Effective Potential of $\lambda \phi^4_{1+3}$ at Zero and Finite Temperature

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abstract
The effective potential of $\lambda \phi^4_{1+3}$ model with both sign of parameter $m^2$ is evaluated at $T = 0$ by means of a simple but effective method for regularization and renormalization. Then at $T \neq 0$, the effective potential is evaluated in imaginary time Green Function approach, using the Plana formula. A critical temperature for restoration of symmetry breaking in the standard model of particle physics is estimated to be $T_c \approx 510$ GeV.

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1 introduction

The symmetry breaking (SB) in quantum field theory (QFT) is an important problem both for particle physics and cosmology. Among various field models, the $\lambda\phi^4_{1+3}$ model attracts much attention. Its Lagrangian reads

$$L\{\phi(x)\} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4.$$ (1)

The coefficient of $\phi^2$ term may be positive ($m^2 > 0$) or negative ($m^2 = -\sigma < 0$, $\sigma > 0$). In the latter case, the model has spontaneous symmetry breaking (SSB) at the tree level, i.e., the lowest ground state (vacuum) will be shifted from $\phi = 0$ to $\phi = (6\sigma/\lambda)^{1/2}$. If the SB is caused by high loop ($L \geq 1$) calculation (so called as quantum radiative correction), then the SB is refered to as dynamical symmetry breaking (DSB). For studying the SB, a systematic method of loop expansion was developed by Coleman and Weinberg [1]. An effective potential (EP) $V(\hat{\phi})$ is derived to show where is the stable vacuum with $\hat{\phi}$ being the constant configuration of $<\phi>$, which is the quantum average (at zero temperature, $T = 0$) or thermodynamic average (at finite temperature, $T \neq 0$) of field $\phi(x)$. The theory of EP method was further elaborated by Jackiw [2]. After the pioneering work of Kirzhnits and Linde [3] and the suggestion of S. Weinberg, the EP at finite temperature was investigated by various authors [4-6]. It was found that SSB may be restored at some critical temperature $T_c$, i.e., $V(\hat{\phi})|_{T=T_c}$ takes a minimum at $\hat{\phi} = 0$ again.

The aim of this paper is to restudy SB by some new method in calculation. In Section II, a simple regularization and renormalization method will be introduced so that the ultraviolet divergence in Feynman diagram integral (FDI), the counter term and the ambiguity between bare and renormalized parameters will disappear. Then in Sections 3, 4 and 5, the EP at $T = 0$ and $T \neq 0$ will be evaluated respectively. For $T \neq 0$ case, basing on imaginary time Green Function method, the Plana Formula is used. For $m^2 < 0$ case, a critical temperature for restoration of SSB in the standard model is estimated to be 510 GeV. In Sec. 6 and 7, we consider the high loop correction and the fermion contribution to EP if a coupling term between a fermion field $\psi(x)$ and $\phi(x)$ is added to Eq. (1.1). The final Section 8 contains a summary and discussion. An Appendix is added to explain the Plana Formula.

2 mass correction – self energy

As a warm up, let us consider the simplest nontrivial FDI in $\lambda\phi^4$ theory. The 1 loop contribution to mass comes from the self energy diagram $\Sigma$ or tadpole diagram:

$$\Sigma = \frac{1}{2} \lambda \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon}$$ (2)

with $1/2$ being the symmetry factor. Turning to Euclidean space with the subscript $E$:

$$\Sigma = \frac{1}{2} \lambda \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + m^2}.$$ (3)

this FDI is quadratically divergent. Various kinds of regularization method have been proposed for handling it. After learning all of them, especially that in Refs. [7-11], begining from J-f Yang, we proposed a simple trick for calculating the chiral anomaly [12], i.e., to differentiate the FDI with respect to the external momentum enough times so that it becomes convergent. Then after integrating back to original one, we got some arbitrary constants as the substitution of the original divergence. We will extend this method to study the problem here with the external momentum replaced by some mass parameter.
Hence we differentiate Eq. (2.2) with respect to \( m^2 \) twice,

\[
\frac{d^2 \Sigma}{d(m^2)^2} = \lambda \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{(k_E^2 + m^2)^3}
\]

\[
= \frac{\lambda}{32\pi^2 m^2}.
\]

Then we obtain after integrating two times:

\[
\Sigma = \frac{\lambda}{32\pi^2} (m^2 \ln m^2 + Cm^2 + C')
\]

where \( C \) and \( C' \) are two arbitrary constants. As we will see immediately below that endowing \( C \) and \( C' \) some values amounts to some renormalization and corresponding explanation of the parameter \( m \). By using the well-known chain approximation, the propagator is modified from the lowest order one, \( G_2 \):

\[
G_2(p) = \frac{i}{p^2 - m^2}
\]

to

\[
\tilde{G}_2(p) = G_2 + G_2(-i\Sigma)G_2 + ...
\]

\[
= \frac{G_2}{1 + i\Sigma G_2} = \frac{i}{p^2 - m^2_R}
\]

with

\[
m^2_R = m^2 + \Sigma = m^2 + \frac{\lambda}{32\pi^2} (m^2 \ln \frac{m^2}{\mu^2} + C')
\]

where we rewrite the constant \( C \) as \((- \ln \mu^2)\) with \( \mu \) being an arbitrary mass scale.

Now our renormalization procedure amounts to setting \( C' = 0 \) so that when \( m^2 = 0 \), \( m^2_R = 0 \) also. Furthermore

\[
m^2_{R|\mu=m} = m^2
\]

defines the observed mass of particle in free motion. It also gives the explanation of parameter \( m \) in the model (1.1). Note that, however, we need not to change the notation of \( m \) whether it is used at tree level calculation or in QFT at higher loop order as in Eq. (2.7). In the latter case, the quantum field effect has been absorbed into the definition of \( m^2 \).

It is at the very most we can do in performing a calculation on self-energy \( \Sigma \). And it is also the best way to express the fact that we cannot obtain full information about mass generation mechanism via simple calculation on some FDI. Let us think the converse, suppose one finds a definite and finite \( \delta m \) via some calculation on \( \Sigma \). then it would be possible to generate mass \( m \) from a “bare” mass \( m_0 \) and the latter could be approaching to zero. Eventually one would claim that he can generate a mass from a massless model. It is incredible. Because if one generates a mass, say 3 grams, he must generates a standard weight 1 gram at the same time. Only the dimensionless ratio 3 can be calculated from a massless model. In short, what can emerge from a massless model is either no mass scale or two mass scales, but never one mass scale. This sceneario was clearly shown in the Gross-Neveu model [13] and a reinvestigation on NJL model [14] in Ref. [15] (see also [16]). The emergence of a massive fermion is accompanied by the change (phase transition) of vacuum which provides another mass scale (a standard weight). To a large extent, the stability and mass of a particle is ensured by its environment and not merely by itself. (The stability is related to the “imaginary part” of mass, i.e., the decay constant. For instance, a neutron has different half-life time in different nuclear environment).
So the mass generation mechanism is a nonperturbative process accompanying the change of environment (vacuum). On the other hand, the perturbative FDI calculation can at most provide some information about the change of mass with some parameter. In the example here, Eq. (2.3) does tell us the knowledge about $\frac{d^2\Sigma}{d(m^2)^2}$, but the FDI cannot tell us definitely about $\Sigma$ or $\frac{d\Sigma}{d(m^2)}$. Thus we understand that the emergence of divergence in FDI is essentially a warning. It warns us that we expected too much. We understand now that it is the indefiniteness or arbitrariness rather than the divergence which is implicitly implied by the appearance of cut off $\Lambda$ or pole $1/\epsilon$ in previous regularization methods.

Basing on above consideration, we are pleased to get rid of the divergence, the counter term and the bare parameter.

3 effective potential at zero temperature

For comparison, let us use the formulas and notation in Ref. [5] while denote $\hat{\phi} = \phi$ for brevity. The EP at tree level ($L = 0$) reads

$$V_0(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$ (10)

The one-loop contribution to EP is evaluated [1,2,4-6] as

$$V_1(\phi) = \frac{1}{2} \int \frac{d^4k_E}{(2\pi)^4} \ln(\frac{k^2_E}{M^2} + m^2 + \frac{1}{2}\lambda\phi^2)$$ (11)

Denoting

$$M^2 = m^2 + \frac{1}{2}\lambda\phi^2$$ (12)

we differentiate $V_1$ with respect to $M^2$ three times until it is convergent

$$\frac{\partial^3 V_1}{\partial (M^2)^3} = \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{(k^2_E + M^2)^3}$$

$$= \frac{1}{2(4\pi)^2} \frac{\lambda^2}{M^2}$$ (13)

Then integrate it with respect to $M^2$ three times with the result

$$V_1 = \frac{1}{2(4\pi)^2} \left[ \frac{M^4}{2} \ln M^2 - \frac{1}{2} M^4 + \frac{1}{2} C_1 M^4 + C_2 M^2 + C_3 \right]$$ (14)

where three arbitrary constants $C_1$, $C_2$ and $C_3$ are introduced. The renormalization simply amounts to fix these constants at our disposal.

For example, if $m^2 = 0$, we take $C_1 = -\ln\mu^2$ with $\mu$ an arbitrary mass scale and $C_2 = C_3 = 0$. Eq. (3.5) is simplified to

$$V_1 = \frac{1}{256\pi^2} \left[ \frac{\lambda^2}{2\mu^2} \ln \frac{\lambda\phi^2}{2\mu^2} - \frac{3\lambda^2}{2}\phi^4 \right]$$ (15)

Combining $V$ with $V_0$ in Eq. (3.1) and imposing the renormalization condition

$$\left. \frac{d^4}{d\phi^4} (V_0 + V_1) \right|_{\phi = M'} = \lambda$$ (16)
one recovers the famous Coleman-Weinberg EP [1]:

\[ V(\phi) = \frac{\lambda}{4!} \phi^4 + \frac{\lambda^2 \phi^4}{256 \pi^2} (\ln \frac{\phi^2}{M^2} - \frac{25}{6}) \] (17)

Now consider \( m^2 > 0 \) case. Choosing \( C_1 = -\ln m^2 \), one has

\[
V_1 = \frac{1}{2(4\pi)^2} \left\{ (m^2 + \frac{1}{2} \lambda \phi^2)^2 \left[ \frac{1}{2} \ln \frac{(m^2 + \lambda \phi^2/2)^2}{m^2} - \frac{3}{4} \right] + C_2 (m^2 + \frac{1}{2} \lambda \phi^2) + C_3 \right\}
\] (18)

If we further choose \( C_2 = m^2 \), \( C_3 = -\frac{1}{4} m^4 \), then

\[
V_1|_{\phi=0} = \frac{\partial V_1}{\partial \phi}|_{\phi=0} = \frac{\partial^2 V_1}{\partial \phi^2}|_{\phi=0} = \frac{\partial^3 V_1}{\partial \phi^3}|_{\phi=0} = \frac{\partial^4 V_1}{\partial \phi^4}|_{\phi=0} = 0
\] (19)

The whole EP combining \( V_0 \) and \( V_1 \) reads

\[
V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 + \frac{1}{2(4\pi)^2} \left\{ (m^2 + \frac{1}{2} \lambda \phi^2)^2 \left[ \frac{1}{2} \ln \frac{(m^2 + \lambda \phi^2/2)^2}{m^2} - \frac{3}{4} \right] + m^2 (\frac{1}{2} \lambda \phi^2 + \frac{3}{4} m^2) \right\}
\] (20)

with

\[
m^2 = \frac{\partial^2 V}{\partial \phi^2}|_{\phi=0}, \quad \lambda = \frac{\partial^4 V}{\partial \phi^4}|_{\phi=0}.
\] (21)

For \( m^2 = -\sigma < 0 \) (SSB case), the expression (3.5) remains valid with \( M^2 = -\sigma + \frac{1}{2} \lambda \phi^2 \). We take \( C_1 = -\ln \mu^2 \), then

\[
V = -\frac{1}{2} \sigma \phi^2 + \frac{\lambda}{24} \phi^4 + \frac{1}{2(4\pi)^2} \left\{ (\frac{1}{2} \lambda \phi^2 - \sigma)^2 \left[ \frac{1}{2} \ln \frac{\lambda \phi^2/2 - \sigma}{\mu^2} - \frac{3}{4} \right] + C_2 (\frac{1}{2} \lambda \phi^2 - \sigma) + C_3 \right\}
\] (22)

and

\[
\frac{dV}{d\phi} = \phi \{ -\sigma + \frac{\lambda}{6} \phi^2 + \frac{\lambda}{2(4\pi)^2} [ (\frac{1}{2} \lambda \phi^2 - \sigma)(\ln \frac{\lambda \phi^2/2 - \sigma}{\mu^2} - 1) + C_2 ] \} = 0
\] (23)

gives two solutions. One is

\[
\phi_0 = 0 \quad \text{(symmetric phase)}
\] (24)

To render the another one

\[
\phi^2_1 = 6\sigma/\lambda \quad \text{(SSB phase)}
\] (25)

formally coinciding with that from tree level. The choice of \( \mu^2 = C_2 = 2\sigma \) is necessary which also makes the mass excited at the broken vacuum has the same expression as that at the tree level:

\[
m^2_\sigma = \frac{d^2 V}{d\phi^2}|_{\phi=\phi_1} = 2\sigma
\] (26)

The derivatives of different order at two phases are summarized in the Table 1. The above assignment of \( C_1, C_2 \) and \( C_3 \) renders the appearance of imaginary part in \( V \) and its derivatives at symmetric phase (\( \phi_0 = 0 \)). It means the unstability of symmetric phase at the presence of stable SSB phase.

It is interesting to see that another choice

\[
C_1 = -\ln(-\sigma), \quad C_2 = -\sigma, \quad C_3 = -\frac{1}{4} \sigma^2
\] (27)

leaves only \( \phi_0 = 0 \) an extremum, i.e., semistable state with

\[
V(0) = \frac{dV}{d\phi}|_{\phi=0} = \frac{d^3 V}{d\phi^3}|_{\phi=0} = 0
\] (28)
and
\[ \frac{d^2V}{d\phi^2}\big|_{\phi=0} = -\sigma, \quad \frac{dV}{d\phi}\big|_{\phi=0} = \lambda \]  

(29)

No real SSB solution \( \phi_0^2 \neq 0 \) exists. Hence we see that two different choices of \( C_i \) lead to two different sectors of EP. The implication will be discussed at the final section.

Actually, we can always perform the high loop renormalization such that Eqs. (3.16) and (3.17) remain valid at any high order. This will be quite beneficial to discuss the recovery of SSB at high temperature (see section 5).

4 Effective potential at finite temperature

As shown clearly in Refs. [4-6], to study the temperature effects in QFT, one of most widely used methods is the imaginary time Green function approach, which amounts to replace the continuous fourth (Euclidean) momentum \( k_4 \) by discrete \( \omega_n \) and integration by a summation (\( \beta = 1/k_B T \)):

\[ k_4 \rightarrow \omega_n = \left\{ \begin{array}{ll} \frac{2\pi n}{\beta}, & n = 0, \pm 1, \ldots \text{(boson)} \\ \frac{\pi(2n+1)}{\beta}, & n = 0, \pm 1, \ldots \text{(fermion)} \end{array} \right. \]

\[ \int \frac{d^4k}{(2\pi)^4} \rightarrow \sum_n \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \]

(30)

Thus instead of Eq. (3.2), the one-loop contribution to EP at \( T \neq 0 \) reads

\[ V_1^\beta(\phi) = \frac{1}{2\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln[k^2 + \left(\frac{2\pi n}{\beta}\right)^2 + m^2 + \frac{1}{2} \lambda \phi^2] \]

(31)

Let us evaluate \( V_1^\beta \) by a rigorous trick. Because \( V_1^\infty(\phi) \equiv V_1(\phi) \) is already known, what we need is the difference, \( V_1^\beta - V_1 \), which turns out to be finite:

\[ V_1^\beta(\phi) - V_1(\phi) = \frac{1}{2\beta} \int \frac{d^3k}{(2\pi)^3} (\sum_n - \int dn) f(n) \]

\[ f(n) = \ln(n^2 + b^2) \]

\[ b^2 = \frac{\beta^2}{4\pi^2}(k^2 + m^2 + \frac{1}{2} \lambda \phi^2) \]

(32)

Since \( (\sum_{n=-\infty}^{\infty} - \int_{n=-\infty}^{\infty} dn) f(n) = 2(\sum_{n=0}^{\infty} - \int_{n=0}^{\infty} dn) f(n) \), where \( \sum_{n=0}^{\infty} f(n) = \frac{1}{2} f(0) + \sum_{n=1}^{\infty} f(n) \), we manage to calculate it by Plana formula [16,17] (see Appendix). Note that \( f(z) \) has two branch points at \( z = -bi \) and \( bi \). Introducing a cut from \(-bi\) to \( bi\), denoting \( \arg(z+bi) = \theta_1 > 0 \) (anticlock wise), \( \arg(z-bi) = \theta_2 < 0 \) (clockwise) and taking \( \theta_1 = \theta_2 = 0 \) at the left side of cut, one has \( z = it \):

\[ f(z) = \left\{ \begin{array}{ll} \ln |b-t| + \ln |-b-t| - i\pi, & (-\infty < t < -b) \\ \ln |b+t| + \ln |b-t|, & (-b < t < b) \\ \ln |b+t| + \ln |t-b| + i\pi, & (b < t < \infty) \end{array} \right. \]

(33)

Hence the formula (A.1) can be used to yield

\[ (\sum_{n=0}^{\infty} - \int_{0}^{\infty} dn) f(n) = -2\pi \int_{b}^{\infty} \frac{dt}{e^{2\pi t} - 1} \]

(34)
Substituting (4.5) into (4.3) and changing the order of integration with respect to \( k \) and \( t \), one has

\[
V_1^\beta(\phi) - V_1(\phi) = -\frac{1}{\pi \beta} \int_0^\infty \frac{dt}{\sqrt{m^2 + \lambda \phi^2}} \frac{k^2 dk}{e^{2\pi t} - 1} \int_0^\infty [(2\pi t)^2 - m^2 - \frac{1}{2} \lambda \phi^2]^{1/2} \kappa^2 dk
\]

\[
= -\frac{8\pi^2}{3\beta^4} \int_{m/\lambda}^{\infty} \frac{t^3 dt}{e^{2\pi t} - 1} (1 - \frac{m^2 + \lambda \phi^2/2}{(2\pi t/\beta)^2})^{3/2}
\]

Eq. (4.6) is rigorous but difficult to evaluate. As a high temperature approximation of (4.6), \( T \to \infty, \beta \to 0 \), we expand the parentheses to second term and set the low limit of integral as zero to yield:

\[
V_1^\beta(\phi) - V_1(\phi) \simeq -\frac{8\pi^2}{3\beta^4} \int_0^\infty \frac{t^3 dt}{e^{2\pi t} - 1} + \frac{m^2 + \lambda \phi^2/2}{\beta^2} \int_0^\infty \frac{tdt}{e^{2\pi t} - 1}
\]

\[
= -\frac{\pi^2}{90\beta^4} + \frac{m^2 + \lambda \phi^2/2}{24\beta^2}, \quad (T \to \infty, \beta \to 0)
\]

which coincides with Eq. (3.16) in Ref. [5] to the second term.

5 Symmetry restoration at finite temperature

In this case the EP at \( T = 0 \) is given by (3.13). For evaluating the EP at \( T > 0 \), we cannot use the results in Sec. 4 directly for small \( \phi \). Because now in Eq. (4.3) \( m^2 = -\sigma < 0 \).

\[
b^2 = -\frac{\beta^2}{4\pi^2} (\sigma - \frac{1}{2} \lambda \phi^2 - k^2) \equiv -a^2 < 0, \quad (if \lambda \phi^2/2 < \sigma, k^2 < \sigma - \lambda \phi^2/2)
\]

So we have to divide the integration with respect to \( k \) into two regions

\[
V_1^\beta(\phi) - V_1(\phi) = \frac{1}{4\pi^2 \beta} (I_1 + I_2)
\]

\[
I_1 = \int_0^{(\sigma - \lambda \phi^2/2)^{1/2}} k^2 dk [\sum_{n=0}^{\infty} (\int_0^\infty dn) \ln(n^2 - a^2)]
\]

\[
I_2 = \int_{(\sigma - \lambda \phi^2/2)^{1/2}}^{\infty} k^2 dk [\sum_{n=0}^{\infty} (\int_0^\infty dn) \ln(n^2 + b^2)]
\]

Evidently, \( I_2 \) can be calculated as that in Sec. 4. Let us concentrate on \( I_1 \).

Fortunately, there is a formula derived by Barton [18]

\[
(\sum_{n=0}^{\infty} - \int_0^\infty dn) \ln|\eta + n| = -2 \int_0^\infty \frac{dt}{e^{2\pi t} - 1} \tan^{-1} \frac{t}{\eta} + \theta(\eta) \ln|2 \sin \pi \eta|
\]

The first term changes sign with that of \( \eta \), so

\[
(\sum_{n=0}^{\infty} - \int_0^\infty dn) \ln(n^2 - a^2) = \ln|2 \sin \pi a|
\]

\[
I_1 = 2 \int_0^{(\sigma - \lambda \phi^2/2)^{1/2}} k^2 dk [2 \sin \beta \sqrt{\sigma - \lambda \phi^2/2 - k^2}]
\]

\[
I_2 = -4\pi \int_0^\infty \frac{dt}{e^{2\pi t} - 1} \int_{(\sigma - \lambda \phi^2/2)^{1/2}}^{[(2\pi t/\beta)^2 + \sigma - \lambda \phi^2/2]^{1/2}} k^2 dk
\]
In (5.3), after integration with respect to \( k \), we can expand it with respect to small \( t \) and keep one term only because the integration is dominant there

\[
I_2 \simeq -4\pi \frac{1}{3} \int_0^\infty \frac{dt}{e^{2\pi t} - 1} \left[ \sigma - \frac{1}{2} \lambda \phi^2 \right]^{3/2} \left[ \frac{3}{2} \frac{4\pi^2 t^2}{\sigma} - \frac{\lambda \phi^2}{\sigma} \right]^{1/2} = -\frac{8\pi^3}{\beta^2} \left( \sigma - \frac{1}{2} \lambda \phi^2 \right)^{1/2} \int_0^\infty \frac{t^2 dt}{e^{2\pi t} - 1} = -\frac{2\zeta(3)}{\beta^2} \left( \sigma - \frac{1}{2} \lambda \phi^2 \right)^{1/2}
\]

where \( \zeta(3) = 1.202 \).

We are interested in finding the critical temperature \( \beta_c \) for restoring the SSB. There are two possible points of view:

(a) Following Ref. [5] and noting Eq. (3.20) or Table 1, we may determine \( \beta_c \) from the following condition

\[
\left[ \frac{\partial^2}{\partial \phi^2} (V_1^\beta - V_1) \right]_{\phi=0} = \sigma
\]

which means that the curvature of EP at \( \phi = 0 \), \( \left[ \frac{\partial^2}{\partial \phi^2} V_1^\beta \right]_{\phi=0} \), turn from negative to positive at \( T_c \).

Combining Eqs. (5.2)-(5.7), we obtain after an integration by part:

\[
\left[ \frac{\partial^2}{\partial \phi^2} (V_1^\beta - V_1) \right]_{\phi=0} = \frac{1}{4\pi^2 \beta} \left\{ -\lambda \int_0^\sqrt{\sigma} dk \ln \left[ \frac{2}{\sqrt{\sigma}} \left( \sqrt{\sigma} - k \right) \right] + \frac{\zeta(3) \lambda}{\beta^2} \sigma^{1/2} \right\}
\]

In the high temperature approximation, the first term can be neglected:

\[
\left[ \frac{\partial^2}{\partial \phi^2} (V_1^\beta - V_1) \right]_{\phi=0} = \frac{\zeta(3) \lambda}{4\pi^2 \beta^3 \sigma^{1/2}}, \quad (T \to \infty, \beta \to 0)
\]

Combining (5.8) and (5.10), we find the critical temperature

\[
\frac{1}{\beta_c} = \left[ \frac{4\pi^2}{\lambda \zeta(3)} \right]^{1/3} \sigma^{1/2} \approx 3.202 \frac{\sigma^{1/2}}{\lambda^{1/3}}, \quad (\beta \to 0)
\]

The result of Eq. (3.17) in Ref. [5] reads

\[
\frac{1}{\beta_c} = \sqrt{\frac{24}{\lambda \sigma}} = 4.899 \frac{\sigma^{1/2}}{\lambda^{1/2}}, \quad (\beta \to 0)
\]

which was derived formally from the expression valid for \( m^2 > 0 \) as can also be seen from Eq. (4.7) in this paper.

(b) But for the restoration of SSB, a better criterion should be found directly from the vanishing of curvature at \( \phi = \phi_1 \). Basing on (4.7) and (3.13), we find the broken vacuum is shifted to \( \phi_1^\beta = \sqrt{\frac{6\sigma^2}{\lambda}} \) with approximately

\[
\sigma^\beta = \sigma - \frac{\lambda}{24\beta^2}
\]

\( \mu^2 = C_2 = 2\sigma \) as before. Then to the order \( O(\lambda) \), the vanishing condition of

\[
\left. \frac{d^2 V_1^\beta}{d\phi^2} \right|_{\phi=\phi_1^\beta} = 2\sigma - \frac{\lambda}{12\beta^2}
\]

yields the critical temperature

\[
T_c = \frac{1}{\beta_c} = \sqrt{\frac{12}{\lambda} m_{\sigma}}
\]
$m_{\sigma}$ being the excitation mass at $T = 0$. It is pleased to see that Eq. (5.15) is conciding with (5.12) (i.e., (3.20) in Ref. [5]).

As an interesting application of (5.15), in the standard model (electro-weak unified theory) of particle physics, the mass square of Higgs particle reads at tree level as

$$m^2_H = \frac{2}{3} \lambda |\phi|^2$$

(52)

while $|\phi>$ can be evaluated from the mass of $W$ boson

$$m_W = \frac{1}{\sqrt{2}} |\phi|$$

(53)

to be of the order

$$|\phi| \approx 180 GeV$$

(54)

combination of (5.15) ($m_{\sigma} \rightarrow m_H$) with (5.16) and (5.18) leads to an estimation that the SSB of standard model would be restored at a critical temperature

$$T_c = \sqrt{8} |\phi| \approx 510 GeV$$

(55)

6 high loop correction

Let us consider the higher loop correction, for example, the dominant two-loop contribution at $T = 0$ in a $O(N) \lambda \phi^4$ model for large $N$ reads (see Eq. (3.28a) in Ref. [5]):

$$V_2(\phi) = \frac{1}{6} \lambda \left( \frac{1}{2} N I \right)^2$$

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \mu^2 + \lambda \phi^2 / 2}$$

(56)

Using the same regularization-renormalization trick as that in Sec. 3, we arrive at

$$V_2(\phi) = \frac{\lambda N^2}{24} \left\{ \frac{1}{(4\pi)^4} \left[ (m^2 + \lambda \phi^2) \ln \frac{m^2 + \lambda \phi^2 / 2}{\mu^2} - 1 \right] + C_2 \right\}^2$$

(57)

One can set $\mu^2 = m^2$ and $C_2 = m^2$ such that

$$V_2(\phi)|_{\phi^2=0} = \frac{dV_2}{d\phi^2}|_{\phi^2=0} = 0$$

(58)

as mentioned at the end of Sec. 3. For $m^2 = -\sigma < 0$ case, similar formula can be found for SSB occurs at $\phi_t^2 = 2\sigma$ ($\mu^2 = C_2 = 2\sigma$). Furthermore, we consider the correction at $T \neq 0$

$$V_2^\beta - V_2 = \frac{\lambda N^2}{24} \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{1}{\beta} \sum_{n=\infty}^{\infty} f(n)^2 \right\} - \int \frac{d^3k}{(2\pi)^3} \frac{1}{\beta} \int_{-\infty}^{\infty} dn f(n)^2 \right\}$$

$$f(n) = \frac{\beta^2}{4\pi^2 [n^2 + \frac{\beta^2}{4\pi^2} (k^2 + m^2 + \lambda \phi^2 / 2)]}$$

(59)

Note that $f(n)$ is now a single-valued even function of $n$, so

$$V_2^\beta - V_2 = \frac{\lambda N^2}{24} \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{1}{\beta} \left( \sum_{n=0}^{\infty} + \int_{-\infty}^{\infty} dn \right) f(n) \right\} - \int \frac{d^3k}{(2\pi)^3} \frac{2}{\beta} \left( \sum_{n=0}^{\infty} - \int_{0}^{\infty} dn \right) f(n) \right\} = 0$$

(60)

due to Eq. (A.2).

We see that in this example there is no temperature dependent correction at two-loop order. It seems to us that this property persists to any higher order. Therefore, all temperature dependent correction to EP comes from one-loop graph as shown in Eq. (4.6)-(4.7) for $m^2 > 0$ or Eq. (5.2)-(5.7) for $m^2 < 0$, at least for $O(N)\lambda \phi^4$ model at large $N$ limit.
7 contribution of fermion field to effective potential

Assuming that there is a fermion field $\psi(x)$ coupling to the multiplet of Bose fields $\phi_a(x)$ (see Eq. (4.6) in Ref. [5]):

$$\mathcal{L}\{\phi_a(x), \psi(x)\} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m_f \bar{\psi}\psi - \bar{\psi} G^n \phi_a + \text{boson terms}$$

then the shifted “free” Lagrangian is

$$\hat{\mathcal{L}}\{\hat{\phi}, \phi(x), \psi(x)\} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - \bar{\psi} M \psi + \text{boson terms}$$

If the mass matrix after SB at $<\phi_a> = \hat{\phi}_a$

$$M = m_f + G_a \hat{\phi}_a$$

has the $i$th eigenvalue $M_i$, then one-loop EP at $T = 0$, apart from the boson contribution reads

$$V_{f1}(\hat{\phi}) = -2 \sum \int \frac{d^4k_E}{(2\pi)^4} \ln(k_E^2 + M_i^2)$$

As in Sec. 3, we obtain

$$V_{f1}(\hat{\phi}) = -2 \frac{1}{(4\pi)^2} \sum \left\{ \frac{1}{2} M_i^4 \ln \frac{M_i^2}{\mu^2} - \frac{3}{4} M_i^4 + C_2 M_i^2 + C_3 \right\}$$

For simplicity, we consider the case of one species: $M = m_f + G\phi_1$, ($\hat{\phi} = \phi$), and take $\mu^2 = 2\sigma = m_\sigma^2$, the mass square of excitation at broken phase $\phi = \phi_1 (\phi_1^2 = 6\sigma/\lambda$ in the absence of fermion). Then the constant $C_2$ can be chosen so that

$$\frac{\partial V_{f1}}{\partial \phi}|_{\phi = \phi_1} = 0$$

which leads to

$$\frac{\partial^2 V_{f1}}{\partial \phi^2}|_{\phi = \phi_1} = -\frac{8G^2}{(4\pi)^2} (m_f + G\phi_1)^2 \ln \frac{(m_f + G\phi_1)^2}{m_\sigma^2}$$

Hence we see that the coupling between $\phi$ field and a fermion with mass $M = m_f + G\phi_1$ may increase or decrease the mass of $\phi$ field depending on the condition $M < m_\sigma$ or $M > m_\sigma$.

An interesting question is the following. If in (7.2) one has only the free boson term

$$V_0 = \frac{1}{2} m_0^2 \phi^2$$

can the DSB occur via the coupling with a fermion?

Suppose the DSB occurs at $\phi = \phi_1 \neq 0$. For simplicity, we consider the case of one species of fermion, so $M = m_f + G\phi_1$ in (7.5). The criterion

$$\frac{d}{d\phi} (V_0 + V_{f1})|_{\phi = \phi_1} = 0$$

determines the constants

$$C_2 = \frac{(4\pi)^2}{4G} \frac{m_0^2 \phi_1}{G\phi_1 + m_f} + (G\phi_1 + m_f)^2$$

with

$$\mu = G\phi_1 + m_f$$
chosen for convenience. Then the mass square of excitation at broken vacuum reads

$$m_0^2 = \frac{d^2V}{d\phi^2} |_{\phi_1} = m_0^2 (1 - \frac{G\phi_1}{G\phi_1 + m_f})$$  \hspace{1cm} (72)

It looks meaningful. However, the value of $\phi_1$ in (7.10) and (7.11) is actually arbitrary. This ambiguity reflects the fact that the DSB is accompanying by the change of vacuum (environment), $\phi_1 \neq 0$, which is far beyond the reach of perturbative QFT. Especially, if $m_f = 0$, $m_0^2 = 0$, irrespective of the value of $\phi_1$.

This situation does happen in NJL model [14], in the simpler version of discrete symmetry

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi + \frac{1}{2}g^2 (\bar{\psi}\psi)^2$$  \hspace{1cm} (73)

By adding a term $-\frac{1}{2}[m_0\phi + g(\bar{\psi}\psi)]^2$ with $\phi$ an auxiliary scalar field (with dimension $[M]$) and an arbitrary mass scale $m_0$, we get

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi - \frac{1}{2}m_0^2\phi^2 - G\phi\bar{\psi}\psi$$  \hspace{1cm} (74)

where $G = gm_0$ is a dimensionless parameter. Then Eq. (7.12) tells us that we cannot discuss the DSB of NJL model in the present scheme. An improved nonperturbative treatment like Ref. [15] is needed.

It will be quite different to put the Lagrangian (7.13) in $1 + 1$ dimensional space, i.e., to discuss Gross-Neveu model [13]. By adding a term $-\frac{1}{2}[\sigma + g(\bar{\psi}\psi)]^2$ with auxiliary field $\sigma (\sim [M])$ and dimensionless $g$, we find the fermion contribution at 1 loop level

$$V_1(\sigma) = -\int \frac{d^2k_E}{(2\pi)^2} \ln(k_E^2 + g^2\sigma^2)$$

$$= \frac{1}{4\pi}g^2\sigma^2(\ln \frac{g^2\sigma^2}{\mu^2} - 1) + C_2$$  \hspace{1cm} (75)

with only two arbitrary constants $\mu^2$ and $C_2$. The DSB criterion ($V = V_0 + V_1$):

$$[\frac{d}{d\sigma}V]|_{\sigma = \sigma_M} = 0$$  \hspace{1cm} (76)

yields a relation between $\sigma_M$ and $\mu$

$$\sigma_M = \frac{\mu}{g}e^{-\pi/g^2}$$  \hspace{1cm} (77)

The mass square excited at $\sigma_M$ reads

$$m_\sigma^2 = \frac{d^2V}{d(\sigma/\sigma_M)^2}|_{\sigma_M} = \frac{g^2}{\pi}\sigma_M^2$$  \hspace{1cm} (78)

This is indeed meaningful as $m_\sigma$ and $\sigma_M$ are the only two massive observables, now relating each other. Note also that in (7.17), if $g^2 \to 0^+$, $\sigma_M \to 0$, $g^2 = 0$ is an essential singular point, so the mass generation mechanism is a nonperturbative process.

After the excursion on DSB, let us return to the finite temperature case. Using formula (4.1) and Eq. (7.4), we can easily write down the temperature correction on 1 loop fermion contribution as ($E_{M_i}^2 = k^2 + M_i^2$)

$$V_{f1}^\beta - V_{f1} = -2\sum_i \int \frac{d^3k}{(2\pi)^3} \frac{1}{\beta} (\sum_{n=-\infty}^{\infty} - \int_{-\infty}^{\infty} dn) \ln[\frac{(2n + 1)^2\pi^2}{\beta^2} + E_{M_i}^2]$$

$$= -\frac{2}{\beta} \sum_i \int \frac{d^3k}{(2\pi)^3} \left\{(\sum_{n=0}^{\infty} f(n) + \int_{0}^{\infty} dn)g(n) + (\sum_{n=0}^{\infty} f(n) + \int_{0}^{\infty} dn)g(n)\right\}$$  \hspace{1cm} (79)
where

\[ f(n) = \ln \left( \frac{1}{4} \left[ (2n + 1)^2 + 4b^2 \right] \right) \]

\[ g(n) = \ln \left( \frac{1}{4} \left[ (2n - 1)^2 + 4b^2 \right] \right) \]

\[ b^2 = \frac{\beta^2 E_{M_i}^2}{4\pi^2} \] (80)

Similar to the trick used in Sec. 4, we first consider the multi-valued function with complex variable \( z \):

\[ f(z) = \ln \left( \frac{1}{4} \left[ (z + \frac{1}{2})^2 + b^2 \right] \right) \] (81)

having two branch points

\[ z_{2,1} = -\frac{1}{2} \pm ib \] (82)

Rewrite

\[ f(z) = \ln(z - z_1)(z - z_2) \equiv \ln \rho_1 \rho_2 + i(\theta_1 + \theta_2) \] (83)

Let \( z = it \) running along the imaginary axis, \((-\infty < t < \infty)\). Then

\[ \rho_1 = \left[ \frac{1}{4} + (t + b)^2 \right]^{1/2}, \quad \rho_2 = \left[ \frac{1}{4} + (t - b)^2 \right]^{1/2} \]

\[ \theta_1 = 2\pi - \alpha_1, \quad \alpha_1 = \tan^{-1} \frac{1/2}{b + t} \]

\[ \theta_2 = -(2\pi - \alpha_2), \quad \alpha_2 = \tan^{-1} \frac{1/2}{b - t} \] (84)

Using Eq. (A.1), we obtain

\[
\left( \sum' - \int_0^\infty dn \right) f(n) = 2 \int_0^\infty \frac{dt}{e^{2\pi t} - 1} \left[ \tan^{-1} \frac{1/2}{b + t} - \tan^{-1} \frac{1/2}{b - t} \right]
\] (85)

But it is just cancelled by the contribution of \( \left( \sum' - \int_0^\infty dn \right) g(n) \), yielding

\[ V_{f_1} - V_{f_1} = 0 \] (86)

So besides the dependence of \( M_i \) via \( \phi_a \) on the temperature, there is no extra temperature dependence in the fermion contribution to EP of \( \phi \) field, Eq. (7.5), at least at the 1 loop level.

8 summary and discussion

(a) Basing on previous experiences in literature, we propose to use a simple but effective method for regularization and renormalization in QFT. Once encountering a superficially divergent FDI, we first take its derivative with respect to a mass parameter (or external momentum) enough times until it becomes convergent. After performing integration, we reintegrate it with respect to the parameter the same times for returning back to the original FDI. Now instead of divergence, several arbitrary constants \( C_i (i = 1, 2, \ldots) \) appear in FDI. Further renormalization amounts to reasonably fix \( C_i \) at one’s disposal.

(b) The physical essence of the appearance of these arbitrary constants \( C_i \), as discussed in Sec. 2, is ascribed to the lack of knowledge about the model at the level of QFT under consideration.
To be precise, the $\lambda\phi^4$ model at classical field theory (CFT) level is described by Lagrangian density $\mathcal{L}$, Eq. (1.1). While at QFT level, it is described by effective Hamiltonian density:

$$\mathcal{H}_{\text{eff}} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + V_{\text{eff}}(\phi)$$

(87)

with EP $V_{\text{eff}}$ given by $V_0 + V_1 + V_2 + \ldots$ in loop expansion as in this paper.

The important thing lies in the fact that $\mathcal{H}_{\text{eff}}$ is not only different from $\mathcal{H}$ (derived from $\mathcal{L}$ at the classical level) in form, but containing these arbitrary constants $C_1$, $C_2$ and $C_3$. While $C_3$ is trivial, the precise information, i.e., the exact value of $C_i$ ($i = 1, 2$) is not contained in $\mathcal{L}$ and cannot be predicted by the perturbative QFT. It is beyond our theoretical ability and so can only be fixed by experiment via suitable renormalization procedure.

(c) Similar situation occurs in the relation between quantum mechanics (QM) and classical mechanics (CM). In CM, the motion of a particle is determined by the classical Hamiltonian $H = T + V$. One can quantize it in QM to solve the Schrödinger Equation (SE)

$$\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$$

(88)

with $\hat{H} = \hat{T} + V$ in configuration space. Usually $V$ does not change. But in the case of singular potential, e.g., $V(r) = -Z \frac{e^2}{4\pi r}$, $Z > \frac{4\pi}{e^2} = 137$ in Dirac Equation or $V(r) \sim -\frac{1}{r}$ in SE, one soon runs into difficulty. After many years research, (see Chapter 2 in Ref. [16]), people realize that in this case the problem in QM is not well defined by Eq. (8.2), i.e., by $\hat{H}$ with suitable boundary conditions. We cannot find a complete orthonormal set $\{\psi_i(x)\}$ unambiguously. What we can do is to find a complete orthonormal set $\{\psi_i(x, \theta)\}$ which contains a (common) phase angle $\theta$. In other words, the Hilbert space is divided into infinite sectors characterizing by a continuous parameter $\theta$, which should be looked as a complement to the usual problem in QM from outside. To fix $\theta$ is beyond the ability of QM. An interesting example of a fermion moving in the field of a dyon was discussed in Ref. [19] (see also [16]) with the value of $\theta$ fixed eventually via the knowledge of QFT.

(d) Now for the QFT of $\lambda\phi^4$ model with $m^2 > 0$, since we are staying at symmetric phase $\phi = 0$, the renormalization (3.12) determine $C_i$. As discussed in Sec. 2, this is the only reasonable choice, because $m$ is the only observable mass parameter.

For the case of SSB, $m^2 = -\sigma < 0$, there are two observable mass scales, the shifted vacuum, $\phi_1 = \sqrt{\frac{\sigma}{\lambda}}$, and the excitation mass on it, $m_\sigma = \sqrt{2\sigma}$. They are related (similar to that in Gross-Neveu model [13]) with $C_i$ shown in Table 1. Note that, however, different choice of $C_i$ leads to different sector of EP. Maybe it is just the counterpart of $\theta$ in $\{\psi_i(x, \theta)\}$ of QM. Perturbative QFT is not well defined by $\mathcal{L}$ given in CFT, it should be complemented by some $C_i$. In this sense, $\lambda\phi^4$ model is nontrivial and renormalizable at QFT level.

(e) Due to the above advantage and the use of Plana formula, we evaluate the EP of $\lambda\phi^4$ model at both zero and finite temperature with the following results:

(i) At $T = 0$, we manage to keep the location of stable vacuum and mass excitation on it unaltered in form. Especially, in SSB case, $\phi_0^2 = 6\sigma/\lambda$ and $m_0^2 = 2\sigma$ remain as two observed quantities up to any loop orders.

(ii) When $T > 0$, the restoration of SSB occurs at some critical temperature $T_c$ given by Eq. (5.15). A value of $T_c$ in the standard model of particle physics is estimated to be $T_c \simeq 510$ GeV.

(iii) For pure $\lambda\phi^4$ model, the one-loop ($L = 1$) contribution to EP is temperature dependent. Whether higher ($L \geq 2$) contribution to EP is temperature independent needs further study.

(iv) Further coupling with fermions may modify the EP of $\phi$ field, but at least at $L = 1$ order, their contributions seem also temperature independent.
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A Plana Formula [17,16]

Theorem. Assuming a function \( f(z) \) is analytic on the right half complex plane, \( \text{Re} z > 0 \), then

\[
\sum_{n=1}^{\infty} f(n) + \frac{1}{2} f(0) - \int_{0}^{\infty} f(x) dx = \int_{0}^{i\infty} \frac{f(z) dz}{e^{2\pi iz} - 1} - \int_{-i\infty}^{0} \frac{f(z) dz}{e^{2\pi iz} - 1} \quad (89)
\]

Here \( C_1 \) and \( C_2 \) are two integration contours along the right side of imaginary axis on one sheet of Riemann surface of complex plane from \((-i\infty)\) to \(0\) and from \(0\) to \((i\infty)\) respectively.

If the integration along \( C_1 \) and \( C_2 \) does not encounter a cut, i.e., if \( f(z) \) is a single-valued function, one may set \( z = it \) in the integrand directly and obtain:

\[
\left( \sum_{n} - \int_{0}^{\infty} dn \right) f(n) \equiv \sum_{n=1}^{\infty} f(n) + \frac{1}{2} f(0) - \int_{0}^{\infty} f(n) dn = i \int_{0}^{\infty} \frac{f(it) - f(-it)}{e^{2\pi it} - 1} dt \quad (90)
\]

To our knowledge, some applications of Eq. (A.2) can be found in Refs. [20,21] whereas Eq. (A.1) was used to derive the Casimir effect [17]. This Plana formula had also been used extensively by Barton [18].

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Table 1: \( \mu^2 = C_2 = 2\sigma, \ C_3 = -\sigma^2 + (4\pi)^2 \frac{3\sigma^2}{\lambda} \)

|                      | SSB phase | Symmetric phase |
|----------------------|-----------|-----------------|
| \( \phi \)           | \( \phi_1 = \sqrt{\frac{6\sigma}{\lambda}} \) | \( \phi_0 = 0 \) |
| \( V \)              | 0         | \( -\frac{\sigma^2}{2(4\pi)^2}\left(\frac{15}{4} + \frac{1}{2}\ln 2 - i\frac{\pi}{2}\right) + \frac{3}{2} \frac{\sigma^2}{\lambda} \) |
| \( \frac{dV}{d\phi} \) | 0         | 0               |
| \( \frac{d^2V}{d\phi^2} \) | 2\sigma   | \( -\sigma[1 - \frac{\lambda}{2(4\pi)^2}(3 + \ln 2 - i\pi)] \) |
| \( \frac{d^3V}{d\phi^3} \) | \( \lambda \sqrt{\frac{6\sigma}{\lambda}} [1 + \frac{3\lambda}{2(4\pi)^2}] \) | 0               |
| \( \frac{d^4V}{d\phi^4} \) | \( \lambda[1 + \frac{9\lambda}{2(4\pi)^2}] \) | \( \lambda[1 - \frac{3\lambda}{2(4\pi)^2}(\ln 2 - i\pi)] \) |