Coordinating vector field equations and diagrams with a serious game in introductory physics

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Abstract

Mathematical reasoning with algebraic and graphical representations is important for a throughout understanding of physics problems and concepts. Many problems require students to fluently move between algebraic and graphical representations. We developed a freely available serious game to challenge the representational fluency of introductory students regarding vector fields. Within the game, interactive puzzles are solved using different types of vector fields that must be configured with the correct mathematical parameters. A reward system implemented in the game prevents from using trial-and-error approaches and instead encourages the player to establish a mental connection between the graphical representation of the vector field and the (algebraic) equation before taking any action. For correct solutions, the player receives points and can unlock further levels. The graphics implemented in the game make the effects of changing the direction of a field or its magnitude visible and thus can help the learner gain a throughout understanding of vector fields. We report about the aim of the game from an educational perspective, describe potential learning scenarios and put in perspective the use of the game in the classroom.

Keywords: serious game, vector fields, multiple representations, introductory physics

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1. Introduction

The difficulties encountered in switching between an algebraic representation (formula) and the graphical representation are addressed in this paper by developing a serious game. After a brief introduction to the topic (vector fields), the paper first provides an overview of serious games before discussing the educational background of multiple representations (MR). Finally, the development of the game is described and how the game should help to improve learning in this domain.

Vector fields are mathematical objects that assign a vector to every point in the space or in a subset of space (e.g., the two-dimensional plane). Vector fields are important in many physics subjects, some of which are part of the introductory physics curriculum. Examples include Newton’s gravitational field, velocity fields of fluids, and electromagnetic fields. Two illustrative examples of vector fields are given in figure 1.

There are decades of research about students’ difficulties associated with vectors [1–8]. Numerous studies have shown that first-year university students have substantial difficulties regarding basic vector concepts, e.g., interpreting graphical properties such as direction, length and component decomposition, vector addition and subtraction, or scalar and cross products [1–3]. Based on the broad body of research, a reliable instrument to assess students’ understanding of vectors was developed, validated, and used by independent research groups [5–7]. Students’ difficulties occur both with and without physics contexts [4, 8], i.e., when vector concepts are treated purely mathematically. A comprehensive knowledge of these concepts is a prerequisite for understanding vector fields, thus, students’ difficulties with single vectors have a direct impact on their understanding of vector fields. In addition, students might understand basic concepts for single vectors but show difficulties applying the concepts to vector fields [9]. For instance, students might succeed to superimpose two force vectors but they might fail to superimpose two force vector fields.

Physics concepts, such as conservation laws or cause-effect relationships, are usually expressed using (one or more) external representations [10]. Common representations of vector fields are both graphical representations, e.g. vector field diagrams (see figure 1), and algebraic expressions using unit vectors in a previously defined coordinate system (see caption of figure 1) [9]. The coordination of both forms of representations (diagram and algebraic expression) represents a further challenge for students in general [11] and, particularly, for vector fields [9]. Among others, the study of Bollen et al suggest that a confident and flexible handling of MR can have a positive impact on learning and problem solving and to develop domain-specific expertise (see section 3 for more details). Indeed, several of the aforementioned learning difficulties regarding vectors were found to originate from a lack of understanding of the connections between the algebraic and geometric aspects of vectors [7].

Serious games in general are games designed to teach skills or knowledge that can be transferred to other activities or tasks. The serious game presented here is intended to foster the skills and routines for changing between representations in the context of vector fields that could complement traditional instructions. The aim of the vector field game consists of guiding moving particles from a source to a goal by choosing appropriate vector fields. The player can...
Figure 1. Two examples of 2D vector field diagrams. The left vector field is described by the equation $F(x, y) = -x\hat{x}$, and the right one by.

Figure 2. The effect of an appropriate vector field on the particles: the particles (yellow dots) move from the sources (left) to the goal (right). The vector field equation and the parameters are present on the upper part of the screen.

manipulate the algebraic expression and directly observe the impact on the field representation and on the movement of the particles, see figure 2.

2. Serious games in physics education

Games and playing in general, as forms of learning, pose an important factor in human development, especially, in early years as a child [12–15]. Play and games follow rules, at least given implicit but often fully formalized [16]. Games are often seen as subset of play, but there are various definitions of what defines a game [17–19]. We will refer to the definition of Salen and Zimmerman (2004) of games as ‘a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome’ [17]. A system is further defined by
objects or parts within the system, attributes of these objects or the system as whole, relationships between the objects, and the environment as the context surrounding the system. Other authors suggest also to include the player’s emotional reaction which could also be seen as a natural reaction to a game rather than a defining trait. As Connolly et al point out in their systematic literature review on game-based learning, games appear to offer many activities that modern theories of learning consider effective for learning, that is, learning being active, experiential, situated, problem-based and providing immediate feedback [20]. Accordingly, Toprac identified several dimensions that are important for games, including ‘challenge’ (having an optimal level of difficulty and uncertain goal attainment), ‘mystery’ (providing an optimal level of informational complexity), active learner control and having clear rules, goals, and feedback on progress toward goals [21]. To sum it up, the potential learning outcomes of games include outcomes relating to learning and skill acquisition (i.e., addressing the cognitive domain) as well as affective and motivational outcomes [20].

The term serious games was first described in the pre-digital age by Abt (1970) as ‘games (that) have an explicit and carefully thought-out educational purpose and are not intended to be played primarily for amusement’ [22] originally not referring to digital games. The term is somewhat contradictory, since it might imply that other games might not be serious or that serious games should not be entertaining. There are many other terms used for specific domains, especially in the education domain (games for learning, edutainment, educational games, etc). We will further use the term game-based learning [20, 23] for referring to a serious game in the education area. Nevertheless, the term serious game is often used as the best available summarizing term that encompassed many different approaches and domains [24]. There have been early (non-digital) examples of game-based learning especially in the military domain, e.g., GO and chess, to train tactical and strategic thinking and decision making. With the ongoing digitalization and transformation of society, digital games have entered the scene and with the cultural and economic success of (entertainment) games [25], a whole generation has been socialized by gaming which in turn may contribute to the success of serious games [24].

The research on serious games in physics education dates back to the work of White in 1986 who designed computer games to teach Newton’s laws of motion [26]. In a controlled study she found a significant effect of playing the game on the students’ conceptional understanding of Newtonian dynamics, especially about circular motion. The positive effect was explained by the fact that the students make connections between their intuitive beliefs about physics and their knowledge about formal physics, encompassing, e.g., vector addition. Students who lack an understanding of Newton’s laws can rely on general problem-solving heuristics combined with feedback to reach the goal. White argues that the serious game should not require an a priori knowledge of physics, since ‘students might find the game dull if they already knew the physics and impossible if they did not’ [26, p 73]. The research on the learning outcome of serious games in general is summarised by meta-analyses such as the one by Clark et al (2016) which found positive effects of game-based learning [27], which was in accordance with several other meta-analyses carried out before [28–30]. In the meta-analysis by Li and Tsai (2013) [31] three studies on game-based learning in the field of physics with undergraduate students were documented [32–34]. In these studies, the games ‘supercharged!’ and ‘circuit game’ are examined. The first is used to teach electrostatics while the second is a puzzle game to learn how electrical circuits work. Furthermore, Echeverría et al (2012) proposed a structured methodology for game design for the conceptual understanding of physics [35]. The three principles include that the independent variables of the simulation are integrated as low-level game atoms, the dependent variable of the simulation is integrated as a scaffolded game atom, and this connects to the goal through an additional game atom that provides an interesting
### Table 1. Integration of representations and game elements within the serious vector field game.

| Learning content       | Implementation                                                                 | Educational consideration                  |
|------------------------|--------------------------------------------------------------------------------|---------------------------------------------|
| Components of vector fields | Selection of x- and y-components from discrete values and unit vectors          | Qualitative evaluation                      |
| Magnitude and direction | Selection of absolute values and unit vectors                                   | Qualitative evaluation                      |
| Vector field diagram    | Adjusts based on parameter input                                                | Coherence between equation and diagram      |
| Field-particle interaction | Trajectory and ghost spots                                                      | Feedback on parameter selection             |
| Superposition          | Sequence of vector fields to pass obstacles                                     | Addition of vector fields                   |
| Special vector fields   | Radial fields and rotational fields                                              | Relevance in physics                        |
| Polarity                | Inverse field-particle interaction for different particles                      | Relevance in physics (cf charge)            |

challenge. Our approach in the vector field game followed basically the work of White and considers also the structure of Echeverría et al (2012), see section 4.3.

Most of the studies which have been carried out on serious games in physics education where on primary and secondary level, while on the higher education level research focused on simulations and virtual lab environments (e.g., [36]). While most of the games have been designed for the computer, advances in technological development implicate an extension to gaming consoles and especially handheld devices. In this context, a commercial video game for the PlayStation 3 console was used to teach kinematics to undergraduate students [37]. The game presented here is available for PC, smartphones, and tablet computers.

### 3. Educational and theoretical background: handling multiple representations of vector fields

Learning processes with MR are a central subject of physics education research. Ainsworth (1999) formulated three central functions of MR that can facilitate learning [38]: (1) MR contain complementary information, i.e. the learner benefits from the advantages from both representations [39]; (2) MR can help the learner to develop a better understanding of the subject by using one representation to limit possible misinterpretations by another; and (3) MR can help the learners to develop a deeper understanding of a concept. Based on these functions, many researchers report a positive effect of using MR on knowledge acquisition and problem-solving skills [11, 40–47]. For example, Nieminen, Savinainen and Viiri (2012) found a strong correlation between learners’ ability to interpret MR consistently (representational consistency) and their learning gain in a study on forces [11]. Their result confirms that judicious use of MR can contribute to a better understanding of physical concepts. In the context of visually interpreting vector field concepts (e.g., the flux concept), Klein et al (2018) found that students’ performance improved when two different strategies were introduced instead of only one strategy, and students performed best when they were free to choose between the two strategies [46]. Their finding supports the idea of introducing MR to foster student understanding. In general, the understanding that learners gain by using MR is broader, deeper [41] more flexible and robust [48] than that which they generate using single representations.
To sum it up, the use of MR allows problems and concepts to be considered from different perspectives. The ability to use different strategies can have synergistic effects on the construction of coherent knowledge structures [39] and might help to evoke varied mental pictures assisting to make new discoveries.

Despite the potential benefits described above, there are numerous studies showing that the use of MR does not per se lead to higher learning outcomes [42, 49, 50]. In order to benefit from the advantages of MR in learning and problem solving, a deeper understanding of the representations is necessary, which is described with the help of two competences, following De Cook (2012) and Nistal et al (2009, 2012) [10, 51, 52]:

(a) Representational fluency: this competence enables the interpretation and construction of representations and enables the correct and quick switching and translation between different forms of representation [53].

(b) Representational flexibility: this competence enables the choice of an appropriate form of representation in a given problem or learning situation and involves the ability to take into account characteristics of the subjects interacting with the representation and the context of the interaction [51, 52].

Bollen et al (2017) investigated students’ errors when switching representations between vector field diagrams, field line diagrams and algebraic expression [9]. When constructing a vector field diagram using an algebraic formula expression, errors were found that were mainly due to problems with vector addition, representing the change in length and direction of the vectors with increasing distance from the origin, and, for the reverse direction (i.e., changing...
Figure 4. In game view of two different levels of the vector field game. The dark (vertical) bars are obstacles which the particles cannot pass. Top: the simulation started but the particles did not enter the goal. The colored icons on the bottom right of each vector field box denote whether the correct type was used. Down: a level with particles of mixed ‘polarity’. The positive and negative particles each have a respective goal.

4. The vector field game

4.1. Educational considerations

The vector field game was designed to complement teaching of vector fields and to train the interpretation and construction of different vector field visualizations. The different learning
content is listed in table 1. For example, players can select different values for the x- and y-component of the vector field (as can be seen in the upper part of figure 2), and the visual representation changes accordingly. This induces a direct link between the algebraic expression and the field diagram. In other examples of vector fields, the player must select the magnitude or the direction of the vector field (instead of explicitly selecting the coordinates). Once the player made the decision, a source spawns particles and the trajectory is shown to the player. In later stages (levels) of the game, more than one vector field must be defined and the particles can have different polarity.

4.2. Game style and gameplay

We chose a casual style game to improve accessibility and so that learners can explore the direct effect of various vector field types (and adjusting their parameters) on particles in playful interaction. A flow-chart for solving a level is depicted in figure 3 and rough overview of the typical gameplay can be seen in figure 2.

The game itself consists of multiple levels where particles spawning at a particle source must be guided to the goal by adjusting the vector fields in highlighted areas. Each level is constructed out of the following elements: in every level, there is at least one source of particles (drop shaped), at least one goal (circle), at least one but usually multiple boxes representing vector fields, and obstacles shown as black bars. The player needs to select and configure the appropriate vector fields to guide the particles to the goal and solve the level. This is performed by graphical construction of the appropriate formulaic representations of vector fields by selecting the correct components from magnitude, polarity and unit vector combination in certain areas (boxes), i.e. selecting its corresponding vector field type and adjusting its parameters.

A box denotes an area where the vector field needs be adjusted. The player selects these by clicking/tapping on them. Afterwards, the top bar is shown to the player (see figure 2) and always displays the vector field formula (as link to the mathematical representation) and possible parameters for it. The game includes four kinds of vector fields: constant vector fields, hook fields, radial fields and rotational fields. When changing the parameters of the vector field, the arrow visualization in the vector field box is instantly updated so that the player can visually explore the link between the vector field formula and the arrow representation and thus encouraging representational fluency. There are two types of boxes: boxes with a light background indicate that the vector field type is already selected and only the parameters of the vector field can be changed whereas grey boxes indicate that the vector field type also needs to be determined. This is made explicit in figure 4.

By selecting the start button on the lower right, the game starts the particle simulation. Particles are spawned at the particle source(s) moving into the direction indicated by the particle source. The particle’s movement is influenced by the vector fields when entering the box areas and particles despawn when colliding with an obstacle and when reaching a goal. The particles leave ghost spots in order to trace their paths which is very useful to identify mistakes in the vector field selection. The simulation speed can be selected with the slider left of the pause/start button. If the particles do not reach the goal the player gets additional visual feedback to the ghost spots in the form of icons displayed on the boxes and can again change the vector fields until they press start again.

If all particles reach the goal, the level is finished and the player is rewarded with a summarizing screen providing an additional ‘star rating’ based on the score (see section 4.3 for more detail). From this view, the player can continue or return to the main screen. To raise the difficulty and make it more interesting, levels may also require the combination of multiple
vector fields, have multiple particle spawners and goals or may include obstacles. To increase the difficulty further, sources may spawn particles of mixed ‘polarity’, denoted by a $+/−$ sign. ‘Positively polarized’ particles are yellow color coded, ‘negatively polarized’ ones red. Red particles interact with the field in opposite direction. Please note, that the vector fields have no physical meaning, so no physical meaning in the polarization is intended here.

On the top right of the screen, a timer counting upwards and the current score are displayed, on the left the numbering of the current level and display representing the current rating where each part represents a star.

4.3. Design principles

Our design approach was in general significantly influenced by the principles of self-determination theory\[55, 56\]. SDT postulates three basic psychological human needs that contribute to motivation: competence, autonomy and (social) relatedness. Especially competence and autonomy contribute significantly to initial and mid-term motivation, while relatedness in general is a facilitator in combination with competence and autonomy and also contributes significantly to long-term motivation. In our case, we tried to address competence by providing feedback on different levels of granularity: (1) ‘seeing’ the particles taking the right or wrong route, (2) the aforementioned graphical indicator showing if the vector fields are correctly parametrized/assigned, (3) the score and the star indicator after a level has been solved. We addressed autonomy (4) by providing an appropriate degree of freedom in assigning and parametrizing the vector fields. In the current iteration, we did not directly address social relatedness, however, through the star indicators for solved levels we provide a way that players could compare themselves to their peers if they so desire.

We describe the design principles (a)–(d) in more detail in the following.

(a) First of all, the level design itself and the progression of levels is set up to start simple and raise complexity slowly: the game introduces simpler vector field types first, with fewer parameters to choose from, and the early levels include overall fewer vector fields. In the design framework of Echeverría et al (2012)\[35\], the vector field and the equation correspond to the low-level game atoms (independent variable that can be controlled by the user), and the track of the particle is the dependent variable. The challenge is to establish a connection between the source of the particle and the goal which reflects an additional game atom.

(b) We designed a graphical indicator as a direct reaction to player testing providing feedback in different tiers: red for wrong field type, amber for correct field type but wrong parameters, and green for correct field type with correct parameters. The type of boxes represent the scaffoldsthat Echeverría et al (2012) describes.

(c) As countermeasure for blindly trying to solve the level by random choices, the number of parameter selections and the overall level time influence the resulting score. We decided to put a mild time pressure on players to provide a motivating challenge and to avoid an overly slow and boring pace of the game. However, every level is still solvable even if the time runs out to reduce player frustration in challenging levels.

(d) Our goal is to facilitate the appreciation of vector field mathematics. Therefore, we refrained from using unconstrained mathematics as it would offer too many degrees of freedom without an additional benefit in terms of intuition as minimal changes of parameters would almost be indiscernible but might lead to unintended results in some cases. However, we also wanted to avoid pure trial and error type play, which was a difficult balance to achieve, and partly, this is still open for evaluation. Following an iterative design and development process, we found a good balance between offering enough degrees of
freedom in the formulas and different types of vector fields—to facilitate actually making errors to learn from—but also provide guidance and scaffolding in the form of different design elements.

In general, the reward design to motivate the players is twofold: on one hand, intrinsic motivation is boosted by good usability and stream-lined, non-distracting interaction as well as graphical aesthetic design. On the other hand, extrinsic motivation is given in the form of points and star ratings for each level. The later also facilitates replayability because the star ratings are also shown in an overview screen of all levels unlocked so far to motivate players to beat their own high-scores.

4.4. Design process and qualitative expert feedback

The game was developed in an iterative design process as proposed by Fullerton (2014) [19]. This enabled us to focus on player experience (‘playcentric design process’) in every development phase. In order to obtain acceptable usability, which we identified as a key quality requirement, we included a tutorial and focused on the development of an intuitive interface.

In this section, we illustrate key aspects of the design process and how user feedback influenced important design decisions. While this does not replace a meaningful study of the game’s effect on student motivation and learning success it provides insight into how the game design is grounded in empirical feedback of experts and representatives of the target group throughout the whole development process. The group of authors consists of an interdisciplinary team of experts in physics, physics education and serious games. Starting with concept drawings and simple prototypes of different game mechanics, we agreed on the general concept of a puzzle/casual game that provides a lot of freedom to ‘play around’ with vector fields and is to some degree intrinsically motivating because players want to observe different outcomes, possibly also extreme behaviours. That approach is inspired by how players make use of games like Bridge Builder3 or Kerbal Space Program4 but is also grounded in the notion of ‘play’ as described by Fullerton et al (2014) [19] or Salen and Zimmerman (2004) [17].

In the further process, we experimented with different game mechanics and elements that are typical for the genre, such as different forms of time pressure and scoring that is at least partly based on the number of tries it took a player to solve a level. Established game design practice but also educational considerations led us to design the progress and pacing of the game in a way that introduces simpler vector field types first, then let’s players explore them in scenarios of increasing complexity and difficulty, then adding additional vector field types to their ‘toolbox’. We regularly brought in (physics) student volunteers every two or three weeks and let them test the current iteration and we collected additional expert feedback one time at about 2/3 of the development. The group of experts consisted of nine participants known to the authors but otherwise not involved in the project or design of the game with a physics or physics education background. Four experts with a bachelor degree at the time of testing, three holding a master degree and two with a doctorate in the respective area.

We discussed all feedback internally and decided on the modification, addition or removal of game elements and mechanics accordingly. It must be noted that the feedback we received both by student volunteers and experts while very important and helpful to us at the same time was not always unequivocal. This is not surprising in game design/development. In general, results from user and focus group testing in this area have to be carefully discussed and reflected, however, typically some prominent strengths and weaknesses emerge that are reported by many

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3 https://youtu.be/3b4xkunkXkY.
4 https://kerbalspaceprogram.com.
participants/experts. In the case of the vector field game, both experts and student volunteers raised concerns about the potential number of tries needed to find the right combination, the integration with the formal representation and the stressful time limit in early prototype versions. This made us reconsider and adapt some design decisions and let for example to the introduction of the additional visual feedback in the game regarding the choices of vector fields and parameters, which is especially helpful for the more complex levels. We also balanced the number of vector fields players can choose from in each level carefully over several iterations and tested several different approaches to presenting the formulas, e.g., we went from the possibility to freely choose parameter values to limiting choices to a certain set of values to facilitate successful ‘educated guesses’ that actually make the formulas useful and not relying only on (visual) trial and error. However, we still wanted to keep the exploratory intrinsically motivating possibility of trying out different combinations on purpose.

During development, we also made several balancing changes to the scoring and how time and the number of tries are weighted to reduce negative stress while still motivating players to avoid too many tries without thinking.

4.5. Vector fields used in the game

Note that the vector fields that were constructed for the serious game and for the educational studies do not necessarily have a physical meaning. For our aims we wanted to avoid that lacking physical knowledge hinders the acquisition of representational fluency. To become knowledgeable physicists, however, students definitely must supplement their mathematical skills with conceptual knowledge of the physical world. For instance, the left vector field in figure 1 could be interpreted as the force field of a one-dimensional spring, \( F(x) = -kx \), that was extended to the two-dimensional plane, \( F(x, y) = -kx\hat{x} \). Even though conceptual knowledge in physics of this kind might be helpful for solving the puzzles, they are no prerequisite. As mentioned above, four kinds of vector fields have been included (constant vector fields, hook fields, radial fields and rotational fields), and the advanced physics student might attribute physical meaning to them.

4.6. Resources

The game is free and available on the desktop\(^5\) for Windows and macOS and on mobile platforms for Android\(^6\) and iOS\(^7\).

4.7. Research potential

In January 2020, the game was distributed to first-term undergraduate physics students and an investigation of the presumed learning potential of the game was prepared. For this purpose, a diagnostic test was designed that addresses the connection between the mathematical representation form (formula and equation) and the corresponding graphical representation of vector fields—a competence that should be promoted by the game from an educational point of view, see section 3.

Initially it turned out that the informal distribution of the game without instructions did not produce the desired response. Of the 68 students to whom the download link for the game was issued with the request to play the game during the upcoming four weeks, only 26 actually

\(^5\) https://tuk-software.procampus.de/de/vektorfeldspiel/
\(^6\) https://play.google.com/store/apps/details?id=de.unikl.eit.sge.vectorfieldgame.android.
\(^7\) https://apps.apple.com/de/app/vektorfeldspiel/id1517002472.
downloaded the game and provided feedback. Of those who downloaded the game, 13 students were engaged with the game for less than 10 min, 9 between 10 min and 1 h, and 6 students said they had played the game to the end. 62% of the students thought that the game was fun and the same number said that they learned something from it.

The total participation rates are consistent with response rates garnered in survey studies among students [57–59]. The low participation rate might be due to several reasons, e.g. feeling ‘bombarded’ with questionnaires during the online semester, demands on students’ time or perceiving no personal relevance of the game for learning [58]. Due to the low student participation rate, a post-test was not carried out to reassess the competence, so no conclusions can be drawn about the change in this competence that the game might cause. In follow-up examinations it is advisable to anchor the game formally and instructionally. Future studies should involve structured tests, and compensation for taking part in the study can be achieved by gaining some additional course credit or passing a tutorial. To avoid unfairness within a study group, but still realizing an experimental design with control groups a cross-over randomization can be used, that is a group is tested following traditional study and a second group is tested using the game. Following that the groups will be reversed so all had the same experience to satisfy any ethical consideration of their participation in the study.

In order to lower the usage threshold for students in the future and to enable testing on a larger scale, the porting and publication of the game to mobile platforms (Android and iOS) was carried out.

5. Concluding remarks and outlook

In times of increasing importance of online learning due to the COVID-19 pandemic, educational technology and digital teaching materials gain in value. We reported about a freely available serious game that offers a contribution for physics students and lectures. The vector field game was developed to challenge the representational fluency of introductory students regarding vector fields. It was discussed that (1) being fluent to make connections between equations and diagrams is an important skill in mathematics and physics education, particularly in the context of vector fields, and (2) that serious games can have the potential to substantially foster learning by a mixture of using intuitions, problem-solving heuristics, and feedback from the game. Combining both lines of research, i.e., learning with MR and serious games engineering, we suppose that the vector field game is beneficial for introductory physics students. The actual impact on student learning has to be tested.

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Informed consent and ethical policy statement

We confirm that the data collection from University students was carried out in accordance with informed consent and with the principles outlined in the ethical policy of IOP. The evaluation was approved by the local data protection office of the Technische Universität Kaiserslautern.
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