Reduced-order models for vertical human-structure interaction

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Abstract. For slender and lightweight structures, the vibration serviceability under crowd-induced loading is often critical in design. Currently, designers rely on equivalent load models, upscaled from single-person force measurements. Furthermore, it is important to consider the mechanical interaction with the human body as this can significantly reduce the structural response. To account for these interaction effects, the contact force between the pedestrian and the structure can be modelled as the superposition of the force induced by the pedestrian on a rigid floor and the force resulting from the mechanical interaction between the structure and the human body. For the case of large crowds, however, this approach leads to models with a very high system order. In the present contribution, two equivalent reduced-order models are proposed to approximate the dynamic behaviour of the full-order coupled crowd-structure system. A numerical study is performed to evaluate the impact of the modelling assumptions on the structural response to pedestrian excitation. The results show that the full-order moving crowd model can be well approximated by a reduced-order model whereby the interaction with the pedestrians in the crowd is modelled using a single (equivalent) SDOF system.

1. Introduction

In the prediction of the structural response induced by groups of pedestrians or crowds, persons crossing the footbridge are often simplified to (moving) loads [1]. However, in the low-frequency range of interest for vibration serviceability (0 < \( f_b < 10 \) Hz), the dynamic behaviour of the human body in vertical direction resembles that of a highly damped single degree of freedom (SDOF) system [2, 3, 4] for which the natural frequency (2.7 < \( f_{h1} < 7.0 \) Hz) and damping ratio (20 < \( \xi_{h1} < 60 \) %) largely depend on the body posture [5, 6, 7]. As a result of the interaction between the human body and the footbridge, the dynamic behaviour of the coupled crowd-structure system can differ significantly from that of the empty structure [8, 9, 10]. The significance of human-structure interaction (HSI) increases with the crowd to structural mass ratio [11, 12, 13, 14] and for frequency ratios (\( f_{h1}/f_b \)) approaching (a value slightly lower than) unity [15, 16, 17]. Hence, the effects of HSI on the structural response to pedestrian excitation are in many cases non-negligible for lightweight structures [17, 18].

The computational cost associated with the prediction of the structural response to pedestrian excitation, depends on the system order of the coupled crowd-structure system. Generally, the number of modes representing the dynamic behaviour of the structure is limited and
much smaller than the number of pedestrians. As the pedestrian density increases, and therefore also the number of unique pedestrians, the model order of the coupled crowd-structure system increases rapidly. To limit the computational effort, this contribution investigates the performance of two equivalent reduced-order models. The objective is to reduce the model order and to approximate the dynamic behaviour of the time-variant moving crowd model by a time-invariant system.

The outline of this paper is as follows. First, the full-order time-variant moving crowd model is presented. Second, two reduced-order models are introduced. Finally, the structural response to pedestrian excitation is evaluated for a range of footbridge parameters and pedestrian densities and the impact of the modelling simplifications is assessed by comparison with the predictions of the full-order moving crowd model.

2. Moving crowd model

Pedestrians in a crowd are active systems for which the walking behaviour is controlled by an internal driving term. In the crowd-structure model presented here, the pedestrian and the footbridge are considered as two linear subsystems coupled at a single contact point. The contact force not only depends on the internal driving term of the pedestrian but also on the dynamic properties of both subsystems [19]. The following subsections subsequently discuss (1) the characteristics of the crowd, (2) the coupled crowd-structure model, and (3) the considered statistical approach for the analysis of the predicted structural response.

2.1. The crowd

Pedestrian excitation is characterized by great randomness, whereby each pedestrian has his own characteristics, such as the weight, the step frequency and the walking speed (inter-subject variability [20]). Furthermore, some parameters such as the step length and the walking speed may vary along the individual trajectories (intra-subject variability [1, 21]). Given the fact that focus is on the resulting structural response, a number of reasonable simplifying assumptions are made and briefly discussed next. For a more comprehensive discussion of the pedestrian flow model applied in this study, the reader is referred to [22].

The crowd-structure model accounts for inter-subject variability but disregards intra-subject variability and all pedestrians are assumed to move at the same walking speed $v_s = 1.5 \text{ m/s}$ [23, 24]. The arrival rate $\lambda$ [persons/s] is computed from the pedestrian density $d$ [persons/m$^2$]:

$$\lambda = \frac{n_{H}}{T} = \frac{A_{eff} d}{T}$$  \hspace{1cm} (1)

with $n_{H}$ [-] the number of pedestrians, $T$ [s] denoting the time taken by a pedestrian to cross the bridge span with length $L$ [m] ($T = L/v_s$) and $A_{eff}$ [m$^2$] the walkable bridge deck surface. The arrival times are assumed to follow a Poisson distribution [25].

The vibration serviceability of footbridges is mostly assessed assuming (near-)resonant loading. Hence, the mean step frequency (or one of its harmonics $n$) is chosen to match the natural frequency of the structure. The fundamental harmonic of the walking load is considered ($n = 1$) for resonant conditions up to 2.5 Hz, while for frequencies between 2.5 Hz and 5.0 Hz, and between 5.0 Hz and 7.5 Hz, (near-)resonance occurs with the second ($n = 2$) and the third ($n = 3$) harmonic of the walking load, respectively. Sparse or spatially unrestricted crowd conditions ($d < 1$) can be simulated assuming a Gaussian distribution of the step frequencies with a standard deviation of 0.175 Hz ($f_s = \mathcal{N}(f_{nj}/n, 0.175)$ Hz [23, 24]). For dense crowd conditions ($1 \geq d \geq 1.5$), the normal walking behaviour gets hindered causing the forward movement of the stream to slow down and the level of synchronisation to increase. In that case, all pedestrians are generally given the same step frequency ($f_s = f_{nj}/n$ [23, 24]). The weight of each pedestrian $G$ is set to 700 N [26]. Together, the pedestrian’s step frequency $f_s$ and weight
G characterise the vertical single foot force as exerted on a rigid laboratory floor [1]. In this study, the latter is determined by the load model defined by Li et al. [27].

To account for the interaction between the pedestrian and the supporting structure, each pedestrian is in addition represented by a linear mechanical system. To this end, this study adopts a SDOF system (see figure 1-a): a spring mass \( m_{d1} \), an unsprung mass \( m_{u1} \) and a spring \( k_{u1} \) and damping element \( c_{u1} \) with \( m_d = m_{d1} + m_{u1} = 70 \text{ kg} \) and \( \mu_{d1} = m_{d1}/m_d = 0.95 \) [5, 6], as this provides a good approximation of the low-frequency (0 – 10 Hz) dynamic behaviour of the human body [2, 6, 28, 29]. The mechanical properties of this SDOF system are set to approximate the low-frequency dynamic behaviour of a person with one or two legs slightly bent \( (f_{d1} \approx 3.25 \text{ Hz}, \xi_{d1} \approx 0.30 [-]) \), i.e. similar to the postures assumed during the walking cycle [6, 7]. To account for the expected inter-subject variability, the natural frequencies and modal damping ratios for the different individuals are sampled from the following distributions: \( f_{d1} = \mathcal{N}(\mu_{d1}, \sigma_{d1}) = \mathcal{N}(3.25, 0.32) \) [Hz], \( \xi_{d1} = \mathcal{N}(\mu_{\xi d1}, \sigma_{\xi d1}) = \mathcal{N}(0.30, 0.05) [-] \) [6, 7]. The Gaussian distributions involved in this study are truncated to exclude infeasible negative values.

2.2. Coupled crowd-structure model

A modally reduced system is derived from the linear dynamic finite element (FE) model of the supporting structure. The governing equations of motion in modal coordinates read:

\[
\ddot{z}(t) + \Gamma \dot{z}(t) + \Omega^2 z(t) = \Phi^T p(t)
\]  

(2)

with \( z(t) \in \mathbb{R}^{n_m} \) the modal coordinate vector, \( n_m \) the number of modes retained in the modally reduced system, \( \Omega^2 \in \mathbb{R}^{n_m \times n_m} \) a diagonal matrix containing the square of the natural frequencies \( \omega_{nj} = 2\pi f_{nj} \) in rad/s, \( \Gamma \in \mathbb{R}^{n_m \times n_m} \) a diagonal matrix containing the terms \( 2c_{nj} \omega_{nj} \) with \( \xi_{nj} [-] \) the modal damping ratios, \( \Phi \in \mathbb{R}^{n_{dof} \times n_m} \) a matrix which has the mass-normalised mode shapes \( \phi_j \) as columns, \( \Phi^T p(t) \) the modal projection of the external forces \( p(t) \in \mathbb{R}^{n_{dof}} \) with \( n_{dof} \) the number of degrees of freedom of the FE model. The vector of the forces \( p(t) \) contains the \( n_{cont} \) contact forces between the pedestrians and the footbridge. For each pedestrian, the contact is reduced to a single point representing at the same time the left and the right foot, an assumption which is justified given the small dimension of the step length relative to the bridge span. The time history of the contact force(s) of the \( n_{cont} \) pedestrians are collected in the force vector \( P_p(t) \in \mathbb{R}^{n_{u}} \). The corresponding time variant location(s) on the bridge deck are indicated by the matrix \( S_p(t) \in \mathbb{R}^{n_{dof} \times n_{u}} \). The vector of forces applied to the bridge deck \( p(t) \) in Eq. (2) now reads:

\[
p(t) = S_p(t)P_p(t)
\]  

(3)

The moving crowd model applied in this study is based on the key assumption that the walking behaviour of the pedestrian is not affected by the vibrating surface, i.e. the internal driving term of the pedestrian is identical to the one in case of a stiff supporting system. This assumption holds when the displacements of the bridge deck are (sufficiently) small. In the case where the perceived motion causes the pedestrians to adapt their walking behaviour as for lock-in, this condition is not met. So far, the latter has only been observed for lateral [30, 31] and not for vertical deck motion [32]. Assuming a fixed driving term allows the body motion and, hence, the contact force of a pedestrian \( p_p(t) \), to be decomposed in a term resulting from the driving term \( p_d(t) \) and a term resulting from the mechanical interaction between the pedestrian and the supporting structure \( p_u(t) \) [19]:

\[
p_p(t) = p_d(t) + p_u(t)
\]  

(4)

For a perfectly rigid floor, i.e. without interaction (\( p_u(t) = 0 \)), the contact force equals the ground reaction forces (GRFs) as registered by the force plates or an instrumented treadmill fixed to a rigid laboratory floor \( (p_p(t) = p_d(t)) \) [1, 33]. This force term depends solely on the pedestrian’s characteristics (the pedestrian’s weight and step frequency, see subsection 2.1).
The key parameters determining the dynamic response of the footbridge are the corresponding vibration serviceability of footbridges is mostly assessed assuming (near-)resonant loading.

### 3. Reduced-order models

Carlo simulations is increased until the desired level of convergence of the statistical quantity of interest is reached [22].

To this end, the present study applies a Monte Carlo procedure whereby the number of Monte Carlo simulations is increased until the desired level of convergence of the statistical quantity of interest is reached [22].

### 2.3. Statistical analysis

Given the inherent variability of pedestrian excitation (see section 2.1), a statistical analysis seems most suited for the evaluation of the predicted structural behaviour and response [27, 34]. To this end, the present study applies a Monte Carlo procedure whereby the number of Monte Carlo simulations is increased until the desired level of convergence of the statistical quantity of interest is reached [22].

The vibration serviceability of footbridges is mostly assessed assuming (near-)resonant loading. The key parameters determining the dynamic response of the footbridge are the corresponding natural frequency $f_n$, modal damping ratio $\xi_n$, and modal mass $m_n$. For the empty footbridge, the maximum steady-state amplitude $\ddot{u}_{n,\text{max}}$ to harmonic excitation is found as:

$$\ddot{u}_{n,\text{max}} \sim \frac{1}{2\xi_n m_n} = |H_n(\omega_n)|$$

Figure 1. Schematic representation of (a) the human body model, (b) a footbridge subjected to pedestrian excitation and the proposed decomposition of the contact force $p_p$ into (c) the fixed-driver term $p_f$ and (d) the interaction term $p_i$. When the supporting structure is flexible, the contact force $p_p(t)$ in addition depends on the interaction between the two subsystems [19]. The interaction term $p_{i1}(t)$ in Eq. (4), can be further elaborated based on the mechanical properties of the linear system representing the human body (see section 2.1 and figure 1-a). When $p_{i1}(t)$ is written in terms of the displacements of the mass $m_{h1}$ ($u_{h1}(t)$) and the bridge at the contact point ($u_h(t)$), Eq. (4) reads:

$$p_{p}(t) = p_{f}(t) - m_{h0} \ddot{u}_{h}(t) + c_{h1}[\dddot{u}_{h1}(t) - \ddot{u}_{h}(t)] + k_{h1}[u_{h1}(t) - u_{h}(t)]$$

The second-order differential equations of motion of the moving crowd model can be written as a first-order continuous-time state equation:

$$\dot{x}(t) = A_c(t)x(t) + B_c(t)x_s(t)p_p(t); \quad \text{with} \quad x(t) = [z(t) \quad u_h(t) \quad \dot{z}(t) \quad \ddot{u}_h(t)]^T$$

where $x(t) \in \mathbb{R}^{n_x}$ represents the modal state vector and $n_x = 2(n_m + n_h)$. The time-variant system matrices $A_c(t) \in \mathbb{R}^{n_x \times n_x}$ and $B_c(t) \in \mathbb{R}^{n_x \times n_d}$ are defined as:

$$A_c(t) = \begin{bmatrix} 0 & \mathbf{I} & \mathbf{0} \\ -M_{hh}^{-1}(t)K_{hh}(t) & -M_{hh}^{-1}(t)C_{hh}(t) \end{bmatrix}; \quad B_c(t) = \begin{bmatrix} \mathbf{T}_p(t) \end{bmatrix}$$

where the $\mathbf{T}_p(t)$ denotes the generalised input transformation matrix and $M_{hh}(t), K_{hh}(t)$ and $C_{hh}(t)$ are the generalised mass-, stiffness and damping matrices of the coupled crowd-structure system, as elaborated in [18].

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The vibration serviceability of footbridges is mostly assessed assuming (near-)resonant loading.

The key parameters determining the dynamic response of the footbridge are the corresponding

$$\ddot{u}_{n,\text{max}} \sim \frac{1}{2\xi_n m_n} = |H_n(\omega_n)|$$

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Figure 2. The FRF between input and output acceleration at the antinode of the considered mode of the empty structure with $f_b = 2.0$ Hz, $\xi_b = 0.4\%$ and $m_b = 25 \times 10^3$ kg (solid) and the structure occupied by a pedestrian density $d$ of 0.5 persons/m$^2$ (dashed), 1.0 persons/m$^2$ (dash-dot) and 1.5 persons/m$^2$ (dotted).

with $H_\omega(\omega)$ the FRF relating the harmonic input excitation to the acceleration response at the antinode of the considered mode of the footbridge, $\omega_n$ the natural frequency of the footbridge in rad/s and $|x|$ an operator applied in this paper to take the absolute value of the element(s) of $x$. The footbridge structure considered in this study is a simply supported beam with a span of 50 m and a bridge deck width of 3 m. Only the contribution of the fundamental mode is considered, with a sinusoidal mode shape, a modal damping ratio $\xi_b$ of 0.4%, a modal mass $m_b$ of $25 \times 10^3$ kg and a natural frequency $f_b$ of 2.0 Hz or 4.0 Hz.

The FRF of the coupled crowd-structure system relating the harmonic input excitation to the acceleration response (in the following referred to as $H_{in}(\omega)$), also taken at the antinode of the considered mode of the footbridge, is examined. Note that for the moving crowd model, the coupled crowd-structure system is time-variant (see section 2). However, in this section where the FRF of the coupled crowd-structure system is at focus, the pedestrian models are considered stationary and uniformly distributed on the structure thus resulting in a time-invariant system. Figure 2 compares the FRF of the empty footbridge to that of the coupled crowd-structure system for two pedestrian densities ($d = \{0.2, 0.4\}$ pedestrians/m$^2$), involving a single sample of human body model characteristics (see section 2.1). This illustrates that the FRF $H_{in}(\omega)$ is characterised by a single peak. The peak value of $H_{in}(\omega)$ highly depends on the frequency ratio $f_{in}/f_b$, and, is at its lowest for frequency ratios $f_{in}/f_b$ slightly lower than unity [7]. In the latter case, the crowd behaves in a similar way as a TMD [15, 18].

The two reduced-order models presented next in this section, aim to approximate the dynamic behaviour of the coupled crowd-structure system as characterised by its FRF $H_{in}(\omega)$ and involve an simplified SDOF system (section 3.1) or an simplified 2DOF system (section 3.2). In both cases, the number of DOF of the equivalent system, i.e. one or two, is significantly lower than that of the full-order crowd-structure system ($1 + n_h$). Note that the number of operations required in the derivation of these simplified models is far less than those involved in the statistical analysis that is required to evaluate the structural response to pedestrian excitation (see section 4). Hence, a significant reduction in the computational effort can be achieved.

3.1. Simplified SDOF system
The simplified SDOF system approximates the dynamic behaviour of the coupled crowd-structure system by a SDOF system with a modal mass that is identical to the one of the empty structure and an effective natural frequency ($f_{eff}$) and effective damping ratio ($\xi_{eff}$) defined
to describe the maximum steady-state acceleration response of the coupled crowd-structure system. A similar approach is often followed in the analysis of structures equipped with tuned mass dampers [35].

The peak value of the FRF $H_{bb}(\omega)$ determines the maximum amplitude of the steady-state acceleration response of the footbridge $\ddot{u}_{bb,max}$ when subjected to harmonic excitation:

$$\ddot{u}_{bb,max} \sim |H_{bb}(\omega_{bb})| , \text{ where } \omega_{bb} = \argmax_{\omega} |H_{bb}(\omega)|$$ (9)

Subsequently, the effective frequency of the coupled crowd-structure system $f_{eff}$ [Hz] is defined as:

$$f_{eff} = \frac{\omega_{bb}}{2\pi}$$ (10)

Accordingly, the effective damping ratio $\xi_{eff}$ [-] is defined as a measure for the change in maximum steady-state acceleration response in relation to that of the empty footbridge:

$$\ddot{u}_{bb,max} \sim \frac{1}{2\xi_{eff} m_{b}} \Rightarrow \xi_{eff} = \frac{\ddot{u}_{bb,max}}{|\ddot{u}_{bb,max}|} \xi_{B} = \frac{|H_{B}(\omega_{B})|}{|H_{bb}(\omega_{bb})|} \xi_{B}$$ (11)

Previous studies have shown that the most significant HSI-effect is in the effective damping ratio of the coupled crowd-structure system which is much higher than the inherent damping of the footbridge, in particular for modes with a natural frequency between 2.5 Hz and 5 Hz which are close to the natural frequency of the human body [6, 17]. This is also observed for the effective damping ratios identified for the simplified SDOF system (see Table 1), that is, for a single sample of human body model characteristics (see section 2.1). In addition, Table 1 also depicts the remaining difference in the FRF of the coupled crowd-structure system and the one simulated with the simplified SDOF system ($\varepsilon_{eff}$), defined as:

$$\varepsilon_{eff} = \frac{\|H_{bb}(\omega_{r}) - H_{eq}(\omega_{r})\|_{2}^{2}}{\|H_{bb}(\omega_{r})\|_{2}^{2}} \text{ with } \omega_{r} = [\omega_{1}, \ldots, \omega_{u}]$$ (12)

with lower bound $\omega_{l} = 2\pi(f_{eff} - 0.175n)$, upper bound $\omega_{u} = 2\pi(f_{eff} + 0.175n)$, and $n = 1$ and $n = 2$ for a natural frequency of the footbridge of 2 Hz and 4 Hz, respectively, i.e. in accordance with the dominant frequency range for (near-)resonant pedestrian excitation. These results show that, despite of the good match of the peak value of the FRF, significant discrepancies as high as 160% and 92% are observed for a natural frequency of the footbridge of 2 Hz and 4 Hz, respectively (see also Figure 3).

3.2. Simplified 2DOF system
As pedestrian excitation is in fact near-harmonic and transient [36, 37, 38], the impact of human-structure interaction may be overestimated on the basis of the corresponding increase in effective damping ratio [22]. To this end, the present subsection introduces an simplified 2DOF system that is composed of a SDOF system representing the resonant mode of vibration of the footbridge and an SDOF system ($f_{eq}, m_{eq}, \xi_{eq}$) to represent the mechanical interaction with the crowd. The parameters of the latter are determined by fitting to the FRF of the coupled crowd-structure system to that of the simplified 2DOF system.

An optimisation problem is formulated where the objective function contains the differences between the full-order ($H_{bb}(\omega)$) and the reduced-order ($H_{eq}(\omega)$) FRF of the coupled crowd-structure system within a frequency range defined by a lower ($\omega_{l}$ [rad/s]) and upper bound ($\omega_{u}$ [rad/s]). The residuals are computed at discrete frequencies $\omega_{k}$ [rad/s]:

$$\omega_{k} = \omega_{1} + k \Delta \omega \text{ for } k = 0, \ldots, N$$ (13)
Table 1. The main characteristics of the simplified SDOF and simplified 2DOF system and the corresponding remaining error $\varepsilon$. 

| $d$ [persons/m$^2$] | $f_{eff}$ [Hz] | $f_b = 2$ Hz | $\xi_{eff}$ [%] | $\varepsilon_{eff}$ [%] | $f_{eff}$ [Hz] | $f_b = 4$ Hz | $\xi_{eff}$ [%] | $\varepsilon_{eff}$ [%] |
|----------------------|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|
| 0.50                 | 1.87           | 1.59           | 155            | 4.03            | 1.59           | 155            | 4.03            | 1.59           |
| 1.00                 | 1.76           | 2.42           | 156            | 4.24            | 1.76           | 2.42           | 156            | 4.24            |
| 1.50                 | 1.68           | 2.65           | 158            | 4.58            | 1.68           | 2.65           | 158            | 4.58            |

with $N = (\omega_u - \omega_l)/\Delta \omega$ the number of samples, $\Delta \omega$ the selected frequency resolution and $\omega_u = \omega_l + N \Delta \omega$ the upper bound [rad/s].

The decision variable $x$ of this optimisation problem contains the natural frequency $f_{eq}$ and modal damping ratio $\xi_{eq}$ of the SDOF model representing the crowd. Based on the assumption that the crowd is uniformly distributed on the footbridge, the total mass of this supplementary
SDOF system is chosen as half of its total mass ($m_{eq} = m_{\text{crowd}}/2$), i.e. in accordance with the effect on the modal mass of the considered mode for a uniformly distributed added mass. Taking into account the physical constraints ($f_{eq} > 0$ and $\xi_{eq} > 0$), the problem is formulated in standard form:

$$\begin{align*}
\text{minimise} & \quad \frac{\| H_{\text{bb}}(\omega_r) - H_{\text{eq}}(\omega_r, x) \|^2}{\| H_{\text{bb}}(\omega_r) \|^2} \\
\text{subject to} & \quad f_{eq} > 0 \\
& \quad \xi_{eq} > 0
\end{align*}$$

and can be categorised as a nonlinear programming problem due to the nature of the objective function. The inequality constrained optimisation problem is solved using the fmincon-solver (MATLAB) by means of the Interior Point algorithm [39, 40].

The results of the optimisation problem are listed in Table 1 together with the remaining difference in the FRF of the coupled crowd-structure system and the one simulated with the simplified 2DOF system ($\varepsilon_{eq}$), defined analogously to $\varepsilon_{eff}$ (see Equation 12). The results in Table 1 and Figure 3 show that in this way, a nearly perfect fit is obtained between the FRF of the full-order crowd-structure system and the simplified 2DOF system ($\varepsilon_{eq} < 1.5\%$).

4. Prediction of the structural response to pedestrian excitation

In this final section, the objective is to evaluate the impact of the modelling assumptions on the structural response to pedestrian excitation. To this end, the response of the footbridge is predicted using the moving crowd model (section 2.2), the simplified SDOF system (section 3.1) and the simplified 2DOF system (section 3.2). The pedestrian flows are generated according to the crowd flow model defined in section 2.1. The impact of the modelling assumptions is evaluated through the comparison of the predicted 95 percentile value of the maximum acceleration level ($\bar{u}_{\text{max}95}$).

The analysis is performed for three pedestrian densities ($d = \{0.5, 1.0, 1.5\} \text{ persons/m}^2$) and two values of the fundamental natural frequency of the simply supported beam representing the footbridge ($f_b = \{2.0, 4.0\} \text{ Hz}$). To evaluate the impact of inter-subject variability, the following two cases are defined:

- **Case 1**: The step frequencies of the pedestrians follow a Gaussian distribution, with a mean value equal to the targeted natural frequency $f_l$ and a standard deviation of 0.175 Hz: $f_s = N(f_l, 0.175) \text{ [Hz]}, i.e. as often assumed for the simulation of spatially unrestricted crowd conditions (see section 2.1);

- **Case 2**: The step frequencies of the pedestrians are identical and equal to the targeted natural frequency $f_l$: $f_s = f_l$, i.e. as often assumed to simulate crowds with an increased level of synchronization (see section 2.1).

In order to achieve resonant conditions, the targeted frequency $f_l$ equals the effective frequency ($f_{\text{eff}}$) or half of the effective frequency ($f_{\text{eff}}/2$) of the coupled crowd-structure system for $f_{\text{eff}} < 2.5 \text{ Hz}$ and $2.5 < f_{\text{eff}} < 5.0 \text{ Hz}$, respectively.

Figure 4 shows the 95 percentile value of the maximum acceleration level ($\bar{u}_{\text{max}95}$) predicted by the moving crowd model, the simplified SDOF model and the simplified 2DOF model, as a function of the considered (i) inter-subject variability (case 1 or case 2), (ii) pedestrian density (0.5, 1.0 or 1.5 persons/m$^2$) and (iii) natural frequency of the empty footbridge (2.0 Hz or 4.0 Hz). The results show that the predictions of the simplified 2DOF model are in all cases within 5% of the corresponding predictions of the full-order moving crowd model. In absence of inter-subject variability (case 2), the deviation between the predictions of the moving crowd model and the simplified SDOF model are easily as high as 10%. Furthermore, when the step
The 95 percentile value of the maximum acceleration at midspan ($\ddot{u}_{\text{max}95}$) for a crowd with step frequencies following a Gaussian distribution (case 1) and identical step frequencies (case 2), and using (light to dark): (1) the moving crowd model, (2) the simplified SDOF model and (3) the simplified 2DOF model.

5. Conclusions
To account for the vertical mechanical interaction with the human body in the prediction of the induced structural response, the present contribution models the contact force between the pedestrian and the structure as the superposition of the force induced by the pedestrian on a rigid floor and the force resulting from the mechanical interaction between the structure and a SDOF system representing the human body. For large crowds, this approach leads to models with a high system order. In order to reduce the associated computational cost, this contribution proposes two reduced-order models to approximate the dynamic behaviour of the full-order coupled crowd-structure system. As far as the structure is concerned, a modally-reduced model is applied in which only the resonant mode is retained.

The first reduced-order model approximates the dynamic behaviour of the coupled crowd-structure system by a SDOF system with a modal mass equal to the one of the empty structure and an effective frequency and damping ratio chosen to match the maximum steady-state acceleration response of the full-order system. The results show that in absence of inter-subject variability, this simplified SDOF system allows for a prediction of the structural response that is within $\pm10\%$ of the corresponding predictions of the full-order moving crowd model. However, when the step frequencies are chosen to follow a Gaussian distribution with a mean value equal to the targeted natural frequency and a standard deviation of 0.175 Hz, the predictions of the simplified SDOF model are on average 40% higher (for $f_B = 2$ Hz) and 30% lower (for $f_B = 4$ Hz) in comparison to those of the moving crowd model.

Alternatively, an simplified 2DOF system is introduced that is composed of an SDOF system representing the resonant mode of vibration of the footbridge and an SDOF system representing the crowd. The parameters of the latter are determined by fitting the FRF of the simplified
2DOF system to that of the full-order coupled crowd-structure system. The results show that the predictions of the simplified 2DOF model are in all cases within 5% of the corresponding predictions of the full-order moving crowd model. Hence, when the prediction of (near-)resonant pedestrian-induced vibrations is of interest, the simplified 2DOF model provides as a good approximation of the full-order coupled crowd-structure model.

Further research will investigate the performance of similar reduced-order models in cases where multiple structural modes significantly contribute to the overall structural response.

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