Tunneling in the moving domain wall

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The mechanism of the parametrical stimulated tunneling in the spectrum of the moving domain wall considered. It is shown that such a mechanism can to cause the initial phase of the parametrical evolution of domain wall’s surface waves. In turn, this evolution leads to an appearance of essential features in domain walls dynamics.

In a middle 80th a new magnetic materials with an unique physical and technological properties became available for the investigations. It is so-called orthoferritin. Its magnetic nature orthoferritin [1] are the weak ferromagnets and may be considered within a framework of a double-sublattice model by introducing the vectors of ferromagnetism and antiferromagnetism so that \( \mathbf{m} = 0 \) and \( l^2 = 1 - m^2 \approx 1 \). In spherical coordinates \((l_x = \sin \theta \cos \varphi, l_y = \sin \theta \sin \varphi, l_z = \cos \theta)\) the Lagrangian density will be given by the expression

\[
\mathcal{L} = \frac{\chi}{2\gamma^2} \mathbf{l}^2(\mathbf{H}\mathbf{l}) - \Phi, \tag{1}
\]

where the thermodynamical potential \( \Phi \) is [2]

\[
\Phi = A(\nabla \mathbf{l})^2 - \frac{\chi}{\gamma} \left( \mathbf{H}^2 - (\mathbf{H}\mathbf{l})^2 \right) - M_x^0 H_x l_x - M_y^0 H_y l_y + K_{an} l_z^2 - K_{ab} l_z^2, \tag{2}
\]

and \( A \) - uniform exchange constant, \( K_{ac} \) and \( K_{ab} \) - anisotropy constants, \( \mathbf{H} \) - total external field, \( \chi \) - transverse susceptibility, \( M_x^0 \) and \( M_y^0 \) are the values of magnetization in the phases \( \Gamma_2(\mathbf{F}_2 \mathbf{C}_y \mathbf{G}_x) \) and \( \Gamma_4(\mathbf{G}_x \mathbf{A}_y \mathbf{F}_z) \), respectively. Corresponding Lagrange equation for the domain wall moving in a weak ferromagnet may be written as

\[
\theta'' \xi = -\sin \theta \cos \theta, \tag{3}
\]

where \( \xi = \frac{v - u}{\Delta}, \quad \Delta = \Delta_0 \sqrt{1 - \frac{u^2}{c^2}}, \quad \Delta_0 = \sqrt{\frac{A_{ac}+A_{ab}}{\alpha}} \) is a width of the resting wall. By \( v \) we denote the velocity of the wall, and \( c = \gamma \sqrt{A/\chi} \) is a limiting velocity for the domain walls motion, which coincides with a spin wave velocity.

One of the major circumstances which made the orthoferritin so unique, is an extremely high value of the limiting speed \( c \), which a several times as much the velocity of sound in such a materials \( (c = 2 \cdot 10^6 \text{ cm/s}) \). It makes possible very exotic processes of the interaction of domain walls with environment such as Cherenkov’s phonon radiation or the surface magnons emission. Recently last phenomena attract again the attention of the experimentalist due to improvement on experimental technique [3].

The early investigations in 80th reveals a very interesting features of the domain walls dynamics in yttrium orthoferritin and iron borate: an anomalous behavior of domain walls mobility was discovered. Domain walls velocity growth up monotonically with increasing of the external field in regular case. But for the some intervals of the field values the saturation of the mobility has been occurred: there was no increasing of velocity with a growth of a field. At the same time, later on, for the biggest magnitude of the field the increasing of velocity recommenced. Such a behavior leads to occurrence of the plateaus on the plots of \( v(H) \).

Some of such mobility anomalies may be easily interpreted as a result of Cherenkov’s processes: the emission of magnons and phonons with a different polarizations by the wall, because of the plateaus has been occurred for the values of walls velocities which coincides with corresponding velocities of the spin or elastic waves [4]. But for the explanation of another finding in experiments similar plateaus there were no appropriate Cherenkov’s processes and one must looking for another physical reasons, which may leads to occurrence of a such anomalous.

A suitable physical mechanism has been proposed by Zvezdin and Popkov [5], and Makhro and Kazakov [6]. Such a mechanism may be described as follows. Let’s domain wall will be moving in the periodically non-uniform magnetic field. Further, let’s some internal degrees of freedom are intrinsical for the domain wall, and we can to characterize theirs by some frequencies \( \omega_i \). These degrees of freedom will excites every time when the frequency of the perturbation arising due to moving through non-uniform media will coincides with one of an eigen frequencies \( \omega_i \): \( \omega_i = 2\pi u_i / a \), where \( a \) is a space period of the non-uniformity. In that way the outflow of the energy of the external field on the internal degrees of freedom will arise when wall will arrive at “resonant” velocities \( u_i \). As a consequence for such a velocities deceleration of the wall will occur.

Lately it became clear that interaction of the wall with media non-uniformities has a parametrical character [7], [10]: the effective elasticity of the wall which moving in periodically non-uniform media may be considered as a function of a time and a velocity of motion.

Indeed, if one use for the description of domain wall in the weak ferromagnet the model of a flat membrane,
than in the terms of the so-called “reduced” description (see Ref. [1]) one may write for the domain wall in periodically non-uniform media the equation of motion as

\[
\frac{\partial}{\partial t} (m \dot{x}) + m \dot{x} \Gamma - \nabla_\perp \phi \nabla_\perp x = 2M_0 H + \frac{U_0}{a} \sin \frac{2\pi x}{a},
\]

where \(x\) - is a coordinate of a center of a wall, \(m\) - the effective mass of the wall, \(\Gamma\) - phenomenological constant of a dissipation and \(\sigma = mc^2\).

Let’s suppose that the wall moving with a constant speed \(\dot{x} = u\) and then let’s denote by \(q\) deflection of the points of a walls surface from the equilibrium position. Then for the \(x = ut + q\) one can write

\[
m_\perp \ddot{q} + m_\perp \dot{q} \Gamma - m_\parallel c^2 \nabla_\perp^2 q = -\frac{4\pi^2 U_0}{a^2} q \cos \frac{2\pi ut}{a},
\]

\[
m_\perp = m_0 (1 - u^2/c^2)^{-3/2}, m_\parallel = m_0 (1 - u^2/c^2).
\]

The solution of Eq. (4) will be

\[
q = q_0 \exp \left( -\frac{\Gamma t}{2} - ik_\perp r_\perp \right) \Phi \left( \frac{\pi ut}{a} \right),
\]

where \(\Phi \left( \frac{\pi ut}{a} \right)\) is a solution of the Mathieu equation

\[
\ddot{\Phi} \left( \frac{\pi ut}{a} \right) + \left( \lambda + b \cos \frac{2\pi ut}{a} \right) \Phi \left( \frac{\pi ut}{a} \right) = 0,
\]

with a parameters

\[
\lambda = \frac{m_\parallel c^2 k^2 a^2}{\pi^2 m_\perp u^2} - \frac{\Gamma^2 a^2}{4\pi^2 u^2} \equiv \frac{a^2 [(c^2 - u^2)k^2 - \Gamma^2/4]}{\pi^2 u^2},
\]

\[
b = 2U_0/m_\perp u^2.
\]

Formally, one can say that for the velocities which determined by the condition \(\lambda = n^2\) (\(n\) - integer) the parametrical resonance of the surface vibrations will occur and the velocity of the wall will decrease. The values of a such resonant velocities are given by the expression

\[
u_n = \sqrt{\frac{a^2 c^2 k^2 - a^2 \Gamma^2 /4}{\pi^2 n^2 + a^2 k^2}}.
\]

Let’s note, however, that the Eq. (4) is not a self-consistent equation. Indeed, for any nonzero value of \(\Gamma\) in resonant case the velocity doesn’t stabilize to the one of \(u_n\) but become to decrease. In this reason the parameters of (6) go off the instability region and parametrical pumping into internal degrees of freedom come to the end. In principle, the problem required the solving of the system of coupled equations as follows

\[
\ddot{x} + \Gamma \dot{x} - \frac{1}{m_0} \left[ 2M_0 H - \frac{U_0}{a} \sin \frac{2\pi x}{a} \left( 1 - \frac{\dot{x}^2}{c^2} \right)^{1/2} \right] = 0,
\]

\[
\ddot{q} + \Gamma \dot{q} - c^2 \nabla_\perp^2 q \left( 1 - \frac{\dot{x}^2}{c^2} \right) + \frac{4\pi^2 q U_0}{m_0 a} \cos \frac{2\pi ut}{a} \left( 1 - \frac{\dot{x}^2}{c^2} \right) = 0,
\]

where \(F(T)\) is a braking force, which can be calculated as \(F_T = \frac{d}{dt} \sum_k m_\parallel k^2 q_k^2\). Numerical solving of the system (8)-(9) has been firstly carried out in Ref. [10]. The results of [11] (especially, a width and disposal of plateau) demonstrate a good qualitatively agreement with experiments.

But the next circumstance remained not clear enough: parametrical resonance for its beginnings required of existence of the initial “inoculating” flexure \(q_0\), without such a flexure the resonance can’t appear. It is clear that script of parametrical evolution of surface vibrations is very sensitive to the magnitude of the initial flexure. In Ref. [10] it was supposed that arising of the initial flexure is conditioned by the interaction of the wall with a non-regular magnetic defects of the media or by the random fluctuation of the external field. But it is difficult to agree with such an explanations. Indeed, in experiments the plateaus on the dependence \(\psi(H)\) has been observed always for the same velocities, it will be impossible if the initial flexure has a random value of a magnitude: the evolution of the surface vibrations must follows to the different scripts for the each time. Thus, the problem of explanation of the regularity of the plateaus disposal now as before is actual.

We propose to discuss the next mechanism which can not only to explain the initial flexure origin but also to be able to describe some another features in high-speed domain walls dynamics. This is a resonant tunneling in the spectrum of excitations of a weak ferromagnet. In such a spectrum the ground state, which corresponds to natural oscillations of the wall as a whole, is separated from the excited states which corresponds to states, with a surface (or Winter’s, [12] magnons, by the energy gap with a width \(\sigma\). The transition from the ground state into the first excited state just corresponds to the occurrence of the initial flexure. However, the width of a gap in orthoferritin as a rule is large enough and both tunneling and thermal activation has a neglected probability even for the room temperatures. Nevertheless, some mechanism exists which can to increase this probability and made it more or less appreciable. Such mechanism is a resonant stimulation of tunneling. For the first time it was described by Lin and Ballentine [14] who found by numerical simulation that a monochromatic external field acting

\footnote{Here and below we give all quantities per unit area of the domain wall.}
on a quartic double-well oscillator can increase the rate of coherent tunneling to values several orders of magnitude higher than those for the undriven system. Later, in Ref. [14] the similar problem of the thermal resonant stimulation of the the tunneling has been considered and it was shown that the frequency of stimulating factor must be close to the eigenfrequency of the inverted barrier’s potential. We propose to apply such an approach for the solving of our problem too.

Let’s map the initial problem of the tunneling between ground and first excited states onto particle problem [13]: such a double-level problem can be reduced to problem of the tunneling in the double-well potential

\[ V = \sigma (x^2 - \eta^2)^2. \]  

(10)

If one take into account the usual for the orthoferritin values \( K_{ac} = 1 \cdot 10^5 \text{ erg/cm}^3 \), \( A = 1 \cdot 10^{-7} \text{ erg/cm} \cdot M_0^0 = 10 \text{ emu}, c = 2 \cdot 10^6 \text{ cm/s}, \) then \( \sigma_0 = mc^2 = 4\sqrt{AK_{ac}} = 0.4 \text{ erg/cm}^2\), and if one accept the usual for the real experiments (see, for example Ref. [14]) values of the walls square \( S \sim 10^{-8} \text{ cm}^2\), then for the \( \sigma \) one obtain the value \( 10^{-15} \text{ erg} \). Such a values as it was mentioned above gives for both the usual tunneling rate and probability of the thermal activation the neglected values.

We shall presume here as in Ref. [13] that parametrical disturbance will be acted on the domain wall moving through a periodical magnetic non-uniform media because of its “elasticity” (and, as consequence \( m \) and \( \sigma \)) becomes a periodical functions of the time and the velocity of moving.

Let’s, for the concreteness, the ground state corresponds the localization of the particle in the left well of the effective potential. The wave function in such a case one can close approximate by the wave function of the ground state of the harmonic oscillator with the eigenfrequency \( \omega_0 = \sqrt{V''(x_{min} = 0)/m} = \sqrt{8\sigma \eta^2/m} \):

\[ \Psi_0 = e^{-1/2(\xi^2 + i\eta)}, \]  

(11)

where

\[ \xi = x\sqrt{\frac{m \omega_0}{h}}, \tau = \omega_0 t. \]

The evolution of Eq. (11) can be described by

\[ \frac{\partial^2 \Psi}{\partial \xi^2} - \xi^2(1 - \beta \cos 2\tau \eta)\Psi = -2i \frac{\partial \Psi}{\partial \tau}, \]  

(12)

where \( \beta \) is coefficient of parametrical modulation, and \( \tau \) is a frequency of parametrical perturbation (with the assumption of the fine adjustment on the main resonance \( \tau = \pi u/a \)). The solution of Eq. (12) one can found as

\[ \Psi = e^{-\eta \xi^2 + k}. \]  

(13)

Eq. (12) has a sense when the next conditions will be fulfilled

\[ k = i \int_0^\tau y d\tau, \]  

(14)

and

\[ 2i \frac{dy}{d\tau} - 4y^2 + 1 + \beta \cos 2\tau = 0. \]  

(15)

By the introduction of the function \( u(\tau) \):

\[ 2iuy = \frac{du}{d\tau}, \]

Eq. (15) may be reduced to the canonical form

\[ \frac{d^2 u}{d\tau^2} + (1 + \beta \cos 2\tau)u = 0. \]  

(16)

In the case of the fine resonance the solution of Eq. (16) is

\[ u = e^{\Delta \tau} \cos(\tau + \frac{1}{4}\pi) + Ce^{-\Delta \tau} \cos(\tau - \frac{1}{4}\pi), \]  

(17)

where \( \Delta = \frac{1}{4} \beta \) and \( C \) is a constant of integration.

After returning to \( y \) one can obtain

\[ y(\tau) = e^{\Delta \tau} \sin(\tau + \frac{1}{4}\pi) + Ce^{-\Delta \tau} \sin(\tau - \frac{1}{4}\pi), \]

(18)

Therefore, the value of \( \mathcal{P} = \Theta \Theta \tau \) may be represented as (see [10])

\[ \mathcal{P}(\xi, \tau) \sim (\xi^2)^{-\infty/\epsilon} \xi^{-\epsilon/\epsilon^2(\xi^2)}, \]

(19)

where

\[ \xi^2 = \frac{1}{Re[4y]} = \frac{1}{2} \left( \frac{p^2 \cos^2 \theta + \frac{1}{p^2} \sin^2 \theta}{\sigma^2} \right), \]  

(20)

\[ p = e^{i\beta \tau}, \theta = \tau - \frac{1}{4}\pi. \]

The interpretation of Eq. (19)-(20) is evident enough: the wave packet, which initially was localized as a whole in left well became vigorously oscillate so that the probability amplitude take the non-zero values in right well. The transition coefficient may be easily found as an integral of probability density \( \mathcal{P}(\xi, \tau) \) between the classical turning points in the right well averaged by infinite time interval

\[ D = \int_{\xi_l}^{\xi_r} d\xi \langle \Psi \Psi^* \rangle \tau \in (0, \infty), \]

(21)

where

\[ \xi_{l, r} = \sqrt{\frac{\theta^2 \pm \sqrt{\theta^2 - 2\sigma^{-1/2} \sqrt{8\pi \eta}}}{2}}. \]

The numerical calculations show that penetrability coefficient will periodically changes with a growing of walls
velocity from zero to value close to 1 for the different values of the velocity. More over, the resonant velocities which determined by the “classical” condition of the parametrical resonance (7) demonstrate resonant behavior in the considered situation too: the magnitude of the penetrability coefficients reach maximal values just for the same velocities.

In Table 1 we show for the example some values of the penetrability coefficients calculated for the \( YFeO_3 \) (\( \beta = 1 \cdot 10^{-4} \)).

In conclusion let’s describe the general script of the parametrical evolution of the surface waves in the moving domain wall.

When the velocities of the wall are far from the resonance, the wall remains flat and no vibrations occur. It can be explained by the fact that ground state which corresponds to the flat wall are separated by the width energy gap from the excited states which corresponds to the wall with a surface waves.

But such a gap can be overcomed due to the parametrical tunneling, which probability reaches the macroscopical values when the wall’s velocity close to one of its resonant values. After excitation of the lowest mode it’s following evolution will be determined by the classical parametrical resonance, and due to the energy conservation the growth of the surface wave’s amplitude will leads to the braking of the wall, and the peculiarities of the mobility may occur likewise there were observed in experiments.

**TABLE I.** The probability of the resonant transitions in spectrum of moving domain wall for some resonant velocities

| Numbers of resonance | Resonant velocities (cm/s) | Probability D |
|----------------------|-----------------------------|---------------|
| 0                    | \( 2.0 \times 10^2 \)       | 1             |
| 1                    | \( 6.0663 \times 10^7 \)    | 0.934         |
| 3                    | \( 2.1102 \times 10^7 \)    | 0.865         |
| 5                    | \( 1.2707 \times 10^7 \)    | 0.649         |
| 10                   | \( 63630.0 \)               | 0.501         |

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