The Quantum Zeno Effect – Evolution of an Atom Impeded by Measurement

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The evolution of a quantum system is supposed to be impeded by measurement of an involved observable. This effect has been proven indistinguishable from the effect of dephasing the system’s wave function, except in an individual quantum system. The coherent dynamics, on an optical $E2$ line, of a single trapped ion driven by light of negligible phase drift has been alternated with interrogations of the internal ion state. Retardation of the ion’s nutation, equivalent to the quantum Zeno effect, is demonstrated in the statistics of sequences of probe-light scattering “on” and “off” detections, the latter representing back-action-free measurement.

The act of measuring an observable of a system that obeys quantum mechanics consists of recording one of the eigenvalues and rejecting all the other ones. This act is accompanied by sudden transition of the system’s wave function into the eigenfunction corresponding to the recorded eigenvalue; the response of the system is known as the "state reduction" [1]. It has been recognized that repeated measurements retard, or even impede, the evolution of a quantum system to the extent that they may inhibit the evolution [2,3]. This consequence, the "quantum Zeno effect" [4] alluding to eleatic ontology, has aroused a great wealth of work devoted to contemplating the subject [5], and an attempt to observe it: An experiment including the drive and probe laser irradiation of an ensemble of some 5000 beryllium ions confined in an ion trap has resulted in complete agreement with quantum-mechanical predictions [6]. However, these predictions based on the deletion, in the acts of measurement, of all superpositions of eigenstates can be identified with the effect of any phase perturbations by the environment upon the multi-particle wave function of the system ("dephasing"). In fact, a perturbation via the back action of the meter on the quantum system has been invoked as the origin of QZE [7-10]. The ambiguity of the initial $t^2$ evolution being set back by the measurements [2,3,11], or dephasing, – i.e. effect of measurement vs dynamical effect – is unresolvable since a decision would require knowledge of the states of all the members (the "micro-state") of any ensemble that remain unknown in a global measurement. Here, both the result of a particular measurement, and the temporal evolution of the ensemble’s state, do not statistically depend on the results of previous measurements; they are deterministic, save the "projection noise" [12,13] that affects measurements of non-commuting observables and vanishes with a large enough ensemble. However, with an individual system, the result of a measurement as well as the system’s evolution do statistically depend on the history, and the results are in general found indeterministic, except after particular preparation of the system, in an eigenstate of the observable to be detected [13]. The statistics of the results will embody the signature of the state reductions by the measurements, and their effect cannot be ascribed to dephasing [14, 15]. This argument has been quantified, by numerical calculation, for a single spin-like quantum system interacting with a light mode whose photon number is measured, and for a corresponding ensemble [16]: With a single quantum system, the evolution is not revealed by reiterated measurements. The state of this system is reduced to an eigenstate, by each precise enough measurement, according to the result of the measurement. As long as the evolution of the quantum system between two subsequent reductions is coherent, there is no base for invoking dephasing. This is so because the configuration space of, say, an individual spin system extends only over the surface of the unit sphere in SU2 symmetry, and the micro-state of this system is completely known from the result of a measurement, in contrast with that of an ensemble of spins.

In the experiment of [6], the state of the system has been interrogated only after sequences of "measurement" pulses, such that back-and-forth transitions go unnoticed and falsify the probability of survival [17]. In contrast, here the result of each measurement is registered. We have demonstrated the retardation of the light-driven quantum evolution of an individual, localized cold ion upon repeated reduction by intermittent probing the ion’s two energy eigenstates involved in the driven resonance. A single ion, $^{172}$Yb$^+$, was localized in the node of the electric field of a 2-mm-sized electrodynamic trap in ultra-high vacuum [18]. The ion was centred in the pseudo-potential of the trap with less than 30nm error, and laser-cooled to the Doppler limit (mean vibrational state $|n\rangle \simeq 10$) in order to minimize the driven micro-motion, and the free harmonic secular motion, respectively. The $E2$ transition $S_{1/2} - D_{5/2}$ was coherently driven, during time intervals $\tau = 2ns$, by shining upon the ion 411-nm quasi-monochromatic light generated by frequency-doubling the light of an 822-nm diode laser (Fig. 1). The spectral width of the blue output did not exceed 500 Hz, with 1s of integration. The pulses of this drive light alternated with pulses of 369-nm probe light, generated by frequency-doubling the output of a cw LD700 dye laser. The probe light excites some $10^8$ events/s of resonance scattering on the $S_{1/2} - P_{1/2}$ line.
of the ion only if the ion is found in the $S_{1/2}$ ground state after the driving pulse has been applied [19]. The probe pulses were 10-ms long such that clear-cut presence or absence of resonance scattering represent measurements of the ion’s internal state: Scattered light proves the ion to reside in the ground state immediately after the detection of each photon, the absence of light scattering instead reduces the ion to the $D_{5/2}$ state.

The decay of the ion’s $D_{5/2}$ state into the metastable state $F_{7/2}$ of extreme lifetime [20] complicates the dynamics on the driven $E2$ line. However, we kept the ion continuously irradiated by the cw output light of a diode laser at 638nm that completely saturates the $F_{7/2} - \frac{5}{2}[5/2]_{5/2}$ line of the ion in order to immediately repump the ion from the $F_{7/2}$ level into its $S_{1/2}$ ground state.

A measurement based on light scattering absent (“off”), i.e., when the ion is in its $D_{5/2}$ state, extends no physical reaction on the "quantum object" (i.e., the spin represented by the $E2$ transition), it is of the quantum non-demolition type [10]: (i) Both the quantum object and the "quantum probe" (the dipole on the resonance line) return to their initial states after the measurement. (ii) The probe light does neither cause any dissipation nor back action in the combination of quantum object and quantum probe. The state of this probe is indirectly measured by the null detection of scattered light, with zero recoil upon the ion. This state is correlated with the upper one of the two alternative eigenvalues of the observable to be measured, the internal energy of the quantum object.

Probe-light scattering makes the ion recoil. Its net effect is spatial and velocity fluctuations, and a random phase shift upon the quantum object, the driven quadrupole. However, the ion remains cooled deep inside the Lamb-Dicke regime (excursion $\ll$ light wavelength), and the corresponding phase variations do not exceed a small fraction of $\pi$.

The condition for strong trapping [21] holds with the driven $E2$ line, such that any net recoil of the drive light is indeed absorbed by the trap since the ion’s vibrational frequency, 1.3 MHz, far exceeds the natural line width.

The crucial issue is the capability, of the two kinds of measurements, to distinguish retarded evolution from the effect of dephasing. The phase of the drive light is found to diffuse, in each 2-ms interval of irradiation, by such a small fraction of $\pi$ only, that there is no risk of quenching the coherence. The ion’s super-position state would be conserved during the breaks of the drive, as long as it is not measured, for $t < \Gamma^{-1} = \text{the decay time of the inversion}$. State reduction by each subsequent probing yields random results, and the coherent dynamics is retrieved only when averaging over an ensemble of measurements, after identical preparation. – Preservation of the ion’s vibronic state under probing the states of an electronic resonance has permitted detection of the corresponding nutational dynamics by a stroboscopic or “sampling” technique [22].

Fig. 2 shows a trajectory of "on" and "off" events. The number of on/off pairs accumulated over 500 measurements and normalized by the number of "on" events yields the probability of excitation, on the $E2$ line, to the metastable $D_{5/2}$ level. With negligible relaxation, this transition probability is

$$p_{01} = \cos^2 \chi \cdot \sin^2 (\theta/2),$$

where $\tan \chi = \Delta/\Omega, \theta = \sqrt{\Omega^2 + \Delta^2} \tau,$ and $\Omega$ and $\Delta = \omega - \omega_0$ are Rabi frequency and detuning of the driving light ($\omega$) off its resonance $\omega_0$, respectively.

![FIG. 2. Part of trajectory of results of measurements each of which consists of a drive and a probe pulse applied to the ion.](image)
detuning of the drive frequency $\omega$, the corresponding mean rate oscillates as a result of sampling the effective Rabi nutation frequency $\theta/\tau$ (Fig. 3). The observation of this modulated excitation spectrum proves the coherent nature of the interaction of ion and drive light. Note the first-order sideband, at 1.3 MHz up-tuning, generated by residual vibrational phase modulation of the ionic quadrupole.

Let us turn to the statistics of trajectories, as in Fig. 2. When starting with the ion in the ground state, another "on" event takes place with probability $p_0 = \cos^2(\Omega \tau/2)$. When starting with the ion in the metastable state, an "off" result would take place with same probability, $p_1 = p_0$, if we neglect relaxation, for the moment. In each driving pulse, the light-driven nutation starts anew thanks to the state reduction by the previous probing, and it extends over the pulse duration $\tau$. The next probe pulse reduces the ion again to one of the two energy eigenstates. Then, the probability of finding either "on" ($i = 0$) or "off" ($i = 1$) $q$ times in a sequence, is

$$U(q) = U(1)V(q-1)$$

where

$$V(q) = p_q^q = \cos^{2q}(\Omega \tau/2).$$

In contrast, with state reduction lacking the ion’s evolution would continue coherently over the total time of driving, as long as the laser-induced quadrupole moment survives ($q\tau < \gamma^{-1} = $ time of residual dephasing). This dynamics would require $V(q) = \cos^2(q\Omega \tau/2)$.

Actually the ion dynamics is modified by relaxation: The two probabilities $p_i$ must include (i) spontaneous decay, and (ii) phase ("transverse") relaxation of the ionic quadrupole. The Bloch equations for a spin system in-clude these processes [23]. From an analytic solution on resonance ($\Delta = 0$) [24] one derives

$$p_i = 1 - f_i B_i \left(1 - \sqrt{1 + \tan^2 \epsilon_i} \ e^{-(a+b)\cos(\theta - \epsilon_i)}\right),$$

where $\tan \epsilon_1 = (a - b)/(\Omega \tau)^2$, $\tan \epsilon_0 = (a + b)/\theta$, and $B_i = B_0(\Omega \tau)^2$, $B_1 = 1 - B_0$, $2a = \gamma \tau = \gamma_{ph} \tau + (\Gamma/2) \tau$, $2b = \Gamma \tau$, $\theta^2 = (\Omega \tau)^2 - (a - b)^2$. The supplementary factor $f_i$ is unity for non-degenerate levels, the net probability of excitation is $p_{01} = 1 - p_0$, and $\gamma_{ph}$ is the rate of phase diffusion, and $\Gamma/2$ is the decay rate of state $D_{5/2}$.

The probabilities $p_0$ and $p_1$ no longer agree. The rate $\gamma_{ph}$ results from residual phase fluctuations of the drive laser. A simple model of phase diffusion [23] yields $\gamma_{ph} = D/2$. The diffusion constant $D$ is related to the phase variance as $\langle (\delta \varphi)^2 \rangle = D \tau$, such that the standard deviation of the drive’s phase is $\delta \varphi = \sqrt{\langle (\delta \varphi)^2 \rangle} = \sqrt{2} \sqrt{a - b}$.

![FIG. 3. Probability of excitation, vs detuning $\nu - \nu_o$ by 20-kHz steps of the drive (top). Note the first-order vibronic sideband at 1.3 MHz. Within a small range close to resonance, detuning replaces variation of the drive-pulse length $\tau$. The spectrum of absorption is superimposed by stroboscopic sampling of the ion’s Rabi modulation, as demonstrated by a simulation with small steps on an expanded scale (bottom).]

The $S_{1/2}$ ground state is resolved in two Zeeman-split sublevels one of which only is excited by the drive light, leaving $f_0 = 1/2$, whereas the less resolved upper state $D_{5/2}$ suggests $f_1 = 1$. From a fit of $p_01$ to its contrast of modulation, i.e., to the extreme values of the data sets 7 and 9 close to resonance in Fig. 3, one derives the values $B_0 = 0.49$ and $a + b \simeq 0.38$. The approximate phase of nutation achieved by the 2-ms-long driving pulse is derived from the contrast on the wing of the power-broadened resonance (data sets no. 15 and 16) using eq. (1): $\theta_{app} \simeq \Omega \tau \simeq 578 \times 2\pi$. A better value is revealed by the increment of nutation per each step of detuning, $\delta \omega = 20k \times z \times 2\pi$, that is $\delta \theta = \sqrt{\theta^2 + (\gamma \delta \omega)^2} - \theta = 0.5 \pi$ mod $2\pi$. Compatible with $\theta_{app}$ is only $\delta \theta = 1.25 \times 2\pi$, and $\theta = 2\pi n + \theta'$, where the integer $n \simeq 640$, and $\theta'$ is a fraction of $2\pi$.

The numbers of sequences made up of $q$ identical results have been evaluated from each trajectory, and they are identified with $U(q)$. Fig. 4 shows data of $U(q)/U(1)$, recorded at the indicated three settings of the frequency detuning close to the principal resonance of Fig. 3. It shows also values of $V(q-1)$ calculated from the solutions of $p_0$ and $p_1$, and multiplied by the factor $(N - q + 1)/N$ that takes care of the finite length of the data sets. From the fit to the "on" sequences of data set 7, $a + b = 0.395$ and $\theta' = (1 + 10^{-4})\pi$ are derived. The constant of total relaxation is inserted when fitting the "on" statistics of the neighbouring data sets for their fractional phases.
\( \theta' \). The same phase values \( \theta' \) attributed to the "off" statistics require \( f_1 \) decreasing from unity with increasing transition probability \( p_{10} \): Cycles of spontaneous decay and reexcitation increasingly contribute to the "off" sequences and redistribute the ion over the \( D_{5/2} \) sublevels such that potential deexcitation becomes selective, and \( f_1 \) is expected to approach 1/6.

The statistical distribution of sequences of such measurements identifies the degree to which the measurements, interrupted by state reductions, and the retardation cannot be attributed to dephasing. The frustrated attempts of detecting fluorescence qualify as quantum non-demolition measurements of the internal ion energy. The statistical distribution of sequences of such measurements agrees with the evolution of the ion's wave function assumed interrupted by state reductions, and the retardation cannot be attributed to dephasing. The frustrated attempts of detecting fluorescence qualify as quantum non-demolition measurements of the internal ion energy. The statistical distribution of sequences of such measurements identifies the degree to which the measurements, intertwined among the pulses of driving, have impeded the quantum evolution of the ion.

In conclusion, we have demonstrated the inhibition of the evolution of a quantum system by repeated measurements on the system, i.e., the quantum Zeno effect, by alternately driving and probing an individual \(^{172}\)Yb\(^{+}\) ion on a weak (E2) and a strong (resonance) transition, respectively. We have revealed the statistics of uninterrupted sequences of observations which find the ion either in the ground state subjected to resonance scattering of the probe light, or in the \( D_{5/2} \) state that does not show resonance scattering since it is decoupled from the probe resonance. The evolution derived from this statistics agrees with the evolution of the ion's wave function assumed interrupted by state reductions, and the retardation cannot be attributed to dephasing. The frustrated attempts of detecting fluorescence qualify as quantum non-demolition measurements of the internal ion energy. The statistical distribution of sequences of such measurements identifies the degree to which the measurements, intertwined among the pulses of driving, have impeded the quantum evolution of the ion.

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FIG. 4. Probability \( U(q)/U(1) \) of uninterrupted sequences of \( q \) "on" results (white dots) and "off" results (black dots). The lines show the distributions of probabilities \( V(q - 1) \) for the ion's evolution on its drive transition, according to Eqs. 2 and 3. \( \theta' \) and \( f_1 \) from fit; values \( f_1 < 1 \) indicate redistribution, over sublevels, by cycles of spontaneous decay and reexcitation.

The agreement of \( U(q)/U(1) \) and \( V(q - 1) \) confirms state reduction by each measurement that goes along with probing the ion's resonance scattering, and it proves the coherent evolution of the ion to set out again after each of these state reductions. The statistics of "no count" sequences provides such a proof even under QND conditions. The overall effect of the reiterated resettings is the ion's quantum dynamics being impeded.

Since \( \theta \gg 2\pi \), the resonance of Fig. 3 is substantially power-broadened, and the other natural mode of relaxation given by \( a - b \) cannot be determined. However, \( a + b \) is compatible with the lifetime of the \( D_{5/2} \) level (5.7ms [20], 7.2ms [25]). It serves also as an upper limit of \( a \): For the standard deviation of the driving phase, the limit yields \( \delta \varphi < 1.2 \approx 2\pi \). This phase fluctuation corresponds to the bandwidth less than 100Hz, which is compatible with the controlled 1-s frequency bandwidth, \( \delta \nu = \delta \varphi/\tau \lesssim 500 \) Hz. This restriction secures the interaction of light and ion being coherent during the pulse length \( \tau \), and it proves that not dephasing but the effect of state reduction in the act of measurement is responsible for the observed values of the joint probabilities \( U(q) \). Moreover, the data demonstrate that an individual atom interacts coherently on the drive transition. So far coherent manipulation has been demonstrated with a Raman [26] or a vibronic [22] transition.

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