CLOSED-LOOP SUPPLY CHAIN NETWORK EQUILIBRIUM MODEL WITH RETAILER-COLLECTION UNDER LEGISLATION

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(Communicated by Aviv Gibali)

ABSTRACT. This paper examines the waste of electrical and electronic equipments (WEEE) and draws on variational inequalities to model the closed-loop supply chain network. The network consists of manufacturers, retailers and consumer markets engaging in a Cournot-Nash game. Retailers are responsible for collecting WEEE in the network. It is assumed that the price of the remanufactured goods is different from that of the newly manufactured ones. The network equilibrium occurs when all players agree on volumes and prices. Several properties of the model are examined and the modified projection method is utilized to obtain the optimal solutions. Numerical examples are provided to illustrate the impact of CLSC parameters on the profits of channel members and consumer benefits, and to provide policy support for governments. We find that it is necessary to regulate a medium collection rate and a certain minimum recovery rate. This is also advantageous to manufacturers in producing new manufactured products. The impact of collection rate and recovery rate on manufacturers are greater than that on retailers. Consumers can benefit from the increase of the recovery rate as well as the collection rate.

1. Introduction. The clock-speed technology advancements have shortened the life cycle of many electronic products causing massive growth in waste electrical and electronic equipment (WEEE). To protect the environment, most governments have enacted laws to compel WEEE collection at the end of a product’s life, such
as Waste Electrical and Equipment Directive (WEEE, 2003), Restriction of Hazardous Substances Directive (RoHS, 2003) in Europe, Integrated Waste Management Board (IWMB) in California in USA and so on. Generally, the legislations regarding WEEE collection require the manufacturers to achieve the minimum collection target and the minimum recovery target. For instance, core elements of the WEEE directive 2002 include a compulsory collection rate of 4 kg/cap/yr and recovery targets differentiated by product categories. In a recast of the WEEE Directive approved by the EU parliament in 2012 the collection target is set at 65% and it must be realized in 2016 (Salhofer et al., 2016)[19]. Without doubt such legislation will have or have already had remarkable impact on supply chain operations.

With the consideration of WEEE collection and reuse, the supply chains are gradually evolving from open-loop unidirectional flows of product-from supplier to end user-to more complex, closed-loop, linked forward and reverse arcs (Guide et al., 2003)[7]. It is found a closed-loop supply chain (CLSC) usually benefits manufacturers, as reusing waste products helps reduce their costs and make more profits.

In practice, some manufacturers collect the waste products from consumer markets through their retailers. In this case, manufacturers offer retailers buy-back price for waste products, and retailers are expected to accept the commission of manufacturers to gather WEEE. For example, Sony has created the Green Fill Program that provides its retailers collection kiosks for used electronics. Savaskan et al. (2004) compare three alternative reverse channel structures and conclude the most efficient CLSC is the one with WEEE collected by the retailer[21]. Still, Savaskan and Van Wassenhove (2006) analyze the impact of competition in a CLSC with competing retailers[20]. Therefore, in this paper we also select the retailers as the collection agents.

1.1. Literature on CLSC. Due to several advantages, CLSC has received wide attention from the scholars in the last decades. Guide et al. (2003) discuss the challenge of CLSC[7]. Georgiadis and Vlachos (2004) analyze the behavior of CLSC through a simulation model based on system dynamics[5]. Calcott and Walls (2005) discuss the roles of markets and policy instruments[1]. Schultmann et al. (2006) study how reverse material flows can be handled by various design options[22]. Webster and Mitra (2007) develop a general two-period model to address concerns of government and industry[25]. Zuidwijk and Krikke (2008) examine four levels of return quality information[28]. Wei et al. (2015) investigate pricing and collection decisions in a CLSC with symmetric and asymmetric information[26]. For a comprehensive understanding of CLSC, we refer readers to some review articles such as Souza (2013)[23] and Govindan et al.(2015)[6].

1.2. The CLSC network. In reality, the CLSC usually exists in the form of network structure, which includes several mutual competitive manufacturers, competitive retailers and consumer markets. Many researchers have focused on CLSC network. For example, Nagurney and Toyasaki (2003) develop a network model for supply chain decision-making with environmental criteria[15]. Later, Nagurney and Toyasaki (2005) develop a reverse supply chain network model using variational inequality[14]. Hammond and Beullens (2007) develop a simple two-tier CLSC network model with manufacturer collection[9]. Yang et al. (2009) construct a five-tier CLSC network including raw material suppliers, manufacturers, retailers,
consumers and third-party collectors and optimize the equilibrium state of the network by variational inequalities [27]. Qiang et al. (2013) investigate a decentralized CLSC, consisting of raw material suppliers, retailers, and manufacturers, who collect the recycled product directly from the demand market. Combined with numerical examples they discuss the effects of competition, distribution channel investment, yield rates and uncertainties in demand on equilibrium quantity transactions and prices [18]. Hamdouch et al. (2017) proposes a decentralized CLSC network model in which the demands for the product and the corresponding returns are assumed to be random and price-sensitive [8].

However, little research considers the case that retailers are responsible for WEEE collection in a CLSC network. Moreover, the aforementioned CLSC network models assume that the remanufactured products are perfect substitutes for new products, but in reality, the consumers’ willing-to-pay for the remanufactured products and their selling prices are lower than new ones. Qiang (2015) proposes a CLSC network model considering competition and design for remanufacturability, and examines the impacts of consumers’ valuation for new and remanufactured products, but the CLSC network only consists of manufacturers and consumer markets [17].

The model we propose examines the effect of the collection rate and recovery rate on the volumes of products and differentiated pricing policies of new and remanufactured products in a three-tier CLSC, including manufacturers, retailers and consumer markets. Each tier engages in a Cournot-Nash game with perfect information. Retailers collect WEEE and sell them to manufacturers. As in Hammond and Beullens (2007) [9], we assume that consumer markets have an inherent aversion to returning products and such aversion is overcome by retailers who offer a collection price. Like Savaskan et al. (2004) [21], we assume the buy-back price does not affect demands for the new and remanufactured products, while demands for these two products are correlated. Numerical examples are used to illustrate the impact of key parameters on the profits of channel members and consumer benefits, and to provide policy support for governments.

The paper is organized as follows. In Section 2, we formalize the model and derive the optimal conditions for the CLSC network equilibrium. Section 3 discusses properties of the variational inequality model. In Section 4, we present numerical study and offer qualitative insights. Finally, Section 5 summarizes the work and offers future research directions.

2. The equilibrium of the closed-loop supply chain network model.

2.1. Formalize the closed-loop supply chain network. The CLSC network investigated is illustrated in Figure 1. There are three tiers in the network, which consists of manufacturer tier, retailer tier and the consumer market tier. Competition occurs horizontally among the members at the same tiers and vertically between adjacent tiers. As shown in Figure 1, there are \( l \) manufacturers, \( m \) retailers and \( n \) consumer markets in the CLSC network.

Each manufacturer \( i (i \in \{1,\ldots,l\}) \) attempts to generate profit by manufacturing and remanufacturing products. The associated costs at manufacturer \( i \)'s include the purchasing cost of original materials, the transaction cost with retailers, the disposing and recovery cost of waste products. We define that the flow of waste products from retailer \( j (j \in \{1,\ldots,m\}) \) to manufacturer \( i \) as \( q_{ij} \), while that from consumer market \( k (k \in \{1,\ldots,n\}) \) to retailer \( j \) as \( q_{jk} \). The flow of new products from manufacturer \( i \) to retailer \( j \) is denoted as \( q_{ij}^f \), and that from retailer \( j \) to market \( k \)
is represented by $q_{ij}^f$. The flow of remanufactured products from manufacturer $i$ to retailer $j$ is expressed by the variable $R_{ij}^r$, and the flow of remanufactured products from retailer $j$ to market $k$ is represented by the variable $R_{jk}^r$.

In Hammond and Beullens (2007)[9], it is assumed the manufacturers need to buy back WEEE from the consumer markets. However, in this paper retailers are commissioned by manufacturers to collect WEEE and sell them back to manufacturers. It is assumed that the recovery rate is a constant fraction. The recycling program is directed by government and manufacturers incur no further setup costs. Retailers also act as intermediaries between manufacturers and markets in the forward chain. They buy new and remanufactured products from manufacturers and sell them to consumers.

When purchasing new or remanufactured products from retailers, consumers face transaction costs as well. They regard products made by different manufacturers as substitutable. At the end of products’ life, they will decide whether to return the waste products or not according to the collection prices offered by the retailers.

The problem is an oligopolistic Cournot-Nash game with perfect information. All the associated cost functions are continuous differentiable convex functions. Nash equilibrium occurs when all players agree on product volumes shipped and prices charged. We examine the behavior of each player, and then identify the optimal condition for each player, and finally the system as a whole.

We first introduce the exogenous parameters necessary for the model formulation. Parameter $\alpha$ denotes the minimum ratio of waste products bought back from retailers to total product sold to consumer markets. The fraction of usable materials that can be recovered from one unit of collected product is $\beta$. Conversely, $\bar{\beta}$ is the fraction of unusable material (equals to $1 - \beta$). As waste products may be of variable quality, the amount of usable material may differ from unit to unit. Thus, we regard $\beta$ as the expected fraction of usable materials.

2.2. The behavior of manufacturers and their equilibrium conditions. In order to maximize his own profit, each manufacturer $i$ must decide $q_{ij}^f$, $R_{ij}^r$, and $q_{ij}^r$. For simplicity, let $Q_{ij}^f$ denote the column vector of all $q_{ij}^f$, and $Q_{ij}^r$ for that of $q_{ij}^r$. Manufacturer $i$ also needs to decide on the appropriate values for (a) $w_i^f$: the selling
price of a new product, \( w^f \); the selling price of a remanufactured product, and \( w^r \); the buy-back price paid to retailer \( j \) for obtaining a unit of waste product.

Let \( f_i(Q^f_{ij}) \) be the cost of producing from original materials, while \( r_i(R^r_{ij}) \) be that of remanufacturing from waste products. Manufacturers compete for original materials. In the forward logistics, manufacturer incurs transaction costs \( C^f_{ij}(q^f_{ij}) \) and \( C^r_{ij}(R^r_{ij}) \) respectively when selling new and remanufactured products to retailer \( j \). In the reverse logistics, manufacturer \( i \) faces inspection and sorting costs \( \phi_i(q^r_{ij}) \) for products bought back from retailer \( j \). We assume new products can only be made from original materials and remanufactured products are only made by returned waste products. Such an assumption is consistent with the practice in many cities (for instance, Pittsburgh in US and Shanghai in China). Moreover, it guarantees the validity of the model presented because different materials cannot be mixed.

Given the above notation, each manufacturer \( i \) wishes to maximize

\[
\Pi_i = \sum_{j=1}^{m} q^f_{ij} w^f_i + \sum_{j=1}^{m} R^r_{ij} w^r_j - \sum_{j=1}^{m} q^r_{ij} w^r_j - f_i(Q^f_{ij}) - r_i(R^r_{ij}) - \sum_{j=1}^{m} C^f_{ij}(q^f_{ij}) - \sum_{j=1}^{m} C^r_{ij}(R^r_{ij}) - \phi_i(q^r_{ij})
\]

Subject to:

\[
\alpha(\sum_{j=1}^{m} q^f_{ij} + \sum_{j=1}^{m} R^r_{ij}) \leq \sum_{j=1}^{m} q^r_{ij}, \tag{2}
\]

\[
\sum_{j=1}^{m} R^r_{ij} \leq \beta \sum_{j=1}^{m} q^r_{ij} \tag{3}
\]

and \( q^f_{ij}, q^r_{ij}, R^r_{ij} \geq 0 \quad \forall j; j = 1, ..., m. \)

Eq. (1) states that the manufacturer’s objective is to maximize his profit, which equals sales revenue minus costs associated with manufacturing, remanufacturing, inspection, transaction and buy-back. Constraint (2) expresses the minimal amount of waste products which need to be bought back from retailers. Constraint (3) states that the number of remanufactured products from the manufacturers shipped to retailers must be less than or equal to the volumes recovered from the waste products.

All manufacturers compete in a non-cooperative manner. Therefore, the objective function of manufacturers can be described by the following variational inequality:

\[
\sum_{i=1}^{l} \sum_{j=1}^{m} [-w^f_i + \frac{\partial f_i(q^f_{ij})}{\partial q^f_{ij}} + \frac{\partial C^f_{ij}(q^f_{ij})}{\partial q^f_{ij}} + \alpha \lambda^r_i] \times [q^f_{ij} - q^f_{ij}^*]
\]

\[
+ \sum_{i=1}^{l} \sum_{j=1}^{m} [-w^r_j + \frac{\partial r_i(R^r_{ij})}{\partial R^r_{ij}} - \lambda^r_i - \beta \gamma^r_i] \times [q^r_{ij} - q^r_{ij}^*]
\]

\[
+ \sum_{i=1}^{l} \sum_{j=1}^{m} [-w^f_i + \frac{\partial f_i(R^r_{ij})}{\partial R^r_{ij}} + \frac{\partial C^r_{ij}(R^r_{ij})}{\partial R^r_{ij}} + \alpha \lambda^r_i + \gamma^r_i] \times [R^r_{ij} - R^r_{ij}^*]
\]

\[
\sum_{i=1}^{l} \sum_{j=1}^{m} [-w^f_i + \frac{\partial f_i(q^f_{ij})}{\partial q^f_{ij}} + \frac{\partial C^f_{ij}(q^f_{ij})}{\partial q^f_{ij}} + \alpha \lambda^r_i] \times [q^f_{ij} - q^f_{ij}^*]
\]

\[
+ \sum_{i=1}^{l} \sum_{j=1}^{m} [-w^r_j + \frac{\partial r_i(R^r_{ij})}{\partial R^r_{ij}} - \lambda^r_i - \beta \gamma^r_i] \times [q^r_{ij} - q^r_{ij}^*]
\]

\[
+ \sum_{i=1}^{l} \sum_{j=1}^{m} [-w^f_i + \frac{\partial f_i(R^r_{ij})}{\partial R^r_{ij}} + \frac{\partial C^r_{ij}(R^r_{ij})}{\partial R^r_{ij}} + \alpha \lambda^r_i + \gamma^r_i] \times [R^r_{ij} - R^r_{ij}^*]
\]
\[
+ \sum_{i=1}^{l} \left[ \sum_{j=1}^{m} q_{ij}^r - \alpha \sum_{j=1}^{m} q_{ij}^f - \alpha \sum_{j=1}^{m} R_{ij}^r \right] \times [\lambda_i - \lambda_i^*] \\
+ \sum_{i=1}^{l} \left[ \beta \sum_{j=1}^{m} q_{ij}^r - \sum_{j=1}^{m} R_{ij}^r \right] \times [\gamma_i - \gamma_i^*] \geq 0 \tag{4}
\]

\forall (Q_{ij}^r, Q_{ij}^f, R_{ij}^r, \lambda_i, \gamma_i) \in R_+^{3m+2l}.

Note that \(\lambda_i\) and \(\gamma_i\) above are the Lagrange multipliers of constraint (2) and (3) respectively, while the Lagrange multiplier vectors correspond to the \(l\)-dimensional column vectors. The price variables \(w_{ij}^f\), \(w_{ij}^r\) and \(w_{ik}^r\) are endogenous and can be determined by the equilibrium solutions.

2.3. The behavior of retailers and their equilibrium conditions. Retailers interact with manufacturers and consumers, that is, they procure new and/or remanufactured products from the manufacturers, and also collect WEEE from markets and sell them back to manufacturers. Retailer \(j\) faces purchasing costs of new and/or remanufactured products and the waste collection costs. These costs are denoted by \(C_f^j(q_{ij}^f)\), \(C_r^j(R_{ij}^r)\) and \(C_r^j(q_{jk}^r)\), respectively.

Retailer \(j\) must establish appropriate values for (a) \(P_f^j\): the selling price of a new manufactured product at markets, (b) \(P_r^j\): the selling price of a remanufactured product at markets, and (c) \(P_k^r\): the collection price paid to consumer market \(k\) in return for waste products.

Given the above notation, each retailer \(j\) wishes to maximize

\[
\Pi_j = \sum_{k=1}^{n} q_{jk}^f P_f^j + \sum_{k=1}^{n} R_{jk}^r P_r^j + \sum_{i=1}^{l} q_{ij}^r w_{ij}^r - \sum_{k=1}^{n} q_{jk}^r P_k^r - \sum_{i=1}^{l} R_{ij}^r w_i^r - \sum_{i=1}^{l} q_{ij}^f w_i^f \\
- \sum_{i=1}^{l} C_f^j(R_{ij}^r) - \sum_{i=1}^{l} C_r^j(q_{ij}^r) - \sum_{k=1}^{n} C_r^j(q_{jk}^r) \tag{5}
\]

Subject to:

\[
\sum_{k=1}^{n} q_{jk}^f \leq \sum_{i=1}^{l} q_{ij}^f, \tag{6}
\]

\[
\sum_{i=1}^{l} q_{ij}^r \leq \sum_{k=1}^{n} q_{jk}^r, \tag{7}
\]

\[
\sum_{k=1}^{n} R_{jk}^r \leq \sum_{i=1}^{l} R_{ij}^r, \tag{8}
\]

and \(q_{jk}^f, q_{jk}^r, R_{jk}^r \geq 0\) \(\forall k; k = 1, \ldots, n\).

Eq.(5) shows the profit the retailer \(j\) wishes to maximize. It is the revenue minus the handling costs, and the payouts to manufacturers and consumers. Constraints (6) and (8) ensure the products purchased by consumers are no more than the volume the retailers buy from the manufacturers. This is true for both the new and the remanufactured products. Constraint (7) expresses that the amount manufacturers buy back from retailers must not exceed the amount consumers return to the retailer.
Retailers compete in a non-cooperative manner so that each retailer aims to maximize his own profit given other retailers’ actions. In equilibrium, all volumes and prices between the tiers of network agents have to coincide.

The optimal solutions for retailers \((Q^f_{jk}, Q^r_{jk}, R^f_{jk}, Q^r_{ij}, R^f_{ij}, R^r_{ij}, b^*_j, c^*_j, d^*_j)\text{\forall}\ R^3_{3m+3mn+3m}\) can be identified by satisfying the following variational inequality:

\[
\sum_{j=1}^{m} \sum_{k=1}^{n} \left[-P^f_j + b^*_j \right] \times \left[q^f_{jk} - q^r_{jk} \right] + \sum_{j=1}^{m} \sum_{k=1}^{n} \left[P^r_k + \frac{\partial C^r_j (q^r_{jk})}{\partial q^r_{jk}} - c^*_j \right] \times \left[q^r_{jk} - q^r_{jk} \right] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left[-P^r_j + d^*_j \right] \times \left[R^r_{jk} - R^r_{jk} \right] + \sum_{j=1}^{m} \sum_{i=1}^{l} \left[-w^r_j + c^*_j \right] \times \left[q^r_j - q^r_{ij} \right] \\
+ \sum_{j=1}^{m} \sum_{i=1}^{l} \left[w^r_i + \frac{\partial C^f_i (R^r_{ij})}{\partial R^r_{ij}} - d^*_j \right] \times \left[R^r_{ij} - R^r_{ij} \right] \\
+ \sum_{j=1}^{m} \sum_{i=1}^{l} \left[w^f_i + \frac{\partial C^f_j (q^f_{ij})}{\partial q^f_{ij}} - b^*_j \right] \times \left[q^f_{ij} - q^f_{ij} \right] \\
+ \sum_{j=1}^{m} \sum_{i=1}^{l} q^f_{ij} - \sum_{k=1}^{n} q^f_{jk} \times \left[b^*_j - b^*_j \right] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} q^r_{jk} - \sum_{i=1}^{l} q^r_{ij} \times \left[c^*_j - c^*_j \right] \\
+ \sum_{j=1}^{m} \sum_{i=1}^{l} R^r_{ij} - \sum_{k=1}^{n} R^r_{jk} \times \left[d^*_j - d^*_j \right] \geq 0
\]

\(\forall(Q^f_{jk}, Q^r_{jk}, R^f_{jk}, Q^r_{ij}, R^f_{ij}, Q^r_{ij}, b^*_j, c^*_j, d^*_j) \in R^3_{3m+3mn+3m}\)

The parameters \(b^*_j, c^*_j\) and \(d^*_j\) above are the Lagrange multipliers of constraints \((6)-(8)\) respectively, and the Lagrange multiplier vectors correspond to \(m\)-dimensional column vectors.

### 2.4. The behavior of consumers and their equilibrium conditions.

In the forward supply chain, consumers purchase new or remanufactured products. They take into account not only the prices charged by retailers but also the transaction costs to obtain the products. Let \(C^f_j (q^f_{jk})\) and \(C^r_j (R^r_{ij})\) be the transaction cost functions for consumers at market \(k\) to procure new and remanufactured products respectively. The unit retail price is denoted by \(P^f_k\) and \(P^r_k\) respectively. The \(n\)-dimensional column vector \(P^f\) contains all values of \(P^f_j\); while that of \(P^r\) contains all values of \(P^r_k\). Consumer demands for the new and remanufactured products are \(d^f_k (P^f, P^r)\) and \(d^r_k (P^f, P^r)\) respectively; and these demand functions are linear. The two types of products are substitutable, and the demands not only depend on both the prices of the new and the remanufactured products, but also the price of
the products at other markets. In addition, the demand for a product is assumed to be negatively correlated to its price and positively associated with competitors’ prices. Thus, the linear demand functions of the two products at market \( k \) take the forms as follows:

\[
d_f^k(P^f, P^{fr}) = M_k - \theta_f^k P^f_k - \theta_f^k P^f_k + \theta_f^{fr} P^{fr}_k, \quad \theta_f^k \geq \theta_f^{fr} \geq 0, \quad \theta_f^k \geq \theta_f^{fr} \geq 0 \tag{10}
\]

\[
d_{fr}^k(P^f, P^{fr}) = m_k - \theta_{fr}^k P^{fr}_k - \theta_{fr}^k P^{fr}_k + \theta_{fr}^f P^f_k, \quad \theta_{fr}^k \geq \theta_{fr}^f \geq 0, \quad \theta_{fr}^k \geq \theta_{fr}^f \geq 0 \tag{11}
\]

where the subscript \( \bar{k} \) represents the markets other than \( k \). \( M_k \) and \( m_k \) represent the primary demands of market \( k \) about new and remanufactured products respectively. \( \theta_f^k \) and \( \theta_f^k \) denote the factors of new product in market \( k \) that influence consumers’ sensitivity to price; while \( \theta_{fr}^k \) and \( \theta_{fr}^k \) represent the substitutive factors of the remanufactured product. Although these definitions have some disadvantages in practice, this demand pattern is widely used in literature (see Choi (1991, 1996); Padmanabhan et al. (1997); Nagurney (2006); Cruz (2008); Yao et al. (2008))

The equilibrium conditions for consumers in market \( k \) facing retailer \( j \) take the following form:

\[
q_{fjk} = \begin{cases} 
P^f_j + C_f^j(q_{fjk}) \geq P^f_j, & \text{if } q_{fjk} > 0, \\
q_{fjk} \geq 0, & \text{if } q_{fjk} = 0 
\end{cases} \tag{12}
\]

\[
P^r_j + C_r^j(R_{rjk}) \geq P^{fr}_j, \quad R_{rjk} \geq 0 \tag{13}
\]

\[
d_f^k(P^f, P^{fr}) = \begin{cases} 
\sum_{j=1}^{m} q_{fjk}^f, & \text{if } P^f_k > 0, \\
\sum_{j=1}^{m} q_{fjk}^f, & \text{if } P^f_k = 0 
\end{cases} \tag{14}
\]

and

\[
d_{fr}^k(P^f, P^{fr}) = \begin{cases} 
\sum_{j=1}^{m} R_{rjk}^r, & \text{if } P^{fr}_k > 0, \\
\sum_{j=1}^{m} R_{rjk}^r, & \text{if } P^{fr}_k = 0 
\end{cases} \tag{15}
\]

In the forward logistics, Eqs.(12)-(13) state that if consumers at market \( k \) purchase products from retailer \( j \), the prices charged by the retailer plus the transaction costs do not exceed the consumers’ willingness to pay (reservation price). Eqs.(14)-(15) express that if the surplus utility is positive, then the amount sold by retailers equals market demands. The equilibrium conditions of consumer markets are identical to the spatial equilibrium conditions as discussed in Nagurney et al. (2002).

In the reverse logistics, the consumer’s behavior can be described by Eq.(16) subject to Eq.(17). Eq.(16) states that the consumer in market \( k \) chooses to return one unit of product based on the collection price. Eq.(17) states that the amount of
consumers return must not exceed the total amount of the new and remanufactured products sold before.

\[ a_k(q_{jk}^r) \begin{cases} = P_k^r, & \text{if } q_{jk}^r > 0, \\ \geq P_k^r, & \text{if } q_{jk}^r = 0, \end{cases} \] (16)

Subject to:

\[ \sum_{j=1}^{m} q_{jk}^r \leq \sum_{j=1}^{m} q_{jk}^f + \sum_{j=1}^{m} R^r_{jk}. \] (17)

Combining consumer’s behavior at markets in both the forward and reverse logistics, the equilibrium conditions of the consumer markets, \((Q_{j,k}^r, Q_{j,k}^f, P_k^r, P_k^f, R^r_{jk}, R^r_{jk}, \eta_k^r) \in R^{3mn+3m}\), can be identified by satisfying

\[
\sum_{j=1}^{m} \sum_{k=1}^{n} [P_j^f + C^r_{jk}(q_{jk}^f) - P_k^r - \eta_k^r] \times [q_{jk}^f - q_{jk}^r] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} [a_k(q_{jk}^r) - P_k^r + \eta_k^r] \times [q_{jk}^r - q_{jk}^r] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} [P_j^r + C^r_{jk}(R^r_{jk}) - P_k^f - \eta_k^r] \times [R_{jk}^r - R_{jk}^r] \\
+ \sum_{k=1}^{n} \sum_{j=1}^{m} [q_{jk}^f - d^r_k(P_j^f, P_j^r)] \times [P_k^r - P_k^r] \\
+ \sum_{k=1}^{n} \sum_{j=1}^{m} [R_{jk}^r - d^r_k(P_j^f, P_j^r)] \times [P_k^f - P_k^f] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} [q_{jk}^f + R_{jk}^r - \sum_{j=1}^{m} q_{jk}^r] \times [\eta - \eta_k^r] \geq 0 \\
\forall (Q_{j,k}^r, Q_{j,k}^f, P_k^r, P_k^f, R^r_{jk}, \lambda_k) \in R^{3mn+3n}. \]

Note that \(\eta_k\) is the Lagrange multiplier of Constraint (17) and \(\eta\) is n-dimensional column vector.

2.5. The equilibrium conditions of the closed-loop supply chain network.

We now define the CLSC network equilibrium in the model and establish a corresponding theorem.

**Definition 2.1.** The equilibrium state of the CLSC network is one where the forward and reverse flows between the distinct tiers of the decision-makers coincide and the product flows and prices satisfy the sum of optimal conditions in Eqs.(4), (9) and (18).

We can establish the following theorem:

**Theorem 2.2.** (*Variational inequality formulation*) The equilibrium conditions governing the CLSC network model with competition are equivalent to solve the variational inequality problem, by identifying the optimal values for \((Q_{ij}^r, Q_{ij}^f, \gamma_i^*, \gamma_i^*, Q_{jk}^r, R_{jk}^r, b_j, \varepsilon^*, d^*_j, P_k^r, P_k^f, \eta_k^r, \eta_k^r) \in K\) that satisfies
\[
\sum_{i=1}^{l} \sum_{j=1}^{m} \left( \frac{\partial f_i(q_{ij}^r)}{\partial q_{ij}^r} + \frac{\partial C_{ij}^f(q_{ij}^r)}{\partial q_{ij}^r} + \frac{\partial C_{ij}^r(q_{ij}^r)}{\partial q_{ij}^r} + \alpha \lambda_i^* - b_j^* \right) \times [q_{ij}^r - q_{ij}^*] \\
+ \sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \frac{\partial f_i(q_{ij}^r)}{\partial q_{ij}^r} - \lambda_i^* - \beta \gamma_i^* + c_j^* \right] \times [q_{ij}^r - q_{ij}^*] \\
+ \sum_{i=1}^{l} \sum_{j=1}^{m} \left( \frac{\partial C_{ij}^r(R_{ij}^r)}{\partial R_{ij}^r} + \frac{\partial C_{ij}^r(R_{ij}^r)}{\partial R_{ij}^r} + \alpha \lambda_i^* + \gamma_i^* - d_j^* \right) \times [R_{ij}^r - R_{ij}^*] \\
+ \sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \sum_{j=1}^{m} q_{ij}^r - \alpha \sum_{j=1}^{m} q_{ij}^r - \alpha \sum_{j=1}^{m} R_{ij}^r \right] \times [\lambda_i - \lambda_i^*] \\
+ \sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \beta \sum_{j=1}^{m} q_{ij}^r - \sum_{j=1}^{m} R_{ij}^r \right] \times [\gamma_i - \gamma_i^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left( C_{j,k}^r(q_{jk}^r) - P_k^r - \eta_k^* + b_j^* \right) \times [q_{jk}^r - q_{jk}^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left( \frac{\partial C_{j,k}^r(q_{jk}^r)}{\partial q_{jk}^r} - c_j^* + a_k(q_{jk}^r) + \eta_k^* \right) \times [q_{jk}^r - q_{jk}^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left( C_{j,k}^r(R_{jk}^r) - P_k^r - \eta_k^* + d_j^* \right) \times [R_{jk}^r - R_{jk}^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left[ \sum_{j=1}^{m} q_{jk}^r - \sum_{k=1}^{m} q_{jk}^r \right] \times [b_j - b_j^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left[ \sum_{j=1}^{m} q_{jk}^r - \sum_{i=1}^{l} q_{ij}^r \right] \times [c_j - c_j^*] \\
+ \sum_{j=1}^{m} \sum_{k=1}^{n} \left[ \sum_{j=1}^{m} R_{ij}^r - \sum_{k=1}^{m} R_{ij}^r \right] \times [d_j - d_j^*] \\
+ \sum_{k=1}^{m} \sum_{j=1}^{n} \left[ \sum_{j=1}^{m} q_{jk}^r - d_k^* (P_k^r, P_k^r) \right] \times [P_k^r - P_k^r] \\
+ \sum_{k=1}^{m} \sum_{j=1}^{n} \left[ \sum_{j=1}^{m} R_{jk}^r - d_k^* (P_k^r, P_k^r) \right] \times [P_k^r - P_k^r] \\
+ \sum_{k=1}^{m} \sum_{j=1}^{n} \left[ \sum_{j=1}^{m} q_{jk}^r + \sum_{j=1}^{m} R_{jk}^r - \sum_{j=1}^{m} q_{jk}^r \right] \times [\eta_k^* - \eta_k^*] \geq 0
\]

\forall (Q_{ij}^f, Q_{ij}^r, R_{ij}^r, \lambda_i, \gamma_i, Q_{jk}^f, Q_{jk}^r, R_{jk}^r, b_j, c_j, d_j, P_k^f, P_k^r, \eta_k) \in K \quad (19)

where

\[
K \equiv \left\{ (Q_{ij}^f, Q_{ij}^r, R_{ij}^r, \lambda_i, \gamma_i, Q_{jk}^f, Q_{jk}^r, R_{jk}^r, b_j, c_j, d_j, P_k^f, P_k^r, \eta_k) | (Q_{ij}^f, Q_{ij}^r, R_{ij}^r, \lambda_i, \gamma_i, Q_{jk}^f, Q_{jk}^r, R_{jk}^r, b_j, c_j, d_j, P_k^f, P_k^r, \eta_k) \in R_{ij}^{3m+3n+2l+3m+3n} \right\}
\]
In fact, inequality (19) can be rewritten in standard variational inequality form as follows. Determine \( X^* \in K \) which satisfies

\[
\langle F(X^*), X - X^* \rangle \geq 0, \forall X \in K,
\]

where \( X \equiv (Q_{ij}^l, Q_{ij}^r, R_{ij}^r, \lambda_i, \gamma_i, Q_{jk}^l, Q_{jk}^r, R_{jk}^r, b_j, c_j, d_j, P_{ik}^l, P_{ik}^r, \eta_k), F(X) \equiv (F_{ij}^l, F_{ij}^r, F_{ik}^l, F_{ik}^r, F_{jk}^l, F_{jk}^r, F^k, F^{fr}, F^m) \ i = 1, \ldots, l, j = 1, \ldots, m, k = 1, \ldots, n. \) The specific components of \( F \) are given by the functional terms preceding the multiplication signs in (19). The term \( \langle \cdot, \cdot \rangle \) represents the inner product in \( N \)-dimensional Euclidean space.

**Proof of Theorem 2.2.** Variational inequality formulation.

We first prove that the equilibrium conditions imply variational inequality (19). Indeed, combining Eqs. (4), (9) and (18) and after algebraic simplification, we obtain inequality (19).

Conversely, we also need to prove that the solution to (19) is really the equilibrium conditions imply variational inequality (19). We first prove that the equilibrium conditions imply variational inequality (19). Note that such terms do not change the inequality since they are equal to zero, and this yields Eq. (21), which can be rewritten as Eq. (22). Eq. (22) equals the sum of Eqs. (4), (9) and (18).

\[
\sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \frac{\partial C^l_{ij}(q_{ij}^r)}{\partial q_{ij}^l} - \lambda_i^* - \beta \gamma_i^* + c_j^* + w_{ij}^r - w_{ij}^r \right] \times [q_{ij}^l - q_{ij}^r] + \sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \frac{\partial C^l_{ij}(q_{ij}^r)}{\partial q_{ij}^l} \right] = 0
\]
\[
\begin{align*}
&\sum_{j=1}^{m} \sum_{k=1}^{n} [\sum_{i=1}^{l} q_{ij}^k - \sum_{i=1}^{l} q_{ij}^r] \times [c_j - c_j^r] \\
&+ \sum_{j=1}^{m} \sum_{i=1}^{l} R_{ij}^* - \sum_{k=1}^{n} R_{ij}^r] \times [d_j - d_j^r] \\
&+ \sum_{k=1}^{n} \sum_{j=1}^{m} q_{ij}^r - d_{ij}^r (P_{ij}^r, P_{ij}^{fr})] \times [P_{ij}^f - P_{ij}^{fr}] \\
&+ \sum_{j=1}^{m} \sum_{k=1}^{n} R_{ij}^r - [P_{ij}^{fr} - P_{ij}^{fr}] \\
&+ \sum_{k=1}^{n} [\sum_{j=1}^{m} q_{ij}^r + \sum_{j=1}^{m} R_{ij}^r - \sum_{j=1}^{m} q_{ij}^r] \times [\eta_k - \eta_k^r] \geq 0
\end{align*}
\]

\[\forall (Q_{ij}^f, Q_{ij}^r, R_{ij}^f, \alpha, \gamma_i, Q_{ij}^f, Q_{ij}^r, R_{ij}^f, b_j, c_j, d_j, P_{ij}^f, P_{ij}^{fr}, \eta_k) \in K, \quad (21)\]

where

\[K = \left\{ (Q_{ij}^f, Q_{ij}^r, R_{ij}^f, \lambda_i, \gamma_i, Q_{ij}^f, Q_{ij}^r, R_{ij}^r, b_j, c_j, d_j, P_{ij}^f, P_{ij}^{fr}, \eta_k) \mid (Q_{ij}^f, Q_{ij}^r, R_{ij}^f, \lambda_i, \gamma_i, Q_{ij}^f, Q_{ij}^r, R_{ij}^r, b_j, c_j, d_j, P_{ij}^f, P_{ij}^{fr}, \eta_k) \in R_+^{3l+3m+2l+3m+3n} \right\}.\]
The price variable \( w_{ij}^{fr} \) can be retrieved by setting the first term of Eq. (4) to zero, and other price variables can be obtained similarly.

\[
{w}_{ij}^{fr} = \frac{\partial f_i(q_{ij}^*)}{\partial q_{ij}^*} + \frac{\partial C_{ij}^f(q_{ij}^*)}{\partial q_{ij}^*} + \alpha \lambda_i^*.
\]  

By setting the second term of Eq. (4) to zero, we have

\[
{w}_{ij}^{fr} = -\frac{\partial \phi_i(q_{ij}^*)}{\partial q_{ij}^*} + \lambda_i^* + \beta \gamma_i^*.
\]  

By setting the third term of Eq. (4) to zero, we find

\[
{w}_{ij}^{fr} = \frac{\partial c_i(R_{ij}^*)}{\partial R_{ij}^*} + \frac{\partial C_{ij}^c(R_{ij}^*)}{\partial R_{ij}^*} + \alpha \lambda_i^* + \gamma_i^*.
\]  

By setting the first term of Eq. (18) to zero, we obtain

\[
{P}_j^{fr} = -C_{jk}^f(q_{jk}^*) + P_k^{fr} + \eta_k.
\]  

By setting the second term of Eq. (18) to zero, we find

\[
{P}_k^{fr} = a_k(q_{jk}^*) + \eta_k.
\]  

By setting the third term of Eq. (18) to zero, we have

\[
{P}_j^{fr} = -C_{jk}^r(R_{jk}^*) + P_k^{fr} + \eta_k.
\]  

From Eqs. (23)-(25), we find that a non-zero value of \( \eta_k \) leads to increasing in the selling price of new and remanufactured products in market, as are the collection and buy-back prices which can be obtained by Eqs. (26)-(28).
3. Qualitative properties. It is essential to know if a solution exists for the model. Variational inequality admits at least one solution if function $F$ is continuous, and the feasible field is compact (Nagurney (2002, 1993))\[13,11\]. Since the feasible set underlying Eq. (19) is not compact, the existence of a solution cannot be simply obtained from the assumption of continuity of $F$. It is necessary to impose a weak condition on $K$ to guarantee its existence. Let

$$K_e = \left\{ (Q^f_{ij}, Q^r_{ij}, R^r_{ij}, \lambda_i, \gamma_i, Q^f_{jk}, Q^r_{jk}, R^r_{jk}, b_j, c_j, d_j, P^f_k, P^{fr}_k, \eta_k) \right\}$$

where $e = (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}) \geq 0$, then $K_e$ is bounded, closed convex subset of $R^{4m+3m+2l+3m+3n}$. So the following variational inequality

$$\langle F(X^e), X - X^e \rangle \geq 0, \forall X_e \in K_e \tag{30}$$

will have at least one solution. Therefore, we have Lemma 3.1:

**Lemma 3.1.** Variational inequality (30) admits a solution if and only if there exists a vector, $e > 0$ such that variational inequality (30) admits a solution in $K_e$.

**Theorem 3.2.** (Existence). Suppose there exists positive constants $M,N,R$ with $R > M$ such that

$$\frac{\partial f_i(q^f_{ij})}{\partial q^f_{ij}} + \frac{\partial C^i_j(q^f_{ij})}{\partial q^f_{ij}} \geq R, \forall Q^f_{ij}, q^f_{ij} \geq N, \forall i, j, \tag{31}$$

$$\frac{\partial \phi_i(q^r_{ij})}{\partial q^r_{ij}} \geq R, \forall Q^r_{ij}, q^r_{ij} \geq N, \forall i, j, \tag{32}$$

$$\frac{\partial r_i(R^r_{ij})}{\partial R^r_{ij}} + \frac{\partial C^i_j(R^r_{ij})}{\partial R^r_{ij}} + \frac{\partial C^r_j(R^r_{ij})}{\partial R^r_{ij}} \geq R, \forall R^r_{ij}, R^r_{ij} \geq N, \forall i, j, \tag{33}$$

$$C^f_{jk}(q^f_{jk}) \geq R, \forall Q^f_{jk}, q^f_{jk} \geq N, \forall j, k, \tag{34}$$

$$C^r_{jk}(q^r_{jk}) \geq R, \forall Q^r_{jk}, q^r_{jk} \geq N, \forall j, k, \tag{35}$$

$$d^k_{ij}(P^f_k, P^{fr}_k) \leq N, d^k_{ij}(P^f_k, P^{fr}_k) \leq N, P^f_k > M, P^{fr}_k > M, \forall k. \tag{36}$$

Then variational inequality (19) admits at least one solution.

**Proof of Theorem 3.2.** For $e_1 = e_{14}$, we can choose large enough values to satisfy condition (29) subject to constraints (31)-(37). Under the precondition of Lemma 3.1, variational inequality (19) has at least one solution.

**Theorem 3.3.** (Uniqueness). Assuming the cost functions, including transaction, handling and production cost functions are strictly convex, the consumers aversion functions are strictly monotone increasing, and the demand functions are strictly monotone decreasing, variational inequality (19) has one unique solution.
Proof of Theorem 3.3. As shown in Eq. (20), the variational inequality (19) can be rewritten in standard variational inequality form as determining \( X^* \in K \) which satisfies: \( \langle F(X), X - X^* \rangle \geq 0 \forall X \in K \), where \( X \equiv (Q^f_{ij}, Q^r_{ij}, r_i^f, \lambda_i, \gamma_i, Q^r_{jk}, R^r_{jk}, b_j, c_j, d_j, P^f_k, P^r_k, \eta_k) \)

Assume (19) has two solutions denoted by \( X^{1*} \) and \( X^{2*} \), then the following inequalities are obtained.

\[
\langle F(X^{1*}), X - X^{1*} \rangle \geq 0
\]

and

\[
\langle F(X^{2*}), X - X^{2*} \rangle \geq 0
\]

Let \( X = X^{2*} \) and substitute the vector into (38). Similarly, let \( x = X^{1*} \) and substitute the vector into (39). Adding the two resulting inequalities yields

\[
\langle F(X^{1*}) - F(X^{2*}), X^{1*} - X^{2*} \rangle \leq 0
\]

By the assumptions, all the cost functions, including transaction, handling and production cost functions are strictly convex and the consumers’ aversion and demand functions are strictly monotone increasing and decreasing, respectively. The left-hand side of the variational inequality (40) is strictly greater than zero, which contradicts the assumptions in this theorem. Hence there is only one solution for (19).

In the next section, the numerical examples have been modified from the examples in Nagurney et al. (2003)[15]. The modified projection method is used in our numerical examples as Nagurney et al. (2002)[13]. \( \square \)

4. Numerical examples and discussion.

4.1. Numerical examples. We implement the algorithm in MATLAB 7.0. The convergence criterion is that the absolute value of the flows and prices between two successive iterations differ by no more than \( 10^{-4} \). The step in the modified projection method is set at 0.02. The solutions discussed below satisfy the equilibrium conditions with high accuracy.

Assume that the CLSC network is composed of two manufacturers, two retailers and two consumer markets. The cost functions are defined as follows.

\[
f_1(Q^f) = 2.5q_i^2 + q_1q_2 + 2q_1, f_2(Q^f) = 3q_2^2 + q_1q_2 + 2q_2 \text{ and } q_i = \sum_{j=1}^{2} q^j_{ij} (i = 1, 2),
\]  

\[
r_i(R_i^f) = 2.5R_i^2 + 2R_i^f \text{ and } R_i^f = \sum_{j=1}^{2} R^r_{ij},
\]  

\[
C^f_{ij}(q^f_{ij}) = 0.5(q^f_{ij})^2 + 3.5q^f_{ij}, C^f_{ij}(R^r_{ij}) = 0.5(R^r_{ij})^2 + 4R^r_{ij},
\]  

\[
\phi_i(q_i^f) = 2.5(q_i^f)^2 + 2q_i^r \text{ and } q_i^r = \sum_{j=1}^{2} q^j_{ij},
\]  

\[
C^f_{jk}(q^f_{jk}) = q^f_{jk} + 5, C^f_{jk}(R^r_{jk}) = R^r_{jk} + 5,
\]  

\[
C^r_j(R^r_j) = 0.5\left(\sum_{i=1}^{2} R^r_{ij}\right)^2, C^f_j(q^f_{ij}) = 0.5\left(\sum_{i=1}^{2} q^j_{ij}\right)^2, C^r_j(q^r_{jk}) = 0.5\left(\sum_{i=1}^{2} q^j_{ij}\right)^2,
\]
The initial values of exogenous variables are given as $\alpha = 1, \beta = 0.5$. In the following Figures and Tables, the trends of volumes and prices when exogenous variables change are shown, in which the volume of each product is the sum in the same tier.

The initial values of exogenous variables are given as $\alpha = 1, \beta = 0.5$. In the following Figures and Tables, the trends of volumes and prices when exogenous variables change are shown, in which the volume of each product is the sum in the same tier.
Figure 3. The volume changes of three kinds of products with the changes of recovery rate while \( \alpha = 1 \) are fixed

Table 1. The prices under different collection rate \( (\alpha) \), given \( \beta = 0.5 \)

| \( \alpha \) | \( w_f^* \) | \( w_r^* \) | \( P_f^* \) | \( P_r^* \) |
|---|---|---|---|---|
| 0  | 352.8712 | 59.8369 | 39.8914 | 393.9832 |
| 0.2 | 352.8712 | 59.8369 | 39.8914 | 393.9832 |
| 0.4 | 358.2704 | 71.6460 | 47.7639 | 388.5579 |
| 0.6 | 383.6992 | 98.1422 | 65.4282 | 388.5579 |
| 0.8 | 414.7223 | 108.6234 | 72.4157 | 378.4611 |
| 1  | 435.5493 | 106.8103 | 71.2072 | 377.7207 |

Therefore, when the recovery rate increases, since the same quantity waste products can yield more remanufactured products, the manufacturers are not willing to invest more in buying back waste products from retailers in order to avoid the yield of excessive remanufactured products; accordingly, the retailers decrease the collection volumes of waste products. Despite the decrease of the collected waste products, the volumes of remanufactured products still increase when the recovery rate increases (\( \beta < 0.5 \)). So increasing recovery rate within a certain region contributes to the boost of remanufacturing activity.

Combined Table 1 with Figure 2, when the collection rate is less than 0.2, the volumes of new products, collected wastes and remanufactured wastes are constant, so all prices are invariable in this case. When the collection rate increases from 0.2 to 0.8, the retailers increase the price of new products and drop the price of remanufactured products to attract more consumers to remanufactured products. Since the retailers need to collect more waste products, the collection difficulty increases and the prices of waste products increase. However, when the collection rate exceeds 0.8, it can be seen from Figure 2 that the volumes of new products and remanufactured products decrease while the collected waste products keep stable. The price of new products increases continually due to the decrease of new products in markets. With the decrease of remanufactured products, the price of remanufactured products increases so as to keep the profits of manufacturers and retailers. Because the retailers needn’t to collect more waste products, the price of the waste products decreases slightly.

Combined Table 2 with Figure 3, when the recovery rate increases (\( \beta < 0.5 \)), the volumes of remanufactured products increase more quickly than the new ones, so the manufacturers and retailers will increase the price of new products and decrease the price of the remanufactured ones to attract more consumers to remanufactured
Table 2. The prices under different recovery rate ($\beta$), given $\alpha = 1$

| $\beta$ | $w_i^f$ | $w_j^f$ | $w_i^r$ | $P_i^f$ | $P_j^f$ | $P_k^r$ | $P_j^r$ |
|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.1     | 422.8170 | 91.0883 | 424.1402 | 450.0756 | 60.7962 | 427.1726 |
| 0.2     | 424.1308 | 97.7935 | 409.5847 | 450.2139 | 65.1957 | 416.1012 |
| 0.3     | 427.8478 | 103.2339 | 394.2371 | 451.9405 | 68.8227 | 394.2371 |
| 0.4     | 433.8971 | 106.4136 | 378.9959 | 455.1846 | 70.9424 | 393.1814 |
| 0.5     | 435.5493 | 106.8103 | 375.6179 | 456.1049 | 71.2072 | 390.6693 |

Table 3. The profits of manufacturers and retailers with different collection rate ($\alpha$), given $\beta = 0.5$

| Total profits | $\alpha \leq 0.2$ | $\alpha = 0.4$ | $\alpha = 0.6$ | $\alpha = 0.8$ | $\alpha = 1$ |
|---------------|-------------------|----------------|----------------|----------------|----------------|
| Manufactures  | 13642.876         | 17247.346      | 15531.509      | 12759.803      | 10108.842      |
| Retailers     | 2995.498          | 2993.549       | 2794.858       | 2376.378       | 1916.658       |

Table 4. The profits of manufacturers and retailers with different recovery rate ($\beta$), given $\alpha = 1$

| Total profits | $\beta = 0.1$ | $\beta = 0.2$ | $\beta = 0.3$ | $\beta = 0.4$ | $\beta \geq 0.5$ |
|---------------|---------------|---------------|---------------|---------------|-----------------|
| Manufactures  | 9082.426      | 9616.632      | 9981.219      | 10112.544     | 10108.842       |
| Retailers     | 1669.838      | 1785.409      | 1944.052      | 2113.821      | 1916.658        |

products. The price of the waste products also increases with the boosting price of new ones. When the recovery rate is higher than 0.5, it can be seen from the Figure3 that the volumes of the new, remanufactured and waste products keep constant, so all the prices keep invariant.

Table 3 shows the profits of manufacturers are maximized at $\alpha = 0.4$ in this example. The profits of retailers are maximized when $\alpha \leq 0.2$. It is safe to conclude that both manufacturers and retailers are not willing to indefinitely increase collection rate when profit is of concern. Furthermore, since the retailers are close to the market and can respond quickly to the change of the market, the impact of collection rate changes on manufacturers is greater than its impact on retailers.

Table 4 shows the profits of manufacturers as well as retailers are maximized at $\beta = 0.4$ when $\beta \geq 0.5$, the volumes and prices of new, remanufactured and waste products keep fixed, so the profits of manufacturers and retailers will not change. In other words, it is not true that the higher the recovery rate is, the more the profits the manufacturers and the retailers will obtain. In addition, the impact of recovery rate changes on manufacturers is greater than that on retailers, which is the same as the collection rate.

4.2. Discussion of results. Some insights can be derived from the above analysis. Since the price of new products increase in collection rate, consumer benefits decrease in collection rate when they buy new products. In contrast, consumer benefits increase when they buy remanufactured products whose price decreases in
collection rate except when the collection target is higher than 80%. This suggests that as long as the collection target by the government is not set too high, the increase of collection rate can induce more consumers to switch to the remanufactured products. Besides, consumers benefit from returning waste products because their volumes and prices increase in collection rate as well as recovery rate. In addition, the price of remanufactured products decrease in recovery rate, hence the consumer benefits increase on condition that consumers buy remanufactured products.

Interestingly, the volume of collected products decreases in recovery rate. In other words, the collection activity is stifled by the increase of recovery rate. This reflects the interdependence of the collection rate and recovery rate. When recovery rate is increasing, both the volumes and the prices of new manufactured products increase (Table 2 and Figure 3). It indicates that manufacturers benefit from the increase of recovery rate by producing more new products. However, consumers benefit from buying remanufactured products with the increase of recovery rate.

Overall, the above results illustrate that even though collection and recovery of products incur additional costs, manufacturers still choose to buy back waste products within CLSC network. Moreover, for the government, some suggestions about how to set the collection and recovery targets can be provided: i) It is essential to regulate an appropriate collection rate target ensuring the volumes of collected waste products, which is prerequisite for the establishment of CLSC network. However, a too high collection rate target may lead to decrease in volumes but increase in prices for new manufactured products, and sometimes the decrease of profits for both manufacturers and retailers (Table 1, Table 3 and Figure 2). Consequently, the manufacturers and retailers may not be willing to abide by the legislation. So the collection rate target should be legislated based on the costs associated with different stakeholders; ii) Legislating that collected products must be recovered at a certain minimum target is necessary. The recovery rate is one of the representative and most widely used indicators for monitoring progress in waste recycling and resource-saving activities, thus many governments have incorporated it into national targets, such as European Union, USA and other developed countries (Hotta et al., 2016)[10]. However, Chinese WEEE legislations put more emphasis on collection but less on recovery. Recently they have not provided the recovery rates target, but only the recoverable rates (Wang and Chen, 2013)[24]. As demonstrated in this paper, although the collection activity is stifled by the increase of recovery rate, it is advantageous to manufacturers in producing more new products and attracting more consumers to remanufactured products. In short, it is indispensable to set the certain minimum recovery rates differentiated by product categories for Chinese government; iii) Retailer collection channel discussed in this paper is common in European Union and other developed countries, whereas only a few retailers in China have a designated role in WEEE collection. Currently WEEE collection in China is dominated by informal structures due to their cheaper labor forces, flexibility and high reimbursements to consumers (Salhofer et al., 2016)[19]. However, once WEEE enters the informal channels, informal dismantling and resale in secondary markets often take place, which significantly lower the recovery rate. Therefore, the Chinese government should take some measures (subsidies to the manufacturer and the retailer or governance mechanisms to control the informal collection agents) to strengthen the retailer collection channel, which has been verified as the most efficient reverse channel in environment performance and resource utilization; iv) To ensure that the CLSC network operates smoothly, the
legislation should also balance the benefits of all stakeholders, including manufacturers, retailers and consumers.

5. Conclusion. This paper has presented an equilibrium model of oligopolistic CLSC network based on the variational inequality approach. Retailers are responsible for collecting WEEE within the network. Focusing attention on the volumes in various tiers, the equilibrium framework provides a benchmark with which product volumes and prices can be compared. Furthermore, the differentiation between new manufactured and remanufactured products is considered. To ensure generality, asymmetric cost functions are provided in the numerical examples.

Qualitative properties of the equilibrium pattern are established, including the existence of a solution, as well as uniqueness under reasonable assumptions. The modified projection method is proposed for the computation of equilibrium prices and volumes. Several illustrative CLSC network examples with different exogenous variables are compared in the computations.

The equilibrium solutions suggest that WEEE legislation should impose a suitable collection target on the manufacturers and retailers. Meanwhile, the legislation should impose some minimum recovery level of collected products. This is also advantageous to manufacturers in producing new manufactured products and attracting the consumers to buy remanufactured products. Consumers can benefit from the increase of recovery rate and collection rate.

There are some limitations in our research. The model only focuses on the optimal decisions of players in a single-period environment and cannot fully describe the dynamic characteristics of the CLSC network. Besides, random factors are not considered in the demand functions. These problems could be viable directions for further research. Notwithstanding its limitations, this study does suggest some valuable insights for WEEE legislation.

Acknowledgments. The work was supported by the Fundamental Research Funds for the Central Universities of China (Project No. 2017XKQY034). The authors would like to thank Professor Qiang Meng and his doctoral student Yikai Huang from the Department of Civil Engineering, National University of Singapore, Singapore, for their assistance with variational inequalities. Also, the authors would like to thank the Editor and the two anonymous referees of this paper for their valuable comments.

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