Cordial Miners: Blocklace-Based Ordering Consensus Protocols for Every Eventuality

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ABSTRACT

Cordial Miners are a family of efficient Byzantine Atomic Broadcast protocols, with instances for asynchrony and eventual synchrony. Their efficiency—almost half the latency of state-of-the-art DAG-based protocols—stems from not using reliable broadcast as a building block. Rather, Cordial Miners use the blocklace—a partially-ordered generalization of the totally-ordered blockchain—for all algorithmic tasks required for ordering consensus: Dissemination, equivocation-exclusion, and ordering.

1 INTRODUCTION

1.1 Overview and Related Work

The problem of reaching consensus on the ordering of acts by participants in a distributed system has been investigated for four decades [38], with efforts in the last decade falling into two categories: Permissioned, where the set of participants is determined by some authority, and permissionless, where anyone may join and participate provided that they pass some ‘sybil-proof’ test, notably proof-of-work [34] or proof-of-stake [29]. Two leaders-of-the-pack in the permissioned category are the State-Machine-Replication protocol (SMR, consensus on an ordering of proposals) for the eventual-synchrony model – Hotstuff [42] and its extensions and variations [13], and the Byzantine Atomic Broadcast protocol (BAB, consensus on an ordering of all proposals made by correct participants) for the asynchronous model – DAG-Rider [28] and its extensions and variations [25]. Since the emergence of Bitcoin [34], followed by Ethereum with its support for smart contracts [8], permissionless consensus protocols have received the spotlight.

Recent conceptual and computational advances, notably stake-based sampling, have allowed permissioned consensus protocols to join the cryptocurrency fray (e.g. Cardano [29] and Algorand [24]), offering much greater efficiency and throughput compared to proof-of-work protocols. According to this approach, in every epoch (which could be measured in minutes or weeks) a new set of miners is chosen in a random auction, where the probability of being an auction winner is correlated with the stake bid by the miner. Mechanism design ensures that miners benefit from performing the protocol well, benefit less if they perform the protocol less well, and lose their stake if they subvert the protocol.

With this in mind, the expectation is that miners will do their best, not their worst, to execute the protocol, and hence the focus of analyses of permissioned consensus protocols has shifted from worst-case complexity to good-case complexity [1, 25], where miners are generally expected to behave as well as they can, given compute and network limitations, as opposed to as bad as they can. Still, standard protections against a malicious adversary are needed, for example to prevent a double-spending, a hostile takeover, or a meltdown of the cryptocurrency supported by the consensus protocol.

The use of a DAG-like structure to solve consensus has been introduced in previous works, especially in asynchronous networks [33]. Hashgraph [3] builds an unstructured DAG, with each block containing two references to previous blocks, and on top of the DAG the miners run an inefficient binary agreement protocol. This leads to expected exponential time complexity. Aleph [23] builds a structured round-based DAG, where miners proceed to the next round once they receive $2f + 1$ DAG nodes from other miners in the same round. On top of the DAG construction protocol a binary agreement protocol decides on the order of vertices to commit. Nodes in the DAG are reliably broadcast. Blockmania [15] uses a variant of PBFT [11] in the eventual synchrony model.

DAG-Rider [28] is a BAB protocol for the asynchronous model. It assumes an adaptive adversary that controls the finite delay of messages between any two correct miners. In DAG-Rider, the miners jointly build a DAG of blocks, with blocks as vertices and pointers to previously-created blocks as edges, divided into strong and weak edges. Strong edges are used for the commit rule, and weak edges are used to ensure fairness. The protocol employs an underlying reliable broadcast protocol of choice, which ensures that eventually the local DAGs of all correct miners converge and equivocation is excluded. Each miner independently converts its local DAG to an ordered sequence of blocks, with the use of threshold signatures to implement a global coin that retrospectively chooses one of the miners as the leader for each round. The decision rule for delivering a block is if the vertex created by the leader is observed by at least $2f + 1$ miners three rounds after it is created. The DAG is divided into waves, each consisting of the nodes of four rounds. When a wave ends, miners locally check whether a decision rule is met, similar to our protocol. DAG-Rider has an expected amortized linear message complexity, and expected constant latency. Tusk [16] is an implementation based on DAG-Rider. Bullshark [25] is the current state-of-the-art dual consensus protocol based on DAG-Rider that offers a fast-track to commit nodes every two rounds in case the network is synchronous. Other DAG-based consensus protocols include [12, 20, 36, 40].

HotStuff [42] is an SMR protocol designed for the eventual synchrony model. The protocol employs all-to-leader, leader-to-all communication: In each round, a deterministically-chosen designated leader proposes a block to all and collects from all signatures on the block. Once the leader has $2f + 1$ signatures, it can combine them into a threshold signature [4] which it sends back to all. The decision rule for delivering a block is three consecutive correct leaders. This leads to a linear message complexity and constant latency in the good case. The protocol delivers a block if there are three correct leaders in a row, which is guaranteed to happen after
GST. HotStuff is based on Tendermint [7] and is also the core of several other consensus protocols [14, 22, 27, 41]. In this model, there are a number of leader-based protocols such as DLS [21], PBFT [11], Zyzzyva [30], and SBFT [26].

It is within this context that we introduce Cordial Miners – a family of simple, efficient, self-contained Byzantine Atomic Broadcast [9] protocols, and present two of its instances: Retrospective random leader selection for the asynchronous model and deterministic leader selection for the eventual synchrony model (See Table 1 for their performance).

In a Cordial Miners protocol, correct miners cooperatively create and share a blocklace, each contributing its own cryptographically-signed blocks, with each new block observing all the blocks previously received or created by the miner. In addition, each miner incrementally converts its local blocklace into an ordered sequence of blocks, which is the output of the protocol. We believe that the simplicity-cum-efficiency of the Cordial Miners protocols stems from the use of the blocklace data structure and its analysis for all key algorithmic tasks (the following refers to correct miners):

1. **The Blocklace** [37] is a partially-ordered generalization of the totally-ordered blockchain (Def. 3.1), that consists of cryptographically-signed blocks, each containing a payload and a finite number of cryptographic hash pointers to previous blocks. The blocklace induces a DAG, as cryptographic hash pointers cannot form cycles by a compute-bound adversary. The DAG induces a partial order > (Def. 3.3) on the blocks that includes Lamport’s ‘happened-before’ causality relation [31] among correct miners. The globally-shared blocklace is constructed incrementally and cooperatively by all miners, who cordially disseminate it to each other. In a run of a Cordial Miners protocol, the local blocklaces of correct miners all converge to the same shared global blocklace (Proposition 5.4), referred to as the blocklace of the run.

2. **Ordering**. The ordering algorithm (Algorithm 2) is used locally by each miner to topologically-sort its local blocklace into a totally-ordered output sequence of blocks, excluding equivocation along the way. This conversion is monotonic (Prop. 4.3) – the output sequence is extended as the miner learns of or produces ever-larger portions of the global blocklace, and in this sense every output block of each miner is final. We say that two sequences are consistent if one is a prefix of the other (Def. 2.2), a notion stronger than the common prefix property of Ouroboros [29]. We assume that less than one-third of the miners are faulty, and prove that the following holds for the correct miners of the Cordial Miners protocols (Theorem 5.1):

   - **Safety**: Outputs of correct miners are consistent.
   - **Liveness**: A block sent by a correct miner is eventually output by every correct miner.

3. **Dissemination**: Any new block created by a miner \( p \) observes blocks known to \( p \) by including pointers to the tips (DAG sources) of \( p \)'s local blocklace. Correspondingly, a miner \( p \) will buffer, rather than include in its blocklace, any received block with dangling pointers – pointers to blocks not known to \( p \). Hence, a block \( b \) by a correct miner \( p \) informs its recipients of blocks known to \( p \) at the time of \( b \)'s creation. Thus \( p \), being cordial, when sending to \( q \) a new \( p \)-block, will include with it blocks \( p \) knows but, to the best of \( p \)'s knowledge, are not yet known to \( q \) and have not already been sent to \( q \) by \( p \), thus ensuring block dissemination (Prop. 5.4).

4. **Equivocation exclusion**. An equivocation (Def. 3.4) is a pair of blocks by the same miner that are not causally-related – have no path of pointers from one to the other; a miner that creates an equivocation is an **equivocator** and is considered faulty. A key observation is that a miner cannot approve (Def. 3.5) both blocks of an equivocation without being itself an equivocator (Ob. 2). Hence, if less than one-third of the miners are equivocators, then no equivocation will ever receive an approval from blocks created by a supermajority (at least two-thirds) of the miners. That’s how blocklace-based is the basis of equivocation-exclusion by the blocklace: A miner finalizes a block \( b \) once its local blocklace includes blocks that approve \( b \) by a supermajority (Algorithm 2).

5. **Cordial Miners**. The depth, or round, of a block \( b \) is the maximal length of any path emanating from \( b \) (Def. 3.1). A **round** is a set of blocks of the same depth. Miners are cordial in two respects. First, as explained above, in informing other miners of blocks they believe the other miner lacks. Second, awaiting a supermajority of round \( d \) before producing a block of round \( d + 1 \) (Def. 3.14).

6. **Leader Selection**. The Cordial Miners protocol for the eventual synchrony model employs prospective leader selection (e.g. via a shared pseudorandom function). In the asynchronous model, the adversary has complete control over the order of message delivery, indefinitely. The panacea to such an adversary, employed for example by DAG-Rider [28], is to use a shared random coin [10] and elect the leader retrospectively.

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| Protocol          | Latency | Message complexity |
|-------------------|---------|--------------------|
|                   | Eventual Synchrony | Asynchrony |
|                   | Good | Expected | Good | Expected |
| Cordial Miners    | 2    | 4.5 | 5 | 7.5 | amortized \( O(n^2) \) |
| Bullshark [25]    | 4    | 9 | 8 | 12 | amortized \( O(n^2) \) |
| DAG-Rider [28] for Asynchrony | 8 | 18 | 16 | 24 | amortized \( O(n) \) |

Table 1: Performance summary. Latency is measured in the number of communication rounds to finalize a correct leader block, which in turn orders the blocks till the previous final leader. The Cordial Miners protocols have better latency than Bullshark and DAG-Rider in both protocols and in both the good case and the expected case. Note that each formal round of Bullshark and DAG-Rider employs reliable broadcast [5], which requires at least two rounds of communication of simple messages [1] \( O(n^2) \) amortized message complexity), or four round with erasure coding [18] \( O(n) \) amortized message complexity).
(7) **Ratified and Super-Ratified Leaders**: A block \( b \) is *ratified* by block \( b' \) if \([b'] \) includes a cordial and non-equivocating supermajority that approves \( b \). A block \( b \in B \) is *super-ratified* in blocklace \( B \) if there is a supermajority of blocks in \( B \) that ratify \( b \) (Def.3.10). Given a wavelength \( (w = 2 \) for eventual synchrony and \( w = 5 \) for asynchrony), a leader block \( b \) is *final* in blocklace \( B \) if the \( \text{depth}(b) + w \) prefix of \( B \) super-ratifies \( b \) (Def. 3.10 and Fig. 1).

(8) **Blocklace Ordering with \( \tau \) and Finality**: We assume a topological sort procedure that takes a blocklace as an input and produces a sequence of its blocks while respecting their causal partial order \( >_\text{c} \) and excluding equivocations along the way. With it, an *ordering function* \( \tau \) that converts a partially-ordered blocklace \( B \) to a totally-ordered output sequence is defined (Def. 4.1). The function \( \tau \), applied to a blocklace \( B \), seeks the highest-depth final leader block \( b \in B \) and produces an ordering of \( [b] \). It has the property that if \([b] \) includes another final leader block \( b' \), then \( \tau(b') \) is a prefix of \( \tau(b) \). Hence, it is safe to compute \( \tau \) starting from any blocklace, and as the blocklace extends and includes a new final leader block, then the output sequence produced by \( \tau \) would only extend the one computed already. Hence any output of \( \tau \) is final.

(9) **Exclusion of faulty miners**: Any faulty \( p \)-block known to some correct miner will eventually be known to all, resulting in correct miners suspending any further communication with \( p \). For example, any miner can easily verify whether another miner \( p \) is cordial in the second sense by examining the blocklace. In addition, an equivocation by \( p \), with each block of the pair known to a different correct miner, will eventually be known to all and result in the exclusion of \( p \).

Miners accomplish all the above by simple and efficient analyses of their local blocklace.

The rest of the paper is organized by the following Sections:

2. Provides an overview of the two models explored—asynchrony and eventual synchrony—the Byzantine Atomic Broadcast problem we aim to solve with the Cordial Miners protocols in the two models, the safety and liveness requirements any solution must satisfy, and the outline of the proof of safety and liveness of the Cordial Miners protocols.

3. Introduces the notions of blocklace safety and liveness. The Blocklace was introduced in reference [37], and its preliminaries are reproduced in Section 3.1.

4. Introduces the blocklace ordering function \( \tau \), which is parameterized by the notion of final leader, and the requirements on a blocklace for the safety and liveness of \( \tau \). It then provides sufficient conditions for a blocklace-based ordering consensus protocol to produce a blocklace that satisfies these requirements.

5. Introduces the Cordial Miners protocols and argue their safety and liveness.

6. Discusses performance analysis, future optimizations and future Cordial Miners protocols.

7. Concludes the paper.

2 **MODELS, PROBLEM, SAFETY AND LIVENESS**

We assume \( n \geq 3 \) miners (aka agents, processors) \( \Pi \), of which at most \( f < n/3 \) may be faulty (act under the control of the adversary, be ‘Byzantine’), each equipped with a single and unique cryptographic key-pair, with a public key known to others. Miners can create, sign, and send messages to each other, where any message sent from one correct miner to another is eventually received. In addition, each miner can sequentially output (aka ‘deliver’) messages (e.g., to a local output device or storage device).

**Definition 2.1.** We consider two models of distributed computing with an adversary:

(1) **Asynchrony** [6], in which an adversary controls up to \( f \) faulty miners and the finite delay of every message.

(2) **Eventual Synchrony** [21]: In addition, there is a point in time, known as the global stabilization time (GST), beyond which message delay among correct miners does not exceed a known bound \( \Lambda \).

Next we define the problem we aim to solve in this paper:

**Definition 2.2 (Prefix, \( \preceq \), Consistent Sequences).** A sequence \( x \) is a *prefix* of a sequence \( x', x \preceq x' \), if \( x' \) can be obtained from \( x \) by appending to it zero or more elements. Two sequences \( x, x' \) are *consistent* if \( x \preceq x' \) or \( x' \preceq x \).

**Definition 2.3 (Safety and Liveness of an Ordering Consensus Protocol).** An ordering consensus protocol is:

- **Safe** if outputs of correct miners are consistent.
- **Live** if a message sent by a correct miner is eventually output by every correct miner with probability 1.

**Problem:** Devise safe and live ordering consensus protocols for the models of distributed computing with an adversary.

We note that safety and liveness, combined with message uniqueness (e.g., a block in a blocklace, see next), imply the standard Byzantine Atomic Broadcast guarantees: Agreement, Integrity, Validity, and Total Order [5, 28]. Hence, the blocklace-based Cordial Miners protocols that address the problem are in fact protocols for Byzantine Atomic Broadcast [9].

3 **THE BLOCKLACE**

A blocklace [37] is a partially-ordered generalization of the totally-ordered blockchain: In a blocklace, each block may contain a finite set of cryptographic hash pointers to previous blocks, in contrast to one pointer (or zero for the initial/genesis block) in a blockchain.

Next we present the basic definitions and results from [37]. Please consult the original reference for explanations and proofs. Blocklace utilities that realize these notions are presented in Algorithm 1. These are followed by definitions and results needed for establishing the safety and liveness of blocklace-based consensus protocols.

3.1 **Blocklace Basics**

In addition to the set of miners \( \Pi \), we assume a given set of payloads \( \mathcal{A} \) and a cryptographic hash function \( \text{hash} \).
Definition 3.1 (Block, Acknowledge). A block over \( \Pi, \mathcal{A} \) and hash is a triple \( b = (p, a, H) \), referred to as a \( p \)-block, \( p \in \Pi \), with \( a \in \mathcal{A} \) being the payload of \( b \), and \( H \) is a finite set of hash pointers to blocks, namely for each \( h \in H \), \( h = hash(b') \) for some block \( b' \) in which case we also say that \( b \) acknowledges \( b' \). If \( H = \emptyset \) then \( b \) is initial.

In a concrete implementation, a \( p \)-block is encoded by a string signed by \( p \). Note that hash being cryptographic implies that a set of blocks that form a cycle cannot be effectively computed. A set of blocks \( B \) induces a finite-degree directed graph \((B, E)\), \( E \subset B \times B \), with blocks \( B \) as vertices and directed edges \((b, b') \in E \) if \( b \in B \) includes a hash pointer to \( b' \in B \). We overload \( B \) to also mean its induced directed graph \((B, E)\).

Definition 3.2 (Blocklace). Let \( B \) be the maximal set of blocks over \( \Pi, \mathcal{A} \) and hash for which the induced directed graph \((B, E)\) is acyclic. A blocklace over \( \mathcal{A} \) is a set of blocks \( B \subseteq \mathcal{B} \).

Note that the directed graph induced by a blocklace \( \mathcal{B} \subset \mathcal{B} \) is acyclic. The two key blocklace notions used in our protocols are observation and approval.

Definition 3.3 (\( \rhd \), Observe). The strict partial order \( \rhd \) is defined by \( b' \rhd b \) if there is a nonempty path from \( b' \) to \( b \). A block \( b' \) observes \( b' \geq b \) if \( b' \rhd b \). Miner \( p \) observes \( b \) in \( \mathcal{B} \) if there is a \( p \)-block \( b' \in \mathcal{B} \) that observes \( b \), and a group of miners \( Q \subseteq \Pi \) observes \( b \) in \( B \) if every miner \( p \in Q \) observes \( b \).

We note that ‘observe’ is the transitive closure of ‘acknowledge’. With this, we can define the basic notion of evacuation (which may result in double-spending when payloads are conflicting financial transactions). See Figure 1.A.

Definition 3.4 (Equivocation, Equivocator). A pair of \( p \)-blocks \( b \neq b' \in B, p \in \Pi \), form an equivocation by \( p \) if they are not consistent wrt \( \rhd \), namely \( b' \not\rhd b \) and \( b \not\rhd b' \). A miner \( p \) is an equivocator in \( B \), equivocator \( p \), if \( B \) has an equivocation by \( p \). Namely, a pair of \( p \)-blocks form an equivocation of \( p \) if neither observes the other. As any \( p \)-block is cryptographically signed by \( p \), an equivocation by \( p \) is a volitional fault of \( p \).

Definition 3.5 (Approval). Given blocks \( b, b' \in B \), the block \( b \) approves \( b' \) if \( b \) observes \( b' \) and does not observe any block \( b'' \) that together with \( b' \) forms an equivocation. A miner \( p \in \Pi \) approves \( b' \in B \) if there is a \( p \)-block \( b \in B \) that approves \( b' \), in which case we also say that \( p \) approves \( b' \) in \( B \). A set of miners \( Q \subseteq \Pi \) approve \( b' \in B \) if every miner \( p \in Q \) approves \( b' \) in \( B \).

Observation 1. Approval is monotonic wrt \( \rhd \).

Namely, if miner \( p \) approves \( b \) in \( B \) it also approves \( b \) in \( B' \supseteq B \).

A key observation is that a miner cannot approve an equivocation of another miner in a blocklace \( B \) without being an equivocator in \( B \) itself (Fig. 1.A):

Observation 2. [Approving an Equivocation] If miner \( p \in \Pi \) approves an equivocation \( b_1, b_2 \) in a blocklace \( B \subseteq \mathcal{B} \), then \( p \) is an equivocator in \( B \).

Definition 3.6 (Closure, Closed, Tip). The closure of \( b \in B \) wrt \( \rhd \) is the set \([b] := \{b' \in B : b \rhd b'\} \). The closure of \( B \subset B \) wrt \( \rhd \) is the set \([B] := \bigcup_{b \in B} [b] \). A blocklace \( B \subseteq \mathcal{B} \) is closed if \( B = [B] \). A block \( b \in B \) is a tip of \( B \) if \( [b] = [B] \cup \{b\} \).

Note that a blocklace \( B \) is closed if its blocks do not contain ‘dangling pointers’ to blocks not in \( B \).

Definition 3.7 (Block Depth/Round, Blocklace Prefix & Suffix). The depth (or round) of a block \( b \in \mathcal{B} \), \( depth(b) \), is the maximal length of any path of pointers emanating from \( b \). For a blocklace \( \mathcal{B} \subseteq \mathcal{B} \) and \( d \geq 0 \), the depth-\( d \) prefix of \( B \) is \( B(d) := \{b \in \mathcal{B} : depth(b) \leq d\} \), and the depth-\( d \) suffix of \( B \) is \( B(d) := \mathcal{B} \setminus B(d) \).

3.2 Blocklace Safety

As equivocation is a fault, at most \( f \) miners may equivocate.

Definition 3.8 (Supermajority). A set of miners \( P \subseteq \Pi \) is a super-majority if \( |P| \geq \frac{n - f}{2} \). A set of blocks \( B \) is a supermajority if the set of miners \( P = \{p \in \Pi : b \in B \text{ is a } p\text{-block}\} \) is a supermajority.

Note that a supermajority is defined to be large enough to necessarily include a majority of the correct miners. Also note that if a faulty miner is exposed and repelled by the correct miners, such a step effectively deduces \( 1 \) from both \( n \) and \( f \), causing the supermajority to decrease, reaching simple majority when all \( f \) faulty miners are exposed and repelled, namely \( f = 0 \). As the Cordial Miners protocol does expose and repel faulty miners, the notion of supermajority is preferred over the standard formulas of \( n - f \) or \( 2f + 1 \), as they converge to all or none, respectively, rather than to majority when exposed faulty miners are deducted from both \( f \) and \( n \).

Lemma 3.9 (No Supermajority Approval for Equivocation). If there are at most \( f \) equivocators in a blocklace \( \mathcal{B} \subset \mathcal{B} \) with an equivocation \( b, b' \in B \), then not both \( b, b' \) have supermajority approval in \( B \).

Next, we introduce the notions of ratification (Figure 1.B) and super-ratification (Figure 1.C), which is the blocklace algebraic counterpart of the conditions for ‘commit & terminate’ of Byzantine reliable broadcast protocols [1].

Definition 3.10 (Ratified and Super-Ratified Block). A block \( b \in B \) is ratified by block \( b' \) if \( b' \rhd b \) includes a supermajority of blocks that approve \( b \). It is super-ratified in blocklace \( \mathcal{B} \subset \mathcal{B} \) if \( B \) includes a supermajority of blocks, each of which ratifies \( b \).

Definition 3.11 (Wavelength, Leader Selection Function, Leader Block). Given a wavelength \( w \geq 1 \), a leader selection function is a partial function \( l : \mathbb{N} \rightarrow \Pi \) satisfying \( l(r) \in P \text{ if } r \mod w = 0 \) else \( l(r) = \bot \). A \( p\)-block \( b \) is a leader block if \( l(depth(b)) = p \).

Note that an equivocating leader can have several leader blocks in the same round. In the following we assume given wavelength \( w \geq 1 \) and a leader selection function \( l \). The following definition has two cases: The first applies to retrospective leader selection, employed by the Cordial Miners protocol for asynchrony using a shared coin; the second to prospective leader selection, employed by the Cordial Miners protocol for eventual synchrony. See Figure 1.B.
Figure 1: Observation, Equivocation, Approval, Ratification, Super-Ratification: (A) Observing an Equivocation: The 'drop shape' depicts a blocklace with a block at the tip of the drop that observes all the blocks inside the drop. Initial blocks are at the bottom, and inclusion among two drops implies $\succ$ among their tips. Assume $b_1, b_2$ are an equivocation (Def. 3.4) by the red miner. According to the figure, the green block approves $b_2$ (Def. 3.5) since it observes $b_2$ (Def. 3.3) and does not observe any conflicting red block, in particular it does not observe $b_1$. However, since the purple block observes the red green block, it observes both $b_1$ and $b_2$ and hence does not approve the equivocating $b_1$ (nor $b_2$). (B) Ratified: The blue block (blue dot) is ratified by the red block (red dot), since the red block observes a supermajority (thick green horizontal line) that approves the blue block. (C) Super-Ratified: The blue block is super-ratified by the red supermajority (thick red horizontal line), each member of which ratifies the blue block by observing a green supermajority (possibly a different one for each member of the red supermajority) that approves the blue block. A leader is final if super-ratified within a protocol-specific wave (number of rounds).

Algorithm 1 Cordial Miners: Blocklace Utilities

pseudocode for miner $p \in \Pi$

```
Local variables:
structure block b:
  b.creator – the miner that created b
  b.payload – a set of transactions
  b.pointers – a possibly-empty set of hash pointers to other blocks
blocklace ← {}

1: procedure create_block(d)
  2: new b
  3: b.payload ← payload()
  4: b.creator ← p
  5: b.pointers ← hash(tips), where tips are the tips of blocklace_prefix(d)
  6: blocklace ← blocklace ∪ {b}
  7: output_blocks()
  8: return b
9: procedure hash(b) return collision-free cryptographic hash pointer to block b
10: procedure b ⪰ b' return
11:    return ∃b_1, b_2, . . . , b_k ∈ blocklace, k ≥ 1, s.t. b_1 = b, b_k = b' and ∀i ∈ [k − 1]: hash(b_{i+1}) ∈ b_i.pointers
12: procedure closure(b) return {b' ∈ blocklace : b ⪰ b'}
  13: procedure equivalence(b_1, b_2) return b_1.creator = b_2.creator ∧ b_1 ⩾ b_2 ∧ b_2 ⩾ b_1
14: procedure equivoocator(q, b) return ∃b_1, b_2 ∈ {b} ∧ b_1.creator = b_2.creator = q ∧ equivalence(b_1, b_2)
15: procedure approves(b, b_1) return b ⪰ b_1 ∧ (b ⪰ b_2 =⇒ ¬equivalence(b_1, b_2))
16: procedure ratifies(b, b_2) return \{b.creator : b_1 ⪰ b ∧ approves(b, b_2)\} is a supermajority
17: procedure depth(b) return max \{k : ∃b' ∈ blocklace with a path from b to b' of length k\}
18: procedure blocklace_prefix(d) return \{b ∈ blocklace : depth(b) ≤ d\}
```
**Definition 3.12 (Final Leader Block).** Let $B \subseteq \mathcal{B}$ be a blocklace. A leader block $b \in B$ of round $r$ is **final** in $B$ if:

1. **Retrospective:** It is super-ratified in $B(r + w - 1)$, or
2. **Prospective:** It is super-ratified in $B(r + w)$ and the leader block of round $r + w$ also ratifies $b$.

The following notion of blocklace safety is the basis for the monotonicity of the blocklace ordering function $\tau$, and hence for the safety of a protocol that uses $\tau$ for blocklace ordering.

**Definition 3.13 (Blocklace Leader Safety).** A blocklace $B$ is **leader-safe** if every final leader block in $B$ is ratified by every subsequent leader block in $B$.

Next, we identify a sufficient condition for blocklace leader safety.

**Definition 3.14 (Cordial Block, Blocklace).** A block $b \in \mathcal{B}$ of round $r$ is **cordial** if it acknowledges blocks by a supermajority of miners. A blocklace $B \subseteq \mathcal{B}$ is **cordial** if all its blocks are cordial.

The following proposition ensures that in a cordial blocklace, a final leader block is ratified by any subsequent leader block. See Figure 2.

**Proposition 3.15 (Cordial Blocklace Leader Safety).** A cordial blocklace is leader-safe.

**Proof of Proposition 3.15.** Let $B$ be a cordial blocklace and $b \in B$ (blue dot) a final leader block of round $r$ in $B$. We have to show $b$ is ratified by any subsequent leader block in $B$. We consider in turn the two cases in Definition 3.12, and in reference to the three cases in Figure 2. Consider the case of a prospective leader, in which the leader of round $r + w$ is included in the ratifying supermajority:

A. The subsequent leader block $b'$ of round $r + w$ (red dot) ratifies $b$ by definition. For a further subsequent leader block $b''$ (purple dot) of a round $r'' > r + w$, there are two cases:

B. There is an overlap between the supermajority observed by $b'$ at round $r'' - 1$ (thick purple line) and the supermajority that ratifies $b$ (thick red line). Then there is a block $b'''$ (black dot) shared by both, hence $b'$ observes $b'''$, which observes a supermajority (thick green line) that approves $b$, hence $b'$ ratifies $b$.

C. The supermajority observed by $b'$ at round $r'' - 1$ (thick purple line) is of a later round than the members of the supermajority that ratifies $b$ (thick red line). By counting, there is a non-equivocating miner $p \in P$ with a block $b_1$ in the purple supermajority and a block $b_2$ in the red supermajority (black dots). Since $p$ is non-equivocating, $b_1$ observes $b_2$. Hence, $b'$ observes $b_1$, which observes $b_2$, which observes the green supermajority that approves $b$, hence $b'$ ratifies $b$.

For the retrospective notion of a final leader, the arguments in cases B and C above apply as is. This completes the proof. □

### 3.3 Blocklace Liveness

The notion of blocklace leader liveness is defined thus:
**Definition 3.16 (Blocklace Leader Liveness).** A blocklace \( B \) is **leader-live** if for every block \( b \in B \) by a miner not equivocating in \( B \) there is a final leader block in \( B \) that observes \( b \).

Next, we discuss conditions that ensure blocklace leader liveness. Given a blocklace, a set of miners \( P \) is (mutually) disseminating if every block by a miner in \( P \) is eventually observed by every miner in \( P \).

**Definition 3.17 (Disseminating).** Given a blocklace \( B \subseteq B \), a set of miners \( P \subseteq \Pi \) is **mutually disseminating** in \( B \), or **disseminating** for short, if for any \( p, q \in P \) and any \( p \)-block \( b \in B \) there is a \( q \)-block \( b' \in B \) such that \( b' > b \). The blocklace \( B \) is **disseminating** if it has a disseminating supermajority.

**Observation 3** (Dissemination is Unbounded). If a set of miners \( P \subseteq \Pi, |P| > 1 \), are disseminating in a blocklace \( B \) that includes a \( p \)-block, \( p \in P \), then \( B \) is infinite and any suffix of \( B \) has blocks by every member of \( P \).

The following liveness condition will be proven for each Cordial Miners protocol, thus establishing their liveness (Figure 3).

**Proposition 3.18 (Blocklace Leader Liveness Condition).** If \( B \subseteq B \) is a cordial blocklace with a non-equivocating and disseminating supermajority of miners, such that for every \( r > 0 \) there is a final leader block of round \( r' > r \), then \( B \) is leader-live.

**Figure 3: Liveness Condition, Proposition 3.18**

Proof of Proposition 3.18. Let \( B \subseteq B \) be a cordial blocklace and \( P \subseteq \Pi \) a supermajority of miners non-equivocating and disseminating in \( B \). Let \( b \) be a \( p \)-block by a miner \( p \in P \) (blue dot in Figure 3). As \( P \) are disseminating in \( B \), then for every \( q \in P \) there is a first \( q \)-block \( b_q \in P \) that observes \( b \); let \( r \) be the maximal round of any of these blocks (thick horizontal red line). By assumption, \( B \) has a final leader block \( \hat{b} \) of round \( \hat{r} > r \) (purple dot). As \( \hat{b} \) is cordial, it must observe a block \( b'_{\hat{q}} \) of depth \( \hat{r} - 1 \) of a miner \( q \in P \) (black dot). As \( q \) is non-equivocating, there is a (possible empty) path from \( b'_{\hat{q}} \) to \( b_q \) (black path among black dots), and from there to \( b \) (blue line). Hence \( \hat{b} \) observes \( b \). □

### 4 BLOCKLACE ORDERING WITH \( \tau \): SAFETY AND LIVENESS

Here we present a deterministic function \( \tau \) that, given a block \( b \), employs final leaders to topologically sort the blocklace \( \{b\} \) into a sequence of its blocks, respecting \( > \). The intention is that in a blocklace-based ordering consensus protocol, each miners would use \( \tau \) to locally convert their partially-ordered blocklace into the totally-ordered output sequence of blocks.

The section concludes with Theorem 4.6, which provides sufficient conditions for the safety and liveness of any blocklace-based ordering consensus protocol that employs \( \tau \). In the following section we prove that the Cordial Miners protocols, which employ Algorithm 2 that realizes \( \tau \), satisfy these conditions, and thus establish their safety and liveness.

We show that if \( \tau \) is called with a sequence of super-ratified blocks increasing \( \text{wrt} \ > \) then its output is monotonic \( \text{wrt} \) to the subset relation. This monotonicity ensures finality, as it implies that the output sequence will only extend while the local blocklace that is the input of \( \tau \) increases over time. With \( \tau \), final leaders are the anchors of finality in the growing chain, each ‘writes history’ backwards till the preceding final leader. We use the term ‘Okazaki fragments’ for the sequences computed backwards from each leader to its predecessor, acknowledging the analogy with the way one of the DNA strands of a replicated DNA molecule is elongated via the stitching of backwards-synthesized Okazaki-fragments [35].

Proposition 4.3 ensures that given a blocklace \( B \), a super-ratified leader \( b \) in \( B \) will be ratified by any subsequent cordial leader, and hence will always initiate an Okazaki-fragment in any execution of \( \tau \) with any subsequent final leader. Hence, the final sequence up to a super-ratified leader \( b \) is fully-determined by \( b \) itself independently of the (continuously-changing) identity of the last super-ratified leader. Hence the final sequence up to a super-ratified leader \( b \) can be ‘cached’ and will not change as the blocklace increases. Proposition ensures that if \( \tau \) is incrementally applied to a leader-live blocklace, then any block in its input blocklace will eventually be included in its output sequence. Together they provide the conditions for Theorem 4.6.

The following recursive ordering function \( \tau \) maps a blocklace into a sequence of blocks, excluding equivocations along the way. Formally, the entire sequence is computed backwards from the last super-ratified leader, afresh by each application of \( \tau \). Practically, a sequence up to a super-ratified leader \( b \) is fully-determined by \( b \) itself independently of the (continuously changing) identity of the last super-ratified leader. Hence the final sequence up to a super-ratified leader \( b \) can be ‘cached’ and will not change as the blocklace increases. Proposition ensures that if \( \tau \) is incrementally applied to a leader-live blocklace, then any block in its input blocklace will eventually be included in its output sequence. Together they provide the conditions for Theorem 4.6.

**Definition 4.1 (\( \tau \)).** We assume a fixed topological sort function \( \text{xsort}(b, B) \) (exclude and sort) that takes a block \( b \) and a blocklace \( B \), and returns a sequence consistent with \( > \) of all the blocks in \( B \) that are approved by \( b \). The function \( \tau : 2_B \rightarrow B^* \) is defined for a
Any leader will eventually have a subsequent final leader (large dot) with probability 1. (C) Leader-Based Equivocation Requirement: A final leader (large dot) is ratified by any subleader does not observe the red equivocation. However, the exclusion has already been computed by the previous invocation of the output sequence up to the previous final leader has already computed. The output form each fragment is a sequence of the current leader but not observed by the previous ratification consists of the portion of the blocklace observed by the previous leader it ratifies. The input to computing the fragment consists of the portion of the blocklace observed by the current leader but not observed by the previous ratified leader. The output form each fragment is a sequence of blocks computed forward by topological sort of the input blocklace fragment, respecting > and using the leader of the fragment to resolve and exclude equivocations. Final leaders are final, hence the backwards computation starting from the last purple final leader need not proceed beyond the recursive call to the previous red final leader, as the output sequence up to the previous final leader has already been computed by the previous invocation of \( \tau \). Safety Requirement: A final leader (large dot) is ratified by any subsequent leader (large or small dot). Liveness Requirement: Any leader will eventually have a subsequent final leader (large dot) with probability 1. (C) Leader-Based Equivocation Exclusion: The green Okazaki fragment created by the green leader includes the \( V \)-marked red block, since the green leader does not observe the red equivocation. However, the red \( X \)-marked red block is excluded from the purple fragment created by the purple leader, since the purple leader observes the equivocation among the two red blocks. (See also Figure 1.A)

blocklace \( B \subset \mathcal{B} \) backwards, from the last output element to the first, as follows: If \( B \) has no final leaders then \( \tau(B) := \Lambda \). Else let \( b \) be the last final leader in \( B \). Then \( \tau(B) := \tau'(b) \), where \( \tau' \) is defined recursively:

\[
\tau'(b) := \begin{cases} 
\text{xsort}(b, [b]) & \text{if } [b] \text{ has no leader ratified by } b, \text{ else} \\
\tau'(b') \cdot \text{xsort}(b, [b] \setminus [b']) & \text{if } b' \text{ is the last leader ratified by } b \text{ in } [b] 
\end{cases}
\]

Note that \( \tau' \) uses the notion of a leader ratified by another leader, not a final leader.

A pseudo-code implementation of \( \tau \) is presented as Algorithm 2. The Algorithm is a literal implementation of the mathematics described above: It maintained outputBlocks that includes the prefix of the output \( \tau \) has already computed. Upon adding a new block to its blocklace (line 19), it computes the most-recent final leader \( b_1 \) according to Definition 3.12, and applies \( \tau \) to it, realizing the mathematical definition of \( \tau \) (Def. 4.1). With the optimization, discussed above, that a recursive call with a block that was already output is returned. Hence the following proposition:

**Proposition 4.2 (Correct implementation of \( \tau \)).** The procedure \( \tau \) in Algorithm 2 correctly implements the function \( \tau \) in Definition 4.1.

### 4.1 \( \tau \) Safety

A safe blocklace ensures a final leader is ratified by any subsequent leader, final or not. Hence the following:

**Proposition 4.3 (Monotonicity of \( \tau \)).** Let \( B \) be a cordial blocklace with a supermajority of correct miners. Then \( \tau \) is monotonic wrt the superset relation among closed subsets of \( B \), namely for any two closed blocklaces \( B_2 \subseteq B_1 \subseteq B \), \( \tau(B_2) \leq \tau(B_1) \).

**Proof of Proposition 4.3.** Let \( B, B_1, B_2 \) be blocklaces as assumed by the Proposition. If \( B_2 \) has no final leader then \( \tau(B_2) \) is the empty sequence and the proposition holds vacuously. Let \( b_2 \) be the last final leader of \( B_2 \) and \( b_1 \) be the last final leader of \( B_1 \). Note that according to Definition 4.1, \( \tau(B_1) \) calls \( \tau'(b_1) \) and \( \tau(B_2) \) calls \( \tau'(b_2) \). Let \( b_1, b_2, \ldots, b_k, k \geq 2 \), be the sequence of ratified leaders in the recursive calls of the execution of \( \tau'(b_1) \), starting with \( b_1 = b_1 \). We argue that \( b_2 \) is called in this execution, namely \( b_2 = b_j \) for some \( j \in [k] \). Note that according to Proposition 3.15, \( b_1 = b_1 \), being cordial, ratifies \( b_2 \). Let \( j \in [k] \) be the last index for which \( b_j \) ratifies \( b_2 \). We argue by way of contradiction that \( b_{j+1} = b_2 \). Consider three cases regarding the relative depths of \( b_{j+1} \) and \( b_2 \):

1. Note that two different blocks of the same depth cannot observe each other: If only one observes the other, it is one deeper than the other; if both observe each other they form a cycle, which is impossible. Since both \( b_{j+1} \) and \( b_2 \) are leader blocks of the same depth, then they must be by the same leader, and hence, being different blocks by the same miner that do not observe each other, they form an equivocation. By assumption, both are ratified, implying that both have supermajority approval, contradicting the assumption that there is a supermajority of correct miners in \( B \).
Algorithm 2 Cordial Miners: Ordering of a Blocklace with $\tau$

**pseudocode for miner $p \in \Pi$, including Algorithms 1 & 4**

| Line | Code | Description |
|------|------|-------------|
| 19.  | procedure output_blocks() |  |
| 20.  | currentLeader $\leftarrow$ last_final_leader() |  |
| 21.  | $\tau$(currentLeader) |  |
| 22.  | procedure $\tau(b_1)$ |  |
| 23.  | if $b_1 \in$ outputBlocks $\lor$ $b_1 = \emptyset$ then return |  |
| 24.  | $b_2 \leftarrow$ previous_ratted_leader($b_1$) |  |
| 25.  | $\tau(b_2)$ |  |
| 26.  | output xsort($b_1$, $\{b_1\} \setminus \{b_2\}$) |  |
| 27.  | outputBlocks $\leftarrow$ outputBlocks $\cup$ xsort($b_1$, $\{b_1\} \setminus \{b_2\}$) |  |
| 28.  | procedure xsort($b$, $\mathcal{B}$) |  |
| 29.  | return topological sort wrt $>$ of the set $\{b' \in \mathcal{B} : approved(b', b)\}$ |  |
| 30.  | procedure previous_ratted_leader($b_1$) |  |
| 31.  | return $\arg\max R$ max depth($b$) |  |
| 32.  | where $R = \{b \in \{b_1\} \setminus \{b_1\} : b.creator = leader(depth(b)) \land ratified(b, b_1)\}$ |  |
| 33.  | procedure last_final_leader() |  |
| 34.  | return $\arg\max U$ max depth($u$) where |  |
| 35.  | $U = \{b \in$ blocklace : $b.creator = leader(depth(b)) \land final_leader(b)\}$ |  |
| 36.  | procedure leader() is defined in Algorithm 4 |  |
| 37.  | procedure final_leader() is defined Algorithm 4 |  |

> If $b_{j+1}$ is deeper than $\hat{b}_2$, then by Definition 4.1, $\tau'$ elects the first leader ratified by the current leader $b_j$, and hence cannot prefer calling $b_{j+1}$ over $\hat{b}_2$, which precedes it by assumption.  
< If $\hat{b}_2$ is deeper, then Proposition 3.15 implies that $b_{j+1}$ ratifies $\hat{b}_2$, in contradiction to the assumption that $b_j$ is the last leader in the list that ratifies $\hat{b}_2$. Hence $\hat{b}_2$ is included in the recursive calls of $\tau'(\hat{b}_1)$, which, according to Definition 4.1 of $\tau$, implies that $\tau(b_2) \preceq \tau(b_1)$.

**Observation 4 (Consistent triplet).** Given three sequences $x, x', x''$, if both $x' \preceq x$ and $x'' \preceq x$ then $x'$ and $x''$ are consistent.

The following Proposition ensures that if there is a supermajority of correct miners, which jointly create a cordial blocklace, then the output sequences computed by any two miners based on their local blocklaces would be consistent. This establishes the safety of $\tau$ under these conditions.

**Proposition 4.4 ($\tau$ SAFETY).** Let $B$ be a blocklace with a supermajority of correct miners. Then for every $B_1, B_2 \subseteq B$, $\tau(B_1)$ and $\tau(B_2)$ are consistent.

**Proof of Proposition 4.4.** By monotonicity of $\tau$ (Prop. 4.3), both $\tau(b_1) \preceq \tau(B_1 \cup B_2)$ and $\tau(b_2) \preceq \tau(B_1 \cup B_2)$. By Observation 4, $\tau(b_1)$ and $\tau(b_2)$ are consistent.  

4.2 $\tau$ Liveness

While $\tau$ does not output all the blocks in its input, as blocks not observed by the last final leader in its input are not in its output, the following observation and proposition sets the conditions for $\tau$ liveness:

**Observation 5 ($\tau$ output).** If a $p$-block $b \in B$ by a miner $p$ not equivocating in $B$ is observed by a final leader in $B$, then $b \in \tau(B)$.

**Proof of Observation 5.** Since $b$ is observed by a final leader in $B$, it is also observed by the last final leader of $B$. Consider the recursive construction of $\tau(B)$. If in its last recursive call $\tau'(b')$, $b'$ observes $b$, then by definition of $\tau$, $b, b' \in \tau(b')$ and hence $b \in \tau(B)$. Otherwise, consider the first recursive call $\tau'(b')$, by $\tau'(b'')$ in which $b''$ observes $b$ but $b'$ does not observe $b$. Then by definition of $\tau'$, $b \in \tau'(b'')$ and hence $b \in \tau(B)$.

**Proposition 4.5 ($\tau$ LIVENESS).** Let $B_1 \subset B_2 \subset \ldots$ be a sequence of finite blocklaces for which $B = \bigcup_{i \geq 1} B_i$ is a cordial leader-live blocklace. Then for every block $b \in B$ by a correct miner in $B$ there is an $i \geq 1$ such that $b \in \tau(B_i)$.

**Proof of Proposition 4.5.** Let $B$ be as assumed and $b \in B$. As $B$ is leader-live, there is a final leader block $b'$ that observes $b$. Let $i \geq 1$ be an index for which $B_i$ includes $b'$. Consider the call $\tau(B_i)$. Since $b$ is a final leader in $B_i$ than according to Observation 5, the output of $\tau(B_i)$ includes $b$.

And based on it, we conclude that the safety and liveness properties of $\tau$ carry over to Algorithm 2.

The following Theorem provides a sufficient condition for the safety and liveness (Def. 2.3) of any blocklace-based ordering consensus protocol that employs $\tau$:
Theorem 4.6 (Sufficient Condition for the Safety and Liveness of a Blocklace-Based Ordering Consensus Protocol). A blocklace-based consensus protocol that employs \(r\) for ordering is safe and live if in every run all correct miners have in the limit the same blocklace \(B\), and \(B\) is leader-safe and leader-live.

Proof of Theorem 4.6. Let \(P \subseteq \Pi\) be the correct miners in a run of the protocol that produce in the limit the blocklace \(B\). The protocol is safe since the local blocklaces of any two miners in \(p, q \in P\) at any time are subsets of \(B\), hence by Proposition 4.4 the outputs of \(p\) and \(q\) are consistent. The protocol is live since by Proposition 4.5, every block \(b \in B\) will be output by every correct miner \(p \in P\).

Next, we prove that the two Cordial Miners consensus protocols—eventual synchrony and asynchrony—satisfy the conditions of Theorem 4.6, and hence are safe and live.

5 THE CORDIAL MINERS PROTOCOLS

The shared components of the Cordial Miners protocols are specified via pseudocode in Algorithms 1 (blocklace utilities), 2 (the ordering function \(r\)) and 3 (dissemination). There are several differences between the Cordial Miners protocols for eventual synchrony and for asynchrony: Wavelength (2 or 5); leader selection (prospective or retrospective); the notion of a final leader (whether it includes ratification by the next leader or not); and when is a round complete (based only on cordiality or also on including or observing the leader or a timeout). These differences are summarized in Table 2 and are specified via pseudocode in Algorithm 4.

The main Theorem we prove here is the following:

Theorem 5.1 (Cordial Miners Protocols Safety and Liveness). The Cordial Miners protocols for eventual synchrony and for asynchrony specified in Algorithms 1, 2, 3 & 4 are safe and are live.

Proof Outline. We prove two propositions that together establish the Theorem: Proposition 5.5 shows that the two Cordial Miners protocols are safe and Proposition 5.6 shows that they are live.

In the Cordial Miners protocols there is a tradeoff between latency and messages complexity, analogously to this same tradeoff in reliable-broadcast protocols [18], and there is a range of possible optimizations and heuristics. These are discussed in Section 6. Here, we present latency-optimal Cordial Miners protocols in which every block is communicated among every pair of correct miners. In a Cordial Miners protocol, each miner maintains a history array that records its communication history with the other miners, and updates it upon accepting (line 42) and sending (line 48) blocks. When a miner \(p\) creates a block \(b\), it includes in it pointers to the tip of its blocklace (line 45), so that \([b]\) is identical to the updated blocklace that includes \(b\). It then sends \(b\) to all other miners (line 48), sends to each miner \(q\) all the blocks that \(p\) knows but \(q\) does not know, based on their communication history. Namely, it sends to \(q\) the blocklace \([b] \setminus [\text{history}(q)]\) (line 48). A recipient of a block defers accepting it until the block has no dangling pointers in its local blocklace (line 39), so that adding the block to the local blocklace would retain it being closed.

5.1 Cordial Miners Safety

We argue next that in the limit the blocklaces of correct miners that participate in a run of a Cordial Miners protocol are identical, are leader-safe and leader-live. Variations and optimizations are discussed in Section 6.

A formal description of blocklace-based protocols in terms of asynchronous multiagent transition systems with faults has been carried out in reference [37]. Here, we employ pseudocode, presented in Algorithms 1, 2, 3 & 4 to describe the correct behaviors of a miner in a protocol, and discuss only informally the implied multiagent transition system and its computations. We assume that the run of the protocol by the miners \(\Pi\) results in a sequence of configurations \(r = e_0, e_1, \ldots,\), each encoding the local state of each miner. A miner is correct in a run \(r\) if it behaves according to the pseudocode during \(r\), faulty otherwise. As stated above, we assume that there are at most \(f \leq n/3\) faulty miners in any run. We use \(B_p(e)\) to denote the local blocklace of miner \(p \in \Pi\) in configuration \(e\), \(\bigcup_{e \in R} B_p(e)\), and \(B(r)\) to denote the unions of the blocklaces of all correct miners in the limit, \(B(r) := \bigcup_{P \subseteq \Pi} B_p(r)\), where \(P \subseteq \Pi\) is the set of miners correct in run \(r\).

Proposition 5.2 (Miner Asynchrony). If a miner can create a block (line 45) then it can create it also after receiving blocks from other miners.

Proof of Proposition 5.2. Examination of the completed_round procedures of Algorithm 4, which gate block creation in Algorithm 3, line 45, shows that if it holds for a blocklace it holds after blocks by other miners are received and buffered or added to the local blocklace.

Miner asynchrony combined with the standard notion of fairness, that a transition that is enabled infinitely often in a run is eventually taken in the run, implies that once a Cordial Miners block creation transition is enabled then it will eventually be taken.

Proposition 5.3 (Miners Liveness). In a fair run of a Cordial Miners protocol with correct miners \(P \subseteq \Pi\), if there is a configuration for which completed_round() \(\geq d\) (line 44) for \(d \geq 0\) and for the blocklace of every miner \(p \in P\), then there is a subsequent configuration for which completed_round() \(\geq d + 1\) for the blocklace of every miner \(p \in P\).

Proof of Proposition 5.3. We show by induction on the round number. Consider a configuration \(c\) in which the depth of the last completed round in the blocklace of all miners be \(d \geq 0\). If \(d = 0\) then the completed_round() call (line 44 in both Cordial Miners protocols (Algorithm 4, line 57 and line 66) returns 0 and \(p\) can create a initial block (line 45) with no predecessors (line 5). Assume \(d > 0\). Consider a miner \(p \in P\) that has not yet created a block of depth \(d + 1\) in \(c\). Then the condition completed_round() \(\geq r\) (line 44) holds for \(r = d\), and the transition to create the next block is enabled. By miner asynchrony (Proposition 5.2) such a transition is enabled indefinitely, and by the fairness assumption it is eventually taken, in which \(p\) sends a new \(p\)-block \(b\) of depth \(d + 1\) to all other miners (line 48), and includes, for each miner \(q\), all the blocks in the closure of \(b\) that \(q\) might not know of, based on the communication history of \(p\) with \(q\). By assumption, all said messages among correct miners
Algorithm 3 Cordial Miners: Blocklace-Based Dissemination

**Pseudocode for miner** \( p \in \Pi \), including Algorithms 1, 2 & 4

**Local variables:**
- array `history(n)`, initially \( \forall k \in [n] : history(k) \leftarrow {} \)
- \( r \leftarrow 0 \)  
  - \( \triangleright \) Communication history of \( p \)
  - \( \triangleright \) The current round of \( p \)
- \( \triangleright \) send and \( \triangleright \) receipt are simple messaging on a reliable link

```plaintext
upon receipt of \( b \) do
  if `cordial_block(b)` then `buffer` \( \leftarrow \) `buffer` \( \cup \) \{\( b \)\}
while true do
  for \( b \in \text{buffer} : (b\text{-pointers} \subseteq \text{hash(blocklace)} \land \forall b' \in \text{blocklace} : \neg \text{equivocator}(b\text{-creator}, b')) \) do
    `buffer` \( \leftarrow \) `buffer` \( \setminus \) \{\( b \)\}
    `blocklace` \( \leftarrow \) `blocklace` \( \cup \) \{\( b \)\}
    `history(q)` \( \leftarrow \) `history(q)` \( \cup \) \{\( b \)\}
    `output_blocks()`
  if \( \text{completed_round()} \geq r \) then
    \( b \leftarrow \text{create_block(completed_round())} \)
    \( r \leftarrow \text{depth}(b) \)
    for \( q \in \Pi \land q \neq p \land \neg \text{equivocator}(q, b) \) do
      `send` \( \{b\} \setminus \{\text{history}(q)\} \) to \( q \)
      `history(q)` \( \leftarrow \) `history(q)` \( \cup \) \{\( b \)\}
  \end{algorithmand}

```plaintext
procedure `cordial_block(b)`
return \{\( b\text{-creator} : b > b' \land \text{depth}(b') = \text{depth}(b) - 1 \)\} is a supermajority.

`procedure output_blocks()` is defined in Algorithm 2

`procedure completed_round()` is defined in Algorithm 4

| Property                       | Eventual Synchrony | Asynchrony |
|-------------------------------|--------------------|------------|
| Wavelength \( w \)            | 2                  | 5          |
| Leader Selection              | Prospective        | Retrospective |
| Leader Final if:              | Super-ratified & Ratified by next leader within \( w \) | Super-ratified within \( w \) |
| Round Completed if:           | Cordial and (includes or ratifies leader, or timeout) | Cordial |
| Probability of Wave Success:  | \( \frac{3}{5} \)    | \( \frac{1}{5} \)  |

Table 2: Differences Between the Cordial Miners Protocols for Eventual Synchrony and for Asynchrony

eventually arrive at their destination. Hence there is some subsequent configuration \( c' \) in which every correct miner has receives a \( d + 1 \)-depth \( q \)-block \( b \) from every other correct miner \( q \), as well as all preceding blocks to \( b \). Hence in \( c' \), \( \text{completed_round()} \geq r \) holds for \( r = d + 1 \) for every correct miner. □

**Proposition 5.4 (Cordial Miners Dissemination).** In a run \( r \) of a Cordial Miners protocol with correct miners \( P \in \Pi \), \( B(r) = B_p(r) \) for every miner \( p \in P \).

**Proof of Proposition 5.4.** Given a run \( r \) of a Cordial Miners protocol with correct miners \( P \subset \Pi \), we have to show that for any \( p, q \in P \), a configuration \( c \) of the run, and a block \( b \) in the blocklace of \( p \) is configuration \( c \), there is a subsequent configuration \( c' \) of the run in which \( b \) is in the local blocklace of \( q \). By miners liveness (Proposition 5.3), for any miner \( q \), there is a subsequent configuration by which miner \( p \) sends a block to miner \( q \). According to the dissemination protocol (line 48), \( b \) is also sent to \( q \), minus any blocks in \( b \) that have been previously communicated between \( p \) and \( q \). Hence there is a subsequent configuration to \( c' \) in which \( b \) is included in the blocklace of \( q \). □

We refer to \( B(r) \) as the **blocklace of run** \( r \), and conclude that every miner \( p \) correct in a run produces the blocklace of the run, namely \( B_p(r) = B(r) \).

We can now argue the safety of the Cordial Miners protocols:

**Proposition 5.5 (Cordial Miners Protocol Safety).** The Cordial Miners protocols for asynchrony and synchrony are safe.

**Proof of Proposition 5.5.** According to Proposition 5.4, in any computation of a Cordial Miners protocol, the local blocklace of any two correct miners \( p, q \in \Pi \) is the same blocklace \( B \). Hence, in any configuration of the computation, the local blocklaces of \( p \) and \( q \) are subsets of \( B \), and hence according to Proposition 4.4, their
We now proceed to argue the liveness of the Cordial Miners protocol. This is the liveness requirement of ordering consensus protocols (Def. 2.3). Hence the Cordial Miners protocols are liveness live.

4.2 Procedures for Asynchrony

```plaintext
62: w ← 5

63: procedure leader(d) return p ∈ Π via a random shared coin tossed at round d + w − 1 if d mod w = 0 else ⊥.

64: procedure final_leader(b)

65: return {b.creator : b ∈ blocklace ∧ depth(b) = r} is a supermajority ∧

66: procedure completed_round()

67: return \( \max r : \{b.creator : b ∈ blocklace ∧ depth(b) = r\} \) is a supermajority ∧
```

Next we prove the required leader-liveness propositions.

5.2 Cordial Miners Liveness

We now proceed to argue the liveness of the Cordial Miners protocols.

Proposition 5.6 (Cordial Miners Protocol Liveness). The Cordial Miners protocols for asynchrony and eventual synchrony are live.

Proof of Proposition 5.6. According to Propositions 5.7 and 5.14 below, the blocklace produced in any computation of the Cordial Miners protocols for eventual synchrony and for asynchrony is leader-live with probability 1. According to Proposition 4.5, if the function \( r \) applied to a sequence of blocklaces that converge to a leader-live blocklace \( B \) then any \( b \in B \) appears eventually in the output of \( r \). If the blocklace \( B \) is leader-live with probability 1 then any \( b \in B \) appears eventually in the output of \( r \) with probability 1, which is the liveness requirement of ordering consensus protocols (Def. 2.3). Hence the Cordial Miners protocols are live.

Next we prove the required leader-liveness propositions.

Proposition 5.7 (Leader-Liveness Cordial of Miners Eventual Synchrony Protocol). The blocklace produced by a run of a Cordial Miners eventual synchrony protocol is leader-live with probability 1.

Proof of Proposition 5.7. Let \( B \) be the cordial blocklace produced by a run of a Cordial Miners eventual synchrony protocol, \( P \subseteq \Pi \) the supermajority of miners correct in the run, and let \( r > 0 \) be any round for which \( r − 1 \) suffix of \( B \), \( B(r − 1) \), is equivocation-free, \( r \) mod \( w = 0 \), where \( w = 2 \) (line 52). Let \( r' > r \) be any even round following network synchronization (GST). Since leader selection is pseudo random (line 53), and \( P \) is a supermajority, there is probability of \( \frac{|P|}{\pi} \) that the \( r' \)-leader \( q \) is correct, namely \( q \in P \). As timeout does not affect correct miners after GST, then for every \( p \in P \), the \( r' \)-depth \( q \)-block \( b \) is approved by the \( r' + 1 \) \( P \)-block (line 59 and ratified by the \( r' + 2 \)-round block of \( P \). Hence, the blocks of the correct miners \( P \) satisfy the conditions of \( b \) being a final leader (line 54) for the Cordial Miners eventual synchrony protocol. As this holds also for the leader of any round following \( r \), the probability that for any depth \( r' \geq r \), a leader in \( B(r') \) has a final leader is 1, hence \( B \) is leader-live with probability 1. Hence the condition of Proposition 3.18, that for every \( r > 0 \), \( B \) has a final leader block of some round \( r' > r \) with probability 1, is satisfied and we conclude that \( B \) is leader-live with probability 1.

We note that, following GST, the probability of a leader block being final is at least \((\frac{|P|}{\pi})^2\), and given that \( w = 2 \), if \( \frac{|P|}{\pi} > \frac{2}{\pi} \) then the expected latency is at most \(2/(\frac{2}{\pi})^2 = 4.5\).

The Cordial Miners asynchrony protocol, for which \( w = 5 \) (line 62), elects leaders retroactively using a shared random coin. To elect the leader of round \( r \), where \( r \) mod \( 5 = 0 \), all correct miners toss the coin in round \( r + 3 \) and know in round \( r + 4 \) the elected leader of round \( r \), as follows. To elect a leader

Definition 5.8 (Blocklace-Based Shared Random Coin). We assume two shared random coin functions, \( toss \_coin \) and \( combine \_tosses \). The function \( toss \_coin(p, d) \) takes the secret key \( p \) of miner \( p \in \Pi \) and a round number \( d > 0 \) as input, and produces \( p \)’s share of the coin of round \( d \), \( s_{p,d} \), as output. If the protocol needs to compute the shared random coin for round \( d \), then \( s_{p,d} \) is incorporated in the payload of the \( d \)-depth \( P \)-block of every correct miner \( p \). The function \( combine \_tosses(S, d) \) takes a set \( S \) of shares \( s_{p,d} \), \( d > 0 \), for which \( |\{p : s_{p,d} \in S\}| > f + 1 \), and returns a miner \( q \in \Pi \).
Assume some \( d > 0 \) and let \( S = \{ \text{toss\_coin}(p, d) : p \in P \} \) for a set of miners \( P \subseteq \Pi \), \( |P| > f + 1 \). Then the functions have the following properties:

**Agreement** If both \( S', S'' \subseteq S \) and both \(|S'|, |S''| > f + 1\), then \( \text{combine\_tosses}(S', d) = \text{combine\_tosses}(S'', d) \)

**Termination** \( \text{combine\_tosses}(S, d) \in \Pi \).

**Fairness** The coin is fair, i.e., for every set \( S \) computed as above and any \( p \in \Pi \), the probability that \( p = \text{combine\_tosses}(S, d) \) is \( \frac{1}{2^{d}} \).

**Unpredictability** If \( S' \subseteq S, |S'| < f + 1 \), then the probability that the adversary can use \( S' \) to guess the value of \( \text{combine\_tosses}(S, d) \) is less than \( \frac{1}{2^{d}} + \epsilon \).

Examples of such a coin implementation using a PKI and threshold signatures [4, 32, 39] are in [10, 28]. See DAG-Rider [28] on details on how to implement such a coin as part of a distributed blocklace-like structure.

The following proposition ensures that in a disseminating cordial blocklace, all correct miners eventually repel all equivocators and stop observing blocks by them.

**Definition 5.9 (Equivocator-Repelling).** Let \( b \in B \) be a \( p \)-block, \( p \in \Pi \), that acknowledges a set of blocks \( B \subseteq B \). Then \( b \) is **equivocator-repelling** if \( p \) does not equivocate in \([b]\) and all blocks in \( B \) are equivocator-repelling. A blocklace \( B \) is **equivocator-repelling** if every block \( b \in B \) is equivocator-repelling.

Note that the recursive definition terminates with an initial block, where \( B = \emptyset \). Also note that a block (or blocklace) that is equivocator-repelling may include equivocations, for example two equivocating blocks each observed by a different block in \( B \). However, once an equivocation by miner \( q \) is observed by a block \( b \), \( q \) would be repelled: Any block that observes \( b \) would not acknowledge any \( q \)-block, preventing any further \( q \)-blocks from joining the blocklace. Also note that the local blocklace created by a correct miner is equivocation-repelling, due to the last condition in line 39.

**Proposition 5.10 (Equivocators-Free Suffix).** Let \( B \) be an equivocator-repelling and cordial blocklace and \( P \subseteq \Pi \) the set of miners that do not equivocate in \( B \) and are disseminating in \( B \). If \( P \) is a supermajority then there is a depth \( d > 0 \) for which the depth-d suffix of \( B \), \( \tilde{B}(d) \), includes only \( P \)-blocks.

The following Lemma is the blocklace-variant of the notion of a common core [2, 19]. Its proof is an adaptation to the cordial blocklace setting of the common core proof in [19], which in turn is derived from the proof of get-core in [2]. Figure 5 illustrates its proof as well as the proof of the following ratified common core Corollary 5.12.

**Lemma 5.11 (Blocklace Common Core).** Let \( B \) be a cordial blocklace, \( P \subseteq \Pi \) the set of miners that do not equivocate in \( B \) and are disseminating in \( B \), and \( r > 0 \) a depth for which the depth-(\( r - 1 \)) suffix of \( B \), \( \tilde{B}(r - 1) \), includes only \( P \)-blocks (\( r \) exists by Proposition 5.10), and hence equivocation free. If \( P \) is a supermajority then there is a supermajority of \( r \)-round blocks \( \tilde{B} \subseteq B \), referred to as **common core**, such that every \( (r + 3) \)-round block approves every block in \( B \).

**Proof of Lemma 5.11.** Let \( B \) and \( r \) be as assumed by the Lemma, and let \( P \) be the set of miners that have \( (r + 2) \)-round blocks in \( B \). Since \( B \) is cordial and disseminating it is infinite of depth \( > r + 3 \) (Observation 3), it follows that \(|P| \geq 2r + 1\). Define a table \( T \) with rows and columns indexed by \( P \). Each \( (r + 2) \)-round \( p \)-block of a miner \( p \in P \) observes \((r + 1) \)-round blocks by at least \( 2f + 1 \) miners, which includes at least blocks by \( f + 1 \) miners of \( P \), represented in \( T \). For \( p, q \in P \), entry \( T[p, q] \) in the table is 1 if the \((r + 2) \)-round \( p \)-block observes the \((r + 1) \)-round \( q \)-block, and 0 otherwise. Observe that if 1 appears in entry \( T[p, q] \), the \((r + 2) \)-round \( p \)-block observes all the \( 2f + 1 \) \( r \)-round blocks observed by the \((r + 1) \)-round block of \( q \).

Since all miners in \( P \) have \((r + 2) \)-round blocks, \( T \) contains at least \((2f + 1)(f + 1) \) entries with 1. This implies that there is a miner in \( P \), say \( \bar{p} \), that appears in at least \( f + 1 \) rows; let \( 
\bar{P} \) be the set of miners indexing \( \bar{p} \) rows and \( \bar{b} \) (blue dot) the \((r + 1) \)-round \( \bar{p} \)-block. Thus, the \((r + 2) \)-round blocks of miners in \( \bar{P} \) (thick green line at round \( r + 2 \)) observe \( \bar{b} \). We argue that \( [\bar{b}] \) is a common core.
First, note that as $\bar{b}$ is cordial, $[\bar{b}]$ includes $2f + 1$ $r$-round blocks. Second, consider any $(r + 3)$-round block $b$ (green dot). It observes $2f + 1$ $(r + 2)$-round blocks (thick red line), so it also observes at least one of the $(r + 1)$-round blocks (black dot) of the $f + 1$ miners of $P$, which in turn observes $\bar{b}$. Thus $[\bar{b}] \subseteq [b]$, with $[\bar{b}]$ satisfying the requirements of a common core.

COROLLARY 5.12 (SUPER-RATIFIED COMMON CORE). Under the same conditions as Lemma 5.11 and assuming $\bar{B}$ is a common core, then every $(r + 4)$-round block in $B$ ratifies every block in $\bar{B}$. Hence every member of the common core $\bar{B}$, is super-ratified in $B(r + 4)$.

PROOF OF COROLLARY 5.12. Under the assumptions of the Corollary, let $\bar{B}$ be a common core and consider any $(r + 4)$-block $b \in B$ (purple dot). Being cordial, $b$ observes $2f + 1$ $(r + 2)$-round blocks (thick purple line). By Lemma 5.11, each of these blocks observes each block in $\bar{B}$. Hence $b$ ratifies every block in $\bar{B}$. □

Note that the Lemma and Corollary require an equivocators-free section of the blocklace, which may be the entire equivocation-free suffix of the blocklace as in the proof. But the proof also holds if there is a long enough stretch of rounds without equivocation, in which case a common core also exists.

We conclude that if a Cordial Miners protocol relies on the common core for liveness, dissemination and cordiality are sufficient to ensure it.

COROLLARY 5.13 (LIVENESS OF COMMON CORE). Let $B$ be a disseminating and cordial blocklace. Then there is a $d > 0$ for which the common core Lemma 5.11 holds for $B$ and for any round $r \geq d$.

PROPOSITION 5.14 (LEADER-LIVENESS OF CORDIAL MINERS ASYNCHRONY PROTOCOL). The blocklace produced by a run of a Cordial Miners asynchrony protocol is leader-live with probability 1.

PROOF OF PROPOSITION 5.14. Let $B$ be the cordial blocklace produced by a run of a Cordial Miners asynchrony protocol, $P \subseteq \Pi$ the supermajority of miners correct in the run, and let $r > 0$ be any round for which the $(r - 1)$ suffix of $B$, $B(r - 1)$, is equivocation-free, $r \mod w = 0$, where $w = 5$ (line 62). According to Corollary 5.12, a supermajority $\bar{B}$ of the round-$r$ blocks are super-ratified by all round-$r + 4$ blocks. As the leader of round $r$ is selected at random, and retroactively after the common core $\bar{B}$ has been established, the probability that the elected leader is super-ratified, and hence final, is at least $\frac{|P|}{n}$. As this holds also for the leader of any round following $r$, the probability that for any depth $d \geq r$, a leader in $B(d)$ has a final leader is 1, hence $B$ is leader-live with probability 1. □

This completes the proof of Proposition 5.6 and hence also the proof of Theorem 5.1.

6 PERFORMANCE ANALYSIS AND POTENTIAL OPTIMIZATIONS

Latency. (See Table 1). Latency is defined as the number of blocklace rounds between every two consecutive ratified leaders.

In the asynchronous instance of the protocol, each wave (rounds till leader finality) consists of 5 rounds. According to Lemma 5.11, the probability that the decision rule is met, namely that a supermajority of the blocks in $r + 4$ each ratify a leader at $r$ is $\frac{1}{4}$. Therefore, in the expected-case, the decision rule is met on average every 1.5 waves, and therefore the expected latency is 1.5$w = 7.5$ rounds of communication.

Note that the adversary can equivocate or not be cordial up to $f$ times, but after each Byzantine process $p$ equivocates, all correct processes eventually detect the equivocation and do not consider $p$'s blocks as part of their cordial rounds when building the blocklace. Thus, in an infinite run, equivocations do not affect the overall expected latency.

In the eventual synchrony case, each wave consists of 2 rounds. The probability that the decision rule is met in each wave is the probability that there are two correct leaders in a row, therefore, it is at least $\frac{1}{4}$. Thus, in the expected-case, the latency is 4.5 rounds of communication.

Bit complexity. Each block in the blocklace is linear in size, since it has a linear number of hash pointers to previous blocks. In the worst-case, each block is sent to all miners by all the other miners, i.e., the bit complexity is $O(n^2)$ per block. But, if we batch per block a linear number of transactions, when the decision rule is met, we can amortize the bit complexity by a linear factor. Thus, the amortized bit complexity per decision is $O(n^2)$. 
We note that DAG-Rider and Bullshark achieve amortized $O(n)$ message complexity per decision by using a reliable broadcast protocol that relies on erasure coding [17]. We plan as part of future work to incorporate similar techniques as used in [17] to the block-face dissemination protocol to achieve the same amortized message complexity.

**Future directions.** Several optimizations are possible to the protocol instances presented. First, as faulty miners are exposed, they are repelled and therefore need not be counted as parties to the agreement, which means that the number of remaining faulty miners, initially bounded by $f$, decreases. As a result, the supermajority needed for finality is not $\frac{n+f}{2n}$ (namely $2f+1$ votes in case $n = 3f+1$), but $\frac{n+f-2f'}{2(n-f')}$, where $f'$ is the number of exposed faulty miners, which converges to simple majority ($\frac{1}{2}$) among the correct miners as more faulty miners are exposed and $f'$ tends to $f$.

Secondly, once faulty miners are exposed and repelled, their slots as leaders could be taken by correct miners, improving good-case and expected complexity.

Thirdly, a hybrid protocol in the spirit of Bullshark [25] can be explored. Such a protocol would employ two leaders per round – deterministic and random, try to achieve quick finality with the deterministic leader, and fall back to the randomly-selected leader if this attempt fails.

**Exclusion of nonresponsive miners:** A miner $p$ need not be cordial to miner $q$ as long as $q$ has not observed a previous block $b$ sent to $q$ by $p$. If $q$ fails-stop, then $p$ should definitely not waste resources on $q$; if $q$ is only suspended or delayed, then eventually it will send to $p$ a block observing $b$, following which $p$—being cordial—will send to $q$ the backlog $p$ has previously refrained from sending it, and is not observed by the new block received from $q$.

**7 CONCLUSIONS**

An important next step towards making Cordial Miners a useful foundation for cryptocurrencies is to design a mechanism that will encourage miners to cooperate—as needed by these protocols—as opposed to compete, which is the current standard in mainstream cryptocurrencies.

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