Two-qubit controlled phase gate based on two nonresonant quantum dots trapped in a coupled-cavity array

Jian-Qi Zhang, Ya-Fei Yu, and Zhi-Ming Zhang

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We propose a scheme for realizing quantum controlled phase gates with two nonidentical quantum dots trapped in two coupled photonic crystal cavities and driven by classical laser fields under the condition of non-small hopping limit. During the gate operation, neither the quantum dots are excited, while the system can acquire different phases conditional upon the different states of the quantum dots. Along with single-qubit operations, a two-qubit controlled phase gate can be achieved.

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I. INTRODUCTION

As a solid state implementation of cavity quantum electrodynamics (CQED) based approaches would open new opportunities for scaling the quantum network into the practical and useful quantum information processing (QIP) systems, many proposals have been presented in this field [1]. In these schemes, the systems of self-assembled QDs embedded in photonic crystal (PC) nanocavities are considered to be a kind of very promising systems to realize the QIP. That is not just because the strong QD-cavity interaction can be achieved in these systems [2], but also because both QDs and PC cavities are suitable for monolithic on-chip integration.

However, there are two main challenges in this kind of systems. One is that the variation in emission frequencies of the self-assembled QDs is large [3], the other is that the interaction between the QDs is difficult to control [4]. Until now, several methods have been used to overcome the first challenge, such as, by using Stark shift tuning [5] and voltage tuning [7]. And several solutions have been also employed to get over the second challenge, for instance, coherent manipulating of coupled QDs [4], and controlling the coupled QDs by Kondo effect [8]. Due to the small line widths of the QDs, cavity modes and the frequency spread of the QD ensemble, the tuning of individual QD frequencies is mainly achieved with two near neighbor QDs trapped in one single PC cavity [7]. In this way, both the controlled interaction and the controlled gate between the QDs also can be realized. On the contrary, there are few papers about how to implement the controlled interaction and the controlled gate with the QDs embedded in a coupled-cavity array.

On the other hand, the quantum gates based on the dynamical phases are sensitive to the quantum fluctuation, which is the main blockage toward a large-scale quantum computing. The ideas that adopt the geometric phase have been utilized for solving this challenge [9–14]. As the geometric phase is determined only by the path area, it is insensitive to the starting state distributions, the path shape, and the passage rate to traverse the close path [10,12]. In this aspect, the geometric phase is better than the dynamical one in realization of quantum computing. So far, there are two methods for realizing the computation based on the geometric phase. The method involving canceling the dynamical phase is often referred to as conventional geometric phase, which is also named Bell phase [10]. In contrast, the method containing the dynamical phase is named unconventional geometric phase [10]. Comparing with the conventional geometric phase, for the dynamical phase in unconventional geometric phase being proportional to the geometric phase, it doesn’t need to eliminate the dynamical phase. As a result, the unconventional geometric phase is better than the conventional one. Moreover, the unconventional geometric phase has been realized in the system of ions [17].

Recently, Lin et al gave a proposal for realizing a tunable and controllable phase shift via the second effective Hamiltonian. But their proposal is based on identical atoms trapped in a cavity. Very recently, Feng et al. proposed a scheme to achieve an unconventional geometric phase with two qubits in decoherence-free subspace by using a dispersive atom-cavity interaction [14]. And this scheme has been extended into the nonidentical QDs system in Ref. [15]. Motivated by these works, we propose a scheme for realizing a controlled phase gate with two different QDs trapped in two coupled PC cavities. In this scheme, the controlled phase gate can be constructed with two methods. One is based on the unconventional geometric phase; the other is dependent on the second effective Hamiltonian.

*Corresponding author Email: zmzhang@scnu.edu.cn
During the gate operation, the QDs undergo no transitions, while the system can acquire different phases conditional upon the states of QDs. With the choice of the appropriate time and single-qubit operations, a quantum controlled phase gate can be realized. The distinct advantage of this scheme is that it could be controlled by the external light fields and realized with nonidentical QDs in the regime of the non-small hopping limit.

The organization of this paper is as follows. In Sec. II we introduce the theoretical model and derive the first effective Hamiltonian. In Sec. III we present how to realize the quantum phase gate based on the unconventional fields and realized with nonidentical QDs in the regime of the non-small hopping limit. In Sec. IV, we show how to deduce the second effective Hamiltonian and construct the quantum effective Hamiltonian. In Sec. V, we prove that our scheme can be realized in the geometric phase. In Sec. VI we also simulate the decoherence of the system and compare the first effective phase gate based on the second effective Hamiltonian. The conclusion is given in Sec. VI.

II. THEORETICAL MODEL

As it is shown in FIG.1, we consider two coupled single-mode PC cavities with the same frequencies. Each cavity contains one QD. And each dot has two lower states (|g⟩ = |↑⟩, |f⟩ = |↓⟩) and two higher states (|e⟩ = |↑↓⇑⟩, |d⟩ = |↓↑⇓⟩), here (|↑⟩, |↓⟩) and (|⇑⟩, |⇓⟩) denote the spin up and spin down for electron and hole, respectively. At zero magnetic field, the two lower states are twofold degenerate, and the only dipole allowed transitions |g⟩ ↔ |e⟩ and |f⟩ ↔ |d⟩ are coupled with σ+ and σ− polarization lights, respectively [7, 18]. With the choice of the fields in the σ+ polarization [19], the transition |g⟩ ↔ |e⟩ is coupled to the cavity mode and classical laser fields, while |f⟩ and |d⟩ are not affected. Then the Hamiltonian describing this model can be written as:

$$\hat{H} = \sum_{j=A,B} (g_j a_j^+ a_j + \Omega_j e^{i\Delta_j t}) + \nu a_j^+ a_j + H.c. $$  

(1)

where $\sigma_j^+ = |e⟩_j⟨g|$, $g_j$ represents the coupling constant between the QD $j$ and the cavity $j$ with the detuning $\Delta_j^C$, $\Omega_j$ and $-\Omega_j^C$ are the Rabi frequencies of the laser fields with the detunings $\Delta_j$ and $\Delta_j^C$, respectively. $a_j^+$ and $a_j$ is the creation and annihilation operator for the cavity $j$, $\nu$ is the hopping strength (cavity-cavity coupling) between the two cavities.

![FIG. 1: Schematic diagram of a system formed by two coupled cavities which includes the configuration of the QDs level structure and relevant transitions. Both the cavity fields and light fields are in the σ+ polarization. The photon can hop between the cavities. The states |g⟩ and |f⟩ correspond to two lower levels, while |e⟩ and |d⟩ are two higher levels. The transition |g⟩ ↔ |e⟩ for each dot is driven by the cavity field and the classical pulses with the detunings $\Delta_j^C$, $\Delta_j$ and $-\Delta_j$, respectively. $g_j$ represents the coupling rate of the QDs to cavity mode, $\Omega_j$ and $\Omega_j^C$ are the Rabi frequency of the classical pulses, and $\nu$ is the hopping strength.

Introducing new annihilation operators $c_1$ and $c_2$ for new two bosonic modes, and defining $a_A = \frac{1}{\sqrt{2}}(c_1 + c_2)$ and $a_B = \frac{1}{\sqrt{2}}(c_2 - c_1)$, the new two bosonic modes are linearly relative to the cavity modes, and the eigen-states for these new bosonic modes are the entangled states of the cavity modes. In this situation, the whole Hamiltonian (1) can be
rewritten as

\[
\hat{H}_i = \hat{H}_c + \hat{H}_{eq}
\]

\[
\hat{H}_c = \nu(c_c^2 c_2 - c_1^2 c_1),
\]

\[
\hat{H}_{eq} = \left[\frac{1}{2} g_A (c_2 + c_1) e^{i(\Delta_A + \delta)t} + \Omega_A e^{i\Delta_A t} + \Omega_B e^{-i\Delta_A t} \sigma_A^+ + \frac{1}{2} g_B (c_2 - c_1) e^{i(\Delta_B + \delta)t} + \Omega_B e^{i\Delta_B t} + \Omega_B e^{-i\Delta_B t} \sigma_B^+ + H.c.\right]
\]  

(2)

With the application of the unitary transformation \(e^{i\hat{H}_c t}\), the free Hamiltonian \(H_c\) for the new two bosonic modes can be removed, and the above Hamiltonian (2) reduces to:

\[
\hat{H}_I = \left[\frac{1}{2} g_A (c_2 e^{i(\Delta_A + \delta - \nu)t} + c_1 e^{i(\Delta_A + \delta + \nu)t}) + \Omega_A e^{i\Delta_A t} + \Omega_B e^{-i\Delta_A t} \sigma_A^+ + \frac{1}{2} g_B (c_2 e^{i(\Delta_B + \delta - \nu)t} - c_1 e^{i(\Delta_B + \delta + \nu)t}) + \Omega_B e^{i\Delta_B t} + \Omega_B e^{-i\Delta_B t} \sigma_B^+ + H.c.\right]
\]  

(3)

Using the method proposed in Ref. [14, 20], the effective Hamiltonian for the system can be derived under the following condition: (1) \(\Delta_j = \Delta_j^*\) and \(|\Omega_j| = |\Omega_j^*|\); (2) the large detuning condition (|\(\Delta_j|, |\Delta_j^*| \gg |g_j|, |\Omega_j|, |\Omega_j^*|\)); (3) \(|\Omega_j| \gg |g_j|\). The first condition can cancel the Stark shifts caused by the classical laser fields completely. Under the large detuning condition, if the initial state of QDs is in the ground state, since the probability for QDs absorbing photons from the light field or being excited is negligible, the excited state of QD can be adiabatically eliminated. The second condition and the final condition ensure that the terms proportional to \(|g_j|^2\) and \(|g_A g_B|\) can be neglected. Thus the effective Hamiltonian takes the form of:

\[
\hat{H}_{eff} = - \sum_{m=1,2} \sum_{j=A,B} (\lambda_{j,m} c_m e^{i\eta_m t} + \lambda_{j,m}^* c_m^+ e^{-i\eta_m t}) |g\rangle \langle g|,
\]  

(4)

where

\[
\lambda_{A,1} = \frac{g_A \Omega_j^*}{4} \left(\frac{\Delta_A + \delta + \nu}{\Delta_A^*} + \frac{\Delta_A^*}{\Delta_A}\right);
\]

\[
\lambda_{B,1} = -\frac{g_A \Omega_j^*}{4} \left(\frac{\Delta_A + \delta - \nu}{\Delta_A^*} + \frac{\Delta_A^*}{\Delta_A}\right);
\]

\[
\lambda_{A,2} = \frac{g_B \Omega_j^*}{4} \left(\frac{\Delta_B + \delta - \nu}{\Delta_B^*} + \frac{\Delta_B^*}{\Delta_B}\right);
\]

\[
\lambda_{B,2} = \frac{g_B \Omega_j^*}{4} \left(\frac{\Delta_B + \delta + \nu}{\Delta_B^*} + \frac{\Delta_B^*}{\Delta_B}\right);
\]

\[
\eta_1 = \frac{\delta + \nu}{\Delta_A} ; \quad \eta_2 = \frac{\delta - \nu}{\Delta_B}.
\]

It describes the couplings between the cavity modes and classical light fields, and these couplings are induced by the virtual QDs.

**III. QUANTUM PHASE GATE BASED ON THE UNCONVENTIONAL GEOMETRIC PHASE**

Now, we will show how to construct the controlled phase gate based on the unconventional geometric phase. First of all, the information of the system is encoded in the states \(|g\rangle\) and \(|f\rangle\). Then the Hamiltonian (1) assumes a diagonal form

\[
\hat{H}_{eff}(t) = diag[H_{ff}(t), H_{fg}(t), H_{gf}(t), H_{gg}(t)]
\]  

(5)

where

\[
\left\{
\begin{array}{l}
H_{ff}(t) = 0; \\
H_{fg}(t) = - \sum_{m=1,2} (\lambda_{A,m} c_m e^{i\eta_m t} + \lambda_{A,m}^* c_m^+ e^{-i\eta_m t}); \\
H_{gf}(t) = - \sum_{m=1,2} (\lambda_{B,m} c_m e^{i\eta_m t} + \lambda_{B,m}^* c_m^+ e^{-i\eta_m t}); \\
H_{gg}(t) = H_{fg}(t) + H_{gf}(t).
\end{array}
\right.
\]  

(6)
And the evolution operator \( \hat{U}(t) \) for states \( \{|ff\}, |fg\}, |gf\), and \( |gg\}\) in the diagonal with displacement operator \( D(\alpha) = e^{\alpha a^\dagger - \alpha^* a} \) can be written as \[21\]:

\[
\hat{U}(t) = \text{diag}[1, U_{ff}(t), U_{gf}(t), U_{gg}(t)]
\]  

(7)

with

\[
U_{\mu\nu}(t) = \hat{T} \exp(-i \int_0^t H_{\mu\nu} dt)
\]  

\[
= U_{\mu\nu-1}(t) U_{\mu\nu-2}(t), (\mu, \nu = f, g)
\]  

(8)

\[
\hat{U}_{\mu\nu-m}(t) = \hat{T} \exp(i \phi^m_{\mu\nu}) D(\int_0^t d\alpha^m_{\mu\nu}),
\]  

(9)

Here, \( \hat{T} \) is the time ordering operator, and

\[
\begin{cases}
\phi^m_{fg} = \text{Im}(\int_0^t \alpha^m_{fg} d\alpha^m_{fg}), \\
\phi^m_{gf} = \text{Im}(\int_0^t \alpha^m_{gf} d\alpha^m_{gf}), \\
\phi^m_{gg} = \text{Im}(\int_0^t \alpha^m_{gg} d\alpha^m_{gg}),
\end{cases}
\]  

(10)

\[
\begin{cases}
d\alpha^m_{fg} = -i \lambda^m_{\alpha, f} e^{-i \eta^m_{\alpha} t} dt, \\
d\alpha^m_{gf} = -i \lambda^m_{\alpha, g} e^{-i \eta^m_{\alpha} t} dt, \\
d\alpha^m_{gg} = d\alpha^m_{fg} + d\alpha^m_{gf}.
\end{cases}
\]  

(11)

Assuming that the cavity mode is initially in the vacuum state, at any time \( t > 0 \), we can get

\[
\begin{cases}
\alpha^m_{fg} = -i \int_0^t \lambda^m_{\alpha, f} e^{-i \eta^m_{\alpha} t} dt = -\frac{\lambda^m_{\alpha, f}}{\eta_m} (e^{-i \eta^m_{\alpha} t} - 1), \\
\alpha^m_{gf} = -i \int_0^t \lambda^m_{\alpha, g} e^{-i \eta^m_{\alpha} t} dt = -\frac{\lambda^m_{\alpha, g}}{\eta_m} (e^{-i \eta^m_{\alpha} t} - 1), \\
\alpha^m_{gg} = -i \int_0^t (\lambda^m_{\alpha, f} e^{-i \eta^m_{\alpha} t} + \lambda^m_{\alpha, g} e^{-i \eta^m_{\alpha} t}) dt = \alpha^m_{gf} + \alpha^m_{fg},
\end{cases}
\]  

(12)

\[
\begin{cases}
\phi^m_{fg} = \text{Im}(\int_0^t \alpha^m_{fg} d\alpha^m_{fg}) = -\frac{\lambda^m_{\alpha, f}}{\eta_m} \frac{1}{\eta_m} (t - \sin(\eta_m t)), \\
\phi^m_{gf} = \text{Im}(\int_0^t \alpha^m_{gf} d\alpha^m_{gf}) = -\frac{\lambda^m_{\alpha, g}}{\eta_m} \frac{1}{\eta_m} (t - \sin(\eta_m t)), \\
\phi^m_{gg} = \text{Im}(\int_0^t \alpha^m_{gg} d\alpha^m_{gg}) = \phi^m_{gf} + \phi^m_{fg} + \theta_m,
\end{cases}
\]  

(13)

\[
\theta_m = \text{Im}(\int_0^t \frac{\lambda^m_{\alpha, f}}{\eta_m} + \lambda^m_{\alpha, g} \frac{\lambda^m_{\alpha, g}}{\eta_m} (1 - e^{-i \eta^m_{\alpha} t}) dt)
\]  

\[
= -\frac{2 \lambda^m_{\alpha, f} \lambda^m_{\alpha, g}}{\eta_m} \cos \theta_m \left( t - \sin(\eta_m t) \right),
\]  

(14)

and \( \theta_m \) is the argument of \( \lambda^m_{\alpha, f} \lambda^m_{\alpha, g} \).

According to Eq.(8) and Eq.(12), at the time \( t_0 = 2 \pi [1/\eta_1, 1/\eta_2] = 2k \pi / \eta_m \) \[25\], for \( k = 1, 2, 3, ... \), the corresponding time evolution matrix \( \hat{U}(t = t_0) \) in the diagonal is:

\[
\hat{U}(t = t_0) = \text{diag}[1, e^{i \phi_{fs}}, e^{i \phi_{sf}}, e^{i(\phi_{fs} + \phi_{sf} + \theta)}]
\]  

(15)
will induce the coupling between the vacuum cavity mode and classical fields. As the Stark shifts are conditional on the laser field acting, QDs will take place the Stark shifts and acquire the virtual excitation, and the virtual excitation of logical states we also assume the initial state of the cavities is in the vacuum state for the Hamiltonian (18). Then the evolutions of the controlled phase gate.

\[
\begin{align*}
\Phi_{fg} &= -2\pi \sum_{m=1,2} \frac{k_m |\lambda_{A,m}|^2}{\eta_m} , \\
\Phi_{gf} &= -2\pi \sum_{m=1,2} \frac{k_m |\lambda_{B,m}|^2}{\eta_m} , \\
\Theta &= -4\pi \sum_{m=1,2} \frac{k_m |\lambda_{A,m} \lambda_{B,m}| \cos \vartheta_m}{\eta_m} .
\end{align*}
\] 

(16)

It means, in the case of \( t = t_0 \), the displacements for the new bosonic modes have finished their closed paths, returned to their original points in the phase space, and generated the unconventional geometric phases conditional upon the states of QDs. In addition, according to Eq.(9) and Eq.(12), when \( 0 < t < t_0 \), although the new bosonic modes are independent with each other, the cavity modes may be in a entangled state. The reason for this is that the eigen-states of the new bosonic modes are the entangled states of the two cavity modes. On the contrary, when \( t = 0 \) and \( t = t_0 \), as both the two cavity modes and the new bosonic modes are in the same state \( 00 \), there is no entanglement between the two cavity modes.

With the application of the single-qubit operations \( |g\rangle_A = e^{-i\Phi_{ff}} |g\rangle_A \) and \( |g\rangle_B = e^{-i\Phi_{ff}} |g\rangle_B \) [19, 22], the evolutions for the logical states \( \{|ff\}, |fg\rangle, |gf\rangle, \) and \( |gg\rangle \) are:

\[
\begin{align*}
|ff\rangle|00\rangle &\rightarrow |ff\rangle|00\rangle , \\
|fg\rangle|00\rangle &\rightarrow |fg\rangle|00\rangle , \\
|gf\rangle|00\rangle &\rightarrow |gf\rangle|00\rangle , \\
|gg\rangle|00\rangle &\rightarrow e^{i\Theta} |gg\rangle|00\rangle .
\end{align*}
\] 

(17)

This transformation corresponds to the quantum controlled phase gate operation, in which if and only if both the controlling and controlled bits are in the states \( |g\rangle \) and \( |g\rangle \), there will be an additional phase \( \Theta \) in this system. With the choice of \( \Theta = (2l + 1)\pi, (l = 1, 2, 3, ...) \), it is a controlled phase \( \pi \) gate.

### IV. QUANTUM PHASE GATE BASED ON THE SECOND EFFECTIVE HAMILTONIAN

The above is the two-qubit controlled phase gate based on unconventional geometric phase. Here, we will show the method based on the second effective Hamiltonian.

#### A. The second effective Hamiltonian

Following the first Hamiltonian (4), if we assume \( |\eta_m| \gg |\lambda_j| \), it means the bosonic modes cannot exchange energy with the classical fields. Since the nonresonant couplings between the new bosonic modes and the classical fields lead to energy shifts depending on the state of QDs, the second effective Hamiltonian takes the form:

\[
\hat{H}_{eff} = \sum_{m=1,2} \sum_{j=A,B} \frac{|\lambda_{j,m}|^2}{\eta_m} |g\rangle_j \langle g| + 2 \sum_{m=1,2} \mu_m \cos \vartheta_m |g\rangle_A \langle g| |g\rangle_B \langle g| ,
\]

(18)

where \( \mu_m = \frac{|\lambda_{A,m} \lambda_{B,m}|}{\eta_m} \) and \( \vartheta_m \) is the argument of \( \lambda_{A,m} \lambda_{B,m}^{\ast} \). This equation can be understood as follows. With the laser field acting, QDs will take place the Stark shifts and acquire the virtual excitation, and the virtual excitation will induce the coupling between the vacuum cavity mode and classical fields. As the Stark shifts are conditional upon the state of QDs, when the state of two QDs is in the state \( |gg\rangle \), the system composed by two QDs can acquire an additional phase \( 2 \sum_{m=1,2} \mu_m \cos \vartheta_m \). For these reasons, the above Hamiltonian (18) can be employed to construct the controlled phase gate.

#### B. Quantum phase gate

Next, we will show how to construct the controlled phase gate based on the second effective Hamiltonian (18). Here we also assume the initial state of the cavities is in the vacuum state for the Hamiltonian (18). Then the evolutions of logical states \( \{|ff\}, |fg\rangle, |gf\rangle, \) and \( |gg\rangle \), under the the effective Hamiltonian (18), are given [21]:
\[
\begin{align*}
|ff\rangle &\rightarrow |ff\rangle \\
|fg\rangle &\rightarrow \exp(-i\phi_{fg}t) |fg\rangle \\
|gf\rangle &\rightarrow \exp(-i\phi_{gf}t) |gf\rangle \\
|gg\rangle &\rightarrow \exp(-i(\phi_{fg} + \phi_{gf} + \phi)t) |gg\rangle
\end{align*}
\]

(19)

with

\[
\begin{align*}
\phi_{fg} &= \sum_{m=1,2} \frac{|\lambda_{A,m}|^2}{\eta_m}, \\
\phi_{gf} &= \sum_{m=1,2} \frac{|\lambda_{B,m}|^2}{\eta_m}, \\
\phi &= 2 \sum_{m=1,2} \mu_m \cos \vartheta_m.
\end{align*}
\]

After the performance of the single-qubit operations \(|g\rangle_A = e^{-i\phi_{fg}t}|g\rangle_A\) and \(|g\rangle_B = e^{-i\phi_{gf}t}|g\rangle_B\), there are:

\[
\begin{align*}
|ff\rangle &\rightarrow |ff\rangle, |fg\rangle &\rightarrow |fg\rangle, \\
|gf\rangle &\rightarrow |gf\rangle, |gg\rangle &\rightarrow e^{-i\phi t}|gg\rangle.
\end{align*}
\]

(20)

This transformation also corresponds to the quantum controlled phase gate operation, in which if and only if both the controlling and controlled bits are in the states \(|g\rangle\) and \(|g\rangle\), there will be an additional phase \(-\phi t\) in this system. With the choice of \(t = t_0\), we can get:

\[
-\phi t_0 = -4\pi \sum_{m=1,2} \mu_m k_m / \eta_m \cos \vartheta_m = \Theta.
\]

(21)

It means that the controlled phase gates for Eqs.(17) and (21) are the same. The second Hamiltonian (18) is the special case of the first Hamiltonian (11).

V. DISCUSSION AND SIMULATION

As the first Hamiltonian (11) is a generate effective Hamiltonian, in the following, we will take the two-qubit operation (7) for controlled phase \(\pi\) gate (17) as an example to discuss that it is possible to experimentally demonstrate our scheme in the regime of the non-small hopping limit, and show the simulation of decoherence in our system. Moreover, a brief comparison between Hamiltonians (4) and (18) will also be given in this section. Here, all the parameters in the simulation refer Refs.[18, 23, 24].

![FIG. 2: Calculated two-qubit operation time \(t_0\) as functions of parameters of \(\nu\) and \(\delta\) for Eq.(23). Here, \(g_A = 0.1meV\), \(g_B = 0.8g_A\), \(\Omega_A = 10g_A\), and \(\Omega_B = \Omega_A g_A / g_B\).](attachment:fig2.png)
A. The regime of realization

First of all, we will show our scheme can be realized in the regime of the non-small hopping limit. Since our model includes two coupling types, one is the QD-cavity coupling $g$, and the other is the hopping strength $\nu$, there are three different relationships between these two coupling types: the large hopping limit ($\nu \gg g$), the small hopping limit ($\nu \ll g$), and the small detuning between $\nu$ and $g$ ($\nu \approx g$). According to Eqs. (10) and (24), the additional phase for the state $|gg\rangle$ is

$$
\Theta = -2t_0 \sum_{m=1,2} \left| \frac{\lambda_{A,m}\lambda_{B,m}}{\eta_m} \right| \cos \vartheta_m 
= -2t_0 \left[ \frac{|\lambda_{A,1}\lambda_{B,1}|}{\delta + \nu} - \frac{|\lambda_{A,2}\lambda_{B,2}|}{\delta - \nu} \right] \cos \vartheta_1.
$$

(22)

where $\vartheta_2 = \pi - \vartheta_1$ for there is a sign different between $\lambda_{A,1}\lambda_{B,1}$ and $\lambda_{A,2}\lambda_{B,2}$. In the case of $\Delta_j \gg |\delta \pm \nu|$, by using the appropriately external light fields, we can get $|\lambda_{A,1}\lambda_{B,1}| \approx |\lambda_{A,2}\lambda_{B,2}|$. For simplicity, we can choose $|\lambda_{A,1}\lambda_{B,1}| = |\lambda_{A,2}\lambda_{B,2}|$, then the additional geometric phase can be rewritten as

$$
\Theta = t_0 \frac{4\nu}{\delta^2 - \nu^2} |\lambda_{A,1}\lambda_{B,1}| \cos \vartheta_1.
$$

(23)

It means, in the regime of the small hopping limit $\nu \ll g$, $4\nu |\lambda_{A,1}\lambda_{B,1}|/(\delta^2 - \nu^2)$ would be so small that the two-qubit operation time $t_0 = (\delta^2 - \nu^2)^2/(4\nu |\lambda_{A,1}\lambda_{B,1}|)$ could be much longer than the effective decay times. On the contrary, in the regimes of the large hopping limit and the small detuning between $\nu$ and $g$, since $4\nu |\lambda_{A,1}\lambda_{B,1}|/(\delta^2 - \nu^2)$ could be large enough by tuning $\delta^2 - \nu^2$, the two-qubit operation time $t_0 = \pi(\delta^2 - \nu^2)/(4\nu |\lambda_{A,1}\lambda_{B,1}|)$ could be smaller than the effective decay times. And this phenomena can be seen form FIG. 2. For the same reason, when the value of $\delta$ is definite, with the increasing of $\nu$, the two-qubit operation time decreases at first, then increases, and vice versa. Therefore, this scheme could be demonstrate in regime of the non-small hopping limit.

B. The simulation of the decoherence

Then, we will confirm the validity of the proposal by using some numerical simulations about the two-qubit operation which is corresponding to the two-qubit controlled phase $\pi$ gate (17). Under the condition of the large detuning, the excited states of QDs is rarely populated, so the influence of the spontaneous emission can be neglected, and the main decoherence effect is due to cavity decays. Then we can write the master equation:

$$
\dot{\rho} = -i[H_1, \rho] + \sum_{j=A,B} \frac{\gamma_j}{2} (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j),
$$

(24)

where $\rho$ is the density operator of the system, $\gamma_j$ is the decay rate for cavity $j$. And the fidelity of the two-qubit operation can be expressed as $F = Tr(\rho \rho')$, with $\rho'$ being the density operator of the system without cavity decays. The numerical calculations for the fidelities of the two-qubit operations with the different parameters versus the cavity decays are given in FIG. 9.

FIG. 9 shows the follows:

Firstly, with the increase of $\gamma/g_A$, the fidelity for the two-qubit operation decreases. It means that the cavity decays affect the fidelity of the two-qubit operation largely. The reason for this is that the states of new bosonic modes evolve between the vacuum state and the coherent state. It is worthy pointing out that the decay of coherent state depends on the mean photon number of coherent state and the cavity decay. On the one hand, when the mean photon number is definite, the decay of coherent state increases with the increase of the cavity decay. On the other hand, when the cavity decay is definite, the decay of coherent state increases with increasing the mean photon number of coherent state.

Secondly, when $\delta + \nu$ is definite, with the increase of $\delta - \nu$, the fidelity of the two-qubit operation increases for the decrease of the mean photon number of coherent state. It can be seen from the dot dash black line ($\delta + \nu = 1.2g_A$ and $\delta - \nu = 0.4g_A$) and the dash blue line ($\delta + \nu = 1.2g_A$ and $\delta - \nu = 0.3g_A$). On the other hand, when $\delta - \nu$ is definite, with the increase of $\delta + \nu$, the fidelity of the two-qubit operation increases for the decrease of both the mean photon number of coherent state and the two-qubit operation time. That can be seen from the solid green line ($\delta + \nu = 20.3g_A$ and $\delta - \nu = 0.3g_A$) and the blue dash line. Moreover, these lines also show our scheme can be achieved in the regime of the non-small hopping limit.
Thirdly, the lowest fidelity for the two-qubit operation is about 98.3% when the cavity decay rate is $\gamma = 0.01g_A$, and it decreases to about 96.8% when $\gamma = 0.02g_A$. Specifically, $\gamma = 0.01g_A$ has been achieved in the experiment [24]. Moreover, according to Eq.(16), since accumulated unconventional geometric phase for one loop would be small, our system has to take multi-loops. Therefore, our scheme needs a good cavity, which can prevent the photons leaking from the cavity mode in the coherent state and ensure the higher fidelity. In addition, according to the parameters in FIG. 3, the longest two-qubit operation time is about 13.5 ns, which is much smaller than the effective decay time of cavity $\gamma(\delta - \nu)^2/(\max(\lambda_{j,m})^2) \sim 400 \text{ ns}$.

As a result, it is possible to realize our scheme in the experiment.

C. Discussion on the effective Hamiltonians

Now, we discuss the relationship between the effective Hamiltonians (4) and (18) in brief. According to the derivation of the Hamiltonians (4) and (18), these two Hamiltonians are in the different order. The first effective Hamiltonian (4) is the 2 order Hamiltonian, which can be derived from the original Hamiltonian directly, while the second effective Hamiltonian (18) is the 4 order Hamiltonian, which is based on the first effective Hamiltonian under the condition of $\eta_m \gg \lambda_{j,m}$. For this reason, not only there is more effective and wider implications of Hamiltonian (4), which can be used in the condition that doesn’t satisfy $\eta_m \gg \lambda_{j,m}$, but also that the Hamiltonian (4) contains more physical means, such as the coherent states of the new bosonic modes, and the entanglement between the modes of cavities and QDs. On the other hand, the Hamiltonian (18) is the limiting case of the Hamiltonian (4). In the case of $\eta_m \gg \lambda_{j,m}$, the highest mean number photon of the coherent state is $|\alpha_{gg}|^2 = |(\lambda_{A,m} + \lambda_{B,m})/\eta_m|^2$. This mean number photon is so small that both the coherent states of the new bosonic modes and the entanglement between the modes of cavities and QDs can be ignored. Moreover, the system also can obtain the longer decoherence time in this way.

VI. CONCLUSION

In summary, we have shown a protocol that, in the regime of non-small hopping limit, two nonidentical QDs trapped in a coupled-cavity array can be used to construct the two-qubit controlled phase gate with the application of the external classical light fields. During the gate operation, none of the QDs is in the excited state, while the system can acquired the phases conditional upon the states of QDs. The advantages of the proposed scheme are as follows: firstly, as evolution of the system is dependent on the laser fields, it is controllable; secondly, during the gate operation, the QDs are always in their ground states; finally, as the QDs are non-identical and the coupling between the two cavities can be much larger than the one between QD and cavity, it is more practical. Therefore, we can use this scheme to construct a kind of solid-state controllable quantum logical devices. In addition, as the controlled phase gate is a universal gate, this system can also realize the controlled entanglement and interaction between the two nonidentical QDs trapped in a coupled-cavity array.
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