Measurement of the CP Violation Parameters in $B^0 \to \pi^+\pi^-$ Decays

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Abstract

We present a measurement of the time-dependent $CP$ violating parameters in $B^0 \rightarrow \pi^+\pi^-$ decays. The results are obtained from the final data sample containing $772 \times 10^6$ $B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We obtain the $CP$ violation parameters

$$A_{CP}(B^0 \rightarrow \pi^+\pi^-) = +0.33 \pm 0.06 \text{ (stat)} \pm 0.03 \text{ (syst)},$$

$$S_{CP}(B^0 \rightarrow \pi^+\pi^-) = -0.64 \pm 0.08 \text{ (stat)} \pm 0.03 \text{ (syst)},$$

where $A_{CP}$ and $S_{CP}$ represent the direct and mixing-induced $CP$ asymmetry, respectively. Using an isospin analysis including results from other Belle measurements, we find $23.8^\circ < \phi_2 < 66.8^\circ$ is disfavored at the $1\sigma$ level, where $\phi_2$ is one of the three interior angles of the CKM unitarity triangle related to $B_{u,d}$ decays.

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FIG. 1: Leading-order Feynman diagrams for $B^0 \to \pi^+\pi^-$ decays. (a) shows the dominant first-order (tree) while (b) shows the second-order loop (penguin) diagram. In the penguin diagram, the subscript $x$ in $V_{xb}$ refers to the flavor of the intermediate-state quark ($x = u, c, t$).

I. INTRODUCTION

Violation of the combined charge-parity symmetry ($CP$ violation) in the standard model (SM) arises from a single irreducible phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1, 2]. A main objective of the Belle experiment at KEK, Japan is to over-constrain the unitarity triangle of the CKM matrix related to $B_{u,d}$ decays. This permits a precision test of the CKM mechanism for $CP$ violation as well as the search for new physics (NP) effects. Mixing-induced $CP$ violation in the $B$ sector has been clearly established by Belle [3, 4] and BaBar [5, 6] in the $\bar{b} \to \bar{c}c\bar{s}$ induced decay $B^0 \to J/\psi K^0$. There are many other modes that may provide additional information on various $CP$ violating parameters.

Decays that proceed predominantly through the $\bar{b} \to \bar{u}u\bar{d}$ transition are sensitive to the interior angle of the unitarity triangle $\phi_2 \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$. This paper describes a measurement of $CP$ violation parameters in $B^0 \to \pi^+\pi^-$ decays, shown in Fig. 1. Belle, BaBar and LHCb have reported time-dependent $CP$ asymmetries in related modes including $B^0 \to \pi^+\pi^-$ [7–9], $(\rho\pi)^0$ [10, 11], $\rho^+\rho^-$ [12, 13] and $a_1^\pm\pi^\mp$ [14, 15].

The decay of the $\Upsilon(4S)$ can produce a $B^0\bar{B}^0$ pair in a coherent quantum-mechanical state, from which one meson ($B^0_{\text{Rec}}$) may be reconstructed in the $\pi^+\pi^-$ decay mode. This decay mode does not determine whether the $B^0_{\text{Rec}}$ decayed as a $B^0$ or as a $\bar{B}^0$. The $b$-flavor of the other $B$ meson ($B^0_{\text{Tag}}$), however, can be identified using information from the remaining charged particles and photons. This dictates the flavor of $B^0_{\text{Rec}}$ as it must be opposite that of the $B^0_{\text{Tag}}$ flavor at the time $B^0_{\text{Tag}}$ decays. The proper time interval between $B^0_{\text{Rec}}$ and $B^0_{\text{Tag}}$, which decay at time $t_{\text{Rec}}$ and $t_{\text{Tag}}$, respectively, is defined as $\Delta t \equiv t_{\text{Rec}} - t_{\text{Tag}}$ measured in the $\Upsilon(4S)$ frame. For the case of coherent $B^0\bar{B}^0$ pairs, the time-dependent decay rate for a $CP$ eigenstate when $B^0_{\text{Tag}}$ possesses flavor $q$ where $B^0$ has $q = +1$ and $\bar{B}^0$ has $q = -1$, is given by

$$\mathcal{P}(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left\{ 1 + q \left[ A_{CP} \cos \Delta m_d \Delta t + S_{CP} \sin \Delta m_d \Delta t \right] \right\}. \quad (1)$$

Here, $\tau_{B^0}$ is the $B^0$ lifetime and $\Delta m_d$ is the mass difference between the two mass eigenstates of the neutral $B$ meson. This assumes $CPT$ invariance, no $CP$ violation in the mixing,

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1 Another notation, $\alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$, also exists in literature.
and that the difference in decay rates between the two mass eigenstates is negligible. The parameter $A_{CP}$ measures the direct $CP$ violation, while $S_{CP}$ is a measure of the amount of mixing-induced $CP$ violation.

In the limit that only the dominant tree amplitude contributes, no flavor-dependent direct $CP$ violation is expected and $S_{CP}$ is sin$2\phi_2$. However, in the $B^0 \rightarrow \pi^+\pi^-$ final state and other $\bar{b} \rightarrow \bar{u}ud$ self-conjugate modes, the value of $\phi_2$ is shifted by an amount $\Delta\phi_2$, due to the presence of additional penguin contributions that interfere with the dominant tree contribution (see Fig. [1]). Thus, the observable mixing-induced $CP$ parameter becomes $S_{CP} = \sqrt{1 - A_{CP}^2} \sin(2\phi_2 + 2\Delta\phi_2)$.

Despite penguin contamination, it is still possible to determine $\phi_2$ in $B^0 \rightarrow \pi^+\pi^-$ with an SU(2) isospin analysis [16] by considering the set of $B \rightarrow \pi\pi$ decays into the three possible charge states for the pions. Here, the two pions in $B^+ \rightarrow \pi^+\pi^0$ decays must have a total isospin of $I = 1$ or $I = 2$, since $I_3^Z = 1$. For the penguin contributions, only $I = 0$ or $I = 1$ is possible because the gluon is an isospin singlet carrying $I = 0$. However, $I = 1$ is forbidden by Bose-Einstein statistics; thus, strong loop decays cannot contribute and hence $B^+ \rightarrow \pi^+\pi^0$ decays only through the tree diagram in the limit of negligible electroweak penguins.

The complex $B^0 \rightarrow \pi\pi$ and $\bar{B}^0 \rightarrow \pi\pi$ decay amplitudes obey the relations

$$A_{+0} = \frac{1}{\sqrt{2}} A_{+-} + A_{00}, \quad \bar{A}_{-0} = \frac{1}{\sqrt{2}} \bar{A}_{+-} + \bar{A}_{00},$$

(2)

respectively, where the subscripts refer to the combination of the pion charges. The decay amplitudes can be represented as the triangles shown in Fig. [2]. As $B^+ \rightarrow \pi^+\pi^0$ is a pure tree mode, these triangles share the same base, $A_{+0} = \bar{A}_{-0}$, and $\Delta\phi_2$ can be determined from the difference between the two triangles. These triangles and $\phi_2$ can be fully determined from the branching fractions, $B(B^0 \rightarrow \pi^+\pi^-)$, $B(B^0 \rightarrow \pi^0\pi^0)$ and $B(B^+ \rightarrow \pi^+\pi^0)$, and the $CP$ violation parameters, $A_{CP}(B^0 \rightarrow \pi^+\pi^-)$, $S_{CP}(B^0 \rightarrow \pi^+\pi^-)$ and $A_{CP}(B^0 \rightarrow \pi^0\pi^0)$. This method has an 8-fold discrete ambiguity in the determination of $\phi_2$, which arises from the 4 triangle orientations about $A_{+0}$ and the two solutions of $\phi_2^{eff}$ in the measurement of $S_{CP}$.

Belle, BaBar and LHCb have reported measurements [7–9], summarized in Table [1] of the $CP$ violation parameters reported here. The previous Belle measurements were based on a sample of 535 million $B\bar{B}$ pairs and are superseded by the analysis presented here.

In Sec. [III] we briefly describe the data set and Belle detector. We explain the selection criteria used to identify signal candidates and suppress backgrounds in Sec. [IV] followed by the fit method used to extract the signal component in Sec. [V]. In Sec. [VI] the results of the fit are presented along with a discussion of the systematic uncertainties in Sec. [VII]. Finally, our conclusions are given in Sec. [VIII].
TABLE I: Summary of $CP$ violation parameters obtained by Belle [7], BaBar [8] and LHCb [9]. For all parameters, the first uncertainty is statistical and the second is systematic. The Belle value for $A_{CP}$ is marginally consistent (1.9σ) with the BaBar and LHCb measurements.

| Parameter $B^0 \to \pi^+\pi^-$ | Belle $(535 \times 10^6 B\bar{B} \text{ pairs})$ | BaBar $(467 \times 10^6 B\bar{B} \text{ pairs})$ | LHCb $(0.7 \text{ fb}^{-1})$ |
|---|---|---|---|
| $A_{CP}$ | $+0.55 \pm 0.08 \pm 0.05$ | $+0.25 \pm 0.08 \pm 0.02$ | $+0.11 \pm 0.21 \pm 0.03$ |
| $S_{CP}$ | $-0.61 \pm 0.10 \pm 0.04$ | $-0.68 \pm 0.10 \pm 0.03$ | $-0.56 \pm 0.17 \pm 0.03$ |

II. DATA SET AND BELLE DETECTOR

This measurement of the $CP$ violation parameters in $B^0 \to \pi^+\pi^-$ decays is based on the final data sample containing $772 \times 10^6 B\bar{B}$ pairs collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ (3.5 on 8 GeV) collider [17]. At the $\Upsilon(4S)$ resonance ($\sqrt{s} = 10.58 \text{ GeV}$), the Lorentz boost of the produced $B\bar{B}$ pairs is $\beta\gamma = 0.425$ nearly along the +z direction, which is opposite the positron beam direction. We also use a 100 fb$^{-1}$ data sample recorded at 60 MeV below the $\Upsilon(4S)$ resonance, referred to as off-resonance data, for continuum ($e^+e^- \to q\bar{q}$, where $q = d, u, s, c$) background studies.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprising CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K^0_L$ mesons and to identify muons (KLM). The detector is described in detail elsewhere [18]. Two inner detector configurations were used. A 2.0 cm radius beampipe and a 3-layer silicon vertex detector (SVD1) were used for the first sample of $152 \times 10^6 B\bar{B}$ pairs, while a 1.5 cm radius beampipe, a 4-layer silicon detector (SVD2) and a small-cell inner drift chamber were used to record the remaining $620 \times 10^6 B\bar{B}$ pairs [19]. We use a GEANT-based Monte Carlo (MC) simulation to model the response of the detector and to determine its acceptance [20].

III. EVENT SELECTION

The decay channel $B^0 \to \pi^+\pi^-$ is reconstructed from two oppositely charged tracks. Charged tracks are identified using a loose requirement on the distance of closest approach with respect to the interaction point (IP) along the beam direction, $|dz| < 4.0 \text{ cm}$, and in the transverse plane, $dr < 0.4 \text{ cm}$. Additional SVD requirements of at least two $z$ hits and one $r - \phi$ hit [21] are imposed on all charged tracks so that a good quality vertex of the reconstructed $B$ candidate can be determined. Using information obtained from the CDC, ACC and TOF, particle identification (PID) is determined from a likelihood ratio $L_{i/j} \equiv L_i/(L_i + L_j)$. Here, $L_i$ ($L_j$) is the likelihood that the particle is of type $i$ ($j$). To suppress background due to electron misidentification, ECL information is used to veto particles consistent with the electron hypothesis. The PID ratios of the two charged tracks $L_{K/\pi}^\pm$, are used in the fit model to discriminate among the three possible two-body channels: $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and $B^0 \to K^+K^-$. 

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Reconstructed $B$ candidates are identified with two nearly uncorrelated kinematic variables: the beam-energy-constrained mass $M_{bc} \equiv \sqrt{(E_{\text{CMS}}^B - p_{B}^\text{CMS})^2}$, and the energy difference $\Delta E \equiv E_{\text{CMS}}^B - E_{\text{beam}}^B$, where $E_{\text{CMS}}^B$ is the beam energy and $E_{B}^\text{CMS}$ ($p_B^\text{CMS}$) is the energy (momentum) of the $B$ meson, all evaluated in the $e^+e^-$ center-of-mass system (CMS). The $B$ candidates that satisfy $M_{bc} > 5.24 \text{ GeV}/c^2$ and $-0.20 \text{ GeV} < \Delta E < 0.15 \text{ GeV}$ are retained for further analysis.

The dominant background in the reconstruction of $B^0_{\text{Rec}}$ arises from continuum production. Since continuum events tend to be jet-like, in contrast to spherical $B\bar{B}$ decays, continuum background can be distinguished from $B\bar{B}$ signal using event-shape variables, which we combine into a Fisher discriminant $F_{bb/q\bar{q}}^{\text{tag}}[22]$. The $B\bar{B}$ training sample is taken from signal MC, while the $q\bar{q}$ training sample is from the off-resonance data sample. The Fisher discriminant is then constructed from the variables described in Ref. [14]. The variable providing the strongest discrimination against continuum is the cosine of the angle between the $B^0_{\text{Rec}}$ thrust direction (TB) and the thrust of the tag side (TO) $|\cos \theta_{\text{TB,TO}}|$. The thrust is defined as the vector that maximizes the sum of the longitudinal momenta of the particles. For a $B\bar{B}$ event, the pair is nearly at rest in the CMS, so the thrust axis of $B^0_{\text{Rec}}$ is uncorrelated with the thrust axis of $B^0_{\text{Tag}}$. In a $q\bar{q}$ event, on the other hand, the decay products align along two nearly back-to-back jets, so the two thrust axes tend to be collinear. Before training, a loose requirement of $|\cos \theta_{\text{TB,TO}}| < 0.9$ is imposed that retains 90% of the signal while rejecting 50% of the continuum background. The range of the Fisher discriminant $-3 < F_{bb/q\bar{q}} < 2$, encompasses all signal and background events.

Backgrounds from charm ($b \to c$) decays are found to be negligible and are thus not considered, while charmless ($b \to u, d, s$) decays of the $B$ meson contribute in the analysis region, though rarely in the signal region.

As the $B^0_{\text{Rec}}$ and $B^0_{\text{Tag}}$ are almost at rest in the $\Upsilon(4S)$ CMS, the difference in decay time between the two $B$ candidates, $\Delta t$, can be determined approximately from the displacement in $z$ between the final state decay vertices as

$$
\Delta t \simeq \frac{(z_{\text{Rec}} - z_{\text{Tag}})}{\beta \gamma c} \equiv \frac{\Delta z}{\beta \gamma c}.
$$

(3)

The vertex of reconstructed $B$ candidates is determined from the charged daughters, with a further constraint coming from the known IP. The IP profile is smeared in the plane perpendicular to the $z$ axis to account for the finite flight length of the $B$ meson in that plane. To obtain the $\Delta t$ distribution, we reconstruct the tag side vertex from the tracks not used to reconstruct $B^0_{\text{Rec}}[21]$. Candidate events must satisfy the requirements $|\Delta t| < 70 \text{ ps}$ and $h_{\text{Rec,Tag}} < 500$, where $h_{\text{Rec,Tag}}$ is the multi-track vertex goodness-of-fit, calculated in three-dimensional space without using the IP profile constraint [4]. To avoid the necessity of also modeling the event-dependent observables that describe the $\Delta t$ resolution in the fit, the vertex uncertainty is required to satisfy $\sigma_{z_{\text{Rec,Tag}}} < 200 \mu\text{m}$ for multi-track vertices and $\sigma_{z_{\text{Rec,Tag}}} < 500 \mu\text{m}$ for single-track vertices.

The flavor tagging procedure is described in Ref. [23]. The tagging information is represented by two parameters, the $B^0_{\text{Tag}}$ flavor $q$ and the flavor tagging quality $r$. The parameter $r$ is determined from MC on an event-by-event basis and ranges from zero for no flavor discrimination to unity for an unambiguous flavor assignment. Due to a non-zero probability of mistagging $w$, the $CP$ asymmetry is diluted by a factor $1-2w$. The measure of the flavor tagging algorithm performance is the total effective tagging efficiency $\epsilon_{\text{eff}} = \epsilon_{\text{Tag}}(1-2w)^2$, rather than the raw tagging efficiency $\epsilon_{\text{Tag}}$, as the statistical significance of the $CP$ parameters is
proportional to \((1 - 2w)\sqrt{\epsilon_{\text{Tag}}}\). These are determined from data to be \(\epsilon_{\text{eff}} = 0.284 \pm 0.010\)
and \(\epsilon_{\text{eff}} = 0.301 \pm 0.004\) for the SVD1 and SVD2 data, respectively.

About 1% of events have more than one \(B\) candidate. For these events, the candidate containing
the two highest momentum tracks in the lab frame is selected.

Differences from the previous Belle analysis include an improved tracking algorithm
that was applied to the SVD2 data sample and the inclusion of the event shape \(F_{\bar{b}\bar{b}/q\bar{q}}\) into
the fit rather than the optimization of selection criteria for this variable. As the latter
strategy results in a large increase of the continuum background level, a reduced fit region
in \(M_{bc}\) and \(\Delta E\) is chosen in order to reduce this background without significant loss of signal
events. These changes increase the detection efficiency by 19% over the previous analysis,
according to MC simulation.

IV. EVENT MODEL

The \(CP\) violation parameters are extracted from a seven-dimensional unbinned extended
maximum likelihood fit to \(M_{bc}, \Delta E, F_{\bar{b}\bar{b}/q\bar{q}}, L_{K/\pi}^+, \Delta t\) and \(q\) from a data sample divided into
7 bins \((l = 0..6)\) in the flavor-tag quality \(r\) and 2 SVD configurations \(s\). Seven categories
are considered in the event model: \(B^0 \rightarrow \pi^+\pi^-\) signal, \(B^0 \rightarrow K^+\pi^-, B^0 \rightarrow K^-\pi^+\) and
\(B^0 \rightarrow K^+K^-\) peaking backgrounds, continuum, charmless neutral and charged \(B\) decays.
For most categories, the linear correlations between fit variables are small, so the probability
density function (PDF) for each category \(j\) is taken as the product of individual PDFs for
each variable: \(P_{l,s}^j(M_{bc}, \Delta E, F_{\bar{b}\bar{b}/q\bar{q}}, L_{K/\pi}^+, \Delta t, q) = P_{l,s}^j(M_{bc}) \times P_{l,s}^j(\Delta E) \times P_{l,s}^j(F_{\bar{b}\bar{b}/q\bar{q}}) \times
P_{l,s}^j(L_{K/\pi}^+, L_{K/\pi}^-) \times P_{l,s}^j(\Delta t, q)\) in each \(l,s\) bin, unless stated otherwise.

A. Peaking Models

The four peaking shapes, including the signal, are determined from reconstructed MC
events. The PDFs for \(M_{bc}\) and \(\Delta E\) are taken to be the sum of three Gaussian functions,
where the two tail Gaussians are parameterized relative to the core, which incorporates
 calibration factors that correct for the difference between data and MC simulation. These
factors calibrate the mean and width of the core Gaussian component. The PDF for \(F_{\bar{b}\bar{b}/q\bar{q}}\)
is taken to be the sum of three Gaussians in each flavor-tag bin \(l\), where the shape parameters
are identical for all peaking channels. Calibration factors that correct for the shape
differences between data and MC are incorporated into the core mean and width. These
factors for \(M_{bc}\) are determined directly in the fit, while for \(\Delta E\) and \(F_{\bar{b}\bar{b}/q\bar{q}}\) these factors
are determined from a large-statistics control sample of \(B^+ \rightarrow \bar{D}^0[K^+\pi^-]\pi^-\) decays. The
\(L_{K/\pi}^\pm\) shape is modeled with a two-dimensional histogram that has been corrected for the
difference between data and MC in PID as determined from an independent study with
inclusive \(D^{\ast +} \rightarrow D^0[K^-\pi^+]\pi^+\) slow decays. The PDF of \(\Delta t\) and \(q\) for \(B^0 \rightarrow \pi^+\pi^-\) is given by

\[
P_{l,s}^{\Delta t,q}(\Delta t, q) \equiv \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left\{ 1 - q\Delta w_{l,s} + q(1 - 2w_{l,s}) \times \left[ A_{CP} \cos \Delta m_D \Delta t + S_{CP} \sin \Delta m_D \Delta t \right] \right\} \otimes R_{B^0D^0}(\Delta t), \tag{4}\]
which accounts for CP dilution from the probability of incorrect flavor tagging $w^{l,s}$ and the wrong tag difference $\Delta w^{l,s}$ between $B^0$ and $\bar{B}^0$, both of which are determined from flavor specific control samples using the method described in Ref. [23]. The physics parameters $\tau_{B^0}$ and $\Delta m_d$ are fixed to their respective current world averages [24]. This PDF is convolved with the $\Delta t$ resolution function for neutral $B$ particles $R_{B^0\bar{B}^0}$, as in Ref. [4]. We consider the $\Delta t, q$ distributions for the flavor-specific $B^0 \rightarrow K^+ \pi^-$ and $\bar{B}^0 \rightarrow K^- \pi^+$ peaking backgrounds separately with

$$\mathcal{P}^{l,s}_{K^{+\pi^-}}(\Delta t, q) \equiv \frac{e^{-|\Delta t/\tau_{B^0}|}}{4\tau_{B^0}} \left\{ 1 - q\Delta w^{l,s} + q(1 - 2w^{l,s}) \cos \Delta m_d \Delta t \right\} \otimes R_{B^0\bar{B}^0}(\Delta t). \quad (5)$$

For the $B^0 \rightarrow K^+ K^-$ peaking background, the $\Delta t, q$ PDF is taken to be the same as that for $B^0 \rightarrow \pi^+ \pi^-$ signal, but as $B^0 \rightarrow K^+ K^-$ has not yet been observed, the CP parameters are set to zero. To account for the outlier $\Delta t$ events not described by the $\Delta t$ resolution function, a broad Gaussian PDF is introduced for every category:

$$\mathcal{P}^{l,s}_{\text{Out}}(\Delta t, q) \equiv \frac{1}{2} G(\Delta t; 0, \sigma_{\text{Out}}^s). \quad (6)$$

### B. Continuum Model

The parameterization of the continuum model is based on the off-resonance data; however, all the shape parameters of $M_{bc}$, $\Delta E$, $\mathcal{F}_{b\bar{b}/q\bar{q}}$ and $\mathcal{L}^\pm_{K/\pi}$ are floated in the fit. As continuum is the dominant component, extra care is taken to ensure that this background shape is understood as precisely as possible, incorporating correlations above 2%. The PDF for $M_{bc}$ is an empirical ARGUS function [25], while $\Delta E$ is modeled by a linear fit in each flavor-tag bin with a slope parameterized by $p_{0}^{l,s}$ and $p_{1}^{l,s}$, depending linearly on $\mathcal{F}_{b\bar{b}/q\bar{q}}$:

$$\mathcal{P}^{l,s}_{q\bar{q}}(\Delta E|\mathcal{F}_{b\bar{b}/q\bar{q}}) = 1 + (p_{0}^{l,s} + p_{1}^{l,s}\mathcal{F}_{b\bar{b}/q\bar{q}}) \Delta E. \quad (7)$$

The $\mathcal{F}_{b\bar{b}/q\bar{q}}$ shape is observed to shift depending on the PID region, so the PDF is a sum of two Gaussian functions in two PID regions, $\mathcal{L}^\pm_{K/\pi} \leq 0.5$ and $(\mathcal{L}^+_{K/\pi} \text{ or } \mathcal{L}^-_{K/\pi}) > 0.5$. A small correlation between the $\mathcal{L}^\pm_{K/\pi}$ shape and flavor-tag $q$ is also observed due to the $s\bar{s}$ component of continuum. As an example, consider the case where two jets are produced in which one contains a $K^+$ and the other contains a $K^-$. If a $B^0_{\text{Rec}}$ candidate is successfully reconstructed with the $K^+$, it inhabits the flavor-specific $K^+ \pi^-$ sector of $\mathcal{L}^+_{K/\pi}$. Then the accompanying $K^-$ could then be used as part of the flavor tagging routine, which leads to a preferred flavor-tag of $B^0$. This enhances the $\mathcal{L}^\pm_{K/\pi}$ distribution in the $K^+ \pi^-$ region and depletes it in the $K^- \pi^+$ region for $q = -1$. To account for this effect, we model $\mathcal{L}^\pm_{K/\pi}$ with an effective asymmetry $A_{q\bar{q}}^{l,s}$ that modifies the two-dimensional PID histogram model $H^{l,s}(\mathcal{L}^\pm_{K/\pi}, \mathcal{L}^\pm_{K/\pi})$, in each $l, s$ bin depending on the flavor-tag:

$$\mathcal{P}^{l,s}_{q\bar{q}}(\mathcal{L}^\pm_{K/\pi}, q) \equiv \frac{1 + qA_{q\bar{q}}^{l,s}(\mathcal{L}^\pm_{K/\pi}, \mathcal{L}^\pm_{K/\pi})}{2} H^{l,s}(\mathcal{L}^\pm_{K/\pi}, \mathcal{L}^\pm_{K/\pi}), \quad (8)$$

where

$$A_{q\bar{q}}^{l,s}(\mathcal{L}^\pm_{K/\pi}, \mathcal{L}^\pm_{K/\pi}) = \begin{cases} +a_{0}^{l,s}[\mathcal{L}^-_{K/\pi} - \mathcal{L}^+_{K/\pi}]^{a_1^{l,s}} & \text{if } \mathcal{L}^-_{K/\pi} - \mathcal{L}^+_{K/\pi} \geq 0 \\ -a_{0}^{l,s}[\mathcal{L}^-_{K/\pi} - \mathcal{L}^+_{K/\pi}]^{a_1^{l,s}} & \text{if } \mathcal{L}^-_{K/\pi} - \mathcal{L}^+_{K/\pi} < 0, \end{cases} \quad (9)$$

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which we hereafter refer to as the “manta ray” function. The $\Delta t$ model

$$P_{l,s}^{B_0 \bar{B}_0}(\Delta t) \equiv \left[ (1 - f_3) \frac{e^{-|\Delta t|/\tau_{\text{tail}}}}{2\tau_{\text{tail}}} + f_4 \delta(\Delta t - \mu_s^\text{tag}) \right] \otimes R_{qq}^s(\Delta t), \quad (10)$$

contains a lifetime and prompt component to account for the charmed and charless contributions, respectively. It is convolved with a sum of two Gaussians

$$R_{qq}^s(\Delta t) \equiv (1 - f_{\text{tail}}^s) G(\Delta t; \mu^s_\text{mean}, S^s_\text{main}, \sigma) + f_{\text{tail}}^s G(\Delta t; \mu^s_\text{mean}, S^s_\text{main}, S^s_\text{tail}), \quad (11)$$

which uses the event-dependent $\Delta t$ error constructed from the estimated vertex resolution

$$\sigma \equiv (\sqrt{\sigma_{\text{Rec}}^2 + \sigma_{\text{Tag}}^2})/\beta \gamma c$$

as a scale factor of the width parameters $S^s_\text{main}$ and $S^s_\text{tail}$.

C. $BB$ Model

The charless $B$ background shape is determined from a large sample of MC events based on $b \to u, d, s$ transitions that is further subdivided into neutral and charged $B$ samples. A sizeable correlation of 18% is found between $M_{bc}$ and $\Delta E$, and is taken into account with a two-dimensional histogram. The PDF for $\mathcal{F}_{bb/qq}$ is taken to be the sum of three Gaussians in each flavor-tag bin $l$, similar to the peaking model. Here, we are able to fix the shape parameters from the peaking model except for the core mean and width. A similar correlation between the flavor-tag and $\mathcal{L}_{K/\pi}^\pm$, similar to that in continuum, is also observed.

Due to $B^0 \bar{B}^0$ mixing in the neutral $B$ background, this effect is correlated with $\Delta t$ and $q$. For the neutral $B$ background, the PDF is given by

$$\mathcal{P}_{B^0 \bar{B}^0}^{l,s}(\mathcal{L}_{K/\pi}^\pm, \Delta t, q) = H^{l,s}(\mathcal{L}_{K/\pi}^+, \mathcal{L}_{K/\pi}^-) \times \frac{e^{-|\Delta t|/\tau_{B^0 \bar{B}^0}}}{4\tau_{B^0 \bar{B}^0}} \left\{ 1 + q A_{B^0 \bar{B}^0}^{l,s} (\mathcal{L}_{K/\pi}^+, \mathcal{L}_{K/\pi}^-) \cos \Delta m d \Delta t \right\} \otimes R_{B^0 \bar{B}^0}^s(\Delta t), \quad (12)$$

and the charged $B$ background PDF is given by

$$\mathcal{P}_{B^+ \bar{B}^-}^{l,s}(\mathcal{L}_{K/\pi}^+, \Delta t, q) = \frac{1 + q A_{B^+ \bar{B}^-}^{l,s} (\mathcal{L}_{K/\pi}^+, \mathcal{L}_{K/\pi}^-)}{2} H^{l,s}(\mathcal{L}_{K/\pi}^+, \mathcal{L}_{K/\pi}^-) \frac{e^{-|\Delta t|/\tau_{B^+ \bar{B}^-}}}{2\tau_{B^+ \bar{B}^-}} \otimes R_{B^+ \bar{B}^-}^s(\Delta t), \quad (13)$$

where $A_{BB}^{l,s}$ are manta ray functions for each $B\bar{B}$ category and $R_{B^+ \bar{B}^-}$ is the $\Delta t$ resolution function for charged $B$ events. As reconstructed background $B$ candidates may borrow a track from the tag side, the average $\Delta t$ lifetime tends to be smaller and is taken into account with the effective lifetime, $\tau_{BB}$.

D. Full Model

The total likelihood for 559797 $B^0 \to h^+ h^-$ candidates in the fit region is

$$\mathcal{L} \equiv \prod_{l,s} \frac{e^{-\sum_{N^j_s} N^j_{l,s} f_{l,s}^{j,s} N_i^{l,s}}}{N_i^{l,s}!} \prod_{i=1}^{N_s} \sum_{j} N^j_i f_{j,l,s}^{l,s} P_{l,s}^{B_0 \bar{B}_0}(M_{bc} \Delta E^i, \mathcal{F}_{bb/qq}^{i}, \mathcal{L}_{K/\pi}^{i}, \mathcal{L}_{K/\pi}^{-i}, \Delta t^i, q^i), \quad (14)$$
which iterates over $i$ events, $j$ categories, $l$ flavor-tag bins and $s$ detector configurations. The fraction of events in each $l,s$ bin, for category $j$, is denoted by $f_{j}^{l,s}$. The fraction of signal events in each $l,s$ bin, $f_{S}^{l,s}$, is calibrated with the $B^{+} \to \bar{D}^{0}[K^{+}\pi^{-}]\pi^{+}$ control sample. Free parameters of the fit include the $B^{0} \to \pi^{+}\pi^{-}$ and $B^{0} \to K^{+}K^{-}$ yields, $N_{q}^{s}$ and $N_{B_{0}^{0}}^{s}$. The individual $B^{0} \to K^{+}\pi^{-}$ and $\bar{B}^{0} \to K^{-}\pi^{+}$ yields are parameterized in terms of their combined yield $N_{K\pi}$ and the $CP$ violating parameter $A_{CP}^{K\pi}$, which are both free in the fit: $N_{K\pi} = N_{K\pi}(1 \mp A_{CP}^{K\pi})/2$. The remaining $N_{B_{0}^{0}}^{s}$ yields are fixed to $N_{B_{0}^{0}}^{SVD1} = (0.269 \pm 0.010)N_{B_{0}^{0}}^{SVD1}$ and $N_{B_{0}^{0}}^{SVD2} = (0.268 \pm 0.004)N_{B_{0}^{0}}^{SVD2}$ as determined from MC simulation. In addition, all shape parameters of the continuum model with the exception of the $\Delta t$ parameters are allowed to vary in the fit. In total, there are 116 free parameters in the fit: 10 for the peaking models, 104 for the continuum shape and 2 for the $B\bar{B}$ background.

To determine the component models and $CP$ violation parameters, in contrast to the previous Belle analysis [7], we fit all variables simultaneously. The previous analysis applied a 2-step procedure where the event-dependent component probabilities were calculated from a fit without $\Delta t$ and $q$. These were then used as input in a fit to $\Delta t$ and $q$ to set the fractions of each component to determine the $CP$ parameters. Our procedure has the added benefit of further discrimination against continuum with the $\Delta t$ variable and makes the treatment of systematic uncertainties more straightforward, at a cost of analysis complexity and longer computational time. A pseudo-experiment study indicates a 10% improvement in statistical uncertainty of the $CP$ parameters over the previous analysis method.

V. RESULTS

From the fit to the data, the following $CP$ violation parameters are obtained:

$$A_{CP}(B^{0} \to \pi^{+}\pi^{-}) = +0.33 \pm 0.06 \text{ (stat)} \pm 0.03 \text{ (syst)}$$

$$S_{CP}(B^{0} \to \pi^{+}\pi^{-}) = -0.64 \pm 0.08 \text{ (stat)} \pm 0.03 \text{ (syst)},$$

where the first uncertainty is statistical and the second is the systematic error (Sec. VI). Signal enhanced fit projections are shown in Figs. 3 and 4. The effects of neglecting the correlation between $M_{bc}$ and $\Delta E$ in the peaking models can be seen there; however, pseudo-experiments show that this choice does not bias the $CP$ violation parameters. These results are the world’s most precise measurements of time-dependent $CP$ violation parameters in $B^{0} \to \pi^{+}\pi^{-}$. The statistical correlation coefficients between the $CP$ violation parameters is +0.10. The peaking event yields including signal are $N(B^{0} \to \pi^{+}\pi^{-}) = 2964 \pm 88$, $N(B^{0} \to K^{+}\pi^{-}) = 9205 \pm 124$ and $N(B^{0} \to K^{+}K^{-}) = 23 \pm 35$ where the uncertainties are statistical only. From the yields obtained in the fit, the relative contributions of each component are found to be 0.5% for $B^{0} \to \pi^{+}\pi^{-}$, 1.6% for $B^{0} \to K^{+}K^{-}$, 97.7% for continuum and 0.2% for $B\bar{B}$ background. For the $CP$ violating parameter $A_{CP}^{K\pi}$, we obtain a value of $-0.061 \pm 0.014$, which is consistent with the latest Belle measurement [20].

Our results confirm $CP$ violation in this channel as reported in previous measurements and other experiments [21, 22], and the value for $A_{CP}$ is in marginal agreement with the previous Belle measurement. As a test of the accuracy of the result, we perform a fit on the data set containing the first $535 \times 10^{6} B\bar{B}$ pairs, which corresponds to the integrated luminosity used in the previous analysis. We obtain $A_{CP} = +0.47 \pm 0.07$ which is in good agreement with the value shown in Table II considering the new tracking algorithm and the 19% increase in detection efficiency due to improved analysis strategy. In a separate fit to
only the new data sample containing $237 \times 10^6$ $B\bar{B}$ pairs, we obtain $A_{CP} = +0.06 \pm 0.10$. Using a pseudo-experiment technique based on the fit result, we estimate the probability of a statistical fluctuation in the new data set causing the observed shift in central value of $A_{CP}$ from our measurement with the first $535 \times 10^6$ $B\bar{B}$ pairs to be 0.5%.

To test the validity of the $\Delta t$ resolution description, we perform a separate fit with a floating $B^0$ lifetime; the result for $\tau_{B^0}$ is consistent with the current world average [24] within 2$\sigma$. As a further check of the $\Delta t$ resolution function and the parameters describing the probability of mistagging, we fit for the $CP$ parameters of our control sample $B^+ \rightarrow \bar{D}^0[K^+\pi^-]\pi^+$; the results are consistent with the expected null asymmetry. Finally, we determine possible fit bias from an MC study in which the peaking channels and $B\bar{B}$ backgrounds are obtained from GEANT-simulated events, and the continuum background is generated from our model of off-resonance data. The statistical errors observed in this study agree with those obtained from our fit to the data.

Using Eq. (2) and input from other Belle publications [26, 27], an isospin analysis is performed to constrain the angle $\phi_2$. A goodness-of-fit $\chi^2$ is constructed for the 5 amplitudes shown in Fig. 2, accounting for the correlations between our measured physics observables used as input. The $\chi^2$ is then converted into a p-value (CL) as shown in Fig. 5. The region $23.8^\circ < \phi_2 < 66.8^\circ$ is disfavored and the constraint on the shift in $\phi_2$ caused by the penguin contribution is $|\Delta \phi_2| < 44.8^\circ$ at the 1$\sigma$ level, including systematic uncertainties.

VI. SYSTEMATIC UNCERTAINTIES

Systematic errors from various sources are considered and estimated with independent internal studies and cross-checks. These are summarized in Table [1] Uncertainties affecting the vertex reconstruction include the IP profile, charged track selection based on track helix errors, helix parameter corrections, $\Delta t$ and vertex goodness-of-fit selection, $\Delta z$ bias and SVD misalignment. The fit model uncertainties including the fixed physics parameters $\tau_{B^0}$ and $\Delta m_d$, parameters describing the difference between data and MC simulation, $\Delta t$ resolution function parameters, as well as the flavor tagging performance parameters $w$ and $\Delta w$, are varied by $\pm 1\sigma$. The parametric and non-parametric shapes describing the background are varied within their uncertainties. For non-parametric shapes (i.e., histograms), we vary the contents of the histogram bins by $\pm 1\sigma$. The fit bias is determined from the difference between the generated and fitted physics parameters using pseudo-experiments. Finally, a large number of MC pseudo-experiments are generated and an ensemble test is performed to obtain possible systematic biases from interference on the tag-side arising between the CKM favored $b\bar{d} \rightarrow (c\bar{u}d)\bar{d}$ and doubly-CKM-suppressed $b\bar{d} \rightarrow (\bar{u}c\bar{d})d$ amplitudes in the final states used for flavor tagging [28].

VII. CONCLUSION

We report an improved measurement of the $CP$ violation parameters in $B^0 \rightarrow \pi^+\pi^-$ decays, confirming $CP$ violation in this channel as reported in previous measurements and other experiments [7, 9]. These results are based on the full Belle data sample after reprocessing with a new tracking algorithm and with an optimized analysis performed with a single simultaneous fit, supersede those of the previous Belle analysis [7]. They are now the world’s
FIG. 3: (color online) Projections of the fit to the data enhanced in the $B^0 \rightarrow \pi^+\pi^-$ signal region. Points with error bars represent the data and the solid black curves or histograms represent the fit results. The signal enhancements, $M_{bc} > 5.27 \text{ GeV}/c^2$, $|\Delta E| < 0.04 \text{ GeV}$, $F_{b\bar{b}/q\bar{q}} > 0$, $L_{K/\pi}^+ < 0.4$ and $r > 0.5$, except for the enhancement of the dimension being plotted are applied to each projection. (a), (b), (c), (d) and (e) show the $M_{bc}$, $\Delta E$, $L_{K/\pi}^+$, $L_{K/\pi}^-$ and $F_{b\bar{b}/q\bar{q}}$ projections, respectively. Blue hatched curves show the $B^0 \rightarrow \pi^+\pi^-$ signal component, green dotted curves show the $B^0 \rightarrow K^\pm\pi^\mp$ peaking background component, dashed red curves indicate the total background, and purple dash-dotted curves show the $B\bar{B}$ background component.
FIG. 4: (color online) Background subtracted time-dependent fit results for $B^0 \rightarrow \pi^+\pi^-$. (a) shows the $\Delta t$ distribution for each $B^0_{\text{tag}}$ flavor $q$. The solid blue and dashed red curves represent the $\Delta t$ distributions for $B^0$ and $\bar{B}^0$ tags, respectively. (b) shows the asymmetry of the plot above them, $(N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, where $N_{B^0}$ ($N_{\bar{B}^0}$) is the measured signal yield of $B^0$ ($\bar{B}^0$) events in each bin of $\Delta t$.

FIG. 5: Difference 1-CL, plotted for a range of $\phi_2$ (a) and $|\Delta \phi_2|$ (b) values as shown by the solid curve. The dashed lines indicate the 1$\sigma$ exclusion level.

most precise measurement of time-dependent $C\bar{P}$ violation parameters in $B^0 \rightarrow \pi^+\pi^-$, disfavoring the range $23.8^\circ < \phi_2 < 66.8^\circ$, at the 1$\sigma$ level.

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TABLE II: Systematic uncertainties of the measured physics parameters.

| Category                        | $\delta A_{CP}(\pi^+\pi^-) \times 10^{-2}$ | $\delta S_{CP}(\pi^+\pi^-) \times 10^{-2}$ |
|---------------------------------|--------------------------------------------|--------------------------------------------|
| IP profile                      | 0.13                                       | 1.19                                       |
| $B^0_{Tag}$ track selection     | 0.30                                       | 0.33                                       |
| Track helix errors              | 0.00                                       | 0.01                                       |
| $\Delta t$ selection            | 0.01                                       | 0.03                                       |
| Vertex quality selection        | 0.37                                       | 0.23                                       |
| $\Delta z$ bias                 | 0.50                                       | 0.40                                       |
| Misalignment                    | 0.40                                       | 0.20                                       |
| $\tau_{B^0}$ and $\Delta m_d$  | 0.12                                       | 0.09                                       |
| Data/MC shape                   | 0.15                                       | 0.19                                       |
| $\Delta t$ resolution function  | 0.83                                       | 2.02                                       |
| Flavor tagging                  | 0.40                                       | 0.31                                       |
| Background Parametric shape     | 0.15                                       | 0.28                                       |
| Background Non-parametric shape | 0.37                                       | 0.57                                       |
| Fit bias                        | 0.54                                       | 0.86                                       |
| Tag-side interference           | 3.18                                       | 0.17                                       |
| Total                           | 3.48                                       | 2.68                                       |

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[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[3] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 87, 091802 (2001).
[4] I. Adachi et al. (Belle Collaboration), Phys. Rev. Lett. 108, 171802 (2012).
[5] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 87, 091801 (2001).
[6] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 79, 072009 (2009).
[7] H. Ishino et al. (Belle Collaboration), Phys. Rev. Lett 98, 211801 (2007).
[8] J. P. Lees et al. (BaBar Collaboration), Phys. Rev. D 87, 052009 (2013).
[9] R. Aaij et al. (LHCb Collaboration), LHCb-CONF-2012-007 (2012).
[10] A. Kusaka et al. (Belle Collaboration), Phys. Rev. Lett 98, 221602 (2007).
[11] J. P. Lees et al. (BaBar Collaboration), arXiv:1304.3503 [hep-ex] (2013).
[12] A. Somov et al. (Belle Collaboration), Phys. Rev. D 76, 011104 (2007).
[13] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 76, 052007 (2007).
[14] J. Dalseno et al. (Belle Collaboration), Phys. Rev. D 86, 092012 (2012).
[15] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 98, 181803 (2007).
[16] M. Gronau and D. London, Phys. Rev. Lett 65, 3381 (1990).
[17] S. Kurokawa and E. Kikutani, Nucl. Instrum. Methods Phys. Res. Sect. A 499, 1 (2003), and other papers included in this Volume; T. Abe et al., Prog. Theor. Exp. Phys. (2013) 03A001 and following articles up to 03A011.
[18] A. Abashian et al. (Belle Collaboration), Nucl. Instrum. Methods Phys. Res. Sect. A 479, 117 (2002); also see detector section in J. Brodzicka et al., Prog. Theor. Exp. Phys. (2012) 04D001.
[19] Z. Natkaniec et al. (Belle SVD2 Group), Nucl. Instrum. Methods Phys. Res. Sect. A 560, 1(2006).
[20] R. Brun et al., GEANT 3.21, CERN DD/EE/84-1 (1984).
[21] H. Tajima et al., Nucl. Instr. and Meth. A 533, 370 (2004).
[22] R. A. Fisher, Annals of Human Genetics 7, 179 (1936).
[23] H. Kakuno et al., Nucl. Instr. and Meth. A 533, 516 (2004).
[24] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[25] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 241, 278 (1990).
[26] Y.-T. Du et al. (Belle Collaboration), Phys. Rev. D 87, 031103(R) (2012).
[27] Y. Chao et al. (Belle Collaboration), Phys. Rev. Lett. 94, 181803 (2005).
[28] O. Long, M. Baak, R. N. Cahn and D. Kirkby, Phys. Rev. D 68, 034010 (2003).