Gravity-induced neutrino–antineutrino oscillation: CPT and lepton number non-conservation under gravity

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Abstract
We introduce a new effect in the neutrino oscillation phase which shows that the neutrino–antineutrino oscillation is possible under gravity even if the rest masses of the corresponding eigenstates are the same. This is due to CPT violation, and is possible to demonstrate if the neutrino mass eigenstates are expressed as a combination of neutrino and antineutrino eigenstates, as of the neutral kaon system, with the plausible breaking of lepton number conservation. For Majorana neutrinos, this oscillation is expected to significantly affect the inner edge of neutrino-dominated accretion discs around compact objects by influencing the neutrino sphere which controls the accretion dynamics, and then the related type-II supernova evolution and the r-process nucleosynthesis. On the other hand, in the early universe, in the presence of various lepton number violating processes, this oscillation, we argue, might have led to neutrino asymmetry which resulted in baryogenesis from the B–L symmetry by electro-weak sphaleron processes.

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1. Introduction
It is well known that the discrepancy between various observed and expected numbers of neutrino flavours, e.g. $\nu_e$, $\nu_\mu$ and $\nu_\tau$, occurs because they transform from one type of flavour to another, namely flavour oscillation, which conserves lepton number. Therefore, if a detector is built and set to detect one type of neutrinos, while that type of neutrinos are coming, some of them convert to another type which is unsuited to the detector, and thus results a shortfall. This anomaly has been found in the cases of solar neutrinos, atmospheric neutrinos and the
neutrino experiments at accelerators or reactors. It is also known that the neutrino flavour states consist of a superposition of different mass eigenstates which evolve differently with time and thus flavour states transform from one to another. However, if CPT is violated, even for a particular flavour the neutrino and the antineutrino state may evolve differently and then the lepton number violating oscillation may take place.

The study of neutrinos has a lot of cosmological and astrophysical implications. For example, neutrinos are an important ingredient of the energy produced and removed from the centre of the Sun, they carry off the largest amount of energy of an exploding star in a supernova, neutrinos in certain accretion discs around compact objects are responsible for the cooling process. Moreover, neutrinos may be considered as a prime dark matter candidate. Although the mass of a neutrino is very small, because of their large number, their density is very great and they would thus exert a very potent gravitational effect helping to determine the rate and the pattern of galaxy formation. Therefore, they may have a significant role in the large scale structure of the universe. Finally, the neutrino asymmetry in the early universe may be responsible for leptogenesis and then baryogenesis, etc.

In flat space, the neutrino oscillation is due to the difference in rest masses between two mass eigenstates. However, Gasperini [1] first pointed out that the presence of a gravitational field affects different neutrino flavours differently which violates the equivalence principle and thus governs oscillation, even if neutrinos are massless or of degenerate mass. Subsequently, the effect of a strong gravitational field on neutrino oscillation was discussed [2]. Later it was shown that the neutrino oscillation with LSND data [3] can be explained by degenerate or massless neutrinos with flavour non-diagonal gravitational coupling. It was further argued [4] that in a weak gravitational field the flavour oscillation is possible with the probability phase proportional to the gravitomagnetic field. The oscillation was also shown to be feasible when the maximum velocities of different neutrinos differ from each other, even if they are massless [5].

All the above results are for flavour oscillation or/and without any rigorous effects of general relativity. However, as we show explicitly below, the nature of curvature plays a very important role in determining whether the oscillation occurs or not, even under strong gravity. Therefore, the goal of the present paper is to address the neutrino–antineutrino oscillation, which violates lepton number conservation, focusing on the nature of spacetime curvature and its special effect. In this case, any oscillation depends on the gravitational coupling strength. The main message is something new and unique compared to that in earlier studies mentioned above and many other under curved spacetime [6].

While the neutrino–antineutrino oscillation under gravity is an interesting issue in its own right, the present result is able to address two long-standing mysteries in astrophysics and cosmology as well as related nuclear physics: (1) the source of the abnormally large neutron abundance to support the r-process nucleosynthesis in astrophysical sites; (2) the possible origin of baryogenesis.

It has recently been shown that under gravity, which may bring the CPT violating interaction, the energy level for the neutrino is split from that of the antineutrino [7, 8]. Similar interactions not preserving Lorentz and CPT symmetry were noticed earlier [9] in different contexts. Now the splitting of the energy level for neutrinos from that of corresponding antineutrinos may create a difference in their evolutionary phases. This CPT violation effect is responsible for the transition over neutrino mass eigenstates, if such states which are the linear combination of a neutrino and an antineutrino state are possible to construct, in the presence of a mechanism to break the lepton number conservation.

The possible violation of local Lorentz symmetry in the kaon sector of the standard model was studied with both CPT conserving and non-conserving cases [10]. It was also pointed
Gravity-induced neutrino–antineutrino oscillation

out [11] that in a CPT non-conserved case the neutrino and the antineutrino acquire different masses and thus there is a possibility of oscillation between them. Later, it was shown [12] in a CPT non-conserved case that in the absence of neutrino decay a neutrino evolves differently from an antineutrino of the same flavour and thus the oscillation between them. Recently, it has been argued [13] that a linear superposition of two opposite helicity states can be described as an eigenstate of the de Sitter Casimirs which actually gives rise to the Majorana state. However, all these works neither explain the explicit physical origin of CPT violation nor demonstrate the dependence of oscillation upon the physically observable quantities. In the present paper, we consider realistic situations when neutrinos are propagating in a strong gravitational field, e.g. the spacetime close to a compact object or in the early universe era, such that the spin of a neutrino couples with the spin connection to the spacetime and thus generates a gravitational interaction even if there is no other interaction present.

In the following section, we describe the basic formalism of the problem starting from the fermion Lagrangian density and then discuss how this generates the oscillation between a neutrino and an antineutrino state. Subsequently, we discuss the oscillation phase and the oscillation length in various natural spacetimes in sections 3 and 4. In section 5, we discuss experimental bounds on CPT violation. Finally, we summarize the results in section 6.

2. Formalism

2.1. Fermion Lagrangian density in curved spacetime

We consider a locally flat coordinate system where the gravitational interaction comes in as an effective interaction. Then, the neutrino Lagrangian density under gravity is [7, 8]

$$\mathcal{L} = \sqrt{-g} \bar{\psi} \left[ (i\gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi = \mathcal{L}_f + \mathcal{L}_I,$$

where

$$B^d = e^{abc} e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{a\mu} e^\mu_c e_a^\lambda \right), \quad e^a_b e^b_d \eta^a_d = g^{a\beta},$$

where the choice of unit is \(c = \hbar = k_B = 1\). Naturally, the Lagrangian density in a local flat spacetime can be split into two parts. One is the free part similar to the flat-space Lagrangian density apart from a multiplicative factor \(\sqrt{-g}\) and other is the gravitational interaction part which comes in due to an effective extension of a flat-space description.

2.2. CPT status of the Lagrangian

\(\mathcal{L}_I\) may be a CPT and Lorentz violating interaction if the background curvature coupling, \(B_a\), is constant in the local frame or CPT even [8]. Earlier, similar interaction terms were also considered in CPT violating theories and string theory without any explicit possible origin in nature [9, 14]. If \(B_a\) does not flip in sign under CPT transformation, then \(\mathcal{L}_I\) violates CPT. Actually, as described in detail in our previous work [15], under the CPT transformation, associated axial-vector or pseudo-vector \((\bar{\psi} \gamma^a \gamma^5 \psi)\) changes sign. If \(B_a\) is treated as a background field in a local frame, then the interaction violates CPT explicitly. However, in the present case, with its functional form we can determine the explicit CPT status of \(B_a\) itself at a spacetime point \((t, x, y, z)\). If \(B_a(-x, -y, -z, -t) = -B_a(x, y, z, t)\), then \(B_a\) is CPT odd. On the other hand, \(B_a\) corresponds to \(B_a(-x, -y, -z, -t) = B_a(x, y, z, t)\). If \(B_a(x, y, z, t)\) is not an odd function under CPT \([B_a(-x, -y, -z, -t) \neq -B_a(x, y, z, t)]\), then \(\mathcal{L}_I\) comes out to be a CPT violating interaction along the spacetime. It is the nature
of background metric which determines whether $B_a$ is odd or even under CPT and then the overall CPT status of the interaction\(^1\).

It is very important to note, and can be easily verified, that the gravitational coupling, $B_a$, is zero when the spacetime is spherically symmetric and the metric does not consist of any off-diagonal element, e.g. for the Robertson–Walker universe, the Schwarzschild black hole, etc. Nevertheless, there may exist the spin–orbit coupling when the helicity of a neutrino is different from its chirality. However, the spin–orbit coupling effect is expected to be small compared to the effect due to $B_a$ arising in a non-spherical symmetric spacetime.

2.3. Energy dispersion relations for neutrinos and antineutrinos

Now we stick to the standard model so that a neutrino is solely left-handed and an antineutrino is solely right-handed. Therefore, when the background gravitational field is constant or $B_a$ is CPT even, $L_{1}$, in Weyl’s representation, can be written explicitly in combination of the neutrino and the antineutrino fields as

$$
\bar{\psi} \gamma^a \gamma^5 \psi B_a = (\bar{\psi}_R \gamma^a \psi_R - \bar{\psi}_L \gamma^a \psi_L) B_a.
$$

Then, the dispersion energy relations for the neutrino and the antineutrino are [8]

$$
E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2 + B_0},
E_\bar{\nu} = \sqrt{(\vec{p} + \vec{B})^2 + m^2 - B_0}.
$$

As $E_\nu \neq E_\bar{\nu}$, the time evolution of a neutrino is different compared to that of an antineutrino.

Let us now consider the Majorana neutrino when the antiparticle is basically the charged conjugated particle, and thus one can construct the Majorana spinor which has only a left-handed component such that $\psi = \psi_L$ and $\psi' = \psi'_L = \Psi_R$, which gives rise to the lepton number violating mass terms in $L$. Therefore, equation (1) can be rewritten explicitly for Majorana neutrinos as

$$
L = \sqrt{-g} \left[ \left( i \bar{\psi}_L \gamma^a \partial_a \psi_L + i \bar{\psi}'_L \gamma^a \partial_a \psi'_L \right) - m \left( \bar{\psi}_L \psi_L + \bar{\psi}'_L \psi'_L \right) + \left( \bar{\psi}_L \gamma^a \psi'_L - \bar{\psi}'_L \gamma^a \psi_L \right) B_a \right].
$$

Thus, the Lagrangian may break both the lepton number conservation and CPT simultaneously, depending on the background, which may lead to oscillation.

2.4. Oscillation between neutrino and antineutrino states

Now motivated by the neutral kaon system, we consider two distinct orthonormal eigenstates $|E_\nu\rangle$ and $|E_\bar{\nu}\rangle$, for a neutrino and an antineutrino type respectively, of the same flavour. Further, we introduce a set of neutrino mass eigenstates at $t = 0$ as

$$
|m_1\rangle = \cos \theta |E_\nu\rangle + \sin \theta |E_\bar{\nu}\rangle, \quad |m_2\rangle = -\sin \theta |E_\nu\rangle + \cos \theta |E_\bar{\nu}\rangle.
$$

Therefore, the oscillation probability for $|m_1(t)\rangle$ at $t = 0$ to $|m_2(t)\rangle$ at a later time $t = t_1$ can be found as

$$
P_{12} = |\langle m_1(t) | - \sin \theta \langle E_\nu | + \cos \theta \langle E_\bar{\nu} | [e^{-iE_\nu t_1} \cos \theta |E_\nu\rangle + e^{-iE_\bar{\nu} t_1} \sin \theta |E_\bar{\nu}\rangle]|^2 = \sin^2 2\theta \sin^2 \delta,
$$

where for ultra-relativistic neutrinos,

$$
\delta = \frac{(E_\nu - E_\bar{\nu}) t_1}{2} = \left[ \left( B_0 - |\vec{B}| \right) + \frac{\Delta m^2}{2|\vec{p}|} \right] t_1.
$$

\(1\) The associated axial-vector, $\bar{\psi} \gamma^a \gamma^5 \psi$, always changes sign under CPT.

\(2\) This is clearly due to the presence of background curvature which appears as gravitational 4-vector $B_a$. 

The term within parentheses on the right-hand side of equation (8) arises due to gravity, while the other term is the flat-space contribution which is zero when the rest masses of the neutrino and the antineutrino are the same and/or in the massless limit. In the rest of the description we set $\Delta m^2 = 0$, as our aim is to establish any gravity effect which is expected to contribute dominantly.

Therefore, a non-zero oscillation phase, due to $B_\alpha \neq 0$, between a neutrino and an antineutrino is expected to appear which is gravitational in nature. The interesting point to note is that the present mechanism explicitly affects the neutrino–antineutrino pair unlike the cases considered earlier (e.g. [1, 6]) when the gravity would mainly affect different flavours. Note that a general field theoretic description of neutrino oscillation including the possible neutrino–antineutrino oscillation has already been worked out and it has been shown that CPT violation provides oscillations without the additional sterile neutrinos [16]. In a similar fashion, our mechanism also explains neutrino–antineutrino oscillation without incorporating sterile neutrinos. In general, it is the non-zero coefficient for CPT violation in the effective Lagrangian which generates neutrino–antineutrino oscillation in both the models, while we explicitly show that the CPT violation coefficient may originate from the background curvature. Both the models mainly emphasize the possibility of oscillation in the absence of neutrino mass difference. The unconventional energy dependence in the oscillation phase, $\delta$, (for massless neutrinos) given by equation (8) also reflects its similarity to the earlier work [16].

3. Oscillation phase in specific spacetimes

It is very clear from equations (1), (4) and (5) that due to gravity the neutrino and the antineutrino acquire different effective masses which finally leads to a non-zero oscillation probability in two mass eigenstates. Therefore, in curved backgrounds, e.g. in the anisotropic phase of the early universe when the spacetime is non-flat, close to a rotating compact object, the neutrino–antineutrino oscillation exists in the presence of a suitable mechanism to violate lepton number conservation. Below we describe the oscillation according to specific spacetimes.

3.1. Early universe

First we consider the anisotropic phase of an axially symmetric early universe, when the GUT processes lead to lepton number non-conservation. With a simplified version of the Bianchi II model,

$$ds^2 = -dr^2 + S(t)^2 dx^2 + R(t)^2 [dy^2 + f(y)^2 dz^2] - S(t)^2 h(y)[2 dx - h(y) dz] dz,$$

where $f(y) = y$ and $h(y) = -y^2/2$. The corresponding orthogonal set of non-vanishing tetrad (vierbien) components can be chosen as

$$e_0^t = 1, \quad e_1^x = f(y)R(t)S(t)/\sqrt{f(y)^2 R(t)^2 + S(t)^2 h(y)^2}, \quad e_2^y = R(t),$$

$$e_3^z = \sqrt{f(y)^2 R(t)^2 + S(t)^2 h(y)^2}, \quad e_4^x = -S(t)^2 h(y)/\sqrt{f(y)^2 R(t)^2 + S(t)^2 h(y)^2}.$$  

We then obtain the components of $B^d$

$$B^0 = \frac{4R^3 S + 3y^2 R S^3 - 2y S^4}{8R^4 + 2y^2 R^2 S^2}, \quad B^1 = 0,$$

$$B^2 = \frac{(4y^2 - 8RS - y^2 S^2)(RS' - R'S)}{8R^4 + 2y^2 R^2 S^2}, \quad B^3 = 0.$$  

(11)
It is very clear from above that \( B^0 \) and \( B^2 \) do not flip sign under space-inversion, i.e. for \( y \rightarrow -y \). Thus, it is not an odd function over the spacetime for the Bianchi model and the form of \( B_0 \) is such that \( B_0(-x, -y, -z, -t) \neq \pm B_0(x, y, z, t) \). An example of a spacetime where \( B_0(-x, -y, -z, -t) = -B_0(x, y, z, t) \) can be found in [17] such that \( \mathcal{L}_I \) under CPT does not change sign overall. Therefore, according to the discussion in section 2.2, \( B_0 \) leads to CPT violation at any spacetime point \((x, y, z, t)\). Hence, one can obtain the oscillation phase \( \delta \) from equation (8). If we consider a special case when \( S(t) = \) arbitrary constant \( = k \), then \( B^0, B^2 \rightarrow 0 \) at \( t \rightarrow \infty \), which verifies that the curvature coupling is insignificant in the present universe to exhibit any such oscillation. It can also easily be checked that in a spherically symmetric case, say for the Robertson–Walker universe, \( B^d \) vanishes at all \( t \) and thus the oscillation probability is zero.

3.2. Black holes

A second example can be given for a spacetime around a compact object when the curvature effect is significant. Early on, the flavour oscillations around rotating black holes (in the case of active galactic nuclei) were studied [19]. The black hole spacetime, namely the Kerr geometry, can be described in Cartesian-type coordinates \((t, x, y, z)\) [20] as

\[
dx^2 = \eta_{ij} dx^i dx^j - \left[ \frac{2\alpha}{\rho^4} s_i v_j + \alpha^2 v_i v_j \right] dx^i dx^j
\]

where

\[
\alpha = \sqrt{2Mr}, \quad \rho^2 = r^2 + a^2 z^2,
\]

\[
v_i = \left( 1, \frac{ay}{a^2 + r^2}, -\frac{ax}{a^2 + r^2}, 0 \right), \quad s_i = \left( 0, \frac{rx}{\sqrt{r^2 + a^2}}, \frac{ry}{\sqrt{r^2 + a^2}}, \frac{z\sqrt{r^2 + a^2}}{r} \right).
\]

Here, \( a \) and \( M \) are respectively specific angular momentum and mass of the Kerr black hole and \( r \) is positive definite satisfying

\[
r^4 - r^2(x^2 + y^2 + z^2 - a^2) - a^2 z^2 = 0.
\]

The corresponding non-vanishing components of tetrad (vierbien) are given as [20]

\[
e_i^0 = 1, \quad e_i^1 = -\frac{a}{\rho}s_1, \quad e_i^2 = -\frac{a}{\rho}s_2, \quad e_i^3 = -\frac{a}{\rho}s_3,
\]

\[
e_i^1 = 1 - \frac{a}{\rho}s_1 v_1, \quad e_i^2 = -\frac{a}{\rho}s_2 v_1, \quad e_i^3 = -\frac{a}{\rho}s_3 v_1,
\]

\[
e_i^1 = -\frac{a}{\rho}s_1 v_2, \quad e_i^2 = 1 - \frac{a}{\rho}s_2 v_2, \quad e_i^3 = -\frac{a}{\rho}s_3 v_2.
\]

From equation (2) we obtain the gravitational scalar potential

\[
B^0 = e_{1i} \left( \partial_i e_j^1 - \partial_j e_i^1 \right) + e_{2i} \left( \partial_i e_j^2 - \partial_j e_i^2 \right) + e_{3i} \left( \partial_i e_j^3 - \partial_j e_i^3 \right) = -\frac{4a\sqrt{Mz}}{\rho^3\sqrt{2r^3}},
\]

where \( \rho^2 = 2r^2 + a^2 - x^2 - y^2 - z^2 \).

Similarly, one can obtain \( B^1, B^2, B^3 \). Clearly, from equation (16), \( B^0(-t, -x, -y, -z) = B^0(t, x, y, z) \); above \( B^0 \) is a CPT even function which makes \( \mathcal{L}_I \) a CPT odd interaction. Therefore, the neutrino–antineutrino oscillation is feasible and the oscillation phase, \( \delta \), can easily be computed. It is then a trivial job to check that the oscillation phase for a non-rotating

\[3\] It is easy to compute that \( R(t) = (At - B)^{1/2} \), when \( A \) and \( B \) are arbitrary constants [18].
(Schwarzschild) compact object is zero, as there $B^d$ itself vanishes, by setting $a=0$ in the expression of $B^d$.

4. Oscillation length and its consequence

The above discussion establishes the fact that the gravity-induced neutrino–antineutrino oscillation is expected to occur provided the metric consists of one off-diagonal term at least. The amplitude of oscillation is zero at $\theta = 0, \pi/2$ and maximum at $\theta = \pi/4$. From equations (7) and (8) the oscillation length, $L_{\text{osc}}$, by appropriately setting dimensions, is obtained as

$$L_{\text{osc}} = ct_1 = \frac{\pi \tilde{B} c}{B} \sim \frac{6.3 \times 10^{-19} \text{GeV}}{B} \text{ km},$$

(17)

where $\tilde{B} = B_0 - |\vec{B}|$ is expressed in GeV unit and the neutrino is moving at the speed of light.

4.1. Black holes

One of the most fruitful situations for an oscillation of this kind to take place is the inner region of an accretion disc around a rotating compact object. Therefore, for a compact object of mass $M = M_\odot$, the oscillation length at an orbit, $r \sim \tilde{r} = xM$ and $z = HM$, of the disc, assuming $\tilde{B} \sim B_0$, is computed from equations (16) and (17):

$$L_{\text{osc}} \sim \frac{1.8 x^{7/2} M}{aH} \text{ km} = \frac{1.2 x^{7/2}}{aH} M.$$  

(18)

This result has a very important implication to the neutrino-dominated accretion flow (NDAF) [21, 22]. Although the NDAF and the related supernova are described in the existing literature ignoring any gravity-induced oscillation effects, it is expected that the oscillation at an inner edge of the disc will be influenced by gravity that affects the neutrino sphere, then the accretion dynamics and outflow which are directly related to the corresponding prediction of the supernova explosion. It appears from equation (18) that at the inner accretion disc, when $x \leq 10$, $L_{\text{osc}}$ varies from a few factors to several hundreds of Schwarzschild radii, depending on the location and the thickness of the disc, for a fast spinning compact object. This is interesting as the accretion disc can be extended up to several thousands of Schwarzschild radii.

One of its important consequences is to the r-process nucleosynthesis. Supernovae are thought to be the astrophysical sites of r-process nucleosynthesis. During supernovae, neutron capture processes for radioactive elements take place in the presence of abnormally large neutron flux. However, how the large neutron flux arises is still an open question. There are two related reactions:

$$n + \nu_e \rightarrow p + e^-, \quad p + \bar{\nu}_e \rightarrow n + e^+.$$  

(19)

If $\bar{\nu}_e$ is over abundant compared to $\nu_e$, then from equation (19) neutron production is expected to be more than proton production in the system. Therefore, the possible conversion of $\nu_e$ to $\bar{\nu}_e$ due to the gravity-induced oscillation explains the overabundance of neutrons.

4.2. Early universe

If the oscillation is considered in the early universe with anisotropic phase, then at the GUT scale $\tilde{B} \sim 10^5 \text{ GeV}$ [15]. From equation (17), this leads to $L_{\text{osc}} \sim 10^{-24} \text{ km}$ which is $10^{14}$ orders of magnitude larger than the Planck length. This has an important implication as the
size of the universe, $\chi = \int_t^0 c \, dt'/R(t')$, at the GUT era is within $\sim 10^{26}$ times of the Planck. Therefore, the oscillation may lead to leptogenesis and then to baryogenesis by electro-weak sphaleron processes due to B–L conservation, as we see today.

4.3. Atmosphere of the Earth

If we consider a case of neutrino–antineutrino oscillation in the atmosphere of the Earth where the curvature effect, $\tilde{B} \sim 10^{-37}$ GeV, can be found on a satellite orbiting the Earth with a velocity $v_\phi \sim 1$ km s$^{-1}$ [17], then $L_{\text{osc}}$ at that orbit comes out to be of the order of $10^{13}$ km. This is quite large given that the satellite revolves about $10^{13}$ times in the time taken to complete only one oscillation. This is because the neutrino–antineutrino oscillation is difficult to observe in the atmosphere of the Earth.

5. Bounds on CPT violation from experiments and our model

Kostelecký and his collaborators [9] already argued that the CPT violation, if present, would be a very small effect and then they argued for very stringent constraints to the corresponding coefficients. The experiment E773 at Fermilab published a bound to the mass difference between particles and antiparticles in the neutral kaon sector as $r_K = |m_{K^0} - m_{\bar{K}^0}|/m_{K^0} < 1.3 \times 10^{-10}$ [23]. It was also argued that any observable effects must be suppressed due to involvement of, perhaps, higher-level fields of Planck scale mass $M_{\text{Pl}}$ [24]. When the electro-weak scale mass $m_{\text{ew}} \sim 10^2$ GeV, $m_{\text{ew}}/M_{\text{Pl}} \lesssim 10^{-10}$ provides the natural dimensionless quantity which governs the suppression. In the neutral kaon system, one could expect $r_K \sim m_{\text{ew}}/M_{\text{Pl}}$ which is just below the present bound. The same authors also pointed out that $m_{\text{ew}}/M_{\text{Pl}}$ might be expected to arise in a gravitational mechanism that violates CPT.

In the present case, we have constructed the neutrino mass eigenstates under gravity described in section 2.4 in the spirit of the neutral kaon mass matrix when the mass difference between neutrinos and antineutrinos is $\sim 2\tilde{B}$. Now from section 4, for black holes of mass range, e.g., $10^{M_\odot} \lesssim M_\bullet \lesssim 10^6 M_\odot$, the oscillation length comes out to be $10$ km $\lesssim L_{\text{osc}} \lesssim 10^7$ km (which is equivalent to a few factors to a few hundred Schwarzschild radii, as mentioned in section 4.1), provided $10^{-19}$ GeV $\lesssim \tilde{B} \lesssim 10^{-28}$ GeV. Therefore, the necessary $\tilde{B}$ to govern any significant oscillation in an accretion disc is very small and is quite compatible with the experimental bound.

In the GUT scale, as described in section 4.2, the physically interesting oscillation length may range, e.g., as $10^{-24}$ km $\lesssim L_{\text{osc}} \lesssim 10^{-9}$ km, provided $10^5$ GeV $\gtrsim \tilde{B} \gtrsim 10^{-10}$ GeV, when the size of the universe is $\lesssim 10^{13}$ km. If we compare this $\tilde{B}$ with the mass scale ($m_{\text{GUT}} \sim 10^{16}$ GeV) of the system, then it comes out to be one part in $10^{11}$ to one part in $10^{26}$.

Much more stringent bounds can be put on the gravitational pseudo 4-vector potential $B_\mu$ (which is equivalent to $\tilde{B}$) from the tests for CPT and Lorentz violation [25]. $\mathcal{L}_I$ in non-relativistic limit is equivalent to $\vec{s} \cdot \vec{B}$ due to the interaction between the fermion spin $\vec{s}$ and the external field $\vec{B}$. One can measure this interaction energy in experiments where a macroscopic number of fermions can be polarized in the same direction. For example, in the Eot-Wash II experiment [26], the spin-polarized torsion balance has $N = 8 \times 10^{22}$ aligned spins (with very negligible net magnetic moment). There will be a torque $\tau = (N/\pi) \Delta E$ on such a torsion balance where $\Delta E = |\vec{B}|$ is the difference in energy between the fermion spins polarized parallel and antiparallel to the external $\vec{B}$ field. From this experiment [26], one can measure up to $|\vec{B}| \sim 10^{-28}$ GeV [27]. The field $\vec{B}$ can also be probed by measuring the net magnetization in a paramagnetic material using a squid [28]. An external field $\vec{B}$ appears as
an effective magnetic field of strength $\vec{B}_{\text{eff}} = (\vec{B}/\mu_B)$. The magnitude of $\vec{B}_{\text{eff}}$ can be probed in this experiment as $|\vec{B}_{\text{eff}}| = 10^{-12}$ G which reflects $B \sim 10^{-20}$ GeV [27, 28].

6. Summary

Although the neutrino flavour oscillation has already been described in several occasions successfully, the underlying mechanism for the neutrino–antineutrino oscillation is still not well understood. While in the first case, lepton number of the system remains conserved, the second case is a lepton number violating phenomenon that causes it to be more difficult to explain and establish. We have argued that in the presence of spacetime curvature violating CPT symmetry, the energy level of a neutrino splits from that of an antineutrino which gives rise to their different effective mass. This affects their distribution and results in differences in their phases of evolution. Then, motivated by the neutral kaon system, we have described two mass eigenstates as a linear combination of a neutrino-type and an antineutrino-type state. Therefore, the oscillation between mass eigenstates is expected.

We have explicitly demonstrated this lepton number violating oscillation with some natural examples. We have shown that in the anisotropic phase of the early universe, when the spacetime is non-flat, the neutrino–antineutrino oscillation is feasible which may lead to the leptogenesis and then to the baryogenesis in the presence of a suitable mechanism to violate the lepton number conservation. This may be the possible origin of the matter–antimatter asymmetry that we see in the universe today.

On the other hand, in the inner region of an NDAF where the general relativistic effect is important, this oscillation may affect the accretion dynamics and outflow. It has recently been argued [22] that the energy carried out in the outflowing wind from such discs could be as high as $10^{51}$ erg which might be sufficient to convert a failed supernova explosion into a successful one. It has also been noted that the neutron to proton ratio is large in several regions of the disc. The neutron-rich regions are thought to be the astrophysical sites for r-process nucleosynthesis. However, it is unclear how the large neutron abundance over protons arises in the first place. Our calculation suggests that this is due to a larger antineutrino abundance, due to oscillation, over neutrinos which favours the decay of protons into neutrons compared to neutrons to protons. One now needs to study the NDAF in detail incorporating oscillation effects and to see how the disc structure including the neutrino sphere changes and then affects the energy budget.

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