Irregular topological indices of certain metal organic frameworks

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Abstract: It is interesting to study the molecular topology that provides a base for relationship of physicochemical property of a definite molecule. The topology of a molecule and the irregularity of the structure plays a vital character in shaping properties of the structure like enthalpy and entropy. In this article, we are interested to calculate some irregular topological indices of two classes of metal organic frameworks (MOFs) namely BHT (Butylated hydroxytoluene) based metal (M = Co, Fe, Mn, Cr) organic frameworks (MBHT) and MiTPyP-M2 (TPyP = 5, 10, 15, 20-tetrakis (4-pyridyl) porphyrin and M1, M2, = Fe and Co) MOFs. Also we compare our results graphically.

Keywords: topological descriptors, irregularity indices, chemical graph theory, metal organic frameworks

1 Introduction

Metal organic frameworks (MOFs) are defined by their three-dimensional frameworks formed of metal ions and organic molecules. In MOFs, all metal ions and organic molecules form networks which can get a variety of guest molecules. MOFs have many applications, such as energy storage devices, gas storage, heterogeneous catalysis, and assessment of chemicals (Awais et al., 2020). In 1959, Kinoshita and coworkers were the first ones to study metal organic frameworks (Kinoshita, 1959). Due to their design and synthesis, MOFs gain attention rapidly (Hoskins, 1989). Till now a large number of MOFs have been synthesized and used in many applications especially in gas catalysis (Hall et al., 2016; Lee et al., 2009; Roy et al., 2012), delivery of drugs (Horcajada et al., 2008; Mandal et al., 2017; Vallet-Regi et al., 2007), sensing (Sarkisov et al., 2012), separation (Kim et al., 2017; Li et al., 2009), storage (Kennedy et al., 2013; Murray et al., 2009; Rosi et al., 2003), and absorption (Czaia et al., 2009; Geier et al., 2013; Mu et al., 2010; Park et al., 2017; Queen et al., 2014). MOFs are capable of capturing industrial gases, such as CO\(_2\), SO\(_2\), NO, CO, NO\(_2\), etc. (Dietzel et al., 2009; Lee et al., 2015; Wu et al., 2009). These gases are very dangerous for our environment. For example, the CO\(_2\) continuously changing our climate and effects greenhouse (Rodhe, 1990), acid rain, and smog is due to the emission of SO\(_2\) and NO\(_x\) (Singh and Agarwal, 2007), and CO and NO are very harmful for humans (Olson and Phillips, 1997). For a healthy environment, it is necessary to control these dangerous gases. MOFs have ability to reduce the quantity of CO\(_2\) at room temperature and low pressure. We can study the ability of MOFs to reduce flue gases in these articles (Chakarvarty et al., 2016; Howe et al., 2017; Tan et al., 2017; Yu et al., 2012). We can study for degree based topological invariants of metal-organic networks (Hong et al., 2020).

Let G(V,E) with vertex set V and edge set E be a connected graph of order n = |V(G)| and size m = |E(G)|. The number of edges associated with a vertex is the degree of that vertex. The quantitative topological categorization of irregularity of graphs has an increasing significance for analyzing the structure of deterministic and arbitrary networks and systems occurring in chemistry, biology and common networks. The idea of topological indices was given by Wiener (1947). He also found a strong relation among weiner index and physicochemical properties of compounds. But mathematicians did not work with interest on it for next 20 years. In the mid of 1970s, Wiener index gain popularity and gave some important research articles. After 1990s, a lot of work has done on other distance based topological indices closely related...
to Wiener index. Till now, thousands of topological indices (distance based and degree based topological indices) have found which plays a vital role in chemical graph theory.

Now we present the irregularity topological indices that is calculated here. Albertson (1997) defined a degree based index called the Albertson index (AL) as

\[ AL(G) = \sum_{uv \in E} |d_u - d_v| \]

Vučičević and Gasparov (2004) defined the irregularity index IRL and IRLU as

\[ IRL(G) = \sum_{uv \in E} |ld_u - ld_v| \quad \text{and} \quad IRLU(G) = \sum_{d \in V} \frac{|ld_u - ld_v|}{\min(d_u, d_v)} \]

Abdoo et al. (2014) defined the total irregularity index (IRRT) as

\[ IRRT(G) = \sum_{uv \in E} |d_u - d_v| \]

Gutman (2018) introduced the IRF(G) irregularity index that is. The Randić index (Li and Gutman, 2006) was described as

\[ IRF(G) = \sum_{uv \in E} \ln|d_u - d_v| \]

We have some more degree based irregularity topological indices that was studied in Reti et al. (2018). These degree based irregularity topological indices are defined as

1. \[ AL(G) = \sum_{uv \in E} |d_u - d_v| = (24cd + 1)[3 – 1] + (6c + 6d – 6)[3 – 2] \]
2. \[ IRL(G) = 28.668 cd + 1.282(2c + d) \]
3. \[ IRLU(G) = 50.6667cd + 1.6667(c + d) + 0.3333 \]
4. \[ IRRT(G) = 28cd + c + d \]
5. \[ IRF(G) = 104cd + 2(c + d) + 2 \]
6. \[ IRA(G) = 4.3336cd + 0.088(c + d) + 0.0906 \]

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Proof: We will use Figure 1 to prove all the above theorems. We can verify the values given in Table 1 for the edges of \( G_1(c,d) \).

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2. \[ IRL(G) = 28.668 cd + 1.282(2c + d) \]
3. \[ IRLU(G) = 50.6667cd + 1.6667(c + d) + 0.3333 \]
4. \[ IRRT(G) = 28cd + c + d \]
5. \[ IRF(G) = 104cd + 2(c + d) + 2 \]
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2 Irregularity topological indices of 2D structure of M1TPyP-M2 metalorganic frameworks

Let \( G_1(c,d) \) be the graph of 2D structure of M1TPyP-M2 metal organic frameworks, where ‘c’ and ‘d’ are the unit cells in a row and column respectively. The molecular graph of \( G_1(2,2) \) is shown below. We can verified that \( G_1(c,d) \) has 74cd number of vertices and 88cd-2c-2d+1 number of edges.

Theorem 2.1

Let \( G_1(c,d) \) be the graph of 2D structure of M1TPyP-M2 metal organic frameworks, then the irregularity indices of \( G_1(c,d) \) are:

1. \[ AL(G) = 56cd + 2(c + d) \]
2. \[ IRL(G) = 28.668cd + 1.2822(c + d) – 0.1836 \]
3. \[ IRLU(G) = 50.6667cd + 1.6667(c + d) + 0.3333 \]
4. \[ IRRT(G) = 28cd + c + d \]
5. \[ IRF(G) = 104cd + 2(c + d) + 2 \]
6. \[ IRA(G) = 4.3336cd + 0.088(c + d) + 0.0906 \]
7. \[ IRDIF(G) = 68.6672cd + 2.6666(c + d) + 0.0001 \]
8. \[ IRLF(G) = 30.0224cd + 1.2944(c + d) – 0.1397 \]
9. \[ LA(G) = 26.2858cd + 1.2572(c + d) – 0.2572 \]
10. \[ IRI(G) = 31.9112cd + 1.3862(c + d) – 0.2876 \]
11. \[ IRGA(G) = 3.5336cd + 10.8314(c + d) – 10.6876 \]
12. \[ IRBG(G) = 6.5744cd + 0.3188(c + d) – 0.0688 \]

Proof: We will use Figure 1 to prove all the above theorems. We can verify the values given in Table 1 for the edges of \( G_1(c,d) \).
4. \( \text{IRRT}(G) = \frac{1}{2} \sum_{(u,v) \in E} |du - dv| \)
\[
= \frac{1}{2} \left[ (24cd + 1)(3-1) + (6c + 6d - 6)(3-2) ight. \\
\left. + (56cd - 4c - 4d + 2)(3-3) \\
+ (8cd - 4c - 4d + 4)(4-3) \right] \\
= \frac{1}{2} \left[ 48cd + 2 + 6c + 6d - 6 + 8cd - 4c - 4d + 4 \right] \\
= 28cd + c + d
\]

5. \( \text{IRFG}(G) = \sum_{(u,v) \in E} |du - dv|^2 \)
\[
= \frac{1}{2} \left[ (24cd + 1)(3-1)^2 + (6c + 6d - 6)(3-2)^2 \\
+ (56cd - 4c - 4d + 2)(3-3)^2 \\
+ (8cd - 4c - 4d + 4)(4-3)^2 \right] \\
= 96cd + 4 + 6c + 6d - 6 + 8cd - 4c - 4d + 4 \\
= 104cd + 2(c + d) + 2
\]

6. \( \text{IRA}(G) = \sum_{(u,v) \in E} \left( \frac{1}{2} - \frac{1}{2} \right)^2 \)
\[
= \frac{1}{2} \left[ \left( \frac{1}{3} - \frac{1}{3} \right)^2 \\
+ (6c + 6d - 6) \left( \frac{1}{3} - \frac{1}{2} \right)^2 \\
+ (56cd - 4c - 4d + 2) \left( \frac{1}{3} - \frac{1}{3} \right)^2 \right] \\
= \frac{1}{2} \left[ \left( \frac{1}{3} - \frac{1}{3} \right)^2 + (6c + 6d - 6) \left( \frac{1}{3} - \frac{1}{2} \right)^2 + (56cd - 4c - 4d + 2) \left( \frac{1}{3} - \frac{1}{3} \right)^2 \right]
\]

**Figure 1:** 2 × 2 supercell of M1TPyP-M2 MOFs (M1, M2 = Fe and Co).

**Table 1:** Edge partition of 2 × 2 supercell of M1TPyP-M2 MOFs (M1, M2 = Fe and Co)

| (deg(x), deg(y)) where xy ∈ E(G1(c, d)) | Total number of edges |
|---------------------------------------|----------------------|
| (1, 3)                                | 24cd + 1             |
| (2, 3)                                | 6c + 6d - 6          |
| (3, 3)                                | 56cd - 4c - 4d + 2   |
| (3, 4)                                | 8cd - 4c - 4d + 4    |

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Table 1: Edge partition of 2 × 2 supercell of M1TPyP-M2 MOFs (M1, M2 = Fe and Co)
\[(8cd - 4c - 4d + 4)]
\[= 2(12c + 0.5 + 0.2(6c + 6d - 6)
+ 0.1429(8cd - 4c - 4d + 4)]
\[= 26.2858cd + 1.2572(c + d) - 0.2572]

10. \(IRD(G) = \sum_{uv \in E} \ln(1 + |du - dv|)\)
\[= (24cd + 1)\ln(1 + |3 - 1|)
+ (6c + 6d - 6)\ln(1 + |3 - 2|)
+ (56cd - 4c - 4d + 2)\ln(1 + |3 - 3|)
+ (8cd - 4c - 4d + 4)\ln(1 + |4 - 3|)
\[= 1.0986(24cd + 1) + 0.6931(6c + 6d - 6)
+ 0.6931(8cd - 4c - 4d + 4)
\[= 31.9112cd + 1 - 3862(c + d) - 0.2876]

11. \(IRGA(G) = \sum_{uv \in E} \ln \frac{du + dv}{2\sqrt{dudv}}\)
\[= (24cd + 1)\ln \frac{3 + 1}{2\sqrt{3 * 1}}
+ (6c + 6d - 6)\ln \frac{3 + 2}{2\sqrt{3 * 2}}
+ (56cd - 4c - 4d + 2)\ln \frac{3 + 3}{2\sqrt{4 * 3}}
+ (8cd - 4c - 4d + 4)\ln \frac{4 + 3}{2\sqrt{4 * 3}}
\[= 0.1438(24cd + 1) + 1.8121(6c + 6d - 6)
+ 0.0103(8cd - 4c - 4d + 4)
\[= 3.5336cd + 10.8314(c + d) - 10.6876]

12. \(IRB(G) = \sum_{uv \in E} \left(\frac{1}{du} - \frac{1}{dv}\right)^2\)
\[= (24cd + 1)\left(\frac{1}{3}^2 - (\frac{1}{3})^2\right)^2
+ (6c + 6d - 6)\left(\frac{1}{3}^2 - (\frac{1}{2})^2\right)^2
+ (56cd - 4c - 4d + 2)\left(\frac{1}{3}^2 - (\frac{1}{3})^2\right)^2
+ (8cd - 4c - 4d + 4)\left(\frac{1}{4}^2 - (\frac{1}{3})^2\right)^2
\[= 0.25(24cd + 1) + 0.1010(6c + 6d - 6)
+ 0.0718(8cd - 4c - 4d + 4)
\[= 6.5744cd + 0.3188(c + d) - 0.0688\]
3 Irregularity topological indices of 2D CoBHT(CO) lattice

Let G2(a,b) be the graph of 2D CoBHT(CO) lattice, where ‘a’ and ‘b’ are the unit cells in a row and column respectively. The molecular graph of G2(2,2) is shown below. We can verified that G2(a,b) has 27ab number of vertices and 36ab-2a-2b number of edges.

Theorem 2
Let G2(a,b) be the graph of 2D structure of CoBHT(CO) lattice, then the irregularity indices of G2(a,b) are:

1. \( AL(G) = 36ab - 2(a+b) \)
2. \( IRL(G) = 13.1832ab \)
3. \( IRLU(G) = 18ab + a + b \)
4. \( IRRt(G) = 18ab - a + b \)
5. \( IRF(G) = 60ab - 2(a+b) \)
6. \( IRA(G) = 0.294ab + 0.3082(a + b) \)
7. \( IRDIF(G) = 27.9996ab + 0.6668(a + b) \)
8. \( IRLFOI(G) = 13.3836ab + 0.0788(a + b) \)
9. \( LA(G) = 12.8ab - 0.0667(a + b) \)
10. \( IRDI(G) = 21.5004ab - 1.3862(a + b) \)
11. \( IRGA(G) = 22.452ab - 3.4544(a + b) \)
12. \( IRB(G) = 5.3292ab - 0.3882(a + b) \)

Proof: We will use Figure 2 to prove all above theorems. We can verify the values given in Table 2 for the edges of G2(a,b).

Table 2: Edge partition of 2 × 2 supercell of 2D CoBHT(CO) lattice

| (deg(x), deg(y)) where y < E(G2(a, b)) | Total number of edges |
|-----------------------------------------|-----------------------|
| (1, 3)                                  | 2a + 2b               |
| (2, 2)                                  | 2a + 2b               |
| (2, 3)                                  | 12ab - 2a - 2b        |
| (2, 4)                                  | 12ab - 2a - 2b        |
| (3, 3)                                  | 12ab                  |

1. \( AL(G) = \sum_{y \in E(G2)} |d_u - d_v| \)
   \( = (2a + 2b)|3 - 1| + (2a + 2b)|2 - 2| \)
   \( + (12ab - 2a - 2b)|3 - 2| \)
   \( + (12ab - 2b - 2b)|4 - 2| + 12ab|3 - 3| \)
   \( = 4a + 4b + 12ab - 2a - 2b + 24ab - 4a - 4b \)
   \( = 36ab - 2(a + b) \)

2. \( IRL(G) = \sum_{y \in E(G2)} |in_u - in_v| \)
   \( = (2a + 2b)|hn3 - hn1| + (2a + 2b)|hn2 - hn2| \)
   \( + (12ab - 2a - 2b)|hn3 - hn2| \)
   \( + (12ab - 2a - 2b)|hn4 - hn2| + 12ab|hn3 - hn3| \)
   \( = 1.0986(2a + 2b) + 0.4055(12ab - 2a - 2b) \)
   \( + 0.6931(12ab - 2a - 2b) = 13.1832ab \)
3. \( IRL(G) = \sum_{uv \in E} \frac{\|d_u - d_v\|}{\min(d_u, d_v)} \)
\[
= (2a+2b) \left[ 3 - \frac{1}{2} \right] + (2a+2b) \left[ 2 - \frac{3}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{2}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{4}{2} \right] + 12ab \left[ 3 - \frac{3}{2} \right] \\
= 4a + 4b + 6ab - a - b + 12ab - 2a - 2b \\
= 18ab + a + b
\]

4. \( IRR(G) = \sum_{uv \in E} |du - dv| \)
\[
= \frac{1}{2} \left[ (2a+2b) \left[ 3 - \frac{1}{2} \right] + (2a+2b) \left[ 2 - \frac{3}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{2}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{4}{2} \right] + 12ab \left[ 3 - \frac{3}{2} \right] \right] \\
= \frac{1}{2} \left[ 4a + 4b + 12ab - 2a - 2b + 24ab - 4a - 4b \right] \\
= 18ab + a - b
\]

5. \( IRF(G) = \sum_{uv \in E} (du - dv)^2 \)
\[
= (2a+2b)(3 - \frac{1}{2})^2 + (2a+2b)(2 - \frac{3}{2})^2 \\
+ (12ab - 2a - 2b)(3 - \frac{2}{2})^2 \\
+ (12ab - 2a - 2b)(4 - \frac{2}{2})^2 + 12ab(3 - \frac{3}{2})^2 \\
= 8a + 8b + 12ab - 2a - 2b + 48ab - 8a - 8b \\
= 60ab - 2(a + b)
\]

6. \( IRA(G) = \sum_{uv \in E} \left( \frac{1}{du - \frac{1}{2}dv} \right)^2 \)
\[
= (2a+2b) \left[ \frac{1}{2} \right] ^2 + (2a+2b) \left[ \frac{1}{2} \right] ^2 \\
+ (12ab - 2a - 2b) \left[ \frac{1}{2} \right] ^2 \\
+ (12ab - 2a - 2b) \left[ \frac{1}{2} \right] ^2 \\
+ 12ab \left[ \frac{1}{2} \right] ^2 \\
= 0.1786(2a+2b) + 0.0186(12ab - 2a - 2b) \\
+ 0.0059(12ab - 2a - 2b) \\
= 0.294ab + 0.3082(a + b)
\]

7. \( IRDF(G) = \sum_{uv \in E} \frac{du}{dv} - \frac{dv}{du} \)
\[
= (2a+2b) \left[ \frac{3 - 1}{2} \right] + (2a+2b) \left[ 2 - \frac{1}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{2}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 4 - \frac{2}{2} \right] + 12ab \left[ 3 - \frac{3}{2} \right] \\
= 2.6667(2a+2b) + 0.8333(12ab - 2a - 2b) \\
+ 1.5(12ab - 2a - 2b) \\
= 27.996ab + 0.6668(a + b)
\]

8. \( IRLF(G) = \sum_{uv \in E} \frac{|du - dv|}{\sqrt{(du - dv)}(du - dv)} \)
\[
= (2a+2b) \left[ \frac{3 - 1}{2} \right] + (2a+2b) \left[ 2 - \frac{1}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{2}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 4 - \frac{2}{2} \right] + 12ab \left[ 3 - \frac{3}{2} \right] \\
= 1.1547(2a+2b) + 0.4082(12ab - 2a - 2b) \\
+ 0.7071(12ab - 2a - 2b) \\
= 13.386ab + 0.0788(a + b)
\]

9. \( LA(G) = \sum_{uv \in E} \frac{|du - dv|}{(du - dv)} \)
\[
= 2 \left[ (2a+2b) \left[ \frac{3 - 1}{2} \right] + (2a+2b) \left[ 2 - \frac{1}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 3 - \frac{2}{2} \right] \\
+ (12ab - 2a - 2b) \left[ 4 - \frac{2}{2} \right] + 12ab \left[ 3 - \frac{3}{2} \right] \right] \\
= 2[2a + b + 2.4ab - 0.4a - 0.4b] \\
+ 4ab - 0.6667a - 0.6667b \\
= 12.8ab - 0.6667(a + b)
\]
10. \[
IRDI(G) = \sum_{u,v \in E} \ln \left\{ 1 + |du - dv| \right\}
= (2a+2b)\ln[1+|3-1|]+(2a+2b)\ln[1+|4-2|]
+(12ab-2a-2b)\ln[1+|3-2|]
+(12ab-2a-2b)\ln[1+|4-2|]
+12ab\ln[1+|3-3|]
= 1.0986(2a+2b)+0.6931(12ab-2a-2b)
+1.0986(12ab-2a-2b)
= 21.5004ab - 1.3862(a+b)
\]

11. \[
IRGA(G) = \sum_{u,v \in E} \ln \left( \frac{du + dv}{2(du dv)} \right)
= (2a+2b)\ln \left( \frac{3+1}{2(3x1)} + (2a+2b)\ln \left( \frac{2+2}{2(2x2)} \right) \right)
+(12ab-2a-2b)\ln \left( \frac{3+2}{2(3x2)} \right)
+(12ab-2a-2b)\ln \left( \frac{4+2}{2(4x2)} \right)
+12(ab)\ln \left( \frac{3+3}{2(3x3)} \right)
= 0.1438(2a+2b) + 1.8121(12ab-2a-2b)
+ 0.0589(12ab-2a-2b)
= 22.452ab - 3.4544(a+b)
\]

12. \[
IRBG(G) = \sum_{u,v \in E} \left( \frac{du}{2} - \frac{dv}{2} \right)^2
= (2a+2b)\left( \frac{1}{(3)^2} - \frac{1}{(3)^2} \right)^2
+ (2a+2b)\left( \frac{1}{(2)^2} - \frac{1}{(2)^2} \right)^2
+(12ab-2a-2b)\left( \frac{1}{(3)^2} - \frac{1}{(2)^2} \right)^2
+(12ab-2a-2b)\left( \frac{1}{(4)^2} - \frac{1}{(2)^2} \right)^2
+12ab\left( \frac{1}{(3)^2} - \frac{1}{(3)^2} \right)^2
= 0.25(2a+2b) + 0.1010(12ab-2a-2b)
+ 0.3431(12ab-2a-2b)
= 5.3292ab - 0.3882(a+b)
\]

| Irregularity indices | \( c = 1, \) | \( c = 2, \) |
|---------------------|------------|------------|
| 1. AL(G) = 56cd + 2(c + d) | 60 | 232 |
| 2. IRL(G) = 28.668cd + 1.2822(c + d) - 0.1836 | 31.2324 | 119.8008 |
| 3. IRLU(G) = 50.6676cd + 1.6667(c + d) + 0.3333 | 54.3334 | 209.6669 |
| 4. IRRt(G) = 28cd + c + d | 30 | 116 |
| 5. IRF(G) = 104cd + 2(c + d) + 2 | 110 | 426 |
| 6. IRA(G) = 4.3336cd + 0.088(c + d) + 0.0906 | 4.6002 | 17.7770 |
| 7. IRDF(G) = 68.6672cd + 2.6666(c + d) + 0.0001 | 74.0005 | 285.3354 |
| 8. IRLF(G) = 30.0224cd + 1.2944(c + d) - 0.1397 | 32.4715 | 125.1275 |
| 9. LA(G) = 26.2858cd + 1.2572(c + d) - 0.2572 | 28.543 | 109.9148 |
| 10. IRL(G) = 31.9112cd + 1.3862(c + d) - 0.2876 | 34.3960 | 132.9020 |
| 11. IRAG(G) = 3.5336cd + 10.8314(c + d) - 10.6876 + 14.5088 + 46.7724 |
| 12. IRBG(G) = 6.5744cd + 0.3188(c + d) - 0.0688 | 7.1432 | 27.5040 |

4 Graphical analysis and conclusions

Here we present the graphical analysis and comparison table (Table 3) of some of the irregularity indices of the graph of 2D structure of M1TPyP-M2 metal organic frameworks. Figure 3 contains the graphical values of Albertson index (AL(G)), irregularity IRL(G) and IRLU(G) indices, total irregularity index (IRRt), IRF(G) irregularity index, randic index (IRA(G)), irregularity index IRDIF(G), irregularity index IRLF(G), irregularity index LA(G), irregularity index IRDI(G), irregularity index IRGA(G), and irregularity index IRB(G). In this table, we can check the values of some irregularity indices for some different values of ‘c’ and ‘d’.

Now we present the graphical analysis and comparison table (Table 4) of some of the irregularity indices of the graph of 2D structure of CoBHT(CO) lattice. Figure 4 contains the graphical values of Albertson index (AL(G)), irregularity IRL(G) and IRLU(G) indices, total irregularity index (IRRt), IRF(G) irregularity index, randic index (IRA(G)), irregularity index IRDIF(G), irregularity index IRLF(G), irregularity index LA(G), irregularity index IRDI(G), irregularity index IRGA(G), and irregularity index IRB(G). In this table, we can check the values of some irregularity indices for some different values of ‘a’ and ‘b’.
Acknowledgements: All the authors are thankful to their respective institutes for providing the research facilities.

Funding information: Authors state no funding involved.

Author contributions: Yu-Ming Chu: writing – review and editing; Muhammad Abid: writing – original draft; Muhammad Imran Qureshi: writing – original draft; Asfand Fahad: writing – review and editing, methodology; Adnan Aslam: writing – review and editing, methodology.

Conflict of interest: Authors state no conflict of interest.

Data availability statement: Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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