Interplay of soft and perturbative correlations in multiparton interactions at central rapidities

B. Blok\textsuperscript{1}, M. Strikman\textsuperscript{2}

\textsuperscript{1} Department of Physics, Technion – Israel Institute of Technology, Haifa, Israel
\textsuperscript{2} Physics Department, Pennsylvania State University, University Park, USA

We study the role of soft/nonperturbative correlations in the multi parton interactions in the central kinematics relevant for double parton scattering (DPS) and underlying event (UE) measurements at ATLAS and CMS. We show that the effect of soft correlations is negligible for DPS regime (typical transverse momenta larger than 10-20 GeV), but may be important for UE (several GeV scale). The characteristic scale where soft correlations become important increases with decrease of $x$ (energy increase) leading to approximately constant $\sigma_{\text{eff}}$ at small $x$.

PACS numbers: 12.38.-t, 13.85.-t, 13.85.Dz, 14.80.Bn

Keywords:
I. INTRODUCTION.

It is widely realized now that hard Multiple Parton Interactions (MPI) occur with a probability of the order one in typical inelastic LHC proton-proton $pp$ collisions and hence play an important role in the description of inelastic $pp$ collisions. MPI were first introduced in the eighties $[1, 2]$ and in the last decade became a subject of a number of the theoretical studies, see e.g. $[3–17]$ and references therein.

Also, in the past several years a number of Double Parton Scattering (DPS) measurements in different channels in the central rapidity kinematics were carried out $[18–23]$, while many Monte Carlo (MC) event generators now incorporate MPIs.

It was pointed out starting with $[4, 24]$ that the rate of DPS can be calculated through the integral over the generalized parton distributions (GPD) under assumption that partons are not correlated and double GPD is simply a product of single GPDs. Information about GPDs at small $x$ is available from the analyses of the HERA $[24, 25]$. Based on these analyses it was demonstrated that the uncorrelated model predicts the DPS rates which are a factor $\sim 2$ too low to explain the data, hence indicating presence of significant parton - parton correlations.

It was pointed out in $[10–12, 16, 17]$ that correlations generated in the course of the DGLAP evolution – $1 \otimes 2$ mechanism – explain the DPS rates in the central rapidity region $[10–12, 16, 17]$ provided the starting scale for the QCD evolution – $Q^2_0 = 0.5 \div 1 \text{GeV}^2$ is chosen.

The role of $1 \otimes 2$ mechanism decreases however once we move to smaller $x$ $[10]$. On the other hand as it was explained in $[9]$ with the decrease of $x$ the relative importance of soft correlations in the nucleon increases. These correlations lead to a non-factorizable initial conditions for the evolution of double GPD at initial scale $Q_0$. In our recent paper $[26]$ we presented a simple way to take into account these correlations for the forward kinematics recently studied by LHCb which corresponds to $x \sim 10^{-4}$. The method is based on the connection between the correlation contribution to the MPI and inelastic diffraction $[26]$. We demonstrated that taking into account the mean field $2 \otimes 2$ contribution, $1 \otimes 2$ contribution and soft correlations leads to a good description of the DPS data in the forward LHCb kinematics $[27, 28]$. (These data are the most accurate data to date on the DPS and have the smallest background from the leading twist processes). Moreover it was pointed out that the account of the soft correlations leads to a strong reduction of the dependence of the predicted $\sigma_{\text{eff}}$ on the incident energy and $p_t$ of a minijet, and to $\sigma_{\text{eff}}$ being approximately constant for minijets contributing to the Underlying event (UE) $[26]$. Moreover, including soft correlations have led to a significant reduction of the sensitivity of the results to the value of the parameter $Q_0$.
separating soft and hard correlations [10].

Naturally, it will be interesting to check the consequences of the soft correlations model developed in [26] for the central kinematics at the LHC relevant for CMS and ATLAS experiments to clarify the role of soft correlations in this kinematics.

Hence in this paper we will extend the model of [26] to the central kinematics, and quantify the role of the soft correlations in central kinematics. We shall see that for large transverse scales \((Q > 20 \text{ GeV})\) DPS, considered in [10–12, 16–17], they constitute a small correction with dominant correlations originating from the perturbative \(1 \otimes 2\) mechanism. At the same time we find that soft correlations are important for the few GeV transverse scales corresponding to UE, leading to a weak dependence of \(\sigma_{\text{eff}}\) on \(Q\) for \(Q\) of the order several GeV followed by a decrease of \(\sigma_{\text{eff}}\) at \(Q > 10 – 15 \text{ GeV}\) largely due to the \(1 \otimes 2\) contribution. Also, similar to the case of the forward kinematics inclusion of the soft contribution makes the result less sensitive to the different initial scales \(Q_0\).

For simplicity we will limit our analysis to production of four jets in the gluon interactions. The paper is organized as follows. In section 2 we summarize the results of our previous analyses of the \(2 \otimes 2, 1 \otimes 2\) mechanisms in the central kinematics. In section 3 we discuss the soft correlations, and in section 4 investigate their role numerically. Our conclusions are presented in section 5.

II. MEAN FIELD APPROXIMATION AND \(1 \otimes 2\) MECHANISM ESTIMATE OF \(\sigma_{\text{eff}}\) IN CENTRAL KINEMATIC.

If the partons are uncorrelated (the mean field approach) the double parton GPDs, describing the DPS, are given by the product of the single parton GPDs:

\[
2 D(x_1, x_2, Q_1^2, Q_2^2, \Delta)) = D(x_1, Q_1^2, \Delta_1) \cdot D(x_2, Q_2^2, \Delta_2), \tag{1}
\]

where the one particle GPDs \(1 D\) are known from the analyses [25–29] of exclusive \(J/\Psi\) photoproduction at HERA. One can parameterize GPDs as

\[
D_1(x, Q^2, \Delta) = D(x, Q^2) F_{2g}(\Delta, x). \tag{2}
\]

Here \(D(x, Q^2)\) is the conventional gluon PDF of the nucleon, and \(F_{2g}(\Delta, x)\) is the two gluon nucleon form factor. The effective cross section \(\sigma_{\text{eff}}\) is then given by

\[
1/\sigma_{\text{eff}} = \int \frac{d^2\Delta}{(2\pi)^2} F^4(\Delta) = \frac{1}{2\pi} \frac{1}{B_g(x_1) + B_g(x_2) + B_g(x_3) + B_g(x_4)} \tag{3}
\]
We shall use exponential parametrization \cite{37} form factor. The production at HERA. They are parametrized as DPS, are here
\begin{equation}
B_0(x) = B_0 + 2K_Q \cdot \log(x_0/x)
\end{equation}
with $x_0 \sim 0.0012$, $B_0 = 4.1$ GeV$^{-2}$ and $K_Q = 0.14$ GeV$^{-2}$ (very weak $Q^2$ dependence of $B_g$ is neglected). Hence we find for the mean field value of $\sigma_{\text{eff}}$ at the LHC energies in the central kinematics $x_1 \sim x_2 = \sqrt{4Q_1^2/s}$, $x_3 \sim x_4 \sim \sqrt{4Q_2^2/s}$, $\sigma_{\text{eff}}^{MF}$ drops from $\approx 43$ mb at $Q_i \sim 2$ GeV to $\approx 37$ mb at $Q_i \sim 20$ GeV due to the increase of $x$ with increase of $Q_i$ and increase of the transverse area occupied by gluons with decrease of $x$, cf. eq.\cite{4}. For simplicity we shall consider here the symmetric case $Q_1 \sim Q_2$.

Presence of the $1 \otimes 2$ mechanism (Fig. 1b) in addition to uncorrelated mechanism of Fig. 1a leads to the enhancement of the rate of DPS (increase of $1/\sigma_{\text{eff}}$) as compared to its mean field value. The $1 \otimes 2$ mechanism was suggested in \cite{4,7,9,10}, where it was demonstrated that taking into account the pQCD DGLAP ladder splittings leads to a decrease of $\sigma_{\text{eff}}$.

We calculate $R_{pQCD}$ - the ratio of the contributions of $1\otimes2$ and mean field mechanisms by solving by iterations the evolution equation for $2GPD$ \cite{7}. The numerical results for the enhancement coefficient $R_{pQCD}$ for $\sigma_{\text{eff}}$ (here $\sigma_{DPS}$ includes $2 \otimes 2$ and $1 \otimes 2$ mechanisms) are summarized in Fig. 4 below.
III. NON-FACTORIZED CONTRIBUTION TO $2D$ AT THE INITIAL $Q_0$ SCALE.

There is an additional contribution to the DPS at small $x$ which was first discussed in \[9\], and in more detail in \[26\] which is related to the soft dynamics.

It was demonstrated in \[26\] that soft dynamics leads to positive correlations between partons at small $x$ which have to be included in the calculation of the DPS cross section. These soft correlations can be calculated using the connection between MPI and inelastic diffraction.

This non-factorized contribution to $2GPD$ is calculated at the initial scale $Q_0^2$ that separates soft and hard physics and which we consider as the starting scale for the DGLAP evolution. One expects that for this scale the single parton distributions at small $x$ are given by the soft Pomeron and soft Reggeon exchange.

\begin{equation}
2D(x_1, x_2, Q_0^2)_{nf} = c_{3IP} \int_{x_m/a}^{1} \frac{dx}{x^2} D(x_1/x, Q_0^2) D(x_2/x, Q_0^2) \left(\frac{1}{x}\right)^{\alpha_P} + c_{IP\Phi\Phi} \int_{x_m/a}^{1} \frac{dx}{x^2} D(x_1/x, Q_0^2) D(x_2/x, Q_0^2) \left(\frac{1}{x}\right)^{\alpha_R}
\end{equation}

Here $x_m = max(x_1, x_2)$. We also introduced an additional factor of $a = 0.1$ in the limit of integration over $x$ (or, equivalently, the limit of integration over diffraction masses $M^2$) to take into account that the Pomeron exchanges should occupy at least two units in rapidity, i.e. $M^2 < \ldots$
0.1 \cdot \min(s_1, s_2) (s_{1,2} = m_0^2/x_{1,2}), or x > \max(x_1, x_2)/0.1, where m_0^2 = \frac{m_N^2}{2} = 1 \text{ GeV}^2 is the low limit of integration over diffraction masses. Here \(c_{3P}\) and \(c_{P.P.R}\) are normalized three Pomeron and Pomeron-Pomeron-Reggeon vertices. We determine \(c_{3P}\) and \(c_{P.P.R}\) from the HERA data [30] for the ratio of inelastic and elastic diffraction at \(t = 0\):

\[
\omega \equiv \left. \frac{d\sigma_{\text{in.dif.}}}{d\sigma_{\text{el.}}} \right|_{t=0} = 0.25 \pm 0.05, \tag{6}
\]

and from analysis of diffraction for large \(x\) carried in [31], which shows that \(c_{P.P.R} \sim 1.5c_{3P}\).

We are considering here relatively low energies (relative large \(x\)) and a rather modest energy interval. Hence we neglect energy dependence of \(c_{3P}\). Numerically, we obtain \(c_{3P} = 0.075 \pm 0.015, c_{P.P.R} \sim 0.11 \pm 0.03\) for \(Q_0^2 = 0.5 \text{ GeV}^2\) and \(c_{3P} = 0.08 \pm 0.015\) and \(c_{P.P.R} = 0.12 \pm 0.03\) for \(Q_0^2 = 1. \text{ GeV}^2\), using the Pomeron intercept values given below.

Note that the intercept of the Pomeron that splits into 2 (region between 2 blobs in fig. 3) is always 1.1 for \(t = 0\), i.e. this Pomeron is by definition soft, and the intercept of Reggeon is 0.5.

Note that in [26] we used the model that contained only dominant Pomeron component. Here we need to consider both Pomeron and Reggeon contributions, since we need to go to larger \(x\) and smaller diffractive masses. Hence we need to take into account the Reggeon contribution which gives dominant contribution to diffraction for small diffractive masses. Hence the value of \(c_{3P}\) here is slightly different from the one in [26]. However in the kinematics we are interested in, once the value of \(\omega\) is fixed, the numerical results depend only weakly on \(c_{3P}/c_{P.P.R}\) ratio.

For the parton density in the ladder we use [26]:

\[
xD(x, Q_0^2) = \left(1 - \frac{x}{x^0(\mu^2)}\right), \tag{7}
\]

where the small \(x\) intercept of the parton density \(\lambda\) is taken from the GRV parametrization [32] for the nucleon gluon pdf at \(Q_0^2\) at small \(x\). Numerically \(\lambda(0.5\text{GeV}^2) \sim 0.27, \lambda(1.0\text{GeV}^2) \sim 0.31\).

Consider now the \(t = -\Delta^2\) dependence of the above expressions. The \(t\)-dependence of elastic diffraction is given by

\[
F(t) = F_2g(x_1, t) = \exp(B_{el}(x_1)t). \tag{8}
\]

Thus the \(t\) dependence of the factorized contribution to \(2D_f\) is given by

\[
F(t) = F_{2g}(x_1, t) \cdot F_{2g}(x_2, t) = \exp((B_{el}(x_1) + B_{el}(x_2))t/2), \tag{9}
\]

where \(F_{2g}\) is the two gluon nucleon form factor.
The t-dependence of the non-factorized term eq. 5 is given by the t-dependence of the inelastic diffraction: \( \exp(B_{\text{in}}t) \). Using the exponential parameterization \( \exp(B_{\text{in}}t) \) for the t-dependence of the square of the inelastic vertex \( pM_{\chi\gamma P} \), the experimentally measured ratio of the slopes \( B_{\text{in}}/B_{\text{el}} \simeq 0.28 \) translates into the absolute value \( B_{\text{in}} = 1.4 \pm 1.7 \text{GeV}^2 \).

The evolution of the initial conditions, eq. 5, is given by

\[
2D(x_1, x_2, Q_1^2, Q_2^2)_{\text{nf}} = \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} G(x_1/z_1, Q_1^2) G(x_2/z_2, Q_2^2)_{\text{2D}}(z_1, z_2, Q_0^2)_{\text{nf}},
\]

where \( G(x_1/z_1, Q_1^2, Q_0^2) \) is the conventional DGLAP gluon-gluon kernel \[34\] which describes evolution from \( Q_0^2 \) to \( Q_1^2 \). In our calculation we neglect initial sea quark densities in the Pomeron at scale \( Q_0^2 \) (obviously Pomeron does not get contribution from the valence quarks).

Numerical calculation of this integral

\[
K(x_1, x_2, Q_1^2, Q_2^2) = \frac{D(x_1, x_2, Q_1^2, Q_2^2)_{\text{nf}}}{D(x_1, Q_1^2) D(x_2, Q_2^2)},
\]

for \( Q_0^2 = 0.5 \text{ GeV}^2 \) and \( Q_0^2 = 1.0 \text{ GeV}^2 \) is presented in Fig.4. (the corresponding \( x_i = \sqrt{4Q_i^2/s} \)).

**FIG. 4:** The ratio of non-factorized and factorized contributions to \( 2D(t=0) \), \( K(t=0) \), as a function of transverse scale \( p_t \equiv Q \) in maximum transverse kinematics for \( \sqrt{s} = 7 \text{ TeV} \) run (left) and \( \sqrt{s} = 13.0 \text{ TeV} \) run (right).

IV. \( \sigma_{\text{eff}} \) IN THE CENTRAL KINEMATICS

Now we are in the position to determine the overall enhancement of \( 1/\sigma_{\text{eff}} \) as compared to the mean field result. It is given by the enhancement coefficient

\[
R = R_{pQCD} + R_{soft}.
\]
Here as it was already explained in section 2, $R_{pQCD}$ corresponds to the contribution of $1 \otimes 2$ pQCD mechanism (Fig. 1b) and was calculated in [10]. The expression for $R_{soft}$ is given by

$$R_{soft} = \frac{4K}{1 + B_{inel}/B_{el}} + \frac{K^2B_{el}}{B_{inel}} + K R_{pQCD} B_{el}/B_{inel},$$  \hspace{1cm} (13)$$

where we calculate all factors for $x_1 = x_2 = x_3 = x_4 = \sqrt{4Q^2/s}$, with $s$ being invariant energy of the collision. We present our numerical results in Figs. 5 – 8. In Fig. 5,6 we present $\sigma_{eff}$ as a function of $p_t$ for two values of the starting evolution scale for the central kinematics in the mean field approach, accounting also for the pQCD $1 \otimes 2$ mechanism and including in addition soft correlations.

In fig. 7,8 we show the corresponding enhancement factors.

FIG. 5: $\sigma_{eff}$ as a function of the transverse scale $p_t$ for $Q_0^2 = 0.5$ (left),1 GeV$^2$ (right)in the central kinematics. We present the mean field, the mean field plus $1 \otimes 2$ mechanism and total $\sigma_{eff}$ for $\sqrt{s} = 7$ TeV.

FIG. 6: $\sigma_{eff}$ as a function of the transverse scale $p_t$ for $Q_0^2 = 0.5$ (left),1 GeV$^2$ (right)in the central kinematics. We present the mean field, the mean field plus $1 \otimes 2$ mechanism and total $\sigma_{eff}$ for $\sqrt{s} = 13$ TeV.
In addition, in order to illustrate the combined picture of $\sigma_{\text{eff}}$ behaviour in both UE and DPS, in Fig.9 we give the example of $\sigma_{\text{eff}}$ behaviour as a function of $p_t$ in the combined transverse momenta region 2-50 GeV for $Q_0^2=0.5$ GeV$^2$ (for $Q_0^2=1$ GeV$^2$ the behaviour is very similar).

We also studied the energy dependence of $\sigma_{\text{eff}}$ for fixed transverse momenta $p_t$ on center of mass.
energy in the UE kinematic region in the energy region from Tevatron to LHC, that is depicted in Fig. 10:

![Energy Dependence of $\sigma_{\text{eff}}$](image)

**FIG. 10:** The characteristic energy dependence of $\sigma_{\text{eff}}$ on c.m.s. energy $\sqrt{s}$

We see that at LHC energies the energy dependence of $\sigma_{\text{eff}}$ practically saturates. In order to understand the evolution of $\sigma_{\text{eff}}$ for higher energies for given transverse scale we shall need the evolution of two-gluon formfactor for small $x \leq 10^{-4}$ which is still not available from experimental data.

**V. CONCLUSIONS**

We used the model of [26] to study nonperturbative parton - parton correlations in the central kinematics at the LHC. Our estimates have been only semiquantitative due to the large uncertainties in diffraction parameters as well as the use of the "effective" values for the reggeon/pomeron parameters (which very roughly included screening corrections). Nevertheless, our results indicate a number of basic features of soft nonperturbative parton - parton correlations relevant for the central LHC dynamics, relevant for ATLAS, CMS and ALICE detectors.

(i) We see that for large transverse momenta, relevant for hard DPS scattering, soft effects are small and essentially negligible, contributing only 5% to the enhancement coefficient $R$ if we start from the scale $Q_0^2 = 0.5 \text{ GeV}^2$, and 10 - 15% from 1 GeV$^2$, for $p_t \sim 15 - 20 \text{ GeV}$. Thus they do not influence detailed hard DPS studies carried in [9, 11]. Our results also indicate that the characteristic transverse momentum $p_{t0}$, for which soft correlations constitute given fixed part of enhancement $R$ rapidly increase with $s$. Indeed, if we look at the scale where soft contribution are say 10% of the rescaling coefficient $R$ changes from 12 to 15 GeV once we change c.m.s. energy from $\sqrt{s_1} = 7$ to $\sqrt{s_2} = 13 \text{ TeV}$. This scale further changes to $\sim 6 \text{ GeV}$ once we go to 2 TeV
c.m.s. energy corresponding to Tevatron. The dependence of $p_t$ on energy starting from Tevatron energies is depicted in Fig.11 for $Q_0^2 = 0.5$ GeV$^2$ (for $Q_0^2 = 1$ GeV$^2$ the qualitative behaviour of $p_t$ is rather similar). We see that the characteristic transverse scales for which soft correlations are important, rapidly increase with energy.

\[ \text{FIG. 11: The characteristic energy dependence of } p_t \text{ on c.m.s. energy } \sqrt{s} \]

(ii) The soft non-factorisable contributions may contribute significantly in the underlying event dynamics, especially at the scales 2-4 GeV where they are responsible for about 50% of the difference between mean field result and full prediction for $\sigma_{\text{eff}}$ for 0.5 GeV$^2$ case, and are dominant up to 4 GeV if we start evolution from 1 GeV$^2$. In UE they lead to stabilization of $\sigma_{\text{eff}}$, that decreases more slowly with increase of $p_t$ than if we include only perturbative correlations,i.e. $1 \otimes 2$ mechanism. For UE dynamics if we use 0.5 GeV$^2$ as a start of the evolution, and take $p_t \sim 2$ GeV, $\sigma_{\text{eff}}$ changes from 43mb (mean field value) to 26 mb, while if we neglect soft correlations the change from mean field value is to 33 mb (for $\sqrt{s} = 7$ TeV). Similarly if we start evolution at $Q_0^2 = 1$ GeV$^2$, the change is to 27 mb, instead of 39 mb, if only pQCD effects are included. For $p_t \sim 4$ GeV the changes are from 42 mb, to 26-27 mb (instead of 30-35 mb if soft correlations are not accounted for). These values for $\sigma_{\text{eff}}$ for UE, especially for scales 2-4 GeV are very close to the ones used by Pythia. The results show that $\sigma_{\text{eff}}$ in UE significantly decreases as compared to the values one obtains including only contributions of the mean field and $1 \otimes 2$ mechanism. Thus for the 2-4 GeV scales, the full value of $\sigma_{\text{eff}}$ that includes soft correlations is 26-27 mb, instead of 30-39 mb that we find with only pQCD and mean field contributions included.

Note that the pQCD contribution $R_{\text{pQCD}}$ was included in the MC calculations in [16]. It was shown that its inclusion leads to a significant improvement of the description of DPS scattering for $p_t > 15 - 20$ GeV. On the other hand for UE regime the best agreement was for the Pythia nonrescaled tune. Our current results for the Underlying event are very close to Pythia, while for DPS with $p_t \geq 15-20$ GeV effectively only mean field+$1 \otimes 2$ pQCD terms contribute. We see
that the new framework can give a decent description of the data over the full transverse momenta range, but with less dependence of the quality of fit on the starting point of the evolution $Q_0$ than in [16]. (Of course more detailed comparisons of the old and new frameworks will be ultimately needed. Such more detailed MC simulations and comparison with experimental data are currently under way [35].)

(iii) The evolution of $\sigma_{\text{eff}}$ with transverse scale stabilizes for UE regime, as it is seen in Fig. 5, leading to almost plateau like picture with only slight decrease with transverse scale.

(iv) The inclusion of soft correlations stabilizes the energy, i.e. $\sqrt{s}$ dependence of $\sigma_{\text{eff}}$. It changes marginally between 3.5 TeV and 6.5 TeV collision energies for the same transverse scale for small $p_t$. In other words the increase of soft correlations compensates the decrease of the relative pQCD contribution with an increase of energy (i.e. decrease of effective $x_i$ in the process).

Overall we conclude that soft correlations do not influence significantly hard DPS dynamics, but are important for the description of MPI in the UE regime.

Note also that inclusion of the soft correlations decreases the difference between $R$ obtained while carrying evolution from $Q_0^2 = 0.5$ and $Q_0^2 = 1$ GeV$^2$ scales, especially for small transverse scales of several GeV.

Our results do not influence our previous results for DPS scattering in the Tevatron [10], where soft correlations give only $2 - 5\%$ contribution, since the corresponding region of $x$ is $x_i \sim 0.01$. This is in accordance with increase of characteristic momenta where soft correlations cease to be important with energy mentioned above.

Finally, we note that an attempt to include soft correlations, which is also based on the ideas of ref. [9], was recently reported [36]. There are several important differences in the used models:

(a) in [36] scale $Q/2$ was used for production of jets rather more commonly used scale $Q$,

(b) the mean field contribution was calculated using the two gluon form factor with the $t$-dependence much harder that the one allowed by the exclusive $J/\psi$ data; (c) In [36] it was assumed that the QCD evolution starts only at $Q^2 \sim 3$ GeV$^2$. Combined these assumptions resulted in an enhancement of nonperturbative contribution to $\sigma_{\text{eff}}$ and suppression of the perturbative contribution.

(c) Another difference is that we used the effective values for Reggeon parameters in the spirit of estimates in ref. [31], and GRV gluon densities for the Pomeron, while the authors of [36] used a version of the full Reggeon Field theory with screenings. Note in passing that the Reggeon Field theory was formulated by Gribov assuming presence of only one (soft) scale in strong interaction. Such an assumption is difficult to justify for the LHC energies where minijet cross section is very
Numerically we find that for the UE our numerical results are rather close (see Fig. 10 and corresponding figure in [36].)

While the authors of ref. [36] find $\sigma_{\text{eff}} = 25 \div 25.5 \text{ mb}$ for energy range considered in our letter, we get, depending on $Q_0$, $\sigma_{\text{eff}} \sim 20 \div 25 \text{ mb}$ for $Q_0^2 = 0.5 \text{ GeV}^2$, and $24 \div 28 \text{ mb}$ for $Q_0^2 = 1 \text{ GeV}^2$, i.e. our results differ by 20\% or less. The pattern of slow increase of $\sigma_{\text{eff}}$ with energy and its almost complete saturation for the LHC highest energies are also similar. Such similarity appears rather accidental, as it results from very different model for the gluon GPD used for the mean field, and different assumptions about the range of QCD evolution.

Indeed, for large scale considered in [36] of order 50 GeV we have a value of $\sigma_{\text{eff}}$ approximately 1.5-1.7 times smaller [10] than in [36], and for such scales soft correlation contribution is negligible.

Note also that we do not extend our calculation of UE beyond the LHC energies, as it was done in ref.[36], since the dependence on $x$ of the two gluon form factor and inelastic diffraction on $x$ are not known so far for $x < 10^{-4}$.

Acknowledgements

M.S.’s research was supported by the US Department of Energy Office of Science, Office of Nuclear Physics under Award No. DE-FG02-93ER40771. We thank Yuri Dokshitzer and Leonid Frankfurt for many useful discussions.

[1] N. Paver and D. Treleani, Nuovo Cim. A 70 (1982) 215.
[2] M. Mekhfi, Phys. Rev. D32, 2371 (1985).
[3] J.R. Gaunt and W.J. Stirling, JHEP 1003, 005 (2010) [arXiv:0910.4347 [hep-ph]];
    J.R. Gaunt, C.H. Kom, A. Kulesza and W.J. Stirling, Eur. Phys. J. C 69, 53 (2010) [arXiv:1003.3953 [hep-ph]].
[4] B. Blok, Yu. Dokshitzer, L. Frankfurt and M. Strikman, Phys. Rev. D 83, 071501 (2011) [arXiv:1009.2714 [hep-ph]].
[5] M. Diehl, PoS D IS2010 (2010) 223 [arXiv:1007.5477 [hep-ph]].
[6] J.R. Gaunt and W.J. Stirling, JHEP 1106, 048 (2011) [arXiv:1103.1888 [hep-ph]].
[7] B. Blok, Yu. Dokshitser, L. Frankfurt and M. Strikman, Eur. Phys. J. C 72, 1963 (2012) [arXiv:1106.5533 [hep-ph]].
[8] M. Diehl, D. Ostermeier and A. Schafer, JHEP 1203 (2012) 089 [arXiv:1111.0910 [hep-ph]].
[9] B. Blok, Yu. Dokshitzer, L. Frankfurt and M. Strikman, [arXiv:1206.5594v1 [hep-ph]] (unpublished).

[10] B. Blok, Y. Dokshitzer, L. Frankfurt and M. Strikman, Eur. Phys. J. C 74 (2014) 2926 [arXiv:1306.3763 [hep-ph]].

[11] J. R. Gaunt, R. Maciula and A. Szczurek, [arXiv:1407.5821 [hep-ph]].

[12] K. Golec-Biernat and E. Lewandowska, Phys. Rev. D 90 (2014) no.9, 094032 [arXiv:1407.4038 [hep-ph]].

[13] M. Diehl, J. R. Gaunt, D. Ostermeier, P. Plobl and A. Schafer, JHEP 1601 (2016) 076 [arXiv:1510.08696 [hep-ph]].

[14] R. Astalos et al., “Proceedings of the Sixth International Workshop on Multiple Partonic Interactions at the Large Hadron Collider,” arXiv:1506.05829 [hep-ph].

[15] ‘Proceedings of the Seventh International Workshop on Multiple Partonic Interactions at the Large Hadron Collider,”

[16] B. Blok and P. Gunnellini, Eur. Phys. J. C 75 (2015) no.6, 282 [arXiv:1503.08246 [hep-ph]].

[17] B. Blok and P. Gunnellini, Eur. Phys. J. C 76 (2016) no.4, 202 [arXiv:1510.07436 [hep-ph]].

[18] F. Abe et al. [CDF Collaboration], Phys. Rev. D 56, 3811 (1997).

[19] V.M. Abazov et al. [D0 Collaboration], Phys. Rev. D 81, 052012 (2010).

[20] V.M. Abazov et al. [D0 Collaboration], Phys. Rev. D 83, 052008 (2011).

[21] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. D 89 (2014) 9, 092010 [arXiv:1312.6440 [hep-ex]].

[22] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 83 (2011) 112001 [arXiv:1012.0791 [hep-ex]].

[23] S. Chatrchyan et al. [CMS Collaboration], JHEP 1403 (2014) 032 [arXiv:1312.5729 [hep-ex]].

[24] L. Frankfurt and M. Strikman, Phys. Rev. D 66 (2002) 031502 [hep-ph/0205223].

[25] L. Frankfurt, M. Strikman and C. Weiss, Phys. Rev. D 69, 114010 (2004)

[26] B. Blok and M. Strikman, [arXiv:1608.00014 [hep-ph]], to be published in Eur. Phys. J. C.

[27] I.M. Belyaev, talk at MPI-2015 conference.

[28] R. Aaij et al. (LHCb collaboration), JHEP, 1206(2012), 141,1403 (2014)108 arXiv: 1205.0975v3.

[29] L. Frankfurt, M. Strikman and C. Weiss, Phys. Rev. D 83 (2011) 054012 [arXiv:1009.2559 [hep-ph]].

[30] C. Adloff et al. [H1 Collaboration], Z. Phys. C 74 (1997) 221

[31] E. G. S. Luna, V. A. Khoze, A. D. Martin and M. G. Ryskin, Eur. Phys. J. C 59 (2009) 1 [arXiv:0807.4115 [hep-ph]].

[32] M. Glick, E. Reya and A. Vogt, Eur. Phys. J. C 5 (1998) 461 [hep-ph/9806404].

[33] F. D. Aaron et al.[H1 Collaboration], JHEP 1005, 032 (2010)

[34] Y. L. Dokshitzer, D. Diakonov and S. I. Troian, Phys. Rept. 58 (1980) 269.

[35] B. Blok and P. Gunnellini, in preparation.

[36] S. Ostapchenko and M. Bleicher, Phys. Rev. D 93 (2016) no.3, 034015 [arXiv:1511.06784 [hep-ph]].