Two-Plasmon Intersubband Absorption in Quantum Wells

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Abstract. Linear light absorption of 2D electrons confined within a biased quantum well is studied theoretically. We demonstrate that for light polarization perpendicular to the 2D plane, in addition to conventional absorption peak at frequency $\hbar \omega \approx \Delta$, where $\Delta$ is the intersubband energy distance, there exists a peak around a double frequency $\hbar \omega \approx 2\Delta$. This additional peak is entirely due to electron-electron interactions, and corresponds to excitation of two electrons by one photon. The magnitude of two-electron absorption is proportional to $U^2$, where $U$ is the applied bias.

1. Introduction

Intersubband absorption of light in a quantum well is studied theoretically and experimentally for more than two decades. Original motivation for such a close attention to this process was its crucial role in design of infrared detectors [1]. Lately, the interest in intersubband transitions is spurred by advances in fabrication of the quantum cascade lasers [2].

In the present paper we point out that, in addition to conventional intersubband absorption, there exists another distinctive interaction-induced effect, namely, two-electron linear absorption of light. Obviously, without interactions, one photon can excite only one electron from the lower subband to the upper subband. Our main point is that interactions allow a photon with energy $\hbar \omega \approx 2\Delta$ to excite simultaneously two electrons from the lower to the upper subband. For weak interaction strength, the amplitude of this process is proportional to the interaction strength. However, it is easy to see that, for a rectangular quantum well, two-electron absorption is prohibited by symmetry. On the other hand, this symmetry selection rule can be lifted by applying a finite bias to the structure. Then the electric field $U/d$, where $U$ is the voltage drop within the well, mixes the subband wave functions with different parities. As a result, the two-electron absorption coefficient contains a small parameter $(U/\Delta)^2$. Unlike single-electron intersubband absorption, the two-electron intersubband peak possesses a finite width already in the lowest order of the perturbation theory in interactions. Below we calculate the shape of this peak perturbatively, and then demonstrate that the many-body effects, taken into account within the RPA, modify the peak significantly only in the vicinity of the threshold.
2. Matrix Element

We start from the standard Hamiltonian of the two-subband system

$$\hat{H} = \sum_{nk} E_n(k) \hat{a}_{nk}^\dagger \hat{a}_{nk} + \frac{1}{2} \sum_{(nkq)} V_{n_1'n_2' n_1 n_2}(q) \hat{a}_{n_1' k+q}^\dagger \hat{a}_{n_2' k'} - \hat{a}_{n_2 k'}^\dagger \hat{a}_{n_1 k},$$

(1)

where $\hat{a}_{nk}$ is annihilation operator for nth subband electron with in-plane momentum $k$ and dispersion $E_n(k) = E_n + \epsilon_k = E_n + \hbar^2 k^2 / 2m ~ (n = 1, 2)$, and $V_{n_1'n_2' n_1 n_2}(q)$ are matrix elements calculated from the size-quantization wave functions, $\varphi_n(z)$. Within the perturbative approach the process of two-electron absorption can be explained as follows. In the absence of interactions, the wave function of a two-electron state $|n_1 k_1, n_2 k_2\rangle$ is a product of single-electron wave functions $|n_1 k_1\rangle$ and $|n_2 k_2\rangle$. Incident light with frequency $\omega$ is polarized along the $z$-axis. Then it is apparent that the matrix element of this perturbation between the states $|1 k_1, 1 k_2\rangle$ and $|2 k_1, 2 k_2\rangle$ is zero due to orthogonality of the size-quantization wave functions, i.e., the two-electron absorption is forbidden.

Our prime observation is that, with interactions, the bare state $|1 k_1, 1 k_2\rangle$ acquires an admixture of the states $|1 k_1 - q, 2 k_2 + q\rangle$ where $q$ is the transferred momentum. Due to this admixture, the dipole matrix element of the two-electron transition becomes finite. To calculate this matrix element, we write the first-order interaction correction to the bare two-electron state in the form

$$\delta |n_1 k_1, n_2 k_2\rangle = \sum_{n_1' n_2'} |n_1' k_1 - q, n_2' k_2 + q\rangle V_{n_1'n_2' n_1 n_2}(q) E_{n_1 n_2}(k_1, k_2) - E_{n_1'n_2'}(k_1 - q, k_2 + q),$$

(2)

where $E_{n_1 n_2}(k_1, k_2) = E_{n_1}(k_1) + E_{n_2}(k_2)$ are the energies of the bare two-electron states. Using Eq. (2), we can express the dipole matrix element of a two-electron transition as

$$M_{k_1 k_2}(q) = \langle 2 k_1 - q, 2 k_2 + q | z_1 + z_2 | 1 k_1, 1 k_2 \rangle = 2 \omega \frac{V_{1222}(q) - V_{1211}(q)}{\Delta + \epsilon_{k_1 - q} + \epsilon_{k_2 + q} - \epsilon_{k_1} - \epsilon_{k_2}},$$

(3)

where $z_{12} = \int dz_1 \varphi_1(z_1) \varphi_2(z_2)$. For a symmetric quantum well, both Coulomb matrix elements $V_{1222}$ and $V_{1211}$ are zero. If, however, a bias, $U$, is applied to the quantum well, the size-quantization wave functions $\varphi_1(z)$, $\varphi_2(z)$ are no longer symmetric or antisymmetric, so that the Coulomb matrix elements in Eq. (3) are proportional to $U$. For rectangular quantum a well, a straightforward calculation yields

$$M(q) = -\left( \frac{2 \omega \Delta}{\omega - \Delta} \right) \left( \frac{U \Delta}{\Delta d} \right) \left( \frac{2 \pi e^2 d}{\kappa} \right) F(qd),$$

(4)

where we used energy conservation $h\omega = E_{22}(k_1 - q, k_2 + q) - E_{11}(k_1, k_2) = 2\Delta + \epsilon_{k_1 - q} + \epsilon_{k_2 + q} - \epsilon_{k_1} - \epsilon_{k_2}$.

3. Absorption Coefficient

A general expression for two-electron absorption coefficient, $\alpha_2(\omega)$, has the form

$$\alpha_2(\omega) \propto \omega \sum_q |M(q)|^2 J(\omega, q),$$

(5)

where we omitted a frequency-independent factor. Here $J(\omega, q)$ is a joint spectral function which, within perturbative description, is given by a convolution,

$$J(\omega, q) = \int \frac{dE}{2\pi} B_0(E, q) B_0(h\omega - E, q),$$

(6)
of single-particle intersubband spectral functions

\[ B_0(E,q) = 2 \sum_k n_{1k} 2\pi \delta \left[ E - E_2(k+q) + E_1(k) \right], \]  

(7)

where \( n_{1k} \) is the occupation of \( n = 1 \) subband electron, and the factor 2 accounts for spin.

It is convenient to present the final result for two-electron absorption peak relative to the single-electron absorption, \( \alpha_1(\omega) \): 

\[ \tilde{\alpha}_2(\omega) = \alpha_2(\omega) \left[ \int d\omega' \alpha_1(\omega') \right]^{-1} \left[ \int d\omega \alpha_1(\omega) = 2\nu E_F \Delta z_{12}^2 \right] \]

(8)

with \( t = (2\Delta - \hbar \omega)/E_F \). Here the dimensionless spectral function \( \Phi(t) \) is given by

\[ \Phi(t) = \frac{1}{\pi^2 k_F^4} \int_{k_1 < k_F} \int_{k_2 < k_F} \theta \left[ \frac{|k_1 - k_2|^2}{2k_F^2} - t \right], \]

(9)

while \( r_s = \sqrt{2m} e^2/\kappa \hbar^2 k_F \) is the interaction parameter (\( \kappa \) is the dielectric constant). For \( U \ll \Delta \) the dipole matrix element, \( z_{12} \), can be evaluated between the nonperturbed functions \( \varphi_1^{(0)} \) and \( \varphi_2^{(0)} \). This yields \( z_{12} = 16d/9\pi^2 \approx 0.18d \), while \( F(0) = -595/144\pi^2 \approx -0.42 \). The function \( \Phi(t) \) calculated numerically is plotted in Fig. 1 together with the asymptotes.

**Figure 1.** Linear absorption spectrum of a quantum well is shown schematically. Dashed line: single-electron peak; Full line: two-electron peak. Sharp feature at the top illustrates plasmon contribution. Inset: threshold behavior of two-electron absorption calculated from Eq. (9).
4. Plasmon contribution
Within RPA, the plasmon dispersion is determined from the intersubband density response function,

$$\Pi(E, q) = \frac{P_0(E, q)}{1 - V_{1212}(q)P_0(E, q)},$$

where the free-electron intersubband polarization has the form

$$P_0(E, q) = \frac{\nu}{2\epsilon_q} \left[ E - \Delta - \epsilon_q - \sqrt{(E - \Delta - \epsilon_q)^2 - 4E_F\epsilon_q} \right].$$

From the pole of $\Pi(E, q)$ we obtain the dispersion law of intersubband plasmon,

$$\hbar \Omega_{pl}(q) = \hbar \Omega_{pl}(0) + (1 + \lambda_0^{-1}) \epsilon_q,$$

with $\hbar \Omega_{pl}(0) = \Delta + \lambda_0 E_F$ and $\lambda_0 = \nu V_{1212}(0) = 10 r_s \sqrt{2\pi E_F/\Delta}$ (we assumed $qd \ll 1$). The bottom of plasmon band is shifted up by $\lambda_0 E_F$ from the intersubband separation $\Delta$.

Turning to the absorption, we recall that the probability Eq. (5) of two-electron transition is given by

$$\Phi - \Delta > 2 \sqrt{E_F \epsilon_q + \epsilon_q}$$

The main plasmon contribution comes from the region $E - \Delta > 2 \sqrt{E_F \epsilon_q + \epsilon_q}$ of the $(E, q)$ plane, where plasmon is not damped by intersubband single-particle excitation. In this region, the spectral function has the form

$$B(E, q) = 2\pi A_q \delta[E - \hbar \Omega_{pl}(q)],$$

where the oscillator strength $A_q$ is given by

$$A_q = \left[ V_{1212}(0) \frac{\partial P_0(E, q)}{\partial E} \right]^{-1} = \nu \left( E_F - \frac{\epsilon_q}{\lambda_0^2} \right),$$

with the condition $E_F > \frac{\epsilon_q}{\lambda_0^2}$ restricting the momentum, $q$, to the domain where plasmon in undamped. Then the absorption is again given by Eq. (8), but with $\Phi$ replaced by

$$\Phi_{pl} \approx \frac{\lambda_0}{1 + \lambda_0} \left[ 1 - \frac{\omega - 2\Omega_{pl}(0)}{\delta \omega_{pl}} \right]^2,$$

where the width, $\delta \omega_{pl}$, is given by $\hbar \delta \omega_{pl} = 2\lambda_0(1 + \lambda_0)E_F$. Note that $\Phi_{pl}$ vanishes outside the interval $2\Omega_{pl}(0) < \omega < 2\Omega_{pl}(0) + \delta \omega_{pl}$. Since $\delta \omega_{pl} \lesssim E_F$, we conclude that the plasmon peak constitutes a sharp feature on top of a wider two-electron band in $\alpha_2(\omega)$ (see Fig. 1). The relative magnitude of the plasmon peak compared to the maximum of $\alpha_2$ is $\lambda_0/(1 + \lambda_0)$ (the single-particle value is $\Phi = 1$ for $\omega = 2\Delta$). Thus, the relative contribution of the plasmon peak to the net oscillator strength is $\sim \lambda_0^3 E_F/\Delta \lesssim 1$. Note finally that the relative weakness of two-plasmon peak oscillator strength can be traced to the smallness of plasmon effective mass, $m_{pl} = m\lambda_0/(1 + \lambda_0) < m$.

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[1] K.K. Choi, The Physics of Quantum Well Infrared Photodetectors (World Scientific, Singapore, 1997).
[2] Intersubband Transitions in Quantum Wells: Physics and Device Application I, edited by H. C. Liu and F. Capasso (Academic Press, San Diego, 2000).