Characterizing the WASP-4 System with TESS and Radial Velocity Data: Constraints on the Cause of the Hot Jupiter’s Changing Orbit and Evidence of an Outer Planet

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Abstract

Orbital dynamics provide valuable insights into the evolution and diversity of exoplanetary systems. Currently, only one hot Jupiter, WASP-12b, is confirmed to have a decaying orbit. Another, WASP-4b, exhibits hints of a changing orbital period that could be caused by orbital decay, apsidal precession, or the acceleration of the system toward the Earth. We have analyzed all data sources from NASA’s Transiting Exoplanet Survey Satellite together with all radial velocity (RV) and transit data in the literature to characterize WASP-4b’s orbit. Our analysis shows that the full RV data set is consistent with no acceleration toward the Earth. Instead, we find evidence of a possible additional planet in the WASP-4 system, with an orbital period of ~7000 days and $M_r \sin(i) = 5.47^{+0.44}_{-0.43} M_{\oplus}$. Additionally, we find that the transit timing variations of all of the WASP-4b transits cannot be explained by the second planet but can be explained with either a decaying orbit or apsidal precession, with a slight preference for orbital decay. Assuming the decay model is correct, we find an updated period of $1.33823587 \pm 0.00000022$ days, a decay rate of $-7.73 \pm 0.71$ ms yr$^{-1}$, and an orbital decay timescale of $\tau = P/|P'| = 15.77 \pm 1.57$ Myr. If the observed decay results from tidal dissipation, we derive a modified tidal quality factor of $Q'' = 5.1 \pm 0.9 \times 10^4$, which is an order of magnitude lower than values derived for other hot Jupiter systems. However, more observations are needed to determine conclusively the cause of WASP-4b’s changing orbit and to confirm the existence of an outer companion.

Unified Astronomy Thesaurus concepts: Star-planet interactions (2177); Exoplanet dynamics (490); Extrasolar gaseous giant planets (509); Exoplanets (498); Radial velocity (1332); Timing variation methods (1703); Transit photometry (1709)

Supporting material: machine-readable tables

1. Introduction

One of the most powerful tools used to study exoplanets is observing them while they transit across the disk of their stars. The transit method can be used to search for temporal variations in the planetary orbital parameters and allows for a direct measurement of the planetary radius and thus its atmosphere (e.g., Charbonneau et al. 2000, 2007). Specifically, light curves of transiting planets can be used to search for transit timing variations (TTVs; Agol et al. 2005; Agol & Fabrycky 2018), transit duration variations (Agol & Fabrycky 2018), and impact parameter variations (Herman et al. 2018; Szabó et al. 2020). The presence of TTVs may indicate additional bodies in the system, a decaying planetary orbit, precession in the orbit, or a variety of other effects (e.g., Applegate 1992; Dobrovolskis & Borucki 1996; Miralda-Escudé 2002; Schneider 2003; Agol et al. 2005; Mazeh et al. 2013; Agol & Fabrycky 2018).

It is theorized that orbital decay might occur on short-period massive planets orbiting stars with surface convective zones due to exchange of energy with their host stars through tidal interactions (e.g., Rasio et al. 1996; Lin et al. 1996; Chambers 2009; Lai 2012; Penev et al. 2014; Barker 2020).

Measurements of orbital decay would expand our understanding of the hot Jupiter population and its evolution (e.g., Jackson et al. 2008; Hamer & Schlaufman 2019). Even though such decay may occur over millions of years, it is possible to search for small changes in the orbital period of hot Jupiter systems since many of these planets have been monitored for decades.

Currently, WASP-12b is one of the few hot Jupiters confirmed to have a varying period. It is an ultra-hot planet around a G0 star with an orbital period of 1.09 days (Hebb et al. 2009). WASP-12b is believed to have an escaping atmosphere (e.g., Lai et al. 2010; Bisikalo et al. 2013; Turner et al. 2016a) as suggested by Hubble Space Telescope near-ultraviolet observations (Fossati et al. 2010; Haswell et al. 2012; Nichols et al. 2015). Maciejewski et al. (2016) were the first to detect its decreasing orbital period, and subsequent studies have confirmed the period change (Patra et al. 2017; Maciejewski et al. 2018; Bailey & Goodman 2019; Balué et al. 2019) and established orbital decay as its cause (Yee et al. 2020). The decaying orbit of WASP-12b was confirmed also using transit and occultation observations with NASA’s Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015; Turner et al. 2021; see also Owens et al. 2021). The decay rate of WASP-12b was found to be $32.53 \pm 1.62$ ms yr$^{-1}$ corresponding to an orbital decay timescale of $\tau = P/|P'| = 2.90 \pm 0.14$ Myr (Turner et al. 2021), shorter than the estimated mass-loss timescale of 300 Myr (Lai et al. 2010; Jackson et al. 2017). Assuming the observed decay results from tidal dissipation, Turner et al. (2021) derived a modified tidal
quality factor, a dimensionless quantity that describes the efficiency of tidal dissipation, of $Q'_e = 1.39 \pm 0.15 \times 10^3$, which falls at the lower end of values derived for binary star systems (Meibom et al. 2015) and hot Jupiters (Jackson et al. 2008; Husnoo et al. 2012; Barker 2020).

Besides WASP-12b, WASP-4b is a well-studied system with hints of a changing period. WASP-4b is a typical hot Jupiter since its discovery in 2008. Bouma et al. (2019) first reported an orbital period variation of WASP-4b using TESS and ground-based observations. Southworth et al. (2019) confirmed that the period was decaying with additional ground-based observations and found a decay rate of $9.2$ ms yr$^{-1}$. They found that orbital decay and apsidal precession could explain the TTVs after ruling out instrumental issues, stellar activity, the Applegate mechanism, and light-time effect. Bouma et al. (2020) obtained additional radial velocity (RV) data on WASP-4b using HIRES on Keck and found that the observed orbital period variation could be explained by the system accelerating toward the Sun at a rate of $-0.0422$ m s$^{-1}$ day$^{-1}$. Recently, Baluven et al. (2020) analyzed a comprehensive set of 129 transits and additional RV data (presented in Baluven et al. 2019) from 2007–2014 (mostly in years not covered by the RV data presented in Bouma et al. 2020). They also confirmed a period change in WASP-4b’s orbit but do not confirm the RV acceleration found by Bouma et al. (2020). However, Baluven et al. (2020) did not include the new RV HIRES data from Bouma et al. (2020) in their analysis. Therefore, the cause of the period variation in WASP-4b is still an open question.

Motivated by the possible changing period of WASP-4b, we analyze all three sectors (Figure 1) from TESS and combine the results with all transit, occultation, and RV measurements from the literature to verify its changing period and derive updated planetary properties. TESS is well suited for our study because it provides high-precision time-series data, ideal for searching for TTVs (e.g., Hadden et al. 2019; Pearson 2019; von Essen et al. 2020; Ridden-Harper et al. 2020; Turner et al. 2021). Altogether, the transit data span 13 yr from 2007–2020 and the RV data span 12 yr from 2007–2019. Using the combined data set we hope to shed light on the cause of WASP-4b’s period variation.

Figure 1. TESS light curve of WASP-4b in Sectors 2, 28, and 29. Top: raw simple-aperture photometry light curves. Bottom: detrended DVT.

2. Observations and Data Reduction

TESS observed WASP-4b in Sector 2 (2018 August 22–2018 September 20), Sector 28 (2020 July 30–2020 August 26), and Sector 29 (2020 August 26–2020 September 22). The TESS observations were processed by the Science Processing Operations Center (SPOC) pipeline$^4$ (Jenkins et al. 2016). The SPOC pipeline produces light curves ideal for characterizing transiting planets since they are corrected for systematics. SPOC produces presearch data conditioning (PDC) and data validation time series (DVT) light curves. The PDC light curves are corrected for instrumental systematics (pointing or focus related), discontinuities resulting from radiation events in the CCD detectors, outliers, and flux contamination. The DVT light curves are created by using a running median filter on the PDC light curves to remove any long-period systematics. We use the DVT light curves (Figure 1) for our analysis because they have less scatter in their out-of-transit (OoT) baseline. As shown in Ridden-Harper et al. (2020) for XO-6b TESS data, the DVT and PDC light curves produce similar results on the timing of the transits. For the Sector 2 data, the light curves produced from the SPOC pipeline have a known issue$^5$ that overestimates their uncertainties. Therefore, we estimated the uncertainties using the scatter in the OoT baseline as we did in our previous study of WASP-12b (Turner et al. 2021) and as recommended (T. Barclay, private communication). The Sectors 28 and 29 data are unaffected by this problem. Therefore, we used the uncertainties provided by the SPOC pipeline.

3. Data Analysis

3.1. Transit Modeling

All the TESS transits of WASP-4b were modeled with the EXOplanet MOdEling Package (EXOMOP; Pearson et al. 2014; Turner et al. 2016b, 2017)$^6$ to find a best fit. EXOMOP creates a model transit using the analytic equations of Mandel & Agol (2002) and the data are modeled using a differential evolution Markov Chain Monte Carlo (DE-MCMC; Eastman et al. 2013) analysis. The residual permutation, time-averaging, and wavelet methods are incorporated into EXOMOP to account for red noise in the light curve; see Pearson et al. (2014) and Turner et al. (2016b) for more detailed descriptions of EXOMOP.

Each TESS transit (Figure 1) was modeled with EXOMOP independently. We used 20$^6$ links and 20 chains for the DE-MCMC model and use the Gelman–Rubin statistic (Gelman & Rubin 1992) to ensure chain convergence (Ford 2006). The mid-transit time ($T_c$), scaled semimajor axis ($a/R_*$), planet-to-

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$^4$ All of the SPOC data products are publicly accessible from the Mikulski Archive for Space Telescopes at https://archive.stsci.edu/.

$^5$ The issue is related to inaccurate uncertainties in the 2D black model, which represents the fixed pattern that is visible in the black level for a sum of many exposures. See TESS Data Release Notes: Sector 27, DR38.38.

$^6$ EXOMOPv7.0; https://github.com/astrojake/EXOMOP.
Table 1

| Parameter | Units | Value | 1σ uncertainty |
|-----------|-------|-------|----------------|
| \(P\)     | Days  | 1.338231587 | 0.000000022 |
| \(R_p/R_a\) |        | 0.15158 | 0.00057 |
| \(\omega/R_a\) |        | 5.410 | 0.088 |
| Inclination | °      | 88.02 | 0.69 |
| Duration | Minutes | 129.594 | 0.097 |
| \(R_p\) | \(R_{\text{stars}}\) | 1.312 | 0.045 |
| \(M_p\) | \(M_{\text{stars}}\) | 1.164 | 0.082 |
| \(\rho_p\) | \(g\text{ cm}^{-3}\) | 0.639 | 0.079 |
| \(\log g_p\) | cgs | 3.216 | 0.035 |
| \(\rho_a\) | \(g\text{ cm}^{-3}\) | 1.67 | 0.16 |
| \(T_{\text{eq}}\) | K | 1641.65 | 27.36 |
| \(a\) | au | 0.02239 | 0.000 84 |

Note. To calculate the planetary mass \(M_p\) of WASP-4b we used a stellar RV amplitude \(K_0\) of 237.3 ± 2.2 m s\(^{-1}\) as listed in Table 4 from the best-fit two-planet RV model (Model #3). The period of WASP-4b was taken from the orbital decay model in Table 6. All the physical parameters for WASP-4c were taken from the best-fit two-planet RV model (Model #3).

star radius \(R_p/R_a\), and inclination \(i\) are set as free parameters for every transit. The linear and quadratic limb-darkening coefficients and period \(P_{\text{orb}}\) are fixed during the analysis. The linear and quadratic limb-darkening coefficients are taken from Claret (2017) and are set to 0.382 and 0.210, respectively.

The parameters derived for every TESS transit event can be found in Tables A1–A3 in Appendix A. The modeled light curves for each individual transit can be found in Figures A1–A7 in Appendix A. All parameters for each transit event are consistent within 2σ of every other transit. The Sector 2 data for WASP-4b was analyzed in Bouma et al. (2019), and our timing analysis for each individual transit is consistent within 1σ of their findings (see Figure B1 in Appendix B).

The light curve of WASP-4b was phase-folded at each derived mid-transit time and modeled with EXOMOP to find the final fitted parameters. The phase-folded light curve and model fit can be found in Figure 2. We use the light-curve model results combined with literature values to calculate the planetary mass \(M_p\); Winn (2010), radius \(R_p\), density \(\rho_p\), equilibrium temperature \(T_{\text{eq}}\), surface gravity \(\log g_p\); Southworth et al. (2007), orbital distance \(a\), inclination \(i\), Safronov number \(\Theta\;\text{Safronov}\;1972\); Southworth (2010), and stellar
### Table 4

Results of the one-planet and two-planet RV Fitting of the WASP-4 System with RadVel

| Parameter   | Model #1                     | Model #2                     | Model #3                     | Model #4                     | Units            |
|-------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------|
| $P_b$       | $1.338231343 \pm 7.2 \times 10^{-7}$ | $1.338232067 \pm 7.2 \times 10^{-7}$ | $1.338231349 \pm 7.3 \times 10^{-7}$ | $1.338232068 \pm 7.3 \times 10^{-7}$ | days             |
| $T_{\text{conj}}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | JD               |
| $e_b$       | $\equiv 0.0$                 | $\equiv 0.0$                 | $\equiv 0.0$                 | $\equiv 0.0$                 |                  |
| $\omega_b$  | $\equiv 0.0$                 | $\equiv 0.0$                 | $\equiv 0.0$                 | $\equiv 0.0$                 |                  |
| $K_b$       | $238.1^{+3.7}_{-3.6}$        | $238.2^{+3.8}_{-3.7}$        | $237.3 \pm 2.2$              | $237.3^{+2.2}_{-2.3}$        |                  |
| $P_c$       | ...                          | ...                          | $7001.0^{+6.0}_{-6.6}$        | $7001.0^{+6.6}_{-6.3}$        | days             |
| $T_{\text{conj}c}$ | ...                         | ...                          | $0 \pm 2300$                 | $0 \pm 2300$                 | JD               |
| $T_{\text{peri}}$ | ...                        | ...                          | $-1750 \pm 2000$             | $801 \pm 2400$               | JD               |
| $e_c$       | ...                          | ...                          | $\equiv 0.0$                 | $0.094 \pm 0.067$            |                  |
| $\omega_c$  | ...                          | ...                          | $\equiv 0.0$                 | $2.7^{+1.1}_{-1.2}$          |                  |
| $K_c$       | $642.2^{+2.3}_{-2.2}$        | $63.7^{+3.7}_{-3.3}$         | ...                          | ...                          | m s$^{-1}$       |

| Orbital Parameters |
|--------------------|
| $P_b$ | $1.338231343 \pm 7.2 \times 10^{-7}$ | $1.338232067 \pm 7.2 \times 10^{-7}$ | $1.338231349 \pm 7.3 \times 10^{-7}$ | $1.338232068 \pm 7.3 \times 10^{-7}$ | days             |
| $T_{\text{conj}b}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | $745.515752 \pm 1.9 \times 10^{-5}$ | JD               |
| $e_b$ | $\equiv 0.0$ | $\equiv 0.0$ | $\equiv 0.0$ | $\equiv 0.0$ |                  |
| $\omega_b$ | $\equiv 0.0$ | $\equiv 0.0$ | $\equiv 0.0$ | $\equiv 0.0$ |                  |
| $K_b$ | $238.1^{+3.7}_{-3.6}$ | $238.2^{+3.8}_{-3.7}$ | $237.3 \pm 2.2$ | $237.3^{+2.2}_{-2.3}$ |                  |
| $P_c$ | ... | ... | $7001.0^{+6.0}_{-6.6}$ | $7001.0^{+6.6}_{-6.3}$ | days             |
| $T_{\text{conj}c}$ | ... | ... | $0 \pm 2300$ | $0 \pm 2300$ | JD               |
| $T_{\text{peri}}$ | ... | ... | $-1750 \pm 2000$ | $801 \pm 2400$ | JD               |
| $e_c$ | ... | ... | $\equiv 0.0$ | $0.094 \pm 0.067$ |                  |
| $\omega_c$ | ... | ... | $\equiv 0.0$ | $2.7^{+1.1}_{-1.2}$ |                  |
| $K_c$ | $642.2^{+2.3}_{-2.2}$ | $63.7^{+3.7}_{-3.3}$ | ... | ... | m s$^{-1}$       |
| $M_c \sin(i)$ | ... | ... | $5.47^{+0.44}_{-0.43}$ | $5.40^{+0.50}_{-0.56}$ |                  |

| Other Parameters |
|-------------------|
| $\gamma_{\text{HIRES}}$ | $\equiv -33.5423$ | $\equiv -33.4244$ | $\equiv -42.8019$ | $\equiv -41.3003$ | m s$^{-1}$       |
| $\gamma_{\text{HARPS}}$ | $\equiv -36.1144$ | $\equiv -36.6269$ | $\equiv -82.2669$ | $\equiv -81.6365$ | m s$^{-1}$       |
| $\gamma_{\text{CORALIE}}$ | $\equiv 57750.7122$ | $\equiv 57751.1433$ | $\equiv 57710.1977$ | $\equiv 57710.7494$ | m s$^{-1}$       |
| $\gamma$ | $\equiv 0.0$ | $0.0001^{+0.0014}_{-0.0013}$ | $\equiv 0.0$ | $\equiv 0.0$ |                  |
| $\sigma_{\text{HIRES}}$ | $65^{+17}_{-13}$ | $64^{+18}_{-15}$ | $4.5^{+2.5}_{-2.1}$ | $4.4^{+2.5}_{-2.2}$ | m s$^{-1}$       |
| $\sigma_{\text{HARPS}}$ | $12.7^{+3.1}_{-2.9}$ | $13.5^{+3.5}_{-2.6}$ | $12.3^{+2.8}_{-2.4}$ | $12.7^{+2.9}_{-2.1}$ | m s$^{-1}$       |
| $\sigma_{\text{CORALIE}}$ | $10.6^{+2.9}_{-2.6}$ | $11.0^{+3.1}_{-2.6}$ | $5.2^{+3.5}_{-2.9}$ | $5.3^{+3.0}_{-2.8}$ | m s$^{-1}$       |

**Note.** Reference epoch for $\gamma$, $\gamma$, $\gamma'$: 2455059. The HARPS and CORALIE data were taken from Balu... (2020).

_density ($\rho_{\text{Jup}}$), Seager & Mou
den-Ormelas 2003). The planet properties we derived for WASP-4b are shown in Table 1. All the planetary parameters are consistent with their discovery values (Wilson et al. 2008) but their precision is greatly improved.

### 3.2. Occultation

We created an occultation light curve by phase-folding all the data about the secondary eclipse using the first TESS transit as the reference transit time. As shown in Figure 3, we do not see an occultation of WASP-4b in the TESS data. We only show the PDC light curve but this is also the case for the DVT light curve. We find a 3σ upper limit on the occultation depth ($\delta_{\text{occ}}$) of $1.34 \times 10^{-5}$. The geometric albedo ($A_{g,\text{occ}}$) of WASP-4b can be calculated assuming no thermal contribution:

$$\delta_{\text{occ}} = A_{g,\text{occ}} \left( \frac{R_p}{a} \right)^2. \tag{1}$$

We find a 3σ upper limit on $A_{g,\text{occ}}$ of 0.017 using Equation (1). Our upper limit on $A_{g,\text{occ}}$ is consistent with the overall trend that hot Jupiters are very dark (e.g., Kipping & Bakos 2011, Močnik et al. 2018, Kane et al. 2020).

### 3.3. Modeling Radial Velocities

We combined all available RV data of WASP-4 in the literature in our analysis. The complete RV data set is given in Table 2. The set includes CORALIE and HARPS data from Balu... (2019), HARPS data from Pont et al. (2011), and HIRES data from Bouma et al. (2020). We did not include the RV data from Triaud et al. (2010) as these data were taken for Rossiter–McLaughlin measurements and were reduced with a nonstandard pipeline.

The RV data were modeled with RadVel (Fulton et al. 2018). We ran four models in total for the RV analysis (Table 3). The priors used in the analysis are summarized in Table C1 in Appendix C. We used Gaussian priors for the
Figure 4. Best-fit Keplerian orbital model for the WASP-4 system for Models #1 (top left), #2 (top right), #3 (bottom left), and #4 (bottom right) using RadVel. (a) panels: the maximum-likelihood model is plotted while the orbital parameters listed in Table 4 are the median values of the posterior distributions. The thin blue line is the best-fit model. We add in quadrature the RV jitter terms listed in Table 4 with the measurement uncertainties for all RVs. (b) panels: residuals to the best-fit model. The error bars of the residuals reflect both the measurement error and the jitter from the MCMC fit. The larger error bars for the one-planet fits reflect the larger amounts of jitter needed to fit the data with only one planet. (c) panels: RVs phase-folded to the ephemeris of WASP-4b. The phase-folded model for WASP-4b is shown as the blue line. The Keplerian orbital models for all other planets (if modeled) have been subtracted. (d) panels: RVs phase-folded to the ephemeris of the second planet, WASP-4c. The Keplerian orbital model for the other planet (panels c), WASP-4b) has been subtracted. Red circles are the velocities binned in 0.08 units of orbital phase.
orbital period and time of inferior conjunction that were set to the values derived by Bouma et al. (2019). We set the eccentricity of the orbit to zero, as indicated by previous upper limit studies (Beeper et al. 2011; Knutson et al. 2014; Bonomo et al. 2017).

As done in previous studies, we first modeled the data with only one planet. The free parameters in the model were the orbital velocity semi-amplitude ($K_b$), the instrument zero-points, white-noise instrument jitter for each instrument ($\sigma$, added in quadrature to its uncertainties), and the linear acceleration ($\dot{\gamma}$). We ran models with and without $\dot{\gamma}$ to check if an acceleration was needed to fit the data (Models #1 and #2). The results of the one-planet models of WASP-4b can be found in Table 4 and Figure 4. We use the Bayesian information criterion (BIC) to assess the preferred model. The BIC is defined as

$$\text{BIC} = \chi^2 + k \ln(N_{\text{pts}}),$$

where $k$ is the number of free parameters in the model fit and $N_{\text{pts}}$ is the number of data points. The power of the BIC is the penalty for a higher number of fitted model parameters, making it a robust way to compare different best-fit models. The preferred model is the one that produces the lowest BIC value. We find BIC values of 675.08 and 682.76 for the model without (Model #1) and with fitting $\dot{\gamma}$ (Model #2), respectively. For two generic models, $i$ and $j$, we can relate the difference in the BIC values between models, $\Delta(\text{BIC}_{ij}) = \text{BIC}_j - \text{BIC}_i$, and the Bayes factor, $B_{ij}$, the ratio of the likelihood between models $i$ and $j$, assuming a Gaussian distribution for the posteriors (e.g., Faulknerberry 2018):

$$B_{ij} = \exp[\Delta(\text{BIC}_{ij})/2],$$
$$B_{y} = \exp[-\Delta(\text{BIC}_{y})/2].$$

Therefore, Model #1 without fitting for $\dot{\gamma}$ is the preferred model with a $\Delta(\text{BIC}_{1,2}) = \text{BIC}_1 - \text{BIC}_2 = -7.68$ and a Bayes factor, $B_{1,2}$, of 46.5. When fitting for $\dot{\gamma}$, we find a linear acceleration term that is positive but is consistent with zero within the uncertainties ($\dot{\gamma} = 0.0001^{+0.0034}_{-0.0036} \text{ m s}^{-1} \text{ day}^{-1}$). Our results are in conflict with the findings by Bouma et al. (2020) that find an acceleration along our line of sight at a rate of $\dot{\gamma} = -0.0422^{+0.0028}_{-0.0025} \text{ m s}^{-1} \text{ day}^{-1}$. We can reproduce the results of Bouma et al. (2020) by modeling only their data (see Figure C5 and Table C2 in Appendix C). Based on this test, we conclude that the acceleration found by Bouma et al. (2020) was caused by modeling only part of the full RV data set. Therefore, we conclude that the changing orbital period detected using the transit data is not caused by the WASP-4 system accelerating toward Earth.

Next, we fit the RV data with a two-planet model because the residuals of the one-planet fit showed some sinusoidal structure (panel b in Figures 4(I)–(II)). This sinusoidal trend is not caused by stellar activity as the S-index time series from the HIRES data shows no signs of secular or sinusoidal trends (Bouma et al. 2020). We performed several different models where we fit for $K_b$, $\sigma$, $\dot{\gamma}$, the orbital velocity semi-amplitude of the second body ($K_c$), and an eccentricity of the second planet ($e_c$). The results of Models #3 and #4 are summarized in Table 3. We find that Model #3 with $K_b$ and $K_c$ set as free parameters and $e_f$, $e_c$, and $\dot{\gamma}$ fixed to zero finds the best fit with a BIC of 628.34. The derived orbital parameters of this model can be found in Table 4 and the two-planet RV fit can be found in Figures 4(III)–(IV). The two-planet model (Model #3) is highly preferred over the one-planet model (Model #1) with a $\Delta(\text{BIC}_{3,1}) = -46.74$ and a Bayes factor, $B_{3,1}$, of $1.41 \times 10^{10}$. For the second body, we find a period ($P_c$) of 7001.0$^{+6.6}_{-6.6}$ days, a semimajor axis of 6.85$^{+0.23}_{-0.25}$ au, and a $M_c$, $\sin(i)$ of 5.47$^{+0.44}_{-0.45}$ $M_{\text{Jup}}$ (Tables 1 and 4). The companion is expected to be much fainter than the planet host star because Wilson et al. (2008) found no evidence for changing spectral line bisectors in their spectroscopic observation. Becker et al. (2017) found that distant exterior companions to hot Jupiters around cool stars ($T_{\text{star}} < 6200$ K) are typically coplanar within 20–30 degrees. Therefore, we find that the mass of WASP-4c is between 5.47–6.50 $M_{\text{Jup}}$ assuming that its inclination is within 30 degrees of WASP-4b’s inclination. However, more RV measurements are needed to verify the existence of this second planet around WASP-4.

### 3.4. Transit Timing Variations

For the timing analysis, we combined the TESS transit data with the prior transit and occultation times. All the transit and occultation times used in this analysis can be found in Table 5. In the table we give the original reference in which the data were reported and the reference for the timing if different from the original source. We combine transit data as tabulated by Bouma et al. (2020) and additional transits reported by amateur observers from Balveu et al. (2020). This table is available in its entirety in machine-readable form online.

Similar to what was done in Yee et al. (2020), Patra et al. (2017), and Turner et al. (2021), we fit the timing data to three different models. The first model is the standard constant-period formalization:

$$t_{\text{tra}}(E) = T_{\text{c},0} + P_{\text{orb}} \times E,$$

$$t_{\text{occ}}(E) = T_{\text{c},0} + \frac{P_{\text{orb}}}{2} + P_{\text{orb}} \times E,$$

where $T_0$ is the reference transit time, $P_{\text{orb}}$ is the orbital period, $E$ is the transit epoch, and $T_{\text{tra}}(E)$ is the calculated transit time at epoch $E$.

The second model assumes that the orbital period is changing uniformly over time:

$$t_{\text{tra}}(E) = T_{\text{c},0} + P_{\text{orb}} \times E + \frac{1}{2} \frac{dP_{\text{orb}}}{dE} E^2,$$

$$t_{\text{occ}}(E) = T_{\text{c},0} + \frac{P_{\text{orb}}}{2} + P_{\text{orb}} \times E + \frac{1}{2} \frac{dP_{\text{orb}}}{dE} E^2,$$

where $dP_{\text{orb}}/dE$ is the decay rate.

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### Table 5.

All WASP-4b Transit and Occultation Times

| Event Type | Midtime | Error | Epoch | Timing Source | Transit Source |
|------------|---------|-------|-------|---------------|---------------|
| tra        | 2458843.00493 | 0.00054 | 2325 | This paper | This paper |

Note. This table is available in its entirety in machine-readable form.

References. Wilson et al. (2008); Winn et al. (2009); Dragomir et al. (2011); Cáceres et al. (2011); Beerer et al. (2011); Sanchis-Ojeda et al. (2011); Nikolov et al. (2012); Hoyer et al. (2013); Zhou et al. (2015); Huíton et al. (2017); Southworth et al. (2019); Balveu et al. (2019); Bouma et al. (2019); Balveu et al. (2020).
The third model assumes the planet is precessing uniformly (Giménez & Bastero 1995):

$$t_{\text{trans}}(E) = T_{c,0} + P_s \times E - \frac{P_a}{\pi} \cos \omega(N),$$

$$t_{\text{occ}}(E) = T_{c,0} + \frac{P_{\text{orb}}}{2} + P_s \times E + \frac{P_a}{\pi} \cos \omega(N),$$

$$\omega(N) = \omega_0 + \frac{d \omega}{dE},$$

$$P_s = P_a \left(1 - \frac{1}{2 \pi \frac{d \omega}{dE}}\right),$$

where $e$ is a nonzero eccentricity, $\omega$ is the argument of pericenter, $P_s$ is the sidereal period, and $P_a$ is the anomalistic period.

For all three models, we found the best-fitting model parameters using a DE-MCMC analysis. We used 20 chains and 2000 links in the model, and again we ensure chain convergence using the Gelman–Rubin statistic.

The results of the timing model fits can be found in Table 6. Figure 5 shows the transit and occultation timing data fit with the orbital decay and apsidal precession models. In this figure, the best-fit constant-period model has been subtracted from the timing data. The orbital decay model fits the transit and occultation data slightly better than the precession model (Table 6; Figure 5).

Our finding that the constant-period model does not fit the data well is consistent with previous studies (Bouma et al. 2019; Southworth et al. 2019; Baluev et al. 2020). The orbital decay and apsidal precession models fit the data with a minimum chi-squared ($\chi^2_{\text{min}}$) of 276.35 and 270.42, respectively. We find that the orbital decay model is the preferred model with a $\Delta(BIC) = -5.93$ and a Bayes factor of 9.3. Therefore, based on our analysis of the observed timing residuals, the orbital decay model is only slightly preferred over apsidal precession.

Due to the RV measurements showing evidence of a possible second planet, we modeled the two-planet system to see if they could reproduce the TTVs. For this analysis, we used the publicly available TTV analysis package OCFit\(^8\) (Gajdoš & Parimucha 2019). Specifically, we used the AgolExPlanet function, which is an implementation of Equation (25) in Agol et al. (2005). The priors in OCFit were set to the values given for Model #4 in Table 4 for both planets where the outer planet has an eccentricity of 0.094. We were not able to fit the TTVs well with an outer planet consistent with the RV constraints. We also produced several forward models with OCFit that show that the expected TTV signal from the outer body is less than 2 s (dependent on the real mass of the body) over the full observational period (Figure 6). We did not use the preferred two-planet model (Model #3) because this model did not produce a detectable signal. The two objects are assumed to be coplanar but relaxing this condition will only decrease the TTV signal.

We can also analytically calculate the expected TTV signal on WASP-4b ($\delta t_b$) using the following equation from Agol et al. (2005) assuming nonresonant planets with large period ratios on circular orbits:

$$\delta t_b = \frac{M_e}{M_{\text{star}}} \left(\frac{P_b}{P_s}\right)^2 P_b,$$

\(^8\) https://github.com/pavolgaj/OCFit

Table 6

| Parameter Symbol | Units | Value | 1σ uncertainty |
|------------------|-------|-------|----------------|
| Period $P_{\text{orb}}$ | days | 1.338231392 | 0.000000014 |
| Mid-transit time $T_{c,0}$ | BJD | 2455804.51545 | 0.000018 |
| Decay rate $\frac{dP}{dE}$ | ms yr$^{-1}$ | -7.33 | 0.71 |
| Decay rate $\frac{dP}{dt}$ | ms yr$^{-1}$ | 0.0011 | 0.00037 |
| Sidereal period $P_s$ | days | 1.338231448 | 0.00000099 |
| Mid-transit time $T_{c,0}$ | BJD | 2455804.51545 | 0.00000099 |
| Eccentricity $e$ | | 0.00090 | 0.000018 |
| Argument of periastron $\omega_0$ | rad | 2.70 | 0.36 |
| Precession rate $\frac{d\omega}{dN}$ | rad/orbit | 0.0011 | 0.00037 |
| Parameter Symbol | Units | Value | 1σ uncertainty |
|------------------|-------|-------|----------------|
| Period $P_{\text{orb}}$ | days | 1.338231578 | 0.000000022 |
| Mid-transit time $T_{c,0}$ | BJD | 2455804.51545 | 0.000000031 |
| Decay rate $\frac{dP}{dE}$ | days/orbit | -0.00000000311 | 0.00000000030 |
| Decay rate $\frac{dP}{dt}$ | ms yr$^{-1}$ | -7.33 | 0.71 |
| Sidereal period $P_s$ | days | 1.338231087 | 0.000000022 |
| Mid-transit time $T_{c,0}$ | BJD | 2455804.51545 | 0.000000031 |
| Eccentricity $e$ | | 0.00090 | 0.000018 |
| Argument of periastron $\omega_0$ | rad | 2.70 | 0.36 |
| Precession rate $\frac{d\omega}{dN}$ | rad/orbit | 0.0011 | 0.00037 |

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where $M_c$ and $P_c$ are the mass and orbital period of the outer companion, $M_{\text{star}}$ is the mass of WASP-4, and $P_b$ is the period of WASP-4b. The assumptions of Equation (12) are all satisfied within the constraints of the best-fit RV model parameters found by RadVel (Table 4). For a $M_c$ between 5.47 $M_{\text{Jup}}$ and 2 $M_{\text{Sun}}$, we find a $\delta t_b$ using Equation (12) to be between $1.9 \times 10^{-10}$ s and $7.4 \times 10^{-8}$ s. Hence, the expected TTV signal is many orders of magnitude below the observed TTVs regardless of the mass of the outer companion. Our results are expected, as resonant perturbations between close planets is the main cause of large ($\sim >$ minutes) TTVs (e.g., Agol et al. 2005; Steffen et al. 2012; Nesvorný et al. 2013; Dawson et al. 2019). Therefore, we conclude that the observed TTVs are caused by orbital decay or apsidal precession and not gravitational perturbations from the outer body.

4. Discussion

From our analysis, we derived updated planetary parameters and find that the orbital decay model is slightly preferred to explain the observed TTVs. We conclusively rule out linear acceleration as the cause of the period change. We also show that TTVs cannot be caused by the second body orbiting WASP-4 regardless of its mass. However, apsidal precession is not ruled out. Further transit and occultation measurements of WASP-4b are needed to disentangle the cause of the variation. The orbital decay and apsidal precession models exhibit very different timing variations in the mid-2020s (Figure 7). Therefore, we predict that it will be possible to conclusively determine the cause of WASP-4b’s changing orbit by then.

Assuming the TTVs can be explained entirely by orbital decay, we derive an updated period of $1.338231587 \pm 0.000000022$ days and a decay rate of $-7.33 \pm 0.71$ ms yr$^{-1}$. Our results indicate an orbital decay timescale of $\tau = P/|\dot{P}| = 15.77 \pm 1.57$ Myr, slightly longer than the value.

Figure 5. WASP-4b transit (panels (a), (b), and (c)) and occultation (panel (d)) timing variations after subtracting the data with a constant-period model. The filled black triangles are the data points from the literature and the square orange points are from the TESS data in this paper. All the transit and occultation times can be found in Table 5. The orbital decay and apsidal precession models are shown as the blue and red lines, respectively.

Figure 6. Forward models of the TTV signal of WASP-4b produced by the possible outer companion. These models were created using OCFit. The models used the priors as listed in Table 4, where the outer planet has an eccentricity of 0.094 (Model #4). The two objects are assumed to be coplanar.
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derived by Bouma et al. (2019) of 9.2 Myr. Assuming that the planet mass is constant, the rate of change of WASP-4b’s orbital period, \( \dot{P} \), can be related to its host star’s modified tidal quality factor by the constant-phase-lag model of Goldreich & Soter (1966), defined as

\[
\dot{P} = -\frac{27\pi}{2Q_*^i} \left( \frac{M_\star}{M_\mathrm{p}} \right) \left( \frac{R_\star}{a} \right)^5,
\]

where \( M_\mathrm{p} \) is the mass of the planet, \( M_\star \) is the mass of the host star, \( R_\star \) is the radius of the host star, and \( a \) is the semimajor axis of the planet. Using our derived value of \( \dot{P} \) and planetary parameters from Table 1, we find a modified tidal quality factor of \( Q_*^i = 5.1 \pm 0.9 \times 10^4 \). This value is higher than the value found by Bouma et al. (2019) of \( Q_*^i = 2.9 \pm 0.3 \times 10^4 \) and by Southworth et al. (2019) of \( Q_*^i = 3.8 \pm 0.3 \times 10^4 \). The cause of this small discrepancy (all \( Q_*^i \) values are within 2\( \sigma \)) is due to our including more transit data in our analysis than previous studies.

It is currently not clear how to account theoretically for all the observed low values of \( Q_*^i \). Our value is an order of magnitude lower than the observed values for binary star systems (10\(^5\)–10\(^7\); Meibom et al. 2015) and hot Jupiters (10\(^5\)–10\(^6.5\); e.g., Jackson et al. 2008; Husnoo et al. 2012; Barker 2020), and theoretically predicted values (10\(^5\)–10\(^10\); e.g., Ogilvie 2014; Essick & Weinberg 2016; Collier Cameron & Jardine 2018; Ma & Fuller 2021). However, our value is consistent with Hamer & Schlaufman (2019), who found that hot Jupiter host stars tend to be young, requiring \( Q_*^i \lesssim 10^7 \). Some possible causes of a large tidal dissipation rate might be that WASP-4 is turning off the main sequence (as suspected for WASP-12; Weinberg et al. 2017) or that an exterior planet could be trapping WASP-4b’s spin vector in a high-obliquity state (also predicted for WASP-12b; Milholland & Laughlin 2018). The latter theory is intriguing as we now have evidence for an additional body in the system (Figure 4). More investigation is needed to understand the low value of \( Q_*^i \).

If confirmed, WASP-4b would be the second exoplanet after WASP-12b (Yee et al. 2020; Turner et al. 2021) to show evidence of tidal orbital decay. Future observations of WASP-4b are needed to verify this possibility. Other planets, such as WASP-103b, KELT-16b, and WASP-18b, are also predicted to exhibit large rates of tidal decay (Patra et al. 2020). Additionally, the planets HATS-2b and WASP-64b are also ideal candidates to search for orbital decay because they are in systems similar to WASP-4 (Southworth et al. 2019). To understand the formation and evolution of the hot Jupiter population, timing observations of additional systems are needed.

The discovery of WASP-4c makes the WASP-4 system unique, as it would be the most widely separated companion of a transiting hot Jupiter known to date. However, our discovery of WASP-4c is not surprising because Knutson et al. (2014) estimated that 27% \pm 6% of hot Jupiters have a planetary companion at a distance of 1–10 au and a mass of 1–13 \( M_{\text{Jup}} \). Similar planets may be common in other systems but could only be detected with long-time-baseline RV data sets. Therefore, we expect that the unique status of the WASP-4 system is a result of observational bias rather than an intrinsic rarity of such systems.

Future observations of the WASP-4 system can help us put the system in context with the overall hot Jupiter population and shed light on the possible formation scenarios of the system. Specifically, stronger constraints on the obliquity, mutual inclinations, and full orbital parameters of WASP-4c will help us understand planetary migration in this system.

5. Conclusions

We analyzed all available sectors of TESS data of WASP-4b to investigate its possible changing orbit. Our TESS transit timing investigations confirm that the planet’s orbit is changing (Figure 5). We conclude that the acceleration of the WASP-4 system toward Earth is not the cause of the period variation after analyzing all available RV data (Figure 4; Table 4). From the RV analysis, we also find evidence of a possible second planet orbiting WASP-4 with a period \( (P_\circ) \) of 7001.0\(^{+10.0}_{-6.6} \) days, semimajor axis of 6.82\(^{+0.23}_{-0.25} \) au, and a \( M_\circ \sin(i) \) of 5.47\(^{+0.44}_{-0.43} M_{\text{Jup}} \) (Figure 4; Table 4). WASP-4c is the most widely separated companion of a transiting hot Jupiter discovered to date. This outer planet is not the cause of the observed TTVs (Figure 6). Our timing analysis slightly favors the orbital decay scenario over apsidal precession as the cause of the TTVs (Table 6; Figure 5). However, apsidal precession cannot be ruled out. We find an updated period of 1.338231587 \pm 0.000000022 days, a decay rate of \(-7.33 \pm 0.71 \) ms yr\(^{-1} \), and an orbital decay lifetime of 15.77 \pm 1.57 Myr assuming the system is undergoing
orbital decay. The planetary physical parameters are also updated with greater precision than previous studies. More transit, occultation, and RV data are needed over the next few years to determine conclusively the cause of WASP-4b’s changing orbit and help place the system in context with the overall hot Jupiter population.

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**Facilities:** TESS (Ricker et al. 2015); Exoplanet Archive.

**Software:** EXOMOP (Pearson et al. 2014; Turner et al. 2016b, 2017); IDL Astronomy Users Library (Landsman 1995); Coyote IDL created by David Fanning and now maintained by Paulo Penteado (JPL); RadVel (Fulton et al. 2018).

### Appendix A

**Transit Fits to Individual TESS Transit Events**

The parameters for each transit fit can be found in Tables A1–A3. The light curves and EXOMOP model fits can be found in Figures A1–A7.
Figure A1. Individual TESS transit events (1–8) from Sector 2 of WASP-4b. The best-fitting model obtained from the EXOplanet MOdeling Package (EXOMOP) is shown as a solid red line. The residuals (light curve – model) are shown below the light curve.
Figure A2. Individual TESS transit events (8–16) from Sector 20 of WASP-4b. Other comments are the same as Figure A1.
Figure A3. Individual TESS transit events (17–18) from Sector 2 of WASP-4b. Other comments are the same as Figure A1.

| Transit | T$_{c}$ (BJD$_{TDB}$-2458350) | $R_p/R_*$ | $a/R_*$ | Inclination (°) | Duration (minutes) | $\chi^2_{\text{reduced}}$ |
|---------|-------------------------------|-----------|---------|----------------|--------------------|----------------------|
| 1       | 5.1848698 ± 0.0003176         | 0.1516 ± 0.0016 | 5.32 ± 0.27 | 87.35 ± 2.64 | 127.97 ± 2.84 | 1.02 |
| 2       | 6.5225253 ± 0.00034           | 0.1523 ± 0.0015 | 5.36 ± 0.16  | 89.93 ± 2.36  | 127.97 ± 2.93 | 1.05 |
| 3       | 7.8609667 ± 0.00032           | 0.1538 ± 0.0016 | 5.46 ± 0.18  | 89.96 ± 2.56  | 126.04 ± 2.85 | 0.98 |
| 4       | 9.1994963 ± 0.00032           | 0.1513 ± 0.0019 | 5.31 ± 0.31  | 90.05 ± 2.87  | 129.90 ± 2.93 | 0.97 |
| 5       | 10.5370341 ± 0.00037          | 0.1505 ± 0.0017 | 5.40 ± 0.21  | 90.00 ± 2.41  | 128.00 ± 2.89 | 1.00 |
| 6       | 11.8754331 ± 0.00037          | 0.1513 ± 0.0016 | 5.01 ± 0.06  | 89.0 ± 4.23   | 127.97 ± 2.90 | 0.97 |
| 7       | 13.2140842 ± 0.00036          | 0.1516 ± 0.0019 | 9.05 ± 2.87  | 127.97 ± 2.99 | 127.97 ± 2.83 | 1.04 |
| 8       | 14.5520029 ± 0.00037          | 0.1505 ± 0.0017 | 9.00 ± 2.41  | 128.00 ± 2.89 | 132.01 ± 2.87 | 1.03 |
| 9       | 15.8906531 ± 0.00037          | 0.1513 ± 0.0016 | 5.01 ± 0.06  | 89.0 ± 4.23   | 127.97 ± 2.90 | 1.00 |
| 10      | 19.9050877 ± 0.00036          | 0.1519 ± 0.0019 | 9.09 ± 3.11  | 127.97 ± 2.85 | 132.01 ± 2.88 | 1.01 |
| 11      | 21.2429237 ± 0.00039          | 0.1525 ± 0.0025 | 9.14 ± 0.32  | 128.00 ± 2.83 | 132.01 ± 2.87 | 0.97 |
| 12      | 22.5812197 ± 0.00038          | 0.1519 ± 0.0016 | 9.09 ± 3.11  | 127.97 ± 2.85 | 127.97 ± 2.84 | 1.09 |
| 13      | 23.9197516 ± 0.00036          | 0.1525 ± 0.0021 | 9.14 ± 0.32  | 127.97 ± 2.85 | 132.01 ± 2.88 | 1.01 |
| 14      | 25.2580051 ± 0.00037          | 0.1519 ± 0.0016 | 9.09 ± 3.11  | 127.97 ± 2.85 | 132.01 ± 2.88 | 1.01 |
| 15      | 26.5961751 ± 0.00038          | 0.1519 ± 0.0016 | 9.09 ± 3.11  | 127.97 ± 2.85 | 127.97 ± 2.84 | 1.07 |
| 16      | 27.9342669 ± 0.00033          | 0.1519 ± 0.0014 | 5.18 ± 0.30  | 132.01 ± 2.90 | 130.08 ± 2.84 | 1.01 |
| 17      | 29.2731926 ± 0.00033          | 0.1530 ± 0.0015 | 5.30 ± 0.32  | 130.08 ± 2.84 | 126.04 ± 2.89 | 1.01 |
| 18      | 30.6110363 ± 0.00035          | 0.1521 ± 0.0018 | 5.46 ± 0.30  | 130.08 ± 2.84 | 127.98 ± 2.95 | 1.03 |

Note. The linear and quadratic limb-darkening coefficient used in the analysis are 0.382 and 0.210 (Claret 2017).
Figure A4. Individual TESS transit events (1–8) from Sector 28 of WASP-4b. Other comments are the same as Figure A1.
Figure A5. Individual TESS transit events (9–15) from Sector 28 of WASP-4b. Other comments are the same as Figure A1.
Figure A6. Individual TESS transit events (1–8) from Sector 29 of WASP-4b. Other comments are the same as Figure A1.
Figure A7. Individual TESS transit events (9–14) from Sector 29 of WASP-4b. Other comments are the same as Figure A1.
Table A2
Individual TESS Sector 28 Transit Parameters for WASP-4b Derived Using EXOMOP

| Transit | 1 | 2 | 3 |
|---------|---|---|---|
| $T_c$ (BJD$_{TDB}$-2459050) | 13.1085457 ± 0.00043 | 14.44734 ± 0.00041 | 15.7854034 ± 0.00043 |
| $R_p/R_*$ | 0.1534 ± 0.0027 | 0.1541 ± 0.0022 | 0.1524 ± 0.0028 |
| $a/R_*$ | 4.92 ± 0.49 | 5.41 ± 0.27 | 5.25 ± 0.33 |
| Inclination (°) | 85.32 ± 3.66 | 90.12 ± 3.35 | 89.80 ± 4.81 |
| Duration (minutes) | 133.95 ± 2.82 | 126.21 ± 2.83 | 127.97 ± 2.82 |
| $\chi^2_{\text{reduced}}$ | 0.88 | 0.98 | 0.85 |

| Transit | 4 | 5 | 6 |
|---------|---|---|---|
| $T_c$ (BJD$_{TDB}$-2459050) | 17.1240632 ± 0.00036 | 18.4619638 ± 0.00035 | 19.8001969 ± 0.00044 |
| $R_p/R_*$ | 0.1532 ± 0.0018 | 0.1511 ± 0.0028 | 0.1507 ± 0.0029 |
| $a/R_*$ | 5.46 ± 0.20 | 5.36 ± 0.38 | 5.12 ± 0.34 |
| Inclination (°) | 90.06 ± 2.65 | 90.12 ± 4.37 | 89.97 ± 5.19 |
| Duration (minutes) | 126.21 ± 2.82 | 127.97 ± 2.86 | 131.84 ± 2.83 |
| $\chi^2_{\text{reduced}}$ | 0.85 | 0.98 | 0.97 |

| Transit | 7 | 8 | 9 |
|---------|---|---|---|
| $T_c$ (BJD$_{TDB}$-2459050) | 21.1379261 ± 0.00049 | 26.4905036 ± 0.00043 | 27.8293474 ± 0.00040 |
| $R_p/R_*$ | 0.1532 ± 0.0018 | 0.1511 ± 0.0028 | 0.1507 ± 0.0029 |
| $a/R_*$ | 5.30 ± 0.40 | 5.05 ± 0.37 | 5.33 ± 0.31 |
| Inclination (°) | 89.96 ± 4.27 | 89.74 ± 5.80 | 89.93 ± 3.78 |
| Duration (minutes) | 126.21 ± 2.82 | 131.84 ± 2.83 | 127.97 ± 2.83 |
| $\chi^2_{\text{reduced}}$ | 0.95 | 0.92 | 0.92 |

| Transit | 10 | 11 | 12 |
|---------|----|----|----|
| $T_c$ (BJD$_{TDB}$-2459050) | 29.1681276 ± 0.00042 | 30.564451 ± 0.00044 | 31.8447896 ± 0.00040 |
| $R_p/R_*$ | 0.1521 ± 0.0025 | 0.1534 ± 0.0036 | 0.1548 ± 0.0021 |
| $a/R_*$ | 5.24 ± 0.32 | 5.23 ± 0.44 | 5.62 ± 0.25 |
| Inclination (°) | 90.13 ± 3.79 | 90.00 ± 4.75 | 90.06 ± 2.89 |
| Duration (minutes) | 131.84 ± 2.83 | 130.08 ± 2.83 | 121.99 ± 2.83 |
| $\chi^2_{\text{reduced}}$ | 0.92 | 0.87 | 0.99 |

| Transit | 13 | 14 | 15 |
|---------|----|----|----|
| $T_c$ (BJD$_{TDB}$-2459050) | 33.1816361 ± 0.00041 | 34.5203618 ± 0.00044 | 38.5347781 ± 0.00034 |
| $R_p/R_*$ | 0.1536 ± 0.0021 | 0.1494 ± 0.0028 | 0.1503 ± 0.0015 |
| $a/R_*$ | 5.47 ± 0.27 | 5.18 ± 0.35 | 5.43 ± 0.16 |
| Inclination (°) | 90.04 ± 3.14 | 90.09 ± 4.41 | 90.00 ± 2.37 |
| Duration (minutes) | 125.86 ± 2.83 | 130.08 ± 2.83 | 127.97 ± 2.83 |
| $\chi^2_{\text{reduced}}$ | 0.95 | 0.87 | 0.89 |

Note. The linear and quadratic limb-darkening coefficient used in the analysis are 0.382 and 0.210 (Claret 2017).
Appendix B

Difference in Mid-transit Times for the Sector 2 TESS Data

The comparison in the derived mid-transit times between our analysis and Bouma et al. (2020) for the Sector 2 TESS data is found in Figure B1. Our results are consistent within 1σ.
Appendix C
Radial Velocity Models

We performed an RV analysis on all the RV data in the literature with RadVel. Table 3 compares all the one-planet and two-planet RV models carried out in our analysis. The priors used in RadVel can be found in Table C1. The best fit of the data is the two-planet model (Model #3 in Table 3) with a BIC of 628.34. In this model, the eccentricities of both bodies ($e_b$ and $e_c$) are fixed to zero and there is no linear acceleration. The planetary parameters derived of all models can be found in Table 4. The posterior distributions for all free parameters of Model #1–4 can be found in Figures C1–C4. The best-fit Keplarian orbital models compared to the RV data can be found in Figure 4.

We also performed an RV analysis only on the data presented in Bouma et al. (2020). We used priors listed in Table C1. The results of that analysis can be found in Table C2 and Figure C5. We find a $\dot{\gamma} = -0.0400^{+0.0037}_{-0.0037}$ m s$^{-1}$ days$^{-1}$, which is consistent with the value found by Bouma et al. (2020) of $\dot{\gamma} = -0.0422^{+0.0028}_{-0.0027}$ m s$^{-1}$ day$^{-1}$.
Figure C1. Posterior distributions for all free parameters for the one-planet RV analysis without (Model #1) fitting for the linear acceleration. The results of this model can be found in Table 4 and Figure 4.
Figure C2. Posterior distributions for all free parameters for the one-planet RV analysis without (Model #2) fitting for the linear acceleration. The results of this model can be found in Table 4 and Figure 4.
Figure C3. Posterior distributions for all free parameters for the best-fit two-planet RV analysis (Model #3). The results of this model can be found in Table 4 and Figure 4.
Figure C4. Posterior distributions for all free parameters for the two-planet RV analysis (Model # 4) fitting for $e_\nu$. The results of this model can be found in Table 4 and Figure 4.
Figure C5. Best-fit one-planet Keplerian orbital model for WASP-4b using RadVel of only the Bouma et al. (2020) RV data. (a) The maximum-likelihood model is plotted while the orbital parameters listed in Table C2 are the median values of the posterior distributions. The thin blue line is the best-fit one-planet model. We add in quadrature the RV jitter terms listed in Table C2 with the measurement uncertainties for all RVs. (b) Residuals to the best-fit one-planet model. (c) RVs phase-folded to the ephemeris of WASP-4b. The small point colors and symbols are the same as in panel (a). Red circles are the same velocities binned in 0.08 units of orbital phase. The phase-folded model for WASP-4b is shown as the blue line.

Table C1
Priors Used in the RadVel Analysis of the RV Data of WASP-4b

| Prior                  | Constraint                               |
|------------------------|------------------------------------------|
| $e_b$ constrained to be | < 0.99                                   |
| Gaussian prior on $T_{\text{conj}}$ (BJD-2455804): | 0.515752 ± 0.000019                       |
| Gaussian prior on $P_b$ (days): | 1.33823147 ± 0.000000023                 |
| Bounded prior:          | 0.0 < $\sigma_{\text{HARPS,Bouma2020}}$ < 50.0 |
| Bounded prior:          | 0.0 < $\sigma_{\text{HARPS,Baluev2019}}$ < 50.0 |
| Bounded prior:          | 0.0 < $\sigma_{\text{CORALIE,Baluev2019}}$ < 50.0 |
| Bounded prior:          | 0.0 < $\sigma_{\text{CORALIE,Bouma2020}}$ < 50.0 |
| Bounded prior:          | 0.0 < $\sigma_{\text{HIRES}}$ < 100       |

Note. The Gaussian priors on $T_{\text{conj}}$ and $P_b$ were taken from the constant-period model of Bouma et al. (2019).
Table C2
Results of Fitting only the Bouma et al. (2020) RV Data of WASP-4b with RadVel

| Parameter | Value |
|-----------|-------|
| $P_b$ | 1.3382320649 ± 0.00000002 days |
| $T_{comb}$ | 745.51572 ± 0.005 days |
| $e_b$ | 0.0 ± 0.0 |
| $\omega_b$ | 0.0 ± 0.0 radians |
| $K_b$ | 243.0 ± 3.8 m s$^{-1}$ |
| $\gamma$ | 48.8703 ± 0.0005 m s$^{-1}$ |
| $\dot{\gamma}$ | -0.0400 ± 0.0002 m s$^{-1}$days$^{-1}$ |

Note. Reference epoch for $\gamma$, $\dot{\gamma}$: 2455059.

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