Moduli fixing in semirealistic string compactifications

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Abstract. Heterotic orbifold compactifications yield a myriad of models that reproduce many properties of the supersymmetric extension of the standard model and provide potential solutions to persisting problems of high energy physics, such as the origin of the neutrino masses and the strong CP problem. However, the details of the phenomenology in these scenarios rely on the assumption of a stable vacuum, characterized by moduli fields. In this note, we drop this assumption and address the problem of moduli stabilization in realistic orbifold models. We study their qualities and their 4D effective action, and discuss how nonperturbative effects indeed lift all bulk moduli directions. The resulting vacua are typically de Sitter and there are generically some quasi-flat directions which can help to deal with cosmological challenges, such as inflation.

1. Introduction
There has been a great effort in deriving models of elementary particle physics from string theory. Promising models have arisen from intersecting D-branes in type IIB string theory, F-theory, Calabi-Yau and orbifold compactifications of the heterotic string. In particular, heterotic orbifolds have been extensively studied and successful models reproducing several features of the minimal supersymmetric extension of the standard model (the MSSM) have been found [1, 2, 3, 4, 5, 6, 7]. Unfortunately, contact between all these different constructions and particle phenomenology requires a full understanding of the details of the vacua that each model offers. These details are associated with the dynamics of the moduli fields that describe all geometrical features of the particular compactification. For instance, the masses of quarks and leptons as well as the dynamics of supersymmetry breakdown depend on the vacuum expectation values (VEVs) of the moduli, which are arbitrary at tree level in many string constructions. Therefore, in order to achieve a prediction from string theory and simultaneously to avoid severe cosmological constraints, a mechanism that fixes the VEVs of the moduli and gives them large masses is needed. This mechanism is usually called moduli fixing or stabilization.

Besides constructions that potentially explain particle physics, moduli stabilization has been independently studied in several scenarios. In type IIB string theory, it has been shown that conjugating fluxes, nonperturbative effects and/or \(\alpha'\) corrections could yield stable vacua [8, 9, 10]. There has also been some progress in fixing Kähler and complex structure moduli in heterotic compactifications on Calabi-Yau [11] and (generalized) half-flat [12] manifolds. In heterotic orbifold compactifications [13, 14], which is the focus of the present note, it is frequently argued that the absence of fluxes (other than the NS flux) can render much more difficult –if not impossible– the search for a stable vacuum. However, it is known that the inherent target
Figure 1: Geometry of the 6D compact space of a $Z_6$–II orbifold. The 6D space is chosen to be the torus $T^6 = T^2 \times T^2 \times T^2$ spanned by the roots $e_i$ of the Lie algebra $G_2 \times SU(3) \times SO(4)$. The smaller/blue arrows denote the noncontractible loops that are embedded as nontrivial Wilson lines $A_i$ in the gauge degrees of freedom. Modding out a $Z_6$ symmetry in this geometry leaves the points $\bullet$, $\star$ and $\blacksquare$ untouched.

space modular symmetries [15, 16, 17] together with nonperturbative effects such as gaugino condensation and string worldsheet instantons can provide a scalar potential with metastable minima, at least in toy models [18, 19, 20, 21, 22].

The purpose of the present note is to review the main results of [23]. We go beyond the usual claims and tackle explicitly the problem of moduli fixing in MSSM candidates arising from heterotic orbifolds. We base our study on a subset of the $O(300)$ realistic $Z_6$–II orbifold models of the minilandscape [24, 25], in which gaugino condensation can successfully yield an acceptable scale of supersymmetry breakdown [26]. The main phenomenological properties of these models shall be discussed in sec. 2, where the details of a sample model are given. We consider all bulk moduli fields, i.e. the dilaton $S$ and those fields describing the deformations in size $T_i$ and shape $U_i$ of the compact dimensions. We analyze the 4D effective action of these models, which is well understood [27, 28, 29]. In particular, the scalar potential consists of various computable, perturbative and nonperturbative contributions, which, as it turns out, lift all bulk moduli directions. This will be the topic of sec. 3. Once moduli stabilization is achieved in these setups, it results immediate to explore some cosmological consequences. We speculate in sec. 4 that inflation might emerge readily, since some of the moduli directions exhibit very small curvature, rendering the associated fields natural inflaton/curvaton candidates.

In the following, we shall work in Planck units, specifically we set $M_{Planck} = 1$.

2. An MSSM candidate

The so-called minilandscape is perhaps the most fertile search of string-derived MSSM candidates. There have been found about 300 models with the exact spectrum of the MSSM (or NMSSM [30]) and no exotics\(^1\) at low energies. Besides, these models exhibit other appealing properties: approximate gauge coupling unification [31, 32], see-saw neutrino masses [33], predominantly TeV gravitino mass [26], gauge-top unification [34], R-symmetries [6, 35] and other discrete symmetries [36] that prevent rapid proton decay and other lepton/baryon-number violating processes.

The minilandscape is based on $Z_6$–II orbifold compactifications of the $E_8 \times E_8$ heterotic string. The 6D torus is chosen to be the product of three 2-tori, $T^6 = T^2 \times T^2 \times T^2$, which are spanned by the simple roots of the Lie algebra $G_2 \times SU(3) \times SO(4)$, as depicted in fig. 1. In this basis, the action of the $Z_6$–II symmetry acts identifying the points related by a rotation (or repeated rotations) by $\pi/3$ in the $G_2$ 2-torus, a rotation $2\pi/3$ in the $SU(3)$ 2-torus, and a reflection through the origin in the last 2-torus. The special points that are left unaffected by this identification, depicted by $\bullet$, $\star$ and $\blacksquare$ in fig. 1, are called fixed points and are of crucial relevance in the minilandscape, as we shall see shortly. The intrinsic modular invariance of the heterotic

\(^1\) The term exotic refers to any field charged under the SM gauge group and which does not appear in the MSSM matter spectrum.
Table 1: The matter spectrum of a realistic $Z_6$–II orbifold model. It corresponds to the spectrum of the MSSM plus standard model singlets and vectorlike exotics, which acquire masses through a Higgs-like effect after some singlets attain VEVs. The bulk moduli are uncharged under the gauge group. Quantum numbers w.r.t. the gauge group $SU(3)_c \times SU(2)_L \times \{SO(8) \times SU(3)\}_{\text{hidden}}$ are given in parenthesis. The hypercharge $U(1)_Y$ appears as subindex.

| Bulk moduli: $S,T_1,T_2,T_3,U$ | 3 (net) generations |
|---------------------------------|---------------------|
| $3$                            | $(3,2,1,1)_{1/6}$   |
| $3$                            | $(\mathbf{3},1,1,1)_{-2/3}$ $u_i$ |
| $3+4$                          | $(1,1,1,1)_{1/3}$   |
| $3+4$                          | $(1,2,1,1)_{1/2}$   |
| Higgses                        | $(1,2,1,1)_{1/2}$   |
| Standard model singlets        | $(1,1,1,1)_{0}$     |
|                                 | $N_i$               |
| Exotics                        | $\delta_i$         |
|                                | $(3,1,1,1)_{1/6}$   |
|                                | $s_i^+$             |
|                                | $m_i$               |

A string demands the orbifold action to be embedded in the gauge degrees of freedom, producing thereby the breakdown of the original $E_8 \times E_8$ gauge group to a subgroup thereof, which in the minilandscape models takes the generic form $SU(3)_c \times SU(2)_L \times U(1)_Y \times G_{\text{hidden}} \times U(1)^n$. All standard model (SM) fields are uncharged under the nonabelian gauge factor $G_{\text{hidden}}$, but some other “hidden” fields form nontrivial representations under this group.

The guiding principle of the minilandscape search is local grand unification, which refers to the possibility that certain fixed points of an orbifold be endowed with the gauge symmetry of a grand unified theory (GUT) like $SU(5)$ or $SO(10)$, while the 4D gauge group be the one of the SM. This indeed happens in the small moduli-space region of the minilandscape and leads to scenarios that preserve the appealing features of GUTs such as gauge coupling unification while solving their common phenomenological drawbacks such as problematic quark/lepton mass relations. The crucial advantage is that matter fields living at points endowed e.g. with an $SO(10)$ local symmetry can transform as 16-plets locally. Since the 16 spinor of $SO(10)$ contains a SM family of quarks and leptons, if ideally there were three of such special local GUTs, one would find a geometric origin of the three SM generations. Moreover, the SM Higgs doublets need not arise from such local GUTs, but from the bulk or a fixed point with no local GUT. Therefore, no doublet-triplet splitting must be enforced in this situation. Unfortunately, the scenario with three degenerate SM families coming from local GUTs is not favored by orbifold constructions [24]. Instead, only two generations arise from equivalent local $SO(10)$ GUTs (see the encircled/red • in fig. 1), what is not necessarily bad news, for we might prefer a setting in which a distinction between the two lightest and the heaviest SM generation emerges naturally.

Let us focus now on one particular minilandscape model. The resulting matter spectrum is presented in tab. 1. The 4D gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y \times \{SO(8) \times SU(3)\}_{\text{hidden}} \times U(1)^6$. Note that no SM field is charged under $\{SO(8) \times SU(3)\}_{\text{hidden}}$, what “hides” this sector, in the sense that it only interacts gravitationally with the observed matter. In addition to the 4D $\mathcal{N} = 1$ supergravity multiplet (which is not displayed in tab. 1), one obtains a net number of three SM generations, the up and down Higgses required in the MSSM, some
SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$ singlets and a bunch of vectorlike exotics that are decoupled at a scale $M_{\phi}$ without breaking supersymmetry. The SM singlets charged under the hidden group can also acquire masses of order $M_d$, breaking spontaneously the additional U(1)$^6$ and leaving a pure Yang-Mills SO(8)$\times$SU(3) hidden sector at lower energies. This situation leads to gaugino condensation in this sector, which renders the scale of supersymmetry breakdown as low as $\sim$ 100 TeV [26], not far from the admissible/expected value.

In the absence of matter parity in this model, some of the SM singlets (those without VEVs) can be considered right-handed neutrinos. We have verified that they become massive when the exotics decouple, i.e. they get masses of order $M_d \sim M_{GUT}$ too. This large right-handed neutrino mass enables the see-saw mechanism to produce left-handed neutrino masses of order $10^{-3} - 10^{-2}$ eV, as suggested by neutrino-oscillation probes [38].

As shown in tab. 1, this model has five bulk moduli. The dilaton $S$ arising from the supergravity multiplet, and four deformation parameters. The latter are obtained from demanding invariance of the metric $g_{\alpha\beta} = e_{\alpha} \cdot e_{\beta}$ under the $\mathbb{Z}_6$-II orbifold action. The free parameters in this case are the “radius” of the cycle $e_1$ in the G$^2$$_2$ torus, $e_3$ in the SU(3) torus, and the cycle radii and angle of the last torus, i.e. the magnitude of $e_5$ and $e_6$ and the angle between them. These deformations are usually collected in three Kähler moduli $T_1, T_2, T_3$ for the radii of each of the tori, and a complex structure modulus $U$ for the angle in the last torus.

As a last remark, we would like to mention that this model possesses some useful symmetries. Like many mininodelandcape models, it presents an SO(2) family symmetry [39] which is the result of the vertical symmetry between the fixed points of the last torus (see e.g. the encircled/red • in fig. 1). This symmetry is particularly useful for particle phenomenology, as it could shed light on the structure of the quark and lepton mass matrices. Besides, all orbifold models exhibit target-space modular symmetries [40, 41] that act as discrete transformations on the moduli, winding numbers and momenta. In the present model, they are $[\text{SL}(2,\mathbb{Z}) \times \Gamma_1(3) \times \Gamma_0(4)]_T \times [\Gamma^0(4)]_U$ transforming the three Kähler moduli $T_i$ and one complex structure $U$ moduli, respectively. These discrete transformations do not only ensure (and help to verify) the consistency of the theory, but are a great tool for moduli fixing, as we shall see.

3. Fixing the moduli

In order to stabilize the moduli, a full knowledge of the effective supergravity theory is required. Remarkably, heterotic orbifold compactifications are the only scenario in which all relevant quantities can be computed explicitly from string theory. Perturbatively, the effective theory receives contributions from nonvanishing couplings between matter states that can be found by applying the so-called selection rules [27, 28], based on the string symmetries that are left unbroken by the orbifold action. The coupling strengths have been worked out using conformal field theory techniques [42, 43, 44] finding that they are exponentially suppressed by the Kähler moduli. Perturbative $\alpha'$ corrections do not contribute to the tree-level superpotential [45]. At nonperturbative level, the main contribution to the effective theory comes from gaugino condensation, which is affected by threshold corrections due to the decoupling of exotic matter and string excitations [46, 47].

To simplify somewhat our discussion, we adopt the following assumptions:

- all (vectorlike) exotics decouple consistently with supersymmetry at a unique scale $M_d$, such that $(M_{GUT} \sim) M_d \lesssim M_{\text{string}} \lesssim 1$;\(^5\)

\(^2\) It has been checked that e.g. operators $n^2 \ell_j \ell_k \bar{\ell}_x$ exist, implying that the additional four (exotic) pairs $\ell_i - \bar{\ell}_i$, attain large masses once some $n_i$ develop VEVs preserving $\cal{N} = 1$ supersymmetry and the three desirable lepton doublets. All other exotic fields $(d, d', \delta, \delta', s^R$ and $m_\nu)$ are decoupled analogously.

\(^3\) We point out that there are many other models which do exhibit matter parity [6],[37, app. E].

\(^4\) Details on these symmetries can be found in [23, app. A.4].

\(^5\) In fact, this assumption has been confirmed in several cases [6, 35].
• at some scale close to \( M_d \), the charged hidden matter \( \bar{N}, N, \bar{N} \) are decoupled too, yielding hidden-sector gaugino condensation at the scale \( \Lambda_{gc} \lesssim M_d \);
• nonrenormalizable couplings among matter fields are negligible;
• some SM singlets (those present in admissible renormalizable couplings) \( n_i \) do not enter in the process of decoupling of unwanted matter and attain VEVs \( A_i \ll 1 \). It follows that matter kinetic terms proportional to \( |A_i|^2 \) in the Kähler potential can be safely ignored;
• no twisted moduli nor blow-up modes appear, so that we can remain at the orbifold point;
• D-term contributions to the scalar potential are negligible.

Under these conditions, the (large-volume) Kähler potential of the model introduced in sec. 2 is computable [29, 41, 48] and given by

\[
K = -\log \left[ S + \bar{S} - \frac{19}{24\pi^2} \log(T_1 + \bar{T}_1) + \frac{7}{24\pi^2} \log(T_2 + \bar{T}_2) \right] - \sum_j \log(\phi_j + \bar{\phi}_j),
\]

where we have included all loop effects and denoted all bulk moduli by \( \phi_j = \{T_1, T_2, T_3, U, S\} \).

The superpotential is split in the perturbative bit coming from matter couplings \( W_{yuk} \) and the gaugino condensation part \( W_{gc} \), so that we obtain for our model

\[
W_{yuk} = 2 N_{255} A_1^2 e^{-2\pi T_2/3} \left( A_2 + A_3 e^{-2\pi T_1/3} \right) + \ldots,
\]

\[
W_{gc} = -\frac{c}{\epsilon} \frac{3}{16\pi^2} e^{-4\pi^2 S/3} M_d^{3/2} \eta(3T_2)^{-16/9} \left[ \eta(4T_3) \eta(U/4) \right]^{1/3} - \frac{c}{\epsilon} \frac{3}{32\pi^2} e^{-8\pi^2 S/3} M_d^{10/3} \eta(3T_2)^{4/9} \left[ \eta(4T_3) \eta(U/4) \right]^2,
\]

where \( c \) is an unknown constant arising from integrating out the condensate, \( \eta(p, \phi) \) is the well-known Dedekind function, and \( N_{255} \approx 1 \) comes from the explicit computation of string worldsheet instantons. Two remarks are in order: 1) the ellipsis in \( W_{yuk} \) contains terms with higher powers in \( e^{-T_i} \) which we neglect; and 2) modular symmetries control the moduli dependence of the eta functions, e.g. only \( \eta(4T_3) \) transforms covariantly under the unbroken \( \Gamma_0(4) \) modular symmetry. The two contributions to \( W_{gc} \) come from the two hidden groups, SO(8) and SU(3) respectively.

The structure of the superpotential is very rich. First, \( W(S) \) takes a racetrack form which can indeed lead to dilaton stabilization [20, 21] (although it must be noticed that the SU(3) contribution is subleading mainly due to the powers of \( M_d \)). Second, \( W(T_1) \) is KKLT-like which is known to yield potentially stable vacua with no or little fine-tuning. Third, \( W(T_2, T_3, U) \) is a complicated combination of cusp forms, which exhibits several critical points. We would like to point out that the inclusion of matter fields in \( W \) could be crucial for arriving to a de Sitter vacuum [49]. It is noticeable that, contrary to what happens in toy models in which there are gazillions of tunable parameters, the only parameters left free to tune in this promising model are \( A_i, M_d \) and \( c \), connected to the decoupling of unwanted states and the gaugino condensates. Therefore, the next task is just to compute the scalar potential and locate a minimum by varying these few free parameters around admissible values: \( c \sim 1, A_i \sim M_d \ll 1 \).

Applying eqs. (1) and (2) to the general expression of the F-term potential, \( V_F = e^K \left( K^{AB} D_A W D_B \bar{W} - 3 |W|^2 \right) \) (the indices \( A, B \) label all chiral supermultiplets present and \( D_A W = \partial_A W + W \partial_A K \)), we have computed the scalar potential of the model. In the limits of our assumptions and with \( c = 1/10, 20A_1 = 100A_2 = A_3 = 1/10, M_d = 1/65 \), we have found numerically dozens of vacua out of which we chose the one shown in tab. 2. The scalar potential is plotted in fig. 2. Unfortunately, all vacua are unstable for they turn out to have at least one
In the context of string theory, there has been an effort mostly in the type IIB string [52, 53, 54, 55] to derive inflation from first principles. Remarkably, it has been found that, in certain scenarios, admissible fluctuations and nongaussianities are more natural in a universe dominated by multi-field dynamics [56], as is always the case in string constructions. However, the heterotic string has not been explored sufficiently in this regard.

It seems plausible that the experience gathered in type IIB will be enriched by the structures found in heterotic orbifolds. In particular, the present study produces generically at least two...
almost flat directions. In the model investigated in this note, they correspond to the directions \(\text{Im} T_3\) and \(\text{Im} U\). Two fields of this kind seem to accommodate perfectly the scenario introduced in [56], yielding more accessible values for cosmological observables. Therefore, together with the question of moduli fixing, the feasibility of a heterotic inflationary scenario must be investigated.

Figure 2: Scalar potential as a function of the bulk moduli of an orbifold MSSM candidate for the vacuum presented in tab. 2. The scalar potential is rescaled by a factor \(10^{19}\).

References
[1] Ibáñez L E, Kim J E, Nilles H P and Quevedo F 1987 *Phys. Lett.* B191 282–286
[2] Faraggi A E, Nanopoulos D V and Yuan K j 1990 *Nucl. Phys.* B335 347
[3] Kobayashi T, Raby S and Zhang R J 2005 *Nucl. Phys.* B704 3–55 (Preprint hep-ph/0409098)
[4] Buchmüller W, Hamaguchi K, Lebedev O and Ratz M 2006 *Phys. Rev. Lett.* 96 121602 (Preprint hep-ph/0511035)
[5] Buchmüller W, Hamaguchi K, Lebedev O and Ratz M 2007 *Nucl. Phys.* B785 149–209 (Preprint hep-th/0606187)
[6] Lebedev O et al. 2008 *Phys. Rev.* D77 046013 (Preprint 0708.2691)
[7] Blaszczyk M et al. 2010 *Phys. Lett.* B683 340–348 (Preprint 0911.4905)
