Learning Physics from the Cosmic Microwave Background

J. Ellis

CERN-TH/98-401 astro-ph/9902242

The Cosmic Microwave Background (CMB) provides a precious window on fundamental physics at very high energy scales, possibly including quantum gravity, GUTs and supersymmetry. The CMB has already enabled defect-based rivals to inflation to be discarded, and will be able to falsify many inflationary models. In combination with other cosmological observations, including those of high-redshift supernovae and large-scale structure, the CMB is on the way to providing a detailed budget for the density of the Universe, to be compared with particle-physics calculations for neutrinos and cold dark matter. Thus CMB measurements complement experiments with the LHC and long-baseline neutrino beams.

1. Why the CMB Might be a Good Physics Teacher

Measurements of the CMB by experiments before COBE, by COBE itself, and by subsequent experiments, have already amassed an impressive amount of data [1], and this is set to grow dramatically with future experiments culminating in the MAP and Planck Surveyor satellites. We already know that the spectrum is very close to black-body, which imposes important constraints on entropy deposition, late particle decays, reionization, etc. [2]. We also know that the CMB is highly isotropic, providing the best evidence for the relevance of Friedman-Robertson-Walker (FRW) cosmological models [3]. This isotropy immediately raises the horizon problem: why is the Universe apparently so homogeneous and isotropic on large scales? It is worth recalling that the scale size of the Universe at the epoch of last scattering is about two orders of magnitude larger than the horizon size $a_H = 2t$ at that epoch $t$, which is the largest distance over which a message could have travelled in a conventional FRW cosmology. So how were the opposite sides of the Universe able to coordinate so precisely? Small anisotropies in the CMB have been seen: the first to be discovered was the dipole anisotropy of about $10^{-3}$, which is conventionally interpreted as a Döppler effect due to the velocity of the Earth in the Machian reference frame provided by the CMB [4]. More recently, COBE and its successors have detected the higher-order anisotropies shown in Fig. 1 [5], which promise to teach us a lot of fundamental physics.

---

**Figure 1.** Compilation of CMB anisotropy measurements.
ing to which, at some very early epoch, the energy density \( \rho \) of the Universe may have been dominated by an (almost) constant term \( V \):

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{a^2} : \rho \simeq V
\]

(1)

It is easy to see that, if this constant term were dominant, it would generate an epoch of exponential growth of the scale factor \( a \):

\[
a \simeq a_I \exp(H(t - t_I)) : H = \sqrt{\frac{8\pi G_N}{3}} V
\]

(2)

where \( a \simeq a_I \) at the initial time \( t_I \) marking the onset of inflation. If so, the horizon size \( a_H \) of the Universe would also have expanded exponentially, and the entire observable Universe would have been born within the pre-inflationary horizon:

\[
a = a_I e^{H(t - t_I)} \gg 2t \text{ even if } a_I \ll 2t_I
\]

(3)

During this epoch of exponential expansion, the (approximate) homogeneity and isotropy of the observable Universe could have been imparted. Moreover, the curvature term \( -k/a^2 \) in (1) would rapidly have become negligible, so that the Universe would become almost critical: \( \Omega \equiv \rho/\rho_c \simeq 1 \), where \( \rho_c \equiv \sqrt{3H/8\pi G_N} \). Furthermore, unwanted particles from the very early Universe, such as GUT monopoles, would have been inflated away beyond the last-scattering surface of the CMB.

In this picture, the CMB anisotropies are ascribed to density fluctuations originating from quantum fluctuations in the scalar field whose potential energy \( V = \mathcal{O}(\mu^2) \) drove inflation. These fluctuations would have induced perturbations in the field energy in different parts of the Universe, evolving subsequently into fluctuations in the temperature of the CMB. These would be (approximately) a Gaussian random field of perturbation \( \delta \rho/\rho \), with similar magnitudes on different scale sizes, as favoured by astrophysicists. The magnitude of these perturbations is related to the field energy density during inflation

\[
\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \propto \mu^2 G_N
\]

(4)

The magnitude \( \delta T/T \sim 10^{-5} \) observed by COBE et seq. favours \( \mu \lesssim 10^{16} \text{ GeV} \), comparable with the unification scale in GUTs \( \Lambda \). Therefore, at the very least, inflation provides with a unique window through which we can look back at an energy scale far beyond the direct reach of current accelerators, and might even provide us with a precious window on GUTs themselves. A challenge which has not yet been fully met, however, is to derive an inflationary potential from some GUT (or string) theory in a natural way.

2. What we Might Hope to Learn

The large mass scale \( \mu \) associated with inflation suggests that observations of the CMB may be sensitive to all mass scales in physics, not excluding that associated with quantum gravity, which may not be so far beyond \( m_{GUT} \), if current \( M \)-theory ideas are right.

Indeed, examples can easily be given of the CMB’s sensitivity to aspects of Quantum Gravity. In addition to the scalar density perturbations \( \delta \) that are expected to dominate the CMB anisotropies, there may also be tensor perturbation modes, which are none other than gravitational waves. String cosmology scenarios have been proposed \( \delta \), of which these may provide a key observational signature. Then we should recall that it is the combination of CMB data with those on high-redshift supernovae \( \delta \), that provides the clearest evidence for a cosmological constant \( \Lambda \), as discussed in more detail later.

The interpretation of these observations corresponds to \( \Lambda \lesssim 10^{-123} m_{\text{P}}^4 \), which is far smaller than the individual contributions to \( \Lambda \) in many particle theories. For example, condensates in the QCD vacuum yield

\[
\delta_{QCD} \Lambda \sim (100 \text{ MeV})^4 \sim 10^{-80} m_{\text{P}}^4,
\]

the Higgs vacuum of the Standard Model contributes

\[
\delta_{EW} \Lambda \sim (100 \text{ MeV})^4 \sim 10^{-68} m_{\text{P}}^4,
\]

and global supersymmetry breaking might contribute

\[
\delta_{S} \Lambda \gtrsim (1 \text{ TeV})^4 \sim 10^{-64} m_{\text{P}}^4,
\]
The discrepancy between these estimates and the (inferred) astrophysical value may be the biggest problem in particle physics, much bigger even than the gauge hierarchy problem. Its resolution certainly requires a consistent quantum theory of gravity that also includes all the other particle interactions.

Personally, I regard the observational indications for non-zero vacuum energy as a tremendous opportunity for theoretical physics, as it provides a number to calculate in one’s candidate theory of quantum gravity. Much effort has been applied to trying to prove that $\Lambda = 0$ [3], but a corresponding exact unbroken symmetry has not been identified. Perhaps $\Lambda \neq 0$ after all? Or perhaps it is merely relaxing towards zero: $\Lambda(t) \to 0$ with a non-trivial equation of state: $P/\rho \equiv \alpha < 0$? Present data require $\alpha \lesssim -0.6$ [3], but do not impose $\alpha = -1$ as required if $\Lambda$ is constant. Models in which $\Lambda(t) \to 0$ include a mobile scalar field $\phi(t)$ (quintessence) whose potential energy $V(\phi(t)) \to 0$ [4], and gradual de-excitation of the quantum-gravity vacuum [5]. The CMB and other data may eventually be able to make interesting distinctions between possible equations of state, and thereby discriminate between different theories of quantum gravity.

As for Grand Unification, a primary hope is that the vacuum energy driving inflation could be related to the GUT scalar potential. The CMB may also cast light on the magnitudes of the neutrino masses expected in GUTs. Laboratory experiments have established that these must be much smaller than the masses ($m$) of the charged leptons and quarks [5]:

$$m_{\nu_e} \lesssim 2.5 \text{ eV},$$
$$m_{\nu_{\mu}} \lesssim 160 \text{ keV},$$
$$m_{\nu_{\tau}} \lesssim 18 \text{ MeV}.$$  

Theorists expect non-zero neutrino masses, because there are no candidate exact gauge symmetries with associated conserved charges to forbid them, by analogy with the $U(1)_{\text{em}}$ of QED, with its associated conserved $Q_{\text{em}}$ and vanishing photon mass. We expect the other apparently conserved quantum numbers such baryon number $B$ and lepton number $L$ eventually to be violated, most likely at some high mass scale $M \approx M_{\text{GUT}}$. Lepton-number violation leads generically to neutrino masses.

Most theorists expect a see-saw mass matrix mixing the known $\nu_L$ with heavy singlet neutrinos $N$ (often called right-handed neutrinos, but I dislike this nomenclature) of the form [6]

$$(\nu_L, N) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

whose diagonalization yields

$$m_{\nu} \sim \frac{m^2}{M} \ll m \sim m_{l,q}$$

For example, if we put $m \sim 100 \text{ GeV}$ and take $m_{\nu} \sim 10^{-1} \text{ eV}$ for the third generation, we estimate $M \sim 10^{13} \text{ GeV}$. Recent evidence for atmospheric neutrino oscillations suggests [7]

$$\Delta m^2_A \sim (10^{-2} \text{ to } 10^{-3}) \text{ eV}^2$$

for the mass-squared difference between one pair of mass eigenstates $m_{\nu}$. The range [1] can be explored with approved and projected long-baseline neutrino experiments with accelerator beams [8]. In addition, solar neutrino data have for some time suggested [9]

$$\Delta m^2_S \sim (10^{-5} \text{ or } 10^{-7}) \text{ eV}^2$$

for the mass-squared difference between another pair. These are not measurements of the absolute scale of neutrino masses, but most models suggest that the neutrinos are not heavy and almost degenerate, and hence that

$$m_{\nu_1} \sim (10^{-1} \text{ to } 10^{-3/2}) \text{ eV} >$$
$$m_{\nu_2} \sim (10^{-5/2} \text{ or } 10^{-5}) \text{ eV} > m_{\nu_3}$$

for the three mass eigenstates. As is discussed below, the CMB and large-scale structure data may eventually provide the best constraint on the expected hierarchy (13).

Another possible output of grand unification that will be constrained by CMB measurements is baryogenesis. Already $\Omega_B$ (and hence $n_B/s$) is being bounded by present CMB measurements [10], and these may eventually provide the most accurate determination of $\Omega_B$, for comparison with baryogenesis scenarios at the GUT or electroweak scale [11], or in between.
Another possible extension of the Standard Model that may be tested by CMB measurements is Supersymmetry [21]. This is invoked [22] by particle theorists to stabilize the gauge hierarchy: \( m_W \ll m_P \), or equivalently \( G_F \sim 1/m_W^2 \gg G_N = 1/m_P^2 \), or equivalently \( V_{\text{Coulomb}} = e^2/r \gg V_{\text{Newton}} = (m_pm_e/m_P^2) \ 1/r \) inside an atom. If one tries to set such a hierarchy by hand, one discovers large quantum corrections:

\[
\delta m^2_W = O \left( \frac{\alpha}{\pi} \right) \Lambda^2 \quad (14)
\]

which are much larger than the physical value of \( m^2_W \) if the cutoff \( \Lambda \) in (14) is \( O(m_P \text{ or } m_{\text{GUT}}) \). An effective cutoff \( \Lambda \) is provided by sparticle masses in supersymmetric models:

\[
\delta m^2_W = O \left( \frac{\alpha}{\pi} \right) |m^2_B - m^2_F| \quad (15)
\]

where the subscripts (\( B, F \)) denote superpartner bosons and fermions, and the remainder is \( \lesssim m^2_W \) if

\[
|m^2_B - m^2_F| \approx 1 \text{ TeV}^2 \quad (16)
\]

This motivates the appearance of superpartners at energies accessible to accelerators such as the LHC [23]. As discussed below, it also suggests the presence of massive supersymmetric relic particles contributing \( O(1) \) to the matter density \( \Omega_m [24] \). CMB measurements already bound \( \Omega_m \), and may soon provide accurate measurements of it, thereby constraining supersymmetric models.

Subsequent epochs of the history of the Universe, such as the electroweak phase transition, the quark-hadron QCD phase transition and cosmological nucleosynthesis will also be constrained by CMB measurements, but we do not go into details here.

3. Density Budget of the Universe

We phrase our subsequent discussion in terms of the density budget of the Universe, expressed relative to the critical density: \( \Omega_i \equiv \rho_i/\rho_c \).

\( \Omega_{\text{tot}} \): Inflation suggests that this is practically indistinguishable from unity: \( \Omega_{\text{tot}} = 1 \pm 0(10^{-4}) \), but there are models that predict \( \Omega_{\text{tot}} < 1 \) [25]. One of these is illustrated in Fig. 2, which has the potential

\[
V = \frac{m^2 \phi^2}{2} Z \left( 1 + \frac{\alpha^2}{\beta^2 + (\phi - v)^2} \right) : \beta \ll v (17)
\]

This looks rather bizarre, but who knows what a priori probability measure the inflationary God uses, or even whether such a concept makes sense [3]? As seen in Fig. 2b, this model predicts a spectrum of density perturbations that is far from flat [25], and hence amenable to test by CMB measurements.

\( \Omega_b \): Measurements of the \( D/H \) ratio in high-redshift Lyman-\( \alpha \) clouds [26] correspond to

\[
\frac{D}{H} = (3.3 \pm 0.3) \times 10^{-5} \quad (18)
\]

This is indeed the correct primordial \( D/H \) ratio, big-bang nucleosynthesis calculations suggest that

\[
\frac{n_B}{s} = (5.1 \pm 0.3) \times 10^{-10} \quad (19)
\]
corresponding to
\[ \Omega_B h^2 = 0.019 \pm 0.001 \]  
(20)
where \( h \) is the present Hubble expansion rate in units of 100 km/s/Mpc. Using the currently favoured range \( h = 0.65 \pm 0.10 \), we see from (20) that \( \Omega_b \approx 0.08 \), which is insufficient to explain all the matter density in the following paragraph.

\( \Omega_m \): The cluster measurements (\( M/L \) ratio, present and past abundances, cluster dynamics and the baryon fractions inferred from X-ray measurements) all suggest \[ \Omega_m \sim 0.2 \text{ to } 0.3 \]  
(21)
Moreover, the combination of CMB measurements and high-redshift supernovae [11] also support independently such a value for \( \Omega_m \).

\( \Omega_{\text{CDM}} \): The theory of large-scale structure formation strongly suggests that most of \( \Omega_m \) is cold dark matter, so that
\[ \Omega_{\text{CDM}} \sim \Omega_m \]  
(22)
as perhaps provided by supersymmetric particles.

The lightest supersymmetric particle is expected to be stable in most models, and hence present in the Universe today as a cosmological relic from the Big Bang [24]. Its stability would be due to a multiplicatively-conserved quantum number, called \( R \) parity, which is related to baryon number \( B \), lepton number \( L \) and spin \( S \):
\[ B = (-1)^{3B+L+2S} \]  
(23)
and takes the value +1 for all conventional particles, changing to -1 for all supersymmetric particles, because they have identical internal properties but spins differing by half a unit. There are three important consequences of \( R \) conservation: (i) sparticles are always produced in pairs, such as \( \bar{p}p \rightarrow q\bar{q} + X \) or \( e^+e^- \rightarrow \mu^+\mu^- \), (ii) heavier sparticles decay into lighter ones, such as \( q \rightarrow q\bar{g} \) or \( \mu \rightarrow \mu\tilde{\gamma} \), and (iii) the lightest sparticle is stable because it has no legal decay mode.

In many models [24], the favoured scandidate for the lightest sparticle is the lightest neutralino \( \chi \), which is a mixture of the photino \( \tilde{\gamma} \), the zino \( \tilde{Z} \) and the neutral Higgsinos \( H^0 \) [25]. At the tree level, the neutralinos are characterized by three parameters: the unmixed gaugino mass \( m_{1/2} \), a Higgsino mixing parameter \( \mu \) and tan \( \beta \), the ratio of Higgs vacuum expectation values. The properties of the \( \chi \) particle simplify in the limit \( m_{1/2} \rightarrow 0 \), where it becomes an almost pure photino \( \tilde{\gamma} \), and in the limit \( \mu \rightarrow 0 \), where it becomes almost a pure Higgsino \( H \). However, the non-observation of supersymmetric particles at LEP excludes these simple limits [30]. The purely experimental limit \( m_\chi \gtrsim 20 \) to 30 GeV may be strengthened by taking other constraints into account [34, 31], as seen in Fig. 3.

There are generic domains of supersymmetric parameter space where an “interesting” cosmological relic density \( 0.1 \lesssim \Omega_h h^2 \lesssim 0.3 \) is possible [24] and it can even be argued that this is the most natural range [32]. If this upper limit is imposed, the lower bound on \( m_\chi \) is strengthened to the dotted line marked \( C \) in Fig. 3. The limit coming from the non-observation of a supersymmetric Higgs boson at LEP is indicated by the dotted line marked \( H \) in Fig. 3, which is strengthened to the solid line marked \( \text{UHM} \) if all the scalar sparticles are assumed to have the same mass as the Higgs fields at the GUT input scale. Finally, combining this assumption with the lower and upper limits on the cosmological relic density yields the lines marked DM + UHM and cosmo + UHM in Fig. 3. These considerations currently yield
\[ m_\chi \gtrsim 42 \text{ GeV} \]  
(24)
and subsequent LEP runs should be able to explore thoroughly the range \( m_\chi \lesssim 50 \text{ GeV} \).

Although theorists of structure formation prefer most of the dark matter to be composed of cold non-relativistic particles, such as neutralinos, they think this may not be the whole story, as seen in Fig. 4 [33]. The plain CDM model would require a very non-flat spectrum of perturbation \( n \ll 1 \), which is disfavoured in most inflationary models, if \( n \approx 0.65 \), as suggested by current data. A model (\( \nu \text{CDM} \)) with decaying dark matter fares somewhat better, but the most promising are the mixed (\( \nu \text{CDM} \)) and the model (ACDM) with a cosmological constant.

\( \Omega_{\text{HDM}} \): The hot dark matter density due to
neutrinos can be predicted accurately as a function of the neutrino masses
\[ \Omega_{HDM} h^2 \sim \sum_i \left( \frac{m_{\nu_i}}{0.98 \text{ eV}} \right) \] (25)

The theory of structure formation suggests that \( \Omega_{HDM} \ll \Omega_{CDM} \), and the indications (11), (12) from atmospheric and solar neutrino data can easily be explained (13) by light neutrinos: \( m_{\nu_i} < 0.1 \text{ eV} \), which would make a small contribution to \( \Omega_{tot} \).

The present and prospective sensitivities of cosmological data to \( m_\nu \) are shown in Fig. 5 (34). So far, \( m_\nu \gtrsim 3 \text{ eV} \) is excluded by the available upper limit on the density of hot dark matter. Comparison of future data on large-scale structure and the CMB are thought to be sensitive to \( m_\nu \gtrsim 0.3 \text{ eV} \). This is very close to the range \( m_\nu \sim 0.1 \) to 0.03 eV favoured by the atmospheric neutrino data, so one should not abandon hope of detecting neutrino masses astrophysically (35).

\( \Omega_\Lambda \): As we have already seen, the largest fraction of the energy density of the Universe may be provided by vacuum energy, if one combines the CMB (3) and high-redshift supernova data (11). It is also required by the dynamical estimates of \( \Omega_m \) and inflation, which requires \( \Omega_{tot} = \Omega_m + \Omega_\Lambda \simeq 1. \)

If one takes at face value the absolute scale of neutrino masses suggested by the atmospheric neutrino data, one would be led to favour the (\( \Lambda \)CDM) option in Fig. 4. In this case, Fig. 6 pieces together the indications concerning \( \Omega_\Lambda \) and \( H_0 \) from different astrophysical and cosmological data excluding those on high-redshift supernovae. We see that these favour independently \( \Omega_\Lambda \sim 0.6, \ h \sim 0.65 \) (33). Thus a remarkably consistent picture of the density budget of the Universe may be emerging:

![Figure 3](image1.png)

**Figure 3.** Lower limits on the lightest neutralino mass (31), obtained under the different assumptions listed in the text.

![Figure 4](image2.png)

**Figure 4.** Allowed ranges (33) of the Hubble expansion rate and the power law for cosmological perturbations, in different dark matter models.
Figure 5. Compilation of indications on neutrino mass-squared differences $\Delta m^2$ and mixing angles $\theta$ from oscillation experiments, compared with cosmological sensitivities to neutrino masses [34].

$$\Omega_{\text{tot}} \simeq 1 = \Omega_m + \Omega_\Lambda : \quad \Omega_m \sim 0.3, \Omega_\Lambda \sim 0.7 \quad (26)$$

where

$$\Omega_m = \Omega_{\text{CDM}} + \Omega_\nu + \Omega_b \quad (27)$$

with

$$\Omega_b < 0.1, \quad \Omega_\nu \ll \Omega_{\text{CDM}} \simeq \Omega_m \quad (28)$$

Let us see whether future data confirm this picture.

4. What we Have Learnt

The first generation of CMB measurements has already taught us a great deal about fundamental physics [1], some of which has already been mentioned in previous sections. Most of the discussion is in terms of inflationary models, but it should not be forgotten that the CMB delivered a death blow to the alternative models based on cosmological defects [36]. These did not predict an acoustic peak, as apparently observed in the data at a harmonic number $\ell \sim 200$, as suggested by the data compiled in Fig. 1. It is mainly the location of this peak that suggests $\Omega_{\text{tot}} = \Omega_m + \Omega_\Lambda \simeq 1$, as seen in Fig. 7. The height of the peak, as seen in Fig. 1, suggests that $\Omega_b \lesssim 0.1$. Moreover, the combination of CMB with large-scale structure suggests that $\Omega_{\text{CDM}} \gg \Omega_{\text{HDM}}$, and the value of $\Omega_{\text{CDM}}$ suggested by combining
the CMB data with high-redshift supernovae is compatible with $\Omega_{CDM} \sim 0.3$ (as also seen in Fig. 7 [37]) as suggested by cluster observations. As has already been mentioned, standard cold dark matter does not fit the CMB and large-scale structure data, but a model with $\Omega_{\Lambda} \sim 0.7$ does. Furthermore, the indications from the CMB and large-scale structure data are that the spectral index of the density perturbations $n \sim 1 \pm 0.2$, in agreement with the Harrison-Zeldovich spectrum and most inflationary models.

Thus, we have reached the stage where alternative theories of structure formation have been, or are being, eliminated, and attention is being focussed on a candidate Standard Model of structure formation. The next step is to test the model, determine its parameters, and try to over-constrain them, with the hope of eventually moving beyond it.

5. What we May Hope to Learn

The next generation of CMB measurements, culminating in the MAP and Planck satellites, will provide us with precision determinations of physical quantities, and probe the emerging Standard Model of structure formation, much as LEP and the SLC have probed the Standard Model of particle physics. For example, $\Omega_{tot}$ may be determined with a precision of 0.1, possibly 0.01 in combination with high-redshift supernova data, in conjunction with a comparable precision in $\Omega_m$. Similarly, $\Omega_b$ will be determined with a small fractional error. In the case of LEP, many quantities such as $m_Z$, $\sin^2 \theta_W$ and the number of light neutrino species $N_\nu$ were eventually determined with errors far smaller than theoretical guesses before the accelerator started. For that reason, I am not going to hazard here many guesses about the eventual errors in cosmological parameters! However, let us consider neutrinos as an example of what may be possible.

These decoupled when the temperature was $O(1)$ MeV. Following reheating by $e^+e^- \rightarrow \bar{\nu}\nu$, we expect a relic density

$$\rho_{\nu} \rho_{\gamma} = \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_\nu \simeq 0.681N_\nu$$

(29)

This is subject to small corrections due to incomplete decoupling: $\delta N_{\nu}^{ID} \simeq 0.03$ to 0.04 and finite-temperature QED corrections: $\delta N_{\nu}^{FT} \simeq 0.01$ [38]. The precise value of the ratio $\rho_{\nu}/\rho_\gamma$ affects the epoch of matter-radiation equality, and can be measured accurately by the Planck satellite, particularly using polarization [38].

Figure 8 shows the predicted sensitivity to $\delta N_\nu$ as a function of the maximum value of $\ell$, demonstrating the advantages gained from polarization data and from measurements at high $\ell$. These may reach the sensitivity required to see the effects of incomplete decoupling and finite-temperature QED [38]. They may even match...
Figure 8. Possible sensitivity of future CMB measurements to the effective number of neutrino species [38]. The lower (upper) pair of lines (do not) assume exact knowledge of other cosmological parameters. Within each pair, the lower (solid) line is the sensitivity obtainable if polarization is measured.

the LEP error $N_{\nu}^{LEP} = 2.994 \pm 0.011$ [29]!

CMB measurements, in conjunction with other observations, will also be able to put interesting constraints on the mass of any stable neutrino, in the range of 1 to possibly $0.1$ eV [34]. As already mentioned, this is getting close to the range indicated by atmospheric neutrino oscillations:

$$m_\nu \sim \sqrt{\Delta m_{\text{Atmo}}^2} \sim 0.1 \text{ to } 0.03 \text{ eV}$$  \hspace{1cm} (30)

as seen in Fig. 5. I would not bet a lot of money against the CMB and large-scale structure data eventually reaching down to the range (30). The present CMB data are already able to exclude decaying neutrinos with $m_\nu \gtrsim 10 \text{ eV}$ and $10^{13} \text{s} \lesssim \tau \lesssim 10^{17} \text{s}$ [40]. Again, the future sensitivity is expected to extend down to about 1 eV, and there will be analogous constraints on other unstable massive particles such as neutralinos, gravitinos, etc.

The future CMB measurements will also make precision tests of inflationary models, much as LEP and the SLC have made precision tests of electroweak models and measured $\sin^2 \theta_W$ very accurately. The observables of interest are the scalar perturbation mode $S$, the tensor mode $T$, and their spectral indices $n, n_T$. Knowledge of them enables the inflationary potential to be reconstructed [21]:

$$V_* \approx 1.65 T m_P^2,$$  \hspace{1cm} (31)

$$V'_* \approx \pm \sqrt{\frac{8 \pi}{T}} \frac{T}{S} \frac{V_*}{m_P},$$

$$V''_* = 4 \pi \left[ (n-1) + \frac{3}{7} \frac{T}{S} \right] \frac{V_*}{m_P^2},$$

where the primes denote derivatives with respect to the inflaton field $\phi$, and the * subscript denotes the scale at which the measurement is made. In addition, there is a consistency condition

$$\frac{T}{S} = -7 n_T$$  \hspace{1cm} (32)

which enables the inflationary paradigm to be checked. Figure 9 shows how the spectral index $n$ and the tensor/scalar ratio $r$ vary in different inflationary models [22]. Also shown are the error ellipses expected from Planck. We see that the latter should be able to distinguish between different power-law potential models, and between many of these and models with an exponential potential.

CMB measurements take inflationary models into the scientific domain: individual models may be falsified, and even very general classes of models, for example by observing strong non-Gaussian correlations. A word of caution is, however, in order. Like any finite-size set of measurements, the CMB measurements alone will not have a unique interpretation – the so-called cosmological degeneracy problem. Specifically, models with the same stress history will give the same pattern of acoustic peaks in the CMB spectrum and the same perturbation power spectrum. Other measurements sensitive to the comoving sound speed and curvature fluctuations would be needed to distinguish between models.

6. Conclusions

The CMB is a powerful probe of fundamental physics, including quantum gravity, inflation, grand unification, cold dark matter, hot dark
matter, decaying particles, Big-Bang baryosynthesis and much else besides, as well as being of
capital importance for astrophysics and cosmology. It is one of the very few probes we have of
physics at the grand unification scale, along with neutrino physics, as can be probed using long-
baseline neutrino experiments [18], and the measurements of gauge couplings and particle masses,
e.g., of sparticles at the LHC [23]. Hence the future generation of CMB experiments, notably
MAP and particularly Planck, is an invaluable complement to the next generation of particle ac-
ccelerator experiments at the LHC. Together, they may not only establish a Standard Model of cos-
mology and structure formation, but also take us beyond the Standard Model of particle physics.

REFERENCES

1. G.F. Smoot, astro-ph/9902027.
2. Particle Data Group, C. Caso et al., Eur.Phys.J., C3 (1998) 1.
3. For a review with a constructively critical at-
titude, see:
G.F.R. Ellis, gr-qc/9812046.
4. A great scientific check of this interpretation is to compute successfully the dipole anisotropy using knowledge of the local distri-
bution of matter. see, e.g.,
M. Hudson, A. Dekel, S. Courteau, S. Faber and J.A. Willick, Mon.Not.Roy.Astron.Soc., 274 (1995) 305.
5. for a recent compilation, see: M. Tegmark,
http://www.sns.ias.edu/~max/r_frames.html.
6. A.A. Starobinsky, Phys.Lett., 91B (1980) 99;
D. Kazanas, Astrophys.J., 241 (1980) L59;
A. Guth, Phys.Rev., D23 (1981) 347.
7. J. Preskill, Phys.Rev.Lett., 43 (1979) 1365;
Ya. Zel’dovich and M. Khlopov, Phys.Lett., 79B (1979) 239.
8. J. Bardeen, P.J. Steinhardt and M.S. Turner,
Phys.Rev., D28 (1983) 679;
A.H. Guth and S.-Y. Pi, Phys.Rev.Lett., 49 (1982) 1110;
A.A. Starobinsky, Phys.Lett., 117B (1982) 175;
S.W. Hawking, Phys.Lett., 115B (1982) 295.
9. J. Ellis, S. Kelley and D.V. Nanopoulos,
Phys.Lett., B249 (1990) 441 and Phys.Lett., B260 (1991) 131;
U. Amaldi, W. de Boer and H. Furstenau,
Phys.Lett., B260 (1991) 447;
P. Langacker and M. Luo, Phys.Rev., D44 (1991) 817.
10. G. Veneziano, hep-th/9902097 and references therein.
11. A.G. Riess et al., astro-ph/9805201;
S. Perlmutter et al., astro-ph/9812133.
12. S. Kachru and E. Silverstein, hep-th/9810129.
13. P.M. Garnavich et al., astro-ph/9806396;
S. Perlmutter, M.S. Turner and M. White,
astro-ph/9901052.
14. I. Zlatev, L.-M. Wang and P.J. Steinhardt,
astro-ph/9807003.
P.J. Steinhardt, L.-M. Wang and I. Zlatev,
astro-ph/9812313.
15. J. Ellis, N. Mavromatos and D. Nanopoulos,
gr-qc/9810088.
16. T. Yanagida, Proc. Workshop on the Unified
Theory and the Baryon Number in the Uni-
verse (KEK, Japan, 1979);
R. Slansky, Talk at the Sanibel Symposium, Caltech preprint CALT-68-709 (1979).
17. Super-Kamiokande collaboration, Y. Fukuda et al., Phys.Rev.Lett., 81 (1998) 1562.
18. Y. Oyama, for the K2K collaboration, hep-ex/9803014;
MINOS collaboration, E. Ables et al., Fermilab proposal P-875 (1995);
G. Acquistapace et al., CERN report 98-02 (1998).
19. J.N. Bahcall, astro-ph/9808162.
20. J.M. Cline, hep-ph/9902329.
21. P. Fayet and S. Ferrara, Phys.Rep., 32, 251 (1977);
H.E. Haber and G.L. Kane, Phys.Rep., 117, 75 (1985).
22. L. Maiani, Proc. Summer School on Particle Physics, Gif-sur-Yvette, 1979 (IN2P3, Paris, 1980) p. 3;
G 't Hooft, in: G 't Hooft et al., eds., Recent Developments in Field Theories (Plenum Press, New York, 1980);
E. Witten, Nucl.Phys., B188 513 (1981);
R.K. Kaul, Phys.Lett., 109B 19 (1982).
23. S. Abdullin and F. Charles, hep-ph/9811402.
24. J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Nucl.Phys., B238, 453 (1984).
25. A. Linde, Phys.Rev., D59 (1999) 023503.
26. D. Tytler, S. Burles, L.-M. Wu, X.-M. Fan, A. Wolfe and B.D. Savage, astro-ph/9810217.
27. S. Burles, K.M. Nollett, J.N. Truran and M.S. Turner, astro-ph/9901303.
28. R.E. Lopez, S. Dodelson, A. Heckler and M.S. Turner, astro-ph/9803095.
29. M. Grünwald and D. Karlen, talks at International Conference on High-Energy Physics, Vancouver 1998,
http://www.cern.ch/LEPEWG/misc.
30. U.-L. Pen, U. Seljak and N.G. Turok, Phys.Rev.Lett., 79 (1997) 1611.
31. A.N. Lasenby, S.L. Bridle and M.P. Hobson, astro-ph/9901168.
32. E.J. Copeland, I.J. Grivell, E.W. Kolb and A.R. Liddle, Phys.Rev., D58 (1998) 043002,
and references therein.
33. W.H. Kinney, Phys.Rev., D58 (1998) 12350.