Research Article

New Nonlinear Cumulative Fatigue Damage Model Based on Ecological Quality Dissipation of Materials

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The failure of many aircraft structures and materials is caused by the accumulation of fatigue damage under variable-amplitude cyclic loading wherein the damage evolution of materials is complicated. Therefore, to study the cumulative fatigue damage of materials under variable-amplitude cyclic loading, a new nonlinear fatigue damage accumulation model is proposed based on the ecological quality dissipation of materials by considering the effects of load interaction and sequence. The proposed new model is validated by the test data obtained for three kinds of material under multilevel fatigue loading. Compared with the Miner model and Kwofi model, the proposed model can more effectively analyse the accumulative damage and predict fatigue life of different materials under variable-amplitude cyclic loading than others. The study provides a basis for predicting fatigue life accurately and determining reasonable maintenance periods of aircraft structures.

1. Introduction

Fatigue damage is one of the major failure modes of engineering structures and materials [1]. A reliable accumulative damage model is very important for assessing the integrity and safety of aircraft structures and materials [2], many of which are actually subjected to variable-amplitude cyclic loading. The damage evolution behaviours of materials under variable-amplitude cyclic loading are complicated. Therefore, studying the cumulative fatigue damage of materials under variable-amplitude cyclic loading, a new nonlinear fatigue damage accumulation model is proposed based on the ecological quality dissipation of materials by considering the effects of load interaction and sequence. The proposed new model is validated by the test data obtained for three kinds of material under multilevel fatigue loading. Compared with the Miner model and Kwofi model, the proposed model can more effectively analyse the accumulative damage and predict fatigue life of different materials under variable-amplitude cyclic loading than others. The study provides a basis for predicting fatigue life accurately and determining reasonable maintenance periods of aircraft structures.
concept of fatigue driving stress. In damage accumulation, the physical properties of materials gradually degrade with the increasing number of load cycles. However, the above model cannot explain the mechanism by which material properties degrade with accumulating fatigue damage. Based on the toughness-exhaustion model, Peng et al. [14] introduced load interaction factors and independently proposed improved fatigue damage accumulation models. On the basis of conducting a comparative study on five nonlinear fatigue damage accumulation models, Zhu et al. [15] presented a new nonlinear fatigue damage accumulation model by considering the effects of load interaction and sequence. Xia et al. [16] proposed a fatigue damage accumulation model considering the exhaustion of static toughness based on iso-damage curves. Böhm et al. [17] proposed a material memory-based fatigue damage accumulation model analogy with the Ebbinghaus forgetting curve [18]. Peng et al. [19] proposed a phenomenological fatigue damage accumulation model by combining the material memory concept and the remaining S-N curve, but the model did not take into account the effects of the average stress and load interaction on materials. On the basis of the model proposed by Peng et al., Zhou et al. [20] proposed a new material memory degradation-based damage model by introducing a load interaction factor and using the Smith–Watson–Topper (SWT) average stress correction criterion [21]. The SWT criterion did not consider the influence of the material type on the average stress correction. Therefore, the corrections for some materials were nonspecific [22]. The Ebbinghaus forgetting curve indicates that the forgetting velocity is first fast and then slow, which is inconsistent with the observation that material physical properties degrade first slow and then fast [23, 24] during fatigue damage accumulation.

We drew an analogy with litter decomposition in an ecosystem during which some organic matter decomposes first slowly and then quickly and regarded the degradation of material performances as the ecological quality dissipation of the materials. On this basis, a nonlinear fatigue damage accumulation model was developed by considering the influences of material type, load interaction, and load sequence. Through the validation of test data, the proposed model can be used to study fatigue damage accumulation and predict the fatigue life effectively and accurately.

2. Formulation of the Nonlinear Fatigue Damage Accumulation Model

During fatigue damage accumulation, the physical properties of materials continue to degrade with the increasing number of load cycles. Just as litter in an ecosystem decomposes over time, material physical properties analogously degrade as the ecological quality dissipation of materials. On this basis, a nonlinear fatigue damage accumulation model was developed by considering the influences of material type, load interaction, and load sequence. Through the validation of test data, the proposed model can be used to study fatigue damage accumulation and predict the fatigue life effectively and accurately.

\[
\frac{dX}{X} = -k dt, \quad (1)
\]

where \(X\) is the amount of litter at a given moment, \(t\) is the time, and \(k\) is the instantaneous decomposition rate.

P. Rovira and R. Rovira [26] replaced \(k\) in Equation (1) with a function of time \((f(t))\) and then integrated the resulting equation to obtain a more general expression describing the decomposition of litter.

\[
X_t = X_0 e^{-\int_0^t f(t) dt}, \quad (2)
\]

where \(X_t\) is the amount of litter at time \(t\) and \(X_0\) is the initial amount of litter.

If the decomposition rate \((f(t))\) follows an exponential law and assuming \(f(t) = e^t\), we can obtain

\[
X_t = X_0 e^{(e^t - 1)}, \quad (3)
\]

Because the double-exponential function changes faster than the single-exponential one for the same base, another
expression can be obtained indicating that the organic matter in the litter decomposes first slow and then fast.

\[ X_t = A - B e^{\frac{t}{N_f}} \]  \hspace{1cm} (4)

where \( A \) and \( B \) are correlation parameters. The curve represented by Equation (4) is depicted in Figure 1.

According to the test data in Reference [23], the curve of degradation of material performance with the increase in the number of load cycles is depicted in Figure 2.

By comparing the curve represented by Equation (4) with the curve obtained based on test data in Figure 2, it is found that the degradation process of material performance has many similarities with the decomposition process of litters in the ecosystem. The variable \( t \) in Equation (4) is replaced by the normalized number of load cycles \( n/N_f \), and the ecological quality function of materials is obtained:

\[ X(n) = A - B e^{\left(\frac{n}{N_f}\right)^{1-\gamma}} \]  \hspace{1cm} (5)

where \( n \) is the number of load cycles and \( N_f \) is the fatigue life.

The ecological quality of materials dissipates gradually under the cyclic loading. The dissipation degree of ecological quality of materials is expressed as

\[ \Delta X(n) = \frac{X(0) - X(n)}{X(0) - X(N_f)} = 1 - e^{\left(\frac{n}{N_f}\right)^{1-\gamma}}. \]  \hspace{1cm} (6)

When \( n = 0 \) in the initial state, the dissipation degree of ecological quality of materials \( \Delta X(0) \) is 0. When \( n = N_f \) in the failure state, the dissipation degree of ecological quality of materials \( \Delta X(N_f) \) is 1.
The fatigue damage [27] considering load interaction and load sequence is defined as

\[ D = \left( \frac{n}{N_f} \right)^{g(\sigma)}, \]  

where \( g(\sigma) \) is a function of cyclic load and is specifically expressed as

\[ g(\sigma) = \zeta_0 (\sigma_{eq})^{\delta_0}, \]  

where \( \sigma_{eq} \) is the equivalent stress amplitude when the average stress is equal to 0 and \( \zeta_0 \) and \( \delta_0 \) are material parameters. \( \zeta_0 \) is cancelled out under multilevel cyclic loading. In Reference [27], according to test data of the 30NiCrMoV12 alloy steel under two-level cyclic loading, the value of \( \delta_0 \) is obtained as -5.78. Through the test data of other materials, it is verified that the value of \( \delta_0 \) as -5.78 is also applicable to the other materials.

Considering the influence of the material type on the average stress correction and the Walker average stress correction criterion [28, 29], Equation (8) changes as

\[ g(\sigma) = \zeta_0 (\sigma_{eq})^{\delta_0} = \zeta_0 (\sigma^2 - \sigma_{max})^{\delta_0}, \]  

where \( \sigma_{eq} \) and \( \sigma_{max} \) are the stress amplitude and maximum stress of the cyclic load, respectively, and \( \gamma \) is the material parameter. In Reference [30], according to the fatigue performance test data of different materials under different conditions, \( \gamma \) can be determined by the following equation:

\[ \gamma = 0.5 + \frac{\sigma_u - \sigma_y}{\sigma_u + \sigma_y}, \]  

where \( \sigma_u \) is the ultimate strength and \( \sigma_y \) is the yield strength.

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**Table 1:** Experimentally obtained and corresponding theoretical remaining life values predicted by the Miner, Kwofie, and proposed new model for the 30NiCrMoV12 alloy steel under two-level cyclic loading.

| Load sequence | Test data | Miner model | Kwofie model | New model |
|---------------|-----------|-------------|--------------|-----------|
| \( \sigma_{max} \) (MPa) | \( n_1 \) (cycle) | \( n_1/N_{f1} \) | \( n_2 \) (cycle) | \( n_2p \) (cycle) | \( n_2p \) (cycle) |
| 485–400 (H–L) | 13749 | 0.25 | 51304 | 109313 | 100352 | 39438 |
| | 27499 | 0.50 | 45765 | 72875 | 66901 | 23908 |
| | 41249 | 0.75 | 16032 | 36438 | 33451 | 11729 |
| | 17013 | 0.25 | 66845 | 85407 | 81630 | 51946 |
| 465–420 (H–L) | 34072 | 0.50 | 30405 | 56938 | 54427 | 31898 |
| | 51040 | 0.75 | 38262 | 28469 | 27210 | 15629 |
| | 20082 | 0.25 | 79372 | 85407 | 82847 | 61988 |
| 450–420 (H–L) | 40165 | 0.50 | 24711 | 56938 | 55231 | 38613 |
| | 60248 | 0.75 | 15943 | 28469 | 27616 | 18960 |
| | 36437 | 0.25 | 53348 | 41250 | 44932 | 54918 |
| 400–485 (L–H) | 72875 | 0.50 | 45373 | 27500 | 29954 | 53202 |
| | 109312 | 0.75 | 46693 | 13750 | 14978 | 39044 |
| | 36440 | 0.25 | 58594 | 51040 | 54533 | 65445 |
| 420–465 (L–H) | 72870 | 0.50 | 56416 | 34027 | 36356 | 54293 |
| | 109310 | 0.75 | 48998 | 17013 | 18177 | 30599 |
| | 28469 | 0.25 | 70530 | 60248 | 62109 | 73665 |
| 420–450 (L–H) | 56938 | 0.50 | 39362 | 40165 | 41406 | 56413 |
| | 85407 | 0.75 | 10523 | 20083 | 20703 | 30010 |
Then, considering the average stress and the load interaction, the load interaction factor is expressed as

$$\alpha_i = \frac{g(\sigma_i)}{g(\sigma_{i+1})} = \left(\frac{\sigma_{i+1}^{\gamma} \sigma_{\text{max},i}^{1-\gamma}}{\sigma_{i}^{\gamma} \sigma_{\text{max},i+1}^{1-\gamma}}\right)^{\delta_i}. \quad (11)$$

By substituting $\Delta X(n)$ for $n_i/N_i$ in Equation (7), the damage model based on the ecological quality dissipation of the materials is given by

$$D = \left(\frac{X(0) - X(n)}{X(0) - X(N_i)}\right)^{g(e)} = \left(\frac{1 - e^{(e^{n_i}-1)}}{1 - e^{(e-1)}}\right)^{g(e)}. \quad (12)$$

In two-level loading, according to the principle of equivalent damage, the damage caused by $n_1$ cycles under the first-stage loading ($\sigma_1$) is equal to that caused by $n_2'$ cycles under...
the second-stage loading ($\sigma_2$), i.e.:

$$\left(1 - \frac{e^{\alpha_1 N_1}}{1 - e^{\alpha_1 e^{-1}}}\right)^{g(\sigma_1)} = \left(1 - \frac{e^{\alpha_2 N_2}}{1 - e^{\alpha_2 e^{-1}}}\right)^{g(\sigma_2)},$$  \hspace{1cm} (13)

where $N_{f1}$ and $N_{f2}$ are the fatigue lives corresponding to $\sigma_1$ and $\sigma_2$, respectively.

According to Equation (13), we can obtain

$$\frac{n_{2'}}{N_{f2}} = \ln \left\{ 1 + \ln \left[ 1 - \left(\frac{1 - e^{\alpha_1 N_1}}{1 - e^{\alpha_1 e^{-1}}}\right)^{\alpha_1} \right] \right\}^{g(\sigma_2)},$$  \hspace{1cm} (14)

Then, the remaining life fraction obtained at the second-stage load is

$$\frac{n_{2,\text{res}}}{N_{f2}} = 1 - \frac{n_2'}{N_{f2}} = 1 - \ln \left\{ 1 + \ln \left[ 1 - \left(\frac{1 - e^{\alpha_1 N_1}}{1 - e^{\alpha_1 e^{-1}}}\right)^{\alpha_1} \right] \right\}^{g(\sigma_2)},$$  \hspace{1cm} (15)

where $n_{2,\text{res}}$ is the number of cycles up to fatigue failure under the second-stage load after $n_1$ cycles under the first-stage load.

Figure 6: Relationships between cycling ratios of the 30NiCrMoV12 alloy steel under two-level cyclic loading.
In three-level loading, according to the principle of equivalent damage, the remaining life fraction obtained at the third-stage load is

\[ n_{3\text{res}} = 1 - \ln \left\{ 1 + \ln \left\{ 1 - e^{\left( \frac{\epsilon^2}{\nu_f^2 + 1} \right)} \right\} \right\}. \tag{16} \]

By logical extension, the remaining life fraction at the \( i \text{th} \)-stage load \((i > 3)\) is

\[ n_{i\text{res}} = 1 - \ln \left\{ 1 + \ln \left\{ 1 - e^{\left( \frac{\epsilon^2}{\nu_f^2 + 1} \right)} \right\} \right\}. \tag{17} \]
The properties of the alloy steel are the ultimate strength $\sigma_f$, the yield strength $\sigma_y$, and the fatigue limit $\sigma_l$. The test data were obtained from References [11, 31–33] for the 30NiCrMoV12 alloy steel and Ti-6Al-4V titanium alloy under two-level cyclic loading and the 6082-T6 aluminium alloy under multilevel cyclic loading and were used to verify the applicability and prediction accuracy of the proposed new model. In addition, the predictions obtained using the proposed new model were compared with those obtained using the Miner model and Kwoe model.

### 3.1. Two-Level Cyclic Loading

Fatigue tests [11] were carried out on the hardened and tempered 30NiCrMoV12 alloy steel under high-low and low-high two-level loading. The geometry of specimens is shown in Figure 4. The mechanical properties of the alloy steel are the ultimate strength $\sigma_u = 1035$ MPa, the yield strength $\sigma_y = 755$ MPa, and the fatigue limit $\sigma_l = 391$ MPa. The stress ratio $R$ is −1. The test data [11] and corresponding theoretical remaining life values predicted by the Miner model, Kwoe model, and proposed new model are tabulated in Table 1.

Fatigue tests [31] were carried out on the Ti-6Al-4V titanium alloy under the two-level (high-low and low-high) loading by using an electrodynamic vibration system. The geometry of specimens is shown in Figure 5. The mechanical properties of the titanium alloy are the ultimate strength $\sigma_u = 1005$ MPa, the yield strength $\sigma_y = 975$ MPa, and the Young modulus $E = 107$ GPa. The stress ratio $R$ is −1. The experimentally obtained data [31] and corresponding theoretical remaining life values predicted by the Miner model, Kwoe model, and proposed new model are tabulated in Table 2.

The relations between the cycling ratios of the 30NiCrMoV12 alloy steel and Ti-6Al-4V titanium alloy under two-level cyclic loading are shown in Figures 6 and 7, respectively. According Reference [15], the load sequence effect is the change in fatigue life due to a reordering of the load spectrum. Figures 6 and 7 show the effect of load sequence on damage accumulation. Under low-high loading, low-amplitude cyclic loading strengthened the fatigue properties of the materials. Afterward, when high-amplitude cyclic loading was applied, the accumulated fatigue damage did not deteriorate significantly. The fatigue lives predicted under low-high loading were longer than those predicted under constant-amplitude loading, and the sum of the damage was greater than 1. Under high-low loading, the materials were damaged, and the damage began to accumulate under high-amplitude cyclic loading. Afterward, when the low-amplitude cyclic loading was applied, the accumulated fatigue damage deteriorated further. The fatigue lives predicted under high-low loading were shorter than those predicted under constant-amplitude loading, and the sum of the damage was less than 1. The proposed nonlinear fatigue damage accumulation model described more accurately the damage accumulated under variable-amplitude cyclic loading. The test data are in good agreement with the values predicted by the proposed new model. The greater the difference between the maximum values of the two loading stress levels, the more obvious the concavity and convexity of the curve corresponding to the proposed new model. This indicates that the greater the change in the rate of damage accumulation under variable-amplitude loading, the greater the impact of the load sequence on the fatigue life. Under high-low loading, when $n_1/N_{f1}$ is small, the greater the difference between the two loading levels, the greater the rate of change for $n_1/N_{f2}$. When $n_1/N_{f1}$ is large, the greater the difference between

### Table 3: Experimentally obtained and corresponding theoretical remaining life values predicted by the Miner, Kwoe, and proposed new model for the 6082-T6 aluminium alloy under multilevel cyclic loading.

| Load sequence $\sigma_{max}$ (MPa) | Test data $n_1$ (cycle) | $n_2$ (cycle) | $n_3$ (cycle) | $n_4$ (cycle) | $n_5$ (cycle) | Miner model $n_{last,p}$ (cycle) | Error (%) | Kwoe model $n_{last,p}$ (cycle) | Error (%) | New model $n_{last,p}$ (cycle) | Error (%) |
|--------------------------------------|--------------------------|---------------|---------------|---------------|---------------|---------------------------------|-----------|---------------------------------|-----------|-------------------------------|-----------|
| 260-275-290-305                      | 210000                   | 110500        | 60000         | 54000         | —             | 33750                           | −37.5     | 44387                           | −17.8     | 94537                         | 75.1      |
| 280-305-260-240                      | 19427                    | 10950         | 26258         | 43400         | —             | 136050                          | 213.5     | 124351                          | 186.5     | 32991                         | −24.0     |
| 260-275-290-305-260                  | 210000                   | 110500        | 60000         | 33750         | 116750        | 0                               | −100.0    | 57314                           | −50.9     | 149580                        | 28.1      |

![Figure 10: Geometry of 6082-T6 aluminium alloy specimens (dimensions in mm).](image-url)
the two loading levels, the smaller the rate of change for \( n_f / N_f \). Under low-high loading, when \( n_f / N_f \) is small, the greater the difference between the two loading levels, the smaller the rate of change for \( n_f / N_f \). When \( n_f / N_f \) is large, the greater the difference between the two loading levels, the greater the rate of change for \( n_f / N_f \).

To further analyse the prediction accuracy of the proposed new model, the experimentally obtained and theoretically predicted remaining life values of the 30NiCrMoV12 alloy steel and Ti-6Al-4V titanium alloy were compared, as shown in Figures 8 and 9.

Figure 8 clearly shows that for the 30NiCrMoV12 alloy steel, 72.2, 77.8, and 88.9\% of the remaining life values predicted by the Miner, Kwofie, and proposed new model fall within scatter bands of 2, respectively. Only two remaining life values predicted by the proposed new model are not within the scatter band of 2. Figure 9 clearly shows that for the Ti-6Al-4V titanium alloy, 55.0, 65.0, and 85.0\% of the remaining life values predicted by the Miner, Kwofie, and proposed new model fall within scatter bands of 2, respectively. The remaining life values predicted by the Miner and Kwofie model are more widely distributed than those predicted by the proposed new model. Therefore, the proposed new model gives more accurate predictions and shows wider applicability.

3.2. Multilevel Cyclic Loading. Fatigue tests [32, 33] were carried out on the 6082-T6 aluminium alloy under multilevel cyclic loading. The geometry of specimens is shown in Figure 10. The mechanical properties of the aluminium alloy are the ultimate strength \( \sigma_u = 370 \text{ MPa} \), the yield strength \( \sigma_y = 347 \text{ MPa} \), and the Young modulus \( E = 73 \text{ GPa} \). The stress ratio \( R \) is 0. The experimentally obtained data [32, 33] and corresponding theoretical remaining life values predicted by the Miner, Kwofie, and proposed new model are tabulated in Table 3, where \( n_{\text{last, p}} \) represents the remaining life values predicted by the theoretical models under the last stage of cyclic loading. All of the remaining life values predicted by the proposed new model fall within a scatter band of 2 and are more accurate than those predicted by the Miner.
model and Kwoifie model. Because the Miner model does not consider the load sequence, the remaining life values predicted by it show a high degree of error. Even in some multilevel loadings, it is impossible to accurately predict the remaining fatigue life under the last stage of cyclic loading.

The load histories of the 6082-T6 aluminium alloy under multilevel cyclic loading and the corresponding accumulated fatigue damage are depicted in Figures 11–13, which clearly show the obvious deviation between the accumulated fatigue damage experimentally obtained and those theoretically predicted by the Miner model and Kwoifie model. In multilevel cyclic loading, the changes in the damage accumulation rates predicted by the Miner model are discontinuous, especially under alternating high and low loadings. We believe that in the proposed new model, the damage accumulates very slowly in the initial stage of fatigue. At the end of fatigue, the damage accumulates fast, and the remaining life values predicted by the proposed new model are closer to the experimentally obtained ones. Therefore, the proposed new model is more suitable for studying the accumulative fatigue damage of engineering materials under multilevel cyclic loading, and the predictions obtained by the proposed new model are more accurate than those obtained by the Miner model and Kwoifie model.

4. Conclusions

In this paper, a material performance degradation-based nonlinear fatigue damage accumulation model is proposed by introducing the concept of the ecology quality dissipation of materials. Test data obtained for three materials under multilevel cyclic loading are used for model validation and comparison. The conclusions are drawn as follows:

1. The degradation of material performance with the increasing number of load cycles is analogous to the decomposition of litter over time. Through this analogy, a nonlinear fatigue damage accumulation model based on ecological quality dissipation of materials is proposed. The proposed model considers the influence of load interaction and sequence

2. By using test data for the 30NiCrMoV12 alloy steel and Ti-6Al-4V titanium alloy under two-level cyclic loading and the 6082-T6 aluminium alloy under multilevel cyclic loading, the model validation and comparison are conducted. The results show that the fatigue lives predicted by the proposed new model are highly accurate. The proposed model provides a theoretical basis for damage analysis and life prediction of aircraft structures

Data Availability

Test data were obtained from References [11, 31–33] for the 30NiCrMoV12 alloy steel and Ti-6Al-4V titanium alloy under two-level cyclic loading and the 6082-T6 aluminium alloy under multilevel cyclic loading and were used to verify the applicability and prediction accuracy of the proposed new model.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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