Formalization of semantic network of image constructions in electronic content

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Abstract — A formal theory based on a binary operator of directional associative relation is constructed in this article. An understanding of an associative formal system of image constructions is introduced. A model of a commutative semigroup, which provides a presentation of a sentence as three components of an interrogative linguistic image construction, is considered.

Keywords — Associative pair, formal theory, graph, image construction, model.

1. Introduction

Recently problems of computational linguistics have become especially important because of a growing demand for a natural language interface in the information technologies available to Internet users. The research is carried out in the development of an approach to modeling creative human thinking [1] and is directed towards solving the problem of increasing the level of recognition and the understanding of natural language constructions. The problems which are covered by this research are associated with support for human-computer dialogue, relevant search of information, e-learning tasks and a wide range of other problems in the realm of artificial intelligence. The purpose of this work lies in the formalization of semantic network of image constructions in electro and electronic content

II. A Formal Theory of a Commutative Semigroup of Image Constructions

A formal theory $Th$ is constructed as an applied theory of the first degree based on known provisions of the formal systems theory set forth in [2-4], taking into account the requirements of a concept of understanding the meaning of image constructions (IC) proposed in [5].

1. We introduce a finite alphabet consisting of symbols to be used as:
   - $Al = \{A, B, ..., Z, x_1, x_2, ..., x_n, t_1, t_2, t_3\}$ – variables;
   - $Con = \{\emptyset, 1, ..., n\}$ – constants;
   - $\{\oplus\}$ – symbols of binary operations defined below;
   - $\{=\}$ – a binary predicate symbol “equality sign” in the sense of the set theory;
   - $\{\neg\}$ – negation,
   - $\rightarrow$ – inference (if …, then …), $\forall$ – a universal quantifier;
   - brackets “(“, “)” and commas “,”.

   In accordance with a concept of understanding the meaning of image constructions [5], we consider that the symbol $\setminus$ denotes a direct relationship between two images in an associative pair $a \in \Omega$, whose meaning is given below, and a symbol $\ominus$ – an operation unifying image constructions «AND IC».

2. Now we will define the procedures for constructing terms (strings of characters) and formulas (acceptable expressions) of the formal theory $Th$. Terms are obtained by concatenating alphabet symbols:

   $\langle Term \rangle := x_j | x_j \in Al, j \in Con,$
   $\langle Term \rangle := \langle Term \rangle \langle Term \rangle,$

   We denote terms constructed that way in the associative normal form (ANF) by the characters $t_1, t_2, t_3 \in Al$.

   $\langle ANF \rangle := x_j | x_j \in Al ;$
   $\langle ANF \rangle := \langle ANF \rangle \langle ANF \rangle ;$
   $\langle ANF-term \rangle := \langle ANF-term \rangle \langle ANF-term \rangle ;$
   $\langle ANF-term \rangle := \langle ANF-term \rangle \langle ANF-term \rangle ,$

   where $\langle ANF \rangle$ is called an elementary term in ANF.

   To simplify understanding we denote individual formulas constructed in that way by characters $A, B, ..., Z \in Al$:

   $\langle Formula \rangle := \langle ANF-term \rangle ,$
   $\langle Formula \rangle := \langle Formula \rangle \langle Formula \rangle ,$
   $\langle Formula \rangle := \neg \langle Formula \rangle ,$
   $\langle Formula \rangle := \langle Formula \rangle \langle Formula \rangle ,$
   $\langle Formula \rangle := (\forall x) \langle Formula \rangle ,

   For convenient use we add to the theory $Th$ alphabet 3 more logical connections, a quantifier $\exists$ and a functional symbol $\times$ to be used as:

   $A \land B := \neg (A \rightarrow \neg B),$
   $A \lor B := \neg A \rightarrow B ,$
   $A \iff B := (A \rightarrow B) \land (B \rightarrow A) ,$
   $(\exists x)(A) := \neg (\forall x)(\neg A) ,

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$x \times y := (x, y) \ominus (y, x)$, \hspace{1cm} (15)

where $\&$ – a logical «AND», $\lor$ – a logical «OR», $\Leftrightarrow$ – if and only if, $\exists$ – an existential quantifier, $\times$ – an applied functional symbol which is defined below by the symbol $\backslash$. Hereafter, a formula $A$, in which a variable

$x \in Al$ or a term $t$, is connected with one of the quantifiers, the formula is denoted by $A(x)$ or $A(t)$.

3. We select a set of formulas that are considered axiom schemes.

Logical axioms (3.1÷3.3 – expressions calculus, 3.4÷3.5 – first-order predicate calculus [3]):

3.1. $A \rightarrow (B \rightarrow A)$.

3.2. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$.

3.3. $(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$.

3.4. $\forall x A(x) \rightarrow A(t_i)$ [where $A(x)$ is a formula from Th and $t_i$ is a term from Th, free for $x$, in $A(x)$].

3.5. $\forall x (A \rightarrow B) \rightarrow (A \rightarrow \forall x B)$ [if a formula $A$ does not include free occurrences $x$].

Proper axioms (3.6÷3.11 – axioms of a commutative semigroup [4], 3.12÷3.14 – applied axioms (products) of the theory):

3.6. $\forall t_1 \forall t_2 \forall t_3 (t_1 \ominus (t_2 \ominus t_3) = (t_1 \ominus t_2) \ominus t_3)$ (associativity).

3.7. $\forall t_i (t_i = t_i)$ (reflexiveness).

3.8. $\forall t_i \forall t_j (t_i = t_j \rightarrow t_i = t_j)$ (symmetry).

3.9. $\forall t_i \forall t_j \forall t_k (t_i = t_j \rightarrow (t_k = t_j \rightarrow t_k = t_i))$ (transitivity).

3.10. $\forall t_i \forall t_j \forall t_k (t_i = t_j \rightarrow (t_k = t_j \rightarrow t_k = t_i))$ (substitution).

3.11. $\forall t_i t_j t_k (t_i = t_j \rightarrow (t_k = t_i \rightarrow t_k = t_j))$ (commutativity).

3.12. $\forall x, y, z, x_i y_i = x_i y_i$ (transformation of a string to terms in ANF).

3.13. $\forall x, x_i y = x_i y$ (finite transformation of a string to a term in ANF).

3.14. $\forall x, y, x_i y_i = x_i y_i$ (reduction of a term in ANF).

4. We define a finite set of rules of inference, which provide a different set of formulas from some finite set of formulas.

$A, A \rightarrow B \rightarrow B$ «Modus ponens»,

$A \rightarrow (\forall t_i) A$ «a generalization rule»,

where the notation $A \rightarrow A$ means that $A$ is a result of the formulas set $\Gamma$.

Besides theorems of the formal theory of the first-order predicates, in the theory $Th$ such proper theorems are true.

Theorem 1. $\forall A \rightarrow <ANFterm> \rightarrow <ANFterm>$.

Proving by induction on a length of derivation $B_1, B_2, ..., B_k = B$:

a) $<Term>$ – a hypothesis;

b) $x, y$ – an induction base: according to the 1st definition of a term (2a);

c) $x_i \backslash x_j - 3.13$ before $b$;

d) $<ANFterm>$ – according to the 1st definition of a term in ANF;

e) $x, j, i - 3.12$ before $c$;

f) $x, \backslash x, \odot x, j - 3.12$ before $e$;

g) $x, \backslash x, \odot x, \backslash x, j - 3.13$ before $f$;

h) $<ANFterm>$ – according to the 2nd definition of a term in ANF;

i) $\prod x_j \prod i - 3.12$ before $c$.

j) $<ANFterm> \odot x, i - 3.12$ before i) $k$-1 times;

k) $<ANFterm> \odot x, \backslash x, k - 3.13$ before j);

l) $<ANFterm>$ – according to the 2nd definition of a term in ANF.

Theorem 2. A similar proof of such a theorem:

$<ANFterm> \rightarrow <ANFterm>$ $\odot <ANFterm>$ $\odot <ANFterm>$; where $<ANFterm> = x, i, x, j \in Al$ for convenience is denoted by $<ANFterm>$.

$l) <ANFterm> \odot <ANFterm> \odot <ANFterm>$ – all elementary terms of $<ANFterm>$, where a symbol $x, j$ is the first, then the next symbol is substituted recursively based on the principle of depth-first search, but if $<ANF?> x, j \backslash x$ is found, then a symbol $x$ and all symbols following it are not taken into account;

$<ANFterm>$ – all other elementary terms that compose $<ANFterm>$.

III. A MODEL OF A COMMUTATIVE SEMIGROUP OF IMAGE CONSTRUCTIONS AND EXAMPLES

We will now consider a model of the formal theory $Th$ as a commutative semigroup of image constructions. Within the model we consider that function symbols denote the following relations between two linguistic image [5]: $\backslash$ – «principal-subordinate» relation, $\times$ – «subject-predicate» relation. Under the term we understand the image construction of a simple sentence (syntagma), and under the formula of the theory – an image analog of a logical natural language expression. We denote individual images from the set $I = \{x, x_1, ..., x_n\}$ by the characters $x, x_1, ..., x_n$, terms in ANF – by characters $t_1, t_2, t_3$, formulas $A, B, ..., X$, an unknown subject – $Y$, an unknown predicate – $Z$. The elementary term in ANF $<ANF?>$ is called an associative pair of images, where $|$ – a denotation of the OR operator in Backus-Naur Form. Terms or image constructions are constructed from natural language sentences based on this rule 1: a sentence of $k$ words is written as a string of $2 \cdot k$ characters, where each $i$-th word in a sentence is put in correspondence to a linguistic image $x, i \in Al$, and after it $j \in Con$ is recorded as an indicator of another image $x, j$ of this sentence that is principal to a subordinate image $x, j$. If homogeneous parts are found in a sentence, then the possible cases are

$(x_i \& x_j) j \rightarrow x, i \& x_j \backslash x, j \backslash x_i \backslash x_j$ \hspace{1cm} (16)
or
\[(x_1 & x_2) j \otimes \text{<ANFterm>} \otimes x_1 \setminus x_j \rightarrow x_j \setminus x_1 \otimes x_2 \rightarrow \text{<ANFterm>} \otimes x_1 \setminus x_j \otimes x_2 \setminus x_j . \quad (17)\]

Limitations of the considered model:
- natural language sentences must have both subject and predicate, otherwise they are included artificially using \(Y\) and/or \(Z\) symbols;
- rule 1 applies only to meaningful words in a sentence that correspond to image constructions, and punctuation marks, prepositions and syncategorematic words in sentences are not accounted for.

Within the model, theorems of the formal theory \(Th\) receive this interpretation.

Theorem 1. Any term that corresponds to a natural language sentence (syntagma) and is based on rule 1, can be represented as a term in ANF:
\[
\text{<Term> } \rightarrow <\text{ANFterm}> . \quad (18)
\]

Theorem 2. If from a sentence represented in the form of a term in ANF \(<\text{ANFterm}>\) one selects one associative pair as an interrogative pronoun, then all elementary terms that directly dependent on this pair in ANF will make an answer, and all other elementary terms from \(<\text{ANFterm}>\) – an interrogative sentence:
\[
<\text{ANFterm}> \rightarrow <\text{ANFq}\otimes <\text{ANFp}}>\otimes <\text{ANFq}> . \quad (19)
\]

For convenient use of the model of the formal theory \(Th\) in content elements we introduce rule 2:
\[
<\text{tQ}> \rightarrow <\text{tA}> ,
\]

where
\[
<\text{tQ} \gg := (x_1 \in <\text{ANFq}>= \emptyset) \bigg| (x_1, x_1, \ldots , x_1, x_1, x_1, \ldots , x_1) ;
\]

\[
<\text{tA} \gg := (x_1 \in <\text{ANFA}>= \emptyset) \bigg| (x_1, x_1, \ldots , x_1, x_1, x_1, \ldots , x_1) ;
\]

– an additional sign that denotes the end of a interrogative part \(<\text{ANFterm}>\).

Strings of characters \(x, x, \ldots , x, x_i\) received for \(<\text{tQ}>\) and \(<\text{tA}>\) are rewritten by removing those characters from left to right, which recur. Formally, for 2ns symbol \(x_i, x_i \rightarrow (\{x_i = x_i, x_i, x_i, x_i\}) , \) for \(k\)-th symbol \(x_i, x_i, x_i \rightarrow (\{x_1 = x_i, x_1 = x_i, \ldots , x_1 = x_i, x_i, x_i, x_i, x_i, \ldots , x_i\}) .\)

To demonstrate the capabilities of the model of IC commutative semigroup of the formal theory \(Th\) we consider examples of sentences in English and Russian.

Example 1. Once I saw (a) little bird \((x, x, x, x, x_i) .\)

According to rule 1, we construct a term
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 2x_5\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 2x_5\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 2x_5\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 2x_5\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

We denote \(<\text{ANF}? > := x_1 \setminus x_i\) by a word \(<\text{when}? > .\)

According to theorem 2, \(<\text{ANF}a> \rightarrow \emptyset ,\)

\(<\text{ANFq}> \rightarrow x_1 \setminus x_1, \otimes x_1, \otimes x_i, \otimes x_i, \otimes x_i, \setminus x_i .\)

Then, according to rule 2, \(<\text{tA}> \rightarrow x_i ,\) and

\(<\text{tQ}> \rightarrow x_i, x_i, x_i .\) Thus, we have the following result:

\[
<\text{when}? > := x_1 \setminus x_i, \setminus x_i .
\]

It is easy to prove an equivalence of a graph model [6] and presented in this paper theory \(Th\) that is a subject of the further research from an applied point of view. This fact allows us to use known search algorithms on graphs for solving applied problems of finding the optimal path, the traversal of the graph and search during processing natural language constructions. Presented in Fig. 1 is the graph of a sentence illustrating an example of using of the theory \(Th\) model. The following notation is used:

\(\text{O} –\) a linguistic image – a part of a sentence;

\(\rightarrow –\) relation between principal and subordinate members of a sentence;

\(\text{word}? –\) an interrogative pronoun of an associative pair that is used to form an interrogative sentence.

Example 2 (Russian). Забытую песню несет ветерок (\(x, x, x, x, x_i\)).

According to rule 1, we construct a term
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]

– a product 3.12 to substring \(x, 3x_2\) leads to
\[
x_1 \setminus x_1, \otimes x_3, x_3, 2x_5, 5x_3 ;
\]
As follows:

- a product $3.12$ to substring $x_3 x_1$ leads to $x_3 \times x_1$.

According to theorem 2, $x_1 \times x_2$ leads to $x_1 \times x_2$.

Thus, the following result:

Thus, we have a term in ANF.

Now we denote $< ANF > := x_1 \times x_2$ by a word $<\text{what?}>$. According to theorem 2, $< ANF a > := x_1 \times x_2$.

Thus, according to rule 2, $< tA > := x_1 \times x_2$, and $< tQ > := x_1 \times x_2$. Thus, we have the following result:

Fig. 2 shows the graph of a sentence with selection of two associative pairs.

IV. Conclusion

Thus, the given examples demonstrate the intuitive intelligibility of the results of applying the model of IC commutative semigroups of the formal theory $Th$ to natural language structures in the form of sentences in English and Russian. Unlike existing formal theories, a binary operator of directional associative relation and the concept of ANF, according to the concept of understanding the sense of an electronic text content, are applied in the formal theory $Th$. A model of image constructions commutative semigroup that, based on the theory $Th$, provides a representation of IC of a natural language syntagma as 3 components of an interrogative construction of linguistic images.

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