On a debate about cosmic censor violation

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(Dated: October 20, 2018)

We review the arguments and counter arguments about the recent proposal for generic censorship violation. In particular the argument made in [3] against our proposal for a possible expanding domain wall that could encompass a large black hole, is shown to have a serious flow. Other problems of the original idea are also discussed.

PACS numbers: 04.20.-q, 04.20.Dw, 95.30.Sf,

I. THE MODEL

There has been a recent upsurge in interest in the possibility of a generic violation of Cosmic Censorship, motivated by the arguments put forward in [1]. This work describes a type of situation that is argued would lead to evolution from smooth initial data to a naked singularity for certain type of scalar field models with asymptotic Anti de Sitter (AdS) space-times. Specifically the model considers a scalar field minimally coupled to gravity and having a self interaction potential with two local minima having negative values: $V(0) = -3 V_0$ and $V(\phi_1) = -3 V_1$ with $0 < V_1 < V_0$. The potential is chosen to have a small positive barrier between these minima and is required to satisfy the positive energy condition among the configurations that are asymptotically AdS corresponding to $\phi = \phi_1$ with $0 < \phi_1 < \phi_0$. The effective cosmological constant, its ADM mass is given by

$$M_{\text{ADM}} = \frac{V_1}{2} R_1^3 + \tilde{M}_V R_1^3 + \tilde{M}_\phi R_1$$

(1.3)

where $M_V$ and $\tilde{M}_\phi$ are expressions (the first related to the contribution of the scalar field potential and the second related only to the scalar field gradients) that depend only on the scalar field configuration expressed in terms of a rescaled radial coordinate $y = r/R_1$ and thus not depend explicitly on $R_1$. Concretely the functionals are given by

$$M_V[\phi(y)] = \frac{1}{2} \int_0^1 e^{-\int_{y_0}^y \frac{1}{2} (\partial_y \phi)^2 dy} V(\phi)y^2 dy$$

$$\tilde{M}_\phi[\phi(y)] = \frac{1}{2} \int_0^1 e^{-\int_{y_0}^y \frac{1}{2} (\partial_y \phi)^2 dy} \frac{1}{2} (\partial_y \phi)^2 y^2 dy$$

Thus, if one keeps the rescaled configuration fixed (i.e. one keeps $\phi(y)$ fixed) the scaling properties of the mass are given by the expression [1,3]. The situation considered in [1] involves adjusting the parameters of the theory so that for the configuration that minimizes $M_V$ (denoted by $\phi_0(y)$) the terms proportional to $R_1^3$ cancel out, i.e. $V_1 = -2 M_V[\phi_0(y)]$. The argument for cosmic censorship violation is the following: The configuration $\phi_0(y)$ has a value $y_0 < 1$ such that for $y < y_0$, the configuration is very close to the true vacuum $\phi = 0$, i.e. $\phi_0(y) < \epsilon$ for a small epsilon. The corresponding region is essentially a region of radius $r_0 = R_1 y_0$ of AdS spacetime with a cosmological constant given by $V_0$ and a scalar field slightly removed from the true minimum. Such a configuration is known to evolve to a singularity. The ADM mass of this configuration is $M_0(R_1) = M(\phi_0)R_1$. Let’s assume that a black hole develops and that it has at asymptotically late times a radius $R_{BH}$. Assuming it is a standard AdS black hole corresponding to the asymptotic value of the effective cosmological constant, its ADM mass is given by $M_{BH} = (1/2)(R_{BH} + V_1 R_{BH}^3)$. Its radius therefore must satisfy $(1/2)(R_{BH} + V_1 R_{BH}^3) < M_0(R_1)$. In particular $R_{BH} < (2M_0(R_1)/V_1)^{1/3} = CR_1^{1/3}$. As the matter fields in this theory satisfy the null energy condition,
the area of the event horizon has to be an increasing quantity in the sense that given two Cauchy hypersurfaces the intersection of the event horizon with the later one has larger area than its intersection with the earlier one. Thus the intersection of the initial data hypersurface with the event horizon has to occur (if at all) at $r_{\text{intersection}} < R_{BH} < CR_1^{1/3}$. Clearly one can choose $R_1$ sufficiently large so that $r_0 = R_1 y_0$ be much larger than $CR_1^{1/3}$ given the fact that the former scales like $R_1$ while the latter scales like $R_1^{2/3}$. In this situation, it is argued that the black hole can not encompass the region that evolves into a singularity. Thus the singularity that results from the evolution should be naked.

II. DISCUSSION

The argument presented by us in [2] suggests the possibility that given the initial conditions set up in [1], a type of domain wall (connecting the local and global minima of the potential) will expand continuously, leading to a condition where a large black hole could form in the interior region (the one corresponding to the minimal area). In [2] it is argued that “It is easy to show that in our case this could not occur” because “the region where $\phi$ is close to the global minimum (of the potential) cannot expand without increasing the total energy. This is because the initial profile $\phi(y)$ is already chosen to minimize the potential contribution to the energy... Any other shape for the wall will have higher energy”. There is a serious flaw in this argument: The initial profile was chosen to minimize the value of $\hat{M}_V$, and not of the total energy: $M_{\text{ADM}}$. In fact the configuration $\phi_0(y)$ could in principle have a total energy that is quite higher than the minimum within $C[R_1]$. If one were to choose as initial configuration the true minimum of the total energy in $C[R_1]$, the scaling properties (as one changes the value of $R_1$) would certainly differ from those above. Thus, there is a clear possibility, as suggested in [2], that the configuration will evolve into a barrier that expands indefinitely (i.e. $R_1$ would expand), while the energy at every “instant” of the corresponding frozen configuration (i.e. one where the kinetic terms are made zero by hand) decreases, the term $\hat{M}_V$ increasing slowly and the term $\hat{M}_0$ decreasing faster (as needed). The true total energy of the configuration ($M_{\text{ADM}}^{\text{true}}$) being of course conserved, after adding the kinetic terms in the actual solution ($M_{\text{ADM}}^{\text{true}} = M_{\text{ADM}}^{\text{frozen}} + M_{\text{kinetic}}$). What is known is therefore that $\hat{M}_V$ has to increase relative to its initial value and that $\hat{M}_0$ has to be positive definite. The issue is then if one can envision an evolution (with some appropriate time parameter $t$ label the spacetime foliation) such that the instantaneous configuration $\phi(r, t)$ differs from $\phi_1$ only for values of $r < R_1(t)$, and where $R_1(t)$ increases without bound. Energetically all we need is to show that it is possible for the corresponding ADM mass of the instantaneously frozen configuration to decrease. It is easy to construct an example compatible with what is known about the functionals $\hat{M}_V$ and $\hat{M}_0$: Let us take as parameter the value of $R = R_1(t)$ rather than $t$. So the instantaneous configuration will be described by $\phi(y, R)$. As a result, through the dependence of the configuration on $R$, the functionals become, when evaluated on $\phi(y, R)$, functions of $R$. About these functionals we know, in principle, only that $\hat{M}_V[R]$ increases relative to its value at $R_0 = R_1(t = 0)$, and that $\hat{M}_0[R]$ is positive definite. Now one can give a simple example of functions satisfying the required behaviour:

$$\hat{M}_V[R] = \hat{M}_V[R_0] + A(R - R_0)R^{-9/2} \quad (2.1)$$

where $A = \hat{M}_0[R_0]R_0^{3/2}$, and

$$\hat{M}_0[R] = \hat{M}_0[R_0] \left( \frac{R_0}{R} \right)^{5/2} \quad (2.2)$$

In this way

$$M_{\text{ADM}}^{\text{frozen}}[R] = V_1 R^3 + \hat{M}_V R^3 + \hat{M}_0 R = \hat{M}_0[R_0] R_0^{3/2} R^{1/2} \quad (2.3)$$

which is clearly a decreasing function of $R$. As mentioned above the true mass of the evolving configuration will be increased relative to the value above by the kinetic terms, leading to a mass that is conserved through out the evolution. Therefore, and in contrast with the claims made in [2] and seconded by [3], these energetic arguments can not be used to exclude the possibility that a domain wall will developed in the situation that has been proposed in [1]. Furthermore, as the arguments in [1] are supposed to refer to a generic situation (i.e. they are presented as evidence of a generic “violation of Cosmic Censorship”), the discussion of the issue at hand can not (if the generic nature of the argument is to be preserved) be based on the detailed properties of a specific configuration (i.e. the absolute minimum of $\hat{M}_V$).

In a recent work [3], the original proponents of the generic cosmic censor violation have argued that the connection between the almost homogeneous region and a singularity is in doubt due to possible influences of the arbitrarily far away regions in finite time in AdS spacetimes.

One final issue that needs to be clarified is the following, even if one knew that the starting configuration had no “bag of gold” present, a situation that would clearly invalidate the initial parametrization of the metric, there is the possibility that a “bag of gold” configuration might appear as a result of the evolution. In such a case all the region containing the singularity could be arbitrarily large and still be contained within a black hole of arbitrary small area, completely circumventing the arguments of [1]. In fact, even in the standard collapse of a spherical distribution of dust leading to the formation of a Schwarzschild black hole, one can encounter the formation of such “bags of gold”: The vacuum region would
eventually include points within the event horizon, which, by virtue of Birkoff’s theorem, must correspond to the extended Schwarzschild metric. Within the event horizon the surfaces of constant value of \( r \) (the radial area parameter of standard Schwarzschild coordinates) are spacelike, thus a given Cauchy hypersurface can easily cross them in both directions. In such situation the Cauchy surfaces would have a bag of gold type of structure: The value of the area of the orbits of isotropic symmetry, would, as we move inwards, change from being a decreasing quantity into an increasing one.

### III. CONCLUSION

In conclusion, there are at this point various issues that cast doubts on the argument of [1]: The one raised in [2], and discussed here in more detail, and the one raised in [3], and the issue regarding the possibility of the generation of bag of gold. However the main issue that is raised as the result of [1]: whether or not a Big Crunch singularity \( \mathbb{2} \) arises in the original proposed situation, remains at this time an important and open question.

### IV. ACKNOWLEDGMENTS

This work was supported in part by CONACyT through grant 149945, by DGAPA-UNAM through grants IN112401, IN122002 and IN108103-3, and by DGEP-UNAM through a complementary grant.

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