Updated parameter limits of the left-right symmetric model

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September 1994

Submitted to Physics Letters B

Abstract

Bounds of the neutral current sector parameters of the left-right symmetric model are investigated taking into account the low-energy data, LEP-data and CDF-result for the top mass $m_t = 174 \pm 10^{13}_{12}$. It is found that in the case of the minimal scalar sector with a left- and a right-handed triplet and a bidoublet Higgses the mass of the heavy neutral gauge boson $M_{Z'}$ should be larger than 1.2 TeV, assuming equal left- and right-handed gauge couplings and a negligible VEV of the left-handed triplet. For larger values of the ratio $g_L/g_R$ smaller values of $M_{Z'}$ are allowed.

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1. Introduction. The left-right symmetric model (LR-model) with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is a very appealing extension of the Standard Model. It has several attractive features. In this model, parity is a symmetry of the lagrangian and it is broken only spontaneously due to the form of the scalar potential providing a natural explanation for the parity violation. Furthermore, the $U(1)$ generator has a physical interpretation as the $B-L$ quantum number. Finally, the seesaw-mechanism can be realized and it leads to very small Majorana masses for the neutrinos which are mainly left-handed and large Majorana masses for the neutrinos which are mainly right-handed. In addition to the Standard Model particle content, there are heavy charged gauge boson $W'$ and neutral gauge boson $Z'$ and three right-handed neutrinos which form as mentioned, together with left-handed neutrinos the six Majorana mass eigenstates.

The purpose of this paper is to update the parameter limits for the LR-model using the latest LEP results, low-energy data and the CDF-result for the top mass $m_t = 174 \pm 10^{+13}_{-12}$ GeV. We shall do this in three cases. First, we do not specify the scalar sector of the LR-model. In this case we have three fitting parameters: the tree level correction $\Delta \rho_0 = M_W^2/(M_Z^2 \cos^2 \theta_w) - 1$ to the parameter $\rho = G_{NC}/G_{CC}$, which measures the relative strength of the neutral and charged current effective four fermion interactions and is unity in the Standard Model at the tree level; the mixing angle $\xi_0$ between $Z$ and $Z'$ and the mass $M_{Z'}$ of the $Z'$-boson. As the second case we consider the minimal LR-model, with left- and right-handed triplets $\Delta_{L,R}$ and a bidoublet $\Phi$ in the scalar sector. In ref. the most general scalar potential of the minimal LR-model was studied\footnote{However, it was assumed that the parameters of the scalar potential are real.}. It was shown that the potential has a minimum with the see-saw relation $v_L v_R = \gamma (k_1^2 + k_2^2)$, where $v_{L,R}$ and $k_i$ are VEV parameters of the left- and right-handed triplets and bidoublet, respectively, while $\gamma$
is a particular combination of the scalar potential parameters and $k_i$'s. By analysing the mass limits of neutrinos it was further shown, abandoning the possibility of fine-tuning the Yukawa couplings and the scalar potential parameters, that, to avoid the need to fine-tune the parameter $\gamma$ very close to zero, the most natural possibility is to have $v_R \gtrsim 10^7$ GeV. Another possibility, to have $v_R$ and thus $M_{Z'}$ in observable range, is to look for a new symmetry to eliminate the relevant terms from the scalar potential to guarantee that $\gamma = 0$ without fine-tuning. In both cases $v_L$ becomes negligible. Thus we assume that the VEV of the left-handed triplet vanishes, $v_L = 0$. In this case the parameter $\Delta \rho_0$ can be expressed in terms of the mixing angle $\zeta$ of the charged gauge bosons and the ratio $M_Z^2/M_{Z'}^2$, and the angle $\xi_0$ can be expressed in terms of the ratio $M_Z^2/M_{Z'}^2$, leaving us with two fitting parameters. Finally we assume that the angle $\zeta$ is negligibly small and perform the data-analysis only with $M_{Z'}$ as the fitting parameter.

The present study differs from the previous ones in the respect that we use the experimental value of the top mass as a constraint and that we study also the case where the gauge couplings $g_L$ and $g_R$ corresponding to the subgroups $SU(2)_L$ and $SU(2)_R$ may differ by performing the analysis with various values of the ratio $\lambda \equiv g_L/g_R$. The motivation for doing this is that if the LR-model is embedded in a grand unified theory, it can happen that the discrete left-right symmetry is broken at much higher energy scale than the weak scale, allowing $g_R \neq g_L$ in the low-energy phenomena. For example, in the case of supersymmetric version of $SO(10)$ grand unified theory, a chain of symmetry breakings can be realized that leads to a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ breaking scale $\approx 1$ TeV and to a value of $\lambda$ as large as 1.2.[4]

2. Basic structure of the LR-model. In the LR-model, with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the left-handed leptons $\psi_L = (\nu, l)_L$ are in the represen-
The quark sector is assigned correspondingly. In the minimal LR-model the scalar sector contains fields $\Phi$, $\Delta_L$ and $\Delta_R$ assigned to the representations $(2, 2, 0)$, $(3, 1, 2)$ and $(1, 3, 2)$, respectively. The vacuum expectation values of the fields are

$$
\langle \Phi \rangle = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 \\ 0 \\ v_L \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}.
$$

(1)

As discussed in the Introduction, we shall set $v_L = 0$. Due to these VEV’s, the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken down to the electromagnetic group $U(1)_Q$ and six gauge bosons $W^\pm, W'^\pm, Z, Z'$ acquire mass. The masses of the charged gauge bosons $W$ and $W'$ are found to be, in the limit $v_R^2 \gg k_1^2 + k_2^2$,

$$
M_{W}^2 = g_L^2 \bar{k}^2 \left(1 - \frac{k_1^2 k_2^2}{k^2 v_R^2}\right),
$$

$$
M_{W'}^2 = g_R^2 v_R^2,
$$

(2)

where $\bar{k}^2 = (k_1^2 + k_2^2)/2$. The masses of the neutral gauge bosons $Z, Z'$ read as

$$
M_{Z}^2 = \frac{g_L^2 \bar{k}^2}{c_w} \left(1 - \frac{y^4 \bar{k}^2}{2 c_w^4 v_R^2}\right),
$$

$$
M_{Z'}^2 = \frac{2 c_w^2 g_R^2 v_R^2}{y^2},
$$

(3)

where shorthand notation $c_w = \cos \theta_W$ for weak mixing angle has been used. In the LR-model the weak mixing angle is defined through

$$
g_L s_w = g' y = e.
$$

(4)

Here $g'$ is the $U(1)_{B-L}$ gauge coupling and

$$
y = \sqrt{c_w^2 - \lambda^2 s_w^2},
$$

(5)

and $\lambda = g_L/g_R$. 


Using Eqs. (2) and (3) one deduces the value of the parameter $\Delta \rho_0$,

$$\Delta \rho_0 = \frac{y^2 \beta}{\lambda^2} - \frac{y^2 \zeta^2}{2\beta \lambda^2},$$

(6)

where $\beta = M_Z^2/M_\nu^2$. The $W-W'$ mixing angle $\zeta$ is in the minimal LR-model given by

$$\zeta = \frac{\lambda k_1 k_2}{\nu_R^2}.$$ 

(7)

One should notice that $\Delta \rho_0$ can be either positive or negative depending on the values of $\beta$ and $\zeta$.

The neutral current lagrangian reads

$$\mathcal{L}_{NC} = g_L j_{3L} \cdot W_{3L} + g_R j_{3R} \cdot W_{3R} + g' J_{B-L} \cdot B,$$

(8)

where $W_{3L,3R}$ are the neutral $SU(2)_{L,R}$ gauge bosons, and $B$ is the gauge boson of $U(1)_{B-L}$. The fermion neutral currents have a form

$$j_{\mu}^{L,R} = \bar{\psi} \gamma^\mu T_{3L,3R} \psi, \quad j_{\mu}^{B-L} = \bar{\psi} \gamma^\mu \frac{1}{2}(B - L) \psi.$$ 

(9)

The lagrangian (8) can be expressed in terms of the photon field $A$ and the fields $Z_L$ and $Z_R$ requiring that photon couples only to the electromagnetic current $j_{em} = j_{3L} + j_{3R} + j_{B-L}$ and defining $Z_R$ to be that combination of $W_{3L}$, $W_{3R}$ and $B$ that does not couple to $j_{3L}$. It follows that $Z_L$ and $Z_R$ couple to the currents $e/(s_w c_w)(j_{3L} - s_w^2 J_{em})$ and $e/(s_w c_w \lambda y)(y^2 j_{3R} - \lambda^2 s_w^2 j_{B-L})$, respectively. After a rotation to the mass eigenstate basis $Z, Z'$,

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos \xi_0 & \sin \xi_0 \\ -\sin \xi_0 & \cos \xi_0 \end{pmatrix} \begin{pmatrix} Z_L \\ Z_R \end{pmatrix},$$

(10)

we can express the neutral current lagrangian in terms of the mass eigenstates $A$, $Z$ and $Z'$. The mixing angle $\xi_0$ measures the deviations of the $Z$-boson LR-model couplings from the Standard Model couplings. Since the Standard Model is tested
to be valid with a good accuracy, we can expand the $Z$-coupling in linear order and, at energy scales much lower than $M_{Z'}$, $Z'$-coupling in zeroth order in $\xi_0$. The neutral current lagrangian then reads

$$L_{NC} = eA \cdot j_{em} + \frac{e}{s_wc_w}Z \cdot \left[ j_{3L} \left( 1 + \frac{s_w^2 \lambda}{y} \xi_0 \right) - s_w^2 j_{em} \left( 1 + \frac{\lambda}{y} \xi_0 \right) + j_{3R} \frac{c_w^2}{\lambda y} \xi_0 \right]$$

$$+ \frac{e}{s_wc_w}Z' \cdot \left[ j_{3L} \frac{s_w^2 \lambda}{y} - s_w^2 j_{em} \frac{\lambda}{y} + j_{3R} \frac{c_w^2}{\lambda y} \right].$$

(11)

In the minimal LR-model $\xi_0$ reads, in the limit $M_{Z'} \gg M_Z$,

$$\xi_0 = \frac{y}{\lambda} \beta.$$ 

(12)

3. The LR-model formulas for the observables. In Standard Model, the analyses of the low-energy data are based on the effective lagrangian of the form

$$L_{eff} = \frac{e^2(q^2)}{q^2} j_{em}(1) \cdot j_{em}(2)$$

$$+ 4\sqrt{2} G_F \rho(q^2)[j_{3L}(1) - s_{eff}^2(q^2)j_{em}(1)] \cdot [j_{3L}(2) - s_{eff}^2(q^2)j_{em}(2)].$$

(13)

Here the loop corrections are collected to form three effective quantities $e^2(q^2)$, $\rho(q^2)$ and $s_{eff}^2(q^2)$, which depend on the energy scale $\sqrt{|q^2|}$, such that $L_{eff}$ preserves the form of the tree level lagrangian. This can be naturally done also in the context of the LR-model. However, one might wonder if the form of the effective quantities $e^2$, $\rho$ and $s_{eff}^2$ is changed when the tree level LR-model corrections are taken into account. It was shown in [3] that, in leading order in quantities $\beta$, $\xi_0$ and $\Delta \rho_0$ the changes can be parametrized with $\Delta \rho_0$ only:

$$\rho = 1 + \Delta \rho_{SM} + \Delta \rho_0$$

$$s_{eff}^2 = s^2(1 + \Delta \kappa_{SM}) + c^2 \Delta \rho_0$$

$$e^2 = e_{SM}^2,$$

(14)

where $\Delta \rho_{SM}$ and $\Delta \kappa_{SM}$ represent the Standard Model loop corrections and $c^2 = 1 - s^2 = M_W^2/M_Z^2$. Thus the low-energy lagrangian for the LR-model can be written
in the form

\[ \mathcal{L}_{\text{eff}} = \frac{e^2}{q^2} j_{\text{em}}(1) \cdot j_{\text{em}}(2) \]

\[ + 4\sqrt{2} G_F \rho \left[ j_{3L}(1) \left( 1 + \frac{s^2 \lambda}{y} \xi_0 \right) - s^2 j_{\text{em}}(1) \left( 1 + \frac{\xi_0}{\lambda y} \right) + j_{3R}(1) \frac{c^2 \xi_0}{\lambda y} \right] \times \]

\[ j_{3L}(2) \left( 1 + \frac{s^2 \lambda}{y} \xi_0 \right) - s^2 j_{\text{em}}(2) \left( 1 + \frac{\xi_0}{\lambda y} \right) + j_{3R}(2) \frac{c^2 \xi_0}{\lambda y} \]

\[ + 4\sqrt{2} G_F \beta \left[ j_{3L}(1) \frac{s^2 \lambda}{y} - s^2 j_{\text{em}}(1) \frac{\lambda}{y} + j_{3R}(1) \frac{c^2}{\lambda y} \right] \times \]

\[ j_{3L}(2) \frac{s^2 \lambda}{y} - s^2 j_{\text{em}}(2) \frac{\lambda}{y} + j_{3R}(2) \frac{c^2}{\lambda y} \].

(15)

Strictly speaking, the Eq. (14) for \( s_{\text{eff}}^2 \) is only valid when \( M_W \) and hence \( s^2 \) is used as an input. The parameter \( s^2 \) is calculable as a function of the other more precisely measured parameters from the expression for the Fermi coupling constant, which reads, when taking into account the LR-model corrections,

\[ \frac{1}{\sqrt{2}} G_F = \frac{\pi \alpha}{2s^2c^2M_Z^2}(1 + \Delta r - \frac{c^2}{s^2}\Delta \rho_0 + \delta_F). \]

(16)

Here \( \Delta r \) represents the Standard Model loop corrections and \( \delta_F \) the LR-model tree level corrections to the muon decay rate. As \( \delta_F \) is a second order correction in the parameters \( \zeta \) and \( M_W^2/M_W'^2 \), it will be neglected in the following. By calculating \( s^2 \) from Eq. (16) and substituting the result to Eq. (14), one then obtains

\[ s_{\text{eff}}^2 = s_{\text{eff,SM}}^2 - \frac{s^2 c^2}{c^2 - s^2} \Delta \rho_0. \]

(17)

In the relation (16), we have included in addition to the \( \mathcal{O}(\alpha) \)-corrections also the \( \mathcal{O}(\alpha \alpha_s) \)-corrections whereas in Eqs. (14) only \( \mathcal{O}(\alpha) \)-corrections are included. This is because the parameter \( s^2 \), calculated from relation (13), enters also in the expressions of the LEP-observables, which are measured with a much greater accuracy than the low-energy observables.

From (15) one can write the model independent low-energy parameters, as defined through the model independent effective lagrangians, in terms of LR-model
parameters. For deep inelastic neutrino-hadron scattering the parameters $\varepsilon_{L,R}(q)$ are defined through the lagrangian

$$L^{\nu H} = \sqrt{2} G_F \bar{\nu}_L \gamma_i \gamma_L \sum_q (\varepsilon_L(q) \nu_L \gamma_\mu q_L + \varepsilon_R(q) \bar{\nu}_R \gamma_\mu q_R),$$

with the LR-model expressions

$$\varepsilon_L(q) = \rho (1 + A_L \xi_0) \left[ T_3 (1 + A_L \xi_0 + A_L^2 \beta) - Q s^2_{\text{eff}} (1 + A_Q \xi_0 + A_L A_Q \beta) \right],$$

$$\varepsilon_R(q) = \rho (1 + A_L \xi_0) \left[ T_3 (A_R \xi_0 + A_L A_R \beta) - Q s^2_{\text{eff}} (1 + A_Q \xi_0 + A_L A_Q \beta) \right],$$

where $A_L = s^2 \lambda / y$, $A_Q = \lambda / y$ and $A_R = c^2 / \lambda y$ and $T_3 \equiv T_{3L} = T_{3R}$. Note that in Eq. (18), neutrinos are assumed to be left-handed. But in the LR-model, neutrinos are most naturally Majorana particles. The see-saw mechanism produces three heavy and three light mass eigenstates, with mass matrices $M_N \approx v_R h_M$ and $M_\nu \approx M_D M_N^{-1} M_D^T$, respectively [8]. Here $h_M$ is the matrix of Yukawa couplings between leptons and right-handed triplet scalar $\Delta_R$ and $M_D = F k_1 + G k_2$ is a Dirac mass term with Yukawa coupling matrices $F$ and $G$. Further, the charged lepton mass matrix has a form $M_l = F k_2 + G k_1$. Assuming that neither of the two terms in $M_l$ is negligible and neglecting the inter-generational mixings between neutrinos, we have the see-saw relation between the light and heavy neutrino masses

$$m_\nu \approx \frac{m_l^2}{m_N}. \quad (20)$$

This implies, together with the experimental limits of the light neutrino masses [9], $m_{\nu_1} < 7.3$ eV, $m_{\nu_2} < 0.27$ MeV and $m_{\nu_3} < 35$ MeV, approximate lower bounds for the heavy neutrinos:

$$m_{N_1} \gtrsim 4 \text{ GeV}, \quad m_{N_2} \gtrsim 40 \text{ GeV}, \quad m_{N_3} \gtrsim 90 \text{ GeV}. \quad (21)$$

Further, the current eigenstates $\nu_L$ and $\nu_R$ can be expressed in terms of the mass eigenstates $\chi$ through a unitary transformation,

$$\nu_{Li} = U_{Li,j} \chi_{l,j} + U_{Lh,i} \chi_{h,j},$$

8
 where $U_{Li}$ etc. are $3 \times 3$ submatrices of a unitary $6 \times 6$ matrix $U$ and $\chi_h, \chi_l$ denote the heavy and light Majorana neutrinos, respectively. The see-saw mechanism implies that $U_{Lh}$ and $U_{Rh}$ are $O(m_l/m_N)$ and $U_{Li}$ and $U_{Rl}$ are $O(1)$ \cite{8}. We can now write the left- and right-handed parts of the neutrino neutral current effectively as

$$\nu_L \gamma^\mu \nu_L = \bar{\chi}_L \gamma^\mu U^\dagger_{LI} U_{LI} \chi_L + \ldots = \bar{\chi}_L \gamma^\mu \chi_L + O(m_l^2/m_N^2) + \ldots$$

$$\nu_R \gamma^\mu \nu_R = \bar{\chi}_R \gamma^\mu U^\dagger_{RI} U_{RI} \chi_R + \ldots = (m_l^2/m_N^2) + \ldots,$$



(23)

where dots represent the contribution where there is at least one heavy neutrino involved. When the limits (21) apply, the production of heavy neutrinos is forbidden at low-energy scales and the lagrangian (18) is applicable.

Note also that, as the parameters $\varepsilon_L$ and $\varepsilon_R$ are determined from the ratios

$$R = \sigma^{NC}_{\nu N} / \sigma^{CC}_{\nu N}$$

and

$$\bar{R} = \sigma^{NC}_{\bar{\nu} N} / \sigma^{CC}_{\bar{\nu} N}$$

of the neutral and charged current cross sections of deep inelastic neutrino scattering, one needs in principle to consider also the charged sector of the LR-model. However, it is straightforward to check that this contribution is of the second order in parameters $\zeta$ and $M_W^2/M_W^2$, and as such, negligible.

The effective lagrangian for the electron-neutrino scattering defines the parameters $g_V^e$ and $g_A^e$ according to

$$\mathcal{L}^{\nu e} = \sqrt{2} G_F \bar{\nu}_L \gamma^\mu \nu_L \tau^\gamma \mu (g_V^e - g_A^e).$$

The LR-model expressions for them are

$$g_V^e = \rho(1 + A_L \xi_0) \left(-\frac{1}{2}(1 + (A_L + A_R)\xi_0 + A_L(A_L + A_R)\beta) + 2s^2_{\text{eff}}(1 + A_Q \xi_0 + A_L A_Q \beta)\right),$$

$$g_A^e = -\frac{1}{2} \rho(1 + A_L \xi_0)(1 + (A_L - A_R)\xi_0 + A_L(A_L - A_R)\beta).$$

(25)
For the $\nu_e-e$ scattering the charged current contribution must be included. Again, it is easy to check that the charged current LR-model contribution to the cross-section is a negligible second order term in parameters $\xi_0$ and $M_W^2/M_W^2$.

The effective parity violating lagrangian in the electron-hadron scattering defines the parameters $C_{iq}$ according to

$$L^{eH} = \frac{G_F}{\sqrt{2}} \sum_q \left( C_{1q} \bar{e}_\mu \gamma^\mu \gamma^5 e_q + C_{2q} \bar{e}_\mu e_q \gamma^\mu \gamma^5 q \right),$$

with the LR-model expressions

$$C_{1q} = \rho (1 + (A_L - A_R)\xi_0) \left( -T_{3q}(1 + (A_L + A_R)\xi_0 + (A_L^2 - A_R^2)\beta) \right.$$
$$+ 2s_{eff}^2 Q_q(1 + A_Q\xi_0 + A_Q(A_L - A_R)\beta) \left. \right),$$

$$C_{2q} = 2T_{3q}\rho (1 + (A_L - A_R)\xi_0) \left( -\frac{1}{2}(1 + (A_L + A_R)\xi_0 + (A_L^2 - A_R^2)\beta) \right.$$
$$+ 2s_{eff}^2 (1 + A_Q\xi_0 + A_Q(A_L - A_R)\beta) \left. \right).$$

(27)

The parameters $\rho$ and $s_{eff}^2$ in the low-energy formulas depend slightly on the process in question. Furthermore, there are some additional terms from the box graphs [6, 10], which should be included. The experimental values of the low-energy parameters are taken from Ref. [10].

In the $Z$-line shape measurement at LEP, the $e^-e^+\to f\bar{f}(\gamma)$ cross-sections are fitted, after subtracting the pure QED effects and the $\gamma-Z$ interference term, to the function

$$\sigma^0(s) = \sigma_f^p \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2},$$

where

$$\sigma_f^p = \frac{12\pi\Gamma_f\Gamma_f}{M_Z^2\Gamma_Z^2}.$$ 

(29)

An additional gauge boson would give a contribution to the cross-section [11]

$$\frac{\delta\sigma_0}{\sigma_0} \approx \delta R_{ZZ} \frac{s - M_Z^2}{M_Z^2},$$

(30)
where
\[
\delta R_{ZZ'} \approx -2 \frac{M_Z^2}{M_{Z'}^2} \frac{v_e v'_e + a_e a'_e v_f v'_f + a_f a'_f}{v_e^2 + a_e^2} \frac{v_e^2 + a_e^2}{v'_f + a'_f}, \tag{31}
\]
where \(v_f\) and \(a_f\) are vector and axial-vector couplings of the fermion \(f\) to the \(Z\)-boson while \(v'_f\) and \(a'_f\) are the corresponding quantities for \(Z'\). Presence of the term \(\delta R_{ZZ'}\) could in principle affect the line shape parameters, but it turns out that this effect is negligible even for modest values of \(M_{Z'}\). For example, the location of the maximum of the cross-section gets shifted by an amount
\[
\frac{\delta s_0}{s_0} \approx \frac{\Gamma_Z^2}{2M_Z^2} \delta R_{ZZ'} \approx 4 \cdot 10^{-4} \delta R_{ZZ'} \tag{32}
\]
Using \(\delta s_0/s_0 \approx 2\delta M_Z/M_Z\) and \(M_Z = (91.1899 \pm 0.0044) \text{ GeV} \tag{12}\) and taking the couplings in \(\delta R_{ZZ'}\) to be equal for \(Z\) and \(Z'\) requires \(M'_{Z'} < 300 \text{ GeV}\) for the additional gauge boson to give a measurable contribution.

Thus the LEP measurements are sensitive only to the parameters \(\Delta \rho_0\) and \(\xi_0\) through the dependence of the couplings \(v_f\) and \(a_f\) of them. The form of the couplings can be read from the lagrangian \(\text{(11)}\) by replacing the bare quantity \(s_w^2\) with the effective quantity \(s_f^2\).

We shall use the following high energy observables in the analysis: the total width of the \(Z\)-boson \(\Gamma_Z\), the hadronic peak cross-section \(\sigma_{p\text{had}}\), the ratio \(R_l\) between the hadronic and leptonic widths and the mass of the \(Z\), the ratio \(R_b\) between the partial width to a b\(\bar{b}\)-pair and the hadronic width, the mass of the \(Z\) and the effective leptonic weak mixing angle defined through
\[
\sin^2 \theta_{ew}^{\text{eff}} = \frac{1}{4} \left(1 - \frac{v_l}{a_l}\right), \tag{33}
\]
which can be extracted from any of the leptonic asymmetries \(A_{FB}, P_\tau, A_{FB}^{\text{pol}(\tau)}\) or \(A_{LR}\). Using mass of the \(Z\) as input leaves us five observables, of which \(\Gamma_Z, \sigma_{p\text{had}}, R_l\) and \(R_b\) can be expressed in terms of the partial fermionic widths. The widths have
the form
\[ \Gamma_f = \frac{G_F M_Z^2 \rho_f}{6\sqrt{2\pi}} (v_f^2 + a_f^2) (1 + \frac{3a_s}{4\pi} Q_f^2) K_{QCD}, \]
where
\[ v_f = T_{3f} (1 + (A_L + A_R) \xi_0) - 2s_f^2 Q_f (1 + A_Q \xi_0), \]
\[ a_f = T_{3f} (1 + (A_L - A_R) \xi_0) \]
and the QCD correction factor is defined by
\[ K_{QCD} = 3(1 + \frac{\alpha_s}{\pi}) \quad \text{for quarks} \]
\[ = 1 \quad \text{for leptons.} \]

The partial width to a $b\bar{b}$-pair has a slightly different behaviour due to the large contribution from the $Zb\bar{b}$-vertex. This is taken into account by a parameter $\delta_{vb}$ defined through
\[ \Gamma_b = \Gamma_d (1 + \delta_{vb}). \]

In the limit of the large top mass it has the form
\[ \delta_{vb} = -\frac{20}{13} \frac{\alpha}{\pi} \left( \frac{m_t^2}{M_Z^2} + \frac{13}{6} \ln \frac{m_t^2}{M_Z^2} \right). \]

The Eqs. (34) and (35) can also be applied to the case of light neutrinos after removing the $A_R \xi_0$ terms. The partial widths to a light and a heavy neutrino and to a heavy neutrino pair can be neglected even if these decays are kinematically allowed. This is because the widths are proportional to
\[ |g_L(\chi_i\chi_j)|^2 + |g_R(\chi_i\chi_j)|^2, \]
where $g_{L,R}(\chi_i\chi_j)$ are left- and right- handed couplings of the $Z$ to the neutrinos $\chi_i$ and $\chi_j$. By substituting Eq. (22) to the lagrangian (11) one deduces that the couplings $g_{L,R}(\chi_i\chi_j)$, except the left-handed couplings of the light neutrinos, are
proportional at least to the first power of the parameters $\xi_0$ or $U_{Lh} = O(m_l/m_N)$ and hence give a negligible second order contribution to the partial widths.

The quantities $\rho_f$ and $s_f^2$ have the same dependence on the parameter $\Delta \rho_0$ as the corresponding low-energy quantities. The Standard Model loop corrections for them differ slightly because of the different energy scale and the non-negligible vertex corrections. In addition to the $O(\alpha)$-corrections [6], we have also included $O(\alpha \alpha_s)$-corrections [7] in the expressions of the parameters $\rho_f$ and $s_f^2$. Note that $\sin^2 \theta_{\text{eff}}^w$ is equal to $s_l^2$ in the absence of LR-corrections. The values of the high energy observables to be used in our analysis are [12]

\begin{align*}
\Gamma_Z &= 2.4974 \pm 0.0038 \text{ GeV}, \\
\sigma_p^{\text{had}} &= 41.49 \pm 0.12 \text{ nb}, \\
R_l &= 20.795 \pm 0.040, \\
R_b &= 0.2192 \pm 0.0018, \\
\sin^2 \theta_{\text{eff}}^w &= 0.2317 \pm 0.0004. \tag{40}
\end{align*}

For the quantities $\Gamma_Z$, $\sigma_p^{\text{had}}$ and $R_l$ we have applied the correlations used by the DELPHI Collaboration [14] (i.e. $c_{12} = -0.20$, $c_{13} = 0.00$ and $c_{23} = 0.14$).

In addition to the low-energy and LEP-data we also use the $W$-mass value $M_W = 80.23 \pm 0.18$ [12] as constraint, theoretical value for $M_W$ being calculable from Eq. (16).

**4. Results and discussion.** We have performed a $\chi^2$-function minimization to fit the LR-model parameters with various values of $\lambda = g_L/g_R$. As input we have used $M_Z = (91.1888 \pm 0.0044)$ GeV [12], $m_t = (174 \pm 17)$ GeV [2], $\alpha_s = 0.118 \pm 0.007$ [15] and $\Delta \alpha^{(5)} = 0.0288 \pm 0.0009$ [16]. Here $\Delta \alpha^{(5)}$ is the contribution of the light quarks to the running of $\alpha$ from low energies up to $M_Z$. It appears in the loop correction factor $\Delta r$ in Eq. (13). In addition to the LR-model parameters, the strong coupling
constant $\alpha_s$ and the top mass $m_t$ were allowed to vary. The experimental values for them cited above were used as constraints. The mass of the higgs was assumed to be between 60 and 1000 GeV with a central value 250 GeV.

The 95% CL results for the case with the unspecified scalar sector are presented in Table 1. The allowed ranges of the parameters are slightly larger for larger values of $\lambda$. The same holds also for the case of minimal LR-model, results for which are presented in Table 2, and for the minimal LR-model with a negligibly small $W-W'$ mixing angle $\zeta$, the results for which are presented in Table 3.

The experimental value of the ratio $R_b$ prefers lower values of $m_t$. Setting $m_t = 174$ GeV causes the theoretical value of $R_b$ to be two standard deviations away from the experimental value. For example, in the case of LR-model with unspecified scalar sector and $\lambda = 1$, excluding the $R_b$-contribution lowers the minimum of $\chi^2$ to $\chi^2_{min} = 8.5$, changes the 95% CL range of the parameter $\Delta \rho_0$ to $\Delta \rho_0 = (1.0 \pm 3.7) \cdot 10^{-3}$, changes the 95% CL range of the top mass from $m_t = 152 \pm 33$ GeV to $m_t = 165 \pm 35$ GeV but leaves the bounds for $\xi_0$ and $M_{Z'}$ unchanged. This behaviour can be explained by noting that the dominant $m_t$-dependence of $R_b$ comes through the term $\delta_{v_b}$ in Eq. \(17\), whereas the other observables receive a significant contribution also from $\Delta \rho \equiv \Delta \rho_0 + \Delta \rho_t$. Here $\Delta \rho_t$ is the top mass dependent part of the Standard Model contribution to the parameter $\rho$ and it reads, in the limit of large top mass, $\Delta \rho_t = \frac{3 G_F m_t^2}{(8\sqrt{2}\pi^2)}$. When $m_t$ decreases to fit better to $R_b$, the Standard Model contribution to the parameter $\Delta \rho$ decreases allowing a larger value of $\Delta \rho_0$.

The best value of the top mass was found to be, almost independently of the model considered, to be around $m_t = 150 \pm 35$ GeV (95% CL). We performed also a Standard Model fit to the parameters $m_t$ and $m_H$. We found the 68% CL result

$$m_t = 157^{+11}_{-11} \text{ GeV},$$
\[ m_H = 77^{+144}_{-48} \text{ GeV} \quad (41) \]

in agreement with a recent study \[17\].

We now compare our results with those obtained in the studies \[18\] and \[19\]. Langacker and Luo \[18\] used low-energy measurements, LEP-measurements and \( M_W \)-measurement to fit the parameters of the extended models. They found the 95\% CL bounds \( \xi_0 = (1.8^{+0.1}_{-0.6}) \cdot 10^{-3} \) and \( M_{Z'} > 387 \text{ GeV} \) in the case of the general LR-model with \( g_L = g_R \); and \( M_{Z'} > 857 \text{ GeV} \) in the case of the minimal LR-model with \( g_L = g_R \). Due to the increased precision of the LEP-measurements the bounds obtained in the present study are considerably tighter except for \( M_{Z'} \) in the general LR-model, for which our bound is 30 GeV lower. This is presumably due to the larger low-energy data set used in \[18\], in addition to the experimental quantities used in the present study Langacker and Luo use also \( e^-e^+ \)-annihilation data below the \( Z \)-pole.

Altarelli et al. \[19\] used the LEP-measurements and \( M_W \)-measurement to fit the parameters of the extended models. In the case of LR-model with \( g_L = g_R \) and unspecified scalar sector they found the 1\( \sigma \) ranges \( \xi_0 = (0.15 \pm 1.58) \cdot 10^{-3} \) with top mass fixed to \( m_t = 150 \text{ GeV} \); and \( \xi_0 = (-0.1 \pm 2.5) \cdot 10^{-3} \), \( m_t < 147 \text{ GeV} \) when letting \( \alpha_s \) and \( m_t \) vary. The low value of the top mass in the latter case is due the use of \( \Delta \rho = \Delta \rho_0 + \Delta \rho_t \) as a fitting parameter, causing the other observables than \( R_b \) being almost independent of the top mass. Our results are in agreement with those cited above. The 1\( \sigma \) range for \( \xi_0 \) in the case of LR-model with unspecified scalar sector and \( \lambda = 1 \) is \( \xi_0 = (0.5 \pm 1.4) \cdot 10^{-3} \). The constraint \( m_t = 174 \pm 17 \text{ GeV} \) used in our analysis raises the central value and reduces slightly the allowed range of \( \xi_0 \).

In the case of minimal LR-model, one obtains from Table 2 a lower bound also
for $M_{W'}$ by using the relation

$$M_{W'} = \frac{y}{\sqrt{2c_w}} M_{Z'}$$

which follows from the Eqs. (2) and (3). For example, in the case of $\lambda = 1$, $M_{W'} > 740$ GeV. This bound is more restrictive than the bound obtained from charged current data [20] in the case when the right-handed quark mixing matrix $U^R$ is unrelated to the left-handed CKM-matrix $U^L$, $M_{W'} > 670$ GeV (90% CL); but is less restrictive in the case of manifest or pseudomanifest left-right symmetry, which implies $|U^R_{ij}| = |U^L_{ij}|$, $M_{W'} > 1.4$ TeV (90% CL).

To conclude, using the latest LEP results and the top mass constraint $m_t = 174 \pm 17$ GeV and assuming the left-right symmetric model, one is able to constrain the $Z-Z'$ mixing to be smaller than 0.5 % and the tree level contribution $\Delta \rho_0$ to the $\rho$-parameter to be smaller than 0.6 %. If one further assumes the LR-model with minimal scalar sector, it is found that the mass of the heavier neutral gauge boson should be larger than 1 TeV.

**Acknowledgement** Author thanks Iiro Vilja and Jukka Maalampi for a critical reading of the manuscript.

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**TABLE CAPTIONS**

**Table 1.** The 95% CL neutral current parameter limits ($\chi^2 < \chi^2_{\text{min}} + 4.8$) for the LR-model with unspecified scalar sector.

**Table 2.** The 95% CL parameter limits for the minimal LR-model.

**Table 3.** The 95% CL lower limits of the mass of the heavy neutral gauge boson in the minimal LR-model with a negligible charged current mixing angle.
Table 1.

| $\lambda$ | $\chi^2_{min}$ | $\Delta \rho_0$ | $\xi_0$ | $M_{Z',min}$ [GeV] |
|-----------|----------------|-----------------|-------|-----------------|
| 1.0       | 14.5           | $(2.1 \pm 3.6) \cdot 10^{-3}$ | $(0.5 \pm 3.1) \cdot 10^{-3}$ | 359 |
| 1.1       | 14.4           | $(2.1 \pm 3.6) \cdot 10^{-3}$ | $(0.6 \pm 3.6) \cdot 10^{-3}$ | 344 |
| 1.2       | 14.3           | $(2.2 \pm 3.7) \cdot 10^{-3}$ | $(0.8 \pm 4.1) \cdot 10^{-3}$ | 333 |

Table 2.

| $\lambda$ | $\chi^2_{min}$ | $|\zeta|_{max}$ | $M_{Z',min}$ [TeV] |
|-----------|----------------|---------------|-----------------|
| 1.0       | 15.7           | $5.5 \cdot 10^{-3}$ | 1.24 |
| 1.1       | 15.6           | $6.7 \cdot 10^{-3}$ | 1.06 |
| 1.2       | 15.6           | $8.1 \cdot 10^{-3}$ | 0.92 |

Table 3.

| $\lambda$ | $\chi^2_{min}$ | $M_{Z',min}$ [TeV] |
|-----------|----------------|-----------------|
| 1.0       | 15.7           | 1.24 |
| 1.1       | 15.6           | 1.06 |
| 1.2       | 15.6           | 0.92 |