Research Article

Balance Harvest from the Forest as a Renewable Resource Using Game Theory

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Forests are among the most important renewable natural resources. While having a great range of benefits, the crucial advantage of forests is the production of oxygen and the absorption of pollutants. Thus, the current article attempts to examine forest exploitation as a source of wood to prevent forest overexploitation. Three smugglers, residents, and timber companies exploit this resource. There is a competition for the perception of this common source between these factors. In this article, these interactions are considered a game in order to obtain the most probable solution in the real competition between these factors by examining this game. This answer is the balance of the game. In order to use this answer, the behavior of the players can be predicted and used in management planning. Since the amount of withdrawals of operating companies depends on the current inventory (t), the player’s behavior is determined by the system’s current state and then Markov equilibrium was obtained for them.

1. Introduction

Forests have direct and indirect impacts on human life and other living organisms and can also serve as an infinite source of economic development if properly exploited. Forests also contribute to the control of surface water, recharge of groundwater aquifers, and soil production and conservation and can be the engine of herb production, wood production, and tourism industries [1–3].

Iran is a country with low forest cover. According to the FAO [4], the world has 3.454 billion hectares of forest cover [3], of which the share of Iran is only 0.4%. Most of Iran’s forests are concentrated in the north of the country. Considering the quantitative and qualitative features of Iran’s northern forests in terms of uniqueness (Hyrcanian forests), age, biodiversity, economic and social values, and the alarming rate of quantitative and qualitative loss of these forests, it is crucial to muster all resources for the preservation of this natural heritage [2].

Industrial-scale wood harvesting with stump removal can profoundly impact forest ecosystems and their elements and is believed to be one of the most important factors affecting the forest structure and plant composition [5]. However, small-scale wood smuggling by residents can also harm forests to the extent that the damage cannot be reversed by natural growth or even tree planting. It has been estimated despite the efforts of Mazandaran’s Natural Resources and Watershed Management Organization, only 30% of wood smuggling in this area is discovered, a deficiency that has been attributed due to the shortage of monitoring and enforcement personnel [3, 6]. Furthermore, some local residents are not willing to leave the forest and are adamant about expanding rural areas into forests [7, 8].
Any interaction between the individuals and groups that involves a conflict of interest can be modeled with the game theory. By modeling the conflict as a game between a set of players, this theory can help us predict and analyze how each person or group (each player) interacts with other parties to the conflict and what outcomes can be expected from these interactions [4, 9, 10]. Thus, this theory can be used to analyze the behavior of the players competing for a common-property renewable resource.

To analyze the behavior of the players, first, the conflict must be modeled as a game. Extensive research has been conducted on this modeling. In the work by Benchekroun et al. [6], an experimental study was conducted on linear and nonlinear Markov perfect equilibria of a game of symmetric competition for a common-property renewable resource. In the study of Sorger [7], this problem was modeled with the amenity value and extraction costs included in calculations, and the cooperative equilibrium and the Markov perfect equilibrium for the noncooperative game were obtained with the assumption of symmetry in resource exploitation. In the study of Antoniadou et al. [8], the Markov perfect Nash equilibrium of this game was obtained under the premise of symmetric exploitation and nonprobabilistic natural replenishment. In the study by Jorgenson et al. [9], a similar model was developed for a common-property fishery where players are n fishing companies.

The present study considers the case of a common-property renewable resource that three competing players are exploiting. Here, the common-property renewable resource is the forest cover (trees), and the players are timber companies, forest inhabitants, and smugglers, which do not have the same extraction conditions [10, 11]. For the timber companies, the extraction rate depends on the initial stock level and will decrease or even stop upon dropping below certain stock levels (implementing the forest recovery plan). Therefore, the players have a Markov strategy (feedback strategy) [12–14].

The results of this study, especially the developed model, can contribute to devising efficient plans for stopping the overexploitation of forests in the study area.

2. Modeling

Considering the relevant studies, it can be easily concluded the catastrophic role of overexploitation of forests in almost all areas of human lives and industry. Hence, it is attempted to develop a model to examine the practical ways of preventing the overexploitation of forests. The issue of competitive extraction from a common-property renewable resource was modeled as a noncooperative game with asymmetric strategies for the players. Then, the Markov equilibrium for this differential game was acquired utilizing HJB and differential equations [14–16].

Overall, this section clarifies the behavior of the players and how it was modeled. However, first, the factors influencing forest stock will be examined.

2.1. Natural Replenishment. Forest was considered a renewable resource with a natural regeneration rate based on the initial stock level. The natural replenishment function for a renewable resource is as follows:

\[
S(0) = S_0, \\
F(s) = \begin{cases} 
\delta S, & \text{for } S \leq S_y, \\
\delta S_y \left( \frac{S - S}{S - S_y} \right), & \text{for } S > S_y,
\end{cases}
\]  

where \( S_0 \) is the initial stock level, \( F(S) \) is the natural replenishment function (representing the recovery of the resource), \( F(S) \) is the stock limit (a stock-based threshold for growth rate), and \( S \) is the maximum carrying capacity of the habitat. The stock naturally grows at a rate of \( \delta \) until reaching the stock limit \( F(S) \), after which the growth continues at a lower or even negative rate [6].

2.2. Resource Extraction by Timber Companies. Resource extraction by timber companies (the first player) reduces the stock level at a rate of \( q_1(t) \), where \( t \) denotes time. However, these companies also plant as many trees as they harvest so that the planted seedlings can be harvested in the coming years. Although one can define different functions for the replenishment of stock at time \( t \) due to seedlings planted in previous years, in this study, this replenishment rate was considered to be 1.

2.3. Resource Extraction by Smugglers. The extraction of forest trees by smugglers (the second player) reduces the stock level at a rate of \( q_2(t) \), where \( t \) denotes time.

2.4. Resource Extraction by Forest Inhabitants. Research has shown that natural causes account for only 13% of the loss of natural resources in Iran, and the remaining 87% can be attributed to anthropogenic causes. Essential anthropogenic causes of the destruction of Iran’s northern forests are overgrazing, the devastation caused by herders, and the land use change by forest inhabitants [11]. Following adopting a policy of preventing grazing in certain forest areas, some of the local residents have resisted leaving the forest and are still actively exploiting the forest cover and reducing its stock level. Here, the rate of reduction of the stock level by these people is \( q_3(t) \), where \( t \) denotes time.

Since the companies’ extraction rate completely depends on the stock level at the extraction time, it can be considered to have the Markovian property. Thus, changes in the stock level over time can be formulated as follows:

\[
\frac{ds}{dt} = F(S(t)) + g_1(t) - q_1(S) - q_2(t) - q_3(t).
\]  

2.5. Payoff Functions. Timber companies: the price function for this player (Figure 1) was considered to be as follows:
Figure 1: The payoff for timber companies [11].

\[ P_1(t) = 1 - \frac{q_1(t)}{2}. \]  \hspace{1cm} (3)

This gives the payoff function as follows:

\[ u_1(q_1(t)) = P_1(t) \cdot q_1(t) = q_1(t) - \frac{q_1(t)^2}{2}. \]  \hspace{1cm} (4)

For timber companies, the payoff function will be maximized when \( q_1(t) = 1 \), i.e., when the first player exploits the entire amount added to the initial stock level. Thus, for this player, we have: (5)–(7)

\[ \max Z_1 = \int_0^\infty u_1(q_1(t))e^{-rt} dt, \]

\[ \text{s.t. } \frac{ds}{dt} = F(S(t)) + g_1(t) - q_1(t) - q_2(t) - q_3(t), \]

\[ S(0) = S_0, \]

\[ q_1(t) \geq 0. \]

Smugglers: smugglers often have to sell their timber at lower prices than legally operating timber companies. Therefore, the price function for smugglers was considered to be as follows:

\[ P_2(t) = 1 - a \cdot \frac{q_2(t)}{2}, \quad a \geq 1. \]  \hspace{1cm} (6)

The most profitable price condition for smugglers is \( a = 1 \), i.e., when smugglers and legal timber companies have the same sales conditions. Thus, the payoff function of smugglers shown in Figure 2 is as follows:

\[ u_2(q_2(t)) = P_2(t) \cdot q_2(t) = q_2(t) - a \cdot \frac{q_2(t)^2}{2}. \] \hspace{1cm} (7)

For smugglers, the payoff function will be maximized when \( q_2(t) = (1/a) \leq 1 \)

It should be noted that smugglers can also access the deep interior parts of the forest. To account for this access, we considered \( \gamma_1 \) to be the ratio of these parts to the total forest area in the region of interest. Thus, the model for smugglers was formulated as follows:

\[ \max Z_2 = \int_0^\infty u_2(q_2(t))e^{-rt} dt, \]

\[ \text{s.t. } \frac{ds}{dt} = F(S(t)) + g_1(t) - q_1(t) - q_2(t) - q_3(t), \]

\[ S(0) = S_0, \]

\[ q_2(t) \geq 0, \]

\[ q_2(t) \leq \gamma_1 S(t). \] \hspace{1cm} (8)

Forest inhabitants: for this player, we defined a utility function instead of the profit function (because forest inhabitants consume the harvested wood rather than sell it). The payoff function of forest inhabitants is increasing up to a certain point and then decreases as in Figure 3.

\[ u_2(q_2(t)) = q_2(t) - \frac{q_2(t)^2}{2b}. \] \hspace{1cm} (9)

This payoff function will be maximized when \( q_3(t) = b \geq 1 \).

Considering that forest inhabitants also have access to moderately interior parts of the forests, \( \gamma_2 \) was defined as the ratio of these parts to the total forest area.

With this definition, the model was formulated as follows:

\[ \max Z_3 = \int_0^\infty u_3(q_3(t))e^{-rt} dt, \]

\[ \text{s.t. } \frac{ds}{dt} = F(S(t)) + g_1(t) - q_1(t) - q_2(t) - q_3(t), \]

\[ S(0) = S_0, \]

\[ q_2(t) \geq 0, \]

\[ q_2(t) \leq \gamma_2 S(t). \] \hspace{1cm} (10)

3. Search for Equilibrium

The search for \( q_1(S) \) is conducted in the following range. Further explanation in this regard is provided in Appendix A.
\[
F(s) = \delta S(t). \tag{12}
\]

Using the Hamilton–Jacobi–Bellman equation \[12\] and the differential calculations \[13\], the equilibrium is obtained as follows (the complete solution is provided in the appendix):

\[
q_1(S) = \begin{cases} 
0, & S < S_s, \\
\delta S + g_1(t) - b - \frac{1}{a} \left( \frac{2\delta - r}{5\delta - 2r} \right) + \frac{3\delta - r}{5\delta - 2r}, & S_s < S < \frac{1 + (1/a) + b - g_1(t)}{\delta}, \\
\delta S + g_1(t) - b - \frac{1}{a} \left( \frac{2\delta - r}{5\delta - 2r} \right), & S > \frac{1 + (1/a) + b - g_1(t)}{\delta}.
\end{cases}
\]

\[
S_s = \frac{1}{\delta} \left( -g_1(t) + b + \frac{1}{a} \right) - \frac{3\delta - r}{(2\delta - r)\delta}.
\]

\[
q_2(t) = (\delta S + g_1(t) - b - 1) \left( \frac{2\delta - r}{5\delta - 2r} \right) + \frac{3\delta - r}{a(5\delta - 2r)} \quad 0 < t < T,
\]

\[
q_3(t) = (\delta S + g_1(t) - 1) \left( \frac{2\delta - r}{5\delta - 2r} \right) + \frac{1}{a} \left( \frac{2\delta - r}{5\delta - 2r} \right) + \frac{b(3\delta - r)}{5\delta - 2r} \quad 0 < t < T,
\]

\[
S(t) = \left( -g_1(t) + \frac{1}{a} + b \right) \left( \frac{1}{\delta} \right) + \left[ S_0 - \left( \frac{1}{a} + b \right) \left( \frac{1}{\delta} \right) \right] e^{-(r - \delta)\delta/5\delta - 2r}.
\]

The range of the extraction rate for the first player is given in Appendix B.

The companies’ extraction rate starts to increase when the stock level reaches \( S_s \) and continues to increase until reaching 1 at \( S = 1 + (1/a) + b \), at which point the companies’ payoff function will be at its highest level.

Based on the results acquired, enforcing this range is crucial for preventing a sharp decline in the forest stock, as the natural growth rate will not be able to compensate for higher extraction rates. Also, the functions \( q_3(t) \) and \( q_3(t) \) were formulated to estimate the rate of decrease in the forest stock at time \( t \) due to the harvest of wood by timber smugglers and forest inhabitants. The developed model can be used to devise plans for stopping the overexploitation of forests in the study area.

4. Conclusions

In this study, the problem of competitive extraction from a common-property renewable resource was modeled as a noncooperative game with asymmetric strategies for the players. Ultimately, the Markov equilibrium for this differential game was obtained by the use of HJB and differential equations. Also, the range of stock levels at which timber companies will be free to extract timber was calculated. Considering the study’s results, implementing this range seems vital to prevent a radical drop in the forest stock, as the natural growth rate cannot compensate for higher extraction rates.

Future studies are recommended to use the model in a case study. It is also recommended to expand the model for uncertain payoff functions and obtain the equilibria for the cooperative game (Pareto frontier) and compare the results with the results of this study.

Appendix

A. The Range of \( S \)

The range of \( S \) for the first player.

If the stock increase is greater than the extraction rate that gives each player the greatest payoff (i.e., 1 for the first player, \( 1/a \) for the second player, and \( b \) for the third player), then the players will adopt the same extraction rates that gives them these payoffs.
\[ \delta S + g_1(t) \geq 1 + \frac{1}{a} + b, \quad (A.1) \]

\[ S \geq \frac{1 + 1/a + b - g_1(t)}{\delta}. \]

In this range, each player gets the highest payoff. However, if the stock increase is lower than this amount, since the first player’s extraction rate is stock-dependent, it will start to compete with other players.

\[ \delta S + g_1(t) \leq 1 + \frac{1}{a} + b. \quad (A.2) \]

Therefore, the range of interest will be equal to

\[ \left[ 0, \frac{1 + (1/a) + b - g_1(t)}{\delta} \right]. \quad (A.3) \]

**B. For the First Player**

Based on the HJB equality [12], it holds that

\[ rV_1(S) = \max \left[ q_1(t) - \frac{q_1(t)^2}{2} + \frac{dV_1}{ds} (\delta S + g_1(t) - q_2(t) - q_3(t)) \right], \quad (B.1) \]

Placing \( q_1(t) \) in the HJB equation gives

\[ rV_1(S) = \frac{(1 - V_1')^2}{\delta} + V_1' (\delta S + g_1(t) - q_2(t) - q_3(t)). \quad (B.2) \]

Taking derivative with respect to \( S \), we arrive at

\[ (r - \delta)V_1' = V_1'(V_1' + \delta S + g_1(t) - q_3(t) - 1). \quad (B.3) \]

For the function to be reversible,

\[ q_1 \neq \delta S + g_1(t) - q_2(t) - q_3(t). \quad (B.4) \]

Putting \( P \) in place of \( V_1' \) in Equation (B.4) gives

\[ (r - \delta)P \]

\[ \frac{dS}{dP} = \frac{P + \delta S + g_1(t) - q_2(t) - q_3(t) - 1}{(r - \delta)}. \quad (B.5) \]

By solving this differential equation, we arrive at [13]

\[ S(P) = \frac{P}{2\delta + r} + \frac{1}{\delta} (g_1(t) - q_2(t) - 1) + CP^{\delta r - \delta}, \]

\[ q_1(S) = \left( \frac{1 - q_1}{1 - \delta} + \frac{1}{-2\delta + r - \delta} \right) (g_1(t) - q_2(t) - q_3(t) - 1) + C(1 - q_1)^{\delta r - \delta}. \quad (B.6) \]

This gives a linear Markov Nash equilibrium if \( C = 0 \) and a nonlinear Markov Nash equilibrium if \( C \neq 0 \). Here, we obtained the linear Markov Nash equilibrium.

\[ q_1(S) = 1 + S(2\delta - r) + (g_1(t) - q_2(t) - q_3(t) - 1) \left( \frac{2\delta - r}{\delta} \right). \quad (B.7) \]

Similarly, for the second and third players,

\[ q_2(t) = \frac{1}{a} + S(t) (2\delta - r) \]

\[ + \left( g_1(t) - q_1(S) - q_3(t) - \frac{1}{a} \left( \frac{2\delta - r}{\delta} \right) \right). \quad (B.8) \]

By solving this system of equations, we arrive at

\[ q_1(S) = \left( \delta S + g_1(t) - b - \frac{1}{a} \left( \frac{2\delta - r}{5\delta - 2r} \right) \right) + \frac{3\delta - r}{5\delta - 2r}. \]
Data Availability

The data used in this study are available from the corresponding author on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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