A Robust SHM Scheme Combining Time Series Models with Dynamic QPSO Algorithm

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Abstract

A novel damage diagnostic algorithm combining time series models with Dynamic Quantum Particle Swarm Optimization (DQPSO) algorithm is proposed. Acceleration time history data is used to detect damage from the ratio of variances of the prediction errors from Autoregressive (AR) and Autoregressive with eXogeneous (ARX) models of baseline and current data. While Vector Autoregressive (VAR) model is used to locate the damage spatially, its intensity is obtained by solving an inverse problem, using dynamically reconfigurable multiple swarm based DQPSO algorithm. Numerical simulation studies on a simply supported beam clearly indicate the robustness to detect, locate and quantify the damage in the presence of environmental variability and measurement noise.

Keywords: Timeseries models, damage diagnosis, Dynamic QPSO, environmental variabilities, noise.

1. Introduction

The goal of SHM is to determine and classify damages (location, type, and severity) for a dynamical system exposed to varying environmental and operational conditions as well as instrumentation noise (i.e., 'real world' conditions) while eliminating false indications. Although the field of SHM has experienced significantly increased research during the last decade, a robust damage diagnostic method that can provide quantitative damage information anywhere in spatially large structures, such as bridges, is still under development. Majority of earlier research in damage diagnostic techniques are based on vibration response measurements and also majority of these techniques rely on finite element modeling processes and/or linear modal properties for damage diagnosis. For practical civil engineering applications these methods have not been shown to be effective in detecting damage at an early state. In view of the limitations associated with modal based SHM schemes and uncertainties associated with the updated FEM models, the non modal based techniques like time series models are gaining popularity in recent
past. The damage diagnostic techniques based on time series models require only the acceleration time history data collected from various sensors on the structure of interest. There is no requirement of physics-based or finite element models and vibration modal information. Sohn and Farrar [1,2] pose the SHM problem in statistical pattern recognition and time series analysis paradigm which composed of a two-stage prediction model, combining an Auto-Regressive (AR) and an Auto-Regressive with eXogenous inputs (ARX) model, constructed with selected and normalized acceleration signals obtained from the undamaged structure. The one-step-ahead error prediction was defined as a damage sensitive index. Carden and Brownjohn [3] classified different states of several structures using AutoRegressive Moving Average (ARMA) models. Zhang [4] used AR and ARX models for identifying the damage location in a steel beam under random loading and different damage scenarios. Bodeux and Golinval [5] used ARMAV models for damage detection in the ‘Steel-Quake’ structure in the framework of a European benchmark test structure. Mattson and Pandit [6] proposed a method based on Autoregressive Vector (ARV) models and used statistical moments of the residuals of these models as damage-sensitive features. Mosavi [7] used the ARV models and a statistical measure called Mahalanobis distance to identify the damage locations and found successful, even when the induced damage was very small. MUSTAFA GUL and Catbas [8] proposed an approach using a modified time series analysis methodology to detect and locate structural changes by using ambient vibration data where, the level of the damage feature gives important information about the relative change of the damage severity, although direct damage quantification is not achieved. Kopsaftopoulos and Fassois [9] introduced a vibration based statistical time series method that is capable of effective damage detection, precise localization, and magnitude estimation within a unified stochastic framework, based on the extended class of vector-dependent functionally pooled (VFP) models. However, the treatment of multiple damage scenarios, as well as operation under varying operating conditions and large uncertainties were not implemented. In this paper, a novel damage diagnostic algorithm is proposed by combining ARX and ARV time series models with Dynamic Quantum Particle Swarm Optimization (DQPSO) algorithm for detection, localization and quantification of damage. The proposed algorithm is shown to be robust with respect to measurement noise, environmental variability and multiple damage scenarios through solving a numerical example of a simply supported beam girder.

2. Damage Diagnostic Methodology

2.1 Damage Detection using AR-ARX Models

Since the nature of the input load on the structure is unknown, a two-stage prediction model i.e., combining auto-regressive (AR) and auto-regressive with exogenous inputs (ARX) techniques, adopted by Zhang [4] is used for damage detection. Healthy (baseline) databases of the structural responses (at specified sensor locations as shown later) are generated by using varied levels of random loads, environmental conditions and measurement noises in the form of Signal to Noise Ratios (SNR). The reference acceleration time history data at each node is partitioned into several subsets. Similarly, the current datasets are generated and partitioned into several subsets, which consists of acceleration time history data of both healthy and damaged structure. All the reference and current datasets will be standardized, prior to generation of smaller subsets as follows.

\[ X = \frac{x - \mu_x}{\sigma_x} \]  

(1)

where \( x \) is the acceleration time history signal, \( X \) is the standardized signal. \( \mu_x \) and \( \sigma_x \) are the mean and standard deviation of \( y \) respectively. An AR(p) model [10] of reference data \( x(t) \), can be written as

\[ x(t) = \sum_{j=1}^{p} \phi_{ij} x(t-j) + \epsilon_x(t) \]  

(2)

where \( p \) is the AR order, \( \Phi \) is the AR coefficients and \( \epsilon_x \) is the prediction error.

The AR order is set to be 25 based on a partial auto-correlation analysis described in Box et al [10]. Now, employing a new current data subset \( y(t) \) obtained from an unknown structural condition of the system, AR model is fitted as in the following equation. Here the new data subset \( y(t) \) has the same length as the signal \( x(t) \).

\[ y(t) = \sum_{j=1}^{p} \phi_{ij} y(t-j) + \epsilon_y(t) \]  

(3)

Then, the reference data subset \( x(t) \), closest to the new current data subset \( y(t) \) is defined as the one that minimizes the difference of their AR coefficients. If the new signal was recorded under a structural condition different from the
conditions, where reference signals were obtained, the prediction model estimated from even the ‘closest’ signal in the reference database would not reproduce the new signal well. For the construction of a two-stage prediction model proposed in this study, it is assumed that the error ($\varepsilon_x(t)$), between the measurement and the prediction obtained by AR model is mainly caused by unknown external input. With this assumption, an ARX model is used to reconstruct the input-output relationship between $\varepsilon_x(t)$ and $x(t)$ as follows.

$$x(t) = \sum_{i=1}^{a} \alpha_i x(t-i) + \sum_{j=1}^{b} \beta_j \varepsilon_x(t-j) + \varepsilon_x(t)$$

(4)

where $\varepsilon_x(t)$ is the residual error after fitting the ARX(a,b) model to the $\varepsilon_x(t)$ and $x(t)$ pair. In the present work both $a$ and $b$ values are taken as 10. Even though the parameters $a$ and $b$ for the ARX model are set arbitrarily keeping $(a+b)$ less than $p$. The ARX(a,b) model estimated in eqn.(5) is used to reproduce the input/output relationship of $y(t)$ and $x(t)$.

$$e_y(t) = y(t) - \sum_{i=1}^{a} \alpha_i y(t-i) - \sum_{j=1}^{b} \beta_j \varepsilon_y(t-j)$$

(5)

where $e_y(t)$ is considered to be an approximation of the system input estimated from eqn.(3). $\alpha_i$ and $\beta_j$ coefficients are associated with $x(t)$ and obtained from eqn.(4). Therefore, if the ARX model obtained from the reference signal block pair $x(t)$ and $e_x(t)$ were not a good representation of the newly obtained signal segment pair $y(t)$ and $e_y(t)$, there is likely to be a significant change in the standard deviation of the residual error, $e_y(t)$, compared to that of $e_x(t)$. In particular, the standard deviation ratio of the residual errors, $\sigma(e_y)/\sigma(e_x)$, is expected to reach its maximum value in the presence of damage. In view of this, the ratio of standard deviation of similar signals $\sigma(e_y)/\sigma(e_x)$ is considered as damage sensitive feature, $D_f$.

$$D_f = \frac{\sigma(e_y)}{\sigma(e_x)}$$

(6)

If the structure is free from damage and of almost similar environmental conditions, the value of $D_f$ is expected to be close to 1, whereas, the presence of damage in the structure will alter the situation quite significantly even with the matched environmental conditions and the value of $D_f$ will exhibit higher values as the variance of $e_y$ will be much higher than $e_x$.

2.2 Damage Localization using ARV Model

AR and ARX models, even though, capable of providing information about natural frequencies and damping ratios, they do not include the spatial information about structural vibration response i.e. modal information. In view of this, we use the vector autoregressive(ARV) model, whose coefficients are expected to be sensitive to structural mode shape changes and hence be considered as ideal candidates for spatial damage location. Once the presence of damage is confirmed in the first phase, vector AR model is used to identify the spatial location of damage in the second phase. The ARV model can be written as

$$y_t = \sum_{i=1}^{q} \Phi_i y_{t-i} + \varepsilon_t$$

(7)

where $q$ is the order of the estimated model, and $\Phi_i$ is the estimated coefficient matrix. The size of the $\Phi_i$ matrices depends on the number of sensors included in the model estimation. After the estimation of the coefficient matrix, it is obvious that the coefficients associated with early response lags are the most crucial in modeling the dynamic response of the structure. Hence, in the present work, the first two coefficients, $\Phi_1$ and $\Phi_2$ were plotted in the Cartesian coordinate system, considering $\Phi_1$ and $\Phi_2$ in x and y-axis respectively, in order to extract damage features for sensor j. The damage feature based on Mahalanobis distances of the coefficients in the Cartesian coordinates[10] are then arrived for all the sensors and can be computed using the Fisher criterion which gives the measure of shift in the probability distributions of two groups of data with normal distributions as given below.

$$F = \frac{(m_y - m_x)^2}{\sigma_x^2 + \sigma_y^2}$$

(8)
where $m_h$ and $\sigma_h^2$ are mean and variance of the Mahalanobis distances computed using the healthy data subsets of current data, $m_d$ and $\sigma_d^2$ are mean and variance of the Mahalanobis distances computed using the data subsets of current data, corresponding to the structure with damages. The spatial damage feature $F$, calculated for all sensors, when plotted, clearly indicates the spatial location of the damage with peak values of $F$.

2.3 Damage Quantification using DQPSO

The identified damage is quantified by formulating it as an optimization problem and an evolutionary algorithm based on swarm intelligence is employed for solving the resulting complex non-linear optimization problem. Most of the earlier works invariably use genetic algorithms (GA) for Structural System Identification[11-13]. On the other hand, PSO has some attractive characteristics and in many cases proved to be more effective when compared with GA and other similar evolutionary techniques[14]. Keeping these things in view, an advanced particle swarm optimization algorithm based on quantum mechanics called Dynamic quantum PSO algorithm[15] is used in this paper, for solving the optimisation problem of damage quantification. Here, the stiffness reduction factors of each elements are considered as the design variables. Element-level modeling of the structural damage is assumed by writing the stiffness matrix of the damaged $j$th element as

$$K_j' = \hat{\beta} K_j$$

(9)

where the scalar $\hat{\beta} \in [0, 1]$ defines the reduction in the stiffness of the $j$th element. The cost function is formulated as the minimization of difference between the sum of the coefficients of the ARX(a) model of all the sensors corresponding to reference and the current scenario as follows

$$f(\hat{\beta}) = \min \sum_{j=1}^{n_s} \sum_{i=1}^{t} \text{abs}(\alpha_{ji}^1 - \alpha_{ji}^2)$$

(10)

Where $n_s$ is the number of sensors, $t$ is the timeseries, $\alpha_{ji}^1$ is the $i$th ARX coefficient of $j$th sensor of the damaged structure and $\alpha_{ji}^2$ is the $i$th ARX coefficient of $j$th sensor of the finite element model constructed with a given trial set $\hat{\beta}$. The DQPSO algorithm is explained in the following section.

2.3.1 Dynamic Quantum PSO Algorithm

In the quantum physics, the state of a particle with momentum and energy can be depicted by its wave function $\psi(x, t)$ instead of the position and velocity used in traditional PSO. The probability of a particle appearing in a certain position $x$, can be obtained from probability density function $|\psi(x, t)|^2$, the form of which depends on the potential field in which the particle lies. The particle move according to the following equation

$$x_{ij}^{t+1} = p_{ij}^t \pm \hat{\beta} |m_{\text{best}}^t_{ij} - x_{ij}^t| \ln (1 / u_{ij})$$

(11)

where $m_{\text{best}}_{ij}$ is the mean best of the $i$th particle in $j$th dimension and $u_{ij}$ is a random number uniformly distributed in (0, 1). This equation is implemented by using Monte-Carlo technique as

$$x_{ij}^{t+1} = p_{ij}^t + \beta |m_{\text{best}}^t_{ij} - x_{ij}^t| \ln (1 / u_{ij}) \quad \text{if} \quad k > 0.50$$

$$x_{ij}^{t+1} = p_{ij}^t - \beta |m_{\text{best}}^t_{ij} - x_{ij}^t| \ln (1 / u_{ij}) \quad \text{if} \quad k \leq 0.50$$

(12)

where $k$ is a random number in the range [0, 1]. The most commonly used controlled strategy of $\hat{\beta}$ is to initially setting it to 1.0 and reducing it linearly to 0.30. In the present work, the parameter $\hat{\beta}$ varied linearly from 1.0 to 0.30 with the iteration as

$$\hat{\beta}^t = \beta_{\text{max}} - \frac{(\beta_{\text{max}} - \beta_{\text{min}}) x \text{ max iterations}}{t}$$

(13)

$p_{ij}^t$ is the local attractor and defined as:

$$p_{ij}^t = \varphi_{ij}^t \text{Pbest}_{ij}^t + (1 - \varphi_{ij}^t) \text{gbest}_{j}^t$$

(14)
where $\phi_{ij}$ is a random number uniformly distributed in $(0, 1)$. $\beta$ is called the contraction-expansion coefficient, which can be tuned to control the convergence speed of the algorithm. The ‘mbest’ is the mean best position and is defined as the center of pbest positions of the swarm and it can be written as:

$$m_{best}^t = (m_{best_1}^t, m_{best_2}^t, ..., m_{best_D}^t) = \frac{1}{M} \sum_{i=1}^{M} P_i^t$$

where $M$ is population size and $P_i$ is the personal best position of particle $i$ with dimension $D$. The details of DQPSO algorithm are as follows.

1. In DQPSO, each possible solution $X_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{iD})$, where $D$ is the number of design variables is considered as a particle. The initial population of $N$ particles (solutions) is generated randomly and it constitutes the swarm. The fitness of each of the solution (frog) is evaluated and the particles are then sorted in descending order according to their fitness.

2. Divide the swarm into ‘$M$’ sub-swarms each holding ‘$K$’ particles such that $N = K \times M$. The division is done in round robin fashion i.e., the first particle is assigned to the first sub-swarm. Second one is assigned to the second sub-swarm, the $M^{th}$ particle to the $M^{th}$ sub-swarm and ($M+1)^{th}$ particle back to the first sub-swarm. This way of distributing particles to sub-swarms preserves diversity among frogs within each sub-swarm.

3. Each sub-swarm works independently in achieving the goal of exploring the search space for optimum solution. Each particle is improved within the subswarms using equations 11-15. After user specified number (say ‘$S$’) of evolutions in each of the sub-swarm, the particles are regrouped and are sorted again. Repeat steps (2) and (3) till the convergence criteria are satisfied.

3. Numerical Studies

Numerical experiments using a simply supported beam girder, are conducted to test and verify the damage diagnostic technique combining timeseries models and DQPSO discussed in this paper. The span of the steel beam is 10 meters and is discretised into 20 elements as shown in Figure 1(a). The material and geometrical properties are also given in the Figure. The beam is excited using random dynamic loading which is stochastic in nature. The acceleration time history response is computed using finite element analysis with Newmark’s time marching scheme. The sampling rate is considered as 2000 samples per second. Initially acceleration time history data for healthy structure is generated with varied load intensities and environmental variabilities. One of the main issues related to structural damage diagnostic techniques, when applied to real situations, is their sensitivity to noise. In view of this, white Gaussian noise in the form of SNR (signal-to-noise ratio) is added to the acceleration time history response generated by the finite element code. SNR defines the amplitude of the noise with respect to that of the clean signal. Moreover the noisy sequences affecting different nodes are uncorrelated and thus, severe experimental conditions were simulated. The reference data generated is partitioned into several subsets and each subset of data is added with varied noise levels (with SNR values ranging between 40 and 60) to exactly simulate the measurement noise. Each subset of reference data consists of 2000 samples. Similarly, the current data is generated with varied load levels and noise levels. The current data may consist of both data of a healthy structure as well as data of a structure with simulated damages in some of the elements. Hypothetical damage scenarios are assumed by means of reduction of stiffness of some elements. The first phase of damage detection procedure involves identifying the damage by employing a two-stage prediction model combining auto-regressive (AR) and auto-regressive with exogenous inputs (ARX) techniques as outlined in the previous section. Once the damage is identified, we use vector autoregressive model to exactly identify the spatial location of damage. The damage is simulated into the beam girder after obtaining 4000 samples (i.e. 4000 time steps) by reducing the stiffness of element 6. Three damage scenarios have been investigated by varying the stiffness reduction levels in element 6 as 20, 30 and 50 percentages indicated as DP1,DP2 and DP3 respectively. The two stage prediction model combining auto-regressive (AR) and auto-regressive with exogenous inputs (ARX) techniques is employed to identify the damage. The damage features i.e. $D_f$ values are evaluated as explained in the earlier section. Each subset of the current data is tested for the presence of damage by calculating the damage feature $D_f$. From Figure 1(b), one can clearly observe from the $D_f$ plot for each sensor that there is a significant variation when compared to the healthy data indicating clearly the presence of damage. This confirms the presence of damage and the current data subsets
identified by AR-ARX model as damaged subsets are preserved along with their ARX coefficients to locate and quantify the damage. These damaged subsets are fitted in the vector autoregressive model and the damage features are obtained based on Mahalanobis distances. Similarly, the damage features are obtained for all the patterns considered. It can be clearly observed from the damage features in Figure 2(a) that the spatial location of damage i.e. 6th sensor location is identified. The noise levels considered are varied from SNR 40 to 60. Figure 2(b), clearly shows that the algorithm is robust in locating the damage even with the varied levels of noise present in the signal. After the location of damage, quantification of its intensity using DQPSO algorithm is carried out as the final stage of diagnostic process. The ARX parameters corresponding to the damaged datasets are used now in the cost function given in eqn 12. In this study, the population for DQPSO algorithm is considered as 30 with 5 subswarms of 6 swarms each. After every 10 generations, the exchange of information takes place dynamically among the sub-swarms till convergence. The final converged stiffness reduction factors for the elements of the beam are shown in Figure 4(a). From the figure it is clear that the proposed DQPSO algorithm is capable of quantifying the stiffness loss accurately from the noisy data.

In order to test the robustness of the algorithm to handle multiple spatial damages, the first damage is simulated in element 2 by reducing the corresponding stiffness by 20% after 3000 time steps (i.e. 3 seconds) and the second damage is simulated in element 15 by reducing the corresponding stiffness by 20% after 16000 time steps (i.e. 8 seconds). The presence of damage is identified using the similar procedure by employing combined AR-ARX models. The first spatial damage is identified using the vector autoregressive model by using the corresponding reference data and the current data collected after the presence of damage. The damage features are presented in Figure 3a for all sensors using vector AR model. Figure 3a., clearly identifies the spatial damage location as element 2. Once the first damage is identified, the remaining current data with first damage is also considered as subsets in the reference data. The combined AR-ARX model is employed on the current data collected after the occurrence of first damage to identify the subsequent damages. The vector ARV model is employed on the updated reference and current data sets, to identify the spatial location of the 2nd damage. Figure 3(a) and 3(b) gives the damage features obtained using vector AR model clearly indicating the first (element 2) and second (element 15) spatial location of damage in the beam girder with varied levels of damage intensities, and noise. Finally, the determination of the intensity of damages is performed by DQPSO algorithm and the values of the stiffness reduction factors for the elements of the beam are obtained as shown in fig 4(b). From fig 4(b), it is clear that the proposed damage diagnostic algorithm combining timeseries models and DQPSO, works robustly for multiple damage scenarios also in the presence of environmental variabilities and measurement noise.

4. Conclusion

In this paper, AR-ARX models which only include global vibration information are used as a damage identifier. After damage is identified using AR-ARX model, time series model with spatial correlations, called ARV model is used for extracting the spatial damage features. Finally, DQPSO algorithm is used to quantify the intensity of damage at their locations. Numerical studies on a simply supported beam considering different damage patterns clearly indicate that the proposed algorithm is robust in identifying the presence, spatial location and the intensity of damage, even with practical measurement noise (SNR) levels and environmental variabilities.

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(a) S.S. Beam with material properties

(b) Damage features extracted using AR-ARX models

Figure 1. Simply supported (S.S.) beam girder.
Figure 2. Damage feature plots of S.S. beam with single damage using ARV model.

(a) Damage at 6th element

(b) Damage at 6th element with different SNR for DP3

Figure 3. Damage feature plots of S.S. beam with multiple damages using ARV model.

(a) First damage at element 2.

(b) Second damage at element 15.

Figure 4. Identified stiffnesses of various scenarios of S.S. Beam using DQPSO algorithm.

(a) Single damage scenario

(b) Multi damage scenario