Glassy dynamics in a disordered Heisenberg quantum spin system

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Understanding the dynamics of strongly interacting disordered quantum systems is one of the most challenging problems in modern science, due to features such as the breakdown of thermalization and the emergence of glassy phases of matter. We report on the observation of anomalous relaxation dynamics in an isolated quantum spin system realized by an ultracold gas of atoms initially prepared in a superposition of two different Rydberg states. The total magnetization is found to exhibit sub-exponential relaxation analogous to classical glassy dynamics, but in the quantum case this relaxation originates from the build-up of non-classical correlations. In both experiment and in semi-classical simulations, we find the evolution towards a randomized state is independent of the strength of disorder up to a critical value. This hints towards a unifying description of relaxation dynamics in disordered isolated quantum systems, analogous to the generalization of statistical mechanics to out-of-equilibrium scenarios in classical spin glasses.

I. INTRODUCTION

The far-from-equilibrium behavior of isolated quantum systems and in particular their relaxation towards equilibrium still evades a unifying description. It has been conjectured that these systems generically relax to a state of local thermal equilibrium according to the eigenstate thermalization hypothesis (ETH) 11. However, the ETH does not explain how the equilibrium state will be reached, or even if it will be reached in experimentally accessible timescales. This appears to be especially important in disordered quantum systems where a strong interplay between interactions and many-body entanglement can give rise to new and intrinsically non-equilibrium effects such as pre-thermalization 2-4, many-body localization 5-7, Floquet time crystals 8-10, and quantum scars 10-11.

In contrast, most natural systems (e.g., in condensed matter) are not fully isolated from their environment and hence always relax to thermal equilibrium imposed by the external bath 12. But it is known that disorder and frustration effects can lead to a dramatic slowdown of thermalization, associated with the onset of glassy behavior 13. A key signature of this behavior is that macroscopic observables relax in characteristically non-exponential way, as encountered for example, in doped semiconductors 14 and organic superconductors 15, quasi crystals 16, atoms in optical lattices 17 or diamond color centers 18, 19. This opens the question whether slow relaxation, which appears to be ubiquitous in open disordered systems, also emerges in isolated quantum systems, and if so, what are the underlying mechanisms responsible for quantum glassiness.

A prototypical system for studying far-from-equilibrium quantum dynamics is the Heisenberg XXZ Hamiltonian for spin-1/2 particles. Compared to the Ising Hamiltonian, this system has fewer conserved quantities and shows complex, chaotic far-from-equilibrium dynamics which are difficult to describe theoretically. Here, we experimentally realize a disordered quantum Heisenberg-XXZ spin-1/2 model in an ultracold atomic gas by encoding the spin degree of freedom into two electronically excited (Rydberg) states of each atom. Spin-spin interactions arise naturally through distance dependent couplings 20. Using a strong microwave field pulse that couples the two Rydberg states, we initialize the spins in a far-from-equilibrium state and probe their time evolution, thus employing our system as a quantum simulator for unitary spin dynamics out of equilibrium 10, 21-25.

Exploring the dynamics in a large ensemble of Rydberg spins, we observe that the magnetisation follows a sub-exponential dependence characterised by a universal exponent that is independent of the strength of disorder up to a critical value. Our experiments and supporting numerical simulations that go beyond the mean-field approximation suggest that slow dynamics described by stretched exponential decay is a generic feature of disordered quantum spin systems, hinting to-
Towards a unifying effective theory description.

The paper is organized as follows: In Sec. II we give a qualitative physical picture by solving the time-dependent Schrödinger equation for a few spins exactly. We then describe in Sec. III how to implement the Heisenberg-XXZ spin model in a gas of ultracold atoms that are excited to Rydberg states. In Sec. IV we experimentally characterize the relaxation dynamics, and we discuss its universal character in Sec. V. Finally, we theoretically investigate the dependence on disorder strength and character in Sec. VI.

II. QUALITATIVE PICTURE OF THE QUANTUM DYNAMICS

We consider an ensemble of spin-1/2 particles randomly positioned in space and all initialized in the $|\uparrow\rangle^\otimes N = 1/\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)^\otimes N$ state, corresponding to an initial magnetization $\langle S^z_i \rangle = 1/2$. Here, $S^{(i)}_a$ ($a = \{x, y, z\}$) refers to the spin-1/2 operator of the $i$th spin. The experimental protocol is illustrated in Figure 1(a). The unitary dynamical evolution of the system is governed by the Heisenberg-XXZ Hamiltonian in the absence of magnetic fields (in units where $\hbar = 1$),

$$H_{\text{XXZ}} = \frac{1}{2} \sum_{i,j} J_{ij}(S^{(i)}_x S^{(j)}_x + S^{(i)}_y S^{(j)}_y + \delta S^{(i)}_z S^{(j)}_z),$$

where $\delta$ is the anisotropy parameter and $J_{ij}$ are the interaction couplings between the spins $i$ and $j$. To remain consistent with the experimental implementation (see Section III), we focus on an anisotropy parameter $\delta = -0.73$ and spin-spin interactions that decay as a power law $J_{ij} = C_6/r_{ij}^6$ with the inter-particle distance $r_{ij}$.

To obtain a qualitative understanding of the quantum dynamics in this system, we perform a full quantum mechanical simulation on a small ensemble of 12 spins $i$. In Fig. 1(b) we show the time evolution of the magnetization $\langle S^z \rangle_k = 1/N \sum_i \langle S^{(i)}_z \rangle_k$ for a single disorder realization $k$ (grey curve). Due to the spatial disorder, spin-spin interactions give rise to complex many-body dynamics on strongly varying energy scales. This is in stark contrast to an effectively classical, mean-field prediction for this Hamiltonian [dotted line in Fig. 1(c)], which assumes each spin to evolve in the average field generated by all other spins thus neglecting quantum correlations. The initial fully magnetized state is an eigenstate of the mean-field Hamiltonian which explains the total absence of relaxation. Therefore, in the many-body case the loss of magnetization is not caused by classical dephasing, but by the build-up of entanglement between spins, witnessed by the decrease of the local purity $\text{Tr}(\rho^2_i)$ for each spin. Since the dynamics are unitary and therefore the full system remains pure ($\text{Tr}(\rho^2) = 1$), we can quantify entanglement by the second order Rényi entropy

$$S_i^{(2)} = -\log_2 \left( \text{Tr}(\rho_i^2) \right),$$

which increases to $S_i^{(2)} = 1$ on a similar timescale as the relaxation of the magnetization (see Fig. 2).

After ensemble and disorder averaging, the magnetization approaches a fully randomized state with $\langle S^z_{\text{ensemble}} \rangle = 0$ [see black curve in Fig. 1(b)], consistent with the ETH prediction. However, this relaxation occurs very slowly compared to the timescales associated with spin-spin interactions, and appears to follow an almost logarithmic dependence (dashed line in...
duced by absorption imaging of the ground-state atoms of the beyond nearest-neighbor interactions (for the theoretical description see Appendix A).

The single photon frequency $\nu = 35.2$ GHz is detuned from the intermediate state $|48P_{3/2}\rangle$ by 170 MHz, far enough to guarantee the population in this state due to off-resonant coupling is smaller than 4%. The atoms can therefore be considered as two-level systems described by a pseudo-spin degree of freedom. In this description the microwave field acts as an external field with adjustable phase $\phi$.

III. REALIZING A HEISENBERG-XXZ SPIN SYSTEM WITH RYDBERG ATOMS

We address this question experimentally using a gas of ultracold rubidium atoms prepared in a superposition of two different Rydberg states: $|\downarrow\rangle = |48s\rangle$ and $|\uparrow\rangle = |49s\rangle$. The electric dipole-dipole coupling between these two states realizes the Hamiltonian (1) [26], with $C_6/2\pi = 59$ GHz $\cdot$ pm$^6$ characterizing the strength of the beyond nearest-neighbor interactions (for the derivation of the Hamiltonian, see Appendix A).

The experimental procedure [Fig. 1(a)] starts with a gas of $^{87}$Rb atoms prepared in their electronic ground state $|g\rangle = |5S_{1/2}, F = 2, m_F = 2\rangle$ in an optical dipole trap and with a temperature $T \sim 50$ $\mu$K, low enough to freeze the motional degrees of freedom over the time scale of the experiment. A laser pulse of variable duration brings a controllable number of atoms $N \leq 1200$ to the $|\downarrow\rangle = |48S_{1/2}, m_j = -1/2\rangle$ Rydberg state. For this we use a two-photon laser excitation at 780 nm and 480 nm, with a detuning $\Delta/2\pi = -100$ MHz from the intermediate state $|e\rangle = |5P_{3/2}, F = 3, m_F = 3\rangle$ and an effective Rabi frequency of $\Omega/2\pi = 150$ kHz. To individually address two specific Rydberg states, including Zeeman substructure, we continuously apply a magnetic field of 6 G. To characterize the resulting three-dimensional Rydberg density distribution we perform depletion imaging, where the Rydberg density is deduced by absorption imaging of the ground-state atoms before and after the Rydberg excitation laser pulse [27].

A two-photon microwave field is then used to couple the $|\downarrow\rangle$ state to the $|\uparrow\rangle = |49S_{1/2}, m_j = +1/2\rangle$ Rydberg state. The single photon frequency $\nu = 35.2$ GHz is detuned from the intermediate state $|48P_{3/2}\rangle$ by 170 MHz, far enough to guarantee the population in this state due to off-resonant coupling is smaller than 4%. The atoms can therefore be considered as two-level systems described by a pseudo-spin degree of freedom. In this description the microwave field acts as an external field described by the Hamiltonian

$$H_{\text{ext}} = \sum_i \Omega \cos \phi S_z^{(i)} + \Omega \sin \phi S_x^{(i)} + \Delta S_z^{(i)},$$

with $\Omega$ the Rabi frequency, $\Delta$ the detuning and $\phi$ the phase of the field. To vary these three parameters at the 10 ns time scale, we use frequency up-conversion with a radio-frequency field of frequency 400 MHz. The Rabi frequency is calibrated from the period of Rabi oscillations between the two spin states.

To perform the Ramsey sequence shown in Fig. 1(a), a resonant microwave $\pi/2$ pulse at an effective Rabi frequency $\Omega/2\pi = 3.00(1)$ MHz rotates all spins to the fully-magnetized state $|\rightarrow\rangle^{\otimes N}$ with all spins pointing along the $x$-direction on the Bloch sphere. Uncertainties in the duration and amplitude of the pulses as well as interaction effects lead to imperfect initial spin state preparation. Based on simulations, we estimate the fidelity to be higher than 96%.

After a free evolution time $t$ in the absence of the microwave field ($\Omega = 0$), a second microwave $\pi/2$ pulse with adjustable phase $\phi$ is applied to rotate the equatorial magnetization components to the detection basis (Fig. 2(c)):

$$\langle S_y \rangle = \cos \phi \langle S_z \rangle + \sin \phi \langle S_y \rangle.$$

In this way we effectively read out the $\langle S_z \rangle$ and $\langle S_y \rangle$ magnetizations from population measurements of the two spin states $|\downarrow\rangle$ and $|\uparrow\rangle$ using electric field ionization (see appendix B).

To ensure unitary Hamiltonian dynamics, we restrict the experimental time scales to a maximum of 10 ns which is short compared to the spontaneous decay time and redistribution by black-body radiation ($113 \mu$s and $121 \mu$s respectively for the chosen Rydberg states [28]). To verify that the single-spin phase coherence is preserved during experimental time, we perform a Ramsey measurement with finite detuning $\Delta$ at low spin densities where interactions can be neglected [29, 30]. The full contrast oscillation shows that the single-spin phase coherence is preserved over the duration of the experiment, as shown in Fig. 3.
IV. EXPERIMENTAL OBSERVATION OF RELAXATION DYNAMICS

We now study the relaxation dynamics due to spin-spin interactions for increasing spin densities. Fig. 4 shows the experimentally observed relaxation of the magnetization using tomographic spin-resolved read-out of \( \langle S_x \rangle \) and \( \langle S_y \rangle \). Starting from the almost fully magnetized state \( \langle S_x \rangle = 1/2 \) we observe that the magnetization decays towards the unmagnetized state within \( \approx 10 \mu s \). This is much shorter than the single-spin phase coherence time measured in Fig. 3 but still slower than the characteristic timescale governing microscopic physics, \( \langle C_6/(2\pi/a_0^6) \rangle^{-1} = 0.7(3) \mu s \). Here, \( a_0 = (4\pi\rho_S^{1/3})^{-1/3} \) is the Wigner-Seitz radius of the Gaussian spin distribution with peak density \( \rho_S^0 \).

The time evolution seen in the experiment is qualitatively similar to the dynamics obtained by exact diagonalisation in Fig. 4 (b). Even when accounting for imperfect preparation of the initial state \( \langle S_x \rangle \leq 1/2 \) the mean field prediction shows essentially no relaxation on the experimentally relevant timescales (see Appendix C). This absence of dynamics at the mean-field level implies that the relaxation seen in the experiment is closely related to build-up of entanglement also apparent in the exact diagonalisation calculations.

We empirically observe that the relaxation is well described by a stretched exponential, in apparent similarity with glassy dynamics in classical disordered media [33, 34, 35]:

\[
\langle S_x(t) \rangle = \frac{1}{2} \exp\left[-(\gamma_J t)^\beta\right]
\]

(5)

where \( \beta \) is the stretching exponent and \( \gamma_J \) defines an effective relaxation rate. The exponent \( \beta \) characterizes the deviation from a simple exponential (\( \beta = 1 \)) towards a purely logarithmic decay (\( \beta \to 0 \)) [see Appendix D]. The experimental data are well described by this phenomenological function (dashed line in Fig. 4) yielding an exponent \( \beta = 0.32(2) \). This value clearly rules out a pure exponential decay, i.e. \( \beta = 1 \), that could be expected on the basis of single-particle dephasing. It is also inconsistent with an exponent of 1/2 that is expected for an Ising model [35, 36] or a fluctuator model for open quantum systems [19].

Our observation therefore indicates a close similarity between classical glassy dynamics and quantum glassy dynamics in the form of subexponential relaxation of the macroscopic magnetization [35].

V. EVIDENCE FOR UNIVERSAL RELAXATION DYNAMICS

To further investigate how slow relaxation and the characteristic exponent depends on microscopic details, we control the degree of spatial disorder by taking advantage of the Rydberg blockade effect in the state preparation stage [36, 37]. During laser excitation the strong van der Waals interactions between Rydberg states prevents two spins to be prepared at distances smaller than the Rydberg blockade radius \( R_{bl} \). The degree of disorder is thus controlled by the ratio between blockade radius \( R_{bl} \) and Wigner-Seitz radius \( a_0 \) (Fig. 5a). For \( a_0 \gg R_{bl} \) the blockade effect has little influence and the spins are randomly distributed, whereas the limit \( a_0 \approx R_{bl} \) corresponds to a strongly
ordered configuration. In between the short distance cutoff imposed by the Rydberg blockade effect effectively reduces the strength of the disorder compared to fully uncorrelated random spin positions.

In the experiment we can tune the disorder strength by changing the peak spin density $\rho_0^3$ and thus the mean number of spins per blockade sphere $(a_0/R_0)^{-3}$ from 0.20(5) to 0.7(2) (two-dimensional representations of corresponding distributions are depicted in Fig. 5(a)). Remarkably, we find the stretching exponent $\beta$ to be almost constant over this range (inset in Figure 5(a)). Furthermore, after rescaling the time axis by the characteristic energy scale $C_0/a_0^3$, the time-dependent data collapse onto a single line (Fig. 5(b)). From this we conclude that the dynamics are universal, i.e. independent of the microscopic details of the system, an observation that will be explored further in numeric simulations.

To simulate the three-dimensional Rydberg gas on a classical computer, we model the system by an ensemble of spin-1/2 particles with interactions described by the Heisenberg XXZ-Hamiltonian. Due to the large number of spins in the experiment, the full simulation of the unitary dynamics of the Hamiltonian Eq. (1) is not possible. Instead, quantum effects can be partially taken into account by applying the semiclassical discrete Truncated Wigner Approximation (dTWA) (Appendix E) [38, 39] which has recently been shown to describe the dynamics of Rydberg interacting spin systems very well [20]. To model the present experiments, all relevant parameters for the simulation are determined through independent measurements, such as the spatial density distribution, total number of spins and the microwave coupling strength $\Omega$ used in the preparation and readout stages. The initial spin distribution is generated from a random excitation model of the Rydberg atoms, including a cut-off distance to account for the blockade effect (see Appendix E).

The numerical simulations describe the glassy dynamics and its universal character very well (solid line in Fig. 4), further confirming the validity of the dTWA approximation for treating the dynamics of disordered quantum systems. The ability to reproduce the experimental observations using dTWA simulation also suggests a possible interpretation in analogy to classical spin glasses: quantum, instead of thermal fluctuations, allow the system to explore a rugged energy landscape leading to a hierarchical separation of timescales [35, 40], resulting in stretched exponential relaxation.

Figure 5. Rescaled magnetization dynamics for different densities and disorder strengths. (a) 2D representation of a spin system with different densities and thus strengths of disorder, characterized by the mean number of spins per blockade radius $(a_0/R_0)^{-3}$. The bar denotes the mean $x$ and standard deviation $\sigma$ of the interparticle-distances $\min_{i\neq j} r_{ij}$. (b) Data points represent measurements of the averaged magnetization $\langle S_x \rangle$ for spin densities $\rho_0^3 = 0.43(11) \times 10^3 \text{cm}^{-3}$ (blue), $0.8(2) \times 10^3 \text{cm}^{-3}$ (green), $1.2(3) \times 10^3 \text{cm}^{-3}$ (orange and red). By rescaling time with the effective interaction strength $C_0/a_0^3$ the data collapses on a single curve described by a stretched exponential function with stretching exponent $\beta = 0.32(2)$. The inset shows $\beta$ as a function of the corresponding ratio $(a_0/R_0)^{-3}$ for the experimental data and discrete Truncated Wigner Approximation (dTWA) calculations (solid gray line).

VI. ROLE OF DISORDER AND BREAKDOWN OF UNIVERSALITY

Theoretical modeling using the dTWA allows to further test the role of disorder on the universal behavior, while excluding possible effects of the inhomogeneous spin density resulting from the optical trap. The simulated spin dynamics for a uniform density distribution $\rho_0^3$ are shown by the solid lines in Fig. 6(a). For early times where $t \leq 2\pi R_0^3/C_0$, the leading order quadratic Hamiltonian evolution is clearly visible. For times beyond the perturbative regime, the relaxation is well described by a stretched exponential [dashed lines in Fig. 6(a), see Appendix D], proving that the glassy dynamics is an intrinsic many-body effect and not a result of the Gaussian spin distribution.

The fitted relaxation rate $\gamma_J$ does not scale with
$C_0/a^6 \ [a = (4\pi \rho_S)^{-1/3}]$ but with the median of the mean field interaction strengths $J_{mf}$ [see Fig. 6(b)], which we conclude is the characteristic energy scale in the system (see Appendix E). This scale $J_{mf} = C_0/a^6$ defines an effective Wigner-Seitz radius $\bar{a}$, that coincides with the usual Wigner-Seitz radius $a$ for small densities but deviates at larger densities when spatial correlations induced by the Rydberg blockade effect become important. Rescaling the time with $C_0/a^6$, the simulated data collapses on a single stretched exponential curve [see Fig. 6(c)], similarly to the experimental observations in Fig. 5(b), hence confirming the universal relaxation dynamics. This is further confirmed by the fitted stretching exponent $\beta$ that is shown in the inset of Fig. 6(c). We find it to be approximately constant for large disorder strengths where spatial correlations are weak (i.e. $\bar{a}/R_{bl} \lesssim 0.7$), with a value of 0.36 close to the experimental one of 0.32(2).

The simulations allow to access even higher densities than those accessible in the experiment, corresponding to more correlated spatial configurations of the atoms. The dTWA simulations shows significantly different dynamics [dotted line in Fig. 6(a)] that translates into a stretching exponent $\beta$ that is sensitive to the strength of the disorder above a threshold (see inset in Fig. 6(c)). This is linked to an increasing importance of the cutoff at large interaction strengths induced by the blockade effect which modifies the spatial distribution function. It can be seen in the probability distribution of nearest-neighbor interaction strengths, which becomes more peaked compared to a random spin distribution without blockade effect (see Fig. 7).

On the other hand, in the regime of large disorder where we observe universal, glassy dynamics, the cutoff is much beyond the median interaction strength, and the functional form of the spin distribution function is significantly modified only at larger interactions strengths. Only short time dynamics is influenced by the existence of this cutoff, and one might be tempted to conjecture that universal glassy dynamics emerges as a consequence of the interaction strengths following distribution functions of similar shape. However, this hypothesis is in stark contrast to the results of a rate model analogue to the fluctuator model [13], which assumes that each spin decays with a local rate $\gamma_i$ sampled from the probability distributions from Fig. 7.

$$\langle S_x^{(i)}(t) \rangle = \frac{1}{2} \exp \left[-\gamma_i t \right]$$ \hspace{1cm} (6)

In this oversimplified model, one finds non-universal relaxation dynamics with a stretched exponential varying as a function of disorder strength, as shown by the dashed line the inset of Fig. 6(c). In this model the difference in distribution functions matters across the whole range of disorder. From this we conclude that the universal stretching exponent $\beta$ we observe in experiment and simulation emerges from many-body effects.

For the experimental conditions the inhomogeneous density distribution leads to an average over a range of disorder strengths. However, the densities are sufficiently low that the experiments fall well within the window where a universal stretching exponent is observed. This is confirmed by additional dTWA simulations performed for the actual inhomogeneous spin distribution used in the experiments, which results in a density-independent stretched exponent $\beta$ which is in excellent agreement with the measurements, as shown.

Figure 6. Numerical simulation of the dynamics for a uniform density distribution using the dTWA. (a) The simulated dynamics before rescaling. After the quadratic onset of dynamics, the time evolution is fitted by the stretched exponential function (dashed lines). (b) The fitted decay rate $\gamma_i$ agrees well with the median interaction strength $J_{mf}$ which is the typical energy scale of the system. In the weak interaction limit, $J_{mf}$ scales linearly with $C_0/a^6$, as indicated by the dotted line. (c) For small densities of $\rho_S = 1.25 \times 10^9 \text{cm}^{-3}$ (blue line), intermediate densities of $3.51 \times 10^8 \text{cm}^{-3}$ (green line) and $8.73 \times 10^9 \text{cm}^{-3}$ (yellow line), the numerical data collapses on one curve after rescaling time with $C_0/a^6$, where $\bar{a}$ plays the role of an effective distance that takes into account the roles of disorder and power-law interactions. For densities as large as $\rho_S = 2.11 \times 10^9 \text{cm}^{-3}$ (dotted line) the spatial order introduced by the blockade is so large, that the dynamics does not follow the universal behavior observed for smaller densities. Inset: Stretching exponent $\beta$ as a function of the order in the system expressed by $(\bar{a}/R_{bl})^{-3}$. Below a critical value of $(\bar{a}/R_{bl})^{-3} \lesssim 0.7$ the exponent becomes constant. The dots denote the exponent $\beta$ derived from the curves depicted in Fig. 6.
Figure 7. Normalized probability distribution of nearest-neighbour interactions \( g(J) \) for the densities given in Fig. 6 (blue line: \( \rho_S = 1.25 \times 10^5 \text{cm}^{-3} \), yellow line: \( \rho_S = 8.73 \times 10^6 \text{cm}^{-3} \), red line: \( \rho_S = 2.11 \times 10^7 \text{cm}^{-3} \)). The dashed line depicts a distribution corresponding to randomly distributed spins. The blockade effect induces correlations in the system and hence the probability distribution at high densities become more peaked. This results in a decreased disorder compared to the completely random case.

VI. CONCLUSION AND OUTLOOK

In this work we observed glassy dynamics in close analogy to the subexponential relaxation known from classical spin glasses. While the latter is driven by thermal fluctuations, the dynamics of the disordered isolated quantum system is governed by quantum fluctuations and spreading of entanglement going beyond mean-field approximations. Remarkably, the stretching exponent \( \beta \) takes on a universal value below a certain disorder strength, as confirmed by both experiment and semi-classical simulations. In the experiment, disorder is changed by controlling of the density of spins, which is much larger than the LeRoy radius \( R_{LR} \) that describes the typical spread of the electron wave function. The leading order term of this expansion is the dipole-dipole interaction Hamiltonian

\[
\hat{H}_{\text{DDI}} = \frac{\hat{d}_i \cdot \hat{d}_j - 3 \left( \hat{d}_i \cdot e_r \right) \left( \hat{d}_j \cdot e_r \right)}{R^3}
\]

(A1)

We interpret the observed independence of the cut-off at large interaction strengths and the validity of classical description as a strong hint that the dynamics of many-body quantum systems might be amenable to a simplified description of the late-time dynamics in terms of effective low energy degrees of freedom. Concretely, this could be approached is the spirit of the strong disorder renormalization group, iteratively integrating out the highest energy degrees of freedom resulting from most strongly interacting spins or clusters of spins [12]. Furthermore, spin glasses in the aging regime have been found to show certain quasi-thermal properties. For examples a fluctuation dissipation theorem has been found to hold [43] and the spin-glass transition shows similarities to a thermal phase transition [44]. Thus, the similarities to classical glassy dynamics observed in this work are encouraging to extend such an effective thermal-like description to quantum spin glass dynamics.

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Appendix A: Calculation of Rydberg interactions and spin model

In order to describe the interaction between two Rydberg excitations, the Hamiltonian is expanded in multipoles. This is well justified, as the minimal distance between the Rydberg atoms that is determined by the blockade radius \( R_{B} \) is much larger than the LeRoy radius \( R_{LR} \) that describes the typical spread of the electron wave function. The leading order term of this expansion is the dipole-dipole interaction Hamiltonian
that couples Rydberg atoms with different angular moment quantum number \(l\). For dipolar forbidden transitions, the second order term in perturbation theory needs to be calculated giving rise to the van der Waals Hamiltonian

\[
H_{\text{vdW}} = -\frac{1}{\hbar} \sum_m H_{\text{DDI}}|m\rangle\langle m|H_{\text{DDI}}^{\dagger} \delta(\omega_{fm} + \omega_{mi}) \tag{A2}
\]

where the Förster defect \(\Delta_F = E_m - E_i\) is the energy difference between the intermediate and initial state. Aiming for a simpler notation, the two different Rydberg states can be identified as spin states \(|\uparrow\rangle\) and \(|\downarrow\rangle\). In the pair state basis \(\langle \uparrow\uparrow|, \langle \uparrow\downarrow|, \langle \downarrow\uparrow|, \langle \downarrow\downarrow|\rangle\), the total Hamiltonian describing the interaction between two atoms \(i\) and \(j\) can be written in matrix form as

\[
\hat{H}_{ij}^{\text{int}} = \begin{pmatrix} E_{i\uparrow} & 0 & 0 & 0 \\ 0 & E_{j\uparrow} & \frac{\Delta_{\text{vdW}}}{2} & 0 \\ 0 & \frac{\Delta_{\text{vdW}}}{2} & E_{j\uparrow} & 0 \\ 0 & 0 & 0 & E_{i\downarrow} \end{pmatrix} \tag{A3}
\]

with the matrix elements \(E_{i\uparrow} = \langle \uparrow\uparrow|H_{\text{vdW}}|\uparrow\uparrow\rangle\), \(E_{i\downarrow} = \langle \downarrow\downarrow|H_{\text{vdW}}|\downarrow\downarrow\rangle\), \(E_{\downarrow\uparrow} = \langle \downarrow\uparrow|H_{\text{vdW}}|\downarrow\uparrow\rangle\) and \(E_{\downarrow\downarrow} = \langle \downarrow\downarrow|H_{\text{vdW}}|\downarrow\downarrow\rangle\). This Hamiltonian can be identified as the Heisenberg XXZ Hamiltonian

\[
H_{\text{XXZ}} = \frac{1}{2} \sum_{i,j} J_{ij}(S_x^{(i)} S_x^{(j)} + S_y^{(i)} S_y^{(j)} + \delta S_z^{(i)} S_z^{(j)}) + \sum_i \Delta_{\text{vdW}} S_z^{(i)}
\]

where \(J_{ij} = 2J_{\text{ex}}, \delta = (E_{\downarrow\downarrow} + E_{\uparrow\downarrow} - 2E_{\uparrow\uparrow})/J_{ij}\) and \(\Delta_{\text{vdW}} = (E_{\downarrow\downarrow} - E_{\uparrow\downarrow})/2\). The additional single-spin detuning \(\Delta_{\text{vdW}}\) is an order of magnitude smaller than the interaction strength \(J_{ij}\) and thus negligible.

The matrix elements \(E_{\uparrow\uparrow}, E_{\uparrow\downarrow}, E_{\downarrow\uparrow}\) and \(J_{\text{ex}}\) were calculated using the python module ARC [23]. For the Rydberg states \(\{48S_{1/2} + \frac{1}{2}\}\) and \(\{49S_{1/2} + \frac{1}{2}\}\) this yields the interaction strength \(J_{ij} = C_0/r_{ij}\) with \(C_0/(2\pi) = 59\,\text{GHz}\,\mu\text{m}^6\) and \(\delta = -0.73\).

Appendix B: Determination of the magnetization

The magnetization is extracted from population measurements of the Rydberg states using electric field ionization [15]. At the end of the sequence, a strong electric field of 100 V cm\(^{-1}\) is switched on and the resulting ions are guided towards a multichannel plate detector. To calibrate the detection efficiency, we combine ionization measurements and depletion imaging. We deduce a detection efficiency \(\eta = 0.173 \pm 0.043\) from four different calibration curves.

At time \(t\) we access the magnetization \(\langle S_\phi \rangle\) by counting after the readout pulse both the population of the \(|\uparrow\rangle\) state \(N_\uparrow(\phi)\), and the total spin number \(N_{\downarrow\uparrow}\), according to

\[
\langle S_\phi \rangle = \frac{N_\uparrow(\phi) - N_\downarrow(\phi)}{2N_{\downarrow\uparrow}} = \frac{N_\uparrow(\phi) - \frac{1}{2}}{N_{\downarrow\uparrow}} \tag{B1}
\]

Since the ionization is not state-selective, \(N_\uparrow(\phi)\) is inferred by counting the spin number after depopulating the \(|\downarrow\rangle\) state. It is performed by optically coupling the \(|\downarrow\rangle\) state to the short-lived intermediate state \(|c\rangle\) during 1.5\,\mu\text{s}.

Due to the finite lifetime of the spin states and microwave transfer, auxiliary Rydberg states might also be populated. This residual population leads to an offset in the measured ion signal, a number \(N_a\) of those atoms being energetically above the ionization threshold (see Fig. 8). As a consequence, what we measure instead of \(N_\uparrow(\phi)\) and \(N_{\downarrow\uparrow}\) are two quantities \(M_\uparrow(\phi)\) and \(M_{\downarrow\uparrow}\) given by

\[
M_\uparrow(\phi) = N_\uparrow(\phi) + N_a, \quad M_{\downarrow\uparrow} = N_{\downarrow\uparrow} + N_a \tag{B2}
\]

The measured quantity \(M_\uparrow\) is a sinusoidal function of \(\phi\), centered around its mean value

\[
\overline{M}_\uparrow = \frac{N_{\downarrow\uparrow} + N_a}{2} \tag{B4}
\]

We determine from a sinusoidal fit the values \(M_\uparrow(\phi)\) and \(\overline{M}_\uparrow\) and thus compute the magnetization \(\langle S_\phi \rangle\) using eqs. (B1) to (B4). The amplitude \(A\) of the sinusoidal fit, normalized by \(N_{\downarrow\uparrow}\), corresponds to the magnetization in the \(xy\)-plane. Following this procedure, we deduce that the number of atoms in the auxiliary states \(N_a\) increases linearly in time with a rate of 7 kHz, consistent with the blackbody decay of the spin states toward Rydberg states above the ionization threshold.

Appendix C: Theoretical models

To compare the experiment to the mean-field prediction, we solve the classical equations of motion that are obtained from the classical Hamiltonian function [39]

\[
H_C = \frac{1}{2} \sum_{i,j} J_{ij}(s_x^{(i)} s_x^{(j)} + s_y^{(i)} s_y^{(j)} + \delta s_z^{(i)} s_z^{(j)}) \tag{C1}
\]

via Hamilton’s equation

\[
s_t^{(i)} = \left\{ s^{(i)}, H_C \right\}. \tag{C2}
\]

Here, \(s^{(i)} = (s_x^{(i)}, s_y^{(i)}, s_z^{(i)})\) are classical spins and \(\{\ldots\}\) denotes the Poisson bracket. The system of ordinary
Appendix D: Quantification of slow dynamics by a stretched exponential

A phenomenological approach to describe slow dynamics in disordered systems is a fit of the magnetization with a stretched exponential

\[
\langle S_x \rangle (t) = \frac{1}{2} \exp\left(-\left(\frac{\gamma J t}{\tau}\right)^\beta\right)
\]

with relaxation rate \(\gamma J\) and stretched exponent \(\beta\). This was already proposed by Kohlrausch in 1847 \[51\], a review on the stretched exponent in numerical simulations and in experimental data of various materials can be found in \[51\].

For \(\beta = 1\) the stretched exponential describes an exponential decay. In the limit \(\beta \to 0\), the stretched exponential approaches the logarithmic decay which can be seen by performing a Taylor expansion at small \(\beta\)

\[
\exp\left(-\left(\frac{t}{\tau}\right)^\beta\right) = \frac{1}{e} \frac{\beta \log \left(\frac{1}{\beta e}\right)}{\beta^2} + \mathcal{O}(\beta^3).
\]

So, the stretched exponent \(\beta\) quantifies how slow a system relaxes: A small value signifies that the dynamics are close to logarithmic and slow, a large value indicates fast dynamics.

The magnetization at early times can be calculated differentially equations is solved by Tsitouras 5/4 Runge-Kutta method \[46\] using the Julia Differential Equations package \[47\]. For a perfect initial state where all spins are aligned in \(x\)-direction, mean-field theory does not predict any dynamics. However, the interactions present during the first \(\pi/2\)-pulse of the Ramsey protocol induce small fluctuations in the initial state. We take these imperfections into account by including the preparation and read-out pulses into the simulations which leads to the dynamics shown by the dotted line in Fig. 4. For the relevant time-scale of the experiment, these dynamics are negligible.

For the considered dynamics, the initial state is an eigenstate of the mean-field Hamiltonian. The relaxation is thus triggered by the initial quantum fluctuations, meaning that mean-field approaches fails in this case. Instead, we use a discrete Truncated Wigner Approximation (dTWA), which still performs classical evolution of the spins following eq. (C2) but includes the initial quantum fluctuations into statistical ensembles of the initial state by sampling Monte-Carlo trajectories on discrete phase-space \[38\] \[39\]. For the simulations in this paper, we sample over 100 initial conditions which is sufficient for the magnetization to be converged. Simulating far-from equilibrium dynamics of disordered spin systems has been successfully applied to spin systems in recent work \[20\]. Imperfections of the preparation and readout are taken into account by simulating the whole Ramsey sequence including those two microwave pulses. We also compared dTWA with an approximate quantum mechanical model, the so-called Moving Average Cluster Expansion \[18\] which qualitatively gives similar results (see Fig. 9).
by the Baker-Campbell-Haussdorff formula
\[ \langle S_x(t) \rangle = \langle e^{iH} S_x e^{-iH} \rangle = \langle S_x \rangle + it \langle [H, S_x] \rangle - t^2/2 \langle [H, [H, S_x]] \rangle + \ldots \] (D3)

Since the initial state \( |\rightarrow \rangle \otimes N \) is an eigenstate of \( S_x \), the expectation value of the commutator \( [H, S_x] \) vanishes for this state and we expect the initial dynamics to be quadratic in time. However, this doesn't hold for the stretched exponential function that is a power low with exponent \( \beta \) for short times \( t \ll \frac{1}{\gamma} \):
\[ \langle S_x \rangle(t) = 1/2 \left( 1 - \beta (\gamma t)^\beta \right) \] (D4)

Therefore, we exclude the very early dynamics from the fit where \( t < 1/J_{\text{max}} \) (see Fig. 6).

**Appendix E: Spatial spin distribution and disorder strength**

To model the experimental 3D spin distribution, we simulate the Rydberg excitation dynamics in a cloud of ground state atoms with Gaussian density distribution (measured radii at 1/e²: \( \sigma_x = 203(3) \) μm, \( \sigma_y = \sigma_z = 35(1) \) μm). We excite each atom randomly one after the other with a certain probability if it is not within the blockade sphere of an already excited atom. The excitation probability takes into account the \( \sqrt{N_b} \) enhancement due to Rydberg interactions, with \( N_b \) the number of atoms within a blockade sphere. It also considers the profile of the laser excitation, characterized by a Gaussian distribution of the two-photon Rabi frequency with measured radius \( \sigma = 70.6(3) \) μm (1/e²). Its amplitude is adjusted such that the total number of excited atoms equals the one measured by field ionization.

For simulations of a homogeneous system, the spins are randomly distributed in a uniform box taking into account a blockade radius of 5 μm until the desired density \( \rho_{GS} \) is reached. In the limit of no blockade effect, the nearest-neighbor distribution for a given Wigner-Seitz radius \( a = (4\pi\rho/a^{3})^{-1/3} \) would be given by
\[ h(r) = \frac{3}{a} \frac{r}{a}^2 \exp \left(-\frac{(r/a)^3}{3}\right), \] (E1)
yielding the distribution of coupling strengths \( h(J) = h(r) \frac{\partial}{\partial r} \). Instead, the blockade effect modifies the nearest-neighbour distribution, resulting in a different coupling distribution \( g(J) \) (see Fig. 7 (a)). We quantify the disorder strength of the spin system with the Kullback-Leibler divergence
\[ D_{KL}(g \parallel h) = \int g(J) \log \left( \frac{g(J)}{h(J)} \right) dJ, \] (E2)
i.e. the amount of information that is gained by updating from the distribution \( h(J) \). Indeed, the Kulback-Leibler divergence increases almost linearly with density (see Fig. 10). This confirms that \( (a/R_b)^{-3} \) is a relevant scale to describe the disorder strength.

When investigating the universal character of the spin dynamics for homogeneous systems, we have concluded that the typical energy scale should be determined by \( C_b/\tilde{a}^6 \) (see Fig. 6). The effective distance \( \tilde{a} \) is defined as the median of the non-Gaussian distribution \( \{\tilde{r}_i\} \) with \( \tilde{r}_i^{-6} = \sum_j r_{ij}^{-6} \). The energy scale \( C_b/\tilde{a}^6 \) is thus the median of the mean-field energies \( C_b/\tilde{r}_i^6 \).

**Appendix F: Summary of parameters**

Table I summarizes the parameters of the individual measurements shown in figure 2 and 3 of the main text.
