The coupled-channel analysis of the $D$ and $D_s$ mesons

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The shift of the $p$-wave $D_s$ meson mass due to coupling to the $DK$ channel is calculated without fitting parameters using the chiral Lagrangian. As a result the original $Q\bar{q}$ mass 2.490 MeV generically calculated in the relativistic quark models is shifted down to the experimental value 2317 MeV. With the same Lagrangian the shift of the radial excited $1^{-}$ level is much smaller, while the total width $\Gamma > 100$ MeV and the width ratio is in contradiction with the $D^*(2632)$ state observed by SELEX group.

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I. INTRODUCTION

The heavy-light $D$ and $D_s$ mesons have extensively been investigated experimentally in the last 20 years [1]. Recently a lot of attention has been paid to the $0^{+}$ state of the $D_s$ meson. It was found by the BABAR group [2] at the mass $M = 2318 \pm 1.3$ MeV and confirmed by CLEO and Belle [3]. This value of mass is $\sim 40$ MeV below the KD threshold and the width is very small. On the theoretical side this state was the object of intensive study (see Ref. [4] for reviews and references). There exists some disparity between the theoretical predictions of the $D_s^*(0^{+})$ mass in the Relativistic Quark Model (RQM) calculations [5]-[10] and experimental results. Indeed the RQM predictions vary from 2380 MeV [6, 7] to 2487 MeV [10], being however substantially larger than measured experimentally.

Most recently a new $D_s$ state was observed in the SELEX experiment [11] at the mass value $M = 2632$ MeV. It is argued in Ref. [12] that this state may be associated with the radial excited $1^{-}$ level of $D_s$ meson, which is shifted down due to coupling to the $\eta D_s$ and $KD$ channels. As one will see both levels $D_s^*(2317)$ and $D_s^*(2632)$ are connected to thresholds via chiral decays and we shall treat them below using the chiral Lagrangian containing only $f_\pi$, $f_K$, $f_\eta$ as parameters.

It is the purpose of the present paper to study the effect of nearby thresholds on the position of resonances first in the most general setting and then to calculate numerically using the chiral quark Lagrangian without fitting parameters. The paper is organized as follows. In section 2 we present a general discussion of channel coupling and level shift using the (relativistic) Hamiltonian formalism, where we also give a classification of possible $S$-matrix poles. In section 3 the chiral quark Lagrangian is written down and used to describe the decay transition $D_s^* \rightarrow DK$, $D_s^* \rightarrow D\eta$. The final equation for the resonance position with account of this decay is explicitly written. In section 4 the numerical solution of this equation is described and the final results are presented. The paper closes with a discussion and comparison with other results.

II. RESONANCE STATES IN THE COUPLED-CHANNEL SYSTEM

The relativistic quark model has been remarkably successful in predicting the $D$ and $D_s$ meson spectrum, apart from some exceptions of a few resonances, which are experimentally found at substantially lower masses. Similar results have been found recently in the relativistic Hamiltonian approach [12] derived on the basis of Field Correlator Method (FCM) [14] and applied to the $D, D_s$ mesons in Ref. [15]. The results for the masses in the FCM analysis are shown in Table 1 and Table 2. (The entries given in the tables are recalculated for parameters given in the Table captions).
Table 1 Masses of $L = 0, 1$ states of $D$ mesons. Input parameters used in the FCM calculations: $\alpha_s = 0.46, \sigma = 0.17$ GeV$^2$, $m_c = 1.44$ GeV, $m_n = 7$ MeV

| State $J^P$ | 0$^-$ | 1$^-$ | 0$^+$ | 1$^+(l)$ | 1$^+(h)$ | 2$^+$ | 1$^{*-}$ |
|-------------|------|------|------|---------|---------|-------|---------|
| Mass (MeV)  |      |      |      |         |         |       |         |
| from [15]   | 1859 | 2047 | 2370 | 2425    | 2455    | 2456  | 2729    |
| Mass (MeV)  |      |      |      |         |         |       |         |
| experiment  | 1869 | 2010 | 2300±60 | 2400  | 2422   | 2459  | (2640?) |
| $\Gamma$ (MeV)| — | < 0.13 | 280 | ~ 250 | 20 | 23÷45 | < 15 |

As is seen from Tables 1 and 2 the overall agreement is reasonably good except for a few states. In particular, the $D^*_s(0^+)$ state is one example of such a discrepancy in the prediction of theoretical models and which can be associated with the KD threshold at 2366 MeV.

Table 2 Masses of $L = 0, 1$ states of $D_s$ mesons. Input parameter used in the FCM calculations: $\alpha_s = 0.46, \sigma = 0.17$ GeV$^2$, $m_c = 1.44$ GeV, $m_s = 0.175$ GeV

| State $J^P$ | 0$^-$ | 1$^-$ | 0$^+$ | 1$^+(l)$ | 1$^+(h)$ | 2$^+$ | 1$^{*-}$ |
|-------------|------|------|------|---------|---------|-------|---------|
| Mass (MeV)  |      |      |      |         |         |       |         |
| from [15]   | 1929 | 2087 | 2404 | 2462    | 2488    | 2494  | 2774    |
| Mass (MeV)  |      |      |      |         |         |       |         |
| experiment  | 1968 | 2112 | 2317 | 2462    | 2536    | 2572  | 2632(?) |
| $\Gamma$ (MeV)| — | < 1.9 | < 10 | < 6.6   | < 2.3   | ~ 15  | < 17    |

For comparison in Table 3 a summary of the results is given of other theoretical quark model predictions for this state. A look at the table tells us that all the theoretical predictions are about ~ 90÷190 MeV higher. So one needs a shift of about this value to get agreement with the experimental value. A similar descrepancy can be seen from Table 2 for the $D^*_s(2632)$ resonance. We explore in this paper whether this disparity can be explained due to the presence of coupled channels with nearby thresholds.

Table 3 Theoretical predictions of the mass $D^*_s(0^+)$ in various quark models

| Ref. | [5]  | [6]  | [7]  | [8]  | [9]  | [10] |
|------|------|------|------|------|------|------|
| Mass (MeV) | 2480 | 2388 | 2380 | 2508 | 2455 | 2487 |

Resonances in the coupled-channel system were considered in numerous papers both in nonrelativistic nuclear physics and in the relativistic Hamiltonian dynamics, see [10] for a review and references. Assuming that a local or nonlocal relativistic Hamiltonian can be written for each channel $H_i$, $i = 1, 2, ...$ and for the Channel Coupling (CC), $V_{ij}$, $i,j = 1, 2, ...$ the time-independent system of equations can be written as

$$[(H_i - E)\delta_{\nu\nu'} + V_{\nu\nu'}]G_{\nu\nu'} = 1.$$ \hfill (1)
For two channels it is

\[(H_1 - E)G_{11} + V_{12}G_{21} = 1, \quad (H_1 - E)G_{12} + V_{12}G_{22} = 0,\]
\[(H_2 - E)G_{22} + V_{21}G_{12} = 1, \quad V_{21}G_{11} + (H_2 - E)G_{21} = 0.\]  \hfill (2)

The system (2) can be reduced to the effective one-channel problem, corresponding to the Feshbach equation [17]

\[(H_1 - E)G_{11} - \frac{1}{H_2 - E}V_{21}G_{11} = 1.\]  \hfill (3)

At this point one can classify all possible poles \(E\) of the Green functions \(G_{ik}\). These poles may originate from the bound states or resonances in a given channel \(i\), located at \(E_i^{(n)}\) and shifted due to CC to a new position, which we will denote by \(E_i^{(n)*}\). Another possibility is that resonance poles appear solely due to the strong CC interaction – the so-called CC poles [16, 18]. These extra poles usually originate from distant dynamical poles in the complex plane, which move close to threshold when the CC coupling increases. The quantitative characteristics of the CC interaction is given by the last term on the l.h.s. of Eq. (3), which can be called the Feshbach potential,

\[V_{121}(E) \equiv -V_{12}G_{22}V_{21} = -V_{12}H_2 - EV_{21}.\]  \hfill (4)

Note that \(V_{121}(E)\) can support bound states or resonances even in the case when diagonal interaction \(V_i, \ i = 1, 2\) vanishes but \(V_{12} = V_{12}^{(n)}\) is large enough.

Of special importance for us is the case when in one channel, e.g. \(i=1\), the spectrum is discrete (see Ref. [18] for a more extensive discussion), and one is interested in the shift of the discrete level due to the coupling to channel 2, where states can be unconfined.

A somewhat similar approach was undertaken in recent papers [19], where in our notations the scattering channel 2 and the corresponding Feshbach potential \(V_{212}\) was modelled to calculate the scattering cross section in channel 2. We shall compare the results of Ref. [19] with ours in the concluding section. Eq. (3) connects in general all states in channel 1 and channel 2. If one separates one state and neglects all other states in channel 1, then one gets the following equation for the position of the pole(s) in the Green function

\[E = E_1^{(n)} - \langle n|V_{12}\frac{1}{H_2 - E}V_{21}|n\rangle,\]  \hfill (5)

where \(E_1^{(n)}\) is the selected unperturbed level in the channel 1. Insertion of the complete set of states \(|m\rangle\langle m|\) with eigenvalues \(E_2^{(m)}\) in the channel 2, yields

\[E = E_1^{(n)} - \sum_m \langle n|V_{12}|m\rangle \frac{1}{E_2^{(m)} - E} \langle m|V_{21}|n\rangle.\]  \hfill (6)

In what follows we shall be using Eq. (6) to calculate the shift \(\Delta E_n = E_1^{(n)*} - E_1^{(n)}\) of the \(c\bar{s}\) levels due to the open channel 2: \(KD\) or \(\eta D_s\) scattering states, neglecting interaction in these states. The most important point is how to find the operators \(V_{12}\). In the next section we shall use the chiral Lagrangian which will provide \(V_{12}\) explicitly without free parameters.

### III. COUPLED CHANNELS AND CHIRAL DECAYS

One starts with the Lagrangian for the flavor SU(3) triplet of quarks in the field of the heavy (\(c\) or \(b\)) quark [20,21]. In the Euclidean notations

\[L = i \int d^4 x \psi^+(\hat{\delta} + m + \hat{M})\psi,\]  \hfill (7)

where the mass operator is

\[\hat{M} = m\hat{U} = M \exp\left(i\gamma_5 \frac{2\phi}{f}\right).\]  \hfill (8)
and \( t_a = \frac{1}{\sqrt{3}} \lambda_a \), \( \lambda_a \) is the Gell-Mann matrix, \( a = 1, \ldots, 8 \), \( f_\pi = 0.093\) GeV and the matrix Nambu-Goldstone SU(3) wave function is

\[
\varphi_a \lambda_a = \sqrt{2} \begin{pmatrix}
\frac{\sigma^0}{\sqrt{6}} + \frac{\sigma^3}{\sqrt{2}} & \pi^+ & K^+ \\
\frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & \pi^- & K^-
\end{pmatrix}.
\]

(9)

\( M \) is the (nonlocal) effective mass operator, which in the local limit has the form (see [22, 23] for discussion and derivation)

\[ M = \sigma |r|, \]

(10)

where \( |r| \) is the distance from the light quark (\( u, d, s \)) to the heavy quark (\( c \) or \( b \)). Thus the Lagrangian (7) contains effects of both confinement and chiral symmetry breaking.

From Eqs. (11) and (12), expanding the exponent in Eq. (13), one can derive the meson emission part of Lagrangian,

\[
\Delta L = - \int \psi^+(x) \sigma |x| \gamma_5 \varphi_a \lambda_a \psi d^4 x.
\]

(11)

This Lagrangian can be expressed as in Ref. [20] in terms of the standard Weinberg Lagrangian [24]. It was used in Ref. [25] to calculate the decay widths of heavy-light mesons with good accuracy.

It is clear that the Lagrangian (7) generates (due to the various Fock components in \( \psi^+ \) or \( \psi \)) in general a many-channel system of equations for the Green functions. It contains the main channel (e.g. the \( D_s \) channel) and in addition the channel(s) for its virtual decay products like the \( (D + K) \) channel or \( (D_s + \eta) \) channel.

In what follows we shall be working with Eq. (11) to apply it first of all to the \( D_s^*(2317) \) state. In this case \( E_1^{(n)} \) refers to the \( 0^+ \) level of the \( D_s^* \) system, and \( E_2^{(m)} \) refers to the \( (D + K) \) channel or \( (D_s + \eta) \) channel.

where \( E_1^{(n)} \) and \( E_2^{(m)} \) are the (continuous) energy of the system \( D + K \) in the orbital \( S \) state. One can neglect the \( DK \) interaction in the first approximation and write for the wave functions Dirac equations

\[
|n\rangle = \Psi(D_s^*) = \frac{1}{r} \begin{pmatrix}
G_n^{(1)} \Omega_{j' M}^{(1)} \\
i F_n^{(1)} \Omega_{j' M}^{(1)}
\end{pmatrix},
\]

(12)

\[
\Omega_{j' M}^{(1)} = \Omega_{j' M}^{(1)} \Omega_{j' M}^{(1)} = \Omega_{j' M}^{(1)} \Omega_{j' M}^{(1)}.
\]

(13)

\[
|m\rangle = \Psi(D) \frac{e^{\text{pr}}}{\sqrt{2\varepsilon_p V_3}},
\]

(14)

\[
\Psi(D) = \frac{1}{r} \begin{pmatrix}
G_2^{(0)} \Omega_{j' M}^{(2)} \\
i F_2^{(0)} \Omega_{j' M}^{(2)}
\end{pmatrix},
\]

(15)

\[
\Omega_{j' M}^{(2)} = \Omega_{j' M}^{(2)} \Omega_{j' M}^{(2)} = \Omega_{j' M}^{(2)} \Omega_{j' M}^{(2)}.
\]

(16)

Therefore the matrix elements in Eq. (17) are

\[
\langle n|V_12|m\rangle = - \int \psi^+(D_s^*) \sigma |r| \gamma_5 \sqrt{2} \frac{\varphi_a \lambda_a}{f_\pi} \psi d^4 x.
\]

(17)

\[
= \sqrt{\frac{2\sigma}{f_\pi}} \int \frac{d^3 r}{r} (G_n^{(1)} + F_2^{(0)} \Omega_{j' M}^{(2)} \Omega_{j' M}^{(2)} - F_n^{(1)} + G_2^{(0)} \Omega_{j' M}^{(2)} \Omega_{j' M}^{(2)}) e^{\text{pr}} \sqrt{2\varepsilon_p V_3}
\]

(18)

\[
= \sqrt{\frac{2\sigma}{f_\pi}} \int \frac{\sin pr}{p\sqrt{2\varepsilon_p V_3}} (G_n^{(1)} + F_2^{(0)} - F_n^{(1)} + G_2^{(0)}) dr.
\]
Now $G^{(1)}$, $F^{(1)}$ and $G^{(2)}$, $F^{(2)}$ are solutions of the Dirac equation

$$\frac{dF^{(i)}}{dt} - \frac{2i}{\hbar} F^{(i)} + (\varepsilon^{(i)} - V_c(r) - m_i) G^{(i)} - MG^{(i)} = 0,$$

$$\frac{dG^{(i)}}{dt} + \frac{2i}{\hbar} G^{(i)} - (\varepsilon^{(i)} - V_c(r) + m_i) F^{(i)} - MF^{(i)} = 0. \quad (19)$$

Here $\kappa_1 = 1$, $\kappa_2 = -1$, $M = \sigma r$, $V_c(r) = - \frac{4m_\pi}{3r}$, and

$$m_1 = m_\pi = 0.15 \div 0.25 \text{ GeV}, \quad m_2 = 0.$$

The connection between $\varepsilon^{(i)}$ and $E_i$ is

$$E_1^{(n)} = \varepsilon^{(1)} + m_c, \quad E_2^{(m)} = \varepsilon^{(2)} + m_K + m_c + \frac{p^2}{2m_K}, \quad \tilde{m}_K = \frac{m_K m_D}{m_K + m_D}. \quad (20)$$

If one neglects higher states of the $D$ meson, the sum in Eq. (6) can be rewritten as

$$E = E_1^{(n)} - \sum V_0 \frac{d^3p}{(2\pi)^3} \frac{|\langle n|V_{12}|m \rangle|^2}{E_2^{(m)}(p) - E}. \quad (21)$$

From Eqs. (27) and (21) it is clear that the free Green function of the KD system has the form (we take into account the fact that only $S$-waves of KD are involved)

$$G_0(k, x, x') = \left( \int \frac{d^3p}{(2\pi)^3} \frac{\exp[ip \cdot (x-x')]}{2\omega(p) (\tilde{m}_K - \Delta E)} \right)_{S-waves} = \frac{1}{4\pi} \frac{\tilde{m}_K}{\omega(k)} \sin(kx, x') \exp(ikx').$$

where

$$k^2 = 2\tilde{m}_K(E - m_D - m_K), \quad \omega(k) = \sqrt{k^2 + m_K^2}. \quad (23)$$

Finally Eq. (21) can be rewritten as

$$E = E_1^{(n)} - \int V(x)V^+(x')d^3xd^3x'G_0(k, x, x), \quad (24)$$

where $V(x)$ is

$$V(x) = \bar{\psi}_1(x) \frac{\gamma_5}{2\gamma_\pi} \gamma_\pi \psi_1(x) = \frac{\gamma_5}{16\pi^2} (G^{(1)} + F^{(2)} - F^{(1)} + G^{(2)}) \Omega^+_{\frac{1}{2}0M_1} \Omega^0_{\frac{1}{2}0M_2} \quad (25)$$

and we have used relation $\Omega^\dagger_{\frac{1}{2}0M} = \sigma n \Omega_{\frac{1}{2}0M}$. As a first approximation one can use the fact that functions $G^{(i)}$, $F^{(i)}$ are concentrated around the middle point $x \equiv b$ and write

$$V(x) \approx C\delta(x - b); \quad C = \int V(x)dx \quad (26)$$

$$b = \frac{\int xV(x)dx}{\int V(x)dx}. \quad (27)$$

As a result one obtains

$$E = E_1^{(n)} - C^2 \frac{4\pi b^2 \tilde{m}_K \sin(kb)e^{ikb}}{\omega(k)k}, \quad (27)$$

where $k = \sqrt{2\tilde{m}_K(E - m_D - m_K)}$ is the relative momentum of the K meson.

In the vicinity of the DK threshold one can replace $\omega(k) \approx m_K$ (this is implied by the form of Eq. (22)). Eq. (27) is a transcendental equation for the position of the pole $E$. Since $C$ does not depend on $k$ Eq. (27) has a simple
square-root threshold at \( E = m_D + m_K \). The starting position of \( E = E_{\text{pole}} \) is at \( E_{1}^{(n)} \). When one takes into account the second term in Eq. (27) with gradually increasing \( C^2 \) the pole moves to the final value in the upper physical \( k \)-sheet. One expects that the trajectory will go down in mass, possibly nearby the final value of \( m(D^*_s) = 2317 \) MeV. In Fig. 1 we display the trajectory of the pole solution of Eq. (27) in \( k \)-plane parameterized by \( C \). One can see that for strong enough channel coupling the resonance pole moves down under the DK threshold, which will be substantiated by the exact calculation of \( V_{12} \), given by Eq. (18).

We now turn to the case of the \( D^*_s(2632) \) assuming after Ref. [12] that it can be associated with the radially excited \( D^*_s(1^-) \) state. From Table 2 one can see that the expected shift should be around 100–150 MeV downwards, and from the channels \( DK, D_s \eta \) the decay is in the \( p \)-wave. As before we shall use Eqs. (12) and (17) where now instead of \( D^*_s(0^+) \) one should write the \( D^*_s(1^-) \) state, i.e.

\[
\Omega_{jM}^{(1)} = \Omega_{\pm 0 M_1}^{(1)}, \quad \Omega_{j' M_1}^{(1)} = \Omega_{\pm 1 M_1}^{(1)}.
\]

The wave function in channel 2 is either the same as in Eqs. (14) - (16), or in the case of the \( \eta \) channel, one should replace \( D \) by \( D_s \), and the \( K \) meson with momentum \( p \) by the \( \eta \) meson with momentum \( p' \). With these assignments of the various states Eq. (17) retains its form, but Eq. (18) becomes

\[
\langle n|V_{12}|m \rangle = \frac{\sqrt{2}\sigma}{2\pi f} \int \frac{d^3 p}{r} (G_n^{(1)} + F^{(2)}(G_n^{(1)} + F^{(2)})\Omega_{\pm 0 M_1}^{(1)} + F^{(1)}(G_n^{(1)} + F^{(2)})\Omega_{\pm 1 M_1}^{(1)} + F^{(2)}(G_n^{(1)} + F^{(2)})\Omega_{\pm 0 M_1}^{(2)}) \frac{e^{i p r}}{2\varepsilon_p v_3}
\]

where \( p \) is the momentum of the \( K \) or \( \eta \) meson.

Here the radial quantum number in Eq. (29) is the first radial excited state \( n = 1 \). To compute the transition potential matrix elements for the \( D^*_s \leftrightarrow DK \) coupling one can use the coordinate representation as in Eqs. (15)-(25), including Eq. (28), or directly calculate the \( d^3 p \) integral in Eq. (21). In the latter way one needs to compute Eq. (21), \( \omega(p) \equiv \varepsilon_p = \sqrt{p^2 + m_K^2} \)

\[
E = E_{1}^{(n)} - \frac{1}{(2\pi)^3} \int \frac{d^3 p |v_{12}^{(K)}(p)|^2}{2\omega(p)(E_2(p) - E)}
\]

and \( v_{12}(p) \) is

\[
v_{12}^{(K)}(p) = V_{12} \cdot \sqrt{2\varepsilon_p}.
\]
Eq. (30) has to be extended to also include the \( D_s \eta \)-channel contribution. Observation of Eqs. 9 and 11 shows that in Eq. 9 only the lowest diagonal term enters in the transition potential matrix element. Hence we get for \( \langle n|V_{12}|m \rangle \) an additional factor \( \left(-\frac{2}{\sqrt{6}}\right) \). As a result we have

\[
v_{12}^{(\eta)}(p) = -\frac{2}{\sqrt{6}} v_{12}^{(K)}(p).
\]

The modification of \( V_{12} \), Eq. (29), for the case of the \( \eta \)-channel is straightforward. Clearly, the assignment of the \( D_s^* \) state in Eq. 28 remains the same and we have to replace in Eq. 20 \( m_k \to m_q \) and \( m_D \to m_{D_s} \). Moreover, the state \( \Psi(D) \) in Eq. (17) has to be replaced by \( \Psi(D_s) \), where \( F^{(2)}, G^{(2)} \) now refer to the \( D_s \) state.

IV. MASS SHIFT IN THE CHIRAL LAGRANGIAN FORMALISM

We may determine the energy shift \( \Delta E \) using the Dirac wave function as found by solving Eq. 15 for the case of an effective quark mass operator in the field correlator method 23. In the leading order it has a nonlocal form and can be parameterized as

\[
M(x, y) \approx \frac{1}{2T_g \sqrt{\pi}} |x + y| \exp\left(-\frac{(x - y)^2}{4T_g^2}\right),
\]

(32)

where \( \sigma \) is the string tension and \( T_g \) the gluon correlation length, characterizing the scale of nonlocality. Note that for \( T_g \to 0 \) one obtains from Eq. (28) the local limit (10). The physical value of \( T_g \) found on the lattice and analytically is small, \( T_g = 0.25 \) fm [12].

For the case of a single coupled channel \( D_s^*_f \leftrightarrow DK \) we may write Eq. (30) as

\[
\Delta E = -\frac{1}{(2\pi)^3} \int \frac{d^3p \left| v_{12}^{(K)}(p)\right|^2}{2\omega(p)(E(p) - E_0 - \Delta E)}.
\]

(33)

where \( \Delta E = E - E_0 \) with \( E_0 = m_{D_s} - m_D - m_K \) is the mass shift and \( E(p) \) the kinetic operator \( E(p) = \frac{p^2}{2m_q} \).

In the considered QCD string model we have taken \( T_g = 0.25 \) fm, in accordance with lattice gauge simulations 14. For a given string tension \( \sigma \) the wave functions of the \( D \) and \( D_s \) system can be found as solutions of the Schwinger-Dyson-Dirac equation 23 for the light-heavy quark system. It is given by Eq. (19) with the quark mass operator 22. The latter is found from the selfconsistent solution of nonlinear equations, (Eqs. (15), (16) in [22]). It exhibits the property of both confinement and chiral symmetry breaking. The states are in general characterized by the quantum numbers \( j, l, \kappa \). In particular, the \( D \) and the orbitally excited state \( D_s^* \) correspond to the solution of the ground state in the \( j, l, \kappa = 1/2, 0, -1 \) and \( j, l, \kappa = 1/2, 1, +1 \) channel respectively.

In this study a value of \( \sigma = 0.18 \) GeV\(^2\) and \( \alpha_s = 0.35 \) is adopted. For convenience, a zero mass is used for the \( u, d \) quark, while for the \( s \)-quark we have taken \( m_s = 200 \) MeV. Having constructed these wave functions with these parameters we determine the matrix elements \( v_{12}^{(K)} \). From this we may then solve the resulting Eq. (33) iteratively. To determine the actual position of the pole we in general have to analytic continue the integral into the second sheet in the case that \( E \) is above the \( KD \)-threshold. Although this can be done, we will assume in this study that the imaginary part of the pole position does not affect the solution substantially, which is certainly true when the pole diverges under the \( KD \)-threshold, and was checked in other situations. Confining ourself to real values of \( E \) and taking the principal value of the integral in Eq. (33) when \( E \) is above threshold the energy shift is determined as a solution of the resulting equation. In case of a solution above threshold, the width of the resonance can be obtained by calculating the discontinuity of the integral at this energy.

In the calculations we have used for the threshold mass of the \( KD \) system \( m_K + m_D = 2.366 \) GeV. A typical value of 2.49 GeV is adopted for the unperturbed \( D_s^* \) meson mass. As is seen in Table 3 this is in accordance with the predictions of FCM and of many quark constituent quark models.

In Fig. 2 is shown the prediction of the shifted mass of the \( D_s^* \) meson due to the channel coupling to the \( KD \)-system as a function of the unperturbed mass \( D_s^{(*)} \). The flavoured symmetric and broken value of \( f_s = 93 \) MeV and \( f_K = 121 \) MeV have been used [1]. The shifted mass is found using

\[
M_{D_s^*} = M_{D_s^{(*)}} + \Delta E,
\]

(34)

where \( \Delta E \) satisfies Eq. (33). Due to the coupling to the \( KD \) system the \( D_s^* \) meson can either become unstable or stable, depending on the sign and magnitude of the mass shift. We find in our model, that it can be substantial and
FIG. 2: The shifted $D_s^*$ meson mass as a function of the unperturbed $M_D^{(0)}$ for two values of the decay constant $f$. The experimentally observed mass is given by the dashed horizontal line.

of the order of hundred MeV. As is seen from the figure we find the experimentally observed value of 2.317 GeV for an unperturbed mass of approximately 2.49 GeV.

In the string model studied we find that the position of the pole moves well down below the $KD$ threshold, yielding a stable state. The channel coupling is found to yield attraction, so that the position of the mass pole shifts downwards.

Similarly as for the $D_s^*(2317)$ meson we may estimate the mass shift of the $D_s^*(2632)$ due to channel coupling. The calculations proceeds in the same way. The bare $D_s^*(2632)$ is assumed to correspond to the first radial excited state with $j, l, \kappa = 1/2, 0, -1$ and to have a mass of 2.76 GeV as found in the FCM [15]. Clearly a mass shift of about $-140$ MeV is needed to get agreement with the observed mass. As discussed, coupling can occur in this case to $KD$ and $\eta D_s$ channels. For the $D$ meson we assume the experimental observed mass of 1.869 GeV. There are two candidates for the $D_s$ state, being the $0^-$ and $1^-$ states with masses of 1.968 and 2.120 GeV respectively. Using the various wave functions obtained from the QCD string model, the interaction matrix elements are calculated from Eq. (29). The two $D_s$ states are degenerate in the considered model and as a result have the same wave function, but differ in the kinematics of the momenta in view of the adopted mass difference, which occurs due to the hyperfine interaction neglected in our heavy-quark approximation. For simplicity we have considered only one $D_s$ state with mass 2.0 GeV. With the obtained potential matrix elements we solve numerically the eigenvalue equation for $\Delta E$. It has essentially the same form as Eq. (33), but has now two terms due to the contributions from the two coupled channels.

We find that the bare mass is shifted downwards by 51 MeV. Furthermore, the contributions from the various inelastic channels are given by

$$\Delta E(KD) = -35 \text{ MeV}, \quad \Delta E(\eta D_s) = -16 \text{ MeV}.$$  

The magnitude of the total mass shift is clearly smaller here than found in the first considered case of the $D_s^*(2317)$ meson as can be explained by the presence of $P$-wave, rather than $S$-wave for the case of $D_s^*(2317)$. It is clearly not sufficient to explain the experimental observed mass. We can also calculate the width, being the discontinuity of the right hand side of Eq. (33). We find $\Gamma = 174$ MeV, decaying predominantly into the $KD$-channel. The corresponding partial decay widths to the various channels are found to be

$$\Gamma(KD) = 139 \text{ MeV}, \quad \Gamma(\eta D_s) = 35 \text{ MeV}.$$  

Clearly the predicted width is considerably larger than found in the SELEX experiment. We have also solved Eq. (33) for the case of the flavour symmetric value $f_K = f_\eta = .093 \text{ GeV}$, which yields again a large value for the width. The above results suggest in this case that the resonance becomes very broad and does not support the SELEX observation, also the ratio of $KD$ to $\eta D_s$ channels quantitatively disagrees with the experiment.
In general, the size of the mass shift clearly depends on structure of the quark wave functions and hence it should be expected to be model dependent. Our study demonstrates, that the size of the shift due to channel coupling is in general large, but that it also can lead to very large widths in case that the resonance is above threshold of the coupled channels. As a result it can accommodate for the discrepancy between the predictions of dynamical quark models and the observed $D_s^*(1237)$ resonance, but there may exist situations where the resonance can become very broad due to inelastic channel coupling, as it is in the case of radial excited $D_s^*$.

V. DISCUSSION

Let us compare our results to the existing in literature. The explanation of $D_s^*(2317)$ as the $c\bar{s}$ $p$-wave level shifted down by the coupling to the decay channel $D + K$ was considered in a series of papers [19], where authors have used a simple phenomenological model similar to our Eq. (26), (27) to describe the $p$-wave mesons.

Another type of phenomenological model for the channel coupling, namely the model of Eichten at al. [26] was used in Ref. [21] to calculate the shift of the $D_s(0^+)$ level and it was shown that the desired mass shift is obtained for a reasonable choice of parameters.

Our results obtained with the parameter free chiral Lagrangian containing full $x$-dependence, qualitatively agree with those in Refs. [19] and [27] and exactly reproduce the experimentally found mass $D_s^*(2318)$.

We now turn to the state $D_s^*(2632)$, found in Ref. [11], but not yet confirmed by other groups [28]. The theoretical prediction for the $2^3S_1$ state vary from 2774 MeV in Ref. [15] to 2737 MeV made using the relativistic Salpeter equation in Ref. [29] and 2716 in Ref. [8], and in principle are subject to the correction due to the global string breaking effect occurring for states of large size [30] (m.s.r. radius of $2^3S_1$ state in Ref. [29] is around 1 fm). The expected correction is around – 20 MeV, which brings the theoretical mass of the $D_s(2^3S_1)$ state to 2700-2720 MeV.

Our calculation for the initial mass $D_s^*(1^-)$, $m = 2710$ MeV, using Eq. (33), yields the shift $\Delta E = -76$ MeV, with the total width $\Gamma = 131$ MeV, $\Gamma_{KD} = 111$ MeV, $\Gamma_{D\eta} = 20$ MeV. Hence also in this case the total width and width ratio contradicts experimental data, implying that $D_s^*(2632)$ cannot be explained as the shifted $1^-$ level.

VI. ACKNOWLEDGEMENTS

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