Magnetic and spin evolution of neutron stars in close binaries

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ABSTRACT
The evolution of neutron stars in close binary systems with a low-mass companion is considered, assuming the magnetic field to be confined within the solid crust. We adopt the standard scenario for the evolution in a close binary system, in which the neutron star passes through four evolutionary phases (‘isolated pulsar’ – ‘propeller’ – accretion from the wind of a companion – accretion resulting from Roche-lobe overflow). Calculations have been performed for a great variety of parameters characterizing the properties of both the neutron star and the low-mass companion. We find that neutron stars with more or less standard magnetic field and spin period that are processed in low-mass binaries can evolve to low-field rapidly rotating pulsars. Even if the main-sequence life of a companion is as long as $10^{10}$ yr, the neutron star can maintain a relatively strong magnetic field to the end of the accretion phase. The model that is considered can account well for the origin of millisecond pulsars.

Key words: binaries: close - stars: magnetic fields - stars: neutron - pulsars: general - X-rays: stars.

1 INTRODUCTION
In recent years, evidence has been obtained for a relatively fast magnetic field decay in neutron stars undergoing accretion in binary systems (see, e.g., Taam & van den Heuvel 1986 and Chattacarya & van den Heuvel 1991). The weak surface magnetic fields of many binary radio pulsars ($B \leq 10^5$ G), for which efficient accretion has taken place, strongly suggest the idea of the accretion-induced magnetic field decay in these neutron stars. Most of the low magnetic field pulsars are believed to be old neutron stars that have experienced mass transfer, in low-mass binary systems where accretion results from Roche-lobe overflow and may last for a relatively long period.

Recently Geppert & Urpin (1994) and Urpin & Geppert (1995) have suggested a simple and efficient mechanism of rapid field decay in neutron stars undergoing accretion. Accretion changes the thermal evolution of the neutron star drastically if the accretion rate is sufficiently high and mass transfer lasts a sufficiently long time (see, e.g., Fujimoto et al. 1984). The energy release resulting from accretion heats the neutron star and reduces the crustal conductivity, because the latter depends on the temperature. A decrease of the conductivity accelerates the ohmic decay of the magnetic field if this field is maintained by currents in the crust. If the mass transfer phase lasts as long as $10^6-10^7$ yr the surface field strength can be reduced by a factor $\sim 10^2-10^4$ depending on the magnetic configuration at the beginning of the accretion phase.

Accretion influences not only the field decay also but the spin evolution of pulsars, and the strength of the magnetic field is one of the dominating factors that determines this evolution. If the total amount of accreted matter is large enough ($\geq 0.01-0.1 M_\odot$), the neutron star in a low-mass binary can be ‘recycled’ to millisecond periods (Alpar et al. 1982).

The accretion phase, however, does not exhaust all possible stages in the evolution of neutron stars entering low-mass binary systems. The evolution of such systems is extremely long, because even the main-sequence lifetime of a low-mass companion exceeds $10^9$ yr. In accordance with the standard scenario, the neutron star in a close binary can pass throughout several evolutionary phases (see, e.g., Pringle & Rees 1972 and Illarionov & Sunyaev 1975).

(i) The initial obscured radio pulsar phase, in which the pressure of the pulsar radiation is sufficient to keep plasma from the stellar wind of the companion away from the
neutron star magnetosphere. The radio emission of the pulsar is partly (or completely) absorbed by the plasma cloud surrounding the binary system, so the pulsar is practically unobservable; the pulsars spin down as a result of magnetodipole radiation.

(ii) The propeller phase, in which the radiation pressure, which is reduced because of field decay and spin-down, cannot prevent the infalling matter from interaction with the magnetosphere. However, the spin rotation is still sufficiently rapid to eject the wind matter as a result of centrifugal force and to transfer the angular momentum from the neutron star to the wind matter.

(iii) The wind accretion phase, in which the matter from the wind falls on to the surface of the neutron star. Nuclear burning of the accreted material heats the neutron star and causes accretion-driven field decay; in the course of this phase the neutron star can spin up slightly, because the accreted matter carries a certain amount of angular momentum.

(iv) The enhanced accretion phase, which starts when the secondary star leaves the main-sequence and fills its Roche lobe. Vigorous mass transfer on to the neutron star can spin up significantly, because the accreted matter can carry a certain amount of angular moment.

The neutron star processed in the above evolutionary picture can probably prolong its life as a radio pulsar with relatively low magnetic field and short spin period after accretion resulting from Roche-lobe overflow is exhausted.

The properties of pulsars produced by evolution in low-mass binaries have not been analysed in detail yet, and we address this problem in the present paper. Note that a similar problem has been considered by Jahan Miri & Bhattacharya (1994), who assumed that the magnetic evolution is determined by the expulsion of the field lines from the neutron star core because of interaction between the fluxoids in the proton superconductor and the vortices in the neutron superfluid. The authors, however, restricted themselves to the case of wide low-mass binaries and considered the evolution only during the main-sequence lifetime.

The paper is organized as follows. We describe the adopted model in detail in Section 2. The results of calculations are presented in Section 3. In Section 4, we discuss our results and compare them with the available observational data on the evolution of neutron stars in low-mass binaries.

2 DESCRIPTION OF THE MODEL

We consider the evolution of neutron star in a binary system with the secondary being a low-mass star. The binary is assumed to be relatively close; thus the companion can fill its Roche lobe after the end of its main-sequence life. Both magnetic and spin evolution of the neutron star may be strongly influenced by accretion from the companion.

During its main-sequence evolution [which lasts as long as \(1-5 \times 10^8\) yr], the secondary loses mass because of the stellar wind, and a certain fraction of the wind plasma can generally be captured and accreted by the neutron star. After the secondary leaves the main sequence and its Roche lobe, the rate of mass loss and hence the accretion rate can be drastically enhanced.

We assume that the magnetic field of the newly born neutron star is created by an unspecified mechanism either in the course of collapse or soon after the neutron star is born. We also assume that the field is maintained by currents in the crust and originally occupies only a fraction of the crust volume. The evolution of the crustal field is determined by the conductive properties of the crust and material motion throughout it (see Sang & Chanmugam 1987 and Urpin & Geppert 1995 for details). Neglecting the anisotropy of conductivity, the induction equation governing the evolution of the magnetic field reads

\[
\frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2}{4\pi} \nabla \times \left( \frac{1}{\sigma} \nabla \times \mathbf{B} \right) + \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

where \(\sigma\) is the conductivity and \(\mathbf{v}\) is the velocity of material movement. The velocity \(\mathbf{v}\) is caused by the flux of the accreted matter throughout the crust and may be non-zero only during those evolutionary phases at which accretion on to the neutron star surface is allowed. The total amount of the accreted mass in most cases does not exceed 0.1–0.2 \(M_\odot\) (see Bhattacharya & van den Heuvel 1991), even for neutron stars undergoing a very extended accretion phase. Therefore, the mass and radius of the neutron star do not change significantly and hence the corresponding changes in the crustal thickness, density profile, etc., are also negligible. The total mass flux is the same throughout the crust and should be taken to be equal to the accretion rate. Assuming spherical symmetry of the mass flow in deep crustal layers, one obtains the radial velocity of material movement.

\[
v = \frac{M}{4\pi\rho r^2}.
\]

where \(M\) is the accretion rate and \(\rho\) is the density. This inward-directed flow tries to push the magnetic field into the deep crustal layers and can generally influence the evolution of the field at a high accretion rate.

Introducing the vector potential for a dipole magnetic field \(\mathbf{A} = (0, 0, A_r)\), where \(A_r = \tilde{5}(r, t) \sin \theta/r\) and \((r, \theta, \phi)\) are spherical coordinates, the induction equation (1) can be transformed to

\[
\frac{\partial \tilde{s}}{\partial t} = \frac{c^2}{4\pi} \frac{\partial^2 \tilde{s}}{\partial r^2} - \frac{2s}{r^2} - \frac{\partial \tilde{s}}{\partial r}.
\]

At the surface \(r = R\), the standard boundary condition for a dipole field should be fulfilled, i.e. \(\tilde{s}_\theta = \tilde{s}_\phi = 0\). We assume the core of the neutron star to be superconductive, so that the magnetic field cannot penetrate into the core. Therefore, the second boundary condition is \(s = 0\) at the crust-core boundary, which lies at the density \(2 \times 10^{17}\) g cm\(^{-3}\).

The conductive properties of the solid crust are determined by the scattering of electrons on phonons and impurities (see, e.g., Yakovlev & Urpin 1980). Scattering on
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accretion rate within the range $10^{-11} \lesssim \dot{M} \lesssim 10^{-16} M_\odot \, \text{yr}^{-1}$. For phase IV, when accretion is caused by Roche-lobe overflow, the internal temperature has been calculated by Fujimoto et al. (1984) for the accretion rate $\dot{M} = 3 \times 10^{-16} M_\odot \, \text{yr}^{-1}$ and by Miralde-Escude et al. (1990) for $M$ within the range $10^{-10} \lesssim M \lesssim 10^{-12} M_\odot$. We match their results and extrapolate the obtained dependence slightly in order to estimate $T$ for $M \sim 10^{-11} M_\odot \, \text{yr}^{-1}$, the maximal accretion rate that we use in calculations.

Note that the accretion rate during phases III and IV is sensitive to the separation between the stars and their masses. In the present study we will not consider the dependence of the evolution on the orbital period of a binary, despite the importance of this dependence for the interpretation of observational data (see van den Heuvel & Blitzarfi 1995). We are planning to address this problem in a forthcoming paper.

As the spin evolution of the neutron star is strongly coupled with its magnetic evolution, it may be essentially different for different evolutionary phases. As already mentioned, the neutron star is scarcely influenced by its companion during phase I, because the wind plasma of the secondary is stopped by the pressure of magnetodipole radiation behind the light cylinder. The stopping radius, $R_s$, is determined by the balance between the dynamical pressure of the wind and the radiative pressure of magnetodipole waves. The dynamical pressure of the wind is of the order of $\rho_w V_w^2$, where $\rho_w$ and $V_w$ are the density and velocity of the wind plasma, respectively, near the stopping radius. The density, $\rho_w$, at the distance $s$ from the secondary may be estimated as $\rho_w \approx M_s/4\pi s^2 V_w$, with $M_s$ being the rate of mass loss of the secondary. The pressure of the magnetodipole radiation at the distance $R_s$ from the neutron star is $\sim (1/4\pi R_s^4)(B^2 R_s^6 \Omega/6c^5)$, where $B$ is the surface field strength at the magnetic pole and $\Omega$ is the spin angular velocity of the neutron star. Equating this pressure to the dynamical pressure of the wind at the nearest point to the neutron star, and assuming that the stopping radius is smaller than separation between stars, a, i.e. $s \sim a$, one obtains the expression for $R_s$.

$$R_s \sim \left( \frac{B^2 R_s^6 \Omega^4 a^{1.5}}{6c \rho_w M_s} \right)^{1/2}.$$  (6)

In our calculations, it is more convenient to use the accretion rate, $M$, instead of the rate of mass loss $M_\odot$. At $a > R_s$, where $R_s \sim 2GM/c^2 < \sqrt{a^3}$ is the radius of gravitational capture (Bondi 1952) and $M$ is the neutron star mass, these quantities are related by $4M/R_s^2 \approx M_\odot/a^2$; we assume that the wind velocity is larger than both the sound velocity of the wind plasma and the velocity of orbital motion. Phase I lasts while the stopping radius is larger than the radius of gravitational capture, $R_s > R_s$.

With the assumption that the spin-down torque on the neutron star is caused by radiation of magnetodipole waves (Ostriker & Gunn 1969), the evolution of the spin period, $P$, is governed by

$$P P = \frac{8\pi B^2 R^6}{3c^9},$$  (7)

where $I$ is the moment of inertia. Note that approximately

$$\frac{1}{\sigma} = \frac{1}{\sigma_{\text{ph}}} + \frac{1}{\sigma_{\text{imp}}},$$  (4)

where $\sigma_{\text{ph}}$ and $\sigma_{\text{imp}}$ are the conductivities resulting from electron–phonon and electron–impurity scattering mechanisms, respectively. The phonon conductivity depends on the temperature, decreasing when $T$ increases, and hence heating caused by accretion accelerates the field decay. The impurity conductivity is practically independent of the temperature, but its magnitude is determined by the so-called impurity parameter,

$$Q = \frac{1}{n_i} \sum_{i} n'(Z - Z_i)^2,$$  (5)

where the dominant background ion species has the number density $n_i$ and charge $Z_i$; the summation is over all species of impurities. In our calculations, we use the numerical data for the phonon conductivity obtained by Itoh, Hayashi & Koyama (1993) and a simple analytical expression for the impurity conductivity derived by Yakovlev & Uryon (1980). As the conductivity depends on the temperature, the magnetic evolution of a neutron star is strongly sensitive to its thermal history. According to the scenario suggested in Section 1, we divide the evolution into four essentially different phases:

(i) the ‘isolated pulsar’ phase (phase I), in which the neutron star scarcely feels the influence of its companion,

(ii) the ‘propeller’ phase (phase II), in which, because of fast spin rotation, the neutron star ejects the infalling wind matter,

(iii) the wind accretion phase (phase III), in which accretion of the wind plasma is allowed, and

(iv) the accretion phase (phase IV), in which the secondary fills its Roche lobe and the neutron star experiences vigorous mass transfer.

Evidently, during phases I and II the thermal evolution of the neutron star in a binary system exactly follows that of an isolated neutron star. For the time dependence of the temperature during these two phases, we use the standard cooling model given by Van Riper (1991) for a 1.4-$M_\odot$ neutron star with the Pandharipande–Smith equation of state. During phases III and IV, however, the neutron star can be substantially heated by accretion. The thermal evolution of accreting neutron stars has been a subject of study of several papers (see, e.g., Fujimoto et al. 1984, Miralde-Escude et al. 1990). It was argued that, after a relatively short initial stage, the temperature distribution within the neutron star reaches a steady state. The temperature is then almost uniform in deep layers of the crust of neutron star with standard neutrino emissivities. The temperature that the crust will attain in the steady state depends on the accretion rate, $\dot{M}$. For phase III, with a low accretion rate, we use the crustal temperature computed by Zdunik et al. (1992) for an
the same braking law applies if the torque is determined by currents, in the model suggested by Goldreich and Julian (1969). At \( R_s < R_\text{A} \), the pressure of the magnetodipole radiation cannot prevent the wind plasma from penetrating behind the light cylinder. The influence of the wind plasma on the spin-down rate becomes appreciable, and further spin evolution of the neutron star departs from that of an isolated pulsar. In our simplified model, the condition \( R_s = R_\text{A} \) determines the transition between phases I and II.

During phase II, the wind matter can directly interact with the neutron star magnetosphere. However, rotation still is rather fast and the magnetosphere acts as a 'propeller', ejecting the infalling wind matter. During this phase, a certain fraction of the spin momentum of the neutron star is transferred to the matter being ejected, and the spin period may increase by a few orders of magnitude. In calculations presented here, we adopt a very simplified model of the interaction between the accretion flow and magnetosphere which is, nevertheless, qualitatively close to that commonly encountered in the literature (see, e.g., Pringle \\& Rees 1972 and Illarionov \\& Sunyaev 1975). We assume that the accreted matter interacts with the magnetosphere at the so-called Alfvén radius, \( R_\text{A} \):

\[
R_\text{A} = \left( \frac{R^2 B^2}{4M \sqrt{\Gamma M}} \right)^{2/7} \approx 1.2 \times 10^{12} \left( \frac{B_{12}}{M_{12}} \right)^{2/7} \text{ cm},
\]

where \( B_{12} = B/10^{12} \text{ G} \) and \( M_{12} = M/10^{12} \text{ M}_\odot \text{ yr}^{-1} \); the numbers given are for a neutron star with \( M = 1.4 \text{ M}_\odot \) and \( R = 1.64 \times 10^{10} \text{ cm} \). If the neutron star rotates rapidly and its angular velocity is larger than the Keplerian angular velocity at the Alfvén radius, \( \Omega (R_\text{A}) \), the wind matter has to be expelled by the magnetosphere, carrying the spin angular momentum out too. In order to be expelled, the wind matter must reach an angular momentum slightly larger than that corresponding to Keplerian rotation. Therefore, the amount of the spin momentum lost by the neutron star per unit time, \( \dot{J}_p \), may be estimated as

\[
\dot{J}_p \approx 4\pi R_\text{A}^4 \mu \Omega_c v_r \approx \dot{\mathcal{M}} \sqrt{\Gamma M R_\text{A}},
\]

where \( v_r \) is the radial velocity of the accreting flow near the magnetospheric boundary. The corresponding spin-down rate of the neutron star is

\[
\dot{\Omega} \approx \frac{\dot{J}_p}{2\pi I} \approx \frac{\dot{\Omega}}{2\pi I} \frac{B^2 R_\text{A}}{4M},
\]

where \( \dot{\Omega} = (GM R_\text{A}^3/2I) \dot{\Omega} \). The neutron star rotation slows down until the angular velocity becomes comparable to \( \Omega_c(R_\text{A}) \). We determine the critical period, \( P_{\text{crit}} \), at which the transition between the 'propeller' and accretion phases takes place from the condition \( \Omega = \Omega_c(R_\text{A}) \):

\[
P_{\text{crit}} = \frac{18.68 \times 10^{12}}{M_{12}^{3/7}} \text{ s}.
\]

This period can be large if the neutron star is accompanied by a secondary star with a low rate of mass loss. Equation (11) determines the so-called spin-up line in the \( B - \dot{P} \) plane. If the spin rotation slows down to the value given by equation (11), it is generally accepted that the accreting matter can flow through the magnetosphere and accrete on to the surface, so that the neutron star enters phase III. During this phase accretion from the stellar wind is allowed and, as mentioned earlier, nuclear burning of the accreted material causes some heating of the neutron star interior and induces accretion-driven field decay. In turn, the decay of the magnetic field decreases the critical period, \( P_{\text{crit}} \). As accretion from the wind in a binary cannot be exactly spherical, the accreting matter carries some of the angular momentum to the neutron star (see Geppert, Urpin \\& Konenkov 1995) and, because of this, the neutron star can experience spin-up to a shorter period. A balance will probably be reached between spin-up and the rate of field decay, so that the neutron star slides down the corresponding spin-up line (see, e.g., Bhattacharya \\& Srinivasan 1991). Phase II of the evolution lasts either until the end of the main-sequence lifetime of the companion (when it fills its Roche lobe and the neutron star enters phase IV) or until the magnetic field becomes weak to such an extent that the accreting angular momentum is insufficient to maintain a balance between spin-up and the rate of field decay.

The amount of angular momentum carried by the accreted matter can be estimated from the condition at the magnetospheric boundary, assuming that the angular momentum there is characterized by its Keplerian value, \( \mu \Omega_c R_\text{A}^2 \), multiplied by some 'efficiency' factor \( \zeta \). This factor is \( \approx 1 \) if the accreted wind matter forms a Keplerian disc outside the magnetosphere. If a disc is not formed, then the efficiency of spin-up is much lower and the parameter \( \zeta \) is probably of the order of 0.1–0.01 (see Jahan Miri \\& Bhattacharya 1994). The amount of spin angular momentum added to the neutron star per unit time, \( \dot{J}_w \), can be estimated as

\[
\dot{J}_w \approx \frac{\pi}{4} R_\text{A}^4 \mu \Omega_c v_r \approx \zeta \dot{\mathcal{M}} \sqrt{\Gamma M R_\text{A}}.
\]

The corresponding spin-up rate is

\[
\dot{P} \approx - \zeta \beta \dot{\Omega} \text{B}^{2/7} \text{M}^{6/7}.
\]

If the magnetic field becomes very weak, \( \dot{P} \) may be small and accretion cannot maintain a balance between the spin up and field decay rates, so that the neutron star leaves the spin-up line.

After the companion fills its Roche lobe and phase IV starts, accretion on to the neutron star is strongly enhanced and heating of the crust leads to rapid field decay. In low-mass binaries, accretion resulting from Roche-lobe overflow can probably last as long as \( (1-5) \times 10^3 \text{ yr} \). During this phase, the accreted matter likely forms a Keplerian disc outside the neutron star magnetosphere. Therefore, the rate of angular momentum transfer and the spin-up rate are given by equations (12) and (13), respectively, with the 'efficiency' parameter \( \zeta \approx 1 \). The neutron star spins up in accordance with these equations until it approaches the spin-up line corresponding to the Roche-lobe accretion rate. The accretion rate probably has to reduce when the neutron star reaches the spin-up line (Bhattacharya \\& Srinivasan 1991) but, nevertheless, the field continues to decay as a result of accretion-driven mechanisms. During further evolution, a balance should be reached between spin-up and the rate of field decay, as in the case of accretion from the wind. Because of this balance, the neutron star slides down the corresponding spin-up line with a rate that is determined by field decay. In reality, the evolution on the spin-up
line may be very complicated, but in our calculations we will adopt a simplified model. Namely, we will neglect possible differences in the accretion rate before and after the neutron star reaches the spin-up line. As mentioned above, the amount of matter accreted on to the surface may be reduced when the star evolves on the spin-up line. However, this decrease of the accretion rate is unlikely to change the internal temperature drastically because at high accretion rates the temperature is close to saturation and depends relatively weakly on the accretion rate (see Miralda-Escude et al. 1990 and Fujimoto et al. 1984). Therefore we hope that our assumption does not change the rate of the magnetic field decay on a spin-up line substantially. Note that in our model the neutron star slides down the corresponding spin-up line with the maximal rate, but in reality the evolution will be slower. The star can leave the spin-up line either if accretion is exhausted or if the field becomes too weak to maintain a balance between spin-up and field decay.

3 NUMERICAL RESULTS AND DISCUSSION

We consider the evolution of the neutron star for a wide range of accretion rates and initial magnetic fields. Calculations are performed for a neutron star model with $M = 1.4 \, M_\odot$ based on the Pandharipande–Smith equation of state (Pandharipande & Smith 1975). This model is representative of stiff equations of state, so the radius and thickness of the crust for the corresponding neutron star are relatively large, $\approx 16.4$ and $4.2$ km, respectively. We choose a stiff model for our calculation because the magnetic evolution of isolated neutron stars based on such models is in good agreement with the available observational data on pulsar magnetic fields (Urpin & Konenkov 1997). The evolution of neutron star models with equations of state softer than that of Friedman–Pandharipande seems to be inconsistent with the B–P distribution of radio pulsars.

The conductive properties of the crust depend on its chemical composition which, unfortunately, is rather uncertain for neutron stars. This dependence is not significant during the initial evolution, when the magnetic field is confined to the layers with rather low density and the difference between the existing models of chemical composition is not very large. However, the effect of composition may be important during phase IV, when the neutron star is strongly heated and the conductivity is determined by phonon scattering, which is strongly sensitive to the chemical composition. Therefore, in our calculations, we adopt the simplest model, in accordance with which the crust is composed of nuclei processed in various nuclear transformations in accreting neutron stars (see Hainesel & Zdunik 1990) during the whole evolution of the neutron star.

In calculations, the impurity parameter, $Q$, is assumed to be constant throughout the crust during the whole evolution and to range from 0.001 to 0.1. The initial spin rotation is assumed to be relatively fast, $P_0 = 0.01 \, $s. Note, however, that the results are not sensitive to the value of $P_0$. On the contrary, the evolution is strongly dependent on the original magnetic configuration of the neutron star. The initial magnetic field is assumed to be confined to the outer layers of the crust with densities $\rho \leq \rho_0$. The calculations presented here are performed for $\rho_0$ in the range $4 \times 10^{11}$ to $10^{13}$ g cm$^{-3}$. These values correspond to a depth from the surface of $\approx 820$ and 1100 m, respectively. The choice of $\rho_0$ is imposed by comparing the calculated magnetic and spin evolution of isolated neutron stars with a crustal magnetic field with the available observational data on $R$ and $P$ for radio pulsars (see Urpin & Konenkov 1997). The initial field strength at the magnetic pole, $B_0$, is taken to be within the range $10^{10} \leq B_0 \leq 10^{11}$ G.

The present study is mainly addressed to neutron stars entering binary systems with a low-mass companion. For such secondary stars, the main-sequence lifetime, $t_{\text{ms}}$, may last as long as $10^9 - 10^{10}$ yr. During the main-sequence evolution of the secondary, the accretion rate on to the neutron star is determined by both the characteristics of the stellar wind and the separation between the stars, and may generally vary within a wide range. In calculations, we suppose that $M = 10^{-13}, 10^{-15}$ and $10^{-17} \, M_\odot \, $yr$^{-1}$ during both the propeller and wind accretion phases; $V_\infty = 500 \, $km s$^{-1}$, $a = 5 \times 10^{12}$ cm. We also assume $\zeta = 0.1$ for the efficiency parameter. Note, however, that our calculations show a weak sensitivity of the evolutionary tracks to the particular choice of $\zeta$. After the secondary leaves the main-sequence and fills its Roche lobe, the accretion rate has to be strongly enhanced, $M \approx 10^{-5} - 10^{-3} \, M_\odot \, $yr$^{-1}$. We do not consider a higher accretion rate because of the lack of calculations on the thermal structure of accreting neutron stars with $M > 3 \times 10^{-10} \, M_\odot \, $yr$^{-1}$.

Figs 1–4 show examples of the magnetic, spin and thermal evolution of a neutron star in a binary system for different values of the parameters. Numbers in the top panels indicate the evolutionary phase. Note a difference in the horizontal scales for different phases. The evolutionary tracks are shown as solid lines if the accretion rate resulting from Roche-lobe overflow is $M = 10^{-10} \, M_\odot \, $yr$^{-1}$ and as dashed lines if $M = 10^{-9} \, M_\odot \, $yr$^{-1}$.

In Fig. 1, we plot the evolution for a relatively strong initial magnetic field, $B_0 = 10^{14}$ G, and for $\rho_0 = 10^{13}$ g cm$^{-3}$. The impurity parameter is $Q = 0.01$ and the duration of the main-sequence lifetime of a low-mass companion is $3 \times 10^9$ yr. Accretion resulting from Roche-lobe overflow is assumed to last as long as $10^8$ yr; thus the total amount of accreted mass can reach $0.1 \, M_\odot$ if $M = 10^{-9} \, M_\odot \, $yr$^{-1}$. During phase III, the accretion rate is $10^{-13} \, M_\odot \, $yr$^{-1}$.

This model gives an example of a neutron star that is processed in all evolutionary stages. Phase I, when the evolution of the neutron star is not influenced by a companion, lasts a little less than $10^7$ yr. During this sort stage the field does not decay significantly, thus its surface strength is about $2 \times 10^{22}$ G when the neutron star enters the ‘propeller’ phase. An efficient spin-down caused by strong magnetodipole radiation leads to a rapid increase of the spin period from $10^{-2}$ to $\approx 6$ s. During the next evolutionary stage, when the neutron star works as a propeller, the field continues to decay slowly because, after $10^7$ yr of the evolution, the temperature falls to a very low value and the conductivity of crustal matter is high. Note that during this stage the conductivity is determined by impurity scattering and depends on the impurity parameter $Q$. Phase II is relatively long ($\approx 230 \, $M yr), but the field is still very strong ($\approx 10^{22}$ G) to the end of this phase. As a result of such a strong field, the neutron star can eject the wind matter, which penetrates into the magnetosphere, as long as the spin period is shorter.
than $\approx 4 \times 10^3$ s. When spin rotation slows down to this period, matter from the wind can fall on to the surface of the neutron star and it becomes a soft and not very bright X-ray source. Nuclear burning of the accreted material heats the neutron star interior to $T \approx 2 \times 10^8$ K. However, this heating is too weak to accelerate the field decay appreciably; thus, the magnetic evolution during the wind accretion phase does not differ essentially from that of an isolated star. Note that wind accretion can accelerate the decay appreciably only if the accretion rate is larger than $10^{-13}$ $M_\odot$ yr$^{-1}$. Until the end of the wind accretion, the field strength remains as high as $2 \times 10^{11}$ G. The neutron star can slowly spin up, obtaining some angular momentum from the accreted wind matter, but this is not enough for the star to spin up to a short period; the period therefore turns out to be as long as $\approx 10^3$ s after $3 \times 10^3$ yr, when the companion ends its main-sequence lifetime. Phase III is the most extended for the model considered, however, as will be shown, it may not be the case for other models. The most dramatic changes are experienced by the neutron star during phase IV, when enhanced accretion resulting from Roche-lobe overflow starts. Accretion with an accretion rate $\dot{M} = 10^{-10} - 10^{-8}$ $M_\odot$ yr$^{-1}$ or higher heats the neutron star to a temperature $T \approx (2-3) \times 10^8$ K and decreases the crustal conductivity substantially. If the field is reduced only by a factor $\sim 50$ during the previous $3 \times 10^3$ yr$^{-1}$, the accretion-driven mechanism leads to much more efficient decay. Thus, in the case $M = 10^{-7} M_\odot$ yr$^{-1}$, the field weakens $\approx 30$ times after $10^6$ yr of accreting, when the neutron star has accreted $0.1 M_\odot$. For a higher accretion rate, the decay is even stronger. As the accreted matter carries a large amount of the angular momentum with it, the neutron star can accelerate its spin rotation to relatively short periods during phase IV. The spin evolution is non-monotonous (this is shown in the small panel in Fig. 1). At the start of accretion the neutron star spins up very rapidly, and after $\approx 10^4$ yr it reaches the corresponding spin-up line. As the magnetic field is still sufficiently strong $[\sim (1-2) \times 10^{11}$ G], the corresponding critical period, $P_{\text{cr}}$, is relatively long ($\approx 1$ s). Further spin evolution proceeds slowly because the neutron star slides down the spin-up line, and the rate of this migration is determined by the field decay. Finally, when accretion is exhausted after $10^6$ yr, the neutron star that has been processed in all the above transformations will operate as a radio pulsar with a low magnetic field $[\sim (1-3) \times 10^{10}$ G] and a short period ($\sim 0.04-0.3$ s).
Fig. 2 shows another example of the evolution, which is typical for stars with a relatively weak initial magnetic field. The initial field is assumed to be $10^{11}$ G; other parameters are the same as in Fig. 1. In this case, the neutron star does not pass through all evolutionary stages: accretion of matter from the wind is impossible because rotation spins down very slowly as a result of a weak magnetic field and, after $3 \times 10^9$ yr, the period is still shorter than the corresponding value of $P_{eq}$. A weak initial field results in a very long duration of phase I for this model: the period slows down to a value of $\sim 0.4$ s, which allows the infalling matter to interact with the magnetosphere only after $\sim 600$ M yr. However, the magnetic field turns out to be reduced only by a factor $\sim 10$ and is $\sim 10^{10}$ G after this long phase. For the remainder of the main-sequence lifetime of the companion (which is about 80 per cent of this lifetime), the neutron star works as a propeller, ejecting the wind plasma. Despite a long duration for this phase, neither the magnetic field nor the spin rotation change significantly before the beginning of Roche-lobe overflow: the surface field strength decreases approximately by a factor of 3, whereas the period increases from 0.4 to 0.46 s. Note the great difference in the periods just before the accretion phase for this model and the model plotted in Fig. 1, where $P \approx 10^3$ s at $t = 3 \times 10^9$ yr. Enhanced accretion, however, can accelerate a spin rotation very rapidly; thus an accreting neutron star forgets quickly about its period before the accretion phase. Recycling is particularly fast during the first $\sim (4-10)$ M yr of accreting, when the neutron star spins up to 10–30 ms depending on the accretion rate and approaches the corresponding spin-up line. Further spin evolution on the spin-up line proceeds slower but, nevertheless, the star can reach millisecond periods if a mass transfer lasts as long as $10^8$ yr. For this model, the ‘recycled’ periods when accretion is exhausted are $\sim 1$ and $\sim 10$ ms for $\dot{M} = 10^{-6}$ and $10^{-12}$ $M_\odot$ yr$^{-1}$, respectively. The magnetic field is also substantially reduced during phase IV under the action of the accretion-driven mechanism. To the end of this phase, the surface field may be as low as $10^8$ G if $\dot{M} = 10^{-5}$ $M_\odot$ yr$^{-1}$ (when the total accreted mass is $0.1 M_\odot$) and $3 \times 10^8$ G if $\dot{M} = 10^{-10}$ $M_\odot$ yr$^{-1}$ (when the accreted mass is $0.01 M_\odot$). When accretion has been exhausted, the neutron star can be observed as a millisecond radio pulsar with a period $\sim 1-10$ ms and a magnetic field $\sim (1-3) \times 10^8$ G.

In order to illustrate the effect of the impurity content, we plot in Fig. 3 the evolution of a neutron star with a strongly polluted crust, $Q = 0.1$. We also assume that for this model the field is initially confined to the layer with the smallest density, $\rho_c = 10^{12}$ g cm$^{-3}$, corresponding to a depth from the surface $\approx 900$ m. Like the model plotted in Fig. 1, this one also passes through all evolutionary stages. However, the field decays faster for this model because the crustal con-
ductivity is lower as a result of a higher impurity content and the field is initially anchored in the layers with lower density (and hence a lower conductivity). A faster decay leads to a slower spin-down and longer duration of the first two phases in comparison with Fig. 1. Thus the ‘isolated pulsar’ and ‘propeller’ phases last \( \approx 12 \) and 650 Myr, respectively, whereas the duration of these phases in Fig. 1 is \( \approx 6 \) and 230 Myr. The maximal spin period that is reached to the end of the propeller phase does not exceed, \( \approx 300 \) s however. An increase of the impurity parameter \( Q \) causes a particularly rapid field decay during phases II and III, when the conductivity is determined by scattering on impurities. During these two phases the field is reduced by two orders of magnitude, and it is a little lower than \( 10^{10} \) G when the companion fills its Roche lobe and enhanced accretion starts. Accretion of matter during \( 10^8 \) yr leads to the formation of a millisecond pulsar with a period within the range \( \approx 2–20 \) ms and with a magnetic field \( \approx (3–10) \times 10^9 \) G, depending on the accretion rate during phase IV.

Fig. 4 represents the evolution of a neutron star with an intermediate initial magnetic field, \( B_0 = 10^{12} \) G, and with a longer main-sequence lifetime of the companion, \( t_{\text{ms}} = 5 \times 10^9 \) yr. Other parameters are the same as in Fig. 1. The evolution does not differ in the main from that plotted in Fig. 1, except for a weaker magnetic field of the neutron star at the beginning of the accretion phase. For this model, \( B \approx 2 \times 10^{10} \) G at \( t = 5 \times 10^9 \) yr. When accretion starts, such a weak magnetic field is probably unable to channel the inflowing matter towards the magnetic poles; thus the rotation of the neutron star will not cause the observed X-ray emission to have a pulsed appearance. Note that the same is true of the models plotted in Figs 2 and 3. In contrast, the neutron star with the evolutionary scenario represented in Fig. 1 can be observed (at least at the beginning of the accretion phase) as a pulsating binary X-ray source. The accretion-driven field decay causes a further decrease of the magnetic field so that by the end of phase IV it may be as weak as \( 10^9 \) G if \( M = 10^{-3} \ M_{\odot} \) yr\(^{-1}\). If accretion lasts \( 10^9 \) yr, the neutron star can be easily recycled to a very short period. For this model, the final spin periods are \( \approx 10 \) and 100 ms after accreting 0.1 and 0.01 \( M_{\odot} \), respectively. Note that the field strengths and periods of the neutron stars processed in accordance with the scenarios shown in Figs 2–4 are very similar to those of millisecond pulsars.

Figs 5–10 show the evolutionary tracks of neutron stars in the \( B-P \) plane for different values of various parameters. We also plot in these figures the observational data on isolated pulsars (dots) and pulsars in binary systems (open diamonds) taken from the catalogue by Taylor, Manchester & Lyne (1993). The dotted lines show spin-up lines corresponding to different accretion rates (numbers near these lines indicate the logarithm of the accretion rate). The solid
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Lines represent the evolutionary tracks. All tracks are calculated for the case of accretion resulting from Roche-lobe overflow with $\dot{M} = 10^{-9} \, M_\odot \, yr^{-1}$. Numbers near the tracks indicate the logarithm of the age; numbers with the subscript (a) mark the time required for the neutron star to reach the corresponding spin-up line after the end of main-sequence evolution. Large crosses show the points on tracks where a transition takes place between phases I and II; small crosses with the corresponding numbers mark the end of tracks after $10^8 \, yr$ of enhanced accretion.

Fig. 5 plots the evolution of neutron stars that accrete matter from the stellar wind with different accretion rates. Calculations are performed for $\dot{M} = 10^{-12}$ (curve 1), $10^{-13}$ (2), $10^{-14}$ (3), $10^{-15}$ (4) and $10^{-17} \, M_\odot \, yr^{-1}$ (5); the duration of the main-sequence lifetime of the companion is $3 \times 10^9 \, yr$. The initial field is $B_0 = 10^{12} \, G$, the density penetrated by the initial field is $\rho_0 = 10^{12} \, g \, cm^{-3}$, and the impurity parameter is $Q = 0.1$. The neutron stars starting the evolution with such parameters typically pass through all evolutionary stages. The only exception is model (5) with a very weak stellar wind, $\dot{M} = 10^{-17} \, M_\odot \, yr^{-1}$. This wind is insufficient to slow down the spin period to the critical value (11) when accretion from the wind becomes possible and, therefore, the neutron star is still in the 'propeller' phase when the companion fills its Roche lobe and enhanced accretion starts. The duration of phases is obviously strongly dependent on the rate of wind accretion. For instance, the initial phase, in
which the neutron star evolves like an isolated neutron star, lasts as long as \( \approx 160 \text{ Myr} \) if \( \dot{M} = 10^{-12} \text{ M}_\odot \text{ yr}^{-1} \) and only \( 10^9 \text{ yr} \) if \( \dot{M} = 10^{-15} \text{ M}_\odot \text{ yr}^{-1} \). To the end of this phase, the magnetic field and period are ranged from \( 2 \times 10^{12} \text{ G} \) and from 6 to 1 s, respectively, for different models. After this phase, the field and spin are reduced sufficiently that the wind can penetrate into the magnetosphere, and the neutron star works as a propeller. The 'propeller' phase lasts until the neutron star, moving to the right and down in the \( P - P_{\text{spin}} \) plane, reaches the corresponding spin-up line. Evidently, a star that experiences stronger accretion from the wind requires a shorter time to approach the spin-up line. Thus, one has \( P = P_{\text{spin}} \) after 54 and 630 M yr of evolution if \( \dot{M} = 10^{-12} \) and \( 10^{-15} \text{ M}_\odot \text{ yr}^{-1} \), respectively. As mentioned above, in the case of very weak accretion with \( \dot{M} = 10^{-17} \text{ M}_\odot \text{ yr}^{-1} \) the spin-up line can never be reached. Accretion from the wind starts when the magnetic field is still rather high for models (1)-(4), \( B = 10^{11-12} \text{ G} \). This field is certainly able to channel the accreting matter towards the magnetic poles; thus, probably, some neutron stars accreting matter from winds in low-mass binaries can be observed as weak pulsating X-ray sources. Further evolution of the neutron star on the spin-up lines is completely determined by the field decay, which can be accelerated as a result of accretion-induced heating for a relatively high accretion rate. The wind-accretion phase is the most extended phase for all models plotted in Fig. 5 with the exception of model (5). The parameters of stars can be drastically changed during this long phase if the accretion rate is sufficiently large. In fact, the field is reduced by a factor \( \sim 10^4 \) and is only \( \sim 10^8 \text{ G} \) for model (1) with accretion rate \( \dot{M} = 10^{-12} \text{ M}_\odot \text{ yr}^{-1} \). Note, however, that at the end of the main-sequence lifetime of the companion the magnetic fields of models (2)-(5), which experience accretion with \( \dot{M} \leq 10^{-13} \text{ M}_\odot \text{ yr}^{-1} \), lie within a relatively narrow range, \( \sim (2-10) \times 10^8 \text{ G} \), despite a large difference in the duration of different phases for these models. The main reason for this is the relatively weak heating caused by accretion with such low accretion rates, which makes the accretion-driven mechanism inefficient at \( \dot{M} \leq 10^{-13} \text{ M}_\odot \text{ yr}^{-1} \).

When the companion fills its Roche lobe and enhanced accretion starts, the neutron star initially spins up very rapidly; it requires only 0.2 M yr to reach the spin-up line for model (5) and 1.6 M yr for model (2). These time intervals are too short for an appreciable decay of the magnetic field; therefore the corresponding pieces of tracks are represented by approximately horizontal plateaus. Note that the magnetic field of model (1) is very weak after \( 3 \times 10^9 \text{ yr} \) of evolution (\( \sim 10^8 \text{ G} \)), and this model will never reach the spin-up line. Like the case of accretion from a stellar wind, in its further evolution the neutron star slides down the spin-up line and the rate of this sliding is determined by the field decay. However, in the case of Roche-lobe overflow, the accretion-driven mechanism is much more efficient and hence the decay is much faster. During phase IV, the magnetic field decreases by a factor \( \sim 30 \) for models (2)-(5); thus, when accretion is exhausted, all these neutron stars will work as pulsars with weak magnetic fields, \( \sim (1-5) \times 10^8 \text{ G} \), and short periods, \( \sim 2-10 \text{ ms} \). Model (1) is also spun up to a millisecond period, but its magnetic field is substantially smaller than \( 10^9 \text{ G} \).

In Fig. 6, we plot the same as in Fig. 5 but for the magnetic field confined initially to deeper layers, \( \rho_0 = 10^{13} \text{ g cm}^{-3} \), and for a crust with a smaller impurity content, \( Q = 0.01 \). Evidently, the magnetic evolution is much slower for a less polluted crust. Like the case shown in Fig. 5, models (1)-(4) pass through all evolutionary phases whereas model (5), with very weak accretion from the wind (\( \dot{M} = 10^{-17} \text{ M}_\odot \text{ yr}^{-1} \)) avoids phase III. As a result of lower field decay and hence faster spin-down, the duration of phases I and II is shorter for the models in Fig. 6 than for the models with the same wind in Fig. 5. Therefore, wind accretion starts when the magnetic field of the neutron star is very strong, \( B \sim 2 \times 10^{22} \text{ G} \) for all models. As in the previous case, wind accretion is the most extended phase for models (1)-(4). To the end of the main-sequence evolution of the companion (\( t_{\text{ms}} = 3 \times 10^9 \text{ yr} \)), the magnetic fields of neutron stars that experience wind accretion with a low accretion rate, \( \dot{M} < 10^{-13} \text{ M}_\odot \text{ yr}^{-1} \), lie in a narrow range from \( 4 \times 10^{10} \) to \( 3 \times 10^{11} \text{ G} \). Only accretion with \( \dot{M} > 10^{-13} \text{ M}_\odot \text{ yr}^{-1} \) can substantially change the magnetic evolution during phase III.

Owing to a higher magnetic field at \( t = 3 \times 10^9 \text{ yr} \), the models plotted in Fig. 6 reach the spin-up line on a much shorter time-scale when enhanced accretion starts. Thus, only \( 4 \times 10^9 \text{ yr} \) is required for model (5) and \( \approx 1 \text{ Myr} \) for model (1). Note that in the previous case model (1) could never reach the spin-up line during the accretion phase. If accretion resulting from Roche-lobe overflow lasts \( 10^9 \text{ yr} \), models (2)-(5), which experienced weak accretion from the wind, evolve to relatively weakly magnetized pulsars with \( B \sim (2-10) \times 10^8 \text{ G} \), rotating with a period \( \sim 0.03-0.1 \text{ s} \). Model (1), with a very high accretion rate during phase III, evolves to a millisecond pulsar with \( P \approx 3 \text{ ms} \) and \( B \approx 10^9 \text{ G} \). Obviously, the magnetic field and period are larger for all pulsars if the accretion phase is shorter.

![Figure 6](https://example.com/figure6.png)

*Figure 6.* Same as Fig. 5, but for \( \rho_0 = 10^{13} \text{ g cm}^{-3} \) and \( Q = 0.01 \).
Models considered lie within a wide range from $10^{-10}$ to $10^{-3}$. To the beginning of wind accretion, magnetic fields for the Roche-lobe overflow are $10^{-9}$ and $10^{-2}$, respectively. These fields are probably too weak to channel accreting matter towards the magnetic poles; thus the neutron star is unable to work as a pulsating X-ray source. Model (4), which initially has a strong magnetic field, $B_0 = 10^{13}$ G, may also be transformed into a weakly magnetized and rapidly rotating pulsar towards the end of the accretion phase. However, the field of this pulsar ($\sim 2 \times 10^9$ G) is a little stronger and the period ($\sim 30$ ms) is a little longer than it is typical for millisecond pulsars. Evidently if accretion lasts longer than $10^9$ yr model (4) can also evolve to a millisecond pulsar with the standard parameters.

In Fig. 8, the dependence of evolutionary tracks on the duration of main-sequence lifetime of the companion, $t_{\text{main}}$, is shown. Since we address mainly the evolution of neutron stars entering binary systems with a low-mass companion, calculations have been performed for a sufficiently long $t_{\text{main}}$, $10^{10} \geq t_{\text{main}} \geq 10^9$ yr. Despite the fact that we plot in this figure tracks for a neutron star with a relatively polluted crust, $Q = 0.03$, and an initial magnetic field $B_0 = 3 \times 10^{12}$ G, which is certainly weaker than the measured fields of many isolated pulsars, it is seen that crustal currents can maintain a sufficiently strong magnetic field during the whole evolution. This conclusion is in contrast to the widely accepted opinion that neutron star models with a crustal magnetic field are not suitable to account for the relatively strong magnetic fields of many long-lived pulsars. Even for a 'polluted' crust, the decay turns out to be very slow; thus the neutron star can possess a field stronger than $10^9$ G after $10^{10}$ yr of evolution and enhanced accretion.

During the 'isolated pulsar' phase, which lasts $\sim 20$ Myr, the field weakens by a factor $\approx 5$, whereas during the 'propeller' phase, which is as long as $\approx 630$ Myr, the field is reduced only by a factor of 2. Working as a propeller at $\sim 610$ Myr, the neutron star spins down to a period of the order of $10^3$ s, which is slow enough to accrete wind matter.
At this time the field is rather strong, \( B \approx 3 \times 10^{11} \) G, and so the accreted matter has to be channeled towards the magnetic poles. Perhaps during the wind accretion phase such a neutron star can be observed as a weak and pulsating X-ray source. When the companion fills its Roche lobe, the magnetic field of the neutron star can range from \( 3 \times 10^{10} \) to \( 2 \times 10^{11} \) G depending on the duration of the main-sequence lifetime. The corresponding spin periods lie within the range 100–500 s. Owing to a relatively strong magnetic field, a spin-up is initially very fast; thus the neutron star approaches the spin-up line shortly after the beginning of Roche-lobe overflow. The field does not change during this short time interval (\( \approx 10^{-9} - 10^{-8} \) yr). In the course of further evolution alongside the spin-up line, the field is decreased by a factor \( \approx 30 \) under the action of the accretion-driven mechanism; thus to the end of the accretion phase a pulsar can be formed with \( B \approx (2-5) \times 10^{8} \) G and \( P \approx 20-70 \) ms. If accretion lasts longer than \( 10^{9} \) yr and the total amount of accreted mass is larger than \( 0.1 \, M_{\odot} \), all the models plotted in Fig. 8 can evolve to ‘standard’ millisecond pulsars with \( B \approx 10^{8} \) G and \( P \approx 1-10 \) ms. Note that if the mass of a low-mass companion is relatively large so that the main-sequence lifetime is closer to \( 10^{9} \) yr rather than \( 10^{10} \) yr, then the field at the beginning of phase IV is sufficiently strong (\( B \approx 2 \times 10^{11} \) G) to channel the accreted matter towards the magnetic poles, and the neutron star can work as a pulsating X-ray source. At the end of accretion, the field has to be reduced to a very low value, insufficient to produce X-ray pulses but sufficient for a neutron star to work as a radio pulsar.

Fig. 9 shows the dependence of the evolution on the impurity ‘pollution’ of the crust. Calculations have been performed for values of the impurity parameter, \( Q \), within the range 0.001 to 0.3. The duration of the main-sequence lifetime of the companion is \( 3 \times 10^{10} \) yr. During the initial evolution, the field decay does not depend on the impurity parameter. This is caused by the fact that the neutron star is relatively hot initially and the crustal conductivity is determined by phonon scattering, which is independent of \( Q \). Later on, when the star cools down and impurity scattering dominates the conductivity, the latter becomes sensitive to ‘pollution’ of the crust and the decay may be essentially different for different \( Q \).

The behaviour of all models plotted in Fig. 9 is more or less standard during the main-sequence evolution of the companion. The only exception is model (1), with extremely large \( Q \). Owing to a large impurity content and, as a consequence, a low conductivity, the magnetic field of this model decays too fast. During phases I and II, which last together \( \approx 1.6 \times 10^{9} \) yr, the field strength has to be reduced more than 1000 times and, when the neutron star approaches the spin-up line corresponding to the wind accretion rate \( \dot{M} = 10^{-15} \, M_{\odot} \, yr^{-1} \), its field is too low (\( B \approx 2 \times 10^{10} \) G) to maintain a balance between spin-up and the rate of field decay. Therefore, the neutron star does not slide down the spin-up line in its further evolution but moves down the \( B-P \) plane much faster. To the beginning of Roche-lobe overflow, the field of the model (1) weakens to a very low value \( \approx 10^{9} \) G. Enhanced accretion causes further field decay; thus when accretion is exhausted the field strength of this model is extremely low (\( \approx 10^{8} \) G) and the pulsar is almost unobservable.
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evolves like an isolated pulsar. The duration of this stage may vary, depending on the initial field strength and the rate of field decay as well as on the intensity of the stellar wind of the companion. The initial phase is obviously shorter for a neutron star with a higher magnetic field and for a stronger wind. Thus, for the initial field as strong as $10^{13}$ G, phase I may last only $\sim 0.1-1$ Myr if the neutron star captures gravitationally $\sim 10^{-12}-10^{-11}$ M$_\odot$ yr$^{-1}$ from the wind plasma. However, this phase may be extremely long ($\sim 100-300$ Myr) if the initial field is weak, $B_0 \sim 10^9$ G or if the rate of gravitational capture is small, $\sim 10^{-15}-10^{-17}$ M$_\odot$ yr$^{-1}$. For most models considered, the field is typically reduced by a factor $\sim 5-10$ during this phase, except for the cases when its duration is extremely long.

When the energy loss caused by magnetodipole radiation slows down rotation to such an extent that the wind matter can interact with the magnetosphere, the neutron star begins to work as a propeller, ejecting the wind plasma. Obviously, the propeller action is more efficient for a stronger magnetic field. The ‘propeller’ phase is usually more extended than phase I. During the ‘propeller’ phase, the magnetic evolution of the neutron star in a binary does not differ from that of an isolated star. As the star is relatively cool at the beginning of phase II, the conductivity of the crustal matter is determined by impurity scattering and may be high for a low impurity content. Typically, because of this, the field decays less during a more extended ‘propeller’ phase than during a shorter ‘isolated pulsar’ stage. The spin evolution proceeds, however, more rapidly because the rate of angular momentum loss is larger for the propeller effect than for magnetodipole braking. During the ‘propeller’ phase, the neutron star can spin down to a very long period, $\sim 10^{2-2.5}$ s. Note that this period may be shorter for stars with low magnetic fields and for a stronger stellar wind.

After further evolution, the spin period of the neutron star can reach the critical value given by equation (11). When rotation becomes slow enough, the centrifugal force is unable to eject the wind plasma that penetrates into the magnetosphere, and matter falls on to the neutron star surface. Nuclear burning of the accreted material heats the neutron star, decreases the crustal conductivity and accelerates field decay. Note that the efficiency of the accretion-driven mechanism of field decay is strongly sensitive to the accretion rate. The effect of accretion is relatively small for a low accretion rate, $\dot{M} < 10^{-17}$ M$_\odot$ yr$^{-1}$, but it may be of great importance for higher accretion rates. During wind accretion, the neutron star slides down the corresponding spin-up line, and the rate of this sliding is determined by the rate of field decay. Depending on the duration of this phase and the rate of accretion, the field can be reduced by a few orders of magnitude. As the accreted wind matter carries some amount of angular momentum, the neutron star can also spin up slightly during this phase but, for most cases considered, the spin period is still longer than 1 s up to the end of the main-sequence evolution of the companion. Usually, the wind accretion phase is the most extended of all evolutionary phases. Note, however, that the star can miss this stage in some cases (low rate of mass loss by the secondary star, rapid decay of the magnetic field), so that the evolution goes directly from the ‘propeller’ phase to enhanced accretion.
In low-mass binaries, the neutron star experiences the most dramatic changes during vigorous mass transfer, which starts when the companion ends its main-sequence evolution and fills its Roche lobe. Mass transfer in such systems can probably last as long as $10^7$–$10^8$ yr. Nuclear burning of the accreted material can heat the neutron star interior to a temperature $T \sim (2-3 \times 10^8$ K and can substantially reduce the crustal conductivity. As a result of this, the accretion-driven mechanism of field decay is especially efficient during phase IV. Typically, the field may be reduced about 20–30 times if accretion lasts $10^8$ yr, but the decrease may be larger for a longer accretion. Depending on the initial magnetic configuration and the duration of the main-sequence evolution, the surface magnetic field can fall to the value $\sim 10^9$–$10^5$ G by the end of mass transfer. As the matter accreted from the disc carries a large angular momentum, the spin evolves very rapidly. The neutron star reaches a spin-up line corresponding to enhanced accretion on a short time-scale, $\sim 0.01$–1 M yr. In its further evolution, the star moves along the spin-up line, while the magnetic field is able to maintain a balance between spin-up and the rate of field decay. During this phase, the accretion torque spins up the neutron star to a very short period, $\sim 1$–100 ms.

Our calculations show that a neutron star with a crustal magnetic configuration can maintain a relatively strong field during its whole evolution in a low-mass binary. Even if the main-sequence life of the companion lasts as long as $10^{10}$ yr and, after that, the neutron star experiences strong accretion with $M \sim 10^{-9}$ M$_{\odot}$ yr$^{-1}$ during $10^8$ yr, the magnetic field can remain sufficiently strong to the end of the accretion phase. When accretion is exhausted, such a neutron star works as a radio pulsar with a weak magnetic field, $\sim 10^5$–$10^6$ G, and a short period, $P \sim 1$–100 ms. These parameters are close to those of millisecond pulsars, and perhaps these objects are formed in accordance with the considered scenario. Note that if the main-sequence evolution is shorter and a mass transfer is not so extended, the neutron star processed in the above evolutionary transformations should have a stronger magnetic field and a longer period at the end of accretion. This gives a natural explanation of the origin of those radio pulsars in binary systems that lie between millisecond pulsars and the main pulsar population in the B–P plane.

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