ANALYZING X-RAY VARIABILITY BY STATE SPACE MODELS

Application to an EXOSAT AGN sample

MICHAEL KÖNIG AND RÜDIGER STAUBERT
Institut für Astronomie und Astrophysik - Astronomie,
Universität Tübingen, Germany
AND
JENS TIMMER
Fakultät für Physik, Albert-Ludwigs-Universität Freiburg, Germany

Abstract. In recent years, autoregressive models have had a profound impact on the description of astronomical time series as the observation of a stochastic process. These methods have advantages compared with common Fourier techniques concerning their inherent stationarity and physical background. If autoregressive models are used, however, it has to be taken into account that real data always contain observational noise often obscuring the intrinsic time series of the object. We apply the technique of a Linear State Space Model which explicitly models the noise of astronomical data and allows to estimate the hidden autoregressive process. As an example, we have analysed a sample of Active Galactic Nuclei (AGN) observed with EXOSAT and found evidence for a relationship between the relaxation timescale and the spectral hardness.

1. Introduction and Mathematical Background

Irregular X-ray variability is a common phenomenon of Active Galactic Nuclei (AGN). The power spectra of the observed lightcurves exhibit an ‘observational noise floor’ at high frequencies, an increase of power towards low frequencies ($1/f^\alpha$ behavior) and, most often in long term lightcurves, a flat top at the low frequency end of the spectrum (McHardy 1988). As a commonly applied method the periodogram is used to estimate part of the power spectrum with a slope $\alpha$ usually between 0 and 2 and a mean of about 1.5 (Lawrence and Papadakis 1993).
When the periodogram is computed, the window function (the Fourier transform of the observational sampling) is convolved with to the true spectrum of the source. This can produce artefacts in the power spectrum, which make a proper interpretation more difficult (Papadakis and Lawrence 1995; Priestley 1992). In the case of AGN spectra the sidebands of the window function yield spectral leakage to higher frequencies which will cause the spectra to appear less steep and the spectral slope will be underestimated (Deeter and Boynton 1982; Deeter 1984). Consequently a model is required which operates in the time domain and avoids any misleading systematic effects. As an alternative approach, we have used the Linear State Space Model (LSSM). The LSSM is a generalization of the autoregressive (AR) model invented by Yule (1927) to model the variability of Wolf’s sunspot numbers.

The AR model expresses the temporal correlations of the time series in terms of a linear function of its past values plus a noise term and is closely related to the dynamics of the system. LSSMs generalize the AR processes by explicitly modelling observational noise. The variable $x(t)$ that has to be estimated cannot be observed directly since it is covered by observational noise $\eta(t)$. The measured observational variables $y(t)$ do not generally agree with the system variables $x(t)$. Thus an LSSM is defined with two equations, the system or dynamical equation (1) and the observation equation (2).

\[
\begin{align*}
\vec{x}(t) &= A \vec{x}(t-1) + \vec{\epsilon}(t) \quad \vec{\epsilon}(t) \sim \mathcal{N}(0, Q) \\
y(t) &= C \vec{x}(t) + \eta(t) \quad \eta(t) \sim \mathcal{N}(0, R)
\end{align*}
\]

This definition is a multivariate description, which means that the AR[p] process is given as a $p$-dimensional AR process of order one, with a matrix $A$ that determines the dynamics. The matrix $C$ maps the unobservable dynamics to the observation. The terms $\vec{\epsilon}(t)$ and $\eta(t)$ represent the Gaussian dynamical noise with covariance matrix $Q$ and the Gaussian observational noise with variance $R$, respectively. We have used the Expectation-Maximization algorithm to estimate the dynamics and a KS test to quantify the order of the hidden AR process (Honerkamp 1993, König and Timmer 1997).

2. The EXOSAT AGN sample

As the X-ray lightcurves from the X-ray satellite EXOSAT are the longest existing observations of AGN, we have used individual observations longer than 25 ks to create an EXOSAT AGN timing sample (see table 1). We use the lightcurve of the Seyfert galaxy NGC 5506 as an example for a brief description of how to apply LSSM and interpret LSSM results. We have found that the X-ray lightcurve of NGC 5506 can be well modelled with a
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Figure 1. a) EXOSAT ME X-ray lightcurve of NGC 5506 (30sec/bin, Jan. 1986), b) Hidden AR[1]-process, estimated with the LSSM fit. Both lightcurves are shown without error bars for clarity.

LSSM AR[1] model, since the residuals between the estimated AR[1] process and the measured data are consistent with Gaussian white noise. The corresponding dynamical parameter of the LSSM AR[1] fit corresponds to a relaxation time of about 1.6 hours. Higher order LSSM AR[p] models cannot improve the fits significantly. Therefore more complex models (with additional relaxators and damped oscillators) are not needed to describe the dynamics of the AGN lightcurve. In addition, an AR[1] model provides all features occurring in AGN power spectra, especially the flat top at low frequencies which cannot be described by the $1/f$ models (König and Timmer 1997).

3. Discussion

The physical interpretation of the discovered AR[1] dynamics is a stochastic superposition of single shots all decaying with the same relaxation time. The assumption of an exponentially decaying shot seems to be reasonable as time-dependent Inverse Comptonisation (IC) models lead to such a pulse profile (Liang and Nolan 1983). In this scenario inhomogeneities in the accretion flow produce single UV flares. These UV photons are then upscattered in the Comptonising medium producing the observed X-ray energy spectrum. Since harder X-ray photons need more interactions in the Compton cloud and thus have experienced a longer stay in the medium, they are temporally delayed with respect to the soft X-ray photons. The LSSM fits of the EXOSAT AGN sample yield a relationship between the
Smirnov test on white noise residuals, derived to (table 1), which might be a fingerprint of the IC process. The relation is spectral hardness (expressed by the slope $\Gamma$ of the power-law fit of the photon spectrum fit). The relation is derived to $\tau = 74890 - 38040/\Gamma$ sec with a corresponding Kendalls tau value of 0.744 and a two-sided significance level of 5.7 $\cdot$ 10$^{-5}$ for the hypothesis of uncorrelated data. Since the $\tau - \Gamma$ relation is independent of the AGN type, a common physical mechanism for the spectral and temporal behavior is needed for all kinds of AGN — possibly in support of the unifying model of Active Galactic Nuclei (Netzer 1990).

### References

Deeter J.E., 1984, ApJ 281, 482-491
Deeter J.E., Boyton P.E., 1982, ApJ 261, 337-350
Honerkamp J., 1993, Stochastic Dynamical Systems, VCH Publ. New York, Weinheim
König M., Timmer J., 1997, A&A, accepted for publication
Lawrence A., Papadakis P., 1993, ApJ Suppl. 414, 85
Liang E.P., Nolan P.L., 1983, Space Sci. Review 38, 353
McHardy I., Czerny B., 1987, Nature 325, 696
Netzer H., 1990, Saas-Fee Advanced Course 20 on Active Galactic Nuclei, eds. R.D. Blandford, H.Netzer, L.Woltjer, 57-160
Papadakis I.E., Lawrence A., 1995, MNRAS 272, 161
Priestley M.B., 1992, Spectral Analysis and Time Series, San Diego, Academic Press
Scargle J.D., 1981, ApJ Suppl. 45, 1
Yule G., 1927, Phil. Trans. R. Soc. A 226, 267

### Table 1. LSSM fit results of the EXOSAT AGN sample

| AGN    | observation | $T_{\text{tot}}$ | $L_{\text{2-10 keV}}^0$ | $\tau$ | $K_{\text{S, Test}}^0$ | $\Gamma_{\text{2-10 keV}}^0$ | $\chi^2_{\text{red}}$ |
|--------|-------------|-----------------|------------------------|-------|------------------------|----------------——-|----------------------|
| NGC 4051 | 337/85      | 143.84          | 41.182                 | 2.38$^{+1.94}_{-0.74}$ | 81.2            | 1.90$^{+0.04}_{-0.04}$ | 0.99                |
| MKN 421  | 338/84      | 26.59           | 44.439                 | 3.41$^{+2.39}_{-1.60}$ | 98.0            | 1.89$^{+0.13}_{-0.09}$ | 1.93                |
| MCG 6-30-15 | 28/86       | 183.69          | 42.659                 | 3.63$^{+2.82}_{-1.44}$ | 76.0            | 1.82$^{+0.14}_{-0.10}$ | -                   |
| NGC 4593 | 176/85      | 32.49           | 42.217                 | 4.24$^{+2.67}_{-1.43}$ | 93.5            | 1.82$^{+0.05}_{-0.04}$ | 0.74                |
| NGC 5506 | 24/86       | 225.56          | 42.777                 | 5.89$^{+1.39}_{-1.71}$ | 99.8            | 1.84$^{+0.05}_{-0.10}$ | 0.95                |
| Fairall 9 | 286/83      | 30.60           | 43.791                 | 6.28$^{+1.60}_{-1.80}$ | 83.6            | 1.87$^{+0.10}_{-0.09}$ | 1.42                |
| NGC 4593 | 9/86        | 95.65           | 42.613                 | 9.72$^{+2.64}_{-0.70}$ | 80.1            | 1.78$^{+0.05}_{-0.05}$ | 1.01                |
| NGC 5548 | 62/86       | 85.88           | 43.430                 | 10.65$^{+3.45}_{-2.14}$ | 85.8            | 1.61$^{+0.04}_{-0.03}$ | 1.47                |
| 3C 120   | 228/83      | 44.61           | 43.896                 | 11.34$^{+3.22}_{-2.24}$ | 84.9            | 1.70$^{+0.06}_{-0.04}$ | 0.74                |
| Com A    | 44/84       | 44.22           | 42.147                 | 13.29$^{+11.48}_{-4.21}$ | 94.4            | 1.75$^{+0.08}_{-0.08}$ | 1.28                |
| 3C 120   | 276/84      | 44.52           | 44.029                 | 13.30$^{+11.42}_{-4.21}$ | 97.0            | 1.71$^{+0.03}_{-0.03}$ | 1.39                |
| NGC 5548 | 19/86       | 63.75           | 43.545                 | 13.54$^{+12.11}_{-4.25}$ | 95.9            | 1.64$^{+0.03}_{-0.03}$ | 0.83                |
| 1068     | 8/85        | 54.35           | 41.262                 | 16.05$^{+8.82}_{-4.11}$ | 83.1            | 1.68$^{+0.07}_{-0.05}$ | 0.71                |
| 3C 273   | 17/86       | 145.20          | 46.402                 | 18.02$^{+11.35}_{-5.03}$ | 98.4            | 1.52$^{+0.02}_{-0.02}$ | 0.90                |
| NGC 4515 | 192/83      | 86.66           | 42.327                 | 28.54$^{+9.91}_{-7.23}$ | 75.0            | 1.23$^{+0.11}_{-0.09}$ | 1.31                |
| NGC 4515 | 27/85       | 94.45           | 42.393                 | 31.77$^{+14.61}_{-11.82}$ | 90.7            | 1.53$^{+0.10}_{-0.10}$ | 0.60                |

a: total observation time, b: X-ray luminosity in the 2-10 keV energy range, c: AR relaxation time, d: Kolmogorov-Smirnov test on white noise residuals, e: slope of the power law fit of the photon spectrum in the 2-10 keV energy range, f: reduced $\chi^2$ of the photon spectrum fit