Pion interferometry for hydrodynamical expanding source with a finite baryon density

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We calculate the two-pion correlation function for an expanding hadron source with a finite baryon density. The space-time evolution of the source is described by relativistic hydrodynamics and the HBT radius is extracted after effects of collective expansion and multiple scattering on the HBT interferometry have been taken into account, using quantum probability amplitudes in a path-integral formalism. We find that this radius is substantially smaller than the HBT radius extracted from the freeze-out configuration.

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The Bose-Einstein correlation of identical bosons produced in high-energy heavy-ion collisions, also known as the HBT effect, is an important tool for the study of the space-time structure of the emitting source. As the source expands, cools, and freezes out, it is important to know what source distribution the HBT interferometry is sensitive to. The validity of the conventional assumption on HBT interferometry have been taken into account, using quantum probability amplitudes in a path-integral formalism. We find that this radius is substantially smaller than the HBT radius extracted from the freeze-out configuration.

\[ T^{\mu \nu}(x) = \left[ \epsilon(x) + p(x) \right] u^\mu(x) u^\nu(x) - p(x) g^{\mu \nu}, \]  

(1)

where \( x \) is the space-time coordinate, \( \epsilon, p, \) and \( u^\mu = \gamma(1, v) \) are respectively the energy density, pressure, and 4-velocity of the cell, and \( g^{\mu \nu} \) is the metric tensor. The local conservation of energy and momentum can be expressed by

\[ \partial_\mu T^{\mu \nu}(x) = 0, \quad (\nu = 0, 1, 2, 3). \]  

(2)

The conservation of baryon number gives

\[ \partial_\mu j_\mu(x) = 0, \]  

(3)

where \( j_\mu = n_b u^\mu \) is the four-current-density of baryon \((n_b \text{ is baryon density}). In the thermalized cell, the density number \( n_i \) of the particle species \( i \), the energy density \( \epsilon_i \), the pressure \( p_i \), and the local velocity of sound \( c_s \) can be obtained as a function of local temperature \( T(x) \) and local chemical potential \( \mu_i(x) \) by assuming an ideal hadron gas. For simplicity, we consider a hadronic gas consisting of nucleons, \( \Delta(1232) \), and pions with spherical geometry as in Ref. [4]. For spherical geometry Eqs. (2) and (3) become

\[ \begin{align*}
\partial_i E + \partial_r [(E + p)v] &= -F, \\
\partial_i M + \partial_r (M v + p) &= -G, \\
\partial_i N + \partial_r (N v) &= -U,
\end{align*} \]  

(4, 5, 6)

where \( E \equiv T^{00}, M \equiv T^{vr}, N \equiv n_b \gamma, \)

\[ F = \frac{2v}{r} (E + p), \quad G = \frac{2v}{r} M, \quad U = \frac{2v}{r} N. \]  

(7)

In order to bring out the salient features of the effects under consideration, we shall carry out model calculations for some idealized space-time configurations. In these model calculations, we assume that the initial velocity \( v(0, r) = 0 \), and the initial energy density is a Gaussian distribution \( \epsilon(0, r) = \epsilon_0 e^{-r^2/2\mu_0^2} \). In a local equilibrium cell, we have \( \mu_\sigma = 0, \mu_N = \mu_\Delta = \mu_\pi \). Using the HLLE scheme [8] and with the relations of \( \epsilon(T(x), \mu_B(x)), p(T(x), \mu_B(x)), n_B(T(x), \mu_B(x)), \) and \( c_s(T(x), \mu_B(x)) \), we can get the solution of the hydrodynamical equations for \( F = G = U = 0 \). Then, using the
Raryon density at spherical expanding source with Sod's operator splitting method [9], we can obtain the expanding source at

\[
\Phi(\kappa_1 \kappa_2 : x_1 x_2 \rightarrow x_{d1} x_{d2}) = \frac{1}{\sqrt{2}} \left\{ A(\kappa_1 (x_1), x_1) A(\kappa_2 (x_2), x_2) \times \exp \left[ -i \int_{x_1}^{x_{f1}} \kappa_1 (x') \cdot dx' - ik_1 \cdot (x_{d1} - x_{f1}) \right] \right. \\
\times \exp \left[ -i \int_{x_2}^{x_{f2}} \kappa_2 (x') \cdot dx' - ik_2 \cdot (x_{d2} - x_{f2}) \right] \\
+ A(\kappa_1 (x_2), x_2) A(\kappa_2 (x_1), x_1) \times \exp \left[ -i \int_{x_2}^{x_{f2}} \kappa_2 (x') \cdot dx' - ik_2 \cdot (x_{d2} - x_{f2}) \right] \\
\left. \times \exp \left[ -i \int_{x_1}^{x_{f1}} \kappa_1 (x') \cdot dx' - ik_1 \cdot (x_{d1} - x_{f1}) \right] \right\},
\]

(12)

where \( x_{f1} \) and \( x_{f2} \) are the freeze-out points corresponding to the particles 1 and 2 produced at \( x_1 \) and \( x_2 \) and detected at \( x_{d1} \) and \( x_{d2} \), respectively. \( x'_{f1} \) and \( x'_{f2} \) are the freeze-out points corresponding to the particles 1 and 2 produced at \( x_1 \) and \( x_2 \) detected at \( x_{d2} \) and \( x_{d1} \), respectively. As we consider mainly two particles whose momenta are nearly parallel, we approximately have \( x'_{f1} = x_{f1} \) and \( x'_{f2} = x_{f2} \).

Figure 1 shows the temperature and velocity for a spherical expanding source with \( \epsilon_0 = 0.5 \) GeV/fm\(^3\) and \( R_0 = 4.0 \) fm. The corresponding initial temperature and baryon density at \( r = 0 \) are \( T_0 = 137 \) MeV and \( n_{00} = 0.36 \) fm\(^{-3}\), respectively. They are obtained by using the state of equation of mixed ideal gas and \( \epsilon_0 \). The temperature \( T_0 = 137 \) MeV is lower than the estimated critical temperature, \( T_c = 160 \) MeV, of the transition between quark-gluon plasma and hadronic gas.

The two-particle Bose-Einstein correlation function is defined as the ratio of the two-particle momentum distribution \( P(k_1, k_2) \) to the product of the single-particle momentum distribution \( P(k_1) P(k_2) \). For an expanding source, \( P(k) \) and \( P(k_1, k_2) \) can be expressed as

\[
P(k) = \int d^4 x e^{-2 \Im \hat{\phi}_s(x)} \rho(x) A^2(\kappa(x), x),
\]

(8)

\[
P(k_1, k_2) = \int d^4 x_1 d^4 x_2 e^{-2 \Im \hat{\phi}_s(x_1)} e^{-2 \Im \hat{\phi}_s(x_2)} \rho(x_1) \rho(x_2) |\Phi(\kappa_1 \kappa_2 : x_1 x_2 \rightarrow x_{d1} x_{d2})|^2,
\]

(9)

where \( A(\kappa(x), x) \) is the magnitude of the amplitude for producing a pion with momentum \( \kappa \) at \( x \), which is proportional to the Bose-Einstein distribution characterized by the local temperature in the local frame of the cell at \( x \). \( \rho(x) \) is the pion-source density, which includes a primary source and the secondary source from \( \Delta \) decay,

\[
\rho(x) = \rho^{\text{prim}}(x) + \rho^{\text{second}}(x) = n_\pi(x) \delta(t) + \Gamma n_\Delta(x),
\]

(10)

where \( \Gamma = 120 \) MeV is the width of \( \Delta \). \( e^{-2 \Im \hat{\phi}_s(x)} \) is the absorption factor due to multiple scattering,

\[
e^{-2 \Im \hat{\phi}_s(x)} = \exp \left( -\int_{x}^{x_{f1}} \sigma_{\text{abs}}(\sqrt{s_{\pi N}}) n_N(x) dl \right),
\]

(11)

where \( \sigma_{\text{abs}}(\sqrt{s_{\pi N}}) \) is the absorption cross section of \( \pi + N \to \Delta \) at the center-of-mass energy \( \sqrt{s_{\pi N}} \) and \( dl \) is the spatial line element along the path of particle propagation. In Eq. (9), \( \Phi(\kappa_1 \kappa_2 : x_1 x_2 \rightarrow x_{d1} x_{d2}) \) is the wave function for two-particles produced at \( x_1 \) and \( x_2 \) with momenta \( \kappa_1 (x_1) \) and \( \kappa_2 (x_2) \) and detected at \( x_{d1} \) or \( x_{d2} \) with momenta \( k_1 \) and \( k_2 \), respectively.

\[
C(k_1, k_2) = 1 + \int d^4 x e^{i(k_1 - k_2) \cdot x + i \hat{\phi}_s(x, k_1, k_2)} \times e^{-2 \Im \hat{\phi}_s(x)} \rho_{\text{eff}}(x; k_1 k_2)^2,
\]

(13)

where \( \rho_{\text{eff}} \) is the effective density

\[
\rho_{\text{eff}}(x; k_1 k_2) = \frac{\sqrt{f_{\text{init}}(\kappa_1 (x), x)} \sqrt{f_{\text{init}}(\kappa_2 (x), x)}}{\sqrt{P(k_1) P(k_2)}},
\]

(14)

\[
f_{\text{init}}(\kappa(x), x) = \rho(x) A^2(\kappa(x), x),
\]

(15)
is the phase space distribution of particle sources, and
\[
\phi_c(x, k_1 k_2) = -\int_x^{x'} \left\{ [\kappa_1(x') - \kappa_2(x')] - [k_1 - k_2] \right\} \cdot dx'.
\]
(16)

It can be seen that the two-pion correlation function is related to the phase space distribution of the pion production source, modified by an absorption factor arising from multiple scattering and a phase factor \( \phi_c \) from the collective expansion.

Knowing the hydrodynamical solution as the space-time variations of density, velocity, and thermodynamical functions of the fluid cells, we can construct the trajectories of a pair of test pions which start initially at \( x_1 \) and \( x_2 \) and emerge finally with momenta \( k_1 \) and \( k_2 \) at freeze-out. The knowledge of the space-time trajectories of the pair of pions allows one to calculate the path integrals in the wave function \( \Phi \) of Eq. (12) and the function \( P(k_1, k_2) \) and \( P(k) \) after summing over all source elements in Eqs. (8) and (9).

The correlation function \( C(q) \) of the relative momentum of the two particles, \( q = |k_1 - k_2| \), can be constructed from \( P(k_1, k_2) \) and \( P(k_1)P(k_2) \) by integrating over the average momentum \( (k_1 + k_2)/2 \). The HBT radius \( R \) can then be extracted by parameterizing the correlation function \( C(q) \) as
\[
C(q) = 1 + \lambda e^{-q^2 R^2}.
\]
(17)

In our model calculation, we shall consider the freeze-out configuration to be characterized by a freeze-out temperature of \( T_f = 0.5 T_0 \). We calculate first the correlation function \( C(q) \) for the case (a) when the detected pions are assumed to originate only from the freeze-out configuration. This is the configuration considered in the conventional description of intensity interferometry with collective expansion where one assumes a chaotic source at freeze-out due to random pion rescattering.

We calculate next the case (b) using the present formalism to take into account the effects of multiple scattering and collective expansion with quantum probability amplitudes in a path-integral framework. We use the experimental absorption cross section [14] represented by the model of the absorption of a pion by a nucleon forming a delta resonance. For further comparison, we calculate \( C(q) \) for the case (c) in which the detected pions originate from the source under the hydrodynamical expansion with no absorption.

Figure 2 shows results of the two-pion interferometry for these cases. The conventional freeze-out configuration [case (a)] leads to an extracted HBT radius of 6.87 fm. When there is no absorption of pions [case (c)], the HBT radius is 4.32 fm. The HBT radius obtained by using the present formalism [case (b)] leads to an HBT radius of 5.31 fm which lies between the radii of case (a) and case (c). The extracted HBT radius depend on the description of the pion source and absorption.

We conclude that for a pion source of the type considered, the HBT radius extracted from the freeze-out configuration is substantially greater than the radius obtained when effects of multiple scattering and collective expansion on the HBT interferometry have been properly taken into account using quantum probability amplitudes in a path-integral formalism. As the latter description is a more realistic description of the HBT interferometry, the conventional assumption that the HBT interferometry measures the distribution of the freeze-out configuration is therefore subject to question.

We have carried out model calculations for some idealized configurations in order to illustrate the main features of the effects under consideration. These features will remain the same for more realistic space-time configurations. In this paper the hadronic gas is taken to be an ideal gas. For a more realistic case, one should consider the volume correction [14,16], which will lead to a slightly lower temperature and lower density of hadronic gas but will hardly affect the main features of the HBT interferometry considered here. The freeze-out temperature \( T_f = 0.5 T_0 \) considered in this paper is lower than the freeze-out temperatures \( T_f = 0.7 T_c \) and \( 0.9 T_c \) (\( T_c = 160 \) MeV) considered for a zero net baryon density case 9 because the cross section between pion and baryon is greater than that between pions. The HBT radii of the three cases considered in this paper will increase when the freeze-out temperature decreases but the main feature concerning the relative magnitudes of the HBT radii is unaltered.

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