Analytic Optimization of the Halbach Array Slotless Motor Considering Stator Yoke Saturation

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Hybrid subdomain analysis is utilized to optimize the design of a high-speed compressor motor. Permanent magnets (PMs) are arranged in Halbach array and bounded by a carbon fiber sleeve. The stator core does not have a slot structure, so that it is advantageous in reducing the iron loss. Since the core saturation takes place, magnetic equivalent circuit (MEC) is used to find the permeability depending on the flux density of stator yoke. Furthermore, since the core is not infinitely permeable in the subdomain analysis, the solution is obtained in the whole subdomain regions: shaft, PM, air gap, coil, and stator yoke. The results of hybrid analysis are compared with those of finite-element analysis (FEA). Very close matching results are obtained in the flux density and torque even under yoke saturation. The stator yoke height is optimized against the rotor outer radius so that torque, power density, and efficiency are maximized, while stator outer radius, stack length, and coil area are fixed.

Index Terms—Electromagnetic performances, Halbach array rotor, slotless motor, subdomain model, vector potential, yoke saturation.

I. INTRODUCTION

The use of an extreme high-speed motor (≥100 krpm) is increasing in the applications such as compressors and micro-turbine generators [1], [2]. In high-speed motors, the iron loss should be considered since the eddy current loss is proportional to the square of frequency. Most iron loss takes place in the stator core. As an effort to minimize the iron loss, a slotless permanent magnet (PM) motor is considered [3]. In such a case, the effective air gap tends to be large. The Halbach PM arrangement is often beneficial in a large air gap machine, since it can steer all PM fields to the air gap side. Further, it is also possible with the Halbach array to make the rotor without a back iron. Lee et al. [4] used the Halbach array slotless motor as a propulsion motor for lightweight aircraft, since it required high-power density and high efficiency.

Subdomain analysis is used to design PM motors of various shapes since we can avoid repeated use of time-consuming finite-element analysis (FEA) [5], [6]. It utilizes a big matrix inversion to obtain the coefficients in the general solution of Laplace and Poisson’s equations. The solving process is relatively simple when the core is assumed to be infinitely permeable. It is because the field penetrates vertically to the iron core boundary. The analysis becomes more complex when core permeability is finite. In such a case, the field components should be considered at the boundaries between teeth and slots. It is called r-edge boundary [7]. Hannon et al. [8] summarized the overall Fourier-based modeling of electrical machines.

To reflect the core saturation in the subdomain analysis, it is necessary to apply different permeability depending on the flux density. It means that iterative matrix calculations are required since permeability and flux density cannot be calculated separately. Thus, hybrid models which incorporated the magnetic equivalent circuit (MEC) were proposed to consider the core saturation problem. Liang et al. [9] reflected the core saturation by decreasing the equivalent air gap permeability which was obtained from an MEC analysis. It required an iteration process in subdomain and MEC calculations until they reach the same field solution. On the other hand, Guo et al. [10] obtained the flux density results without using the iteration process. Instead, three different methods were used to analyze the interior PM motors: the FEA for the complex rotor configuration, the conformal mapping for the slotting effect, and the subdomain analysis for load conditions. However, the rotor geometry was oversimplified in the subdomain model.

In this work, a high-speed PM motor is designed using a Halbach array and a slotless stator. A hybrid analysis is proposed to account for the stator yoke saturation. The electromagnetic results of the hybrid analysis are very close to the FEA results. In the optimal design, the split ratio, defined as the air gap to the outer radii, is determined.

II. HYBRID ANALYSIS

Fig. 1 shows a sectional view of a high-speed motor for a turbo compressor. The design goals of maximum speed and power are 120000 rpm and 4.0 kW, respectively. The motor has a two-pole structure with a three-phase distributed winding.

In this work, the hybrid analysis is utilized to design the Halbach array slotless motor with the stator yoke saturation. The roles of the MEC and subdomain methods are determined by advantage of each method. The MEC method is used to find the relative permeability of the stator yoke ($\mu_i$). The detailed flux density of each region can be obtained by the subdomain analysis with the $\mu_i$ provided by the MEC analysis. Fig. 2 shows the whole design process. Note that it consists of two loops: the stator permeability is calculated in the inner
loop of the MEC analysis, whereas overall motor performances are evaluated using the subdomain analysis in the outer loop.

A. MEC Analysis for Stator Permeability

Before establishing the subdomain analysis, it is necessary to find a proper permeability, \( \mu_i \) of the stator yoke. The stator permeability depends on the non-linear \( B-H \) curve of material. Thereby, the core flux density and its permeability form iterative relations in solving the MEC: \( \mu_i \) is required for \( B \) calculation, and a value of \( B \) is necessary to read out \( \mu_i \) from the \( B-H \) curve. Kano et al. [11] solved the MEC repeatedly to find a proper set of \( (B, \mu_i) \) using the following update algorithm:

\[
\mu_i^2(k) = \frac{B_y(k-1)}{\mu_0 H_y(k-1)} \quad (1)
\]

\[
\mu_i(k) = \mu_i^2(k)^d \mu_i(k-1)^{1-d} \quad (2)
\]

where \( B_y \) and \( H_y \) are the magnetic flux density and field intensity of the stator yoke, respectively, \( d \) is the damping constant, and \( k \) is the iteration index.

Once the motor dimensions are determined, the stator permeability is calculated using (1) and (2). A simple MEC is depicted on the section diagram. Each PM is denoted by a current source along with a parallel resistance. The reluctance and MMF are calculated using the geometric dimensions and magnetic permeability [12]. Here, the stator coil MMF is not considered as the armature reaction is relatively minor. Details about determining reluctance and MMF in the MEC model are omitted here.

B. Subdomain Analysis

Fig. 1(b) shows a subdomain model of the Halbach slotless motor. The regions of the subdomain model are divided into five domains: stator yoke with finite permeability, coil, air (air gap and carbon fiber sleeve), PM, and non-magnetic material (shaft). The governing equations are

\[
\begin{align*}
\text{Region 1 (yoke)} & : \quad \nabla^2 A_{z1} = 0 \\
\text{Region 2i (coil)} & : \quad \nabla^2 A_{z2i} = -\mu_0 J_i \\
\text{Region 3 (air)} & : \quad \nabla^2 A_{z3} = 0 \\
\text{Region 4 (PM)} & : \quad \nabla^2 A_{z4} = -\frac{\mu_0}{r} \left( M_\theta - \frac{\partial M_\theta}{\partial \theta} \right) \\
\text{Region 5 (shaft)} & : \quad \nabla^2 A_{z5} = 0
\end{align*}
\]

where \( A \) is the vector potential in each region, \( \mu_0 \) is the vacuum permeability, \( J_i \) is the current density in the \( i \)th coil region, and \( M_\theta = \sum_n M_{\theta n} \sin(\theta_1 - \phi_{1n}) \) and \( M_\phi = \sum_n M_{\phi n} \cos(\theta_1 - \phi_{1n}) \) are the radial and circumferential magnetizations of PM, respectively, [6]. General solutions are written in Appendix. The unknown coefficients in the solutions are determined by the boundary conditions. There are two types of boundary conditions: One is over the angle interval at each \( \theta \)-edge boundary (\( r = r_{si}, r_c, r_m, r_{ro} \)) and the other is over the radius interval at each \( r \)-edge boundary (\( \theta = \phi_{ci} \pm (\phi_{ei}/2) \)). It should be noted that the \( r \)-edge boundary must be set on the boundary of different phase coil regions. It is solved by adding the series in \( r \) [7].

1) \( \theta \)-Edge Boundary: At \( r = r_{si} \) and \( \theta \in [\phi_{ei} - \phi_{ci} / 2, \phi_{ci} + \phi_{ci} / 2] \), continuity of vector potential, \( A_{z1} = A_{z2i} \) yields

\[
\begin{align*}
&b_{1i} \ln \left( \frac{r_{si}}{r_{ro}} \right) + \sum_{u}^{U} b_{1u} (1 - Z_{1u}^2) \eta_i + \sum_{u}^{U} d_{1u} (1 - Z_{1u}^2) \zeta_i \\
&= a_{2i0} + b_{2i0} \ln r_{si} - \frac{\mu_0}{4} J_i r_{si}^2 \\
&\sum_{u}^{U} b_{1u} (1 - Z_{1u}^2) \eta_i + \sum_{u}^{U} d_{1u} (1 - Z_{1u}^2) \zeta_i \\
&= a_{2im} + b_{2im} Z_{2m}
\end{align*}
\]

where \( Z_{1u} = (r_{si} / r_{ro})^{pp} \) and \( Z_{2m} = (r_c / r_{si})^{pp} \). In the same way, it follows from \( H_{\theta 1} = \sum_i Q_i H_{2i} \) that:

\[
\begin{align*}
&b_{1i} \mu_i \frac{Q_i}{\mu_i} = \sum_{k} b_{2i0} - \mu_0 J_r r_{si}^2 \frac{Q_i}{2} + \sum_{k} c_{2ik} \cos(k \pi) G_k \frac{\cos(k \pi) G_k}{\sinh(G_k \phi_c) K_{okp}} \\
&+ \sum_{k} d_{2ik} \frac{\cos(k \pi) G_k}{\sinh(G_k \phi_c) K_{okp}}
\end{align*}
\]
\[ b_{1u} \left( \frac{-\mu p}{\mu_i} \right) (Z_{1u}^2 + 1) \]
\[ = \sum_i \left[ \left( b_{2io} - \frac{\mu_0 J_i r_i^2}{2} \right) \frac{p}{\pi} \phi_c \eta_{1o} \right. \]
\[ + \sum_m (a_{2im} F_m - b_{2im} F_m Z_{2m}) \frac{p \phi_c}{2\pi} \eta_i \]
\[ + \sum_k c_{2ik} \left( \frac{\cos(k\pi) G_k}{\sinh(G_k \phi_c)} \right) K_{kcp} \]
\[ + \sum_k d_{2ik} \left( \frac{\cos(k\pi) G_k}{\sinh(G_k \phi_c)} \right) K_{ksp} \]
\[ \left. \right] \]
\[ d_{1u} \left( \frac{-\mu p}{\mu_i} \right) (Z_{1u}^2 + 1) \]
\[ = \sum_i \left[ \left( b_{2io} - \frac{\mu_0 J_i r_i^2}{2} \right) \frac{p}{\pi} \phi_c \xi_{1o} \right. \]
\[ + \sum_m (a_{2im} F_m - b_{2im} F_m Z_{2m}) \frac{p \phi_c}{2\pi} \xi_i \]
\[ + \sum_k c_{2ik} \left( \frac{\cos(k\pi) G_k}{\sinh(G_k \phi_c)} \right) K_{ksp} \]
\[ + \sum_k d_{2ik} \left( \frac{\cos(k\pi) G_k}{\sinh(G_k \phi_c)} \right) K_{kss} \] (11)

where \( r_c \) and \( \theta \in [\phi_c - (\phi_c/2), \phi_c + (\phi_c/2)] \), continuity of vector potential, \( A_{c2i} = A_{c3} \) yields

\[ a_{2io} + b_{2io} \ln r_c - \frac{\mu_0 J_i r_i^2}{4} \]
\[ = a_{3o} + b_{3o} \ln r_c + \sum_U (a_{3u} + b_{3u} Z_{3u}) \eta_{1o} \]
\[ + \sum_U (c_{3u} + d_{3u} Z_{3u}) \xi_{1o} \]
\[ a_{2im} Z_{2m} + b_{2im} \]
\[ = \sum_U (a_{3u} + b_{3u} Z_{3u}) \eta_i + \sum_U (c_{3u} + d_{3u} Z_{3u}) \xi_i \] (13)

At \( r = r_c \) and \( \theta \in [\phi_c - (\phi_c/2), \phi_c + (\phi_c/2)] \), continuity of vector potential, \( A_{c2i} = A_{c3} \) yields

\[ a_{3o} + b_{3o} \ln r_m \]
\[ = a_{3o} + b_{3o} \ln r_m + \sum_U (a_{3u} + b_{3u} Z_{3u}) \eta_{1o} \]
\[ + \sum_U (c_{3u} + d_{3u} Z_{3u}) \xi_{1o} \]
\[ a_{2im} Z_{2m} + b_{2im} \]
\[ = \sum_U (a_{3u} + b_{3u} Z_{3u}) \eta_i + \sum_U (c_{3u} + d_{3u} Z_{3u}) \xi_i \] (14)

where \( Z_{3u} = (r_m/r_c)^{up} \). Also, the boundary condition \( \sum_i Q_i H_{\theta 2i} = H_{\theta 3} \) results in

\[ \sum_i \left[ (b_{2io} - \frac{\mu_0 J_i r_i^2}{2}) \frac{p}{\pi} \phi_c \eta_{1o} + \sum_m (a_{2im} Z_{2m} - b_{2im}) \right. \]
\[ \left. \int F_m p \phi_c \xi_{1o} + \sum_m (a_{2im} Z_{2m} - b_{2im}) \right. \]
\[ \left. \times \frac{F_m p \phi_c}{2\phi} \eta_i + \sum_k c_{2ik} \left( \frac{G_k}{\sinh(G_k \phi_c)} \right) K_{kcp} \right] \]
\[ \left. + \sum_k d_{2ik} \left( \frac{G_k}{\sinh(G_k \phi_c)} \right) K_{ksp} \right] \]
\[ = a_{3o} \eta_{1o} - b_{3o} \eta_{1o} Z_{3u} \]
\[ + \sum_m (a_{2im} Z_{2m} - b_{2im}) \]
\[ \times \frac{F_m p \phi_c}{2\phi} \eta_i + \sum_k c_{2ik} \left( \frac{G_k}{\sinh(G_k \phi_c)} \right) K_{ksp} \] (16)

\[ = c_{3o} \eta_{1o} - d_{3o} \eta_{1o} Z_{3u} \] (17)

At \( r = r_m \) and \( \theta \in [0, (2\pi/p)] \), continuity of vector potential, \( A_{c3} = A_{c4} \) yields

\[ a_{3o} + b_{3o} \ln r_m = a_{4o} + b_{4o} \ln r_m \] (18)
\[ a_{3u} Z_{3u} + b_{3u} = a_{4u} + b_{4u} Z_{4u} \] (19)
\[ a_{3u} Z_{3u} + d_{3u} = c_{4u} + d_{4u} Z_{4u} \] (20)

where \( Z_{3u} = (r_m/r_c)^{up} \). Also, from boundary condition \( H_{\theta 3} = H_{\theta 4} \) it follows that:

\[ b_{3o} \mu_m = b_{4o} \] (21)
\[ a_{3u} \mu_m Z_{3u} - b_{3u} \mu_m Z_{3u} = a_{4u} \mu_m Z_{4u} + (E_{ms} - \mu_0 M_{\theta 4}) r_m \] (22)
\[ c_{3u} \mu_m Z_{3u} - d_{3u} \mu_m Z_{3u} = c_{4u} \mu_m Z_{4u} + (E_{mc} + \mu_0 M_{\theta 4}) r_m \] (23)

where \( \mu_m \) is the relative permeability of PM.

At \( r = r_{ro} \) and \( \theta \in [0, (2\pi/p)] \), continuity of vector potential, \( A_{c3} = A_{c4} \) yields

\[ a_{4o} + b_{4o} \ln r_{ro} = a_{5o} \ln (r_{ro}/r_{ci}) \] (24)
\[ a_{4u} Z_{4u} + b_{4u} + E_{ms} r_{ro} = a_{5u} (1 - Z_{5u}^2) \] (25)
\[ c_{4u} Z_{4u} + d_{4u} + E_{mc} r_{ro} = c_{5u} (1 - Z_{5u}^2) \] (26)

where \( Z_{5u} = (r_{ro}/r_{ci})^{up} \). Also, boundary condition \( H_{\theta 4} = H_{\theta 5} \) results in

\[ b_{5o} = b_{5o} \mu_m \] (27)
\[ a_{5u} \mu_m Z_{4u} - b_{5u} \mu_m Z_{4u} = a_{5u} \mu_m Z_{4u} + (E_{ms} - \mu_0 M_{\theta 4}) r_{ro} \] (28)
\[ c_{5u} \mu_m Z_{4u} - d_{5u} \mu_m Z_{4u} = c_{5u} \mu_m Z_{4u} + (E_{mc} + \mu_0 M_{\theta 4}) r_{ro} \] (29)

The above equations summarize \( \theta \)-edge boundary conditions. Note that \( \eta_{1o}, \xi_{1o}, \eta_i, \xi_i, K_{kcp}, K_{kkn}, K_{kscp}, K_{kssn}, K_{kissn}, K_{kissc}, \) and \( K_{kissp} \) are calculations for Fourier series expansion. Due to the page limit, only two definitions are written in the following:

\[ \eta_i = \frac{2}{\phi_c} \int_{\phi_i - \frac{\phi_c}{2}}^{\phi_i + \frac{\phi_c}{2}} \cos(u \theta) \cos \left( \frac{F_m (\theta - \phi_i + \frac{\phi_c}{2})}{2} \right) d\theta \] (27)

\[ K_{kissc} = \frac{p}{\pi} \int_{\phi_i - \frac{\phi_c}{2}}^{\phi_i + \frac{\phi_c}{2}} \sinh \left( \frac{G_k (\theta - \phi_i + \frac{\phi_c}{2})}{2} \right) \cos(u \theta) d\theta \]
2) r-Edge Boundary: At \( r \in [r_{ci}, r_{e}] \) and \( \theta = \phi_{ci} + \frac{\phi_i}{2} \), continuity of vector potential, \( A_{2i} = A_{2(i+1)} \) yields
\[
(a_{2(i+1)} - a_{2i})v_r + (b_{2(i+1)} - b_{2i})v_l - \frac{\mu_0}{4} (J_{i+1} - J_i) v_r + \sum_{m} d_{2(i+1)m} e_{ni} v_r F_m p + b_{2(i+1)m} e_{ni} v_r F_m n - a_{2im} \times \cos(m \pi) e_{ni} v_r F_m p - b_{2im} \cos(m \pi) e_{ni} v_r F_m n \]
\[
= c_{2ik} \ln(r_{s1}/r_{c}) + d_{2ik} \frac{\ln(r_{s1}/r_{c})}{2}.
\]
(30)

Also, boundary condition \( H_{02i} = H_{02(i+1)} \) result in
\[
c_{2ik} \cosh(G_k \phi_c) + d_{2ik} = c_{2(i+1)k} + d_{2(i+1)k} \cosh(G_k \phi_c).
\]
(31)

The above equations summarize r-edge boundary conditions. As in the \( \theta \)-edge problem, \( v_r, v_l, v_{rs}, v_{rs} p, \) and \( v_{rs} F_m p \) are calculations for series expansion [7]. Due to the page limit, only two are shown as follows:
\[
v_r = \int_{r_c}^{r_s} \frac{\sin(G_k \ln(r/r_c))}{r} dr
\]
\[
v_{rs} F_m p = \int_{r_c}^{r_s} \frac{r_{s1}}{r} \ln(r_{s1}/r_{e}) dr.
\]

The vector potential is solved by putting all the regional boundary equations, (8)–(31) into a single matrix equation
\[
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
0 & Q_{22} & Q_{23} & 0 & 0 \\
0 & 0 & Q_{33} & Q_{34} & 0 \\
0 & 0 & 0 & Q_{44} & Q_{45} \\
0 & 0 & 0 & 0 & Q_{52}
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5
\end{bmatrix}
= \begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 \\
P_5
\end{bmatrix}
\]
(32)

where \( C_1 = [b_{10}, b_{11} \ldots b_{11U}, d_{11} \ldots d_{1U}]^T \), \( C_2 = [d_{20}, b_{20}, a_{1} \ldots a_{1U}, b_{11} \ldots b_{11U}, c_{1} \ldots c_{1U}, d_{1} \ldots d_{1U}]^T \) are coefficient vectors. The \( \theta \)-edge conditions between Region 1 and Region 2 are condensed in \( Q_{11}, Q_{12}, \) and \( P_1 \). The same conditions between Region 2 and Region 3 are summarized in \( Q_{22}, Q_{23}, \) and \( P_2 \), and the conditions between Region 3 and Region 4 in \( Q_{33}, Q_{34}, \) and \( P_3 \). The conditions between Region 4 and Region 5 are summarized in \( Q_{44}, Q_{45}, \) and \( P_4 \), and the \( r \)-edge conditions between Region 2 in \( Q_{52} \) and \( P_5 \).

III. FEA VALIDATION

The motor parameters for validation are listed in Table I. The analytic results of air gap flux density are compared with the 2-D FEA results under a loaded condition in Fig. 3(b).

| Parameters | Values | Parameters | Values |
|-----------|--------|-----------|--------|
| Pole/slot combination | 2/6 | Air-gap [mm] | 0.6 |
| Yoke outer radius [mm] | 24 | Stack length [mm] | 30.0 |
| Coil turns per pole per phase | 6 | Coil area [mm²] | 127.3 |
| Magnet height [mm] | 7.0 | Sleeve height [mm] | 1.0 |
| Max. current [Ams] | 85 | Max. r_m [mm] | 13 |

Note that the two results agree well in both the radial and circumferential directions.

Fig. 4(a) and (c) shows the field intensity \( H \) and flux density \( B \) along the middle arc of the stator yoke. The result shows that the FEA and analytic results of \( B \) are fully agreed. However, the two results are quite different at \( H \). It is because the stator yoke was not segmented into many pieces and different permeability of each segment is not assigned depending on the level of saturation. It is clear when we look at the relative permeability shown in Fig. 4(b). The FEA result is 8000 in the non-saturation region but drops to 30 in the high saturation area. However, in the subdomain analysis, a constant value 60 is assigned to the entire stator. It is surprising that the \( B \) values match well despite the differences in \( H \) and \( \mu_i \). It is because the stator has no slots and the back yoke is very narrow. As a result, it is mostly saturated. Then it looks like an air core machine, leaving only the fundamental sinusoidal flux component. Accordingly, the single-domain solution with an average \( \mu_i \) obtained from the MEC yields similar \( B \) even in the stator yoke.

IV. DESIGN OPTIMIZATION

Fig. 6(a) shows the design constraints and variable. It is assumed that the stator outer diameter, the magnet height, the stack length, and carbon sleeve height are fixed.
Here, the coil current density, i.e., the coil area is also fixed. However, the rotor radius is considered as a design variable and is limited by maximum speed ($r_{m} \leq 13 \text{ mm}$ [1]). The split ratio is defined as $\gamma = (r_{m} + 1) / r_{so}$, where “1” is the height of the carbon sleeve, $r_{m}$ is the magnet outer radius, and $r_{so}$ is the stator outer radius [2]. Therefore, when the split ratio changes, the yoke height is traded with the PM area as the coil area is fixed. The yoke is saturated with a large $\gamma$, and the air gap flux density and torque will be reduced with a low $\gamma$.

Fig. 5(a) shows a comparison of the time stepping torque calculated by Maxwell stress tensor [5] when the split ratio is 0.56. Here, “subdomain” refers a subdomain result with $\mu_{i} = \infty$ in the stator yoke. On the other hand, “hybrid” means the subdomain result with a finite $\mu_{i}$ obtained from the MEC. It shows that the proposed hybrid analysis yields the identical torque to the FEA result. Note also from Fig. 5(b) that the average torque increases along with the split ratio until $\gamma = 0.56$. After that, it drops due to the stator saturation as the stator yoke height rapidly decreases. It should be emphasized that the FEA and this hybrid subdomain analysis match well in all ranges.

Power density is defined as the mechanical power divided by the total weight of the active material

$$\chi_{p} = T \omega_{r} / (m_{y} + m_{c} + m_{s} + m_{m})$$  \hspace{1cm} (33)$$

where $T$ is the magnetic torque, $\omega_{r}$ is the mechanical speed, $m_{y}$, $m_{c}$, $m_{s}$, and $m_{m}$ are the weights of iron yoke, coil, sleeve, and PM, respectively.

The electrical power applied to the motor is equal to

$$P_{e} = T \omega_{r} + 3R_{ph} I_{rms}^{2} + (k_{h} B_{y}^{2} \alpha_{r} + k_{c} B_{y}^{2} \omega_{r}^{2}) v_{y}$$  \hspace{1cm} (34)$$

where $R_{ph}$ is the phase resistance, $k_{h}$ is a hysteresis constant, $k_{c}$ is an eddy current constant, $\beta$ is the Steinmetz constant [13], and $v_{y}$ is a volume of the stator yoke. Now the efficiency is defined as $\chi_{e} = T \omega_{r} / P_{e}$. Note that the copper loss is constant since the coil area is kept the same independently of split ratio change. On the other hand, $B_{y}$ and $\gamma$ are affected by the split ratio according to the change in the yoke height.

Fig. 6 shows the change in power density and efficiency as the split ratio changes. Note that the efficiency steadily decreases as the yoke height reduces. However, the power density increases until $\gamma = 0.56$. It is because torque is maximized at $\gamma = 0.56$ as shown in Fig. 5, while the yoke mass decreases. After that, the power density also drops due to yoke saturation.

The two design criteria can be combined with a weighting factor $W$ such that $\delta = p_{n} W + e_{n} (1 - W)$, where $p_{n}$ and $e_{n}$ are the normalized power density and efficiency by some target values. The bar graph in Fig. 6(b) shows the evaluation factor when $W = 0.5$, the target power density is 12 kg/kW, and the target efficiency is 0.95. Based on it, the optimal design is determined as $\gamma = 0.56$.

Fig. 7 shows the flux density contours of two case designs for $\gamma = 0.5$ and 0.56. Design 1 has a greater yoke height, thus it is efficiency oriented. On the other hand, Design 2 has a larger rotor radius, thus it is a power-density-oriented design.

V. CONCLUSION

A high-speed compressor motor is designed with the Halbach PM arrangement. The Halbach arrangement is a good combination with the slotless motor, since it can steer all radial fields to the air gap via circumferentially magnetized PM. As a design tool, the hybrid subdomain analysis was utilized. When the yoke saturation takes places, the assumption of infinite permeability is no longer valid. It requires to solve the finite core region, which makes the solving process much more complex. Since the permeability decreases with saturation, a proper value of permeability should be found for each flux density. To this end, the MEC was used recursively. The hybrid subdomain analysis yielded very similar results to the FEA results in both field and torque. While determining the rotor size and height of the stator yoke, both power density and efficiency are considered. Design study shows that an optimal design is found with a larger rotor radius that allows for slight efficiency reduction. Based on this optimized design, the real motor is now manufactured.
APPENDIX

Region 1 (Stator Yoke):

\[ A_{z1} = b_{1a} \ln \left( \frac{r}{r_{so}} \right) + \sum_{u} \left[ a_{1u} \left( \frac{r}{r_{so}} \right)^{up} + \frac{r}{r_{si}} \right] - up \times [b_{1a} \cos (up \theta) + d_{1u} \sin (up \theta)]. \]

Region 2i (Coil): For the r-edge, the coil region includes a term expanded in \( r \).

\[ A_{z2i} = a_{2i0} + b_{2i0} \ln r - \frac{\mu_0}{4} I_i r^2 \]

\[ + \sum_{m} \left[ a_{2im} \left( \frac{r}{r_{si}} \right)^{F_m} + b_{2im} \left( \frac{r}{r_c} \right)^{-F_m} \right] \]

\[ \times \cos \left( F_m \left( \theta - \phi_{ci} + \frac{\varphi}{2} \right) \right) \]

\[ + \sum_{k} \left[ c_{2ik} \frac{\sinh (G_k (\theta - \phi_{ci} + \frac{\varphi}{2}))}{\sinh (G_k \varphi)} \right] \]

\[ \times \sin (G_k \ln (r/r_{ci})). \]

Region 3 (Air Gap & Carbon Fiber):

\[ A_{z3} = a_{30} + b_{30} \ln r + \sum_{u} \left[ a_{3u} \left( \frac{r}{r_c} \right)^{up} + b_{3u} \left( \frac{r}{r_m} \right)^{-up} \right] \]

\[ \times \cos (up \theta) + \sum_{u} \left[ c_{3u} \left( \frac{r}{r_c} \right)^{up} + d_{3u} \left( \frac{r}{r_m} \right)^{-up} \right] \]

\[ \times \sin (up \theta). \]

Region 4 (PM):

\[ A_{z4} = a_{40} + b_{40} \ln r \]

\[ + \sum_{u} \left[ a_{4u} \left( \frac{r}{r_m} \right)^{up} + b_{4u} \left( \frac{r}{r_{ro}} \right)^{-up} + E_{ms} \right] \]

\[ \times \cos (up \theta) + \sum_{u} \left[ c_{4u} \left( \frac{r}{r_m} \right)^{up} + d_{4u} \left( \frac{r}{r_{ro}} \right)^{-up} + E_{mc} \right] \]

\[ \times \sin (up \theta) \]

where \( E_{ms} = (-\mu_0/(up)^2 - 1)(M_{0u} + upM_{ru}) \sin (up \phi_1) \)

and \( E_{mc} = (\mu_0/(up)^2 - 1)(M_{0u} + upM_{ru}) \cos (up \phi_1) \).

Region 5 (Rotor Yoke):

\[ A_{z5} = a_{50} \ln \left( \frac{r}{r_{ri}} \right) + \sum_{u} \left[ \left( \frac{r}{r_{ro}} \right)^{up} - Z_{5u} \left( \frac{r}{r_{ri}} \right)^{-up} \right] \]

\[ \times [a_{5u} \cos (up \theta) + c_{5u} \sin (up \theta)]. \]

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