Smooth crossing of $w_\Lambda = -1$ line in a single scalar field model

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Abstract

Smooth double crossing of the phantom divide line $w_\Lambda = -1$ has been found possible with a single minimally coupled scalar field for a most simple form of generalized k-essence cosmological model, in the presence of background cold dark matter. Such crossing is a sufficiently late time transient phenomena and thus does not have any pathological behaviour.

1 Introduction

Recent analysis of the three year WMAP data [1, 2, 3] provides no indication of any significant deviations from gaussianity and adiabaticity of the CMBR power spectrum and therefore suggests that the universe is spatially flat to within the limits of observational accuracy. Further, the combined analysis of the three-year WMAP data with the supernova Legacy survey (SNLS), in [1], constrains the equation of state $w_\Lambda$, corresponding to almost 74% of dark energy present in the currently accelerating Universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favour a dark energy equation of state, $w_\Lambda < -1$ [4]. The marginalized best fit values of the equation of state parameter are given by $-1.14 \leq w_\Lambda \leq -0.93$ at 68% confidence level. In case, one considers a flat universe a-priori, then the combined data leads to $-1.06 = w_\Lambda = -0.90$. Thus, it is realized that a viable cosmological model should admit a dynamical equation of state that might have crossed the value $w_\Lambda = -1$, in the recent epoch of cosmological evolution.

So far, it has been administered by Vikman [5] and accepted almost by all [6], except perhaps by Andrianov et-al [7], and more recently by Cannata and Kamenshchik [8] that smooth crossing of $w_\Lambda = -1$ line is not possible in minimally coupled theories, even through a generalized k-essence Lagrangian [9] in the form $L = g(\phi)\phi^2 - V(\phi)$. It is clear that something went wrong with the analysis of Andrianov et-al [7], since, it is not difficult to understand that the standard minimally coupled theory can not go smoothly over to the phantom [10] domain without violating the stability both at the classical [11] and the quantum mechanical levels [12] (although it has recently been inferred [13] that quantum Effects which induce the $w < -1$ phase, are stable in the $\phi^4$ model). However, Vikman [5], in particular, argued that transitions from $w_\Lambda \geq -1$ to $w_\Lambda < -1$ (or vice versa) of the dark energy described by a general scalar-field Lagrangian $(\rho(\phi), \nabla(\phi))$, are either unstable with respect to the cosmological perturbations or realized on the trajectories of measure zero, even in the presence of k-essence Lagrangian. It is not really clear why a single field minimally coupled theory with a k-essence Lagrangian does not permit a transient phantomization. The treatment carried out by Vikman [5] is rather complicated. As a consequence, it has given birth to further complicated models to establish a smooth crossing. Particularly, it requires hybrid models composed of at least two scalar fields [14], one-the quintessence and the other a phantom and is usually dubbed as quintom models [15]. Others, even further complicated models like lessence [16], nonminimal scalar tensor theories of gravity [25], Gauss-Bonnet gravity [15], have also been invoked for the purpose.

In the present work we have been able to show that the so called phantom divide line corresponding to the state parameter, $w_\Lambda = -1$, can indeed be crossed in a single minimally coupled scalar field model, only by invoking the generalized k-essence Lagrangian [9] in the above mentioned form, without requiring higher order curvature invariant terms [19]. Dark energy cosmological models with an equation of state parameter $w_\Lambda$ less than $-1$ violates the null energy condition and show unstable behaviour [11, 12]. In our model the crossing is transient and thus such violation is only a momentary phenomena. Further, the velocity of sound remains always positive $(c_s^2 = 1)$ in the medium under consideration, except at the two points of transitions $w_\Lambda = -1$. As a result, the theory does not develop any instabilities or other pathological features during the cosmological evolution.

The essential feature of the model is a solution of the scale factor in the form, $a = a_0 e^{(\frac{L}{2f})}$, with $0 < f < 1$ and
n > 0. Such a solution was dubbed as intermediate inflation in the nineties [20]. Recently, it has been observed [21] that Gauss-Bonnet interaction in four dimensions with dynamic dilatonic scalar coupling admits such solution leading to late time cosmic acceleration rather than inflation at the very early Universe. Under this consequence, a comprehensive analysis has been carried out [22] with such solution in the context of a generalized k-essence model. It has been observed that it admits scaling solution with a natural exit from it at a later epoch of cosmic evolution, leading to late time acceleration with asymptotic de-Sitter expansion. The corresponding scalar field has also been found to behave as a tracker field [23]. Unfortunately, we have not analyzed the behaviour of the state parameter in the intermediate region. In this work, we show that such solution in the presence of background matter leads to late time cosmic acceleration with a transient double crossing of the phantom divide line.

2 The Model

As mentioned in the introduction, we start with generalized k-essence [9] Lagrangian in the form,

\[ L = g(\phi)F(X) - V(\phi), \]

where, \( X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \), which, when coupled to gravity may be expressed in the following most simplest form

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - g(\phi) \phi, \phi - V(\phi) \right], \]

(1)

where, a coupling parameter, \( g(\phi) \) appears with the kinetic energy term. \( g(\phi) \) has got a Brans-Dicke origin, \( g = \frac{1}{\omega(\phi)} \) too, \( \omega(\phi) \) being the Brans-Dicke parameter. This is the simplest form of an action in which both canonical and non-canonical forms of kinetic energies can be treated and a possible crossing of the phantom divide line may be expatiated. For the spatially flat Robertson-Walker space-time

\[ ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2(\theta) d\phi^2)], \]

the field equations are

\[ 2 \frac{\dot{a}}{a} + \frac{\dot{a}^2}{a^2} = 2 \dot{H} + 3H^2 = -[g \dot{\phi}^2 - V(\phi) + p_m], \]

(2)

and

\[ 3 \frac{\dot{a}^2}{a^2} = 3H^2 = g \dot{\phi}^2 + V(\phi) + \rho_m, \]

(3)

together with the \( \phi \) variation equation

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{1}{2} \frac{g'}{g} \dot{\phi}^2 + \frac{V'}{2g} = 0, \]

(4)

in the units \( \kappa^2 = 8\pi G = \hbar = c = 1 \). In the above equations, \( H = \dot{a}/a \), is the Hubble parameter, while \( p_m \) and \( \rho_m \) stand for pressure and the energy density of the background matter. The above field equations may also be written in a more convenient form as,

\[ \dot{H} = -[g \dot{\phi}^2 + \frac{\rho_m + p_m}{2}], \]

(5)

\[ \dot{H} + 3H^2 = V(\phi) + \frac{\rho_m - p_m}{2}. \]

(6)

So, altogether, we have got three independent equations, viz., (4) through (6), corresponding to six variables of the theory, viz., \( a \) or \( H, \phi, g(\phi), V(\phi), \rho_m \) and \( p_m \). Therefore, we need three physically reasonable assumptions to
obtain complete set of solutions. Our first assumption is to neglect the amount of radiation present in the present day Universe, and to consider the background matter is filled with luminous along with baryonic and non-baryonic cold dark matter with equation of state $w_m = \frac{p_m}{\rho_m} = 0$. So, the continuity equations for the background matter, 

$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = 0,$$

leads to,

$$\rho_m = \rho_m^{(0)} a^{-3},$$

where, $\rho_m^{(0)}$ is a constant. To find a solution viable for crossing the phantom divide line, we present our second assumption, which is the following ansatz for the Hubble parameter,

$$H = \frac{f}{n^{1-f}}.$$

with $n > 0$ and $f > 0$. It is clear that $f = 1$, leads to exponential expansion. However, we choose, $f$ in between, i.e., $0 < f < 1$. The scale factor, in view of such a form of the Hubble parameter, takes the form as mentioned in the introduction.

$$a = a_0 e^{\frac{ft}{n}},$$

This solution, as already mentioned in the introduction, has been found to admit scaling behaviour with a natural exit from it at the later epoch of cosmological evolution, leading to late time acceleration [22]. Thus, the kinetic energy ($g\dot{\phi}^2$), corresponding to the scalar field and the potential energy $V(\phi)$ are found in view of equations (5) and (6) respectively as,

$$g\dot{\phi}^2 = \frac{f(1 - f)}{n\phi^{2(1-f)}} - \frac{\rho_m}{2};$$

$$V = \frac{3f^2}{n^2\phi^{2(1-f)}} - \frac{f(1 - f)}{n\phi^{2(1-f)}} - \frac{\rho_m}{2},$$

together with the matter energy density $\rho_m$ from (8) as,

$$\rho_m = \frac{\rho_m^0}{[a_0 e^{\frac{(f/3)!}n}]^3}.$$

Now to express $g(\phi)$ and $V(\phi)$ as functions of $\phi$, we put forward our third assumption by choosing the scalar field $\phi$ as a monotonically increasing function of time, in a most simple form,

$$\phi = t.$$

Thus, the potential $V(\phi)$ and the coupling parameter $g(\phi)$ in (11) may be finally expressed as,

$$V = \frac{3f^2}{n^2\phi^{2(1-f)}} - \frac{f(1 - f)}{n\phi^{2(1-f)}} - \frac{\rho_m^{(0)}}{2[a_0^3 e^{\frac{(f/3)!}n}]^3},$$

$$g(\phi) = \frac{f(1 - f)}{n\phi^{2(1-f)}} - \frac{\rho_m^0}{2a_0^3 e^{\frac{(f/3)!}n} e^f}.$$

It can now be trivially checked that all the field equations (4) through (6) are satisfied in view of the solutions (9) and (12) through (15). We would like to mention that the above form of the potential has been found in an earlier
work \cite{22} to behave as a tracker field \cite{23}. The effective energy density, $\rho_\phi = g(\phi)\dot{\phi}^2 + V(\phi)$ and the pressure, $p_\phi = g(\phi)\dot{\phi}^2 - V(\phi)$ of the scalar field are now expressed as,

$$
\rho_\phi = \frac{3f^2}{n^2t^{2(1-f)}} - \frac{\rho_m^{(0)}}{|a_0^3\exp\left\{\frac{3}{n}t^f\right\}|},
$$

and

$$
p_\phi = \frac{2f(1-f)}{nt^{2(1-f)}} - \frac{3f^2}{n^2t^{2(1-f)}},
$$

which ultimately lead to the effective equation of state $w_\phi = \frac{p_\phi}{\rho_\phi}$ corresponding to the scalar field as,

$$
w_\phi = a_0^3\left(\frac{2nf(1-f) - 3f^2t^f}{3a_0^3t^2t^n - \rho_m^0 n^2 t^{2(1-f)} \exp\left(-\frac{3}{n}t^f\right)}\right).
$$

The above form state parameter $w_\phi$ appearing in (18), has been found in an earlier work \cite{22}, where, we just mentioned that it goes over to $-1$ value asymptotically. Here, our attempt is to analyze it’s behaviour in the interim region. For this purpose let us express the state parameter $w_\phi$ as a function of the red-shift parameter. For simplification, we choose $a_0 = 1$, without loss of generality. As a result, the constant $\rho_m^0$, appearing in equation (12) stands for the amount of matter density present in the Universe at $t = 0$. The red-shift parameter $z$ is defined as,

$$
1 + z = \frac{a(t_o)}{a(t)} = \exp\left[\frac{1}{n}(t_f^o - t_f^t)\right],
$$

where, $a(t_o)$ is the present value of the scale factor, while $a(t)$ is that value at some arbitrary time $t$, when the light was emitted from a cosmological source. Thus,

$$
t_f^t = t_f^o - n\ln(1+z).
$$

In view of equation (19), $w_\phi$ can now be expressed as,

$$
w_\phi = \left(\frac{2nf(1-f) - 3f^2t^f - n\ln(1+z)}{3f^2t^o - n\ln(1+z)} - \rho_m^0 n^2 t^o - n\ln(1+z)\right)\frac{2}{3}\frac{t^f}{\exp\left(-\frac{2}{3}t^f - n\ln(1+z)\right)}.
$$

For a graphical representation of the state parameter versus the red-shift parameter, we need to select a few parameters of the theory. Firstly, let us choose $f = 0.5$ to find $n$. The motivation of choosing the value of $f$ in the middle is simply to set a comfortable dimension of time for $n^2$ and to obtain a reasonably better form of the potential $V(\phi)$. Taking the present value of the Hubble parameter ($H_o^{-1}$) and the age of the Universe ($t_o$) as,

$$
H_o^{-1} = \frac{9.78}{h} \text{Gyr}, \quad t_o = 13 \text{Gyr},
$$

and with, $h = 0.65$, $n$ can be found from the ansatz (9) as

$$
n = 0.5\left(\frac{H_o^{-1}}{\sqrt{t_o}}\right) = 2.08.
$$

To estimate the amount of matter density present at the time $t = 0$, we take the present value of the matter density parameter $\Omega_{m0} = 0.26$, and so in view of solution (12),

$$
\Omega_{m0} = \frac{\rho_{m0}}{\rho_{co}} = \rho_m\left(\frac{H_o^{-2}}{3}\right) = 0.26,
$$
Figure 1: State parameters \( w_\phi(z) \) has been plotted against the red-shift parameter \( z \), (with, \( a_0 = \kappa^2 = 1, f = 0.5, h = 0.65, t_0 = 13 \text{ Gyr} \)). Smooth double crossing of the Cosmological constant barrier is observed at sufficiently later epoch, \( z \approx 1.8 \) from above and \( z \approx 0.44 \) from below.

Figure 2: The coupling parameters \( g(\phi) \) clearly demonstrates a smooth transient phantomization.

where, \( \rho_{mo} \) and \( \rho_{co} \) are the present values of the matter density and the critical density respectively. Thus, we find,

\[
n^2 \rho_{mo}^0 = 2.72.
\]

With these values we have plotted the state parameter \( w_\phi(z) \), the coupling parameter \( g(\phi) \), the Hubble parameter \( H(z) \) and the potential \( V(\phi) \), noting that in the present model we have started from the value of the scale factor \( a = 1 \), at \( t = 0 \), corresponding to which the red-shift parameter is approximately \( z = 4.66 \).

Figures 1 and 2 clearly demonstrate a smooth double crossing, one from above at \( z \approx 1.8, t \approx 2.2 \text{ Gyr} \) and other from below \( z \approx 0.44, t \approx 8.2 \text{ Gyr} \). Since there is a dynamical transition of the equation of state from below (phantom-like) to \( w_\phi > -1 \), so it avoids big-rip singularity [11] and also prevents undesirable quantum mechanical negative energy graviton and phantom particle production [12]. As a result both the classical and quantum mechanical stability are guaranteed. Thus, the present model does not exhibit any pathological behaviour either at classical or at quantum mechanical level. Andrianov et-al [7] believed that such double crossing is possible for an ordinary lagrangian with minimally coupled scalar field and is depicted through a minima and maxima of the Hubble parameter. Figure 3 however shows no such indication. On the other hand, Cannata and Kamenshchik [8] could demonstrate such crossing with some exceptional initial condition together with a cusp in the form of the potential. Neither do we require such initial condition nor do the potential in Figure 4 shows such cusp. Rather it has been shown in an earlier work [22] that the solution in the form given by equation (10) has a scaling behaviour which has got a natural exit from it exhibiting asymptotic de-Sitter expansion, with cosmological evolution. Further, the scalar field carrying a potential as in (14) has been found to behave as a tracker field, which naturally
Figure 3: The Hubble parameters $H(z)$ does’nt show any maxima or minima to indicate phantomization or dephantomization as demonstrated by Andrianov-et al. [7].

Figure 4: There is no indication of any cusp in the form of the Potential $V(\phi)$ as required by Cannata and Kamenschik, for crossing the $w = -1$ line [8].
avoids the coincidence problem. At the end, it is better to see how far our model fits with the standard $\Lambda CDM$ model. In connection with $\Lambda CDM$ model, the luminosity-redshift relation is,

$$H_0dL = (1 + z) \int_0^z \frac{dz}{\sqrt{0.74 + 0.26(1 + z)^3}},$$

while in the present model it is,

$$H_0dL = \frac{(1 + z)}{\sqrt{t_0}} \int_0^z [\sqrt{t_0} - n \ln(1 + z)]dz,$$

with, $t_0 = 13$ Gyr., and $n = 2.08$. Now the combined plot of the two models, is shown in figure 5. It is really a pleasure to observe that the two models fit perfectly up-to the red-shift value $z = 3$ and thus they are indistinguishable. However, for $z > 3$, there appears slight discrepancy. The curve (blue) corresponding to $\Lambda CDM$ model, overtakes one related to the present model (red). For further clarification, we also plot the distance modulus - redshift graph in figure 6. The relation is given by,

$$m - M = 5\log_{10}(\frac{dL}{Mpc}) + 25,$$

where, $m$ and $M$ are the apparent and absolute bolometric magnitudes respectively. Since we have $H_0dL$ instead, so the relation modifies a little bit and becomes,
\[ m - M = 5 \log_{10}(DL) + 31, \]

where, \( DL = H_0 dL \). Figure 6 shows almost a perfect fit between the two models. This creates some problem in terms of identifiability between the two. The only way out is to determine independently the value of the state parameter and to see if it is dynamical and has gone through a very recent crossing.

Finally, we should check if the ongoing model is stable, by satisfying the positivity criteria of the velocity of sound propagating in the medium under consideration. The velocity of sound in the medium (here it corresponds to the scalar field \( \phi \)) is given by \( 25 \).

\[ c_s^2 = \frac{\partial p}{\partial \rho} = \frac{p_{\phi,X}}{\rho_{\phi,X}} \]

Here, comma followed by \( X \) stands for derivative with respect to \( X \). In the ongoing model it is

\[ c_s^2 = \frac{g(\phi)F_X}{g(\phi)F_X + 2XF_{XX}}. \]

As for the present model, \( F(X) = X = \frac{1}{2} \dot{\phi}^2 \), so, \( F_X = 1 \) and \( F_{XX} = 0 \). Thus, \( c_s^2 = 1 \) always, except at the two points of phantomization and de-phantomization \( (w_\phi = -1) \), where it is apparently undefined. However, one can trivially check that in the limit \( c_s^2 = 1 \), always. Thus, stability of the model has been established unambiguously.

3 Concluding remarks

In summary, we have been able to demonstrate that smooth double crossing of the phantom divide line is indeed possible in a minimally coupled single scalar field model in view of a simplest type of k-essence or age-old Brans-Dicke lagrangian. The phantomization is found to be a transient phenomena and thus the model does not suffer from any sort of pathological behaviour and also it is stable as the velocity of sound propagating in the medium \( \phi \) is always positive. The present best fit shows a crossing at \( z \approx 0.2 \) \( 24 \), while, it is at \( z \approx 0.44 \) in our model. However, we understand that such results are essentially model dependent, but then, even if such value of the red-shift parameter corresponding to the recent crossing is established by some other method, in a model independent way, we think, one can fit the presently available data, with some intelligent choice of the parameters \( n \) or \( f \) of the theory. The luminosity-redshift and distance modulus-redshift curves show perfect fit between the present and the \( \Lambda \)CDM models. Thus the two models can be distinguished only by measuring in a model independent way, the present value of the state parameter together with observing if it is dynamical and has gone through a recent crossing of the phantom divide line. As, mentioned earlier, the coupling parameter \( g = g(\phi) \) has a Brans-Dicke origin, since, \( \omega(\phi) = g(\phi) \dot{\phi}. \) Brans-Dicke theory leads to Einstein’s theory in the limit \( \omega \rightarrow \infty \) and so \( \omega \) is constrained by classical tests of general relativity. The light deflection and the time delay experiments demand \( \omega > 500 \), while the bounds on the anisotropy of the microwave background radiation demands \( \omega < 30 \), on the other hand. Since \( \omega \) in the present model grows with \( \phi \), so it passes the bounds required by microwave background radiation and, \( \omega \rightarrow \infty \) asymptotically leads to Einstein’s theory. Thus, we believe that the present work will give some relief from studying complicated cosmological models unnecessarily.

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