Investigation of the Influence of the Cutout Dimensions on the Stress-strain State of Three-layer Shells with Load-bearing Layers of Composite Materials

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Abstract

The description of the model for investigation of the influence of rectangular dimensions in terms of cutouts on the stress-strain state (SSS) in the layers of three-layer conical shells is given. The finite element model is based on the layer-by-layer analysis approach and allows to perform calculations of SSS in shell layers in a wide range of changes in the physical and mechanical characteristics of the layers in the presence of cut-outs. The equations of the classical theory of shells at reception of effective approximations (leading to high speed of convergence of received results) and construction of finite-element model of moment bearing layers are used. The approach with application of the elasticity theory relations for a three-dimensional body in curvilinear coordinates and obtained approximations for construction of a finite element model of the filler layer is described. Influence of the cut-out mortar angle on the parameters of the three-layer shell with load-bearing layers of composite materials is investigated.

Keywords: three-layer conical shells, composite materials, rectangular cut-outs, finite element model, stress-strain state

1. Introduction

When designing aviation and aerospace equipment, three-layer shells [1], including composite materials, are spread. This is due to the high weight efficiency and rigidity, high thermal, acoustic and vibration resistance of three-layer structures. An example of the application of three-layer and especially cellular structures can serve as a supersonic bomber firm "Convair" B-58. The surface of the glider of this aircraft (80–90% of the total streamlined area) is made of structures with filler. These structures are used in the power elements of the wing, fuselage.

The Dynamic-Grumman F-111B aircraft has about 300 cellular panels (basically all nodes except the nose fairing, the small area behind the cockpit, the middle areas of the wing and the keel).

The construction with the filler of C-141 and C-5A aircrafts by Lockheed is of certain interest. The total area occupied by cellular panels on C-141 aircraft is 557 m².

Experience of manufacturing and operation of panels with the filler of C-M1 aircraft was considered by the company in the construction of the American strategic military transport aircraft C-5L "Galaxy". The total area of cellular structures on
this aircraft is increased by 4.5 times and is 2320 m² with an average panel thickness of 25.4 mm.

Filler panels on C-5L aircraft are used in the following designs: nose and tail parts of the wing and nose parts of the fuselage, tail part of the flaps, engine pylons, cargo hatches, elements of the tail part of the fuselage, fairings of the main landing gear, etc.

Structures with filler are also used in space equipment for power units of apparatuses and heat-insulating screens.

Aeroika’s multi-layer design, used for atmospheric re-entry vehicles, deserves consideration. The multi-layer filler ensures a high thermal insulation coefficient with a minimum weight of the structure.

Aircraft and aerospace engineering structures need to have different purpose cut-outs, hatches that weaken the load-bearing capacity. This explains the continuous improvement of methods for calculating the stress-strain state (SSS) of structural elements, including three-layer shells, with cut-outs.

2. Description of the research

Conical shells, including three-layer structures, are widely used in the design of aviation and aerospace equipment. This work is devoted to the influence of the rectangular solution angle on the stress-strain state in the layers of three-layer conical shells. It should be noted that the problem of analyzing the stress-strain state of three-layer shells, considering the heterogeneity of the structure and the presence of cut-outs, rectangular in terms of shape, is insufficiently studied.

There is a lot of work to build theories of layered and three-layer shells. Among them are the works of S.A. Ambartsumian, V.V. Bolotin, V.V. Vasiliev, N.K. Galimov, E.I. Grigolyuk, Y.M. Grigorenko, M.A. Ilgavos, H.M. Mustari, V.N. Paymushin, S.N. Sukhinin, E. Reissner, L. Heebep and many others.

In order to calculate the VAT of three-layer shells with cut-outs under different conditions of fixing and loading, variable properties of layers, it is necessary to develop models based on numerical methods and, first, the finite element method (FEM). Monographs are devoted to the construction of finite element models (FEM) of natural curvature for the filler layer. Thus, the methodology of finite element construction is based on the development of the layer-by-layer analysis approach for the creation of block models of three-layer, in general, irregular shells; the use of equations and relations of the classical theory of shells in obtaining effective finite-element approximations (FEA), resulting in high convergence rate of numerical procedures and obtained results, and construction of finite element model of natural curvature for momentary bearing layers; the proposed approach with application of elasticity theory for three-dimensional body in curvilinear coordinates and effective FEA in the construction of FEM of natural curvature for the filler layer. Thus, the simulation of the stress-strain state in the bearing layers will be carried out with the help of two-dimensional cladding finite elements (FE) and with the help of three-dimensional cladding FE modeling of SSS in the filler layer.
Approximating functions of displacement of the median surface of the finite elements of the bearing layers are composed of displacements as a solid body and displacements caused by deformation of FE. Displacements as a solid body are determined by the integration of Cauchy relations [13] at zero deformation values and are recorded similarly [7, 10, 14] through six undefined coefficients, which are constant integration. The efficiency of approximations leading to a high rate of convergence of numerical procedures and obtained results, and the constructed finite element model of natural curvature for the moment-bearing layers of three-layer conical shells are shown in [12].

A block of two-dimensional and three-dimensional shell finite elements is assembled from the thickness of the three-layer shell (block three-layer conical element), obtained by the cross section of two planes passing through the shell axis and two conical surfaces formed by rotation around the shell axis to the median surface at the node points of FE.

Application of two-dimensional and three-dimensional envelope finite elements for modeling accordingly momentary bearing layers and a layer of a filler will lead to rupture of the generalized moving on contact surfaces of these finite elements if they are constructed with use of various approximating functions of moving. In order to avoid errors caused by such a gap in generalized displacements, the same number of nodes is selected for the finite elements of the filler layer on conical surfaces as for the finite elements of the bearing layers, and the same generalized displacements and approximating functions as for the finite elements of the bearing layers are used as nodal unknown and approximating functions. As the nodal generalized displacements for the finite elements of the bearing layers and, consequently, the finite elements of the filler layer, there are three linear displacements and two normal rotation angles to the median surface around the meridional and circumferential coordinate axes in each of the four knots of the bearing layers' FE and eight knots of the finite elements of the filler layer.

The displacements caused by deformation of the finite elements of the bearing layers are approximated by an incomplete bicubic polynomial deflection polynomial and bilinear polynomial deflection polynomials for meridional and circular displacements. Having obtained the approximating functions of the displacements of the finite elements of the bearing layers (recorded with twenty undefined coefficients) and substituting them for the Cauchy relations linking strains with displacements, we express the dependences for generalized strains through these twenty undefined coefficients.

The Cauchy relations that bind deformations with displacements for a three-dimensional body in curvilinear coordinates [15] are applicable for the final element of the filler layer, which is a thick-walled conical shell.

\[ e_{11} = \partial_\varphi \mu, e_{22} = r^{-1}(\partial_\varphi v + \mu \sin \gamma + w \cos \gamma), e_{33} = \partial_\varphi w, \]

\[ \gamma_{12} = r^{-1}\partial_\varphi \mu + \partial_\varphi v - r^2 v \sin \gamma, \gamma_{23} = r^{-1}\partial_\varphi w + \partial_\varphi v - r^2 v \cos \gamma, \gamma_{31} = \partial_\varphi w + \partial_\varphi \mu, \]

where \( r = R_2 \cos \gamma, R_2 = (R_1 + R_3) / 2 \), \( R_1 \) and \( R_3 \), radius of the inner and outer conical surfaces of the final element of the filler layer, respectively.

To obtain approximating displacement functions of the finite elements of the bearing layers on the surfaces of contact with the filler layer, we use the transition matrix from the middle surface of the bearing layers to the interfaces with the filler layer in a similar way [16].

Having determined the approximating functions of the movement of the finite elements of the bearing layers, and therefore approximating functions of the movement on the conical surfaces of the finite element of the filler layer, and applying the linear law of displacement change in the thickness of the final element of the filler layer from (1), we obtain the dependences for generalized strains, recorded through forty undefined coefficients.

Knowing the relations for the physical law for the bearing layers and the filler layer, with the help of the obtained dependencies of generalized deformations we write down the expressions for stresses in the finite elements of the layers of the block three-layer conical element. Further, using procedures such as [17], we define the matrices of finite element stiffness of the layers and proceed to the construction of a finite element model of a three-layer conical shell.

4. Investigation of the influence of rectangular cutouts on the stress-strain state of the three-layer shell

As noted above, the problem of the analysis of the stress-strain state of three-layer shells, considering the heterogeneity of the structure and the presence of rectangular cut-outs in terms of shape, has not been sufficiently investigated. Not many works in this direction are known, for example, [12, 18, 19]. Obtaining an analytical solution to study the stress-strain state of three-layer shells, taking into account the heterogeneity of the structure and the presence of cut-outs, rectangular in terms of shape, is associated with great difficulty. This results in the application of numerical methods and, first of all, the finite element method.

Let's consider the problem about the influence of rectangular cutout sizes on the stress-strain state of the three-layer conical shell. A three-layer conical compartment with two diametrically opposed rectangular cut-outs, loaded with uniformly distributed internal pressure, was chosen as the object of study. The bearing layers are made of fiberglass and the filler is made of polyurethane foam. The boundary conditions at the ends of the casing are considered to
correspond to the case of a rigid seal (axial movement is permitted at the end of the smaller diameter).

The geometric parameters of the shell are as follows:

\[ R = 1.5 \text{ m}, \quad L = 1.5 \text{ m}, \quad h_1 = 0.43 \text{ cm}, \quad h_2 = 0.21 \text{ cm}, \]
\[ H = 5.63 \text{ cm}, \quad \gamma = 20.556^\circ, \]

where \( R \) — inner radius of the larger base; \( L \) — bay height; \( h_1, h_2 \) — the thickness of the inner and outer load-bearing layers accordingly; \( H \) — three-layer package thickness, \( \gamma \) — taper angle.

The cutouts are located at an equal distance from the ends of the casing. The length of the cutouts is 1/3 of the length of the cutouts forming. Cutout corner angle \( \alpha \) was changing within the limits of \( 30^\circ \leq \alpha \leq 60^\circ \).

The physical and mechanical characteristics of the three-layer shell are as follows:

- for the inner bearing layer: \( E_1 = 2.5 \times 10^3 \text{ kg/cm}^2, \quad E_2 = 2.2 \times 10^3 \text{ kg/cm}^2, \quad G_{12} = 0.35 \times 10^3 \text{ kg/cm}^2, \quad \mu_2 = 0.1; \)
- for the outer load-bearing layer: \( E_1 = 2.1 \times 10^3 \text{ kg/cm}^2, \quad E_2 = 1.9 \times 10^3 \text{ kg/cm}^2, \quad G_{12} = 0.36 \times 10^3 \text{ kg/cm}^2, \quad \mu_2 = 0.1; \)
- for the filler: \( E_1 = E_{22} = E_{33} = 240 \text{ kg/cm}^2, \quad G_{13} = G_{23} = 100 \text{ kg/cm}^2. \)

Due to symmetry, the calculation considers the 1/4 symmetrical part of the shell, which was divided into 24 elements in the axial direction and 30 elements in the circumferential direction.

Fig.1, 2 shows graphs of changes in the main parameters of the stress state of membrane tensions \( \sigma_{y_1}, \sigma_{y_2} \) in the bearing layers along the meridional (Fig.1) and circumferential (Fig.2) coordinates for two values of the cutout angle: \( \alpha = 30^\circ \) and \( \alpha = 60^\circ \). Figures 1, 2 in the figures show graphs of voltage distribution in the inner and outer load-bearing layers, respectively, for the case \( \alpha = 30^\circ \), with the numbers 3, 4 for the case \( \alpha = 60^\circ \). Fig. 1 shows diagrams in the section of the plane passing through the shell axis and the meridional line located at a distance of half the size of the finite elements in the circumferential direction from the rectilinear edge of the cut (let's mark this line \( \text{A} \rightarrow \text{A} \)), where \( N_{ax} \) — FE numbers on the line \( \text{A} \rightarrow \text{A} \), counted from the base of the large-diameter cone.

Fig. 2 shows diagrams of the cross-section of the plane perpendicular to the axis of the shell and passing through a line parallel to the curvilinear edge of the large-diameter cutout and located at a distance of half the size of the finite elements from this edge of the cut (let's mark this line \( \text{B} \rightarrow \text{B} \)), where \( N_{ax} \) — FE numbers on the line \( \text{B} \rightarrow \text{B} \), the countdown is taken from the symmetry plane passing through the middle of the curved edges of the cut.

As can be seen from the above graphical dependences, the stress-strain state of the shell weakened by the cutouts is characterized by a pronounced edge effect in the vicinity of the corner points of the cutouts, which quickly fades away as the distance from them. The maximum values of membrane and torque stresses in this problem are observed in the internal bearing layer. In the outer load-bearing layer, the maximum stress values are considerably lower than in the inner load-bearing layer. The largest absolute values in the bearing layers are diaphragm stresses \( \sigma_{y_1}, \sigma_{y_2} \). The maximum values of shear voltages are 2–2.5 times less than the maximum stresses \( \sigma_{y_1}, \sigma_{y_2} \).

**Figure 1.** Diagrams of diaphragm stress changes \( \sigma_{y_1}, \sigma_{y_2} \) in the bearing layers along the meridional coordinate in the section by the plane separated from the rectilinear edge of the rectangular cutout at a distance of half the size of the FE, for two cutout angles: 1, 2 in the inner and outer support layers, respectively, for the case \( \alpha = 30^\circ \), 3, 4 — in case of \( \alpha = 60^\circ \).
Figure 2. Diagrams of diaphragm stress changes $\sigma_{12}, \sigma_{22}$ in the bearing layers at the circumferential coordinate in the section of the plane separated from the curved edge with a large diameter of a rectangular cut at a distance of half the size of the FE, for two cutout angles: 1, 2 in the inner and outer support layers, respectively, for the case $\alpha = 30^\circ$, 3, 4 — in case of $\alpha = 60^\circ$.

It should be noted that the maximum voltage values in the filler are $\sigma_{11}, \sigma_{22}$, which in most cases are neglected, are comparable to the maximum values of voltages $\sigma_{13}, \tau_{13}, \tau_{23}$, usually taken into account in the calculations. And only for stresses $\tau_{12}$ the maximum values are several times lower than for the other components of the core stress state.

Fig. 3 shows graphical dependences of the stress state parameters in the inner bearing layer near the corner point of the cut on the size of the cut. Figures 1 to 6 indicate the stress dependencies $\sigma_{1x}, \sigma_{1y}, \sigma_{3x}, \sigma_{3y}, \sigma_{6x}, \sigma_{6y}$, respectively. The presented parametric dependences show that the change in the cutout size within the specified limits has little effect on the maximum sheath stress values. So when you increase the angle of the cutout by a factor of 2 (from 30° to 60°) maximum values of diaphragm stresses $\sigma_{11}, \sigma_{22}$ in the inner bearing layer are increased by 5–15%.

Figure 3. Dependence of the stress state parameters in the inner bearing layer near the corner point of the cut on the cutout dimensions. The numbers 1–6 correspond to the voltages $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{66}, \sigma_{12}, \sigma_{23}$.

Thus, from the given results it is possible to draw a conclusion that change of an angle of a solution of a cut out in the specified limits practically does not render appreciable influence on parameters of the SSS of a three-layer conic cover.

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