Weak magnetic field corrections to light vector or axial mesons mixings and vector meson dominance

Fábio L. Braghin
Instituto de Física, Federal University of Goias
Av. Esperança, s/n, 74690-900, Goiânia, GO, Brazil

April 20, 2020

Abstract

Weak magnetic field induced corrections to effective coupling constants describing light vector mesons mixings and vector meson dominance (VMD) are derived. The magnetic field must be weak with respect to an effective quark mass \( M^* \) such that: \( eB_0/M^*^2 < 1 \) or \( eB_0/M^*^2 << 1 \).

For that, a flavor SU(2) quark-quark interaction due to non perturbative one gluon exchange is considered. By means of methods usually applied to the Nambu Jona Lasinio (NJL) and Global Color Models (GCM), leading light vector/axial mesons couplings to a background electromagnetic field are derived. The corresponding effective coupling constants are resolved in the structureless mesons and longwavelength limits. Some of the resulting coupling constants are redefined such as to become magnetic field induced corrections to vector or axial mesons couplings. Due to the approximated chiral symmetry of the model, light axial mesons mixings induced by the magnetic field are also obtained. Some numerical estimates are presented for the coupling constants and for some of the corresponding momentum dependent vertices. The contributions of the induced VMD and vector mesons mixing couplings for the low momentum pion electromagnetic form factor and for the (off shell) charge symmetry violation potential at the constituent quark level are estimated. The relative overall weak magnetic field-induced anisotropic corrections are of the order of \( (eB_0/M^*^2)^n \), where \( n = 2 \) or \( n = 1 \) respectively.

1 Introduction

Light vector mesons mixings \([1]\) and vector meson dominance (VMD) \([2,3]\) are interesting effects considered in hadron and nuclear strong interacting systems which are believed to play relevant role in different processes. Vector mesons \( \rho - \omega \) mixing is usually attributed to isospin violation from different quark or nucleon masses. It shows up for example in the pion form factor \([4,5,6,7,8]\) and it can be responsible, at least in part, for the charge symmetry violation (CSV) component of the nuclear potential \([9,10,11,12,13,14]\). An experimental value of mixing strength, that is usually associated to the energy scale of the rho or omega mass \([5,14]\), is given by \(<\rho^0|H|\omega> = -4520 \pm 600 \text{ MeV}^2\). In the VMD assumption a photon \( A_\mu \) fluctuates into a quark-antiquark pair with the quantum numbers
of a neutral rho $V^3_{\mu}$ or omega $V_\mu$ meson at intermediary energies. Different scenarios that aim to
describe vector mesons dynamics and the electromagnetic couplings of light hadrons were envisaged
to incorporate VMD \[4, 15, 16, 17, 18, 19\]. From a dynamical point of view, these two effects can be
accounted by effective phenomenological Lagrangian terms. By considering the $\omega$
to incorporate VMD \[4, 15, 16, 17, 18, 19\]. From a dynamical point of view, these two effects can be
described vector mesons dynamics and the electromagnetic couplings of light hadrons were envisaged
written as

\[
L = g_{\text{mix}1} F_{\mu\nu}^3 F_{\mu\nu}^3 + g_{\text{mix}2} V_{\mu} V_{\mu}^3 + g_{\text{vmd}1} F_{\mu\nu}^3 A_{\mu} V_{\mu}^3 + g_{\text{vmd}2} A_{\mu} V_{\mu}^3 + g_{\text{vmd}2} A_{\mu} V_{\mu}^3,
\]

where $F_{\mu\nu}^\omega, F_{\mu\nu}^i$ and $F_{\mu\nu}^V$ are the Abelian strength tensors for the photon, rho and omega fields respectively. There are therefore two types of phenomenological mixing ($g_{\text{mix}1}, g_{\text{mix}2}$) and VMD couplings ($g_{\text{vmd}1}, g_{\text{vmd}2}$ for the $\rho$ and $g_{\text{vmd}1}^\omega, g_{\text{vmd}2}^\omega$ for the $\omega$): the momentum dependent ones and the momentum
independent ones. They are usually supposed to be considered separately. The VMD couplings
$g_{\text{vmd}2}$ and $g_{\text{vmd}2}^\omega$ break gauge invariance. They induce a nonzero photon mass and therefore they should
be avoided. However, $g_{\text{vmd}2}$ and $g_{\text{vmd}2}^\omega$ have been shown to be equivalent to $g_{\text{vmd}1}$ and $g_{\text{vmd}1}^\omega$ by field
redefinitions \[20\]. Concerning the two different mixing couplings, several works adopting different frame-
works and methods have shown the momentum dependent coupling turns out to be more appropriated
\[21, 9, 10, 22, 23\]. Although the $\rho$ and $\omega$ mesons were found to be the main vector excitations in the
chiral SU(2) meson sector of hadrons there are axial mesons often associated to chiral partners. For
instance, the $A_1(1260)$ and the $f_1(1285)$ are usually considered as chiral partners respectively of the $\rho$
and of the $\omega$ \[24, 25\]. So far no attempt to investigate eventual mixings of such axial mesons has been
done.

Many different effects in hadron structure and reactions have been found to emerge in the presence
of intense magnetic fields expected to appear in non central heavy ions collisions and magnetars \[26, 27, 28, 29, 30\]. These might be relatively weak magnetic fields with respect to hadron mass scales
of the order of $eB_0 \approx 0.5m_\pi^2 \approx 0.1M^* \approx 10^{17}$G, for $M^* \approx 0.33$ GeV. Although in peripheral heavy
ions collisions $B_0$ is expected to last a short time interval \[29\], vector mesons dynamics might also be
expected to have relevant effects in these reactions. Among the effects produced by magnetic fields
vector mesons rho-omega mixing has already been estimated \[31, 32\]. The light vector mesons mixing,
and also their chiral axial mesons partners mixings, due to a weak external magnetic field is addressed in
the present work from a dynamical approach. The method considered below also provides $B_0$
dependent corrections to the VMD phenomenological coupling simultaneously.

In this work a quark-quark interaction due to non perturbative one gluon exchange, as one of the
leading terms for the QCD effective action, is considered. The one loop background field method is
applied and light quark-antiquark vector and axial mesons are introduced by means of auxiliary fields
for which the structureless mesons limit is taken. This approach was able to produce the complete
Weinberg’s Large Nc effective field theory (EFT) \[33\] for pions and constituent quarks, with leading
and next leading symmetry breaking terms, and their couplings to background photons \[34, 35\]. Besides
that, vector/axial mesons couplings to constituent quarks and their couplings to the photon have also
been derived \[36, 37\]. The starting Global Color Model (GCM) is given by the following normalized
generating functional \[38, 39\]:

\[
Z = \mathcal{N} \int \mathcal{D}[\bar{\psi}, \psi] e^{i \int \bar{\psi} \left( i\slashed{D} - m \right) \psi - \frac{g^2}{2} \int \sum_i j^h_i (x) R_{\mu\nu}^h (x - y) j^h_{\nu i} (y) + \bar{\psi} J + J^* \psi},
\]
where $\mathcal{N}$ is a normalization constant, $\int_x$ stands for $\int d^4x$ and $a,b... = 1,...(N_c^2 - 1)$ stands for color in the adjoint representation, $N_c = 3$. $g^2$ is the quark-gluon coupling constant squared, and the color quark currents are given by $j_a^\mu = \bar{\psi} \gamma^\mu \psi$, being $\lambda_a$ the Pauli matrices for SU(2) isospin. The sums in color, isospin and Dirac indices are implicit. The covariant quark derivative includes the minimal coupling to an external background electromagnetic field: $\mathcal{D} = \gamma^\mu (\partial^\mu - i e Q \delta_{ij} A_\mu)$. With the diagonal matrix $\hat{Q} = \text{diag}(2/3, -1/3)$. Explicit multiquark interactions in the QCD effective action due to gluon self interactions [40] are therefore neglected being outside the scope of this work. However to account for non Abelian structure of the gluon sector, the gluon propagator, $\tilde{R}_{\mu\nu}^{ab}(x - y)$, and the quark-gluon coupling constant will be required to be non perturbative such that they are expected to provide strength enough to produce DChSB. In several gauges the gluon kernel can be written in terms of a transversal and a longitudinal components, $R^T_T(x - y)$ and $R^L_L(x - y)$, for momentum operators in coordinate space as:

$$\tilde{R}_{\mu\nu}^{ab}(x - y) = \delta_{ab} \left[ \left( g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{2^2} \right) R^T_T(x - y) + \frac{\partial^\mu \partial^\nu}{2^2} R^L_L(x - y) \right].$$

The method employed below was developed at length in Refs. [34, 36, 37, 35, 41, 42], so that it will not be discussed with details in this work. In the following section the method will be schematically described and the sea quark determinant will be presented for the limit of structureless vector mesons. The determinant will be expanded in the absence of constituent quarks in the limit of large quark effective mass and small electromagnetic background field. The leading couplings between vector/axial mesons and photons will be obtained and the corresponding effective coupling constants will be resolved in the longwavelength limit. The leading electromagnetic couplings are responsible for weak magnetic field induced corrections to the vector/axial mesons couplings and the corresponding $B_0$-dependent coupling constants will be defined. These coupling constants will be expressed in terms of the parameters of the GCM, Eq. (2), and components of the quark propagator. However the momentum independent VMD terms disappear by redefining the vector mesons and photon fields as shown in section (2.2) in the same way proposed in [20]. The resulting effective model becomes gauge invariant. Up and down quark masses will be considered to be equal along the work to emphasize the different nature of the mechanism investigated in the present work. In section (2.3) the magnetic field dependent couplings are defined. Numerical estimates for effective coupling constants and some of the corresponding momentum dependent form factors will be shown in section (3) as well as some simple fittings. Two further applications are presented. Firstly the contributions of $B_0$ dependent VMD and induced mixing for the pion electromagnetic form factor is exhibited in section (3.2). And secondly, the corresponding weak $B_0$ induced mixing contribution to the (off-shell) charge violation potential at the constituent quark level is presented in section (3.3). An overall discussion will be presented in the final section.

2 Effective couplings for light vector/axial mesons and photons

The main steps of the whole calculations are briefly described in the following. The flavor structure of the model can be suitably investigated by means of a flavor SU(2) Fierz transformation and by picking up the leading color singlet interaction terms as it is usually done. The color non singlet are smaller at least by a factor $1/N_c$ besides the fact that they only generate higher order corrections that are
numerically smaller \[35\]. The quark field is splitted into sea quark (\(\psi_2\)) that form light quark-antiquark states, mesons and the chiral condensate, and background quark (\(\psi_1\)) that gives rise to constituent quarks eventually to compose baryons. At the one loop Background Field Method (BFM) level this can be done by splitting the quark bilinears. For a particular Dirac flavor-color operators \(\Gamma\) the following shift is enough \[33, 34, 35\]:

\[
\bar{\psi}\Gamma\psi \rightarrow (\bar{\psi}\Gamma\psi)_1 + (\bar{\psi}\Gamma\psi)_2.
\]  

(4)

The sea quarks can be integrated completely by means of the auxiliary field method. Besides that, this work concerns only the vector and axial light quark-antiquark mesons and therefore only the corresponding vector and axial quark currents must be taken into account. The scalar quark interactions, however, is also important since it is responsible for the emergence of the quark-antiquark condensate and DChSB with the large contribution for the quark (hadrons) effective mass. The details for the scalar and pseudoscalar sector have been discussed in a large variety of papers and will be omitted below.

The following set of bilocal auxiliary fields (a.f.) is considered: \(V^i_\mu, V_\mu, A^i_\mu, \bar{A}_\mu\), for vectors and axial-vectors, isospin singlets and triplets, being that the indices \(i, j, k \equiv 0, \ldots (N_f^2 - 1)\), with \(N_f = 2\), will be used. They correspond to the color singlet vector channels of the bilocal quark currents. The following unit Gaussian integrals for bilocal auxiliary fields are introduced in the generating functional \[38, 39, 44\]:

\[
\exp\left\{-\frac{\alpha}{4} \int x,y \left( \bar{R}^{\mu\nu} V^i_\mu V^i_\nu + \bar{R}^{\mu\nu} A^i_\mu A^i_\nu \right) + \frac{\alpha}{4} \int x,y \left( \bar{R}^{\mu\nu} V^i_\mu V^i_\nu + \bar{R}^{\mu\nu} A^i_\mu A^i_\nu \right) \right\},
\]

(5)

where \(N\) is a normalization constant that does not show up in observables and

\[
R^{\mu\nu}(x - y) = g^{\mu\nu}(R_T(x - y) + R_L(x - y)) + 2 \frac{\partial^\mu \partial^\nu}{\partial^2}(R_T(x - y) - R_L(x - y)),
\]

(6)

and \(\alpha = 4/9\). Now it is possible to introduce the renormalization constants and to perform a shift in each of the auxiliary field with the corresponding quark-current with the same quantum number given by:

\[
\begin{align*}
V^i_\mu &\rightarrow (Z_{V}^{\frac{1}{2}}V^i_\mu - g Z_g Z_\psi j^{i,(2)}_{V,\mu}), & A^i_\mu &\rightarrow (Z_{A}^{\frac{1}{2}}A^i_\mu - g Z_g Z_\psi j^{i,(2)}_{A,\mu}), \\
V_\mu &\rightarrow (Z_{V}^{\frac{1}{2}}V_\mu - g Z_g Z_\psi j^{(2)}_{V,\mu}), & A_\mu &\rightarrow (Z_{A}^{\frac{1}{2}}A_\mu - g Z_g Z_\psi j^{(2)}_{A,\mu}),
\end{align*}
\]

(7)

where the usual quark flavor currents were considered \[36\]: \(j^{i,(2)}_{V,\mu} = \bar{\psi} \gamma_\mu \lambda^i \psi\), \(j^{(2)}_{V,\mu} = \bar{\psi} \gamma_\mu \psi\), \(j^{i,(2)}_{A,\mu} = \bar{\psi} \gamma_5 \gamma_\mu \lambda^i \psi\) and \(j^{(2)}_{A,\mu} = \bar{\psi} \gamma_5 \gamma_\mu \psi\). These shifts have clearly unity Jacobian. The wavefunction renormalization constants will not however be written explicitly along the development below until section (2.2).

The (sea) quark vector/axial quark-current interactions from the Fierz transformation are canceled out and it becomes possible to integrate out sea quarks.

However for low energy regime these bilocal fields can be expanded in an infinite basis of local meson fields \[38\]. By picking up only the lowest energy modes and making the form factors to reduce to constants in the zero momentum limit it yields the structureless mesons limit analysed in Refs.
The vector mesons and their chiral partners were assumed to develop the same normalization constants that provide the corresponding canonical normalization. With this, the components of the gluon propagator in Eq. (5) are absorbed in the structureless vector/axial mesons normalization constants. The saddle point equations for the auxiliary fields are the usual gap equations and only the scalar one might have non trivial solution corresponding to the quark-antiquark scalar condensate from DChSB in the vacuum, \(< S >\). By taking into account this constant, the free quark propagator with a suitable implicit regularization, by omitting the quark wavefunction renormalization constant, can then be written as:

\[ S_{0,c}(x - y) = (iD - M^*)^{-1}\delta(x - y) \]

where \(M^* = m + < S >\). Its behavior in the vacuum and finite energy density systems has been investigated in many works. In particular, the chiral condensate and consequently the quark effective mass \(M^*\) have strong dependence on an external magnetic field \([26, 27, 28]\). This \(B_0\) dependence will be investigated in the numerical analysis done below by choosing two different values for the effective mass \(M^*\), one in the vacuum (\(M^* = 330\) MeV) and another larger value for finite (weak) \(B_0\).

The Gaussian integration of the sea quark field is performed and the resulting determinant can be written as:

\[ S_{\text{eff}} = i \text{Tr} \ln \left\{ -i \left( S^{-1}_c(x - y) + \sum_q a_q \Gamma_q j_q(x, y) \right) \right\}, \tag{8} \]

where \(Tr\) stands for traces of all discrete internal indices and integration of spacetime coordinates and the quark kernel can be written as

\[ S^{-1}_c(x - y) = S^{-1}_{0,c}(x - y) + \Xi_v(x - y), \tag{9} \]

where the following quantity with the (chiral) structureless vector/axial mesons fields was defined above:

\[ \Xi_v(x, y) = -\frac{\gamma^\mu}{2} \left[ \sigma_i \left( V^{\mu}_\mu(x) + i\gamma_5 \bar{A}^{\mu}_i(x) \right) + \left( V^{\mu}_\mu(x) + i\gamma_5 \bar{A}_\mu(x) \right) \right] \delta(x - y), \tag{10} \]

with canonically normalized fields. All the constituent quark currents were included in following quantity \(\sum_q a_q \Gamma_q j_q(x, y)\), for the particular (Dirac, flavor) \(q\)-channel. Their couplings to the light vector mesons and to the pseudoscalar/scalar sector were investigated respectively in Refs.\([36, 37]\) and \([34, 35, 45, 42]\) and they will be neglected in this work.

### 2.1 Leading photon-vector mesons from expansion

Consider a large quark effective mass of the determinant above, within a zero order derivative expansion \([46]\) for the longwavelength regime such that the local limit is reached. The following leading and next leading local terms are obtained:

\[
\mathcal{L}_{VMD} = -g_{\rho A} V^\rho_{\mu} A^\mu - g_{\omega A} V^\mu_{\mu} A^\mu - g_{F\rho} \mathcal{F}_{\mu \nu}^3 \mathcal{F}_{\mu \nu}^\rho - g_{F\omega} \mathcal{F}_{\mu \nu}^\omega \mathcal{F}_{\mu \nu}^\omega,
\]

\[
\mathcal{L}_{F} = g_{F\rho\omega} (F^\mu_{\rho} F^\nu_{\mu} F^\omega_{\nu}) + F^\mu_{\rho} \mathcal{G}^3_{\rho \mu} + g_{F\mu \omega} F^\mu_{\nu} \mathcal{F}_{\rho}^\nu + g_{FF\rho} F^\mu_{\nu} F^\nu_{\rho} F^\rho_{\mu} + \epsilon_{ij3} \left[ g_{m1} F^i_{\mu\nu} V^\mu_{\nu} + g_{m1A} \bar{A}^i_{\mu} \bar{A}^\mu_{\nu} \right] - \epsilon_{ij3} \left[ g_{m1} F^i_{\mu\nu} A^\mu_{\nu} V^\nu_{\mu} + g_{m1A} A^i_{\mu} A^\mu_{\nu} \bar{A}^\nu_{\mu} \right],
\]
where the following Abelian strength tensors have been defined:

\[ F^{\mu\nu} = \partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu} , \quad G^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} , \quad (13) \]

\[ F_i^{\mu\nu} = \partial_i^{\mu} V^{\nu} - \partial_i^{\nu} V^{\mu} , \quad G_i^{\mu\nu} = \partial_i^{\mu} A^{\nu} - \partial_i^{\nu} A^{\mu} . \quad (14) \]

Although the present approach gives rise to non Abelian corrections for the mesons field strength [17], these interaction terms are outside of the scope of the present work. The effective coupling constants in [11,12] were resolved in the long wavelength limit by calculating the traces in Dirac, color and isospin indices. After a Wick rotation to the Euclidean momentum space, they can be written as:

\[ g_{\mu A} = 3 g_{\omega A} = 4 e N_c d_1 T r' (((\tilde{S}_3(k)))), \quad (15) \]

\[ g_{F\rho} = 3 g_{F\omega} = 8 e N_c d_1 T r' (((\tilde{S}_0^3(k)))), \quad (16) \]

\[ g_{FF\rho} = \frac{3}{5} g_{FF\omega} = \frac{4e}{3} g_{F\rho\omega} = \frac{8}{3} e^2 N_c d_1 T r' (((\tilde{S}_0^3(k)))), \quad (17) \]

\[ g_{m1} = 4 e N_c d_1 T r' (((\tilde{S}_0(k)\tilde{S}_2(k)))), \quad (18) \]

\[ g_{m1A} = g_{m1} + M^2 d_1 8 N_c T r' (((\tilde{S}_0^3(k)))) = g_{m1} + 4 M^2 g_{F\rho\omega}, \quad (19) \]

where \( d_n = \frac{(-1)^{n+1}}{2n} \) and \( T r'(\ldots) \) are integrals in internal momenta for the zero momentum exchange limit. The following functions were used:

\[ \tilde{S}_0(k) = \frac{1}{k^2 + M^2}, \quad \tilde{S}_2(k) = \frac{k^2 - M^2}{(k^2 + M^2)^2}. \quad (20) \]

The interactions in Eq. (11) correspond to the two different types of vector meson dominance coupling. The first coupling constants \( g_{\mu A}, g_{\omega A} \), with dimension \( M^2 \) where \( M \) is a mass scale, are ultraviolet (UV) divergent and they are related by: \( f_r \equiv g_{\omega A}/g_{\mu A} = 1/3 \). This ratio is close to usual fittings \( 1/f_r \sim 3.5 \) [21]. Note that the definition obtained in the present work is the inverse of the usual convention for these couplings, i.e. \( g_{\mu A} \sim 1/g_{\rho} \). The same ratio holds for the momentum dependent coupling constants \( g_{F\omega}/g_{F\rho} = 1/3 \) being that these couplings are dimensionless and logarithmic UV divergent. The way to handle UV divergences will be discussed below. The dimensionless coupling constants \( g_{m1} \) and \( g_{m1A} \) are also logarithmic divergent and they correspond to electromagnetic couplings of vector and axial mesons. The coupling constants \( g_{F\rho\omega}, g_{FF\rho} \) and \( g_{FF\omega} \) have dimension \( M^{-2} \) and they can be seen as mixings induced by an external photon and effective couplings of each of the vector mesons to two photons. These issues will be reminded in the conclusions and they might have interesting consequences. These last coupling constants are UV finite and proportional among each other and they will provide the most important results of this work. In Figures (1-a) and (1-b) the Feynman diagrams corresponding to the couplings [15,16] are shown. The photon is represented by a wavy line, the vector mesons by a dotted-dashed line, and internal quarks by solid lines. The couplings for the strength tensors of vector mesons or photon are indicated by a triangle in the corresponding vertex. In Figures (2-a), (2-b) and (2-c) the Feynman diagrams for all the interaction terms [12] are exhibited.

### 2.2 Equivalence of VMD couplings and zero photon mass

The momentum dependent VMD couplings in Eq. (11) have been found more suitable to describe phenomenology [21]. Besides that it has been pointed out by Kroll et al [20] that a field redefinition
Figure 1: In these diagrams, an internal solid line represents a sea quark, the wavy and dot-dashed lines stand for a photon and a vector meson respectively. The wavy (dot-dashed) line with a triangle in a vertex stands for the electromagnetic (vector mesons) strength tensor $F^{\mu\nu}$ ($F^{\mu\nu}_i$, $F^{\mu\nu}$).

Figure 2: In these diagrams, the same convention used in Figure 1 is considered for the three-leg vertices. eliminates the momentum independent ones. To see that, consider the resulting free vector/axial mesons terms $L_{\text{free}}$ [36, 37] and the leading photon correction terms $L_{A,M}$ from the expansion of the determinant above [16, 47]:

\begin{align}
L_{\text{free}} &= -\frac{g_f^{(0)}}{4} \left( F^{\mu\nu}_i F_{\mu\nu}^i + G^{\mu\nu}_i G^{\mu\nu}_i + F^{\mu\nu} F_{\mu\nu} + G^{\mu\nu} G_{\mu\nu} \right), \\
L_{A,M} &= -\frac{5}{9} \frac{M_v^{(0)^2} e^2}{f_v^2} A_\mu A^\mu - \frac{g_F}{4} F_{\mu\nu} F^{\mu\nu} - \frac{M_v^{(0)^2}}{2} \left( V^{(0)^2}_i + V^{(0)^2}_\mu \right) - \frac{M_A^{(0)^2}}{2} (\tilde{A}_{\mu i} + \tilde{A}^{(0)^2}_\mu),
\end{align}

where $f_v$ is the vector mesons field normalization and the following effective parameters have been defined in the same long wavelength and zero momentum limit considered before:

\begin{align}
g_f^{(0)} &= d_{\text{A}}^1 N_c Tr' \left( \tilde{S}_1^2(k) \right), \\
M_v^{(0)^2} &= d_{\text{A}}^1 N_c Tr' \left( \tilde{S}_2(k) \right), \\
M_A^{(0)^2} &= M_v^{(0)^2} + 4g_f^{(0)} M^{*^2}.
\end{align}

By considering field redefinitions of the type:

\begin{align}
A_\mu &= c\tilde{A}_\mu, \\
V^{(0)^2}_\mu &= \tilde{V}^{(0)^2}_\mu + \frac{c e}{2 f_v} \tilde{A}_\mu, \\
V_\mu &= \tilde{V}_\mu + \frac{c e}{6 f_v} \tilde{A}_\mu,
\end{align}

where $c$ is a constant, the photon mass reduces to zero and the momentum independent VMD couplings $g_{\rho A}, g_{\omega A}$ disappear from the Lagrangian in the same way it was shown in Ref. [20].
Several of the expressions found above, Eqs. (15,16,18,22,23,24), are ultraviolet (UV) divergent and the following procedure was adopted to render results finite. There are only two types of divergences, quadratic and logarithmic, and they show up in Eqs. (22,23). These two quantities are simply expected to provide the free vector mesons (ρ or ω) tree level parameters, mass and field normalization. By fixing the values of $M_{v}^{(0)}$, $g_{f}^{(0)}$, one may expect to attribute finite values for the two types of integrals in all the expressions above. All the other UV divergent coupling constants can be written in terms of these integrals and with that they become finite. Therefore, let us consider finite fixed values:

\[ g_{f}^{(0)} = f_{v}^{2}, \]
\[ M_{v}^{(0)2} = m_{V}^{2}. \]  

Initially one could take $F_{v} = 1$ and $M_{v}^{(0)} \simeq 0.77$ GeV. However smaller values should be expected because to obtain Eqs. (22) and (23) the vector mesons structureless limit has been assumed. With this, the axial mesons masses are different from the vector mesons ones by a finite positive factor as shown in Eq. (24). By considering the values used below ($g_{f}^{(0)} \simeq 0.5$ and $M^{*} = 0.33$ GeV), the axial meson-vector mesons mass difference is of the order of $\Delta M \sim \sqrt{2} \times 0.33 = 0.467$ GeV that provides the correct experimental mass differences values [24]. When plugging these relations in the coupling constants with UV divergences, Eqs. (15,16,18) and (19), it can be written that:

\[ g_{\rho A} = 3g_{\omega A} = \frac{e}{2}m_{V}^{2}, \]
\[ g_{F \rho} = 3g_{F \omega} = 2e f_{v}, \]
\[ g_{m1} = e f_{v} + \frac{3M^{*2}}{e}g_{FF \rho} = e f_{v} + 4M^{*2}g_{F \rho \omega}, \]
\[ g_{m1A} = e f_{v} + \frac{6M^{*2}}{e}g_{FF \rho} = e f_{v} + 8M^{*2}g_{F \rho \omega}. \]

This procedure might be viewed as a (partial) renormalization of the model and the reason is the following. The renormalized quantities are obtained with implicit $Z_{q}, Z_{\rho}, Z_{\omega}, Z_{A}$ (quark, rho, omega and axial meson wavefunction renormalizations) and eventually $Z_{g}$ for the quark-gluon coupling constant. These renormalization constants make all the divergent integrals finite in the same way. Eqs. (22) and (23) may be considered renormalization conditions for the isospin symmetric case analysed in the present work with $Z_{\rho} = Z_{\omega} = Z_{A}$. The effective coupling constants depend on the relevant renormalization constants basically in the same way at this order of the quark determinant expansion, so that it is enough to consider the two renormalization conditions above to render all the UV-divergent integrals finite. Eqs. (27-29) establish well definite scales or order of magnitude for the resulting effective coupling constants in terms of $g_{f}^{(0)}$ and $M_{v}^{(0)}$.

### 2.3 Weak magnetic field dependent effective couplings

For a magnetic field, $B_{0}$ along the $-\hat{z}$ direction one might choose $A^{\mu} = B_{0}(0,0,x,0)$. The three-leg coupling constants in Eq. (12) can be then rewritten to incorporate $B_{0}$. By identifying explicitly the
Lorentz and isospin components index of the tensors and fields, the following terms arise:

\[
\mathcal{L}_{F}^{B} = \left( \frac{eB_{0}}{M^{*2}} \right) g_{F \rho}^{B} (\mathcal{F}_{\mu=2,\rho}^{i=3, \mathcal{F}_{\nu=1}^{\rho}} + g_{\mu=2,\nu=1}^{i=3} G_{\rho}^{\mu} ) + \left( \frac{eB_{0}}{M^{*2}} \right) g_{FF\rho}^{B} F_{\mu=2,\rho}^{\nu=3} \mathcal{F}_{\nu=1}^{\rho} + \left( \frac{eB_{0}}{M^{*2}} \right) g_{FF\omega}^{B} F_{\mu=2,\rho}^{\nu=1} \mathcal{F}_{\nu=1}^{\rho}
\]

\[
+ \left( \frac{eB_{0}}{M^{*2}} \right) \epsilon_{ij} [-g_{m1}^{B} F_{\mu=2,\nu}^{i=3} V_{j}^{\nu} - g_{m1A}^{B} G_{\mu=2,\nu}^{i} A_{j}^{\nu} + g_{m2}^{B} V_{i}^{\mu=1} V_{j}^{\nu=2} + g_{m2A}^{B} A_{i}^{\mu=1} A_{j}^{\nu=2}], \quad (31)
\]

where some of these effective coupling constants were simply redefined as:

\[
g_{FF\rho}^{B} = 3 \left( \frac{e}{3} g_{F \rho}^{B} = \frac{A^{*2}}{g_{FF\rho}}, \right. (32)
\]

\[
g_{m2}^{B} = \frac{M^{*2}}{g_{m1}}, \quad (33)
\]

\[
g_{m2A}^{B} = \frac{M^{*2}}{g_{m1A}}, \quad (34)
\]

\[
g_{m1}^{B} = 4 N_c d_1 M^{*2} T r' \left( (\delta_{kx} [\bar{S}_0(k) S_2(k, k + q)]) \right)_{q=0}, \quad (35)
\]

\[
g_{m1A}^{B} = g_{m1}^{B} + 4 M^{*} g_{F \rho \omega}, \quad (36)
\]

where \( \partial_{q} = \frac{\partial}{\partial q^{x}} \) and

\[
\bar{S}_2(k, k + q) = \frac{(k^2 + k \cdot q - M^{*2})}{(k^2 + M^{*2})(k + q)^{2} + M^{*2}}. \quad (37)
\]

The effective coupling constants \( g_{m1}^{B} \) and \( g_{m2}^{B} \) have dimensions \( M \) and \( M^{2} \) respectively, while the others \( g_{F \rho \omega}^{B} \) and \( g_{FF\rho}^{B}, g_{FF\omega}^{B} \) are dimensionless. The coupling constants \( g_{FF\rho}^{B} \) is a magnetic field induced anisotropic vector mesons mixing term. It is also in the axial mesons mixing coupling in Eq. \( (36) \).

Note that the difference in the \( \omega \) and \( \rho \) vector mesons couplings to the photon in the vacuum \( (g_{F \rho}, g_{F \omega}) \) is not the same under weak magnetic field due to the isospin/chiral symmetry breakings induced by the magnetic field. According to expressions above, the ratio of the anisotropic corrections to VMD effective couplings due to weak \( B_{0} \) is

\[
\frac{g_{FF\rho}^{B}}{g_{FF\omega}^{B}} = \frac{3}{5}.
\]

Note however that the strengths of the axial meson mixing coupling constants are larger than the corresponding ones from the vector mesons mixings.

\( g_{m1}^{B}, g_{m1A}^{B} \) and \( g_{m2}^{B}, g_{m2A}^{B} \) correspond respectively to anisotropic magnetic field corrections to the propagation of charged rho and A1 mesons that mix Dirac components \( x, y \). The terms from \( g_{m2}^{B} \) and \( g_{m2A}^{B} \) might be identified as types of mass-corrections that can be written as:

\[
\Delta M_{\rho^{*}}^{x,y} = \sqrt{\frac{eB_{0}}{M^{*2}}} \sqrt{\frac{2g_{m2}^{B} M^{*2}}{e}}, \quad \Delta M_{A^{*}}^{x,y} = \sqrt{\frac{eB_{0}}{M^{*2}}} \sqrt{\frac{2g_{m2A}^{B} M^{*2}}{e}}. \quad (38)
\]

It is interesting to emphasize that the weak magnetic field corrections to mixing and VMD couplings are simply proportional among themselves. Therefore the magnetic field induced coupling constants have two contributions, one from the quark effective mass \( M^{*} \) that is dependent on the magnetic field from the gap equation and the multiplicative factor \( eB_{0}/M^{*2} \). The dependence of all these quantities on the up and down quark mass differences will not be considered at length here.
3 Numerical estimates and form factors

Exact relations between some of the effective coupling constants were already exhibited in Eqs. (15,16) and (17) and the corresponding $B_0$-dependent ones (32,33). Further relations for resulting effective parameters and the regularization parameters, $M_v^{(0)}$, $g_f^{(0)}$, were shown in Eqs. (27,28) and (29). Some simple approximated ratios can also be estimated in the limit of very large quark effective mass by noting the dependence of the quark kernels on $M^*$, i.e.

$$S_0 \sim \frac{1}{M^*}, \quad \tilde{S}_0 \sim \frac{1}{M^{*2}}.$$  

This yields:

$$\frac{g_{F\rho\omega}}{g_{m1}} \sim \frac{1}{2M^{*2}}, \quad \frac{g_{\rho A}}{g_{F\rho}} \sim \frac{M^{*2}}{2}, \quad \frac{g_{F\rho\omega}}{g_{F\rho}} \sim \frac{B_0}{4M^{*2}}.$$ (39)

These ratios make explicit the relative order of magnitude of the effective coupling constants in the limit of very large $M^*$. The second of these ratios, for the VMD momentum dependent and independent coupling constants, have the same order of magnitude of the corresponding ratio calculated with values fitted from phenomenology.

In the Table I several estimations for coupling constants in the vacuum and under weak $B_0$ from Eqs. (15,16,18) and (33,38) are presented for different values of the normalization integrals (26). The only coupling constant with explicit normalization dependence on the effective mass $M^*$ is $g_{m2}$, from Eq. (29). In this case, values for two different effective mass were presented. The structureless limit of the vector mesons can be expected to be responsible for limited account of the vector mesons mass and normalization constant, by means of the parameters $M_v^{(0)}$ and $g_f^{(0)}$. Therefore smaller values than the expected ones for a tree level Lagrangian terms to describe vector mesons dynamics were also considered. To compare with results, usual values for the VMD phenomenological coupling constants are the following:

$$g_{vmd1} = \frac{e}{g_\rho} \sim 6.2 \times 10^{-2}, \quad g_{vmd2} \sim \frac{em^2_\rho}{g_\rho} \sim 3.7 \times 10^4 \text{ MeV}^2.$$ 

[19]. The best values for the two normalization parameters can be expected to be $m_V \simeq 0.5 \text{ GeV}$ and $f_\pi \simeq 0.1$ for Eqs. (27) and (28) to reproduce respectively $g_{vmd2}$ and $g_{vmd1}$ from phenomenology.

The couplings $g_{F\rho\omega}^B$ and $g_{FF\omega}^B$ represent corrections to vector meson dominance terms induced by the external magnetic field and it contains an anisotropic contribution. These terms can be added to the terms in (11) for the VMD coupling constant. Similarly the emerging magnetic field induced anisotropic mixing term has a coupling with $g_{F\rho\omega}^B$. The resulting coupling constants, with anisotropic corrections, can be written as:

$$g_{\rho}^{vmd1}(B_0) = g_{F\rho} + \left( \frac{eB_0}{M^{*2}} \right) g_{FF\rho}^B,$$ (40)

$$g_{\omega}^{vmd1}(B_0) = g_{F\omega} + \left( \frac{eB_0}{M^{*2}} \right) g_{FF\omega}^B,$$ (41)

$$g_{mix1}(B_0) = \left( \frac{eB_0}{M^{*2}} \right) g_{F\rho\omega}^B.$$ (42)
Table 1: Numerical results for several of the coupling constants in the vacuum and induced by weak $B_0$ from Eqs. (15,16) and (18) and (33,38) respectively. Different values of the normalization values for the integrals from $f_v, m_V$ given by Eq. (26).

| coupling constants | $m_V$ (GeV) | $f_v = 1.0$ | $f_v = 0.1$ | $f_v = 0.1$ | $f_v = 1.0$ | $f_v = 1.0$ |
|-------------------|-------------|------------|------------|------------|------------|------------|
| $g_{\rho A}$ (MeV$^2$) | $8.9 \times 10^4$ | $8.9 \times 10^4$ | $3.8 \times 10^4$ | $3.8 \times 10^4$ | $1.5 \times 10^3$ | |
| $g_{F \rho}$ | 0.606 | 0.061 | 0.061 | 0.606 | 0.606 | |
| $g_{m_1}$ | 0.315 | 0.042 | 0.042 | 0.315 | 0.315 | |
| $g_{m_2}^B$ (GeV$^2$) | 0.113 | 0.015 | 0.015 | 0.113 | 0.113 | |
| ($M^* = 0.33$ GeV) | | | | | | |
| $g_{m_2}^B$ (GeV$^2$) | 0.210 | 0.032 | 0.032 | 0.210 | 0.210 | |
| ($M^* = 0.45$ GeV) | | | | | | |
| $\frac{\Delta M_{\rho - \omega}^{e - g}}{\sqrt{(e B_0/M^*)^2}}$ (MeV) | 475 | 173 | 173 | 475 | 475 | |

where the resulting coupling constants are dimensionless and they still present the implicit magnetic field dependence of $M^*$.

Considering two different values for the quark effective mass $M^*$ the following values are obtained:

$$M^* = 0.33 \text{ GeV} \quad \rightarrow \quad g_{FF \rho} = 1.07 \times 10^{-2} \text{ GeV}^{-2},$$
$$g_{BFF \rho} = 3.83 \times 10^{-3}, \quad g_{BFF \omega} = 2.48 g_{FF \rho},$$

$$M^* = 0.45 \text{ GeV} \quad \rightarrow \quad g_{FF \rho} = 5.74 \times 10^{-3} \text{ GeV}^{-2},$$
$$g_{BFF \rho} = 3.83 \times 10^{-3}, \quad g_{BFF \omega} = 2.48 g_{FF \rho}.$$  
(43) \quad \text{(44)}

The weak magnetic field induced corrections to the momentum dependent VMD and vector mesons mixings are therefore finite and basically independent of $M^*$. These coupling constants, $g_{FF \rho} \propto g_{FF \omega}$, are of the order of $(e B_0/M^*)^2$, i.e. therefore small with respect to the zero magnetic field value $g_{FF \rho}$.

After the estimates done above, some further numerical ratios between coupling constants presented in this work will be exhibited next. In particular ratios between magnetic field induced corrections to rho and omega mesons dominance and also vector mesons and axial mesons coupling constants. By taking numerical values for $M^* = 0.33 \text{ GeV}$ and $e B_0/M^* = 0.1$ one has:

$$R_1 \equiv \frac{g_{\text{vmd1}}(B_0)}{g_{\text{vmd1}}(B_0)} = \frac{g_{F \rho} + \frac{e B_0}{M^*}}{g_{F \omega} + \frac{e B_0}{M^*}} \sim 2.99 \quad \text{(for } f_v = 1.0)$$
$$\sim 2.93 \quad \text{(for } f_v = 0.1),$$

$$R_3 \equiv \frac{g_{m_A}^B}{g_{m_2}} = \frac{g_{m_A}}{g_{m_1}} = \frac{e f_v + \frac{6 M^*}{e} g_{FF \rho}}{e f_v + \frac{3 M^*}{e} g_{FF \rho}} \sim 1.04 \quad \text{(for } f_v = 1),$$
$$\sim 1.27 \quad \text{(for } f_v = 0.1).$$

(45) \quad (46)
The ratio $R_1$ can be compared to its value in the vacuum discussed after Eq. (20) that is $R_1(B_0 = 0) = 3$. From ratio $R_3$ the momentum independent axial mesons mixing coupling constant ($g_{m2A}$) induced by weak magnetic field is larger than the corresponding vector mesons mixing coupling constant ($g_{m2V}$). The momentum dependent mixings $g_{F_{ho
u}}$ were noted to be the same at this level of calculation in Eq. (31).

### 3.1 Form factors

Next, it is interesting to write the Fourier transformation of complete non local expressions for some of the terms obtained from the quark determinant expansion [11][12] and also (31). Consider the following terms:

\[
\mathcal{L}_{ff} = -g_{\rho A}(x-y)V_{i=3}^\mu(x)A_{\mu}(y) - \tilde{g}_{F_{\rho \nu}}(x-y)F_{\mu\nu}^i(x)F_{\mu\nu}(y) + \tilde{g}_{F_{\rho \nu}}(x, y, z)F_{\mu\nu}(x)F_{\nu\rho}(y)F_{\rho}^{i=3,\mu}(z) + \epsilon_{ij3} \left[ \tilde{g}_{m1A}(x, y, z)F_{\mu\nu}(x)V_{i}^\nu(y)V_j^\mu(z) + \tilde{g}_{m1A}(x, y, z)F_{\mu\nu}(x)\tilde{A}_i^\nu(y)\tilde{A}_j^\mu(z) \right] + \frac{\sqrt{B}}{M^{*2}} F_{\mu=2,\rho}(x)F_{\mu=1,\nu}^{i=3}(y). \tag{47}
\]

The resulting expressions are the momentum dependent vertices that correspond to the Feynman diagrams of Figures [12]. The following Fourier transformed terms are obtained:

\[
\tilde{\mathcal{L}}_{ff} = -g_{\rho A}(Q^2)V_{i=3}^\mu(Q)A_\mu(-Q) - g_{F_{\rho \nu}}(Q^2)F_{i=3,\mu}^\nu(Q)F_{\mu\nu}^\nu(-Q) + g_{F_{\rho \nu}}(Q, Q_2)F_{\mu\nu}(Q_2)F_{\nu\rho}^\nu(Q)F_{\rho}^{i=3,\mu}(Q + Q_2) + \epsilon_{ij3} \left[ g_{m1A}(Q, Q_2)F_{\mu\nu}(Q_2)V_{i}^\nu(Q + Q_2) + g_{m1A}(Q, Q_2)F_{\mu\nu}(Q_2)\tilde{A}_i^\nu(Q + Q_2) \right] + \frac{\sqrt{B}}{M^{*2}} F_{\mu=2,\rho}(Q)F_{\mu=1,\nu}^{i=3}(-Q). \tag{48}
\]

The corresponding form factors, in Euclidean momentum space, are given by:

\[
g_{\rho A}(Q^2) = 4eN_c d_1 \int \frac{k^2 + k \cdot Q - M^{*2}}{(k^2 + M^{*2})((k + Q)^2 + M^{*2})^4}, \tag{49}
\]

\[
g_{F_{\rho \nu}}(Q^2) = 8eN_c d_1 \int \frac{1}{(k^2 + M^{*2})((k + Q)^2 + M^{*2})^4}, \tag{50}
\]

\[
g_{m1A}(Q, Q_2) = 4eN_c d_1 \int \frac{(k^2 + k \cdot Q + k \cdot Q_2 - M^{*2})}{(k^2 + k \cdot Q + k \cdot Q_2 - M^{*2})}, \tag{51}
\]

\[
g_{m1A}(Q, Q_2) = g_{m1}(Q, Q_2) + \frac{3M^{*2}}{e} g_{F_{\rho \nu}}(Q, Q_2), \tag{52}
\]

\[
g_{F_{\rho \nu}}(Q, Q_2) = \frac{8}{3} e^2 N_c d_1 \int \frac{1}{(k^2 + M^{*2})((k + Q)^2 + M^{*2})((k + Q + Q_2)^2 + M^{*2})^4}, \tag{53}
\]

\[
g_{F_{\rho \nu}}(Q, Q_2) = 8eM^{*2}N_c d_1 \int \frac{1}{(k^2 + M^{*2})((k + Q)^2 + M^{*2})}, \tag{54}
\]

where $\int_k = \int d^4k/(2\pi)^4$. The form factors were presented as functions of $Q^2$ or $(Q, Q_2)$ being this second case functions of all the scalars formed by the momenta $Q$ and $Q_2$. The coupling constants
were defined in the zero momentum exchange limit of these expressions \((Q = Q_2 = 0)\). Although the integrals of Eqs. \((49,50)\) and \((51)\) have quadratic or logarithmic UV divergences, their values at \(Q = 0\) are normalized according to the prescriptions \((26)\) as discussed above. All the curves shown below for the form factors do not include the linear momentum \(Q\) from the strength tensors in Eq. \((48)\).

In Figure \((3)\) the following form factors \(g_{\rho A}(Q^2), g_{F\rho}(Q^2)\) and also \(g_{F\rho\omega}(Q^2)\) are drawn for \(M^* = 330\) MeV. The coupling \(g_{F\rho\omega}(Q^2)\) represents the weak magnetic field correction to VMD and it is proportional to the induced vector mesons mixing strength

\[
g_{F\rho\omega}(Q^2) = \frac{3}{4e} g_{F\rho\omega}(Q^2).\]

Simple approximated fittings for the three form factors of figure \((3)\), \(g_{\rho A}(Q^2), g_{F\rho}(Q^2)\) and \(g_{F\rho\omega}(Q^2)\), were found with the following simple functions parameterized by two constants each of them \((k_{\rho A}, k_{F\rho}, k_{FF\rho}\) and \(g_{\rho A}(0), g_{F\rho}(0), g_{F\rho\omega}(0)):\)

\[
Fit_1(Q^2) = k_{\rho A}^A \frac{g_{\rho A}(0)}{(Q^2 + k_{\rho A}^2)^2},
\]

\[
Fit_2(Q^2) = k_{F\rho}^2 \frac{g_{F\rho}(0)}{(Q^2 + k_{F\rho}^2)^2},
\]

\[
Fit_3(Q^2) = k_{FF\rho}^4 \frac{g_{F\rho\omega}(0)}{(Q^2 + k_{FF\rho}^2)^2}.
\]

The zero momentum values in these fittings are those from the Table and \((43)\) and approximated values for the other constants are the following: \(k_{\rho A} \simeq 2.2\) GeV, \(k_{F\rho} \simeq 1.8\) GeV and \(k_{FF\rho} \simeq 1.5\) GeV.

In Figure \((4)\) the form factor \(g_{m1}(Q, Q_2)\) is shown for \(M^* = 330\) MeV and Euclidean spacelike momenta as function of \(|Q| = \sqrt{|Q|^2}\). The regularization point shown is the one for \(f_v = 0.1\) of the Table. Since there are two external momenta, \(Q\) and \(Q_2\), different arbitrary choices for the spacelike momentum \(Q_2\) are exhibited. However the final values for these form factor must be multiplied by \(Q\) due to the linear momentum from the strength tensor, and therefore it reduces to zero for zero momentum transfer. Firstly, \(Q_2 \cdot Q\) is considered to be positive representing a vector meson and a photon incoming to the vertex, and vector meson outgoing from the vertex. The following different values were adopted: \(Q_2 = +Q;\) \(Q_2 = +2Q;\) \(Q_2 = +Q/2\). Secondly, \(Q_2 \cdot Q < 0\) was taken to represent one incoming vector meson to the vertex, and a photon and the other vector meson outgoing from the vertex. The following values were adopted: \(Q_2 = -Q;\) \(Q_2 = -2Q;\) \(Q_2 = -Q/2\). There are stronger differences in the momentum dependence for different values of \(M^*\) for the cases \(Q_2 = \pm Q/2\) and \(Q_2 = \pm 2Q\). There is a non monotonic behavior for the \(Q_2 = +2Q\), and larger.

### 3.2 Pion form factor, VMD and rho-omega mixing

In this section some of the above corrections of the \(B_0\) to vector meson dominance coupling and of the \(B_0\)-induced vector mesons mixing are verified on pion electromagnetic form factors. The pion form factor has been parameterized within the VMD momentum assumption by the following Eq. \((4)\):

\[
F_\pi(Q^2) = 1 - Q^2 \frac{G_{vmd}^\pi \rho(\Gamma_{\rho}(Q^2))}{Q^2 - m_\rho^2 + im_\rho \Gamma_{\rho}(Q^2)} g_{\rho\pi\pi}.
\]
Figure 3: The form factors $G_i(Q^2)$ are the following: $g_{\rho A}(Q^2)$ (MeV$^2$), $g_{F\rho}(Q^2)$ and $g_{FF\rho}(Q^2)$. They were presented in Eqs. (49) and (50) and (54) and they are shown for $M^* = 330$ MeV as functions of spacelike $|Q| = \sqrt{|Q^2|}$. Dashed line is used for $g_{\rho A}$, continuous line for $g_{F\rho}$ and dotted line for $g_{FF\rho}(Q^2)$.

where $m_\rho$, $\Gamma_\rho(Q^2)$, $g_{\rho\pi\pi}$ are respectively the rho mass and width and the rho coupling to pions. In phenomenological models the VMD strength is simply described by $G_{vmd} = g_\rho \simeq \sqrt{4\pi \times 2}$. The rho width can also be considered as momentum dependent, by incorporating the rho threshold, as [4]:

$$
\Gamma_\rho(Q^2) = \Gamma_\rho \left( \frac{\sqrt{Q^2 - 4m_\pi^2}}{\sqrt{m_\rho^2 - 4m_\pi^2}} \right)^3 \left( \frac{m_\rho}{\sqrt{Q^2}} \right)^\lambda,
$$

(59)

where $\Gamma_\rho = 146.2$ MeV [24] and $\lambda = 1$.

The effect of the weak $B_0$ induced correction to VMD momentum dependent coupling from Eq. (48) can be verified in the expression above. For this, the VMD strength will be considered to be composed by $B_0$ independent and dependent components. It will be parameterized by:

$$
G_{vmd}(Q^2) = g_{F\rho} + g_{FF\rho}(Q^2) eB_0 M^{*2},
$$

(60)

Besides that, a correction due to rho omega mixing in the vacuum has also been found by considering the isospin breaking up and down quark mass difference [4]. In the present work this mixing is obtained
Figure 4: The form factor $g_{m1}(Q, Q_2) \times 10^{-3}$ presented in Eq. (51) is exhibited by considering the zero momentum value of the Table for $f_\pi = 0.1$. for $M^* = 330$ MeV as functions of spacelike $|Q|$ = $\sqrt{|Q^2|}$. Different particular choices for spacelike momentum $Q_2$ were considered to be related to $Q$ as: $Q_2 = +Q$ means $Q_2 \cdot Q = Q^2$; $Q_2 = -Q$ means $Q_2 \cdot Q = -Q^2$; and so one for $Q_2 = +2Q$; $Q_2 = -2Q$; $Q_2 = +Q/2$; and $Q_2 = -Q/2$. due to a weak external magnetic field and its isolated effect will be investigated below. Its contribution to be added to $F_\pi(q)$ can be written as:

$$\Delta F_\pi(Q^2) = -\epsilon Q^2 \frac{g_{\rho\pi\rho} g_\omega}{Q^2 - m_\rho^2 + i m_\omega \Gamma_\omega},$$

(61)

where

$$\epsilon = \frac{G_{\omega\rho}(Q^2)}{m_\omega^2 - m_\rho^2 - i(m_\omega \Gamma_\omega - m_\rho \Gamma_\rho(Q^2))},$$

(62)

in this expression, as discussed above, $g_\omega = g_\rho/3.5$ according to usual fits (considered in the numerical estimation below) or $g_\omega = g_\rho/3$ according to Eqs. (15) and (16). The omega width is considered to be constant and the rho-omega momentum dependent mixing has a $B_0$ induced component with a form factor $g_{F\rho\omega}^B(q)$ that is proportional to $g_{F\rho\omega}^B$. It will be considered that

$$G_{\omega\rho}(Q^2) = g_{mix} + g_{F\rho\omega}^B(Q^2) \frac{eB_0}{M^*}.$$  

(63)

As reminded above the isospin breaking mixing strength is usually fitted to $g_{mix} = -4.52 \times 10^{-3}$ GeV$^2$. 

15
Some of the values considered below are the following \[24\]:

\[
m_\rho = 775 \text{ MeV}, \quad m_\omega = 783 \text{ MeV}, \quad g_{\rho\pi\pi} = \sqrt{4\pi \times 2.9}, \quad \Gamma_\omega = 8.5 \text{ MeV}.
\]

To show the corrections to the pion form factor induced by the \(B_0\) dependent mixing and VMD couplings, the following differences will be used:

\[
D_1 F_\pi^2(Q^2) = |F_\pi(Q^2)|^2 - |F_{\pi,B}(Q^2)|^2,
\]

\[
D_2 F_\pi^2(Q^2) = |F_{\pi,mix,B}(Q^2)|^2 - |F_\pi(Q^2)|^2,
\]

where these functions were defined above. In Figure 5 these functions \(D_1 F_\pi^2(Q^2)\) and \(D_2 F_\pi^2(Q^2)\) are presented for two different quark effective masses, \(M^* = 0.33 \text{ GeV}\) and \(0.45 \text{ GeV}\), and for \(eB_0/M^* = 0.1\). Since the correction of the weak magnetic field is of the order of \(eB_0/M^* = 0.1\) their contributions to \(F_\pi^2(Q^2)\) are at most of the order of \(10^{-2}\), and therefore small. It can be seen that \(D_1 F_\pi^2(Q^2)\), that contains the VMD dependence on \(B_0\), presents a larger deviation. The \(B_0\) dependence of the mixing form factor is considerably less important.

### 3.3 Charge violation potential from weak magnetic field

The charge symmetry violation nucleon potential has been attributed to isospin breaking as a source to the light vector mesons mixing \([13, 9, 10, 14, 23, 4]\). Consider a \(\rho - \omega\) mixing matrix element \(<\rho|H_{mix}|\omega>\) that has been shown to be better described by a momentum dependent interaction. Instead of the rho-nucleon form factor \(F_{\rho N}(Q^2)\) and omega-nucleon form factor \(F_{\omega N}(Q^2)\) it will be considered the corresponding constituent quark couplings \(F_{\rho qq}(Q^2)\) and \(F_{\omega qq}(Q^2)\). The CSV contribution in momentum space, for spacelike Euclidean momenta \(Q^2 > 0\), can be written as \([13, 9, 10]\):

\[
V(Q) = -\frac{F_{\omega qq}(Q^2)F_{\rho qq}(Q^2)<\rho|H_{mix}|\omega>}{(Q^2 + m_\rho^2)(Q^2 + m_\omega^2)}.
\]

where the form factors \(F_{\rho qq}(Q^2)\) and \(F_{\omega qq}(Q^2)\) were considered to be monopole or quadrupole fittings \([48, 49, 13]\) shown below. In position space the CSV potential is given by:

\[
V_{\text{csv}}(r) = -\frac{1}{2\pi^2 r} \int_0^\infty dQ \sin(Qr) Q V(Q).
\]

The effect of weak magnetic field on the mixing amplitude will be shown below by considering its contribution by means of the following quantity:

\[
D_B V_{\text{csv}}(r) = -(V_{\text{csv}}^{(B)}(r) - V_{\text{csv}}^{(B=0)}(r)),
\]

being that \(V_{\text{csv}}^{(B)}(r)\) is calculated by adding the magnetic field correction to the mixing amplitude, i.e.

\[
\Delta_B <\rho|H_{mix}|\omega> = g_{F_{\rho\omega}}^B(Q).
\]
Figure 5: The difference between zero and finite weak magnetic field corrections to VMD and mixing strength for the fit of the pion form factor given by Eq. (58) as functions of $Q^2$, timelike ($Q^2 > 0$) and spacelike ($Q^2 < 0$) momenta by means of the quantities defined in Eq. (65). It was considered $(eB_0/M^*^2) = 0.1$ and $M^* = 0.33$ GeV and $0.45$ GeV. $m_\rho = 775$ MeV, $m_\omega = 783$ MeV, $g_{\rho\pi\pi} = \sqrt{4\pi \times 2.9}$, $\Gamma_\omega = 8.5$ MeV.

Two different parameterizations for the form factors $F_{\rho qq}(Q^2)$ and $F_{\omega qq}(Q^2)$ were considered: one with a monopole shape and another with a quadrupole shape both presented in the literature [13, 48, 49]. They are given respectively by:

$$F_{\text{Mono}}(Q^2) = \frac{g_\rho}{1 + \frac{Q^2}{\Lambda_\rho^2}}, \quad F_{\text{Quad}}(Q^2) = \frac{g_\rho}{(1 + \frac{Q^2}{\Lambda_\rho^2})^3},$$

(71)

where $g_\rho$ was given above with an analogous expressions for the $\omega$ form factors. The values for the constants are [13, 48, 49]: $\Lambda_\rho = 1.4$ GeV and $\Lambda_\omega = 1.5$ GeV for the monopole form and $\Lambda_\rho \simeq \Lambda_\omega = 1.12$ GeV for the quadrupole form.

In Figure 6 the magnetic field induced contribution to the (off-shell) CSV potential, Eq. (69), $D_B V_{\text{csv}}(r)$ is presented for $(eB_0/M^*^2) = 0.1$ by considering the two shapes for the vector mesons form factors (71) and two quark effective masses $M^* = 0.33$ GeV and $M^* = 0.45$ GeV. For a given (fixed) effective mass the effect of the weak magnetic field reduces mostly to the multiplicative factor $(eB_0/M^*^2)$ according to the expressions shown in the previous sections. Therefore for $(eB_0/M^*^2) = 0.2$
or \((eB_0/M^*) = 0.3\) one can basically multiply the values of Fig. 6 by 2 or 3 respectively, with smaller correction due to the \(B_0\) dependence of the effective mass. The quadrupole form factors induces stronger suppression of the resulting potential as it could be expected. Therefore the difference between the two curves for \(M^* = 0.33\) GeV and \(M^* = 0.45\) GeV is considerably smaller for the quadrupole form factors. Several previous analysis suggested values in the range \(V_{csv}(0) \sim 0.1 - 2.0\) MeV for the (total) isospin symmetry breaking contribution, but a smaller (and negative) strength for off-shell case, of the order of \(-V_{csv}(0) \sim 0.01 - 0.1\) MeV \([4, 13, 50, 23, 9]\). This is of the same order of magnitude of the resulting \(V_B(0) - V_{B=0}(0)\) found in Fig. (6) for \((eB_0/M^*) = 0.1\). In the case of the monopole form factors, there is a change of sign in the potential around \(r \simeq 0.6\) fm. This effect had been found in different works also for the off-shell potential with \(V(r \simeq 0.75 - 0.9\) fm) = 0 \([22, 13, 50, 23, 9]\).

Figure 6: Contribution of the weak \(B_0\)-induced vector mesons mixing for the CSV potential from Eq. (69) for \((eB_0/M^*) = 0.1\) and two values of the quark effective mass \(M^* = 0.33\) GeV and \(M^* = 0.45\) GeV. Both shapes for the vector mesons form factors, Monopole (Mono) and Quadrupole (Quad), are compared with Eq. (71).

4 Final remarks

The limit of structureless vector and axial-vector mesons considered in the present work made possible to derive simple expressions for their leading couplings to a background electromagnetic field in the large
quark effective mass expansion of the sea quark determinant. The two simplest types of phenomenological photon couplings to vector mesons (VMD), the momentum dependent and independent ones, and several other electromagnetic couplings were obtained. From them, weak magnetic field-induced corrections to usual phenomenological coupling constants for VMD and vector mesons mixings were obtained. A magnetic field in the \( \hat{z} \) direction was considered to be weak with respect to a hadron mass scale such as the quark effective mass \( (eB_0/M^*)^2 \) < 1. Being some of the effective couplings anisotropic, one can choose a slightly more symmetric vector potential such as to symmetrize the role of the parallel/transversal momentum components and vector/axial mesons components in Eq. (31). All the resulting coupling constants were expressed in terms of components of the quark propagator, obtained with DChSB, and of the parameters of the GCM (quark-gluon coupling constant and quark masses). The non degeneracy of up and down constituent quark masses has been investigated in several papers in the literature and it was left outside the scope of the work to emphasize the effect of the weak magnetic field. To render the ultraviolet divergent momentum integrals finite a renormalization scheme was adopted by fixing vector mesons masses and normalization constants as renormalization conditions. Although the main aim of the work is to derive photon and weak magnetic field induced couplings between light vector and axial mesons, magnetic field independent couplings and free terms were also presented for the sake of completeness. It turns out they provide relevant and complementary information as summarized below. The corresponding values of the VMD effective coupling constants in the vacuum, \( g_{\rho A} \), \( g_{\rho \rho} \), were also calculated for the sake of completeness and they were found to be of the order of magnitude of the values accepted in the literature being related to the vector meson mass and normalization \( M_v^{(0)} \) and \( g_f^{(0)} \) in Eqs. (27) and (28). This comparison with usual accepted values shows it is reasonable to expect that \( M_v^{(0)} < 0.770 \text{ GeV} \) and \( g_f^{(0)} < 1 \). The structureless mesons limit and isospin symmetric case \( m_u = m_d \) should be responsible for the lower values of these parameters. The momentum independent gauge non invariant omega and rho VMD effective coupling constants were eliminated by means of a field redefinition in the same way proposed in Ref. [20]. This procedure involves the cancelation of a resulting photon effective mass from the expansion of the determinant. Light axial mesons, eventually \( A_1(1260) \) and \( f_1(1285) \) as chiral partners of the vector mesons, were found to develop similar mixing couplings induced by a weak magnetic field whose consequences are outside the scope of this work. The strength of their mixing coupling constant was found to be slightly larger than the vector mesons’ one.

Several simple relations between the resulting photon and magnetic field induced coupling constants were found and they go along the lines of the Universality hypothesis [2] and they present very reasonable orders of magnitude and values. In particular, the \( B_0 \) correction to momentum dependent VMD coupling constant \( g_{\rho A}^{B_{\rho A}} \) is proportional to the \( B_0 \) induced vector meson mixing momentum dependent coupling constant, \( g_{\rho A}^{B_{\rho A}} \), seen in Eqs. [17,32] and (39). These magnetic field induced corrections are UV finite. Some simple fittings for some of the momentum dependent form factors, \( g_{\rho A}(Q^2) \), \( g_{\rho \rho}(Q^2) \) and \( g_{\rho \rho}(Q^2) \), were presented. The weak electromagnetic field expansion, as performed in this work, accounts the leading Landau orbits. Higher order terms of the expansion must provide their complete account corresponding to strong \( B_0 \) [31] and then comparisons with different methods for the induced vector meson mixing [31,32] could be performed.

The effect of the \( B_0 \)-dependent VMD and mixing couplings were investigated in two very different processes. Firstly in the low momenta region of the electromagnetic pion form factor for \( (eB_0/M^*^2) = 0.1 \). Quite small contributions of the order of \( (eB_0/M^*^2)^2 \) were found. Whereas the \( B_0 \)-correction
for the mixing yields very small contribution in timelike and spacelike momenta, the $B_0$-correction to VMD yields a sizeable contribution for timelike momenta around $0.15 < Q^2 < 0.25 \text{ GeV}^2$. No further comparison with experimental data was presented because they might be obtained from processes in which magnetic field should not show up, in particular in $e^+e^-$ collisions. However, the two photon coupling to vector mesons, $g_{FF}$, might be viewed as inducing further strength for VMD in Eq. (12) and this process might contribute in processes like $e^+e^-$ collisions. A further evaluation of how this further photon coupling would contribute by (off shell) scattering is left outside the scope of the work. Secondly, the effect of the weak magnetic field on the off-shell charge symmetry violation potential at the constituent quark level was found to be sizeable and of the order of $eB_0/M^2$. It is also considerably larger for monopole rho and omega form factors instead of quadrupole ones, as it could be expected. Besides that, the monopole form factors also favor a change of sign around $r \simeq 0.75 \text{ fm}$ that had been found in several works for the off shell CSV potential [9, 22, 13, 23, 50]. Since hadrons structure and interactions undergo changes under magnetic fields, further sizeable effects on other different components of the nucleon and nuclear potentials in magnetars might be expected.

**Acknowledgments**

F.L.B. thanks short discussions with F.S. Navarra, S. Schramm and G.I. Krein. F.L.B. is member of INCT-FNA, Proc. 464898/2014-5 and acknowledges partial support from CNPq-312072/2018-0 and CNPq-421480/2018-1.

**References**

[1] S. Coleman, S.L. Glashow, Phys. Rev. **134**, 671 (1964). A.S. Goldhaber, G.C. Fox, C. Quigg, Phys. Lett. **30B**, 249 (1969).

[2] J. J. Sakurai, Ann. of Physics **11**, 1 (1960).

[3] Y. Nambu, Phys. Rev. **106**, 1366 (1957)

[4] H.B. O’Connell, B.C.Pearce, A.W. Thomas, A.G. Williams, Prog. Nucl. Part. Phys. **39**, 201 (1997); and references therein.

[5] H.B. O’Connell, B.C. Pearce, A.W. Thomas, A.G. Williams, Phys. Lett. B **354**, 14 (1995).

[6] E.B. Dally *et al*, Phys. Rev. Lett. **48**, 375 (1982).

[7] S.R. Amendolia *et al*, Nucl. Phys. **B 277**, 168 (1986).

[8] L.M. Barkov *et al*, Nucl. Phys. **B 256**, 365 (1985).

[9] J. Piekarewicz, A. G. Williams, Phys. Rev. C **47**, R2462 (1993).
[10] S. Biswas, P. Roy, A. K. D.-Mazumder, Phys. Rev. C 78, 045207 (2008).
[11] P. C. McNamee, M. D. Scadron, and S. A. Coon, Nucl. Phys. A249, 483 (1975).
[12] S. A. Coon, and M. D. Scadron, Nucl. Phys. A287, 381 (1977).
[13] G. Krein, A.W. Thomas, A.G. Williams, Phys. Lett. B 317, 293 (1993).
[14] S. A. Coon, R. C. Barrett, Phys. Rev. C 36, 2189 (1987).
[15] D. Schildknecht, Acta Phys.Polon. B 37, 595 (2006).
[16] M. C. Birse, Z. Phys. A 355, 231 (1996).
[17] M. Benayoun, H.B. O’Connell, A.G. Williams, Phys. Rev. D 59, 074020 (1999).
[18] D. Ebert, H. Reinhardt, Nucl. Phys. B 271, 188 (1986).
[19] C.M. Shakin and W.-D. Sun, Phys. Rev. D 55, 2874 (1997).
[20] N.M. Kroll, T.D. Lee, B. Zumino, Phys. Rev. 157, 1376 (1967).
[21] H.B. O’Connell, B.C. Pearce, A.W. Thomas, A.G. Williams, Phys. Lett. B 336, 1 (1994).
[22] T. Goldman, J.A. Henderson, A.W. Thomas, Few Body Systems 12, 193 (1992). K.L. Mitchell et al, [hep-ph/9403223]
[23] T. Hatsuda, E.M. Henley, Th. Meissner, G. Krein, Phys. Rev. C 49, 452 (1994).
[24] C. Patrignani et al (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
[25] J. Eser, M. Grahl, D. H. Rischke Phys. Rev. D 92, 096008 (2015) . D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D 87, 014011 (2013). D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82, 054024 (2010).
[26] V. A. Miransky and I. A. Shovkovy, Phys. Rep. 576, 1 (2015).
[27] J. O. Andersen et al, Rev. Mod. Phys. 88, 025001 (2016).
[28] For example: V.A. Miransky, I.A. Shovkovy, Phys. Rev. D 66 (2002) 045006. G.S. Bali et al, Phys. Rev. D 86, 071502 (2012). D. E. Kharzeev, Prog. Part. Nucl. Phys. 75, 133 (2014).
[29] K. Tuchin, Advances in High Energy Physics 2013, 490495 (2013).
[30] V.V. Skokov et al, Mod. Phys. Lett. A 24, 5925 (2000). K. Tuchin, Phys. Rev. C 93, 014905 (2016).
[31] M. Mandal, A. Mukherjee, S. Ghosh, P. Roy, S. Sarkar, Eur. Phys. J. A 54, 99 (2018).
[32] Y-H Chen, D-L Yao, H-Q Zheng, [arXiv:1710.11448]
[33] S. Weinberg, Phys. Rev. Lett. 105, 261601 (2010).
[34] F.L. Braghin, Eur. Phys. Journ. A 52, 134 (2016).
[35] F.L. Braghin, Eur. Phys. Journ. A 54, 45 (2018).
[36] F.L. Braghin, Phys. Rev. D 97, 054025 (2018).
[37] F.L. Braghin, Phys. Rev. D 97, 014022 (2018).
[38] C.D. Roberts, R.T. Cahill, J. Praschifka, Ann. of Phys. 188, 20 (1988).
[39] D. Ebert, H. Reinhardt, M.K.Volkov, Pr. Part. Nucl. Phys. 33, 1 (1994).
[40] Q. Wang et al, Phys. Rev. D 61, 054011 (2000).
[41] F.L. Braghin, Phys. Rev. D 94, 074030 (2016).
[42] F.L.Braghin, W.F. de Sousa, Journ. of Phys. G 47 (2020) 045110. arXiv:1809.10823.
[43] S. Weinberg, The Quantum Theory of Fields Vol. II, Cambridge, (1996).
[44] H. Kleinert, in Erice Summer Institute 1976, Understanding the Fundamental Constituents of Matter, 289, Plenum Press, New York, ed. by A. Zichichi (1978).
[45] F.L. Braghin, Phys. Rev. D 99, 014001 (2019).
[46] U. Mosel, (2004) Path Integrals in Field Theory, An Introduction, Springer.
[47] U.G. Meissner, Phys. Rept. 161, 213 (1988).
[48] M. S. Bhagwat, P. Maris, Phys.Rev. C77, 025203 (2008).
[49] J.C.R. Bloch, C.D. Roberts, S.M. Schmidt, Phys.Rev.C61, 065207 (2000).
[50] J. L. Friar, U. Van Kolck, G. L. Payne, S. A. Coon, Phys. Rev. C 68, 024003 (2003).
[51] T.-K. Chyi, et al Phys. Rev. D 62, 105014 (2000).