Shot noise of spin current and spin transfer torque

Yunjin Yu\textsuperscript{1}, Hongxin Zhan\textsuperscript{1}, Langhui Wan\textsuperscript{1}, Bin Wang\textsuperscript{1,3}, Yadong Wei\textsuperscript{1}, Qingfeng Sun\textsuperscript{2} and Jian Wang\textsuperscript{3}

\textsuperscript{1} College of Physics Science and Technology and Institute of Computational Condensed Matter Physics, Shenzhen University, Shenzhen 518060, People's Republic of China
\textsuperscript{2} Institute of Physics, Chinese Academy of Sciences, Beijing, People's Republic of China
\textsuperscript{3} Department of Physics and The Center of Theoretical and Computational Physics, The University of Hong Kong, People's Republic of China

E-mail: ywei@szu.edu.cn

Received 21 December 2012, in final form 8 February 2013
Published 22 March 2013
Online at stacks.iop.org/Nano/24/155202

Abstract

We report the theoretical investigation of the shot noise of the spin current \( S_\sigma \) and the spin transfer torque \( S_\tau \) for non-collinear spin polarized transport in a spin-valve device which consists of a normal scattering region connected by two ferromagnetic electrodes (MNM system). Our theory was developed using the non-equilibrium Green's function method, and general nonlinear \( S_\sigma - V \) and \( S_\tau - V \) relations were derived as a function of the angle \( \theta \) between the magnetizations of two leads. We have applied our theory to a quantum dot system with a resonant level coupled with two ferromagnetic electrodes. It was found that, for the MNM system, the auto-correlation of the spin current is enough to characterize the fluctuation of the spin current. For a system with three ferromagnetic layers, however, both auto-correlation and cross-correlation of the spin current are needed to characterize the noise of the spin current. For a quantum dot with a resonant level, the derivative of spin torque with respect to bias voltage is proportional to \( \sin \theta \) when the system is far away from resonance. When the system is near resonance, the spin transfer torque becomes a non-sinusoidal function of \( \theta \). The derivative of the noise of the spin transfer torque with respect to the bias voltage \( N_\tau \) behaves differently when the system is near or far away from resonance. Specifically, the differential shot noise of the spin transfer torque is a concave function of \( \theta \) near resonance while it becomes a convex function of \( \theta \) far away from resonance. For certain bias voltages, the period \( N_\tau (\theta) \) becomes \( \pi \) instead of \( 2\pi \). For small \( \theta \), it was found that the differential shot noise of the spin transfer torque is very sensitive to the bias voltage and the other system parameters.

(Some figures may appear in colour only in the online journal)

1. Introduction

Electronic shot noise describes the fluctuation of current and is an intrinsic property of quantum devices due to the quantization of electron charge. In the past decade, the study of shot noise has attracted increasing attention [1] because it can give additional information that is not contained in the conductance or charge current. It can be used to probe the kinetics of electrons [2] and investigate correlations of electronic wavefunctions [3]. In the study of shot noise \( \langle (\Delta \hat{I})^2 \rangle \) with \( \Delta \hat{I} = \hat{I} - \langle \hat{I} \rangle \), the Fano factor \( F = \langle (\Delta \hat{I})^2 \rangle / 2 q \langle \hat{I} \rangle \) is often used, where \( \langle \hat{I} \rangle \) is the current. When \( F > 1 \) it is referred as super-Poissonian noise, while \( F < 1 \) corresponds to sub-Poissonian behavior. In general, for a quantum device, Pauli exclusion suppresses the shot noise and hence reduces the Fano factor [4–6], but Coulomb interaction can either suppress or enhance shot noise depending on the system details [7–11]. The suppression of shot noise has been
confirmed experimentally in quantum point contacts [12, 13], the single electron tunneling regime [14, 15], graphene nano-ribbons [16, 17], and atom-size metallic contacts [18, 19]. The enhancement of shot noise was also observed in GaAs based quantum contacts when the system is in the negative differential conductance region [20]. Recently, with the development of spintronics, polarized spin current, especially pure spin current, has received much more attention. Though the spin current correlation for double quantum dots in the spin blockade regime was investigated by Sanchez et al. [21], however, less attention has been paid to the polarized spin current correlation compared with the charge current correlation [22–27]. One may also define the spin resolved Fano factor as \( F^s = \langle (\Delta I_n^s)^2 \rangle / \langle I_n^s \rangle^2 \), since the spin transport is also a stochastic process and full counting statistics is necessary to characterize the spin transport. The shot noise of polarized spin current has been studied in several quantum devices including the MNM (ferromagnet–normal–ferromagnet) [28] and NMN (normal–magnetic–normal) [29]. In these devices, shot noise is provided to additional information about the spin-dependent scattering process and spin accumulation. It was shown that shot noise can be used to probe attractive or repulsive interactions in mesoscopic systems and to measure the spin relaxation time [30]. Furthermore, the spin transfer torque (STT) and the torque noise were studied for the MNM system. For a two-probe normal system (NNN system) it is well known that the charge current correlation between different probes (cross-correlation noise) is negative definitely [31], but for a magnetic junction the spin cross-correlation noise between different probes is not necessarily negative due to the spin flip mechanism. For example, [32] showed that the cross-correlation can be positive at special Fermi energy due to Rashba interaction.

Recently, STT, predicted by Slonczewski [33, 34] and Berger [35], has been the subject of intensive investigations [36–39]. Spin current can transfer spin angular momentum and be used to switch the magnetic orientation of the magnetic moment and be used to switch the magnetic orientation of ferromagnetic layers in GMR and TMR devices. Therefore, STT has potential applications [40] such as hard-disk read heads [41], magnetic detection sensors [42], magnetic random access memory (MRAM) [43], etc. It comes from the absorption of the itinerant flow of angular momentum components normal to the magnetization direction and relies on the system spin polarized current. The noise of the STT drastically affects the magneto-resistance behavior [44]. Many studies have focused on the STT in various materials and under dc or ac condition. The correlation effect or quantum noise of STT has not been studied so far. It is the purpose of this paper to fill this gap. In this paper, we have calculated the shot noise of particle current and spin current as well as STT in the nonlinear regime for a magnetic quantum dot connected with two non-collinear magnetic electrodes. We found that, for a MNM spin-valve system, the spin auto-correlation is enough to characterize the fluctuation of spin current. For a system with three ferromagnetic layers (MNMMN), however, both auto-correlation and cross-correlation are needed to characterize the fluctuation of spin current. For the quantum dot with a resonant level, the behavior of differential STT depends on whether the system is in resonance or off resonance. When the system is off resonance, the differential STT reduces to the familiar result of the tunneling barrier \( \frac{1}{2} (P(\pi) - P(0)) \sin \theta \), where \( \theta \) is the angle between magnetic moments of ferromagnetic leads. If it is on resonance, the dependence of differential STT on \( \theta \) becomes non-sinusoidal. The resonance also has influence on the noise of the STT. If the system is near resonance the noise of the STT is a concave function of \( \theta \), while it becomes a convex function far away from resonance.

This paper is organized as follows. First, we derive the general formulas of spin auto-correlation shot noise, spin cross-correlation shot noise and STT shot noise from the non-equilibrium Green’s function method. Then we analyze the spin transport properties for the MNM system. Finally, we give the conclusions.

2. Theory formalism

We start from the Hamiltonian of the quantum dot which is connected by two magnetic leads. We assume that the current flows in the \( y' \) direction and the lead magnetic moment \( \vec{M}_l \) always points in the \( z \) direction, while the right lead magnetic moment \( \vec{M}_R \) points at an angle \( \theta_R \) to the \( z \) direction in the \( x-z \) plane (see figure 1).

In the second quantized form, the Hamiltonian is

\[
\hat{H} = \hat{H}_{\text{lead}} + \hat{H}_{\text{dot}} + \hat{H}_T,
\]

where \( \hat{H}_{\text{lead}} \) is the Hamiltonian of the leads,

\[
\hat{H}_{\text{lead}} = \sum_{k,\sigma} (\epsilon_{k,\sigma} - \sigma M_0 \cos \theta_0) \hat{c}_{k,\sigma}^{\dag} \hat{c}_{k,\sigma} - \sum_{k,\sigma} M_0 \sin \theta_0 \hat{C}_{k,\sigma}^{\dag} \hat{C}_{k,\sigma},
\]

and \( \hat{C}_{k,\sigma}^{\dag} \) creates an electron in lead \( \sigma \) with energy level \( k \) and spin \( \sigma, \sigma = \pm 1 \) and \( \bar{\sigma} = -\sigma \). The second term \( \hat{H}_{\text{dot}} \) is the Hamiltonian of the isolated quantum dot,

\[
\hat{H}_{\text{dot}} = \sum_{n} \epsilon_{n} \hat{d}_{n}^{\dag} \hat{d}_{n}.
\]

The third term \( \hat{H}_T \) is the Hamiltonian describing the coupling between the quantum dot and the leads with the coupling
The current operator of the lead \( \alpha \) can be written as [46]

\[
\Gamma_\alpha = R_\alpha \begin{pmatrix} I_{\alpha\uparrow} & 0 \\ 0 & I_{\alpha\downarrow} \end{pmatrix} R_\alpha^T,
\]

where

\[
R_\alpha = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}
\]

is the rotational matrix.

The current operator of the lead \( \alpha \) with spin \( \sigma \) is defined as

\[
\hat{I}_{\alpha\sigma}(t) = -i\frac{q}{\hbar} \sum_{k_m} [t_{k_m\alpha\sigma} \hat{C}_{k_m\sigma}^\dagger \hat{d}_{m\sigma}(t)] + h.c.
\]

The average current can be expressed in terms of Green’s function,

\[
\langle \hat{I}_{\alpha\sigma}(t) \rangle = -\frac{q}{\hbar} \sum_{k_m} [t_{k_m\alpha\sigma} G_{k_m\sigma}^{<}(t,t) + h.c.].
\]

If we consider the total charge current flowing through the lead \( \alpha \), the charge current operator can be expressed as

\[
\hat{I}_{\alpha} = \hat{I}_{\alpha\uparrow} + \hat{I}_{\alpha\downarrow}
\]

and the spin current operator in the \( z \) direction is

\[
\hat{I}^z_{\alpha\sigma} = \frac{\hbar}{2\beta} (\hat{I}_{\alpha\uparrow} - \hat{I}_{\alpha\downarrow}).
\]

Since the local spin current is not conserved, the loss of the spin angular momentum is transferred to the magnetization of the free layer. For STT, we are interested in the MNM system and we assume that the electron is coming from the left lead which is pinned and the right lead is the free layer. The STT can be calculated as follows. The total spin of the right ferromagnetic electrode is [47]

\[
\hat{S}^\alpha = \frac{\hbar}{2} \sum_{k_{R\mu}\sigma} C_{k_{R\mu}}^\dagger C_{k_{R\mu}} (R^{-1} \chi_{\mu})^\dagger \hat{\sigma} (R^{-1} \chi_{\nu}).
\]

Here, \( \hat{\sigma} \) is the Pauli matrix and the spin up state \( \chi_{\nu}(v) = (\hat{i}) \) for \( \mu(v) = 1 \) or the spin down state \( \chi_{\mu}(v) = (\hat{i}) \) for \( \mu(v) = -1 \). Note that the equation above is written in the \( xyz \) coordinate frame while \( \hat{S}^\alpha \) are quantized in the \( x’y’z’ \) frame. Because \( \hat{S}^\alpha(t) \) is along the direction of \( \hat{z} \), the total spin torque \( \hat{t} = \frac{\partial \hat{S}^\alpha}{\partial t} = \frac{i}{\hbar} [\hat{H}_T, \hat{S}^\alpha] \) should be along the direction of \( \hat{x} \) (see figure 1). So we need the expression of the spin operator of the right lead along the \( x \) direction, \( \hat{S}_x^\alpha \), which can be obtained from equation (14),

\[
\hat{S}_x^\alpha = \frac{\hbar}{2} \sum_{k_{R\sigma}} (\hat{C}_{k_{R\sigma}}^\dagger \hat{C}_{k_{R\sigma}} \cos \theta - \sigma \hat{C}_{k_{R\sigma}}^\dagger \hat{C}_{k_{R\sigma}} \sin \theta).
\]

According to the Heisenberg equation of motion, the spin transfer torque operator is

\[
\hat{t}_R = \hat{t}_x = \frac{i}{\hbar} [\hat{H}_T, \hat{S}_x^\alpha] = -\frac{i}{\hbar} \left[ \sum_{l_{R\sigma}} (\hat{C}_{l_{R\sigma}}^\dagger \hat{R}_{\sigma\sigma} \hat{d}_{\sigma\sigma} l_{R\sigma\sigma} - \hat{l}_{R\sigma\sigma}^\dagger \hat{R}_{\sigma\sigma} \hat{C}_{l_{R\sigma}}) \right]
\]

where

\[
\mathcal{R} = \begin{pmatrix} \hat{R}_{\uparrow\uparrow} & \hat{R}_{\uparrow\downarrow} \\ \hat{R}_{\downarrow\uparrow} & \hat{R}_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\sin \theta R & \cos \theta R \\ \cos \theta R & \sin \theta R \end{pmatrix},
\]

\[
t_{R\sigma\sigma} = \begin{pmatrix} t_{R\sigma\sigma}^\uparrow & 0 \\ 0 & t_{R\sigma\sigma}^{\downarrow} \end{pmatrix}.
\]

The average STT is [47]

\[
\langle \hat{t}_R \rangle = \text{Re} \left\{ \sum_{k_{R\sigma}} \text{Tr}_{\sigma} [t_{k_{R\sigma}}^\dagger \mathcal{R} G_{k_{R\sigma}}^{<}] \right\} = \frac{\hbar}{2\pi} \int \frac{dE}{f_\nu - f_R} \text{Tr}[G^{\dagger} \Gamma L G^{\dagger} (i\Sigma^a_R \mathcal{R} - i\mathcal{R} \Sigma^a_R)],
\]

where \( \text{Tr}_{\sigma} \) is over spin space.

The correlation of the charge current is given by

\[
\langle \Delta \hat{I}_{\alpha\sigma}(t_1) \Delta \hat{I}_{\beta\sigma}(t_2) \rangle = \sum_{\sigma\sigma'} \langle \Delta \hat{I}_{\alpha\sigma}(t_1) \Delta \hat{I}_{\beta\sigma'}(t_2) \rangle
\]

and the shot noise of the spin current is

\[
\langle \Delta \hat{I}_{z\sigma}^2(t_1) \Delta \hat{I}_{z\sigma}^2(t_2) \rangle = \frac{\hbar}{4} \sum_{\sigma\sigma'} \sigma\sigma' \langle \Delta \hat{I}_{\alpha\sigma}(t_1) \Delta \hat{I}_{\beta\sigma'}(t_2) \rangle,
\]

where

\[
\Delta \hat{I}_{z\sigma} = \hat{I}_{z\sigma} - \langle \hat{I}_{z\sigma} \rangle
\]
where $\Delta \hat{\tau}_k = \hat{r}_k - \langle \hat{r}_k \rangle$.

We now derive the correlation of charge current, spin current, and STT. Clearly, all correlation functions contain the term

$$\langle \hat{I}_{\alpha \sigma}(t) \hat{I}_{\beta \sigma^*}(t) \rangle = -\frac{q^2}{\hbar} \sum_{k_{\alpha \beta}n_{mn}} [t_{k_{\alpha \beta} \sigma \sigma^*} \langle \hat{C}_{k_{\alpha \sigma}}(t) \hat{C}_{k_{\beta \sigma^*}}(t) \rangle \delta_{\alpha \beta} + t_{k_{\alpha \beta} \sigma \sigma^*} \langle \hat{C}_{k_{\beta \sigma^*}}(t) \hat{C}_{k_{\alpha \sigma}}(t) \rangle \delta_{\alpha \beta}].$$

Using Wick’s theorem [48], we have

$$\langle \hat{C}_{k_{\alpha \sigma}}(t) \hat{C}_{k_{\beta \sigma^*}}(t) \rangle = \langle \hat{C}_{k_{\alpha \sigma}}(t) \hat{C}_{k_{\beta \sigma^*}}(t) \rangle \delta_{\alpha \beta} + \langle \hat{C}_{k_{\alpha \sigma}}(t) \hat{C}_{k_{\beta \sigma^*}}(t) \rangle \delta_{\alpha \beta}.$$  

The shot noise can be expressed in terms of Green’s function

$$\langle \Delta \hat{I}_{\alpha \sigma}(t) \Delta \hat{I}_{\beta \sigma^*}(t) \rangle = -\frac{q^2}{\hbar} \sum_{k_{\alpha \beta}n_{mn}} [t_{k_{\alpha \beta} \sigma \sigma^*} \langle \hat{G}_{n_{m\sigma \sigma^*}} G_{m_{n\sigma \sigma^*}} + t_{k_{\alpha \beta} \sigma \sigma^*} \langle \hat{G}_{n_{m\sigma \sigma^*}} G_{m_{n\sigma \sigma^*}} \rangle \delta_{\alpha \beta} + t_{k_{\alpha \beta} \sigma \sigma^*} \langle \hat{G}_{n_{m\sigma \sigma^*}} G_{m_{n\sigma \sigma^*}} \rangle \delta_{\alpha \beta}].$$

From the Langreth theorem of analytic continuation, we have

$$G_{n_{m\sigma \sigma^*}}(t_1, t_2) = \frac{\delta(t_2 - t_1)}{2\pi} \int dt \{ G_{n_{m\sigma \sigma^*}}(t_1, t) G_{n_{m\sigma \sigma^*}}(t_1, t_2) \}.$$  

and

$$G_{k_{\alpha \sigma}k_{\beta \sigma^*}}(t_1, t_2) = \frac{\delta(t_2 - t_1)}{2\pi} \int dt \{ G_{k_{\alpha \sigma}k_{\beta \sigma^*}}(t_1, t) G_{k_{\alpha \sigma}k_{\beta \sigma^*}}(t_1, t_2) \}.$$
For spin current noise, the situation is very different. We combine equation (20) with (31), taking Fourier transformation, and using the relation

$$\sum_{\sigma'\sigma} \sigma' A_{\sigma'\sigma} B_{\sigma'\sigma} = \text{Tr}[A_{\sigma} B_{\sigma}]$$  

(40)

$\sigma$ is the Pauli matrix. We can obtain the autospin current shot noise (zero-temperature limit),

$$S'^{\dagger}_{LL} = \frac{\hbar^2}{2\pi} \left\{ \text{Tr}[\sigma_z T \sigma_z(1 - T)] \right\}$$  

(41)

Similarly, the cross spin current shot noise can be obtained:

$$S'^{\dagger}_{LR} = -\frac{\hbar^2}{4\pi} \left\{ \text{Tr}[(G^T G^a)(\sum_{\sigma}^z \sigma_z - \sigma_z \sum_{\sigma}^z) + G^T \Gamma_R \sigma_z]][G^T \Gamma_L \sigma_z + G^T \Gamma_L G^a(\sum_{\sigma}^z \sigma_z - \sigma_z \sum_{\sigma}^z)] \right\}$$  

(42)

Now, we derive the shot noise of the STT $S(t_1, t_2)$,

$$S(t_1, t_2) = -\frac{1}{2} \left\{ \sum_{k_b} \sum_{\sigma_a \sigma_a'} \left( \hat{C}^d_{k_b \sigma_a} \sigma_a \sigma_a' \hat{d}^a_{k_b \sigma_a} \sigma_a' \right) - \hat{C}^d_{k_b \sigma_a} \sigma_a \sigma_a' \hat{d}^a_{k_b \sigma_a} \sigma_a' \right\}$$  

and

$$\langle \hat{r}_R(t) \rangle = \frac{-\hbar^2}{4\pi} \left\{ \sum_{k_b} \sum_{\sigma_a \sigma_a'} \left( \hat{C}^d_{k_b \sigma_a} \sigma_a \sigma_a' \hat{d}^a_{k_b \sigma_a} \sigma_a' \right) - \hat{C}^d_{k_b \sigma_a} \sigma_a \sigma_a' \hat{d}^a_{k_b \sigma_a} \sigma_a' \right\}$$  

(43)

(44)

Similarly, we define the shot noise of the STT like the shot noise of spin current. It can be written as

$$\pi \delta(0) S^x = \int dt_1 dt_2 S(t_1, t_2),$$  

(45)

where $S(t_1, t_2)$ is defined in equation (22).

By using Wick’s theorem, and after Fourier transformation, we can obtain the expression of $S'$,

$$\frac{\hbar^2}{4\pi} \left\{ \text{Tr}[G^T \Gamma_L G^a \Gamma_R R + G^T \Gamma_L G^a(\sum_R^z \sigma_z) \Gamma_L G^a(\sum_R^z \sigma_z) - \sigma_z \sum_R^z) + G^T (i \partial \sum_R^z - i \sum_R^z \sigma_z + \Gamma_R R)] G^a \Gamma_L G^a (\sum_R^z - \sigma_z \sum_R^z) + (\Gamma_R R - i \sum_R^z \Gamma_L G^a (\sum_R^z \sigma_z) + (\sum_R^z \sigma_z) - (\sum_R^z \sigma_z) G^a (\sum_R^z \sigma_z) \right\}.$$  

(46)

3. Shot noise of spin current and spin current for MNM system

In this paper, we consider a normal quantum dot connected by two ferromagnetic leads (see figure 1) (MNM system). The magnetic moment of the left lead is pointing in the $z$ direction, while the moment of the right lead is at an angle $\theta$ to the $z$ axis in the $x$-$z$ plane. Hence the Hamiltonian of quantum dot can be written as

$$H_{dot} = \left( \begin{array}{cc} \epsilon_0 & 0 \\ 0 & -\epsilon_0 \end{array} \right).$$  

(47)

First, we set the direction of magnetization of the right lead to be along the $z$ direction, i.e. $\theta_0 = 0$, and calculate the charge and spin current according to the Landauer-Büttiker formula

$$I_e = -\frac{q}{\hbar} \int \frac{dE}{2\pi} \text{Tr}[\tilde{F}(E)](f_L - f_R),$$  

(48)

and the expression of the spin current is

$$I_s = \frac{1}{2} \int \frac{dE}{2\pi} \text{Tr}[\sigma_z \tilde{F}(E)](f_L - f_R).$$  

(49)

The charge and spin transmission coefficients are depicted in figure 2. In the calculation, we have chosen $\Gamma_{L\uparrow} = \Gamma_{R\downarrow} = 0.8$ eV and fix the energy unit to be eV. Letting $\Gamma_{L\downarrow} = 0.2$ eV (here, we let $\Gamma_{L\uparrow} \neq \Gamma_{R\downarrow}$ due to the presence of ferromagnetic leads), we found that the total charge transmission coefficient $(T_T + T_T')$ reaches two at the resonant energy level $E = \epsilon_0$ of the quantum dot (solid line in the left panel of figure 2), while the spin transmission coefficient $(T_T - T_T')$ is zero at the resonant energy point (solid line in the right panel of figure 2). For the parallel situation ($\theta_0 = 0$) there is no spin flip so different spin channels can be treated separately. For a symmetric coupling from the lead, both spin up and spin down electrons have complete transmission at the resonance. For total charge current they add together while for total spin current they cancel each other. When we break this symmetry and change $\Gamma_{L\downarrow}$ while keeping $\Gamma_{L\uparrow}$ constant, the spin down transport will be partially blocked, so the total charge transmission coefficient will decrease and the spin transmission coefficient will increase (see the solid lines in dotted lines in figure 2). Figure 3 gives a comparison between the charge current and spin current versus $\theta_0$ under the small bias voltage 0.05 V. From the figure, we find that, for the symmetric coupling with $\Gamma_{L\uparrow} = \Gamma_{R\downarrow}$ and $\Gamma_{L\downarrow} = \Gamma_{R\uparrow}$, both charge current and spin current decrease as $\theta_0$ increases from zero to $\pi$ (see the solid lines in figures 3(a) and (b)). But if we fix $\Gamma_{L\downarrow}$ and change $\Gamma_{R\downarrow}$, although the charge current still decreases when $\theta_0$ changes from zero to $\pi$, the spin current increases when $\Gamma_{R\downarrow} > \Gamma_{L\downarrow}$, decreases when $\Gamma_{R\downarrow} < \Gamma_{L\downarrow}$ and changes sign at $\theta_0 = \pi$. To understand the behavior, we plot the spin up current in panel (c) and spin down current in panel (d). One can clearly see that spin up current always decreases with $\theta_0$ from zero to $\pi$, but spin down current always increases though it is negative. So the competition between spin up and down channels determines how the total spin current varies with $\Gamma_{R\downarrow}$. Another interesting result is
Figure 2. (a) The total charge transmission coefficient \((T^\uparrow + T^\downarrow)\) and (b) the spin transmission coefficient \((T^\uparrow - T^\downarrow)\) versus Fermi energy when \(\Gamma_R = 0.1\) (red dashed line), \(\Gamma_R = 0.2\) (black solid line), \(\Gamma_R = 0.4\) (blue dotted line). The other parameters are \(\theta_R = 0\), \(\epsilon_0 = 0\), \(\Gamma_{L\downarrow} = 0.2\), \(\Gamma_{L\uparrow} = \Gamma_{R\uparrow} = 0.8\). The energy unit is eV.

Figure 3. The charge current (a), total spin current (b), spin up current (c) and spin down current (d) versus \(\theta_R\) for the MNM system at \(\Gamma_R = 0.1\) (red dashed line), \(\Gamma_R = 0.2\) (black solid line) and \(\Gamma_R = 0.4\) (blue dotted line). The other parameters are \(V_{\text{bias}} = 0.05\) V, \(\epsilon_0 = 0\), \(\Gamma_{L\uparrow} = \Gamma_{R\uparrow} = 0.8\), \(\Gamma_{L\downarrow} = 0.2\).

that at \(\theta_R = \pi\), i.e., when the magnetic moments of the two leads are antiparallel, the spin down current does not change when we change \(\Gamma_{R\downarrow}\) (see figure 3(d)) while keeping the other parameters the same. In fact, when we change \(\Gamma_{R\downarrow}\) at \(\theta_R = \pi\), we actually change the right coupling line-width constant of spin up but not spin down because \(\Gamma_R(\pi) = R_R(\pi)\left(\begin{array}{cc} 0 & \Gamma_{R\downarrow} \\ \Gamma_{R\uparrow} & 0 \end{array}\right) R_R(\pi) = \left(\begin{array}{cc} 0 & \Gamma_{R\downarrow} \\ \Gamma_{R\uparrow} & 0 \end{array}\right)\). So we can find that at \(\theta_R = \pi\) the spin up current is different with different \(\Gamma_{R\downarrow}\), but spin down stays unchanged.

To study the shot noise of the spin current, we first examine the differential shot noise versus bias voltage \(V_L = V_{\text{bias}}\) and \(V_R = 0\). At zero temperature, they can be calculated from equations (41) and (42) (AC means auto-correlation and CC means cross-correlation).

\[
N_{\text{AC}} = \frac{4\pi}{g\hbar^2} \frac{\partial S_{\text{LL}}}{\partial V} = \text{Tr}[[\sigma_z T(E)\sigma_z (1 - T(E))]|E = qV_{\text{bias}}\rangle (50)
\]

and

\[
N_{\text{CC}} = \frac{4\pi}{g\hbar^2} \frac{\partial S_{\text{LR}}}{\partial V} = -\text{Tr}[[G^2 \Gamma_R G^a (\Sigma_R^a - \sigma_z \Sigma_R^a) \\
+ G^a \Gamma_R \sigma_z (G^2 \Gamma_L G^a + G^a \Gamma_L G^a) \\
\times (\Sigma_L^a \sigma_z - \sigma_z \Sigma_L^a)]|E = qV_{\text{bias}}\rangle (51)
\]
For equation (51), we see that if the directions of the magnetic moments of the two leads are parallel the off-diagonal matrix elements of all the physical quantities including the line-width function $\Gamma_{\alpha\sigma}$ are zero, so $\sigma_z$ commutes with other matrices in equation (51). Using this property and $G^\alpha - G^\beta = iG^\alpha \Gamma_L G^\beta + iG^\beta \Gamma_R G^\alpha$, we find

$$N_{CC} = -\text{Tr}[\sigma_z T \sigma_z (I - T)] = -N_{AC}. \quad (52)$$

Now we calculate $N_{AC}$ from equations (50). Figure 4(a) gives $N_{AC}$ versus $\theta_R$. One can find that the differential spin shot noise $N_{AC}$ is small for the parallel situation and reaches a maximum when the magnetizations of the leads are antiparallel. We also plot differential spin shot noise versus bias voltage at parallel and antiparallel configurations in figures 4(b) and (c). When the magnetizations of the two leads are parallel, $N_{AC}$ increases abruptly with the bias voltage and reaches a flat plateau between about $V_{bias} = (0.3, 0.7) \text{ V}$, then decreases gradually upon further increasing bias voltage. However, for the antiparallel case, $N_{AC}$ starts at a large value compared with that of the parallel case and increases slightly to a maximum value at $V_{bias} = \pm 0.26 \text{ V}$. For large bias voltage, $N_{AC}$ decreases and gradually approaches zero. To understand the behavior of figure 4(b), we examine the expression of $N_{AC}$ given in equation (50) for the case of $\theta = 0$ (i.e., magnetic momenta of left and right electrodes are parallel). For $\theta = 0$ we have $\Gamma_{L\uparrow} = \Gamma_{R\uparrow} = \Gamma_{\uparrow}$ and $\Gamma_{L\downarrow} = \Gamma_{R\downarrow} = \Gamma_{\downarrow}$, so the system is symmetric and the Green’s functions are diagonal. Just like the charge current shot noise, we find that the contribution to $N_{AC}(\theta = 0)$ is given by $\sum_\sigma T_\sigma (1 - T_\sigma)$ where $T_\sigma = \frac{\Gamma_\sigma^2}{\Gamma_0^2 + \Gamma_\sigma^2}$ and $\sigma = \uparrow, \downarrow$. When the system is near the resonance (corresponding to $q V_{bias} \sim \epsilon_0 = 0$ in this context), we have $T_\sigma \sim 1$ and $1 - T_\sigma \sim 0$. Physically, this means that the partition noise $T_\sigma (1 - T_\sigma)$ is zero. When the system is far away from the resonance (corresponding to large $V_{bias}$), we have $T_\sigma \sim 0$ and $1 - T_\sigma \sim 1$. For both situations $N_{AC}$ is approaching zero. This is what we saw in figure 4(b). The plateau in figure 4(b) is due to the competition between $T_\sigma$ and $1 - T_\sigma$ when we change the energy (or $q V_{bias}$) from zero to a finite value (from near resonance to off resonance). Since $N_{AC}(\theta = 0) = \frac{(q V_{bias})^2 \Gamma_{\uparrow}^2}{(q V_{bias})^2 \Gamma_{\uparrow}^2 + (q V_{bias})^2 \Gamma_{\downarrow}^2}$, it is easy to see that for either $q V_{bias} = \Gamma_{\uparrow}$ or $q V_{bias} = \Gamma_{\downarrow}$ (the starting and ending points of the plateau) we have the same value of $N_{AC}$: $N_{AC} = \frac{1}{4} \frac{\Gamma_{\uparrow}^2 + \Gamma_{\downarrow}^2}{\Gamma_{\uparrow}^2 + \Gamma_{\downarrow}^2}$. When $q V_{bias}$ is between $\Gamma_{\uparrow}$ and $\Gamma_{\downarrow}$, $N_{AC}$ is not very sensitive to $V_{bias}$. For this reason, we get a plateau when $q V_{bias}$ changes from $\Gamma_{\uparrow}$ to $\Gamma_{\downarrow}$. In the situation where the magnetizations of the two electrodes are antiparallel, i.e., $\theta = \pi$, the system is asymmetric since $\Gamma_{\sigma\uparrow} \neq \Gamma_{\sigma\downarrow}$, where $\alpha = L, R$. The transmission coefficient is found to be $T_\sigma = \frac{\Gamma_\sigma}{\Gamma_\sigma^2 + (\Gamma_{\uparrow} + \Gamma_{\downarrow})^2}$. In addition, we have $N_{AC}(\theta = \pi) = \frac{\Gamma_\sigma}{8(\Gamma_{\uparrow} - \Gamma_{\downarrow})^2 + (\Gamma_{\uparrow} - \Gamma_{\downarrow})^2}$. Therefore, no resonant tunneling occurs in this case and we cannot have $T_\sigma = 1$ anymore. As a result, $N_{AC}$ is nonzero for small $V_{bias}$ (see figure 4(c)).

We have shown that $N_{AC} + N_{CC} = 0$ in the case of parallel and antiparallel situations. It is found that this relation is still valid when $\theta_R$ is not equal to 0 or $\pi$. In general, the relation $N_{AC} + N_{CC}$ is not satisfied. For instance, if we study a system MNMNM with three ferromagnetic layers or the MM interface where coupling matrix elements $\Gamma_{\sigma\hat{\sigma}} \neq 0$, one can get $N_{AC} + N_{CC} \neq 0$.

Now we analyze the STT and its auto-correlation function. From equations (18) and (46), we calculate the
This behavior can be understood as follows. When we set
\[ T \] (figure 5(a)). This gives very good agreement with
the system. When the system is near resonance, however,
\[ T \] was derived for a non-resonant tunneling
behaviors depending on whether it is near resonance or far
away from it. When the bias voltage is tuned far away from the resonant
is a convex function of
\[ \theta \] (figure 6(b)). Concerning this figure, we focus
0 but close to \[ \pi \] (figure 6(a)). In the intermediate
range of bias voltage, the differential noise of the STT behaves
like sin(2θR) (figure 6(b)). Concerning this figure, we focus
our attention on two special cases of \[ \theta = 0 \] and \[ \pi \], where
the analytic solution becomes simple. From equation (54) we
find analytic expressions of the differential shot noise of the
STT \( N_{\tau} \) for these two cases,
\[ N_{\tau}(0) = \frac{4(\Delta E)^2 + (\Gamma_1 - \Gamma_2)^2[i \Gamma_1 \Gamma_2]}{2[(\Delta E)^2 + \Gamma_1^2]} \] (57)

We examine the denominator of this equation. Since \( \Gamma_1 \neq \Gamma_2 \), it is clear that, near the resonance \( qV_{\text{bias}} \sim \epsilon_0 \), the term
\[ \sin^2(\theta/2) \] in the denominator cannot be neglected, so \( T_{\tau} \) in the upper panel of figure 5 does not have the sin\( \theta_R \) dependence.
But when \( qV_{\text{bias}} - \epsilon_0 \) is large enough that the term \sin\( ^2(\theta/2) \) is small compared with the term \( qV_{\text{bias}} - \epsilon_0 \)\( ^2 \), we obtain
\[ T_{\tau} \approx T'_{\tau} \] Actually, we can derive \( T'_{\tau} \) by differentiating \( F(\pi) \) and \( F(0) \) according to equation (49) and obtain
\[ T'_{\tau} = \frac{1}{2} \frac{(qV_{\text{bias}} - \epsilon_0)^2(\Gamma_1^2 - \Gamma_2^2)}{[qV_{\text{bias}} - \epsilon_0]^2 + \Gamma_1^2][qV_{\text{bias}} - \epsilon_0]^2 + \Gamma_1^2]} \] (56)

One can easily find that, if we neglect the term \[ \sin^2(\theta/2) \] in the denominator of \( T_{\tau} \), \( T_{\tau} \) will equal \( T'_{\tau} \).

Finally, we calculated the derivative of the noise of the
STT with respect to the bias voltage by equation (54). From
figure 6, we see that \( N_{\tau} \) as a function of \( \theta_R \) gives very different
behaviors depending on whether it is near resonance or far
away from it. When the bias voltage is close to \( \epsilon_0/q \), i.e. when
the system is near resonance (figure 6(c)), \( N_{\tau} \) is a concave
function of \( \theta_R \), which is very large at \( \theta_R = 0 \) but close to
zero at \( \theta_R = \pi \). However, when the system is far away from
resonance, \( N_{\tau} \) is a convex function of \( \theta_R \), that is small at
\( \theta_R = 0 \) but large at \( \theta_R = \pi \) (figure 6(a)). In the intermediate

Since most of calculations for the STT were obtained using the formula \[ [49–51] \] \( \tau_0 = \frac{L}{\tau} \) for comparison. In order to see clearly
the resonant and off-resonant behaviors of the spin torque
and the differential torque noise at different bias voltages,
especially small bias voltages, we set \( \epsilon_0 = 1.0 \) eV in the following discussions. In figure 5, we plot \( T_{\tau} \) and \( T'_{\tau} \) versus \( \theta_R \). When the bias voltage is tuned far away from the resonant
point \( \epsilon_0 \), the profile of \( T_{\tau} \) versus \( \theta_R \) obeys the sin\( \theta_R \) function (figure 5(a)). This gives very good agreement with \( T'_{\tau} \), which is expected since \( T'_{\tau} \) was derived for a non-resonant tunneling
system. When the system is near resonance, however, \( T_{\tau} \) deviates away from the sinusoidal dependence \[ [47, 52] \].
This behavior can be understood as follows. When we set
\( \Gamma_L = \Gamma_R = \Gamma_1 \) and \( \Gamma_L = \Gamma_1 = \Gamma_1 \), equation (53) can be simplified as
\[ T_{\tau} = \frac{1}{2}((qV_{\text{bias}} - \epsilon_0)^2(\Gamma_1^2 - \Gamma_2^2) ^\sin \theta) \left[ (qV_{\text{bias}} - \epsilon_0)^2 \right] \times \left[ \frac{(qV_{\text{bias}} - \epsilon_0)^2 - \Gamma_1 \Gamma_2}{4(\Gamma_1^2 - \Gamma_2^2) ^\sin \frac{\theta}{2} \frac{1}{2}} \right] \] (55)
and

\[ N_s(\theta = \pi) = \frac{16(\Delta E)^2(\Gamma^2 + \Gamma^2)}{[4(\Delta E)^2 + (\Gamma^1 + \Gamma^1)]^2} \]  \hspace{1cm} (58)\]

where \( \Delta E = qV_{\text{bias}} - \epsilon_0 \). Clearly when \( qV_{\text{bias}} \rightarrow \epsilon_0 \), \( N_s(\theta = 0) \) approaches \( \frac{\Gamma^2}{\Delta E^2} \), but \( N_s(\theta = \pi) \) approaches zero. This means that, near resonance, symmetric coupling (parallel) gives large shot noise for STT while for asymmetric coupling (antiparallel) \( N_s \) is almost zero. This gives rise to the concave behavior in figure 6(c). For the case of off resonance, the situation is different. For large \( \Delta E \), we have \( N_s(\theta = 0) = \frac{2\Gamma^2 + \Gamma^2}{\Delta E^2} \) and \( N_s(\theta = \pi) = \frac{\Gamma^2 + \Gamma^2}{\Delta E^2} \), so we always have the relation \( N_s(\theta = 0) < N_s(\theta = \pi) \), which corresponds to convex behavior in figure 6(a). When we change \( \Gamma_{R1} \) and keep the other parameters the same, we find that the noise of the STT is very sensitive to \( \Gamma_{R1} \) when \( \theta_q \) is near zero (the red dashed lines and the blue dotted lines in figures 6(a) and (c)).

4. Conclusions

In conclusion, based on the Green’s function approach, the spin current and spin noise of a quantum dot coupled by two ferromagnetic leads were investigated. The spin auto-correlation function is always positive, while the spin cross-correlation noise is negative definite. Due to the existence of the spin flip, their sum can be nonzero for the existence of the spin flip, their sum can be nonzero for any resonant level, the spin cross-correlation noise are needed to characterize the existence of the spin flip, their sum can be nonzero for the existence of the spin flip, their sum can be nonzero for any resonant level, the spin cross-correlation noise are needed to characterize the convex behavior in figure 6(c). For the case of off resonance, the situation is different. For large \( \Delta E \), we have \( N_s(\theta = 0) = \frac{2\Gamma^2 + \Gamma^2}{\Delta E^2} \) and \( N_s(\theta = \pi) = \frac{\Gamma^2 + \Gamma^2}{\Delta E^2} \), so we always have the relation \( N_s(\theta = 0) < N_s(\theta = \pi) \), which corresponds to convex behavior in figure 6(a). When we change \( \Gamma_{R1} \) and keep the other parameters the same, we find that the noise of the STT is very sensitive to \( \Gamma_{R1} \) when \( \theta_q \) is near zero (the red dashed lines and the blue dotted lines in figures 6(a) and (c)).

Acknowledgments

We gratefully acknowledge the support by the National Natural Science Foundation of China through grant No 10947018 (Y-JY), No 11074171 (Y-DW) and No 11274364 (Q-FS), and a GRF grant from HKSAR (HKU 705611P) (JW).

References

[1] Blanter Y M and Büttiker M 2000 Phys. Rep. 336 1
[2] Landauer R 1998 Nature 392 658
[3] Gomespacher T and Büttiker M 1998 Phys. Rev. Lett. 81 2763
[4] Khlus V A 1987 Sov. Phys.—JETP 66 1243
[5] Büttiker M 1990 Phys. Rev. Lett. 65 2901
[6] Büttiker M 1992 Phys. Rev. B 46 12485
[7] González T, González C, Mateos J and Pardo D 1998 Phys. Rev. Lett. 80 2901
[8] Beenakker C W 1999 Phys. Rev. Lett. 82 2761
[9] Iannaccone G, Lombardi G, Macucci A and Pwelligrini B 1998 Phys. Rev. Lett. 80 1054
[10] Kuznetsov V V, Mendez E E, Bruno J D and Pham J T 1998 Phys. Rev. B 58 R10159
[11] Chen Q and Zhao H K 2008 Europhys. Lett. 82 68004
[12] Reznikov M, Heiblum M, Shtrikman H and Mahalu D 1995 Phys. Rev. Lett. 75 3340
[13] DiCarlo L, Zhang Y, McClure D T, Reilly D J, Marcus C M, Pfeiffer L N and West K W 2006 Phys. Rev. Lett. 97 036810
[14] Birk H, de Jong M J M and Schonenberger C 1995 Phys. Rev. Lett. 75 16160
[15] Nauen A, Hapke-Wurst L, Hohls F, Zeiter U, Haug R J and Pierz K 2002 Phys. Rev. B 66 161303(R)
[16] Danneau R, Wu F, Craciun M F, Russo S, Tomi M Y, Salmilehto J, Morpurgo A F and Hakonen P J 2008 Phys. Rev. Lett. 100 196802
[17] Danneau R, Wu F, Tomi M Y, Oostinga J B, Morpurgo A F and Hakonen P J 2010 Phys. Rev. B 82 161405(R)
[18] Van den Brom H E and Van Ruitenbeek J M 1999 Phys. Rev. Lett. 82 1526
[19] Wang B and Wang J 2011 Phys. Rev. B 84 165401
[20] Safonov S S, Savchenko A K, Bagrets D A, Jouravlev O N, Nazarov Y V, Lintheld E H and Ritchie D A 2003 Phys. Rev. B 67 134406
[21] Sánchez R, Kohler S and Platero G 2008 New J. Phys. 10 115013
[22] Chen Y C and Di Ventra M 2003 Phys. Rev. B 67 153304
[23] Wang B G, Wang J and Guo H 2004 Phys. Rev. B 67 153301
[24] Chen Y C and Di Ventra M 2005 Phys. Rev. Lett. 95 166802
[25] Zhao H K, Zhao L L and Wang J 2010 Eur. Phys. J. B 77 441
[26] Wang B and Wang J 2011 Phys. Rev. B 84 165401
[27] Zhang Q, Fu D, Wang B G, Zhang R and Xing D Y 2008 Phys. Rev. Lett. 101 047005
[28] Zhao H K and Wang J 2008 Front. Phys. China 3 280
[29] Ouyang S H, Lam C H and You J Q 2008 Eur. Phys. J. B 64 67
[30] Sauret O and Feinberg D 2004 Phys. Rev. Lett. 92 106601
[31] Sanchez D, Lopez R, Samuelsson P and Buttiker M 2003 Phys. Rev. B 68 214501
[32] Shangguan M and Wang J 2007 Nanotechnology 18 154401
[33] Slonczewski J 1989 Phys. Rev. B 39 6995
[34] Slonczewski J 1996 J. Magn. Magn. Mater. 159 L1
[35] Berger L 1996 Phys. Rev. B 54 9353
[36] Tserkovnyak Y, Brataas A, Bauer G E W and Halperin B I 2005 Rev. Mod. Phys. 77 1375
[37] Xu Y, Wang S and Xia K 2008 Phys. Rev. Lett. 100 226602
[38] Haney P M and Stiles M D 2010 Phys. Rev. Lett. 105 126602
[39] Mahfouzi F, Nagaosa N and Nikolic B K 2012 Phys. Rev. Lett. 109 166602
[40] Katine J A and Fullerton E E 2008 J. Magn. Magn. Mater. 320 1217
[41] Takagishii M, Yamada K, Iwasaki H, Huku H N and Hashimoto S 2010 IEEE Trans. Magn. 46 2086–9
[42] Bragancia P M, Gurney B A, Wilson B A, Katine J A, Maat S and Childress J R 2010 Nanotechnology 21 235202
[43] Sun Z Y, Li H, Chen Y R and Wang X B 2012 IEEE Trans. VLSI 20 2020–30
[44] Chudnovsky A, Siewboodzinski J and Kamenev A 2008 Phys. Rev. Lett. 101 066601
[45] Bogoliubov N N 1947 J. Phys. USSR 11 23
[46] Wang B G, Wang J and Guo H 2001 J. Phys. Soc. Japan 70 2645
[47] Zhu Z G, Su G, Zheng Q R and Jin B 2003 Phys. Rev. B 68 224413
[48] Mahan G D 2000 Many-Particle Physics (New York: Kluwer Academic/Plenum)
[49] Ioannis T, Nicholas K, Alan K, Mairbeck C and Butler W H 2006 Phys. Rev. Lett. 97 237205
[50] Jia X T, Xia K, Ke Y Q and Guo H 2011 Phys. Rev. B 84 014401
[51] Liu D P, Han X F and Guo H 2012 Phys. Rev. B 85 245436
[52] Chen X, Zheng Q R and Su G 2008 Phys. Rev. B 78 104410