THE SPONTANEOUS BREAKDOWN OF THE VACUUM

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We discuss the spontaneous breakdown of the vacuum by a strong electromagnetic field as observed in SLAC experiment E-144. We show that the data follow the Schwinger non-perturbative result obtained for a static field.

1 Pair Production in a Strong em Field

In his 1951 paper, J. Schwinger predicted that an intense static electric field will break down the vacuum to produce $e^+e^-$ pairs. This occurs when the field-strength approaches the critical value $E_c = m^2c^3/\bar{e}h = 1.3 \times 10^{16}$ V/cm.

We argue that this effect has been observed in the scattering of high energy electrons from the focus of an intense laser pulse.

The experimental conditions differ from the original Schwinger premise in two respects:

(a) The field strength in the laboratory is only $E \sim 3 \times 10^{10}$ V/cm but reaches near-critical value in the rest frame of the 46.6 GeV incident electrons ($E^* = 2\gamma E \simeq 1.8 \times 10^{5}E$).

(b) The field in the laboratory is not static but a well-defined coherent wave field. However, Brezin and Itzykson have shown that spontaneous $e^+e^-$ pair production will also occur in a time-dependent field. They derive the probability for pair production in both the perturbative (low field, $E^* < E_c$) and non-perturbative ($E^* \gtrsim E_c$) regimes.

It is convenient to introduce the normalized vector potential of the field

$$\eta = \left[ \frac{e|\langle A_\mu A^\mu \rangle|}{m^2} \right]^{1/2} = \frac{eE_{\text{rms}}}{\omega_0 mc}$$  \hspace{1cm} (1)

where $\omega_0$ is the angular frequency of the field which is assumed to be monochromatic and sinusoidal. The probabilities per unit time – unit volume derived are

$$\eta \ll 1 \quad w \simeq \frac{\alpha E^2}{4\hbar} \left( \frac{eE}{2m_0^2c^3/\hbar} \right)^{4m_0c^2/\hbar\omega_0}$$  \hspace{1cm} (2)

$$\eta \gg 1 \quad w \simeq \frac{\alpha E^2}{\pi\hbar} \exp \left( \frac{-\pi m^2c^3}{e\hbar E} \right)$$  \hspace{1cm} (3)
Eqs. (2,3) have an immediate interpretation in physical terms. When $\eta \ll 1$ we are in the perturbative regime and $n = 2mc^2/\hbar \omega_0$ is the number of photons required to produce the pair. Thus the probability is proportional to the $n^{th}$ power of the square of the normalized vector potential: $(\eta^2)^n$. When $\eta \gg 1$ the probability depends on the electric field strength through the singular expression $\exp(-\pi E_c/E)$. In the static case this behavior can be interpreted as quantum-mechanical tunneling through a potential $V_0 \sim 2mc^2$.

In the intermediate case, $\eta \sim 1$,

$$w = \frac{\alpha E^2}{\pi \hbar} \frac{1}{g(\eta) + \frac{1}{2}g'(\eta)/\eta} \exp \left[ -\frac{\pi mc^3}{\hbar E} g(\eta) \right]$$

where the function

$$g(\eta) = \frac{4}{\pi} \int_0^1 dy \left[ \frac{1 - y^2}{1 + y^2/\eta^2} \right]^{1/2}$$

smoothly interpolates between the two regimes. In the non-perturbative regime it is customary to introduce the dimensionless parameter

$$\Upsilon = \frac{E}{E_c} = \frac{e\hbar E}{mc^3}$$

If an electron moves through the electric field with 4-momentum $p$ ($\gamma = p_0/mc$) then in the electron’s rest frame the parameter $\Upsilon$ takes the value

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu}p_{\nu})^2 \rangle mc^2}}{E_c}$$

(6')

where $F^{\mu\nu}$ is the field tensor. Eq. (6') can also be used when a high energy photon of 4-momentum $k_{\nu}$ traverses the field. For head-on collisions we can write

$$\Upsilon = 2\gamma \frac{e\hbar E}{mc^3} = \frac{\eta^2(c_p_0)/(\hbar \omega_0)}{(mc^2)^2}$$

(6'')

where $\gamma$ was defined previously and $\omega_0$ is the frequency of the em field. A few comments are in order: (1) $\Upsilon$ is a relativistic invariant describing the interaction of a particle with the electric field (2) $\Upsilon$ is well defined for a static field (3) It is a measure of the cm energy in the collision of the incoming particle with one laser photon (in units of the electron mass) multiplied by the normalized potential $\eta$.

In the experiment reported in ref. [1] electron-positron pairs were produced when 46.6 GeV/c electrons crossed the focus of a laser pulse of wavelength...
\( \lambda = 527 \text{ nm} \). This observation can be interpreted as a two-step process in which first a photon backscatters off an electron to become a high energy \( \gamma \)-ray (\( \omega \gamma \sim 29 \text{ GeV} \)) and subsequently the \( \gamma \)-ray scatters from at least four laser photons to produce the pair. The photon density in the focus is adequately high (\( n \omega \sim 2.5 \times 10^{20} \text{ cm}^{-3} \)) so that multiphoton processes up to \( n = 5 \) could be observed over the course of the experiment. Support for this interpretation comes from plotting the positron yield as a function of \( \eta \) and observing that it varies as \( \eta^{2n} \) with \( n = 5.1 \pm 0.2 \) as expected from Eq. (2) if one replaces \( \omega_0 \) by \( 2\gamma \omega_0 \). In fact the data agree with an exact calculation of the multiphoton Breit-Wheeler equation as shown in Fig. 1.

To examine the alternative interpretation in terms of the spontaneous breakdown of the vacuum we wish to test Eq. (3). Note that in the experiment, \( \eta \sim 0.3 \) namely one is between the two regimes. We plot the data as a function of \( 1/\Upsilon \) as shown in Fig. 2 where we use the form of Eq. (6') for \( \Upsilon \); here we have also included data obtained at 49.1 GeV. A fit to the \( \Upsilon \) dependence of Eq. (4) yields for the factor in the exponent

\[
\pi g(\eta) = 2.01 \pm 0.12 \pm 0.4
\]

the first error being statistical and the second systematic. The prediction of Eqs. (4,5) for \( \langle \eta \rangle = 0.25 \) is \( g(\eta) = 0.58 \) and thus \( \pi g(\eta) = 1.83 \).

However, the result of Eq. (7) must be corrected for two factors. In Fig. 2 we used the rms value of the electric field to define \( \Upsilon \) and \( \eta \), whereas in ref. [3] the peak values are used. Secondly the frame in which the high energy gamma and one photon collide should be used in the definition of \( \gamma \) entering Eq. (6'); namely \( \Upsilon = \Upsilon_{\text{rms}} \sqrt{2(29.2/46.6)} \). We then find that \( \Upsilon \) has to be reduced by a factor of 1.14, while \( \eta \) must be increased by \( \sqrt{2} \). This leads to

\[
\pi g(\eta) = 1.77 \pm 0.35 \quad \text{observed}
\]
\[
\pi g(\eta) = 2.12 \quad \text{predicted (7')}
\]

We see that the dependence of the pair production rate on the field strength agrees with the predictions of refs. [1,3].

To estimate the positron yield predicted by Eq. (3) we must integrate the probability over volume and time. We associate a volume equal to \( \lambda^3 \) (\( \lambda \) is the Compton wavelength of the electron) for each electron crossing the focus and use \( \Delta t = (\ell/c)(1/\gamma) \) for the time of interaction in the electron rest-frame; here \( \ell \) is the length of the focus, \( \ell = 2d/\sin \theta \sim 20 \mu \text{m} \). We then obtain

\[
w = \frac{\alpha}{\pi} \frac{E_e^2}{\hbar} \Upsilon^2 e^{-\pi g/\Upsilon} \frac{1}{g + \frac{1}{2} g'/\eta}
\]
with
\[ \frac{\alpha E^2}{\pi \hbar} = 3.45 \times 10^{50} \text{ cm}^{-3} \text{s}^{-1} \]  
(8')

For \( \Upsilon = 0.24 \) and \( \eta = 0.35 \) we find for the probability per incident high energy \( \gamma \)-ray
\[ \int \omega d^3x dt \simeq 4 \times 10^{-4} \]  
(9)

However per laser shot only \( \sim 10^6 \) \( \gamma \)-rays cross the laser focus and we must account for the fraction of \( \gamma \)-rays of sufficient energy to produce a pair (\( \sim 10^{-2} \) of the total spectrum). These qualitative arguments predict a pair production rate of \( \sim 4/\text{laser shot} \) as compared to the observed rate of \( 0.1/\text{laser shot} \).

We make two additional remarks. First, that eventhough the electric field seen in the electron rest-frame is time-dependent, the period is longer than the formation time of the pair by a factor of fifteen. In the rest-frame
\[ T^* = \frac{1}{\gamma} T = \frac{1}{\gamma} \frac{\lambda}{c} \sim 2 \times 10^{-20} \text{s} \]

whereas the quantum-mechanical uncertainty time associated with an energy fluctuation of the order of an electron mass is
\[ \Delta t = \frac{\lambda}{c} \sim 1.3 \times 10^{-21} \text{s} \]

Thus one can treat the fields seen in the electron rest-frame as static as considered in ref. [2]. Note however that this assumption is not needed in deriving Eqs. (2,3).

In the experiment of ref. [1] the energy of the electron and positron in the pair is provided by the incident high energy \( \gamma \)-ray. The presence of an incident particle resolves the issue of energy-momentum balance since it is known that a plane wave (for which \( E^2 - B^2 = 0 \)) cannot produce pairs. On the other hand in a focused wave, there are regions near the focus where \( E^2 - B^2 > 0 \). The value of the invariant is approximately \( \frac{1}{2} (E/f^\#)^2 \) where \( f^\# \) is the \( f \)-number of the focussing optics.

It would be of considerable interest to observe the breakdown of the vacuum without the participation of an incident particle. This would require an intensity of the em flux \( I = 5 \times 10^{29} \text{ W/cm}^2 \) to reach \( \Upsilon = 1 \). Some future FEL’s are planned to operate in the 1 Å region and one could consider extremely tight focussing (say to 10 Å) to reach high intensity. The peak power will be at best \( P = 10^{11} \text{ W} \) so that \( I = 10^{26} \text{ W/cm}^2 \) which is still short of producing critical field in the laboratory frame.
2 Breakdown of the Vacuum and the Fine Structure Constant

The condition for the breakdown of the vacuum is that the electric field $E_c$ be such that an electron gains energy equal to its rest mass in one Compton wavelength

$$eE_c \lambda_c = mc^2$$  \hspace{1cm} (10)

Eq. (10) leads to the definition of the critical field $E_c$ introduced previously

$$E_c = \frac{m^2 c^3}{e \hbar}$$  \hspace{1cm} (11)

The above is a statement on the interaction of the electron with the field. However for the pair to be produced there must also be sufficient energy in the field in a volume of order $V = \lambda_c^3$. We therefore obtain a necessary condition on the field energy

$$\frac{1}{2} \epsilon_0 E_c^2 \lambda_c^3 > 2mc^2$$  \hspace{1cm} (12)

Combining (9) and (10) leads to an upper limit on the value of the fine structure constant

$$\alpha = \frac{e^2}{(4\pi \epsilon_0) \hbar c} < \frac{1}{16\pi}$$  \hspace{1cm} (13)

This inequality is satisfied in nature and appears to be a necessary condition for the spontaneous breakdown of the vacuum.

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References

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Fig. 1 Dependence of the positron rate on the laser field-strength parameter $\eta$. The rate is normalized to the number of Compton scatters inferred from the EC37 monitor. The solid line is the prediction based on the numerical integration of the two-step process of laser backscattering followed by multiphoton Breit-Wheeler pair production. From ref. [1], 46.6 GeV data.
Fig. 2 Number of positrons per laser shot as a function of $1/\Upsilon$. The solid line is a fit to the data of the form $R_{e^+e^-} \propto \exp(-a/\Upsilon)$ and yields $a = 2.01 \pm 0.12 \pm 0.4$. Circles are for the 46.6 GeV data, squares for the 49.1 GeV data. From Th. Koffas “Positron Production in Multiphoton Light by Light Scattering” Ph.D Dissertation University of Rochester (1998) to be published.
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