Parametric synthesis of a robust controller on a base of mathematical programming method

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Abstract. Considered paper is dedicated to deriving sufficient conditions, linking root indices of robust control quality with coefficients of interval characteristic polynomial, on the base of mathematical programming method. On the base of these conditions, a method of PI- and PID-controllers, providing aperiodic transient process with acceptable stability degree and, subsequently, acceptable setting time, synthesis was developed. The method was applied to a problem of synthesizing a controller for a depth control system of an unmanned underwater vehicle.

1. Introduction
Real control objects are commonly described with a set of parameters, which vary on the base of unknown laws or their accurate measuring is unavailable. Control systems, which manipulate such objects, are considered as interval control systems (ICS) or systems with interval parameters [66]. ICS can be described via its interval characteristic polynomial (ICP), which coefficients include interval parameters of a control object. ICP are widely used to perform ICS controller synthesis on a base of maximal operating speed criteria. It is known, that in order to maximize operating speed of the system, it is necessary to maximize robust stability degree. The problem of robust stability degree maximization was considered in papers [1-5], [8, 9].

There are many control systems, which require an aperiodic transient process. This makes a problem of providing simultaneously a maximal operating speed and an aperiodic transient process in ICS highly relevant. Aperiodic transient process in an ICS can be provided by allocating one migrating real pole closely to imaginary axis and allocating all other poles away of imaginary axis. This approach is based on a pole domination principle and is described in [10-13].

However, these papers describe a synthesis procedure for systems with interval ICP coefficients uncertainty. This makes relevant a problem of parametric synthesis of controllers for systems with affine uncertainty of ICP coefficients, in which primary interval parameters are included. Considering this, the main aim of the research is to develop a method of parametric synthesis of a robust controller, which would provide a maximal aperiodic stability degree for an ICS with affine uncertainty of ICP coefficients.

2. Algorithm of synthesizing a robust controller for an ICS
Let us consider an ICS with unity feedback, which includes a control object

\[ W_c(s) = \frac{B(s)}{A(s)} = \frac{\sum b_h([\bar{T}]) s^h}{\sum a_h([\bar{T}]) s^h}, \]

here \([\bar{T}]\) – vector of interval, \(s\) – Laplace operator. Also, the system includes a controller \(W_c(s, \vec{k}) = \frac{F(s, \vec{k})}{s}\), here \(\vec{k}\) – vector of controller parameters. Considering this, an ICP of the system can be written as follows:

\[ D(s, [\bar{T}], \vec{k}) = B(s, [\bar{T}]) F(s, \vec{k}) + sA(s, [\bar{T}]) = \sum_{i=0}^n d_i([\bar{T}], \vec{k}) s^i. \]  

(1)

It is proposed to solve the synthesis problem with the help of mathematical programming theory. Such approach is described in [5-7] and allows to maximize a stability degree \(\alpha\) of a time-invariant system by synthesizing a vector of controller parameters \(\vec{k}\) on the base of characteristic polynomial. Interval expansion of a mathematical programming method is proposed to be based on examining vertices characteristic polynomials (VCP), which is ICP in vertices of its parametric polytope:

\[ D^q(s) = \sum_{i=1}^m T_i^c \cdot A_i(s) + B(s), \]  

(2)

here \(q\) – number of vertex.

Let us substitute \(s\) in (2) with \(s = \alpha + j\beta\) and rewrite it as a complex number:

\[ D^q(\vec{k}, \alpha, \beta) = \text{Re } D^q(\vec{k}, \alpha, \beta) + j \text{Im } D^q(\vec{k}, \alpha, \beta) \]  

(3)

In order to find the maximal stability degree and parameters of a controller, a system of equations can be derived on the base of (3):

\[
\begin{align*}
\text{Re } D^q(\vec{k}, \alpha, \beta) &= 0; \\
\text{Im } D^q(\vec{k}, \alpha, \beta) &= 0; \\
\frac{\partial \text{Re } D^q(\vec{k}, \alpha, \beta)}{\partial \alpha} &= 0; \\
\frac{\partial \text{Im } D^q(\vec{k}, \alpha, \beta)}{\partial \alpha} &= 0; \\
\vdots \\
\frac{\partial^r \text{Re } D^q(\vec{k}, \alpha, \beta)}{\partial \alpha^r} &= 0; \\
\frac{\partial^r \text{Im } D^q(\vec{k}, \alpha, \beta)}{\partial \alpha^r} &= 0.
\end{align*}
\]

Let us notice, that in order to provide an aperiodic transient process, one of system poles, which is the closest to imaginary axis, should have zero imaginary part \(\beta = 0\). Considering this, to provide an aperiodic transient process in considered ICS, a following system of equations must be solved:

\[
\begin{align*}
\text{Re } D^q(\vec{k}, \alpha) &= 0; \\
\frac{\partial \text{Re } D^q(\vec{k}, \alpha)}{\partial \alpha} &= 0; \\
\vdots \\
\frac{\partial^r \text{Re } D^q(\vec{k}, \alpha)}{\partial \alpha^r} &= 0.
\end{align*}
\]  

(4)
Considered research resulted in two methods of robust stability degree maximization for systems with different number of interval parameters, determining a number of parametric polytope $P_T$ vertices. If number of interval parameters is less or equal to 3, than the synthesis algorithm can be formulated as follows:

1. Define ICP of the system in a form (1)
2. On the base of ICP, with the help of $s = \alpha + j\beta$ substitution, derive a VCP in a form (2)
3. Rewrite VCP (2) as (3)
4. Solve systems of nonlinear equations (4) in each vertex of parametric polytope $P_T$ and determine values of aperiodic stability degree $\alpha$ and according parameters of controller $\tilde{k}$.
5. Choose maximal aperiodic stability degree $\alpha^* = \max_i \alpha_i$ and according controller parameters $\tilde{k}$ from the set of $(\alpha_i, \tilde{k})$ in each vertex of $P_T$.

If the number of interval parameters exceeds 3, than the method of synthesis is based on combining synthesis procedures for different types of ICP coefficients uncertainty. It includes following steps:

1. With the help of coefficient method [9] find parameters of controller $\tilde{k}$, providing a quasimaximal stability degree $\eta_{\max}$.
2. Plot a boundary vertex route of parametric polytope [14], considering previously found parameters of the controller.
3. Find projections of a vertex route on a complex plane and determine a vertex, which projection is the closest to the imaginary axis.
4. Calculate values of maximal stability degree $\alpha^*$ and according parameters of a controller $\tilde{k}$ by solving the system of nonlinear equations in a vertex, found in the step 5.
5. If the closest to imaginary axis projection is a projection of an inner point of the parametric polytope edge and not a vertex, than it is proposed to substitute coordinates of this point in an ICP in step 3 instead of vertex coordinates a calculate a value of interval parameter $T_i$, $i \in 1, m$.

3. Mathematical Model of the Submerging Velocity Control Loop

Considered AUV is shown in the figure 1.

![Figure 1. Hull of the considered AUV](image)

Submerging control is performed with the help of two vertical thrusters. The structure of depth control channel is shown in the figure 2.

![Figure 2. Structure of the submerging control channel](image)

In the figure 2 following designations are accepted: $h_0$ — depth setpoint; $h$ — actual value of depth; $\varepsilon_i$ — a difference between a setpoint and an actual value of the depth; $P$ — P-controller of the outer
control loop; \( u_1 \) – output of the P-controller; \( \varepsilon_2 \) – a difference between a P-controller output signal and a submerging velocity signal \( \nu_y \); PID – PID-controller of an inner control loop; \( K_1, T_i \) – transfer coefficient and time constant of the thruster; \( T_s \) – thrust of vertical steering thrusters; \( m \) – AUV mass; \( \lambda_{22} \) – additional mass of water; \( c_y \) – hydrodynamic lift force coefficient; \( k \) – interval linearization coefficient; \( \nu_y \) – submerging velocity; \( K_{VS} \) – transfer coefficient of the velocity sensor; \( K_{DS} \) – transfer coefficient of the depth sensor.

The problem of synthesizing a P-controller of the outer control loop will not be considered in the paper. Let us now consider the mathematical model of the inner control loop of the submerging control channel, which controls submerging velocity.

Considering previous designations, transfer function of the submerging velocity control loop can be written as follows:

\[
W(s) = \frac{2 \cdot K_1 \cdot (K_p \cdot s^2 + K_k \cdot s + K_i)}{a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0}
\]

\[
a_0 = 2 \cdot K_1 \cdot K_p \cdot K_{VS};
\]

\[
a_t = k \cdot c_y + 2 \cdot K_1 \cdot K_p \cdot K_{VS};
\]

\[
a_2 = m + \lambda_{22} + k \cdot T_i \cdot c_y + 2 \cdot K_1 \cdot K_p \cdot K_{VS};
\]

\[
a_3 = G_1 \cdot (m + \lambda_{22}).
\]

Here \( K_p, K_i, K_{DS} \) – proportional, integral and differential coefficients of the PID-controller.

The model considers an interval uncertainty of the AUV hydrodynamic parameters and interval uncertainty of the transfer coefficient of its thrusters.

Expression (5) shows us, that parametric polytope \( T \) includes two interval parameters and, subsequently, 4 vertices. For each vertex of \( T \) let us derive a system (4). From set of solutions of all these systems, let us choose a maximal aperiodic stability degree \( \alpha^* = 3.5 \) and according parameters \( \bar{k} = (11.367 13.758 0.118) \) of a controller.

4. PID-controller parametric synthesis

Let us perform a PID-controller synthesis via developed method. To do this, let rewrite (5) in an affine form:

\[
D(s) = d_3 \cdot s^3 + [d_2] \cdot s^2 + [d_1] \cdot s + [d_0],
\]

here \( d_3 = a_3, \ d_2 = a_2, \ d_1 = a_1, \ d_0 = a_0 \)

5. Conclusion

Considered research resulted in a method of synthesizing a PI- or PID-controllers, providing aperiodic transient process in a control system, despite its interval parametric uncertainty. The method is applicable for synthesizing controllers of ICS with affine uncertainty of ICP coefficients and various numbers of interval parameters.

The method was tested on a problem of synthesizing a controller for a depth control system of a UUV.
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Acknowledgments

The reported study is supported by the Ministry of Education and Science of Russian Federation (project #2.3649.2017/PCh).