BDS triple-frequency carrier phase combination smoothing pseudo-range algorithm

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Abstract. The carrier phase smoothing pseudo-range algorithm can effectively reduce the influence of multi-path effect and observation noise on pseudo-range positioning and improve the accuracy of pseudo-range positioning. Aiming at the problem that the existing classic Hatch filter smoother is susceptible to the cumulative interference of ionospheric delay in the pseudo-range smoothing process, the ionospheric delay and observed noise of the original double-difference observation are further weakened by constructing the optimal Beidou triple-frequency linear combination. The triple-frequency double-difference carrier phase combination are used to smooth the pseudo-range combination to replace the original observations. Finally, four groups of Beidou triple-frequency measured data with different baseline lengths, including 4m, 14.4km, 61.7km and 131.6km, are used to verify the algorithm. The theoretical derivation and effectiveness of the algorithm are verified by comparing and analyzing the smoothing effect of the algorithm in the following four groups of baseline conditions.

1. Introduction
Pseudo-range observations and carrier phase observations are the two most basic distance observations of GNSS receiver, which are complementary and have their own advantages and disadvantages [1]. Pseudo-range positioning has been applied as the basis of GNSS navigation and positioning due to its advantages such as fast positioning speed, easy implementation and no ambiguity. However, compared with the carrier phase observation, the pseudo-range observation’s total noise level (TNL) is two orders of magnitude higher, especially for the low-frequency part, which makes it far from meeting the requirements of high-precision positioning [2,3]. Moreover, previous studies have shown that, compared with GPS system, pseudo-range observations of BDS system, especially GEO satellite, are more seriously affected by multipath noise, and the peak-to-peak value of partial single-frequency multipath noise can even reach 4 meters.

In order to improve the accuracy of pseudo-range positioning, a lot of research has been done by the predecessors, and the accuracy of pseudo-range positioning is improved by adopting various combinations and filtering smoothing methods, among which the carrier phase smoothing pseudo-range algorithm based on Hatch filtering is the most classical and widely used one. Based on dual-frequency ionospheric-free combination, this algorithm extracts both the low frequency band of pseudo-range observations with lower accuracy through arithmetic average operation, and the high frequency band of carrier phase observations with higher accuracy by time difference operator. The pseudo-range observation values can be effectively smoothed by using the difference between carrier phase epoch elements [6-10].
2. The basic principle of triple-frequency carrier phase combination smoothing pseudo-range

The original BDS triple-frequency observations can weaken or eliminate most of the observation errors through double-difference and triple-frequency linear combination, providing better preconditions for pseudo-range smoothing [11-13]. The algorithm structure used in this paper is shown in Figure 1. Firstly, several sets of Beidou triple-frequency observations with different baseline lengths are collected. Secondly, the original pseudo-range observations and the carrier phase observations are combined with the triple-frequency double-difference linear combination to obtain the triple-frequency double-difference carrier phase observation and pseudo-range observation. Finally, the final pseudo-range smoothing solution was obtained with the help of the classical Hatch filter.

![Figure 1. Basic principle of triple-frequency carrier phase combination smoothing pseudo-range](image)

1. Triple-frequency double-difference observation combination

The original BDS observation can basically eliminate the satellite clock error and receiver clock error through station and satellite double difference, and greatly attenuate the atmospheric propagation errors such as ionospheric delay and tropospheric delay. At the moment k of epoch, considering a triple-frequency GNSS receiver whose triple-frequency double-difference pseudo-range observation and triple-frequency double-difference carrier phase observation can be expressed as:

\[
P_{i,k} = \rho_k + T_k + I_{i,k} + \varepsilon_{P,i,k}
\]

\[
\phi_{i,k} = \lambda_i \phi_{i,k} = \rho_k + T_k - I_{i,k} + \lambda_i N_i + \varepsilon_{\phi,i,k}
\]

For convenience of expression, the double-difference symbol as well as the receiver and satellite identification are omitted in the above two formulas, where \(i = 1, 2, 3\), represent the three frequency bands B1, B2 and B3 of the BDS system; \(\lambda_i\) corresponds to the wavelength of three frequency bands; \(P_{i,k}\) represents the double-difference pseudo-range observation of the \(i\)-frequency band in the epoch \(k\) time; \(\phi_{i,k}\) represents the double-difference carrier phase observation in meters of the \(i\)-frequency band in the epoch \(k\) time; \(\phi_{i,k}\) is the corresponding double-difference carrier phase observation in cycles; \(\rho_k\) represents the geometric distance from the satellite to the receiver; \(T_k\) is the double difference tropospheric delay; \(I_{i,k}\) is the double difference ionospheric delay; \(N_i\) is the double difference ambiguity; \(\varepsilon_{P,i,k}\) is the double-difference carrier phase observation noise of the \(i\)-frequency band in the epoch \(k\) time; \(\varepsilon_{\phi,i,k}\) is the double-difference pseudo-range observation noise of the \(i\)-frequency band in the epoch \(k\) time.

By linearly combining the carrier phase observations of the three frequency bands, the combined triple-frequency carrier phase observations in cycles can be obtained:

\[
\phi_c = j \phi_1 + k \phi_2 + l \phi_3
\]

In the formula, \(a, b,\) and \(c\) are triple-frequency linear combination coefficients, and \(a, b,\) and \(c\) should be integers to guarantee the whole-cycle characteristic of the combined ambiguity.

Correspondingly, the combined triple-frequency carrier phase observations in meter can be expressed as:

\[
\phi_c = \frac{j f_1 \phi_1 + k f_2 \phi_2 + l f_3 \phi_3}{j f_1 + k f_2 + l f_3}
\]
The corresponding combined triple-frequency pseudo-range observations can be expressed as:

\[ P = \frac{if_1 P_1 + kf_2 P_2 + if_3 P_3}{if_1 + kf_2 + if_3} \quad (5) \]

Theoretically, when the combination coefficient satisfies the integer condition, the BDS triple-frequency observations can form countless combinations satisfying the integer-cycle characteristics of ambiguity. However, the optimal linear combination can further weaken the ionospheric delay, the tropospheric delay, and the observed noise, thereby obtaining a combined carrier phase observation with higher accuracy for smoothing the pseudo-range. Literature [13–15] gives a number of optimal combinations for different positioning conditions. The combination coefficients selected in this experiment are [1, 4, -5].

2.2. Carrier phase smoothing pseudo-range algorithm

As can be seen from the above, the combined triple-frequency carrier phase observations at epoch \( k \) can be expressed as:

\[ \phi_{c,k} = \lambda_c \varphi_{c,k} = \rho_{c,k} + T_{c,k} - I_{c,k} + \lambda_c N_c + \varepsilon_{\phi,c,k} \quad (6) \]

\[ P_{c,k} = \rho_{c,k} + T_{c,k} + I_{c,k} + \varepsilon_{P,c,k} \quad (7) \]

It should be noted that it is assumed here that the receiver continuously tracks satellite signals and does not have a cycle slip. In this case, the ambiguity does not change once it is fixed, and the carrier phase combination observation can be eliminated by making a difference between adjacent epochs. Moreover, because the variation of ionospheric delay between adjacent epoch is very small and negligible, that is, \( I_{c,k} - I_{c,k-1} \approx 0 \). Therefore, the above two equations can be differentiated between adjacent epoch to obtain the following solution:

\[ \phi_{c,k} - \phi_{c,k-1} = \rho_{c,k} - \rho_{c,k-1} + \varepsilon_{\phi,c,k} - \varepsilon_{\phi,c,k-1} \quad (8) \]

\[ P_{c,k} - P_{c,k-1} = \rho_{c,k} - \rho_{c,k-1} + \varepsilon_{P,c,k} - \varepsilon_{P,c,k-1} \quad (9) \]

By solving the above two equations simultaneously, the following equation can be obtained:

\[ P_{c,k} - P_{c,k-1} = \phi_{c,k} - \phi_{c,k-1} + \varepsilon_{\phi,c,k} - \varepsilon_{\phi,c,k-1} \quad (10) \]

Since the variation of observation error between adjacent epoch is also relatively small, the above equation can be approximately expressed as:

\[ P_{c,k} - P_{c,k-1} \approx \phi_{c,k} - \phi_{c,k-1} \]

\[ P_{c,k} \approx P_{c,k-1} + \phi_{c,k} - \phi_{c,k-1} \quad (11) \]

\[ P_{c,k} \approx P_{c,k-1} + \phi_{c,k} - \phi_{c,k-1} \quad (12) \]

The combination symbol \( c \) is omitted, and the pseudo-range smoothing value at epoch \( k \) obtained by the combined triple-frequency double-difference carrier phase observation can be expressed as:

\[ \tilde{P}_k = P_{c,k} - \phi_1 - \phi_{c,k-1} \quad (13) \]

There are \( k \) epochs observations, where \( \tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_k \) are the pseudo-range smoothing values of each epoch, and then at the epoch \( k \), the pseudo-range smoothing values can be expressed as:

\[
\begin{aligned}
\tilde{P}_1 &= P_1 + \phi_1 - \phi_1 \\
\tilde{P}_2 &= P_2 + \phi_1 - \phi_2 \\
&\quad \vdots \\
\tilde{P}_k &= P_k + \phi_1 - \phi_{c,k-1}
\end{aligned}
\quad (14) \]

Add the above equations and let \( \tilde{P}_k = \frac{1}{k} \sum_{i=1}^{k} P_i \), you can get:

\[ \tilde{P}_k = P_k + \phi_1 - \frac{1}{k} \sum_{i=1}^{k} \phi_i \quad (15) \]
Assume that the standard deviation of pseudo-range observation noise is $\sigma_p$, the standard deviation of carrier phase observation error is $\sigma_\phi$, and the standard deviation of pseudo-range smoothing value error is $\tilde{\sigma}_p$. Assuming the pseudo-range observation accuracy of each epoch is the same, and the carrier phase observation accuracy is the same, and the two are unrelated. According to the error propagation law, it can be obtained:

$$\sigma_{\tilde{p}}^2 = \frac{1}{k} \sigma_p^2 + \frac{k+1}{k} \sigma_{\phi}^2 \tag{16}$$

Since $\sigma_\phi$ is much smaller than $\sigma_p$, so $\sigma_{\tilde{p}} = \frac{1}{\sqrt{k}} \sigma_p$. That is to say, the accuracy of pseudo-range observation value after smoothing is improved by $\sqrt{k}$ times.

Substitute in the Hatch filtering formula, the smoothing formula that smoothes pseudo-range observations with the help of carrier phase observations can be expressed as follows:

$$P_k = \frac{1}{M} P_k + \frac{M-1}{M} [P_{k-1} + \phi_k - \phi_{k-1}] \tag{17}$$

Combined with the algorithm architecture in this paper, $\phi_k$ and $\phi_{k-1}$ in equation (16) are the combined observation values of triple-frequency double-difference carrier phase at epoch $k$ and $k-1$ respectively, rather than the original carrier phase observations; $M$ is the time smoothing factor, i.e, the size of the smoothing window, whose value is determined by the filter smoothing time sample interval, which is generally between 20-100 epochs.

3. Experiment and analysis

Four sets of Beidou triple-frequency observations with different baseline lengths were collected in this paper, as is shown in table 1.

| Data sets | Baseline | Date         | Receiver | Location | Intervals | Observed epochs |
|-----------|----------|--------------|----------|----------|-----------|-----------------|
| A         | 4m       | 20160606     | Trimble  | Australia| 1s        | 2880            |
| B         | 14.4km   | 20150409     | CHCNAV   | Hang Zhou| 1s        | 1194            |
| C         | 61.7km   | 20131214     | SINO     | Shang Hai| 1s        | 1440            |
| D         | 131.6km  | 20160906     | Trimble  | Australia| 1s        | 1440            |

Data sets A-D represent the four groups of Beidou triple-frequency observations from short baseline to medium-long baseline. According to the algorithm structure in the paper, the first 100 epoch elements were selected in the experiment for pseudo-range smoothing. Since the original pseudo-range value is large, in order to show the smoothing effect of the proposed algorithm, the pseudo-range difference between the epochs is used in the result graph rather than the combined original pseudo-range observation.

![Figure 2(a). The pseudo-range difference of 4m baseline before smoothing](image1)

![Figure 2(b). The pseudo-range difference of 4m baseline after smoothing](image2)
Figure 3(a). The pseudo-range difference of 14.4 km baseline before smoothing

Figure 3(b). The pseudo-range difference of 14.4 km baseline after smoothing

Figure 4(a). The pseudo-range difference of 61.7 km baseline before smoothing

Figure 4(b). The pseudo-range difference of 61.7 km baseline after smoothing

Figure 5(a). The pseudo-range difference of 131.6 km baseline before smoothing

Figure 5(b). The pseudo-range difference of 131.6 km baseline after smoothing

Figure 2-5 figure respectively give the difference between adjacent epoch of the combined pseudo-range before and after smoothing under different baseline conditions. C01 satellite was selected as the reference satellite, and double-difference was made with C02, C03 and C04 satellites that were always visible, and then triple-frequency linear combination was carried out. Finally, the combined double-difference carrier phase observations were used to smooth the combined double-difference pseudo-range observations. As can be seen from the figure, the pseudo-range smoothing needs about 20 epochs to complete under different baseline conditions. Figure 2(b) and Figure 3(b) represent the smoothing effect under short baseline conditions. Under the condition of short baseline, the...
ionospheric delay residual can be basically eliminated by the double difference of observations, so the smoothing result is also better. The difference between the pseudo-range smoothing solution in the adjacent epoch is basically controlled in the interval [-5, 5]. As the baseline length becomes longer, the atmospheric delay residuals such as ionospheric delay cannot be eliminated by double difference, so pseudo-range smoothing is also affected. Figure.4 (b) shows that under the baseline condition of 61.7km, the observed values of C03 and C04 are well smoothed, while the observed values of C02 are poorly smoothed. Figure.5 (b) shows that the smoothing effect is poor at 131.6km baseline, especially for C02 satellite.

4. Conclusion
By analyzing the principle of Beidou triple-frequency linear combination and the principle of carrier phase smoothing pseudo-range algorithm based on Hatch filtering, a smoothing method using triple-frequency double-difference carrier phase combination observations to replace original carrier phase observations was presented, which effectively solves the problem that the traditional method is greatly affected by ionospheric delay interference in the process of calculation. Meanwhile, the pseudo-range smoothing effect under different baseline conditions was given. The results show that the algorithm can achieve pseudo-range smoothing within 20 epochs, and the pseudo-range smoothing effect decreases with the increase of the baseline length.

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