1. INTRODUCTION

The opening of a pseudogap below a strong doping \( \delta \) dependent crossover temperature \( T^*(\delta) \), already above the superconducting critical temperature \( T_c(\delta) \), is an intriguing feature of the underdoped cuprates \( 1 \). One possibility is that the pseudogap arises from strong scattering processes in the particle-hole channel. In this framework, spin fluctuations have been considered \( 2 \). A similar outcome would arise from scattering by charge fluctuations near a charge instability for stripe formation \( 3–6 \). It was indeed suggested that the tendency to spatial charge order (which evolves into a mixed spin-charge stripe phase by lowering the doping) gives rise to an instability line ending in a Quantum Critical Point (QCP) at \( T = 0 \) near optimal doping. By approaching the instability line, the quasi-critical stripe fluctuations affect the states on the FS in a quite anisotropic way. The fermionic states around the M points [i.e. \( (\pm \pi, 0), (0, \pm \pi) \)] interact strongly (these are the so-called “hot spots”), while the states around the \( \Gamma-X \) or \( \Gamma-Y \) diagonals are weakly interacting (“cold spots”). Well below the instability line, deep in the underdoped phase, a local (quasi-static) stripe order takes place, which can strongly affect the spectral distribution of the quasiparticles \( 7 \). This mechanism for pseudogap formation will be discussed in the next section. On the other hand a second possibility is that the pseudogap arises from pairing in the particle-particle channel with \( T^* \) being a mean-field-like temperature where electrons start to form local pairs without phase coherence. Lowering the temperature, the phase coherence and hence superconductivity are established at the critical temperature \( T_c \). In this context it is still debated whether the superconducting transition is intermediate between a BCS transition and a Bose-Einstein condensation of preformed pairs \( 8 \), or is due to a more intricate interplay between preformed bosonic pairs and fermions \( 9,10 \). Within the Stripe-Quantum-Critical-Point (Stripe-QCP) scenario a two-gap model can be considered, where strongly paired fermionic states can coexist and interplay with weakly coupled pairs in different regions of the Fermi surface (FS). This is a natural description of the cuprates near the instability line, when quasiparticles have very different dispersions and effective interactions in different regions of the Fermi surface. In this case the strong momentum dependent singular scattering arising in the proximity of the stripe instability acts in the particle-particle channel and leads to tightly
bound (strongly phase fluctuating) pairs near the hot spots (close to the M points) coexisting with weakly interacting quasiparticles near the Γ-X or Γ-Y directions. In this case the pseudogap will find a natural interpretation in terms of the incoherent Cooper-pair formation around the M-points. We will discuss this model in Section 3.

2. CHARGE ORDERING IN THE UNDERDOPED CUPRATES

Within the Stripe-QCP scenario the underdoped region of the cuprates corresponds to the nearly ordered stripe phase where a local (quasi-static) charge ordering takes place. In this framework, a mean-field analysis will likely capture the main qualitative features of the (locally) charge-ordered phase. In particular, in Ref. [2] the spectral properties of an incommensurate charge-density-wave system were investigated within a standard Hartree-Fock approach. Both a purely one-dimensional ordering with an order parameter \( \rho_q > 1D = \sum_n \rho_q > \delta_{q,nq} \) and a two-dimensional “eggbox” structure with \( \rho_q > egg = \sum_n \rho_q > [\delta_{q,nq_x} + \delta_{q,nq_y}] \) were considered. The effective density-density interaction and the critical wavevectors \( q_c \) were derived from microscopic calculations based on the frustrated phase-separation mechanism for an Hubbard-Holstein model with long-range Coulomb interaction. A first generic outcome was that charge ordering tends to substantially enhance the van Hove singularities near the M points. Pseudogaps are also formed. However the charge ordering modulated along only one single direction, \( x \) or \( y \), naturally produces pseudogaps along one direction only. The superposition of two one-dimensional CDW modulated along perpendicular directions simply fills up the pseudogap in all four M-points. On the other hand, in the eggbox case a leading-edge gap arises near these points, leaving finite arcs of the Fermi surface gapless. This latter non-trivial feature might account for recent ARPES results [1], but does not seem to be robust under the disordering of the eggbox modulation [6]. Therefore, although the particle-hole scattering definitely seems to affect the electronic spectra, particularly around the M points, additional mechanisms, likely related to the particle-particle pairing discussed in the next section, seem to be in order to fully account for the observed pseudogap. On the contrary, in the specific cases where commensuration effects couple the stripes to the underlying lattice structure, charge ordering becomes particularly strong and it can alone open a full insulating gap. This is the case of various hole-doped compounds at doping \( \delta = 1/8 \), where stripes in the (1,0) or (0,1) directions with half an hole in excess per site (half-filled vertical stripes, HFVS) are observed [2]. In this regard it was recently established [12] within a joint (slave-boson)-(unrestricted Hartree-Fock) approach, that both a proper treatment of the strong local hole-hole interaction and the presence of a sizable long-range Coulomb repulsion are needed to obtain a ground state with HFVS. The same conclusion was reached within a realistic three-band extended Hubbard model [14], where also the electron doped case was investigated. For electron doping the most stable configuration was predicted to have the stripes along the (1,1) or (1,-1) directions with one additional electron per site (filled diagonal stripes).

3. THE TWO-GAP MODEL

As discussed above, the formation of a pseudogap in the metallic underdoped phase is likely related to the strongly anisotropic attractive potential arising in the proximity of the instability. In order to capture the relevant physical effects of the anisotropy of both the pairing interaction and the Fermi velocity, we introduce a simplified two-band model for the cuprates [14].

We describe the quasiparticle arcs of FS about the nodal points by a free electron band (labelled below by the index 1) with a large Fermi velocity \( v_{F1} = k_{F1}/m_1 \) and the hot states about the M points with a second free electron band, displaced in momentum and by an energy \( \varepsilon_0 \) from the first, with a small \( v_{F2} = k_{F2}/m_2 \). The energy \( \varepsilon_0 \) is introduced to allow the chemical potential to cross both bands: \( E_{F1} = \varepsilon_0 + E_{F2} \). Moreover, since our main interest is the interplay between strongly and weakly coupled pairs irrespectively of their symmetry, for simplicity we assume a s-
wave pairing interaction \[ [16]. \]

The model Hamiltonian for pairing in the two-band system is then

\[ H_{\text{pair}} = - \sum_{kk'\rho\sigma} g_{\alpha\beta} c_{k\rho\sigma}^+ c_{k'\beta}^+ c_{-k'\beta} c_{-k\alpha} \]

where \( \alpha \) and \( \beta \) run over the band indices 1 and 2. We also introduce a BCS-like energy cutoff \( \omega_0 \) to regularize the pairing interaction. The \( 2 \times 2 \) scattering matrix \( \hat{g} \) accounts for the strongly \( q \)-dependent effective interaction in the p-p channel of the original single-band system. Its elements \( g_{ij} \) are the different coupling constants which couple the electrons in the p-p channel within the same band (\( g_{11} \) and \( g_{22} \)) and between different bands (\( g_{12} = g_{21} \)). The ladder equation for the superconducting fluctuation propagator is given by \( \tilde{L} = \tilde{g} + \tilde{g}IL, \) where the particle-particle bubble operator for the two-band spectrum has a diagonal \( 2 \times 2 \) matrix form with elements \( \Pi_{11}(q) \) and \( \Pi_{22}(q) \). The resulting fluctuation propagator is given by

\[ \tilde{L}(q) = \begin{pmatrix} \tilde{g}_{11} - \Pi_{11}(q) & \tilde{g}_{12} \\ \tilde{g}_{22} - \Pi_{22}(q) & \tilde{g}_{22} \end{pmatrix}^{-1} \tag{2} \]

where we have defined \( \tilde{g}_{ij} = (\tilde{g}^{-1})_{ij}. \) It turns out useful to define the temperatures \( T_{c1}^0 \) and \( T_{c2}^0 \) as \( g_{11} - \Pi_{11}(0, T) = \rho_1 \ln \left[ \frac{T}{T_{c1}^0} \right], \) \( g_{22} - \Pi_{22}(0, T) = \rho_2 \ln \left[ \frac{T}{T_{c2}^0} \right], \) where \( \rho_i = m_i/(2\pi)(i = 1, 2) \) is the density of states of the \( i \)-th band. To emulate the hot and cold points we assume \( g_{22} >> g_{11} \approx g_{12}. \) In this limit \( T_{c1}^0 \) and \( T_{c2}^0 \) (with \( T_{c2}^0 >> T_{c1}^0 \)) give the two BCS critical temperatures for the two decoupled bands (i.e. for \( g_{12} = 0 \)). For the coupled system \( (g_{12} \neq 0) \) the mean-field BCS superconducting critical temperature \( T_c^0 \) is defined by the equation \( \det L^{-1}(q = 0, T_c^0) = 0. \) We then obtain \( T_c^0 > T_{c2}^0 \) given by

\[ T_c^0 = \sqrt{T_{c1}^0 T_{c2}^0} \exp \left[ \frac{1}{2} \ln^2 \left( \frac{T_{c1}^0}{T_{c2}^0} \right) + \frac{4\tilde{g}_{12}^2}{\rho_1 \rho_2} \right]. \tag{3} \]

The role of fluctuations can be investigated within a standard Ginzburg-Landau (GL) scheme, when both \( g_{22} < E_F^2 \) and \( \omega_0 < E_F^2. \) We will assume that the chemical potential is not affected significantly by pairing and that fluctuations from the BCS result are not too strong. The relevance of the space fluctuations of the order parameter is assessed by the gradient term coefficient \( \eta, \) which provides the momentum dependence of the propagator. This calculation requires the expansion of the fluctuation propagator in Eq. (3) in terms of \( q. \) In particular the expansion of the particle-particle bubbles reads \( \Pi_{11}(q) \approx \Pi_{11}(0) - \rho_1 \eta_1 q^2 \) and \( \Pi_{22}(q) \approx \Pi_{22}(0) - \rho_2 \eta_2 q^2. \) Here \( \eta_i (i = 1, 2) \) is given by \( \eta_i = (7/3)/(2\pi^2) \) \( v_F^2 1/\rho_i. \) With \( \eta_1 \gg \eta_2. \) We obtain \( \eta = \alpha_1 \eta_1 + \alpha_2 \eta_2 \) with

\[ \frac{\alpha_1}{\alpha_2} = \frac{\tilde{g}_{12}^2}{\rho_1 \rho_2 \ln^2 (T_{c1}^0 / T_{c2}^0)}, \]

and \( \alpha_1 + \alpha_2 = 1. \) The presence of a fraction of electrons with a large \( \eta_1 \) increases the stiffness of the whole electronic system \( \eta \) with respect to \( \eta_2. \) However when the mean-field critical temperature \( T_{c1}^0 \) is much larger than \( T_{c1}^0 \) the correction to \( \eta_2 \) due to the interband coupling is small. At the same time the Ginzburg number is large implying a sizable mass correction \( \delta \epsilon(T) \) due to fluctuations to the “mass” \( \epsilon(T) \) of the bare propagator \( \tilde{L}(q). \) The renormalized critical temperature \( T_c^0, \) given by the equation \( \epsilon(T_c^0) + \delta \epsilon(T_c^0) = 0, \) is lower than \( T_{c1}^0 \) [18]. We find that the renormalized gradient term coefficient \( \eta^*, \) in the presence of the mass correction is still given by Eq. (4) with \( T_{c1}^0 \) replaced by \( T_c^0. \) Therefore, while mass renormalizations of the fluctuation propagator tend to lower \( T_c, \) at the same time, this increases the gradient term coefficient \( \eta \) by increasing the coupling to \( \eta_1. \) As a consequence the effective Ginzburg number is reduced and the system is stabilized with respect to fluctuations allowing for a coherent superconducting phase even in the extreme limit \( \eta_2 = 0. \) Within the GL approach we associate the temperature \( T_c \sim T_{c2}^0 \) to the crossover temperature \( T_* \) and \( T_c^0 \) to the superconducting critical temperature \( T_c \) of the whole system.

Within the Stripe-QCP scenario the coupling \( g_{22} \) is related to the singular part of the effective interaction mediated by the stripe fluctuations. \( g_{22} \) is the most doping dependent coupling and attains its largest value in the underdoped regime. \( g_{11} \) and \( g_{12} \) are instead less affected by doping. In the region of validity of the GL approach, the explicit calculations show that
\( r(\delta) \equiv \frac{T^* - T_c}{T_1 - T_1^*} \) is increasing by increasing \( g_{22} \), \textit{i.e.}, by decreasing doping. For small values of \( r \) we find that both \( T^* \) and \( T_c \) increases. This regime corresponds to the overdoped and optimally doped region. For \( r \approx 0.25 \div 0.5 \), \( T_c \) is instead decreasing while \( T^* \) is always increasing by decreasing doping. The large values of \( r \), which are attained in the underdoped region show that we are reaching the limit of validity of our GL approach. We think however that the behavior of the bifurcation between \( T^* \) and \( T_c \) represents correctly the physics of the pseudogap phase, at least qualitatively, while a quantitative description would require a more sophisticated approach like a RG analysis.

In the very low doping regime, where \( T^* \) has increased strongly, the value of \( g_{22} \) can be so large to drive the system in a strong coupling regime for the fermions in band 2 (\( g_{22} > E_F2 \)). In this case the chemical potential is pulled below the bottom of the band 2. The GL scheme must be abandoned and, in the limit of tightly bound 2-2 pairs, the propagator \( L_{22}(\mathbf{q}) \) assumes the form of a single pole for a bosonic particle (similarly to the single-band strong-coupling problem \[15\]). The critical temperature of the system is again obtained by the vanishing of the inverse of \( \det L^{-1} \) at \( q = 0 \) where, however, the chemical potential is now self-consistently evaluated including the self-energy corrections to the Green function in band 2 and the fermions left in band 1. One gets

\[
\frac{\tilde{g}_{12}^2}{\rho_1 \ln (T_c/T_{c1})} = \frac{\rho_2 \omega_0 (|\mu_2| - |\mu_B|)}{|\mu_2||\mu_B|} \tag{5}
\]

where \( \mu_2 = \mu_2(T_0^c) \) is the chemical potential measured with respect to the bottom of the band 2 and \( \mu_B = \rho_2 \omega_0 g_{22} \) represents the bound-state energy. In the present case most of the fluctuation effect has been taken into account by the formation of the bound state occurring at a very high \( T^* \sim g_{22} \). In this new physical situation \( \eta \sim \eta_1 \) stays sizable and the fluctuations will not strongly further reduce \( T_c^* \) with respect to \( T_0^c \). \( T_c \sim T_c^* \sim T_0^c \). In this low doping regime \( T^* - T_c \) approaches its largest values before \( T_c \) vanishes.

The strong-coupling limit of our model shares some similarities as well as some important differences with phenomenological models of interacting fermions and bosons \[10\]. In particular, we believe that the model considered here is more suitable to describe the crossover to the optimal and overdoped regime, where no preformed bound states are present and the superconducting transition is quite similar to a standard BCS transition.

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