Maintenance Strategy Choice Supported by the Failure Rate Function: Application in a Serial Manufacturing Line

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Received: 22 May 2021, Accepted: 30 July 2021, Published online: 04 February 2022

Abstract
The purpose of this article is to choose a maintenance procedure for the critical equipment of a forging production line with five machines. The research method is quantitative modelling and simulation. The main research technique includes retrieving time between failure and time to repair data and find the most likely distribution that has produced the data. The most likely failure rate function helps to define the maintenance strategy. The study includes two kinds of maintenance policies, reactive and anticipatory. Reactive policies include emergency and corrective procedures. Anticipatory policies include predictive and preventive ones combined with a total productive maintenance management approach. The most suitable combination for the first three machines is emergency and corrective choice. For the other machines, a combination of total productive maintenance and a predictive approach is optimal. The study encompasses the case of a serial production manufacturing line and maximum likelihood estimation. The failure rate function defines a combination of strategies for each machine. In addition, the study calculates the individual and systemic mean time to failure, mean time to repair, availability, and the most likely number of failures per production order, which follows a Poisson process. The main contribution of the article is a structured method to help define maintenance choices for critical equipment based on empirical data.

Keywords
maintenance strategy, Weibull analysis, reliability, maintainability, predictive maintenance, corrective maintenance, preventive maintenance

1 Introduction
In the past, competition has forced manufacturing operations to achieve at the same time high dependability and low cost (Ward et al., 1996). More recently, also flexibility has been recognised as an essential target for manufacturing (Kanovska and Doubravsky, 2021). Such stringent requirements have increased the relevance of the strategic management of industrial maintenance, as low availability of critical shop floor equipment or excessive stoppage can undermine the three objectives (Patil et al., 2017). Therefore, if one is to meet at the same time high availability, low cost, and flexibility requirements, formulating a maintenance strategy for critical equipment is essential (Alaswad and Xiang, 2017).

Scheduling maintenance activities may require estimating reliability parameters (Chakherloo et al., 2017; Chemweno et al., 2016). Overestimation of time between failures may compromise the availability, while underestimation may increase the cost (Leoni et al., 2021). Reliability-centred maintenance (RCM) is a management technique that uses reliability models (Gharahasanlou et al., 2017) to support decisions on maintenance strategies aiming at increase availability without increase cost (Deshpande and Modak, 2002). However, adopting RCM for supporting strategic choice may require investment and their desired effects may take many years to materialise (Rausand, 1998). Computational methods may be useful in evaluating maintenance strategies derived from empirical data under different scenarios (Sellitto, 2020).

A search in SCOPUS and Web of Science databases with the keywords "maintenance strategy" and "strategic decision" between 2016 and 2020 identified articles consistent with our study. The search was limited to peer-reviewed articles in journals, written in English, already cited by other peer-reviewed articles. Seecharan et al. (2018) define
the maintenance strategy according to the downtime and failure frequency. To identify the maintenance strategy, Han et al. (2020) use ageing models and failure modes, while Liu et al. (2020) and Gao and Xie (2018) use respectively game models and simulation in maintenance strategy decision-making processes. A larger number of articles use multicriteria methods, mainly AHP and ANP (Alrabghi et al., 2017; Antosz et al., 2020; Azadeh and Zadeh, 2015; Borjalilu and Ghambari 2018; Dachyar et al., 2018; Emovon et al., 2018; Firouz and Ghadimi, 2016; Ge et al., 2017; Jeang et al., 2019; Karthik et al., 2017; Kurian et al., 2019; Özcan et al., 2017). The search retrieved only one article employing time to failure data in the decision-making process (Gharahaslanlou et al., 2017).

This study merges quantitative and qualitative concerns. It evaluates quantitatively the failure risk behaviour and classifies it according to qualitative criteria. To the best of our knowledge, no recent, peer-reviewed, cited article presents a purely quantitative approach based only on the failure risk function to help to decide on maintenance choices in manufacturing according to qualitative prescriptions. This is the research gap this study aims to bridge.

The research question is how to choose a maintenance procedure for critical equipment in a manufacturing plant. The purpose of this article is to choose a maintenance procedure for the critical equipment of a forging production line, comprising five machines. The research method is quantitative modelling and simulation. The main research techniques are the probabilistic modelling of time between failures ($TBF$) and time to repair ($TTR$) of machines as well as simulation of the entire system for the calculation of necessary reliability parameters. The use of field data and failure-based decision models can reduce the inherent risks and uncertainties in decision-making on maintenance strategies (Ge et al., 2017; Panchal et al., 2017; Seiti et al., 2017; Seiti et al., 2018a; Seiti et al., 2018b). The study employs the failure rate function, which can be considered as an indication of the equipment's reliability over its entire lifecycle (Jónás et al., 2018). The main novelty is demonstrated to be a powerful tool to model the failure rate function. Specifically, the asymptotic omega distribution is just the Weibull distribution (Dombi et al., 2019). Despite there being other models besides the omega distribution, such as gamma (parallel), q-Weibull (Jia, 2021; Xu et al., 2021), or Marshall-Olkin extended uniform (Jónás et al., 2018), estimating the shape factor of the Weibull model is still widely used to directly reveal the position in the bathtub curve (Jia, 2021). The second limitation concerns to scarcity of data in one of the machines. Due to the large MTBF, the machine failed only eight times. Finally, if data does not fit, two possibilities arise:

- mixture of data retrieved from more than one breakdown process; or
- data are contaminated with shortages not owing to failures.

The rest of the article comprises a theoretical background, methodology, results, discussion, and conclusions.

2 Background

The study focuses on emergency, corrective, preventive, and predictive maintenance strategies and total productive maintenance (TPM) approaches for shop floor management. Emergency and corrective strategies support reactive policies that apply after breakdown (run-to-failure policies). Preventive and predictive strategies and TPM approaches support anticipatory policies that apply to avoid or at least reduce breakdown intensity (Alsyouf, 2007).

Emergency maintenance requires immediate actions to restore a unit in a failure state. Emergency maintenance cannot be scheduled either because there is no redundancy or no inventory enough to ensure deliveries (De Faria et al., 2015; Shafiee, 2015; Tee and Ekpiwhre, 2019).
Corrective maintenance fits when the repair of an unexpected fault can be scheduled. A repair can be postponed due to equipment redundancy or inventory exists to avoid a shortage. The maintenance staff has the chance to remove the root cause and correct the defect that caused the failure. Preventing recidivism, the overall failure rate decreases over time (Higgins and Mobley, 2001).

Preventive maintenance requires systematic activities that prevent or delay failures, such as cleaning, lubrication, calibration, detection of incipient defects, and replacement of fragile parts. Procedures occur before breakdowns (Higgins and Mobley, 2001; Tee and Ekpiwhre, 2019). Preventive maintenance may take place at predetermined intervals, or according to performance criteria, such as the number of deliveries, kilometres, or machine cycles (Carnero and Gómez, 2017).

While preventive strategies execute the planned activities unconditionally, that is, regardless of the machine condition, predictive strategy actions occur only upon a prognostic review indicating a failure in progress. Without such an indication, no further action applies (De Faria et al., 2015; Carnero and Gómez, 2017). The prognostic review can rely on physical inspection based on checklists, mathematical models based on the physics of the degradation process (Do et al., 2015), or probabilistic models based on failure data (Lim and Mba, 2015). A TPM management approach for the shop floor may also be useful. The application of TPM principles at the shop floor usually reduces unexpected incidents that cause random failures (Ahmad et al., 2018; Habidin et al., 2018). Therefore, TPM can also contribute to reducing the failure rate.

Fig. 1 shows the typical behavior of the strategies regarding time-to-failure (Alrabghi and Tiwari, 2015). Given a certain failure effect pattern (for instance, a vibration level), run-to-failure policies (corrective or emergency strategies), react when the effect reaches the breakdown threshold, causing a failure. Predictive strategies employ an alert threshold that ensures a period $\Delta t$ to plan and execute a repair action, anticipating the failure. Finally, the preventive strategy does not require information on ongoing failure processes, relying on other methods to schedule repairs and anticipate failures. If the forecast fails, a failure occurs.

Feedback life data can be useful in defining or at least optimizing maintenance policies. This study relies on a prognostic review technique based on two functions, $R(t)$ and $h(t)$. The reliability function $R(t)$ is the probability that a part or system will perform its intended mission free of failures until time $t$, under specified conditions. The hazard function $h(t)$ is the conditional probability that, given that a failure does not occur until time $t$, it will occur within the interval $[t; t + \Delta t]$ when $\Delta t$ approaches 0 (Gharahasanlou et al., 2017).

Recently, authors have proposed advanced tools and methods to deal with empirical failure data and the failure rate function. Among others, we focus on the q-Weibull distribution, the omega distribution (both related to the Weibull distribution), and fuzzy methods to forecast a failure rate function. Failure rates are time-dependent and can be managed as time series retrieved from empirical observations (Jónás et al., 2018).

The q-Weibull model is a generalization of the Weibull model (Jia, 2021), obtained by multiplying the original expression of the Weibull density function by $(q - 2)$, $q < 2$. When $q \rightarrow 1$, the q-model converges to the Weibull model (De Assis et al., 2020). The q-model is especially useful in modeling failure rates in a series system formed by dependent components (Xu et al., 2021). This is not the case of our study, as the involved equipment is composed of independent, autonomous machines. Assis et al. (2021) compared results obtained with the Weibull and q-Weibull models for the same field data. The q-Weibull model allows new formats for the bath-tube curve, such as the unimodal and U-shaped formats.

The omega distribution overcomes a weakness of the Weibull distribution, as it can model non-monotonic functions, such as a complete bath-tube-shaped failure rate. While the Weibull distribution requires three models, one for each phase, the omega distribution provides a unique model. Dombi et al. (2018) call it all-in-one model. The omega distribution requires three parameters, $\alpha$, $\beta$, and $d$. While $\alpha$ and $\beta$ are similar to the shape and scale parameters of the two-parameter Weibull model, $d$ delimits the range of the omega function $(0, d)$. In practice, the omega probability distribution with parameters $\alpha$, $\beta$, and $d$ can replace the two-parameter Weibull model with parameters $\alpha$, $\beta$.

Finally, Jónás et al. (2018) developed a method based on historical time series of failure rates, analytic curve fitting, and soft computing techniques to provide standard failure
rates. The next step is to classify, by a fuzzy clustering process, a target failure rate series according to one of the standardized functions, which may be considered a prediction for the reliability of an item. The method requires field data comprising the entire life cycle of items, which implies in a wide time lapse or a high failure rate. This is not the case of our study. None of the five machines completed the entire life cycle. Moreover, two machines have high MTBF, offering a small number of failures for a complete analysis.

Even recognising the value of the above-mentioned tools, this study focuses only on the negative exponential and the three parameters Weibull models, represented, respectively, by Eqs. (1) and (2) (Choudhary et al., 2019):

\[ R(t) = e^{-\lambda t}, \quad (1) \]
\[ R(t) = e^{-\left(\frac{t}{\theta}\right)^\gamma}, \quad (2) \]

where:
- \( \lambda \) = constant failure rate;
- \( t_0 \) = time free of failure or shelf-life period;
- \( \gamma \) = shape factor, and
- \( \theta \) = scale factor.

The negative exponential model can be considered a particular case of the Weibull model when \( \gamma = 1 \), which implies \( \lambda = 1/\theta \) (Xie et al., 2000). If both succeed in fitting life data, the exponential model is preferable by parsimony (Hashemian, 2011). Eqs. (3) and (4) represent respectively the MTBF for the exponential and Weibull models (\( \Gamma \) is the gamma function).

\[ MTBF = \frac{1}{\lambda}, \quad (3) \]
\[ MTBF = t_0 + \theta \Gamma \left( \frac{1}{\gamma} \right) \quad (4) \]

The hazard function \( h(t) \) has a homologous behaviour as the failure rate function \( \lambda(t) \), the failure rate observed in the stochastic failure process of a repairable item (Rausand and Hoyland, 2004). The higher the function, the more imminent an equipment failure is. The function can follow three patterns: decreasing, constant, and increasing. From now on, we use the failure rate function \( \lambda(t) \) rather than \( h(t) \).

A decreasing \( \lambda(t) \) indicates that the equipment is in the infant mortality period, subject to early failures. Most failures relate to early design deficiencies, fragile components, and installation errors. Any failure must be addressed by a corrective maintenance strategy that requires determining the root cause of the failure. Emergency maintenance can also be helpful in the case of components with manufacturing failures that should be replaced with perfect parts. Nonetheless, design or installation errors will not be corrected by an emergency, predictive, or preventive maintenance strategy, only by a corrective strategy.

A constant \( \lambda(t) \) is associated with the maturity period, in which random failures occur, following a homogeneous Poisson process. Failures are due to external causes, such as operating errors, accidents with extraordinary power release, or misuse of the equipment. Design and installation errors have already been amended in the infant mortality phase and wear-out failure processes are not in progress yet. In this phase, TPM-based management policies are especially helpful. Organising the shop floor can reduce the incidence of random failures or accidents and reduce the constant failure rate. A predictive maintenance strategy that monitors key equipment signals is also helpful by detecting if the maturity phase approaches the end. A corrective maintenance strategy is unnecessary, as there are no more design or installation errors to correct. A preventive maintenance strategy is contraindicated. Such a strategy replaces strong, reliable parts that have not yet failed and will not fail in the short term as design or installation errors no longer exist.

Finally, an increasing \( \lambda(t) \) indicates the wear-out or degradation phase. As the risk increases, the intervals between failures become smaller due to the wear or end of materials' useful life. A good choice is preventive maintenance. Since failures are inevitable, even if the ttf is not known, scheduled, opportunistic replacement may bring equipment back to the maturity period (Hashemian, 2011). A predictive maintenance strategy can be partially helpful. As failure processes are now in progress, monitoring of key equipment signals can improve predicting ttf, which extends component usage by efficient replacement. Corrective maintenance strategies are useless as design or installation errors no longer exist. An emergency strategy is still necessary as the preventive strategy reduces but not eliminates breakdowns.

The next step is to associate the most likely \( \lambda(t) \) and the shape factor \( \gamma \) of the Weibull distribution (Pandey et al., 2018). Given that a Weibull model fits life data if \( \gamma < 1 \), \( \lambda(t) \) is decreasing, the equipment is in the infant mortality phase, and a combination of corrective and emergency maintenance strategies applies. If \( \gamma = 1 \), \( \lambda(t) \) is constant, the equipment is in the maturity phase and a combination of TPM policy with predictive maintenance strategy applies. Finally, if \( \lambda(t) \) is increasing, \( \gamma > 1 \) and a preventive maintenance strategy combined with a predictive strategy should
be chosen (Hashemian, 2011; Chakerloo et al., 2017).

Another function of interest in maintenance strategy is the maintainability function $M(t)$, the probability that a repair will be completed until time $t$. Lognormal and normal distributions fit well $TTR$ when the same repair strategy tackles the various failure mechanisms that affect equipment. This is the case in our study. Otherwise, mixture distributions may be used (Zaki et al., 2019). The expected value for $M(t)$, the $MTTR$, is calculated analogously to the $MTBF$. If no distribution fits, a reasonable alternative is to use the $TTR$ average value (Tsarouhas, 2018; Choudhary et al., 2019). $MTBF$ and $MTTR$ allow calculating the long-run average availability $AV$, the average proportion of the period in which the item is available. Increasing $AV$ requires increasing $MTBF$, decreasing $MTTR$, or both. Equation (5) expresses $AV$ under perfect repair conditions (Rausand and Høyland, 2004).

$$AV = \lim_{t \to \infty} \int_0^t A(t) \, dt = \frac{MTBF}{MTBF + MTTR},$$

where $A(t)$ is the probability that the item is functioning at time $t$, also referred to as the point availability of the item.

3 Research

The research method adopted here is quantitative modeling combined with computer simulation. Modelling is the use of mathematical techniques to describe the functioning of a production system or part of it. Computer simulation is the use of computational techniques to simulate the operation of a system according to a set of variables in a given domain to investigate causal and quantitative relationships among variables (Jeang et al., 2019). The methodology is:

1. retrieve failure and repair data ($TBF$ and $TTR$) from the maintenance management system of the company during twelve months;
2. find distributions and parameters that better fit data and calculate individual and systemic $MTBF$, $MTTR$, and $AV$;
3. choose the maintenance strategy for each machine; and
4. discuss implications and retrieve lessons from the study.

The study considers only failures that provoked breakdowns, ruling out human failure, lack of scheduling, and interruptions without loss of production such as utility stoppage. Fig. 2 shows a block diagram of the studied manufacturing line.

The software package ProConf/ProSys (conceived for single machines and systems analysis, respectively) supported the study, estimating parameters, confidence intervals, expected values, and fitting functions. ProConf tests the exponential, Weibull, gamma, lognormal, and normal models by maximum likelihood estimation (MLE), verifying validities by the Chi-Square and Kolmogorov-Smirnov (K-S) tests with a significance level greater than 5% (Villarreal et al., 2016; Gharahasanlou et al., 2017). Minitab software provided the Anderson-Darling (AD) test. ProSys provides systemic $R(t)$ and generates random life data according to the individual MLE.

The fitting process requires identifying the distribution and parameters that are most likely to have produced the data $t = (t_1, \ldots, t_n)$. Goodness-of-fit tests determine if a distribution fits or not data. The MLE finds the parameter vector $\Theta$ that most likely has generated the data. The MLE maximises the likelihood function $L(t, \Theta) = \prod_{i=1}^{n} f(t_i, \Theta)$, that is, the joint probability of the data, given by the product of the probability density function evaluated at each $t_i = (t_1, \ldots, t_n)$. The exponential model requires a constant failure rate. The normal and lognormal require the mean and standard deviation or variance and the logarithm of the data, respectively. The Weibull model requires shift, scale, and shape factors. These distributions are sufficient for this study.

Usually, it is easier to maximize a log-likelihood function $\log L(t, \Theta) = \sum_{i=1}^{n} \log f(t_i, \Theta)$, taking partial derivatives $\frac{\partial L(t, \Theta)}{\partial \Theta}$ of the log-likelihood function and equating to zero produces a set of equations whose resolution yields the maximum likelihood estimators for the parameters (Usher and Hodgson, 1988). Leemis (1995, p.172–179) and Rausand and Høyland (2004, p.587–590) provide a comprehensive explanation for MLE.

For the exponential, normal, and lognormal distributions, the procedure results in the well-known parameters for mean and standard deviation. For the Weibull distribution, the log-likelihood function requires an iterative procedure to solve simultaneous Eqs. (6) to (8) to find $\Theta = [\alpha, \gamma, \theta]$.

$$\hat{\theta} = \left[ \sum_{i=1}^{n} (t_i - \bar{t}_i)^{\gamma} / n \right] ^{1/\gamma}$$ (6)
where ∆t = [t_{i-1} − t_i] is a fixed step for each execution of Eq. (11) and t_i ∈ [0, 1, 2, …, 200] hours.

A [0–200] hours interval ensures that R(t) ranges between 1 and 0.000. When ∆t = 10 hours (and consequently n = 200/10 = 20), MTBF = 22.52 hours. When ∆t = 1 hour (n = 200), MTBF = 16.92 hours. When ∆t = 0.5 hour (n = 400), MTBF = 16.65 hours. When ∆t = 0.1 hour (n = 2,000), MTBF = 16.44 hours, evidencing that the MTBF converges to some point near 16.4 hours.

To verify the calculation of the MTBF, we used data provided by ProSys to find a suitable model with the support of the CurveExpert® software. The best-fitting model is the Bleasdale model R(t) = (a + bt)^c. Although a high correlation (> 0.999), there is no physical meaning in the parameters. Despite providing an accurate data description, it is not an external validation for the MTBF. Therefore, a negative exponential model, whose mean may inform the systemic MTBF, is required. CurveExpert® found a reasonable model (correlation > 0.97) with λ = 0.06065, whose MTBF = 1/0.06065 = 16.49 hours is consistent with the previous estimation. Fig. 3 shows the simulated and the modelled systemic reliability function. The thin line represents the data provided by ProSys whereas the thick line represents the model adjusted by the CurveExpert® software.

We proceed to the analysis of systemic availability. The systemic availability AV is the probability that all five machines will be available upon request, i.e., the intersection of the five AVi. Therefore, AV = ΠAVi = 92.4%. Ribeiro et al. (2019) reported a case in which the overall Av of a serial production line of an auto parts manufacturer ranged between 94% and 98%. Saari et al. (2019) reported a study in which the Av of five individual balling drums in the mining industry ranged between 92% and 96%. As the five machines form a parallel array, the systemic AV overcomes 99%. In our case, assuming a systemic
Table 1 TBF and TTR (in hours) and availability of equipment

| Failure number | FI-FO-008 | MH-FO-001 | PE-FO-051 | PF-FO-052 | PF-FO-016 |
|----------------|-----------|-----------|-----------|-----------|-----------|
|                | TBF       | TTR       | TBF       | TTR       | TBF       | TTR       | TBF       | TTR       | TBF       | TTR       |
| 1              | 0.98      | 0.82      | 1.288     | 2.7       | 8.7       | 2.12      | 45        | 6.9       | 0.7       | 0.12      |
| 2              | 1.47      | 0.4       | 2.772     | 1.46      | 9         | 0.52      | 54        | 0.5       | 5.4       | 0.18      |
| 3              | 7.27      | 2.53      | 4.802     | 4.24      | 18        | 0.4       | 531       | 3.4       | 27.0      | 1.62      |
| 4              | 11.51     | 7.4       | 5.95      | 0.64      | 18        | 0.57      | 1,243     | 0.7       | 32.4      | 0.14      |
| 5              | 18        | 0.38      | 6.86      | 5.66      | 21.3      | 0.4       | 288       | 0.6       | 32.4      | 0.14      |
| 6              | 18        | 0.45      | 7.882     | 12.06     | 45        | 0.23      | 820       | 1.0       | 43.2      | 0.18      |
| 7              | 18        | 2.35      | 8.372     | 2.46      | 54        | 1.12      | 81        | 6.0       | 43.2      | 0.14      |
| 8              | 27        | 1.38      | 11.34     | 1.54      | 54        | 0.35      | 460       | 3.1       | 54.0      | 0.22      |
| 9              | 27        | 1.55      | 12.6      | 1.74      | 72        | 0.88      |           |           | 75.6      | 0.38      |
| 10             | 90        | 1.48      | 12.6      | 1.7       | 198       | 0.38      |           |           | 113       | 1.36      |
| 11             | 108       | 0.98      | 12.6      | 1.36      | 441       | 0.15      |           |           | 113       | 0.30      |
| 12             | 126       | 0.78      | 16.52     | 1.16      | 1,206     | 0.92      |           |           | 146       | 0.26      |
| 13             | 153       | 1.12      | 25.2      | 2.76      | 2,079     | 2.78      |           |           | 257       | 0.28      |
| 14             | 198       | 1.13      | 25.2      | 1.16      |           |           |           |           | 254       | 0.30      |
| 15             | 792       | 2.38      | 37.8      | 3.1       |           |           |           |           | 297       | 13.5      |
| 16             | 4.02      | 2.13      | 37.8      | 0.7       |           |           |           |           | 5.4       | 0.38      |
| 17             | 4.07      | 0.77      | 50.4      | 2.16      |           |           |           |           | 5.4       | 0.60      |
| 18             | 4.38      | 1.68      | 50.4      | 1.46      |           |           |           |           | 27.0      | 0.18      |
| 19             | 9         | 1.78      | 63        | 0.7       |           |           |           |           | 32.4      | 0.40      |
| 20             | 18        | 2.75      | 63        | 1.36      |           |           |           |           | 32.4      | 17.4      |
| 21             | 18        | 5.3       | 63        | 2.2       |           |           |           |           | 43.2      | 2.62      |
| 22             | 18        | 1.82      | 75.6      | 4.94      |           |           |           |           | 75.6      | 0.54      |
| 23             | 27        | 0.88      | 75.6      | 3.36      |           |           |           |           | 75.6      | 0.90      |
| 24             | 27        | 0.85      | 75.6      | 2.3       |           |           |           |           | 113       | 1.24      |
| 25             | 45        | 0.55      | 100.8     | 1.9       |           |           |           |           | 113       | 0.70      |
| 26             | 54        | 4.03      | 100.8     | 0.8       |           |           |           |           | 140       | 3.60      |
| 27             | 81        | 0.68      | 100.8     | 1.74      |           |           |           |           | 146       | 5.44      |
| 28             | 108       | 0.48      | 113.4     | 1.26      |           |           |           |           | 227       | 0.08      |
| 29             | 153       | 1.13      | 113.4     | 4.96      |           |           |           |           | 254       | 8.40      |
| 30             | 351       | 0.42      | 126       | 4.36      |           |           |           |           | 297       | 11.4      |
| 31             | 9         | 1.33      | 138.6     | 5.06      |           |           |           |           | 1.9       | 15.6      |
| 32             | 27        | 0.57      | 138.6     | 2         |           |           |           |           | 16.2      | 5.04      |
| 33             | 27        | 0.53      | 138.6     | 2.46      |           |           |           |           | 43.2      | 1.44      |
| 34             | 36        | 0.53      | 138.6     | 1.74      |           |           |           |           | 135       | 2.56      |
| 35             | 36        | 0.75      | 151.2     | 5.96      |           |           |           |           | 151       | 3.12      |
| 36             | 90        | 2.27      | 151.2     | 1.36      |           |           |           |           | 184       | 10.0      |
| 37             | 144       | 0.45      | 163.8     | 2         |           |           |           |           | 292       | 6.64      |
| 38             | 387       | 1.63      | 176.4     | 2.24      |           |           |           |           | 289       | 8.61      |
| 39             | 675       | 0.53      | 541.8     | 1.74      |           |           |           |           | 289       | 11.4      |
| 40             | 90        | 0.43      | 768.6     | 1.64      |           |           |           |           | 249       | 10.4      |
| 41             | 54        | 1.32      |           |           |           |           |           |           |           |           |

\[ T_{off}/T_{eff} = 4.092 \]
\[ T_{total} = 4.153 \]
\[ Av = \frac{T_{eff}}{T_{total}} \]
\[ 98.5\% \]
consistent with the previous estimation. Finally, systemic $\lambda = 0.05812$, whose $1/\lambda = 17.21$ hours is also found a suitable model (correlation > 0.98) with curve converges to some point nearby 17.4 hours. CurveExpert® MTBF = 17.4 hours. The sequence indicates that MTBF = 17.61 hours. When $\Delta t = 17.88$ hours. When $\Delta t = 200)$, MTBF = 23.46 hours. When $\Delta t = 1$ hour ($n = 2000$), $MTBF = 17.61$ hours. When $\Delta t = 0.1$ hour ($n = 400$), $MTBF = 17.4$ hours. The sequence indicates that $MTBF$ converges to some point nearby 17.4 hours. CurveExpert® also found a suitable model (correlation > 0.98) with $\lambda = 0.05812$, whose $MTBF = 1/0.05812 = 17.21$ hours is consistent with the previous estimation. Finally, systemic $Av = 92.9\%$, which implies an $MTTR = 1.33$ hour.

![Fig. 3 Simulated and modeled systemic reliability function](image)

MTBF = 16.4 hours and $Av = 92.4\%$ implies an equivalent overall $MTTR = 1.34$ hour.

A similar development is valid for orders that require four machines (the fourth machine is deactivated). When $\Delta t = 10$ hours ($n = 20$), $MTBF = 23.46$ hours. When $\Delta t = 1$ hour ($n = 200$), $MTBF = 17.88$ hours. When $\Delta t = 0.5$ hour ($n = 400$), $MTBF = 17.61$ hours. When $\Delta t = 0.1$ hour ($n = 2000$), $MTBF = 17.4$ hours. The sequence indicates that $MTBF$ converges to some point nearby 17.4 hours. CurveExpert® also found a suitable model (correlation > 0.98) with $\lambda = 0.05812$, whose $MTBF = 1/0.05812 = 17.21$ hours is consistent with the previous estimation. Finally, systemic $Av = 92.9\%$, which implies an $MTTR = 1.33$ hour.

Unexpected breakdown stoppages influence order execution. As exponential models may fit both $TBF$ (five and four machines), the expected number of interruptions during an order follows a homogeneous Poisson process. The process intensity $\lambda = 1/MTBF$ and $t = lead$ time of the order. Equation (12) presents the model.

$$ P\left[N(t) = k\right] = \frac{e^{-\frac{t}{MTBF}}\left(\frac{lead-time}{MTBF}\right)^{k}}{k!}, $$

where $k = [0, 1, 2, \ldots, n]$.

During this period, the manufacturing system executed 188 orders requiring five machines and 42 orders requiring four machines. Applying Eq. (12) to each order and selecting the most likely number of interruptions in the order caused by machine breakdown produces Table 3.

### Table 2 MLE for data

| Machine     | Data | Model and p-value | Parameters | 95% confidence intervals | Expected value $(MTBF, MTTR)$ | Availability |
|-------------|------|-------------------|------------|--------------------------|-----------------------------|--------------|
| FI-FO-008   | TBF  | Weibull           | $t_c = 0$  | $\gamma = 0.68$          | $0.50 < \gamma < 0.82$      | $MTBF = 97.05$|
|             |      |                   | $\theta = 74.3$ |                          |                             |              |
| TTR         | Lognormal |               | $\mu = 0.09$ | $-0.13 < \mu < 0.32$     | $MTTR = 1.44$              | $Av = 98.5\%$|
|             |      |                   | $\sigma^2 = 0.54$ |                         |                             |              |
| MH-FO-001   | TBF  | Weibull           | $t_c = 0$  | $\gamma = 0.79$          | $0.58 < \gamma < 0.96$      | $MTBF = 97.2$|
|             |      |                   | $\theta = 84.8$ |                          |                             |              |
| TTR         | Lognormal |               | $\mu = 0.74$ | $0.55 < \mu < 0.96$      | $MTTR = 2.56$              | $Av = 97.4\%$|
|             |      |                   | $\sigma^2 = 0.40$ |                     |                             |              |
| PE-FO-051   | TBF  | Weibull           | $t_c = 0$  | $\gamma = 0.50$          | $0.28 < \gamma < 0.69$      | $MTBF = 322.8$|
|             |      |                   | $\theta = 158.6$ |                         |                             |              |
| TTR         | Lognormal |               | $\mu = -0.52$ | $-1.00 < \mu < -0.03$    | $MTTR = 0.82$              | $Av = 99.7\%$|
|             |      |                   | $\sigma^2 = 0.64$ |                     |                             |              |
| PF-FO-052   | TBF  | Exponential       | $l = 0.0023$ | $0.001 < l < 0.0041$     | $MTBF = 440.3$             |              |
| TTR         | Lognormal |               | $\mu = 0.55$ | $-0.26 < \mu < 1.39$    | $MTTR = 2.91$              | $Av = 99.3\%$|
|             |      |                   | $\sigma^2 = 1.00$ |                     |                             |              |
| PF-FO-016   | TBF  | Exponential       | $l = 0.0095$ | $0.0064 < l < 0.0125$    | $MTBF = 105.5$             |              |
| TTR         | Lognormal |               | $\mu = -0.09$ | $-0.54 < \mu < 0.5$    | $MTTR = 3.59$              | $Av = 96.7\%$|
|             |      |                   | $\sigma^2 = 2.60$ |                     |                             |              |

### 5 Choice of maintenance intervention

The shape of $\lambda(t)$ indicates how an equipment age and can help to choose the type of intervention in the equipment. Preventive intervention, there is, unconditional replacement of parts that still not failed after a calculated time interval, is viable only in the wear-out phase when $\gamma > 1$. This is not the case in the current study. Nonetheless, in predictive inspections, there are, inspections able to detect ongoing or
imminent problems are due to useful life periods. Table 4 synthesises $R(t)$, the failure rate $\lambda(t)$ behaviour, the choices for each machine, and the most likely time between inspections for a 90% safety, there is, a time $t$ so that $R(t) \approx 90\%$. As each working shift lasts eight hours, the impact of scheduling inspections for the operation team is acceptable.

For a shape factor $\gamma < 1$, the number of failures per unit time decreases over time. Initial failures due to equipment design or installation errors, or critical part manufacturing failures, have not yet been corrected. Initial failures may still be hidden and may manifest in the future. If initial failures still exist, a good choice is corrective maintenance.

A good decision is to take advantage of a failure to identify the root cause and remove it. Usually, such an investigation requires time and machine availability for testing and eventual refurbishment. If there is no equipment redundancy or production inventory, the alternative is to adopt an emergency approach by immediately replacing parts with identical ones, starting a maintenance engineering process to identify the most likely root cause, and remove it as soon as possible. A reactive maintenance policy combines corrective and emergency maintenance procedures and is suitable for the first three machines. Alrabghi et al. (2017) and Carnero and Gómez (2017) report applications of this type of strategy.

The lack of a combined corrective and emergency maintenance policy entails risks. By replacing parts without correcting the failure, the equipment may take a longer time to overcome the infant mortality phase, as the failures persist and repeat over time. As industrial equipment can quickly become obsolete due to technological advances, it may never leave the infant mortality phase. A preventive policy will unconditionally replace critical parts. As emergency actions have already replaced the weak parts (the parts that do not resist the service), the parts to replace are the strong parts (those that resist). If the incoming part is equal to or stronger than the replaced part, there will be no gain or loss. However, if the incoming part is weaker, failure will appear where it previously was not, causing the equipment to retreat into the life cycle, extending the infant mortality phase.

For a shape factor $\gamma = 1$, the average number of failures per unit of time is constant and owing to random, external causes. No more initial failures exist and there are no wear-out failures yet. In this case, a good choice is a predictive maintenance approach combined with TPM principles-based shop floor management. The predictive approach monitors critical points to alert incoming wear-out processes, which will require changing the strategy. Monitoring can be done by techniques such as vibration measurement, measurement of electrical parameters in motors and drives, ferrography, thermography, or periodic inspections based on checklists (Hashemian, 2011; Carnero, 2005). Additionally, adopting shop floor management practices, such as TPM, can reduce random failures originating from inefficiencies in the shop floor. This choice is suitable for the last two machines. De Faria et al. (2015), Kirubakaran and Ilangkumaran (2016), and Carnero and Gómez (2017) report similar applications.

For constant failure rates, a preventive maintenance approach may entail risk. If a non-perfect part replaces a part that did not fail yet, new defects may appear and the equipment may return to the early failure phase. A

| Machines       | Number of breakdowns per order | Frequency of orders free of stoppage | Mean number of breakdowns per order |
|----------------|-------------------------------|-------------------------------------|-------------------------------------|
| 1, 2, 3, 4, 5  | 142                           | 75.5%                               | 0.69                                |
| 1, 2, 3, 5     | 30                            | 71.4%                               | 0.43                                |

| Machine       | $R(t)$                         | failure rate behavior | Type of intervention and time between inspections |
|---------------|--------------------------------|-----------------------|--------------------------------------------------|
| FI-FO-008     | $R(t) = e^{\frac{-t}{\theta_{10}}}$ | Decreasing            | Corrective maintenance                           |
| MH-FO-001     | $R(t) = e^{\frac{-t}{\theta_{100}}}$ | Decreasing            | Corrective maintenance                           |
| PE-FO-051     | $R(t) = e^{\frac{-t}{\theta_{100}}}$ | Decreasing            | Corrective maintenance                           |
| PF-FO-052     | $R(t) = e^{\frac{-t}{\theta_{100}}}$ | Constant              | Predictive maintenance and TPM - 48 hours (one inspection every six shifts) |
| PF-FO-016     | $R(t) = e^{\frac{-t}{\theta_{100}}}$ | Constant              | Predictive maintenance and TPM - 12 hours (two inspections each three shifts) |
preventive maintenance procedure is more suitable to an increasing failure rate \( (\gamma > 1) \), which was not observed in the system. With an increasing failure rate, a critical part that has not already failed will fail in an unknown moment, which may jeopardize the dependability of the manufacturing. Therefore, the lack of a preventive option in the wear-out phase implies an increase in unplanned shutdowns and an additional difficulty to meet orders’ due dates. To avoid unexpected stoppages, optimization models can help choose the best time for an unconditional exchange of fragile or critical parts. Such models are beyond the current scope of this paper as they do not apply to the case.

An analysis of how to predict the beginning of the forthcoming phase of the bathtub curve can also be helpful. Usually, practitioners use the information provided by the failure rate to propose actions aiming to anticipate the transition from infant mortality to useful life and postpone from useful life to wear-out. Indeed, if exist, the transitions can sometimes be predicted by plotting the accumulated number of failures by time intervals. Fig. 4 (a)–(e) respectively depicts the function for the five machines. The numbers on the abscissa axis correspond to the final instant of the interval. In machines two and three, the early failures (EF) phase has finished during the study. The time duration for the phase could be established in 1,000 hours circa for both machines. The first machine seems to persist in EF whereas the fourth and fifth do not yet enter the wear-out phase, which means that the useful life (UL) persists. In the fifth machine, despite the residual oscillation, the exponential model fits, indicating a constant failure rate.

Regarding the second and third machines, the maintenance policy may be adapted to encompass the transition from early failures to useful life. A corrective choice may apply before \( t = 1,000 \) hours when a combined TPM with predictive procedure should start. Under this combination, a time interval for inspection applies. For the second and third machines, the new failure rate and the time interval between inspections are, respectively, \( \lambda = 0.005 \) failure per hour and \( \Delta t = 8 \) hours (one inspection per shift), and \( \lambda = 0.000615 \) failure per hour and \( \Delta t = 64 \) hours (one inspection every eight shifts). The unexpected oscillation in the failure rate of the first and fifth machines is owing to inappropriate preventive, unconditional replacement of strong parts, currently performed by maintenance staff.

Finally, another method for revealing the incoming end of the useful life requires physical measurement, as vibration, oscillation in electric current, and metallic parts in

![Fig. 4 Accumulated number of failures per time intervals](image-url)
the oil, usually embedded in a widely predictive strategy. As belonging to the next stage of the decision-making process, details of specific measurements are outside the current scope.

Table 5 summarizes the relationship between the maintenance choices involved in this study and the failure rate behavior. The table synthesizes the main practical implications of the study, which may serve as a guideline to practitioners on maintenance management.

6 Conclusion

The purpose of this article was to choose a maintenance strategy for the critical equipment of a forging production line comprising five machines. Three out of the five machines were in the infant mortality period and two were in their useful lifetime. In the first case, the study indicates that a corrective maintenance strategy is appropriate; in the second, a predictive maintenance strategy is needed. The company currently adopts a preventive maintenance strategy combined with emergency maintenance, in which the crew immediately responds to unexpected failures, and periodically and unconditionally replaces critical parts and subsystems. The consequence of unconditionally replacing critical parts is the unexpected behavior of the failure rate function observed in two out of the five machines, the first and the fifth.

It is important to note that the first three machines were outdated models, with more than thirty years of use, which had been adapted for the current application. The three machines had not yet surpassed the infant mortality period, confirming the inadequacy of the current preventive maintenance strategy. Generally, every six months, the maintenance crew replaces unconditionally critical components, which prevents more robust parts from having a longer life by reaching wear-out. Their replacement time is underestimated, which explains the partial restarting of the early failures phase observed in Fig. 4(a) and 4(e). In short, four out of the five machines, indicate some form of anomaly in the failure rate function due to inadequacy in the maintenance strategy.

Similarly, in emergency maintenance, there is no study to identify if failed components are undersized or if there is another cause for failure. Identical parts replace faulty ones, not eliminating failure modes and preventing equipment evolution throughout the life cycle. After a breakdown, corrective maintenance must require an engineering study to certify if a design or assembly error or if another root cause exists. Confirmed the error, the component must be replaced by a stronger one or the assembly must be amended, which is not done in the current case. The same is true for the root cause of failure.

The last two machines are in the useful life period. Both are updated models based on advanced technology, and specifically designed for the service. The preventive maintenance strategy is also useless, as wear-out failures still do not exist. The consequences are a partial recurrence of early failures introduced by useless interventions and a constant failure rate still high for the fifth machine, more required than the fourth. Applying a TPM policy could lower the failure rate and reduce the expected

| Policies and choices | Decreasing | Failure rate | Increasing |
|----------------------|------------|--------------|------------|
| Emergency            | Necessary. Unless there is redundancy or sufficient inventory, whenever there is an unexpected failure, an emergency strategy is required for the immediate repair of critical equipment. | Very useful, as it finds and removes the root causes of early failures. | Useless, as early failures are no longer expected. |
| Corrective           | Useless, as the main issues are early failures, not wear-out failures. | Very useful, as it can alert on the beginning of the wear-out process | Useful, as it can help to define the time for exchanging critical parts. |
| Predictive           | Useless, as the main issues do not arise from shop floor inefficiency but design, installation, or manufacturing failures in parts | Very useful, as it prevents random failures and consequently reduces the constant failure rate by organizing the shop floor. | Useless, as the main issues are due to internal wear-out, not from the inefficiency of the shop floor. |
| Preventive           | Very harmful, as unconditional replacement focus on parts that did not fail, i.e., the strong parts | Harmful, as unconditional replacement focus on parts that did not enter the wear-out phase yet | Very useful, as it anticipates the replacement of parts that will fail, preventing unexpected stoppage and ensuring dependability. |

Table 5 Relationships between failure rate and maintenance choices in manufacturing.
number of breakdowns during orders. A predictive maintenance strategy should regularly assess the current operational condition, generating data that will be helpful to define the time to replace critical components. Currently, replacement occurs unconditionally every six months.

In short, selecting an appropriate maintenance strategy is a complex process that should rely on failure data to capture changes in machine operating conditions.

Further research should overcome current limitations. First, it would be important to employ advanced models, such as the q-Weibull, the Gumpel, or the extreme-value model. Although these would require more effort, they might provide different perspectives supporting the formulation of maintenance strategy. Second, the scarcity of data is a recurrent problem when dealing with life-long products or high MTBF equipment. Recent references (Zhang et al., 2019; Leoni et al., 2021, Yousif et al., 2020) calculate point and interval estimation for the Weibull model according to Bayesian, MLE, and LSE methods. Although the differences fall within a reasonable range for limited censored times, the first performs better, which suggests its use in further research. Finally, a safety analysis (FTA – failure tree analysis), as well as an FMEA (failure mode and effect analysis), should include the rest of the production process, such as raw material cutting, blasting, and the heat treatment equipment. Due to the relevance to the plant throughput, further research should also target a reliability analysis of the utility unit.

Acknowledgment
The authors would like to thank CNPq, the Brazilian research agency, for supporting this research (grant number 302570/2019-5).

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