Redefining the Axion Window

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A major goal of axion searches is to reach inside the parameter space region of realistic axion models. Currently, the boundaries of this region depend on somewhat arbitrary criteria, and it would be desirable to specify them in terms of precise phenomenological requirements. We consider hadronic axion models and classify the representations $R_Q$ of the new heavy quarks $Q$. By requiring that i) the $Q$ are sufficiently short lived to avoid issues with long lived strongly interacting relics, ii) no Landau poles are induced below the Planck scale, fifteen cases are selected, which define a phenomenologically preferred axion window bounded by a maximum (minimum) value of the axion-photon coupling about twice (four times) larger than commonly assumed. Allowing for more than one $R_Q$, larger couplings, as well as complete axion-photon decoupling, become possible.

Introduction. In spite of its indisputable success, the standard model (SM) is not completely satisfactory: it does not explain unquestionable experimental facts like dark matter (DM), neutrino masses, and the cosmological baryon asymmetry, and it contains fundamental parameters with highly unnatural values, like the the Higgs potential term $\mu^2$, the first generation Yukawa couplings $h_{e,u,d}$, and the strong CP violating angle $\theta < 10^{-10}$. This last quantity is somewhat special: its value is stable with respect to higher order corrections [1] (unlike $\mu^2$) and (unlike $h_{e,u,d}$ [2]) it evades explanations based on environmental selection [3]. Thus, seeking explanations for the smallness of $\theta$ independently of other “small values” problems is theoretically motivated. Basically, only three types of solutions exist. The simplest possibility, a massless up-quark, is now ruled out [4, 5]. The so-called Nelson-Barr type of models [6, 7] either require a high degree of fine tuning, often comparable to setting $\theta \lesssim 10^{-10}$ by hand, or rather elaborated theoretical structures [8]. The Peccei-Quinn (PQ) solution [9–12], although it is not completely free from issues [13–15], arguably stands on better theoretical grounds.

Setting aside theoretical considerations, the question whether the PQ solution is the correct one could be set experimentally by detecting the axion. In order to focus axion searches, it is then very important to identify as well as possible the region of parameter space where realistic axion models live. The vast majority of search techniques are sensitive to the axion-photon coupling $g_{a\gamma\gamma}$, which is inversely proportional to the axion decay constant $f_a$. Since the axion mass $m_a$ has the same dependence, theoretical predictions and experimental exclusion limits can be conveniently presented in the $m_a$-$g_{a\gamma\gamma}$ plane. The commonly adopted axion band corresponds roughly to $g_{a\gamma\gamma} \sim m_a/(2\pi f_a m_e) \sim 10^{-10} (m_a/eV)^{-1}$ with a somewhat arbitrary width, chosen to include representative models [16, 18]. In this Letter we put forth a definition of a phenomenologically preferred axion window as the region encompassing hadronic axion models which i) do not contain cosmologically dangerous relics; ii) do not induce Landau poles (LP) below some scale $\Lambda_{LP}$ close to the Planck mass $m_P = 1.2 \cdot 10^{19}$ GeV. While all the cases we consider belong to the KSVZ type of models [19, 20], the resulting window encompasses also the DFSZ axion [21, 22] and many of its variants [17].

Hadronic axion models. The basic ingredient of any renormalizable axion model is a global $U(1)_{PQ}$ symmetry. The associated Noether current $J^{PQ}_\mu$ must have a color anomaly and, although not required for solving the strong CP problem, in general it also has an electromagnetic anomaly: 

$$\partial^\mu J^{PQ}_\mu = \frac{N_\alpha}{4\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \frac{e_\alpha}{4\pi} F^\mu \tilde{F}^\mu$$,

where $G^a_{\mu\nu} (F_{\mu\nu})$ is the color (electromagnetic) field strength tensor, $\tilde{G}^{a\mu\nu} (\tilde{F}^\mu)$ is its dual, and $N$ and $E$ the respective anomaly coefficients. In a generic axion model of KSVZ type $\chi^{(1)}$ the anomaly is induced by pairs of heavy fermions $Q_L, Q_R$ which must transform non-trivially under $SU(3)_C \times SU(2)_L \times U(1)_Y$ so that

$$N = \sum_Q (X_{Q_L} - X_{Q_R}) T(C_Q),$$

$$E = \sum_Q (X_{Q_L} - X_{Q_R}) Q_Q^2,$$

where the sum is over irreducible color representations (for generality we allow for the simultaneous presence of more $R_Q$). The color index is defined by $\text{Tr} T^a Q^b = T(C_Q) \delta^{ab}$ with $T_Q$ the generators in
$L_Q$ and $Q_Q$ is the $U(1)_{em}$ charge. The scalar field $\Phi$ can be parametrized as

$$\Phi(x) = (1/\sqrt{2}) [\rho(x) + V_u] e^{i(\omega(x)/V_u)}.$$  \hspace{1cm} (4)

The mass of $\rho$ is of order $V_u > (\sqrt{2} G_F)^{-1/2} = 247$ GeV, while a tiny mass for the axion $a(x)$ arises from nonperturbative QCD effects which explicitly break $U(1)_{PQ}$. The SM quarks $q = q_L, d_R, u_R$ do not contribute to the QCD anomaly, and thus their PQ charges can be set to zero. The renormalizable Lagrangian for a generic hadronic axion model can be written as:

$$L_Q = L_{SM} + L_{PQ} - V_{HQ} + L_{Q_Q},$$  \hspace{1cm} (5)

where $L_{SM}$ is the SM Lagrangian,

$$L_{PQ} = |\vec{\rho}|^2 + \overline{Q_i} B_i Q_j - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c.}),$$  \hspace{1cm} (6)

with $Q = Q_L + Q_R$. The new scalar terms are:

$$V_{HQ} = -\mu_3^2 |\Phi|^3 + \lambda_\Phi |\Phi|^4 + \mu_{H_K}|H|^2|\Phi|^2.$$  \hspace{1cm} (7)

Finally, $L_{Q_Q}$ contains possible renormalizable terms coupling $Q_L R$ to SM quarks which can allow for $Q$ decays [23]. Note, however, that SM gauge invariance allows for $L_{Q_Q} \neq 0$ only for a few specific $R_Q$.

**PQ quality and heavy $Q$ stability.** The issue whether the $Q$ are exactly stable, metastable, or decay with safely short lifetimes, is of central importance in our study, so let us discuss it in some detail. The gauge invariant kinetic term in $L_{PQ}$ features $U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_{\Phi}$ symmetry corresponding to independent rephasings of the $Q_{L,R}$ and $\Phi$ fields. The PQ Yukawa term ($y_Q \neq 0$) breaks $U(1)^3$ to $U(1)^2$. One factor is the anomalous $U(1)_{PQ}$, the other one is a non-anomalous $U(1)_Q$, that is the $Q$-baryon number of the new quarks [19], under which $Q_L, R \rightarrow e^{i\beta} Q_L, R$ and $\Phi \rightarrow \Phi$. If $U(1)_Q$ were an exact symmetry, the new quarks would be absolutely stable. For the few $R_Q$ for which $L_{Q_Q} \neq 0$ is allowed, $U(1)_Q \times U(1)_R$ is further broken to $U(1)_{PQ}$, a generalized baryon number extended to the $Q$, which can then decay with unsuppressed rates. However, whether $L_{Q_Q}$ is allowed at the renormalizable level, does not depend solely on $R_Q$, but also on the specific PQ charges. For example, independently of $R_Q$, the common assignment $X_L = -X_R = \frac{1}{2}$ would forbid PQ invariant decay operators at all orders. $U(1)_{PQ}$ violating decays could then occur only via PQ-violating effective operators of dimension $d > 4$. Both $U(1)_{PQ}$ and $U(1)_{Q}$ are expected to be broken at least by Planck-scale effects, inducing PQ violating contributions to the axion potential $V_{\phi^{d=4}}$ as well as an effective Lagrangian $L_{\phi^{d=4}}$. In particular, in order to preserve $|\theta| < 10^{-10}$, operators in $V_{\phi^{d=4}}$ must be of dimension $d \geq 11$ [13][15]. Clearly, if $R_{Q_Q}$ had to respect $U(1)_Q$ to a similar level of accuracy, the $Q$‘s would behave as effectively stable. However, a scenario in which $U(1)_Q$ arises as an accident because of specific assignments for the charges of another global symmetry $U(1)_{PQ}$, seems theoretically untenable. A simple way out is to assume a suitable discrete (gauge) symmetry $Z_{\tilde{n}}$ ensuring that i) $U(1)_{PQ}$ arises accidentally and is of the required high quality; ii) $U(1)_{Q}$ is either broken at the renormalizable level, or it can be of sufficient bad quality to allow for safely fast $Q$ decays.

Table I gives a neat example of how such a mechanism can work (see also [23]). We choose $R_Q = R_{d_B} = (3, 1, -1/3)$ so that $G_{SM}$ invariance allows for $L_{Q_Q} \neq 0$, and we assume the following transformations under $Z_{\tilde{n}}$: $Q_L \rightarrow Q_L$, $Q_R \rightarrow \omega^{-1} Q_R$, $\Phi \rightarrow \omega \Phi$, with $\omega \equiv e^{2\pi i/3}$. This ensures that the minimum dimension of the PQ breaking operators in $V_{\phi^{d=4}}$ is $3$. The dimension of $U(1)_Q$ breaking decay operators depends on the $Z_{\tilde{n}}$ charges of the SM quarks. Table I lists different possibilities for $d \leq 4$ and $d = 5$. The last column gives the PQ charges that one has to assign to $Q_{L,R}$ so that $U(1)_{PQ}$ can be defined also in the presence of the operators in columns 2 and 3.

**Cosmology.** We assume a post-inflationary scenario ($U(1)_{PQ}$ broken after inflation). Then, requiring that the axion energy density from vacuum realignment does not exceed $\Omega_{SM}$ implies $V_a/N_{DW} \equiv f_a \lesssim f_a^{\text{max}}$, with $f_a^{\text{max}} = 5 \cdot 10^{11}$ GeV [24][25], where $N_{DW} = 2N$ is the vacuum degeneracy corresponding to a $Z_{2N} \subset U(1)_{PQ}$ left unbroken by non-perturbative QCD effects. We further assume $m_Q \lesssim T_{\text{heating}}$ so that a thermal distribution of $Q$ provides the initial conditions for their cosmological history, which then depends only on the mass $m_Q$ and representation $R_Q$. For some $R_Q$, only fractionally charged $Q$-hadrons can appear after confinement, which also implies that decays into SM particles are forbidden [27]. These $Q$-hadrons must then exist today as stable relics. However, dedicated searches constrain the abundances of fractionally charged particles relative to ordinary nucleons to $n_Q/n_b \lesssim 10^{-20}$ [28], which is orders of magnitude below any reasonable estimate of the relic abundance and of the resulting concentrations in bulk matter. This restricts the viable $R_Q$ to the much smaller subset for which $Q$-hadrons are integrally charged or neutral. In this case decays into SM particles are not forbidden, but the lifetime $\tau_Q$ is severely constrained by cosmological observations. For $\tau_Q \sim (10^{-2} - 10^{-5})$ s $Q$ decays would affect Big Bang Nucleosynthesis (BBN) [29][30]. The window

| $Z_{\tilde{n}}(q)$ | $d = 4$ | $d = 5$ | $(X_L,X_R)$ |
|-----------------|--------|--------|---------------|
| 1 $Q_L d_R$    | $Q_L \gamma_{QL}(D^0H^1)$ | $(0,-1)$ |
| $\omega$ | $Q_L d_R \Phi^2$ | $(-1,-2)$ |
| $\omega^{-1}$ | $Q_L d_R \Phi^4$, $Q_R Q_L H^1 \Phi$ | $(2,1)$ |
| $\omega^{-1}$ | $Q_L d_R H^1$, $Q_L d_R \Phi$ | $(1,0)$ |
effects of the composite finiteness \((10^6 - 10^{12})\) s is strongly constrained also by limits on CMB spectral distortions from early energy release \([31, 53]\), while decays around the recombination era \((\tau_Q > 10^{13})\) s would leave clear traces on CMB anisotropies. Decays after recombination would produce free-streaming photons visible in the diffuse gamma ray background \([34]\), and Fermi LAT limits \([35]\) allow to exclude \(\tau_Q > (10^{13} - 10^{20})\) s. For lifetimes longer than the age of the Universe \(\tau_Q \geq 10^{17}\) s the \(Q\) would contribute to the present energy density, and we must require \(\Omega_Q \leq \Omega_{DM} \approx 0.12 h^{-2}\). However, estimating \(\Omega_Q\) is not so simple. Before confinement the \(Q\)'s annihilate as free quarks. Perturbative calculations are reliably giving, for \(n_f\) final state quark flavors:

\[
\langle \sigma v \rangle_{Q\overline{Q}} = \frac{\pi \alpha_s^2}{16 m_Q^2} (c_f n_f + c_g),
\]

with, e.g., \((c_f, c_g) = (2 \frac{5}{3}, \frac{22}{27})\) for triplets and \((\frac{3}{7}, \frac{27}{14})\) for octets. Free \(Q\) annihilation freezes out around \(T_f \sim m_Q/25\) when (for \(m_Q > \text{few TeV}\) there are \(g_s \sim 106.75\) effective degrees of freedom in thermal equilibrium. Together with Eq. \((8)\) this gives:

\[
(\Omega_Q h^2)^\text{Free} \approx \frac{8}{3} \cdot 10^{-3} \left(\frac{m_Q}{\text{TeV}}\right)^2.
\]

The upper lines in Fig. 1 give \((\Omega_Q h^2)^\text{Free}\) as a function of \(m_Q\) for SU(3)C triplets (dotted) and octets (dashed). Only a small corner at low \(m_Q\) satisfies \(\Omega_Q \leq \Omega_{DM}\), and future improved LHC limits on \(m_Q\) might exclude it completely. However, after confinement \((T_C \approx 180\) MeV\), because of finite size effects of the composite \(Q\)-hadrons annihilation could restart. Some controversy exists about the possible enhancements for annihilations in this regime. For example, a cross section typical of inclusive hadronic scattering \(\sigma_{\text{ann}} \sim (m_T^2 v)^{-1} \sim 30 v^{-1}\) mb was assumed in Ref. \([36]\) yielding \(n_Q/n_b \sim 10^{-11}\). It was later remarked \([37]\) that the relevant process is exclusive (no \(Q\) quarks in the final state) with a cross section quite likely smaller by a few orders of magnitude. Ref. \([38]\) suggested that bound states formed in the collision of two \(Q\)-hadrons could catalyse annihilations. This mechanism was reconsidered in \([39, 40]\) which argued that \(\Omega_Q\) could indeed be efficiently reduced. Their results imply:

\[
(\Omega_Q h^2)^\text{Bound} \approx 3 \cdot 10^{-7} \left(\frac{m_Q}{\text{TeV}}\right)^{3/2},
\]

which corresponds to the continuous line in Fig. 1. Ref. \([41]\) studied this mechanism more quantitatively, and concluded that Eq. \((10)\) represents a lower limit on \(\Omega_Q\), but much larger values are also possible. Refs. \([39, 40]\) in fact did not consider the possible formation of \(QQ\)... bound states which, opposite to \(Q\overline{Q}\), would hinder annihilation rather than catalyse it. Then, if a sizeable fraction of \(Q\)'s gets bounded in such states, the free quark result eq. \((8)\) would give a better estimate than eq. \((10)\). If instead the estimate eq. \((10)\) is correct, energy density considerations would not exclude relics with \(m_Q \lesssim 5.4 \cdot 10^3\) TeV, nevertheless, present concentrations of \(Q\)-hadrons would still be rather large \(10^{-8} \lesssim n_Q/n_b \lesssim 10^{-6}\). While it has been debated if concentrations of the same order should be expected also in the Galactic disk \([42, 43]\) searches for anomalously heavy isotopes in terrestrial, lunar, and meteoritic materials yield limits on \(n_Q/n_b\) many orders of magnitude below the quoted numbers \([44]\). Moreover, even a tiny amount of heavy \(Q\)'s in the interior of celestial bodies (stars, neutron stars, Earth) would produce all sorts of effects like instabilities \([45]\), collapses \([46]\), anomalously large heat flows \([47]\). Therefore, unless an extremely efficient mechanism exists that keeps \(Q\)-matter completely separated from ordinary matter, stable \(Q\)-hadrons would be ruled out.

**Selection criteria.** The first criterium to discriminate hadronic axion models is: i) *Models that allow for lifetimes \(\tau_Q \lesssim 10^{-2}\) s are phenomenologically preferred with respect to models containing long lived or cosmologically stable \(Q\)'s.* All \(R_Q\) allowing for decays via renormalizable operators satisfy this requirement. Decays can also occur via operators of higher dimensions. We assume that the cutoff scale is \(m_P\) and write \(\sigma_{Q\overline{Q}}^{d=4} = m_P^{-d} \mathcal{P}(Q, \varphi^n)\) where \(\mathcal{P}\) is a \(d\)-dimensional Lorentz and gauge invariant monomial linear in \(Q\) and containing \(n\) SM fields \(\varphi\). For \(d = 5, 6, 7\) the final states always contain \(n \geq d - 3\) particles. Taking conservatively \(n = d - 3\) we obtain:

\[
\Gamma_d \lesssim \frac{\pi g_f m_Q}{(d-4)!(d-5)!} \left(\frac{m_Q^2}{16\pi^2 m_P^2}\right)^{d-4},
\]

with \(g_f\) the final degrees of freedom, and we have
integrated analytically the $n$-body phase space neglecting $\varphi$ masses and taking momentum independent matrix elements (see e.g. \[18\]). For $d = 5, 6, 7$ we obtain $\gamma_Q^{(d)} \geq (4 \cdot 10^{-20}, 7 \cdot 10^{-3}, 4 \cdot 10^{15}) \times (f_{\text{max}}/m_Q)^{2d-7}$. For $d = 5$, as long as $m_Q \gtrsim 800$ TeV decays occur with safe lifetimes $\gamma_Q^{(5)} \lesssim 10^{-2}$ s. For $d = 6$, even for the largest values $m_Q \sim f_{\text{max}}$ decays occur dangerously close to BBN \[19\].

Operators of $d = 7$ and higher are always excluded. This selects the $R_Q$ which allow for $\mathcal{L}_{\varphi} \neq 0$ (the first seven in Table \[1\]), plus other thirteen which allow for $d = 5$ decay operators. Some of these representations are, however, rather large, and can induce LP in the SM gauge couplings $g_1, g_2, g_3$ at some uncomfortably low-energy scale $\Lambda_{LP} < m_P$. Gravitational corrections to the running of gauge couplings become relevant at scales approaching $m_P$, and can delay the emergence of LP \[50\]. We then specify our second criterion choosing a value of $\Lambda_{LP}$ for which these corrections can presumably be neglected: ii) $R_Q$‘s which do not induce LP in $g_1, g_2, g_3$ below $\Lambda_{LP} \sim 10^{18}$ GeV are phenomenologically preferred. We use two-loop beta functions to evolve the couplings \[48\] and set (conservatively) the threshold for $R_Q$ at $m_Q = 5 \cdot 10^{14}$ GeV. The $R_Q$ surviving this last selection are listed in Table \[1\].

Other features can render some $R_Q$ more appealing than others. For example problems with cosmological domain walls \[51\] are avoided for $N_{DW} = 1$, while specific $R_Q$ can improve gauge coupling unification \[52\]. We prefer not to consider these as crucial discriminating criteria, since solutions to the DW problem exist (see e.g. \[23\] \[53\]), while improved unification might be accidental because of the many $R_Q$ we consider. Nevertheless, we have studied both these issues. The values of $N_{DW}$ are included in Table \[1\] while, as already noted in \[52\], gauge coupling unification gets considerably improved only for $R_3$.

**Axion coupling to photons.** The most promising way to veil the axion is via its interaction with photons $g_{a\gamma\gamma} \propto E \cdot B$ where \[10\]:

$$g_{a\gamma\gamma} = \frac{m_a}{e\nu} \frac{2.0}{10^{10}\text{GeV}} \left( \frac{E}{N} - 1.92(4) \right) ,$$

with $N, E$ the anomaly coefficients in eqs. (2)-(3) (the uncertainty comes from the NLO chiral Lagrangian \[54\]). The last column in Table \[1\] gives $E/N$ for the selected $R_Q$‘s. We have sketched in Fig. \[2\] the “density” of preferred hadronic axion models, drawing with oblique lines (only at small $m_a$) the corresponding couplings. The strongest coupling is obtained for $R_9 = R_8$ and the weakest for $R_9^o = R_3$. They delimit a window $0.25 \leq |E/N - 1.92| \leq 12.75$ encompassing all axion models in Table \[1\]. The corresponding couplings $g_{a\gamma\gamma}$ fall within the band delimited in Fig. \[2\] by the lines $E/N = 5/3$ and $44/3$. With respect to the usual window $0.07 \leq |E/N - 1.92| \leq 7$ \[5\] (delimited by the two dashed lines) the upper (lower) limit is shifted upwards approximatively by a factor of 2 (3.5). It is natural to ask if $g_{a\gamma\gamma}$ could get enhanced by allowing for more $R_Q$‘s ($N_Q > 1$). Let us consider the combined anomaly factor for $R_Q^o \oplus R_Q$:

$$E_c / N_c = \frac{E + E_s}{N + N_s} = \frac{E_c}{N_c} \left( 1 + E/E_s \right) \left( 1 + 1/N/N_s \right) .$$

Since by construction the anomaly coefficients of all $R_Q$‘s in our set satisfy $E/N \leq E_s/N_s$, the factor in parenthesis is $\leq 1$ implying $E_c/N_c \leq E_c/N_s$. This result is easily generalized to $N_Q > 2$. Therefore, as long as the sign of $\Delta X = X_L - X_R$ is the same for all $R_Q$‘s, no enhancement is possible. However, if we allow for $R_Q$‘s with PQ charge differences of opposite sign (we use the symbol $\oplus$ to denote reducible representations of this type) $E/E_s$ and $N/N_s$ in Eq. (13) become negative and $g_{a\gamma\gamma}$ can get enhanced. For $N_Q = 2$ the largest value is $E_c/N_c = 122/3$ obtained for $R_9^o \oplus R_9^x$. For $N_Q > 2$ even larger couplings can be obtained. However, contributions to the $\beta$-functions also become large and can induce LP. This implies that there is a maximum value $g_{a\gamma\gamma}^{\text{max}}$ for which our second condition remains satisfied. We find that $R_9^x \oplus R_9 \oplus R_9$, giving $E_c/N_c = 170/3$, yields the largest possible coupling. The uppermost oblique line in Fig. \[2\] depicts the corresponding $g_{a\gamma\gamma}^{\text{max}}$. More $R_Q$‘s can also suppress $g_{a\gamma\gamma}$ and even produce a complete decoupling. This requires an ad hoc choice of $R_Q$‘s, but no numerical fine tuning. With two $R_Q$‘s there are three cases yielding $g_{a\gamma\gamma} = 0$ within theoretical errors \[27\] (e.g. $R_8 \oplus R_9$ giving $E_c/N_c = 23/12 \simeq 1.92$). This provides additional motivations for search techniques which do not rely on the axion coupling to photons \[55\] \[59\]. Finally,

| $R_Q$ | $C_{Q\tilde{Q}}$ | $N_{\text{max}}^{\text{Q}_{\text{L}}}(\text{GeV})$ | $E/N$ | $N_{\text{DW}}$ |
|------|------------|----------------|--------|---------------|
| $R_1$: $(3, 1, -1)$ | $Q_{l,d}^{LR}$ | $9.3 \cdot 10^{38}$ | $1/3$ | $1$ |
| $R_2$: $(3, 1, +1)$ | $Q_{l,u}^{LR}$ | $5.4 \cdot 10^{34}$ | $1/3$ | $1$ |
| $R_3$: $(3, 2, +1)$ | $Q_{Ll}^{LR}$ | $6.5 \cdot 10^{39}$ | $1/3$ | $1$ |
| $R_4$: $(3, 2, -1)$ | $Q_{l,d}^{LR}$ | $4.3 \cdot 10^{34}$ | $1/3$ | $1$ |
| $R_5$: $(3, 3, +1)$ | $Q_{l,u}^{LR}$ | $5.6 \cdot 10^{32}$ | $1/3$ | $1$ |
| $R_6$: $(3, 3, -1)$ | $Q_{Ll}^{LR}$ | $5.1 \cdot 10^{30}$ | $1/3$ | $1$ |
| $R_7$: $(3, 3, +1)$ | $Q_{l,d}^{LR}$ | $6.6 \cdot 10^{27}$ | $1/3$ | $1$ |
| $R_8$: $(3, 3, -1)$ | $Q_{l,u}^{LR}$ | $3.5 \cdot 10^{18}$ | $1/3$ | $1$ |
| $R_9$: $(6, 1, -1)$ | $Q_{Ll}^{LR}$ | $7.3 \cdot 10^{38}$ | $1/3$ | $1$ |
| $R_{10}$: $(6, 1, +1)$ | $Q_{l,d}^{LR}$ | $1.6 \cdot 10^{15}$ | $1/3$ | $1$ |
| $R_{11}$: $(6, 2, +1)$ | $Q_{l,u}^{LR}$ | $7.6 \cdot 10^{34}$ | $1/3$ | $1$ |
| $R_{12}$: $(6, 2, -1)$ | $Q_{Ll}^{LR}$ | $6.7 \cdot 10^{27}$ | $1/3$ | $1$ |
| $R_{13}$: $(15, 1, -1)$ | $Q_{Ll}^{LR}$ | $8.3 \cdot 10^{23}$ | $1/3$ | $1$ |
| $R_{14}$: $(15, 1, +1)$ | $Q_{l,d}^{LR}$ | $7.6 \cdot 10^{21}$ | $1/3$ | $1$ |

**Table II.** $R_Q$ allowing for $d \leq 4$ and $d = 5$ decay operators ($\sigma \cdot G \equiv \sigma_{\mu\nu} G^{\mu\nu}$) and yielding LP above $10^{36}$GeV. The anomaly contribution to $g_{a\gamma\gamma}$ is given in the fourth column, and the DW number in the fifth one.
FIG. 2. The window for preferred axion models. The green band encompasses models with a single $R_Q$. With more $R_Q$’s the region below the line $E/N = 170/3$ becomes allowed. The two dashed lines enclose the usual window $|E/N| < 1.92$ in [0.07, 7] [8]. Current (projected) exclusion limits are delimited by solid (dashed) lines.

since $T(8) = 3$ and $T(6) = 5/2$, by combining with opposite PQ charge differences $R_{12}$ with $R_9$ or $R_{10}$, new models with $N_{DHI} = 1$ can be constructed.

We have classified hadronic axion models using well-defined phenomenological criteria. The window of preferred models is shown in Fig. 2.

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[1] J. R. Ellis and M. K. Gaillard, Nucl. Phys. B150, 141 (1979).
[2] L. J. Hall, D. Pinner, and J. T. Ruderman, JHEP 12, 134 (2014), arXiv:1409.0551 [hep-ph].
[3] L. Ibaldii, Phys. Rev. D81, 025011 (2010), arXiv:0811.1599 [hep-ph].
[4] S. Aoki et al. (2016), arXiv:1607.00299 [hep-lat].
[5] C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016).
[6] A. E. Nelson, Phys. Lett. B136, 387 (1984).
[7] S. M. Barr, Phys. Rev. Lett. 53, 329 (1984).
[8] M. Dine and P. Draper, JHEP 08, 132 (2015), arXiv:1506.05433 [hep-ph].
[9] R. D. Peccei and H. R. Quinn, Phys. Rev. D16, 1791 (1977).
[10] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).
[11] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978).
[12] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).
[13] M. Kamionkowski and J. March-Russell, Phys. Lett. B282, 137 (1992), arXiv:hep-th/9202003 [hep-th].
[14] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, Phys. Lett. B282, 132 (1992), arXiv:hep-ph/9203206 [hep-ph].
[15] S. M. Barr and D. Seckel, Phys. Rev. D46, 539 (1992).
[16] D. B. Kaplan, Nucl. Phys. B260, 215 (1985).
[17] S. L. Cheng, C. Q. Geng, and W. T. Ni, Phys. Rev. D52, 3132 (1995), arXiv:hep-ph/9506295 [hep-ph].
[18] J. E. Kim, Phys. Rev. D58, 055006 (1998), arXiv:hep-ph/9802220 [hep-ph].
[19] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979).
[20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B166, 493 (1980).
[21] A. R. Zhitnitsky, Sov. J. Nucl. Phys. 31, 260 (1980), [Yad. Fiz.31,497(1980)].
[22] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. B114, 199 (1981).
[23] A. Ringwald and K. Saikawa, Phys. Rev. D93, 085031 (2016), arXiv:1512.06436 [hep-ph].
[24] C. Bonati, M. D’Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo, and G. Villadoro, JHEP 03, 155 (2016), arXiv:1512.06746 [hep-lat].
[25] P. Petreczky, H.-P. Schadler, and S. Sharma, Phys. Lett. B762, 498 (2016), arXiv:1606.03145 [hep-lat].
[26] S. Borsanyi et al., Nature 539, 69 (2016), arXiv:1606.07494 [hep-lat].
[27] L. Di Luzio, F. Mescia, and E. Nardi, In preparation.
[28] M. L. Perl, E. R. Lee, and D. Loomba, Ann. Rev. Nucl. Part. Sci. 59, 17 (2009).
[29] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D71, 083502 (2005), arXiv:astro-ph/0408426 [astro-ph].
[30] K. Jedamzik, JCAP 0803, 008 (2008), arXiv:0710.5153 [hep-ph].
[31] W. Hu and J. Silk, Phys. Rev. Lett. 70, 2661 (1993).
[32] J. Chluba and R. A. Sunyaev, Mon. Not. Roy. Astron. Soc. 419, 1294 (2012), arXiv:1109.6552 [astro-ph.CO].
[33] J. Chluba, Mon. Not. Roy. Astron. Soc. 436, 2232 (2013), arXiv:1304.6121 [astro-ph.CO].
[34] G. D. Kribs and I. Z. Rothstein, Phys. Rev. D55, 4435 (1997), Erratum: Phys. Rev.D56,1822(1997), arXiv:hep-ph/9610468 [hep-ph].
[35] M. Ackermann et al. (Fermi-LAT), Phys. Rev. D86, 022002 (2012), arXiv:1205.2739 [astro-ph.HE].
[36] C. B. Dover, T. K. Gaisser, and G. Steigman, Phys. Rev. Lett. 40, 1117 (1978).
[37] E. Nardi and E. Roulet, Phys. Lett. B245, 105 (1990).
[38] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce, and J. G. Wacker, Phys. Rev. D72, 075011 (2005), arXiv:hep-ph/0504210 [hep-ph].
[39] J. Kang, M. A. Luty, and S. Nasri, JHEP 09, 086 (2006), arXiv:hep-ph/0611322 [hep-ph].
[40] C. Jacoby and S. Nussinov, (2007), arXiv:0712.2681 [hep-ph]

[41] M. Kusakabe and T. Takesako, Phys. Rev. D85, 015005 (2012) arXiv:1112.0860 [hep-ph].

[42] S. Dimopoulos, D. Eichler, R. Esmailzadeh, and G. D. Starkman, Phys. Rev. D41, 2388 (1990).

[43] L. Chuzhoy and E. W. Kolb, JCAP 0907, 014 (2009) arXiv:0809.0436 [astro-ph].

[44] S. Burdin, M. Fairbairn, P. Mermod, D. Milstead, J. Pinfold, T. Sloan, and W. Taylor, Phys. Rept. 582, 1 (2015) arXiv:1410.1374 [hep-ph].

[45] P. Sikivie, Phys. Rev. Lett. 48, 1156 (1982).

[46] A. Gould, B. T. Draine, R. W. Romani, and S. Nussinov, Phys. Lett. B238, 337 (1990).

[47] G. D. Mack, J. F. Beacon, and G. Bertone, Phys. Rev. D76, 043523 (2007), arXiv:0705.4298 [astro-ph].

[48] L. Di Luzio, R. Gröber, J. F. Kamenik, and M. Nardecchia, JHEP 07, 074 (2015) arXiv:1504.00359 [hep-ph].

[49] Since \( m_Q \sim y_Q N_{DW} f_a \), if \( y_Q N_{DW} > 1 \) we can have \( m_Q > f_a \) and a window could open up also for some \( d = 6 \) operators. This case will be addressed in [27].

[50] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006) arXiv:hep-th/0509050 [hep-th].

[51] P. Sikivie, Phys. Rev. Lett. 48, 1156 (1982).

[52] G. F. Giudice, R. Rattazzi, and A. Strumia, Phys. Lett. B715, 142 (2012) arXiv:1204.5465 [hep-ph].

[53] J. E. Kim, Phys. Rept. 150, 1 (1987)

[54] G. Grilli di Cortona, E. Hardy, J. P. Vega, and G. Villadoro, JHEP 01, 034 (2016) arXiv:1511.02867 [hep-ph].

[55] D. Budker, P. W. Graham, M. Ledbetter, S. Rajendran, and A. Sushkov, Phys. Rev. X4, 021030 (2014) arXiv:1306.6089 [hep-ph].

[56] A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. 113, 161801 (2014) arXiv:1403.1290 [hep-ph].