The Matter Plus Black Hole Problem in Axisymmetry

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We present preliminary results in our long-term project of studying the evolution of matter in a dynamical spacetime. To achieve this, we have developed a new code to evolve axisymmetric initial data sets corresponding to a black hole surrounded by matter fields. The code is based on the coupling of two previously existing codes. The matter fields are evolved with a 2D shock-capturing method which uses the characteristic information of the GR hydro equations to build up a linearized Riemann solver. The spacetime is evolved with a 2D ADM code designed to evolve a wormhole in full general relativity. An example of the kind of problems we are currently investigating is the on axis collision of a star with a black hole.

1 Introduction

We report on progress in our project of coupling a general relativistic hydrodynamical code with a code to solve the Einstein field equations in axisymmetric spacetimes. This code will be an important tool to study the evolution of matter in a dynamical spacetime. One of our first applications will be the study of head-on star-black hole collisions. This is an important step toward the simulation of more complex scenarios, such as coalescing binaries, one of the most promising sources of gravitational radiation to be detected by LIGO and VIRGO interferometers.

2 The Hydro Code

The hydro part of the code makes use of modern high-resolution shock-capturing schemes in order to handle discontinuities in the solution. It has the capability of using different linearized Riemann solvers and different cell-reconstruction procedures to accurately solve the Riemann problem at every cell interface. Mathematically, this code relies on the knowledge of the characteristic fields of the Jacobian matrices of the system of equations of general relativistic hydrodynamics when written, explicitly, as a hyperbolic system of balance laws. Further details about the mathematical structure of the equations can be found in [2]. The hydro code has been extensively described in [3, 4].

3 The Black Hole Code

The black hole code is the same as the one developed at NCSA [3] and is based on the standard ADM formalism [1]. The metric is evolved for the full set of Einstein equations using a 3+1 explicit leap-frog scheme with centered differencing. The shift variables are used to eliminate some off-diagonal metric terms. Having a black hole built into the spacetime avoids possible coordinate problems at the origin of the spherical coordinate system, as there is an isometric surface at a finite radius.
This code is also capable of evolving spacetimes with angular momentum, and/or gravitational waves in combination with the matter fields. It also has a number of utilities built-in, e.g., routines to extract the quasi-normal gravitational radiation modes, and to track the motion of apparent and event horizons.

4 Coupling and Preliminary Results

The addition of matter to the initial value problem is a straightforward application of the York procedure. There are two basic configurations which we have considered thus far, a donut of matter distributed about the equator and a Gaussian ball of matter on the axis. This latter configuration represents a three body collision (since we have only used equatorial plane symmetry thus far), but we intend to generalize this in future work and evolve two body collisions.

The codes were coupled by allowing each to alternatively update the variables. First the hydro code takes a step, treating the spacetime metric as fixed, then the black hole code takes a step treating the matter fields as fixed. We have shown that our results compare well with an independently written 1D code that uses a hyperbolic formulation of the Einstein equations. The codes agree at early times, confirming the correctness and accuracy of both codes, but disagree slightly at late times.

Our 2D code can run either with matter fields turned off, or with a fixed general relativistic background, or with a dynamic evolving background which does not react to the presence of matter. The code can, therefore, reproduce results of tests that each independent code (the matter and metric codes) passed. In addition, we have successfully compared our numerically evolved spacetimes against analytic steady state accretion results. Specific details about the structure of the code and the equations being integrated will be reported elsewhere.

As an example of what sort of problems this code can be used to study, consider the collision of the black hole with two sharply peaked blobs of dust that fall onto the black hole along the axis. In Fig. 1 we see the collapse of the lapse. We show the lapse as a function of $\eta$, the logarithmic radial coordinate, along the axis at several different and progressively later times. Since the dust is already sharply peaked when the evolution begins, we expect a singularity to be in the near future of the evolution, and since the spacetime is maximally sliced ($K^a_a = 0$) the fact that the lapse is collapsing there confirms this.

In Fig. 2 we see the growth of the radial metric function with time, essentially due to the tidal forces of the black hole, leading to the phenomenon known as “grid stretching.” Again, the plot shows the value of the function along the axis through a sequence of time steps. In addition to the spike from the black hole, we see two additional spikes growing on either side of the dust ball as a result of the tidal forces it creates. This growth in the radial metric function normally proceeds without limit, and eventually crashes the evolution code. We would expect this problem to be exacerbated in the region surrounding the dust ball as the peaks are narrower and harder to resolve.

This problem can be cured, however, simply by making the dust ball more diffuse. In Fig. 3 we see the result of such an evolution which proceeds stably until
about 70M, at which time the bulk of the gravitational wave packet has propagated passed the detector. Also shown in the plot is a fit against the first two harmonics of the $\ell = 2$ gravitational wave mode. Clearly this agrees quite closely with the wave one expects to see for a black hole with a mass comparable to the dust ball plus black hole system.

5 Future directions

In the near future we plan to use the code for studying a number of astrophysical applications, as well as extending our set of code tests. We plan to compare the code with results from perturbation theory, to make an exhaustive study of the parameter space of the head-on collision of a star with a black hole (the star will be modeled by both dust and a polytrope), dust shells imploding onto a black hole, accretion disks, and a fully dynamical spacetime version of the relativistic Bondi-Hoyle accretion onto a moving black hole. Some of these projects are currently under way.

References

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