Is there a “Δ-isobar puzzle” in the physics of neutron stars?

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We discuss the formation of Δ isobars in neutron star matter. We show that their threshold density strictly correlates with the density derivative of the symmetry energy of nuclear matter, the L parameter. By restricting L to the range of values indicated by recent experimental and theoretical analysis, i.e. 40 MeV ≲ L ≲ 62 MeV, we find that Δ isobars appear at a density of the order of 2 ± 3 times nuclear matter saturation density, i.e. the same range for the appearance of hyperons. The range of values of the couplings of the Δs with the mesons is restricted by the analysis of the data obtained from photoabsorption, electron and pion scattering on nuclei. If the potential of the Δ in nuclear matter is close to the one indicated by the experimental data then the equation of state becomes soft enough that a “Δ puzzle” exists, similar to the “hyperon puzzle” widely discussed in the literature.

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Since the seminal paper of Ref. [1], the possible formation in the core of neutron stars of baryons heavier than the nucleon is one of the most interesting open issue in nuclear astrophysics. While a huge literature is available concerning the appearance of hyperons in neutron stars (see for instance Ref. [2] and references therein) only little work has been done to assess whether Δ(1232) isobars can also take place in those stellar objects [3–9]. The reason why Δ resonances have been neglected is maybe connected with the outcome of Ref. [1] indicating that these particles would appear at densities much higher than the typical densities of the core of neutron stars and they are therefore irrelevant for astrophysics. On the other hand, hyperons could appear already at 2 ± 3 times the density of nuclear matter, n_0 = 0.16 fm^{-3}, and one has to include these degrees of freedom when modeling the equation of state of dense nuclear matter. The consequent softening of the equation of state reduces the maximum mass of neutron stars which in many calculations drops below the 2M⊙ limit imposed by the precise measurements of the masses of PSR J1614-2230 and PSR J0348+0432 [11,12]. This inconsistency between astrophysics (mass measurements) and hadron physics (the necessary appearance of new degrees of freedom at large densities) is re-known as the ”hyperon puzzle”. However the uncertainties on the hyperons’ interactions in dense matter are such that it is still possible to tune the parameters, within phenomenological models, in order to fulfill the 2M⊙ limit also when hyperons are included in the equation of state [2,12–14]. On the other hand in microscopic models based on the Brueckner-Hartree-Fock approach, even three body forces are not enough to allow for the existence of massive stars [15], although more sophisticated calculations based e.g. on Monte Carlo techniques are needed before a firm conclusion can be drawn [16].

In principle, also the appearance of Δ isobars at some critical density n_0Δ, softens the equation of state thus reducing the maximum mass with respect to the case in which those particles are simply neglected. The crucial question, that we investigate in this paper, concerns the value of n_0Δ in beta-stable matter. We will show, in particular, that a significant correlation exists between n_0Δ and the density derivative of the symmetry energy S, the parameter L = 3n_0(dS)/(dn_B). It will be clear in the following that the appearance of the Δ isobars is affected by the value of S at a density close to n_0Δ. Since the value of S at n_0 is determined with a good precision, the crucial quantity becomes L. Only recently it has been possible to strongly constrain the value of L both from terrestrial and astrophysical data [17] with the result that 40.5 ≲ L ≲ 61.9 MeV.

First we will show, within a toy model equation of state based on the GM3 model of Ref.[18], the existence of a correlation between L and n_0Δ; then, by using a “state of the art” equation of state built upon several experimental nuclear physics information [19], we will calculate n_0Δ and show that Δ isobars appear actually before the hyperons and they should be included in the dense matter equation of state. We will also discuss the experimental constraints on the Δ couplings in nuclear matter obtained from photoabsorption, electron and pion scattering data and finally we will compute the effect of the appearance of these additional baryons on the maximum mass and on the radii of compact stars.

We adopt here the scheme of relativistic mean model in which the interaction between baryons is mediated by the exchange of a scalar meson σ, an isoscalar vector meson ω and an isovector vector ρ. The threshold for the formation of the i-th baryon is given by the following relation:

\[ \mu_i \geq m_i - g_{\sigma i} + g_{\omega i} + g_{\rho i} \]

where \( \sigma, \omega, \) and \( \rho \) are the expectation values of the corresponding fields, \( g_{\sigma i}, \, g_{\omega i}, \, g_{\rho i} \) are the couplings between the mesons and the baryons, \( \mu_i, \, m_i, \) and \( \ell_{3i} \) are the chemical potential, the mass and the isospin charge of the baryons. The baryon chemical potential \( \mu_i \) are obtained...
by the β-equilibrium conditions: \( \mu_i = \mu_B + c_i \mu_C \), where \( \mu_B \) and \( \mu_C \) are the chemical potentials associated with the conservation of the baryon number and the electric charge respectively and \( c_i \) is the electric charge of the \( i \)-th baryon.

As already extensively discussed in Ref. [1], among the four \( \Delta \) isobars, the \( \Delta^- \) is likely to appear first, in \( \beta \)-stable matter, because it can replace a neutron and an electron at the top of their Fermi seas. However, this particle is “isospin unfavored” because its isospin charge \( t_3 = -3/2 \) has the same sign of the isospin charge of the neutron. For large values of the symmetry energy \( S \) and, therefore, of \( g_{\rho \Delta} \), the \( \Delta^- \) appears at very large densities or it does not appear at all in dense matter thus playing no role in compact stars. Indeed, in Ref. [1] the \( \Delta \) isobars could appear in neutron stars only for not physical small values of the symmetry energy, obtained by setting \( g_{\rho \Lambda} = 0 \) for all the baryons.

The lagrangian adopted in [1] is a Walecka-type model with minimal coupling terms between baryons and the \( \omega \) and \( \rho \) mesons and linear and non-linear interaction terms for the scalar meson \( \sigma \). In such a scheme the symmetry energy reads \( S = S_{\text{kinetic}} + S_{\text{interaction}} \), where the interaction term \( S_{\text{interaction}} = \frac{g_{\rho N}^2}{2m_\rho^2} n_B \), \( m_\rho \) is the mass of the \( \rho \) meson and \( n_B \) is the baryon density. The coupling \( g_{\rho N} \) (where the label \( N \) stands for the nucleon) is fixed by using the experimental value of the symmetry energy, the most recent estimates ranging in the interval \( 29 \lesssim S \lesssim 32.7 \text{ MeV} \) [17]. In this scheme no experimental information on the density dependence of the symmetry energy can be incorporated and in particular the \( L \) parameter is automatically fixed once a specific value of \( S \) is adopted. It turns out, that in the models introduced in Refs. [1, 18], \( L \sim 80 \text{ MeV} \) and is thus higher than the values suggested by the most recent analysis [17]. There are two ways to modify the lagrangian adopted in the GM models in order to include the new experimental information: to introduce density dependent couplings or to introduce non minimal couplings also for the vector mesons, the two approaches being basically equivalent.

In our first analysis we adopt the GM3 model but we consider a density dependent baryon-\( \rho \) meson coupling i.e. \( g_{\rho \Lambda} = g_{\rho N}(n_0) e^{-\alpha(n_B/n_0 - 1)} \) (see Ref. [20]). In this way we introduce a single parameter \( \alpha \) which affects only the value of \( L \) leaving untouched the other properties of nuclear matter at saturation. As customary, for the couplings of hyperons and \( \Delta \) isobars with the mesons, we introduce the ratios \( x_{\sigma i} = g_{\sigma i}/g_{\sigma N} \), \( x_{\omega i} = g_{\omega i}/g_{\omega N} \) and \( x_{\rho i} = g_{\rho i}/g_{\rho N} \), where the index \( i \) runs over all the hyperons and \( \Delta \) isobars. For simplicity, we start by fixing these ratios to 1 for the \( \Delta \) isobars, as in Ref. [1]. We will later show that \( x_{\sigma \Lambda} \simeq x_{\omega \Lambda} \simeq 1 \) are compatible with the experimental data coming from electron and pion scattering on nuclei and photoabsorption nuclear reactions. For the hyperons we use the same values as in Ref. [8] obtained by reproducing the binding energies of the hyperons in ipernuclei and by imposing the SU(6) symmetry.

We can now study how the values of \( n_{\Lambda \text{crit}}^B \) for the different baryons change as a function of the new parameter \( \alpha \) or, equivalently, as a function of \( L \). We limit our discussion to the case of the \( \Lambda \), \( \Delta^- \) and \( \Xi^- \) which are the first heavy baryons appearing as the density increases. The results are displayed in Fig. [1]. One can notice the different behavior of the thresholds: the larger the value of \( L \) the larger \( n_{\Lambda \text{crit}}^\Lambda \) and the smaller \( n_{\Xi \text{crit}}^- \). Indeed, for growing values of \( L \), the isospin term in Eq. [11] also increases and the \( \Delta \) isobar becomes more and more isospin unfavored. Even though the \( \Lambda \) is not directly coupled to the \( \rho \) meson (\( t_{3\Lambda} = 0 \)), the value of \( L \) still affects \( n_{\Lambda \text{crit}}^\Lambda \) defined by the equation.
\[ \mu_A(k^2 = 0) = \mu_n(n_B = n_A^{\text{crit}}). \]

More explicitly this equation reads

\[ x_{\omega A} g_{\omega n} \omega + m^*_A = g_{\omega n} \omega - \frac{1}{2} g_{\rho n} \rho + \sqrt{k^2_\rho + m^2_n}, \]

The SU(6) symmetry implies \( x_{\omega A} = 2/3 \) and the equation simplifies to:

\[ m^*_A = g_{\omega n} \omega/3 - \frac{1}{2} g_{\rho n} \rho + \sqrt{k^2_\rho + m^2_n}, \]

where the mean field value \( \omega \) is positive being proportional to the baryon density. The mean field \( \rho \) is proportional to the difference between protons and neutrons and it is therefore negative. Clearly larger values of \( g_{\rho n} \) (or equivalently of \( L \)) imply smaller values of \( n_A^{\text{crit}} \).

Similarly for the \( \Xi^- \) the threshold equation reads:

\[ \mu_{\Xi}((k^2 = 0) = \mu_n(n_B = n^{\text{crit}}_A) + \mu_e. \]

Again by using SU(6), \( x_{\omega A} = 1/3 \) and \( x_{\Xi^{-}} = 1 \), and the threshold reads:

\[ \frac{1}{2} g_{\omega n} \omega + \sqrt{k^2_\rho + m^2_n} + \mu_e = m^{*}_{\Xi^-}. \]

Larger values of \( L \) imply larger amounts of protons and electrons, thus \( \mu_e \) increases as a function of \( L \) and the appearance of the \( \Xi^- \) is favored. Finally for the \( \Delta^- \), \( \mu_A(-k^2) = \mu_n(n_B = n^{\text{crit}}_A) + \mu_e \) and the threshold conditions (assuming all the ratios \( x_{\Delta} = 1 \)) reads:

\[ g_{\rho n} \rho + \sqrt{k^2_\rho + m^2_n} + \mu_e = m^{*}_{\Delta} \]

and, contrary to the case of the \( \Lambda \) and \( \Xi^- \), larger values of \( L \) lead to larger values of \( \mu_e \) but, at the same time, also to larger values of \( \mu_A \). Notice that this term is twice as large but with the opposite sign of the similar term appearing in the equation for the \( \Lambda \). The \( L \) dependence of \( n^{\text{crit}}_A \) is therefore dominated by \( g_{\rho n} \rho \).

The dashed lines in Fig. 1 correspond to the \( n^{\text{crit}}_A \) in the cases in which either the hyperons or the \( \Delta \) isobars are artificially excluded in the computation of the equation of state. In particular one can notice, that at high values of \( L \), larger than about 65 MeV, the threshold of the \( \Delta^- \) increases very rapidly with \( L \). This corresponds to the values of \( L \) for which the \( \Xi^- \) appears before the \( \Delta^- \) thus completely suppressing those particles. Indeed within the GM3 model, for which \( L \sim 80 \text{ MeV} \), the \( \Delta^- \) do not appear at all as already found in Ref. [1]. Similarly, one can notice that if the isobars are formed before the hyperons, what happens at \( L \sim 56 \text{ MeV} \), \( n^{\text{crit}}_A \) and \( n^{\text{crit}}_\Xi \) are shifted to larger densities, as already noticed in Ref. [8]. Similar results have been found in [9], where two cases are analyzed, corresponding to a finite and to a vanishing value of \( g_{\rho n} \), with the result that in the case of \( g_{\rho n} = 0 \) the isobars are favored. Finally, the blue lines mark the range of the values of \( L \) indicated by the analysis of Ref. [13]: the recent constraints on \( L \) imply that at densities close to three times \( n_0 \) both the hyperons and the isobars must be included in the equation of state and for the lower allowed values of \( L \), the isobars appear even before the hyperons. This will have consequences both for cold and catalyzed neutron stars, as we will show in the following, and for protoneutron stars evolution.

Finally let us stress that all the previous analysis are based on a rather conservative choice for the couplings between \( \Delta \)s and mesons. If higher values of \( x_{\Delta} \) and or lower values for \( x_{\omega A} \) are adopted, \( n^{\text{crit}}_{\Delta} \) can result to be smaller than \( n^{\text{crit}}_A \) and \( n^{\text{crit}}_{\Xi^{-}} \) for all the reasonable values of \( L \).

Let us turn now to the more sophisticated model for the equation of state proposed in Ref. [13]. In the corresponding lagrangian, self interaction terms for the vector mesons and mixing terms between the scalar and the vector sectors are added [22]. There are 17 parameters (only 5 parameters characterize the GM models [18]) which are fixed by means of a global fit on nuclear matter and finite nuclei’s properties. For our discussion the crucial quantity is the symmetry energy and its derivative with respect to the density: here we adopt the parametrization called SFHo for which \( S = 32 \text{ MeV} \) (very close to the GM3 value) and \( L = 47 \text{ MeV} \). We have included in the lagrangian hyperons (assuming SU(6) symmetry) and \( \Delta \) resonances (assuming \( x_{\Delta} = x_{\rho A} = 1 \) and three different values for \( x_{\omega A} \)). Results for the particles’ fractions as functions of the baryon density in \( \beta \)-stable matter are displayed in Fig. 2. In the upper panel, we have included only hyperons: the \( \Lambda \) and the \( \Xi^- \) appear at a density of about 0.5 fm$^{-3}$ and then the \( \Xi^0 \) at a density of about 1.1 fm$^{-3}$. The \( \Sigma \) hyperons are strongly suppressed because of their repulsive optical potential and are basically irrelevant for the structure of neutron stars. In the lower panel we include also the \( \Delta \) isobars. In agreement with what found from the previous analysis, for values of \( L \) smaller than about 65 MeV the \( \Delta \)s also appear at densities relevant for neutron stars and actually, in the SFHo model, they appear even before the hyperons with the \( \Delta^- \) formed at a density of about 0.4 fm$^{-3}$. The appearance of these particles delays the appearance of hyperons: the threshold for the \( \Xi^- \) is shifted to higher densities by about 0.15 fm$^{-3}$. The \( \Lambda \) is also slightly shifted to higher densities in agreement with the results of Fig. 1. It is important to remark that, within the SFHo model, even using \( x_{\omega A} = 1.1 \), the \( \Delta^- \) appear before hyperons.

Let us now discuss the uncertainties on the couplings between \( \Delta \)s and mesons. Qualitatively, it has been possible to establish that the \( \Delta \)s inside a nucleus feel an
attractive potential. There are several purely theoretical studies on the properties of the isobars in the nuclear medium: for instance, in Ref. [23], from QCD sum rules, it has been found that \( x_{\omega\Delta} \) is significantly smaller than 1. In the many body analysis of Ref. [24], the real part of the \( \Delta \) self-energy has been evaluated to be around \(-30\) MeV at \( n_0 = 0.75n_\text{0} \). Notice that this self energy is relative to the one of the nucleon and the total potential felt by the \( \Delta \) is the sum of its self energy and of the nucleon potential, a number of the order of \(-80\) MeV [30]. Also phenomenological analysis have been performed of data from electron-nucleus [21, 27, 28], photoabsorption [27] and pion-nucleus scattering [28, 29]. When discussing pion scattering data, a value for the real part of the \( \Delta \)–nucleus potential of \(-30\) MeV is extracted [28]. Since pions interact mainly with the nuclear surface, larger values are expected for the binding at \( n_0 \). More recently a global analysis of pion-nucleus scattering and of pion photo-production has been performed in Ref. [29] where the experimental data are correctly described by assuming a \( \Delta \) potential equal to the nucleon potential. From the data analysis of electron-nucleus scattering, either density or momentum dependent potentials have been deduced. In Ref. [23] the binding potential is parameterized as \(-75\,n_B(r)/n_0\) MeV. In Ref. [28] they obtain an optical potential which, at a momentum of about \(400\) MeV (quite typical for electron scattering), gives a binding in agreement with the one of [25]. Electromagnetic excitations of the \( \Delta \) baryon have been also analyzed within a relativistic quantum hadrodynamics scheme with the result that \( 0 \lesssim x_{\sigma\Delta} - x_{\omega\Delta} \lesssim 0.2 \) [21]. The conclusion one can draw from all these analysis is that the potential of the \( \Delta \) falls within the range \(-30\) MeV +\( V_N \lesssim V_{\Delta} \lesssim V_N \) where \( V_N \) is the nucleon potential.

In the relativistic mean field model [18] the potential of the \( \Delta \) (which coincides with the binding energy of the lowest \( \Delta \) level) is given by: \( V_{\Delta} = x_{\sigma\Delta}g_{\omega n}\omega - x_{\sigma\Delta}g_{\pi n}\sigma \) where the mean fields are calculated at \( n_0 \). By fixing a value for \( V_{\Delta} \) a relation between \( x_{\sigma\Delta} \) and \( x_{\omega\Delta} \) is obtained, shown in Fig. 3 together with the experimental constraints on \( x_{\sigma\Delta} - x_{\omega\Delta} \). New analysis, and possibly new experiments, aiming at a better determinations of these couplings would be extremely important. Notice also that no information is available for \( x_{\rho\Delta} \) which in principle could be extracted by analyzing scattering on neutron rich nuclei.

Let us now analyze the effect of including \( \Delta s \) on the structure of neutron stars. We calculate the equation of state of \( \beta \)-stable matter by use of the SFHo model for different values of \( x_{\omega\Delta} \) at fixed values of \( x_{\sigma\Delta} = x_{\pi\Delta} = 1 \) (similar results are found by varying \( x_{\sigma\Delta} \)). From the upper panel of Fig. 4 one can notice that the inclusion of the \( \Delta \) dramatically reduces the maximum mass: if \( x_{\omega\Delta} \lesssim 1 \) as indicated by the experimental data, the maximum mass does not satisfy the \( 2M_\odot \) limit [11]. Concerning the radii, we notice that if only \( \Delta \) resonances are included the maximum mass configurations are very compact, with a radius \( R \lesssim 10.5 \) km. Concerning hyperons, we have not taken into account possible mechanisms making the equation of state stiffer at high densities such as the inclusion of the \( \phi \) meson [2, 12]. The reason is that in this work we are interested in showing that already the appearance of \( \Delta s \) can lead to a problem with astrophysical measurements. The implementation of additional repulsion between hyperons would shift the green curves towards the blue ones which correspond to the case in which hyperons are not present at all.

![Fig. 4: (Color online) Properties of hadronic stars (with and without hyperons) as functions of \( x_{\omega\Delta} \): the maximum mass is displayed in the upper panel while the radii of the \( 1.4M_\odot \) stellar configurations and the radii of the maximum mass configurations are displayed in the lower panel. The labels \( N \), \( \Delta \) and \( \Delta H \) in the legend stand for purely nucleonic stars, for hadronic stars with only \( \Delta s \) and for hadronic stars in which \( \Delta s \) and hyperons are present. The radii of the \( 1.4M_\odot \) \( \Delta H \) hadronic stars coincide with the ones of \( \Delta \) hadronic stars because hyperons do not appear in those stellar configurations. Since the maximum mass of the \( \Delta H \) configuration is smaller than the one of the \( \Delta \) configurations, the corresponding radius is larger (see Fig.1 of Ref. [8]).](image)
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