Generalized Kerr/CFT correspondence with electromagnetic field

Cheng-Yong Zhang\textsuperscript{1}, Yu Tian\textsuperscript{2,3} and Xiao-Ning Wu\textsuperscript{3,4,5}

\textsuperscript{1} Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
\textsuperscript{2} School of Physics, University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
\textsuperscript{3} State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People’s Republic of China
\textsuperscript{4} Institute of Mathematics, Academy of Mathematics and System Science, Chinese Academy of Sciences, Beijing 100190, People’s Republic of China
\textsuperscript{5} Hua Loo-Keng Key Laboratory of Mathematics, CAS, Beijing 100190, People’s Republic of China

E-mail: zhangcy@sjtu.edu.cn, ytian@ucas.ac.cn and wuxn@amss.ac.cn

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Abstract

The electrovac axisymmetric extreme isolated horizon (IH)/conformal field theory (CFT) correspondence is considered. By expansion techniques under the Bondi-like coordinates, it is proved that the near-horizon geometry of electrovac axisymmetric extreme IH is unique. Furthermore, explicit coordinate transformation between the Bondi-like coordinates and the Poincare-type coordinates for the near-horizon metric of the extreme Kerr–Newmann spacetime is found. Based on these analyses and the thermodynamics of the IH, then, the Kerr/CFT correspondence is generalized to nonstationary extreme black holes with electromagnetic fields.

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1. Introduction

The discovery of Bekenstein–Hawking entropy was a great break through in theoretical physics in the last decades, but the microscopic origin of it still remains unclear. In [1, 2], the entropy of five and four-dimensional extreme black holes in string theory was derived by counting the degeneracy of BPS soliton bound states. Based on the demonstration that any consistent
theory of quantum gravity on AdS$_3$ is holographically dual to a two-dimensional conformal field theory (CFT) did not invoke string theory [3], entropy of the black holes whose near-horizon geometry is locally AdS$_3$ was counted in a dual two-dimensional CFT without using string theory or supersymmetry [4].

Paralleling to the AdS$_3$/CFT$_2$ duality, the correspondence between the near-horizon extreme Kerr (NHEK) geometry and CFT was found in [5], via a near-horizon limiting procedure. Given the suitably chosen boundary conditions at the asymptotic infinity of NHEK, they found that the asymptotic symmetry group extends to a Virasoro algebra upon Noether charge realizations. The central charge of this algebra was calculated. Then, with the Frolov–Thorne temperature and the Cardy formula, the microscopic entropy of extreme Kerr was computed, which is found exactly the same as the macroscopic Bekenstein–Hawking entropy of the black hole. Thus, the extreme Kerr black hole is holographically dual to a chiral two-dimensional CFT.

This duality has been generalized to many situations, including other black holes, diverse dimensions, supergravities etc [6–16]. For a nice review, see [17, 18]. We would like to emphasize that Hartman et al [19] generalized the Kerr/CFT correspondence to the Kerr–Newman (KN)-AdS-dS/CFT correspondence in four dimensions since it will be closely related to this paper. Nevertheless, most of the situations that have been considered are all stationary. In fact, Wu and Tian [20] have generalized the Kerr/CFT correspondence to the vacuum extreme isolated horizon (IH)/CFT correspondence. It is well known that all stationary horizons satisfy the definition of IH [21, 22], but IH contains some nonstationary cases [23, 24]. It should also be emphasized that the spacetime itself in the IH framework does not need to be axisymmetric, though the intrinsic horizon data of it is axisymmetric.

On the other hand, based on the result of Lewandowski and Pawlowski [25], we know that the intrinsic geometry of axisymmetric electrovac extreme IH coincides with the extreme KN event horizon. Thus, it is natural to ask whether this correspondence could be generalized further to electrovac extreme IH/CFT correspondence.

The paper is organized as follows. In section 2, we review the properties of KN spacetime. In section 3, the isolated horizon is introduced and we obtain the metric of its near-horizon limit by expansion techniques under the Bondi-like coordinates. We find that the near-horizon geometry of axisymmetric electrovac extreme IH is the same as that of the extreme KN spacetime. In section 4, we discuss the electrovac extreme IH/CFT correspondence. Section 5 is devoted to the summary and some discussions.

2. Review of Kerr–Newman spacetime

The KN solution of the Einstein equation describes the stationary and axisymmetric rotational charged black hole. The metric of KN spacetime in Boyer–Lindquist coordinates is

$$ds^2 = -\frac{\Delta}{\rho^2}(d\tilde{t} - a\sin^2 \theta d\tilde{\phi})^2 + \frac{\rho^2}{\Delta} d\tilde{r}^2 + \rho^2 d\theta^2 + \frac{\sin^2 \theta}{\rho^2} (a d\tilde{r} - (\tilde{r}^2 + a^2) d\tilde{\phi})^2$$

with

$$\rho^2 = \tilde{r}^2 + a^2 \cos^2 \theta,$$

$$\Delta = \tilde{r}^2 - 2M\tilde{r} + a^2 + Q^2,$$

$$Q^2 = Q_e^2 + Q_m^2.$$  \(2\)

Here $Q_e$ is the electric charge and $Q_m$ the magnetic charge.

The electromagnetic potential and field strength of KN are

$$A = -\frac{Q_e}{\rho^2} [d\tilde{t} - a\sin^2 \theta d\tilde{\phi}] - \frac{Q_m \cos \theta}{\rho^2} [a d\tilde{r} - (\tilde{r}^2 + a^2) d\tilde{\phi}].$$  \(3\)
\[ F = -\frac{Q_e (\hat{r}^2 - a^2 \cos^2 \theta)}{\rho^4} + \frac{2Q_m \hat{r}a \cos \theta}{\rho^4} (\hat{d} \cdot a \sin^2 \theta \, d\phi) \wedge d\hat{r} + \frac{Q_m (\hat{r}^2 - a^2 \cos^2 \theta) - 2Q_m \hat{r}a \cos \theta}{\rho^4} \sin \theta \, d\theta \wedge [a \, d\hat{t} - (\hat{r}^2 + a^2) \, d\hat{\phi}]. \] (4)

The outer horizon of KN is at
\[ r_+ = M + \sqrt{M^2 - (a^2 + Q^2)}. \] The Hawking temperature
\[ T_H = \frac{r_+}{4\pi} \left( \frac{r_+^2}{r_+^2 + a^2} \right). \] (5)

For extreme KN black hole, \( T_H = \kappa = 0 \). We have
\[ M^2 = a^2 + Q^2. \]

The outer horizon then becomes
\[ r_+ = M, \]
which coincides with the inner horizon, and the area
\[ A = 4\pi (M^2 + a^2). \] (6)

The Bekenstein–Hawking entropy at extremity is
\[ S_{BH} = \frac{A}{4} = \pi (a^2 + M^2). \] (7)

We consider only the extreme KN black hole from now on.

To find the near horizon limit geometry of extreme KN, the following coordinate transformation can be introduced [26]:
\[ \hat{r} = M + \epsilon r_0 r, \]
\[ \hat{t} = \frac{r_0 t}{\epsilon}, \]
\[ \hat{\phi} = \phi + \frac{\Omega_H r_0}{\epsilon} t, \] (8)

where \( r_0^2 = M^2 + a^2 \) and \( \Omega_H = \frac{a}{r_+ a} \) is the angular velocity of the horizon. After taking the limit \( \epsilon \to 0 \), metric (1) becomes
\[ ds^2 = (M^2 + a^2 \cos^2 \theta) \left( -r^2 \, dt^2 + \frac{dr^2}{r^2} + d\theta^2 \right) + \left( M^2 + a^2 \right) \sin^2 \theta \left( d\phi + \frac{2Ma}{M^2 + a^2} r \, dt \right)^2. \] (9)

Besides, the near-horizon limit of the electromagnetic field becomes
\[ A = f(\theta) \left( d\phi + \frac{2aM}{M^2 + a^2} r \, dt \right), \]
\[ f(\theta) = \frac{(M^2 + a^2) [Q_e (M^2 - a^2 \cos^2 \theta) + 2Q_m aM \cos \theta]}{2(M^2 + a^2 \cos^2 \theta)Ma}. \] (10)

According to [19], the asymptotic symmetries of near-horizon geometry can be worked out for some suitable boundary conditions. All asymptotic Killing vectors form an algebra and can be expressed as linear combinations of the following bases:
\[ \xi_n = \epsilon_n(\phi) \partial_\phi - r \epsilon_n^r(\phi) \partial_r, \] (11)

with mode \( \epsilon_n(\phi) = e^{-i n \phi} \). Considering the charge realization of the above algebra [19], one can obtain a Virasoro algebra with a central charge
\[ c = 12Ma = 12J. \] (12)

Besides, the asymptotic symmetries of KN spacetime also include a \( U(1) \) gauge transformation. However, it has no contribution for the central charge.

Following [5], the Frolov–Thorne vacuum for the extreme KN spacetime was adopted. The quantum fields can be expanded in eigenmodes with energy \( \omega \) and angular momentum \( m \).
\[ e^{-i\omega \hat{t} + i\omega \hat{\phi}}. \] Here, \( \hat{t}, \hat{\phi} \) are the Boyer–Lindquist coordinates. In Poincare-type coordinates (9), we obtain

\[ e^{-i\omega \hat{t} + i\omega \hat{\phi}} = e^{-i\omega \hat{t} + i\omega \hat{\phi}}. \] (13)

Comparing the coefficients in (13), we obtain the left and right charges associated with \( \partial_\phi \) and \( \partial_\tau \) in the near-horizon region [12]:

\[ n_L = m, \quad n_R = \frac{(r_h^2 + a^2)\omega - am}{\epsilon r_h}. \] (14)

The Boltzmann factor observed by a canonical observer can be reexpressed as

\[ e^{-\omega \hat{t} + \omega \hat{\phi}} = e^{-\frac{n_L}{\pi} - \frac{n_R}{\pi}}. \] (15)

So the dimensionless left and right temperatures are

\[ T_L = \frac{T_H}{a(M^2 + a^2)^{-1} - \Omega_H}, \quad T_R = \frac{(M^2 + a^2)T_H}{\epsilon M} \] (16)

with \( T_H \) given by (5). In the extreme limit, we obtain

\[ T_L = \frac{M^2 + a^2}{4\pi M}, \quad T_R = 0. \] (17)

This means that the quantum fields outside the horizon are thermally distributed with temperature \( T_L \), not in a pure state [5].

According to the Cardy formula, the microscopic entropy of the dual CFT to extreme IH is

\[ S_{micro} = \frac{\pi^2}{3} c T_L = \pi (M^2 + a^2). \] (18)

This is just the same as the macroscopic Bekenstein–Hawking entropy (7) of extremal KN black hole.

Now we have reviewed the properties of KN. In the following sections, we will prove that the near-horizon geometry of electrovac extreme isolated horizons is the same as that of the extreme KN. Then the calculation of microscopic origin of the Bekenstein–Hawking entropy of electrovac extreme IHs can be reduced to that of extreme KN.

3. The near-horizon geometry of extreme isolated horizons

In the last section, we briefly review the Kerr/CFT correspondence. It is clear that the near-horizon limit of spacetime geometry plays an important role in such correspondence. For stationary black holes, the classification of near-horizon limit has been reviewed in [27]. In this section, in order to generalize the Kerr/CFT correspondence to more general cases, we consider the near-horizon limit of spacetime which contains Ashtekar’s isolated horizon [22]. It has been proved that all the stationary horizon belongs to IH, including the Schwarzschild and Kerr horizon. It also contains many nonstationary cases [23, 24].

The definition of isolated horizon can be found in [22]. Roughly speaking, IH is a null hypersurface \( \Delta \) with a pair \((h, D)\), where \( h \) is the induced metric tensor and \( D \) the induced derivative operator. By definition, the generator \( l \) of \( \Delta \) is shear free because of the Raychaudhuri equation and \( h_{ab} \) and \( D \) are preserved by \( l \). There also exists a rotation 1-form potential \( \omega_a \) satisfying \( D_a \epsilon^b = \omega_a \epsilon^b \). (Here ‘\( \equiv \)’ means equality holds only on the horizon \( \Delta \).) By definition, \( \omega_a \) satisfies

\[ L_l \omega_a \equiv 0, \] (19)

i.e., \( \omega_a \) is time independent.
Now we choose the Bondi-like coordinates to describe the near-horizon geometry of IH [22, 28, 29]. In this coordinate system, we have a complex null tetrad \([n, l, m, \bar{m}]\) which could be expanded as
\[
\begin{align*}
n &= \partial_r, \\
l &= \partial_t + U \partial_r + X \partial_\vartheta + \bar{X} \partial_{\bar{\vartheta}}, \\
m &= W \partial_r + \xi \partial_\vartheta + \xi \partial_{\bar{\vartheta}}, \\
\bar{m} &= \bar{W} \partial_r + \bar{\xi} \partial_\vartheta + \bar{\xi} \partial_{\bar{\vartheta}}.
\end{align*}
\]
(20)
The vectors \(l, m, \bar{m}\) span the tangent space to \(\Delta\), and \(l\) is the generator of \(\Delta\). The metric could be written as
\[
g^{ab} = -f^n b^n - a^n l^n + m^a \bar{m}^b + \bar{m}^b m^a.
\]
(21)
To obtain the near-horizon limit of IH, the near-horizon behavior of the metric in (21) should be worked out. We will complete this work by using the Newman–Penrose (NP) formalism [30]. Here, the Bondi gauge \(\nabla_n (n, l, m, \bar{m}) = 0\) is chosen, which means that the tetrad is parallel transported along \(n\). The connection between the unknown functions in (20) and the spin coefficients in NP formalism will be built first.

Because \((l, m, \bar{m})\)|\(\Delta\) are tangent vectors on \(\Delta\), from equation (20), it means
\[
U \hat{=} X \hat{=} W \hat{=} 0.
\]
(22)
Under some suitable rotation, \([m^n, \bar{m}^n]\) could always be chosen, such that \(\mathcal{L}_m m^n \hat{=} 0\). The chosen gauge then imply that the spin coefficients satisfy
\[
\tau = \nu = \gamma = \alpha + \bar{\beta} - \pi = \mu - \bar{\mu} = 0.
\]
(23)
The Raychaudhuri equations and the definition of IH lead to [29]
\[
\Psi_0 \hat{=} 0, \quad \Psi_1 \hat{=} 0, \quad \rho \hat{=} 0, \quad \sigma \hat{=} 0.
\]
(24)
In the existence of electromagnetic field, Raychaudhuri equations also give
\[
F_{lm} \hat{=} 0.
\]
(25)
In terms of NP notations, the above result can be expressed as \(\Phi_0 \hat{=} 0\). This leads to \(\Phi_{10} \hat{=} 0\) because of the Einstein equation and the definition of energy–momentum tensor of the Maxwell field.

Based on the results of [29], the extreme condition of IH implies
\[
\epsilon \hat{=} 0.
\]
(26)
After these analyses, the near-horizon metric and electromagnetic field can be solved. The first Cartan structure equations lead to [20]
\[
\begin{align*}
\partial_r W &= \tilde{\pi} - \mu W - \tilde{\lambda} W \hat{=} \tilde{\pi}, \\
\partial_r U &= (\epsilon + \bar{\epsilon}) - \pi W - \bar{\pi} W \hat{=} 0, \\
\partial_r X &= -\pi \xi - \bar{\pi} \bar{\xi}, \\
\partial_r \xi &= -\mu \xi - \bar{\lambda} \bar{\xi}, \\
\partial_r \bar{\xi} &= -\mu \bar{\xi} - \bar{\lambda} \xi.
\end{align*}
\]
(27)
To obtain the second-order derivative of function \(U\), we need the first-order derivative of connection coefficient \(\epsilon\). This can be achieved by using the second Cartan structure equations, which give
\[
-\partial_r (\epsilon + \bar{\epsilon}) = 2|\pi|^2 + 2\text{Re}(\Psi_2) - \frac{R}{12} + 2\Phi_{11}.
\]
(28)
\(^6\) The notations adopted here are in accord with [30]. For example, \(-\tau = \mathcal{L}_l m^l n^n, \pi = n^\rho \rho m^n l^n, \Psi_0 = C_{abcd} m^a l^b n^c m^d, \Phi_{10} = \frac{1}{4} R_{12}, \text{ etc.}\)
Since we consider the Einstein–Maxwell system, we know the scalar curvature vanishes. The above equation also contains the energy–momentum tensor \( \Phi_{11} \) of the Maxwell field, so one has to consider the near-horizon Maxwell field. Based on the NP formulism of Maxwell equations [30] and previous analysis, the Maxwell equations on the horizon are

\[
\begin{align*}
\partial_t \Phi_1 &\equiv 0, \\
\partial_t \Phi_2 &\equiv \delta \Phi_1 + 2\pi \Phi_1, \\
\nabla_m \Phi_1 - \partial_t \Phi_0 &\equiv 0, \\
\nabla_m \Phi_2 - \partial_t \Phi_1 &\equiv 2\beta \Phi_2.
\end{align*}
\]

To work out the unknown functions in (20), we need the near-horizon behavior of the Ricci tensor component \( \Phi_{11} \). It is given by \( \Phi_{11} = 2|\Phi_1|^2 \) where \( \Phi_1 = \frac{1}{4} F_{ab}(l_n n^b + m^m m^b) \) according to [30]. On the other hand, we know the definitions of the electric charge \( Q_e \) and magnetic charge \( Q_m \) of a black hole:

\[
\begin{align*}
Q_e &= \frac{1}{4\pi} \int_{\Delta} *F = \frac{1}{2\pi} \int F_{ln} = \frac{1}{\pi} \int \text{Re}(\Phi_1), \quad (30) \\
Q_m &= \frac{1}{4\pi} \int_{\Delta} F = \frac{1}{2\pi} \int F_{lm} = \frac{1}{\pi} \int \text{Im}(\Phi_1), \quad (31)
\end{align*}
\]

in which \( \Delta \) is a two-dimensional space section surrounding the black hole, and \( * \) means restricting a function on \( \Delta \). Since \( Q_e \) and \( Q_m \) are well defined, \( \Phi_1 \) and \( \Phi_{11} \) are regular on the horizon.

Thus, we obtain

\[
\begin{align*}
U &= -[2|\hat{\pi}|^2 + \text{Re}(\hat{\Psi}_2) + \hat{\Phi}_{11}] r^2 + O(r^3), \\
W &= \hat{\pi} r + O(r^2), \\
X &= -\left(\hat{\pi} \hat{\xi} + \hat{\xi} \hat{\pi}\right) r + O(r^2), \quad (32)
\end{align*}
\]

In order to consider the near-horizon limit, we need the time-dependent relations of \( U, X \) and \( W \) further. The commutative relation of \( l \) and \( m \) gives \( \partial_l \xi = \partial_m \xi = 0 \). Recall the IH condition (19), which means \( \partial_l \pi = 0 \), then combining the Bianchi identity and equation (24), we obtain \( \partial_l \Psi_2 = 0 \). Combining the above results with equation (32), we know that the leading order terms of \( U, W, X, \xi, \xi \) in (20) are all time independent.

In the Einstein–Maxwell case, following the method of [19], we also need to consider the near-horizon limit of the gauge potential \( A \). Because \( A \) has gauge freedom, by solving Lorentz gauge condition and the relation between \( A \) and \( F_{ab} \), one can show that there exists a gauge choice, such that the following equation holds:

\[
\begin{align*}
A_t &\equiv 0, \\
A_r &\equiv 0, \\
\hat{D}^2 \hat{A} &\equiv \hat{D} \cdot \hat{F} - \hat{d} F_{ln}, \quad (33)
\end{align*}
\]

where \( \hat{A} \) is the tangent part of \( A \) on the section of horizon, \( \hat{F} \) is the tangent part of \( F_{ab} \) on the section of horizon, \( \hat{D} \) is the induced connection on the section of horizon and \( \hat{D}^2 \) is the associated Laplace operator. It is clear that the third equation is a Laplace equation on \( \hat{D}^2 \) so \( \hat{A} \) is fixed by the intrinsic horizon data \( \Phi_1 \) uniquely. From the condition \( F_{lm} = 0 \), one can also find \( \partial_t A_m = 0 \). Since \( \Phi_1 = F_{ln} + F_{mmb} \) belongs to intrinsic data of horizon, one can obtain the first derivative of \( A_t \) on horizon as \( \partial_t A_t \equiv F_{ln} = \text{Re}(\Phi_1) \). From Maxwell equation (29), we
also know that $\delta_\epsilon\partial_\epsilon A_\epsilon = 0$ is also time independent. So we obtain that the leading terms of $A$ are all time independent.

Now the inverse metric could be written as

$$g^{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & f_1 r^2 + O(r^3) & f_2 r + O(r^2) & \tilde{f}_2 r + O(r^2) \\ 0 & f_2 r + O(r^2) & 2\xi & \bar{\xi} \\ 0 & \bar{\xi} & |\xi|^2 + |\bar{\xi}|^2 & 2\xi \end{pmatrix}, \quad (34)$$

where

$$f_1 = 6|\pi|^2 + 2\Re \Psi_2 + 2\Phi_1, \quad f_2 = 2(\pi \xi + \bar{\pi} \bar{\xi}). \quad (35)$$

They are only the functions of the coordinate $\theta$. In NP tetrad, $l$ and $n$ are future pointing. The outside region of horizon corresponds to the region $r < 0$. We take a transformation $r \to -r$ for later convenience. The metric becomes

$$g_{\mu\nu} = \begin{pmatrix} (h_{ab}f_2^a f_2^b - f_1) r^2 + O(r^3) & -1 & -h_{ab}f_2^a r + O(r^2) \\ -1 & 0 & 0 \\ -h_{ab}f_2^a r + O(r^2) & 0 & h_{ab} \end{pmatrix} \quad (36)$$

with

$$f_2^a = \pi m^a + \bar{\pi} \bar{m}^a, \quad (37)$$

$$h_{ab} = m_a m_b + m_b m_a. \quad (38)$$

Here, $a, b = \partial, \bar{\partial}$. $h_{ab}$ is the intrinsic metric of section $\Delta$.

By introducing a coordinate rescaling $r \to \epsilon \hat{r}$, $t \to \frac{t}{\epsilon}$ and taking the near-horizon limit $\epsilon \to 0$, we obtain the near-horizon metric and gauge potential of extreme electrovac IH:

$$\begin{align*}
\text{d}x^2 &= (-f_1 + h_{ab}f_2^a f_2^b) r^2 \text{d}\hat{r}^2 - 2 \hat{r} \text{d}r - 2h_{ab}f_2^a r \text{d}x^b \text{d}r + h_{ab} \text{d}x^a \text{d}x^b. \\
A &= -\text{Re}(\hat{\Phi}_1) r \text{d}\hat{r} + \tilde{A}_\hat{\partial} \text{d}\hat{\partial} + \tilde{A}_\partial \text{d}\partial.
\end{align*} \quad (39)$$

Here the tilde was omitted for conveniences.

Based on the analysis of this section, an important fact is that the near-horizon limit of the metric and the gauge potential only depends on the intrinsic horizon data of IH, i.e., $(h, D)$ and $\hat{\Phi}_1$. In other words, any two spacetimes that each contains an extreme IH will have the same near-horizon limit if they share the same intrinsic horizon data.

4. The electrovac extreme IH/CFT correspondence

In the last section, the general near-horizon metric and gauge field of extreme electrovac IH have been derived. In this section, we will work out the explicit form of the metric of axisymmetric electrovac extreme IHs and compare it with that of the extreme KN spacetime.

It has been proved that all the axisymmetric electrovac extreme isolated horizons coincide with the extreme KN event horizon [25]. For axisymmetric extreme IH, there is a vector field $(\partial_\phi)^a$ that generates the proper symmetry group $O(2)$ of the IH. Instead of the auxiliary coordinate $\theta$, Lewandowski and Pawlowski introduced a function $x$. It is defined by $2\epsilon_{ab}(\partial_\phi)^b = 2(\text{d}x)_a$. The coordinate $x$ is in region $[-\frac{A}{8\pi}, \frac{A}{8\pi}]$ where $A$ is equal to the area of $\Delta$ and a constant. In coordinate $(\phi, x)$, the null 2-frame on $\Delta$ takes the following form:

$$m = \frac{1}{2} \left( \frac{1}{P} \partial_\phi + i P \partial_\phi \right). \quad (39)$$
The 1-form $\omega_a$ can be decomposed with two functions $U$ and $B$ which are globally defined on $\hat{\Sigma}$:
\begin{equation}
\omega = \star dU + d\ln B.
\end{equation}
Here $\star$ stands for the intrinsic Hodge dual on section $\hat{\Sigma}$. In NP notations, we have
\begin{equation}
\omega_a = -(\varepsilon + \bar{\varepsilon}) n_a + (\alpha + \bar{\beta}) \bar{m}_a + (\bar{\alpha} + \beta) m_a.
\end{equation}
So, the spin coefficient $\pi$ becomes, for extreme IH,
\begin{equation}
\pi = -i \delta U + \delta \ln B.
\end{equation}

In terms of $U$, $B$ and $P$, based on the result of Lewandowski and Pawlowski [25], the solution of axisymmetric electrovac extremal IH could be expressed by three real parameters $A$, $\alpha$ and $\theta_0$:
\begin{align*}
P^2 &= \frac{4\pi (1 + \alpha^2)}{A} \frac{1 + b^2 x^2}{1 - \left(\frac{8\pi}{A} x\right)^2}, \\
U &= \pm \arctan(bx), \\
B &= (1 + b^2 x^2)^{1/2}, \\
\Phi_1 &= e^{i\theta_0} \sqrt{\frac{2\pi}{A 1 + \alpha^2 (1 \pm ibx)^2}}.
\end{align*}
Here $b^2 = \frac{\frac{A}{1 + \alpha^2} - \alpha^2}{A}$, $\alpha \in [0, 1]$, $\theta_0 \in [0, 2\pi)$. Sign $\pm$ in $U$ indicates the rotating direction of the black hole. Sign $\pm$ in $\Phi_1$ can be absorbed by the electric and magnetic charge. It has no substantial influence on the final result. For convenience, we take $U = -\arctan(bx)$ and $\Phi_1 = e^{i\theta_0} \sqrt{\frac{2\pi}{A 1 + \alpha^2 (1 \pm ibx)^2}}$.

By definition, the electric and magnetic charges are given by the real and imaginary part of the integral, respectively:
\begin{equation}
\frac{1}{4\pi} \int_{\hat{\Sigma}} * F + i F = e^{i\theta_0} \alpha \sqrt{\frac{A}{4\pi}}.
\end{equation}
Thus, $Q_e = Q \cos \theta_0$ and $Q_m = Q \sin \theta_0$ where $a^2 = \frac{A}{4\pi}$.

Now, we could work out the parameters in (35). From (42) and (43), we have
\begin{equation}
\pi = -i \frac{1}{2P} \frac{b}{1 + b^2 x^2} + \frac{1}{2P} \frac{b^2 x}{1 + b^2 x^2}.
\end{equation}
Combining with (37), there are
\begin{align*}
h_{ab} f^a_i \, dx^b &= \frac{2b^2 x}{1 + b^2 x^2} \, dx - \frac{1}{P^2} \frac{2b}{1 + b^2 x^2} \, d\phi, \\
h_{ab} f^b_i \, dx^a &= \frac{2b^2}{P^2 (1 + b^2 x^2)}.
\end{align*}
We come to $f_1$. For electrovac IH, $R = 0$. The NP equations and Einstein–Maxwell equation lead to
\begin{equation}
f_1 = 6 |\pi|^2 - K + 8 |\Phi_1|^2.
\end{equation}
Here, the Gaussian curvature $K$ of $\hat{\Sigma}$ is
\begin{equation}
K = \frac{16\pi}{A(1 + \alpha^2)^2 (1 + b^2 x^2)^3}.
\end{equation}
Plugging (43), (45), (48) into (47), we finally obtain

$$f_1 = \frac{3Ab^2}{8\pi(1 + \alpha^2)} \frac{1 - \left(\frac{2\pi}{\alpha} x\right)^2}{(1 + b^2 x^2)^2} - \frac{16\pi}{A(1 + \alpha^2)^2} \frac{1 - 3b^2 x^2}{(1 + b^2 x^2)^3} + \frac{8\pi}{A} \left(\frac{2\alpha}{1 + \alpha^2}\right)^2 \frac{1}{(1 + b^2 x^2)^2}.$$

(49)

The metric could be reorganized as

$$ds^2 = -f_1 r^2 \, dt^2 - 2 \, dt \, dr + 2p^2 \left(dx - \frac{1}{p^2} \frac{b^2 x}{1 + b^2 x^2} r \, dt\right)^2 + \frac{2}{p^2} \left(d\phi + \frac{b}{1 + b^2 x^2} r \, dt\right)^2.$$

(50)

Let us turn to the electromagnetic field. From (43), we obtain the gauge field strength on $\Delta$:

$$F_{\rho\tau} = \frac{4\pi}{A(1 + \alpha^2)} \frac{Q_e (1 - b^2 x^2) + 2Q_{ab} bx}{(1 + b^2 x^2)^2},$$

$$F_{\phi\rho} = \frac{8\pi}{A(1 + \alpha^2)} \frac{Q_{ab} (1 - b^2 x^2) - 2Q_e bx}{(1 + b^2 x^2)^2}.$$

(51)

Since the gauge field is time independent and rotationally symmetric, the components of gauge potential $A$ are only functions of $x, r$. On the other hand, gauge condition $A_t = 0$ could always be chosen. $A_r$ is only the functions of $x$ on $\Delta$ which could also be gauge fixed to $A_r = 0$. So, we obtain

$$A_r = 0, \quad A_t = 0$$

$$A_r = \frac{4\pi}{A(1 + \alpha^2)} \frac{Q_e (1 - b^2 x^2) + 2Q_{ab} bx}{(1 + b^2 x^2)^2} + O(r^2),$$

$$A_{\phi} = \frac{4\pi}{Ab(1 + \alpha^2)} \frac{Q_e (1 - b^2 x^2) + 2Q_{ab} bx}{1 + b^2 x^2} + O(r).$$

(52)

By introducing a coordinate rescaling $r \rightarrow \epsilon \tilde{r}, t \rightarrow \frac{t}{\epsilon}$ and taking the near-horizon limit $\epsilon \rightarrow 0$, we obtain the near-horizon geometry and gauge field of extreme electrovac IH. They have the same form as (50) and (52) if the tilde was omitted.

Now, the metric of near-horizon extreme IH is determined by two parameters: the horizon area $A$ and charge $Q$. The metric of extreme KN is also determined by two parameters: the mass $M$ and angular momentum $a$. For extreme KN, $M^2 = a^2 + Q^2$ and the area of event horizon $A = 4\pi(M^2 + a^2)$. Thus, the extreme KN could also be described by the horizon area $A$ and the electric charge $Q$. Based on the uniqueness theorem, the near-horizon geometry of extreme IH and extreme KN should be the same. In fact, if the coordinate $x = \frac{\sqrt{2}}{\pi} \cos \theta$ was introduced in extreme KN, and the two parameters $M$ and $a$ were replaced by $A = 4\pi(M^2 + a^2)$ and $a^2 = \frac{\sqrt{2}}{\pi} Q = \frac{M^2 - a^2}{2\pi^2}$, we will find that the near-horizon limit metric of extreme KN is exactly the same as that of extreme IH. To make this clearer, we proceed to make a coordinate transformation for line element (9):

$$\rho = \frac{M^2 + a^2 \cos^2 \theta}{M^2 + a^2} r,$$

$$v = (M^2 + a^2) \left(t + \frac{1}{r}\right),$$

$$\phi = \varphi + \frac{2Ma}{M^2 + a^2} \ln r.$$
Then the metric of near-horizon limit extreme KN becomes

\[
ds^2 = -\frac{(M^2 + a^2 \cos^2 \theta)^2 + 4a^4 \cos^2 \theta \sin^2 \theta + 2a^2 \cos^2 \theta \rho^2 d\nu^2}{(M^2 + a^2 \cos^2 \theta)^3} - 2d\nu d\rho
\]

\[
+ (M^2 + a^2 \cos^2 \theta) \left( d\theta + \frac{2a^2 \cos \theta \sin \theta}{(M^2 + a^2 \cos^2 \theta)^2} \rho d\nu \right)^2
\]

\[
+ \frac{(M^2 + a^2 \cos^2 \theta) \sin^2 \theta}{M^2 + a^2 \cos^2 \theta} \left( d\phi + \frac{2Ma}{(M^2 + a^2)(M^2 + a^2 \cos^2 \theta)} \rho d\nu \right)^2.
\]

This is the Bondi coordinate for KN. It is easy to shown that

\[
\frac{(M^2 + a^2 \cos^2 \theta)^2 + 4a^4 \cos^2 \theta \sin^2 \theta + 2a^2 \cos^2 \theta \rho^2 d\nu^2}{(M^2 + a^2 \cos^2 \theta)^3} = f_1,
\]

\[
\frac{(M^2 + a^2 \cos^2 \theta) \sin^2 \theta}{M^2 + a^2 \cos^2 \theta} = \frac{2}{P^2}.
\]

Thus, the near-horizon metric of extreme (54) is the same as that of extreme IH (50).

Besides, the gauge field (10) under coordinate transformation (53) becomes

\[
A = \frac{k\rho}{M^2 + a^2 \cos^2 \theta} d\nu,
\]

which is exactly the same as (52). Here \( k = \frac{2Ma}{M^2 + a^2 \cos^2 \theta} \).

So, it has been proved that the near-horizon limit of a general electrovac axisymmetric extreme IH is exactly the same as that of the extreme KN black hole. According to the construction of the Bondi-like coordinate, \( \partial_t \) is an asymptotic Killing vector of IH up to some terms of high order. This enables us to take Fourier transformation to define the Frolov temperature, which is similar to the case of Kerr spacetime. It has been shown that IHs have the thermodynamic properties similar to traditional black hole horizons. Hawking radiation is emitted by IHs as well. So, IHs also have an entropy. On the other hand, the correspondence between KN-AdS and CFT was uncovered in [19]. Since we have shown that the near-horizon metric and gauge field of electrovac extreme IHs are the same as those of extreme KN, the electrovac extreme IH/CFT reduces to the case of KN-AdS/CFT by vanishing the cosmological constant, as we have presented in section 2. Thus, the electrovac extreme IHs are holographically dual to a chiral two-dimensional CFT with central charge \( c = 12J \). The macroscopic Bekenstein–Hawking entropy of electrovac extreme IHs can be reproduced by computing the microscopic entropy from the Cardy formula for the two-dimensional CFT.

5. Summary and discussions

The Kerr/CFT correspondence is generalized to the electrovac axisymmetric extreme IH/CFT correspondence. The macroscopic Bekenstein–Hawking entropy of extreme IH is reproduced by computing the microscopic entropy for the dual two-dimensional conformal field theory.

We first reviewed the near-horizon geometry of KN spacetime, then derived the general near-horizon limit of spacetimes containing IHs. It was shown that, in the near horizon limit, any two spacetimes each contains an extreme IH will have the same near-horizon structure if they have the same intrinsic horizon data. For axisymmetric electrovac extreme IH, the uniqueness theorem states that it coincides with the extreme KN event horizon under the near-horizon limit. We built the near-horizon limit of spacetimes containing IHs explicitly and found that it is exactly the same as the near-horizon limit of KN spacetime after a coordinate transformation between the Bondi-like coordinates and Poincare-type coordinates, where the
explicit form of this coordinate transformation is found. Then, the Kerr/CFT correspondence was generalized to the electrovac extreme IH/CFT correspondence.

As stressed in section 3, the IH geometry is intrinsically determined. This implies that the microscopic origin of IH entropy is determined by the intrinsic geometry. We expect that further study on this aspect will more clearly outline a geometric picture of the Kerr/CFT correspondence.

To be even more general, the electrovac spacetimes containing IHs with cosmological constant can be considered, whose near-horizon geometry is expected to be exactly the same as that of the KN-(A)dS spacetime. In order to do that, however, one needs to generalize Lewandowski and Pawlowski’s uniqueness theorem to the case with cosmological constant, which will be left for future works.

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