Simultaneous driving of semiconductor spin qubits at the fault-tolerant threshold

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The promise of quantum information technology hinges on the ability to control large numbers of qubits with high-fidelity. Quantum dots define a promising platform due to their compatibility with semiconductor manufacturing. Moreover, high-fidelity operations above 99.9% have been realized with individual qubits [1–3], though their performance has been limited to 98.67% when driving two qubits simultaneously [4]. Here we present single-qubit randomized benchmarking in a two-dimensional array of spin qubits, for one, two and four simultaneously driven qubits. We find that by carefully tuning the qubit parameters, we achieve native gate fidelities of 99.9899(4)% and 99.904(4)% respectively. We also find that cross talk with next-nearest neighbour pairs induces errors that can be imperceptible within the error margin, indicating that cross talk can be highly local. These characterizations of the single-qubit gate quality and the ability to operate simultaneously are crucial aspects for scaling up germanium based quantum information technology.

INTRODUCTION

Quantum computing with spin qubits has made remarkable progress in the last few years. In both silicon and germanium [5, 6], two-dimensional approaches to scale up quantum dots have been the focus of multiple recent efforts [7–8], as they provide greater qubit connectivity and enable the use of surface code architectures for quantum error correction [9, 10]. For electrons in silicon, scaling qubits in the second dimension is challenging due to the need for components such as striplines and nanomagnets to enable qubit driving, and qubit realizations have been limited to linear arrays [11–13]. Instead, spin qubits based on holes in germanium can be driven all-electrically through the intrinsic strong spin-orbit coupling [14–16]. Furthermore, advances in strained germanium (Ge/SiGe) have yielded low charge noise and percolation density [17] and high hole mobility [18], indicative of a highly uniform platform. These advantages have advanced the Ge/SiGe platform rapidly over the last few years and led to demonstrations of long spin relaxation times [19], single hole qubits and singlet triplet qubits [16, 20] and universal operation on a 2x2 qubit array [3].

However, as spin qubits expand into two dimensions, the growing number of possible qubit cross talk interactions motivates careful characterization [21]. In the present work, we make use of a 2x2 quantum dot array of hole spin qubits, to characterize the single-qubit fidelities of the system, when driving one, two and four qubits simultaneously. We perform randomized benchmarking in the single-qubit Clifford space, and investigate the dependence of fidelity on qubit Rabi period, finding that single-qubit fidelities can be as high as 99.9899(4)%. We then investigate the individual single-qubit performance while benchmarking two and four qubits simultaneously and show elementary gate fidelities of $F_{2Q}^{\pi/2} = 99.904(4)\%$ and $F_{4Q}^{\pi/2} = 99.00(4)\%$ respectively. We compare these experiments at two magnetic fields $B = 1\, \text{T}$ and $B = 0.65\, \text{T}$ and find that while individual qubit operation is best at the lower magnetic field, simultaneous-qubit operation performs better at a higher magnetic field, due to the relevance of qubit addressability and qubit cross-talk. Finally we explore avenues for further improvement of qubit performance with respect to controlling the exchange interaction, preferred driving configurations, magnetic field amplitude and direction, and isotopic enrichment.

RESULTS

Figure 1a shows a false-coloured scanning electron microscope (SEM) image of the device used in the experiment [3]. It consists of two gate layers and an ohmic layer, where ohmic contacts to the quantum well are created by diffused Al [22]. By applying potentials to the plunger gates $P_1$–$P_4$, we can define four quantum dots, each filled with a single hole spin such that we operate in the $(1,1,1,1)$ charge regime (see supplementary information section IIa). The tunnel couplings between these quantum dots can be tuned with the dedicated interdot barrier gates $B_1$–$B_4$. We also define two larger quantum dots using $P_{512}$ and $P_{314}$ in the multiple hole regime, which we utilize as charge sensors. By applying an rf tone to the ohmic gates $O_1$ and $O_3$ via two off-chip inductors bonded in-line, we form a resonant tank circuit allowing to perform fast rf charge sensing. To read out the spin states, we perform spin-to-charge conversion in the form of Pauli spin blockade (PSB). Figure 1b depicts the two PSB readout pairs in the system, with the Q1Q2 system comprising of the two qubits Q1 and Q2 (orange, yellow), and the Q3Q4 system containing qubits Q3 and Q4 (purple, green). We make use of a latched readout mechanism [3, 23], whereby the dot-reservoir tunnel rate is limited significantly for one quantum dot per readout pair, which is depicted by a dashed arrow. We tune the tunnel rates of quantum dots Q2 and Q4 to be $\Gamma_{Q2} = 5\, \text{kHz}$ and $\Gamma_{Q4} = 0.416\, \text{kHz}$ respectively, and read out in an integration time of $10\, \mu\text{s}$ such that we are well within the dot-reservoir limiting timescales. We apply an in-plane magnetic field $B_{\text{ext}}$ to split the spin states. Two different magnetic fields were used in this experiment, of $B_{\text{ext}} = 1\, \text{T}$ and $B_{\text{ext}} = 0.65\, \text{T}$ (Fig. 1c). These fields are chosen to provide a comparison between the different regimes of coherence, qubit response, and qubit resonance frequency spacing.
Figures 2i-l show the randomized benchmarking data for the optimal Rabi periods. We extract generator fidelities above 99% for each qubit in the array, with qubit 3 performing the best with $F_{Q3}^{\pi/2} = 99.9899(4)\%$. For single-qubit randomized benchmarking, we expect a fully decohered state to exhibit a read out spin down probability of about 1/2. However, rapid qubit manipulation is not always optimal for coherent qubit control, with high powers leading to enhanced systematic errors in qubit operation arising from effects such as sample heating or pulse imperfections. Indeed we find a strong dependence of the single-qubit fidelities on the drive speed. Figures 2e-h show the generator infidelities $(1-F_{Qj}^{\pi/2})$ as a function of qubit drive speed. Despite being able to drive qubit rotations in as fast as 10 ns, we find that the associated single-qubit fidelity suffers as a result, visible by a sharp decrease in the fidelity for qubits Q1 and Q3. Fidelity in these cases could be limited by a number of mechanisms, such as quantum dot anharmonicities [28, 29] or systematic Pauli errors due to gate tuning. We also observe a non-linear response of the Rabi period to the applied microwave power, at high powers where the single-qubit fidelity is observed to be lower. From the analysis we conclude that driving in a linear regime yields better gate quality (See supplementary information section III).

In order to get a baseline reading of the individual qubit fidelities, we performed randomized benchmarking to estimate the quality of each qubit. Randomized benchmarking provides the average fidelity of a gate set applied to each qubit. Operations randomly selected from the Clifford group are applied to each single-qubit initialised in a known state. A final recovery Clifford $C^{-1}$ is applied to bring the qubit back to its original state. Imperfections in the applied gates and gradual qubit decoherence result in a decay of the recovered state probability as the number of applied Clifford operators is increased, allowing the extraction of a fidelity by fitting the decay [24]. Each element of the Clifford group can be constructed from a variety of generator gates. We construct a Clifford group from a minimal generator set $G_i \in \{X_{\pi/2}, Y_{\pi/2}\}$. We find this set advantageous since it contains on average 3.217 qubit $\pi/2$ rotations (generators) per Clifford, that differ only by a software phase shift. This means the estimated Clifford fidelity is a direct indicator of the generator fidelity, by equally weighting the generators of the same length [25].

Figures 2a-d show the randomized benchmarking sequences for Q1-4 respectively. A red (blue) measurement window indicates PSB readout on the Q1Q2 (Q3Q4) double quantum dot pair. Each qubit is initialised in the spin down state. For each sequence length N, 32 random permutations of N Clifords are averaged to give the final trace, each of which is composed of 1500 single shot measurements. An exponential decay is fit to the resulting trace (see methods), from which an average generator fidelity $F_{Qj}^{\pi/2}$ can be extracted for each qubit.

Holes in germanium allow for very fast electrical driving, with Rabi frequencies exceeding hundreds of MHz [26, 27]. However, rapid qubit manipulation is not always optimal for coherent qubit control, with high powers leading to enhanced systematic errors in qubit operation arising from effects such as sample heating or pulse imperfections. Indeed we find a strong dependence of the single-qubit fidelities on the drive speed. Figures 2e-h show the generator infidelities $(1-F_{Qj}^{\pi/2})$ as a function of qubit drive speed. Despite being able to drive qubit rotations in as fast as 10 ns, we find that the associated single-qubit fidelity suffers as a result, visible by a sharp decrease in the fidelity for qubits Q1 and Q3. Fidelity in these cases could be limited by a number of mechanisms, such as quantum dot anharmonicities [28, 29] or systematic Pauli errors due to gate tuning. We also observe a non-linear response of the Rabi period to the applied microwave power, at high powers where the single-qubit fidelity is observed to be lower. From the analysis we conclude that driving in a linear regime yields better gate quality (See supplementary information section III).

Single-Qubit Benchmarking

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Figures 2i-l show the randomized benchmarking data for the optimal Rabi periods. We extract generator fidelities above 99% for each qubit in the array, with qubit 3 performing the best with $F_{Q3}^{\pi/2} = 99.9899(4)\%$. For single-qubit randomized benchmarking, we expect a fully decohered state to exhibit a read out spin down probability of about 1/2. However, in the presence of finite exchange and classical cross-talk between the active qubit and the readout qubit in the spin blockade pair, state leakage can occur to all four states in the two-qubit subspace, resulting in a readout signal of about $P_{\text{Blocked}} = 0.329$ (see Supplementary information section II). We find that for the case of qubit Q4, the plateau of the spin blocked probability approaches the expected value of the fully depolarized two-qubit subspace for all driving powers. This is likely due to the high power required to drive Q4 via plunger gate $P_2$ as a consequence of the larger dis-
Figure 2. single-qubit randomized benchmarking at 0.65 T. (a-d) Random Clifford sequences applied to each qubit. Each qubit in the array is prepared in the spin down state. N-1 randomly selected Cliffords are applied to a single-qubit, after which a recovery Clifford (C−1) is applied bringing the system back to the |↓↓⟩ state. Each sequence is repeated 32 times with different random permutations of Cliffords. Readout occurs via PSB on one of the two readout pairs Q1Q2 (red) or Q3Q4 (blue). (e-h) Dependence of qubit fidelity on Rabi period. An optimal Rabi period occurs due to a trade-off between decoherence (high tR) and errors introduced at low Rabi period including gate calibration errors and driving non-linearities. Error bars reflect the statistical error of the fitting. (i-l) Best single-qubit benchmarks for each qubit. All native π/2 fidelities except Fπ/2Q4 exceed 99.9 %, with Fπ/2Q3 approaching four nines. These traces correspond to the respective highlighted points in (e-h).

tance between qubit and drive-gate, resulting in a large degree of cross-talk on qubit Q3. To account for state leakage, a second exponential decay is added for fitting randomized benchmarking traces for qubit Q4, yielding two characteristic decay constants (see methods). From this analysis, we calculate a qubit fidelity Fπ/2Q4 = 99.88(1)%, and a leakage rate of Lπ/2Q4 = 0.023(12)% per generator.

Simultaneous Driving

The operation of practical quantum computers will require multiple qubits to be controlled simultaneously [30]. Doing so however can affect performance to the detriment of the qubits due to qubit cross-talk. Simultaneous benchmarking [31] is a technique used to characterise the effects of qubit cross-talk in quantum processors with multiple qubits and is routinely implemented on superconducting qubits [32], which share comparable control schemes with spin qubits. These techniques have also recently been performed on electron spin qubits in a two-qubit quantum processor [4]. We now turn to the characteriza-

tion of single-qubit fidelities while driving multiple qubits simultaneously. Here, the same generation procedure described in the single-qubit case is used to create a random sequence of single-qubit Cliffords, which is applied to all qubits in the experiment. The recovery Clifford always brings the system back to the |↓↓↓↓⟩ state. Since all qubits experience the same generators simultaneously, this methodology probes the even parity qubit subspace, and decays to the fully depolarized qubit subspace (see Supplementary Information section V for experiments where we probe the odd parity subspace). Figures 3a-b show a characterization of the single, two and four qubit simultaneous benchmarking at a consistent Rabi period of tR = 96 ns and magnetic field Bext = 0.65 T, where we measured all possible permutations. From this dataset, we can approximate the relative loss in fidelity for each qubit due to the additional driving of another qubit by defining the quantity δi,j/Qi,j = ϵQi,j - ϵQi,i, where ϵQi,i = 1 - FQi,i is the infidelity of qubit Qi and ϵQj,j = 1 - FQi,j is the infidelity of qubit Qi while simultaneously driving qubit Qi (Full numerical data available in supplementary section VI). We observe that qubits driven simultaneously with their nearest neighbours typically result in relatively larger
Two different magnetic field settings were applied in this work to clarify the importance of spin coherence and gate speed on the gate fidelities (see Fig. 1c). At low magnetic fields, decoherence due to charge noise coupling in via the spin-orbit interaction is minimized, but also the Rabi speed is reduced. In order to retain the same Rabi period, the drive power needs to be increased to compensate. This leads to increased qubit cross-talk when driving simultaneously and suggests the existence of an optimal magnetic field strength. Figure 3e summarises the randomized benchmarking results of this work, providing detailed information on the extracted average generator as a function of number of simultaneously driven qubits. On average, the individual single-qubit fidelities are higher at lower field, as are the results for driving two qubits simultaneously. The power required to drive the systems at their optimal Rabi period however, was found to be inversely proportional with B, as expected, and as such the position of these points was also observed to shift to longer Rabi periods. We observed that at the stronger magnetic field of 1 T, higher fidelities can be achieved for the case of four simultaneously driven qubits. We attribute this result to the faster achievable Rabi frequencies as well as larger qubit frequency splittings outweighing the relative loss in coherence. We expect that the magnitude of these classical cross-talk effects can be
DISCUSSION

While the individual and multi-qubit control results shown here define benchmarks for quantum dot qubit systems, we envision several strategies can be followed to further improve the fidelity. Precise control over the exchange interactions between adjacent qubits is extremely important for high fidelity quantum operations. While the overlapping gate structure and tight quantum dot definition [9] used here, have proven essential in silicon and in particular SiMOS devices [11], the small effective mass of holes in germanium [36] increases the exchange interaction significantly. While gate voltage pulsing may be used to turn off the exchange, here we operate in a dynamical mode where we program single and multi-qubit gates [3] as required for universal quantum computation and device stability limits the maximal pulsing amplitude. The low disorder in strained germanium, however, enables to relax the gate pitch and the gate structure may thus be optimized to obtain larger on/off ratios for the exchange interaction.

In the present work, we observe high single-qubit fidelities, using direct (Q2, Q3), nearest neighbour (Q1), and next-nearest neighbour (Q4) plunger driving strategies. For the first two, the relative power required to achieve similar driving speeds is comparable, and result in very high single-qubit fidelities, as well as no discernible state leakage. Driving Q4 via its next-nearest neighbour plunger P2 however requires significantly larger powers to achieve comparable driving speeds, and results in larger qubit cross talk to the direct and nearest neighbour qubits of P2, resulting in clear state leakage. We also observe that the fidelity of Q4 is the lowest of the system. We therefore expect that an important parameter for optimal qubit control is thus the placement of the qubit driving gate. A particularly promising direction may be the engineering of barrier gates that can efficiently modulate the in-plane electric field. The device used here is based on a design where the barrier gates are patterned after the plunger gates and have a reduced pitch, which lead to a limited lever arm of the barrier gates. By widening the barrier gates and by patterning these gates before the plunger gates it may therefore be possible to both enhance the on/off ratio of the exchange coupling as well as to realise efficient qubit driving gates. The operation amplitude and angle of the external magnetic field is an important factor. In this work, we benchmark single-qubit gate fidelities at in-plane external magnetic fields of 0.65 T and 1 T. Spin dephasing times are higher at 0.65 T, however driving speeds and resonance frequency spacings are lower. We observe that trying to drive faster at lower field results in more frequency crowding due to the additional power required. This leads to larger cross-talk effects that ultimately limit fidelities when driving all qubits simultaneously, but allows for the highest single-qubit fidelities when driven sparsely owing to the enhanced coherence. Conversely, at higher field, we observe higher fidelities when driving simultaneously due to lower required powers, and higher frequency spacing, but lower fidelities when driving sparsely owing to the shorter coherence times.

Architectures targeting large-scale quantum computing with germanium can thus benefit from high-fidelity operation and fast and simultaneous quantum control. Pulse shaping techniques to overcome qubit crosstalk may be relevant in a dense qubit arrays, while the locality of the electric field may already be sufficient for sparse qubit arrays. Furthermore, germanium quantum technology offers a rich and diverse platform that can take advantage of integrated superconductivity for long-range links and where standard semiconductor technology may be incorporated [37], providing an exciting pathway for scalable and high-fidelity quantum technology [6].

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COMPETING INTERESTS

The authors declare no competing interests. Correspondence should be addressed to M.V. (M.Veldhorst@tudelft.nl).

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METHODS

A. Device Fabrication

The Ge/SiGe wafer is fabricated on a silicon substrate. We use reduced-pressure chemical vapour deposition to grow a 1.6 μm strain-relaxed Ge layer, a 1 μm reverse graded Si1−xGe x (x varies from 1 to 0.8), a 500 nm constant composition Si0.2Ge0.8, and a 16 nm compressively strained Ge quantum well. Finally on the quantum well we grow a 55 nm Si0.2Ge0.8 barrier followed by an oxidised Si cap layer (<2 nm). An ohmic contact layer is created by first defining it using electron beam lithography, etching away the oxidised cap, then depositing 30 nm of Al. This layer is then covered in 7 nm of Al2O3 via atomic layer deposition at 300 °C. The gate stack is in two overlapping layers of Ti:Pd (3/37 nm), separated by 7 nm of Al.

B. Generation of the Single-Qubit Clifford Set

We quantify the quality of the single-qubit gates on all qubits by performing randomized benchmarking using the Clifford group C1 = {Cn ∈ U1 | CnPCn† = P} with the single-qubit Pauli group P = {I, X, Y, Z}. All 24 elements of the Clifford group are generated from a minimal set, Cn = ∏i∈G gi with G = {Xπ/2, Yπ/2}. The average number of elementary gates per Clifford is 3.217. All 24 Clifford gates are provided in the supplementary material section III. While not the most efficient choice for elementary gates, this particular choice of set leads to uniform Clifford gates, whose elementary gates vary only by a θ = π/2 phase-shift in the MW signal and are identical otherwise, beneficial for comparison and performing simultaneous
benchmarking protocols.

C. Fitting

Standard fitting of randomized benchmarking decays assumes a single exponential decay of the form $P_{\text{Blocked}} = A \times e^{-mx} + c$, where $c$ should be equal to half the visibility between the relevant two qubit subspace. However, for qubit Q4, and for the high power regime of qubits Q1-3, we observe that the blocked signal plateaus to values corresponding to the fully decohered two-qubit subspace, indicating state leakage. In this case, we fit with two exponentials $P_{\text{Blocked}} = A_1 \times e^{-m_1x} + A_2 \times e^{-m_2x} + c$, where $c$ is set to the average signal of the four two-qubit states in the readout pair. Here, $m_1$ and $m_2$ correspond to the two time constants from which a fidelity and a leakage rate can be approximated. For simultaneous driving decays involving qubits from the same readout pair, eg. Q1Q2 or Q3Q4, we fit a single exponential, giving a result that is representative of the average fidelity of both qubits in the system. For figure 3a and figure 3b, we fit a single exponential when the plateau corresponds to a fully depolarized single-qubit subspace, and a double exponential when it decays to a fully depolarized two-qubit subspace. In the latter case, the fidelity is always calculated from the faster of the two exponents.

D. Simultaneous Elementary Gate Tuning

In order to tune the system for the case of two-simultaneously driven qubits, we first determine the non-simultaneously driven resonance frequency and Rabi frequency by applying a $X_{\pi}$ pulse to each qubit separately and perform readout out via PSB. We then tune the Rabi periods $t_{\pi}$ within 1 ns of each other and then set the desired simultaneous Rabi period as the average of the two single-qubit Rabi periods. This constitutes a rough calibration of the qubits. To fine tune, we apply a $X_{2\pi}$ rotation on both qubit simultaneously, and sweep the frequency of both qubits. Fitting a two-dimensional Gaussian then gives the first iteration correction to the two qubits’ resonance frequencies. We then perform the same experiment, but varying the amplitude of the MW power of both qubits. When driving four qubits simultaneously, the tuning procedure is extended to include both systems Q1Q2 and Q3Q4 in the tuning iteration.

DATA AVAILABILITY

All data underlying this study will be made available on the 4TU ResearchData repository.
Supporting Information

I. READOUT

Supporting Figure 1. (a) Charge stability diagram of the 2x2 quantum dot array charge filling. The DC voltage point of the Digital-Analogue converters (DACs) is positioned within the (1,1,1,1) charge filling regime. Anticrossings between the (1,1,1,1)/(0,2,1,1) and (1,1,1,1)/(1,1,0,2) charge transitions are shown in red and blue respectively. We use these anticrossings for spin to charge conversion and thus for readout of the qubit state. The axes \( \epsilon_{12,34} \) and \( U_{12,34} \) are virtual gates describing the combined detuning and energy of the two qubit pairs Q1Q2 and Q3Q4. (b) Readout calibration of the Q1Q2 system. While in the (1,1,1,1) charge state, we prepare the qubit system in the \(| \downarrow\downarrow\downarrow\downarrow >\) state and apply a \( \pi \)-pulse to qubit Q2, bringing the system to the \(| \downarrow\uparrow\downarrow\downarrow >\) state. We then sweep the virtual detuning (V_{\epsilon_{12}}) and on-site energy (V_{U_{12}}) around the (1,1,1,1)/(0,2,1,1) anticrossing. We then repeat the experiment without applying the \( \pi \)-pulse to Q2, and plot the difference of the two measurements, and pick the point of greatest visibility as the readout position. (c-d) Single shot readout calibration. (c) Sensor value plot as a function of virtual sensor plunger gate potential (V_{PS12}) for sensors one (upper, red) and two (lower, blue), for the Q1Q2 system. Qubit Q2 is brought into a superposition state by applying a \( X_{\pi/2} \)-pulse, and the system is pulsed to the readout point from (b). We repeat 1000 single shot measurements, for each value of V_{PS12} and plot the resulting histogram of sensor response. A double-gaussian is fitted to the line cuts to find the sensor response to the blocked and non-blocked signals, and the plunger gate voltage for maximal separation between the gaussian peaks, is set. (d) The same experiment performed at the given V_{PS12}. A double gaussian is fit to the resulting histogram, and the single-shot threshold is calculated as the point of equal separation between the gaussian peaks. (e) Visibility of the two qubit subspace of the Q1Q2 system. Rabi oscillations are performed on Q1 with Q2 prepared either in the down state (red trace) or the up state (yellow trace). There is a small blocked fraction difference between the \(| \downarrow\uparrow >\) and \(| \uparrow\downarrow >\) states, and the \(| \uparrow\uparrow >\) state is less blocked than the \(| \downarrow\uparrow >\). (f-i) The same procedure as (b-e), but pertaining to the Q3Q4 system, (1,1,1,1)/(1,1,0,2) anticrossing, with \( X_\pi \)- and \( X_{\pi/2} \)-pulses on qubit Q4, instead.
II. EFFECTS OF DRIVING POWER

Supporting Figure 2. Resonance frequency dependence at 0.65 T on applied microwave drive power. (a-d) correspond to the resonance frequency of qubits Q1-4 respectively. Power dependence of Rabi period. The lines are guides for the eye to indicate a linear dependence. (e-h) correspond to the extracted times $t_\pi$ to perform a $X_\pi$ rotation of qubits Q1-4 respectively. A deviation from a linear dependence of $t_\pi$ on the square root of power $P^{1/2}$ can be observed in all qubits except qubit Q4 (h), where we have not probed further to see at what point the non-linear regime begins. The deviation in resonance frequency is likely due to an average change in position of the quantum dot due to the electric field modulation during driving, which in combination with a strong Rashba spin orbit interaction, can alter the frequency eigenstates of the qubits.
### III. CLIFFORD GATE SET

Table I. single-qubit Clifford sequences and their composition via the minimal generator set. We benchmark by selecting a random sequence of Cliffords from the table below excluding C₁, and calculate the recovery Clifford that projects the system back into its original state. We only use a gate set containing π/2 rotations around the Bloch Sphere, so the gates Xπ/2 and Yπ/2 are explicitly referring to a rotation of π/2 around the x-axis and y-axis of the Bloch sphere of a single-qubit respectively. A Yπ/2 rotation is just an Xπ/2 rotation with a π/2 software phase correction. There are on average 3.217 generators per Clifford composition. The extracted fidelity then corresponds exactly to the π/2 rotation fidelity.

| Clifford | Composition |
|----------|-------------|
| C₁       | I           |
| C₂       | Yπ/2        |
| C₃       | Xπ/2        |
| C₄       | Yπ/2Xπ/2    |
| C₅       | Xπ/2Yπ/2    |
| C₆       | Yπ/2Yπ/2    |
| C₇       | Xπ/2Xπ/2    |
| C₈       | Yπ/2Xπ/2Xπ/2|
| C₉       | Yπ/2Yπ/2Xπ/2|
| C₁₀      | Yπ/2Xπ/2Yπ/2|
| C₁₁      | Yπ/2Yπ/2Yπ/2|
| C₁₂      | Xπ/2Xπ/2Xπ/2|
| C₁₃      | Xπ/2Xπ/2Yπ/2|
| C₁₄      | Xπ/2Yπ/2Yπ/2|
| C₁₅      | Yπ/2Xπ/2Yπ/2Yπ/2|
| C₁₆      | Yπ/2Yπ/2Xπ/2Yπ/2|
| C₁₇      | Xπ/2Xπ/2Xπ/2Yπ/2|
| C₁₈      | Yπ/2Xπ/2Xπ/2Xπ/2|
| C₁₉      | Yπ/2Yπ/2Xπ/2Xπ/2|
| C₂₀      | Xπ/2Xπ/2Yπ/2Yπ/2|
| C₂₁      | Yπ/2Yπ/2Xπ/2Xπ/2Yπ/2|
| C₂₂      | Xπ/2Yπ/2Yπ/2Xπ/2Yπ/2|
| C₂₃      | Yπ/2Xπ/2Xπ/2Yπ/2Xπ/2Yπ/2|
| C₂₄      | Yπ/2Xπ/2Yπ/2Xπ/2Yπ/2Xπ/2Yπ/2|
Supporting Figure 3. Simultaneous randomized benchmarking of qubits Q1 and Q2 (a) and qubits Q3 and Q4 (b) at $B_{ext} = 0.65$ T comparing the difference in measured single-qubit gate fidelity for the cases where the even parity single-qubit space is probed (red, blue) and when the odd parity single-qubit space is probed (orange, cyan). To probe the odd parity qubit space, we prepare the system in the $|\uparrow\downarrow>$ by applying a $X_\pi$ pulse to flip $Q1$($Q3$) in the $Q1Q2$($Q3Q4$) system before performing randomized benchmarking, then applying a second $X_\pi$ pulse to the same qubit to project the final system back to the $|\downarrow\downarrow>$ state. We find that it is challenging to turn off these exchange interactions due to the closely designed pitch.
VI. SIMULTANEOUS BENCHMARKING CHARACTERIZATION

Table II. Extracted fidelities from figure 3a-b in the main text, a simultaneous randomized benchmarking experiment with all single, two and four qubit simultaneous driving error rates and leakage rates (when applicable) reported. Qubits measured simultaneously in the same readout system are not measured independently, so the reported number is a combined system fidelity ($F_{Q1|Q2} = F_{Q2|Q1}$ and $F_{Q3|Q4} = F_{Q4|Q3}$ in the table). The Rabi period for this dataset is $t_\pi = 96$ ns. Leakage rates $L_{Q_i|Q_j}$ are reported when the decay plateau approaches the expected value of the fully depolarized two-qubit subspace, and are taken to be the longer of the two decay constants (see methods).

|       | Q1            | Q2            | Q3            | Q4            |
|-------|---------------|---------------|---------------|---------------|
| 1-F_{Q1} | 0.000329 ± 0.000042 | 0.003155 ± 0.000310 | 0.000308 ± 0.000019 | 0.003224 ± 0.000446 |
| L_{Q1}  | -             | -             | -             | 0.000153 ± 0.000041 |
| 1-F_{Q1|Q1} | 0.004018 ± 0.000130 | 0.0004018 ± 0.000130 | 0.0000249 ± 0.000022 | 0.003087 ± 0.000690 |
| 1-F_{Q1|Q2} | 0.000301 ± 0.000048 | -             | 0.003791 ± 0.000300 | 0.004692 ± 0.000978 |
| 1-F_{Q1|Q3} | 0.0002676 ± 0.0000565 | 0.007324 ± 0.000388 | 0.002778 ± 0.000298 | -             |
| L_{Q1|Q1}  | -             | -             | -             | 0.000175 ± 0.000065 |
| L_{Q1|Q2}  | -             | -             | 0.000210 ± 0.000034 | 0.000741 ± 0.000213 |
| L_{Q1|Q3}  | -             | -             | -             | -             |
| L_{Q1|Q4}  | -             | -             | -             | -             |
| 1-F_{Q4}  | 0.098565 ± 0.009031 | 0.020007 ± 0.000428 | -             | -             |

Supporting Figure 4. Table of infidelities and relative fidelity losses of the two-simultaneously driven qubit cases reported in Table II of the supplementary information and Figure 3a-b of the main text. The plots can be interpreted by considering the qubit in question to be the column, and the qubit to be driven with to be the row. Qubits are represented diagrammatically by a coloured sphere, while the plunger gate from which they are driven is depicted above them by a coloured cylinder. (a) Absolute infidelity of the simultaneously driven qubits. Values on the diagonals represent the single-qubit fidelity in the non-simultaneously driven space. (b) relative fidelity loss $\delta$ of each qubit pair. Diagonals have no significance and are set to 0. Here we observe that typically nearest neighbour qubit pairs exhibit higher values of $\delta$, while next-nearest neighbour pairs exhibit lower values of $\delta$. We observe some exceptions to this trend, typically when Q4 is involved in the driving. Q4 is driven via next-nearest plunger gate $P_2$, requiring greater power achieve a similar Rabi period. This is detrimental to the simultaneous driving fidelity of qubit Q2.