Methods for the determination of effective management decisions in insufficient information conditions

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Abstract. The article analyzes the methods of removing uncertainty in conditions of insufficient information for making effective management decisions in complex organizational systems. In such tasks, the choice of an effective solution depends on the state of the external environment of the study and implies the use of a number of methods or mathematical models for solving multi-criteria problems. The main methods used in cases of occurrence of a situation of uncertainty due to insufficient information are the a priori ranking methods and multiple regression analysis. However, methods based on expert judgment have a significant degree of subjectivism, and methods formed on the experimental basis and mathematical planning of research are laborious and generate uncertainty with a large number of objects or elements involved in the situation. An alternative to the above is the method of zoning, which implies dividing the set of possible states of the research environment (nature) into subsets of the dominance of individual actions. The zoning operation is an inverse parametric linear programming problem. Consequently, for solving problems in conditions of insufficient information, one can use vector optimization methods.

1. Introduction

In principle, a decision-making situation in the context of insufficient information is a situation with varying degrees of uncertainty. The decision in this case has to be taken under risk. The authors consider the main decision-making methods in the context of insufficient information. In this case, one of the management tasks is to organize the goals or rank the goals of each level according to their importance. The most simple and operational method of ranking is the examination based on the identification and systematization of the opinion of qualified professionals, repeatedly considered in various scientific papers [1-3]. Another way to identify the significance of factors and assess their relationship in the context of insufficient information is the method of multiple regression analysis. Mathematical planning of an experiment allows studying the complex phenomena and processes with a significant number of factors in a limited period of time according to the results of a limited number of experiments.

The essence of the method of mathematical planning of an experiment is the procedure for choosing the number and conditions for conducting experiments that are necessary and sufficient to establish a reliable quantitative relationship between factors and optimization parameters. To find the optimum of
the optimization parameter, experiments are usually carried out in several stages, in small series, analyzing the results of the previous series when building a new series of experiments. It is necessary to repeat this procedure until the model becomes adequate to the process under study, that is, until it describes the change in the values of the optimization parameter with the required accuracy.

The main difficulty of applying the method of multiple regression analysis to the situation under study lies in the fact that a necessary experimental base must always be the foundation of mathematical planning - a set of observations, empirical data obtained in practical activities and directly related to the process or phenomenon under study. In this case, there appears an uncertainty generated by a large number of objects or elements included in the situation.

2. Problem Statement
The simplest and operational method of expert evaluation based on identifying and systematizing the opinions of qualified specialists is the a priori ranking of factors. An a priori analysis of the factors and the elimination of less significant ones from them can be done by analyzing the available data and logical conclusions based on it. However, this does not exclude the danger of subjective errors, which rises with increasing complexity and little knowledge of processes and phenomena. In order to avoid this, for a priori analysis it is advisable to involve several specialists and use certain variants of expert assessment methods. The essence of the method lies in ranking of factors according to a particular system, in compliance with the expected degree of influence on the optimization parameter, and excluding after that some of the factors with the worst ranks from the first series of experiments. First of all, when using this method, experts are selected and a questionnaire is developed in which questions for them are formulated. Specific literature provides recommendations on the selection of experiments and the preparation of questionnaires. The results of the questionnaire are subjected to mathematical processing. The main goal of such processing is to identify the factors that can be excluded from the subsequent experiment or to establish them during the experiment as controlled ones (that is, with fixed, constant values). In addition, the tasks of mathematical processing of materials of the questionnaires are:

1) assessment of the consistency of expert opinions;
2) assessment of the non-random nature of coincidence of opinions of specialists (significance of the coefficients of concordance);
3) construction of a graph of the ranks of factors.

In addition to the above mentioned aspects, other expert assessment methods are also used in practice: the method of sequential preferences and the method of pairwise comparisons, which are rather widely described in special literature [4,5]. These methods have both indisputable advantages and a number of disadvantages.

The advantages of using expert assessment methods are as follows:
1. Relative simplicity of organizing and conducting the procedure.
2. Efficiency of obtaining results.

The authors also define the disadvantages of using expert assessment methods:
1. A large dependence of the results on the quality of the organization of expertise and selection of experts, that is, a fairly high subjectivity.
2. When assessing certain factors (measures) for this system, experts use their previous experience or views (this is why the expertise is called an a priori one). Therefore, a correct formulation of questions and the choice of factors for this system are of particular importance and significantly affect the results of the examination [6-8].

It can be concluded that methods based on expert assessment represent a subjective apparatus for making subjective decisions.

3. Research Questions
The authors consider another method used to solve the problem posed - the method of zoning [9], focused on sequential clarification of information situations in the process of solving it. Situations in which all or nothing is known about the input parameters of the research environment are extreme. In practice, as a rule, intermediate situations arise in which one of the parameters is specified precisely, others are known in some ranges of possible changes, and there are certain statistics regarding knowledge of the third, etc. From the general theory of operations research, it is known that solutions are stable with respect to changes in the initial information [10-12]. This means that it is not necessary to know exactly all the input parameters of the problem being solved. In other words, it is wrong to proceed from complete ignorance of the situation, and there is no need to strive for the complete removal of uncertainty.

Purposeful clarification of only really necessary information is required, for which determination zoning methods can be used. The essence of the method of zoning is to divide the set of vectors characterizing the state of nature into subsets of dominance of individual actions. Such a partition can be done in various ways. One of such methods in the 60s of the last century was proposed by I Ya Diner. The idea of the method is to partition the set of possible states of nature into subsets of the dominance of individual actions. The development of this method was an essential step towards solving the problems of games with nature.

However, in a number of cases, the use of this method does not allow obtaining the desired optimal solution, although it significantly simplifies its search. Somewhat later, another method was suggested by B.D. Prudovskiy. In the works [13,14] it was proved that under the condition of continuity of the function of the efficiency index from changing the state vector of nature, the matrix game with nature can be reduced to a linear vector optimization problem and vice versa.

The achievements of these authors laid the foundation for the development of the zoning method in order to improve the efficiency of solving problems in the context of insufficient information.

4. Research Methods
In these situations, any problem of optimizing the decision being made is characterized by three main concepts:
- set of possible solutions (actions);
- many types of environment or state of the environment (SE);
- effectiveness of any decision for each type of situation.

Below we will use the following notation:
- \( m \) – number of possible options for an action;
- \( n \) – number of possible SEs;
- \( a_{ij} \) – efficiency of the \( i \)-th action for the \( j \)-th state of SE, \( i = 1, m, j = 1, n \).

The matrix of the efficiencies of various actions at different SEs is:

\[
\|a_{ij}\| = \begin{pmatrix}
  a_{11} & a_{12} & \ldots & a_{1n} \\
  a_{21} & a_{22} & \ldots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix}
\]

(1)

Let the state of the study environment be characterized by \( n = 3 \) states acting at a given time, that is, the efficiency of any solution is known for each type of environment and the efficiency matrix is given.

\[
A = \begin{pmatrix}
  0.20 & 0.24 & 0.22 \\
  0.76 & 0.18 & 0.34 \\
  0.18 & 0.80 & 0.26 \\
  0.84 & 0.02 & 0.46
\end{pmatrix}
\]

(2)

Consider the solution to problem (2) using the zoning method:

\[
\begin{align*}
  k_1 &= 0.20x_1 + 0.76x_2 + 0.18x_3 + 0.84x_4 \rightarrow \text{max}, \\
  k_2 &= 0.24x_1 + 0.18x_2 + 0.80x_3 + 0.02x_4 \rightarrow \text{max}, \\
  k_3 &= 0.22x_1 + 0.34x_2 + 0.26x_3 + 0.46x_4 \rightarrow \text{max},
\end{align*}
\]

(3)
\[ x_1 + x_2 + x_3 + x_4 = N \]
\[ x_i = \begin{cases} N, & i = j \\ 0, & i \neq j \end{cases} \]

If the authors consider this task as a game with nature, then in this case they have \( n = 3 \) states of nature and \( m = 4 \) options for possible actions. The efficiency matrix is as follows:

\[
\|a_{ij}\| = \begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{14} \\
  a_{21} & a_{22} & \cdots & a_{24} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{41} & a_{42} & \cdots & a_{44}
\end{pmatrix}, \quad i = 1, m \ , \ j = 1, n.
\]

If we use the well-known subjective criteria in this example [15], the authors obtain the following results:

1. Using the Laplace criterion in this problem leads to the choice of the fourth action \((x_4 = N)\).
2. The maximin Wald criterion recommends using the first \((x_3 = N)\) action.
3. The minimax Savage criterion shows preference for the second one \((x_2 = N)\).
4. The criterion of pessimism – optimism by Hurwitz with coefficient \( \alpha = 0.5 \) recommends the third \((x_3 = N)\) action.

The authors illustrate the idea of zoning in Figure 1 and the example given above (2). They denote the probability of occurrence of the \( j \) – th state of nature \( P_j, \ j = 1, 2, 3 \).

It appears natural that

\[
\sum_{j=1}^{3} P_j = 1, \ P_j \geq 0, \ j = 1, 2, 3.
\]

In this example \( n = 3 \), so this system takes the following form:

\[
P_1 + P_2 + P_3 = 1, \ P_1 \geq 0, P_2 \geq 0, P_3 \geq 0.
\]

5. Results of Research

The axes of coordinates here are the probabilities of the appearance of the first \( P_1 \) and the second \( P_2 \) states of nature. Any point of the triangle shown in the figure uniquely determines the probability of occurrence of each of the state of nature

\[
(P_3 = 1 - (P_1 + P_2)) \]

Figure 1. Graphic solution of the problem (2).
All possible solutions of the system (5) are in the triangle $P_1 O P_2$ and on its borders - the segments $P_1 O$, $O P_2$ and $P_1 P_2$. As it can be seen, the entire set of vectors characterizing the state of nature is divided into three subsets:

- in the first of which the third action dominates;
- in the second one dominates the second action;
- in the third - the fourth action.

- the first action is not optimal under any probability distribution, therefore, it can be excluded from consideration as noncompetitive (recall that it was recommended when using the maximin Wald criterion).

The plot of domination of the second action is very small. In addition, the values of the efficiency index achieved when using the second action in this area do not differ much from the efficiency obtained using the fourth action. This gives grounds to exclude the second action from the number of competitive ones, and to add a part of its domination to the area of domination of the fourth action. The field of vectors characterizing the state of nature is thus divided into only two subsets, in one of which the third action is optimal, and the fourth action is optimal in the other (recall that the second action we excluded was recommended as the optimal Savage criterion).

If now the polygonal chain separating these subsets is approximated by a straight line, then the border on which the efficiency of the third and fourth actions is the same is analytically written as

$$P_1 = 0.378 + 1.860 \cdot P_2.$$  

(7)

This means that when $P_1 > 0.378 + 1.860 \cdot P_2$, the fourth action should be applied, and when $P_1 < 0.378 + 1.860 \cdot P_2$, the third action is optimal.

If now the border between the areas of the third and fourth actions perpendicular to the $O P_2$ axis is drawn (make another approximation – the dashed line), then a very simple approximate rule is obtained: if $P_2 < \frac{1}{3}$, then it is necessary to use the fourth action, otherwise if $P_2 > \frac{1}{3}$, the third action is optimal.

The authors analyze the obtained solution of the example. Obviously, the most critical is the probability of the second state of nature $P_2$. The lack of information about the probability of the first and third states of nature is completely insignificant and does not affect the choice of the optimal solution. But if there is at least a strictly oriented idea of the probability $P_2$, then the choice of the solution can not be considered objectively justified. Therefore, it is possible to recommend the decision-maker, direct efforts to clarify the probability of the second state of nature $P_2$ and use the third action for $P_2 > \frac{1}{3}$ and apply the fourth action for $P_2 < \frac{1}{3}$.

6. Conclusion

The method of zoning consists in dividing the set of possible states of nature into subsets of domination of individual actions. The zoning operation is an inverse parametric linear programming problem. It can be shown that any matrix game with nature is reduced (assuming the continuity of the function of performance indicators from a change in the state vector of nature) to a linear vector optimization problem and vice versa. Consequently, to solve games with nature, one can use vector optimization methods, and multi-criteria problems in many cases can be solved using the apparatus of the theory of games with nature. In the transition from a multi-criteria problem to a "game with nature", the probabilities of the states of nature $p_j$ are adequate to the coefficients of the relative importance of the criteria $c_j$, i.e.

$$p_j \equiv c_j.$$  

(8)

Therefore, if $c_1 > 0.378 + 1.860 \cdot c_2$, then the solution of the example is

$$x_1 = \begin{cases} N, & i = 4, \\ O, & i = 1,2,3 \end{cases}.$$  

(9)

And if $c_1 < 0.378 + 1.860 \cdot c_2$, then

$$x_1 = \begin{cases} N, & i = 3, \\ O, & i = 1,2,4 \end{cases}.$$  

(10)
where $c_1$ and $c_2$ – coefficients reflecting in relative units the importance, respectively, of the first ($k_1$) and second ($k_2$) criteria in the task in question [16].

Thus, in order to increase the reliability of decisions made in case of a situation of uncertainty in the state of the external environment or the system functioning environment caused by a lack of information due to technical, social and other reasons, it is extremely important to choose the method of zoning.

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