Probing a D6 + D0 state with D6-branes:
SYM—supergravity correspondence at subleading level

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Abstract. We probe a non-supersymmetric D0 + D6 state with D6-branes and find agreement at subleading order between the supergravity and super-Yang–Mills description of the long-distance, low-velocity interaction.

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1. Introduction

During the last year and a half there has been an intensive study of the correspondence between supergravity and super-Yang–Mills (SYM) descriptions of the long-distance interactions between D-branes and their bound states [1–6]. Most of this work considered only the leading $F^4$ terms in the 1-loop SYM effective action, for which there is a known expression for a general gauge field background $F$. For the specific case of D0-brane–D0-brane interaction, the subleading terms were computed directly in SYM theory, and once again agreement was found with the supergravity calculations [7, 8].

This led to the conjecture [5, 9] (see also [10, 11]) that the leading part of the $L$-loop term in the SYM effective action in $D = p + 1$ dimensions has a universal $F^{2L+2}/M^{(7-p)L}$ structure, and that this term, when computed for a SYM background representing a configuration of interacting branes and in the large-$N$ limit, should reproduce the $1/r^{(7-p)L}$ term in the corresponding long-distance supergravity potential. The authors of [9] then proposed a specific form for the $F^6$ terms in the 2-loop effective action, and proceeded to show that it reproduces the subleading supergravity potentials for a variety of configurations. However, only configurations that still preserved some (1/2, 1/4 or 1/8) supersymmetry were considered. The proposed $F^6$ term has not yet been tested for a non-supersymmetric configuration.

The existence of extreme but non-supersymmetric black holes was demonstrated in [12], where a supergravity background was found that after compactification to 10 dimensions describes a dyonic non-supersymmetric black hole. A different compactification scheme was used in [13] to obtain an extremal non-supersymmetric solution to low-energy-type IIA string theory carrying D0- and D6-brane charges. This solution, for non-zero values of both charges, breaks all supersymmetries. The scattering of D0- and D6-branes from these states was studied in [14, 15], and considering a more general supergravity background in

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‡ This is by no means an exhaustive list. For more complete references see, e.g., [6].
[16] (applying the SYM results of [5, 17]). Once again agreement was found between the leading terms in supergravity and the SYM expressions at 1-loop†.

Our aim in this paper is to extend the supergravity calculation [16] of the source–probe potential to subleading terms in $1/r$, and compare these with those obtained using the ansatz of [9] for the 2-loop SYM effective action.

2. Supergravity description

In this section, we use the same normalization as in [16], $\alpha' = 1$. In the supergravity calculation, one considers a probe moving in the background created by a source. Both the probe and source can be clusters of branes or of bound states of branes. The relevant part of the action of a $D_p$-brane (containing no lower-dimensional branes) in a supergravity background is

$$S = T_p \int d^{p+1}x \left[ e^{-\phi} \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu})} + \frac{1}{(p + 1)!} \varepsilon^{\mu_1\mu_2...\mu_{p+1}} C_{\mu_1\mu_2...\mu_{p+1}} \right], \quad (1)$$

where $G_{\mu\nu}$, $B_{\mu\nu}$ and $C_{p+1}$ are the pullbacks to the worldvolume of the background 10D metric, 2-form and R–R $(p + 1)$-form (for ‘electric’ branes) or the Hodge dual of the R–R $(7 - p)$-form (for ‘magnetic’ branes), respectively, the brane tension is [20]

$$T_p = n_p g_s^{-1} (2\pi)^{(1-p)/2} T^{(p+1)/2}, \quad (2)$$

with $g_s$ the string coupling constant and the string tension is $T = (2\pi\alpha')^{-1}$.

2.1. The supergravity background

In [16], to which we refer the reader for further details, a solution to the low-energy effective action of type IIA string theory,

$$S = \frac{1}{(2\pi)^7 g_s^2} \int d^{10}x \sqrt{-g_{10}} \left[ e^{-2\phi} \left( R_{10} + 4(\nabla \phi)^2 \right) - \frac{1}{2} F_{\mu\nu}^2 \right], \quad (3)$$

was found describing a system with D0- and D6-branes. In this action we have, besides the 10D metric, the dilaton field $\phi$ and a 2-form field $F = dA$.

The spherically symmetric, time-independent solutions are parametrized by the mass ($M$), electric charge ($Q$) and magnetic charge ($P$). The dilaton charge ($\Sigma$) is related to these by

$$\frac{8}{3} \Sigma = \frac{Q^2}{\Sigma + \sqrt{3}M} + \frac{P^2}{\Sigma - \sqrt{3}M}. \quad (4)$$

The explicit form of the fields is

$$e^{4\phi/3} = \frac{B}{A}, \quad (5)$$

$$A_\mu \ dx^\mu = \frac{Q}{B} (r - \Sigma) \ dt + P \cos \theta \ d\phi, \quad (6)$$

$$g_{\mu\nu} \ dx^\mu \ dx^\nu = - \frac{F}{\sqrt{AB}} \ dr^2 + \frac{B}{A} (dx_1^2 + \cdots + dx_6^2) + \sqrt{\frac{AB}{F}} \ dr^2$$

$$+ \sqrt{AB} (d\theta^2 + \sin^2 \theta \ d\phi^2). \quad (7)$$

† A very similar supergravity background was found in [18], describing a four-dimensional black hole carrying D0- and D6-brane charges. The black hole was probed with D0-branes both in the supergravity and M(atrix) theory formalism, with the by now expected agreement at 1-loop.
where
\[ F = (r - r_+)(r - r_-), \]
\[ A = (r - r_{A+})(r - r_{A-}), \]
\[ B = (r - r_{B+})(r - r_{B-}), \]  
(8)

and
\[ r_\pm = M \pm \sqrt{M^2 + \Sigma^2 - \frac{1}{3} P^2 - \frac{1}{4} Q^2}, \]
\[ r_{A\pm} = \frac{\Sigma}{\sqrt{3}} \pm \sqrt{\frac{P^2 \Sigma / 2}{\Sigma - \sqrt{3} M}}, \]
\[ r_{B\pm} = -\frac{\Sigma}{\sqrt{3}} \pm \sqrt{\frac{Q^2 \Sigma / 2}{\Sigma + \sqrt{3} M}}. \]  
(9)

The extremality condition, \( r_+ = r_- \), is equivalent to
\[ M^2 + \Sigma^2 - \frac{1}{3} P^2 - \frac{1}{4} Q^2 = 0. \]  
(10)

When imposing this condition it turns out to be convenient to perform the coordinate change \( r \rightarrow r' = r - M \). The extremal solution is then written as (we drop the prime on \( r \))
\[ ds^2 = -f_1(r) dt^2 + f_2(r) dx_i dx_i + f_1^{-1}(r)(dr^2 + r^2 d\Omega^2), \]  
(11)

where
\[ f_1(r) = \frac{r^2}{\sqrt{AB}}, \quad f_2(r) = \sqrt{\frac{B}{A}}. \]  
(12)

It is easy to see that the pure magnetic solution
\[ P = 4M, \quad Q = 0, \quad \Sigma = -\sqrt{3} M, \]  
(13)
solves the extremality condition (10). Plugging these values in (9), (8) we see that (5)–(7) now describe a D6-brane.

At the same time, the pure electric solution
\[ P = 0, \quad Q = 4M, \quad \Sigma = \sqrt{3} M, \]  
(14)
also solves (10) and describes D0-branes smeared over a 6-torus.

We expect that solutions with both electric and magnetic charge interpolate between these and so describe a system with both D0- and D6-branes.

It turns out ([16] and references therein) that the mass, electric and magnetic charge are related to the number of branes as
\[ M = \frac{g_s N_6}{8}, \]
\[ P = \frac{g_s N_6}{2}, \]
\[ Q = \frac{g_s N_0 (2\pi)^2}{2 V_6}. \]  
(15)

We can now describe a system with a large number of D6-branes and a relatively small number of D0-branes, i.e. \( P \gg Q \). To do so we consider some fixed \( M \), and move away from the pure magnetic solution, by taking
\[ \Sigma = M(-\sqrt{3} + \epsilon), \quad \epsilon \ll 1. \]  
(16)
Note that by doing this we are moving away from a $\frac{1}{2}$ supersymmetric solution, a D6-brane, towards one that breaks all supersymmetries.

The dilaton charge equation (4) and the extremality condition (10) allow us to determine the electric and magnetic charge of this system as

\[ P = \frac{1}{\sqrt{2}} \sqrt{72 M^2 - 36 \sqrt{3} \epsilon M^2 + 18 \epsilon^2 M^2 - \sqrt{3} \epsilon^3 M^2}, \]

\[ Q = \frac{\sqrt{2} \epsilon^{3/2} M}{3^{1/2}}. \]

or, to leading order in $\epsilon$,

\[ P = 4 M - \sqrt{3} M \epsilon + \frac{1}{8} M \epsilon^2 + O(\epsilon^3), \]

\[ Q = \frac{\sqrt{2} \epsilon^{3/2} M}{3^{1/2}}. \]

For these values of the parameters our solution is an extremal (in the sense of (10)), near-supersymmetric one, with the parameter $\epsilon$ measuring deviation from supersymmetry.

2.2. Supergravity calculation

We now consider a D6-brane probe moving in this background. We take the D6 probe to be parallel to the D6 source, and assume the static gauge, i.e. that the worldvolume coordinates of the D6 probe are the same as those of the source, and that the transverse coordinates do not depend on the spatial worldvolume coordinates.

With these assumptions, the long-distance, low-velocity action of the D6 probe can be obtained from (1) by plugging in the background (5), (6), (11) for the values of the parameters (19), (20) and expanding in $1/r$ and $v$, with $r$ the transverse distance between branes and $v$ the transverse velocity.

We obtain

\[ S = \frac{n_6}{g_s (2\pi)^6} V_6 \int dt \left[ 1 + \frac{\epsilon^2 M}{8 r} - \left( \frac{1}{2} + \frac{3 \epsilon^2 M^2}{4 r^2} + \frac{\sqrt{3} \epsilon M}{2 r} \right) v^2 \\
- \left( \frac{1}{8} + \frac{\sqrt{3} \epsilon M^2}{r^2} - \frac{\epsilon^2 M^2}{16 r^2} + \frac{M}{2 r} + \frac{\sqrt{3} \epsilon M}{8 r} \right) v^4 \\
- \left( \frac{1}{16} + \frac{M^2}{r^2} + \frac{\sqrt{3} \epsilon M^2}{r^2} - \frac{5 \epsilon^2 M^2}{32 r^2} + \frac{M}{2 r} + \frac{\sqrt{3} \epsilon M}{16 r} \right) v^6 \right], \]

where $n_6$ is the number of D6 branes in the probe.

Note that in the limit $\epsilon \to 0$ the static term of the potential vanishes, and the corrections to the energy start only at $v^4$. This is what we should expect [21], since in this limit our background reduces just to a collection of overlapping D6-branes, and so the full system is just D6-branes parallel to D6-branes, which is a BPS configuration.

† We have dropped here a term linear in $v$, since we will not compare it with the SYM results. A term of this type was recently discussed in [19] for the potential between a D0- and a D6-brane in relative motion.
3. SYM description

In this section, we use the normalization of [9], \( T = 1 \).

The low-energy dynamics of \( N \) D-branes is described by the dimensional reduction to \( p + 1 \) dimensions of \( \mathcal{N} = 1 \) SYM in 10 dimensions with \( U(N) \) gauge symmetry (for a recent review and references see [22]).

In [9, 23] it was argued that the sum of leading large-\( N \) IR contributions to the effective action can be written as

\[
\Gamma = \sum_{L=1}^{\infty} \Gamma^{(L)} = \frac{1}{2} \sum_{L=1}^{\infty} \int \frac{a_p}{M^{7-p}} \left( \frac{g_{\text{SYM}}^2}{L^{-1}} \right)^L \hat{C}_{2L+2}(F),
\]

where \( F \) is a background field, the coefficients \( a_p \) are given by (\( T = 1 \))

\[
a_p = 2^{2-p}(p+1)^{2-p}(p+2)\left( \frac{7-p}{2} \right),
\]

and the coefficients \( \hat{C}_{2L+2} \) are polynomials of \( F \). The only term which was explicitly computed is the 1-loop one, with the result

\[
\hat{C}_4 = S\text{Tr} C_4,
\]

where \( S\text{Tr} \) is the symmetrized trace in the adjoint representation, and \( C_4 \) is as given below.

For the corresponding 2-loop term the authors of [9] have proposed the ansatz

\[
\hat{C}_6 = \hat{S}\text{Tr} C_6,
\]

where \( \hat{S}\text{Tr} \) is a modified symmetrized trace,

\[
\hat{S}\text{Tr}(X_{i_1}...X_{i_k}) = 2N \text{Tr}(X_{i_1}...X_{i_k}) + 60 \text{Tr}(X_{i_1} X_{i_2}) \text{Tr}(X_{i_3} X_{i_4})
\]

\[
-50 \text{Tr}(X_{i_1}...X_{i_3}) \text{Tr}(X_{i_4} X_{i_5}) - 30N^{-1} \text{Tr}(X_{i_1} X_{i_2}) \text{Tr}(X_{i_3} X_{i_4}).
\]

The coefficients \( C_4 \) and \( C_6 \) are the same as the polynomials appearing in the expansion of the Abelian Born–Infeld action (\( T = 1 \)),

\[
\sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})} = \sum_{n=0}^{\infty} C_{2n}(F),
\]

with

\[
C_0 = 1, \quad C_2 = -\frac{1}{4} F^2, \quad C_4 = -\frac{1}{8} [F^4 - \frac{1}{6}(F^2)^2],
\]

\[
C_6 = -\frac{1}{12} F^6 - \frac{3}{8} F^4 F^2 + \frac{1}{32} (F^2)^3, \ldots
\]

where \( F^k \) is the trace of the matrix product over Lorentz indices,

\[
F^2 = F_{\mu\nu} F^{\nu\mu}, \ldots, F^{2k} = F_{\mu_1\mu_2} F^{\mu_2;\mu_3} \ldots F_{\mu_{2k-1}\mu_{2k}} F^{\mu_{2k;\mu_1}}.
\]

3.1. The SYM background

A SYM background describing a system with D0- and D6-branes but no D2- nor D4-branes was found in [24].

Let\( ^\dagger \)

\[
F_0^S = F_0 J_1^S, \quad F_1^S = F_0 J_2^S, \quad F_0^S = F_0 J_3^S.
\]

\( ^\dagger \) We use the superscript ‘S’ for ‘source’, i.e. the background describing just the D0 + D6 system, in order to avoid confusion with the full background (33) and (34).
where $F_0$ is an arbitrary constant, the $J_i^S$'s are $N_6 \times N_6$ block-diagonal matrices built out of $\frac{1}{4}N_6$ copies of $\mu_i$, $N_6$ a multiple of four,

$$J_i^S = \text{diag}(\mu_i, \ldots, \mu_i), \quad i = 1, 2, 3, \quad (29)$$

and $\mu_i$ are the $su(4)$ Cartan subalgebra matrices

$$
\begin{align*}
\mu_1 &= \text{diag}(1, 1, -1, -1), \\
\mu_2 &= \text{diag}(1, -1, -1, 1), \\
\mu_3 &= \text{diag}(1, -1, 1, -1).
\end{align*}
\quad (30)
$$

They have the properties [24]

$$
\begin{align*}
\text{tr}(\mu_i^2) &= 4, \\
\text{tr}(\mu_i) &= 0, \\
\mu_i\mu_j &= |\epsilon_{ijk}|\mu_k, \quad i \neq j \quad \Rightarrow \quad \text{tr}(\mu_i\mu_j) = 0, \quad i \neq j.
\end{align*}
\quad (31)
$$

Using these we easily evaluate the D0-, D2- and D4-brane charges induced by the flux of the background in the worldvolume,

$$
\begin{align*}
N_4 &\propto \int d^2x \text{tr}(FS) = 0, \\
N_2 &\propto \int d^4x \text{tr}(FS \wedge FS) = 0, \\
N_0 &= \frac{T^3}{6(2\pi)^3} \int d^6x \text{tr}(FS \wedge FS \wedge FS) \\
&= \frac{F_3^3 N_6 V_6 T^3}{(2\pi)^3},
\end{align*}
\quad (32)
$$

so that, as required, there are no D2- or D4-brane charges, but only D0- and D6-brane ones. It can easily be seen [24] that, for $F_0 \neq 0$, this is a non-supersymmetric state. This state is also not a bound state, since its energy is larger than that of the separated constituents. It is expected nevertheless that it should represent a metastable state.

We will probe this system with D6-branes. We consider a D6-brane probe parallel to the D6 source, and for simplicity consider them aligned with the coordinate axis (0)123456. The probe will have transverse velocity $v$, and we will take its direction to be along the 9-axis.

The SYM background describing the full system (source + probe) is (see, e.g., [9]),

$$
F_{12} = F_0 J_1, \quad F_{34} = F_0 J_2, \quad F_{56} = F_0 J_3, \quad F_{09} = v J_0, \quad (33)
$$

where the $J$'s are block-diagonal matrices,

$$
\begin{align*}
J_i &= \text{diag}(0_{n_6 \times n_6}, (J_i^S)_{N_6 \times N_6}), \quad i = 1, 2, 3, \\
J_0 &= \frac{1}{N} \text{diag}(N_6 I_{n_6 \times n_6}, -n_6 I_{N_6 \times N_6}), \quad N \equiv n_6 + N_6.
\end{align*}
\quad (34, 35)
$$

3.2. SYM calculation

Note that the $F$ matrices in (33) are in the fundamental representation of $su(N)$, and the trace in (23), (24) is in the adjoint representation. We therefore need to know how to relate traces in the two representations. To that effect, we collect some useful formulae in the appendix.
We start by evaluating the $F^4$ term. Plugging the SYM background (34), (35) into (27) we obtain

\[ C_4 = -\frac{1}{8} \left[ F_0^4 (J_1^4 + J_2^4 + J_3^4) - 2F_0^4 (J_1^2 J_2^2 + J_1^2 J_3^2 + J_2^2 J_3^2) \
+ 2F_0^2 (J_1^2 + J_2^2 + J_3^2) J_0^2 v^2 + J_0^4 v^4 \right], \tag{36} \]

from which, using the trace results of the appendix,

\[ \hat{C}_4 = \frac{1}{2} n_6 N_6 (3F_0^4 - 6F_0^2 v^2 - v^4). \tag{37} \]

From (22) we read the expression for the 1-loop effective action,

\[ \Gamma^{(1)} = \frac{a_6}{2M} \int d^7x \hat{C}_4 (F) \]
\[ = \frac{n_6 N_6}{16 (2\pi)^3} V_6 \int dt (3F_0^4 - 6F_0^2 v^2 - v^4). \tag{38} \]

Repeating the procedure for the $F^6$ term, we obtain

\[ C_6 = \frac{1}{16} F_0^6 (J_1^6 + J_2^6 + J_3^6 - J_1^2 J_2^2 - J_1^2 J_3^2 - J_2^2 J_3^2 - J_1^4 J_2^2 - J_1^4 J_3^2 - J_2^4 J_3^2 + 2J_1^2 J_2^2 J_3^2) \
+ \frac{1}{16} F_0^4 [J_0^4 (J_1^4 + J_2^4 + J_3^4) - 2J_0^2 (J_1^2 J_2^2 + J_1^2 J_3^2 + J_2^2 J_3^2)] v^2 \
- \frac{1}{16} F_0^2 [J_0^2 (J_1^2 + J_2^2 + J_3^2)] v^4 - \frac{1}{16} J_0^4 v^6. \tag{39} \]

A straightforward but lengthy calculation of the several traces in (24) results in

\[ \hat{C}_6 = -F_0^6 N_6 (n_6^2 + 4n_6 N_6 + 32N_6^2) - F_0^4 3n_6 N_6 (n_6 + 3N_6) v^2 \
- F_0^2 3n_6 N_6 (n_6 + 2N_6) v^4 - n_6 N_6 v^6. \tag{40} \]

Reading from (22) the 2-loop effective action, we have

\[ \Gamma^{(2)} = \frac{1}{2} \left( \frac{a_6}{M} \right)^2 N g_{YM}^2 \int d^7x \hat{C}_6 (F) \]
\[ = \frac{g_s}{2^9 (2\pi)^{7/2} r^2} V_6 \int dt \left[ -F_0^6 N_6 (n_6^2 + 4n_6 N_6 + 32N_6^2) - 3F_0^4 n_6 N_6 (n_6 + 3N_6) v^2 \
- 3F_0^2 n_6 N_6 (n_6 + 2N_6) v^4 - N_6 N_6 v^6 \right]. \tag{41} \]

4. Comparison

We proceed to compare the supergravity and SYM results. In the two pictures there is a parameter that measures deviation from a supersymmetric state, $\epsilon$ in supergravity and $F_0$ in SYM. We expect them to be related.

In fact, from (32) we have

\[ N_6 = \frac{F_0^3 N_6 V_6}{(2\pi)^6}. \tag{42} \]

Then (15) imply

\[ F_0^3 = \frac{Q}{P}, \tag{43} \]

or, after replacing (19) and (20), to leading order in $\epsilon$.

\[ F_0^3 = \frac{\epsilon}{2\sqrt{3}} + \frac{\epsilon^2}{12} + O(\epsilon^3). \tag{44} \]
So far we have used different normalizations in the supergravity and SYM calculations. In order to compare those results, we will from now on express the SYM results in $\alpha' = 1$ normalization.

At 1-loop, restoring $T$ in (38) and replacing the results (15) and (44), we have, to leading order in $\epsilon$ for each order in $v$ and $1/r$,

$$
\Gamma^{(1)} = \frac{n_6}{g_s(2\pi)^6} V_6 \int dr \left( \frac{\epsilon^2 M}{8r} - \frac{\epsilon \sqrt{3} M}{2r} v^2 - \frac{M^2}{2r} v^4 \right).
$$

Comparing with the supergravity result (21) we see that the SYM result reproduces exactly the $1/r$ and $v^2/r$ terms, a result already obtained in [16].

At 2-loop level, repeating this procedure and considering the ‘source much heavier than probe’ limit, $N_6 \gg n_6$, we obtain, to leading order in $\epsilon$ in each order in $v$ and $1/r$,

$$
\Gamma^{(2)} = \frac{n_6}{g_s(2\pi)^6} V_6 \int dr \left( -\frac{3\epsilon^2 M^2}{4r^2} v^2 - \frac{3\epsilon M^2}{r^2} v^4 - \frac{M^2}{r^2} v^6 \right).
$$

Comparing with (21), we see that the SYM result reproduces the $v^2/r^2$, $v^4/r^2$ and $v^6/r^2$ terms to leading order in $\epsilon$.

5. Conclusion

We studied the long-distance, low-velocity interaction potential between a D6-brane probe and a non-supersymmetric source containing D0- and D6-branes. We extended the supergravity calculation of [16] to subleading order in $1/r$, and compared the resulting potential with the one obtained using the ansatz of [9] for the 2-loop SYM effective action. We found agreement at subleading order, thus providing a further non-trivial check for this ansatz.

It would be interesting to have a direct SYM calculation of the 2-loop terms, and to see how they compare with those obtained here.

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Appendix. Tr versus tr

This section is based in [4, 9] and references therein.

Let $\text{Tr}$ denote the trace in the adjoint representation, and $\text{tr}$ denote trace in the fundamental representation. For $su(N)$ generators, $T_a$, and an element of the algebra, $X = X^a T_a$, the traces are related as

$$
\text{Tr}(T_a T_b) = N \delta_{ab},
$$

$$
\text{tr}(T_a T_b) = \frac{1}{2} \delta_{ab},
$$

$$
\text{Tr}(X^2) = 2N \text{tr}(X^2),
$$

$$
\text{Tr}(X^4) = 2N \text{tr}(X^4) + 6[\text{tr}(X^2)]^2,
$$

$$
\text{Tr}(X^6) = 2N \text{tr}(X^6) + 30 \text{tr}(X^4) \text{tr}(X^2) - 20[\text{tr}(X^3)]^2.
$$
Similar relations apply to symmetrized products of generators,

\[ \text{STr}(X_{i_1}X_{i_2}X_{i_3}X_{i_4}) = \text{Tr}(X_{i_1}X_{i_2}X_{i_3}X_{i_4}) \]

\[ = 2N \text{tr}(X_{i_1}X_{i_2}X_{i_3}) + 6 \text{tr}(X_{i_1}X_{i_2}) \text{tr}(X_{i_3}X_{i_4}) \]

\[ \text{STr}(X_{i_1}X_{i_2} \ldots X_{i_6}) = \text{Tr}(X_{i_1}X_{i_2} \ldots X_{i_6}) \]

\[ = 2N \text{tr}(X_{i_1}X_{i_2} \ldots X_{i_6}) + 30 \text{tr}(X_{i_1} \ldots X_{i_6}) \text{tr}(X_{i_7}X_{i_8}) \]

\[ - 20 \text{tr}(X_{i_1}X_{i_2}X_{i_3}) \text{tr}(X_{i_4}X_{i_5}X_{i_6}). \]

Note that even for commuting backgrounds, where obviously \( \text{STr} = \text{Tr} \), it is often convenient to keep \( \text{STr} \) in order to change from \( \text{Tr} \) to \( \text{tr} \).

We list some results that will be used. For arbitrary **commuting** \( X_0, X_1 \)

\[ \text{STr}(X_0^2X_1^2) = 2N \text{tr}(X_0^2X_1^2) + 2 \text{tr}(X_0^2) \text{tr}(X_1^2) + 4[\text{tr}(X_0X_1)]^2 \]

Another common term is of the form \( X_0^6X_1^2 \). Let \( X_i \) represent either \( X_0 \) or \( X_1 \), such that \( X^6 = X_0^4X_1^2 \). Then

\[ \text{sym tr} \left( X^4 \right) \text{tr} \left( X^2 \right) = \frac{1}{15} \left[ \text{tr} \left( X_0^4 \right) \text{tr} \left( X_1^2 \right) + 8 \text{tr} \left( X_0^3X_1 \right) \text{tr}(X_0X_1) + 6 \text{tr} \left( X_0^2X_1^2 \right) \text{tr} \left( X_1^2 \right) \right] \]

\[ \text{sym tr} \left( X^3 \right) \text{tr} \left( X^2 \right) = \frac{1}{2} \left[ 2 \text{tr} \left( X_0^3 \right) \text{tr}(X_0X_1X_2) + 3 \left[ \text{tr} \left( X_0^2X_1 \right) \right]^2 \right] \]

\[ \text{sym tr} \left( X^2 \right) \text{tr} \left( X^2 \right) = \frac{1}{2} \left[ \left[ \text{tr} \left( X_0^2 \right) \right]^2 \text{tr} \left( X_1^2 \right) + 4 \text{tr} \left( X_0^2 \right) \text{tr}(X_0X_1) \right]^2 \]

where \( \text{sym} \) denotes the symmetrization operator.

Yet another common term is of the form \( X_0^2X_1^2X_2^2 \), all \( X_i \) commuting. Let \( X_i \) represent one of the \( X_i, i = 1, 2, 3 \), such that \( X^6 = X_1^2X_2^2X_3^2 \). Then

\[ \text{sym tr} \left( X^4 \right) \text{tr} \left( X^3 \right) = \frac{1}{15} \left[ \left[ \text{tr} \left( X_0^2 \right) \text{tr} \left( X_1^2X_2^2 \right) + \text{two terms} \right] \right. \]

\[ + 4 \left[ \text{tr}(X_1X_2) \text{tr}(X_1X_2X_3^2) + \text{two terms} \right] \left[ \text{tr} \left( X_1^4X_2^2X_3^2 \right) + \text{two terms} \right] \]

\[ \text{sym tr} \left( X^3 \right) \text{tr} \left( X^3 \right) = \frac{1}{2} \left[ 2[\text{tr}(X_1X_2X_3)]^2 + \left( \text{tr} \left( X_1^2X_2X_3 \right) + \text{two terms} \right) \right] \]

\[ \text{sym tr} \left( X^2 \right) \text{tr} \left( X^2 \right) = \frac{1}{15} \left[ \left( \text{tr} \left( X_0^2 \right) \right)^2 \text{tr} \left( X_1^2 \right) \right. \]

\[ + 2 \left( \text{tr} \left( X_0^2 \right) \text{tr}(X_1X_2) + \text{two terms} \right) \]

\[ + 8 \text{tr}(X_1X_2) \text{tr}(X_2X_3) + \text{two terms} \]

A common matrix appearing in these calculations is

\[ J_0 = \frac{1}{N} \left( \begin{array}{cc} N_6I_{n_6 \times n_6} & 0 \\ 0 & -n_6I_{N_6 \times N_6} \end{array} \right) \]

where \( N = n_6 + N_6 \).

The adjoint trace of its even powers is given by

\[ \text{Tr}(J_0^{2k}) = 2n_6N_6. \]

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