INTERMEDIATE-MASS STARS: UPDATED MODELS

INMA DOMINGUEZ,1 ALESSANDRO CIEFFI,2 MARCO LIMONGI,3 AND OSCAR STRANIERO4

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ABSTRACT

A new set of stellar models in the mass range 1.2–9 $M_\odot$ is presented. The adopted chemical compositions cover the typical Galactic values, namely, 0.0001 ≤ Z ≤ 0.02 and 0.23 ≤ Y ≤ 0.28. A comparison of the most recent compilations of similar stellar models is also discussed. The main conclusion is that the differences among the various evolutionary results are still rather large. For example, we found that the H-burning evolutionary time may differ up to 20%. An even larger disagreement is found for the He-burning phase (up to 40%–50%). Since the connection between the various input physics and the numerical algorithms could amplify or counterbalance the effect of a single ingredient on the resulting stellar model, the origin of these discrepancies is not evident. However most of these discrepancies, which are clearly found in the evolutionary tracks, are reduced on the isochrones. By means of our updated models we show that the ages inferred by the theory of stellar evolution is in excellent agreement with those obtained by using other independent methods applied to the nearby open clusters. Finally, the theoretical initial/final mass relation is revised.

Subject headings: open clusters and associations: general — stars: evolution — stars: interiors

1. INTRODUCTION

The comprehension of the evolutionary status of the various stellar systems, from the simplest stellar clusters up to the more complex galaxies, is mainly based on the comparison between theoretical stellar models and observational data. It follows that any improvement and/or assumptions in the basic input physics (equation of state, opacity, and the like) included in the computations of the stellar models directly influence the interpretation of the observed data. Moreover, since population synthesis requires the availability of stellar models in a large range of masses and chemical compositions, and since such a homogeneous database is still missing, people involved in such studies have been forced to collect models computed by different authors.

In this paper we present new evolutionary sequences for stellar masses ranging between 1.2 and 9 $M_\odot$; these models have been included in our database of stellar evolution, which is available by anonymous ftp. This database also includes fully homogeneous models for low-mass stars (Straniero, Chieffi, & Limongi 1997a) and for massive stars (Chieffi, Limongi, & Straniero 1998; Limongi, Straniero, & Chieffi 1999). All these models have been obtained by means of the Frascati Raphson Newton Evolutionary Code (FRANEC; Chieffi & Straniero 1989; Chieffi et al. 1998). The input physics (equation of state, opacity, neutrino losses, and the like) we adopt here are discussed in Straniero et al. (1997a).

Several papers described sets of models of intermediate-mass stars with various chemical compositions. However, owing to the continuous improvement of the input physics, the (re)computation of the same stellar models becomes necessary. Major contributions are from Kippenhahn, Thomas, & Weigart (1965); Iben (1967a, and references therein); Paczynski (1970a, 1970b, 1971); Trimble, Paczynski, & Zimmerman (1973); Alcock & Paczynski (1978); Becker & Iben (1979); Becker (1981); Vandenbergh (1985); Maeder & Meynet (1987, 1988, 1989, 1991); Castellani, Chieffi, & Straniero (1990, 1992); Lattanzio (1991); Stothers & Chin (1990, 1991a, 1991b, 1992); Vassiliadis & Wood (1993); Aloni et al. (1991, 1992), Bressan et al. (1993); and Schaller et al. (1992). Thus, our new models will be compared with the most recent compilations of similar evolutionary sequences. Whenever possible, we analyze the origin of the resulting differences. The main goal of this study is to constraint the present level of uncertainty of the stellar evolution in the range of intermediate-mass stars. In doing so, we provide a basic tool to check the reliability of our understanding of Galactic history emerging from the study of population synthesis.

This is the plan of the paper: in the next section we revise the possible sources of uncertainties for H- and He-burning intermediate-mass stellar models; in § 3 we describe our latest evolutionary computations for intermediate mass stars, from the zero-age main sequence (ZAMS) up to the asymptotic giant branch (AGB); in § 4 we discuss the comparisons of different evolutionary models; and we present selected tests of the evolutionary sequences in § 5. Final remarks follow.

2. INPUT PHYSICS AND CONVECTION

Although the theoretical investigation of low-mass stars appears well anchored to the results of helioseismology (see Straniero et al. 1997a), this is not the case for intermediate-mass stars. It is commonly believed that many uncertainties in the theory of turbulent convection still affect our understanding of the internal structures of these kinds of stars. Owing to the lack of a conclusive test for the adequacy of the current theory of convection, the astrophysical literature presents a variety of different approaches to the computation of stellar models. It was recognized early that the mixing of material in the core of a given star significantly alters its lifetime and, in turn, could modify the age estimates of various Galactic components. The instability...
against turbulent convection is classically handled by means of a thermodynamical criterion (the Schwarzschild criterion for a chemically homogeneous fluid). As is well known, this criterion is based on the evaluation of the expected gradient of temperature produced by the radiative transport of energy: when the required gradient is too high, radiative flux cannot account for the whole energy transport so convection is needed.

First of all, let us emphasize that the correct evaluation of the size of an unstable region is primarily dependent on the accuracy of the input physics. Any improvement of the stellar physics (equation of state [EOS], opacity, cross section, and the like) could imply a variation of the estimated value of the temperature gradient, and it in turn could modify the location of the borders of the convective regions, with sizable consequences for the computed stellar lifetime.

A second question concerns the possibility that the convective motion is not drastically inhibited in a stable region located just outside the convective core. In fact, although out of the Schwarzschild border a moving element of matter is subject to a strong deceleration, it might be possible that a nonzero velocity is maintained for a certain path. In such a case this mechanical overshoot might induce a mixing of material in a radiatively stable region and might also contribute to energy transport. A large number of papers have been devoted to the inclusion of such phenomenon in the computation of stellar models. Major contributions are from Shaviv & Salpeter (1973), Maeder (1975), Cloutmann & Whitaker (1980), Bressan, Bertelli, & Chiosi (1981), Sto-

**TABLE 1**

| $M$ ($M_\odot$) | $\Delta t_{\text{ff}}$ (Myr) | $M_{\text{ff}}^\ast$ ($M_\odot$) | He$_{\text{surf}}$ (4) | log $L_{\text{top}}^{\text{GB}}$ | $M_{\text{He}}^1$ (Myr) | $\Delta t_{\text{He}}$ (Myr) | $M_{\text{He}}^2$ ($M_\odot$) | He$_{\text{surf}}^2$ (5) | $M_{\text{He}}^3$ ($M_\odot$) | He$_{\text{surf}}^3$ ($M_\odot$) |
|----------------|-----------------|-----------------|----------------|-----------------|----------------|----------------|-----------------|----------------|----------------|----------------|
| 1.2........ | 3220            | 0.084           | 0.251          | 3.222           | 0.495           |               |                 |                 |                 |                 |
| 1.5........ | 1509            | 0.143           | 0.254          | 3.122           | 0.475           | 98.4           | 0.577           | 0.255           | 0.255           | 0.592           |
| 1.8........ | 832             | 0.219           | 0.254          | 2.927           | 0.444           | 104            | 0.595           | 0.255           | 0.255           | 0.609           |
| 2.0........ | 598             | 0.266           | 0.252          | 2.795           | 0.411           | 109            | 0.608           | 0.253           | 0.253           | 0.625           |
| 2.1........ | 519             | 0.300           | 0.251          | 2.789           | 0.407           |               |                 |                 |                 |                 |
| 2.2........ | 457             | 0.323           | 0.250          | 2.533           | 0.373           | 118            | 0.635           | 0.251           | 0.251           | 0.651           |
| 2.3........ | 417             | 0.372           | 0.232          | 2.130           | 0.334           | 121            | 0.664           | 0.240           | 0.240           | 0.681           |
| 2.4........ | 367             | 0.400           | 0.231          | 2.148           | 0.340           | 109            | 0.684           | 0.240           | 0.240           | 0.700           |
| 2.5........ | 335             | 0.444           | 0.231          | 2.174           | 0.349           | 103            | 0.700           | 0.240           | 0.240           | 0.716           |
| 2.7........ | 277             | 0.531           | 0.230          | 2.354           | 0.375           | 79.5           | 0.739           | 0.240           | 0.240           | 0.752           |
| 3.0........ | 217             | 0.642           | 0.230          | 2.505           | 0.383           | 61.6           | 0.803           | 0.240           | 0.240           | 0.806           |
| 4.0........ | 117             | 1.038           | 0.230          | 2.926           | 0.514           | 27.5           | 1.030           | 0.278           | 0.278           | 0.870           |
| 5.0........ | 74.6            | 1.419           | 0.230          | 3.279           | 0.683           | 14.5           | 1.272           | 0.301           | 0.301           | 0.935           |
| 6.0........ | 52.5            | 1.822           | 0.230          | 3.507           | 0.813           | 9.3            | 1.539           | 0.317           | 0.317           | 1.032           |
| 7.0........ | 39.6            | 2.180           | 0.230          | 3.722           | 0.983           | 6.5            | 1.826           | ...*            |                 |                 |
| 9.0........ | 25.7            | 3.071           | 0.230          | 4.102           | 1.414           | 3.6            | 2.416           | ...*            |                 |                 |

* Off-center C burning.

b Central C burning.
the convective core size must increase into C within the core, the opacity increases, and, in turn, measured from above the stability border defined by means of the Schwarzschild criterion. In fact, since the assumed input physics affects the size of the unstable regions, the calibration of the overshooting depends on these assumptions. For example, the larger the opacity, the larger the convective region is and, in turn, the lower the required amount of overshoot. However, the opacity coefficients are likely underestimated rather than overestimated. For this reason, although at the beginning of the 1980s Becker & Methews (1983) claimed a relatively strong overshoot in order to reconcile the theory with the observed distribution of stars in the young globular clusters in the Magellanic Clouds, the latest attempts to derive the size of the convective core overshoot for H-burning stars indicate that, if it is present, it should be mild (see, e.g., Stothers & Chin 1992; Castellani et al. 1992; Schaller et al. 1992; Bressan et al. 1993; Demarque, Sarajedini, & Guo 1994; Mermilliod, Huestmendia, & del Rio 1994; Schröder, Pols, & Eggleton 1997). In particular these studies generally found that the best reproduction of the various indicators of the convective core size is obtained with models including an overshoot roughly confined between 0 and 0.3 (in units of pressure scale height and measured from above the stability border defined by means of the Schwarzschild criterion).

The situation is still more controversial for the central He-burning phase. In such a case, when He is converted into C within the core, the opacity increases, and, in turn, the convective core size must increase (Paczynski 1970b; Castellani, Giannone, & Renzini 1971a). As the He burning proceeds, a minimum in the radiative gradient develops, so mixing occurs in two separated regions: an internal one, which is fully convective, and an external one, in which the resulting mixture of C and He is just that needed to allow convective neutrality (Castellani, Giannone, & Renzini 1971b; see also Iben 1986, and references therein). Such a phenomenon is called He-burning semiconvection. Close to He exhaustion (when the central He becomes lower than approximately 0.1), some instabilities at the border of the convective core appear in stellar model computations (Castellani et al. 1985a, 1985b; Iben 1986). These instabilities are called breathing pulses (BPs). As a consequence of both semiconvection and BPs, a larger amount of fuel is available for central He burning. There is some debate concerning the actual occurrence of BPs in real stars (Renzini & Fusi Pecci 1988; Caputo et al. 1989). In any case, the inclusion of these phenomena might affect some important results of stellar evolution: the estimated central He-burning and AGB evolutionary times, the final amount of C and O in the core, and the final WD mass. Note that the efficiency of both semiconvection and BPs also depends on the adopted input physics. For example, as first discussed by Iben (1972), the use of different prescriptions for the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction rate alters the duration of the final part of the He-burning phase, during which the BPs occur. Therefore, a larger value of this rate will enhance the effects of the BPs.

In summary, when comparing the evolutionary results obtained by different authors, the connection between the assumed mixing scheme and the adopted input physics must be taken into account. This will be done in § 4.

3. THE NEW MODELS

Models from 1.2–9 $M_{\odot}$ and metallicity ranging between $10^{-4}$ and $2 \times 10^{-2}$ have been computed from the ZAMS up to the end of the early asymptotic giant branch (E-AGB) phase. The evolutionary tracks in the H-R diagram are reported in Figures 1, 2, 3, and 4. The runs of the central temperature versus the central density are shown in Figures 5, 6, 7, and 8. Some examples of the evolution of the fully convective regions are illustrated in Figures 9, 10, 11, and 12.

In Tables 1, 2, 3, and 4 we have reported the fundamental properties of the evolutionary sequences: by column in each table, (1) the total mass, (2) the central H-burning lifetime (in Myr), (3) the maximum size (in solar mass units) of the convective core during central H burning, (4) the surface He mass fraction after the first dredge-up, (5) the tip luminosity of the first red giant branch (RGB), (6) the He core mass at the beginning of the He burning, (7) the central He-burning lifetime (in Myr), (8) the He core mass (in solar units) at the end of the He burning, (9) the surface He mass fraction after the second dredge-up, and (10) the He core mass (in solar units) at the beginning of the thermally pulsing asymptotic giant branch (TP-AGB) phase.

In the following subsections, we briefly summarize the main features of the computed sequences of models, revising the dependence of the various evolutionary phases on the stellar mass and on the chemical composition. As already recalled in the introduction of this paper, the evolutionary history of an intermediate-mass star, crossing the H-R

![Fig. 3.—Evolutionary tracks for $Z = 0.001, Y = 0.23$](image-url)
Fig. 4.—Evolutionary tracks for $Z = 0.0001$, $Y = 0.23$

Fig. 5.—Evolution of the central temperature vs. the central density for $Z = 0.02$ and $Y = 0.28$. 

diagram from the main sequence up to the AGB, is well known. For a more accurate description of the various evolutionary phases and an exhaustive list of references, we direct the reader to the review paper by Iben (1991).

3.1. The Central H Burning

All sequences of models having mass larger than $1.2 M_\odot$ develop convective cores during central H burning, independently of initial chemical composition. As a consequence, at variance with low-mass stars, for which H burning occurs in a radiative environment, the evolutionary tracks evolve off the ZAMS toward lower temperature and larger luminosity. As the H is converted into He in the central region of the star, the opacity decreases and the convective core recedes (in mass). The convective instability in the core is retained until the H mass fraction is reduced to about 0.1. Then an overall contraction occurs, and the star rapidly moves toward the radiative main sequence. A maximum in luminosity is reached at the time of H exhaustion.

One interesting quantity characterizing the H-burning phase is the maximum extension of the convective core. As already recalled, this maximum is attained just after the ZAMS (see Figs. 9, 10, 11, and 12). In the last 30 years the computed values for this quantity have systematically increased, mainly because of the increasing values of the adopted radiative opacity coefficients. Our present results are listed in column (3) of Tables 1, 2, 3, and 4. Note that the size of the convective core decreases monotonically as the

### Table 2

**Properties of the Models with $Z = 10^{-3}$, $Y = 0.23$**

| $M$ ($M_\odot$) | $\Delta t_{\text{He}}$ (Myr) | $M_{\text{He}}^*$ ($M_\odot$) | $\log L_{\text{He}}^\text{RGB}$ (5) | $M_{\text{He}}^*$ (Myr) | $\Delta t_{\text{He}}$ (Myr) | $M_{\text{He}}^*$ (Myr) | $\log L_{\text{He}}^\text{RGB}$ (5) | $M_{\text{He}}^*$ (Myr) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.2........    | 3267            | 0.103           | 0.253           | 3.343           | 0.491           | 97.4            | 0.558           | 0.250           | 0.571           |
| 1.5........    | 1502            | 0.141           | 0.250           | 3.331           | 0.488           | 95.7            | 0.575           | 0.253           | 0.587           |
| 1.8........    | 873             | 0.241           | 0.247           | 3.298           | 0.482           | 104             | 0.580           | 0.246           | 0.592           |
| 2.0........    | 659             | 0.321           | 0.246           | 3.152           | 0.456           | 144             | 0.564           | 0.245           | 0.579           |
| 2.1........    | 514             | 0.395           | 0.244           | 2.637           | 0.387           | 152             | 0.592           | 0.244           | 0.609           |
| 2.2........    | 462             | 0.433           | 0.244           | 2.402           | 0.339           | 135             | 0.615           | 0.243           | 0.629           |
| 2.3........    | 412             | 0.472           | 0.243           | 2.369           | 0.339           | 152             | 0.592           | 0.244           | 0.609           |
| 2.4........    | 373             | 0.511           | 0.241           | 2.372           | 0.346           | 118             | 0.630           | 0.241           | 0.644           |
| 2.5........    | 309             | 0.581           | 0.236           | 2.412           | 0.364           | 93.8            | 0.674           | 0.236           | 0.687           |
| 3.0........    | 240             | 0.686           | 0.233           | 2.504           | 0.398           | 68.1            | 0.741           | 0.235           | 0.751           |
| 4.0........    | 127             | 1.065           | 0.230           | 2.845           | 0.526           | 29.0            | 0.988           | 0.269           | 0.853           |
| 5.0........    | 78.9            | 1.425           | 0.230           | 3.132           | 0.672           | 16.1            | 1.257           | 0.299           | 0.920           |
| 6.0........    | 54.5            | 1.822           | 0.230           | 3.464           | 0.809           | 10.0            | 1.504           | 0.317           | 0.991           |
| 7.0........    | 40.6            | 2.202           | 0.230           | 3.597           | 1.013           | 7.2             | 1.776           | ...            | ...             |
| 9.0........    | 26.2            | 3.071           | 0.230           | 4.025           | 1.442           | 3.6             | 2.392           | ...            | ...             |

* Off-center C burning.

* Central C burning.
 initial He increases, while it initially increases as the metallicity decreases (up to $Z = 0.001$), and then it decreases at smaller metallicities. The corresponding H-burning evolutionary times are reported in column (2).

### 3.2. The H-burning Shell

When the H-burning shell settles on, the convective envelope penetrates to more than 80% in the mass of the star, bringing to the surface the products of H burning. As firstly pointed out by Iben (1964, 1967b), the main consequences of this first dredge-up are the increase of the surface abundances of $^4\text{He}$, $^3\text{He}$, and $^{14}\text{N}$ and a decrease of $^{12}\text{C}$ and $^{16}\text{O}$. The modification of the surface composition is stronger in low-mass stars because of the smaller size of the envelope. The surface amount of He resulting after the first dredge-up in our models is listed in column (4) of Tables 1, 2, 3, and 4. The subsequent evolution up to the onset of the He-burning phase is mainly characterized by the equation of state governing the He core. In the strong degenerate regime, a quite large He core mass is necessary to ignite He (about 0.5 $M_\odot$, but it depends on the chemical composition). This is the case in a low-mass star (i.e., $M \leq 1.5 M_\odot$). For more massive stars the degree of degeneracy in the core is reduced, and, in turn, He ignition is more rapidly attained. In the asymptotic limit of nondegenerate matter, the minimum mass needed to ignite He is about 0.35 $M_\odot$. For this reason, the luminosity of the RGB tip and the He core mass attained at He ignition (cols. [5] and [6] of Tables 1, 2, 3, and 4) decrease when the total mass increases from 1.5 to 2.5 $M_\odot$.

### TABLE 3

**Properties of the Models with $Z = 6 \times 10^{-3}$, $Y = 0.26$**

| $M$ ($M_\odot$) | $\Delta t_H$ (Myr) | $M_H$ ($M_\odot$) | $M_{\text{He}}$ ($M_\odot$) | $\log \ell_{\text{RGB}}$ (5) | $M_{\text{He}}$ ($M_\odot$) | $\Delta t_{\text{He}}$ (Myr) | $M_{\text{He}}$ ($M_\odot$) | $\text{He}_{\text{surf}}$ (9) | $M_{\text{He}}$ ($M_\odot$) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.2           | 3558           | 0.061          | 0.281          | 3.412          | 0.483          | 118            | 0.547          |                 |                 |
| 1.5           | 1656           | 0.158          | 0.276          | 3.408          | 0.483          | 116            | 0.554          | 0.276          | 0.558          |
| 1.8           | 974            | 0.260          | 0.273          | 3.364          | 0.474          | 119            | 0.554          | 0.273          | 0.558          |
| 2.0           | 728            | 0.327          | 0.271          | 3.173          | 0.441          | 144            | 0.540          | 0.271          | 0.545          |
| 2.2           | 561            | 0.397          | 0.272          | 2.463          | 0.334          | 253            | 0.506          | 0.272          | 0.520          |
| 2.5           | 500            | 0.419          | 0.273          | 2.502          | 0.340          | 237            | 0.518          | 0.273          | 0.537          |
| 2.7           | 403            | 0.511          | 0.275          | 2.464          | 0.342          | 194            | 0.543          | 0.275          | 0.550          |
| 3.0           | 332            | 0.565          | 0.275          | 2.496          | 0.359          | 150            | 0.578          | 0.275          | 0.580          |
| 3.7           | 255            | 0.676          | 0.275          | 2.575          | 0.391          | 98.6           | 0.649          | 0.275          | 0.649          |
| 4.0           | 128            | 1.011          | 0.266          | 2.898          | 0.519          | 35.1           | 0.911          | 0.283          | 0.848          |
| 5.0           | 77.1           | 1.412          | 0.261          | 3.184          | 0.659          | 19.5           | 1.152          | 0.313          | 0.902          |
| 6.0           | 52.8           | 1.754          | 0.261          | 3.451          | 0.824          | 11.0           | 1.441          | 0.335          | 0.990          |
| 7.0           | 38.9           | 2.159          | 0.261          | 3.675          | 1.004          | 7.8            | 1.739          |                 |                 |
| 8.0           | 30.3           | 2.587          | 0.261          | 3.871          | 1.197          | 5.2            | 2.044          |                 |                 |
| 9.0           | 24.5           | 3.044          | 0.262          | 4.043          | 1.425          | 4.0            | 2.371          |                 |                 |

* Off-center C burning.

b Central C burning.
larger than 2.5–3 $M_\odot$, the off–main-sequence evolution is not further controlled by the growth of the He core. In such a case, owing to the internal mixing occurring during the main sequence, the H-burning shell at the beginning of the RGB settles well outside the minimum mass needed to ignite He. Hence, the RGB tip and the mass of the He core at He ignition rise as the total mass increases.

The minimum resulting from the combination of these two behaviors marks the so-called RGB phase transition (Iben 1967c). Such an occurrence is illustrated in Figure 13. According to the classical results, we found that the minimum core mass is attained for a total mass of 2.3–2.5 $M_\odot$, with the value slightly increasing with metallicity. Note that the almost constant minimum core mass at very different metallicities is the consequence of the opposite influence of $Z$ and $Y$ on this quantity. In fact, as pointed out by Sweigart, Greggio, & Renzini (1990), the transition mass increases as the metallicity increases and decreases as the He increases.

![Figure 8](image-url)

**Fig. 8.—**Evolution of the central temperature vs. the central density for $Z = 0.0001$ and $Y = 0.23$.

![Figure 9](image-url)

**Fig. 9.—**Evolution of the convective regions (hatched regions) for 2.5 $M_\odot$ and solar chemical composition.

![Figure 10](image-url)

**Fig. 10.—**Evolution of the convective regions (hatched regions) for 4 $M_\odot$ and solar chemical composition.

### Table 4

Properties of the Models with $Z = 2 \times 10^{-2}$, $Y = 0.28$

| $M$ ($M_\odot$) | $\Delta t_H$ (Myr) | $M_{H}^*$ ($M_\odot$) | He$_{\text{core}}^{\text{final}}$ | log $t_{\text{tipRGB}}$ | $M_{\text{He1}}^*$ (Myr) | $M_{\text{He2}}^*$ ($M_\odot$) | $M_{\text{He3}}^*$ ($M_\odot$) | $\Delta t_{\text{He}}$ (Myr) | $M_{\text{He1}}^*$ ($M_\odot$) | $M_{\text{He2}}^*$ ($M_\odot$) | $M_{\text{He3}}^*$ ($M_\odot$) | He$_{\text{core}}^{\text{final}}$ | $M_{\text{H}}^{\text{final}}$ ($M_\odot$) |
|--------------|-------------------|---------------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.2…………  | 5121              | 0.056               | 0.300           | 3.440          | 0.477          | 110            | 0.525          | 0.300           | 2.530          | 0.525          | 0.300           | 2.530          | 0.525          | 0.300           |
| 1.5…………  | 2233              | 0.141               | 0.294           | 3.445          | 0.477          | 111            | 0.536          | 0.294           | 0.557          | 0.536          | 0.294           | 0.557          | 0.536          | 0.294           |
| 1.8…………  | 1307              | 0.242               | 0.290           | 3.413          | 0.471          | 119            | 0.540          | 0.290           | 0.550          | 0.540          | 0.290           | 0.550          | 0.540          | 0.290           |
| 2.0…………  | 949               | 0.298               | 0.289           | 3.311          | 0.451          | 130            | 0.527          | 0.289           | 0.550          | 0.527          | 0.289           | 0.550          | 0.527          | 0.289           |
| 2.3…………  | 636               | 0.388               | 0.291           | 2.856          | 0.388          | 192            | 0.501          | 0.291           | 0.530          | 0.501          | 0.291           | 0.530          | 0.501          | 0.291           |
| 2.4…………  | 575               | 0.427               | 0.291           | 2.427          | 0.331          | 259            | 0.487          | 0.291           | 0.517          | 0.487          | 0.291           | 0.517          | 0.487          | 0.291           |
| 2.5…………  | 505               | 0.448               | 0.292           | 2.490          | 0.339          | 234            | 0.495          | 0.292           | 0.534          | 0.495          | 0.292           | 0.534          | 0.495          | 0.292           |
| 2.7…………  | 409               | 0.513               | 0.294           | 2.510          | 0.350          | 196            | 0.510          | 0.294           | 0.539          | 0.510          | 0.294           | 0.539          | 0.510          | 0.294           |
| 3.0…………  | 307               | 0.608               | 0.296           | 2.560          | 0.378          | 141            | 0.545          | 0.296           | 0.571          | 0.545          | 0.296           | 0.571          | 0.545          | 0.296           |
| 4.0…………  | 145               | 0.928               | 0.296           | 2.880          | 0.505          | 46.0           | 0.744          | 0.297           | 0.752          | 0.744          | 0.297           | 0.752          | 0.744          | 0.297           |
| 5.0…………  | 84.5              | 1.275               | 0.296           | 3.187          | 0.655          | 20.8           | 1.024          | 0.325           | 0.859          | 1.024          | 0.325           | 0.859          | 1.024          | 0.325           |
| 6.0…………  | 55.8              | 1.650               | 0.297           | 3.459          | 0.799          | 10.3           | 1.278          | 0.344           | 0.937          | 1.278          | 0.344           | 0.937          | 1.278          | 0.344           |
| 7.0…………  | 39.9              | 2.048               | 0.297           | 3.685          | 1.001          | 7.3            | 1.591          | 0.366           | 1.005          | 1.591          | 0.366           | 1.005          | 1.591          | 0.366           |
| 8.0…………  | 30.3              | 2.490               | 0.300           | 3.889          | 1.156          | 5.1            | 1.883          | …*             | …*            | …*            | …*            | …*            | …*            | …*            |
| 9.0…………  | 24.2              | 2.869               | 0.301           | 4.052          | 1.415          | 4.0            | 2.223          | …*            | …*            | …*            | …*            | …*            | …*            | …*            |

* Off-center C burning.

b Central C burning.
Fig. 11.—Evolution of the convective regions (*hatched regions*) for 7 $M_\odot$ and solar chemical composition.

Fig. 12.—Evolution of the convective regions (*hatched regions*) for 9 $M_\odot$ and solar chemical composition.

Fig. 13.—He core mass at the He ignition as a function of the total mass.
3.3. The Central He Burning

As is well known, the larger the core mass at the He ignition the brighter the star is during the central He-burning phase and the shorter the central He-burning lifetime. Hence a maximum in the He-burning lifetime, corresponding to the minimum He core mass occurring at the RGB phase transition, is expected. In Figure 14 we report the He-burning lifetime vs. the stellar mass for $Z = 0.02$ and $Y = 0.28$. They are listed in column (7) of Tables 1, 2, 3, and 4 for the full set of models.

During central He burning, the evolutionary tracks move toward the blue part of the H-R diagram on a Kelvin-Helmoltz timescale and then move back to the red giant branch when the central He vanishes. The extension of this loop depends on both the stellar mass and the chemical composition (see Alcock & Paczynski 1978, and references therein). The larger the mass, the hotter the left border of the loop is, but an interesting exception is worth noting (Castellani et al. 1990). At the lower metallicities ($Z = 0.0001, 0.001, 0.006$), the more massive stars ignite He before they can reach the RGB, thus skipping the first dredge-up. This occurs for stellar masses $M \geq 2.7 M_\odot$, $M \geq 4 M_\odot$, and $M \geq 5 M_\odot$ at $Z = 0.0001, 0.001$, and $0.006$ respectively. Since the larger the amount of surface He, the lower the opacity is, the fate of the first dredge-up affects the blue loop extension: those models in which the first dredge-up does not occur have narrower blue loops. Such an occurrence is clearly shown in Figure 2, which shows the H-R diagrams for $Z = 0.006$: note that the He-burning evolutionary track of the $4 M_\odot$ sequence has a significantly larger blue loop than the $5 M_\odot$ sequence.

3.4. The Double Shell Phase

During the whole central He-burning phase the H-burning shell moves outward, so the longer the central He burning lifetime, the greater the increment of the He core mass will be (see col. [8] in Tables 1, 2, 3, and 4). For this reason, the minimum in the $M_{\text{He}}$—initial mass relation, which is evident at the beginning of central He burning, is smoothed away at the end of the E-AGB phase (see Fig. 15). As a consequence, all stars with $M \leq 3 M_\odot$ starting the thermally pulsing AGB phase have rather similar He core masses, namely, $0.55 \pm 0.05 M_\odot$ depending on the metallicity (col. [10] of Tables 1, 2, 3, and 4). Such a value provides us a lower limit to the expected mass of a CO WD.

For a sufficiently high initial stellar mass, a second dredge-up occurs during the early AGB (Kippenhahn et al. 1965; Paczynski 1970a, 1970b; Becker & Iben 1979). In such a case, the convective envelope penetrates the H discontinuity located just below the H-burning shell, so the resulting He core mass is lowered with respect to the value attained at the end of central He burning (cols. [10] and [8], respectively, of Tables 1, 2, 3, and 4). The second dredge-up takes place only if the H-burning shell extinguishes. This is not the case for a low-mass star, in which the expansion induced by the He-burning shell does not induce a sufficient cooling of the H-burning shell. We found that the minimum mass for the occurrence of the second dredge-up is $4 M_\odot$ for the three lower metallicities and a bit larger (about $5 M_\odot$) in the case of $Z = 0.02$. The surface He mass fraction after the second dredge-up is listed in column (9) of Tables 1, 2, 3, and 4.

3.5. The Final Masses and $M_{\text{up}}$

The white dwarf (WD) masses resulting from the evolution of low- and intermediate-mass stars are very important quantities for the purpose of the study of population synthesis, planetary nebula, novae, supernovae, and the like (see, e.g., Iben 1991).

In Figure 16 we compare our theoretical final masses with the relation reported by Weidemann (1987) and the updated one by Herwig (1995). Squares represent the He core masses at the end of the E-AGB, while arrows show the growth of the He core masses during the TP-AGB phase, as derived using our thermally pulsing models (Straniero et al. 1997b). The number reported at the top of each arrow is the number of thermal pulses computed up to the end of the AGB phase. It was determined according to the mass-loss rate prescriptions of Groenewegen & de Jong (1994).

Note that the Weidemann predictions (as well as the recent improvements incorporated by Herwig) are based on a semiempirical approach. They make use of various methods to evaluate the WD masses in nearby open clusters, whose initial mass is derived from the turnoff age. Our final masses are instead the result of a pure theoretical calculation and, hence, they are mainly dependent on the adopted input physics. Such a difference should be kept in mind when comparing our results with those of Weidemann (or Herwig), as we do in Figure 16. Despite the two different approaches, there is an acceptable agreement between our theoretical final masses and those of Weidemann (Herwig). In the next section we will discuss our final core masses in comparison with the ones obtained by other authors by means of alternative stellar evolutionary codes.

Stars with higher masses ignite carbon before the onset of the thermally pulsing phase (Paczynski 1971; Alcock & Paczynski 1978; Becker & Iben 1979, 1980; Castellani et al. 1990; Bressan et al. 1993; García-Berro, Ritossa, & Iben 1997). In Tables 1, 2, 3, and 4 we have distinguished among...
the models that experience an off-center carbon ignition and those with a central carbon ignition. We found that $M_{\text{up}}$, i.e., the maximum mass for which the concurrent action of the pressure of a strong degenerate electron component and the neutrino energy loss in the core prevent the onset of the C burning, ranges between 6.5 and 8 $M_\odot$, the lower and the larger values being obtained for $Z = 0.0001$ and 0.02, respectively.

4. THE PRESENT LEVEL OF UNCERTAINTY

Owing to the large number of numerical algorithms and physical ingredients commonly used in the computation of stellar models, the evaluation of their reliability is not trivial. A first idea of the possible sources of uncertainties can be obtained by comparing the evolutionary sequences obtained by different authors by means of different evolutionary codes and/or input physics. This might also be useful to evaluate the correctness of merging different sets of stellar models. As already recalled, there exists a rather large number of papers that present set of models for intermediate-mass stars. In the following we will compare our results with the most recent and widely adopted collections of these stellar models.

4.1. Old and New Physics

Let us firstly compare the present computations to those of Castellani et al. (1990, 1992), which were obtained by means of almost the same evolutionary code, but by adopting an “old” physics. Such a comparison will provide us with an evaluation of the importance of the most recent (last decade) improvements of the input physics. In Figure 17 we have compared the evolutionary tracks for $Z = 0.02$. The two sets of models appear rather similar except for some (important) details. Concerning the main sequence, the new

![Graphs showing He core mass as a function of the total mass for different Z values and initial Y values.](image-url)
tracks are slightly brighter and the convective path (i.e., from the ZAMS up to the beginning of the overall contraction) is longer.

The new H-burning lifetimes are generally lower (~5%) than the old ones, but this is mainly because of the slightly lower amount of He used in our old computations (namely, \( Y = 0.27 \)). Concerning He burning, the most striking difference is the extension of the blue loop in the more massive sequences, the new ones being significantly wider. The He-burning lifetime is substantially unchanged.

4.2. Different Evolutionary Codes

The second step in the evaluation of the reliability of the evolutionary sequences will be the comparison with the most recent and widely adopted compilations of stellar models. Let us distinguish between models with and without mechanical convective core overshoot.

The most recent sets of intermediate-mass stellar models without overshoot have been published by Lattanzio (1991, hereafter L91) and Vassiliadis & Wood (1993, hereafter VW93). Despite the differences in the chemical composition and in the input physics, there is good agreement between our H-burning models and those in the two papers cited above. For example, by interpolating on the grid published by Lattanzio, we derive for a 2.5 \( M_\odot \) star (\( Z = 0.02 \) and \( Y = 0.28 \)) an H-burning lifetime of 512 Myr, as compared with our result, 505 Myr. For the same stellar mass, but \( Y = 0.25 \), Vassiliadis & Wood found 619 Myr. Since in this range of mass and metallicity we found that by increasing the original helium of \( \delta Y = 0.1 \) the corresponding \( t_H \) must be reduced of about 25 Myr, the quoted value corresponds to about 545 Myr at \( Y = 0.28 \). Similar differences are found for other masses and other chemical compositions. For the He-burning models, the situation is more complicated. The He-burning lifetime is strongly dependent on both the assumed scheme for convection and the \( ^{12}C(\alpha, \gamma)^{16}O \) reaction rate. As in VW93, we allow convection and suppress breathing pulses, whereas both are allowed in Lattanzio’s computation. On the other hand, we use the \( ^{12}C(\alpha, \gamma)^{16}O \) reaction rate of Caughlan et al. (1985), which is about 3 times larger than the rates adopted by Lattanzio (1991) and Vassiliadis & Wood (1993). We recall that the

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**Fig. 16.**—Final masses: the He core masses at the beginning of the TP-AGB phase (squares); the residual masses at the end of the AGB (triangles); initial/final mass relation by Weideman (1987) for the Galactic disk (dotted line) and for the Magellanic Clouds (dashed line); the initial/final mass relation updated by Herwig (1995) for the Galactic disk (solid line). The numeric labels indicate the number of thermal pulses occurring before the end of the AGB phase.

**Fig. 17.**—Comparison of the present evolutionary tracks and those presented by Castellani et al. (1990, 1992).

**Fig. 18.**—Comparison of the present H-burning lifetimes (p-p) and those by Bressan et al. (1993; “B93”) and Shaller et al. (1992; “S92”). The differences (in percent) for each mass are reported.
larger this reaction rate, the larger the He-burning lifetime. Bearing in mind these differences in the input physics, for a $5 M_\odot$ star with $Z = 0.02$ (0.016 in VW93), one finds 20.8, 23.5, and 30.6 in the present paper, VW93, and L91, respectively. For lower masses and/or metallicities, the differences are similar. For example, for $M = 1.5 M_\odot$ and $Z = 0.001$ we found $t_{\text{He}} = 97.4$ Myr, as compared to 122.2 Myr in VW93 and 118.3 in L91. In summary, our H-burning lifetimes appear to be in good agreement with those obtained in other studies, the differences in the evolutionary time-scales being always lower than 10%. Note that similar differences were found with respect to our old computations. On the contrary, the present uncertainty in the theoretical evaluation of the He-burning lifetime is definitely larger. Differences of up to 30% are found in $t_{\text{He}}$. In principle they should be primarily attributed to the uncertainties in the convective algorithm and/or in the major He-burning reaction rates. In practice, because of the connection between input physics and mixing efficiency, it is rather complicated to disclose the origin of such differences.

Recent models including a moderate amount of convective core overshoot have been published in a series of papers by the Padua group (see Bressan et al. 1993, hereafter B93, and reference therein) and the Geneva group (Schaller et al. 1992, hereafter S92, and reference therein). We recall that the B93 models were obtained by extending, in practice, the mixed central region of an H-burning or He-burning intermediate-mass star by approximately 0.25 $H_p$ over the unstable zone, while Schaller et al. assume 0.2 $H_p$.

For H burning, when the convective core overshoot is taken into account, a larger amount of fuel is available in the burning region of the star. However, since the larger the mixed region is, the brighter the star, this additional fuel is more rapidly burned, so the effect of the core overshoot on the H-burning lifetime is partially counterbalanced. In Figure 18 we compare our H-burning lifetimes to those resulting from the B93 and S92 models. As expected, overshoot models are generally older than the corresponding classical ones. Note, however, that despite the similar amount of overshoot assumed by Bressan et al. and Schaller et al., the differences between these two sets of models are comparable to those found with respect to our (no-overshoot) models.

Another important consequence of convective core overshoot during central H burning is the reduction of the mass at which the RGB transition occurs. Because the RGB evolution is faster if the star has a nondegenerate He core, the number of stars lying on the RGB of Galactic open clusters might be used, in principle, to derive the value of the transition mass and, in turn, to discriminate between models with and without core overshoot (see, e.g., Mermilliod et al. 1994, and references therein). The comparison between our models and those of the Padua group shows that the difference is presently quite small. For example, at $Z = 0.02$ we found a transition mass of 2.4 $M_\odot$, while Bressan et al. (1993) found 2.2 $M_\odot$. Thus, minor differences are expected in the synthetic RGB populations. Note that a similar comparison cannot be made with the Schaller et al. models because their set is not spaced enough in mass.

At variance with the H-burning phase, the inclusion of a moderate overshoot in computing He-burning models does not significantly alter the resulting He-burning lifetime. In fact, if a moderate overshoot is taken into account, the semiconvective layer is hidden by this extra mixing, but the total amount of fuel (He) available for the central nuclear burning should be practically the same as that found in models without core overshoot but including semiconvection. Our He-burning evolutionary times and those obtained by Bressan et al. and Schaller et al. are compared in Figure 19. Note that B93 adopt the rather low Caughlan & Fowler (1988) rate for the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction, whereas S92 use our preferred rate (i.e., Caughlan et al. 1985). The differences in the stellar lifetimes are larger than those found in the case of the H-burning phase (up to 60%). Also, in this case, the origin of the disagreement is not easily recognized.

Let us conclude by noting that the typical differences that we found when comparing our models (no overshoot) with those by L91, VW93, B93, and S92 are of the same order of magnitude as those found in the comparisons between the two set of models with convective core overshoot. In other words, in the range of intermediate-mass stellar models, the current uncertainty due to a possible nonnegligible occurrence of convective core overshoot appears less severe than, or of the same order of magnitude as, those induced by other input physics.

4.3. The Reliability of the Theoretical Core Masses

In the previous section we have compared the final masses obtained by evolving our models up to the end of
Fig. 21.—Isochrones fitting to the Pleiades by using (left panel) our isochrones and (right panel) those of Bertelli et al. (1994)

Fig. 22.—Isochrones fitting to NGC 2420

the AGB to the semiempirical initial/final mass relation (Weidemann 1987). These quantities depend on the core mass attained at the beginning of the thermally pulsing AGB phase and on the AGB mass-loss rate (see, e.g., Iben & Renzini 1983). For the more massive stars, the efficiency of the second dredge-up should also be taken into account (Paczynski 1971; Becker & Iben 1979, 1980). In the following we compare our evolutionary core masses at the first thermal pulse (TP) with the ones obtained by other authors. Let us recall that the larger the duration of the He burning phase is, the larger the time available for the shell H burning to advance in mass. Hence, the large uncertainty on the current estimation of the stellar lifetime (as illustrated in § 4.2) might affect the theoretical previsions of the final masses. Concerning the convective algorithm, the lowest lifetime and, in turn, the smallest He core mass are obtained when semiconvection, BPs and overshooting are neglected, while an approximate doubling of the He-burning lifetime is found when, as in our models, only the semiconvection is taken into account. However by comparing the RGB, hydrogen burning (HB), and AGB theoretical lifetime ratios to the observed stellar number ratios of well-studied Galactic globular clusters, it is possible to discriminate among the
various mixing hypothesis (see, e.g., Renzini & Fusi Pecci 1988). Thus, there is a support for the classical semiconvection scheme (no BPs), but a moderate overshooting, which could mimic the effect of semiconvection, cannot be ruled out. The current uncertainty in the resonant contribution to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate might also change the estimated He-burning lifetime (Iben 1972). In such a case, by varying the astrophysical factor in the range of value compatible with the available measurements for this reaction rate (see Buchmann 1996, 1997), we have obtained a variation of the He-burning lifetime of about 5%-10%.

In Figure 20 we compare our core masses at the end of the E-AGB to those computed by Lattanzio (1991) and Bressan et al. (1993). In spite of the rather large discrepancies found in the He-burning evolutionary timescales, there is a good agreement between the core masses obtained by the different authors. Only the core masses of the more massive models by Bressan et al. are rather larger than ours (i.e., for $M \geq 5 M_\odot$). From Table 5 of B93 we see that the maximum core masses attained before the onset of the second dredge-up in the 5 and 7 $M_\odot$ sequences are slightly lower than in our models. Thus their larger core masses at the first TP should be a consequence of a less efficient second dredge-up. This is also confirmed by the smaller changes induced by the dredge-up on the surface composition. Note that B93 even include 0.7 $H_p$ of undershoot in their computations.

5. TEST OF THE EVOLUTIONARY SEQUENCES

Although a detailed comparison of our stellar models with the observed properties of different stellar systems is well beyond the purpose of the present paper, let us discuss two interesting tests of the evolutionary sequences recently proposed by different authors, which allow us to check the reliability of the current theoretical scenario. To do that we have computed selected isochrones and synthetic diagrams based on the present set of stellar models.

5.1. The Pleiades and the Brown Dwarf Test

The certain identification of brown dwarfs would provide important informations on the star formation rate of very low mass stars and shed some light on the dark matter problem. For this reason, in the last few years, the search for these objects in nearby stellar clusters has been intensified. Brown dwarfs are (quasi) stars for which H burning does not occur or, at least, it does not reach full equilibrium. How might one verify such an occurrence? A low-mass star approaching the main sequence is fully convective, so that the products of the internal nuclear burning should appear at the surface. Thus, the best indicators of the occurrence of the internal nuclear burning are the secondary elements of the $p$-$p$ chain. In this context, Li is a good tracer, since it is a very volatile element and is easily observable in faint objects. Stars with $M \leq 1 M_\odot$ deplete Li even in the pre-main sequence. However, in very low mass stars, in which the internal temperature never exceeds $2-3 \times 10^6 K$, the Li remains unburned. Therefore, the presence of observable Li lines in stellar spectra is a certain identification of a brown dwarf (Rebolo, Martin, & Magazzù 1992). In turn the reappearance of Li in the lower main sequence of stellar clusters provides an interesting test for the age estimated by means of stellar models. In fact the upper luminosity for which Li is measured depends on the age of the cluster: the larger the age is, the fainter the Li cutoff. Basri, Marcy, & Graham (1996), by means of accurate infrared photometry coupled to high-resolution spectroscopy of brown dwarf candidates in the Pleiades, have been able to identify the Li cutoff. By using this Li test, they found an age of $\sim 115$ Myr. They conclude that this value is definitely larger than the age estimated by comparing the turnoff luminosity with that predicted by canonical stellar models.

In Figure 21 we report the isochrones fitting obtained by using our present stellar models (no overshoot) and that derived by Bertelli et al. (1994) by using the B93 models. Accordingly to the Hipparcos parallaxes, we adopt a true distance modulus of 5.33. A reddening of 0.04 has been assumed. In both cases the isochrones are computed for $Z = 0.02$ and $Y = 0.28$. From the canonical models, by excluding the isolated bright star at $V = 2.87$ or $M_V = -2.58$ (a blue straggler?), one can get an age of at least 120 Myr and not more than 140 Myr, i.e., a value in very good agreement with the brown dwarf test. It is worth noting that owing to the small number of stars in the turnoff region, a precise age cannot be derived by means of the isochrone fitting. However, even taking into account such an uncertainty, we can exclude ages lower than 100 Myr. Similarly, with the Bertelli et al. (1994) isochrones one may get an age between 150 and 200 Gyr, which is a bit larger than that implied by the brown dwarf Li cutoff. Obviously, in this case, too, uncertainty due to the low statistical significance of the number of turnoff stars might be claimed, so we cannot definitely rule out the presence of a moderate overshoot. Let us finally note that this canonical estimate of the Pleiades age, based on the new distance, removes the old controversy of the lack of an evident lower main-sequence turn-on for those stars approaching the ZAMS (Herbig 1962; Stauffer 1984). In fact the corresponding lower limit claimed by Stauffer (i.e., 100 Myr) is well in agreement with the present determination.

5.2. The White Dwarf Luminosity Function

An intermediate-mass star ($M \sim 5-6 M_\odot$) ends its life as a CO white dwarf. Thus, if this star is a member of an old stellar system (say 1 Gyr or older), it spends most of its life as a WD, so its cooling time might be used as an age indicator for the stellar system. A search for the cutoff of the WD luminosity function has been recently performed by Von Hippel, Gilmore, & Jones (1995) by means of the Hubble Space Telescope with the Wide Field Planetary Camera 2. They were able to identify this cutoff in two old open clusters, NGC 2420 and NGC 2477. Then, by means of the theoretical cooling sequences computed by Wood (1994), they estimated the ages of these two clusters and concluded that they are in contrast with all the available stellar models. Owing to the existence of accurate CCD photometry for only one of these two clusters, NGC 2420 (Anthony-Twarog et al. 1990), we will focus our attention on this one. From the paper by Von Hippel et al., we derive an age of 1.5–1.6 Gyr based on the WD luminosity function cutoff.

In Figure 22 we show the isochrones fit to the color magnitude diagram of NGC 2420. According to Anthony-Twarog et al. (1990), we have assumed a metallicity of $Z = 0.008$, which corresponds to about $[M/H] = -0.4$, and an $E(B-V) = 0.05$. Then the distance modulus was derived by reproducing the clump of the He-burning stars,
which is a feature almost independent of age (see Castellani et al. 1992). The resulting age is of the order of 1.6 (±0.2) Gyr, a value in very good agreement with the age derived from the WD cooling sequence. A slightly larger value would be obtained by adopting isochrones including a moderate amount of convective core overshoot. For example, Carraro & Chiosi (1994) found 2.1 Gyr, while Friel (1995) reported 2.8 Gyr. However, as already noted by Demarque et al. (1994), the canonical isochrones cannot account for the distribution of stars near the turnoff in NGC 2420. In particular the path of the isochrones just before the overall contraction appears shorter than that observed. Demarque et al. showed that a moderate amount of convective core overshoot (λ = 0.23H_p) make the isochrones path longer, but they are forced to use an age of 2.4 Gyr.

We argue that the isochrones provide us just the locus "permitted" to single stars in the color-magnitude diagram (CMD). In order to understand if and how this permitted locus is really populated or not (and to what extent), a comparison between observed and synthetic CMDs is absolutely required. When a suitable mass function as well as binary stars are considered, a very good reproduction of the observed sequences of NGC 2420 is obtained (see Fig. 23). In particular the contribution of the binaries leads to a larger spread in the main sequence. Note the effect on the turnoff region: the convective path of single stars seems prolonged by the presence of the binary main sequence and the bluer region after the overall contraction gap is depopulated. Also in this case we obtain an age of 1.6 Gyr, which is in very good agreement with the value derived from the WD luminosity function cutoff.
6. CONCLUSIONS

In this paper we have illustrated the main properties of our latest set of stellar models for intermediate-mass stars as obtained by means of the FRANEC code (Chieffi & Straniero 1989). By comparing the most recent evolutionary sequences computed by using different evolutionary codes and/or input physics, we found a rather large disagreement, which is partially due to the influence of the theoretical assumptions on the estimated extension of the convective regions. We again remark that it is not only the difference in the adopted convective algorithm (Schwarzschild criterion, overshooting, semiconvection and the like) that is responsible for such a disagreement. The connection among the various ingredients of the model recipe must be understood in order to recognize the origin of the theoretical uncertainties. In some cases we found that models obtained by using very different schemes for the treatment of the convective instabilities are more similar than models obtained by using the same algorithm but different input physics (EOS, opacity, nuclear reaction rate, and the like).

In spite of this disagreement, since the brighter a model is, the lower the lifetime, many differences are smoothed away when transposing the evolutionary tracks onto the isochrones. This has been already shown in the previous section, where we compared our theoretical isochrones and those of Bertelli et al. (1994) with the color magnitude diagrams of some well-studied open clusters (see Fig. 21). Although the evolutionary features of these two sets of models are rather different, the resulting isochrone fittings are quite similar in the two cases. This means that a quite similar isochrone path may be obtained simply by rescaling the mass (or the age). Such an occurrence is clearly illustrated in the example reported in Figure 24. In this figure we compare our isochrones of 0.7 Gyr with the ones of Bertelli et al. having 0.8 Gyr.

We recall that the Bertelli isochrones were obtained by assuming a moderate amount of overshoot, whereas our models do not include any extra mixing with respect to the instability boundary. As discussed in the previous section, the only evident difference is in the shape of the turnoff.

Another quantity that is well established in the framework of the current theory of the stellar evolution is the core mass attained at the beginning of the AGB phase. In fact, since the luminosity of an off-main-sequence star is mainly controlled by the size of its He-core mass, the H-burning shell has more time to advance in mass in a model with a lower core mass.

Let us finally comment that the use of color magnitude diagrams to check the reliability of a particular set of models can hardly be made by means of the isochrone fitting. In fact the best photometric studies of open clusters include few thousand of stars. Then the “permitted” locus does not necessarily coincide with the “populated” locus. In addition many open clusters have a huge population of binary stars that contribute to determining the shape of the observed color magnitude diagrams. The case of NGC 2420, as discussed in the previous section, is an example of such a situation.

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Fig. 24.—Comparison of our isochrone for an age of 0.7 Gyr (no overshoot) and that of Bertelli et al. (1994) for 0.8 Gyr (moderate overshoot). Both these isochrones have $Z = 0.02$ and $Y = 0.28$. 

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