TURBINE MAP EXTENSION – THEORETICAL CONSIDERATIONS AND PRACTICAL ADVICE

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ABSTRACT
Physically sound compressor and turbine maps are the key to accurate aircraft engine performance simulations. Usually, maps only cover the speed range between idle and full power. Simulation of starting, windmilling and re-light requires maps with sub-idle speeds as well as pressure ratios less than unity.

Engineers outside industry, universities and research facilities may not have access to the measured rig data or the geometrical data needed for CFD calculations. Whilst research has been made into low speed behavior of turbines, little has been published and no advice is available on how to extrapolate maps.

Incompressible theory helps with the extrapolation down to zero flow as in this region the Mach numbers are low. The zero-mass flow limit plays a special role; its shape follows from turbine velocity triangle analysis.

Another helpful correlation is how mass flow at a pressure ratio of unity changes with speed. The consideration of velocity triangles together with the enthalpy-entropy diagram leads to the conclusion that in these circumstances flow increases linearly with speed.

In the incompressible flow region, a linear relationship exists between torque/flow and flow. The slope is independent of speed and can be found from the speed lines for which data are available. This knowledge helps in extending turbine maps into the regions where pressure ratio is less than unity.

The application of the map extension method is demonstrated with an example of a three-stage low pressure turbine designed for a business jet engine.

INTRODUCTION
In by far the biggest area of a turbine map both the stator exit Mach number and the rotor exit Mach number (in the relative system) are high. In this map region the similarity laws for incompressible flow are of little value for extrapolation purposes.

However, for extending a turbine map to the low mass flow region - where turbines operate in a windmilling engine - correlations derived from incompressible flow theory are very helpful.

THERY
Work and Flow Coefficient
If we consider the form of the $\psi$-$\Phi$ relationship for a single stage turbine with symmetrical velocity triangles in simple terms, we can conclude that the work coefficient $\psi$ is a linear function of the flow coefficient $\Phi$. This follows from the fact that the flow leaves a blade or vane row in the direction given by the trailing edge geometry (see figure 1).

![Figure 1: Turbine work and flow coefficient](image)

The work output $H$ is a straight line when plotted as $\Psi$ over $\Phi$, whereas work input $H_{is}$ - calculated from pressure ratio - is not (see figure 2).
\[ \psi = \Phi \times (\tan \beta_2 - \tan \alpha_1) - 1 = c_1 \times \Phi - 1 \quad (1) \]

The losses in the expansion process, described by efficiency \( \eta = H/H_{in} \), are smallest at the peak efficiency point.

**Figure 2: Peak efficiency point of the \( \psi \)-\( \Phi \) correlation**

The only prerequisite for the validity of equation 1 is that the (relative) flow direction downstream of the blades and vanes is enforced by the geometry of the blades and vanes. Furthermore, in an incompressible fluid (constant density), there is only one curve for \( \psi_\infty = f(\Phi) \) and this is valid for any speed.

**Figure 3: \( \Psi \)-\( \Phi \) correlations from a turbine**

Figure 3 shows data published by Broichhausen (1994) with lines for \( \Psi \) plotted against flow coefficient \( \Phi \). The lower the speed, the more the \( \Psi \)-\( \Phi \) correlation is a straight line.

**Figure 4: Efficiency corresponding to figure 3**

**Efficiency**

Figure 4 shows the efficiency values corresponding to figure 3. For the relative speed values 0.4, 0.6 and 0.8 the efficiency lines collapse in the region where flow coefficients are less than \( \Phi \approx 1.2 \), where the peak efficiency values occur.

Another efficiency correlation originates from theoretical considerations about impulse turbines. In such a turbine the inlet guide vane works like a nozzle which produces the jet velocity \( V_1 \).

There is no velocity change in the rotor of an impulse turbine (\( W_2 = W_1 \)) as can be seen in figure 5. Maximum efficiency is achieved when the direction of the absolute velocity \( V_2 \) at the turbine exit is axial. The ratio of \( U/V_1 \) for optimum efficiency is approximately 0.47.

**Figure 5: Impulse turbine**

Experience from working with the maps of many single- and multi-stage turbines has shown that efficiency generally correlates well with the speed / jet-velocity ratio. Figure 6 demonstrates this for the same turbine as used for figure 3.

**Figure 6: Efficiency = f(\( N/V_{jet} \))**

Note that figure 6 does not employ the absolute circumferential speed \( U \) – it uses relative speed \( N/N_{ref} \). The x-axis numbers in figure 6 therefore differ in magnitude from the value shown in figure 5.

The correlations of efficiency with flow coefficient and speed ratio are useful only in the region with positive effective work \( H_{eff} \). Efficiency is zero if effective work is zero and drops to minus infinity when pressure ratio is 1, see figure 7.
The torque can be expressed as the product of flow and specific work as well as the product of angular speed and torque:

\[ PW = m \cdot H = \omega \cdot Trq \]  \hspace{1cm} (2)

Rearrangement and insertion of equation 1 yields

\[ \frac{Trq}{m^2} = \frac{c_2 \cdot H}{U^2} \cdot m = c_2 \cdot \frac{c_1 \cdot \Phi - 1}{\Phi} \]  \hspace{1cm} (3)

This equation is valid where flow velocity \( V_{ax} \) is proportional to mass flow \( m \), in the incompressible flow region. Under this condition, \( Trq/m^2 \) is a linear function of \( 1/\Phi \) and \( Trq/m \) is – for a given circumferential speed \( U \) - a linear function of \( m \):

\[ \frac{Trq}{m} = c_1 \cdot c_2 \cdot m - c_2 \cdot U \]  \hspace{1cm} (4)

The zero-speed line

For a locked rotor, the turbine may be considered as a pipe with restrictions. Total temperature is constant because no work is transferred. Total pressure decreases from the inlet to the exit of the turbine. For incompressible flow, the zero-speed line – i.e. the pressure ratio = f(flow) correlation - is a parabola. The locked rotor changes the direction of the fluid and downstream of the rotor this is determined by the rotor blade exit geometry. Torque is proportional to the force in the circumferential direction which is exerted by the fluid on the locked rotor. Equation 4 is also valid for the locked rotor.

The zero-flow line

Reverse flow never happens in turbines during starting and windmilling simulations. The zero-flow line is a lower pressure ratio limit for the turbine map extension discussed.

The velocity triangles yield interesting insights about operation at zero flow. Figure 9 shows how the design point velocity triangles and the enthalpy-entropy diagram look typically.

If the mass flow is reduced at the same speed to very low values and finally to zero, then the incidence to the rotor becomes highly negative, see figure 10.

The enthalpy-entropy diagrams for the two low mass flow velocity triangles are shown in figure 11. From the diagram on the right one can conclude that for the zero-flow case, the effective specific work \( H_{ef} \) is...
twice as big as the isentropic specific work $H_1$ since both $W_1$ and $V_2$ are equal to circumferential speed $U$.

Corrected isentropic work $H_1/T_1$ is proportional to corrected speed squared:

$$\frac{H_\text{gs}}{T_1} \propto U^2 \quad \frac{T_2}{T_1} \propto \frac{N}{\sqrt{T_1}} \tag{5}$$

$H_1/T_1$ relates to pressure ratio:

$$\frac{H_\text{gs}}{T_1} = c_p \frac{T_2}{T_1} - T_1 = c_p \left[ \left( \frac{P_2}{P_1} \right)^{\gamma-1} - 1 \right] \tag{6}$$

Thus, pressure ratio relates to corrected speed:

$$\left( \frac{P_2}{P_1} \right)^{\gamma-1} - 1 = const \left( \frac{N}{\sqrt{T_1}} \right)^2 \tag{7}$$

Relating the true corrected speed to a reference value makes it easy to determine the constant in this equation because then the speed term equals unity. The difference between specific enthalpy for the temperatures $T_{2,\text{gs}}$ and $T_1$ for zero flow is $W_i/2$ as the right part of figure 11 shows.

Since $W_i$ equals $U$ (figure 10) it holds that

$$\left( \frac{P_2}{P_1} \right)^{\gamma-1} - 1 = const \left( \frac{N}{\sqrt{T_1}} \right)^2$$

The circumferential Mach number $M_{i,\text{ref}}$ at the map reference point needs to be estimated. The constant in equation 7 follows from equation 9:

$$\text{const} = \frac{\gamma - 1}{2} M_{i,\text{ref}}^2 \tag{8}$$

Pressure ratio for any other corrected speed follows from

$$\frac{P_2}{P_1} = 1 + \text{const} \left( \frac{N}{\sqrt{T}} \right)^2 \left( \frac{N}{\sqrt{T}_{\text{ref}}} \right)^2 \tag{9}$$

Flow at pressure ratio 1

Figure 12 shows the velocity triangles and the enthalpy – entropy diagram for pressure ratio 1. The rotor entry triangle is symmetrical which makes the total temperature in the relative system $T_{1,\text{rel}}$ equal to the inlet temperature $T_1$. The total pressure losses of the inlet guide vane $P_0 - P_{1,\text{rel}}$ are compensated by the rotor, which works as a compressor.

Figure 12: Pressure ratio 1

The shape of the velocity triangles remains the same when spool speed changes, the triangles remain similar. All enthalpy differences change in proportion to circumferential speed squared and mass flow changes in proportion to circumferential speed. Therefore, the pressure ratio 1 line is linear in a plot of flow vs. speed.

Map calculation

The preceding sections describe simple correlations based on incompressible theory for a single stage axial turbine. These correlations will next be compared with the results from the NASA turbine map calculation program published by Flagg (1967).

This program is applicable to turbines having any number of stages up to eight. It allows for a change in mean-section radius between blade rows and includes provisions for radial variation in loss and flow conditions in up to six radial sectors. Each sector is a quasi-one-dimensional element and the radial centres are joined utilizing simple radial equilibrium at the stator and rotor exits. The analysis is applicable from zero to high speed, and the work done may be negative up to the maximum work condition limited by discharge annulus area choking.

Glassman (1994) describes modifications to this program and makes comparisons between computed performances and experimental data. The loss model was improved by revising the blade-row efficiency calculation and calibrating the incidence-loss law.

Using the revised off-design performance model, computed flows and efficiencies were compared with experimental values for seven aircraft-type turbines of
various design characteristics and operating over wide ranges of speed and pressure ratio. The experimental values were generally within 1 percent of the computed values and seldom beyond 2 percent. Maximum discrepancies between computation and experiment were reduced to about half of those from the original performance model.

The quality of the user interface of the original FORTRAN program published by Flagg does not comply with modern standards. Therefore, the program was translated into a Delphi program. The program enhancements which Glassman describes were implemented and checked by running the example input data.

The turbine operating point is selected by specifying speed and the pressure ratio $P_0/P_{s1}$, i.e. total pressure at turbine inlet over static pressure at the exit of the first stator. This ratio specifies the Mach number at the stator exit and thus the mass flow indirectly. With respect to map extension, low mass flow values down to zero are of special interest.

There are three input data sets available for checking the program: one in the Flagg report and two more in the report of Glassman. All of them consider five radial sectors. It was found that the program does not yield reasonable results for low input values of $P_0/P_{s1}$ because computationally reverse flow then exists in one or more of the sectors. For the map extension down to mass flow zero, however, it must be possible to input $P_0/P_{s1}$ values very near to 1.0.

Switching from five radial sectors to one sector solves this problem. This introduces - as a side result - a step change in the calculated overall performance numbers. The mass flow step is taken care of by a correction factor which reconciles the mass flow values for $P_0/P_{s1}$=1.1 +/-e. The revised program works for any value of $P_0/P_{s1}$>1.0.

When calculating a whole map, very small steps in $P_0/P_{s1}$ are needed in the region of small pressure ratios, otherwise the mass flow changes from point to point become too big. Larger $P_0/P_{s1}$ steps are adequate when the pressure ratio is higher. A map calculation algorithm with specified step-size for inlet guide vane Mach number yields an optimal point distribution in the map.

**Properties of a calculated map**

The following figures show a map calculated with the Flagg/Glassman method and how well this agrees with the theory presented here.

Figure 13 demonstrates that the torque/flow = f(flow) lines are truly linear as derived from equation 4 in the incompressible flow range (flow<30…40).

Figure 14 shows flow as function of pressure ratio for relative speed values from 0.1 to 1.2. All lines begin at zero flow and their shapes vary as predicted. The line for the lowest speed ($N_{rel}$=0.1) nearly passes through the point {pressure ratio 1; mass flow 0} and its shape is a parabola as postulated in the section dealing with the locked rotor.

**APPLICATION**

The turbine map extension method described here is implemented in the most recent version of the program Smooth T. This software is a specialized plotting routine which helps a performance engineer generate smooth and meaningful lines through a cloud of measured or calculated data points. Many graphs show the measured data together with the lines in figures with physically meaningful parameters to allow checking whether the result makes sense or not.

The efficiency contour lines in a pressure ratio – speed plot for a turbine define a landscape with a broad peak region at high speed (figure 15). This high efficiency region becomes smaller in the mid speed range and deforms to a narrow ridge at low speed. Small differences in pressure ratio coincide with big changes in efficiency in this map region. Capturing the details of the efficiency ridge at low speed accurately would require a huge number of equally distributed pressure ratio grid points. Using a rectangular grid marked with the grey circles in figure 15 would lead to severe accuracy problems in the low speed area. Moreover, we would have to store much useless data – especially in the region of high pressure ratios and low speed. We never need to know the turbine performance in that part of the map when we simulate standard gas turbine performance.
We can achieve high accuracy with a minimum number of grid points if we focus on the region of interest, which is the peak efficiency zone. The blue circles in figure 15 are distributed in such a way that we get adequate accuracy for any point of interest in the map. The blue circles are at the crossing between the speed lines with so-called β-lines which serve as auxiliary coordinates for reading data from the map. (Kurzke, Halliwell, 2018)

**Baseline map**

The turbine map shown previously in figures 3, 4, 6 and 8 serves to illustrate the map extension method. This map is from a three-stage low pressure turbine designed for a business jet engine.

**Figure 15: The boundaries of the β-line grid**

The map as shown in figure 16 has been created using 30 β-lines with the standard procedure described in the Smooth T manual. The β-line grid encloses all the given data points. The β-numbers (between 0 and 1) have no physical meaning. The map reference point is properly placed at the peak efficiency location of the speed line 1.

**Figure 16: Baseline map**

**Extension to zero flow**

A new β-line grid is required for extending the map to zero flow. The upper β-line (β=1) remains unchanged, the lower β-line, however, will now get a physical meaning: it will represent zero-flow. This line passes through the origin (relative speed=0, pressure ratio=1) and bends downwards when speed increases. The shape of the zero-flow line is given by equation 10 and is defined implicitly by the circumferential Mach number $M_U$ for relative speed 1.

**Figure 17: Revised β-line grid**

Note that the number of β-lines needs to be increased because the pressure ratio range is bigger. 50 β-lines have been used in figure 17.

Start the map extension with a guess for the β=0 line by choosing $M_U=0.5$, for example. All the extended flow lines begin with mass flow zero on the β=0 line and approach the measured data points smoothly.

**Figure 18: Flow=f(pressure ratio)**

Modify the assumption for $M_U$ if the lines flow=f(pressure ratio) do not merge smoothly with the part of the line which is defined by measured data. Special attention should be paid to comparing the shape of the line for the lowest speed with that for zero speed.

The discussion of figure 12 lead to the conclusion that in a plot flow=f(speed) with contour lines for pressure ratio, the line 1.0 must be a straight line. As figure 19 shows, this is true.
Also check the correlation between isentropic work coefficient and flow coefficient (figure 20). All speed lines must collapse in the low flow coefficient region.

Extension to low speed

Extending the map to low speed uses the same ß-line grid as before. Add one speed line after the other and adapt the mass flow line first in such a way that it matches figure 23 reasonably well.

Equation 1 indicates that in incompressible flow the correlation between work and flow coefficients is linear and all lines collapse. Figure 22 shows that this is truly the case in our map example – compare the data with the dashed straight line.

If it is not feasible to get both efficiency and torque/flow lines right, then the form of the function flow=f(pressure ratio) needs to be modified. Go back and modify the flow lines in such a way that the torque/flow lines have the desired shape.
To get all the correlations right requires only a few iterations. The final result of the map extension process is shown in figures 24, 25 and 26.

CONCLUDING REMARKS

The turbine map extension towards low mass flow is based on incompressible flow theory. The lower limit of the map is the zero-flow line – negative flow will not happen during aircraft engine start and windmilling.

The advantage of the current method over using map calculation programs is that no geometry information is needed. The accuracy of the result is more than adequate for starting and windmilling performance simulations.

The approach is implemented in the software Smooth T and has been applied successfully to many maps from the open literature. Starting and windmill relight simulations have been demonstrated with GasTurb. Deviations from full engine test data are expected due to uncertainties in the modeling of oil viscosity effects on gearbox drag and bearing losses. Also, combustor light up and efficiency immediately after ignition are in some degree random effects, and that makes an absolute agreement between model and reality improbable.

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NOMENCLATURE

c1, c2, …constant values
c_p specific heat at constant pressure
eff effective
H specific work
is isentropic
m (corrected) mass flow
M Mach number
M_U circumferential Mach number
N rotational speed
P total pressure
P_s static pressure
S specific entropy
T total temperature
Trq torque
U circumferential speed
V absolute velocity
V_ax axial component of V
V_U circumferential component of V
W relative velocity
W_U circumferential component of W
\alpha_1 stator exit flow angle
\beta_2 rotor blade exit flow angle
\beta map coordinate
\gamma isentropic exponent
\eta efficiency
\Phi flow coefficient V_{in}/U
\Psi work coefficient H/U^2
\Psi_{is} isentropic work coefficient H_{in}/U^2
\Pi pressure ratio
\omega angular speed

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