Interference in interacting quantum dots with spin

Daniel Boese
Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

Walter Hofstetter
Theoretische Physik III, Elektronische Korrelationen und Magnetismus, Universität Augsburg, D-86135 Augsburg, Germany

Herbert Schoeller
Institut für Theoretische Physik A, RWTH Aachen, D-52056 Aachen, Germany

(Dated: October 30, 2018)

We study spectral and transport properties of interacting quantum dots with spin. Two particular model systems are investigated: Lateral multilevel and two parallel quantum dots. In both cases different paths through the system can give rise to interference. We demonstrate that this strengthens the multilevel Kondo effect for which a simple two-stage mechanism is proposed. In parallel dots we show under which conditions the peak of an interference-induced orbital Kondo effect can be split.

I. INTRODUCTION

Interference is one of the key phenomena of quantum physics. The prototype experiment is the famous double slit experiment where interference between two possible paths leads to an oscillatory pattern on the detection screen. In those experiments the phase difference is of geometrical nature, i.e. one of the paths is longer. A phase difference can also be introduced due to an enclosed magnetic flux. In mesoscopic physics such an experiment is referred to as Aharonov-Bohm (AB) ring, where the current through the AB ring shows oscillations as function of the magnetic field threading the ring.

An AB ring can be used as an interferometer, where the object under consideration is placed in one of the rings’ arms, and the phase is tuned by changing the object’s parameters. In this way one can measure the transmission phase of an interacting system, like a quantum dot (QD), which in general (and especially when tuned to the Kondo regime) has a complicated many-body ground state. In recent experiments quantum dots have been put into both arms, in some cases so close that a strong capacitive Coulomb interaction between the two dots has been introduced (see Fig. 1 upper right for an illustration). The two paths are no longer independent, but influence each other considerably. In a naive classical picture one could imagine that interaction would destroy interference, as making use of one path effectively closes the other. To answer this question the phase dependence of the current needs to be studied, and it turns out that the current indeed can be modulated. For completely equivalent paths ($\delta \epsilon = 0$ and $T_1 = T_2$) the system can be tuned opaque by setting $\phi = \pi$. In this case the Hamiltonian corresponds to a model of two capacitively coupled QDs, each of which is coupled to a different reservoir (this can be seen from the Hamiltonian in the form given in Eq. 3 and will be made explicit in Sec. V). Hence there is no way for an electron to traverse from left to right (schematically shown in Fig. 2). Note that such systems are of fundamental interest also because they can be viewed as artificial molecules where e.g. entangled states can be observed in transport and noise.

The coherence of quantum mechanical states has recently become a topic of broad interest, as it is fundamental to applications like quantum computing and to many phenomena, such as the Kondo effect. In AB interferometers coherence is essential as otherwise interference would not take place. Therefore they constitute good test-grounds to study the gain and loss of coherence in nanoscale devices, as was demonstrated by Buks and coworkers who demonstrated controlled dephasing by intentionally introducing dephasing in one of the arms.

Single quantum dots can constitute interacting inter-
ferometers by themselves. The capacitive Coulomb interaction between two dots is replaced by the on-site interaction between different levels. The tunability of the phase with magnetic fields, however, is lost, although some tunability using gates is still present. Nevertheless it is instructive to study interference effects in single quantum dots, since in general many dot levels participate in the transport, see Fig. 1 bottom left. A prominent example is the occurrence of the Fano effect 11,12,13 with its characteristic lineshape, which is due to interference between a resonant and a non-resonant transport channel. Moreover, it is often assumed that one level dominates the transparency, while the others are only very weakly coupled. We show that such a situation, although not present in the beginning, can be created dynamically.

In most quantum dots the levels are spin degenerate in the absence of a magnetic field. The effect of this degeneracy is manifold. As electrons with different spin can not interfere with each other their role is contrary to interference. The difference is indeed drastic, as on one side parallel QDs can be opaque due to destructive interference, while on the other hand the spin in a single QD can form a Kondo ground state leading to perfect transparency. Accounting for the spin degree of freedom is therefore a necessary step towards more realistic models of QDs.

In the course of this work we will show that the combination of interference and Kondo physics in multilevel QDs leads to a stronger Kondo effect. However, this effect is caused by a new, effective level and thus resembles single level Kondo physics.

Parallel QDs can be tuned to an interference–induced orbital Kondo effect by using the AB–phase. We demonstrate that the corresponding Kondo peak is split only if both a magnetic field and a level splitting are present. Interference can be described by a tunneling Hamiltonian with at least one non-conserved index. Therefore the tunneling part takes the general form $H_T = \sum_{krn} T_{kn}^T a_{krn}^\dagger c_{rn} + h.c.$. The quantum number $l$ is present only in the QD Hamiltonian, it is the analog of the paths. The index must not be conserved in tunneling, as otherwise the electrons would not know of each other (as if they would be in different reservoirs), ruling out any interference. $k$ denotes the wavevectors and $n$ an additional conserved quantum number in reservoir $r$. The conserved index $n$ can be due to symmetries present in the leads and dot, such as a rotational symmetry in some vertical quantum dots giving rise to an angular momentum quantum number. As seen from the structure of the tunneling Hamiltonian, they play a similar role as the spin and can consequently increase a Kondo effect (orbital Kondo effect). In lateral quantum dots such symmetries are typically not present and we suppress those indices from now on.

Interference is also interesting from a technical and fundamental point of view. The non-conservation of quantum numbers leads to non-vanishing off-diagonal elements of the reduced density matrix of the local system, which describe the coherence of states. Their presence explains why transport in first order, which usually is referred to as sequential tunneling, can still be coherent. Moreover, non-equilibrium one-particle Green’s functions are needed, even to describe the linear response regime.

The coupling to the leads can be so strong that perturbation theory may not be sufficient anymore. For the Anderson model this is referred to as the regime where Kondo correlations develop. Also for a simple model of two spinless dot levels it has been shown that near destructive interference the model can be mapped onto an effective Kondo model showing strong-coupling behavior in a peculiar way. A phase transition of the type RKKY vs Kondo tunable by a magnetic flux has been predicted 11,12,13.

In this work we study interference effects in strongly interacting quantum dot systems with spin. In the next section we introduce and discuss the model. In a qualitative discussion we summarize conclusions drawn from a spinless model and generalize them to the present case. We then focus on the Kondo effect multilevel QDs in Sec. IV and on the interference-induced orbital Kondo effect in parallel QDs in Sec. V.

## II. MODEL

We introduce the following model Hamiltonian of two parallel, interacting QDs connected to two electron reservoirs $r \in \{R, L\}$ via tunnel barriers, see also Fig. 1 bottom right. Each quantum dot (labeled $l \in \{1, 2\}$) is modeled by an Anderson-type Hamiltonian of a single spin-degenerate level

$$H = \sum_{kr\sigma} \epsilon_{kr\sigma} a_{kr\sigma}^\dagger a_{kr\sigma} + \sum_{l\sigma} \epsilon_l c_{l\sigma}^\dagger c_{l\sigma} + \sum_{(l\sigma) \neq (l'\sigma')} U_{ll'} n_{l\sigma} n_{l'\sigma'} + \sum_{krn} \left( T_{kn}^T a_{krn}^\dagger c_{rn} + h.c. \right).$$

FIG. 2: Destructive interference leads to a Kondo like situation. A geometric (left/right) pseudospin is introduced. The quantum dots interact capacitively.
we can write \[ \epsilon \]

Together with the definition \[ \epsilon_{1/2} = \epsilon \pm \delta \epsilon / 2 \] this yields

the new Hamiltonian

\[
H = \sum_{\sigma} \epsilon (n_{f_1 \sigma} + n_{f_2 \sigma}) - \frac{\delta \epsilon}{\tau^2} \left( f_{1 \sigma}^\dagger f_{2 \sigma} + f_{2 \sigma}^\dagger f_{1 \sigma} \right) + U \sum_{(f_1 \sigma) \neq (f_2 \sigma')} n_{f_1 \sigma} n_{f_2 \sigma'} + \sum_{\kappa} \left[ \tau a_{kR\sigma}^\dagger f_{1 \sigma} \right. \\
+ \left. a_{kL\sigma} \left( \frac{T_1^2 + T_2^2 e^{i\phi}}{\tau} f_{1 \sigma} + \frac{T_1 T_2}{\tau} (1 - e^{i\phi}) f_{2 \sigma} \right) \right] + \text{h.c.}] + \sum_{k\sigma} \epsilon_{k\sigma} a_{k\sigma}^\dagger a_{k\sigma}. \tag{3}
\]

This makes clear that for \( \delta \epsilon = 0 \) the cases \( \phi = 0 \) and \( \phi = \pi \) plus \( T_1 = T_2 \) are special and should be considered separately. Note that it is the DOS of the \( f_{1\sigma} \) level that is relevant for the transport.

It is useful to compare the above Hamiltonian Eq. (3) to that of a single, lateral, multilevel QD (see Fig. 1b). In this case the index \( l \) labels the dot states and the sum runs in general over many such states. Yet, for large level spacing one may approximate the situation by taking only two states. A generalization to many levels will be given in Section IV. The interaction parameters \( U_{ll} \) now corresponds to intra-dot interactions. Taking them all equal is a standard assumption (constant interaction treatment). This allows to restrict the discussion on two charge states, i.e., when \( U \) is the largest energy of the system, requiring an explicit argument. Their model of strongly and weakly coupled levels is related to the Fano effect studied in Ref. 12 and \( \delta \epsilon \) dependence we take explicitly) and \( \phi = \pi \) plus \( T_1 = T_2 \) are special and should be considered separately. Note that it is the DOS of the \( f_{1\sigma} \) level that is relevant for the transport.

We introduce another set of dot states that simplifies the discussion later on (see Fig. 3 for an illustration of the physical meaning of these states). With \( T_{1/2} \) being real (the \( \phi \) dependence we take explicitly) and \( \tau = \sqrt{T_1^2 + T_2^2} \) we can write

\[
f_{1/2 \sigma} = \frac{T_{1/2 \sigma}}{T_{1/2 \sigma}} \left( a_{1/2 \sigma}^\dagger \right) + \frac{T_{1/2 \sigma}}{T_{1/2 \sigma}} \left( a_{1/2 \sigma} \right).
\]

Together with the definition \( \epsilon_{1/2} = \epsilon \pm \delta \epsilon / 2 \) this yields

\[
H = \sum_{\sigma} \epsilon (n_{f_1 \sigma} + n_{f_2 \sigma}) - \frac{\delta \epsilon}{\tau^2} \left( f_{1 \sigma}^\dagger f_{2 \sigma} + f_{2 \sigma}^\dagger f_{1 \sigma} \right) + U \sum_{(f_1 \sigma) \neq (f_2 \sigma')} n_{f_1 \sigma} n_{f_2 \sigma'} + \sum_{\kappa} \left[ \tau a_{kR\sigma}^\dagger f_{1 \sigma} \right.
\]

\[
+ \left. a_{kL\sigma} \left( \frac{T_1^2 + T_2^2 e^{i\phi}}{\tau} f_{1 \sigma} + \frac{T_1 T_2}{\tau} (1 - e^{i\phi}) f_{2 \sigma} \right) \right] + \text{h.c.}] + \sum_{k\sigma} \epsilon_{k\sigma} a_{k\sigma}^\dagger a_{k\sigma}. \tag{3}
\]

III. QUALITATIVE DISCUSSION OF GENERAL PROPERTIES

We start with a discussion of multilevel dots with no phase, i.e., \( \phi = 0 \). It is well known that QDs with a single level (the two-lead Anderson model) display Kondo

\[ \text{states.} \]

This is not the case for \( N > 1 \), where interesting new physics can be observed.
physics for temperatures below the Kondo scale

$$T_K \sim \frac{\sqrt{U \Gamma}}{2} \exp\left(\frac{\pi \epsilon (\epsilon + U)}{\Gamma U}\right). \tag{4}$$

The manifestation of this is an increased density of states at the Fermi edge resulting in an increased conductance of the dot, which for \( T \to 0 \) even may reach the unitary value of \( 2e^2/h \). It is a priori not clear if and how this prevails when more orbitals participate.

The physics of two and more orbitals without spin has been addressed before, and it was found that instead of Kondo physics a hybridization

$$\Delta \sim \frac{\Gamma}{2\pi} \ln \frac{E_C}{\omega_c} \tag{5}$$

of the two levels is introduced. This scale \( \Delta \) is much larger than the exponentially small Kondo scale, and it leads to a shoulder in the DOS of order \( \Delta \) above the Fermi edge. The weight of this shoulder is related to the level splitting and vanishes for \( \delta \epsilon \to 0 \) and its width is roughly half the width of the main excitation, i.e. \( \Gamma/2 \).

In order to understand what happens for two orbitals with spin we perform a Schrieffer-Wolff transformation (see App. A for details), followed by a poor man’s scaling approach. In this transformation the hybridization is created and thus the level splitting increases until it becomes of the same order as the flow parameter \( \omega_c \). Then the upper \( f_{2\sigma} \) level is too high in energy, decouples, and thus does not participate anymore. The scaling proceeds with the renormalized single \( f_{1\sigma} \) level. Hence we have found a two-stage situation: First one level is pushed upwards until it is out of reach, then in the second step the remaining, renormalized level makes the Kondo effect alone.

The picture is slightly different for the parallel QDs. The flux enclosed leads to destructive interference and the current can even go to zero. The energy scale \( \Delta \) is modified by a factor \((1+\exp[i\phi])/2\) and thus vanishes for \( \phi = \pi \). In this case the model can be mapped onto an effective Kondo model. When the spin is included this is still the case and a more strong Kondo effect takes place as will be discussed in Sec. VI A.

**IV. MULTILEVEL QUANTUM DOTS**

We now discuss multilevel QDs in detail. Quantum dots have in general many levels that can participate in transport. In contrast to vertical QDs, the states in lateral QDs are labeled by a non-conserved quantum number. Furthermore, a multilevel structure is also relevant to other systems, like single atom contacts, heavy fermion compounds (e.g. studied by photo-emission) or general molecular electronics setup, where many channels can interfere. We focus on the interesting regime of levels below the Fermi edge and low temperatures. This is the regime of the Kondo effect, where correlation effects dominate and the dot’s spin is screened by the electrons in the leads. For clarity we mention again that \( \phi = 0 \) in this section.

In a first step we look at the case of two degenerate levels in the dot. In Fig. 4 we show results for the total spectral density. There are four possible states an electron can occupy in the dot, characterized by a spin index, which is conserved in tunneling, and an orbital index, which is not conserved. As discussed before, this is equivalent to one strongly coupled level and one decoupled one. Hence we see single-level Kondo physics with greatly increased \( T_K \). The big increase of \( T_K \) compared to the factor of \( \sqrt{2} \) in the tunneling matrix element can be easily understood from the definition of \( T_K \) which involves the coupling \( \Gamma \) exponentially.

In the second step we allow the two orbitals to be different in energy. One might speculate that this should lead to the appearance of side or satellite Kondo peaks. However, in Fig. 5 we demonstrate that single-level Kondo physics is effectively seen for split levels as well. With increasing splitting the Kondo peak becomes narrower, signaling a decreasing \( T_K \). At the same time the shoulder discussed in the previous section becomes visible and progressively moves to higher frequencies. This can be understood from the Schrieffer-Wolff transformed Hamiltonian in the \( f \)-basis. Equation (A10) shows that only the \( f_{1\sigma} \) level generates the Kondo resonance. In the scaling language it can be thought of as a two-step process. First the tunnel-splitting is created from integrating out the very high energies. This stops at an intermediate energy scale \( \omega_c \) where diagonalization shifts one level above \( \omega_c \). It can no longer contribute to scaling, while the other one—the broad \( f_{1\sigma} \) level—stays in the window. The scaling now gives the usual Kondo physics of a single, but modified level. It should be noted that this reflects the strong coupling behavior of the problem, i.e., all energy scales are important and contribute equally. In the in-

![FIG. 4: Effective density of states for the Kondo effect with one and two orbitals. The Kondo temperature increases strongly with the number of levels. Parameters for the symmetric dot are in units of \( \Gamma \): \( 2\pi U = 50 \), \( \epsilon_1 = \epsilon_2 = -25/2\pi \), \( 2\pi D = 25 \), \( \phi = 0 \), \( T = 0 \).](image-url)
parameters are $2\pi U = 50$, $2\pi D = 25$, $\phi = 0$, $T = 0$.

![Figure 5](image-url)  
**FIG. 5:** Effective density of states for a multilevel Kondo dot with increasing level splitting. The lower level sits at $2\pi \epsilon_1 = -25$ and the upper level at $2\pi \epsilon_2 = -25, -23.75, -22.5$ and $-20$ (outermost to innermost curve, everything in units of $\Gamma$). The inset shows the spectral densities of the lower (solid) and upper level (dashed) for $2\pi \epsilon_2 = -20$. Common parameters are $2\pi U = 50, 2\pi D = 25, \phi = 0, T = 0$.

This mechanism can be generalized to many ($N$) levels, where the role of the $f_{1\sigma}$ level is played by the ‘sum’ over or the superposition of all levels. One level after the other is shifted to higher energies, and only one broad ($\sim NT$) level remains, as sketched in Fig. 6. This new, broad level alone participates in the Kondo effect, which shows a strongly increased $T_K$, making it much easier to observe. We suggest that this mechanism explains the observed single-level Kondo physics in QDs.

We conclude that even for many spin-degenerate levels (with non-conserved orbital index) only one single Kondo peak is seen. The Kondo temperature depends on the level splittings. The other excitations can be traced back to shoulders as discussed in Refs. 21, 22 and 24. In two parallel QDs the level splitting is easily tunable, which allows to directly measure the change of $T_K$.

**V. PARALLEL QUANTUM DOTS**

In this section we study the physics of two parallel, interacting quantum dots as previously introduced, which can be tuned by an AB phase. We focus on the special case $\phi = \pi$, which corresponds to a Kondo–like situation. Note that this does not necessarily require parallel QDs but can also be realized in multilevel dots, when for instance one level is symmetric and the other antisymmetric.

**A. Interference-induced orbital Kondo effect**

As mentioned before, the case $\phi = \pi$ corresponds to a model where one level couples only to the left and the other one only to the right, as shown in Fig. 2. Evidently there are two conserved quantities: the spin and a geometrical pseudo-spin (left/right). Introducing symmetric and antisymmetric combinations of the lead states $b_{k\sigma} = a_{kL\sigma} - (-1)^i a_{kR\sigma}$, we can rewrite the tunneling part of the Hamiltonian as

$$H_T = \sum_{k\sigma} T_{k\sigma} b_{k\sigma}^\dagger c_{i\sigma} + \text{H.c.}. \quad (6)$$

This has the form of an Anderson Hamiltonian with the two conserved quantities discussed before. One therefore finds an enhanced Kondo effect for a low lying level at low temperatures. In other words, the state of complete destructive interference is a strong coupling state. Such models have been studied for instance for multi-level quantum dots, where the orbital momentum is conserved in tunneling, or in double-layer QD systems where the index $i$ corresponds to the upper or lower plane. In such cases the Kondo temperature is enhanced with respect to a pure spin Kondo model, as the second quantum number – the pseudospin – can give rise to Kondo correlations alone. This is true also in our case, where strong correlations can be expected even without spin. In Fig. 7 we show the spectral density corresponding to $c_{1\sigma}$. For zero phase a weak Kondo peak and a second broader peak at higher frequencies are visible. The broad peak (essentially the shoulder discussed before) moves to lower frequencies when the phase is increased towards $\pi$ and merges with the Kondo resonance for $\phi = \pi$. This strengthens the peak and thus enhances the Kondo temperature $T_K$ as can be seen more clearly in the inset, where the density of states of the $f_{1\sigma}$ level is shown. Note that one of the special features of this Kondo effect is that the tunneling matrix elements are

---

2 For this level splitting the lower and the $f_{1\sigma}$ level have significant overlap.
tunable for each (pseudo)-spin, as the individual levels can be controlled.

We remark that the Kondo effect discussed here is qualitatively different from an orbital Kondo effect as discussed in Ref. [17] and also from two-channel Kondo physics.

B. Splitting the Kondo peak

The ordinary Kondo effect in quantum dots can be destroyed by the application of either a magnetic field that splits the level by the Zeeman energy \( \Delta_Z \) or by a bias voltage introducing dephasing \( \Delta_Z \) (where the latter might under certain conditions open the door for two-channel Kondo physics again). In our case the orbital Kondo effect can be destroyed by the analog of the Zeeman term which is the level splitting, by different tunneling amplitudes (not accessible in ordinary QDs), by a bias voltage in the usual sense, and via a detuning of the phase, i.e. away from \( \phi = \pi \).

An interesting question is whether a splitting of the levels leads to a splitting of the Kondo peak, the development of satellite peaks or if only a weakening and destruction of the Kondo peak is observed. In Fig. 8 we find that a peak splitting can only be observed if both, the Zeeman and the orbital level splitting, are introduced. No side peaks appear if only one of them is present, which only leads to a reduction of \( T_K \). The suppression of side peaks has been attributed to an enhanced dephasing rate, such as produced by spinflip-cotunneling.

Note that this result also applies to other geometries like double-layer QDs.

The detection of an interference-induced orbital Kondo effect is more difficult than for the usual spin Kondo effect. Nevertheless it is possible by probing the resonance properties of a quantum point contact which is in the vicinity of the double dot system. In contrast to the spin Kondo effect, the up and down pseudospins correspond to charges in the upper or lower dot, which are weak enough one can perform spectroscopic measurements on the spectral densities in the individual dots. Another method is to measure the transport and noise properties of a quantum point contact which is in the vicinity of the double dot system. In contrast to the spin Kondo effect, the up and down pseudospins correspond to charges in the upper or lower dot, which are much easier to detect. The strong fluctuations in the Kondo regime will therefore influence the transmission properties of the point contact allowing an indirect measurement of the Kondo resonance, in a way which is not accessible for the usual spin Kondo effect. The measurement of charge fluctuations thus provides a direct handle on spin fluctuations.

In real QD systems complete destructive interference, where the dots become opaque, is not achieved experimentally. The reasons are the difficulty to realize exactly equal QDs, as well as effects not captured in our model, such as more levels (at higher energy) or processes that break the phase coherence of an otherwise coherent process (less relevant at low temperatures). Yet, more than 50% contrast is possible in today’s experiments and the effect is therefore observable.

VI. CONCLUSIONS

We studied coherence in two interacting quantum dot systems. First we investigated multilevel QDs with spin. We discussed the relevant excitations and energy scales. The multilevel Kondo effect has been analyzed. We demonstrated that single-level Kondo physics essentially prevails, and that the corresponding Kondo temperature
can be strongly enhanced. We have also investigated a very similar system, namely two single-level (but spin-degenerate) QDs in parallel. Their behavior can be tuned by an enclosed magnetic flux. We showed that coherence persists when the two dots interact with each other. In the case of destructive interference, the system exhibits novel Kondo behavior (interference-induced orbital Kondo effect) that is due to the spin degree of freedom and allows to access Kondo correlations via charge fluctuations. Side peaks in the density of states appear only if a Zeeman and a level splitting are introduced together.

Acknowledgments

We would like to thank S. Kleff, J. König, J. Kroha, T. Pohjola, A. Rosch, G. Schön, and D. Vollhardt for useful discussions. This work was supported by the DFG through Graduiertenkolleg "Kollektive Phänomene im Festkörper" and the CFN (D.B.), as well as through SFB 484 and a postdoctoral research grant (W.H.).

APPENDIX A: SCHRIEFFER-WOLFF TRANSFORMATION

We perform a unitary transformation on the Hamiltonian Eq. (1) such that the un- and doubly-occupied states are projected out

\[ H' = e^S H e^{-S} = H_0 + \frac{1}{2} [S, H_T] + \ldots, \]  

(A1)

where \( S \) has been chosen to fulfill \([S, H_0] = -H_T\). In our case this operator is given by

\[ S = \sum_{kr,\sigma} T_{kr,\sigma}^{r,s} \left( \frac{1}{\epsilon_{s\sigma} - \epsilon_{kr}} \right) \left( n_{\bar{s}\sigma} + n_{\bar{\bar{s}}\sigma} + n_{s\sigma} \right) \]  

(A2)

To avoid cluttering the notation we suppress the indices on the tunneling matrix elements and local energies from now on, and take \( U \to \infty \). We introduce the two new coupling constants

\[ J_K = -\frac{|T|^2}{\epsilon - \epsilon_k}, \]  

(A3)

\[ \Delta_0 = \sum_{kr} J_K, \]  

(A4)

The new Hamiltonian is finally given by

\[ H = H_0 + \sum_{ss',\sigma} \epsilon_{s\sigma} c_{s\sigma}^\dagger c_{s'\sigma} \]  

\[ + \sum_{kr,\sigma} J_K \left( c_{s\sigma}^\dagger c_{\bar{s}\sigma} + c_{\bar{\bar{s}}\sigma}^\dagger c_{\bar{s}\sigma} + c_{\bar{s}\sigma} c_{\bar{s}\sigma}^\dagger \right) \]  

\[ + \sum_{kr,\sigma} \left( c_{s\sigma}^\dagger c_{\bar{s}\sigma} \right) \]  

Replacing the dot operators by the (anti)-symmetric combinations \( f_{1/2\sigma} \) we obtain

\[ H = H_0^\text{sm} + \frac{\epsilon_1 + \epsilon_2}{2} \left( f_{1\sigma}^\dagger f_{1\sigma} + f_{2\sigma}^\dagger f_{2\sigma} \right) + \sum_{\sigma} J_K \left( f_{1\sigma}^\dagger f_{1\sigma} f_{2\sigma} + h.c. \right) + \sum_{\sigma} \left( f_{1\sigma}^\dagger f_{1\sigma} \right) \]  

\[ + \sum_{kr,\sigma} J_K \left( a_{kr,\sigma}^\dagger a_{kr,\sigma} + a_{kr,\sigma} a_{kr,\sigma}^\dagger \right). \]  

(A5)

(A6)
D. Boese, *Quantum transport through nanostructures: Quantum dots, molecules and quantum wires* (Shaker Verlag, Aachen, 2002).

W. Hofstetter and H. Schoeller, Phys. Rev. Lett. **88**, 016803 (2002).

T. Inoshita et al., Phys. Rev. B **48**, 14725 (1993).

J. König, Y. Gefen, and G. Schön, Phys. Rev. Lett. **81**, 4468 (1998).

J. König and Y. Gefen, Phys. Rev. B **65**, 045316 (2002).

P. G. Silvestrov and Y. Imry, Phys. Rev. Lett. **85**, 2565 (2000).

P. G. Silvestrov and Y. Imry, cond-mat/0102088.

W. Izumida, O. Sakai, and Y. Shimizu, J. Phys. Soc. Jpn. **66**, 717 (1997).

W. Izumida, O. Sakai, and Y. Shimizu, J. Phys. Soc. Jpn. **67**, 2444 (1998).

A. L. Chudnovskiy and S. E. Ulloa, Phys. Rev. B **63**, 165316 (2001).

K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).

T. A. Costi, A. C. Hewson, and V. Zlatić, J. Phys.: Condens. Matter **6**, 2519 (1994).

W. Hofstetter, Phys. Rev. Lett. **85**, 1508 (2000).

S. Kirchner, J. Kroha, and E. Scheer, in *Proceedings of the NATO Advanced Research Workshop “Size dependent magnetic scattering”*, Pecs, Hungary, (2000), Kluwer Academic Publishers; cond-mat/0010103.

F. Reinert et al., Phys. Rev. Lett. **87**, 106401 (2001).

T. Pohjola et al., Europhys. Lett. **40**, 189 (1997).

K. A. Matveev, Zh. Éksp. Teor. Fiz. **99**, 1598 (1991). [Sov. Phys. JETP **72**, 892 (1991)].

K. Vladar and A. Zawadowski, Phys. Rev. B **28**, 1564 (1983).

A. Zawadowski, J. v. Delft, and D. C. Ralph, Phys. Rev. Lett. **83**, 2632 (1999).

D. L. Cox and A. Zawadowski, Adv. Phys. **47**, 599 (1998).

A. Kaminski, Y. V. Nazarov, and L. I. Glazman, Phys. Rev. Lett. **83**, 384 (1999).

A. Kaminski, Y. V. Nazarov, and L. I. Glazman, Phys. Rev. B **62**, 8154 (2000).

A. Rosch, J. Kroha, and P. Wölfle, Phys. Rev. Lett. **87**, 156802 (2001).

P. Coleman, C. Hooley, and O. Parcolle, Phys. Rev. Lett. **86**, 4088 (2001).

Y. Meir, N. S. Wingreen, and P. A. Lee, Phys. Rev. Lett. **70**, 2601 (1993).

Q. Sun and H. Guo, Phys. Rev. B **64**, 153306 (2001).

E. Lebanon and A. Schiller, cond-mat/0105488.

S. di Francesco, private comm.

D. Sprinzak, Y. Ji, M. Heiblum, D. Mahalu, and H. Shtrikman, cond-mat/0109402.