{ \xi^{i}, \bar{\xi} \}^{*} = \mp \frac{1}{2} \delta^{i}_{j} \). The variables \( \xi^{i} \) and \( \bar{\xi} \) is canonically conjugate each other. This construction generate automatically corresponding kinetic term for \( \xi^{i} \) and \( \bar{\xi} \) in Lagrangian lower.

Thus, we exclude completely the space–time variables and obtain the system with variables \( \lambda_{\alpha}^{i}, \bar{\lambda}_{\alpha i}, \omega^{i}, \bar{\omega}^{\alpha i}, \xi^{i}, \bar{\xi} \) and constraints

\[ h \equiv \lambda^{\alpha i} \lambda_{\alpha i} + 2m \approx 0 , \]  

\[ \bar{h} \equiv \bar{\lambda}_{\alpha i} \bar{\lambda}^{\alpha i} + 2m \approx 0 , \]
\[ D_0 = D_0 + 2\bar{\xi}_i \xi^i = i(\bar{\omega}_i^a \lambda_i^a - \bar{\lambda}_{ai}\bar{\omega}^{\bar{\alpha}i}) + 2\bar{\xi}_i \xi^i \approx 0 , \]  
\[ D_r = D_r \pm (\sigma_r)i_j \bar{\xi}_j \xi^i = (\sigma_r)i_j \left[ \frac{1}{2}(\bar{\omega}_i^a \lambda_i^a - \bar{\lambda}_{ai}\bar{\omega}^{\bar{\alpha}i}) + \bar{\xi}_j \xi^i \right] \approx 0 , \]  
\[ S \equiv S - j \equiv \pm \bar{\xi}_i \xi^i - j \approx 0 , \]  
\[ B_0 = B_0 = \omega_i^a \lambda_i^a + \bar{\lambda}_{ai}\bar{\omega}^{\bar{\alpha}i} \approx 0 . \]  

The last constraint \( B_0 \approx 0 \) (41) and constraint \( h + \bar{h} \approx 0 \) form pair of self-conjugated second class constraints. We can consider the constraint \( B_0 \approx 0 \) as gauge fixing condition for constraint \( h + \bar{h} \approx 0 \). Thus we have equivalent system which has the constraints (37)-(40). Of course we can impose the constraint \( B_0 \approx 0 \) in arbitrary moment.

Let us verify the equality of number of physical degrees of freedom in twistor system with constraints (37)-(40) and massive spinning particle with Lagrangian (2). The constraint (40) \( S - j \approx 0 \) and traceless parts (39) \( D_r \approx 0 \) are first class constraints. Also first class constraints is the constraint \( h - \bar{h} \approx 0 \), whereas constraint \( h + \bar{h} \approx 0 \) and trace part (38) \( D_0 \approx 0 \) are conjugate each other second class constraints. Thus in twistor variables we have 5 of first class constraints and 2 of second class ones. These constraints exclude 12 degrees of freedom. Since phase space, which contain \( \lambda_i^a \), \( \omega^{\bar{\alpha}i} \) and \( \xi^i \), have 20 variables, number of the physical degrees of freedom in twistor model with constraints (37)-(40) are 8. This coincides with number of the physical degrees of freedom in space–time formulation of massive spinning particle with Lagrangian (2). Here we have 16 variables in \( x^\mu \), \( p_\mu \), \( \zeta^\alpha \) and \( p_{\zeta^\alpha} \) and 2 of first class constraints (spin constraint and mass–shell constraint) and 4 of second class constraints (spinor constraints). Thus the number of of physical degrees of freedom is also 8.

Note that constraints (38) and (39) are inscribed in form of constraints
\[ D_0 \equiv \bar{\xi}_i \xi^i \approx 0 \]  
Then \( D_0 \approx 0 \) is defined trace part of \( D_i^j \approx 0 \), \( D_0 = D_i^i \), whereas \( D_r \approx 0 \) are proportional traceless parts \( D_i^j - \frac{1}{r}h_i^j D_k^k \approx 0 \). We consider arbitrary moment. We can impose the constraint \( B_0 \approx 0 \) in arbitrary moment.

\section*{Twistor Transformation and Lagrangian of Massive Spinning Particle in Twistor Formulation}

Expressions obtained in previous section give us directly full set of equations defined twistor transformation.

Using completeness conditions for harmonics \( v_i^i (48) \) or for spinors \( \lambda_i^a (37) \) we have
\[ p_{\alpha \dot{\alpha}} = - \frac{1}{m} \lambda_i^a \dot{p}_r^j \bar{\lambda}_{aj} = - \frac{1}{m} \lambda_i^a (p_{0j} \delta_i^j + p_{(r)}i^j) \bar{\lambda}_{aj} . \]  
Then after gauge fixing (adaptation of harmonic basis to space–time one) (32) \( p_{(r)} = 0, r = 1, 2, 3 \) and mass–shell condition (33) \( p^{(0)} = -p_{(0)} = \pm m \) we obtain twistor–like representation for four–momentum
\[ p_{\alpha \dot{\alpha}} = \pm \lambda_i^a \bar{\lambda}_{ai} . \]  
From expressions (17) and using (19) we have expressions for new variables \( \xi^i \), \( \bar{\xi}_i \) in term of spinning variables \( \lambda_{ai} \), \( \bar{\lambda}_{ai} \)
\[ \xi^i = \zeta^\alpha \lambda_i^a , \quad \bar{\xi}_i = \bar{\lambda}_{ai} \bar{\zeta}^\dot{\alpha} . \]  
From canonical transformation (21), (22) and using conditions (32), (33) and (9), we obtain incidence conditions
\[ \bar{\omega}_i^a = \pm \bar{\lambda}_{ai} x^{\dot{\alpha}a} \pm i \xi_i^a \zeta^\alpha , \quad \omega^{\dot{\alpha}i} = \pm \frac{1}{2} x^{\dot{\alpha}a} \lambda_i^a \mp i \xi_i^a \bar{\zeta}^\dot{\alpha} , \]  
which defined \( \omega \)-spinors by another variables. Thus we obtain twistor transformations (44)-(46) of massive spinning particle from space–time formulation in terms of the variables \( x^\mu \), \( p_\mu \), \( \zeta^\alpha \), \( \bar{\zeta}^\dot{\alpha} \) to twistor formulation in terms of the variables \( \lambda_i^a \), \( \bar{\lambda}_{ai} \), \( \omega^{\dot{\alpha}i} \), \( \omega^{\dot{\alpha}i} \), \( \xi^i \), \( \bar{\xi}_i \). Let us summarize the obtained twistor transition.

In twistor formulation of massive particle we use two spinors
\[ \lambda_i^a , \quad \bar{\lambda}_{ai} = \overline{(\lambda_i^a)} , \quad i = 1, 2 \]  
in form of them the four–momentum of particle has the resolved form (44). For fulfilment of the mass–shell condition
\[ p_{\alpha \dot{\alpha}} p^{\dot{\alpha} \alpha} - 2m^2 = -2(p^2 + m^2) \approx 0 \]
The spinors $\lambda^i_a$ are subjected to the conditions (37)

$$\lambda^{ai}\lambda_{ai} + 2m \approx 0, \quad \bar{\lambda}_{\bar{a}i}\bar{\lambda}^{\bar{a}i} + 2m \approx 0,$$

where $\lambda^i_a = \epsilon_{ij} \lambda^j_t$, $\bar{\lambda}_{\bar{a}i} = \epsilon^{ij} \bar{\lambda}_{t \bar{a}j}$ and components of skew-symmetric tensor $\epsilon^{ij}$ are equal matrix elements of matrix $i\sigma_2$, $\epsilon^{ij}\epsilon_{jk} = \delta_k^i$. The conditions can be written in the equivalent form

$$\lambda^{ai}\lambda^i_i - m\epsilon^{ij} \approx 0, \quad \bar{\lambda}_{\bar{a}i}\bar{\lambda}^{\bar{a}i} - m\epsilon_{\bar{a}\bar{j}} \approx 0$$

or also in form

$$\lambda^{ai}\lambda^i_i - m\epsilon^{\alpha\beta} \approx 0, \quad \bar{\lambda}_{\bar{a}i}\bar{\lambda}^{\bar{a}i} - m\epsilon_{\bar{a}\bar{b}} \approx 0.$$  

For each spinor $\lambda^i_a$, $i = 1, 2$ it is canonically conjugated spinor

$$\omega^{\bar{a}i}, \quad \bar{\omega}^i = (\omega^{\bar{a}i}), \quad i = 1, 2,$$

which play the role of the second spinor component of corresponding twistor. The incidence conditions which defined $\omega$-spinors by another variables have the form (46). After contractions with $\bar{\lambda}_{\bar{a}i}$ and $\lambda^i_a$ the incidence conditions (46) give us the following constraints (42) (or, equivalently, (38) and (39)).

In twistor formulation, particle spin is described by means of two complex scalar variables

$$\xi^i, \quad \bar{\xi}^i = (\xi^i), \quad i = 1, 2.$$

Connection of them with index spinor $\zeta$ (spinning variables in space–time formulation) is defined by expressions (45). The variables (52) satisfy the constraint (40).

Such form of twistor transformations (44)-(46) gives desired form of kinetic terms in twistor variables. Precisely, using (44)-(46) the kinetic terms in twistor variables are

$$-\frac{1}{2} p_{\alpha\alpha} \Pi^{\alpha\alpha} = \mp \frac{1}{2} (d\omega^\alpha_i \lambda^i_\alpha - d\bar{\lambda}_{\bar{a}i}\omega^{\bar{a}i} - \omega^{\bar{a}i} d\lambda^i_\alpha + \bar{\lambda}_{\bar{a}i} d\omega^{\bar{a}i}) \pm i (d\xi_i \xi^i - \bar{\xi}_i d\xi^i).$$

Spinors $\lambda^i_a$ and $\omega^{\bar{a}i}$ are combined in twistors $Z^i_a$ by

$$Z^i_a = (\lambda^i_a, \omega^{\bar{a}i}).$$

If we introduce in standard way conjugate twistors for ones (54) by

$$\bar{Z}_{\bar{a}i} = (Z^i_a) = (\bar{\lambda}_{\bar{a}i}, \bar{\omega}^i_a), \quad \bar{Z}^a = g_{ab} Z_b = (\bar{\omega}^a_i, -\bar{\lambda}_{\bar{a}i}),$$

where

$$g_{ab} = \begin{pmatrix} 0 & \delta_{\alpha\beta} \\ -\delta_{\alpha\bar{\beta}} & 0 \end{pmatrix}$$

then the kinetic terms (53) can be rewritten as

$$p_{\mu\nu} = -\frac{1}{2} p_{\alpha\alpha} \Pi^{\alpha\alpha} = \mp \frac{1}{2} (d\bar{Z}^a_i Z^a_i - \bar{Z}^a_i d\bar{Z}^a_i) \pm i (d\bar{\xi}^i \bar{\xi}_i - \bar{\xi}^i d\bar{\xi}^i).$$

Thus the twistor formulation of massive spinning particle is described by Lagrangian

$$L = -\frac{1}{2} \left( \bar{Z}^a_i Z^a_i - \bar{Z}^a_i \bar{Z}^\alpha_i \right) - i \left( \bar{\dot{\xi}}^i \xi^i - \bar{\dot{\xi}}_i \bar{\xi}^i \right) - M (S - j) - N_j iD_j^3 - K\bar{h} - \bar{K} h,$$

where $M, N_j^i, K$ and $\bar{K}$ are Lagrange multipliers for constraints (40), (42) and (37).

It is noted that twistor mass–shell constraints (37) can be rewritten in form

$$h = Z^a_i I_{ab} Z^b_i + 2m \approx 0, \quad \bar{h} = \bar{Z}^a_i I_{ab} \bar{Z}^{\bar{a}i} + 2m \approx 0,$$

if we use so-called infinity twistors (asymptotic twistors)

$$I_{ab} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & 0 \end{pmatrix}, \quad I_{ab} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon^{\bar{\alpha}\bar{\beta}} \end{pmatrix}.$$
Also the constraints (42) are represented in covariant contractions of twistors

\[ D^{\lambda}_j = \frac{i}{2} \bar{Z}^a_j Z^j_a \mp \bar{\xi}_j \xi^j \approx 0. \]  

We can introduce also so-called ‘bosonic supertwistors’

\[ Z^i_\lambda = (Z^i_\lambda ; \xi^i) , \quad \bar{Z}^A_i = (\bar{Z}^A_i ; \mp 2i\bar{\xi}_i) \]

in form of them the kinetic terms of Lagrangian (57) are rewritten as

\[ \mp \frac{i}{2} \left( \bar{Z}^A_i \bar{Z}^i_\lambda - \bar{Z}^A_i \bar{Z}^i_\lambda \right) \]

and the constraints (59) are

\[ D^{\lambda}_j = \frac{i}{2} \bar{Z}^A_i Z^j_\lambda \approx 0. \]

**QUANTIZATION OF THE TWISTORIAL SPINNING PARTICLE**

Let us carry out canonical quantization a la Dirac of massive spinning particle in twistor formulation. For definiteness, we consider case with upper sign in Lagrangian (57). The system is described by twistor variables \( \bar{\omega}^i , \omega^{\alpha i} , \lambda_\alpha , \lambda_{\dot{\alpha}} , \xi^i , \bar{\xi}_i \) and constraints (37)–(40), i.e.

\[ S - j \equiv \bar{\xi}_i \xi^i - j \approx 0 , \]  

**SU(2)–constraints**

\[ D_\sigma (\sigma_r)^j D_j = \frac{i}{2} [\lambda^i_\alpha (\sigma_r)^j \bar{\omega}^i_\alpha - \omega^{\alpha i} (\sigma_r)^j \bar{\lambda}_{\dot{\alpha} i}] + \bar{\xi}_i (\sigma_r)^j \xi^j \approx 0 , \]

the normalization conditions

\[ h \equiv \lambda^{\alpha i} \lambda_{\dot{\alpha} i} + 2m \approx 0 , \quad \bar{h} \equiv \bar{\lambda}_{\dot{\alpha} i} \bar{\lambda}^{\alpha i} + 2m \approx 0 \]  

and **U(1)–constraint**

\[ D_0 = 2D_i^j = i(\lambda^i_\alpha \bar{\omega}^a_\beta - \omega^{\alpha i} \bar{\lambda}_{\dot{\alpha} j}) + 2\bar{\xi}_i \xi^i \approx 0 . \]

The constraint \( D_0 \) is gauge fixing condition for constraint \( h - \bar{h} \approx 0 \). We impose also gauge fixing condition (41)

\[ B_0 = \lambda^i_\alpha \bar{\omega}^a_\beta + \omega^{\alpha i} \bar{\lambda}_{\dot{\alpha} i} \approx 0 \]  

for constraint \( h + \bar{h} \approx 0 \) and regard that the constraint (65)–(67) are fulfilled in strong sense. Introduction of Dirac brackets for them do not change commutation relations of another constraints [23], [24] and we may consider the twistor variables \( \omega \) and \( \lambda \) with standard canonical relations

\[ \{ \lambda^i_\alpha , \bar{\omega}^j_\beta \} = \delta^i_\alpha \delta^j_\beta , \quad \{ \bar{\lambda}^{\dot{\alpha} i} , \omega^{\beta j} \} = \delta^i_\beta \delta^j_\alpha \]  

in **SU(2)–constraints** (64). The kinetic terms in (57) lead to following Dirac brackets for \( \xi \)–variables

\[ \{ \bar{\xi}_i , \xi^j \}^* = -\frac{1}{2} \delta^j_i . \]

We use the realization of spinor variables, quantum commutators of which are

\[ [\bar{\omega}^i_\alpha , \lambda^j_\beta] = -i\delta^i_\alpha \delta^j_\beta \delta^j_\beta , \quad [\omega^{\alpha i} , \bar{\lambda}^{\dot{\alpha} j}] = -i\delta^\alpha_\beta \delta^\beta_\jmath \delta^j_\beta , \]

as differential operators. For definiteness we take representation with diagonal \( \lambda \)–spinors whereas realization of \( \omega \)–spinors in constraints (64) is

\[ \bar{\omega}^i_\alpha = -i\partial / \partial \lambda^i_\alpha , \quad \omega^{\alpha i} = -i\partial / \partial \bar{\lambda}^{\dot{\alpha} i} . \]

Quantum algebra of \( \xi \)–variables are

\[ [\xi^i , \bar{\xi}_j] = -\frac{1}{2} \delta^i_\jmath . \]
The variables

\[ a_i \equiv \sqrt{2\xi_i}, \quad a_i^+ \equiv \sqrt{2\xi^i} \]  

are usual annihilation and creation operators of two-dimensional oscillator

\[ [a_i, a_j^+] = \delta^i_j. \]  

The wave function will be taken in filling numbers space of these operators.

Thus the wave function \( \Psi(\lambda, \bar{\lambda}) \) is subjected the first class constraints

\[ (S - J) \Psi \equiv \left( \frac{1}{2} a_i^+ a_i - J \right) \Psi = 0, \]  

\[ D_r \Psi = (D_r + \Delta_r) \Psi = 0, \quad r = 1, 2, 3, \]  

where

\[ D_r \equiv \frac{1}{2} \left[ \lambda^i_{\alpha_i}(\sigma_r)^j_i \frac{\partial}{\partial \lambda^j_{\alpha_i}} - \frac{\partial}{\partial \bar{\lambda}^j_{\dot{\alpha}_j}}(\sigma_r)^i_j \bar{\lambda}_{\dot{\alpha}_j} \right], \]  

\[ \Delta_r \equiv \frac{1}{2} a_i^+(\sigma_r)^i_j a_j. \]  

The constant \( J \) in constraint (71) is classical constant \( j \) in (63) renormalized ordering constants.

The operators \( D_r \) and \( \Delta_r \) form \( SU(2) \)-algebras

\[ [D_r, D_s] = i\epsilon_{rsp}D_p, \quad [\Delta_r, \Delta_s] = i\epsilon_{rsp}\Delta_p. \]  

The possible ordering constants in operators \( D_3 \) and \( \Delta_3 \) mutually compensate each other. Otherwise the quantum algebra of first class constraints \( D_r \)

\[ [D_r, D_s] = i\epsilon_{rsp}D_p \]

will not be closed.

**ANALYSIS OF SPECTRUM**

Let us find the possible values of spin in spectrum. The direct method for finding of particle spin in spectrum is determination of eigenvalues of Casimir operators of Poincare group. The operator of the four-translations in realization on the space of wave function \( \Psi(\lambda, \bar{\lambda}) \) has the form

\[ P_{\alpha\dot{\alpha}} = \lambda^i_{\alpha_i} \bar{\lambda}_{\dot{\alpha}_i} \]  

whereas the operator of Lorentz transformations is

\[ M_{\alpha\dot{\alpha}\beta\dot{\beta}} = 2i(\epsilon_{\alpha\beta} M_{\alpha\beta} + \epsilon_{\alpha\dot{\beta}} M_{\alpha\dot{\beta}}), \]  

where

\[ M_{\alpha\beta} = \lambda^i_{\alpha_i} \frac{\partial}{\partial \lambda^i_{\beta_i}}, \quad \bar{M}_{\alpha\dot{\beta}} = \bar{\lambda}_{\dot{\alpha}_i} \frac{\partial}{\partial \bar{\lambda}^j_{\dot{\beta}_j}}. \]

In consequence of normalization conditions (65) of \( \lambda \)-spinors we have on physical states

\[ P^2 = -m^2 \]  

i.e. the physical states describe the particle of mass \( m \).

By means direct calculations we obtain that Pauli–Lubanski pseudovector

\[ W_{\alpha\dot{\alpha}} = P^\beta_{\alpha} \bar{M}_{\beta\dot{\alpha}} - P^\beta_{\dot{\alpha}} M_{\beta\alpha} \]

take the form

\[ W_{\alpha\dot{\alpha}} = i u_{ra\dot{\alpha}} D_r, \]  

where

\[ u_{ra\dot{\alpha}} = \lambda^i_{\alpha_i}(\sigma_r)^j_i \bar{\lambda}_{\dot{\alpha}_j}, \quad u_r \cdot u_s = -m^2 \delta_{rs}, \]
and operators $\mathcal{D}_r$ are the same as in (73). Note that $[D_r, u_{s1a1}] = \epsilon_{rsp}u_{pari}$. Right now we obtain

$$W^2 = m^2D_rD_r.$$  \hfill (79)

But from (72) we see that on physical states $\mathcal{D}_r = D_r - \Delta_r$ and $\mathcal{D}_rD_r = D_rD_r - 2\Delta_rD_r + \Delta_r\Delta_r$, i.e. on states of spectrum

$$W^2 = m^2\Delta_r\Delta_r.$$  \hfill (80)

But direct calculation gives us that

$$\Delta_r\Delta_r = \frac{1}{2}a^+a_i(\frac{1}{2}a^+a_i + 1) = S(S + 1).$$

In consequence of constraint (71) the operator $S$ is equal $J$ on physical states. Therefore on states of spectrum

$$W^2 = m^2S(S + 1)$$  \hfill (81)

i.e. in spectrum we have massive particle with fixed spin which equal $J$.

**WAVE FUNCTION OF TWISTORIAL MASSIVE PARTICLE**

The operators $\Delta_r$ form $SU(2)$–algebra which realized by operators of two oscillators. Let integer non–negative numbers $n_1$ and $n_2$ are corresponding filling numbers i.e. $n_1$ and $n_2$ are the eigenvalues of operators $a^{+1}a_1$ and $a^{+2}a_2$. The constraints (71) gives us that $\frac{1}{2}(n_1 + n_2) = J \geq 0$. Then the number $\frac{1}{2}(n_1 - n_2) \equiv M$ takes $(2J + 1)$ values $M = -J, -J + 1, ..., J - 1, J$. In normalized basis the action of operators $\Delta_{\pm} = \Delta_1 \pm i\Delta_2$, $\Delta_3$ on wave function $\Psi(\lambda, \bar{\lambda})$, which has index $M$,

$$\Psi_M(\lambda, \bar{\lambda}), \quad M = -J, -J + 1, ..., J - 1, J,$$

is

$$\Delta_3\Psi_M = M\Psi_M, \quad \Delta_{\pm}\Psi_M = \sqrt{(J \mp M)(J \pm M + 1)}\Psi_{M \pm 1}.$$  \hfill (82)

Then the action of constraints (72) on $(2J + 1)$–component wave function $\Psi_M(\lambda, \bar{\lambda})$ takes the form

$$\mathcal{D}_3\Psi_M = -M\Psi_M, \quad \mathcal{D}_{\pm}\Psi_M = -\sqrt{(J \mp M)(J \pm M + 1)}\Psi_{M \pm 1},$$  \hfill (83)

where $\mathcal{D}_{\pm} = \mathcal{D}_1 \pm i\mathcal{D}_2$. All $(2J + 1)$ components of wave function are obtained from one component, for example from component of highest weight $\Psi_{+J}$ or lowest one $\Psi_{-J}$

$$\Psi_M = \sqrt{\frac{(J \pm M)!}{(J \mp M)!(2J)!}}(-1)^{M}(\mathcal{D}_{\pm})^{J\mp M}\Psi_{\pm J}.$$  \hfill (84)

These components $\Psi_{\pm J}$ are defined by equations

$$\mathcal{D}_3\Psi_{\pm J} = \pm J\Psi_{\pm J}, \quad \mathcal{D}_{\pm}\Psi_{\pm J} = 0, \quad (\mathcal{D}_{\pm})^{2J+1}\Psi_{\pm J} = 0.$$  \hfill (85)

The operators $\mathcal{D}_r$ are generators of $SU(2)$–transformations, acting on indices $i, j, k, ...$ of $SL(2, C)$–matrix $\lambda_i^a$. The constraints (82) state that the wave function $\Psi_M(\lambda, \bar{\lambda})$ is defined up to local transformations acting on index $M$

$$\Psi'M(\lambda') = \mathcal{D}_{MN}(h)\Psi_N(\lambda),$$  \hfill (86)

where $h \in SU(2)$ and $\lambda_i^a = h_i^j\lambda_j^a$. The $\mathcal{D}_{MN}$ is matrix of $SU(2)$–transformations of weight $J$. Thus the wave function is defined in fact on homogeneous space $\mathcal{M} = G/H = SL(2, C)/SU(2)$.

The harmonic expansion of function defined on $SL(2, C)$ is [25]

$$\Phi(\lambda) = \frac{-1}{8\pi^2} \int \text{Tr} \left( F(\chi)T^{-1}_\chi(\lambda) \right) c(\chi)d\chi$$

$$= \frac{-1}{32\pi^2} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d\rho(\rho^2 + \rho^2) \text{Tr} \left( F(\chi)T^{-1}_\chi(\lambda) \right),$$  \hfill (87)

9
where Fourier transformation \( F(\chi) \) acts on space of function \( \varphi(z) \), defined on two–dimensional complex plane with coordinates \( z = z^\alpha, \alpha = 1, 2 \), by means of

\[
F(\chi)\varphi(z) \equiv \int \Phi(\lambda)T_\chi(\lambda)\varphi(z)d\lambda
\]

and \( T_\chi \) is operator \( SL(2,C)–\)transformations

\[
T_\chi\varphi(z) = \varphi(z\lambda).
\]

In decomposition (85) it is taken only representations of basic series \( \chi = ((n+i\rho)/2, (-n+i\rho)/2), c(\chi) = n^2 + \rho^2. \)

For \( SU(2)–\)covariant function (84)

\[
n = M.
\]

Therefore wave function of massive particle of spin \( J \) has harmonic decomposition on basic series of following form

\[
\Psi_M(\lambda) = \frac{1}{32\pi^2} \int_{-\infty}^{\infty} d\rho(M^2 + \rho^2)\text{Tr}(F_M(\chi)T^{-1}_\chi(\lambda)) ,
\]

where

\[
\chi = ((M + i\rho)/2, (-M + i\rho)/2).
\]

Thus as result quantization of the massive twistorial particle with Lagrangian (57) we obtain in spectrum the particle with fixed mass and fixed spin. The wave function of it is defined by equations (83).

**CONCLUSION**

In this work we presented the twistor formulation of massive particle with arbitrary spin. This formulation is obtained from massive spinning particle in index spinor formulation by means introducing pure gauge harmonic variables. After partial fixing of gauges we obtain the model described two twistors (bitwistor) and two complex scalars. As result of canonical transformation we obtain the conditions of twistor transformation. It is carried out quantization of the twistorial spinning particle. On physical states Casimir operators of Poincare group hae the value corresponding to the massive particle of fixing nonzero spin. The wave function of twistorial massive particle is defined on homogeneous space \( SL(2,C)/SU(2) \) and has harmonic expansion in representations of basic series with one fixing weight.

This work was supported in part by INTAS Grant INTAS-2000-254 and by Ukrainian National Found of Fundamental Researches under the Project N 02.07/383. We would like to thank I.A. Bandos, A. Frydryzhak, E.A. Ivanov, S.O. Krivonos, J. Lukierski, A.J. Nurmagambetov and D.P. Sorokin for interest to the work and for many useful discussion.

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