Magnetic field dependence of the penetration depth of d-wave superconductors with strong isolated impurities

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Abstract

A d-wave superconductor with isolated strong non-magnetic impurities should exhibit an upturn in the penetration depth at low temperatures [1]. Here we calculate how an external magnetic field supresses this effect.

Key words: d-wave superconductivity, impurities, current response

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In a recent paper [1] we presented a calculation of current response of a d-wave superconductor containing a single impurity and showed that it is singular in the low-temperature limit, leading in the case of strong scattering to a 1/T term in the penetration depth. For a small number of such impurities, this low-T upturn could be observable in cuprate superconductors. An estimate of the size of this effect and of the temperature range in which it should occur agrees with experiments reported by Bonn et al. [2] in which upturns correlated with disorder introduced by Zn atoms in YBCO have been observed. Here we consider the influence of a magnetic field on this effect.

Since the low-temperature upturn in the penetration depth is due to the large number of quasiparticle excitations near the nodal directions, as is the similar upturn in the case of Andreev surface states [3–6], we might expect any physical effect which smears the gap nodes to cut off the upturn. In particular, the orbital coupling to an applied magnetic field (nonlinear electrodynamics) will suppress the upturn as it does in the Andreev case. We need to add the Doppler shift of the quasiparticle energy iω → iω + vs ⋅ k and follow the same steps as in Ref. [1] to calculate the magnetic field dependence of the upturn. Here vs is the local superfluid velocity and a typical shift vs kF⊥ ≈ (H/H0)Δ0, where H is the applied field and H0 = 3Φ0/(π2ξ0λ0) is of the order of the thermodynamic critical field.

The Nambu propagator for a pure d-wave superconductor in the presence of an external magnetic field is Gk(ωn) = [(iωn + vs ⋅ k)τ0 + ξkτ3 + Δkτ1]/Dk, where Dk = (ωn − ivs ⋅ k)2 + ξ2 + Δ2, ξk = εk − μ, and the τi are Pauli matrices. In the presence of one single δ-function impurity, the Green’s function can be expressed exactly in terms of the Green’s function of the pure system and the T-matrix for the impurity. The τ0-component of the T-matrix is T0(ω) = (πN0)−1G0(ω)/(c2 − G0(ω)2) where
$c$ is cotangent of the $s$-wave scattering phase shift ($c = 0$ corresponds to infinitely strong scattering) and $G_0 = (1/2\pi N_0) \text{Tr} \sum_\omega G_0^0(\omega)$. Here we consider the case in which the superfluid velocity is in the $x$-direction, so the Doppler shift $v_x \cdot k_F$ at the four nodes is $\omega_{1,4} = -\omega_{2,3} = \omega_s = v_s k_F / \sqrt{2}$. The integrated Green's function $G_0(\omega) = (\pi N_0 \omega_s^2 + \Delta_0^2)K(\Delta_0 / \sqrt{\omega_s^2 + \Delta_0^2}) + (\pi N_0 \omega_s^2 + \Delta_0^2)K(\Delta_0 / \sqrt{\omega_s^2 + \Delta_0^2})$ where $\omega_{\pm} = \omega \pm i \omega_s$ and $K$ is the complete elliptic integral of the first kind [7]. We now use Eq. 3 of Ref. [1] to calculate the paramagnetic response $\delta K^{xx}(p, q)$ and from it obtain the change in the penetration depth $\delta \lambda$ following the procedure as in Ref. [1]. Fig. 1 shows the result for a unitary scatterer ($c = 0$) and $\omega_s / \Delta_0 = 0, 0.01, 0.03, 0.05, 0.09$. The integral over momentum is performed for $p, q \sim 1 / \lambda < k_F$ and nodal expansion of the order parameter. The Matsubara sum has been performed numerically.

In order for the upturn to be observable, a finite number of isolated strong scatterers must be present in the sample. An estimate of the likelihood of occurrence of these rare configurations for randomly distributed impurities in order to extend the theory to finite disorder can be found in Ref. [1]. In real systems clustering may take place and we expect dependence not only on the concentration of impurities but also on the nature of the statistical distribution of the impurities in the sample. Indeed measurements on YBCO single crystals doped with the same amount (0.31%) of Zn show strong sample-to-sample dependence [2].

In the absence of the external magnetic field, the parameter $\eta_p = v_F / \lambda$ sets a lower bound for the temperature in which the upturn occurs. A rough estimate for the value of the external magnetic field in which suppression of the upturn should occur is given by $\omega_s \sim \eta_p$, i.e., $H / H_0 \sim \xi_0 / \lambda$. Typical values for YBCO ($\xi_0 \approx 5 \AA$, $\lambda \approx 1500 \AA$, $H_0 \approx 2.5 T$) give $H \approx 8 m T$.

Observation of this effect would represent a remarkable example of the strong influence that statistically rare impurity distributions can sometimes have on the macroscopic properties of a condensed matter system. It can be distinguished from the similar effect in a $d$-wave superconductor due to Andreev surface states by its disorder dependence and by its independence of field orientation and crystal shape.

References

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