Coherent Control of Collective Spontaneous Emission through Self-interference

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As one of the central topics in quantum optics, collective spontaneous emission such as superradiance has been realized in a variety of systems. This work proposes an innovative scheme to coherently control collective emission rates via a self-interference mechanism in a nonlinear waveguide setting. The self-interference is made possible by photon backward scattering incurred by quantum scatterers in a waveguide working as quantum switches. Whether the interference is constructive or destructive is found to depend strongly on the distance between the scatterers and the emitters. The interference between two propagation pathways of the same photon leads to controllable superradiance and subradiance, with their collective decay rates much enhanced or suppressed (also leading to hyperradiance or population trapping). Furthermore, the self-interference mechanism is manifested by an abrupt change in the emission rates in real time. An experimental setup based on superconducting transmission line resonators and transmon qubits is further proposed to realize controllable collective emission rates.

\textbf{Introduction.}—Waveguide quantum electrodynamics has recently been a growing area in quantum optics with important applications in quantum information processing [1–5]. Different integrations of quantum emitters (QEs) with nanophotonic structures have been achieved, such as guided surface plasmons confined on a conducting nanowire with individual optical emitters [6, 7], photonic nanowire coupled by embedded quantum dots [8, 9], or superconducting transmission line with superconducting qubits [10–12]. These physical platforms make it possible to let QEs interact with one-dimensional bosonic modes with nontrivial dispersions, leading to intriguing dynamics such as persistent quantum beats [13, 14], supercorrelated radiance [15], and single photons by quenching the vacuum [16]. Indeed, enhanced light-matter interaction because of dimensionality reduction in waveguide quantum electrodynamics continues to attract a great deal of attention [17–27]. Advances in designing and probing light-matter interactions have allowed the investigation of collective phenomena such as superradiance, subradiance [28–36], cavity antiresonance spectroscopy [37], and nonequilibrium collective phase transition [38, 39].

As a typical collective emission, Dicke superradiance has been demonstrated in systems of hot atoms [40, 41], cold atoms [42, 43], trapped ions [44–46], and superconducting qubits [47, 48], etc. Its counterpart with reduced emission rate is termed subradiance. Transitions between superradiant and subradiant states have been realized in superconducting circuits by initially applying phase gate on each qubit [49]. Superradiance and subradiance are highly relevant to quantum memory as their roles can be important in the writing and reading of quantum information [50, 51]. However, to date continuously controllable collective emission rate without using external driving fields [52–54] remains a challenge for almost all QE systems.

In this letter we reveal an unknown aspect of spontaneous emission in a nonlinear waveguide setting. We consider quantum scatterers in addition to general quantum emitters in the same waveguide. The emitted photon propagating in the waveguide can be bounced back by the scatterers as quantum switches. The backward scattered photon then interferes with the other branch of the photon propagating in the opposite direction. Such self-interference is exploited to achieve continuous and extensive control of the spontaneous emission rate of QEs. As shown below, even the transition from superradiance to subradiance can be readily achieved if we control certain features of the quantum switches, such as its resonance frequency and the QE-scatterer distance. The underlying self-interference mechanism is analyzed both qualitatively and quantitatively, with excellent agreement between physical insights and theoretical studies. In particular, the QE-scatterer distance is found to be a crucial parameter to induce constructive or destructive interference. An experimental setup based on superconducting transmission line resonators and transmon qubits is further proposed to realize continuously controllable collective emission rates.

\textbf{Model.}—The system we consider consists of a one-dimensional array of tunneling-coupled cavities which accommodate one assembly of QEs at position $x = x_1$, as well as a second collection of two-level atoms at $x_2$ respectively. A schematic plot of this configuration is shown in Fig. 1. Atoms at $x_2$ play the role of quantum scatterers, through which the spontaneous emission dynamics of QEs at $x_1$ is to be manipulated. Though playing two different roles, these two collections of atoms will be treated with similar notation, indexed by $A$ and $B$ respectively, and assumed to have excited states $|e^A\rangle$, $|e^B\rangle$ and ground states $|g^A\rangle$, $|g^B\rangle$, separated in energy by frequencies $\Omega_A$ and $\Omega_B$ (we set $\hbar = 1$ throughout). The tunneling-coupled photonic waveguide forms a lattice, modeled by the following tight-binding Hamiltonian

$$H_{\text{ph}} = \sum_x \omega_x a_x^\dagger a_x + J \sum_x \left( a_{x+1}^\dagger a_x + a_x^\dagger a_{x+1} \right), \quad (1)$$

where $a_x^\dagger$ is the creation operator of the waveguide mode.
at position $x$, and $\omega_c$ is the resonance frequency of a single cavity. For convenience, we assume the lattice constant to be $a = 1$ throughout. The total Hamiltonian describing the system is then

$$H = H_{\text{ph}} + \sum_j \Omega_{j,A} |e_j^A\rangle \langle e_j^A| + \sum_j \Omega_{j,B} |e_j^B\rangle \langle e_j^B| + \sum_j \left( V_{j,A} \sigma_{j,A}^+ a_{2x} + V_{j,B} \sigma_{j,B}^+ a_{2x} + h.c. \right),$$

(2)

where $\sigma_{j,A}^+$ and $\sigma_{j,B}^+$ are the creation operators for the $j$th atom in each assembly and $V_{j,A}$ and $V_{j,B}$ are the respective coupling strengths. For the case of only one QE with weak coupling ($V \ll J$), the QE decays with the radiation rate $\Gamma = V^2/J$ if the emitter is near resonance with the frequency of a single cavity [55]. In the single excitation subspace, the spectrum of Eq. (2) comprises discrete localized bound states and a continuum of delocalized dressed states with energy $\omega_k = \omega_c + 2J \cos k$ vs the mode wavevector $k$, thus forming a scattering band with $\omega_c$ being the band central frequency with bandwidth $4J$ ($J > 0$). The said bound states result in the known fractional trapping of an emitted photon and nonexponential dynamics of the spontaneous emission [56, 57]. If the excitation frequency of QEs is equal to the single-cavity frequency $\omega_k$ (a condition assumed below), then the momentum of a radiated photon is around $k_m = \pm \pi/2$. Note also that the peak photon group velocity is $|v_{2x}^m| = 2J$ reached by the wavevector $k_m$.

General theoretical considerations.—Let us now assume that the QEs and the quantum scatterers are separated by a distance $\Delta x \equiv |x_2 - x_1|$, which is less than the half of the coherence length $\sim v_{2x}^m/\Gamma$ of a spontaneously emitted photon. The initial state of the whole system is that one of the QEs is excited, or a superpositions of such configurations, with the photon field in vacuum. This hence places the whole wavefunction in the single-excitation invariant subspace. The time-evolving state at time $t$ can be written as

$$|\psi(t)\rangle = \sum_{i=1}^{M_i} \sum_{j=1}^{C_j^o(t)} \sigma_{j,i}^+ + \sum_k C_k(t) a_k^\dagger \right| g,\text{vac}\rangle,$$

(3)

where $i = A, B$ and $a_k^\dagger = (1/\sqrt{N}) \sum_x e^{ikx} a_x^\dagger$. $M_i$ is the number of the QEs or of the scatterers. $C_j^o$ is the excitation amplitude for the $j$th atom in each collection of atoms, $C_k$ is the amplitude of the waveguide mode with momentum $k$. Without loss of generality, we assume that only QEs with indices $j_n$ may be excited at time zero, i.e., $C_{j_n}^o(0) = 0, C_{j}^o(0) = 0 (j \neq j_n)$, and $C_{j_1}^o(0) = C_{j_2}^o(0) = 0$. If initially at most two excited QEs indexed by $j_1$ and $j_2$ are involved in the initial excitation, then exact results about their ensuing time dependence can be obtained:

$$C_{j_1}^o(t) = \frac{L_1(s) C_{j_1}^o(0) + L_2(s) C_{j_2}^o(0)}{G(s)} e^{st}|_{s = -i\omega_A}$$

$$+ \sum_m \frac{iL_2(s) \left[ C_{j_1}^o(0) + C_{j_2}^o(0) \right]}{(is - \Omega_A) [G(s)]'} e^{st} \bigg|_{s = x_m}$$

$$- \sum_{\alpha = \pm} \int_{-1}^{1} \frac{[Q_1^\alpha C_{j_1}^o(0) + Q_2^\alpha C_{j_2}^o(0)]}{2\pi i\alpha (y + \Omega_A) [Q_1^\alpha - Q_2^\alpha]} dy,$$

(4)

where $L_{1,2}(s)$ and $G(s)$ strongly depend on the separation parameter $\Delta x$, $Q_{1,2}$ are explicit functions of the integration variable $y$, and $x_m$ is the roots of the equation $G(s) = 0$ [58]. The imaginary part of each $x_m$ corresponds to the inverse of system’s eigenenergies of the localized photon-QE dressed states. In fact, the second term on the right-hand side of Eq. (4) originates from the system’s photon-QE bound states with nonzero field amplitudes. $C_{j_1}^o(t)$ can be obtained by exchanging the positions of $C_{j_1}^o(0)$ and $C_{j_2}^o(0)$ in Eq. (4). In obtaining the analytical expressions above, it has been assumed that each collection of atoms (namely, among the QEs, or among the scatterers) are identical and thus $\Omega_{j,i}$ and $V_{j,i}$ are independent of the atom index $j$, denoted by $\Omega_A, \Omega_B, V_A, \text{and} V_B$.

Cases with one single emitter.—To gain some important insights first, we consider a single QE indexed by $j_1$ coupled with multiple scatterers through the waveguide, with the initial state $|\psi(0)\rangle = |\sigma_1^A(0)\rangle|g,\text{vac}\rangle$. The time evolution of the excited state population $P_t(t) = |C_{j_1}^o(t)|^2$ is shown in Fig. 2(a) vs the detuning parameter $\Delta_B = \Omega_B - \omega_c$ depicting the scatterers, with the QE-scatterer separation $\Delta x = 7$ (i.e., 7 lattice constants) as an example. At early times, the emission dynamics matches well with that of a normal decay process $P_t(t) \approx e^{-\Gamma_1 t}$ with $\Gamma_1 = V_A^2/J$ (assuming $V_A \ll J$), without feeling the presence of the scatterers. Later the scatterers makes a dramatic difference during the spontaneous emission process. In particular, as the main reason to introduce the scatterers in the first place, the scatterers as two-level systems can extensively control the coherent transport of a single photon in the waveguide, including a complete reflection of the emitted photon [5]. Hence, once part or even the whole of the propagating photon towards the
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FIG. 2. (a) Excited state population $P_e(t)$ vs detuning $\Delta_B$ for $\Delta x = 7$. (b) Excited state population $P_e(t)$ vs detuning $\Delta_B$ for $\Delta x = 8$. Time is in units of $1/(2J)$. The parameters are $V_A/(2J) = 0.08$, $V_B/(2J) = 1.8$, $\Delta_A/(2J) = 0$, the number of emitters is $M_A = 1$ and that of scatterers is $M_B = 2$. In both (a) and (b) the vertical dashed lines indicate the time of arrival of a reflected photon.

scatters comes back to the decaying QE, it will interfere with the other branch of the photon propagating along the other direction. Supporting this physical picture, Fig. 2(a) depicts an abrupt change of the radiation rate once $t$ reaches $t_0 \approx 2\Delta x/(2J)$, yielding a cascade of stimulated emission. In terms of the real-time dynamics, we are witnessing an intriguing scenario that an emitted photon, when being bounced back, can further boost the emission process that has not been completed yet. The degree of enhancement is continuously controlled by the frequency of the scatterers, as evidenced by the shown strong dependence of the emission rates vs the detuning parameter $\Delta_B$.

Continuing our investigations of the cases with $\Delta x = 2n + 1$ ($n$ being an integer), let us further assume that the quantum scatterers are near resonance with each single cavity with small $\Delta x$, plus the conditions $V_A \ll 2J$ and $\sqrt{M_B} V_B \sim 2J$. The enhanced emission rate under these conditions are found to be $P_e(t) \approx (M_B V_B^2/(4J^2) - \alpha_1/2)^2 e^{-2J\alpha_1 t/\beta_1}$ with $\alpha_1 = V_B^2/(J^2 - \Delta x V_B^2/2)$ and $\beta_1$ is a time-independent quantity [58]. As can be seen in Fig. 2(a), our approximate theoretical results agree well with the exact results obtained from Eq. (4). For $\Delta x$ being small enough, the radiation rate is two times the normal decay rate. Although there is only one QE here, the emission rate is much enhanced and even beyond two-QE Dicke superradiance. The physical understanding is the following: The QE interferes with itself via a delayed photon, thus it can effectively realize collective interference and hence achieve superradiance.

Next we consider cases with $\Delta x = 2n$. A few computational examples with $\Delta x = 8$ are shown in Fig. 2(b), where suppressed emission rates are clearly observed. As the frequency of the scatterers is tuned from $\omega_c$ to the values far away from the photon band, the suppression becomes weak and ultimately the emission comes back to the normal decay. It is curious to qualitatively understand why superradiance and subradiance are observed for $\Delta = 2n + 1$ and $\Delta = 2n$ in Fig. 2(a) and Fig. 2(b), respectively. If $\Delta x = 2n + 1$, the phase difference incurred by the round travel of the bounced photon can be estimated as $|k_m|(2\Delta x) = (2n + 1)\pi$, if considering the main wave component around $k_m = \pm \pi/2$ with the largest group velocity. Also accounting for the $\pi$ shift associated with a complete photon reflection, the overall phase difference between the bounced photon and the original photon is thus $2n\pi$, yielding constructive interference and hence enhanced emission. By contrast, if $\Delta x = 2n$ is chosen, then the overall phase difference between the two interfering pathways is $(2n + 1)\pi$, thus producing destructive interference and leading to suppression of the emission and thus subradiance. Confirming this understanding, in Fig. 3(a), we further show how the emission rates for weak coupling $V_B$ is changed over a wide range if the QE-scatterer distance $\Delta x$ is adjusted.

Our results above have clearly indicated the important role of the backward scattering in the self-interference mechanism. It is hence useful to examine some details of the scattering process. When the radiation field reaches the scatterers, part of the field is reflected with the reflection amplitude $r_k$ given by $r_k = M_B V_B^2/[2J \sin(\omega_k - \Omega_B)] - M_B V_B^2$ [58]. Around the resonance, the reflection spectrum yields the so-called Breit-Wigner line shape with the spectrum width given by $M_B V_B^2/[J \sin(k_0)]$, where $k_0$ is determined by the relation $4J^2 \sin^2(\omega_k - \Omega_B) = M_B^2 V_B^2$ [5]. In particular, if the photon energy $\omega_k$ is under resonance with the two-level scatterers, namely, $\omega_k = \Omega_B$, one then obtains complete reflection with $r_k = -1$ (hence the above-mentioned $\pi$ phase shift). Under the parameter setting $\omega_c = \Omega_B$, the resonance scattering condition $\omega_k = \omega_c + 2J \cos(k) = \Omega_B$ occurs for $k = \pi/2$. From the expression of $r_k$, it is also seen that if $k = 0$ or $k = \pi$, complete reflection happens also. However, this is irrelevant to our self-interference mechanism because such components have a vanishing group velocity in the waveguide. In Fig. 3(b), we show the reflection and transmission spectra vs momentum $k$.

Cases with two and more emitters.—We now examine how collective emission rates with two QEs can be manipulated by exploiting self-interference. The two emitters are prepared in typical single-photon entangled states $|\psi^\pm\rangle = (1/\sqrt{2})(|\sigma^+_A, \sigma^-_B\rangle + |\sigma^-_A, \sigma^+_B\rangle)_{g, vac}$. For $|\psi^\pm\rangle$, the decay of QEs is completely suppressed since it is a dark state that cannot emit a photon. Thus we focus on the emission dynamics emanating from $|\psi^+\rangle$. When $V_B = 0$ and the frequencies of QEs lie outside the scattering band with $|\Delta_A \pm 2J| \gg V_A$, the evolution of $|\psi^+\rangle$ is dominated by a trapping regime due to the presence of bound states [56]. For the case $V_A \ll 2J$, the decay of the amplitudes $C^A(t) = C^A_2(t) \equiv C_2(t)$ is basically exponential, with a very slowly changing radiation rate as $\Omega_A$ is tuned from from $\omega_c \pm J$ to $\omega_c$. This is what one expects from the Wigner-Weisskopf and Markovian perturbative theories, which predict $|C_2(t)|^2 \approx (1/2)e^{-T_J(\Delta A)}t$, with
Here the radiance rate $\Gamma_h = 4J V_b^2/(J^2 - \Delta x V_b^2)$ with $\chi = M_B V_b^2 - \Delta x M_B V_b^2 V_b^2/J^2$. The dotted line shown in Fig. 3(c) is obtained from Eq. (5), in excellent agreement with exact results obtained directly from Eq. (4). Under the limit $\Delta x$ being small enough, $\Gamma_h$ is found to be just two times the superradiance rate $\Gamma_s$. On the other hand, Eq. (5) indicates that $\Gamma_h$ becomes large as $\Delta x$ increases. As such, tuning $\Delta x$ allows us to further boost hyperradiance rates.

Finally, we investigate how the self-interference mechanism works when there are multiple QEs. In this case, we rely fully on computational studies since it becomes tedious to find analytical results with more than two QEs being initially excited. To investigate if the above hyperradiance dynamics can be extended to cases with multiple QEs, we consider the following initial amplitudes $C_{A_n}^J(0) = \cdots = C_{M A_n}^J(0) = 1/\sqrt{M_A}$. Fig. 3(d) depicts the results with $M_A = 5$ QEs for different values of $\Delta x$. In the absence of scatterers, under the condition $\sqrt{M_A} V_b \ll 2J$, the amplitudes $C_{A_n}^J(t) = \cdots = C_{M A_n}^J(t) = C_{M A_n}(t)$ can be approximately described by $|C_{M A_n}(t)|^2 \approx (1/M_A) e^{-\Gamma_h t}$ with $\Gamma_h = M_A V_b^2/J$, which is nothing but the Dicke superradiance.

However, for cases with $\Delta x = 2n + 1$, Fig. 3(d) shows that the self-interference mechanism further boosts the collective emission rates by a factor of two for relatively small $\Delta x$. Furthermore, as $\Delta x$ increases, the emission rate continues to be enhanced and hence surpasses $2\Gamma_s$. This echoes with our observation in the case of two emitters. To understand this intriguing trend due to increasing $\Delta x$, we first note that the wavevectors of the photon that will be backward scattered are spread around $k = \pi/2$ (in the direction towards the scatterers), but only the component with precisely $k = \pi/2$ of the largest group velocity can optimally induce the constructive self-interference. If $\Delta x$ increases, the potential phase dispersion among these components along the propagation pathway increases. This imposes a more strict selection on the wave components that can contribute to the self-interference. These selected wavecomponents also tend to induce the self-interference more synchronously.

For cases with $\Delta x = 2n$, it is clearly observed in Fig. 3(d) that the emission rates are much suppressed due to the destructive self-interference. For suppressed subradiance, the asymptotic values of the excited state population is finite at sufficiently long time. Indeed, under $\Delta B_0 = 0$, the emitted photon is first bounced back by the scatterers. Once the scattered photon meets the QEs, destructive self-interference suppresses the collective emission and as such the QEs tend to reflect the emitted photon as well, thus also dynamically trapping the photon between the QEs and the scatterers. These results and insights indicate the role of bound states in fully explaining the population trapping on the excited state.
**Discussion and conclusions.**—In a one-dimensional nonlinear waveguide setting, we have shown that the self-interference incurred by a backward scattered photon originally emitted from quantum emitters can dramatically change the emission rate. The backward scattering is caused by a collection of quantum scatterers with a preselected distance. For an initial state as a superposition of excitation of not more than two emitters, we obtain an exact analytical solution to predict how quantum scatterers can be used to control the emission rate. In both our theoretical treatments and our qualitative analysis, it is found that the distance between the emitters and the scatterers plays a critical role in deciding whether constructive or destructive interference occurs. This self-interference mechanism then leads to extensive control of the collective emission dynamics, ranging from hyperradiance to strongly suppressed subradiance. The theory and the physics communicated in this work are rather general. Indeed, the considered quantum emitters can be of different types, including both natural and artificial ones.

It might not be straightforward to tune the separation between the emitters and the scatterers if they are already grown in a nanophotonic structure. We propose to install several different groups of quantum scatterers at different positions in the waveguide. When the transition frequencies of the scatterers are tuned to be outside the band and far away from the resonance frequency of a single cavity, this group of scatterers can be considered to be turned off and hence irrelevant to our self-interference mechanism. For this reason, one can effectively realize the position tuning of the scatterers by adjusting their resonance frequencies.

Finally, we discuss how the main idea of this work can be realized on an experimental platform consisting of superconducting transmission line resonators and transmon qubits. The coplanar transmission line resonators, which offer the continuum for the coherent transport of nonlinear photons, can be constructed by many equal transmission-line segments coupling with each other by dielectric materials [5, 60]. The transmon qubit is viewed as a Cooper pair box shunted by a capacitor that is large relative to the stray capacitance of the Josephson junction [61]. The frequencies of transmon qubits can be manipulated via the external magnetic flux intersecting the loop formed by the SQUID [61, 62]. The practicable values for the coupling strength between resonator and transmon qubits range from a few to hundreds of MHz while the qubit frequency can be controlled from a few to tens of GHz that is similar to the range of the resonance frequency of each resonator [2, 63].

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DYNAMICS OF COLLECTIVE SPONTANEOUS EMISSION

The time evolution of the amplitudes defined in Eq. (3) of the main text is given by

\[
i \frac{\partial}{\partial t} C_j^i (t) = \Omega_{j,i} C_j^i (t) + \sum_k \frac{V_{j,i} e^{i k x_i}}{\sqrt{N}} C_k (t), \quad (SM1)
\]

\[
i \frac{\partial}{\partial t} C_k (t) = \omega_k C_k (t) + \sum_{j,i} \frac{V_{j,i} e^{-i k x_i}}{\sqrt{N}} C_j^i (t), \quad (SM2)
\]

where \(x(A) = x_1\) and \(x(B) = x_2\). To proceed further one may invoke the Wigner-Weisskopf and Markovian theories by neglecting the contributions of bound states, but this treatment would not be able to capture the fractional trapping [1, 2]. To go beyond these approximations, we take the Laplace transform of Eqs. (SM1) and (SM2), yielding

\[
i \left[ -C_j^i (0) + s C_j^i (s) \right] = \Omega_{j,i} \tilde{C}_j^i (s) + \sum_k \frac{V_{j,i} e^{i k x_i}}{\sqrt{N}} \tilde{C}_k (s), \quad (SM3)
\]

\[
i \left[ -C_k (0) + s \tilde{C}_k (s) \right] = \omega_k \tilde{C}_k (s) + \sum_{j,i} \frac{V_{j,i} e^{-i k x_i}}{\sqrt{N}} \tilde{C}_j^i (s). \quad (SM4)
\]

The expression of \(\tilde{C}_j^i (s)\) is written as

\[
i \left[ -C_j^A (0) + s \tilde{C}_j^A (s) \right] = \Omega_A \tilde{C}_j^A (s) + \frac{V_A}{N} \sum_k \frac{1}{i s - \omega_k} \left[ \sum_{j'} V_A \tilde{C}_j'^A (s) + \sum_{j'} V_B e^{-i k \Delta x} \tilde{C}_j'^B (s) \right], \quad (SM5)
\]

\[
i s \tilde{C}_j^B (s) = \Omega_B \tilde{C}_j^B (s) + \frac{V_B}{N} \sum_k \frac{1}{i s - \omega_k} \left[ \sum_{j'} V_A e^{i k \Delta x} \tilde{C}_j'^A (s) + \sum_{j'} V_B \tilde{C}_j'^B (s) \right], \quad (SM6)
\]

Substituting \(\tilde{C}_j^B (s)\) in Eq. (SM5) into Eq. (SM6) and denoting the initial excited QEs by \(j_n\), so \(\tilde{C}_j^A (s)\) can be found as the following:

\[
\tilde{C}_{j_1}^A (s) = i \left\{ \frac{K_A (s, M_A - 1) K_B (s, M_B) - (M_A - 1) M_B [V_A V_B F (s, \Delta x)]^2}{(is - \Omega_A) \left[ K_A (s, M_A) K_B (s, M_B) - M_A M_B [V_A V_B F (s, \Delta x)]^2 \right]} C_{j_2}^A (0) \right. \\
+ i \left. \frac{K_B (s, M_B) V_A^2 F (s, 0) + M_B [V_A V_B F (s, \Delta x)]^2}{(is - \Omega_A) \left[ K_A (s, M_A) K_B (s, M_B) - M_A M_B [V_A V_B F (s, \Delta x)]^2 \right]} C_{j_2}^A (0) \right\}, \quad (SM7)
\]

where \(K_i (s, M_i) = i s - \Omega_i - M_i V_i^2 F (s, 0)\). \(F(s, x)\) are given by

\[
F(s, x) = \frac{(is - is \sqrt{(s^2 + 4 J^2) / s^2})^{\lfloor x \rfloor}}{is \sqrt{(s^2 + 4 J^2) / s^2 \lfloor 2 J \rfloor}}. \quad (SM8)
\]

In the calculations above, we have used the formula [3]

\[
\frac{1}{2 \pi} \int dk \frac{e^{i k x}}{z + 2 J \cos (k)} = \frac{- \frac{i}{2} \pi + \frac{i}{2} \pi \sqrt{1 - \left( \frac{2 J}{z} \right)^2}^{\lfloor z \rfloor}}{z \sqrt{1 - \left( \frac{2 J}{z} \right)^2}}.
\]
Without loss of generality, we set $\omega_c$ to be zero as reference energy. By exchanging the positions of $C_j^A(0)$ and $C_j^A(0)$ in Eq. (SM7), one can analogously obtain the time evolution expression of $\tilde{C}^A_j(t)$. The amplitude $C_j^A(t)$ is given by the inverse Laplace transform $C_j^A(t) = (1/2\pi i) \int_{-\infty}^{\infty} \tilde{C}^A_j(s)e^{st}ds$. To evaluate this integral, we consider the analytic behavior of $\tilde{C}^A_j(s)$ in the whole complex plane except a branch cut along the imaginary axis from $-i2J$ to $i2J$.

With the residue theorem, the time dependence of $\tilde{C}^A_j(t)$ can then be obtained:

$$C_j^A(t) = \frac{L_1(s)C_j^A(0) + L_2(s)C_j^0(0)}{G(s)}e^{st}|_{s=-\Omega_A} + \sum_{m} iL_2(s) \frac{[C_j^A(0) + C_j^0(0)]}{(is - \Omega_A)|G(s)|'}e^{st}|_{s=x^2_{m}}$$

$$- \sum_{\alpha=\pm} \int_{-\infty}^{\infty} \left[ Q_1^\alpha C_j^A(0) + Q_2^\alpha C_j^0(0) \right] e^{2iJyt} \frac{2\pi i\alpha (y + \Omega_A)}{(Q_1^\alpha - Q_2^\alpha)}dy,$$

where $L_1(s)$ and $L_2(s)$ are defined as

$$L_1(s) = K_A(s, M_A - 1)K_B(s, M_B) - (M_A - 1)M_B[V_AV_BF(s, \Delta x)]^2$$

and

$$L_2(s) = K_B(s, M_B)V_A^2F(s, 0) + M_B[V_AV_BF(s, \Delta x)]^2.$$ 

The function $G(s) = L_1(s) - L_2(s)$ and $[G(s)]'$ means the derivative of $G(s)$ with respect to $s$. Here $x_m$ is pure imaginary and the equation $G(-iE) = 0$ is nothing but the system's eigenenergy equation of the localized photon-QE dressed states. The functions $Q_1^\pm$ and $Q_2^\pm$ are given by

$$Q_1^\pm = U_1^\pm(y, M_A - 1)U_2^\pm(y, M_B) - (M_A - 1)M_B[V_AV_Bf^\pm(y, \Delta x)]^2$$

and

$$Q_2^\pm = U_2^\pm(y, M_B)V_A^2f^\pm(y, 0) + M_B[V_AV_Bf^\pm(y, \Delta x)]^2,$$

where $U_i^\pm(y, M_i) = -2Jy - \Omega_i - M_iV_i^2f_i\pm(y, 0)$ and $f_i\pm(y, x)$ are given by

$$f_i\pm(y, x) = \pm i \frac{(-y \pm i\sqrt{1-y^2})^{\pm-x}}{2\sqrt{1-y^2}}.$$  

By exchanging the positions of $C_j^A(0)$ and $C_j^A(0)$ in Eq. (SM9), one can get the time evolution of $C_j^A(t)$.

For $M_A = 1$ and $V_A \ll 2J$ with $\Delta x = 2n + 1$ ($n$ being in integer), the first two terms in Eq. (SM9) are found to be numerically much smaller than the third term when both type-A and type-B QEs are near resonance with a single cavity. Dropping then first two terms, $C_j^A(t)$ can thus be approximately obtained as the following:

$$C_j^A(t) \approx \frac{J}{\pi i} \int_{-\infty}^{\infty} \left[ \frac{2Jy - i \frac{M_BV_B^2}{2J\sqrt{1-y^2}}}{2Jy - i \frac{V_A^2}{2\sqrt{1-y^2}}} \left( \frac{2Jy - i \frac{M_BV_B^2}{2J\sqrt{1-y^2}}} {2Jy - i \frac{M_BV_B^2}{2J\sqrt{1-y^2}}} + M_BV_A^2V_B^2 (\frac{\Delta x}{2\sqrt{1-y^2}})^2 \right) \right] e^{2iJyt}dy.$$  

After the radiation field comes back to the decaying QE and under the condition $\sqrt{MBVB} < \sim 2J$, the oscillating term $e^{2iJyt}$ in the above expression indicates that the above integrand is insensitive to the lower or upper limit of this integration with small $\Delta x$ (see the following comparison with different values of $\Delta x$ between the approximate result and exact result in Fig. SM1). As such, the integration range can be safely extended to infinity. Then $C_j^A(t)$ can be written as

$$C_j^A(t) \approx \frac{J}{\pi i} \int_{-\infty}^{\infty} \left[ \frac{2Jy - i \frac{M_BV_B^2}{2J}}{2Jy - i \frac{V_A^2}{2\sqrt{1-y^2}}} + M_BV_A^2V_B^2 (\Delta x)(\frac{\Delta x}{2\sqrt{1-y^2}})^2 \right] e^{2iJyt}dy.$$  

$$C_j^A(t) \approx \frac{J}{\pi i} \int_{-\infty}^{\infty} \left[ \frac{2Jy - i \frac{M_BV_B^2}{2J}}{2Jy - i \frac{V_A^2}{2\sqrt{1-y^2}}} + M_BV_A^2V_B^2 (\Delta x)(\frac{\Delta x}{2\sqrt{1-y^2}})^2 \right] e^{2iJyt}dy.$$  

$$C_j^A(t) \approx \frac{J}{\pi i} \int_{-\infty}^{\infty} \left[ \frac{2Jy - i \frac{M_BV_B^2}{2J}}{2Jy - i \frac{V_A^2}{2\sqrt{1-y^2}}} + M_BV_A^2V_B^2 (\Delta x)(\frac{\Delta x}{2\sqrt{1-y^2}})^2 \right] e^{2iJyt}dy.$$
where the emission rate $\Gamma_h$ can also be found with small $\Delta x$, namely

$$\Gamma_h = 4JV_A^2/(J^2 - \Delta xV_A^2)$$

and

$$C_{\text{sup}}(t) \approx \frac{M_BV_B^2(J^2 - \Delta xV_A^2) - 4J^2V_A^2}{(J^2 - \Delta xV_A^2)\sqrt{2(\chi^2 - 16M_BV_A^4V_B^2)}e^{-\frac{\chi}{2}t}},$$

where $\chi = M_BV_B^2 - \Delta xM_BV_A^2V_B$.

FIG. SM1. Excited state population $P_e(t) = |C_{ij}(t)|^2$ with $\Delta x = 1$ in (a), $\Delta x = 5$ in (b), $\Delta x = 9$ in (c), $\Delta x = 13$ in (d), $\Delta x = 17$ in (e), $\Delta x = 21$ in (f). The parameters are $V_A/(2J) = 0.07$, $V_B/(2J) = 1.8$, $\Delta_A/(2J) = 0$, $\Delta_B/(2J) = 0$, $M_A = 1$ and $M_B = 2$. Time is in units of $1/(2J)$. 

The poles of the first term in the integrand are

$$y_{0,\pm} = i\frac{1}{2} \left( \frac{M_BV_B^2}{4J^2} + \frac{V_A^2}{4J^2} - 2\Delta xM_BV_A^2V_B^2 \right),$$

$$\pm \frac{1}{2} \sqrt{- \left( \frac{M_BV_B^2}{4J^2} + \frac{V_A^2}{4J^2} - 2\Delta xM_BV_A^2V_B^2 \right)^2 + 8M_BV_A^2V_B^2}. \quad (SM13)$$

For $\sqrt{M_BV_B} \sim 2J$, $y_{0,\pm}$ are pure imaginary. Using the residue theorem, one reduces $C_{j_1}(t)$ to the following:

$$C_{j_1}(t) \approx \frac{M_BV_B^2/(4J^2) - \alpha_1/2}{\sqrt{\beta_1}} e^{-\alpha_1t}, \quad (SM14)$$

where $\alpha_1 = V_A^2/[J^2 - \Delta xV_A^2/2]$ and $\beta_1 = \{(2J^2M_BV_B^2 - \Delta xM_BV_A^2V_B^2)/(8J^4)\}^2 - M_BV_A^2V_B^2/(2J^4)$.

In Fig. SM1, we compare the approximate results (dotted lines) in Eq. (SM14) with the exact results (solid lines) in Eq. (SM9) for different values of $\Delta x$. As $\Delta x$ decreases, the approximate results agree better with the exact results.
THEORY OF PHOTON SCATTERING IN A WAVEGUIDE

To describe the radiation field propagating towards and interacting with the quantum scatterers, we start with the Green’s function defined in terms of the system Hamiltonian $H_{\text{sca}}$ (assuming there are only scatterers but no emitters)

$$ G(z) = \frac{1}{z - H_{\text{sca}}} \quad (\text{SM16}) $$

where $z$ is a complex energy variable and the expression of $H_{\text{sca}}$ is

$$ H_{\text{sca}} = H_{\text{ph}}^0 + V_{\text{ph}} \quad (\text{SM17}) $$

and

$$ H_{\text{ph}}^0 = \sum_k \omega_k a_k^\dagger a_k + \sum_j \Omega_B \langle e_j^B \rangle \langle e_j^B \rangle, \quad (\text{SM18}) $$

$$ V_{\text{ph}} = \sum_j \sum_k V_B \sqrt{N} \left( \sigma_j^B a_k e^{ikx_2} + \sigma_j^B a_k^\dagger e^{-ikx_2} \right). \quad (\text{SM19}) $$

The Green’s function $G(z)$ is an analytic function on the complex $z$ plane except at those points and branch cut of the real axis (which correspond to the eigenvalues of bound states and scattering states). $G(z)$ satisfies the resolvent equations [3]

$$ G(z) = G_0(z) + G_0(z) V_{\text{ph}} G(z). \quad (\text{SM20}) $$

Here $G_0(z)$ is the free Green’s function and is written as

$$ G_0(z) = \frac{1}{z - H_{\text{ph}}^0}. \quad (\text{SM21}) $$

The S-matrix element in this scattering process is given by

$$ S_{p,k} = \delta_{p,k} - i2\pi \delta (\omega_p - \omega_k) T_{p,k} (\omega_k + i\epsilon), \quad (\text{SM22}) $$

where $T(z) = V_{\text{ph}} + V_{\text{ph}} G(z) V_{\text{ph}}$ is the T-matrix and $T_{p,k} (\omega_k + i\epsilon) = \langle g,\text{vac} | a_p T (\omega_k + i\epsilon) a_k^\dagger | g,\text{vac} \rangle$. With the method of self-consistent equations [2], $T_{p,k} (\omega_k + i\epsilon)$ is found to be

$$ T_{p,k} (\omega_k + i\epsilon) = \frac{M_B V_B^2}{\omega_k - \Omega_B - \frac{M_B V_B^2}{N} \sum_{k'} \frac{1}{\omega_k - \omega_{k'} + i\epsilon}}. \quad (\text{SM23}) $$

So the S-matrix element is

$$ S_{p,k} = (1 + r_k) \delta_{p,k} + r_k \delta_{-p,k} \quad (\text{SM24}) $$

with the reflection amplitude

$$ r_k = \frac{-iM_B V_B^2}{|\partial \omega_k / \partial k| (\omega_k - \Omega_B) + iM_B V_B^2}. \quad (\text{SM25}) $$

For the case of $M_B = 1$, Eq. (SM25) comes back to the result obtained in [4]. The transmission amplitude can be obtained from relations $t_k = 1 + r_k$ and $|r_k|^2 + |t_k|^2 = 1$.

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