Metallic ferromagnetism in the Kondo lattice

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Metallic magnetism is both ancient and modern, occurring in such familiar settings as the lodestone in compass needles and the hard drive in computers. Surprisingly, a rigorous theoretical basis for metallic ferromagnetism is still largely missing. The Stoner approach perturbatively treats Coulomb interactions when the latter need to be large, whereas the Nagaoka approach incorporates thermodynamically negligible holes into a half-filled band. Here, we show that the ferromagnetic order of the Kondo lattice is amenable to an asymptotically exact analysis over a range of interaction parameters. In this ferromagnetic phase, the conduction electrons and local moments are strongly coupled but the Fermi surface does not enclose the latter (i.e., it is “small”). Moreover, non-Fermi-liquid behavior appears over a range of frequencies and temperatures. Our results provide the basis to understand some long-standing puzzles in the ferromagnetic heavy fermion metals, and raise the prospect for a new class of ferromagnetic quantum phase transitions.

Fermi surface | itinerant magnetism | non-Fermi liquid

A contemporary theme in quantum condensed matter physics concerns competing ground states and the accompanying novel excitations (1). With a plethora of different phases, magnetic heavy fermion materials should reign supreme as the prototype for competing order. So far, most of the theoretical scrutiny has focused on antiferromagnetic heavy fermions (2, 3). Nonetheless, the list of heavy fermion metals that are known to exhibit ferromagnetic order continues to grow. An early example subject to extensive studies is CeRu₂Ge₂ (ref. 4 and references therein). Other ferromagnetic heavy fermion metals include CePt (5), CeSi₂ (6), CeAgSb₂ (7), and URu₂₃Re₂Si₂ at x > 0.15 (8, 9). More recently discovered materials include CeRuPO (10) and UIr₂Zn₂₀ (11). Finally, systems such as UGe₂ (12) and URhGe (13) are particularly interesting because they exhibit a superconducting dome as their metallic ferromagnetism is tuned toward its border. Some fascinating and general questions have emerged (14, 15, 16), yet they have hardly been addressed theoretically. One central issue concerns the nature of the Fermi surface: Is it “large,” encompassing both the local moments and conduction electrons as in paramagnetic heavy fermion metals (17, 18), or is it “small,” incorporating only conduction electrons? Measurements of the de Haas–van Alphen (dHvA) effect have suggested that the Fermi surface is small in CeRu₂Ge₂ (14–16), and have provided evidence for Fermi surface reconstruction as a function of pressure in UGe₂ (19, 20). At the same time, it is traditional to consider the heavy fermion ferromagnets as having a large Fermi surface when their relationship with unconventional superconductivity is discussed (12, 13, 21); an alternative form of the Fermi surface in the ordered state could give rise to a new type of superconductivity near its phase boundary. All these points to the importance of theoretically understanding the ferromagnetic phases of heavy fermion metals, and this will be the focus of the present work.

We consider the Kondo lattice model in which a periodic array of local moments interact with each other and with a conduction-electron band. Kondo lattice systems are normally studied in the paramagnetic state, where Kondo screening leads to heavy quasiparticles in the single-electron excitation spectrum (17). The Stoner (22) mean field treatment of these heavy quasiparticles may then lead to an itinerant ferromagnet (23). With the general limitations of the Stoner approach in mind, here we carry out an asymptotically exact analysis of the ferromagnetic state. We are able to do so by using a reference point that differs from either the Stoner or Nagaoka approach (24), and accessing a ferromagnetic phase whose excitations are of considerable interest in the context of heavy fermion ferromagnets. We should stress that a ferromagnetic order may also arise in different regimes of related models, such as in one dimension (25) or in the presence of mixed-valency (26).

Our model contains a lattice of spin-½ local moments (S_i) for each site i with a ferromagnetic exchange interaction (J > 0), a band of conduction electrons (c_k), and a characteristic bandwidth W, and an on-site antiferromagnetic Kondo exchange interaction (J_K > 0) between the local moments and the spin of the conduction electrons. The corresponding Hamiltonian is

$$H = \sum_k \epsilon_k c_k^\dagger c_k + I \sum_i \sum_{\sigma} S_i^\sigma S_i^{-\sigma} + \sum_k p_k \sigma \epsilon_k c_k^\dagger c_{in} + \frac{t_{out}}{2} c_{in} \sigma. \quad [1]$$

The symbol $\tau$ represents the Pauli matrices, with indices $a \in \{x,y,z\}$ and $\sigma \in \{\uparrow, \downarrow\}$. Here $I$ represents the sum of direct exchange interaction between the local moments and the effective exchange interaction generated by the conduction electron states that are not included in Eq. 1. Incorporating this explicit exchange interaction term allows the study of the global phase diagram of the Kondo lattice systems, and tuning a control parameter in any specific heavy fermion material represents taking a cut within this phase diagram. The Hamiltonian above is to be contrasted with models for double-exchange ferromagnets in the context of, for example, manganites, where it is the “Kondo” coupling that is ferromagnetic due to Hund’s rule.

The parameter region we will focus on is $J_K \ll |I| \ll W$. Here we can use the limit $J_K = 0$ as the reference point, which contains the local moments, representing the f-electrons with strong repulsions, and conduction electrons. As illustrated in Fig. 1, the local moments order in a ferromagnetic ground state because $I < 0$, whereas the conduction electrons form a Fermi sea with a Fermi surface. A finite but small $J_K$ will couple these two components, and its effect is analyzed in terms of a fermion + boson renormalization group (RG) procedure (27–29). We will use an effective field theory approach, which we outline below and describe in detail in SI Text. Though our analysis will focus on this weak $J_K$ regime, the results will be germane to a more extended parameter regime through continuity.

The Heisenberg part of the Hamiltonian, describing the local moments alone, is mapped to a continuum field theory (30) in the form of a Quantum Nonlinear Sigma Model (QNLσM). In this framework, the local moments are represented by an O(3) field,

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Fig. 1. An illustration of the Kondo lattice. Local moments from f-orbitals are in green, and are depicted here to be spin down. Spin-up conduction electrons are in red, which have a higher probability density than the spin-down conduction electrons in blue. The Hamiltonian for the model is given in Eq. 1 where \( \sigma \) is the spin index and a refers to the three spin directions. Note that the Einstein summation convention is used on indices. For simplicity, we assume \( \epsilon_{\sigma} = \frac{\omega}{2} \). The characteristic kinetic energy, \( W \), is defined as \( W \equiv 1/\rho_0 \), where \( \rho_0 \equiv \sum_{\sigma}(E_{\sigma} - \epsilon_{\sigma}) \) is the single-particle density of states at the Fermi energy (\( E_F \)). Both \( E_F \) and the chemical potential, \( \mu \), scale like \( W \). We use the Shankar notation with \( K = |K| \) measured from the center of the Brillouin zone.

\[ m, \text{ which is constrained nonlinearly with a continuum partition function. Combining the local moments with the conduction electrons, we reach the total partition function: } Z = \int D\bar{\psi} D\psi \delta(m^2 (\bar{\psi}, \psi) - 1) e^{-\delta}, \text{ where } \delta = \delta_m + \delta_c. \text{ The action for the conduction electrons, } \delta_c, \text{ is standard. Defining } m^m = m + im, \text{ and } m^m = m - im, \text{ the low energy action for the local moments is expressed in terms of a single complex scalar:} \]

\[ \delta_m^\ast = \frac{1}{2} \int d^4 \psi_d \bar{\psi}_d (\bar{\psi}_d, \psi_d)(-m^2 \mu + \rho_1 \gamma^2) m^m (-\bar{\psi}_d, \psi_d) + g \int (\bar{m} m)^4. \]

Here, \( M_0 \) is the magnetization density, and \( \rho_1 \) the magnon stiffness constant. The magnon–magnon coupling \( g \) is defined as a constant. As expected from the RG sense, the action for the conduction electrons, \( \delta_c \), is standard. Defining \( m^m = m + im \), and \( m^m = m - im \), the low energy action for the local moments is expressed in terms of a single complex scalar:

\[ \delta_c = \int d^4 K d\bar{\psi}_\sigma (\bar{K}, \sigma) \left( -i e + \frac{K^2}{2m_c} - \mu + \sigma \Delta \right) \psi_\sigma (\bar{K}, \sigma). \]

The dynamical part couples the magnons with the conduction electrons, leading to

\[ \delta_c^\ast = \frac{J}{4K} \int d^4 q d\bar{\psi}_\sigma (\bar{q}, \sigma) \left( m^m \psi_\sigma (q, \sigma) + m^m \bar{\psi}_\sigma (q, \sigma) \right) \]

\[ \delta_K = \frac{J}{4K} \int d^4 q d\bar{\psi}_\sigma (\bar{q}, \sigma) \left( m^m \bar{\psi}_\sigma (q, \sigma) + m^m \psi_\sigma (q, \sigma) \right). \]

The mapping from the microscopic model in Eq. 1 to the field theory in Eqs. 2–5 is similar to the antiferromagnetic case (27), but differs from the latter in several important ways. One simplification is that the translational symmetry is preserved in the ferromagnetic phase. At the same time, two complications arise. Ferromagnetic order breaks time-reversal symmetry, which is manifested in the Zeeman splitting of the spin up and down bands. In addition, the effective field theory for a local-moment quantum ferromagnet involves a Berry phase term (30) such that Lorentz invariance is broken, even in the continuum limit; the effective exponent, connecting \( \omega \) and \( q \) in Eq. 2, is \( z = 2 \) instead of 1. The effective field theory, comprising Eqs. 2–5, is subjected to a two-stage RG analysis as detailed in SI Text.

**Results**

For energies and momenta above their respective cutoffs, \( \omega_0 \sim (J/W)^2 \Delta \) and \( q_0 \sim (K_F/W) \Delta \), the magnons are coupled to the continuum part of the transverse spin excitations of the conduction electrons, see Fig. 2. Here, the Kondo coupling is relevant in the RG sense below three dimensions. This implies strong coupling between the conduction electrons and the local moments, and both the QNLσM as well as the action for the conduction electrons will be modified. Explicitly, the correction to the quadratic part of the QNLσM is

\[ \Pi(\bar{q}, q) \equiv J_K \rho_0 \left( 1 + \frac{q^2 \alpha}{q^2} \right) \]

where \( \gamma \) is a dimensionless constant prefactor. At the same time, the conduction electrons acquire the following self-energy

\[ \Sigma(K_F, \epsilon) = \begin{cases} -A_2 (\rho_0 K_F / I)^{(3/3)} (-ie^{2/3}) & d = 2 \\ -A_3 (\rho_0 K_F / I)^{d/3} \log(-ie) & d = 3 \end{cases} \]

where \( A_2 \) and \( A_3 \) are dimensionless constants of order unity. The self energies \( m^m \gamma m^m \) and \( \bar{\gamma}_0 \) add directly to the quadratic parts of the action, \( \delta_m^\ast \) and \( \delta_c \), respectively. Similar forms for the self-energies appear in other contexts, notably the gauge-fermion problem and the spin-fluctuation-based quantum critical regime. The formal similarities as well as some of the important differences are discussed in SI Text.

With these damping corrections incorporated, the effective transverse Kondo coupling, \( J_K \), becomes marginal in the RG sense in both two and three dimensions; the marginality is exact in the sense that it extends to infinite loops, as detailed in SI Text. This signals the stability of the form of damping for both the magnons and conduction electrons (28, 31). At the same time, the effective longitudinal Kondo coupling, \( J_K^L \), as well as the nonlinear coupling among the magnons, \( g \), are irrelevant in the RG sense.

![Fig. 2](https://example.com/fig2.png)

**Fig. 2.** Phase space for the Kondo coupling. (A) The spin-splitting of the conduction electron band, which kinetically suppresses interband processes associated with the Kondo spin-flip coupling to the local-moment magnons. (B) The kinematics for the spin-flip Kondo coupling. The low-lying excitations of the local-moment system are the magnons that enter the continuum atfinite \( \omega_0 \) and \( q_0 \). Those of the conduction electrons are expressed in terms of the spin-flip continuum, whose Kondo-coupling to the local-moment magnons is cut off below the cutoff energy, \( \omega_0 = (J/W)^2 \Delta \), and the cutoff momentum, \( q_0 \approx K_F - K_F \approx (K_F/W) \Delta \).
The exactly marginal nature of the Kondo coupling in the continuum part of the phase space implies that the effective coupling remains small as we scale down to the energy cutoff $\omega \sim \omega$, and, correspondingly, the momentum cutoff $q \sim q$. Below these cutoffs, the transverse Kondo coupling, which involves spin flips of the conduction electrons, cannot connect two points near the up-spin and down-spin Fermi surfaces; see Fig. 2. Although there is no gap in the density of states, as far as the spin-flip Kondo coupling is concerned, the system behaves as if the lowest energy excitations have been gapped out. The important conclusion, then, is that the effective transverse Kondo coupling renormalizes to stronger values as the energy screened (34, 35). The difference is that, in the latter case, the log of the electrical resistivity \(\rho\) corresponds to zero in the zero-energy and zero-momentum limit. This establishes the absence of static Kondo screening. Hence, the Fermi surface is small, and this is illustrated in Fig. 3.

The region of validity of Eqs. 6 and 7 corresponds to $\omega \ll \omega \ll |\mu|$ and $q \ll 2K_F$. This range is well-defined, given that $\Delta \approx J_K/(m^2) \leq J_K$ and that we are considering $J_K \ll |\mu| \ll W$. In this same energy and, correspondingly, temperature ranges, other physical properties also show a non-Fermi-liquid behavior. In two dimensions, the specific heat coefficient, $C/T \sim T^{-1/3}$ and the electrical resistivity $\rho \sim T^{2/3}$. These non-Fermi-liquid features have form similar to those of the quantum critical ferromagnets (32, 33), although here we are deep inside the ferromagnetically ordered part of the phase diagram.

**Discussion**

Our result is surprising given that the ratio $J_K/\omega \sim W^2/(J_K/(m^2)^2) \gg 1$. By contrast, the standard Kondo impurity problem with a pseudogap of order $\Delta_{pg} \ll J_K$ in the conduction electron density of states near the Fermi energy would be Kondo-screened (34, 35). The difference is that, in the latter case, the Kondo coupling renormalizes to stronger values as the energy is lowered in the range $\Delta_{pg} \ll \omega \ll W$; for $J_K/\Delta_{pg} \ll 1$, the renormalized Kondo coupling is already large by the time the energy is lowered to $\omega \sim \Delta_{pg}$.

The small Fermi surface we have established is to be contrasted with the large Fermi surface of a ferromagnetic heavy fermion metal in the Stoner treatment, illustrated in Fig. 3B. In the latter case, the local moments become entangled with the conduction electrons as a result of the static Kondo screening. Kondo resonances develop and the local moments become incorporated into a large Fermi surface. This Fermi surface comes from a Zeeman-splitting of an underlying Fermi surface for the paramagnetic phase; the latter is large, as seen through a non-perturbative proof (18) that relies upon time-reversal invariance.

Our result of a stable ferromagnetic metal phase with a small Fermi surface provides the basis to understand the dHvA-measured (14–16) Fermi surface of CeRu$_2$Ge$_2$, which is ferromagnetic below $T_c = 8$ K. Our interpretation rests on a dynamical Kondo screening effect that turns increasingly weak at lower energies. This is supported by the observation of the collapsing quasielastic peak measured in the inelastic neutron-scattering cross section as the temperature is reduced (36). It will be very instructive if the Fermi surface of UGe$_2$ (19) is further clarified and if systematic dHvA measurements are carried out in other ferromagnetic heavy fermion metals as well. With future experiments in mind, we note that our conclusion of a small Fermi surface also applies to ferromagnetic order.

In the parameter regime we have considered, the non-Fermi-liquid features are sizable. For instance, the non-Fermi-liquid contribution to the self-energy (Eq. 7) is, at the cutoff energy $\omega_1$, larger than the standard Fermi-liquid term associated with the interactions among the conduction electrons. It remains to be fully established whether the non-Fermi-liquid terms in the electrical resistivity and specific heat can be readily isolated from contributions of other processes. Still, there is at least one family of materials, URu$_2$Re$_2$Si$_2$ at $x > 0.15$, in which non-Fermi-liquid features have been shown to persist deep inside the ferromagnetic regime (8, 9). Whether this observed feature is indeed a property of the ferromagnetic phase, or if it is related to some quantum critical fluctuations or even certain disorder effects, remains to be clarified experimentally. We hope that our theory will provide motivation for the experimental search of non-Fermi-liquid behavior in ferromagnetic heavy fermion metals as well.

The existence of a ferromagnetic phase with a small Fermi surface raises the prospect of a direct quantum phase transition from a Kondo-destroyed ferromagnetic metal to a Kondo-screened paramagnetic metal. This, like its antiferromagnetic counterpart (2, 37, 38), in turn raises the possibility of a new type of superconductivity; the underlying quantum fluctuations would be associated with not only the development of the ferromagnetic order (12) but also the transformation of a large-to-small Fermisurface. Theoretically, accessing the quantum phase transition requires that our analysis be extended to the regime where the Kondo coupling is large compared to the RKKY interaction, and this represents an important direction for the future. Experimentally, in the case of CeRu$_2$Ge$_2$, applying pressure or doping Ge with Si fails to reach a ferromagnetic-to-paramagnetic quantum phase transition due to the intervention of antiferromagnetic order; other control parameters or other ferromagnetic heavy ferromagnets should be explored.

Finally, it is instructive to compare our study of the Kondo lattice Hamiltonian with traditional studies of itinerant ferromagnetism based on a one-band Hubbard model. We have taken advantage of the separation of energy scales of the Kondo lattice Hamiltonian and derived our results from an asymptotically exact RG analysis. By contrast, the one-band Hubbard model does not feature a separation of energy scales at the Hamiltonian level and the corresponding theoretical studies (39) have been based on mean-field (random-phase) approximations. Furthermore, the separation of energy scales in the Kondo lattice Hamiltonian is crucial for our conclusion that the non-Fermi-liquid behavior exists over a large energy window, which does not happen in the one-band case. Finally, the issue of small Fermi surface, which represents a major conclusion of our study, is absent in the case of the one-band Hubbard model.

To summarize, we have shown that the ferromagnetic Kondo lattice has a parameter range where the Kondo screening is destroyed and the Fermi surface is small. This conclusion is important for heavy fermion physics. It allows us to understand a long-standing puzzle on the Fermi surface, as epitomized by the dHvA measurements in CeRu$_2$Ge$_2$. It also sharpens the analogy with the extensively studied antiferromagnetic heavy fermion metals, where the dichotomy between Kondo breakdown and conventional quantum criticality is well established. More broadly, the present work has led to one of the very few asymptotically exact results for metallic ferromagnetism whose rigorous understanding has remained elusive for many years (40). Our findings highlight an important lesson, namely that correlation effects can lead...
to qualitatively new properties even for magnetism occurring in a metallic environment. This general lesson could very well be relevant to a broad array of magnetic systems, including the extensively debated iron pnictides (41).

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