Dalitz-plot Analysis of $B^0 \to D^0 \pi^+ \pi^-$

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We report preliminary results from a study of the decay $B^0 \to D^0 \pi^+ \pi^-$ using a data sample of 470.9 ± 2.8 million $B\bar{B}$ events collected with the BABAR detector at the $T(4S)$ resonance. Using the Dalitz-plot analysis technique, we find contributions from the intermediate resonances $D_s^*(2400)^-, D_0^*(2400)^-, \rho(770)^+$ and $f_2(1270)$ as well as a nonresonant $S$-wave term, a $D^0 \pi^-$ nonresonant $S$-wave term and a virtual $D^*(2010)^-$ amplitude. We measure the branching fractions of the contributing decays.

INTRODUCTION

The study of the Dalitz plot $D^0 \to D^0 \pi^+ \pi^-$ decays is motivated by several factors. The branching fractions of $B \to D^{**}$ transitions are of interest to help address a conflict between theoretical predictions and experimental results in semileptonic $B \to D^{**} l \nu$ decays. The $D^0 \pi^+ \pi^-$ final state allows relatively clean studies of the $J^{PC} = 0^+$ and $2^+ D^{**}$ states, since the $1^+$ mesons cannot decay to $D\pi$. Measurements of these decays test theoretical models including quark models, QCD sum rules and lattice QCD. Similarly, measurement of the branching fraction of the $B^0 \to D^0 \phi^0$ decay will help to test the dynamics of “color-suppression” in $B$ decays (related to the fact that the color quantum numbers of the quarks produced from the virtual $W$ boson must match that of the

1 $D^{**}$ mesons are $P$-wave excitations of states containing one charmed and one light ($u, d$) quark.
spectator quark in order for a $\rho^0$ meson to be formed\cite{10,13}. Moreover, using isospin symmetry to relate the decay amplitudes of $B^0 \to \bar{D}^0\rho^0$, $B^0 \to D^-\rho^+$ and $B^+ \to \bar{D}^0\rho^+$, it is possible to study effects of final state interactions in these decays\cite{11,14}.

Another motivation is that the $B^0 \to \bar{D}^0\rho^0$ decay can be used to measure $\sin 2\beta$, where $\beta$ is the CKM unitarity triangle angle\cite{13,14}, if the $\bar{D}^0$ meson is reconstructed in a $CP$ eigenstate. The measurement of this angle in the $b \to c\bar{q}d$ quark-level transition is theoretically cleaner than the commonly used $b \to c\bar{q}s$ decays (such as $B^0 \to J/\psi K^0_{s}$)\cite{17,18} and comparisons of the values measured in different quark-level transitions can be used to search for the influence of physics beyond the Standard Model\cite{19}. The time-dependent analysis of the $B^0 \to \bar{D}^0\pi^+\pi^-$ Dalitz plot not only allows a proper handling of effects due to interference between broad resonances, but also enables an improved measurement of $\beta$ since terms proportional to $\cos 2\beta$ as well as $\sin 2\beta$ can be measured\cite{20,21}. For such an analysis, it is necessary to have a good understanding of the population of the $B^0 \to \bar{D}^0\pi^+\pi^-$ Dalitz plot. This can be best studied in the $\bar{D}^0 \to K^+\pi^-$ decay, which is the subject of this study.

The $B^0 \to \bar{D}^0\pi^+\pi^-$ decay has been previously studied by Belle\cite{22} and the related $B^+ \to D^-\pi^+\pi^+$ decay has been studied by both BABAR\cite{23} and Belle\cite{24}. In this paper we present preliminary results from the first study of the $B^0 \to \bar{D}^0\pi^+\pi^-$ decay by BABAR. The data used in the analysis, collected with the BABAR detector\cite{25} at the PEP-II asymmetric energy $e^+e^-$ collider at SLAC, consist of an integrated luminosity of 429 fb$^{-1}$ recorded at the $\Upsilon(4S)$ resonance ("on-peak") and 45 fb$^{-1}$ collected 40 MeV below the resonance ("off-peak"). The on-peak data sample contains the whole BABAR dataset of 470.9 ± 2.8 million $B\bar{B}$ events.

**SELECTION**

We reconstruct $B^0 \to \bar{D}^0\pi^+\pi^-$ candidates (the inclusion of charge conjugate reactions is implied throughout this paper) by combining a $\bar{D}^0$ candidate with two oppositely charged pion candidates. The charged pion candidates are required to satisfy particle identification requirements that have efficiency above 97% and kaon misidentification probability below 20%. We reconstruct $\bar{D}^0$ mesons in the decay channel $K^+\pi^-$. For the $\bar{D}^0$ daughters, the charged kaon candidates are required to satisfy particle identification requirements that have efficiency above 97% and pion misidentification probability below 15%, while the charged pion candidates are required to pass slightly looser criteria than those for the bachelor pions. The $D$ candidates are required to have an invariant mass within 15 MeV/$c^2$ of the nominal $\bar{D}^0$ mass\cite{26}; this requirement is 85% efficient for signal Monte Carlo (MC) events.

The $\bar{D}^0$ candidate and the two bachelor pion candidates are required to originate from a common vertex. Signal events are distinguished from background using two almost uncorrelated kinematic variables: the difference $\Delta E$ between the CM energy of the $B$ candidate and $\sqrt{s}/2$, and the beam-energy-substituted mass $m_{ES} = \sqrt{s}/4 - \vec{p}_B$, where $\sqrt{s}$ is the total CM energy and $\vec{p}_B$ is the momentum of the candidate $B$ meson in the CM frame. We apply preselection criteria of $-0.075\text{ GeV} < \Delta E < 0.075\text{ GeV}$ and $5.272\text{ GeV}/c^2 < m_{ES} < 5.286\text{ GeV}/c^2$; these requirements are 86% efficient for signal MC events. We make further use of these kinematic variables to discriminate signal from background in the fit described below. We exclude candidates consistent with the abundant $B^0 \to D^*(2010)^-\pi^+$ decay by rejecting events which contain a candidate with $\bar{D}^0\pi^-$ invariant mass below 2.02 GeV/$c^2$ (to maintain the symmetry of the Dalitz plot, we also remove the region with $\bar{D}^0\pi^+$ invariant mass below the same value). These events are used as a control sample to monitor differences between data and MC.

To suppress the background contribution from continuum $e^+e^- \to q\bar{q} (q = u, d, s, c)$ events, we construct a neural network (NN) discriminant that combines four variables commonly used to separate jet-like $q\bar{q}$ events from the more spherical $B\bar{B}$ events. These are: the 0th order momentum-weighted monomial moment,\cite{2} $L_0^q$; the ratio of the 2nd order momentum-weighted monomial ($L_2^q$) to that of 0th order ($L_0^q$), $L_2/L_0^q$; the absolute value of the cosine of the angle between the $B$ direction and the beam ($z$) axis, $|\cos \theta_{B\text{mon}}|$; and the absolute value of the cosine of the angle between the $B$ thrust axis and the beam ($z$) axis, $|\cos \theta_{B\text{thel}}|$. All these variables are evaluated in the $e^+e^-$ center-of-mass frame. We apply a requirement on the NN output that retains approximately 88% of the signal and rejects $\sim 52\%$ of the continuum background. Most of the remaining background originates from $B$ decays, and is discussed below.

After applying all selection criteria, we retain 26334 events with candidate $B^0 \to \bar{D}^0\pi^+\pi^-$ decays. Around 20% of these events have multiple candidates. When an event has multiple candidates we retain the candidate with the best geometrical $B$-vertex probability.

\footnote{The momentum-weighted monomial moments are defined $L_i = \sum_j p_j |\cos \theta_j|^i$, where $\theta_j$ is the angle of the track or neutral cluster $i$ with respect to the signal $B$ thrust axis, $p_j$ is its momentum, and the sum excludes the daughters of the $B$ candidate.}
The efficiency for signal events to pass all the selection criteria is determined as a function of position in the Dalitz plot (DP). Using a Monte Carlo (MC) simulation in which events uniformly populate the phase-space, we obtain an average efficiency of approximately 35%. The efficiency is shown as a function of phase-space in Figure 1 both in terms of the conventional DP (for which we choose axes \(m_2^2 = m_{D_0^{*+}}^2\) and \(m_2^2 = m_{D_0^{*-}}^2\)), and in terms of the “square Dalitz plot” (SDP). The latter is described by the variables 
\[
M \equiv \frac{1}{\pi} \arccos \left( \frac{2m_{\pi^+\pi^-} - m_{\pi^+\pi^-}^{\text{min}} - m_{\pi^+\pi^-}^{\text{max}}}{m_{\pi^+\pi^-}^{\text{max}} - m_{\pi^+\pi^-}^{\text{min}}} \right) \quad \text{and} \quad \Theta \equiv \frac{1}{\pi} \theta_{\pi^+\pi^-},
\]
(1)
where \(m_{\pi^+\pi^-}\) is the invariant mass of the two pions, \(m_{\pi^+\pi^-}^{\text{max}} = m_B - m_{D_0}\) and \(m_{\pi^+\pi^-}^{\text{min}} = 2m_\pi\) are the kinematical limits of \(m_{\pi^+\pi^-}\), and \(\theta_{\pi^+\pi^-}\) is the angle between the \(D_0^+\) and the \(\pi^+\) in the \(\pi^+\pi^-\) rest frame. While the conventional DP representation provides a useful visual representation of the physics of the signal decay, the SDP allows closer scrutiny of the most densely populated regions of the phase-space, and hence is appropriate for studies of background distributions, for example.

**FIG. 1:** Variation of the signal reconstruction efficiency over the phase-space. In these plots the \(D^*(2010)^\pm\) veto is not applied.

**BACKGROUNDS**

In addition to the background from continuum processes, we expect backgrounds from other \(B\bar{B}\) decays. These are studied using large MC samples in which the \(B\) mesons decay generically according to our current knowledge of their branching fractions. We classify backgrounds from \(B\bar{B}\) decays in six categories based on their \(\Delta E\) and \(m_{ES}\) distributions as determined from MC samples. The different \(B\bar{B}\) background categories also have different DP distributions.

Table I lists the expected number of events and the dominant contributing mode for each category. Background categories 1 and 2 have four track final states, and peak in both \(\Delta E\) and \(m_{ES}\) – category 1 has signal-like peaks in both, while category 2 has a \(\Delta E\) peak shifted to positive values due to pion→kaon misidentification. The decay modes that contribute to these categories do not contain real \(D\) mesons, with the exception of \(D_0^{*0}K_0^\ast\), which contributes to category 1. Background categories 3–6, which are dominant, do contain real \(D\) mesons. Category 3 peaks in \(m_{ES}\) and has a \(\Delta E\) distribution that is shifted to negative values due to kaon→pion misidentification. Categories 4–6 do not peak strongly in either \(\Delta E\) or \(m_{ES}\). Category 4 has a broad \(m_{ES}\) distribution and a slight peak in \(\Delta E\), and includes background from \(D^*(2010)^\pm\pi^\pm\) events that escape the veto due to misreconstruction. Category 5 has a broad \(m_{ES}\) distribution (similar to category 4) and an approximately linear \(\Delta E\) shape. Category 6 has combinatorial distributions for both \(m_{ES}\) and \(\Delta E\). The continuum background shape is combinatorial and does not peak strongly in either \(\Delta E\) or \(m_{ES}\). A summary of the backgrounds in given in Table I.

**MAXIMUM LIKELIHOOD FIT**

We perform an extended unbinned maximum likelihood fit using the variables \(\Delta E\), \(m_{ES}\) and the DP co-ordinates in order to determine the signal yield and the properties of the Dalitz plot. The complete likelihood function is given
TABLE I: Summary of backgrounds. For each category the dominant contributing mode and the expected number of events after all selection requirements are applied to the data are given.

| Category | Dominant contribution | Total # Expected |
|----------|-----------------------|------------------|
| $B\bar{B}$ 1 | $J/\psi K^+\pi^-$ | 444 ± 24 |
| $B\bar{B}$ 2 | $a_1^{\pm} \pi^+$ | 32 ± 7 |
| $B\bar{B}$ 3 | $D^0 K^+\pi^-$ | 240 ± 18 |
| $B\bar{B}$ 4 | $D^0\rho^+$ | 7415 ± 101 |
| $B\bar{B}$ 5 | $D^\ast_0 \pi^+$ | 1475 ± 44 |
| $B\bar{B}$ 6 | Combinatoric | 7336 ± 99 |
| $q\bar{q}$ | | 5352 ± 226 |

by:

$$L = \exp \left( -\sum_k N_k \right) \prod_i \left[ \sum_k N_k \mathcal{P}_k(m^2_{+i}, m^2_{-i}, m_{\text{ES}i}, \Delta E^i) \right],$$

(2)

where $N_k$ is the event yield for species $k$, the index $i$ runs over the $N_e$ events in the data sample and $\mathcal{P}_k$ is the probability density function (PDF) for species $k$, which consists of a product of the DP, $m_{\text{ES}}$ and $\Delta E$ PDFs. The different species $k$ are signal, $q\bar{q}$ background and six $B\bar{B}$ background categories. The function $-\ln L$ is minimized to obtain the preferred values of the free parameters of the fit.

For each of the $B\bar{B}$ background categories, the $\Delta E$, $m_{\text{ES}}$ and DP PDFs are described with histograms obtained using MC. For $q\bar{q}$ background, the $\Delta E$ and $m_{\text{ES}}$ PDFs are a 1st-order polynomial and an ARGUS function [27], respectively. The parameters of the ARGUS function are fixed to values determined using off-peak data, while the slope of the $q\bar{q}$ $\Delta E$ PDF is a free parameter of the fit. The continuum background DP PDF is modelled with a histogram obtained from data in a sideband region of $m_{\text{ES}}$, after subtraction of the (MC-based) expected contribution from $B\bar{B}$ decays in this region. We have verified the consistency of our background PDFs in off-peak data, in background MC samples, and in on-peak data sidebands. All histograms used in the fit are in the square Dalitz plot format.

The signal component is composed of two parts which are distinguished by whether or not the kinematics of the daughter particles are well reconstructed. We refer to the well reconstructed events as “correctly reconstructed” (CR) and the misreconstructed events as “self-cross-feed” (SCF). The fraction of SCF events as a function of DP position $f_{\text{SCF}}(m^2_{+}, m^2_{-})$ is determined from MC, and is shown in Figure 2. Its value is typically below 10% but is larger in the corners of the Dalitz plot where one of the pions has low momentum.

Both CR and SCF events have the same underlying physics PDF, but due to misreconstruction SCF events have reconstructed DP positions that differ from their true values. This smearing is implemented by convoluting the PDF with a resolution function $R_{\text{SCF}}(m^2_{+}, m^2_{-}; \tilde{m}^2_{+}, \tilde{m}^2_{-})$ that gives the probability that an event with true DP position $(\tilde{m}^2_{+}, \tilde{m}^2_{-})$ is reconstructed at $(m^2_{+}, m^2_{-})$, and is described by a histogram in the square Dalitz plot co-ordinates that is itself a function of position in the phase-space. For correctly reconstructed events, DP resolution effects are negligible.
The signal Dalitz plot PDF is thus written as

$$P_{\text{sig}}(m_+^2, m_-^2) = \frac{1}{N} \left\{ P_{\text{phys}}(m_+^2, m_-^2) \epsilon(m_+^2, m_-^2) (1 - f_{\text{SCF}}(m_+^2, m_-^2)) + \right\}$$

$$\int_{\text{DP}} \left[ P_{\text{phys}}(\tilde{m}_+^2, \tilde{m}_-^2) (1 - f_{\text{SCF}}(\tilde{m}_+^2, \tilde{m}_-^2)) R_{\text{SCF}}(m_+^2, m_-^2; \tilde{m}_+^2, \tilde{m}_-^2) d(\tilde{m}_+^2) d(\tilde{m}_-^2) \right] ,$$

where $P_{\text{phys}}(m_+^2, m_-^2)$ is the underlying physics PDF (discussed below), $\epsilon(m_+^2, m_-^2)$ is the efficiency (Figure 1), and $f_{\text{SCF}}(m_+^2, m_-^2)$ is the SCF fraction (Figure 2). The integral is over the Dalitz plot. The normalization factor $N$ ensures that $P_{\text{sig}}(m_+^2, m_-^2)$ gives unity when integrated over the phase-space.

The CR and SCF signal events have different distributions in $\Delta E$ and $m_{\text{ES}}$. For $m_{\text{ES}}$, both CR and SCF PDFs are described by double Gaussian functions where the widths of the two Gaussians are constrained to be the same. For $\Delta E$, the CR PDF is again a double Gaussian function (in which the two Gaussians have different widths) while the SCF PDF is represented by an asymmetric Gaussian with power-law tails. The two Gaussian widths of the $\Delta E$ PDF for the CR component are given by linear functions of $(m_{\text{ES}}^2)^{\text{min}} = \min(m_+^2, m_-^2)$ to account for the momentum dependence of the resolution across the DP. All SCF PDF parameters are fixed to values determined from MC, while CR PDF parameters are floated in the fit where possible. The CR PDF parameters that cannot be determined from the fits are determined from MC. Data/MC correction factors determined from the $D^*(2010)^-\pi^+$ control sample are applied to all such parameters, except for the slopes of the dependence of the $\Delta E$ widths on $(m_{\text{ES}}^2)^{\text{min}}$.

We determine a nominal signal DP model using information from previous studies of $B^0 \rightarrow \overline{D}^0 \pi^+\pi^-$ and $B^+ \rightarrow D^-\pi^+\pi^+$, and the change in the fit likelihood value observed when omitting or adding resonances. We use the isobar model [22,30], which models the total amplitude as resulting from a sum of amplitudes from the individual decay channels:

$$P_{\text{phys}}(m_+^2, m_-^2) = |A(m_+^2, m_-^2)|^2 \quad \text{where} \quad A(m_+^2, m_-^2) = \sum_{j=1}^{N} c_j F_j(m_+^2, m_-^2) ,$$

where $F_j(m_+^2, m_-^2)$ are the dynamical amplitudes and $c_j$ are complex coefficients describing the relative magnitude and phase of the different decay channels. All the weak phase dependence is contained in the $c_j$ coefficients, which we express in terms of their real and imaginary parts: $c_j = x_j + iy_j$, so $F_j(m_+^2, m_-^2)$ contains kinematics and strong dynamics only. We treat the $\overline{D}^0 \rightarrow K^+\pi^-$ decay as flavour-specific and neglect contributions from the doubly-Cabibbo-suppressed $D^0 \rightarrow K^+\pi^-$ decay. We assume direct CP violation is negligible and hence use the same model for $B^0 \rightarrow \overline{D}^0\pi^+\pi^-$ and its conjugate decay. We also neglect possible contributions from $b \rightarrow u$ mediated, and hence highly suppressed, transitions (e.g. $B^0 \rightarrow D(2460)^+\pi^+$).

In the $D\pi$ spectrum previous studies [22,24] have observed contributions from $D_s^+(2460)$ and $D_s^0(2400)$, as well as the effect of a virtual $D^*$ ($D_s^*(2010)$) amplitude. The latter amplitude is described as virtual since although the region around the narrow $D^*(2010)$ pole is vetoed, off-shell production can contribute to the amplitude – the effect is similar to a nonresonant P-wave term. We find that an additional nonresonant (S-wave) $D\pi$ contribution is necessary to fit the data; we describe the nonresonant (NR) term using an empirical shape, first introduced in Ref. 31, proportional to $e^{-\alpha m^2}$, where the shape parameter is determined from the data to be $\alpha = 0.60 \pm 0.15$ (statistical uncertainty only). In the $\pi^+\pi^-$ spectrum previous studies [22] have observed contributions from $\rho(770)^0$ and $f_2(1270)$. We find it is necessary to include S-wave terms and hence include a contribution using the K-matrix formalism [32,33], described in more detail in the Appendix. To our knowledge, this is the first use of the K-matrix formalism in $B$ meson decays. All other resonances are described using relativistic Breit–Wigner (RBW) shapes, with Blatt–Weisskopf barrier form factors [33] and angular distributions given in the Zemach tensor formalism [36,37]. The Dalitz plot formalism used in this analysis is the same as that described in more detail in several previous publications [38,41]. The masses and widths of all resonances are constrained to world-average values [26], while K-matrix parameters are fixed to the values tabulated in the Appendix.

In total there are 43 free parameters of the fit. These are the yields of signal, $q\overline{q}$ and the 6 $B\overline{B}$ background categories; the real and imaginary parts of 5 intermediate contributions to the signal DP model (not counting those of $D_s^+(2460)^-\pi^+$ which are fixed as a reference); the real and imaginary parts of 10 complex coefficients in the production vector of the K-matrix parametrization of the $\pi^+\pi^-$ S-wave; 2 parameters each of the CR signal $\Delta E$ and $m_{\text{ES}}$ PDFs and the slope of the continuum $\Delta E$ distribution.
RESULTS

The fit returns $5098 \pm 102$ signal events. For this and all other quantities the statistical uncertainties are calculated from an MC study where the events are generated from the PDFs and the PDF parameters are the central values from the fit to data. Yields of the various background categories are broadly in line with expectation, although there appears to be some cross-feed between $B\bar{B}$ categories. Projections of the fit result onto $m_{ES}$ and $\Delta E$ are shown in Figure 3 while projections onto each of the two-particle invariant masses are shown in Figure 4 and projections onto the cosines of the helicity angles, defined as the direction of one of the two daughters of the resonance relative to the direction of the third particle in the rest frame of the resonance, are shown in Figure 5. The signal distribution across the phase-space, in both conventional and square Dalitz plot co-ordinates, calculated using the $\chi^2$Plot technique [42], is shown in Figure 6. Structures due to the $D_s^0(2460)^-$, $\rho(770)^0$ and $f_2(1270)$ resonances are clearly visible.

Figures 4 and 5 show that our DP model gives an excellent representation of the data in most regions of the Dalitz plot. The only region where discrepancies between the data and the fit result are apparent is at low values of $m_-$, where a sharp rise near threshold is observed. This structure also appears as a reflection in the $m_+$ and $\cos \theta_-$ distributions. We discuss this further when we consider model uncertainties, below.

We calculate the fit fractions and interference fit fractions, shown as a matrix in Table III. The fit fractions are the elements along the diagonal, and are given by

$$FF_j = \frac{\int_{DP} |c_j F_j(m^2_+, m^2_-)|^2 d(m^2_+)d(m^2_-)}{\int_{DP} \left| \sum_j c_j F_j(m^2_+, m^2_-) \right|^2 d(m^2_+)d(m^2_-)}, \tag{5}$$

while the interference fraction are the off-diagonal elements and are given by

$$FF_{ij} = \frac{\int_{DP} 2 \text{Re} \left[ c_i c^*_j F_i(m^2_+, m^2_-) F^*_j(m^2_+, m^2_-) \right] d(m^2_+)d(m^2_-)}{\int_{DP} \left| \sum_j c_j F_j(m^2_+, m^2_-) \right|^2 d(m^2_+)d(m^2_-)}, \tag{6}$$

for $i < j$ only. Note that, with this definition, $FF_{jj} = 2FF_j$. These give a convention independent representation of the population of the DP. Although the sum of fit fractions can be greater than unity – in this case it is $(148 \pm 5)\%$ (statistical uncertainty only) – the sum including interference fit fractions must be identically equal to one. The largest interference effect is between $D_0^0(2400)^- \pi^+$ and the $D\pi$ nonresonant amplitude.

In Table III we give results for the branching fractions. The inclusive $B^0 \to D^0 \pi^+\pi^-$ branching fraction is calculated by dividing the signal yield by the average efficiency determined from the nominal model, by the number of $B\bar{B}$ pairs in the data sample, and by the branching fraction for the $D$ decay ($B(D^0 \to K^+\pi^-) = (3.91 \pm 0.05) \times 10^{-2}$ [26]). The average efficiency is found to be $30.6\%$ and is further corrected for the measured data/MC differences (discussed under systematic uncertainties below). Our result compares well to that of Belle: $B(B^0 \to D^0 \pi^+\pi^-) = (8.4 \pm 0.4 \pm 0.8) \times 10^{-4}$ [22]. The product branching fractions for the contributing decay modes are obtained by multiplying the
the total background, the black dot-dashed lines show the signal, and the blue solid lines show the total fit result. The points with error bars show the data, the red dotted lines show the continuum background, the green dashed lines show and a much larger branching fraction for $D_{0}^{*}(2400)^{-}$ since, although decay modes other than $D\pi$ have been seen, the relative branching fractions are not known. The $D_{0}^{*}(2400)$ has only been observed to decay into $D\pi$, but it may be presumptuous to conclude that its branching fraction is 100%. Our results for $D_{0}^{*}(2400)^{-}\pi^{+}$ and $f_{2}(1270)D^{0}$ are consistent with those of Belle, while we see a somewhat larger branching fraction for $D_{1}^{*}(2010)^{-}\pi^{+}$ and a much larger branching fraction for $D_{0}^{*}(2400)^{-}\pi^{+}$ (Belle measures $B(B^{0} \rightarrow D_{0}^{*}(2400)^{-}\pi^{+}) \times B(D_{0}^{*}(2400)^{-} \rightarrow \bar{D}^{0}\pi^{-}) = (0.60 \pm 0.13 \pm 0.15 \pm 0.22) \times 10^{-4}$).

### TABLE II: Matrix of fit fractions and interference fractions (central values only without uncertainties).

|                  | $D_{0}^{*}(2400)^{-}\pi^{+}$ | $D_{0}^{*}(2400)^{0}\pi^{+}$ | $\rho(770)^{0}D^{0}$ | $f_{2}(1270)D^{0}$ | $D_{0}^{*}(2010)^{-}\pi^{+}$ | $D\pi$ NR | K matrix |
|------------------|------------------------------|------------------------------|----------------------|---------------------|-----------------------------|-----------|----------|
| $D_{0}^{*}(2400)^{-}\pi^{+}$ | 0.2047                       | -                            | -                    | -                   | -                           | -         | -        |
| $D_{0}^{*}(2400)^{0}\pi^{+}$ | 0.0000                       | 0.2481                       | -                    | -                   | -                           | -         | -        |
| $\rho(770)^{0}D^{0}$       | -0.0133                      | 0.0264                       | 0.3343               | -                   | -                           | -         | -        |
| $f_{2}(1270)D^{0}$         | -0.0130                      | 0.0223                       | 0.0000               | 0.0983              | -                           | -         | -        |
| $D_{0}^{*}(2010)^{-}\pi^{+}$| 0.0000                       | -0.0001                      | -0.0565              | -0.0347             | 0.1579                      | -         | -        |
| $D\pi$ NR                 | 0.0000                       | -0.2471                      | -0.0246              | -0.0458             | 0.0001                     | 0.1844    |          |
| K matrix                  | 0.0019                       | -0.0672                      | 0.0000               | -0.0003             | -0.0016                    | -0.0303   | 0.2559   |

We consider the following systematic effects on the values of the fit fractions.

- **Fixed shapes of the efficiency, $q\overline{q}$ and $B\overline{B}$ Dalitz-plot histograms:**
  The contents of all bins of square Dalitz plot histograms used to describe these shapes are fluctuated in accordance with the uncertainties. This procedure is repeated many times and the RMS of the distribution of the change in the fit results is taken as the associated systematic uncertainty.

- **Fixed $m_{BS}$ and $\Delta E$ PDF parameters (or histograms):**
  We vary any fixed parameters in the PDF descriptions by their uncertainties, taking correlations into account.

### SYSTEMATIC UNCERTAINTIES

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FIG. 5: Projections onto cosines of helicity angles. The signal component has been enhanced in all plots by additional cuts $(5.270\text{ GeV}/c^2 < m_{ES} < 5.282\text{ GeV}/c^2$ and $|\Delta E| < 20\text{ MeV}$). The top row shows the projection onto the cosine of the $\pi^+\pi^-$ helicity angle in the regions (left) around the $\rho(770)$, and (right) around the $f_2(1270)$. The bottom row shows the projection onto the cosine of the $D\pi$ helicity angle in the regions (left) below and (right) around the $D^*_s(2460)$. The points with error bars show the data, the red dotted lines show the continuum background, the green dashed lines show the total background, the black dot-dashed lines show the signal, and the blue solid lines show the total fit result.

FIG. 6: $s$Plots of the signal distribution in the (left) Dalitz plot and (right) square Dalitz plot.

The variation in the fit results is taken as the systematic uncertainty. For most parameters, their values and uncertainties are determined from data control samples. An exception is the self-cross-feed fraction which is obtained from Monte Carlo. To conservatively allow for possible data/MC differences in the behaviour of the SCF component, we apply a Dalitz-plot independent scale factor that alternately increases and decreases the SCF fraction by a factor of two, and take the larger difference compared to the nominal result as the uncertainty. The contents of the histograms used to describe the $B\bar{B}$ background $m_{ES}$ and $\Delta E$ PDFs are varied using the same prescription as described above.

• Fit bias:
  We generate large ensembles of pseudo-experiments, containing fully simulated signal events, using the parameters returned by the fit to data. From the distribution of results of these ensembles, we evaluate biases on the fit parameters. All biases are found to be small compared to the statistical uncertainties. We assign systematic uncertainties of the sum in quadrature of half the bias and its uncertainty.
A summary is given in Table IV. The total model uncertainty is obtained by combining all sources in quadrature.

These sources of systematic uncertainty are summarized in Table IV. The total is obtained by combining all sources in quadrature.

We consider additional systematic effects on the values of the branching fractions. These are uncertainties on the differences between the efficiencies of selection requirements on data and MC for tracking (1.0%), particle identification (4.0%), the neural network cut (3.2%), and the number of $B\bar{B}$ pairs (0.6%). Furthermore, where we have divided by a daughter branching fraction in order to isolate the $B$ decay branching fraction, any uncertainty in the world average value used in the division also contributes systematic uncertainty.

An additional source of uncertainty in Dalitz-plot analyses arises due to the composition of the Dalitz plot. We consider the following sources of model uncertainty:

- Fixed parameters of contributing amplitudes:
  We vary the masses and widths of all resonances described by RBW shapes according to the uncertainties of the world average values \(20\) (with the exception of the $D_0^*(2400)$ mass, which we vary by $\pm100$ MeV/$c^2$ to account for the discrepancy in the measured masses of charged and neutral isospin partners). We vary the $\alpha$ parameter of the $D\pi$ nonresonant contribution within its uncertainty. We change the radius parameter of the $\omega$ peak in the $D\pi$ background category, and include the deviation in the results as a source of model uncertainty.

- Alternative parameterisations:
  We use the Gounaris–Sakurai lineshape \(13\) as an alternative description for the $\rho(770)$ resonance. We replace the $\pi^+\pi^-$ S-wave K-matrix term with contributions used in the analysis of $B^0 \rightarrow D^0\pi^+\pi^-$ by Belle \(22\), namely $\sigma$ (described as in Ref. \(44\)), $f_0(980)$ (described by the Flatté distribution \(13\)) and $f_0(1370)$ (RBW). To address the possible discrepancy between the data and the fit result at low values of $m_\pi$, we replace the $D\pi$ nonresonant contribution with a functional form proposed for a putative “dabba” state \(40\). We have also performed a fit in which the background from $D^{*-}(2010)\pi^+$ events escaping the veto is treated as a separate (seventh) $B\bar{B}$ background category, and include the deviation in the results as a source of model uncertainty.

- Additional possible contributions:
  We repeat the fit adding states to the model: $\omega(782)$, $\rho(1450)$, $D(2600)$ (both as scalar and vector) and $D(2760)$ (vector) \(17\).

A summary is given in Table IV. The total model uncertainty is obtained by combining all sources in quadrature.
Isospin symmetry can be used to relate the decay amplitudes of $B^0 \to \bar{D}^0 \rho^0$, $B^0 \to D^- \rho^+$ and $B^+ \to \bar{D}^0 \rho^+$:\n
\begin{align}
A(\bar{D}^0 \rho^+) &= \sqrt{3} A_{3/2}, \\
A(D^- \rho^+) &= \sqrt{1/3} A_{3/2} + \sqrt{2/3} A_{1/2}, \\
\sqrt{2} A(\bar{D}^0 \rho^0) &= \sqrt{4/3} A_{3/2} - \sqrt{2/3} A_{1/2},
\end{align}

where $A_{3/2}$ and $A_{1/2}$ are the amplitudes for isospin 3/2 and 1/2 final states respectively. These equations give the triangle relation

\begin{equation}
A(\bar{D}^0 \rho^+) = A(D^- \rho^+) + \sqrt{2} A(\bar{D}^0 \rho^0).
\end{equation}

This relation can be used to determine $\cos \delta_{D\rho}$, where $\delta_{D\rho}$ is the phase between the $A_{3/2}$ and $A_{1/2}$ amplitudes, and $R_{D\rho} = |A_{1/2}/\sqrt{2} A_{3/2}|$. In QCD factorization, both of these are expected to be unity up to corrections due to final state interactions of $\mathcal{O}(A_{QCD}/m_Q)$, where $A_{QCD}$ is the QCD scale and $m_Q$ is either $m_c$ or $m_b$\cite{11}. We obtain constraints on these parameters using the same approach previously used in the $D^{(*)}\pi$ system\cite{50,51}. Using our result for $\mathcal{B}(B^0 \to \bar{D}^0 \rho^0)$, together with world average values of $\mathcal{B}(B^0 \to D^- \rho^+)$, $\mathcal{B}(B^+ \to \bar{D}^0 \rho^+)$ and the ratio of lifetimes $\tau(B^+)/\tau(B^0)$\cite{24}, we find

$$
\cos \delta_{D\rho} = 0.998^{+0.133}_{-0.062},
$$

$$
R_{D\rho} = 0.68^{+0.15}_{-0.16},
$$

where all sources of uncertainty are combined. These results suggest the presence of non-factorizable final state interaction effects that, in contrast to the $D^{(*)}\pi$ system, do not introduce a significant non-zero phase difference between the isospin amplitudes.

**SUMMARY**

We have performed a Dalitz-plot analysis of $B^0 \to \bar{D}^0 \pi^+ \pi^-$ decays using the whole BaBar dataset of 470.9 ± 2.8 million $B\bar{B}$ events. We measure the inclusive branching fraction

$$
\mathcal{B}(B^0 \to \bar{D}^0 \pi^+ \pi^-) = (8.81 \pm 0.18 \pm 0.76 \pm 0.78 \pm 0.11) \times 10^{-4}
$$
where the first uncertainty is statistical, the second is systematic, the third is due to the Dalitz-plot model, and the fourth is due to secondary branching fractions. We find the Dalitz plot to be composed of contributions from $D_s^+(2460)^-, D_s^0(2400)^-, \rho(770)^0$ and $f_2(1270)$ as well as a $\pi^+\pi^-\pi^-$ S-wave, a $D\pi$ nonresonant S-wave term and a virtual $D_s^0(2010)^-$ contribution. We determine their branching fractions:

$$
B(B^0 \to D_s^+(2460)^-\pi^+) \times B(D_s^+(2460)^- \to D^0\pi^-) = (1.80 \pm 0.09 \pm 0.19 \pm 0.37 \pm 0.02) \times 10^{-4},
$$

$$
B(B^0 \to D_s^0(2400)^-\pi^+) \times B(D_s^0(2400)^- \to D^0\pi^-) = (2.18 \pm 0.23 \pm 0.33 \pm 1.15 \pm 0.03) \times 10^{-4},
$$

$$
B(B^0 \to \rho(770)^0 D^0) = (2.98 \pm 0.19 \pm 0.53 \pm 0.93 \pm 0.04) \times 10^{-4},
$$

$$
B(B^0 \to f_2(1270) D^0) = (1.02 \pm 0.12 \pm 0.18 \pm 0.36 \pm 0.03) \times 10^{-4}.
$$

Our Dalitz plot model differs from that obtained in a previous study of $B^0 \to D^0\pi^+\pi^-$ by Belle \cite{22} in that (i) we use the K-matrix description of the $\pi^+\pi^-\pi^-$ S-wave, instead of including separate contributions from the $f_0(600)$ ($\sigma$), $f_0(980)$ and $f_0(1370)$ scalar resonances; (ii) we include an additional $D\pi$ nonresonant S-wave term. Our results for the inclusive branching fraction and for the color-suppressed decays $B^0 \to \rho(770)^0 D^0$ and $B^0 \to f_2(1270) D^0$ are consistent with those from Belle and (for $\rho(770)^0 D^0$) with theoretical predictions \cite{12,52}. However, we find the product branching fractions for the broad and narrow $D^{*+}$ states ($D_s^0(2400)$ and $D_s^2(2460)$, respectively) to have similar values. This result disagrees with the analysis by Belle, which found a much smaller value for the $D_s^0(2400)$ branching fraction.

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Appendix: K-matrix description of $\pi^+\pi^- S$ wave

The K-matrix formalism gives a physical description of broad overlapping states – i.e. it does not violate unitarity, unlike the more conventional "sum of Breit–Wigners" approach. The K-matrix formalism can be shown to reduce to more familiar forms (the Breit–Wigner lineshape for single resonances, the Flatté lineshape [45] for coupled channels, the K-matrix formalism can be shown to reduce to the LASS formula [53] for broad resonances interfering with nonresonant terms). Detailed descriptions of the K-matrix formalism can be found in various references [32–34, 54]. Here we give an outline of the salient features.

The scattering ("S") matrix describes transitions from initial states $|i\rangle$ into final states $|f\rangle$ ($S_{if} = \langle f|S|i\rangle$), and can be written

$$S = I + 2i\{\rho^i\}^{1/2}T\{\rho\}^{1/2},$$

where $I$ is the identity matrix, $\rho$ is a diagonal phase-space matrix, with elements $\rho_{ii} = 2q_i/m$ with $q_i$ the threshold momentum, and $T$ is the transition matrix. The unitarity requirement ($SS^\dagger = S^\dagger S = I$) gives

$$(T^{-1} + i\rho)^\dagger = T^{-1} + i\rho, \quad \Rightarrow \quad K^{-1} = T^{-1} + i\rho,$$

where $K$ is a Lorentz-invariant and Hermitian matrix which describes the decay process. This formalism was developed for scattering processes, but can also be applied to Dalitz plot analyses, with the assumption that the two "scattering" products do not interact with the third bachelor particle [53]. However, it is also necessary to include a process-dependent production vector, which accounts for the relative production rates of the different states $|i\rangle$. We refer to the "K-matrix amplitude" as a product of the production vector $P$ and the (matrix) propagator $(I - iK\rho)^{-1}$:

$$A_i = (I - iK\rho)^{-1}_{ij} P_j$$

(A.13)
The K matrix is expressed as

\[
K_{ij}(s) = \left[ f_{ij}^{\text{scatt}} \frac{1 - s_{ij}^{\text{scatt}}}{s - s_{ij}^{\text{scatt}}} + \sum_{\alpha} g_{ij}^{(\alpha)} \frac{g_{ij}^{(\alpha)}}{m_{\alpha}^2 - s} \right] \left\{ \frac{1 - s_{\pi 0}}{s - s_{\pi 0}} \left( s - \frac{s_{\pi 0} m_{\pi}^2}{2} \right) \right\},
\]  
(A.14)

where the factor \( g_{ij}^{(\alpha)} \) is the real coupling constant of the K matrix pole \( \alpha \) (with mass \( m_{\alpha} \)) to meson channel \( i \), the parameters \( f_{ij}^{\text{scatt}} \) and \( s_{ij}^{\text{scatt}} \) describe a smooth part for the K-matrix elements, and the last factor accounts for the so-called “Adler zero”, and suppresses kinematically fake singularities near \( \pi^+\pi^- \) production threshold \( s \) represents the square of the \( \pi^+\pi^- \) invariant mass). The K-matrix parameters are determined from global fits to scattering data experiments below 1900 MeV/c^2 [34]. Note that the phase-space for \( B^0 \to \bar{D}^0 \pi^+\pi^- \) extends beyond this limit, and that the K-matrix amplitude in this high-\( \pi^+\pi^- \) invariant mass region is therefore an extrapolation.

The parameters unique to the production vector, by contrast, must be determined from our data. The \( P \) vector is given by

\[
P_j(s) = \left[ f_{ij}^{\text{prod}} \frac{1 - s_{ij}^{\text{prod}}}{s - s_{ij}^{\text{prod}}} + \sum_{\alpha} \beta_{ij} g_{ij}^{(\alpha)} \right] \left( s - \frac{s\pi 0 m_{\pi}^2}{2} \right),
\]  
(A.15)

where as before the first term in the square brackets is nonresonant-like (“slowly varying”), and the second term is resonant-like. Hence the free parameters in the Dalitz plot fit are the complex coupling and production vector parameters \( \beta_{ij} \) and \( f_{ij}^{\text{prod}} \) (we use a fixed value of \( s_{ij}^{\text{prod}} \)). The index \( j \) runs over the open channels for the \( \pi\pi \) S-wave, which are: \( \pi\pi, KK, \eta\eta, \eta'\eta' \) and \( 4\pi \) (or multi-meson). At higher masses there are in principle more open channels, but this is not expected to affect the results significantly. Global fits to the scattering data determine the number of poles and their parameters. We use a 5 pole approximation, and give the values of all fixed parameters in the K-matrix model in Table VI. Note that all \( f_{ij}^{\text{prod}} = 0 \) for \( i \neq 1 \) since we are interested only in the \( \pi\pi \) final state.

**TABLE VI:** K-matrix parameters from a global analysis of the available \( \pi\pi \) scattering data from threshold up to 1900 MeV/c^2 [34, 55]. Masses and coupling constants are given in GeV/c^2.

| \( m_{\alpha} \) | \( g_{\pi\pi}^{\alpha} \) | \( g_{KK}^{\alpha} \) | \( g_{\eta\eta}^{\alpha} \) | \( g_{\eta'\eta'}^{\alpha} \) | \( g_{4\pi}^{\alpha} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.65100        | 0.22889        | -0.55377       | 0.00000        | -0.39899       | -0.34639       |
| 1.20360        | 0.94128        | 0.55095        | 0.00000        | 0.39065        | 0.31503        |
| 1.55817        | 0.36856        | 0.23888        | 0.55639        | 0.18340        | 0.18681        |
| 1.21000        | 0.33650        | 0.40907        | 0.85679        | 0.19906        | -0.00984       |
| 1.82206        | 0.18171        | -0.17558       | -0.79658       | -0.00355       | 0.22358        |
| \( f_{ij}^{\text{scatt}} \) | \( f_{ij}^{\text{prod}} \) | \( f_{ij}^{\text{scatt}} \) | \( f_{ij}^{\text{prod}} \) | \( f_{ij}^{\text{scatt}} \) | \( f_{ij}^{\text{prod}} \) |
| 0.23399        | 0.15044        | -0.20545       | 0.32825        | 0.35412        |
| \( s_{ij}^{\text{scatt}} \) | \( s_{ij}^{\text{prod}} \) | \( s_{\pi 0} \) | \( s_{\pi 0} \) | \( s_{\pi 0} \) | \( s_{\pi 0} \) |
| -3.92637       | -3.0           | -0.15          | 1              | 1              | 1              |