Stochastic Process Simulation of Soil Displacement in Calculations of Seismic Resistant Buildings

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Abstract. The solution to the problem of finding the statistical characteristics for a random process of seismic displacement of the foundation soil is given. Analytical methods of the theory of random functions and the method of statistical tests are used. The initial data for solving this problem are the characteristics of the random process of accelerating the soil of the base. The preliminary adjustment of the original accelerogram used for the subsequent calculation is shown. The implementations (accelerogram and seismogram) of the considered processes are constructed for two types of spectral acceleration densities in the form of a polyextreme function and an averaged function with one extreme. The obtained results are intended for carrying out probabilistic calculations of seismic resistant buildings as nonlinear systems and assessing their level of reliability.

1. Introduction
Random process simulation of seismic displacements of the soil foundation is directly related to the reliability assessment of structures for seismic effects, as well as their probabilistic calculation taking into account the physical and geometric nonlinearity [1]. When constructing a computational model of a seismogram, an adjustment of the initial accelerogram is necessary [2]. This is due to the fact that the integration of seismic acceleration recording of soil foundation leads to the appearance of low-frequency trends, which qualitatively changes the picture of the actual displacements. This is due to the uncertainty of the initial integration conditions, which, as a rule, are assumed to be zero, and the instrumentation errors that give low-frequency distortions in the accelerogram spectrum [3]. Thus, for a more accurate construction of a computational model of a seismogram, it is necessary to release the second accelerogram integral from the indicated disadvantages.

2. Main part
We simulate accelerogram as a product of a normal stationary random process \( \tilde{X}(t) \) and deterministic enveloping curve \( A(t) \) in accordance with [4]
\[ \tilde{a}(t) = A(t) \tilde{X}(t). \]  

Random process \( \tilde{X}(t) \) consists of such statistical characteristics as:

- Expected value \( m_{i}(t) = 0; \)
- Correlation function \( K_{s}(\tau); \)
- Dispersion \( D_{i}; \)
- Spectral density \( S_{i}(\omega). \)

The task is to build realizations for a random process of seismic displacement of the foundation soil \( \tilde{u}(t) \) by given characteristics of a random acceleration process \( \tilde{a}(t) \) and finding its statistical characteristics [5-8].

Let random process \( \tilde{X}(t) \) be in the form of canonical decomposition

\[ \tilde{X}(t) = \sum_{i=1}^{n} \left( \tilde{A}_{i} \cos(\omega_{i} t) + \tilde{B}_{i} \sin(\omega_{i} t) \right), \]  

where \( \tilde{A}_{i}, \tilde{B}_{i} \) – random normally distributed quantities with zero expectation and pair wise equal variances.

\[ D_{a_{i}} = D_{b_{i}} = D_{i} \]  

Decomposition (2) corresponds to dispersion decomposition \( D_{i} \) of random function \( \tilde{X}(t) \) by frequencies \( \omega_{i} \).

\[ D_{i} = D_{s} \cdot d_{s}(\omega_{i}) = D_{s} \int_{\frac{\omega_{i} - \Delta\omega}{2}}^{\frac{\omega_{i} + \Delta\omega}{2}} S_{s}^{*}(\omega)d\omega. \]  

Thus, the original random process \( \tilde{X}(t) \) is reduced to the sum of uncorrelated band pass white noise, with a constant over the interval \( \left[ \frac{\omega_{i} - \Delta\omega}{2}, \frac{\omega_{i} + \Delta\omega}{2} \right] \) of spectral density.

Correlation function of \( i \) elementary random function of decomposition (2) is a simple cosine with frequency \( \omega_{i} \)

\[ \rho(\tau) = \cos(\omega_{i} \tau). \]  

Then, in view of (5), the correlation function \( K_{s}(\tau) \) can be represented as

\[ P(\tau) = D_{i} \sum_{i=1}^{n} d_{i}(\omega_{i}) \cdot \cos(\omega_{i}\tau). \]  

Let us integrate function (2) twice, taking into account that, in the case of a linear transformation of a random function defined by a canonical decomposition, we also obtain a canonical decomposition

\[ \tilde{u}(t) = L\left\{ \tilde{X}(t) \right\} = \sum_{i=1}^{n} \left[ \tilde{A}_{i} \psi_{i}(t) + \tilde{B}_{i} \psi_{i}(t) \right] \]  

with coordinate functions
\begin{align}
\psi_1'(t) &= \int_0^t \cos(\omega t)d\tau d\tau = \frac{1 - \cos(\omega t)}{\omega^2}, \\
\psi_i'(t) &= \int_0^t \sin(\omega t)d\tau d\tau = \frac{\sin(\omega t) - \omega t}{\omega^3}.
\end{align}

Eliminating the trend appearing in the double integration of coordinate functions, we obtain the displacement function in the form of

\[ \ddot{u}(t) = \sum_{i=1}^n \left[ \tilde{A}_i \frac{\cos(\omega t)}{\omega^2} + \tilde{B}_i \frac{\sin(\omega t)}{\omega^3} \right]. \]

At double differentiation of function (10) we arrive at the process \( \ddot{X}_0(t) \), which is different from the process \( \ddot{X}(t) \) by amplitude sign \( \tilde{B}_i \). Random variables \( \tilde{A}_i \) and \( \tilde{B}_i \) have a zero expectation and the appearance of their positive and negative values are equiprobable. Dispersions and correlation functions of processes \( \ddot{X}(t) \) and \( \ddot{X}_0(t) \) are equal to each other, so in a probabilistic sense \( \ddots \).

The correlation function of a random process (10) is found by the formula

\[ K_s(t_1, t_2) = D_s \sum_{i=1}^n d_i(\omega) \left[ \frac{\cos(\omega t_1) \cos(\omega t_2)}{\omega^4} + \frac{\sin(\omega t_1) \sin(\omega t_2)}{\omega^4} \right] = D_s \sum_{i=1}^n d_i(\omega) \frac{\cos[\omega_i(t_1 - t_2)]}{\omega_i^4} \]

This approach is applicable to stationary random processes with any kind of spectral density and envelope function [10, 11].

Stochastic construction of structures for seismic loads involves either using averaged spectral density parameters [12, 13], or seismic spectra are considered as polyextremal functions [14, 15], or the seismic impact model is represented by a set of random processes whose spectral densities fill in some given (known from experience) carrier interval frequencies [16, 17]. As an example, consider the simulation of the implementation of a random process \( \ddots \) with normalized input spectral density \( S_n^s(\omega) \), containing \( N = 7 \) main extremes, and random process \( \ddots \), normalized input spectral density of which \( S_n^s(\omega) \) contains one main extreme (\( N = 1 \)). The spectral density parameters were obtained when processing components of seismic soil motions during the Gazli earthquake (1976) [18-21]. The spectral density of seismic acceleration is approximated by the function

\[ S_n^s(\omega) = \frac{2}{N \pi} \sum_{i=1}^7 \frac{m_k + \omega^2}{m_k^2 + 2a_k \omega^2 + \omega^4}, \]

where \( m_k = \alpha_k^2 + \beta_k^2 \); \( a_k = \alpha_k^2 - \beta_k^2 \); \( k = 1, \ldots, N \).

The spectral density parameters (12) for the two processes considered are listed in Table 1.

**Table 1.** The coefficients for the approximation of the spectral densities of the processes \( \ddots \) and \( \ddots \).

| Processes | \( \ddots \) | \( \ddots \) |
|-----------|--------------|--------------|
| \( \# \) of extreme | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 1 |
| \( \alpha \), sec\(^{-1}\) | 0.208 | 0.494 | 0.291 | 0.745 | 0.583 | 0.812 | 1.378 | 3.21 |
| \( \beta \), sec\(^{-1}\) | 0.69 | 5.86 | 8.62 | 17.24 | 23.1 | 33.45 | 45.17 | 9.0 |
Figure 1. Process spectral density $\tilde{X}(t)$ and $\tilde{X}_1(t)$.

Figure 1 shows the graphs of spectral densities, and Figure 2 – their representation in the form of discrete spectra $dx(\omega)$ and $dx_1(\omega)$ of uncorrelated band white noises.

Figure 2. Discrete spectral density of processes $\tilde{X}(t)$ and $\tilde{X}_1(t)$.

Normalized correlation functions corresponding to spectral densities (12) with $N = 7$ and $N = 1$, have the form (Figure 3).

$$K_s^n(\tau) = \frac{1}{N} \sum_{i=1}^{N} e^{-\omega_i \tau} \cos(\beta_i \tau) \approx \sum_{i=1}^{N} dx(\omega_i) \cos(\omega_i \tau), \quad (13)$$

$$K_{s1}^n(\tau) = e^{-\alpha \tau} \cos(\beta \tau) \approx \sum_{i=1}^{N} dx_1(\omega_i) \cos(\omega_i \tau). \quad (14)$$

Figure 3. Normalized correlation functions of processes $\tilde{X}(t)$ and $\tilde{X}_1(t)$.
We take the function as the envelope [2]

\[ A(t) = A_0 \gamma t e^{-\gamma t}. \]  

(15)

Modeling realizations of random processes of seismic acceleration and displacement of the soil foundation, shown in the form of canonical decomposition (2) and (10), is reduced to modeling random numbers \( \tilde{A}_i \) and \( \tilde{B}_j \) with known statistical characteristics (3). Process implementations \( \tilde{a}(t) \) (Figure 4) and \( \tilde{u}(t) \) (Figure 5) are calculated with the following calculated parameters: earthquake intensity 7 points, average peak acceleration \( A_{max} = 1 \text{m/sec}^2 \); standard deviation \( \sigma_{A_{max}} = 0.408 \text{m/sec}^2 \); \( A_0 = 2.718 \); \( \gamma = 0.3 \).

Figure 4. Process implementation \( \tilde{a}(t) \) (accelerogram).

Figure 5. Process implementation \( \tilde{u}(t) \) (seismogram).

3. Conclusion

The shown solution to the problem of modeling seismic loads is intended for carrying out a probabilistic calculation of buildings and structures based on realizations of random processes of acceleration or displacement of soil by analytical methods of the theory of random functions, as well as by the method of statistical tests using a computer.

The method of canonical decompositions for modeling random processes allows to obtain a solution for different parameters of seismic soil motion: the envelope function, spectral density, process attenuation rate, intensity and duration of an earthquake. The calculation is carried out at the level of correlation approximations taking into account the expectation and correlation function.
(spectral density). The conducted studies allow asserting about the admissibility of using the obtained implementations for further calculations of the structures of seismic resistant buildings and structures as nonlinear stochastic systems, including for assessing the level of their operational reliability.

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