Chaos in short-range spin glasses

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Abstract

The nature of static chaos in spin glasses is studied. For the problem of chaos with magnetic field, scaling relations in the case of the SK model and short-range models are presented. Our results also suggest that if there is de Almeida-Thouless line then it is similar to that of mean-field theory for $d = 4$ and close to the $h = 0$ axis for $d = 3$. We estimate the lower critical dimension to be in the range $2.7 - 2.9$. Numerical studies at $d = 4$ show that there is chaos against temperature changes and the correlation length diverges like $\xi \sim (T - T^*)^{-1}$. 
The nature of the spin-glass phase is poorly understood in short-range spin glasses\[1\]. One very interesting topic is the static chaos problem. By this we mean how the free energy landscape is modified when a small perturbation is applied to the system.

Most generally one is interested in the problem of chaos when a small magnetic field is applied or when the temperature of the system is slightly changed. The interest of this problem is twofold. On the one hand, it is important to discover what is the nature of the spin-glass phase in short-range models. The study of chaos can give interesting predictions regarding this question. On the other hand, it is relevant for the understanding of some recent cycling temperature experiments in spin glasses \[2, 3\]. Under the hypothesis that in the dynamical experiment one is probing some kind of equilibrium states \[4\] it is of the utmost importance to understand the effect of changing the temperature on the free energy landscape.

In this letter we shall present results on the problem of chaos in a magnetic field and will see how scaling arguments may be used within the spin-glass phase. From these arguments one can predict the nature of the spin-glass phase in finite dimensions. Also in the case when the temperature is changed we present some results which show that in finite dimensions the system is more chaotic to temperature changes than in mean-field theory.

The study of chaos with a magnetic field was addressed by I. Kondor in case of mean-field theory \[5\]. Let us suppose two copies of the same system (i.e. with the same realization of bonds): one at zero magnetic field, the other one at finite magnetic field \(h\) (in the general case one could study different non-zero magnetic fields but for simplicity we will focus on the particular case in which one of the magnetic fields is zero.)

The hamiltonian can be written as:

\[
H[\sigma, \tau] = -\sum_{i<j} J_{ij} \sigma_i \sigma_j - \sum_{i<j} J_{ij} \tau_i \tau_j - h \sum_i \tau_i \tag{1}
\]
One defines the order parameter
\[ q = \frac{1}{N} \sum_i \sigma_i \tau_i \] (2)
and its corresponding correlation function
\[ C(i-j) = \langle \sigma_i \sigma_j \tau_i \tau_j \rangle \] (3)
where \( \langle \cdot \rangle \) means average over disorder and \( \langle (\cdot) \rangle \) is the usual thermal average over the Hamiltonian eq.(1).

It has been shown by I. Kondor that this correlation function decays to zero at large distances with a finite correlation length which diverges for \( h \to 0 \). For a finite field \( q = 0 \) is a stable solution and the system is chaotic. In fact, the propagator \( G(p) \) (i.e. the Fourier transform of \( C(i-j) \)) can be exactly computed within the Gaussian approximation. Its singular part is given by [5]:
\[ G(p) = \int_0^{q_{\max}} dq \int_{Q_{\min}}^{Q_{\max}} dQ \frac{p^2 + 1 + \lambda(q)\lambda(Q)}{(p^2 + 1 - \lambda(q)\lambda(Q))^{\frac{3}{2}}} \] (4)
with
\[ \lambda(q) = \beta(1 - q_{\max} + \int_q^{q_{\max}} dq \, x(q)) \] (5)
where \( \beta \) is the inverse of the temperature. The same expression applies in the case of \( \lambda(Q) \). Here \( q(x) \) and \( Q(x) \) are the order parameter functions for the spin glass at zero and \( h \) field respectively.

It was found that the correlation length \( \xi \) diverges like \( (1 - \lambda(Q_{\min}))^{-\frac{1}{2}} \). Close to \( T_c = 1 \) we have \( Q_{\min} \sim h^{\frac{2}{3}} \). This gives \( \xi \sim h^{-\frac{2}{3}} \). These are the results already obtained in reference [5].

Now we want to extract more information on the transition when \( h \to 0 \). We can define a certain kind of non-linear susceptibility by:
\[ \chi_{nl} = \sum_i C(i) \] (6)
It can be shown that it is also given by \( \chi_{nl} = N \langle q^2 \rangle \) with \( q \) given in eq. (2). In terms of eq. (3) we have \( \chi_{nl} = G(0) \). Using the known expressions [6] for \( q(x) \) and \( Q(x) \) close to \( T_c \) in eq. (4) we obtain a divergent expression for \( G(0) \). Its most divergent part is given by

\[
G(0) \sim \int_{Q_{min}}^{Q_{max}} \frac{dQ}{(1 - \lambda(Q))^{5/2}}
\]

which gives \( \chi_{nl} \sim \xi^{4} \sim h^{-\frac{5}{2}} \). In the Gaussian approximation this gives \( G(x) \sim \frac{1}{x^{\mu}} \) for \( h \to 0 \) with \( \mu = d - 4 \). This means that correlation functions decay more slowly in the spin-glass phase than they do at the critical point \( (\mu = d - 2) \) and this is a consequence of the existence of a large number of marginal states in the spin-glass phase.

In general we can introduce exponents \( \gamma \) and \( \nu \) in the spin-glass phase such that \( \chi_{nl} \sim h^{-\gamma} \) and \( \xi \sim h^{-\nu} \). Now we are interested in deriving some finite-size scaling relations in order to establish (if it exists) the upper critical dimension below which we expect scaling to be satisfied. Also scaling relations can be very helpful to test predictions using numerical simulations.

The derivation of scaling relations within the spin-glass phase proceeds analogously as is done at the critical point. At the critical point (below six dimensions) we can write \( \chi_{nl} \sim L^{2-\eta} f(N h^2 q) \) because \( h^2 \sum_{a,b} Q_{ab} \) is the singular part of the free energy per site (\( Q_{ab} = \langle \sigma_a \sigma_b \rangle \) with \( a, b \) replica indices). At the critical point \( (Q_{ab} = q) \) we introduce the exponent \( \delta \) such that \( q = h^\delta \). From these results one gets \( \xi \sim h^{-\frac{2(d+1)}{d-2+\eta}} \) and the scaling relation

\[
\chi_{nl} = L^{2-\eta} f(L h^{\frac{2(d+1)}{d-2+\eta}})
\]

This gives the usual hyperscaling relations \( \beta (\delta + 1) = d\nu \) and \( \delta = \frac{d+2-\eta}{d-2+\eta} \). In mean-field theory [7] we have \( q \sim h \) i.e \( \delta = 2, \nu = \frac{1}{2} \). Hyperscaling relations give \( \eta = 0 \) and \( d = 6 \) which is the upper critical dimension.

In the spin-glass phase, the derivation is slightly different because \( Q_{ab} \neq 0 \). To obtain the singular part of the free energy one has to substract from \( h^2 \sum_{a,b} Q_{ab} \) that part corresponding to zero magnetic field. In mean-field
theory this is given by $h^2 (\int_{Q_{\text{min}}}^{Q_{\text{max}}} Q(x) dx - \int_0^{q_{\text{max}}} q(x) dx)$ which is proportional to $h^2 Q_{\text{min}} x_{\text{min}}$ with $x_{\text{min}}$ equal to the first breakpoint of the function $Q(x)$. Because $Q_{\text{min}} \sim x_{\text{min}} \sim h^{\frac{2}{3}}$ we obtain

$$\chi_{\text{nl}} = N^\frac{4}{3} f(N h^{\frac{10}{3}})$$

(9)

If an upper critical dimension exists then it should be $d_u = 5$ because for that dimension $\xi \sim h^{-\frac{2}{3}}$.

We have performed Monte Carlo numerical simulations of the SK model in order to test the prediction eq.(9). The results for different magnetic fields ranging from 0.2 up to 1.0 at $T = 0.6$ are shown in figure 1 for several sizes (up to $N = 2016$). There is agreement with the prediction eq.(9) and $\chi_{\text{nl}}$ versus the field $h$ does not change its behaviour when crossing the de Almeida-Thouless line ($h_{AT} \sim 0.45$ for $T = 0.6$).

Now we present the results of our simulations and our predictions for short-range models. We expect that scaling is satisfied in the spin-glass phase below five dimensions. In general, one has:

$$\chi_{\text{nl}} = L^\lambda f(L^d h^2 Q_{\text{min}} x_{\text{min}})$$

(10)

The critical point is a particular case of eq.(11). Since there is not replica symmetry breaking one has $x_{\text{min}} \sim 1$ and $Q_{\text{min}} = q \sim h^{\frac{2}{3}}$. Putting $\lambda = 2 - \eta$ one recovers eq.(8). To test this expression at the critical point we have performed Monte Carlo numerical simulations of the 4d $\pm J$ Ising spin glass (with periodic boundary conditions) which is known to have a transition at $T = 2.06 \pm 0.02$ [8, 9]. In this case the known values $\eta \simeq -0.25$, $\delta \simeq 3.6$ fit the data reasonably well but even though the scaling is very sensitive to the precise value of the critical temperature. This shows that, at the critical point, the critical behavior of eq.(3) is the same as that of the correlation function of two identical copies of the system at the same field $h$.

Now we present our results at $T = 1.5$ in the spin-glass phase for the 4d Ising spin glass. One has to be sure that samples are well thermalized. To this end we have performed a simulated annealing algorithm which reaches
equilibrium in a reasonable time. Our results in figure 2 fit well a scaling law \( \chi_{nl} \sim L^{3.25} f(h L^{1.45}) \). This gives \( \lambda \simeq 3.25, \nu \simeq 0.69 \) and \( \gamma = \lambda \nu \simeq 2.24 \). We should draw attention to the fact the value found for \( \lambda \) is close to that found in mean-field theory eq.(9) putting \( N = L^4 \).

We have also studied the 3d \( \pm J \) Ising spin glass which has a transition close to \( T = 1.2 \) \([10, 11]\). Simulations in the critical temperature give exponents in eq.(8) in agreement with those already known. We have also performed simulations for small sizes at \( T = 0.8 \) in the spin-glass phase. Our results are compatible with \( \lambda \sim 2.4, \nu \sim 0.7 \) and \( \gamma = \lambda \nu \simeq 1.9 \).

From our results at \( d = 3, 4 \) it seems that \( \lambda = \frac{4d}{5} \) and \( \nu = \frac{2}{5} \) are a good approximation to the exponents at least for \( d \leq 5 \).

Now we can adress the question of the existence of a phase transition line in finite magnetic field in the 4d Ising spin glass (the so called AT line \([12]\)). From a theoretical point of view the problem remains unsolved \([15]\). Recent numerical studies suggest that this line really exists \([13, 14]\). If this is the case and there is also mean-field behaviour in the spin-glass phase at zero magnetic field \([16, 17]\) then it is natural to suppose that (as happens in mean-field theory) \( Q_{min} \sim x_{min} \). This is a very plausible hypothesis which agrees with the fact that \( q(x) \sim x \) for small \( x \), or that \( P(0) = (\frac{dx}{dq})_{q=0} \) has a finite value \([18]\). If \( Q_{min} \sim h^{\frac{2}{3}} \) (\( \delta = 3 \) in the mean-field case) we obtain, from eq.(10) the result \( \xi \sim h^{-\frac{2(2+\delta)}{3\delta}} \). For \( d = 4 \) this gives \( \delta \simeq 5.27 \) and \( Q_{min} \sim h^{0.38} \). In three dimensions \( \delta \) is uncertain but is a very large value (of order 20).

We expect the transition line in magnetic field to occur when \( Q_{min} \sim Q_{max} \), i.e. \( h^{\frac{2}{\beta}} \sim \tau^{\beta} \) or \( h \sim \tau^{\frac{\beta}{\delta}} \) with \( \tau = T_c - T \). Because \( \beta \sim 0.6, 0.5 \) in \( d = 4 \) \([16, 17]\), respectively this gives \( h \sim \tau^{1.58} \) in four dimensions and \( h \sim \tau^{r} \) with \( r \) of order five in three dimensions. This means that in three dimensions the AT line is very close to the \( h = 0 \) axis and very difficult to see numerically, at least not very far from \( T_c \) \([19, 20]\).

From the scaling relation eq.(10) we can also estimate the lower critical dimension. If we define the exponent \( \theta \) as that exponent for which \( (\xi^4 h^2 q)^{\frac{1}{2}} \sim \)
where \( q \) is finite then this gives the thermal exponent introduced in droplet models [21, 22, 23]. The exponent \( \theta \) vanishes for \( d = d_l \) where \( d_l \) is the lower critical dimension. We have obtained \( \theta = 1, 0.55, 0.05 \) in \( d = 5, 4, 3 \) respectively. Extrapolating to \( \theta = 0 \) we estimate \( d_l = 2.7 - 2.9 \) which is in agreement with perturbative calculations in the spin-glass phase [24] but higher than the value reported in [25].

We have also investigated the problem of chaos against temperature changes. The outline of the ideas follow that presented above in the case of a magnetic field. Now one couples two copies of the system at different temperatures. In mean-field theory the problem has not yet been fully solved and chaos could be marginal [5, 26]. In finite dimensions a interesting behaviour is expected in low dimensions [25]. Our numerical results for the SK model indicate that, if there is chaos, it is very small (details will be presented elsewhere). We have performed simulations in the 4d Ising spin glass. Figure 4 shows the non-linear susceptibility defined in eq.(6) using eq.(2) which is the overlap obtained by coupling two identical copies of the system at different temperatures. Our results are consistent with a correlation length which diverges like \((T - T^*)^{-1}\) where \( T^* \) is the reference temperature of one of the two copies. This results are in agreement with perturbative calculations in the range of dimensionalities \( 6 < d < 8 \) [27].

Summarizing, in the case of chaos with a magnetic field we find that there is scaling behaviour in the spin-glass phase in mean-field theory, the main result being eq.(7). Short-range systems also satisfy scaling relations from which we can extract the exponents associated to the correlation length. We derive that if there is an AT line then in \( d = 4 \) it is similar to that of mean-field theory and in \( d = 3 \) it is very close to the \( h = 0 \) axis and more difficult to see using numerical simulations. The lower critical dimension is also predicted to be in the range \( d_l \sim 2.7 - 2.9 \). We also reported some results of chaos in temperature which show that short-range models are more chaotic than the SK model and the correlation length diverges like \((T - T^*)^{-1}\).
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FIGURE CAPTION

Fig. 1 Chaos with magnetic field in the SK model at $T = 0.6$. Field values range from $h = 0.2$ up to $h = 1.0$ for the smaller sizes and up to $h = 0.4$ for the largest ones. The number of samples range from 200 for $N = 32$ down to 25 for $N = 2016$.

Fig. 2 Chaos with magnetic field in the 4d Ising spin glass at $T = 1.5$. Magnetic field values range from $h = 0.1$ up to $h = 1.0$. The number of samples is approximately 100 for all lattice sizes.

Fig. 3 Chaos with temperature changes in the 4d Ising spin glass. The reference temperature of one copy is $T^* = 1.5$. Temperature values of the other copy range from 1.6 up to 2.2. The number of samples is approximately 100 for all lattice sizes.