Slow Magnetic Monopole: Interaction with Matter and New Possibility of Their Detection.

I.V. Kolokolov, P.V. Vorob’ev
Budker Institute of Nuclear Physics (BINP) *
V.V. Ianovski
Petersburg Nuclear Physics Institute (PNPI) †

Abstract

The possibility of existence of a magnetic monopole has been surveyed by P. Dirac even in 1931, and then from the point of view of the modern theory by A.M. Polyakov and G. 'tHooft in 1974. Numerous and unsuccessful attempts of experimental search for monopole in cosmic rays and on accelerators in high energy particle collisions have been done. Also the searches have been carried out in mica for monopole tracks as well as for relict monopoles, entrapped by ferromagnetic inclusions in iron-ores, moon rock and meteorites. These entrapped monopoles, when released, would have the lowest velocities $\beta < 10^{-6}$ and do not yield ionization at all, and are hard to detect. Therefore it is necessary to examine thoroughly the mechanisms of slow heavy monopole interaction with matter and their scale of energy loss.

We discuss here the interaction of a massive slow magnetic monopole with magnetically ordered matter, with conductors, superconductors and with condensed matter in general. Our results indicate that the energy loss of a slow supermassive monopole reach $10^8$ eV/cm and more if we take into consideration the Cherenkov radiation of magnons or phonons and conductivity of the media. A new method of search for cosmic and relict monopoles by magnetically ordered film is considered too. This approach resembles the traditional method of nuclear emulsion chamber. Apparently the proposed method is particularly attractive for detection of relict monopoles, released from melting iron ore.

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1 Introduction

A concept of a magnetic monopole has been introduced into modern physics in 1931 by Paul Dirac [1]. He postulated existence of an isolated magnetic

*RU-630090 Novosibirsk Academician Lavrentiev Prospect 11 Russia. P.V.VOROBYOV@inp.nsk.su
†RU-188350 Gatchina St.Petersburg Russia. IANOVSKI@lnpi.spb.su
charge \( g \). Using general principles of quantum mechanics, he has related the electric and magnetic charge values: 
\[
ge = \frac{e}{2}\hbar c,
\]
where \( e \) is the electron electric charge, \( \hbar \) is the Plank constant, \( c \) is the speed of light, \( n = \pm 1, 2... \) is an integer. Numerous but unsuccessful attempts of experimental search for this magnetic monopole at accelerators and in cosmic rays \[2, 3, 4, 5\] have been done since then.

The new interest to this problem has arisen in 1974, when Polyakov \[6\] and \'t Hooft \[7\] have shown that such objects exist as solutions in a wide class of models with spontaneously broken symmetry. The nature of their monopoles is absolutely different from the nature of other elementary particles, since they represent a non-trivial topological construction of finite size, which originates from non-Abelian fields. So registration of the non-Dirac monopoles or estimation of their flux limit could be an essential contribution to construction of the Grand Unified Theory, and as well it would give incentives for solutions of various problems in astrophysics.

The magnetic charge of the Polyakov–’t Hooft monopole is a multiple of the Dirac one \( g = 2ne/\alpha \), and the value of its mass \( M_g \) lies in the range of \( 10^8 \) — \( 10^{16} \) GeV. It is completely clear, that the Polyakov–’t Hooft massive monopoles can not emerge at accelerators, therefore we do not consider accelerator experiments. Moreover, we assume that the monopoles that reach the surface of the Earth are gravitationally bound up either with the Galaxy (if \( \beta = v/c < 10^{-3} \)) or with the Sun (when \( \beta < 10^{-4} \)).

The monopole ionization loss in matter has been evaluated by many authors (look at the reviews: \[2, 3, 8\]). For fast monopole the ionization loss appreciably (about 4700 times!) exceeds the loss for the minimum ionizing particles — MIPs, which is \( dE/dl \approx 2 \) MeV/g.

In units of the ionizing loss of particle with charge \( e \), the monopole ionization loss is given by:
\[
\left( \frac{dE}{dl} \right)_g = \left( \frac{dE}{dl} \right)_e \left( \frac{g}{e} \right)^2 \beta^2.
\]

If we recollect that ionization loss of a charged particle is proportional to \( 1/\beta^2 \), then it is clear, that the loss of a monopole does not depend on velocity. It should be pointed out here that in GUT the monopole is a very heavy particle. Therefore supermassive monopole is practically always non-relativistic.

When \( \beta \sim 10^{-3} \) the ionization loss of a monopole decreases to level of energy loss of MIP, at \( \beta < 10^{-4} \) the ionization mechanism of the energy loss
is switched off practically, because in this case the energy of monopole-atom collision already is not enough for ionization of the latter.

To estimate the maximum of monopole velocity $v$, it is natural to take the velocity of the Sun relatively to the background radiation

$$\beta = \frac{v}{c} \simeq 10^{-3}.$$ (2)

We shall remind here, that the virial velocity for our Galaxy does not exceed $10^{-3}c$ too.

Some expansion of the ionizing detector sensitivity for slow monopoles is reached due to the Drell effect [9]. The essence of the Drell effect is in following. Let us assume, that a monopole passes by a helium atom. In a strong magnetic field of the monopole occurs a crossing of main and excited levels, due to Zeeman effect. In the varying magnetic field, due to non-adiabaticity of the process, it is possible that an electron transits to an excited level. After the monopole passes, some of helium atoms remain in excited states with energies of about 20 eV. An admixture of a gas with ionization potential lower than the excitation energy of helium atom gets ionized by collision with excited helium (quenching gases - $CH_4$, $CO_2$ or $n-\text{pentane}$) — the Penning effect. Therefore the ionization can be detected either by direct methods, or by the radiative recombination. The gas counters for cosmic monopole detection on the base of Drell effect were created and used in a number of experiments [3]. The ionization and Drell effect are discussed in detail in [2]. There is also a review of experiments on search of massive monopoles.

For detection of a slow monopole with efficiency close to 1, the superconducting induction detectors were designed and constructed. The pioneer in this field was B. Cabrera [10]. The single event, registered by the Cabrera detector, has originated a burst of experimental activity of searches for monopoles of cosmic origin. However, the significant progress in sensitivity (and corresponding limits on the monopole flux) has been achieved only for ionizing detectors [2, 11]. Sensitivity of these detectors is close to the Parker limit [12]

$$\mathcal{F} \leq 1 \cdot 10^{-16}(m/10^{17}\text{ GeV})\ cm^{-2}s^{-1}\ sr^{-1}.$$ (3)

The results, obtained recently by the induction experiments, are more modest. At the flux of $10^{-15}\ cm^{-2}sr^{-1}s^{-1}$, the observation of only one magnetic monopole per year would require the effective area of a detector of about 1000 $m^2$. The modern superconducting inductive detectors with superconducting quantum interference devices (SQUID) and using magnetometer techniques
to register the particle have an effective area only of the order of 1 m$^2$. Moreover, there are attractive features of traditional detectors of slow moving monopoles $\beta < 10^{-4}$ since normally the surface of composite cryogenic induction detectors is only few square meters.

Another possibility of detection of a slow GUT-monopole is the search for proton decay induced by a heavy monopole [13], [14]. Recently the group working with the Baikal lake Cherenkov detector [15] has set the following limit on the flux of heavy magnetic monopoles and the Q-balls, which are able to induce the proton decay

$$\mathcal{F} < 3.9 \cdot 10^{-16} cm^{-2}s^{-1}sr^{-1}.$$  

In this paper we discuss the interaction of slow moving supermassive magnetic monopole with ferromagnetic, conducting and other media, and some new features of this process. Actually we consider a possibility of building-up of a new type of detector of slow monopoles. Our idea is based on registration of interaction of a slow cosmic-ray-related monopole with the film of easy-axis and high coercitivity ferromagnet [16]. As a sensitive element of such a detector one can use an advanced high density storage media, namely the magneto-optical disk (MO disk) [17]. (To our knowledge for modern MO disks an areal density of 45 Gbit/in$^2$ has been demonstrated using near-field techniques, with a theoretical possibility in excess of 100 Gbit/in$^2$). The slow monopole which is passing through the magnetic coat of MO disk, leaves a distinctive magnetic track, and this track can be detected by the standard polarimetric equipment. It is important to note that considerable surface can be covered by such MO disks. They can be exposed any reasonable time without any maintenance, like in emulsion chamber experiments or the CR39 nuclear track subdetector of MACRO [18].

Apparently, the use of such passive detectors will be especially effective in search of the relict monopoles, entrapped in ferromagnetic inclusions of iron ore. Such monopoles should be extracted from the ore during the melting process. These monopoles are extracted at relatively small cross-section of the furnace and freely fall downwards. Such slow moving monopoles can be detected by a passive detector with MO disks. These disks are to be placed e.g. in a cavity under the furnace. The effective exposition time, normalized to the mass of ore, from which the monopoles are extracted, can be very large. We must note, that during the exposition no detector service is required. After exposition the
MO disks should further be placed into specialized device to find the traces of the magnetic monopole.

Let us consider now the main mechanisms of energy loss of slow monopole in matter.

## 2 Cherenkov energy loss

The Cherenkov loss in matter was considered by many authors [19]-[21]. The charged particle energy loss due to Cherenkov radiation per unit length is

\[
\frac{dE}{dl} = \frac{e^2}{c^2} \int_{v > c_{\text{med}}} \left(1 - \frac{c_{\text{med}}^2}{v^2}\right) \omega d\omega ,
\]

(3)

where \(e\) is the particle charge, \(c_{\text{med}}(\omega)\) is the velocity of electro-magnetic wave with the frequency \(\omega\) in a matter.

For \(v >> c_{\text{med}}\) Eq.(3) gives

\[
\frac{dE}{dl} = \frac{2e^2}{c^2} \omega^2 = \frac{4\pi e^2}{\lambda^2} ,
\]

(4)

here \(\lambda\) is the length of a Cherenkov radiation wave in vacuum. The Cherenkov radiation of charge \(e\) in isotropic ferromagnet (ferrite) is given by

\[
\frac{dE}{dl} = \frac{e^2}{c^2} \int_{v > c_{\text{med}}} \mu \left(1 - \frac{1}{\beta^2 \epsilon \mu}\right) \omega d\omega .
\]

(5)

Here \(\mu(\omega)\) and \(\epsilon(\omega)\) are electrical and magnetic permeabilities of medium, \(c_{\text{med}}(\omega) = c/\sqrt{\mu(\omega)\epsilon(\omega)}\).

What should be expected in case of monopole moving in a magnetic medium? If

\[
\nabla \cdot \mathbf{B} = 4\pi \mu \rho_g ,
\]

(6)

then for the radiation by the magnetic charge we obtain the expression [19, 21]

\[
\frac{dE}{dl} = \frac{g^2 \mu \epsilon}{c^2} \int_{v > c_{\text{med}}} \mu \left(1 - \frac{1}{\beta^2 \mu \epsilon}\right) \omega d\omega = \frac{g^2}{c_{\text{med}}^2} \int_{v > c_{\text{med}}} \mu \left(1 - \frac{c_{\text{med}}^2}{v^2}\right) \omega d\omega .
\]

(7)

Hence, for the ratio of magnetic \(E_g\) to electric \(E_e\) radiation we have

\[
\frac{E_g}{E_e} = \frac{g}{e} \frac{c^2}{c_{\text{med}}^2} \simeq 4700 \cdot \frac{c^2}{c_{\text{med}}^2} .
\]

(8)
Such an approach meets the idea of duality

\[ E \leftrightarrow B \]

and we shall adhere just such an agreement herein.

A heavy slow monopole cannot emit usual Cherenkov radiation in a ferromagnet, because of high phase velocity of electro-magnetic waves in ferromagnetic, about \( c/10 \), which is always much faster than the monopole. However, photons are not the only particles, which can be radiated during the monopole movement in media. Besides the photons, there are magnons (spin waves) in magnetically ordered media, and the phonons (sound waves) in any condensed matter. Now we shall demonstrate that the dispersion law of these excitations is such that they can be radiated like Cherenkov photons. And the coupling of a magnon (phonon) to a monopole is not small and hence this Cherenkov loss is big enough.

## 3 Excitation of spin wave Cherenkov radiation by the heavy magnetic monopole

It is well known, that a slowly moving heavy monopole cannot emit usual Cherenkov radiation in ferromagnetic media, because the phase speed of electro-magnetic waves is of the order \( c/10 \) and is always much faster than the monopole speed.

We shall consider the slow monopole movement in an ordered magnetic matter \[22\]. In such case the main mechanism of kinetic energy loss is the Cherenkov radiation of magnons. This is because the magnon phase velocity reaches zero and the coupling of monopole to magnons is linear in the magnon field (or in creation and annihilation operators) and large.

For definiteness, we shall consider a ferromagnet, but the estimations below are of more general character.

The magnon’s Hamiltonian in presence of magnetic field of a moving monopole can be written in the form

\[
H = \sum_k \hbar \omega_k a_k^{\dagger} a_k + \sum_k \left( f_k e^{-i\Omega_k t} a_k^{\dagger} + c.c \right),
\]

(9)

where \( a_k^{\dagger} \) is the operator of a magnon creation with a wave vector \( k \), \( \omega_k \) is its dispersion law, \( \Omega_k = k v \), \( v \) is the vector of monopole velocity and \( f_k \) is the
coupling factor of the monopole magnetic field \( B = g \nabla \frac{1}{r} \) to the magnon. The magnon energy, radiated in a unit of time, is

\[
\epsilon = \frac{2 \pi}{\hbar} \sum_k \omega_k |f_k|^2 \delta(\Omega_k - \omega_k) .
\]  

(10)

Let the monopole velocity \( v \) be directed along the spontaneous magnetization, along \( z \)-axis. The general case is investigated absolutely similarly and the results will not differ much. Then

\[
f_k = \frac{4 \pi g \mu_B}{a^{3/2} \sqrt{V}} \sqrt{\frac{S}{2} k_x - i k_y} \frac{k_y^2}{k^4} \delta(k_z v - \omega_k) ,
\]  

(11)

here \( a \) is the lattice constant, \( V \) is the sample volume, \( S \) is the spin size on the node and \( \mu_B \) is the Bohr magneton.

Taking into consideration Eq.(11) the equation for \( \epsilon \) can be written as

\[
\epsilon = \frac{2 g^2 \mu_B^2 S}{a^3 \hbar} \int d^3k \omega_k \frac{k_x^2 + k_y^2}{k^4} \delta(k_z v - \omega) .
\]  

(12)

The integration in Eq.(12) is performed over the first Brillouin zone.

If \( v \geq u \), where \( u \) is the magnon velocity near the border of Brillouin zone, then the magnons with large \( k \) are essential. Then

\[
\epsilon \simeq \bar{\omega} g^2 \omega_M v ,
\]  

(13)

where the frequency \( \omega_M = \frac{4 \pi g^2 \mu_B^2 S}{a^3 \hbar} \) characterizes the magnetization of media [23],

\[
\bar{\omega} = \frac{1}{2 \pi} \int \frac{d^2k_\perp}{k_\perp^2} \omega_{k_\perp} ,
\]  

(14)

here \( k_\perp = (k_x, k_y) \), and \( \bar{\omega} \) is close to the maximal frequency of magnons.

For \( g^2 \simeq 4700 \cdot e^2 \) we obtain

\[
\epsilon \simeq 10^3 \cdot Ry \cdot \omega_M \bar{\omega} \tau ,
\]  

(15)

where \( \tau = a/v \) is the characteristic time of interaction, and \( Ry \) is the Rydberg constant.

The typical values for magneto-ordered dielectrics are such: \( \bar{\omega} \simeq 10^{-13} s^{-1} \), \( \omega_M \simeq 10^{-11} s^{-1} \) and for \( v/c \simeq 10^{-4} \) we have \( \epsilon \simeq 10^{14} eV/s \), that makes the loss per unit of length:

\[
\frac{dE}{dl} \simeq 10^8 \text{ eV/cm}
\]  

(16)
From Eq. (13) it is clear, that the energy loss $\epsilon$ and $dE/dl$ grow with slowing down of the monopole. When the speed $v$ becomes $v < u$, the main contribution to the loss is done by the magnons from "the bottom" of the spectrum. For them, $\omega_k = \omega_{ex}(ak)^2$, where $\omega_{ex}$ is the frequency, characterizing the exchange interaction \[23, 24\], and the expressions for loss become:

$$\epsilon = g^2 \frac{\omega_M v}{4\omega_{ex} a^2} ;$$

$$\frac{dE}{dl} = \frac{\epsilon}{\nu} = g^2 \frac{\omega_M}{4\omega_{ex} a^2} .$$

As one can see, the energy loss per unit of length has approached a constant with decrease of the monopole velocity. The characteristic velocity values will be: $\omega_M/\omega_{ex} \simeq 10^{-2}$, $a \simeq 10^{-8} \text{ cm}$ and for $v/c \simeq 10^{-4}$ we have again the estimation (10).

We’d like to stress, that the square-law for a magnon dispersion leads to a non-trivial spatial structure of the Cherenkov radiation field of the spin waves. The structure of the radiation field is similar to a shock wave, but there is no radiation in front of the charge. The radiation field for the square-law dispersion overtakes the charge and is non-zero in front of the charge. This is because the quadratic dispersion makes the group velocity of the wave faster than the phase velocity and faster than the velocity of the charge.

From these estimations it is clear, that the level of energy loss of a slow magnetic monopole in magnetically ordered matter can be comparable to the ionization loss of a fast monopole. This circumstance opens new opportunities for detection of slow monopoles in the velocity region $v/c < 10^{-4}$. Let us note, that the conversion mechanism of spin waves to electromagnetic ones \[24\] permits to detect a monopole passing through a magnetic layer by traditional techniques.

4 Excitation of Cherenkov acoustic radiation by the magnetic monopole

For estimation of energy loss by radiation of sound waves (excitation of phonons) by the monopole moving through isotropic matter, we shall write the Hamilto-
nian of an elastic system in an external field as follows

\[ H = \sum_n \frac{p_n^2}{2M} + \frac{A}{2} \sum_{n,\Delta} (x_n - x_{n+\Delta})^2 + \sum_n F_n(t)x_n. \tag{19} \]

Here \( n + \Delta \) numbers the closest neighbors to the node \( n \), \( M \) is the mass of an ion in a lattice site, \( A \) is an elastic constant and

\[ F_n(t) = F(r_n - vt). \tag{20} \]

We shall estimate the strength of the force \( F(r_n) \), acting from the monopole to the given node, as follows. First, we shall assume that this force is located on one node (due to its short range):

\[ F(r_n) = F\delta_{n0}. \tag{21} \]

Secondly, at rest this force causes deformation, and the affected node is shifted by

\[ \delta a \sim \frac{F}{A}, \tag{22} \]

and, assuming the deformation energy to be \( \epsilon_{\text{def}} \sim A\delta a^2 \), we have the force as

\[ F \sim A\sqrt{\frac{\epsilon_{\text{def}}}{A}} \sim \sqrt{\epsilon_{\text{def}}A}. \tag{23} \]

The Hamiltonian Eq.(19) can be expressed similar to Eq.(9)

\[ H = \sum_k \hbar\omega_k a_k^\dagger a_k + \sum_k \left( f_k e^{-i\Omega_k t} a_k^\dagger + c.c \right), \tag{24} \]

but now: \( a_k^\dagger \) is the operator of phonon creation with the wave vector \( k \), \( \omega_k \) is its dispersion, \( \Omega_k = kv \), \( v \) is the vector of monopole velocity, and \( f_k \) is the coupling factor of the magnetic monopole field \( B = g\nabla \frac{1}{r} \) to the phonon. We shall write the following expressions for them:

\[ f_k = \frac{1}{2i\hbar^{1/2}} \frac{1}{\sqrt{N}} \frac{F}{(D_k M)^{1/4}}, \tag{25} \]

\[ \omega_k = \sqrt{\frac{2D_k}{M}}, \tag{26} \]

\[ D_k = \frac{A}{2} \sum_{\Delta} \left| 1 - e^{ik\Delta} \right|. \tag{27} \]
Here $N$ is the number of lattice sites in the crystal.

Accordingly, the energy of phonons, radiated in one unit of time, is equal to

$$\epsilon = \frac{a^3}{4(2\pi)^2} \int d^3k \frac{F^2}{(D_k M)^{1/2} \omega_k \delta(k_z v - \omega_k)} ,$$  \hspace{1cm} (28)

where $a$ is the lattice constant $a^3 = V/N$, $V$ is the sample volume.

For the essentially supersonic monopoles, integration over $dk_z$ gives the factor $1/v$, and the Eq.(28) results to:

$$\epsilon = \frac{a^3}{4(2\pi)^2} \frac{1}{v} \int d^2k_\perp \frac{1}{(D_{k_\perp} M)^{1/2} \omega_{k_\perp}} = \frac{a}{2\sqrt{2}v} \overline{F^2} M .$$  \hspace{1cm} (29)

Using the evaluation of Eq.(23), for $F$ we shall obtain:

$$\epsilon \simeq \epsilon_{def} \frac{a}{v} \frac{A}{M} .$$  \hspace{1cm} (30)

Now from decomposition of Eq.(27) at small $k$ and using Eq.(26) it is possible to express $A/M$ through the speed of sound. As a result Eq.(30) acquires the form

$$\epsilon \simeq \epsilon_{def} \frac{1}{Z} \frac{c_s}{v} \frac{A}{a} \simeq \epsilon_{def} \frac{c_s}{v} \overline{\omega_a} ,$$  \hspace{1cm} (31)

where $\overline{\omega_a}$ is the frequency limit for phonons, $Z$ is the number of nearest neighbors.

If $\epsilon_{def} \sim R_y$, $c_s/v \sim 0.1$ and $\overline{\omega_a} \sim 10^{13} \text{ s}^{-1}$ is about the Debye temperature in energy units, then:

$$\epsilon \sim 10^{13} \text{ eV/s} ,$$

$$\frac{dE}{dl} \sim 10^7 \text{ eV/cm} ,$$

which is a little less than the loss by radiation of magnons.

5 Interaction of a massive monopole with a superconductor

The monopole leaves ”a tail”- a string of magnetic field, as it traverses a superconductor. It is the Abrikosov vortex, which is filled by the normal phase and by the monopole magnetic flux. The vortex core radius is determined by the
coherent length $\xi$, and the effective radius of magnetic flux tube, which is the London penetration depth $\lambda_L$

$$\lambda_L = \sqrt{\frac{mc^2}{\mu n_s e^2}}. \quad (32)$$

Thus, the energy loss of a slow monopole in a superconductor is made up of two components:
- moderation by magnetic string tension,
- loss due to destruction of the condensate of Cooper pairs and formation of the normal-phase core.

The magnetic string loss is

$$\frac{dE_m}{dl} = \frac{1}{8\pi} \frac{\Phi_0^2}{\pi \lambda_L^2}. \quad (33)$$

Using a typical value of $\lambda_L = 10^{-5} \text{cm}$, and $\Phi_0 = 4 \cdot 10^{-7} \text{ G cm}^2$ we obtain

$$\frac{dE_m}{dl} \simeq 10^{-5} \text{ erg/cm} \simeq 10^7 \text{ eV/cm}. \quad (34)$$

The loss due to formation of the core of normal phase is

$$\frac{dE_n}{dl} = \Delta E n_s \pi \xi^2, \quad (35)$$

here $\Delta E$ is the gap width, $n_s$ is the density of superconducting carriers. For typical superconductor parameters we estimate the value of the energy loss to be of the order of $10^7 \text{ eV/cm}$.

To be accurate, it is necessary to include the surface energy on the core border of the Abrikosov vortex into $\frac{dE_n}{dl}$. However it is clear, that the result will remain within the same order of magnitude.

At the London length $\lambda_L = 10^{-5} \text{ cm}$ the pressure of the magnetic field is relatively small, and the loss for the Cherenkov acoustic radiation can be neglected.

6 Monopole interaction with conductors

When the monopole transverses a normal massive conductor, finally it loses energy on behalf of the Joule heat. Besides that, the moderating force acts on
it in the same manner as in a superconductor, from the magnetic field of the induced currents.

The thickness of a skin layer in a conductor is described by the expression
\[ \delta = \sqrt{\frac{c^2 \tau}{2\pi \sigma \mu}}, \]  
(36)

where \( \sigma \) is the conductivity, \( \mu \) the permeability, and \( \tau \) is the period (the pulse duration) of the magnetic field.

We can express \( \tau \) as
\[ \tau = \frac{\delta}{v}, \]  
(37)

where \( v \) is the monopole velocity. Substituting Eq.(37) into Eq.(36) we obtain the expression
\[ \delta_c = \frac{c^2}{2\pi \sigma \mu v}. \]  
(38)

This length has a simple physical meaning – at distances less than \( \delta_c \), the monopole field can be considered to be free, with an accuracy to the permeability of the matter, but at the distances of \( \delta_c \) and more, it will form the magnetic tail of the monopole, like the string in a superconductor. Due to the finite conductivity of the matter, the tail gradually dissipates, and energy of the magnetic field transforms into Joule heat.

It is easy to show, that for a typical conductor, the maximal current-density of a metal layer with thickness \( \delta_c \) is large enough to keep the magnetic flux of the monopole in a tube with a radius of the order of the critical length, \( \delta_c \).

With the known critical length, it is easy to evaluate the monopole energy loss in a conductor. Within the cross section of the diameter equal to the critical length, the magnetic field is
\[ B_c = \frac{\Phi_0}{\pi \delta_c^2}. \]  
(39)

Accordingly, the monopole energy loss due to the tail formation (friction in the magnetic field) is
\[ \frac{dE_m}{dl} = \frac{\Phi_0^2}{8\pi^2 \delta_c^2} = \frac{\Phi_0^2 \sigma^2 \mu^2 v^2}{2c^4}. \]  
(40)

Examining Eq.(40), we can see that the monopole energy loss in the conductor is proportional to the square of the monopole velocity. At \( v = 10^{-4}c \) the
loss is
\[ \frac{dE_m}{dl} \simeq 10^{-5} \text{ erg/cm} \simeq 10^7 \text{ eV/cm} . \] (41)

7 Track formation by the slowly moving monopole in a ferromagnetic

It is expected that a slow monopole, moving transversely through a magnetized ferromagnetic film, should leave a distinctive track of magnetization in it. We can use this phenomenon to design an effective detector of supermassive cosmic monopoles. For this purpose we shall consider now in detail the mechanism of magnetic track formation by such a type of monopoles.

7.1 Quasistatic approximation.

Let us consider a thin layer of easy-axis hard ferromagnetic magnetized perpendicularly to the surface along an easy axis. It is easy to see that the external magnetic field is absent (double layer!), but the surface density of a magneto-static energy of such configuration is rather large.

Therefore such a magnetization configuration appears to have too high energy, and the magnetic film is split into a system of magnetic domains. Magnetizations of the domains are also orthogonal to the surface and are directed along the easy axis, and have opposite signs in neighboring domains [25].

The domain characteristic size is determined by a condition of minimal total energy per unit of the film surface. For films of ferromagnetic garnets a characteristic scale of domain structure is about 1/100 mm. And the total energy of the film decreases more than 1000 times due to creation of the domain structure.

However, if the anisotropy constant \( K_u \) is reasonably large, and the effective field of anisotropy exceeds the value of demagnetizing field
\[ \frac{2K_u}{I_s} \geq \frac{I_s}{\mu_0} , \quad K_u \geq \frac{I_s^2}{2 \mu_0} , \] (42)
then the system is in a metastable state. Therefore for domain formation, it is necessary to have a magnetic bubble with magnetization opposite to the film.

\(^{1}\)Let us recall that the term "easy-axis" means that the anisotropy energy of a magnet has the form \( \epsilon_a = -K \sum_n (S_n^z)^2 \), where the easy axis is z-axis, \( S_n^z \) is z–component of the ion spin placed on the lattice site \( n \) and the anisotropy constant \( K \) is positive. The spontaneous magnetization of such a magnet is oriented along the easy axis.
magnetization and parallel to the demagnetizing field. In Eq. (42) the parameter $I_s$ is magnetization of the film material and $\mu_0$ is permeability.

It turned out, that the size of such magnetic bubble should be finite and not very small. For simplicity we consider a magnetic bubble of cylindrical shape of radius $r$ and with axis orthogonal to the film surface and its magnetization opposite to the film magnetization.

Let us introduce the characteristic length $r_0$:

$$r_0 = 2\mu_0\gamma/I_s$$

Then the total energy of the magnetic bubble can be found as

$$E = 2\pi\gamma rh \left( (1 - 2N)\frac{r}{r_0} - 1 \right),$$

where $N$ is a demagnetization factor of the domain, $r$ is the radius of the domain, $h$ is the film thickness, $\gamma$ is the density of surface energy of the domain wall.

The magnetic bubble size can be evaluated from the condition

$$\frac{\partial E}{\partial r} = 0.$$  \hspace{1cm} (45)

If we use for the demagnetization factor the following approximation:

$$N = \frac{2r}{3h},$$

then we find two values $r$ of the domain radius as solutions Eq.(45).

The first value is the radius of collapse $r_c$, below which the domain-magnetic bubble is unstable and collapses

$$r_c = \frac{h}{4} \cdot \left[ 1 - \left( 1 - \frac{4r_0}{h} \right)^{1/2} \right].$$

And the second solution gives us the equilibrium radius $r_{eq}$

$$r_{eq} = \frac{h}{4} \cdot \left[ 1 + \left( 1 - \frac{4r_0}{h} \right)^{1/2} \right].$$

At $h >> r_0$ we can find from (47) and (48)

$$r_c|_{h>r_0} \to \frac{r_0}{2};$$

$$r_{eq}|_{h>r_0} \to \frac{r_0}{2};$$

$$r_{eq}|_{h<r_0} \to h/4;$$

$$r_{eq}|_{h<r_0} \to h/4;$$

$$r_{eq}|_{h=r_0} \to h/4;$$
and

\[ r_{eq \mid h > r_0} \rightarrow \frac{h}{2}. \]  

We see that \( r_0 \) defines the minimal radius of collapse. From non-negativity of the radicand in (47) we obtain, that the minimal width of a ferrolayer, in which a cylindrical domain (further - magnetic bubble) can exist, is

\[ h_{\text{min}} = 4r_0 = \frac{\mu_0 \gamma}{I_s^2}. \]  

(51)

It is necessary to emphasize, that in a zero external magnetic field, the magnetic bubble is unstable and turns into a stripe domain [26].

Let us evaluate the orders of magnitudes:

\[ \gamma \simeq 1 \text{ erg/cm}^2, \]  

\[ I_s^2 \simeq 10^6 \text{ erg/cm}^3. \]

So \( r_0 \) will be of the order of \( 10^{-6} \text{ cm} \), or 10 nm. And the collapse radius is yet less for ”thick” enough films! Thus, at film thickness about 100 nm we shall have a magnetic bubble with characteristic size of the order of 30 nm.

What is the field of a monopole at such distance?

\[ H = \frac{\Phi_0}{4\pi r_0^2} \simeq 2 \cdot 10^3 \text{ Oe}, \]  

(52)

that is enough without any doubt for re-magnetization of a material with coercitivity of the order of 1 Oe.

Let us note, that the existence of the minimal radius hinders the fluctuational creation of microscopic domains, the magnetic bubbles. In this sense, a homogenically magnetized film can be quite stable against splitting into a structure of magnetic domains. All abovementioned is true for films with high mobility of domain walls. Films with low wall mobility are even more stable, and at the same time, domains with radius less than the collapse radius can exist in them, in principle. So, in the Co/Pt films the movement of domain walls is suppressed. And in the 20 nm film the transverse domains of cylindrical form with diameter of the order of 50-100 nm are obtained. We wish to note, that the coercitivity of the easy-axis Co/Pt film is of the order of 1-2 kOe [27].


7.2 Track dynamics — domain formation

However, our speculations in previous subsection are true only in static, for very slow monopoles only. As we have noted before, the characteristic velocity of a monopole is \( v \simeq 10^{-4} - 10^{-3}c \), and for our consideration let us assume \( v = 10^{-4}c \). The time of monopole interaction with an electron \( \tau \) can be defined as the time, during which a field higher than some critical field \( H_c \) interacts with the electron

\[
\tau \simeq \frac{r_c}{v} \simeq \frac{1}{v} \sqrt{\frac{\Phi_0}{4\pi H_c}}.
\]  

At \( H_c \) of the order \( 3 \cdot 10^3 \) Oe we have \( \tau \sim 3 \cdot 10^{-12} \) s. It means, that the spin-flip of the magnetic in the ”track” takes place during the interaction time.

For such spin-flip, the adiabatic condition is necessary since the frequency of spin precession in the magnetic field of the moving monopole should be much larger than the inverse time of the interaction

\[
\omega = \frac{\mu_B H}{\hbar} \gg \frac{1}{\tau}.
\]  

It is possible to derive from here the minimal magnetic field which is appropriate for the adiabatic mode, and the track radius:

\[
H \gg H_c = \frac{\hbar}{\mu_B \tau} = \frac{4\pi \hbar^2 v^2}{\mu_B^2 \Phi_0},
\]  

\[
R_t \ll r_c = \sqrt{\frac{\Phi_0}{4\pi H_c}}.
\]  

In our case at \( v \simeq 10^{-4}c \) we get

\[
H_c \simeq 10^7 Oe ; \quad r_c \simeq 10^{-7} cm,
\]  

and for \( v \simeq 10^{-6}c \), we have

\[
H_c \simeq 10^3 Oe ; \quad r_c \simeq 10^{-5} cm.
\]  

It is obvious, that the conditions of adiabatic and even resonant spin flip are not fulfilled, while \( r_c < r_0 \), that corresponds to the monopole speed \( v \simeq 10^{-6}c \).

We shall consider the influence of the conductivity of the film material now. The reason is that the monopole magnetic flux is being frozen into the cylindrical area around the track axis and then spreads radially. The radius \( \delta_c \) of
the flux pipe and the diffusion factor of the flux are determined by the film conductivity. The value of $\delta_c$ we can estimate as

$$\delta_c = \frac{c^2}{2\pi \cdot \sigma \mu v}.$$  \hfill (57)

As it was noted before, this length has a simple physical meaning. At distances less than $\delta_c$ the monopole field can be considered as free. At distances of the order of $\delta_c$ and more, the magnetic tail of the monopole is formed, which is an analogue of a string in a superconductor. Due to the finite conductivity of material, the tail spreads gradually, and the energy of the magnetic field converts into heat.

At the monopole velocity about $v \simeq 10^{-4} c$, the flux pipe has the radius of the order of $10^{-5} cm$. The flux pipe blows with time as:

$$R(t) = \delta_c \sqrt{\frac{t}{\tau}}.$$  \hfill (58)

Thus the magnetic moment of the track is conserved, as well as the frozen flux. It is easy to calculate the average intensity of the magnetic field in the flux pipe immediately after monopole flight

$$H = \frac{\Phi_0}{\pi \delta_c^2} \simeq 10^3 Oe.$$  \hfill (59)

The typical time of the monopole interaction with an electron in a conductor is:

$$\tau \simeq \frac{\delta_c}{v} \simeq 10^{-11} - 10^{-10} \text{ s},$$  \hfill (60)

and the field strength, providing the adiabatic inversion of the magnetic spin in a track, will be:

$$H_c \simeq \frac{\hbar}{\mu_B \tau} \simeq 10^4 Oe.$$  \hfill (61)

Thus, the frozen field in the conductor $H_c$ can affect appreciably the process of spin-flip in the track and provide the adiabatic spin-flip of electrons in the magnetic at monopole speeds below $10^{-4} c$. Besides, it can render a certain influence on dynamics of the domain walls in conducting films with high mobility.
7.3 Detection of the monopole track

As it was shown in this paper, the domain induced by the moving monopole has the typical size of the order of 50 nm and magnetization about several thousand Gauss. Then the domain magnetic flux $\Phi_d$ will be of the order:

$$\Phi_d = \pi r^2 \cdot I_s \simeq \Phi_0 = 2 \cdot 10^{-7} \, \text{G} \cdot \text{cm}^2$$

For detection of such a flux we can use the high sensitive fluxmeter on the basis of superconducting quantum interference device such as SQUID, or magneto-optical device on the basis of Kerr effect (rotation of the polarization plane of light reflected by a surface of a ferromagnet which is magnetized perpendicular to the surface). It is clear that the second way is technically easier and does not require a cryogenic maintenance. In the later case, the realization of such a detector requires a surface covered with a thin layer of easy-axis magnetized magnetic media, plus a magneto-optic device to detect the spots of transverse magnetization of the film (to detect the domain with opposite direction of magnetization!) with a system of precise positioning.

A similar technique has emerged recently in an almost ready shape, suitable for the detector design with minimal adjustment. It is the magneto-optic recording technology used in modern magneto-optic disks (MO disks) and their readout devices. Already there are MO disks with multilayer coating of $Sm/Co$ and $Pt/Co$. The coercitivity of multilayer coats $Pt/Co$ is about $1K\text{Oe}$ at 10 layers of total thickness about 15 $nm$ [27]. The size of a magnetic bubble which can be detected in such a coat by the magneto-optical method is about 60 $nm$. This techniques using the near-field optics has been designed, f.i. in Bell Laboratories [28].

The coercitivity of coats with $Sm/Co$ is in the interval $3 - 5 \, k\text{Oe}$, and the reference size of magnetic bubble is 50 $nm$ [27]. For detection of the magnetic track of the monopole it is possible to use slightly modified standard MO-drives, as we have mentioned before. Having covered a large enough surface with such MO disks, we can obtain an effective and relatively nonexpensive detector of slow moving space monopoles, which we can expose during unlimited time.

However, it is more effective to use the MO-detector to search for relict monopoles, entrapped in ferromagnetic inclusions of Fe ore [29]. Naturally, the melted iron ore becomes paramagnetic and the ferromagnetic trap disappears. Then the monopole is likely to be surrounded by a cluster of several dozens of
iron atoms. The size of a complex is determined by thermodynamic equilibrium:

$$\frac{\mu_{Fe} \cdot g}{r_{Fe}^2} = \frac{3}{2} kT$$

(62)

and the radius of the iron atomic complex, paramagnetically bound to the monopole at $T \approx 1200^0 C$ is:

$$r_{Fe} \simeq 6 \cdot 10^{-8} cm.$$ 

(63)

Apparently the complex is small and contains about 30 atoms of Fe. Considering, that the movement of such small blob is determined by the Stokes law:

$$F_v = 6\pi \cdot r_{Fe} \cdot \eta v,$$

(64)

where $v$ is the monopole velocity, $\eta$ is the dynamic viscosity of liquid foundry iron, $\eta = 2 \cdot 10^{-3} kg/(m \cdot sec)$ at $T = 1250^0 C$.

Equating the force of friction to the gravity $F_g = mg$, we find the velocity $v$ of the monopole falling through the melt

$$v = \frac{mg}{6\pi \cdot r_{Fe} \cdot \eta},$$

(65)

that makes $v \simeq 3 \cdot 10^{-1} m/sec$ for a monopole with mass about $10^{15} GeV$.

This corresponds to kinetic energy of the complex about $10^6 eV$, which is large enough for a skinning off of the complex at the solid bottom of the furnace. Let’s remark, that from a formal point such an approach is quite acceptable, as the Reynolds number in our case it is not large enough: $Re < 10^{-3}$. This grain (complex) should sink in the liquid iron at 10-100 cm/s velocity until it reaches the bottom of the blast furnace. Then the atoms of iron are stripped off the monopole in the material of the oven bottom, and the monopole falls further, accelerating up to the velocity of sound in the matter.

Usually the blast furnace melts about 10 000 tons of Fe ore per day, and it could be easily expose the MO-detector, for example, during one year. Thus we hope that such MO-detector can improve significantly the experimental limit on the density of relict monopoles entrapped in Fe ore, which today is equal to $\rho_M < 2 \cdot 10^{-7}/g$ [30].

Furthermore, in a sinter machine the ore is also heated above the Curie temperature, but not up to the melting point. So, to shake off the iron atoms, we have to kick the iron ore pieces with acceleration of $10 - 100 g$. Clearly, in this case the probability of monopole release is considerably lower.
8 Conclusion

In this paper we analyze the mechanisms of energy loss of supermassive slowly moving monopole in matter. In general, this study concerns the magneto-ordered materials, e.g. ferromagnets, as well as conductors in normal and in superconducting phases. The new effective mechanism of energy loss of slow magnetic monopole is considered: the Cherenkov excitation of spin waves during monopole movement through the magneto-ordered magnetic.

The interaction of monopoles with films of magnetic materials is considered. In particular, the interaction of slow monopoles with thin films of easy-axis magnetics with high and low mobility of domain walls (materials with magnetic bubbles) is discussed. It is shown, that during the movement of a slow monopole through the magneto-hard magnetic film, a track-domain can be formed with typical size of about 50 nm and with magnetization of about several thousand Oe. Thus the magnetic flux of the track appears to be about the value of the flux quantum. For detection of such a flux, the detectors using fluxmeter on the basis of already widely known SQUID can be used.

However, in our opinion, for registration of traces of slow cosmic monopoles in magnetic matter, the experimental devices using the Kerr magneto-optical effect are more appropriate. They have emerged recently in a shape suitable for detector design, with an appropriate adjustment.

Apparently, such passive detectors will be especially effective in search of relict monopoles, entrapped in ferromagnetic inclusions of iron ore. These monopoles should be extracted from the ore during melting process. Then these slow moving monopoles can be detected by a passive MO detector. We can expose MO disks in a cavity under a blast furnace exactly under the bath with melting metal, where the temperature does not exceed +50°C. In the melting process the temperature of ore exceeds the Curie point and its ferromagnetic properties disappear. Hence the ferromagnetic traps which hold the monopoles are ”switched off” and the released monopoles fall through the melting metal to the bottom of the bath and finally through the MO disks. While a monopole, moved downwards by the gravity force crosses the surface of one of the MO disks, it leaves a magnetic track in its coat. It is possible to obtain the slow moving relict monopole also by the sinter machine.

The modern blast furnace have capacity of the order of 10 000 tons of pig-iron per day. The arrangement of the MO-detector under such furnace allow us
to considerably improve the recent experimental limit on the relict monopole density entrapped in Fe ore.

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