Liquefaction and post-liquefaction of granular material under multi-directional cyclic loading

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Introduction

It is well accepted that soil liquefaction induced by natural events is one of the major contributors to construction damages; thus, soil liquefaction and soil behaviour after liquefaction are of great interest to researchers. To investigate the liquefaction and post-liquefaction behaviour of soil, numerous studies have been carried out by using various experimental techniques and testing apparatuses (Ishihara and Yamazaki 1980; Boulanger and Seed 1995; Porcino and Caridi 2007; Wang, Yang, and Onyejekwe 2013; Hubler, Athanasopoulos-Zekkos, and Zekkos 2017; Castelli et al. 2019; Shahnazar, Rezvani, and Tutunchian 2019; Kumar, Dey, and Krishna 2020). On account of technical difficulties and apparatus limitations, most of these studies only focus on shear behaviour in a single direction. However, in reality, natural events, such as earthquakes, involve complicated loading directions and magnitudes. With the assistance of advanced facilities, a handful of researchers with the multi-directional simple shear devices have investigated shear behaviour under such conditions (Ishihara and Yamazaki 1980; Boulanger and Seed 1995; Kammerer, Pestana, and Seed 2002; Matsuda et al. 2011). Kammerer, Pestana, and Seed (2002) found that the pore water pressure generation under the multi-directional condition is far more rapid and complicated than that under the uni-directional cyclic loading. Matsuda et al. (2011) clarified that the cyclic shear direction and shear strain amplitude have significant effects on the post-earthquake settlements.

Most of the studies mentioned above use the stress-controlled method to conduct experiments. As a result, the liquefaction resistance of tested material is interpreted based on the influence of cyclic stress ratio (cyclic shear stress amplitude/effective vertical stress) within a preset number of cycles. In this method, the criterion for liquefaction is the normalized pore water pressure reaching 1.0, or the single cycle of the induced shear strain reaching 3% to 3.75% (Ishihara and Yamazaki 1980; Boulanger and Seed 1995; Wijewickreme, Sriskandakumar, and Byrne 2005). However, for the loose and some medium dense samples, the induced shear strain can be significantly large when liquefaction initiates. In comparison to stress controlling, a few researchers believe that strain controlling is more intuitive, since liquefaction is a displacement-dependent phenomenon. It is clarified that shear strain is the key parameter that allows control over the ground settlements and the development of pore water pressure during the cyclic loading (Dobry et al. 1982; Talaganov 1992; Vucetic 1992; Matsuda et al. 2011; Kang et al. 2016; Kumar, Krishna, and Dey 2018; Dammala et al. 2019). More importantly, unlike the stress controlling method, cyclic tests controlled by shear strain do not reach uncontrolled strain condition. In other words, a full liquefaction process can be obtained, which is necessary to undertake the post-liquefaction investigation.

Although it is reported that soils are more prone to liquefaction under multi-directional condition than under unidirectional condition, most of the comparison is made based on one type of multi-directional path (circular shape).
Likewise, the interpretation of the data so obtained is expected to be given from other aspects, for example, in the view of energy dissipation. Consequently, the authors believe that our knowledge of the mechanism characterizing multidirectional cyclic liquefaction is still limited. Studies that investigate post-liquefaction and consider multi-directional loading history are even more limited. Therefore, the main objective of this work is to provide an understanding of cyclic liquefaction and post-liquefaction behaviour of granular materials under multi-directional simple shearing. To meet this end, a series of strain-controlled uni- and multi-directional cyclic simple shear tests were conducted on uniformly sized glass beads. Accordingly, the following influencing factors, namely shear strain amplitude, relative density and loading path, on liquefaction resistance, were investigated. To analyze the exact impact of these factors, an energy-based method is adopted. Lastly, the undrained monotonic shearing test was conducted on liquefied and virgin samples (without experiencing liquefaction) to study their post-liquefaction shear behaviours.

**Multi-directional cyclic simple shear tests in constant volume condition**

*Description of the testing apparatus*

The testing apparatus, known as the variable direction dynamic cyclic simple shear (VDDCSS) apparatus, is shown in Figure 1(a). Three electro-mechanical actuators control the apparatus: One vertical actuator (z-direction) provides the device with vertical load or displacement. Conversely, the other two orthogonal actuators (x- and y-direction) allow the device to exert shear stress or strain in any direction on the horizontal plane.

In this device, a cylindrical specimen (Figure 1(b,c)), which is 70 mm in diameter and 22.6 mm in height, is laterally confined by several circular Teflon coated rings. Each ring is 71 mm in diameter and 1 mm in height. A latex membrane is placed between the rings and the specimen to protect the rings from being damaged and to ensure the uniform shearing of the specimen. During the test, the smooth surface of the rings allows the specimen to be sheared freely with minimum friction. The VDDCSS can perform monotonic and cyclic shear tests, and more detailed information of this apparatus can be found in Li (2016).

**Testing material and testing procedure**

It is reported that the factors, such as grading curve, angularity and content of fine particles can affect the cyclic behaviour of the tested material (Vaid et al. 1990; Choobast, Ghalandarzadeh, and Esmaeili 2014; Hubler, Athanasopoulos-Zekkos, and Zekkos 2017). Since in this work, the prior aim is to investigate the influence of multi-directional loading path on the liquefaction and post-liquefaction behaviour, the
uniform-distributed glass beads are selected as the tested material. Moreover, in discrete element method (DEM), simulations are often implemented on the spherical particles, for example, Zhang et al. (2019) employed uniform-sized glass beads to investigate static simple shear behaviour in bi-direction. Thus the testing results obtained from glass beads in this work may provide a reference for future research in understanding the behaviour of granular material either from the experimental aspect or the numerical aspect. In this research, the glass beads consist of spherical particles that primarily contain silicon dioxide with some other silicates. It is uniformly graded with a mean diameter ($D_{50}$) of 0.8 mm (Figure 2). Its specific gravity, maximum and minimum dry void ratios were measured according to the American Society for Testing and Materials (ASTM) (2014, 2016, 2019) standards D854, D4253 and D4254. The physical properties of this material are summarized in Table 1.

| Property                      | Value          |
|-------------------------------|----------------|
| Mean diameter ($D_{50}$) (mm) | 0.8            |
| Uniformity coefficient ($C_u = D_{60}/D_{10}$) | 1              |
| Specific gravity ($G_s$ (g/cm³)) | 2.5            |
| Maximum void ratio ($e_{\text{max}}$) | 0.716          |
| Minimum void ratio ($e_{\text{min}}$) | 0.523          |

Each specimen was prepared as follows: oven-dried glass beads samples with predetermined weight were poured into the cylindrical shear box by employing the dry funnel method. This method is believed to be a proper way to duplicate the densified soil sample in earthquake regions (Li et al. 2018). To obtain higher relative densities, a low-energy and high-frequency shaking table was used for providing even striking (vibration). The amplitude and frequency of the vibration are 0.5 mm and 2 Hz.

After the sample preparation, the specimens were tested following the process shown in Figure 3. During the stage of consolidation, the specimen was $K_0$ consolidated for 30 min under the effective vertical stress $\sigma'_{\text{vc}} = 50$ kPa. The relative densities obtained at the end of consolidation are $D_r = 45\%$, $D_r = 70\%$ and $D_r = 81\%$, which denote medium dense, dense and very dense category respectively. Subsequently, the undrained cyclic simple shear tests were conducted on these specimens with various shear strain amplitudes, relative densities and loading paths. During the cyclic shearing, the vertical displacement was fixed to keep the volume of the specimen constant and obtain an equivalent undrained shearing condition. In this condition, the decrease of the vertical stress that is attained in a dry sample is equal to the increase of the pore water pressure that is found in a saturated sample under the true undrained shearing test (Dyvik et al. 1987). The details of the performed tests are summarized in Table 2. In particular, the tests employed three different cyclic paths, namely the uni-directional Path-X, multi-directional Path-O and Path-8. The Path-X represents the conventional cyclic loading in a single direction. Figure 4(a) shows the shear waveform and plan view of the strain path for this type of loading, in which the shear strain was exerted on a specimen along the x-direction of the VDDCSS. The Path-O and Path-8 represent the complex cyclic loadings with the same or different frequencies in two directions. Their typical shear waves and shear strain movement are illustrated in Figure 4(b) and (c), respectively. For the multi-directional tests conducted in this study, the same shear strains were simultaneously applied to the specimens with the phase difference of 90 degrees in the x- and y-direction. Specifically, in the Path-O, the frequencies of the x- and y-direction are the same, while in the Path-8, the frequency in the x-direction is half of the one employed in the y-direction. Since liquefaction is a deformation-based phenomenon, it is insensitive to loading frequency (Wong 1971; Jong and Seed 1988). Consequently, the loading frequency was set as 0.1 Hz in this study for ease of control. The cycle number used in this paper denotes the cycles in the x-direction, and the number of liquefaction cycles is recorded when the equivalent pore water pressure reaches over 95% of the effective vertical stress.

After reaching the first liquefaction, the shear displacement was returned to the original point before the specimens were re-consolidated under the same conditions as in the first consolidation. Since the cyclic shear strain amplitudes used during the tests are relatively small, the nominal relative density for each category of specimens (medium dense, dense, and very dense) after re-consolidation is table:}

| Property                      | Value          |
considered to be the same, and they are $D_r^0 = 50\%$, $D_r^0 = 73\%$ and $D_r^0 = 82\%$ respectively. Finally, the monotonic undrained shearing test was conducted on the specimens in the x-direction. This stage is controlled in displacement with a speed of 0.01 mm/min to ensure the accurate recording of the equivalent pore water pressure generation. To provide a reference, the samples with no liquefaction history were also examined with the same monotonic undrained shearing test.

Test results and discussion

Initial liquefaction

In this study, the liquefaction resistance of the specimens, which were tested by the strain-controlled method, has been estimated based on the number of liquefaction cycles and the magnitude of shear strain amplitude. Taking the tests with the shear strain amplitude of 0.1% under the effective vertical stress of 50 kPa and relative density of 45% as examples, Figures 5–7 illustrate the cyclic shear behaviour of the specimens with different loading paths when tested with these conditions. Figure 5 shows the shear stress–strain hysteresis loops in the x- and y-direction. From this image, it can be seen that, as the cycle increases, the secant shear moduli (the ratio of cyclic shear stress and strain amplitude) decrease gradually to nearly zero. To compare the variation of shear stress under different loading paths more easily, Figure 6 shows the development of shear stress against the number of loading cycles in the x- and y-direction. It proves that at the first few cycles, the cyclic loading paths have little impact on the sample stiffness since their respective shear stress magnitudes are almost the same. However, as the tests continue, the decreasing rates of shear stress for Path-X, Path-O and Path-8 in the x-direction are in ascending order. Similar results are also found in the y-direction, where the specimen under the Path-8 loses its shear stress faster than under the Path-O.

From a more direct point of view, as shown in Figure 7, the pore water pressure, which is one of the key indicators of cyclic liquefaction, is plotted against the number of cycles for different cyclic loading paths. During the cyclic shearing, the specimens tend to decrease the volume, and in a drained condition, this is called densification. However, the decreasing of this volume is prevented by the undrained condition. Similarly, the tendency of the volume to change throughout successive cycles determines the magnitude and rate of pore water pressure generation in the specimens. From Figure 7, it could also be observed that the specimen under uni-directional cyclic path develops pore water pressure most slowly and the specimen under multi-directional cyclic Path-8 develops pore water pressure most quickly.

To closely compare the liquefaction behaviour of tested material under uni- and multi-directional loading paths, Figure 8 displays the effects that any given cyclic loading path and relative density had on the liquefaction resistance. In this figure, the number of cycles that is necessary to reach liquefaction $N$ is plotted against the cyclic shear strain amplitude $\gamma_c$ in a logarithmic format. Figure 8(a) presents the result obtained under the medium dense state and Figure 8(b,c) show the corresponding results for the dense and very dense states.

Firstly, it can be seen that under the same relative density and shear strain amplitude, tests that were conducted under Path-X require far more cycles to reach liquefaction than the tests sheared multi-directionally. At the same time, the tests run under the multi-directional Path-O only require slightly more cycles than the tests performed under the Path-8. The trend of the curves is the same as the results previously presented in Figures 5–7. The difference of

| Initial liquefaction | Post-liquefaction |
|----------------------|------------------|
| $\sigma'_{vc}$ (kPa) | $\sigma'_{vc}$ (kPa) |
| Relative density $D_r$ (%) | Relative density $D_r$ (%) |
| Cyclic loading path | Static loading path |
| $A_c$ (%) | $A_c$ (%) |
| Path-X | 0.1, 0.2, 0.3 | 50 |
| Path-O | 0.05, 0.1, 0.2 | 50 |
| Path-8 | 0.05, 0.1, 0.2, 0.25 | 73 |
| Path-X | 0.1, 0.2, 0.3 | 82 |
| Path-O | 0.05, 0.1, 0.2, 0.25 | Undrained in the x-direction |
| Path-8 | 0.05, 0.1, 0.2, 0.25 | |

Figure 4. Cyclic shear waves and plan view of test strain paths for: a, uni-directional Path-X; b, multi-directional Path-O; c, multi-directional Path-8.
liquefaction cycle numbers resulted from the different tendencies of volume change, which may be related to the rearrangement of particles due to different loading paths. Secondly, as each graph of Figure 8 shows, the specimens are sheared under different relative densities but by the same loading path. It is consistent with all loading paths, where increasing the relative density will increase the liquefaction resistance. It is because the specimens tend to contract less under the higher relative density, and a less contractive specimen is more difficult to liquefy.

Lastly, in Figure 8, it can be observed that the curves are almost parallel to each other. Moreover, any increase in the shear strain amplitude reduces the number of cycles to reach liquefaction. When the shear strain amplitude reduces significantly, the effects of cyclic loading path and relative density on the liquefaction resistance are noticeable. However, when the shear strain amplitude increases, the differences previously found between liquefaction cycle numbers for different cyclic loading paths or relative densities decline.

**Energy dissipation**

Nemat-Nasser and Shokooh (1979) first established an energy approach to understanding the liquefaction potential of sand under cyclic shearing. Their work indicates that the rearrangement of sand particles engendered by cyclic loading dissipates a certain amount of energy. Verified by experiments and models only considering single-directional shearing, the energy dissipated inside of the specimen is proved to be directly related to the rate of liquefaction (Figueroa et al. 1994; Liang 1995; Baziar and Jafarian 2007; Sonmezer 2019). In this approach, the energy per unit volume (kJ/m³) dissipated in one cycle is denoted by the area in the corresponding shear stress-strain loop (for example, in Figure 5(a)).

Because of considering multi-directional cyclic loading, in this study, the total energy per unit volume (\(\delta W\)) necessary to liquefy a specimen is given by:

\[
\delta W = \sum_{i=1}^{n-1} \frac{1}{2} (\tau_{x_{i+1}} - \tau_{x_i})(\gamma_{x_{i+1}} - \gamma_{x_i}) + \frac{1}{2} (\tau_{y_{i+1}} - \tau_{y_i})(\gamma_{y_{i+1}} - \gamma_{y_i})
\]

where \(n\) is the total number of recorded data before liquefaction, \(\tau_x\) and \(\tau_y\) are the \(\delta\) shear stresses in the \(x\)- and \(y\)-direction, \(\gamma_x\) and \(\gamma_y\) are the shear strains in the \(x\)- and \(y\)-direction.

To investigate the mechanism of liquefaction under uni- and multi-directionally cyclic conditions, the total energy...
per unit volume calculated from Equation (1) is shown in Figure 9. It is presented against the shear strain amplitude of different cyclic loading paths. Figure 9(a) shows the curves obtained under the medium dense state. Similarly, Figure 9(b,c) show the curves found under the dense and very dense state, respectively. It is demonstrated that in Figure 9(a,b), the curves of the energy dissipated for different loading paths are relatively flat, while Figure 9(c) shows a similar trend except for a slight rise appearing at the beginning of the curve for the Path-X. According to Figure 8(c), when the very dense specimen is liquefied under the Path-X and shear strain amplitude of 0.1%, the number of liquefaction cycles is almost 100 for this case, and the friction error induced inside of the apparatus may be enlarged significantly to affect the energy calculation. However, in the big picture, Figure 9 illustrates that under a specific loading path and a certain relative density, the total energy per unit volume for liquefaction is generally a constant value, despite the shear strain amplitude that is exerted on the specimen. Moreover, this amount of energy grows in accordance to the increase registered in the relative density. This result is consistent with previous research findings (Figueroa et al. 1994; Baziar and Jafarian 2007; Jafarian et al. 2012; Millen et al. 2020), which claimed that the total dissipated energy per unit volume up to liquefaction rises in concomitance with the increase of relative density. However, it is less dependent on the shear strain amplitude. More importantly, from this figure, it can also be observed that when the relative density is the same, the total energy dissipated per unit volume under different loading paths is, by and large, the same. A similar study conducted by Polito et al. (2013) with stress-controlled cyclic triaxial tests showed that the accumulated energy per unit volume at the onset of liquefaction is not affected by the loading shape (sinusoidal, square, triangular, irregular symmetric and irregular asymmetric).

To discuss the difference of liquefaction resistance that was found between uni- and multi-directional simple shearing, the set of tests with the shear strain amplitude of 0.1% under the effective vertical stress of 50 kPa and relative density of 45% is chosen as an example. Figure 10, for which data is calculated based on Equation (1), illustrates the development of the energy dissipated per cycle as well as the accumulated energy. This figure shows that the energy dissipated drops quickly in the last few cycles, which, in turn, indicates the occurrence of liquefaction.

Since the total energy dissipated per unit volume for liquefaction is a constant value independent of the cyclic loading path, Figure 10 evidently shows that the energy dissipated in each cycle of the specimen sheared under the Path-X is considerably lower than that under the multi-directional paths. The primary reason for this can be found in Figure 10(a), which shows that the energy dissipated per cycle in the Path-X is contributed by a single direction. While Figure 10(b,c) show that the energy dissipated per cycle in the Path-O and Path-8 is contributed by the x- and y-direction simultaneously. The energy dissipated per cycle

![Figure 6. Decay of shear stress during cyclic shear testing for specimens under different loading paths for \( \sigma'_vc = 50kPa, D_r = 45\% \) and \( A_{\gamma} = 0.1\% \): a, in the x-direction; b, in the y-direction.](image6)

![Figure 7. Variation of pore water pressure during cyclic shear testing for specimens with different loading paths under \( \sigma'_vc = 50kPa, D_r = 45\% \) and \( A_{\gamma} = 0.1\% \).](image7)
in one direction is much lower than the energy dissipated in two directions. Although both in the Path-O and Path-8 energy is dissipated in two directions, the number of cycles necessary to achieve liquefaction in the Path-8 is slightly lower than in the Path-O as compared in Figure 10(b) and Figure 10(c). This is the case in that the frequency of the Path-8 in the y-direction is twice that of the one of the Path-O in the y-direction. In conclusion, more energy in one cycle is dissipated in the Path-8 than in the Path-O in the y-direction.

**Post-liquefaction**

After the first liquefaction happened, the specimens were re-consolidated under the same condition as the first consolidation. Then, they were sheared in the x-direction monotonically in an undrained manner. In this study, this process is called post-liquefaction. Since the Path-O is a representative of multi-directional loading path, the set of tests, which have experienced liquefaction in the Path-O, is taken as the main example to investigate the impact that the history of the initial liquefaction on the post-liquefaction stage. Figures
11 and 12 display the volume changes that occur throughout the re-consolidation stage, and the relative density of each specimen after re-consolidation is also provided in these two figures. Figure 11 shows the volumetric strain development of the specimens with a liquefaction history in the Path-O under the medium dense, dense and very dense state. It can be seen from the figure that the amount of volume reduction so observed is highest under the medium dense state, but lowest under the very dense state. More importantly, the volumetric strain and the relative density increase in accordance to the amount of the initial liquefaction shear strain amplitude, although the increase of the relative density is not significant. Figure 12 demonstrates the volumetric strain development of specimens with the shear strain amplitude of 0.1% under different initial liquefaction loading paths. By doing so, it could be observed that the multi-directionally liquefied specimens generate more volumetric strain than the uni-directionally ones to achieve higher relative densities. Same results are presented here. To facilitate the analysis, the data of the virgin samples are also provided as references. Figure 13(a) shows the development of shear stress and stress path of specimens that liquefied in the Path-O with different strain amplitudes under the medium dense state. Figure 13(b,c) show the specimens that liquefied under the dense and very dense state, respectively. From these data, it can be observed that the specimens with liquefaction history, despite the amount of strain amplitude, have greater peak strength than the virgin samples. This phenomenon is most obvious for the medium dense specimens. As shown in Figure 13(a), the specimens with liquefaction history shift shear behaviour from strain-softening to strain-hardening. It may be caused by the densification process that is necessary for the liquefied specimens and virgin sample to achieve the same nominal relative density. To do so, the virgin sample underwent consolidation only once, while the liquefied specimens experienced it twice. This means that, in terms of structure, the liquefied specimens are more uniformly distributed than the virgin sample. Moreover, similar conclusions have been reported in the previous studies (Porcino, Marciano, and Nicola Ghionna 2009; Bastidas et al. 2017) that were conducted through post-liquefaction undrained cyclic tests of sand and by using uni-directional simple shear devices. They showed that, although there is no significant change in the relative density, the sand samples manifest higher post-liquefaction resistance if they have experienced small cyclic pre-shearings. The concept of small cyclic pre-shearings
Figure 13. Post-liquefaction undrained shear behaviour of Path-O liquefied specimens under $\sigma'_{uv}$ =50kPa: a, for $D'_r$=50%; b, for $D'_r$=73%; c, for $D'_r$=82%.
mentioned above was first introduced by Ishihara and Okada (1978), who conducted undrained cyclic triaxial compression tests on sands. According to this concept, the small pre-shearing was defined as a stress history where the stress path is restricted to the domain bounded by two PT-lines (lines of phase transformation). According to this definition, the initial liquefaction loading histories used in this study in the post-liquefaction stage are all confirmed to be the small pre-shearing. The stress paths of specimens in the x- and y-direction under the effective vertical stress of 50 kPa, relative density of 45% and strain amplitude of 0.1%, are shown in Figure 14 as an example. Thus it is reasonable that the rest of the tests agree with the trend shown in Figure 13.

Apart from the above observations, there is another important finding in Figure 13. It indicates that for each category of the specimens, as the initial liquefaction shear strain amplitude increases, the peak strength in the post-liquefaction stage raises as well. These results are also seen in the specimens liquefied in the Path-X and Path-8. It is because the liquefied specimens with higher shear strain amplitude have higher relative densities and greater dilatancy than those with lower shear strain amplitude. Furthermore, a similar study is conducted by Wahyudi et al. (2015) with stacked-ring simple shear. The results confirmed by the image analysis show that different levels of pre-strain can induce different fabric inside of the tested material. Thus, it is supposed that, within the range of small pre-shearing, the larger shear strains can eliminate more local instabilities in the sample structure than the smaller ones.

Figure 15 shows the undrained shear behaviour of specimens with different liquefaction loading paths. Data presented in this figure are extracted from the tests conducted with a shear strain amplitude of 0.1% under the dense state. The results show that, the specimens liquefied multi-directionally, in general, exhibit higher peak strength than those liquefied uni-directionally. For the multi-directionally liquefied samples, the result of the specimen liquefied with the Path-8 history is slightly higher than the one liquefied with the Path-O history. The same results are also observed in the tests which were run under the medium dense and very dense state. Since the relative density of the multi-directionally liquefied specimen is slightly higher than the uni-directional one, the observation presented in Figure 15 indicates that the post-liquefaction shear strength is obviously affected by the fabric anisotropies, which are induced by different loading paths. The uni-directional path eliminates local instabilities of the specimen through shearing in one direction, while the multi-directional paths eliminate

Figure 14. Illustration of small cyclic pre-shearing for specimens under $\sigma'_{vc}=50\text{kPa}, D_r=45\%$ and $A_{\gamma}=0.1\%$: a, for Path-X; b, for Path-O; c, for Path-8.

Figure 15. Post-liquefaction undrained shear behaviour of liquefied specimens under $\sigma'_{vc}=50\text{kPa}, A_{\gamma}=0.1\%$ and $D_{\gamma}=73\%$.  

more through shearing in two directions. As a result, the multi-directionally liquefied specimens are in a more uniform state of structure than the uni-directionally ones, and the specimens undergoing liquefaction under multi-directional loading paths are stronger in the post-liquefaction stage than those subjected to liquefaction in the uni-directional loading path.

Conclusions

To study the liquefaction behaviour of granular material under multi-directional conditions, a series of cyclic simple shear tests have been conducted on the uniform-sized glass beads. Two types of multi-directional loading paths, namely the Path-O and Path-8, are employed and compared with the uni-directional loading Path-X. After initial liquefaction, the monotonic undrained simple shearing was conducted on the re-consolidated specimens to investigate their post-liquefaction behaviour. The effects of relative density and strain amplitude have also been considered. The main conclusions are summarized below:

1. In the initial liquefaction stage, the liquefaction resistance of specimen is higher under the multi-directional path. Moreover, the number of cycles necessary to reach liquefaction are increased by an increasing relative density or a decreasing shear strain amplitude.

2. The total energy dissipated to liquefy a specimen is dependent on the relative density but independent of the shear strain amplitude and loading path. Nevertheless, during one cycle of loading, the multi-directional Path-8 dissipates energy most quickly. In contrast, the uni-directional Path-X dissipates energy most slowly. Therefore, this finding explains the difference in liquefaction resistance that a specimen exhibits under different loading paths.

3. In the re-consolidation stage, the settlements obtained with multi-directionally liquefied specimens are higher than those accrued with uni-directionally liquefied specimens. Likewise, the settlements gained with higher initial shear strain amplitudes are larger than those attained with lower initial shear strain amplitudes.

4. In the post-liquefaction stage, the initially liquefied specimens are generally stronger than the virgin specimens. The larger the initial shear strain amplitude is, the stronger the specimen becomes. Furthermore, the specimens liquefied with the Path-8 shows the highest peak strength. In contrast, the specimens liquefied with the Path-X shows the lowest one.

Disclosure statement

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