Determination of heat transfer criterial equation when cooling aluminum ingots

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Abstract. A big problem when casting aluminum ingots is the uneven structure formation, which leads to an increased rejection of products. Nonequilibrium structure elimination is carried out by heat treatment. To obtain the required aluminum ingots’ physicochemical properties, it is necessary to know the conditions of heat transfer between the ingots and the cooling air, i.e. a mathematical model of conjugate heat transfer is needed. The mathematical model obtained by the authors makes it possible to analytically investigate the ingots temperature and cooling air during heat treatment. This mathematical model assumes the heat transfer coefficient calculation. The existing criterion equations for determining the heat transfer coefficient have a drawback - the heat transfer coefficient according to these equations is calculated in circular channels, while heat transfer between aluminum ingots and air occurs in rectangular channels. The article describes the criterion equation identification for heat transfer, used in the analytical study, by the data of the experimental study.

1. Introduction
One of the main problems in casting aluminum ingots is the formation of micropores, cracks, and inhomogeneous structure of the ingot. The elimination of these defects is carried out by heat treatment, one of the types of which is homogenization annealing. The technology for carrying out this type of heat treatment requires a certain cooling rate after heating and holding the ingots in a homogenizing furnace. To form the required properties of aluminum ingots, it is necessary to have a mathematical model of heat transfer between the ingots and the cooling air, taking into account the thermophysical properties, geometry, process time, etc. A heat transfer model satisfying these conditions is described in [1, 14, 16].

2. Theoretical part
The solution of the resulting model of conjugate heat transfer between the cooling air and aluminum ingots involves the heat transfer coefficient \( \alpha \) determination. The heat transfer coefficient characterizes the heat exchange between the surface of a solid and the heat carrier and depends on a large number of different factors, such as: the physical properties of the heat transfer agent; the speed of its movement; shape, size and orientation in space of the heat exchange surface; temperature head.

In the mathematical sense, \( \alpha \) is the ratio of the heat flux density on the wall related to the temperature difference between the wall and the liquid.
\[ \alpha = \frac{q}{(t_c - t_1)} \]  

where \( \alpha \) - average heat transfer coefficient, W/m\(^2\)-K, \( q \) - heat flux density, W/m\(^2\), \( t_c \) - wall temperature, K, \( t_1 \) - liquid temperature, K.

A widely accepted method for determining the heat transfer coefficient is the application of similarity equations for the criterion Nusselt number. These equations make it possible to determine the heat transfer coefficient for various heat transfer conditions, including free and forced convection, and for movement in channels of various shapes. As applied to the problem under consideration, air movement occurs along channels bounded by horizontal rows of cylindrical aluminum ingots. Therefore, for this problem, the criterion equations are applicable to determine the heat transfer coefficient when washing the wave surface and in the transverse flow around the cylinder.

Heat transfer in a turbulent regime when flowing around a wavy surface and a single cylinder has been well studied [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The studies carried out show that the Nusselt criterion when moving in such channels depends on the Reynolds criteria \( Re \), Prandtl \( Pr \) and changes in the physical properties of the flow. Based on the generalization of numerous experimental data Gomelauri V.I. [6] proposed a criterion equation for heat transfer when washing a wave surface [6, 7]

\[ Nu_{zhd_{eq}} = 0.021 \cdot Re_{zhd_{eq}}^{0.8} \cdot Pr_{zh}^{0.43} \cdot \left( \frac{Pr_z}{Pr_c} \right)^{0.25} \cdot \varepsilon_w \]

where \( Nu_{zhd_{eq}} \) - is the average value of the Nusselt criterion along the length of the channel; \( \alpha \) - heat transfer coefficient, W/m\(^2\)-K; \( \lambda \) - coefficient of thermal conductivity of air, W/m·K; \( Re_{zhd_{eq}} = \frac{W}{d_{eq} \cdot v} \) - the average value of the Reynolds criterion along the length of the channel; \( d_{eq} \) - equivalent channel diameter, m; \( v \) - coefficient of kinematic viscosity of air, m\(^2\)/s; \( Pr_{zh} \) - the average value of the Prandtl criterion along the length of the channel; \( Pr_c \) - the average value of the Prandtl criterion; \( \varepsilon_w = 1,04 \cdot Pr_{zh}^{0.4} \cdot \exp \left[ 0.85 \cdot f \cdot \left( \frac{S}{h} \right) \right] \), where \( \frac{S}{h} \) is the relative step of the waves, \( S \) - is the distance between adjacent waves, m; \( h \) - is the height of the wave surface, m.

In the studies of E.P. Dyban, E. Ya. Epik, L.G. Kozlova [12], the local heat transfer coefficient in the frontal part of the cylinder is expressed by the dependence

\[ Nu_{fx} = 0.65 \cdot Re_{fx}^{0.5} \cdot Pr_f^{0.33} \cdot \left( \frac{Pr_f}{Pr_c} \right) \]

and in the aft part of the cylinder

\[ Nu_{fx} = 0.065 \cdot Re_{fx}^{0.73} \cdot Pr_f^{0.4} \cdot \left( \frac{Pr_f}{Pr_c} \right)^{0.25} \]

In equation (3), the local velocity is taken as the determining velocity, and the cylinder arc length from the frontal critical point to the considered point is taken as the determining size, and in (4) the velocity in the smallest flow area and the cylinder diameter are taken as the determining ones.

From the above criterion equations for analytical research, it is more expedient to use equation (2) intended for calculating the average heat transfer coefficient from the wave surface, in contrast to equations (3), (4), which describe the local heat transfer coefficients in the frontal and aft parts of a single cylinder. The criterion equation (2) is intended to determine the heat transfer coefficient in circular channels, while heat transfer during ingot cooling occurs in rectangular channels bounded by rows of aluminum ingots, which is a disadvantage of using equation (2) for this problem. Experiments will make it possible to obtain the speed of the cooling air and approximate them with the criterion equation (2).

3. Experiment
The developed model of conjugate heat transfer [1, 14, 16] and criterion equation (2) for calculating the heat transfer coefficient made it possible to conduct an analytical study of heat transfer during cooling of aluminum ingots. Research has shown that:

1. maintaining a variable cooling rate allows you to maintain a given cooling rate;
2. an increase in the speed of the cooling air after cooling the ingots below allows to speed up the cooling process and reduce energy costs for the heat treatment process;
3. a higher channel between the rows of ingots results in a lower final temperature;
4. the height of the channel does not affect the cooling time of the ingots to the temperature.

The results of the analytical study are described in [13, 15]. To check the adequacy of the analytical data, an experimental study was carried out in a cooling chamber. The purpose of the experimental study was to obtain the values of the temperatures of the ingots in the process of their cooling for comparison with the results obtained by the mathematical model, as well as to obtain the values of the velocities of the cooling air.

Aluminum ingots were placed in pallets in three rows. The study was carried out on ingots with a diameter, the height of the channel between the rows was chosen according to the results of an analytical study [13,15]. The ingot temperature was measured with TCA thermocouples; the secondary device was a 2TRM1 meter-regulator. The cooling air velocity was determined by recalculating the dynamic pressure measured by the Pitot tube using the formula

\[ W = 4.43 \cdot K_{P,T} \cdot \frac{P_D}{\gamma} \cdot \frac{m}{s} \]  

where \( K_{P,T} \) is the coefficient of the pressure tube, for the L-shaped tube we take equal to [5] \( K_{P,T} = 0.98 \); \( P_D \) - dynamic pressure according to the indication of the secondary device, Pa; \( \gamma \) - specific gravity of air, kg/m³. The secondary instrument was a Comarc C9557 electronic pressure gauge. The location of the thermocouples in the cage is shown in Figure 1, connecting thermocouples to the secondary in Figure 2.

![Figure 1. Charge with thermocouples in ingots (1-thermocouples, 2-aluminum ingots, 3-place for forming a charge)](image)

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4. Practical application
As noted above, the disadvantage of using equation (2) for this problem formulation is that the

criterion equation (2) is intended to study heat transfer in circular channels, while in this case the
channel is flat, bounded by aluminum ingots.

The study made it possible to obtain the values of the cooling air velocities during the ingots
cooling, which allow approximating the criterion equation by the obtained experimental data, in which
the height of the channel between the ingots $\delta$ will be used as the equivalent diameter. To approximate
criterion equation (2) with experimental data, one can use a widespread criterion - the least squares
method. The essence of this method is to minimize the sum of deviations of the experimental data
from the data obtained by the mathematical model.

The obtained experimental data are approximated to a criterion equation connecting the Nusselt
$Nu$, Reynolds $Re$ and Prandtl $Pr$ criteria

$$Nu = A \cdot Re^B \cdot Pr^C$$

where $A$, $B$, $C$ – are constant dimensionless numbers. This criterion equation describes the
Nusselt criterion in general form, and for its use in relation to the problem of convective heat transfer
in the cooling chamber, it is necessary to determine the unknown coefficients. The function to be
minimized to determine the coefficients by the least squares method has the form

$$F(A, B) = \sum_{i=1}^{n} (Nu_{\delta_i} - A \cdot Re_{\delta_i}^B)^2,$$  \hspace{1cm} (6)

where $Nu_{\delta_i}$ is the value of the dependent variable in the $i$-experiment, $A \cdot Re_{\delta_i}^B$ is the value of the
independent variable, $A$ and $B$ are the unknown coefficients to be determined. Since the value of the
Prandtl criterion $Pr$ remains almost unchanged in the temperature range 0-600°C, we will neglect it to
simplify the calculations. As a result of minimizing function, we obtain a system of two equations for
determining the coefficients

$$A = \frac{\sum_{i=1}^{n} (Nu_{\delta_i} \cdot Re_{\delta_i}^B)}{\sum_{i=1}^{n} (Re_{\delta_i}^B)},$$

$$B = \frac{\sum_{i=1}^{n} (Nu_{\delta_i} \cdot Re_{\delta_i}^{B-1})}{\sum_{i=1}^{n} (Re_{\delta_i}^B - Re_{\delta_i}^{B-1})},$$  \hspace{1cm} (7)

The system of equations (7) was solved by the substitution method in the Excel program. In the
initial data, the ingot diameter $d$ was specified; the height of the channel between the rows of ingots $\delta$;
ingot cooling time $\tau$; the speed of the cooling air determined from an experimental study; the Nusselt
and Reynolds criteria Re were calculated; heat transfer coefficient $\alpha$. As a result, the necessary
coefficients were determined and, taking into account the Prandtl criterion Pr, the equation was finally obtained

\[ \text{Nu}_{zh \text{eq}} = 0.0352 \cdot Re^{0.65}_{zh \text{eq}} \cdot Pr_{zh}^{0.43} \cdot \left( \frac{Pr_{zh}}{Pr_c} \right)^{0.25} \]  

(8)

5. Conclusion

Expression (8) is a criterion equation for determining the heat transfer coefficient in the channel bounded by the surface of aluminum ingots, where the height of the channel between the ingots \( \delta \) is used as the equivalent diameter.

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