Nonlinear force-free field extrapolation in spherical geometry: improved boundary data treatment applied to a SOLIS/VSM vector magnetogram

T. Tadesse1,2, T. Wiegelmann1, B. Inhester1, and A. Pevtsov3

1 Max-Planck-Institut für Sonnensystemforschung, Max-Planck-Strasse 2, 37191 Katlenburg-Lindau, Germany
e-mail: [tadesse;wiegelmann;inhester]@mps.mpg.de
2 Addis Ababa University, College of Education, Department of Physics Education, PO Box 1176, Addis Ababa, Ethiopia
e-mail: tilaye.tadesse@gmail.com
3 National Solar Observatory, Sunspot, NM 88349, USA
e-mail: apevtsov@nso.edu

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ABSTRACT

Context. Understanding the 3D structure of coronal magnetic field is important to understanding: the onset of flares and coronal mass ejections, and the stability of active regions, and to monitoring the magnetic helicity and free magnetic energy and other phenomena in the solar atmosphere. Routine measurements of the solar magnetic field are mainly carried out in the photosphere. Therefore, one has to infer the field strength in the upper layers of the solar atmosphere from the measured photospheric field based on the assumption that the corona is force-free. Meanwhile, those measured data are inconsistent with the above force-free assumption. Therefore, one has to apply some transformations to these data before nonlinear force-free extrapolation codes can be applied.

Aims. Extrapolation codes in Cartesian geometry for modelling the magnetic field in the corona do not take the curvature of the Sun’s surface into account and can only be applied to relatively small areas, e.g., a single active region. Here we apply a method for nonlinear force-free coronal magnetic field modelling and preprocessing of photospheric vector magnetograms in spherical geometry using the optimization procedure.

Methods. We solve the nonlinear force-free field equations by minimizing a functional in spherical coordinates over a restricted area of the Sun. We extend the functional by an additional term, which allows us to incorporate measurement errors and treat regions lacking observational data. We use vector magnetograph data from the Synoptic Optical Long-term Investigations of the Sun survey (SOLIS) to model the coronal magnetic field. We study two neighbouring magnetically connected active regions observed on May 15 2009.

Results. For vector magnetograms with variable measurement precision and randomly scattered data gaps (e.g., SOLIS/VSM), the new code yields field models that satisfy the solenoidal and force-free condition significantly better as it allows deviations between the extrapolated boundary field and observed boundary data within the measurement errors. Data gaps are assigned an infinite error. We extend this new scheme to spherical geometry and apply it for the first time to real data.

Key words. magnetic fields – Sun: corona – Sun: photosphere – methods: numerical

1. Introduction

Observations have shown that physical conditions in the solar atmosphere are strongly controlled by the solar magnetic field. The magnetic field also provides the link between different manifestations of solar activity such as, for instance, sunspots, filaments, flares, or coronal mass ejections. Therefore, the information about the 3D structure of magnetic field vector throughout the solar atmosphere is crucially important. Routine measurements of the solar vector magnetic field are mainly carried out in the photosphere. Therefore, one has to use numerical modelling to infer the field strength into the upper layers of the solar atmosphere from the measured photospheric field based on the assumption that the corona is force-free. Owing to the low value of the plasma \( \beta \) (the ratio of gas pressure to magnetic pressure) (Gary 2001), the solar corona is magnetically dominated. To describe the equilibrium structure of the coronal magnetic field when non-magnetic forces are negligible, the force-free assumption is then appropriate:

\[
( \nabla \times \mathbf{B} ) \times \mathbf{B} = 0, \tag{1}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{2}
\]

\[
\mathbf{B} = \mathbf{H}_{\text{obs}} \quad \text{on photosphere}, \tag{3}
\]

where \( \mathbf{B} \) is the magnetic field and \( \mathbf{H}_{\text{obs}} \) is the 2D observed surface magnetic field in the photosphere. Extrapolation methods have been developed for different types of force-free fields: potential field extrapolation (Schmidt 1964; Semel 1967), linear force-free field extrapolation (Chiu & Hilton 1977; Seehafer 1978, 1982; Semel 1988; Clegg et al. 2000), and nonlinear force-free field extrapolation (Sakurai 1981; Wu et al. 1990; Cuperman et al. 1991; Demoulin et al. 1992; Mikic & McClymont 1994; Roumeliotis 1996; Amari et al. 1997, 1999; Yan & Sakurai 2000; Valori et al. 2005; Wheatland 2004; Wiegelmann 2004; Amari et al. 2006; Inhester & Wiegelmann 2006). Among these, the nonlinear force-free field has the most realistic description of...
the coronal magnetic field. For a more complete review of ex-
isting methods for computing nonlinear force-free coronal mag-
netic fields, we refer to the review papers by Amari et al. (1997),
Schrijver et al. (2006), Metcalf et al. (2008), and Wiegelmann
(2008).

The magnetic field is not force-free in the photosphere,
but becomes force-free roughly 400 km above the photosphere
(Metcalf et al. 1995). Nonlinear force-free extrapolation codes
that can be applied only to low plasma-β regions, where the force-
free assumption is justified. The preprocessing scheme used until
now modifies observed photospheric vector magnetograms with
the aim of approximating the magnetic field vector at the bottom
of the force-free domain (Wiegelmann et al. 2006; Fuhrmann
et al. 2007; Tadesse et al. 2009). The resulting boundary values
are expected to be more suitable for an extrapolation into a force-
free field than the original values. Preprocessing is important to
NLFF-codes that use the magnetic field vector on the boundary
directly. Consistent computations for the Grad-Rubin method,
which use $B_0$ and $J_0$ (or $J$) as boundary conditions were carried
out by Wheatland & Régnier (2009).

In this paper, we use a larger computational domain that ac-
commodates most of the connectivity within the coronal region.
We also take the uncertainties of measurements in vector mag-
etograms into account as suggested in De Rosa et al. (2009).
We implement the preprocessing procedure of Tadesse et al. (2009)
to SOLIS data in spherical geometry by considering the cur-
vature of the Sun’s surface within the large field of view con-
taining two active regions. We use a spherical version of the
optimization procedure implemented in Cartesian geometry in
Wiegelmann & Inhester (2010) for synthetic boundary data.

2. Method

2.1. The SOLIS/VSM instrument

In this study, we use vector magnetogram observations from
the Vector Spectromagnetograph (VSM, see Jones et al. 2002),
which is part of the Synoptic Optical Long-term Investigations
of the Sun (SOLIS) synoptic facility (SOLIS, see Keller et al.
2003). VSM/SOLIS currently operates at the Kitt Peak National
Observatory, Arizona, and has provided magnetic field observa-
tions of the Sun almost continuously since August 2003.

VSM is a full disk Stokes polarimeter. As part of daily
synoptic observations, it takes four different observations in
three spectral lines: Stokes $I$ (intensity), $V$ (circular polar-
ization), $Q$, and $U$ (linear polarization) in photospheric spec-
tral lines Fe I 1630.15 nm and Fe I 1630.25 nm, Stokes $I$ and
$V$ in Fe I 1630.15 nm and Fe I 1630.25 nm, similar observations
in chromospheric spectral line Ca II 854.2 nm, and Stokes $I$ in
the He I 1083.0 nm line and the near by Si I spectral line.
Observations of $I$, $Q$, $U$, and $V$ are used to construct full disk
vector magnetograms, while $I - V$ observations are employed to
create separate full disk longitudinal magnetograms in the pho-
tosphere and the chromosphere.

In this study, we use a vector magnetogram observed on
15 May 2009. The data were taken with 1.25 arcsec pixel size
and 2.71 pm spectral sampling. In December 2009, SOLIS/VSM
upgraded its cameras from Rockwell (90 Hz, 18 micron pixels)
to Sarnoff (300 Hz, 16 micron pixels). This camera upgrade has
resulted in improved spatial and spectral sampling. The noise
level for a line-of-sight component is about 1 Gauss. However,
noise due to atmospheric seeing may be much higher, and the fi-
nal measurement error depends on the measured flux, its spatial
distribution, as well as the seeing conditions. A rough estimate
suggests a noise level of a few tens of Gauss for areas with a
strong horizontal gradient of magnetic field and about 1 arcsec
atmospheric seeing.

To create a single magnetogram, the solar disk image is
scanned from terrestrial south to north; it takes about 20 min
to complete one vector magnetogram. After the scan is done,
the data are sent to an automatic data reduction pipeline that
includes dark and flat field correction. Once the spectra are
properly calibrated, full disk vector (magnetic field strength, in-
clination, and azimuth) magnetograms are created using two dif-
ferent approaches. Quick-look (QL) vector magnetograms are
created based on an algorithm by Auer et al. (1977). The al-
gorithm uses the Milne-Eddington model of solar atmosphere,
which assumes that the magnetic field is uniform (no gradients)
through the layer of spectral line formation (Unno 1956). It also
assumes symmetric line profiles, disregards magneto-optical ef-
fects (e.g., Faraday rotation), and does not distinguish the con-
tributions of magnetic and non-magnetic components in spec-
tral line profiles (i.e., magnetic filling factor is set to unity).
A complete inversion of the spectral data is performed later us-
ing a technique developed by Skumanich & Lites (1987).
This latter inversion (called ME magnetogram) also employs Milne-
Eddington model of atmosphere, but solves for magneto-optical
effects and determines the magnetic filling factor i.e., (the frac-
tional contribution of magnetic and non-magnetic components
to each pixel). The ME inversion is only performed for pixels with
spectral line profiles above the noise level. For pixels below the
polarimetric noise threshold, the magnetic field parameters are
set to zero.

From the measurements, the azimuths of transverse mag-
netic field can be determined with 180-degree ambiguity. This
ambiguity is resolved using the non-potential field calculation
(NPFC, see Georgoulis 2005). The NPFC method was selected
on the basis of a comparative investigation of several methods
for 180-degree ambiguity resolution (Metcalf et al. 2006). Both
QL and ME magnetograms can be used for potential and/or
force-free field extrapolation. However, in strong fields inside
sunspots, the QL field strengths may exhibit an erroneous de-
crease inside the sunspot umbra due to so-called magnetic satu-
ration. For this study, we choose to use fully inverted ME mag-
etograms. Figure 1 shows a map of the radial component of
the field as a contour plot with the transverse magnetic field
depicted as black arrows. For this particular dataset, about 80% of
data pixels are undetermined and as a result the ratio of data
gaps to total number of pixels is large.

2.2. Preprocessing of SOLIS data

The preprocessing scheme of Tadesse et al. (2009) involves
minimizing a two-dimensional functional of quadratic form in
spherical geometry similar to

$$H = \text{argmin}(L_p),$$

$$L_p = \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4,$$  \hspace{1cm} (4)

where $H$ is the preprocessed surface magnetic field from the in-
put observed field $H_{obs}$. Each of the constraints $L_p$ is weighted
by an as yet undetermined factor $\mu$. The first term ($n = 1$)
corresponds to the force-balance condition, the next ($n = 2$) to
the torque-free condition, and the last term ($n = 4$) controls
the smoothing. The explicit form of $L_1$, $L_2$, and $L_4$ can be found
in Tadesse et al. (2009). The term ($n = 3$) ensures that the
optimized boundary condition agrees with the measured pho-
tospheric data. In the case of SOLIS/VSM data, we modified $L_3$
Fig. 1. Surface contour plot of radial magnetic field vector, and vector field plot of transverse field indicated by black arrows.

with respect to the one in Tadesse et al. (2009), to treat those data gaps, to become

\[
L_3 = \sum_p (H - H_{\text{obs}}) \cdot W(\theta, \phi) \cdot (H - H_{\text{obs}}).
\]  

In this integral, \( W(\theta, \phi) = \text{diag}(w_{\text{radial}}, w_{\text{trans}}, w_{\text{trans}}) \) is a diagonal matrix which gives different weights to the different observed surface field components depending on their relative measurement accuracy. A careful choice of the preprocessing parameters \( \mu_n \) ensures that the preprocessed magnetic field \( H \) does not deviate from the original observed field \( H_{\text{obs}} \) by more than the measurement errors. As a result of the parameter study in this work, we found that \( \mu_1 = \mu_2 = 1.0, \mu_3 = 0.03, \) and \( \mu_4 = 0.45 \) as optimal values.

2.3. Optimization principle

Equations (1) and (2) can be solved with the help of an optimization principle, as proposed by Wheatland et al. (2000) and generalized by Wiegelmann (2004) for Cartesian geometry. The method minimizes a joint measure of the normalized Lorentz forces and the divergence of the field throughout the volume of interest, \( V \). Throughout this minimization, the photospheric boundary of the model field \( B \) is matched exactly to the observed \( H_{\text{obs}} \) and possibly preprocessed magnetogram values \( H \). Here, we use the optimization approach for functional \( L_{\omega} \) in spherical geometry (Wiegelmann 2007; Tadesse et al. 2009) along with the new method, which instead of an exact match enforces a minimal deviation between the photospheric boundary of the model field \( B \) and the magnetogram field \( H_{\text{obs}} \) by adding an appropriate surface integral term \( L_{\text{photo}} \) (Wiegelmann & Inhester 2010). These terms are given by

\[
B = \text{argmin}(L_{\omega})
\]

\[
L_{\omega} = L_f + L_d + vL_{\text{photo}}
\]

\[
L_f = \int_V \omega_f(r, \theta, \phi) B^{-2} |(\nabla \times B) \times B|^2 r^2 \sin \theta dr d\theta d\phi
\]

\[
L_d = \int_V \omega_d(r, \theta, \phi) |\nabla \cdot B|^2 r^2 \sin \theta dr d\theta d\phi
\]

\[
L_{\text{photo}} = \int_S (B - H_{\text{obs}}) \cdot W(\theta, \phi) \cdot (B - H_{\text{obs}}) r^2 \sin \theta d\theta d\phi
\]

where \( L_f \) and \( L_d \) measure how well the force-free Eqs. (1) and divergence-free (2) conditions are fulfilled, respectively, and both \( \omega_f(r, \theta, \phi) \) and \( \omega_d(r, \theta, \phi) \) are weighting functions. The third integral, \( L_{\text{photo}} \), is the surface integral over the photosphere which allows us to relax the field on the photosphere towards force-free solution without too much deviation from the original surface field data, and the term \( W(\theta, \phi) \) is the diagonal matrix in Eq. (5).

Numerical tests of the effect of the new term \( L_{\text{photo}} \) were performed by Wiegelmann & Inhester (2010) in Cartesian geometry for a synthetic magnetic field vector generated from Low & Lou model (Low & Lou 1990). They showed that this new means of incorporating the observed boundary field allows us to cope with data gaps as they appear in SOLIS and other vector magnetogram data. Within this work, we use a spherical geometry for the full disk data from SOLIS. We adopt a spherical grid \( r, \theta, \phi \) with \( n_r, n_\theta, n_\phi \) grid points in the direction of radius, latitude, and longitude, respectively. The method works as follows:
We compute an initial source surface potential field in the computational domain from \( H_{\text{obs}} \), the normal component of the surface field at the photosphere at \( r = 1 R_\odot \). The computation is performed by assuming that a currentless (\( J = 0 \) or \( \nabla \times B = 0 \)) approximation holds between the photosphere and some spherical surface \( S_n \) (source surface where the magnetic field vector is assumed radial). We computed the solution of this boundary-value problem in a standard form of harmonic expansion in terms of eigen-solutions of the Laplace equation written in a spherical coordinate system, \((r,\theta,\phi)\).

We minimize \( L_w \) (Eqs. (6)) iteratively without constraining \( H_{\text{obs}} \) at the photosphere boundary as in a previous version of Wheatland algorithm (Wheatland et al. 2000). The model magnetic field \( B \) at the surface is gradually driven towards the observations, while in the field in the volume \( V \) relaxes to be charge-free. If the observed field is inconsistent, the difference \( B - H_{\text{obs}} \) or \( B - H \) (for preprocessed data) remains finite depending on the control parameter \( v \). At data gaps in \( H_{\text{obs}} \), we set \( w_{\text{radial}} = 0 \) and \( w_{\text{trans}} = 0 \), and the respective field value is automatically ignored.

The state \( L_w = 0 \) corresponds to a perfect charge-free and divergence-free state and exact agreement of the boundary values \( B \) with observations \( H_{\text{obs}} \) in regions where \( w_{\text{radial}} \) and \( w_{\text{trans}} \) are greater than zero. For inconsistent boundary data, the force-free and solenoidal conditions can still be fulfilled, but the surface term \( L_{\text{photo}} \) will remain finite. This results in some deviation of the bottom boundary data from the observations, especially in regions where \( w_{\text{radial}} \) and \( w_{\text{trans}} \) are small. The parameter \( v \) is tuned so that these deviations do not exceed the local estimated measurement error.

The iteration stops when \( L_w \) becomes stationary as \( \Delta L_w/L_w < 10^{-4} \).

### 3. Results

We use the vector magnetograph data from the Synoptic Optical Long-term Investigations of the Sun survey (SOLIS) to model the coronal magnetic field. We extrapolate by means of Eq. (6) both the observed field \( H_{\text{obs}} \) measured above two active regions observed on May 15 2009 and preprocessed surface field \( (H \) obtained from \( H_{\text{obs}} \), applying our preprocessing procedure). We compute 3D magnetic field in a wedge-shaped computational box \( V \), which includes an inner physical domain \( V' \) and the buffer zone (the region outside the physical domain), as shown in Fig. 3 of the bottom boundary on the photosphere. The wedge-shaped physical domain \( V' \) has its latitudinal boundaries at \( \theta_{\text{min}} = 3^\circ \) and \( \theta_{\text{max}} = 42^\circ \), longitudinal boundaries at \( \phi_{\text{min}} = 153^\circ \) and \( \phi_{\text{max}} = 212^\circ \), and radial boundaries at the photosphere \((r = 1 R_\odot)\) and \( r = 1.75 R_\odot \).

The weighting functions \( \omega_i \) and \( \omega_j \) in \( L_d \) and \( L_d \) in Eq. (6) are chosen to be unity within the inner physical domain \( V' \) and decrease with a cosine profile in the buffer boundary region (Wiegelmann 2004; Tadesse et al. 2009). They reach a zero value at the boundary of the outer volume \( V \). The distance between the boundaries of \( V' \) and \( V \) is chosen to be \( m = 10 \) grid points wide. The framed region in Figs. 3a–i corresponds to the lower boundary of the physical domain \( V' \) with a resolution of \( 132 \times 196 \) pixels in the photosphere. The original full disc vector magnetograph has a resolution of \( 1788 \times 1788 \) pixels out of which we extracted \( 142 \times 206 \) pixels for the lower boundary of the computational domain \( V \), which corresponds to \( 550 \) \( \text{Mm} \times 720 \text{ Mm} \) on the photosphere. The main reason for the implementation of the new term \( L_{\text{photo}} \) in Eq. (6) is that we need to work with boundary data of different noise levels and qualities or even neglect some data points completely. SOLIS/VSM provides full-disk vector magnetograms, but for some individual pixels the inversion from line profiles to field values may not have been successfully inverted and field data there will be missing for these pixels. Since the old code without the \( L_{\text{photo}} \) term requires complete boundary information, it cannot be applied to this set of SOLIS/VSM data.

In our new code, these data gaps are treated by setting \( w = 0 \) for those pixels, for which \( H_{\text{obs}} \) was successfully inverted, we allow deviations between the model field \( B \) and the input fields either observed \( H_{\text{obs}} \) or preprocessed surface field \( H \) using Eq. (6), so that the model field can be iterated closer to a force-free solution even if the observations are inconsistent. This balance is controlled by the Lagrangian multiplier \( v \) as explained in Wiegelmann & Inhester (2010), where we used \( w_{\text{radial}} = 100 w_{\text{trans}} \) for the surface fields both from data with preprocessing and without.

Figure 2 shows the position of the active region on the solar disk for both the SOLIS full-disk magnetogram1, and the SOHO/EIT image of the Sun observed at 195 Å on the same day at 16:00 UT2. As stated in Sect. 2.3, the potential field is used as the initial condition for iterative minimization required in Eq. (6). The respective potential field is shown in the rightmost panel of Fig. 2. During the iteration, the code forces the photospheric boundary of \( B \) towards observed field values \( H_{\text{obs}} \) or \( H \) (for preprocessed data) and ignores data gaps in the magnetogram. A deviation between surface vector field from model \( B \) and either \( H_{\text{obs}} \) or \( H \) (for preprocessed data) occurs where \( H_{\text{obs}} \) is inconsistent with a force-free field. In this sense, the term \( L_{\text{photo}} \) in Eq. (6) acts on \( H_{\text{obs}} \) as in the preprocessing, generating a surface field \( B \) instead of \( H \) from \( H_{\text{obs}} \), which is close to \( H_{\text{obs}} \), but consistent with a force-free field above the surface. In Fig. 3, we therefore compare the difference between the preprocessing procedure and the new extrapolation code (Eq. (6)) on \( H_{\text{obs}} \). The figure shows the surface magnetic field differences of the preprocessed, un-preprocessed, and potential surface fields.

To identify the similarity of vector components on the bottom surface, we calculate their pixel-wise correlations. The correlation were calculated from

\[
C_{\text{vec}} = \frac{\sum (v_i - u_i)(w_i - u_i)}{\left(\sum (v_i^2 - u_i^2)\right)^{1/2}},
\]

where \( v_i \) and \( u_i \) are the vectors at each grid point \( i \) on the bottom surface. If the vector fields are identical, then \( C_{\text{vec}} = 1 \); if \( v_i \perp u_i \), then \( C_{\text{vec}} = 0 \). Table 1 shows the correlations between the surface fields from \( B_{\text{pre}} - H_{\text{obs}} \) (where \( B_{\text{pre}} \) is the model field obtained from the preprocessed surface field \( H \) using Eq. (6)) and \( B_{\text{unpre}} - H_{\text{obs}} \), where \( B_{\text{unpre}} \) is the model field obtained from observed surface field \( H_{\text{obs}} \) using Eq. (6). We computed the vector correlations of the two surface vector fields for the three components at each grid point to compare how well they are aligned in each direction. From those values in Table 1, one can see that the preprocessing and extrapolation with Eq. (6) act on \( H_{\text{obs}} \), in a similar way. In Fig. 4, we plot magnetic field lines for the three configurations in which the vector correlations of potential field lines in the 3D box for both the extrapolated NLFF with and without preprocessing data are 0.741 and 0.793, respectively.

To understand the physics of solar flares, including the local reorganization of the magnetic field and the acceleration of

1 http://solis.nso.edu/solisdatal.html
2 http://sohowww.nascom.nasa.gov/data/archive
energetic particles, one has to estimate the free magnetic energy available for these phenomena. This is the free energy that can be converted into kinetic and thermal energy. From the energy budget and the observed magnetic activity in the active region, Régnier & Priest (2007a) and Thalmann et al. (2008) investigated the free energy above the minimum-energy state for the flare process. We estimate the free magnetic energy to be the difference between the extrapolated force-free fields and the po-

Fig. 2. Left: full disc vector magnetogram of May 15 2009 at 16:02 UT. Middle: SOHO/EIT image of the Sun on the same day at 16:00 UT. Right: potential magnetic-field line plot of SOLIS vector magnetogram at 16:02 UT, which has been computed from the observed radial component.

Fig. 3. Top row: radial surface vector field difference between a) modelled $B$ without preprocessing and $H_{obs}$, b) modelled $B^{pre}$ and $H_{obs}$, e) initial potential and $H_{obs}$. Middle row: latitudinal surface vector field difference between d) modelled $B$ without preprocessing and $H_{obs}$, e) modelled $B^{pre}$ and $H_{obs}$, and f) initial potential and $H_{obs}$. Bottom row: longitudinal surface vector field difference between g) modelled $B$ without preprocessing and $H_{obs}$, h) modelled $B^{pre}$ and $H_{obs}$, and i) initial potential and $H_{obs}$. The vertical and horizontal axes show latitude, $\theta$ and longitude, $\phi$ in degree on the photosphere respectively.
The correlations between the components of surface fields from \((B^\text{pre} - H_{\text{obs}})\) and \((B^\text{unpre} - H_{\text{obs}})\).

| \(\nu\) | \(u\) | \(C_{\text{corr}}\) |
|-------|-------|---------|
| \((B^\text{unpre} - H_{\text{obs}})_x\) | \((B^\text{pre} - H_{\text{obs}})_x\) | 0.930 |
| \((B^\text{unpre} - H_{\text{obs}})_y\) | \((B^\text{pre} - H_{\text{obs}})_y\) | 0.897 |
| \((B^\text{unpre} - H_{\text{obs}})_z\) | \((B^\text{pre} - H_{\text{obs}})_z\) | 0.875 |

Table 2. The magnetic energy associated with extrapolated NLFF field configurations with and without preprocessing.

| Model | \(E_{\text{eff}}(10^{32}\text{ erg})\) | \(E_{\text{ref}}(10^{32}\text{ erg})\) |
|-------|----------------|----------------|
| No preprocessing | 37.456 | 4.915 |
| Preprocessed | 37.341 | 4.800 |

density above the volume of the active region. There are strong current configurations above each active regions. This becomes clear if we compare the total current in-between each active region with the current moving from the left to the right active region. These currents were added up from the surface normal currents emanating from the pixels that are magnetically connected inside or across active regions, respectively. The result is shown in Table 3. The active regions share a decent amount of magnetic flux compared to their internal flux from one polarity to the other. In terms of the electric current, they are much more isolated. The ratio of shared to intrinsic magnetic flux is of the order of unity, while for the electric current those ratios are much less, 1.58/49.6 and 1.58/32.17, respectively. We can similarly calculate the average value of \(\alpha\) on the field lines with the respective magnetic connectivity. The averages are shown in the second row of Table 3. The two active regions are magnetically connected but are much less by electric currents.

4. Conclusion and outlook
We have investigated the coronal magnetic field associated with the AR 11017 on 2009 May 15 and a neighbouring active region...
by analysing SOLIS/VSM data. We have used the optimization method for the reconstruction of nonlinear force-free coronal magnetic fields in spherical geometry by restricting the code to limited parts of the Sun (Wiegelmann 2007; Tadesse et al. 2009). In contrast to previous implementation, our new code allows us to deal with a lack of data and regions with poor signal-to-noise ratio in the extrapolation in a systematic manner because it produces a field that is closer to a force-free and divergence-free field and tries to match the boundary only where it has been reliably measured (Wiegelmann & Inhester 2010).

For vector magnetograms with a lack of data points and where zero values have been replaced for the signal below a certain threshold value, the new code relaxes the boundary and allows us to fulfill the solenoidal and force-free conditions more reliably as it allows deviations between the extrapolated boundary field and an inconsistent observed boundary data. With the new term, $L_{\text{photo}}$ extrapolation from $H_{\text{obs}}$ and $H$ yields almost the same 3D field. However, in the latter case the iteration to minimize Eq. (6) saturates in fewer iteration steps. At the same time, preprocessing does not affect the overall configuration of magnetic field and its total energy content.

We plan to use this newly developed code for upcoming data from SDO (Solar Dynamics Observatory)/HMI (Helioseismic and Magnetic Imager) when full disc magnetogram data become available.

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