An interferometric complementarity experiment in a bulk Nuclear Magnetic Resonance ensemble

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Abstract

We have experimentally demonstrated the interferometric complementarity, which relates the distinguishability $D$ quantifying the amount of which-way (WW) information to the fringe visibility $V$ characterizing the wave feature of a quantum entity, in a bulk ensemble by Nuclear Magnetic Resonance (NMR) techniques. We primarily concern on the intermediate cases: partial fringe visibility and incomplete WW information. We propose a quantitative measure of $D$ by an alternative geometric strategy and investigate the relation between $D$ and entanglement. By measuring $D$ and $V$ independently, it turns out that the duality relation $D^2 + V^2 = 1$ holds for pure quantum states of the markers.

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I. INTRODUCTION

Bohr complementarity [1] expresses the fact that quantum systems possess properties that are equally real but mutually exclusive. This is often illustrated by means of Young’s two-slit interference experiment, where “the observation of an interference pattern and the acquisition of which-way (WW) information are mutually exclusive” [2]. As stated by Feynman, the two-slit experiment “has in it the heart of quantum mechanics. In reality it contains the only mystery” [3]. Complementarity is often superficially identified with the ‘wave-particle duality of matter’. As its tight association with the interference experiment, the terms of the “interferometric duality” or “interferometric complementarity” are more preferable. Two extreme cases, “full WW information and no fringes when measuring the population of quantum states” and “perfect fringe visibility and no WW information” have been clarified in textbooks and demonstrated with many different kinds of quantum objects including photons [4], electrons [5], neutrons [6], atoms [7] and nuclear spins in a bulk ensemble with NMR techniques [8]. In Ref.[8] we further proved theoretically and experimentally that full WW information is exclusive with population fringes but compatible with coherence patterns.

In order to describe the duality in the intermediate regime “partial fringe visibility and partial WW information”, quantitative measures for both the fringe visibility $V$ and WW information are required. The definition of the former is the usual one. In variants of two-slit experiments different WW detectors or markers, such as microscopic slit and micromaser, are used to label the way along which the quantum entity evolves. A quantitative approach to WW knowledge was first given by Wootters and Zurek [11], and then by Bartell [12]. Some relevant inequalities to quantify the interferometric duality can be found in a number of other publications [2,13–16]. Among them, Englert [2] presented definitions of the predictability $P$ and the distinguishability $D$ to quantify how much WW information is stored in the marker, and derived an inequality $D^2 + V^2 \leq 1$ at the intermediate stage which puts a bound on $D$ when given a certain fringe visibility $V$. Although the quantitative aspects
of the interferometric complementarity have been discussed by a number of theoretical papers, there are just a few experimental studies, i.e., the neutron experiments [17, 18], the photon experiments [19, 20] and the atom interferometer [21]. Recently, a complementarity experiment with an interferometer at the quantum-classical boundary [22] was also testified.

In this paper, we experimentally investigate the interferometric complementarity of the ensemble-averaged spin states of one of two kinds of nuclei in NMR sample molecules for the intermediate situations. We follow our approach detailed in Ref. [8] but use two non-orthogonal spin states of another nuclei in the sample molecules as the path markers. By entangling the observed spin with the marker one, interference is destroyed because it is in principle possible to determine the states the observed spin possesses by performing a suitable measurement of the marker one [2]. However, in this paper, an alternative geometric strategy of measuring $D$ is given and the relationship between $D$ and the entanglement of the spin states is clarified. And finally the duality relation $D^2 + V^2 = 1$ for various values of $D$ and $V$ is testified.

II. SCHEME AND DEFINITION

Our experimental scheme can be illustrated by a Mach-Zehnder interferometer (shown in Fig. 1), a modified version of the two-slit experiment. The observed and marker quantum objects, represented by B and A respectively, compose a bipartite quantum system BA. Suppose the input state of BA to be $|\psi_0\rangle = |0\rangle_B |0\rangle_A \equiv |00\rangle$, with $|0\rangle$ being one of two orthonormal basis $|0\rangle$ and $|1\rangle$ of B and A. Firstly, a beam splitter (BS) splits $|0\rangle_B$ into $\frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)$, meaning that the observed system B evolves along two paths $|0\rangle_B$ and $|1\rangle_B$ simultaneously with equal probabilities. In the meantime, path markers (PM) label the different paths $|0\rangle_B$ and $|1\rangle_B$ with the marker states $|m_+\rangle_A$ and $|m_-\rangle_A$ correspondingly. The joint action of the BS and PM denoted by operation $U_1$, thus transforms $|\psi_0\rangle$ into

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B |m_+\rangle_A + |1\rangle_B |m_-\rangle_A).$$

(1)
Secondly, phase shifters (PS) add a relative phase difference between the two paths, which are then combined into the output state $|\psi_2\rangle$ by a beam merge (BM). The joint action of the PS and BM, which is applied on B solely, is accomplished by a unitary operation

$$U_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi} \\ -e^{-i\phi} & 1 \end{pmatrix}. \quad (2)$$

And the output state $|\psi_2\rangle = U_2|\psi_1\rangle$ could be read as

$$|\psi_2\rangle = \frac{1}{2} \left[ |0\rangle_B \left( |m_+\rangle_A + e^{i\phi}|m_-\rangle_A \right) + |1\rangle_B \left( |m_-\rangle_A - e^{-i\phi}|m_+\rangle_A \right) \right]. \quad (3)$$

Finally, measuring the population, $I$, of B in the state $|0\rangle_B$ and $|1\rangle_B$ gives

$$I(\phi) = \frac{1}{2} \left( 1 \pm \text{Re}(A \langle m_+|m_-\rangle_A e^{i\phi}) \right). \quad (4)$$

where “±” correspond to the population in $|0\rangle_B$ and $|1\rangle_B$, respectively. Repeating the measurements at different $\phi$ might produce population fringes. Suppose the marker states $|m_{\pm}\rangle_A = \cos\varphi_{\pm}|0\rangle_A + \sin\varphi_{\pm}|1\rangle_A$, and from the usual definition of the fringe visibility $V = (I_{\text{max}} - I_{\text{min}}) / (I_{\text{max}} + I_{\text{min}})$ and Eq. (4) one gets

$$V = |A \langle m_+|m_-\rangle_A| = |\cos \varphi|, \quad (5)$$

where $\varphi = \varphi_- - \varphi_+$. 

Englert [2] proposed a quantitative measure for $D$ by introducing a physical quantity $L_W$—the “likelihood for guessing the right way”, which depends on the choice of an observable $W$,

$$L_W = \sum_i \max \left\{ p(W_i, |0\rangle_B), p(W_i, |1\rangle_B) \right\}. \quad (6)$$

where $p(W_i, |0\rangle_B)$ and $p(W_i, |1\rangle_B)$ denote the joint probabilities that the eigenvalue $W_i$ of $W$ is found and the observed object takes path $|0\rangle_B$ or $|1\rangle_B$. For example, for the state of Eq. (1), an optimal observable $W_{\text{opt}}$ can be found to maximize $L_W = (1 + |\sin \varphi|) / 2$ in the experiments [21] and by the definition of the distinguishability $D$ of paths $D = -1 + 2 \max W \{L_W\}$ [2], one gets

4
Here, we present an expression for $D$ in an intuitively geometric way. To this end, one projects the marker states $|m_\pm\rangle_A$ into an appropriate orthonormal basis $\{ |\beta_+\rangle_A, |\beta_-\rangle_A \}$,

$$
|m_+\rangle_A = \gamma_+ |\beta_+\rangle_A + \gamma_- |\beta_-\rangle_A,
$$
$$
|m_-\rangle_A = \delta_+ |\beta_+\rangle_A + \delta_- |\beta_-\rangle_A, 
$$

where $|\gamma_+|^2 + |\gamma_-|^2 = |\delta_+|^2 + |\delta_-|^2 = 1$. In the two-path case the criterion of choosing $\{ |\beta_+\rangle_A, |\beta_-\rangle_A \}$ is to make the difference of probabilities of measuring the two states $|m_+\rangle_A$ and $|m_-\rangle_A$ on the basis $|\beta_+\rangle_A$ to be equal to that while measuring $|m_+\rangle_A$ and $|m_-\rangle_A$ on $|\beta_-\rangle_A$. These probability differences are then defined as the distinguishability

$$
D = | |\gamma_+|^2 - |\delta_+|^2 | = | |\delta_-|^2 - |\gamma_-|^2 |. 
$$

(9)

The basis $\{ |\beta_+\rangle_A, |\beta_-\rangle_A \}$ can be rewritten into in the computational basis:

$$
|\beta_+\rangle_A = \cos\theta |0\rangle_A + \sin\theta |1\rangle_A, 
$$
$$
|\beta_-\rangle_A = \sin\theta |0\rangle_A - \cos\theta |1\rangle_A, 
$$

where $\theta$ is the angle of the state vector $|\beta_+\rangle_A$ with respect to the basis $|0\rangle_A$. In order to satisfy Eq. (9), from Fig. 2 and by the geometric knowledge $\theta = \frac{\varphi_+ + \varphi_-}{2} - \frac{\varphi}{4}$ must be held, which yields

$$
\gamma_+ = \delta_- = \cos\left( \frac{\varphi}{4} - \frac{\varphi}{2} \right), 
$$
$$
\gamma_- = \delta_+ = \sin\left( \frac{\varphi}{4} - \frac{\varphi}{2} \right). 
$$

(11)

Here, $\varphi = \varphi_- - \varphi_+$ is the angle between the two marker state vectors in the Hilbert space. So from Eqs. (9) and (11), the distinguishability is equally given by Eq. (7). It can also be seen that the desired basis $\{ |\beta_+\rangle_A, |\beta_-\rangle_A \}$ deduced by our geometric strategy is just the eigenvectors of the optimal observable $W_{opt}$ [21].

These expressions for $V$ and $D$ are consistent with those in Ref. [21] and lead to the duality relation

$$
D^2 (\varphi) + V^2 (\varphi) = 1. 
$$

(12)
Eqs. (5) and (7) reveal the sinusoidal and cosinusoidal behaviors of $D$ and $V$, respectively, on the angle $\varphi$ between $|m_+\rangle_A$ and $|m_-\rangle_A$ in the Hilbert space $H_A$. $D$ and $V$, therefore, are determined by the feature of $|m_+\rangle_A$ and $|m_-\rangle_A$, especially by the value of $\varphi$. However, for any value of $\varphi$ the duality relation (13) holds when two evolution paths, $|0\rangle_B$ and $|1\rangle_B$, are labeled by quantum pure state $|m_+\rangle_A$ and $|m_-\rangle_A$. Generally Eq. (12) should be replaced by $D^2 + V^2 \leq 1$ [2,15,16].

As WW information of the observed system B is stored in the states of the marker system A through the interaction and correlation of A and B, the distinguishability of the B’s paths depends on the feature of the marker states, or more exactly, the correlation property of the combined system AB. It would be natural to examine the relationship between the entanglement of the system AB and the distinguishability. For a bipartite pure state the entanglement $E$ can be denoted by the von Neumann entropy $S [24]$, $S = S (\rho^{(A)}) = S (\rho^{(B)})$, with $S (\rho^{(B)}) = -Tr (\rho^{(B)} \log_2 \rho^{(B)})$ and $\rho^{(B)} = Tr_B (\rho_{AB})$ for each subsystem. The entanglement $E$ for the pure state $|\psi_1\rangle$ shown in Eq. (1) is then derived as

$$E (\varphi) = -\frac{1 - \cos \varphi}{2} \log_2 \left( \frac{1 - \cos \varphi}{2} \right) - \frac{1 + \cos \varphi}{2} \log_2 \left( \frac{1 + \cos \varphi}{2} \right).$$

(13)

It can be obtained from Eq. (13) that $E = 0$ for $\varphi = k\pi$ and $E = 1$ for $\varphi = (2k + 1) \pi/2$ with $k = 0, 1, 2, \cdots$, which correspond to $D = 0$ and 1, respectively. A detailed quantitative analysis of $E$ will be given later (see Fig. 3 below).

III. EXPERIMENTAL PROCEDURE AND RESULTS

The scheme stated above was implemented by liquid-state NMR spectroscopy with a two-spin sample of carbon-13 labeled chloroform $^{13}$CHCl$_3$ (Cambridge Isotope Laboratories, Inc.). We made use of the hydrogen nucleus ($^1$H) as the marker spin A and the carbon nuclei ($^{13}$C) as the observed spin B in the experiments. Spectra were recorded on a BrukerARX500 spectrometer with a probe tuned at 125.77MHz for $^{13}$C and at 500.13MHz for $^1$H. The spin-spin coupling constant $J$ between $^{13}$C and $^1$H is 214.95 Hz. The relaxation times were
measured to be $T_1 = 4.8 \text{ sec}$ and $T_2 = 3.3 \text{ sec}$ for the proton, and $T_1 = 17.2 \text{ sec}$ and $T_2 = 0.35 \text{ sec}$ for carbon nuclei.

At first, we prepared the quantum ensemble in an effective pure state $\rho_0$ from the thermal equilibrium by line-selective pulses with appropriate frequencies and rotation angles and a magnetic gradient pulse [25]. $\rho_0$ has the same properties and NMR experimental results as the pure state $|\psi_0\rangle = |00\rangle$. Then we transferred $\rho_0$ to another state $\rho_1$ equivalent to the state $|\psi_1\rangle$ shown in Eq. (1) for accomplishing the BS and PM actions by applying a Hadamard transformation $H_B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ on spin B and two unitary transformations

$$
P_1 = \exp(-iE^A_+ \sigma^B_+ \varphi_+),$$
$$
P_2 = \exp(-iE^A_- \sigma^B_- \varphi_-),$$

where $\sigma^i_\eta (\eta = x, y, z)$ are Pauli matrices of the spin $i$, $E^i_\pm = \frac{1}{2}(1 \pm \sigma^i_z)$ and $I_2$ is the 2×2 unit matrix. These operations were implemented by the NMR pulse sequence $Y_A(\varphi_+ + \varphi_-)X_A(\frac{\pi}{2})J_{AB}(\varphi_- - \varphi_+)X_A(-\frac{\pi}{2})X_B(\pi)Y_B(\frac{\pi}{2})$ to be read from left to right, where $Y_A(\varphi_+ + \varphi_-)$ denotes an $\varphi_+ + \varphi_-$ rotation about the $\hat{y}$ axis on spin $A$ and so forth, and $J_{AB}(\varphi_- - \varphi_+)$ represents a time evolution of $(\varphi_- - \varphi_+)/\pi J_{AB}$ under the scalar coupling between spins $A$ and $B$. Finally, the PS and BM operations were achieved by the transformation $U_2$, which was realized by the NMR pulse sequence $X_B(-\theta_1)Y_B(\theta_2)X_B(-\theta_1)$ with $\theta_1 = \tan^{-1}(-\sin \phi)$, and $\theta_2 = 2\sin^{-1}(-\cos \phi/\sqrt{2})$.

In our experiments, two sets of experiments for a given value of $\varphi = \varphi_- - \varphi_+$ were performed to measure the fringe visibility $V$ and the distinguishability $D$. In the experiment of a quantitative measure for $D$, whether it is defined by the geometric way or the maximum likelihood estimation, the joint probabilities $p(|\beta_\pm\rangle_A, |0\rangle_B)$ and $p(|\beta_-\rangle_A, |1\rangle_B)$ must firstly be measured. We performed the joint measurements by a two-part procedure inspired by Brassard et al. [26]. Part one of the procedure is to rotate from the basis $\{|0\rangle_B|\beta_+\rangle_A, |0\rangle_B|\beta_-\rangle_A, |1\rangle_B|\beta_+\rangle_A, |1\rangle_B|\beta_-\rangle_A\}$ into the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ (omitting the subscripts A and B), which was realized by the unitary
\[ R_B = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \]

where \( \alpha = \frac{\pi}{4} - \frac{\varphi_1 + \varphi}{2} \), corresponding to the NMR pulse \( Y_B(2\alpha) \). Part two of the procedure is to perform a projective measurement in the computational basis which could be mimicked by a magnetic gradient pulse along \( z \)-axis [27]. Accordingly, the joint probabilities \( p (|\beta_\pm\rangle_A, |0\rangle_B) \) and \( p (|\beta_\pm\rangle_A, |1\rangle_B) \) were obtained with reconstructing the diagonal elements of the deviation density matrix by quantum state tomography [28]. The results are shown in Fig. 3. In our geometric strategy, it can be obtained from Eqs. (1) and (8) that, the information of \( \gamma_+, \gamma_- \) or \( \delta_+, \delta_- \) are determined by the population probabilities, i.e., \( |\gamma_+|^2 = 2p(|0\rangle_B|\beta_+\rangle_A), |\gamma_-|^2 = 2p(|0\rangle_B|\beta_-\rangle_A) \) and \( |\delta_+|^2 = 2p(|1\rangle_B|\beta_+\rangle_A) = |\delta_-|^2 = 2p(|1\rangle_B|\beta_-\rangle_A) \). Finally, we used Eq. (9) and took the average value of \( (|\gamma_+|^2 - |\delta_+|^2 + |\delta_-|^2 - |\gamma_-|^2) / 2 \) to give data points of \( D \) which shown in Fig. 4. On the other hand, utilizing data points of Fig. 3, we achieved the experimental values of the likelihood \( L_W \) from Eq. (6) and obtained the \( D \) measure with the maximum likelihood estimation strategy, which is the same outcomes as that in our geometric strategy. Therefore, the intuitively geometric strategy gives the equally effective measure of the distinguishability \( D \).

For measuring \( V \), we repeatedly applied the NMR pulse sequence \( X_B(-\theta_1) Y_B(\theta_2) X_B(-\theta_1) \) that represents the \( U_2(\phi) \) operation for various values of \( \phi \) and detected the population of \( \rho_2 \) in the state equivalent to the output state \(|\psi_2\rangle\). A set of appropriate values \( \theta_1 \) and \( \theta_2 \) were chosen to vary the values for \( \phi \) from 0 to \( 2\pi \). Using the same reading-out pulses and tomography method as in the measurement of \( D \), we reconstructed the populations of \( B \) for various values of \( \phi \). The variation of the normalized populations versus \( \phi \) showed a desirable interference fringe, from which the value of \( V \) was extracted. Care should be exercised in processing the spectra data of the different experimental runs in order to get the normalized populations of the deviation density matrix.
The objective of the present paper is to study the interferometric complementarity in the intermediate regime with two non-orthogonal marker states, so the experimental procedure mentioned above was repeated for different \( \phi \). Without loss of generality, we assumed \( \phi_+ = \frac{\pi}{2} \) and changed the \( \phi \) values from 0 to \( 5\pi/4 \) by varying the \( \phi_- \) value with the increment of \( \pi/16 \). The measured values of \( V(\phi) \) and \( D(\phi) \) in two sets of independent experiments were plotted in Fig. 3, along with the theoretical curves of \( V(\phi) \), \( D(\phi) \) and \( E(\phi) \). The experimental data and theoretical curve for \( D^2(\phi) + V^2(\phi) \) were depicted in Fig. 4.

From Figs. 3 and 4 some remarks can be made as follows.

1) For \( \phi = k\pi, (k = 0, 1, 2, \cdots) \) which means \( |A\langle m_+|m_-\rangle_A| = 1 \), two marker states are identical (differing with an irrelevant phase factor possible), and the state of the system AB is completely unentangled \( (E = 0) \). In this case no WW information of system B is stored in system A so that two evolution paths of B is indistinguishable \( (D = 0) \) and perfect fringe visibility is observed \( (V = 1) \). For \( \phi = (2k+1)\pi/2 \), i.e., \( |A\langle m_+|m_-\rangle_A| = 0 \), the marker states are orthogonal, and the state of the system AB is completely entangled \( (E = 1) \). This leads to full WW information \( (D = 1) \) and no interference fringes \( (V = 0) \). These two extremes are exactly the same examples that we have studied in Ref. [8] with a NMR bulk ensemble by population measurements.

2) When \( \phi \) equals other values than \( k\pi \) and \( (2k+1)\pi/2 \), which corresponds to \( 0 < |A\langle m_+|m_-\rangle_A| < 1 \), the marker states are partially orthogonal, and the state of the AB system is partially entangled \( (0 < E < 1) \). In this intermediate situations partial fringe visibility \( (0 < V < 1) \) and partial WW information \( (0 < D < 1) \) are resulted. Nevertheless, the interferometric duality still holds as in the extreme cases.

3) In the whole range of \( \phi \), \( E \) varies synchronously with \( D \). The reason is that the increase of \( E \) means more correlation between system B and A and more WW information of B stored in A, so \( D \) rises, and vice versa. On the contrary, the variation trend of \( E \) versus \( \phi \) is opposite to that of \( V \) versus \( \phi \). As the function of \( E \) versus \( \phi \) has a complicated form there is no similar relation between \( E \) and \( V \) to the duality of \( D^2 + V^2 = 1 \).

4) The measured values of \( V \), \( D \) and thus the derived values of \( D^2 + V^2 \) are fairly in
agreement with the theoretical expectation. The discrepancies between the experimental and theoretical values of $V$, $D$ and $D^2 + V^2$ in some data points, estimated to be less than ±10%, are due to the inhomogeneity of the RF field and static magnetic field, imperfect calibration of RF pulses, and signal decaying during the experiments.

**IV. CONCLUSION**

In conclusion, we have experimentally tested the interferometric complementarity in a spin ensemble with NMR techniques. In addition to two extremes, the intermediate cases that the fringe visibility $V$ reduces due to the increase of the storage of WW information are emphasized. The measured data of $D$ and $V$ in our NMR experiments are in consistent with the duality relation. In particular, the close link among $D$, $V$ and the entanglement of the composite system consisting of the observed and marker states is explicitly revealed and explained. Though the experiment was not strictly limited in the one-photon-at-a-time fact, it was performed on a quantum ensemble whose dynamical evolution is still quantum mechanical. Therefore, our experiment provides a test of the duality relation in the intermediate situations.

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Figure Captions

Fig. 1 Schematic diagram of a two-way interferometer. The input, say $|0\rangle_B|0\rangle_A$, is split to two ways by the beam splitter (BS), then labeled by the path markers (PM), phase shifted by the phase shifters (PS) and finally recombined into the output by the beam merger (BM).

Fig. 2 The state vectors in Hilbert space for defining $D$. $\{|0\rangle, |1\rangle\}$ represents the standard orthonormal basis. Two marker states $\{|m_{\pm}\rangle\}$ and another orthonormal basis $\{|\beta_{\pm}\rangle\}$ are determined by the angle $\varphi_+, \varphi_-$ and $\theta, \frac{\pi}{2} + \theta$, respectively. The dashed line denotes the angle bisector between the two states $\{|m_{\pm}\rangle\}$. From the map, one can get the relation $\theta = \frac{\varphi_+ + \varphi_-}{2} - \frac{\pi}{4}$.

Fig. 3 Normalized populations versus the angle $\varphi$, between two marker states in the experiments to measure $D$. Data points $+, \bigcirc, \ast$ and $\blacksquare$ denote the joint probabilities $p(|\beta_+\rangle_A, |0\rangle_B)$, $p(|\beta_-\rangle_A, |0\rangle_B)$, $p(|\beta_+\rangle_A, |1\rangle_B)$ and $p(|\beta_-\rangle_A, |1\rangle_B)$, respectively. Theoretical curves expressed by $p(|\beta_+\rangle_A, |0\rangle_B) = p(|\beta_-\rangle_A, |1\rangle_B) = |\cos\left(\frac{\pi}{4} - \varphi\right)|^2 / 2$ and $p(|\beta_-\rangle_A, |0\rangle_B) = p(|\beta_+\rangle_A, |1\rangle_B) = |\sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)|^2 / 2$ are depicted with the solid line and the dashdotted line, respectively.

Fig. 4 Visibility $V$ (denoted by $\bigcirc$) and distinguishability $D$ (denoted by $\ast$) as a function of $\varphi$. The solid lines are the theoretical expectations of $V$ and $D$ and the dashed line denotes $E$ expressed by Eq. (11).

Fig. 5 Experimental test of the duality relation based on the data from Fig. 3. $D^2 + V^2$ is plotted as a function of $\varphi$. The solid line represents the theoretical expectation.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5