GZK Horizons and the Recent Pierre Auger Result on the Anisotropy of Highest-energy Cosmic Ray Sources

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Abstract

Motivated by recent Pierre Auger result on the correlation of the highest-energy cosmic rays with the nearby active galactic nuclei, we explore possible ultrahigh energy cosmic ray (UHECR) source distributions and their effects on GZK horizons. Effects on GZK horizons by local over-density of UHECR sources are examined carefully with constraints on the degree of local over-density inferred from the measured UHECR spectrum. We include the energy calibration effect on the Pierre Auger data in our studies. We propose possible local over-densities of UHECR sources which are testable in the future cosmic ray astronomy.

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I. INTRODUCTION

Recently, Pierre Auger observatory published results on correlation of the highest-energy cosmic rays with the positions of nearby active galactic nuclei (AGN) \cite{1, 2}. Such a correlation is confirmed by the data of Yakutsk \cite{3} while it is not found in the analysis by HiRes \cite{4}. In the Auger result, the correlation is maximal for the threshold energy of cosmic rays at $5.7 \times 10^{19}$ eV, the maximal distance of AGN at 71 Mpc and the maximal angular separation of cosmic ray events at $\psi = 3.2^\circ$. With the same threshold energy, and the angular separation $\psi \leq 6^\circ$, the correlation remains strong for a range of maximal AGN distance between 50 Mpc and 100 Mpc. Due to increasing efforts on verifying the Auger result, it is worthwhile to examine the above correlation from a phenomenological point of view.

Since the angular scale of the observed correlation is a few degrees, one expects that these cosmic ray particles are predominantly light nuclei. The effect of GZK attenuations on these cosmic ray particles \cite{5, 6} can be described by a distance scale referred to as “GZK horizon” which is a function of the selected energy threshold for the arriving cosmic ray particles. By definition, the GZK horizon associated with a threshold energy $E_{\text{th}}$ is the radius of a spherical region which is centered at the Earth and produce 90% of UHECR events arriving on Earth with energies above $E_{\text{th}}$. With continuous energy loss approximation, the GZK horizon for protons with $E_{\text{th}} = 57$ EeV is about 200 Mpc by assuming a uniformly distributed UHECR sources with identical cosmic ray luminosity and spectral index \cite{7}. The calculations based upon kinetic equation approach or stochastic energy loss also reach to similar conclusions \cite{8, 9}.

The departure of theoretically calculated GZK horizon to the maximum valid distance of the V-C catalog \cite{10} employed in Pierre Auger’s analysis, which is around 100 Mpc, can be attributed to several factors. As mentioned in \cite{2}, such a deviation may arise from non-uniformities of spatial distribution, intrinsic luminosity and spectral index of local AGN. In addition, the energy calibration also plays a crucial role since the GZK horizon is highly sensitive to the threshold energy $E_{\text{th}}$. Energy values corresponding to the dip and the GZK cutoff of UHECR spectrum were used to calibrate energy scales of different cosmic ray experiments \cite{11, 12}. It has been shown that all measured UHECR energy spectra can be brought into good agreements by suitably adjusting the energy scale of each experiment \cite{11}. Keeping the HiRes energy scale unchanged, the energy-adjustment factor $\lambda$ is found to
be 1.2, 0.75, 0.83 and 0.625 respectively for Auger, AGASA, Akeno and Yakutsk. Furthermore, it has been shown that a different shower energy reconstruction method infers a 30% higher UHECR energy than that determined by Auger’s fluorescence detector-based shower reconstruction [13].

In this paper, we investigate the consistency between Auger’s UHECR correlation study and its spectrum measurement. As just stated, the V-C catalog used by Pierre Auger for the correlation study is complete only up to 100 Mpc while the GZK horizon for $E_{th} = 57$ EeV is generally of the order 200 Mpc. We first consider the local over-density of UHECR sources as a possible resolution to the above discrepancy. It is motivated by the existence of Local Supercluster (LS) which has a diameter of the order 60 Mpc. In LS, the over-density of galaxies has been estimated to be $\sim 2$ [14].

The local over-density of UHECR sources has been invoked [15, 16, 17] to account for AGASA data [18, 19]. Such a density distribution naturally leads to a smaller GZK horizon. However, it also significantly affects the UHECR energy spectrum in $(5 \cdot 10^{19} - 10^{20})$ eV region. Hence fittings to the measured UHECR spectrum [20] can provide information on the degree of local over-density. Subsequently, the magnitude of GZK horizon can be better estimated.

We next study the energy calibration effect on the estimation of GZK horizon and the spectrum of UHECR. Certainly a 20% – 30% upward shift on UHECR energies reduces the departure of theoretically calculated GZK horizon to the maximum valid distance of V-C catalog [2]. The further implications of this shift will be studied in fittings to the shifted Auger spectrum.

We fit the UHECR spectrum for events with energies above $10^{19}$ eV. This is the energy range where the GZK attenuation exhibits its effect. It is also the energy range where the local over-density of UHECR sources shows significant effects. In our analysis, we take the UHECR as protons, which is hinted in the Auger events with energies $\geq 57$ EeV although the composition study by the same group suggests a heavier composition for $E \leq 40$ EeV [21]. The HiRes experiment measures the composition up to 50 EeV [22] and obtains a composition lighter than that of Auger. For $E > 50$ EeV, the event number is still too small for the composition study. To fit the UHECR spectrum at the highest energy, it is more appropriate to treat the cosmic ray energy loss as a stochastic process [23]. There are numerical packages available for treating stochastic energy loss of cosmic ray particles.
We employ the latter package for our calculations. Although UHECR loses its energy mostly by scattering off CMB photons, it also loses some amount of energy by scattering off infrared background photons \[26, 27, 28, 29, 30\]. Thus we include the infrared photon contribution to the UHECR energy attenuation. Source evolution \(n(z) = n_0(1 + z)^3\) is adopted in the calculation of GZK horizon and spectrum, where \(n_0\) is the source number density at the present epoch. It is from the generally-accepted soft evolution model which traces the star formation history and has been adopted in previous works \[31\].

We discuss about GZK horizons in Sec. II. We calculate the accumulated event probabilities of UHECR for \(E_{th} = 57\) EeV, 70 EeV, 80 EeV and 90 EeV respectively. GZK horizons corresponding to different \(E_{th}\) are tabulated. We also calculate GZK horizons with local over-density of UHECR sources taken into account. In Sec. III, we fit the measured UHECR spectrum with various local over-densities of UHECR sources and obtain information on the degree of local over-density. To study the energy calibration effect, we also perform fittings to the shifted UHECR spectrum. Sec. IV contains discussions and conclusions.

## II. THE ACCUMULATIVE EVENT PROBABILITIES OF UHECR

For single UHECR source, the cosmic-ray energy attenuation is governed by the equation

\[
\frac{\partial \phi_N(E,t)}{\partial t} = \frac{\partial}{\partial E} \left[ \left( -\frac{dE}{dt} \right) \phi_N(E,t) \right],
\]

in the continuous energy loss approximation. This equation results from the number conservation of cosmic-ray particles in the energy attenuation process. The cosmic-ray energy loss per unit time \(-dE/dt\) is due to the cosmic expansion and its scattering with cosmic microwave background photons through photo-pion production process \(P\gamma \rightarrow N\pi\) and pair production process \(P\gamma \rightarrow Pe^+e^-\). The above attenuation equation is well known \[32\]. In the current context, the solution of Eq. (1) can be expressed in terms of the red-shift variable \[17\]

\[
\phi_N(E,z) = \phi_N(\bar{E},z_s) \times \exp \left[ \int_z^{z_s} dz' \left( \frac{(1 + z')}{H(z')} \times \frac{\partial b_0((1 + z')\bar{E})}{\partial \bar{E}} + \frac{1}{1 + z'} \right) \right],
\]

where \(z_s\) is the red-shift of the UHECR source and the function \(b_0\) is related to the rate of cosmic-ray energy loss at the present epoch by

\[
-\frac{dE}{dt}(z = 0) = b_0(E) + H_0 E,
\]
where $H_0$ is the present value of Hubble constant. The UHECR has an energy $E$ at the source with red-shift $z_s$ and its energy is downgraded to $E$ at the red-shift $z$. The energy $E$ is a function of $E$ and $z$ so that $\tilde{E}(E, z_s) = E$ and

$$
\frac{d\tilde{E}}{dz} = -\frac{b_0 (1 + z) \tilde{E}}{H(z)} (1 + z) - \frac{\tilde{E}}{1 + z}.
$$

(4)

Due to the non-trivial form of $b_0$, one resorts to numerical methods for computing the function $\tilde{E}$ and the flux $\phi_N(E, z)$.

We have mentioned that the stochastic nature of UHECR energy loss can not be overlooked for shorter propagation distances [23]. One then treats the energy attenuation by photo-pion production as a stochastic process while treating other attenuations as continuous processes.

FIG. 1: The accumulative event probability $P(D, E_{\text{th}})$ as a function of $D$ for $E_{\text{th}} = 57$ EeV, 70 EeV, 80 EeV and 90 EeV respectively. The horizontal dash line in each panel denotes $P(D, E_{\text{th}}) = 0.9$. The red, green, blue and black curves represent results from models with over-density $n(l < 30\text{Mpc})/n_0 = 1, 2, 4, 10$ respectively. The intrinsic spectrum index $\gamma = 2.4$, energy cut $E_{\text{cut}} = 1000$ EeV and the source evolution model $n(z) = n_0(1 + z)^3$ are used for calculations.

To facilitate our discussions, we define the accumulative event probabilities of UHECR
as
\[ P(D, E_{\text{th}}) = \frac{\int_0^D dl \cdot N(l, E_{\text{th}})}{\int_0^\infty dl \cdot N(l, E_{\text{th}})}, \]  
(5)

where \( N(l, E_{\text{th}}) \cdot dl \) is the number of cosmic ray events which are originated from sources at distances between \( l \) and \( l + dl \) from the Earth and arrive at the detector with energies above \( E_{\text{th}} \). We calculate \( P(D, E_{\text{th}}) \) for various local over-densities of UHECR sources. The source distribution over the red-shift is taken as \( n(z) = n_0(1 + z)^3 \) and the energy spectrum of each source is taken to be the form, \( \phi_N(E) \equiv dN/dE = AE^{-\gamma} \), with the maximal energy \( E_{\text{cut}} = 1000 \) EeV. We choose \( \gamma = 2.4, 2.5 \) and 2.6 where \( \gamma = 2.5 \) gives the best fitting to the measured UHECR spectrum as will be shown in the next section. The accumulative event probability \( P(D, E_{\text{th}}) \) for \( E_{\text{th}} = 57 \) EeV, 70 EeV and 90 EeV are shown in Fig. 1 for \( \gamma = 2.4 \). Results for \( \gamma = 2.5 \) and \( \gamma = 2.6 \) are not distinguishable from those for \( \gamma = 2.4 \). In each panel, the red, green, blue, and black curves represent local over-density \( n(l < 30\text{Mpc})/n_0 = 1, 2, 4, \) and 10 respectively. The local over-density \( n(l < 30\text{Mpc})/n_0 = k \) is defined explicitly as
\[
\begin{align*}
n(l < 30\text{Mpc})/n_0 &= k(1 + z)^3, \\
n(l \geq 30\text{Mpc})/n_0 &= (1 + z)^3.
\end{align*}
\]  
(6)

The horizontal dash line in each panel denotes \( P(D, E_{\text{th}}) = 0.9 \). The intersection of this line with each color curve gives the GZK horizon corresponding to a specific local over-density characterized by the ratio \( n(l < 30\text{Mpc})/n_0 \).

| \( n(l < 30\text{Mpc})/n_0 \) | \( E_{\text{th}} = 57 \) EeV | \( E_{\text{th}} = 70 \) EeV | \( E_{\text{th}} = 80 \) EeV | \( E_{\text{th}} = 90 \) EeV |
|---|---|---|---|---|
| 1 | 220 | 150 | 115 | 90 |
| 2 | 210 | 140 | 105 | 75 |
| 4 | 195 | 120 | 85 | 60 |
| 10 | 155 | 85 | 50 | 30 |

TABLE I: GZK horizons of UHECR calculated with the local over-density \( n(l < 30\text{Mpc})/n_0 = 1, 2, 4, \) and 10, and arrival threshold energy \( E_{\text{th}} = 57 \) EeV, 70 EeV, 80 EeV and 90 EeV respectively. The listed numbers are in units of Mpc.

GZK horizons corresponding to different local over-densities and \( E_{\text{th}} \) are summarized in Table I. It is seen that local over-densities up to \( n(l < 30\text{Mpc})/n_0 = 4 \) do not alter GZK
horizons significantly for a given $E_{\text{th}}$. One could consider possibilities for higher local over-densities. However, there are no evidences for such over-densities either from astronomical observations [14] or from fittings to the measured UHECR spectrum. We note that GZK horizons are rather sensitive to $E_{\text{th}}$. Table I shows that GZK horizons are $\sim 100$ Mpc or less for $E_{\text{th}} \geq 80$ EeV.

III. FITTINGS TO THE UHECR SPECTRUM MEASURED BY PIERRE AUGER

As mentioned earlier, the local over-density of UHECR sources affects the cosmic-ray spectrum at the highest energy, especially at energies higher than $5 \cdot 10^{19}$ eV. Hence the degree of local over-density can be examined through fittings to the measured UHECR spectrum as will be shown momentarily.

Fittings to the Auger spectrum have been performed in [33, 34, 35, 36]. In our work, we take into account the over-density of UHECR sources in the distance scale $l \leq 30$ Mpc. As stated previously, we take the UHECR to be protons. Figure 2 shows our

| $n(l < 30\text{Mpc})/n_0$ | 1   | 2   | 4   | 10  |
|--------------------------|-----|-----|-----|-----|
| $\gamma = 2.5$           | 14.12(9.34) | 14.61(9.93) | 17.09(10.50) | 28.09(13.93) |
| $\gamma = 2.6$           | 16.64(12.28) | 15.56(11.90) | 16.01(11.83) | 20.76(11.67) |

fittings to the Auger measured UHECR spectrum with $\gamma = 2.5$ and 2.6 respectively. We take the red-shift dependence of the source density as $n(z) = n_0(1 + z)^m$ with $m = 3$. We have fitted 12 Auger data points beginning at the energy $10^{19}$ eV. We make a flux normalization at $10^{19}$ eV while varying the power index $\gamma$ and the the degree of local over-density, $n(l < 30\text{Mpc})/n_0$. Part of $\chi^2$ values from our fittings are summarized in Table II. We found that $\gamma = 2.5$, $n(l < 30\text{Mpc})/n_0 = 1$ gives the smallest $\chi^2$ value with $\chi^2$/d.o.f. = 1.57. For the same power index, the large local over-density $n(l < 30\text{Mpc})/n_0 = 10$ is ruled out at the significance level $\alpha = 0.001$. For $\gamma = 2.6$, $n(l < 30\text{Mpc})/n_0 = 10$ is ruled out at the significance level $\alpha = 0.02$. 

TABLE II: The values of total $\chi^2$ from fittings to the Auger measured UHECR spectrum. Numbers in the parenthesis are $\chi^2$ values from fittings to the 8 data points in the energy range $19.05 \leq \log_{10}(E/\text{eV}) \leq 19.75$. The last 4 data points record events with energy greater than 71 EeV.
FIG. 2: Fittings to the Auger measured UHECR spectrum where the red, green, blue and black curves denote the model with the local over-density \( n(l < 30 \text{Mpc})/n_0 = 1, 2, 4, \) and 10 respectively. Solid curves correspond to \( \gamma = 2.6 \) while dash curves correspond to \( \gamma = 2.5 \). We take the source evolution parameter \( m = 3 \) throughout the calculations.

We note that, for both \( \gamma = 2.5 \) and \( \gamma = 2.6 \), the GZK horizon with \( n(l < 30 \text{Mpc})/n_0 = 10, \) \( E_{\text{th}} = 57 \text{ EeV}, m = 3 \) and \( E_{\text{cut}} = 1000 \text{ EeV} \) is about 155 Mpc. Since \( n(l < 30 \text{Mpc})/n_0 = 10 \) is clearly disfavored by the spectrum fitting, one expects a GZK horizon significantly larger than 155 Mpc for \( E_{\text{th}} = 57 \text{ EeV} \).

We next perform fittings to the shifted Auger spectrum. The results are shown in Fig. 3 where the cosmic ray energy is shifted upward by 30%. Part of \( \chi^2 \) values are summarized in Table III. The smallest \( \chi^2 \) value occurs approximately at \( \gamma = 2.4, n(l < 30 \text{Mpc})/n_0 = 2 \) with \( \chi^2/\text{d.o.f} = 0.82 \). For \( \gamma = 2.5, \chi^2/\text{d.o.f} = 1.31, 0.96 \) and 0.87 for \( n(l < 30 \text{Mpc})/n_0 = 1, 2 \) and 4 respectively. It is seen that \( \chi^2 \) values from current fittings are considerably smaller than those from fittings to the unshifted spectrum. Given a significance level \( \alpha = 0.1 \), it is seen that every local over-density listed in Table III except \( n(l < 30 \text{Mpc})/n_0 = 10 \) is consistent with the measured UHECR spectrum. It is intriguing to test such local over-densities as will be discussed in the next section.

We note that, with a 30% upward shift of energies, the cosmic ray events analyzed in Auger’s correlation study would have energies higher than 74 EeV instead of 57 EeV. The GZK horizon corresponding to \( E_{\text{th}} = 74 \text{ EeV} \) is 120 Mpc for \( n(l < 30 \text{Mpc})/n_0 = 2 \) and 105 Mpc for \( n(l < 30 \text{Mpc})/n_0 = 4 \).

We have so far confined our discussions at \( m = 3 \). In the literature, \( m \) has been taken as
FIG. 3: Fittings to the Auger measured UHECR spectrum with a 30% upward shift on UHECR energies where the red, green, blue and black curves denote the model with the local over-density $n(l < 30\text{Mpc})/n_0 = 1, 2, 4,$ and 10 respectively. Solid curves correspond to $\gamma = 2.4$ while dash curves correspond to $\gamma = 2.5$. We take the source evolution parameter $m = 3$ throughout the calculations.

TABLE III: The total $\chi^2$ values from fittings to the Auger measured UHECR spectrum with a 30% upward shift on UHECR energies. Numbers in the parenthesis are $\chi^2$ values from fittings to the 8 data points in the energy range $19.16 \leq \log_{10}(E/\text{eV}) \leq 19.86$. The last 4 data points record events with energy greater than 92 EeV.

| $n(l < 30\text{Mpc})/n_0$ | 1     | 2     | 4     | 10
|--------------------------|-------|-------|-------|-----|
| $\gamma = 2.4$           | 8.65(4.30) | 7.39(4.67) | 10.26(6.35) | 27.31(13.34) |
| $\gamma = 2.5$           | 11.82(6.16) | 8.67(5.49) | 7.78(5.23) | 16.18(7.39) |

any number between 0 and 5. It is demonstrated that the effect on UHECR spectrum caused by varying $m$ can be compensated by suitably adjusting the power index $\gamma$. Since GZK horizons are not sensitive to $\gamma$ and $m$, results from the above analysis also hold for other $m$'s.
IV. DISCUSSIONS AND CONCLUSIONS

We have investigated the consistency between Auger’s latest result on the correlation of UHECR sources with positions of nearby extra-galactic AGN and its measured UHECR spectrum. As stated before, this investigation is motivated by the fact that the V-C catalog used by Pierre Auger for the correlation study is reliable only up to 100 Mpc while the GZK horizon for $E_{\text{th}} = 57$ EeV is generally of the order 200 Mpc. We have explored the possibility for local over-density of UHECR sources, which is expected to shorten the GZK horizon for a given threshold energy of arrival cosmic-ray particles. This is indeed the case as can be seen from Table I. On the other hand, the effect is far from sufficient to shorten the GZK horizon at $E_{\text{th}} = 57$ EeV to $\sim 100$ Mpc for a local over-density of UHECR sources consistent with the measured UHECR spectrum.

We have performed an upward energy shift to the Auger measured UHECR spectrum. As said, an upward energy shift is motivated by simulations of shower energy reconstructions as well as the requirement of reproducing the theoretically predicted GZK cutoff energy. With a 30% energy shift, each cosmic ray event used by Auger for the correlation study would have an energy above 74 EeV instead of 57 EeV. GZK horizons corresponding to $E_{\text{th}} = 74$ EeV then match well with the maximum valid distance of V-C catalog. Fittings to the shifted Auger spectrum indicate a possibility for the local over-density of UHECR sources.

We point out that the local over-density of UHECR sources is testable in the future cosmic ray astronomy where directions and distances of UHECR sources can be determined. Table IV shows percentages of cosmic ray events that come from sources within 30 Mpc for different values of $E_{\text{th}}$ and $n(l < 30\text{Mpc})/n_0$. We take $\gamma = 2.4$, $m = 3$ and $E_{\text{cut}} = 1000$ EeV for calculating these percentages. We note that these percentages are not sensitive to the above parameters. For $E_{\text{th}} = 57$ EeV, only 17% of cosmic ray events come from sources less than 30 Mpc away for $n(l < 30\text{Mpc})/n_0 = 1$. For $n(l < 30\text{Mpc})/n_0 = 2$ and the same threshold energy, 30% of cosmic ray events are originated from sources in the same region.

It should be stressed that we have focused only on resolving the apparent discrepancy between the GZK horizon at $E_{\text{th}} = 57$ EeV and the maximum valid distance of V-C catalog. The statistics analysis for establishing the source correlation is an independent issue beyond the scope of the current paper. We have found that the above discrepancy can not be resolved by merely introducing the local over-density of UHECR sources. On the other hand,
TABLE IV: Percentages of cosmic ray events that come from sources within 30 Mpc for different values of $E_{\text{th}}$ and local over-density $n(l < 30\text{Mpc})/n_0$.

| $n(l < 30\text{Mpc})/n_0$ | $E_{\text{th}} = 57\text{ EeV}$ | $E_{\text{th}} = 70\text{ EeV}$ | $E_{\text{th}} = 80\text{ EeV}$ | $E_{\text{th}} = 90\text{ EeV}$ |
|--------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1                        | 0.17                          | 0.27                          | 0.36                          | 0.46                          |
| 2                        | 0.30                          | 0.43                          | 0.53                          | 0.63                          |
| 4                        | 0.46                          | 0.60                          | 0.70                          | 0.77                          |
| 10                       | 0.68                          | 0.79                          | 0.85                          | 0.89                          |

if Auger’s energy calibration indeed underestimates the UHECR energy, such a discrepancy can be reduced. More importantly, fittings to the shifted Auger spectrum indicate a possible local over-density of UHECR sources, which is testable in the future cosmic ray astronomy.

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