Relations between inclusive decay rates of heavy baryons

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Abstract

The dependence of inclusive weak decay rates of heavy hadrons on the flavors of spectator light quarks is revisited with application to decays of charmed and $b$ hyperons. It is pointed out that the differences in the semileptonic decay rates, the differences in the Cabibbo suppressed decay rates of the charmed hyperons, and the splitting of the total decay rates of the $b$ hyperons are all related to the differences in the lifetimes of the charmed hyperons independently of poor knowledge of hadronic matrix elements. The approximations used in these relations are the applicability of the expansion in the inverse of the charmed quark mass and the flavor SU(3) symmetry.
1 Introduction

The problem of unequal inclusive weak decay rates of hadrons containing a heavy quark ($c$ or $b$) attracts considerable experimental and theoretical interest ever since the first experimental indication [1] of substantially different lifetimes of the charmed $D^0$ and $D^\pm$ mesons. The total weak decay rates of charmed hadrons are presently known to be vastly different, with the ratio of the longest known lifetime to the shortest: $\tau(D^\pm)/\tau(\Omega_c) \sim 20$, while the differences among the $b$ hadrons are much smaller. The relation between the magnitude of the lifetime differences in the charmed hadrons versus that in the $b$ hadrons reflects the fact that these effects, associated with the light quark/gluon “environment” of the heavy quark $Q$ in a hadron, vanish as an inverse power of the heavy mass $m_Q$, so that in the limit $m_Q \to \infty$ the ‘parton’ picture sets in, where the inclusive decay rate of a heavy hadron is given by that of an isolated heavy quark.

Thus the problem of the differences in the rates can be approached theoretically in the limit of large $m_Q$ in terms of a systematic expansion [2 -6] in $m_Q^{-1}$ for the decay rates of the hadrons. The leading term in this expansion is the ‘parton’ decay rate $\Gamma_{\text{part}} \propto m_Q^5$, which sets the overall scale for the decay rates of hadrons containing the quark $Q$ and which does not depend on the specifics of the spectator light quarks (in baryons) or the antiquark (in mesons). The first non-perturbative term, suppressed with respect to the leading one by $m_Q^{-2}$, arises due to the kinetic energy of the heavy quark in a hadron (time dilation effect) and due to the chromomagnetic interaction of the heavy quark [3]. This term generally gives a splitting of decay rates between heavy mesons and baryons, and between baryons of different spin structure, however it generates no splitting depending on the flavors of the spectator quarks or antiquarks (e.g. between different $D$ mesons, or within the triplet of baryons Λ$_c$, Ξ$^0_c$ and Ξ$^+_c$). The flavor dependence arises in the next order, $m_Q^{-3}$ relative to $\Gamma_{\text{part}}$, and can be interpreted as due to two mechanisms: the weak scattering (WS) and the Pauli interference (PI). The weak scattering corresponds to a cross-channel of the decay, generically $Q \to q_1 q_2 \overline{q}_3$, where either the quark $q_3$ is a spectator in a baryon and can undergo a weak scattering off the heavy quark: $q_3 Q \to q_1 q_2$, or an antiquark in meson, say $\overline{q}_1$, weak-scatters (annihilates) in the process $q_1 Q \to q_2 \overline{q}_3$. The Pauli interference effect arises when one of the final (anti)quarks in the decay of $Q$ is identical to the spectator (anti)quark in the hadron, so that an interference of identical particles should be taken into account. The latter interference can be either constructive or destructive, depending on the relative spin-color arrangement of the (anti)quark produced in the decay and of the spectator one, thus the
sign of the PI effect is found only as a result of specific dynamical calculation.

Although the WS and PI effects carry the relative suppression by $m_Q^{-3}$, they are found to be greatly enhanced by a large numerical factor, typically $16 \pi^2/3$, reflecting mainly the difference of the numerical factors in the two-body versus the three-body final phase space, which makes these effects overwhelmingly essential in the charmed hadrons, while greatly reduced in the heavier $b$ hadrons, as is confirmed by the experimental data. In effect, the contribution of the $O(m_Q^{-3})$ terms is significantly smaller than that of the $O(m_Q^{-1})$ terms in the charmed hadrons and is slightly smaller in the $b$ hadrons \cite{6}. In particular the $O(m_Q^{-2})$ terms split the decay rate of the $\Lambda_b$ from that of the $B$ mesons by only about $2\%$, which is by far insufficient to explain the current data \cite{7} on the ratio of the lifetimes: $\tau(\Lambda_b)/\tau(B) = 0.79\pm0.05$ (for a recent theoretical discussion see e.g. Ref. \cite{8}). A quantitative estimate of the effects of the $O(m_Q^{-3})$ terms runs into a problem of evaluating matrix elements over the hadrons of four-quark operators of the type $(\bar{Q} \Gamma_1 Q)(\bar{q} \Gamma_2 q)$ with certain spin-color matrix structures $\Gamma_1$ and $\Gamma_2$. Although simple estimates within a non-relativistic picture of the light quarks in hadrons (where these operators reduce to the density of the light quarks at the location of the heavy quark) allowed to correctly predict \cite{4, 5} the hierarchy of the lifetimes of the charmed hadrons, these estimates are obviously very unreliable for a quantitative description of the effect. Neither can this approach explain the ratio $\tau(\Lambda_b)/\tau(B)$ to be less than approximately 0.9. In view of this difficulty it is quite worthwhile to have a better understanding of the spectator flavor dependent differences of the rates in a possibly more model-independent way.

The purpose of this paper is to point out relations between some inclusive decay rates of the charmed and $b$ baryons in the $(\Lambda_Q, \Xi_Q)$ triplets, which do not require explicit knowledge of the matrix elements of the four-quark operators, and rely only on the general expansion in $m_Q^{-1}$ for the rates and on the flavor SU(3) symmetry. Certainly, the latter symmetry is known to be not very precise, however arguably the uncertainties due to the SU(3) violation are substantially less than those brought in by the current model assumptions about the hadronic matrix elements. Thus it may be expected that an experimental verification of these relations can serve as a test of the whole method based on the operator product expansion for weak decay rates. To an extent, such approach was pursued in the prediction \cite{9} of a significant enhancement of the total semileptonic decay rates of the $\Xi_c$ baryons over the same rate for the $\Lambda_c$ and even greater enhancement of this rate for $\Omega_c$. That analysis was further extended \cite{10, 11} to include the enhancement of the Cabibbo suppressed semileptonic decays of $\Lambda_c$. Although those papers used model considerations for the matrix elements of
the four-quark operators, in fact one can obtain, as shown in the present paper, quantitative results for the \((\Lambda_c, \Xi_c)\) triplet without resorting to models of quark dynamics in the baryons. Namely, it will be shown that the difference of the semileptonic rates within the \((\Lambda_c, \Xi_c)\) baryon triplet, both the dominant and the Cabibbo suppressed, as well as the difference of the non-leptonic Cabibbo suppressed decay rates, can all be expressed in terms of the total lifetime differences within the same triplet in a model-independent fashion, modulo the assumption of the flavor SU(3) symmetry. In addition the differences of the lifetimes within the triplet of the \(b\) baryons \((\Lambda_b, \Xi_b)\) are also expressed through the same differences for the charmed baryons, with a possible extra uncertainty due to the quark mass ratio \(m_b^2/m_c^2\).

Using the currently available data on lifetimes of the charmed hyperons, the discussed effects are estimated to be quite large. In particular, the conclusion of the previous analyses is confirmed that the semileptonic decay rates of the \(\Xi_c\) baryons should exceed by a factor 2 to 3 the same rate for the \(\Lambda_c\) hyperon. It is also found that the lifetime of the \(\Xi_b^-\) baryon can be longer than that of \(\Lambda_b\) by about 14\%, which is a very large effect for \(b\) hadrons.

2 Effective Lagrangian for spectator-flavor dependent effects in decay rates

The systematic description of the leading as well as subleading effects in the inclusive decay rates of heavy hadrons arises \([2, 4, 5]\) through application of the operator product expansion in powers of \(m_Q^{-1}\) to the ‘effective Lagrangian’ \(L_{eff}\) related to the correlator of two weak-interaction terms \(L_W\):

\[
L_{eff} = 2 \text{Im} \left[ i \int d^4x e^{ix} T \{L_W(x), L_W(0)\} \right].
\]

In terms of \(L_{eff}\) the inclusive decay rate of a heavy hadron \(H_Q\) is given by the mean value\(^1\)

\[
\Gamma_H = \langle H_Q | L_{eff} | H_Q \rangle.
\]

The leading term in \(L_{eff}\) describes the ‘parton’ decay rate. For instance, the term in the non-leptonic weak Lagrangian \(\sqrt{2} G_F V(\bar{q}_{1L} \gamma_\mu Q_L)(\bar{q}_{2L} \gamma_\mu q_{3L})\) with \(V\) being the appropriate combination of the CKM mixing factors, generates through eq.\(^3\) the leading term in the effective Lagrangian

\[
L_{eff}^{(0)}_{nl} = |V|^2 \frac{G_F^2 m_Q^5}{64 \pi^3} \eta_{nl} \left(\bar{Q}Q\right),
\]

\(^1\)The non-relativistic normalization for the heavy quark states is used throughout this paper: \(\langle Q|Q^\dagger Q|Q\rangle = 1\).
where \( \eta_{\text{eff}} \) is the perturbative QCD radiative correction factor. In the limit \( m_Q \to \infty \) this expression reproduces the ‘parton’ inclusive non-leptonic decay rate associated with the underlying process \( Q \to q_1 q_2 \overline{q}_3 \), due to the relation \( \langle H_Q|\overline{Q}Q|H_Q \rangle \approx \langle H_Q|Q^1 Q|H_Q \rangle = 1 \) with the approximate equality being valid up to corrections of order \( m_Q^2 \). In order to reproduce the complete expression of order \( m_Q^2 \) one should also include the first non-trivial term of the OPE for \( L_{\text{eff}} \) containing the dimension 5 chromomagnetic operator \( (\overline{Q} \sigma_{\mu \nu} G_{\mu \nu} Q) \) with the gluonic field tensor \( G_{\mu \nu} \). However neither this operator nor the leading term \( L_{\text{eff}}^{(0)} \) involve light quark operators, thus at this level no splitting arises between the decay rates within flavor \( SU(3) \) multiplets.

The dependence on the flavor of spectator quarks arises in the next term of OPE for \( L_{\text{eff}} \): \( L_{\text{eff}}^{(3)} \). For the dominant Cabibbo unsuppressed non-leptonic decays of the charmed quark, generated by the underlying process \( c \to s u \overline{d} \), this term reads as [5]:

\[
L_{\text{eff}, m}^{(3,0)} = c^4 \frac{G_F^2 m_c^2}{4\pi} \left\{ C_1 (\overline{\tau} \Gamma_\mu c)(\overline{d} \Gamma_\mu d) + C_2 (\overline{\tau} \Gamma_\mu d)(\overline{d} \Gamma_\mu c) + C_3 (\overline{\tau} \Gamma_\mu c + \frac{2}{3}\overline{\tau} \gamma_\mu \gamma_5 c)(\overline{\tau} \Gamma_\mu s) + C_4 (\overline{\tau} \Gamma_\mu c k + \frac{2}{3}\overline{\tau} \gamma_\mu \gamma_5 c)k + \sigma_0 c k) \right\} \tag{4}
\]

with the coefficients \( C_A, A = 1, \ldots, 6 \) given by

\[
\begin{align*}
C_1 &= C_+^2 + C_-^2 + \frac{1}{3}(1 - \kappa^{1/2})(C_+^2 - C_-^2), \\
C_2 &= \kappa^{1/2}(C_+^2 - C_-^2), \\
C_3 &= -\frac{1}{4} \left[ (C_+ - C_-)^2 + \frac{1}{3}(1 - \kappa^{1/2})(5C_+^2 + C_-^2 + 6C_+ C_-) \right], \\
C_4 &= -\frac{1}{4} \kappa^{1/2}(5C_+^2 + C_-^2 + 6C_+ C_-), \\
C_5 &= -\frac{1}{4} \left[ (C_+ + C_-)^2 + \frac{1}{3}(1 - \kappa^{1/2})(5C_+^2 + C_-^2 - 6C_+ C_-) \right], \\
C_6 &= -\frac{1}{4} \kappa^{1/2}(5C_+^2 + C_-^2 - 6C_+ C_-). \tag{5}
\end{align*}
\]

In Equations (4) and (5) the following notation is used: \( \Gamma_\mu = \gamma_\mu (1 - \gamma_5) \), \( C_+ \) and \( C_- \) are the standard coefficients in the QCD renormalization of the non-leptonic weak interaction from \( m_W \) down to the charmed quark mass: \( C_- = C_+^{-2} = (\alpha_s(m_c)/\alpha_s(m_W))^{4/b} \), where \( b \), the coefficient in the one-loop beta function in QCD, can be taken as \( b = 25/3 \) for the case of the charmed quark decay (see e.g. in the textbook [12]). Furthermore, the
powers of the parameter $\kappa = (\alpha_s(\mu)/\alpha_s(m_c))$ describe the so called ‘hybrid’ QCD renormalization of the operators from the normalization scale $m_c$ down to a low scale $\mu$, and $j^a_\mu = \bar{u} \gamma_\mu t^a u + d \gamma_\mu t^a d + s \gamma_\mu t^a s$ is the color current of the light quarks with $t^a = \lambda^a/2$ being the generators of the color SU(3). Finally, the elements $V_{cs}$ and $V_{ud}$ of the CKM weak mixing matrix are approximated here by the cosine of the Cabibbo angle, $c \equiv \cos \theta_c$, hence the overall coefficient in eq. (3) is written as $c^4$.

The terms in eq. (4) with the flavor singlet operator $j^a_\mu$ produce no splitting of the decay rates within the flavor SU(3) multiplets and thus are not of immediate relevance to the present discussion. The rest of the terms containing operators with the $u$, $d$ and $s$ quarks with different coefficients are responsible for those splittings. In terms of the physical interpretation of the effect the operators with the $d$ quark describe the WS in charmed hyperons, while the terms with the $u$ and $s$ quarks correspond to the PI effects.

In order to discuss further in the paper the spectator effects on the Cabibbo suppressed non-leptonic decays as well as on the semileptonic ones, we will also need the expressions for the corresponding parts of the effective Lagrangian $L^{(3)}_{\text{eff}}$. For the non-leptonic decays we limit the present discussion to those suppressed by only one extra factor of $s \equiv \sin \theta_c$, i.e. those generated by the quark processes $c \to s u \bar{s}$ and $c \to d u \bar{d}$. The Cabibbo-suppressed part of the non-leptonic effective Lagrangian then can be found in the form:

$$L^{(3,1)}_{\text{eff, nl}} = c^2 s^2 \frac{G_F^2 m_c^2}{4\pi} \left\{ C_1 (\bar{q} \Gamma_\mu c)(\bar{q} \Gamma_\mu q) + C_2 (\bar{q} \Gamma_\mu c k)(\bar{q} k \Gamma_\mu q k) + 
C_3 (\bar{q} \Gamma_\mu c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c)(\bar{q} \Gamma_\mu q) + C_4 (\bar{q} \Gamma_\mu c k + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c k)(\bar{q} k \Gamma_\mu q k) + 
2C_5 (\bar{q} \Gamma_\mu c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c)(\bar{q} \Gamma_\mu u) + 2C_6 (\bar{q} \Gamma_\mu c k + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c k)(\bar{q} k \Gamma_\mu u k) + 
\frac{2}{3} \kappa^{1/2} (\kappa^{-2/9} - 1) \left\{ 2 (C_+^2 - C^2) (\bar{q} \Gamma_\mu t^a c j^a_\mu - (5C_+^2 + C^2)(\bar{q} \Gamma_\mu t^a c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 t^a c j^a_\mu) \right\} \right\}$$

with the notation $(\bar{q} \Gamma q) = (\bar{d} \Gamma d) + (\bar{s} \Gamma s)$.

The corresponding term in the effective Lagrangian for semileptonic decays, generated by the quark-lepton process $c \to s \ell^+ \nu$ and the Cabibbo-suppressed one: $c \to d \ell^+ \nu$ is given by

$$L^{(3)}_{\text{eff, sl}} = \frac{G_F^2 m_c^2}{12\pi} \left\{ c^2 \left[ L_1 (\bar{q} \Gamma_\mu c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c)(\bar{q} \Gamma_\mu s) + L_2 (\bar{q} \Gamma_\mu c k + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c k)(\bar{q} k \Gamma_\mu s k) \right] + 
\right\}$$

\[2\] In mesons the same term describes the Pauli interference of the $\bar{d}$ quark in the decays of $D^+$, which is considered to be the dominant reason for the observed suppression of the $D^+$ total decay rate.
\[ s^2 \left[ L_1 (\overline{c} \gamma_\mu c + \frac{2}{3} \overline{c} \gamma_\mu \gamma_5 c) (\overline{d} \Gamma_\mu d) + L_2 (\overline{c}_i \gamma_\mu c_k + \frac{2}{3} \overline{c}_i \gamma_\mu \gamma_5 c_k) (\overline{d}_k \Gamma_\mu d_i) \right] - \\
\kappa^{1/2} (\kappa^{-2/9} - 1) (\overline{c} \Gamma_\mu t^a c + \frac{2}{3} \overline{c} \gamma_\mu \gamma_5 t^a c) j_\mu^a \right\}, \]  

where the coefficients \( L_1 \) and \( L_2 \) are

\[ L_1 = (\kappa^{1/2} - 1), \quad L_2 = -3 \kappa^{1/2}. \]  

In the next Section the general expressions in equations (4), (7) and (7) are used for an analysis of the relations between the splittings of various inclusive decay rates within the triplet of charmed baryons.

3 Differences of inclusive decay rates for charmed baryons

Estimates of the absolute effect of the flavor-dependent operators in the effective Lagrangian \( L^{(3)}_{\text{eff}} \) require evaluation of the matrix elements of the four-quark operators over charmed hadrons. One approach (for a review see e.g. Ref. [8]) is to use a low normalization point \( \mu \) of the order of the confinement scale and invoke a constituent quark model, simplifying it further to a non-relativistic model, with possible additional (also non-relativistic) input about the wave functions of the light quarks at the origin (see e.g. Refs. [14, 15] and a rather general consideration in Refs. [16, 17]). Needless to mention that such approach can be used only for qualitative, or very approximate semi-quantitative estimates, since it inevitably involves poorly controllable approximations. In order to be able to find arguably more reliable relations we do not attempt here an absolute evaluation of those matrix elements, but rather use the flavor SU(3) properties of the operators in \( L^{(3)}_{\text{eff}} \) to relate the measurable splittings of the semileptonic and the Cabibbo-suppressed inclusive decay rates in the charmed baryon triplet to the splittings of the total decay rates. Namely, assuming the flavor SU(3) symmetry, and the applicability of the heavy quark limit to the charmed quark, it will be shown that the discussed splittings are determined by only two independent matrix elements, which can be expressed in terms of the differences in the measured total decay rates: \( \Delta_1 = \Gamma(\Xi^0_c) - \Gamma(\Lambda_c) \) and \( \Delta_2 = \Gamma(\Lambda_c) - \Gamma(\Xi^+). \)

Proceeding with derivation of the relations we first notice that for the triplet of the baryons \((\Lambda_c, \Xi_c)\) in the heavy quark limit the spin of the charmed quark is not correlated with spinorial characteristics of its light ‘environment’. Thus the operators with the axial current of the \( c \) quark give no contribution to the matrix elements. The remaining flavor non-singlet structures in \( L^{(3)}_{\text{eff}} \) involve only operators of the types \( (\overline{c} \gamma_\mu c) (\overline{q} \Gamma_\mu q) \) and \( (\overline{c}_i \gamma_\mu c_k) (\overline{q}_k \Gamma_\mu q_i) \)
with $q$ being $d$, $s$ or $u$. Due to the SU(3) symmetry the flavor non-singlet part of the matrix elements of each of these types of operators in the baryon triplet is expressed in terms of only one parameter. Indeed, the difference of the matrix elements between the components of each of these types of operators in the baryon triplet is expressed in terms of \( \Delta V = 1 \) combination of the operators, proportional to \( (\bar{u} \Gamma u) - (\bar{s} \Gamma s) \), while the difference between the components of a \( U \)-spin doublet: \( \Lambda_c \) and \( \Xi_c^+ \), receives the same contribution only from the \( \Delta U = 1 \) operator \( (\bar{s} \Gamma s) - (\bar{d} \Gamma d) \).

Thus if one introduces two parameters \( x \) and \( y \) as

\[
x = \left\langle \frac{1}{2} (\bar{c} \gamma_{\mu} c) \left[ (\bar{u} \Gamma_{\mu} u) - (\bar{s} \Gamma_{\mu} s) \right] \right\rangle_{\Xi^0 - \Lambda_c} = \left\langle \frac{1}{2} (\bar{c} \gamma_{\mu} c) \left[ (\bar{s} \Gamma_{\mu} s) - (\bar{d} \Gamma_{\mu} d) \right] \right\rangle_{\Lambda_c - \Xi^+_c},
\]

\[
y = \left\langle \frac{1}{2} (\bar{c} \gamma_{\mu} c_k) \left[ (\bar{u}_k \Gamma_{\mu} u_i) - (\bar{s}_k \Gamma_{\mu} s_i) \right] \right\rangle_{\Xi^0 - \Lambda_c} = \left\langle \frac{1}{2} (\bar{c} \gamma_{\mu} c_k) \left[ (\bar{s}_k \Gamma_{\mu} s_i) - (\bar{d}_k \Gamma_{\mu} d_i) \right] \right\rangle_{\Lambda_c - \Xi^+_c}
\]

with the shorthand notation for the differences of the matrix elements: \( \langle \mathcal{O} \rangle_{A-B} = \langle A | \mathcal{O} | A \rangle - \langle B | \mathcal{O} | B \rangle \), the splitting of the inclusive decay rates within the baryon triplet are expressed in terms of \( x \) and \( y \) as follows. The differences of the dominant Cabibbo unsuppressed non-leptonic decay rates are given by

\[
\delta_{1}^{nl,0} \equiv \Gamma_{\Delta S=0}^{nl}(\Xi^0_c) - \Gamma_{\Delta S=0}^{nl}(\Lambda_c) = c^4 \frac{G_F^2 m_c^2}{4 \pi} \left[ (C_5 - C_3) x + (C_6 - C_4) y \right],
\]

\[
\delta_{2}^{nl,0} \equiv \Gamma_{\Delta S=0}^{nl}(\Lambda_c) - \Gamma_{\Delta S=0}^{nl}(\Xi^+_c) = c^4 \frac{G_F^2 m_c^2}{4 \pi} \left[ (C_3 - C_1) x + (C_4 - C_2) y \right].
\]

The once Cabibbo suppressed decay rates of \( \Lambda_c \) and \( \Xi^+_c \) are equal, due to the \( \Delta U = 0 \) property of the corresponding effective Lagrangian \( L_{eff,nl}^{(3,1)} \) (eq.(4)). Thus the only difference for these decays in the baryon triplet is

\[
\delta_{1}^{nl,1} \equiv \Gamma_{\Delta S=0}^{nl}(\Xi^0_c) - \Gamma_{\Delta S=0}^{nl}(\Lambda_c) = c^2 s^2 \frac{G_F^2 m_c^2}{4 \pi} \left[ (2 C_5 - C_1 - C_3) x + (2 C_6 - C_2 - C_4) y \right].
\]

The dominant semileptonic decay rates are equal among the two \( \Xi_c \) baryons due to the isotopic spin property \( \Delta I = 0 \) of the corresponding interaction Lagrangian, thus there is only one non-trivial splitting for these decays:

\[
\delta_{1}^{sl,0} \equiv \Gamma_{\Delta S=0}^{sl}(\Xi_c) - \Gamma_{\Delta S=0}^{sl}(\Lambda_c) = -c^2 \frac{G_F^2 m_c^2}{12 \pi} \left[ L_1 x + L_2 y \right].
\]

Finally, the Cabibbo suppressed semileptonic decay rates are equal for \( \Lambda_c \) and \( \Xi_c^0 \), due to the \( \Delta V = 0 \) property of the corresponding interaction. Thus the only difference for these is

\[
\delta_{1}^{sl,1} \equiv \Gamma_{\Delta S=0}^{sl}(\Lambda_c) - \Gamma_{\Delta S=0}^{sl}(\Xi^+_c) = -s^2 \frac{G_F^2 m_c^2}{12 \pi} \left[ L_1 x + L_2 y \right].
\]
Using the relations (10) - (13) one can express the splittings of the total decay rates in terms of the two parameters $x$ and $y$ as

\[
\Delta_1 = \delta_{n1}^{nl,0} + \delta_{n1}^{nl,1} + 2\delta_{s1}^{sl,0},
\]
\[
\Delta_2 = \delta_{n2}^{nl,0} - 2\delta_{s2}^{sl,0} + 2\delta_{s1}^{sl,1},
\]

(14)

where the factor 2 for the semileptonic splittings takes into account the decays with both $e\nu$ and $\mu\nu$ leptonic pairs, and the small contribution of double Cabibbo suppressed non-leptonic decays is neglected. Thus the unknown parameters $x$ and $y$ can be found in terms of the measured differences $\Delta_1$ and $\Delta_2$ and used to predict the splittings of inclusive decay rates for each experimentally identifiable type of decays, described by the equations (10) - (13).

It is quite satisfying to see that although the parameters $x$ and $y$ as well as the coefficients $C_A$ and $L_1$ and $L_2$ depend on the arbitrarily chosen low normalization point $\mu$, in the resulting relations between the physically measurable splittings the dependence on $\mu$ cancels out, as it should be expected. In fact in the expression for $\delta_{n1}^{nl,1}$ in terms of the dominant non-leptonic splittings the dependence on the QCD radiative effects cancels out altogether:

\[
\delta_{n1}^{nl,1} = \frac{s^2}{c^2} \left( 2\delta_{n1}^{nl,0} + \delta_{n2}^{nl,0} \right),
\]

(15)

while the relation between the splitting of the dominant semileptonic decay rates and the dominant non-leptonic splittings, emerging after excluding $x$ and $y$ in the equations (12) and (10), reads as

\[
\delta_{s1}^{sl,0} = \frac{1}{c^2} \left[ \frac{5}{12} C_+^2 + \frac{5}{2} C_+ - 2 C_+ C_- (C_+^2 + 2 C_-^2) \delta_{n1}^{nl,0} \right] - \frac{1}{3} \left( \frac{C_+^2 + 2 C_-^2}{3} \right) \delta_{n2}^{nl,0}.
\]

(16)

The coefficients in this relation depend only on the physical ratio of the couplings $r = (\alpha_s(m_c)/\alpha_s(m_W))$. Moreover, this dependence is in fact rather weak: in the absence of the QCD radiative effects, i.e. with $r = 1$, the coefficients in the square brackets in eq.(16) are $2/9 \approx 0.22$ and $-1/9 \approx -0.11$, while with a realistic value $r \approx 2.5$ they are respectively $0.23$ and $-0.09$. Similar relative insensitivity of the numerical results to the exact value of $r$ also holds for other relations between the observable splittings. In subsequent numerical estimates the numerical values $r = 2.5$ and $s^2 = 0.05$ are used. We also use the data from Ref. [7] on the lifetimes of the charmed baryons in the form: $\Gamma(\Lambda_c) = 4.85 \pm 0.28 \text{ps}^{-1}$, $\Gamma(\Xi_c^+o) = 2.85 \pm 0.5 \text{ps}^{-1}$ and $\Gamma(\Xi_c^0) = 10.2 \pm 2 \text{ps}^{-1}$. (The error bars on the lifetimes for the $\Xi_c$ hyperons are in fact not symmetric. However the symmetry improves for the inverse quantities, i.e. for the total widths, and a close approximation to the errors in the widths is used here in a symmetric fashion.)
Solving the equations (14) for the Cabibbo dominant splitting of the semileptonic widths yields
\[ \delta^{s,0} = 0.13 \Delta_1 - 0.065 \Delta_2 \approx 0.59 \pm 0.32 \text{ps}^{-1}, \]
and, obviously, for the Cabibbo suppressed semileptonic decays one has \( \delta^{s,1} = (s^2/c^2) \delta^{s,0} \approx 0.030 \pm 0.016 \text{ps}^{-1} \). The solution of eqs.(14) for the splitting of the Cabibbo suppressed non-leptonic rates similarly gives
\[ \delta^{nl,1} = 0.082 \Delta_1 + 0.054 \Delta_2 \approx 0.55 \pm 0.22 \text{ps}^{-1}. \]

4 Splitting of lifetimes in the triplet of \( b \) baryons

Once the unknown baryonic matrix elements \( x \) and \( y \) are phenomenologically determined through the differences of the total decay rates in the charmed baryon triplet, one can use these parameters for evaluating the differences of decay rates in the triplet of the \( b \) baryons: \( \Lambda_b, \Xi^0_b, \) and \( \Xi^-_b \). Indeed, in the limit where both the \( c \) and the \( b \) quarks are heavy the matrix elements of the four-quark operators over the \( b \) hyperons should be the same as for the charmed ones, provided that the operators are normalized at a low point \( \mu \) which does not depend on the masses \( m_c \) or \( m_b \). For proceeding in this manner we write here the expression [5] for the corresponding effective lagrangian for non-leptonic \( b \) decays, neglecting small kinematical effects \( O(m^2_c/m^2_b) \) in the relevant expressions for the two-body phase space of the pair \( c \bar{c} \) or \( c q \). \[ L^{(3,b)}_{\text{eff, nl}} = c^2 |V_{bc}|^2 \frac{G_F^2 m^2_b}{4\pi} \left\{ \tilde{C}_1 (\bar{b}\Gamma \mu b)(\bar{u}\Gamma \mu u) + \tilde{C}_2 (\bar{b}\Gamma \mu u)(\bar{u}\Gamma \mu b) + \tilde{C}_5 (\bar{b}\Gamma \mu b + \frac{2}{3} b \gamma_\mu \gamma_5 b)(\bar{q}\Gamma \mu q) + \tilde{C}_6 (\bar{b}_i \Gamma \mu b_k + \frac{2}{3} b_i \gamma_\mu \gamma_5 b_k)(\bar{q}_k \Gamma \mu q_i) + \frac{1}{3} \tilde{k}^{-1/2} (\tilde{k}^{-2/9} - 1) \left[ 2 (\tilde{C}_+^2 - \tilde{C}_-^2) (\bar{b}\Gamma \mu t^a b)(\bar{t}_\mu j^a) - (5\tilde{C}_+^2 + \tilde{C}_-^2 - 6 \tilde{C}_+ \tilde{C}_-) (\bar{b}\Gamma \mu t^a b + \frac{2}{3} b \gamma_\mu \gamma_5 t^a b) j^a \right] \right\}, \] where again the notation \( (\bar{q}\Gamma q) = (\bar{d}\Gamma d) + (\bar{s}\Gamma s) \) is used, and the renormalization coefficients are determined by \( \alpha_s(m_b) \) instead of \( \alpha_s(m_c) \): \( \tilde{C}_- = \tilde{C}_+^{-2} = (\alpha_s(m_b)/\alpha_s(m_W))^{4/b} \),

\[ 3 \text{The expression with these small terms included can be found in Ref. 18. Also only the CKM-dominant processes } b \to c \bar{c} d \text{ and } b \to c \bar{c} s \text{ are taken into account in order to keep the formulas simple. The contribution of sub-dominant processes to the total rates is below the expected uncertainty.} \]
\[ \tilde{\kappa} = (\alpha_s(\mu)/\alpha_s(m_b)) \]. The coefficients \( \tilde{\kappa} \) are related to \( \tilde{C}_- \), \( \tilde{C}_+ \), and \( \tilde{\kappa} \) in the same way as in eqs. (23).

The dominant semileptonic decay \( b \rightarrow c \ell \nu \) does not involve light quarks, thus one expects no substantial splitting of the semileptonic decay rates within flavor SU(3) multiplets of \( b \) hadrons. The non-leptonic effective lagrangian in eq.(20) is symmetric with respect to \( s \leftrightarrow d \), i.e. it has \( \Delta U = 0 \) (this property is broken if the small kinematical effects of the \( c \) quark mass are kept in \( L_{\text{eff}}^{(3,b)} \)). Thus at this level there is no splitting between the non-leptonic decay rates of \( \Lambda_b \) and \( \Xi_b^0 \). The splitting of the decay rate between either of these and \( \Xi_b^- \) is given in terms of \( x \) and \( y \) by

\[ \Delta_b = \Gamma(\Lambda_b) - \Gamma(\Xi_b^-) = c^2 |V_{bc}|^2 \frac{C^2_2 m_b^2}{4\pi} \left[ (\tilde{C}_5 - \tilde{C}_1) x + (\tilde{C}_6 - \tilde{C}_2) y \right]. \]  

One can notice that in the absence of any QCD radiative effects the latter difference is simply related to \( \delta_1^{\text{nl},0} \) and \( \delta_2^{\text{nl},0} \) for the charmed baryons:

\[ \Delta_b = \frac{|V_{bc}|^2 m_b^2}{c^2 m_c^2} \left( \delta_1^{\text{nl},0} + \delta_2^{\text{nl},0} \right). \]

The QCD correction coefficients make the full expression somewhat more lengthy:

\[ \Delta_b = \frac{|V_{bc}|^2 m_b^2}{c^2 m_c^2} \frac{1}{4 C_+ C_- (C^2_2 + 2 C^2_+)} \left\{ \left[ C^2_+ \left( (3 + 2 \xi) \tilde{C}^2_2 + 4 \xi \tilde{C}_- \tilde{C}_+ + 6 (1 - \xi) \tilde{C}^2_\mp \right) + 2 C_+ C_- \left( \tilde{C}^2_2 + 2 \tilde{C}^2_+ \right) - C^2_2 \left( (1 - \xi) \tilde{C}^2_- - 2 \xi \tilde{C}_- \tilde{C}_+ + (2 + 3 \xi) \tilde{C}^2_\mp \right) \right] \delta_1^{\text{nl},0} + 4 C_+ C_- \left( \tilde{C}^2_- + 2 \tilde{C}^2_+ \right) \delta_2^{\text{nl},0} \right\}, \]

where \( \xi = (\tilde{\kappa}/\kappa)^{1/2} = (\alpha_s(m_c)/\alpha_s(m_b))^{1/2} \). One can again notice that the relation (22) between physically measurable quantities does not contain dependence on the low normalization point \( \mu \). Numerically, however the full expression is not far from the simple approximation in eq.(21): with a realistic value \( (\alpha_s(m_c)/\alpha_s(m_b)) \approx 1.25 \) one finds from eq.(22):

\[ \Delta_b \approx \frac{|V_{bc}|^2 m_b^2}{c^2 m_c^2} (0.91 \delta_1^{\text{nl},0} + 0.93 \delta_2^{\text{nl},0}). \]

When expressed in terms of the differences in the total decay rates \( \Delta_1 \) and \( \Delta_2 \) for the charmed baryons, using eq.(14), the splitting of the decay rates within the \( b \) baryon triplet reads numerically as

\[ \Delta_b \approx |V_{bc}|^2 \frac{m_b^2}{m_c^2} (0.85 \Delta_1 + 0.91 \Delta_2) \approx 0.015 \Delta_1 + 0.016 \Delta_2 \approx 0.11 \pm 0.03 \text{ ps}^{-1}, \]  

which represents the estimate from the present analysis of the expected suppression of the total decay rate of \( \Xi_b^- \) with respect to that of \( \Lambda_b \), or \( \Xi_b^0 \).
5 Discussion

The relative differences of the lifetimes for charmed particles are large, even within one flavor SU(3) triplet of the hyperons. Therefore the assumption that these differences in the triplet are described by just one term of the expansion in $m_Q^{-1}$ certainly requires additional study. It should be noted however, that this assumption is not necessarily flawed, since the discussed $O(m_Q^{-3})$ terms are singled out by large numerical coefficient, and there is no reason for recurrence of such anomaly further in the expansion. Thus the relations between the splittings of the decay rates within the triplet charmed hyperons ($\Lambda_c$, $\Xi_c$) as well as of the $b$ hyperons ($\Lambda_b$, $\Xi_b$) may present a good testing point for experimental study of this issue. As a consequence of large difference in the lifetimes the additional effects, discussed here, are also estimated to be quite large. The predicted difference in the semileptonic decay rates between the $\Xi_c$ and $\Lambda_c$ (eq. (17)) can be compared with the current data on the semileptonic width of $\Lambda_c$: $\Gamma_{sl}(\Lambda_c) = 0.22 \pm 0.08 \text{ ps}^{-1}$. This comparison confirms the conclusion of a previous analysis [1], that the semileptonic decay rates of the $\Xi_c$ hyperons can be larger than that of $\Lambda_c$ by a factor of 2 or 3. Similarly, the small Cabibbo suppressed semileptonic decay rate of $\Lambda_c$, should be enhanced by the same factor [11] and may in fact constitute 10% to 15% of all semileptonic decays of $\Lambda_c$. The effect in the Cabibbo suppressed non-leptonic decays evaluated in eq. (18) can amount to more than 10% of the difference in the total non-leptonic decay rates of $\Xi_c^+$ and $\Lambda_c$ and should be quite prominent, provided that it would be possible to separate and measure the inclusive Cabibbo suppressed rates experimentally.

Finally, the prediction of eq. (23) for the difference of the total decay rates of $\Lambda_b$ and $\Xi_b^-$ can be interesting in relation to the mentioned earlier problem of the ratio $\tau(\Lambda_b)/\tau(B)$. Indeed, the central number in eq. (23) amounts to about 14% of the total decay rate $\Gamma(\Lambda_b) = 0.81 \pm 0.05 \text{ ps}^{-1}$, and a difference of such relative magnitude is undoubtedly to be considered as very large for the $b$ hadrons. If confirmed, this would indicate that the spectator effects in heavy hyperons can be substantially larger, than usually expected, and may shed some light on the problem of the $\Lambda_b$ versus $B$ lifetime.

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