Dielectric Boosted Axion Haloscope Sensitivity In Cylindrical Azimuthally Varying Transverse Magnetic Resonant Modes

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Axions are a popular dark matter candidate which are often searched for in experiments known as “haloscopes” which exploit a putative axion-photon coupling. These experiments typically rely on Transverse Magnetic (TM) modes in resonant cavities to capture and detect photons generated via axion conversion. We present a study of a novel resonant cavity design for application in haloscope searches, of particular use in the push to higher mass axion searches (above ∼60µeV). In particular, we take advantage of azimuthally varying TM_{10} modes which, whilst typically insensitive to axions due to field non-uniformity, can be made axion sensitive (and frequency tunable) through strategic placement of dielectric wedges, becoming a type of resonator known as a Dielectric Boosted Axion Sensitivity (DBAS) resonator. Results from Finite Element Modelling (FEM) are presented, and compared with a simple proof-of-concept experiment. The results show a significant increase in axion sensitivity for these DBAS resonators over their empty cavity counterparts, and high potential for application in high mass axion searches when benchmarked against simpler, more traditional designs relying on fundamental TM modes.

I. INTRODUCTION

The composition and nature of dark matter continues to elude physicists, despite decades of observations implying its existence [1–3]. However, the search for compelling candidates is narrowing through various experimental and theoretical efforts. In particular, the class of particles known as WISPs (weakly interacting sub-eV particles) are becoming increasingly favoured as dark matter candidates [4]. The axion is one such particle, widely considered amongst the most compelling dark matter candidates, which arises as a consequence of an elegant solution to the strong CP problem in QCD [5].

The proposal of the axion haloscope by Sikivie in 1983 was one of the first plausible methods of detecting axions in the lab by way of exploiting their expected coupling with photons [6]. The inverse Primakoff effect is the mechanism by which an axion decays into a real photon through the absorption of another photon. Traditionally, a strong DC magnetic field is used to saturate a resonant cavity in a sea of virtual photons. Dark matter axions may scatter off these virtual photons, producing detectable photons with a frequency corresponding to the mass of the axion. If the cavity contains a geometrically appropriate resonant mode at the correct frequency, these photons will be captured in the cavity and the signal will be resonantly enhanced. The power in the cavity can then be read out via a low-noise receiver chain. However, because the axion mass and the strength of its coupling to photons is unconstrained by theory, there exists a very large parameter space to be searched, and many experiments are required to span the range. Several such experiments exist [7, 8], with many focused around the microwave frequency band (corresponding to masses in the µeV range). However, many experiments are increasingly interested in lower [9] and higher [10] mass axions.

The majority of the physics of the axion is determined by a parameter known as the Peccei-Quinn symmetry breaking scale, f_a, which arises in the solution to the strong CP problem which motivates the axions. f_a is what most axion experiments ultimately hope to measure or constrain. This parameter is unconstrained by theory (although some cosmological constraints exist [11, 12]). f_a determines the axion mass and the strength of its coupling to photons according to

\[ m_a \sim \frac{4.51 \times 10^{15}}{f_a} \text{eV} \]

\[ g_{a\gamma\gamma} = \frac{g_\gamma \alpha}{f_a \pi} \]  

(1)

Here, m_a is the mass of the axion, g_{a\gamma\gamma} is the two photon coupling constant of the axion and α is the fine structure constant [13–15]. The dimensionless axion-model dependent parameter g_γ is of order one, and takes different values in different axion models. In the most popular two models, the KimShifmanVainshtein-Zakharov (KSVZ), and DineFisher-SrednickiZhitnisky (DFSZ) models, g_γ takes values of -0.97 and 0.36 respectively [13–15].

To date, The Axion Dark Matter eXperiment (ADMX) is the most sensitive and mature haloscope experiment, placing impressive exclusion limits on the searchable pa-
rameter space [16–18]. However, current ADMX cavity designs are limited to probing masses of the order of a few μeV at KSVZ and DFSZ sensitivity.

Currently, the high axion mass regime (> 60μeV or 15GHz) is largely inaccessible using traditional haloscope designs, attributed to the substantial decrease in sensitivity in this mass range owing to a range of technical factors which will be discussed below. Interestingly, despite the lack of sensitive experimental constraints, this high mass region has benefited from a recent surge in theoretical and observational motivation [19–22]. For example, the SMASH model favours axions with mass ~ 100μeV [21].

As discussed, haloscopes operate on the principle that axions from the galactic dark matter halo are resonantly converted into detectable photons in a cavity. The signal power due to axion-photon conversion for a critically-coupled cavity, with axion conversion occurring on resonance is given by [23]

\[ P_a \propto g_{a\gamma\gamma}^2 B^2 CV Q_L \frac{\rho_a}{m_a} \]  

(2)

The parameters \( g_{a\gamma\gamma} \), \( m_a \) and the local axion halo dark matter density \( \rho_a \) are beyond experimental control. However, the external magnetic field strength \( B \), cavity volume \( V \), mode dependent form factor (of order 1) \( C \), and loaded quality factor \( Q_L \) are parameters within experimental control [24]. The form factor for a given mode in a cylindrical cavity, with a homogeneous static magnetic field aligned in the \( \hat{z} \) direction can be defined as

\[ C = \left| \frac{\int \overline{dV \cdot E_c \cdot \hat{z}}} {\int \overline{dV \cdot \epsilon_r |E_c|^2}} \right|^2. \]  

(3)

Here \( \overline{E_c} \) is the cavity electric field and \( \epsilon_r \) is relative dielectric constant of the medium. For a non-zero form factor, there must exist some degree of overlap between the electromagnetic field of the axion induced photon and electromagnetic field of the resonant cavity mode, and the integral of this overlap must be non-zero. Thus, in an empty cylindrical cavity, only TM\(_{0n0} \) modes will couple to axions in the experimental context outlined above. The highest form factor belongs to the TM\(_{010} \) mode, which is consequently the mode of choice for most haloscope searches.

The mode quality factor \( Q \) can be calculated through the mode dependent geometry factor \( G \).

\[ Q = \frac{G}{R_s} \quad G = \frac{\omega \mu_0 \int |\vec{H}|^2 dV}{\int |\vec{H}|^2 dS} \]  

(4)

Here \( R_s \) is the surface resistance of the material, \( \vec{H} \) the cavity magnetic field, \( \omega \) the resonant angular frequency of the cavity mode and \( \mu_0 \) the vacuum permeability. It is assumed throughout this work that resistive wall losses are the dominant loss mechanism, far greater than any losses in low-loss dielectric materials.

As mentioned, poor constraints on the axion mass and photon coupling strength create a large searchable parameter space. This places a high premium on axion-sensitive haloscopes with frequency tuning mechanisms. We therefore define the scanning rate of a haloscope as [25]

\[ \frac{df}{dT} \propto \frac{1}{SNR_{goal}^2} \frac{g_{a\gamma\gamma}^4 B^4 V^2 \rho_a^2 Q_L Q_a}{m_a^2 (k_B T_S)^2}. \]  

(5)

Where \( SNR_{goal} \) denotes the chosen signal-to-noise ratio, \( T_s \) represents the total system noise temperature, largely due to the noise of the first stage amplifier, and \( Q_a \sim 10^6 \) is the effective axion signal quality factor, owing to the velocity distribution of dark matter. The sensitivity of an experiment is therefore measured by the rate at which a haloscope can scan through a frequency range, at a desired level of axion-photon coupling and signal-to-noise ratio. The figure of merit for resonator design is then given by the quantity \( C^2 V^2 Q_L \), or equivalently \( C^2 V^2 G \), as these are the controllable parameters which explicitly depend on the chosen resonator.

Now we can see why axion haloscopes become increasingly difficult at high masses. The volume, \( V \) of resonant cavities scales by \( V \propto f^{-3} \), and the expression contains an explicit dependence on \( m_a^{-2} \). Furthermore, the noise temperature, \( T_S \) of amplifiers increases at higher frequency, and the surface resistances of materials increase leading to a decrease in \( Q_L \). All of these factors conspire to decrease \( \frac{df}{dT} \) rapidly with increasing axion mass, making haloscope searches extremely difficult, requiring careful resonator design. Some suggestions on how to mitigate this problem at high frequencies include multiple cavity designs [8, 10].

### II. DIELECTRIC HALOSCOPES

Dielectric embedded haloscopes have been of growing interest in recent times. Since axion conversion has a high dependence on field geometry, the addition of dielectric in suitable regions can alter the geometry to favour axion conversion. Experiments such as The Electric Tiger [26], Orpheus [27] and MADMAX [28] incorporate dielectrics to facilitate their axion searches, for various reasons.

Traditional, tuning rod cavity haloscope designs, like the ones used by ADMX [16, 17], exploit the TM\(_{010} \) mode for its superior form factor of \( \sim 0.69 \). However, resonators that utilise lower order modes are ineffective at higher frequencies due to the dramatic decrease in volume, since the cavity dimensions must be of order \( \lambda/2 \), where \( \lambda \) is the axion’s Compton wavelength (which decreases with mass). Higher order resonances are thus attractive in the push to probe higher axion frequencies, allowing for higher cavity volumes at a given frequency. The cost of using higher order modes is the large de-
FIG. 1: The $E_z(\phi)$ field of a TM$_{m10}$ mode shown before and after the addition of dielectric wedges (blue regions), as viewed in the azimuthal direction.

gree of field variation, resulting in degraded form factors, cancelling out the sensitivity benefit from the increased volume. For example, compared to a TM$_{010}$ mode in an empty cavity, a TM$_{020}$ has a significant portion of its $E_z$ field out of phase with the applied $B$ field, reducing the coupling between the cavity mode and the axion, and degrading the form factor to $\sim 0.13$. However, this issue can be addressed with the use of carefully placed dielectric materials to alter the field structure of the higher order modes, as has been previously considered for the case of TM$_{0n0}$ modes with $n > 1$.

As shown by McAllister et al. (2017) [29], careful placement of dielectric “rings” in out of phase regions of the $E_z$ field of higher order TM$_{0n0}$ modes successfully mitigates this loss in form factor, while keeping the cavity volume high. This is possible due to the fact that dielectric structures effectively suppress electric field. Additionally, TM$_{010}$ modes are highly uniform, which makes frequency tuning difficult due to the high degree of symmetry. The use of dielectrics can be exploited to create “built-in” tuning mechanisms as a result of more free parameters and broken symmetries in the cavity. Such resonators were named Dielectric Boosted Axion Sensitivity (DBAS) resonators in the context of TM$_{0n0}$ modes. This has been further confirmed by some recent experiments by Kim et. al. [30], who introduced further ways to tune such TM$_{0n0}$ modes with reasonable frequency tunability. Also, Alesini et. al. [31] recently realised a fixed frequency prototype for axion searches, with boosted quality factor. In this work we consider a new type of DBAS resonator, in the context of TM$_{m10}$ modes with $m > 0$, and will refer to them as the Wedge DBAS resonators. This resonator appears visually similar to a dielectric equivalent to the multiple cell “Pizza” resonator proposed recently [32]. In this case, the wedges act like the boundaries of the individual “Pizza” cells.

III. WEDGE DBAS RESONATORS

The cavity mode electric field, $\vec{E}_c$ for a given TM$_{m10}$ mode inside a hollow cylindrical resonator of radius $R$, parametrised in cylindrical coordinates $r$, $\phi$ and $z$ is defined as

$$\vec{E}_c = E_0 e^{i\omega t} J_m \left( \frac{\varsigma_{m1}}{R} r \right) \cos(m\phi) \hat{z}. \quad (6)$$

Where $E_0$ is some constant denoting the amplitude of the field, $J_m$ is a Bessel J function of order $m$, with $\varsigma_{m1}$ denoting its 1st root (i.e. the cavity wall). The field is in one phase in the $r$ direction, but alternates in phase $m$ times in the $\phi$ direction over the $2\pi$ range. Therefore implementing the DBAS method for a given TM$_{m10}$ mode would require placement of $m$ dielectric wedges in the $m$ lobes of one of the phases, suppressing their contribution to the form factor integral shown in equation 3 by suppressing the field amplitude in these regions.

Maximising the out of phase $E_z$ field confinement inside the dielectric wedges is done by placing the dielectric boundaries of the wedges between nodes of the field. For a TM$_{m10}$ mode, each of the $m$ total azimuthal variations occurs over a range of $\frac{2\pi}{m}$ radians, in this range the field must alternate between maxima in both phases. We denote the optimal dielectric region size by $\theta$ (the angular size of each wedge) and the region without dielectric by $\bar{\theta}$ (vacuum). Hence we can find $\theta$ by demanding that

$$\theta + \bar{\theta} = \frac{2\pi}{m}. \quad (7)$$

Introducing dielectric material reduces the speed of light within it by a factor of $\sqrt{\epsilon_r}$. This is tantamount to the space inside the dielectric increasing by a factor of $\sqrt{\epsilon_r}$, and so the physical size of the dielectric wedge must be decreased by this factor to meet our optimal condition. In the empty cavity structure, the angular size of the two phases is equal, and here we are reducing only one of them, such that $\theta = \bar{\theta}/\sqrt{\epsilon_r}$. Considering (7), the optimal
dielectric wedge thickness, $\theta$, can then found to be

$$\theta = \frac{2\pi}{m(1 + \sqrt{\varepsilon})}. \quad (8)$$

Figure 1 shows the implementation of the Wedge DBAS method by placing dielectric (blue regions) of appropriate thickness in the out of phase parts of the $E_z$ field. It should be noted that this sketch is not to scale and only serves to show the effects of adding dielectric; namely the suppressed amplitude of the $E_z$ field and the reduced size of the dielectric region as compared to the empty region (vacuum). Integrating $E_z \cdot \hat{z}$ over the entire range now produces a non-zero value, and hence a non-zero form factor.

IV. MODELLING

A. 4 Wedge DBAS cavity

Using Finite Element Modelling (FEM) in COMSOL Multiphysics, we investigated the axion sensitive TM modes in a 4 wedge resonator, with sapphire chosen as the dielectric, and wedge sizes as per (8) with $m = 4$. Potential axion haloscope mode candidates must be highly tunable whilst retaining a sufficiently high scan rate, as indicated by the product $C^2V^2G$, computed via the FEM. The “built-in” frequency tuning mechanism for the wedge-type cavities explored in this work relies on tuning via the relative angular separation of the wedges from one another. This can be achieved in an $m$-wedge cavity with $m/2$ wedges which are stationary, and $m/2$ wedges which can move relative to the others. We denote the tuning parameter (degree of wedge rotation) by $\phi$ and define $\phi = 0$ as the starting symmetric position where the angular separation between all wedges is the same. The maximum possible tuning is given by $\theta$ radians, defined from $\phi = 0$ to the angular position where the wedges are touching. We find $\theta \sim 1.21$ using equation 8, where for sapphire, $\varepsilon_r \sim 11.349$. Various axion sensitive, tunable modes exist in the structure, and will now be named, discussed and compared in turn.

1. The $TM_{410}$ mode

Shown in the top panel of figure 2 is the field structure of the $TM_{410}$ mode as the wedges tune together from left to right. Although reminiscent of a $TM_{410}$ mode, strictly speaking, this mode is not a true $TM_{410}$ due to the intruding dielectric. Using the results from FEM and equation 3, we determine the form factor for this mode at each $\phi$ position. We find that the $TM_{410}$ mode successfully confines out of phase lobes of the $E_z$ field to produce a non-zero form factor, $C \sim 0.37$ at the $\phi = 0$ position, decreasing to $C \sim 0.0055$ at the $\phi = 1.2$ position.

2. The $TM_{410}$-like mode

In an empty cavity, there exist degenerate doublet $TM_{m10}$ modes that are a quarter period out of phase, or $\frac{\pi}{2m}$. However, when dielectric is added and azimuthal symmetry is broken, the modes break degeneracy and move to different frequencies. Through FEM, we find the formerly degenerate doublet of the $TM_{410}$ mode, with a similar but distinctly different field structure. This mode is therefore referred to as the $TM_{410}$-like mode, whose field structure is shown in the second panel of figure 2. It should be noted from our discussion on form factor that it is only after perfect symmetry between the wedges breaks ($\phi \neq 0$), that this mode becomes axion sensitive. Past this initial position, the form factor gradually increases to $\sim 0.2$ at the $\phi = 1.2$ position.

3. The $TM_{210}$ mode

DBAS $m$-wedge resonators that have $m \geq 4$ and even can once again exploit azimuthal symmetry to find $TM_{210}$ modes, that tune in the same way as the previous two modes. These fractional modes only show significant sensitivity when the wedges are close together. This is an intuitive result, since we can think of an $m$-wedge resonator with its wedges tuned together as effectively being a $\frac{m}{2}$-wedge resonator, with an axion sensitive $TM_{210}$ mode. Indeed, the optimal wedge angle for the $TM_{210}$ mode is exactly double that of the $TM_{m10}$ mode. Through FEM we find the field structure of the $TM_{210}$ mode in a 4 wedge cavity as shown in the bottom panel of figure 2. It is clear that this mode begins with a form factor of zero and gradually becomes more sensitive as tuning progresses, increasing to have $C \sim 0.13$ at the $\phi = 1.2$ position.

4. Sensitivity

The relevant axion sensitivity and frequency tuning results of FEM for a 4 wedge DBAS cavity with a radius of 20mm, a height of 60mm, and an angular wedge thickness ($\theta$) of $\sim 0.36$ rads are shown in fig. 3. Although the total tuning range is shown, it is useful to define a so-called “sensitive” tuning range, which only considers a given mode when it is within an order of magnitude of the maximum $C^2V^2G$. The $TM_{410}$ mode shows the greatest peak sensitivity, however tuning of this mode is poor, with a starting frequency of $\sim 13.7$ GHz, we observe a total and sensitive tuning of $\sim 720$ and $\sim 70$MHz respectively. The “doublet” $TM_{410}$-like mode however is a much more promising candidate for axion searches, offering substantial sensitivity over a broad tuning range, with a total and sensitive tuning of $\sim 2.4$ and $\sim 1.7$GHz respectively. Analogous to the $TM_{410}$ mode, the $TM_{210}$, although sensitive to axion detection, also suffers from a
FIG. 2: The $E_z$ profile of the TM$_{410}$ (upper), TM$_{410}$-like (middle) and TM$_{210}$ (lower) modes in a 4 sapphire wedge cavity as the wedges move together. It should be noted that the dielectric cylinder at the center acts only to increase the minimum mesh size in that area and has little to no impact on the field structure. Each mode is shown at tuning angles of $\phi = 0$, 0.4, 0.8 and 1.2 rads from left to right respectively.

FIG. 3: Left: Resonant frequencies of the TM$_{410}$ (blue), TM$_{410}$-like (orange) and TM$_{210}$ (green) modes shown as a function of tuning angle ($\phi$). Right: $C^2V^2G$ product as a function of frequency for the three modes of interest.

poor degree of tuning, reporting only a total and sensitive tuning of $\sim$340 and $\sim$320MHz respectively.

B. 8 Wedge DBAS cavity

Successfully finding 3 axion sensitive modes in the 4 wedge configuration, we now investigate higher order TM modes in an 8 wedge configuration, whilst using the same cavity dimensions. The angular size of each wedge will then be exactly half the size used in the 4 wedge resonator, as shown from equation 8. The results of FEM in an 8 wedge DBAS cavity are shown in figures 4 and 5. Whilst not surprising, it is clear from the field profiles, that the same three modes of interest (with twice the number of azimuthal variations) also exist in the 8 wedge cavity. Where the fractional TM$_{210}$ mode is now a TM$_{410}$, as shown in the bottom panel of figure 4. We can then think of the TM$_{410}$ mode as having two tuning regimes; one in the earlier presented 4 wedge cavity and another in the 8 wedge configuration. Importantly, this mode is axion sensitive across different regions of frequency space for the two tuning regimes, effectively extending the mode’s sensitive tuning range. Again, the fractional TM$_{410}$ mode starts with a form factor of zero, increasing to have $C \sim 0.37$ at the $\phi = 1.2$ position, equivalent to the $\phi = 0$ position in the 4 wedge cavity. Interestingly, the TM$_{410}$ mode performs better in the 8 wedge configuration when it comes to the total ($\sim 1.6$GHz) and sensitive ($\sim 1.3$GHz) tuning.

In similarity to the modes presented in the previous 4-wedge iteration, the TM$_{810}$ has $C \sim 0.33$ to be maximal
FIG. 4: The $E_z$ profile of the TM$_{810}$ (upper), TM$_{810}$-like (middle) and TM$_{410}$ (lower) modes in a 8 sapphire wedge cavity as the wedges move together. Each mode is shown at tuning angles of $\phi = 0$, 0.2, 0.4 and 0.6 rads from left to right respectively.

FIG. 5: Left: Resonant frequencies of the TM$_{810}$ (brown), TM$_{810}$-like (purple) and TM$_{410}$ (red) modes shown as a function of tuning angle ($\phi$) in an 8 sapphire wedge cavity. Right: $C^2V^2G$ product as a function of frequency for the three modes of interest.

at the starting $\phi = 0$ position, whereas the TM$_{810}$-like mode is completely axion insensitive at this position. As tuning progresses, the TM$_{810}$ becomes less sensitive with $C$ decreasing to $\sim 0.003$, while the TM$_{810}$-like mode increases to have a maximum $C \sim 0.11$. The total and sensitive tuning for the TM$_{810}$ mode is poor, with $\sim 740$ and $340\text{MHz}$ respectively. In contrast the TM$_{810}$-like mode has a more impressive tuning of $\sim 2.5$ and $\sim 1.8\text{GHz}$ respectively.

C. Practicalities and Mode Crossings

As previously discussed, novel cavity design is an essential step in the push towards searching the higher frequency axion parameter space. However, there is an inherent trade off between cavity volume and the use of higher order modes. The DBAS method seeks to rectify this by mitigating the downside of higher order modes (reduction in form factor), whilst keeping the cavity volume high. However, higher order modes should be approached with caution, as they introduce significant mode crowding and risk “avoided level crossings”, resulting in degraded axion sensitivity in those regions of frequency space. To combat this, we opted for a relatively low aspect ratio of 3, thus preventing higher order length dependent mode crowding.
V. POSSIBLE IMPLEMENTATION AND COMPARISON

In principle, it would be possible to combine the 8 and 4 wedge cavities discussed here. If we we began with the 8 wedge configuration, and tuned the wedges until they were touching, each pair of 2 wedges would be the same size as the wedges in the 4 wedge cavity. We could then tune 2 of these new, thicker wedges relative to the other 2, and recreate the tuning of the 4 wedge cavity. In this way, all 6 axion sensitive modes would become accessible within a single cavity. Since FEM modelling for both configurations was done using the same cavity dimensions, we plot $C^2V^2G$ against frequency for all 6 modes, as shown in fig. 6. Although possible in principle, a modular Wedge DBAS design is highly conceptual and would face significant practical challenges in its implementation. Foremost of which is an intricate tuning mechanism such that the 8 wedge configuration can “fold” into 4 wedges, and then be tunable after. Alternatively, one could avoid significant engineering and complexity by simply inserting the desired wedge configuration, since the cavity radius is the same for both regimes.

To assess the viability of new haloscope designs, it is common practice to compare against a reference cavity that tunes in the same frequency range. We have chosen to benchmark against a TM$_{810}$ mode tuned by radially moving a conducting rod, resulting in subtle changes to the mode geometry, thus altering the resonant frequency, this is the type of resonator used by world-class haloscopes, and thus a good comparison for a novel design. Overlaid in black (dashed) in fig. 6 is the $C^2V^2G$ data for this benchmark cavity, constructed and additionally scaled such that the frequency tuning ranges are comparable with the other wedge cavity designs presented. To create a clear comparison, an aspect ratio of 3 was also chosen for the benchmark cavity. There are ultimately many free parameters, and much optimisation possible in the design of both schemes, and thus the compared designs should be thought of as a relatively simple one. However it should be noted that the benchmark cavity, although comparable in sensitivity in these regions, becomes increasingly impractical to implement in the high frequency regime, attributed to the significantly reduced cavity and tuning rod dimensions.

As shown, almost all of the modes of interest have a peak $C^2V^2G$ greater than the benchmark cavity, with some modes sustaining an improved scan rate over their entire sensitive tuning range. If implemented, this modular cavity design is very attractive as an axion haloscope in the hard to reach but well motivated high mass regime, due to its broadband tuning and high sensitivity, but we can take it even further.

Using the inverse relationship between radius and frequency ($\omega \propto R^{-1}$), we can simply scale the results from the modelled cavity to imitate the results of a second cavity with a slightly different radius, so that the gaps in the previous sensitivity plot (fig. 6) are filled by a second resonator of the same type. As a result, 2 cavities of slightly different radii can be used to almost completely cover a frequency range between 9.5-21.5GHz with a high degree of sensitivity. In the case of uniform rescaling, the volume changes with the cube of the radius, whereas mode dependent factors \(C \) and \(G \) remain unchanged. Therefore increasing the resonant frequency of a particular mode by a factor \(f \) results in $C^2V^2G$ decreasing by factor \(f^6 \). The second cavity was scaled such that the resonant frequencies for the modes of interest increased by a factor \(f = 1.11 \), degrading $C^2V^2G$ by $f^{-6} \approx 0.53$. Once again it is clear why many experiments have so far been unable to probe the higher frequency parameter space. As shown in figure 7, the combination of two multi-stage cavities can almost completely cover a 12GHz region with $C^2V^2G$ greater than $10^{-8}$.
FIG. 8: The $E_z$ profile of the TM$_{410}$-like (upper) and TM$_{410}$ (lower) modes in a 4 teflon wedge cavity as the wedges move together. Each mode is shown at tuning angles of $\phi = 0$, 0.2, 0.6 and $\sim$0.93 rads from left to right respectively.

VI. PROOF OF CONCEPT EXPERIMENT

To assess the viability of a Wedge DBAS type resonator, a prototype with 4 teflon wedges was first considered. As a proof of concept this cavity was expected to tune the TM$_{410}$ and doublet TM$_{410}$-like modes in line with expectations from the COMSOL modelling (within experimental uncertainty). Teflon wedges were an ideal choice due to their relatively low cost and ease of production, unlike more expensive, harder to machine, low-loss crystals such as sapphire. Based on the success of the teflon proof of concept, a sapphire resonator will be constructed and tested.

We use equation 8 to once again find the optimal teflon wedge thickness. The copper cavity had a radius of 13.47mm and a height of 22.5mm. The field profiles for the modes of interest in the teflon cavity are shown in fig. 8 and closely resemble what is seen in the sapphire iteration, albeit with a significant reduction in the degree of out of phase field suppression and the presence of a central lobe, attributed to teflon’s comparatively low permittivity, $\epsilon_r \sim 2.1$. The TM$_{210}$ mode is not shown here and was not further investigated due to an absence of tuning, as indicated by the initial FEM results.

The proof of concept measurements were done at room temperature using a Thorlabs stepper motor and rotation stage. The lid of the cavity was clamped down, such that the base of the cavity was able to rotate relative to the lid. Two of the wedges were affixed to the base, and two to the lid, meaning that as the base tuned with respect to the lid, two of the wedges tuned. The cavity was coupled to with coaxial antennae, and transmission measurements were made with a Vector Network Analyzer as a function of wedge angular position. The

FIG. 9: The teflon Wedge DBAS cavity used in the proof of concept experiment. As discussed in the text, a pair of diametrically opposed wedges are mounted to the moveable lid, while another pair are affixed to the base.

FIG. 10: Colour density plot of the transmission coefficient as a function of resonant frequency and tuning angle $\phi$. Of specific interest are the TM$_{410}$ (lower) and TM$_{410}$-like modes (upper), identified by hand-taken measurements (orange) and the predicted tuning from FEM (blue). Darker regions represent less transmission while lighter regions represent greater transmission.
two modes were first tracked by hand using a step size of 0.02 radians, and later via automated transmission coefficient measurements that used a 0.0087 rad step size. The modelled and measured frequencies are shown as a function of tuning angle in figure 10. Horizontal error bars placed on the measured data are due to the Thorlabs rotation stage quoting an accuracy in angular position of ±820 µrad. This being an open-loop system, the horizontal error compounds for each subsequent measurement. Additionally, deviations from perfect symmetry can significantly perturb the mode field structure and hence frequency. Unequal wedge sizes, wedge tilt, crude measurements of their thickness (within ±0.02 rad) and the addition of probes greatly effect the resonant frequency of the mode. Modelling these small perturbations in COMSOL in conjunction with other uncertainties resulted in a total uncertainty of approximately ±150 MHz in the modelled frequencies (<1% of the central starting frequency). The vertical error bars on the FEM data represent this uncertainty.

Importantly, the overall shape of the two modelled modes match almost perfectly what is seen experimentally. We also observe highly responsive frequency tuning as a result of the novel “built-in” tuning mechanism. Furthermore, the modes in the proof of concept cavity were at even higher frequencies than the modelled sapphire cavity, owing to the diameter of the available teflon stock - and very few avoided level crossings were observed over the experimental tuning range. These factors demonstrate the viability of this promising resonator design.

VII. CONCLUSION

This work presents a theoretical and experimental study of a novel Wedge DBAS cavity resonator for use in high mass axion haloscopes. Through strategic placement of dielectric structures, these resonator designs were shown to significantly boost the form factors of various TM_{m,10} modes. The results of FEM for both 8 and 4-wedge cavity configurations are presented, and show 6 axion sensitive modes with varying widths of frequency tuning. We compare their performance with a conventional conducting rod resonator, and find the DBAS cavity modes to boast superior C^2V^2G products, albeit each over reduced tuning ranges. However, the viability of this resonator design is enhanced when both the 8 and 4 wedge regimes are combined into a single 2-stage resonator, effectively broadening its sensitive tuning range by allowing access to all 6 modes within a single cavity. These modes are especially promising for applications at higher frequencies than those accessible with traditional rod-tuned haloscopes.

Also undertaken was a proof of concept experiment using a prototype teflon Wedge DBAS cavity. The cavity’s built-in tuning mechanism was successful in altering the frequency for the modes of interest in a highly responsive and reliable way, demonstrating the feasibility of such a design. Currently, plans are in place to commission a cryogenic-compatible 4-wedge cavity using less lossy, higher permittivity sapphire for possible implementation in The ORGAN Experiment.

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