Research Article

Fuzzy $C_e$-$I$(ec, eo) and Fuzzy Completely $C_e$-$I$(rc, eo) Functions via Fuzzy e-Open Sets

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We introduced the notions of fuzzy $C_e$-$I$(ec, eo) functions and fuzzy completely $C_e$-$I$(rc, eo) functions via fuzzy e-open sets. Some properties and several characterizations of these types of functions are investigated.

1. Introduction

With the introduction of fuzzy sets by Zadeh [1] and fuzzy topology by Chang [2], the theory of fuzzy topological spaces was subsequently developed by several fuzzy topologist based on the concepts of general topology. In 2014, the concept of fuzzy e-open sets and fuzzy e-continuity and separations axioms and their properties were defined by Seenivasan and Kamala [3]. In this paper, we introduce the notion of fuzzy $C_e$-$I$(ec, eo) functions, fuzzy $C_e$-continuous, fuzzy completely $C_e$-$I$(rc, eo) functions, and fuzzy e-kernel via fuzzy e-open sets and studied their properties and several characterizations of these types of functions are investigated. In this paper, we denote fuzzy e-open, fuzzy e-closed, and fuzzy regular closed as, eo, ec, and rc, respectively.

2. Preliminaries

Throughout this paper, $(X, τ)$ and $(Y, σ)$ (or simply $X$ and $Y$) represent nonempty fuzzy topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let $μ$ be any fuzzy set of $X$. The fuzzy closure of $μ$, fuzzy interior of $μ$, fuzzy $δ$-closure of $μ$, and the fuzzy $δ$-interior of $μ$ are denoted by $cl(μ)$, $int(μ)$, $cl_δ(μ)$, and $int_δ(μ)$, respectively. A fuzzy set $μ$ of $X$ is called fuzzy regular open [4] (resp., fuzzy regular closed) if $μ = int(cl(μ))$ (resp., $μ = cl(int(μ))$).

The fuzzy $δ$-interior of fuzzy set $μ$ of $X$ is the union of all fuzzy regular open sets contained in $μ$. A fuzzy set $μ$ is called fuzzy $δ$-open [5] if $μ = int_δ(μ)$. The complement of fuzzy $δ$-open set is called fuzzy $δ$-closed (i.e., $μ = cl_δ(μ)$). A fuzzy set $μ$ of $X$ is called fuzzy $δ$-preopen [6] (resp., fuzzy $δ$-semi open [7]) if $μ \leq int(cl_δ(μ))$ (resp., $μ \leq cl(int_δ(μ))$). The complement of a fuzzy $δ$-preopen set (resp., fuzzy $δ$-semiopen set) is called fuzzy $δ$-preclosed (resp., fuzzy $δ$-semiclosed).

Definition 1. A fuzzy set $μ$ of a fuzzy topological space $X$ is called fuzzy e-open [3] if $μ \leq cl(int(μ)) \lor int(cl(μ))$. Fuzzy e-closed if $μ \geq cl(int_δ(μ)) \land int_δ(cl(μ))$.

The intersection of all fuzzy e-closed sets containing $μ$ is called fuzzy e-closure of $μ$ and is denoted by $fe-cl(μ)$ and the union of all fuzzy e-open sets contained in $μ$ is called fuzzy e-interior of $μ$ and is denoted by $fe-int(μ)$.

Definition 2. A mapping $f : X \to Y$ is said to be fuzzy e*-open [8] if the image of every fuzzy e-open set in $X$ is fuzzy e-open set in $Y$.

Definition 3. A function $f : X \to Y$ is called fuzzy e-irresolute [3]. $f^{-1}(λ)$ is fuzzy e-open in $X$ for every fuzzy e-open set $λ$ of $Y$.

Definition 4. A fuzzy set $μ$ is quasicoincident [9] with a fuzzy set $λ$ denoted by $μ\lambda$ iff there exist $x \in X$ such that $μ(x) + λ(x) > 1$. If $μ$ and $λ$ are not quasicoincident, then we write $μ\lambda < λ$. $μ \leq λ$ iff $μ\lambda$.

Definition 5. A fuzzy point $x_p$ is quasicoincident [9] with a fuzzy set $λ$ denoted by $x_p\lambda$ iff there exist $x \in X$ such that $p + λ(x) > 1$. 

Definition 6. A fuzzy topological space $(X, \tau)$ is said to be fuzzy $e\text{-}T_1$ [3] if for each pair of distinct points $x$ and $y$ of $X$ there exist fuzzy e-open sets $\mu$ and $\mu_2$ such that $x \in \mu_1$ and $y \notin \mu_2$ and $x \notin \mu_2$ and $y \notin \mu_1$.

Definition 7. A fuzzy topological space $(X, \tau)$ is said to be fuzzy $e\text{-}T_2$ [3] if for each pair of distinct points $x$ and $y$ of $X$ there exists disjoint fuzzy e-open sets $\eta$ and $\rho$ such that $x \in \eta$ and $y \notin \rho$.

Definition 8. A fuzzy topological space $X$ is said to be fuzzy weakly Hausdorff [10] if for each element of $X$ is an intersection of fuzzy regular closed sets.

Definition 9. A fuzzy topological space $X$ is said to be fuzzy e-normal [3] if for every two disjoint fuzzy closed sets $\rho$ and $\eta$ of $X$ there exist two disjoint fuzzy e-open sets $\mu$ and $\lambda$ such that $\eta \leq \mu$ and $\rho \leq \lambda$ and $\mu \wedge \lambda = 0$.

Definition 10. A fuzzy topological space $X$ is said to be fuzzy strongly normal [10] if for every two disjoint fuzzy closed sets $\eta$ and $\rho$ of $X$ there exist two disjoint fuzzy e-open sets $\mu$ and $\lambda$ such that $\eta \leq \mu$ and $\rho \leq \lambda$.

Definition 11. A fuzzy topological space $X$ is said to be fuzzy Urysohn [11] if for every distinct points $x$ and $y$ in $X$ there exist fuzzy open sets $\mu$ and $\lambda$ in $X$ such that $x \in \mu$ and $y \in \lambda$ and $\text{cl}(\mu) \wedge \text{cl}(\lambda) = 0$.

Definition 12. A space $(X, \tau)$ is called fuzzy S-closed [2] (resp., fuzzy e-compact [3]) if every fuzzy regular closed (resp., fuzzy e-open) cover of $X$ has a finite subcover.

Definition 13. A function $f : X \to Y$ is called fuzzy completely continuous [12] if $f^{-1}(\lambda)$ is fuzzy regular open in $X$ for every fuzzy open set $\lambda$ in $Y$.

Definition 14. A fuzzy filter base $\xi$ is said to be fuzzy rc-convergent [10] to a fuzzy point $x_\epsilon$ in $X$ if for any fuzzy regular closed set $\eta$ in $X$ containing $x_\epsilon$ there exists a fuzzy set $\rho \in \xi$ such that $\rho \leq \eta$.

Definition 15. A collection of fuzzy subsets $\Delta$ of a fuzzy topological spaces $X$ is said to form fuzzy filterbases [13] iff for every finite collection $\{\lambda_\alpha : \alpha = 1, 2, \ldots, n\}$, $\bigwedge_{\alpha=1}^{n} \lambda_\alpha \neq 0_X$.

3. Fuzzy $C_e\text{-}I$ (ec, eo) Functions

In this section, the notion of fuzzy $C_e\text{-}I$(ec, eo) functions is introduced and some characteristics and properties are studied.

Definition 16. A mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is called fuzzy $C_e\text{-}I$(ec, eo) if the inverse image of every fuzzy e-open set of $Y$ is fuzzy e-closed in $X$.

Remark 17. The concepts of fuzzy $C_e\text{-}I$(ec, eo) and fuzzy e-irresolute are independent notions as illustrated in the following example.

Example 18. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$ and the fuzzy sets $\mu_1, \mu_2, \nu$ be defined as follows:

\[
\begin{align*}
\mu_1 (a) &= 0.1, \\
\mu_2 (a) &= 0.7, \\
\nu (x) &= 0.2, \\
\mu_1 (b) &= 0.6, \\
\mu_2 (b) &= 0.5, \\
\nu (y) &= 0.2, \\
\mu_1 (c) &= 0.4, \\
\mu_2 (c) &= 0.6, \\
\nu (z) &= 0.4.
\end{align*}
\]

Let $\tau = \{0, 1, \mu_1, \mu_2, \mu_1 \vee \mu_2, \mu_1 \wedge \mu_2\}$ and $\sigma = \{0, \nu, 1\}$. Then, the mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is defined by $\varphi(a) = x$, $\varphi(b) = y$, $\varphi(c) = z$. Then, $\varphi$ is fuzzy $C_e\text{-}I$(ec, eo) but not fuzzy e-irresolute.

Example 19. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$ and the fuzzy sets $\eta_1, \eta_2, \rho$ are defined as follows:

\[
\begin{align*}
\eta_1 (a) &= 0.4, \\
\eta_2 (a) &= 0.3, \\
\rho (x) &= 0.4, \\
\eta_1 (b) &= 0.7, \\
\eta_2 (b) &= 0.5, \\
\rho (y) &= 0.5, \\
\eta_1 (c) &= 0.8, \\
\eta_2 (c) &= 0.2, \\
\rho (z) &= 1.
\end{align*}
\]

Let $\tau = \{0, 1, \eta_1, \eta_2\}$ and $\sigma = \{0, \rho, 1\}$. Then, the mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is defined by $\varphi(a) = x$, $\varphi(b) = y$, $\varphi(c) = z$. Then, $\varphi$ is fuzzy e-irresolute but not fuzzy $C_e\text{-}I$(ec, eo).

Definition 20. A mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is called fuzzy $C_e$-continuous if the inverse image of every fuzzy open set of $Y$ is fuzzy e-closed in $X$.

Remark 21. Every fuzzy $C_e\text{-}I$(ec, eo) function is fuzzy $C_e$-continuous, but not conversely from the following example.
Example 22. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$ and the fuzzy sets $\mu_1, \mu_2, \nu, v$ are defined as follows:

\[
\begin{align*}
\mu_1(a) &= 0.6, \\
\mu_2(a) &= 0.4, \\
\nu(a) &= 0.8, \\
\nu(x) &= 0.3, \\
\mu_1(b) &= 0.5, \\
\mu_2(b) &= 0.7, \\
\nu(b) &= 0.5, \\
\nu(y) &= 0.4, \\
\mu_1(c) &= 0.3, \\
\mu_2(c) &= 0.5, \\
\nu(c) &= 0.3, \\
\nu(z) &= 0.6.
\end{align*}
\]

Let $\tau = \{0, 1, \mu_1, \mu_2, \mu_1 \lor \mu_2, \mu_1 \land \mu_2\}$ and $\sigma = \{0, \nu, 1\}$. Then, the mapping $\varphi : (X, \tau) \rightarrow (Y, \sigma)$ is defined by $\varphi(a) = x$, $\varphi(b) = y$, and $\varphi(c) = z$. Then, $\varphi$ is fuzzy $C_e$-continuous but not fuzzy $C_e-I(\text{ec}, \text{eo})$ as the fuzzy set $v$ is fuzzy $e$-open in $Y$ but $\varphi^{-1}(v)$ is not fuzzy $e$-closed set in $X$.

Theorem 23. For a fuzzy function $\varphi : X \rightarrow Y$, if $\varphi(x_\epsilon) \vert \mu$, the inverse image of every fuzzy $e$-closed set of $Y$ is fuzzy $e$-open in $X$ if for any $x_\epsilon \in X$, if $\varphi(x_\epsilon) \vert \mu$, then $x_\epsilon \lnot \varphi^{-1}(\varphi^{-1}(\mu))$.

Proof. Let $\mu \leq Y$ be a fuzzy $e$-closed set and $\varphi(x_\epsilon) \vert \mu$. Then, $x_\epsilon \varphi^{-1}(\mu)$ and, by hypothesis, $\varphi^{-1}(\mu) = \text{fe-int}(\varphi^{-1}(\mu))$. We obtain, $x_\epsilon \varphi^{-1}(\varphi^{-1}(\mu))$. Converse can be shown easily.

Theorem 24. For a fuzzy function $\varphi : X \rightarrow Y$, if $\varphi(x_\epsilon) \vert \mu$, for any fuzzy $e$-closed set $\mu \leq Y$ and for any $x_\epsilon \in X$, $x_\epsilon \varphi^{-1}(\varphi^{-1}(\mu))$ if there exists a fuzzy $e$-open set $\delta$ such that $x_\epsilon \delta$ and $\varphi(\delta) \leq \mu$.

Proof. Let $\mu \leq Y$ be any fuzzy $e$-closed set and let $\varphi(x_\epsilon) \vert \mu$. Then, $x_\epsilon \varphi^{-1}(\varphi^{-1}(\mu))$. Take $\delta = \text{fe-int}(\varphi^{-1}(\mu))$ then $\varphi(\delta) = \varphi(\text{fe-int}(\varphi^{-1}(\mu))) \leq \varphi(\varphi^{-1}(\mu)) \leq \mu$, and $\delta$ is fuzzy $e$-open in $X$ and $x_\epsilon \delta$.

Conversely, let $\mu \leq Y$ be any fuzzy $e$-closed set and let $\varphi(x_\epsilon) \vert \mu$. By hypothesis, there exists fuzzy $e$-open set $\delta$ such that $x_\epsilon \delta$ and $\varphi(\delta) \leq \mu$. This implies, $\delta \leq \varphi^{-1}(\mu)$ and then $x_\epsilon \varphi^{-1}(\varphi^{-1}(\mu))$.

Theorem 25. For a fuzzy function $\varphi : X \rightarrow Y$, the following statements are equivalent:

1. $f$ is fuzzy $C_e-I(\text{ec}, \text{eo})$.
2. For every fuzzy $e$-closed set $\mu$ in $Y$, $\varphi^{-1}(\mu)$ is fuzzy $e$-open in $X$.
3. For every fuzzy open set $\mu$, $\varphi^{-1}(\text{fe-int}(\mu))$ is fuzzy $e$-closed.
4. For every fuzzy closed set $\eta$, $\varphi^{-1}(\text{fe-cl}(\eta))$ is fuzzy $e$-open.
5. For each $x_\epsilon \in X$ and each fuzzy $e$-closed set $\mu$ in $Y$ containing $\varphi(x_\epsilon)$, there exists a fuzzy $e$-open set $\rho$ in $X$ containing $x_\epsilon$ such that $\varphi(\rho) \leq \mu$.
6. For each $x_\epsilon \in X$ and each fuzzy $e$-open set $\mu$ in $Y$ noncontaining $\varphi(x_\epsilon)$, there exists a fuzzy $e$-closed set $\nu$ in $X$ noncontaining $x_\epsilon$ such that $\varphi^{-1}(\nu) \leq \nu$.

Proof. (1) $\Leftrightarrow$ (2): let $\rho$ be a fuzzy $e$-open set in $Y$. Then, $1_Y - \rho$ is fuzzy $e$-closed. By (2), $\varphi^{-1}(1_Y - \rho) = 1_X - \varphi^{-1}(\rho)$ is fuzzy $e$-open. Thus, $\varphi^{-1}(\rho)$ is fuzzy $e$-closed. Converse can be shown easily.

(1) $\Leftrightarrow$ (3): let $\mu$ be a fuzzy open set. Since $\text{fe-int}(\mu)$ is fuzzy $e$-open, then by (1) it follows that $\varphi^{-1}(\text{fe-int}(\mu))$ is fuzzy $e$-closed. The converse is easy to prove.

(2) $\Leftrightarrow$ (4): let $\eta$ be a fuzzy closed set. Since $\text{fe-cl}(\eta)$ is fuzzy $e$-closed set, then by (2) it follows that $\varphi^{-1}(\text{fe-cl}(\eta))$ is fuzzy $e$-open. The converse is easy to prove.

(2) $\Leftrightarrow$ (5): let $\mu$ be any fuzzy $e$-closed set in $Y$ containing $\varphi(x_\epsilon)$. By (2), $\varphi^{-1}(\mu)$ is fuzzy $e$-open set in $X$ and $x_\epsilon \in \varphi^{-1}(\mu)$. Take $\rho = \varphi^{-1}(\mu)$. Then, $\varphi(\rho) \leq \mu$. The converse can be shown easily.

(5) $\Leftrightarrow$ (6): let $\mu$ be any fuzzy $e$-open set in $Y$ noncontaining $\varphi(x_\epsilon)$. Then, $1 - \mu$ is a fuzzy $e$-closed set containing $\varphi(x_\epsilon)$. By (5), there exists a fuzzy $e$-open set $\rho$ in $X$ containing $x_\epsilon$ such that $\varphi(\rho) \leq 1 - \mu$. Hence, $\rho \leq \varphi^{-1}(1 - \mu) = 1 - \varphi^{-1}(\mu)$ and $\varphi^{-1}(\mu) \leq 1 - \rho$. Take $\nu = 1 - \rho$. We obtain that $\nu$ is a fuzzy $e$-closed set in $X$ noncontaining $x_\epsilon$. The converse can be shown easily.

Theorem 26. Let $\phi : X \rightarrow Y$ be a function and let $\varphi : X \rightarrow Y \times Y$ be the fuzzy graph function of $\phi$, defined by $\varphi(x_\epsilon) = (x_\epsilon, \phi(x_\epsilon))$ for every $x_\epsilon \in X$. If $\phi$ is fuzzy $C_e-I(\text{ec}, \text{eo})$, then $\phi$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$.

Proof. Let $\mu$ be a fuzzy $e$-closed set in $Y$; then, $1_X \times \mu$ is a fuzzy $e$-closed set in $X \times Y$. Since $\phi$ is fuzzy $C_e-I(\text{ec}, \text{eo})$, then $\phi^{-1}(\mu) = \phi^{-1}(1_X \times \mu)$ is fuzzy $e$-open in $X$. Thus, $\phi$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$.

Theorem 27. Let $\{Y_\lambda : \lambda \in \Lambda\}$ be a family of product spaces. If a function $\varphi : X \rightarrow \prod Y_\lambda$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$, then $P_\lambda \circ \varphi : X \rightarrow Y_\lambda$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$ for each $\lambda \in \Lambda$ where $P_\lambda$ is the projection of $\prod Y_\lambda$ onto $Y_\lambda$.

Proof. Let $\delta$ be any fuzzy $e$-open set in $Y_\lambda$. Since $P_\lambda$ is a fuzzy continuous and fuzzy open set, it is a fuzzy $e$-open set. Now $P_\lambda : \prod Y_\lambda \rightarrow Y_\lambda$, $P_\lambda^{-1}(\delta)$ is a fuzzy $e$-open in $\prod Y_\lambda$. Therefore, $P_\lambda$ is a fuzzy $e$-irresolute function. Now $(P_\lambda \circ \varphi)^{-1}(\delta) = \varphi^{-1}(P_\lambda^{-1}(\delta))$, since $\varphi$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$. Hence $\varphi^{-1}(P_\lambda^{-1}(\delta))$ is a fuzzy $e$-closed set, since $P_\lambda^{-1}(\delta)$ is a fuzzy $e$-open set. Hence, $P_\lambda \circ \varphi$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$.

Theorem 28. If the function $\varphi : \prod X_\lambda \rightarrow \prod Y_\lambda$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$, then $\varphi_\lambda : X_\lambda \rightarrow Y_\lambda$ is fuzzy $C_e-C_e-I(\text{ec}, \text{eo})$ for each $\lambda \in \Lambda$. 
Proof. Let $\lambda_0 \in \Lambda$ be an arbitrary fixed index and let $\nu_{\lambda_0}$ be any fuzzy e-open set of $Y_{\lambda_0}$; then, $[Y_{\lambda_0} \times \nu_{\lambda_0}]$ is fuzzy e-open in $[Y_{\lambda_0}]$, where $\lambda_0 \neq \mu \in \Lambda$. Since $\varphi$ is fuzzy $C_e$-I (ec, eo) function, then $\varphi^{-1}(Y_{\lambda_0} \times \nu_{\lambda_0}) = \bigsqcap X_{\mu} \times \nu_{\lambda_0}$ is fuzzy e-closed in $[X_{\lambda_0}]$ and hence $\varphi^{-1}_Y(Y_{\lambda_0})$ is fuzzy e-closed in $X_{\lambda_0}$. This implies $\varphi^{-1}_Y$ is fuzzy $C_e$-I (ec, eo).

Theorem 29. If $\varphi : X \to Y$ is fuzzy $C_e$-I (ec, eo) and $\delta$ is fuzzy closed set of $X$, then $\varphi^{-1}_Y : \delta \to Y$ is fuzzy $C_e$-I (ec, eo).

Proof. Let $\lambda$ be a fuzzy e-open set of $Y$; then, $(\varphi^{-1}_Y)^{-1}(\lambda) = \varphi^{-1}(\lambda \land \delta)$. Since $\varphi^{-1}(\lambda)$ and $\delta$ are fuzzy closed, hence $(\varphi^{-1}_Y)^{-1}(\lambda)$ is fuzzy e-closed in the relative topology of $\delta$.

Definition 30. The intersection of all fuzzy e-open set $\eta$ of a fuzzy topological space $(X, \tau)$ containing $\mu$ is called the fuzzy e-kernel of $\mu$ (briefly, fe-$K_{\mu}$), fe-$K_{\mu} = \bigsqcap \{ \eta : \mu \leq \eta \land \eta$ is fuzzy e-open set of $X \}$. The following properties hold for fuzzy sets $\mu, \lambda$ of $X$:

1. $x \in$ fe-$K_{\mu}$ iff $\mu \land \eta \neq 0$ for any fuzzy e-closed set $\eta$ containing $x$.
2. $\mu \leq$ fe-$K_{\mu}$ and $\mu =$ fe-$K_{\mu}$ if $\mu$ is fuzzy e-open in $X$.
3. $\mu \leq \tau$; then, fe-$K_{\mu} \subseteq$ fe-$K_{\lambda}$.

Theorem 31. For a fuzzy function $\varphi : X \to Y$, the following statements are equivalent:

1. $\varphi$ is fuzzy $C_e$-I (ec, eo).
2. $\varphi^{-1}(fe-cl(\mu)) \subseteq fe-K_{\varphi(\mu)}$ for every fuzzy set $\mu$ of $X$.
3. $fe-cl(\varphi^{-1}(\eta)) \subseteq \varphi^{-1}(fe-K_{\eta})$ for every fuzzy set $\eta$ of $Y$.

Proof. (1) $\Rightarrow$ (2): Let $\mu \leq \mu \leq Y \neq fe-K_{\varphi(\mu)}$. There exists a fuzzy e-closed set $\gamma$ in $Y$, such that $\gamma \neq \gamma \land \gamma = 0$. Therefore, $\varphi^{-1}(\varphi(\mu) \land \gamma) = 0$. This implies that $\mu \land \gamma \neq 0$ and fe-$K(\mu) \varphi^{-1}(0) = 0$. Thus, $\varphi^{-1}(\mu \land \gamma) = 0$ and $\varphi \neq \varphi^{-1}(fe-cl(\mu))$. Hence, $fe-cl(\varphi^{-1}(\eta)) \neq fe-K_{\varphi(\mu)}$.

(2) $\Rightarrow$ (3): Let $\eta \leq \mu$; then, $\varphi^{-1}(\eta) \leq \varphi^{-1}(\mu) \land \gamma$. By hypothesis, $\varphi^{-1}(fe-cl(\eta)) \neq fe-K_{\varphi^{-1}(\eta)} \leq fe-K_{\varphi^{-1}(\mu)}$. Hence, $fe-cl(\varphi^{-1}(\eta)) \neq \varphi^{-1}(fe-K_{\eta})$.

(3) $\Rightarrow$ (1): Let $\eta$ be any fuzzy e-open set of $Y$; we have $fe-cl(\varphi^{-1}(\eta)) \neq \varphi^{-1}(fe-K_{\eta}) = \varphi^{-1}(\eta)$, since $\eta$ is fuzzy e-open and fe-$K(\varphi^{-1}(\eta)) = \varphi^{-1}(\eta)$. This implies that $\varphi^{-1}(\eta)$ is fuzzy e-closed in $X$.

Definition 32. The fuzzy e-Frontier of a fuzzy set $\gamma$ of a fuzzy topological space $X$ is given by $fe-Fr(\gamma) = fe-cl(\gamma) \land fe-cl(1_X - \gamma)$.

Theorem 33. The fuzzy point $x_\gamma \in X$ such that $\varphi : X \to Y$ is not fuzzy $C_e$-I (ec, eo) is exactly the union of fuzzy e-Frontier if the inverse image of the fuzzy e-closed set in $Y$ contains $\varphi(x_{\gamma})$.

Proof. Suppose that $\varphi$ is not fuzzy $C_e$-I (ec, eo) at the point $x_{\gamma} \in X$; then there exists a fuzzy e-closed set $\gamma$ such that $\varphi^{-1}(\gamma) \neq \varphi(\mu) \land (1_Y - \gamma) \neq 0$ for all fuzzy e-open set $\mu$ such that $x_{\gamma} \in \mu$. It follows that $\mu \land \varphi^{-1}(1_Y - \gamma) \neq 0$ and hence $x_{\gamma} \in fe-cl(\varphi^{-1}(1_Y - \gamma)) = fe-cl(1_X - \varphi^{-1}(\gamma))$. Thus, $x_{\gamma} \in \varphi^{-1}(\gamma) \subseteq fe-cl(\varphi^{-1}(\gamma))$ and hence $x_{\gamma} \in fe-Fr(\varphi^{-1}(\gamma))$. Conversely, suppose that $x_\gamma \in fe-Fr(\varphi^{-1}(\gamma))$, $\gamma$ is fuzzy e-closed set of $Y$ containing $\varphi(x_{\gamma})$, and $\varphi$ is fuzzy $C_e$-I (ec, eo) at $x_{\gamma} \in X$. There exists fuzzy e-open set $\mu$ such that $x_{\gamma} \in \mu$ and $\varphi(\mu) \leq \varphi(\gamma)$. Thus, $x_\gamma \in fe-int(\varphi^{-1}(\gamma))$ and hence $x_\gamma \in fe-Fr(\varphi^{-1}(\gamma))$ for each fuzzy e-closed set $\gamma$ of $Y$ containing $\varphi(x_{\gamma})$, a contradiction. Therefore, $\varphi$ is not fuzzy $C_e$-I (ec, eo).

Theorem 34. The following hold for functions $\varphi : X \to Y$ and $\eta : Y \to Z$:

(a) If $\varphi : X \to Y$ is fuzzy $C_e$-I (ec, eo) and $\varphi : Y \to Z$ is fuzzy $C_e$-continuous then $\varphi \circ \varphi : X \to Z$ is fuzzy $C_e$-continuous.

(b) If $\varphi : X \to Y$ is fuzzy $C_e$-I (ec, eo) and $\varphi : Y \to Z$ is fuzzy e-irresolute then $\varphi \circ \varphi : X \to Z$ is fuzzy $C_e$-I (ec, eo).

Theorem 35. If $\varphi : X \to Y$ is a fuzzy e-irresolute surjective function and $\varphi : Y \to Z$ is a fuzzy function such that $\varphi \circ \varphi : X \to Z$ is fuzzy $C_e$-continuous, then $\varphi$ is fuzzy $C_e$-I (ec, eo).

Proof. Let $\eta$ be any fuzzy closed set in $Z$. Since $\varphi \circ \varphi$ is fuzzy $C_e$-I (ec, eo), $(\varphi \circ \varphi)^{-1}(\eta)$ is fuzzy e-open in $X$. Therefore, $(\varphi \circ \varphi)^{-1}(\eta) \subseteq (\varphi \circ \varphi)^{-1}(\eta)$ is fuzzy e-open in $X$. Since $\varphi$ is fuzzy e-irresolute, surjection implies $(\varphi^{-1} \varphi^{-1}(\eta)) = \varphi^{-1}(\eta)$ is fuzzy e-open in $Y$. Thus, $\varphi$ is fuzzy $C_e$-I (ec, eo).

Theorem 36. If $\varphi : X \to Y$ is a fuzzy $e^*$-open surjective function and $\varphi : Y \to Z$ is a fuzzy function such that $\varphi \circ \varphi : X \to Z$ is fuzzy $C_e$-continuous, then $\varphi$ is fuzzy $C_e$-I (ec, eo).

Proof. Let $\eta$ be any fuzzy closed set in $Z$. Since $\varphi \circ \varphi$ is fuzzy $C_e$-continuous, $(\varphi \circ \varphi)^{-1}(\eta)$ is fuzzy e-open in $X$. Therefore, $(\varphi \circ \varphi)^{-1}(\eta) \subseteq (\varphi \circ \varphi)^{-1}(\eta)$ is fuzzy e-open in $X$. Since $\varphi$ is fuzzy $e^*$-open, surjection implies $(\varphi^{-1} \varphi^{-1}(\eta)) = \varphi^{-1}(\eta)$ is fuzzy e-open in $Y$. Thus, $\varphi$ is fuzzy $C_e$-I (ec, eo).

4. Fuzzy Completely $C_e$-I (rc, eo) Functions

In this section, the notion of fuzzy completely $C_e$-I (rc, eo) functions is introduced and the relation between other functions is studied and further some structure preservation properties are investigated.

Definition 37. A mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is called fuzzy completely $C_e$-I (rc, eo) if inverse image of every fuzzy e-open set in $Y$ is fuzzy regular closed in $X$. Example 38. Let $X = \{x, y, z\}$ and the fuzzy sets $\mu_1, \mu_2$ are defined as follows:

$$
\begin{align*}
\mu_1(x) & = 0.4, \\
\mu_2(x) & = 0.7, \\
\mu_1(y) & = 0.6, \\
\mu_2(y) & = 0.3, \\
\mu_1(z) & = 0.1, \\
\mu_2(z) & = 0.5.
\end{align*}
$$
Let $\tau = \{0, 1, \mu_1, \mu_2, \mu_1 \vee \mu_2, \mu_1 \wedge \mu_2\}$ and $\sigma = \{0, \mu_1, \mu_1 \wedge \mu_2, 1\}$. Then, the mapping $\varphi : (X, \tau) \to (X, \sigma)$ is defined by $\varphi(x) = 1 - x$. Then, $\varphi$ is fuzzy completely $C_e(I)(rc, eo)$. 

**Remark 39.** Every fuzzy completely $C_e(I)(rc, eo)$ function is fuzzy $C_e(I)(ec, eo)$ and fuzzy $C_e$-continuous, but the converse is not true, which can be seen in the following example.

**Example 40.** Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$ and the fuzzy sets $\mu_1, \mu_2$ and $\nu$ are defined as follows:

$$
\begin{align*}
\mu_1(a) &= 0.6, \\
\mu_2(a) &= 0.3, \\
\nu(x) &= 0.4, \\
\mu_1(b) &= 0.5, \\
\mu_2(b) &= 0.7, \\
\nu(y) &= 0.3, \\
\mu_1(c) &= 0.2, \\
\mu_2(c) &= 0.8, \\
\nu(z) &= 0.1.
\end{align*}
$$

Let $\tau = \{0, 1, \mu_1, \mu_2, \mu_1 \vee \mu_2, \mu_1 \wedge \mu_2\}$ and $\sigma = \{0, \nu, 1\}$. Then, the mapping $\varphi : (X, \tau) \to (Y, \sigma)$ is defined by $\varphi(a) = x$, $\varphi(b) = y$, $\varphi(c) = z$. Then, $\varphi$ is fuzzy $C_e$-continuous and also fuzzy $C_e(I)(ec, eo)$ but not fuzzy completely $C_e(I)(rc, eo)$ as the fuzzy set $\nu$ is fuzzy e-open in $Y$ but $\varphi^{-1}(\nu)$ is not fuzzy regular closed set in $X$.

From the above examples, we have the following implications.

- **Fuzzy completely $C_e(I)(rc, eo)$**
- **Fuzzy $C_e(I)(ec, eo)$**
- **Fuzzy $C_e$-continuous**

None of these implications is reversible.

**Theorem 41.** For a fuzzy function $\varphi : X \to Y$, if $\varphi(x)_{\mu} \mu$, the inverse image of every fuzzy e-closed set of $Y$ is fuzzy $\delta$-open in $X$ iff for any $x \in X$ if $\varphi(x)_{\mu} \mu$, then $x, qint_3(\varphi^{-1}(\mu))$.

**Proof.** Let $\mu \leq Y$ be a fuzzy e-closed set and $\varphi(x)_{\mu} \mu$. Then, $\nu, q\varphi^{-1}(\mu)$ and, by hypothesis, $\varphi^{-1}(\mu) = int_3(\varphi^{-1}(\mu))$. From here, $x, qint_3(\varphi^{-1}(\mu))$. The converse can be shown easily.

**Theorem 42.** For a fuzzy function $\varphi : X \to Y$, if $\varphi(x)_{\mu} \mu$, for any fuzzy e-closed set $\mu \leq Y$ and for any $x \in X$, $x, qint_3(\varphi^{-1}(\mu))$ iff there exists a fuzzy $\delta$-open set $\theta$ such that $x, q\theta$ and $\varphi(\theta) \leq \mu$.

**Proof.** Let $\mu \leq Y$ be any fuzzy e-closed set and let $\varphi(x)_{\mu} \mu$. Then, $x, qint_3(\varphi^{-1}(\mu))$. Take $\theta = int_3(\varphi^{-1}(\mu))$; then, $\varphi(\theta) = \varphi(int_3(\varphi^{-1}(\mu))) \leq \varphi^{-1}(\mu) \leq \mu$; $\theta$ is fuzzy $\delta$-open in $X$ and $x, q\theta$.

Conversely, let $\mu \leq Y$ be any fuzzy e-closed set and let $\varphi(x)_{\mu} \mu$. By hypothesis, there exists fuzzy $\delta$-open set $\theta$ such that $x, q\theta$ and $\varphi(\theta) \leq \mu$. This implies $\theta \leq \varphi^{-1}(\mu)$ and then $x, qint_3(\varphi^{-1}(\mu))$.

**Theorem 43.** For a fuzzy function $\varphi : X \to Y$, the following statements are equivalent:

1. $f$ is fuzzy completely $C_e(I)(rc, eo)$.
2. For every fuzzy e-closed set $\mu$ in $Y$, $\varphi^{-1}(\mu)$ is fuzzy regular open in $X$.
3. For every fuzzy open set $\mu$, $\varphi^{-1}(fe-int(\mu))$ is fuzzy regular closed.
4. For every fuzzy closed set $\eta$, $\varphi^{-1}(fe-cl(\eta))$ is fuzzy regular open.
5. For each $x_\epsilon \in X$ and each fuzzy e-closed set $\mu$ in $Y$ containing $\varphi(x_\epsilon)$, there exists a fuzzy regular open set $\rho$ in $X$ containing $x_\epsilon$ such that $\varphi(\rho) \leq \mu$.
6. For each $x_\epsilon \in X$ and each fuzzy e-open set $\mu$ in $Y$ noncontaining $\varphi(x_\epsilon)$, there exists a fuzzy regular closed set $\nu$ in $X$ noncontaining $x_\epsilon$ such that $\varphi^{-1}(\nu) \leq \mu$.

**Proof.** (1) $\Leftrightarrow$ (2): let $\mu$ be a fuzzy e-open set in $Y$. Then, $1-\mu$ is fuzzy e-closed. By (2), $\varphi^{-1}(1-\mu) = 1-\varphi^{-1}(\mu)$ is fuzzy regular open. Thus, $\varphi^{-1}(\rho)$ is fuzzy regular closed. Thus, $\varphi$ is fuzzy completely $C_e(I)(rc, eo)$. The converse can be shown easily.

(1) $\Leftrightarrow$ (3): let $\mu$ be a fuzzy open set. Since $fe-int(\mu)$ is fuzzy e-open, then by (1) it follows that $\varphi^{-1}(fe-int(\mu))$ is fuzzy regular closed. The converse is easy to prove.

(2) $\Leftrightarrow$ (4): let $\eta$ be a fuzzy closed set. Since $fe-cl(\eta)$ is fuzzy e-closed, then by (2) it follows that $\varphi^{-1}(fe-cl(\eta))$ is fuzzy regular open. The converse is easy to prove.

(2) $\Leftrightarrow$ (5): let $\mu$ be any fuzzy e-closed set in $Y$ containing $\varphi(x_\epsilon)$. By (2), $\varphi^{-1}(\mu)$ is fuzzy regular open set in $X$ and $x_\epsilon \in \varphi^{-1}(\mu)$. Take $\rho = \varphi^{-1}(\mu)$. Then, $\varphi(\rho) \leq \mu$. The converse can be shown easily.

(5) $\Leftrightarrow$ (6): let $\mu$ be any fuzzy e-open set in $Y$ noncontaining $\varphi(x_\epsilon)$. Then, $1-\mu$ is a fuzzy e-closed set containing $\varphi(x_\epsilon)$. By (5), there exists a fuzzy regular open set $\rho$ in $X$ containing $x_\epsilon$ such that $\varphi(\rho) \leq 1-\mu$. Hence, $\rho \leq \varphi^{-1}(1-\mu) = 1-\varphi^{-1}(\mu)$ and $\varphi^{-1}(\mu) \leq 1-\rho$. Take $\nu = 1-\rho$. We obtain that $\nu$ is a fuzzy regular closed set in $X$ noncontaining $x_\epsilon$. The converse can be shown easily.

**Theorem 44.** Let $\varphi_1 : X \to Y$ be a function and let $\varphi_2 : X \to X \times Y$ be the fuzzy graph function of $\varphi_1$, defined by $\varphi_2(x_\epsilon) = (x_\epsilon, \varphi_1(x_\epsilon))$ for every $x_\epsilon \in X$. If $\varphi_2$ is fuzzy completely $C_e(I)(rc, eo)$, then $\varphi_1$ is fuzzy completely $C_e(I)(rc, eo)$.

**Proof.** Let $\mu$ be a fuzzy e-closed set in $Y$; then, $1_X \times \mu$ is a fuzzy e-closed set in $X \times Y$. Since $\varphi_2$ is fuzzy completely $C_e(I)(rc, eo)$, then $\varphi_1^{-1}(\mu) = \varphi_2^{-1}(1_X \times \mu)$ is fuzzy regular open in $X$. Thus, $\varphi_1$ is fuzzy completely $C_e(I)(rc, eo)$.
Theorem 45. The following holds for functions $\varphi_1: X \rightarrow Y$ and $\varphi_2: Y \rightarrow Z$:

(a) If $\varphi_1: X \rightarrow Y$ is fuzzy $C_e^{-1}(rc, eo)$ and $\varphi_2: Y \rightarrow Z$ is fuzzy completely $C_e^{-1}(rc, eo)$, then $\varphi_2 \circ \varphi_1: X \rightarrow Z$ is fuzzy e-irresolute.

(b) If $\varphi_1: X \rightarrow Y$ is fuzzy completely $C_e^{-1}(rc, eo)$ and $\varphi_2: Y \rightarrow Z$ is fuzzy $C_e^{-1}$-continuous, then $\varphi_2 \circ \varphi_1: X \rightarrow Z$ is fuzzy completely continuous.

Definition 46. A fuzzy filter base $\xi$ is said to be fuzzy e-convergent to a fuzzy point $x_\xi$ in $X$ if for any fuzzy e-open set $\eta$ in $X$ containing $x_\xi$ there exists a fuzzy set $\rho \in \xi$ such that $\rho \leq \eta$.

Theorem 47. If a fuzzy function $\varphi: X \rightarrow Y$ is fuzzy completely $C_e^{-1}(rc, eo)$ for each fuzzy point $x_\xi \in X$, and each fuzzy filter base $\xi$ in $X$ is fuzzy rc-convergent to $x_\xi$, then the fuzzy filter base $\varphi(\xi)$ is fuzzy e-convergent to $\varphi(x_\xi)$.

Proof. Let $\xi = \{x_\xi\} \subseteq X$ be any fuzzy filter base in $X$ which is fuzzy rc-convergent to $x_\xi$. Since $\varphi$ is fuzzy completely $C_e^{-1}(rc, eo)$, then for any fuzzy e-open set $\eta$ in $Y$ containing $\varphi(x_\xi)$, there exists a fuzzy regular closed set $\rho \in \rho$ in $X$ containing $x_\xi$ such that $\varphi(\rho) \subseteq \eta$. Since $\xi$ is fuzzy rc-convergent to $x_\xi$, there exists a $\delta \in \xi$ such that $\delta \subseteq \rho$. This means that $\varphi(\delta) \subseteq \rho$ and therefore the fuzzy filter base $\varphi(\xi)$ is fuzzy e-convergent to $\varphi(x_\xi)$.

Theorem 48. If $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ surjection and $X$ is fuzzy S-closed, then $Y$ is fuzzy e-compact.

Proof. Suppose that $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ surjection and $X$ is fuzzy S-closed. Let $\{\eta_i\}_{i \in I}$ be a fuzzy e-closed cover of $Y$. Since $\varphi$ is a fuzzy completely $C_e^{-1}(rc, eo)$, then $\varphi^{-1}(\eta_i)$ is fuzzy regular closed cover of $X$ and hence there exists finite set $I_0 \subseteq I$ such that $X = \bigcup \{\varphi^{-1}(\eta_i) : i \in I_0\}$. Therefore, we have $Y = \bigcup \{\eta_i : i \in I_0\}$ and $Y$ is fuzzy e-compact.

Theorem 49. If $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ injection and $Y$ is fuzzy $e^{-1}T_1$, then $X$ is fuzzy weakly Hausdorff.

Proof. Suppose $Y$ is fuzzy $e^{-1}T_1$. For any distinct fuzzy points $x_\xi$ and $y_\mu$ in $X$, there exist fuzzy e-open sets $\eta$ and $\rho$ in $Y$. Since $\varphi$ is injective, $\varphi(x_\xi) \in \eta$, $\varphi(y_\mu) \notin \eta$, $\varphi(x_\xi) \in \rho$, and $\varphi(y_\mu) \notin \rho$. Since $\varphi$ is fuzzy completely $C_e^{-1}(rc, eo)$, $\varphi^{-1}(\eta)$ and $\varphi^{-1}(\rho)$ are fuzzy regular closed sets of $X$ such that $x_\xi \in \varphi^{-1}(\eta)$, $y_\mu \notin \varphi^{-1}(\eta)$, $x_\xi \notin \varphi^{-1}(\rho)$, and $y_\mu \in \varphi^{-1}(\rho)$. This shows that $X$ is fuzzy weakly Hausdorff.

Theorem 50. If $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ injection and $Y$ is fuzzy $e^{-1}$ normal, then $X$ is fuzzy strongly normal.

Proof. Let $\eta$ and $\rho$ be disjoint nonempty fuzzy closed sets of $X$. Since $\varphi$ is injective, $\varphi(\eta)$ and $\varphi(\rho)$ are disjoint fuzzy closed sets. Since $Y$ is fuzzy $e$-normal, there exist fuzzy $e$-open sets $\mu$ and $\lambda$ such that $\varphi(\eta) \subseteq \mu$ and $\varphi(\rho) \subseteq \lambda$ and $\mu \cap \lambda = 0$. This implies that $fe-cl(\mu)$ and $fe-cl(\lambda)$ are fuzzy $e$-closed sets in $Y$. Then, since $\varphi$ is fuzzy completely $C_e^{-1}(rc, eo)$, $\varphi^{-1}(fe-cl(\mu))$ and $\varphi^{-1}(fe-cl(\lambda))$ are fuzzy regular open sets. Then, $\eta \subseteq \varphi^{-1}(fe-cl(\mu))$ and $\rho \subseteq \varphi^{-1}(fe-cl(\lambda))$ and $\varphi^{-1}(fe-cl(\mu))$ and $\varphi^{-1}(fe-cl(\lambda))$ are disjoint; hence $X$ is fuzzy strongly normal.

Definition 51. A fuzzy topological space $(X, \tau)$ is said to be fuzzy $e^{-1}T_0$ ($r-T_0$) [14] if for every fuzzy set $\lambda$ of $X$ can be written in the form $\lambda = \bigvee_{i \in I} \lambda_i$, where $\lambda_i$ are fuzzy e-open (fuzzy regular open) or fuzzy e-closed (fuzzy regular closed) sets of $Y$.

Theorem 52. If $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ injection and $Y$ is fuzzy $e^{-1}T_0$, then $X$ is fuzzy $r-T_0$.

Proof. Let $\eta$ be a any fuzzy set of $X$. Since $Y$ is fuzzy $e^{-1}T_0$, $\varphi(\eta)$ is fuzzy e-open set of $Y$. Then, $\varphi(\eta) = \bigvee_{i \in I} \lambda_i$, where $\lambda_i$ are fuzzy e-open set or fuzzy e-closed sets of $Y$. Since $\varphi$ is completely $C_e^{-1}(rc, eo)$ injection we have $\eta = \varphi^{-1}(\varphi(\eta)) = \varphi^{-1}(\varphi(\varphi^{-1}(\varphi^{-1}(\eta_i)))) = \bigvee_{i \in I} \lambda_i$, where $\lambda_i$ are fuzzy regular open sets or fuzzy regular closed sets of $Y$. Thus, $X$ is fuzzy $r-T_0$.

Theorem 53. If $\varphi: X \rightarrow Y$ is a fuzzy completely $C_e^{-1}(rc, eo)$ injection and $Y$ is fuzzy $e^{-1}T_2$, then $X$ is fuzzy Urysohn.

Proof. Let $x_\xi$ and $y_\mu$ be any two distinct fuzzy points in $X$. Since $\varphi$ is injective, $\varphi(x_\xi) \neq \varphi(y_\mu)$. In $Y$ $\varphi$ is fuzzy $e^{-1}T_2$, there exist fuzzy e-open sets $\eta$ and $\rho$ such that $\varphi(x_\xi) \in \eta$ and $\varphi(y_\mu) \in \rho$. This implies that $fe-cl(\eta)$ and $fe-cl(\rho)$ are fuzzy e-closed sets in $Y$. Then, since $\varphi$ is fuzzy completely $C_e^{-1}(rc, eo)$, there exists fuzzy regular open sets $\delta$ and $\gamma$ in $X$ containing $x_\xi$, and $y_\mu$, respectively, such that $\varphi(\delta) \subseteq fe-cl(\eta)$ and $\varphi(\gamma) \subseteq fe-cl(\rho)$. This implies that $\delta \subseteq \varphi^{-1}(fe-cl(\eta))$ and $\gamma \subseteq \varphi^{-1}(fe-cl(\rho))$, we have that $\varphi^{-1}(fe-cl(\eta))$ and $\varphi^{-1}(fe-cl(\rho))$ are disjoint and hence $cl(\delta) \cap cl(\gamma) = 0$; by definition, $X$ is fuzzy Urysohn.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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