Computational Experiment of Non-Isothermal Filtration of Real Gas during at Injection through a Short Borehole

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Abstract. In this paper, a computational experiment of injecting real gas through a single borehole, revealing 80% of the capacity of the gas-bearing formation, taking into account the heat exchange of the reservoir formation with the host rocks. The experiment is performed within the modified mathematical model of non-isothermal gas filtration, which is derived from the laws of mass and energy conservation, as well as from the Darcy law. Physical and caloric equations of state and Newton-Richmann's law describing the heat exchange of the gas-bearing layer with the host rocks were used as closing relations. The results obtained in this experiment were compared with the results obtained earlier in the case when the borehole revealed the entire formation capacity. When injecting gas through a short borehole, significant changes in the dynamics of the temperature field in the bottomhole zone were revealed.

1. Research objective

The complete system of equations describing the thermohydrodynamic state of the non-deformable layer consists of the equations of continuity, energy, state and Darcy law [1]. To describe the process of injection of natural gas into a circular formation through a single borehole, it is advisable to write this system in cylindrical coordinates, where the vertical axis coincides with the axis of the borehole. In the case when the borehole does not fully disclose the entire capacity of the gas-bearing layer, it is necessary to take into account the pressure and temperature gradients on the vertical coordinate, while the angular gradients of the latter can be neglected. Since the borehole radius is an order of magnitude smaller than the radius of the outer contour of the gas-bearing layer, the calculated area is represented as a rectangle for convenience. Under such assumptions, the original system will take the form of:

\[
\frac{\partial \frac{\rho}{\partial t}}{\partial T} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\rho \frac{\partial \rho}{\partial T}}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{\rho \frac{\partial \rho}{\partial T}}{\partial z} \right),
\]

(1)

\[
\frac{\partial T}{\partial t} = \delta \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\rho \frac{\partial T}{\partial r}}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + \left( 1 + \frac{\rho \frac{\partial T}{\partial r}}{R \frac{\partial T}{\partial r}} + \frac{\rho \frac{\partial T}{\partial z}}{\frac{\partial T}{\partial z}} \right) - \frac{T}{\frac{\partial T}{\partial r}} \left( \frac{\partial \rho}{\partial r} \right)^2 + \left( \frac{\partial \rho}{\partial z} \right)^2,
\]

(2)

where

\[
r = \frac{r}{l}, \quad \bar{r} = \frac{r}{l}, \quad \bar{r}_w = \frac{r_w}{l}, \quad \bar{r}_k = \frac{r_k}{l}, \quad \bar{z} = \frac{z}{l}, \quad \bar{h} = \frac{h}{l}, \quad \bar{\tau} = \frac{k_p \tau}{\bar{l}^2},
\]

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\[ \bar{p} = \frac{p}{p_0}, \quad \bar{T} = \frac{c_v T}{m_p p_0}, \quad \delta = \frac{\kappa}{\kappa_p}, \quad \kappa = \frac{\lambda_r}{c_r}, \quad \kappa_p = \frac{k p_0}{m_k}. \]

In the future, the bar above the dimensionless variables is omitted for convenience. Here, the following notation is used: \( p \) – pressure, \( T \) – temperature, \( r \) – radial coordinate, \( z \) – height coordinate, \( t \) – time, \( R \) – gas constant, \( Z \), \( c_p \), \( \mu \) – imperfection coefficient, specific heat capacity and dynamic viscosity of gas, respectively, \( h \) – formation capacity, \( k \) – formation permeability coefficient, \( m \) – porosity, \( \kappa \), \( \kappa_p \), \( c_r \), \( \lambda_r \) – thermal diffusivity, piezo conductivity, volumetric heat capacity and thermal conductivity of a gas-saturated formation, respectively, \( l \) – are a characteristic size; subscripts mean: 0 – the initial state, \( w \) – on the borehole wall, \( k \) – on the outer contour of the reservoir.

The mode of gas injection with a constant mass flow rate and constant temperature is considered.

\[-\frac{p}{\bar{T}} \frac{\partial p}{\partial r} = A, \quad T = T_w, \quad 0 \leq z \leq h_w, \quad r = r_w, \quad (3)\]

where, \( A = \frac{m \mu l R M}{2 \pi r \alpha_w h_w p_0 k r} \), \( M \) – is the mass flow rate of gas, \( h_w \) – is the reservoir capacity disclosed by the borehole.

On the strip \( h_w < z \leq h \), condition (3) is supplemented by the symmetry condition at a radial coordinate equal to the borehole radius

\[ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad h_w < z \leq h, \quad r = r_w. \quad (4)\]

The outer boundary of the reservoir is gas-tight and thermally insulated.

\[ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad r = r_k. \quad (5)\]

The rocks bedding and covering the gas-bearing formation are considered to have the same thermal-physical properties, the roof and the bottom of the formation are impermeable to gas, and the heat exchange of the formation with the host rocks takes place according to Newton's law.

\[ \frac{\partial p}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = \bar{a}(T - T_0), \quad z = 0, \quad (6) \]

\[ \frac{\partial p}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = -\bar{a}(T - T_0), \quad z = h, \quad (7) \]

where \( \bar{a} = \frac{a l}{\lambda_z} \) – the Biou number, \( a \) – is the heat transfer coefficient, \( \lambda_z \) – is the thermal conductivity of the host rock.

At the initial moment of time, the temperature and pressure of the gas are considered constant.

\[ p(r, z, 0) = 1, \quad T(r, z, t) = T_0, \quad r_w \leq r \leq r_k, \quad 0 \leq z \leq h. \quad (8) \]

The Latonov-Gurevich equation of state is taken as the equation of state Gurevich [2]

\[ Z = \left( 0.17376 \ln \left( \frac{c_v T_e}{m_p p_0} \right) + 0.73 \right) \frac{p_0}{p_c} + 0.1 \frac{p_0}{p_c} p, \quad (9) \]

where, \( T_e, p_c \) – critical temperature and pressure of natural gas, depending on its composition.

To solve the initial boundary value problem (1)-(9) we used a purely implicitly stable differential scheme similar to the scheme used in [3], which considers the process of gas filtration in the case when the borehole reveals the entire reservoir capacity. Difference analogues of the first boundary condition (3), conditions (4)-(7) were recorded with the second order of approximation. The method of simple iterations was used for the numerical realization of the difference problem.

### 2. Computational experiment

Calculations were made for natural gas from the Middle Vilyuysk field in the Republic of Sakha (Yakutia) with the following input data: \( \alpha = 2 \) and \( 10 \) Wt/(m\(^2\)-K), \( \lambda_r = 2.326 \) Wt/(m-K), \( r_w = 0.1 \) m, \( r_k = 100.1 \) m, \( h = 10 \) m, \( h_w = 8, 9 \) and \( 90 \) m, \( l = 1 \) m, \( T_0 = T_w = 323 \) K, \( p_0 = 240 \) atm, \( c_p = 2300 \) J/(kg-K), \( R = 449.4 \) J/(kg-K), \( \mu = 2 \cdot 10^{-5} \) Pass-s, \( c_r = 6 \cdot 10^6 \) J/(m\(^3\)-K), \( m = 0.2 \), \( k = 10^{-13} \) m\(^2\), \( M = 1 \) kg/s. Critical parameters of \( T_e = 205.022 \) K and \( p_c = 46.596 \) atmospheres were determined by the method presented in [4], with known gas composition (vol. %): \( \text{CH}_4 \) – 90.34, \( \text{C}_2\text{H}_6 \) – 4.98, \( \text{C}_3\text{H}_8 \) – 1.74, \( i\text{C}_4\text{H}_{10} \) – 0.22, \( n\text{C}_4\text{H}_{10} \) – 0.41, \( \text{C}_5\text{H}_{12}+ \) – 1.55, \( \text{CO}_2 \) – 0.28, \( \text{N}_2 \) – 0.48.
The duration of the experiment was in dimensionless values \( t = 1.6 \cdot 10^6 \), which corresponds to 731.1 h.

3. Results discussion
The computational experiment was carried out in order to reveal the influence of the borehole length on the dynamics of changes in the fields of temperature and pressure in the gas-bearing layer during gas injection. Hereinafter, the borehole length is understood as the length of that part of the borehole, which is located in the formation and is indicated through \( h_w \). Preliminary results have shown that this influence is most typical at higher value of heat exchange coefficient of the reservoir with the host rocks. Therefore, the results obtained at the heat exchange coefficient \( \alpha = 10 \text{ Wt/(m}^2\text{K)} \) are given below.

The results of the computational experiment are illustrated in Figures 1-10. The analysis started at the gas-bearing layer pressure field dynamics. It is obvious from physical considerations that the flow rate per unit of borehole surface area is inversely proportional to the borehole length when injecting gas in the constant flow mode. Correspondingly, in the initial start-up period, the pressure in the bottomhole zone is higher than in the bottomhole zone at the length of the borehole than in the bottomhole zone, which is confirmed by Fig. 1 (comparing surfaces 1 and 2). At the same time, in the part of the reservoir that has not been opened by the borehole this effect is reverse - the pressure grows more slowly. Over time, the vertical pressure distribution is smoothed and becomes almost constant. In the case when the borehole reveals the entire capacity of the gas-bearing formation (surface 3 in Fig. 1), the vertical pressure distribution remains constant, the same results were obtained earlier in the works [3, 5], [3, 5]

![Figure 1](image-url). Spatial pressure distribution at the time \( t = 9533 \): surface 1 at \( h_w = 8 \text{ m} \), surface 2 – \( h_w = 9 \text{ m} \), surface 3 – \( h_w = 10 \text{ m} \).
Figure 2. Dynamics of temperature change at \( h_w = 9 \) m (curves 1 and 2) and at \( h_w = 9 \) m (curves 3 and 4) on the plane of \( z = 5 \) m. Odd numbers correspond to \( h_w = 8 \) m, even numbers to \( h_w = 9 \) m.

The dynamics of pressure changes at all formation points, including the borehole boundary, is monotonically increasing in nature, which is explained by the mode of gas injection with a constant mass flow rate. That is, this mode provides at all times a sufficiently high rate of pressure growth in the bottomhole zone, which will also affect the dynamics of temperature, namely, near the borehole almost at all times, the effect of throttling will be maximum than, for example, in injection mode with constant bottomhole pressure. Thus, in Fig. 2, near the injection point in the initial start-up period due to adiabatic compression gas heating is observed (curves 1 and 2), but with time here begins to dominate the Joule-Thomson effect - the gas cools down, and this effect is preserved during the rest of the calculated time. In some distance from the injection point, for example, at \( r = 8.002 \) m (curves 3 and 4 in Fig. 2), the gas temperature is determined mainly by adiabatic compression, and here it has a monotonically increasing character. In Fig. 1 it is clearly seen that at \( z = 5 \) m the influence of borehole length on the temperature field is localized by the bottomhole zone. This conclusion is confirmed by Fig. 3, where the temperature distribution by radial coordinates is presented at different time values (time is dimensionless).
Figure 3. Temperature distribution by radial coordinate at \( t = 2863 \) (curves 1 and 2) and at \( t = 270600 \) (curves 3 and 4) on the plane of \( z = 5 \) m. Odd numbers correspond to \( h_w = 8 \) m, even numbers to \( h_w = 9 \) m.

At the gas-bearing stratum strip \( h_w < z < h \), uncovered borehole, the temperature distribution along the vertical coordinate in the bottomhole zone has some resemblance to the radial distribution. First, the temperature increases due to adiabatic expansion, then, as the pressure equalizes vertically, the gas begins to cool down (see Fig. 4 a). This is explained by the fact that in the course of time the throttling effect begins to dominate, and this effect is most pronounced in the vertical direction than in the radial direction, and, as a consequence, the strongest cooling of gas near the injection point is observed on the strip \( h_w < z < h \) (see curves 3 and 4 in Fig. 5). The increase in borehole length leads to a noticeable increase in temperature in the bottomhole zone on the strip \( h_w < z < h \) (compare curves 1 and 2, 3 and 4 in Fig. 5). This is explained by the fact that the borehole maintains a constant temperature and reduces the distance from the injection point to the bottom of the formation, through which the heat exchange with the host rocks takes place.

Figure 4. Temperature on vertical coordinate on bottomhole (a) and at \( r = 0.641 \) (b): curves 1 and 2 – at \( t = 835 \), curves 3 and 4 – at \( t = 2863 \). Odd numbers correspond to \( h_w = 8 \) m, even numbers correspond to \( h_w = 9 \) m.
Figure 5. The same curves as in Fig. 4: curves 1 and 2 – at $t = 95333$, curves 3 and 4 – at $t = 1.6 \times 10^6$.

Limitation of the zone of influence of the borehole length on the dynamics of temperature field changes is well seen in Fig. 6, where spatial temperature distributions at different time values are presented, and the size of this zone increases with time. Also from the presented figure it is possible to notice that the given influence increases in process of decrease in length of a borehole.

Figure 6. Temperature distribution by spatial variables at the moment $t = 95333$ (a) and at $t = 1.6 \times 10^6$ (b): surface 1 - at $h_w = 8$ m, surface 2 - at $h_w = 9$ m

4. Conclusions
Thus, the length of the borehole (within the limits of 80-90% of the gas-bearing layer capacity) at gas injection into the formation saturated with the same gas, has a weak impact on the dynamics of changes in the reservoir pressure field, and this effect is most pronounced in the initial start-up period, but the length of the borehole is quite strongly influenced by the dynamics of changes in the temperature field in the zone between the soles of the borehole and the formation, the size of this zone with time increases.

5. References
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