SCALING BEHAVIOUR OF TENSOR ANALYSING POWER ($A_{yy}$) IN THE INELASTIC SCATTERING OF RELATIVISTIC DEUTERONS.

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Abstract

We suggest a new dimensionless relativistic invariant variable $R = \Delta m_X/\nu$ which may be interpreted as the ratio of the excitation energy to the full transferred energy; therefore this variable measures a ”degree of inelasticity” of the scattering.

Existing data on the tensor analysing power of the $p(\vec{d}, d')X$ and $^{12}C(\vec{d}, d')X$ inelastic scattering at momenta from 4.2 to 9 GeV/c are analysed in terms of this variable.

We observe that $A_{yy}$ taken as a function of $R$ does not depend upon the incident energy, the scattering angle (up to the angles of $\vartheta_{cm} \sim 30^\circ$); there is no noticeable difference between the proton and nuclear targets as well.

It is remarkable that $A_{yy}$ is maximal (of $\sim 0.5$) when $R \sim 0.5 - 0.6$ and is small in absolute value when $R$ is close to its limiting values of 0 and 1.

Key-words: inelastic scattering, tensor analysing power, scaling, proton, deuteron, carbon.

1 Introduction.

The set of experiments measuring the tensor analyzing power ($A_{yy}$) of inelastic ($\vec{d}, d'$) scattering at the lab. angles of $0^\circ$ and $5^\circ$ off protons and carbon nuclei in the deuteron momentum range from 4 to 9 GeV/c was performed in Dubna at 1994 - 97 (refs.[1, 2, 3]). It was stressed in refs.[1, 2, 3] that the region of initial deuteron momentum of $\sim 3$ to 9 GeV/c is the optimal one for studies of the lowest baryon resonances such as $\Delta(1232)$ and $N^*(1440)$. Data on polarization characteristics of these reactions are of a special interest because of the ”spin-isospin filtering” (see ref.[1] and references therein) of different mechanisms, what can be used for better understanding of the mechanisms of the resonance excitations and properties of the relevant resonances.

The $A_{yy}$ data published in refs.[1, 2, 3] were obtained at different energies and angles; therefore $t$, the 4-momentum transfer squared, was used in order to analyse and compare data obtained at different kinematical conditions. It was noticed that $A_{yy}$ plotted versus $t$ demonstrates an approximate scaling (see Fig.1 and refs.[1, 2]). It means that at different momenta of initial deuterons the behaviour of the $A_{yy}(t)$ is approximately the same.

At the lab. scattering angle of $0^\circ$ the tensor analyzing power $T_{20} = -\sqrt{2} \cdot A_{yy}(t)$ is negative in the explored $t$-interval ($0 < -t < 0.6 \text{ GeV}^2/c^2$). It is small in absolute value at small $-t$ and at $-t > 0.4 \text{ GeV}^2/c^2$; the absolute value reaches it’s maximum at $0.2 < -t < 0.4 \text{ GeV}^2/c^2$. Moreover the approximately universal behaviour of $T_{20}(t)$, or the scaling, was observed not only at different momenta of initial deuterons but when the deuteron is scattered on proton or carbon targets.

Still, it was noticed that the scaling is not perfect: data sets taken at different energies but at the same scattering angle are systematically shifted as a whole relative to each other.
This shift is small and comparable with the error bars, but noticeable. On the other hand, existence of the approximate scaling pointed on a possibility that a better scaling relative to a better variable might be found.

In this paper we suggest such variable. It is relativistic invariant, dimensionless and has rather clear interpretation; these features make this variable rather attractive.

Let us define the relativistic invariant dimensionless quantity

\[ R = \frac{\Delta m_X}{\nu}, \quad \nu = \frac{1}{m_t} P_t (P_d - P_{d'}) = m_d u_t (u_d - u_{d'}), \]  

where \( P_d, P_{d'} \) and \( P_t \) are 4 - momenta of the projectile, the ejectile and the target respectively; \( u_d, u_{d'} \) and \( u_t \) are the 4 - velocities of these particles. The \( \Delta m_X = m_X - m_t \) is the difference between masses of the recoiled system in the final state (the missing mass, \( m_X \)) and the initial state (the target mass, \( m_t \)) respectively. In other words, this difference is the energy absorbed by internal degrees of freedom of the colliding particles (obviously, for elastic scattering one has \( \Delta m_X = 0 \), hence \( R = 0 \) in this special case).

Figure 1: \( T_{20}(t) \) for \( p(d,d')X \) from ref.[2]. Open circles: 4.2-4.5 GeV/c; full circles: 5.53 GeV/c; stars: 9 GeV/c.
It is interesting that $\mathcal{R}$ can be rewritten in the form which reminds variables widely used in analysis of lepton deep inelastic scattering:

$$m_X^2 = m_t^2 + t + 2m_t\nu ; \quad \mathcal{R} + \frac{\Delta m_X^2 - t}{2m_t\nu} = 1 \quad \text{or} \quad \mathcal{R}(1 + \frac{\Delta m_X}{2m_t}) = 1 + \frac{t}{2m_t\nu}$$

![Figure 2: Data from the Fig.1 plotted versus $\mathcal{R}$.](image)

It is easy to see that in the target rest frame

$$\mathcal{R} = \frac{\Delta m_X}{Q} = 1 - \frac{T_X}{Q},$$

(where $Q$ is the energy transfer from the projectile to the target and $T_X$ is kinetic energy of the recoiled system).

Therefore it is possible to interpret $\mathcal{R}$ as the part of transferred energy which was absorbed by constituents of the target system. In other words, this variable can be considered as a measure of the inelasticity of the scattering: it differs from zero only for an inelastic scattering.
2 Behaviour of the analysing power as a function of the inelasticity variable $\mathcal{R}$.

First, it is necessary to emphasize that no assumptions about reaction mechanisms, structure of fragments and so on are made in this paper.

The tensor analyzing power $T_{j\mu}$ can be defined for reactions where the incident particle has spin larger than $1/2$ as follows:

$$T_{j\mu} = \frac{Tr\{MS_{j\mu}M^+\}}{Tr\{MM^+\}},$$

where the $M$ is the scattering amplitude. For the case of spin 1 it can be written in terms of the reaction cross sections $\sigma_+, \sigma_-$ and $\sigma_0$ for states with the spin projections onto the quantization axis $S_z = +1, 0, -1$ respectively as follows:

$$T_{20} = \frac{1}{\sqrt{2}} \frac{\sigma_+ + \sigma_- - 2 \cdot \sigma_0}{\Sigma}; \quad \Sigma = \sigma_+ + \sigma_- + \sigma_0$$

The cross section can be expressed in terms of the spherical tensor operators according Madison Convention [5].

The data on $T_{20}$ published in refs.[1, 2] for the inelastic $p(d, d')X$ scattering of relativistic deuterons at $\vartheta_{\text{lab}} = 0^\circ$ are plotted on the Figure 1 versus $\mathcal{R}$.

In contrast with the same data plotted versus $t$ on Figure 1, there is no visible tendency of a systematic shift between data taken at different incident energies on the Figure 1. All the data show an universal dependence on $\mathcal{R}$; the absolute value of $T_{20}$ has a maximum at $\mathcal{R} \sim 0.5 - 0.6$.

Apart from the data taken at $\vartheta_{\text{lab}} = 0^\circ$ from refs.[1, 2], recently a new set of the data on the analysing power for inelastic scattering of deuterons off carbon nuclei at 9 GeV/c were published in ref.[3]. These data were taken at $\vartheta_{\text{lab}} \sim 85$ mrad (i.e. $\vartheta_{\text{cm}} \sim 27^\circ - 35^\circ$).

Because at the scattering angles larger than $\vartheta_{\text{lab}} = 0^\circ$ not only $T_{20}$ enters in the cross sections for polarized particles of spin 1, it is more convenient to use so-called ”cartesian” representation of the analysing powers. In the experiment of ref.[3] the analysing power $A_{yy}$ was actually measured. Fortunately, at $\vartheta_{\text{lab}} = 0^\circ$ the $A_{yy}$ is related with $T_{20}$ in a rather simple way:

$$A_{yy} = -\frac{1}{\sqrt{2}}T_{20}$$

what makes it possible to plot all the available data versus $\mathcal{R}$ on Figure 3. Calculating $\mathcal{R}$, we assume quasifree $d + p$ kinematics, i.e $m_t$ in eq. (1) is the nucleon mass as was in the case of $p(d, d')X$ scattering.

As one can see from the Figure 3, the data taken at 85 mrad support the general tendency in the behaviour of tensor analysing power, which was observed at zero angle.

3 Conclusion.

We have suggested a new relativistic invariant and dimensionless variable for inelastic processes, which takes a constant value equal to zero for any elastic scattering process. This variable may be interpreted as a ratio of the excitation energy to the full transferred
energy taken in the target rest frame. Therefore it is a measure of the degree of ”inelasticity” of the scattering process; in this aspect it reminds the similar parameter introduced in the ref.[6].

We see that $A_{yy}$ taken as function of $\mathcal{R}$ does not dependent upon the initial momentum, the scattering angle and sort of the target. We see also that when the transferred energy is shared in almost equal proportions between the internal degrees of freedom of the collidind particles and the kinetic motion of the recoiled system as whole, the $A_{yy}$ is maximal.

This observation inspires an assumption that this might be a general feature of the inelastic reactions with polarized particles: when the ratio between the ”absorbed” and transferred energies is close to 0.5 - 0.6, the polarization effects are strong, while when this ratio is close to its limits (0 and 1), the polarization effects are weak.

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4 Appendix.

In the case of elastic scattering, the ratio $R$ is zero identically. Therefore this is "genuine inelastic" variable. On the other hand, it is easy to see that when one starts from the inelastic scattering at $\vartheta_{\text{lab}} = 0$ and approaches to the elastic limit $m_X \to m_t$ keeping the scattering angle fixed, one gets $R \to 1$ but crosses the unphysical region where $m_X - m_t < m_{\text{min}}$, i.e. where inelastic processes are kinematically forbidden because the $m_{\text{min}}$ is the mass of the lightest particle which can be created in the reaction under consideration. At the same time, $R \to 0$ if $m_X = m_t$ (fixed) and $\vartheta_{\text{lab}} \to 0$.

That means that there exist a "singular point", because the value of the limit depends upon the way of approaching that point.

On the other hand, in the completely inelastic limit (the missing mass is at its maximal value allowed by the conservation laws at given energy, i.e. $m_X = \sqrt{S} - m_{d'}$), the lab. scattering angle is zero. Therefore

$$R = 1 - \frac{(\sqrt{S} - m_{d'})(\frac{E}{\sqrt{S}} - 1)}{(\sqrt{S} - m_{d'})\frac{E}{\sqrt{S}} - m_t} \to \frac{\sqrt{S}}{E} \quad \text{(when } \sqrt{S} \gg m_{d'}, m_t) \quad (2)$$

in the target rest frame ($E$ is full energy). The ratio $R$ goes to zero when initial momentum increases to infinity.

Except for the completely inelastic limit, at fixed initial energy and scattering angle (in the target rest frame) the $R$ as function of $m_X$ (or $\Delta m_X$) has two branches, which correspond to the "forward" and "backward" (in the center of mass frame) scattering. It is clear that the "forward" value of $R$ must be larger than "backward" one at given $\Delta m_X$.

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