MAGNETOFLUID COUPLING: ERUPTIVE EVENTS IN THE SOLAR CORONA

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ABSTRACT

By modeling the coronal structures by “slowly” evolving magnetofluid equilibrium states (double Beltrami states) created by the interaction of the magnetic and velocity fields, the conditions for catastrophic transformations of the original state are derived. It is shown that, at the transition, much of the magnetic energy of the original state is converted to the flow kinetic energy.

Subject headings: Sun: corona — Sun: coronal mass ejections (CMEs) — Sun: flares — Sun: magnetic fields — Sun: prominences

1. INTRODUCTION

The latest Transition Region and Coronal Explorer (TRACE) and Solar and Heliospheric Observatory/EUV Imaging Telescope observations reveal that the structures that constitute the solar corona are in constant motion. They are heated in a few minutes primarily in the first 10–20 thousand kilometers above the surface, i.e., in a rather small fraction of the bright part of the anchored structure. The loops are not, as believed before, static bodies supported by interior gas pressure and heated along their lengths (Rosner, Tucker, & Vaiana 1978). They fill and drain so quickly that the gas in them must be moving nearly ballistically rather than undergoing “quiescent heating” (Schrijver et al. 1999). From a detailed study of the loops with different characteristic parameters, one also concludes that the heating is quite nonuniform (Aschwanden, Nightingale, & Alexander 2000). Transient brightenings, with their associated flows of cool and hot material, are also a very common phenomenon in the TRACE movies. These observations pose a new challenge for the theories of quiescent as well as non-quiescent coronal structures and events. In this Letter, we explore the possibility that the formation and primary heating of the coronal structures as well as the more violent events (possibly flares, erupting prominences, and coronal mass ejections [CMEs]) may be expressions of different aspects of the same general global dynamics operating in a given coronal region (Mahajan et al. 2001). The basic conjecture of the model is that the wide variety of magnetofluid states (Mahajan et al. 2001; Mahajan & Yoshida 1998; Yoshida & Mahajan 1999) that arise from the interaction of plasma flows with the magnetic field may be responsible for the immense diversity of the coronal structures. Important aspects of the observed dynamics of the coronal structures may, then, be revealed by examining the nature of these magnetofluid states and their response to slow changes in their defining parameters.

After giving a summary of the physics of the relevant magnetofluid states, we will be seeking answers to the following: (1) Can the basic framework of this model predict the possibility of, and the pathways for, the occurrence of sudden, eruptive, and catastrophic events (such as flares, eruptive prominences, and CMEs) in the solar atmosphere? (2) Does the eventual fate, possibly a catastrophic reorganization, of a given structure lie in the very conditions of its birth? (3) Is it possible to identify the range and relative values of identifiable physical quantities that make a given structure prone to eruption (flaring)? (4) Will an eruption be the result of the conversion of the excess magnetic energy into heat and bulk plasma motion, as is generally believed to happen in the solar atmosphere (Sakurai 1989; Parker 1996; Birn et al. 2000; Chen & Shibata 2000; Choe & Cheng 2000; Roald, Sturrock, & Wolfson 2000)?

2. MODEL

Within the framework of our approach, there are two distinct scenarios for eruptive events: (1) when a “slowly” evolving structure finds itself in a state of no equilibrium and (2) when the process of creating a long-lived hot structure is prematurely aborted; the flow shrinks/distorts the structure that suddenly shines and/or releases energy or ejects particles. The latter mechanism requires a detailed time-dependent treatment and is not the subject matter of this Letter. The following semi-equilibrium, collisionless magnetofluid treatment pertains only to the former case.

To derive the magnetofluid states, we begin with the two-fluid system (neglecting electron inertia and transport processes) written in dimensionless variables:

\[
\frac{\partial}{\partial t} A = (V - \nabla \times B) \times B - \nabla (\phi - p),
\]

(1)

\[
\frac{\partial}{\partial t} (V + A) = V \times (B + \nabla \times V) - \nabla (V^2/2 + p + \phi),
\]

(2)

where the magnetic field \( B \) is normalized to an appropriate measure \( B_0 \), the fluid velocity \( V \) to the corresponding Alfvén speed, and distances to the collisionless ion skin depth \( l_i \); \( A \) and \( \phi \) are the vector and scalar potentials, and \( p_i \) and \( p \) are the normalized electron and ion pressures. This set of equations can be cast in the vortex dynamics form in terms of a pair of generalized vorticities (Mahajan & Yoshida 1998 and references therein) \( \Omega = B, \Omega_s = B + \nabla \times V \) and effective flows...
\( U_1 = V - \nabla \times B, \quad U_2 = V: \)

\[
\frac{\partial}{\partial t} \Omega_j - \nabla \times (U_j \times \Omega_j) = 0 \quad (j = 1, 2). \tag{3}
\]

We have used a constant density assumption for simplicity; the extension to varying density is straightforward (Mahajan et al. 2001). The system allows general steady state solutions \( U_j \times \Omega_j = -\nabla \phi (j = 1, 2) \), where the scalar fields \( \phi \) correspond to the fluid energy densities: \( \phi_1 = p_1, \quad \phi_2 = V^2/2 + \phi + p_1. \)

The simplest and perhaps the most fundamental equilibrium solution to equation (3) is given by aligning the vorticities and the ions follow the field lines modified by fluid vorticity. In this context, the Beltrami conditions also imply “generalized Beltrami state” \( \url{Arnold & Khesin 1998, p. 72} \) is an example of a double Beltrami (DB) state:

\[
B = a(V - \nabla \times B), \quad B + \nabla \times V = bV, \tag{4}
\]

with \( a, b = \text{const.} \) The two Beltrami conditions represent very simple physics: the inertialless electrons follow the field lines, and the ions follow the field lines modified by fluid vorticity. In this context, the Beltrami conditions also imply “generalized Bernoulli conditions” \( \phi_j = \text{const.} \), which allow pressure confinement [taking \( \phi_2 - \phi_1 \) we obtain \( V^2 + \beta = \text{const.} \), where \( \beta = 2(p_1 + p_2) \)].

The DB field encompasses a wide class of steady states of mathematical physics—from the force-free paramagnetic to the fully diamagnetic states (Mahajan et al. 2001); its defining equation is obtained by combining the Beltrami conditions given by equation (4):

\[
(\nabla \times -\lambda_+)(\nabla \times -\lambda_-)B = 0, \tag{5}
\]

where \( \tilde{a} = 1/a \) and

\[
\lambda_\pm = \frac{1}{2} \left( b - \tilde{a} \pm \sqrt{(b + \tilde{a})^2 - 4} \right). \tag{6}
\]

For sub-Alfvenic flows (the flows generally encountered in the solar atmosphere), the length scales \( (\lambda_\pm) \) are quite disparate. We assume \( \lambda_+ \gg \lambda_- \) without loss of generality. The structure of equation (5) implies that its general solution is simply a linear combination of two single Beltrami fields \( G_x \) satisfying \( \nabla \times G_x = 0, \) i.e. \( (C_x \) are arbitrary constants),

\[
B = C_x G_x + C_x G_x, \tag{7}
\]

with the corresponding velocity field given by \( V = (\lambda_+ + \tilde{a}) C_x G_x + (\lambda_- + \tilde{a}) C_x G_x \). An illustrative example of a Beltrami field for periodic boundary conditions is given in Figure 1.

We begin by identifying the quasi-equilibrium state of a typical coronal structure with a slowly changing DB state. The slow changes may be due to changes in the Sun that affect the local fields due to the interaction of various nearby structures or due to disturbances in the solar atmosphere. The parameter change is assumed to be sufficiently slow that, at each stage, the system can find its local DB equilibrium (adiabatic evolution). It is worthwhile to remark here that Steinhauser & Ishida (1997) proposed a variational principle using the total energy \( E \) and two helicities and derived equation (4) as an Euler-Lagrange equation describing the relaxed state in two-fluid MHD. We remind the reader that \( E = \int (B^2 + V^2) dr/2, \) the magnetic helicity \( h_1 = \int (A \cdot B) dr/2, \) and the generalized helicity \( h_2 = \int (A + V) \cdot (B + \nabla \times V) dr/2 \) are the three rugged bilinear invariants of the collisionless two-fluid dynamics, and their conservation will provide three algebraic relations between the four parameters \( \lambda_+, \lambda_- \) (eigenvalues), and \( C_x, C_y \) (amplitudes) characterizing the DB field (Yoshida et al. 2001).

The problem of predicting sudden events via a slow evolution, conserving the dynamical invariants \( h_1, h_2, \) and \( E, \) must then reduce to finding the range of invariants (if any) in which the slowly evolving structure may suffer a loss of equilibrium. For the DB states, the signature of a catastrophic change is quite easy to identify. One can consider the following two possible ways to the transition: (1) the roots of the quadratic equation, determining the length scales for the field variation, go from being real to complex, resulting in a change in the topology of the magnetic (from para to diamagnetic) and the velocity fields or (2) the amplitude of either of the two component states ceases to be real, reducing a DB state to a single Beltrami state. For our current problem (sub-Alfvenic flows), the sudden change is likely to follow the second route. To predict the possibility of an eruptive event, we analytically investigate this system using the macroscale \( (\lambda_-^1) \) of the closed structure as a control parameter and observe the trajectory of the other three parameters as \( (\lambda_+^1) \) varies. This choice is physically sensible and is motivated by observations because in the process of structure-structure interactions, “initial” shapes do undergo deformations/distortions with rates strongly dependent on the initial and boundary conditions. Note that this class of investigations, albeit in different contexts, has been carried out by numerous authors (see, for example, Forbes & Isenberg 1991; Kusano, Suzuki, & Nishikawa 1995).

For simplicity, we explicitly work out the details in a Cartesian cube of length \( L \). We take each of \( G_x \) to be a simple two-dimensional Beltrami \( ABC \) field (Arnold & Khesin 1998, p. 72),

\[
G_x = g_{x \pm} \begin{pmatrix} 0 & \cos \lambda_x x \\ \sin \lambda_x x \end{pmatrix} + g_{y \pm} \begin{pmatrix} \cos \lambda_y y \\ \sin \lambda_y y \end{pmatrix}, \tag{8}
\]

with the normalization \( (g_{x \pm})^2 + (g_{y \pm})^2 = 1. \) For real \( \lambda_x, \) equation (8) represents an arcade magnetic field structure resembling interacting coronal loops (Fig. 1). Assuming \( L = n_\perp (2\pi/\lambda_x) = n_\parallel (2\pi/\lambda_\perp) \) \( (n \) are integers), \( G_x \) satisfy the following relations: \( \int G_x^2 \cdot dr = L \) and \( \int G_x \cdot G_x \cdot dr = 0, \) where \( \int dr = \int_0^L dx \). The invariants can now be readily evaluated:
\[ h_1 = \frac{L^2}{2} \left( \frac{C_1^2}{\lambda_+} + \frac{C_2^2}{\lambda_-} \right), \quad (9) \]

\[ \tilde{h}_2 = \frac{L^2}{2} \left[ (2 + \lambda_+ (\lambda_+ + \tilde{a})) (\lambda_+ + \tilde{a}) \lambda_+^2 \right. \]

\[ + [2 + \lambda_- (\lambda_- + \tilde{a})] (\lambda_- + \tilde{a}) \lambda_-^2, \quad (10) \]

\[ E = \frac{L^2}{2} [1 + (\lambda_+ + \tilde{a})^2] C_1^2 + [1 + (\lambda_- + \tilde{a})^2] C_2^2, \quad (11) \]

providing three promised algebraic relations that may be conveniently written as \( \tilde{h}_2 = \tilde{h}_1 + \tilde{h}_2, \tilde{h}_2 = bE - \lambda_+ \lambda_- h_1 \):

\[ \tilde{h}_2 = \frac{E}{2} [\lambda_+ + \lambda_-] \]

\[ \pm \sqrt{(\lambda_+ - \lambda_-)^2 + 4} - \lambda_+ \lambda_- h_1, \quad (12) \]

\[ C_1^2 = D^{-1} [E - [1 + (\lambda_+ + \tilde{a})^2] \lambda_- h_1] \lambda_+, \quad (13) \]

\[ C_2^2 = -D^{-1} [E - [1 + (\lambda_- + \tilde{a})^2] \lambda_- h_1] \lambda_-, \quad (14) \]

where the common factor \( L^2/2 \) has been normalized out and

\[ D = [1 + (\lambda_+ + \tilde{a})^2] \lambda_+ - [1 + (\lambda_- + \tilde{a})^2] \lambda_- \]

\[ = (\lambda_+ - \lambda_-) b (b + \tilde{a}). \quad (15) \]

For given \( h_1, E, \tilde{h}_2 (h_2) \) (invariants), and \( \lambda_+ \) (control parameter), we can solve the preceding system to determine the physical quantities \( \lambda_- \) and \( C_\ast \), which must all remain real for the equilibrium to exist. Before we give an analytic derivation for the bifurcation conditions (leading to loss of equilibrium), we display in Figure 2 the plots of \( \lambda_- \) and \( C_\ast \) as functions of \( \lambda_+ \) for two distinct sets for the values of the invariants: we choose \( h_1 = 1, \tilde{h}_2 = 1.5, E = 0.4 \) for Figure 2a and \( h_1 = 1, \tilde{h}_2 = 1.5, E = 1.3 \) for Figure 2b (dashed lines correspond to the region of imaginary \( \lambda_- \)). We find that the behavior of the solution changes drastically with \( E \). For the parameters of Figure 2a, \( \lambda_- \) and \( C_\ast \) remain real and change continuously with varying \( \lambda_+ \), implying that as the macroscopic scale of the structure \((1/\lambda_-)\) changes continuously, the equilibrium expressed by equation (8) persists—there is no catastrophic or qualitative change. With \( E \) changing from 0.4 to 1.3 but with same \( h_1 \) and \( \tilde{h}_2 \), we face a fundamentally different situation in Figure 2b: when \( \lambda_+ \) exceeds a critical value \( \lambda_+^{crit} \), i.e., the macroscale becomes smaller than a critical size, the physical determinants of the equilibrium cease to be real; the sequence of equilibria is terminated. Note that since we assume that changes are slow and transport processes can be ignored in our model, the representation of a changing structure by a local DB field persists, and during these changes the length scales do remain vastly separated.

The condition for catastrophe must turn out to be a constraint (involving \( h_1, \tilde{h}_2 \), and \( E \)) derived from the extremum condition for the curve \( \lambda_(\lambda_+) \). It is straightforward to show that

\[ E^2 \geq E_1^2 = 4\left(h_1 \pm \sqrt{h_1 \tilde{h}_2}\right)^2 \]

(16)

and the critical \( \lambda_+ \) comes out to be

\[ \lambda_+^{crit} = \frac{1}{2h_1} \left( \frac{E}{\pm \sqrt{E^2 - E_1^2}} \right). \]

Thus, for \( E > E_1 \) (determined by helicities \( h_1 \) and \( \tilde{h}_2 \)) when the macroscopic size of a structure shrinks below a critical value, it can suffer a severe reorganization.

At the critical point, the expected but most remarkable transition occurs. Using the value of \( \lambda_+^{crit} \), we find from equation (14) that the coefficient \( C_\ast \), which measures the strength of the short-scale fields, identically vanishes, and the equilibrium changes from a DB to a single Beltrami state defined by \( \lambda_+ = \lambda_+^{crit} \), i.e., \( B = C, G \), \((G \times B = \lambda_+ B)\), with \( V \) parallel to \( B \). The transition leads to a magnetically more relaxed state with the magnetic energy reaching its minimum with appropriate gain in the flow kinetic energy (see Fig. 3).

The sudden transformation of a DB state to a single Beltrami state (the standard one-parameter, one–scale length relaxed state) with qualitatively different physical properties from the original

![Figure 2](image-url)

Fig. 2.—(a) Plots of \( \lambda_- \) and \( C_\ast \) vs. \( \lambda_+ \) for \( E = 0.4 < E_c = 0.45 \), the critical energy, no catastrophe. (b) Plots of \( \lambda_- \) and \( C_\ast \) vs. \( \lambda_+ \) for \( E = 1.3 > E_c \). There is a critical point at \( \lambda_+ = 0.041 \).
discussed in Klimchuk & Sturrock (1989), where it was argued of the system. This is quite different from the thought experiment catastrophe—a sudden change in the defining physical attributes (the magnetic and the velocity fields, etc.) signifies a genuine state (kinetic and magnetic energies, the relative orientation of the flow causing an eruption. Notice that for coronal plasmas, the skin depth $l$ is small, $\sim 100$ cm ($l/\lambda_\perp \sim 10^6$ km), for a typical density of $\sim 10^9$ cm$^{-3}$.

state (kinetic and magnetic energies, the relative orientation of the magnetic and the velocity fields, etc.) signifies a genuine catastrophe—a sudden change in the defining physical attributes of the system. This is quite different from the thought experiment discussed in Klimchuk & Sturrock (1989), where it was argued that the specification of appropriate photospheric boundary conditions for the magnetic field governed by the force-free equation could lead to a well-behaved evolutionary sequence without exhibiting any catastrophic behavior.

In our model, even at the critical point of the catastrophe, we can define physical parameters like the flow kinetic energy and the magnetic energy. The assumptions of the model like the vastly separated scales hold throughout and up to the critical point. It should also be stressed that energy transformations do take place during the slow-evolution era (Fig. 3) for both the catastrophe-free and the catastrophe-prone cases. Only the rates and direction of the transformation are dictated by the initial conditions of the system. An appropriate choice of initial values of the invariants can lead to a desired transformation.

A word of caution is necessary: as we approach the critical point, the quasi-equilibrium considerations are just an indicator of what is happening. They must be replaced by a full time-dependent treatment including transport processes to capture the dynamics; the changes are no longer slow.

3. Conclusions

By modeling quasi-equilibrium, slowly evolving coronal structures as a sequence of DB magnetofluid states in which the magnetic and the velocity fields are self-consistently coupled, we have shown the possibility of, and derived the conditions for, catastrophic changes leading to a fundamental transformation of the initial state. The critical condition comes out as an inequality involving three invariants of the collisionless magnetofluid dynamics. When the total energy exceeds a critical energy, the DB equilibrium suddenly relaxes to a single Beltrami state corresponding to the large macroscopic size. All of the short-scale magnetic energy is lost, having been transformed to the flow energy and partly to heat via the viscous dissipation of the flow energy.

This general mechanism in which the flows (and their interactions with the magnetic field) play an essential role could certainly help in advancing our understanding of a variety of sudden (violent) events in the solar atmosphere like the flares, the erupting prominences, and the CMEs. The connection of flows with eruptive events is rather direct: it depends on their ability to deform (in specific cases distort) the ambient magnetic field lines to temporarily stretch (shrink, destroy) the closed field lines so that the flow can escape the local region with a considerable increase in kinetic energy in the form of heat/bulk motion.

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