Flavor-dependent Neutrino Angular Distribution in Core-collapse Supernovae

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Received 2017 January 31; revised 2017 March 24; accepted 2017 March 24; published 2017 April 25

Abstract

According to recent studies, the collective flavor evolution of neutrinos in core-collapse supernovae depends strongly on the flavor-dependent angular distribution of the local neutrino radiation field, notably on the angular intensity of the electron lepton number carried by neutrinos. To facilitate further investigations of this subject, we study the energy and angle distributions of the neutrino radiation field computed with the VERTEX neutrino-transport code for several spherically symmetric (1D) supernova simulations (of progenitor masses 11.2, 15, and 25 $M_\odot$) and explain how to extract this information from additional models of the Garching group. Beginning in the decoupling region (“neutrino sphere”), the distributions are more and more forward peaked in the radial direction with an angular spread that is largest for $\nu_e$, smaller for $\bar{\nu}_e$, and smallest for $\nu_x$, where $x = \mu$ or $\tau$. While the energy-integrated $\nu_e$ minus $\bar{\nu}_e$ angle distribution has a dip in the forward direction, it does not turn negative in any of our investigated cases.

Key words: hydrodynamics – neutrinos – supernovae: general

Supporting material: animation, tar.gz file

1. Introduction

Neutrinos are the main agents of energy and lepton-number transport in core-collapse supernovae (SNe). Within the delayed explosion mechanism of Bethe and Wilson, neutrinos cause the explosion by shock reheating and determine nucleosynthesis yields (Janka 2012; Burrows 2013). Neutrinos will also allow us to monitor core-collapse phenomenology when high event statistics will be collected by existing and future large-volume detectors in the event of a nearby SN explosion (Scholberg 2012).

The energies and densities in core-collapse environments are of typical nuclear-physics scales, i.e., $\mu$ and $\tau$ leptons do not exist except perhaps for some muons in the hottest regions. Hence, heavy-lepton neutrinos, often collectively called $\nu_x$, mostly interact by neutral-current processes, whereas $\nu_e$ and $\bar{\nu}_e$ interact predominantly by $\beta$ reactions. Neutrino transport, emission, and their detection therefore have a pronounced flavor dependence.

Developing a reliable theoretical understanding of neutrino flavor evolution in astrophysical environments with high neutrino density has been surprisingly difficult because of neutrino–neutrino refraction (Duan et al. 2006, 2010; Chakraborty et al. 2016a; Mirizzi et al. 2016). Flavor evolution is a dynamical phenomenon as neutrinos feed back upon themselves, i.e., one needs to study collective flavor degrees of freedom of the entire ensemble, not only of individual neutrinos or individual momentum modes. These collective modes can show instabilities such that flavor conversion can occur even in regions where the large matter effect suppresses normal flavor oscillations of individual modes. One generic type of process is pair conversion $\nu_e\bar{\nu}_e \leftrightarrow \nu_x\bar{\nu}_x$ on the forward-scattering (refractive) level. It does not violate the flavor-lepton number and proceeds anyway by normal non-forward scattering, but can be collectively enhanced even without neutrino flavor mixing.

For the neutrino energies relevant in collapsed stars, the weak processes are usually described by the four-fermion interaction proportional to the Fermi constant $G_F$. The space-time structure is of current–current form and implies that the interaction energy between relativistic neutrinos is proportional to $(1 - \cos \vartheta)$, where $\vartheta$ is their relative angle of propagation. In particular, parallel-moving neutrinos have no mutual refractive effect at all. Therefore, in this context, we cannot treat neutrinos as flowing in a purely radial direction.

To account for the crucial neutrino angle distribution, one often adopted a simple “bulb model” of neutrino emission (Duan et al. 2006). It consists of an emitting surface, the “neutrino sphere,” as a source of flavor-dependent neutrino spectra with a common angle distribution. The latter was often taken to be blackbody like, i.e., isotropic into the outer half-space, or in the form of “single-angle emission,” i.e., only one zenith angle of emission relative to the local radial direction with local axial symmetry. Some tentative studies used different local angle distributions between $\nu_x$ and a common one for $\nu_e$ and $\bar{\nu}_e$ in a schematic way (Mirizzi & Serpico 2012). On the other hand, nontrivial angle distributions, especially those between $\nu_e$ and $\bar{\nu}_e$, may be crucial for a full appreciation of flavor evolution (Sawyer 2005, 2009, 2016; Chakraborty et al. 2016b; Dasgupta et al. 2017; Izaguirre et al. 2017; Wu & Tamborra 2017).

At large distances from the SN core, the neutrino flux is essentially a narrow “beam” with a small opening angle. In addition, residual scattering provides a wide-angle “halo” which has low intensity, yet it is responsible for a non-negligible contribution to the $(1 - \cos \vartheta)$ term (Cherry et al. 2012; Sarikas et al. 2012b). By the same token, a “backward” flux toward the SN core exists and is particularly important near the decoupling region where neutrinos flow in all directions with different intensities. Therefore, a simple treatment of flavor evolution as a function of radius with only an inner boundary condition at the arbitrarily defined “neutrino
sphere” does not necessarily capture this physical situation (Izaguirre et al. 2017). Moreover, if the $\nu_e$ and $\bar{\nu}_e$ distributions are sufficiently different, “fast flavor instabilities” can ensue. The latter does not depend on neutrino mass differences and, specifically, are driven by the angle distribution of the electron lepton number (ELN) carried by neutrinos.

Despite the role played by neutrino angle distributions in the flavor evolution, it is difficult to glean enough insight about them from the published literature on numerical SN simulations. The basic picture of how neutrinos interact and decouple tells us that the $\nu_e$ distribution should be broader than that of $\bar{\nu}_e$, and that both $\nu_e$ and $\bar{\nu}_e$ should be broader than that of $\nu_x$. However, to mimic the local radiation field at some distance, $r$, by assuming that these flavors are emitted by different neutrino spheres, i.e., a separate “bulb model” for each species, is overly simplistic, especially close to the SN core where backward propagating neutrinos are important as well.

The main point of our paper is to fill this gap in the literature and to provide a first overview of neutrino energy and angle distributions. In this way we hope to provide crucial input for further studies of collective neutrino flavor evolution. In particular, we present angle-dependent distributions from hydrodynamical simulations of three SN progenitors with masses of 11.2, 15, and 25 $M_\odot$, and characterize their variation as a function of the distance from the core. These models were developed by Hüdepohl (2013) with the 1D version of the Garching group’s PROMETHEUS-VERTEX code.

In principle, it would be desirable to provide neutrino distributions from 3D models. One expects an even richer phenomenology and a greater diversity of cases regarding the local neutrino distributions. However, the numerical approximations used in state-of-the-art 3D simulations (such as ray-by-ray or two-moment closure schemes) are not qualified to provide reliable neutrino angle distributions. Solutions of the Boltzmann transport equation in 3D are needed to obtain full phase-space information for the neutrinos. Corresponding methods, however, are still being developed. First results for static and stationary conditions in 3D (Sumiyoshi et al. 2015) and time-dependent simulations in 2D (Ott et al. 2008; Brandt et al. 2011; Nagakura et al. 2017) with multi-angle transport are available, but still suffer from major shortcomings, e.g., the lack of energy-bin coupling (Ott et al. 2008; Brandt et al. 2011) and coarse resolution, especially in the momentum space. Therefore, studies similar to the one presented here, but for 3D models, will have to wait for the next generation of 3D SN simulations including Boltzmann neutrino transport, which will require exascale computing.

In Section 2 we introduce our models and in Section 3 we describe the angle-integrated features of the neutrino field and their variations as functions of radius. In Section 4 we study the neutrino distributions as functions of zenith-angle, radius, and post-bounce time. Conclusions and an outlook are provided in Section 5. Appendix A reports a glossary of the definitions usually adopted in neutrino radiative transport versus the terminology that is more common in flavor oscillation studies. The neutrino angle distributions for our models are provided as supplementary material and instructions on how to use these data are provided in Appendix B.

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2. Numerical Supernova Models

We will explore the characteristics of the neutrino radiation field during the accretion phase of three spherically symmetric SN simulations with progenitor masses of 11.2, 15, and 25 $M_\odot$. The nuclear equation of state is from Lattimer & Swesty (1991) with compressibility modulus $K = 220$ MeV (Hüdepohl 2013). The simulations were performed with the 1D version of the PROMETHEUS-VERTEX code. It couples an explicit third-order Riemann-solver-based Newtonian hydrodynamics code with an implicit three-flavor, multi-energy group two-moment closure scheme for neutrino transport. The neutrino transport applied here used three species, $\nu_e$, $\bar{\nu}_e$, and $\nu_x$ (with $\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$), and the variable Eddington-factor closure for the two-moment equations is obtained from a model Boltzmann equation, whose solution is based on a tangent-ray discretization and also provides information on the angle-dependent neutrino intensities (Rampp & Janka 2002), which are of central importance for the present work.

General relativistic (GR) corrections are accounted for using an effective gravitational potential (case A of Marek et al. 2006) and by including GR redshift and time dilation in the transport. The relatively small effects of GR ray bending in the NS environment, however, are ignored in the neutrino treatment, i.e., the tangent-ray method assumes that neutrinos will propagate along straight paths instead of curved geodesics. Tests showed good overall agreement until several 100 ms after core bounce (Liebendörfer et al. 2005; Marek et al. 2006) with fully relativistic simulations of the Basel group’s AGILE-BOLTZTRAN code. A more recent comparison with a GR program (Müller et al. 2010) that combines the CoCoNuT hydro solver (Dimmelmeier et al. 2002) with the VERTEX neutrino transport, revealed almost perfect agreement except for a few quantities with deviations of 7%–10% at most until several seconds. Our models presented here include the full set of neutrino reactions described in Appendix A of Buras et al. (2006) with the original references given there. (The simulation setup is analog to the “full” opacity case discussed in Hüdepohl et al. 2010.) In particular, we account for nucleon recoils and thermal motions, nucleon–nucleon (NN) correlations, weak magnetism, a reduced effective nucleon mass and quenching of the axial-vector coupling at high densities, NN bremsstrahlung, $\nu\nu$ scattering, and neutrino-antineutrino-pair conversions between different flavors (Burasc et al. 2003). In addition, we include electron capture and inelastic neutrino scattering on nuclei (Langanke et al. 2003, 2008).

The progenitor models employed for our stellar core-collapse simulations are taken from Woosley & Weaver (1995) in the case of the 15 $M_\odot$ progenitor model s15s7b2 and from Woosley et al. (2002) for all of the other investigated cases. Figure 1 shows the evolution of the shock radius as a function of the post-bounce time. All of the simulated models exhibit similar features; see also the angle-integrated neutrino emission properties shown in Figure 7 of Janka et al. (2012). The 11.2 and the 25 $M_\odot$ progenitors gauge the maximum variation of the neutrino light curves of the broader simulated mass range of SN progenitors shown in Figure 7 of Janka et al. (2012), while the 15 $M_\odot$ progenitor is an intermediate case.

For each of the three progenitors discussed here, we inspect the neutrino angle and energy distributions at three representative post-bounce times. These are chosen as to represent the neutrino-field properties soon after the shock-breakout burst (early accretion phase), just before the drop in the luminosity.
On the other hand, if we consider the neutrino-matter decoupling region, it may be more appropriate to use Lagrangian coordinates comoving with the local matter flow. The output quantities of the Garching simulations are given in this comoving frame and we will use it here as well.

### 3. Radial Variation of Neutrino Radiation Field

As a first overview, we show the radial variation of several global properties of the neutrino radiation field in our benchmark model, the 280 ms snapshot of the 15 $M_\odot$ simulation. The flux density of energy (MeV cm$^{-2}$ s$^{-1}$) for a given species $\nu_\alpha$, at a given radius $r_i$ is

$$F(r_i) = 2\pi \sum_{k=1}^{N_k} \sum_{l=1}^{N_l} \Delta \mu_{lj} I_{lj} \mu_{lj},$$

(2)

where $\Delta \mu_{lj}$ is the width of the $\mu = \cos \theta$ bin centered on $\mu_{lj}$ and the factor $2\pi$ arises from the azimuth integration $\int d\varphi$. The corresponding number flux density $\tilde{F}(r_i)$ is the same expression with $\tilde{I}_{lj} = I_{lj}/E_k$ under the sum. The overall neutrino luminosity of species $\nu_\alpha$ (MeV s$^{-1}$) at radial position $r_i$ is $L(r_i) = 4\pi r_i^2 F(r_i)$ and the analogous expression for the number luminosity or number flux (s$^{-1}$) is $\tilde{L}(r_i) = 4\pi r_i^2 \tilde{F}(r_i)$. The local neutrino number density (cm$^{-3}$) is

$$n(r_i) = 2\pi e^{-1} \sum_{k=1}^{N_k} \sum_{l=1}^{N_l} \Delta \mu_{lj} \tilde{I}_{lj}.$$

(3)

Figure 2 shows the radial variation of these and other characteristics, where quantities for $\nu_e$ are shown in blue, for $\bar{\nu}_e$ in red, and for $\nu_\tau$ in green. The upper left panel shows the flavor-dependent neutrino luminosities. Within the proto-neutron star, they show fast variations and are also negative in some regions, indicating inward-flowing energy that is in agreement with the temperature profile shown in Figure 3 that has a maximum at around 10 km. The radial variation depends strongly on flavor in agreement with the usual picture of flavor-dependent neutrino production and transport. The sudden increase at the shock-wave radius of around 75 km is an artifact of expressing the luminosities in the comoving fluid frame. The large infall velocity of around $-0.2 c$ outside of the shock wave (upper right panel) introduces a significant blueshift of both the neutrino energies and their rate-of-flow. From the perspective of a distant observer in the laboratory frame, the luminosities remain continuous through the shock-wave region and are essentially constant beyond the decoupling region except for smaller changes (of the order of a few percent) due to energy deposition in the gain layer below the shock wave at around 75 km.

The $\nu_\tau$ flavor decouples at a smaller radius than $\nu_e$ and $\bar{\nu}_e$ and it reaches its shoulder in the luminosity profile at around 20 km. Also in this case, a further luminosity decline of a few percent (in the distant-observer frame) occurs from the energy transfer by inelastic neutrino-nucleon scattering of neutrinos on the cooler stellar plasma between the energy sphere and the transport sphere (Raffelt 2001; Keil et al. 2003).

The neutrino number flux (or number luminosity) $\tilde{L}$ is shown in the left middle panel of Figure 2. Qualitatively, its radial variation follows the luminosities. However, while the $\nu_e$ and $\bar{\nu}_e$ luminosities are nearly equal, the number fluxes differ by the $\nu_\tau$...
The deleptonization flux, compensated by lower average $\nu_e$ energies.

The local electron and neutrino number densities and the electron–neutrino lepton ($\nu_e - \bar{\nu}_e$) number density are shown in the lower left panel. From this plot one concludes that within the shock-wave radius all matter effects are certainly dominated by electrons, which, however, does not necessarily preclude neutrino self-induced flavor conversion. The electron fraction (bottom right panel) is locally defined as the ratio of the net electron number density to the proton plus neutron number density. Its profile reflects the deleptonization evolution of the infalling matter of the stellar core with the well-known trough around the neutrinospheric region and the continuous rise from the corresponding minimal value to the value of 0.5 (reached at 55 km) of the surrounding layers of the progenitor star. A quantity that nicely illustrates neutrino decoupling is the effective outward velocity of the neutrino fluid,

$$v_{\text{eff}}(r_i) = \frac{\dot{F}(r_i)}{n(r_i)},$$

which is the number flux density divided by the number density. The ratio of $v_{\text{eff}}$ to the speed of light, $f = v_{\text{eff}}/c$ (displayed in the upper panel of Figure 3), is often considered as the “flux factor” in neutrino-transport discussions. Figure 3 (upper panel) shows this quantity as a function of $r$ for the three species and also the fluid velocity of matter, which is the same as in the upper right panel of Figure 2. Deep in the core, where the neutrino distribution is basically isotropic, $v_{\text{eff}}$ is zero, whereas at large radii, where all neutrinos stream essentially in the radial direction with the speed of light, it approaches $c$. In agreement with Figure 2, the decoupling happens in a flavor-dependent way and neutrinos start to decouple from the matter fluid at around 22 km. However, while $\nu_x$ decouple almost instantaneously, $\nu_e$ and $\bar{\nu}_e$ decouple over a much broader radial interval, in agreement with the upper left panel of Figure 2.

Another quantity that illustrates neutrino decoupling is the average neutrino energy at a given radius, where we mean the average $\langle E \rangle_0$ defined in Equation (17), which is the local neutrino energy density divided by the local number density.
without weighting with the angular projection factors $\mu$. For $\nu_x$ with vanishing chemical potential we define an effective temperature as

$$T (r_i) = \frac{180 \xi_3}{7 \pi^4} \langle E \rangle_0 \simeq \frac{\sum_{k=1}^{N_k} \sum_{j=1}^{N_j} \Delta \mu_{yj} I_{ijk}}{3.15 \sum_{k=1}^{N_k} \sum_{j=1}^{N_j} \Delta \mu_{yj} I_{jk}}.$$  \hfill (5)

This quantity and the matter temperature are plotted in the bottom panel of Figure 3. At $r \simeq 22$ km, the $\nu_x$ start to decouple from the stellar material and their temperature differs relative to that of the surrounding matter. The effective $T$ of the final $\nu_x$ flux is considerably lower than it is at the “energy sphere” where $\nu_x$ begins to decouple; see Keil et al. (2003) for more details.

4. Angular Variation of Neutrino Radiation Field

In this section, we finally explore the neutrino angle distributions for all flavors, and in particular, how they vary with radius and evolve in time. We also observe that the local neutrino spectrum in a given direction of propagation is well described by a Gamma distribution.

4.1. Radial Variation and Temporal Evolution of the Neutrino Angle Distributions

In order to grasp the general trend of how the neutrino angular distributions vary in space and time, we introduce the local number intensity at a given radius (see also Equation (12)),

$$\tilde{I}_\mu = 2\pi \sum_{k=1}^{N_k} I_{ijk},$$ \hfill (6)

which is the local number density of streaming neutrinos, integrated over energy and azimuth angle, but differential with regard to $\mu = \cos \theta$, and has units of cm$^{-2}$ s$^{-1}$. More specifically, in the following, we will show the corresponding number density (not the number intensity), i.e., $\tilde{I}_\mu / c$, which has units cm$^{-3}$.

Figure 4 shows $\tilde{I}_\mu / c$ for $\bar{\nu}_e$ as a function of $\theta$ for the indicated radial distances. The upper panel shows the distributions at small radii. As expected, inside the decoupling region the distribution is almost isotropic (see the magenta curve at $r = 20$ km). At larger radii, the angle distributions become more and more forward peaked, corresponding to $\theta = 0$. However, even at $r = 37$ km (black curve), a non-negligible backward contribution remains clearly visible in this linear plot, confirming that neutrinos stream fully in the forward direction only for $r > 40$ km, as speculated from Figure 2. For
r = 54 km (blue curve), the distribution already becomes similar to a beam with a broad opening angle that becomes smaller with distance from the source, corresponding to the angular size of the SN core as seen from the given distance.

A logarithmic plot (bottom panel of Figure 4), however, reveals that this picture is not complete. As neutrinos stream outward, they suffer residual collisions with the matter layers. As a consequence, the “neutrino beam” emerging from the source is accompanied by a “halo” which extends to all directions. The neutrino halo populating the large θ phase space is visible in the bottom panel of Figure 4. One can see that the number intensity of the neutrino halo is small with respect to the neutrino beam propagating forward (θ ≈ 0), but its broad angular distribution allows it to dominate the neutrino–neutrino interaction energy as discussed by Sarikas et al. (2012b).

Figure 5 is a polar map of the normalized $\bar{\nu}_e$ monochromatic intensity $I_{E,\Omega}$ introduced in Equation (1) for $E = 10$ MeV and at different radii. For larger radii, the distributions become more forward peaked, and outside of the neutrino-decoupling region the intensity is essentially conserved along radiation paths that point outward from the radiating neutrinosphere. A similar behavior was also found by Thompson et al. (2003), see their Figure 4 and Figures 10–12.

Figure 6 shows the flavor dependent angle distributions for our three progenitors. The width of the distributions decreases in the sequence $\nu_x$, $\bar{\nu}_x$, and $\nu_\tau$ (Ott et al. 2008; Sarikas et al. 2012a) as expected from the decreasing interaction rates in the medium. Qualitatively, this behavior is the same for all progenitors. At some radii, the angular distributions of $\nu_x$ and $\nu_\tau$ or the ones of $\bar{\nu}_e$ and $\bar{\nu}_x$ may cross as assumed in Miririzi & Serpico (2012).

### 4.2. Neutrino Electron Lepton Number

A new issue in the context of self-induced neutrino flavor conversion is the question of the angular distribution of the electron lepton number (ELN) carried by neutrinos and in particular, the question of possible crossings of the $\bar{\nu}_e$ with the $\nu_x$ angle distributions (Sawyer 2016; Izaguirre et al. 2017). Such situations could trigger fast flavor conversion, i.e., self-induced flavor conversion where the instability scale is not set by the neutrino mass differences, but rather by the neutrino–neutrino interaction energy, which is a much larger scale for typical SN conditions.

The ELN carried by neutrinos, defined as $(\bar{I}_{\mu,\nu_e} - \bar{I}_{\mu,\bar{\nu}_e})/c$, is shown in magenta in Figure 6 for the selected snapshots of our three progenitors. It is comparable in intensity for all three progenitors, but the dip in the forward direction is less pronounced for the 25 $M_\odot$ SN model for the selected post-bounce time and radius. Figure 7 shows the ELN as a function of θ for different radii for our benchmark model. It develops a dip in the forward direction as the radius increases. We also provide an animation of the ELN variation with radius for the 15 $M_\odot$ progenitor here.\(^7\) In all of our models we have found the ELN to always be positive. In particular, the universal dip in the forward direction never turns negative, so there is no crossing between the $\nu_e$ and $\bar{\nu}_e$ angular distributions. On the contrary, crossings in the ELN distribution naturally appear in compact merger remnants because of the different emission geometry with respect to SNe and the $\bar{\nu}_e$ flux being larger than the $\nu_e$ one (Wu & Tamborra 2017).
This behavior could change in 3D models, for example, in the presence of LESA (Lepton-Emission Self-sustained Asymmetry; Tamborra et al. 2014a). LESA manifests itself in a pronounced large-scale dipolar pattern in the ELN emission. This is a consequence of large-scale convection modes inside the newly formed neutron star with a strong dipolar flow component, which grows during the contraction phase (Janka et al. 2016) and has significant feedback also on the accretion flow around the neutron star (Tamborra et al. 2014a). LESA naturally implies a change of sign in \( \langle I_{\mu_\nu} - I_{\mu_\nu,0} \rangle \). It is therefore conceivable that, especially in the regions where the ELN changes its sign, crossings in the ELN angular distributions may occur.

Existing 3D simulations employ, however, the ray-by-ray transport approximation (Takiwaki et al. 2014; Lentz et al. 2015; Melson et al. 2015a, 2015b) or multi-dimensional two-moment treatments with algebraic closure relations such as the “M1 methods” Roberts et al. (2016). None of these can provide reliable angle distributions. M1 solvers use the neutrino energy and momentum equations to evolve the angle-integrated moments of the neutrino intensity (i.e., the energy-dependent energy and flux densities). The ray-by-ray approximation, even if based on a two-moment solver with Boltzmann closure (Rampp & Janka 2002; Buras et al. 2006), makes use of the assumption that the neutrino intensity is locally axially symmetric around the radial direction. It thus ignores nonradial flux components and off-diagonal elements in the pressure tensor. For certain questions (e.g., for rough estimates of observable emission asymmetries) one can work around the missing phase-space information using low-order multipole approximations of the intensity based on energy-density and flux-density information (Müller et al. 2012; Tamborra et al. 2014b). Detailed, accurate phase-space information, however, requires the solution of the time-dependent Boltzmann transport equation for the neutrino intensity in dependence on all three spatial and momentum-space variables.

Existing supercomputers can hardly tackle this high-dimensional problem in two spatial dimensions despite severe restrictions with respect to resolution and sophistication of the employed weak-interaction physics (Ott et al. 2008; Brandt et al. 2011; Nagakura et al. 2017). Time-dependent numerical solutions of the Boltzmann transport equation in 3D will require exascale computing capability, but will still be very challenging when high resolution in momentum space is required. The results based on a tangent-ray integration of the energy and angle-dependent Boltzmann equation in 1D presented in this work are therefore likely to set the benchmark for neutrino-oscillation studies for the coming years.

### 4.3. Spectral Fit

Flavor oscillation effects generally depend both on neutrino energy and angle. Therefore, in our data files we also provide information about the energy distribution. It has been observed previously that SN neutrino energy spectra often can be well approximated by a Gamma distribution (Keil et al. 2003; Tamborra et al. 2012), sometimes also referred to as “\( \alpha \) fit.” In our context it means that we will express the spectral intensities in the form

\[
I_{\mu,\theta} \propto \left( \frac{E}{E_0} \right)^{\alpha_\theta} e^{-\alpha_\theta E/E_0},
\]

where the energy \( E_0 = \langle E \rangle_\theta \) is the average energy of the neutrinos streaming in direction \( \theta \), whereas the shape parameter \( \alpha_\theta \) measures the amount of spectral pinching. It is related to the first and second energy moments through

\[
\langle E^2 \rangle_\theta = \frac{2 + \alpha_\theta}{1 + \alpha_\theta}.
\]

Besides the energy-integrated intensity \( I_\mu \), we provide\(^8\) the first two energy moments (Equations (19), (20)) for any \( \mu \) and in this way characterize the spectra with reasonable accuracy.

As a representative example, we show in Figure 8 the \( \bar{\nu}_e \) energy spectrum at \( r = 30 \) km for our benchmark SN model. We consider the outward direction (\( \mu = 1 \) in blue) and a nearly backward direction (\( \mu = -0.9 \) in red). For the former we show both the numerical distribution as a histogram and the fit by a Gamma distribution (top panel). We then compare the smooth fits for both directions (bottom panel of Figure 8), illustrating that the energy distributions depend significantly on direction simply because of the energy-dependent neutrino scattering cross sections. A similar comparison was provided by Sarikas et al. (2012b) at large distances to compare the spectrum of the neutrinos coming directly from the SN core versus the halo flux which arises from residual scattering. In our case, we find that the \( \bar{\nu}_e \) streaming in the forward direction have \( E_0 = 17.1 \) MeV and \( \alpha_\theta = 2.7 \) and therefore are cooler and less pinched than the quasibackward direction, where \( E_0 = 22.1 \) MeV and \( \alpha_\theta = 4 \). More energetic neutrinos are typically more isotropically distributed in energy than less energetic ones.

### 5. Conclusions

Recent developments in the context of neutrino–neutrino interactions in dense media have highlighted the role of the neutrino angular distributions. For the first time, we have carried out a careful analysis of the neutrino angular distributions.

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\(^8\) http://wwwmpa.mpa-garching.mpg.de/ccsarchive/data/Tamborra2017/
The Astrophysical Journal, 839:132 (10pp), 2017 April 20

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National Research Foundation (DNRF91), the Deutsche Forschungsgemeinschaft under the Excellence Cluster Universe (Grant No. EXC 153), and the European Union under the Innovative Training Network “Elusives” (Grant No. H2020-MSCA-ITN-2015/674896) and under the ERC Advanced Grant No. 341157-COCO2CASA. We thank the Institute for Nuclear Theory at the University of Washington for hospitality and the DOE for partial support during early stages of this work.

Appendix A
Neutrino Radiation Field

In the context of neutrino flavor oscillation studies, the local neutrino radiation field for a given species $\nu_e$, $\bar{\nu}_e$, or $\nu_\mu$ is traditionally described by occupation numbers (classical phase-space densities) $f(p)$ for every momentum mode $p$. These can be extended to $3 \times 3$ matrices $\rho(p)$ to capture flavor coherence on the off-diagonal elements. On the other hand, in the traditional treatment of neutrino radiative transfer, one uses the spectral intensity $I_{E,\Omega}$ as the fundamental quantity, i.e., the energy carried by the given neutrino species per unit area and unit time, differential with respect to neutrino energy $E$ and solid angle $\Omega$ (see Equation (1)). Moreover, in the particle-physics tradition one uses natural units with $\hbar = c = 1$, although we will keep $\hbar$ and $c$ explicitly here. In this appendix, we provide a brief dictionary between these languages because simple issues of definition, normalization, or units can be a source of confusion.

Taking neutrinos to be massless, their energy is $E = c|p|$. Therefore, the differential local number density is

$$dn = \frac{f_p}{\hbar^3} \frac{d^3p}{(2\pi)^3} = \frac{f_{E,\Omega}}{(\hbar c)^3} \frac{E^2 dE d\Omega}{(2\pi)^3},$$

where we show the dependence on $p$, $E$, or $\Omega$ as indices. In the second expression the neutrino momentum is represented by its energy and direction of motion. Noting that massless neutrinos stream with the speed of light $c$ in a given direction $\Omega$, the spectral intensity is

$$I_{E,\Omega} = cE \frac{dn}{dE d\Omega} = \frac{f_{E,\Omega}}{(\hbar c)^3} \frac{E^3}{(2\pi)^3},$$

in units of $\text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}$. Integrating over all energies, the intensity is $I_\Omega = \int_0^\infty I_{E,\Omega} dE$ in units of $\text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}$.

In the neutrino flavor evolutions context, the relevant quantities are number densities and number fluxes, not energy densities or fluxes. To develop a systematic notation, we use a tilde on a symbol for the corresponding number quantity. In particular, we define the spectral number intensity

$$\tilde{I}_{E,\Omega} = \frac{I_{E,\Omega}}{c} = c \frac{dn}{dE d\Omega} = \frac{f_{E,\Omega}}{(\hbar c)^3} \frac{E^2}{(2\pi)^3}$$

in units of $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1} \text{ster}^{-1}$.

For producing a weak potential on other neutrinos, a more intuitive quantity is the local number density, differential with regard to energy and solid angle. Therefore, without introducing a special symbol, our real quantity of interest is $I_{E,\Omega}/c$ in units of $\text{cm}^{-3} \text{MeV}^{-1} \text{ster}^{-1}$. Of course, in natural units where $c = 1$, both quantities are the same. Still, there is a conceptual difference between a flux-like quantity ($I_{E,\Omega}$) and a density, even though for relativistic particles they are equivalent except for units.

Figure 8. Representative example for the $\nu_e$ spectrum (energy distribution) at $r = 30 \text{ km}$ for the 280 ms snapshot of the 15 $M_\odot$ model, where blue is for $\cos(\theta) = 1$ and red for $\cos(\theta) = 0.9$. Top panel: a normalized histogram of the numerical spectrum and fit by a Gamma distribution (smooth line). Bottom panel: same fit (blue line) as well as a similar fit for a nearly backward direction, both of them normalized to the same value at maximum. The backward flux has larger energies caused by the energy dependence of the neutrino cross sections.

distributions from one-dimensional hydrodynamical SN simulations with sophisticated neutrino transport.

To facilitate dedicated studies of flavor conversion, we provide data of the energy and angle distributions as supplementary material. We specifically explore the angle distributions of neutrinos in three SN progenitors with masses of 11.2, 15, and 25 $M_\odot$, and scan three different post-bounce times during the accretion phase for each of them.

The neutrino radiation field ranges from being completely isotropic inside the proto-neutron star, where neutrinos frequently scatter, to being forward peaked at large distances from the proto-neutron star (50 km for the analyzed models) where neutrinos stream almost freely. However, while propagating further, neutrinos scatter on nuclei in the stellar envelope, generating a broad “halo” and backward distribution, becoming relevant at larger radii. The neutrino radiation flowing in the backward direction is in general characterized by an energy spectrum hotter and more pinched than the forward streaming component.

The question if fast flavor conversion occurs in the neutrino decoupling region will require a better theoretical understanding of the behavior of the neutrino radiation field in the SN environment. Our present study provides the input information that is required for such studies.

We acknowledge support from the Knud Højgaard Foundation, the Villum Foundation (Project No. 13164), the Danish 9 http://www.wmpa.mpa-garching.mpg.de/ccsnarchive/data/Tamborra2017/
Henceforth we assume that the radiation field is axially symmetric around the local radial direction. Moreover, we assume that the overall SN model is spherically symmetric. We use local polar coordinates $\Omega = (\theta, \varphi)$ with the differential $d\Omega = d\mu \, d\varphi$ with $\mu = \cos \theta$. The azimuthal integration $\int d\varphi \to 2\pi$ is trivial because of the assumed symmetry.

In particular, we are interested in the number intensity, integrated over energy and azimuth angle, which is

$$\tilde{I}_\mu = \int dE \, \frac{dE \, I_{E, \mu}}{c} = \int dE \, \frac{f_{E, \mu}}{(\hbar c)^3} \, \frac{E^2}{(2\pi)^2}$$  \hspace{1cm} (12)$$

in units of $\text{cm}^{-3}$, a quantity which remains differential with regard to $\mu$. Integrating it over $d\mu$ gives us the local number density $n$ of the given neutrino species.

We define the local specific flux density (of energy) in the radial direction in the form

$$F_E = \int d\Omega \, \mu \, I_{E, \Omega} = \int d\Omega \, \mu \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}$$  \hspace{1cm} (13)$$
in units of $\text{cm}^{-2}\text{s}^{-1}$. The corresponding energy density is

$$F = \int dE \, d\Omega \, \mu \, I_{E, \Omega} = \int dE \, d\Omega \, \mu \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}$$  \hspace{1cm} (14)$$
in units of $\text{MeV cm}^{-2}\text{s}^{-1}$. Finally, the luminosity of the entire SN at a given radius $r$ is

$$L = 4\pi r^2 F = 4\pi r^2 \int dE \, d\Omega \, \mu \, I_{E, \Omega}$$  \hspace{1cm} (15)$$
in units of $\text{MeV s}^{-1}$. The corresponding number-quantities (symbols with tildes) are the same expressions with $E^{-1}$ under the integrals. In particular, $\tilde{L}$, the “number luminosity,” is the total number of neutrinos per second of the given species streaming outward through a surface of radius $r$.

At large distances from the SN, we usually define the average neutrino energy in the form

$$\langle E \rangle = \frac{F}{\tilde{F}} = \frac{\int dE \, d\Omega \, \mu \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}}{\int dE \, d\Omega \, \mu \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}}$$  \hspace{1cm} (16)$$

which is the average energy of the neutrino flux at distance $r$. In deeper regions around and below the weak decoupling region, we may also consider the local average energy, without weighting it with the angular projection factor $\mu$, so

$$\langle E \rangle_0 = \frac{\int dE \, d\Omega \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}}{\int dE \, d\Omega \, \frac{f_{E, \Omega}}{\hbar^2 c^2} \, \frac{E^3}{(2\pi)^3}}.$$  \hspace{1cm} (17)$$

It is this quantity that we have shown in Figure 3 as temperature ($T_0 = \langle E \rangle_0/3.15$).

Appendix B

Data Structure

The quantity provided by SN simulations is the monochromatic neutrino intensity for each flavor $\nu_{\alpha}$, integrated over the energy bin centered on $E_0$, for each radial point $r_i$ and zenith-angle $\mu_j$. Those data can be downloaded from the Garching SN Archive\footnote{http://wwwmpa.mpa-garching.mpg.de/ccsnarchive/index.html} upon request.

We here provide data useful for neutrino oscillation studies for three progenitors and three selected post-bounce times as supplementary material\footnote{http://wwwmpa.mpa-garching.mpg.de/ccsnarchive/data/Tamborra2017/} and give brief instructions on how to read the data files. All data refer to quantities in a coordinate frame that moves with the matter fluid.

The files named as “radial-neutrino-properties-time-SNmass.dat” list the angle-integrated neutrino emission properties as a function of the radius ($r_i$). The first column lists the radius. The luminosities $L(r_i)$ of $\nu_\alpha$, $\bar{\nu}_\alpha$, and $\nu_\tau$ are stored in the second, third, and fourth columns, respectively. The first energy moments, defined as in Equation (16), for $\nu_\alpha$, $\bar{\nu}_\alpha$, and $\nu_\tau$ are stored in the fifth, sixth, and seventh columns. The second energy moments for these flavors are stored in the eighth, ninth, and tenth columns. Using these data and Equation (7) one can reconstruct the variation of the neutrino energy spectra in the SN comoving frame as a function of the distance from the proto-neutron star radius.

The files named as “angular-quantities-nualpha-time-SNmass.dat” list the angle- and radius-dependent neutrino emission properties for each flavor $\nu_{\alpha}$, and for each selected post-bounce time and progenitor mass. The radius $r_j$ is listed in the first column and $\mu_j$ is reported in the second column. Note that the binning in $\mu_j$ is not uniform as a function of $r_j$ because of the tangent-ray discretization of the Boltzmann transport equation (see Rampp & Janka 2002 for more details). Moreover, degenerate angular zones of measure zero may be present around $\theta = \pi/2$ due to the peculiar angular-grid formulation.

The third column of the file “angular-quantities-nualpha-time-SNmass.dat” lists the differential (with respect to $\mu$, i.e., per unit interval of the cosine of the zenith angle) neutrino number luminosity (0th moment, in units of $s^{-1}$) defined as

$$\tilde{L}_\mu(r_j, \mu_j) = 4\pi r_j^2 \sum_{k=1}^{N_E} 2\pi I_{jk}.$$  \hspace{1cm} (18)$$

We recover the neutrino number luminosity $\tilde{L}(r_j)$ by integrating the equation above over $\mu_j$. The fourth column represents the first energy moment (in units of $\text{MeV}^{-1}$) as

$$L_\mu(r_j, \mu_j) = 4\pi r_j^2 \sum_{k=1}^{N_E} 2\pi I_{jk} E_k.$$  \hspace{1cm} (19)$$

while the second energy moment (in units of $\text{MeV}^2 s^{-1}$) is reported in fifth column and it is defined as

$$S_\mu(r_j, \mu_j) = 4\pi r_j^2 \sum_{k=1}^{N_E} 2\pi I_{jk} E_k.$$  \hspace{1cm} (20)$$

The differential neutrino density is stored in the sixth column (in units of $\text{cm}^{-3}$) and it is defined as

$$\tilde{I}_\mu(r_j, \mu_j) = 2\pi e^{-1} \sum_{k=1}^{N_E} I_{jk}.$$  \hspace{1cm} (21)$$

For all SN models, we used 21 nearly geometrically spaced energy bins up to 380 MeV and a number tangent rays and radial grid points that vary as functions of post-bounce time: $N_r = 510$, $N_t = 235$ for $t = 61$ ms, $N_r = 836$, $N_t = 398$ for $t = 256$ ms, and $N_r = 838$, $N_t = 399$ for $t = 550$ ms for the 11.2 $M_\odot$ model; $N_r = 672$, $N_t = 316$ for $t = 150$ ms, $N_r = 824$, $N_t = 392$ for $t = 280$ ms, and $N_r = 914$.
$N_e = 437$ for $t = 500$ ms for the $15 M_\odot$ SN progenitor; $N_{\mu} = 510, N_e = 235$ for $t = 63$ ms, $N_{\mu} = 780, N_e = 370$ for $t = 252$ ms, and $N_{\mu} = 784, N_e = 372$ for $t = 352$ ms for the $25 M_\odot$ SN progenitor. For all studied progenitors and post-bounce times, we also provide data on the matter density profile, the electron abundance, the velocity profile, central and boundary values of the energy bins, and the radial grid.

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