NEW STRATEGIES FOR EXTRACTING $V_{ub}$

Zoltan Ligeti
Department of Physics, University of California San Diego
La Jolla, CA 92093–0319, USA

Abstract

The prospects for a precise and model independent determination of $|V_{ub}|$ from inclusive and exclusive semileptonic $B$ decays are reviewed.

I. INTRODUCTION

The next generation of $B$ decay experiments will test the flavor sector of the standard model with high precision. The basic approach is to determine the elements of the CKM matrix using different methods and then check for the consistency of these results. In practice this amounts to determining the sides and angles of the unitarity triangle from $CP$ conserving decays and from $CP$ asymmetries, respectively. For these checks to be meaningful, a precise and model independent determination of the $b \to u$ CKM matrix element, $|V_{ub}|$, is very important.

$CP$ violation has only been observed in kaon decay arising from $K^0 – \bar{K}^0$ mixing. This observation can be accommodated in the three generation standard model using the otherwise free parameter $\delta$ in the CKM matrix, but this description of $CP$ violation has not yet been tested. Moreover, to explain the baryon asymmetry of the universe, other sources of $CP$ violation are needed [1]. Many extensions of the standard model have new particles with weak scale masses, and could give large contributions to $CP$ asymmetries or flavor changing neutral current processes (like $K^0 – \bar{K}^0$ mixing, $B^0 – \bar{B}^0$ mixing, $B \to K^{*} \gamma$, etc.).

At the present time, the magnitude of $V_{ub}$ is determined by comparing experimental results on the endpoint region of the electron spectrum in inclusive $B$ decays with phenomenological models [2], or by comparing experimental results on $B \to \rho \ell \bar{\nu}$ and $B \to \pi \ell \bar{\nu}$ with phenomenological models and lattice QCD results [3]. These two approaches yield remarkably consistent determinations of $|V_{ub}|$, but have large theoretical uncertainties [1].

$V_{ub}$ is one of the least precisely known parameters of the standard model. The other poorly known CKM matrix element, $V_{td}$, is related to $V_{ub}$ through the unitarity triangle.
To reduce these uncertainties, it will be advantageous to consider different observables. The problem is that there is almost no overlap between quantities sensitive to $|V_{ub}|$ which can be reliably calculated theoretically and those which can be measured experimentally. Here I discuss two proposals which bridge this gap to some extent. In Section II the possibility of extracting $|V_{ub}|$ from the hadron invariant mass spectrum in inclusive semileptonic $B$ decay is reviewed. Section III concerns extracting $|V_{ub}|$ from a double ratio of form factors in exclusive semileptonic $B$ and $D$ decays to $\rho$ and $K^*$. Section IV contains some conclusions.

II. $V_{ub}$ FROM INCLUSIVE $B$ DECAYS

The traditional method for extracting $|V_{ub}|$ from experimental data involves a study of the electron energy spectrum in inclusive charmless semileptonic $B$ decay [2]. In the $B$ rest frame, electrons with energies in the endpoint region $E_e > (m_B^2 - m_D^2)/2m_B$ must arise from $b \rightarrow u$ transitions. There has been considerable theoretical progress recently in our understanding of inclusive semileptonic $B$ decay [1-3], based on the use of the operator product expansion (OPE) and heavy quark effective theory. At leading order in the $\Lambda_{QCD}/m_b$ expansion the $B$ meson decay rate is equal to the $b$ quark decay rate. There are no nonperturbative corrections of order $\Lambda_{QCD}/m_b$. In the electron endpoint region our calculational ability is lost, since the size of this region $m_D^2/2m_B \approx 330$ MeV is comparable to $(m_B - m_b)/2$. An infinite set of higher order terms in the OPE, which extend the endpoint from $m_b/2$ to $m_B/2$, yield singular contributions to $d\Gamma/dE_e$ that are equally important integrated over such a small endpoint region.

In the future, it may be possible to determine $|V_{ub}|$ from a comparison of the measured hadronic invariant mass spectrum in the region $s_H < m_D^2$ with theoretical predictions [4,5]. Here $s_H = (p_B - q)^2$, where $p_B$ is the $B$ meson four-momentum, and $q = p_e + p_\nu$ is the sum of the lepton four-momenta. An obvious advantage to studying this quantity rather than the lepton energy spectrum is that most of the $B \rightarrow X_u \, e\bar{\nu}$ decays are expected to lie in the region $s_H < m_D^2$, while only a small fraction of the $B \rightarrow X_u \, e\bar{\nu}$ decays have electron energies in the endpoint region. Both the invariant mass region, $s_H < m_D^2$, and the electron endpoint region, $E_e > (m_B^2 - m_D^2)/2m_B$, receive contributions from hadronic final states with invariant masses that range up to $m_D$. However, for the electron endpoint region the contribution of the states with masses nearer to $m_D$ is kinematically suppressed. This region is dominated by the $\pi$ and the $\rho$ in the ISGW model [6], with higher mass states making only a small contribution. The situation is very different for the low invariant mass region, $s_H < m_D^2$. Now all states with invariant masses up to $m_D$ contribute without any preferential weighting towards the lowest mass ones. In the ISGW model the $\pi$ and the $\rho$ mesons comprise only about a quarter of the $B$ decays to states with $s_H < m_D^2$. Consequently, it is much more likely that the first few terms in the OPE will provide an accurate description of $B$ semileptonic decay in the region $s_H < m_D^2$ than in the endpoint region of the electron energy spectrum. In fact, from a theoretical point of view, the cut $s_H < m_D^2$ provides the optimal kinematical separation between inclusive $b \rightarrow u$ and $b \rightarrow c$ decays. A modest cut on the electron energy, which will probably be required...
experimentally for the direct measurement of $s_H$ via the neutrino reconstruction technique, will not destroy this conclusion.

To begin with, consider the contribution of dimension three operators in the OPE to the hadron mass squared spectrum in $B \to X_u e \bar{\nu}$ decay. This is equivalent to $b$ quark decay and implies a result for $d\Gamma/dE_0 ds_0$ (where $E_0 = p_b \cdot (p_b - q)/m_b$ and $s_0 = (p_b - q)^2$ are the energy and invariant mass of the strongly interacting partons arising from the $b$ quark decay) that can easily be calculated using perturbative QCD up to order $\alpha_s^2\beta_0$. Even at this leading order in the OPE there are important nonperturbative effects that come from the relation between the $b$ quark mass and the $B$ meson mass, $m_B = m_b + \bar{\Lambda} + \mathcal{O}(\Lambda_{QCD}^2/m_b)$. The most significant effect comes from $\bar{\Lambda}$, and it relates the hadronic invariant mass $s_H$ to $s_0$ and $E_0$ via

$$s_H = s_0 + 2\bar{\Lambda}E_0 + \bar{\Lambda}^2. \quad (1)$$

Changing variables from $(s_0, E_0)$ to $(s_H, E_0)$ and integrating $E_0$ over the range

$$\sqrt{s_H} - \bar{\Lambda} < E_0 < \frac{1}{2m_B(s_H - 2\bar{\Lambda}m_B + m_B^2)}, \quad (2)$$

gives $d\Gamma/ds_H$, where $\bar{\Lambda}^2 < s_H < m_B^2$. Feynman diagrams with only a $u$-quark in the final state contribute at $s_0 = 0$, which corresponds to the region $\bar{\Lambda}^2 < s_H < \bar{\Lambda}m_B$.

Although $d\Gamma/ds_H$ is integrable in perturbation theory, powers of $\alpha_s\ln^2[(s_H - \bar{\Lambda}m_B)/m_B^2]$ occur in the invariant mass spectrum. This shows that perturbative and nonperturbative corrections are both important for $s_H \lesssim \bar{\Lambda}m_B$. (In the $m_b \to \infty$ limit perturbative corrections are important in a slightly larger region since $\alpha_s\ln(s_H/m_B^2) \sim 1$ for $s_H \sim \bar{\Lambda}m_B$.) While $d\Gamma/ds_H$ cannot be reliably predicted for $s_H \lesssim \bar{\Lambda}m_B$, the behavior of the spectrum for $s_H \lesssim \bar{\Lambda}m_B$ becomes less important for observables that average over larger regions of the spectrum, such as $d\Gamma/ds_H$ integrated over $s_H < \Delta^2$, with $\Delta^2$ significantly greater than $\bar{\Lambda}m_B$.

In Fig. 1 we plot the quantity $\hat{\Gamma}(\Delta^2, \bar{\Lambda})$ defined by

$$\int_0^{\Delta^2} ds_H \frac{d\Gamma(B \to X_u e \bar{\nu})}{ds_H} = \frac{G_F^2m_b^5}{192\pi^3}|V_{ub}|^2 \left(1 - \frac{\bar{\Lambda}}{m_B}\right)^5 \hat{\Gamma}(\Delta^2, \bar{\Lambda}), \quad (3)$$

as a function of $\Delta^2$ for $\bar{\Lambda} = 0.2, 0.4$ and 0.6 GeV in the range $\bar{\Lambda}m_B < \Delta^2 < 4.5\text{GeV}^2$, including terms up to order $\alpha_s^2\beta_0$ (using $\alpha_s(m_b) = 0.2$). These curves approach $\hat{\Gamma}(m_B^2, \bar{\Lambda}) \approx 0.73$ as $\Delta^2 \to m_B^2 \bar{\Lambda}$. $\hat{\Gamma}(\Delta^2, \bar{\Lambda})/\hat{\Gamma}(m_B^2, \bar{\Lambda})$ is the fraction of events with hadronic invariant mass less than $\Delta^2$. It is mostly the ability to compute $\hat{\Gamma}(\Delta^2, \bar{\Lambda})$ and our knowledge of the value of $\bar{\Lambda}$ which determine the uncertainty from theory in a value of $|V_{ub}|$ extracted from the invariant mass spectrum in the region $s_H < \Delta^2$.

In the low mass region, $s_H \lesssim \bar{\Lambda}m_B$, nonperturbative corrections from higher dimension operators in the OPE are very important. Just as in the case of the electron spectrum in the endpoint region, the most singular terms can be identified and summed into a shape function, $S(s_H)$. Neglecting perturbative QCD corrections, we write

$$\frac{d\Gamma}{ds_H} = \frac{G_F^2m_b^5}{192\pi^3}|V_{ub}|^2 S(s_H). \quad (4)$$
The function $\hat{\Gamma}(\Delta^2, \bar{\Lambda})$ defined in Eq. (3) as a function of $\Delta^2$ for $\bar{\Lambda} = 0.2 \text{ GeV}$ (dotted curve), $0.4 \text{ GeV}$ (solid curve), and $0.6 \text{ GeV}$ (dashed curve). Note that $m_D^2 = 3.5 \text{ GeV}^2$.

It is convenient to introduce the scaled variable $y = s_H / \bar{\Lambda} m_b$. Then

$$
\hat{S}(y) = \sum_{n=0}^{\infty} \frac{(-1)^n A_n}{n! \bar{\Lambda}^n} \frac{d^n}{dy^n} \left[ 2y^{n+2} (3 - 2y) \theta(1 - y) \right],
$$

where $\hat{S}(y) = \bar{\Lambda} m_b S(s_H)$ is dimensionless. The matrix elements $A_n$ are the same ones that determine the shape functions for the semileptonic $B$ decay electron energy spectrum in the endpoint region, and also the photon energy endpoint region in weak radiative $B$ decay,

$$
\langle B(v) | \bar{h}_v^{(b)} iD_{\mu_1} \ldots iD_{\mu_n} h_v^{(b)} | B(v) \rangle / 2m_B = A_n v_{\mu_1} \ldots v_{\mu_n} + \text{(terms involving } g_{\mu_i\mu_j}) \rangle.
$$

The $A_n$’s have dimension of $[\text{mass}]^n$, and hence the coefficients $A_n / \bar{\Lambda}^n$ are dimensionless numbers of order one. The first few $A_n$’s are $A_0 = 1$, $A_1 = 0$, $A_2 = -\lambda_1 / 3$, etc.

The shape function $\hat{S}(y)$ is an infinite sum of singular terms which gives an invariant mass spectrum that leaks out beyond $y = 1$ (i.e., $s_H = \bar{\Lambda} m_b$). For $y \sim 1$, all terms in Eq. (5) are formally of equal importance. Since $\bar{\Lambda} m_b \sim 2 \text{ GeV}^2$ is not too far from $m_D^2$, it is necessary to estimate the influence of the nonperturbative effects on the fraction of $B$ decays with invariant hadronic mass squared less than $\Delta^2$. It is difficult to obtain a model-independent estimate of the leakage of events above an experimental cutoff $s_H = \Delta^2$, given that we can estimate only the first few moments, $A_n$. However, in the ACCM model with reasonable parameters, the shape function $\hat{S}(y)$ causes a small (i.e., $\sim 4\%$ with $\bar{\Lambda} = 0.4 \text{ GeV}$, and perturbative QCD corrections neglected) fraction of the events to have $s_H > m_D^2$. Moreover, this leakage depends primarily on $\bar{\Lambda}$, and only to a lesser extent on other ingredients of the model.

Thus the analysis of both perturbative and nonperturbative corrections implies that the uncertainty in the determination of $|V_{ub}|$ from the hadronic invariant mass spectrum in the region $s_H < m_D^2$, is largely controlled by the uncertainty in $\bar{\Lambda}$, or equivalently, by that in the $b$
quark mass. (There is a so-called renormalon ambiguity in $\bar{\Lambda}$ and in the $b$ quark pole mass. For the physically measurable quantity in Eq. (3), this is cancelled by a similar ambiguity in the perturbative series in $\hat{\Gamma}$.) To measure $|V_{ub}|$ with a theoretical uncertainty below $\sim 10\%$, $\bar{\Lambda}$ has to be determined \[4\] (with better than $\sim 100\text{ MeV}$ uncertainty), and the cut on $s_H$ has to be as close to $m^2_{D}$ as possible. If the experimental resolution forces one to consider a significantly smaller region then the theoretical uncertainties will be larger.

III. $V_{ub}$ FROM EXCLUSIVE $B$ DECAYS

Heavy quark symmetry \[13\] is much less predictive for heavy to light decays than it is for heavy to heavy transitions. In the infinite mass limit not all form factors are related to one another, and their normalization is not fixed at any kinematic point. There are still relations between semileptonic $B$ and $D$ decays to the same charmless exclusive final state \[16\], such as between $B \to \rho \ell \bar{\nu}$ and $D \to \rho \ell \bar{\nu}$, or between $B \to \pi \ell \bar{\nu}$ and $D \to \pi \ell \bar{\nu}$. The order $1/m_{c,b}$ corrections to the infinite mass limit may be sizable, however, and for final state pions there are additional complications since $m_c$ is comparable to the mass difference between the vector and pseudoscalar mesons \[17\]. The question is whether we can construct an observable sensitive to $V_{ub}$ that is (almost) free of $1/m_{c,b}$ corrections.

The basic idea \[18,16\] is to use heavy quark symmetry to relate the $SU(3)$ violation between $D \to K^* \ell \nu$ and the Cabibbo suppressed decay $D \to \rho \ell \bar{\nu}$ to those that occur in a comparison of $B \to K^* \ell \bar{\nu}$ (or $B \to K^* \nu \bar{\nu}$) with $B \to \rho \ell \bar{\nu}$. Then experimental data on $B \to K^* \ell \bar{\nu}$ in conjunction with data on $D \to \rho \ell \bar{\nu}$ and $D \to K^* \ell \bar{\nu}$ can be used to determine $|V_{ub}|$. This proposal is complementary to other approaches for determining $|V_{ub}|$, since it relies on the standard model correctly describing the rare flavor changing neutral current process $B \to K^* \ell \bar{\nu}$.

We denote by $g^{(H \to V)}$, $f^{(H \to V)}$, and $a_{\pm}^{(H \to V)}$ the form factors relevant for semileptonic transitions between a pseudoscalar meson containing a heavy quark $H$ ($H = B, D$), and a member of the lowest lying multiplet of vector mesons $V$ ($V = \rho, K^*, \omega$),

\begin{align}
\langle V(p', \epsilon) | \bar{q} \gamma_\mu Q | H(p) \rangle &= i g^{(H \to V)} \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p + p')^\lambda (p - p')^\sigma, \\
\langle V(p', \epsilon) | \bar{q} \gamma_\mu \gamma_5 Q | H(p) \rangle &= f^{(H \to V)} \epsilon_\mu^{*} + a_{+}^{(H \to V)} (\epsilon^* \cdot p) (p + p')_\mu + a_{-}^{(H \to V)} (\epsilon^* \cdot p) (p - p')_\mu.
\end{align}

(7)

We view the form factors as functions of the dimensionless variable $y = v \cdot v'$, where $p = m_H v$, $p' = m_V v'$, and $q^2 = (p - p')^2 = m^2_{H} + m^2_{V} - 2m_{H} m_{V} y$. (Although we are using the variable $v \cdot v'$, we are not treating the quarks in $V$ as heavy.) Assuming nearest pole dominance for the $q^2$ dependences, the $D \to K^* \ell \nu$ form factors are \[19\]

\begin{align}
f^{(D \to K^*)}(y) &= \frac{(1.9 \pm 0.1) \text{ GeV}}{1 + 0.63 (y - 1)}, \\
a_{+}^{(D \to K^*)}(y) &= -\frac{(0.18 \pm 0.03) \text{ GeV}^{-1}}{1 + 0.63 (y - 1)}, \\
g^{(D \to K^*)}(y) &= -\frac{(0.49 \pm 0.04) \text{ GeV}^{-1}}{1 + 0.96 (y - 1)}.
\end{align}

(8)
The shapes of these form factors are beginning to be probed experimentally [19]. The form factor $a_-$ is not measured because its contribution to the $D \to K^* \bar{\ell} \nu$ decay amplitude is suppressed by the lepton mass. These form factors are measured over the kinematic region $1 < y < (m_D^2 + m_{K^*}^2)/(2m_D m_{K^*}) \simeq 1.3$. Note that $f(y)$ changes by less than 20% over this range. The full kinematic region for $B \to \rho \ell \bar{\nu}$ is much larger, $1 < y < 3.5$. In the following analysis we will extrapolate the measured $D \to K^*$ form factors to $1 < y < 1.5$. (The validity of this extrapolation can be tested [20].)

The differential decay rate for semileptonic $B$ decay (neglecting the lepton mass, and not summing over the lepton type $\ell$) is

$$
\frac{d\Gamma(B \to \rho \ell \bar{\nu})}{dy} = \frac{G_F^2}{48\pi^3} m_B m_{\rho}^2 S^{(B \to \rho)}(y) .
$$

Here $S^{(H \to V)}(y)$ is the function

$$
S^{(H \to V)}(y) = \sqrt{y^2 - 1} \left[ f^{(H \to V)}(y) \right]^2 (2 + y^2 - 6 y r + 3 r^2)
+ 4 \text{Re} \left[ a_+^{(H \to V)}(y) f^{(H \to V)}(y) \right] m_H^2 r (y - r)(y^2 - 1)
+ 4 \left| a_+^{(H \to V)}(y) \right|^2 m_H^4 r^2 (y^2 - 1)^2 + 8 \left| g^{(H \to V)}(y) \right|^2 m_H^4 r^2 (1 + r^2 - 2 y r)(y^2 - 1)
= \sqrt{y^2 - 1} \left[ f^{(H \to V)}(y) \right]^2 (2 + y^2 - 6 y r + 3 r^2) [1 + \delta^{(H \to V)}(y) ]
$$

with $r = m_V/m_H$. The function $\delta^{(H \to V)}$ depends on the ratios of form factors $a_+^{(H \to V)}/f^{(H \to V)}$ and $g^{(H \to V)}/f^{(H \to V)}$, $S^{(B \to \rho)}(y)$ can be estimated using combinations of $SU(3)$ flavor symmetry and heavy quark symmetry. $SU(3)$ symmetry implies that the $\bar{B}^0 \to \rho^+$ form factors are equal to the $B \to K^*$ form factors and the $B^- \to \rho^0$ form factors are equal to $1/\sqrt{2}$ times the $B \to K^*$ form factors. Heavy quark symmetry implies the relations [13]

$$
(f, a_+, g)^{(B \to K^*)} = \left( \frac{m_B}{m_D} \right)^{1/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} (f, a_+, g)^{(D \to K^*)},
$$

where we used $a_-^{(D \to K^*)} = -a_+^{(D \to K^*)}$, valid in the large $m_c$ limit.

Using Eq. (12) and $SU(3)$ symmetry to get $\bar{B}^0 \to \rho^+ \ell \bar{\nu}$ form factors (in the region $1 < y < 1.5$, corresponding to $q^2 > 16$ GeV$^2$) from those for $D \to K^* \bar{\ell} \nu$ given in Eq. (8) yields $S^{(B \to \rho)}(y)$ plotted in Fig. 1 of Ref. [18]. (The numerical values in Eq. (8) differ slightly from those used in Ref. [18].) This prediction for $S^{(B \to \rho)}$ can be used to determine $|V_{ub}|$ from the $B \to \rho \ell \bar{\nu}$ semileptonic decay rate in the region $1 < y < 1.5$. We find that about 20% of $\bar{B}^0 \to \rho^+ \ell \bar{\nu}$ decays are in the range $1 < y < 1.5$, and $\mathcal{B}(B^0 \to \rho^+ \ell \bar{\nu}) \bigg|_{y<1.5} = 5.9 |V_{ub}|^2$. This method is model independent, but cannot be expected to yield a very accurate value of $|V_{ub}|$. Typical $SU(3)$ violations are at the 10–20% level and similar violations of heavy quark symmetry are expected. In this region $|\delta^{(B \to \rho)}(y)|$ defined in Eq. (10) is less than 0.06, indicating that $a_+^{(B \to \rho)}$ and $g^{(B \to \rho)}$ make only a small contribution to the differential rate in this region. Thus the main uncertainties are $SU(3)$ and heavy quark symmetry violations in the $f^{(H \to V)}$ form factor only; these are precisely the ones we can eliminate.
FIG. 2. Feynman diagram that gives the leading contribution to \( R(1) - 1 \). The dashed line is a \( \pi \) or an \( \eta \). The black square indicates insertion of the weak current.

Ref. [18] proposed a method for getting a value of \( S^{(B \to \rho)}(y) \) with small theoretical uncertainty using a “Grinstein-type” [21] double ratio

\[
R(y) = \left[ \frac{f^{(B \to \rho)}(y)}{f^{(B \to K^*)}(y)} \right]/\left[ \frac{f^{(D \to \rho)}(y)}{f^{(D \to K^*)}(y)} \right],
\]

which is unity in the limit of \( SU(3) \) symmetry or in the limit of heavy quark symmetry. Corrections to the prediction \( R(y) = 1 \) are suppressed by \( m_s/m_{c,b} \) \((m_{u,d} \ll m_s)\) instead of \( m_s/\Lambda_{QCD} \) or \( \Lambda_{QCD}/m_{c,b} \). The leading deviation of \( R(1) \) from unity arising from the Feynman diagrams in Fig. 2 has been estimated using chiral perturbation theory. These yield a calculable non-analytic \( \sqrt{m_q} \) dependence on the light quark masses, and such corrections cannot arise from other sources. The result is \( R(1) = 1 - 0.035 g g_2 \) [23], where \( g \) is the \( DD^*\pi \) coupling and \( g_2 \) is the \( \rho\omega\pi \) coupling. Experimental data on \( \tau \to \omega \pi \nu_\tau \) decay gives \( g_2 \simeq 0.6 \) [23]. Estimates of \( g \) vary between near unity and much smaller values [24]. There may be significant corrections to \( R(1) \) from higher orders in chiral perturbation theory. However, the smallness of our result lends support to the expectation that \( R(1) \) is very close to unity. There is no reason to expect any different conclusion over the kinematic range \( 1 < y < 1.5 \).

Since \( R(y) \) is very close to unity, the relation

\[
S^{(B \to \rho)}(y) = S^{(B \to K^*)}(y) \left| \frac{f^{(D \to \rho)}(y)}{f^{(B \to K^*)}(y)} \right|^2 \left( \frac{m_B - m_\rho}{m_B - m_{K^*}} \right)^2,
\]

and measurements of \( |f^{(D \to K^*)}|, |f^{(D \to \rho)}|, \) and \( S^{(B \to K^*)} \) will determine \( S^{(B \to \rho)} \) with small theoretical uncertainty. The last term on the right hand side makes Eq. (13) equivalent to Eq. (12) in the \( y \to 1 \) limit. The ratio of the \((2 + y^2 - 6y\tau + 3\tau^2) [1 + \delta^{(B \to V)}(y)] \) terms makes only a small and almost \( y \)-independent contribution to \( S^{(B \to \rho)}/S^{(B \to K^*)} \) in the range \( 1 < y < 1.5 \). Therefore, corrections to Eq. (13) are at most a few percent larger than those to \( R(y) = 1 \).

\( |f^{(D \to K^*)}| \) has already been determined. \( |f^{(D \to \rho)}| \) may be obtainable in the future, for example from experiments at \( B \) factories, where improvements in particle identification help reduce the background from the Cabibbo allowed decay. The measurement \( \mathcal{B}(D \to \rho^0 \bar{\ell} \nu)/\mathcal{B}(D \to \bar{K}^{*0} \bar{\ell} \nu) = 0.047 \pm 0.013 \) [23] already suggests that \( |f^{(D \to \rho)}/f^{(D \to K^*)}| \) is close to unity. Assuming \( SU(3) \) symmetry for the form factors, but keeping the explicit \( m_\tau \)-dependence in \( S^{(D \to V)}(y) \) and in the limits of the \( y \) integration, the measured form factors in Eq. (8) imply \( \mathcal{B}(D \to \rho^0 \bar{\ell} \nu)/\mathcal{B}(D \to \bar{K}^{*0} \bar{\ell} \nu) = 0.044 \) [22].

\( S^{(B \to K^*)} \) is obtainable from experimental data on \( B \to K^* \ell \bar{\ell} \) or \( B \to K^* \nu \bar{\nu} \). While the latter process is very clean theoretically, it is very difficult experimentally. A more realistic goal is to use \( B \to K^* \ell \bar{\ell} \), since CDF expects to observe \( 400 - 1100 \) events in the Tevatron Run II (if
the branching ratio is in the standard model range) \cite{26}. The uncertainties associated with long distance nonperturbative strong interaction physics in this extraction of $S^{(B\to K^*)}(y)$, averaged over the region $1 < y < 1.5$, are probably less than 10\% \cite{22}. Consequently, a determination of $|V_{ub}|$ from experimental data on $D \to K^* \ell \bar{\nu}$, $D \to \rho \ell \bar{\nu}$, $B \to K^* \ell \bar{\nu}$ and $B \to \rho \ell \bar{\nu}$ with an uncertainty from theory of about 10\% is feasible. If a precise value of $|V_{ub}|$ is available before $B \to K^* \ell \bar{\nu}$ is measured, then we get an accurate standard model prediction for the $B \to K^* \ell \bar{\nu}$ decay rate in the region $1 < y < 1.5$. Comparison with data may signal new physics or provide stringent constraints on extensions of the standard model.

**IV. CONCLUSIONS**

The present determinations of $|V_{ub}|$ rely on comparing experimental data with model calculations, and therefore suffer from theoretical uncertainties of order 30\%. (This is hard to quantify, and such a number is necessarily ad hoc.) To reduce these uncertainties one needs to consider somewhat different observables for which the theoretical predictions are less model dependent than those for the endpoint region of the inclusive electron spectrum and for the total exclusive $B \to \pi \ell \bar{\nu}$ or $B \to \rho \ell \bar{\nu}$ decay rates. In this talk I reviewed two ideas which seem promising to me: i) extracting $|V_{ub}|$ from the hadronic invariant mass spectrum in inclusive semileptonic $B$ decays; and ii) using heavy quark and chiral symmetries for form factors of exclusive semileptonic $B$ and $D$ decays to vector mesons. These may lead to model independent determinations of $|V_{ub}|$ with an uncertainty from theory of about 10\%. Lattice calculations \cite{27} and dispersion relation constraints on form factors \cite{20} will also be important.

There is not really one “gold-plated” observable for extracting $|V_{ub}|$. To reduce the strong interaction model dependence, several measurements will be needed to guide us which approximations and expansions have smaller uncertainties. At the 10\% level consistency between different determinations of $|V_{ub}|$ will be necessary to have confidence that the uncertainties are indeed so small. I am hopeful that this will be achieved within the next few years.

**ACKNOWLEDGMENTS**

I am grateful to Adam Falk, Iain Stewart, and Mark Wise for collaboration on the topics discussed in this talk. I thank the organizers for the invitation, and for putting together a very interesting and enjoyable conference. This work was supported in part by the U.S. Dept. of Energy under grant no. DOE-FG03-97ER40506 and by NSF grant PHY-9457911.

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