GETTING AT THE CP ANGLES

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In anticipation for intensive experimental effort on $B$ physics, theorists have introduced many ingenious ways to measure properties of the unitarity triangle. We review some of these methods. We will be critical in the hope that, if there is any defect in them, it can be remedied either theoretically or experimentally.

1 Introduction

It has been known for more than thirty years that there exists $CP$ violation in the $K^0 \leftrightarrow \bar{K}^0$ transition. This effect can be naturally interpreted by a non-trivial phase of the Kobayashi-Maskawa (KM) matrix for quark flavor mixing in the standard electroweak model. So far, no other evidence for $CP$ violation has been unambiguously established. Some intensive experimental efforts, such as the $B$ factory programs at KEK and at SLAC, are underway to search for large signals of $CP$ asymmetries and to test the KM mechanism of $CP$ violation in the $B$-meson system. It is also expected that the study of $B$ physics can provide a unique opportunity to discover new physics beyond the standard model, in particular, that responsible for the origin of quark masses and $CP$ violation.

Unitarity of the $3 \times 3$ KM matrix allows a geometrical description of $CP$ violation in the complex plane, the so-called unitarity triangle. To meet various possible measurements of $CP$ asymmetries at the forthcoming $B$ factories, theorists have introduced many ingenious ways to determine properties of the unitarity triangle. The aim of this talk is to review some of those methods proposed in the past few years in a realistic manner.

To date, $K^0\bar{K}^0$ mixing ($\Delta S = 2$) and $B^0_d\bar{B}^0_d$ mixing ($\Delta B = 2$) are the only second-order weak transition effects that have been actually observed. If some new physics existed in the $K$-meson or $B$-meson system, it is most likely to show itself in the $\langle K^0|\mathcal{H}|\bar{K}^0 \rangle$ or $\langle B^0|\mathcal{H}|\bar{B}^0 \rangle$ amplitude. New physics is less likely to compete with the first-order weak interactions of the standard model. The presence of new physics in $B^0\bar{B}^0$ mixing will, in general, provide an additional weak phase to $CP$ asymmetries in neutral $B$ decays (due to the interplay of decay and mixing). Thus the measurement of $CP$ violation and the KM unitarity triangle at $B$ factories may serve as a useful approach towards getting at the possible new physics beyond the standard model.
2 Unitarity Triangle

One of the constraints among the KM matrix elements due to unitarity is:

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \]  

(2.1)

This relation corresponds to a triangle in the complex plane, the well-known unitarity triangle (see Fig. 1). Its three inner angles are denoted as

\[ \phi_1 = \text{arg} \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right), \]

\[ \phi_2 = \text{arg} \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right), \]

\[ \phi_3 = \text{arg} \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right). \]  

(2.2)

If penguin and new physics effects are neglected, then \( \phi_1, \phi_2 \) and \( \phi_3 \) can be measured from \( CP \) asymmetries in \( B_d \to \psi K_S, B_d \to \pi^+ \pi^- \) and \( B_s \to \rho^0 K_S \), respectively.

Beyond the standard model, new physics may introduce an additional \( CP \)-violating phase into \( B_d^0-B_d^0 \) or \( B_s^0-B_s^0 \) mixing. In this case, the phases to be measured from the above decay modes (denoted by \( \phi_{\psi K_S}, \phi_{\pi^+ \pi^-} \) and \( \phi_{\rho^0 K_S} \), respectively) may deviate to some extent from the geometrical ones defined in (2.2). For the purpose of simplicity and instruction, we only consider the kinds of new physics that do not violate unitarity of the \( 3 \times 3 \) KM matrix. Then \( B_d^0-B_d^0 \) and \( B_s^0-B_s^0 \) mixing phases can be written as

\[ \left( \frac{q}{p} \right)_{B_d} = \frac{V_{td}^* V_{ub}}{V_{td}^* V_{ub}} e^{2i\phi_{NP}^d}, \quad \left( \frac{q}{p} \right)_{B_s} = \frac{V_{ts}^* V_{tb}}{V_{ts}^* V_{tb}} e^{2i\phi_{NP}^s}, \]  

(2.3)

where \( \phi_{NP}^d \) and \( \phi_{NP}^s \) denote the \( CP \)-violating phases induced by new physics. Neglecting penguin effects and tiny \( CP \) violation in \( K^0-K^0 \) mixing, we arrive
at three angles from three $CP$ asymmetries:

\[
\begin{align*}
\text{Im} \left[ -\left( \frac{q}{p} \right) \frac{V_{cb}V_{cs}^*}{V_{cb}V_{cs}} \right] &= \sin 2(\phi_1 - \phi_{\text{NP}}^d), \\
\text{Im} \left[ +\left( \frac{q}{p} \right) \frac{V_{ub}V_{ud}^*}{V_{ub}V_{ud}} \right] &= \sin 2(\phi_2 + \phi_{\text{NP}}^d), \\
\text{Im} \left[ -\left( \frac{q}{p} \right) \frac{V_{ub}V_{ud}^*}{V_{ub}V_{ud}} \right] &= \sin 2(\phi_3 - \phi_{\text{NP}}^s),
\end{align*}
\]

where the sign “+” (or “−”) comes from the $CP$ even (or odd) final state. For simplicity in subsequent discussions, we define

\[
\phi_{\psi K_S} = \phi_1 - \phi_{\text{NP}}^d, \quad \phi_{\pi^+\pi^-} = \phi_2 + \phi_{\text{NP}}^d, \quad \phi_{\rho^0 K_S} = \phi_3 - \phi_{\text{NP}}^s
\]

as three measurable angles. Two sides of the unitarity triangle, $|V_{ub}V_{ud}|$ and $|V_{cb}V_{cd}|$, have been model-independently measured, and will be improved in the future. A determination of the side $|V_{cb}V_{td}|$ depends on the data of $B_d^0 - \bar{B}_d^0$ mixing which might be affected by the presence of new physics.

Within the standard model, a detailed analysis of presently available data yields the following constraints on three angles of the unitarity triangle:

\[
9^\circ \leq \phi_1 \leq 35^\circ, \quad 45^\circ \leq \phi_2 \leq 148^\circ, \quad 36^\circ \leq \phi_3 \leq 144^\circ.
\]

(2.6)

Note that $\phi_1 + \phi_2 + \phi_3 = 180^\circ$ is a natural consequence of the above geometrical description. If a sum of $\phi_{\psi K_S}$, $\phi_{\pi^+\pi^-}$ and $\phi_{\rho^0 K_S}$ does not amount to $180^\circ$ at an acceptable precision level, then $\phi_{\text{NP}}^s \neq 0$. However, an experimental confirmation of the angle sum rule may not exclusively test the standard model as it is not sensitive to the presence of $\phi_{\text{NP}}^d$. We therefore stress that accurate measurements of both angles and sides are necessary in order to fully construct the unitarity triangle and pin down underlying new physics in the $B$ system.

3 Penguin Pollution

The time-dependent rates of $B_d$ (or $B_s$) transitions to a $CP$ eigenstate $f$ can be given as

\[
\Gamma \left[ B^0(t) \to f \right] \propto e^{-\tau} \left[ \frac{1 + |\bar{\rho}_f|^2}{2} - \frac{1 - |\bar{\rho}_f|^2}{2} \cos(x\tau) \right. \\
\left. - \text{Im} \left( \frac{q}{\rho_f} \bar{\rho}_f \right) \sin(x\tau) \right],
\]

(3.1)
where \( x = \Delta m / \Gamma, \tau = \Gamma t, \tilde{\rho}_f = \langle f|\mathcal{H}|\bar{B}^0|/\langle f|\mathcal{H}|B^0|, and q/p = e^{-2i\Phi_M} \) denotes the mixing phase. Then the \( CP \) asymmetry between these two \( CP \)-conjugate processes reads

\[
A_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}
= A_f^r \cos(x\tau) + A_f^i \sin(x\tau) \tag{3.2}
\]

with

\[
A_f^r = \frac{1 - |\tilde{\rho}_f|^2}{1 + |\tilde{\rho}_f|^2}, \quad A_f^i = \frac{-2}{1 + |\tilde{\rho}_f|^2} \text{Im}(e^{-2i\Phi_M} \tilde{\rho}_f) \tag{3.3}
\]
denoting direct and indirect \( CP \) asymmetries, respectively.

Due to the presence of penguin pollution, which may cause \( A_f^r \neq 0 \), a \( CP \) angle cannot be straightforwardly extracted from the \( CP \) asymmetry \( A_f^i \). To see this point more clearly, we decompose the decay amplitudes as

\[
\langle f|\mathcal{H}|B^0 \rangle = e^{i\Phi_1} e^{i\delta_1} A_1 + e^{i\Phi_2} e^{i\delta_2} A_2, \\
\langle f|\mathcal{H}|\bar{B}^0 \rangle = n_f \left[ e^{-i\Phi_1} e^{i\delta_1} A_1 + e^{-i\Phi_2} e^{i\delta_2} A_2 \right], \tag{3.4}
\]

where \( n_f = \pm 1 \) denotes the \( CP \) parity of \( f \), \( \Phi_1 \) and \( \Phi_2 \) are weak phases, \( \delta_1 \) and \( \delta_2 \) are strong phases, \( A_1 \) and \( A_2 \) are magnitudes of hadronic matrix elements and KM matrix elements. Then we obtain

\[
\tilde{\rho}_f = n_f e^{-2i\Phi_1} \left[ 1 - \frac{2A_2}{A_1} \frac{\sin(\Phi_2 - \Phi_1) e^{i(\delta_2 - \delta_1)}}{1 - e^{i(\Phi_2 - \Phi_1)} e^{i(\delta_2 - \delta_1)} A_2 / A_1} \right]. \tag{3.5}
\]

Without loss of generality one can take \( A_1 > A_2 \), i.e., the decay amplitude is primarily governed by the \( A_1 \) component. Calculating \( \text{Im}(e^{-2i\Phi_M} \tilde{\rho}_f) \) up to the leading terms of \( A_2 / A_1 \), we arrive at

\[
\text{Im}(e^{-2i\Phi_M} \tilde{\rho}_f) \approx -n_f \left[ \sin(2(\Phi_M + \Phi_1) + \Delta_f) \right], \tag{3.6}
\]

where

\[
\Delta_f = 2 \frac{A_2}{A_1} \sin(\Phi_2 - \Phi_1) \cos [(\delta_2 - \delta_1) - 2(\Phi_M + \Phi_1)]. \tag{3.7}
\]

Obviously the correction term \( \Delta_f \) vanishes if \( \Phi_2 = \Phi_1 \) or \( A_2 / A_1 = 0 \).

For illustration, we estimate the size of \( A_2 / A_1 \) within the standard model and link the phase combinations to the \( CP \) angles for three typical decay modes, as listed in Table 1. Clearly \( \Delta_{\phi KS} \) is safely negligible, while \( \Delta_{x+\pi^-} \) and \( \Delta_{\rho KS} \) may significantly contaminate the extraction of \( \phi_2 \) and \( \phi_3 \) from \( A_3^{x+\pi^-} \).
Table 1: Rough estimation of penguin pollution in the standard model ($\lambda \approx 0.22$).

| Example | $A_2/A_1$ | $\sin(\Phi_2 - \Phi_1)$ | $\sin(2\Phi_M + \Phi_1)$ |
|---------|-----------|--------------------------|--------------------------|
| $B_d \to \psi K_S$ | $\sim \lambda^2$ | $\sim \lambda^2$ | $\sin(2\phi_1)$ |
| $B_d \to \pi^+\pi^-$ | $\sim \lambda$ | $\sin \phi_2$ | $\sin(2\phi_2)$ |
| $B_s \to \rho^0 K_S$ | $\sim 1$ | $\sin \phi_3$ | $\sin(2\phi_3)$ |

and $\mathcal{A}_{\rho' K_S}^s$, respectively. As an example, the correlation between values of $\sin(2\phi_2)$ and $\Delta_{\pi^+\pi^-}$ is illustrated in Fig. 2, where $A_2/A_1 = 0.2$ and $\delta_2 - \delta_1 = 0$ have been typically taken. One can see that the penguin pollution in such decay channels have to be resolved\[10,11\], in order to determine the relevant weak angles reliably from their $CP$ asymmetries.

### 4 Cleanup of Penguin Pollution

To cleanly extract the $CP$ angle $\phi_2$ from $B_d \to \pi^+\pi^-$ or other charmless $B_d$ transitions, the relevant penguin and tree-level effects should be disentangled. One can get around the problem of penguin pollution by making use of isospin relations. Subsequently we take two examples to illustrate this method.

**Example 1:** $B \to \pi\pi$. For $B \to 2\pi$ decays, an isospin analysis of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$, $B_u^+ \to \pi^+\pi^0$ and their charge-conjugate processes is possible\[5\]. The amplitudes of these decay modes are related by isospin symmetry as follows:

$$
A_{\pi^+\pi^-} = \sqrt{2}(A_2 - A_0), \quad A_{\pi^0\pi^0} = 2A_2 + A_0, \quad A^{+0} = 3A_2,
\begin{align*}
\bar{A}_{\pi^+\pi^-} &= \sqrt{2}(A_2 - \bar{A}_0), \quad \bar{A}_{\pi^0\pi^0} = 2\bar{A}_2 + \bar{A}_0, \quad \bar{A}^{-0} = 3\bar{A}_2, \\
A^{+0} &= 3A_2, \quad A^{+0} = 3\bar{A}_2.
\end{align*}
(4.1)

where $A_I$ and $\bar{A}_I$ ($I = 1, 0$) are isospin amplitudes. There exist two triangular relations in the complex plane:

$$
A^{+0} + \sqrt{2}A^{00} = \sqrt{2}A^{+0}, \quad \bar{A}^{+0} + \sqrt{2}\bar{A}^{00} = \sqrt{2}\bar{A}^{+0}.
(4.2)
$$

In terms of isospin amplitudes, $\tilde{\rho}_{\pi^+\pi^-}$ and $\tilde{\rho}_{\pi^0\pi^0}$ read

$$
\tilde{\rho}_{\pi^+\pi^-} = \frac{\bar{A}^{+0}}{A^{+0}} = \frac{\bar{A}_2 1 - \bar{z}}{A_2 1 - z}, \quad \tilde{\rho}_{\pi^0\pi^0} = \frac{A^{00}}{\bar{A}^{00}} = \frac{\bar{A}_2 2 + \bar{z}}{A_2 2 + z},
(4.3)
$$

where $z = A_0/A_2$ and $\bar{z} = \bar{A}_0/\bar{A}_2$ can be solved from the isospin triangles in (4.2). Neglecting the electroweak penguin effect, which is expected to be small.
enough for our present purpose, we get $\bar{A}_2/A_2 = (V_{ub}V_{ud}^\ast)/V_{ub}V_{ud}$ as a pure weak phase from the tree-level quark diagrams. It turns out that

$$\text{Im} \left( e^{-2i\phi_{\pi^-}} \bar{\rho}_{\pi^+\pi^-} \right) = \text{Im} \left( e^{2i\phi_{\pi^+\pi^-}} \frac{1 - \bar{z}}{1 - z} \right),$$

$$\text{Im} \left( e^{-2i\phi_{\pi^-}} \bar{\rho}_{\pi^0\pi^0} \right) = \text{Im} \left( e^{2i\phi_{\pi^+\pi^-}} \frac{2 + \bar{z}}{2 + z} \right),$$

where $\phi_{\pi^+\pi^-} = \phi_2 + \phi_{NP}^d$, given in (2.5). Thus the $CP$ angle $\phi_2$ can be sorted out in spite of the penguin pollution, if there is no new physics contribution to $B_d^0 - \bar{B}_d^0$ mixing (i.e., $\phi_{NP}^d = 0$).

The feasibility of this isospin method to extract $\phi_{\pi^+\pi^-}$ relies on the observation of $B_d^0 \to \pi^0\pi^0$ and $\bar{B}_d^0 \to \pi^0\pi^0$, whose branching ratios are expected to be very small ($\sim 10^{-6}$) due to color suppression. If the branching ratio of $B_d^0 \to \pi^0\pi^0$ were too small (e.g., $\leq 10^{-7}$) to be measured, one could take $A^0 \approx \bar{A}^0 \approx 0$ as an effective approximation. In this limit, $z \approx \bar{z} \approx -2$ or $\bar{\rho}_{\pi^+\pi^-} \approx 1$ would hold, leading to $\text{Im}(e^{-2i\phi_{\pi^-}} \bar{\rho}_{\pi^+\pi^-}) \approx \text{sin}(2\phi_{\pi^+\pi^-})$ with little penguin contamination.

In practice, a model-dependent constraint on the branching ratio of $B_d^0 \to \pi^0\pi^0$...
\( \pi^0 \pi^0 \) may be obtained from the above isospin analysis:

\[
\mathcal{B}(B_d^0 \to \pi^0 \pi^0) \geq \frac{1}{2} \left( \sqrt{2} - \frac{\mathcal{B}(B_d^+ \to \pi^+ \pi^-)}{\mathcal{B}(B_d^0 \to \pi^+ \pi^0)} \right)^2 \mathcal{B}(B_u^+ \to \pi^+ \pi^0), \tag{4.5}
\]

once the modes \( B_u^+ \to \pi^+ \pi^0 \) and \( B_d^0 \to \pi^+ \pi^- \) are reliably measured. This lower bound can be used to check those model-dependent calculations for \( \mathcal{B}(B_d^0 \to \pi^0 \pi^0) \), and to examine if the approximation \( \mathcal{B}(B_d^0 \to \pi^+ \pi^-) \ll \mathcal{B}(B_d^0 \to \pi^+ \pi^0) \) (or \( A^{00} \approx \bar{A}^{00} \approx 0 \)) is acceptable in reality. As the \( B_d^0 \to \pi^0 \pi^0 \) transition is of crucial importance in determining \( \phi_2 \), it is worthwhile to make all possible experimental efforts to detect it.

**Example 2:** \( B \to \rho \pi \). For \( B \to \rho \pi \) transitions, the final states include \( I = 0, 1 \) and 2 isospin configurations and thus are more complicated than those in \( B \to \pi \pi \) decays. A detailed isospin analysis of \( B_u^+ \to \rho^+ \pi^0 \), \( B_u^+ \to \rho^0 \pi^+ \), \( B_d^0 \to \rho^+ \pi^- \), \( B_d^0 \to \rho^- \pi^+ \) and \( B_d^0 \to \rho^0 \pi^0 \) has been made by Lipkin et al.[4] For simplicity, one may distinguish between the tree-level (T) and penguin (P) contributions to each overall decay amplitude, as the penguin effect is purely of \( I = 1/2 \) transition. Then five decay amplitudes can be written as

\[
\begin{align*}
A^{+0} & = (T^{+0} + 2P_1)/\sqrt{2}, \\
A^{0+} & = (T^{0+} - 2P_1)/\sqrt{2}, \\
A^{+} & = T^{+} + P_1 + P_0, \\
A^{-} & = T^{-} - P_1 + P_0, \\
A^{00} & = (T^{+0} + T^{0+} - T^{++} - T^{--} - 2P_0)/2, \tag{4.6}
\end{align*}
\]

in which \( T^{00} \) is eliminated in terms of \( T^{+0}, T^{0+}, T^{++} \) and \( T^{--} \) due to isospin constraints. For the charge-conjugate channels \( B_u^- \to \rho^- \pi^0 \), \( B_u^- \to \rho^0 \pi^- \), \( B_d^0 \to \rho^- \pi^+ \), \( B_d^0 \to \rho^+ \pi^- \) and \( B_d^0 \to \rho^0 \pi^0 \), we define their amplitudes as \( \bar{A}^{+0}, \bar{A}^{-0}, \bar{A}^{+}, \bar{A}^{-} \) and \( \bar{A}^{00} \), respectively. The corresponding tree-level and penguin amplitudes can be denoted by \( T \) and \( \bar{p} \), which differ from \( T \) and \( P \) only in the sign of the KM phases. Then we arrive at two pentagonal relations:

\[
\begin{align*}
\sqrt{2}(A^{+0} + A^{0+}) &= A^{+} + A^{-} + 2A^{00}, \\
\sqrt{2}(\bar{A}^{-0} + \bar{A}^{0-}) &= \bar{A}^{+} + \bar{A}^{-} + 2\bar{A}^{00}, \tag{4.7}
\end{align*}
\]

contrasting with the simple triangular relations of \( B \to \pi \pi \) given in (4.2).

Measurements of the above ten decay rates can, (at least) in principle, determine the ten sides of both pentagons in (4.7). Furthermore, observation of \( CP \) asymmetries in \( B_d^0 \) vs \( \bar{B}_d^0 \to \rho^+ \pi^- \), \( \rho^- \pi^+ \) and \( \rho^0 \pi^0 \) will allow one to
determine the following three quantities (here again the electroweak penguin effects are negligible [14]):

\[
\text{Im} \left( e^{2i\phi_{\pi^+\pi^-}} \frac{\bar{A}^{+-}}{A^{+-}} \right), \quad \text{Im} \left( e^{2i\phi_{\pi^+\pi^-}} \frac{\bar{A}^{-+}}{A^{-+}} \right), \quad \text{Im} \left( e^{2i\phi_{\pi^+\pi^-}} \frac{\bar{A}^{00}}{A^{00}} \right).
\]  

(4.8)

Thus the CP angle \( \phi_{\pi^+\pi^-} \) can be extracted without the (strong) penguin pollution. This method, however, may be plagued with multiple discrete ambiguities [15, 16]. To avoid this drawback Quinn and Snyder have considered a maximum-likelihood fit of the parameters to the full Dalitz plot distribution, which is possible to successfully extract \( \phi_{\pi^+\pi^-} \) and other parameters with as few as \( 10^3 \) Monte-Carlo-generated events [17]. Of course, such an estimate relies on the assumption that \( B \to 3\pi \) events are fully dominated by \( B \to \rho \pi \) [18]. If the detector efficiency and background effects are taken into account, for the practical purpose, perhaps about \( 10^4 \) \( B \to \rho \pi \) events are required. This implies that we need to accumulate as many as \( 10^9 \) \( B\bar{B} \) events, which may be beyond what can be achieved in the first-round experiments of a \( B \) factory.

5 Extraction of \( \phi_1 - \phi_2 \) and \( \phi_3 \)

Some neutral \( B \) decays to hadronic non-CP eigenstates can also be used to extract the CP angles. In particular, the angle difference \( \phi_1 - \phi_2 \) is associated with the CP-violating quantity in \( B^0 \) vs \( \bar{B}^0 \to D^{(*)0}K_S \) or \( \bar{D}^{(*)0}K_S \); while \( \phi_3 \) is associated with the CP-violating quantity in \( B^0_s \) vs \( \bar{B}^0_s \to D^{(*)0}\phi \) or \( \bar{D}^{(*)0}\phi \). In addition, \( \phi_3 \) is responsible for CP violation in the transitions \( B^0 \) vs \( \bar{B}^0_s \to D_s^{(*)\pm}K^{(*)\mp} \). Subsequently we shall analyze these three types of decays in some detail.

(a) \( B^0 \) vs \( \bar{B}^0 \to D^{(*)0}K_S \) or \( \bar{D}^{(*)0}K_S \). Such decay modes can only occur through the tree-level quark transitions \( b \to ucs \) and \( b \to cus \); thus each decay amplitude involves only a single weak phase [19] i.e., \( \text{arg}(V_{ub}V^*_{cs}) \) or \( \text{arg}(V_{ub}V^*_{us}) \). Neglecting tiny CP violation from \( K^0 - \bar{K}^0 \) mixing in the final state, we parametrize the four transition amplitudes as follows:

\[
\begin{align*}
\langle D^{(*)0}K_S | \mathcal{H} | B^0 \rangle &= (V^*_{ub}V_{cs}) A_1 e^{i\delta_1}, \\
\langle \bar{D}^{(*)0}K_S | \mathcal{H} | \bar{B}^0 \rangle &= (V_{ub}V_{cs}) A_1 e^{i\delta_1}, \\
\langle D^{(*)0}K_S | \mathcal{H} | B^0_s \rangle &= (V^*_{cb}V_{us}) A_2 e^{i\delta_2}, \\
\langle \bar{D}^{(*)0}K_S | \mathcal{H} | \bar{B}^0_s \rangle &= (V_{cb}V_{us}) A_2 e^{i\delta_2}.
\end{align*}
\]  

(5.1)

where \( A_i \) and \( \delta_i \) \((i = 1, 2)\) are the real hadronic matrix element and the corresponding strong phase. A time-dependent measurement of the above decay
modes will allow one to determine two quantities:

\[
\begin{align*}
\text{Im} \left( e^{-2i\Phi_M} \bar{\rho}_{D(\ast)0} K_S \right) & \approx \left| \bar{\rho}_{D(\ast)0} K_S \right| |\sin(\phi_{\psi K_S} - \phi_{\pi^+\pi^-} - \Delta\delta)|, \\
\text{Im} \left( e^{-2i\Phi_M} \bar{\rho}_{D(\ast)0} K_S \right) & \approx \left| \bar{\rho}_{D(\ast)0} K_S \right| |\sin(\phi_{\psi K_S} - \phi_{\pi^+\pi^-} + \Delta\delta)|,
\end{align*}
\]

where \( \Delta\delta = \delta_2 - \delta_1 \), and \( \phi_{\psi K_S} = \phi_1 - \phi_{NP}^\psi \) and \( \phi_{\pi^+\pi^-} = \phi_2 + \phi_{NP}^\pi \) have been given in (2.5). Since \( \left| \bar{\rho}_{D(\ast)0} K_S \right| \) and \( \left| \bar{\rho}_{D(\ast)0} K_S \right| \) can also be determined from the time-dependent measurement (see (3.1) for illustration), one may extract both \( \phi_{\psi K_S} - \phi_{\pi^+\pi^-} \) and \( \Delta\delta \) from (5.2). Note that the strong phase shifts \( \Delta\delta(D^{0}K_S) \) and \( \Delta\delta(D^{*0}K_S) \) are expected to be different in general, thus it is possible to resolve the discrete ambiguity associated with the determination of \( \phi_{\psi K_S} - \phi_{\pi^+\pi^-} \). In the absence of new physics (i.e., \( \phi_{NP}^\psi = 0 \)), this weak phase difference amounts to \( \phi_1 - \phi_2 \).

The feasibility of this method depends mainly on branching ratios of relevant decay modes. As we have shown before, \( \phi_1 \) and \( \phi_2 \) will be separately determined from some \( B_d \) decays to \( CP \) eigenstates like \( \psi K_S \) and \( \pi^+\pi^- \). Thus a comparison between the angle difference \( \phi_3 - \phi_1 \) obtained from such measurements and that obtained from (5.2) will be helpful in order to check the self-consistency of the standard model predictions.

(b) \( B^0_s \) vs \( B^0_d \rightarrow D_s^{(\ast)+0} K^{(\ast)+} \). These decay modes are also governed by the tree-level quark transitions \( b \rightarrow ucs \) and \( b \rightarrow cus \), hence they can be analyzed in a similar way as done for \( B^0_s \) vs \( B^0_d \rightarrow D^{(\ast)+0} K_S \) or \( D^{(\ast)+0} K_S \). We find that the \( CP \)-violating quantities in \( B^0_s \) vs \( B^0_s \rightarrow D_s^{(\ast)+0} K^{(\ast)+} \) are associated with the \( CP \) angle \( \phi_3 \); i.e.,

\[
\begin{align*}
\text{Im} \left( e^{-2i\Phi_M} \bar{\rho}_{D_s^{(\ast)+0} K_S} \right) & \approx \left| \bar{\rho}_{D_s^{(\ast)+0} K_S} \right| |\sin(2\phi_{NP}^\pi - \phi_3 + \Delta\delta)|, \\
\text{Im} \left( e^{-2i\Phi_M} \bar{\rho}_{D_s^{(\ast)+0} K_S} \right) & \approx \left| \bar{\rho}_{D_s^{(\ast)+0} K_S} \right| |\sin(2\phi_{NP}^\pi - \phi_3 - \Delta\delta)|,
\end{align*}
\]

where \( \phi_{NP}^\pi \) denotes the \( CP \) phase from new physics in \( B^0_s - B^0_s \) mixing (see (2.3) for illustration), and \( \Delta\delta \) stands for the strong phase difference. We see that \( \phi_3 - 2\phi_{NP}^\pi \) can be extracted from (5.3). The feasibility of this method has been discussed in detail by Aleksan, Dunietz and Kayser. It might suffer from the rapid rate of \( B^0_s - B^0_s \) oscillation during the time-dependent measurement of relevant decay modes. As \( B_s \) mesons cannot be produced at the KEK and
SLAC $B$ factories, more time is needed to realize the above method at high-luminosity hadron machines.

(c) $B_s^0$ vs $\bar{B}_s^0 \to D_s^{(*)0} \phi$ or $\bar{D}_s^{(*)0} \phi$. The analysis of these decay modes is the same as that of $B_s^0$ vs $\bar{B}_s^0 \to D_s^{(*)+} K^{(*)+}$. The result similar to (5.3) can be obtained, allowing one to extract the $CP$ angle $\phi_3 - 2\phi_{NP}$. In comparison between methods (b) and (c), we find that the former is more promising in practice, because the relevant (color-favored) transitions have larger branching ratios and the final-state (charged) particles are easier to detect.

6 $B \to (D^0, \bar{D}^0, D_{1,2}) + X$ and $\phi_3$

There is a variety of two-body $B$ decays involving $D^0$, $\bar{D}^0$ or $D_{1,2}$ in the final states, where $D_{1,2} \equiv (D^0 \pm \bar{D}^0)/\sqrt{2}$ denotes the $CP$ eigenstates of neutral $D$ mesons. Such decay modes are interesting as they can be used to determine the $CP$ angles $\delta_a, \delta_b$. For example, the $CP$-violating quantities in $B_d \to (D^0, \bar{D}^0, D_{1,2}) + K_S$ and $B_s \to (D^0, \bar{D}^0, D_{1,2}) + \phi$ are associated with $\phi_1 - \phi_2$ and $\phi_3$, respectively, as discussed above. In the following we shall concentrate on the charged $B$ decays $B^+_u \to (D^0, \bar{D}^0, D_{1,2}) + K^\pm$ to outline the main feature of such transitions as well as the possibility to extract $\phi_3$.

The decay modes $B^+_u \to (D^0, \bar{D}^0, D_{1,2}) + K^\pm$ occur only through the tree-level quark processes $b \to u\bar{c}s$ and $b \to c\bar{u}s$. Neglecting tiny $D^0, \bar{D}^0$ mixing in the final states $D_{1,2} K^\pm$, one can parametrize all six transition amplitudes in the Wolfenstein phase convention:

\[
A(D^0 K^+) = \langle D^0 K^+|H|B^+_u \rangle = |V_{ub}V_{cs}|A_a e^{i(\delta_a + \phi_3)},
\]

\[
A(\bar{D}^0 K^-) = \langle \bar{D}^0 K^-|H|B^-_u \rangle = |V_{ub}V_{cs}|A_a e^{i(\delta_a - \phi_3)}; \quad (6.1)
\]

\[
A(\bar{D}^0 K^+) = \langle \bar{D}^0 K^+|H|B^+_u \rangle = |V_{ub}V_{us}|A_b e^{i\delta_b},
\]

\[
A(D^0 K^-) = \langle D^0 K^-|H|B^-_u \rangle = |V_{ub}V_{us}|A_b e^{i\delta_b}; \quad (6.2)
\]

and

\[
A(D_{1,2} K^+) = \langle D_{1,2} K^+|H|B^+_u \rangle = \frac{1}{\sqrt{2}} \left[ A(D^0 K^+) - A(\bar{D}^0 K^+) \right],
\]

\[
A(D_{1,2} K^-) = \langle D_{1,2} K^-|H|B^-_u \rangle = \frac{1}{\sqrt{2}} \left[ A(D^0 K^-) - A(\bar{D}^0 K^-) \right]; \quad (6.3)
\]

where $A_i$ and $\delta_i$ ($i = a, b$) are the real hadronic matrix element and the strong phase, respectively. Clearly $A(D^0 K^+) = A(\bar{D}^0 K^-) e^{2i\phi_3}$ and $A(D^0 K^-) = A(\bar{D}^0 K^+)$ hold. In the factorization approximation, $A_a/A_b \sim a_1/a_2 \approx 0.26$, \ldots
Table 2: Rough estimation of $|A(D^0X)/A(D^0X)|$ for three types of $B$ decays.

| Transitions | $|A(D^0X)/A(D^0X)|$ |
|-------------|----------------------|
| $B_u^+ \rightarrow (D^0, D^0, D_{1,2}) + K^{(*)\pm}$ | $\sim \frac{V_{ub} V_{cs} a_2}{V_{cb} V_{us} a_1}$ $\sim 0.1$ |
| $B_d^- \rightarrow (D^0, D^0, D_{1,2}) + K^{(*)0}$ | $\sim \frac{|V_{ub} V_{cs} a_2|}{|V_{cb} V_{us} a_2|}$ $\sim 0.4$ |
| $B_c^+ \rightarrow (D^0, D^0, D_{1,2}) + D_s^{(*)\pm}$ | $\sim \frac{|V_{ub} V_{cs} a_1|}{|V_{cb} V_{us} a_2|}$ $\sim 1.4$ |

where $a_1$ and $a_2$ are the Bauer-Stech-Wirbel factors. One can see from (6.3) that $A(D^0K^+), A(D^0K^+), A(D_{1,2}K^+)$ and $A(D^0K^0), A(D^0K^-), A(D_{1,2}K^-)$ form two correlated triangles in the complex plane, as illustrated by Fig. 3. Since all sides of these two triangles can be determined from the decay rates of $B_u^+ \rightarrow (D^0, D^0, D_{1,2}) + K^{(*)\pm}$, the $CP$ angle $\phi_3$ is then resolved \cite{24}. Of course, the above discussion can be trivially extended to $B_d^- \rightarrow (D^0, D^0, D_{1,2}) + K^{(*)0}$ and $B_c^+ \rightarrow (D^0, D^0, D_{1,2}) + D_s^{(*)\pm}$ decays.

In order to fully construct two correlated triangles in the complex plane, their six sides should be comparable in magnitude. By use of the factorization approximation, we estimate the ratio $|A(D^0X)/A(D^0X)|$ for three types of transitions mentioned above ($X = K^{(*)\pm}, K^{(*)0}$ or $D^{(*)\pm}$) and list the rough result in Table 2. For $B_u^+ \rightarrow D^0K^+$ and $A(D^0K^-)$ are too short in comparison with $A(D^0K^+)$ and $A(D^0K^-)$. Thus it will be very hard, if not even practically impossible, to extract $\phi_3$ from these $B_u^\pm$ channels. Also, it is very difficult to identify $D^0$ in the suppressed decay mode $B_u^+ \rightarrow D^0K^+$; e.g., if one identifies $D^0$ by use of its leptonic decay, the
same lepton from direct $B_u^\pm$ decay will swamp the signal $\phi_3$. A determination of $\phi_3$ from $B_u^\pm$ decays might be possible at LHC-B.

To get around the above-mentioned problem associated with the extraction of $\phi_3$ from $B_u^\pm \rightarrow (D^0, \bar{D}^0, D_{1.2}) + K^{(*)}\pm$, Atwood, Dunietz and Soni have proposed the measurement of $B_u^+ \rightarrow D^0 K^+ \rightarrow (K^-\pi^+)_{D^0} K^+$ and their charge-conjugate channels. The point is that $D^0 \rightarrow K^-\pi^+$ is Cabibbo-allowed while $\bar{D}^0 \rightarrow K^-\pi^+$ is doubly Cabibbo-suppressed, hence the amplitudes $A[(K^-\pi^+)_{D^0} K^+]$ and $A[(K^-\pi^+)_{\bar{D}^0} K^+]$ become comparable in magnitude:

$$\left| \frac{A[(K^-\pi^+)_{D^0} K^+]}{A[(K^-\pi^+)_{\bar{D}^0} K^+]} \right| \approx \frac{|V_{ub} V_{cs}|}{|V_{cb} V_{us}|} \times \left| \frac{a_2}{a_1} \right| \times \frac{\langle K^-\pi^+|H|D^0 \rangle}{\langle K^-\pi^+|H|\bar{D}^0 \rangle} \sim 1. \quad (6.4)$$

where $B(\bar{D}^0 \rightarrow K^-\pi^+)/B(D^0 \rightarrow K^-\pi^+) \approx 0.0077$ reported by CLEO has been used. Then the overall amplitude of $B_u^+ \rightarrow (K^-\pi^+)_{D^0} K^+$ involves large interference between its two components. Experimentally one may also measure $B_u^+ \rightarrow (K^-\pi^+)_{D^0} K^+$, $(K^-\rho^+)_{D^0} K^+$, $(K^-\rho^+)_{\bar{D}^0} K^+$ and their charge-conjugate processes, which have the same weak interaction but different strong final-state interactions. This allows the extraction of $\phi_3$ from four different decay rates, e.g.,

$$B[B_u^+ \rightarrow (K^-\pi^+)_{D^0} K^+] = B(B_u^+ \rightarrow D^0 K^+)B(D^0 \rightarrow K^-\pi^+) + B(B_u^+ \rightarrow \bar{D}^0 K^+)B(\bar{D}^0 \rightarrow K^-\pi^+) + 2\sqrt{B(B_u^+ \rightarrow D^0 K^+)B(D^0 \rightarrow K^-\pi^+)B(B_u^+ \rightarrow \bar{D}^0 K^+)B(\bar{D}^0 \rightarrow K^-\pi^+)} \times \cos(\delta_{ab}^K + \phi_3),$$

$$B[B_u^- \rightarrow (K^+\pi^-)_{D^0} K^-] = B(B_u^- \rightarrow D^0 K^-)B(D^0 \rightarrow K^+\pi^-) + B(B_u^- \rightarrow \bar{D}^0 K^-)B(\bar{D}^0 \rightarrow K^+\pi^-) + 2\sqrt{B(B_u^- \rightarrow D^0 K^-)B(D^0 \rightarrow K^+\pi^-)B(B_u^- \rightarrow \bar{D}^0 K^-)B(\bar{D}^0 \rightarrow K^+\pi^-)} \times \cos(\delta_{ab}^K - \phi_3) \quad (6.5)$$

with $\delta_{ab}^K = \delta_{ab}^{K^*} = \delta_{0}^{K^*}$, and those associated with $(K^*\pi)_D$ in the final state. Note that $B(B_u^+ \rightarrow D^0 K^+) = B(B_u^- \rightarrow \bar{D}^0 K^-)$, $B(B_u^+ \rightarrow \bar{D}^0 K^-) = B(B_u^- \rightarrow D^0 K^-)$, $B(D^0 \rightarrow K^{(*)}\pi^-) = B(\bar{D}^0 \rightarrow K^{(*)}\pi^-)$ and $B(\bar{D}^0 \rightarrow K^{(*)}\pi^+) = B(\bar{D}^0 \rightarrow K^{(*)}\pi^-)$. Therefore the four unknown quantities $B(B_u^+ \rightarrow D^0 K^+)$, $\delta_{ab}^K$, $\delta_{ab}^{K^*}$ and $\phi_3$ can all be extracted from the measurement. To resolve the discrete ambiguities in this method, more decay modes of $D^0$ and $\bar{D}^0$ mesons should be taken into account. It is expected that this method works in the second-round experiments of a $B$ factory.
7 SU(3) Analysis

Finally let us comment briefly on the SU(3) method of extracting the CP angles from B decays, which has recently attracted a lot of theorists’ attention. Under flavor SU(3) symmetry, amplitudes of a variety of two-body mesonic decays, such as $B \to \pi\pi$, $B \to K\pi$ and $B \to KK$, are related to one another, allowing the possibility to determine the associated weak and strong phases. Intuitively the SU(3)-reduced matrix elements can be described in terms of a set of quark diagrams. However, SU(3) symmetry is expected to be broken by effects of order 20% (e.g., $f_K/f_\pi \approx 1.2$), hence one has to introduce appropriate SU(3) breaking terms in explicit analyses. In most of SU(3) analyses, it is usually argued that the decay amplitudes via $W$-exchange, annihilation and penguin annihilation diagrams are formfactor suppressed and thus negligible as a good approximation. This assumption can experimentally be checked if one detects the expected suppression $B(B^+_d \to K^+K^-)/B(B^0_d \to \pi^+\pi^-)$.

Many possibilities to extract three CP angles from various SU(3) relations of charmless B decays have been proposed. For example, the triangular relation

$$\sqrt{2} A(B^+_u \to \pi^0K^+) + A(B^+_u \to \pi^+K^0) = \sqrt{2} \left| \frac{V_{us}}{V_{ud}} \right| \frac{f_K}{f_\pi} A(B^+_u \to \pi^+\pi^0) \quad (7.1)$$

and its charge-conjugate relation can be used to determine the angle $\phi_3$ in the standard model. Since several comprehensive reviews on these approaches have existed in the literature, we shall not go into any detail here.

Of course there are several sources of uncertainties associated with the SU(3) analysis (e.g., it is difficult to estimate the SU(3) breaking effect in relevant strong phases). For this reason, this method cannot be used to look for new physics effects; but it should be useful for self-consistency checks.

8 Concluding Remarks

In this mini-review, we have discussed various ways to get at properties of the unitarity triangle. We believe that $B^0-\bar{B}^0$ mixing is the best bet to look for new physics. As CP asymmetries in most of neutral B decays involve the interplay of decay and $B^0-\bar{B}^0$ mixing, some special attention has been paid to the relationship between the geometrical angles ($\phi_1$, $\phi_2$, $\phi_3$) and the measurable ones ($\phi_{K^0\pi^+\pi^0}$, $\phi_{\rho^0K^0}$, etc). The latter may contain some information about the underlying new physics in B decays. Thus it is worthwhile to confront different methods of extracting $\phi_i$ with the forthcoming measurements at B-meson factories, in order to fully test the KM mechanism of CP violation and pin down possible new physics.
Eventually an accurate measurement of $|V_{ub}|$ and $|V_{cb}|$ will be available to fix two sides of the unitarity triangle without much interference from new physics. The element $|V_{td}|$ can be well determined (or constrained) from observation of $K^+ \to \pi^+ \nu \bar{\nu}$. When the hadronic matrix element associated with $\langle B^0 | H | B^0 \rangle$ is measured from more delicate lattice-QCD computation, there will be a more reliable (and independent) constraint on $|V_{td}|$ from the data of $B_d^0 - \bar{B}_d^0$ mixing. These two measurements of $|V_{td}|$ may not agree to each other, however, if there is substantial new physics.

It is quite clear that while a search for new physics in $B$ decays represents a great experimental challenge, it might yield great reward. Also it is a good time to think about more delicate $B$-physics experiments beyond asymmetric $B$ factories.

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