On the Age of Information for Multicast Transmission with Hard Deadlines in IoT Systems

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Abstract—We consider the multicast transmission of a real-time Internet of Things (IoT) system, where a server transmits time-stamped status updates to multiple IoT devices. We apply a recently proposed metric, named age of information (AoI), to capture the timeliness of the information delivery. The AoI is defined as the time elapsed since the generation of the most recently received status update. Different from the existing studies that considered either multicast transmission without hard deadlines or unicast transmission with hard deadlines, we enforce a hard deadline for the service time of multicast transmission. This is important for many emerging multicast IoT applications, where the outdated status updates are useless for IoT devices. Specifically, the transmission of a status update is terminated when either the hard deadline expires or a sufficient number of IoT devices successfully receive the status update. We first calculate the distributions of the service time for all possible reception outcomes at IoT devices, and then derive a closed-form expression of the average AoI. Simulations validate the performance analysis, which reveals that: 1) the multicast transmission with hard deadlines achieves a lower average AoI than that without hard deadlines; and 2) there exists an optimal value of the hard deadline that minimizes the average AoI.

I. INTRODUCTION

Internet of Things (IoT) provides ubiquitous wireless connectivity and automated information delivery for massive devices that have the capabilities of monitoring, processing, and communication [1]. Many emerging IoT applications, including environment monitoring, smart metering, and autonomous transportation, require timely information delivery, i.e., the statuses observed at the receivers are always fresh. The conventional performance metrics (e.g., throughput and delay) cannot adequately capture the information freshness. Fortunately, the information freshness can be well characterized by a recently proposed performance metric termed as Age of Information (AoI) [2]. The AoI at a receiver is defined as the time difference between the current time and the generation time of the most recently received status update.

The analysis and optimization of the AoI performance for various systems have recently attracted considerable attention [3–9]. Relying on queueing theory, the authors in [3] analyzed the average AoI for different queueing models. Results in [3] showed that minimizing the AoI is different from minimizing the delay, because the delay does not capture the inter-delivery time of status updates. This seminal analysis was further extended in [4] and [5] to show that the average AoI can be decreased by reducing the buffer size and increasing the server number, respectively. With heterogeneous service time distributions, the authors in [6] derived the average AoI for an $M/G/1$ queueing system. The authors in [7] proposed age-minimal scheduling schemes that jointly optimize the status sampling and updating for IoT networks. The authors in [8] studied the minimum AoI with non-uniform status packet sizes in IoT networks. Besides, the tradeoff between AoI and energy efficiency of IoT monitoring systems was characterized in [9], where a limited number of retransmissions are allowed for each status update. We note that these studies mainly focused on the status update systems with unicast transmission.

Multicast transmission can simultaneously serve multiple receivers that are interested in the same information. The timely delivery of these popular information requested by multiple IoT devices is critical for many emerging IoT applications. For example, in a connected vehicular network, the status update of an autonomous vehicle, especially the safety warning message, needs to be timely disseminated to the nearby vehicles and pedestrians. In a smart parking lot, a server continuously collects the occupancy information of all parking spaces and reports the locations of the vacant parking spaces to the nearby drivers. The authors in [10], [11] derived the average AoI of a multicast system, where a status update is dropped as soon as it has been successfully received by a certain number of receivers. The tradeoff between the energy efficiency and the average AoI in multicast systems was studied in [12], where a scheduling strategy based on the optimal stopping theories was proposed. In [13], the authors studied the average AoI in a two-hop multicast network. In addition, the authors in [14] proposed several scheduling policies to minimize the average AoI for broadcast transmission over unreliable channels. However, the aforementioned studies on multicast transmission did not take into account the hard deadline. This is crucial for many real-time IoT applications, where the outdated status updates have little value to be delivered. It has been demonstrated in [15], [16] that the packet deadline has a significant impact on the average AoI for unicast transmission. Specifically, the authors in [15], [16] derived the closed-form expressions of the average AoI for $M/M/1$ and $M/G/1$ queueing systems, respectively, where the waiting time of each packet is subject to a hard deadline but the service time can be arbitrary large. Motivated by the
The time required to successfully deliver status update stamped and transmitted by the server once it is generated. We denote the sets of status updates and IoT devices as server transmitting multicast information to multiple IoT devices. Different from the existing studies that considered either multicast transmission without hard deadlines [10] or unicast transmission with hard deadlines [15], we enforce a hard deadline for the service time of multicast transmission. Once a status update is generated, it is time-stamped and transmitted by the server. The server terminates the transmission of a status update if either the deadline expires or a sufficient number of IoT devices successfully receive the status update. It is worth noting that the instantaneous AoI evolution is more complicated for multicast transmission with hard deadlines than that for the existing studies [11], [15]. This is because, the instantaneous AoI evolution in this paper depends on both the reception outcomes of multiple IoT devices and the hard deadline, whereas that in the existing studies only depends on one of these important factors. We explicitly show that the average AoI depends on the service time of multiple devices, the hard deadline, and the total number of IoT devices. The main contributions of this paper are summarized as follows.

- We derive the probability density functions (PDFs) of the service time by using order statistics for all possible reception outcomes at the receiving IoT devices, and obtain the first and second moments of the inter-generation time of two consecutive status updates.
- We derive the closed-form expression of the average AoI for multicast transmission with hard deadlines, which includes multicast transmission without hard deadlines, broadcast transmission with hard deadlines, and unicast transmission with hard deadlines as special cases.
- Simulations validate the theoretical analysis, which illustrates the impact of various parameters on the average AoI. Results also reveal that the average AoI of multicast transmission with hard deadlines is lower than that without hard deadlines, and the deadline can be further optimized to minimize the average AoI.

The rest of this paper is organized as follows. In Section II, we describe the system model and the AoI evolution. The average AoI of multicast transmission with hard deadlines is analyzed in Section III. The numerical results are presented in Section IV. Finally, Section V concludes this paper.

II. System Model

Consider a real-time IoT system consisting of a single server transmitting multicast information to N IoT devices. We denote the sets of status updates and IoT devices as \( \mathcal{J} = \{1, \ldots, j, \ldots \} \) and \( N = \{1, \ldots, n, \ldots, N \} \), respectively. The status updates are generated by the server and there is no random status update arrival. Each status update is time-stamped and transmitted by the server once it is generated. The time required to successfully deliver status update \( j \) to IoT device \( n \) is denoted as \( T_{n,j} \). We follow [10], [11] and assume that \( \{T_{n,j}\} \) are independent and identically distributed (i.i.d.) shifted exponential random variables with rate \( \lambda_x \) and positive constant shift \( c \). Hence, the PDF of \( T_{n,j} \) is \( f_T(t) = \lambda_x e^{-\lambda_x(t-c)} \), where \( t > c \). A status update is considered to be served when it is successfully received by at least \( K \) IoT devices, where \( K \leq N \). Once a device successfully receives a status update, it sends an acknowledgement (ACK) packet back to the server via an error-free and delay-free control channel. In addition, we consider that each status update subjects to a hard deadline, which is denoted as \( T_D \). Specifically, if a status update is not successfully received by \( K \) intended devices when the deadline expires, then this status update is considered to be useless to the IoT devices and immediately dropped by the server. The server terminates the transmission of the current status update (e.g., \( j \)) if it is either served or dropped, and meanwhile generates and time-stamps a new status update (e.g., \( j + 1 \)).

The instantaneous AoI of device \( n \) at time \( t \) is defined as \( \Delta_n(t) = t - u_n(t) \), where \( u_n(t) \) denotes the generation time of the most recently received status update at device \( n \) as of time \( t \). We depict the evolution of the instantaneous AoI at device \( n \) over time as the sawtooth pattern in Fig. 1. As can be observed, the instantaneous AoI increases linearly with time \( t \) and drops to a smaller value once a new status update containing fresher information is received.

To better describe the AoI evolution, we present the following definitions. We denote \( t_j \) as the time instant that the server generates status update \( j \in \mathcal{J} \). We define \( X_{n,j}^{K} = t_{j+1} - t_j \) as the inter-generation time of two consecutive status updates \( j \) and \( j + 1 \) if status update \( j \) fails to be received by device \( n \). Similarly, we define \( X_{n,j}^{S} = t_{j+1} - t_j \) as the inter-generation time of two consecutive status updates \( j \) and \( j + 1 \) if status update \( j \) is successfully received by device \( n \). Due to the hard deadline (i.e., \( T_D \)) and the randomness of service time \( T_{n,j} \), it is possible that some status updates cannot be successfully received by device \( n \). Thus, we further denote \( t'_{n,q} \) as the termination time of the \((q-1)\)-th status update that has been successfully received by device \( n \). As shown in Fig. 1, \( t'_{n,q} = t_j \) implies that the \((q-1)\)-th status update transmitted by the server is the \((q-1)\)-th status update successfully received by device \( n \), where \( j \geq q \). Note that we use \( j \) and \( q \) to index the status updates transmitted by the server and successfully

![Fig. 1: Age evolution of device \( n \) over time. The time instances that device \( n \) successfully receive status updates are marked by ●.](image-url)
received by the IoT device, respectively.

As \( \{T_{n,j}, n \in \mathcal{N}, j \in \mathcal{J} \} \) are i.i.d., the evolution processes of the instantaneous AoI for all IoT devices are statistically identical and hence each device ends up having the same average AoI. This allows us to focus on analyzing the average AoI of any device. We denote \( Q(T) = \max(q' | n_q' \leq T) \) as the number of status updates that have been successfully received by device \( n \) as of time \( T \). As in [3], the average AoI of device \( n \) is given by

\[
\bar{\Delta}_n = \lim_{T \to \infty} \frac{1}{T} \int_0^T \Delta_n(t) \, dt = \lim_{T \to \infty} \frac{Q(T)}{T} \frac{1}{Q(T)} \sum_{q=1}^{Q(T)} A_{n,q} = \frac{\mathbb{E}[A_{n,q}]}{\mathbb{E}[Y_{n,q}]},
\]

(1)

where \( \frac{Q(T)}{T} \) is the steady-state rate of the update delivery, \( A_{n,q} \) is the area of the shaded polygon under the sawtooth curve in Fig. [1] and \( Y_{n,q} = t_{n,q+1} - t_{n,q} \) denotes the time duration starting from the termination time of the \( (q-1) \)-th update to that of the \( q \)-th update at device \( n \). Based on Fig. [1] we have

\[
A_{n,q} = (X_{n,j-1}^S + W)\hat{T}_{n,q} + (2X_{n,j-1}^S + W)\frac{W}{2} + \frac{1}{2} \left( X_{n,j-M+1}^S \right)^2,
\]

(2)

where \( \hat{T}_{n,q} \) denotes the service time of the \( q \)-th status update successfully delivered to device \( n \). \( M \) is the number of status updates transmitted by the server during \([t_{n,q}, t_{n,q+1})\) and \( W = \sum_{j=i+1}^{j+M-2} X_{n,j}^F \). As \( \{X_{n,j}^F, j \in \mathcal{J}\} \) are i.i.d., we denote \( \mathbb{E}[X_{n,j}^F] = \mathbb{E}[X_n^F] \). As \( X_{n,j-1}^S, \hat{T}_{n,q} \), and \( X_{n,j}^F \) are independent of each other, the expectation of \( A_{n,q} \) can be expressed as

\[
\mathbb{E}[A_{n,q}] = \mathbb{E}[X_{n,j-1}^S] \mathbb{E}[\hat{T}_{n,q}] + \mathbb{E}[W] \mathbb{E}[\hat{T}_{n,q}] + \mathbb{E}[X_{n,j-M+1}^S] \mathbb{E}[W] + \frac{1}{2} \mathbb{E}[W^2] + \frac{1}{2} \mathbb{E} \left[ \left( X_{n,j-M+1}^S \right)^2 \right] \mathbb{E}[W^2] + \frac{1}{2} \mathbb{E}[W] \mathbb{E}[X_{n,j-M+1}^S] \mathbb{E}[W] + \frac{1}{2} \mathbb{E}[X_{n,j-M+1}^S] \mathbb{E}[W] + \frac{1}{2} \mathbb{E}[\left( X_{n,j-M+1}^S \right)^2].
\]

(3)

As \( X_{n,j-1}^S \) and \( X_{n,j-M+1}^S \) are identically distributed, we denote

\[
\mathbb{E} \left[ X_{n,j-1}^S \right] = \mathbb{E} \left[ X_{n,j-M+1}^S \right] = \mathbb{E} \left[ X_{n,j}^S \right]
\]

and rewrite (3) as

\[
\mathbb{E}[A_{n,q}] = \frac{1}{2} \mathbb{E} \left[ W^2 \right] + \left( \mathbb{E}[\hat{T}_{n,q}] + \mathbb{E}[X_n^S] \right) \mathbb{E}[W] + \mathbb{E}[X_n^S] \mathbb{E}[\hat{T}_{n,q}] + \frac{1}{2} \mathbb{E} \left[ \left( X_n^S \right)^2 \right].
\]

(4)

The time duration of the shaded polygon can also be written as \( Y_{n,q} = W + X_{n,j-M+1}^S \), and hence its expectation is

\[
\mathbb{E}[Y_{n,q}] = \mathbb{E}[W] + \mathbb{E} \left[ X_n^S \right].
\]

(5)

By substituting (4) and (5) into (1), we have

\[
\bar{\Delta}_n = \frac{\mathbb{E}[A_{n,q}]}{\mathbb{E}[Y_{n,q}]} = \frac{\mathbb{E} \left[ W^2 \right] + 2 \left( \mathbb{E}[\hat{T}_{n,q}] + \mathbb{E}[X_n^S] \right) \mathbb{E}[W] + 2 \mathbb{E}[X_n^S] \mathbb{E}[\hat{T}_{n,q}] + \frac{1}{2} \mathbb{E} \left[ \left( X_n^S \right)^2 \right]}{2 \mathbb{E}[W] + 2 \mathbb{E}[X_n^S]}
\]

(6)

To obtain the closed-form expression of the average AoI, we calculate all the expectation terms in (6) in Section III.
\[
\mathbb{E}[T_N^2(K)|C_{F,1}] = \frac{1}{1 - \mathcal{Z}_K} \sum_{j=0}^{K-1} B_{K,j} \left( (1 + c \lambda_s U_{K,j})^2 + 1 \right) V_{K,j},
\]
where \( B_{K,j} = K \binom{K}{j} (K-1)^{-1} (-1)^{j-1} \), \( U_{K,j} = N - K + 1 + j \), \( V_{K,j} = \frac{1}{\lambda_s U_{K,j} (T_d - c)} \), and \( \mathcal{Z}_K = \mathbb{P}(T_d < T_N(K)) = \frac{1}{\lambda_s} \sum_{j=0}^{K-1} B_{K,j} V_{K,j}. \)

Proof. See Appendix A.

The occurrence probability of Case \( C_{F,2} \) is given in the following Proposition.

**Proposition 2.** The probability that Case \( C_{F,2} \) occurs can be expressed as
\[
\mathbb{P}(C_{F,2}) = \frac{\left( N - K \right) \mathcal{Z}_K + N \sum_{h=1}^{K} \mathcal{Z}_h}{N e^{-\lambda_s (T_d - c)} + (N - K) - \sum_{h=K+1}^{N} \mathcal{Z}_h}. \tag{12}
\]

Proof. See Appendix B.

By definition, we have \( \mathbb{P}(C_{F,1}) = 1 - \mathbb{P}(C_{F, 2}). \) By substituting \( \mathbb{P}(10) - \mathbb{P}(12) \) into \( \mathbb{E} \left[ X_n^{2} \right] \) and \( \mathbb{E} \left[ (X_n^{2})^2 \right] \).

**B. First and Second Moments of Inter-Generation Time \( X_n^{2} \)**

In this subsection, we derive the first and second moments of the inter-generation time of two consecutive status updates when the former status update is successfully received by device \( n \), i.e., \( \mathbb{E} \left[ X_n^{2} \right] \) and \( \mathbb{E} \left[ (X_n^{2})^2 \right] \). Note that device \( n \) successfully receives status update \( j \) if \( T_{n,j} \leq \min(T_{D,j}, T_N(K)) \). We denote the case that device \( n \) successfully receives the status update as \( C_{S} \). We observe that \( X_n^{2} \) behaves differently for the following two cases: (1) \( C_{S,1} \) - Event II occurs earlier than Event I (i.e., \( T_N(K) < T_D \)); (2) \( C_{S,2} \) - Event I occurs earlier than Event II (i.e., \( T_D < T_N(K) \)). When Case \( C_{S,1} \) occurs, the instantaneous AoI of device \( n \) increases by \( T_N(K) \) (i.e., \( X_n^{2} = T_N(K) \)). When Case \( C_{S,2} \) occurs, the instantaneous AoI of device \( n \) increases by \( T_D \) (i.e., \( X_n^{2} = T_D \)).

The first and second moments of \( X_n^{2} \) are given by
\[
\mathbb{E}[X_n^2] = \mathbb{P}(C_{S,1}) \mathbb{E}[T_N^2(K)|C_{S,1}] + \mathbb{P}(C_{S,2}) \mathbb{E}[T_D^2], \quad \mathbb{E}\left[ (X_n^2)^2 \right] = \mathbb{P}(C_{S,1}) \mathbb{E}[T_N^4(K)|C_{S,1}] + \mathbb{P}(C_{S,2}) (T_D^2)^2, \tag{13, 14}
\]
where \( \mathbb{P}(C_{S,1}) \) and \( \mathbb{P}(C_{S,2}) \) denote the probabilities of the occurrence of Cases \( C_{S,1} \) and \( C_{S,2} \) when device \( n \) successfully receives the status update, respectively, with \( \mathbb{P}(C_{S,1}) = 1 \). To obtain \( \mathbb{E}[X_n^2] \) and \( \mathbb{E}\left[ (X_n^2)^2 \right] \), we need to calculate \( \mathbb{P}(C_{S,1}) \), \( \mathbb{E}[T_N(K)|C_{S,1}] \), and \( \mathbb{E}[T_N^2(K)|C_{S,1}] \). The following proposition gives the first and second moments of \( T_N(K) \) conditioning on the occurrence of Case \( C_{S,1} \).

**Proposition 3.** The first and second moments of the time that K IoT devices successfully receive a status update (i.e., \( T_N(K) \)) conditioning on the occurrence of Case \( C_{S,1} \) are given by \( \mathbb{E}[T_N(K)|C_{S,1}] = \mathbb{E}[T_N(K)|C_{F,1}] \) and \( \mathbb{E}[T_N^2(K)|C_{S,1}] = \mathbb{E}[T_N^2(K)|C_{F,1}] \), respectively, where \( \mathbb{E}[T_N(K)|C_{F,1}] \) and \( \mathbb{E}[T_N^2(K)|C_{F,1}] \) are given in Proposition 1.

**Proof.** The proof can be readily obtained by following the steps as in Appendix A. Due to space limitation, the detailed proof is omitted.

By definition, the occurrence probability of Case \( C_{S,1} \) can be calculated by
\[
\mathbb{P}(C_{S,1}) = \frac{K(1 - \mathcal{Z}_K)}{N \mathbb{P}(C_{S})}, \tag{15}
\]
where \( \mathbb{P}(C_{S}) \) denotes the probability that device \( n \) successfully receives the status update and can be calculated by \( \mathbb{P}(C_{S}) = \mathbb{P}(T_{n,j} < \min(T_{D,j}, T_N(K))) = \frac{1}{N} \sum_{h=1}^{K} (1 - \mathcal{Z}_h) \).

By substituting the derived expressions of \( \mathbb{P}(C_{S,1}) \), \( \mathbb{E}[T_N(K)|C_{S,1}] \), and \( \mathbb{E}[T_N^2(K)|C_{S,1}] \) into (13) and (14), we obtain \( \mathbb{E}[X_n^2] \) and \( \mathbb{E}\left[ (X_n^2)^2 \right] \).

**C. First and Second Moments of W**

Recall that \( W \) is the summation of \( M - 1 \) consecutive inter-generation time \( X_{n,j}^2 \), i.e., \( W = \sum_{j=1}^{M-1} X_{n,j}^2 \). As the probability that device \( n \) successfully receives each status update is the same, \( M \) is a geometric random variable. As a result, the probability mass function (PMF) of \( M \) is given by \( \mathbb{P}(M = m) = (1 - \mathbb{P}(C_{S}))^{m-1} \mathbb{P}(C_{S}), m \geq 1 \). Obviously, we have \( \mathbb{E}[M] = \frac{1}{\mathbb{P}(C_{S})} \) and \( \mathbb{E}[M^2] = \mathbb{E}[M] + \mathbb{E}[M] = \frac{2}{\mathbb{P}(C_{S})} \). As \( M \) and \( X_n^2 \) are independent, the first moment of \( W \) can be calculated by
\[
\mathbb{E}[W] = (\mathbb{E}[M] - 1) \mathbb{E}[X_n^2]. \tag{16}
\]

To derive the expression of \( \mathbb{E}[W^2] \), we first calculate the variance of \( W \) as follows
\[
\text{Var}[W] = \mathbb{E}[W^2] + \mathbb{E}[W] \mathbb{E}[M] - \mathbb{E}[W]. \quad \text{Var}[W] = \left( \mathbb{E}[X_n^2] \right)^2 \mathbb{E}[M] + \mathbb{E}[X_n^2] \mathbb{E}[M] - \mathbb{E}[X_n^2], \tag{17}
\]
where \( \mathbb{E}[X_n^2] = \left( \mathbb{E}[X_n^2] \right)^2 \mathbb{E}[M] - \mathbb{E}[X_n^2] \). Based on (16 and 17), we obtain \( \mathbb{E}[W^2] = (\mathbb{E}[W])^2 + \text{Var}[W] \).

**D. First Moment of Service Time \( \hat{T}_{n,q} \)**

Recall that \( \hat{T}_{n,q} \) is the service time of the \( q \)-th status update successfully delivered to device \( n \). Conditioning on the occurrence of Case \( C_{S} \), the CDF of the service time is
\[
\mathbb{P}(T_{n,q} < t | C_{S}) = \sum_{h=1}^{K} \left[ \mathcal{Z}_h - \mathbb{E}[U_{h,j}] \right] b_{h,j} e^{-\lambda_s (t - c) U_{h,j}} / b_{h,j}, \tag{18}
\]
where \( U_{h,j}, V_{K,j}, b_{h,j}, \) and \( \mathcal{Z}_h \) are defined in Proposition 1. Based on (18), the expectation of \( \hat{T}_{n,q} \) can be calculated by
\[
\mathbb{E}[\hat{T}_{n,q}] = \int_{c}^{T_d} t \, d\mathbb{P}(T_{n,q} | C_{S}) = \sum_{h=1}^{K} \frac{b_{h,j}}{\mathbb{P}(C_{S})} \mathbb{E}[\hat{T}_{n,q} | C_{S,1}] = \sum_{h=1}^{K} \mathbb{P}(C_{S,1}) \mathbb{E}[\hat{T}_{n,q} | C_{S,1}] = \sum_{h=1}^{K} \mathbb{E}[\hat{T}_{n,q} | C_{S,1}], \tag{19}
\]
where \( \mathbb{P}(C_{S}) = \frac{1}{N} \sum_{h=1}^{K} (1 - \mathcal{Z}_h) \).
E. Average AoI

Finally, by substituting (13), (14), (16), and (19) into (6), we obtain the average AoI of the multicast transmission with hard deadlines. It is worth pointing out that the results presented in this paper are general and can be easily extended to the scenarios for broadcast transmission with hard deadlines by replacing $K$ with $N$, for multicast transmission without hard deadlines by setting $T_D = \infty$, and for unicast transmission with hard deadlines by setting $N = K = 1$.

IV. Performance Evaluation and Discussions

In this section, we present the simulation and numerical results of the considered multicast transmission with hard deadlines. Unless specified otherwise, we set the total number of IoT devices $N = 10$, the number of IoT devices required to successfully receive each status update $K = 7$, and $c = 0.1$.

Fig. 2 shows the impact of hard deadline $T_D$ on the average AoI for different values of average service rate $\lambda_s$. The simulation results match well with the theoretical results, which validates the performance analysis presented in Section III. With the variation of deadline $T_D$, the average AoI first decreases to a minimum value and then increases to a saturation value. Specifically, when $\lambda_s = 1/3$ and deadline $T_D$ is small, the probability that each device can successfully receive a status update within a transmission interval (i.e., $\min(T_N(K), T_D)$) is small. As such, it takes each IoT device many transmission intervals to successfully receive a status update. Note that the average AoI is proportional to the average number of transmission intervals required to successfully receive a status update and the average length of transmission intervals. Hence, the average AoI of the considered system is large when the deadline is small (e.g., $0.2$). By increasing the value of deadline $T_D$ to $0.9$, the average AoI declines quickly until reaching its minimum value. This is due to the fact that the probability of successful status update reception within each transmission interval increases. By further increasing the value of deadline $T_D$, the average length of transmission intervals increases and it starts to play a more important role in the AoI evolution than the average number of transmission intervals required to successfully a status update, leading to the increase of the average AoI. When deadline $T_D$ is sufficiently large, the average AoI reaches a saturation value. The saturation value corresponds to the average AoI of multicast transmission without hard deadlines and is plotted with dashed lines in Fig. 2. In addition, the average AoI decreases with the value of $\lambda_s$. This is because the average service time affects the average length of transmission intervals. Moreover, we can also observe that the value of deadline $T_D$ that minimizes the average AoI becomes larger as the value of $\lambda_s$ decreases.

Fig. 3 illustrates the impact of $K$ on the average AoI of the considered system for different values of $\lambda_s$ when $T_D = 3$. When $K$ is small (e.g., $K = 1$), the probability that a specific device is one of the first $K$ devices that successfully receive the status update is low, and hence the average AoI is relatively large. When $\lambda_s = 1/2$, by increasing the value of $K$ to 3, the probability of successful status update reception increases, which reduces the number of transmission intervals that are required to successfully receive a status update and in turn reduces the average AoI. By further increasing the value of $K$, the average length of transmission intervals increases as more devices are required to successfully receive each status update. As the average length of transmission intervals increasingly dominates the AoI evolution when $K \geq 4$, the average AoI increases. Therefore, with the variation of $K$, there exists a value of $K$ that balances the tradeoff between these two effects and minimizes the average AoI. Similarly, we can observe that the average AoI increases as the value of $\lambda_s$ decreases.

V. Conclusions

In this paper, we studied the average AoI of multicast transmission with hard deadlines in IoT systems. We characterized the instantaneous AoI evolution and derived the first and second moments of the inter-generation time of two consecutive status updates for both successful and unsuccessful reception cases. We also derived the first and second moments of the time duration within which all the status updates transmitted by the server are not successfully received by a specific device. Based on these derived expressions, we obtained the closed-form expression of the average AoI. Simulations validated the theoretical analysis and illustrated the impact of system
parameters on the average AoI. Results showed that the hard
deadline has a significant impact on the average AoI of
multicast transmission and there exists an optimal value of
the hard deadline that minimizes the average AoI.

APPENDIX

A. Proof of Proposition 1

When Case $C_{F,1}$ occurs, we have $T_N(K) < T_D$ and
$T_{n,j} > \min\{T_D,T_N(K)\}$, which can be simplified as $T_N(K) < \min\{T_D,T_{n,j}\}$. As a result, the CDF of the time that $K$ IoT
devices successfully receive a status update conditioning on
the occurrence of Case $C_{F,1}$ can be expressed as

$$F_{T_N(K)|C_{F,1}}(t) = \frac{\Pr(T_N(K) < t|C_{F,1})}{\Pr(T_N(K) < \min\{T_D,T_{n,j}\})}.$$  \hfill (20)

The numerator of (20) can be calculated as

$$\Pr(T_N(K) < t, T_N(K) < \min\{T_D,T_{n,j}\})$$
$$= \frac{N-K}{N} (1 - \sum_{j=0}^{K-1} B_{K,j} e^{-\lambda_s (t-c) U_{K,j}} - V_{K,j}).$$  \hfill (21)

where $V_{K,j}, B_{K,j}, U_{K,j}$, and $Z_K$ are defined in Proposition 1. On the other hand, the denominator of (20) is given by

$$\Pr(T_N(K) < \min\{T_D,T_{n,j}\}) = \frac{N-K}{N} (1 - Z_K).$$  \hfill (22)

By substituting (21) and (22) into (20), we obtain the conditional CDF of $T_N(K)$, i.e., $F_{T_N(K)|C_{F,1}}(t)$, as follows

$$F_{T_N(K)|C_{F,1}}(t) = 1 - \frac{1}{1 - Z_K} \sum_{j=0}^{K-1} B_{K,j} e^{-\lambda_s (t-c) U_{K,j}} - V_{K,j}.$$  \hfill (23)

As a result, the conditional first and second moments of $T_N(K)$, denoted as $\mathbb{E}[T_N(K)|C_{F,1}]$ and $\mathbb{E}[T_N(K)^2|C_{F,1}]$, can be written as

$$\mathbb{E}[T_N(K)|C_{F,1}] = \int_0^{T_D} t f_{T_N(K)|C_{F,1}}(t) dt$$
$$= \sum_{j=0}^{K-1} B_{K,j} U_{K,j} + 1 + \lambda_s U_{K,j} - (1 + T_D \lambda_s U_{K,j}) V_{K,j},$$  \hfill (24)

$$\mathbb{E}[T_N^2(K)|C_{F,1}] = \int_0^{T_D} t^2 f_{T_N(K)|C_{F,1}}(t) dt$$
$$= \frac{1}{1 - Z_K} \sum_{j=0}^{K-1} B_{K,j} U_{K,j}^3$$
$$\left\{ \frac{(1 + \lambda_s U_{K,j})^2}{K_j} + \left(1 + T_D \lambda_s U_{K,j}\right)^2 + 1 \right\} V_{K,j},$$  \hfill (25)

where $Z_K, B_{K,j}, U_{K,j}$, and $V_{K,j}$ are given in Proposition 1, and $f_{T_N(K)|C_{F,1}}(t)$ is the first derivative of $F_{T_N(K)|C_{F,1}}(t)$ and denotes the conditional PDF of $T_N(K)$.

B. Proof of Proposition 2

The occurrence probability of Case $C_{F,2}$ is given by

$$\mathbb{P}(C_{F,2}) = \frac{\Pr(T_D < \min\{T_N(K),T_{n,j}\})}{\Pr(T_{n,j} > \min\{T_D,T_N(K)\})},$$  \hfill (26)

where the denominator $\mathbb{P}(T_{n,j} > \min\{T_D,T_N(K)\}) = \Pr(T_D > T_D) + \Pr(T_{n,j} > T_N(K) - \Pr(T_{n,j} > T_D,T_N(K) > T_N(K))).$

By definition, we have $\mathbb{P}(T_{n,j} > T_D) = e^{-\lambda_s (T_D-c)}$ and

$$\mathbb{P}(T_{n,j} > T_N(K)) = \frac{N-K}{N}. $$

Besides, the probability that the $T_{n,j}$ is greater than both $T_D$ and $T_N(K)$ is given by

$$\mathbb{P}(T_{n,j} > T_D, T_{n,j} > T_N(K)) = \sum_{h=K+1}^{N} \mathbb{P}(T_{n,j} > T_D, T_{n,j} = T_N(h)) = \frac{N-K}{N} Z_K + \sum_{h=K+1}^{N} Z_h.$$  \hfill (27)

By substituting (27) and (28) into (26), we obtain $\mathbb{P}(C_{F,2})$.

REFERENCES

[1] T. Taleb and A. Kunz, “Machine type communications in 3GPP networks: Potential, challenges, and solutions,” IEEE Commun. Mag., vol. 50, no. 3, pp. 178–184, Mar. 2012.
[2] S. K. Kaul, R. D. Yates, and M. Gruteser, “Status updates through queues,” in Proc. IEEE CISS, Princeton, NJ, Mar. 2012.
[3] S. Kaul, R. Yates, and M. Gruteser, “Real-time status: How often should one update?” in Proc. IEEE INFOCOM, Orlando, FL, Mar. 2012.
[4] M. Costa, M. Codreanu, and A. Ephremides, “On the age of information in status update systems with packet management,” IEEE Trans. Inf. Theory, vol. 62, no. 4, pp. 1897–1910, Apr. 2016.
[5] R. D. Yates and S. Kaul, “Real-time status updating: Multiple sources,” in Proc. IEEE ISIT, Boston, MA, Jul. 2012.
[6] L. Huang and E. Modiano, “Optimizing age-of-information in a multi-class queueing system,” in Proc. IEEE ISIT, Hong Kong, China, Jun. 2015.
[7] B. Zhou and W. Saad, “Joint status sampling and updating for minimizing age of information in the internet of things,” IEEE Transactions on Communications, vol. 67, no. 11, pp. 7468–7481, 2019.
[8] ———, “Minimum age of information in the internet of things with non-uniform status packet sizes,” arXiv preprint arXiv:1901.07069, 2019.
[9] Y. Gu, H. Chen, Y. Zhou, Y. Li, and B. Vucetic, “Timely status update in Internet of Things monitoring systems: An age-energy tradeoff,” IEEE Internet Things J., vol. 6, no. 3, pp. 5324–5335, Jun. 2019.
[10] J. Zhong, R. D. Yates, and E. Soljanin, “Multicast with prioritized delivery: How fresh is your data?” in Proc. IEEE SPAWC, Kalamata, Greece, Jun. 2018.
[11] J. Zhong, E. Soljanin, and R. D. Yates, “Status updates through multicast networks,” in Proc. IEEE Allerton, Monticello, IL, Oct. 2017.
[12] S. Nath, J. Wu, and J. Yang, “Optimum energy efficiency and age-of-information tradeoff in multicast scheduling,” in Proc. IEEE ICC, Kansas, MO, May 2018.
[13] B. Buyukates, A. Soysal, and S. Ulukus, “Age of information in two-hop multicast networks,” in IEEE ACSSC, Pacific Grove, CA, Nov. 2018.
[14] I. Kadota, A. Sinha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, “Scheduling policies for minimizing age of information in broadcast wireless networks,” IEEE/ACM Trans. Netw., vol. 26, no. 6, pp. 2637–2650, Dec. 2018.
[15] C. Kam, S. Kompella, G. D. Nguyen, J. E. Wieselthier, and A. Ephremides, “On the age of information with packet deadlines,” IEEE Trans. Inf. Theory, vol. 64, no. 9, pp. 6419–6428, Sep. 2018.
[16] Y. Inoue, “Analysis of the age of information with packet deadline and infinite buffer capacity,” in Proc. IEEE ISIT, Vail, CO, Jun. 2018.
[17] H. A. David and H. N. Nagaraja, “Order Statistics,” Encyclopedia of Statistical Sciences, 2004.