Beam shaping system design using double freeform optical surfaces

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Abstract: A numerical double-freeform-optical-surface design method is proposed for beam shaping applications. In this method, both the irradiance distribution and the wavefront of the output beam are taken into account. After numerically obtaining the input-output ray mapping based on Energy conservation using the variable separation method, the two freeform optical surfaces can be constructed simultaneously and point by point corresponding to the ray mapping based on Snell’s law and the constancy of the optical path length. The method is only applicable for separable irradiance distributions. However, such a restriction is fulfilled by many practical laser beam shaping examples. Moreover, the restriction can simplify the computation considerably. Therefore, the method may be quite useful in practice, although it is not applicable to more general cases. As an example, the method was applied to design a two-plano-freeform-lens system for transforming a collimated 20 mm Gaussian laser beam (beam waist: 5mm) into a uniform 10 × 40 mm2 rectangular one without changing the wavefront. Simulation results show that we can obtain a dual lens beam shaping system with the relative root mean square deviation of the irradiance ranging from 0.0652 to 0.326 and the power ratio concentrated on the desired region ranging from 97.5% to 88.3% as the output beam transfers from 0mm to 1000mm.

OCIS codes: (080.4298) Nonimaging optics; (080.1753) Computation methods; (080.4225) Nonspherical lens design; (140.3300) Laser beam shaping.

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1. Introduction

One challenge to the practical aspects of optics is beam shaping i.e. the redistribution of the beams of rays emitted from a given source. Synthesis of a beam shaping optical system usually involves determining the refractive or reflective optical surfaces with desired transformation capabilities. For example, to create a collimated light beam in medium $n_2$ from a point light source in medium $n_1$, the interface must be ellipsoidal when the refractive index $n_1 > n_2$ and hyperboloidal when $n_1 < n_2$ based on the constancy of the optical path length (OPL) [1]. Furthermore, since it involves finding an exact solution, the design of the optical surfaces must balance the system freedoms and the performance requirements. Thus, in most cases, a single optical surface suffices to produce a given wavefront or desired irradiance distribution but doesn’t suffice to meet these two requirements simultaneously.

Traditional beam shaping optics design techniques usually utilize rotational or translational symmetries, where the calculated cross section curve is swept around or along its symmetry axis to generate the 3-D geometry shape. However, the need for beam shaping optics that can distribute light beam in a non-rotationally or non-translationally manner has strongly increased. This leads to the concept of freeform optical surfaces, which can provide much more controlling freedom. The rapidly advancing manufacture technologies of freeform surfaces stimulate the development of freeform beam shaping elements.

In illumination optics, the typical problem encountered is to produce a desired irradiance distribution on a given target surface, where the direction of the rays of the output beam is irrelevant in many cases. A multitude of freeform-optical-surface design methods has been proposed [2–10]. Among these methods, Wang’s method [5] is fast and effective especially for transforming irradiance distributions that can be factorized in two orthogonal transverse coordinates (separable). It first establishes separately the correspondence of the coordinate-variables between the light source and the target plane. Then, the freeform surface is generated point by point corresponding to the above ray mapping i.e. the next point is obtained by intersecting the next input ray to the tangent plane at the last point [11]. However, this single freeform-optical-surface design method doesn’t leave any room for an additional requirement.

In contrast to illumination optics, laser beam shaping optics is usually required to control the output wavefront as well. In this case, at least two freeform optical surfaces are needed to implement the two requirements. To the authors’ knowledge, there are few papers dealing with two freeform optical surfaces [12–14]. The most closely related to our work is Ref. 14, where two off-axis reflectors are tailored simultaneously to convert a Gaussian laser beam into a circular flat top one without changing the wavefront. However, it remains entirely silent on explaining how to carry out the procedure. We can get a hint from his previous work [3] that the two freeform reflective surfaces may be calculated by solving a complex second-order non-linear partial differential equation of Monge–Ampère type.

A new numerical two-freeform-optical-surface design method is proposed for producing a desired irradiance distribution whilst forming a prescribed wavefront from a given input beam. To simplify the computation, the input-to-output irradiance mapping is firstly obtained based on Energy conservation using the variable separation method [5]. Thus, both the input and output irradiance distributions must fulfill the restriction that they can be factorized in two orthogonal transverse coordinates, which is still suitable for many laser beam shaping applications. Then, the two freeform optical surfaces are generated simultaneously and point by point corresponding to the input and output ray sequences defined by the first step. The method is fast and can be easy understood by optical engineers since it avoid solving the underlying Monge–Ampère equation. The detailed design outline is given in section 2. In section 3, a two-plano-freeform-lens beam shaping system is designed as an example of the
method, wherein the simulation results are included. Finally, a short summary is given in section 4.

2. Design method

Consider a two-freeform-refractive-surface optical system shown schematically in Fig. 1 (one or both of the two optical surfaces could also be reflective). The input and output beam are supposed as propagating toward the positive \( z \) direction and their wavefronts can be represented as position vectors \( S = (x_s, y_s, z_s) \) and \( T = (x_t, y_t, z_t) \), respectively. The points on the two freeform surfaces are denoted by \( P = (x_p, y_p, z_p) \) and \( Q = (x_q, y_q, z_q) \), respectively. \( n_1, n_0, \) and \( n_2 \) are set as the refractive indices of the mediums on the right side of the first freeform surface, between the two freeform surfaces and on the left side of the second freeform surface, respectively.

Let \( I_{in} \) and \( I_{out} \) denote the prescribed irradiance distributions over the planes perpendicular to the \( z \)-axis near the input and output wavefronts, respectively. Their relationship can be described by Energy conservation written as Eq. (1):

\[
I_{in}(x_s, y_s)dx_s dy_s = I_{out}(x_t, y_t)dx_t dy_t \tag{1}
\]

Assume that \( I_{in} \) and \( I_{out} \) can be separated into a product of two one-dimensional irradiance distributions: \( I_{in}(x_s, y_s) = I_{in,x}(x_s)I_{in,y}(y_s) \) and \( I_{out}(x_t, y_t) = I_{out,x}(x_t)I_{out,y}(y_t) \). Thus, both \( (x_s, y_s) \) and \( (x_t, y_t) \) can be numerically specified based on “source-to-target” [5] or “target-to-source” [7,8] variable separation mapping strategies if one of them is predefined. Take “source-to-target” variable separation mapping strategy for example, if \( (x_s, y_s) \) are equidistantly divided into \( n \times m \) rectangular grids: \( x_s = x_{s,i} \) and \( y_s = y_{s,j} \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \), then \( x_{t,i,j} \) and \( y_{t,i,j} \) can be calculated by satisfying Eq. (2) and Eq. (3), respectively.

\[
\int_{x_{s,i}}^{x_{s,i+1}} I_{in,x}(x_s)dx_s \int_{y_{s,j}}^{y_{s,j+1}} I_{in,y}(y_s)dy_s = \int_{x_{t,i}}^{x_{t,i+1}} I_{out,x}(x_t)dx_t \int_{y_{t,i,j}}^{y_{t,i,j+1}} I_{out,y}(y_t)dy_t \tag{2}
\]

\[
\int_{x_{s,i}}^{x_{s,i+1}} I_{in,x}(x_s)dx_s \int_{y_{s,j}}^{y_{s,j+1}} I_{in,y}(y_s)dy_s = \int_{x_{t,i}}^{x_{t,i+1}} I_{out,x}(x_t)dx_t \int_{y_{t,i,j}}^{y_{t,i,j+1}} I_{out,y}(y_t)dy_t \tag{3}
\]

Then, the input and output unit ray vectors \( \text{In}_{i,j} \) and \( \text{Out}_{i,j} \) can be obtained as the unit normal vectors at \( S_i(x_{s,i}, y_{s,j}, z_{s,i,j}) \) and \( T_i(x_{t,i}, y_{t,i,j}, z_{t,i,j}) \), respectively, as shown in Eq. (4) and Eq. (5):

\[
\text{In}_{i,j} = \left[-\left(\frac{\partial z_s}{\partial x}\right)_{x_{s,i},y_{s,j}} \frac{\partial z_s}{\partial y}\right]_{x_{s,i},y_{s,j}} + 1 \left(\frac{\partial z_s}{\partial x}\right)_{x_{s,i},y_{s,j}}^2 + \left(\frac{\partial z_s}{\partial y}\right)_{x_{s,i},y_{s,j}}^2 \tag{4}
\]
\[
\text{Out}_{ij} = \left[ -\frac{\partial z}{\partial x}_{h_{ij}, g_{ij}, 1} - \frac{\partial z}{\partial y}_{h_{ij}, g_{ij}, 1} \right] \sqrt{1 + \left( \frac{\partial z}{\partial x}_{h_{ij}, g_{ij}, 1} \right)^2 + \left( \frac{\partial z}{\partial y}_{h_{ij}, g_{ij}, 1} \right)^2}
\]

(5)

Thus, the input ray sequence is defined both by \( \text{In}_{ij} \) and \( \text{S}_{ij} \), and the output ray sequence is defined both by \( \text{Out}_{ij} \) and \( \text{T}_{ij} \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \).

The next step is to calculate the data points \( \text{P}_{i,j} \) and \( \text{Q}_{i,j} \) on the two freeform optical surfaces which are desired to transform the input ray sequence into the output ray sequence. From two starting points \( \text{P}_{1,1} \) and \( \text{Q}_{1,1} \), e.g., the central points of the first and second freeform surfaces, the normal vector \( \text{N}_{1,1} \) at \( \text{P}_{1,1} \) is firstly calculated using the vector form of Snell’s law so that the ray emitted from \( \text{P}_{1,1} \) can reach \( \text{Q}_{1,1} \), as shown in Eq. (6):

\[
\text{N}_{1,1} = \left( n_i \text{R}_{1,1} - n_i \text{In}_{1,1} \right) / \sqrt{n_i^2 + n_i^2 + 2n_i n_i (\text{R}_{1,1} \cdot \text{In}_{1,1})}
\]

(6)

wherein \( \text{R}_{1,1} \) denotes the unit vector of the ray through \( \text{P}_{1,1} \) and \( \text{Q}_{1,1} \). \( \text{P}_{2,1} \) can be obtained by intersecting its corresponding input ray to the tangent plane of \( \text{P}_{1,1} \) [11], which can be formulated as Eq. (7) and Eq. (8):

\[
(\text{P}_{2,1} - \text{P}_{1,1}) \cdot \text{N}_{1,1} = 0
\]

(7)

\[
\left( \frac{\text{P}_{2,1} - \text{S}_{2,1}}{\text{P}_{2,1} - \text{S}_{2,1}} \right) = \text{In}_{2,1}
\]

(8)

Then, \( \text{Q}_{2,1} \) is obtained by equaling the OPL of the ray through \( \text{P}_{2,1} \) and itself to the OPL of the ray through \( \text{P}_{1,1} \) and \( \text{Q}_{1,1} \), as shown in Eq. (9) and Eq. (10):

\[
n_i [\text{S}_{2,1}, \text{P}_{2,1}] + n_i [\text{P}_{2,1}, \text{Q}_{2,1}] + n_i [\text{Q}_{2,1}, \text{T}_{2,1}] = n_i [\text{S}_{1,1}, \text{P}_{1,1}] + n_i [\text{P}_{1,1}, \text{Q}_{1,1}] + n_i [\text{Q}_{1,1}, \text{T}_{1,1}]
\]

(9)

\[
\left( \text{T}_{2,1} - \text{Q}_{2,1} \right) / \left[ \text{T}_{2,1} - \text{Q}_{2,1} \right] = \text{Out}_{2,1}
\]

(10)

wherein \( [X, Y] \) represents the distance between two arbitrary points \( X \) and \( Y \). Then, the required normal vector \( \text{N}_{2,1} \) at \( \text{P}_{2,1} \) can be computed so that the ray through \( \text{P}_{2,1} \) can be refracted to \( \text{Q}_{2,1} \). The entire starting curves on the two surfaces can be generated by repeating the above process.

All the points of the second curve on the first freeform surface can be obtained on the tangent planes of the starting curve’s points, written as Eq. (11) and Eq. (12):

\[
(\text{P}_{1,2} - \text{P}_{1,1}) \cdot \text{N}_{1,1} = 0
\]

(11)

\[
\left( \frac{\text{P}_{1,2} - \text{S}_{1,2}}{\text{P}_{1,2} - \text{S}_{1,2}} \right) = \text{In}_{1,2}
\]

(12)

Then, points of the second curve on the second freeform surface can be computed by equaling their OPLs to that of the ray through \( \text{P}_{1,1} \) and \( \text{Q}_{1,1} \), expressed as Eq. (13) and Eq. (14):

\[
n_i [\text{S}_{1,2}, \text{P}_{1,2}] + n_i [\text{P}_{1,2}, \text{Q}_{1,2}] + n_i [\text{Q}_{1,2}, \text{T}_{1,2}] = n_i [\text{S}_{1,1}, \text{P}_{1,1}] + n_i [\text{P}_{1,1}, \text{Q}_{1,1}] + n_i [\text{Q}_{1,1}, \text{T}_{1,1}]
\]

(13)

\[
\left( \text{T}_{1,2} - \text{Q}_{1,2} \right) / \left[ \text{T}_{1,2} - \text{Q}_{1,2} \right] = \text{Out}_{1,2}
\]

(14)

All the normal vectors at the points of the second curve on the first freeform surface can then be obtained. We can repeat this process to generate all the surface points and interpolate them to get the entire freeform surfaces [15]. The flow diagram of the proposed design method is shown in Fig. 2.
3. Design example

As an example of the proposed method, a dual lens beam shaping system is designed to transform a Gaussian laser beam into a uniform rectangular one as shown in Fig. 3. To simplify the calculation and simulation, the prescribed input and output beam are supposed as plane waves. In the system, each lens has one freeform surface, designed to redistribute the rays, and one flat surface. Such a system can be considered as a three-dimensional extension of the popular Galilean refractive beam shaping system with rotational symmetry. The design parameters are shown in Table 1.

![Fig. 2. The flow diagram of the proposed design method](image)

![Fig. 3. The prescribed irradiance distributions of the (a) input and (b) output beams.](image)
**Table 1. Design parameters**

| Parameter                              | Value     |
|----------------------------------------|-----------|
| Input beam waist                       | 5mm       |
| Input beam radius                      | 10mm      |
| Input beam power                       | 100W      |
| Desired output beam dimension          | 10mm × 40mm |
| Refractive index of the two lenses     | 1.5083    |
| Distance of the two lenses             | 86mm      |
| Thickness of the first lens            | 6mm       |
| Thickness of the second lens           | 8mm       |

In the design, the “target-to-source” strategy is adopted, wherein the output plane coordinates \((x_t, y_t)\) are equidistantly divided into \(401 \times 401\) points. The resulted two freeform surfaces are shown in Fig. 4, where the first surface is uniformly interpolated for better visualization. We can see from Fig. 4 that the two freeform surfaces are bended mainly in \(x\) direction and very close to cylindrical surfaces. Figure 5 shows the final dual lens beam shaping system, where the freeform surfaces are reconstructed with \(100 \times 100\) points based on NURBS [15].

![Fig. 4. The pseudo-color images of the (a) first and (b) second freeform surfaces.](image)

![Fig. 5. Designed two-plano-freeform-lens beam shaping system (Not all the grids are displayed).](image)
Ray tracing is implemented with the dual lens system in Fig. 5 based on the Monte-Carlo method. Figure 6 shows the simulated irradiance distributions (represented by $I_{\text{simulate}}$) on six receivers placed at 0mm, 200mm, 400mm, 600mm, 800mm and 1000mm away from the flat emitted surface of the secondary lens. Each receiver plane has a dimension of $30 \times 50 \text{ mm}^2$ and the irradiance values are sampled on the grids of $1 \times 1\text{ mm}^2$ spacing. The relative root-mean-square-deviation (RRMSD) and the ratio (represented by $P_r$) of the power on the desired region to the total power $P$ are adopted to evaluate the designed dual lens system (see Eq. (15) and Eq. (16)). The RRMSD of the simulated irradiance at the emitted surface of the secondary lens is 0.065 and $P_r$ is 97.5% (Regardless of the Fresnel losses). As the receiver distance increases from 0mm to 1000mm, the RRMSD monotonically increases to 0.326 and $P_r$ is reduced to 88.3% (see Fig. 7). The probably major reason for the deformations is that only the surface slope is considered to compute the adjacent points on the first freeform surface. The performance could be improved by introducing an approximate second-order scheme since the first surface curvature is related to the relative change in irradiance along the ray.

$$\text{RRMSD} = \sqrt{\frac{\sum_{i=1}^{20} \sum_{j=1}^{50} (I_{\text{out}}(x_{i,j}, y_{i,j}) - I_{\text{simulate}}(x_{i,j}, y_{i,j}))^2}{\sum_{i=1}^{20} \sum_{j=1}^{50} (I_{\text{out}}(x_{i,j}, y_{i,j}))^2}}$$

(15)

$$P_r = \frac{\iint_{0:400\text{mm}} I_{\text{simulate}} \, dx \, dy}{P} = \frac{\sum_{i=11}^{20} \sum_{j=6}^{45} I_{\text{simulate}}(x_{i,j}, y_{i,j}) \times 1\text{mm}^2}{P}$$

(16)

Fig. 6. The simulated irradiance distributions ($30 \times 50$ grids) on the receivers placed at (a) 0 mm, (b) 200mm, (c) 400mm, (d) 600mm, (e) 800mm and (e) 1000mm away from the emitted surface of the second lens.
4. Conclusion

To summarize, a two-freeform-surface design method was proposed for transforming separable irradiance distributions whilst forming a prescribed wavefront. In combination with the input-output ray mapping obtained by the variable separation method, the two surfaces can be generated simultaneously and point by point based on Snell’s law and the constancy of the optical path length. A dual lens beam shaping system was designed as an example of the proposed method. The simulation results show that the dual lens beam shaping system can effectively transform a collimated Gaussian laser beam into a “flat-top” rectangular one with a long depth of field and small divergence.

The computation algorithm is very fast and can also be applied to the design of LED illumination and solar concentration optical systems under certain conditions. Future work will focus on reducing the freeform surface construction errors and generalizing this method to work for non-separable irradiance distributions.

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