A simple picture of intermittency in turbulent flows places rare, hot regions that dissipate energy inside a cold laminar sea. As the hot regions evolve, they maintain a clustered structure, and the dissipation, occurring at small scales, remains correlated at large ones. For instance, examining the sky at night, one sees stars, galaxies and clusters of galaxies against a dark background. Many striking examples of turbulent intermittency occur in astrophysical, space or laboratory plasmas, such as flaring events in the solar corona, magnetic substorms, bursty bulk flows, auroral emissions, turbulence in the solar wind, or bursts observed in RFX and RFP experiments. Important parameters include the Reynolds number(s), $R = V l_0 / \nu$ (or $R_m = V l_0 / \eta$, etc.), where $\nu$ is the viscosity, $\eta$ is the magnetic diffusivity, and $V$ is the velocity difference over the integral scale $l_0$. For increasing $R$, characteristic widths of the dissipating regions decrease, while their intensity increases. Thus turbulence becomes an “on/off” phenomena as the appropriate Reynolds number(s) become large.

The controversial hypothesis that turbulent intermittency may be a manifestation of self-organized criticality (SOC) has been discussed by Bak and others. In this scenario, intermittent energy dissipation is a stick-slip or threshold process. Each slip can trigger further slips – either through short or long range interactions. Eventually, a regime materializes where sparse, sporadic avalanches of intense energy dissipation interrupt laminar regions of space-time which rest near static equilibrium with little dissipation despite the continuous, global input of energy.

Up to now, SOC has mostly been studied with vividly plain models, such as the original Bak-Tang-Wiesenfeld sandpile (BTW), that ignore many features of hydrodynamic, plasma or other kinds of turbulence. Rather, they generate patterns of rapid energy dissipation in space and time, which is the hallmark of intermittency. On the other hand, most studies of turbulence examine structure functions and multiscaling phenomena, and/or nonlinear instabilities and coherent structures, etc., all of which are associated with intermittency but do not directly characterize the bursts of energy dissipation in space and time. As a result, some comparisons that have been made are superficial and should be made more definitive. Further, one argument used so far to distinguish SOC from turbulence is misleading and erroneous.

While BTW and other SOC models exhibit a broad distribution of avalanche sizes and durations, which are comparable to e.g. solar flare data, a marked difference has been noted regarding the time intervals between bursts. For instance, Boffetta et al. found that the distribution of times between flares exhibits power law statistics, while intervals between subsequent avalanches in BTW are (approximately) Poissonian. Further, they and other groups have found that shell or reduced magnetohydrodynamics (MHD) models gave a better description of the waiting time statistics, and some used this to distinguish SOC from turbulence, or to question the applicability of the SOC paradigm for magnetically confined plasmas in thermonuclear research.

We make three key points. First, intermittent bursts can never be detected, nor distinguished from the background, at arbitrarily low thresholds. For instance, the studies above comparing reduced MHD or shell models with flares or bursts in man-made plasmas use a threshold for defining bursts. Such a threshold is realistic because the emission associated with e.g. flares decays slowly after a local peak, allowing overlaps with subsequent peaks. Although a threshold is unavoidably connected to the precise definition of events, robust features may be observed with rescaled distributions measured at different thresholds. For such time series, event durations and quiet times between them are measured on...
and its next fall below constitute measured durations. Additive noise, respectively. Shaded regions indicate a detection threshold. Dark (red online) areas and the black line represent the actual signal and the signal with some number of topplings, \( n_c = 10 \) (yellow, dashed line) and \( n_c = 100 \) (blue, dot-dashed line); any signal below \( n_c \) is considered undetectable. Intervals between a rise of the signal above \( n_c \) and its next fall below constitute measured durations \( t_d \) of events, followed by a “quiet time” \( t_q \) until the next rise. The “waiting time” between two consecutive rises is \( t_w = t_d + t_q \).

A single clock with equal precision. Thus one can consider the hypothesis that the sequence of bursts arises from a single avalanche observed at finite detection threshold. The bursts within an avalanche in a SOC system can be expected to be correlated in space and time, being part of the same, critical process.

To test this idea, we study a BTW sandpile [14] in which the time unit is that of a parallel update step, and consider both slow driving (A) and running sandpile [20, 30, 31] (B) conditions. In the time series of the activity, \( n(t) \), bursts are defined as consecutive intervals during which \( n(t) > n_c \), where the detection threshold for events is \( n_c \geq 0 \) (see Fig. 1). A data analysis technique analogous to that recently developed by Baiesi et al. [28] to examine inter-occurrence statistics of flares provides a direct comparison between results from numerical simulations of BTW and solar flares. It turns out that BTW shares several features with the intermittent statistics of flares, as shown below.

The BTW sandpile consists of a \( L \times L \) lattice with a discrete number \( z_i \) of sand grains occupying each site \( i \). We study two versions: (A) in the slow driving limit, a grain is added to the pile at a randomly chosen location when the previous \( n_c = 0 \) avalanche ends. The durations \( t_d \) and quiet times \( t_q \) then refer to intervals between local peaks within each avalanche and statistics are obtained over many avalanches. In the second case (B), one grain of sand is dropped every \( \Delta T \) update steps at a randomly chosen site. In both cases, at each update step, \( t \), all sites that exceed a threshold for stability, \( z_i > z_c = 3 \), topple in parallel by distributing a single grain of sand to each of their four nearest neighbors or, for boundary sites, over the edge of the lattice. Taken as the instantaneous dissipation signal, the activity \( n(t) \) is the number of unstable sites toppling at each parallel update step. In model B, if \( \Delta T > \langle t_d \rangle L \), the sequence of topplings is also interrupted by instances where the activity completely stops \( (n(t) = 0) \) [31]. The quantity \( \langle t_d \rangle L \) is the average duration of avalanches on a lattice of scale \( L \) in the stationary state of model A. For both A and B, consecutive stopping points separated by intervals where \( n(t) > 0 \) delimit \( n_c = 0 \) avalanches. The time series of Model B has similar character to the solar flare data studied in Ref. [28], with a broad distribution of events that exceed each threshold \( n_c \), albeit with a finite-size cutoff curtailing the power-law tail observed in Fig. 2a of Ref. [28].

Consider an observer who measures the global activity sequence with a finite error, so that the time series she records is \( n_{\text{obs}}(t) = n(t) + \eta(t) \). For instance, let \( \eta(t) \) be an independent random number uniformly distributed between 0 and 15. The effect of this noise is shown in Fig. 1 for model B. One way for the observer to separate the signal from the noise is to increase her threshold for detecting events, and coarse-grain her unit of measurement. In observing natural phenomena, such as flares, these detection thresholds are an intrinsic and unavoidable part of the measurement. Hence, we study the original time series together with a finite threshold \( n_c > 0 \) (e.g. \( n_c = 100 \) as in Fig. 1) to distinguish bursts, considering all instances with \( n(t) \leq n_c \) to have no activity. For sufficiently large \( n_c \), the event statistics that the observer measures (at large times) are the same as the actual statistics without noise.

As Fig. 2 shows, on increasing the threshold \( n_c \) from zero, the distribution of quiet times \( t_q \) for model B switches from an (approximately) exponential distribution to a power law,

\[
P_{\text{quiet}}(t_q) \sim t_q^{-\gamma_q} \quad \text{with} \quad \gamma_q^{\text{BTW}} = 1.67 \pm 0.05. \tag{1}
\]

For model A one observes an even cleaner and broader scaling regime, with the same \( \gamma_q^{\text{BTW}} \). In model A, the power law tail arises from the correlations of bursts within each avalanche. Further, the Abelian property of BTW assures that the power law behavior of quiet times in model B cannot be due to overlapping avalanches. Thus, the natural introduction of detection thresholds leads to the discovery of a hierarchical sequence of correlated bursts (or sub-avalanches) within a large avalanche.

A similar data analysis by Baiesi et al. [28] for flares, detected as intervals during which the emission intensity in the GOES time series exceeds \( I \), used a particularly simple scaling ansatz for the quiet time distribution,

\[
P_{\text{quiet}}(t_q|I) = \frac{1}{\langle t_q \rangle I} f_{\text{flare}} \left( \frac{t_q}{\langle t_q \rangle I} \right). \tag{2}
\]
De Menech [33] found multiscaling.

In the latter case, Lübeck and Usadel [32] determined a critical exponent at high thresholds, \( \gamma_{BTW} \). This suggests a critical exponent at high thresholds, \( \gamma_{BTW} = 1.67(5) \) in Eq. (1).

Event durations, \( t_d \), observed at different thresholds in BTW and for flares occurring during solar minimum, are shown in Fig. 3 of Ref. [28]. One parallel update step in BTW has been divided by the average quiet time \( t_q \) at each threshold, and the distributions rescaled to preserve normalization. BTW and flare data are similar in the intermediate regime. The former are shifted up by one unit on the log scale.

The inset of Fig. 4 shows how the apparent power law behavior for burst durations in BTW changes with increasing threshold \( n_c \). At sufficiently large \( n_c \), a plateau appears in the function \( t_q^{5/3} P_{dur}(t_d) \). This suggests a critical exponent at high thresholds, \( \gamma_{dur} = 2.0 \pm 0.1 \) [28], while this exponent the \( n_c \gg 1 \) bursts in BTW is smaller.

If the sequence of quiet times \( (t_q)_i \) are uncorrelated, the cumulative variable \( y_l(j) = \sum_{i=j-l}^{i=j} (t_q)_i \) exhibits diffusive behavior, with an average variance scaling as \( \sigma \sim \langle y^2 \rangle - \langle y \rangle^2 \sim t^H \), with \( H = 1/2 \) [34]. Our measurements of this quantity confirm that quiet time inter-
vals for BTW at different thresholds are uncorrelated, in all cases giving $H = 1/2$. However, at solar minimum, we get $H \simeq 0.62$ (see also [35]). Therefore, BTW does not reproduce correlations between quiet times for flares.

In fact, a variety of SOC models exhibit power laws in times between events. Previous analyses [36, 37, 38] using a detection threshold considered a completely different limit than that discussed here, namely that of an infinite time scale separation between the driving rate and the durations of events. Consequently, a threshold was imposed on a variable related to the total size or duration of each $n_e = 0$ avalanche, rather than its instantaneous dissipation. However, as explained earlier our limit of overlapping time scales for durations and quiet times may be the correct one to describe intermittency in turbulence. Indeed, cellular automata models of laboratory plasmas are running sandpiles; see e.g. Newman et al. [39]. Within this scheme, some works [20, 21], also using a threshold, have found power law quiet times for the Hwa-Kardar [30] running sandpile driven at a sufficiently high rate. This was claimed to be due to interactions between overlapping avalanches. In contrast, here we show that the BTW model, considering one avalanche at a time, generates a power law distribution of quiet times when a finite detection threshold is used. Its Abelian property assures that this correlation remains the same when the model is driven at a finite rate, in agreement with the results shown here. It is unlikely, though, that the Abelian property of BTW is essential to getting a scale free distribution of quiet times, although this remains to be clarified by studying other models in a similar way.

In conclusion, we have demonstrated that SOC remains a viable alternative for the explanation of intermittent dissipation in turbulence by comparing results from numerical simulations of a BTW sandpile with solar flare statistics. By including an inevitable detection threshold in the analysis of BTW, as well as allowing an overlap of time scales between burst durations and quiet times, qualitative correspondence is obtained for the power law statistics of inter-occurrence times for solar flares. Studies of more physically realistic SOC including detection thresholds for short time dissipation in the whole system may improve quantitative agreement.

M. P. thanks Peter Grassberger for comments on the manuscript. M. B. acknowledges support from an FWO post-doctoral position (Flanders). S. B. thanks the Perimeter Institute for its hospitality.

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[41] Error bars quoted here and elsewhere in the text represent our estimate of statistical and systematic errors associated with finite size effects.
[42] For flares, the distribution of burst durations is independent of threshold, but depends on the phase of the solar cycle.