Next-to-leading order QCD corrections to the top quark decay via model-independent FCNC couplings

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D0 and CDF collaborations at the Fermilab Tevatron have searched for non-standard-model single top-quark production and set upper limits on the anomalous top quark flavor-changing neutral current (FCNC) couplings $\kappa^{q}_{tG}/\Lambda$ and $\kappa^{q}_{tA}/\Lambda$ using the measurement of total cross section calculated at the next-to-leading order (NLO) in QCD. In this Letter, we report on the effect of anomalous FCNC couplings to various decay branching ratios of the top quark, calculated at the NLO. This result is not only mandatory for a consistent treatment of both the top quark production and decay via FCNC couplings by D0 and CDF at the Tevatron but is also important for the study of ATLAS and CMS sensitivity to these anomalous couplings at the CERN LHC. We find that the NLO corrections to the partial decay widths of the three decay channels $t \rightarrow q + g, t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ are at the order of 10% in magnitude and modify their branching ratios by about 20%, 0.4% and 2%, respectively, as compared to their leading order predictions.

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The top quark FCNC processes $t \rightarrow q + V$ ($V = g, \gamma, Z$) have tiny branching ratios in the standard model (SM) and are probably unmeasurable at the CERN Large Hadron Collider (LHC) and future colliders. On the other hand, any positive signal of these rare decay events would definitely imply some new physics beyond the SM. For the LHC, ATLAS collaboration has presented its sensitivity to studying FCNC top decays [5]. These results show that studying top quark FCNC couplings will provide a good probe to new physics beyond the SM. Although there are many discussions in the literature on rare decay processes involving model-independent top quark FCNC couplings [2, 3, 6, 9, 10, 11, 12], most of them were based on LO calculations. However, especially for $t \rightarrow q + g$, due to the large uncertainties from the renormalization scale dependence in its LO prediction through the strong coupling constant $\alpha_s$, it is necessary to improve the theoretical prediction to NLO in order to match the expected experimental accuracy at the LHC.

In this Letter, we present the analytic results of the NLO QCD corrections to the partial decay widths and decay branching ratios of top quark via anomalous FCNC couplings for the three processes $t \rightarrow q + g, t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, which are the most commonly studied decay channels by the experimentalists at the Tevatron and the LHC.

New physics effects involved in top quark FCNC processes can be incorporated in a model-independent way into an effective Lagrangian which includes the dimension-5 operators as listed below [10]

\[
\mathcal{L}_{\text{eff}} = -\frac{e}{\sin 2\theta_W} \sum_{q=u,c} \frac{\kappa_{tq}^g}{\Lambda} \bar{q} a^{\mu\nu} (f_{tq}^g + i h_{tq}^g \gamma_5) t Z_{\mu\nu} - e \sum_{q=u,c} \frac{\kappa_{tq}^a}{\Lambda} \bar{q} a^{\mu\nu} (f_{tq}^a + i h_{tq}^a \gamma_5) t A_{\mu\nu} - g_s \sum_{q=u,c} \frac{\kappa_{tq}^g}{\Lambda} \bar{q} a^{T^a} (f_{tq}^g + i h_{tq}^g \gamma_5) t G_{\mu\nu}^a + \text{H.c.},
\]

where $\Lambda$ is the new physics scale, $\theta_W$ is the Weinberg angle, and $T^a$ are the Gell-Mann matrices. $\kappa_{tq}^V$, with $V = g, \gamma$ and $Z$ are normalized to be real and positive, while $f_{tq}^V$ and $h_{tq}^V$ are complex numbers in general. Since the partial widths in our calculation depend only on the product of $\kappa$ and $|f|^2 + |h|^2$, we could set $|f|^2 + |h|^2 = 1$ by redefining the parameter $\kappa$.

From the effective Lagrangian given by Eq. (1), we obtain the following LO partial decay width of the FCNC top decays in $D = 4 - 2\epsilon$ dimension,

\[
\Gamma_0(t \rightarrow q + g) = \frac{8\alpha_s \Lambda^2}{m_t^2} \frac{(\kappa_{tq}^g)}{\Lambda}^2 C_\epsilon,
\]
\[ \Gamma_0(t \to q + \gamma) = 2\alpha m_t^3 \left( \frac{\gamma^2}{\Lambda^2} \right)^2 C_\gamma, \]

\[ \Gamma_0(t \to q + Z) = \frac{\alpha m_t^3 \beta_Z Z^2}{\sin^2 \theta W} \left( \frac{\gamma Z}{\Lambda} \right)^2 (3 - \beta_Z - 2\epsilon) C_\epsilon, \]

where the masses of light quarks \(q (q = u, c)\) have been neglected, \(\beta_Z \equiv (1 - M_Z^2/m_t^2)^{-1/2}\) and \(C_\epsilon = \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \left( \frac{4\pi^2}{m_t^2} \right)^{\epsilon}. \)

Below, we present in details our calculation for the inclusive decay width of the top quark, decaying into hadrons, up to the NLO with the LO partonic process denoted as \(t \to q + g\). The final results of \(t \to q + \gamma\) and \(t \to q + Z\) are also given for completeness.

At the NLO, we need to include both one-loop virtual gluon corrections and real gluon emission contribution. We use \(D = 4 - 2\epsilon\) dimensional regularization to regulate both ultraviolet (UV) and infrared (IR) (soft and collinear) divergences so that all divergences appear as \(1/\epsilon^\alpha\) with \(\alpha = 1\) and 2. The UV singularities cancel after summing up the contributions from the one-loop virtual diagrams and counterterms according to the same convention used in Ref. [3]. The soft-singularities cancel after adding up the virtual and real radiative corrections. To cancel collinear singularities, we need to also include contribution induced from gluon splitting to light quark pairs at the same order in QCD coupling.

The renormalized virtual corrections to the partial decay width of \(t \to q + g\) is

\[ \Gamma_{\text{virt}}^g = \frac{\alpha_s}{6\pi} \Gamma_0(t \to q + g) \left\{ -\frac{13}{\epsilon_{IR}} + \frac{1}{\epsilon_{IR}} \left[ -13 \ln \frac{4\pi^2}{m_t^2} + 13\gamma_E + N_f - \frac{53}{2} \right] + \left[ -\frac{13}{2} \left( \ln \frac{4\pi^2}{m_t^2} - \gamma_E \right)^2 - 12 \ln \frac{\mu^2}{m_t^2} \right] + \left( N_f - \frac{53}{2} \right) \left( \ln 4\pi - \gamma_E \right) + \frac{55\pi^2}{12} - 23 \right\}. \]

Here \(\mu\) is a renormalization scale and \(N_f\) denotes the number of active flavors at the scale \(\mu\). All the ultraviolet divergences have canceled in \(\Gamma_{\text{virt}}^g\), as they must, but the infrared divergent piece is still present.

The contribution from real gluon emission \((t \to q + g + g)\) is denoted as \(\Gamma_{\text{real}}(t \to q + g + g)\). In order to cancel all the collinear singularities in the sum of virtual and real radiative corrections, we also need to include the contributions from gluon splitting into a pair of quark and anti-quark in the collinear region, which is denoted as \(\Gamma_{\text{real}}(t \to q + q' + \bar{q'})\). Note that there are two configurations of final states when the flavor of the light quark coming from gluon splitting is the same as the light quark directly from the FCNC coupling, and only one configuration when they are different. We find that at the NLO

\[ \Gamma_{\text{real}}(t \to q + g + g) = \frac{\alpha_s}{6\pi} \Gamma_0(t \to q + g) \left( -\frac{13}{\epsilon_{IR}} + \frac{1}{\epsilon_{IR}} \left[ -13 \ln \frac{4\pi^2}{m_t^2} - 13\gamma_E + \frac{53}{2} \right] + \frac{13}{2} \left( \ln \frac{4\pi^2}{m_t^2} - \gamma_E \right)^2 \right) \]

\[ + \gamma_E \left( \ln \frac{4\pi^2}{m_t^2} - \gamma_E \right) - \frac{53}{2} \left( \ln \frac{4\pi^2}{m_t^2} - \gamma_E \right) - \frac{31\pi^2}{4} + \frac{171}{2} \right\}. \]

After adding them together, the total real contributions can be written as

\[ \Gamma_{\text{real}}^g = \Gamma_{\text{real}}(t \to q + g + g) + \Gamma_{\text{real}}(t \to q + q' + q') \]

\[ = \frac{\alpha_s}{6\pi} \Gamma_0(t \to q + g) \left\{ -\frac{13}{\epsilon_{IR}} - \frac{1}{\epsilon_{IR}} \left[ -13 \ln \frac{4\pi^2}{m_t^2} + 13\gamma_E + N_f - \frac{53}{2} \right] + \left[ -\frac{13}{2} \left( \ln \frac{4\pi^2}{m_t^2} - \gamma_E \right)^2 - 12 \ln \frac{\mu^2}{m_t^2} \right] + \left( N_f - \frac{53}{2} \right) \left( \ln 4\pi - \gamma_E \right) + \frac{55\pi^2}{12} - 23 \right\}. \]

Combining the real and virtual contributions, we obtain the full NLO corrections for \(t \to q + g\) as

\[ \Gamma_{\text{NLO}}(t \to q + g) = \Gamma_{\text{virt}}^g + \Gamma_{\text{real}}^g \]

\[ = \frac{\alpha_s}{72\pi} \Gamma_0(t \to q + g) \left[ -12 N_f \ln \left( \frac{\mu^2}{m_t^2} \right) - 36 N_f - 38\pi^2 + 749 \right]. \]

Note that all the infrared divergences have been canceled in Eq. 7 as they should.

As for \(t \to q + \gamma\) and \(t \to q + Z\), the NLO QCD corrections to the partial decay widths are given by

\[ \Gamma_{\text{NLO}}(t \to q + \gamma) = \Gamma_0^\gamma + \Gamma_{\text{real}}^\gamma \]

\[ = \frac{2\alpha_s}{9\pi} \Gamma_0(t \to q + \gamma) \left[ -3 \ln \left( \frac{\mu^2}{m_t^2} \right) - 2\pi^2 + 8 \right]. \]
TABLE I: Branching ratios as functions of $\kappa_{tq}^V/\Lambda$. Here $\mu = m_t$.

| BR [in unit of ($\kappa_{tq}^V/\Lambda$)] | LO | NLO | NLO/LO |
|----------------------------------------|----|-----|--------|
| $t \to q + g$                         | 1.0010 | 1.1964 | 1.195 |
| $t \to q + \gamma$                    | 0.0544 | 0.0542 | 0.996 |
| $t \to q + Z$                         | 0.0448 | 0.0458 | 1.022 |

and

$$
\Gamma_{\text{NLO}}(t \to q + Z) = \Gamma_{\text{LO}}^2 + \Gamma_{\text{NLO}}^2
= \frac{\alpha_s}{3\pi} \Gamma_0(t \to q + Z) \left[ -2 \ln \left( \frac{m_t^2}{\Lambda^2} \right) - \frac{4(9 - \beta_Z^2)}{3 - \beta_Z^2} \ln(1 - \beta_Z^2) 
\right. \\
\left. + \frac{(1 - \beta_Z^2)(1 + 6\beta_Z^2 - 3\beta_Z^4)}{\beta_Z^2(3 - \beta_Z^2)} \ln(1 - \beta_Z^2) \right]
+ 4\text{Li}_2 \left( -\frac{1 - \beta_Z^2}{\beta_Z^2} \right) + \frac{1 + 3\beta_Z^2}{\beta_Z^2(3 - \beta_Z^2)} - \frac{4\pi^2}{3} + \frac{10}{3} \right],
$$

(9)

respectively. Hence, up to the NLO, the partial decay widths of the three FCNC decays can be obtained by $\Gamma(t \to q + V) = \Gamma_0(t \to q + V) + \Gamma_{\text{NLO}}(t \to q + V)$.

In order to study the effect of NLO corrections to decay branching ratios, we define the following branching ratio for latter numerical analysis:

$$
BR_{\text{LO}}(t \to q + V) = \frac{\Gamma_0(t \to q + V)}{\Gamma_0(t \to W + b)},
$$

(10)

$$
BR_{\text{NLO}}(t \to q + V) = \frac{\Gamma(t \to q + V)}{\Gamma(t \to W + b)}.
$$

(11)

The partial decay width for the dominant top quark decay mode of $t \to W + b$ at the NLO can be found in Ref. [13], namely,

$$
\Gamma(t \to W + b) = \Gamma_0(t \to W + b) \left\{ \begin{array}{c} 1 \\
\frac{2\alpha_s}{3\pi} \left[ 2 \left( (1 - \beta_W^2)(2\beta_W^2 - 1)(\beta_W^2 - 2) \right) \ln(1 - \beta_W^2) 
\right. \\
9 - 4\beta_W^2 \ln \beta_W^2 + 2\text{Li}_2(\beta_W^2) \\
\left. - 2\text{Li}_2(1 - \beta_W^2) - \frac{6\beta_W^4 - 3\beta_W^2 - 8}{2\beta_W^2(3 - 2\beta_W^2)} - \pi^2 \right] \right\},
$$

(12)

with

$$
\Gamma_0(t \to W + b) = \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \beta_W^4 (3 - 2\beta_W^2),
$$

(13)

where $\beta_W \equiv (1 - m_W^2/m_t^2)^{1/2}$.

For the numerical calculation of the branching ratios, we take the top quark width given in Ref. [13] with the following parameters [14],

$$(m_W, m_Z, m_t) = (80.398, 91.187, 171.2) \text{GeV},$$

$N_f = 5, \quad \alpha = 1/128, \quad \sin^2 \theta_W = 0.231,$

$$V_{tb} = 1, \quad G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2}.$$ We analyze our results as functions of FCNC couplings $\kappa_{tq}^V/\Lambda$ and renormalization scale $\mu$. For $\mu = m_t$, our numerical results are listed in Table II and Table III, showing the NLO effect to various decay branching ratios and partial decay widths, respectively. From Table II we see that the NLO correction increases the LO branching ratio by about 20% for $t \to q + g$, while the NLO corrections are much smaller for the other two decay modes. But we note that the NLO results modify the LO widths by about 10% in magnitude for all the three modes.

For convenience, we show the branching ratio of $t \to g + g$ as function of $\kappa_{tq}^V/\Lambda$ in Fig. 1 where we have set $\mu = m_t$ as before. Using the upper limits measured by the D0 Collaboration at the Tevatron [11], we get the following results:

$$
\frac{\kappa_{tq}^V}{\Lambda} < 0.15 \text{ TeV}^{-1} \Rightarrow BR(t \to c + g) < 2.69 \times 10^{-2},
$$

$$
\frac{\kappa_{tq}^V}{\Lambda} < 0.037 \text{ TeV}^{-1} \Rightarrow BR(t \to u + g) < 1.64 \times 10^{-3}.
$$

Following the analysis in Ref. [3], we plot the coupling $\kappa_{tq}^V/\Lambda$ as a function of the branching ratio in Fig. 2 where the ATLAS sensitivities for the two different expected integrated luminosities are also exhibited. From Fig. 2 it
is palpable that the NLO prediction improves the sensitivities of the LHC experiments to measuring the top quark FCNC couplings. With an integrated luminosity of 10 fb\(^{-1}\), the ATLAS experiment sensitivities can be translated to the more stringent relations on FCNC couplings:

\[
BR(t \to q + g) \geq 1.3 \times 10^{-3} \Rightarrow \frac{\kappa_{tq}^g}{\Lambda} \geq 0.033 \text{ TeV}^{-1},
\]

\[
BR(t \to q + \gamma) \geq 4.1 \times 10^{-5} \Rightarrow \frac{\kappa_{tq}^\gamma}{\Lambda} \geq 0.028 \text{ TeV}^{-1},
\]

\[
BR(t \to q + Z) \geq 3.1 \times 10^{-4} \Rightarrow \frac{\kappa_{tq}^Z}{\Lambda} \geq 0.082 \text{ TeV}^{-1},
\]

and with an integrated luminosity of 100 fb\(^{-1}\), they can be translated to the more stringent relations:

\[
BR(t \to q + g) \geq 4.2 \times 10^{-4} \Rightarrow \frac{\kappa_{tq}^g}{\Lambda} \geq 0.019 \text{ TeV}^{-1},
\]

\[
BR(t \to q + \gamma) \geq 1.2 \times 10^{-5} \Rightarrow \frac{\kappa_{tq}^\gamma}{\Lambda} \geq 0.015 \text{ TeV}^{-1},
\]

\[
BR(t \to q + Z) \geq 6.1 \times 10^{-5} \Rightarrow \frac{\kappa_{tq}^Z}{\Lambda} \geq 0.036 \text{ TeV}^{-1}.
\]

Lastly, we illustrate the fact that the NLO prediction reduces the theoretical uncertainty in its prediction on the decay branching ratios and partial decay widths of the top quark. We define \(R_{\text{LO}}(\mu) = \Gamma(\mu)/\Gamma(\mu = m_t)\) and \(R_{\text{NLO}}(\mu) = \Gamma(\mu)/\Gamma(\mu = m_t)\), and show the value of \(R(\mu)\) as functions of \(\mu\) for \(t \to q + g\) in Fig. 3. It shows that the theoretical uncertainty from the renormalization scale dependence can be largely reduced to a couple of percent once the NLO calculation is taken into account.

In conclusion, in order to achieve consistent studies for both the top quark production and decay via FCNC couplings, we have calculated the NLO QCD corrections to the three decay modes of the top quark induced by model-independent FCNC couplings of dimension 5 operators. For \(t \to q + g\), the NLO results increase the experimental sensitivity to the anomalous couplings. Our results show that the NLO QCD corrections enhance the LO branching ratio about 20%. Moreover, the NLO QCD corrections vastly reduce the dependence on the renormalization scale, which leads to increased confidence in our theoretical predictions based on these results. For \(t \to q + \gamma\) and \(t \to q + Z\), the NLO corrections are minuscule in branching ratios, albeit they can decrease the LO widths by about 9% and 7%, respectively.

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