Bias-controllable intrinsic spin polarization in a quantum dot

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Abstract

We propose a novel scheme to efficiently polarize and manipulate the electron spin in a quantum dot. This scheme is based on the spin-orbit interaction and it possesses following advantages: (1) The direction and the strength of the spin polarization is well controllable and manipulatable by simply varying the bias or the gate voltage. (2) The spin polarization is quite large even with a weak spin-orbit interaction. (3) Both electron-electron interaction and multi-energy levels do not weaken but strengthen the spin polarization. (4) It has the short spin flip time. (5) The device is free of a magnetic field or a ferromagnetic material. (6) It can be easily realized with present technology.

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How to efficiently control and manipulate the spin is an important and challenging issue in spintronics. Normally, the spin is difficult to be manipulated by a voltage-bias because the bias (or an electric field) does not act on the spin. Alternative methods, such as using a magnetic field or polarized light to manipulate the spin have been suggested, but they are far from in real use.

The quantum dot (QD) is an elementary cell of nano-electronic devices. The electron spin in the QD has been suggested as an ideal candidate for the qubit. Electron spin automatically comprises two levels that is a natural representation of a qubit, moreover the spin has a long decoherent time. However, in order to utilize the electron spin in the QD as a qubit, one first has to figure out how to efficiently polarize and manipulate the spin in the QD, i.e. writing a spin into the QD. One natural idea is to couple the QD to a ferromagnetic (FM) lead, such that the polarized spin in the FM can be injected into the QD. Another idea is to use an external magnetic field to polarize the spin in the QD. But both methods are not feasible in current experiments, because first it is very difficult to inject the spin from a FM into a semiconductor and for the second proposal to succeed, one needs a very strong external magnetic field confined to a small region of a QD.

Recently, based on the spin-orbit (SO) interaction, several theoretical studies have proposed that spontaneous spin accumulation can take place. For example, in the confined spin Hall devices, the opposite spin accumulations emerge at the boundaries of the samples. Can one achieve an effective spin manipulation in a QD by using the SO interaction?

In this Letter, we propose a new scheme to polarize and manipulate the spin in a QD by using the SO interaction. The main idea is as follows. Consider a QD coupled to two (left and right) leads and there also exists a direct bridge coupling between the two leads (see Fig.1a). In this system, an electron from the QD tunnelling to the left lead or vice versa has two paths: one path is through the direct tunnelling, the other is for the electron to first travel to the right lead and follow up with a tunnelling to the left lead through the bridge coupling (see Fig.1a). Let $t_{ij}$ describes the transmission coefficient from $j$ to $i$, with $i, j = L$ (left lead), $R$ (right lead), and $d$ (QD). Assume that the QD or the bridge arm contains the Rashba SO interaction, a spin-dependent extra phase $\sigma \varphi$ is generated in the path, thus, $t_{Rd}$ changes into $t_{Rd} e^{i\sigma \varphi}$ (the phase $\varphi = -k_R L = -\alpha_R m^* L / \hbar^2$ also describes the spin precession angle, with $\alpha_R$ being the Rashba SO interaction constant and $L$ being the size of...
Then the total effective coupling (or tunnelling) strength $T_{L\sigma}$ between the QD and the left lead is:

$$T_{L\sigma} = |t_{Ld} + t_{LR}(-i\pi\rho)t_{Rd}e^{i\sigma\phi}|^2$$

$$= |t_{Ld}|^2 + |\pi\rho t_{LR}t_{Rd}|^2 + 2\pi\rho|\tilde{t}|\sin(\phi_0 + \sigma\phi),$$

(1)

where $\tilde{t} = t_{Ld}t_{Rd}^*$, $\phi_0$ is the phase of $\tilde{t}$, and $\rho$ is the density of states in the lead. Similarly, the total effective coupling strength $T_{R\sigma}$ for the QD and the right lead is:

$$T_{R\sigma} = |t_{Rd}e^{i\sigma\phi} + t_{LR}^*(-i\pi\rho)t_{Ld}|^2$$

$$= |t_{Rd}|^2 + |\pi\rho t_{LR}t_{Ld}|^2 - 2\pi\rho|\tilde{t}|\sin(\phi_0 + \sigma\phi).$$

(2)

In general $T_{\alpha\uparrow}$ ($\alpha = L, R$) is different from $T_{\alpha\downarrow}$. If $T_{L\uparrow} > T_{L\downarrow}$, [then $T_{R\uparrow}$ must be $< T_{R\downarrow}$ from Eqs.(1,2)], it is easier for the spin-up electron to tunnel from the left lead into the QD than for the spin-down electron, but it is more difficult for it to tunnel out from the QD to the right lead because $T_{R\uparrow} < T_{R\downarrow}$. Therefore the QD should be spin polarization in the ‘up’ (or ‘down’) direction when the left lead is the high (or low) voltage terminal. Our following detailed numerical investigation indeed shows that the QD is spin polarized under the non-zero bias. The spin polarization is quite large even with a weak SO interaction and in a small QD. Particularly, the strength and the direction of the spin polarization are easily controllable and manipulatable by varying the bias or the gate voltage.

Our device is described by the following Hamiltonian:

$$H = \sum_{k,\sigma,\alpha} \epsilon_{\alpha k} a_{\alpha k\sigma}^\dagger a_{\alpha k\sigma} + \sum_{\sigma} \epsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma} + U d_{\uparrow}^\dagger d_{\downarrow}^\dagger d_{\uparrow} d_{\downarrow} + \sum_{k,k',\sigma} t_{LR} \left[ a_{Lk\sigma}^\dagger a_{Rk'\sigma} + a_{Rk'\sigma}^\dagger a_{Lk\sigma} \right]$$

$$+ \sum_{k,\sigma} \left[ t_{Ld} a_{Lk\sigma}^\dagger d_{\sigma} + t_{Rd} e^{i\sigma\phi} a_{Rk\sigma}^\dagger d_{\sigma} \right] + H.c.$$ 

(3)

where $d_{\sigma}$ and $a_{\alpha k\sigma}$ are annihilation operators in the QD and the lead $\alpha$, respectively. The QD consists a single energy level and an electron-electron (e-e) interaction $U$. Consider there exists the Rashba SO interaction in the QD, an extra phase $i\sigma\phi$ is added in the hopping term of $t_{Rd}$. We emphasize that the system contains no magnetic field and the sample is not a FM material.

The intradot spin-up (or spin-down) electronic occupation number $n_{\sigma}$ can be solved by using the standard Keldysh nonequilibrium Green’s function method. Following the
 procedure of our previous paper, the retarded Green function \( G_{d\sigma}^r \) is obtained as:

\[
G_{d\sigma}^r(\omega) = G_{d\sigma}^r(\tilde{t}_{d\sigma} + \tilde{t}_{d\sigma}^* g_{\alpha}^\dagger g_{\alpha}^r) g_{\alpha}^r / A, 
\]

where \( G_{d\sigma}^r(\omega) = 1/\{g_{d\sigma}^r - i\sum_{\alpha}(\tilde{t}_{d\sigma} + \tilde{t}_{d\sigma}^* g_{\alpha}^\dagger t_{\alpha\sigma}^r) g_{\alpha}^r + \omega\} \). \( A = 1 - g_{RL}^r t_{LR} G_{\sigma}^r t_{LR}^* / \), \( \tilde{t}_{d\sigma}^* = t_{d\sigma}^* = t_{d\sigma}^* = t_{d\sigma}^* \cdot g_{\alpha}^r \). 

The Green functions \( g^r \) are for the decoupled system (i.e. when \( t_{LR} = t_{Ld} = t_{Rd} = 0 \)), with \( g_{d\sigma}^r(\omega) = \frac{\omega - \epsilon_d + i\pi\rho}{\omega - \epsilon_d + \Gamma_{d\sigma}} \) and \( g_{\alpha}^r(\omega) = g_{\alpha}^r(\omega) = -i\pi\rho \). Then the occupation numbers are:

\[
n_{\sigma} = -i \int \frac{d\omega}{2\pi} G_{d\sigma}^r(\omega), \quad G_{d\sigma}^r(\omega) = \sum_{\alpha} |G_{d\sigma}^r(\omega)|^2 2i f_\alpha(\omega)/(\pi\rho), \quad f_\alpha(\omega) = [e^{\omega/(\mu_\alpha - \mu_\alpha)}] \sim 1 \]

is the Fermi distribution function in the lead \( \alpha \).

Next we present our numerical investigation. In all numerical calculations, we take \( \rho = 1 \) and symmetric coupling strengths \( t_{Ld} = t_{Rd} = 0.4 \) (the corresponding line-width \( \Gamma = 2\pi\rho t_{LR} / |\omega| \)). The chemical potential \( \mu_L = -\mu_R = V/2 \) with the bias \( V \). Fig.2 shows the occupation number \( n_{\sigma} \) and the spin accumulation \( \Delta n \) versus the intradot level \( \epsilon_d \) (Fig.2a-d) and the bias \( V \) (Fig.2e-h). The non-zero \( \Delta n \), i.e. the spin polarization in the QD, indeed emerges under a finite bias. \( \Delta n \) has the following features: (1) When the bias \( V = 0 \), \( \Delta n \) is identically zero for any \( \epsilon_d \) because of the time reversal invariance. (2) With the bias \( V \) increasing from 0, \( \Delta n \) increases. While \( V/2 > |\epsilon_d| \), i.e. \( \mu_L > \epsilon_d > \mu_R \), \( \Delta n \) is already large and the QD is well spin polarized (see Fig.2f,h). If the bias is reversed, \( \Delta n \) changes its sign, i.e. the spin polarized direction is reversed. This means that the direction and the strength of the spin polarization are easily controlled and tuned by changing of the external bias. (3) For a fixed bias \( V \) with varying \( \epsilon_d \) by tuning the gate voltage, \( \Delta n \) can also be modulated (see Fig.2b,d). When \( \epsilon_d \) is above both \( \mu_L \) and \( \mu_R \), \( n_\uparrow \) and \( n_\downarrow \) are almost zero. On the other hand, if \( \epsilon_d < \mu_L, \mu_R, n_\uparrow, n_\downarrow \approx 1 \). But when the energy level \( \epsilon_d \) is in the bias window with \( \mu_L > \epsilon_d > \mu_R \), \( \Delta n \) is quite big and the QD is largely spin polarized. (4) Even for a small \( \varphi \), \( \Delta n \) is large. For example, \( \varphi = 0.2 \), \( \Delta n \) is near 0.2 (see Fig.2b,f). While \( \varphi = \pi/4 \), \( \Delta n \) can be over 0.5 (see Fig.2d,h), which is quite large for spin polarization.

In Fig.3a,b we show \( \Delta n \) dependence on the phase \( \varphi \) and the bridge coupling strength \( t_{LR} \). \( \Delta n \) versus \( \varphi \) exhibits a periodic function with the period of \( 2\pi \). While \( \varphi = \pm \pi/2 \), \( \Delta n \) is near \( \pm 1 \) and the spin polarization can reach almost 100%. \( \Delta n \) versus \( t_{LR} \) is shown in Fig.3b, in which \( \Delta n = 0 \) at \( t_{LR} = 0 \) because of the shut-down of the bridge coupling. With a gradual opening of the bridge coupling (i.e. the gradual raising of \( t_{LR} \)), \( \Delta n \) increases first and follows by a slight reduction. But \( \Delta n \) still is over 0.1 even at quite large values of \( t_{LR} \).
Following, we investigate the effect of the e-e interaction (i.e. $U \neq 0$), which is shown in Fig. 4a-d. In general, the interaction $U$ increases $\Delta n$ because of the repulsive interaction between two electrons, namely strengthens the spin polarization. In particular, it has the following features: (1) For a wide range of $\epsilon_d$, $\Delta n$ can maintain large values (see Fig. 4b). In fact, $\Delta n$ is large as soon as $\epsilon_d$ or $\epsilon_d + u$ is within the bias window. (2) $\Delta n$ is larger than the value with $U = 0$. For example for the case of $\varphi = 0.2$, $\Delta n$ only reaches 0.18 at $U = 0$ (see Fig. 2b,f). However, at $U = 10$, $\Delta n$ is about 0.24 for a large range of $\epsilon_d$. Furthermore, $\Delta n$ can reach up to 0.33 at some special values of $\epsilon_d$ (see Fig. 4b). Correspondingly, the spin polarization $p = \Delta n/ (n^\uparrow + n^\downarrow)$ can reach 30% for that range of $\epsilon_d$ and 42% at those special values of $\epsilon_d$. Note this spin polarization is indeed fairly large, although $\varphi$ is only 0.2. (3) With the bias $V$ increasing from 0, $\Delta n$ can quickly increase as shown in Fig. 4d. While $U = 0$, $\Delta n$ reaches 0.17 until $V = 5$. However, when $U \neq 0$ (e.g. $U = 3$ or 5), $\Delta n$ has exceeded 0.18 at $V = 1$ (see Fig. 4d).

In the above model [or the Hamiltonian (3)], only one energy level in the QD is considered. How is the spin accumulation $\Delta n$ affected by the multi-levels in the QD? In fact, if there is only one level in the bias window and the others are outside the bias window, the outside levels do not affect the spin accumulation, the system acts as if it is a one-level system. On the other hand, if there are $N$ ($N > 1$) energy levels in the bias window, then each level will contribute a $\Delta n$ because the mechanism mentioned in the introduction [see Eq. (1, 2)] is effective for each level. So the total spin accumulation $\Delta n_T$ is approximatively $N\Delta n$ and is strongly enhanced.

How is time required for the spin flip to take place under a reversed bias? In other words, to consider that the bias is positive $V$ in the time $t < 0$ (so the QD has the spin polarization in $+z$ direction with a positive $\Delta n$), and the bias is reversed at $t = 0$ and it keeps the value $-V$ all along in the time $t > 0$. After this bias reversal, how long does it take for $\Delta n$ to change its sign? In order to answer this question, we have to solve the time-dependent occupation number $n_\sigma(t)$. From the Keldysh equation, we have:

$$n_\sigma(t) = -iG^<_{d\sigma}(t,t) = \sum_\alpha \int \int \int \int dt_1 dt_2 dt_3 dt_4 G^r_{d\alpha \sigma}(t,t_1)g^r_\alpha(t_1, t_2)g^<_\alpha(t_2, t_3)g^a_\alpha(t_3, t_4)G^a_{\alpha d\sigma}(t_4, t). \tag{5}$$

In the present case, although the bias is reversed at time $t = 0$, the retarded (advanced)
Green functions $G^r(a)(t, t_1)$ and $g^r(a)(t_1, t_2)$ are not affected (at $U = 0$), and they are still functions of the time difference. For example, $G^r_{daa}(t, t_1) = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t_1)} G^r_{daa}(\omega)$ and $G^r_{daa}^\dagger(\omega)$ are identical with the ones in Eq.(4) that are for a constant-biased case. The Keldysh Green function $g^\le_{L/R}(t_1, t_2)$ for the decoupled lead in Eq.(5) is: $g^\le_{L/R}(t_1, t_2) = i\rho \int d\omega f(\omega)e^{-i\omega(t_1-t_2)} e^{i(+/-)(|t_1|-|t_2|)V/2}$, with $f(\omega) = 1/\{exp(\omega/k_BT) + 1\}$.

The numerical results of $n_\sigma(t)$ and $\Delta n(t) = n_\uparrow(t) - n_\downarrow(t)$ versus the time $t$ are shown in Fig.5. $\Delta n(t)$ shows a quick reversal when the bias is reversed. For example, for the parameters of Fig.5a, $\Delta n(t) \approx 0.564$ for $t \leq 0$. When the bias is reversed at $t = 0$, $\Delta n(t)$ quickly reverses. When $t = 3/\Gamma$, $\Delta n(t) \approx -0.549$, thus, well reversed. If to take $\Gamma = 0.1 meV \mu m$ the reversal time $t = 3/\Gamma \approx 2 \times 10^{-11}\text{s}$, that is very short.

Before summary, we discuss the realizability. We suggest a possible experimental setup as shown in Fig.1b. The device is fabricated with a two-dimensional electron gas (e.g. the one in Ref.18). The dark region is the etching region or the deposited metal split gate with applied negative voltage to control the coupling coefficients $t_{LR}$, $t_{Ld}$, and $t_{Rd}$. The electrons are not present in the dark region and a QD is formed in the lower arm. A gate voltage $V_g$ is applied above the QD to control the Rashba SO interaction constant $\alpha_R$ as well the intradot energy levels. This device with its size within the phase coherent length can be easily realized with today’s semiconductor technology.17,18 The parameters of the bias $V$, the coupling coefficients $t_{LR}$, $t_{Ld}$, and $t_{Rd}$, can be conveniently tuned to satisfy the condition for substantial spin polarization. Next we discuss the phase parameter $\varphi$ (i.e. the spin precession angle) and the temperature effects. In our proposed scheme, even for quite small $\varphi$ (e.g. 0.2), $\Delta n$ is already large. In a recent experiment,18 $\varphi$ was successfully modulated over $0.75\pi$ with size $L = 1.5\mu m$ (correspondingly $\alpha_R \approx 2 \times 10^{-12}\text{eV}\mu m$). If the size of our QD is 200nm, $\varphi$ should be tunable in the range $0.1\pi \approx 0.3$. Moreover, some experiments have measured that $\alpha_R$ can reach $3 \times 10^{-11}\text{eV}\mu m$19 then $\varphi = 0.2$ for a QD as small as $L \approx 10\text{nm}$. So the parameter of $\varphi = 0.2$ can be realized.20 The temperature $k_BT$ is not a problem with this scheme. Even with $k_BT/e = V/2$, the results is almost unchanged. If one takes the bias $V = 2mV$, $T = eV/2k_BT \approx 10K$. Finally, we compare this proposal to some recent works on the spin Hall effect.9,10,11,12 The opposite spin accumulations emerge at two opposite boundaries in a confined spin Hall system. In contrast with those works, the size of the present system is $L = \frac{\varphi}{2\pi}L_{SO} = \frac{0.2}{2\pi}L_{SO} \approx 0.03L_{SO}$ ($L_{SO} \equiv 2\pi\hbar^2/\alpha_Rm^*$) and this size is much smaller than the confined spin Hall system for which the size is usually several times
of $L_{SO}$.

In summary, we have proposed a new method to generate the spin polarized electrons in a quantum dot by utilizing the spin-orbit (SO) interaction. A large spin polarization can be produced even with a weak SO interaction and in a small dot. In particular, the direction and the strength of the spin polarization can be controlled and tuned by varying the bias or the gate voltage.

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In order to formed the QD, some split gates are added. Those split gates perhaps affect the potential in the vertical direction of 2DEGs, so it maybe reduces or destroy the Rashba SO interaction in the QD and then $\varphi$. We emphasize that it is nothing even the Rashba SO interaction in the QD is completely destroyed. Because that in our scheme it only need the phase (i.e. spin precession) different for the electron travelling the QD and the bridge arm. So even the phase $\varphi_{QD}$ of passing the QD is zero, the spin polarization still emerges and the proposed scheme can work as long as the existence of the phase in the bridge arm.
FIG. 1: (Color online) (a) Schematic diagram for the system of two leads coupled to a QD as well as a bridge coupling between two leads. (b) Schematic diagram for our proposed experimental device fabricated in a 2DEGs. The dark regions are the split gate to control the coupling coefficients $t_{LR}$, $t_{Ld}$, and $t_{Rd}$. The inclined lattice region is the gate that controls the Rashba SO interaction constant $\alpha_R$ and the level $\epsilon_d$.

FIG. 2: (Color online) (a-d) are $n^\uparrow$ [solid curves in (a) and (c)], $n^\downarrow$ [dotted curves in (a) and (c)], and $\Delta n$ [in (b) and (d)] vs the level $\epsilon_d$ for the bias $V = 2, 4$, and $8$ along the arrow direction. (e-h) are $n^\uparrow$ [solid curves in (e) and (g)], $n^\downarrow$ [dotted curves in (e) and (g)], and $\Delta n$ [in (f) and (h)] vs the bias $V$, where in (e) and (g) $\epsilon_d = -1, 0, 1$, and $2$ from top to bottom, in (f) and (h) $\epsilon_d = 0, 1, -1$, and $2$ along the arrow direction. Notice that in (f) the curve of $\epsilon_d = 0$ ($-1$) almost overlaps with one of $\epsilon_d = 1$ ($2$) so that they cannot be seen in the figure. The other parameters are: $t_{LR} = 0.3$ and $\varphi = 0.2$ [in (a), (b), (e), and (f)], $t_{LR} = 0.2$ and $\varphi = \pi/4$ [in (c), (d), (g), and (h)]. The temperature $k_B T = 0$ and $U = 0$.

FIG. 3: (Color online) (a) $\Delta n$ vs $\varphi$ for $t_{LR} = 0.3$ and (b) $\Delta n$ vs $t_{LR}$ for $\varphi = 0.2$. The parameters are $\epsilon_d = U = k_B T = 0$.

FIG. 4: (Color online) (a) and (b) are $n^\uparrow$ [the thick curves in (a)], $n^\downarrow$ [the thin curves in (a)], and $\Delta n$ vs $\epsilon_d$ for the bias $V = 6$, and $U = 3$ (the dotted curves) and $10$ (the solid curves). (c) and (d) are $n^\uparrow$ [the thick curves in (c)], $n^\downarrow$ [the thin curves in (c)], and $\Delta n$ vs the bias $V$ for $\epsilon_d = 0$. The other parameters are $t_{LR} = 0.3$, $\varphi = 0.2$, and $k_B T = 0$.

FIG. 5: (Color online) $n^\uparrow$, $n^\downarrow$, and $\Delta n$ vs the time $t$ when the bias $V$ is reversed at $t = 0$, where the parameters are: $\epsilon_d = U = k_B T = 0$, $V = 8$ in $t < 0$ and $V = -8$ in $t > 0$. In (a) $t_{LR} = 0.2$ and $\varphi = \pi/4$, and in (b) $t_{LR} = 0.3$ and $\varphi = 0.2$. 

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