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Special wave solutions in the theory of waves of a mixed spectrum in a planar gradient bianisotropic nanocrystalline structure

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Abstract. The propagation of surface and pseudosurface waves of a mixed spectrum in a symmetric nanocrystalline film structure with continuously-inhomogeneous anisotropic material characteristics was considered. The substrate and coating of the compositional structure had the same scalar dielectric and magnetic permeabilities. The artificial medium of the waveguide layer was made from a bianisotropic material with an anisotropy of the magneto-dielectric bond, characterized by four tensors of the dielectric, magnetic, and magneto-dielectric permeabilities with nonzero diagonal elements. They depended on spatial coordinates with a biquadratic law. A system of coupled quasi-differential wave equations was obtained toward to the transverse components of the electric and magnetic fields from Maxwell's equations with taking into account specific material relationships. It had linearly independent solutions, which were represented in the form of generalized series. Recurrence relations were determined for the determination of their terms. Taking into account the continuity of the tangential components of the electric and magnetic fields, a dispersion equation for calculating the transverse wave numbers was found. We obtained the existence of four proper surface and pseudosurface waves with different polarizations, which possess a whole series of fundamentally new characteristics, from the structure of the system of wave equations and their solutions.

1. Introduction
At present time, we ascertain the rapid development of research on the synthesis of materials that have previously unknown properties[1,2]. This type of materials includes gradient anisotropic magnetodielectric and particulary nanostructured gyromagnetic media[3,4]. A natural consequence of this technology development is a steady trend towards the development of optoelectronic elements and subsequent generation nodes that have a number of parameters and characteristics that are fundamentally new, in comparison with homogeneous and ordinary anisotropic dielectric structures. As a result, the possibilities for creating various control devices for the waveguide properties of monochromatic optical waves, light beams and pulses are deepened. In this sphere, bianisotropic and bi-isotropic media of a rather wide class are distinguished [5].And we considered their applications depending on the frequency range used. The study of wave propagation and resonance mode interactions in regular and irregular compositional hybrid anisotropic nanostructured waveguides presents significant mathematical and computational difficulties, since it requires the development of an adequate mathematical apparatus and the attraction of modern computer technology[6,7].
2. Formulation of the problem

We proceed from the assumption that the surface and pseudosurface waves of a waveguide compositional structure with a complex internal medium may be find in the form of an expansion in the complete system of waves of a mixed spectrum of a waveguide comparison structure that satisfies given boundary conditions. The success of this approach depends on the successful choice of the approximating system of the cross-sectional functions of this structure. Therefore, the complex task of a reasonable choice of representing functions is constantly faced. As one of ways to solve this problem is to find such functions from the solution of the same boundary value problems for a simpler compositional structure of comparison. We can choose a planar waveguide in which the coating and the substrate are homogeneous for that structure of comparison. Earlier, we investigated the waveguide layer, which has tensors of dielectric and magnetic permeabilities, in which only diagonal elements differ from zero, and they depend on the transverse coordinate according to the generalized laws. We discussed such waveguide problems, when the directed wave propagates along the longitudinal coordinate and there is no dependence on the Oy axis. The Maxwell's equations broke up into two independent systems of quasi-differential equations for determining the transverse components of the eigenvectors of the discrete and continuous spectra. We obtained the equations with polynomial coefficients. They were not Fuchsian. Nevertheless, we used the Frobenius method for their analytical solution. As a result, we obtained the transverse and longitudinal components of the electric and magnetic fields of the characteristic waves in the compositional structure. We analyzed the entire variety of waves existing in such a structure in the general case. This allowed us to obtain the eigenwaves corresponding to the discrete part of the spectrum. The wave numbers satisfy the special dispersion equation. In the general case, we have found the longitudinal complex wave numbers (complex waves). The waves became into the surface waves, when the transverse wave numbers were imaginary. We found the natural waves of the continuous part of the spectrum. The wave numbers of these waves were real or imaginary and allow us to describe the radiation fields, as well as the separation of resonances (outgoing waves).

We also researched the synthesis of materials with special waveguide properties. This investigation included the synthesis of gradient anisotropic magnetodielectric and different nanostructured gyromagnetic media. However, the assessment of the limits of applicability of this approach involves obtaining higher approximations for the solution of the integral equation of the field. We used an approximate distribution of variables, which were based on the special characteristics and structure conditions of the falling laser beam. For example, we have taken into account an effective variation of the waves with infinite phase front entering into the wave vector. Then slowly varying amplitude was defined from the expression of the Fourier transform of the reflected beam. Then we simplified the integral operator using the partial integration of the transverse and longitudinal wave numbers. And differentiating it for the longitudinal coordinate, we got the special differential equation for the amplitudes of modes of discrete wave spectrum in the constructed beam. As a result, the possibilities for creating various control devices for the waveguiding properties of monochromatic optical waves, light beams and pulses were deepened. We defined the presence of a magnetodielectric connection in waveguide structures in the absence of anisotropy, for example, in a bisisotropic medium. If the reciprocity restriction is added to such a medium, then the directing structure becomes chiral[8]. These waveguides include artificial media with metal spirals, liquid crystals, a variety of biological objects (DNA, and proteins-collagens), molecules in which are oriented in the form of spiral chains. A rigorous and correct electrodynamic study of compositional structures for optoelectronics based on artificial media of this class is of great interest [9]. In recent years, carbon nanoelectronics has been intensively developing [10]. The exotic electrical properties of graphics, fullerene, nanotubes, graphite and diamond make it possible later to replace the elements and sites of flint optoelectronics with nanostructures [11]. One of the most promising allotropic carbon species is graphene, which is a planar monolayer of carbon atoms packed in a two-dimensional hexagonal lattice, which, under certain conditions, has great thermal conductivity, carrier mobility, and mechanical strength [12, 13]. The profound research and physical applications of interactions of different types of waves with
medium are very important. Particularly the most interesting sphere in that area is investigation of different complex environments, which may be characterized by the increasing number of mathematical methods. Their objective analysis increased because of great applied significance. For the optical range it comes to considering the structure, the dimensions of the spatial inhomogeneity of which are comparable to the wavelength of the radiation.

Separate attempts were made to apply the method of functional transformations and perturbation methods. These are the most universal methods, which are suitable for solving of any equations of linear and nonlinear finite, in ordinary and partial derivatives, integral, integro-differential, functional, with deviating argument, stochastic and determinate for any boundary and initial conditions. Many scientists used the great spectrum of various special methods for solving equations based on one or another modification of the perturbation method: widely known methods of small (large) parameter, boundary layer, iterative methods, linearization, regular and singular, and other approximate and analytical methods. It is possible to use integral methods for an approximate solution of the wave equations for the structures of this class. These methods reduce the solutions of quasi-differential equations for given boundary conditions to the integration of equations in ordinary derivatives obtained from the initial ones by averaging them.

There are a lot of works in which the matrix methods are used for homogeneous bianisotropic structures, for example, the matrix method “4x4”. However, we noted the main drawback of these methods: the so-called matrix of material parameters turns out to depend on the longitudinal wave number and frequency. This incorrect, because, firstly, the waveguide refractive index is determined by the matrix of material parameters, and secondly, the form of this matrix should not depend on the frequency of the wave, if we do not take into account the dispersion of the waveguide structures. Analytic solutions of the designated problems can be carried out up to the end by the Fourier method, finite integral transformations or by the Green’s function method, etc. We can obtain the numerical solution of all the wave tasks by the method of grids or variational methods, etc. The solution of the equations by iterative and direct grid methods also is not complete. One of the direct methods is the reduction method, it was called a direct method of reduction or decomposition in the literature. The gradient anisotropy of the dielectric and magnetic permeability in nanomaterials suggests that the intrinsic waves of thin films are mixed hybrid surface and pseudosurface waves.

Despite significant progress in creating and using waveguide structures based on gradient anisotropic nanomaterials in optoelectronics and integrated optics, few studies have been devoted to the propagation of hybrid surface waves in such structures. An analysis of the discrete spectrum of optical waves in thin-film nanostructures of such type were made for homogeneous anisotropic media and individual particular cases of orientations of the principal axes of the permittivity tensor with respect to the $x$, $y$, $z$ axes related to the geometry of the waveguide structure. Dielectric and magnetic permeabilities and optical anisotropy of nanocrystalline films were experimentally obtained in [14].

3. The material equations of the waveguide medium

We studied the propagation of electromagnetic waves in continuously inhomogeneous (gradient) bianisotropic structures in this work. There is a particular interest to such type of the waveguide media because of the possibilities of revealing the features of the displacement of the spectra of hybrid modes and their mutual bonds in it. We reviewed the changes of the gradient parameters in the elements of four tensors of the dielectric $\varepsilon$, magnetic $\mu$ and magnetoelectric permeabilities $\alpha$ and $\nu$:

$$\begin{align*}
\bar{D} &= \varepsilon(x)\bar{E} + \alpha(x)\bar{H}, \\
\bar{B} &= \mu(x)\bar{H} + \nu(x)\bar{E},
\end{align*}$$

(1)

The material characteristics of a metamaterial were determined by thirty-six elements, each of which has its own spatial distribution profile. The form of the gradient tensors (the form of the dependence on the transverse or longitudinal coordinates) was determined by the mutual diffusion of the components of the metamaterial of the waveguide layer, the symmetry of its crystal lattice, the continuous change in the direction of the polarization vector or the magnetic moment, the mechanical stresses accompanying the crystal growth, etc. [15].

Analytic solutions of Maxwell’s equations in known functions for generalized gradient bianisotropic media of the above type (1) include great mathematical difficulties and are currently
absent. Here we considered one of the methods for finding models with which you can set the properties of real metamaterials. The method was based on the analysis of wave equations with diagonal gradient tensors of a bianisotropic medium [16,17]. We have found fairly general analytical solutions. So, the most well known in the literature models of metastructures are particular cases of the results, which we obtained in this work. Exact analytical solutions are of particular importance in the mathematical theory of waves in complex structures, but they do not cover a wide variety of real practical applications. Because of this, exact methods have to be supplemented by approximations, which are based on perturbation theory, variational methods, and direct numerical calculations.

4. The main part of the study
We considered the propagation of electromagnetic waves in a waveguide structure, which consists of a film of metamaterial, substrate and coating in this paper. The coating and the substrate had scalar material characteristics of the dielectric and magnetic permeabilities, respectively $\varepsilon_1, \mu_1; \varepsilon_2, \mu_2$. The waveguiding layer was characterized by the gradient tensors of dielectric, magnetic permeabilities $\epsilon_2$, $\mu_2$ and of the magnetoelectric permeabilities $\epsilon_2$, $\mu_2$:

$$
\begin{align*}
\epsilon_2 &= \begin{bmatrix}
\varepsilon_{xx}(q_0, q_1, \ldots, q_n, x) & 0 & 0 \\
0 & \varepsilon_{yy}(g_0, g_1, \ldots, g_n, x) & 0 \\
0 & 0 & \varepsilon_{zz}(a_0, a_1, \ldots, a_n, x)
\end{bmatrix}, \\
\mu_2 &= \begin{bmatrix}
\mu_{xx}(b_0, b_1, \ldots, b_n, x) & 0 & 0 \\
0 & \mu_{yy}(c_0, c_1, \ldots, c_n, x) & 0 \\
0 & 0 & \mu_{zz}(d_0, d_1, \ldots, d_n, x)
\end{bmatrix}, \\
\alpha_2 &= \begin{bmatrix}
\alpha_{xx}(e_0, e_1, \ldots, e_n, x) & 0 & 0 \\
0 & \alpha_{yy}(h_0, h_1, \ldots, h_n, x) & 0 \\
0 & 0 & \alpha_{zz}(f_0, f_1, \ldots, f_n, x)
\end{bmatrix}, \\
\nu_2 &= \begin{bmatrix}
\nu_{xx}(\tau_0, \tau_1, \ldots, \tau_n, x) & 0 & 0 \\
0 & \nu_{yy}(\lambda_0, \lambda_1, \ldots, \lambda_n, x) & 0 \\
0 & 0 & \nu_{zz}(\chi_0, \chi_1, \ldots, \chi_n, x)
\end{bmatrix},
\end{align*}
$$

where $a_0, a_1, \ldots, a_n, b_0, b_1, \ldots, b_n, c_0, c_1, \ldots, c_n, d_0, d_1, \ldots, d_n, e_0, e_1, \ldots, e_n, f_0, f_1, \ldots, f_n, g_0, g_1, \ldots, g_n, h_0, h_1, \ldots, h_n,$ $q_0, q_1, \ldots, q_n, \tau_0, \tau_1, \ldots, \tau_n, \lambda_0, \lambda_1, \ldots, \lambda_n, \chi_0, \chi_1, \ldots, \chi_n$ - gradient parameters of spatial profiles of tensor elements for material characteristics in waveguidelayer.

We presented the Maxwell’s equations in the limits of a waveguide film:

$$
\begin{align*}
\beta H_y &= j\omega \varepsilon_{xx}(q_0, q_1, \ldots, q_n, x) E_x + \alpha_{xx}(e_0, e_1, \ldots, e_n, x) H_x, \\
- j\gamma H_x &= -j\omega \varepsilon_{yy}(g_0, g_1, \ldots, g_n, x) E_y + j\omega \alpha_{yy}(h_0, h_1, \ldots, h_n, x) H_y, \\
\frac{dH_y}{dx} &= j\omega \varepsilon_{zz}(a_0, a_1, \ldots, a_n, x) E_z + j\omega \alpha_{zz}(f_0, f_1, \ldots, f_n, x) H_z, \\
\gamma E_y &= -j\omega \mu_{xx}(b_0, b_1, \ldots, b_n, x) H_x - j\omega \nu_{xx}(\tau_0, \tau_1, \ldots, \tau_n, x) E_x, \\
j\gamma E_x + \frac{dE_z}{dx} &= j\omega \mu_{yy}(c_0, c_1, \ldots, c_n, x) H_y + j\omega \nu_{yy}(\lambda_0, \lambda_1, \ldots, \lambda_n, x) E_y, \\
\frac{dE_y}{dx} &= -j\omega \mu_{zz}(d_0, d_1, \ldots, d_n, x) H_z - j\omega \nu_{zz}(\chi_0, \chi_1, \ldots, \chi_n, x) E_z.
\end{align*}
$$

(12)
We took into account the equations (6) - (9) and found the relationship between the transverse and longitudinal components of the electric and magnetic fields:

\[
E_x = \frac{\gamma \left[ \mu_{xx} (b_0, b_1, ..., b_n, x) H_y + \alpha_{xx} (e_0, e_1, ..., e_n, x) E_y \right]}{j \omega \left[ e_{xx} (q_0, q_1, ..., q_n, x) \mu_{xx} (b_0, b_1, ..., b_n, x) - \nu_{xx} (r_0, r_1, ..., r_n, x) \alpha_{xx} (e_0, e_1, ..., e_n, x) \right]},
\]

(12)

\[
H_y = -\frac{\gamma \left[ \varepsilon_{xx} (q_0, q_1, ..., q_n, x) E_x + \nu_{xx} (r_0, r_1, ..., r_n, x) H_x \right]}{j \omega \left[ e_{xx} (q_0, q_1, ..., q_n, x) \mu_{xx} (b_0, b_1, ..., b_n, x) - \nu_{xx} (r_0, r_1, ..., r_n, x) \alpha_{xx} (e_0, e_1, ..., e_n, x) \right]},
\]

(13)

\[
E_z = \mu_{zz} (d_0, d_1, ..., d_n, x) \frac{dH_y}{dx} + \alpha_{zz} (f_0, f_1, ..., f_n, x) \frac{dE_y}{dx},
\]

(14)

\[
H_z = -\frac{\varepsilon_{zz} (a_0, a_1, ..., a_n, x) \mu_{zz} (d_0, d_1, ..., d_n, x) - \alpha_{zz} (f_0, f_1, ..., f_n, x) \nu_{zz} (\chi_0, \chi_1, ..., \chi_n, x)}{j \omega \left[ \varepsilon_{zz} (a_0, a_1, ..., a_n, x) \mu_{zz} (d_0, d_1, ..., d_n, x) - \alpha_{zz} (f_0, f_1, ..., f_n, x) \nu_{zz} (\chi_0, \chi_1, ..., \chi_n, x) \right]},
\]

(15)

Then we renamed:

\[
\theta_{xx}(x) = \frac{\mu_{xx}}{\varepsilon_{xx} \mu_{xx} - \nu_{xx} \alpha_{xx}}, \quad \theta_{zz}(x) = \frac{\mu_{zz}}{\varepsilon_{zz} \mu_{zz} - \nu_{zz} \alpha_{zz}}, \quad Q_{xx}(x) = \frac{\alpha_{xx}}{\varepsilon_{xx} \mu_{xx} - \nu_{xx} \alpha_{xx}}, \quad Q_{zz}(x) = \frac{\alpha_{zz}}{\varepsilon_{zz} \mu_{zz} - \nu_{zz} \alpha_{zz}},
\]

(16)

\[
\theta_{xx}(x) = \frac{\mu_{xx}}{\varepsilon_{xx} \mu_{xx} - \nu_{xx} \alpha_{xx}}, \quad \theta_{zz}(x) = \frac{\mu_{zz}}{\varepsilon_{zz} \mu_{zz} - \nu_{zz} \alpha_{zz}}, \quad Q_{xx}(x) = \frac{\alpha_{xx}}{\varepsilon_{xx} \mu_{xx} - \nu_{xx} \alpha_{xx}}, \quad Q_{zz}(x) = \frac{\alpha_{zz}}{\varepsilon_{zz} \mu_{zz} - \nu_{zz} \alpha_{zz}},
\]

(17)

We obtained:

\[
E_x = \frac{\gamma}{j \omega} \left[ \theta_{xx}(x) H_y + Q_{xx}(x) E_y \right],
\]

(18)

\[
H_y = -\frac{\gamma}{j \omega} \left[ Q_{xx}(x) E_y + T_{xx}(x) H_y \right],
\]

(19)

\[
E_z = \frac{1}{j \omega} \left[ Q_{zz}(x) \frac{dH_y}{dx} + T_{zz}(x) \frac{dE_y}{dx} \right],
\]

\[
H_z = -\frac{1}{j \omega} \left[ Q_{zz}(x) \frac{dE_y}{dx} + T_{zz}(x) \frac{dH_y}{dx} \right],
\]

We substituted the expressions (12) - (15) into equations (7) - (10) and obtained the main system of quasidifferential wave equations for surface and pseudosurface waves of a mixed spectrum in a symmetric nanocrystalline film structure with tensor characteristics of the internal waveguide medium:

\[
\frac{j \gamma^2}{\omega} Q_{xx}(x) E_y + \frac{j \gamma^2}{\omega} T_{xx}(x) H_y + \frac{1}{j \omega} \frac{dQ_{xx}(x)}{dx} \frac{dE_y}{dx} + \frac{1}{j \omega} Q_{zz}(x) \frac{d^2 E_y}{dx^2} + \frac{1}{j \omega} \frac{dE_y}{dx} \frac{dH_y}{dx} + \frac{1}{j \omega} T_{zz}(x) \frac{d^2 H_y}{dx^2} = j \omega \varepsilon_{yy} (g_0, g_1, ..., g_n, x) E_y + j \omega \alpha_{yy} (h_0, h_1, ..., h_n, x) H_y,
\]

(20)

\[
\frac{j \gamma^2}{\omega} Q_{xx}(x) \frac{d^2 E_y}{dx^2} + T_{zz}(x) \frac{d^2 H_y}{dx^2} + \frac{dQ_{xx}(x)}{dx} \frac{dE_y}{dx} + \frac{dQ_{zz}(x)}{dx} \frac{dH_y}{dx} + \frac{dT_{zz}(x)}{dx} \frac{dH_y}{dx} + \left[ \omega^2 \varepsilon_{yy} (g_0, g_1, ..., g_n, x) + j \gamma^2 Q_{xx} (x) \right] E_y + \left[ \omega^2 \alpha_{yy} (h_0, h_1, ..., h_n, x) + j \gamma^2 T_{xx} (x) \right] H_y = 0,
\]

(21)
\[
\begin{align*}
\frac{j\gamma^2}{\omega} \theta_{zz}(x) H_y + \frac{j\gamma^2}{\omega} \theta_{x}(x) E_y + & \frac{1}{j\omega} \frac{d\theta_{zz}(x)}{dx} \frac{dH_y}{dx} + \frac{1}{j\omega} \theta_{xx}(x) \frac{d^2H_y}{dx^2} + \frac{1}{j\omega} \frac{d\theta_{xx}(x)}{dx} \frac{dE_y}{dx} + \\
& + \frac{1}{j\omega} \frac{d^2E_y}{dx^2} = j\omega \mu_{yy}(e_{0},e_{1},...,e_{n},x) H_y + j\omega \nu_{yy}(\lambda_{0},\lambda_{1},...,\lambda_{n},x) E_y,
\end{align*}
\]

\[
(22)
\]

\[
\begin{align*}
\theta_{xz}(x) \frac{d^2E_y}{dx^2} + & \theta_{zz}(x) \frac{d^2E_y}{dx^2} + \frac{d\theta_{zz}(x)}{dx} \frac{dH_y}{dx} + \frac{d\theta_{xx}(x)}{dx} \frac{dE_y}{dx} + \left[ \omega^2 \mu_{yy}(e_{0},e_{1},...,e_{n},x) + \\
& + j\gamma^2 \theta_{xx}(x) \right] H_y + \left[ \omega^2 \nu_{yy}(\lambda_{0},\lambda_{1},...,\lambda_{n},x) + j\gamma^2 \theta_{xz}(x) \right] E_y = 0.
\end{align*}
\]

\[
(23)
\]

In the componentwise form, without isolating the dependence on the parameters of the gradient and the transverse coordinate, the system of coupled equations (20) - (23) have taken a form:

\[
e_{zz}(e_{zz} \mu_{zz} - \alpha_{zz} \nu_{zz})(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{d^2E_y}{dx^2} + v_{zz}(e_{zz} \mu_{zz} - \alpha_{zz} \nu_{zz})(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{d^2H_y}{dx^2} + \\
+ A(x)(e_{zz} \mu_{zz} - \alpha_{zz} \nu_{zz}) \frac{dE_y}{dx} + B(x)(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{dH_y}{dx} + \left[ \omega^2 \mu_{yy}(e_{0},e_{1},...,e_{n},x) + \\
+ j\gamma^2 \theta_{xx}(x) \right] H_y + \left[ \omega^2 \nu_{yy}(e_{0},e_{1},...,e_{n},x) + j\gamma^2 \theta_{xz}(x) \right] E_y = 0,
\]

\[
(24)
\]

\[
\mu_{zz}(e_{zz} \mu_{zz} - \alpha_{zz} \nu_{zz}) \frac{d^2H_y}{dx^2} + \mu_{xx}(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{d^2E_y}{dx^2} + \\
+ A_1(x)(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{dH_y}{dx} + B_1(x)(e_{xx} \mu_{xx} - \alpha_{xx} \nu_{xx}) \frac{dE_y}{dx} + \left[ \omega^2 \mu_{yy}(e_{0},e_{1},...,e_{n},x) + \\
+ j\gamma^2 \theta_{xx}(x) \right] H_y + \left[ \omega^2 \nu_{yy}(e_{0},e_{1},...,e_{n},x) + j\gamma^2 \theta_{xz}(x) \right] E_y = 0,
\]

\[
(25)
\]

where

\[
A(x) = \alpha_{zz}' e_{zz} + \alpha_{zz} e_{zz}' - \alpha_{zz} \nu_{zz}' - \nu_{zz} e_{zz}' - \nu_{zz} e_{zz}' , B(x) = \nu_{zz}' e_{zz} \mu_{zz} + \alpha_{zz}' \nu_{zz}' - \nu_{zz} \mu_{zz}' - \nu_{zz} \mu_{zz}' , \\
A_1(x) = \alpha_{zz}' e_{zz} \mu_{zz} + \alpha_{zz} e_{zz} \mu_{zz}' - \alpha_{zz} \mu_{zz} e_{zz}' - \alpha_{zz} \nu_{zz} e_{zz}' , B_1(x) = \nu_{zz}' \alpha_{zz} \mu_{zz} + \alpha_{zz}' \nu_{zz}' - \mu_{zz} \nu_{zz}' - \mu_{zz} \nu_{zz}' .
\]

and hatch \(\alpha'\) means derivative on the transverse coordinate \(x\).

We used power, exponential, polynomial, and a number of other functions to approximate the material characteristics of the artificial waveguide media. We can always use a Taylor series in powers of the argument in the case of an arbitrary functional analytic dependence of the elements of the tensors of material characteristics on the coordinates or time. We have found that the first few terms of the series approximate the properties of the experimentally determined characteristic of the gradient bianisotropic medium in the course of performing numerical calculations. Therefore, it was sufficient for us to confines to a polynomial in carrying out further research instead of the series. This result is the primary one in the strict theory of waves in gradient bianisotropic regular and irregular structures, because it allows us to obtain all the necessary wave characteristics.

5. Some particularly significant results of the research

Differential equations for artificial media with polynomial approximations contain variable coefficients in the form of polynomials. Some of them have already been studied well, others lead to new special functions [18, 19]. As an example, Figure 1 shows the dependence of the longitudinal wave numbers of the first hybrid modes on the angle \(\varphi\) for various parameters of the gradient of the spatial profiles of the elements of the permittivity tensor in the compositional medium.

It is clear from them that the degree of dependence of the longitudinal wave numbers for the hybrid modes on the angle \(\varphi\) is determined by the comparative composition of the ordinary and extraordinary modes of the discrete spectrum in them for the waveguide structure of comparison. It can be shown from equations (24) - (25) that \(\gamma\) of the hybrid mode, which is formed mainly by extraordinary waves, decreases monotonically with an increasing of an angle \(\varphi\).
This is mainly due to the fact that the surface of the change in the elements of the permittivity tensor \( \varepsilon_{q,q}(x,q_{1},q_{n},x) \) was a rotational ellipsoid with variable semiaxes. If the modes of the waveguide structure prevail in the composition, then the angular dependence is much weaker, since the surface of the variation of the corresponding elements of the tensor \( \varepsilon_{\mu,\nu} \) is a sphere with a varying radius and is determined only by the gradient of the anisotropy of the artificial internal environment of the structure. It is clear from Fig. 1 that when the angle \( \phi \) and the gradient parameters are varied, the curves of the variation of the longitudinal wave numbers of the hybrid modes \( HE_{\mu,\nu}, EH_{\mu,\nu} \) both with the same and with different indices \( \nu, \mu \) can intersect, that is, in certain directions of mode propagation. So we noted the fact that an effective transformation of the intrinsic hybrid modes of the compositional structure is possible. We proved in our results that it is possible to set the problems of an optimal synthesis of the tensor of material characteristics in nanocrystalline films by the given parameters of the guided hybrid modes.

The resulting system of differential equations includes many well-studied equations depending on the sign of the coefficients of the polynomial and its degree. The system of coupled equations (24), (25) is general, but it contains a large number of equations, which have not yet been studied, in addition to a number of the well-known equations. We can use a method of solution in the form of generalized power series or the Frobenius method for such systems of equations, which was further developed in our works. Therefore, we have written the tangential components of the electric and magnetic fields inside the waveguide structure for the biquadratic profiles of the elements of the tensors of the dielectric, magnetic and magnetoelectric permeabilities in the form:

\[
E_y = D_1 \Phi_1(q_0,q_1,\ldots,q_n,x) + D_2 \Phi_2(q_0,q_1,\ldots,q_n,x),
\]

\[
H_y = C_1 \Psi_1(q_0,q_1,\ldots,q_n,x) + C_2 \Psi_2(q_0,q_1,\ldots,q_n,x).
\]

Then we had to solve a task of tabulating of these new special solutions. We studied their dependence on the input gradient parameters, root distribution, asymptotic behavior of solutions for the practical application.

**Figure 1.** The dependence for the longitudinal wave number of the first hybrid modes in the compositional structure on the angle \( \phi \) for various parameters of the gradient profiles of the elements of the permittivity tensor:

\[
\varepsilon = \frac{\varepsilon_{\alpha,\alpha}}{\varepsilon_{\alpha,\alpha}} = 0.931, \quad \varepsilon = \frac{\varepsilon_{\alpha,\alpha}}{\varepsilon_{\alpha,\alpha}} = 0.97, \quad \varepsilon = \frac{\varepsilon_{\alpha,\alpha}}{\varepsilon_{\alpha,\alpha}} = 0.9804;
\]

\[
1 - q_i = q_{i} = 0; \quad 2 - q_i = 6 \times 10^{-2}, q_i = 36 \times 10^{-4}; \quad 3 - q_i = 10^{-2}; \quad q_i = 10^{-4}.
\]

We found the other components of the electric and magnetic vectors from the equations (12) - (15). Fields in the surrounding layers are represented as a sum of electric and magnetic waves. We have
found the transcendental equations for the constant propagation of intrinsic hybrid modes in the waveguide structure taking into account the boundary conditions. We obtained the results, which make it possible to create the foundations of a rigorous theory of waves in gradient bianisotropic regular and irregular structures.

6. References

[1] Kadomina E A, Bezus E A and Doskolovich L L 2016 Resonant photonic-crystal structures with a diffraction grating for refractive index sensing Computer Optics 40(2) 164-172 DOI: 10.18287/2412-6179-2016-40-2-164-172.

[2] Storozhenko D V, Dzyuba V P, Kulchin Y N and Amosov A V 2016 Excitonic optical nonlinearity of dielectric nanocomposites in weak optical fields Computer Optics 40(6) 855-662 DOI: 10.18287/2412-6179-2016-40-6-855-862.

[3] Chernozatonsky L A, Sorokin P B and Artyukh A A 2014 New nanostructures based on graphene: physical and chemical properties and applications Advances in Chemistry 83(3) 251-279.

[4] Fan Z 2010A three – dimensional corbon nanotube/grapheme sandwich and its application as electrode in supercapacitors Advanced Materials 22(33) 3723-3728.

[5] Labunov V A 2010 Composite nanostructure of vertically aligned carbon nanotube array and planar graphite layer obtained by the injection CVD method Quantum Electronics and Optoelectronics 13(2) 137-141.

[6] Ivakhnik V V and Nikonov V I 2017 Six-wave interaction with double wavefront reversal on thermal nonlinearity in a medium with a nonlinear absorption coefficient Computer Optics 41(3) 315-321 DOI: 10.18287/2412-6179-2017-41-3-315-321.

[7] Soifer V A, Korotkova O, Khonina S N and Shchepakina E A 2016 Vortex beams in turbulent media: review Computer Optics 40(5) 605-624 DOI: 10.18287/2412-6179-2016-40-5-605-624.

[8] Tkachev A and Zolotuhin I 2007 Equipment and methods for the synthesis of solid-state nanostructures (Moscow: Mashinostroenie) p 316 (in Russian).

[9] Hanson G W 2000 Dyadic Green’s functions and guided surface waves for a surface conductivity model of graphene J. Appe. Phys. V 103 197-203.

[10] Gonzalez J, Gtuinea F and Herrero J 2009 Propagating, evanescent, and localized states in carbon nanotube-graphene junctions Phys. Rev. B 79(16) 165-434.

[11] Bakeeva I 2008 Nanostructures: basic concepts, classification, methods of obtaining (Moscow: MITHT named after Lomonosov MV) p 68.

[12] Hauschild R and Kalt H 2006 Guided modes in ZnOnanjrods Appl. Phys. Lett. 89 123107.

[13] Chern R L 2013 Spatial dispersion and nonlocal effective permittivity for periodic layered metamaterials Opt. Express 21(14) 16514-16527.

[14] Gomez-Diaz J, Mosig J and Perruisseau-Carrier J 2013 Effect of Spatial dispersion on surface waves propagating along graphene sheets IEEE Trans. Antennas Propag 61(7) 3589-3596.

[15] Kireeva A I and Rudenok I P 2017 To the theory of diffraction of surface waves at the open end of a planar composite structure with an artificial medium Inzhenernyj Vestnik Dona 1.

[16] Kireeva A I and Rudenok I P 2017 Some elements of investigation the transformation of surface and pseudosurface modes in an open asymmetric thin-film structure with a synthetic medium SPIE Proc. 104662A DOI: 10.1117/12.2288778.

[17] Rudenok I P and Kireeva A I 2017 On a system of coupled equations of mode interactions in composite structures with a complex internal medium Thesis of XXIII Int.symp.Optical Technologies in Telecommunications 90-93.

[18] Rudenok I P and Kireeva A I 2017 Peculiar waves in planar continuously heterogeneous structures with optical bianisotropy Thesis of XIII Int. conf. Pulsed lasers and laser applications p 56.

[19] Rudenok I P and Kireeva A I 2017 On transformations of guided modes in composite nanostructures with gradient tensors of dielectric and magnetic permeabilities Thesis and reports of XVIInt. scientific and technical conf. The physics and technology of waveprocesses 19-22.