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Gauge-invariant theory of pion photoproduction with dressed hadrons

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Abstract

Based on an effective field theory of hadrons in which quantum chromodynamics (QCD) is assumed to provide the necessary bare cutoff functions, a gauge-invariant theory of pion photoproduction with fully dressed nucleons is described. The formalism provides consistent dynamical descriptions of $\pi N \to \pi N$ scattering and $\gamma N \to \pi N$ production mechanisms in terms of nonlinear integral equations for fully dressed hadrons. The dressed hadron currents and the pion photoproduction current satisfy gauge invariance in a self-consistent manner. Approximations are discussed that make the nonlinear formalism manageable in practice and yet preserve gauge invariance.
INTRODUCTION

It is the purpose of the present contribution to describe a comprehensive theory for the production of pions due to the interactions of incident photons with nucleons \([1]\). The history of such descriptions goes back to the fifties, and indeed many of the more general basic relations have been well-known for about forty years (see, e.g., \([2,3]\) and references therein). In recent years, the attention has focused on approaches attempting to take into account the fact that all hadrons involved in the reaction have an internal structure \([6–12]\). The work reported here adds to the latter approaches by providing a detailed theoretical framework for the gauge-invariant interactions of physical — i.e., fully dressed — hadrons with photons. The description is based on an effective field theory where the (at present, in detail unknown) quark and gluon degrees of freedom are parametrized by the bare quantities of the effective Lagrangian. We assume here that QCD provides us with cutoff functions for the \(N-N\pi\) vertices that make all integrations convergent. These vertices of the effective Lagrangian are “bare” at the hadronic level, i.e., they can still be dressed by purely hadronic mechanisms.

The physical currents for all processes contributing to the pion production amplitude are derived via their corresponding hadronic \(n\)-point Green’s functions by employing a mathematical operation called a “gauge derivative” which allows one to obtain currents directly from the momentum-space versions of the respective Green’s functions. For local fields, this is equivalent to the usual minimal-substitution procedure. However, for the present nonlocal case, where we assume bare vertex functions originating from QCD, this goes beyond minimal substitution. For lack of space, we cannot go into any details here regarding the gauge derivative. For the same reason, we rely entirely on a — hopefully largely self-explanatory — diagrammatic exposition of the formalism. Complete details are given in Ref. \([1]\).

PIioneer PHOTOPRODUCTION

Generically, at any level of dynamical sophistication, there are four distinctly different contributions to the current \(M^\mu\) for the pion photoproduction process \(\gamma N \rightarrow \pi N\): Three contributions from the photon interacting with the three external hadronic legs of the \(N-N\pi\) vertex and a fourth one arising from the photon attaching itself within the vertex itself. At the simplest level where one has only (bare) tree graphs, this last contribution vanishes identically for a local pseudoscalar vertex; for a bare \(N-N\pi\) vertex with derivative coupling, it is equal to the Kroll–Ruderman \([3]\) contact term obtained from minimal substitution.

This generic picture, which is shown in Fig. \([1]\), remains true even if we consider fully dressed, physical hadrons. What becomes more complicated then are the ingredients contributing to the various mechanisms depicted in Fig. \([1]\). For the fully dressed case, the internal

\[ \text{FIG. 1. Basic diagrams describing pion photoproduction. Solid and dashed lines describe nucleons and pions, respectively.} \]
FIG. 2. Interaction current. The first graph on the right-hand side is the bare contact term present already in the simplest tree-level case.

FIG. 3. Auxiliary current $b^\mu$ required in Fig. 2. The exchange current $U^\mu$ appearing here and in Fig. 2 is shown in Fig. 4.

details of the right-most graph of Fig. 1 — now called the interaction current (where, except for the bare contact term, there is at least one hadronic $N-N\pi$ vertex before or after the photon interacts with the hadronic system) — are found [1] to be given by the mechanisms shown in Fig. 2. The interaction current is seen to contain a bare Kroll–Ruderman-type contact term which is present already at the tree-level. In addition, it contains an exchange current contribution $U^\mu$ and an auxiliary current $b^\mu$ dressed by hadronic final-state interactions denoted by $X$. The details of $b^\mu$ are given in Fig. 3. The remaining current pieces are depicted in Figs. 4 and 5, where the latter shows the (on and off-shell) current for a composite nucleon.

As alluded to in the Introduction, the basis for this dynamical structure is the gauge-derivative formalism of Ref. [1]. The underlying effective hadronic field theory describing $\pi N$ scattering, which supplies the required $n$-point Green’s functions, is summarized in Fig. 6. The formalism is seen to be highly nonlinear at both the hadronic and electromagnetic levels.

GAUGE INVARIANCE

The general condition for gauge invariance of all physical currents can be formulated very succinctly: All current contributions $R^\mu$ associated with the photon’s interaction within the interaction region of any hadronic reaction $R$ must satisfy the equation [1]

$$k_\mu R^\mu + R\hat{Q}_i - \hat{Q}_f R = 0,$$

(1)

where $Q_i$ is the sum of all charge operators for all incoming and $Q_f$ for all outgoing particle legs; the caret signifies that for all subsequent reaction mechanisms the particle described by the respective individual charge operator will have the photon’s four-momentum $k^\mu$ added to its own. The situation is illustrated in Fig. 7. This is an off-shell generalization of the well-known Ward-Takahashi identity for propagators [2,4] to the case of several incoming and outgoing particles. (Transverse magnetic moment contributions are contained in $R^\mu$; they are conserved by themselves, of course.)

As has been shown in Ref. [1], all reaction elements appearing in Figs. 1–5 do satisfy the gauge condition (1) and therefore all physical currents are indeed gauge-invariant.
FIG. 4. (a) Exchange current $U^\mu$ due to driving term $U$ and (b) current $X^\mu$ appearing in (a).

FIG. 5. Various equivalent representations of the fully dressed electromagnetic current for the nucleon.
FIG. 6. Summary of coupled, nonlinear equations for $\pi N \rightarrow \pi N$ in the $P_{11}$ channel [1] (a similar set obtains for the $P_{33}$ channel): (a) Fully dressed $N-N\pi$ vertex (filled circle) given in terms of bare vertex (open circle) and final-state interaction mediated by the nonpolar $\pi N$ amplitude $X$. (b) Nonlinear equation for the dressed nucleon propagator (line with solid circle) determined by the bare propagator (line with open circle) and the $\pi N$ self-energy bubble. (c) Integral equation for the auxiliary amplitude $X$ given in terms of the $\pi N$ irreducible driving term $U$ and intermediate propagation of noninteracting $\pi N$ system. (d) Full $\pi N$ Bethe–Salpeter amplitude $T$, expressed as the sum of the polar $s$-channel term (with fully dressed $\pi N$ vertices and $N$ propagator) and the solution $X$ of (c). Some of the lowest-order contributions for the driving term $U$ of the nonpolar amplitude $X$ are given in the figure on the right-hand side. The corresponding first graph subsumes the $u$-channel exchanges of both $N$ and $\Delta$, i.e., it also incorporates the crossing-symmetric partner of the $s$-channel graph of (d). The other three box graphs depict intermediate scattering processes of the nonpolar $P_{11}$ $\pi N$ amplitude $X$ dressed by a pion, of the $\pi\pi$ amplitude dressed by a baryon, and of the full $P_{33}$ $\pi N$ amplitude with pion dressing, respectively.

FIG. 7. Generic representation of $R\hat{Q}_i - \hat{Q}_f R$ of Eq. (5). $R$ is an arbitrary hadronic reaction mechanism where all incoming and outgoing uncharged particles have been subsumed in the upper lines and all incoming and outgoing charged particles in the lower, thicker lines. The first graph on the left sums up all contributions where the photon is attached to an incoming particle whereas the one on the right depicts the sum of all contributions from the photon being attached to an outgoing particle. $R\hat{Q}_i - \hat{Q}_f R$ is the difference between the purely hadronic contributions enclosed in the dashed boxes; it measures the change brought about in the hadronic reaction when a photon momentum is transmitted through the hadronic interaction region entering and leaving via charged particles.
FIG. 8. Effective treatment of the bare current as an example of the gauge-invariant current approximation given in Eq. (2), where the current pieces due to a photon attaching itself within a hadronic interaction region is replaced by a sum of suitably constructed currents over all incoming and outgoing hadron legs. Each of the three leg currents is given by an entire diagram on the right-hand side and can be interpreted as a product of a particle current, a propagator, and a bare vertex function, in the manner shown here. For a constant vertex with derivative coupling, the result is identical to the Kroll–Ruderman term (see Eq. (93) of [1]).

From a practical perspective, in view of the high nonlinearity of the present formalism, the question arises how this gauge invariance can be maintained if some elements of the reaction mechanisms can only be treated by approximations. Two well-known methods of ensuring gauge invariance in situations where the proper interaction currents are not available are due to Gross and Riska [6] and Ohta [8]. The recent work of [12] suggests that there may be a theoretical problem with Ohta’s method having to do with the fact that, in general, it requires knowledge of the vertex functions at unphysical values of its arguments which can be traced to an underlying non-Hermitian effective Lagrangian.

Haberzettl [1] has suggested another, quite general, method of maintaining gauge invariance when approximating interaction currents which essentially replaces the full interaction current \( R^\mu \) by a sum of single-particle currents over all incoming and outgoing hadron legs of the reaction \( R \),

\[
R^\mu \rightarrow R^\mu_{\text{approx}} = \sum_{x_f} j^\mu_{\text{leg, } x_f} + \sum_{x_i} j^\mu_{\text{leg, } x_i},
\]

(2)

where the individual leg currents \( j^\mu_{\text{leg, } x} \) are obtained from a straightforward procedure using only charge conservation across the hadronic reaction in question (for details, see [1]). When applied to a nonlocal vertex as, e.g., the bare \( N-N\pi \) vertex (see Fig. 8) whose associated bare (QCD-based) interaction current is undetermined at the hadronic level, the resulting expressions are reminiscent of Ohta’s [8] for an extended vertex. However, they are different in detail and require only physical values of the vertex functions and, therefore, do not suffer from any of the problems discussed in Ref. [12]. We add that, of course, the approximate current (2) can be amended by transverse pieces which are conserved by themselves.
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