SWIFT and BATSE bursts’ classification

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Two classes of gamma-ray bursts were identified in the BATSE catalogs characterized by their durations. There were also some indications for the existence of a third type of gamma-ray bursts. Swift satellite detectors have different spectral sensitivity than pre-Swift ones for GRBs. Therefore in this paper we analyze the bursts’ duration distribution and also the duration-hardness bivariate distribution, published in The First BAT Catalog. Similarly to the BATSE data, to explain the BAT GRBs’ duration distribution three components are needed. Although, the relative frequencies of the groups are different than they were in the BATSE GRB sample, the difference in the instrument spectral sensitivities can explain this bias in a natural way. This means theoretical models may have to explain three different type of gamma-ray bursts.

I. INTRODUCTION

The discovery of the third type of GRBs goes back as early as 1998 [1, 2]. After that many research groups studied the BATSE bursts’ sample and concluded the third group of the GRBs statistically exists [3, 4, 5, 6, 7, 8, 9, 10]. Later several papers were published analysing different data sets [11, 12]. In this paper we analyze the bursts’ duration distribution and also the duration-hardness bivariate distribution, published in The First BAT and The BATSE Catalog.

II. THE ONE DIMENSIONAL GAUSSIAN FITS

We used the Maximum Likelihood (ML) method for the analysis of the (Swift) BAT and BATSE bursts. The ML method assumes that the probability density function of an x observable variable is given in the form of $g(x, p_1, ..., p_k)$ where $p_1, ..., p_k$ are parameters of unknown value. Having N observations on x one can define the likelihood function in the following form:

$$l = \prod_{i=1}^{N} g(x_i, p_1, ..., p_k)$$

or in logarithmic form (the logarithmic form is more convenient for calculations):

$$L = \log l = \sum_{i=1}^{N} \log (g(x_i, p_1, ..., p_k))$$

The ML procedure maximizes L according to $p_1, ..., p_k$. Since the logarithmic function is monotonic the logarithm reaches the maximum where l does it as well. The confidence region of the estimated parameters is given by the following formula, where $L_{max}$ is the maximum value of the likelihood function and $L_0$ is the likelihood function at the true value of the parameters [13]:

$$2(L_{max} - L_0) \approx \chi^2_k$$

Therefore one can fit the log $T_{90}$ distribution using ML with a superposition of k log-normal components, each of them having 3 unknown parameters to be fitted with N measured points. Our goal is to find the minimum value of k suitable to fit the observed distribution. Assuming a weighted superposition of k log-normal distributions one has to maximize the following likelihood function:

$$L_k = \sum_{i=1}^{N} \log \left( \sum_{j=1}^{k} w_l f_l(x_i, \log T_l, \sigma_l) \right)$$

where $w_l$ is a weight, $f_l$ a log-normal function with log $T_l$ mean and $\sigma_l$ standard deviation having the form of
III. ONE DIMENSIONAL GAUSSIAN FITS FOR THE BATSE BURSTS

A fit to the duration distribution of the BATSE bursts was taken using a maximum likelihood method with the superposition of two log-normal distributions. This can be done by a standard search for 5 parameters with \( N = 1929 \) measured points. Both log-normal distributions have two parameters; the fifth parameter defines the weight \( (w_1) \) of the first log-normal distribution. The second weight is \( w_2 = (N - w_1) \) due to the normalization. Therefore we obtain the best fit to the 5 parameters through a maximum likelihood estimation. The maximum of the likelihood was \( L_2 = 12320.11 \).

Secondly, a three-Gaussian fit was made (Figure 1. shows both the two and the three component fits) with three \( f_k \) functions with eight parameters (three means, three standard deviations and two weights). The logarithm of the best likelihood \( (L_3) \) is 12326.25. According to the mathematical theory, twice the difference of these numbers follows the \( \chi^2 \) distribution with three degrees of freedom because the new fit has three more parameters. The difference is 6.14 which gives us a 0.5% probability. Therefore the three-Gaussian fit is better and there is only a 0.005 probability that it is caused by statistical fluctuation.

IV. ONE DIMENSIONAL GAUSSIAN FITS FOR SWIFT GRBS

In the Swift BAT Catalog [14] there are 237 GRBs, of which 222 have duration information. Fig. 2. shows the log \( T_{90} \) distribution. We made fits for this distribution. Assuming only one log-normal component the fit gives \( L_{1\text{max}} = 951.666 \) but in the case of \( k = 2 \) one gets \( L_{2\text{max}} = 983.317 \).

Based on Eq. (3) we can infer whether the addition of a further log-normal component is necessary to significantly improve the fit. We make the null hypothesis that we have reached already the true value of \( k \). Adding a new component, i.e. moving from \( k \) to \( k + 1 \), the ML solution of \( L_{k\text{max}} \) has changed to \( L_{(k+1)\text{max}} \), but \( L_0 \) remained the same. In the meantime we increased the number of parameters with 3 \( (w_{k+1}, \log T_{k+1} \) and \( \sigma_{(k+1)} \)). Applying Eq. (4) on
both $L_{\text{max}}^{k}$ and $L_{(k+1)\text{max}}$ we get after subtraction

$$2(L_{(k+1)\text{max}} - L_{k\text{max}}) \approx \chi_{3}^{2} \quad (7)$$

For $k = 1$ $L_{2\text{max}}$ is greater than $L_{1\text{max}}$ by more than 30, which gives for $\chi_{3}^{2}$ an extremely low probability of $5.88 \times 10^{-13}$. It means the two log-normal fit is really a better approximation for the duration distribution of GRBs than one log-normal.

Thirdly, a three-log-normal fit was made combining three $f_{k}$ functions with eight parameters (three means, three standard deviations and two weights). The highest value of the logarithm of the likelihood ($L_{3\text{max}}$) is 989.822. For two log-normal functions the maximum was $L_{2\text{max}} = 983.317$. The maximum thus improved by 6.505. Twice of this is 13.01 which gives us the probability of 0.46% for the difference between $L_{2\text{max}}$ and $L_{3\text{max}}$ is being only by chance. Therefore there is only a small chance the third log-normal is not needed. Differently said the three-log-normal fit (see Figure 2.) is better and there is a 0.0046 probability that it was caused only by statistical fluctuation.

One should also calculate the likelihood for four log-normal functions. The best logarithm of the ML is 990.323. It is bigger with 0.501 than it was with three log-normal functions. This gives us a low significance (80.1%), therefore the fourth component is not needed.

V. THE TWO DIMENSIONAL GAUSSIAN FITS

When studying a GRB distribution, one can assume that the observed probability distribution in the parameter space is a superposition of the distributions characterizing the different types of bursts present in the sample. Using the notations $x$ and $y$ for the variables (in a 2D space), and using the law of full probabilities, one can write

$$p(x, y) = \sum_{l=1}^{k} p(x, y|l)p_{l} \quad (8)$$

In this equation $p(x, y|l)$ is the conditional probability density assuming that a burst belongs to the $l$-th class. $p_{l}$ is the probability for this class in the observed sample ($\sum_{l=1}^{k} p_{l} = 1$), where $k$ is the number of classes. In order to decompose the observed probability distribution $p(x, y)$ into the superposition of different classes we need the functional form of $p(x, y|l)$. The probability distribution of the logarithm of durations can be well fitted by Gaussian distributions, if we restrict ourselves to the short and long GRBs [2]. We assume the same also for the $y$ coordinate. With this assumption we obtain, for a certain $l$-th class of GRBs,
FIG. 3: The duration - hardness distribution of the BATSE bursts and the three components.

\[ p(x, y|l) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-r^2}} \times \exp \left[ -\frac{1}{2(1-r^2)} \left( \frac{(x-a_x)^2}{\sigma_x^2} + \frac{(y-a_y)^2}{\sigma_y^2} - \frac{2r(x-a_x)(y-a_y)}{\sigma_x \sigma_y} \right) \right], \quad (9) \]

where \( a_x, a_y \) are the means, \( \sigma_x, \sigma_y \) are the dispersions, and \( r \) is the correlation coefficient. Hence, a certain class is defined by 5 independent parameters, \( a_x, a_y, \sigma_x, \sigma_y, r \), which are different for different \( l \). If we have \( k \) classes, then we have \((6k - 1)\) independent parameters (constants), because any class is given by the five parameters of Eq.(9) and the weight \( p_l \) of the class. One weight is not independent, because \( \sum_{l=1}^{k} p_l = 1 \). The sum of \( k \) functions defined by Eq.(9) gives the theoretical function of the fit.

By decomposing \( p(x, y) \) into the superposition of \( p(x, y|l) \) conditional probabilities one divides the original population of GRBs into \( k \) groups. Decomposing the left-hand side of Eq.(8) into the sum of the right-hand side, one needs the functional form of \( p(x, y|l) \) distributions, and also \( k \) has to be fixed. Because we assume that the functional form is a bivariate Gaussian distribution (see Eq.(9)), our task is reduced to evaluating its parameters, \( k \) and \( p_l \).

Balázs et al. [15] used this method for \( k = 2 \), and gave a more detailed description of the procedure. However, that paper used fluence instead of hardness, and used the BATSE data. Here we will make similar calculations for \( k = 2, k = 3 \) and \( k = 4 \) using CGRO and Swift observations.

VI. THE TWO DIMENSIONAL GAUSSIAN FITS FOR BATSE BURSTS

The data from The Final BATSE GRB Catalog have been used, in which there are 2702 GRBs, for 1956 of which both the hardnesses and durations are measured. These 1956 GRBs define the sample studied in here. The hardness duration distribution can be seen in Figure 3.

Moving from \( k = 2 \) to \( k = 3 \) the number of parameters \( m \) increases by 6 (from 11 to 17), and \( L_{\text{max}} \) grows from 1193 to 1237. Since \( \chi_{11}^2 = \chi_{11}^2 + \chi_{6}^2 \) the increase in \( L_{\text{max}} \) by a value of 44 corresponds to a value of 88 for a \( \chi_{6}^2 \) distribution. The probability for \( \chi_{6}^2 \geq 88 \) is extremely low (< 10^{-10}). Therefore we may conclude that the inclusion of a third class into the fitting procedure is well justified by a very high level of significance.

Moving from \( k = 3 \) to \( k = 4 \), however, the improvement in \( L_{\text{max}} \) is only 6 (from 1137 to 1143) corre-
responding to $\chi^2_6 \geq 12$, which can happen by chance with a probability of 6.2%. Hence, the inclusion of the fourth class is not justified. We may conclude from this analysis that the superposition of three Gaussian bivariate distributions - and only these three ones - can describe the observed distribution of the BATSE data Figure 3. also shows the three components.

VII. THE TWO DIMENSIONAL GAUSSIAN FITS FOR THE SWIFT BURSTS

In the Swift BAT Catalog [14] there are 237 GRBs, of which 222 have duration information. Following the same procedure of data reduction we have extended this sample with all the bursts detected until mid December 2008 (ending with GRB 081211a). Our total sample thus comprises the first four years of the Swift satellite (since the detection of its first burst GRB 041217) and includes 342 bursts. 222 from Sakamoto et al. [14] and 120 reduced by us. The data reduction was done by using HEAsoft v.6.3.2 and calibration database v.20070924. For lightcurves and spectra we ran the batgrbproduct pipeline. We fitted the spectra integrated for the duration of the burst with a power law model and a power law model with an exponential cutoff. As in Sakamoto et al. [14] we have chosen the cutoff power law model if the $\chi^2$ of the fit improved by more than 6.

For calculating the hardness ratio we have chosen fluence 2 (25–50keV) and fluence 3 (50–100keV) and the hardness is defined by the $HR = F3/F2$ ratio. The duration - hardness distribution can be seen in Figure 4. The two dimensional fits have been made in this plane.

Moving from $k = 2$ to $k = 3$ the number of parameters $m$ increases by 6 (from 11 to 17), and $L_{\text{max}}$ grows from 506.6 to 531.4. Since $\chi^2_{17} = \chi^2_{11} + \chi^2_6$ the increase in $L_{\text{max}}$ by a value of 25 corresponds to a value of 50 for a $\chi^2_6$ distribution. The probability for $\chi^2_6 \geq 50$ is very low ($10^{-8}$), therefore we conclude that the inclusion of a third class into the fitting procedure is well.
justified by a very high level of significance.

Moving from $k = 3$ to $k = 4$, however, the improvement in $L_{\max}$ is 3.4 (from 531.4 to 534.8) corresponding to $\chi^2_6 \geq 6.8$, which can happen by chance with a probability of 33.9%. Hence, the inclusion of the fourth class is not justified. We conclude again that the superposition of three Gaussian bivariate distributions - and only these three ones - can describe the observed distribution of the Swift BAT bursts.

One can see from the fits and also from Figure 4, that the mean hardness of the intermediate class is very low - the third class is the softest one. This is in a good agreement with the BATSE fits (for more details see Horváth et al. [9]), where we found that the intermediate duration class is the softest in the BATSE database. In that database 11% of all GRBs belonged to this group. In our analysis $p_2 = 0.296$, therefore 30% of the Swift bursts belong to the third (intermediate duration) group.

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