Spectroscopy of \(a_1\) mesons from lattice QCD with the truncated overlap fermions

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We study the ground state and next radial excitation of the \(a_1\) mesons from a quenched lattice QCD simulation with the truncated overlap fermion formalism based on the domain wall fermion formalism\(^2,3\). Our objective is to reveal the relationship between the nature of the \(a_1\) meson associated with the chiral partner of the \(\rho\) meson and dynamical chiral symmetry breaking, as for the \(\pi\) and the chiral partner of the \(\sigma\) meson\(^6\). In the conventional constituent quark model, the \(\sigma\) meson and the \(a_1\) meson are assigned excited states.

Lattice simulations of \(a_1\) mesons have been previously conducted.

1. Introduction

In hadron physics, determining the properties of excited light mesons will provide important information on the chiral dynamics of quantum chromodynamics (QCD). We study the structures of \(a_1\) mesons determined using the truncated overlap fermion (TOF) formalism by Boriçi\(^1\) based on the domain wall fermion formalism\(^2,3\). The TOF formalism is classified into lattice chiral fermions\(^2-5\), and exhibits good chiral symmetry. Our objective is to reveal the relationship between the nature of the \(a_1\) meson associated with the chiral partner of the \(\rho\) meson and dynamical chiral symmetry breaking, as for the \(\pi\) and the chiral partner of the \(\sigma\) meson\(^6\). In the conventional constituent quark model, the \(\sigma\) meson and the \(a_1\) meson are assigned excited states.

Lattice simulations of \(a_1\) mesons have been previously conducted.
Wingate et al. were the first to measure the mass of the $a_1$ meson using two-flavor lattice QCD. Their result agrees with the experimental value for $a_1(1260)$. A decade later, a study of the $a_1$ meson was performed with the Lüscher–Weisz gauge action and the chirally improved Dirac operator in the quenched approximation. Although the ground state of the $a_1$ meson was improved by using various interpolators including derivative quark sources in the simulations, the obtained mass of the ground state is close to $a_1(1420)$, instead of $a_1(1260)$. Moreover, the mass of the first excited state of $a_1$ was observed to be above 2 GeV.

Recently, Prelovšek et al. presented results for the mass of the $a_1$ meson and its coupling constant. They performed a simulation for a full QCD lattice with clover-improved Wilson quarks. This work was continued in Ref., in which they extracted the resonance mass of the ground state of the $a_1$ meson $m_{a_1} = 1.435(53)(^{+0}_{-109})$ GeV and the coupling $g_{a_1\pi\rho} = 1.71(39)$ GeV by simulating the corresponding scattering channel $\pi\rho$. Their obtained value of the $a_1$ meson mass is higher than the experimental result of $a_1(1260)$.

In our previous work, we investigated the mass of the ground state of the $a_1$ meson by a quenched lattice QCD using TOF. Our result is consistent with the experimental value of $a_1(1260)$.

2. **Truncated overlap fermions**

The TOF are defined by

$$D_{TOF} = \epsilon_1 P^I \overline{D}^{-1}_{PV} D_{DW F} P \epsilon ,$$

where the five dimensional projection operator $P$ is constructed from the four-dimensional projection operators $P_{R/L} = (1 \pm \gamma_5)/2$. The indexes $x_5$ and $y_5$ represent the fifth-dimensional lattice sites, which are defined in $x_5,y_5 \in [1,N_5]$. The domain wall fermion operator $D_{DF}^{2,3}$ is defined by

$$D_{DF}(x,y) = (4 - M_5)\delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1} (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu},y} ,$$

The parameters of TOF are $m_f$ and $M_5$, which correspond to the bare quark mass and the height of the domain wall, respectively. The Pauli-Villars matrix $D_{PV}$ is given by $D_{PV} = D_{DF}(m_f = 1)$. In the $N_5 \to \infty$ limit, the lattice chiral symmetry is exactly reproduced in TOF.
3. Simulation setup and Lattice QCD results

In this work, we simulate the spectroscopy of $a_1$ mesons on $8^3 \times 24$ quenched lattice with the plaquette gauge action with $\beta = 5.7$. Gauge configurations are generated with the pseudo-heat-bath method. After 20000 thermalization iterations, we start to save gauge configurations every 1000 sweeps. The propagators of the $\pi$, $\rho$ and $a_1$ mesons are calculated with TOF. The fermion parameters are set to $N_5 = 32$, $M_5 = 1.65$ and $m_f a = 0.04–0.08$.

We calculate the meson propagators and their effective masses (see Fig. 1). We estimate the statistical errors using the jackknife method. By performing a single- or double-pole fit to the effective masses, we obtain the meson masses as listed in Table 1.

Fig. 1. Time dependences of the propagators (left) and the effective masses (right) for $m_f a = 0.06$. In the right figure, the dotted lines and dotted curve represent the single-pole and double-pole fitting results, respectively.

| $m_f a$ | $m_\pi a$ | $m_\rho a$ | $m_\pi / m_\rho$ | Confs. |
|--------|-----------|------------|------------------|-------|
| 0.08   | 0.6668(7) | 0.9496(18) | 0.702(2)         | 3000  |
| 0.07   | 0.6283(7) | 0.9249(21) | 0.679(2)         | 3000  |
| 0.06   | 0.5895(8) | 0.9042(24) | 0.652(3)         | 3000  |
| 0.05   | 0.5478(8) | 0.8816(27) | 0.621(3)         | 3600  |
| 0.04   | 0.5028(6) | 0.8614(24) | 0.584(2)         | 7864  |
Fig. 2 shows the quark mass dependence of the meson masses. We linearly extrapolate the meson masses to the chiral limit, \((m_\pi a)^2 = 0\). By tuning the \(\rho\) meson mass in the chiral limit to \(m_\rho = 775\) MeV, we obtain a lattice spacing of \(a = 0.1893(15)\) fm. Note that calculations for TOF have a residual mass such as \(m_{\pi a} = -m_{\text{res}} a\) in the chiral limit due to the finite \(N_5\) effect. In our calculation with \(N_5 = 32\), we obtain \(m_{\text{res}} a = 1.29(4) \times 10^{-2}\), that is \(m_{\text{res}} = 13.4(5)\) MeV, which is negligible. We estimate the masses of the ground state and first excited states of the \(a_1\) mesons to be \(m_{a_1} = 1158(42)\) MeV and \(m_{a_1}^{\text{ext}} = 1667(202)\) MeV. Our results are consistent with the experimental values of \(a_1(1260)\) and \(a_1(1640)\), \(m_{a_1(1260)} = 1230(40)\) MeV and \(m_{a_1(1640)} = 1654(19)\) MeV.

4. Conclusion

We have investigated the masses of the ground and first excited states of \(a_1\) mesons from a quenched lattice QCD with the TOF action. We have obtained the masses of the \(a_1\) mesons to be 1158(42) MeV and 1667(202) MeV, which are in good agreement with the experimental values of \(a_1(1260)\) and \(a_1(1640)\). Since our simulation was performed in the quenched approximation and the \(\bar{q}q\) source and sink, in which virtual intermediate states such as \(\bar{q}qqq\) states are highly suppressed, our results suggest that \(a_1(1260)\) and \(a_1(1640)\) are the simple \(\bar{q}q\) states whereas \(a_1(1420)\) may have a more
complicated structure than the $\bar{q}q$ state.

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