ON THE UNDERSTANDING AND USE OF “UNINTEGRATED” PARTON DISTRIBUTIONS IN SMALL-X QCD

EMIL AVSAR
Department of Physics, Penn State University, University Park, PA 16802, USA

We review and discuss the use of TMD, or “unintegrated”, gluon distributions in the domain of small-x physics. The definitions employed, and the hazards of the naive applications of the TMD factorization and the associated gluon distributions are discussed.

Keywords: Quantum Chromodynamics; Hadron Colliders; High Energy QCD

PACS numbers: 12.38.Aw, 12.38.Bx, 24.85.+p

1. Introduction

The concept of transverse-momentum-dependent (TMD), or $k_T$-dependent, or “unintegrated”, parton distributions is frequently encountered in the QCD literature. This is so particularly in the small-$x$ domain where they play a prominent role. These distributions are to be contrasted with the integrated parton distributions (parton distribution functions, pdfs) which play a fundamental role in global QCD fits.

For the $k_T$-dependent distributions there is no unique definition, and as a consequence it is for the phenomenology of these quantities very important to relate the different definitions. We shall therefore here analyze some of the most commonly found definitions in the literature.

Before going any further, we should mention that there exist two different and distinct sets of users of these distributions, each with corresponding domains of application:

- One area we might characterize as “hard scattering factorization”, where the primary object of attention is a hard scattering factorization property. An example is the Drell-Yan process at low transverse momentum.
- The other area is that of small $x$, DIS with $x \ll 1$ being a typical example. Here the emphasis starts with the BFKL formalism (or a generalization of it) for scattering processes in the Regge region.

We are here primarily interested in the small-$x$ domain. The theory of the “hard scattering” domain is in great detail explained in [1] (see also [2] for a short review), and a compact presentation is also given in the talk by John Collins [3]. We
also note that the terminology “TMD” distribution is usually used in the “hard scattering factorization” while traditionally in the small-$x$ domain one speaks of “$k_\perp$-dependent”, or more commonly of “unintegrated” parton distributions.

2. The parton model as background

In the parton model, the concept of a parton distribution quantifies the intuitive expectation of a number density of partons of given flavor in the target hadron. A mathematically exact definition can be given in the light-front quantization as

$$f_{j/h}(x, k_\perp) = \sum_\alpha \frac{1}{2x(2\pi)^3} \frac{\langle P, h | a^\dagger_{k_\perp, \alpha, j} a_{k_\perp, \alpha, j} | P, h \rangle}{\langle P, h | P, h \rangle}.$$ (1)

Here $j$ and $h$ label parton and hadron flavor, $\alpha$ is a parton helicity index, $\langle P, h \rangle$ is the target state of momentum $P$, and $a$ and $a^\dagger$ are parton light-front annihilation and creation operators. We use light-front coordinates, the hadron target has zero transverse momentum, and $x = k^+/P^+$. Integrating Eq. (1) over all $k_\perp$ one obtains the integrated distribution

$$f_{j/h}(x) = \int_{\text{all } k_\perp} d^2k_\perp f_{j/h}(x, k_\perp).$$ (2)

The exactness of the relation, however, depends on the specific assumptions of the parton model which no longer hold in full QCD.

A superficial glance over the relevant literature reveals that the number density interpretation of the TMD, $k_\perp$-dependent, and unintegrated parton distributions is taken rather literally also in QCD. The actual technical definitions in the different cases, however, may or may not conform to the idea of a number density. We also note that the parton model relation between the integrated and the TMD distributions in (2) is taken to be true in QCD as well (with the difference that the $k_\perp$ integral is performed only up to the hard scale $Q$). The question then is what exactly the justifications are for these assumptions.

3. The gluon distribution at small-$x$

3.1. BFKL and the dipole picture

The prototype of the QCD applications in the small-$x$ domain is the BFKL formalism \cite{4,5}. Here the $\gamma^*\gamma^*$ scattering amplitude (for $A + B \rightarrow A' + B'$) is in the Regge limit $(s/t \gg 1$ with $t = -q^2_\perp$) written as

$$\frac{\text{Im} A(s,t)}{s} = \int \frac{d^2k_\perp}{k^2_\perp} \frac{d^2k'_\perp}{(k'_\perp - q_\perp)^2} I_A(k_\perp, q_\perp) I_B(k'_\perp, q_\perp) F(s, k_\perp, k'_\perp, q_\perp),$$ (3)

\*Strictly speaking the momentum states $|P\rangle$ do not belong to the ordinary Hilbert space of states since they are not normalizable. One can remedy this by replacing them with wave packets, and then taking appropriate limits as these packets approach the momentum states, but we need not worry about such level of rigor here.
where \( A \) and \( B \) denote the two virtual photons. The dimensionless objects \( I_A \) and \( I_B \) are called the impact factors, while \( F \) is the object satisfying the BFKL equation (Eq. (15) below), and is commonly referred to as the “BFKL Greens’ function”. If by \( \phi_A \) and \( \phi_B \) one denotes the couplings of gluons to the external photons \( A \) and \( B \) (via quark boxes), then to lowest order in the singlet channel in Feynman gauge (two gluon exchange) the amplitude can be written in shorthand notation as

\[
\phi_A^{\mu\nu} \phi_B^{\alpha\beta} g_{\mu\nu} g_{\alpha\beta'} |0\rangle T A^{\mu'} A^{\nu'} A^{\alpha'} A^{\beta'} |0\rangle
\]

\[
\approx_{eik} I_A I_B \langle 0 | T (p_A \cdot A) (p_A \cdot A) (p_B \cdot A) (p_B \cdot A) |0\rangle. \tag{4}
\]

Here \( \approx_{eik} \) indicates the eikonal approximation in which the numerator of the Feynman gauge propagator \( g^{\mu\nu} \) is replaced by \( p_B^\mu p_B^\nu / p_A \cdot p_B \) for gluons coupling to \( A \) (\( B \)). Therefore to lowest order \( F \) is given by the vacuum expectation value of four off-shell gluons. To all orders, the gluon fields have to be summed into eikonal gauge links, i.e. Wilson lines, which make \( F \) gauge invariant.

In the space-time picture by Balitsky \[7\], the \( q\bar{q} \) pair emerging from the electromagnetic currents \( J \) from one of the particles (say \( A \)) travels along a straight line trajectory and scatters off the gluon field created by \( B \) which has a delta function shape, i.e. a shockwave, in the high energy approximation. The propagation of the \( q\bar{q} \) in the external gluon field created by \( B \) is represented by the Wilson lines

\[
U_A(x_\perp) = \exp(-ig \int_{-\infty}^{\infty} d\lambda n_A \cdot A^\alpha (x_\perp + \lambda n_A) t_F^\alpha), \tag{5}
\]

where the color matrix \( t_F^\alpha \) is in the fundamental representation, and \( n_A \) is a unit vector in the direction of \( p_A \). This picture also leads to the so-called dipole formalism, as the \( q\bar{q} \) pair emerging from the current \( J \) is in a color singlet state and thus can be seen as a “color dipole”. The lowest order result in Eq. (4) is in the formulation by Balitsky generalized to

\[
\frac{\text{Im} A(s, t)}{s} = \int d^2k_\perp I_A(k_\perp, q_\perp) \left\langle \text{Tr} \{ U_A^\dagger U_A \} (k_\perp, q_\perp) \right\rangle \tag{6}
\]

where

\[
\left\langle \text{Tr} \{ U_A^\dagger U_A \} (k_\perp, q_\perp) \right\rangle \equiv \int d^2x_\perp e^{-ik_\perp \cdot x_\perp} \left\langle \text{Tr} \{ U_A^\dagger(x_\perp) U_A(0_\perp) \} \right\rangle_{q_\perp}. \tag{7}
\]

The exact definition of the bracket \( \langle O \rangle \) is given in \[7\]. Here it suffices to say that it involves a convolution of the Wilson lines with the currents \( J_B \) and \( J_{B'} \). Notice that the Wilson lines here correspond to the all order summation of the factors \( p_A \cdot A \) in (4), while the fields \( p_B \cdot A \) are kept implicitly in the definition of the bracket. When made explicit they should give rise to an additional pair of Wilson lines \( U_B \) in the direction of \( p_B \).

In formula (3) the main object is the BFKL function \( F \). From (4) we see that one has

\[
\left\langle \text{Tr} \{ U_A^\dagger U_A \} (k_\perp, q_\perp) \right\rangle = \int \frac{d^2k_\perp'}{k_\perp^2 (k_\perp' - q_\perp)^2} I_B(k_\perp', q_\perp) F(s, k_\perp, k_\perp', q_\perp). \tag{8}
\]
The expectation value of the Wilson line pair is essentially taken to be the “gluon distribution” (also called the “dipole gluon distribution”) in the small-$x$ domain. While the formulas here are derived for $\gamma^*\gamma^*$ scattering, a similar identification of the “gluon distribution” is made for other processes as well, such as DIS off a proton or a nucleus, and also in proton-proton ($pp$), proton-nucleus ($pA$), and nucleus-nucleus ($AA$) collisions. For these latter processes we have not been able to find any proofs showing that this is indeed fully legitimate. The definition of the “dipole gluon distribution” is then generally taken to be

$$F_{\text{dip}} = C \int d^2 x_\perp k_\perp^2 e^{-ik_\perp \cdot x_\perp} \langle P, h | \text{Tr}\{U^\dagger(x_\perp)U(0_\perp)\}|P, h \rangle$$  \hspace{1cm} (9)

where we leave the prefactor $C$ unspecified since there does not seem to be a universally accepted value in the literature.\footnote{In fact we have find several incompatible values of $C$ used in different studies of the same processes. Moreover, even the form of (9) varies from case to case so it is hard to pin down any exact universal definition.} We should also mention that one needs to insert transverse gauge links at infinity to make the operators in (6) and (9) exactly gauge invariant.

We notice that the structure of (9) does not really conform to the parton model definition (1) of a number density. Therefore, despite its name, this object does not correspond to the direct generalization of the parton model definition (1). Moreover, the Wilson lines in the definition are always take in the fundamental representation while the generalization of the parton model result to full QCD naturally leads to Wilson lines in the adjoint representation for the gluon distribution (11). It is, however, still common to assert the relation (2) to the integrated pdf, by writing

$$G(x, Q^2) = \int_{Q^2}^{\infty} d^2 k_\perp F_{\text{dip}}(x, k_\perp).$$  \hspace{1cm} (10)

By $G$ we here denote the gluon pdf. For an explanation of the meaning of the parameter $x$ in $F_{\text{dip}}$, see below.

A reason for asserting this result is that the leading twist approximation of the BFKL evolution, which $F_{\text{dip}}$ satisfies, reproduces for $G$, via (10), the DGLAP evolution in the approximation where $n_f = 0$ and where only the $1/z$ term in the splitting function $P_{gg}$ is kept.

Despite this agreement in the simplified limit, it is, however, a bad practice to hold on to the relation (10) for several reasons. One reason is that the partial agreement of the evolution equations does not generally imply that the relation (10) is a priori consistent with the respective operator definitions. The integrated pdf, $G$, which follows from the collinear factorization approach has a clear operator definition (see for example (11)) that cannot simply be reduced to the integral of (9) (or some variant of it). Moreover, it is generally for phenomenology important to include the non-singular parts of the DGLAP splitting function, even at rather small-$x$. Therefore the simplified limit mentioned above is not in practice very useful.

bIn fact we have find several incompatible values of $C$ used in different studies of the same processes. Moreover, even the form of (9) varies from case to case so it is hard to pin down any exact universal definition.
There are also conceptual problems. It is known that the use of light-like Wilson lines in the operator definitions for the TMD distributions gives rise to rapidity divergences which have to be somehow cut-off. A convenient way to cut the divergences is to take the Wilson lines to be non-light-like \[1, 7\], i.e. to let them have finite rapidity. It is actually this cut-off which gives rise to the evolution in rapidity, just as the cut-off \(\mu\) in momenta gives rise to the standard DGLAP evolution with respect to \(\mu\). If we denote the cut-off in rapidity by \(\zeta\), then \(F_{dip}\) depends on \(k_T\) and \(\zeta\), \(F_{dip}(k_T; \zeta)\). It is important to realize that \(\zeta\) is conceptually different than the variable \(x\) appearing in \(f_j/h(x, k_T)\) in the parton model definition (1). In that case \(x\) is an “intrinsic” or kinematical variable, and is literally the light-cone momentum fraction \(x = k^+/P^+\) of the parton which participates in the hard scattering. On the other hand \(\zeta\) is an arbitrary cut-off variable which determines the total rapidity range for the soft gluons associated with \(F_{dip}\), and has no counterpart in the parton model. Now, in the small-\(x\) literature one always sets \(\zeta = x\) (we remind that in DIS \(x = Q^2/2q \cdot P\)) . This is the reason behind the notation \(F_{dip}(x, k_T)\).

The variable \(x\) in the integrated distribution is not really related to the rapidity cut-off in \(F_{dip}\). It is in the integrated distribution a kinematical variable which has the meaning of the total light-cone momenta \(k^+ = xP^+\) exchanged in the \(t\)-channel between the hard scattering and the target hadron. On the other hand in \(F_{dip}\) it arises as the rapidity cut-off which can be implemented by taking the Wilson lines \(U\) to be non-light-like. In fact, the corresponding kinematical variable \(x\) has in the derivation of \(6, 9\) and \(11\) already been set to 0 (that is, the direct analogue of \(x\) in \(11\) has already been set to 0 in \(9\)). A more correct notation for \(F_{dip}\) would therefore be \(F_{dip}(x = 0, k_T; \zeta = x)\). The integrated distribution \(G\) does not have any \(\zeta\) dependence because it does not contain any rapidity divergences. We therefore see that one generally has to be more careful when relating \(G\) and \(F_{dip}\).

### 3.2. The “Weizsacker-Williams” distribution

There is also a different type of gluon distribution found in the small-\(x\) literature which is meant to literally be the analogue of the parton model definition \(11\). This distribution has been dubbed the “Weizsacker-Williams” (WW) gluon distribution, and is in a sense somewhat more closely related to the gluon distribution in the hard scattering factorization approach. A definition is provided in \(8\) which starts from the relation

\[
\frac{dN}{dk^+} = \langle a^\dagger_a(x^+, k) a^\dagger_a(x^+, k) \rangle = \frac{2k^+}{(2\pi)^3} \langle A^+_a(x^+, k) A^+_a(x^+, -k) \rangle
\]

(11)

where \(a\) and \(a^\dagger\) are the light-front gluon field annihilation and creation operators. Noticing that in light-cone gauge, \(A^+ = 0\), one has \(F^{++} = ik^+A^+_a\), it is seen that this definition thus corresponds to the expectation value \(\langle F^{++}_a F^{++}_a \rangle\). Indeed this is the direct generalization of the parton model definition \(11\). The result in a generic
The standard reference to $k_\perp$-factorization in small-$x$ physics is the work by Catani, Ciafaloni and Hautmann (CCH) \cite{bib:CCH}. One studies here the production of heavy $q\bar{q}$ pairs in photoproduction, DIS, and in hadron-hadron collisions, and the main goal is to formulate a TMD factorization formula at small-$x$ which at the same time can in the collinear limit be related to the standard collinear factorization formula.

The factorization formula, based on the properties of the light-cone gauge, is written as (in case of photoproduction)

$$
\sigma_{\gamma g} = \frac{1}{2M^2} \int \frac{dz}{z} \int d^2k_\perp \hat{s}(\rho/z, k_\perp^2/M^2) \mathcal{F}(z, k_\perp),
$$

(13)

where $\rho = 4M^2/s$, and $M$ is the invariant mass of the heavy quark. This formula
is somewhat different than the BFKL formula (3), and is more close in spirit to the hard factorization approach since $\hat{\sigma}$ here plays the role of a hard scattering coefficient. The object referred to as the “unintegrated gluon distribution” can in the light-cone gauge used in [10] be written as [11]

$$F(x, k_{\perp}) = \int \frac{dx^2 dx_{\perp}}{(2\pi)^3} \frac{1}{4P^+} e^{ixP^+x^- - ik_{\perp}x_{\perp}} (P|F_a^{i+}(0^+, x^-, x_{\perp})F_a^{i+}(0)|P). \tag{14}$$

This definition exactly corresponds to the parton model one in (1) to $x_f g/h$, and is also the analogue of the WW distribution in the CGC. If taken literally, however, it leads to rapidity divergences (in this case due to the singular denominator of the light-cone gauge propagator), and consequently it must be modified. The point is, however, that the definition is never actually used in [10]. It is instead stated that $F(z, k_{\perp})$ in (13) is “defined” by the BFKL equation

$$F(x, k_{\perp}, Q_0^2) = \frac{1}{\pi} \delta(k_{\perp}^2 - Q_0^2) + \bar{\alpha}_s \int \frac{d^2q_{\perp}}{\pi q_{\perp}^2} \int \frac{dz}{z} \left( F(x/z, k_{\perp} + q_{\perp}, Q_0^2) - \theta(k_{\perp}^2 - q_{\perp}^2)F(x/z, k_{\perp}, Q_0^2) \right), \tag{15}$$

where $\bar{\alpha}_s \equiv \alpha_s N_c/\pi$. This switch implies that the rapidity divergence is cut off since there is an implicit cut in the BFKL formalism. Effectively one introduces a cut-off $\zeta$, and then sets $\zeta = z$ in (13).

This switch implies a conceptual change which is rather important to understand. As we have noticed, the would be definition (14) in light-cone gauge corresponds to the WW definition. However, once it is instead declared that $F$ is “defined” via the BFKL equation (15), one comes back to (3) and essentially to the dipole definition (9), and the latter does not conform to the parton model idea of a parton distribution. There is therefore an implicit change in the meaning of $F$. Actually, $F$ is now instead replaced by $F$ in (3), that is the “BFKL Green’s function” which itself is not a “gluon distribution”. It is in fact stated in [10] that for any realistic calculation one should rather provide a non-perturbative $Q_0$-distribution of the gluon in the hadron to be convoluted with $F(x, k_{\perp}; Q_0^2)$. What this simply means is that one needs to perform a convolution just as in the right hand side of Eq. (8) ($Q_0$ here corresponds to $k_{\perp}^2$ in (3)).

It is clear, however, that this cannot be directly compatible with the definition (14) that follows from (13), because the operator definition of (9) cannot be reduced to (14) in light-cone gauge. There is moreover another problem. In the set up of [10] which gives (13), the photon actually scatters off a single gluon with momentum $P$. The state $|P\rangle$ in (9) therefore strictly speaking corresponds to an on-shell gluon with momentum $P$, and not to a hadron. The idea is based on the so-called “factorization of mass singularities” which has been used when dealing with collinear factorization

---

$^c$ The cross section in Eq. (13) is therefore really a partonic cross section which is indicated by the subscript $\gamma g$. 
(see for example [12]). In this approach it is first asserted that a hadronic structure function, \( W_{\text{hadron}} \), is a convolution of the corresponding partonic structure function \( W_{\text{parton}} \) and a so-called “bare parton density” \( G_{\text{bare}} \),

\[
W_{\text{hadron}}(q, P) = W_{\text{parton}}(q, p = \xi P) \otimes \xi G_{\text{bare}}(\xi).
\]  \tag{16}

The convolution here in the longitudinal variable \( \xi \) is the same as the \( z \) convolution in \( (13) \). Factorization is then understood as the procedure of extracting out a divergent factor \( D \) from \( W_{\text{parton}} \) to “define” a “renormalized parton distribution”

\[
W_{\text{hadron}} = (\hat{\sigma} \otimes D) \otimes G_{\text{bare}} = \hat{\sigma} \otimes (D \otimes G_{\text{bare}}) = \hat{\sigma} \otimes G_{\text{ren}}.
\]  \tag{17}

For the conceptual problems of this approach in collinear factorization we refer the reader to [1]. In this case the procedure is thus extended to TMD factorization. However, the parameter \( Q_0^2 \) in \( (15) \) implies that the incoming gluon \( P \) is no longer on shell, and the type of convolution \( (8) \) which is supposed to give a definition of the “unintegrated gluon distribution” clearly is rather different than the procedure in Eqs. \( (16) \) and \( (17) \). It is also clear that \( F \) in \( (3) \) is different than \( F \) defined in \( (14) \).

The question here should be: Can we in the small-\( x \) region indeed formulate a formula like in \( (13) \) and then show that it leads to a precise operator definition of the TMD gluon distribution which is related to the WW distribution, with all the subtleties regarding the rapidity divergences taken properly into account? Additionally, can we then properly relate this distribution to the integrated gluon distribution? Moreover, what evolution equation in rapidity will this distribution satisfy? In the TMD approach the rapidity evolution is given by the so-called CSS evolution [1], while in the formalisms studied here by the BFKL equation. One should investigate the exact connection between the two.

4. Speakable and unspeakable

As we have seen above, the idea and concept of the “unintegrated gluon distribution” in small-\( x \) QCD is still intuitively based very much on the parton model definition of a number density, but we have also seen that the “dipole gluon distribution” does not conform to this idea, even if it is intuitively thought of being so (and repeatedly said being so). The parton model obviously provides a very useful intuitive guidance in full QCD, but unnecessary confusion, and even wrong results, can easily arise if one is not careful.

In the historical development of QCD it was of course the approximative Bjorken scaling observed in the SLAC-MIT data, together with the successes of the quark-gluon model, which eventually led to the identification of partons with quarks and gluons, thus providing the latter an ontological commitment as real entities (rather than simply being abstract mathematical objects).

The scaling violations of QCD modify the naive parton model and the definition of the parton distribution functions which in the parton model are strict number
On the understanding and use of “unintegrated” parton distributions in small-$x$ QCD

9

The parton model is, however, not completely thrown away, but the full QCD result rather corresponds to an improvement, without too dramatic or violent differences. Very important in this is, however, the existence of factorization theorems in QCD (for a number of elementary processes). For assume that factorization did not hold (its existence requires after all non-trivial proofs). In that case the concept of parton distributions would not be useful at all, and it would not be possible to gain any real knowledge of the underlying entities, the partons, using them. In particular since partons are not directly observable due to confinement, we would be in a rather difficult situation in the detailed exploration of the inner structure of hadrons. In that case it would obviously be very questionable to commit any ontology to the partons, for example speaking of a “number density”, or a \( k_{\perp}\)-distribution” or the “probability of finding partons with momentum fraction between \( xP^+ \) and \( (x+dx)P^+ \)” and so on. Even when factorization does hold, however, the strict number density interpretation is lost due to the UV renormalization in QCD, and the legitimacy of the statements made upon the parton distributions must be solely based on the mathematical rules of QCD.

Quantum mechanics teaches us to speak of only what we can measure. In accelerator experiments, what we measure are a bunch of hadrons and leptons in tracking chambers and calorimeters. To gain any knowledge of the partons from the observed hadrons we employ factorization and parton distributions. The former is, however, the crucial ingredient in this procedure, which is the moral of the story. In short we may say that parton distributions have no ontological priority over the factorization theorems.

5. Summary and Outlook

The problem of factorization is more intriguing and complex in the case of TMD factorization which contains more information on the underlying entities, but which also poses more difficulties. The fact that factorization appears to be broken in $pp$ collisions \[13,14\] (and mostly likely so in nucleus collisions as well) calls for caution in the applications of naive factorization. As the LHC is a $pp$ (and $AA$) machine at very high energies, it is particularly crucial to understand the issue in the small-$x$ domain. A preliminary overview of the literature shows a lack of proofs, and also some confusion as to the meaning of the “unintegrated gluon distribution”.

There has recently been many applications of the “dipole gluon distribution” in single inclusive hadron production (or rather gluon production), see for example \[15–17\]. The recent multiplicity measurements of ALICE \[18\] has shown that the central multiplicity grows faster with $s$ in $AA$ collisions than in $pp$ collisions. The question is of course why this is the case. We must then be very cautious, however, since applications of small-$x$ physics all assume the existence of factorization in these processes, and this is highly questionable. Moreover it is not at all clear that it is the “dipole gluon distribution” which is the relevant object to use in these cases. Given the enormous complexity especially of the $AA$ collisions, it is no wonder that...
the naive application of “dipole gluon distribution” does not work.

We have undertaken a comprehensive study in the hope of provoking further work to address the issues mentioned here, so that the important question regarding factorization and the TMD parton distributions may be hopefully clarified.

Acknowledgments

I would like to thank Anna Stasto and Bowen Xiao for useful discussions.

References

1. J. C. Collins, “Foundations of perturbative QCD, Cambridge University Press 2011.”.
2. J. C. Collins, “What exactly is a parton density?,” Acta Phys. Polon. B34 (2003) 3103, hep-ph/0304122.
3. J. C. Collins, “Talk given at the QCD evolution workshop: from collinear to non collinear case, 8–9 April 2011.”.
4. V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, “On the Pomeranchuk Singularity in Asymptotically Free Theories,” Phys. Lett. B60 (1975) 50–52.
5. E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, “The Pomeranchuk Singularity in Nonabelian Gauge Theories,” Sov. Phys. JETP 45 (1977) 199–204.
6. I. I. Balitsky and L. N. Lipatov, “The Pomeranchuk Singularity in Quantum Chromodynamics,” Sov. J. Nucl. Phys. 28 (1978) 822–829.
7. I. Balitsky, “Operator expansion for high-energy scattering,” Nucl. Phys. B463 (1996) 99–160, hep-ph/9603448.
8. E. Iancu, A. Leonidov, and L. McLerran, “The colour glass condensate: An introduction,” hep-ph/0202270.
9. F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, “Universality of Unintegrated Gluon Distributions at small x,” 1101.0715.
10. S. Catani, M. Ciafaloni, and F. Hautmann, “High-energy factorization and small x heavy flavor production,” Nucl. Phys. B366 (1991) 135–188.
11. E. Avsar and J. C. Collins, “In preparation.”.
12. R. Ellis, J. Stirling, and B. Webber, “QCD and Collider Physics, Cambridge University Press 2003.”.
13. J. Collins and J.-W. Qiu, “k_T factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions,” Phys. Rev. D75 (2007) 114014, 0705.2141.
14. T. C. Rogers and P. J. Mulders, “No Generalized TMD-Factorization in Hadro-Production of High Transverse Momentum Hadrons,” Phys.Rev. D81 (2010) 094006, 1001.2977.
15. D. Kharzeev, E. Levin, and M. Nardi, “Color glass condensate at the LHC: Hadron multiplicities in pp, pA and AA collisions,” Nucl.Phys. A747 (2005) 609–629, hep-ph/0408050.
16. J. L. Albacete and A. Dumitru, “A model for gluon production in heavy-ion collisions at the LHC with rcBK unintegrated gluon densities,” 1011.5161.
17. E. Levin and A. H. Rezaeian, “Gluon saturation and energy dependence of hadron multiplicity in pp and AA collisions at the LHC,” 1102.2385.
18. ALICE Collaboration, K. Aamodt et al., “Charged-particle multiplicity measurement in proton-proton collisions at sqrt(s) = 7 TeV with ALICE at LHC,” Eur. Phys. J. C68 (2010) 345–354, 1004.3514.