Mexican Relaxation

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A new relaxion mechanism is proposed, where a small electroweak scale is preferably selected earlier than the larger one due to potential instability, different from previously proposed stopping mechanisms by either Hubble friction from increasing periodic barrier or thermal friction from gauge boson productions. A sub-Planckian field excursion of axion can be achieved without violating the bound on the e-folding number from quantum gravity consideration, and our relaxion can be identified as QCD axion, preserving the Peccei-Quinn solution to the strong CP problem as well as making up all the cold dark matter in our current Universe.

Introduction. — Recently an enlightening observation [1,2] is made for reducing the cosmological hierarchy problem into the electroweak (EW) hierarchy problem, which is naturally realized in a simple model of quintessential Starobinsky inflation [2]. The proposed model [2] is capable of achieving the currently observed magnitude of dark energy density without fine-tuning, and curing the Higgs instability problem soon after along with the relaxion mechanism.

The essential idea behind relaxion mechanism is to dynamically relax the electroweak scale via cosmological evolutions. The potential for the Higgs $h$ and axion $a$ contains three parts, $V(a, h) = V_{	ext{Higgs}} + V_{\text{scan}} + V_{\text{stop}}$, of form

$$V_{\text{Higgs}} = \frac{\lambda}{4} h^4 + \frac{1}{2}(\Lambda^2_h - \Lambda_h^2) a^2;$$

$$V_{\text{scan}} = c_1 \Lambda_h^4 \frac{a}{f} + c_2 \Lambda_h^4 \left(\frac{a}{f}\right)^2 + \cdots;$$

$$V_{\text{stop}} = \Lambda_a (\langle h \rangle)^4 \left(1 - \cos \frac{a}{f_a}\right).$$

The Higgs part $V_{\text{Higgs}}$ contains a axion-dependent mass-square triggering EW symmetry breaking (EWSB) when it scans from a large cutoff scale $\Lambda_h$ to a critical point at $a = f$ where it becomes negative. The scanning part $V_{\text{scan}}$ could generally include the higher-order terms in $a/f$ that gradually become unimportant once axion rolls down to $a < f$ [13]. The dimensionless positive parameters $c_n$ is of order unity from naturalness argument [13]. The stopping part $V_{\text{stop}}$ quickly stops axion from further rolling due to Higgs backreaction soon after it passes through $a = f$, where the growing barrier amplitude $\Lambda^4_h = \Lambda^4_c - n \langle h \rangle^n$ can be either generated from QCD dynamics $(a/32\pi^2 f)GG$ with $n = 1$ or other hidden strong dynamics with $n = 2$ [14]. $\Lambda_c$ is the strong coupling condensation scale of some gauge group with gauge field strength $G$. The current vacuum-expectation-value (VEV) $v$ of Higgs is selected from stopping condition that the slop from linear scanning term balances the slop from periodic potential at current axion value $a_0$, namely

$$\left(\frac{v}{\Lambda_h}\right)^n \sim \frac{f_a}{f} \left(\frac{\Lambda_h}{\Lambda_c}\right)^{4-n} \frac{1}{\sin(a_0/f_a)}.$$  

Therefore, the relaxion mechanism solves the EW hierarchy problem by trading the EW hierarchy $v \ll \Lambda_h$ with axion hierarchy $f_a \ll f$ [14], which is technically natural since those terms involving $f$ in the potential that explicitly break shift symmetry of axion should be highly suppressed by sufficiently large enough $f$.

Despite the elegance in the original relaxion mechanism [11], there are some unusual features for model building. For the scanning part, one generally expects super-Planckian field excursion of axion with a gigantic e-folding number during ultra low-scale inflation. These requirements come from scanning enough Higgs mass range slowly so that being independent from initial condition during slow-roll era dominated by classical evolutions other than quantum fluctuations. For the stopping part, if the relaxion is QCD axion, one only obtains an EW cutoff scale as well as an inflation scale below QCD confinement, and the final misalignment angle of QCD vacuum is of order unity, which destroys the Peccei-Quinn (PQ) solution to the strong CP problem. See [15-18], however, for alternative realizations of QCD relaxion.

Besides the dynamical stopping mechanism by Hubble friction from an increasing periodic barrier as a result of backreaction of Higgs after EWSB, there is an alternative stopping mechanism by thermal friction from gauge particle productions as a result of an axion couplings to weak gauge bosons [19, 20] (see also [21, 22]). The energy dissipation during gauge particle production is most efficient when the Higgs VEV is small, hence a small EW scale is thus selected. The primary motivation for this thermal stopping mechanism is to eliminate the need of an ultra low-scale inflation with a gigantic e-folding number over a super-Planckian field excursion. However, the...
picture of axion coupling to only weak gauge bosons but not to photon might be lost at higher-order corrections, where dissipation to photon could stop the axion from approaching small EW scale. See \[25\]-\[26\] for alternative realizations of relaxation with particle productions.

In this letter, we propose a new relaxation mechanism with a stochastic stopping mechanism different from the dynamical or thermal stopping mechanisms we mention above. The current EW scale is naturally selected due to potential instability. Furthermore, a consistent choice of parameters can be identified to render a QCD relaxation as all the cold dark matter (CDM).

**General picture.** — Our new relaxation mechanism admits a Mexican-hat alike potential \[27\] (hence the name Mexican relaxion mechanism)

\[
V(a, h) = \frac{\lambda}{4} \left( h^2 - \frac{\Lambda_h^2 - ga^2}{\lambda} \right)^2
\]

\[
= \frac{\lambda}{4} h^4 + \frac{1}{2} \left( \frac{\Lambda_h^2 - ga^2}{\lambda} \right) h^2 + \frac{\lambda}{4} \left( \frac{\Lambda_h^2 - ga^2}{\lambda} \right)^2
\]

(5)

where \(\Lambda_h\) is the cutoff scale of standard-model (SM) loops, \(g\) is a technically natural small coupling that explicitely breaks the shift symmetry of axion \(a\). We include the appearance of a periodic potential \(\Lambda_h^4 (1 - \cos(a/f_a))\) later below the confinement scale \(\Lambda_c\) with axion decay constant \(f_a\). The sketch of this Mexican-hat alike potential is presented in Fig[1] where all the local minimums are degenerated and form a hyperbola of form

\[
\frac{a^2}{\Lambda_h^2/g} + \frac{h^2}{\Lambda_h^2/\lambda} = 1.
\]

(6)

Since \(g\) is generally much smaller than \(\lambda\), this Mexican-hat alike potential is therefore highly squeezed along the Higgs axis direction and extremely stretched along the very flat relaxation axis direction.

The general picture of our Mexican relaxion mechanism is very simple to state. Since Higgs VEV is zero before EWSB, the axion can thus be released along axion axis direction from any point \(a \gtrsim 0\) (called release point) and specified as a red point in the left panel of Fig[1]. The rolling axion will scan the mass-square of Higgs from \(O(1)\Lambda_h/\sqrt{g}\) down to zero until passing through \(a = \Lambda_h/\sqrt{g}\) (called critical point) and specified as a magenta point in the left panel of Fig[1] where EW symmetry starts to be broken. It is obvious to see that, the axion axis direction is stable against Higgs fluctuations in the range \(a > \Lambda_h/\sqrt{g}\) before EWSB, while unstable against Higgs fluctuations in the range \(a < \Lambda_h/\sqrt{g}\) after EWSB. Therefore, the relaxation certainly cannot climb up the ridge further far away the critical point, until Higgs fluctuations kick in at some point (called deviation point and specified as a green point in the left panel of Fig[1]) where relaxation falls off the hillside to a point (called relaxation point and specified as a blue point in the left panel of Fig[1]) in the valley of ellipse, thus the current VEV of Higgs is accidentally selected due to the potential instability, and it is further securely locked by the periodic potential of axion below confinement scale.

There is also a hyperbolic version of our Mexican relaxion mechanism when axion is released from small field regime instead of large field regime in \([5]\), of which the potential

\[
V(a, h) = \frac{\lambda}{4} \left( h^2 - ga^2 - \Lambda_h^2 \right)^2,
\]

\[
= \frac{\lambda}{4} h^4 + \frac{1}{2} \left( \frac{\Lambda_h^2 - ga^2}{\lambda} \right) h^2 + \frac{\lambda}{4} \left( \frac{\Lambda_h^2 - ga^2}{\lambda} \right)^2
\]

(7)

is shown in the right panel of Fig[1]. All the local minimums are also degenerated and form a hyperbola of form

\[
\frac{a^2}{\Lambda_h^2/g} - \frac{h^2}{\Lambda_h^2/\lambda} = 1.
\]

(8)

The physical picture of \([7]\) is essentially the same as \([5]\), while the only difference is that the release point is now at \(a = 0\). All of following discussions on Mexican relaxion can also be carried over to its hyperbolic version, which will not be repeated below. The general picture above is lacking in specific background cosmological evolution, of which each stage will be constrained below.

**Scanning stage.** — Suppose that the critical point where EW symmetry starts to be broken is around the \(T_{EW} \sim 10^2\) GeV within the radiation dominated era so that the leading-order thermal corrections should not be important below the critical point to destroy the valley of ellipse, and the scanning stage from release point to critical point covers all the background inflationary era up to current observable scale and the reheating era as well as a part of radiation dominated era. To be independent of initial condition, the part of scanning stage during inflation with \(a \sim O(1)\Lambda_h/\sqrt{g}\) should be slow-rolling,

\[
m_a^2(h = 0) \simeq O(1) \frac{g}{\Lambda_h^4} \ll H_{inf}^2 \Rightarrow g\Lambda_h^4 \ll \lambda H_{inf}^2.
\]

(9)

Meanwhile, the classical evolution should dominate the quantum fluctuations in the first place,

\[
\left. \frac{V_a'}{3H_{inf}^2} \right|_{h=0} > \frac{H_{inf}}{2\pi} \Rightarrow \sqrt{g}\Lambda_h^3 \gtrsim \lambda H_{inf}^3,
\]

(10)

and the energy density of relaxation must not disturb the background inflation,

\[
V(a, h = 0) \ll M_P^2 H_{inf}^2 \Rightarrow \Lambda_h^4 \ll \lambda M_P^2 H_{inf}^2.
\]

(11)

and the field excursion of axion should also be large enough for sufficient scanning,

\[
\Delta a \simeq \left. \frac{V_a'}{3H_{inf}^2} \right|_{h=0} \Delta N \simeq O(1) \frac{\Lambda_h}{\sqrt{g}} \Rightarrow \Delta N \simeq \frac{\lambda H_{inf}^2}{g\Lambda_h^2}.
\]

(12)
leads to $g$ serves the e-folding bound (13). Combining (9) with (10) a sub-Planckian axion excursion (13) automatically pre-
and our cutoff scale could be as large as the GUT scale. Therefore, our Mexican relaxion mechanism could natu-
survive the selecting and securing stages, which impose further constraints in order to eliminate any fine-tuning.

Selecting stage. — When axion passes through the critical point, the scanning mass term of Higgs in (5) be-
comes negative, namely the EW symmetry starts to be broken. After that, the relaxation continues to climb up the
ridge, which is unstable against either quantum or thermal fluctuations along Higgs direction. After a small field
excursion $\delta a$ below critical point $a = \Lambda_h/\sqrt{g}$, the relaxation is accidentally kicked off the ridge by fluctuations along
Higgs direction, and eventually rolls down the hillside to a local minimum in the ellipse $g(\Lambda_h/\sqrt{g} - \delta a)^2 + \lambda(h) = \Lambda_h^2$. The obtained VEV of Higgs is therefore technically small,
\[ \langle h \rangle^2 \sim \frac{\sqrt{g} \delta a}{\lambda \Lambda_h} \ll 1, \tag{17} \]
due to a technically small $g$ or a naturally small hierarchy $\delta a \ll \Lambda_h/\sqrt{g}$, which is another way of saying that, the deviation point $\langle a \rangle = \Lambda_h/\sqrt{g} - \delta a$ where Higgs fluctuations kick in should be very close to the critical point. However, the relaxed Higgs VEV $\langle h \rangle$, or equivalently $\delta a$, cannot be predicted as an established value, because one simply cannot predict exactly when Higgs fluctuations kick in and then relaxion rolls down the hillside, whereas it most likely falls off the ridge soon after it passes through the critical point. In fact, the probability for relaxion to stay on the ridge at some time (e-folding number) $N$ is

Note that two extra constraints from quantum gravity can be imposed, namely a sub-Planckian field excursion [28] and a not-too-large e-folding number [29],
\[ \Delta a \lesssim M_{Pl}, \quad \Delta N \lesssim M_{Pl}^2/H_{inf}^2. \tag{13} \]
With help of (10) for a sub-Planckian inflationary scale,
\[ \Delta a \approx \frac{\sqrt{g} \Lambda_h^3}{\lambda H_{inf}^2} \Delta N \approx M_{Pl} \Rightarrow \Delta N \lesssim \frac{\lambda H_{inf}^2 M_{Pl}^2}{\sqrt{g} \Lambda_h^3} \lesssim \frac{M_{Pl}^2}{H_{inf}^2}, \]
a sub-Planckian axion excursion (13) automatically pre-
leads to $g^2 \ll \lambda$. Combining (10) with (11) leads to $\lambda H_{inf}^6 \ll g^2 M_{Pl}^6$. Therefore, $g$ is constrained by
\[ \sqrt{\lambda}(H_{inf}/M_{Pl})^3 \ll g \ll \sqrt{\lambda}, \tag{14} \]
Combining (10) with $g^2 \ll \lambda$ leads to $\Lambda_h^3 \gg g^{3/2} H_{inf}^3$. Therefore, $\Lambda_h$ is constrained by
\[ g^2 H_{inf} \ll \Lambda_h \approx \lambda^{1/4} M_{Pl}^{1/2} H_{inf}^{1/2}, \tag{15} \]
It turns out as a nice surprise that, we could meet all the constraints above by taking,
\[ g \approx 10^{-4}, \quad \Lambda_h \approx 10^{15} \text{ GeV}, \quad H_{inf} \approx 10^{14} \text{ GeV}, \quad \Delta N \approx O(10), \Delta a \approx 10^{17} \text{ GeV}. \tag{16} \]
Therefore, our Mexican relaxion mechanism could naturally avoid the super-Planckian field excursion during a ultra low-scale inflation with gigantic e-folding number that usually plague the original relaxion mechanism [11], and our cutoff scale could be as large as the GUT scale.
$P(N,h = 0)$, then the probability to find Higgs still on the ridge after some time (e-folding number) $\delta N$ can be solved from the Fokker-Planck equation

$$\frac{\partial P}{\partial N} = \frac{\partial}{\partial h} \left( \frac{V'}{3H^2} P + D^2 \frac{\partial P}{\partial h} \right),$$

(18)

If Higgs could classically slow-rolls down the hillside over the fluctuation $D$ term after an accidental kick (and eventually obtain a VEV ($h$)), then a crude estimation reads

$$\frac{P(N + \delta N, h = 0)}{P(N, h = 0)} \approx 1 + \frac{m_h^2}{3H^2} \approx e^{\frac{\lambda v^2}{m_h^2} \delta N}. \quad \text{(19)}$$

Therefore, the probability to stay on the ridge is exponentially suppressed, because the would-be Higgs VEV becomes larger and larger when climbing up the ridge, hence a technically and accidentally small Higgs VEV is expected. However, it actually becomes more and more difficult to slow-roll down the hillside, thus the slow-roll approximation made in (18) only serves as a conservative estimation, and we expect the qualitative conclusion unchanged for more rigorous analysis [30] [31].

For our naive choice [16], one expects $\delta a \sim 10^{-10}$ GeV for an EW-scale Higgs VEV $\langle h \rangle \sim 10^2$ GeV. Such small $\delta a$ is actually fine-tuning, because that, if the relaxation climbs up the ridge further by even a small amount of $\delta a \sim 10^{-9}$ GeV, then the corresponding deviation in Higgs VEV would be as large as $\delta h = (g\langle a \rangle/\lambda \langle h \rangle)\delta a \sim 10^8$ GeV, resulting in an even much smaller probability to stay there. In this view, it is reasonable to expect such seemingly fine-tuned $\delta a$. However, thermal fluctuations of order $T_{EW} \sim 10^2$ GeV at onset of EWSB could easily bump the relaxation up along the ridge so that shift the relaxed Higgs VEV by amount of $\delta h = (g\langle a \rangle/\lambda \langle h \rangle)\delta a \sim 10^6$ GeV. To avoid this problem, one necessarily encounters a suspicious coincidence problem that why the relaxation rolls down the hillside even before thermal fluctuations are ever developed along the ridge. Fortunately, one can easily acquire a larger $\delta a$ at the price of allowing for a larger e-folding number of inflation at lower scale, which can be seen from combining (12) with (17), namely

$$\frac{\lambda H_{\text{inf}}^2}{\Delta N \Lambda_h^2} \leq g \leq \frac{\lambda^2 v^4}{\delta a^2 \Lambda_h^2} \Rightarrow \delta a^2 \leq \frac{\lambda v^4 \Lambda_h^2}{H_{\text{inf}}^2} \Delta N \ll \frac{\lambda v^4 M_{\text{pl}}^2}{H_{\text{inf}}^2}. \quad \text{(20)}$$

Hence an EW-scale $\delta a \sim 10^2$ GeV requires an inflationary scale below $10^{10}$ GeV. With $\langle h \rangle \sim 10^3$ GeV, one has

$$g \lesssim 10^{-16}, \Lambda_h \approx 10^9 \text{GeV}, H_{\text{inf}} \approx 10^6 \text{GeV},$$

$$\Delta N \approx 10^9, \Delta a \approx 10^{17} \text{GeV}, \delta a \sim 10^2 \text{GeV}, \quad \text{(20)}$$

that meets all the constraints we have discussed so far as well as evading the fine-tuning and coincidence problems of $\delta a$, because thermal fluctuations of order $T_{EW} \sim 10^6$ GeV now lead to a shift for Higgs VEV of order $\delta h = (g\langle a \rangle/\lambda \langle h \rangle)\delta a \sim 10^2$ GeV, which is sufficient to solve the EW hierarchy problem with most conservative precision.

**Securing stage.** — After relaxation climbs up the ridge until deviation point and then rolls down the hillside, the Higgs is found itself in a local minimum ($\langle a \rangle, \langle h \rangle$) along both directions from the ellipse equation [6].

$$\frac{\partial V}{\partial a} = g\langle a \rangle^2 + \frac{g}{\lambda} (g\langle a \rangle^2 - \Lambda_h^2)\langle a \rangle = 0; \quad \text{(21)}$$

$$\frac{\partial V}{\partial h} = \lambda \langle h \rangle^3 + (g\langle a \rangle^2 - \Lambda_h^2) \langle h \rangle = 0, \quad \text{(22)}$$

where the evaluation of partial derivative at ($\langle a \rangle, \langle h \rangle$) is understood. Since different regions of our Universe could roll down the hillside at different deviation points with width of thermal fluctuations $T_{EW} \sim 10^2$ GeV, the achieved Higgs VEVs also admits some distribution with width $\delta h \sim 10^2$ GeV under (20), leading to an inhomogeneous distribution of Higgs VEVs in our Universe. Furthermore, before the non-perturbative effect at strong confinement scale $\Lambda_c$ takes place (if it is lower than EW scale), the drift of VEV due to fluctuations along flat direction in the valley of ellipse is of order $\sqrt{\delta a^2 + \delta h^2} \approx \delta a \sim 10^2$ GeV, which, under (20), results in a shift of Higgs VEV of order $\delta h \sim 10^2$ GeV that also leads to an inhomogeneous distribution of Higgs VEVs in our Universe. This can be solved after strong confinement with the presence of the periodic potential, $V_{\text{tot}} = V + \Lambda_h^4 (1 - \cos (a/f_a))$, where the inhomogeneous distribution of Higgs VEVs rolls down to the new minimum set by

$$\frac{\partial V_{\text{tot}}}{\partial a} = \frac{\partial V}{\partial a} + \frac{\Lambda_h^4 \langle h \rangle}{f_a} \sin \langle a \rangle = 0; \quad \text{(23)}$$

$$\frac{\partial V_{\text{tot}}}{\partial h} = \frac{\partial V}{\partial h} + \frac{\partial \Lambda_h^4}{\partial h} \left( 1 - \cos \langle a \rangle \right) = 0, \quad \text{(24)}$$

namely a set of discrete points in the original ellipse with interval $\delta a = 2n\pi f_a$. The final misalignment angle would be exactly zero at these discrete vacuums, which preserves the PQ solution [32] [33] to the strong CP problem if our relaxation is chose as the QCD axion. It worth noting that, the exactly vanishing energy density of axion potential at these discrete vacuums also evades the argument from de Sitter quantum breaking bound [34] [35] (see also [37] [38] for recent arguments on the existence of axion). Nevertheless, the way out of swampland [39] at these discrete vacuums should be discussed separately.

However, for QCD axion with decay constant $f_a \gtrsim 10^9$ GeV bounded from below by supernova cooling observations [40] [41], the improved choice [40] would cause another fine-tuning problem that, unless a minimum of periodic potential is coincided with the relaxed minimum during selecting stage, namely a fine-tuning relation $\Lambda_h^2 - g(2n\pi f_a)^2 \approx \lambda v^2$ is conspired, the obtained relaxation during selecting stage would generally roll down the periodic potential to the new minimum with axion shift of order $\delta a \sim f_a$. Since the Higgs VEVs...
of two adjacent minimums would differ by an amount of \( \delta h = (g(a)/\lambda(h))f_a \simeq 10^6 \text{ GeV} \), this would necessarily destroy the desired solution obtained from selecting stage. Fortunately, there exists a parameter space for achieving \( \delta a \) as large as the decay constant \( f_a \) of QCD axion that also meets all the constraints we discussed so far as well as a desirable Higgs VEV \((h) \sim 10^2 \text{ GeV}\).

\[
\Lambda_h \simeq \frac{\lambda(h)\delta h}{\sqrt{g}} \simeq 10^6 \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1} \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{-1};
\]
\[
H_{\text{inf}} \simeq \frac{g}{\lambda^2} \Lambda_h \simeq 10^2 \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1} \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{-1};
\]
\[
\Delta a \simeq \frac{\Lambda_h}{\sqrt{g}} \simeq 10^{18} \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1} \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{-1};
\]
\[
\Delta N \simeq \frac{\lambda H_{\text{inf}}^2}{g \Lambda_h^2} \simeq 10^{15} \left(\frac{g}{10^{-24}}\right)^{-2};
\]

(25)

Now that the width of deviation point is much smaller than the decay constant, all the relaxed Higgs VEVs will be distributed within the same period of the periodic potential closest to the critical point, and eventually roll down to the same minimum below confinement scale, thus there is no inhomogeneous problem of Higgs VEVs. The current Higgs VEV can be achieved with a precision better than \( \delta h = (g(a)/\lambda(h))f_a \simeq 10^2 \text{ GeV} \) under the typical values of (25). Our solution to EW hierarchy problem is thus accomplished here.

Furthermore, the configuration (25) render an inflation scale lower than the decay constant, therefore, PQ symmetry is already broken during inflation, hence there is also no inhomogeneous problem for the QCD axion phase \( \theta_{\text{PQ}} = a/f_a \), since the inflation stretches out each path of \( \theta_{\text{PQ}} \) and dilutes corresponding topological defects (if any), so that our observable Universe ends up with a single uniform value of \( \theta_{\text{PQ}} \) everywhere. It turns out as a nice surprise that, the parameter space of (25) also embraces an appealing choice of \( f_a \simeq 10^{11} \text{ GeV} \) with

\[
\Lambda_h \simeq 10^4 \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1/2};
\]
\[
H_{\text{inf}} \simeq 1 \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1/3};
\]
\[
\Delta a \simeq 10^6 \text{ GeV} \left(\frac{g}{10^{-24}}\right)^{-1};
\]
\[
\Delta N \simeq 10^5 \left(\frac{g}{10^{-24}}\right)^{-2/3};
\]

(26)

that our QCD relaxion could also make up all CDM without fine-tuning the initial misalignment angle and without violating current upper bound on the fraction of isocurvature perturbation among adiabatic perturbation. In this case, our QCD relaxion mass is \( 2g^2a^2/\lambda \simeq 2gA_h^2/\lambda \simeq 10^{-15} \text{ GeV} \) before confinement and \( 10^{-14} \text{ GeV} \) after confinement \([44, 45]\), and we also expect no gravitational waves from inflation at observable level. Note that both e-folding numbers in (25) and (26) satisfy (13), which can be further reduced by considering realistic inflationary background with Hubble flow parameter \( \epsilon_H = -\dot{H}/H^2 \simeq O(10^{-2}) \), namely

\[
\Delta N \simeq \log \left(1 + 3\epsilon_H \frac{\lambda H_{\text{inf}}^2}{g \Lambda_h^2}\right)^{1/\epsilon_H} \simeq O(10^3). \tag{27}
\]

Conclusions. — A new relaxion mechanism is proposed with most economic manner that selects our current EW scale with a stochastic stopping mechanism, different from previous proposed either dynamical or thermal stopping mechanisms. Not only a small EW scale is naturally obtained, but also a comparable precision is achieved, therefore, no fine-tuning is needed for our new relaxion mechanism. Without violating the bounds from quantum gravity on the field excursion and e-folding number, a consistent choice of parameters can be identified for our QCD relaxion that not only preserves the PQ solution to the strong CP problem but also makes up all the CDM in our current Universe.

Discussions. — Recently it has been concretely shown that two major hierarchy problems in the SM of particle physics and modern cosmology are intimately related by the cosmological relaxation of EW hierarchy \([11]\) and the EW relaxation of cosmological hierarchy \([2]\). The early-time acceleration helps us to relax the EW scale, and the relaxed EW scale helps us to relax the late-time acceleration scale (namely the cosmological constant though it is actually dynamical according to swampland conjectures \([3–5, 46]\)). The primary reason to choose relaxion mechanism as the solution to EW hierarchy problem is the use of the axion-like particles (ALPs) that could also offer solutions to various other problems beyond the EW hierarchy problem. For example, ALPs not only provide the PQ solution \([32, 33]\) to the strong CP problem by QCD axion, but also play a role in explaining the baryon asymmetry of our Universe (BAU) by baryogenesis \([47, 51]\) as well as a promising candidate for CDM \([52, 53]\). However, it is difficult to build the relaxion mechanism into \([1]\). The initial motivation of this letter is to construct a relaxion mechanism that could be embedded into \([2]\) so that both EW and cosmological hierarchy problems could be solved. However, a naive embedding of our Mexican relaxion into \([2]\) is failed, because two fine-tuning problems during selecting and securing stages could not be circumnavigated unless lowering down the inflation scale, thus calls for further investigation on a successful embedding of our Mexican relaxion into \([2]\). Nevertheless, our EW relaxation mechanism \([2]\) could also work with other solutions to the EW hierarchy problem regardless the use of our Mexican relaxion mechanism, for example, \(N\)aturalness \([12]\). It is also interesting to explore whether BAU can be realized in our Mexican relaxion mechanism.
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