Extensions of Birkhoff’s Theorem in 6D Gauss-Bonnet Gravity

Charalampos Bogdanos
Laboratoire de Physique Theorique
Universite de Paris-Sud 11
Bateau 210, 91405 Orsay CEDEX, France
E-mail: Charalampos.Bogdanos@th.u-psud.fr

Abstract. We present a generalization of Birkhoff’s theorem in the context of six-dimensional Gauss-Bonnet theory. Contrary to what we encounter in ordinary General Relativity, the presence of the higher curvature terms leads to novel constraints on the geometry of the admissible black hole horizons. As a result, we can have static solutions which depart from spherical symmetry, but must satisfy specific constraints involving the Weyl tensor of the four-dimensional horizon. After obtaining the general profile of the static solutions in the transverse space, we give explicit examples of such black hole horizons, which are in general of an anisotropic nature.

1. Introduction
Higher dimensional theories of gravity have become a very prominent research field in theoretical physics over the last decade[1]. With the introduction of novel ways to incorporate extra dimensions to explain puzzling phenomena such as the hierarchy problem[2, 3, 4, 5], a considerable effort has been devoted to the study and clarification of the rich phenomenology of these models. Braneworlds and other higher dimensional modifications of ordinary General Relativity, like Gauss-Bonnet and Lovelock[6] theory have been considered as possible extensions in the quest for a more complete theory of gravity. The absence of a consistent theory of quantum gravity, as well as mounting evidence from the front of observational cosmology seem to suggest that General Relativity may indeed need some non-trivial modification along these lines. In particular, the observed accelerated expansion of the universe[7, 8], as well as the problem of a very small cosmological constant[9] responsible for this expansion cannot be addressed in a satisfying manner within the context of pure GR. Higher dimensional theories like Gauss-Bonnet gravity thus seem as a likely candidate for an infrared completion of GR, accounting for accelerated expansion in geometrical terms (for a review see[10]).

One of the most well known results in ordinary GR is Birkhoff’s theorem: A spherically symmetric solution to Einstein’s equations is necessarily static. As a result, the Schwarzschild solution is the unique static and spherically solution in four dimensions and any other static solution will be necessarily (locally) isomorphic to Schwarzschild[11, 12], with the possibility of a non-trivial overall topological difference. This theorem is in fact amplified in strength when we consider higher dimensional GR with just the ordinary Einstein-Hilbert action[13, 14]. As it turns out in this case, spherical symmetry is not necessary in order to ensure the staticity of the
solutions. In fact, the horizon of the higher dimensional black hole, and consequently any other hypersurface in the transverse direction can be any kind of Einstein space. We are thus allowed to take the horizon geometry of a higher dimensional Schwarzschild black hole, substitute it with any Einstein space of our liking and the result will be a new, static solution to the full Einstein’s equations.

When new terms are introduced in the gravitational action of a higher dimensional theory, not surprisingly, non-trivial modifications to Birkhoff’s theorem occur. This has already been studied in five dimensions for the Gauss-Bonnet theory[15], as well as in arbitrary dimensions for a more general Lovelock theory in the context of spherically symmetric solutions[16]. As it turns out, for the Gauss-Bonnet case, there are two classes of solutions emerging, with only one of them leading to static solutions. Within the static category, we encounter two distinct branches, characterized by the sign of the Gauss-Bonnet coupling. Only one of them is perturbatively connected to pure GR, while the other is found to be unstable. Similar results also emerge in the Lovelock case.

Here we consider the case of six-dimensional Gauss-Bonnet theory, where the assumption of spherical symmetry is relaxed. Our goal is to study whether a generalization of Birkhoff’s theorem is possible and what are the restrictions on the admissible horizon geometry of a six-dimensional black hole. As we will see, the presence of the Gauss-Bonnet term leads to a number of non-trivial constraints on the geometry of the horizon, in order to have a static solution. We can thus prove a version of Birkhoff’s theorem in six-dimensional Gauss-Bonnet theory, which is stronger than in four-dimensional GR (we can also have static solutions which are not spherically symmetric), but more constrained than its higher dimensional GR counterpart (not every Einstein space is a good horizon solution). The square of the Weyl tensor of the horizon, $C_{\alpha \beta \gamma \delta}C^{\alpha \beta \gamma \delta}$, figures prominently in the constraint. This is also one of the reasons why the six-dimensional model is qualitatively different from the five-dimensional one. In five dimensions, the horizon of the black hole is three-dimensional and the Weyl tensor is automatically zero. In six dimensions the horizon becomes four-dimensional and the Weyl tensor (and its square) is in general non-vanishing. So six dimensions is the least we need to have a non-trivial contribution from the horizon geometry. When considered in the context of Lovelock gravity, this is also the last case for which we can safely neglect any higher curvature corrections in the action. For $d = 7$ and above, we should also include the next Lovelock density $L_3$ and even more densities as we keep increasing the dimensionality of the construction.

That the Weyl tensor of the horizon enters the solution should come as no surprise. Contrary to ordinary GR, the Gauss-Bonnet term exposes the entire Riemann tensor to the equations of motion, hence the dependence on $C_{\alpha \beta \gamma \delta}$. As it was recently shown, these higher curvature terms can constrain the form of the solution for a priori static configurations[17]. We show that this is a general characteristic which can be used to classify solutions obeying Birkhoff’s theorem. The static solutions have as an additional parameter the square of the Weyl tensor. We also give some explicit examples of horizon geometries compatible with the constraints. In general, these are topologically non-trivial and of an anisotropic nature.

2. Equations of Motion and Staticity Conditions

We begin by considering the Einstein-Gauss-Bonnet action with a cosmological constant in six dimensions

$$S^{(6)} = \frac{M^{(6)}_6}{2} \int d^6x \sqrt{-g^{(6)}} \left[ R - 2\Lambda + \alpha \hat{G} \right], \quad (1)$$

where $M^{(6)}_6$ is the fundamental mass scale in six-dimensional spacetime, $\hat{G}$ the Gauss-Bonnet density defined as

$$\hat{G} = R_{ABCD}R^{ADCB} - 4R_{AB}R^{AB} + R^2. \quad (2)$$
and the cosmological constant $\Lambda = -10k^2 < 0$ and $\Lambda = 10a^2 > 0$. Using these conventions we can vary the action with respect to the field metric to derive the equations of motion

$$\mathcal{E}_{AB} = G_{AB} + \Lambda g_{AB} + \alpha H_{AB} = 0,$$

(3)

where $G_{AB}$ stands for the Einstein tensor. Uppercase indices refer to six-dimensional coordinates. We have also introduced the Gauss-Bonnet tensor,

$$H_{AB} = \frac{g^{AB}}{2} \hat{G} - 2R_{RA}R_{BC} + 4R_{AC}R_{BD} + 2R_{ACDE}.$$

(4)

We now choose an appropriate ansatz to look for static solutions of a black hole profile,

$$ds^2 = e^{2\nu(t,z)}B(t,z)^{-3/4}(-dt^2 + dz^2) + B(t,z)^{1/2}h^{(4)}_{\mu\nu}(x)dx^\mu dx^\nu.$$

(5)

The coordinates $t, z$ parametrize the “transverse space”. Lowercase greek indices correspond to internal coordinates of the 4-space, which also characterizes the black hole horizon. The two functions $B(t, z)$ and $\nu(t, z)$ determine the profile of the transverse geometry. We then switch the coordinates of the transverse space to light-cone coordinates,

$$u = \frac{t - z}{\sqrt{2}}, \quad v = \frac{t + z}{\sqrt{2}},$$

(6)

in terms of which the metric reads

$$ds^2 = -2e^{2\nu(u,v)}B(u,v)^{-3/4}dudv + B(u,v)^{1/2}h^{(4)}_{\mu\nu}(x)dx^\mu dx^\nu.$$  

(7)

Using the above prescription, we are now able to write down the equations of motion. The $uu$ and $vv$ equations yield

$$\mathcal{E}_{uu} = \frac{2\nu_u B_{,u} - B_{,uu}}{B} \left(1 + \alpha \left(B^{-1/2}R^{(4)} + \frac{3}{2}e^{-2\nu}B^{-5/4}B_{,u}B_{,u}\right)\right),$$

(8)

$$\mathcal{E}_{vv} = \frac{2\nu_v B_{,v} - B_{,vv}}{B} \left(1 + \alpha \left(B^{-1/2}R^{(4)} + \frac{3}{2}e^{-2\nu}B^{-5/4}B_{,u}B_{,u}\right)\right).$$

(9)

The off-diagonal equation reads

$$\mathcal{E}_{uv} = \frac{B_{,uv}}{B} - \Lambda e^{2\nu}B^{-3/4} + \frac{\alpha}{2}e^{2\nu}B^{-7/4}\hat{G}^{(4)} + R^{(4)} \left(\frac{1}{2}e^{2\nu}B^{-5/4} - \alpha B^{-3/2} \left(\frac{1}{2}B_{,u}B_{,v} - B_{,uv}\right)\right) + \alpha e^{-2\nu}B^{-5/4} \left(-\frac{15}{16} \left(\frac{B_{,u}B_{,v}}{B}\right)^2 + \frac{3}{2}B_{,u}B_{,v}B_{,uv}\right).$$

(10)

We also have the $\mu\nu$ equations, which can be brought into the form

$$\mathcal{E}_{\mu\nu} = G^{(4)}_{\mu\nu} - e^{-2\nu}B^{1/4} \left(\frac{3}{4}B_{,\mu\nu} + 2B_{,\mu\nu}\right)h^{(4)}_{\mu\nu} + \Lambda B^{1/2}h^{(4)}_{\mu\nu}$$

$$+ \frac{3}{2}\alpha e^{-4\nu}(B_{,\mu\nu} - 2\nu_{,\mu}B_{,\nu})(B_{,\mu\nu} - 2\nu_{,\nu}B_{,\mu})h^{(4)}_{\mu\nu}$$

$$- \alpha e^{-4\nu} \left(\frac{45}{32} \left(\frac{B_{,u}B_{,v}}{B}\right)^2 - \frac{21}{8}B_{,u}B_{,v}B_{,uv} + \frac{3}{2}B_{,u}B_{,\mu\nu}B_{,\mu\nu} + 3B_{,u}B_{,\mu\nu}B_{,\mu\nu}\right)h^{(4)}_{\mu\nu}$$

$$- \alpha e^{-2\nu}B^{-1/4} \left(\frac{3}{4}B_{,u}B_{,v} - \frac{1}{2}B_{,uv} + 4B_{,\mu\nu}\right) \left(R^{(4)}h^{(4)}_{\mu\nu} - 2R^{(4)}_{\mu\nu}\right).$$

(11)
The classification of solutions proceeds by inspecting the so-called integrability conditions, (8) and (9). We see that we can set these equal to zero provided that we either choose

\[ 1 + \alpha \left( B^{-1/2} R^{(4)} + \frac{3}{2} e^{-2\nu} B^{-5/4} B_u B_v \right) = 0 , \]  

(12)

or

\[ 2\nu_u B_u - B_{uu} = 2\nu_v B_v - B_{vv} = 0 . \]  

(13)

Choosing the former leads to the class of solutions which we call class I. These are new solutions specific to the Gauss-Bonnet case (since for \( \alpha = 0 \) this equation cannot be satisfied). The later choice gives rise to the class II solutions. These are essentially the same integrability conditions that we also encounter in ordinary GR. As we will see, class I solutions are generally time-dependent and thus do not obey Birkhoff’s theorem, however their physical relevance is ambiguous. Class II solutions on the other hand are the ones obeying Birkhoff under certain restrictions on the internal geometry of the horizon.

We deal with class I solutions first. From the integrability condition (12) we can solve for \( \nu(u,v) \) as

\[ \nu(u,v) = \frac{1}{2} \ln \left( \frac{-3\alpha}{2} B^{5/4} \left( 1 + \alpha B^{-1/2} R^{(4)} \right) \right) . \]  

(14)

Restrictions are already imposed on the four-dimensional geometry of the internal space, as only spaces of constant scalar curvature \( R^{(4)} \) allow for separation of variables. Plugging the above expression for \( \nu(u,v) \) into the \( uv \) equation leads to the following constraints

\[ 5 + 12\alpha \Lambda = 0, \quad \hat{G}^{(4)} = \frac{1}{6} R^{(4)} , \]  

(15)

and the \( \mu\nu \) equation reduces to

\[ \mathcal{E}_{\mu\nu} = \left( R^{(4)}_{\mu\nu} - \frac{1}{4} R^{(4)} h^{(4)}_{\mu\nu} \right) \left[ 1 + 2\alpha e^{-2\nu} B^{-1/4} \left( \frac{3}{4} B_u B_v \right) - \frac{1}{2} B_{uu} + 4B\nu_{uv} \right] . \]  

(16)

There are obviously two ways to satisfy this final equation and get a solution. Setting the first parenthesis equal to zero is practically equivalent to the definition of an Einstein space for the internal geometry. Coupled with (15), this leads to the additional condition \( C^{(4)}_{\alpha\beta\mu\nu} C^{(4)}_{\alpha\beta\mu\nu} = 0 \), so the Weyl-squared of the internal space must also be zero. Under these conditions, we do have a valid solution, with no other equation left to satisfy. But the function \( B(u,v) \) is undetermined up to this point! We thus realise that the system of equations is in this case underdetermined and since \( B(u,v) \) is arbitrary, it can also be time-dependent and Birkhoff’s theorem will not hold in general for this kind of solutions.

On the other hand, by requiring the second parenthesis in (16) to be zero, we end up with a third order differential equation for \( B(u,v) \), which can in principle be solved and thus completely determine the transverse geometry. Again, these solution can in general be time-dependent, so Birkhoff’s theorem is again violated. Still, there is now no additional equation constraining the internal space, which may now not be of the Einstein variety. The underdeterminacy of the system is in this sense shifted from the transverse to the internal space. The presence of an underdetermined system is a sign of pathology, so although these solutions are not static, they may be in the end physically irrelevant. As a result, we will now focus on the class II solutions, for which no such problems emerge.

For the class II solutions the integrability conditions promptly lead to an equation of the form

\[ e^{2\nu} = B_{uu} = B_{vv} , \]  

(17)
which, in turn, yields $B = B(U + V)$, with $U = U(u)$ and $V = V(v)$. In this way, under a change of variables of the form of (6) the function $B$ does not depend on time and thus Birkhoff’s theorem is satisfied as well. Additionally, rewriting (17), $\nu(u,v)$ is now defined as

$$e^{2\nu} = B'U'V', \quad (18)$$

where primes denote differentiation with respect to the single argument of each function. The $uu$ and $vv$ equations thus determine the staticity of the metric, as well as the relation between $B$ and $\nu$. We can then determine $B(u,v)$, or equivalently the form of the black hole potential, from the $uv$ equation. Taking advantage of the already deduced staticity, we can express this as

$$B'' + \frac{1}{2} R^{(4)}B^{-1/4}B' - \frac{15}{16} \alpha B^{-9/4}B'' + \frac{3}{2} \alpha B^{-5/4}B''B'$$

$$- \frac{1}{2} \alpha R^{(4)}B^{-3/2}B'' + \alpha B^{-1/2}R^{(4)}B'' + \frac{1}{2} \alpha B^{-3/4}B' G^{(4)} - \Lambda B^{1/4}B' = 0. \quad (19)$$

Inspection of the above expression leads to the conclusion that a priori only solutions with a constant Ricci scalar and Gauss-Bonnet density for the internal space are permissible. Upon integration, this leads to a quadratic equation for $B'$. We can then solve for $B'$ and determine the black hole potential $V(r)$

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 h_{\mu\nu}(x) dx^\mu dx^\nu, \quad (20)$$

using the change of variables $r = B^{1/4}$. The corresponding potential turns out to be

$$V(r) = \frac{R^{(4)}}{12} + \frac{r^2}{12\alpha} \left[ 1 \pm \sqrt{1 + \frac{12\alpha \Lambda}{5} + \frac{\alpha^2 (R^{(4)} - 6G^{(4)})}{r^4} + 24\frac{\alpha M}{r^3}} \right], \quad (21)$$

where $M$ is an integration constant independent of $x$, related to the mass of the six-dimensional black hole.

As in the five-dimensional case, we encounter within the static class II solutions two branches, corresponding to the two possible signs in the potential (21). The plus sign corresponds to the so-called Gauss-Bonnet branch and is not perturbatively connected to the usual six-dimensional black hole solutions of general relativity, as can be easily inferred. This branch has been shown to be unstable[19]. The minus sign gives rise to the Einstein branch, which has a well-defined limit for $\alpha \to 0$. We will consider this to be the physical branch from now on.

We will now clarify the significance of the constraints on the internal geometry. We have already seen that the horizon should first have constant Ricci scalar. Furthermore, the combination $R^{(4)} - 6G^{(4)}$ appearing in the expression for the potential is indeed related to the square of the four-dimensional Weyl tensor and must also be a constant. To illustrate this better, we consider a horizon geometry of the Einstein kind with a constant curvature $R^{(4)}_{\mu\nu} = 3\kappa h_{\mu\nu}$. Coupled to the requirement of constant Gauss-Bonnet scalar, this leads to the relation $C^{\alpha\beta\gamma\mu}C_{\alpha\beta\gamma\mu} = \Theta$ where $\Theta$ is a positive constant. Thus we get the condition

$$C^{\alpha\beta\gamma\mu}C_{\alpha\beta\gamma\mu} = \Theta \delta^\mu_\nu, \quad (22)$$

1 There are, however, special cases which should be treated separately. For a more detailed discussion and connection with black strings, see [18].
i.e. the square of the Weyl tensor of the internal space must be constant. The potential in this case reduces to

\[ V(r) = \kappa + \frac{r^2}{12\alpha} \left( 1 \pm \sqrt{1 - 24k^2\alpha - 24\frac{\alpha^2\Theta}{r^4} + 24\frac{\alpha M}{r^5}} \right), \]  

(23)

and in general the parameter \( \Theta \) acts now as a classifier for the admissible horizons. Static solutions that obey Birkhoff’s theorem have to satisfy the constraints of constant \( R(4) \) and \( \Theta \), which is a more restrictive situation compared to higher dimensional GR. Still, the theorem remains stronger than in four-dimensional GR, since spherical symmetry is not a necessary condition for staticity. In fact, as we will discuss in the following section, we can have horizon geometries which significantly depart from spherical symmetry.

3. Horizon Geometries

As we have already discussed, the presence of the Gauss-Bonnet term leads to non-trivial constraint on the Weyl tensor of the internal geometry. We will now present some explicit examples of such admissible geometries. In looking for such four-dimensional spaces of constant \( \Theta \), we immediately realize that these spaces cannot be asymptotically flat, as this would lead us to the admissible but trivial case of \( \Theta = 0 \). In general, we have to consider constructions of a non-trivial topology and a shift from asymptotically flat to asymptotically locally flat. A first example is the case of an \( S^2 \times S^2 \) geometry, the product of two spheres, bearing the ansatz

\[ ds^2 = \rho_1^2 \left( d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 \right) + \rho_2^2 \left( d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 \right), \]  

(24)

where we consider the (dimensionless) radii \( \rho_1 \) and \( \rho_2 \) of the spheres to be constant. The entire six-dimensional space has the form

\[ ds^2 = -V(r) \, dt^2 + \frac{dr^2}{V(r)} + r^2 \rho_1^2 \left( d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 \right) + r^2 \rho_2^2 \left( d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 \right), \]  

(25)

with the potential

\[ V(r) = \frac{R(4)}{12} + \frac{r^2}{12\alpha} \left( 1 \pm \sqrt{1 - 24k^2\alpha - 24\frac{\alpha^2\Theta}{r^4} + 24\frac{\alpha M}{r^5}} \right). \]  

(26)

In order for (25) to be a solution to the Gauss-Bonnet equations of motion, we are led to the condition of equal sphere radii, \( \rho_1 = \rho_2 \). The Ricci scalar and \( \Theta \) are in this case \( R(4) = \frac{4}{\rho_1^2} \), \( \Theta = \frac{4\alpha^2}{3\rho_1^2} \). Clearly, this is an anisotropic, non-asymptotically flat space.

Another example of topologically non-trivial space is the Taub-NUT/Bolt geometry. This is a homogeneous but anisotropic space, which is asymptotically locally flat. It is not globally flat because of the non-trivial topology. The general metric describing Taub-NUT/Bolt spaces of Euclidean signature is

\[ ds^2 = V(r) \left( d\tau^2 + 2n \cos\theta d\phi \right)^2 + \frac{dr^2}{V(r)} + \left( r^2 - n^2 \right) \left( d\theta^2 + \sin^2\theta d\phi^2 \right). \]  

(27)

Here, \( n \) is the nut parameter. Depending on the form of the potential \( V(r) \), we can either have a nut, i.e. a zero-dimensional fixed point set of the Euclidean time \( U(1) \) isometry, or a bolt, which is a two-dimensional fixed point set. Still, this is not yet a good candidate for our horizon.
What we need is the AdS variant of the Taub-NUT/Bolt space\[20\]. Written in the Pedersen form\[21\], it is

\[
ds^2 = \frac{4}{(1 - k^2 r^2)^2} \left[ \frac{1 - \mu^2 r^2}{1 - k^2 \mu^2 r^4} dr^2 + r^2 \left( 1 - \mu^2 r^2 \right) \left( \sigma_1^2 + \sigma_2^2 \right) + r^4 \left( 1 - k^2 \mu^2 r^4 \right) \sigma_3^2 \right] ,
\]

where the mass parameter $\mu$ is defined in terms of $k$, the curvature of AdS space and the nut parameter by $\mu^2 = k^2 - \frac{1}{4\nu^2}$. By calculating $\Theta$ for this space we find that $\Theta = 6\mu^4 \left( 1 - k^2 r^2 \right)^6$, which only becomes constant at infinity. But we notice that taking the $n \to \infty$ limit, or $\mu = k$, $\Theta$ becomes constant everywhere. Surprisingly, there is already a name for this geometry, known as Bergman space. In these coordinates it can be written as

\[
ds^2 = \frac{4}{(1 - k^2 r^2)^2} \left[ \frac{1}{1 + k^2 r^2} dr^2 + r^2 \left( 1 - k^2 r^2 \right) \left( \sigma_1^2 + \sigma_2^2 \right) + r^4 \left( 1 + k^2 r^2 \right) \sigma_3^2 \right] .
\]

It describes the coset space $SU(2,1)/U(2)$, which is a Kähler-Einstein manifold with Kähler potential

\[
K(z_1, \bar{z}_1, z_2, \bar{z}_2) = 1 - z_1 \bar{z}_1 - z_2 \bar{z}_2 , \quad \text{for } z_1 \bar{z}_1 + z_2 \bar{z}_2 < 1 ,
\]

and the topology of the open ball in $C^2$. Setting $z_1 = k\rho \cos(\theta/2)e^{i(\phi+\psi)/2}$ and $z_2 = k\rho \sin(\theta/2)e^{i(\phi-\psi)/2}$ the metric $g_{\alpha\bar{\beta}} = -\partial_\alpha \partial_{\bar{\beta}} \ln K^{1/k^2}$ reproduces exactly (29) after a change of coordinate $\rho^2 = 2r^2/(1 + k^2 r^2)$. The Bergman metric (29) has an isometry group of $SU(2,1)$. In practice, the choice $\mu = k$ corresponds to infinite “squashing” of the 3-sphere at the boundary $r \to 1/k$, such that only a one-dimensional circle remains intact. By comparing the terms multiplying $\sigma_1^2 + \sigma_2^2$ (2-sphere) and $\sigma_3^2$, we see that as we approach the boundary, the $\sigma_3^2$ part blows up faster and becomes dominant. The space has this circle as its conformal boundary. This space is asymptotically locally flat. It is now possible to see from the expression for $\Theta$ in AdS Taub-NUT that the Bergman space has $\Theta = 6k^4$ and is thus a suitable horizon solution. Substituting (29) as the metric of the internal space $h^{(4)}_{\mu\nu}$, we verify that it is a solution to the equations of motion. To do so, we first rescale the radial coordinate as $\rho = r/l$, with $l$ having dimensions of mass$^{-1}$ in order to make the metric dimensionless. As a result, we identify the dimensionless curvature scale $\kappa = kl$. The resulting black hole potential is then

\[
V (r) = -\kappa^2 + \frac{r^2}{12\alpha} \left( 1 \pm \sqrt{1 - 24k^2 \alpha - 144\kappa^4 \frac{\alpha^2}{r^4} + 24\alpha \frac{M}{r^3}} \right) .
\]

Bergman space is known to be unstable in ordinary GR\[22\], but here we consider its Euclidean version, for which the same conclusion doesn’t necessarily hold. It is also not known how the presence of the Gauss-Bonnet corrections modifies the stability properties, so it is hard to argue on the physical relevance of this construction.

4. Conclusions
We studied the staticity properties of six-dimensional black hole solutions in Gauss-Bonnet theory and proved a version of Birkhoff’s theorem for such setups. As we saw, we can have four-dimensional spaces of constant Ricci scalar and $\Theta$ as possible horizon geometries. Non-spherically symmetric solutions are admissible and we gave explicit examples in the form of $S^2 \times S^2$ and the Bergman space. In general, the internal space has a non-trivial topology and should not be asymptotically flat in order to obtain novel solutions. Geometries which exhibit asymptotic flatness only locally are good candidates. Intuitively, we also expect similar
conclusions to hold for even higher dimensional scenarios, in the context of the full Lovelock theory. There, the presence of higher curvature terms in the action will potentially lead to further constraints, involving quantities similar to $\Theta$. It is an open question whether a more systematic classification of such spaces can be established.

5. Acknowledgments
The author is indebted to the organisers of the First Mediterranean Conference in Classical and Quantum Gravity. This presentation is based on work done in collaboration with C. Charmousis, B. Gouteraux at LPT Orsay and R. Zegers at Durham University. The author is supported by the CNRS and the Université de Paris-Sud XI.

References
[1] Rubakov V A and Shaposhnikov M E 1983 Phys. Lett. B125 136–138
[2] Antoniadis I 1990 Phys. Lett. B246 377–384
[3] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 Phys. Lett. B429 263–272 (Preprint hep-ph/9803315)
[4] Randall L and Sundrum R 1999 Phys. Rev. Lett. 83 3370–3373 (Preprint hep-ph/9905221)
[5] Randall L and Sundrum R 1999 Phys. Rev. Lett. 83 4690–4693 (Preprint hep-th/9906064)
[6] Lovelock D 1971 J. Math. Phys. 12 498–501
[7] Riess e A G et al. (Supernova Search Team) 1998 Astron. J. 116 1009–1038 (Preprint astro-ph/9805201)
[8] Astier P et al. (The SNLS) 2006 Astron. Astrophys. 447 31–48 (Preprint astro-ph/0510447)
[9] Weinberg S 1989 Rev. Mod. Phys. 61 1–23
[10] Charmousis C 2009 Lect. Notes Phys. 769 299–346 (Preprint 0805.0568)
[11] Israel W 1968 Commun. Math. Phys. 8 245–260
[12] Israel W 1967 Phys. Rev. 164 1776–1779
[13] Gibbons G and Hartnoll S A 2002 Phys. Rev. D66 064024 (Preprint hep-th/0206202)
[14] Gibbons G W, Hartnoll S A and Pope C N 2003 Phys. Rev. D67 084024 (Preprint hep-th/0208031)
[15] Charmousis C and Dufaux J F 2002 Class. Quant. Grav. 19 4671–4682 (Preprint hep-th/0202107)
[16] Zegers R 2005 J. Math. Phys. 46 072502 (Preprint gr-qc/0505016)
[17] Dotti G and Gleiser R J 2005 Phys. Lett. B627 174–179 (Preprint hep-th/0508118)
[18] Bogdanos C, Charmousis C, Gouteraux B and Zegers R 2009 (Preprint 0906.4953)
[19] Charmousis C and Padilla A 2008 JHEP 0812 038 (Preprint 0807.2864)
[20] Chamblin A, Emparan R, Johnson C V and Myers R C 1999 Phys. Rev. D59 064010 (Preprint hep-th/9808177)
[21] Zoubos K 2002 JHEP 12 037 (Preprint hep-th/0209235)
[22] Kleban M, Porrati M and Rabadán R 2005 JHEP 08 016 (Preprint hep-th/0409242)