Instanton Contribution to Polarized and Unpolarized Gluon Distributions in Nucleon

N.I. Kochelev
DESY-Zeuthen Platanenallee 6, D-15735 Zeuthen, Germany, and
JINR, Dubna, Moscow region, 141980, Russia

Abstract

The contribution of the anomalous quark-gluon interaction induced by instantons to the polarized and unpolarized gluon distributions in nucleon is estimated. It is shown that this interaction leads to negative gluon polarization in nucleon.

1 Introduction

In recent years the interest in polarized hadron-hadron and lepton-hadron interactions at high energies has grown strongly. This interest stems from sensational result of measurement by the EMC(CERN) Collaboration [1] in polarized DIS of the part of the proton spin carried by quarks. It has been measured that this value is very small. As a result, a “spin crisis” of the naive parton picture of the spin-dependent structure functions (see review [2]) arose.

One of the way to resolve this problem is based on an assumption of the large gluon polarization in nucleon [3]. Recently, NLO analysis of the polarized DIS world data on $g_1(x, Q^2)$ was performed to extract the polarized parton densities in nucleon [4]. The result suggests a positive value for the gluon polarization. Still, this result is sensitive to the input shapes of the polarized parton densities as well as to the marginal statistical strength inherent in the $Q^2$ dependence of existing polarized DIS data.

A positive gluon polarization in the proton is actually expected in the framework of perturbative QCD due to conservation of helicity in perturbative quark-gluon vertex [5]. However, in QCD, the gluon distribution function is a nonperturbative object and therefore one should take into account the nonperturbative contribution to $\Delta G$. Up to now only one calculation of the nonperturbative gluon contribution to $\Delta G$ was presented in framework of MIT bag model and the nonrelativistic model [6]. It was shown that the sign of the gluon polarization is negative.

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which is in contradiction with the positive value that one would expect to explain the “spin crisis” by a gluon contribution only.

Here we estimate the gluon polarization in proton induced by nonperturbative vacuum fluctuations of the gluon fields, so-called instantons [7].

2 Anomalous Quark-Gluon Chromomagnetic Interaction Induced by Instantons

One of the models for the description of non-perturbative effects in QCD is the instanton liquid model (see reviews [8], [9]). In this model many properties of the hadrons as masses, decays widths etc., have been described rather well.

The existence of instantons leads to a specific spin-dependent t’Hooft’s quark-quark interaction through QCD vacuum [10], which determines the spin-spin mass splitting in hadron multiplets [11] and gives rise to a negative sea quark polarization and valence quark depolarization in nucleon [12] (see also [13]).

Recently, it was shown that instantons induce the anomalous chromomagnetic quark-gluon interaction [14]:

\[ \Delta \mathcal{L}_A = -i\mu_\alpha \sum_q \frac{g}{2m_q^*} \bar{q} \sigma_{\mu\nu} t^\alpha q G^a_{\mu\nu}. \]  

(1)

The value of the quark anomalous chromomagnetic moment can be estimated in the liquid instanton model for QCD vacuum [8] as

\[ \mu_\alpha = -\frac{f_\pi}{2\alpha_s}, \]  

(2)

where \( f = n_c \pi^2 \rho^4 \) is the so-called packing fraction of instantons in vacuum, \( m_q^* = m_q - 2\pi^2 \rho_c^2 \) is the effective quark mass, and \( n_c \) is the number of instantons in QCD vacuum. The value of \( n_c \) is connected with the value of the gluon condensate by the formula:

\[ n_c = <0 | \alpha_s G_{\mu\nu} G^a_{\mu\nu} | 0 > / 16\pi \approx 7.5 \times 10^{-4} \text{ GeV}^4. \]  

(3)

One can obtain (1) from the effective Lagrangian induced by instantons [15]

\[ \mathcal{L}_{\text{eff}} = \int \prod_q (m_q \rho - 2\pi^2 \rho^3 \bar{q} \rho_G (1 + \frac{i}{4} \tau^a U_{a'\alpha} \bar{q} \rho_G \sigma_{\mu\nu} q_L) + \epsilon^\alpha_\beta U_{\alpha'\beta' \alpha' \beta' \alpha' \beta'} \frac{d\rho}{\rho^4} d\beta_0(\rho) d\beta + R \leftrightarrow L, \]  

(4)
where $\rho$ is the instanton size, $\tau^a$ are the matrices of the $SU(2)_c$ subgroup of the $SU(3)_c$ colour group, $d_0(\rho)$ is the density of the instantons, $d\hat{\sigma}$ stands for integration over the instanton orientation in colour space, $\int d\hat{\sigma} = 1$, $U$ is the orientation matrix of the instanton, $\tilde{\eta}_{\mu\nu}$ is the numerical t’Hooft symbol and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$.

The quark–gluon vertex (1) follows from (4) by expanding it in the set of powers of the gluon strength and by integrating over $d\hat{\sigma}$.

For the numerical calculation, we use the NLO approximation for the strong coupling constant

$$\alpha_s(\rho) = -\frac{2\pi}{\beta_1 t} \left( 1 + \frac{2\beta_2 t}{\beta_1^2} \right),\quad \beta_1 = \frac{33}{6} - 2 N_f, \quad \beta_2 = \frac{153 - 19 N_f}{12},$$

and

$$t = \log\left( \frac{1}{\rho^2 \Lambda^2 + \delta} \right).$$

In Equation (7), the parameter $\delta \approx 1/\rho^2 \Lambda^2$ provides a smooth interpolation of the value of $\alpha_s(\rho)$ from the perturbative ($\rho \rightarrow 0$) to the nonperturbative region ($\rho \rightarrow \infty$) [17].

For $N_f = 3$, $\Lambda = 230\text{MeV}$, the following estimate for the value of the anomalous quark chromomagnetic moment has been obtained [14]

$$\mu_\mu = -0.2 \text{ for } \rho_c = 1.6\text{GeV}^{-1}.$$ (8)

### 3 Gluon Polarization Induced by Instantons

The contribution of the interaction (1) the gluon distribution functions can be estimated by using the Altarelli-Parisi method [18]. The matrix element for the transition of the initial positive polarized quark state to the final quark-gluon state has the following form (see Fig.1):

$$|V(q_+ \rightarrow G\pm q)|^2 = \frac{C_F \mu_a^2}{4 m^2 q} \text{Tr}(\hat{k}_c \sigma_{\mu\nu} \hat{k}_a \sigma_{\rho\tau}(1 - \gamma_5)) \cdot \left( q_\mu \epsilon_\nu^\pm - q_\nu \epsilon_\mu^\pm \right)^\dagger \left( q_\rho \epsilon_\tau^\pm - q_\tau \epsilon_\rho^\pm \right),$$

where $\epsilon_\mu^\pm$ is gluon polarization vector.

The calculation yields the result

$$|V_{q+ \rightarrow G\lambda q}|^2 = \frac{8 p_1^4 \mu_a^2}{3 m^2 q^2 (z^2(1 - z))}(1 - \lambda),$$

(10)
where \( z \) is the part of the initial quark momentum carried by the gluon, \( p_\perp \) is its transverse momentum, \( \lambda \) is the gluon polarization. To obtain (10), we have used the following quark and gluon momenta in the infinite momentum frame (see [18]):

\[
\begin{align*}
    k_A &= (P, P, 0) \\
    k_C &= ((1-z)P + \frac{p_\perp^2}{2(1-z)P}, (1-z)P, -p_\perp) \\
    q &= (zP + \frac{p_\perp^2}{2zP}, zP, p_\perp).
\end{align*}
\]

The result of the calculation of the quark splitting function averaged over the transverse momentum is

\[
P_{G,\lambda,q_+}(z, Q_0) = \frac{|\mu_a|(1 - \lambda)}{8z} \int_0^{Q_0^2\rho^2} d\beta, \tag{11}
\]

where \( \beta = p_\perp^2 \rho^2 \), and the relation \( 2n_c = <0\mid \bar{q}q \mid 0> \) \( m_q^* \) for the light quarks \( (m_q = 0) \) have been used.

It should be mentioned that due to the spin-flip of the quark at the instanton vertex the quark splitting function (11) has a very specific dependence on the gluon helicity. It is non-zero only if the emitted gluon has the opposite helicity compared to the initial quark helicity. This is in contrast to the case of the perturbative quark-gluon vertex [4], where due to helicity conservation the probability is larger to emit the gluon with the same helicity as the initial quark. As a result we anticipate a \textit{positive} gluon polarization induced by the perturbative quark-gluon vertex and \textit{negative} gluon polarization induced by the non-perturbative instanton-quark vertex.
To estimate the instanton contribution to the polarized and unpolarized gluon distribution, the convolution formula will be used

$$\Delta G(x, Q_0) = \int_x^1 \frac{dy}{y} \Delta P_{G,q}(y, Q_0) \Delta q_V(y),$$  \hspace{1cm} (12)$$

$$G(x, Q_0) = \int_x^1 \frac{dy}{y} P_{G,q}(y, Q_0) q_V(y),$$ \hspace{1cm} (13)

where

$$\Delta P_{G,q} = P_{G,+q} - P_{G,-q}, \hspace{1cm} P_{G,q} = P_{G,+q} + P_{G,-q}. \hspace{1cm} (14)$$

For the unpolarized and polarized valence quark distributions a simple shapes were utilized:

$$u_V(x) = 2.18x^{-0.5}(1-x)^3, \hspace{1cm} d_V(x) = 1.09x^{-0.5}(1-x)^3$$

$$\Delta u_V(x) = 3.7(1-x)^3, \hspace{1cm} \Delta d_V(x) = -1.3(1-x)^3. \hspace{1cm} (15)$$

The unpolarized distributions have been normalized to the number of $u$– and $d$– quarks in proton and the polarized ones have been normalized to the experimental data on the weak decay coupling constants of hyperons:

$$g_A^3 = \Delta u_V - \Delta d_V = 1.25; \hspace{1cm} g_A^8 = \Delta u_V + \Delta d_V = 0.6. \hspace{1cm} (16)$$

In the beginning we will estimate the instanton contribution at low value of $Q_0 \approx 1/\rho_c = 600 MeV$.

The results of the calculation of the polarized gluon distribution is presented in Fig. 2. The most remarkable feature is that the sign of the polarization is negative over all the kinematical range and its size is steadily increasing for decreasing $x$ values. The total instanton contribution to the gluon polarization in the nucleon is $\Delta G = -0.42$.

We can also estimate the instanton contribution to the unpolarized gluon distribution function in the same approach. In this case due to the vanishing of the splitting function for the same helicities of the initial quark and emitted gluon we have

$$P_I(x, Q_0) = -\Delta P_I(x, Q_0), \hspace{1cm} (17)$$

for the instanton contribution.

The result of the calculation is presented in Fig.3. The instanton contribution to the momentum of proton carried by gluons is

$$\int_0^1 dx x G_I(x, Q_0) = 0.017. \hspace{1cm} (18)$$
One can take into account the $Q^2$ dependence of the instanton contribution by using the simple formula (see Eq. (11) in which $\mu_a \propto 1/\alpha_s$):

$$
\Delta G(x, Q^2) \approx \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \Delta G(x, Q_0), \quad G(x, Q^2) \approx \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} G(x, Q_0), \quad (19)
$$

with result at $Q^2 = 4 GeV^2$

$$
\Delta G_I(4 GeV^2) = -0.67, \quad \int_0^1 dx xG_I(x, 4 GeV^2) = 0.027. \quad (20)
$$

Therefore, the instanton induced quark-gluon interaction leads to a rather large negative integral of the gluon polarization in nucleon. Furthermore, instanton induced glue gives about 5% to the value of the proton momentum carried by the gluons.

The negative gluon polarization induced by instantons should gives, through axial anomaly, the positive contribution to the spin-dependent structure functions $g_1(x)$. Therefore we expect

\[ \text{In Refs. [19] and [20] some contribution from instantons to the coefficient functions of unpolarized DIS has been taken into account in the dilute instanton gas approximation.} \]
positive values for neutron and proton structure functions $g_1^{p,n}(x)$ at low $x$ region where the instanton contribution dominates.

As it was mentioned, recent NLO fits [4] of the experimental data on $g_1^{p,n}(x,Q^2)$ show some indication on the positive value of $\Delta G$. To explain the same experimental data with the negative gluon polarization, one should introduce rather large negative sea quark polarization at $x \geq 0.01$. The fundamental mechanism for this polarization can be quark-quark t’Hooft interaction induced by instantons (see [12]).

4 Summary

In summary, we have shown that the instanton induced quark-gluon interaction leads to a large negative gluon polarization in nucleon. The instanton contribution to the unpolarized gluon distribution is approximately 5% at $Q^2 = 4GeV^2$. The sign of a gluon polarization can be checked in HERMES and COMPASS experiments by measurement of the double spin asymmetry in the charm quarks production [21].
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