Higgs portal dark matter in the minimal gauged $U(1)_{B-L}$ model

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Abstract

We propose a scenario of the right-handed neutrino dark matter in the context of the minimal gauged $U(1)_{B-L}$ model by introducing an additional parity which ensures the stability of dark matter particle. The annihilation of this right-handed neutrino takes place dominantly through the $s$-channel Higgs boson exchange, so that this model can be called Higgs portal dark matter model. We show that the thermal relic abundance of the right-handed neutrino dark matter with help of Higgs resonance can match the observed dark matter abundance. In addition we estimate the cross section with nucleon and show that the next generation direct dark matter search experiments can explore this model.

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I. INTRODUCTION

The nonvanishing neutrino masses have been confirmed by various neutrino oscillation phenomena and indicate the evidence of new physics beyond the Standard Model. The most attractive idea to naturally explain the tiny neutrino masses is the seesaw mechanism [1], in which the right-handed (RH) neutrinos singlet under the SM gauge group are introduced. The minimal gauged $U(1)_{B-L}$ model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ [2] is an elegant and simple extension of the SM, in which the RH neutrinos of three generations are necessarily introduced because of the gauge and gravitational anomaly cancellations. In addition, the mass of RH neutrinos arises associated with the $U(1)_{B-L}$ gauge symmetry breaking.

Although the scale of the $B-L$ gauge symmetry breaking is basically arbitrary as long as phenomenological constraints are satisfied, one interesting option is to take it to be the TeV scale [3]. It has been recently pointed out [4] that when the classical conformal invariance is imposed on the minimal $U(1)_{B-L}$ model, the symmetry breaking scale appears to be the TeV scale naturally. If this is the case, all new particles, the $Z'$ gauge boson, the $B-L$ Higgs boson $H$ and the RH neutrinos appear at the TeV scale unless the $U(1)_{B-L}$ gauge coupling is extremely small, and they can be discovered at Large Hadron Collider [5–8]. Then we may be able to understand the relation between the gauge symmetry breaking and the origin of neutrino masses.

Although such a TeV scale model is interesting and appealing, one might think that the absence of dark matter (DM) candidate is a shortcoming of this model. A sterile RH neutrino with mass of the order of MeV is one possibility [9]. In this paper, we propose a very simple idea to introduce the DM candidate in the minimal gauged $U(1)_{B-L}$ model. We introduce the $Z_2$ parity into the model and impose one of three RH neutrinos to be odd, while the others even. In this way, the $Z_2$-odd RH neutrino becomes stable and the DM candidate. Note that two RH neutrinos are enough to reconcile with the observed neutrino oscillation data, with a prediction of one massless light neutrino. Therefore, without introducing any additional new dynamical degrees of freedom, the DM particle arises in the minimal gauged $U(1)_{B-L}$ model.

The paper is organized as follows. In the next section, we briefly describe our model. In section III, we estimate the thermal relic density of the RH neutrino and identify the model
parameter to be consistent with the current observations. We also calculate the scattering cross section between the DM particle and nucleon and discuss the implication for the direct DM search experiments. We summarize our results in the section IV. Our notations and the formulas used in our analysis are listed in Appendix.

II. THE MINIMAL GAUGED $U(1)_{B-L}$ MODEL WITH $Z_2$ PARITY

The model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. Additional fields besides the standard model fields are a gauge field $Z'_\mu$ of the $U(1)_{B-L}$, a SM singlet $B-L$ Higgs boson $\Psi$ with two $U(1)_{B-L}$ charge, and three RH neutrinos $N_i$ which are necessary for the gauge and gravitational anomaly cancellations. In describing the RH neutrinos, we use the four component representation of RH neutrino constructed from the Weyl spinor $\nu_{Ri}$,

$$N_i \equiv \begin{pmatrix} \nu_{Ri} \\ \epsilon \nu_{Ri}^* \end{pmatrix}, \quad (1)$$

For the two RH neutrinos, $N_1$ and $N_2$, we assign $Z_2$ parity even, while odd for $N_3$, so that the RH neutrino $N_3$ is stable and, hence, the DM candidate.

Due to the additional gauge symmetry $U(1)_{B-L}$, the covariant derivative for each fields is given by

$$D_\mu = D_\mu^{(SM)} - iq_{B-L} g_{B-L} Z'_\mu, \quad (2)$$

where $D_\mu^{(SM)}$ is the covariant derivative in the SM, and $q_{B-L}$ is the charge of each fields under the $U(1)_{B-L}$ with its gauge coupling $g_{B-L}$.

Yukawa interactions relevant for the neutrino masses are given by

$$L_{int} = \sum_{\alpha=1}^3 \sum_{i=1}^2 y_{\alpha i} \bar{L}_\alpha \tilde{\Phi} N_i - \frac{1}{2} \sum_{i=1}^3 \lambda_{Ri} \bar{N}_i \Psi P_R N_i + h.c., \quad (3)$$

where $\tilde{\Phi} = -i\tau_2 \Phi^*$ for $\Phi$ being the SM Higgs doublet, and without loss of generality we have worked out in the basis where the second term in the right-hand-side is in flavor diagonal for RH neutrinos. Because of the $Z_2$ parity, the DM candidate $N_3$ has no Yukawa couplings with the left-handed lepton doublets.

The general Higgs potential for the $SU(2)_L$ doublet $\Phi$ and a singlet $B-L$ Higgs $\Psi$ is generally given by

$$V(\Phi, \Psi) = m_1^2 |\Phi|^2 + m_2^2 |\Psi|^2 + \lambda_1 |\Phi|^4 + \lambda_2 |\Psi|^4 + \lambda_3 |\Phi|^2 |\Psi|^2. \quad (4)$$
The Higgs fields $\phi$ and $\psi$ are obtained by expanding $\Phi$ and $\Psi$ as

$$
\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \phi) \end{pmatrix},
$$

(5)

$$
\Psi = \frac{1}{\sqrt{2}}(v' + \psi),
$$

(6)

around the true vacuum with the vacuum expectation values $v$ and $v'$. These are related with the mass eigenstates $h$ and $H$ through

$$
\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix},
$$

(7)

with $\theta$ being the mixing angle. Their masses are given by

$$
M^2_h = 2\lambda_1 v^2 \cos^2 \theta + 2\lambda_2 v'^2 \sin^2 \theta - 2\lambda_3 vv' \sin \theta \cos \theta,
$$

(8)

$$
M^2_H = 2\lambda_1 v^2 \sin^2 \theta + 2\lambda_2 v'^2 \cos^2 \theta + 2\lambda_3 vv' \sin \theta \cos \theta.
$$

(9)

The mass of the new neutral gauge boson $Z'$ arises by the $U(1)_{B-L}$ gauge symmetry breaking,

$$
M^2_{Z'} = 4g^2_{B-L}v'^2.
$$

(10)

Associated with the $U(1)_{B-L}$ gauge symmetry breaking, the RH neutrinos $N_i$ acquire masses

$$
M_{N_i} = -\lambda_R \frac{v'}{\sqrt{2}}.
$$

(11)

From LEP experiment, the current lower bound on the $Z'$ boson mass has been found to be $[10, 11]$

$$
\frac{M_{Z'}}{g_{B-L}} = 2v' \gtrsim 6 - 7 \text{ TeV}.
$$

(12)

Two $Z_2$-even RH neutrinos $N_1$ and $N_2$ are responsible for light neutrino masses via the seesaw mechanism,

$$
m_{\nu_{\alpha\beta}} = -\sum_{i=1,2} y_{\alpha i} y_{\beta i} \frac{v^2}{2M_{N_i}}.
$$

(13)

Note that the rank of this mass matrix is two, so that the lightest neutrino is massless.

### III. RIGHT-HANDED NEUTRINO DARK MATTER

Due to the $Z_2$ parity, one of RH neutrino $N_3$ (we denote it as $N$ hereafter) in our model can be the DM candidate. We first estimate its relic abundance and identify the model
parameters to be consistent with the current observations. Next we calculate the scattering cross section between the DM particle and a proton and discuss the implication for the direct DM search experiments.

A. Thermal relic density

The DM RH neutrino interacts with the SM particles through couplings with $B - L$ gauge and $B - L$ Higgs bosons. Note that neutrino Dirac Yukawa interactions are absent because of the $Z_2$ parity. The most of annihilation of the RH neutrinos occurs via $Z', H$ and $h$ exchange processes in the s-channel. In practice, the dominant contributions come from the Higgs ($h$ and $H$) exchange diagrams, because the $Z'$ exchange processes are suppressed by the inverse square of the $B - L$ Higgs VEV $v' \gtrsim 3$ TeV. Thus, we obtain Higgs portal DM of RH neutrino effectively. The relevant annihilation modes are the annihilation into $f \bar{f}$, $W^+W^-$, $ZZ$, and $h(H)h(H)$. Since RH neutrino DM couples to only $B - L$ Higgs $\Psi$ while a SM particle does to SM Higgs $\Phi$, the DM annihilation occurs only through the mixing between these two Higgs bosons. Although it is not so severe, the precision electroweak measurements [12] as well as the unitarity bound [13] give constraints on the mixing angle and mass spectrum of the Higgs bosons.

The thermal relic abundance of DM

$$\Omega_N h^2 = 1.1 \times 10^9 \frac{m_N/T_d}{\sqrt{g^*} M_P \langle \sigma v \rangle} \text{GeV}^{-1},$$

with the Planck mass $M_P$, the thermal averaged product of the annihilation cross section and the relative velocity $\langle \sigma v \rangle$, the total number of relativistic degrees of freedom in the thermal bath $g^*$, and the decoupling temperature $T_d$, is evaluated by solving the Boltzmann equation for the number density of RH neutrino $n_N$:

$$\frac{dn_N}{dt} + 3Hn_N = -\langle \sigma v \rangle (n_N^2 - n_{EQ}^2),$$

and the Friedmann equation

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3M_P^2} \rho,$$

with $n_{EQ}$ and $a(t)$ being the equilibrium number density and the scale factor, under the radiation dominated Universe with the energy density $\rho = \rho_{rad}$ [14].
Fig. 1 shows the relic density $\Omega_N h^2$ as a function of the DM mass $m_N$ for a set of parameters: $(v', M_h, M_H, M_{Z'}, \sin \theta) = (4000 \text{ GeV}, 120 \text{ GeV}, 200 \text{ GeV}, 1000 \text{ GeV}, 0.7)$, for example. Wilkinson Microwave Anisotropy Probe measured the value of DM abundance as $\Omega_{DM} h^2 \simeq 0.1 \text{ [15]}$. The figure shows that a desired DM relic abundance can be obtained for only near Higgs resonances, $m_N \approx M_h/2$ or $M_H/2$.

Fig. 2 shows the relic density $\Omega_N h^2$ as a function of the DM mass $m_N$ for a smaller Higgs mixing $\sin \theta = 0.3$ (others are the same as in Fig. 1). Compared with Fig. 1, for $m_N \lesssim M_W$ where the DM particles dominantly annihilate into $f \bar{f}$, the relic density further increases because of the small mixing angle. When the DM is heavier, the annihilation mode into Higgs boson pairs is opened and the relic density slightly decreases, but the reduction is not enough to reach $\Omega_N h^2 \simeq 0.1$.

![Figure 1: The thermal relic density of RH neutrino DM as a function of its mass for a parameter set: $(v', M_h, M_H, M_{Z'}, \sin \theta) = (3000 \text{ GeV}, 120 \text{ GeV}, 200 \text{ GeV}, 1000 \text{ GeV}, 0.7)$.

Our model is quite analogous to the so-called gauge singlet scalar dark matter [16–18]. Some recent studies can be found in Refs. [19, 20]. In the gauge singlet scalar DM model, the thermal abundance is mainly controlled by the interactions between the SM Higgs boson and the DM particle. In our model, $B - L$ Higgs VEV $v'$ can play the same role for $m_N < M_W$, namely a larger $v'$ corresponds to weaker coupling between DM and Higgs for a fixed DM mass. On the other hand, for $m_N > M_W$ the difference appears. Even if the annihilation
mode into $W$-boson pair becomes kinematically available, it is not possible to obtain the desired DM abundance without the Higgs resonant annihilation because the bound on $v'$ given by Eq. (12) is stringent.

B. Direct detection of dark matter

Our RH neutrino DM can elastically scatter off with nucleon, unlike another RH neutrino DM model has been proposed by Krauss et. al. \[21\] and studied \[22, 23\]. The main process is Higgs exchange and the resultant cross section for a proton is given by

$$\sigma_{SI}^{(p)} = \frac{4}{\pi} \left( \frac{m_p m_N}{m_p + m_N} \right)^2 f_p^2,$$

with the hadronic matrix element

$$f_p = \sum_{q=u,d,s} f_{Tq} \alpha_q m_q + \frac{2}{27} f_{TG} \sum_{c,b,t} \alpha_q m_q,$$

and the effective vertex (see Appendix for notations)

$$\alpha_q = -\lambda_N y_q \left( \frac{\partial \Phi}{\partial h} \frac{1}{M_h^2} \frac{\partial \Psi}{\partial h} + \frac{\partial \Phi}{\partial H} \frac{1}{M_H^2} \frac{\partial \Psi}{\partial H} \right),$$

where $m_q$ is a mass of a quark with a Yukawa coupling $y_q$, and $f_{Tq}^{(p)}$ and $f_{TG}^{(p)}$ are constants.
From Eq. (19), one can see that $\sigma^{(p)}_{SI} \propto (\sin 2\theta/v')^2$ for a given DM mass $m_N$. Fig. 3 shows the spin-independent cross section of RH neutrino with a proton. The resultant cross section is found to be far below the current limits reported by XENON10 \[24\] and CDMSII \[25\]: $\sigma_{SI} \lesssim 4 \times 10^{-8} - 2 \times 10^{-7}$ pb, for a DM mass of 100 GeV-1 TeV. Future experiments such as XENON1T \[26\] can reach the cross section predicted in our model.

![Graph showing spin-independent scattering cross section with a proton.](image)

**FIG. 3:** The spin independent scattering cross section with a proton. All parameters are same as those used in the previous section. The upper and lower lines correspond to $\sin \theta = 0.7$ and 0.3, respectively.

### IV. SUMMARY

We have proposed a scenario of the RH neutrino dark matter in the context of the minimal gauged $U(1)_{B-L}$ model. We have introduced a discrete $Z_2$ parity in the model, so that one RH neutrino assigned as $Z_2$-odd can be stable and, hence, the DM candidate, while the other two RH neutrinos account for neutrino masses and mixings through the seesaw mechanism. No additional degrees of freedom are necessary to be added. We have evaluated the relic density of the dark matter particle. The dominant annihilation modes are via the Higgs boson exchange processes in the $s$-channel and thus, our model can be called Higgs portal DM model. It has been found that the relic density consistent with the current observation
can be achieved only when the annihilation processes are enhanced by Higgs resonances. Therefore, the mass of the RH neutrino DM should be around a half of Higgs boson masses. We have also calculated the elastic scattering cross section between the DM particle and a proton and found it within the reach of future experiments for the direct DM search.

Appendix A: The Higgs sector

The Higgs potential (4) contains five parameters: \( m_1^2, m_2^2, \lambda_1, \lambda_2 \) and \( \lambda_3 \). These parameters can be rewritten in terms of two Higgs VEVs, two physical Higgs masses and the mixing angle between them. The stationary conditions are

\[
m_1^2 + \lambda_1 v^2 + \frac{1}{2} \lambda_3 v'^2 = 0, \tag{A1}
\]
\[
m_2^2 + \lambda_2 v^2 + \frac{1}{2} \lambda_3 v'^2 = 0. \tag{A2}
\]

The physical Higgs masses are given by Eqs. (8) and (9) with the mixing angle that \( \theta \) satisfies

\[
\tan 2\theta = -\frac{\lambda_3 vv'}{(\lambda_1 v^2 - \lambda_2 v'^2)}. \tag{A3}
\]

Higgs self interaction terms are expressed as

\[
\mathcal{L}_{int} = \lambda_1 v \phi^3 + \lambda_2 v' \psi^3 + \frac{1}{2} \lambda_3 (v \phi^2 + v' \psi^2) + \frac{1}{4} (\lambda_1 \phi^4 + \lambda_2 \psi^4 + \lambda_3 \phi^2 \psi^2), \tag{A4}
\]
in terms of \( \phi \) and \( \psi \). With Eq. (7), these are rewritten in terms of \( h \) and \( H \) with \( \theta \) as

\[
\mathcal{L}_{int} = \lambda_1 v \cos^3 \theta - \lambda_2 v' \sin^3 \theta + \frac{1}{2} \lambda_3 (v \cos \theta \sin^2 \theta - v' \sin \theta \cos^2 \theta) \] hh\h
\[
+ \left[ 3\lambda_1 v \cos^2 \theta \sin \theta + 3\lambda_2 v' \sin^2 \theta \cos \theta + \frac{1}{2} \lambda_3 (v \sin^3 \theta - 2 \cos^2 \theta \sin \theta) \\
+ v' (\cos^3 \theta - 2 \sin^2 \theta \cos \theta) \right] hhH
\]
\[
+ \left[ 3\lambda_1 v \cos \theta \sin^2 \theta - 3\lambda_2 v' \sin \theta \cos^2 \theta + \frac{1}{2} \lambda_3 (v \cos^3 \theta - 2 \sin^2 \theta \cos \theta) \\
+ v' (\cos^3 \theta - 2 \sin \theta \cos^2 \theta) \right] hHH
\]
\[
+ \left[ \lambda_1 v \sin^3 \theta + \lambda_2 v' \cos^3 \theta + \frac{1}{2} \lambda_3 (v \sin \theta \cos^2 \theta + v' \sin^2 \theta \cos \theta) \right] HHH
\]+four point interactions. \tag{A5}

We can read off a Higgs three point vertex from Eq. (A5).
In the expression of annihilation cross section, we used the following notations:

\[
\begin{align*}
\frac{\partial \Phi}{\partial h} &= \frac{1}{\sqrt{2}} \cos \theta, \\
\frac{\partial \Phi}{\partial H} &= \frac{1}{\sqrt{2}} \sin \theta, \\
\frac{\partial \Psi}{\partial h} &= -\frac{1}{\sqrt{2}} \sin \theta, \\
\frac{\partial \Psi}{\partial H} &= \frac{1}{\sqrt{2}} \cos \theta.
\end{align*}
\] (A6)

Appendix B: Amplitude

We give explicit formulas of the invariant amplitude squared for the pair annihilation processes of the RH neutrinos.

1. Annihilation into charged fermions

\[
|M|^2 = 
32 \left| \frac{g_{B-L}^2 q_f q_N}{s - M_{Z'}^2 + i M_{Z'} \Gamma_{Z'}} \right|^2 (s - 4m_N^2) \left( \frac{3}{8} s - \frac{1}{2} \left( \frac{s}{2} - m_f^2 \right) + \frac{1}{2} \left( \frac{s}{4} - m_f^2 \right) \cos^2 \theta \right)
+ 16 \lambda_N^2 |y_f| \left( \frac{\partial \Phi}{\partial h} - M^2 h + i M_h \Gamma_h \frac{\partial \Psi}{\partial h} \right) \left( \frac{\partial \Phi}{\partial H} - M^2 H + i M_H \Gamma_H \frac{\partial \Psi}{\partial H} \right) \right|^2
\] (B1)

2. Annihilation into neutrinos

a. Annihilation into \( \nu_a, \nu_a \) (light active-like neutrinos)

\[
|M|^2 = 
32 \left| \frac{g_{B-L}^2 q_f q_N}{s - M_{Z'}^2 + i M_{Z'} \Gamma_{Z'}} \right|^2 (s - 4m_N^2) \left( \frac{3}{8} s - \frac{1}{2} \left( \frac{s}{2} + m_{\nu_0}^2 \right) + \frac{1}{2} \left( \frac{s}{4} + m_{\nu_0}^2 \right) \cos^2 \theta \right) (B2)
\]
b. Annihilation into $\nu_s, \nu_s$ (heavy sterile-like neutrinos)

$$|\mathcal{M}|^2 = 32 \left| \frac{g_{hL}^2 q_N}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2 (s - 4m_N^2) \left( \frac{3}{8} s - \frac{1}{2} \left( \frac{s}{2} + m_{\nu_s}^2 \right) + \frac{1}{2} \left( \frac{s}{4} + m_{\nu_s}^2 \right) \cos^2 \theta \right)$$

$$+ 4\lambda_N^2 \lambda_{\nu_s}^2 \left| \frac{\partial \Psi}{\partial h} \frac{i}{s - M_h^2 + i M_h \Gamma_h} \frac{\partial \phi}{\partial h} + \frac{\partial \Psi}{\partial H} \frac{i}{s - M_H^2 + i M_H \Gamma_H} \frac{\partial \phi}{\partial H} \right|^2 (s - 4m_N^2)(s - 4m_{\nu_s}^2).$$

(B3)

3. Annihilation into $W^+W^-$

$$|\mathcal{M}|^2 = 8\lambda_N^2 \left( \frac{1}{2} g^2 v \right)^2 \left| \frac{\partial \Psi}{\partial h} \frac{1}{s - M_h^2 + i M_h \Gamma_h} \frac{\partial \phi}{\partial h} + \frac{\partial \Psi}{\partial H} \frac{1}{s - M_H^2 + i M_H \Gamma_H} \frac{\partial \phi}{\partial H} \right|^2$$

$$(s - 4m_N^2) \left( 1 + \frac{1}{2 M_W^4} (\frac{s}{2} - M_W^2)^2 \right).$$

(B4)

4. Annihilation into $ZZ$

$$|\mathcal{M}|^2 = 8\lambda_N^2 \left( \frac{1}{4} (g^2 + g'^2) v \right)^2 \left| \frac{\partial \Psi}{\partial h} \frac{1}{s - M_h^2 + i M_h \Gamma_h} \frac{\partial \phi}{\partial h} + \frac{\partial \Psi}{\partial H} \frac{1}{s - M_H^2 + i M_H \Gamma_H} \frac{\partial \phi}{\partial H} \right|^2$$

$$(s - 4m_N^2) \left( 1 + \frac{1}{2 M_Z^4} (\frac{s}{2} - M_Z^2)^2 \right).$$

(B5)

5. Annihilation into $hh$

$\mathcal{M}_1$ denotes the amplitude by $s$-channel Higgs bosons $h$ and $H$ exchange, while $\mathcal{M}_2$ does that for $t(u)$-channel $N$ exchange diagram. The formulas for $NN \rightarrow hH$ and $HH$ can be obtained by appropriate replacement of the vertexes, e.g., $\lambda_{hhh} \rightarrow \lambda_{hhH}$.

$$|\mathcal{M}_1|^2 = |\mathcal{M}_1 + \mathcal{M}_2|^2,$$

$$|\mathcal{M}_1|^2 = \lambda_N^2 \left( \frac{s}{2} - 2m_N^2 \right) \left| \frac{\partial \Psi}{\partial h} \frac{i}{s - M_h^2 + i M_h \Gamma_h} i \lambda_{hhh} + \frac{\partial \Psi}{\partial H} \frac{i}{s - M_H^2 + i M_H \Gamma_H} i \lambda_{hhH} \right|^2,$$

(B6)
\[
\int \frac{d\cos \theta}{2} |\mathcal{M}_2|^2 = \lambda_N^4 \left( \frac{\partial \Psi}{\partial h} \right)^4 \left( -8 - I_{22} + J_{22} \ln \left| \frac{A + 2b}{A - 2b} \right| \right), \tag{B8}
\]
\[
\int \frac{d\cos \theta}{2} \mathcal{M}_1 \mathcal{M}_2^* = 4m_N \lambda_N^3 \left( \frac{\partial \Psi}{\partial h} \right)^2 \left( \frac{\partial \Psi}{\partial h} s - M_h^2 + iM_h \Gamma_h \right) i\lambda_{hh} + \frac{\partial \Psi}{\partial H} s - M_H^2 + iM_H \Gamma_H i\lambda_{Hhh} \right) \left( -4 + \frac{s - 4m_N^2 + A}{2b} \ln \left| \frac{A + 2b}{A - 2b} \right| \right), \tag{B9}
\]
where \( \theta \) is the scattering angle in the center of mass frame. The auxiliary functions appear above are defined as
\[
I_{22}(s) \equiv 4 \left( A + 2a \right)^2 - 2(s + 4m_N^2)A - s(A + m_N^2) - 3m_N^2(s - 4m_N^2) \left( A^2 - 4b^2 \right), \tag{B10}
\]
\[
J_{22}(s, m_h) \equiv \frac{1}{Ab} \left( 2A(A + 2a) - A(s + 4m_N^2) + A^2 - 4a^2 - (s - 2m_N^2)(m_N^2 - m_h^2) \right), \tag{B11}
\]
\[
A(s, m_h) \equiv -\frac{s}{2} + m_h^2, \tag{B12}
\]
\[
b(s, m_N, m_h) \equiv \sqrt{\frac{s}{4} - m_h^2} \sqrt{\frac{s}{4} - m_N^2}. \tag{B13}
\]

Appendix C: Thermal averaged annihilation cross section

In partial wave expansion, the thermal averaged cross section is given by
\[
\langle \sigma v \rangle = \frac{1}{m_N^2} \left[ w(s) - \frac{3}{2} \left( 2w(s) - 4m_N^2 \frac{dw}{ds} \right) \frac{T}{m_N} \right] \bigg|_{s=4m_N^2} \tag{C1}
\]
\[
= 6 \frac{dw}{ds} \bigg|_{s=4m_N^2} \frac{T}{m_N}, \tag{C2}
\]
with
\[
4w(s) \equiv \int d\text{LIPS} \sum |\mathcal{M}|^2 = \frac{1}{8\pi} \sqrt{\frac{s - 4m_{\text{final}}^2}{s}} \int \frac{d\cos \theta}{2} \sum |\mathcal{M}|^2, \tag{C3}
\]
where \( m_{\text{final}} \) is the mass of final state particle.

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