Higher order QED corrections to deep inelastic scattering

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\footnote{For QED corrections to polarized lepton scattering off polarized nucleons see [1].}

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We calculate the leptonic $O(\alpha^2 L)$ QED corrections for unpolarized deeply inelastic $ep$ scattering using mixed variables.

1. INTRODUCTION

Deep inelastic electron–nucleon scattering allows for fundamental QCD tests investigating the scaling violations of structure functions in the perturbative regime of large values of $Q^2$. The detailed knowledge of the structure functions enables to study various aspects of the dynamics of non-Abelian gauge theory, and is necessary for the future experimental search for the Higgs–boson and new particles at TEVATRON and LHC.

One of the major goals of the experiments H1 and ZEUS at the $ep$–collider HERA at DESY is to perform a QCD test at large space–like virtualities $Q^2$ at high precision. This presumes to know the QED radiative corrections to the double–differential scattering cross sections of deeply inelastic $ep$ corrections as precisely as possible. Previous calculations of the radiative corrections for the unpolarized cross sections at leading order [1–5], the leading–log level [7–12] to leading and higher orders, and QED–resummations of small–x terms [11,13] revealed that these corrections are very large, in a wide kinematic range of $x$ and $Q^2$. The corrections are, moreover, complicated by a new type of sizeable contributions as the Compton–peak [14]. This makes it necessary to extend the calculations to higher orders.

The higher order leading–logarithmic contributions $O[(\alpha L)^k]$ to QED corrections are obtained as the leading order solution of the associated renormalization group equations [13] for mass factorization. These corrections are universal, process–independent w.r.t. their structure, and are given in terms of Mellin–convolutions of leading order QED splitting functions. The next–to–leading order (NLO) corrections can be obtained along the same line. However, besides the splitting functions to NLO also the respective process–dependent Wilson coefficients and operator matrix elements in the on-mass-shell (OMS) scheme contribute. In the past this method was applied to calculate the $O(\alpha^2)$ initial–state QED radiative corrections to $e^+e^-\rightarrow \mu^+\mu^-$ in Ref. [16].

In the present paper we summarize results of a recent calculation [17] of the leptonic QED corrections to deeply inelastic $ep$ scattering defining the double–differential scattering cross section for mixed variables [8,2,5] to $O(\alpha^2 L)$. If compared to the case dealt with in [16] to $O(\alpha^2 L)$ the present calculation is more complicated due to the emergence of final state radiation, the double–differential cross section and the relevant rescaling, which implies non–Mellin type convolutions in general. We first summarize main kinematic aspects and present then the different contributions to the leptonic NLO QED corrections.

2. Mixed variables

$ep$ collider experiments allow to measure the kinematic variables defining the inclusive deep–inelastic scattering cross sections in various ways since in principle four kinematic variables are available with the energies and angles of both the outgoing lepton and the struck–quark, see e.g. [15]. At the Born level all methods are equivalent, however, resolution effects as a consequence of the detector’s structure, differ in certain kinematic regions. The Bremsstrahlung–effects...
of QED radiative corrections change this picture drastically and the QED correction factors depend on the way the kinematic variables, as e.g. Bjorken–y and the virtuality \( Q^2 \) are measured.  

In the present paper we calculate the NLO radiative corrections in the case of neutral current deep–inelastic scattering for mixed variables, i.e. that \( Q^2 = Q_1^2 \) is measured at the leptonic and \( y = y_h \) is measured at the hadronic vertex, and \( x_m = Q_1^2/(S y_h) \). The Born cross section for \( \gamma–\) exchange is given by:

\[
\frac{d^2\sigma^{(0)}}{dydQ^2} = \frac{2\pi\alpha^2}{yQ^4} \left[ y^2 \left( 2xF_1 + 2(1-y) \right) F_2 \right],
\]

with

\[
F_1(x, Q^2) = \frac{1}{2} \sum_{k=1}^{N_f} \left[ q_k(x, Q^2) + \bar{q}_k(x, Q^2) \right],
\]

\[
F_2(x, Q^2) = 2xF_1(x, Q^2) + F_L(x, Q^2).
\]

Here, \( F_{1,2,L}(x, Q^2) \) denote the nucleon structure functions for photon exchange, and \( q(x, Q^2) \) and \( \bar{q}(x, Q^2) \) are the quark– and antiquark distribution functions. The sub–system variables obey the following rescaling relations for initial– and final–state radiation:

\[
isr : \tilde{y} = \frac{y_h}{z}, \tilde{Q}^2 = zQ_1^2, \tilde{S} = zS, \tilde{x} = zx_m,
\]

\[
J^I(z) = 1, \quad z^I = \min \left\{ \frac{Q_1^2}{Q_0^2}, \frac{Q_0^2}{Q_1^2} \right\}, \quad (4)
\]

\[
\fsr : \tilde{y} = y_h, \tilde{Q}^2 = \frac{Q_1^2}{z}, \tilde{S} = S, \tilde{x} = \frac{x_m}{z},
\]

\[
J^F(z) = \frac{1}{z}, \quad z^F = x_m. \quad (5)
\]

Here, \( J^{I,F}(z) \) are the initial– and final–state Jacobians \( d^2\tilde{y}/d^2(y_h, Q_1^2) \), and \( z_0 \) marks the lower bound of the sub–system rescaling variable \( z \in [z_0,1] \). The rescaling in Eqs. (4,5) was chosen such that both the initial– and final–state operator matrix elements can be expressed with a variable \( z \in [0,1] \). \( Q_0^2 \) is introduced as a scale to cut away contributions of the Compton peak. Although these terms do formally belong to the QED radiative corrections, they stem from a kinematic domain of low virtualities and are therefore not being associated to deep inelastic scattering. The scale \( Q_0^2 \) can be chosen by experiment accepting only those events in the sample to be analyzed for which the hadronic \( Q^2 \) is larger than \( Q_0^2 \). By this measure the Compton peak is cut away widely and the QED–correction factor is dominated by a deep–inelastic sub–process by far. In the case of mixed variables the leptonic QED radiative corrections can be easily grouped into those for the initial and final state. The separation scale between the two kinematic regions is \( Q_1^2 \).

3. NLO corrections

In this paper we limit the consideration to the calculation of the NLO corrections to leptonic variables for one–photon exchange in electron–nucleon scattering. This approach is widely model independent and allows to refer to general non–perturbative parameterizations of the structure functions which describe the hadronic tensor. In this way a direct unfolding of the experimentally measured structure functions is possible down to the range in \( Q^2 \) and \( x \) in which partonic approaches fail to provide a description of structure functions. The radiative corrections calculated are thus valid as well for inclusive diffractive \( ep \)-scattering, see e.g. [19]. Here the k-th order cross section is denoted by,

\[
\frac{d^2\sigma^{(k)}}{dy_h dQ_1^2} = \sum_{l=0}^{k} \left( \frac{\alpha}{2\pi} \right)^k \ln^{k-l} \left( \frac{Q_1^2}{m_e^2} \right) C^{(k,l)}(y, Q^2)
\]

with \( C^{(0,0)}(y_h, Q_1^2) = d^2\sigma^0/dy_h dQ_1^2 \). The \( O([\alpha L]) \) and \( O([\alpha L]^2) \) corrections were calculated in Ref. [22]. The term \( C^{(1,1)}(y_h, Q_1^2) \) was derived in Ref. [3] completing the \( O(\alpha) \) corrections. We re–calculated these corrections and agree with the previous results.

The NLO–correction \( C^{(2,1)}(y_h, Q_1^2) \) can be obtained representing the scattering cross section using mass–factorization. Although the differential scattering cross section does not contain any mass singularity, one may decompose it in terms of Wilson coefficients and operator-matrix elements being convoluted with the Born cross

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\(^3\)See e.g. Refs. [8,2,5] for a comparison a wide range of different choices of measurement.
section. In this decomposition both the operator matrix elements and the Wilson coefficients depend on the factorization scale $\mu^2$. One writes the scattering cross section as, see also \cite{20},

$$
\frac{d^2 \sigma}{d y_h d Q_t^2} = \left( \frac{d^2 \sigma^0}{d y_h d Q_t^2} \right) \otimes \sum_{i,j} \Gamma_{ei}^I \otimes \tilde{\sigma}_{ij} \otimes \Gamma_{je}^F \tag{7}
$$

with $\Gamma_{ij}^{I,F}(z, \mu^2/m_e^2)$ the initial and final state operator matrix elements and $\tilde{\sigma}_{kl}(z, Q^2/\mu^2)$ the respective Wilson coefficients. $\otimes$ denotes a convolution, which depends on specific rescalings of the chosen kinematic variables for the differential cross sections. Both the operator matrix elements and the Wilson coefficients obey the representations

$$
\Gamma_{ij}^{I,F} = \delta(1-z) + \sum_{m \geq n} \tilde{a}^{m, I,F(m,n)} L(m-n) \tag{8}
$$

$$
\tilde{\sigma}_{kl} = \delta(1-z) + \sum_{m \geq n} \tilde{a}^{m, \sigma_{kl}(m,n)} \tilde{L}(m-n), \tag{9}
$$

where $\tilde{a} = \alpha/(2\pi)$ and the sequences $\{ij\}$ and $\{kl\}$ in the above do always denote $j(l)$ for the incoming and $i(k)$ the outgoing particle, and $L$, $\tilde{L}$ denote $\ln(\mu^2/m_e^2)$, $\ln(Q^2/\mu^2)$ respectively. As the differential cross section is $\mu$–independent, the cross section is expressed by convolutions of the functions $\Gamma_{ij}^{I,F(m,n)}(z)$ and $\tilde{\sigma}_{kl}^{(m,n)}(z)$ such that the $\mu^2$–dependence cancels and a structure like in Eq. \cite{3} is obtained. The present treatment in the OMS scheme assumes that the light fermion mass, $m_e$, is kept everywhere it is giving a final answer in the scattering cross section if compared to the large scale $Q^2$, i.e. the only terms being neglected are power corrections which are of $O\left[(m_e^2/Q^2)^k\right]$, $k \geq 1$ and therefore small. The last step is necessary to maintain the anticipated convolution structure which, in parts, is of the Mellin–type, as also in a massless approach.

In the subsequent relations we make frequent use of the rescaling \cite{4,4}. For this purpose we introduce the following short–hand notation for a rescaling a function $F(y, Q^2)$

$$
\tilde{F}_{I,F}(y, Q^2) = F\left(y = \tilde{y}_{I,F}, Q^2 = \tilde{Q}_{I,F}^2\right), \tag{10}
$$

\footnote{Very recently also the electron energy spectrum in muon decay was calculated \cite{20} using this method.}

where $I, F$ label the respective type of rescaling. The NLO–corrections may be grouped into the following contributions:

i. LO initial and final state radiation off $C_{ee}^{(1,1)}(y, Q^2)$

ii. coupling constant renormalization of $C_{ee}^{(1,1)}(y, Q^2)$

iii. LO initial state splitting of $P_{\gamma e}$ at $C_{ee}^{(1,1)}(y, Q^2)$

iv. LO final state splitting of $P_{\gamma e}$ at $C_{ee}^{(1,1)}(y, Q^2)$

v. NLO initial and final state radiation off $C_{ee}^{(0,0)}(y, Q^2)$

The function $C^{(2,1)}(y, Q^2)$ is given by

$$
C^{(2,1)}(y, Q^2) = \sum_{i=1}^{V} C_{i}^{(2,1)}(z, y_h). \tag{11}
$$

The contribution $C_{i}^{(2,1)}(y, Q^2)$ is

$$
C_{i}^{(2,1)}(y, Q^2) = \int_{0}^{1} dz P_{ee}^{0} \left[ \theta(z - z_{0}^{i}) J_{I}^{(1,1)} - C_{I}^{(1,1)} \right] \tag{12}
$$

$$
+ \int_{0}^{1} dz P_{ee}^{0} \left[ \theta(z - z_{0}^{F}) J_{F}^{(1,1)} - C_{F}^{(1,1)} \right].
$$

where $C_{I}^{(1,1)}(y, Q^2)$ denotes the non–logarithmic part of the $O(\alpha)$ correction \cite{3,3} and $P_{ee}^{0}(z)$ is the fermion–fermion LO splitting function

$$
P_{ee}^{0}(z) = \frac{1 + z^2}{1 - z}. \tag{13}
$$

Also the LO off-diagonal splitting functions

$$
P_{ee}^{0}(z) = z^2 + (1 - z)^2 \tag{14}
$$

$$
P_{ee}^{0}(z) = \frac{1 + (1 - z)^2}{z} \tag{15}
$$

occur in other contributions to $C^{(2,1)}$. Here both for LO and NLO splitting functions for equal particle transitions we write the contributions for $z < 1$ and account for the +/-functions in explicit form below.
We express the final result in terms of \( \alpha(m_e^2) \) and therefore rewrite the coupling constant by

\[
\alpha(\mu^2) = \alpha(m_e^2) \left[ 1 - \frac{\beta_0}{4\pi} \alpha(m_e^2) \left( \frac{\mu^2}{m_e^2} \right) \right],
\]

(16)

with \( \beta_0 = -4/3 \). Due to this \( C^{(1,1)} \) receives the running coupling correction

\[
C_{ii}^{(2,1)}(y, Q^2) = -\frac{\beta_0}{2} C^{(1,1)}(y, Q^2).
\]

(17)

The contributions \( C_{iii,iv}^{(2,1)}(y, Q^2) \) refer to two new \( O(\alpha) \) cross sections : \( d^2\sigma^{\gamma_e(1)}/dydQ^2 \) and \( d^2\sigma^{\gamma_e(1)}/dydQ^2 \). The corrections are

\[
C_{iii}^{(2,1)}(y, Q^2) = \int_{z_0}^1 dz P_{\gamma_e}^0(z) J^i(z) C_{e\gamma,i}^{(1,1)}(y, Q^2)
\]

(18)

\[
C_{iv}^{(2,1)}(y, Q^2) = \int_{z_0}^1 dz P_{\gamma_e}^0(z) J^F(z) C_{\gamma,e,F}^{(1,1)}(y, Q^2).
\]

(19)

The \( O(\alpha) \) sub–system cross sections read, see Ref. [17] :

\[
\frac{d^2\sigma^{\gamma_e(1)}}{dydQ^2} = \frac{\alpha}{2\pi} \sum_{n=0,1} \ln^{1-n} \left( \frac{Q^2}{m_e^2} \right) C_{\gamma e}^{(1,n)}(y, Q^2)
\]

(20)

\[
\frac{d^2\sigma^{\gamma_e(1)}}{dydQ^2} = \frac{\alpha}{2\pi} \sum_{n=0,1} \ln^{1-n} \left( \frac{Q^2}{m_e^2} \right) C_{\gamma e}^{(1,n)}(y, Q^2)
\]

(21)

Here, the functions \( C_{ij}^{(1,0)}(y, Q^2) \) are given by

\[
C_{e\gamma}^{(1,0)}(y, Q^2) = \int_{z_0}^1 dz P_{\gamma_e}^0(z) J^i(z) C_{i}^{(0,0)}(y, Q^2)
\]

(22)

\[
C_{\gamma e}^{(1,0)}(y, Q^2) = \int_{z_0}^1 dz P_{\gamma_e}^0(z) J^F(z) C_{F}^{(0,0)}(y, Q^2).
\]

(23)

The contribution \( C_{\nu}^{(2,1)}(y, Q^2) \) reads :

\[
C_{\nu}^{(2,1)}(y, Q^2) = \int_{z_0}^1 P_{ee,S}^{1,NS,OM} J^i C_i^{(0,0)} - C^{(0,0)} + \int_{z_0}^1 P_{ee,S}^{1,PS,OM} J^i C_i^{(0,0)}
\]

(24)

The splitting functions in the OMS are obtained from the \( \overline{\text{MS}} \)–splitting functions [21] by

\[
P_{ee,S,T}^{1,NS,OM}(z) = P_{ee,S,T}^{1,NS,\overline{\text{MS}}}(z) + \frac{\beta_0}{2} \Gamma_{ee,S,T}^{0,\text{SR}}(z),
\]

(25)

where

\[
\Gamma_{ee,S,T}^{0,\text{SR}}(z) = -2 \left[ \frac{1 + z}{1 - z} \left( \ln(1 - z) + \frac{1}{2} \right) \right],
\]

(26)

and \( P_{ee,S,T}^{1,PS,OM}(z) = P_{ee,S,T}^{1,PS,\overline{\text{MS}}}(z) \). The details of the calculation are given in [17].

4. Conclusions

We calculated the \( O(\alpha^2) \) leptonic QED corrections to deep inelastic \( ep \) scattering for the case of mixed variables. The corrections are given in terms of double–differential distributions to be compared to the double differential Born cross section. The calculation was performed using the renormalization–group decomposition of the 2–loop corrections to the differential cross section w.r.t. mass factorization in the OMS scheme for the light fermion mass. By this method an artificial factorization scale \( \mu^2 \) is introduced on which the physical cross section does not depend. Its elimination leads to a re–organization of the cross section which allows to assemble it in terms of pieces which can be calculated first individually. We grouped the NLO correction into five terms : the LO ISR and FSR radiation correction of the non–logarithmic \( O(\alpha) \) contribution, a respective term due to charge renormalization, two new terms containing \( e - \gamma - e \) initial and final state transitions, and the ISR and FSR OMS–NLO radiation correction to the Born term.

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