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Simulation of multi-agent interaction in the system “labor exchange - enterprise”

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Abstract. The work simulates a game with full information with two agents in the supply chain: one labor exchange - one enterprise. The strategy of the enterprise is the demand for labor resources. The labor exchange strategy is the party of labor resources. The company provides the labor exchange with information on the demand for workers, the labor exchange decides on the procurement and delivery of batches of workers. Labor resources as stocks are managed by the labor exchange. Thus, the model proposed in the work obeys the VMI (Vendor Managed Inventory) concept and is a model of cooperation between the labor exchange and the enterprise. A multi-product EOQ (Economic Order Quantity) model for a two-tier supply chain is proposed. The task is to minimize the value of the supply chain of labor resources, in which the optimization variable is the maximum level of labor shortages. The task takes into account the shortage of labor resources, monetary fines for the deficit, restrictions on the space of the labor exchange and the number of orders.

1. Introduction

With globalization of the economy and liberalization of the labor market, the need for firms to optimize the management of supply chains of labor resources is increasing. In particular, it is required to develop new concepts for manpower management systems that take into account financial risks and operational uncertainties. Manpower management models have been studied over the past few decades to reduce the costs of enterprises that are trying to achieve the necessary inventory levels to satisfy stochastic consumer demand and thereby improve their image on the labor market. One of the key factors for improving the level of service for enterprises is the management of the level of labor reserves of each participant within the supply chain [1].

A Supply Chain (SC) is a network of firms that manufacture, sell, and deliver goods or services to a specific market segment [2]. The concept of SC is associated with the movement of goods from goods to manufacturers, then from distributors to goods and, finally, to customers along the chain [3]. Customers and their consumer demand are the initiators of the supply chain. The main goal of SC is to
minimize system-wide costs based on service level requirements [4]. One of the best-known concepts used in SC is the Vendor Managed Inventory (VMI) [5]. In recent years, researchers and developers of inventory management models have been trying to make their models as applicable as possible in the real labor market. Factors such as the cooperation of SC participants, information exchange, labor shortages, the impact of demand, contract terms, the participation of several labor exchanges and enterprises are taken into account discounts, the use of information technology. Thus, mathematical models become more complex, and the search for optimal solutions is complicated. To achieve an acceptable solution time, various heuristic algorithms are developed. In some cases, such algorithms are able to give an optimal solution, but often you have to be content with only an approximate solution (quasi-optimal) [6].

In work in VMI models, the enterprise provides information to the labor exchange about the level of its labor resources, and the labor exchange, based on this information, determines the number of required workers in each period. A cycle is the time interval between deliveries. The use of VMI models gives advantages to both the labor exchange and the enterprise, and also helps to increase the level of consumer service in terms of reliable access to labor resources. Because the labor exchange can use customer stock information, future demand is better predicted and supplies are better coordinated [7]. Thus, it is required to develop VMI models that are more applicable in the real world [8].

2. Statement of the problem
In this problem, the game of two agents is simulated: labor exchange - enterprise. The strategy of the enterprise is the demand for workers. The labor exchange strategy is manpower. Labor resources are managed by the labor exchange; thus, the proposed model is subject to the VMI concept (Vendor Managed Inventory) and is a cooperation model between the labor exchange and the enterprise. The company provides the labor exchange with information on the demand for workers, the labor exchange decides on registration and delivery of workers [9].

We define the conditions of the game as follows:

- one labor exchange and one enterprise interact;
- the labor exchange selects the number of workers ordered and determines the time parameters of the SC, minimizing its total costs, which are equal to the costs of the entire SC;
- the enterprise acquires \( n \) types of workers of various qualifications;
- the shortage of workers is allowed and taken into account in the form of monetary fines;
- there are no discounts for labor;
- prices (wages) for all types of workers are considered fixed during the planning horizon;
- demand for workers is considered deterministic during the planning horizon;
- the labor exchange has a limitation \( F \) the amount for all types of labor resources;
- the total number of orders for employees should not exceed \( M \);
- when an employee’s inventory level drops to \( R \), a batch of size is ordered \( Q \);
- Deliveries are made only in whole batches of the same qualifications;
- batches are delivered at regular intervals \( \tau \) (cycle length);
- batch order planning is carried out on a time horizon \( T \).

For the mathematical formulation of the problem, we introduce the following notation [10]:

\[ j = 1, 2, \ldots, n \] types of workers; \( T \) planning period; \( \tau \) cycle length in units
Planning period measurements $t$ cycle number; $K = \left\lceil \frac{T}{\tau} \right\rceil$ the number of orders for the entire planning period; $Q_j^t$ order quantity of workers of the type $j$ in a batch in a loop $t$; $D_j$ demand for workers of the type $j$ in the cycle $T$; $b_j^t$ maximum deficit of workers of the type $j$ in the cycle $t$ (unit / time); $A_s$ fixed costs of the labor exchange for placing a batch order (including delivery to the enterprise); $A_R$ fixed costs of the enterprise for placing a batch order; $\pi$ fixed fines for shortages of workers ($/unit$); $h_{jR}$ labor exchange costs for collecting information about employees of the type $j$; $f_j$ Exchange space occupied by employee type information $j$; $F$ the total available space of the labor exchange for all types of labor resources; $M$ total number of orders for all types of labor resources; $n$ the number of different types of labor resources in the ordered party; $TC_{VMI}$ total supply chain costs VMI; $KR_{VMI}$ employee procurement costs in VMI chains; $KR_{VMI}$ labor exchange procurement costs in VMI chains; \\
\forall t, j = 1, 2, ..., n : \frac{Q_j^t}{D_j} = \tau.

Since the proposed model obeys the concept VMI, then the costs of the labor exchange and the enterprise, as well as the total costs of the entire chain during the planning period, are as follows [11]:

$$KR_{VMI} = 0;$$

$$KS_{VMI} = \left[ \frac{1}{\tau} \right] \left( A_s + A_R \right) + \sum_{t=1}^{K} \sum_{j=1}^{n} \left[ \frac{\left(D_j - b_j^t\right)^2}{2D_j \tau} + \frac{\pi b_j^t}{2D_j \tau} + \frac{\pi b_j^t}{\tau} \right];$$

$$TC_{VMI} = \left[ \frac{1}{\tau} \right] \left( A_s + A_R \right) + \sum_{t=1}^{K} \sum_{j=1}^{n} \left[ \frac{\left(D_j - b_j^t\right)^2}{2D_j \tau} + \frac{\pi b_j^t}{2D_j \tau} + \frac{\pi b_j^t}{\tau} \right].$$

(Model of cooperation - common interests are pursued)
The restriction on the available space of the labor exchange is formulated as follows

way: $\sum_{j=1}^{n} f_j \left(D_j \tau - b_j^t\right) \leq F$.

Restriction on labor shortage type $j$: $b_j^t \leq \tau D_j$.

Limit on the number of orders: $K \leq M$.

Minimize the following objective function:

$$\left[ \frac{1}{\tau} \right] \left( A_s + A_R \right) + \sum_{t=1}^{K} \sum_{j=1}^{n} \left[ \frac{\left(D_j \tau - b_j^t\right)^2}{2D_j \tau} + \frac{\pi b_j^t}{2D_j \tau} + \frac{\pi b_j^t}{\tau} \right] \rightarrow_{\tau, b_j^t} \min;$$

with restrictions:
\[ \sum_{j=1}^{n} f_j \left( D_j \tau - b_j \right) \leq F \]  \hspace{1cm} (2)

\[ b_j \leq \tau D_j \]  \hspace{1cm} (3)

\[ 1 \leq \left[ \frac{T}{\tau} \right] \leq M \]  \hspace{1cm} (4)

\[ b_j \geq 0, \in \mathbb{Z} \]  \hspace{1cm} (5)

\[ \tau > 0, \in \mathbb{R}, \forall t, j = 1,2,...,n \]

The posed problem belongs to the class of convex mixed integer nonlinear optimization problems, which is, as you know, NP–difficult [12].

We consider a modification of problem (1) - (5), assuming that \( b_1 = b_2 = ... = b_n \) Then the objective function will take the following form [13]:

\[ \left[ \frac{1}{\tau} \right] \left( A_x + A_R \right) + \frac{1}{\tau} \sum_{j=1}^{n} \left( D_j \tau - b_j \right)^2 h_{j,t} + \frac{\pi b_j^2}{2D_j} + \tau b_j, \]  \hspace{1cm} (6)

\[ \sum_{j=1}^{n} f_j \left( D_j \tau - b_j \right) \leq F, \]  \hspace{1cm} (7)

\[ b_j \leq \tau D_j, \]  \hspace{1cm} (8)

\[ 1 \leq \left[ \frac{T}{\tau} \right] \leq M, \]  \hspace{1cm} (9)

\[ b_j \geq 0, \in \mathbb{Z}, \]  \hspace{1cm} (10)

\[ \tau > 0, \in \mathbb{R}, \ j = 1,2,...,n \]

It turns out the task of minimizing with the same level of labor shortage in all supply cycles.

The requirement to simultaneously meet the restrictions on the available space of the labor exchange and the number of orders significantly complicates the task [14]. The presence of an optimization variable \( \tau \) in the upper limit of the sum in the objective function (1) also complicates the search for a solution. From the assumption \( b_1 = b_2 = ... = b_n \), it follows that the solutions to problems (1) - (5) and (6) - (10) are equivalent, therefore, to simplify the search for a solution, we will further consider problem (6) - (10) [15].

3. The solution to the problem

Solutions of problem (6) - (10) in the medium are given MATLAB. Test data is taken from the article [16]: \( F = 18000 \), \( M = 12 \), \( \pi = 3 \), \( \pi = 0 \), \( A_x = 200 \), \( A_R = 250 \).

To speed up the algorithm, the real variable \( \tau \) the algorithm for finding the initial solution from the article is fixed and applied [17].

Listing of the Inventory's script without a description of the code of the functions used:

```matlab
// initialization of task parameters
% Parameters:
```
// create the target function
% Objective function
// initialize linear constraints
% Linear Constraints: Ax = c
// initialize the upper and lower bounds for the solution
% Bounds
// restriction on the integer value of the optimization variable (I stands for Integer)
% Integer Constraints
// initial solution (for test data, it is enough to check the first step of the heuristic algorithm, since
the value is quite large)
% Initial Guess
// create task (6) - (10)
% Create Object
// solving problem (6) - (10) with calling up information about the results
% Solve the MINLP problem
// calculation of the value of the objective function (6) and the vector of values of Q, ceil is applied
% the rounding function to the nearest smaller integer
% Total cost and vector Q
Results:
Inventory
// target function
Problem Properties:
# Decision Variables: 10 // number of task variables
# Constraints: 31 // total number of restrictions
# Linear Inequality: 1 // number of linear constraints
# Bounds: 20 // number of borders
# Integer Variables: 10 // number of integer variables

--------------------------------------------------
// calculation of Jacobian
Objective Gradient: @ (x) mkJac (prob.fun, x, 1) [numdiff]

--------------------------------------------------
// solution - the found integer values of the vector b
b = [60 67 94 39 69 85 83 48 43 105] T - labor exchange strategies
// value of the objective function
fval = 260.0139
// the algorithm came down to the optimal solution
exitflag = 1
// information about the result of the algorithm
info =
// applied solution method: LP / NLP based on the branch and bound method), a nonlinear problem
is solved at each node using the internal point algorithm [18], and linear — using the Branch and Cut
algorithm [19].
// status of the solution found is optimal
Status: 'Optimal'
// value of the objective function (6)
TC = 2.8401e + 03 - the value of the overall objective function of the game
// solution - the found integer values of the vector Q
Q = [15 17 24 10 18 22 21 12 11 27] T

--------------------------------------------------
The convergence of the LP / NLP algorithm based on the branch and bound method was proved in [20]. From this work, the fulfilled assumptions and the theorem are applied to the problem (6) - (10):

1. Problem (6) - (10) is convex.
2. The objective function (6) and constraint (7) are continuously differentiable with respect to $b_j$.
3. Constraint (7) is satisfied for each solution nlp - subtasks obtained by fixing integer variables $b_j$.
4. Nlp - subtasks obtained by fixing integer variables $b_j$ can be solved for sure.

Theorem. If assumptions 1 - 4 are satisfied, and the set of values of variables $b_j$ less than infinity, then the LP / NLP algorithm based on the branch and bound method converges in a finite number of steps to the optimal solution or stops when the solution is invalid. Thus, it should be noted that based on a specific economic statement of the problem, one can choose a set of the most acceptable values of the supply cycle $\tau$. Since in a real market, the supply chain depends on many factors: the distance of the enterprise from the labor exchange, the need to cross the state border (working with customs), transport, etc. Therefore, the proposed solution algorithm does not lose its applicability, despite the fact that one of optimization variables ($\tau$) was fixed. Therefore, a solution to problem (6) - (10) is found [21-23].

4. Conclusion
In this paper, a mathematical model of the labor exchange-enterprise game in the context of cooperation is proposed. The EOQ model proposed in this paper can be applied in a real labor market. For the developed model, you can choose the length of the supply cycle of labor resources; consider the most important limitations; take into account the penalties incurred by the labor exchange under the terms of the VMI - contract; if there is a wide nomenclature on the labor exchange, it is advantageous to combine different types of labor resources in the party; consider the integer size of the order. The problem of minimizing the overall objective function is solved. The strategies of both agents are found. For further research, it is possible to propose a process for sharing profits (the topic of contracting).

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