Percolation of New Product Critical Market Penetration

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Abstract

A simulation of new product market penetration in a social environment is performed, using a spintronic model, where each element of a 3D network interacts with its first neighbors. Agents are assumed to be rational, with a perfect market foresight. Unitary production cost decreases when consumption is increased. Simulations indicate that social interaction is the most important factor for new product market penetration as compared to the consumer readiness to pay a higher price. Besides the critical exponent of the nucleation of new consumers is computed, signing a phase transition characterized by the build up of new consumers intermediate clusters and this critical exponent is compared to others belonging to several critical phenomena.

1 Introduction

The importance of building up groups of consumers of brand new products has originated different strategies such as viral marketing, where inexpensive products as well as free services can be offered, generating streams of new consumers. In regards to Internet, this can be expressed in the form of new buttons providing either free mails or demo software, as dictated by Wilson marketing law: It is a question of giving and receiving [1]. This way aggregate demand is sustained and therefore new Internet markets may survive, though market survival has been regarded as problematic even since the last century and the absence of aggregate demand has led to Web Portals. Digital segregation inhibits access to Digital Economy, decreasing aggregate demand [2] indicating future limitations for Internet expansion. Also aggregate demand is very sensible to State intervention [3] as well as to the induction of both incorrect prices and investors incorrect expectations in the banking system [4]. Social interaction is noticed through the influence received by individuals in the social networks, where an individual typically interacts with about eight to twelve members [5]. Notwithstanding there is a great variety of mass media, such as Internet and telephone, where high complexity communication is transmitted via face to face communication [6].

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Basu has developed algorithms to detect conversations among members of a group, in terms of synchronization, duration and frequency of the proximity between members of a group. This way, group connectors can be detected, namely those concentrating the transmission of information as well establishing some grounds to prove the influence of the environment on information diffusion. Nevertheless, there are fine effects affecting message transmission such as audience nodding frequency.

Using a simplified version of Asavathiratham model to reproduce first-neighbors interaction, Basu et. al. have developed a model that describes human interaction with the influence model, considering constant first-neighbors influence intensity as well as depending on first-neighbors state. Campbell and Ormerod have studied the influence of social interaction on crime dynamics, with a model using a non linear system of differential equations. Due to the bifurcation of solutions emerging from the non linearity introduced by social interaction, time and location factors are more important than socio-economic status. Employing a similar model, based on a system of differential equations, Campbell and Ormerod studied social interaction influence on English family structural evolution and found that the determinant factor for the English family structural evolution in the last three decades was the increase of real salaries as well as the economic independence of female labor force. This behavior was explained as a result of non linearities introduced by social interaction in an equilibrium attractive solution.

Bonabeau et. al. have developed a first-neighbors interaction diffusion model, where social hierarchy arises from population density fluctuations. Though initially every neighbor has the same probability to own the adjacent node, later a memory of the victories achieved is kept, periodically reducing up to a ten percent the value of this memory to reproduce the lack of social memory. This way the probability of winning a confrontation is computed, taking on account the number of previous victories and the associated random origin of social hierarchy. Stauffer et. al. studied the number of elements whose opinion remains unaltered, in Sznejd consensus model, using a multiple spin model where the sublattices are simultaneously actualized. They observed that the number of elements whose opinion remains unaltered decays on time following a power law \[ t^{-\theta} \], with an exponent \( \theta \sim 3/8 \) in the case of a linear chain and \( \theta \sim 1/2 \) in the case of a square or 3D network. This is compared with the results obtained by Derrida et. al. for a squared network, with a value of \( \theta \sim 0.2 \).

In this study, new product market penetration is analyzed, assuming that the market is already saturated by a given product, using a spintronic model that considers first-neighbors interaction. Social interaction and new product demand established by pre-campaign turn out to be the most important factors to ensure new product market penetration. This results in the creation of a new market, as new consumers are incorporated, originating an aggregate demand for a lapse of time.

2 Methodology.

In this study, social environment is simulated by one million individuals and first-neighbors interaction. Scale factor is taken on account considering that when new product production is increased, its production price is decreased. Besides, when the new product is used by more costumers,
the price they are ready to pay is increased. Each element of the spherical
network may assume three states: -1 when the element consumes the new
product, 0 when the element does not consume the new product and 1
when the element refuses the new product. Initially, every element is in
state 0, namely not using the new product and is randomly distributed in
a certain percentage.

This way system evolves randomly selecting an element, which consid-
ers the new product acceptance among its first-neighbors and this way
takes a rational decision in terms of the maximum cost at which people
is ready to pay for the new product and the price at which the new pro-
duct is offered. Agents rationality and their market foresight are modeled
following the model introduced by Hohnisch et. al. [17]. A parabolic de-
pendence is supposed between the minimum price at which the pro-
ducer is ready to sell with and the number of produced units:
\[ p_t(x) = p_0 - qx + \alpha x^2 \]

Besides, consumers are ready to pay a higher price when the new
product has more consumers:
\[ p_a(x) = p_0 + \mu x \quad (1) \]

Each potential consumer behaves as a rational agent, choosing the
product with the best price and satisfying:
\[ p_a(x) \geq p_t(x) \quad (2) \]

This dynamics is followed to detect the determinant factor for new
product penetration, comparing the number of initial consumers and
the acceptation coefficient \( \mu \). Model constants were fixed elsewhere in a
study on the adoption of a new corn variety at the state of Iowa [18].

When market penetration is simulated, new consumers clusters are built
up. For infinite systems, there is a critical percolation probability, above
which the probability of finding a percolation cluster is equal to 1 and
for a value lower than this critical percolation the probability of finding
a percolation cluster is 0. For finite networks, this transition is soft, i.e.,
the probability of finding a percolation cluster is not equal to 0 at any
probability. Chayes et.al. have shown that the critical exponent for the
scaling of finite clusters is equal to: \( \tau - 2 = 1/2 \) and a value of the critical
exponent for the scaling of infinite clusters is equal to: \( \tau - 2 = 1/3 \) [19].

In the percolation model de weight of a given configuration \( C \) with n
binding is: \( W(C) = p^n (1 - p)^{N-n} \) where N is the number of vertices in
the network. Close to the percolation threshold, the critical behavior is
characterized by the critical exponents:
\[ P_{\infty} = 1 - \sum s n(s, p) \sim |(p - p_c)|^\beta \quad (3) \]

and:
\[ S(p) = \sum s^2 n(s, p) \sim |p - p_c|^\gamma \quad (4) \]

The cluster distribution satisfies the following scaling relation:
\[ n(s, p) = s^{-\sigma} f(p - p_c) s^\sigma \quad (5) \]

therefore a power law is expected close to the critical point:
\[ n(s, p) = s^{-\sigma} f(0) \quad (6) \]

Using these relations, the following equation can be obtained:
\[ \tau = 2 + \frac{\beta}{\beta + \gamma} \quad (7) \]
and:

\[ \sigma = \frac{1}{\beta + \gamma} \]  \hspace{1cm} (8)

In 3D, the best estimation is \( \tau = 2.18 \) and \( \sigma = 0.45 \) \[20\]. In previous studies on percolation, critical exponents have been estimated for finite physical systems with values of the critical exponent in the range \( \tau \sim 2 - 3 \) \[21\]. In the case of new product market penetration, a value of \( \tau = 2.88 \) was obtained hereby, indicating the influence of social interaction. In the Fisher Liquid Droplet Model \[22\], the probability of obtaining a critical cluster, is given by:

\[ n_A = q_0 A^{-\tau} \] \hspace{1cm} (9)

with a proportionality constant \( q_0 \) that can be obtained using the first moment:

\[ M_1 = \sum n_A \] \hspace{1cm} (10)

of the normalized distribution, i.e.:

\[ M_1 = 1 \] \hspace{1cm} (11)

Hence, \( q_0 \) can be obtained from:

\[ q_0 = \frac{1}{\sum A^{1-\tau}} \] \hspace{1cm} (12)

3 Results

As shown in Fig. 1, as the number of new product consumers is increased, the maximum price that the consumers are ready to pay for the new product is also increased. And in a similar fashion, the price that the consumers are ready to pay for the old product is decreased. This happens for a given new costumers threshold generated from a pre-campaign where the new product is given as a present or promoted by viral marketing techniques. The results obtained indicate that the most important factor is the social interaction, as shown by the linearity of the Arrhenius curves with respect to the percentage of initial new consumers (Fig. 2). Therefore, new product market penetration depends on the capacity of previously introducing the new product in the population. This can be achieved using techniques as those used by viral marketing, where a new product acceptance threshold is reached, independently of its technical characteristics. Arrhenius plot has been fitted in terms of new consumers cluster size, obtaining a critical exponent equal to 2.88 (Fig. 3). Since Arrhenius plot is fitted selecting intermediate size clusters, the phase transition is related to the onset of new consumers clusters with an intermediate size, which is known to be of approximately 8 to 12 consumers.

4 Conclusions

The simulation suggests that a pre-campaign previous to the introduction of the new product is necessary to ensure new product market penetration, independently of its technical characteristics. The importance of social interaction is expressed in terms of the need to reach an adequate threshold of new consumers to displace the older product. This indicates the convenience of using viral marketing techniques with the purpose of attaining this consumption threshold before the new product distribution. This study on the built up of new consumer clusters indicates a
phase transition with a critical exponent equal to 2.88, which is different of those belonging to other physical systems. Author acknowledges partial support from UAM-Azcapotzalco and the access to computational facilities of Intensive Computation Lab of UAM-A.
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$\langle \text{Clients} \rangle$ vs $P$
