Study of $\tau^- \to VP^-\nu_\tau$ in the framework of resonance chiral theory

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In this paper we study two kinds of $\tau$ decays: (a) $\tau^- \to (\rho^0\pi^-, \omega\pi^-, \phi\pi^-, K^*0K^-)\nu_\tau$, which belong to $\Delta S = 0$ processes and (b) $\Delta S = 1$ processes, like $\tau^- \to (\rho^0K^-, \omega K^-, \phi K^-, K^*0\pi^-)\nu_\tau$, in the framework of resonance chiral theory (R$\chi$T). We fit the $\tau^- \to \omega\pi^-\nu_\tau$ spectral function and the invariant mass distribution of $\omega K$ in the process of $\tau^- \to \omega K^-\nu_\tau$ to get the values of unknown resonance couplings. Then we make a prediction for branching ratios of all channels.

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I. INTRODUCTION

As the only lepton with the ability of decaying into hadrons, $\tau$ decay provides an excellent environment to study the nonperturbative dynamics of QCD. In these decays, the intermediate resonances may play an important role. On the other side, due to the improvement of statistically significant measurements, the branching ratios and spectral functions of the processes containing resonances in the final states of $\tau$ decays have also been determined in recent experiments [1][2][3][4][5]. Some theoretical discussions have been devoted to the study of a $\tau$ decaying into a resonance plus a pseudo-Goldstone meson and a tau neutrino in literature. Angular decay distribution for $\tau \to \omega\pi\nu_\tau$ was studied in [6]. A chiral lagrangian derived in a similar way to the Nambu Jona-Lasinio model, has been used to investigate mesonic $\tau$ decays in [7]. Vector meson dominance model has also been applied to the studies of $\tau$ decaying into $\phi(\omega)$ plus one pseudoscalar meson recently.

Chiral perturbation theory ($\chi$PT) is a powerful tool to describe the interaction for pseudo-Goldstone mesons, which is based on the chiral symmetry and the momentum expansion [9]. When the typical energy scale of the process reaches around $M_\rho$, $\chi$PT has been extended to resonance chiral theory (R$\chi$T), where all of the symmetry allowed operators, consisting of specific number of multiplets of resonances and pseudo-Goldstone mesons, can be introduced in a systematic way.
To build a more realistic QCD-like effective theory, large-$N_C$ techniques and short-distance constraints from QCD have been implemented into the resonance effective theory to constraint resonance couplings. Therefore, resonance chiral effective theory can be a perfect tool to study hadronic $\tau$ decays. Indeed it has already been employed in the studies of $\tau \rightarrow \pi K \nu$, $\tau \rightarrow \pi\pi\pi \nu$, and $\tau \rightarrow K\bar{K} \pi \nu$.

In this paper, we will make a comprehensive analysis for $\tau$ decaying into a vector resonance plus a pseudo-Goldstone meson and a tau neutrino: (a) $\Delta S = 0$ processes, such as $\tau^- \rightarrow (\rho^0\pi^-, \omega\pi^-, \phi\pi^-, K^*0 K^-) \nu$, and (b) $\Delta S = 1$ processes, like $\tau^- \rightarrow (\rho^0 K^-, \omega K^-, \phi K^-, K^*0 \pi^-) \nu$, in the frame of $R\chi T$.

II. THEORETICAL FRAME FOR TAU DECAYS

The amplitude for $\tau^-(p) \rightarrow P^-(p_1)V(p_2)\nu\tau(q)$, where $P^-$ can be $\pi^-, K^-$ and $V$ can be $\rho^0, \omega, \phi, K^*0, \bar{K}^*0$, has the general structure

$$-G_F V_{uQ}(q)\gamma^\mu(1-\gamma_5)u\tau(p)[v\epsilon_{\mu\nu\rho\sigma}(a_1 g_{\mu\nu} + a_2 p_1\mu p_1\nu + a_3 p_2\mu p_1\nu)]\epsilon^{\nu\nu}(p_2),$$

where $G_F$ is the Fermi constant; $V_{uQ}$ is the CKM matrix element; $\epsilon_{\mu\nu\rho\sigma}$ is the anti-symmetric Levi-Civita tensor; $\epsilon^{\nu\nu}(p_2)$ is the polarization vector for the vector resonance; $v$ denotes the form factor of the vector current and $a_1, a_2, a_3$ are the corresponding axial-vector form factors.

We will evaluate the form factors in Eq. (1) using $R\chi T$. The leading $O(p^2)$ lagrangian of $\chi$PT is

$$\mathcal{L}_2 = \frac{F^2}{4}(u_\mu u^\mu + \chi_+) .$$

The kinematic term for the (axial) vector resonance, in the antisymmetric tensor formalism, is

$$\mathcal{L}_{kin}(R) = -\frac{1}{2} R^\mu_{\alpha\beta} R^\mu_{\alpha\beta} - \frac{1}{2} R^\mu_{\alpha\beta} R_{\mu\nu}, \quad R = V, A$$

and the relevant interaction lagrangian only including one multiplet of resonances are given by

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}}(V_{\mu\nu} f^\mu_{\mu+}) + \frac{iG_V}{2\sqrt{2}}(V_{\mu\nu}[u^\mu, u^\nu]),$$

$$\mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}}(A_{\mu\nu} f^\mu_{\mu-}).$$

The interaction operators containing two multiplets of resonances have also been written down in [15,16]:

$$\mathcal{L}_{VV} = d_1\epsilon_{\mu\nu\rho\sigma}(\{V^{\mu\nu}, V^{\rho\sigma}\}\nabla_\alpha u^\alpha) + id_2\epsilon_{\mu\nu\rho\sigma}(\{V^{\mu\nu}, V^{\rho\sigma}\}\chi_-) + d_3\epsilon_{\mu\nu\rho\sigma}(\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\} u^\sigma) + d_4\epsilon_{\mu\nu\rho\sigma}(\{\nabla_\sigma V^{\mu\nu}, V^{\rho\alpha}\} u_\alpha),$$
\[ \mathcal{L}_{VJP} = \frac{c_1}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{V^\mu, f_+^{\rho\sigma}\} \nabla_\alpha u^\alpha \rangle + \frac{c_2}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{V^\mu, f_+^{\rho\sigma}\} \nabla_\alpha u^\sigma \rangle \\
+ \frac{ic_3}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{V^\mu, f_+^{\rho\sigma}\} \chi_- \rangle + \frac{ic_4}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle V^{\mu\nu}[f_-, \chi^+] \rangle \\
+ \frac{c_5}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\sigma}\} u^\sigma \rangle + \frac{c_6}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\} u^\nu \rangle \\
+ \frac{c_7}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\sigma}\} u_\alpha \rangle, \tag{7} \]

\[ \mathcal{L}_{VAP} = \lambda_1^V \langle [V^{\mu\nu}, A_{\mu\nu}] \chi_- \rangle + i \lambda_2^V \langle [V^{\mu\nu}, A_{\nu\alpha}] h^{\alpha}_\mu \rangle + i \lambda_3^V \langle [\nabla_\mu V^{\mu\nu}, A_{\nu\alpha}] u^\alpha \rangle + \]

\[ + i \lambda_4^V \langle [\nabla_\alpha V^{\mu\nu}, A_{\nu\alpha}] u_\mu \rangle + i \lambda_5^V \langle [\nabla_\alpha V^{\mu\nu}, A_{\mu\nu}] u^\alpha \rangle, \tag{8} \]

where \langle \ldots \rangle is short for the trace in flavor space and as usual the chiral fields \[ u_\mu, f^{\mu\nu}_\pm, \chi_\pm, h_{\mu\nu} \]

are defined in terms of the pseudo-Goldstone mesons and external source fields \[ \text{[10]} \]. The SU(3) matrices for vector and axial-vector resonances are given by

\[
V_{\mu\nu} = \begin{pmatrix}
\frac{a_0}{\sqrt{2}} + \frac{a_1}{\sqrt{6}} + \frac{a_3}{\sqrt{3}} & \rho^+ & \rho^- \\
\rho^+ & -\frac{a_0}{\sqrt{2}} + \frac{a_1}{\sqrt{6}} + \frac{a_3}{\sqrt{3}} & K^{*0} \\
K^{*0} & -\frac{2a_0}{\sqrt{6}} + \frac{a_3}{\sqrt{3}} & -\frac{2a_1}{\sqrt{6}} + \frac{a_3}{\sqrt{3}}
\end{pmatrix}_{\mu\nu},
\]

\[
A_{\mu\nu} = \begin{pmatrix}
\frac{a_0}{\sqrt{2}} + \frac{f_0^s}{\sqrt{6}} & a_1^+ & a_1^- \\
a_1^- & -\frac{a_0}{\sqrt{2}} + \frac{f_0^s}{\sqrt{6}} & K_{1A}^+ \\
K_{1A}^- & -\frac{2f_0^s}{\sqrt{6}} + \frac{f_1^s}{\sqrt{3}} & -\frac{2f_0^s}{\sqrt{6}} + \frac{f_1^s}{\sqrt{3}}
\end{pmatrix}_{\mu\nu},
\]

and \[ K_{1A} \] is related to the physical states \[ K_1(1270), K_1(1400) \] through:

\[ K_{1A} = \cos \theta \ K_1(1400) + \sin \theta \ K_1(1270). \tag{9} \]

About the nature of \[ K_1(1270) \] and \[ K_1(1400) \], it has been proposed in \[ \text{[20]} \] that they result from the mixing of \[ K_{1A} \] and \[ K_{1B} \], where \[ K_{1A} \] denotes the strange partner of the axial vector resonance \[ a_1 \] with \[ J^{PC} = 1^{++} \] and \[ K_{1B} \] is the corresponding strange partner of the axial vector resonance \[ b_1 \] with \[ J^{PC} = 1^{+-} \]. However in this paper, we will not include the nonet of axial vector resonances with \[ J^{PC} = 1^{+-} \]. As argued in \[ \text{[20]} \], the contributions from these kind of resonances to tau decays are proportional to the \[ SU(3) \] symmetry breaking effects. Moreover, as one can see later, we will assume the \[ SU(3) \] symmetry for pseudo-Goldstone masses and also \[ SU(3) \] symmetry for both vector and axial vector resonances in deriving the T-matrix throughout this article. Physical masses will be taken into account in the kinematics. For the vector resonances \[ \omega \] and \[ \phi \], we assume
the ideal mixing for them throughout this paper:

\[ \omega_1 = \sqrt{\frac{2}{3}} \omega - \sqrt{\frac{1}{3}} \phi , \]
\[ \omega_8 = \sqrt{\frac{2}{3}} \phi + \sqrt{\frac{1}{3}} \omega. \] (10)

In summary the R\(\chi\)T lagrangian that we will use in this paper is found to be

\[ \mathcal{L}_{R\chi T} = \mathcal{L}_2 + \mathcal{L}_{kin}(V, A) + \mathcal{L}_{2V, A} + \mathcal{L}_{VV, P} + \mathcal{L}_{VJP} + \mathcal{L}_{VAP} \cdot \] (11)

Using the R\(\chi\)T lagrangian above, we obtain the form factors \(v, a_1, a_2, a_3\) for \(\tau^- (p) \rightarrow K^- (p_1) \rho^0 (p_2) \nu_{\tau} (q)\):

\[
v = -i \frac{2 F_V}{F_K M_\rho} [(d_1 + 8 d_2) m_K^2 + d_3 (-m_K^2 + M_\rho^2 + s)] D_K^*(s)
+ i \frac{\sqrt{2}}{F_K M_V M_\rho} [(c_1 + c_2 + 8 c_3 - c_5) m_K^2 + (c_2 + c_5 - c_1 - 2 c_6) M_\rho^2 + (c_1 - c_2 + c_5) s],
\]

\[
a_1 = \frac{1}{4 M_\rho F_K} \left\{ F_V (m_K^2 - M_\rho^2 - s) - 2 G_V (m_K^2 + M_\rho^2 - s) + 2 \sqrt{2} F_A [\lambda_0 m_K^2 + (M_\rho^2 - s)(\lambda M_\rho^2 - \lambda'' s) - m_K^2 (\lambda_0 M_\rho^2 + \lambda M_\rho^2 + \lambda s + \lambda'' s)]
\times [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)] \right\},
\]

\[
a_2 = \frac{-G_V M_\rho}{F_K} D_K(s) + \frac{\sqrt{2} F_A M_\rho}{F_K} (\lambda' + \lambda'' [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)]) ,
\]

\[
a_3 = \frac{F_V - 2 G_V}{2 F_K M_\rho} - \frac{G_V M_\rho}{F_K} D_K(s) + \frac{\sqrt{2} F_A}{F_K M_\rho} (\lambda_0 m_K^2 + \lambda'' M_\rho^2 - \lambda'' s) [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)],
\] (12)

where \(s = (p_1 + p_2)^2\) is the invariant mass of \(\rho\) and \(K\); \(c_0, s_0\) are short for \(\cos \theta, \sin \theta\); \(F\) and \(F_K\) are the decay constants for pion and kaon; \(K, \rho, K^*, K_{1L}, K_{1H}\) are used for kaon, \(\rho (770), K^* (892), K_1 (1270)\) and \(K_1 (1400)\) respectively. \(D_K(s)\) denotes the propagator for kaon

\[ D_K(s) = \frac{1}{m_K^2 - s} \] (13)

and the corresponding definition for the resonance \(R\) is

\[ D_R(s) = \frac{1}{M_R^2 - s - i M_R \Gamma_R(s)} , \] (14)
where the energy dependent decay width $\Gamma_R(s)$ will be discussed in detail later. For convenience, some useful combinations for $\lambda^V_A$ have been used:

$$\sqrt{2}\lambda_0 = -4\lambda^1_A - \lambda^2_A - \frac{\lambda^4_A}{2} - \lambda^5_A,$$

$$\sqrt{2}\lambda' = \lambda^3_A - \lambda^2_A - \frac{\lambda^4_A}{2} + \lambda^5_A,$$

$$\sqrt{2}\lambda'' = \lambda^2_A - \frac{\lambda^4_A}{2} - \lambda^5_A,$$

(15)

which have already been determined in [16]. In deriving the expressions above, we have assumed $SU(3)$ symmetry for the pseudoscalar masses in the T-matrix. Physical masses are taken into account in the kinematics. The kaon decay constant $F_K$ have been used in the form factors, instead of the parameter $F$ appearing in the resonance chiral lagrangian. Likewise for the processes containing pion in the final states, $F_\pi$ will be introduced. The $O(p^4)$ corrections to $F_\pi, F_K$ in resonance chiral theory have been studied in [21]. Throughout this paper, instead of stepping into the detail of the high order corrections to $F_\pi, F_K$ in R$\chi$T, we will take the phenomenological values for them in the numerical discussion. However when discussing the high energy constraints, $SU(3)$ symmetry will be imposed to the pseudo-Goldstone meson, i.e. only $F$ will enter the discussion of QCD short distance constraints [16]. For the value of the pseudo-Goldstone meson decay constant $F$ in $SU(3)$ limit, we will use the pion decay constant to estimate it. $F$ will be also used to denote the pion decay constant in the remaining part of this paper. The form factors for $\tau^- \to \omega K^- \nu_\tau$, $\tau^- \to \phi K^- \nu_\tau$ and $\tau^- \to K^0 \pi^- \nu_\tau$ are very similar to the ones of $\tau^- \to \rho^0 K^- \nu_\tau$. We give the explicit expressions for these channels in the Appendix.

All of the processes above are driven by $\Delta S = 1$ currents, where both vector and axial-vector currents can take part in each channel. In $\Delta S = 0$ processes, such as $\tau^- \to \rho^0 \pi^- \nu_\tau$, $\tau^- \to \omega \pi^- \nu_\tau$, $\tau^- \to \phi \pi^- \nu_\tau$ and $\tau^- \to K^0 \pi^- \nu_\tau$, not every channel can get contributions from both vector and axial-vector currents. For $\tau^- \to \rho^0 \pi^- \nu_\tau$, only the axial-vector current contributes, while $\tau^- \to \omega \pi^- \nu_\tau$ is only driven by the vector current. $\tau^- \to \phi \pi^- \nu_\tau$ vanishes in our model, since it belongs to the next to leading order of $1/N_C$ expansion, which is beyond our scope. So in this paper we will not discuss this process. The explicit expressions for the form factors of $\Delta S = 0$ processes are also given in the Appendix.

Besides the lowest multiplet of resonances one can also introduce heavier multiplets in R$\chi$T. However, including another multiplet, though trivial, may affect the well established relations of the couplings for the lowest multiplet [22].
The contribution to $\tau^- \rightarrow \rho^0 K^-\nu_\tau$ from the new vector multiplet $V_1$ is

$$v_{V_1} = -i \frac{F_{V_1}}{F_K M_\rho} \left[ d_m m_K^2 + d_M M_\rho^2 + d_s s \right] D_{K^*}(s),$$

(16)

where $K^{*'}$ corresponds to the physical resonance $K^*(1410)$ and we have used the following lagrangian, given in [22], to get Eq.(16):

$$L_{2V_1} = \frac{F_{V_1}}{2\sqrt{2}} (V_{1\mu\nu} f_{\mu\nu}^\rho),$$

(17)

$$L_{VV_1} = d_a \epsilon_{\mu\nu\rho\sigma} \langle [V_1^{\mu\nu}, V_1^{\rho\sigma}] \nabla_\alpha u^\sigma \rangle + d_b \epsilon_{\mu\nu\rho\sigma} \langle [V_1^{\mu\alpha}, V_1^{\rho\sigma}] \nabla_\nu u^\rho \rangle +$$

$$d_c \epsilon_{\mu\nu\rho\sigma} \langle [\nabla_\alpha V_1^{\mu\nu}, V_1^{\rho\sigma}] u^\sigma \rangle + d_d \epsilon_{\mu\nu\rho\sigma} \langle [\nabla_\nu V_1^{\mu\alpha}, V_1^{\rho\sigma}] u^\rho \rangle +$$

$$d_e \epsilon_{\mu\nu\rho\sigma} \langle [\nabla_\sigma V_1^{\mu\nu}, V_1^{\rho\alpha}] u_\alpha \rangle + id_f \epsilon_{\mu\nu\rho\sigma} \langle [V_1^{\mu\nu}, V_1^{\rho\sigma}] \chi_- \rangle.$$  \hspace{1cm} (18)

For the sake of simplicity, some combinations of $d_i$ from Eq.(18) have been defined in Eq.(16)

$$d_m = d_a + d_b - d_c + 8d_f,$$

$$d_M = d_b - d_a + d_c - 2d_d,$$

$$d_s = d_c + d_a - d_b.$$  \hspace{1cm} (19)

The corresponding contributions from the new vector multiplet to other channels will be included in the form factors given in the Appendix.

To fulfill the QCD short-distance behavior [23], the vector form factor $v$ defined in Eq.(11) should vanish at high energy limit, which gives us a new constraint to the resonance couplings

$$c_6 - c_5 = \frac{2d_3 F_V + d_4 F_{V_1}}{2\sqrt{2} M_V},$$

(20)

and also allows us to recover the constraint already given in [15]:

$$c_1 - c_2 + c_5 = 0.$$  \hspace{1cm} (21)

Both the chiral symmetry and $SU(3)$ symmetry for vector resonances have been assumed in deriving the above constraints. Another constraint from [15] that will be useful for us is

$$c_1 + 4c_3 = 0,$$  \hspace{1cm} (22)

which, like the constraint in Eq.(21), will not be influenced by including the new multiplet [22].

For the axial-vector current, since we do not include extra operators for axial-vector resonances in this paper, the constraints on the axial-vector resonance couplings are the same as those in [16].
III. PHENOMENOLOGICAL DISCUSSION

To perform the numerical discussion, the values of related parameters will be taken from [16]:

\[ F_V^2 = F^2 \frac{M_A^2}{M_A^2 - M_V^2}, \quad F_A^2 = F^2 \frac{M_V^2}{M_A^2 - M_V^2}, \quad G_V^2 = F^2 \frac{M_A^2 - M_V^2}{M_A^2}, \]  

(23)

where \( M_V \) and \( M_A \) denote the masses for vector and axial-vector resonances in large-\( N_C \) limit. As pointed in [22], \( M_V \) can be safely estimated by the mass of \( \rho(770) \) meson, while for the axial-vector resonance, \( M_A \) is apparently different from the mass of \( a_1(1260) \) and \( M_A = 0.998(49) \)GeV is obtained using the experiment value for the axial-vector form factor in [22]. Throughout this paper, we will use \( M_V = M_\rho \) and \( M_A = 0.998\)GeV to evaluate resonance couplings, but masses appearing in the kinematics will take their values from [1]. The explicit values for resonance couplings that we use in the fit are given by

\[ F = 0.0924, \quad F_K = 0.113, \quad F_V = 0.147, \quad F_A = 0.114, \quad G_V = 0.058, \]  

(24)

in units of GeV and the values of the pion decay constant \( F \) and kaon decay constant \( F_K \) are taken from [24]. Using the decay widths of \( \rho \to e^+e^- \), \( a_1 \to \pi\gamma \) and \( \rho \to \pi\pi \), one can respectively estimate the values for \( F_V, F_A, G_V \):

\[ F_V = 0.156, \quad F_A = 0.122, \quad G_V = 0.066, \]  

(25)

which are given in units of GeV. One can see that they are reasonably consistent with the theoretical determinations given in Eq. (24).

The resonance couplings \( \lambda', \lambda'' \) and \( \lambda_0 \), which are related to axial-vector resonances, have also been given in terms of the masses of resonances in [16]  

\[ \lambda' = \frac{M_A}{2\sqrt{2}M_V}, \quad \lambda'' = \frac{M_A^2 - 2M_V^2}{2\sqrt{2}M_A M_V}, \quad \lambda_0 = \frac{\lambda' + \lambda''}{4}. \]  

(26)

One can easily get the values:

\[ \lambda' = 0.455, \quad \lambda'' = -0.0938, \quad \lambda_0 = 0.0904. \]  

(27)

However in later discussion, to test the stability, we will also perform our fit using another set of values for \( \lambda_i \):

\[ \lambda' = 0.5, \quad \lambda'' = 0, \quad \lambda_0 = 0.125, \]  

(28)

which can be derived by assuming the original KSRF relation \( F_V = 2G_V \) [16].
For the heavier vector multiplet parameter $F_{V_1}$, we determine its value using the decay widths of $V_1 \to e^+ e^-$, which will be discussed in detail in the following subsection. Other parameters related to the heavier vector multiplet $d_m, d_M, d_s$, will be fit later. Due to the inclusion of the new vector multiplet, some of the well established constraints in [15], such as $d_1 + 8d_2$, $d_3$, will get corrections [22]. However, since the combination of $d_1 + 8d_2$ is the coefficient of the mass of pseudo-Goldstone mesons, the final results should be rather insensitive to the value of this combination. So we will still take the value from [15] as an approximation. While for $d_3$, we will fit its value in the $\tau^- \to \omega \pi^- \nu_\tau$ spectral function. The mixing angle $\theta$ in Eq.(9) will be fit in the invariant mass distribution of $\omega K$ system from $\tau^- \to \omega K^- \nu_\tau$. So in total, we have 5 free parameters to fit: $d_3$, which is a resonance coupling related to the lowest vector multiplet; $d_m, d_M, d_s$, which are the couplings for the excited vector multiplet $V_1$; the mixing angle $\theta$, which is a parameter for the axial-vector resonances defined in Eq.(9). For the remaining parameters, which have not been mentioned above, we will take their values from [1].

A. Determination of the parameter $F_{V_1}$ for the heavier vector multiplet

Since we have included a set of new multiplet for vector resonances, a corresponding set of physical states has to be assigned to this multiplet and a natural choice from the particle lists presented in [1] should be

$$\{\rho(1450), \omega(1420), \phi(1680), K^*(1410)\} \in V_1.$$  \hspace{1cm} (29)

As for $\omega$ and $\phi$, ideal mixing will also be assumed for $\omega'$ and $\phi'$.

For $F_{V_1}$ we use the decay widths of $V_1 \to e^+ e^-$ to determine its value. The decay width for a vector resonance $V \to e^+ e^-$ can be deduced using $F_{V} \sqrt{2} \langle V_{\mu\nu} F^\mu_+ \rangle$. The expression for the decay width of $V \to e^+ e^-$ is found to be

$$\Gamma_{V}^{e^+e^-} = \frac{1}{3 \times 8 \pi} \left( \frac{M_V^2 - 4m_e^2}{2M_V^2} \right) \frac{64\alpha^2 \pi^2 F_V^2 (2m_e^2 + M_V^2)}{M_V^2}, \hspace{1cm} (30)$$

where $\alpha$ is the fine structure constant; $V$ is a vector resonance with isospin $I = 1$. For $I = 0$ states, such as $\omega$ and $\phi$ like vector resonances, one has to multiply the above formula with $\frac{1}{9}$ and $\frac{2}{9}$ respectively due to their different couplings with the photon.

The available decay widths of the vector resonances in the new multiplet to $e^+ e^-$ can be extracted from [1]: $\Gamma_{\rho(1450)} \sim 1.8$ KeV, $\Gamma_{\omega(1420)} \sim 0.12$ KeV, where we have used $\Gamma_F \Gamma_{e^+e^-}/\Gamma_{total} = 0.12$ KeV, $\Gamma_F/\Gamma_{e^+e^-} = 0.32$, $\Gamma_{e^+e^-}/\Gamma_{total} = 0.21$ to get $\Gamma_{\rho(1450)} \sim 1.8$ KeV, $\Gamma_{\omega(1420)} \sim 0.12$ KeV, $\Gamma_{e^+e^-}/\Gamma_{total} = 0.21$.
0.081KeV, $\Gamma_{\rho\pi}/\Gamma_{\text{total}} = 0.7$ to extract the value for $\Gamma_{\omega(1420)\to e^+e^-}$. Using this set of data allows us to make an estimate for $F_{V_1}$:

$$\Gamma_{\rho(1450)} = 1.8\text{KeV} \Rightarrow |F_{V_1}| = 0.11\text{GeV},$$
$$\Gamma_{\omega(1420)} = 0.12\text{KeV} \Rightarrow |F_{V_1}| = 0.08\text{GeV},$$

(31)

where one can see that the assignment (29) seems reasonable. Although due to the poor knowledge for $\rho(1450)$ and $\omega(1420)$, different sets of values for $\Gamma_{\rho(1450)\to e^+e^-}$ and $\Gamma_{\omega(1420)\to e^+e^-}$ can be extracted from different experimental groups results [1], most of them lead to a prediction for $|F_{V_1}|$ around 0.1GeV. About the sign of $F_{V_1}$, since in our case what appears in the T-matrix is always the combination of $F_{V_1}(d_m m_\pi^2 + d_M M_V^2 + d_s s)$, this allows us to fix the sign of $F_{V_1}$ and to leave $d_m, d_M, d_s$ free. So in later discussion, we will set $F_{V_1} = -0.1\text{GeV}$. To determine the free parameters $d_m, d_M, d_s$, which are related to the new vector multiplet, we will fit the $\tau^- \to \omega\pi^-\nu_\tau$ spectral function and the invariant mass distribution for $\omega K$ in the decay of $\tau^- \to \omega K^-\nu_\tau$.

B. Introduce the energy dependent decay widths for intermediate resonances

Before stepping into the fit, some points about the decay widths of the intermediate resonances appearing in $\tau$ decays will be stressed. Since most of the intermediate resonances have wide decay widths, the off-shell widths of these resonances may play an important role in the dynamics of $\tau$ decays. To introduce the finite decay widths for the resonances implies that the corrections from the next-to-leading order of $1/N_C$ expansion are taken account into our game. This issue has been discussed in [25] for the decay width of $\rho(770)$ and we take the result of that article

$$\Gamma_{\rho}(s) = \frac{sM_V}{90\pi F^2} \left[\sigma_\pi^3 \theta(s - 4m_\pi^2) + \frac{1}{2}\sigma_K^3 \theta(s - 4m_K^2)\right],$$

(32)

where $\sigma_P = \sqrt{1 - 4m_P^2/s}$ and $\theta(s)$ is the step function. About the energy dependent widths for $\rho', K^*, K^{*+}, K_1(1270), K_1(1400), a_1(1260)$, we follow the similar way introduced in [17, 26] to
construct them:

\[
\Gamma_{\rho^0}(s) = \frac{s}{M_{\rho^0}^2} \left[ \frac{\sigma_{\pi\pi}^3(s) + \frac{1}{2}\sigma_{KK}^3}{\sigma_{\pi\pi}^3(M_{\rho^0}^2) + \frac{1}{2}\sigma_{KK}^3(M_{\rho^0}^2)} \right], \\
\Gamma_{K^*}(s) = \frac{s}{M_{K^*}^2} \left[ \frac{\sigma_{K\pi}^3(s) + \sigma_{K\eta}^3(s)}{\sigma_{K\pi}^3(M_{K^*}^2) + \sigma_{K\eta}^3(M_{K^*}^2)} \right], \\
\Gamma_{K^{*0}}(s) = \frac{s}{M_{K^{*0}}^2} \left[ \frac{\sigma_{K\pi}^3(s) + \sigma_{K\eta}^3(s)}{\sigma_{K\pi}^3(M_{K^{*0}}^2) + \sigma_{K\eta}^3(M_{K^{*0}}^2)} \right], \\
\Gamma_{K_{1L}}(s) = \frac{s}{M_{K_{1L}}^2} \left[ \frac{\sigma_{K\rho}^3(s) + \sigma_{K^{*}\pi}^3(s)}{\sigma_{K\rho}^3(M_{K_{1L}}^2) + \sigma_{K^{*}\pi}^3(M_{K_{1L}}^2)} \right], \\
\Gamma_{K_{1H}}(s) = \frac{s}{M_{K_{1H}}^2} \left[ \frac{\sigma_{K\rho}^3(s) + \sigma_{K^{*}\pi}^3(s)}{\sigma_{K\rho}^3(M_{K_{1H}}^2) + \sigma_{K^{*}\pi}^3(M_{K_{1H}}^2)} \right], \\
\Gamma_{a_1}(s) = \frac{s}{M_{a_1}^2} \left[ \frac{\sigma_{\pi\rho}^3(s) + \frac{1}{2}\sigma_{KK^*}^3(s)}{\sigma_{\pi\rho}^3(M_{a_1}^2) + \frac{1}{2}\sigma_{KK^*}^3(M_{a_1}^2)} \right],
\]

where

\[
\sigma_{PQ}(s) = \frac{1}{8} \sqrt{(s - (m_P + m_Q)^2)(s - (m_P - m_Q)^2)} \theta (s - (m_P + m_Q)^2); \\
\]

and the constant parameter \( \Gamma_R \) appearing in the energy dependent width will be fixed at the central value of the corresponding resonance decay width given in \([\text{1}])\). For \( a_1(1260) \), \( \Gamma_{a_1} = 0.5 \text{ GeV} \) will be taken. In the following sections, the energy dependent widths will be always implemented into our discussion.

C. Predictions for branching ratios only including the lowest multiplet

In this subsection, we perform our discussion only including the lowest multiplet resonances for \( \tau \) decays. If the values for vector resonance couplings are taken from \([\text{15,16}])\) and for axial-vector resonance couplings we take \( \lambda' = \frac{1}{2}, \lambda'' = 0, \lambda_0 = \frac{\lambda'+\lambda''}{4}, \theta = 45^\circ \) (assuming ideal mixing), the theoretical predictions for branching ratios are summarized in Table \([\text{I}]\) for \( \Delta S = 0 \) process and Table \([\text{II}]\) for \( \Delta S = 1 \) process, which are presented in the next subsection.

We can see that the largest gap between the theoretical prediction and experiment data happens in \( \tau^- \rightarrow \omega \pi^- \nu_{\tau} \) channel. Since only vector current enters into \( \tau^- \rightarrow \omega \pi^- \nu_{\tau} \), it implies that only including the lowest vector resonance in this channel is not enough and \( d_3 \) determined in \([\text{15}])\) will get non-negligible corrections when extra multiplet is introduced. Therefore it can be a perfect channel to investigate vector resonances. Moreover \( \tau^- \rightarrow \rho^0 \pi^- \nu_{\tau} \) can be an excellent process to study the axial-vector resonance, since only the axial-vector current takes part in this channel. However the data for this channel is absent.
In the following discussion, we will always include two multiplets for vector resonances and fit unknown couplings in the \( \tau^− \to \omega \pi^− \nu_\tau \) spectral function and the invariant mass distribution for \( \omega K \) in \( \tau^− \to \omega K^− \nu_\tau \).

### D. Fitting results

The experiment data of the \( \tau^− \to \omega \pi^− \nu_\tau \) spectral function is given in [2] and the invariant mass distribution of \( \omega K \) system in \( \tau^− \to \omega K^− \nu_\tau \) can be found in [3].

Following the definition of spectral functions for \( \tau \) decays first given in [27] and recently summarized in [28], the explicit expression of the \( \tau^− \to \omega \pi^− \nu_\tau \) spectral function in our model can be written as

\[
V(s) = \frac{1}{6F^2M_\omega^2\pi^2S_{EW}} \left[ m_\pi^4 + (M_\omega^2 - s)^2 - 2m_\pi^2(M_\omega^2 + s) \right]^{3/2} \\
\times \left| \left( 2d_3F_V + d_sF_{V_1} \right) \frac{M_\omega^2}{M_V^2} + F_{V_1}(d_m m_\pi^2 + d_M M_\omega^2 + d_s s)D_\rho'(s) \right|^2 + 2F_V [(d_1 + 8d_2)m_\pi^2 + d_3(s + M_\omega^2 - m_\pi^2)]D_\rho(s),
\]

where Eq.(20), Eq.(21) and Eq.(22) have been used and the value of the electroweak correction factor \( S_{EW} \), which has been analyzed in [29], will be taken as \( S_{EW} = 1.0194 \) (at the scale \( m_\tau \)).

In the fit of the \( \tau^− \to \omega \pi^− \nu_\tau \) spectral function, we set \( F_{V_1} = -0.1 \text{GeV}, d_m = -1.0 \) and fit the parameters \( d_3, d_M, d_s \). The fitting results are

\[
\begin{align*}
d_3 &= -0.25 \pm 0.01, \\
d_M &= 0.99 \pm 0.08, \\
d_s &= -0.29 \pm 0.03,
\end{align*}
\]

with \( \chi^2/d.o.f = 25.8/13 \simeq 2.0 \).

The decay width for \( \rho^\prime \) given in [1] is \( 0.4 \pm 0.06 \text{ GeV} \). The fitting results above are based on \( \Gamma_{\rho^\prime} = 0.4 \text{ GeV} \). If we take \( \Gamma_{\rho^\prime} = 0.34 \text{ GeV} \), we find the \( \chi^2 \) in the fit will get better. The fitting results then are

\[
\begin{align*}
d_3 &= -0.25 \pm 0.01, \\
d_M &= 0.97 \pm 0.08, \\
d_s &= -0.27 \pm 0.03,
\end{align*}
\]

with \( \chi^2/d.o.f = 21.8/13 \simeq 1.7 \).
In the fit of the invariant mass distribution of $\omega K$, we take Eq. (37) as our inputs. So we set $d_m = -1.0, d_s = -0.27, d_3 = -0.25$ and take different values of $\lambda', \lambda'', \lambda_0$ to fit $d_M, \theta$:

- In case of $\lambda' = \frac{M_A^2}{2\sqrt{2}M_V}, \lambda'' = \frac{\sqrt{2}M^2_A - 2M^2_V}{2\sqrt{2}}, \lambda_0 = \frac{\lambda' + \lambda''}{4}$:

  The results are

  \[
  d_M = 0.64 \pm 0.93, \\
  \cos^2 \theta = 0.26 \pm 0.11, 
  \]

  (38)

  with $\chi^2/d.o.f \simeq 3.4/5 = 0.68$.

- In case of $\lambda' = 0.5, \lambda'' = 0, \lambda_0 = 0.125$:

  The results are

  \[
  d_M = 0.64 \pm 0.87, \\
  \cos^2 \theta = 0.28 \pm 0.12, 
  \]

  (39)

  with $\chi^2/d.o.f \simeq 3.3/5 = 0.66$.

Since what we can fit in the two processes for the couplings $d_m, d_M$ are actually the combinations of $(d_mm^2_\pi + d_M M^2_\omega)$ in $\tau^- \rightarrow \omega \pi^- \nu_\tau$ and $(d_mm^2_K + d_M M^2_\omega)$ in $\tau^- \rightarrow \omega K^- \nu_\tau$, the true values for $d_m, d_M$ can be solved using the values we have got in the two processes:

- For $\lambda' = \frac{M_A^2}{2\sqrt{2}M_V}, \lambda'' = \frac{\sqrt{2}M^2_A - 2M^2_V}{2\sqrt{2}}, \lambda_0 = \frac{\lambda' + \lambda''}{4}$:

  \[
  d_m = -1.90, \quad d_M = 1.00. 
  \]

- For $\lambda' = 0.5, \lambda'' = 0, \lambda_0 = 0.125$:

  \[
  d_m = -1.90, \quad d_M = 1.00. 
  \]

(40)

(41)

With the above values of resonance couplings, the predictions for branching ratios we get are summarized in Table II and Table III.

At the end of this section, some comments about the fitting results are given below:

1. For $\rho K^-, \omega K^-$ and $K^{0}\pi^-$ channels: The branching fractions for $\tau^- \rightarrow \omega K^- \nu_\tau$ with different choices for $\lambda_i$ are both consistent with the experimental value. However in our model we
One multiplet

3

8

(1

1

4

Fit 1

Fit 2

K

K

φK

φK

2. For ∆S = 1 processes. The meaning of numbers in different columns is the same to Table I. The experimental values for φK− channel are taken from [4] and [5]. The remaining experimental data is taken from [1].

B(τ− → ρ0K−ντ)
(B(τ− → ωπ−ντ))
B(τ− → K*0K−ντ)

(1.6 ± 0.6) × 10−3
(1.95 ± 0.08) × 10−2
(2.1 ± 0.4) × 10−3

3.9 × 10−4
0.17 × 10−2
1.4 × 10−3

4.7 × 10−4
2.1 × 10−2
1.5 × 10−3

3.5 × 10−4
2.1 × 10−2
1.5 × 10−3

TABLE II: Branching ratios for ∆S = 1 processes. The experimental values for φK− channel are taken from [4] and [5]. The remaining experimental data is taken from [1].

2. For φK− channel: As we will see later, although our prediction for the invariant mass distribution of φK in the decay of τ− → φK−ντ seems reasonably consistent with the experimental data, the prediction for the branching ratio is around 50% of the experimental value.

3. For ∆S = 0 channel: Since we fit the spectral function for ωπ process, the branching ratio for τ− → ωπ−ντ is always perfect. The corresponding prediction for τ− → K*0K−ντ is also reasonable comparing with the experimental data and it is insensitive to the values of axial-vector resonance couplings λi. Although the branching ratio for τ− → ρ0π−ντ is absent in [1],

Assumptions: only including the lowest multiplet, the fitting results with experimental data is taken from [1]. The values from the third column to the fifth column denote our predictions under different assumptions: only including the lowest multiplet, the fitting results with \( \lambda' = \frac{M_A}{2\sqrt{2}M_V} \), \( \lambda'' = \frac{M_A^2 - 2M_V^2}{2\sqrt{2}M_V} \), \( \lambda_0 = \frac{\lambda' + \lambda''}{4} \) (Fit 1) and the fitting results with \( \lambda' = 0.5, \lambda'' = 0, \lambda_0 = 0.125 \) (Fit 2).

| | Exp | One multiplet | Fit 1 | Fit 2 |
|---|---|---|---|---|
| \( B(\tau^- \rightarrow \rho^0\pi^-\nu_\tau) \) | (1.6 ± 0.6) × 10−3 | 3.9 × 10−4 | 4.7 × 10−4 | 3.5 × 10−4 |
| \( B(\tau^- \rightarrow \omega\pi^-\nu_\tau) \) | (1.95 ± 0.08) × 10−2 | 0.17 × 10−2 | 2.1 × 10−2 | 2.1 × 10−2 |
| \( B(\tau^- \rightarrow K^{*0}\pi^-\nu_\tau) \) | (2.1 ± 0.4) × 10−3 | 1.4 × 10−3 | 1.5 × 10−3 | 1.5 × 10−3 |

TABLE I: Branching ratios for ∆S = 0 processes. The second column denotes experimental values, which are taken from [1]. The values from the third column to the fifth column denote our predictions under different
the branching fraction of $\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$ can be a reference for the $\tau^- \to \rho^0 \pi^- \nu_\tau$ process. Comparing the experimental data $B(\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau(ex.K^0, \omega)) = (8.99 \pm 0.08) \times 10^{-2}$ [1], our predictions indicate that the branching ratio of $\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$ is dominantly contributed by the $\tau^- \to \rho^0 \pi^- \nu_\tau$ channel.

4. In the fit of the invariant mass distribution of $\omega K^-$, we get a rather large error for the coupling $d_M$. To understand this, we have made a detail analysis for this process. If we switch off all of the other contributions except $K^{*}\ell$ resonance in $\tau^- \to \omega K^- \nu_\tau$, the remaining branching ratio is only around 12\% of the total. Hence the large error we get for $d_M$, a resonance coupling related with $K^{*\prime}$, is not hard to understand. If the contributions from $K_1(1270)$ and $K_1(1400)$ are turned off, we find the remaining parts contribute around 38\% of the total branching ratio and the situations in $\rho K^-$ and $\overline{\rho} K^0 \pi^-$ channels are similar to $\omega K^-$ channel, which indicates that the axial-vector resonances play a rather important role in these processes. However this phenomenon does not happen in $\phi K^-$ channel. In contrast, if we switch off all of the other contributions except $a_1$ resonance in $\tau^- \to K^{*0} K^- \nu_\tau$ channel, which is driven by the $\Delta S = 0$ current, only 23\% of the total branching ratio can be reached.

5. In case of $\lambda' = 0.5, \lambda'' = 0, \lambda_0 = 0.125$, our fitting result for $\theta$ is $\cos^2 \theta = 0.28 \pm 0.12$, which indicates $|\theta| = 58.1^{+8.4}_{-7.3}$ degrees. The result in the other case is quite similar. Our result for $\theta$ is consistent with $|\theta| = 37^0$ and $58^0$ recently determined also in $\tau$ decays [30]. Our fit favors the larger value.

6. About the uncertainties of our predictions for the branching ratios, taking into account the approximation we have made throughout this paper, such as working in the leading order of $1/N_C$ expansion and $SU(3)$ symmetry for vector and axial-vector resonances in the T-matrix, and also considering the large error of the resonance couplings that we get from the fit of $\tau^- \to \omega K^- \nu_\tau$, a conservative estimate of the uncertainties of our predictions for the branching ratios should be around thirty percent.

7. Comparison of the figures we have obtained for the $\tau^- \to \omega \pi^- \nu_\tau$ spectral function and the invariant mass distributions for $\omega K^-, \phi K^-$ between the experimental data are given in Fig.(1-3) respectively. Although different choices for $\lambda_i$ affect the branching ratios, the invariant mass distributions are barely influenced. So we only plot the figures with $\lambda' = 0.5, \lambda'' = 0, \lambda_0 = 0.125$. 
FIG. 1: Spectral function for $\tau^- \to \omega \pi^- \nu_\tau$. The experimental data are taken from [2].

FIG. 2: Invariant mass distribution for $\omega K^-$ in the process of $\tau^- \to \omega K^- \nu_\tau$. The experimental data are taken from [3].

IV. CONCLUSION

In this work, the resonance chiral effective theory is exploited to study a $\tau$ decaying into a vector resonance plus a pseudo-Goldstone meson and a $\tau$ neutrino. Two multiplets of vector resonances have been introduced in our discussion. We fit the $\tau^- \to \omega \pi^- \nu_\tau$ spectral function and decay distribution of $\tau^- \to \omega K^- \nu_\tau$ to get unknown resonance couplings. Then we make a prediction for branching ratios for all channels. Taking into account the approximation we have made in this paper, like taking the leading order of $1/N_C$ expansion and $SU(3)$ symmetry for vector and axial-vector resonances in the T-matrix, we conclude that resonance chiral effective theory can describe the experimental data reasonably, although the issue of the small ratio $\frac{B(\tau^- \to \omega K^- \nu_\tau)}{B(\tau^- \to \rho K^- \nu_\tau)}$ raised in [3] is
FIG. 3: Invariant mass distribution for $\phi K^-$ in the process of $\tau^- \rightarrow \phi K^- \nu_\tau$. The experimental data are taken from [4], where only the data up to $m_{\phi K} = 1.75$ GeV are quoted in the plot.

still there. Our fit for the mixing angle $\theta$, which is defined in Eq.(9) to describe the mixture between the flavor eigenstates of the axial-vector resonances and physical states $K_1(1270), K_1(1400)$, leads to $|\theta| \simeq 58^\circ$, which is consistent with previous determination with $|\theta| = 37^\circ$ and $58^\circ$ also from tau decays [30].

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APPENDIX: Explicit expressions of form factors for other channels

The form factors of $v, a_1, a_2, a_3$ for other channels derived by $\Delta S = 1$ currents are given by
\[ \tau^-(p) \rightarrow K^-(p_1)\nu(p_2)\nu_\tau(q) : \]

\[
v = -i\frac{F_{V_1}}{F_K M_\omega} [d_m m_K^2 + d_M M^2_\phi + d_s s] D_{K^*}(s) \\
- i\frac{2F_V}{F_K M_\omega} [(d_1 + 8d_2)m_K^2 + d_3(-m_K^2 + M^2_\phi + s)] D_{K^*}(s) \\
+ i\frac{\sqrt{2}}{F_K M_V M_\omega} [(c_1 + c_2 + 8c_3 - c_5)m_K^2 + (c_2 + c_5 - c_1 - 2c_6)M^2_\phi + (c_1 - c_2 + c_5)s], \]

\[
a_1 = \frac{1}{4M_\omega F_K} \left\{ F_V(m_K^2 - M^2_\omega - s) - 2G_V(m_K^2 + M^2_\omega - s) + \\
+ 2\sqrt{2} F_A [\lambda_0 m_K^4 + (M^2_\phi - s)(\lambda M^2_\omega - \lambda'' s) - m_K^2(\lambda_0 M^2_\omega + \lambda M^2_\phi + \lambda_0 s + \lambda'' s)] \\
\times [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)] \right\}, \]

\[
a_2 = -\frac{G_V M_\omega}{F_K} D_{K}(s) + \frac{\sqrt{2} F_A M_\omega}{F_K} (\lambda + \lambda'') [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)], \]

\[
a_3 = \frac{F_V - 2G_V}{2F_K M_\omega} \frac{G_V M_\omega}{F_K} D_{K}(s) + \\
+ \frac{\sqrt{2} F_A}{F_K M_\omega} (\lambda_0 m_K^2 + \lambda M^2_\omega - \lambda'' s) [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)]. \quad (A-1) \]

\[ \tau^-(p) \rightarrow K^-(p_1)\phi(p_2)\nu_\tau(q) : \]

\[
v = i\frac{\sqrt{2} F_{V_1}}{F_K M_\phi} [d_m m_K^2 + d_M M^2_\phi + d_s s] D_{K^*}(s) \\
+ i\frac{2\sqrt{2} F_V}{F_K M_\phi} [(d_1 + 8d_2)m_K^2 + d_3(-m_K^2 + M^2_\phi + s)] D_{K^*}(s) \\
- i\frac{\sqrt{2}}{F_K M_V M_\phi} [(c_1 + c_2 + 8c_3 - c_5)m_K^2 + (c_2 + c_5 - c_1 - 2c_6)M^2_\phi + (c_1 - c_2 + c_5)s], \]

\[
a_1 = \frac{1}{2\sqrt{2}M_\phi F_K} \left\{ F_V(m_K^2 - M^2_\phi - s) - 2G_V(m_K^2 + M^2_\phi - s) + \\
+ 2\sqrt{2} F_A [\lambda_0 m_K^4 + (M^2_\phi - s)(\lambda M^2_\phi - \lambda'' s) - m_K^2(\lambda_0 M^2_\phi + \lambda M^2_\phi + \lambda_0 s + \lambda'' s)] \\
\times [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)] \right\}, \]

\[
a_2 = -\frac{\sqrt{2} G_V M_\phi}{F_K} D_{K}(s) + \frac{2F_A M_\phi}{F_K} (\lambda + \lambda'') [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)], \]

\[
a_3 = \frac{F_V - 2G_V}{\sqrt{2}F_K M_\phi} \frac{\sqrt{2} G_V M_\phi}{F_K} D_{K}(s) + \\
+ \frac{2F_A}{F_K M_\phi} (\lambda_0 m_K^2 + \lambda'' M^2_\phi - \lambda'' s) [c^2_\theta D_{K1H}(s) + s^2_\theta D_{K1L}(s)]. \quad (A-2) \]
• $\tau^-(p) \rightarrow \pi^- (p_1) \bar{K}^{*0}(p_2) \nu_\tau(q)$:

$v = -i\frac{\sqrt{2}F_V}{F M_{K^*}}[d_m m^2_\pi + d_M M^2_{K^*} + d_s s]D_{K^*}(s) - i\frac{2\sqrt{2}F_V}{FM_{K^*}}[(d_1 + 8d_2)m^2_\pi + d_3(-m^2_\pi + M^2_{K^*} + s)]D_{K^*}(s) + i\frac{\sqrt{2}FM_V M_{K^*}}{2}[(c_1 + c_2 + 8c_3 - c_5)m^2_\pi + (c_2 + c_5 - c_1 - 2c_6)M^2_{K^*} + (c_1 - c_2 + c_5)s],$

\[a_1 = -\frac{1}{2\sqrt{2}M_{K^*} F} \left\{ F_V (m^2_\pi - M^2_{K^*} - s) - 2G_V (m^2_\pi + M^2_{K^*} - s) + 2\sqrt{2}F_A [\lambda_0 m^4_\pi + (M^2_{K^*} - s) (\lambda' M^2_{K^*} - \lambda'' s) - m^2_\pi (\lambda_0 M^2_{K^*} + \lambda' M^2_{K^*} + \lambda_0 s + \lambda'' s)] \times [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)] \right\},\]

\[a_2 = \frac{\sqrt{2}G_V M_{K^*}}{F} D_{K}(s) - \frac{2F_A M_{K^*}}{F} (\lambda' + \lambda'') [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)],\]

\[a_3 = -\frac{F_V - 2G_V}{\sqrt{2}FM_{K^*}} + \frac{\sqrt{2}G_V M_{K^*}}{F} D_{K}(s) - \frac{2F_A}{FM_{K^*}} (\lambda_0 m^2_\pi + \lambda'' M^2_{K^*} - \lambda'' s) [c_0^2 D_{K1H}(s) + s_0^2 D_{K1L}(s)].\]  

(A-3)

The form factors for the $\Delta S = 0$ processes are:

• $\tau^-(p) \rightarrow \pi^- (p_1) \rho^0(p_2) \nu_\tau(q)$

$v = 0, \quad$ (A-4)

\[a_1 = \frac{1}{2M_\rho F} \left\{ F_V (m^2_\pi - M^2_\rho - s) - 2G_V (m^2_\pi + M^2_\rho - s) + 2\sqrt{2}F_A D_{a_1}(s) \times \times [\lambda_0 m^4_\pi + (M^2_\rho - s) (\lambda' M^2_\rho - \lambda'' s) - m^2_\pi (\lambda_0 M^2_\rho + \lambda' M^2_\rho + \lambda_0 s + \lambda'' s)] \right\},\]

\[a_2 = -\frac{2G_V M_\rho}{F} D_{\pi}(s) + \frac{2\sqrt{2}F_A M_\rho}{F} (\lambda' + \lambda'') D_{a_1}(s),\]

\[a_3 = \frac{F_V - 2G_V}{FM_\rho} - \frac{2G_V M_\rho}{F} D_{\pi}(s) + \frac{2\sqrt{2}F_A}{FM_\rho} (\lambda_0 m^2_\pi + \lambda'' M^2_\rho - \lambda'' s) D_{a_1}(s).\]  

(A-5)
• $\tau^-(p) \to \pi^-(p_1)\omega(p_2)\nu_\tau(q)$

$$v = -i \frac{2F_V}{F_{M_\omega}} [d_m m_\pi^2 + d_M M_\omega^2 + d_s s] D_\rho(s)$$

$$-i \frac{4F_V}{F_{M_\omega}} [(d_1 + 8d_2)m_\pi^2 + d_3(-m_\pi^2 + M_\omega^2 + s)] D_\rho(s)$$

$$+ i \frac{2\sqrt{2}}{F_{M_V} M_\omega} [(c_1 + c_2 + 8c_3 - c_5)m_\pi^2 + (c_2 + c_5 - c_1 - 2c_6)M_\omega^2 + (c_1 - c_2 + c_5)s],$$

(A-6)

$$a_i = 0.$$  

(A-7)

• $\tau^-(p) \to K^-(p_1)K^{*0}(p_2)\nu_\tau(q)$

$$v = -i \frac{2\sqrt{2}F_V}{F_K M_{K^*}} [(d_1 + 8d_2)m_K^2 + d_3(-m_K^2 + M_{K^*}^2 + s)] D_\rho(s)$$

$$+ i \frac{2}{F_{M_V} M_{K^*}} [(c_1 + c_2 + 8c_3 - c_5)m_K^2 + (c_2 + c_5 - c_1 - 2c_6)M_{K^*}^2 +$$

$$(c_1 - c_2 + c_5)s] - i \frac{\sqrt{2}F_V}{F_K M_{K^*}} [d_m m_K^2 + d_M M_{K^*}^2 + d_s s] D_\rho(s),$$

$$a_1 = -\frac{1}{2\sqrt{2}M_{K^*} F_K} \left\{ F_V (m_K^2 - M_{K^*}^2 - s) - 2G_V (m_K^2 + M_{K^*}^2 - s) + 

+ 2\sqrt{2}F_A [\lambda_0 m_K^4 + (M_{K^*}^2 - s)(\lambda' M_{K^*}^2 - \lambda'' s) 

- m_K^2 (\lambda_0 M_{K^*}^2 + \lambda' M_{K^*}^2 + \lambda_0 s + \lambda'' s)] D_{a_1}(s) \right\},$$

$$a_2 = \frac{\sqrt{2}G_V M_{K^*}}{F_K} D_\pi(s) - \frac{2F_A M_{K^*}}{F_K} (\lambda' + \lambda'') D_{a_1}(s),$$

$$a_3 = -\frac{F_V + 2G_V}{\sqrt{2}F_K M_{K^*}} + \frac{\sqrt{2}G_V M_{K^*}}{F_K} D_\pi(s) - \frac{2F_A}{F_K M_{K^*}} (\lambda_0 m_K^2 + \lambda' M_{K^*}^2 - \lambda'' s) D_{a_1}(s),$$

(A-8)

where the contributions from the new vector multiplet $V_1$ have been included in the expressions above. We have used the same notation $a_1$ for the axial vector resonance $a_1(1260)$ and the axial vector form factor. This should not cause any confusion.

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