Assessing the effect of proppant compressibility on the conductivity of heterogeneously propped hydraulic fracture

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Abstract. Hydraulic fracturing is a technology that is widely used in the development of oil and gas formations. Given that the fracture closure has a strong impact on production, quantifying the resulting fracture conductivity is critical for optimizing treatment design. The goal of this paper is to better understand the influence of the closing stress on the fracture conductivity when the proppant distribution is heterogeneous. In addition to the spatial proppant distribution, the conductivity of the propped fracture is affected by proppant deformation and embedment. Numerical results indicate that compressibility of proppant can significantly change the residual fracture aperture and, consequently, production performance in oil and gas reservoirs.

1. Introduction
Hydraulic fracturing (HF) technology is based on pumping fluid into the reservoir at high pressure through the well to create extended fractures that serve to enhance hydrocarbon production. To prevent the fracture from closing after the injection is stopped, it is filled with a proppant, which is sand or ceramic balls with a diameter of ~1 mm. The optimal fracture geometry is achieved by varying the pumping schedules with the selection of suitable fluids and proppants. The proppant pack inside the fracture has high permeability and is responsible for the filtration of hydrocarbons back to the surface. Both numerical simulations [1] and laboratory testing [2] are used to determine correlations for proppant pack conductivity for various conditions. The aim of this study is to determine the effective conductivity of a fractured fracture under heterogeneous proppant placement, taking into account the compressibility of proppant clusters and the elasticity of the fracture walls. It is assumed that due to the good bearing capacity of the hydraulic fracturing fluid, the placement of the proppant in the closed fracture coincides with the placement at the time of stopping the injection.

2. Model for fracture closure on incompressible proppant
In this study, a coupled model of hydraulic fracture propagation and proppant transport [3] is used to model heterogeneous proppant distribution inside the fracture. Then, a non-local stationary model of fracture closure under compressive stress is employed. Finally, after the complete fracture closure, the inflow from the fracture to the well is calculated based on the difference between reservoir and borehole pressures.
To model fracture closure, the quasi-static elastic equilibrium of the medium around the fracture can be written as:  
\[
\text{div}\sigma = 0, \quad \sigma = \lambda \text{div}\mathbf{u} + 2\mu \varepsilon(\mathbf{u}),
\]
(1)
where \(\sigma\) is the stress tensor, \(\lambda\) and \(\mu\) are the elastic moduli, \(I\) is the identity tensor, and \(\varepsilon(\mathbf{u})_{ij} = \frac{1}{2}(\partial u_i / \partial x_j + \partial u_j / \partial x_i)_{i,j=1,2,3}\) are components of strain tensor. These equations are solved using Finite Element Method (FEM).

In the case of incompressible proppant pack, the condition for closing requires that  
\[
w > w_{p,0},
\]
(2)
where \(w\) is the fracture width and \(w_{p,0}\) is the width of proppant pillars. Later on, for the case of compressible proppant, the width \(w_{p,0}\) will be changed with respect to elastic properties of the proppant particles and the whole pack.

In order to simultaneously account for filtration through both the proppant pack and the open channel, the fracture conductivity \(\Lambda(x,y)\) is computed as  
\[
\Lambda(x,y) = \begin{cases} 
\frac{w(x,y)k(\sigma)(x,y)}{\mu}, & \text{if } \Delta w(x,y) \leq a, \\
\frac{w^3(x,y)}{12\mu}, & \text{if } \Delta w(x,y) > a,
\end{cases}
\]
(3)
where \(k(\sigma)\) is the proppant pack permeability, \(\mu\) is the effective viscosity, \(\Delta w(x,y) = w(x,y) - w_{p,0}(x,y)\) is the fracture clearance and \(a = 0.01\) mm is the clearance threshold value. The first term in Eq. (3) describes filtration through a proppant pack, while the second term defines the fluid flow in narrow open channels between the proppant pillars. The dependence of proppant pack permeability versus stress is taken in the empirical form \([4]\)  
\[
k(\sigma) = k_0 \cdot \exp(-\gamma \cdot (\sigma - \sigma_0)),
\]
(4)
where \(k_0\) is the pack permeability for stress \(\sigma_0\), and \(\gamma\) is the sensitivity coefficient.

In order to compute the production influx, the following equations are solved:  
\[
\text{div}\mathbf{q} = 0, \quad \mathbf{q} = \begin{cases} 
\frac{k_r}{\eta_r}w\nabla p, & \text{at } \Omega, \\
\Lambda(x,y)\nabla p, & \text{at } \Gamma_{HF},
\end{cases}
\]
(5)
where \(\Omega\) is the 3D domain surrounding the fracture, \(\Gamma_{HF}\) is the boundary of \(\Omega\) in which the fracture propagates, \(q, p\) are the flow and pressure and \(k_r\) is the rock permeability. Finally, the total volume of the fluid flowing through the perforation interval back to the surface is found as  
\[
Q_{in} = \int_{\Gamma_{perf}} \Lambda(x,y) \cdot \left(-\frac{\partial p}{\partial x}\right) dy,
\]
(6)
where \(\Gamma_{perf}\) is the boundary of the perforation interval.
3. Proppant pack compressibility

The elastic reaction of the proppant pack consists of the proppant embedment into fracture walls, the elastic interaction between proppant particles, and the compression of voids between the grains. These three terms are denoted as $\Delta w_{\text{embed}}, \Delta w_{\text{pp}},$ and $\Delta w_{\text{pack}}$, respectively, so the final proppant pack width is equal to

$$w_{\text{p,c}} = w_{\text{p,0}} - \Delta w_{\text{pack}} - \Delta w_{\text{embed}} - \Delta w_{\text{pp}},$$  \hspace{1cm} (7)

3.1. Embedment and particle-particle compression

Fig. 1a depicts an elastic sphere of radius $r$ in contact with an elastic half-space. Force $F$ is applied to the sphere. The elastic sphere indents the half-space with the depth $d$, and creates a contact area of radius $a$. The value of $d$ and the apparent elastic modulus $E^*$ can be expressed using the Hertzian contact theory [5]

$$d = \frac{a^2}{r} = \left(\frac{9F^2}{16rE^*}\right)^{\frac{1}{3}}, \quad \frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2},$$  \hspace{1cm} (8)

where $E_1$ and $E_2$ are the Young’s moduli of the elastic sphere and the half-space, respectively. $\nu_1$ and $\nu_2$ are the Poisson’s ratios of the elastic sphere and the half-space, respectively.

If the rock matrix is an elastic half-space, then the average force on a single elastic sphere is $F = \sigma \cdot D^2 = 4R^2 \sigma$. Substituting this relation into Eq. (8) gives the depth of indentation $\Delta w_{\text{embed}} = d$ for proppant pack embedment into the rock matrix as

$$\Delta w_{\text{embed}} = \left(\frac{9(4R^2 \sigma)^2}{16RE^{*2}}\right)^{\frac{1}{3}} = \left(\frac{3\sigma}{E^*}\right)^{\frac{1}{3}}, \quad \frac{1}{E^{*}} = \frac{1 - \nu_{pp}^2}{E_{pp}} + \frac{1 - \nu_{rock}^2}{E_{rock}},$$  \hspace{1cm} (9)

where $E_{pp}$ and $E_{rock}$ are the Young’s moduli of the proppant pack and the rock matrix, respectively. $\nu_{pp}$ and $\nu_{rock}$ are the Poisson’s ratios of the proppant pack and the rock matrix, respectively. $E^*$ represents an apparent Young’s modulus, which describes the mechanical interaction between the proppant pack and the rock matrix. In this study, we use Eq. (9) to describe the proppant embedment depth during the fracture closure process.

Fig. 1b shows schematics of a propped fracture during fracture closure. We use a layer of proppant pack with characteristic radius $R$ to represent proppant grains inside the fracture. The distance between...
the centres of two adjacent spheres is $2R$. The variation of the deformation between two extruded spheres under the action of stress $\sigma$ can be expressed as [6]

$$
\Delta w_{p+p} = \frac{3\sigma}{4E^{*}} \left( \frac{R_p}{R_1 + R_p} \right)^{1/3}, \quad \frac{1}{E^{*}} = \frac{1 - \nu_1^2}{E_{p1}} + \frac{1 - \nu_2^2}{E_{p2}},
$$

(10)

where $R$, $E$ and $\nu$ are the particle radius, Young’s modulus, and Poisson’s ratio for the first ($p1$) or second ($p2$) sphere, respectively. Since in this study all proppant particles are assumed to be identical, their size and elastic properties are equal, then $R_1 = R_2$, $E_1 = E_2$ and $\nu_1 = \nu_2$. Since there may be several proppant layers in the considered pack, it is suitable to take into account the elastic deformation between all layers except the last one, where embedding occurs. Then the proppant pack width change due to the particle-particle interaction is

$$
\Delta w_{2p} = N_l \cdot \Delta w_{p+p} = \left( \frac{w_{p0}}{R} - 1 \right) \cdot \Delta w_{p+p},
$$

(11)

where $N_l$ is the number of proppant layers.

3.2. Proppant pack compression

The compressive behavior of proppant pack due to interparticle void loss is described using Terzaghi’s one dimensional soil consolidation model [7]. The empirical relationship between the deformation of the proppant pack and the compressive stress acting on the proppant pack may be written as

$$
\Delta w_{pack} = C \cdot w_0 \cdot \ln \left( \frac{\sigma}{\sigma_0} \right),
$$

(12)

where $\sigma_0$ is the compressive stress at which the thickness of the pack is $w_0$, and $C$ is the compressibility coefficient:

$$
C = C_c \left( \frac{1}{e_0} \right),
$$

(13)

where $e_0$ is the void ratio and $C_c$ is the compression index. Coduto [8] suggested a classification of the particle assembly compressibility based on the $C$ value, and in this study compressibility coefficient is considered to be “very slightly compressible”, and therefore can be approximately taken as constant $C = 0.05$.

4. Numerical results

In this section, the effects of proppant compressibility and proppant pack permeability are investigated via numerical examples. Two problems are considered: test problem in which a gap between two proppant columns is considered, and a realistic propped fracture that occurs by alternating the pumping of clean fluid and slurry with proppant. Mechanical parameters for both considered cases are given in Table 1.

| $\sigma_0$ | $E_{rock}$ | $E_{pack}$ | $E_p$ | $\nu$ | $\mu$ | $R$ |
|------------|------------|------------|-------|-------|-------|-----|
| 500 Atm    | 9.5 GPa    | 20 GPa     | 72 GPa| 0.2   | 0.01 Pa · s| 0.279 mm |
Figure 2. Numerical results for two symmetric proppant pillars for $\sigma_0 = 500$ Atm. Panel (a): compressive stress $\sigma_z$ in the (x; y) plane; panel (b): cross-section of $\sigma_z$ along the y-axis. Panel (c): initial proppant pack width $w_{p,0}$ (solid black line); final $w_{p,c}$ (dashed black line); and impact of all three components: $w_{p,0} - \Delta w_{pack}$ (red dashed line), $w_{p,0} - \Delta w_{embed}$ (brown dashed line) and $w_{p,0} - \Delta w_{\Sigma p}$ (blue dashed line), respectively.

In the first numerical simulation, the proppant pack compression is considered for the test example of a gap between two proppant pillars adjacent to the perforation interval. The width of the incompressible proppant pack $w_{p,0}$ is 1 mm, and the dimension of each pillar along the y-axis direction is 46 mm.

Fig. 2a-2b shows the stress distribution along the pillars and y-section of the stress along the perforation interval. The value of the applied external stress is 500 Atm. It is interesting to see how stress values oscillate and reach maximum at the outer pillar’s border. This oscillating behavior becomes the reason for the non-uniform proppant compression, since all three components depend on the stress value. Fig. 2c depicts the initial proppant width $w_{p,0}$, the impact of every compression term and the total compressed width $w_{p,c}$. It can be noted that the greatest contribution was made by the term $\Delta w_{pack}$, followed by the sedimentation term $\Delta w_{embed}$, and the smallest contribution is provided by the particle elasticity $\Delta_{\Sigma p}$. The proportion may vary depending on the problem parameters, however, a full parametric study requires a more accurate model and is beyond the scope of the paper.

In the second simulation, hydraulic fracture is modeled by the coupled model [3] with a pulse pumping schedule, where alternating fluids of different rheologies/viscosities allow creating high-conductive channels due to flow instabilities. Fig. 3a shows the arc-shaped proppant distribution according to such a pumping schedule, and Fig. 3b illustrates the difference in the proppant pack width with or without compressibility. The value of applied stress is also 500 Atm, so the impact of the interparticle void loss, embedment, and particle elasticity remain approximately the same as for the previous case shown in Fig. 2c.

The total value of inflow according to the Eq. (6) for the case of a compressed and incompressible proppant pack differs by nearly 20%. This means that even in the case of a relatively small fracture with a short proppant pack and modest stress magnitude, neglecting the proppant compressibility may lead to the performance overestimation by almost 20%.
Figure 3. Numerical results for fracture closing on spatially heterogeneous proppant pack. Panel (a): proppant distribution with package width shown by colour. The white arrow indicates the cross-section for Fig. 3b. Panel (b): cross-section along the x axis or the first 20 m from the perforation interval. The cases of incompressible proppant \( w_{p,0} \) (black solid line) and compressible proppant \( w_{p,c} \) (red solid line) are displayed.

Conclusions
The described method for calculating proppant cluster compressibility may be employed to estimate the change of the production rate of hydraulically stimulated reservoirs due to the effects associated with proppant compressibility. This method provides the dependence of the propped fracture conductivity on the magnitude of compressive stresses and elastic properties of the proppant and the reservoir. Overall, the results highlight the importance of accurately modeling the change of proppant properties before and during the production stage.

Acknowledgements
The work was supported by the Russian Ministry of Education and Science (grant No.14.W03.31.0002). The author thanks the Science & Technology Centre of Gazprom Neft (St Petersburg) and the Gazprom Neft-NSU Science and Education Centre (SEC) for supporting the research.

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