Fully-heavy pentaquark states

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Developing the calculation techniques for fivefold heavy hadrons, we perform the study of novel fully-heavy $QQQQ\bar{Q}$ pentaquark states by the QCD sum rule approach that is firmly based on the QCD basic theory. Numerically, masses of fully-heavy pentaquark states are calculated to be $7.38^{+0.20}_{-0.22}$ GeV for $cccc\bar{c}$ and $21.56^{+0.17}_{-0.15}$ GeV for $bbbb\bar{b}$, respectively. In experiment, these predicted all-heavy pentaquark states could be searched for in the $\Omega_{QQQ}\eta$ invariant mass spectrum.

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Introduction.— In recent years, hunting for evidences of the multiquark states composed of more than three quarks have attracted one’s great interest. Not long ago, LHCb Collaboration observed a broad structure just above twice the $J/\psi$ mass and a narrower structure $X(6900)$ by using proton-proton collision data, for which could possibly be all-charmed tetraquark states [1]. Before this, LHCb Collaboration discovered a narrow state $P_{c}(4312)^+$ in the $J/\psi p$ invariant mass spectrum, and separated the formerly reported $P_{c}(4450)^+$ to two narrow overlapping peaks, $P_{c}(4440)^+$ and $P_{c}(4457)^+$, for which could be some pentaquark state candidates [2]. On the investigations of these exotic states, there are actually large numbers of related works and one could see some recent reviews [4–11] and references therein. In addition, there newly appeared a systematical study on the mass spectra of ground pentaquark states in a modified chromo-magnetic interaction model [12]. Making a synthetical consideration of all these observations, it seems very promising that pentaquark states consist of fully-charmed quarks could be found in the $\Omega_{cccc}\eta_c$ invariant mass spectrum experimentally.

Without any light quark contamination, fully-heavy pentaquark states are quite ideal prototypes to refine one’s knowledge on heavy quark dynamics. To probe a fully-heavy pentaquark state, one inevitably has to confront the very intricate nonperturbative QCD problem. As one reliable way for calculating nonperturbative effects, the QCD sum rule [13] is firmly founded on the QCD theory, and has been successfully applied to plenty of hadronic states [14–17]. Referring to fully-heavy pentaquark states, the related operator product expansion (OPE) calculations are quite complicated as one has to treat many multi-loop massive propagator diagrams. Making the development of corresponding calculation techniques, we devote to investigating fully-heavy pentaquark states with the help of trustworthy QCD sum rule method in this letter.

Fully-heavy pentaquark states in QCD sum rules.— Consulting interpolating currents for heavy mesons and baryons in full QCD, one can construct the following form of current

\[ j_\mu = (e_{abc} Q_a^T C \gamma_\mu Q_b Q_c) (Q_e i \gamma_5 \bar{Q}_e), \]

for $QQQQ\bar{Q}$ pentaquark states. Here $T$ denotes matrix transposition, $C$ means the charge conjugation matrix, $Q$ could be the heavy charm or bottom quark, and the subscript $a, b, c, e$ are color indices.

Generally, the two-point correlator

\[ \Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0| T[j_\mu(x)j_\nu(0)]|0\rangle, \]

can be parameterized as

\[ \Pi_{\mu\nu}(q^2) = -g_{\mu\nu} g_1(q^2) + g_2(q^2) + \ldots. \]

Concerning with the part proportional to $-g_{\mu\nu} g_1$, matching its two descriptions at the hadron level and at
the quark level, and applying a Borel transform, one arrives at
\[ \lambda^2 e^{-M_H^2/M^2} = \int_{25m_Q^2}^{s_0} \frac{ds}{s} \rho e^{-s/M^2}, \]  
(4)
in which \( M_H \) is the studied hadron’s mass and the spectral density \( \rho = \frac{1}{\pi} \operatorname{Im} \Pi_1(s) \). Taking the derivative of Eq. (4) with respect to \( -\frac{s}{M^2} \) and then dividing the result by Eq. (4) itself, one could acquire the mass
\[ M_H = \sqrt{\int_{25m_Q^2}^{s_0} \frac{ds}{s} \rho e^{-s/M^2}/\int_{25m_Q^2}^{s_0} \frac{ds}{s} \rho e^{-s/M^2}}. \]  
(5)

In the OPE calculation, one works at the momentum-space with the heavy-quark propagator, and then the result is dimensionally regularized at \( D = 4 \), by extending the interrelated techniques to fully-heavy pentaquark systems. Concretely, the spectral density \( \rho = \rho_{\text{pert}} + \rho(g^2 G^2) + \rho(g^3 G^2) \) is expressed as
\[
\rho_{\text{pert}} = \frac{3}{5 \cdot 2^{14} \pi^8} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{\beta_{\max}} \frac{d\beta}{\beta^3} \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{d\gamma}{\gamma^3} \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\xi^3} \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ \begin{array}{c} h^3 (m_Q^2 - h s)^3 + 8 h^2 (m_Q^2 - h s)^2 \\ + (35 h^3 s + 25 \alpha \beta m_Q^2) (m_Q^2 - h s) - 20 h^4 s^2 - 40 \alpha \beta h^2 m_Q^2 s - 20 \alpha \beta \xi m_Q^4 \end{array} \right],
\]
\[
\rho(g^2 G^2) = \frac{m_Q^2 g^2 G^2}{2^{14} \pi^8} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{\beta_{\max}} \frac{d\beta}{\beta^3} \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{d\gamma}{\gamma^3} \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\xi^3} \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ \begin{array}{c} h^3 (m_Q^2 - h s)^3 + 8 h^2 (m_Q^2 - h s)^2 \\ + (35 h^3 s + 25 \alpha \beta m_Q^2) (m_Q^2 - h s) - 20 h^4 s^2 - 40 \alpha \beta h^2 m_Q^2 s - 20 \alpha \beta \xi m_Q^4 \end{array} \right],
\]
and
\[
\rho(g^3 G^2) = \frac{g^3 G^2}{2^{16} \pi^8} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{\beta_{\max}} \frac{d\beta}{\beta^3} \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{d\gamma}{\gamma^3} \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\xi^3} \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ (1 - \alpha - \beta - \gamma - \xi)^3 \right] \left[ \begin{array}{c} h^3 (m_Q^2 - h s)^3 + 8 h^2 (m_Q^2 - h s)^2 \\ + (35 h^3 s + 25 \alpha \beta m_Q^2) (m_Q^2 - h s) - 20 h^4 s^2 - 40 \alpha \beta h^2 m_Q^2 s - 20 \alpha \beta \xi m_Q^4 \end{array} \right],
\]
It is defined as \( h = \frac{1}{\pi \beta \gamma \xi} \), and the integration limits of \( \alpha, \beta, \gamma, \) and \( \xi \) are given by
\[
\alpha = \frac{1}{2} \left[ (1 - \frac{15 m_Q^2}{s}) \pm \sqrt{(1 - \frac{15 m_Q^2}{s})^2 - \frac{4 m_Q^2}{s}} \right],
\]
\[
\beta = \frac{1}{2} \left[ (1 - \alpha - \frac{8 \alpha m_Q^2}{as - m_Q^2}) \pm \sqrt{(1 - \alpha - \frac{8 \alpha m_Q^2}{as - m_Q^2})^2 - \frac{4 \alpha (1 - \alpha) m_Q^2}{as - m_Q^2}} \right],
\]
\[
\gamma = \frac{1}{2} \left[ (1 - \alpha - \beta - \frac{3}{m_Q^2} - \frac{1 - \beta}{\beta}) \pm \sqrt{(1 - \alpha - \beta - \frac{3}{m_Q^2} - \frac{1 - \beta}{\beta})^2 - \frac{4}{m_Q^2} \frac{1 - \alpha - \beta}{\beta}} \right],
\]
and

\[ \xi = \frac{1}{2} \left[ (1 - \alpha - \beta - \gamma) \pm \sqrt{(1 - \alpha - \beta - \gamma)^2 - 4 \frac{1 - \alpha - \beta - \gamma}{m_Q^2} \frac{1}{\alpha - \beta - \gamma}} \right]. \]

**Numerical analysis.**— The input parameters are taken as \( \langle g^2 G^2 \rangle = 0.88 \pm 0.25 \text{ GeV}^4 \) and \( \langle g^3 G^3 \rangle = 0.58 \pm 0.18 \text{ GeV}^6 \) [13, 17], and \( m_Q \) is first set as the running charm mass \( m_c = 1.27 \pm 0.02 \text{ GeV} \) [23]. It is needed to find appropriate work windows for the threshold \( \sqrt{s_0} \) and the Borel parameter \( M^2 \). The lower bound of \( M^2 \) could be obtained by considering the OPE convergence, and the upper one is get from the pole dominance restriction. Besides, the threshold \( \sqrt{s_0} \) characterizes the beginning of continuum state in phenomenology.

At the start, the inputs are kept at their central values. Through comparing the relative contributions of various condensates from sum rule (4), the OPE convergence is analyzed to find the lower bound of \( M^2 \), and it is noted that the relative contributions of two-gluon condensate \( \langle g^2 G^2 \rangle \) and three-gluon condensate \( \langle g^3 G^3 \rangle \) are very small. In view of the OPE convergence analysis, it is taken as \( M^2 \geq 3.5 \text{ GeV}^2 \) numerically. Phenomenologically, one could fix the upper bound of \( M^2 \) according to the pole dominance requirement. By making the comparison between pole and continuum contribution from sum rule (4) for \( \sqrt{s_0} = 8.0 \text{ GeV} \), one notes that the relative pole contribution is around 50% at \( M^2 = 4.8 \text{ GeV}^2 \) and it descends with \( M^2 \). Accordingly, the pole dominance condition could be satisfied when \( M^2 \leq 4.8 \text{ GeV}^2 \), and the Borel window is chosen as \( M^2 = 3.5 \sim 4.8 \text{ GeV}^2 \) for \( \sqrt{s_0} = 8.0 \text{ GeV} \). In the similar analysis, they are determined to be \( M^2 = 3.5 \sim 4.5 \text{ GeV}^2 \) for \( \sqrt{s_0} = 7.9 \text{ GeV} \) and \( M^2 = 3.5 \sim 5.0 \text{ GeV}^2 \) for \( \sqrt{s_0} = 8.1 \text{ GeV} \), respectively. For the \( cccc \bar{c} \) pentaquark state, its Borel curves are shown in FIG. 1, and in the chosen work windows its mass is extracted to be \( 7.38 \pm 0.14 \text{ GeV} \). After varying all the input values, the achieved mass is \( 7.38 \pm 0.14^{+0.06}_{-0.08} \text{ GeV} \) (the first uncertainty from work windows, and the second one due to the uncertainty of QCD parameters) or compactly \( 7.38^{+0.20}_{-0.22} \text{ GeV} \).

![FIG. 1: The mass \( M^H \) dependence on \( M^2 \) for the fully-heavy \( cccc \bar{c} \) pentaquark state from sum rule (4) is shown. The Borel windows of \( M^2 \) are \( 3.5 \sim 4.5 \text{ GeV}^2 \) for \( \sqrt{s_0} = 7.9 \text{ GeV} \), \( 3.5 \sim 4.8 \text{ GeV}^2 \) for \( \sqrt{s_0} = 8.0 \text{ GeV} \), and \( 3.5 \sim 5.0 \text{ GeV}^2 \) for \( \sqrt{s_0} = 8.1 \text{ GeV} \), respectively.](image)

Replacing the heavy \( m_Q \) by the running bottom mass \( m_b = 4.18^{+0.03}_{-0.02} \text{ GeV} \) [23], one could straightway put forward the affiliated analysis for fully-bottomed \( bbbb \) pentaquark state, and its Borel curves are displayed in FIG. 2. Having taken into the uncertainty of QCD parameters, the mass value is computed to be \( 21.56^{+0.17}_{-0.15} \text{ GeV} \) for the \( bbbb \) pentaquark state.

**Summary.**— By making the development of calculation techniques to fivefold heavy pentaquark states, we present the investigation of fully-heavy \( QQQQ \bar{Q} \) pentaquark states from trustable QCD sum rules. Eventually, their mass spectrums are predicted to be \( 7.38^{+0.20}_{-0.22} \text{ GeV} \) for the \( cccc \bar{c} \) state, and \( 21.56^{+0.17}_{-0.15} \text{ GeV} \) for the \( bbbb \) state, respectively. It is proposed that these states could be experimentally looked for in the
FIG. 2: The mass $M_H$ dependence on $M^2$ for the fully-heavy $bb\bar{b}b\bar{b}$ pentaquark state from sum rule (5) is shown. The Borel windows of $M^2$ are $8.0 \sim 9.1\text{ GeV}^2$ for $\sqrt{s_0} = 21.9\text{ GeV}$, $8.0 \sim 9.6\text{ GeV}^2$ for $\sqrt{s_0} = 22.0\text{ GeV}$, and $8.0 \sim 10.0\text{ GeV}^2$ for $\sqrt{s_0} = 22.1\text{ GeV}$, respectively.

$\Omega_{QQQ}\eta_Q$ invariant mass spectrum. For the future, one can expect that further theoretical studies and experimental efforts may shed more light on the nature of exotic fully-heavy pentaquark states.

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