Formation of sub-horizon black holes from preheating

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We study the production of primordial black holes (PBHs) during the preheating stage that follows a chaotic inflationary phase. The scalar fields present in the process are evolved numerically using a modified version of the HLATTICE code. From the output of the numerical simulation we compute the probability distribution of curvature fluctuations paying particular attention to sub-horizon scales. We find that in some specific models these modes grow to large amplitudes developing highly non-Gaussian probability distributions. We then calculate PBH abundances using the standard Press-Schechter criterion and find that overproduction of PBHs is likely in some regions of the chaotic preheating parameter-space.

I. INTRODUCTION

During primordial inflation space-time expands exponentially for about 60 e-folds, producing the homogeneous, isotropic and almost flat Universe we observe today. Small fluctuations of the inflationary field are stretched to scales larger than the cosmological horizon and reenter in subsequent epochs to source the cosmic structures. After inflation ends, the energy stored in the dominant field must decay into relativistic particles to create the radiation-dominated environment required by nucleosynthesis, transition phase referred to as reheating. Modelling reheating remains a challenge since one must deal with the evolution of highly inhomogeneous fields in an expanding background, including non-linear phenomena up to the time of thermalisation of the Universe.

Of particular interest in recent years has been a reheating model in which resonant amplification by the inflaton field leads to particle production, a process known as preheating [1]. Here a spectator field $\chi$ is non-minimally coupled with the dominant inflaton $\phi$ which oscillates at the bottom of the potential. The quantum fluctuations of $\chi$ experience a resonant amplification, causing an exponential growth of its occupation numbers and an explosive production of relativistic particles. This parametric-resonant mechanism of reheating is known as preheating [2] and it has proved to be extremely efficient for a range of parameters.

To improve our understanding of preheating models we can use the gravitational instability that will amplify the matter fluctuations. Large matter over-densities may form due to the highly non-linear physics of the preheating mechanism [3, 4], and the highest density concentrations may collapse and form black holes. Consequently, the observable constraints on the abundance of these Primordial Black Holes (PBHs) could help us to constrain models of preheating. The goal of this paper is to determine the production rate of PBHs during the preheating stage driven by a chaotic inflation model with two scalar fields and a “four-legs interaction”. The evolution of the scalar fields is calculated numerically with a simplified version of the HLATTICE code [5], which performs a 3D integration of the equations of motion for the full non-linear variables. We find that the amplitude of fluctuations increases rapidly inside the Hubble horizon even when the background energy component behaves almost like radiation. At the same time the probability distribution of curvature inhomogeneities develops a skewed profile. We provide an example where both effects conspire to produce a considerable number of PBHs at sub-horizon scales (a possibility explored in Ref. [6]).

Our paper is organised as follows. The following section presents the elements of the preheating model we work with, including a description of the numerical setup in the present study. Section [III] describes the criteria used to account for the formation of PBHs from overdensities at sub-horizon scales in the preheating phase. In Section [IV] we present the probability density distribution for inhomogeneities in our model and estimate the probability of PBH formation. We conclude in Section [V] with a summary and where extensions of our present study are discussed.
II. THE PREHEATING MODEL

The preheating model we consider is given by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \phi, \phi' + \frac{1}{2} \chi, \chi' - \frac{1}{2} m^2 \phi'^2 - \frac{1}{2} g^2 \phi'^2 \chi \phi^2,$$  \hspace{1cm} (1)

where $\phi(x, t)$ is the inflaton, and $\chi(x, t)$ is the auxiliary, spectator field; both are non-linear functions of time and space. The index $\alpha$ denotes the spacetime coordinates $(0, 1, 2, 3)$. $m = 10^{-6} m_{Pl}$ is the inflaton mass, and $g^2$ is the coupling constant, which is a free parameter in our study.

To study the evolution of the matter fields together with that of the spacetime, we have used a modified version of the HLATTICE code $[5]$, with a flat Friedmann-Lemaître-Robertson-Walker metric, whose dynamics is governed by the homogeneous scale factor $a(t)$ only. Note that this implies the suppression of spacetime fluctuations in our simulations, but it can be regarded as the choice of a flat gauge in the context of cosmological perturbation theory $[2]$. Each hypersurface of constant time is thus conformal to flat space (we shall discuss this point further in Section III).

Our numerical simulation starts from about one e-fold before the end of inflation (defined here as the time where the Hubble scale finds a global minimum). We choose background values for the fields at the initial time as $\phi_{\text{init}} = \langle \phi \rangle = 0.3 M_{Pl}$ and $\chi_{\text{init}} = \langle \chi \rangle = 0$, with a Gaussian distribution of perturbations around these mean values, defined in Fourier space as $|f_k|^2 = 1/(2 \omega_k)$ and $|\dot{f}_k| = \omega_k / 2$. Here $\omega_k = \sqrt{k^2 + m^2}$ for each one of the fields $f : (\phi, \chi)$. From the Lagrangian in Eq. (1) we can read the effective masses of the fields $m_{\phi}^2 = m^2 + g^2 \chi^2$ and $m_{\chi}^2 = g^2 \phi^2$. In particular we will test for PBH formation considering two values of $g^2$ commonly used in preheating studies: Case I takes $g^2 = 6.5 \times 10^{-8}$ and in Case II we consider $g^2 = 2.5 \times 10^{-7}$ (more details of the simulations can be found in Refs. $[2, 8, 9]$).

The growth of the modes $\chi_k$ in Fourier space yields an increase in the number of the created particles. Indeed, the number density $n_k$ of particles with momentum $k$ can be evaluated as the energy of that mode divided by the energy of each particle, obtaining

$$n_k = \frac{\omega_k^2}{2} \left( \frac{|\chi_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}. \hspace{1cm} (2)$$

Figure 1 shows the exponential increase in $n_k$ for modes in the resonance band with the characteristic step-like profile observed in previous works (e.g. $[2]$). We confirm the exponential growth of the occupation number but in a dynamic background, an improvement over $[2]$ which follows the evolution of scalar field in a spacetime dominated by dust (c.f. Figure 4 of Ref. $[2]$). As indicated in $[10]$, $n_k$ helps us to understand the energy distribution for each mode. Once the distribution hits the highest $k$-mode resolved by the simulation, the energy is reflected back to the infrared modes. For the model and resolution considered here, the numerical simulation can be trusted up to a final time $t_f$ where $a(t_f) = 30$ (well within the broad parametric resonance stage), just before the energy reflection effect kicks in and develops spurious modes.

Because of the exponential growth in the occupation number shown in Figure 1 we can treat the energy density in classical terms nearly after the end of inflation. Thus we are free to study the formation of PBHs following the usual methods. The total energy density and pressure are defined by

$$\rho(x, t) = \frac{1}{2} \phi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} \chi^2 + \frac{1}{2} (\nabla \chi)^2 + V(\phi, \chi) \hspace{1cm} (3)$$

and can be split in an homogeneous part and inhomogeneous fluctuations. The homogeneous part of the energy density and pressure, $\bar{\rho}(t)$ and $\bar{p}(t)$, are defined through their spatial average at every time step during the simulations. Instead of plotting the behaviour of these homogeneous quantities we follow the evolution of the equation of state $w \equiv \bar{p}/\bar{\rho}$, which plays a significant role on both the analysis of matter-radiation transition and in determining the criterion of gravitational collapse of overdense regions into PBHs $[4]$.

The oscillations of fields and their derivatives, which characterise the preheating epoch, can be observed in the oscillations of $w$ for which time average is $w = 0$ at the

\footnotesize \begin{itemize}
  \item 1 The conditions to produce gravitational collapse have been analysed at Hubble scales for cosmological epochs characterised by a constant state equation in the range $0 < w < 1/3$ $[11, 12]$.
\end{itemize}
III. PBH FORMATION CRITERIA

Primordial Black Holes form from over-dense regions in high density environments. Fluctuations with large density contrast $\delta = \delta \rho / \bar{\rho}$ overcome the pressure of the environment and detach from the background expansion. If the amplitude of an over-density of size $L$ is large enough, it will re-collapse as soon as it enters in causal contact, that is, at the scale of the cosmological horizon $L_H = 1 / H$. As a result of the gravitational collapse, a black hole of mass comparable to the Hubble mass is formed right after horizon crossing. The threshold amplitude for the density contrast $\delta_{th}$ to reach the collapse was first estimated to be $\delta_{th} \approx w$ in a barotropic fluid with pressure $p = w \rho$ using the Jeans criterion for overdensities in a Friedmann background [13]. Recently, simulations considering inhomogeneous cosmologies have determined a threshold amplitude for the comoving matter over-density as $\delta_{th} \approx 0.41$ in a pure radiation background [13]. Extensions to general barotropic fluid backgrounds are considered in [11, 12].

The threshold for collapse is best expressed in terms of the gauge-invariant curvature perturbation $\zeta$ [12, 16]. This is defined in the uniform density gauge as

$$\zeta = -\psi - H \frac{\delta \rho}{\bar{\rho}}. \tag{4}$$

For the cases that concern us, we note from Figure 2 that during preheating the fluctuations grow in amplitude inside the cosmological horizon (as we shall see in detail in Section IV). This indicates that PBHs are more likely to form inside the cosmological horizon, and a different criterion is required to set the threshold of amplitudes that collapse and form PBHs. The formation of PBHs at scales inside the Hubble horizon has been largely ignored due to the linear Jeans’ instability criterion, which for a radiation-dominated universe sets the scale of instability close to the Hubble scale. Furthermore, in the dust-like environment, typical of phase transitions in unification theories, PBHs are thought to form at small scales and from overdensities of arbitrarily small amplitude because there is no pressure to prevent the collapse [17]. However, scattering of small black holes could prevent the formation of larger PBHs [e.g. 18]. In Ref. [6] the formation of PBHs at sub-horizon scales is studied. The authors show that the threshold amplitude for the metric fluctuation to form a black hole at scales well inside the horizon can be taken to be equivalent to the known $\zeta_{th} = 0.7$ of the radiation background. Starting from the matter over-density in the flat gauge $\delta_{flat}$, we can compute the curvature fluctuation as Eq. (2),

$$\zeta = -H \frac{\delta \rho_{flat}}{\bar{\rho}} = \frac{1}{3} \frac{\delta_{th}}{1 + w}. \tag{5}$$

In the following we use the threshold value $\zeta_{th} = 0.7$ to select those over-densities that will inevitably form black holes at sub-horizon scales and look into the abundance of PBHs.

beginning and, in a ideal preheating model, $w = 1/3$ at end of this process.\footnote{Strictly speaking, the average of equation state is $w = 0$ only when $V = \frac{1}{2} m^2 \phi^2$, during preheating the energy transference from the inflaton to the spectator field yield $w > 0$, in average.}

The time averaging may be unnecessary if the energy transference from inflaton to the spectator field breaks the oscillations and stabilises the equation of state. Unfortunately, the high non-linearity of the process indicates that an a priori selection of the value of the parameters of the model (i.e. $g^2$) is required in order to produce a well defined constant value of $w$. Moreover, the complete damping of the oscillations of $w$ and its value at the end of the preheating process is highly dependent on the nature of the interaction. Figure 2 shows that the damping on the oscillations of $w$ at the end of preheating can be clearly observed only in the Case II, with the asymptotic value $w \lesssim 1/3$. However, neither Case I nor Case II are able to produce an ideal complete preheating, (more general analysis about the effectiveness of the model are reported in [11]).

FIG. 2. Time evolution of the equation of state $w$ for CASE I (top) and CASE II (bottom). Only for the latter $w$ steadily converges towards a constant value. The trend is not too obvious for the Case I but it is still present. See the tex for more details.
IV. THE PROBABILITY OF PBH FORMATION IN PREHEATING

The numerical simulation generates a distribution of field amplitudes $\langle \phi(x,t), \chi(x,t) \rangle$ reliable up to a final time $t_f$, set right before the appearance of the spurious rescattering effects. A key result of the simulation is presented in Figure 3 where we plot the evolution of the mean amplitude of the Fourier space $\zeta_k(t)$ over the range of comoving scales covered by the simulation box. The figure shows how the curvature perturbation grows and peaks at scales below the Hubble horizon, contrary to the effect observed in a purely radiation-dominated environment. This motivates the consideration of PBH formation at sub-horizon scales.

In order to estimate the mass fraction of black holes at $t = t_f$, $\beta_{\text{PBH}}(M) \equiv \rho_{\text{PBH}}/\rho(t_f)$, we first construct the probability distribution of the curvature perturbation $\zeta$ at a specific scale $k$ (or the equivalent mass $M \propto \bar{\rho}(t_f)k^{-3}$), namely $P_M(\zeta)$. As shown in Figures 4 and 5, the PDF develops an appreciable skewness below the Hubble horizon, contrary to the effect observed in a purely radiation-dominated environment. This motivates the consideration of PBH formation at sub-horizon scales.

We have computed density profiles $P(\zeta)$ for several modes by looking at the over-densities of size a few times the resolution scale. In the plots of Figure 4 for the Case I, the mean amplitude of $\zeta_k$ shows a negatively-skewed distribution for modes inside the horizon, while for Case II, the distribution is positively-skewed. This leads to an enhancement in PBH formation (with respect to the Gaussian) for the model of Case II, as shown in Table I.

\begin{align}
\beta_{\text{PBH}}(M) &= \frac{\rho_{\text{PBH}}}{\rho(t_f)} = 2 \int_{\zeta_{\text{th}}}^{\infty} P_M(\zeta) d\zeta. \quad (6)
\end{align}

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The value of $t$ compute upper and lower bounds to the true can easily evaluate the integral Eq. (6). As a result we and below (see Figure 6). For these analytical PDFs we contributions which bound the true distribution from above and below the Landau function is the best approximation to the complete probability of large amplitude fluctuations, we complete sample of modes translates into a truncated of the numerically generated PDF. To estimate the tail of the PDF. To estimate the complete probability of large amplitude fluctuations, we carefully match the numerical PDF with analytic distributions which bound the true distribution from above and below (see Figure 6). For these analytical PDFs we can easily evaluate the integral Eq. (6). As a result we compute upper and lower bounds to the true $\beta_{\text{PBH}}$ in Table I. The numerical values of $\beta_{\text{PBH}}$ are calculated at $t_f$ for fluctuations of size a fraction of the Hubble scale. The value of $\beta_{\text{PBH}}$ increases with the $k$-number as a consequence of the non-Gaussian evolution of fluctuations inside the cosmological horizon. The development of non-Gaussianity of the PDFs is evident from the snapshots of evolution illustrated in Figures 4 and 5.

Let us finish this section by analysing the results of Table I in light of the bounds to the PBH mass fraction at the smallest scales. It is known that the Hawking evaporation process of PBHs may halt at the Planck scale. The remnant Planck-mass black holes would survive behaving as a component of the dark matter until the present time. Since their mass fraction cannot exceed that of dark matter, one can impose a bound to their abundance, that is

$$\beta_{\text{PBH}}(M_{\text{Pl}}) < 10^{-28} \left( \frac{M}{M_{\text{Pl}}} \right)^{3/2}$$

(7)

$\beta_{\text{PBH}}$ estimated in $k/H(t_f) = 2$. The true distribution is approximated by two analytical PDFs. From below, the closest approximation is a Moyal function, while from above the Landau function is the best approximation to the tail of the distribution.

| $L_H/L$ | Case | $\beta_{\text{gauss}}$ | $\beta_{\text{max}}$ | $\beta_{\text{min}}$ |
|--------|------|----------------|----------------|----------------|
| 46     | I    | $2.94 \times 10^{-14}$ | $2.94 \times 10^{-14}$ | $6.05 \times 10^{-14}$ |
| 46     | II   | $5.53 \times 10^{-16}$ | $4.73 \times 10^{-13}$ | $1.54 \times 10^{-14}$ |
| 23     | I    | $4.59 \times 10^{-24}$ | $4.59 \times 10^{-24}$ | $8.63 \times 10^{-26}$ |
| 23     | II   | $2.26 \times 10^{-26}$ | $2.6 \times 10^{-4}$ | $6.5 \times 10^{-25}$ |
| 16     | I    | $2.20 \times 10^{-30}$ | $2.20 \times 10^{-30}$ | $1.23 \times 10^{-30}$ |
| 16     | II   | $1.37 \times 10^{-28}$ | $2.3 \times 10^{-5}$ | $4.52 \times 10^{-28}$ |
| 9      | I    | $3.19 \times 10^{-41}$ | $3.19 \times 10^{-41}$ | $7.93 \times 10^{-41}$ |
| 9      | II   | $2.37 \times 10^{-64}$ | $2.6 \times 10^{-7}$ | $5.49 \times 10^{-64}$ |
| 8      | I    | $5.14 \times 10^{-81}$ | $5.14 \times 10^{-81}$ | $5.87 \times 10^{-81}$ |
| 8      | II   | $2.58 \times 10^{-80}$ | $5.32 \times 10^{-80}$ | $3.98 \times 10^{-80}$ |
| 6      | I    | $2.38 \times 10^{-99}$ | $2.38 \times 10^{-99}$ | $5.95 \times 10^{-100}$ |
| 6      | II   | $7.18 \times 10^{-99}$ | $1.15 \times 10^{-9}$ | $3.39 \times 10^{-99}$ |

TABLE I. $\beta_{\text{PBH}}$ estimations for modes inside the horizon in both cases I and II computed from Eq. (6) with a threshold value $\zeta_{\text{th}} = 0.7$. The statistics of smaller modes presents a considerable uncertainty due to the resolution of our numerical simulation.

where $M_{\text{Pl}}$ is the Planck mass. When we compute the physical mass enclosed in a region of radius $L/L_H = 1/16$ at $t_f$ we note that the PBHs formed at that scale would have $M_{\text{PBH}} \approx 10^9 M_{\text{Pl}}$. For this scale we note that the minimum value $\beta_{\text{min}}$ reported in Table I exceeds the bound imposed by Eq. (7). Table I also shows that the abundance of PBHs on larger scales falls below the observational bounds (a complete set of bounds is reported in [24] and see also [25] for specific bounds to $\zeta_{\text{th}}$). As for smaller scales, they lie too close to the resolution scale of our simulation and thus we do not consider them physical. The implications and limitations of the results derived are discussed in the following last section.
V. DISCUSSION

In this paper we have considered the possibility of forming sub-Hubble scale black holes from the numerical solution to the classical equations of the chaotic preheating model. We have focused in computing the distribution of the curvature fluctuation $\zeta_k$ from the sample of modes in the simulation box of size smaller than the Hubble scale $L_H$. For the typical values of the coupling parameter $g^2 = 6.5 \times 10^{-7}$ in the reheating model (Case II of this paper), we find that the small size inhomogeneities grow in amplitude. When constructing the probability distribution function (PDF) we note that large skewness is developed in the probability profile as the modes enter the cosmological horizon (c.f. Figure 4). We find that, at late enough times, primordial black holes of mass about one gram could be substantially produced, saturating the observational bounds when a Planck mass remnant survives the evaporation process. Interestingly enough, a smaller value of the field coupling $g^2 = 6.5 \times 10^{-8}$ (Case I) shows no such overproduction of PBHs. This is because of the development of a negatively-skewed PDF as shown in Figure 4.

One can argue that dependence of the PBH formation rate on the coupling parameter $g^2$ is due to the existence of resonance bands for the enhancement of fluctuations inside the cosmological horizon [2], and some studies have argued that PBH overproduction is not a generic feature of preheating. Indeed, Ref. [20] finds an insignificant mass fraction $\beta_{\text{PBH}}$ assuming matter fluctuations collapsing at horizon scales and with a Gaussian distribution of probability. Our results show that the Gaussian distribution underestimates the true probability for the Case II (see the first column of Table I for a $\beta_{\text{PBH}}$ computed from the Gaussian PDF). If we relax the assumptions of [20] and consider the possibility of PBH formation below the Hubble scale, we find a clearly non-Gaussian PDF from the distribution of fluctuations in the numerical simulation. Our estimations of the PBH mass fraction in Table I indicate that the overproduction of PBH is a likely feature in preheating and this possibility must be studied in more detail.

Cosmologically PBHs represent a unique tool to probe the small scales which are hard to reach by observations of the CMB and large scale structure. In the present paper we show that, while strong couplings between the fields are favoured to reheat the Universe efficiently, it is important to test the viable models for PBH overproduction due to the highly non-linear physics involved in the preheating stage. In this work we have put forward a method to account for the mass fraction of PBHs and we shall eventually use it to constrain preheating models.

Motivated by the results presented here, we aim to evaluate a much larger sample of coupling values $g^2$ in a subsequent work, where we shall also take into account the evolution of metric perturbations and explore a larger range of scales. Another aspect that deserves further study is the criterion used to determine which over-densities will collapse to form PBHs. The threshold values reported in the literature for scalar fields or barotropic fluids consider mostly collapsing inhomogeneities as soon as they enter the cosmological horizon. The growth of large inhomogeneities due to the non-linear interactions inside the horizon might eventually require a criterion for PBH formation at sub-horizon scales beyond that presented in [2], in order to constrain the parameter space with even higher accuracy.

We finally note that the overproduction of PBHs could alter the mechanism of transition to a radiation-dominated stage: It is known that preheating in chaotic models cannot drive the universe to a radiation stage, and one has to consider other higher couplings in the fields to achieve that [10, 27]. According to our results, the overproduction of PBHs during preheating opens the possibility to reconsider PBH evaporation as an auxiliary mechanism to reheat the Universe [28]: such mechanism may benefit unification models [29, 30].

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