Vacuum oscillation solution to the solar neutrino problem
in standard and non-standard pictures

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Abstract

The neutrino long wavelength (just-so) oscillation is revisited as a solution to the solar neutrino problem. We consider just-so scenario in various cases: in the framework of the solar models with relaxed prediction of the boron neutrino flux, as well as in the presence of the non-standard weak range interactions between neutrino and matter constituents. We show that the fit of the experimental data in the just-so scenario is not very good for any reasonable value of the $^8B$ neutrino flux, but it substantially improves if the non-standard $\tau$-neutrino–electron interaction is included. These new interactions could also remove the conflict of the just-so picture with the shape of the SN 1987A neutrino spectrum. Special attention is devoted to the potential of the future real-time solar neutrino detectors as are Super-Kamiokande, SNO and BOREXINO, which could provide the model independent tests for the just-so scenario. In particular, these imply specific deformation of the original solar neutrino energy spectra, and time variation of the intermediate energy monochromatic neutrino ($^7Be$ and $pep$) signals.

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1. Introduction

The deficit of the solar neutrinos, dubbed the Solar Neutrino Problem (SNP), was observed more than 20 years ago in the Homestake Cl – Ar experiment. The 1970-93 average of the chlorine experiment result reads as

\[ R_{Cl} = 2.32 \pm 0.26 \text{ SNU} \]  

(1)

whereas the Standard Solar Model (SSM) by Bahcall and Pinsonneault (BP) \[ \] implies \[ R_{Cl} = 8 \text{ SNU} \], where 6.2 SNU comes from \(^8\!B\) neutrinos, 1.2 SNU from \(^7\!Be\) neutrinos and the remaining 0.6 SNU from the other sources. The predictions of the other SSM \[ \] do not differ strongly. However, the chlorine result alone does not seem sufficient to pose the problem, since the predicted flux of the boron neutrinos has rather large uncertainties. These are mainly due to the poorly known nuclear cross sections \( \sigma_{17}, \sigma_{34} \) at low energies, some other astrophysical uncertainties which could change the solar central temperature, the plasma effects etc. (see e.g. \[ \] and refs. therein). All these, working coherently, may decrease \( \phi^B \) by more than a factor 2 compared to the SSM prediction. Also the \(^7\!Be\) neutrino flux can have uncertainties up to 20%. Therefore, for a comprehensive analysis, it is suggestive to consider these fluxes as free parameters: \( \phi^B = f_B \phi^B_0, \phi^{\text{Be}} = f_{\text{Be}} \phi^{\text{Be}}_0 \), where \( \phi_0 \) are the BP model fluxes and the factors \( f \) reflect the uncertainties.

However, the direct observation of solar \(^8\!B\) neutrinos by Kamiokande detector \[ \] brings another evidence to the SNP. The Kamiokande signal is less than that is expected from the SSM by BP, unless \( f_B \leq 0.6 \). However, more important is that the signal/prediction ratio

\[ Z_K = \frac{R_{K}^{\text{exp}}}{R_{K}^{\text{pred}}} = \frac{1}{f_B} (0.51 \pm 0.07) \]  

(2)

for any \( f_B \) is incompatible to the one of the chlorine experiment

\[ Z_{Cl} = \frac{R_{Cl}^{\text{exp}}}{R_{Cl}^{\text{pred}}} = \frac{1}{0.78f_B + 0.22f_{\text{Be}}} (0.29 \pm 0.03) \]  

(3)

unless \( f_{\text{Be}} \ll f_B \) (for the simplicity, we have extended the factor \( f_{\text{Be}} \) also to other sources contributing the \( Cl – Ar \) signal). However, such a situation is absolutely improbable from the astrophysical viewpoint: whatever effect (e.g. diminishing the central temperature) kills \(^7\!Be\) neutrinos, it should kill more the \(^8\!B\) ones.\[ \]

One could even assume that the uncalibrated Homestake experiment has some uncontrollable systematical error and the true value of \( \phi^B \) is measured by Kamiokande (i.e. \( f_B \approx 0.5 \)). However, the data of the Ga – Ge experiment show that in doing so the SNP will not disappear. Indeed, the weighted average of the GALLEX \[ \] and SAGE \[ \] results is:

\[ R_{Ga} = 78 \pm 10 \text{ SNU} \]  

(4)

as compared with the BP prediction 131 SNU. The bulk of this signal (71 SNU) comes from the \( pp \) source. The latter is essentially determined by the solar luminosity and, therefore, cannot be seriously altered by astrophysical uncertainties. On the other hand, the contribution of about 7 SNU is granted by the \(^8\!B\) neutrinos as measured by Kamiokande. Therefore,\[ \]

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there is not much room left for the $^7$Be neutrinos which, according to BP model, have to provide 36 SNU: $\phi^{Be}$ should be suppressed much stronger than $\phi^{B}$ ($f_{Be} < 0.25$). Thus, the SNP which arised initially as the boron neutrino problem, now has become the problem of the beryllium neutrinos.

All these arguments are strong enough to believe that the astrophysical solutions to the SNP are excluded [1]. It is more conceivable that in the way to the earth the solar $\nu_e$'s are partially converted into the other neutrino flavours. Moreover, the experimental data require the conversion mechanism capable to suppress differently neutrinos of different energies. According to a general paradigm, following from the experimental results, it should lead to a moderate reduction of the $pp$ and $^8$B neutrino fluxes and to a strong depletion of the intermediate energy $^7$Be flux.

The neutrino oscillation picture can provide the necessary energy dependence in two regimes, which are known as the MSW [11] and the just-so [12, 13] scenarios. The MSW resonant conversion in matter is the most attractive and elegant solution, requiring $\delta m^2$ of about $10^{-5}$ eV$^2$ and small mixing angle, $\sin^2 2\theta \sim 10^{-2}$. It provides a very good fit of the experimental data, due to the selective strong reduction of the $^7$Be neutrinos [16, 17].

Another attractive possibility is offered by the just-so oscillation, i.e. vacuum oscillation $\nu_e \rightarrow \nu_x (\nu_x = \nu_\mu, \nu_\tau)$ with the wavelength comparable to the sun-earth distance [12, 13]. This solution needs $\delta m^2$ of about $10^{-10}$ eV$^2$ and large mixing angles [18], which parameter range can be naturally generated by non-perturbative quantum gravitational effects [19, 20].

The just-so scenario, due to the energy dependence of the survival probability, can provide an acceptable fit of the solar neutrino data (not as good, however, as the MSW does). The recent analysis of this scenario is given in refs. 21.

As it was pointed out in ref. [22], this scenario faces the difficulty being confronted with the SN 1987A neutrino burst [23]. The original $\bar{\nu}_{\mu,\tau}$ energy spectrum from the supernova has a larger average energy (about 25 MeV) than the spectrum for $\nu_e$ (about 12 MeV), due to the smaller opacities of $\bar{\nu}_{\mu,\tau}$. The neutrino conversion $\bar{\nu}_e \rightarrow \bar{\nu}_x$ induced by the neutrino mixing results in a partial permutation of the original $\bar{\nu}_e$ and $\nu_x$ spectra. If the permutation is strong, it would significantly alter the energy spectrum of the supernova $\bar{\nu}_e$-signal. The analysis [22], derived by using the SN 1987A data and different models of the neutrino burst, shows that for $\delta m^2 \sim 10^{-10} - 10^{-11}$ eV$^2$ the range of mixing excluded at 99% CL is $\sin^2 2\theta \geq 0.7$, which covers the range required by the just-so scenario, $\sin^2 2\theta \geq 0.7$. Nevertheless, we do not consider the SN 1987A argument as a sharp evidence against the large neutrino mixing. Moreover, as we will discuss below, this constraint can be removed by assuming some non-standard neutrino interactions which could increase the $\bar{\nu}_x$ opacity in the supernova core, reducing thereby its average energy.

In the present paper we address certain issues in the context of the long wavelength neutrino oscillation as a possible solution to the SNP. In Sect. 2 we study how this scenario fits the experimental data in various cases: (i) SSM+SM: in the reference SSM by Bahcall

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2 According to a cliché, the neutrino oscillation is regarded as a non-standard property. However, from the viewpoint of the modern particle physics, the existence of the neutrino mass and mixing should be considered as a rather standard feature. In the framework of the Standard Model (SM) the neutrino mass can arise through the higher order operators of the type $\frac{1}{M}(i\partial \chi)HH$, where $l$ and $H$ are respectively the lepton and Higgs doublets and $M$ is some regulator scale. In particular, the neutrino mass range needed for the just-so scenario corresponds to the Planck scale, $M \sim 10^{19}$ GeV, whereas the MSW scenario requires $M$ to be of the order of the supersymmetric grand unification scale, $M \sim 10^{16}$ GeV. As for the adjective "non-standard", it should be rather reserved for the really non-standard neutrino properties, implied by the SNP solutions based on the magnetic moment transition [14] or on the fast neutrino decay [13].
and Pinsonneault [2], (ii) NSSM+SM: in the context of models with relaxed prediction of \( \phi^B \) (which we conventionally refer to as non-standard solar models). In both cases the neutrinos are supposed to have only the standard interactions, (iii) SSM+NSM: in the SSM framework, assuming however that neutrinos have some additional non-standard interactions with matter constituents.

Sect. 3 is devoted to the model independent analysis of the just-so scenario. This essentially implies the modification of the solar neutrino spectrum due to the energy and time dependence of the survival probability. We focus our attention on the advantages inherent in the future real-time neutrino detectors like Super-Kamiokande [24], SNO [25] and BOREXINO [26]. All these experiments can measure the recoil electron spectrum, which could provide specific signatures allowing to discriminate the just-so scenario, in particular from the MSW one.

At the end, we give a brief summary of our conclusions.

2. Data fit in standard and non-standard pictures

For the simplicity, we consider the vacuum oscillations in the case of two neutrino flavours: \( \nu_e \to \nu_x \), where \( \nu_x \) can be \( \nu_\mu \) or \( \nu_\tau \). The survival probability for solar \( \nu_e \)'s with energy \( E \) is given by:

\[
P(L_t, E) = 1 - \sin^2 2\theta \sin^2\left(\pi \frac{L_t}{L}\right)
\]

where \( l = \frac{4\pi E}{\delta m^2} = \frac{E\text{[MeV]}}{\delta m^2[10^{-10}\text{eV}^2]} \cdot 2.47 \cdot 10^{10} \text{m} \) is the oscillation wavelength. The sun-earth distance \( L \) depends on time as \( L_t = \bar{L}[1 - \varepsilon \cos(2\pi t/T)] \), where \( \bar{L} = 1.5 \cdot 10^{11} \text{m}, T = 365 \text{days}, \) and \( \varepsilon = 0.0167 \) is the ellipticity of the orbit.

The time averaged signals predicted in the radiochemical experiments is given by:

\[
R = \int dE \sigma(E) \sum_i \langle P(E)\phi^i \rangle_T \lambda_i(E)\]

Here \( \sigma(E) \) is the detection cross section, \( \phi^i \) are the fluxes of the relevant components of the solar neutrinos (\( i = B, Be, \) etc.), \( \lambda_i(E) \) are their energy spectra normalized to 1, and \( \langle . . . \rangle_T \) stands for the average over the whole time period \( T \). In this way, the time dependence of the original flux (\( \phi(t) \propto L_t^{-2} \)) is also taken into the account.

For the Kamiokande detector, since we consider the \( \nu_e \) conversion into an active neutrino, the expression for the signal becomes

\[
R_K = \int_{E_{\text{th}}} dE \lambda_B(E) \left[ \langle P(E)\phi^B \rangle_T \sigma_{\nu_e}(E) + \left( \langle \phi^B \rangle_T - \langle P(E)\phi^B \rangle_T \right) \sigma_{\nu_y}(E) \right]
\]

Here \( \sigma_{\nu_y} (y = e, x) \) is the \( \nu_y e^- \) scattering cross section and \( E_{\text{th}} = \frac{1}{2}(T_e + \sqrt{T_e(T_e + 2m_e)}) \), where \( T_e = 7.5 \text{ MeV} \) is the recoil electron kinetic energy threshold.

Below we examine the just-so scenario in view of the recent status of the solar neutrino problem. We accept the hypothesis that the solar neutrino luminosities are constant in time,
and use the averaged data of the chlorine, gallium and Kamiokande experiments to perform the standard $\chi^2$ analysis for various cases (for the run-by-run analysis see ref. [21].)

(i) SSM+SM. We use as reference SSM the BP model, without taking into account the underlying theoretical uncertainties. The case of the other SSM will be effectively recovered by relaxing $\phi^B$ and $\phi^{Be}$ (see below).

The fit is not so good: the minimal $\chi^2$ obtained is 4.4. Thus, the just-so oscillation is allowed as a SNP solution at the 3.6% confidence level. Once this solution is assumed, the parameter regions containing the true values with the 68% and 95% probability are given by $\chi^2 \leq \chi^2_{min} + \delta\chi^2$, where $\delta\chi^2 = 2.28, 5.99$ respectively. These regions are shown in Fig. 1. They are limited by the values $\delta m^2 = (5 - 8) \cdot 10^{-11}$ eV$^2$, $\sin^2 2\theta = 0.7 - 1$. Our results are essentially in agreement with the recent analysis [21], where a somewhat different way of the data fitting is used.

In the same figure, we have also shown the $\delta m^2$ dependence of the time averaged $\nu_e \rightarrow \nu_x$ transition probability for the monochromatic $^7Be$ and $\text{pep}$ neutrinos. For the best fit point these probabilities are large, in agreement with the general paradigm implying a strong suppression for the intermediate energy neutrinos. However, as we see, in the wide range of the CL parameter regions there is no definite behaviour and even the ratio of the signals (which can be measured in BOREXINO detector – see below) is unpredictable. On the other hand, the same effect of the strong oscillation leads to the significant time variations of these monochromatic neutrino lines (see below).

(ii) NSSM + SM. Here $\phi^B$ and, to a less extent, also $\phi^{Be}$ are considered as free parameters. So, we describe the $^8B$ neutrino flux as $\phi^B = f_B \cdot \phi^B_0$, where evidently $\phi^B_0$ is the prediction of the BP SSM and the factor $f_B$ accounting for the uncertainty is varied in the range $0.4 - 1.6$ (for example, by taking $f_B = 0.8, f_{Be} = 0.9$ the case of the Turk-Chi`eze and Lopez SSM [4] is reproduced). The lower limit $f_B = 0.4$ is actually set by the Kamiokande measurement of the boron neutrino flux.

We have repeated the $\chi^2$ analysis for various values of $f_B$ and $f_{Be}$. The corresponding best fit points and 68% CL parameter areas are given in Fig. 2. The relevant range of $\delta m^2$ remains rather stable against variation of $f_B$, whereas the $\sin^2 2\theta$ becomes smaller with decreasing $f_B$. The lowering (increasing) of $f_B$ results in a weakening (strengthening) of the neutrino oscillations. Therefore, with smaller values of $f_B$ the model could be in agreement with the SN 1987A bound $\sin^2 2\theta \leq 0.7$ [22]. However, as a general tendency, by decreasing $f_B$ the fit becomes worse, whereas it slightly improves for $f_B > 1$. E.g., for $f_B = 0.4$ the high value of $\chi^2_{min} = 11.7$ indicates a poor fit (this solution is excluded at more than 99.9% CL). On the contrary, for $f_B = 1.3$ we have $\chi^2_{min} = 3.0$ which is acceptable at 8.3% CL. In this case the boron neutrino flux must be depleted stronger so that the larger mixing is required, what reconciles mutually the chlorine and the Kamiokande data. On the other hand, the large mixing contradicts the supernova bound. The decreasing of the beryllium flux (see Fig. 2b) does not alter significantly the previous results.

(iii) SSM+NSM. Here we still take the BP model as reference SSM but assume that neutrinos have some non-standard interactions in addition to the SM ones. Namely, we suppose that the $\nu_x$ state in which the solar $\nu_e$ is converted is just $\nu_x$ and it has extra weak interactions.

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4 The analysis of ref. [27] shows that the MSW scenario reacts in the same way by varying $f_B$, but the best $\chi^2$ fit is achieved for $f_B = 1$.

5 It is interesting to note that for $f_B \approx 0.6$ even the one parameter ($\sin^2 2\theta$) fit of the averaged short-wavelength oscillation provides somewhat better CL, $\chi^2_{min} = 11$ at $\sin^2 2\theta = 1$. 

range interaction with the electron:

\[ \mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\tau \gamma^\mu (1 - \gamma_5) \nu_\tau \left[ \epsilon \bar{e} \gamma_\mu (1 + \gamma_5)e + \epsilon' \bar{e} \gamma_\mu (1 - \gamma_5)e \right] \]  

(8)

Here \( \epsilon \) and \( \epsilon' \) parametrize the strength of new interactions with respect to the Fermi constant \( G_F \). The first term in this lagrangian, with positive \( \epsilon \), can be effectively obtained (after the Fierz transformation) from the exchange of some additional electroweak doublet scalar \( \varphi \) (the relevant Yukawa coupling is \( \bar{l}_{\tau L} e_R^{\nu} \varphi \), where \( l_{\tau L} \) is the lepton doublet including \( \tau \) and \( e_R \) is the right handed component of the electron). The second term could be due to the exchange of some charged singlet Higgs \( \eta \). However, the same exchange of the charged singlet unavoidably contributes the \( \tau \to e \nu \tau \bar{\nu}_e \) decay width, which sets the strong bound \( \epsilon' < 0.05 \). As for the strength of the first interaction \( \epsilon \), its value is not seriously constrained by any laboratory limit, while the astrophysical bounds on stellar evolution in the most conservative case imply \( \epsilon \leq 1 \) \( [28] \).

The extra neutral current interaction of \( \nu_\tau \) with the electron contributes to the \( \nu_\tau - e \) elastic scattering together with the standard neutral current and, as far as \( \epsilon > 0 \), it increases the \( \sigma_{\nu_\tau} \) cross section (see below, Fig. 9), and thus the signal in the Kamiokande detector. This implies a larger suppression of the boron neutrino flux, what leads to a better agreement between the Kamiokande and Homestake data.

In order to study the impact of these extra NC coupling on the just-so scenario\(^6\) we have repeated the \( \chi^2 \) analysis for the interval \( \epsilon = 0 - 1 \). The results of the fitting are shown in Fig. 3. One can observe that the allowed region of the parameters \( \delta m^2 \) and \( \sin^2 2\theta \) is rather stable against the variation of \( \epsilon \). However, as it was expected the data fit improves by increasing \( \epsilon \), since now the Kamiokande signal requires larger mixing angles. E.g., for \( \epsilon = 1 \) we achieve \( \chi^2_{\text{min}} = 1.8 \), which implies that in this case the just-so oscillations can be regarded as a solution of the SNP at the 18% CL.

Certainly, along with the interactions (8) one can consider also the analogous non-standard interactions of \( \nu_\tau \) with protons and neutrons. They could be induced due to the exchange of some scalar leptoquark with mass of about 100 GeV. These interactions do not contribute the signal in the detectors under operation. Nevertheless, they can be relevant for the signal in the future real-time detectors, especially SNO and BOREX.

Let us conclude this section with the following remarks. As we have seen, the just-so picture can be relevant for SNP only for the following mass and mixing range

\[ \delta m^2 = (0.5 - 0.8) \cdot 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta = 0.7 - 1 \]  

(9)

for any reasonable values \( f_{B,\text{Be}} \) and \( \epsilon \) (see Figs. 2,3). Moreover, for the plausible interval \( f_B = 0.7 - 1.3 \) the best fit area is essentially located in the very narrow band around \( \delta m^2 \approx 0.6 \cdot 10^{-10} \text{ eV}^2 \), rather independently on the concrete values of \( f_{B,\text{Be}} \) and \( \epsilon \), while \( \sin^2 2\theta \) varies from 0.7 to 1 depending on the concrete values of these parameters. The data fit for certain cases of the simultaneous variation of \( f_B \) and \( \epsilon \) is shown in the Table 1.

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\(^6\)For \( \nu_\mu \) such interactions are severely restricted by laboratory limits (see [23] and refs. therein.)

\(^7\)The effects of such non-standard interactions (flavour diagonal as well as flavour changing) for the MSW picture were studied in ref. [29]. However, the altering of the neutrino propagation in the solar interior due to the interaction (8) has no importance in the case of just-so oscillation. This interaction is relevant only for the detection cross-section in the \( \nu - e \) scattering experiment.
3. Predictions for the future solar neutrino experiments

Although the data fit in the just-so scenario is somewhat worse than in the MSW picture, it cannot be ruled out as a SNP solution. On the other hand, these solutions cannot be discriminated by the recent experiments. However, the next generation of the solar neutrino detectors will shed more light on the situation. The novel detectors like Super-Kamiokande, SNO and BOREXINO/BOREX could provide tests, almost independent of the SSM details. In particular, these real time detectors will be able to observe the seasonal time variations of the various neutrino components, due to the ellipticity of the earth orbit and sufficiently strong (but not very strong to be averaged) oscillation effects in the just-so regime. On the contrary, the MSW mechanism can exhibit only the standard 7% simultaneous variation of all signals from June to December, since in this case all neutrino conversions take place in the sun interior and the small oscillation effects in the way from sun to earth are negligible.

As we have seen, the just-so picture can be relevant for SNP only for a narrow interval, rather independently on the values \( f_{B,Be} \) and \( \epsilon \) (see Figs. 2,3). Moreover, for the moderate values \( f_B = 0.7 - 1.3 \) the best fit area is essentially located at \( \delta m^2 \approx 0.6 \cdot 10^{-10} \) eV\(^2\). Then it is easy to see that for the \( \delta m^2 \) in the range (9) the monochromatic \(^7\)Be neutrinos \((E = 0.861 \text{ MeV})\) oscillate along the distance \( L = 1.5 \cdot 10^{11} \text{ m} \) about 3 \(-\) 5 times, pep neutrinos \((E = 1.442 \text{ MeV})\) about 2 \(-\) 3 times and the boron neutrinos (with typical energy of about 10 MeV) do not undergo even one full oscillation. Therefore, since the value \( \epsilon \pi L/l \) is a small parameter (e.g., for \(^7\)Be neutrinos it is about 0.2), from eq. (5) we obtain for the \( \nu_e \) survival probabilities at June and December \((L = L(1 \pm \epsilon))\):

\[
P_{\pm}(E) \approx P(E) \mp \left[ 1 - P(E) \right] \frac{2 \epsilon \pi L/l_E}{\tan(\pi L/l_E)}
\]

where the quantity \( P(E) = P(L, E) \) essentially is the average survival probability of the \( \nu_e \) with energy \( E \). This formula demonstrates that the seasonal variations should be stronger for the neutrinos with smaller energies, and it can be dramatic for the monochromatic neutrino lines. Namely, in the best fit region (\( \delta m^2 \approx 0.6 \cdot 10^{-10} \) eV\(^2\)) we have \( P_{Be} \sim 0.5 \) for the \(^7\)Be neutrinos (see Fig. 1) while the phase factor \( \tan(\pi L/l_{Be}) \sim 1 \). Therefore, one should expect up to 50\% seasonal variations for the beryllium neutrino signal in BOREXINO detector (see below, Table 1). The standard 7\% variations are negligible in this case. At the same time, for this range of \( \delta m^2 \) the variation of the pep signal is expected to be smaller, less then 10\%, essentially due to large \( \tan(\pi L/l_{pep}) \) – see Fig. 1. However, for the wider range of parameters (9) also the pep neutrino signal variation can be significant. As for the \(^8\)B neutrinos, one cannot expect strong time variations (up to 10\%), due to large oscillation length as well as smoothing effects due to continuous spectrum.

Another possibility to discriminate the just-so scenario is related to the spectral distortion of the various solar neutrino components. The original energy spectra \( \lambda_i(E) \) \((i = B, Be, \text{ etc.})\) are independent of the details of the solar models. They are determined only by the nuclear reactions producing the neutrinos. The neutrino energy dependent conversion mechanisms for the SNP solution can strongly modify the initial neutrino spectra, offering thereby specific signatures for their discrimination. Below we consider the “just-so” spectral predictions for the planned experiments.

\[\text{8}\]The feasibility of the Super-Kamiokande and SNO detectors for the observation of the boron neutrino signal variations was studied in details in the recent paper by Krastev and Petcov.
**Super-Kamiokande.** This detector is expected to measure the spectrum of the high energy \(^8B\) neutrinos. The original neutrino distribution can be reproduced from the recoil electron spectrum due to \(\nu - e\) scattering, though it is somewhat smeared due to the integration over the neutrino energy:

\[
F(T) = \int dE \lambda_B(E) \left[ \langle P(E) \phi^B \rangle_T \frac{d\sigma_{\nu_e}}{dT}(E, T) + \left( \langle \phi^B \rangle_T - \langle P(E) \phi^B \rangle_T \right) \frac{d\sigma_{\nu_e}}{dT}(E, T) \right]
\]

\[
\frac{d\sigma_{\nu_y}}{dT}(E, T) = \frac{2G_F^2 m_e}{\pi} \left[ g_{yL}^2 + g_{yR}^2 (1 - T/E) - g_{yL} g_{yR} m_e T/E^2 \right], \quad y = e, \mu, \tau
\]

where \(T\) is the recoil electron energy. For the \(\nu_e - e\) scattering we adopt the Standard Model values for the NC coupling constants \(g_{eL} = \frac{1}{2} + \sin^2 \theta_W\) and \(g_{eR} = \sin^2 \theta_W\), whereas for the \(\nu_x\) state we also account for the possible non-standard couplings given by eq. (8): \(g_{xL} = -\frac{1}{2} + \sin^2 \theta_W + \epsilon'\) and \(g_{xR} = \sin^2 \theta_W + \epsilon\).

We calculate the ratio of the distorted spectrum \(F(T)\) to that is predicted by the SSM \(F_0(T)\). For the definiteness we normalize \(\xi(T) = F(T)/F_0(T)\) to 1 at \(T = 10\) MeV. Clearly, this ratio does not depend on the SSM details, as far as \(F_0(T)\) is essentially determined by the boron beta decay spectrum \(\lambda_B(E)\).

The shape of \(\xi(T)\) for various couples of the parameters \(\delta m^2\) and \(\sin^2 2\theta\) from the allowed area is given in Fig. 4a for \(\epsilon = 0\) and Fig. 4b for \(\epsilon = 1\). The present sensitivity of Kamiokande (long error bars) is not enough to discriminate the just-so solution, whereas Super-Kamiokande (short error bars) could distinguish it from the MSW picture, especially due to the characteristic distortion in the lower energy part of the spectrum (for the recoil electron spectrum in MSW case see ref. [27]). The deformation of the energy spectrum can alter the average energy \(\overline{T}\) of the recoil electrons as compared to the standard spectrum prediction \(\overline{T}_0 = 7.44\) MeV (with an electron energy threshold \(T_{th} = 5.5\) MeV). In Fig. 5 the iso-curves for the variation of \(\overline{T}\) as compared to \(\overline{T}_0\) (in percents) are plotted in the \((\delta m^2, \sin^2 2\theta)\) plane. As we see, \(\overline{T}\) can change up to 4%. In the case \(\epsilon = 0\) (Fig. 5a) the variation is rather positive than negative, whereas for \(\epsilon = 1\) (Fig. 5b) it is dominantly negative. In particular, for the best fit solutions the variation is 2.6 % for \(\epsilon = 0\), and -0.6% for \(\epsilon = 1\).

**SNO.** This heavy water real-time detector will measure the \(^8B\) neutrino flux through the charged-current (CC) and neutral-current (NC) processes:

\[
CC : \quad \nu_e d \rightarrow e^- p p
\]

\[
NC : \quad \nu_y d \rightarrow \nu_y p n, \quad y = e, \mu, \tau
\]

The ratio \(\eta = R_{CC}/R_{NC}\) in the SSM (i.e. when no neutrino conversion takes place) is independent of the value of \(f_B\). If the neutrino conversion occurs, the flux of the survived solar \(\nu_e\) is directly measured by the CC signal:

\[
R_{CC} = \int_{E_{th}} dE \sigma_{\nu_e}(E) \lambda_B(E) \langle P(E) \phi^B \rangle_T
\]

where \(E_{th} = 7\) MeV and for the cross section \(\sigma_{CC}\) we use the data presented in [31].

If the solar \(\nu_e\)'s are converted into active neutrinos \(\nu_x = \nu_\mu, \nu_\tau\) having only the SM neutral current couplings to nucleons (Z-boson exchange), then the probability conservation guarantees that the NC signal is the same as in the reference SSM: \(R_{NC}\) directly measures the original \(\phi^B\) flux. Therefore, if the measured ratio \(\eta = R_{CC}/R_{NC}\) is less than that is predicted
by SSM ($\eta_0 = 1.8$ for $E_{th} = 7$ MeV, independently of $f_B$), this would unambiguously indicate the deficit of the boron $\nu_e$, caused by the neutrino conversion. In Fig. 7 the iso-signal curves are given for the ratio $Z_{SNO} = R_{CC}^{pred}/R_{CC} = \eta/\eta_0$. As we see, in the parameter region relevant for the just-so scenario this ratio varies in the range 0.2 – 0.35.

The CC signal will allow to clearly discriminate the just-so picture by measuring the recoil electron spectrum $F(T)$. In fact, the latter reproduces the energy spectrum of the $\nu_e$'s survived the conversion, i.e. $\langle P(E)\phi_B(T)\lambda_B(E) \rangle$, shifted by an amount equal to the small recoil energy left to the nuclei: $T = E - 1.44$ MeV. Therefore, the ratio of the distorted spectrum to the SSM predicted one does not depend on $f_B$ and it directly characterizes the energy dependence of the survival probability.

In Fig. 6 the ratio $\xi(E) = F(E)/F_0(E)$, normalized to 1 at $E = 10$ MeV, is plotted for the same parameters as in the Fig. 4. The presence of the pronounced minimum discriminates the just-so solution from the MSW one, which instead provides characteristic monotonic shape of this ratio \cite{27}. The effect is manifested stronger than in the case of Super-Kamiokande, since now the spectral distortion is not smoothed by the integration over the neutrino energy. In Fig. 7 we show the iso-curves of the recoil electron average energy deviation from the SSM prediction ($T_0 = 8.42$ MeV with the electron energy threshold of 5.5 MeV). It ranges up to 12%, stronger than in Super-Kamiokande. For the best fit points it is $\sim 8\%$ for $\epsilon = 0$ and $11.5\%$ for $\epsilon = 1$ (see Table 1). The energy variation in the MSW picture has the same sign \cite{17}, but it is considerably smaller.

The non-standard interactions \cite{3} of $\nu_\tau$ with electrons do not contribute the signal neither in CC nor NC channels. However, the presence of the analogous non-standard $\nu_\tau$ interactions with quarks, violating universality of the neutrino interactions with nucleons, could be relevant. In this case the neutral current signal becomes

$$R_{NC} = R_{NC}^{SM} + \int_{E_{th}} dE \Delta \sigma_{NC}^{NSM}(E) \lambda_B(E) \left(\langle \phi^B \rangle_T - \langle P(E)\phi^B \rangle_T \right)$$

(14)

where $\Delta \sigma_{NC}^{NSM}$ is the additional (to the SM) contribution to the $\nu_xd \rightarrow \nu_xp$ n cross section arising due to the non-standard interactions. This extra contribution can differently affect the ratio $\eta$ expected, depending on the sign of $\Delta \sigma_{NC}^{NSM}$. In particular, in the case of sterile $\nu_x$ (i.e. when the extra contribution exactly cancels the standard one), we have $\eta \approx \eta_0$ independently of whether the conversion occurs or not \cite{31}.

**BOREXINO.** Due to the high radiopurity of this scintillator, the detection threshold is low: $T = 0.25$ MeV. This allows to have enough statistics to detect the $^7$Be and $pep$ neutrino lines through the $\nu - e$ scattering. In fact, the beryllium neutrino flux can be measured by exploring the energy window $T = 0.25 - 0.7$ MeV for the recoil electrons. In this window, according to BP SSM, about 50 events are expected per day, versus about 10 events provided by the natural radioactivity background \cite{20}. As for the $pep$ neutrinos whose contribution dominates the recoil electron energy range $T = 0.7 - 1.3$ MeV, their detection is less feasible, since the predicted signal (about 3 events per day) is comparable with the internal background.

As already anticipated, in the just-so picture the strong oscillations of the intermediate energy neutrinos prevent to make some definite prediction for the time averaged signals of $^7$Be and $pep$ neutrinos: in the relevant parameter region the signal to the SSM prediction ratios $Z_{Be}$ and $Z_{pep}$ can be rather arbitrary (see Fig. 1). In the MSW case, no precise prediction can be obtained as well \cite{17}, however, the relation between the $^7$Be and $pep$ signals remains
close to that is expected in SSM\footnote{As we commented above, the best possibility to distinguish MSW and just-so scenarios is provided by strong seasonal variations of $^7\text{Be}$ and $\text{pep}$ neutrino signals in the later case.}. On the contrary, in the case of just-so solution no definite prediction can be given neither for these signal ratio $Z_{\text{Be}}/Z_{\text{pep}} = \frac{R_{\text{Be}}/R_{\text{pep}}}{[R_{\text{Be}}/R_{\text{pep}}]_0}$: in the relevant parameter regions it can be much less or more than 1 (see Fig. 1).

The high sensitivity of the BOREXINO detector will allow to measure the recoil electron energy spectrum due to the $^7\text{Be}$ neutrinos and, to some extent, also due to the $\text{pep}$ ones. In this respect it is of interest to study how these spectra are affected in the just-so oscillation picture. The typical curves of the $\nu - e$ event distribution for some parameter values are plotted in Fig. 8a,b for the cases $\epsilon = 0$ and $\epsilon = 1$. In the former case, when $\nu_\tau$ has only SM interactions with the electron, the energy spectrum appears generally depleted throughout the relevant energy interval. However, the shape of the spectrum is not substantially changed and it essentially repeats the one of SSM (see Fig. 8a). In the case of NSM the rate of events is less depleted in the $^7\text{Be}$ energy window: in the presence of new interactions the $\nu_\tau$ contribution becomes very effective for the lower energies, which compensates the deficit of the original $\nu_e$'s. Moreover, for $\epsilon \simeq 1$ the signal can be even larger than that is expected in SSM: $Z_{\text{Be}} > 1$ (see Fig. 8b). Also, the shape of the spectrum becomes steeper as compared to the SSM predicted one. Let us remark also that the compensating effects of the $\tau$--neutrino NSM interactions can smear the time variations of $^7\text{Be}$ and $\text{pep}$ neutrino signals (compare the Tables 1A and 1B).

4. Discussion

We have confronted the just-so oscillation scenario with the recent experimental data on the solar neutrinos experiments in the context of non-standard solar models. Namely, we studied the response of this scenario to possible changes of the boron and beryllium neutrino fluxes. In the framework of the BP SSM the data fit is not excellent: $\chi^2_{\text{min}} = 4.4$, while it becomes worse for $f_B < 1$ and slightly improves for $f_B > 1$. The better data fit can be achieved by assuming that the $\nu_e$ state, emerged from the oscillation, has some non-standard neutral current coupling to the electron. The existing laboratory and astrophysical bounds indeed allow the $\tau$--neutrino to have such NSM interactions in the weak range, with $\epsilon \simeq 1$. In this case, also with moderate increasing of $f_B$ (up to 1.3), one can achieve quite reasonable $\chi^2$ fit (see Table 1B). It is interesting to note that the relevant mass range is rather stable against the variation of $f_{\text{Be}}$ and $\epsilon$: for the best fit area we have $\delta m^2 \approx 6 \cdot 10^{-11}$ eV$^2$.

The new generation of the real-time solar neutrino detectors can test the just-so scenario independently of the SSM details, and distinguish it from other candidates to the SNP solution. Even more, the possible NSM neutrino interactions can be also tested. Indeed, these detectors will be able to measure the spectra of various solar neutrino components, as well as to detect the effects of their seasonal variations. This will allow to determine unambiguously all unknown parameters, namely the SSM ones ($f_{\text{Be}}$ etc.), possible NSM ones ($\epsilon$, etc.) as well as neutrino mass and mixing range itself. In Table 1 we show the average rates and their seasonal variations in the chlorine, gallium and Kamiokande experiments, as well as in the future detectors (Super-Kamiokande, SNO and BOREXINO), for the best fit points corresponding to different values of $f_B$ and $\epsilon$.

In the case of $\nu_e \rightarrow \nu_x$ just-so oscillation the recoil electron energy spectra appear to be specifically altered, and different from the one expected from the MSW conversion.
us imagine that the SNO and/or Super-Kamiokande spectral measurements really point to the just-so oscillation. These spectra separately cannot tell us anything about the presence of the NSM interactions of $\nu_e$ with the electron (compare the curves in Figs. 4a and 4b). However, both the CC and NC reactions in SNO provide the measurements of the boron neutrino energy spectrum on the earth, which also constitutes the only contribution to the Super-Kamiokande signal. Therefore, the presence of non-standard $\nu_x-e$ interactions could be determined by confronting the spectra measured by SNO and Super-Kamiokande. In fact, the CC reaction in SNO directly measures the energy spectrum of the survived boron $\nu_e$'s reaching the earth, i.e. essentially the value $P(E)\phi_B$. Substituting this value in the eq. (11) for the Super-Kamiokande signal, one can unambiguously deduce the only 'unknown' quantity, the differential cross-section $d\sigma/dT(E,T)$, and confront it to the SM prediction. (As we mentioned above, the non-standard $\nu_x-e$ couplings can be also tested by the spectral shape of the recoil electrons in BOREXINO.) By confronting the CC and NC signals in an analogous manner, one can extract also the information on possible NSM couplings of $\nu_x$ with nucleons (see eq. (14)). Thus, as far as we believe that SNP is related to some conversion mechanism of solar $\nu_e$'s into the other neutrino flavours, the sun appears to be quite a strong and cheap source of the latter. Then the measurement of the recoil electron energy spectra in the novel real-time detectors offers not only a test for any possible SNP solution, but it can also be considered as a test for the neutrino NSM interactions, or in other words, as a test for the Standard Model of the electroweak interactions itself.

Last but not least we wish to emphasize that the non-standard neutrino interactions, besides improving the data fit in the just-so picture, could also resolve its potential conflict with the SN 1987A $\nu$-signal, pointed out in ref. [22]. Namely, these interactions would increase the $\bar{\nu}_\tau$ opacity in the supernova core, and thereby reduce their average energy. This could occur due to the dramatic increase of the $\bar{\nu}_\tau-e$ cross-section, as compared with the $\nu_e-e$ one, for large values of $\epsilon$ (see Fig. 9, where these cross-sections are plotted versus the neutrino energy for different values of $\epsilon$). Then the interference of the original $\bar{\nu}_e$ and $\bar{\nu}_\tau$ spectra due to the neutrino mixing will less affect the expected $\bar{\nu}_e$ signal in the terrestrial detectors. According to ref. [22], the problem will be dissolved if the average energy of $\bar{\nu}_e$ drops below $17-20$ MeV: then even the maximal mixing, $\sin^22\theta = 1$, cannot be excluded. Moreover, in this case the partial permutation between the $\bar{\nu}_e$ and $\bar{\nu}_\tau$ spectra could explain the certain excess of the higher energy $\bar{\nu}_e$ events from SN 1987A following from the comparison of the IMB and Kamiokande data [23]. On the other hand, the difference between $\bar{\nu}_\tau$ and $\nu_\tau$ opacities can provide a significant asymmetry in their average energies, which, due to the strong oscillation, can result in an asymmetry between $\bar{\nu}_e$ (isotropic) and $\nu_e$ (directional) signals in the terrestrial detectors. Obviously, for the precise evaluation of the effects from the non-standard neutrino interactions it is necessary to consistently include them into a detailed computer analysis of the stellar core collapse at the beginning.

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References

[1] R. Davis et al., Prog. Part. Nucl. Phys. 32 (1994).

[2] J.N. Bahcall and M.H. Pinsoneau, Rev. Mod. Phys. 64 (1992) 885.

[3] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. 60 (1989) 297.

[4] S. Turk-Chièze and I. Lopez, Ap. J. 408 (1993) 347; S. Turk-Chièze et al., Phys. Rep. 230 (1993) 57.

[5] V. Castellani, S. Degl’Innocenti and G. Fiorentini, A. & A. 271 (1993) 601.

[6] J.N. Bahcall and H.A. Bethe, Phys. Rev. Lett. 65 (1993) 2233; J.N. Bahcall et al., preprint IASSNS-AST 94/13 (1994); J.N. Bahcall, preprints IASSNS-AST 94/14, IASSNS-AST 94/37 (1994); N. Hata and P. Langacker, preprint UPR-0592T (1993); S. Bludman et al., Phys. Rev. D 47 (1993) 2220; Phys. Rev. D 49 (1994) 3622; V. Castellani, S. Degl’Innocenti and G. Fiorentini, Phys. Lett. B 303 (1993) 68; V. Castellani et al., Phys. Lett. B 324 (1994) 425; preprint INFNFE-3-94 (1994); V. Berezinsky, preprints LNGS-93/86 (1993), LNGS-94/101 (1994); A.Yu. Smirnov, preprint DOE/ER/40561-136-INT94-13-01 (1994); A. Dar and G. Shaviv, preprint Technion-Ph-94-5 (1994); V.N. Tsytovich, Proc. Int. Workshop ”Solar neutrino problem: astrophysics or oscillations”, eds. V. Berezinsky and E. Fiorini, INFN Gran Sasso, 1994.

[7] Y. Suzuki, in TAUP-93, Nucl. Phys. B (Proc. Suppl.) 35 (1994) 407.

[8] V. Castellani et al., in ref. [3].

[9] GALLEX Collaboration, P. Anselmann et al., Phys. Lett. B 327 (1994) 377.

[10] V.N. Gavrin, Proc. Int. Symposium ”Neutrino Telescopes”, Venice, 1994.

[11] S.P. Mikheyev and A.Yu. Smirnov, Nuovo Cimento 9C (1986) 17; Prog. Part. Nucl. Phys. 23 (1989) 41; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369.

[12] B. Pontecorvo, ZhETF 53 (1967) 1717; I.Ya. Pomeranchuk, (1967) unpublished; V. Gribov and B. Pontecorvo, Phys. Lett. B 28 (1969) 493; R. Ehrlich, Phys. Rev. D 18 (1978) 2323.

[13] V. Barger, R.J.N. Phillips and K. Whisnant, Phys. Rev. D 24 (1981) 538; S.L. Glashow and L.M. Krauss, Phys. Lett. B 190 (1987) 199.

[14] M. Voloshin and M. Vysotsky, Yad. Fiz.44 (1986) 845; M. Voloshin, M. Vysotsky and L.B. Okun, ZhETF 91 (1986) 745 [JETP 64 (1986) 446]; E. Akhmedov, Phys. Lett. B 213 (1988) 64; C.S. Lim and W. Marciano, Phys. Rev. D 37 (1988) 1368.

[15] J.N. Bahcall, N. Cabibbo and A. Yahill, Phys. Rev. Lett. 28 (1972) 316; S. Pakvasa and K. Tennakone, ibid 1415; J.N. Bahcall et al., Phys. Lett. B 181 (1986) 369; Z. Berezhiani and M. Vysotsky, Phys. Lett. B 199 (1987) 281; Z. Berezhiani et al., Z. Phys. C 54 (1992) 581.
[16] N. Hata and P. Langacker, Phys. Rev. D 48 (1993) 2937; P. Krastev and S. T. Petcov, Phys. Lett. B 299 (1993) 99.

[17] G. Fiorentini et al., Phys. Rev. D 49 (1994) 6298.

[18] V. Barger, R.J.N. Phillips and K. Whisnant, Phys. Rev. Lett. 65 (1990) 3084; Phys. Rev. D 43 (1991) 1110; A. Acker, S. Pakvasa and J. Pantaleone, Phys. Rev. D 43 (1991) 1754.

[19] R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. B 90 (1980) 249.

[20] E. Akhmedov, Z. Berezhiani and G. Senjanović, Phys. Rev. Lett. 69 (1992) 3013.

[21] P. Krastev and S. Petcov, Phys. Lett. B 285 (1992) 85; Phys. Rev. Lett. 72 (1994) 1960; N. Hata, preprint UPR-0605T, 1994.

[22] J.N. Bahcall, A.Yu. Smirnov and D.N. Spergel, Phys. Rev. D 49 (1994) 1389.

[23] K. Hirata et al., Phys. Rev. Lett. 58 (1987) 1490; R.M. Bionta et al. ibid 1494.

[24] Y. Totsuka, ICCR report 227-90-20, (1990); Y. Suzuki, in TAUP-93, Nucl. Phys. B (Proc. Suppl.) 35 (1994) 273.

[25] SNO Collaboration, G. Aartsma et al., Phys. Lett. B 194 (1987) 321.

[26] C. Arpesella et al. (Borexino Collaboration), in Proposal of BOREXINO (1991).

[27] P.I. Krastev and A.Yu. Smirnov, preprint DOE/ER/40561-137-INT94-13-02 (1994); V. Berezinsky, G. Fiorentini and M. Lissia, preprint LGNS-94-104 (1994).

[28] S. Degl’Innocenti and B. Ricci, Mod. Phys. Lett. A 8 (1993) 471.

[29] L. Wolfenstein, in ref. [11]; J.W.F. Valle, Phys. Lett. B 199 (1987) 432; M. Guzzo, A. Masiero and S.T. Petcov, Phys. Lett. B 260 (1991) 154; E. Roulet, Phys. Rev. D 44 (1991) R935; V. Barger, R.J.N. Phillips and K. Whisnant, Phys. Rev. D 44 (1991) 1629.

[30] S.M. Bilenky and C. Giunti, preprint DFTT 62/93 (1993).

[31] K. Kubodera and S. Nozawa, preprint USC-NT-93-6 (1993).

[32] P. Krastev and S. Petcov, preprint SISSA 41/94/EP (1994).

[33] S.M. Bilenky and C. Giunti, preprint DFTT 32/94 (1994).
### Table A: $\epsilon = 0$

| Detector | $f_B = 1$ ($\chi^2_{\text{min}} = 4.4$) | $f_B = 0.7$ ($\chi^2_{\text{min}} = 6.4$) | $f_B = 1.3$ ($\chi^2_{\text{min}} = 3.0$) |
|----------|--------------------------------------|--------------------------------------|--------------------------------------|
|          | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ |
| Cl − Ar  | 0.32 ($\pm 10\%$) | 2.55 | 0.43 ($\pm 4.7\%$) | 2.63 | 0.27 ($\pm 12\%$) | 2.65 |
| Ga − Ge  | 0.50 ($\pm 10\%$) | 66  | 0.55 ($\pm 4.3\%$) | 70  | 0.51 ($\pm 10\%$) | 69  |
| Kamiokande ($T_{th} = 7.5$ MeV) | 0.41 ($\pm 2.4\%$) | [0.31+0.1] | 0.41 | 0.52 ($\pm 2.0\%$) | [0.44+0.08] | 0.36 | 0.34 ($\pm 3.5\%$) | [0.23+0.11] | 0.44 |
| SK ($T_{th} = 5.5$ MeV) | 0.37 ($\pm 2.0\%$) | [0.26+0.11] | 0.37 | 0.49 ($\pm 1.1\%$) | [0.40+0.09] | 0.34 | 0.28 ($\pm 3.2\%$) | [0.16+0.12] | 0.36 |
| SNO      | 0.26 ($\pm 3.7\%$) | 0.26 | 0.41 ($\pm 1.5\%$) | 0.29 | 0.17 ($\pm 6.5\%$) | 0.22 |
| BOREXINO: $^7$Be ($T = 0.25 − 0.7$ MeV) | 0.61 ($\pm 22\%$) | [0.45+0.16] | 32 | 0.55 ($\pm 11\%$) | [0.39+0.16] | 29 | 0.79 ($\pm 18\%$) | [0.67+0.12] | 41 |
| BOREXINO: pep ($T = 0.7 − 1.3$ MeV) | 0.41 ($\pm 4.7\%$) | [0.28+0.13] | 1.0 | 0.58 ($\pm 6.2\%$) | [0.48+0.10] | 1.5 | 0.32 ($\pm 3.2\%$) | [0.16+0.16] | 0.8 |
| $\delta(T)_{SK}$ | 2.6% | 1.4% | 4.0% |
| $\delta(T)_{SNO}$ | 8.0% | 3.8% | 14.4% |

### Table B: $\epsilon = 1$

| Detector | $f_B = 1$ ($\chi^2_{\text{min}} = 1.8$) | $f_B = 0.7$ ($\chi^2_{\text{min}} = 4.2$) | $f_B = 1.3$ ($\chi^2_{\text{min}} = 1.0$) |
|----------|--------------------------------------|--------------------------------------|--------------------------------------|
|          | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ | $Z$ | $R$ |
| Cl − Ar  | 0.31 ($\pm 13\%$) | 2.47 | 0.41 ($\pm 5.5\%$) | 2.51 | 0.25 ($\pm 8.0\%$) | 2.45 |
| Ga − Ge  | 0.54 ($\pm 10\%$) | 71  | 0.55 ($\pm 7\%$) | 70  | 0.55 ($\pm 10\%$) | 75  |
| Kamiokande ($T_{th} = 7.5$ MeV) | 0.44 ($\pm 2.0\%$) | [0.26+0.18] | 0.44 | 0.56 ($\pm 1.3\%$) | [0.42+0.14] | 0.39 | 0.35 ($\pm 2.8\%$) | [0.14+0.21] | 0.46 |
| SK ($T_{th} = 5.5$ MeV) | 0.46 ($\pm 1.3\%$) | [0.20+0.26] | 0.46 | 0.58 ($\pm 0.7\%$) | [0.37+0.21] | 0.41 | 0.41 ($\pm 0.2\%$) | [0.11+0.30] | 0.53 |
| SNO      | 0.21 ($\pm 4.5\%$) | 0.21 | 0.38 ($\pm 1.5\%$) | 0.27 | 0.11 ($\pm 5.4\%$) | 0.14 |
| BOREXINO: $^7$Be ($T = 0.25 − 0.7$ MeV) | 1.02 ($\pm 2.0\%$) | [0.68+0.34] | 53 | 1.04 ($\pm 1.0\%$) | [0.42+0.62] | 54 | 1.02 ($\pm 1.5\%$) | [0.80+0.21] | 53 |
| BOREXINO: pep ($T = 0.7 − 1.3$ MeV) | 0.72 ($\pm 0.7\%$) | [0.20+0.52] | 1.8 | 0.80 ($\pm 1.3\%$) | [0.42+0.38] | 2.1 | 0.92 ($\pm 2.7\%$) | [0.77+0.15] | 2.4 |
| $\delta(T)_{SK}$ | −0.6% | −0.6% | −2.6% |
| $\delta(T)_{SNO}$ | 11.5% | 4.5% | 13.0% |

**Table 1.** The expected signals in different detectors, for the best fit points corresponding to different values of $f_B$. The Tables A,B are for the cases $\epsilon = 0, 1$, respectively. $Z$ is the ratio of the calculated signal to the one expected in the solar model with the given $f_B$ (clearly, $Z$ does not depend on $f_B$). Within round brackets the percentage seasonal variation of the signal, compared to the time averaged value $Z$, is reported, where the upper sign refers to June and the lower one to December. For the $\nu − e$ scattering experiments the individual contributions from the survived $\nu_e$ and emerged $\nu_x$ are also shown (within the square brackets). $R$ are the annual average signals predicted for each detector. For the radiochemical experiments $R$ is given in SNU, whereas for BOREXINO in the number of events per day, for the recoil electron energy intervals indicated. For (Super) Kamiokande and SNO $R$ is given in units of the BP SSM prediction: $R = f_B \cdot Z$. The quantity $\delta(T)$ stands for the variations of the recoil electron average energy with respect to the one predicted in SSM.
Figure Captions

Fig. 1. Confidence regions in the parameter space $\delta m^2$ and $\sin^2 2\theta$, for the case SSM+SM. The diamond marks the best fit point to the experimental data ($\chi^2_{\text{min}} = 4.4$). Solid and dotted curves delimit the 68 % CL and 95 % CL regions, respectively. On the right axis, the time averaged transition probabilities (modulo $\sin^2 2\theta$) are shown as a function of $\delta m^2$ for the $^7$Be and $pep$ neutrinos (dashed and dot-dashed curves, respectively).

Fig. 2. The best fit points (diamonds) and the 68 % CL regions in the case NSSM+SM for different $f_B$, where $f_{Be} = 1$ (Fig. 2a) or $f_{Be} = 0.8$ (Fig. 2b). The $\chi^2_{\text{min}}$ corresponding to values $f_B = 0.4, 0.7, 1.0, 1.3, 1.6$ are 11.7, 6.4, 4.4, 3.0, 2.8 in Fig. 2a, and 10.3, 5.7, 4.3, 3.1, 2.8 in Fig. 2b.

Fig. 3. The best fit point (marked as 2, $\chi^2_{\text{min}} = 1.8$) and the 68 % CL regions in the case SSM+NSM, for $\epsilon = 1$ (solid curves) confronted with the case SSM+SM, $\epsilon = 0$ (dotted curves, best fit point marked as 1). In the following these points, as well as the other typical points 3 and 4, will be used to demonstrate the effects of spectral distortion.

Fig. 4. Super-Kamiokande: the ratio $\xi(T)$ of the recoil electron energy spectrum, distorted due to the just-so oscillation, to the undistorted one (normalized to 1 at 10 MeV), given for the points shown in Fig. 3. Here Fig. 4a,b refer to the cases $\epsilon = 0$ and $\epsilon = 1$, respectively. The longer error bars indicate the present sensitivity of the Kamiokande detector and the shorter ones represent the expected sensitivity in Super-Kamiokande.

Fig. 5. The iso-signal curves for $Z_{SK}$ expected at Super-Kamiokande, with 5.5 MeV threshold (solid). The curves for the iso-percentage variations of the average electron energy compared with the SSM value are also shown (dashed). Fig. 5a refers to the case $\epsilon = 0$ and Fig. 5b to that $\epsilon = 1$. The corresponding 68% CL regions are also shown (dotted curves).

Fig. 6. SNO: the ratio $\xi(E)$ of the distorted boron neutrino energy spectrum to that expected in absence of solar neutrino conversion, normalized to 1 at 10 MeV. The curves correspond to the points marked in Fig. 3. The error bars indicate the expected sensitivity of the detector.

Fig. 7. The iso-signal contours due to the CC reaction at SNO with 5.5 MeV threshold (solid). The dashed curves represent the iso-percentage variations of the average electron energy as compared to that expected in SSM.

Fig. 8. Distribution of the $\nu - e$ scattering events expected at BOREXINO as a function of the recoil electron energy $T$, for the cases $\epsilon = 0$ (Fig. 8a) and $\epsilon = 1$ (Fig. 8b). These are given for the typical points shown in Fig. 3 (solid, long dashed, dot-dashed and short dashed curves, respectively). For comparison, the dotted curve corresponds the electron spectrum expected in BP SSM, in the absence of neutrino conversion.

Fig. 9. The energy dependence of the $\bar{\nu}_\tau - e$ and $\nu_\tau - e$ scattering cross-sections (dashed and solid, respectively), normalized to $\sigma_0 = 2G_F^2 m_\tau^2/\pi$, for different values of $\epsilon$. 
