Modeling quantum information dynamics achieved with time-dependent driven fields in the context of universal quantum processing

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Modeling quantum information dynamics achieved with time-dependent driven fields in the context of universal quantum processing

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Abstract. Quantum information is a useful resource to set up information processing. Despite physical components are normally two-level systems, their combination with entangling interactions becomes in a complex dynamics. Already studied for piecewise field pulses, this work now analyzes the modeling for quantum information operations with fields affordable technologically towards a universal quantum computation model.

1. Introduction
Quantum information takes advantage of quantum physical systems and their properties as superposition and entanglement, where two-level systems are combined and scaled becoming in complex behaviors. Instead of the original states of isolated systems, the use of more advisable non-local basis to set the grammar lets exceed the complexity (polarization or spin). We focus the analysis on magnetic systems for two qubits following the Heisenberg-Ising model with magnetic fields in a fixed direction $h = 1, 2, 3$:

$$H_h = \sum_{k=1}^{3} J_k \sigma_1^{(k)} \sigma_2^{(k)} - B_{1h} \sigma_1^{(h)} - B_{2h} \sigma_2^{(h)} \quad (1)$$

here, Bell states recover the binary dynamics [1]. The $SU(4)$ dynamics is split in two subspaces with $SU(2)$ dynamics $(U(1) \times SU(2))^2$ as $U_h(t) = s_{h1} \oplus s_{h2})$. In terms of Hilbert space: $\mathcal{H}_h^\otimes 2 = \mathcal{H}_{h,1} \oplus \mathcal{H}_{h,2}$, thus the states are split in the subspaces $|\psi(t)\rangle = \alpha_1 |\psi_1(t)\rangle + \alpha_2 |\psi_1(t)\rangle$, $|\alpha_1|^2 + |\alpha_2|^2 = 1$ and $|\psi_k(t)\rangle = s_{hk} |\psi_k(0)\rangle$. Then, $H_h = H_{1,h} \oplus H_{2,h}$ [1] with:

$$H_{k,h} \equiv \tilde{H}_{k,h}^0 + \tilde{H}_{k,h}$$

with: $\tilde{H}_{k,h}^0 = -s_0 J_h \sigma_0^{(h,k)}$, $\tilde{H}_{k,h} = s_1 J_{(h)} s_0 \sigma_3^{(h,k)} + s_2 B_{h,s} \sigma_q^{(h,k)}$

$\sigma_{i,k} = 0, ..., 3$ are the Pauli matrices for a space generated by paired Bell states. $\sigma_q^{(h,k)} = -(q-2)\sigma_1^{(h,k)} + (q-1)\sigma_2^{(h,k)}$, with $s_0 = (-1)^{h+k+1}, s_1 = s_0^p, s_2 = (-1)^p s_0^{p+q}$ and $p =$
1 + \frac{1}{2}(h-1)(h-2), q = 2 - h \text{mod} 2 \text{ depending on } h, k, \text{ together with the arrangement of pairs of Bell states generating each subspace [1] (not relevant for this development). } J_h, J_{[h]}_{s_0}, B_{h-s_0} \\
are obtained from strengths and fields. The block structure of \tilde{H}_{k,h} \text{ for } k = 1, 2 \text{ is inherited to } U_h(t) = s_{h1} \oplus s_{h2} \text{ through the time ordered integral [2] fulfilling the Schrödinger equation. As } \tilde{H}^0_{k,h} \text{ commutes with } \tilde{H}_{k,h}, \text{ by defining } s_{hk} \equiv \exp(-\frac{i}{\hbar} \tilde{H}^0_{k,h}) s_{kh}^0 \text{ (revealing the } U(2) = U(1) \times SU(2) \text{ structure), we get } s_{hk}^0 \text{ fulfilling the Schrödinger equation with } \tilde{H}^0_{k,h}. \text{ Then, we will work with } \tilde{H}_{k,h} \text{ and } s_{hk}^0. \text{ Analytical solutions for time-independent or stepwise fields exist [3, 4] but they are few feasible because resonant effects.}

This work sets a procedure a control procedure with fields as those in resonant cavities, ion traps and laser beams [5, 6, 7, 8]. In the second section, we benchmark linear and quadratic models to solve numerically the time-dependent problem. Third section presents few feasible because resonant effects.

2. Evolution in the time-dependent regime

Baker-Campbell-Hausdorff formula rarely provides closed analytical solutions for time-dependent problems. Alternatively, numerical approaches are necessary. Here, we combine the SU(2) reduction with linear and quadratic approaches to solve the time-dependent problem getting the generic block \( s_h \) for composite quantum systems. A comparative benchmark is presented at the end. By defining the differential evolution operator \( |\psi_k(t_0 + \delta t)\rangle = s_{hk}^0(t_0 + \delta t, t_0) |\psi_k(t_0)\rangle \) and \( J_h = \hbar J_0, -s_1 J_{[h]}_{s_0} = \hbar J_{s_0}, -s_2 B_{h-s_0} = \hbar B_{-s_0} \):

\[
s_{hk}^0(t_0 + \delta t, t_0) \approx 1_k - \frac{i}{\hbar} \tilde{H}_{k,h}(t_0)\delta t - \frac{i}{2\hbar} \left( \frac{\partial \tilde{H}_{k,h}(t_0)}{\partial t} + \frac{\tilde{H}_{k,h}^2(t_0)}{\hbar} \right) \delta t^2 + \ldots
\]

\[
\approx \sigma_0^{(h,k)} + iL^{(h,k)}\delta t - \frac{1}{2}Q^{(h,k)}\delta t^2 + \ldots
\]

with: \( L^{(h,k)} = J_{s_0} \sigma_3^{(h,k)} + B_{-s_0}(t_0)\sigma_q^{(h,k)} \) and \( Q^{(h,k)} = (J_{s_0}^2 + B_{-s_0}(t_0))\sigma_0^{(h,k)} - iB_{-s_0}(t_0)\sigma_q^{(h,k)} \).

By splitting \([0, t]\) in \(n\) intervals \([0, \delta t] \cup \ldots \cup [(n-1)\delta t, n\delta t = t]\) and using (2):

\[
s_{hk} = s_{hk}(t, 0) \approx \prod_{i=1, \ldots, n} s_{hk}(i\delta t, (i-1)\delta t) = e^{i\sigma_0 J_0 t} \prod_{i=1, \ldots, n} s_{hk}^0(i\delta t, (i-1)\delta t)
\]

where \(\leftarrow\) means factors stack on the left in the product and \(\delta t \approx 0\) is assumed. As in the independent-time case, \( J_0 \) is only responsible from the weak \( U(1) \) link between the two \( SU(2) \) blocks through \( e^{i\sigma_0 J_0 t} \). (3) is a second order approximation in \(\delta t\) for \(s_{hk}\). We have reduced the dependence \(s_{hk} \ (k = 1, 2) \) on the parameters \( J_0, J_\pm, B_\pm(t) \).

3. Construction of universal operations

We fix \( B_{h\pm}(t) \) in a model applicable for resonant cavities, laser beams, ion traps or superconducting circuits [5, 6, 7, 8] . Despite possible modes \((m = 1, 2, \ldots, )\), we will take one single mode \(m\). Absorbing physical quantities: \( t' = \frac{m}{\hbar}t \ (d\ an\ effective\ wavelength\ or\ length\ in\ the\ system) \) and \( A_\pm \equiv A_\pm \frac{d}{mc} \), \( J'_\pm = J_\pm \frac{d}{mc} \), we assume (dropping the apostrophe):

\[
B_{h\pm}(t) = A_\pm \sin(\pi t)
\]
For the numerical approach of $s_{hk}$, we develop a benchmark comparing linear and quadratic approximations in (3, 5) (Figure 1a). By running $5 \times 10^4$ random experiments distributed uniformly on $J_{\pm}, A_{\pm}$, we track both the average time of computer processing and the $p$ correct figures reached. The process was repeated for $n$ ranging from 10 to $10^4$, showing at least one figure of improved performance for the quadratic approach in (3), then reducing $n$ from thousands to hundreds. Our implementation (second order and $n = 100$) reaches at least five figures of precision. Introducing (6) in (3), we are interested in the reduction of $s_{hk}$ into:

$$s_{hk} = e^{i\varphi} s_{hk}^0 = e^{i\varphi} \begin{pmatrix} A e^{i\phi} & B e^{i\theta} \\ -B e^{-i\theta} & A e^{-i\phi} \end{pmatrix}$$

with $A^2 + B^2 = 1$. It reduces the dynamics in several traditional quantum processing operations as $C^a NOT_b$, $NOT$, Hadamard, etc. under the $SU(2)$ reduction scheme [4]. It is affordable imposing restrictions on $A$, and some concrete prescriptions for $\phi, \theta$ and $\varphi$ during $t \in [0, 1]$. Figure 1b shows a set of solutions for $A_\pm, J_\pm$ with $\phi$ in the color chart and $\theta$ as an inset for a set of values for $A$.

**Figure 1.** a) Benchmarking for linear and quadratic approximation as function of $n$ (partition), $p$ (digits) and time computer processing (Intel Core i7 2.6GHz); and b) Solutions for $A_{\pm}, J_{\pm}$ with $\phi$ in the color chart and $\theta$ as an inset for a set of values for $A$.

4. Conclusions

Solutions obtained for the generic processing operations could be combined under the scheme proposed in [4] for affordable fields as (6). The numerical process developed shows to be efficient to obtain the prescriptions. Then, multi-mode implementations are possible for a common period or otherwise the semi-pulse presented here could be obtained as a superposition of infinite modes as a Fourier series in the set-ups depicted before, letting establish a sequence of semi-pulses corresponding to different gates one followed by another.

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References

[1] Delgado F 2015 *Int. J. Quant. Info.* 13 1550055

[2] Grossman M and Katz R 1972 *Non-Newtonian calculus* (Pigeon Cove, MA: Lee Press)

[3] Delgado F 2016 *J. Phys.: Conf. Series* 648 012024

[4] Delgado F 2017 *J. Phys.: Conf. Series* 839 012014

[5] Serikawa T, Shiozawa Y, Ogawa H, Takanashi N, Takeda S, Yoshikawa J and Furusawa A 2018 QIP with a travelling wave of light *Proc. SPIE-OPTO 2018* 10535

[6] Britton J, Sawyer B, Keith A, Wang J, Freericks J, Uys H, Biercuk M and Bollinger J 2012 *Nature* 484 489

[7] Bohnet J, Sawyer B, Britton J, Wall M, Rey A, Foss-Feig M and Bollinger J 2016 *Science* 352 6291

[8] de Sa Neto O and de Oliveira M 2011 *Hybrid Qubit gates in circuit QED* (quant-ph/1110.1355)