N=1 NO-SCALE SUPERGRAVITY FROM IIB ORIENTIFOLDS

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ABSTRACT

We describe the low-energy effective theory of N=1 spontaneously broken supergravity obtained by flux-induced breaking in the presence of $n$ D3 branes. This theory can be obtained by integrating out three massive gravitino multiplets in the hierarchical breaking $N = 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$ of a N=4 orientifold. This integration also eliminates the IIB complex dilaton. The resulting theory is a no-scale supergravity model, whose moduli are the three chiral multiplets that correspond to the three radii of $T_2 \times T_2 \times T_2$ in $T_6$, together with the $6n$ brane coordinates. The $U(n)$ gauge interactions on the branes respect the no-scale structure, and the N=1 goldstino is the fermionic partner of the $T_6$ volume.
1 Introduction

Type IIB orientifolds with flux-induced supersymmetry breaking \[1, 2\] offer interesting examples of models with moduli stabilization \[3, 4, 5, 6\] and hierarchical supersymmetry breaking with vanishing cosmological constant at the classical level.

They can be regarded as N-extended no-scale supergravities \[7, 8\] as far as the supersymmetry breaking does not stabilize all moduli, thus offering the possibility of creating a large hierarchy of scales \[4\].

String and M–theoretical constructions of these models have been established \[10, 11, 12\] but their low-energy description is only known in particular cases. Most of them involve \(N = 2 \rightarrow N = 1\) partial supersymmetry breaking, as in the case of models with R-R fluxes in Calabi-Yau compactifications \[4, 6, 13, 14, 15, 16, 17\].

Recently, the effective, spontaneously broken \(N=4\) supergravities corresponding to a type IIB orientifold with arbitrary fluxes have been investigated \[18, 19\]. They have \(0 \leq N' < 4\) surviving supersymmetries. Those Lagrangians offer the possibility to obtain effective theories with lower supersymmetry, by integrating out massive multiplets. For example, the \(N=3\) effective theory, obtained by integrating out a single massive gravitino multiplet is completely predicted by supersymmetry, and its symmetry breaking terms arise by the gauging of its (Abelian) isometries \[20\]. These isometries are related to the R-R scalars coming from the type IIB four form \[1, 2\].

In this paper, we would like to exhibit the particular simple form of the \(N = 1 \rightarrow N = 0\) supergravity action, inclusive of D3 brane non-Abelian gauge interactions, and show that it provides, as expected, an exact no-scale \(N=2\) supergravity model of the kind constructed long ago in the literature \[7, 8, 21\].

The main distinction from the case previously studied, is that the supersymmetry breaking does not involve the IIB complex dilaton, \(S\), since that field can be integrated out already at the level of the \(N = 4 \rightarrow N = 3\) breaking. Therefore, the effective theory contains just the three chiral multiplets associated with the three radii (volumes) of \(T_2 \times T_2 \times T_2 = T_6\), together with the brane coordinates, the \(U(n)\) gauge couplings, and the flux parameter \(\mu\).

2 Integrating out Massive Multiplets

The apparently puzzling feature arising from integrating out the massive gravitino multiplets, in the presence of \(n\) D3 branes, is that the scalar manifold of the truncated theory is not a submanifold of the original theory, as it is, instead, in the absence of D3 branes, when the original theory contains only moduli coming from the bulk, 10-d sector \[22, 23\]. The reason for this phenomenon is that the massless modes of the brane couple to massive bulk fields. So, for
instance, if $A^m_\mu$ is a massive bulk field, the truncated theory in the presence of branes is not obtained by setting $A^m_\mu = 0$, but rather by setting $A^m_\mu \sim \phi \partial_\mu \phi$, where $\phi$ denotes some brane coordinate. The effect of this procedure is a distortion of the original manifold, rather than a submanifold. This phenomenon was implied by the discussion of Frey and Polchinski, in the derivation of the $N = 3$ sigma-model metric in the presence of D3 brane coordinates. It becomes evident by writing out the gauge covariant derivative of the R-R axions,

$$b^{ij} = (b^{ij}, b^{i\bar{j}}), \quad I = (i, \bar{i}), \quad J = (j, \bar{j}).$$

The covariant derivative reads

$$D_\mu b^{ij} = \partial_\mu b^{ij} + C^{[i}_I \partial_\mu C^{j]}_I + \epsilon^{ijk} A_k,$$

and results in the elimination of the term

$$\left| C^{[i}_I \partial_\mu C^{j]}_I \right|^2.$$

Also, the D3 brane action term

$$g_{ij} \partial_\mu C^{i}_I \partial^\mu C^{j}_I, \quad g_{IJ} = (g_{ij}, g_{i\bar{j}}),$$

is eliminated altogether from the two-derivative action, when, in the presence of fluxes, the $g_{ij}$ components of the metric acquire a mass term (from the potential). The effect of these changes is that the original $N=4$ sigma model

$$SU(1,1) \times U(1) \times SO(6,6 + n) \times SO(6) \times SO(6 + n),$$

becomes

$$U(3, 3 + n) \times U(3 + n),$$

This is not a submanifold of the former, unless $n = 0$.

If one further integrates out the second gravitino multiplet, in the presence of D3 branes, one finds, using the same reasoning,

$$U(1, 1 + n) \times U(1 + n) \times U(2, 2 + n) \times U(2 + n),$$

One could say that the manifold in Eq. (5) is obtained by modding out an $R^6$ isometry of the manifold in Eq. (6), where the $R^6$ is gauged by the six vectors that become massive.

The complex type IIB dilaton is part of the massive $N=3$ gravitino multiplet. This is the reason why the $SU(1,1)/U(1)$ factor drops out after integrating out the massive modes.

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Here, the first factor is the manifold of vector multiplets, and the second is the manifold of hypermultiplets. By integrating out also the third gravitino multiplet (and a chiral multiplet) one arrives at a N=1 theory with scalar manifold
\[
\frac{U(1, 1 + n)}{U(1) \times U(1 + n)} \times \frac{U(1, 1 + n)}{U(1) \times U(1 + n)} \times \frac{U(1, 1 + n)}{U(1) \times U(1 + n)}, \quad N=1.
\] (8)

Let us confine ourselves to the N=1 theory, with a residual gravitino mass \(\mu\), related to the flux that breaks N=1 to N=0. The Kähler potential for the manifold in Eq. (8) is
\[
K = \sum_{i=1}^{3} K_i = - \sum_{i=1}^{3} \log(t_i + \bar{t}_i - C_i^j \bar{C}_j^i).
\] (9)

If we turn on the \(U(n)\) gauge interactions for the \(n\) D3 branes, the superpotential becomes
\[
W = \mu + f^{IJK} C_i^j C_j^k C_K^l \epsilon_{ijk}, \quad I = 1, \ldots, n^2,
\] (10)

while the D term is
\[
D^K = \sum_{i=1}^{3} \frac{1}{t_i + t_i - C_i^j \bar{C}_j^i} C_i^j f^{IJK} \bar{C}_j^i.
\] (11)

In the two previous equations, \(f^{IJK}\) are the \(SU(n)\) structure constants, and \(\mu\) is the flux.\footnote{In supergravity, this is a free parameter, but string theory imposes on it a quantization condition.} This theory is a no-scale model with scalar potential
\[
V = \sum_{i=1}^{3} e^K \left[ K C_i^j \frac{\partial W}{\partial C_i^j} \frac{\partial W}{\partial \bar{C}_j^i} \right] + \sum_I (D^I)^2
\]
\[
= e^K \left[ \sum_{i=1}^{3} \frac{1}{t_i + t_i - C_i^j \bar{C}_j^i} \frac{\partial W}{\partial C_i^j} \frac{\partial W}{\partial \bar{C}_j^i} \right] + \sum_I (D^I)^2.
\] (12)

The gravitino mass term is
\[
e^{K/2}|W| = \prod_{i=1}^{3} (t_i + \bar{t}_i - C_i^j \bar{C}_j^i)^{-1/2} |\mu + f^{IJK} C_i^j C_j^k C_K^l \epsilon_{ijk}|.
\] (13)

The vacuum is at \(V = 0\), i.e. \(\partial W/\partial C_i^j = 0\). Therefore, all \(C\)s belong to the Cartan algebra of \(U(n)\), and the gravitino mass is
\[
m = \mu \frac{1}{R_1 R_2 R_3},
\] (14)

with \(R_i\) the radius of the \(i\)-th 2-torus in \(T_2 \times T_2 \times T_2\).\footnote{In supergravity, this is a free parameter, but string theory imposes on it a quantization condition.}
3 Computation of the Scalar Potential

In this Section we collect all formulas used to derive Eqs. (12,13).

\[ V = e^K \left[ K^{a\bar{a}} D_a W \bar{D}_{\bar{a}} \bar{W} - 3|W|^2 \right], \]
\[ K^{a\bar{a}} = (K)_{a\bar{a}}^{-1}, \quad K_{a\bar{a}} = \partial_a \partial_{\bar{a}} K. \] (15)

\( K \) is given in Eq. (9), \( W \) is given in Eq. (10).

For each \( K \) we have

\[ K_{t_i \bar{t}_i} = \frac{1}{(t_i + \bar{t}_i - C_i^j \bar{C}_i^j)^2}, \quad K_{t_i} = -(t_i + \bar{t}_i - C_i^j \bar{C}_i^j)^{-1}, \]
\[ K_{t_i \bar{C}_i^j} = -C_i^j (t_i + \bar{t}_i - C_i^j \bar{C}_i^j)^{-2}, \quad K_{\bar{C}_i^j} = C_i^j (t_i + \bar{t}_i - C_i^j \bar{C}_i^j)^{-1}, \]
\[ K_{C_i^j \bar{C}_j^i} = \frac{\delta_{IJ}}{t_i + \bar{t}_i - C_i^j \bar{C}_i^j} + \frac{\bar{C}_i^j C_i^j}{(t_i + \bar{t}_i - C_i^j \bar{C}_i^j)^2}. \] (16)

The entries of the inverse Kähler metric are

\[ K^{t_i \bar{t}_i} = (t_i + \bar{t}_i - C_i^j \bar{C}_i^j)(t_i + \bar{t}_i), \]
\[ K^{t_i \bar{C}_i^j} = \bar{C}_i^j (t_i + \bar{t}_i - C_i^j \bar{C}_i^j), \]
\[ K^{C_i^j \bar{C}_j^i} = \delta_{IJ} (t_i + \bar{t}_i - C_i^j \bar{C}_i^j). \] (17)

Notice the important identity

\[ K_{t_i} K^{t_i} = -K_{C_i^j} K^{C_i^j} C_i^j = -\bar{C}_i^j. \] (18)

In computing the scalar potential, one must remark that the terms proportional to \(|W|^2\) in \( D_a W \bar{D}_{\bar{a}} \bar{W} K^{a\bar{a}} \) cancel the gravitino mass term \(-3|W|^2\). The cross terms proportional to \( \bar{W} \partial W / \partial C_i^j \), cancel out because of Eq. (18).

4 Symmetries of the Model

The no-scale supergravity so far described has some global symmetries besides the obvious \( U(n) \) gauge symmetry.

For \( \mu = 0 \), even in the presence of gauge interactions, the theory is invariant under T-duality transformations

\[ t_i \to 1/t_i, \quad C_i^j \to \frac{1}{t_i} C_i^j. \] (19)

This symmetry is lost for \( \mu \neq 0 \), unless \( \mu \) is replaced with \( \mu / t_1 t_2 t_3 \).

For any \( \mu \), the theory is always invariant under \( t_i \to t_i + i\beta_i \), and \( C_i^j \to \exp(i\alpha^i) C_i^j \), \( \alpha_i, \beta_i \in R \), \( \sum_{i=1}^3 \alpha^i = 0 \).
Notice that the above symmetries are sensitive to the cubic form of the superpotential. On the other hand, the no-scale structure of the model only depends on the particular form of the Kähler potential, as long as \( W \) is independent of the moduli \( t_i \). Some higher order \( \alpha' \) corrections have recently been computed, and they seem to affect the no-scale structure of these models \[25]\.

We finally notice that the \( N = 1 \to N = 0 \) phase is the only one that does not result in a loss of moduli. Indeed, starting with 21 metric deformations in the \( N=4 \) theory (without fluxes), 12 are lost in the \( N = 4 \to N = 3 \) breaking, together with the complex dilaton. Four more metric moduli are lost in the breaking \( N = 3 \to N = 2 \), and two more metric moduli are lost in the \( N = 2 \to N = 1 \) breaking. Therefore, in all these phases, some of the metric moduli acquire a nontrivial potential. They precisely correspond to the superpartners of the charged axions. The latter, are absorbed by the vector particles by the Higgs mechanism \[26]\.

The \( n \) D3 brane coordinates, together with their superpartners, are moduli in all phases, independently of the value of the four gravitino masses.

5 Conclusions

The interesting feature of the supergravity effective action of bulk fields coupled to \( n \) branes, is that it allows us to compute terms due to the gravitational back-reaction of the probe on the metric, as well as terms due to the non-abelian nature of the probe-brane action \[27]\.

These effects, which are taken into account automatically by the requirement of local supersymmetry, may be very hard to compute in a string calculation.

Interestingly enough, it turns out that these effects, at least in the approximation we are working on (two-derivative action), respect the exact no-scale structure, including the non-abelian interactions of the D3 branes.

An example of these effects is that the \( U(1) \) part of the \( U(n) \) gauge group, which decouples in the flat limit, has non-trivial gravitational couplings to the other modes, because it enters in the Kähler potential. The contribution of the non-Abelian brane coordinates to the gravitino mass term,

\[
e^{K/2} \gamma^{\mu \nu} \psi_{\mu} \psi_{\nu} f^{IJK} C_i^I C_j^J C_k^K \epsilon_{ijk},
\]

(20)

is also another effect of the bulk-brane couplings requested by local supersymmetry.

Needless to say, these theories may also possess vacua where \( \partial_C W \neq 0 \). If this is the case, these vacua have a positive cosmological constant so that they give rise to de Sitter spaces.

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