INTRODUCTION

At latest from the Hellenistic Period onwards, transport amphorae constituted the standard containers for a large variety of commodities, such as wine, oil, fish, or grains, on long-distance transport. They were produced in large numbers usually close to the production places of the trading goods and their manufacture reached a high level of standardization in terms of material quality as well as in terms of vessel shapes, which signified origin and content. The traditional main scope of scientific studies of transport amphorae is the study of distribution of specific amphora types through investigation of ceramic compositions either by elemental analysis or by petrography. On the basis of these data, transport routes of amphorae can be investigated providing information about trade not only of the vessels but also indeed rather of their contents. During recent years, however, the focus of research has moved or at least has been broadened toward other topics such as production technology or vessel function. Production technology, for instance, concerns raw material selection, clay paste
preparation, design and standardization of vessel shapes, and firing technology. The examination of these parameters provides comprehensive reference data for provenance studies, following the above-mentioned traditional approach. On the other hand, though, additional information is provided concerning the organization of production and its diachronic development. Eventually, technical aspects of amphorae and their use as transport containers can be studied, such as packaging of the amphorae during transport, their performance under mechanical loads, and the interpretation of damage and failure patterns. Such studies provide information about the use and function of transport amphorae as well as an understanding of technical choices in terms of production parameters, such as raw material selection, clay paste preparation, and firing technology.

In order to address diverse research questions, suitable sample selection is essential. Amphora assemblages can be studied at different stages of their supposed life cycle. The life cycle started with the production. In order to investigate production, either amphorae from an individual kiln can be studied or the study is extended to specific production sites or entire regions manufacturing specific amphora types. Another possible focus can be on assemblages from transport infrastructures, such as sunken cargo ships, which depending on trading route and progress, in terms of ports called at, comprise a more or less diverse mix of amphorae from various production regions. An even larger diversity within a studied amphora assemblage can be expected when investigating market places where large numbers of amphorae from different regions were traded and eventually places of consumption. Hence, the variation of amphorae assemblages increases according to the stage of their life cycle as at every stage amphorae from different production regions were mixed. On the other hand, it can be assumed that amphorae were produced basically for one-time usage. Even though vessels were reused, for example, for storage or other purposes, it was impractical to use them again for their primary purpose, which was the transport of specific goods from a specific place of production to the place of consumption. Probably, a considerable part of the vessels was simply discarded. In the Roman Period, however, the practices might have been different. By then, the reuse of amphorae for transport of wine was apparently common. This might have been related to more standardized vessel shapes not any longer characteristic for a specific place of production. However, it was apparently not common to transport empty amphorae.

Assessing primary purpose and production design, however, the manufacture of a transport amphora can be considered as a design problem toward meeting specific operational principles, analogous to other functional objects such as pottery kilns. The primary functions, the transport amphora had to fulfill, can be specified as follows:

1. Steady containment of the commodity during trade, transport, and consumption: The actual value was the content or trading good. Failure of the container was equivalent to the loss of its content.
2. Appropriate use of space in storing places: Particularly, during marine transport efficient packing of amphorae in the cargo hold was important.
3. Feasibility to lift and move the container and its content usually by one person: It can be assumed that loading and unloading was done by hand.
4. External representation of the content: Vessel shape and possible inscriptions or stamps represented the content of the amphora, in terms of type of commodity and region of origin.

As for the steady containment, the production design comprises two groups of parameters, each affecting the mechanical performance characteristics. The first group concerns material properties of the ceramics, such as elasticity, mechanical strength, and toughness. These depend on the utilized raw materials, the clay paste preparation, and the firing of the ceramics, and they can be examined by material testing of laboratory specimens either cut from amphora fragments or manufactured as replicates based on the observed compositional and microstructural parameters. The second group concerns the vessel design including shape parameters such as wall thickness, curvature, or ridges. Shape, however, in contrast to material properties, is more difficult to examine by material testing as it is substantially more complex. Testing of authentic archaeological vessels is categorically impossible because the tests are commonly destructive. Replicates, on the other hand, will introduce additional uncertainty in terms of level of details in comparison with the original vessel design and accurateness of the copy. Alternatively, however, two- or three-dimensional digital models of different vessel shapes can be generated and the respective material properties determined in laboratory tests can be attributed to these models. The generated digital models can be tested under different simulated constraints using the finite element method (FEM). This approach allows for investigating the performance of different vessel shapes under simulated loads and for assessing the effect of specific design parameters. It allows furthermore for assessment of potential failure and critical loads and for interpretation of actual damages observed in archaeological finds. This risk assessment appears to be important. First of all, failure of a single amphora caused loss of the content, which was essentially more valuable than the actual ceramic vessel. Concerning an entire stack of amphorae, furthermore, the primary damage of a single amphora, located in one of base layers, could have implicated destabilization of the stack and a chain reaction of further damages resulting eventually in calamitous consequences for a cargo ship during marine transport.
In the present paper, the three-dimensional modeling of transport amphorae and the simulation of their mechanical performance by using FEM are introduced. The approach is demonstrated with examples of Hellenistic and Roman amphora types from the East Aegean, including estimations of their material properties. The amphora models are tested under different loads and constraints simulating their use during packaging and transport in a cargo ship. The simulation results are discussed with regard to archaeological evidence, in terms of original wear and damages of amphorae.

2 | METHODOLOGICAL APPROACH

2.1 | Generating a three-dimensional digital model

Three-dimensional models of transport amphorae can be generated following different approaches. A straightforward approach is to start from a two-dimensional profile of the amphora body, which can be extracted for example from photographs or detailed drawings. Based on the scaled digitalized profile, a three-dimensional vase body can be generated by rotation around the symmetry axis using common CAD software. Wall thickness is either included in the initial profile or it is added as parameter during the rotation procedure. Specific parts or features of the amphora, such as the two handles, the shaped rim and the pointed base, are modeled separately and merged with the body. A more accurate three-dimensional model, considering also potential deviations from the rotational symmetry, can be achieved by 3D scanning for example using laser triangulation. If an entire vessel is scanned, though, the nonvisible internal space cannot be recorded so that information about the wall thickness in particular is missing. In order to investigate the internal structure of a vessel, quasi to look inside, X-ray radiography can be applied. While conventional X-ray radiography provides two-dimensional images, which again have to be digitally rotated for generating a three-dimensional model, X-ray computer tomography can be applied in order to capture straightforwardly the complete three-dimensional structure of the amphora body.

In the present study, digital models are generated from profiles (Figure 1). The advantage is that without considering complete three-dimensional information specific design features can be investigated without possible interferences by random deviations from the ideal shape design, such as variation of wall thickness or rotational symmetry. This allows for a general understanding of observed technological choices by the craftsman disregarding level of skills during the manufacture. The profile of an amphora is created by defining series of two-dimensional design points describing the actual amphora body, the pointed base, the rim, one of the two handles, and a cross section of the handle. For this, images, either photographs or drawings, are scaled and gauged with the public domain image processing software ImageJ (https://imagej.nih.gov/). The design point series are imported in the DesignModeler of the ANSYS 19.0 Workbench (ANSYS, Inc, Canonsburg) as basis for smooth three-dimensional curves generated by spline fit. The open curve describing the external profile of the amphora body is rotated around the main symmetry axis applying a predefined uniform wall thickness. Closed curves of the pointed base and the rim are rotated separately around the symmetry axis and merged with the amphora body. The closed curve describing the cross section of the handle is swept along the defined curve of the handle. On the amphora body and on its neck, areas for the joins of the handle are defined and joined with the constructed handle. The digital models allow for estimating the weight of the amphorae taking into account an assumed wall thickness.

FIGURE 1  Scaled drawing of a Roman Dressel 2 4 amphora from Kos (after Grigoropoulos) and digital model, which was created based on this drawing: For the main amphora body, a profile was defined with design points connected with a spline fit. The upper part was modeled including wall thickness, while for the lower part a wall thickness of 10 mm was assumed. The profile was rotated around the symmetry axis (Y). Handles, lip, and pointed base were modeled separately and merged with the amphora body.
average wall thickness and density of the ceramics as well as for estimating the potential volume of the amphora. Even though the amphorae were probably not filled up to the rim, the maximum capacity of the amphora, estimated by closing the vessel model with a simulated cap, indicates the upper limit of possible standardized amounts of the content, in the present case study assumedly wine, which can be compared with ancient volume measures.

Variation of the design points or the predefined wall thickness provides the possibility to investigate slight changes in design, concerning for example the height of the shoulder or the maximum diameter of the vessel. The models are investigated for different loads and constraints simulating their supposed use. In order to simulate for example the packing of amphorae in a cargo ship, the initial amphora model is replicated and a series of several amphora models is placed in assumed arrangements (Figure 2). The simulated arrangement provides information about the spatial efficiency of the packing and about potential contact points among amphora bodies, for example, on shoulder and base, but also between handles and amphora bodies. Remarkably, based on some of the hitherto generated amphora models, the simulated packing indicates an additional function of handles serving potentially as support for amphorae placed in the next higher layer.

2.2 | Finite element Method (FEM)

Mathematical models for the performance of a solid object under external loads can be set up as boundary value problems. The deformation of bodies with basic shapes exposed to mechanical loads for example can be fully described with partial differential equations taking into account explicit boundary conditions. These equations, however, are commonly not trivial to evaluate analytically, particularly when two- or three-dimensional systems are examined. For complex shapes, an analytical solution becomes practically impossible. A numerical approximation of the solution, however, can be achieved through discretization of the structural system. Multiscale modeling allows for virtual material testing on different levels: from microstructures of composite materials over components manufactured from these materials up to entire structures assembled of diverse components. Complex microstructures comprising multiple phases can be investigated for their mechanical response on external loads. In the case of ceramic materials and their prevalently brittle fracture behavior, the investigation of stress and strain distributions provides evaluation of the strength of components taking into account Weibull statistics and stochastic crack initiation originating from inherent flaws. In this way, the mechanical performance and reliability of arbitrary designs of ceramic components can be assessed, as it has been implemented for example in the CARES code (Ceramic Analysis and Reliability Evaluation of Structures).

For the present case study, the ANSYS Workbench 19.0 is used, a software platform providing an integrated environment for diverse types of simulation approaches. First step in the FEM simulation is the definition of the geometry digital amphora models are arranged in groups simulating the above described packaging in a cargo hold. For the interaction between different bodies, contact conditions have to be defined considering for example possible friction of the surface. Suitable element types and sizes are selected balancing spatial resolution and accuracy of the FEM simulation and on the other hand total numbers of elements and nodes. For this, a coarser mesh size can be predefined for the major part of the model, while for regions, in which larger deformation or stress is expected, such as contact zones, a refined mesh size is defined. Another way to reduce the number of elements is making use of symmetries. By dividing the space through symmetry planes, only a part of the entire problem is necessary to be examined and the load simulation can eventually be reduced to one single contact zone.

The performance of the meshed bodies is simulated by applying specific constraints to the model, such as mechanical loads and fixed supports. In the case of vertical loads in an amphora stack, a fixed support is defined for the pointed
base of the lower amphora, while on the internal surface of the amphora on top a vertical weight load is applied. The solution is computed in a series of time steps increasing exponentially the weight load applied to the upper amphora. Stepwise loading is necessary because the contact area between the amphora bodies, which depends essentially on the mechanical load, affects severely the effective compressive stress in the amphora surfaces. The FEM simulation results provide estimations of deformation and spatial distribution of strain and stress in the ceramic bodies. The calculated strain and stress are provided as tensors, comprising the Cartesian components of normal and shear strains or stresses. In order to assess stress particularly in comparison with critical stress or strength as estimation for potential failure of the material, an equivalent stress has to be defined. Common strength hypotheses are based on the von Mises stress, a combination of normal and shear stresses suitable rather for assessing ductile materials, and the principal stress, providing maximum and minimum values of the normal stress vector. In the present case study, the main focus is on the maximum or first principal stress, which is basically tensile stress. Brittle fracture of ceramic components originates from flaws in the ceramic body and is triggered eventually by tensile stress. After initiation, cracks propagate perpendicular to the maximum principal stress. The minimum principal stress on the other hand provides negative values and an estimation of compressive stress, which emerges mainly in the contact zone and can potentially result in crushing and local collapse of the ceramics’ microstructure. However, an expansion of the damaged areas as in the case of crack propagation under tensile stress is less probable but rather a local densification, which can be considered as a quasi-elastoplastic deformation. Failure of the ceramics considering their ultimate tensile and compressive strength, estimated in laboratory experiments of test specimens, can be assessed with the Mohr-Coulomb criterion.

3  |  STUDIED AMPHORA MODELS

The present case study is focused on the development of vessel shapes in the East Aegean amphora production centers of Kos and Rhodes during the Hellenistic Period (Figure 3). The models have been generated using published drawings of characteristic amphora types produced in the two islands. Apart from the Hellenistic amphora types, one model of a Roman Koan Dressel 2 4 amphora is included in the case study. In Table 1, the studied amphora models are listed including their dimensions and estimated weights and capacities. The indicated weights of the empty vessels correspond to a uniform wall thickness of 10 mm and an assumed density of the ceramics of 1.7 g/cm³. The estimated maximum capacity corresponds to the internal volume up to the

FIGURE 3  3D models of amphorae from Kos (top) and Rhodes (bottom)
The actual volume of the wine filled in the amphora was assumedly smaller. While the capacity of four of the Rhodian amphorae corresponds approximately to eight Attic choes, c. 26.3 L, some of the Koan amphorae appear to present higher capacity of c. 12 Attic choes, c. 39.4 L, and maybe even more.

In terms of material properties, Young's modulus of 18 GPa and a Poisson ratio of 0.27 is assumed for the present FEM simulations of the amphora models. Concerning damage assessment, the assumed critical tensile stress is based on material tests, which were carried out on fragments of Hellenistic and Roman amphorae from the two islands. According to these tests, which will be presented separately, the ceramics present a flexure strength of typically between 20 and 35 MPa. The ultimate compressive strength, which has not been measured separately, is supposed to be typically about 2-3 times higher. Simulations are examined considering pure elastic deformation as well as quasi-elastoplastic deformation due to assumed local crushing of the material. For this, a compressive yield strength of 20 MPa is assumed and a tangent modulus of 2 GPa, which corresponds to the performance of ceramic specimens cut from the amphorae, which has been observed in bi-axial flexure tests. Finally, even though no considerable displacement of the amphora surfaces against each other is expected a friction coefficient of 0.6 in the contact zone is assumed. Direct contact between the two amphorae is assumed without any padding or wedging.

The vessel shapes are assessed for their mechanical performance during transport in a cargo ship, in which they are packed in several layers, stacking the amphora bases of one layer between the shoulders of the amphorae in the layer below. According to these tests, which will be presented separately, the ceramics present a flexure strength of typically between 20 and 35 MPa. The ultimate compressive strength, which has not been measured separately, is supposed to be typically about 2-3 times higher. Simulations are examined considering pure elastic deformation as well as quasi-elastoplastic deformation due to assumed local crushing of the material. For this, a compressive yield strength of 20 MPa is assumed and a tangent modulus of 2 GPa, which corresponds to the performance of ceramic specimens cut from the amphorae, which has been observed in bi-axial flexure tests. Finally, even though no considerable displacement of the amphora surfaces against each other is expected a friction coefficient of 0.6 in the contact zone is assumed. Direct contact between the two amphorae is assumed without any padding or wedging.

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Table 2: Simulated stresses in the modeled amphora bodies under a horizontal load of 1000 N applied in incremented steps: Pure elastic deformation with Young's modulus of 18 GPa is considered.

| Type          | Contact area in mm² | Amphora (bottom) | Amphora (top) |
|---------------|---------------------|------------------|---------------|
|               |                     | Ext. surface     | Int. surface  | Max. tensile stress in MPa | Max. compressive stress in MPa | Mohr-Coulomb criterion | Ext. surface | Int. surface | Max. compressive stress in MPa | Mohr-Coulomb criterion |
| Kos           |                     |                  |               |                           |                             |                             |              |              |                             |                             |
| Type I        | 4th BC              | Solid shell      | 20.1          | 30.8                      | 19.0                        | 132.2                       | 0.90          | 26.1          | 25.5                      | 135.6                       | 1.00                      |
|               |                     | Tetrahedron      | 15.3          | 28.4                      | 20.2                        | 160.3                       | 0.89          | 25.0          | 21.3                      | 143.0                       | 1.06                      |
| Type II       | 4th BC              | Solid shell      | 21.1          | 24.9                      | 11.0                        | 98.1                        | 1.17          | 23.7          | 16.2                      | 102.3                       | 1.17                      |
|               |                     | Tetrahedron      | 17.3          | 29.3                      | 12.0                        | 106.4                       | 0.96          | 26.3          | 13.4                      | 101.7                       | 1.05                      |
| Type I        | Late 3rd BC         | Solid shell      | 21.5          | 27.5                      | 11.0                        | 105.0                       | 1.06          | 26.7          | 16.3                      | 97.3                        | 1.01                      |
|               |                     | Tetrahedron      | 17.4          | 21.4                      | 11.2                        | 104.0                       | 1.30          | 23.6          | 13.1                      | 103.3                       | 1.16                      |
| Type I        | 1st BC              | Solid shell      | 16.7          | 32.6                      | 10.9                        | 144.2                       | 0.86          | 31.8          | 15.0                      | 139.2                       | 0.82                      |
|               |                     | Tetrahedron      | 15.4          | 42.1                      | 11.0                        | 159.5                       | 0.62          | 42.8          | 13.1                      | 173.7                       | 0.67                      |
| Dressel 2 4   | Roman               | Solid shell      | 10.0          | 49.8                      | 8.5                         | 156.6                       | 0.59          | 46.6          | 13.1                      | 180.7                       | 0.63                      |
|               |                     | Tetrahedron      | 9.7           | 51.7                      | 9.3                         | 228.0                       | 0.55          | 57.2          | 12.1                      | 234.7                       | 0.51                      |
| Rhodes        |                     |                  |               |                           |                             |                             |              |              |                             |                             |
| Type I A      | Late 4th to early 3rd BC | Solid shell | 17.3          | 44.3                      | 11.1                        | 155.8                       | 0.66          | 43.1          | 18.5                      | 164.5                       | 0.65                      |
|               |                     | Tetrahedron      | 14.8          | 53.7                      | 11.9                        | 199.2                       | 0.53          | 45.4          | 16.7                      | 207.6                       | 0.63                      |
| Type I B      | 3rd BC              | Solid shell      | 17.1          | 44.9                      | 10.3                        | 151.8                       | 0.64          | 40.5          | 15.8                      | 157.1                       | 0.68                      |
|               |                     | Tetrahedron      | 19.0          | 37.4                      | 10.9                        | 155.2                       | 0.74          | 35.2          | 14.0                      | 148.5                       | 0.78                      |
| Type I C      | 3rd BC              | Solid shell      | 17.0          | 41.3                      | 9.8                         | 146.9                       | 0.69          | 51.3          | 17.6                      | 166.4                       | 0.50                      |
|               |                     | Tetrahedron      | 15.5          | 37.9                      | 10.4                        | 162.4                       | 0.68          | 40.1          | 13.8                      | 177.1                       | 0.70                      |
| Type I E      | 2nd BC              | Solid shell      | 17.7          | 36.8                      | 10.5                        | 146.9                       | 0.77          | 43.7          | 16.8                      | 139.5                       | 0.64                      |
|               |                     | Tetrahedron      | 13.7          | 44.5                      | 10.7                        | 165.0                       | 0.58          | 47.9          | 13.8                      | 172.2                       | 0.61                      |
| Type I E2     | 2nd BC              | Solid shell      | 15.8          | 42.3                      | 10.0                        | 152.1                       | 0.67          | 58.7          | 18.9                      | 213.4                       | 0.47                      |
|               |                     | Tetrahedron      | 13.7          | 44.5                      | 10.7                        | 165.0                       | 0.72          | 47.9          | 13.8                      | 172.2                       | 0.53                      |

Note: The simulated contact areas are listed and maximum and minimum principal stresses in MPa, corresponding to tension and compression. Furthermore, the Mohr-Coulomb criterion at a maximum load of 1000 N is indicated, assuming an ultimate tensile strength of 30 MPa and an ultimate compressive stress of 200 MPa. For the FEM simulation, two different element types were tested solid shell and ten-node tetrahedron, providing comparable results.
loads can emerge due to the swell the cargo ship is exposed to. The second case is a dynamic horizontal load at the contact point of two amphorae in the same layer, simulating rolling of the cargo ship in rough sea.

4 | RESULTS AND DISCUSSION

4.1 | Vertical load on shoulder

For the two amphora parts, an initial mesh size of 10 mm is selected. Because the most considerable deformation is expected in the proximate contact region, the mesh size is refined in spheres around the contact zone. Two different element types are tested: ten-node tetrahedral elements and solid shell elements. While tetrahedral elements in the contact zone are refined to sizes of 3 mm and 1.5 in spheres with diameters of 100 and 30 mm, respectively, solid shell elements are refined to 1.25 mm in a sphere with 40 mm diameter with 20 layers over the 10 mm wall thickness. For the simulations, the foot of the lower amphora body is fixed in vertical direction, restricting any displacement, while vertical load is applied to the internal surface of the upper amphora body. The load is exponentially increased reaching in twenty main steps, including up to three sub-steps, a maximum value of 1000 N. This maximum vertical load corresponds roughly to the weight load of eight layers of amphorae with an average weight of 50 kg each weighing on a single contact point.

The FEM solver provides apart from simulated structural deformation estimations for strain and stress of the individual elements (Table 2). The contact area increases up to a range of c. 15-20 mm² under maximum load corresponding to circular areas with diameters of c. 4-5 mm. An exception is the tested Dressel 2 4 amphora model, for which a comparably smaller contact area is indicated, assumedly because of the apparent ridge on the shoulder of the lower amphora on which the upper amphora is placed. As expected, the highest stresses emerge in the external surfaces of the amphorae in the region of the contact areas (Figure 5). Compressive stress prevails primarily in the immediate contact area, while tensile stresses emerge close

| Type         | Mesh size in mm | Contact area in mm² | Amphora (bottom) |          | Amphora (top) |          |
|--------------|-----------------|---------------------|-------------------|----------|---------------|----------|
|              |                 |                     | Max. tensile stress in MPa |            | Max. tensile stress in MPa |            |
|              |                 |                     | Ext. surface | Int. surface | Ext. surface | Int. surface |
| Koan I       | 2.00            | 21.9                | 27.1          | 10.3       | 91.4          | 1.07      |
| 1st BC       | 1.75            | 18.8                | 36.3          | 10.8       | 122.0         | 0.80      |
|              | 1.50            | 18.1                | 27.5          | 10.4       | 122.1         | 0.97      |
|              | 1.25            | 16.7                | 32.6          | 10.9       | 144.2         | 0.86      |
|              | 1.00            | 16.0                | 44.6          | 11.1       | 148.7         | 0.65      |
|              |                 |                     | Max. compressive stress in MPa | Mohr-Coulomb criterion | Max. compressive stress in MPa | Mohr-Coulomb criterion |
|              |                 |                     |              |            |              |            |
|              |                 |                     | Ext. surface | Int. surface | Ext. surface | Int. surface |
| Koan I       | 20              | 21.9                | 27.1          | 10.3       | 91.4          | 1.07      |
| 1st BC       | 17.5            | 18.8                | 36.3          | 10.8       | 122.0         | 0.80      |
|              | 15.0            | 18.1                | 27.5          | 10.4       | 122.1         | 0.97      |
|              | 12.5            | 16.7                | 32.6          | 10.9       | 144.2         | 0.86      |
|              | 10.0            | 16.0                | 44.6          | 11.1       | 148.7         | 0.65      |

FIGURE 5  Section through the contact area between two Rhodium amphorae (Type IE): In this model, both amphora bodies were meshed with solid ten-node tetrahedral elements with a refined mesh size of 1.5 mm in the contact area. The colors of the elements indicate the maximum principal stress estimated for a vertical load of 1000 N. Negative values (blue) correspond to compressive stress and positive values (red) to tensile stress.
### Table 4
Simulated stresses in the modeled amphora bodies under a horizontal load of 1000 N applied in incremented steps: Elastoplastic deformation with Young's modulus of 18 GPa and a yield strength of 20 MPa is considered, assuming local crushing.

| Type      | Contact area in mm² | Amphora (bottom) | Amphora (top) |
|-----------|---------------------|------------------|---------------|
|           |                     | Max. tensile stress in MPa | Max. compressive stress in MPa | Mohr-Coulomb criterion | Max. tensile stress in MPa | Max. compressive stress in MPa | Mohr-Coulomb criterion |
|           |                     | Ext. surface | Int. surface | Mohr-Coulomb criterion | Ext. surface | Int. surface | Mohr-Coulomb criterion |
| Kos       |                     |               |              |                          |               |              |                          |
| Type I    | 4th BC              | Solid shell   |              |                          |               |              |                          |
|           |                     | 24.1          | 21.0         | 19.5                     | 85.9          | 1.37         |                          |
|           |                     | Tetrahedron   | 22.5         | 12.9                     | 21.1         | 77.9         | 1.46         |
| Type II   | 4th BC              | Solid shell   |              |                          |               |              |                          |
|           |                     | 23.5          | 23.2         | 11.0                     | 80.0          | 1.25         |                          |
|           |                     | Tetrahedron   | 21.6         | 21.3                     | 12.1         | 80.5         | 1.33         |
| Type I    | Late 3rd BC         | Solid shell   |              |                          |               |              |                          |
|           |                     | 25.8          | 24.7         | 11.0                     | 82.6          | 1.20         |                          |
|           |                     | Tetrahedron   | 21.0         | 18.1                     | 11.3         | 77.3         | 1.59         |
| Type I    | 1st BC              | Solid shell   |              |                          |               |              |                          |
|           |                     | 22.2          | 22.9         | 10.7                     | 87.5          | 1.29         |                          |
|           |                     | Tetrahedron   | 19.5         | 17.5                     | 10.9         | 86.1         | 1.65         |
| Dressel 2 4 Roman | Solid shell | 14.6  | 41.2  | 8.2  | 110.2 | 0.72  | 24.8  | 12.8  | 110.2 | 0.81  |
|           |                     | Tetrahedron   | 14.2  | 24.8  | 9.2  | 105.0 | 1.19  | 24.8  | 12.0  | 112.6 | 1.14  |
| Rhodes    |                     |               |              |                          |               |              |                          |
| Type I A  | Late 4th to early 3rd BC | Solid shell | 22.7  | 34.3  | 10.8  | 104.5 | 0.87  | 32.7  | 18.4  | 99.9  | 0.92  |
|           |                     | Tetrahedron   | 19.7  | 23.2  | 11.5  | 100.0 | 1.19  | 20.4  | 16.5  | 93.3  | 1.41  |
| Type I B  | 3rd BC              | Solid shell   | 20.6  | 25.2  | 10.0  | 95.4  | 1.08  | 24.6  | 15.5  | 92.4  | 1.19  |
|           |                     | Tetrahedron   | 23.4  | 22.2  | 11.2  | 87.8  | 1.76  | 23.3  | 14.6  | 88.8  | 1.86  |
| Type I C  | 3rd BC              | Solid shell   | 22.4  | 23.5  | 9.5   | 89.0  | 1.18  | 27.1  | 17.4  | 91.8  | 1.09  |
|           |                     | Tetrahedron   | 19.2  | 19.3  | 10.4  | 83.4  | 1.49  | 16.8  | 13.8  | 83.3  | 1.68  |
| Type I E  | 2nd BC              | Solid shell   | 21.5  | 25.2  | 10.3  | 87.0  | 1.23  | 25.6  | 16.5  | 89.2  | 1.13  |
|           |                     | Tetrahedron   | 20.5  | 17.3  | 10.5  | 90.1  | 1.64  | 18.1  | 13.6  | 84.4  | 1.60  |
| Type I E2 | 2nd BC              | Solid shell   | 20.5  | 23.4  | 9.7   | 94.6  | 1.23  | 30.4  | 18.6  | 98.8  | 0.97  |
|           |                     | Tetrahedron   | 20.5  | 17.9  | 10.3  | 91.5  | 1.58  | 17.5  | 14.1  | 86.3  | 1.43  |

**Note:** The simulated contact areas are listed and maximum and minimum principal stresses in MPa, corresponding to tension and compression. Furthermore, the Mohr-Coulomb criterion at a maximum load of 1000 N is indicated, assuming an ultimate tensile strength of 30 MPa and an ultimate compressive stress of 200 MPa. For the FEM simulation, two different element types were tested solid shell and ten-node tetrahedron, providing comparable results.
to the compressive contact zones of the amphorae as the surface is strained due to the deformation. In the case of the lower amphora, the tensile stress is located several mm above the contact zone, while in the case of the amphora on top several mm below the contact zone. Considerable tensile stresses emerge also at the internal surfaces of both amphorae opposite the contact zone, as the internal concavities are strained. In order to take this into account, modeling with solid shell elements is tested as alternative approach to use of tetrahedral elements. The simulation results, however, indicate no substantial deviation among the two element types. Considerable divergence is indicated only for tensile stress at the external surfaces in some cases. The convergence of the simulation results is tested by refining the mesh size for solid shell elements in the contact zone of a Hellenistic Koan amphora model in a range from 2 to 1 mm (Table 3).

Particularly, the tensile stresses calculated for the external surfaces reach or even exceed the critical stress limit potentially resulting in crack initiation. The calculated compressive stresses are quite high as well but restricted to the immediate contact zone. The damage risk is assessed with the Mohr-Coulomb criterion, based on ultimate tensile strength of 30 MPa and ultimate compressive strength of 200 MPa. In most cases, the criterion indicates a value of 1 and below under a full load of 1000 N indicating critical conditions.

In the case of compression, however, local crushing or maybe spalling of the ceramics has to be considered before total failure when reaching ultimate compressive stress. As mentioned above, local collapse of the ceramic structure can be taken into account with refined simulations basically assuming quasi-elastoplastic deformation and a decreased modulus once a predefined yield stress is reached. For this, a simple bi-linear model for isotropic hardening was applied, with an assumed compressive yield strength of 20 MPa and a tangent modulus of 2000 MPa (Table 4). The modified material properties result in an increase in the contact area of c. 20%–70% and in some cases even more than 100%. While the simulated tensile stresses in the internal surfaces remain essentially at the same level, the tensile stresses at the external surfaces are clearly reduced. The simulated maximum compressive stresses are also clearly reduced due to the increased contact area typically approximately at half. Also, the estimated Mohr-Coulomb criterion indicates that the effectual increase in the contact area through considering elastoplastic deformation reduces stresses in the contact zone.

Comparisons with damages observed in archaeological vessels indicate practical validity of the simulations.
TABLE 5  Simulated stresses in the modeled amphora bodies under a horizontal load of 1000 N applied in incremented steps: Pure elastic deformation with Young’s modulus of 18 GPa is considered

| Type     | Contact area in mm$^2$ | Int. surface | Ext. surface | Max. tensile stress in MPa | Max. compressive stress in MPa | Mohr-Coulomb criterion | Int. surface | Ext. surface | Max. compressive stress in MPa | Mohr-Coulomb criterion |
|----------|------------------------|--------------|--------------|-----------------------------|-------------------------------|------------------------|--------------|--------------|-------------------------------|------------------------|
| Kos      |                        |              |              |                             |                               |                        |              |              |                               |                        |
| Type I   | 4th BC                 | 13.9         | 14.3         | 13.9                        | 115.8                         | 1.58                   | 14.6         | 13.7         | 122.9                         | 1.63                   |
| Type II  | 4th BC                 | 13.4         | 9.2          | 12.8                        | 106.0                         | 1.89                   | 9.4          | 15.2         | 113.9                         | 1.76                   |
| Type I   | Late 3rd BC            | 18.0         | 9.2          | 11.7                        | 75.7                          | 2.13                   | 9.3          | 9.9          | 74.6                          | 2.38                   |
| Type I   | 1st BC                 | 18.0         | 9.3          | 14.9                        | 83.3                          | 1.72                   | 9.3          | 11.0         | 86.0                          | 2.10                   |
| Dressel 2 4 | Roman            | 33.6         | 13.4         | 3.3                         | 69.5                          | 2.24                   | 15.0         | 2.9          | 63.8                          | 2.01                   |
| Rhodes   |                        |              |              |                             |                               |                        |              |              |                               |                        |
| Type IA  | Late 4th to early 3rd BC | 21.9       | 9.3          | 5.4                         | 59.5                          | 3.23                   | 10.1         | 4.1          | 55.4                          | 2.97                   |
| Type IB  | 3rd BC                 | 19.4         | 10.8         | 5.7                         | 89.7                          | 2.23                   | 10.9         | 6.7          | 108.7                         | 1.84                   |
| Type IC  | 3rd BC                 | 21.4         | 10.5         | 6.6                         | 72.5                          | 2.76                   | 10.5         | 4.0          | 78.1                          | 2.56                   |
| Type IE  | 2nd BC                 | 13.7         | 10.5         | 10.1                        | 137.3                         | 1.46                   | 10.8         | 8.5          | 138.4                         | 1.44                   |
| Type IE2 | 2nd BC                 | 18.0         | 11.0         | 10.7                        | 88.7                          | 2.26                   | 11.5         | 8.1          | 94.8                          | 2.11                   |

Note: The simulated contact areas are listed and maximum and minimum principal stresses in MPa, corresponding to tension and compression. Furthermore, the Mohr-Coulomb criterion at a maximum load of 1000 N is indicated, assuming an ultimate tensile strength of 30 MPa and an ultimate compressive stress of 200 MPa.
(Figure 6). Occasionally, kidney-shaped holes are detected in amphora bodies, which are potentially related to crack initiation close to the contact zones indicated in the present simulations. The convex edges of the fractures in the two examples appear to be oriented perpendicularly to the simulated first principal stresses, which are directed parallel to the surface toward the contact zones.\textsuperscript{35} For further investigation of crack propagation, though, the FEM model has to be extended, which will be subject of future investigation. A potential approach could be also to couple the FEM model with peridynamics theory.\textsuperscript{46}

Concerning the chronological development of vessel shapes, comparison of tensile stresses is of particular interest because any crack initiation is expected to be caused primarily by tension in the ceramic body. In the case of the Rhodian amphorae, indeed a reduction in simulated tensile stresses is indicated at least comparing the type IA amphora model of the 4th century BC with later amphora types. Also, the Mohr-Coulomb criterion indicates an improvement and effectual technological development after the 4th century BC. In the case of the Koan amphorae, however, the simulations indicate no apparent improvement of the vessel shapes in terms of tensile stresses at the external surface, only a clear reduction in tensile stresses at the internal surface, which were remarkably high in the earliest amphora model (type I 4th BC). It has to be considered, though, that technological development in this case concerns the increase in the capacity of the amphorae, while their mechanical performance remained at approximately the same level. An exception to this is the only Roman amphora type investigated in the present case study. Probably, due to the above-mentioned small contact area simulated tensile stresses in the external surfaces appear to be considerably high, particularly for the upper amphora, while the capacity is smaller compared with later Hellenistic amphorae. For this, maybe a different arrangement or packing of Roman amphorae has to be considered. The simulated packing indicated for example that there could be potentially a second contact point slightly above the distinct shoulder of the lower amphora, which would eventually spread the load.

4.2 | Horizontal load on body

For the simulation of horizontal loads, ten-node tetrahedral elements and the same mesh sizes are selected for the two amphora parts as in the above discussed case of vertical loads: 10 mm in general and a refined mesh of 3 and 1.5 mm, respectively, in 50 and 15 mm distance from the closest contact point. Accordingly, a horizontal load of 1000 N is applied stepwise to the internal surface of one amphora, while the body of the other is fixed. Simulated stresses in general appear to be lower compared with vertical loads of the same amount (Table 5). Taking into account that horizontal loads were intermittent rather than permanent as in the case of vertical weight loads and assumedly also smaller, at least in normal sea conditions, they can be considered as less critical for crack initiation and propagation. Similarly, as in the case of vertical loads maximum compressive stresses emerge at the external surface in proximity of the contact zones. The maximum tensile stresses are distributed more or less uniformly around the contact zone at the external surface and at the internal surface opposite the contact zone. Under horizontal loads, the tensile stresses at the internal surface are at the same level as the tensile stresses around the contact zone at the external surfaces. Potential failure and crack initiation can be expected at the internal surface as well as at the external surface.

At the same time, the calculated stresses provide an estimation of statistical uncertainties of the present simulations. Under the investigated horizontal load, the geometry of the two amphorae at the contact point is indeed consistent so that stresses are expected to be uniformly distributed and observed deviations are related basically to modeling parameters. Taking this into account, there are no significant differences among the specific amphora types. It can be assumed that, if mechanical performance was considered in the development of vessel shapes after all, the focus was probably on weight loads during packaging rather than horizontal loads emerging due to rough sea.

5 | CONCLUSIONS

The presented simulations of vertical and horizontal loads on amphora bodies demonstrate the effective application of FEM in the investigation of use and functionality of transport amphorae used for shipping commodities, such as wine or oil, in antiquity. Particular focus in the present study is on their packaging in the hold of a cargo ship. They were reportedly placed in several layers and exposed to vertical weight loads as well as horizontal loads due to potential rolling of the ship during voyage. It is assumed that the ceramic surfaces were in direct contact during shipping so that simulated contact points as well as contact zones affect substantially the calculated stresses in the ceramic matrix. The simulated compressive stresses have been calculated to be considerably high, but they induce presumably rather local damages, which can be taken into account by defining elastoplastic deformation once a compressive yield stress is reached. Severe failure of an amphora, on the other hand, is presumably caused by critical tensile stresses. The present simulations indicate that these are particularly high under vertical loads at contact zones on the shoulder or the lower body, respectively, of amphorae in different package layers. In the case of failure, crack initiation can be expected most likely at the external surface in proximity of the contact zone. Damages, occasionally observed in archaeological amphorae, support
this assumption of potential failure. Stresses and failure risk at the external surfaces, though, are reduced by increasing the contact area as it is demonstrated by considering elastoplastic deformation in the present study. Effectual increase in the contact area, though, could have been potentially achieved by padding and wedging as well in antiquity, not affecting, however, tensile stresses at the internal surfaces.

Comparison of the chronological development of amphora types during the Hellenistic Period indicates a slight improvement of mechanical performance in the case of Rhodian amphorae. In the case of Koan amphorae, on the other hand, the mechanical performance under simulated loads remained approximately at the same level, while the capacity of the amphora has been apparently increased over time. Thus, it can be assumed that mechanical performance was indeed a criterion considered in the development and design of transport amphorae, particularly in view of vertical weight loads causing potential damages during packaging and transport in cargo ships.

Concerning specific modeling aspects, the used digital models are still simplified assuming for example constant wall thickness. Improved and more realistic models are planned to be investigated in future based on 3D scans and computer tomography. The study and determination of material properties will be advanced as well, particularly in view of stress and strain development, considering for example the discussed elastoplastic deformation, and in terms of crack propagation in the ceramic matrix. In this way, simulations can be further improved allowing for comprehensive comparison with observed damages. Another aspect worth to be investigated in future concerns dynamic or impact loads potentially caused in rough sea. Nevertheless, the present pilot study provides fundamental insights into the mechanical performance and potential failure of Hellenistic and Roman transport amphorae and allows for basic assessment of vessel design.

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