Measurements of Sub-Surface Velocity Fields in Faraday Flows

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Faraday waves are capillary ripples that form on the surface of a fluid being subject to vertical agitation. Although it is well known that the form and shape of the waves pattern depends on driving amplitude and frequency, only recent studies discovered the existence of a horizontal velocity field at the surface, called Faraday flow, which exhibits attributes of two-dimensional turbulence. However, despite the increasing attention towards the well-validated inverse energy flux in the Faraday flow and other not strictly 2-dimensional systems, very little is known about the velocity fields developing beneath the fluid surface. In this study planar velocity fields are measured by means of particle image velocimetry (PIV) with high spatial and temporal resolution on the water surface and below. A sharp reduction of velocity and vorticity values is observed immediately below the water surface, such that at 5 mm below the water surface the mean absolute velocities are 7 times smaller than the surface velocity. Additionally, the flow structures below the surface are found to comprise much larger spatial scales than those on the surface. These large structures are also found to be slow and temporarily persistent. At 5 mm from the container bottom a slight velocity recovery is observed which goes along with more ordered streak like structures in the velocity fields.

I. INTRODUCTION

Faraday waves are capillary ripples that form on the surface of a fluid being subject to vertical agitation. The resulting waves are known to form patterns that vary depending on driving amplitude and frequency. Because of the strong influence of boundary conditions, Faraday waves are subject to studies for a large variety of applications, ranging from bio-medicine to material sciences (e.g. controlled pattern formation, walking and orbiting of droplets). In capillary ripples, a complex and random transport of floating particles is generated by non-linear interactions at the surface of the Faraday wavefield, such as imperfections and traveling waves. However, only recent studies proved the existence of a horizontal velocity field at the surface, called Faraday flow, which was shown to exhibit attributes of two-dimensional (2D) turbulence. Recently the Faraday flow has also been used to control the dispersion of floaters with distinct geometries, opening up a new field of applications. One of the main features of Faraday flows is the presence of an inverse energy cascade. For 3D isotropic turbulence, energy is injected in the flow at large scales, and consequently transported to smaller scales through the vortex stretching mechanism, and finally dissipated through viscous effects. However, numerical and experimental results confirmed the presence of a dual energy cascade in case of 2D-turbulence (and references therein), as theoretically predicted by Kraichnan in 1967.

Energy is introduced at intermediate forcing scales and transferred upwards to larger scales, resulting in a net inverse energy flux. Under particular conditions, this phenomenon can even lead to energy condensation, by which large and ordered flow structures emerge from the seemingly disordered motion at small scales. In principle, this energy could be exploited as a potential source of renewable energy. Inversely, for wavelengths smaller than the forcing scale, an enstrophy cascade transfers enstrophy to the smaller wavelengths. Despite the increasing attention towards the well-validated inverse energy flux in Faraday flow and other not strictly 2-dimensional systems, very little is known about the flow structures developing beneath the surface.

This study aims at shedding light on the flow characteristics of the Faraday experiment, with particular focus on the velocities below the fluid surface. The velocity fields are measured by means of planar PIV with high spatial and temporal resolution at 6 horizontal planes at different heights.

II. MATERIALS AND METHODS

A. Water Container and Shaker Set-Up

Faraday waves are investigated in a circular container of acrylic glass (diameter 290 mm), similar to those used in pioneering studies filled with water at 21.5 °C. A depth of 30 mm is chosen for a deep water approximation, such that the depth is larger than the wavelength of the ripples at the surface. The container is vertically shaken by an electromagnetic shaker (TIRA vib). A schematic representation of the experimental set-up is shown in Fig. 1.

Monochromatic forcing at \( f_0 = 50 \text{ Hz} \) is imposed to the shaker from a function generator (RIGOL). The acceleration of the container is measured with an accelerometer (Kistler) and read out by a high-frequency digitalizer (Spectrum). The forcing is carried out with acceleration \( a = 0.47 \text{ g} \), whereas the threshold for the onset of Faraday waves is observed at \( a_{\text{th}} = 0.42 \text{ g} \). The resulting value for the supercriticality is thus, as defined by Francois et al. in 1972: \( \varepsilon = (a - a_{\text{th}})/a_{\text{th}} = 0.11 \). In this study, measurements are carried out for a much weaker shaking compared to the previous experiments, where a supercriticality of \( \varepsilon = 1.7 \) was chosen.
B. Camera and Image Acquisition

A second signal from the function generator is used to trigger the high-speed camera (PCO dmax HS2). The camera is synchronised with the dominant frequency of the waves, which is found at the first subharmonic of the driving frequency \( f = f_0/2 = 25 \text{ Hz} \), for a rate corresponding to 400 fps, or eight wave periods. The phase difference between the two signals from the function generator was then carefully monitored through the digitizer and tuned in order to capture the point of zero amplitude in the waves (flat surface).

The camera is placed at the side of the shaker supports, and an optical prism-mirror is used to deflect the camera line of sight in the vertical direction. At the chosen working distance, the camera resolution of 1400×1050 pixels. Images are saved in 16 bit format (.b16), and subsequently converted back to a 12 bit format, which corresponds to the actual bit depth of the camera.

C. PIV Measurements

Two PIV techniques are used for the measurements at and below the surface respectively, which mainly differ in the choice of illumination light source and particles employed. For the measurements beneath the waver level, red fluorescent polyethylene microspheres are used (diameter of 10-45 \( \mu \text{m} \), Cospheric), illuminated by a continuous wave argon laser (wavelength of 457-515 nm, Ion Technologies). An optical arrangement is used to deflect the laser beam (first upwards and later again horizontally) in order to generate a light sheet (60 mm wide, 1 mm thick) and to adjust the measuring plane height. The particles have a density of 0.995 g/cm\(^3\) - and uniformly disperse in the water volume, when additionally treated with a surfactant, as described below. A high-precision longpass filter is used to capture the fluorescence of the particles (peak at 607 nm) and simultaneously shield the camera sensor from the laser light. This technique was used to measure the velocity fields at five horizontal planes with height \( h \) from the container floor (\( h \) in \([5, 25] \text{ mm}\) in steps of 5 mm, see Fig. 1(b)).

However, due to total light refraction at the water surface (\( h = 30 \text{ mm} \)), the combination of laser and fluorescent particles could not be used to measure the velocity field at the water surface. In this case, a combination of floating hollow glass microspheres (diameter of approx. 70 \( \mu \text{m} \), Fibre Glast) and back-light (LED panel) was employed instead.

For both PIV techniques, 0.3 g of particles are wetted in a 10%-solids solution with a surfactant (1% Tween 80 solution, Polysorbate 80, non-ionic surfactant). This helps to uniformly disperse the naturally buoyant particles (fluorescent) in the water volume, and the same surfactant is used in all measurements in order to avoid differences in the waves (e.g. due to changes in surface tension).

With the available camera resolution (1400×1050 px), the resulting conversion factors for the spatial calibration of the field of view at \( h = [5, 10, 15, 20, 25, 30] \text{ mm} \) are respectively [19.495, 19.316, 19.15, 19.009, 18.957, 18.943] px/mm.

Fig. 1 provides a schematic representation of the experimental set-up for the two PIV techniques described above.

FIG. 1: (a) Schematic representation of the experimental set-up. A function generator (1) triggers the high-speed camera (2) and drives the shaker (3). The acceleration of the water container (4) is measured with accelerometers (5). A prism-mirror (6) is used to deflect the camera field of view. All the signals are monitored with a digitizer (7), whereas data is saved on a laboratory laptop. (8a) and (9a) depict the laser and its optics, whereas (8b) shows the LED panel for the backlight PIV. (b) Reference for the height \( h \) of the measurement planes, measured from the bottom.

III. RESULTS AND DISCUSSION

PIV measurements of the Faraday flow are carried out on the surface and at different depths in the water. Subsequently, PIV data is computed on grids with different refinement levels and time intervals. The analysis of the results is focused on the evolution of velocity fields, the size of recirculation and vortex structures, as well as gradient-based variables (e.g. vorticity) at different water heights. The \( z \)-component (normal to the planes) of velocity could not be reconstructed from the available set-up. For the following figures and diagrams, the notation \( \mathbf{u} = (u, v, w) \) will be used to denote the velocity field and its components in \( x \)- and \( y \)-direction respectively, and \( h \) will be used for the height of the measurement plane with respect to the container bottom (see Fig. 1(b)).

A. Velocity Fields

Fig. 2 shows an example of a velocity field at the water surface \( h = 30 \text{ mm} \), Fig. 2(a), whereas Fig. 2(b) shows the case for the sub-surface measurements \( h = 25 \text{ mm} \). The background image is an average of 6 successive experimental frames and provides visual validation of the PIV calculations. In Fig. 2(a) the background image corresponds to the actual raw images captured by means of backlight shadography,
whereas in 2(b) the original raw images have been inverted for better visualization of the velocity arrows. From the velocity field, a few characteristics of the Faraday flow on the water surface can easily be recognised, namely the presence of multiple vortices with variable length scales, as it was observed by von Kameke et al. in (5), as well regions of jet-like flow in which the flow is strongly accelerated, similar to the riverlike structures defined as “trajectory bundles” by Francois et al. in (8). By contrast, it is evident from the velocity-fields evaluated that larger and slower structures persist below the surface (also note the difference in velocity magnitude).

Fig. 3 depicts the root-mean-square (RMS) values of $u$, $v$-velocity components and the absolute velocity $|u|$ against height $h$. The values are averaged over all the available time steps, which vary with the height. At the surface, 624 time steps are available, whereas this decreases to 154, 103, 61, 62, 62 for $h = 25, 20, 15, 10, 5$ mm respectively. A dramatic difference can be appreciated for the RMS values of velocity at the surface and beneath it for both $u$ and $v$. In fact, the mean velocity magnitude drops by a factor of 7 in a thin layer of 5 mm right underneath the surface. Furthermore, it can be observed that the RMS velocities are symmetric at the surface (approximately 5 mm/s), whereas more pronounced differences appear for $h = 5, \ldots, 25$ mm, with the $u$-component being generally smaller than $v$. However, this is probably related to the chosen averaging time and field-of-view.

Unexpectedly, the analysis also revealed that the velocity RMS values show a recovery below the half height of the water level ($h = 15$ mm). For example, the averaged RMS values of $|u|$ increase by a factor close to 2. This phenomenon could be attributed to the vertical direction of the shaking, causing vertical streams in the normal direction being forced to deviate to the horizontal one as they impinge on the container bottom (as in a stagnation point flow), similar to the streaming phenomenon described by Schlichting and Gersten in (18).

The error bars in Fig. 3 show the standard deviation of the time-averaged signals, which confirm how much larger the velocity fluctuations are on the turbulent flow at the surface compared to the sub-surface fields.

The time-averaged velocity distributions are depicted in Fig. 4 for the water surface and the deepest plane ($h = 5$ mm from the bottom). A rather symmetric flow condition can be seen at the surface, where both $u$ (red) and $v$ (blue) are symmetric to 0 and show similar deviation and peak values and follow a Gaussian distribution. There is a substantial difference at the bottom of the container for $h = 5$ mm, where $v$ shows a clear bias towards positive values, and for which the bin counter is considerably larger than for $u$-velocities, which could be attributed to the presence of even larger structures that could not be captured with the current field of view and averaging time.
FIG. 3: Evolution of the RMS of velocity components at different heights. Values averaged over all the time steps, with error bars showing the standard deviation. In the 5 mm region right below the surface a dramatic drop in velocity magnitude occurs, resulting in mean RMS values being 7 times smaller than on the surface. At depths below the container half-height, the RMS velocities are more than doubled from the minimum at $h = 15$ mm.

FIG. 4: Time-averaged velocity distributions for $u$ (red) and $v$ (blue), at the water surface (a, b) and close to the container bottom (c, d). The symmetry of the velocity distribution can be appreciated for the measurements at the wave surface, and asymmetry prevails below the surface. All distributions show velocity values divided in 100 bins. Magnitude and number of counters vary according to height and grid spacing.

B. Vorticity Fields

The relative size and behaviour of the ordered structures in the velocity fields is further investigated by analyzing contours of vorticity, computed for the 2D case as $\omega = \partial v / \partial x - \partial u / \partial y$. In Fig. 5 an example of vorticity contours is presented for an instantaneous time step at four different measurement heights. The results reflect the considerations regarding the RMS values of velocities, and the expectations regarding the behaviour of the sizes of typical structures and vorticity intensity. On the wave surface (Fig. 5(a)), regions of alternating vorticity are densely distributed across the entire field of view. Immediately below the surface, the vorticity intensity drops significantly (more than one order of magnitude), as shown in Fig. 5(b).
Structures with local peaks in vorticity are still present, but more sparsely and in a less coherent order. Right below the surface, the planar velocity field is strongly impacted by the vertical oscillation at the surface.

The Faraday experiment has been recreated in a circular container (diameter 290 mm) filled up to a height of 30 mm with water. The container was vertically agitated with monochromatic forcing at a frequency of $f_0 = 50$ Hz and a forcing amplitude $a = 0.47$ g ($\varepsilon = 0.11$). 2D-velocity fields have been measured with PIV techniques on the surface and at different horizontal planes in the water. The results of this experiment have highlighted interesting flow features developing beneath the surface of a Faraday wavefield, which guide the attention for future investigations.

A lattice of counter-rotating vortices has been found on the water surface. The vortices have a diameter ranging from one to two Faraday wavelengths. Furthermore, jet-like structures have been found between these vortices, where the flow is accelerating and shows peaks in absolute velocity.

By analysing the mean RMS values of the velocity components and magnitude at different heights, it has been shown that within the water volume the flow is considerably slower than on the surface (which shows that most of the energy flow is localized on the surface or in a small layer below it). Below half depth of the water, an unexpected recovery in RMS velocity can be appreciated, which might be related to vertical components of the flow being forced in the horizontal direction by the presence of the bottom wall.

The analysis of the mean velocity distribution showed, as ex-

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**FIG. 5:** Instantaneous vorticity fields at four different heights. Vorticity computed as $\omega_z = \partial v / \partial x - \partial u / \partial y$ from reconstructed gradients in physical units (mm). Note the different scales between values at the surface (a) and below the surface (b)-(d). Black arrows qualitatively depict the local velocity field (size is not scaled across the four figures).
pected, a rather symmetric flow condition on the water surface. At further depths however, asymmetric distributions in velocity support the claim that larger and slower structures develop, which are not entirely resolved with the selected field of view and temporal interval of observation.

Results for instantaneous vorticity distribution and overlapping velocity fields highlight the presence of rotational flow at different depths. However, it has been shown that ordered structures are localised at the surface, and that the vorticity intensity drastically decreases right below it. Nevertheless, a strong vortical motion is present at the immersed planes, which has so far been neglected.

3D PIV/PTV measurements will be carried out in order to study the role of the third velocity component.

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