Prospects for the measurement of $B_s^0$ oscillations with the ATLAS detector at LHC

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Abstract. The capabilities of the ATLAS detector to measure the $B_s^0$ oscillations in proton-proton interactions at the Large Hadron Collider were evaluated. $B_s^0$ candidates in the $D_s^+\pi^+$ and $D_s^0\phi$ decay modes from semileptonic exclusive events were fully simulated and reconstructed, using a detailed detector description. The sensitivity and the expected accuracy for the measurement of the oscillation frequency were derived from unbinned maximum likelihood amplitude fits as functions of the integrated luminosity. A detailed treatment of the systematic uncertainties was performed. The dependence of the measurement sensitivity on various parameters was also evaluated.

1 Introduction

The observed $B_s^0$ and $\bar{B}_s^0$ states are linear combinations of two mass eigenstates, denoted here as $H$ and $L$. Due to the non-conservation of flavour in charged weak-current interactions, transitions between $B_s^0$ and $\bar{B}_s^0$ states occur with a frequency proportional to $\Delta m_s = m_H - m_L$.

Experimentally, these $B_s^0$–$\bar{B}_s^0$ oscillations have not yet been observed directly. In the Standard Model, their frequency is predicted in Ref. [1] to be between $12.0 \text{ ps}^{-1}$ and $17.6 \text{ ps}^{-1}$ with $68\%$ CL, and lower than $20 \text{ ps}^{-1}$ at $95\%$ CL, significantly larger than the corresponding value $\Delta m_d$ in the $B_d^0$–$\bar{B}_d^0$ system. From measurements done by the ALEPH, DELPHI and OPAL experiments at LEP, by SLD at SLC, and by CDF at the Tevatron, a combined lower bound of $\Delta m_s > 18.0 \text{ ps}^{-1}$ has been established at $95\%$ CL [2]. In the $B_s^0$–$\bar{B}_s^0$ system, the oscillations have been directly observed and a rather precise value of $\Delta m_d = 0.472 \pm 0.017 \text{ ps}^{-1}$ [3] has been measured.

The values for $\Delta m_d$ and $\Delta m_s$ predicted in the Standard Model by computing the corresponding box diagrams, with the top-quark contribution assumed to be dominant, are proportional to $|V_{td}|^2$ and $|V_{ts}|^2$ respectively. The direct determination of $V_{td}$ and $V_{ts}$ from $\Delta m_d$ and $\Delta m_s$ is, however, hampered by hadronic uncertainties. These uncertainties partially cancel in the ratio:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{M_{B_d} \bar{B}_d f_B^2}{M_{B_s} \bar{B}_s f_B^2} \frac{|V_{ts}|^2}{|V_{td}|^2},$$

where $M_B$ are the $B$-meson masses, $\bar{B}_B$ are the bag parameters, and $f_B$ are the $B$-meson form factors. Using the experimentally-measured masses and a value for the ratio $\xi = \sqrt{\bar{B}_s f_B^2} / \sqrt{\bar{B}_d f_B^2} f_B$ which can be computed in lattice QCD, a better constraint for $V_{ts}/V_{td}$ can be obtained, which can then be converted into a constraint of $|V_{td}|$, the worst-measured side of the unitarity triangle.

In this note, an evaluation of the capability of the ATLAS detector to measure the $B_s^0$ oscillations in proton-proton interactions at the Large Hadron Collider (LHC) is presented. Some quantities involved in the measurement are still uncertain (cross sections, shape of the background, the final characteristics of the detector), therefore the dependence of the measurement sensitivity and of the expected accuracy on these parameters is also evaluated.

2 Event selection

The signal channels considered in this analysis for the measurement of $B_s^0$–$\bar{B}_s^0$ oscillations are $B_s^0 \to D_s^+\pi^+$ and $B_s^0 \to D_s^0\phi$, with $D_s \to \phi\pi$ followed by $\phi \to K^+K^-$. The event samples were generated using PYTHIA 5.7 [4], passed then through the ATLAS full GEANT-based simulation program DICE (Inner Detector (ID) only) and reconstructed using an algorithm based on the Kalman filter implemented in the xKalman package from the ATLAS reconstruction program ATRECON. The physics model used for simulation, the description of the detector and of the reconstruction program are presented in detail in Ref. [5].

In the simulation, $b$-quark pairs were produced in $pp$-collisions at $\sqrt{s} = 14$ TeV by including direct production, gluon splitting and flavour excitation processes for $b\bar{b}$ production. The $b$-quark was forced to decay semileptonically giving a muon with transverse momentum $p_T > 6$ GeV

1 The coordinate system has the $z$ direction along the beam axis, with $x$-axis pointing to the centre of the accelerator ring and $y$-axis pointing upwards. The transverse momenta are computed with respect to the $z$ axis.
and pseudo-rapidity $|\eta| < 2.4$ which is used by the level-1 trigger to select the $B$ hadronic channels in ATLAS. The associated $\bar{b}$ was forced to produce the required $B$-decay channels. All the charged final-state particles from the $B$ decay were required to have $p_T > 0.5$ GeV and $|\eta| < 2.5$.

The reconstruction of the $B^0_s$ vertex proceeded via the following steps (charge-conjugate states are implicitly included). The $\phi$ decay vertex was first reconstructed by considering all combinations of pairs of oppositely-charged tracks with $p_T > 1.5$ GeV for both tracks. Kinematic cuts on the angles between the two tracks $\Delta\varphi_{KK} < 10^\circ$ and $\Delta\theta_{KK} < 10^\circ$ were also imposed. Here $\varphi$ denotes the azimuthal angle and $\theta$ the polar angle in the coordinate system defined previously. The two-track vertex was then fitted assigning the kaon mass to both tracks. Combinations passing a fit-probability $[6]$ cut of 1% with the invariant mass within $3\sigma_{\phi}$ of the nominal $\phi$ mass were selected as $\phi$ candidates. To all accepted $\phi$ candidates, a third negative track with $p_T > 1.5$ GeV from the remaining ones was added. The pion mass was assigned to the third track and a three-track vertex was refitted. Combinations of three tracks which had a fit probability greater than 1% and an invariant mass within $3\sigma_{D_s}$ of the nominal $D_s$ mass were selected as $D^-_s$ candidates.

For each reconstructed $D^-_s$ meson, a search was made for $a^+_1$ candidates in three-particle combinations of the remaining charged tracks. In a first step, $\rho^0$ mesons were reconstructed from all combinations of two tracks with opposite charges and with $p_T > 0.5$ GeV for both tracks, each particle in the combination being assumed to be a pion. Kinematic cuts $\Delta\theta_{\pi\pi} < 15^\circ$ and $\Delta\varphi_{\pi\pi} < 35^\circ$ were used to reduce the combinatorial background. The two selected tracks were then fitted as originating from the same vertex; from the combinations passing a fit probability cut of 1%, those with an invariant mass within 1.5 $\Gamma_{BW}$ of the nominal $\rho^0$ mass were selected as $\rho^0$ candidates. Next, a positive track with $p_T > 0.5$ GeV from the remaining charged tracks was added to the $\rho^0$ candidate, assuming the pion hypothesis for the extra track. The three tracks were then fitted as originating from a common vertex, without any mass constraints. Combinations with a fit probability greater than 1% and with an invariant mass within 300 MeV of the nominal $a_1$ mass were selected as $a^+_1$ candidates.

For the $B^0_s \rightarrow D^-_s \pi^+$ channel, the $B^0_s$ decay vertex was reconstructed by considering all $D^-_s$ candidates and adding a fourth track from the remaining tracks in the event. This track was required to have opposite charge with respect to the pion track from the $D^-_s$ and $p_T > 1$ GeV. The four-track decay vertex was refitted including $\phi$ and $D^-_s$ mass constraints, and requiring that the total momentum of the $B^0_s$ vertex pointed to the primary vertex (within the primary vertex spatial resolutions of $\sigma_x = \sigma_y = 28$ $\mu$m and $\sigma_z = 46$ $\mu$m) and the momentum of $D^-_s$ vertex pointed to the $B^0_s$ vertex.

For the $B^0_s \rightarrow D^-_s a^+_1$ channel, the $B^0_s$ candidates were reconstructed combining the $D^-_s$ candidates with the $a^+_1$ candidates. A six-track vertex fit was then performed with mass constraints for the tracks from $\phi$ and $D_s$; due to the large $a_1$ natural width, the three tracks from the $a^+_1$ were not constrained to $a_1$ mass. As in the $B^0_s \rightarrow D^-_s \pi^+$ channel, the total momentum of the $B^0_s$ vertex was required to point to the primary vertex and the momentum of $D^-_s$ vertex was required to point to the $B^0_s$ vertex.

In order to be selected as $B^0_s$ candidates, the four-track and six-track combinations were required to give a probability greater than 1% for the vertex fit. The signed separation between the reconstructed $B^0_s$ vertex and the primary vertex, and between the $D^-_s$ and $B^0_s$ vertex were required to be positive (the momentum should not point backward to the parent vertex). To improve the purity of the sample, further cuts were imposed: the accepted $B^0_s$ candidates were required to have a proper decay time greater than 0.4 ps, an impact parameter smaller than 55 $\mu$m and $p_T > 10$ GeV.

Background to the channels being considered for the measurement of $\Delta m_u$ can come from two sources: from other four- or six-body $B$-hadron decay channels, and from combinatorial background (random combinations with some or all particles not originating from a $B$ decay). For $B^0_s \rightarrow D^-_s \pi^+$, the following four-body decay channels were considered as potential sources of background: $B^0_s \rightarrow D^-_s \pi^+, B^0_s \rightarrow D^- \pi^+ (\pi^- \rightarrow \phi \pi^-)$ and $B^0_s \rightarrow K^+ K^- (\phi \rightarrow K^+ K^-)$ and $A^0_b \rightarrow A^+_s \pi^-$ followed by $A^+_s \rightarrow pK^- \pi^+$. The similar six-body decay channels considered as potential sources of background for $B^0_s \rightarrow D^-_s a^+_1$ were: $B^0_s \rightarrow D^- a^+_1, B^0_s \rightarrow D^- a^+_1 (\pi^- \rightarrow \phi \pi^-)$ and $B^0_s \rightarrow K^+ K^- (\phi \rightarrow K^+ K^-)$ and $A^0_b \rightarrow A^+_s \pi^-$ followed by $A^+_s \rightarrow pK^- \pi^+ \pi^-$. The simulated four- and six-body background events were passed through the detailed detector-simulation program, reconstructed and analysed using the same programs, the same conditions and the same cuts as the signal events.

In order to study the combinatorial background, a very large sample of simulated inclusive-muon events is needed. The results presented here are based on a sample of 1.1 million $b\bar{b} \rightarrow \mu X$ events, with $p_T > 6$ GeV and $|\eta| < 2.4$ for the muon corresponding to the trigger conditions.

The $b\bar{b} \rightarrow \mu X$ sample was analysed in the framework of a fast-simulation program ATLFAST++ [5], applying the same algorithms and the same cuts as the signal events. Reasonable agreement was obtained for the number of reconstructed events and the widths of the mass peaks for the reconstructed particles.

A multi-level trigger is used to select the events for this analysis. The level-1 trigger is the inclusive muon trigger mentioned before. The level-2 trigger [7] confirms the muon from level-1 trigger, then in an un-guided search for tracks in the Inner Detector reconstructs a $\phi$ meson and, adding a new track, a $D_s$ meson. The level-2 trigger uses dedicated online software. 63% of the signal events selected offline pass the level-2 trigger cuts; from the $b\bar{b} \rightarrow \mu X$ sample, $(3.4 \pm 0.2)$% of the events are selected. The level-3 trigger (the event filter) uses a set of loose offline
The first cross-section column in Table 1 gives the channel cross-section, without any cuts on final-state particles, assuming a cross-section of 2.3 μb for the process $b \bar{b} \rightarrow \mu(p_T > 6 \text{ GeV}, |\eta| < 2.4)X$. The effective cross-section is the cross-section after the cuts on charged final-state particles were applied during simulation.

The events reconstructed from the samples for the exclusive decay modes were counted in a $\pm 2\sigma$ window around the nominal $B_s^0$ mass. Using the fraction of events reconstructed in the simulated sample and the number of events expected for an integrated luminosity of $10 \text{ fb}^{-1}$, the expected number of reconstructed events was estimated. Corrections for muon efficiency (on average 0.82) and for level-2 trigger efficiency (0.63) were also applied.

A total of 3240 reconstructed events is expected for the $B_s^0 \rightarrow D_\pi$ and $B_s^0 \rightarrow D_{a_1}^0$ decay channels for an integrated luminosity of $10 \text{ fb}^{-1}$.

The only significant background comes from the $\bar{B}_d^0 \rightarrow D_{a_1}^0$ channels, and from the combinatorial background. Note that the number of reconstructed events from the two decay channels is conservative since the branching-ratio values used are upper limits. As expected, due to the combination of the $D$-mass shift ($M_{D^+} - M_{D_s^+} = 90 \text{ MeV}$) and $B_d^0$ mass shift ($M_{B_d^0} - M_{B_s^0} \approx 100 \text{ MeV}$), very few $B_d^0 \rightarrow D^- a_1^+$, $D^- \pi^+$ cuts, reducing the rate to $(0.26 \div 0.41)\%$ of the $\mu X$ rate, depending on the actual values which are set for the cuts.

The reconstructed $B_s^0$ invariant-mass distributions in the decay channels $B_s^0 \rightarrow D_\pi$ and $B_s^0 \rightarrow D_{a_1}^0$ are shown in Figure 1 and Figure 2, respectively, for an integrated luminosity of $10 \text{ fb}^{-1}$.

The numbers of events expected for the various signal and background channels that have been analysed are given in Table 1 for an integrated luminosity of $10 \text{ fb}^{-1}$. The first cross-section column in Table 1 gives the channel cross-section, without any cuts on final-state particles, assuming a cross-section of 2.3 μb for the process $b \bar{b} \rightarrow \mu(p_T > 6 \text{ GeV}, |\eta| < 2.4)X$. The effective cross-section is the cross-section after the cuts on charged final-state particles were applied during simulation.

The events reconstructed from the samples for the exclusive decay modes were counted in a $\pm 2\sigma$ window around the nominal $B_s^0$ mass. Using the fraction of events reconstructed in the simulated sample and the number of events expected for an integrated luminosity of $10 \text{ fb}^{-1}$, the expected number of reconstructed events was estimated. Corrections for muon efficiency (on average 0.82) and for level-2 trigger efficiency (0.63) were also applied.

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events are reconstructed in a \( \pm 2\sigma \) window around \( B_s^0 \) nominal mass. Due to the different decay topology, the \( \Lambda_b^0 \rightarrow \Lambda^+_s \pi^- \) channel does not give any contribution to the background.

The statistics available for estimating the combinatorial background are very limited, despite the large size (1.1 million events) of the \( \mu X \) sample. Each simulated event was therefore passed 50 times through the fast-simulation program, different random smearing of the track parameters being applied each time. The number of background events was counted in a \( \sigma \) mass window around the \( B_s^0 \) nominal mass. On average, 0.12 events per pass were reconstructed in the mass window, summing the background.

The proper time of the reconstructed \( B_s^0 \) candidates was computed from the reconstructed transverse decay length, \( d_{xy} \), and from the \( B_s^0 \) transverse momentum, \( p_T \):

\[
t = \frac{d_{xy} M_{B_s^0}}{c p_T} \equiv d_{xy} g
\]

where \( g = M_{B_s^0}/(c p_T) \).

The transverse decay length is the distance between the interaction point and the \( b \)-hadron decay vertex, projected onto the transverse plane. Figure 3 shows, for the example of the \( B_s^0 \rightarrow D_s^- (\phi \pi^-) \pi^+ \) decay mode, the difference \( d_{xy} - d_{xy}^0 \) fitted with two Gaussian functions, where \( d_{xy}^0 \) is the true transverse decay length. For each event, the decay-length uncertainty, \( \sigma_{d_{xy}} \), was estimated from the covariance matrices of the tracks associated with the vertices. The pull of the transverse decay length, \( (d_{xy} - d_{xy}^0)/\sigma_{d_{xy}} \), was found to have a Gaussian shape with a width of \( S_{d_{xy}} = 0.959 \pm 0.017 \) for the \( B_s^0 \rightarrow D_s^- \pi^+ \) channel and \( S_{d_{xy}} = 0.954 \pm 0.020 \) for the \( B_s^0 \rightarrow D_s^- a_1^+ \) channel.

The distributions for \( (g - g_0)/g_0 \) also have a Gaussian shape for both \( B_s^0 \) decay channels, with a width of \( S_g = (0.715 \pm 0.014)\% \) for the \( B_s^0 \rightarrow D_s^- \pi^+ \) channel and \( S_g = (0.636 \pm 0.013)\% \) for the \( B_s^0 \rightarrow D_s^- a_1^+ \) channel. Here \( g_0 = t_0 / \sigma_{d_{xy}} \), with \( t_0 \) being the true proper time. The \( (g - g_0)/g_0 \) distribution for the \( B_s^0 \rightarrow D_s^- (\phi \pi^-) \pi^+ \) decay mode is shown in Figure 4.

The proper-time resolution function \( \text{Res}(t | t_0) \) was parametrised with the sum of two Gaussian functions, with parameters given in Table 2:

\[
\text{Res}(t | t_0) = f_1 \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left( -\frac{(t - t_0)^2}{2 \sigma_1^2} \right) + f_2 \frac{1}{\sigma_2 \sqrt{2\pi}} \exp \left( -\frac{(t - t_0)^2}{2 \sigma_2^2} \right)
\]

Figure 5 and 6 show, for the decay channels \( B_s^0 \rightarrow D_s^- \pi^+ \) and \( B_s^0 \rightarrow D_s^- a_1^+ \), the proper-time resolution together with the parametrization obtained from the function given above. The distributions show deviations from the Gaussian shape, illustrated by the significant fraction of the second, broader Gaussian function in the parametrization function. For \( B_s^0 \) mesons, the ratio of the two Gaussian functions was fixed to the ratio to \( B_s^0 \) parametrization.

The parametrization chosen above reproduces well the tails seen in the distribution for reconstructed events. It has the advantage that the integrals in the log-likelihood function given in Eq. (3) can be computed analytically, much faster than the numerical computation. The depen-
Table 2. Proper-time resolution function $\text{Res}(t|t_0)$ parameterization with the sum of two Gaussian functions.

| $\alpha$ | Fraction $f_\alpha$ | Width $\sigma_\alpha$ | Fraction $f_\alpha$ | Width $\sigma_\alpha$ |
|----------|----------------------|-----------------------|----------------------|-----------------------|
| $B_j^0 \to D_s^- \pi^+$ | $1$ | $59.6 \pm 6.6$ | $1$ | $51.5 \pm 4.0$ |
| $B_j^0 \to D_s^- a_1^+$ | $2$ | $40.4 \pm 6.6$ | $2$ | $107.3 \pm 8.5$ |

The probability density to observe an initial decay (with the sum of two Gaussian functions. For the unmixed case (an initial $B$ decay), the width difference in the $\Delta \Gamma$ is very small.

4 Likelihood function

The probability density to observe an initial $B_j^0$ meson ($j = d, s$) decaying at time $t_0$ after its creation as a $B_j^0$ meson is given by:

$$ p_j(t_0, \mu_0) = \frac{\Gamma_j^2 - \left(\frac{\Delta \Gamma_j}{2}\right)^2}{2 \Gamma_j^2} e^{-\Gamma_j t_0} \times$$

$$ \left(\cosh \frac{\Delta \Gamma_j t_0}{2} + \mu_0 \cos \Delta m_j t_0\right) $$

where $\Delta \Gamma_j = \Gamma_j^H - \Gamma_j^L$, $\Gamma_j = (\Gamma_j^H + \Gamma_j^L)/2$ and $\mu_0 = -1$. For the unmixed case (an initial $B_j^0$ meson decays as a $B_j^0$ meson at time $t_0$), the probability density is given by the above expression with $\mu_0 = +1$. The small effects of CP violation are neglected in the above relation. Unlike $\Delta \Gamma_d$, which is safely neglected, the width difference in the $B^0_s - B^0_s$ system $\Delta \Gamma_s$ could be as much as 20% of the total width [8].

The above probability is modified by experimental effects. The probability as a function of $\mu_0$ and the reconstructed proper time $t$ is obtained as the convolution of $p_j(t_0, \mu_0)$ with the proper time resolution $\text{Res}_j(t | t_0)$:

$$ q_j(t, \mu_0) = N \int_{t_{\text{min}}}^{t_{\text{max}}} p_j(t_0, \mu_0) \text{Res}_j(t | t_0) \, dt_0 $$

with $N$ a normalization factor. Assuming a fraction $\omega_j$ of wrong tags at production or decay, the probability becomes:

$$ q_j(t, \mu) = (1 - \omega_j) q_j(t, \mu) + \omega_j q_j(t, -\mu) $$

For each signal channel, the background is composed of oscillating $B_j^0$ mesons with probability given by the expression (2) and of combinatorial background, with probability given by the reconstructed proper time distribution. For a fraction $f_j^k$ of the $j$ component ($j = s, d$, and combinatorial background $cb$) in the total sample of type $k$, one obtains the probability density:

$$ \text{pdf}_k(t, \mu) = \sum_{j=s,d,cb} f_j^k [(1 - \omega_j) q_j(t, \mu) + \omega_j q_j(t, -\mu)] $$

The index $k = 1$ denotes the $B_s^0 \to D_s^- \pi^+$ channel and $k = 2$ the $B_s^0 \to D_s^- a_1^+$ channel. The likelihood of the
total sample is written as
\[
\mathcal{L}(\Delta m_s, \Delta \Gamma_s) = \prod_{i=1}^{N_{ch}} \prod_{k=1}^{N_{ev}} \text{pdf}_k(t_i, \mu_i) \tag{3}
\]
where \(N_{ev}^k\) is the total number of events of type \(k\), and \(N_{ch} = 2\). Each pdf\(_k\) was properly normalized to unity.

5. Extraction of \(\Delta m_s\) measurement limits and accuracy

The maximum value of \(\Delta m_s\) measurable in ATLAS was estimated using a simplified Monte Carlo model. The input parameters of this model were: for each signal channel \(k\) the number of signal events, \(N_{sig}^k\), the number of background events from \(B_s^0\) decays, \(N_{B_s^0}^k\), and the number of events for the combinatorial background, \(N_{cb}^k\), the characteristics of the events involved in the computation of the proper-time resolution (see below); the wrong-tag fraction. The wrong-tag fraction was assumed to be the same for both \(B_s^0\) and \(B_d^0\) mesons: \(\omega_d = \omega_d = 0.22\) (see Ref. [9]). A Monte Carlo sample with \(N_{sig} = N_{sig}^1 + N_{sig}^2\) signal events oscillating with a given frequency \(\Delta m_s\) together with \(N_{B_s^0} = N_{B_s^0}^1 + N_{B_s^0}^2\) background events oscillating with frequency \(\Delta m_d\) and \(N_{cb} = N_{cb}^1 + N_{cb}^2\) combinatorial events (no oscillations), was generated in the following way. For each event with an oscillating \(b\) hadron, the true proper time was generated according to an exponential distribution with the slope \([-\Gamma(1 + \Delta \Gamma/(2\Gamma))]^{-1}\) which takes into account the contribution of a non-zero \(\Delta \Gamma\). Here \(\Gamma = \Gamma_s\), \(\Delta \Gamma = \Delta \Gamma_s\) for \(B_s^0\) events and \(\Gamma = \Gamma_d\), \(\Delta \Gamma = 0\) for \(B_d^0\) events. The sign in the slope is chosen according to the contribution of each proper time, corresponding to \(\Gamma(1 + \Delta \Gamma/(2\Gamma))\), to the exponential decay from Eq. (1), \(e^{-\Gamma t_0} \cosh(\Delta \Gamma s t_0/2)\), where the cosh term was factorized from the expression in parentheses and included in the exponential part. The uncertainty on the measurement of the transverse decay length, \(\sigma_{d_{xy}}\), and the true value of the \(g\)-factor, \(g_0\), were generated at random according to the distributions obtained from the simulated samples, fitted with appropriate combinations of Gaussian and exponential functions. From the computed true decay length, \(d_{xy}^0 = t_0/g_0\), the corresponding reconstructed decay length was generated as \(d_{xy} = d_{xy}^0 + S_{d_{xy}} \sigma_{d_{xy}} \Omega\). The reconstructed \(g\)-factor was generated as \(g = g_0 + g_0 S_{g} \Omega\). Both \(\Omega\) and \(\Omega'\) are random numbers distributed according to the normal distribution. From the transverse decay length and \(g\)-factor, the reconstructed proper time was then computed as \(t = g d_{xy}\). The probability for the event to be mixed or unmixed was determined from the \(t_0\) and \(\Delta m_s\) values (\(\Delta m_d\) value if the event was a \(B_s^0\) event) using the expression \((1 - \cos(\Delta m_d t_0)/\cosh(\Delta \Gamma_s t_0/2))/2\) left out from Eq. (1) after the exponential part is extracted. For a fraction of the events, selected at random, the state was changed between mixed and unmixed, according to the wrong-tag fraction, \(\omega_{\text{tag}}\). For the combinatorial background, the reconstructed proper time was generated assuming that it has the same distribution as the one for \(B_s^0\) mesons. Half of the combinatorial events were added to the mixed events and half to the unmixed events.

5.1 \(\Delta m_s\) measurement limits

The \(\Delta m_s\) measurement limits were obtained applying the amplitude fit method [10] to the ‘data sample’ generated as above. In this method a new parameter, the \(B_s^0\) oscillation amplitude \(A\), is introduced in the likelihood function by replacing the term ‘\(\mu_0 \cos(\Delta m_s t_0)\)’ with ‘\(\mu_0 A \cos(\Delta m_s t_0)\)’ in the \(B_s^0\) probability density function given by Eq. (1). For each value of \(\Delta m_s\), the new likelihood function is minimized with respect to \(A\), keeping all other parameters fixed, and a value \(A \pm \sigma_{\text{stat}}^A\) is obtained. One expects, within the estimated uncertainty, \(A = 1\) for \(\Delta m_s\) close to its true value, and \(A = 0\) for \(\Delta m_s\) far from the true value. One defines a 5σ measurement limit as the value of \(\Delta m_s\) for which \(1/\sigma_A = 5\), and a sensitivity at 95% confidence limit as the value of \(\Delta m_s\) for which \(1/\sigma_A = 1.645\). Limits are computed with the statistical uncertainty \(\sigma_{\text{stat}}^A\), and, in some cases, with the total uncertainty \(\sigma_{\text{total}}^A = \sigma_{\text{stat}}^A + \sigma_{\text{syst}}^A\). The systematic uncertainty \(\sigma_{\text{syst}}^A\) is described in the next section.

5.2 Systematic uncertainties

The systematic uncertainties on the \(B_s^0\) oscillation amplitude \(\sigma_{\text{syst}}^A\) are computed as
\[
\sigma_{\text{syst}}^A = \Delta A + (1 - A) \frac{\Delta \sigma_{\text{stat}}^A}{\sigma_A} \tag{4}
\]
where \(\Delta A\) is the difference between the value of the amplitude when a single parameter is changed and the analysis repeated, and the value for the ‘nominal’ set of parameters; \(\Delta \sigma_{\text{stat}}^A\) is defined in a similar way.

The following contributions to the systematic uncertainties were considered:

- A relative error of 5% was considered for the wrong-tag fraction for both \(B_s^0\) and \(B_d^0\).
- The widths of the Gaussian functions from the parameterization of the proper time resolution given in Table 2 were varied by \(\pm 1\sigma\).
- The fraction \(f_{B_s^0} = BR(\bar{b} \rightarrow B_s^0)\), the \(B_s^0\) lifetime and the \(\Delta m_d\) value were varied separately by the uncertainty quoted in Ref. [11].
- An uncertainty of 5% was assumed for the decay time \(\tau_{cb}\) of the combinatorial background. The shape remained exponential, only the decay time was modified.

These contributions were added in quadrature to give the systematic uncertainty.

Table 3 shows the dependence of the amplitude and its statistical and systematic uncertainties on \(\Delta m_s\), as well as the contribution of each component to the systematic uncertainties.
uncertainty for an integrated luminosity of 10 fb⁻¹. In the generated event samples, the value of Δmₐ was set to Δmₐ^{gen} = ∞.

From Table 3, it can be seen that the dominant contributions to the systematic uncertainty come from the uncertainty on f_{B^{0}} fraction and from the parametrization of the proper time resolution.

The uncertainty on f_{B^{0}} fraction has the value given in Ref. [11]. It is, however, expected that the uncertainty will be much smaller at the time this analysis will be done with data. Even now (October 2001), the most up-to-date preliminary value [12] of f_{B^{0}} is 0.099 ± 0.011, the uncertainty being at the level of 11%, to be compared with the value of 17% used here for the consistency of the data. If one considers the intense activity in B⁻⁻physics, one can assume that an uncertainty of ~5% will be achieved at the time of data analysis.

The uncertainties on the parameters from the parametrization of the proper time resolution depend on the MC statistics. A larger MC sample can be generated than the one used in this work, in order to reduce these uncertainties. It is therefore reasonable to assume that the uncertainty on the widths from Table 2 can be reduced to half of the actual values.

The systematic uncertainties computed with these ‘projected’ uncertainties are given in Table 4. For the other contributions, the values from Table 3 were used.

The value of the systematic uncertainty as computed here should be considered with caution, as a rough estimate.

### 5.3 Results

The amplitude as a function of Δmₐ for the nominal set of parameters defined in the previous sections, ΔΓ_s = 0 and an integrated luminosity of 10 fb⁻¹ is shown in Fig. 7. Fig. 8 shows the significance of the measurement in units of σₐ. The 5σ measurement limit is 22.5 ps⁻¹ and the 95% CL sensitivity is 36.0 ps⁻¹, when computed with the statistical uncertainty only. Computed with the total uncertainty, the 5σ measurement limit is 16.0 ps⁻¹ and the 95% CL sensitivity is 34.5 ps⁻¹ for the actual systematic uncertainties, and 21 ps⁻¹ and 35.5 ps⁻¹ for the projected systematic uncertainties.

For an integrated luminosity of 30 fb⁻¹, the 5σ measurement limit is 29.5 ps⁻¹ and the 95% CL sensitivity is 41.0 ps⁻¹, computed with the statistical uncertainty only. The 5σ measurement limit and the 95% CL sensitivity become 18.5 ps⁻¹ and 37.5 ps⁻¹, respectively, for the actual set of systematic uncertainties and 27.0 ps⁻¹ and 40.5 ps⁻¹ for the projected systematic uncertainties.

### 6 Dependence of Δmₐ measurement limits on experimental quantities

Some quantities involved in the measurement of the B^{0} oscillations are not yet known with enough precision. The cross section for b̄b production could be more than twice the assumed cross section [5]; some decay branching ratios are also not well determined. The value of ΔΓ_s is not yet measured. The characteristics of the detector could be slightly different, depending on the final configuration of the detector. The shape and the fraction of the background will depend, among other factors, on the accelerator luminosity and on the characteristics of the detector. It is therefore necessary to estimate the dependence of the measurement sensitivity for various values of the parameters. The limits presented in this section are computed with the statistical error only.

The dependence of the Δmₐ measurement limits on ΔΓ_s/Γ_s was determined for an integrated luminosity of 30 fb⁻¹, other parameters having the nominal value. The

| Δmₐ | 0 ps⁻¹ | 5 ps⁻¹ | 10 ps⁻¹ | 15 ps⁻¹ | 20 ps⁻¹ | 25 ps⁻¹ | 30 ps⁻¹ | 35 ps⁻¹ | 40 ps⁻¹ |
|-----|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| A   | 0.0450| -0.1146| 0.1892 | -0.0337| 0.0420 | 0.0072 | -0.2906| 0.4318 | -0.5406|
| σₐ^{at} | ±0.0476| ±0.0707| ±0.0895| ±0.1183| ±0.1673| ±0.2456| ±0.3570| ±0.5640| ±0.9300|
| σₐ^{syst} | +0.0967| +0.1285| +0.1297| +0.1468| +0.1796| +0.2035| +0.2980| +0.3674| +0.3636|
| -0.0838| -0.1018| -0.0960| -0.1144| -0.1424| -0.1580| -0.2261| -0.2658| -0.2592|

**Table 3.** The oscillation amplitude A and its statistical and systematic uncertainties as a function of Δmₛ for an integrated luminosity of 10 fb⁻¹. The contribution of each component to the systematic uncertainty is also given.
for an integrated luminosity of 10 fb\(^{-1}\).

Table 4. The oscillation amplitude \(A\) and its statistical and systematic uncertainties as a function of \(\Delta m_s\) for an integrated luminosity of 10 fb\(^{-1}\), computed with the ‘projected uncertainties’ on proper time parametrization and on \(f_s\). See the text for details.

| \(\Delta m_s\) (ps\(^{-1}\)) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|--------------------------|---|---|----|----|----|----|----|----|----|
| \(A\)                      | 0.0450 | -0.1146 | 0.1892 | -0.0337 | 0.0420 | 0.0072 | -0.2906 | 0.4318 | -0.5406 |
| \(\sigma_{stat}^A\)        | ±0.0476 | ±0.0707 | ±0.0895 | ±0.1183 | ±0.1673 | ±0.2456 | ±0.3570 | ±0.5640 | ±0.9300 |
| \(\sigma_{syst}^A\)        | ±0.0494 | ±0.0564 | ±0.0596 | ±0.0679 | ±0.0848 | ±0.0976 | ±0.1370 | ±0.1660 | ±0.1747 |

Systematic contributions

- proper time resolution

Table 5. The dependence of \(\Delta m_s\) measurement limits on \(\Delta \Gamma_s/\Gamma_s\).

\(\Delta \Gamma_s/\Gamma_s\) was used as a fixed parameter in the amplitude fit method. The results are shown in Fig. 9 and the numerical values are given in Table 5. No sizeable effect is seen up to \(\Delta \Gamma_s/\Gamma_s < 30\%\), therefore \(\Delta \Gamma_s\) was set to zero for all other cases.

$\Delta \Gamma_s/\Gamma_s \text{ limit } 95\% \text{ CL sensitivity}$

| \(\Delta \Gamma_s/\Gamma_s\) (\%) | 5\(\sigma\) limit | 95\% CL sensitivity |
|-------------------------------|-------------------|---------------------|
| 0                            | 29.5              | 41.0                |
| 10                           | 29.5              | 41.0                |
| 20                           | 29.5              | 41.0                |
| 30                           | 29.0              | 41.0                |
| 40                           | 29.0              | 40.5                |
| 80                           | 27.0              | 39.0                |
| 100                          | 25.5              | 38.0                |

Fig. 7. The \(B_s^0\) oscillation amplitude as a function of \(\Delta m_s\) for an integrated luminosity of 10 fb\(^{-1}\).

Fig. 8. The measurement significance as a function of \(\Delta m_s\) for an integrated luminosity of 10 fb\(^{-1}\).

Fig. 9. The dependence of \(\Delta m_s\) measurement limits on \(\Delta \Gamma_s/\Gamma_s\).
The dependence of the $\Delta m_s$ measurement limits on the integrated luminosity is shown in Fig. 10, with the numerical values given in Table 6.

| Luminosity (fb$^{-1}$) | 5$\sigma$ limit (ps$^{-1}$) | 95% CL sensitivity (ps$^{-1}$) |
|------------------------|----------------------------|-------------------------------|
| 5                      | 17.5                       | 32.0                          |
| 10                     | 22.5                       | 36.0                          |
| 20                     | 27.0                       | 39.0                          |
| 30                     | 29.5                       | 41.0                          |

Table 6. The dependence of $\Delta m_s$ measurement limits on the integrated luminosity.

The shape and the fraction of the combinatorial background were varied within reasonable values. It was assumed that the shape remains exponential and only the decay time was modified. The number of events for the combinatorial background was varied within $\pm 50\%$ of the number determined in Section 2, keeping the number of $B_d^0$ and $B_s^0$ events to the nominal values. The dependence of the $\Delta m_s$ measurement limits on the decay time of the combinatorial background is shown in Fig. 11, with the numerical values given in Table 7, while the dependence on the number of events is given in Fig. 12 and Table 8.

| $\tau_{bkg}$ (ps) | 5$\sigma$ limit (ps$^{-1}$) | 95% CL sensitivity (ps$^{-1}$) |
|-------------------|----------------------------|-------------------------------|
| 0.25              | 31.5                       | 42.5                          |
| 0.50              | 30.5                       | 42.0                          |
| 1.00              | 29.5                       | 41.5                          |
| 1.54              | 29.5                       | 41.0                          |

Table 7. The dependence of $\Delta m_s$ measurement limits on the decay time of the combinatorial background, assuming an exponential decay.

7 Accuracy of the $\Delta m_s$ measurement

For $\Delta m_s$ values smaller than the 5$\sigma$ measurement limit, the expected accuracy is estimated using the log-likelihood method, with the likelihood function given by Eq. (3). In the fit, the $\Delta m_s$ value was free, while the other parameters were fixed to their nominal values. An example of the

Fig. 10. The dependence of $\Delta m_s$ measurement limits on the integrated luminosity.

Fig. 11. The dependence of $\Delta m_s$ measurement limits on the decay time of the combinatorial background, assuming an exponential decay.

Fig. 12. The dependence of $\Delta m_s$ measurement limits on the number of events for the combinatorial background.
The dependence of $\Delta m_s$ measurement limits on the number of events for the combinatorial background.

![Fig. 13. The negative log-likelihood function for an integrated luminosity of 10 fb$^{-1}$ and a generated $\Delta m_s^{\text{gen}} = 22.5$ ps$^{-1}$.](image)

The accuracy was determined for different values of the integrated luminosity. The results are given in Table 9.

| Luminosity (fb$^{-1}$) | $\Delta m_s^{\text{gen}}$ (ps$^{-1}$) | $\Delta m_s^{\text{meas}}$ ± stat ± syst (ps$^{-1}$) | Obs. |
|------------------------|--------------------------------------|-------------------------------------------------|------|
| 5                      | 17.5                                 | 17.689 ± 0.083 ± 0.002                           | 5$\sigma$ limit |
| 10                     | 15.0                                 | 15.021 ± 0.049 ± 0.002                           | 5$\sigma$ limit |
| 20                     | 20.0                                 | 20.041 ± 0.068 ± 0.005                           | 5$\sigma$ limit |
| 30                     | 25.0                                 | 25.078 ± 0.083 ± 0.007                           | 5$\sigma$ limit |

Table 9. The accuracy of $\Delta m_s$ measurement as a function of the integrated luminosity. $\sigma_{\text{stat}}$ represents the statistical uncertainty, $\sigma_{\text{syst}}$ the systematic uncertainty.

Carlo simulated events, propagated through a detailed simulation of the detector. For an integrated luminosity of 10 fb$^{-1}$, a 5$\sigma$ measurement will be possible up to 22.5 ps$^{-1}$, the experiment being 95% CL sensitive up to 36.0 ps$^{-1}$. These limits increase to 29.5 ps$^{-1}$ and 41.0 ps$^{-1}$, respectively, for an integrated luminosity of 30 fb$^{-1}$. The effect of the systematic uncertainties on the measurement limits is rather limited, if the precision of the $f_{B_s^0}$ fraction is improved with respect to the actual value, and if enough Monte Carlo events are generated for the parametrization of the proper time.

If the $\Delta m_s$ value will be within the ATLAS reach, the measurement will be dominated by the statistical error. A total uncertainty of the order of $\sim 0.07$ ps$^{-1}$ is expected for a value of $\Delta m_s = 22.5$ ps$^{-1}$ for an integrated luminosity of 10 fb$^{-1}$, the total uncertainty decreasing to $\sim 0.04$ ps$^{-1}$ for a luminosity of 30 fb$^{-1}$.

The dependence of the measurement sensitivity on various parameters which will be known only at the time of the data analysis was also evaluated.

The values obtained in this note for the measurement limits and accuracies should be re-evaluated at a later time, taking into account the changes in the detector geometry and in the simulation and reconstruction software.

Acknowledgements. This work has been performed within the ATLAS collaboration, and we thank collaboration members for helpful discussions. We have made use of the physics analysis framework and tools which are the result of collaboration-wide effort. The authors would like to thank P. Eerola and N. Ellis for fruitful discussions, and E. Kneringer for computing support. The work was supported by the Federal Ministry of Education, Science and Culture, Austria.

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8 Conclusions

In this note, the performance of the ATLAS detector to measure the $B_s^0$ oscillations was estimated using Monte Carlo simulated events, propagated through a detailed simulation of the detector. For an integrated luminosity of 10 fb$^{-1}$, a 5$\sigma$ measurement will be possible up to 22.5 ps$^{-1}$, the experiment being 95% CL sensitive up to 36.0 ps$^{-1}$. These limits increase to 29.5 ps$^{-1}$ and 41.0 ps$^{-1}$, respectively, for an integrated luminosity of 30 fb$^{-1}$. The effect of the systematic uncertainties on the measurement limits is rather limited, if the precision of the $f_{B_s^0}$ fraction is improved with respect to the actual value, and if enough Monte Carlo events are generated for the parametrization of the proper time.

If the $\Delta m_s$ value will be within the ATLAS reach, the measurement will be dominated by the statistical error. A total uncertainty of the order of $\sim 0.07$ ps$^{-1}$ is expected for a value of $\Delta m_s = 22.5$ ps$^{-1}$ for an integrated luminosity of 10 fb$^{-1}$, the total uncertainty decreasing to $\sim 0.04$ ps$^{-1}$ for a luminosity of 30 fb$^{-1}$.

The dependence of the measurement sensitivity on various parameters which will be known only at the time of the data analysis was also evaluated.

The values obtained in this note for the measurement limits and accuracies should be re-evaluated at a later time, taking into account the changes in the detector geometry and in the simulation and reconstruction software.

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