Spin accumulation with spin-orbit interaction

Henri Saarikoski\textsuperscript{1,2} and Gerrit E. W. Bauer\textsuperscript{1}

\textsuperscript{1}Kavli Institute of Nanoscience, Delft University of Technology, 2628-CJ Delft, The Netherlands
\textsuperscript{2}Mathematical Physics, Lund Institute of Technology, SE-22100 Lund, Sweden

(Dated: February 11, 2009)

Spin accumulation is a crucial but imprecise concept in spintronics. In metal-based spintronics it is characterized in terms of semiclassical distribution functions. In semiconductors with a strong spin-orbit coupling the spin accumulation is interpreted as a superposition of coherent eigenstates. Both views can be reconciled by taking into account the electron-electron interaction: a sufficiently strong self-consistent exchange field reduces a spin accumulation to a chemical potential difference between the two spin bands even in the presence of spin-orbit coupling. We demonstrate the idea on a clean two-dimensional electron gas (2DEG) by showing how the exchange field protects a spin accumulation from dephasing and introduces an easy-plane anisotropy.

PACS numbers: 85.75.-d, 71.10.Ca, 71.15 Mb

Metal-based spintronics [1] has evolved into a mature field in which spin phenomena are routinely exploited in versatile applications [2]. However, integration of spin-based functionalities into semiconductor circuits is still a pressing challenge. Much of recent research in this area has been motivated by device concepts, such as the seminal Datta-Das transistor [3], which requires injection and detection of spins by ferromagnetic contacts to a narrow channel of a two-dimensional electron gas with gate-controlled spin-orbit interaction (SOI). In spite of progress to inject, modulate, transport, and detect spin polarization all-electrically [4, 5, 6, 7, 8, 9] as well as time-dependent Kerr and Faraday rotation spectroscopy [10], the route to a working spin transistor appears to be still full of obstacles. In the meantime, many insights have been obtained on the spin accumulation and its dynamics by optical methods, especially time-dependent Kerr and Faraday rotation spectroscopy [11, 12, 13, 14, 15, 16]. Large Rashba splitting have been observed at metal surfaces by angle-resolved photoemission [17], which attracts a lot of attention recently [18, 19].

We define a spin accumulation as a non-equilibrium spin-polarized state injected optically or electrically into a non-magnetic material. In metal-based spintronics a spin accumulation is synonymous to a chemical potential difference between spin up and down bands [20]. However, in semiconductors the SOI prominently affects the electronic structure and transport properties [21]. A spin accumulation is then interpreted as an intrinsically time-dependent quantum superposition of coherent eigenstates. This difference is not just a semantic question but essential for the functionality of spintronic devices. Here we offer a unified mean-field theory for the spin accumulation in both metals and semiconductors.

Spin can be injected either slowly, \textit{e.g.} by a ferromagnetic contact with small electric bias, or rapidly, \textit{e.g.} by pulsed optically induced excitation. We start below with a description of spin-accumulation eigenstates that are accessible by adiabatic excitation followed by a brief discussion of the spin accumulation dynamics of rapidly excited states. We illustrate the general ideas at the hand of a 2DEGs with Rashba SOI [22], in which the disorder-scattering lifetime broadening is much smaller than the spin-orbit splitting at the Fermi-level.

Let us consider an infinitely extended homogeneous 2DEG. To leading order in the electron wave vector \( \mathbf{k} = -i \nabla \) the Hamiltonian including the SOI is [21]

\[
H_0 = \hbar^2 k^2 / 2m^* + \alpha (\sigma_y k_x - \sigma_x k_y),
\]

where \( m^* = m_e m_h / (m_e + m_h) \) is the effective electron mass, \( m_e \) is the bare electron mass, \( \sigma_i \) are the Pauli matrices, and \( \alpha \) is the Rashba SOI strength parameter [22]. Electron-electron interactions are treated within the density-matrix functional theory (DMFT) [23] which is a generalization of the Hohenberg-Kohn-Sham density-functional theory [24] that can handle excited states. Compared to the Hartree-Fock (HF) method, exchange and correlation effects in the DMFT can be treated within local approximations. The reduced density matrix is \( \Gamma(z, z') = \sum_{n} n_i \chi_i(z) \chi_i(z') \), where \( n_i \) are eigenstates of the Kohn-Sham Hamiltonian (natural orbitals), \( 0 < n_i < 1 \) are the corresponding eigenvalues (natural occupation numbers), and \( z = (\mathbf{r}, \sigma) \) is space-spin coordinate. We define \( \Gamma_\sigma \) as the subset of all density matrices which correspond to a given electron density \( \rho(\mathbf{r}) = \sum_{\sigma} n_i \chi_i(z) \chi_i(z) \) and spin-polarization \( s(\mathbf{r}) = \sum_i n_i \chi_i(z) \sigma \chi_i(z) / N \), where \( N \) is the total number of electrons. The density-matrix functional is defined via minimization of the total energy \( E[\Gamma_\sigma] = \min_{\Gamma_\sigma} \langle \Psi[\Gamma_\sigma] | H | \Psi[\Gamma_\sigma] \rangle \) in the space of all many-body wave functions that correspond to a given \( \Gamma_\sigma \).

We now assume that the exact density matrix can be generated by a non-interacting system of pseudo particles

\[
[H_0 + V_{\text{ext}} + V_H + V_{\text{xc}}] \phi_i = \epsilon_i \phi_i,
\]

where \( V_{\text{ext}}, V_H, \) and \( V_{\text{xc}} \) are the external, the Hartree, and the exchange-correlation potential, respectively, such that \( \Gamma_\sigma(z, z') = \sum_i f_i \phi_i(z) \phi_i(z') \) with \( f_i = \{ 0, 1 \} \) and \( V_{\text{xc}}(z, z', [\Gamma_\sigma]) = \delta E_{\text{xc}}[\Gamma_\sigma] / \delta \Gamma_\sigma(z, z') \), where \( E_{\text{xc}} \) is
the exchange-correlation energy. HF-calcultions for the Rashba Hamiltonian, following Ref. [20], confirm that the effect of the SOI on the exchange potential is negligible for small spin polarizations. With the local approximation, we finally arrive at $V_{\text{xc}}(z,z',|\Gamma_s|) \approx \delta(r-r')(V_0(p,s) + J_{\text{xc}}(p,s)\hat{s} \cdot \sigma)$, where $s = s\hat{s}$, and $J_{\text{xc}}(p,s) (\approx J(p)s$ for small $s$) is the modulus of the exchange-correlation field vector. The scalar $V_0$ can be dropped in homogeneous systems. We may approximate $J$ by the HF exchange energy of the strictly 2DEG [25]: $V_{\text{xc}} \approx J_0(p,s)\hat{s} \cdot \sigma \approx J(p)\hat{s} \cdot \sigma = -\sqrt{\frac{2m_e}{e^2}}R_{\text{y}}s \cdot \sigma$, where $r_s = m_e/(a_B \sqrt{\sigma})$ is the dimensionless density parameter, $\epsilon$ is the relative dielectric constant of the medium, $R_{\text{y}} = 13.6$ eV, and $a_B = 0.53$ Å. The effective Hamiltonian is then

$$H(s) = H_0 + Js \cdot \sigma,$$

where $J < 0$ is the effective exchange potential, in which correlations can be included using published parameterizations of the correlation energy for a non-SO coupled 2DEG [27]. For typical electron densities $\rho$ the exchange energies (a few meV) are of the same order of magnitude as SOI energies at the Fermi-level in III–V [10] and II–VI [28] semiconductor-based 2DEG’s.

The eigenstates of the Hamiltonian in the non-polarized ground state are split into two bands with a chiral spin pattern (Fig. 1 (a-b)). The exchange field deforms the electronic bands and spinors as shown in Fig. 1 (c) for in-plane and in Fig. 2 (a-b) for perpendicular direction of an injected spin accumulation. SO split bands of the surface states of in-plane magnetized Gd films have been found to be deformed by the exchange potential [18] similar to Fig. 1 (c). Our task is to find the self-consistent single-determinant eigenfunction of $H(s)$, which according to the DMFT is unique. Introducing the occupation numbers $f_{k\lambda} = \{0, 1\}$ of the spin-split states $\phi_{k\lambda}$, the spin polarization reads

$$s = \sum_{k\lambda} f_{k\lambda} \sigma_{k\lambda} / N,$$

where $\lambda = \{+, -\}$ is the band index. The state we are looking for minimizes the energy under the constraint $\sum_{k\lambda} f_{k\lambda}$ as constrained variational variables. We solve the problem either analytically in limiting cases or numerically by a stochastic minimization method, which uses penalty functions to fix the spin polarization and the Metropolis sampling method to find the global energy minimum. An unequal occupation of spin bands, $N_- = \sum_{k-} f_{k-} > N_+$ can be parameterized by a chemical potential difference or a band polarization $p_b = (N_- - N_+)/N > 0$. Occupations can also shift in momentum space (Fig. 1 (d)).

A spin accumulation in the plane of the 2DEG can be generated at minimized energy by shifting the Fermi circles, which induces currents [29] via a “spin-galvanic Hall effect”: the minimum energy state at fixed $s_x$ is associated with a charge current in the perpendicular $y$-

**FIG. 1:** (Left panel) Spins (a) and energies (b) of the ground-state spin-split bands in a non-interacting Rashba 2DEG. The inner and outer circles correspond to the Fermi energies of the spin-split bands. (Right panel) Modulation of the electronic structure in the presence of an in-plane spin accumulation $s$ by its the exchange field. Circles in (c) are fixed-energy countours. The shifted occupation number distributions that minimize the energy are shown in (d).

**FIG. 2:** (a-b) Spin direction of the lower spin-split ($-$) band in the presence of a spin polarization normal to the 2DEG plane. The total spin polarization is determined by the shaded area between $k_{F-}$ and $k_{F+}$. c) The ground state (solid line) and excited eigenstates (dashed line) with non-zero, small spin polarization perpendicular to the 2DEG surface are separated with an energy gap. d) Band polarization of spin-accumulation states at fixed $J = -2$ meV. Material parameters are $\epsilon = 12.7$, $m^* = 0.067m_e$, $\rho = 2 \times 10^{13}/\text{m}^2$. 

The eigenvalues of the surface states of in-plane magnetized Gd films have been found to be deformed by the exchange potential [18] similar to Fig. 1 (c). Our task is to find the self-consistent single-determinant eigenfunction of $H(s)$, which according to the DMFT is unique. Introducing the occupation numbers $f_{k\lambda} = \{0, 1\}$ of the spin-split states $\phi_{k\lambda}$, the spin polarization reads

$$s = \sum_{k\lambda} f_{k\lambda} \sigma_{k\lambda} / N,$$

where $\lambda = \{+, -\}$ is the band index. The state we are looking for minimizes the energy under the constraint $\sum_{k\lambda} f_{k\lambda}$, with the $f_{k\lambda}$ as constrained variational variables. We solve the problem either analytically in limiting cases or numerically by a stochastic minimization method, which uses penalty functions to fix the spin polarization and the Metropolis sampling method to find the global energy minimum. An unequal occupation of spin bands, $N_- = \sum_{k-} f_{k-} > N_+$ can be parameterized by a chemical potential difference or a band polarization $p_b = (N_- - N_+)/N > 0$. Occupations can also shift in momentum space (Fig. 1 (d)).

A spin accumulation in the plane of the 2DEG can be generated at minimized energy by shifting the Fermi circles, which induces currents [29] via a “spin-galvanic Hall effect”: the minimum energy state at fixed $s_x$ is associated with a charge current in the perpendicular $y$-direction. 

$\delta(r-r')(V_0(p,s) + J_{\text{xc}}(p,s)\hat{s} \cdot \sigma)$, where $s = s\hat{s}$, and $J_{\text{xc}}(p,s) (\approx J(p)s$ for small $s$) is the modulus of the exchange-correlation field vector. The scalar $V_0$ can be dropped in homogeneous systems. We may approximate $J$ by the HF exchange energy of the strictly 2DEG [25]: $V_{\text{xc}} \approx J_0(p,s)\hat{s} \cdot \sigma \approx J(p)\hat{s} \cdot \sigma = -\sqrt{\frac{2m_e}{e^2}}R_{\text{y}}s \cdot \sigma$, where $r_s = m_e/(a_B \sqrt{\sigma})$ is the dimensionless density parameter, $\epsilon$ is the relative dielectric constant of the medium, $R_{\text{y}} = 13.6$ eV, and $a_B = 0.53$ Å. The effective Hamiltonian is then

$$H(s) = H_0 + Js \cdot \sigma,$$

where $J < 0$ is the effective exchange potential, in which correlations can be included using published parameterizations of the correlation energy for a non-SO coupled 2DEG [27]. For typical electron densities $\rho$ the exchange energies (a few meV) are of the same order of magnitude as SOI energies at the Fermi-level in III–V [10] and II–VI [28] semiconductor-based 2DEG’s.

The eigenstates of the Hamiltonian in the non-polarized ground state are split into two bands with a chiral spin pattern (Fig. 1 (a-b)). The exchange field deforms the electronic bands and spinors as shown in Fig. 1 (c) for in-plane and in Fig. 2 (a-b) for perpendicular direction of an injected spin accumulation. SO split bands of the surface states of in-plane magnetized Gd films have been found to be deformed by the exchange potential [18] similar to Fig. 1 (c). Our task is to find the self-consistent single-determinant eigenfunction of $H(s)$, which according to the DMFT is unique. Introducing the occupation numbers $f_{k\lambda} = \{0, 1\}$ of the spin-split states $\phi_{k\lambda}$, the spin polarization reads

$$s = \sum_{k\lambda} f_{k\lambda} \sigma_{k\lambda} / N,$$

where $\lambda = \{+, -\}$ is the band index. The state we are looking for minimizes the energy under the constraint $\sum_{k\lambda} f_{k\lambda}$, with the $f_{k\lambda}$ as constrained variational variables. We solve the problem either analytically in limiting cases or numerically by a stochastic minimization method, which uses penalty functions to fix the spin polarization and the Metropolis sampling method to find the global energy minimum. An unequal occupation of spin bands, $N_- = \sum_{k-} f_{k-} > N_+$ can be parameterized by a chemical potential difference or a band polarization $p_b = (N_- - N_+)/N > 0$. Occupations can also shift in momentum space (Fig. 1 (d)).

A spin accumulation in the plane of the 2DEG can be generated at minimized energy by shifting the Fermi circles, which induces currents [29] via a “spin-galvanic Hall effect”: the minimum energy state at fixed $s_x$ is associated with a charge current in the perpendicular $y$-direction.
direction
\[ j_{x,y} = -\frac{e}{\hbar} \left( 1 + \frac{Jm^*}{\pi \rho \hbar^2} \right) s_x. \] (4)

Since electrons move with constant drift velocity, there are no intrinsic spin-Hall currents \([30]\). Non-equilibrium spin currents are induced, but vanish to first order in \(s_x\).

A spin accumulation perpendicular to the 2DEG surface can be generated by the exchange field that pops the in-plane spin textures out of the plane (Fig. 2(b)). However, the SOI counteracts the spin-alignment and the band polarization \(p_0\) must be increased from that of the ground state to support a spin accumulation \([31]\). Consequently, excited eigenstates corresponding to a finite \(s_z\) are, in contrast to the in-plane case, separated from the non-polarized ground state by a finite energy gap
\[ E_g = \left( \frac{\alpha m^*}{\hbar} \right)^2 \left( J + \frac{\hbar^2 \pi \rho}{m^*} \right) + \mathcal{O}(\alpha^4). \] (5)

The divergence in \(E_g\) at \(J \to 0\) reflects the absence of an \(s_z\) component in the non-interacting Rashba model. The gap is shown in Fig. 2(c) in which Eq. (6) corresponds to the low \(\alpha\) limit of the energy difference. This gap energy must be overcome to achieve spin-polarized eigenstates at arbitrarily small \(s_z \neq 0\). Except for this singular behavior at \(s_z = 0\) we find that the energy of eigenstates is isotropic (to second order in \(s\)) to the numerical accuracy for \(s_z \neq 0\). We suspect that there may be a more general physical reason behind this out-of-plane isotropy.

The divergence of the spin accumulation to \(E(s)\) is not significant for material parameters shown in Fig. 2.

The maximum spin accumulation that can beaccommodated perpendicular to the 2DEG surface is determined by the total spin polarization of a single occupied band \((p_0 = 1)\). In the exchange-only approximation the self-consistency criterion \([33]\) can be fulfilled only when
\[ \alpha < 2|J|/k_F = 2|J|/\sqrt{4\pi \rho} = 0.32 \text{ eV nm}/\epsilon, \] (6)

Figure 2(d) shows the stability limit as a function of \(s_z\).

Per definition, eigenstates do not dephase. The dynamics of the semiclassical spin accumulations discussed above is therefore governed by the Bloch equation \(\dot{s} = -\gamma s \times \mathbf{B}_{\text{eff}} - s/T_1\), in which \(\mathbf{B}_{\text{eff}} = -\partial E(s)/\partial s\) and \(T_1\) is the spin relaxation time. Due to the singular anisotropy of \(E(s)\) the Bloch equation is mathematically not well defined in the mean-field theory employed here. Qualitatively, the absence of an angle dependence of \(E(s)\) (for \(s_z \neq 0\)) implies that exchange-stabilized spin accumulations do not feel an internal SOI field and precess only when an external magnetic field is applied. A spin accumulation exactly in the 2DEG plane is trapped and precesses around an in-plane external magnetic field that exceeds a threshold value governed by the energy gap \([35]\).

A large phase space available for scattering processes makes a large spin accumulation susceptible to fast decay. Therefore the stability limit of eigenstates \([33]\) is not a sharp phase boundary. For not too highly excited systems, the Dyakonov-Perel \([32]\) mechanism by random scattering at defects is believed to be the dominant source of finite \(T_1\) spin life times in clean systems.

Since in an exchange stabilized 2DEG the precession in the SO field is suppressed, the efficiency of the Dyakonov-Perel mechanism is strongly diminished for systems in the clean limit considered here. The opposite (dirty) regime can be handled by spin-coherent kinetic \([33, 34]\) and diffusion \([35, 36, 37]\) equations or numerical simulations \([14, 38]\).

The Datta-Das transistor is a spin valve consisting of a 2DEG spacer with transparent ferromagnetic contacts \([2]\). Even when the magnetizations of the two electrodes are parallel to each other, transport depends on the magnetization direction when exchange is taken into account. When magnetizations are oriented in the current direction, the spin accumulation can be injected into the 2DEG as an eigenstate with shifted distributions (Fig. 1(c)), which does not precess and, hence, does not react to a gate voltage that modulates the SOI. For magnetizations not in the 2DEG plane spin accumulation eigenstates are separated from the ground state by an energy gap and spin cannot be injected adiabatically at low energies. A non-adiabatic spin injection, on the other hand, could lead to a coherent superposition of eigenstates and spins would precess in the SOI field, as envisioned by Datta and Dns.

In optical pump and probe experiments, spin-polarized electrons and holes are generated by short resonant pulses of circularly polarized light, followed by fast thermalization and spin relaxation of the holes. A fast excitation creates a coherent superposition of individual spin eigenstates which dephases with time. We calculate the dynamics of the spin accumulation in the initial state \(\psi_0\) for times \(t > 0\) from \(\langle \psi(t) \rangle = \int dt e^{iH(t)/\hbar} |\psi_0\rangle\), where the Hamiltonian depends on \(t\) by the exchange field \(J_s(t)\) at \(t\). The state at \(t\) is solved iteratively
\[ \langle \psi(t) \rangle \approx e^{iH(-\Delta t)/\hbar} \ldots e^{iH(-\Delta t)/\hbar} e^{iH(0)/\hbar} |\psi_0\rangle \] (7)

for short time steps \(\Delta t \simeq 1\) fs. We assume that dephasing is fast compared to the spin relaxation processes so that the occupation numbers are unchanged. Figure 3 shows the time evolution of the spin accumulation, which has had been excited at time \(t = 0\) into a coherent superposition of eigenstates. A single spin oscillates in the SO-field by a frequency \(\omega_{SO} = 2ak/h\). The spin polarization excited over a finite band width is therefore expected to decay on the scale of the dephasing time \(T_2\), that decreases with increasing \(s\). However, a strong exchange field aligns spins along a common axis and synchronizes spin precession which protects the spin polarization from dephasing. The exchange-induced enhancement of \(T_2\) becomes significant when the exchange-splitting, which is proportional to \(s\), becomes of the same order of magni-
We acknowledge useful discussions with İnanç NWO. H.S. acknowledges support from the Academy of Finland. W. Wegscheider, and D. Weiss, arXiv:0809.1736.

J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).

J. M. Kikkawa, I. P. Smorchkova, N. Samarth, and D. D. Awschalom, Science 277, 1284 (1997).

S. A. Crooker, D. D. Awschalom, and N. Samarth, IEEE J. Sel. Top. Quantum Electron. 1, 1082 (1995).

A. Pugžlys, P. J. Rizo, K. Ivanin, A. Schlacher, D. Reuter, A. D. Wieck, C. H. van der Wal, and P. H. M. Loosdrecht, J. Phys.: Condens. Matter 19, 295206 (2007).

W. J. H. Leyland, R. T. Harley, M. Henini, A. J. Shields, I. Farrer, and D. A. Ritchie, Phys. Rev. B 77, 205321 (2008).

L. Meier, G. Salis, I. Shorubalko, E. Gini, S. Schönenberger, and K. Ensslin, Nature Phys. 3, 651 (2007).

Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science 306, 1910 (2004).

S. LaShell, B. McDougall, and E. Jensen, Phys. Rev. Lett. 77, 3419 (1996).

O. Krupin et al., Phys. Rev. B 71, 201403(R) (2005).

K. He, T. Hirahara, T. Okuda, S. Hasegawa, A. Kakizaki, and Iwao Matsuda Phys. Rev. Lett. 101, 107604 (2008); J. Hugo Dil, F. Meier, J. Lobo-Checa, L. Patthey, G. Bihlmayer, and J. Osterwalder, Phys. Rev. Lett. 101, 266802 (2008); E. Frantzeskakis, S. Pons, H. Mirhosseini, J. Henk, C. R. Ast, and M. Grioni, Phys. Rev. Lett. 101, 196805 (2008).

A. G. Aronov, Pis’ma Zh. Eksp. Teor. Fiz. 24, 37 (1976) [JETP Lett. 24, 32 (1976)]; M. Johnson and R. H. Silsbee, Phys. Rev. Lett. 55, 1790 (1985); F. J. Jedema, A. T. Filip, and B. J. van Wees, Nature 410, 345 (2001).

Roland Winkler, Spin-orbit coupling effects in two-dimensional electron and hole systems, Springer Tracts in Modern Physics 191, Springer-Verlag (2003).

Y. A. Bychkov and E. I. Rashba, J. Phys. C: Solid State Phys. 17, 6039 (1984) [Y. A. Bychkov, E. I. Rashba, JETP Lett. 39(2), 78 (1984)].

G. Zumbrach and K. Maschke, J. Chem. Phys. 82, 5604 (1985).

W. Kohn and L. J. Sham, Phys. Rev. 140, A1133 (1965).

U. von Barth and L. Hedin, Phys. C: Solid State Phys. 5, 1629 (1972).

S. Chesi, G. Simion, and G. F. Giuliani, cond-mat/0702060 (2007).

C. Attaccalite, S. Moroni, P. Gori-Giorgi, and G. B. Bachelet, Phys. Rev. Lett. 88, 256601 (2002).

Y. S. Gui, J. Liu, V. Daumer, C. R. Becker, H. Buhmann, and L. W. Molenkamp, Physica E 12, 416 (2002).

S. D. Ganichev et al., Nature 417, 153-156 (2002).

J. J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).

We find that at large SOI it becomes energetically favorable to instead depopulate a large fraction of states in the majority spin band around \( k = 0 \).

M. I. Dyakonov and V. I. Perel, Fiz. Tverd. Tela 13, 3581 (1971) [Sov. Phys. - Solid State 13, 3023 (1971)].

FIG. 3: Oscillation and dephasing of a spin ensemble in a SO-field. The state is a coherent superposition of spin eigenstates, \( s(t = 0) = 10\% \), and \( \alpha = 4 \cdot 10^{-11} \text{ eV m.} \)

We treat three-dimensional, inhomogeneous, and finite systems as well as the Dresselhaus SOI [40]. The electronic structures of other non-magnetic conductors with significant SOI, e.g. hole gases in doped semiconductors or non-magnetic transition metals, are more complicated, but still amenable to a computational implementation of our method.

This work has been supported by Stichting FOM and NWO. H.S. acknowledges support from the Academy of Finland. We acknowledge useful discussions with İnanç Adagideli and Klaus Capelle.

* Electronic address: h.m.saarikoski@tudelft.nl
† Electronic address: g.e.w.bauer@tudelft.nl

[1] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
[2] D. A. Awschalom and M. E. Flatté, Nature Phys. 3(3), 153 (2007).
[3] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[4] X. Lou, C. Adelmann, S. A. Crooker, E. S. Garlid, J. Zhang, K. S. Madhukar Reddy, S. D. Flexner, C. J. Palmstrom, and P. A. Crowell, Nature Phys. 3, 197 (2007).
[5] P. Chen, J. Moser, P. Kotissek, J. Sadowski, M. Zenger, D. Weiss, and W. Wegscheider, Phys. Rev. B 74, 241302(R) (2006).

[6] E. J. Koop, B. J. van Wees, D. Reuter, A. D. Wieck, and C. H. van der Wal, arXiv:0801.2609
[7] S. M. Frolov, A. Venkatesan, W. Yu, S. Luescher, W. Wegscheider, and J. A. Folk, arXiv:0801.4021
[8] S. A. Crooker, M. Furis, X. Lou, C. Adelmann, D. L. Smith, C. J. Palmstrom, and P. A. Crowell, Science 309, 2191 (2005).
[9] M. Ciorga, A. Einwanger, U. Wurstbauer, D. Schuh, W. Wegscheider, and D. Weiss, arXiv:0809.1736.
[33] M. M. Glazov and E. L. Ivchenko, JETP Lett. 75, 403 (2002).
[34] M. Q. Weng and M. W. Wu, Phys. Rev. B 68, 075312 (2003).
[35] A. A. Burkov, A. S. Núñez, and A. H. MacDonald, Phys. Rev. B 70, 155308 (2004).
[36] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. 93, 226602 (2004).
[37] ˙I. Adagideli and G. E. W. Bauer, Phys. Rev. Lett. 95, 256602 (2005).
[38] E. J. Koop, B. J. van Wees, and C. H. van der Wal, arXiv:0804.2968.
[39] D. Stich et al., Phys. Rev. Lett. 98, 176401 (2007).
[40] G. Dresselhaus, Phys. Rev. 100, 580 (1955).