Dynamic Safety Assessment of Aircraft Mechanism using Extremum Framework with Fault Tree Analysis

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Abstract. In this paper, a hybrid two-level inventory allocation method based on multi-echelon technique for recoverable item control (METRIC) was applied to the spare parts allocation of amphibious aircraft. Firstly, the principle of the spare parts inventory configuration model of amphibious aircraft based on hybrid two-level repair was expounded. The mathematical model was established with maintenance cost as objective function and availability and volume of spare parts as constraints. Secondly, marginal analysis was used to help allocate spare parts, the process of solving the mathematical model was discussed, and the calculation formulas of the quantities involved were briefly stated. Finally, with the line replaceable units of landing gear as the research object, the spare parts inventory allocation of a amphibious aircraft was solved according to the method proposed in this paper. This paper provides an approach to allocate spare parts of amphibious aircraft.

1. Introduction
Aircraft is an integrated body composed of many mechanisms [1], for instance, aircraft fuselage, wings. Aircraft components are affected by dynamic load, which leads to the whole mechanism dysfunction and affects the safety when the aircraft is in operation [2]. In addition, different from the static analysis of a single structure, the mechanism is a dynamic movement process under the condition of driving force or driving speed. Because of the complex relationship between the components in the mechanism, each component has different deformation and stress under operation. Therefore, to ensure the normal operation function of the whole machine mechanism, it is necessary to consider the relationship between the components and analyze its operation reliability and safety.

With the development of reliability and safety analysis technology, many scholars have already carried out corresponding research on these mechanisms. Wang et al. proposed a reliability analysis method of hybrid size reduction mechanism in order to better deal with relevant joint clearance variables [3]. Lin et al. adopted the reliability analysis method combining simulation technology and approximate model in order to solve the nonlinear relationship between input variables and response for joint clearance mechanism [4]. Shi et al proposed a mechanism reliability analysis method based on polynomial chaos expansion, in order to approach the limit function. The calculation efficiency of reliability analysis could be improved while reaching the same accuracy level [5]. Yang et al. put the randomness of each error component in the mechanism motion into consideration before...
deducing the inverse kinematics and established the calculation model of delta mechanism position error, so as to calculate the mechanism motion reliability [6]. According to the characteristics of reliability analysis of small sample structure mechanism in practical engineering, Ma et al. put forward the reliability method of structure mechanism combining bootstrap, support vector regression method and second-order cumulative measurement method [7]. In order to improve the accuracy and efficiency of reliability analysis of flexible mechanisms, Zhang et al. proposed a two-step extreme response surface method for reliability analysis [8]. Wang et al. put forward a hybrid reliability model based on hybrid Bayesian network, which included random, fuzzy and non-probabilistic uncertainties, aiming at targeting the mixed uncertainty in complex mechanism system[9]. In order to reduce the computational complexity, Lu et al. proposed an improved Kriging algorithm with extreme response surface method [10]. Although this research studies the safety and reliability of the structure mechanism from different perspectives, the calculation efficiency and accuracy of the dynamic mechanism safety analysis cannot meet its needs, so it is necessary to find feasible methods to carry out the dynamic safety evaluation of the mechanism, especially for the aircraft mechanism.

To address the above issues, extremum framework with fault trees analysis(FTAEF) is proposed to derive dynamic safety analysis of aircraft mechanism. Firstly, this paper introduces the theory of the method and the process of safety analysis, considering the randomness of working conditions and material parameters, combining the relationship network between the components of mechanism with the dynamic processing of each component. Then, taking the typical four-bar linkage as an example, the safety analysis of deformation failure is carried out by using the proposed FTAEF. Finally, effectiveness of the developed methods are verified by comparing these methods.

2. Basic theory

2.1. Fault tree analysis, FTA

Being one of the analysis tools, fault tree analysis can be used to determine and correctly explain all reasonably possible events. These undesirable events are caused by one or combined reasons, usually referring to the critical safety situation. In addition, it is also used to evaluate the impact of design or environmental changes on system security and determine the probability of top events. The final result of fault tree can be used to produce corresponding corrective measures and improve the safety of mechanism or system [11-13].

In this paper, the target mechanism is divided by fault tree, starting from the top event, and then determines the possible bottom event and the relationship among the bottom events.

![Figure 1. Sample of fault tree analysis.](image-url)
2.2. Extreme response surface method, ERSM

In the extremum response surface method, the random input variables of the structural system are firstly selected. Assuming that the sample of the input variables of the group is \( X^{(i)} \), correspondingly, the output response in the determined time domain is \( y^{(i)}(t, X^{(i)}) \). According to this set of sample points, the output response function representing the relationship between input variables and output variables can be fitted. The function form is as follows [14]:

\[
y = a_0 + B X + X^T C X
\]

(1)

Where \( a_0 \) is constant term, \( B \) is primary undetermined coefficient vector and \( C \) is secondary undetermined coefficient matrix.

\[
B = \begin{bmatrix} b_1 & b_2 & \ldots & b_j \end{bmatrix}
\]

(2)

\[
C = \begin{bmatrix} c_1 & 0 & 0 & \ldots & 0 \\ 0 & c_2 & 0 & \ldots & 0 \\ 0 & 0 & c_3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & c_j \end{bmatrix}
\]

(3)

\[
X^{(i)} = \begin{bmatrix} x_1^{(i)} & x_2^{(i)} & \ldots & x_j^{(i)} \end{bmatrix}
\]

(4)

In these functions, \( i = 1, 2, 3, \ldots, N \); \( N \) is the number of sample points, is the number of input random variables.

Due to the influence of environment, service conditions and other factors, the material parameters are not a certain amount, and the deformation of the structure will change accordingly. At this time, the deformation will interfere with the strain strength, which will lead to the failure of the structure. The relationship between them is also a random variable, which can be expressed by equation (5).

\[
Z = R - D
\]

(5)

When \( Z > 0 \), the structure is in a safe state; when \( Z < 0 \), the structure is in a failure state; when \( Z = 0 \), the structure is in a critical state. Assuming that the deformation and strength are two independent random variables and obey a certain distribution, the expression of structural deformation safety is:

\[
P = P(Z > 0) = P(R - D > 0)
\]

(6)

The dynamic security probability of the corresponding bottom event can be obtained by the extreme response surface method. Combined with the fault tree analysis method mentioned above, the dynamic safety analysis of the mechanism under the extreme condition can be obtained.

2.3. Extremum framework with fault tree analysis, FTAEF

FTAEF method starts from two aspects of multi-component failure logic relationship and dynamic extreme value analysis. FTA, being the medium of multi-component failure analysis, provides the complex logic relationship between the components in the mechanism. ERSM is used for dynamic processing of each component. Taking the mechanism failure as the final analysis objective, the correlation analysis of the above results is carried out to output the dynamic safety evaluation results of the mechanism.
2.4. Analysis process of extremum framework with fault tree analysis
First, the combination relationship among the components is combed out, which is used to explain the failure relationship of the components when the whole mechanism fails, in accordance with the structure and motion relationship of the mechanism studied; Second, dynamic analysis on each component is carried out to find out the maximum failure time and corresponding maximum failure state by input random variables such as material parameters and working conditions of the mechanism. The dynamic extreme value response surface function is obtained by combining the input random variables and output response; Third, the corresponding safety probability can be obtained by large-scale sampling of the dynamic extreme value response surface function of each component through Monte Carlo sampling method; Finally, the failure of the whole mechanism is taken as the evaluation objective, and the combination relationship and safety probability of each component are analyzed based on the correlation analysis of the subordinate relationship between the mechanism failure and the component failure in order to obtain the safety probability of the whole mechanism.

![Logical relation model of component](image1.png)

Building logical relation model of component
Dynamic extreme value analysis of each component
Correlation analysis
Take the final mechanism failure as the evaluation objective
Dynamic safety probability of mechanism

Figure 2. Sample of fault tree analysis.

3. Dynamic safety assessment of four-bar mechanism

3.1. Dynamic deterministic analysis
The calculation formula of nouns involved in the above process is shown as the following. In this paper, a typical four-bar linkage in an aircraft is taken as an example for dynamic safety analysis based on extreme condition and fault tree analysis. The four-bar linkage consists of four parts: the driving part, the connecting rod, the driven part and the frame. First, the material of four-bar linkage in the workbench is set as 7050-T7451. Parameters related to the material performance parameters and working conditions of the connecting rod are selected and are shown in Table 1 as the input random variables. The above input random variables are independent of each other and obey normal distribution.

| Parameter                  | Variable | Distribution type  | Mean      | standard |
|----------------------------|----------|--------------------|-----------|----------|
| Young’s Modulus pa Density | $E$      | Normal distribution| $7.1 \times 10^9$ | $1.05 \times 10^9$ |
| Poisson’s Ration Rotational Velocity | $\rho$  | Normal distribution | 2820 | 378 |
|                           | $u$      | Normal distribution | 0.33 | 0.045 |
|                           | $w$      | Normal distribution | 7 | 0.864 |

Table 1. Random input variables ERSM reliability analysis.
The tetrahedral network of the four-bar linkage is divided into 5134 units and 1637 nodes. The mesh division model of the four-bar linkage is shown in Figure 1.

![Figure 1. Mesh division model of the four-bar linkage.](image)

Based on the basic finite element equation of the following structures, the mean value in Table 1 is substituted into the finite element equation of the following four-bar structure for dynamic simulation, and the corresponding deformation distribution and deformation curve can be obtained as shown in Fig. 2 and Fig. 3. It can be seen from Figure 3 that the maximum deformation moment of the mechanism [15-17].

\[
N_i = \frac{1}{6v} (a_i + b_i x + c_i y + d_i z) (i = 1, 2, 3, 4)
\]  

\[
\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}
\]  

\[
\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}
\]  

\[
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
\]
3.2. Building network relationship among components
According to the function and structure of four-bar mechanism, the failure mode of four-bar mechanism may be caused by deformation. The four-bar mechanism is composed of four parts: the driving part, the connecting rod, the driven rod and the frame which is rigid. The fault tree of the four-bar mechanism can be obtained as follows.

![Deformation characteristic curve of the four-bar linkage.](image)

Figure 5. Deformation characteristic curve of the four-bar linkage.

![FTA of the four-bar linkage.](image)

Figure 6. FTA of the four-bar linkage.
Because of the working condition of the driving rod, the material parameters and rotating speed are random variables which obey the normal distribution. Therefore, the rotating speed of the connecting rod and the driven rod are also random variables that obey the normal distribution. Their rotational speed can be measured in the simulation process, and the mean value and standard deviation can be obtained by analyzing the measured values. As shown in the table below.

**Table 2.** The rotational velocity of components (rad/s).

| Name            | Distribution type   | Mean | Standard deviation |
|-----------------|---------------------|------|--------------------|
| The driving rod | Normal distribution | 7    | 0.864              |
| The connecting rod | Normal distribution  | 10.64 | 2.862             |
| The driven rod  | Normal distribution | 1.98 | 0.823              |

Due to the state of the four-bar mechanism in dynamic motion, the maximum deformation time and the maximum deformation characteristics of a single bottom event must be analyzed in the determined time domain to provide input for the establishment of the dynamic extreme response surface model. The deformation curve of each structure is as follows.

**Figure 7.** Deformation characteristic curve of the driving rod.

**Figure 8.** Deformation characteristic curve of the connecting rod.
3.3. Mathematical model of extremum framework with fault tree analysis

At the moment of the deformation from driving rod, connecting rod and driven rod are the largest. The random variables such as input material and working condition are sampled by Latin hypercube sampling technology, and they are substituted into the dynamic model for calculation, so as to obtain the maximum output response of ERSM model of different members. Taking the input random variable and the output maximum deformation response value as the input of function fitting, the regression analysis of dynamic extreme value response surface is carried out and the fitting data can obtain the ERSM mathematical models of different structures, as follows.

\[
\hat{y}_1 = 0.0623 \times 10^{-5} E - 0.0824 u - 5.5081 \times 10^{-6} \rho - 2.4523 \times 10^{4} \omega + 1.4281 \times 10^{-12} E u \\
+ 8.6356 \times 10^{-17} E \rho + 3.3725 \times 10^{-15} E \omega - 2.8407 \times 10^{-6} u \rho - 1.1175 \times 10^{4} u \omega + 1.2557 \times 10^{-7} \rho \omega \\
+ 4.3196 \times 10^{-25} E^{2} - 0.0168 u^{2} - 1.1429 \times 10^{-10} \rho^{2} - 2.2230 \times 10^{-5} \omega^{2} \\
\hat{y}_2 = 0.05023 - 3.4771 \times 10^{-13} E - 0.0577 u - 4.8478 \times 10^{-6} \rho - 9.9283 \times 10^{4} \omega + 1.1064 \times 10^{-12} E u \\
+ 7.7145 \times 10^{-11} E \rho + 1.4820 \times 10^{-14} E \omega - 2.6489 \times 10^{4} u \rho + 3.0074 \times 10^{4} u \omega + 2.6976 \times 10^{8} \rho \omega \\
- 3.0461 \times 10^{-24} E^{2} - 0.0183 u^{2} - 2.3623 \times 10^{-11} \rho^{2} - 1.7985 \times 10^{6} \omega^{2} \\
\hat{y}_3 = 0.0074 + 1.0138 \times 10^{-14} E - 1.4521 \times 10^{4} u + 7.9687 \times 10^{-8} \rho + 1.5565 \times 10^{-5} \omega \\
- 3.9601 \times 10^{18} E u - 1.5347 \times 10^{18} E \rho - 1.6608 \times 10^{16} E \omega + 6.1601 \times 10^{8} u \rho - 2.5385 \times 10^{5} u \omega \\
+ 2.5920 \times 10^{-9} \rho \omega - 6.0932 \times 10^{-26} E^{2} + 5.0327 \times 10^{4} u^{2} + 1.1955 \times 10^{-12} \rho^{2} - 1.0433 \times 10^{6} \omega^{2}
\] (10)

3.4. Dynamic safety analysis

Through regression analysis, the ERSM mathematical model of deformation of driving bar, connecting bar and driven bar is obtained. After 10000 times of random sampling by MCM, the sampling history and frequency distribution histogram of the above three bars can be obtained. \( U_{\text{max}} \) of these figures show the maximum deformation value of the bar. It can be seen from the figure that the deformation frequency distribution of the driving bar follows the normal distribution with mean value and variance of 0.016239 and 1.1062e-06 respectively, the deformation frequency distribution of the connecting bar follows the normal distribution with mean value and variance of 0.013995 and 9.215e-07 respectively, and the deformation frequency distribution of the driven bar follows the normal distribution with mean value and variance of 0.007246 and 2.351e-09 respectively.
It is assumed that the maximum allowable deformation of the driven rod is \[ u = 0.02\text{m} \], the maximum allowable deformation of the connecting frame rod is \[ u = 0.016\text{m} \], and the maximum allowable deformation of the driven rod is \[ u = 0.00782\text{m} \]. According to the calculation formula of safety probability, the safety probability of each member is 99.98\%, 99.91\% and 99.37\% respectively.

![Deformation samples of the driving rod.](image)

**Figure 10.** Deformation samples of the driving rod.

![Deformation samples of the connecting rod.](image)

**Figure 11.** Deformation samples of the connecting rod.

![Deformation samples of the driven rod.](image)

**Figure 12.** Deformation samples of the driven rod.
According to the analysis of four-bar linkage fault tree structure, each bottom event and top event is OR gate relation, which means each bottom event can lead to the occurrence of top event, so the probability of top event of deformation failure of the four-bar linkage is the sum of the probability values of each minimum cut set. It is deducted the reliability probability of deformation failure of four-bar linkage of top event is 99.26%.

4. Method validation of extremum framework with fault tree analysis

In order to verify the effectiveness of the dynamic safety evaluation of fault tree based on the extreme value condition, the input random variables shown in Table 1 are selected in this paper. Under the same calculation condition, the safety analysis of the four-bar linkage is carried out by using MCM [18] and the method proposed in this paper. The maximum allowable deformation
condition is described above. The safety evaluation results of the two methods are shown in the table below.

**Table 3.** The results of dynamic probability calculation of the driving rod with different methods.

| Sample frequency | computing time/s | calculation accuracy |
|------------------|------------------|----------------------|
|                  | MCM              | ERSM                 |
| $10^2$           | 20920            | 0.584516             |
|                  | 99.98%           | 99.98%               |
| $10^3$           | 46436            | 1.130039             |
|                  | 98.21%           | 97.85%               |
| $10^4$           | 252730           | 7.331397             |
|                  | 99.48%           | 99.39%               |

**Table 4.** The results of dynamic probability calculation of the connecting rod with different methods.

| Sample frequency | computing time/s | calculation accuracy |
|------------------|------------------|----------------------|
|                  | MCM              | ERSM                 |
| $10^2$           | 10489            | 0.255273             |
|                  | 99.62%           | 99.99%               |
| $10^3$           | 47823            | 1.418740             |
|                  | 98.15%           | 99.87%               |
| $10^4$           | 292175           | 7.404436             |
|                  | 99.91%           | 99.84%               |

**Table 5.** The results of dynamic probability calculation of the driven rod with different methods.

| Sample frequency | computing time/s | calculation accuracy |
|------------------|------------------|----------------------|
|                  | MCM              | ERSM                 |
| $10^2$           | 18985            | 0.485910             |
|                  | 99.89%           | 99.99%               |
| $10^3$           | 53829            | 1.719658             |
|                  | 99.13%           | 98.46%               |
| $10^4$           | 213977           | 7.286398             |
|                  | 99.69%           | 99.37%               |

**Table 6.** The results of dynamic probability calculation of the four-bar mechanism with different methods.

| Sample frequency | computing time/s | calculation accuracy |
|------------------|------------------|----------------------|
|                  | MCM              | ERSM                 |
| $10^2$           | 50394            | 1.325699             |
|                  | 99.49%           | 99.97%               |
| $10^3$           | 147995           | 4.268437             |
|                  | 95.48%           | 96.18%               |
| $10^4$           | 758882           | 21.999813            |
|                  | 99.09%           | 96.6%                |

It can be seen from the above table that there is a big difference in the calculation time between the method and MCM proposed in this paper: the calculation time of MCM is longer, and the difference between them increases with the increase of the number of samples. There is not much difference between the two methods in calculation accuracy, and the error of the proposed method is within the acceptable range. It can be seen that the method proposed in this paper is a high-precision and efficient dynamic safety analysis method of mechanism.

5. Conclusions

(1) Through the fault tree analysis, the deformation top event of the four-bar linkage is caused by the deformation bottom event of driving rod, connecting rod and driven rod, and their relationship is OR gate relationship. Using fault tree for fault analysis can get the cause of fault and the relationship between the causes clearly and quickly.

(2) The dynamic deformation curve and the maximum deformation of each component of the mechanism are obtained by the deterministic analysis under the influence of the material parameters of the working condition. The extremum response surface method and the least square method are combined to fit in the limit state function of the member, and the function with high precision can be obtained.
(3) Through Monte Carlo sampling method and dynamic safety analysis based on FTAEF, the safety probability of deformation failure of four-bar linkage is 99.26%, which can basically meet the requirements of reliability and safety. The advantages of this method are fast calculation speed and high accuracy, and it can be applied to the safety analysis of mechanism in complex relationship.

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