Upper limit on first-order electroweak phase transition strength

Shehu AbdusSalam*, Layla Kalhor1, and Mohammad Javad Kazemi1
1Department of Physics, Shahid Beheshti University, Tehran, Islamic Republic of Iran

Abstract. For a cosmological first-order electroweak phase transition, requiring no sphaleron washout of baryon number violating processes leads to a lower bound on the strength of the transition. The phases interface, the so-called bubble wall, velocity can become ultra-relativistic if the friction on the wall due to primordial plasma of particles surrounding it is not sufficient to retard the wall acceleration down to a steady speed. This bubble “runs away” should not occur if a successful baryon asymmetry generation due to the transition is required. Using Boedeker-Moore criterion for bubble wall runaway, within the context of an extension of the standard model of particle physics with a real gauge-single scalar field, we show that a non runaway transition requirement puts an upper bound on the strength of the first-order phase transition.

1 Introduction

The universe we observe and understand is mainly made of matter and has very little antimatter component. There are two important shortcomings of the standard model of particle physics due to this observation – insufficient source for CP violation [1] and scalar sector interactions to allow for a good phase transition [2, 3] at some early time of the cosmos. This is because the observed matter-antimatter asymmetry could be due to strong first order EW phase transition (EWPT) [4, 5] that allows for primordial baryon number and CP violating processes [6]. This mechanism is called electroweak baryogenesis (EWBG) [7–10]. For a first-order EWPT, EWBG will work only if the baryon asymmetry created at the expanding bubble wall is not washed out by sphaleron [11] processes inside the broken phase. A strong enough first-order EWPT is required for EWBG to be successful.

Another consequence for a strong first-order EWPT is that it could generate stochastic background of gravitational waves (GW) [12–15] which (see, e.g. [16] for prospects studies within the context of the model we considered) could be detected at future satellite GW interferometers [17]. The velocity of the expanding bubbles is one of the essential parameters that characterises the EWBG and GW dynamics related to the transition [14, 15, 18–28]. Observable gravity waves require very strong phase transition but this leads to fast-moving bubbles. In general, the speed of the accelerating bubble wall(6,6),(994,994) can be retarded by friction due the collisions with particles in its surrounding plasma. This way, the speed could reach a steady state after some finite time. Otherwise, if the friction is not sufficient, it keeps increasing towards ultra-relativistic magnitudes for very strong first-order phase transitions [29]. The latter scenario leads to the so-called “run away” of the bubble wall for which EWBG will not work.

*Emails: abdussalam@sbu.ac.ir, l_kalhor@sbu.ac.ir, mj_kazemi@sbu.ac.ir
So two trends can be spotted. On one hand, for EWBG to explain the matter-antimatter asymmetry of the universe a strong first-order EWPT is required. There is no precise quantification \[30, 31\] for how strong the transition must be but conventionally a certain lower limit is usually assumed, \(v/T > 1\) where \(v\) is the vacuum expectation value (VEV) of the SM Higgs field at the transition temperature \(T\). Very strong EWPT, on the other hand, could yield stronger GW signals but may also lead to the runaway scenario and conflict with EWBG. Many work have been done \[16, 32–41\] along these directions and mostly focusing on the combination of imposing a no sphaleron washout condition \(v/T \gtrsim 1\) and possibility of the GW observation. In this article we address the question: how arbitrary large could \(v/T \gtrsim 1\) be? This is done within the context of an inert singlet \[42, 43\] extension of the SM. The model parameters can readily be found capable to generate strong phase transitions with high bubble wall velocity. We found that requiring no runaway of bubble walls, using the Boedeker-Moore condition \[29\], puts an upper bound on how strong the EWPT could be. The analyses were made using cosmoTransitions \[44\] package for finding transition and nucleation temperatures, and computing the GW wave power spectra.

2 Model and analyses setup

In this section we set the model, context and notations for analysing the correlations between the requirements for no sphalerons washout and no bubble walls runaway following the primordial electroweak phase transition. The beyond the SM theory we consider is one with a scalar singlet \(S\) added to the Higgs sector. We explore the parameter space of the model and identify regions a first order transition occur. For the sample of parameter points generated we compute the critical and nucleation temperatures and the GW spectrum that could arise from the EWPT.

The tree level potential with a \(\mathbb{Z}_2\) symmetry that forbids Higgs-singlet mixing is:

\[
V_{\text{tree}}(H, S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \lambda_{HS} |H|^2 S^2 + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{4} \lambda_S S^4, \tag{1}
\]

where

\[
H = \frac{1}{\sqrt{2}} \left( \chi_1 + i\chi_2 \right)
\]

and \(\chi_{\{1,2,3\}}\) are the Goldstone bosons. So, the potential in terms of the physical Higgs \(h\) and singlet scalar \(S\) is:

\[
V_{\text{tree}}(h, S) = -\frac{1}{2} \mu_h^2 h^2 + \frac{1}{4} \lambda_h h^4 + \frac{1}{2} \lambda_{HS} h^2 S^2 + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{4} \lambda_S S^4. \tag{3}
\]

The Higgs physical mass and self coupling are fixed at \(m_H = 125\) GeV and \(\lambda_H = m_H^2/2v^2 \approx 0.129\) respectively while its VEV is set to \(v = 246\) GeV. The physical mass of the new scalar \(S\) after electroweak symmetry breaking (EWSB) is \(m_S^2 = \mu_S^2 + \lambda_{HS} v^2\). The full effective potential used for our analyses consists of three parts, namely the tree level, one-loop correction, and thermal corrections terms as presented in appendix A. For our analyses we consider the case of \(\mu_S^2\) positive. In this case the symmetry of singlet is not spontaneously broken and thus we address only the one dimensional potential along the \(h\) direction, leading to a one-step phase transition when the field tunnels through the energy barrier between the zero minimum and the electroweak minimum. In addition, \(\lambda_{HS}\) and \(\lambda_S\) were required to be positive, allowing of stable minimum of the potential energy. Hence there are three inert single free parameters, \(\mu_S\), \(\lambda_S\), and \(\lambda_{HS}\) which will varied simultaneously. For each parameter space point, we used CosmoTransitions \[44\] for determining the critical (\(T_c\)) and nucleation (\(T_n\)) temperatures and the corresponding Higgs field values, \(v_{c,n}\), at the critical point of the EWPT. Only parameter regions with \(\frac{v}{T_c} > 1\) were accepted for further analyses. Next we address the characteristics
of the bubble wall velocity and gravitational waves spectrum that could arise due to the strong first-order EWPT.

In order to analyse the EWPT bubble wall velocity, \( v_w \), and estimate the gravitational wave power spectrum that could result from the inert singlet model, two other quantities need to be determined. These are \[17, 45\] the ratio of released latent heat from the transition to the energy density of the plasma background, \( \alpha \), and the time scale of the phase transition, \( H/\beta \). Using of effective potential and it’s derivative at nucleation temperature, \( T_n \), the parameters \( \alpha \) reads as \[17\],

\[
\alpha = \frac{1}{\rho_R} \left[ -(V_{EW} - V_f) + T_n \left( \frac{dV_{EW}}{dT} - \frac{dV_f}{dT} \right) \right]_{T=T_n},
\]

where \( V_f \) is the value of the potential in the unstable vacuum and \( V_{EW} \) is the value of the potential in the final vacuum. The time scale of the phase transition can be calculate from the derivative of the Euclidean action at nucleation temperature \[17\]:

\[
\frac{H}{\beta} = \left[ T \frac{d}{dT} \left( \frac{S_3(T)}{T} \right) \right]^{-1} \bigg|_{T=T_n}.
\]

The exact calculation of the bubble wall velocity is complicated and for which there is need to consider the interaction between bubble wall and its surrounding plasma (see e.g. \[28\] and references therein), but there are approximate expressions in terms of \( \alpha \) such as in \[18\]:

\[
v_w = \frac{1}{\sqrt{3}} + \frac{\sqrt{\alpha^2 + \frac{2\alpha}{3}}}{1 + \alpha}.
\]

This expression for the bubble wall velocity provides only a lower bound on the true wall velocity \[24\]. In this work, we use this approximation together with the expressions for \( \alpha \) and \( H/\beta \) for calculating the GW signals produced during the phase transition.

Depending on the bubble wall velocity there are two main regimes; when the wall velocity is relativistic or not. In addition, in the relativistic regime, there are two qualitatively different scenarios. First, if bubble wall reaches a terminal velocity (non-runaway scenario), second, the bubble wall accelerates without bound (runaway scenario). In order to calculate the GW spectrum, it is important to know which of the aforementioned scenarios apply. To this aim following critical alpha value, \( \alpha_{\infty} \), can be used to distinguish between these scenarios \[17, 46\],

\[
\alpha_{\infty} \simeq \frac{30}{24\pi^2} \sum_a c_a \Delta m_a^2 \simeq 0.49 \times 10^{-3} \left( \frac{v_n}{T_n} \right)^2,
\]

where \( c_a = n_a/2 \) \( (c_a = n_a) \) and \( n_a \) is the number of degrees of freedom for boson (fermion) species and \( \Delta m_a^2 \) is the squared mass difference of particles between two phases. For non-runaway scenarios, \( \alpha < \alpha_{\infty} \), the wall velocity \( v_w \) remains subliminal and the available energy is transformed into fluid motion.

Another criterion for determining whether the bubble walls runaway or reach steady speed goes back to the in \[47, 48\]. The pressure on the wall come from two sources with opposite directions. One is outwards and due to the difference in energy densities of the symmetric and broken vacuum, \( V_{sym} - V_{br} \). The other is inwards and due to the pressure \( P \) from the thermal plasma of particles surrounding the wall. For each point in the parameter space of the inert singlet model we compute the Boedeker-Moore (BM) criterion and require that

\[
p_{\text{runaway}} = V_{sym} - V_{br} - P < 0,
\]

where \( c_a = n_a/2 \) \( (c_a = n_a) \) and \( n_a \) is the number of degrees of freedom for boson (fermion) species and \( \Delta m_a^2 \) is the squared mass difference of particles between two phases. For non-runaway scenarios, \( \alpha < \alpha_{\infty} \), the wall velocity \( v_w \) remains subliminal and the available energy is transformed into fluid motion.

Another criterion for determining whether the bubble walls runaway or reach steady speed goes back to the in \[47, 48\]. The pressure on the wall come from two sources with opposite directions. One is outwards and due to the difference in energy densities of the symmetric and broken vacuum, \( V_{sym} - V_{br} \). The other is inwards and due to the pressure \( P \) from the thermal plasma of particles surrounding the wall. For each point in the parameter space of the inert singlet model we compute the Boedeker-Moore (BM) criterion and require that

\[
p_{\text{runaway}} = V_{sym} - V_{br} - P < 0,
\]
Figure 1. (Left) The Scatter plot of $\mu_S$ and $\lambda_{HS}$ parameters. The Gray points represent the points which don’t lead to first order EWPT, The black points represent the point which leads to first order EWPT but non detectable GW, The other points represent the points which leads to detectable GW using future space-based GW detectors; Lisa (blue), BBO (magenta) and DECIGO (green). (Right) The Scatter plot of $p_{\text{runaway}}$ versus $\frac{v}{T_n}$. For a successful EWBG, the following conditions must be satisfied: (i) the phase transition must be strongly first order, i.e. $\frac{v}{T_n} > 1$, and (ii) The bubble wall must not runaway, i.e. $p_{\text{runaway}} < 0$. This result shows that for the inert singlet model, the second condition is equivalent with $\frac{v}{T_n} < 5$; an EWPT with $\frac{v}{T_n} > 5$ leads to a runaway bubble wall scenario.

where

$$P \approx \sum_i \left(m_{i,br}^2 - m_{i,sym}^2\right) g_i T_n^2 \int_i \left(m_{i,sym}^2 T_n^2\right)$$

and

$$\tilde{J}_i(x) = \int_0^\infty \frac{y^2 dy}{\sqrt{y^2 + x e^{\sqrt{y^2 + x} - 1} F_i}}.$$ (9)

Here $i$ runs over the considered SM and inert singlet scalar particles, while $g_i$ and $F_i$ are the particle multiplicity and fermion number respectively.

3 Results

The scatter plots on $(\mu_S, \lambda_{HS})$ plane in figure [left] shows a sample of the inert singlet model parameter points indicating regions where the strong phase transition could lead to GW accessible to promising future GW detectors, specifically eLISA, DECIGO and BBO [49]. Typically, the magnitude of the GW signal increases with the strength of the phase transition as shown in figure 2. In figure [right], we show the correlations between $p_{\text{runaway}}$ and $\frac{v}{T_n}$. For a successful EWBG the primordial phase transition must be strong first-order, i.e. $\frac{v}{T_n} > 1$, the bubble wall must not runaway, i.e. $p_{\text{runaway}} < 0$. Requiring these reveals that for an inert singlet model there is an upper bound on the strength of the first-order EWPT, $\frac{v}{T_n} < 5$. 
Figure 2. Spectra of GWs from the electroweak phase transition for randomly sampled examples from the coloured points in figure 1, i.e. the points with strong first-order EWPT. The sensitivity region for prospective GW detectors such as eLISA, BBO and DECIGO are also shown. It can be seen that the intensity of GW signal increases with the strength of the phase transition, i.e. $\frac{v_n}{T_n}$. For comparison we also show the sensitivity regions for SKA and EPTA detectors which cannot probe any part of the inert singlet parameter space.

Acknowledgements:

Thanks very much to C.L. Wainwright for his attention concerning cosmoTransitions package and to S. Sadat Gousheh, Mohammad Mohammadi-Doust, Safura Sadeghi, Mehrodkht Sasanpour, H. Hashamipour, A. Kargaran, Ankit Beniwal, Marek Lewicki, and Philipp Basler for conversations, useful discussions, or comments while working on this project.

A Effective potential

The inert singlet model effective potential for our work is based on [50]. It is composed of the tree level terms, $V_{\text{tree}}(h, S)$, the Coleman-Weinberg zero-temperature quantum correction terms, $V_{\text{1-loop}}(h, S)$, and thermal correction terms, $V_T(h, S, T)$ [51, 52]:

$$V_{\text{eff}}(h, S, T) = V_{\text{tree}}(h, S) + V_{\text{1-loop}}(h, S) + V_T(h, S, T).$$

(10)

The zero temperature one-loop correction [50, 53] in the on-shell renormalisation scheme with cutoff regularisation is given by:

$$V_{\text{1-loop}}(h, S) = \sum_{h,k,w,z,t} \frac{n_i}{64\pi^2} \left[ m_i^4(h, S) \left( \log \frac{m_i^2(h, S)}{m_i^2(v, 0)} - \frac{3}{2} \right) + 2m_i^2(h, S) m_i^2(v, 0) \right]$$

(11)
where $n_{i=h,x,W,Z,S} = \{1, 3, 6, 3, -12, 1\}$ and $m_0(v, 0)$ are masses calculated at the electroweak VEV $S = 0, h = v$. The field-dependent masses are:

$$m_w^2 = \frac{g^2}{4} h^2, \quad m_\gamma^2 = \frac{g^2 + g'^2}{4} h^2, \quad m_\chi^2 = \frac{y_i^2}{2} h^2, \quad m_\chi^2 = -\mu^2 + \lambda_H h^2 + \lambda_H S^2. \quad (12)$$

For the Higgs and the inert scalar singlet, the field-dependent masses are the eigenvalues of the $h$ and $S$ mass mixing matrix:

$$M^2_{HS} = \begin{pmatrix} -\mu^2 + 3\lambda_H h^2 + \lambda_H S^2 & 2\lambda_H h S \\ 2\lambda_H h S & 2\lambda_H h^2 + \lambda_H S^2 \end{pmatrix}. \quad (13)$$

Lastly, the thermal correction terms in the inert singlet effective potential are [50][53]:

$$V_T(h, S, T) = \sum_{h,x,W,Z,S} \frac{n_i T^4}{2\pi^2} J_0 \left( \frac{m_i^2(h, S)}{T^2} \right) + \sum_{i=1}^3 \frac{n_i T^2}{2\pi^2} J_f \left( \frac{m_i^2(h, S)}{T^2} \right) \quad (14)$$

where

$$J_{b/f}(x) = \int_0^\infty dk k^2 \log \left[ 1 \mp \exp \left( \sqrt{k^2 + x} \right) \right]. \quad (15)$$

Expand to leading order in $(\frac{T}{v})^4$, the thermal corrections to the scalar masses in the inert singlet model are can be determined as the eigenvalues of the mass matrix

$$M^2_{HS} + \begin{pmatrix} \Pi_h(T) & 0 \\ 0 & \Pi_S(T) \end{pmatrix} \quad (16)$$

where

$$\Pi_h(T) = \Pi_x(T) = T^2 \left( \frac{g^2}{16} + \frac{3g}{16} + \frac{\lambda_H}{2} + \frac{y_i^2}{4} + \frac{\lambda_S}{12} \right), \quad \Pi_x(T) = T^2 \left( \frac{\lambda_H}{3} + \frac{\lambda_S}{4} \right), \quad \Pi_W(T) = \frac{11}{6} g^2 T^2. \quad (17)$$

The corrected masses of Z-boson and and photon, $\gamma$, are the eigenvalues of the mass matrix

$$\begin{pmatrix} \frac{1}{4} g^2 h^2 + \frac{11}{6} g^2 T^2 & -\frac{1}{2} g' g h^2 \\ -\frac{1}{2} g' g h^2 & \frac{1}{4} g^2 h^2 + \frac{11}{6} g^2 T^2 \end{pmatrix}. \quad (18)$$

References

[1] M. B. Gavela, P. Hernandez, J. Orloff and O. Pene, Mod. Phys. Lett. A 9 (1994) 795 doi:10.1142/S0217732940000629 [hep-ph/9312215].

[2] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B 466 (1996) 189 doi:10.1016/0550-3213(96)00052-1 [hep-lat/9510020].

[3] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Phys. Rev. Lett. 77 (1996) 2887 doi:10.1103/PhysRevLett.77.2887 [hep-ph/9605288].

[4] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 43 (1993) 27 doi:10.1146/annurev.ns.43.120193.000331 [hep-ph/9302210].

[5] D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. 14 (2012) 125003 doi:10.1088/1367-2630/14/12/125003 [arXiv:1206.2942 [hep-ph]].

[6] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32 [JETP Lett. 5 (1967) 24] [Sov. Phys. Usp. 34 (1991) no.5, 392] [Usp. Fiz. Nauk 161 (1991) no.5, 61]. doi:10.1070/PU1991v034n05ABEH002497
