Comparing blade-element theory and vortex computations intended for modelling of yaw aerodynamics of a tethered rotorcraft

Jean-Lou Pfister, Frédéric Blondel
IFP Energies nouvelles, 1 et 4 Avenue de Bois-Préau, 92852 Rueil-Malmaison, France
E-mail: jean-lou.pfister@ifpen.fr

Abstract. The development of airborne wind energy systems involves many design compromises, which encourages the use of systematic optimization approaches, that rely on simple engineering models for each component. In this paper, we present results concerning rotary-based airborne wind energy devices, that are operating like autogiros tethered to the ground. This configuration results in operating conditions at unusual rotor inclinations with respect to the incoming flow. We therefore focus on the description of the aerodynamics of the rotor, comparing blade-element momentum and free-vortex approaches, so as to determine if simple aerodynamic models are adapted for being used in initial design phases.

1. Introduction
In the ecosystem of airborne wind energy systems (AWES), a few concepts exploiting the phenomenon of autorotation have been proposed [1]. These prototypes are based on one or several rotors put in autorotation in the wind, and attached to the ground by a tether. Compared to autogyros [2, 3, 4] that use a propeller to generate the relative velocity required to sustain lift, autorotation-based devices are maintained quasi-steady thanks to the tether and can thus exploit the natural wind velocity for generating lift. It is then hoped that in good enough wind conditions and with the help of an appropriate control law, the device can generate energy either on the ground by pulling the tether or directly on board using generators (electric energy is then transmitted to the ground via a conductive tether) applying a resistive torque on the rotor shaft, or even using a combination of both strategies. Control may be achieved by dynamically varying the pitch of the blades, the span of the airfoils or using side (or top) propellers.

Among the precursors in the field is the Gyromill concept [5, 6]. This prototype has been tested experimentally; and it was claimed that on-board energy generation is possible for wind velocities as low as 10 m s\(^{-1}\) at an altitude of 4600 m [7, 8]. Variants with two or more rotors have been patented [9]. Another concept, similar in principle, is that of the RotoKite [10]. Recently, a prototype comprising at three supporting wings and a linking device was also proposed by the company BladeTips Energy [11]. However, the feasibility of such technology is perhaps even more debated than that of “conventional” airborne wind energy devices such as those based on crosswind kites or rigid gliders. While some authors recognize strong potentialities [12, 8], other authors are much more skeptical [13, 14].
1.1. Generic model for a rotary AWES
Like for other AWES, an evaluation of the potentialities of rotary-based technologies is difficult, because of the complex phenomena at play, the presence of many antagonist effects and strongly variable operational conditions. For instance, an increase of the size of the device enables in principle more energy to be harvested, but may also result in too heavy a device. Operating at higher altitude allows to benefit from a more concentrated wind resource, but also results in an increased tether mass and higher structural constraints. Determining the optimal size is thus not an easy task. Requirements for ground-based energy generation also differ substantially from those for on-board power generation. Thus, it is clear that an optimization-based design is required if one wants to assess the economic potentialities of this class of AWES [15]. In order to remain as generic as possible, we have chosen an approach based on interacting system blocks implemented within the Python MDO framework OpenMDAO [16]. Each block describes a specific sub-system coupled to other sub-systems, as reported in Fig. 1. Solving for the enclosed (by the dashed lines) aero-mechanical sub-system results in an equilibrium configuration, like for instance that depicted in Fig. 2. The rotary wing is sketched by the black, straight oblique line that makes an angle $\alpha$ with the horizontal. It is subject to the aerodynamic load (blue vector) that is almost perpendicular to the rotor plane. The weight (lumped for the craft and the tether) is materialized by the red, vertical arrow. Finally, the restraining force exerted by the tether is represented in green. A small sag is visible because of the weight of the tether. The ultimate outcome — the annual energy production — depends on the history of different equilibriums computed in the different operating conditions. This value thus strongly depends on how accurate the description of each individual equilibrium is.
1.2. Focus on the aerodynamic sub-model

Previous studies [24, 25, 12, 10, 26, 8] have shown that the operating configuration of the considered device is likely to lie somewhere between the wind-turbine case (rotor almost perpendicular to the wind) and the case of an helicopter in forward flight (rotor close to be parallel to the incoming wind). This specific configuration was less studied than the wind-turbine or helicopter cases, and it is therefore useful to know how models designed for the above mentioned “limit” cases behave in between. The objective of the present study is therefore to investigate more specifically the aerodynamic model. Hence, we do not consider the coupling with other sub-systems; neither we consider the aero-elastic coupling with the blade dynamics.

In order to assess the fidelity of the aerodynamic description, a hierarchy of approaches is compared in the present study. Specifically, we will compare the following approaches (ordered by increasing accuracy, complexity and hence computational cost):

(i) Closed-form, analytic Blade-Element Momentum (BEM) models with linear aerodynamics,
(ii) BEM model with numerical integration and non-linear aerodynamic data,
(iii) Free-vortex computations.

For each of these approaches, the resulting rotor forces are computed and commented, for various operating conditions.

2. Methods

2.1. Blade-element momentum model implemented in OpenMDAO

The lowest-fidelity rotor aerodynamic models are based on BEM, that couples a momentum balance across an actuator disk and the blade element theory.

The air passing through a rotor in autorotation or a wind turbine is significantly slowed down — compared to the relative freestream velocity — as a result of the partial transfer of energy from the air to the rotor. As a consequence, the effective velocity that should be taken into account in the computation of the aerodynamic forces within blade-element models is not the freestream relative velocity at the rotor but, rather, the velocity corrected by the slowdown induced by the presence of the rotor. This induced velocity is not known a priori, because it is
based on the knowledge of the rotor wake, which in turn depends on the rotor thrust and the distribution of air loads over the blades.

The total aerodynamic force acting on the entire rotor is computed as the average aerodynamic force during one revolution of the blades, assuming that each section of the blades is subject to an isolated 2d flow perpendicular to the leading edge, with an inflow velocity composed of the relative wind speed, the rotation speed of the rotor, and the induced velocity corrections. Loads are thus obtained by integrating over the rotor plane in the radial ($r$) and azimuthal ($\psi$) directions — we adopt here the same conventions as Bramwell [19, §3.7].

The computational power available today allows to integrate numerically the elementary loads along the chord of an airfoil and obtain the average loads developing over the entire rotor without any difficulty. Actually, numerical integration is needed as long as the chord or the twist of the blades varies in some non-specific way with the radial position $r$ along the blade, and when non-linear aerodynamic lift and drag data are considered for each section. When the blade deformations — including the lagging/flapping/feathering motions — are neglected, as is done here, the steady-state aerodynamic forces acting on the rotor may be computed by solving the following non-linear system of equations, giving the thrust $T_r$ (perpendicular to the rotor plane), the lateral side force $H_r$ (tangent to the rotor plane), the torque $Q_r$ and the mean induced velocity $\overline{v_i}$ (counted positively when it is directed downwards):

$$
T_r - B \int_0^{2\pi} \frac{1}{2\pi} \int_{hR}^R \left( \rho c(r) U(r, \psi)^2 \left( C_l(\theta + \phi) \cos(\phi) + C_d(\theta + \phi) \sin(\phi) \right) \right) r \, dr \, d\psi = 0, \tag{1}
$$

$$
H_r - B \int_0^{2\pi} \frac{1}{2\pi} \int_{hR}^R \left( \rho c(r) U(r, \psi)^2 \left( C_d(\theta + \phi) \cos(\phi) - C_l(\theta + \phi) \sin(\phi) \right) \right) \sin(\psi) r \, dr \, d\psi = 0, \tag{2}
$$

$$
Q_r - B \int_0^{2\pi} \frac{1}{2\pi} \int_{hR}^R \left( \rho c(r) U(r, \psi)^2 \left( C_d(\theta + \phi) \cos(\phi) - C_l(\theta + \phi) \sin(\phi) \right) \right) r \, dr \, d\psi = 0, \tag{3}
$$

$$
\overline{v_i} \sqrt{V^2 \cos^2 \alpha + (V \sin \alpha - \overline{v_i})^2} - \frac{T_r}{2\rho A} = 0. \tag{4}
$$

Here $U_i$ is the inflow velocity component perpendicular to each blade section, $U_l$ the tangential component, $U^2 = U_l^2 + U_p^2$ is the velocity magnitude (the radial flow component is neglected in the blade-element approach), $c(r)$ is the local chord, $\theta$ is the pitch angle (that may also depend on $r$ if the blade is twisted) $B$ denotes the number of blades, and $h$ the radius of a center void region. Finally, $C_l$ and $C_d$ denote the lift and drag coefficients, whose evolution with angle of attack can be obtained by interpolating from tables of airfoil data. The angle of attack is the pitch angle plus the inflow angle $\phi$ given by

$$
\tan \phi = \frac{U_p}{U_l} = \frac{V \sin(\alpha) - v_i(r, \psi)}{V \cos(\alpha) \sin(\psi) + \Omega r}, \tag{5}
$$

where $v_i(r, \psi)$ is the induced velocity distribution over the rotor disk. Equations (1), (2) and (3) correspond to the integration of elementary axial and side aerodynamic forces, and aerodynamic moment. Finally, equation (4) corresponds to Glauert’s induction formula [2] that gives the mean induced velocity $\overline{v_i}$. In reality, the induced velocity is far from uniform and $v_i(r, \psi) \neq \overline{v_i}$. One would in particular expect an upwash (decrease of induced velocity) at the leading edge and a downwash (increase of induced velocity) at the trailing edge. We have considered here the inflow model by Mangler & Squire [27] modified by Bramwell [19, p.82]. For the numerical integration, we consider 20 stations distributed homogeneously along the blades to integrate along $r$, and 90 stations along the $\psi$ direction. Note also that we did not consider any high-thrust correction in the induction equation (4), since the device is unlikely to operate at inclination angles where this feature is necessary. These models have been implemented in OpenMDAO as our general BEM.
Assuming in addition that angles of attack are small everywhere, it is possible under some assumptions on the geometry (typically, a constant chord and no twist) to obtain very simple closed-form formulas for the aerodynamic forces. These models were initially derived for the study of autogyros [2, 20, 21], then for helicopters [22, 23]. Since these models are analytic and simple to implement, they have therefore been used in many sizing studies of rotary-wing wind-energy systems [24, 25, 12, 10, 26, 8]. Within the small-angles, constant-chord and no twist approximation, the expression for the aerodynamic forces can be dramatically simplified. In the case where $h = 0$, we obtain the very simple expressions

\begin{align}
T_r - \sigma \rho A \Omega^2 R^2 \frac{C_0^l}{4} \left( \frac{2}{3} \theta \left( 1 + \frac{3}{2} \mu^2 \right) + \lambda \right) &= 0, \\
H_r - \sigma \rho A \Omega^2 R^2 \left( \frac{\mu C_0^l}{4} - \frac{C_0^d}{4} \mu \lambda \theta \right) &= 0, \\
Q_r - \sigma \rho A \Omega^2 R^3 \frac{C_0^d}{8} \left( 1 + 3 \mu^2 \right) + R \left( \lambda T_r + \mu H_r \right) &= 0,
\end{align}

where $\lambda = (V \sin(\alpha) - \bar{v}_i)/(\Omega R)$ and $\mu = (V \cos(\alpha))/(\Omega R)$ are the non-dimensional perpendicular and tangential (with respect to the rotor plane) inflow velocities, $C_0^l$ is the linear lift slope and $C_0^d$ a constant drag coefficient, $\rho = Bc/(\pi R)$ is the rotor solidity and $A = \pi R^2$ is the rotor disk area. These formulas are used in what is referred in the following to as the closed-form, linear-aerodynamics BEM model.

Whatever the assumptions considered, we end up with an implicit, nonlinear system of four equations of type

\[ f(T_r, Q_r, H_r, \bar{v}_i, \alpha, \Omega) = 0. \]

This system is underdetermined, because the aerodynamic sub-system is decoupled from the axial, radial and moment equilibrium equations of the whole system (which would for instance add a constraint on the angle of attack $\alpha$). We can then set values for $\alpha$ and the rotation speed $\Omega$, and then solve for $T_r, Q_r, H_r$, and $v_i$, that is:

\[ (T_r, H_r, Q_r, v_i) = g_1(\alpha, \Omega). \]

This problem is solved using a Newton-Raphson method — available as a black-box solver in OpenMDAO, with the Jacobians being calculated with finite-differences. Less than 5 iterations are required in general to make the residuals norm converge below $10^{-8}$. Another approach consists in setting a value for the torque $Q_r$ and solve for $T_r, H_r, \Omega$ and $\bar{v}_i$, i.e. compute

\[ (T_r, H_r, \Omega, \bar{v}_i) = g_2(\alpha, Q_r). \]

In this second case, equation (3) becomes implicit as well. The perfect autorotation case is achieved when $Q_r = 0$. This case can also be considered in OpenMDAO by adding an implicit component having the torque residual computed from (3) as an input, and the rotation speed $\Omega$ as an output, in addition to the explicit components calculating $Q_r$ and $T_r, H_r$ and the implicit component used to solve Glauert’s inflow equation. The great modularity offered by OpenMDAO allows to switch very easily between both cases.

2.2. Vortex computations with the solver CASTOR

A drawback of blade-element momentum approach is its inability to accurately account for the vortex wake developing downstream to the rotor. This is especially true when the rotor is not perpendicular to the incoming wind (yawed wind turbines, or the rotor in the present case). Free-vortex methods allow to resolve the complete vortex wake and thus to improve the
description of the yawed rotor [28]. Computations with IFPEN’s in-house code CASTOR (Code Aerodynamique pour la Simulation de Turbines OffshoRe) [29] have therefore been performed. CASTOR is a free-wake vortex filament lifting-line solver based on the generalized Prandtl lifting-line theory [30], based on airfoil polars — so that blade surfaces do not have to be modelled.

In the present case, we consider 60 blade elements along the span distributed with a cosine law (finer close to the blade tips), which helps for maintaining a relatively well resolved wake for the vortex computations. The rolling of vortices at the tip of the blades induces indeed strong distortions that are kept tracked all the better as there are many discretization points at this location of the blade. The eventually collapses into itself further downstream, but this was not found to create numerical stability problems.

The simulation time corresponds to 20 revolutions of the blades. This duration is determined by a convergence study: if the simulation is too short, there are not enough vortices in the wake to correctly reproduce the feedback of the wake structure onto the aerodynamics. On the other hand, the longer the simulation, the heavier the computational effort, since at each time-step a new set of vortices is shed, that interact with all other vortices. We observed that a variation of the thrust by less by 1% occurred when the number of revolutions was increased from 20 to 30.

Note that only a problem of type (9) can be handled in this case, so far. We thus control the rotation speed rather than the torque. Starting with a zero velocity, the speed of the rotor is increased linearly during two revolutions, then kept fixed to its nominal value. The time-step between the shedding of two successive vortices is set to 0.01T, where $T = \frac{2\pi}{\Omega}$ is the period of rotation of the rotor.

3. Results

3.1. Parameters & operating conditions
In all what follows, we consider a rotor made with rectangular blades of tip radius $R = 1$ m and a constant chord $0.075R$, without twist, having a central void of radius $0.2R$. This very non-optimal design allows to use the closed-form, analytic formulas, but is obviously limited to validation purposes. Future work could be devoted to the optimization of the blades within the OpenMDAO platform. With this geometry, and combined with linearised aerodynamics, closed-form, simple analytic formulas for the aerodynamic forces can be computed. A symmetric Göttingen 429 profile is taken for each airfoil section along the span. Data over all angles of attack are available in [31], from which the aerodynamic lift slope coefficient $C_{l0}$ and drag coefficient $C_{d0}$ are found to be 6.19 and 0.0102, respectively. The inflow velocity is set to the constant, uniform value $V = 10$ m s$^{-1}$ and directed along the horizontal axis, while air density is fixed to $\rho = 1.22$ kg m$^{-3}$. We also introduce the tip-speed ratio $\text{TSR} = \Omega R/V$.

We then solve for the steady-state aerodynamic response to these operating conditions. Earlier studies on rotary-based devices have shown that they may vary over a wide range, as reported in Tab. 1. The study is therefore conducted by varying the tip-speed ratio between 1 and 15, and for inclination angles $\alpha$ of 90$^\circ$ (rotor perpendicular to the incoming flow, i.e. like a wind turbine without yaw), 65$^\circ$, 40$^\circ$ and 15$^\circ$. We first conduct an analysis of type (9) that allows to compare with vortex computation, then present an analysis of type (6) that allows to study the autorotating state.

3.2. Thrust and power for varying rotor speed
In this section, we vary the rotor speed and compute the resultant aerodynamic forces. We focus on the thrust, which is largely the dominant contribution. Results for tip-speed ratios (TSR) varying between 0 and 12 are reported in Fig. 3, for four angles of inclination $\alpha$ of the rotor relative to the incident wind, and two pitch angles $\theta$ of the blades. For all cases, we report results from the vortex solver, closed-form BEM method with linear aerodynamics and BEM
Study | range for $\alpha$ [°] | range for TSR [-]
---|---|---
Mackertich [25] | 6 – 32 | 3.5 – 8.5
Rancourt et al. [15] | 10 – 80 | 4 – 11
Rimkus & Das [26] | 5 – 50 | 1.4 – 6.4
Roberts [8] | 20 – 70 | 5 – 7
Rong, Peters & Fei [12] | 20 – 50 | 2.5 – 15

Table 1. Operation range obtained in simulation analyses of rotary-based AWES. High TSR’s are obtained at high angles of attacks.

Figure 3. Comparison of the rotor thrust coefficient $C_t = T_r/(1/2 \rho A V^2)$ obtained from the closed-form, linear-aerodynamics BEM (dashed and dotted lines), numerical BEM with non-linear aerodynamics (+ and × symbols) and vortex (■ and ▼ symbols) computations for various angles of attack $\alpha$ (upper left: $\alpha = 90^\circ$, upper right: $\alpha = 65^\circ$, bottom left: $\alpha = 40^\circ$, bottom right: $\alpha = 15^\circ$), and pitch angles $\theta$ of 0 (red) and 5 degrees (blue).

with numerical integration and non-linear airfoil data. Let us focus first on the case $\alpha = 90^\circ$ — wind-turbine configuration without yaw. The curve obtained with the closed-form BEM (---) describes the well-known parabola predicted by momentum theory. When the pitch angle is increased up to 5 degrees (···), due to the lack of physical foundations of the momentum theory at high TSR and no yaw, the thrust suddenly decreases dramatically from TSR > 6. The numerical, non-linear BEM (+ and ×) follow closely the results obtained from the vortex solvers, except in the 90° case where it is also limited by the inherent limits of momentum-based approaches. In this regime, the alterations of the vortex structure are known to be better captured by vortex methods [28]. At low TSR, less thrust is generated than in the linear case. In this range, a large stalled region is indeed observed because the blades have no twist to counteract the smaller relative speed due to rotation than closer to the blade tips, and hence the non-linear aerodynamic data come with a much smaller lift as what is prescribed by the linear
Figure 4. Comparison of the rotor torque (power) coefficient $C_p = Q_r/(1/2 \rho AV^3)$ obtained from the closed-form, linear-aerodynamics BEM (dashed and dotted lines), numerical BEM (+ and $\times$ symbols) and vortex (■ and ▼ symbols) computations for various angles of attack $\alpha$ (upper left: $\alpha = 90^\circ$, upper right: $\alpha = 65^\circ$, bottom left: $\alpha = 40^\circ$, bottom right: $\alpha = 15^\circ$), and pitch angles $\theta$ of 0 (red) and 5 degrees (blue).

For the three other cases ($\alpha = 65^\circ$, $\alpha = 40^\circ$ and $\alpha = 15^\circ$), there is on average a good agreement between vortex computations and the numerical, non-linear BEM. The closed-form, linear-aerodynamics BEM is remarkably accurate for the case $\alpha = 15^\circ$, probably because in this case the stalled region in the center of the rotor does only a small contribution to the total thrust. For the intermediate cases $\alpha = 65^\circ$ and $40^\circ$ — which are likely to be that of the operating machine — the agreement is less good.

The power coefficient (which actually measures the resultant torque) is reported in Fig. 4. Here a much larger discrepancy is observed between the different computation approaches. The closed-form BEM largely overpredicts the resultant power. On the other hand, a reasonably good agreement is found when it comes to comparing numerical BEM with vortex calculations. Actually, although the thrust was only weakly dependent of the choice of model for the spatial distribution of induced velocity, the Mangler and Squire model appeared to be clearly superior (compared to constant induced velocity or a linear variation along the rotor disk) when it comes to the torque. A better agreement could perhaps be obtained by adding more harmonics (we only considered the leading-order terms) and/or using a combination of type-I and type-III components, as suggested by Leishman [4, §3.5.2].

3.3. Vortex structure in the wake
Let us describe the structure of the wake, as computed by the vortex solver. Four snapshots have been reproduced, for an angle of attack of $\alpha = 40^\circ$ (a,b) and a lower angle of attack of $15^\circ$ (c,d), for a TSR value of 5 in the left side and 10 in the right side. A slice passing through the
Figure 5. Side cuts along the vortex wake in the plane $z = 0$ (black lines) and wake topology (semi-transparent background colors), for (a) $\alpha = 40^\circ$ and TSR = 5, (b) $\alpha = 40^\circ$ and TSR = 10, (c) $\alpha = 15^\circ$ and TSR = 5 and (d) $\alpha = 15^\circ$ and TSR = 10.

Figure 6. Comparison of the rotor thrust obtained from the BEM approaches, as a function of the tip-speed ratio (left) and of the rotor’s inclination angle $\alpha$.

middle of the rotor and of the wake has been done, in order to visualize the vortex structure there. We see clearly the roll of vortices once they are shed by the passing of one blade. The rolling effect in the tip ends closed to the center of the rotor is less marked, because the velocity is slower there.

The vortices then grow downstream and form a complex topology. For the cases at high angles of attack (a,b), we clearly observe the overall downwards deflection of the wake, that ensures the downwards momentum transfer required to generate lift. At lower angles of attack, the vortices shed in the wake tend to merge, and become very intricate quite quickly.

3.4. Autorotative case
So far, we have considered the case where the rotor speed is imposed, which results in general in a non-zero torque, as clearly observable in Fig. 4. This case allowed to compare the BEM results to results obtained with the vortex solver, that can currently only be operated with rotation speed setpoints. On the other hand, the implementation of the BEM within OpenMDAO allows for determining at which condition a net zero torque is experienced by the rotor shaft — problem of type (10). In this case, we vary the angle $\alpha$, since the angular rotor speed is now an unknown of the problem, which in some way adapts itself so as to generate just as much forces to get a zero overall torque.
Like in the previous case, the thrust increases with the rotation speed of the rotor (i.e. the tip-speed ratio when wind velocity and rotor radius are kept fixed), but only up to a certain point, then slightly decreases again. This is barely visible on the left graph but can be observed clearly on the right side: an increasing thrust comes with an increasing rotor inclination angle, but then at some point the thrust will saturate as the configuration approaches the wind turbine configuration. Large differences are observed when the thrust is computed as a function of the tip-speed ratio. We have actually seen previously that the point of autorotation is pretty badly predicted with the linear-aerodynamics BEM (Fig. 4), which is reflected here: a given thrust in the autorotative state corresponds to very different rotor speeds whether we consider the closed-form or numerical-integration BEM. On the other hand, the rotor angle is less sensitive to this and shows similar evolutions for both approaches.

4. Conclusion
Preliminary aerodynamic computations of a rotor operating in a steady-state regime across a large range of angles of attacks and tip-speed ratios have been performed. We showed in this modest study that there is a reasonable agreement between vortex and blade-element momentum computations, as far as early design phases are concerned. A good compromise between speed and accuracy is probably to use a rotor-plane-averaged BEM approach with numerical integration of the non-linear aerodynamic data, and a Mangler & Squire inflow model.

Many further developments are to be undertaken. Aeroelastic computations including the blade deformations will be mandatory in more advanced design phases. Allowing the blades to flap is indeed practically needed so as to avoid structural vibrations [32], which in turn modifies somewhat the aerodynamic forces. This can be added very simply to the present BEM model by considering a harmonic balance approach, i.e. decomposing the flapping degree(s) of freedom into Fourier series of the azimuthal variable, then solving for each harmonic coefficient and couple this dynamics to the inflow and blade-element aerodynamical model.

Finally, the present study was restricted to steady operation conditions and without considering the tether dynamics. Earlier studies showed that the steady tether + craft equilibrium (that must be understood here as the craft autorotating at a constant rotor speed at a fixed position) is actually likely to be unstable to unsteady disturbances [33, 34]. Control strategies are thus likely to be needed. A recent study on a simplified rotary-wing device showed that pitch control seems to be sufficient to achieve stabilization [35]. However, this has still to be confirmed with more realistic models. For instance, it would be worth studying the transient response to attitude modifications of the craft with vortex computations, modified so as to handle the autorotative case. This would for instance allow to fine-tune dynamic inflow models [36, 37] commonly used in conjunction with momentum-based models to capture transient dynamics of helicopter, autogyro or wind-turbine rotors.

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