Vector Gauge Boson Dark Matter for the $SU(N)$ Gauge Group Model

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Abstract  The existence of dark matter is explained by a new neutral vector boson, $C$-boson, of mass (900 GeV), predicted by the Wu mechanisms for mass generation of gauge field. According to the Standard Model (SM) $W$, $Z$-bosons normally get their masses through coupling with the SM Higgs particle of mass 125 GeV. We compute the self-annihilation cross section of the vector gauge boson $C$-dark matter and calculate its relic abundance. We also study the constraints suggested by dark-matter direct-search experiments. The problem on the stability of $C$-particle is left as an open question for future research.

Keywords  Field theory · Dark matter · Gauge boson

1 Introduction

Dark matter was proposed in 1933 to explain why galaxies in some clusters move faster than the speed predicted if they contained only baryonic matter [1]. The nature of dark matter remains one of the open questions of modern physics. Several models of dark matter have been suggested, such as Light Supersymmetric Particles [2–9]; heavy fourth generation neutrinos [10, 11]; Q-Balls [12, 13]; mirror particles [14–18]; and axion particles, introduced in an attempt to solve the Charge-Parity (CP) violation problem in particle physics [19, 20].

Recently, the Brane World idea has been used to furnish new solutions to old problems in particle physics and cosmology [21–35]. Scenaria in which all fields are allowed to propagate in the bulk are called Universal Extra Dimensions (UED) models [36, 37]. UED models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP) [38, 39].

Nevertheless, it has been realized [40–42] that a conventional model [42, 43] based on superstring-inspired by the exceptional group $E_6$ has exactly the ingredients which allow it to become a model of vector-boson dark matter. In such a model, the vector boson itself ($X$) comes from an $SU(2)_W$ gauge extension of the Standard Model [41]. Constraints to such
a model posed by dark-matter direct-search experiments have been studied [41] and possible signals at the Large Hadron Collider (LHC) have been considered [44, 45].

The goal of the present paper is to investigate the possibility that a neutral vector $C$-boson of mass 900 GeV (as predicted by the Wu mechanisms for mass generation of gauge fields: [46–52]), proposed recently by the author [53], explains the existence of dark matter. According to the SM $W, Z$-bosons normally acquire their masses through their coupling with the SM Higgs boson of mass 125 GeV [54–58]. Here we compute the self-annihilation cross section of the vector gauge $C$-boson-dark matter, calculate $C$-boson’s relic abundance and study the modeling constraints generated by dark-matter direct-search experiments.

2 The Lagrangian of the Model

Let us suppose that the gauge symmetry of the theory is $SU(N) \times U(2)$ group, which is written specifically as follows [53]:

$$G = SU(N) \times U(2),$$

where $SU(N)$ is the special unitary group of $N$-dimensions, $\psi(x)$ is a $N$-component vector in the fundamental representative space of $SU(N)$ group, and $T_i$ ($i = 1, 2, \ldots, N^2 - 1$) denotes the representative matrices of the generators of $SU(N)$ group. The latter are Hermit and traceless. They satisfy the condition:

$$[T_i, T_j] = if_{ijk}T_k, \quad \text{Tr}(T_iT_j) = \delta_{ij}K,$$

where $f_{ijk}$ are structure constants of the $SU(N)$ group, and $K$ is a constant independent of the indices $i$ and $j$ but dependent on the representation of the group. The representative matrix of a general element of the $SU(N)$ group is expressed as:

$$U = e^{i\alpha iT_i},$$

with $\alpha^i$ being the real group parameters. In global gauge transformations, all $\alpha^i$ are independent of space-time coordinates, while in local gauge transformations $\alpha^i$ are functions of space-time coordinates. $U$ is a unitary $N \times N$ matrix.

In order to introduce the mass term of gauge fields without violating local gauge symmetry at energy scale close to 2 TeV, two kinds of gauge fields are required: $a_{\mu}$ and $b_{\mu}$ [46, 53].

In this version of the Wu gauge model, the gauge fields $a_{\mu}$ and $b_{\mu}$ are introduced. Gauge field $a_{\mu}$ is introduced to ensure the local gauge invariance of the theory. The generation of gauge field $b_{\mu}$ is a purely quantum phenomenon: $b_{\mu}$ is generated through non-smoothness of the scalar phase of the fundamental spinor fields [53, 59].

From the viewpoint of the gauge field $b_{\mu}$ generation described here, the gauge principle is an “automatic” consequence of the non-smoothness of the field trajectory in the Feynman path integral [53, 59].

$a_{\mu}(x)$ and $b_{\mu}(x)$ are vectors in the canonical representative space of $SU(N)$ group. They can be expressed as linear combinations of generators, as follows:

$$a_{\mu}(x) = a_{\mu}^i(x)T_i$$

$$b_{\mu}(x) = b_{\mu}^i(x)T_i,$$

where $a_{\mu}^i(x)$ and $b_{\mu}^i(x)$ are component fields of the gauge fields $a_{\mu}(x)$ and $b_{\mu}(x)$, respectively.
Corresponding to these two kinds of gauge fields, there are two kinds of gauge covariant derivatives:

\[ D_\mu = \partial_\mu - ig_2 a_\mu \]

\[ D_{b\mu} = \partial_\mu + icg_2 b_\mu. \]  

The strengths of gauge fields \( a_\mu(x) \) and \( b_\mu(x) \) are defined as

\[ a_{\mu\nu} = \frac{1}{i} [D_\mu, D_\nu] = \partial_\mu a_\nu - \partial_\nu a_\mu - ig_2 [a_\mu, a_\nu] \]  

\[ b_{\mu\nu} = \frac{1}{icg} [D_{b\mu}, D_{b\nu}] = \partial_\mu b_\nu - \partial_\nu b_\mu + icg_2 [b_\mu, b_\nu], \]

respectively.

Similarly, \( a_\mu(x) \) and \( b_\mu(x) \) can also be expressed as linear combinations of generators:

\[ a_{\mu\nu} = a^{i}_{\mu\nu} T_i \]

\[ b_{\mu\nu} = b^{j}_{\mu\nu} T_j. \]

Using relations (2) and (8), (9), we obtain

\[ a^{i}_{\mu\nu} = \partial_\mu a^{i}_\nu - \partial_\nu a^{i}_\mu + g_2 f^{ijk} a^{j}_\mu a^{k}_\nu \]

\[ b^{j}_{\mu\nu} = \partial_\mu b^{j}_\nu - \partial_\nu b^{j}_\mu - cg_2 f^{ijk} b^{j}_\mu b^{k}_\nu. \]

The Lagrangian density of the model is

\[ \mathcal{L}_{Wu} = -\bar{\psi} (\gamma^\mu D_\mu + m) \psi - \frac{1}{4K} \text{Tr}(a^{\mu\nu} a_{\mu\nu}) - \frac{1}{4K} \text{Tr}(b^{\mu\nu} b_{\mu\nu}) - \frac{\mu^2}{2(1 + c^2)} \text{Tr}[(a^\mu + cb^\mu)(a_\mu + cb_\mu)] \]  

where \( c \) is a constant.

The space-time metric is selected as \( \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \), \((\mu, \nu = 0, 1, 2, 3)\). According to relation (2), Lagrangian density \( L \) can be rewritten as:

\[ \mathcal{L}_{Wu} = -\bar{\psi} \left( \gamma^\mu (D_\mu + m) \right) \psi - \frac{1}{4K} \text{Tr}(a^{\mu\nu} a_{\mu\nu}) - \frac{1}{4K} \text{Tr}(b^{\mu\nu} b_{\mu\nu}) - \frac{\mu^2}{2(1 + c^2)} (a^\mu + cb^\mu)(a_\mu + cb_\mu) \]

This Lagrangian has strict local gauge symmetry \[46, 53\].

In Eq. (1), the gauge group \( U(2) = SU_L(2) \times U(1)_Y \) is the known SM of electroweak (EW) interactions \[54–57\]. The generators of \( SU_L(2) \) correspond to the three components of weak isospin \( T_a \) \((a = 1, 2, 3)\). The \( U(1)_Y \) generator corresponds to the weak hypercharge \( Y \). These are related to the electric charge by \( Q = T_3^2 + Y \).

The \( SU_L(2) \times U(1)_Y \) invariant Lagrangian is given as follows:

\[ \mathcal{L}_{EW} = \bar{\psi} \gamma^\mu \left( \partial_\mu - ig_1 \frac{1}{2} \tau \cdot A_\mu + ig_\prime \frac{1}{2} B_\mu \right) \psi + \bar{e}_R \gamma^\mu \left( \partial_\mu + i g_1 B_\mu \right) e_R - \frac{1}{4} F^\alpha_{\mu\nu} F^\alpha_{\mu\nu} - \frac{1}{4} B^\alpha_{\mu\nu} B^\alpha_{\mu\nu}, \]

with field strength tensors:
\[ F_{\mu \nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_1 \epsilon^{\alpha \beta \gamma} A^a_\mu A^\alpha_\nu \]  
(17)

\[ B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]  
(18)

for the three non-Abelian fields of \( SU(2)_L \) and the single Abelian gauge field associated with \( U(1)_Y \), respectively.

The covariant derivative is:

\[ D_\mu = \partial_\mu - \frac{1}{2} i g_1 A^a_\mu T_a + \frac{1}{2} i g'_1 B_\mu, \]  
(19)

with \( g_1, g'_1 \) being the \( SU(2)_L \), and \( U(1)_Y \) the coupling strength, respectively.

The Lagrangian (16) is invariant under the infinitesimal local gauge transformations for \( SU(2)_L \) and \( U(1)_Y \) independently. Being in the adjoined representation, the \( SU(2)_L \) massless gauge fields form a weak isospin triplet, with the charged fields being defined by

\[ W^\pm_\mu = (A^1_\mu \mp i A^2_\mu)/\sqrt{2}. \]  
(20)

The neutral component of \( A^3_\mu \) mixes with the Abelian gauge field \( B_\mu \) to form the physical states:

\[ Z_\mu = A^3_\mu \cos \theta_w - B_\mu \sin \theta_w, \]  
(21)

\[ A^\mu = B_\mu \cos \theta_w + A^3_\mu \sin \theta_w, \]  
(22)

where \( \tan \theta_w = g'_1/g_1 \) is the weak mixing angle.

Based on the gauge group \( SU(N) \times U(2) \), the final Lagrangian of the model is given as follows [53]:

\[ \mathcal{L}_{\text{Model}} = \bar{\psi} \gamma^\mu \left[ \partial_\mu - i g_1 A^a_\mu \tau^a + i g'_1 B_\mu \right] \psi + \bar{e}_R \gamma^\mu \left[ \partial_\mu + i g'_1 B_\mu \right] e_R - \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \frac{1}{4} a_{\mu \nu} a^{\mu \nu} - \frac{1}{4} b_{\mu \nu} b^{\mu \nu} - \frac{\mu^2}{2(1 + c^2)} (a^{\mu \nu} + c b^{\mu \nu}) (a_{\mu \nu} + c b_{\mu \nu}) \]  
(23)

where \( c \) is a constant.

### 3 The Masses of Gauge Fields

Two obvious characteristics of the Wu Lagrangian equation (15) is that the mass term of the gauge fields is introduced into the Lagrangian, and that this term does not affect the symmetry of the Lagrangian. It has been proved that this Lagrangian has strict local gauge symmetry [46]. Since both vector fields \( a_\mu \) and \( b_\mu \) are standard gauge fields, this model is a gauge field model which describes gauge interactions between gauge fields and matter fields [46].

The mass term of gauge fields can be written as follows [46]:

\[ \frac{1}{2} (a^\mu, b^\mu) M \left( \begin{array}{c} a^\mu \\ b^\mu \end{array} \right), \]  
(24)

where \( M \) is the mass matrix

\[ M = \frac{1}{1 + c^2} \left( \begin{array}{cc} \mu^2 & c \mu^2 \\ c \mu^2 & c^2 \mu^2 \end{array} \right). \]  
(25)
Physical particles generated from gauge interactions are eigenvectors of mass matrix, and the corresponding masses of these particles are eigenvalues of mass matrix. The mass matrix $M$ has two eigenvalues:

$$m_1^2 = \mu^2, \quad m_2^2 = 0.$$  \hspace{1cm} (26)

The corresponding eigenvectors are:

$$
\begin{pmatrix}
\cos \theta_{wu} \\
\sin \theta_{wu}
\end{pmatrix}
\quad \begin{pmatrix}
-\sin \theta_{wu} \\
\cos \theta_{wu}
\end{pmatrix}
\hspace{1cm} (27)
$$

where

$$
\cos \theta_{wu} = \frac{1}{\sqrt{1 + c^2}}, \quad \sin \theta_{wu} = \frac{c}{\sqrt{1 + c^2}}.
\hspace{1cm} (28)
$$

We define

$$
C_{\mu} = \cos \theta_{wu} a_{\mu} + \sin \theta_{wu} b_{\mu},
$$

$$
F_{\mu} = -\sin \theta_{wu} a_{\mu} + \cos \theta_{wu} b_{\mu}.
\hspace{1cm} (29)
$$

$C_{\mu}$ and $F_{\mu}$ are eigenstates of mass matrix: they describe the particles generated from gauge interactions. The inverse transformations of (29) are:

$$
a_{\mu} = \cos \theta_{wu} C_{\mu} - \sin \theta_{wu} F_{\mu},
$$

$$
b_{\mu} = \sin \theta_{wu} C_{\mu} + \cos \theta_{wu} F_{\mu}.
\hspace{1cm} (30)
$$

Taking Eqs. (29) and (30) into account, the Wu Lagrangian density $L$ given by (15) changes into:

$$
S_{Wu} = S^{(0)}_{Wu} + S^{(I)}_{Wu},
\hspace{1cm} (31)
$$

where

$$
S^{(0)}_{Wu} = -\overline{\psi} \left( \gamma^{\mu} \partial_{\mu} + m \right) \psi - \frac{1}{4} C_{0}^{\mu \nu} C_{0}^{\nu \mu} - \frac{1}{4} K_{0}^{\mu \nu} K_{0}^{\nu \mu} - \frac{\mu^2}{2} C_{i}^{\mu} C_{i}^{\mu},
\hspace{1cm} (32)
$$

$$
S^{(I)}_{Wu} = ig_2 \overline{\psi} \gamma^{\mu} \left( \cos \theta_{wu} C_{\mu} - \sin \theta_{wu} F_{\mu} \right) \psi - \frac{\cos 2\theta_{wu}}{2} g_2 f^{ijk} C_{0}^{j \mu \nu} C_{i}^{\mu} C_{i}^{\nu} + g_2 \sin \theta_{wu} f^{ijk} C_{i}^{j \mu \nu} C_{i}^{\mu} F_{k}^{\nu}
+ \frac{1}{4} \sin^2 2\theta_{wu} g_2^2 f^{ijk} f^{ilm} C_{i}^{j \mu} C_{k}^{l \nu} F_{m}^{\nu} + \frac{\sin^2 \theta_{wu}}{2} g_2^2 f^{ijk} f^{ilm} F_{\mu}^{j} F_{k}^{l} F_{m}^{\nu}
+ \frac{1}{4} \cos^2 2\theta_{wu} g_2^2 f^{ijk} f^{ilm} C_{i}^{j \mu} C_{k}^{l \nu} F_{m}^{\nu} - \frac{\sin^2 \theta_{wu}}{2} g_2^2 f^{ijk} f^{ilm} C_{i}^{j \mu} C_{k}^{l \nu} F_{m}^{\nu}.
\hspace{1cm} (33)
$$

In the above relations, we have used the following simplified notations:

$$
C_{0}^{\mu \nu} = \partial_{\mu} C_{\nu}^{i} - \partial_{\nu} C_{\mu}^{i},
$$

$$
K_{0}^{\mu \nu} = \partial_{\mu} F_{\nu}^{i} - \partial_{\nu} F_{\mu}^{i}.
\hspace{1cm} (34)
$$

From Eq. (32) it is deduced that the mass of field $C_{\mu}$ is $\mu$ and the mass of gauge field $F_{\mu}$ is zero. That is:

$$
M_C = \mu, \quad M_F = 0.
\hspace{1cm} (36)
$$
Transformations (29) and (30) are pure algebraic operations which do not affect the gauge symmetry of the Lagrangian [46]. They can, therefore, be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In other words, the symmetry of the Lagrangian before transformations is absolutely the same with the symmetry of the Lagrangian after transformations. We do not introduce any kind of symmetry breaking at energy scales close to 2 TeV [53].

Fields $C_\mu$ and $F_\mu$ are linear combinations of gauge fields $a_\mu$ and $b_\mu$. The forms of local gauge transformations of fields $C_\mu$ and $F_\mu$ are, therefore, determined by the forms of local gauge transformations of gauge fields $a_\mu$ and $b_\mu$. Since $C_\mu$ and $F_\mu$ consist of gauge fields $a_\mu$ and $b_\mu$ and transmit gauge interactions between matter fields, for the sake of simplicity we also call them gauge fields, just as $W$ and $Z$ are called gauge fields in the electroweak model [54–57]. This gauge field theory, therefore, predicts the existence of two different kinds of force transmitting vector fields: a massive, and a massless one.

Taking the Higgs mechanism [54–57] into account, in the vacuum energy scale of 179 GeV $W^\pm$ and $Z^0$ become massive, while the photon $A$ remains massless. The symmetry $SU(N)$ does not break down, since the gauge bosons $C^0$ and $F$ derive their masses by the Wu mechanisms [46–53] in the vacuum energy scale of 2 TeV.

The most general Lagrangian consistent with $SU(2) \times U(1)$ gauge invariance, Lorentz invariance, and renormalizability is [60]:

$$\mathcal{L}_\phi = -\frac{1}{2} \left| \left( \partial_\mu - i A_\mu \cdot t^{(\phi)} - i B_\mu y^{(\phi)} \right) \phi \right|^2 - \frac{\mu^2}{2} \phi^* \phi - \frac{\lambda}{4} (\phi^* \phi)^2, \quad (37)$$

where

$$t^{(\phi)} = \frac{g_1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad y^{(\phi)} = -\frac{g'_1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (38)$$

are the generators of the $(\phi^+, \phi^0)$, $\lambda > 0$, and

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (39)$$

For $\mu^2 < 0$, there is a tree-approximation vacuum expectation value at the stationary point of the Lagrangian

$$\langle \phi^* \phi \rangle = \nu^2 = |\mu^2|/\lambda. \quad (40)$$

We can always perform a $SU(2) \times U(1)$ gauge transformation to a unitary gauge, in which $\phi^+ = 0$ and $\phi^0$ is Hermitian, with positive vacuum expectation value. In unitarily gauge, the vacuum expectation values of the components of $\phi$ are:

$$\langle \phi^+ \rangle = 0, \quad \langle \phi^0 \rangle = \nu > 0. \quad (41)$$

The scalar Lagrangian (37) then yields a vector boson mass term:

$$-\frac{1}{2} \left| \left( \partial_\mu - i A_\mu \cdot t^{(\phi)} - i B_\mu y^{(\phi)} \right) \phi \right|^2 = -\frac{1}{2} \left| \begin{pmatrix} g_1 \frac{1}{2} A_\mu T^a - \frac{g'_1}{2} B_\mu \\ 0 \end{pmatrix} \nu \right|^2 = -\nu^2 g^2_1 W^* W^\mu - \frac{\nu^2}{8} (g^2_1 + g'^2_1) Z^*_\mu Z^\mu. \quad (42)$$

The masses are given as follows:

$$M_A = 0, \quad M_W = \frac{1}{2} g_1 \nu, \quad M_Z = \frac{1}{2} \sqrt{g^2_1 + g'^2_1} \nu. \quad (43)$$

The gauge fields and masses predicted by this model are summarized in Table 1.
Table 1  Gauge fields and masses predicted by the proposed model [53]

| Gauge fields | Masses (M)-GeV | Symmetry pattern |
|--------------|----------------|------------------|
| C (New)      | 900            | Without symmetry breaking |
| F (New)      | 0              | Without symmetry breaking |
| W            | 81             | With symmetry breaking |
| Z            | 91             | With symmetry breaking |
| A            | 0              | With symmetry breaking |

4 Renormalization of the Model

In this paper we use two mechanisms that can make gauge field to gain nonzero mass. One is the Wu mechanism [46–53], by which the mass term of the gauge field is introduced by using another set of gauge fields. In this mechanism, the mass term of the gauge field does not affect the symmetry of the Lagrangian. We can imagine the new interaction picture: when matter fields take part in gauge interactions, they emit or absorb one kind of gauge field which is not eigenstate of mass matrix. Perhaps this is the gauge field consisted with the existence of dark matter in our Universe. This gauge field would appear in two states, a massless and a massive one, which correspond to two kinds of vector fields.

The second mechanism of mass generation is the Higgs mechanism [54–58], which can make gauge field—W, Z to gain nonzero mass and to guarantee renormalizability by means of the interactions of the Higgs boson (h) with gauge bosons W, Z, C [53]:

\[
\frac{h^2 + 2\nu h}{v^2} (Z_\mu Z^\mu M_Z^2 + C_\mu C^\mu M_C^2 + W^\mu_+ W^{\mu-} M_W^2)
\]

\[
= \left( \frac{M_Z^2}{v^2} \right) h^2 Z_\mu Z^\mu + \left( \frac{M_C^2}{v^2} \right) h^2 C_\mu C^\mu
\]

\[
+ \left( \frac{M_W^2}{v^2} \right) h^2 W^\mu_+ W^{\mu-} + 2 \left( \frac{M_Z^2}{v} \right) h Z_\mu Z^\mu + 2 \left( \frac{M_C^2}{v} \right) h C_\mu C^\mu + 2 \left( \frac{M_W^2}{v} \right) h W^\mu_+ W^{\mu-}
\]

(44)

where

\[
M_Z^2/v^2, \quad M_C^2/v^2, \quad M_W^2/v^2, \quad 2M_Z^2/v, \quad 2M_C^2/v, \quad 2M_W^2/v
\]

are the dimensionless and dimension of mass coupling constants.

For instance, the C-boson readily derives its mass through the Wu mechanism; yet renormalizability is ensured via the Higgs mechanism [53].

However, as we have stated above, the Wu gauge field theory has maximum local $SU(N)$ gauge symmetry [46, 53]. When we quantise the Wu gauge field theory in the path integral formulation, we have to select gauge conditions first [46, 53]. To fix the degree of freedom of the gauge transformation, we must select two gauge conditions simultaneously: one for the massive gauge field $C_\mu$, and another for the massless gauge field $F_\mu$. For instance, if we select temporal gauge condition for massless gauge field $F_\mu$,

\[
F_4 = 0,
\]

(46)

there still exists a remainder gauge transformation degree of freedom, because the temporal gauge condition is unchanged under the following local gauge transformation:

\[
F_4 \rightarrow U F_\mu U^\dagger + (1/ig_2 \sin \theta) U \partial_\mu U^\dagger,
\]

(47)
where

\[ \partial_t U = 0, \quad U = U(\tilde{x}). \]  

(48)

To render this remainder gauge transformation degree of freedom completely fixed, we have to select another gauge condition for gauge field \( C_\mu \), for instance:

\[ \partial_\mu C_\mu = 0. \]  

(49)

If we select two gauge conditions simultaneously, when we quantise the theory in path integral formulation, there will be two gauge fixing terms in the effective Lagrangian. The effective Lagrangian can then be written as:

\[ \mathcal{I}_{\text{eff}} = \mathcal{I} - \frac{1}{2} \alpha_1 f_1 a f_1 a - \frac{1}{2} \alpha_2 f_2 a f_2 a + \bar{\eta}_1 M_{f_1} \eta_1 + \bar{\eta}_2 M_{f_2} \eta_2, \]  

(50)

where

\[ f_1 a = f_1 a (F_\mu), \quad f_2 a = f_2 a (C_\mu). \]  

(51)

If we select

\[ f_2 a = \partial_\mu C_\mu, \]  

(52)

then the propagator for massive gauge field \( C_\mu \) is:

\[ \Delta_{C_\mu \nu}^{ab}(k) = -i \delta^{ab}/(k^2 + \mu^2 - i \varepsilon) \times [g_{\mu \nu} - (1 - \alpha_2)k_\mu k_\nu/(k^2 + \alpha_2 \mu^2)]. \]  

(53)

If we let \( k \) approach infinity, then

\[ \Delta_{C_\mu \nu}^{ab}(k) = \frac{1}{k^2}. \]  

(54)

In this case, according to the power-counting law, the Wu gauge field theory suggested in this paper is a renormalizable theory [46, 53].

5 Annihilation Cross Section of Vector Gauge Boson \( C \)-Dark Matter

The annihilation modes of \( C \)-dark matter particles differ from that of other dark matter candidates such as neutralinos in supersymmetric models [2–9].

Neutralino annihilations to fermions are chirality-suppressed by a factor of \( m_f^2 / m_\chi^2 \), and thus do not produce electrons-positrons \( e^+ e^- \) pairs directly [37]. By contrast, \( C \)-dark matter, being a boson, is not similarly suppressed and can annihilate directly to leptons \( e^+ e^- \), \( \mu^+ \mu^- \) and \( \tau^+ \tau^- \) pairs. Each of these particles yield a large number of high energy electrons and positrons.

Other dominant modes include annihilations to up-type quarks, \( (u) \), \( (c) \) and \( (t) \), neutrinos \( (\nu_e) \), \( (\nu_\mu) \), \( (\nu_\tau) \), Higgs bosons \( (h) \) and down-type quarks, \( (d) \), \( (s) \), \( (b) \). Following, Bhattacharya et al. [42], the total averaged annihilation cross section times the relative velocity of \( C \)-dark matter is given by

\[ \langle \sigma v_{\text{rel}} \rangle = \frac{41 g_c^4}{576 \pi m_c^2} \approx \frac{0.77 \text{ pb}}{(m_c/\text{TeV})^2} = 0.86 \text{ pb}, \]  

(55)

where \( g_c^2 = g_2^2 \approx 0.28 \), for weakly interacting \( C \)-dark matter particle and \( m_c = 900 \text{ GeV} \).
Let us now review the standard calculation of the relic abundance of a particle species, denoted $C$-particle, which was at thermal equilibrium in the early universe and decoupled when it was non-relativistic [61, 62]. The evolution of its number density $n$ in an expanding universe is governed by the Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma u_{rel} \rangle (n^2 - n_{eq}^2), \quad (56)$$

where $\langle \sigma u_{rel} \rangle$ is the total annihilation cross section multiplied by velocity. Brackets denote thermal average, $H = (8\pi \rho / 3M_{pl})^{1/2}$ is Hubble constant, $M_{pl} = 10^{19}$ GeV is the Planck mass, and $n_{eq}$ is the number density at thermal equilibrium. For $C$-particle, in the non-relativistic limit, and in the Maxwell Boltzmann approximation, it is

$$n_{eq} = g \left( \frac{m_C T}{2\pi} \right)^3 / 2 e^{-m_C / T}, \quad (57)$$

where $m_C$ is the $C$-particle mass and $T$ is temperature.

Next we introduce the variables

$$Y = \frac{n}{s}, \quad Y_{eq} = \frac{n_{eq}}{s}, \quad (58)$$

where $s$ is the entropy density $s = 2\pi^2 g_* T^3 / 45$ and $g_*$ counts the number of relativistic degrees of freedom. Using the conservation of entropy per co-moving volume ($sa^3$ = constant), it follows that $\dot{n} + 3Hn = s \dot{Y}$. Equation (56), therefore, reads:

$$s \dot{Y} = -\langle \sigma u \rangle s^2 (Y^2 - (Y_{eq})^2). \quad (59)$$

If we further introduce the variable $x = m / T$, Eq. (59) can be expressed as

$$\frac{dY}{dx} = -\frac{\langle \sigma u \rangle s}{Hx} (Y^2 - (Y_{eq})^2). \quad (60)$$

For heavy states, we can approximate $\langle \sigma u_{rel} \rangle$ with the non-relativistic expansion in powers of $u_{rel}^2$:

$$\langle \sigma u_{rel} \rangle = a + b[u^2] + O([u^4]) = a + \frac{b}{x}, \quad (61)$$

which leads to our final version of Eq. (60) in terms of the variable $\Delta = Y - Y_{eq}$:

$$\Delta' = -Y_{eq} - f(x) \Delta (2Y_{eq} + \Delta), \quad (62)$$

where prime denotes $d/dx$ and

$$f(x) = \sqrt{\frac{\pi g_*}{45}} M_{pl}(a + 6b / x)x^{-2}. \quad (63)$$

Following [61], we introduce the quantity $x_F = m / T_F$, where $T_F$ is the freeze-out temperature of the relic particle. We notice that Eq. (62) can be solved analytically in the two extreme regions $x \ll x_F$ and $x \gg x_F$:

$$\Delta = -\frac{Y_{eq}'}{2f(x)Y_{eq}} \quad \text{for} \quad x \ll x_F \quad (64)$$

$$\Delta' = -f(x)\Delta^2 \quad \text{for} \quad x \gg x_F. \quad (65)$$
These regions correspond to conditions long before freeze-out and long after freeze-out, respectively. Integrating the last equation between \( x_F \) and \( \infty \) and using \( \Delta x_F \gg \Delta \infty \), we can derive the value of \( \Delta \infty \) and arrive at

\[
Y^{-1}_\infty = \sqrt{\frac{\pi g_*}{45}} M_{pl} m x_F^{-1} (a + 3b/x_F). \tag{66}
\]

The present density of a generic relic, \( C \), is simply given by

\[
\rho_c = m_{c} n_{c} = m_{c} s_{0} Y_{\infty}, \tag{68}
\]

where \( s_{0} = 2889.2 \text{ cm}^{-3} \) is the present entropy density. The relic density can finally be expressed in terms of the critical density (\( \Omega_c = \rho_c/\rho_{\text{crit}} \)):

\[
\Omega_{c} h^2 \approx 1.07 \times 10^{9} \text{ GeV}^{-1} \frac{x_F}{M_{pl} \sqrt{g_{*} a + 3b/x_F}}, \tag{67}
\]

where \( a \) and \( b \) are expressed in \( \text{GeV}^{-2} \), and \( g_{*} \) is evaluated at the freeze-out temperature. It is conventional to express the relic density in terms of the Hubble parameter: \( h = H_{0}/100 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

To estimate the relic density, one is thus left with the calculation of the annihilation cross sections (in all of the possible channels) and the extraction of the parameters \( a \) and \( b \), which depend on the particle mass. The freeze-out temperature \( x_F \) can be estimated through the iterative solution of the equation

\[
x_F = \frac{m}{T} \ln \left[ c(c + 2) \sqrt{\frac{45}{8}} \frac{g_{*} M_{pl}(a + 6b/x_F)}{2\pi^3} \right], \tag{68}
\]

where \( c \) is a constant of order one, determined by matching the late-time and early-time solutions.

It is sometimes useful to perform an order-of-magnitude estimate using an approximate version of Eq. (67) [63]. For vector gauge boson \( C \) mass of 900 GeV, predicted by the Wu mechanisms for mass generation of gauge field, the relic density should be

\[
\Omega_c h^2 \approx 3 \times 10^{-27} \text{ cm}^{3} \text{ s}^{-1} \frac{\langle \sigma_{u_{rel}} \rangle}{\langle \sigma_{u_{rel}} \rangle} \approx 0.1 \text{ pb} = 0.11, \tag{69}
\]

where \( \langle \sigma_{u_{rel}} \rangle \) is calculated by Eq. (55).

Analysis of the three-year Wilkinson Microwave Anisotropy Probe (WMAP) data tells us that the density of dark matter is \( \Omega_{dm} h^2 = 0.102 \pm 0.009 \), where \( \Omega_{dm} \) is \( \rho_{dm}/\rho_{\text{crit}} \), \( \rho_{\text{crit}} \) is the density corresponding to a flat universe [64] and \( h \) is the Hubble constant in units of \( 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) [65]. A cold dark matter candidate produced at the Large Hadron Collider (LHC) should, therefore, have this annihilation cross section.

This quantity leads us to the second method of measuring the coupling of dark matter from Standard Model particles: through the search for the annihilation or decay products of dark matter coming from high-density regions of the Universe, such as the centre of galaxies [66]. Since WMAP results provide good information about \( \langle \sigma_{u_{rel}} \rangle \), uncertainties in this approach stem from our sketchy knowledge of the exact density of dark matter in the centre of galaxies and the difficulty in separating the signal from dark-matter annihilation from possible background signals.

In the SM there already exists an example of a particle which is accidentally stable: the proton [67]. There is, therefore, no reason why the \( C \)-particle could not also be stable [67–69].

It thus follows that no dimension five operator between the dark matter vector field candidate and SM fields is allowed by the SM (and hidden sector) gauge symmetries [67–70]. If, on the contrary, they allow dimension 6 operators, it turns out that the lifetime of the
vector dark matter candidate is of the order $10^{26}$ sec if $10^{14}$ GeV [68], which is close to the GUT scale. In other words, an accidentally stable C-dark matter candidate which can be destabilized by a dimension six GUT scale induced interaction results in a flux of cosmic rays of the order of the observed order; therefore, potentially to a rich phenomenology [67].

If the C-particle is accidentally stable, and since it also interacts weakly with baryonic matter, it can be a good Weakly Interacting Massive Particles (WIMP) candidate. We do not discuss the stability of C-particle further in this article; we leave this intriguing topic as an open question for future research.

In any case, however, the C-particle of mass around 900 GeV predicted by the C-dark matter model, also provides the correct abundance of dark matter in the universe. This encouraging theoretical suggestion is testable through LHC studies.

7 Direct Search for C-Dark Matter

Following, J. Hisano et al. [71], the elastic cross section is calculated in terms of effective coupling constants, which are given by coefficients of effective interactions of vector dark matter with light quarks ($q = u, d, s$) and gluons. Since the scattering process is non relativistic, all terms that depend on the velocities of C-dark matter particles and nuclei are subdominant in the velocity expansion. In this study, therefore, we neglect the operators suppressed by the velocities of the C-dark matter particles or nuclei. Since $\partial^\mu C_\mu = 0$, physical degrees of freedom of the C-dark matter particle (assumed to be a real vector field) are then restricted to their spatial components. As a consequence, in the expansion of a strong coupling constant and in the non relativistic limit of the scattering process, the leading interaction of the vector field with quarks and gluons is given as

$$\mathcal{I}^{\text{eff}} = \sum_{q=u,d,s} \mathcal{I}^{\text{eff}}_q + \mathcal{I}^{\text{eff}}_G \ [71],$$

where

$$\mathcal{I}^{\text{eff}}_q = f_q m_q C^\mu C_\mu \bar{q} q + f_q^{i\Phi} C^\mu C_\mu \bar{q} i \Phi q + \frac{d_q}{m_\epsilon} \epsilon_{\mu\nu\rho\sigma} C^\mu i \partial^\nu C^\rho \bar{q} \gamma^\sigma \gamma^5 q$$

and

$$\mathcal{I}^{\text{eff}}_G = f_G C^\rho C_\rho G^{\mu\nu} G_\mu^a G_\nu^a + \frac{g_G}{m_\epsilon} \epsilon_{\mu\nu\rho\sigma} C^\rho i \partial^\nu C_\rho O^{\mu\nu}_{q},$$

where $m_q$ is the mass of a quark, $m_\epsilon$ the mass of a C-particle and $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor defined as $\epsilon^{0123} = +1$. $f_q$, $d_q$ and $g_q$ are the scalars-, axial vector-, and twist 2-type couplings of C-dark matter particle with quarks, respectively.

Here, the covariant derivative is defined as $D_\mu = \partial_\mu + ig A_\mu^a T_a$, with $g_s$, $T_a$, and $A_\mu^a$ being the $SU(3)_C$ coupling constant, generator, and gluon field, respectively. $O^{q}_{\mu\nu}$ and $O^{g}_{\mu\nu}$ are the twist-2 operators (traceless parts of the energy-momentum tensor) for quarks and gluons, respectively. These are given by:

$$O^{q}_{\mu\nu} = \frac{1}{2} \bar{q} i \left( D_\mu \gamma_\nu + D_\nu \gamma_\mu - \frac{1}{2} g_{\mu\nu} \Phi \right) q,$$

$$O^{g}_{\mu\nu} = \left( G^{\rho\sigma}_{\mu} G^\rho_{\nu} + \frac{1}{4} g_{\mu\nu} G^{a\rho}_{\sigma} G^{a\rho}_{\mu} \right).$$

$G_{\mu\nu}^a$ is the field strength tensor of a gluon.
To derive the effective Lagrangian, we use the equations of motion for vector field, i.e., 
\((\partial^2 - m^2)c_\mu = 0\). The third term in the right-hand side of Eq. (71) contributes to the spin-dependent interaction in the scattering of dark matter with nuclei, whereas the other terms in the effective Lagrangian contribute to the spin-independent interaction. The scattering amplitude is given by the matrix element of the effective interaction between initial and final states. In this evaluation, we use the equation of motion for light quarks.

To prove the validity of the application we evaluate the operator using on-shell hadron states \([72]\). In this case, denoting \(|N\rangle (N = p, n)\) as the on-shell nucleon state, we can evaluate the second term in Eq. (71) as
\[
\langle N|\bar{q}i\gamma^\mu q|N\rangle = \langle N|m_q\bar{q}q|N\rangle.
\] (75)

Now, let us first examine the spin-independent scattering. As in the scattering process momentum transfer is negligible; the matrix elements between initial and final nucleon states with the mass \(m_N\) \((N = p, n)\) are obtained by:
\[
\langle N(p)\epsilon |O_{\mu\nu}^q|N(p)\epsilon \rangle = \frac{1}{m_N}(p_\mu p_\nu - \frac{1}{4}m_N^2g_{\mu\nu})\left(q(2) + \bar{q}(2)\right),
\]
\[
\langle N(p)\epsilon |O_{\mu\nu}^g|N(p)\epsilon \rangle = \frac{1}{m_N}(p_\mu p_\nu - \frac{1}{4}m_N^2g_{\mu\nu})G(2).
\] (76)

In the matrix elements of twist-2 operators, \(q(2), \bar{q}(2),\) and \(G(2)\) are the second moments of the Parton Distribution Functions (PDFs) of a quark \(q(x)\), antiquark \(\bar{q}(x)\) and gluon \(g(x)\), respectively,
\[
q(2) + \bar{q}(2) = \int_0^1 dx x[q(x) + \bar{q}(x)],
\]
\[
G(2) = \int_0^1 dx x g(x).
\] (77)

Then, after the trivial calculation of matrix elements for vector dark matter states, we obtain a spin-independent effective coupling of vector dark matter with nucleons, \(f_N\) as
\[
f_N/m_N = \sum_{q=u,d,s} f_q f_{T_q} + \sum_{q=u,d,s,c,b} \frac{3}{4}(q(2) + \bar{q}(2))g_q
\]
\[
- \frac{8\pi}{9\alpha_s} f_{T_G} f_G + \frac{3}{4}G(2)g_G,
\] (78)

where \(f_q = f_q^m + f_q^{\phi}\) and \(\alpha_s = g_s^2/4\pi\). Note that the effective scalar coupling of dark matter with gluons, \(f_G\), gives the leading contribution to the cross section even if it is suppressed by a one-loop factor compared with those of light quarks.

Taking into account twist-2 operators and gluonic contributions calculated recently \([71]\), we find that:
\[
\frac{f_p}{m_p} = 0.052\left[-\frac{g_c^2}{m_h^2} - \frac{g_c^2m_q^2}{16(m_q^2 - m_c^2)^2}\right] + \frac{3}{4}(0.222)\left[-\frac{g_c^2m_q^2}{4(m_q^2 - m_c^2)^2}\right]
\]
\[- (0.925)\left(1.19\right)\frac{g_c^2}{54m_h^2} + \frac{g_c^2}{36}\left(1.19\right)\frac{m_q^2}{6(m_q^2 - m_c^2)^2} - \frac{1}{3(m_q^2 - m_c^2)^2}\right].
\] (79)
To obtain $f_n/m_n$, the numerical coefficients (i.e. 0.052, 0.222, 0.925) in the above equation are replaced with (0.061, 0.330, 0.922) [42]. The spin-independent elastic cross section for C-dark matter scattering off a nucleus of $Z$ protons and $A - Z$ neutrons normalized to one nucleon is then given by:

$$\sigma_0 = \frac{1}{\pi} \left( \frac{m_N}{m_c} \right)^2 \left| \frac{Z f_p + (A - Z) f_n}{A} \right|^2,$$

where $m_N$ and $m_c$ are the masses of the nucleon and C-dark matter, respectively.

Here we compare $^{73}$Ge (with $Z = 32$ and $A - Z = 41$) with the recent Cold Dark Matter Search (CDMS) result [73]. For a C-particle of mass $m_c = 900$ GeV, the experimental upper bound is very well approximated by the following equation [74]

$$\sigma_0 \approx 2.2 \times 10^{-7} \text{pb}(m_c/1 \text{TeV})^{0.86} \quad [42].$$

Another possible dark matter candidate is scalar ($\eta$), predicted by the Wu mechanisms for mass generation of gauge field [47–49].

For the scalar ($\eta$) scattering off a nucleus by the Higgs exchange ($h$) [43, 75], the terms $f_p$ and $f_n$ in cross section (70) become:

$$\frac{f_p}{m_p} = \left( -0.075 - \frac{0.925(3.51)(2)}{21} \right) \frac{\sqrt{2}}{m_\eta^2},$$

$$\frac{f_n}{m_n} = \left( -0.078 - \frac{0.922(3.51)(2)}{27} \right) \frac{\sqrt{2}}{m_\eta^2} \quad [43].$$

Assuming an effective $m_\eta = 125$ GeV and using $Z = 54$ and $A - Z = 77$ for $^{131}$Xe, we find that $\sigma_0 \approx 3.2 \times 10^{-10}$ pb. This value is far below the current upper bound, calculated by the 2011 XENON100 experiment [76].

8 Conclusion

We suggest that a new, neutral C-boson of mass 900 GeV, predicted by the Wu mechanisms for mass generation of gauge field, can explain the dark matter in our Universe.

In the suggested model, the Standard Model $W$ and $Z$-bosons acquire their masses through coupling with the Standard Model Higgs boson of mass 125 GeV.

C-dark matter readily derives its mass by the Wu mechanism; yet renormalizability occurs only via the Higgs mechanism.

We also compute the thermally averaged cross section for C-dark matter self-annihilation into SM particles, calculate the relic abundance of C-dark matter and study the constraints suggested by dark-matter direct-search experiments. In view of these constrains, scalar ($\eta$) is not a suitable dark-matter candidate.

The problem on the stability of C-particle is left as an open question for future research.

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