Analytic Results for Schwinger–Dyson Equations with a Mass Term

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Abstract. Using kinematic renormalization, we derive the Schwinger–Dyson equations for a massive Yukawa model and a Wess–Zumino-like one. Both have linear Schwinger–Dyson equations and a massive renormalized particle. An explicit solution is found in the IR limit of the non-supersymmetric case. Parametric solutions are found in the UV limit of the same model and for the supersymmetric model.

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1. Introduction

As a way to reach non-perturbative information of a quantum field theory (QFT), the Schwinger–Dyson equations have been quite extensively studied over the past few decades. The most common approach to tackle them was (and maybe still is) by a numerical study of simplified versions of the equations that, however, display important physical features, e.g. of QCD. For a recent paper in this domain, the reader can referred to [1].

On the other hand, some progress has been made these last few years to analytically study some Schwinger–Dyson equations. The anomalous dimension (from which the dressed propagator could be extracted using the renormalization group equation, as shown in [2]) has been computed order by order in [3]. Its asymptotic has also been extracted in the same article. Corrections to this asymptotic behaviour have been computed in [4]. This has been rephrased in the Borel plane in [5], where some number-theoretical results could be shown to hold at every order in the corrections to the asymptotic behaviour of the anomalous dimension.

However, there is still an important lack of exact known solutions of Schwinger–Dyson equations. To our knowledge, the only one is for a massless and linear equation and was found by David Broadhurst and Dirk Kreimer in [6]. This paper aims to be a step toward filling this gap in the field of Schwinger–Dyson equations by applying the method of [6] to more general cases.
The result of [6] rests upon three crucial steps. First, the integrodifferential Schwinger–Dyson equation is written as a differential one. This equation, when written with the right functions, can be integrated in the second step. Then, as a third step, it can be solved for some functions having the integrated functions as arguments. Using the initial conditions, we end up with a parametric solution of the initial Schwinger–Dyson equation.

The first of this steps is made possible by the linearity of the initial equation. A Schwinger–Dyson equation is said to be linear if the equation for the two-point function (or equivalently for the self-energy) is an integrodifferential equation with an integrand linear in the propagator. In this paper we will only study linear equations. In order to perform the angular integration in the loop integrals it is needed to have the field without corrections to its two points function to be massless. Here, we will keep that constraint. However, the second and third steps can give results for theories more general than the one studied in [6]. In some cases (infrared limit of massive Yukawa model) we end up with equations simple enough to be solve explicitly (although only at a given impulsion of reference in the soft IR case).

This paper is organized as follows: in Sections 1 and 2, we derive the Schwinger–Dyson equations for the massive Yukawa model and for a linear version of a Wess–Zumino model (a massive version of the model studied in [7]). In both cases, we use kinematic renormalization to do so. In Section 3, a parametric solution to the massive Yukawa model in the ultraviolet limit is found. Section 4 is devoted to the study of the massive Yukawa model in the infrared limit. Finally, in Section 5, a parametric solution to the massive supersymmetric model is found. In order to keep the length of this article within reasonable size, a full presentation of the method of [6] is not included. Nevertheless, a short summary of this method can be found in Section 3, leaving aside any technical details.

2. Schwinger–Dyson Equations of the Massive Yukawa Model

For the Yukawa models, the Schwinger–Dyson equations are linear:

\[
\left( \frac{1}{G(q^2)q + M(q^2)m} \right)^{-1} = 1 - a
\]

with 1 denoting the free propagator and the dressed fermionic propagator being

\[
P_{nSUSY} = \frac{1}{G(q^2)q + M(q^2)m}.
\]

In order to find back the free propagator at a the impulsion of reference \( \mu \), we have the initial conditions \( G(\mu^2) = M(\mu^2) = 1 \). So the equation (1) can be written