CP Violation in the Cabbibo-suppressed Decay $\tau \rightarrow K\pi\nu_\tau$ with Polarized $\tau$ Leptons

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Abstract

CP violation from physics beyond the Standard Model (SM) is investigated in the Cabbibo-suppressed decay $\tau \rightarrow K\pi\nu_\tau$ with polarized $\tau$ leptons, to which both the $J^P = 1^- \text{ resonance } K^*$ and the $J^P = 0^+ \text{ resonance } K^*_0$ contribute. In addition to the CP-odd rate asymmetry, $\tau$ polarization enables us to construct three additional CP-odd polarization asymmetries that can be enhanced due to the interference between the $K^*$ and $K^*_0$, and whose magnitudes depend crucially on the $K^*_0$ decay constant, $f_{K^*_0}$. Taking a QCD sum rule estimate of $f_{K^*_0} = 45$ MeV and the present experimental constraints on the CP-odd parameters into account, we estimate quantitatively the maximally-allowed values for the CP-odd rate and polarization asymmetries in the multi-Higgs-doublet (MHD) model and the scalar-leptoquark (SLQ) models consistent with the SM gauge symmetry where neutrinos are massless and left-handed as in the SM. We find that the CP-odd rate and polarization asymmetries are of a similar size for highly-polarized $\tau$ leptons and, for their maximally-allowed values, CP violation in the MHD model and two SLQ models may be detected with about $10^6$ and $10^7 \tau$'s at the $1\sigma$ level.

(to be submitted to PLB)
The decay of the $\tau$, the most massive of the known leptons, can serve not only as a useful tool in the investigation of some aspects of the SM but also as a powerful experimental probe of new physics phenomena \([1]\). One phenomenon where new physics can play a crucial role is CP violation. In light of this aspect the $\tau$ decays into hadrons have been recently studied as probes of CP violation in the scalar sector of physics beyond the SM \([2, 3, 4, 5]\).

In the present paper we extend the previous work \([5]\) by Choi, Hagiwara and Tanabashi to probe CP violation in the Cabbibo-suppressed $\tau$ decay $\tau \to K\pi\nu_\tau$ with polarized $\tau$’s. The decay mode is dominated by the contributions of the two lowest vector and scalar resonances, $K^*$ and $K_0^*$, with different spins and relatively large width-to-mass ratios \([6]\):

\[
\begin{align*}
K^*: & \quad J^P = 1^-, \quad m_{K^*} = 892 \text{ MeV}, \quad \Gamma_{K^*} = 50 \text{ MeV}, \\
K_0^*: & \quad J^P = 0^+, \quad m_{K_0^*} = 1430 \text{ MeV}, \quad \Gamma_{K_0^*} = 287 \text{ MeV},
\end{align*}
\]

and the mode is expected to have larger scalar contributions than the $2\pi$ or $3\pi$ modes due to the $s$ quark mass much larger than the $d$ quark mass. In the light of these aspects, the Cabbibo-suppressed mode is worthwhile to be investigated in detail.

Including possible contributions from new physics with massless left-handed neutrinos, we can write the matrix element for the decay $\tau^- \to (K\pi)^-\nu_\tau$ in the general form

\[
M = \sqrt{2} G_F \left[ (1 + \chi)\bar{u}(k, -)\gamma^\mu P_- u(p, \sigma) J_\mu + \eta\bar{u}(k, -)P_+ u(p, \sigma) J_S \right],
\]

where $P_\pm = (1 \pm \gamma_5)/2$, $G_F$ is the Fermi constant, and $p$ and $k$ are the four momenta of the $\tau$ lepton and the tau neutrino, respectively. The parameters $\chi$ and $\eta$, which parametrize contributions from physics beyond the SM, are in general complex. In eq. (2), the helicity of the $\tau^-$ is denoted by $\sigma$ ($\sigma = \pm 1$) with its spin quantization direction along its neutrino momentum direction. The tau neutrino is assumed to be massless and left-handed as in the SM so that the helicity value is $-1/2$ as indicated by the negative sign in its spinor $u(k, -)$. The vector and scalar hadronic matrix elements

\[
\begin{align*}
J_\mu &= \sin \theta_C \langle (K\pi)^-|\bar{s}\gamma_\mu u|0\rangle, \\
J_S &= \sin \theta_C \langle (K\pi)^-|\bar{s}u|0\rangle,
\end{align*}
\]

with $\sin \theta_C = 0.23$ for the Cabbibo angle $\theta_C$ are related through the Dirac equations to the $\bar{s}$ and $u$ quarks at the quark level and their explicit form can be parametrized in terms of two form factors $F_K(q^2)$ and $F_S(q^2)$:

\[
\begin{align*}
J_\mu &= \sqrt{2} \sin \theta_C \left[ F_K(q^2) \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (q_1 - q_2)\gamma^\nu + \frac{m_{K^*}^2}{q^2} C_K F_S(q^2) q_\mu \right], \\
J_S &= \sqrt{2} \sin \theta_C \left( \frac{m_{K_0^*}^2}{m_s - m_u} \right) C_K F_S(q^2),
\end{align*}
\]

where $q_1$ and $q_2$ are the four-momenta of $\pi$ and $K$, respectively, $m_s$ and $m_u$ the $s$ and $u$ current quark masses, and $q$ is the four-momentum of the $K\pi$ system; $q = q_1 + q_2$. 

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The coupling strength $C_K$ denoting the scalar contributions is determined by the $K_0^*$ decay constant $f_{K_0^*}$ and the coupling strength $g_{K_0^*K\pi}$ of the $K_0^*$ to $K\pi$ under the assumption of $\text{Br}(K_0^* \to K\pi) = 100%$:

$$C_K = \frac{f_{K_0^*} g_{K_0^*K\pi}}{\sqrt{3}m_{K_0^*}^2}. \quad (5)$$

The value of $g_{K_0^*K\pi}$ is 4.9 GeV from the measured $K_0^* \to K\pi$ decay width $\Gamma(K_0^* \to K\pi) \approx 287$ MeV. Even though the $K_0^*$ decay constant $f_{K_0^*}$ is not experimentally measured, it has been estimated by several model-dependent methods. The QCD sum-rule estimate in the $K_0^*$ narrow-width approximation is $f_{K_0^*} \approx 31$ MeV \[7\] and an effective Lagrangian estimate of $f_{K_0^*}$ including the width effects leads to a larger value of about 45 MeV \[8\], which approaches the pole dominance result of 50 MeV \[9\]. In light of these present crude estimates of the $K_0^*$ decay constant, we adopt for the $K_0^*$ decay constant the effective Lagrangian estimate \[8\]

$$f_{K_0^*} = 45 \text{ MeV}, \quad (6)$$

anticipating that the decay constant $f_{K_0^*}$ is measured more precisely in future experiments.

![Figure 1: Definition of the angular variables in the $\tau$ and $(K\pi)$ rest frames for the Cabbibo-suppressed $\tau$ decay $\tau^- \to (K\pi)^-\nu_\tau$. The $(K\pi)$ momentum direction in the $\tau$ rest frame is denoted by the polar angle $\Theta$ and the azimuthal angle $\Phi$ with respect to the $\tau$ momentum direction defined as the positive z-axis, and the $K$ momentum direction in the $(K\pi)$ rest frame denoted by the angles $\theta$ and $\phi$ with respect to the same positive z direction.](image)

In spite of the several resonance contributions to $F_K$ and $F_S$, the form factors are approximated to be the propagators of the lowest-level resonance states $K^*$ and $K_0^*$:

$$F_K(q^2) = B_{K^*}(q^2), \quad F_S(q^2) = B_{K_0^*}(q^2), \quad (7)$$
respectively, where $B_{K^*}$ and $B_{K_0^*}$ are parametrized in the Breit-Wigner form with the momentum-dependent widths [10]:

$$B_X(q^2) = \frac{m_X^2}{m_X^2 - q^2 - im_X \Gamma_X(q^2)}, \quad \Gamma_X(q^2) = \Gamma_X D_X(q^2),$$

for $X = K^*$ or $K_0^*$. We adopt for the momentum-dependent widths the parametrizations of the $\tau$-decay program package TAUOLA [10]

$$D_{K^*}(q^2) = \begin{cases} \frac{m_{K^*}}{\sqrt{q^2}} \left( \frac{P_{K}(q^2)}{P_{K}(m_{K^*})} \right)^3 & \text{for } q^2 > (m_{K^*} + m_\pi)^2, \\ 0 & \text{for } q^2 \leq (m_{K^*} + m_\pi)^2, \end{cases}$$

$$D_{K_0^*}(q^2) = \begin{cases} \frac{m_{K_0^*}}{\sqrt{q^2}} \left( \frac{P_{K}(q^2)}{P_{K}(m_{K_0^*})} \right)^3 & \text{for } q^2 > (m_{K_0^*} + m_\pi)^2, \\ 0 & \text{for } q^2 \leq (m_{K_0^*} + m_\pi)^2. \end{cases}$$

Let us now calculate the helicity amplitudes of the Cabbibo-suppressed $\tau$ decay. In general, the $\tau$ three-body decay into $(K\pi)\nu_\tau$ is described by five independent kinematic variables. Because the hadronic $(K\pi)$ system is solely determined by the lepton momentum transfer, it is convenient to consider two reference frames, namely the $\tau$ rest frame and the $(K\pi)$ rest frame as shown in Fig. [1]. We define the momentum direction of the virtual $K^*$ or $K_0^*$ in the $\tau$ rest frame by the polar angle $\theta$ and the azimuthal angle $\phi$. The rotational invariance of the total system with respect to the $\tau$ momentum direction allows us to take $\Phi$ to be zero in the calculation of the helicity amplitudes, while the azimuthal angle $\Phi$ is employed to describe $\tau$ polarization, especially the transverse polarization of the $\tau$. Setting $\Phi$ to be zero and using the 2-component spinor technique [11], one can obtain the helicity amplitudes $M_\nu$ of the Cabbibo-suppressed decay $\tau^- \to (K\pi)^0\nu_\tau$ in the reference frames defined in Fig. [1] as

$$M_+ = 2G_F \sin \theta_C(1 + \chi) \sqrt{m_\tau^2 - q^2} \left[ m_\tau \cos \frac{\Theta}{2} \left( \frac{m_{K_0^*}^2}{q^2} + \xi \right) C_K F_S(q^2) - 2P_K \left\{ \cos \frac{\Theta}{2} + \sin \frac{\Theta}{2} \cos \phi \sin \frac{\Theta}{2} - \frac{m_\tau}{\sqrt{q^2}} - 1 \right\} \cos \phi \cos \frac{\Theta}{2} \right] F_K(q^2),$$

$$M_- = 2G_F \sin \theta_C(1 + \chi) \sqrt{m_\tau^2 - q^2} \left[ m_\tau \sin \frac{\Theta}{2} \left( \frac{m_{K_0^*}^2}{q^2} + \xi \right) C_K F_S(q^2) + 2P_K \left\{ \cos \frac{\Theta}{2} - \sin \cos \frac{\Theta}{2} - \frac{m_\tau}{\sqrt{q^2}} - 1 \right\} \cos \phi \cos \frac{\Theta}{2} \right] F_K(q^2),$$

where $\vartheta$ is the angle between the $K$ momentum in the $(K\pi)$ rest frame and the $(K\pi)$ momentum in the $\tau$-rest frame, i.e. $\cos \vartheta = \sin \theta \cos \phi \sin \Theta + \cos \theta \cos \Theta$, and $P_K$ is the size of the $K$ momentum in the virtual $K^*$ and $K_0^*$ rest frame;

$$P_K(q^2) = \frac{1}{2\sqrt{q^2}} \lambda^{1/2}(q^2, m_\pi^2, m_K^2),$$

(12)
with \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz \). The dimensionless parameter \( \xi \) determines the relative size of the scalar contributions to the vector ones:

\[
\xi = \frac{m_{K_0^*}^2}{(m_\nu - m_\mu)m_\tau} \left( \frac{\eta}{1 + \chi} \right). \tag{13}
\]

Clearly, the initial \( \tau^- \) system is not CP self-conjugate and so any genuine CP-odd observable can be constructed only by considering both the Cabibbo-suppressed \( \tau^- \) decay and its charge-conjugated \( \tau^+ \) decay, and by identifying the CP relations of their kinematic distributions. Before constructing all the possible CP-odd asymmetries explicitly, we calculate the helicity amplitudes for the charge-conjugated process \( \lambda^- \) with \( \theta \)

where \( \tau \) only the difference \( \Phi - \theta \) direction has been used to relate the first and second expressions. It is noteworthy that the rotational invariance of the total system with respect to the \( \tau \) momentum direction in the laboratory frame. In this case the decay amplitude to the final state \( |\Theta, \Phi\rangle \) of the virtual \( K^*/K_0^* \) system is expressed in terms of the helicity amplitudes as

\[
\langle \Theta, \Phi | \theta_p, \phi_p \rangle = \langle \Theta, 0 | \theta_p, \phi_p - \Phi \rangle = \frac{\theta_p}{2} M_+ + \frac{\theta_p}{2} e^{i(\phi_p - \Phi)} M_- , \tag{17}
\]

where the rotational invariance of the total system with respect to the \( \tau \) momentum direction has been used to relate the first and second expressions. It is noteworthy that only the difference \( \Phi - \phi_p \) between the azimuthal angles \( \Phi \) and \( \phi_p \) appears.

\[
M_+ = 2G_F \sin \theta_C (1 + \chi^*) \sqrt{m_\tau^2 - q^2} \left[ m_\tau \sin \frac{\Theta}{2} \left( \frac{m_{K_0^*}^2}{q^2} + \xi^* \right) C_K F_S(q^2) \right. \\
- 2P_K \left\{ - \cos \theta \sin \frac{\Theta}{2} + \sin \theta \cos \frac{\Theta}{2} e^{i\phi} + \left( \frac{m_\tau}{\sqrt{q^2}} - 1 \right) \cos \bar{\theta} \sin \frac{\bar{\Theta}}{2} \right\} F_K(q^2) ,
\]

\[
M_- = - 2G_F \sin \theta_C (1 + \chi^*) \sqrt{m_\tau^2 - q^2} \left[ m_\tau \cos \frac{\Theta}{2} \left( \frac{m_{K_0^*}^2}{q^2} + \xi^* \right) C_K F_S(q^2) \right. \\
- 2P_K \left\{ \cos \bar{\theta} \cos \frac{\bar{\Theta}}{2} + \sin \bar{\theta} \sin \frac{\bar{\Theta}}{2} e^{-i\phi} + \left( \frac{m_\tau}{\sqrt{q^2}} - 1 \right) \cos \bar{\theta} \cos \frac{\bar{\Theta}}{2} \right\} F_K(q^2) . \tag{14}
\]

It is easily shown that, if the parameters \( \eta \) and \( \chi \) are real, the helicity amplitudes (14) for the \( \tau^- \) decay and (14) for the \( \tau^+ \) decay satisfy the CP relation:

\[
M_+ (\Theta; q^2; \theta, \phi) = \mp M_\mp (\Theta; q^2; \theta, -\phi) . \tag{15}
\]

With the results in eqs. (14) and (14) for the \( \tau^\pm \) decay helicity amplitudes, one can describe the decay of an arbitrary polarized \( \tau \) lepton by superposing the two helicity states. Generally, a pure \( \tau \) polarization state, which is polarized in the direction \( (\theta_p, \phi_p) \), is given in terms of the helicity states by

\[
|\theta_p, \phi_p\rangle = \cos \frac{\theta_p}{2} |+\rangle + \sin \frac{\theta_p}{2} e^{i\phi_p} |-\rangle , \tag{16}
\]

where \( \theta_p \) and \( \phi_p \) are the polar and azimuthal angles in the \( \tau \) rest frame, respectively, with respect to the \( \tau \) momentum direction in the laboratory frame. In this case the decay amplitude to the final state \( |\Theta, \Phi\rangle \) of the virtual \( K^*/K_0^* \) system is expressed in terms of the helicity amplitudes as

\[
\langle \Theta, \Phi | \theta_p, \phi_p \rangle = \langle \Theta, 0 | \theta_p, \phi_p - \Phi \rangle = \frac{\theta_p}{2} M_+ + \frac{\theta_p}{2} e^{i(\phi_p - \Phi)} M_- , \tag{17}
\]

where the rotational invariance of the total system with respect to the \( \tau \) momentum direction has been used to relate the first and second expressions. It is noteworthy that only the difference \( \Phi - \phi_p \) between the azimuthal angles \( \Phi \) and \( \phi_p \) appears.
Taking into account the polarization degree $P$ for a partially-polarized $\tau^-$ beam, one can decompose the differential decay rate into four independent terms:

$$
d\Gamma = \frac{1}{2} d(\Gamma_{++} + \Gamma_{--}) + \frac{1}{2} P \cos \theta_p d(\Gamma_{++} - \Gamma_{--}) + P \sin \theta_p \cos(\phi_p - \Phi) \text{Re}(d\Gamma_{+-}) - P \sin \theta_p \sin(\phi_p - \Phi) \text{Im}(d\Gamma_{+-}),$$

where the helicity-dependent terms are defined as

$$
d\Gamma_{\sigma\sigma'} = \frac{1}{(2\pi)^5 32 m_\tau} \left(1 - \frac{q^2}{m_\tau^2}\right) M_\sigma M_{\sigma'}^* P_K d\sqrt{q^2} d\cos \theta d\phi d\cos \Theta.$$

(18)

For the sake of notational convenience, we introduce $d\Phi_3$ for the phase space element $d\sqrt{q^2} d\cos \theta d\phi d\cos \Theta$ and denote the four independent terms in eq. (18) as

$$
d\Gamma_1 d\Phi_3 = \frac{d(\Gamma_{++} + \Gamma_{--})}{d\Phi_3}, \quad d\Gamma_2 d\Phi_3 = \frac{d(\Gamma_{++} - \Gamma_{--})}{d\Phi_3},$$

$$
d\Gamma_3 d\Phi_3 = 2 \text{Re} \left(\frac{d\Gamma_{+-}}{d\Phi_3}\right), \quad d\Gamma_4 d\Phi_3 = 2 \text{Im} \left(\frac{d\Gamma_{+-}}{d\Phi_3}\right).$$

(20)

Note that (a) the $\Gamma_1$ term is the unpolarized differential decay rate and the other three terms $\Gamma_i$ ($i = 2, 3, 4$) are polarization-dependent; the $\Gamma_2$ term is due to longitudinal polarization, the $\Gamma_3$ term due to transverse polarization and the $\Gamma_4$ due to normal polarization, and (b) the transverse and normal components after integrating the differential decay rate over the azimuthal angle $\Theta$ or $\phi_p$ vanish, which is consistent with the so-called “null transverse-polarization theorem” [12]. Clearly, to utilize the polarization-dependent terms effectively, the initial $\tau$ beam should be highly polarized, that is to say, $P$ should be relatively large. Deferring the potential impact of the value of $P$ to our later discussion, we assume $P=1$ for the time being. In this case, each $d\Gamma_i/d\Phi_3$ ($i = 1$ to 4) term can be decomposed into a CP-even part $\Sigma_i$ and a CP-odd part $\Delta_i$:

$$
d\Gamma_i d\Phi_3 = \frac{1}{2} (\Sigma_i + \Delta_i).$$

(21)

The four CP-even parts $\Sigma_i$ and four CP-odd parts $\Delta_i$ can be easily identified by use of the CP relation (15) between the $\tau^-$ and $\tau^+$ decay helicity amplitudes and they are expressed in terms of the $\tau^\mp$ helicity-dependent terms $d\Gamma_i/d\Phi_3$ and $d\bar{\Gamma}_i/d\Phi_3$ as

$$
\Sigma_1 = \frac{d(\Gamma_1 + \bar{\Gamma}_1)}{d\Phi_3}, \quad \Sigma_2 = \frac{d(\Gamma_2 - \bar{\Gamma}_2)}{d\Phi_3},$$

$$
\Sigma_3 = \frac{d(\Gamma_3 - \bar{\Gamma}_3)}{d\Phi_3}, \quad \Sigma_4 = \frac{d(\Gamma_4 + \bar{\Gamma}_4)}{d\Phi_3},$$

$$
\Delta_1 = \frac{d(\Gamma_1 - \bar{\Gamma}_1)}{d\Phi_3}, \quad \Delta_2 = \frac{d(\Gamma_2 + \bar{\Gamma}_2)}{d\Phi_3},$$

$$
\Delta_3 = \frac{d(\Gamma_3 + \bar{\Gamma}_3)}{d\Phi_3}, \quad \Delta_4 = \frac{d(\Gamma_4 - \bar{\Gamma}_4)}{d\Phi_3}.$$

(22)
where we have used the same kinematic variables \( \{q^2, \Theta, \theta\} \) for the \( d\Gamma_i/d\Phi_3 \) except for the replacement of \( \hat{\phi} \) by \(-\phi\). We have numerically estimated the four CP-even terms in the allowed phase-space points and have found that the other three \( \Sigma_i \) (\( i = 2, 3, 4 \)) terms are negligible compared to the \( \Sigma_1 \) term. Therefore we neglect those terms in the following. The CP-even \( \Sigma_1 \) term and the CP-odd \( \Delta_i \) (\( i = 1 \) to 4) can be obtained from the \( \tau^\pm \) decay helicity amplitudes and their explicit form is listed in Appendix A. All the CP-odd terms are proportional to the imaginary part of the parameter \( \xi \) in eq. (13).

An appropriate real weight function \( w_i(\Theta; q^2, \theta, \phi) \) is usually employed to separate the \( \Delta_i \) contribution and to enhance its analysis power for the CP-odd parameter \( \text{Im}(\xi) \) through the CP-odd quantity:

\[
\langle w_i \Delta_i \rangle \equiv \int [w_i \Delta_i] d\Phi_3.
\] (23)

of which the analysis power is determined by the parameter

\[
\varepsilon_i = \frac{\langle w_i \Delta_i \rangle}{\sqrt{\langle \Sigma_1 \rangle \langle w_i^2 \Sigma_1 \rangle}}.
\] (24)

For the analysis power \( \varepsilon_i \), the number \( N_i \) of the \( \tau \) leptons needed to observe CP violation at the 1-\( \sigma \) level is

\[
N_i = \frac{1}{Br \cdot \varepsilon_i^2},
\] (25)

where \( Br \) is the branching fraction of the relevant \( \tau \) decay mode. Certainly, it is desirable to find the optimal weight function with the largest analysis power. It is known [13] that, when the CP-odd contribution to the total rate is relatively small, the optimal weight function is approximately given by

\[
w_{i,\text{opt}} = \frac{\Delta_i}{\Sigma_1}.
\] (26)

We adopt these optimal weight functions in the following numerical analyses with several concrete models beyond the SM introduced in the following.

Although there is no CP violation in the \( \tau \) decays within the SM, it is possible to conceive several new sources of CP violation in the \( \tau \) decays. Among them we will consider models with new scalar-fermion interactions, which still preserve the SM gauge symmetries and have only the massless and left-handed neutrinos as in the SM. In this case, only four types of scalar-fermion interactions can contribute to the Cabibbo-suppressed decay \( \tau \to (K\pi)\nu_\tau \) [14]; the MHD model [15] and three SLQ models [16].

In the MHD model CP violation can arise in the charged Higgs sector with more than two Higgs doublets [17] and when not all the charged scalars are degenerate. As in most previous phenomenological analyses, we also will assume in this MHD model that all but the lightest of the charged scalars effectively decouple from fermions. The effective Lagrangian for the decay \( \tau \to K\pi\nu_\tau \) in the assumption is then given at energies
considerably low compared to $M_H$ by

$$\mathcal{L}_{\text{MHD}} = 2\sqrt{2}G_F \sin \theta_C \left( \frac{m_s m_u}{M_H^2} \right) \left[ X^* Z (\bar{s}_R u_L)(\bar{\nu}_{\tau_L} \tau_R) + \left( \frac{m_u}{m_s} \right) Y^* Z (\bar{s}_L u_R)(\bar{\nu}_{\tau_L} \tau_R) \right] + \text{h.c.},$$

(27)

where $X, Y$ and $Z$ are complex coupling constants which can be expressed in terms of the charged Higgs mixing matrix elements. From the effective Lagrangian, one obtain for the MHD CP-violation parameter $\text{Im}(\xi_{\text{MHD}})$

$$\text{Im}(\xi_{\text{MHD}}) = - \left( \frac{m_s}{m_u - m_s} \right) \left( \frac{m_u^2}{M_H^2} \right) \left[ \text{Im}(X Z^*) + \left( \frac{m_u}{m_s} \right) \text{Im}(Y Z^*) \right].$$

(28)

On the other hand, the effective Lagrangians for the three SLQ models [14] contributing to the decay $\tau \to \bar{K}\pi\nu_{\tau}$ are written in the form after a few Fierz rearrangements:

$$\mathcal{L}^{I}_{\text{SLQ}} = - \frac{x_{23} z_{13}^{*}}{2 M_{\phi_1}^2} \left[ (\bar{s}_L u_R)(\bar{\nu}_{\tau_L} \tau_R) + \frac{1}{4} (\bar{s}_L \sigma^{\mu \nu} u_R)(\bar{\nu}_{\tau_L} \sigma_{\mu \nu} \tau_R) \right] + \text{h.c.},$$

$$\mathcal{L}^{II}_{\text{SLQ}} = - \frac{y_{23} z_{13}^{*}}{2 M_{\phi_2}^2} \left[ (\bar{s}_L u_R)(\bar{\tau}_{\tau_L} \nu_{\tau_L}) + \frac{1}{4} (\bar{s}_L \sigma^{\mu \nu} u_R)(\bar{\tau}_{\tau_L} \sigma_{\mu \nu} \nu_{\tau_L}) \right] + \frac{y_{23} z_{13}^{*}}{2 M_{\phi_2}^2} (\bar{s}_L \gamma_{\mu} u_R)(\bar{\tau}_{\tau_L} \gamma_{\mu} \nu_{\tau_L}) + \text{h.c.},$$

$$\mathcal{L}^{III}_{\text{SLQ}} = - \frac{z_{23} z_{13}^{*}}{2 M_{\phi_3}^2} (\bar{s}_L \gamma_{\mu} u_R)(\bar{\tau}_{\tau_L} \gamma_{\mu} \nu_{\tau_L}) + \text{h.c.}$$

(29)

Here the coupling constants $x_{ij}^{(t)}$, $y_{ij}^{(t)}$ and $z_{ij}$ ($i,j = 1, 2, 3$) are in general complex so that CP is violated in the scalar-fermion Yukawa interaction terms. The superscript $c$ in the Lagrangians $\mathcal{L}^{II}_{\text{SLQ}}$ and $\mathcal{L}^{III}_{\text{SLQ}}$ denotes charge conjugation, i.e. $\psi_{R,L}^c = i \gamma^0 \gamma^2 \bar{\psi}_{R,L}^T$ in the chiral representation. Although the tensor parts as well as the scalar parts appear in Model I and Model II, we do not have the tensor contributions to $\tau \to (K \pi) \nu_{\tau}$ because we concentrate on the vector and scalar resonance contributions in the present work. In the approximation that all the CP-even contributions from new interactions are neglected, the size of the SLQ CP-violation effects is dictated by the CP-odd parameters

$$\text{Im}(\xi^{I}_{\text{SLQ}}) = - \frac{m_{K_0}^2}{(m_s - m_u) m_{\tau}} \frac{\text{Im}[x_{23} z_{13}^{*}]}{4 \sqrt{2} G_F \sin \theta_C M_{\phi_1}^2},$$

$$\text{Im}(\xi^{II}_{\text{SLQ}}) = - \frac{m_{K_0}^2}{(m_s - m_u) m_{\tau}} \frac{\text{Im}[y_{23} z_{13}^{*}]}{4 \sqrt{2} G_F \sin \theta_C M_{\phi_2}^2},$$

$$\text{Im}(\xi^{III}_{\text{SLQ}}) = 0.$$  

(30)

This approximation is justified because the contributions from new physics are expected to be very small compared to those from the SM.

The experimental constraints on the CP-violation parameters in [28] and [30] depend on the values for $u$ and $s$ current quark masses, which are not well determined. Here, we
use for the light $u$ and $s$ quark masses $m_u = 5$ MeV and $m_s = 320$ MeV, which satisfy the mass relation $m_s - m_u = 7 f_{K^0}^2$. Inserting the quark mass values into (28) and (30) yields

$$\text{Im}(\xi_{\text{MHD}}) \simeq -3.2 \times 10^{-4} \left(\frac{M_W}{M_H}\right)^2 \left[\text{Im}(XZ^*) + \frac{1}{64}\text{Im}(YZ^*)\right],$$

$$\text{Im}(\xi_{\text{SLQ}}^I) \simeq -39.3 \left(\frac{M_W}{M_{\phi_1}}\right)^2 \text{Im}[x_{23} x_{13}^*],$$

$$\text{Im}(\xi_{\text{SLQ}}^II) \simeq -39.3 \left(\frac{M_W}{M_{\phi_2}}\right)^2 \text{Im}[y_{23} y_{13}^*],$$

where the $W$-boson mass $M_W$ is retained to show the $M_W$ dependence of the parameters explicitly, but 80 GeV will be used for the $W$-boson mass in the actual numerical analysis.

The couplings $X, Y$ and $Z$ in the MHD model can be constrained through the processes such as $B$-meson semileptonic decays. Because these experimental constraints have been extensively reviewed in Ref. [15], we simply follow the analysis from which the combined constraint on $\text{Im}(\xi_{\text{MHD}})$ is obtained to be

$$|\text{Im}(\xi_{\text{MHD}})| < 0.48,$$

when $M_H$ is set to be 45 GeV. Although there are at present no direct constraints on the SLQ CP-odd parameters in (30), a rough constraint to the parameters can be provided on the assumption [18] that $|x_{13}^*| \sim |x_{13}|$ and $|y_{13}^*| \sim |y_{13}|$, that is to say, the leptoquark couplings to quarks and leptons belonging to the same generation are of a similar size; then the experimental upper bound for the $D\bar{D}$ mixing yields

$$|\text{Im}(\xi_{\text{SLQ}}^I)| < 0.15, \quad |\text{Im}(\xi_{\text{SLQ}}^{II})| < 0.14,$$

which are stronger than the constraint (33) on the MHD CP-odd parameter $\text{Im}(\xi_{\text{MHD}})$. Based on the constraints (33) and (34) to the CP-odd parameters, we quantitatively estimate the number of the Cabbibo-suppressed $\tau$ decays to detect CP violation for the maximally-allowed values of the CP-odd parameters:

$$\text{Im}(\xi_{\text{MHD}}) = 0.48, \quad \text{Im}(\xi_{\text{SLQ}}^I) = 0.15, \quad \text{Im}(\xi_{\text{SLQ}}^{II}) = 0.14.$$
asymmetries require a similar number of \( \tau \) decay events. But, we note that the results for the CP-odd polarization asymmetries have been obtained for completely-polarized \( \tau \) leptons. Therefore, in a realistic experiment with partially-polarized \( \tau \) leptons the analysis power of the polarization-dependent observables will be reduced. In his recent works [3], Tsai has claimed that \( \tau \) polarization can play a crucial role in probing P, CP and T violation in \( \tau \) decays. However, we see that at least in the Cabbibo-suppressed \( \tau \) decay \( \tau \to (K\pi)\nu_\tau \) it is crucial to highly polarize the \( \tau \) leptons to fully utilize the CP-odd polarization-dependent observables. Numerically, we find that CP violation in the MHD model and the two SLQ models may be detected with about 10\(^6\) and 10\(^7\) \( \tau \) leptons for the maximally-allowed CP-odd MHD and SLQ parameters, respectively.

### Table 1.
The number of \( \tau \) leptons, \( N_i \), needed for detection with the \( \varepsilon_i \) at the 1\( \sigma \) level, are determined for \( f_{K_0^*} = 45 \) MeV with \( \text{Im}(\xi_{\text{MHD}}) = 0.48 \) in the MHD model, and \( \text{Im}(\xi_{\text{SLQ}}^I) = 0.15 \) and \( \text{Im}(\xi_{\text{SLQ}}^II) = 0.14 \) in the two SLQ models.

| Model | \( N_1 \)   | \( N_2 \)   | \( N_3 \)   | \( N_4 \)   |
|-------|-------------|-------------|-------------|-------------|
| MHD   | 3.19 \times 10^5 | 3.20 \times 10^5 | 3.19 \times 10^5 | 3.21 \times 10^5 |
| SLQI  | 3.26 \times 10^6 | 3.28 \times 10^6 | 3.27 \times 10^6 | 3.29 \times 10^6 |
| SLQII | 3.75 \times 10^6 | 3.76 \times 10^6 | 3.75 \times 10^6 | 3.78 \times 10^6 |

In summary, we have investigated CP violation from the MHD model and SLQ models in the Cabbibo-suppressed decay \( \tau \to K\pi\nu_\tau \) with polarized \( \tau \) leptons to which both the \( J^P = 1^- \) resonance \( K^* \) and the \( J^P = 0^+ \) resonance \( K_0^* \) contribute. In addition to the CP-odd rate asymmetry, \( \tau \) polarization enables us to construct three additional CP-odd polarization asymmetries whose magnitudes depend crucially on the \( K_0^* \) decay constant, \( f_{K_0^*} \). Taking a QCD sum rule estimate of \( f_{K_0^*} = 45 \) MeV and the present experimental constraints on the CP-odd parameters into account, we have quantitatively estimated the maximally-allowed values for the CP-odd rate and polarization asymmetries in the MHD and SLQ models consistent with the SM gauge symmetry with massless left-handed neutrinos. We have found that the CP-odd rate and polarization asymmetries are of a similar size for highly-polarized \( \tau \) leptons and, for their maximally-allowed values, new scalar-fermion interactions may be detected with about 10\(^6\) or 10\(^7\) \( \tau \) leptons at the 1\( \sigma \) level. Consequently, we conclude that since the \( \tau \) leptons of the order of 10\(^7\) or more are expected to be produced yearly at the planned \( B \) factories [19] and the proposed \( \tau \)-charm factories [20], it is important to look for CP violation in the Cabbibo-suppressed \( \tau \) decay \( \tau \to K\pi\nu_\tau \) even without polarized \( \tau \) leptons.
Acknowledgments.

JS gratefully acknowledges support from the Korean Science and Engineering Foundation (KOSEF) through the Center for Theoretical Physics (CTP). The work was supported in part by the KOSEF-DFG large collaboration project, Project No. 96-0702-01-01-2.

Appendix

A. CP-even and CP-odd observables

In the appendix the explicit form of the CP-even term $\Sigma_1$ and the CP-odd terms $\Delta_i$ ($i = 1$ to 4) is presented:

$$\Sigma = F(q^2) \left[ 2C_K^2 m_\tau^2 \frac{m_K^2}{q^2} + \xi \right]^2 |F_S|^2 + 8P_K^2 |F_K|^2 \left\{ 1 + \left( \frac{m_\tau^2}{q^2} - 1 \right) \cos^2 \vartheta \right\}$$

$$-8C_K P_K \frac{m_\tau^2}{q^2} \left( \frac{m_K^2}{q^2} + \text{Re}(\xi) \right) \text{Re}(F_K F_S^* \cos \vartheta) ,$$

(36)

$$\Delta_1 = -8F(q^2) C_K m_\tau P_K \left( \frac{m_\tau}{\sqrt{q^2}} \right) \cos \vartheta \text{Im}(\xi) \text{Im}(F_K F_S^*) ,$$

(37)

$$\Delta_2 = -8F(q^2) C_K m_\tau P_K \text{Im}(\xi) \left[ \left\{ \cos \vartheta + \left( \frac{m_\tau}{\sqrt{q^2}} - 1 \right) \cos \Theta \cos \vartheta \right\} \text{Im}(F_K F_S^*)$$

$$+ \sin \Theta \sin \vartheta \sin \phi \text{Re}(F_K F_S^*) \right] ,$$

(38)

$$\Delta_3 = -8F(q^2) C_K m_\tau P_K \text{Im}(\xi) \left[ \left\{ \sin \vartheta \cos \phi + \left( \frac{m_\tau}{\sqrt{q^2}} - 1 \right) \sin \Theta \cos \vartheta \right\} \text{Im}(F_K F_S^*)$$

$$- \cos \Theta \sin \vartheta \sin \phi \text{Re}(F_K F_S^*) \right] ,$$

(39)

$$\Delta_4 = -8F(q^2) C_K m_\tau P_K \text{Im}(\xi) \left[ \sin \vartheta \sin \phi \text{Im}(F_K F_S^*)$$

$$+ \left( \cos \Theta \sin \vartheta \cos \phi - \sin \Theta \cos \vartheta \right) \text{Re}(F_K F_S^*) \right] ,$$

(40)

where the overall function $F(q^2)$ is given by

$$F(q^2) = \frac{G_F^2 m_\tau \sin^2 \theta_e}{2^7 \pi^4} \left( 1 - \frac{q^2}{m_\tau^2} \right)^2 |1 + \xi|^2 ,$$

(41)
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