Dendrochronology Regression Models in Aufeis Formation Analysis

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Abstract. The paper examines the historical behaviour of such unique natural phenomena as the sheets of aufeis in the North East of Russia. They are studied using statistical methods for analyzing dendrochronological data. Dendrochronology is a scientific method based on comparing quantitative indicators of tree ring growth and the dynamics of aufeis formation in the past. Tree ring growth is an integral indicator which reflects the diversity of conditions in which trees grow and interact with the environment. Time series were obtained for the Ulakhan-Taryn and Buluus aufeis deposits located in the Middle Lena basin in Central Yakutia. Regression models were created using dendrochronological data, with the majority of the models being nonlinear. The sample mean, sample variance, and sample correlation coefficient were calculated for the time series. The regression model of the time series under study was represented as a combination of a non-random component, including the age trend and a climatic component, and a random component with zero expectation, which is the sum of factors unaccounted for, including autocorrelation. Depending on the locations where data were collected, both strong and moderate and weak dependencies of pine growth on aufeis influence were revealed. If there was strong or moderate aufeis influence, humidity had a minor impact on tree growth. If aufeis influence was weak, tree growth was moderately influenced by precipitation. There is a moderate dependence of tree growth on air temperature in all cases.

1. Introduction
In the North East of Russia, there are unique natural formations – sheets of aufeis, which are large masses of ice which do not melt even in summer at temperatures up to +35 degrees Celsius [1].

The issue of aufeis formation in Yakutia’s permafrost has long attracted the attention of researchers since aufeis can affect a variety of constructions and structures, especially roads, railways, bridges, power lines, and pipelines. It is important to find repeating patterns in how different factors influence aufeis deposits in space and time as the results of such studies can be used to predict aufeis growth in the future. This important task seems to be impossible to solve without analyzing how aufeis deposits changed in the past. Dendrochronology is the only method which relies on comparable quantitative indicators and can help researchers arrive at definite conclusions on the historical behaviour of aufeis deposits. The complexity of the issue lies in the fact that tree ring growth is an integral indicator. It reflects the diversity of conditions in which trees grow and interact with the environment.

2. Problem statement and initial data
The dendrochronological method is based on the ability of trees to store for hundreds or even thousands of years information about the environmental conditions under which tree ring formation took place in a particular period of time. This property of trees opens up an opportunity for reconstructing the historical behaviour of aufeis deposits using mathematical statistics [3, 4] in application to dendrochronological data [5-7].

Long-term studies of relationships in such systems as ‘climate – aufeis formation’, ‘climate – tree ring growth’, and “tree ring growth – aufeis” [8, 9] were carried out in the ecosystems of the Ulakhan-Taryn and Buluus aufeis deposits located in the Middle Lena basin in Central Yakutia. The seven tree-ring growth chronologies were selected to describe Buluus tree-ring chronologies: BULN1, BULN2, BULN3, BULS1, BULS2, BULS3, UT. The tree-ring chronologies used in this study are based on the materials collected by O.A. Pomortsev in Ulakhan-Taryn (more than 50 samples) and also by V.S. Efremov, A.N. Nikolaev and O.A. Pomortsev in the Buluus valley (more than 120 samples). Local transects in the Ulakhan-Taryn valley were established with a detailed coverage of the cross-section of the valley, and in the Buluus valley, which is narrow, they were referenced to the brow, foot and middle part of the slope. The following factors affecting tree growth was carried out using the Kolmogorov-Smirnov criterion and the analysis of variance method. It showed that BULN1, BULS1 and UT tree-ring chronologies are homogeneous and at the same time different (with 95% confidence) from other chronologies for the areas which are located father from aufeis. BULS2 and BULS3 chronologies are heterogeneous.

Let us turn to the regression analysis of tree-ring chronologies. Let \( y_1, \ldots, y_T \), where \( T \) is the observation period, be the time series of the statistical data under study with the following characteristics:

\[
\overline{y} = \frac{1}{T} \sum_{t=1}^{T} y_t \quad \text{– sample mean,} \quad \sigma_y^2 = \frac{1}{T} \sum_{t=1}^{T} (y_t - \overline{y})^2 \quad \text{– sample variance,} \quad \sum_{t=1}^{T} (y_t - \overline{y})(t - \overline{t}) \quad \text{– sample correlation coefficient, where} \quad \overline{t} = \frac{1}{T} \sum_{t=1}^{T} t \quad \text{and} \quad \sigma_t^2 = \frac{1}{T} \sum_{t=1}^{T} (t - \overline{t})^2.
\]

Let us describe the regression model of the time series studied as a combination of components \( \hat{y}(t) = f(t) + \varepsilon(t) \), where \( f(t) \) is a non-random component, including the age trend and a climatic component, and \( \varepsilon(t) \) is a random component, which is the sum of factors unaccounted for, including autocorrelation, with \( M\varepsilon(t) = 0 \) and \( D\varepsilon(t) = \sigma_{\varepsilon(t)}^2 \).

The type of function \( f(t) \) is set depending on the correlation field of the statistical data, and the unknown function parameters for \( f(t) \) are calculated by the least squares method [10].

The random component \( \varepsilon(t) \) will be found using the first-order autoregressive model [11]:

\[
\varepsilon(t) = c\varepsilon(t-1) + \delta(t),
\]

where \( c \) is a non-random value, \( |c| \leq 1 \), \( \delta(t) \) is white noise, \( M\delta(t) = 0 \),

\[
\text{cov}(\delta(t), \delta(t + \tau)) = \begin{cases} D\delta(t), & \tau = 0, \\ 0, & \tau \neq 0. \end{cases}
\]

The process \( \varepsilon(t) \) is a homogeneous Markov process; its autocorrelation function is therefore determined by the ratio

\[
r(\tau) = r(\varepsilon(t), \varepsilon(t + \tau)) = M \left( \frac{\varepsilon(t)}{\sigma_{\varepsilon(t)}}, \frac{\varepsilon(t + \tau)}{\sigma_{\varepsilon(t + \tau)}} \right) = \alpha^\tau
\]
where $\alpha = r(1)$ is the correlation coefficient between the adjacent cross-sections of the series, and the white noise dispersion is $D\delta(t) = (1 - \alpha^2)\sigma^2_{\sigma(t)}$.

In its final form, the regression model of the series under study will be written as

$\hat{y}(t) = f(t) + \alpha c(t-1) + \delta(t)$.

### Table 1. Characteristics of the regression models of the tree-ring chronologies

| Tree-ring chronology | Time series characteristics | Linear regression characteristics | Nonlinear regression characteristics |
|----------------------|-----------------------------|----------------------------------|-------------------------------------|
| BULN1                | $\bar{y} = 0.65$, $\sigma_y = 0.22$, $r = -0.63$ | $f(t) = -0.004t + 0.238$ | $f(t) = -2.56v^3 - 0.448v^2 +$  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.65$ |  
 $\sigma^2_0 = 0.02$, $R = 0.63$ |  
 $\alpha = 0.7$, $\sigma^2_0 = 0.02$, $R = 0.41$ |  
 $v = t - 58$, $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.73$ |
| BULS1                | $\bar{y} = 0.67$, $\sigma_y = 0.19$, $r = -0.41$ | $f(t) = -0.002t + 0.13$ | $f(t) = -2.954v^3 - 0.16v^2 +$  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.59$ |  
 $\sigma^2_0 = 0.02$, $R = 0.61$ |  
 $\alpha = 0.44$, $\sigma^2_0 = 0.02$, $R = 0.61$ |  
 $v = t - 58$, $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.73$ |
| UT                  | $\bar{y} = 0.67$, $\sigma_y = 0.22$, $r = -0.58$ | $f(t) = -0.004t + 0.216$ | $f(t) = 0.16v^3 - 0.14v^4 -$  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.7$ |  
 $\sigma^2_0 = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.81$ |  
 $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.73$ |
| BULN2                | $\bar{y} = 0.49$, $\sigma_y = 0.12$, $r = 0.1$ | $f(t) = -0.004t + 0.238$ | $f(t) = -2.275v^3 - 0.603v^2 +$  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.7$ |  
 $\sigma^2_0 = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.81$ |  
 $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.73$ |
| BULN3                | $\bar{y} = 0.56$, $\sigma_y = 0.19$, $r = -0.07$ | $f(t) = -0.005t + 0.319$ | $f(t) = -3.03v^3 - 0.699v^2 +$  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.42$ |  
 $\sigma^2_0 = 0.03$, $R = 0.74$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |  
 $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |
| BULS2                | $\bar{y} = 0.78$, $\sigma_y = 0.25$, $r = -0.74$ | $f(t) = -0.005t + 0.319$ | $f(t) = 0.76$ |  
 $\sigma^2_{\sigma(t)} = 0.03$, $\alpha = 0.42$ |  
 $\sigma^2_0 = 0.03$, $R = 0.74$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |  
 $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |
| BULS3                | $\bar{y} = 1.27$, $\sigma_y = 0.56$, $r = -0.61$ | $f(t) = -0.01t + 0.518$ | $f(t) = 0.65$ |  
 $\sigma^2_{\sigma(t)} = 0.21$, $\alpha = 0.81$ |  
 $\sigma^2_0 = 0.07$, $R = 0.61$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |  
 $\sigma^2_{\sigma(t)} = 0.02$, $R = 0.58$ |  
 $\alpha = 0.61$, $\sigma^2_0 = 0.01$, $R = 0.61$ |
The boundaries of the confidence interval \( \hat{y}(t) - \delta_\beta < y(t) < \hat{y}(t) + \delta_\beta \) for the regression model will be found using the equation \( \delta_\beta = t_\beta \sigma_y \), where \( t_\beta \) is the critical value of the Student’s t-distribution with the corresponding degree of freedom and confidence probability (\( \beta \)).

Using the method described above, regression models \( \hat{y}(t) \) were created for the tree-ring chronologies of the Buluus and Ulakhan-Taryn areas covering the periods from 1888 to 2004 and from 1888 to 2002 respectively. Regression line graphs \( f(t) \) were also made (Figure 1, Table 1).

4. Discussion of the results
The regression models show that there are particular patterns in tree ring growth among the pines studied:

1. Homogeneous tree-ring chronologies BULN1, BULS1, UT are found near aufeis deposits, where they are strongly influenced by aufeis, moderately influenced by temperature (correlation coefficients ranging from \(-0.43\) to \(-0.63\)), and weakly influenced by precipitation (coefficients ranging from \(-0.05\) to \(-0.31\)). Their regression series have average linear correlation coefficients (\(0.41-0.63\)) and rather high nonlinear correlation coefficients (\(0.61-0.76\)).

2. BULN2 and BULN3 tree-ring chronologies (the slope with a northerly aspect) are close to being homogeneous. They are moderately influenced by aufeis, moderately influenced by temperature (\(-0.39;-0.41\)) and weakly influenced by precipitation (from \(0.04\) to \(-0.15\)). Their regression series are distinguished by very low linear (\(0.07-0.1\)) and high nonlinear coefficients (\(0.61-0.73\)).

3. Non-homogeneous BULS2 and BULS3 tree-ring chronologies (the slope with a southerly aspect) are weakly influenced by aufeis, moderately influenced by precipitation (\(-0.53; -0.54\)) and moderately influenced by temperature (\(-0.49; -0.54\)), which is reflected by high linear (\(0.61-0.76\)) and non-linear (\(0.76-0.65\)) coefficients.

![Figure 1. Tree growth graphs shown as deviations from the mean and regression graphs.](image)
5. Conclusion

1) The results of creating mathematical models for studying tree-ring chronologies of pine trees growing in the Ulakhan-Taryn and Buluus areas show that aueis formation can be analyzed using linear and nonlinear correlations in regression models.

2) In analyzing aueis deposits, the most informative regression models are those of homogeneous tree-ring chronologies obtained right near an aueis deposit regardless of the aspect of a slope.

3) According to the regression models discussed, the influence of aueis on trees growing in aueis areas extends everywhere regardless of the degree of homogeneity of tree-ring chronologies and the distance between model trees and aueis deposits.

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