ON THE STUDY OF THE SUBSTRUCTURE OF N-BODY SYSTEMS

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Statistical properties of nonlinear systems revealed in an unexpected way already in Fermi-Pasta-Ulam paper, are responsible for the appearance of substructures. We represent a method within S-tree diagram formalism for investigation of the substructures of N-body gravitating systems in a defined time interval. This approach enables one to consider the link between substructures of the system and its relative instability and is efficient for the study of the substructures in clusters of galaxies.

1 Introduction

The study of hierarchical distribution of galaxies is one important issues of observational cosmology since the properties of substructure of galaxy clusters have to carry information on the mechanisms of their formation and evolution.

Various methods are developed for the study of the substructures in galaxy clusters, e.g. two and higher-point correlation functions, Li statistics, topological measures, wavelets, etc which are using various descriptors and assumptions.

S-tree method is using both the positional and kinematical information about the system in self-consistent way, by means of the consideration of the geometrical properties of the phase space of N-body system and the definition of the criterion of boundness between particles. That method enabled to reveal the substructures in various galaxy clusters, i.e. Perseus, Virgo, Coma, Abell ENACS clusters, etc. The existing versions of the S-tree are dealing with the substructures at the fixed time.

In this paper we will study the behavior of substructure within certain time interval i.e. to try to trace the future and past evolution of the subsystems within that interval.

The approach is based on the generalized S-scheme, and therefore, first, we will briefly review the main steps of the approach.

2 N-body bounded systems: S-diagram method

The key idea of the method, as already mentioned above, is the introduction of the concept of the degree of boundness $\rho$.

Consider a set of $N$ points...
\[ X = \{x_1, \ldots, x_N\}, \]

the function \( P \)

\[ P : X \times X \to R_+ \quad \text{and} \quad \rho \in R_. \]

The basic definitions are as follows.

**Definition 1**
We say that \( \forall x \in X \) and \( \forall y \in X \) are \( \rho \)-bounded, if \( P(x, y) \geq \rho \).

**Definition 2**
We say that \( U \subset X (U \neq \emptyset) \) is a \( \rho \)-bounded subgroup, if:

1. \( \forall x \in U \) and \( \forall y \in \bar{U} \Rightarrow P(x, y) < \rho; \)
2. \( \forall x \in U \) and \( \forall y \in U \exists x = x_{i_1}, x_{i_2}, \ldots, x_{i_k} = y, \) that \( P(x_{i_l}, x_{i_{l+1}}) \geq \rho; \) \( \forall l = 1, \ldots, k - 1. \)

**Definition 3**
We say that \( U_1, \ldots, U_d \) is the distribution of the set \( X \) via \( \rho \)-bounded groups, if:

1. \( \bigcup_{i=1}^d U_i = X; \)
2. \( i \neq j \quad (i, j = 1, \ldots, d) \Rightarrow U_i \cap U_j = \emptyset; \)
3. \( U_i (i = 1, \ldots, d) \) is a \( \rho \)-bounded group.

The function \( P \). Different physical quantities can be assigned to the function \( P \) as listed in \ref{5}, such as the distances, the energy, the force, the perturbations of potential, momentum, and so on. For example, the choice of the mutual distance of the particles in order to define the subgrouping is equivalent to the corresponding correlation functions (spatial or angular, related with each other via the Limber equation), which is, obviously, a rather incomplete characteristic for that aim. It can be shown that, at least for astrophysical problems, among the most informative ones is the Riemannian curvature of the configuration space \ref{5}, which determines the behavior of close geodesics, as known from basic courses on classical mechanics \ref{10}.

So, by this, S-tree algorithm we obtain the substructure of the N-body system, i.e. the degree of boundness of each subgroup of particles, for any given function \( P \) and \( \forall \rho \). This found distribution will satisfy Definitions 1, 2 and 3. The final result can be represented either through tables or by tree-diagrams, graphs (S-tree).
The next important aspect is that the method allows to use simultaneously various boundness criteria between the particles of the system for fixed moment of time.

Consider the $N$-body system: $X = \{x_1, x_2, \ldots, x_N \}$ and $P_1, \ldots, P_H$ functions. Consider also $D_1, \ldots, D_H$ matrices, where $D_\alpha = (d^\alpha_{ij}), \alpha = 1, \ldots, H$; $i, j = 1, \ldots, N; \quad d^\alpha_{ii} = 0$. If $i \neq j \quad d^\alpha_{ij} = P_\alpha(x_i, x_j)$. Constructing matrix $D$ in the following way:

$$D = (\bar{d}_{ij}), \quad i, j = 1, \ldots, N,$$

where

$$\bar{d}_{ij} = (d^1_{ij}, d^2_{ij}, \ldots, d^H_{ij}), \quad i, j = 1, \ldots, N.$$

$D$ matrix contains the whole information about matrices $D_1, \ldots, D_H$. For any function $P_\alpha \ (\alpha = 1, \ldots, H)$ exists its own boundness vector $\rho_\alpha = (\rho_1^\alpha, \ldots, \rho_Q^\alpha)$. We construct for the $D$ matrix a vector of boundness

$$\bar{\rho}_D = (\bar{\rho}_1^D, \ldots, \bar{\rho}_Q^D)$$

and for $k = 1, \ldots, Q_D$: $\bar{\rho}_k^D = (\rho_1^k, \ldots, \rho_Q^k)$, where $k = 1, \ldots, Q$ and $\alpha = 1, \ldots, H$. Vector $\bar{\rho}_D$ is constructed by all possible combinations of components of $\bar{\rho}_k^D$ vector. For any function $P_\alpha \ (\alpha = 1, \ldots, H)$ we introduce "the degree of influence" $W_\alpha \in R^+$. The next steps are realized for fixed value of $\bar{\rho}_k^D$. We transit from the matrix $D$ to the matrix $D_u$ in a following way

$$D_u = (\bar{u}_{ij}); \quad i, j = 1, \ldots, N,$$

where $\bar{u}_{ij} = (u^1_{ij}, \ldots, u^H_{ij})$ and $u^\alpha_{ij} \ (\alpha = 1, \ldots, H$ defined as

$$u^\alpha_{ij} = \begin{cases} W_\alpha, & \text{if} \quad d^\alpha_{ij} \geq \rho^\alpha_k \\ 0, & \text{if} \quad d^\alpha_{ij} < \rho^\alpha_k \end{cases}$$

The following $D_v$ matrix we construct as

$$D_v = (v_{ij}); \quad i, j = 1, \ldots, N, \quad v_{ij} = \sum_{\alpha=1}^{H} u^\alpha_{ij}.$$

Defining $\bar{\mu} = (\mu_1, \ldots, \mu_S)$, the vector of boundness for the $D_v$ matrix and, using the $S$-method for $D_v$ and for any given component of $\bar{\mu}$ vector, we obtain the distribution of the initial system on the bounded subgroups. Note finally, that according to the $S$-generalized scheme with fixed values of $\bar{\rho}_k^D$ and $\mu_l$, we obtain the distribution of system on $\bar{\rho}$-bounded subgroups, where $l = 1, \ldots, S; \quad \bar{\rho} = (\bar{\rho}_1, \mu_l)$. Consider a dynamical time interval or including $z$ dynamical interval $[t_a, t_b]$ of an initial system. Divide this interval on $H - 1$ equal parts.
For any \( t_\alpha; \alpha = 1, \ldots, H \) it is possible to find all sets of \((L_i(t_\alpha), V_i(t_\alpha))\), and where \( i = 1, \ldots, N; \quad L_i, V_i - \) coordinates and velocities of \( i \)-th particle. Consider the \( P \) function at different moments of time. Represent the function \( P_1, \ldots, P_H \) as \( P_1 = P(t_1) = P(t_\alpha), \quad P_2 = P(t_2), \ldots, P_H = P(t_H) = P(t_b) \).

As in the previous section, we will obtain \( D \) and \( D_u \) matrices.

For \( W_\alpha = 1, \alpha = 1, \ldots H, W_\alpha = 1. \) For the matrix \( D_v \) and \( \mu \in R^+ \) using the \( S \)-tree method we obtain the splitting of the system into \( \mu \)-bounded subgroups within the about time interval \([t_a, t_b] \).

As a prefered floor of distribution \( \mu_\ast \) we chose the larger components of the vector \( \bar{\mu} \), because \( \bar{\mu} \)-connection is a quantitative estimation of the sequence of numbers, where

\[
u_{ij}^\alpha = 1, \quad i, j = 1, \ldots, N, \quad \alpha = 1, \ldots, H
\]

After splitting the system into bounded groups by the suggested model, \( \Phi/N \) value may present a define physical interest, where \( \Phi - \) is number of those particles, which are not single in their groups \( \mathbb{C} \).

Consider two different systems \( X_1, X_2 \) with \( N_1 \) and \( N_2 \) particles. For both systems a distribution is analyzed, \([t_1^a, t_1^b] \) and \([t_2^a, t_2^b] \) intervals have to be considered in a self-consistent way. The difference of \( \Phi_1/N_1 \) and \( \Phi_2/N_2 \) can be considered as a criterion of relative instability of these systems. Note, that the presented approach allows to link directly the substructure properties and the relative instability of the N-body systems.

In order to apply this scheme for physical systems the correspondence between the chosen time intervals \([t_1^a, t_1^b] \) and \([t_2^a, t_2^b] \) have to be defined taking into account the formulation of the criterion of relative instability for systems \( X_1 \) and \( X_2 \).

3 Conclusion

The study of the substructure of nonlinear N-body systems is an interesting theoretical problem, as well has a direct link with the astrophysical systems. Particularly, the S-tree approach had proved its practical efficiency in the study of the substructure of clusters of galaxies. Particularly, the subgroups of the Coma cluster detected by S-tree \( 8 \) have been later studied observationally and certain predicted trends in the properties of galaxies of one of subgroups (subgroup 2) have found their apparent confirmation \( 12 \). The further development of the method therefore, seems of particular interest.

Here we proposed a generalization of the approach enabling the investigation of evolution of the substructuring properties of a nonlinear system within a defined time interval. Application of this scheme to the clusters of galaxies
can lead to a valuable insight on the evolution and maybe on the origin of the substructures.

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