QCD corrections to associated Higgs boson production with a W boson pair at the LHC

Song Mao, Ma Wen-Gan, Zhang Ren-You, Guo Lei, Wang Shao-Ming, and Han Liang
Department of Modern Physics, University of Science and Technology of China (USTC), Hefei, Anhui 230026, P.R.China

Abstract

The Higgs boson production in association with a pair of W-bosons at the Large Hadron Collider (LHC) can be used to probe the coupling between Higgs boson and vector gauge bosons and discover the signature of new physics. We describe the impact of the complete QCD NLO radiative corrections and the gluon-gluon fusion subprocess to the cross section of this process at the LHC, and investigate the dependence of the leading order (LO) and the QCD corrected cross sections on the factorization/renormalization energy scale and Higgs boson mass. We present the LO and QCD corrected distributions of the invariant mass of W-boson pair and the transverse momenta of final W and Higgs boson. We find that the QCD NLO corrections and the contribution from gluon-gluon fusion subprocess significantly modify the LO distributions, and the scale dependence of the QCD corrected cross section is badly underestimated by the LO results. Our numerical results show that the K-factor of the QCD correction varies from 1.48 to 1.64 when $m_H$ goes up from 100 GeV to 160 GeV. We find also the QCD correction from $gg \rightarrow H^0 W^+ W^-$ subprocess at the LHC is significant, and should be considered in precise experiment.

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I. Introduction

The Higgs mechanism plays a crucial role in the standard model (SM). The existence of the Higgs boson makes the breaking of the electroweak (EW) symmetry and generates the masses for the fundamental particles [1][2]. Therefore, to study the Higgs mechanism is one of the main goals of the LHC. The LEP experimental data from direct search for Higgs boson in association with $Z^0$ boson provide the exclusion of the Higgs boson in the mass range up to 114.4 GeV at 95% confidence level (CL) [3]. The current SM fit of all electroweak parameters produced by the LEP Electroweak Group predicts $m_H = 84^{+34}_{-26}$ GeV [4], or the one-sided 95% CL limit $m_H < 154$ GeV. Including the LEP direct search results, this upper limit increases to $m_H \lesssim 185$ GeV [5]. It is also interesting that recent combined results from the Tevatron experiments have, for the first time, excluded the hypothesis of a Higgs boson mass around 170 GeV [6] at 95% CL. Although the expected sensitivity of Tevatron experiments is not enough to make a $5\sigma$ discovery of the SM Higgs boson [7], it is enough to exclude it out up to $m_H \sim 200$ GeV at 95% CL, or to make a $3\sigma$ observation. While for the coupling properties, such as the couplings between Higgs boson and gauge bosons, the precise data provide only little information about them.

The CERN Large Hadron Collider (LHC) is a machine with the entire proton-proton colliding energy of 14 TeV and a luminosity of 100 $fb^{-1}$ per year. If the Higgs boson really exists, it will be discovered at the LHC, which can provide a measurement of the Higgs mass at the per-mille level, and of the Higgs boson coupling at the 5 – 20% level. At this machine, the Higgs boson production is dominated by the gluon-gluon fusion process, described at the leading order through a heavy-quark loop. The next-to-leading order cross section for this process is 37.6 $pb$, for $m_H = 120$ GeV. The Higgs boson can also be produced by Vector Boson Fusion (VBF) with a cross section of 4.25 $pb$, or by associated production with a $W^\pm$, a $Z^0$, or a $t\bar{t}$ quark pair, with 3.19 $pb$ for the three processes and $m_H = 120$ GeV (cross sections calculated at next-to-leading order) [8].

After the discovery of Higgs boson, our main task is to probe its properties, such as
spin, CP, and couplings. However, these measurements require accurate theoretical predictions for both signal and background. The process \( pp \rightarrow H^0 W^+ W^- + X \) is one of the important processes in providing the detail information about the coupling between Higgs-boson and vector gauge bosons. As we will see in this work, the QCD corrections increase the \( H^0 W^+ W^- \) cross section significantly, and thus in the quantitative measurement of the coupling \( H^0 W^+ W^- \) we have to take the QCD corrections into account.

At the LHC, most of the important processes will involve multi-particle final states, either through the direct multi-particle production or the decay of resonances. It is known that the theoretical predictions beyond the LO for these processes with more than two final particles are necessary from the data analysis point of view in order to probe the SM and find new physics, but the calculations for these processes involving the NLO corrections are very intricate. In the last few years, the phenomenological results including the QCD NLO corrections for tri-boson production processes at the LHC, such as \( pp \rightarrow W^+ W^- Z^0 \), \( H^0 H^0 H^0 \), \( Z^0 Z^0 Z^0 \), have been provided [9, 10, 11, 12]. The QCD NLO corrections to the weak boson fusion processes, like \( pp \rightarrow WW jj, WZjj \) [13, 14], \( pp \rightarrow Hjj \) with effective gluon-Higgs coupling, [15] \( gg \rightarrow Hqq \) [16], and \( pp \rightarrow t\bar{t}j \) [17] have been studied.

In this paper, we make a precise calculation for the process \( pp \rightarrow H^0 W^+ W^- + X \) at the LHC including the contributions of the QCD NLO corrections and the gluon-gluon fusion subprocess, for the purpose of avoiding a possible experimentally observed deviation from the LO prediction due to the QCD effects being misinterpreted. As we shall see from the following investigation that these QCD NLO corrections and the contribution from the gluon-gluon fusion process turn out to be potentially important in observations of the signal of \( pp \rightarrow H^0 W^+ W^- + X \) process and should be taken into account in experimental data analysis. In section II we give the calculation description of the LO cross section of \( pp \rightarrow H^0 W^+ W^- + X \) process, and the calculations of the complete QCD NLO radiative contribution and the correction from gluon-gluon fusion subprocess are provided in section III. In section IV we present some numerical results and discussion, and finally a short
II. The LO cross section of the $pp \rightarrow H^0W^+W^- + X$ process

In the LO and higher order calculations we employ FeynArts3.4 package\cite{18} to generate Feynman diagrams and their corresponding amplitudes. The amplitude calculations are implemented by applying FormCalc5.4 programs\cite{19}.

The leading order contribution to the cross section of the parent process $pp \rightarrow H^0W^+W^- + X$ comes from the subprocess of $H^0W^+W^-$ production via quark-antiquark($q = u, d, s, c$) annihilation. We denote the subprocess as

$$q(p_1) + \bar{q}(p_2) \rightarrow H^0(p_3) + W^+(p_4) + W^-(p_5), \quad (q = u, d, s, c). \quad (2.1)$$

where $p_1$, $p_2$ and $p_3$, $p_4$, $p_5$ represent the four-momenta of the incoming partons and the outgoing $H^0$, $W^\pm$ bosons, respectively. We use the 't Hooft-Feynman gauge in our LO calculations, if there is no other statement. We ignore the contribution from the Feynman diagrams which involve the couplings between fermions($u$-, $d$-, $s$-, or $c$-quarks) and Higgs boson, since the Yukawa coupling strength is proportional to fermion mass and the masses of $u$-, $d$-, $s$-, and $c$-quark are relatively small and can be negligible. The Feynman diagrams for the subprocess $q\bar{q} \rightarrow H^0W^+W^-$ at the LO are depicted in Fig.1.

The expression for the LO cross section for the subprocess $q\bar{q} \rightarrow H^0W^+W^-$ has the form as

$$\hat{\sigma}_{q\bar{q}}^0 = \frac{1}{4} \frac{1}{9} \frac{(2\pi)^4}{2\hat{s}} \int \sum_{spin} |\mathcal{M}_{LO}|^2 d\Omega_3 \quad (2.2)$$

where the factors $\frac{1}{4}$ and $\frac{1}{9}$ come from the averaging over the spins and colors of the initial partons respectively, $\hat{s}$ is the partonic center-of-mass energy squared, and $\mathcal{M}_{LO}$ is the amplitude of all the tree-level diagrams shown in Fig.1. The summation is taken over the spins and colors of all the relevant particles in the $q\bar{q} \rightarrow H^0W^+W^-$ subprocess. The integration
Figure 1: The tree-level Feynman diagrams for the \( q\bar{q} \rightarrow H^0 W^+ W^- \) \((q = u, d, s, c, U = u, c, D = d, s)\) subprocess, which are considered in our LO calculations.
is performed over the three-body phase space of the final particles $H^0$, $W^+$ and $W^-$. The phase-space element $d\Omega_3$ in Eq.(2.2) is expressed as

$$d\Omega_3 = \delta^{(4)} \left( p_1 + p_2 - \sum_{i=3}^5 p_i \right) \prod_{j=3}^5 \frac{d^3 p_j}{(2\pi)^3 2E_j}. \quad (2.3)$$

Within the framework of the QCD factorization, the LO cross section for the process $pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X$ at the LHC can be obtained by performing the following integration of the cross section for the subprocess $q\bar{q} \rightarrow H^0W^+W^-$ over the partonic luminosities (see Eq.(2.4)).

$$\sigma_{LO} = \sum_{ij=u\bar{u},d\bar{d},s\bar{s},c\bar{c}} \int_0^1 dx_1 \int_0^1 dx_2 \left[ G_{ij/P_1}(x_1, \mu_f)G_{j/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right] \hat{\sigma}_{ij}^0(\hat{s} = x_1x_2s), \quad (2.4)$$

where $G_{ij/A}(x, \mu_f)$ is the parton($i = u, d, s, c$) distribution function of proton $A(= P_1, P_2)$ which describes the probability to find a parton $i$ with momentum $xp_A$ in proton $A$, $s$ is defined as the total colliding energy squared in proton-proton collision, $\hat{s} = x_1x_2s$, and $\mu_f$ is the factorization energy scale. In our LO calculations, we adopt the CTEQ6L1\textsuperscript{[20]} parton distribution functions.

### III. QCD corrections

At the leading order, the parent process $pp \rightarrow H^0W^+W^- + X$ involves four subprocesses, i.e., $q\bar{q} \rightarrow H^0W^+W^-$, where $q = u, d, c$ and $s$. Due to the poor luminosities for charm- and strange-quarks in protons, the contribution to the LO cross section for the parent process $pp \rightarrow H^0W^+W^- + X$ from the subprocesses $s\bar{s}, c\bar{c} \rightarrow H^0W^+W^-$ is relatively small. Our calculation shows their contribution part to the LO cross section is less than 10% at the LHC. Therefore, in the calculations beyond the LO we consider reasonably only the QCD corrections to the processes $pp \rightarrow u\bar{u}, d\bar{d} \rightarrow H^0W^+W^- + X$.

Our QCD correction to the $pp \rightarrow H^0W^+W^- + X$ process at the LHC can be divided into two parts: One is the QCD virtual correction, which should be considered together with
the contribution from the real gluon/light-quark emission subprocesses in order to cancel the
soft/collinear IR singularities appeared in the virtual correction. Actually, there still exists
remaining collinear divergency which can be absorbed by the parton distribution functions.
Another part is from the gluon-gluon fusion subprocess which gives the contribution to the
cross section of the \(pp \rightarrow H^0W^+W^- + X\) process \(\mathcal{O}(\alpha_s)\) order higher than that of the
previous subprocess at the QCD NLO.

III..1 Virtual corrections to the subprocess \(q\bar{q} \rightarrow H^0W^+W^-\)

In our calculations, all the divergences are regularized by using the dimensional regular-
ization method in \(D = 4 - 2\epsilon\) dimensions and the modified minimal subtraction (\(\overline{\text{MS}}\))
scheme is applied to renormalize the relevant fields. There are 171 virtual QCD NLO dia-
grams contributing to the subprocess \(q\bar{q} \rightarrow H^0W^+W^-\) in the SM, including self-energy(94),
vertex(35), box(5) and counterterm(37) diagrams. We present part of these diagrams in
Fig\[2\]. There exist both ultraviolet(UV) and soft/collinear infrared(IR) singularities in the
calculation of the one-loop diagrams, but the total QCD NLO amplitude of subprocess
\(q\bar{q} \rightarrow H^0W^+W^-\) is UV finite after performing renormalization procedure. Nevertheless, it
still contains soft/collinear IR singularities as shown in Eq.(3.1).

\[
d\hat{\sigma}_V^{q\bar{q}} = d\hat{\sigma}_0^{q\bar{q}} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{s} \right)^\epsilon \left( \frac{A_2^V}{\epsilon^2} + \frac{A_1^V}{\epsilon} + A_0^V \right) \right], \quad (q = u, d) \quad (3.1)
\]

where

\[
A_2^V = -2C_F, \quad A_1^V = -3C_F, \quad C_F = 4/3. \quad (3.2)
\]

As we shall see later that the soft/collinear IR singularities can be cancelled by adding
the contributions of the \(q\bar{q} \rightarrow H^0W^+W^- g\) and \(q(\bar{q})g \rightarrow H^0W^+W^- q(\bar{q})\) subprocesses, and
redefining the parton distribution functions at the NLO. In the numerical calculations of the
virtual corrections, we use the expressions in Refs.[21] [22] [23] to implement the numerical
evaluations of IR safe one-point, 2-point, 3-point, 4-point and 5-point integrals.
III..2 Real gluon emission subprocess $q\bar{q} \rightarrow H^0 W^+ W^- g$

We denote the $q - \bar{q}(q = u, d)$ annihilation subprocess with a real gluon emission as

$$q(p_1) + \bar{q}(p_2) \rightarrow H^0(p_3) + W^+(p_4) + W^-(p_5) + g(p_6).$$

The real gluon emission subprocess $q\bar{q} \rightarrow H^0 W^+ W^- g$ (shown in Fig.3) produces both soft and collinear IR singularities which can be conveniently isolated by adopting the two cutoff phase space slicing (TCPSS) method[24]. The soft IR singularity in the subprocess $q\bar{q} \rightarrow H^0 W^+ W^- g$ at the LO cancels the analogous singularity arising from the one-loop level virtual corrections to the $q\bar{q} \rightarrow H^0 W^+ W^-$ subprocess.

In performing the calculations with the TCPSS method, we should introduce arbitrary small soft cutoff $\delta_s$ and collinear cutoff $\delta_c$. The phase space of the $q\bar{q} \rightarrow H^0 W^+ W^- g$ subprocess can be split into two regions, $E_6 \leq \delta_s \sqrt{s}/2$ (soft gluon region) and $E_6 > \delta_s \sqrt{s}/2$ (hard gluon region) by soft cutoff $\delta_s$. The hard gluon region is separated as hard collinear (HC) and hard non-collinear (HC) regions by cutoff $\delta_c$. The HC region is the phase space where $-\hat{t}_{16}$ (or $-\hat{t}_{26}) < \delta_c \hat{s}$ ($\hat{t}_{16} \equiv (p_1 - p_6)^2$ and $\hat{t}_{26} \equiv (p_2 - p_6)^2$). Therefore, the cross section for this real
Figure 3: The tree-level Feynman diagrams for the real gluon emission subprocess $q\bar{q} \rightarrow H^0W^+W^- g$ ($q = u, d$).
The differential cross section for the subprocess \( q\bar{q} \rightarrow H^0W^+W^-g \) in the soft region is given as

\[
d\hat{\sigma}^S_g(q\bar{q} \rightarrow H^0W^+W^-g) = d\hat{\sigma}_{qg}^0 \left[ \alpha_s \frac{\Gamma(1-\epsilon)}{2\pi} \left( \frac{4\pi\mu_F^2}{s} \right)^\epsilon \right] \left( \frac{A_2^S}{\epsilon^2} + \frac{A_1^S}{\epsilon} + A_0^S \right),
\]

with

\[
A_2^S = 2C_F, \quad A_1^S = -4C_F \ln \delta_s, \quad A_0^S = 4C_F \ln^2 \delta_s.
\]

The differential cross section for the process \( pp \rightarrow q\bar{q} \rightarrow H^0W^+W^-g + X \), \( d\hat{\sigma}^{HC}_g \) in the hard collinear region, can be written as

\[
d\hat{\sigma}^{HC}_g = d\hat{\sigma}_{qg}^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{(1-2\epsilon)} \left( \frac{4\pi\mu_F^2}{s} \right)^\epsilon \right] \frac{1}{\epsilon} \delta_c^{\epsilon} \left\{ P_{qq}(z, \epsilon)[G_{q/P_1}(x_1/z)G_{\bar{q}/P_2}(x_2)] + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\} \frac{dz}{z} \left( \frac{1-z}{z} \right)^{-\epsilon} dx_1 dx_2,
\]

where \( G_{q(\bar{q})/P}(x, \mu_F) \) is the bare parton distribution function of quark(anti-quark) and \( P \) refers to proton. \( P_{qq}(z, \epsilon) \) is the D-dimensional unregulated \( z < 1 \) splitting function which can be written explicitly as

\[
P_{qq}(z, \epsilon) = P_{qq}(z) + \epsilon P'_{qq}(z), \quad P_{qq}(z) = C_F \left( \frac{1+z^2}{1-z} \right), \quad P'_{qq}(z) = -C_F(1-z).
\]

### III..3 Real light-quark emission subprocess \( q(\bar{q})g \rightarrow H^0W^+W^-q(\bar{q}) \)

Beside the real gluon emission subprocess discussed above, there is another kind of contribution called the real light-quark emission subprocess which has the same order contribution with previous real gluon emission subprocess in perturbation theory. The corresponding Feynman diagrams of the subprocesses \( q(\bar{q})g \rightarrow H^0W^+W^-q(\bar{q}) \) \( (q = u, d) \) at the tree-level are shown in Fig[4].
Figure 4: The tree-level Feynman diagrams for the real light-quark emission subprocesses $q(\bar{q})g \rightarrow H^0W^+W^-q(\bar{q})$ ($q = u, d$).
These subprocesses contain only the initial state collinear singularities. Using the TCPSS method described above, we split the phase space into collinear region and non-collinear region by introducing a cutoff $\delta_c$. Then the cross sections for the subprocesses $qg \rightarrow H^0W^+W^-q$ and $\bar{q}g \rightarrow H^0W^+W^-\bar{q}$ can be expressed as

$$\hat{\sigma}^R(qg \rightarrow H^0W^+W^-q) = \hat{\sigma}^q = \hat{\sigma}^{HC} + \hat{\sigma}^{HC}_q$$

(3.9)

$$\hat{\sigma}^R(\bar{q}g \rightarrow H^0W^+W^-\bar{q}) = \hat{\sigma}_{\bar{q}} = \hat{\sigma}^{HC}_{\bar{q}} + \hat{\sigma}^{HC}_{\bar{q}}$$

(3.10)

The cross sections $\hat{\sigma}^{HC}_q$ and $\hat{\sigma}^{HC}_{\bar{q}}$ in the non-collinear region are finite and can be evaluated in four dimensions using Monte Carlo method. The differential cross section in the collinear region for the processes $pp \rightarrow qg \rightarrow H^0W^+W^-q + X$, $d\sigma_q^{HC}$, can be expressed as

$$d\sigma_q^{HC} = d\hat{\sigma}_q^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi \mu^2}{\hat{s}} \right)^{\epsilon} \right] \left( \frac{1}{\epsilon} \right) \delta_c^{\epsilon} P_{qg}(z, \epsilon) \left[ G_{q/P_1}(x_1/z) G_{q/P_2}(x_2) \right]$$

(3.11)

The expression of the $d\sigma_q^{HC}$ for the $pp \rightarrow \bar{q}g \rightarrow H^0W^+W^-\bar{q} + X$ process, can be obtained by doing the replacement of $G_{q/P_2}(x_2) \rightarrow G_q/P_2(x_2)$ in the right-handed side of Eq.(3.11). In above equation $G_{q(\bar{q})/P(x)}$ is the bare parton distribution function of quark(anti-quark) in proton and

$$P_{qg}(z, \epsilon) = P_{qg}(z) + \epsilon P'_{qg}(z), \quad P_{qg}(z) = \frac{1}{2}[z^2 + (1-z)^2], \quad P'_{qg}(z) = -z(1-z).$$

(3.12)

### III.4 Gluon-gluon fusion subprocess $gg \rightarrow H^0W^+W^-$

The lowest order contribution of the $gg \rightarrow H^0W^+W^-$ subprocess is at the one-loop level. This contribution to the process $pp \rightarrow H^0W^+W^- + X$ is $O(\alpha_s)$ order higher than the QCD NLO corrections from the one-loop process $pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X$, the production rate of the $pp \rightarrow gg \rightarrow H^0W^+W^- + X$ could be non-negligible, due to the large gluon luminosity in TeV-scale proton-proton collision at the LHC. Here we include the contribution of the gluon-gluon fusion subprocess in the calculations of the QCD corrections to the $pp \rightarrow H^0W^+W^- +$
$X$ process. We neglect again the Feynman diagrams involving the interaction between light fermions and Higgs boson. Among all the 292 QCD one-loop Feynman diagrams, there are 63 self-energy, 148 vertex, 69 box and 12 pentagon diagrams. All the pentagon diagrams for the $gg \to H^0W^+W^-$ subprocess are depicted in Fig.5 as a presentation.

Again we employ the aforementioned dimensional regularization to isolate the UV and IR divergences in one-loop calculation. Since there is no tree-level diagram for the $gg \to H^0W^+W^-$, the calculation for this subprocess can be simply carried out by summing all unrenormalized reducible and irreducible one-loop diagrams, and we find the numerical results are UV and IR finite. We get the lowest order differential cross section of the subprocess $gg \to H^0W^+W^-$ expressed as:

$$d\hat{\sigma}_{gg} = \frac{1}{4} \frac{1}{64} \frac{(2\pi)^4}{p_1^4} \sum_{\text{spin}, \text{color}} |M_{gg}|^2 d\Omega_3.$$ (3.13)

where factors $1/4$ and $1/64$ are obtained by taking averages of the initial spins and colors,
and the phase space element of three-body final states, $d\Omega_3$, is defined as in Eq. (2.3).

After integration of $d\hat{\sigma}_{gg}$ over the partonic luminosities, we can see from the numerical results that although the contributions from the subprocess $gg \to H^0W^+W^-$ are much smaller than the QCD NLO corrections to the $pp \to q\bar{q} \to H^0W^+W^- + X$ process, the QCD relative correction from the $pp \to gg \to H^0W^+W^- + X$ at the LHC is significant and can even reach 24% when $m_H = 160\text{ GeV}$.

III.5 QCD corrected cross section for the $pp \to H^0W^+W^- + X$ process

After adding the renormalized virtual corrections and the real gluon/light-quark emission corrections to the subprocess $q\bar{q} \to H^0W^+W^-$, the partonic cross sections still contain the collinear divergences, which can be absorbed into the redefinition of the distribution functions at the NLO. Using the $\overline{\text{MS}}$ scheme, the scale dependent NLO parton distribution functions are given as

$$G_{i/P}(x, \mu_f) = G_{i/P}(x) + \sum_{j=q,\bar{q},g} (-1)^j \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_f^2}{\mu_f^2} \right)^\epsilon \right] \int_1^1 \frac{dz}{z} P_{ij}(z) G_{j/P}(x/z),$$

$$q = u, d, i = u, \bar{u}, d, \bar{d}, g). \quad (3.14)$$

By using above definition, we get the QCD counter-terms of parton distribution function which are combined with the hard collinear contributions to result in the $O(\alpha_s)$ expression for the remaining collinear contributions:

$$d\sigma^{coll} = \sum_{q=u,d} d\hat{\sigma}_{q\bar{q}}^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_f^2}{\hat{s}} \right)^\epsilon \right] \left\{ \tilde{G}_{q/P_1}(x_1, \mu_f) G_{\bar{q}/P_2}(x_2, \mu_f) + \tilde{G}_{q/P_1}(x_1, \mu_f) G_{\bar{q}/P_2}(x_2, \mu_f) + \left[ A^{sc}_{1}(q \to qg) \right] \right\} dx_1 dx_2,$$

$$+ \tilde{G}_{q/P_1}(x_1, \mu_f) G_{\bar{q}/P_2}(x_2, \mu_f) + \left[ A^{sc}_{0}(q \to qg) \right] \times \left[ G_{q/P_1}(x_1, \mu_f) G_{\bar{q}/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right] \left\{ dx_1 dx_2, \quad (3.15) \right\}$$

where

$$A^{sc}_{1}(q \to qg) = C_F(2 \ln \delta_s + 3/2), \quad A^{sc}_{0} = A^{sc}_{1} \ln(\frac{\hat{s}}{\mu_f^2}). \quad (3.16)$$
and

\[ G_{q/P}(x, \mu_f) = \sum_{j=q,g} \int_x^{1-\delta_q} \frac{dy}{y} G_{j/P}(x/y, \mu_f) \tilde{P}_{qj}(y), \]  

(3.17)

with

\[ \tilde{P}_{ij}(y) = P_{ij} \ln \left( \frac{1 - \frac{s}{y}}{\mu_f^2} \right) - P'_{ij}(y). \]  

(3.18)

We can find that the sum of the soft (expressed in Eq. (3.5)), collinear (expressed in Eq. (3.15)), and ultraviolet renormalized virtual correction (expressed in Eq. (3.1)) terms is finite, i.e.,

\[ A^S_2 + A^V_2 = 0, \quad A^S_1 + A^V_1 + 2A^{sc}_1(q \rightarrow qg) = 0. \]  

(3.19)

The final result for the total QCD correction (\( \Delta \sigma^{QCD} \)) consists of a three-body term \( \Delta \sigma^{(3)} \) and a four-body term \( \Delta \sigma^{(4)} \).

\[
\Delta \sigma^{(3)} = \frac{\alpha_s}{2\pi} \sum_{q=u,d} \int dx_1 dx_2 d\hat{\sigma}_{qq} \left\{ G_{q/P_1}(x_1, \mu_f) G_{q/P_2}(x_2, \mu_f) \right. \\
+ \sum_{j=q,g} \int dx_1 dx_2 d\tilde{\sigma}_{qj} \left\{ G_{q/P_1}(x_1, \mu_f) G_{j/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\} \\
+ \frac{1}{2} \sum_{q=u,d} \int dx_1 dx_2 d\hat{\sigma}_{gg} \left\{ G_{g/P_1}(x_1, \mu_f) G_{g/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\}. 
\]

(3.20)

And

\[
\Delta \sigma^{(4)} = \sum_{q=u,d} \int dx_1 dx_2 \left\{ G_{q/P_1}(x_1, \mu_f) G_{q/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\} \hat{\sigma}_{q;q}^{HC}(\hat{s} = x_1 x_2 s) \\
+ \sum_{q=u,d} \int dx_1 dx_2 \left\{ G_{q/P_1}(x_1, \mu_f) G_{g/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\} \hat{\sigma}_{q;g}^{HC}(\hat{s} = x_1 x_2 s). 
\]

(3.21)

where \( \hat{\sigma}_{q;q}^{HC}(\hat{s} = x_1 x_2 s) \) is the cross section for the subprocess \( qq \rightarrow H^0W^+W^{-}q \) (\( q = u, d \)) in the hard non-collinear phase space region at the colliding energy \( \hat{s} = x_1 x_2 s \) in the partonic center-of-mass system. \( \hat{\sigma}_{q;g}^{HC}(\hat{s}) \), where \( q = u, d, \bar{u}, \bar{d} \), represent the cross sections in the non-collinear phase space regions for the subprocesses \( ug \rightarrow H^0W^+W^{-}u, dg \rightarrow H^0W^+W^{-}d, \bar{u}g \rightarrow H^0W^+W^{-}\bar{u} \) and \( \bar{d}g \rightarrow H^0W^+W^{-}\bar{d} \), respectively.
Finally, the QCD corrected total cross section for the \( pp \rightarrow H^0W^+W^- + X \) process is

\[
\sigma^{QCD} = \sigma^0 + \Delta \sigma^{QCD} = \sigma^0 + \Delta \sigma^{(3)} + \Delta \sigma^{(4)}.
\]  

where the LO cross section part of the parent process \( pp \rightarrow H^0W^+W^- + X \) is expressed as

\[
\sigma^0 = \sum_{q=u,d} \int dx_1 dx_2 d\sigma^0_{q\bar{q}} \left\{ G_{q/P_1}(x_1, \mu_f) G_{\bar{q}/P_2}(x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right\}.
\]  

IV. Numerical results and discussion

In this section we describe and discuss the numerical results of our calculations for the \( pp \rightarrow H^0W^+W^- + X \) process at the LO, the corrections at the QCD NLO and the contribution from the gluon-gluon fusion subprocess. We take one-loop and two-loop running \( \alpha_s(\mu) \) for the LO and the higher order calculations, respectively\([25]\). We set the factorization scale and the renormalization scale being equal, and take \( \mu \equiv \mu_f = \mu_r = (m_H + 2m_W)/2 \) by default unless otherwise stated, the CKM matrix being a unit matrix. We adopt \( m_u = m_d = m_g = 0 \) and employ the following numerical values for the relevant input parameters: \([25]\)

\[
\begin{align*}
\alpha(m_Z)^{-1} &= 127.918, \quad m_W = 80.398 \text{ GeV}, \quad m_Z = 91.1876 \text{ GeV}, \\
m_t &= 171.2 \text{ GeV}, \quad m_s = 104 \text{ MeV}, \quad m_c = 1.27 \text{ GeV}, \\
m_b &= 4.2 \text{ GeV}.
\end{align*}
\]  

By taking \( m_H = 120 \text{ GeV} \) and the CTEQ6L1 parton distribution functions, we perform a check for the correctness of the LO calculation of the process \( pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X \). We use the FeynArts3.4/FormCalc5.4\([18, 19]\) packages and CompHEP-4.4p3 program\([26]\), and apply the Feynman and unitary gauges, separately. The numerical results are listed in Table 1. We can see that all those results are in good agreement.

Figs. 6(a,b) show that our total QCD correction to the \( pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X \) process does not depend on the arbitrarily chosen value of the cutoff \( \delta_s \) with the fixed value of \( \delta_c = \).
Table 1: The numerical results of the LO cross sections for the process $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ by using FeynArts3.4/FormCalc5.4 packages and CompHEP-4.4p3 program, adopting the Feynman and unitary gauges separately, with the CTEQ6L1 parton distribution functions and $m_H = 120$ GeV.

|     | $\sigma_{LO}(\text{fb})$ | $\sigma_{LO}(\text{fb})$ | $\sigma_{LO}(\text{fb})$ | $\sigma_{LO}(\text{fb})$ |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|
| CompHEP Feynman Gauge | 5.902(4) | 5.903(4) | 5.898(6) | 5.898(6) |
| CompHEP unitary gauge | 5.902(4) | 5.903(4) | 5.898(6) | 5.898(6) |

The three-body correction ($\Delta \sigma^{(3)}$, see Eq. (3.20)) and four-body correction ($\Delta \sigma^{(4)}$, see Eq. (3.21)) and the total QCD correction ($\Delta \sigma^{QCD}$) for the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process at the LHC, are depicted as the functions of the soft cutoffs $\delta_s$ by taking $m_H = 120$ GeV and $\delta_s$ running from $10^{-4}$ to $10^{-2}$ in Fig.6(a). The amplified curve for $\Delta \sigma^{QCD}$ is presented in Fig.6(b) together with calculation errors. While Figs.7(a,b) show the independence of the total QCD correction to the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process on the cutoff $\delta_c$ where we take $\delta_s = 10^{-3}$. In Fig.7(b) the amplified curve for $\Delta \sigma^{QCD}$ of the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process is depicted. The fact that the total QCD correction $\Delta \sigma^{QCD}$ for the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process is independent of these two cutoffs, not only proofs the cancelation of soft/collinear IR divergency in the total QCD correction for the process $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$, but also partially verifies the correctness of our calculation. In the following numerical calculations, we fix $\delta_s = 10^{-3}$ and $\delta_c = \delta_s/50$.

In Figs.8(a,b) we assumed $\mu \equiv \mu_r = \mu_f$ and defined $\mu_0 = (m_H + 2m_W)/2$. Fig.8(a) shows the dependence of the LO and the total QCD corrected cross-sections for the process $pp \rightarrow H^0W^+W^- + X$ on the factorization/renormalization scale ($\mu/\mu_0$). We can see that the curve for LO cross section has a tiny variation being less than one percent, but the variation of the QCD corrected cross section is relative large by approximately 10% when the energy scale $\mu$ runs from $0.5\mu_0$ to $4\mu_0$. It demonstrates that the LO curve drastically underestimates the energy scale dependence of the QCD correction. That is because there is no strong interaction...
Figure 6: (a) The dependence of QCD NLO correction parts to the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process on the soft cutoff $\delta_s$ at the LHC with $m_H = 120 \text{ GeV}$, the collinear cutoff $\delta_c = 2 \times 10^{-6}$ and $\sqrt{s} = 14 \text{ TeV}$. (b) The amplified curve for the total QCD correction $\Delta\sigma_{QCD}$ to the process $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$, where it includes the calculation errors.

Figure 7: (a) The dependence of the QCD NLO correction parts to the $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$ process on the collinear cutoff $\delta_c$ at the LHC with $m_H = 120 \text{ GeV}$, $\delta_s = 10^{-3}$ and $\sqrt{s} = 14 \text{ TeV}$. (b) The amplified curve for the total QCD correction $\Delta\sigma_{QCD}$ to the process $pp \rightarrow u\bar{u} \rightarrow H^0W^+W^- + X$, where it includes the calculation errors.
Figure 8: (a) The dependence of the LO and the QCD corrected cross-sections for the process $pp \rightarrow H^0W^+W^- + X$ on the factorization/renormalization scale ($\mu/\mu_0$). (b) The total QCD relative correction to the process $pp \rightarrow H^0W^+W^- + X$ ($\Delta K \equiv \Delta \sigma^{QCD}/\sigma_{LO}$), the QCD relative correction parts from the $pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X$ process ($\Delta K_q \equiv \Delta \sigma^{QCD}_q/\sigma_{LO}$) and the $pp \rightarrow gg \rightarrow H^0W^+W^- + X$ process ($\Delta K_g \equiv \sigma^{QCD}_g/\sigma_{LO}$) versus the factorization/renormalization scale ($\mu/\mu_0$). Here we assume $\mu \equiv \mu_f = \mu_r$ and define $\mu_0 = (m_H + 2m_W)/2$.

In the LO diagrams of the $q\bar{q} \rightarrow H^0W^+W^-$ subprocess, and its weak energy scale dependence is the consequence of the parton distribution functions being related to the factorization scale ($\mu_f$). The similar behavior is demonstrated in the Z production at the Tevatron\cite{27} and the production of three Z-bosons at the LHC\cite{28}. Fig.8(b) describes the total QCD relative correction to the process $pp \rightarrow H^0W^+W^- + X$ defined as $\Delta K \equiv \Delta \sigma^{QCD}/\sigma_{LO}$, the QCD relative corrections from the NLO $pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X$ and the LO $pp \rightarrow gg \rightarrow H^0W^+W^- + X$ processes, defined as $\Delta K_q \equiv \Delta \sigma^{QCD}_q/\sigma_{LO}$ and $\Delta K_g \equiv \sigma^{QCD}_g/\sigma_{LO}$ respectively, as the functions of the factorization/renormalization scale ($\mu/\mu_0$). It demonstrates that the energy scale $\mu$ dependence of the cross section for the $pp \rightarrow H^0W^+W^- + X$ process is mainly related to the contributions of the QCD corrections to the $pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X$ process, and the dependence of the $\Delta K_g$ on the energy scale $\mu$ is obviously weaker than the $\Delta K_q$.

In Fig.9 we present the plot of the LO and the QCD corrected (including $pp \rightarrow gg \rightarrow$
Figure 9: The LO and the QCD corrected cross sections for the process $pp \to H^0 W^+ W^- + X$ as the functions of the Higgs-boson mass $m_H$ at the LHC.

$H^0 W^+ W^- + X$ (contribution) cross sections for the process $pp \to H^0 W^+ W^- + X$ as the functions of the Higgs boson mass $m_H$ at the LHC. From the figure we can see the cross sections at the LO and including the QCD corrections are all sensitive to the Higgs boson mass. We find the LO cross section decreases from 15.93 fb to 5.03 fb and the QCD corrected cross section decreases from 23.50 fb to 8.27 fb when $m_H$ goes up from 100 GeV to 160 GeV. And the corresponding K-factor ($K \equiv \sigma_{QCD}/\sigma_{LO}$) varies in the range from 1.48 to 1.64.

In Table 2 we list some of the numerical results used in Fig.9. They are the data for the tree-level, the QCD corrected (including the $pp \to gg \to H^0 W^+ W^- + X$ contribution) cross sections, the total K-factor ($K \equiv \sigma_{QCD}/\sigma_{LO}$) of the process $pp \to H^0 W^+ W^- + X$, the K-factor part contributed by the $pp \to q\bar{q} \to H^0 W^+ W^- + X$ process up to $\mathcal{O}(\alpha^3)\alpha_s$ order ($K_q \equiv \frac{\sigma_{QCD}^{q\bar{q}}}{\sigma_{LO}}$) and the K-factor part contributed by the $pp \to gg \to H^0 W^+ W^- + X$ process at the $\mathcal{O}(\alpha^3\alpha_s^2)$ order ($\Delta K_g \equiv \frac{\sigma_{QCD}^{gg}}{\sigma_{LO}}$) with the Higgs-boson mass value being in the range from 100 GeV to 160 GeV at the LHC. From Table 2 we can see the LO and the QCD corrected cross sections are all sensitive to the Higgs-boson mass, but the total K-factor is not sensitive to the Higgs-boson mass except in the vicinity where $m_H$ approaches to $2m_W \sim 160$ GeV. The contribution from the $pp \to gg \to H^0 W^+ W^- + X$ process to the total QCD corrections can be remarkable at the LHC, and the QCD relative correction from the process $pp \to gg \to$
Table 2: The LO and the QCD corrected cross sections for the \( pp \rightarrow H^0W^+W^- + X \) process, the total K-factor (\( K \equiv \frac{\sigma_{QCD}}{\sigma_{LO}} \)) for the process \( pp \rightarrow H^0W^+W^- + X \), the K-factor part contributed by the \( pp \rightarrow q\bar{q} \rightarrow H^0W^+W^- + X \) process up to \( \mathcal{O}(\alpha^3\alpha_s) \) order (\( K_q \equiv \frac{\sigma_{QCD}^q}{\sigma_{LO}^q} \)) and the K-factor part contributed by the \( pp \rightarrow gg \rightarrow H^0W^+W^- + X \) process at the \( \mathcal{O}(\alpha^3\alpha_s^2) \) order (\( \Delta K_g \equiv \frac{\sigma_{QCD}^g}{\sigma_{LO}^g} \)) with the Higgs boson mass value varying from 100 GeV to 160 GeV at the LHC.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
m_H (GeV) & \sigma_{LO} (f b) & \sigma_{QCD} (f b) & K_q & \Delta K_g & K \\hline
100 & 15.93(1) & 23.50(9) & 1.435 & 0.040 & 1.475 \\hline
110 & 12.763(8) & 18.75(7) & 1.427 & 0.042 & 1.469 \\hline
120 & 10.366(7) & 15.23(6) & 1.424 & 0.045 & 1.469 \\hline
130 & 8.522(6) & 12.53(5) & 1.420 & 0.051 & 1.471 \\hline
140 & 7.082(5) & 10.42(4) & 1.413 & 0.059 & 1.472 \\hline
150 & 5.941(4) & 8.83(3) & 1.408 & 0.078 & 1.486 \\hline
160 & 5.028(3) & 8.27(3) & 1.403 & 0.241 & 1.644 \\hline
\end{array}
\]

\( H^0W^+W^− + X \) is generally about 4%, and can reach the value of 24% in the vicinity of \( m_H \sim 160 \text{ GeV} \), which is about 37% of the total QCD corrections. That large correction enhancement at the position around 160 \( \text{GeV} \), is mainly induced by the resonance effect of \( m_H \sim 2m_W \) occurring in those Feynman diagrams for the subprocess \( gg \rightarrow H^0W^+W^- \), which involves a internal Higgs-boson line interacting with two external W-bosons.

Since the distribution of the transverse momenta of \( W^- \) boson is the same as that of \( W^+ \) in the CP-conserving SM, we show only the results for the transverse momentum distribution of \( W^+ \)-boson here. The differential cross sections of the \( p_T \) for \( W^+ \)-boson at the LO and including the QCD corrections (QCD NLO correction to the \( pp \rightarrow H^0W^+W^- + X \) and the contribution of the \( pp \rightarrow gg \rightarrow H^0W^+W^- + X \) process), i.e., \( d\sigma_{LO}/dp_T^{W^+} \) and \( d\sigma_{QCD}/dp_T^{W^+} \), are depicted in Fig.10(a), and the distributions of \( d\sigma_{LO}/dp_T^{H^0} \) and \( d\sigma_{QCD}/dp_T^{H^0} \) for \( H^0 \)-boson are plotted in Fig.10(b) separately, by taking \( m_H = 120 \text{ GeV} \). In both figures there exist peaks for the curves of \( p_T^{W^+} \) and \( p_T^{H^0} \) at the LO and including QCD corrections. All the peaks are located at the position around \( p_T \sim 50 \text{ GeV} \). And we can see from Figs.10(a-b) that both the differential cross sections at the LO for \( W^± \)- and \( H^0 \)-boson (\( d\sigma_{LO}/dp_T^{W^±}, d\sigma_{LO}/dp_T^{H^0} \)),
Figure 10: The distributions of the transverse momenta of $W^+$- and $H^0$-boson for the $pp \rightarrow H^0W^+W^- + X$ process at the LO and including QCD corrections at the LHC, by taking $m_H = 120 \text{ GeV}$. (a) for the $W^+$-boson, (b) for the $H^0$-boson.

are significantly enhanced by the QCD corrections.

The curves for the distributions of W-pair invariant mass, denoted as $M_{WW}$, at the LO and including the QCD corrections(involving $pp \rightarrow gg \rightarrow H^0W^+W^- + X$ contribution), are drawn in Fig[11] respectively, by taking $m_H = 120 \text{ GeV}$. The two curves show clearly that the QCD correction including the QCD NLO correction part and contribution from gluon-gluon fusion subprocess, enhances the LO differential cross section $d\sigma_{LO}/dM_{WW}$ obviously in the plotted range of $M_{WW}$, and the differential cross sections reach their maximal values around the vicinity of $M_{WW} \sim 200 \text{ GeV}$.

V. Summary

In this paper we investigate the phenomenological effects due to the QCD NLO corrections and the gluon-gluon fusion subprocess in the Higgs-boson production associated with a W-boson pair at the LHC. We study the dependence of the LO and the QCD corrected cross sections on the factorization/renormalization energy scale and Higgs boson mass. We present the LO and the QCD corrected distributions of the transverse momenta of final particles
Figure 11: The distributions of the invariant mass of $W$-pair at the LO and including QCD corrections at the LHC, when $m_H = 120 \, GeV$.

and the differential cross section of the $W$-pair invariant mass. We find that the QCD NLO radiative corrections and the contribution from the $gg \rightarrow H^0W^+W^-$ subprocess obviously modify the LO distributions, and the scale dependence of the QCD corrected cross section is badly underestimated by the LO results. Our numerical results show that the K-factor of the QCD correction varies from 1.48 to 1.64 when $m_H$ goes up from 100 $GeV$ to 160 $GeV$. We find also the cross section of the $pp \rightarrow H^0W^+W^- + X$ process receives a remarkable QCD correction from the contribution of $gg \rightarrow H^0W^+W^-$ subprocess at the LHC, and we should consider this correction part in precise experimental data analyse.

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