Convergence analysis of beetle antennae search algorithm and its applications

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Abstract

The beetle antennae search algorithm was recently proposed and investigated for solving global optimization problems. Although the performance of the algorithm and its variants were shown to be better than some existing meta-heuristic algorithms, there is still a lack of convergence analysis. In this paper, we provide theoretical analysis on the convergence of the beetle antennae search algorithm. We test the performance of the BAS algorithm via some representative benchmark functions. Meanwhile, some applications of the BAS algorithm are also presented.

Keywords: Beetle antennae search (BAS) algorithm, Meta-heuristic algorithm, Convergence analysis, Successful rate

1. Introduction

As a meta-heuristic algorithm, the beetle antennae search (BAS) algorithm was proposed by Jiang and Li \cite{1}. The design of the algorithm was inspired by the behaviors of beetles when seeking for a mate. The performance of the BAS algorithm has been evaluated in various applications. Zhu

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et al. \cite{2} applied BAS algorithm to multiobjective energy management in microgrids which adopts minimum operation cost and minimum pollutant treatment cost as its objectives under the constraints of time-of-use price and energy storage status. Yin and Ma \cite{3} proposed an aggregation service chain mapping plan based on an improved BAS algorithm for network resources allocation, which consumes less computing resources and has excellent performance in key mapping costs and network latency. Wang et al. \cite{4} applied the BAS algorithm to improve the accuracy of spatial straightness assessment, showing a faster convergence and better accuracy. Sun et al. \cite{5} used the BAS algorithm to train a neural network, which was further applied to the prediction of the unconfined compressive strength of jet grouting coalcretes, which showed a better performance than multiple regression, logistic regression, and support vector machine. Lin et al. \cite{6} utilized the BAS algorithm to the tuning of a PID controller for DC motors, which led to a smaller overshooting and a faster responding speed when the load and disturbance changes compared with a traditional PID controller. Sun et al. \cite{7} used the BAS algorithm to tune the hyperparameters of support vector machine for the determination of Young’s modulus of jet grouted coalcretes. Compared with other algorithms, the method proposed by Sun et al. is less time-consuming and more accurate with a lower cost. Sun et al. \cite{8} adopted the BAS algorithm to tune a support vector regression model for the prediction of permeability and unconfined compressive strength of pervious concretes, leading to a high prediction accuracy. The above works showed that the convergence of the BAS algorithm is fast, the implementation of the BAS algorithm is simple, and the probability of the BAS algorithm to be trapped in local optimum is small. Recently, the combinations of BAS with particle swarm optimization (PSO) were also reported. Chen et al. \cite{9} proposed a beetle swarm optimization (BSO) algorithm by combining the beetle antennae search (BAS) algorithm with the standard PSO algorithm, where the update rule of each particle follows BAS. The algorithm was also adopted to solve the wireless sensor network coverage problem, showing a better performance than the standard PSO \cite{10}. The BSO algorithm was then adopted to solve an investment portfolio problem. The combination of BAS with BSO was also proposed in \cite{11}, which has a better performance than standard BSO.

While the BAS algorithm has been found to be efficient and effective in solving many optimization problems, there is still a lack of theoretical guarantee. Motivated by this fact, in this paper, we aim at providing convergence
analysis on the BAS algorithm. We will also validate the performance of the
algorithm with some typical examples. The contributions of this paper are
listed as follows:

1) The theoretical guarantee for the performance of the BAS algorithm is
provided.

2) The quantitative analysis on the performance of the BAS algorithm for
seven representative test functions are conducted based on the success-
ful rate measure.

3) The performance of the BAS algorithm in engineering applications is
tested.

The rest of this paper is organized as follows. In Section 2, we revisit the
BAS algorithm, followed by the theoretical analysis on Section 3. Then, we
test the performance of the BAS algorithm through numerical experiments
in Section 4. The performance of the BAS algorithm is also tested by three
engineering problems in Section 5. Conclusions are given in Section 6.

2. Algorithm description

In this section, we review the BAS algorithm.

Consider the minimization problem of function $f(x) \in \mathbb{R}$ with the decision
variable being $x = [x_1, x_2, \cdots, x_n]^T$.

Assumption 1: The optimal solution to the minimization problem of $f(x)$
exists.

The BAS algorithm treats the decision variable as the location of the
centroid position of a beetle in the $n$-dimensional space. To minimize the
function $f$, the behavior of the beetle is described as follows according to the
BAS algorithm [1]:

$$x^{k+1} = x^k - \delta^k b \, \text{sgn}(f(x^k_l) - f(x^k_r)),$$

where $x^k_l$ and $x^k_r$ denote the location of the left tentacle and the right tentacle
of the beetle at time instant $k$, respectively; $\delta^k$ denotes the step size of
searching; $b$ denotes a direction vector, which is random, and set as follows:

$$b = \frac{\text{rnd}(n, 1)}{\|\text{rnd}(n, 1)\|_2},$$

where $\text{rnd}(n, 1)$ is a random vector of length $n$.
with \| \cdot \| denoting the two-norm operator and \text{rnd}(n, 1) denotes a randomly generated \(n\)-dimensional vector; \text{sgn}(\cdot) is the sign function. The locations of left and right tentacles are given as follows:

\[
\begin{align*}
  \mathbf{x}_l &= \mathbf{x}^k + d^k \mathbf{b}, \\
  \mathbf{x}_r &= \mathbf{x}^k - d^k \mathbf{b}.
\end{align*}
\]

(3)

In addition, in the BAS algorithm, it is suggested to set

\[
\begin{align*}
  \delta^k &= \alpha \delta^{k-1} + 0.001, \\
  d^k &= c \ast d^{k-1} + d_0
\end{align*}
\]

(4)

with \(c > 0 \in \mathbb{R}, \alpha \in (0, 1), \delta^0 > 0 \in \mathbb{R}, \) and \(d_0 > 0 \in \mathbb{R}.

If the searching ranging is defined in a closed set \(\Omega \in \mathbb{R}^n\), then the BAS algorithm is modified as [1]:

\[
\mathbf{x}^{k+1} = P_\Omega(\mathbf{x}^k - \delta^k \mathbf{b} \ \text{sgn}(f(\mathbf{x}^k_l) - f(\mathbf{x}^k_r))),
\]

(5)

where \(P_\Omega(\cdot)\) denotes the projection operator. Evidently, [1] is a special case of [5] by setting \(\Omega = \mathbb{R}^n\).

The basic BAS algorithm is given in Algorithm 1.

---

**Algorithm 1** BAS algorithm for global minimization

**Require:** Objective function \(f(\mathbf{x})\), and values of parameters \(\alpha, c, \delta_0, d_0, \mathbf{x}^0\), and searching set \(\Omega\)

**Ensure:** Optimal solution \(\mathbf{x}_{bst}\) and optimal function value \(f_{bst}\).

1. Initialize \(f_{bst}\) to be \(f(\mathbf{x}^0)\)
2. Initialize \(\mathbf{x}_{bst}\) to be \(\mathbf{x}^0\)
3. while \((k < K_{max})\) or (stop criterion) do
4.   Generate \(\mathbf{b}\) according to (2)
5.   Calculate \(\mathbf{x}^k_l\) and \(\mathbf{x}^k_r\) according to (3)
6.   Calculate \(\mathbf{x}^{k+1}\) according to (5)
7.   if \(f(\mathbf{x}^{k+1}) < f_{bst}\) then
8.     \(f_{bst} = f(\mathbf{x}^{k+1}), \mathbf{x}_{bst} = \mathbf{x}^{k+1}\)
9.   end if
10. end while
3. Convergence analysis

In this section, convergence analysis for the BAS algorithm is provided. We first give the definition of convergence as follows.

**Definition 1 [12]:** (Convergence with probability 1) Convergence with probability 1 means that with probability 1 a monotone sequence \( \{f(x)\}_{k=1}^{\infty} \) which converges to the infimum of \( f \) is obtained on \( \Omega \).

The convergence analysis is based on Definition 1. Before moving to the analysis, for the sake of illustration, let \( x_{bst}^k = \min_{x_j} \{f(x_j)\} \) with \( j = 0, 1, \ldots, k \) and \( f_{bst}^k = f(x_{bst}^k) \).

**Lemma 1:** For the BAS algorithm, \( f_{bst}^k \) is not increasing.

**Proof:** According to Algorithm 1 at each instant \( k \), if \( f(x^{k+1}) < f_{bst} \), then \( f_{bst} = f(x^{k+1}) \). Note that the initial value of \( f_{bst} \) to be extremely large. As a result, the BAS algorithm guarantees that \( f_{bst}^k \) is not increasing.

Lemma gives a determined conclusion that the BAS algorithm will not diverge in the long term.

**Theorem 1:** Given that the parameters are properly set, the BAS algorithm is convergent with probability 1.

**Proof:** Suppose that the parameters of the BAS algorirthm are properly set such that at each time instant \( k \), the probability of \( P_{\Omega}(x^k + \delta_k \text{sgn}(f(x^k) - f(x^*)|) \) located on the optimal solution \( x^* \) to the minimization problem of \( f \) is larger than 0. Let \( p_k \) denotes the probability that at time instant \( k \), \( x^k \) is not located on \( x^* \). Then, we have

\[
p(x_{bst}^k = x^*) >= 1 - p_0 p_1 \cdots p_k.
\]

Note that \( 0 \leq p_k < 1 \) by the above assumption. Thus,

\[
\lim_{k \to +\infty} (1 - p_0 p_1 \cdots p_k) = 1 - \lim_{k \to +\infty} p_0 p_1 \cdots p_k = 1.
\]

Note that

\[
p(x_{bst}^k = x^*) \leq 1.
\]

Thus, by the squeeze theorem, we further have

\[
\lim_{t \to +\infty} p(x_{bst}^k = x^*) = 1.
\]

The proof is complete. \( \square \)

Theorem 1 shows that by properly choosing the step size, we can guarantee that the BAS algorithm is asymptotic convergent will probability 1.
This conclusion is important. Firstly, it shows that the BAS algorithm can converge under a condition about its step size. Secondly, in practice, this theorem also helps us identify the problem about why the BAS algorithm may not have a good solution performance when facing certain functions, which is a general issue in most bio-inspired algorithms.

4. Illustrative examples

In this section, we provide some illustrative examples to show the performance of the BAS algorithm.

There are many criteria for evaluating the performance of bio-inspired algorithms for solving optimization problem, such as the success rate and number of function evaluations. In this paper, we adopt the success rate to evaluate the performance of the BAS algorithm, which is defined as follows [13]:

\[
\text{success rate} = \frac{N_{\text{success}}}{N_{\text{all}}},
\]

where \(N_{\text{success}}\) denotes the number of successful trials and \(N_{\text{all}}\) denotes the total number of trials. A trial is considered to be successful if the following inequality is satisfied:

\[
\sum_{i=1}^{n} (x_{\text{best}_i} - x^*_i)^2 \leq (UB - LB) \times 10^{-4},
\]

where \(UB\) denotes the identical upper bound and \(LB\) denotes the identical lower bound of the elements in \(x\).

Seven test functions adopted from [13, 14, 15] are considered in this paper. The function expressions, dimensions, and the corresponding global optima are listed on Table 1. The variable bounds for the optimum searching of each function are -10 to 10 (i.e., \(LB = -10\) and \(UB = 10\)) for each variable for all the functions, except that, for function \(f_6\), we have \(LB = -2\pi\) and \(UB = 2\pi\). ALL the test functions have a unique global optimum in the given search regions such that we can easily use (6) to evaluate the performance of the BAS algorithm. These test functions are selected due to their representative properties. For example, \(f_4\) called Griewank’s function is highly multimodal, meaning that it has many local minima. For each function, the maximum number of iterations in each run is set to \(10^5\) (i.e., \(K_{\text{max}}\) is set to \(10^5\)) and each function is tested for 100 runs by using the BAS algorithm. The initial
| function | dimension | global minima |
|----------|-----------|---------------|
| $f_1(x) = \sqrt{n \sum_{i=1}^{n} x_i^2}$ | $n = 30$ | $f_1^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
| $f_2(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$ | $n = 20$ | $f_2^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
| $f_3(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$ | $n = 10$ | $f_3^* = 0$ at $x^* = 1 \in \mathbb{R}^n$ |
| $f_4(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^{n} \cos(2 \pi x_i)) + 20 + \exp(1)$ | $n = 10$ | $f_4^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
| $f_5(x) = 1 + \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \frac{x_i}{\sqrt{i}}$ | $n = 10$ | $f_5^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
| $f_6(x) = (\sum_{i=1}^{n} |x_i|) \exp(-\sum_{i=1}^{n} \sin x_i^2)$ | $n = 5$ | $f_6^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
| $f_7(x) = \sum_{i=1}^{n} x_i^2 + (0.5 \sum_{i=1}^{n} ix_i)^2 + (0.5 \sum_{i=1}^{n} ix_i)^4$ | $n = 20$ | $f_7^* = 0$ at $x^* = 0 \in \mathbb{R}^n$ |
value of each element in \( x \) for each test function is randomly generated with a uniformly distribution. The initial values of \( \delta \) is set to 10 and the initial value of \( d \) is set to \( UB \) for all test functions.

The test results and parameter settings are shown in Table 2. As seen from Table 2, the successful rate of the BAS algorithm is relatively high for the test functions. For example, for functions \( f_1, f_5, f_6, \) and \( f_7 \), the successful rate is 100. The lowest successful rate of the BAS algorithm is 80, which is for function \( f_4 \). This is due to the aforementioned fact, i.e., \( f_4 \) is highly multimodal. It is worth pointing out that the successful rate depends on the parameter setting. However, currently, the parameters are set manually. Thus, better results could be obtained if some automatic parameter tuning methods are used. As seen from Table 2, the standard deviation of the obtained optimal function value is relatively row except for functions \( f_2 \) and \( f_3 \). The reason for this could be that there are some sharp regions in the two functions. Regarding the best function optima obtained by the BAS algorithm, we can see that the differences between the obtained ones and the theoretical ones are about \( 10^{-2} \) for most functions. This is related to the setting of step size. Normally, if we want to have a more accurate optimum, we need to have a smaller step size, which generally will lead to larger consumption of computational resources. In other words, there is a trade-off between accuracy and efficiency. Here, our evaluation criterion is the successful rate, which serves as a trade-off criterion. To sum up, the BAS algorithm has a good performance for finding global optima of functions, regardless of whether they are multimodal or not.

5. Applications

In this section, we show the application of the BAS algorithm to some engineering problems.
| function | parameter setting | successful rate | best $f_{bst}$ | average $f_{bst}$ | standard deviation of $f_{bst}$ |
|-----------|-------------------|-----------------|---------------|------------------|-------------------------------|
| $f_1(x)$  | $\alpha = 0.94, d_0 = 0.001, c = 0.94$ | 100             | 0.0271        | 0.0311           | 0.0016                        |
| $f_2(x)$  | $\alpha = 0.95, d_0 = 0.001, c = 0.94$ | 96              | 0.0708        | 2.8432           | 15.0748                       |
| $f_3(x)$  | $\alpha = 0.7, d_0 = 0.001, c = 0.7$  | 82              | $5.3561e-04$  | 0.5659           | 1.4076                        |
| $f_4(x)$  | $\alpha = 0.97, d_0 = 0.01, c = 0.97$ | 80              | 0.0122        | 0.4377           | 0.8607                        |
| $f_5(x)$  | $\alpha = 0.94, d_0 = 0.001, c = 0.94$ | 100             | 0.9995        | 0.9995           | $8.0206e-09$                 |
| $f_6(x)$  | $\alpha = 0.96, d_0 = 0.1, c = 0.96$  | 100             | 0.0032        | 0.0085           | 0.0019                        |
| $f_7(x)$  | $\alpha = 0.8, d_0 = 0.01, c = 0.8$   | 100             | $3.3053e-04$  | $6.3522e-04$     | $1.1920e-04$                 |
Table 3: Best result of the BAS algorithm among the 1000 runs for solving the spring design problem and the best result obtained by the Bat algorithm in [16].

| Algorithm       | $f(x)$  | $W$     | $D$     | $L$     | $g_1(x)$ | $g_2(x)$ | $g_3(x)$ | $g_4(x)$ |
|-----------------|---------|---------|---------|---------|----------|----------|----------|----------|
| BAS algorithm   | 0.010894| 0.050000| 0.360419| 10.090624| -0.052996| -4.357457| -0.726387| -0.035687|
| Bat algorithm   | 0.012665| 0.051690| 0.356750| 11.287126| /         | /        | /        | /        |
5.1. Spring design problem

The optimal design problem of a tensional and compressional spring is described as follows [16, 17]:

\[
\begin{align*}
\min f(x) &= (L + 2)W^2D, \\
\text{subject to} & \quad g_1(x) = 1 - \frac{D^3L}{71785W^4} \leq 0, \\
& \quad g_2(x) = 1 - \frac{1404.5W}{D^2L} \leq 0, \\
& \quad g_3(x) = \frac{2(W + D)}{3} - 1 \leq 0, \\
& \quad g_4(x) = \frac{D(4D - W)}{W^3(12566D - W)} + \frac{1}{5108W^2} - 1 \leq 0,
\end{align*}
\]

where \(f(x)\) is the weight of the spring which needs to be minimized, \(W\) denotes the wire diameter, \(D\) denotes the mean coil diameter, and \(L\) denotes the length or the number of coils. The contraints are related to the maximum shear stress, minimum deflection, etc. The details can be found in [17]. We first convert the problem to a form that can be addressed by the BAS algorithm by using the penalty method:

\[
\begin{align*}
\min f(x) &= (L + 2)W^2D + \rho h_i(x), \\
\text{subject to} & \quad 0.05 \leq W \leq 2.0, \\
& \quad 0.25 \leq D \leq 1.3, \\
& \quad 2.0 \leq L \leq 15.0,
\end{align*}
\]

where

\[h_i(x) = \max(0, g_i(x)), \quad (8)\]

and \(\rho\) is called the penalty parameter. In the numerical experiment, we set \(\rho = 10^5\), and the parameters of the BAS algorithm is set as \(\alpha = 0.8, d_0 = 0.01,\) and \(c = 0.8\). We run the BAS algorithm for 1000 times and in each run the initial values of \(x\) are set by following the rules in the previous section with \(K_{\text{max}} = 1000\). The best result among the 1000 runs are shown in Table 3, where all the constraints are satisfied. Obviously, the result is better than
the best result attained by the Bat algorithm discussed in [16], for which the optimum is 0.012665.
Table 4: Best result of the BAS algorithm among the 100 runs for solving the speed reducer problem and existing results

|       | BAS algorithm | bat algorithm [16] | deterministic technique [18] |
|-------|---------------|--------------------|-----------------------------|
| $B$   | 3.501597128660806 | 3.5                | 3.5                         |
| $H$   | 0.7           | 0.7                | 0.7                         |
| $Z$   | 17            | 17                 | 17                          |
| $L_1$ | 8.10455092323999 | 7.3                | 7.3                         |
| $L_2$ | 8.021701619497760 | 7.8                | 7.7153190                   |
| $D_1$ | 3.353618456239036 | 3.3436445         | 3.350282                    |
| $D_2$ | 5.291060245756827 | 5.285350625       | 5.286654                    |
| $g_1(x)$ | -0.074337680917883 | -0.073915280397873 | -0.073915280397873 |
| $g_2(x)$ | -0.198364331513853 | -0.19798527141949  | -0.19798527141949  |
| $g_3(x)$ | -0.317436155693268 | -0.49505034120807  | -0.499212509394955  |
| $g_4(x)$ | -0.89318330622976  | -0.90137291570736  | -0.904643904804176   |
| $g_5(x)$ | -0.001627492443412 | 0.00615929578992   | -6.02927303829612e-05  |
| $g_6(x)$ | -0.002436220272286 | 7.56586008787696e-04 | 2.636860652049933e-07 |
| $g_7(x)$ | -0.702500000000000 | -0.702500000000000 | -0.702500000000000  |
| $g_8(x)$ | -4.561143392921574e-04 | 0               | 0                          |
| $g_9(x)$ | -0.583143198968952 | -0.583333333333333 | -0.583333333333333  |
| $g_{10}(x)$ | -0.144872530890374 | -0.05273332191781 | -0.051311917808219  |
| $g_{11}(x)$ | -0.037589948301284 | -0.011040296474359 | 5.184490747822679e-08 |
| $f(x)$ | 3.012610927770214e+03 | 2.993758748042880e+03 | 2.994487910428388e+03|

Note: * means that the constraint is violated.
|       | BAS algorithm | Cricket algorithm | [19] | [20] | [17] | [21] |
|-------|---------------|-------------------|------|------|------|------|
| $x_1$ | 0.788511192166172 | 0.788633           | 0.79500 | 0.78867 | 0.78863 |
| $x_2$ | 0.408717503699073 | 0.408368           | 0.39500 | 0.40902 | 0.40838 |
| $g_1(x)$ | -4.026245777222215e-06 | -3.954291896146600e-07 | -0.00169 | -0.00029 | -3.057141794382545e-06 |
| $g_2(x)$ | -1.463570340396164 | -1.463965733302426 | -0.26124 | -0.26853 | -1.463953424351428 |
| $g_3(x)$ | -0.536433685849614 | -0.536034662126764 | -0.74045 | -0.73176 | -0.536049632790367 |
| $f(x)$  | 263.8963947787828 | 263.895896869962 | 264.3000 | 263.9716 | 263.896248338589 |
5.2. Speed reducer problem

The speed reducer design optimization problem is described as follows [16]:

\[
\begin{align*}
\text{min } f(x) &= 0.7854BH^2(3.3333Z^2 + 14.9334Z - 43.0934) \\
&\quad - 1.508B(D_1^2 + D_2^2) + 7.4777(D_1^2 + D_2^2) \\
&\quad + 0.7854(L_1 D_1^2 + L_2 D_2^2), \\
\text{subject to } &
g_1(x) = \frac{27}{BH^2Z} - 1 \leq 0, \\
&\quad g_2(x) = \frac{397.5}{BH^2Z^2} - 1 \leq 0, \\
&\quad g_3(x) = \frac{1.93L_3^3}{HZD_1^3} - 1 \leq 0, \\
&\quad g_4(x) = \frac{1.93L_3^3}{HZD_2^3} - 1 \leq 0, \\
&\quad g_5(x) = \frac{1}{110D_1^3} \sqrt{\left(\frac{745L_1}{HZ}\right)^2 + 16.9 \times 10^6} - 1 \leq 0, \\
&\quad g_6(x) = \frac{1}{85D_2^3} \sqrt{\left(\frac{745L_2}{HZ}\right)^2 + 157.5 \times 10^6} - 1 \leq 0, \\
&\quad g_7(x) = \frac{HZ}{40} - 1 \leq 0, \\
&\quad g_8(x) = \frac{5H}{B} - 1 \leq 0, \\
&\quad g_9(x) = \frac{B}{12H} - 1 \leq 0, \\
&\quad g_{10}(x) = \frac{1.5D_1 + 1.9}{L_1} - 1 \leq 0, \\
&\quad g_{11}(x) = \frac{1.1D_2 + 1.9}{L_2} - 1 \leq 0,
\end{align*}
\]

\[2.6 \leq B \leq 3.6,\]
\[0.7 \leq H \leq 0.8,\]
\[17 \leq Z \leq 28,\]
\[7.3 \leq L_1 \leq 8.3,\]
\[7.8 \leq L_2 \leq 8.3,\]
\[2.9 \leq D_1 \leq 3.9,\]
\[5.0 \leq D_2 \leq 5.5,\]
where $B$ denotes the face width, $H$ denotes the module of the teeth, $Z$ denotes the number of teeth on pinion, $L_1$ denotes the length of the first shaft between bearings, $L_2$ denotes the length of the second shaft between bearings, $D_1$ denotes the diameter of the first shaft, and $D_2$ denotes the diameter of the second shaft.

We adopt the same approach as in the previous subsection to convert the problem into a form that can be addressed by the BAS algorithm. With $\rho = 10^6$, $\alpha = 0.8$, $d_0 = 0.001$, and $c = 0.8$, and the other settings being the same as in the previous subsection, the best result obtained by the BAS algorithm among 100 runs with $K_{\text{max}} = 10,000$ is shown in Table 4. As seen from Table 4, the solution given by the BAS algorithm can guarantee the compliance with all the constraints with an optimal function value being $3.012610927770214e+03$. Although the other two algorithms can generate better function values, some constraints are violated, which means that the solutions are not feasible. From this point of view, the BAS algorithm is better than the other two for solving this problem.

### 5.3. Three bar truss problem

The three bar truss problem considered in this paper is described as follows [19]:

$$
\begin{align*}
\min f(x) &= 100(2\sqrt{2}x_1 + x_2), \\
\text{subject to} & \quad g_1(x) = 2 - \frac{x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \leq 0, \\
& \quad g_2(x) = 2 - \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} - 2 \leq 0, \\
& \quad g_3(x) = 2 - \frac{1}{x_1 + \sqrt{2}x_2} - 2 \leq 0,
\end{align*}
$$

where $0 < x_1 < 1$ and $0 < x_2 < 1$. We employ the BAS algorithm to solve the problem with $\alpha = 0.8$, $d_0 = 0.01$, and $c = 0.8$. The comparison of the obtained best result with existing ones is shown in Table 5. As seen from the table, the best result obtained by the BAS algorithm is very close to the those obtained by the state-of-the-art, and all the constraints are satisfied.

### 6. Conclusions

In this paper, theoretical guarantee for the BAS algorithm has been provided via the concept of convergence with probability 1. We have also pro-
vided a quantitative analysis on the performance of the BAS algorithm for finding global optima of seven representative test functions based the measure called successful rate. The BAS algorithm has been applied to solve three problems arising from engineering applications, and the results have shown that the BAS algorithm has a good performance.

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