Alcubierre warp drive in Bohmian Quantum Gravity

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Abstract

In the realm of theoretical physics, the Alcubierre warp drive metric has long been a topic of fascination. However, the feasibility of this seemingly miraculous method of faster-than-light travel has been limited by the requirement of exotic matter, which has proven elusive to obtain. This research has revealed a potential breakthrough in the quest for exotic matter, through the coupling of the Alcubierre warp drive metric to a quantum mechanical scalar matter field. By mapping the requirement of exotic matter into a conformal wave equation, it becomes possible to express the problem as a fourth-order partial differential equation in terms of the quantum mechanical density. This research reveals that the key to solving the exotic matter problem lies in finding a proper quantum mechanical density that satisfies the proposed partial differential equation.

1. Introduction

Generalization of Einstein’s theory of gravity is under intense theoretical exploration. Main motivation behind this research involves both mathematical and physical reasons. In a purely mathematical perspective, the Lagrangian density $L$ of Einstein’s gravity is simply linear to the Ricci scalar $R$. One is always tempted to generalize this simple Einstein-Hilbert Lagrangian to more general ones, resulting into $f(R)$ theories and $f(R, T)$ theories etc [6]. In addition, there are several physical motivations to study generalized Einstein theory, for example, the dark energy problem is one among them [7]. Quantum gravity considerations is also there, which hints the extension of Einstein’s theory for a concise physical picture. The classical generalization of general relativity serves as a gravitational alternative for describing both early-time inflation and late-time cosmic acceleration. Various modified theories, including traditional $F(R)$ and scalar–tensor theory, are already explored for their cosmological properties and potential compatibility with local tests [8]. In addition, [9] discusses various theoretical proposals, including $F(R)$, $F(G)$, and $F(T)$ gravity, and their viability as descriptions for our Universe, exploring their ability to explain inflation, bouncing cosmologies, and the dark energy era.

Even though there are several mathematical route for the generalization of Einstein’s theory, $f(R)$ theory is mathematically more straightforward. Later it was shown that $f(R)$ theory of gravity [10, 11] is equivalent to scalar tensor theory of gravity [12]. Based on the nature of the covariant derivative of the metric tensor, Einstein’s gravity can also be generalized in a different manner. Metric compatibility ($\nabla_a \varepsilon_{\mu \nu} = 0$) and zero torsion ($\Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu} = 0$) are the two essential conditions in Einstein’s theory. Both of these conditions can be relaxed and each kind of extensions Einstein’s theory can be obtained. Relaxing the metric compatibility condition, one is naturally lead to Weyl differential geometry [13]. Due to the research in the last few decades, it is identified that the Weyl type of differential geometry is very useful to incorporate quantum mechanical scalar matter fields as a correction to Einstein’s theory [14]. Based on the deBroglie-Bohm version of quantum theory [15–17], one is able to couple quantum theory with classical gravity. There are several research work along this interesting direction.

Usual quantum gravity approaches like Loop Quantum Gravity focuses on the quantization of gravity [18, 19] while Bohmian Quantum Gravity gives only a geometrical way to couple quantum mechanical scalar matter fields with Einstein’s gravity [14, 20, 21]. Note that gravity is still treated as classical and we are only focusing on how classical field theories are coupled each other. We will first focus on the essential equations in
Bohmian Quantum Gravity and more specifically the stress-energy tensor is analyzed in connection with warp drive. Then we will explore Alcubierre warp drive metric, taking it as the background space-time coupled to quantum mechanical scalar matter field. Here we search for the properties of the quantum system in order to support Alcubierre metric as the background space-time.

2. Bohmian Quantum Gravity

In previous research works [22–24], the following action is taken into account, where the quantum mechanical scalar matter field is coupled to gravity in a geometric manner (de Broglie-Bohm manner),

\[
A[g_{\mu\nu}, \Omega, S, \rho, \lambda] = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left( R\Omega^2 - 6\nabla_\mu \Omega \nabla^\mu \Omega \right) + \int d^4x \sqrt{-g} \left( \frac{\rho}{m} \Omega^2 \nabla_\mu S \nabla^\mu S - m^2\rho^2 \right) + \int d^4x \sqrt{-g} \left[ \ln \Omega^2 - \left( \frac{h^2}{m^2} \frac{\nabla_\mu \nabla^\mu \sqrt{\rho}}{\sqrt{\rho}} \right) \right].
\]

(1)

It is already shown that, minimizing the action with respect to \( \rho \) and \( S \) leads to real and imaginary parts of the Generalized Klein–Gordon equation. Here equation (2) describes the real part of the Klein–Gordon equation and it is given by,

\[
\nabla_\mu S \nabla^\mu S - m^2\Omega^2 + \frac{h^2}{2m\Omega^2\sqrt{\rho}} \left[ \left( \frac{\lambda}{\sqrt{\rho}} \right) - \lambda \frac{\nabla \sqrt{\rho}}{\rho} \right] = 0.
\]

(2)

Here equation (3) gives the generalized continuity equation which is the imaginary part of the more general Klein–Gordon equation (see [25] for more details),

\[
\nabla_\mu (\rho \Omega^2 \nabla^\mu S) = 0.
\]

(3)

One can also see that the constraint equation is given by,

\[
\Omega^2 = \exp \left( \frac{h^2}{m^2} \frac{\nabla_\mu \nabla^\mu \sqrt{\rho}}{\sqrt{\rho}} \right).
\]

(4)

where the conformal factor is identified as a quantum mechanical quantity i.e. the exponential of the Bohmian quantum potential. In varying the action with respect to \( \Omega \), one obtains an equation of motion for the scalar curvature \( R \)

\[
R\Omega + 6 \Box \Omega + \frac{2\kappa}{m} \rho \Omega (\nabla_\mu S \nabla^\mu S - 2m^2\Omega^2) + \frac{2\kappa \lambda}{\Omega} = 0.
\]

(5)

Finally, varying the action with respect to the inverse metric tensor \( g^{\mu\nu} \), the conformally transformed Einstein equation, along with its stress-energy tensors, are obtained

\[
G_{\mu\nu} = T_{\mu\nu}^{\text{matter}} (S, \rho) + T_{\mu\nu}^{\text{qm}} (\Omega) + T_{\mu\nu}^{\text{med}} (\lambda, \rho)
\]

(6)

where the stress-energy tensors are given by,

\[
T_{\mu\nu}^{\text{matter}} (S, \rho) = - \frac{2\kappa}{m} \rho \nabla_\mu \nabla_\nu S + \frac{\kappa}{m} \rho g_{\mu\nu} \nabla_\alpha S \nabla^\alpha S - \kappa m \rho \Omega^2 g_{\mu\nu}.
\]

(7)

\[
T_{\mu\nu}^{\text{qm}} (\Omega) = \left( \frac{g_{\mu\nu} \Box \Omega^2 - \nabla_\mu \nabla_\nu \Omega^2}{\Omega^2} \right) + 6 \nabla_\mu \Omega \nabla_\nu \Omega \frac{\Omega^2}{\Omega^2} - 3g_{\mu\nu} \nabla_\alpha \Omega \nabla^\alpha \Omega \frac{\Omega^2}{\Omega^2}.
\]

(8)

\[
T_{\mu\nu}^{\text{med}} (\lambda, \rho) = - \frac{\kappa h^2}{m^2\Omega^2} \left[ \nabla_\mu \sqrt{\rho} \nabla_\nu \left( \frac{\lambda}{\sqrt{\rho}} \right) - g_{\mu\nu} \nabla_\nu \left( \frac{\lambda \nabla \sqrt{\rho}}{\rho} \right) \right].
\]

(9)

Here \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \) is the Einstein tensor. Einstein tensor appears from the geometrical part of the action which is a conformally transformed Einstein-Hilbert action \( S_{\text{EH}} = \frac{1}{2\kappa} \int \sqrt{-g} d^4x \left( R - 6\nabla_\alpha \Omega \nabla^\alpha \Omega \right) \).

Note that, the resulting Einstein’s equation is composed of stress-energy tensors related to the matter \( T_{\mu\nu}^{\text{matter}} (S, \rho) \), quantum \( T_{\mu\nu}^{\text{qm}} (\Omega) \) and mediating \( T_{\mu\nu}^{\text{med}} (\lambda, \rho) \) contributions. The vacuum density \( \lambda \) obeys a first order differential equation coupled to \( \sqrt{\rho} \), which is given by,
\[ \lambda = \frac{\hbar^2}{m^2(1-Q)} \nabla_{\mu} \left( \frac{\lambda \nabla_{\nu} \sqrt{\rho}}{\sqrt{\rho}} \right). \]  

(10)

This \( \lambda \) equation is obtained by comparing trace equation given in equation (5) with the trace of the tensor equation given in equation (6). It is found that \( \lambda \) term can give additional corrections to standard quantum theory (see [23, 24] for more details).

3. Alcubierre warp drive

Usually Alcubierre metric is studied using Einstein’s General Theory of Relativity and one is encountered with the problem of the exotic matter [26]. Later, considerable amount of research works were carried out in relation to the physical feasibility of the Warp drive solution [27], its energy requirements [28, 29]. Advancing further, some of the modifications to Alcubierre warp drive were also suggested [30–32]. Recently various matter–energy sources is explored in order to see whether it supports Alcubierre warp drive metric solution and a relation to Burgers’ equation is found [33–36].

Here we focus on the more generalized theory of Einstein’s gravity, i.e. the Bohmian Quantum Gravity which incorporate quantum mechanical matter field coupling to gravity using conformal factor. In order to explore the warp drive in the context of Bohmian Quantum Gravity, we use the following assumption that the background space-time is Alcubierre warp-drive metric which is coupled to a quantum mechanical scalar matter field \( \phi = \sqrt{\rho} \exp \left( \frac{\lambda}{\hbar} S \right) \). In Bohmian Quantum Gravity, this complex scalar matter-field can be coupled to gravity and the total metric of the system is given by \( g_{\mu \nu}^{gr} = \Omega^2 g_{\mu \nu}^{fr} \) where \( \Omega^2 \) is the quantum mechanical conformal factor. Here the gravitational background metric \( g_{\mu \nu}^{fr} \) is simply represented as \( g_{\mu \nu} \). In \((+++−)\) signature, Alcubierre metric can be written as,

\[ ds^2 = g_{\mu \nu} dx^\mu dx^\nu = (1 - v_i(t)^2 f(r_i(t))^2) dt^2 + 2v_i(t)^2 f(r_i(t)) dx^i dt - dx^2 - dy^2 - dz^2, \]

where \( v_i(t) = \frac{dx_i(t)}{dt} \) is the warp bubble velocity. While the metric tensor \( g_{\mu \nu} \) with \((+++−)\) signature can be written as,

\[ g_{\mu \nu} = \begin{pmatrix} 1 - v_i^2 f(r_i)^2 & v_i f(r_i) & 0 & 0 \\ v_i f(r_i) & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \]

(12)

The inverse metric tensor \( g^{\mu \nu} \) can be evaluated as,

\[ g^{\mu \nu} = \begin{pmatrix} 1 & v_i f(r_i) & 0 & 0 \\ v_i f(r_i) & -1 + v_i^2 f(r_i)^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \]

(13)

Here \( f(r_i) \) is termed as the shape function of the warp metric since it describes the shape of the warp bubble.

\[ f(r_i) = \frac{\tanh[\sigma(\tau_i + R)] - \tanh[\sigma(\tau_i - R)]}{2 \tanh(\sigma R)}, \]

(14)

where \( \sigma \) is inversely related to the thickness of the warp bubble and \( R \) is the warp bubble radius. The variable \( r_i(t) \) is defined as the distance from the center of the bubble \([x_i(t), 0, 0]\) to an arbitrary point \((x, y, z)\) on the surface of the bubble and it is given by, \( r_i(t) = \sqrt{(x - x_i(t))^2 + y^2 + z^2} \). Alcubierre already had mentioned that this shape function can approach a step function as \( \sigma \to \infty \) i.e. \( |r_i(t)| < R \) then \( f(r_i) = 1 \), whereas for distances where \( |r_i(t)| \gg R \) then \( f(r_i) \to 0 \). The components for the Eulerian (normal) observers 4-velocities are given by, \( u^\alpha = [1, v_i(t)f(r_i), 0, 0] \) and \( u_\alpha = [0, 0, 0, 0] \). Note that these vectors are considered in \((+++−)\) signature. Here also one can obtain,

\[ G_{\mu \nu} u^\mu u^\nu = -\frac{v_i^2(y^2 + z^2)}{4r_i^2} \left( \frac{df}{d\sigma} \right)^2. \]

(15)

Using the relation \( G_{\mu \nu} \equiv T_{\mu \nu} \) (see equation (6)) the following expression is obtained,

\[ T_{\mu \nu} u^\mu u^\nu = -\frac{v_i^2(y^2 + z^2)}{4r_i^2} \left( \frac{df}{d\sigma} \right)^2. \]

(16)
Note that this expression is derived using the geometric arguments only. Alcubierre had started with a warp drive metric and found a condition for the energy density in order to have a solution that he desires [26]. Then one needs to look into the proper stress-energy tensor $T_{\mu\nu}$, which can support the desired warp drive space-time structure. In usual general relativity one is compelled to accept the requirement of the exotic matter in order to get the desired warp drive solution. Warp drive metric is no longer a solution to Einstein’s General Relativity since it violate the energy conditions allowed in the theory. Hence one is instructed to look into a more generalized theory which can easily support the warp drive solution without much difficulty and Bohmian Quantum Gravity is a good candidate for that.

Since we are taking into account the extended theory of Einstein General Relativity i.e a Scalar-Tensor Theory which contains an additional scalar degrees of freedom apart from the Einstein’s gravitational tensor field. Note that the scalar field is quantum mechanical in origin. There are many different types of scalar tensor theories but the Bohmian Quantum Gravity impose a special physical meaning to the scalar field and it is intimately related to the conformal factor in the theory which is quantum mechanical in nature. The quantum mechanical conformal factor is getting coupled to gravity. Once quantum mechanical matter field couples with gravity, one can find the exact expression for the stress energy tensor $T_{\mu\nu}$, and it contains three contributions (see equations (7), (8) and (9)). Substituting the expression of $T_{\mu\nu}$ in the equation, one can find the condition which needs to be satisfied in the Bohmian Quantum Gravity.

Even though the space-time is rigid in the Riemannian sense, it is not so in Weyl sense. In Riemannian geometry, the length of a vector remains constant and only the orientation changes during parallel transport. While in Weyl Geometry the length of a vector and orientation changes during parallel transport.

$$\nabla_{\alpha} g_{\mu\nu} = 0 \implies \text{Riemannian Geometry}$$

$$\nabla_{\alpha} g_{\mu\nu} = \alpha_{\alpha} g_{\mu\nu} \implies \text{Weyl Geometry}$$

In Riemannian geometry the covariant derivative of the metric is zero while it is not true in the Weyl Geometry. Since the length changes during the parallel transport, the covariant derivative of the metric gives a nonzero contribution. In Weyl geometry, one can think about different conformal frames where the metric is different by a conformal scaling but the physics remains the same. In Bohmian Quantum Gravity frame-work, one can see that the quantum potential $Q_{\alpha}$ appears as the Weyl scalar field $\sigma$ since $\nabla_{0} g_{\mu\nu} = Q_{\alpha} g_{\mu\nu}$ and it is purely a quantum mechanical factor. Parallel transporting a vector along a Weyl manifold will make a change in the length of the vector and such a freedom is not there in Riemannian manifold. In other words, Space-time is rigid in the Riemannian sense but it is not so rigid in Weyl sense, hence using quantum effects one is able to manipulate space-time structure easily. This interesting fact is reflected while analyzing the contribution coming from $T_{\mu\nu} u^{\mu} u^{\nu}$ and major contribution is from $T^{qm}_{\mu\nu}(\Omega) u^{\mu} u^{\nu}$. Note that the inverse gravitational constant $1/\kappa$ appearing as a term in front of the usual expression for the exotic matter is not present while considering $T^{qm}_{\mu\nu}(\Omega) u^{\mu} u^{\nu}$ term. Since we adopt the Weyl Geometry, it can be seen that the Weyl length change is associated to conformal factor $\Omega$ which is purely quantum mechanical in origin. Hence quantum mechanical contribution can make space-time manipulation much easier than the usual gravitational bending of space-time.

From our previous studies [22–24], it is found that there is a dynamical cosmological term which appears along with the $g_{\mu\nu}$ term. The dynamical cosmological term is written in terms of the quantum potential $Q$ and Lagrange multiplier $\lambda$. Hence, focusing on $T^{qm}_{\mu\nu}(\Omega) u^{\mu} u^{\nu}$ term’s $g_{\mu\nu}$ part and ignoring other contributions (they are smaller due to the presence of $\kappa$), one is lead to

$$T_{\mu\nu} u^{\mu} u^{\nu} \approx T^{qm}_{\mu\nu}(\Omega) u^{\mu} u^{\nu}$$

$$T^{qm}_{\mu\nu}(\Omega) u^{\mu} u^{\nu} \approx \frac{\Box \Omega^{2} - 3 \nabla_{\alpha} \Omega \nabla^{\alpha} \Omega}{\Omega^{2}} g_{\mu\nu} u^{\mu} u^{\nu}$$

Since $g_{\mu\nu} u^{\mu} u^{\nu} = 1$ in $\begin{pmatrix} + & - & - & - \end{pmatrix}$ signature, one can obtain,

$$T_{\mu\nu} u^{\mu} u^{\nu} \approx \frac{\Box \Omega^{2} - 3 \nabla_{\alpha} \Omega \nabla^{\alpha} \Omega}{\Omega^{2}}.$$

This is the result from Bohmian Quantum Gravity and combining this with our previous expression for the exotic matter, one can get,

$$\frac{\Box \Omega^{2} - 3 \nabla_{\alpha} \Omega \nabla^{\alpha} \Omega}{\Omega^{2}} = - \frac{v_{i}^{2}(y^{2} + z^{2})}{4r_{i}^{2}} \left( \frac{df}{dr_{i}} \right)^{2}$$

Note that, while establishing equation (22), one is utilizing more general gravitational theory. Rearranging the terms, one is lead to a Klein–Gordon type wave equation in terms of the conformal factor $\Omega$, i.e
Let us recall the expression of exotic matter as Alcubierre exotic function $\text{Alc}(x, y, z)$ and it is given by

\[
\text{Alc}(x, y, z) = \frac{v^2(y^2 + z^2)}{4c^2} \left( \frac{df}{dx} \right)^2,
\]

then equation (22) becomes,

\[
\Box \Omega^2 - 3 \nabla^2 \Omega + \frac{v^2(y^2 + z^2)}{4c^2} \left( \frac{df}{dx} \right)^2 \Omega^2 = 0
\] (23)

Here the Alcubierre exotic function appear like the positive mass of the conformal wave equation. Note that, the conformal wave equation is purely a quantum mechanical feature, since $\Omega^2$ is intimately related to the quantum potential $Q$ which in turn related to the square root of the quantum mechanical matter wave density. The constraint equation indicates that

\[
\Omega^2 = \exp \left( \frac{h^2 \Box \sqrt{\rho}}{m^2} \right) = \exp(Q).
\] (24)

Note that $\Omega^2$ has a double property, in one hand, it can be written as a purely a quantum mechanical quantity and on the other hand it has a definite geometrical meaning as a conformal factor which can be written in terms of a Weyl scalar field. Substituting the expression of $\Omega^2$ in equation (23), one can write the equation in terms of the quantum potential $Q$. Hence the equation becomes,

\[
\Box Q + \frac{1}{4} \nabla^2 Q + \text{Alc}(x, y, z) = 0
\] (25)

Substituting the expression for $Q$ in terms of the quantum mechanical matter wave density, i.e using the following relation,

\[
Q = \frac{h^2}{m^2} \left( \Box \sqrt{\rho} \right).
\] (26)

One can get,

\[
\frac{h^2}{m^2} \Box \left( \sqrt{\frac{\rho}{\rho}} \right) - \frac{h^4}{4m^4} \nabla^2 \left( \sqrt{\frac{\rho}{\rho}} \right) \nabla^2 \left( \sqrt{\frac{\rho}{\rho}} \right) + \text{Alc}(x, y, z) = 0
\] (27)

Ignoring the higher order terms i.e $O(h^4)$ terms, one can obtain,

\[
\Box \left( \sqrt{\frac{\rho}{\rho}} \right) + \frac{m^2}{h^2} \text{Alc}(x, y, z) = 0
\] (28)

Laplace-Beltrami operator on a curved space-time is given by,

\[
\Box \sqrt{\rho} = \frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu \nu} \partial_{\nu} \sqrt{\rho} \right).
\] (29)

Since $\sqrt{-g} = 1$ (for Alcubierre metric), in our context, the fourth order contribution $\Box \left( \sqrt{\frac{\rho}{\rho}} \right)$ becomes,

\[
\Box \left( \sqrt{\frac{\rho}{\rho}} \right) = \partial_{\alpha} \left( g^{\alpha \beta} \partial_{\beta} \left( \frac{\partial_{\mu} (g^{\mu \nu} \partial_{\nu} \sqrt{\rho})}{\sqrt{\rho}} \right) \right).
\] (30)

Thus equation (28) becomes,

\[
\partial_{\alpha} \left( g^{\alpha \beta} \partial_{\beta} \left( \frac{\partial_{\mu} (g^{\mu \nu} \partial_{\nu} \sqrt{\rho})}{\sqrt{\rho}} \right) \right) + \frac{m^2}{h^2} \text{Alc}(x, y, z) = 0
\] (31)

Expanding this equation in terms of the components of the metric tensor (given in equation (12)), one is lead to the expression containing the fourth order space-time derivative of $\sqrt{\rho}$ with other terms involving the Alcubierre shape function $f(r)$ and the velocity of the warp bubble $v_r$. It is difficult to find the analytical solutions of these kinds of fourth order differential equations but one can take very simplified toy versions of the wave equation in order to have an idea of the new physics emerging out from the equation presented here. The main achievement we have made in equation (31) is mapping the problem of the exotic matter into finding a suitable quantum density for the matter field as a solution to the fourth order partial differential equation on curved space-time. If the square root of the quantum density ($\sqrt{\rho}$) satisfy the wave equation given in equation (31), it is equivalent to having an exotic matter in usual general relativity. Thus, one needs to expand the idea of having exotic matter into the determination of proper quantum mechanical density. Hence the gravitational problem is mapped into a purely quantum mechanical problem. In the Bohmian Quantum Gravity sense, these quantum mechanical density will result into a conformal wave equation. Since the exotic matter problem is addressed by mapping it into a fourth order quantum wave
equation, leading to noticeable physical effects described by equation (31) or equation (22). The presence of a proper quantum mechanical density can induce conformal fluctuations in space-time, resulting in ripples generated by the conformal factor. These quantum mechanical conformal ripples correspond to the exotic matter problem within the framework of Einstein’s general relativity.

Now let us assume a simple fourth order wave equation where the space-time is flat and the quantum density is confined along the x-direction and instead of $\text{Alc}(x, y, z)$, we have just a constant $\beta^3$. Hence we can present a toy equation as follows,

$$\partial_t^4 \sqrt{\rho(t, x)} - \partial_x^4 \sqrt{\rho(t, x)} + \beta^4 \sqrt{\rho(t, x)} = 0$$

(32)

By the method of separation of variables, it is possible to find interesting solutions to the quantum density. Apart from the usual trigonometric functions, hyperbolic functions also appear as the solutions which is not allowed in usual quantum theory, since the wave-equation is only second order. But for this toy equation, one can see that the wave equation is fourth order. Hence the final solution consists of the product of trigonometric and hyperbolic functions. The presence of the hyperbolic terms arises from the fourth order derivative of the toy equation and these are extra solutions only from the conformal waves appearing in the Bohmian quantum gravity. Trignometric solutions are already there in usual quantum theory but the hyperbolic solutions might be bringing the new physics here, due to the fact that they are the extra solutions appearing in the theory. Exotic matter that we perceive in usual General Relativity is a simple effect arises from the complicated conformal wave equations presented here (see equation (31)).

According to our theory, achieving warp drive might be feasible by establishing a quantum density governed by a nonlinear partial differential equation as given in equation (31). Utilizing quantum theory could allow for the modification of local space-time. Investigating the energy density within a Casimir cavity, H White et al had discovered a micro/nano-scale structure that closely matched the requirements for the Alcubierre warp drive metric [37]. Still we lack a theoretical framework which explains such a coincidence. But there are exiting opportunities to test the feasibility of warp drive using quantum theory [38]. Warpdrive can be achieved by creating a proper quantum mechanical density, which can be achieved either through Casimir cavities or Bose–Einstein condensate. Our theory is useful to understand the structure of the space-time due to the generated quantum density. In [38], it is suggested that scalar gravitational waves may arise from Bohmian quantum gravity theory, but the experimental verification is required to confirm the validity of such a proposal.

4. Conclusions

In this study, we have investigated the Alcubierre warp drive within the framework of Bohmian Quantum Gravity. By mapping the exotic matter problem onto a fourth-order quantum mechanical density equation (equation (31)), we have shown that the fourth-order wave equation can lead to additional solutions of quantum density that are not typically present in the standard quantum theory, which is second-order in nature. Our analysis has revealed that hyperbolic functions of quantum density may arise as a result of this fourth-order wave equation, potentially leading to new physics beyond what is currently understood. While we have presented a simple toy model to illustrate this point, the exact expression of quantum density remains a subject for further research. If a viable quantum mechanical density can be found that satisfies the wave equation presented in this study, we may be able to resolve the long-standing problem of exotic matter for the Alcubierre warp drive within the framework of Bohmian Quantum Gravity.

Data availability statement

No new data were created or analysed in this study.

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References

[1] Capozziello S and Faraoni V 2011 *Beyond Einstein Gravity* Ist edn (Berlin: Springer)
[2] Bergmann P G 1968 Comments on the scalar–tensor theory *Int. J. Theor. Phys.* 1 25–36
[3] Deruelle N, Sasaki M, Sendouda Y and Yamauchi D 2010 Hamiltonian formulation of f(riemann) theories of gravity Progress. Theor. Phys. 123 169–85
[4] Shin’ichi N and Odintsov S D 2007 Introduction to modified gravity and gravitational alternative for dark energy Int. J. Geom. Methods Mod. Phys. 04 115–45
[5] Atazadeh K, Khaleghii A, Sepangi H R and Tavakoli Y 2009 Energy conditions in f(r) gravity and brans-dicke theories Int. J. Mod. Phys. D 18 1101–11
[6] Harko T, Lobo F S N, Nojiri S and Odintsov S D 2011 f(R, T) gravity Phys. Rev. D 84 024020
[7] Capozziello S, Nojiri S and Odintsov S D 2006 Dark energy: the equation of state description versus scalar-tensor or modified gravity Phys. Lett. B 634 93–100
[8] Nojiri S, Odintsov S D and Oikonomou V K 2017 Modified gravity theories on a nutshell: Inflation, bounce and late-time evolution Phys. Rep. 692 1–104
[9] Nojiri S and Odintsov S D 2011 Unified cosmic history in modified gravity: From f(R) theory to Lorentz non-invariant models Phys. Rep. 505 59–144
[10] Oikonomou V K and Karagiannakis N 2014 Astrophys. Space Sci. 354 583
[11] Capozziello S and de Laurentis M 2011 Extended theories of gravity Phys. Rep. 509 167–321
[12] Ntahompagaze J, Abebe A and Mbonye M 2017 On f(r) gravity in scalar-tensor theories Int. J. Geom. Methods Mod. Phys. 14 1750107
[13] Scholz E 2017 ‘The Unexpected Resurgence of Weyl Geometry in late 20th-Century Physics Beyond Einstein. Einstein Studies’ (New York: Birkhäuser) 14 (https://doi.org/10.1007/978-1-4939-7708-6_11)
[14] Shojaei F and M Golshani 1998 On the geometrization of bohmian mechanics: a new approach to quantum gravity Int. J. Mod. Phys. A 13 677–93
[15] Bohm D J and Hiley B J 1975 On the intuitive understanding of nonlocality as implied by quantum theory Found. Phys. 5 93–109
[16] Bohm D 1952 A suggested interpretation of the quantum theory in terms of ‘hidden’ variables. i Phys. Rev. 85 166–79
[17] Bohm D 1952 A suggested interpretation of the quantum theory in terms of hidden variables ii Phys. Rev. 85 180–93
[18] Ashtekar A and Pullin J 2017 ed gravity and gravitational alternative for dark energy Phys. Rep. 690 1–11
[19] Rowelli C and Vidotto F 2014 Covariant Loop Quantum Gravity (Singapore: World Scientific)
[20] Shojaei F and Shojaei A 2000 Nonminimal scalar-tensor theories and quantum gravity Int. J. Mod. Phys. A 15 1859–68
[21] Shojaei F 1999 Perturbative solutions of bohmian quantum gravity Phys. Rev. D 60 124001
[22] Gabay D and Joseph S K 2018 On the Mediating Field in a Conformally Transformed Einstein Equation arXiv:1801.00161 [gr-qc]
[23] Joseph S K 2020 Quantum-gravity in a dynamical system perspective arXiv:2005.03996 [physics.gen-ph]
[24] Joseph S K 2021 Weyl Geometry and Quantum Corrections arXiv:2112.12964 [gr-qc]
[25] Gabay D and Joseph S K 2018 On a Modified Klein–Gordon Equation with Vacuum-Energy Contributions arXiv:1802.07678 [gr-qc]
[26] Alcubierre M 1994 The warp drive: hyper-fast travel within general relativity Class. Quantum Gravity 11 L73–7
[27] Van Den Broeck C 1999 On the (im)possibility of warp bubbles arXiv:gr-qc/9906050
[28] Van Den Broeck C 1999 A warp drive with more reasonable total energy Class. Quant. Grav. 16 3973
[29] Santiago J, Schuster S and Visser M 2022 Generic warp drives violate the null energy condition Phys. Rev. D 105 064038
[30] Natario J 2002 Warp drive with zero expansion Class. Quant. Grav. 19 1157
[31] White H G 2003 A discussion of space–time metric engineering Gen. Relat. Grav. 35 3525
[32] White H G 2013 Warp field mechanics 101 J. Br. Interplanet. Soc. 66 242–7
[33] Santos-Pereira O L, Abreu E M C and Ribeiro M B 2020 Dust content solutions for the alcubierre warp drive spacetime Eur. Phys. J. C 80 786
[34] Santos-Pereira O L, Abreu E M C and Ribeiro M B 2021 Charged dust solutions for the warp drive spacetime Gen. Relativ. Gravit. 53 23
[35] Santos-Pereira O L, Abreu E M C and Ribeiro M B 2021 Perfect fluid warp drive solutions with the cosmological constant Eur. Phys. J. Plus 136 902
[36] Santos-Pereira O L, Abreu E M C and Ribeiro M B 2021 Fluid dynamics in the warp drive spacetime geometry Eur. Phys. J. C 81 133
[37] White H, Vera J, Han A, Brucolieri A R and MacArthur J 2021 Worldline numerics applied to custom Casimir geometry generates unanticipated interaction with Alcubierre warp metric Eur. Phys. J. C 81 677
[38] Joseph S K 2023 Quantum origins of scalar gravitational waves: exploring the conformal factor in scalar–tensor theory of gravity Int. J. Mod. Phys. D 32 2350051