CONTROLLING STATIONARY FRONTS IN
TWO-DIMENSIONAL REACTION-DIFFUSION SYSTEMS

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Abstract

This paper considers new approach to control a stationary inhomogeneous planar
front solution of a nonlinear parabolic two-dimensional distributed (reaction-diffusion)
system, by using a gain point-sensor control with actuators that have the simplest
possible spatial dependence. The method is based on multivariable root-locus tech-
nique for the finite-dimensional approximation of the original PDE model and use the
concepts of finite and infinite zeros of linear multidimensional system.

Keywords: Reaction-diffusion processes; Front stabilization; Root-locus method,
System zeros

1 Introduction

Nonlinear parabolic partial differential equations (PDEs), which typically describe reaction-
diffusion systems, may admit spatially-dependent solutions like stationary fronts as well as
spatiotemporal patterns. The latter can often be described as composed of slow-moving
fronts, separated by domains of moderate changes. This article is part of a research aimed to
develop control theory for reaction-diffusion and reaction-convection-diffusion systems for
which a certain patterned state is advantageous. Propagating fronts and patterned states
may emerge in several technologies including catalytic reactors [1], distillation processes
[2], flame propagation and crystal growth [3] (see also [4] for references) as well as in
physiological systems like the heart [5]. Our interest lies in catalytic reactors, in which
stationary or moving fronts and spatiotemporal patterns have been observed and simulated
in various systems like flow through a catalyst [6], fixed-bed reactors [7, 8], reactors with
flow reversal [9] and loop reactors [10]. The instabilities emerge due to the thermal effects in exothermic reactions, due to self-inhibition by a reactant and due to slow reversible modifications of the surface. The construction of a controller that enables to stabilize some inhomogeneous solutions in one-dimensional (1-D) reaction-diffusion and reaction-convection-diffusion systems is currently a subject of intensive investigation [11, 12, 13, 14, 15, 16, 17]. Yet, most catalytic reactors, as well as physiological systems like the heart, exhibit a behaviour that can be described by two- or even three-dimensional models.

In the present work we are interested in stabilizing stationary (planar) fronts in a rectangular two-dimensional (2-D) domain in which a chemical reaction-diffusion process occurs. Previous studies of 1-D systems demonstrated that the simplest approach is by applying the point-sensor control, in which a single space-independent actuator responds to a sensor that is located at the front position [16]. In the 2-D problem the point-sensor control with a single space-independent actuator cannot stabilize the front in a wide system. In that case it is necessary to use control with several point sensors and actuators (space-independent and space-dependent). Here we apply the extension of the point-sensor control to multivariable case by explore the root-locus technique for multivariable system [21]. The main advantage of this control is its insensitivity to parameter uncertainties (robustness) and also small number of evaluated gain coefficients. We apply the root-locus method to determine the minimal number of actuators, their spatial form and the location of the corresponding sensors that will assure the linear stability of the planar front. The method uses the concepts of finite [20, 19] and infinite zeros [21] of a linear multidimensional system; these notions are briefly explained below. The proposed approach is best suited for a systematic computer-aided search of the regulator form.

2 Statement

Consider the reaction-diffusion problem, in the \((z,r)\) rectangular domain of length \(L\) and width \(R\), which is described by a pair of coupled nonlinear parabolic PDEs

\[
y_t - y_{zz} - y_{rr} = P(y, \theta) + \lambda, \quad \theta_t = \epsilon Q(y, \theta)
\]  

subject to no-flux boundary conditions:

\[
y_z(0, r) = 0, \quad y_z(L, r) = 0, \quad y_r(z, 0) = 0, \quad y_r(z, R) = 0
\]  

where the variable \(y = y(z, r, t)\) typically represents the activator, \(\theta = \theta(z, r, t)\) is the slow variable (localized inhibitor), \(\lambda\) is a control variable that is introduced additively; \(\epsilon < 1\) is
the ratio of time scales.

We use the polynomial source functions

\[ P(y, \theta) = -y^3 + y + \theta, \quad Q(y, \theta) = -\gamma y - \theta \quad (3) \]

since they adequately simulate the phenomenon of multiple steady states (bistable kinetics) due to thermal and autocatalytic effects which, in many chemical systems, induce the instabilities [22] and since several analytical results are available [23]. The instability stems from the anticlinal arrangement of \( y \) and \( \theta \) [4]. In Eqn. (3) \( \gamma > 0 \) is a constant parameter. System (1-3) has often been used for studying pattern-formation in chemically reacting systems [1]; similar systems were employed to describe pattern formation and control in cardiac systems.

In this work we address the problem of stabilizing the planar front, in a 2-D reaction-diffusion system (1-3), at the position which in an open-loop system is stationary but unstable (i.e., at the middle, \( z = L/2 \) of the domain). Obviously, narrow systems behave like a one-dimensional system (Eqn. (1-3), \( y_{rr} = 0 \)) and admit a stationary front solution that is unstable and typically oscillates or travels out of the system (see details in [17]). In a sufficiently wide 2-D system, the front-line may undergo symmetry breaking so that in part of it the upper state expands while in other parts the lower state propagates. In that case we suggest to use control with several actuators located along the front at the points \((z, r) = (L/2, r_d), d = 1, \ldots, \eta\) and apply a general feedback control law of the form

\[ \lambda = k \sum_{d=1}^{\nu} [y(L/2, r_d, t) - y^*_d] \psi_d(z, r) \quad (4) \]

where \( k < 0 \) is a scalar gain coefficient, \( y(L/2, r_d, t) - y^*_d \) are deviations of the sensors from the set points; \( y^*_d = y^*(L/2, r_d) \); \( \psi_d(z, r) \) are some space-dependent functions that may imitate the eigenfunctions. We will seek control (4) with the simplest space-independent or space-dependent actuator functions \( \psi_d(z, r) \) and minimal number of sensors.

### 3 Linear analysis

For design of control (4) we use a linearized truncated version of PDEs (1-3). Linearization of (1-3) for \( \lambda = 0 \) around the steady state solution \( y_s = y_s(z, r), \theta_s = \theta_s(z, r) \) yields

\[ \ddot{y} - \dddot{y} - y_{rr} = (-3y_s^2 + 1)\dot{y} + \dot{\theta} + \lambda, \quad \ddot{\theta} = -\epsilon \gamma \ddot{y} - \epsilon \dot{\theta} \quad (5) \]
which is lumped by the Galerkin method: We expand the deviations \( \bar{y} = \bar{y}(z, r, t), \bar{\theta} = \bar{\theta}(z, r, t) \) and \( \bar{y}(z^*, r_d, t) = y(z^*, r_d, t) - y_s(z^*, r_d) \) as

\[
\bar{y}(z, t) = \sum_e a_e(t) \phi_e(z, r), \quad \bar{\theta}(z, t) = \sum_e b_e(t) \phi_e(z, r), \quad \bar{y}(z^*, r_d, t) = \sum_e a_e(t) \phi_e(z^*, r_d)
\]

where \( z^* = L/2 \) and the functions \( \phi_e(z, r) \) are the eigenfunctions of the problem

\[
\phi_{zz}(z, r) + \phi_{rr}(z, r) = -\lambda \phi(z, r), \quad \phi_z(0, r) = \phi_z(L, r) = \phi_r(z, 0) = \phi_r(z, R) = 0
\]

with the corresponding eigenvalues

\[
\lambda_{e(ij)} = \pi^2 [(i - 1)^2/L^2 + (j - 1)^2/R^2]
\]

and the eigenfunctions

\[
\phi_{e(ij)} = \frac{\rho}{\sqrt{LR}} \cos(\frac{i-1}{L}\pi z) \cos(\frac{j-1}{R}\pi z), \quad e = e(ij) = 1, 2, \ldots
\]

(\( \rho = 1 \) when \( i = j = 1 \), \( \rho = 2 \) when \( i, j > 1 \) and \( \rho = \sqrt{2} \) when \( i = 1, j > 1 \) or \( j = 1, i > 1 \), see [4] for derivation ). Substituting (6) into (5),(4) with set points \( y_s^* = y_s(L/2, r_d) \) and integrating with a weight eigenfunctions \( \phi_e(z, r) \) results in the spectral representation of closed-loop linearized system (5),(4)

\[
\dot{a}_e = -\lambda_e a_e + \sum_m J_{em} a_m + b_e + k \sum_{d=1}^\eta (\int_0^L \int_0^R \psi_d(z, r) \phi_f(kl) (L/2, r_d) \phi_e(kl)(z, r) dz dr)
\]

\[
\dot{b}_e = \epsilon(\gamma a_e + b_e)
\]

where

\[
J_{em} = \int_0^L \int_0^R \phi_{m(ij)} \phi_{e(kl)} dz dr, \quad e, m = 1, \ldots
\]

Denoting

\[
h_{df} = \phi_f(L/2, r_d)
\]

\[
\beta_{ed} = \int_0^L \int_0^R \psi_d(z, r) \phi_e(kl)(z, r) dz dr
\]

we can rewrite (10) as follows

\[
\dot{a}_e = -\lambda_e a_e + \sum_m J_{em} a_m + b_e + k \sum_{d=1}^\eta \beta_{ed} \sum_f a_f h_{df}, \quad e, f = 1, 2, \ldots
\]

This system (Eqn. 15, 11) may be presented in the usual vector-matrix form as the linear infinite-dimensional dynamical system with \( \eta \)-dimensional input \( v \) and output \( w \) vectors

\[
\begin{bmatrix}
\dot{a} \\
\dot{b}
\end{bmatrix} = 
\begin{bmatrix}
-A + J & I \\
-\epsilon \gamma I & -\epsilon I
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix} + 
\begin{bmatrix}
\beta \\
O
\end{bmatrix} v
\]

(16)
\[ w = Ha \]  

(17)

closed by the finite-dimensional output feedback

\[ v = kI_\eta w \]  

(18)

where \( a(t) = [a_e], b(t) = [b_e] \) are the infinite dimensional vectors \( (e = 1, 2, \ldots) \); \( v \) and \( w \) are finite-dimensional \( \eta \) vectors, the matrix \( \beta = [\beta_{ed}] \) has \( \eta \) infinite-dimensional columns \( (e = 1, 2, \ldots; d = 1, \ldots, \eta) \) and the matrix \( H = [h_{df}] \) has \( \eta \) infinite-dimensional rows \( (d = 1, \ldots, \eta; f = 1, 2, \ldots) \); \( I_\eta \) is unity \( \eta \times \eta \) matrix; \( \Lambda = diag(\lambda_1, \lambda_2, \ldots) \), \( I = diag(1, 1, \ldots) \), \( J = [J_{em}], e, m = 1, 2, \ldots \) are infinite-dimension matrices and \( k \) is scalar gain coefficient.

Let us study the correlation between the form of original control (4) and the input and output structures of Eqns.(16-17): If all \( r_d, d = 1, 2, \ldots, \eta \) in (4) have different values, then the parameter \( \eta \) assigns the dimension of input and output vectors of Eqns. (16), (17) and the sensor positions stipulate the structure of the output matrix \( H \) (see Eqn.13). Besides, the form of the actuator functions \( \psi_d(z, r) \) influences the matrix \( \beta \) (see Eqn 14). Hence, in view of the original statement the problem can be stated as follows:

**Problem 1.** For the linearized infinite-dimensional ODEs system (16-17) it is necessary to find the matrices \( \beta \) and \( H \) with the minimal number of columns and rows, respectively, such that the finite-dimensional output feedback control (18) stabilizes the closed-loop system.

The obvious way of designing of a finite-dimensional control (18) for an infinite-dimensional system (16-17) is to use a truncated (finite-dimensional) approximation of the PDEs. To truncate the system we capitalize on the dissipative nature of the parabolic PDEs: the truncation order \( N \) is estimated by calculating the leading eigenvalues of the dynamics matrix (16) (see [4] for details). As follows from [24], for a sufficiently large truncated order \( N \), with actuator functions \( \psi_d(z, r) \) that coincide with eigenfunctions \( \phi_e(z, r) \), the finite-dimensional control guarantees the stability of infinite-dimensional system. Below we imply that system (16-17) is the finite-dimensional analog of the original (16-17) with truncated order \( N \).

## 4 Root-locus control design

Design of control (4) implies the determination of the minimal number of actuators to be employed, their spatial form and the location of the corresponding sensors that will assure the linear stability of the planar front. We can manipulate the number and r-coordinate positions \( (r_1, r_2, \ldots, r_\eta) \) of the sensors and the form of the actuator functions \( \psi_d(z, r) \). The former affects the matrix \( H \) (Eqn. 13) and the latter influences the matrix \( \beta \) (Eqn. 14).
For simplification of the search for a matrix $\beta$ structure we assign the eigenfunctions (9) as actuator functions, i.e. $\psi_d(z,r) \sim \phi_e(z,r)$, $e = 1, 2, \ldots$. Thus from the relation between the form of the actuator functions $\psi_d(z,r)$ and the structure of the matrix $\beta$ (see Eqn.14) it follows that every $d$-th column of the matrix $\beta$ will contain only a single non-zero element $\beta_{ed}$. Moreover, such a structure of $\beta$ satisfies the above-mentioned Balas’s restrictions [24] that ensure the validity of control of an infinite-dimensional system by a finite-dimensional controller.

Therefore, we propose the following steps for design of control (4): (i) Assign sensor positions along the front $(z,r) = (L/2, r_d)$, $d = 1, \ldots, \eta$ and calculate the matrix $H$. (ii) Seek the matrix $\beta$ and gain $k$ that assures linear stability of the closed-loop system (Eqns.16-18). (iii) Finally, find the eigenfunctions $\phi_e(z,r)$ that corresponds to the matrix $\beta$ obtained.

For fulfilling step (ii) we use a root-locus technique [21] which is based on an analysis of the finite zeros and infinite zeros of the open-loop system (16-17). This method uses the following known property of closed-loop linear system with feedback (18) (see [25]): as the feedback gain increases towards infinity a part of the closed-loop eigenvalues remain finite and approaches the values which are referred to as finite system zeros [19],[20] (see also Appendix for definitions) while the remainder are located at the points at infinity and are known as infinite zeros [21]. Therefore, we propose to seek a suitable matrix $\beta$ by the repeatedly calculating the finite zeros of open-loop ODEs (16-17), with different input matrices, and finding the one that ensures that the leading finite system zeros are negative. Then we assign a sufficiently large negative gain coefficient $k$.\(^1\)

Finally, Problem 1 may be reformulated as follows:

**Problem 2.** For the linearized truncated system (16-17) with assigned output matrix $H$ it is necessary to find the matrix $\beta$ that has a single nonzero element in every column such that the above-mentioned system has leading finite system zeros in the left-half of the complex plane (‘negative’ zeros)\(^2\).

Let us note that such input and output matrices $H$ and $\beta$ result a minimum phase control system because this system contains finite zeros in the left-half of the complex plane.

The following assertion is needed to avoid cases when Problem 2 has no solution.

\(^1\)Here we must note that large perturbations of the front will cause the control variable to exceed the bistability domain of $P = 0$. So the gain value coefficient $k$ must be restricted below the value ($k_o$) that ensures stability of the closed-loop system.

\(^2\)Similarly, we denote zeros in the right-half of the complex plane by ‘positive’ ones.
Assertion 1. For assigned matrix $H$ the problem has no solution for any matrix $\beta$ if and only if 'positive' finite system zeros of (16), (17) are output-decoupling zeros [26].

The proof follows from definition of decoupling zeros (see Appendix).

Remark 1. If the shape of actuator distribution functions $\psi_d(z,r), d = 1, 2, \ldots$ are preassigned from technical constraints (i.e. the matrix $\beta$ is given) then we need to find $\eta$ sensor locations (i.e. the matrix $H$) which provide the 'negative' leading finite system zeros of system (16-17). The solvability of this problem is formulated as follows.

Assertion 2. For assigned matrix $\beta$ the problem has no solution for any matrix $H$ if and only if 'positive' finite system zeros of (16), (17) are input-decoupling zeros [26].

Remark 2. It is necessary to choose the sensor locations in $r$-direction so that the infinitely increasing eigenvalues of the closed-loop high gain system (16-18) (infinite zeros) tend to infinity along asymptotics with a negative real angle. This condition is guaranteed if $\det(H\beta) \neq 0$ and all eigenvalues of the $\eta \times \eta$ matrix $H\beta$ are positive [21]. If $H\beta$ has several negative eigenvalues then we need to introduce a nonsingular precompensator $M$ to (18) such that the new control

$$v = kI_\eta M w$$

ensures above property for $MH\beta$. Such operation does not changes the finite system zeros of (16), (17) because they are invariant to any nonsingular transformation of output [19],[20].

Therefore, the general strategy of the method is as follows: At first we need to check that assigned sensor positions (or matrix $H$) ensure that output-decoupling zeros of a pair $(A, H)$ are 'negative' (see Assertion 1). Then we seek a suitable matrix $\beta$ by repeating calculations of the finite system zeros of open-loop ODEs (16), (17) with different matrices $\beta$ and finding one that ensures that the leading finite system zeros are negative and $\det(H\beta) \neq 0$.

If necessary, we find the precompensator $M$ which rearranges the sensor position in $r$-direction so that the infinite zeros tend to infinity along asymptotes with a negative real axis angle.

To demonstrate this procedure we apply it for design of control (4) for PDEs (1-3) of length $L$ and various widths.

We start by analyzing the effectiveness of the simplest control law, a single space-independent actuator (Eqn. 4, $\eta = 1, \psi_1(z,r) = 1$)

$$\lambda(t) = ky((L/2, r_1, t) - y_s^*)$$

where $y_s^* = y_s(L/2, r_1)$ coincides with the steady-state value of the problem. The spectral representation of the closed-loop system (1-3),(20) is a single-input, single-output Eqns.(16-
17) with a column vector $\beta = [\beta_{11}, 0, \ldots]$ with $\beta_{11} = 1/\sqrt{LR}$ and row output vector $H = h = [h_1, h_2, \ldots]$. Thus here the shape of the actuator is assigned. Then if input-decoupling zeros of a pair $(A, \beta)$ are 'negative' we need to find a sensor position. At first consider a sensor situated at the domain center ($z^* = L/2, r_1 = R/2$). The analysis of leading system zeros of the related linearized truncated system obtained shows that they possess negative real parts for $0 < R < R_{cr}$ for some value $R_{cr}$ and positive real parts for $R > R_{cr}$. So, control with one space-independent actuator is effective only for systems of width $R \leq R_{cr}$ (narrow systems) that have 'negative' zeros. Changing the sensor position in the $r$-direction alters the vector $h$ (Eq.13 ) and as a consequence may move the finite system zeros (for details see [27]) and $R_{cr}$. However, the position at the domain center assures the maximal value of $R_{cr}$.

For wider systems ($R > R_{cr}$) it is necessary to use space-dependent actuators. We try to apply control in the form

$$\lambda = k \sum_{d=1}^{2} [y(L/2, r_d, t) - y_s^*] \psi_d(z, r)$$  \hspace{1cm} (21)$$

with one space-independent actuator ($\psi_1(z, r) = 1$) and another space-dependent one, $\psi_2(z, r) \sim \phi_e(z, r)$ where $\phi_e(z, r)$ is eigenfunction (9) chosen from the series eigenfunctions ordered in an increasing order of the appropriate eigenvalues. Introducing two sensors at positions $(L/2, r_1)$ and $(L/2, r_2)$ we calculate the $2 \times N$ matrix $H$ by Eqn. (13). If output-decoupling zeros of a pair $(A, H)$ are 'negative' then we need to evaluate the finite system zeros of systems (16-17) with above output matrix $H$ and different $N \times 2$ matrices $\beta = [\beta_{ed}]$, $e = 1, \ldots, N, d = 1, 2$ with assigned elements of the first column $(\beta_{11} = 1/\sqrt{LR}, \beta_{e1} = 0, e = 2, \ldots, N )$ and undetermined elements of the second column.

Thus it is necessary to find a single nonzero element from the second column of $\beta$ which ensures that the leading finite zeros of system (16-17) are 'negative' ones. If such $\beta$ does not exist we need to change the matrix $H$ by shifting the positions of sensors in $r$-direction and begin the search of $\beta$ once more. If an appropriate matrix $H$ does not exist then control (21) is not effective and it is necessary to increase the number of sensors (and actuators) to three and so on.

5 Application

Let us apply the above method for stabilization of planar front solution of system (1-3) (with $L = 20, \gamma = 0.45$, $\epsilon = 0.1$ ) of various widths. The leading eigenvalues of the truncated version (Eqn. 16, $N = 23$) show two real unstable eigenvalues (0.35 and 0) for
all $R$’s and two complex eigenvalues with real parts that becomes positive for $R \geq 5.6$ (see Fig. 4a in [4]). At first we try to apply control (20) with one space-independent actuator and the sensor at $(z^* = 10, r_1 = R/2)$. The analysis of zeros of open-loop system (16-17) discovers two leading (and complex) finite zeros with negative real parts for $0 < R < 5.6$ and positive real parts for $R \geq 5.6$ (see Fig.4b in [4]). Hence, system width $R_{cr} = 5.5$ is a critical one for our ability to do control with a space-independent actuator.

For wider systems ($R > 5.5$) we apply the two actuator control (21). Assigning two sensors in some positions $(z^* = 10, r_1, r_2)$ and calculating the finite zeros of relevant two input, two-output systems (16-17) with different input matrices we find that when $\beta_{24} \neq 0$ the leading finite zeros have negative real parts ($-0.1024$). This corresponds to the fourth ($e = 4$) eigenfunction $\phi_4 \sim \cos(\pi r/R)$ in series of the ordered eigenfunctions: The first six ordered eigenfunctions are $\phi_1 = 1$, $\phi_2 \sim \cos(\pi z/L)$, $\phi_3 \sim \cos(2\pi z/L)$, $\phi_4 \sim \cos(\pi r/R)$, $\phi_5 \sim \cos(3\pi z/L)$, $\phi_6 \sim \cos(2\pi z/L)\cos(\pi r/R)$. Consequently control (21) becomes

$$\lambda(t) = k\{[y(10, r_1, t) - y_s(10, r_1)] + [y(10, r_2, t) - y_s(10, r_2)]\cos(\pi r/R)\} \quad (22)$$

Then, it is necessary to choose $r_1, r_2$ in (22) so that Remark 2 is satisfied. For our case we need to use the sensor locations with $r_1 > r_2$ which is equivalent to introducing the $2 \times 2$ precompensator $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

We verify the proposed methodology by simulating system (1-3) using one space-independent actuator (20) for narrow ($R < R_{cr}$) and two-actuator control (22) for wider system (see [4] for plots).

6 Conclusion remarks

The stabilization of planar stationary fronts in a two-dimensional rectangular domain, in which a diffusion-reaction systems occurs, is studied using a two-variable PDEs model for which some analytical results are available. We consider the simplest control strategy based on sensors placed at the designed front line position and measure deviations from a local state, and actuators that are spatially-uniform or space dependent. We present a systematic control design that determines the number of required sensors and actuators, their position and their form. The control design is corroborated by linear analysis of a lumped truncated model and concepts of finite and infinite zeros of linear multidimensional systems. The method is best suited for a systematic computer-aided search of the regulator form.
APPENDIX

Consider a general linear multivariable finite-dimensional dynamic system described by the set of state-space equations

\[ \dot{a}(t) = Aa(t) + Bv(t) \quad w(t) = Ha(t) \]

with \( a \), \( v \) and \( w \) are the \( n \), \( r \) and \( l \) dimensional state, input and output vectors and \( A \), \( B \) and \( H \) are constant matrices of appropriate dimensions.

**Definition 1.** The finite zeros (system zeros) of the above system are determined as the set of complex \( s_i \) for which the rank of the system matrix

\[
P(s) = \begin{bmatrix} s_i I - A & -B \\ H & O \end{bmatrix}
\]

is reduced.

**Definition 2.** The input-decoupling zeros are defined as the set of the complex variable \( s_i \) at which the row rank of the matrix \([s_iI - A, -B]\) is reduced.

**Definition 3.** The output-decoupling zeros are defined as the set of the complex variable \( s_i \) at which the row rank of the matrix \([s_iI - A^T, H^T]\) is reduced.

**Evaluation of input-decoupling zeros.** The set of input-decoupling zeros may be calculated from Definition 2. To avoid the operations with complex numbers we can apply an alternative method which uses the property: input-decoupling zeros coincide with the uncontrollable eigenvalues of the matrix \( A \). Since the latter eigenvalues are invariant under a proportional state feedback: \( v = Ka \) then they may be calculated as those eigenvalues of the closed-loop matrices \( A + BK_j \) which are invariant with respect to any gain matrices \( K_j \) with finite elements. Let us analyze the leading eigenvalues of the \( N \)-truncated system (16) acted by the state control \( v = K_ja \) with a random \( N \) row vector \( K_j \). It is evident that leading eigenvalues, which are invariant with respect to this control, are input-decoupling zeros of (16), (17).

**ACKNOWLEDGMENT**

M.S. acknowledges the Minerva Center of Nonlinear Dynamics for support.

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