Dynamic LES of the magnetohydrodynamic flow in a square duct with the varied wall conductance parameters

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Abstract. The current study is focused on the magnetohydrodynamics and demonstrates how electrical conductivity of the wall can affect the turbulent flow in the square duct. Different variations of the boundary walls have been considered including arbitrary conductive walls. The Large Eddy Simulations method with the dynamic Smagorinsky sub-grid scale model have been used for the turbulent structures resolving. Results show the significant impact of the wall conductance parameters for both Hartmann and side walls.

1. Introduction
Turbulent flow in magnetohydrodynamics (MHD) remains to be a topic with a lot of unstudied effects as it was demonstrated by Kenjereš [1]. In particular, it is still not clear how the electrical conductivity of the boundary conditions can influence the turbulence flow of the liquid metal. The proper understating and application of these effects can benefit in such industry sectors as continuous metal casting.

Currently, there are a few works which were focused on understanding the MHD turbulence effects using Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES). It was shown by Chaudhary et al. [2] how the transverse magnetic field with the varied Hartmann number \( (Ha = BL\sqrt{\sigma/\rho V}) \) could influence the turbulence characteristics in the square duct flow. Krasnov et al. [3] investigated the dumping properties of the magnetic field for the high Reynolds number \( Re = 10^5 \). DNS as a numerical method has been used in both studies which in turn, obviously leads to higher computations costs. However, using LES can be difficult in terms of a sub-grid scale (SGS) model choice. Krasnov et al. [4] compared different sub-grid scale models for the MHD LES channel flow and showed the ability of the dynamic Smagorinsky model to properly predict the influence of the magnetic field.

In all studies mentioned above, electrically insulated walls have been used as boundary conditions. The objective of this study is to investigate how the wall electrical conductivity will affect the flow in the square duct with the transverse magnetic field.

2. Methods and computational details
We assume that our fluid is the electrically conductive incompressible liquid and fulfills the condition that the magnetic Reynolds number \( Re_m = UL/\lambda << 1 \). Based on this assumption, a one-way MHD coupling approach can be used:

\[
\nabla^2 \Phi = \nabla \cdot (U \times B)
\]
where \( \Phi \) is the electrical potential, \( \mathbf{B} \) is the magnetic field and \( \mathbf{U} \) is the velocity field. The Lorentz force term added into the momentum equation is treated with the Four Steps Protection Method developed by Ni et al. [5]:

\[
(J \times B)^{k+1}_c = -\frac{1}{\Omega} \sum_{f=1}^{nf} \left( J^n_f \right)^{k+1} \left( B^{k+1}_f \times \mathbf{r}_f \right)s_f - \mathbf{r}_c \times \left( J^n_f \right)^{k+1} \left( B^{k+1}_f \right)_{sf}
\]

where \( J \) is the current density, \( \Omega \) is the cell volume, \( J^n_f \) is the normal component of the current density, \( s_f \) is the cell surface area, \( \mathbf{r}_c \) is the cell center distance vector and \( \mathbf{r}_f \) is the face center distance vector.

In order to model the arbitrary electrically conductive walls, an additional solid domain with finite thickness has been used. The equation (1) has been resolved in both domains, fluid and solid simultaneously. The specific boundary conditions have been imposed on the fluid-solid interface:

\[
-\sigma_l \frac{\partial \phi_l}{\partial n} = -\sigma_w \frac{\partial \phi_w}{\partial n}
\]

where the subscripts \( l \) and \( w \) mean the the liquid and the solid value respectively.

We used the LES method for the turbulence resolving and the dynamic Smagorinsky SGS model including an averaging procedure for the Smagorinsky constant, performed over cell faces. This algorithm and equations (1) - (3) have been implemented into OpenFOAM-extend.

The geometry is a square duct with the transverse applied magnetic field and periodic inlet-outlet boundary conditions. The amount of nodes was equal to \( N_x \times N_y \times N_z = 240 \times 120 \times 120 \) respectively.

Three electrical types of the wall boundary conditions have been considered: (i) fully electrically insulated walls \( (\partial \phi / \partial n = 0) \), (ii) arbitrary conductive walls with the wall conductance parameter \( C_d = (\sigma_w d_w)/(\sigma_L) = 0.1 \) and (iii) fully conductive walls \( (\phi = 0) \). The non-dimensional flow parameters are \( Re = 5602 \) and \( Ha = 21.2 \). Additionally, the simulation with the \( Ha = 0 \) has been done in order to provide the reference comparison.

### 3. Results

All simulations were statistically averaging for at least 100 flow-through times. Additionally, the spatial averaging procedure has been performed in order to speed up the convergence.

Considering the velocity and turbulent kinetic energy fields (figures (2) - (3)) one can conclude while the wall conductance parameters is increasing the symmetry is breaking down. That happens because the current density streamlines are gradually changing their behavior and starting to go through the walls.
Figure 2: Velocity averaged fields. (a) $Ha = 0$; (b) $Ha = 21.2$, fully insulated walls ($C_d = 0$); (c) $Ha = 21.2$, arbitrary conductive walls ($C_d = 0.1$); (d) $Ha = 21.2$, fully conductive walls ($C_d \rightarrow \infty$).

Figure 3: Turbulent kinetic energy fields. (a) $Ha = 0$; (b) $Ha = 21.2$, fully insulated walls ($C_d = 0$); (c) $Ha = 21.2$, arbitrary conductive walls ($C_d = 0.1$); (d) $Ha = 21.2$, fully conductive walls ($C_d \rightarrow \infty$).

Our results have been validated with two DNS reference duct flow cases: Gavrilakis [6] without the magnetic field and Chaudhary et al. [2] where $Ha = 21.2$ and surrounding walls are fully insulated. Figures (4) - (5) show this comparison for the dimensionless velocity profiles and RMS profiles. A good agreement is obtained between the reference DNS and the present dynamic LES.

![Graph of dimensionless velocity profiles](image)

Figure 4: Dimensionless velocity profiles

Analyzing the wall conductance parameters influence provides us with interesting results. For the horizontal bisector (side walls, parallel to the magnetic field) the increasing of the wall conductivity leads to a gradual decrease of the RMS values. Meanwhile, the distribution along the vertical bisector (Hartmann walls, perpendicular to the magnetic field) is very different. The fully conductive case has the
highest RMS values among other MHD cases and even close to the case without magnetic influence. The arbitrary conductive case has the lowest RMS peak value near the wall but this value is getting slightly higher when moving to the core. Potentially, this fact means the possibility of achieving the lowest RMS values near the Hartmann wall by varying the conductance parameter.

![RMS profiles](a) horizontal bisector, z-axis  
![RMS profiles](b) vertical bisector, y-axis

**Figure 5: RMS profiles**

### 4. Conclusion

In order to discuss the wall conductance parameters influence on the MHD turbulent duct flow, the LES with the dynamic Smagorinsky SGS model has been performed. Obtained results showed that the RMS values along horizontal and vertical bisector are differently affected. The future investigation of additional characteristics such as turbulent kinetic energy budgets can improve the understanding of this mechanism.

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