Properties of Defect Modes in Periodic Lossy Multilayer with Negative-Index-Materials

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Abstract Employing the characteristic matrix method, this study investigates transmission properties of one-dimensional defective lossy photonic crystals composed of negative and positive refractive index layers with one lossless defect layer at the center of the crystal. The results of the study show that as the refractive index and thickness of the defect layer increase, the frequency of the defect mode decreases. In addition, the study shows that the frequency of the defect mode is sensitive to the incidence angle, polarization, and physical properties of the defect layer, but it is insensitive to the small lattice loss factor. The peak of the defect mode is very sensitive to the loss factor, incidence angle, polarization, refractive index, and thickness of the defect layer. This study also shows that the peak and the width of the defect mode are affected by the numbers of the lattice period and the loss factor. The results can lead to designing new types of narrow filter structures and other optical devices.

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Key words: one-dimensional photonic crystal, defect mode, loss factor, negative index material

1 Introduction

Photonic crystals (PCs) were first introduced theoretically by Yablonovitch,[1] and experimentally by John,[2] PCs, constructed with periodic structure of artificial dielectrics or metallic materials, have attracted many researchers in the past two decades for their unique electromagnetic properties and scientific and engineering applications. Bragg scattering of the electromagnetic waves passing through these periodical structures has made them capable of indicating a range of forbidden frequencies, called photonic band gaps (PBGs).[3–4] It has been proved that in PCs the properties of the PBG play an important role in many of their interesting applications in the field of photonics and optical engineering such as resonance cavities, laser applications, high reflecting omnidirectional mirrors, tunable optical filters, waveguides, and the optoelectronic circuits.[5–8] When the periodicity of the conventional PC structure is broken, a defective crystal is produced. The break in the periodicity of the conventional PC structure can be achieved by changing physical parameters, such as changing the thickness of one of the layers, adding another medium to the structure, or removing a layer from PCs.[9–12] Introducing a layer with different optical properties and changing the interference behavior of light can generate within the PBG localized defect modes, which are also called resonant transmission peaks,[12–13] very similar to the defect states that are generated in the forbidden band of doped semiconductors. Enormous potential applications of defective PCs in different areas, such as light emitting diodes, filters and fabrication of lasers have made such structures interesting topics for research in the field.

About forty years ago, Veselago[14] introduced negative refractive index materials, or negative index materials (NIMs) with simultaneously negative permittivity and permeability, known as left-handed materials or metamaterials. After the experimental realization of metamaterials by Smith et al.,[15] such materials have received attention for their very unusual electromagnetic properties.[16–19] Recently, by possibility of manufacturing PCs with metamaterials, called metamaterial photonic crystals (MPCs), a new research area has emerged, and numerous interesting results have been reported by researchers so far. Among the papers written and published on PCs and MPCs properties, there are a number of reports on defective structures. The properties of the defect modes in different one-dimensional (1D) conventional PC and 1D MPC defective structures have been reported by several authors.[20–35] Several papers have investigated the dependency of the defect modes on the incidence angle, polarization, lattice constant, geometric structure, and thickness of the layers in 1D PC and 1D lossless MPC.

In this paper, the characteristic matrix method is used to investigate the effects of the refractive index and thick-
ness of the lossless defect layer, incidence angle, polarization, number of the lattice period, and also loss factor on the defect modes in 1D defective lossy MPC transmission spectra. The outline of this paper is as follows: Section 2 deals with the geometric MPC structure, the characteristic matrix method and its formulation, and also the permittivity and permeability of NIM. Section 3 presents the numerical results and discussions, and Sec. 4 is the conclusion.

2 MPC Structure and Characteristic Matrix Method

A schematic diagram of a 1D defective MPC structure in air with a defect layer at the center of the structure is shown in Fig. 1. The MPC structure is composed of alternative layers of NIM (layers A) and PIM (layers B), where NIM is dispersive and dissipative. The thickness and the refractive index of the layers are \( d_A \) and \( d_B \), \( n_A \) and \( n_B \), respectively. Layer C is a defect layer and is assumed to be a lossless and non dispersive material with thickness, and refractive index of \( d_C \) and \( n_C \), respectively.

Fig. 1 Schematic of 1D MPC structure with a defect layer. Layer A is NIM and layer B is PIM. \( N \) is the number of lattice period and \( \theta_0 \) is the incidence angle.

The calculations are performed using the characteristic matrix method \([36]\) which is the most effective technique to analyze the transmission properties of PCs. The characteristic matrix of the structure is given by:

\[
M[d] = (M_A \, M_B)^{N/2} \, M_C \, (M_A \, M_B)^{N/2},
\]

where \( M_A, M_B, \) and \( M_C \) are the characteristic matrices of the layers \( A, B, \) and \( C \). The characteristic matrix \( M_i \) for TE waves at incidence angle \( \theta_0 \) in vacuum is given by\([36]\)

\[
M_i = \begin{bmatrix} \cos \gamma_i & -i \mu_i \sin \gamma_i \\ 1/n_i \sin \gamma_i & \cos \gamma_i \end{bmatrix},
\]

where \( \gamma_i = (\omega/c) \, n_i \, d_i \, \cos \theta_i \), \( c \) is speed of light in vacuum, \( \theta_i \) is the ray angle inside the layer \( i \) with refractive index \( n_i \) and \( \mu_i \) is \( \sqrt{\varepsilon_i/\mu_i} \, \cos \theta_i \), where \( \cos \theta_i = \sqrt{1 - (n_0^2 \sin^2 \theta_0/n_i^2)} \), in which \( n_0 \) is the refractive index of the environment wherein the incidence wave tends to enter the structure. The refractive index is given as \( n_i = \pm \sqrt{\varepsilon_i/\mu_i} \) \([32,37]\) where the positive and negative signs are assigned for the PIM and NIM layers, respectively.

The final characteristic matrix for an \( N \) period structure is given by

\[
[M(d)]^N = \prod_{i=1}^{N} M_i = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},
\]

In this equation, \( m_{ij} (i, j = 1, 2) \) are the matrix elements of \([M(d)]^N\). The transmission coefficient of the multilayer is given by

\[
t = \frac{2p_0}{(m_{11} + m_{12} \, p_s) \, p_0 + (m_{21} + m_{22} \, p_s)},
\]

where \( p_0 = n_0 \cos \theta_0 \) and \( p_s = n_s \cos \theta_s \) for TE waves. The transmissivity of the multilayer is given by

\[
T = \frac{p_1}{p_0} |t|^2.
\]

The transmissivity of the multilayer for TM waves can be obtained by changing \( p_i = \sqrt{\mu_i/\varepsilon_i} \, \cos \theta_i \), \( p_0 = \cos \theta_0/\varepsilon_0 \) and \( p_s = \cos \theta_s/\varepsilon_s \). The permittivity and permeability of layer \( A \) with negative refracting index in the microwave region are complex and are defined as\([38–39]\):

\[
\varepsilon_A(f) = 1 + \frac{5^2}{0.92^2 - f^2 - 1/\gamma} + \frac{10^2}{11.5^{2} - f^2 - 1/\gamma},
\]

\[
\mu_A(f) = 1 + \frac{3^2}{0.90^2 - f^2 - 1/\gamma},
\]

where \( f \) and \( \gamma \) are the frequency and damping frequency in GHz, respectively. The behaviours of the real parts of the permittivity and permeability of layer \( A, \varepsilon_A', \) and \( \mu_A' \), versus frequency have been discussed in our previous report\([39]\).

3 Numerical Results and Discussion

The transmission spectrum of the lossy defective MPC structure was calculated based on the theoretical model described in the previous section. The calculations are carried out in the region wherein \( \varepsilon_A' \) and \( \mu_A' \) are simultaneously negative. This region is where the zero-\( \tilde{\eta} \) gap will appear\([38–41]\). Layer \( B \) is assumed to be the vacuum layer with \( \varepsilon_B = \mu_B = 1 \). The thickness of layers \( A \) and \( B \) are chosen as \( d_A = 6 \) mm and \( d_B = 12 \) mm, respectively. The defect layer \( C \) is either NIM or PIM non dispersive and non dissipative. Also, the total number of the lattice period is selected to be \( N = 16 \) \([39]\).

The transmission spectrum for \( d_C = 24 \) mm with positive refractive index of \( n_C = 0.75, 1.25, 1.75, \) and \( 2.25 \), for two different loss factors \( (\gamma = 0.2 \times 10^{-3} \) GHz and \( \gamma = 8 \times 10^{-3} \) GHz) and for normal incidence are shown in Fig. 2. As it is clearly seen, the rate of the transmittance and the peak of the defect modes are affected by the loss factor. Figure 3 shows the frequency of the defect modes as a function of the refractive index of PIM and NIM defect layers for normal incidence, and for two different small lattice’s loss factors. As it is seen, the frequency of the defect modes decreases as \( n_C \) increases and the modes are nearly insensitive to the small loss factors. In addition, for the PIM defect layer (Fig. 3(a)) two defect modes appear for some specified refractive indices. The behaviour is quite different in the NIM defect layer (Fig. 3(b)) where no defect modes are observed in some specific regions.
Fig. 2  Transmission spectra of 1D MPC structure at normal incidence for different positive refractive indices of the lossless defect layer, with $d_C = 24$ mm and for two different lattice loss factors (a) $\gamma = 0.2 \times 10^{-3}$ GHz (b) $\gamma = 8 \times 10^{-3}$ GHz.

Fig. 3  Dependence of the defect mode frequency on the (a) positive and (b) negative refractive index of the lossless defect layer ($n_C$) for two different lattice loss factors ($\gamma = 0.2 \times 10^{-3}$ GHz and $\gamma = 8 \times 10^{-3}$ GHz).

The effects of $n_C$ on the peak of the defect modes for regions I, II, and III are shown in Fig. 3(a) for PIM defect layer and for two different loss factors in Fig. 4. It can be seen that the peak of the defect modes almost quadratically depends on the refractive index. The minimum peak decreases for the regions with large $n_C$. Moreover, the peak of the modes is very sensitive to the loss factor and decreases as $\gamma$ increases. The transmission spectrum for $n_C = 1$ and $d_C = 4, 16, 24$, and $32$ mm and for two different loss factors ($\gamma = 0.2 \times 10^{-3}$ GHz and $\gamma = 8 \times 10^{-3}$ GHz) are presented in Fig. 5. As can be observed, the frequency of the defect modes move toward the lower frequencies (Fig. 5(a)), and the peak of modes decreases as the thickness of the defect layer increases (Fig. 5(b)). The behaviors of the frequency and the peak of the defect modes versus $d_C$ for two different $\gamma$ are shown in Figs. 6(a) and 6(b), respectively. As shown in Figs. 6(a) and 6(b), the frequency of the defect modes is nearly insensitive to the loss factors but the peak is very sensitive to $\gamma$ and decreases as the loss factor increases.

In this section, the effects of the incidence angle on the defect transmission spectra are investigated. The transmission spectra of TE and TM polarized waves for $0^\circ$, $25^\circ$, $50^\circ$, and $75^\circ$ incidence angles for $d_C = 8$ mm, $n_C = 2.5$, and for two different loss factors ($\gamma = 0.2 \times 10^{-3}$ GHz and $\gamma = 8 \times 10^{-3}$ GHz) are shown in Fig. 7.

As it is observed from Figs. 7(a) and 7(b), and reported in [39, 41], the width, the depth, and the central frequency of the band gap for TE waves increase as the incidence angle increases. In contrast, for TM waves, the gap disappears for the incidence angles greater than $\approx 50^\circ$. As $\theta_0$ increases the width and the depth of gap decrease, and the central frequency shifts toward higher frequencies (Figs. 7(c) and 7(d)). The behaviour of the defect mode’s frequency for TE and TM waves as a function of the incidence angle for two different loss factors are shown in Fig. 8(a). Here it is clearly seen that the frequency of the defect mode is nearly insensitive to the small loss factor, and it disappears for TM waves for the incidence angle greater than $\approx 50^\circ$. The change of the peak of the defect mode for both polarizations as a function of the incidence angle for two different loss factors is shown in Fig. 8(b). It can be seen that the peak of the defect modes decreases for TE polarized waves while it increases for TM polarized waves as the incidence angle increases. It is also seen that the peak of the modes are very sensitive to the loss factor for both polarizations.
Fig. 4  Peak of the defect modes versus the positive refractive index of the defect layer, with $d_C = 24$ mm and for two different lattice loss factors (a) $\gamma = 0.2 \times 10^{-3}$ GHz (b) $\gamma = 8 \times 10^{-3}$ GHz.

Fig. 5  Transmission spectra of 1D MPC structure at normal incidence, for different thicknesses of the lossless defect layer, and for two different lattice’s loss factors (a) $\gamma = 0.2 \times 10^{-3}$ GHz (b) $\gamma = 8 \times 10^{-3}$ GHz with $n_C = 1$.

Fig. 6  (a) Frequency and (b) peak of the defect mode versus thickness of the defect layer for two different lattice loss factors, for $n_C = 1$.

In the last part, the effects of the number of lattice period, $N$, and the loss factor on the transmission properties of the defect mode are studied. So far $N = 16$ was assumed. The results are presented in Figs. 9 and 10 for normal incidence angle. As it is seen from Fig. 9(a) and as reported in [39], the width of the band gap for $N > 16$ is almost independent of $N$. On the contrary, the depth and the peak of the defect mode reduce as $N$ increases for $\gamma = 0.2 \times 10^{-3}$ GHz. It is interesting to note that the frequency of the defect mode remains unchanged, but the width of the mode decreases as the number of the lattice period increases (Fig. 9(b)). The transmission spectra of the structure for four different loss factors and for $N = 16$ are shown in Fig. 10. As it is seen from Fig. 10(a), the width and the depth of the band gap, and also the frequency of the defect mode are nearly insensitive to small loss factor, but the transmission is affected by $\gamma$. Moreover, the peak of the defect mode decreases as the loss factor increases (Fig. 10(b)).
4 Conclusion

In this paper the transmission properties of one-dimensional defective lossy photonic crystal with negative and positive refractive index layers, and a lossless defect layer at the center of the crystal were investigated by the characteristic matrix method. It was shown that by adding a lossless defect layer to a 1D lossy MPC, a localized defect mode appears inside the band gap. Our numerical results show that the position of the defect mode depends on the physical properties of the defect layer such as the refractive index and thickness, but it is independent of the loss factor. As the refractive index and thickness of the defect layer increase, the frequency of the defect mode decreases. In addition, it was shown that the frequency and the peak of the defect mode are sensitive to the incidence angle and polarization. For a PIM defect layer with the refractive index between 4.75 to 6.25, two defect modes appear in a specific range of the transmittance spectrum, while for an NIM defect layer the spectrum is quite different where no defect modes is observed in a certain range of the refractive index (\( n_C = -2 \) to \(-3.5\)). The reason for the absence of the defect mode in some spec-
ified regions is not clear and is the subject of our future studies. The following facts can also be seen. i) By increasing the incidence angle, the frequency of the defect mode increases. ii) By increasing the thickness of the defect layer, the frequency of the defect mode moves toward lower frequencies. iii) The peak of the defect modes is very sensitive to the loss factors. iv) The depth of the band gap increases and consequently the peak and the width of the defect mode decrease, so the mode becomes narrower for \( N > 16 \). v) The width and the depth of the band gap and the frequency of the defect modes are nearly insensitive to small loss factors, but the peak of the defect mode is very sensitive to small \( \gamma \) and decreases as the loss factor increases. Such properties are quite useful in designing new types of optical devices in microwave engineering.

\[ \gamma = 0.2 \times 10^{-3} \text{ GHz}. \]

**Fig. 9** Transmission spectra of 1D MPC structure at normal incidence (a) in the frequency range of 1.5 to 3.0 GHz, and (b) in the small frequency range of 2.308 to 2.362 GHz, for different number of the lattice period and for \( \gamma = 0.2 \times 10^{-3} \text{ GHz} \).

\[ \gamma = 0.2 \times 10^{-3} \text{ GHz}, \gamma = 8 \times 10^{-3} \text{ GHz}, \gamma = 20 \times 10^{-3} \text{ GHz}, \gamma = 40 \times 10^{-3} \text{ GHz} \]

**Fig. 10** Transmission spectra of 1D MPC structure at normal incidence (a) in the frequency range of 1.5 to 3.0 GHz, and (b) in the small frequency range of 2.316 to 2.352 GHz, for different lattice loss factors for \( N = 16 \).

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