A precise new KLOE measurement of $|F_\pi|^2$ with ISR events and determination of $\pi\pi$ contribution to $a_\mu$ for $0.592 < M_{\pi\pi} < 0.975$ GeV

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Abstract. The KLOE experiment at the DAΦNE φ-factory has performed a new precise measurement of the pion form factor using Initial State Radiation events, with photons emitted at small polar angle. Results based on an integrated luminosity of 240 pb$^{-1}$ and extraction of the $\pi\pi$ contribution to $a_\mu$ in the mass range $0.35 < M_{\pi\pi}^2 < 0.95$ GeV$^2$ are presented. The new value of $a_\pi^\pi$ has smaller (30%) statistical and systematic error and is consistent with the KLOE published value (confirming the current disagreement between the standard model prediction for $a_\mu$ and the measured value).

Keywords: Hadronic cross section, initial state radiation, pion form factor, muon anomaly

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INTRODUCTION

The anomalous magnetic moment of the muon has recently been measured to an accuracy of 0.54 ppm [1]. The main source of uncertainty in the value predicted [2] in the Standard Model is given by the hadronic contribution, $a_\mu^{hlo}$, to the lowest order. This quantity is estimated with a dispersion integral of the hadronic cross section measurements. In particular, the pion form factor, $F_\pi$, defined via $\sigma_{\pi\pi} \equiv \sigma_{e^+e^- \rightarrow \pi^+\pi^-} = \frac{\pi\alpha^2}{3\pi}\beta_\pi^3(s)|F_\pi(s)|^2$, accounts for $\sim 70\%$ of the central value and for $\sim 60\%$ of the uncertainty in $a_\mu^{hlo}$.

The KLOE experiment already published [3] a measurement of $|F_\pi|^2$ with the method described below, using an integrated luminosity of 140 pb$^{-1}$, taken in 2001, henceforth referred to as KLOE05, with a fractional systematic error of 1.3%.

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MEASUREMENT OF $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma)$ AT DAΦNE

DAΦNE is an $e^+e^-$ collider running at $\sqrt{s} \simeq M_\phi$, the $\phi$ meson mass, which has provided an integrated luminosity of about 2.5 fb$^{-1}$ to the KLOE experiment up to year 2006. In addition, about 250 pb$^{-1}$ of data have been collected at $\sqrt{s} \simeq 1$ GeV, in 2006. Present results are based on 240 pb$^{-1}$ of data taken in 2002 (3.1 Million events) [4].

The KLOE detector consists of a drift chamber [5] with excellent momentum resolution ($\sigma_p/p \sim 0.4\%$ for tracks with polar angle larger than 45$^\circ$) and an electromagnetic calorimeter [6] with good energy ($\sigma_E/E \sim 5.7\%/\sqrt{E}$[GeV]) and precise time ($\sigma_t \sim 54\text{ ps}/\sqrt{E}$[GeV] $\oplus$ 100 ps) resolution.

At DAΦNE, we measure the differential spectrum of the $\pi^+\pi^-$ invariant mass, $M_{\pi\pi}$, from Initial State Radiation (ISR) events, $e^+e^- \rightarrow \pi^+\pi^-\gamma$, and extract the total cross section $\sigma_{\pi\pi} \equiv \sigma_{e^+e^- \rightarrow \pi^+\pi^-}$ using the following formula [7]:

$$s \frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \sigma_{\pi\pi}(M_{\pi\pi}^2) \ H(M_{\pi\pi}^2),$$

where $H$ is the radiator function. This formula neglects Final State Radiation (FSR) terms (which are properly taken into account in the analysis).

In the small angle analysis, photons are emitted within a cone of $\theta_\gamma < 15^\circ$ around the beam line (narrow blue cones in Fig. 1 left). The two charged pion tracks have $50^\circ < \theta_\pi < 130^\circ$. The photon is not explicitly detected and its direction is reconstructed by closing the kinematics: $\vec{p}_\gamma \simeq \vec{p}_{\text{miss}} = - (\vec{p}_{\pi^+} + \vec{p}_{\pi^-})$. The separation of pion and photon selection regions greatly reduces the contamination from the resonant process $e^+e^- \rightarrow \phi \rightarrow \pi^+\pi^-\pi^0$, in which the $\pi^0$ mimics the missing momentum of the photon(s) and from the final state radiation process $e^+e^- \rightarrow \pi^+\pi^-\gamma_{\text{FSR}}$. Since ISR-photons are mostly collinear with the beam line, a high statistics for the ISR signal events remains. On the other hand, a highly energetic photon emitted at small angle forces the pions also to be at small angles (and thus outside the selection cuts), resulting in a kinematical suppression of events with $M_{\pi\pi}^2 < 0.35$ GeV$^2$. Residual contamination from the processes $\phi \rightarrow \pi^+\pi^-\pi^0$ and $e^+e^- \rightarrow \mu^+\mu^-\gamma$ are rejected by cuts in the kinematical variable track-
A particle ID estimator, based on calorimeter information and time-of-flight, is used to suppress the high rate of radiative Bhabhas.

**EVALUATION OF $|F_\pi|^2$ AND $a_\mu^{\pi\pi}$**

The $\pi\pi\gamma$ differential cross section is obtained from the observed spectrum, $N_{ob}$, after subtracting the residual background events, $N_{bkg}$, and correcting for the selection efficiency, $\varepsilon_{sel}(M_{\pi\pi}^2)$, and the luminosity, $\mathcal{L}$:

$$
\frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \frac{N_{ob} - N_{bkg}}{\Delta M_{\pi\pi}^2} \frac{1}{\varepsilon_{sel}(M_{\pi\pi}^2) \mathcal{L}}.
$$

In order to correct for resolution effects, the differential cross section is unfolded using the Bayesian method described in [8]. The integrated luminosity, $\mathcal{L}$, is obtained from the observed number of Bhabha events, divided by the effective cross section evaluated from the Monte Carlo generator Babayaga@NLO [10, 11].

The cross section $\sigma_{\pi\pi}(M_{\pi\pi}^0)$ is obtained by accounting for final state emission (which shifts $M_{\pi\pi}$ to the virtual photon mass $M_{\pi\pi}^0$) and dividing the $\pi^+\pi^-\gamma$ cross section by the radiator function $H$ (obtained from Phokhara [12, 13, 14, 15, 16] by setting pion form factor $F_\pi = 1$ as in Eq. 1).

The bare cross section $\sigma_{\pi\pi}^0$, inclusive of FSR, needed for the $a_\mu^{\pi\pi}$ dispersion integral, is obtained after removing vacuum polarization, VP, effects [17]. Tab. 1 left shows the list of fractional systematic uncertainties of $a_\mu^{\pi\pi}$ in the mass range $0.35 < M_{\pi\pi}^2 < 0.95$ GeV$^2$.

Tab. 1 right shows the good agreement amongst KLOE results, and also with the published CMD-2 and SND values. They all agree within one standard deviation.

![FIGURE 2](image-url) Left: Comparison of the pion form factor measured by CMD-2, SND and KLOE, where for this latter only statistical errors are shown. Right: Absolute difference between the dispersion integral value (in each energy bin) evaluated by CMD-2 or SND respect to KLOE. The light (dark) band represents KLOE statistical (statistical + systematic) errors.

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2 Defined under the hypothesis that the final state consists of two charged particles with equal mass $M_{Trk}$ and one photon.
TABLE 1. Left: Systematic errors on the extraction of $a_{\mu\pi\pi}$ in the mass range $0.35 < M_{\pi\pi}^2 < 0.95$ GeV$^2$. Right: Comparison among $a_{\mu\pi\pi}$ values.

| Systematic Error                  | Value                  |
|-----------------------------------|------------------------|
| Reconstruction Filter             | negligible             |
| Background subtraction            | 0.3 %                  |
| Trackmass/Miss. Mass              | 0.2 %                  |
| $\pi/e$-ID                        | negligible             |
| Tracking                          | 0.3 %                  |
| Trigger                           | 0.1 %                  |
| Unfolding                         | negligible             |
| Acceptance ($\theta_{\text{miss}}$) | 0.2 %                  |
| Acceptance ($\theta_{\pi}$)      | negligible             |
| Software Trigger (L3)             | 0.1 %                  |
| Luminosity ($0.1_{\text{int}} + 0.3_{\text{exp}}$)% | 0.3 %                |
| $\sqrt{s}$ dependence of $H$      | 0.2 %                  |
| Total experimental systematics    | 0.6 %                  |

| Systematic Error                  | Value                  |
|-----------------------------------|------------------------|
| Vacuum Polarization               | 0.1 %                  |
| FSR resummation                   | 0.3 %                  |
| Rad. function $H$                 | 0.5 %                  |
| Total theory systematics          | 0.6 %                  |

$\alpha_{\mu\pi\pi} \times 10^{10} \times 0.35 < M_{\pi\pi}^2 < 0.95$ GeV$^2$

- KLOE05 [3, 18] 384.4 ± 0.8 stat ± 4.6 sys
- KLOE08 [4] 387.2 ± 0.5 stat ± 3.3 sys

$\alpha_{\mu\pi\pi} \times 10^{10} \times 0.630 < M_{\pi\pi} < 0.958$ GeV

- CMD-2 [19] 361.5 ± 5.1
- SND [20] 361.0 ± 3.4
- KLOE08 [4] 356.7 ± 3.1

Fig. 2 left shows a comparison of $|F_{\pi}|^2$ (obtained by $\sigma_{\pi\pi\pi}$ after subtraction of FSR (assuming pointlike pions) between CMD-2 [19], SND [20] and KLOE (with only statistical errors). For the energy scan experiments, whenever there are several data points falling in one 0.01 GeV$^2$ bin, we average the values. Fig. 2 right shows the absolute difference the $a_{\mu\pi\pi}$ values for each energy bin obtained in this analysis and the energy scan experiments. All the experiments are in rather good agreement within errors.

**CONCLUSIONS AND OUTLOOK**

KLOE has measured the dipion contribution to the muon anomaly, $a_{\mu\pi\pi}$, in the interval $0.592 < M_{\pi\pi} < 0.975$ GeV, with negligible statistical error and a 0.6% experimental systematic uncertainty. Theoretical uncertainties in the estimate of radiative corrections increase the systematic error to 0.9%. Combining all errors KLOE gives:

$$a_{\mu\pi\pi} (0.592 < M_{\pi\pi} < 0.975 GeV) = (387.2 \pm 3.3) \times 10^{-10}.$$ 

This result represents an improvement of 30% on the systematic error with respect to the previous published value from KLOE. The new result confirms the current disagreement between the standard model prediction for $a_{\mu\pi\pi}$ and the measured value, as shown in Fig. 3.

Independent analyses are in progress to:

- extract the pion form factor from data taken at $\sqrt{s} = 1$ GeV, off the $\phi$ resonance, where $\pi^+\pi^-\pi^0$ background is negligible, by using detected photons emitted at large angle. This analysis, which is very close to be finalized, allows to measure $\sigma_{\pi\pi\pi}$ down to the 2-pion threshold;
• measure the pion form factor directly from the ratio, bin-by-bin, of $\pi^+\pi^-\gamma$ to $\mu^+\mu^-\gamma$ spectra \cite{22};
• measure $\sigma_{\pi\pi(\gamma)}$ using the large angle analysis at the $\phi$ peak, which would improve the knowledge of the FSR interference effects (in particular the $f_0(980)$ contribution \cite{23,24}).

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