New Flocking Models Apply for UAV Formation

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Abstract. In this paper some new flocking models are introduced. The contact mechanism between agents is from the famous Cooker-Smile model. In these models, the information exchange in the system is fully connected and the agents can adjust their velocities to form a flock. The important innovation of this paper is the flocking agents will automatic slow their velocities to avoid collision when they detects their relatively velocities so quickly and they are too closed with other agents. Using these models, the information interaction between the agents is completely automated which means they do not need human help to finish the flock. We only need to give the initial velocity for each agents they will complete the flocking and not run into each other. Also, we give the final velocity of the system and from the expression it show this velocity is only depended on their agent’s initial velocity.

1. Introduction
Recently Multi-agent cluster control study has attracted great attention of researchers. The mechanism of the multi-agent form a flock is coming from the nature phenomenon, such as birds swarming, fish flocking and so on [1-3]. The Multi-agent cluster has widely application. This study can help robots coordinate to complete a command. Unmanned Aerial Vehicles (UAV) synergies a research hotspot. While people have to control each UAV so that the UAV can form a flock to complete a team task. Now we want to introduce the artificial intelligence to help the UAV form a flock. In order to realize this target we should study the mathematical mechanism of forming a flock.

In 1986, Reynolds proposed a swarming model and simulation the birds’ motion on computer. In this model they introduce three rules: separation, alignment and cohesion. In 1995, Vicsek [4-5] proposed a flocking model which the speed of the agents is a constant, the agents will adjust their direction of movement then complete the flocking. The model is following:

The agent “i” has a constant speed \( v \), and position \( x_i(t) \). The speed direction of the agent “i” is \( \theta_i(t) \) satisfies:

\[
\theta_i(t+1) = \frac{1}{n_i(t)} \sum_{j \in n_i(t)} \theta_j(t)
\]  

(1)

Here \( n_i(t) = \left\{ j : k \| x_j(t) - x_i(t) \| \leq r \right\} \), and \( n_i(t) = \# n_i(t) \) (Here \( \# \) represents the number of elements in a collection), constant \( r > 0 \).

Based on Vicsek’s model, so many scientists do lots of work to study the flocking behavior. In 2007, Cooker and Smile [6-7] propose a classical flocking model, which the information of the agents’
positions and velocities are both considered. The agents will adjust the velocity to keep flocking. They consider N agents in the model. The velocity and position of the agent “i” are \((x_i(t), v_i(t))\), which satisfy:

\[
\frac{dx_i(t)}{dt} = v_i(t), \\
\frac{dv_i(t)}{dt} = \alpha \sum_{j \in N, j \neq i} a_j (\|x_j(t) - x_i(t)\|) (v_j(t) - v_i(t)).
\]

Here the influence function \(a_j(t) = \frac{1}{N} \phi(\|x_j(t) - x_i(t)\|)\) and \(\phi(r) = \frac{1}{(1 + \|r\|^2)^{\beta}}\), the parameters \(\alpha > 0, \beta > 0\).

Also many researcher team continue study this field. Such as Prof. Ha and Li have proposed many classical flocking models. For example the study the flocking system with a leader [8-10]. Also they study the rank relation in the flocking system [11-13]. Based on the flocking study which is introduced above, this paper is focus on how the agents can realize collision avoidance during the flocking behavior. Inspired by the atomic mutual exclusion principle, this paper proposed a novel model. The agents automatic adjust their relative velocities to avoid collision. This model introduce the artificial intelligence (AI) to help the UAV finish the flock. The UAV receive the information by electromagnetic wave which contain the relative position and velocity. Then these information will be received and input to model algorithm to help the AI flocking flying.

In this paper, the fist section is the introduction. The second section will given the new flocking model. The third section is the simulation results of the UAV flocking model. The last section will introduce the following work.

2. New flocking models for UAV

2.1 A flocking model with the AI avoid collision proper.

In order add the AI to the UAV flight, this paper mainly focus on how the agents can form a flock after they receive the information of other agents’ velocities and positions. We proposed a new model. In this model we consider N agents. The agent “i” has its position and velocity as \((x_i(t), v_i(t))\) which satisfy:

\[
\frac{dx_i(t)}{dt} = v_i(t), \\
\frac{dv_i(t)}{dt} = \alpha \sum_{j \in N, j \neq i} b_j (\|x_j(t) - x_i(t)\|) (v_j(t) - v_i(t)) + \sum_{j \in N} \left(\frac{c(x_i(t) - x_j(t))}{\|x_i(t) - x_j(t)\|^2} - \frac{x_i(t) - x_j(t)}{\|x_i(t) - x_j(t)\|}\right).
\]

Here the influence function \(b_j(t) = \frac{1}{N} \phi(\|x_j(t) - x_i(t)\|)\) and \(\phi(r) = \frac{1}{(1 + \|r\|^2)^{\beta}}\), the parameters \(\alpha > 0, \beta > 0\). Let

\[
f(t) = \sum_{j \in N} \left(\frac{c(x_i(t) - x_j(t))}{\|x_i(t) - x_j(t)\|^2} - \frac{x_i(t) - x_j(t)}{\|x_i(t) - x_j(t)\|}\right).
\]

The set of \(N_i\) represent the collection agent “j” which the position of agent “j” satisfy \(\|x_i(t) - x_j(t)\| \leq c\). The function \(f(t)\) represents the exclusive repulsion when the distance of any two agents is less than the constant \(c > 0\). We add this function in the model is to keep agents avoid collision. When the agents are more close, there will be more and more repulsion. This is similar with the intermolecular repulsion in physics.
Definition 1: Flocking there are \( N \) agents in the system. The position and velocity of the agents “i” are \((x_i(t), v_i(t))\). Let \( dX(t) = \max \| x_i(t) - x_j(t) \| \) and \( dV(t) = \max \| v_i(t) - v_j(t) \| \). If \( dX(t), dV(t) \)

Satisfy: \( dX(t) < \infty \) for all \( t > 0 \). and \( \lim_{t \to \infty} dV(t) = 0 \), then the system is a flock.

Theory 1: For a system with \( N \), the position and velocity of the agent “i” are \((x_i(t), v_i(t))\). They satisfy the system \((3)\). If the parameter \( \beta > \frac{1}{2} \), this system will form a flock.

Remark 1: For the length of proof on Theory 1 is too long and very complex, here we won’t give the proof of this Theory. In order to proof it, we will using Lasalle invariance principles to proof the difference of any two agents’ velocities will converge to zero. Also we will introduce an energy function (V-function) to proof the difference of any two agents’ position is bounded. We want to point out that the last velocity of this system is the average the sum of all agents’ initial velocities. Next we will proof this.

Corollary 1 For a system with \( N \), the position and velocity of the agent “i” are \((x_i(t), v_i(t))\). They satisfy the system \((1)\). If the parameter \( \beta > \frac{1}{2} \), the final velocities of all agents will converge to a constant velocity \( \bar{v}(t) = \frac{1}{N} \sum_{i \in \mathbb{N}} v_i(0) \).

Proof: Let \( \bar{v}(t) = \sum_{i \in \mathbb{N}} v_i(t) \); from the definition of \((3)\), we have

\[
\frac{d}{dt} \bar{v}(t) = \sum_{i \in \mathbb{N}} (\alpha \sum_{j \in \mathbb{N}, j \neq i} b_j \| x_j(t) - x_i(t) \| (v_j(t) - v_i(t)) + \sum_{j \in \mathbb{N}} \left( \frac{c(x_i(t) - x_j(t))}{\|x_i(t) - x_j(t)\|^2} - \frac{x_i(t) - x_j(t)}{\|x_i(t) - x_j(t)\|^2} \right) = 0
\]

From the definition it has influence function \( b_j \| x_j(t) - x_i(t) \| = b_j \| x_i(t) - x_j(t) \| \), and if the agent “j” in the set of \( \mathbb{N}, \) the agent “i” must also in the set the \( \mathbb{N}, \). Using the symmetry of the influence function. We can deduce

\[
\frac{c(x_i(t) - x_j(t))}{\|x_i(t) - x_j(t)\|^2} - \frac{x_i(t) - x_j(t)}{\|x_i(t) - x_j(t)\|^2} = 0
\]

For any time \( t > 0 \), so we can easily get

\[
\frac{d}{dt} \bar{v}(t) = \sum_{i \in \mathbb{N}} (\alpha \sum_{j \in \mathbb{N}, j \neq i} b_j \| x_j(t) - x_i(t) \| (v_j(t) - v_i(t)) + \sum_{j \in \mathbb{N}} \left( \frac{c(x_i(t) - x_j(t))}{\|x_i(t) - x_j(t)\|^2} - \frac{x_i(t) - x_j(t)}{\|x_i(t) - x_j(t)\|^2} \right) = 0
\]

Then,

\[
\bar{v}(t) = \sum_{i \in \mathbb{N}} v_i(0)
\]

From the theory 1, we have \( \lim_{t \to \infty} \|v_i(t) - v_j(t)\| = 0 \), which means \( v_i(t) = v_j(t) \) for \( t \to \infty \). then for enough big t, we know \( v_i(t) = \frac{1}{N} \bar{v}(t) \). we can prove \( \bar{v}(t) = \frac{1}{N} \sum_{i \in \mathbb{N}} v_i(0) \).

Remark 2: Form the corollary 1, we can control the final velocity of the system only by the given initial velocity. This will be automatic implementation. We add this arithmetic to the UAV control to realize the velocity of automatic control.

Remark 3: Here there are \( N \) agents in the group, the position and the velocity of the agent “i” are \((x_i(t), v_i(t))\), the force f represent the repulsion which can keep the UAV avoid collision.
In order to finish flock, the function of controller in this model should satisfy some conditions.

2.2 A new hierarchical flocking model with the AI avoid collision

There is a leader in this model. If we want to control the whole UAV group, we only need give the instruction to the leader. The followers will follow the leader to fly and complete the cluster. This model also has practical significance and application value. It can save the costs of human resources and increase efficiency. For example, in a massive drone show there may have thousands of UAV. When we use this model to the UAV group, there will need several workers to operate the leaders. The UAV group will finish the flock. The model is as following:

The leader’s position and velocity \((x_0(t), v_0(t))\) satisfy:

\[
\frac{dx_0(t)}{dt} = v_0(t), \\
\frac{dv_0(t)}{dt} = v, 
\]

Here \(v\) is a constant.

The position and velocity of the follower “i” are \((x_i(t), v_i(t))\) satisfy

\[
\frac{dx_i(t)}{dt} = v_i(t), \\
\frac{dv_i(t)}{dt} = \alpha \sum_{j \neq i, j \neq 0} b_i \left(\|x_j(t) - x_i(t)\| \right) (v_j(t) - v_i(t)) + \sum_{j \neq i, j \neq 0} \frac{c(x_i(t) - x_j(t))}{\left(\|x_j(t) - x_i(t)\|\right)^2} - \frac{x_i(t) - x_j(t)}{\|x_j(t) - x_i(t)\|},
\]

The agents which are in the set of \(\mathbb{N}\) are all the followers. “0” is the leader.

**Theory 2:** For a system with \(\mathbb{N}+0\), the position and velocity of the leader “0” are \((x_0(t), v_0(t))\) and the position and velocity of the agent “i” \((i \in \mathbb{N})\) are \((x_i(t), v_i(t))\). They satisfy the system (10)(11). If the parameter \(\beta > \frac{1}{2}\), this system will form a flock and the final velocity of the system will converge to \(v\).

**Remark 4:** Similar with **Theory 1**, we won’t give the proof of **theory 2**. This theory will provide a useful algorithm which help us to control the whole system only by control the leader. The leader will transfer the information to other followers then form a flock.

3. Simulation

Here we will give two simulations to illustrate the validity of our two models. The mechanism of communication between the agents in the model is studied to help system form a flock. Then we add these arithmetic the UAV to control the UAV be a flock.

3.1 Example 1 (system without a leader)

We consider an UAV system in a two dimension-plane. There three UAV in the system. The initial positions and velocities of the agents “i” are \((x_i(t), v_i(t))\) and their satisfy the system (3). The influence function \(b_i(t)\) is given from the definition 1. The constant \(c = 10\) and the parameter \(\alpha = 0.5, \beta = \frac{1}{3}\). The simulation results are as following:
Figure 1. Three UAV from a flock

Figure 2. The velocity of the UNV flocking system

The figure 1 shows the positions of the three UAV. From the figure 1 and figure 2, it is easy to know the simulation results can check the theory 1. The system form a flock and they avoid to collide each other.

3.2 Example 2 (system with a leader)
This UAV system are in a two dimension-plane. There exists a leader(UAV) “0” who has a constant velocity. The leader’s position and velocity are \((x_0(t), v_0(t))\) which satisfy (10). Also there are three followers (UAV). The position and the velocity of the follower “\(i\)” are \((x_i(t), v_i(t))\) which satisfy the system (11). The followers will automatic adjust their positions and velocities, then the system will form a flock. The simulation results are as following:

Figure 3. UAV flocking system with a leader
Figure 4. UAV system with a leader

From the figure 3 and figure 4, we know this system form a flock. Also they agents can automatically avoid to collide each other. The green line in these two figure represents the leader. From the simulation results, it is easy to know that the followers’ velocities will match the leaders velocity, although the initial velocities of the followers are different from velocity of the leader.

4. Conclusion
This paper proposes two flocking models. From the simulation, the UAV group will form a flock. Next work is how realize this algorithm on the UAV group. If the UAV can keep continue communication, the UAV may complete the flock. While we should consider a practical problem during the flying, the communication maybe briefly stop or there may exist the signal interference.
of the these elements will break the flock. Next work we will add these factors into the flocking model so that the system would keep flocking in more complex environment.

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