Quark deconfinement and Gluon condensate in a weak magnetic field from QCD Sum Rules

Cesareo A. Dominguez and Luis A. Hernandez
Department of Physics, University of Cape Town
cesareo.dominguez@uct.ac.za and hrlnlui001@myuct.ac.za

Introduction.

The behaviour of strongly interacting matter in the presence of external magnetic fields is a very active research field. It has a strong impact on experiments at the LHC (peripheral collisions of heavy nuclei at high energy), as well as on astronomical objects like neutron stars, magnetars, and the early universe. In addition, lattice QCD (LQCD) has shown that the critical temperature for deconfinement/chiral symmetry restoration decreases with increasing field strength. This behaviour is dubbed inverse magnetic catalysis, and it reveals an unexpected, non-trivial phenomenon.

Given the dual nature of the QCD phase transition, a pertinent question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to find a relation between deconfinement and chiral symmetry restoration parameters, and/or of deconfinement. One way to address this question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement.

Finite Energy QCD Sum Rules with magnetic fields.

The charged axial-vector current correlator in the absence of a magnetic field can be written as

\[ \Pi_{\mu\nu}(q^2) = -i \int d^4x e^{iqx} \langle 0 | \langle T \gamma^\mu A_\nu \rangle(x) \langle 0 | 0 \rangle > \]

where \( A_\nu(x) = \frac{1}{2} i \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}(x) \) is the charged axial-vector current, and \( x = x^0 + \textbf{x} \) is the Lorentz index.

The Wilson coefficients \( C_n \) depend on the Lorentz indices and quantum numbers of the currents, and on the local gauge invariant operators \( \mathcal{O}_n \) built from the quark and gluon fields in the QCD Lagrangian.

Hadrionic Sector.

The axial-vector current correlator in the presence of a magnetic field can be interpolated by the charged pion current

\[ A_\mu = -f_\pi \partial_\mu \phi + f_\pi (\partial_\mu - icA_\mu) \bar{\psi} \gamma^\mu \psi \]

where \( f_\pi \) is the pion decay constant, \( n^a \) the pion field, and \( A_\mu = (B/2)(0, -y, 0, x) \) the vector potential in the symmetric gauge. Therefore, the axial-vector correlator in the hadronic sector can be written as

\[ \Pi_{\mu\nu}^{\text{had}}(q^2) = i \mathcal{G}_{\mu\nu}(q^2) \]

where \( \mathcal{G}_{\mu\nu}(q^2) \) is the charged pion propagator in presence of magnetic fields, when it is expressed in terms of a sum over Landau levels and setting \( q_0^2 = 0 \).

The imaginary part of Eq. (9) in the weak field limit \( eB < s_0 \) is given by

\[ \text{Im} \Pi_{\mu\nu}^{\text{had}}(q^2) = \frac{-i f_\pi^2}{4\pi^2} \int_0^{s_0} \frac{dz}{z} \left( \frac{\gamma^\mu \gamma^\nu \gamma^5}{4} - \frac{\gamma^\nu \gamma^5}{4} \right) \pi \delta(z) \]

Substituting Eq. (10) into the QCD sum rules Eqs. (5)-(6) gives

\[ 0 = f_\pi^2 \int_0^{s_0} \frac{dz}{z} \left( \frac{\gamma^\mu \gamma^\nu \gamma^5}{4} - \frac{\gamma^\nu \gamma^5}{4} \right) \pi \delta(z) \]

\[ -C_2 (O_2) = \frac{f_\pi^2}{8} \int_0^{s_0} \frac{dz}{z} \left( \frac{\gamma^\mu \gamma^\nu \gamma^5}{4} - \frac{\gamma^\nu \gamma^5}{4} \right) \pi \delta(z) \]

The pQCD contribution to the axial-vector current correlator in the presence of a magnetic field is depicted in Fig. 1, where we also define the kinematics. The thick internal lines represent the full quark propagators in the magnetic field background.

Conclusions.

We studied QCD FESR for the axial-vector current correlator in the presence of a magnetic field in the weak field limit \( eB < s_0 \). We have shown that the presence of the field modifies both the pQCD as well as the hadronic sectors of the FESR. The magnetic field dependence of \( s_0 \) is thus proportional to the magnetic field dependence of the absolute value of the light-quark condensate. Therefore the magnetic field both helps the formation of the condensate and acts against deconfinement. The gluon condensate also grows as a function of the field strength which goes hand in hand with the behavior of the magnetic field, both as a catalyst of chiral symmetry breaking and confinement.

Forthcoming Research.

The results obtained here should serve as a basis for studies at finite temperature in an external magnetic field.

References

[1] A. Ayala, C. A. Dominguez, L. A. Hernandez, M. Loewe, J. C. Rojas and C. Villavicencio. arXiv:1504.01308 [hep-ph].