Cosmological Constant Influence On Cosmic String Spacetime

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Abstract

We are going to investigate the line-element of spacetime around a linear cosmic string in the presence of cosmological constant. We obtain a static form of the metric and argue how it should be discarded because of some asymptotic considerations. Then a time dependent and consistent form of the metric is obtained and its properties are discussed. This may be considered as an example of a preferred frame in physics.

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I. INTRODUCTION

The most reliable measurement of the redshift-magnitude relation uses supernovae of Type Ia [1]. Two groups the High-z Supernova Search Team [2] and the Supernova Cosmology project [3] working independently and using different methods of analysis each have found evidence for accelerated expansion of the universe. Type Ia supernovae are characterized by the absence of hydrogen lines in the spectra and they are thought to be the result of thermonuclear disruption of white dwarf stars [4]. The data require $\Lambda > 0$ at two or three standard deviations, depending on the choice of data and method of analysis [5, 6]. The measurements agree with the relativistic cosmological model with $\Omega_{k0} = 0$, meaning no space curvature and $\Omega_{\Lambda0} \sim 0.7$ meaning we are living in a cosmological constant dominated universe [7, 8, 9]. Also the latest data of the Wilkinson Microwave Anisotropy Probe are in favour of a flat $\Lambda$-dominated universe [10]. These observationally confirmed results and the other theoretical believes in a non-zero cosmological constant [11] make us to consider the effects of this parameter on different parts of our studies in cosmology. In this article we intend to find out its effect on the solution of field equations of a cosmic string. Cosmic strings are topologically stable objects which may have formed during the breaking of a local $U(1)$ gauge symmetry in the very early universe [12, 13, 14, 15]. First we find out the static solutions of the Einstein’s field equations with non-zero cosmological constant for a straight cosmic string. Then we show that in the limiting case when $\mu \to 0$ the static form of the solutions do not satisfy the required prediction of the observed slope 5 in the magnitude-redshift relation for low redshifts ($z \leq 0.2$). Finally time dependent form of the solutions which have consistent asymptotic behaviour are obtained.

II. THE STATIC LINE-ELEMENT

In this study we consider an infinitely long, thin, straight, static string laying along the $z$-axis with the following stress-energy tensor:

$$T_{\mu}^\nu = \mu \delta(x)\delta(y)diag(1, 0, 0, 1)$$

(1)

where $\mu$ is the mass per unit length of the string in the $z$-direction.

For such a gravitating cosmic string the spacetime possesses the same symmetry and is invariant under time translations, spatial translations in the $z$-direction, rotation around
the z-axis and Lorentz boosts in the z-direction. These special symmetries of the problem, according to the special form of the stress-energy tensor introduced by Eq.(1) guide us to choose the following form of the line-element in the cylindrical coordinate system \((\rho, \phi, z)\):

\[
ds^2 = e^{a(\rho)}(dt^2 - dz^2) - d\rho^2 - e^{b(\rho)}d\phi^2
\]

(2)

For the case \(\Lambda = 0\), the solutions of \(a(\rho)\) and \(b(\rho)\) are well-known \([13, 16, 17]\):

\[
ds^2 = dt^2 - dz^2 - d\rho^2 - (1 - 4G\mu^2)\rho^2d\phi^2
\]

(3)

Einstein’s field equations with cosmological constant \(\Lambda\), in units of \(c = 1\), are:

\[
R_{\mu\nu} = -8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) - \Lambda g_{\mu\nu}
\]

(4)

In the case of \(\Lambda > 0\), \(a(\rho)\) and \(b(\rho)\) must satisfy the following equations:

\[
2b'' + b' + 2a'' + 2a'^2 = -4\Lambda
\]

(5)

\[
2b'' + b' + 2a' = -4\Lambda
\]

(6)

\[
2a'' + 2a'^2 + a'b' = -4\Lambda
\]

(7)

where prime stands for derivation with respect to \(\rho\). These are \(\rho\rho\), \(\theta\theta\) and \(zz\) components of the field equations for the exterior part of the string, where the stress-energy tensor and its trace are zero. Actually \(tt\) component makes the same equation as the \(zz\) component. Combination of Eqs. (5),(6)and (7) yields to the result:

\[
4a'' + \frac{3}{4}a'^2 + \Lambda = 0
\]

(8)

\[
b' = 2\frac{a''}{a'} + a'
\]

(9)

General solutions of \(a(\rho)\) and \(b(\rho)\) have the form

\[
a(\rho) = \frac{4}{3}\ln(\cos(\frac{\sqrt{3}\Lambda}{2}(\rho + \alpha))) + \beta
\]

(10)

\[
b(\rho) = \frac{4}{3}\ln(\cos(\frac{\sqrt{3}\Lambda}{2}(\rho + \alpha))) + \ln(\frac{4\Lambda}{3}\tan^2(\frac{\sqrt{3}\Lambda}{2}(\rho + \alpha))) + \gamma
\]

(11)

where \(\alpha, \beta\) and \(\gamma\) are constants. To fix these constants it is sufficient to impose the condition that the metric (2) should match the form (3) in the limiting case when \(\Lambda \to 0\). With doing this the consistent form of the metric(2) is:

\[
ds^2 = \cos^2(\frac{\sqrt{3}\Lambda}{2}\rho)(dt^2 - dz^2) - d\rho^2 - \frac{4(1 - 4G\mu^2)^2}{3\Lambda}\cos^2(\frac{\sqrt{3}\Lambda}{2}\rho)\tan^2(\frac{\sqrt{3}\Lambda}{2}\rho)d\phi^2
\]

(12)
When $\mu \to 0$ i.e. in the absence of string, Eq.(12) gives the form of Schwarzschild de Sitter spacetime with $m = 0$ in the cylindrical coordinates. We have previously shown that in the case of $\Lambda \neq 0$ the so called static isometry of de Sitter solution i.e.
\[
ds^2 = (1 - \frac{\Lambda}{3} \rho^2) dt^2 - (1 - \frac{\Lambda}{3} \rho^2)^{-1} d\rho^2 - \rho^2 (d\theta^2 + \sin^2 \theta d\phi^2)
\]
(13) does not fulfill the requirement to predict the observed relation of magnitude redshift for low redshifts $z \leq 0.2$ [18]. Now let us check this for the metric (12). Evidently in a static spacetime like Eq.(12) the gravitational redshift of a source located at a point with coordinates $(\rho, \theta, 0)$ is proportional to the 00-component of the metric at that point [19]. In this case calculation of the difference between apparent and absolute magnitudes $m - M$ as a function of redshift $z$ yields:
\[
m - M = 2.5log(z(2 + z)) \right\} - 2.5log(\frac{\Lambda}{8\pi})
\]
(14) Inspecting the logarithmic slope of Eq.(14) for small values of $z$, it turns out to be 2.5. Then a comparison of Eq.(14) with the so called redshift-magnitude relation in a FRW model reveals a shortcoming of the static metric (12) in predicting the experimentally tested value of slope 5 [18]. So it provides enough motivation to seek for a nonstatic solution that overcomes this deficiency. Now we continue to find nonstatic solutions of the field equations for cosmic strings.

III. THE NON-STATIC LINE-ELEMENT

To investigate the nonstatic solution of the cosmic strings we may choose the metric Eq.(2) multiplied by a scaling factor, i.e.
\[
ds^2 = S^2(t) \left\{ e^{a(\rho)}(dt^2 - dz^2) - d\rho^2 - e^{b(\rho)} d\phi^2 \right\}
\]
(15) To find the unknown functions of the metric it reveals to be more simple if we rescale the time coordinate to write the line-element in the form
\[
ds^2 = e^{a(\rho)} d\tau^2 - R^2(\tau) \left\{ d\rho^2 + e^{b(\rho)} d\phi^2 + e^{a(\rho)} dz^2 \right\}
\]
(16) It remains to solve the field equations to determine the functions $a$, $b$ and $R$. Direct calculations of the Ricci tensor lead to the following field equations:
\[
2a'' + 2a'^2 - 12\dot{R} R e^{-a} + a'b' = -4\Lambda R^2
\]
(17)
\[-4\ddot{R}\mathcal{R}e^{-a} + 4a'' + 2a'^2 + 2b'' - 8\dot{R}^2 e^{-a} + b'^2 = -4\Lambda R^2 \quad (18)\]
\[-4\ddot{R}\mathcal{R}e^{-a} + 2a'b' + 2b'' + b'^2 - 8\dot{R}^2 e^{-a} = -4\Lambda R^2 \quad (19)\]
\[-4\ddot{R}\mathcal{R}e^{-a} + 2a'^2 + 2a'' - 8\dot{R}^2 e^{-a} + a'b' = -4\Lambda R^2 \quad (20)\]
\[\dot{R}a' = 0 \quad (21)\]

where prime and dot indicate differentiation with respect to $\rho$ and $\tau$ respectively. A physically nontrivial solution of Eq.(21) may be treated as $a$ to be constant. To be consistent with Eq.(3) when $\Lambda \to 0$ this constant should be equal to zero. In this case the Eq.(17) yields the following result for $\mathcal{R}$:
\[3\ddot{R}\mathcal{R} = \mathcal{R}^2 \Lambda \quad (22)\]

So that
\[\mathcal{R} = e^{\sqrt{\frac{2}{3}}} \tau \quad (23)\]

This in turn results that:
\[2b'' + b'^2 = 0 \quad (24)\]

Evidently the solution $b = \text{const.}$ satisfies the Eq.(24). If we accept that the metric (3) should be recovered in the limiting case $\Lambda \to 0$, consequently this constant should be equal to $(1 - 4G\mu)^2$. Therefore the nonstatic line-element of the cosmic string is:
\[ds^2 = dt^2 - e^{2\sqrt{\frac{2}{3}}\tau} \left\{d\rho^2 + (1 - 4G\mu)^2 d\phi^2 + dz^2 \right\} \quad (25)\]

By a transformation of polar angle, $\theta \to (1 - 4G\mu)\theta$, the metric becomes the flat-space deSitter metric. As expected, spacetime around a cosmic string is that of empty space. However, the range of the flat-space polar angle $\theta$ is only $0 \leq \theta \leq 2\pi(1 - 4G\mu)$ rather than $0 \leq \theta \leq 2\pi$. Due to this and nonstatic nature of (25), the observer will see two images of the source, with the angle separation $\delta \alpha$ between the two images defined by
\[\delta \alpha = 8\pi G\mu \frac{l}{l+d} (1 - \sqrt{\frac{\Lambda}{3}}) \quad (26)\]

Here $l(d)$ is the distance from string to the source (observer) at the observation epoch. It is valid for the approximation range of $\sqrt{\frac{\Lambda}{3}}(l+d)$ to be small compared to 1. Thus the existence of cosmological constant will cause to weaken the double images of objects located behind the string. The other important point to mention concerning (25) is that like deSitter spacetime
it possesses an event horizon at the distance \((\sqrt{\frac{2}{\Lambda}})\). We conclude that in the presence of cosmological constant this special frame is preferred by the cosmological considerations.

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