CP violating phase $\gamma$ and the partial widths asymmetry in $B^- \to \pi^+\pi^-K^-$ and $B^- \to K^+K^-K^-$ decays

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ABSTRACT

Motivated by recent Belle measurement of the branching ratios $B^- \to \pi^+\pi^-K^-$ and $B^- \to K^+K^-K^-$ and $B^\pm \to \chi_{c0} K^\pm$, we investigate the CP violating asymmetry in the partial widths for the decays $B^- \to \pi^+\pi^-K^-$ and $B^- \to K^+K^-K^-$, which results from the interference of the nonresonant decay amplitude with the resonant amplitude $B^\pm \to \chi_{c0} K^\pm \to \pi^+\pi^-K^\pm$ and $B^\pm \to \chi_{c0} K^\pm \to K^+K^-K^-$. By taking $\gamma \simeq 58^\circ$ we predict that the partial widths asymmetry is reaching 10% for the $B^- \to \pi^+\pi^-K^-$ decay and 16% for the $B^- \to K^+K^-K^-$ decay.
The extraction of the CP violation phases $\alpha$, $\beta$ and $\gamma$ has stimulated many studies during last decade (see e.g. [1]). Among them the phase $\gamma$ is most difficult to constrain. Numerous proposals were aimed to determine its size (see e.g. [2]). The partial width asymmetry in charmless three-body decays offers a chance to get more information on $\sin \gamma$ [3] - [10].

The basic idea was that the nonzero asymmetry results from the interference of the nonresonant three-body decay amplitude and the resonant $B^- \to \chi_{c0}\pi^- \to \pi^+\pi^-\pi^-$ decay amplitude [3, 4, 8, 10]. All these approaches were based on a theoretical prediction [3] for the decay rate of $B^- \to \chi_{c0}\pi^-$ which is difficult to obtain in the factorization model. Instead of relying on this prediction for this decay rate, we can now use the recent Belle collaboration measurement of the decay rate [11, 12]

$$\text{BR}(B^- \to \chi_{c0}K^-) = (6.0^{+2.1}_{-1.8}) \times 10^{-4}. \quad (1)$$

which is surprisingly large, comparable to the $B^- \to J/\psi K^-$ decay rate. In addition, the Belle collaboration has announced the measurement of the decay rates

$$\text{BR}(B^- \to K^+K^-K^-) = (37.0 \pm 3.9 \pm 4.4) \times 10^{-6}, \quad (2)$$

and

$$\text{BR}(B^- \to \pi^+\pi^-K^-) = (58.5 \pm 7.1 \pm 8.8) \times 10^{-6}. \quad (3)$$

Although these measurements do not distinguish yet the contributions to the branching ratios of the nonresonant decays, they constrain their magnitudes. On the other hand, the charmless two-body decays of B mesons are stimulating many research projects. The factorization approximation is usually used and recently great improvement has been made in the understanding of $B \to \pi\pi$ and $B \to \pi K$ decay modes (e.g. [13]). The decay mechanism of the nonleptonic charmless three-body B decays is much less understood. The intermediate resonance states might be simpler to understand due to better knowledge of the two-body B decays. Due to the lack of understanding of the three-body nonresonant decay modes of B mesons, we will adopt the existing approaches [4, 8]. The first is the use of the factorization approach. Then we reduce the problem to the calculation of the matrix elements of currents (or density operators). In the
evaluation of these matrix elements, the main contribution to the nonresonant \( B^\pm \rightarrow M\bar{M}K^\pm \), \( M = K^+, \pi^+ \), amplitude comes from the product \( < M\bar{M}|(\bar{u}b)V_{-A}|B^- > < K^-(\bar{s}u)V_{-A}|0 > \) where \((\bar{q}_1 q_2)V_{-A}\) denotes \(\bar{q}_1\gamma_\mu(1-\gamma_5)q_2\), or from \(\sum_q < M\bar{M}|(\bar{q}(1-\gamma_5)b|B^- > < K^-(\bar{s}(1+\gamma_5)q)|0 >\). For the calculation of the matrix element \( < M\bar{M}|(\bar{u}b)V_{-A}|B^- > \) we extend the results obtained in [14], where the nonresonant \( D^+ \rightarrow K^-\pi^+l\bar{\nu} \) decay was analyzed. In this analysis the experimental result for the branching ratio of the nonresonant \( D^+ \rightarrow K^-\pi^+l\bar{\nu} \) decay was successfully reproduced within a hybrid framework which combines the heavy quark effective theory (HQET) and the chiral Lagrangian (CHPT) approach.

The combination of heavy quark symmetry and chiral symmetry has been quite successful in the analysis of D meson semileptonic decays [14] - [21]. Heavy quark symmetry is expected to be even better for the heavier B mesons [18, 19]. However, CHPT could be worse in B decays due to the large energies of light mesons in the final state [5]. In order to adjust the model to be applicable in the whole kinematic region we retain the usual HQET Feynman rules for the vertices near and outside the zero-recoil region, as in [18, 19], but we include the complete propagators instead of using the usual HQET propagator [14, 21, 22].

In the following we systematically use this model [14, 21] to calculate the nonresonant \( B^\pm \rightarrow M\bar{M}K^\pm \), \( M = K^+, \pi^+ \), decay amplitude. We find that contrary to the previous cases [4] penguin contributions are very important.

First we discuss the contributions to the nonresonant decay amplitude and then we discuss the partial width asymmetry.

The effective weak Hamiltonian for the nonleptonic Cabibbo-suppressed \( B \) meson decays is given by [23] - [28]

\[
H_{eff} = \frac{G_F}{\sqrt{2}} [V_{us}^* V_{ub} (c_1 O_{1u} + c_2 O_{2u}) + V_{cd}^* V_{cb} (c_1 O_{1c} + c_2 O_{2c})] \\
- \sum_{i=3}^{10} ([V_{ub}^* V_{ud}^* c_i^u + V_{cb}^* V_{cd}^* c_i^c + V_{tb}^* V_{td}^* c_i^t] O_i) + h.c. \tag{4}
\]

where the superscripts \( u, c, t \) denote the internal quark. The tree-level operators are defined as

\[
O_{1q} = \bar{q}\gamma_\mu(1-\gamma_5)b\bar{s}\gamma^\mu(1-\gamma_5)q, \tag{5}
\]

with \( q = u, c \) and

\[
O_{2q} = \bar{q}\gamma_\mu(1-\gamma_5)q\bar{s}\gamma^\mu(1-\gamma_5)b. \tag{6}
\]
We rewrite $O_3 - O_6$, using the Fierz transformations, as follows:

\[ O_3 = \sum_{q=u,d,s,c,b} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{q} \gamma^\mu (1 - \gamma_5) q, \quad (7) \]

\[ O_4 = \sum_{q=u,d,s,c,b} \bar{s} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b, \quad (8) \]

\[ O_5 = \sum_{q=u,d,s,c,b} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{q} \gamma^\mu (1 + \gamma_5) q, \quad (9) \]

\[ O_6 = -2 \sum_{q=u,d,s,c,b} \bar{s} (1 + \gamma_5) q \bar{q} (1 - \gamma_5) b, \quad (10) \]

The operators $O_7, \ldots, O_{10}$ denote the electroweak penguin operators. Due to the smallness of their Wilson coefficients we do not include them in our analysis [5, 24].

The factorization approximation is obtained by neglecting in the Lagrangian terms which are the product of two color-octet operators after Fierz reordering of the quark fields. The effective Lagrangian for non-leptonic decays are then given by (4) with $c_i$ replaced by $a_i$. In our calculations we use next-to-leading Wilson coefficients (e.g. [24] [27]). In [24, 25, 26] it was found that $a_3$ and $a_5$ are one order of magnitude smaller than $a_4$ and $a_6$ and therefore we can safely neglect the contributions from $O_3$ and $O_5$ operators.

For $N_c = 3, m_b = 5$ GeV, we use [26, 27]:

\[
\begin{align*}
    a_1 &= 1.05, \\
    a_2 &= 0.07, \\
    a_4 &= -0.043 - 0.016i, \\
    a_6 &= -0.054 - 0.016i. \\
\end{align*}
\]

(11)

It is important to notice that the imaginary parts of the Wilson coefficients for the penguin operators are due to the internal charm- and up-quark loop exchanges. They introduce additional strong phases, affecting the CP asymmetries.

The use of factorization gives for the matrix element of $O_1$ operator for the nonresonant $B^- \to \pi^+ \pi^- K^-$ decay, following the results of [4]

\[
< \pi^+(p_1) \pi^-(p_2) K(p_3) | O_1 | B^-(p_B) >_{nr} = \\
< K^-(p_3) | (\bar{s} u)_{V-A} | 0 > < \pi^- (p_2) \pi^+ (p_1) | (\bar{u} b)_{V-A} | B^- (p_B) >_{nr}. \quad (12)
\]
To evaluate the matrix element $\langle \pi^- (p_1) \pi^+ (p_2) \rangle |(u\bar{b})_{V-A}| B^- (p_B) \rangle_{nr}$ we will also use the results obtained previously in the analysis of the nonresonant $D^+ \to \pi^+ K^- \nu$ decay width [13]. We write the matrix element $\langle \pi^- (p_1) \pi^+ (p_2) \rangle |(u\bar{b})_{V-A}| B^- (p_B) \rangle$ in the general form

$$
< \pi^- (p_1) \pi^+ (p_2) |(u\bar{b})_{V-A}| B^- (p_B) > = i r (p_B - p_2 - p_1)_{\mu} + iw_+ (p_2 + p_1)_{\mu} + iw_- (p_2 - p_1)_{\mu} - 2 \mu \epsilon_{\alpha \beta \gamma} p_{B} B_{p_2} \gamma^\alpha \gamma^\beta \gamma^\gamma .
$$

(13)

The form factors $w_{\pm}^{nr}$ and $r^{nr}$ for the nonresonant decay are given in [4, 3, 14] together with a detailed description of our hybrid model. Using the following set of Mandelstam’s variables: $s = (p_B - p_3)^2$, $t = (p_B - p_1)^2$ and $u = (p_B - p_2)^2$ the formfactors can be written as

$$
w_{+}^{nr} (s, t) = - \frac{g}{f_1 f_2} \frac{f_B m_{B*} m_B^{1/2}}{t - m_{B*}^{2}} \left[ 1 - \frac{m_B^{2} - m_{1}^{2} - t}{2 m_{B*}^{2}} \right] + \frac{f_B}{2 f_1 f_2} \frac{\sqrt{m_{B*}^{2}}}{} \frac{1}{2 m_{B}^{2}} (2 t + s - m_{B}^{2} - m_{3}^{2} - 2 m_{1}^{2}) .
$$

(14)

$$
w_{-}^{nr} (s, t) = \frac{g}{f_1 f_2} \frac{f_B m_{B}^{3/2} m_{B*}^{1/2}}{t - m_{B*}^{2}} \left[ 1 + \frac{m_B^{2} - m_{1}^{2} - t}{2 m_{B*}^{2}} \right] + \frac{\sqrt{m_{B*}^{2}}}{} \frac{1}{f_1 f_2} .
$$

(15)

$$
r^{nr} (s, t) = - \frac{1 + \tilde{\beta}}{f_1 f_2} \frac{1}{2} (2 t + s - m_{B}^{2} - m_{3}^{2} - 2 m_{1}^{2}) \sqrt{m_{B'}^{2}} \frac{f_{B'}}{m_{B}^{2} - m_{B'}^{2}} - \frac{1}{\sqrt{m_{B'}^{2}}} \frac{1}{f_1 f_2} \frac{m_{B'}^{2}}{m_{3}^{2} - m_{B'}^{2}} [\frac{1}{2} (s - m_{1}^{2} - m_{2}^{2}) - \frac{1}{4 m_{B'}^{2}} (t + m_{2}^{2} - m_{3}^{2})(m_{B}^{2} - m_{1}^{2} - t)]
$$

$$
\times \frac{1}{t - m_{B*}^{2}} + \frac{g}{f_1 f_2} \sqrt{m_{B}^{2}} \frac{f_{B'}}{m_{B}^{2} - m_{B'}^{2}} (m_{B}^{2} - m_{1}^{2} - t) + \frac{f_B}{2 f_1 f_2} + \frac{\sqrt{m_{B*}^{2}}}{} \frac{1}{2 m_{B}^{2}} (2 t + s - m_{B}^{2} - m_{3}^{2} - 2 m_{1}^{2})
$$

(16)
With the use of these results we write down
\[< K^- (p_3) \pi^+ (p_1) \pi^- (p_2) | O_1 | B(p_B) >_{nr} = \]
\[- [f_3 m_3^2 r_{nr} + \frac{1}{2} f_3 (m_B^2 - m_3^2 - s) w_{nr}]
\[\frac{1}{2} f_3 (s + 2t - m_B^2 - 2m_1^2 - m_3^2) w_{nr} \]
(17)
The parameters \(\alpha_{1,2}\) are defined in [21], \(g\) is the \(B^* B \pi\) strong coupling, discussed in [18, 19, 21]. Here \(B', B'^*, B''\) denote the relevant \(B\) meson poles, and \(f_{1,2}\) denotes the pseudoscalar meson decay constants. The coupling \(\beta\) has been analyzed in [22] and found to be close to zero and therefore will be neglected.

The operator \(O_4\) has the same kind of decomposition as \(O_1\), while the operator \(O_6\) can be rewritten as the product of density operators. The evaluation of the matrix elements like \(< K | \bar{q}(1 + \gamma_5) b | B >\) and \(< \pi \pi | \bar{q}(1 - \gamma_5) b | B >\) can then be reduced to the evaluation of the matrix elements of the weak currents \(< \pi \pi | \bar{q} \gamma_\mu \gamma_5 b | B >\) and \(< K | \bar{q} \gamma_\mu b | B >\) following the procedure described in detail in [3]. For the \(\bar{s}(1 + \gamma_5) q\) scalar and pseudoscalar quark density operator we use the CHPT result [20] and obtain
\[\bar{s}(1 + \gamma_5) q = - \frac{f_\pi^2}{2} B U_{qs}^\dagger\]
(18)
where \(B\) is a real constant expressed in terms of quark and meson masses; e.g., to lowest order \(m_{K^0}^2 = B (m_s + m_d)\) and \(U = \exp(i 2 \Pi / f)\) where \(\Pi\) is a pseudoscalar meson matrix [4]. We use the value \(m_s = 150\) MeV for the \(s\) quark mass at the scale \(m_B\) mass [4], giving \(B = 1.6\) GeV. For the calculation of the density operator \(\bar{q}(1 - \gamma_5) b\) we use the relations [23]
\[\bar{q} \gamma_5 b = - \frac{i}{m_b} \partial_\alpha (\bar{q} \gamma^\alpha \gamma_5 b),\]
(19)
and
\[\bar{q} b = \frac{i}{m_b} \partial_\alpha (\bar{q} \gamma^\alpha b),\]
(20)
where \(m_q\) has been dropped since \(m_q < m_b\).

The factorization assumption in the case of the \(O_6\) operator results in:
\[< K^- (p_3) \pi^+(p_1) \pi^- (p_2) | O_6 | B(p_B) > = \]
\[-2 \sum_{u,d,s,c,b} < K | \bar{s}(1 + \gamma_5) u | 0 > < \pi \pi | \bar{u}(1 - \gamma_5) b | B > .\]
(21)
Since the density operator in \( \langle \pi K | \bar{s}(1 + \gamma_5) \bar{d} | 0 \rangle \) can be related to the matrix element of the corresponding current, and the form factors describing the matrix element of the current operator are dominated by light vector meson resonances, contributions from terms of the type \( \langle \pi K | \bar{s}(1 + \gamma_5) \bar{d} | B \rangle \) is suppressed in the high energy region for the nonresonant amplitude, and is neglected here for our purpose. There is also the contribution \( \langle K \pi \pi | \bar{s}(1 - \gamma_5) \bar{b} | 0 \rangle \langle 0 | \bar{u}(1 - \gamma_5) \bar{u} | B \rangle \) is suppressed in the high energy region for the nonresonant amplitude, and is neglected here for our purpose. There is also the contribution \( \langle K \pi \pi | \bar{s}(1 + \gamma_5) \bar{u} | 0 \rangle \langle 0 | \bar{u}(1 - \gamma_5) \bar{u} | B \rangle \). Here \( L_s \) is the chiral Lagrangian for the light pseudoscalar mesons. These two terms give negligible contribution to the branching ratio as expected from annihilation graphs in \([23]\). The contribution \( \langle K \pi \pi | \bar{s}(1 - \gamma_5) \bar{b} | B \rangle \) does not contribute due to the momentum conservation.

Using the \( s \) and \( t \) variables we derive:

\[
\frac{B}{m_B} \left\{ -2 \frac{f_1 f_2}{f_3} m_3^{nr} + \frac{f_1 f_2}{f_3} (m_B^2 - m_3^2 - s) \right\} \]

\[
- \frac{f_1 f_2}{f_3} (s + 2t - m_B^2 - 2m_1^2 - m_3^2) w^{nr} \}.
\]

The dominant contributions to the amplitude for \( B^- \to K^- \pi^+ \pi^- \) decays are then

\[
\mathcal{M}_{nr} = \frac{G}{\sqrt{2}} [V_{ub} V_{us}^* a_1 < K \pi \pi | O_1 | B >_{nr} - V_{tb} V_{ts}^* a_4 < K \pi \pi | O_4 | B >_{nr} + a_6 < K \pi \pi | O_6 | B >_{nr}].
\]

The decay width of the nonresonant \( B^- \to \pi^- \pi^+ K^- \) can be found with the help of

\[
\Gamma_{nr}(B^- \to \pi^- \pi^+ K^-) = \frac{1}{(2\pi)^3} \frac{1}{32m_B^3} \int |\mathcal{M}_{nr}|^2 \, ds \, dt.
\]
The lower and the upper bounds are given by \( s_{\text{min}} = (m_1 + m_2)^2, s_{\text{max}} = (m_B - m_3)^2 \), while for \( t \) they are given by

\[
t_{\text{min, max}}(s) = m_2^2 + m_3^2 - \frac{1}{s}[(s - m_B^2 + m_3^2)(s + m_2^2 - m_1^2)]
\]

\[
\mp \lambda^2(s, m_B^2, m_3^2) \lambda^2(s, m_2^2, m_1^2),
\]

where \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + ab) \).

The Wolfenstein parametrization of the CKM matrix \([29]\) gives

\[
V_{ub} = A\lambda_3 (\bar{\rho} - i\bar{\eta}),
\]

\[
V_{us} = \lambda, V_{cs} = 1 - \lambda^2/2, V_{cb} = A\lambda^2, V_{ts} = -A\lambda^2 \text{ and } V_{tb} = 1.
\]

The values \( A = 0.82, \lambda = 0.224 \), are well known, while for \( \bar{\rho} \) and \( \bar{\eta} \) there are bounds given in \([30]\). This parametrization indicates \( V_{ub} \) contains the phase, while the combination of \( V_{*}^t V_{tb} \) has none. We denote \( V_{ub} = |V_{ub}| e^{i\gamma} \) following \([3]\). In our numerical calculation we use the recent value \( f_B = 175 \text{ MeV} \) (see e.g. \([31]\)). The strong coupling \( g \) is determined by the CLEO experimental results on the \( D^* \) decay width \([32]\). The value \( g = 0.57 \) reproduces this decay width. Heavy quark symmetry requires that this parameter be the same for B mesons. However, it has been pointed out that the off-mass-shell effects are very important in view of the higher B meson energy scale \([33]\) and therefore we will use the lower value of \( g = 0.23 \) \([4, 5, 34]\). This smaller value of \( g \) correctly reproduces the form factor for the \( B \rightarrow \pi \) transition at zero momentum transfer, calculated in this model, \([21]\) in agreement with results obtained in other approaches (e.g. \([24]\)). We fix the parameters \( \alpha_1, \alpha_2 \) to reproduce the value \( |A_{1,2}^{DK^*}(0)| \) in \([35]\). These results are obtained by assuming a pole behaviour of form factors. We follow this procedure and by using the values of these form factors at \( Q_{\text{max}}^2 \), we obtain \( \alpha_{1}^{DK^*} = 0.16[\text{GeV}^{1/2}] \text{ and } \alpha_{2}^{DK^*} = 0.05[\text{GeV}^{1/2}] \), which after soft scaling described in \([4, 18]\) give for the B mesons \( \alpha_{1}^{DK^*} = \alpha_{1}^{B\rho} \) and \( \alpha_{2}^{DK^*}/m_D = \alpha_{2}^{B\rho}/m_B \) resulting in the values \( \alpha_1 = 0.16[\text{GeV}^{1/2}], \alpha_2 = 0.15[\text{GeV}^{1/2}] \) which we will use here. These values give \( A_{1,2}(0) \) for the \( B \rightarrow \rho \) transition in agreement with the values used in \([23]\). The branching ratio can be written as a sum of tree level contribution \( T \) (in which the operator \( O_1 \) contributes), the penguin contributions \( P \) (operators \( O_4 \) and \( O_6 \)) and there are two interference terms proportional to \( \cos \gamma \) (denoted by \( I_1 \)) and \( \sin \gamma \) (denoted by \( I_2 \)):

\[
\text{BR}(B^- \rightarrow K^+\pi^+\pi^-)_{nr} = T + P + I_1 \cos \gamma + I_2 \sin \gamma.
\]

After performing the numerical integrations using the parameters described
above, we obtain \( T = 7.0 \times 10^{-6}, P = 7.5 \times 10^{-5}, I_1 = -4.3 \times 10^{-5} \) and \( I_2 = -1.5 \times 10^{-5} \).

In the case of \( B^- \to K^-K^+K^- \) decay we find for \( T = 3.4 \times 10^{-6}, P = 3.7 \times 10^{-5}, I_1 = -2.1 \times 10^{-5} \) and \( I_2 = -7.4 \times 10^{-6} \). Taking a larger value \( g \approx 0.4 \), as some of typical smaller value of \( g \) as mentioned in \[36\], we find moderate increase of all values \( T, P, I_{1,2} \) by about factor 20%. Note that while the penguin contributions dominate in these decays, the size of \( \gamma \) is very important for the magnitude of the branching ratio.

The results obtained by Belle collaboration \[12\] are \( \text{BR}(B^- \to K^-\pi^+\pi^-)_{\text{exp}} = (55.6 \pm 5.8 \pm 7.7) \times 10^{-6} \) and \( \text{BR}(B^- \to K^-K^+K^-)_{\text{exp}} = (35.3 \pm 3.7 \pm 4.3) \times 10^{-6} \). Comparing our results with these data we conclude that larger values of \( g \) (e.g. \( g \approx 0.4 \)) should be discounted in these cases; they give rates for nonresonant decays larger than the experimental results.

Due to the strong phases in \( a_4 \) and \( a_6 \) one can generate the CP violating asymmetry as

\[
A = \frac{|\Gamma(B^- \to \pi^+\pi^-K^-) - \Gamma(B^+ \to \pi^+\pi^-K^+)|}{|\Gamma(B^- \to \pi^+\pi^-K^-) + \Gamma(B^+ \to \pi^+\pi^-K^+)|},
\]

which can be written in the form

\[
A = \frac{\sin \gamma N_1}{N_2 + \cos \gamma N_3},
\]

where

\[
N_1 = -2G^2|V_{us}^*||V_{ub}||V_{ts}^*||V_{tb}|\int(dPS)a_1 < O_1 > [\text{Im}(a_4) < O_4 > + \text{Im}(a_6) < O_6 >],
\]

\[
N_2 = G^2|V_{us}^*|^2|V_{ub}|^2\int(dPS)\{|a_1| < O_1 > |^2 + |V_{ts}^*|^2|V_{tb}|^2[|a_4|^2 < O_4 > + |a_6|^2 < O_6 >]\},
\]

\[
N_3 = -2G^2|V_{us}^*||V_{ub}||V_{ts}^*||V_{tb}|\int(dPS)a_1 < O_1 > [\text{Re}(a_4) < O_4 > + \text{Re}(a_6) < O_6 >].
\]
The numerical calculation gives for the $B^- \to \pi^+\pi^-K^-$ decay $N_1 = -3.0 \times 10^{-5}$, $N_2 = 16.4 \times 10^{-5}$ and $N_3 = -8.6 \times 10^{-5}$. For the $B^- \to K^+K^-K^-$ decay it becomes $N_1 = -1.5 \times 10^{-5}$, $N_2 = 8.2 \times 10^{-5}$ and $N_3 = -4.2 \times 10^{-5}$. One might think that the nonresonant decay amplitude might have more complicated structure than what we found within this model and then the CP violating asymmetry will be changed. However, in the factorization model, which we use in the above calculation, the CP violating asymmetry in the nonresonant partial or integrated decay rates, is essentially model independent, since the matrix elements of $O_4$ and $O_6$ are practically proportional to that of $O_1$, as can be seen from Eqs (17) and (22). Even in the presence of low-energy resonances contribution in the nonresonant amplitude, the CP asymmetry, for the total decay rates integrated over the whole phase space, or for the differential decay rates measured as a function of the two-pion or two-kaon invariant mass, can be computed in terms of the Wilson coefficients $a_4$ and $a_6$, and are independent of the form factors. The CP violating asymmetry Eq. (27) depends on the strong phase generated by the absorptive part of the Wilson coefficients.

On the other hand, the partial widths asymmetry at the charmonium resonance discussed in [3, 4, 5] seems to be more reliable for a further constraint of the CP violating phase $\gamma$.

In order to obtain the partial width CP asymmetry, one also needs to calculate the resonant decay amplitude $B^- \to \chi_cK^- \to \pi^+\pi^-K^-$. This amplitude can be easily determined using the narrow width approximation, as in [5]:

$$M_r(B^- \to \chi_cK^- \to \pi^+\pi^-K^-) = \frac{1}{s - m_{\chi_c}^2 + i\Gamma_{\chi_c}m_{\chi_c}}M(\chi_c \to \pi^+\pi^-).$$

In our numerical calculations we will use the recent Belle measurement of the branching ratio [11] $\text{BR}(B^\pm \to \chi_cK^\pm) = (6.0 \pm 1.1) \times 10^{-4}$. The amplitude is then $M(B^- \to \chi_cK^-) = 3.37 \times 10^{-7} \text{ GeV}$. The $\chi_c$ decay data [32] then fix the decay amplitudes for $|M(\chi_c \to \pi^-\pi^+)| = 0.113 \text{ GeV}$ and $|M(\chi_c \to K^-K^+)| = 0.126 \text{ GeV}$.

The partial decay width $\Gamma_p$ for $B^- \to M\bar{M}K^-$, $M = \pi^+, K^+$, which contains both the nonresonant and resonant contributions, is obtained then
by integration from \( s_{\text{min}} = (m_{\chi_c^0} - 2\Gamma_{\chi_c^0})^2 \) to \( s_{\text{max}} = (m_{\chi_c^0} + 2\Gamma_{\chi_c^0})^2 \), where \( m_{\chi_c^0} = 3.415 \) GeV and \( \Gamma_{\chi_c^0} = 0.0149^{+0.0026}_{-0.0023} \) GeV is the width of the \( \chi_{c0} \):

\[
\Gamma_p = \frac{1}{(2\pi)^3} \frac{1}{32m_B^3} \int_{s_{\text{min}}}^{s_{\text{max}}} ds \int_{t_{\text{min}}(s)}^{t_{\text{max}}(s)} dt |M_{nr} + M_r|^2. \tag{33}
\]

Similarly, \( \Gamma_\bar{p} \), the partial decay width for \( B^+ \to M\bar{M}K^+ \), \( M = \pi^+ \), \( K^+ \) also contains both the nonresonant and resonant contributions. The CP nonconserving asymmetry is defined by

\[
A_p = \frac{\Gamma_p - \Gamma_\bar{p}}{\Gamma_p + \Gamma_\bar{p}}. \tag{34}
\]

In calculation of the \( \Gamma_p - \Gamma_\bar{p} \) we derive, assuming that \( V_{ub} = |V_{ub}| e^{i\gamma} \):

\[
\Gamma_p - \Gamma_\bar{p} = \sin \gamma \frac{4m_{\chi_c^0}\Gamma_{\chi_c^0}}{(2\pi)^3 32m_B^3} \times \int_{s_{\text{min}}}^{s_{\text{max}}} ds \int_{t_{\text{min}}(s)}^{t_{\text{max}}(s)} dt \frac{G}{\sqrt{2}} |V_{ub}| |V_{us}| a_1 < K\pi \pi |01| B >_{nr} \]

\[
\times |\mathcal{M}(B^- \to \chi_{c0}K^-)| \frac{1}{(s - m_{\chi_c^0}^2)^2 + (m_{\chi_c^0}\Gamma_{\chi_c^0})^2} |\mathcal{M}(\chi_{c0} \to \pi^-\pi^+)|. \tag{35}
\]

The denominator is given by

\[
\Gamma_p + \Gamma_\bar{p} = 2 \frac{1}{(2\pi)^3 32m_B^3} \int_{s_{\text{min}}}^{s_{\text{max}}} ds \int_{t_{\text{min}}(s)}^{t_{\text{max}}(s)} dt \times \{ |\mathcal{M}_{nr}|^2 + |\mathcal{M}(B^- \to \chi_{c0}K^-)| \frac{1}{s - m_{\chi_c^0}^2 + im_{\chi_c^0}\Gamma_{\chi_c^0}} |\mathcal{M}(\chi_{c0} \to \pi^-\pi^+)|^2 \}. \tag{36}
\]

Then we can write

\[
A_p = \frac{A_1 \sin \gamma}{A_2 + A_3 \cos \gamma}, \tag{37}
\]

where \( A_1 \) is determined by (35), while \( A_2 \) contains the sum of the resonant decay amplitude as well as \( \gamma \)-independent part of (36), corresponding to \( N_2 \) with the integration over the \( \chi_{c0} \) resonance region, \( s_{\text{min}} = (m_{\chi_c^0} - 2\Gamma_{\chi_c^0})^2 \).
\[ s_{\text{max}} = (m_{\chi_{c0}} + 2\Gamma_{\chi_{c0}})^2 \]. The part which is \( \gamma \) dependent is given in \( A_3 \) (corresponding to \( N_3 \) with the integration over the \( \chi_{c0} \) resonance region). Note that the term proportional to \( \cos \gamma \) arising from the interference with the resonance in the denominator does not contribute. Namely, in the integration region of \( s \) in (36), the real part of the resonance amplitude, being antisymmetric in \( s \) gives no contribution to the integrated decay rates over the \( \chi_{c0} \) resonance.

In Fig.1 and Fig.2 we present the distribution \( d\Gamma_p/ds \) in the region \( [s_{\text{min}}, s_{\text{max}}] \). It is clear from these figures that the nonresonant contribution is approximately constant over this region. For the given set parameters, with \( g = 0.23 \), the CP violating asymmetries are

\[ A_p(B^\pm \to K^{\pm}\pi^+\pi^-) = \frac{7.9 \sin \gamma}{73 - 1.2 \cos \gamma}, \tag{38} \]

and

\[ A_p(B^\pm \to K^{\pm}K^+K^-) = \frac{7.2 \sin \gamma}{41 - 5.6 \cos \gamma}. \tag{39} \]

Following recently given bound for the phase \( \gamma = (58 \pm 24)^\circ \) \( ^1 \) we find that the partial widths asymmetry for the \( B^\pm \to K^{\pm}\pi^+\pi^- \) decay is reaching 10\% and 16\% for the \( B^\pm \to K^{\pm}K^+K^- \).

The uncertainties due to the errors in the remaining input parameters have not been included here, but we can roughly estimate that the error in the asymmetry can be as large as 40\%.

In summary, we can compare the previously considered cases \( B^\pm \to M\bar{M}\pi^\pm, M = \pi^+, K^+ \) \( ^1 \) \( ^2 \) \( ^3 \) with the present cases \( B^\pm \to M\bar{M}K^\pm, M = \pi^+, K^+ \). Due to the different CKM matrix elements the penguin contributions dominate in the nonresonant \( B^\pm \to M\bar{M}K^\pm, M = \pi^+, K^+ \) decay rate. Moreover, the phase \( \gamma \) crucially influences their magnitudes. The appearance of the strong phases in the \( a_4 \) and \( a_6 \) Wilson coefficients of the penguin operators can generate the CP violating asymmetry in the nonresonant decay modes \( B^\pm \to M\bar{M}K^\pm, M = \pi^+, K^+ \).

The CP violating partial widths asymmetry is smaller in the cases \( B^\pm \to M\bar{M}K^\pm, M = \pi^+, K^+ \) than \( B^\pm \to M\bar{M}\pi^\pm, M = \pi^+, K^+ \). One might expect that, since now the CP violating partial widths asymmetry comes from the interference of the resonant amplitude and the tree-level nonresonant
amplitude, which is given by $V_{ub}V_{us}^{*}$, which is CKM suppressed, relative to $V_{ub}V_{ud}^{*}$, for the decays $B^\pm \rightarrow M\bar{M}\pi^\pm$, $M = \pi^+, K^+$. One could also measure the differential decay rates and CP asymmetry as a function of the two-pion and two-kaon invariant mass squared as shown in Fig.1 and Fig.2 to obtain more informations on the CP asymmetry as well as possible determination of the nonresonant amplitudes at the $\chi_{c0}$ mass region.

The determination of these partial width asymmetries provides useful guidance for experimental searches of CP violating effects and further constraints on the phase $\gamma$.

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Figure 1: Differential branching ratios for $B^- \rightarrow \pi^+ \pi^- K^-$ vs. $s$. The curves (a), (b), (c) are $d(B)(NR)/ds$, $d(B)/ds + d(\bar{B})/ds$, $d(B)/ds - d(\bar{B})/ds$ against the two-pion invariant mass squared $s$, for the nonresonant, CP symmetric and CP antisymmetric differential branching ratios respectively.
Figure 2: Differential branching ratios for $B^- \to K^+K^-K^-$ vs. $s$. The curves (a), (b), (c) are $d(B)(NR)/ds$, $d(B)/ds + d(\bar{B})/ds$, $d(B)/ds - d(\bar{B})/ds$ against the two-kaon invariant mass squared $s$, for the nonresonant, CP symmetric and CP antisymmetric differential branching ratios respectively.