Entanglement concentration of microwave photons based on Kerr effect in circuit QED

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In recent years, superconducting qubits show great potential in quantum computation. Hence, microwave photons become very interesting qubits for quantum information processing assisted by superconducting quantum computation. Here, we present the first protocol for the entanglement concentration on microwave photons, resorting to the cross-Kerr effect in circuit quantum electrodynamics (QED). Two superconducting transmission line resonators (TLRs) coupled to superconducting molecule with the $N$-type level structure induce the effective cross-Kerr effect for realizing the quantum nondemolition (QND) measurement on microwave photons. With this device, we present a two-qubit polarization purity QND detector on the photon states of the superconducting TLRs, which can be used to concentrate efficiently the nonlocal non-maximally entangled states of microwave photons assisted by several linear microwave elements. This protocol has a high efficiency and it may be useful for solid-state quantum information processing assisted by microwave photons.

I. INTRODUCTION

Quantum entanglement plays an extremely important role in quantum communication, such as quantum teleportation [1], quantum dense coding [2,3], quantum key distribution [4,5], quantum secret sharing [6], and quantum secure direct communication [7,8]. The maximally entangled photons are usually used to act as the information carriers in quantum communication. However, they unavoidably interact with the environment when they are transmitted over a noisy channel or stored in artificial atomic systems, which will lead their decoherence and make them in a partially entangled state or a mixed entangled one. To accomplish the quantum communication effectively, some interesting methods are used to depress the effect of environment noise, such as faithful qubit transmission [9], error-rejecting with decoherence free subspaces [10,11], entanglement purification [12,13], and entanglement concentration [14,15].

Entanglement concentration is aimed at transforming a less-entangled state into a maximally entangled state and it becomes an indispensable part in long-distance quantum communication. By far, there are some interesting entanglement concentration protocols for photon systems and atomic systems. For instance, in 1996, Bennett et al. [20] proposed the original entanglement concentration protocol (ECP) by means of the Schmidt projection. In 2001, two ECPs were proposed for ideal entanglement sources with polarizing beam splitters [21,22]. In 2008, Sheng et al. [23] proposed a repeatable ECP and it improves the efficiency largely by iteration of the entanglement concentration process three times. In fact, the existing ECPs can be divided into two categories depending on whether the parameters of the less-entangled states are known [24,27] or not [20–23]. When the parameters are known, a nonlocal photon system is enough for entanglement concentration [24,27], and it is far more efficient than those with unknown coefficients [20,23]. In 1999, Bose et al. [24] proposed an ECP for partially entangled pure state with known coefficients based on the entanglement swapping of two EPR pairs. In 2012, Sheng et al. [25] proposed two efficient ECPs for the less-entangled states with known parameters with the help of an ancillary single photon. In 2013, Ren et al. [26] presented the parameter-splitting method for the concentraton of nonlocally partially entangled states with known coefficients, and this fascinating method can accomplish the concentration with the maximal success probability by executing the protocol only one time, just resorting to linear-optical elements. They also presented the pioneering hyper-ECP for unknown polarization-spatial less-hyperentangled states with linear-optical elements only. In 2014, Ren and Long [27] gave a general hyper-ECP and another high-efficiency hyper-ECP [28]. Li and Ghose [29] proposed an interesting hyper-ECP with linear optics and another two efficient hyper-ECPs [30,31] with nonlinearity in 2015. Wang et al. [32,33] and Cao et al. [34] gave two good hyper-ECPs with or without local entanglement resource, respectively. Also, Wang et al. [35,36] presented two interesting ECPs for electron-spin systems.

Circuit quantum electrodynamics (QED) is composed of a superconducting qubit coupled to a superconducting resonator [37,38]. It plays an important role in quantum information processing as it has the good scalability [39,40]. Many achievements have been accomplished in circuit QED [41,42]. In recent years, Kerr effect has been generally concerned and researched in circuit QED [43,44]. For example, Rebić et al. [45] presented the giant Kerr effect in 2009. In 2011, Hu et al. [46] proposed a scheme for implementing cross-Kerr effect in circuit QED. In 2013, Kirchmair et al. [47] observed quan-
tum state collapse and revival due to the single-photon Kerr effect and Hoi et al. [60] demonstrated the giant cross-Kerr effect for propagating microwaves experimentally. In 2015, Holland et al. [61] demonstrated single-photon resolved cross-Kerr effect between two superconducting cavities. Due to the low loss and strong anti-interference during transmission, microwave photons hold good prospects for both classical and quantum communication. In recent years, superconducting quantum computation shows great potential in quantum information processing. Combining the microwave quantum communication with quantum computation is an interesting research area. Because of dissipation in the process of microwave photon transmission and storage, the maximally entangled state can not keep all the time. Entanglement concentration of microwave-photon states is an extremely significant and necessary task. However, the research of this area has been in blank. The cross-Kerr effect can be used for quantum nondemolition (QND) measurement. Polarization is an important degree of freedom of microwave and one can manipulate it by adjusting parameters of particular materials [62], for example, anisotropic metamaterials [63] and photonic crystal [64]. Some polarization linear elements have been implemented for microwave photon theoretically and experimentally [64]–[66], such as microwave polarization beam splitter (PBS) and polarization rotator.

In this paper, we propose the first physically feasible protocol to achieve the concentration on the polarization state of nonlocal entangled microwave photons in circuit QEDs. The crucial step of our protocol is the construction of a two-qubit polarization parity QND detector to check the polarization parity of two microwave photons. The QND measurement is accomplished by cross-Kerr effect and other polarization linear elements. This protocol has a high efficiency and it maybe has good application in solid-state quantum computation assisted by microwave photons, especially the combination of quantum computation and quantum communication.

This article is organized as follows: In Sec. II we introduce the physical implementation of a superconducting molecule in Sec. II A and the process of inducing cross-Kerr effect in circuit QED in Sec. II B. In Sec. II C we present the scheme for making QND measurement on two cascade TLRs. In Sec. III we design a two-qubit polarization parity QND detector and present the principle for the concentration of microwave photon pairs. A discussion and a summary are given in Sec. IV.

II. QND MEASUREMENT OF TOTAL PHOTON NUMBER OF RESONATORS

A. Cross-Kerr effect in circuit QED

The schematic diagram for realizing the cross-Kerr effect between the microwave photons in the two TLRs capacitively coupled to an artificial superconducting molecule is shown in Fig. I(a). Two color bars represent two TLRs and the square on the middle is the superconducting molecule whose structure and energy levels are depicted in Fig. I(b) and (c), respectively. The control line on the left is used to adjust the character of the superconducting molecule which is made up of two superconducting transmon qubits [65] and a superconducting quantum interference device (SQUID) [68]. The two qubits couple to each other through the SQUID. The two loops on the left and one loop on the right stand for two transmon qubits and SQUID, respectively. Crosses and circles of every loop represent the Josephson junctions and the external flux biases, respectively. The Josephson junctions in each loop are identical. $E_j$ (i = c, 1, 2) and $C_j/2$ (j = m, 1, 2) represent the energy and capacitance of Josephson junctions, respectively. External fluxes la-

FIG. 1: (a) Scheme for the proposed cross-Kerr effect between two TLRs. The TLR A (upper, red) and the TLR B (lower, blue) are capacitively coupled to a superconducting molecule (middle, green). The superconducting molecule is described in Fig. I(b) in detail and it can be controlled by external coils (left, black). (b) Detailed schematic circuit of the proposed superconducting molecule [55]. The middle circuit stands for coupled transmon qubits. The crosses and the circles in the SQUID loops represent the josephson junctions and the external flux biases, respectively. (c) N-type level structure of the molecule which is described in Fig. I(b).
beled with $\Phi_{e1}$, $\Phi_{e2}$, and $\Phi_{ec}$ are applied to SQUID loops of the two qubits and coupling SQUID, respectively. The additional external flux $\Phi_{st}$ is applied to the center loop of the molecule. Each loop has two arrows on their both sides and they stand for the gauge-invariant phases across the Josephson junctions, marked with $\varphi_{ct}$, $\varphi_{c1}$, $\varphi_{t1d}$, $\varphi_{2a}$, and $\varphi_{d2}$, respectively. $V_{g1}$ and $V_{g2}$ near the capacitors are the gate voltages which bias the two corresponding qubits via the gate capacitors $C_{g1}$ and $C_{g2}$, respectively.

Under the condition of fluid quantization, the Hamiltonian of the superconducting molecule is given by \[ H_0 = \sum_{i=1,2} [4E_{ci}(n_i - n_m) - 2E_{J1}\cos(\pi\Phi_{ci}/\Phi_0)\cos(\Phi_1)] + 4E_m(n_1 - n_g)(n_2 - n_g) - E_{Jm}\cos(\Phi + \phi), \] (1)

where $E_{c1,2} = e^2C_{21}/(2(C_{21}C_{22} - C_m^2))$ are the effective Cooper-pair charging energies and $C_{21} = C_{g1} + C_{J1} + C_m$ is the sum of all capacitances around the $i$-th qubit. $n_i$ ($i = 1, 2$) denotes the canonical conjugate variable to the phase of the superconducting islands $\Phi_i$ and $\Phi_i = (\varphi_{ci} + \varphi_{d2})/2$. $n_g = C_{g1}V_{g1}/2\varepsilon$ is the gate-induced charge number. $\Phi_0 = h/2e$ is the flux quanta. $E_m = e^2C_m/(C_{21}C_{22} - C_m^2)$ and $E_{Jm} = 2E_{Jc}\cos(\pi\Phi_{ec}/\Phi_0)$ are the capacitive coupling strength between the transmon qubits and the tunable effective Josephson tunnel energy of the coupling SQUID, respectively. $\Phi = \Phi_1 - \Phi_2$ and $\phi = \pi(\Phi_{c1} + \Phi_{c2} + \Phi_{ec} + \Phi_{st})/\Phi_0$. Here we assume that $\phi \equiv 0$ and two transmon qubits are identical, and then $2E_{J1}\cos(\pi\Phi_{c1}/\Phi_0) = 2E_{J2}\cos(\pi\Phi_{c2}/\Phi_0) = E_j$, $E_{c1} = E_{c2} = E_c$, $C_{J1} = C_{J2} = C_J$, and $C_{21} = C_{22} = C_{\Sigma}$. One can utilize the two-level language in the region $E_J \ll E_c$ to get the N-type level form \[ \{1\} = \cos\theta|\uparrow\uparrow\rangle - \sin\theta|\downarrow\downarrow\rangle, \]
\[ \{2\} = \langle|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\rangle\sqrt{2}, \]
\[ \{3\} = \langle|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\rangle/\sqrt{2}, \]
\[ \{4\} = \sin\theta|\uparrow\uparrow\rangle + \cos\theta|\downarrow\downarrow\rangle, \] (2)

and the corresponding eigenvalues are given as

\[ E_1 = -E_m(1 - \omega^2 + E_{m-1})^{1/2}, \]
\[ E_2 = E_m - E_{m+}, \]
\[ E_3 = E_m + E_{m+}, \]
\[ E_4 = -E_m(1 - \omega^2 + E_{m-1})^{1/2}, \] (3)

with the symbols $E_{m\pm} = \exp(\pm\alpha^2)E_{Jm}\alpha^4/4$, $E_{m\pm} = E_{Jm}\alpha^2\exp(\pm\alpha^2)\pm E_m\alpha^{-2}$, and $\alpha = \sqrt{2E_c/E_j}$ and $\omega = (8E_cE_j)^{1/2} - E_c$. One can modify the rough energy scale of the molecule and the energy structure by tuning $E_j$ and $E_{Jm}$, shown in Fig. II(c).

The Hamiltonian of the system composed of the TLRs A and B is (with $\hbar = 1$)

\[ H_{TLR} = \omega_Aa\dagger a + \omega_Bb\dagger b, \] (4)

where $\omega_{A,B} = 2\pi/(L_{A,B}\sqrt{F_0})$ and $L_{A,B}$ is the length of corresponding TLR. $F$ and $c$ are the inductance and capacitance of the TLRs per unit length, respectively. $a$ ($a\dagger$) and $b$ ($b\dagger$) are the annihilation (creation) operators of two TLR modes, respectively. $C_{A_j}$ ($C_{B_j}$) with $j = 1, 2$ is the coupling capacitance between TLR A (B) and the $j$-th qubit. Their magnitudes are set to be $C_{A1} = C_{A2}$ and $C_{B1} = C_{B2}$. The locations of these capacitors are $x_{A1} = x_{A2} = 0$ and $x_{B1} = L_{B}/2 - x_{B2} = L_{B}/8$. We add a classical pump field to connect the energy level between \langle 2 \rangle$ and \langle 3 \rangle$ and the form is expressed with \[ H_p = -\Omega_c\exp(i\omega_p\ell)\langle 2\rangle\langle 3 \rangle + \exp(-i\omega_p\ell)\langle 3\rangle\langle 2 \rangle, \] (5)

where $\Omega_c$ and $\omega_p$ are the intensity and frequency of classical pump field. When the classical pump field is tuned in dark resonance with TLR A, the Hamiltonian can be expressed with the form

\[ H_{cs} = ig_{A1}(\sigma_{33}a\dagger - \sigma_{31}a) + ig_{B2}(\sigma_{34}b\dagger - \sigma_{32}b) - \Omega_c\exp(i\omega_p\ell)\langle 2\rangle\langle 3 \rangle + \exp(-i\omega_p\ell)\langle 3\rangle\langle 2 \rangle, \] (6)

where $g_{A1}$ and $g_{B2}$ are the coupling factors. In the interaction picture, the Hamiltonian of the whole system composed of the two TLRs and the superconducting molecule is \[ \frac{g_{A1}}{\Omega_c} \] (7)

Here $\sigma_{ij} = |i\rangle\langle j|$. With the limit that $|g_{A1}/\Omega_c|^2 \ll 1$, $|g_{B2}|\ll |\delta_2|$, and adiabatically eliminating the atomic degrees of freedom, one obtain the ultimate effective cross-Kerr interaction Hamiltonian \[ H_{eff} = -\frac{g_{A1}g_{B2}}{\delta_2\Omega_c}a\dagger ab\dagger b, \] (8)

where the detuning $\delta_2 = E_{d2} - \omega_B$ and cross-Kerr coefficient $\chi = -g_{A1}g_{B2}/(\delta_2\Omega_c^2)$. Here we neglect the subscript letters A and B for convenience.

B. Physical implementation for the QND measurement on TLRs based on cross-Kerr effect

Here, we present a scheme for the QND measurement on the photon number of the cascade TLR $A_1$ and TLR $A_2$. The schematic diagram is depicted in Fig. 2. It is realized based on the cross-Kerr effect in circuit QED. The Hamiltonian for each cross-Kerr medium is assumed to be

\[ H = \chi a\dagger b b\dagger b, \] (9)

where the cross-Kerr coefficient $\chi = -g_{A1}g_{B2}/(\delta_2\Omega_c^2)$, $a\dagger (a)$ and $b\dagger (b)$ are the creation (annihilation) operators of TLR $A_i$ and TLR $B_j$ ($i = 1, 2$, respectively. The
We derive the reflection coefficients which are given by

$$r_j = \frac{b_j^{\text{out}}}{b_j^{\text{in}}} = \frac{i\chi n_j - \frac{\kappa_2}{2}}{i\chi n_j + \frac{\kappa_2}{2}}.$$  \hfill (11)

First, let us consider the case in which TLR $A_1$ is in the Fock state $|n_1\rangle$. We introduce a probe field in the coherent state $|\alpha\rangle = D(\alpha)|0\rangle$, to interact with the cross-Kerr system through TLR $B_1$. The state of the system composed of the signal photons and the probe light will evolve from $|n_1\rangle|\alpha\rangle$ to

$$|\psi_1\rangle = |n_1\rangle D(\alpha/r_1)|0\rangle^{\text{out}} = |n_1\rangle e^{i\theta_{n_1} n_1^{\text{out}}} |\alpha\rangle^{\text{out}},$$  \hfill (12)

where $\theta_{n_1} = \arg(\frac{1}{r_2 r_1})$ which depends on the photon number $n_1$. If one measures this phase shift via an $X$ homodyne measurement, he can infer the Fock state $|n_1\rangle$.

Now, let us analyze joint Fock state of the whole cascaded system with cross-Kerr effect. Here the input field of resonator $B_2$ is just the output field of resonator $B_1$, that is, $b_{2}^{\text{in}} = b_{1}^{\text{out}}$. Therefore the output of resonator $B_2$ connects with input of $B_1$ through $b_{2}^{\text{out}} = r_2 r_1 b_{1}^{\text{in}}$. When the coherent state $|\alpha\rangle_p$ passes through the TLRs $B_1$ and $B_2$, its state becomes

$$|\alpha\rangle_2^{\text{out}} = D(\frac{\alpha}{r_2 r_1}) |\alpha\rangle_2^{\text{out}} = |\alpha\rangle_2^{\text{out}} e^{i\theta_{n_1+n_2}}.$$  \hfill (13)

where

$$\theta_{n_1+n_2} = \arg(\frac{1}{r_2 r_1}) = \arg(\frac{1}{r_1}) + \arg(\frac{1}{r_2}).$$  \hfill (14)

According to the magnitude of total phase shift $\theta_{n_1+n_2}$ which depend on $n_1$ and $n_2$ simultaneously, one can distinguish the different joint Fock states of the system composed of the TLRs $A_1$ and $A_2$.

The principle of our polarization parity QND detector on the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3. Let us assume that the cross-Kerr effects in the microwave photons in two TLRs is shown in Fig. 3.
TABLE I: Corresponding relation between the phase shift and the states of the signal light. The range of coefficients \(a\) and \(b\) are \([0, 1]\). But \(a\) and \(b\) cannot be zero simultaneously.

| Phase shift | State \(|b_1b_2\rangle\) |
|-------------|-------------------|
| \(\theta_0\)  | \(|VV\rangle\)     |
| \(\theta_1\)  | \(a|HV\rangle + b|VH\rangle\) |
| \(\theta_2\)  | \(|HH\rangle\)     |

result and they are undistinguishable in this QND measurement system. The states \(|HH\rangle\) and \(|VV\rangle\) result in the phase shifts \(\theta_2\) and \(\theta_0\), respectively. With an X homodyne measurement on the probe light, one will get the relation between the phase shifts and the states of the signal light, shown in TABLE I. This is a two-qubit polarization parity QND detector on the microwave photons, which is shown in Fig. 4. Suppose that there are two microwave-photon pairs in TLRs is shown in Fig. 4. Now Bob sends the microwave photons \(u_2\) and \(d_2\) into the two-qubit polarization parity QND detector shown in Fig. 3. Bob can get the state of the system composed of the microwave photons and the probe light as

\[
|\psi\rangle_{d_1d_2} = x|H\rangle_{d_1}|H\rangle_{d_2} + y|V\rangle_{d_1}|V\rangle_{d_2},
\]

where \(x^2 + y^2 = 1\). The state of the system composed of the four photons is described as follow

\[
|\psi\rangle_1 = |\psi\rangle_{u_1u_2} \otimes |\psi\rangle_{d_1d_2} = x^2|H\rangle_{u_1}|H\rangle_{u_2}|H\rangle_{d_1}|H\rangle_{d_2} + xy|H\rangle_{u_1}|H\rangle_{u_2}|V\rangle_{d_1}|V\rangle_{d_2} + xy|V\rangle_{u_1}|V\rangle_{u_2}|H\rangle_{d_1}|H\rangle_{d_2} + y^2|V\rangle_{u_1}|V\rangle_{u_2}|V\rangle_{d_1}|V\rangle_{d_2}.
\]

Now Alice and Bob make \(d_1\) and \(d_2\) pass through a 45° microwave polarization rotator, respectively. The transformations of this rotation on a microwave photon are given by

\[
|H\rangle_{d_i} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{d_i} + |V\rangle_{d_i}), \quad |V\rangle_{d_i} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{d_i} - |V\rangle_{d_i}),
\]

where \(i = 3, 4\). Hence, after the 45° rotation, the state \(|\psi\rangle_4\) will turn to

\[
|\psi\rangle_4 = \frac{1}{2\sqrt{2}}(|H\rangle_{u_1}|H\rangle_{u_2} + |V\rangle_{u_1}|V\rangle_{u_2} \otimes (|H\rangle_{d_1}|H\rangle_{d_2} + |V\rangle_{d_1}|V\rangle_{d_2}) + \frac{1}{2\sqrt{2}}(|V\rangle_{u_1}|V\rangle_{u_2} - |H\rangle_{u_1}|H\rangle_{u_2} \otimes (|H\rangle_{d_1}|V\rangle_{d_2} + |V\rangle_{d_1}|H\rangle_{d_2}).
\]

At last, Alice and Bob use a microwave PBS to pass through \(|H\rangle\) and reflect \(|V\rangle\) photons, respectively. Combining Eq. (20) and Fig. 4 one can find that if the...
detectors $D_1$ and $D_3$ or $D_2$ and $D_4$ are in response, Alice and Bob can get the state the two microwave photons from the up spatial modes $u_1 u_2$ with the form

$$|\Psi^+\rangle_{u_1 u_2} = \frac{1}{\sqrt{2}}(|H\rangle_{u_1}|H\rangle_{u_2} + |V\rangle_{u_1}|V\rangle_{u_2}).$$  \hspace{1cm} (21)

If the responsive detectors are $D_1$ and $D_4$ or $D_2$ and $D_3$, the state of the two microwave photons from the up spatial modes $u_1 u_2$ becomes

$$|\Psi^-\rangle_{u_1 u_2} = \frac{1}{\sqrt{2}}(|V\rangle_{u_1}|H\rangle_{u_2} - |H\rangle_{u_1}|V\rangle_{u_2}).$$  \hspace{1cm} (22)

After Alice or Bob rotates the phase on his or her photon, they will get the maximally entangled state $|\Psi^+\rangle$.

![Diagram](image)

FIG. 5: Schematic of making a concentration on less entangled tripartite GHZ microwave photon states. The process of this protocol is similar to the situation with two users. The meanings of marks of elements are the same as those in Fig. 3. $S_1$ and $S_2$ represent two identical ideal tripartite entanglement microwave photon sources.

### IV. DISCUSSION AND SUMMARY

Our ECP could be extended to the multi-user situation. For a less entangled tripartite Greenberg-Horne-Zeilinger (GHZ) type microwave photon state, we suppose it has the form

$$|\Phi^+\rangle_1 = x|H_1\rangle|H_2\rangle|H_3\rangle + y|V_1\rangle|V_2\rangle|V_3\rangle,$$

$$|\Phi^+\rangle_2 = x|H_4\rangle|H_5\rangle|H_6\rangle + y|V_4\rangle|V_5\rangle|V_6\rangle,$$  \hspace{1cm} (23)

where $|x|^2 + |y|^2 = 1$. 1 and 4 belong to Alice, 2 and 5 belong to Bob, 3 and 6 belong to Charlie. As shown in Fig. 3, the state of the composite system composed of two three-photon subsystems can be written as

$$|\Phi\rangle = |\Phi^+\rangle_1 \otimes |\Phi^+\rangle_2$$

$$= x^2|H_1\rangle|H_2\rangle|H_3\rangle|H_4\rangle|H_5\rangle|H_6\rangle + xy(|H_1\rangle|H_2\rangle|V_3\rangle|V_4\rangle|V_5\rangle|V_6\rangle + |V_1\rangle|V_2\rangle|H_3\rangle|H_4\rangle|H_5\rangle|H_6\rangle + y^2|V_1\rangle|V_2\rangle|V_3\rangle|V_4\rangle|V_5\rangle|V_6\rangle.$$  \hspace{1cm} (24)

Now Bob checks the parity of his two polarization microwave photons with his two-qubit polarization parity QND detector. The state of the composite system composed of the six microwave photons and the probe light will evolve to

$$|\Phi\rangle_3 = x^2|H_1\rangle|H_2\rangle|H_3\rangle|H_4\rangle|H_5\rangle|\alpha e^{i\theta_2}\rangle_p$$

$$+ xy(|H_1\rangle|H_2\rangle|V_3\rangle|V_4\rangle|V_5\rangle|V_6\rangle + |V_1\rangle|V_2\rangle|H_3\rangle|H_4\rangle|H_5\rangle|\alpha e^{i\theta_1}\rangle_p$$

$$+ y^2|V_1\rangle|V_2\rangle|V_3\rangle|V_4\rangle|V_5\rangle|V_6\rangle|\alpha e^{i\theta_0}\rangle_p.$$  \hspace{1cm} (25)

Bob makes an $X$ homodyne measurement and tells Alice and Charlie to keep their photons when the result is $\theta_1$. Otherwise, they discard the photons. When Bob obtains the phase shift $\theta_1$, the state of the six-photon system collapses to

$$|\Phi\rangle_4 = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle|V_4\rangle|V_5\rangle|V_6\rangle + |V_1\rangle|V_2\rangle|V_3\rangle|H_4\rangle|H_5\rangle|H_6\rangle$$

with the probability $2|xy|^2$. After all the parties rotate their second photon by $45^\circ$ with their microwave polarization rotators, the state becomes

$$|\Phi\rangle_5 = -\frac{1}{\sqrt{2}}3(|H_1\rangle|H_2\rangle|H_3\rangle|H_4\rangle - |V_1\rangle|V_2\rangle|V_3\rangle|H_4\rangle + |H_5\rangle|V_6\rangle + |H_6\rangle|V_5\rangle|V_4\rangle).$$  \hspace{1cm} (27)

After the second photons pass through the microwave PBSs and obtain an even number of $|V\rangle$ with measurement, the state of the three-photon system 123 becomes

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle).$$  \hspace{1cm} (28)

If the number of $|V\rangle$ is odd, the state is given by

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|V_1\rangle|V_2\rangle|V_3\rangle - |H_1\rangle|H_2\rangle|H_3\rangle).$$  \hspace{1cm} (29)

Similar to above process, one can extend this concentration protocol to the situation of $N$-photon less entangled states.

Now, let us discuss the feasibility of our QND on microwave photons. According to the protocol proposed by Hu et al. [58], when the parameters of the coupled transmon qubits are set with $E_c/2\pi = 0.5$ GHz, $E_J/2\pi = 16$ GHz and $E_m/2\pi = 0.2$ GHz. When $E_{jm}/E_J$ changes from 0 to 1, the level spacing $E_{31}$, $E_{32}$, and $E_{31} - E_{42}$ are proportional to $E_{jm}/E_J$ and their value ranges are about
8 \sim 12 \text{ GHz}, 1 \sim 7 \text{ GHz}, \text{and } 0 \sim 1 \text{ GHz}, \text{respectively. The coupling strength between TLRs and superconducting molecule could be set with } g_{1}/2\pi \sim g_{2}/2\pi \sim 300 \text{ MHz} [53]. \text{In order to satisfy the adiabatic conditions } | g_{1}/\Omega_{c} |^{2} \ll 1 \text{ and } | g_{2} | \ll | \delta_{2} |, \text{the detuning } \delta_{2} \text{ and the classical pump strength } \Omega_{c} \text{ are designed with } \delta_{2}/2\pi \sim \Omega_{c}/2\pi \sim 1.5 \text{ GHz} [53]. \text{Hence the cross-Kerr coefficient } | \chi |/2\pi \sim 2.4 \text{ MHz. We set } \kappa_{1}^{-1} \sim 20 \text{ ms and } \kappa_{2}^{-1} \sim 10 \text{ ns. Large } \kappa_{2} \text{ can promise the probe light pass through the resonator quickly and low } \kappa_{1} \text{ can keep the signal photon in resonator for enough long time. Recent experiments have demonstrated the giant cross-Kerr shift in circuit QED experimentally. For example, in 2013, Hoi et al. [60] observed the average cross-Kerr phase shifts of up to 20 degrees per photon with both coherent microwave fields at the single-photon level by using a transmon qubit embedded in a one-dimensional open transmission line. In 2015, Holland et al. [61] inferred a state dependent shift } \chi_{sc}/2\pi = 2.59 \pm 0.06 \text{ MHz according to the experimental parameters and observed the single-photon-resolved cross-Kerr interaction between the two three-dimensional cavities which connected by transmon qubit.}

In summary, we have proposed the first protocol to concentrate the quantum state of nonlocal microwave photons. The protocol aims at entanglement on the degree of freedom of polarization, assisted by the cross-Kerr effect in circuit QED. The principle of constructing N-type level artificial molecule has been analyzed. Coupling to superconducting molecule induces the effective cross-Kerr effect between two TLRs. We make use of the effective cross-Kerr effect to implement a QND measurement on Fock state of microwave photons in TLRs. The QND measurement system plays a key role which can check the polarization parity of photons in the process of entanglement concentration. With this device, our ECP has high efficiency and it maybe has good applications in quantum communication and quantum computation.

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