Magnetic excitations in the helical Rashba superconductor

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Abstract
We investigate the magnetic excitation spectrum in the helical state of a noncentrosymmetric superconductor with inversion symmetry breaking and strong Rashba spin–orbit coupling. For this purpose we derive the general expressions of the dynamical spin response functions under the presence of strong Rashba splitting of conduction bands, superconducting gap and external field which lead to stabilization of Cooper pairs with finite overall momentum in a helical state. The latter is characterized by momentum space regions of paired and unpaired states with different quasiparticle dispersions. The magnetic response is determined by i) excitations within and between both paired and unpaired regions ii) anomalous coherence factors and iii) additional spin matrix elements due to helical Rashba spin texture of bands. We show that as a consequence typical correlated real space and spin space anisotropies appear in the dynamical susceptibility which would be observable as a characteristic fingerprint for a helical superconducting state in inelastic neutron scattering investigations.

Keywords: magnetic, excitations, helical, Rashba, superconductors, FFLO

(Some figures may appear in colour only in the online journal)

1. Introduction

In this work we investigate the signatures of the helical state of noncentrosymmetric superconductors (NCSs) in the magnetic excitation spectrum obtained from inelastic neutron scattering (INS). NCS compounds have broken inversion symmetry and consequently the gap functions in principle are mixtures of singlet and triplet components [1–3] where the protected triplet admixture has its spin structure perfectly adapted to the spin texture of the underlying split Rashba bands [2]. The NCS state was predominantly investigated for heavy-fermion superconductors (SCs) like CeTX$_3$ (T = transition metal, X = Si, Ge), for some like T = Pt, X = Si it exists already under ambient conditions [4, 5] for others like T = Ir, Rh, X = Si applied pressure is required [6]. This mechanism is also possible in 2D layered SCs like SrPtAs where inversion symmetry is broken in each layer although it is preserved overall in the 3D crystal [7, 8].

In the normal state the inversion symmetry breaking leads to an odd Rashba spin orbit coupling that results in two nondegenerate split bands ($\lambda = \pm 1$) with a momentum-locked spin texture characterized by opposite helicities. The corresponding Fermi wave numbers differ by an amount proportional to the size of Rashba spin–orbit coupling. The application of a field shifts the two Fermi spheres perpendicular to field direction and proportional to field strength.

In the superconducting state in an applied field this means that not only are singlet and triplet components mixed but also the Cooper pairs ($\mathbf{k} + \mathbf{q} \sigma, -\mathbf{k} + \mathbf{q} \sigma'$) will acquire a common

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pair momentum $2\mathbf{q}$ proportional to the shift vector of Fermi surfaces and characterized by a gap function $\Delta_{\mathbf{k},\mathbf{q}} \exp(2i\mathbf{q} \cdot \mathbf{r})$. In this picture the orbital pair breaking is assumed to be small. This commonly called ‘helical’ state [9] is therefore of the Fulde–Ferrell (FF) [10] type but has a different composition of the condensation energy due to the effect of Rashba coupling than the original Zeeman-energy dominated FF case. The advantage of the helical state is that the finite pair momentum appears already at moderate fields due to the shifting of Rashba Fermi surface spheres. The modulus of its gap amplitude is, as in the FF state, constant in real space unlike the Larkin–Ovchinnikov (LO) state [11] which has nodal planes.

Some aspects of the helical state including Rashba coupling and Zeeman term have been studied before, concerning mostly critical field curves [12–16]. Microscopically this state is characterized by a segmentation of Fermi surface sheets into paired and unpaired regions determined by the balance of kinetic, Rashba and Zeeman energies. The observation of this central aspect in the helical (and also in the original FF) state requires spectroscopic means. It has been proposed that surface tunneling microscopy (STM)-based quasiparticle interference (QPI) spectroscopy may be used for FF [17] and helical Rashba NCS [18] cases. The STM spectroscopy which can probe surface, edge or end states would also be the preferred technique to investigate the topological aspects [19] of NCS where a spin-triplet, e.g. p-wave admixture due to the inversion symmetry breaking. For such gap functions with primarily triplet character in 2D topological edge states [20] may exist and 1D constrained geometries like wires or vortices may lead to the formation of Majorana zero-modes (MZM) that are located at the ends of these 1D structures [20, 21] which do not appear for s-wave gap functions, providing in principle a way to identify the triplet content. However, this interesting topological aspects of NCS are not in the focus here.

The complementary method of magnetic INS has also been suggested for the FF state [22]. It allows to investigate the signatures of finite momentum Cooper pairs with paired/unpaired segmentation of Fermi surface sheets on the magnetic excitation spectrum, in particular on the collective spin resonance formation within the superconducting gap and regarding the point group symmetry breaking anisotropy due to finite pair momentum. In this work we investigate exclusively the bulk magnetic excitations for the helical Rashba SC case as they may be probed by INS which is only bulk-sensitive. The scattering cross section (dynamical structure function) is proportional to the imaginary part of the dynamical volume susceptibility. This method can therefore not be employed to identify geometrically constrained edge or end states with MZM character. We first give a brief account of the tight binding (TB) Rashba band model in the normal state. Then we discuss the simplest helical superconducting gap model introduced by Kaur et al [9] and its condensation energy which determines the pair momentum $2\mathbf{q}$ and gap size $\Delta_{\mathbf{k},\mathbf{q}}$ as function of field strength. In the main part of this work we derive the expressions for the dynamical magnetic susceptibility tensor which contains the static response as well as the finite frequency magnetic spectrum. We investigate the field and momentum and polarization dependence of the latter which are in principle accessible by INS experiments. Finally we discuss the temperature dependence of the static homogeneous susceptibility in view of the nuclear magnetic resonance (NMR) Knight shift in the helical SC phase. For these purposes developed in sections 4 and 5 of the present work it is necessary to first briefly recapitulate the ingredients of the Rashba band structure (section 2) and the helical SC ground state and associated quasiparticle excitations (section 3). For more details we refer to [18].

2. Normal state Rashba bands and states

Here we introduce the widely used bandstructure model including the Rashba coupling originating from inversion-symmetry breaking [2, 18]. We use the TB form in view of the later calculations of magnetic response functions but occasionally discuss the features of Rashba bands in the convenient parabolic approximation. In the spin representation, the 2D Rashba Hamiltonian in an external field is characterized by the following [9]

$$H_0 = \sum_{\mathbf{k},\mathbf{q}} \Psi_{\mathbf{k}}^\dagger h_{\mathbf{k}} \Psi_{\mathbf{k}}; \quad h_{\mathbf{k}} = \varepsilon_{\mathbf{k}} s_0 + (\alpha g_{\mathbf{k}} + \mathbf{b}) \cdot \mathbf{\sigma}. \quad (1)$$

Here $\Psi_{\mathbf{k}} = (a_{\mathbf{k} \uparrow}, a_{\mathbf{k} \downarrow}^\dagger)$ represent conduction electrons with the TB dispersion $\varepsilon_{\mathbf{k}} = -2(\cos k_x + \cos k_y) - \mu$ with $-\pi \leq k_x, k_y \leq \pi$. Furthermore $\mu > 0$ is the hopping element corresponding to a conduction band half-width $D_c = 4t$ and $\varepsilon_{\mathbf{k}} \equiv \varepsilon_{\mathbf{k}} - \mu_{TB}$. The chemical potential $\mu_{TB}$ falls in the interval $-D_c \leq \mu_{TB} \leq D_c$ and is counted from the band center $\varepsilon_0 = 0$. It is useful to connect this to the 2D parabolic band model for $\mu_{TB} \leq 0$ with $\varepsilon_{\mathbf{k}} = \varepsilon_0 + k^2/2m$. With $\varepsilon_0 = -D_c$ denoting the band bottom and $m = 2/D_c$ the effective mass. The chemical potential counted from the band bottom is then obtained by $\mu_{TB} - \varepsilon_0 > 0$. Furthermore $\mathbf{b} = \mu_B \mathbf{B}$ is the energy scale of the applied magnetic field $\mathbf{B}$.

The Rashba spin–orbit coupling is odd under inversion with $g_{\mathbf{k}} = -g_{-\mathbf{k}}$, explicitly $g_{\mathbf{k}} = (k_x, -k_y, 0)/k_0 = (\sin k_x - \cos k_y, 0)$ in the parabolic model. Here $\theta_k$ is the azimuthal angle of $\mathbf{k}$ measured in relation to the $k_x$-axis. Moreover, $\theta_k = (2m \mu_{TB})^{1/2}$ is the Fermi wave number and $v_F = k_F/m$ is the corresponding velocity. To retain consistency with the TB dispersion we will also employ the TB form $g_{\mathbf{k}}^{TB} = (\sin k_x, -\sin k_y, 0)$ for the Rashba term. Both forms are normalized according to $|g_{\mathbf{k}}^0|^2 = 1$ and $|g_{\mathbf{k}}^{TB}|_{\max} = \sqrt{2}$. Equivalence for $k_x, k_y \ll \pi$ requires that $\alpha_{TB} = \theta_k / \theta_{\mathbf{k}}$. We will discard the indices $\mathbf{T}, \mathbf{P}$ from now on and mostly rely on the context. The diagonalized Hamiltonian of equation (1) reads

$$H_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \mathbf{c}_{\mathbf{k}} \cdot \mathbf{c}_{\mathbf{k}}; \quad \varepsilon_{\mathbf{k}}(\mathbf{b}) = \varepsilon_0 + \lambda |\alpha g_{\mathbf{k}} + \mathbf{b}|. \quad (2)$$

Here $\varepsilon_{\mathbf{k}}(\mathbf{b})$ are Rashba- split and Zeeman- shifted bands (energies counted from $\mu$) with band states corresponding to helicities $\lambda = \pm 1$. For vanishing field the two Rashba bands may be written as

$$\varepsilon_{\mathbf{k}}^{0}(\mathbf{b}) = \varepsilon_0 + \lambda |\alpha g_{\mathbf{k}}| = \frac{1}{2m} (k + \lambda k_0)^2 - \mu, \quad (3)$$
with \( k_0 = \frac{1}{2} \frac{a}{\tau} k_F \); and \( \tilde{\mu} = \mu(1 + \frac{1}{2} \frac{a^2}{\tau^2}) \). These two parabolic dispersions are shifted by an amount \( k_0 \). The two resulting Fermi surfaces have approximate radii \( k_F^2 = k_F - \lambda k_0 = k_F(1 - \frac{1}{2} \frac{a^2}{\mu^2}) \) for moderately small Rashba coupling strength \( |\alpha| < \mu \).

Then their relative deviation \( (k_F^2 - k_F^2) / k_F = |\alpha| / \mu \) is a direct measure for the size of \( \alpha \). We assumed a physically appropriate hierarchy of energy scales described by \( b < |\alpha| < \mu < D_c \).

For finite but small field \( b \parallel \) parallel to the plane the Rashba Fermi surfaces resulting from equation (2) are shifted perpendicular to the field orientation in opposite directions [2, 18] by a shift vector

\[
q_s = \frac{2b}{2\mu} k_F = \frac{m_{1B} B}{k_F} = \frac{b}{v_F},
\]

(4)

In brief, the splitting of Rashba sheets is a direct measure for the Rashba coupling strength \( |\alpha| \) while their shifting perpendicular to \( B \) is a determined only by field strength. We will use \( b = b x \) and \( q_s = q_s y \) for the geometric configuration. Finally we give the unitary transformation from spin states to the helical Rashba eigenstates \( |k\lambda\rangle = c_{k\lambda}^c|0\rangle \) which are created by operator pairings \( \Phi_k^1 = (c_{k+q\lambda}^c c_{k-q\lambda}) (\lambda = \pm) \). It is defined by \([23]\)

\[
\Phi_k^1 = \Psi_k^c S_k;
\]

\[
S_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i e^{i\theta_k} \\ i e^{-i\theta_k} & 1 \end{bmatrix},
\]

(5)

where the angle is obtained from

\[
\theta_k = -\tan^{-1}(g_{kx} x / g_{ky}) = \tan^{-1}(\sin k_x / \sin k_y) \rightarrow \tan^{-1}(k_x / k_y).
\]

Here the second and last expressions correspond to TB and parabolic bands, respectively. In the latter \( \theta_k \) is simply the azimuthal angle of \( \mathbf{k} \). The shifting of Rashba FS sheets by \( q_s \) out of the BZ center naturally favors the formation of Cooper pairs with finite pair momentum \( 2q \) [12, 14, 18, 24] and one may expect a monotonic correlation. The true size of \( q \) can only be determined by minimization of the ground state energy as it is described in the next section.

3. Helical SC: segmentation into paired and unpaired regions

In the present work we do not investigate possible mechanisms of superconductivity in NCS compounds, for an extended review see [2]. In the compounds with Rashba spin–orbit coupling phonons [25] and alternatively spin-fluctuations [3, 26, 27] may provide a mechanism for Cooper pair formation.

Here we investiagate the possibility of a superconducting state with common momentum \( 2q \) of Cooper pairs due to the pairing effect of the external field in combination with Rashba spin–orbit coupling and how this will influence the dynamical magnetic response. The actual \( q = q\hat{y} \) has to be evaluated by the minimization of the condensation energy in the helical SC phase given below, one may expect it to be correlated with the Rashba shift vector \( \mathbf{q}_s = q_s \hat{y} \) in equation (4) [16, 24]. More general inhomogeneous SC phases with several \( q_s \), e.g. the ‘stripe phase’ [9] with the pair \( \{ q, -q \} \) will not be discussed here. Furthermore we restrict the choice of the helical SC gap functions \( \Delta_{q\lambda}(\mathbf{k}) \) to the minimal model proposed by Kaur et al [9] whose essentials we briefly mention here: it assumes that in the limit of \( \alpha \rightarrow 0 \) the SC gap is of the singlet type (e.g. s-,d-wave) characterized by an orbital basis function \( f_r(\mathbf{k}) \) belonging to the irreducible representation of the tetragonal symmetry group \( D_{4h} \). Turning on a finite \( \alpha \) leads to an additional \( \mathbf{k} \)-dependence of effective interaction and gap function due to the admixture of triplet components enforced by the helical spin structure. For small fields \( b \ll |\alpha| \) this \( \mathbf{k} \)-dependence may be eliminated by a phase transformation \( \Delta_{q\lambda}^9 \rightarrow \pm \exp(\pm i q\theta_k) \Delta_{q\lambda}^\pm \) (for simplicity we keep the same symbol for the transformed gap function). In this helicity representation the individual singlet and triplet components are hidden and one deals with an effective two (Rashba)-band SC. The associated and correspondingly transformed effective two-band interaction [9] in helicity representation is then given by

\[
\hat{V} = -\frac{V_{f\lambda}(\mathbf{k}k^\prime)}{2}(\sigma_0 - \sigma_\lambda);
\]

\[
V_{f\lambda}(\mathbf{k}k^\prime) = V_0 f_r(\mathbf{k}) f_r(\mathbf{k}^\prime).
\]

(6)

Inserting this two-band pairing interaction into the gap equation leads to the condition \( \Delta_{q\lambda}^\pm = -\Delta_{q\lambda}^\pm [9, 18] \). The opposite sign of the two gaps is enforced by the opposite spin texture on the two Rashba bands. For the later numerical discussion we will consider the isotropic s-wave case \( f_r(\mathbf{k}) = 1 \) and the d-wave case \( f_r(\mathbf{k}) = (\cos k_x - \cos k_y) \) corresponding to \( \Delta_{q\lambda}^\pm = \Delta_{q\lambda}^\pm f_r(\mathbf{k}) \).

Introducing the Nambu spinors \( \psi_{q\lambda}^\dagger = (\psi_{k+q\lambda}^c, \psi_{k-q\lambda}^c) \) the total BCS Hamiltonian is given by

\[
\hat{H}_{BCS} = \frac{1}{2} \sum_{k\lambda} \psi_{q\lambda}^\dagger \hat{h}_{q\lambda} \psi_{q\lambda} + \frac{1}{2} \sum_{k\lambda} \varepsilon_{k+q\lambda} + \frac{1}{2} \sum_{k\lambda} \Delta_{q\lambda}^2 / V_0
\]

(7)

with the Hamilton matrix represented by

\[
\hat{h}_{q\lambda} = \varepsilon_{q\lambda}^d + \frac{\Delta_{q\lambda}^\dagger}{\Delta_{q\lambda}} - \frac{\Delta_{q\lambda}}{\Delta_{q\lambda}^\dagger}.
\]

(8)

Considering the symmetries \( \varepsilon_k = \varepsilon_k \) and \( \mathbf{g}_k = -\mathbf{g}_k \) the diagonal matrix elements may be written as

\[
\varepsilon_{k+q\lambda}(\mathbf{b}) = \varepsilon_k + \lambda |\mathbf{g}_k + \mathbf{b}|, \]

\[
\varepsilon_{k-q\lambda}(\mathbf{b}) = \varepsilon_k - \lambda |\mathbf{g}_k - \mathbf{b}|.
\]

(9)

It is convenient to introduce symmetric (s) and antisymmetric (a) expressions according to

\[
\varepsilon_{k+q\lambda}^{sd} = \frac{1}{2}(\varepsilon_{k+q\lambda} + \varepsilon_{k+q\lambda}^a)
\]

(10)

They have even/odd symmetry \( \varepsilon_{k+q\lambda}^s = \varepsilon_{k+q\lambda}^a \) and \( \varepsilon_{k-q\lambda}^s = -\varepsilon_{k-q\lambda}^a \) with respect to inversion. The latter enforces the property \( \sum_{k\lambda} \varepsilon_{k+q\lambda}^a = 0 \) where the sum over \( k \) covers both paired and unpaired regions explained below.

The second term in equation (8) can be diagonalized by a Bogoliubov transformation [18, 28] leading to quasiparticle
states created by $\alpha_{k\lambda}, \beta_{k\lambda}$ and a corresponding quasiparticle Hamiltonian
\[ H_{BCS} = E_G + \frac{1}{2} \sum_{k\lambda} \left( |E_{k\lambda}^\dagger|^2 |\alpha_{k\lambda}|^2 + |E_{k\lambda}^\dagger|^2 |\beta_{k\lambda}|^2 \right). \] (11)

To visualize the quasiparticle sheets we will use the spectral function
\[ \tilde{A}_{k\lambda}^\pm (\omega > 0) = \delta (\omega - |E_{k\lambda}^\pm|) + \delta (\omega - |E_{k\lambda}^\mp|) \] (12)
in subsequent figures. Here the (positive) quasiparticle energies $|E_{k\lambda}^{\pm}|$ are given as $(\tau = \pm, \bar{\tau} = \mp)$:
\[ E_{k\lambda}^\pm = E_{k\lambda}^\pm + \tau \varepsilon_{k\lambda}^\pm = E_{k\lambda}^\mp, \]
\[ E_{k\lambda}^\mp = |\varepsilon_{k\lambda}^\pm|^2 + \Delta_{k\lambda}^\mp = E_{k\lambda}^\pm. \] (13)

If both $E_{k\lambda}^\pm > 0$ or $E_{k\lambda}^\mp < 0$ the pair state is stable and only unpaired quasiparticle states exist for the wave vectors $k + q, -k + q$. Although for such wave vectors $|E_{k\lambda}^{\pm}|$ are normal quasiparticle excitations their energy nevertheless depends on the gap size $\Delta_{k\lambda}$ determined only by the paired FS sections. This is due to the fact that in the coherent helical SC ground state the unpaired electrons and holes also experience the pairing potential supported by the pairedv electrons, although they do not contribute to it.

The constant $E_c = (H_{BCS})$ appearing in equation (11) is equal to the total ground state energy $E_G(q, \Delta_{q\pm})$ of the helical state. Subtracting the normal state ground state energy $E_0 = (1/2) \sum_{k\lambda} (\varepsilon_{k\lambda}^0 - |\varepsilon_{k\lambda}^0|)$ we obtain the superconducting condensation energy $E_c = E_G - E_0$ as [18]
\[ E_c(q, \Delta_{q\pm}) = \frac{1}{2} \sum_{\lambda} \left\{ \sum_{k} N \left( \frac{\Delta_{k\lambda}^\mp}{V_0} \right)^2 \right\} - \sum_{k\lambda} \left( |E_{k\lambda} - |\varepsilon_{k\lambda}^0| | \right) + (\varepsilon_{k\lambda}^0 - |\varepsilon_{k\lambda}^0|) + |E_{k\lambda}^\mp| \theta(-E_{k\lambda}^\mp) \]
\[ + \sum_{k\lambda} (\varepsilon_{k\lambda}^0 - |\varepsilon_{k\lambda}^0|) \} \right], \] (14)

where $\Delta_{k\lambda}^\pm = \Delta_{q\pm} f_{\pm}(k)$. In both s.d-wave cases the normalization is $(1/N) \sum_{k} f_{\pm}(k)^2 = 1$ in the first k-sum of the above equation so that this term is equal to $N \Delta_{k\lambda}^\mp^2 / V_0$. Because of the separation of equation (10) the odd $\varepsilon_{k\lambda}^\pm$ Rashba energies enter only in the last term of equation (15) but not under the square root. The above energy functional must be minimized with respect to $q$ and $\Delta_{q\pm}$ for a given size of the Rashba coupling $\alpha$ and as function of field $b$. Possible ground states are the helical SC state ($q \neq 0, |\Delta_{q\lambda}| > 0$), the BCS state ($q = 0, |\Delta_{q\lambda}| > 0$) or the unpolarized normal ($b = 0, q = 0, \Delta_{q\lambda} = 0$) states. We note again that we are restricted to the small field range $b \ll |\alpha|$ due to the assumption of a field-independent spin texture only determined by the Rashba term.

The minimization problem is much simplified by the equal gap magnitude $|\Delta_{q\pm}| = \Delta_q$ in the model of equation (6). Strictly speaking this holds only when $q = 0$ but this minimization constraint will also be kept for the helical SC case. The effective interaction strength $V_0$ in equations (6) and (14) is connected to the BCS gap amplitude $\Delta_0$ by the gap equation ($b = 0$):
\[ \frac{1}{V_0} = \frac{1}{2N} \sum_{k\lambda} f_{\pm}(k)^2 \] (15)

Here the BCS zero-field quasiparticle energy is simply $E_{k\pm} = |\varepsilon_{k\lambda}^\pm + \Delta_0^\pm|$. Minimization of $E_c(q, \Delta_q)$ with respect to $\Delta_q$ and $q$ determines the true gap $\Delta_q(b, \alpha)$ and wave vector $q(b, \alpha)$ that characterize the helical SC state. One must keep in mind, however, that the model defined in equation (6) is only valid in the low field limit $b/|\alpha| < 1$. An example of the resulting $\Delta_q(b, \alpha)$ dependence for small fields and fixed $\alpha$ is shown in figures 1(a) and 1(d) for the two gap symmetries. These curves depend considerably on the chemical potential $\mu_F$ and Rashba coupling $\alpha$; this is because we had to choose a gap amplitude $\Delta_0$ which is not negligible compared to $\mu_F, \alpha$. When a much smaller $\Delta_0$ is used the numerical accuracy is no longer sufficient to resolve the details of the frequency spectrum of the magnetic response function.

4. The dynamical magnetic response function for the helical Rashba superconducting state

Now we come to the main objective of this work. The calculation of the dynamical magnetic response function of a helical Rashba SC is considerably more involved than in the simple BCS state [29, 30] or even the centrosymmetric FF SC [22, 31] due to the complicated spin textures of Rashba bands which is encoded in the unitary transformation matrix to helical states given in equation (5). Here we give the details of its derivation. We stress that we only included the magnetic field via the Zeeman term in equation (2) and do not include the orbital part leading to vortex formation. This would lead to inhomogeneous state with spatially varying gap amplitudes and cannot be treated using the Nambu-Green’s functions approach employed here. This technical issue has already been discussed in [22]. For the noninteracting helical Rashba quasiparticles the dynamical magnetic susceptibility is obtained from the bubble diagram without vertex corrections. Because of the helical spin structure in principle all cartesian elements $(\alpha, \beta = x, y, z)$ of the susceptibilities may be nonzero and different. They are defined by

\[ \chi_{\alpha\beta}(\mathbf{q}, \omega) = \frac{1}{N} \int_0^\beta d\bar{\tau} e^{i\omega\bar{\tau}} \langle T_{\bar{\tau}} S_{\alpha}(\mathbf{q}, \bar{\tau}) S_{\beta}^\dagger(\mathbf{q}, 0) \rangle, \] (16)

where $\bar{\beta} = 1/kT$ and the spin operators are given in spin $(\alpha_{\kappa\sigma})$ and $(\alpha_{\kappa\lambda})$ helical bases as

\[ S_{\alpha}(\mathbf{q}) = \frac{1}{2} \sum_{k \sigma \kappa \sigma'} \alpha_{k \sigma \kappa}^\dagger \sigma_{\alpha} \sigma_{\alpha'} \alpha_{k \sigma'} \]
\[ = \frac{1}{2} \sum_{k \lambda \lambda'} \hat{c}_{k \lambda}^\dagger \sigma_{\alpha} \sigma_{\alpha'} \alpha_{k \lambda} \] (17)

Here the latter presentation has to be chosen as it forms the eigenbasis of the Rashba Hamiltonian. Therefore the effective spin operators $\frac{1}{2} \sigma_{\alpha} \sigma_{\alpha'} \alpha_{k \lambda}$ in this basis are now
momentum dependent and they are obtained by the transformation of equation (5) according to
\[
\hat{\sigma}_{\lambda}(k',k) = \sum_{\sigma} S_{\sigma,\lambda}^{\alpha}(k') \sigma_{\alpha,\sigma} S_{\sigma,\lambda}(k).
\]

(18)

The matrices (with \(\lambda'\)) indices have the conjugation property \(\sigma^\dagger (k', k)^\dagger = \sigma^{\dagger \lambda}(k, k')\) and obey the commutation rules
\[
[\hat{\sigma}^\dagger (k', k), \hat{\sigma}^\dagger (k', k')] = \delta_{\alpha\beta\gamma\delta} (\hat{\sigma}^\dagger (k') + \hat{\sigma}^\dagger (k))
\]
where \(\delta_{\alpha\beta\gamma\delta}\) is the fully antisymmetric tensor and \(\hat{\sigma}^\dagger (k) \equiv \hat{\sigma}^{\dagger \lambda}(k, k)\) etc. The cartesian spin expectation values in each Rashba state \(|k\lambda\rangle\) are given by \(\langle \hat{\sigma}^\dagger (k) | = (-\lambda \sin \theta_k, \lambda \cos \theta_k, 0)\). Therefore the spins of Rashba states are perpendicular to the momentum direction \(k = (\cos \theta_k, \sin \theta_k, 0)\), i.e. \(\hat{\sigma}^\dagger (k) \times k = \lambda z\). Furthermore they are opposite on the two Rashba bands \(\lambda = \pm\).

Using the helical eigenbasis and its corresponding quasiparticle Green’s functions and effective spin operators the dynamical susceptibility may now be written as
\[
\chi^{\alpha}_{\alpha,\beta}(q, \tau) = \frac{-T}{4N} \sum_{k,\lambda,\lambda'} \hat{\sigma}_{\lambda}^\dagger (k', k) \hat{\sigma}_{\lambda'}^\dagger (k', k') \times \sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)],
\]
where \(\omega_n = \omega_{n'} + \imath \tau\) with \(n' = n + m\). Here \(\tau\) is the Nambu index in particle-hole space and \(\text{Tr}_\tau\) denotes the corresponding trace of the product of Nambu Green’s function matrices given in equation (20). The latter are obtained from equation (8) as
\[
\hat{G}_q (k, \imath \omega_n) = (\imath \omega_n - \hat{E}_{qk\lambda}^{-1})^{-1} \times \frac{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}.
\]

(19)

Using the definitions
\[
M^{\alpha}_{\lambda\lambda'}(k', k) = \hat{\sigma}_{\lambda}^\dagger (k', k) \hat{\sigma}_{\lambda'}^{\dagger \lambda}(k, k'); \\
\chi_{\lambda\lambda'}(k'k'; \imath \omega_n) = \frac{-T}{4N} \sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)],
\]
the response function may be written as a product of helical state spin matrix elements and a dynamical kernel, respectively, according to \((\imath \omega_n \rightarrow \omega + \imath \eta)\)
\[
\chi^{\alpha}_{\alpha,\beta}(q, \omega) = \frac{1}{N} \sum_{\lambda,\lambda'} \sum_{k} M^{\alpha}_{\lambda\lambda'}(k', k) \chi_{\lambda\lambda'}(k'k'; \omega).
\]

(22)

Performing the sum over the Matsubara frequencies \(\omega_n\) and analytically continuing to the real axis according to \(\imath \omega_n \rightarrow \omega + \imath \eta\) a lengthy calculation leads to the final result for the kernel in the dynamical susceptibility of equation (22)
\[
\chi_{\lambda\lambda'}(k'k'; \omega) = \frac{1}{2} \hat{C}^\dagger_{\lambda\lambda'}(k', k') \times \frac{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}.
\]

(24)

where \(\hat{f}(E) = (\exp(E/T) + 1)^{-1}\) is the Fermi function. We may also obtain a different presentation of the last two terms by using
\[
\chi^{\alpha}_{\alpha,\beta}(q, \omega) = \frac{1}{2} \hat{C}^\dagger_{\lambda\lambda'}(k', k') \times \frac{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}{\sum \text{Tr}_\tau [\hat{G}_q (k, \imath \omega_n) \hat{G}_q (k', \imath \omega'_n)]}.
\]

(25)

Furthermore we have to compute the matrix elements in equation (21) for the evaluation of the susceptibility of equation (22). It is useful to note that they satisfy Hermitian symmetry which derive from the fact that the effective spin operators in helical representation are also Hermitian, i.e. full
\[
\hat{\sigma}_{\lambda\lambda'}^\dagger (k', k) = \hat{\sigma}_{\lambda'\lambda}^\dagger (k', k).
\]

(26)

so that cartesian diagonal elements are real symmetric. Using the explicit form given by equations (18) and (5) we derive the latter \((\lambda'\lambda)\) matrices as
\[
\begin{align*}
M_{\lambda\lambda'}^{\alpha\beta}(k, k') &= M_{\lambda\lambda'}^{\alpha\beta}(k, k'); \\
M_{\lambda\lambda'}^{\alpha\beta}(k, k') &= M_{\lambda\lambda'}^{\alpha\beta}(k, k') = |\hat{\sigma}_{\lambda\lambda'}^\dagger (k')|^2,
\end{align*}
\]

(27)

where \(xx,yy\) elements correspond to upper or lower sign, respectively. Furthermore the diagonal elements of \(M_{\lambda\lambda'}^{\alpha\beta}\) matrices describe intraband \((\lambda' = \lambda)\) and the nondiagonal ones interband \((\lambda' \neq \lambda)\) dipolar transitions between the Rashba bands. The matrix elements for the nondiagonal cartesian indices are given in appendix A. The complete dynamical response functions may now be obtained from equation (22) using equations (24), (25) and (27) and for the nondiagonal case equation (A.1) as input.

The above generalized helical expressions reduce to the wellknown BCS results for the magnetic response [29, 30, 32,
33] in the BCS limit \( b, q = 0 \) which are given in appendix B for comparison. If we restrict to the case where \( \mathbf{k}, \mathbf{k}' \) lie both in the segment with paired states (e.g. \( E_{kq\lambda} > 0, E_{k'q'\lambda'} > 0 \)) then the terms in equation (24) may be consecutively interpreted as: quasiparticle scattering (first two terms), pair annihilation (third) and pair creation (fourth) terms. For general \( \mathbf{k}, \mathbf{k}' \) one has to consider processes involving quasiparticles from the paired (p) as well as the unpaired (u) Fermi surface segments. To simplify matters in this general case we consider the zero temperature limit when the Fermi function may be expressed by the step function according to \( f(E) = 1 - \theta(E) = \theta(-E) \). Then we obtain

\[
\chi_{0q}(\mathbf{q}, \omega) = \frac{1}{N} \sum_{k, \lambda, \lambda'} M_{\lambda, \lambda'}^{\alpha\beta}(\mathbf{k}, \mathbf{k}') \times \left\{ \frac{1}{2} \mathcal{C}_{\alpha\beta}(\mathbf{k} \mathbf{k}' \lambda \lambda') \left[ \frac{\theta(E_{kq\lambda}) - \theta(E_{k'q'\lambda'})}{\omega - (E_{k'q'\lambda'} - E_{kq\lambda} + i\eta)} + \frac{\theta(-E_{k'q'\lambda'}) - \theta(-E_{kq\lambda})}{\omega - (E_{k'q'\lambda'} + E_{kq\lambda} + i\eta)} \right] \right\}. \tag{28}
\]

If we look at the numerators of the four terms in this equations we realize that the first two correspond to quasiparticle scattering processes \( \mathbf{k} \leftrightarrow \mathbf{k}' \) from paired (p) to unpaired (u) FS segments and vice versa (p-u, u-p) whereas the third and fourth term are quasiparticle annihilation and creation respectively, containing only processes between the paired (p-p) or unpaired (u-u) segments. The dynamical structure function for low temperature (without the Bose factor) which is proportional to the INS cross section [34] is then obtained as

\[
S(\mathbf{Q}, \omega) = \sum_{\alpha\beta} (\delta_{\alpha\beta} - \hat{Q}_\alpha \hat{Q}_\beta) \frac{1}{\pi} \text{Im} \chi_{0q}^{\alpha\beta}(\mathbf{Q}, \omega), \tag{29}
\]

where \( \hat{Q} = \mathbf{k}' - \mathbf{k} = \mathbf{q} + \mathbf{K} \) is the total momentum transfer with \( \mathbf{K} \) denoting a reciprocal lattice vector and \( \mathbf{Q} = \mathbf{Q}/|\mathbf{Q}| \) denoting the unit vector or direction of total momentum transfer. The prefactor projects out only scattering processes where the magnetic moment is perpendicular to \( \mathbf{Q} \). By choosing various appropriate values of the latter the individual susceptibility components, in particular the diagonal ones (\( \alpha = \beta \)) can be accessed by INS on which we will focus in the discussion of figures 2-4 in section 5. Here we already mention the general arrangement of results presented in these figures: the spectrum \( \text{Im} \chi_{0q}^{\alpha\alpha}(\mathbf{q}, \omega) \) for all \( \mathbf{q} \in \text{BZ} \) with a typical \( \omega < 2 \Delta_0 \) is presented in figure 2. For d-wave case we show both BCS and helical phase (first

**Figure 1.** Field dependence of pair momentum \( q \) and gap amplitude \( \Delta_q \) for small fields \( b \ll \Delta_0 \) (a) and (d). Spectral functions i.e. quasiparticle sheets \( |E_{kq\lambda}| = \omega \) in BCS case \( q = 0 \) (b) and (e) for \( \omega = 0.06 \Delta_0 < \Delta_0 = 0.2 \). In (b) quasiparticle sheets are absent while they appear around nodal directions in (e). In the helical case (c) and (f) \( b = 0.9 \Delta_0 \) extended sheets appear for both gap symmetries due to depairing centered around \( k_z \) direction perpendicular to \( b = B \). The blue/red sheets correspond to \( \tau = +/\cdot \) and outer/inner sheets to \( \lambda = +/\cdot \). In all figures the designations ‘s,d-wave’ refer to the gap form factors \( f_\tau(k) \) of the inversion symmetric limit where \( \alpha = 0 \) (section 3).
Figure 2. Brillouin-zone cuts of spectral functions (a), (e) and (i) and corresponding dynamical magnetic structure function $\sim \text{Im} \chi_{\alpha\alpha}^{00}(\tilde{q}, \omega)$ for $x, y, z$ polarization in the BCS $d$-wave (first row, b–d), helical $d$-wave (second row, f–h) and helical $s$-wave (third row, j–l) cases for $\omega = 0.12 < \Delta_0 = 0.2\tau$. The BCS- $s$-wave case is not shown as it has no spectral features for $\omega$ below threshold $2\Delta_0$ (see also figures 3(a), (d) and 4(a)). In the BCS cases (b)–(d) the $xx$ and $yy$ response is related by a rotation $R_{\pi/2}$ around the $z$ axis while the $zz$ response is by itself fourfold symmetric (equation (34)). These symmetries are lost in the helical phase (second and third row) for both gap models due to the distinguished common pair momentum oriented along $\tilde{q}_y$.

Figure 3. Axis cuts along (10) and (01) of $xx$, $yy$ dynamical magnetic structure functions (imaginary part) and real part of response function for $s$-wave case at $\omega = 0.12 < \Delta_0 = 0.2\tau$. In the BCS case ($q = 0$) the $xx(10)$ and $yy(01)$ pairs (a) and likewise $yy(10)$ and $xx(01)$ pairs (d) are equivalent due to $R_{\pi/2}$ rotational symmetry. In the helical phase this symmetry is lost due to distinguished $\tilde{q}_y$ direction of common pair momentum which introduces combined momentum space and spin space anisotropy. This is obvious from comparison of (a) with (b), (c) and (d) with (e), (f) (see also figure 2).

and second row, respectively); for $s$-wave case only helical one (third row) is shown because in BCS case the spectrum has no features below $2\Delta_0$. Alternatively in figure 3 we show (only for BCS and helical $s$-wave case) the BZ cuts for constant $\omega$ of the spectrum $\text{Im} \chi_{\alpha\alpha}^{00}(\tilde{q}, \omega)$ for in-plane polarization ($\alpha = x, y$) and in addition the corresponding real part $\text{Re} \chi_{\alpha\alpha}^{00}(\tilde{q}, \omega)$. Furthermore a third complementary way presenting the dispersive behavior of $\text{Im} \chi_{\alpha\alpha}^{00}(\tilde{q}, \omega), \alpha = x, y$ in the $\tilde{q}, \omega$ plane for chosen momentum direction is shown in figure 4.
Finally we may also consider the special case of the static staggered susceptibility components by setting $\omega = 0$ in equation (24). After some rearrangements we obtain ($\tilde{\tau} = -\tau$):

$$
\chi^{\alpha\beta}_{0q}(\tilde{q}) = \frac{1}{4N} \sum_{k\lambda \lambda', \tau} M^{\alpha\beta}_{\lambda'\lambda}(k; k') \left\{ \hat{C}^{\alpha}_{\lambda}(k\lambda'\lambda') \times \frac{\tanh \frac{\tilde{q}}{2} E^x_{k'q'\lambda'} - \tanh \frac{\tilde{q}}{2} E^x_{kq'\lambda'}}{E^x_{k'q'\lambda'} - E^x_{kq'\lambda'}} + \hat{C}^{\alpha}_{\lambda}(k\lambda'\lambda') \frac{\tanh \frac{\tilde{q}}{2} E^y_{k'q'\lambda'} + \tanh \frac{\tilde{q}}{2} E^y_{kq'\lambda'}}{E^y_{k'q'\lambda'} + E^y_{kq'\lambda'}} \right\}.
$$

(30)

We may further specify to $\tilde{q} = 0$ which is the homogeneous spin susceptibility. In this case the nondiagonal ($\lambda \neq \lambda'$) contributions are interband vanVleck terms with a large energy denominator whose modulus is $2|\epsilon_{k'}| \gg 2\Delta_0$. Therefore they may be neglected compared to intraband terms ($\lambda' = \lambda$). Then using $\hat{C}^{\alpha}_{\lambda}(k\lambda\lambda) = 1$ and $\hat{C}^{\alpha}_{\lambda}(k\lambda\lambda') = 0$ we arrive at

$$
\chi^{\alpha\beta}_{0q}(0) = \frac{1}{2N} \sum_{k\lambda \lambda', \tau} M^{\alpha\beta}_{\lambda'\lambda}(k) - \frac{\partial f^\lambda}{\partial E^x_{kq'\lambda'}} \left( \frac{1}{\cosh \frac{\tilde{q}}{2} E^x_{kq'\lambda'}} \right).
$$

(31)

Where the diagonal helical matrix elements now simplify to

$$
M^{xx}(k\lambda) = \frac{1}{2} (1 - \cos \theta_k); \quad M^{yy}(k\lambda) = \frac{1}{2} (1 + \cos \theta_k);
$$

$$
M^{zz}(k\lambda) = 0.
$$

(32)

In the parabolic band approximation (for $\mu \ll D_k$) with a 2D DOS $N^x_k = m^* k^2_b / 2\pi$ and effective mass $m^* = 2/D_k$ and Fermi vector $k^2_b = k^2_F - \lambda k_0$, $k^2_F = (2m^* \mu)^2$ the homogeneous susceptibility may approximately be written as

$$
\chi^{\alpha\beta}_{0q}(0, T) = \sum_{k\lambda, \tau} N^x_k \int \frac{d\theta_k}{2\pi} M^{\alpha\beta}_{\lambda'\lambda}(\theta_k, \lambda) Y_{q\lambda}(\theta_k, T);
$$

$$
Y_{q\lambda}(\theta_k, T) = \frac{1}{4\pi} \int \frac{dE_{\lambda}}{\cosh \frac{\tilde{q}}{2} E^x_{kq'\lambda'}}.
$$

(33)

where $Y_{q\lambda}(\theta_k, T)$ is a generalized angular resolved Yosida function [35]. The static homogeneous susceptibility $\chi^{\alpha\beta}_{0q}(q)$ describes the temperature dependence of the NMR Knight shift [22, 36] of the singlet SC (for $\alpha = 0$) in the helical phase. Because of equation (32) $xx$ and $yy$ components are equivalent and the $zz$ component vanishes. For plotting the temperature dependence of the intra-band contribution we use a phenomenological temperature dependence of the helical gap $\Delta_0(t)$ given by $\Delta_0(t) = \Delta_0 \tan \left[ 1.74 \sqrt{1 - \frac{t}{T_c}} \right]$ where $t = T/T_c(B)$ is the reduced temperature with $T_c(B)$ denoting the field-dependent SC transition temperature. The comparison of $\chi^{\alpha\beta}_{0q}(0, T)$ in the BCS ($q = 0$) and helical ($q \neq 0$) case in the interval $t \in [0, 1]$ is shown in figure 5.
5. Discussion of numerical results: the magnetic spectral functions

Here we discuss typical numerical results for the magnetic spectrum that may be obtained from the theory developed in the previous sections. In particular we focus on the resulting various momentum-space and cartesian spin space anisotropies of the response function with respect to the direction of the field \( \mathbf{b} = b \mathbf{k} \) and the corresponding overall pair momentum direction \( \mathbf{q} = q \mathbf{y} \). In this discussion we have to restrict to results for the small field range \( b \ll \alpha \) where the assumption of field-independent spin texture of Rashba states is still acceptable. This is the basis for the simplified gap models [9] employed here. For the feasibility of numerical computations we had to use a sizable gap amplitude of \( \Delta_0/t = 0.2 \). We use the s- and d-wave gap models. We stress that this designation refers to the limiting case of \( \alpha = 0 \) as mentioned in section 3. In the majority of results presented here we use a Rashba parameter value \( \alpha = 0.6t \) and field strength \( b = 0.9 \Delta_0 \) (i.e. \( b/\alpha = 0.3 \)). Furthermore the chemical potential is set to \( \mu = \mu_{TB} = -2.8t \) (or \( \mu_{TB} = 1.2t \)) to achieve quasi-circular Rashba Fermi surfaces. For \( b = 0.9 \Delta_0 \), we found \( (q/\pi = 0.011; \Delta_q = 0.75 \Delta_0) \) and \( (q/\pi = 0.015; \Delta_q = 0.85 \Delta_0) \) for s-wave and d-wave, respectively.

In figure 1 we show the field dependence of \( q \Delta_q \) in the small field region \((a,b)\) and the associated change in the quasiparticle sheets represented by the spectral function for both gap symmetries. In the BCS case the fully gapped s-wave state has none while in the Rashba case the sheets for both Rashba bands (inner and outer circle) appear first around the nodal directions. In the helical phase with finite overall parallel momentum \( q \) large quasiparticle sheets exist around the depaired momentum space regions perpendicular to the field direction. It is obvious that the fourfold symmetry of the BCS state is broken by the distinguished \( \mathbf{q} \)-direction and this should be visible in the dynamical magnetic structure function as a momentum-space anisotropy of the response. Because in the Rashba state with dominating \( \alpha \) spin and momentum directions are locked this should also be transferred to a spin-space anisotropy, expressed by non-equivalence of \( \chi_{\alpha \mathbf{q}}(\mathbf{q}, \omega) \) for the cartesian directions \( \alpha = x, y, z \). While the former effect is already observed in the centrosymmetric FF state [22], the latter is characteristic for the helical Rashba SC due to the spin-momentum locking.

This is nicely illustrated by the constant-\( \omega \) (= 0.12t = 0.6\( \Delta_0 \)) cut of the spectrum (imaginary part of the dynamical susceptibility) in the panels of figure 2. It shows the BCS and helical d-wave case in the top and center row and the helical s-wave case in the bottom row (we do not show the BCS s-wave case in this figure as there is no intensity at finite \( \omega < 2 \Delta_0 \) due to the full gapping (appendix B), this can indeed be seen in the first row (blue lines) of figure 3 and in figure 4(a)). The spectral functions are depicted on the left column and the \( xx, yy, zz \) dynamical magnetic response in the three consecutive columns. We notice that in the BCS case the combined spin/momentum space rotational symmetries

\[
\chi^{\alpha \mathbf{q}}(\mathbf{q}, \omega) = \chi^{\alpha \mathbf{q}}(\mathbf{q}, \omega); \quad \chi^{x \mathbf{q}}(\mathbf{q}, \omega) = \chi^{x \mathbf{q}}(\mathbf{q}, \omega); \quad \chi^{y \mathbf{q}}(\mathbf{q}, \omega) = \chi^{y \mathbf{q}}(\mathbf{q}, \omega),
\]

hold for any \( \mathbf{q} \) in the BZ where \( R_{\mathbf{q}} \) denotes the rotation by \( \mathbf{q} \) around \( \mathbf{q} \)-axis. These symmetries are all violated in the helical phases where the common pair momentum \( \mathbf{q} = q \mathbf{y} \) leads to the anisotropic response depicted in \((f-h)\) and \((j-l)\). This spin/momentum space anisotropy in the \( xx \) and \( q_x q_y \) planes is a fingerprint of the helical SC phase. It should be accessible experimentally by constant-\( \omega \) scans of the INS intensity in the BZ. The various cartesian spin channels for \( \alpha = x, y, z \) may be selected by using a suitable total momentum transfer according to equation (29).

There are two more complementary ways of presenting these anisotropies characteristically appearing in the helical phase.

One possibility is to make cuts through the BZ and plot the dynamical response along that direction. This is shown (only for the s-wave case) in figure 3 for \((10)\) and \((01)\) directions and in-plane \( xx, yy \) polarization components, but now for both real and imaginary (spectrum) parts of the dynamical susceptibility. For BCS limit we have again pairwise equivalence of the \( xx,(10) \) and \( yy,(01) \) response according to equation (34) as evident from the real parts in \((a)\) and \((d)\). The imaginary part for BCS case in \((a)\) and \((d)\), i.e. the spectrum, is featureless below the gap threshold \( \omega < 2 \Delta_0 \), therefore this case was not presented in figure 2. In the helical s-wave phase in figures 3(a), (c) and (e), these directional symmetries are destroyed as is shown by comparing the cuts \((b), (f)\) and \((c), (g)\), which are no longer equivalent as they were in BCS case \((a)\) and \((d)\), respectively.

Another possible presentation of the helical phase anisotropies in the s-wave case is shown in the dispersive plots in the \((\mathbf{q}, \omega)\) plane of figure 4 for \( \mathbf{q} \) oriented along axes parallel \((10)\) and perpendicular \((01)\) to the field direction. In the BCS limit of \((a)\) the dynamical response in \( xx(10) \) \( yy(01) \) is equivalent and vanishing for \( \omega < 2 \Delta_0 \), in agreement with figures 3(a) and (d). For the helical case in the same configurations the dispersive magnetic excitations are clearly present due to low energy quasiparticles in the depaired momentum space sectors but strongly anisotropic for the two different directions of \((b)\) and \((c)\) in agreement with complementary figures 2 and 3.

A further interesting result of this investigation is the temperature dependence of the static spin susceptibility which in principle determines the NMR Knight shift. It is shown in figure 5 in comparison for BCS and helical states. Note that according to section 4 \( xx \) \( yy \) susceptibility are equivalent for \( \mathbf{q} = 0 \) for both BCS and helical state, therefore we plot only the \( xx \) component. In the BCS case the well known exponential and power law behavior of the susceptibility for the \( s- \) and \( d \)-wave cases are observed. The appearance of large sheets of low energy quasiparticles in the helical phase leads to large residual low temperature susceptibility which should be observable. We note that this happens although the helical order parameter does not have nodes in real space as it is true,
e.g. for the LO phase in the case without inversion symmetry breaking [37].

6. Summary and conclusion

This work completes our previous investigations of spectroscopic (QPI, INS) properties of superconducting states with finite momentum pairing [17, 18, 22]. We have derived the general expressions of dynamical magnetic response in NCSs with Rashba spin orbit coupling that may stabilize the helical phase in an applied field. We have shown which type of quasiparticle excitations appear in the response functions and derived the corresponding anomalous coherence factors and matrix elements due to helical spin textures of the Rashba bands. The obtained expressions of the response function generalize the known results for the centrosymmetric BCS case to the two-band Rashba case with spin-momentum locking under the presence of applied fields which are small compared to the Rashba coupling energy.

As a major result we demonstrated that the combined spin/momentum space symmetries of the BCS phase response is broken in the helical phase. It leads to characteristic spin/momentum asymmetries and anisotropies which can in principle be detected by INS experiments. In particular the constant-ω cuts of the magnetic spectrum by scanning through the BZ should give a fingerprint of the helical phase. From the type of observed anisotropies it should be possible to conclude about the direction of the q-pair momentum vector and observe its dependence on applied field direction. A further possibility is to look for the frequency dependence at certain q momentum transfer and whether spin resonance excitations may form as a consequence of quasiparticle interactions similar as has been predicted for the centrosymmetric case [22].

Finally we investigated the generic temperature dependence of the static spin susceptibility important to know for NMR experiments. We demonstrated that in the helical phase a large residual low temperature spin susceptibility remains which signifies the appearance of unpaired states in this phase, although these are not due to real space nodal planes of the order parameter itself. In summary our investigation gives a solid theoretical foundation for magnetic spectroscopy of the helical Rashba superconducting state.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Spin operator matrix elements for the nondiagonal cartesian cases

In a similar manner as in section 4 the matrix elements $M^{\alpha\beta}_{\lambda\lambda'}$ for $\alpha \neq \beta$ may be derived as

$$M^{\alpha\beta}(k',k) = \frac{1}{2} \begin{pmatrix} -\sin(\theta_k + \theta_{k'}) & \sin(\theta_k + \theta_{k'}) \\ \sin(\theta_k + \theta_{k'}) & -\sin(\theta_k + \theta_{k'}) \end{pmatrix} ,$$

$$M^{\alpha\beta}(k',k) = \frac{1}{2} \begin{pmatrix} i(\cos \theta_k - \cos \theta_{k'}) & -i(\cos \theta_k + \cos \theta_{k'}) \\ i(\cos \theta_k + \cos \theta_{k'}) & -i(\cos \theta_k - \cos \theta_{k'}) \end{pmatrix} ,$$

$$M^{\alpha\beta}(k',k) = \frac{1}{2} \begin{pmatrix} i(\sin \theta_k - \sin \theta_{k'}) & -i(\sin \theta_k + \sin \theta_{k'}) \\ i(\sin \theta_k + \sin \theta_{k'}) & -i(\sin \theta_k - \sin \theta_{k'}) \end{pmatrix} .$$

(A.1)

They appear in the expressions for the nondiagonal susceptibility components $\chi^{\alpha\beta}_{0}(\mathbf{q},\omega)$.

Appendix B. Magnetic response function in the BCS limit

The diagonal cartesian susceptibilities ($\alpha = \beta$) of equations (22) and (24) may be rewritten by using the symmetry of matrix elements $M^{\lambda\lambda'}_{\alpha\alpha}(k',k)$ and coherence factors $\hat{C}_{\pm}(k\mathbf{k}'\lambda')$ against interchange of primed and unprimed arguments. Furthermore, since the summation over both is identical, the first two terms of equations (22) and (24) may be contracted into one term. Then in the BCS case ($\hbar = q = 0$) when quasiparticle bands are given by $E_{\mathbf{k}\lambda} = E_{\mathbf{k}\lambda} = (\epsilon_0^\mathbf{k} + \Delta_0^\lambda)^2$ with $\epsilon_0^\mathbf{k}$ defined by equation (3) we obtain the simplified result

$$\chi^{\alpha\alpha}_{0}(\mathbf{q},\omega) = \frac{1}{N} \sum_{\lambda\lambda'} \sum_{\mathbf{k}} M^{\alpha\alpha}_{\lambda\lambda'}(k',k) \begin{pmatrix} \hat{C}_{+}(k\mathbf{k}'\lambda') \\ \frac{1}{\omega - (E_{\mathbf{k}'\lambda'} - E_{\mathbf{k}\lambda}) + i\eta} \end{pmatrix} + \frac{1}{2} \hat{C}_{-}(k\mathbf{k}'\lambda') \left[ \frac{1}{\omega - (E_{\mathbf{k}'\lambda'} + E_{\mathbf{k}\lambda}) + i\eta} \right] .$$

(B.1)

This is a generalization of BCS expressions given in [29, 30, 32, 33] to the magnetic response for the two-band Rashba-BCS SC. The anomalous coherence factors of equation (25) now simplify to

$$\tilde{C}_{\pm}(k\mathbf{k}'\lambda') = \frac{1}{2} \left[ 1 \pm \frac{\epsilon_0^\mathbf{k} + \Delta_0^\lambda + \Delta_0^\lambda'}{E_{\mathbf{k}\lambda} E_{\mathbf{k}'\lambda'}} \right] .$$

(B.2)

In the low temperature limit and for positive frequencies only the last term survives leading to

$$\chi^{\alpha\alpha}_{0}(\mathbf{q},\omega) = -\frac{1}{N} \sum_{\lambda\lambda'} \sum_{\mathbf{k}} M^{\alpha\alpha}_{\lambda\lambda'}(k',k) \frac{i}{\omega - (E_{\mathbf{k}'\lambda'} + E_{\mathbf{k}\lambda}) + i\eta} .$$

(B.3)

The spectrum of the $T = 0$ diagonal BCS response functions is then given by ($\mathbf{q} \in 1$st BZ)

$$\hat{S}_{\alpha\alpha}(\mathbf{q},\omega) = -\frac{i}{\pi} \text{Im} \chi^{\alpha\alpha}_{0}(\mathbf{q},\omega) = \frac{1}{N} \sum_{\lambda\lambda'} \sum_{\mathbf{k}} M^{\alpha\alpha}_{\lambda\lambda'}(k',k) \times \frac{i}{2} \hat{C}_{-}(k\mathbf{k}'\lambda') \delta(\omega - (E_{\mathbf{k}\lambda'} + E_{\mathbf{k}\lambda})),$$

(B.4)
which contains both intra- and interband transitions between the Rashba-split quasiparticle bands. For $\tilde{q} = 0$ i.e. $k = k'$ we obtain

$$
\tilde{S}_{\alpha\alpha}(\omega) = \frac{1}{N} \sum_{\lambda\lambda'} \sum_{k} M_{\lambda\lambda'} \left( k, k' \right) \left[ \frac{1}{2} \tilde{C}_{-}(k\lambda k') \times \delta\left[ \omega - (E_{k\lambda'} + E_{k\lambda}) \right] \right],
$$

(B.5)

The threshold values for this spectrum are different for intra- and interband transitions. For the former ($\lambda' = \lambda$) it is given by $2\Delta_0$ for the latter ($\lambda' \neq \lambda$) by $\omega_0^\pm = |\alpha| k_F^\pm \gg 2\Delta_0$. Therefore the threshold of the total spectrum is $2\Delta_0$.

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