Study of Sparsity-Aware Set-Membership Adaptive Algorithms with Adjustable Penalties

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Abstract—In this paper, we propose sparsity-aware data-selective adaptive filtering algorithms with adjustable penalties. Prior work incorporates a penalty function into the cost function used in the optimization that originates the algorithms to improve their performance by exploiting sparsity. However, the strength of the penalty function is controlled by a scalar that is often a fixed parameter. In contrast to prior work, we develop a framework to derive algorithms that automatically adjust the penalty function parameter and the step size to achieve a better performance. Simulations for a system identification application show that the proposed algorithms outperform in convergence speed existing sparsity-aware algorithms.

Keywords—Adaptive filtering, sparsity-aware algorithms, set-membership algorithms.

I. INTRODUCTION

A system is considered to be sparse if only a few of its elements are nonzero values. A sparse signal can be represented as a vector of a finite-dimensional space which can be expressed as a linear combination of a small number of basis vectors of the related space. There are many applications, such as echo cancellation, channel equalization, and system identification, where sparse signals and systems are found. However, traditional adaptive algorithms, including the least-mean square (LMS), the affine projection (AP), and the recursive least squares (RLS) do not exploit the sparsity of the model. When dealing with learning problems, we attempt to extract as much as possible useful information from the system to obtain better results. Under this scope, the sparsity of systems has been the focus of many research works that are devoted to improving the performance of adaptive algorithms.

One of the first approaches used to exploit sparsity was the proportionate family of algorithms. These algorithms assign proportional step sizes to different weights depending on their magnitudes. These algorithms include the proportionate normalized LMS (PNLMS) [2] and the improved PNLMS (IPNLMS) [3]. Several versions of proportionate algorithms have been proposed such as the λ-law PNLMS (MPNLMS) [4] and improved MPNLMS (IMPNLMS) [5] algorithms. In [6] an individual activation factor PNLMS (IAF-PNLMS) algorithm was presented to better distribute the adaptation over the coefficients. Additionally, the set-membership NLMS (SM-NLMS) [7, 8, 9, 10, 11, 12, 13] and PNLMS (SM-PNLMS) [14] algorithms have been developed. The proportionate algorithms were also extended to the AP algorithm, giving rise to the proportionate AP (PAP) and the improved PAP (IPAP) [15] algorithms. The PAP algorithm has also been discussed in [16]. The main advantage of these algorithms is that they accelerate the speed of convergence by reusing multiple past inputs as a single input. Moreover, a data-selective version, the set-membership PAP (SM-PAP) algorithm has been introduced in [17].

In recent years, another approach to deal with sparsity based on penalty functions has been adopted. In this context, a penalty function is added to the cost function to take into account the sparsity of the model and then a gradient-based algorithm is derived. In [18], the zero-attracting LMS (ZA-LMS) and the reweighted zero-attracting LMS (RZA-LMS) have been presented and used for sparse system identification and other applications [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. This idea has been extended to the AP algorithm in [41], where the zero-attracting AP (ZA-AP) and the reweighted zero-attracting AP (RZA-AP) algorithms have been proposed. Another example of this kind of algorithm is the zero-attracting RLS (ZA-RLS) [42]. Other versions of the RLS algorithm that deal with sparsity in systems have been studied in [43] and [44]. There are also data-selective versions of adaptive algorithms that incorporate a penalty function [45]. A review of common penalty functions used in the literature and another scheme to treat sparsity has been reported in [46].

In general, adaptive algorithms that use a penalty function are computationally less expensive and they also achieve a better trade-off between performance and complexity [47] than proportionate algorithms. However, a critical step in these algorithms is the selection of the value of the regularization term. In this paper, we propose a novel framework to derive data-selective algorithms with adjustable penalties and develop algorithms to automatically adjust the regularization term and the step-size. In particular, we devise a framework for set-membership algorithms that can adjust the step-size and the penalty based on the error bound. We then develop sparsity-aware set-membership algorithms with adjustable penalties using commonly employed penalty functions. Simulations show that the proposed algorithms outperform prior art.

This paper is organized as follows. In Section II, the problem formulation is presented. In Section III the proposed algorithms are derived. Section IV presents the simulations.
and results of the algorithms developed in an application involving system identification. Finally, Section V presents the conclusions of this work.

II. SET-MEMBERSHIP FILTERING AND PROBLEM STATEMENT

In set-membership filtering, the filter \( w(i) \) is designed to achieve a specified bound on the magnitude of an estimate \( y(i) \). As a result of this constraint, set-membership adaptive algorithms will only perform filter updates for certain data, resulting in data-selective or sparse updates. Let \( \Theta(i) \) represent the set containing all possible \( w(i) \) that yield estimates upper bounded in magnitude by an error bound \( \gamma \). Thus, we can write

\[
\Theta(i) = \bigcap_{x(i) \in S} \{ w \in \mathbb{R}^M : |y(i)| \leq \gamma \},
\]

where \( x(i) \) is the input vector, \( S \) is the set of all possible data pairs \( (d(i), x(i)) \) and the set \( \Theta(i) \) is referred to as the feasibility set, and any point in it is a valid estimate \( y(i) = w^T(i)x(i) \). Since it is not practical to predict all data pairs, adaptive methods work with the membership set \( s_i = \cap_{m=1}^{M} H_m \) provided by the observations, where \( H_m = \{ w(i) \in \mathbb{R}^M : |y(i)| \leq \gamma \} \). In order to devise an effective set-membership algorithm, the bound \( \gamma \) must be appropriately chosen. Prior work has considered data-selective or sparse updates, time-varying bounds \( 49 \) and exploited sparsity in \( w(i) \). We review the standard SM-NLMS algorithm next.

Let us consider the M-dimensional input vector expressed by

\[
x(i) = \left[ x(i) \ x(i-1) \ \cdots \ x(i-M+1) \right]^T
\]

The output of the adaptive filter is given by

\[
y(i) = w^T(i)x(i),
\]

and the error is computed as follows:

\[
e(i) = d(i) - y(i)
\]

Let us consider a gradient descent approach, where our model is updated by the recursive equation defined by

\[
w(i) = w(i-1) - \mu(i) \frac{\partial J}{\partial w(i-1)},
\]

where \( J \) is the cost function expressed by

\[
J = \frac{1}{2} \mathbb{E} [\| e(i) \|^2]
\]

The gradient of \( J \) is given by

\[
\frac{\partial J}{\partial w(i-1)} = -e(i)x(i)
\]

Replacing this expression in the update equation leads to:

\[
w(i) = w(i-1) + \mu(i)e(i)x(i)
\]

An update of a set-membership algorithm takes place only if the absolute value of the error exceeds the error bound so we have

\[
\gamma = |d(i) - w^T(i)x(i)|
\]

\[
= |d(i) - (w(i-1) + \mu(i)e(i)x(i))^T x(i)|
\]

\[
= |e(i) - (1 - \mu(i) \| x(i) \|^2),
\]

which leads to the final step-size of the algorithm given by

\[
\mu(i) = \begin{cases} 
\frac{1}{\|x(i)\|^2} (1 - \frac{\gamma}{\|e(i)\|}) & |e(i)| > \gamma \\
0 & \text{otherwise}
\end{cases}
\]

resulting in the SM-NLMS update recursion:

\[
w(i) = w(i-1) + \mu(i)e(i)x(i),
\]

where \( \mu(i) \) is given by \( 10 \).

Set-membership adaptive algorithms have sparse updates and variable step-size, which are useful to ensure a fast learning. Prior work on set-membership algorithms that exploit sparsity includes the studies in \( 14, 17, 45 \). However, the problem of adjusting the regularization term and the resulting penalty imposed on the cost function remains open. In this sense, we are interested in developing algorithms capable of performing sparse updates and exploiting sparsity in signals and systems. However, there has been no attempt to devise a strategy based on the error bound to automatically adjust the regularization term together with the step size.

III. PROPOSED SPARSITY-AWARE SM ALGORITHMS WITH ADJUSTABLE PENALTIES

In this section we introduce a framework for deriving sparsity-aware set-membership adaptive algorithms with adjustable penalties using arbitrary penalty functions. Then, we derive the proposed sparsity-aware set-membership algorithms with adjustable penalties based on a gradient descent approach.

A. Derivation framework

Let us consider a mean-square error cost function with a general penalty function as described by

\[
J[w(i-1)] = \frac{1}{2} \mathbb{E} [\| e(i) \|^2] + \alpha(i)f_1[w(i-1)],
\]

where the function \( f_1(w[i-1]) \) is a general penalty function used to improve the performance of adaptive algorithms in the presence of sparsity and \( \alpha(i) \) is a regularization term that imposes the desired penalty. The cost function can rewritten as follows:

\[
J[w(i-1)] = \frac{1}{2} \mathbb{E} [\| d(i) - w^T(i-1)x(i) \|^2]
\]

\[+ \alpha(i)f_1[w(i-1)] \]

Taking the instantaneous gradient of the cost function with respect to \( w(i-1) \), we obtain

\[
\frac{\partial J[w(i-1)]}{\partial w(i-1)} = -e(i)x(i) + \alpha(i)p_f(i),
\]
where we define $p_f(i) = \frac{\partial l_f(w(i-1))}{\partial w(i-1)}$. Replacing the result in the update equation, we get
\[ w(i) = w(i-1) - \mu(i) (\epsilon(i) \mathbf{x}(i) + \alpha(i) p_f(i)). \tag{15} \]

Note that we employ a time index in $\mu$ to designate a variable step-size following the SM-NLMS approach and that the updates are performed only if $|\epsilon(i)| > \gamma$, which leads to the general equation to update the weights:
\[ w(i) = w(i-1) + \mu(i) \epsilon(i) \mathbf{x}(i) - \rho(i) p_f(i), \tag{16} \]
where $\rho(i) = \mu(i) \alpha(i)$. Using an equality constraint, i.e., the a posteriori error $|\epsilon_p(i)| = \gamma$ we obtain,
\[ \gamma = |d(i) - w^T(i) \mathbf{x}(i)| = |d(i) - w^T(i-1) \mathbf{x}(i) - (\mu(i) \epsilon(i) \mathbf{x}(i) - \rho(i) p_f(i))^T \mathbf{x}(i)|. \tag{18} \]

Multiplying both sides of the last equation by $\frac{\epsilon_p(i)}{|\epsilon_p(i)|}$ results in
\[ \gamma \frac{\epsilon_p(i)}{|\epsilon_p(i)|} = d(i) - w^T (i-1) \mathbf{x}(i) - \mu(i) \epsilon(i) \| \mathbf{x}(i) \|^2 + \rho(i) [p_f(i)]^T \mathbf{x}(i) \tag{19} \]
\[ \gamma \text{sign}(\epsilon_p(i)) = d(i) - w^T (i-1) \mathbf{x}(i) - \mu(i) \epsilon(i) \| \mathbf{x}(i) \|^2 + \rho(i) [p_f(i)]^T \mathbf{x}(i) \tag{20} \]

Since the constraint forces that $|\epsilon_p(i)| = \gamma$, then the function $\text{sign}(\epsilon_p(i))$ generates two possible equations given by,
\[ \gamma = \epsilon(i) - \mu(i) \epsilon(i) \| \mathbf{x}(i) \|^2 + \rho(i) [p_f(i)]^T \mathbf{x}(i) \tag{21} \]
\[ -\gamma = \epsilon(i) - \mu(i) \epsilon(i) \| \mathbf{x}(i) \|^2 + \rho(i) [p_f(i)]^T \mathbf{x}(i) \tag{22} \]

We can express equations (21) and (22) as a single equation as follows:
\[ \epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right) = \mu(i) \epsilon(i) \| \mathbf{x}(i) \|^2 - \alpha(i) \mu(i) [p_f(i)]^T \mathbf{x}(i), \tag{23} \]
where we take into account that the term $(1 + \frac{\gamma}{|\epsilon(i)|})$ would produce a growing step-size, leading to a divergent algorithm. Isolating the step-size from the last equation we obtain
\[ \mu(i) = \frac{\epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right)}{\epsilon(i) \| \mathbf{x}(i) \|^2 - \alpha(i) \mu(i) [p_f(i)]^T \mathbf{x}(i)}. \tag{24} \]

We then use equation (24) to update $\alpha(i)$ as follows:
\[ \alpha(i+1) = \alpha(i) [p_f(i)]^T \mathbf{x}(i) = \epsilon(i) \frac{\gamma}{|\epsilon(i)|} + \epsilon(i) \mu(i) \| \mathbf{x}(i) \|^2 - \epsilon(i), \tag{25} \]

\begin{table}[h]
\centering
\caption{Penalty Functions}
\begin{tabular}{|c|c|}
\hline
Function & Partial derivative \\
\hline
$f_1[w(i)] = \|w(i)\|_1$ & $\epsilon$ \\
\hline
$f_1[w(i)] = \sum_{m=1}^{M} \log \left( 1 + \frac{\|w_m(i)\|}{\epsilon} \right)$ & $\frac{\text{sign}[w(i)]}{\epsilon + |w(i)|}$ \\
\hline
$f_1[w(i)] = \|w(i)\|_0 \approx \sum_{m=1}^{M} (1 - e^{-\beta |w_m(i)|})$ & $\beta e^{-\beta |w(i)|} \text{sign}[w(i)]$ \\
\hline
$\alpha(i+1) = \epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right) + \mu(i) \| \mathbf{x}(i) \|^2 - 1 \right) \tag{26} \]
\end{tabular}
\end{table}

Equations (26), (24), (26) fully describe the proposed sparsity-aware SM-NLMS algorithm with adjustable penalties. We can easily show that if we set $\alpha(i)$ to zero, which means that there is no penalty function being applied, then we get the conventional step size of the SM-NLMS algorithm. Table I summarizes the penalty functions used and their derivatives.

\subsection*{B. Proposed ZA-SM-NLMS-ADP algorithm}

In this section, we employ the previous derivation framework and the $f_1[w(i)] = \|w(i)\|_1$ penalty function to devise the proposed zero-attracting SM-NLMS with adjustable penalties algorithm (ZA-SM-NLMS-ADP). Substituting the l1 regularization function and its derivative, we obtain the recursion and the step-size:
\[ w(i) = w(i-1) + \mu(i) \epsilon(i) \mathbf{x}(i) - \rho(i) \text{sign}[w(i-1)], \tag{27} \]
\[ \mu(i) = \frac{\epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right)}{\epsilon(i) \| \mathbf{x}(i) \|^2 - \alpha(i) \text{sign}[w^T(i-1) \mathbf{x}(i)]} \tag{28} \]

The regularization parameter that applies the adjustable penalties is given by
\[ \alpha(i+1) = \frac{\epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right) + \mu(i) \| \mathbf{x}(i) \|^2 - 1 \right) \tag{29} \]

\subsection*{C. Proposed RZA-SM-NLMS-ADP algorithm}

Here, we consider the derivation framework and use the log-sum penalty function $f_1[w(i)] = \sum_{n=1}^{N} \log \left( 1 + \frac{w(i)}{\epsilon} \right)$ to develop the reweighted zero-attracting SM-NLMS with adjustable penalties (ZA-SM-NLMS-ADP) algorithm whose recursions are given by
\[ w(i) = w(i-1) + \mu(i) \epsilon(i) \mathbf{x}(i) - \rho(i) \text{sign}[w(i-1)] \left( \frac{\epsilon(i) \| \mathbf{x}(i) \|^2}{\epsilon + |w(i)|} + |w(i)| \right), \tag{30} \]
\[ \mu(i) = \frac{\epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right)}{\epsilon(i) \| \mathbf{x}(i) \|^2 - \alpha(i) \text{sign}[w^T(i-1) \mathbf{x}(i)]} \tag{31} \]
\[ \alpha(i+1) = \frac{\epsilon(i) \left( 1 - \frac{\gamma}{|\epsilon(i)|} \right) + \mu(i) \| \mathbf{x}(i) \|^2 - 1 \right) \tag{32} \]
D. Proposed EZA-SM-NLMS-ADP algorithm

Finally, we consider the derivation framework and an approximation to the \( l_0 \) regularization function given by

\[ f_i^M [w(i)] = \sum_{m=1}^M (1 - e^{-|\beta w_m(i)|}) \]

to devise the exponential zero-attractor SM-NLMS with adjustable penalties (EZA-SM-NLMS-ADP) algorithm. Consider the vector \( z(i) \) defined by

\[ z(i) = \beta \rho(i) e^{-\beta |w(i)|}. \] (33)

Then, the update equation, the step size and the regularization term are given by

\[ w(i) = w(i-1) + \mu(i) e(i) x(i) - z(i) (\text{sign}(w(i))), \] (34)

\[ \mu(i) = \frac{e(i) (1 - n_{\beta |x(i)|})}{e(i) \| x(i) \|^2 - z^T(i) (\text{sign}(w^T(i-1))) x(i)}, \] (35)

\[ \alpha(i+1) = \frac{e(i) (1 - n_{\beta |x(i)|})}{e(i) \| x(i) \|^2 - 1} \cdot \mu(i) z^T(i) (\text{sign}(w^T(i-1))) x(i). \] (36)

IV. SIMULATIONS

In this section we assess the performance of the proposed algorithms for a sparse system identification task. For this purpose we consider a system modeled by a finite impulse response (FIR) filter with 64 taps in three different scenarios. The first scenario represents a sparse system where only four taps have values different from zero. In the second case, a semi-sparse model with 32 equispaced nonzero coefficients is considered. In the last scenario, we explore the case where there is no sparsity in the system, so that all taps contribute to calculate the output. The input signal follows a Gaussian distribution with a signal to noise ratio of 20 dB. The desired signal is corrupted by white additive Gaussian noise with \( \sigma_n = 0.04 \). The step-size for the NLMS and the PNLMS algorithms was set to 0.5 and the error bound was fixed to \( \gamma = \sqrt{5} \sigma_n \). A maximum value of \( \alpha(i+1) = 10^{-3} \) was set to maintain the stability of the algorithms.

In the first example, we compare the performance of NLMS-type algorithms without adjustable penalties. Each algorithm runs for 3500 iterations, where the first 1000 corresponds to the first scenario described, the next 1000 iterations corresponds to the second scenario and the last 1500 iterations considered the third scenario. A total of 3000 runs were performed and then averaged to obtain the final learning curves. The results shown in Fig. 1 indicate that the sparsity-aware SM-NLMS algorithms with different penalty functions outperform the conventional NLMS and the PNLMS algorithms.

In the second example, we evaluate the performance of the proposed RZA-SM-NLMS-ADP algorithm. For this comparison we also considered the oracle SM-NLMS algorithms \[46\] that assumes the knowledge of the positions of the nonzero coefficients. In this sense, the oracle algorithm fully exploits the sparsity of the system, being considered as the optimal algorithm. In this example, a total of 4000 iterations were performed, where the first 2000 iterations corresponds to the sparse scenario and the last 2000 iterations considered the semi-sparse scenario. All other parameters remain the same. The results depicted in Fig. 2 show that the adjustable penalties \( \alpha(i) \) can provide a small but consistent gain over the fixed penalty approach. Table II summarizes the update rate performed by the proposed algorithms in a sparse scenario.

![Fig. 1. Learning curves of the NLMS-based algorithms](image)

Table II: % of updates

| Algorithm                  | Update Rate |
|----------------------------|-------------|
| ZA-SM-NLMS-ADP             | %           |
| RZA-SM-NLMS-ADP            | %           |
| EZA-SM-NLMS-ADP            | %           |

Fig. 2. Learning curves of the RZA-SM-NLMS based algorithms

In the third example, we assess the proposed EZA-SM-NLMS-ADP algorithm against the other proposed and existing techniques. The results shown in Fig. 3 demonstrate that EZA-SM-NLMS-ADP has the fastest convergence speed among the conventional and sparsity-aware algorithms.

Finally, we consider two different correlated inputs to evaluate the performance of the proposed EZA-SM-NLMS-ADP algorithm. The input is generated by a white Gaussian sequence \( v(i) \), uncorrelated with the noise. Then this signal
is passed through two different IIR filters described by

$$x_1(i) = 0.7x(i-1) + v(i)$$

$$x_4(i) = 0.8x(i-1) + 0.19x(i-2) + 0.09x(i-3) - 0.5x(i-4) + v(i),$$

which corresponds to first- and fourth-order autoregressive (AR) processes, respectively. For the learning curves, we consider a total of 5000 iterations, where the first 5000 iterations correspond to the sparse scenario and the last set of iterations represent the semi-sparse scenario. The results in Fig. 4 show that a correlated input slows the convergence speed and increases the steady-state MSE. In such cases, applying a penalty function improves both results, the convergence speed and the steady-state MSE.

V. CONCLUSIONS

In this paper data selective sparsity-aware algorithms with adjustable penalty functions have been presented, namely, the ZA-SM-NLMS-ADP, the RZA-SM-NLMS-ADP and the EZA-SM-NLMS-ADP adaptive algorithms. These algorithms have a faster convergence speed than conventional algorithms that implement fixed penalty functions. In addition, the data-selective updates performed by these algorithms can save computational resources. Future work will focus on the statistical analysis of the proposed algorithms.

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