Using modified isolation forest machine learning algorithm to study the neutral triple gauge couplings at an $e^+e^-$ collider

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Abstract
Anomaly detection algorithms have been proved to be useful in the search of new physics beyond the Standard Model. However, a prerequisite for using an anomaly detection algorithm is that the signal to be sought is indeed anomalous. This does not always hold true, for example when interference between new physics and the Standard Model becomes important. In this case, the search of new physics is no longer an anomaly detection. To overcome this difficulty, we propose a modified isolation forest algorithm, which appears to be useful in the study of neutral triple gauge couplings at the CEPC, the ILC and the FCC-ee.

Keywords: neutral triple gauge coupling, machine learning, CEPC, ILC, FCC-ee

1. Introduction
The search for signals from new physics (NP) beyond the Standard Model (SM) is one of the most frontier topics in the field of high energy physics (HEP) [1]. In order to avoid dealing with the huge number of various NP models, a model-independent approach known as the SM effective field theory (SMEFT) has become popular in the phenomenological studies [2]. Since the SM is very successful with only few exceptions [3], it can be expected that the NP signal that has not yet been observed must be very small. Moreover, from the SMEFT point of view, the NP signal comes from new interactions, so it can be expected that the kinematic features of NP signals are different. As a consequence, looking for NP signals is anomaly detection (AD), which is well suited to machine learning (ML) algorithms.

ML algorithms have been widely used in HEP [4], for example in the studies that implement the search for NP as AD [5, 6]. One of the advantages is that AD is not only model-independent, but also operator-independent. From the point of view of the operator geometric space, it is not sufficient to study only dimension-6 operators [7], and phenomenological study of dimension-8 operators has been gaining attention [8, 9]. If one has to consider dimension-8 or even dimension-10 operators, the number of
operators is very large, e.g., 993 for dimension-8 and 15456 for dimension-10 \cite{8}. Using an AD algorithm, one can avoid studying the kinematics for specific operators. Besides, AD algorithms such as the isolation forest (IF) algorithm \cite{10} are efficient and easy to implement, which are suitable for the rapidly growing volume of data collected at colliders. The IF algorithm has been shown to be useful in the study of anomalous quartic gauge couplings (aQGCs) in vector boson scattering processes \cite{6}.

However, NP signals are not always very different. For example we find that IF algorithm is not applicable to study the natural triple gauge couplings (nTGCs) at the CEPC, the ILC and the FCC-ee. nTGCs have received a lot of attentions recently because they do not present in the SM and do not receive contributions from dimension-6 operators \cite{11–13}. It has been shown that $e^+ e^-$ colliders are suitable for studying nTGCs \cite{11,13}. However, when the energy scale is not large, the kinematics of NP signal is not sufficiently distinct from the SM, and the interference becomes important. In this case, the search for NP signals is no longer AD, and improved algorithms are needed.

In this letter, we propose a modified IF (MIF) event selection strategy (ESS), to use the variation of anomaly score to discriminate signal events. It inherits the advantages of IF, with a transparent mechanism and almost no tunable parameters, it is an unsupervised ML algorithm that does not need to tag the source of events, it is model-independent and does not depend on the operator to be studied. We find that, MIF is useful in the study of nTGCs at the CEPC, the ILC and the FCC-ee.

The remainder of this letter is organized as follows, in Sec. 2, we briefly discuss the problem of IF algorithm in the study of nTGCs, the MIF algorithm with numerical results are presented in Sec. 3, finally Sec. 4 is a summary.

2. The problem of isolation forest with nTGCs

The nTGCs receive no tree level contribution neither from the SM nor from the dimension 6 operators. There are 4 CP-conserving dimension-8 operators contributing to nTGCs, the Lagrangian is \cite{11,13}

$$L_{nTGC} = \frac{\text{sign}(c_{\tilde{B} BW})}{\Lambda_{BW}^4} O_{BW} + \frac{\text{sign}(c_{\tilde{W} BW})}{\Lambda_{BW}^4} O_{BW} + \frac{\text{sign}(c_{\tilde{W} BW})}{\Lambda_{WW}^4} O_{WW} + \frac{\text{sign}(c_{\tilde{B} BB})}{\Lambda_{BB}^4} O_{BB},$$

with the operators

$$O_{\tilde{B} BW} = iH^\dagger \tilde{B}_{\mu\nu} W^{\mu\rho} \left[D_\rho, D^\nu\right] H + h.c., \quad O_{\tilde{W} BW} = iH^\dagger B_{\mu\nu} \tilde{W}^{\mu\rho} \left[D_\rho, D^\nu\right] H + h.c.,$$

$$O_{\tilde{W} WW} = iH^\dagger \tilde{W}_{\mu\nu} W^{\mu\rho} \left[D_\rho, D^\nu\right] H + h.c., \quad O_{BB} = iH^\dagger \tilde{B}_{\mu\nu} B^{\mu\rho} \left[D_\rho, D^\nu\right] H + h.c.,$$

where $H$ denotes the SM Higgs doublet, $\tilde{B}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}$, $\tilde{W}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} W^{\alpha\beta}$ with $W^{\mu\nu} \equiv W^{\sigma} / 2$ where $\sigma$ are Pauli matrices, $c_X$ are dimensionless coefficients. The $\Lambda_X$ are related with the cutoff scale as $\Lambda_X = \Lambda / |c_X|^{1/4}$. For simplicity, we define $f_X \equiv \text{sign}(c_X) / \Lambda_X$.

At tree level, the processes $e^+ e^- \rightarrow Z\gamma$ can be affected by those operators via $ZV\gamma$ couplings where $V$ is a $Z$ boson or a photon. It is only necessary to consider the $O_{BW}$
operator, because $\mathcal{O}_{\tilde{W}W}$ and $\mathcal{O}_{\tilde{B}B}$ do not contribute when $Z$ boson and $\gamma$ are on-shell, and $\mathcal{O}_{\tilde{B}W}$ has the same contribution as the $\mathcal{O}_{\tilde{B}W}$ in this case [13].

The dominant contribution from operators in Eq. (2) to the process $e^+e^- \rightarrow \ell^+\ell^-\gamma$ where $\ell$ is $e$ or $\mu$ is through process $e^+e^- \rightarrow Z\gamma$. Therefore, one can use the process $e^+e^- \rightarrow \ell^+\ell^-\gamma$ at $e^+e^-$ colliders to study $\mathcal{O}_{\tilde{B}W}$. The signal of $\mathcal{O}_{\tilde{B}W}$ are induced by Feynman diagrams shown in Fig. 1. In this letter, the events of the processes $e^+e^- \rightarrow \mu^+\mu^-\gamma$ and $e^+e^- \rightarrow e^+e^-\gamma$ are combined. The background is the process $e^+e^- \rightarrow \ell^+\ell^-\gamma$ in the SM. The typical Feynman diagrams are shown in Fig. 2.

The numerical results are studied by using the MadGraph5_aMC@NLO toolkit [14]. A fast detector simulation is applied by using Delphes [15] with the CEPC detector card. The events are generated at $\sqrt{s} = 250$ GeV which is the energy scale of ‘Higgs factory’ and is close to or covered by the CEPC [16], the ILC [17] and FCC-ee [18]. We use the basic cuts same as the default settings except for $\Delta R_{\ell\ell}$ which is defined as $\sqrt{\Delta \eta_{\ell\ell}^2 + \Delta \phi_{\ell\ell}^2}$ where $\Delta \eta_{\ell\ell}$ and $\Delta \phi_{\ell\ell}$ are the differences between pseudo-rapidities and azimuth angles of the charged leptons, respectively. As explained in Ref. [11], the $\Delta R_{\ell\ell}$ is small for an energetic $Z$ boson, so we use $\Delta R_{\ell\ell} > 0.2$ [11, 19] to avoid losing too much signal events.

To highlight the signal significance, an ESS based on IF algorithm has been proposed in Ref. [6] and was found useful in the study of aQGCs. IF algorithm makes use of the fact that the anomalies are ‘few and different’, and can be applied for multi-dimensional data efficiently. In IF algorithm, a dimensionless anomaly score (denoted as $a$) can be calculated for each event where $0 < a < 1$. Details on how the anomaly score is calculated can be found in Refs. [6, 10].

To study the signal of nTGCs, 15 data sets are generated with $\mathcal{L}_{SM} + \mathcal{L}_{nTGCs}$ and with $\int_{\mathcal{B}W} = -700 + 100k$ TeV$^{-4}$ where $0 \leq k \leq 14$ are integers. Each data set consists of $N = 500000$ events. These data sets are in fact those used in Ref. [11], so that we can compare our result with Ref. [11]. We require that there are at least two charged leptons, and the two hardest leptons are with the same flavor and different charges.

Figure 1: Typical Feynman diagrams for signal events.

Figure 2: Typical Feynman diagrams for background events.
Except for that, we also require that there are at least one photon in the final state. These requirements are the same as in Ref. [11]. After these requirements there are about 330000 events left in each data set. The one piece of data corresponding to each event is a 12-dimensional vector consisting of components of 4-momenta of two hardest charged leptons and the photon.

![Figure 3](image.png)

Figure 3: The normalized distributions of $\epsilon_a$ for $k = 0, 7, 14$ when $n = 2000$.

Once data sets are determined, the only parameter in IF algorithm is the number of isolation trees (denoted as $n$) which controls the statistical error of the anomaly scores. In principle, $n$ should be as large as possible as far as the computational power allows. The standard error of anomaly score (denoted as $\epsilon_a$) can be calculated for each event. With $n = 2000$, the normalized distributions of $\epsilon_a$ for the events in the data sets with $k = 0, 7, 14$ are shown in Fig. 3. It can be seen that, generally $\epsilon_a < 0.004$ and 2000 trees are sufficient.

One can use an ESS to select the event with $\alpha$ greater than a threshold score (denoted as $a_{th}$) to highlight the signal events. However, in the case of nTGCs at $\sqrt{s} = 250$ GeV, the outcomes are not satisfactory. The cross-sections with different $a_{th}$ are shown in Fig. 4, no obvious pattern can be seen. The problem is that when the energy scale is not large, the kinematic features of signal events are not sufficiently distinct from the SM.

3. Modified isolation forest algorithm

Since the anomaly score calculated by the IF algorithm can be regarded as a representation of the density of events in the phase space, it can be conjectured that anomaly score can also measure the density variation. In order to achieve this without increasing a large amount of computation, one needs to first build an anomaly score distribution by the SM as a reference. This can be done by building an IF using the data set from the SM. Then the MIF ESS can be summarized as follows:
1. Using data set from MC simulation with the SM as a training data set, and build an isolation forest, the anomaly score of each event is obtained as $a_{\text{ref}}$.

2. For the data set to be investigated (from MC simulations or experiments), build another isolation forest, the anomaly score of each event is obtained as $a_{\text{inv}}$.

3. For each event in the data set to be investigated, find the nearest event in the training data set, the variation of anomaly score can be calculated as $\Delta a = a_{\text{inv}} - a_{\text{ref}}$.

For this algorithm to work, the distance between the events need to be defined. In this letter, the distance is simply defined as the distance in the phase space as $d = \sqrt{\sum_{ij}(p_i^j)^2}$, where $p_i^j$ is the $i$-th component of the 4-momentum of the particle $j$ in the final state, and $j$ runs over the two hardest charged leptons and the photon.

Such a method inherits the advantages of IF algorithm. The mechanism behind MIF algorithm is also transparent, there are almost no parameters to be tuned. The whole procedure is still model independent and operator independent, which works as an automatic ESS. Moreover, as an unsupervised machine learning algorithm, it is not necessary to tag the events. This is important because it is in principle impossible to specify the origin of an event when there is a negative interference term presented.

When the nTGCs-induced events overlap with those of the SM in the phase space, it mainly leads to a decrease in anomaly score. Therefore, we select events where $\Delta a$ is less than a certain threshold (denoted as $\Delta a_{th}$). This simple strategy is the only one we use in this letter, and is denoted as ‘MIF cut’. To build the reference anomaly score distribution, another data set with $N = 500000$ are generated with the SM Lagrangian.

The ESS based on kinematic analysis are discussed in Refs. [11, 13]. We compare the results of MIF cut with the results in Ref. [11] which uses an ESS as

$$|M_{\ell\ell} - M_Z| < 15 \text{ (GeV)} \quad |\cos \theta_f| < 0.9, \quad |\cos \theta_\gamma| < 0.8,$$

Figure 4: The cross-sections as functions of $f_{\tilde{B}W}$ after the ESS that selecting events with $a > a_{th}$.
Figure 5: The cross-sections as functions of $f_{BW}$ after the traditional ESS and MIF cut.

Table 1: The constraints on $f_{BW}$ (TeV$^{-4}$) at $L = 2$ ab$^{-1}$.

| $S_{stat}$ | traditional ESS | $\Delta a < -0.02$ |
|------------|-----------------|--------------------|
| 2          | $[-25.7, 85.4]$ | $[-18.9, 114.0]$   |
| 3          | $[-34.9, 94.6]$ | $[-26.6, 121.7]$   |
| 5          | $[-50.1, 109.8]$| $[-40.0, 135.1]$   |

where $M_{\ell\ell}$ is the invariant mass of two hardest charged leptons, $\theta_\gamma$ is the zenith angle of the photon, $\theta_\ell$ is the zenith angle of $\ell^-$ when the $\ell^-$ is boosted into the c.m. frame of two hardest charged leptons whose z-axis lays along the direction of $\vec{p}_{\ell^+} + \vec{p}_{\ell^-}$. The cross-sections after traditional ESS (which is the same as Ref. [11]) and after MIF cut with $\Delta a_\ell = -0.02$ are shown in Fig. 5. We find that the cross-section after MIF cut can still be approximated by a bilinear function within the range of coefficients in use. The fitted cross-section after MIF cut is $\sigma = (217.5 - 0.018f_{BW} + 0.00030f_{BW}^2)$ (fb). Compare our result with Ref. [11]), it can be seen that, with $\sigma(f_{BW} \neq 0) - \sigma(f_{BW} = 0)$ close to each other, the $\sigma(f_{BW} = 0)$ using MIF cut is smaller, indicating a compatible or even better signal significance.

Using the signal significance defined as $S_{stat} = N_s/\sqrt{N_s + N_{bg}}$, the expected constraints on $f_{BW}$ at luminosity $L = 2$ ab$^{-1}$ are presented in Table 1. The results of Ref. [11] are also listed. It can be seen that, the MIF cut, which is independent of the operator to be studied, still shows better discriminative ability for $f_{BW} < 0$ than the traditional ESS.

4. Summary

Although ML algorithms for AD have been widely used in the study of NP, there are cases that the search for NP signals is no longer AD. This happens when the NP signals
are no longer very different, for example when the energy scale is not very large, or when the interference term is important. In this letter, an MIF algorithm is proposed for such cases.

As a case study, the nTGCs at $\sqrt{s} = 250$ GeV are investigated in this letter. It can be shown that while the IF algorithm is not satisfactory, the MIF is useful in this case. As an automatic ESS, MIF can achieve competitive discriminative ability compared to the traditional ESS without knowing at all what operator is under study. For $f_{\tilde{B}W} < 0$, a stronger constraint can be set with the help of MIF cut. On the other hand, in the region that the negative interference is dominant, i.e. $\sigma(f_{\tilde{B}W} \neq 0) < \sigma(f_{\tilde{B}W} = 0)$, the traditional ESS is better. In this region, the pattern how the anomaly score changes is complicated and needs more exploration. In this case, MIF cut is still competitive because it achieves a discrimination capacity close to that of traditional ESS without kinematic analysis.

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