Effects of characteristic decomposed modes of the internal flow of a circular 90-degree bent nozzle on the behavior of the oil jet interface

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Abstract
Methods of decreasing the CO₂ emissions of the internal combustion engine have been suggested. For example, an engine can be designed with a high compression ratio and/or a downsizing turbocharger. However, these methods generate high combustion temperatures that increase the heat load. The piston cooling gallery has been proposed as a system for cooling the engine piston. The piston cooling gallery is an oil flow path that is set internal to the piston. An oil jet injected from a nozzle placed under the piston flows into the piston cooling gallery through an entrance hall. It may thus be desirable to control the shape of the oil jet such that it is stable and straight. However, the interface of the ambient air and oil jet may have unstable waviness because of Kelvin–Helmholtz instability and/or Rayleigh–Taylor instability. In addition, we investigated the flow and found that the propagation of the flow speed fluctuation of the nozzle internal flow results in the waviness of the oil jet in a previous study. To further clarify the relationship between oil jet interface instability immediately after nozzle exit and flow in nozzle, this paper reports on two types of particle image velocimetry (PIV), namely two-dimensional two-velocity-component PIV and two-dimensional three-velocity-component PIV, in addition to two-component and three-component snapshot proper orthogonal decompositions, and analyzes turbulence propagation adopting a cross-correlation method. We find a characteristic basis vector with large energy that propagates the fluctuation downstream under the condition that the interface between the oil jet and air has strong waviness.

Keywords: Internal combustion engine, Piston cooling, Oil jet, Multi-phase flow, Curved circular nozzle, Interfacial fluctuation, Particle image velocimetry, Snapshot proper orthogonal decomposition

1. Introduction

CO₂ emissions are considered a major cause of global warming. In response to global warming, national governments have established regulations for the emissions of CO₂. In particular, governments in the European Union require any new vehicle to have an average CO₂ emission below 95 g/km from 2021 (Schulz, M. and Kourkoulas D., 2014). The thermal efficiency of the internal combustion engine will need to be increased to meet these regulations by, for example, designing an engine with a high compression ratio (Yamakawa, M. et al., 2012) and/or a downsizing turbocharger (Petitjean, D. et al., 2004). Both designs generate a high combustion temperature, which increases the heat load.
As shown in Fig. 1, the piston cooling gallery has been proposed as a cooling system of the engine piston that could solve the above problem of a high heat load. The piston cooling gallery is an oil flow path internal to the piston. An oil jet that is injected from a nozzle, see Fig. 2, placed under the piston flows into the piston cooling gallery through an entrance hall (Deng, X. et al., 2018). Several studies have investigated the piston cooling gallery. Kajiwara, H et al. (2003) suggested a method of predicting the temperature of a piston with a cooling gallery. Peng, W. et al. (2018) investigated the effect of the oil filling ratio of the piston cooling gallery on the heat transfer and found that the coefficient of heat transfer increases at a filling ratio of ~50%. Xiaoli, Y. et al. (2019) investigated the effects of the piston speed on the Nusselt number and flow pattern within the piston cooling gallery and reported that the Nusselt number plateaued above 700 rpm under their experimental conditions. A premise of the above studies was that the oil jet supplies the oil to the piston cooling gallery appropriately. Such an appropriate supply requires the shape of the oil jet to be stable and straight.

However, the behavior of an oil jet is wavy and unstable owing to Kelvin–Helmholtz instability and/or Rayleigh–Taylor instability at the gas (ambient air)–liquid (oil jet) interface (Rayleigh, L., 1878). This interface instability of a jet ejected from a straight nozzle has been investigated in numerous studies. In experimental studies, Arai, M. et al. (1999) measured the wave velocity and frequency and found the deformation behavior of wave breakup whereas Xu, G. and Antonia, R. (2002) investigated the jet behavior for a fully developed turbulent state in a straight pipe and suggested the effect of turbulence on the jet behavior. In a numerical study, Tomotika, S. (1935) analyzed the jet wavelength having the maximum growth rate under the condition of a cylindrical thread of viscous fluid surrounded by another viscous fluid. Funada, T. et al. (2002, 2004) developed Tomotika’s work further and proposed the concept of viscous potential flow. This concept considers the interface viscosity of the jet and surrounding fluid but not the viscosity of the fluid internal jet and surrounding fluid. The approach is easier to implement than Tomotika’s method yet yields results that are in good agreement with Tomotika’s calculations. In other numerical work, Mullyadzhanov, R. et al. (2017) investigated the effect of the internal turbulent flow of a straight nozzle on the behavior of a jet and found characteristic modes of the jet and internal turbulent flow of the nozzle.

In addition to the interface instability of the jet, many studies have investigated the internal flow of a curved pipe. Twin vortices are generated in a curved pipe because of secondary flow. Eustice, J. (1910) found twin vortices generated by secondary flow in the stream pattern in a circular bend. Dean, W. R. (1927, 1928) analyzed the internal flow of a circular bend using incompressible Navier–Stokes equations and found a twin vortex structure, called the Dean vortex, in secondary flow that, theoretically, agrees with the results of the experiments conducted by Eustice. Oki, J. et al. (2018) investigated pulsatile turbulent flow in a square-sectioned curved duct adopting two-dimensional three-velocity-component (2D3C) time-resolved particle image velocimetry (PIV) and two-velocity-component (2C) snapshot proper orthogonal decomposition (POD). They found the Dean vortex structure and the characteristic POD modes that cause a swirl switching pattern despite a high Reynolds number and pulsatile turbulent flow condition.

Taira et al. (2017) performed POD analysis on the variable component obtained by subtracting the average flow velocity from the original flow field for the wake of a flat-plate wing and reproduced the original flow by summing the average flow and a small number of modes. This achievement made it easier to extract the characteristics of the flow in high-energy modes.

However, few studies have investigated the effect of the instability of a liquid jet ejected from a curved nozzle to the atmosphere at ambient pressure. Nakashima, A. et al. (2019) and Kawaguchi, M. et al. (2021) showed that an ejected oil jet has strong waviness when the nozzle has a small radius of curvature and the Reynolds number is high. Kawaguchi, M. et al. (2021) presented the results of time-averaged two-dimensional two-velocity-component (2D2C) PIV from the nozzle exit to 20 mm upstream and found that the propagation speed of the fluctuation of the internal flow of the nozzle was the same as the propagation speed of the oil jet wave. They consequently suggested that the fluctuating internal flow of the nozzle causes the waviness of the oil jet. The present paper extracts the characteristics of the flow field that affect the waviness of the jet interface through the modal analysis of PIV results using snapshot POD. Knowledge of these characteristics is needed to control the behavior of the oil jet or optimize the nozzle design.
Nomenclature

- **d**: Inner diameter of the nozzle (m)
- **R**: Radius of curvature of the nozzle
- **X, Y, Z**: Coordinate axes (m)
- **ρ**: Density of the silicone oil (kg/m$^3$)
- **ν**: Kinetic viscosity of the silicone oil (m$^2$/s)
- **$d_p$**: Average size of the tracer particles (m)
- **$\rho_p$**: Density of the tracer particles (kg/m$^3$)
- **Re**: Reynolds number (–)
- **$\bar{W}$**: Area-averaged streamwise velocity at the nozzle exit (m/s)
- **Q**: Volume flow rate (m$^3$/s)
- **$\bar{W}_{2D2C, PIV}$**: 2D2C PIV area-averaged streamwise velocity at a position 6 mm before the curved bend (m/s)
- **Δr**: Resolution of the calculation grid (m)
- **$U_{x, y, z}$**: Velocity components (m/s)
- **St**: Stokes number (–)
- **$u_x', u_y', u_z'$**: Turbulence velocity components (m/s)
- **$\bar{u}_x, \bar{u}_y, \bar{u}_z$**: Time-averaged velocity components (m/s)
- **λ**: Eigenvalue (m$^2$/s$^2$)
- **B**: Eigenvector (–)
- **a**: POD coefficient (–)
- **$U_{mag}$**: Flow speed (m/s)
- **σ_{U_{mag}}**: Standard deviation of the velocity magnitude (m/s)
- **$S_z$**: Normalized correlation coefficient (–)
- **A**: Area of the base window and searched window in the calculation of $S_z$ (m$^2$)
- **dA**: Area of one cell in a PIV image (m$^2$)
- **Δz**: Offset length for calculating the cross-correlation coefficient (m)
- **τ'**: Interval time for calibration $S_z$ (s)
- **$U_L$**: Lagrangian speed
- **$C_k$**: Cumulative contribution rate (–)
- **ΔT**: Offset time of compared POD pairs (s)
- **M**: Mode number used for summation (–)
- **n**: Mode number (–)

Fig. 1 Configuration of piston cooling gallery

Fig. 2 Actual oil jet nozzle assay (Mazda Motor Co.)
2. Experimental setup and conditions

We used two types of acrylic curved nozzle and silicone oil (KF-56A, Shin-Etsu Silicones, Tokyo, Japan) for the working fluid in all experiments. The experimental methods were described by Nakashima, A. et al. (2018) and Kawaguchi, M. et al. (2021).

The two nozzles, shown in Kawaguchi, M. et al. (2021), have the same shape except for the radius of curvature. Each nozzle has a 90-degree bend and an inner diameter \( d = 6 \) mm. The length before the bend was 300 mm whereas the length after the bend to the exit was 30 mm. One nozzle had a radius of curvature \( R = 15 \) mm whereas the other nozzle had \( R = 60 \) mm. We denote these nozzles as \( R15 \) and \( R60 \) respectively.

Table 1 compares the physical properties of the silicone oil used in this paper (hereinafter referred to as KF-56A) and engine oil (hereinafter referred to as SAE 0W-30). KF-56A at a temperature of 298 K has a kinematic viscosity and surface tension similar to those of SAE 0W-30 at 353 K. In addition, KF-56A has a refractive index similar to that of acryl, allowing internal velocimetry of the nozzle through index matching, as shown in Kawaguchi, M. et al. (2021).

Figure 3 shows the axes of the coordinate system used in this paper, namely \( X \), \( Y \), and \( Z \) axes. The positive \( X \) direction is from the center of the nozzle to the inner side. The \( Y \) direction is from the center of the nozzle to the front side. The \( Z \) coordinate is positive outside the nozzle from \( Z = 0 \) to \( 23.0 \) mm and negative inside the nozzle from \( Z = 0 \) to \( -19.6 \) mm.

| Physical properties of the silicone oil | KF-56A (298 K) | SAE 0W-30 (353 K) |
|---------------------------------------|---------------|------------------|
| Density [kg/m³]                       | 995           | 846              |
| Kinematic viscosity [m²/sec]          | 1.5×10⁻⁵      | 1.44×10⁻⁵        |
| Surface tension [N/m]                 | 28.1×10⁻³     | 26.0×10⁻³        |

**Table 1**

2.1 2D2C and 2D3C time-resolved PIV

We used two PIV methods in investigating the internal flows of the nozzles. One method was 2D2C time-resolved PIV, that is, 2D2C PIV for the velocimetry of the nozzle internal flow on the \( XZ \) plane. We used the same 2D2C PIV system as we proposed in the previous work (Kawaguchi, M. et al. 2021), and the visualization area in the nozzle to observe internal flow from the exit to 20mm upstream is shown in Fig. 4. The 2D2C PIV system had four components: a laser, camera, digital delay generator, and computer. The digital delay generator (VSD2000, Flowtech Research, Kanagawa, Japan) was used for the time synchronization of the Q-switch of the Nd:YAG high-frequency double-pulse laser (MESA-PIV, Amplitude Systemes, Bordeaux, France) and the exposure of a high-speed camera (SA-Z, Photron, Tokyo, Japan).

The thickness of the laser sheet was set to 50 μm using a slit. Fluorescent tracer particles (Fluostar0459, average particle size \( d_p = 15 \) μm, particle density \( \rho_p = 1100 \) kg/m³, EBM Corporation, Tokyo, Japan) excited at a wavelength of 532 nm to emit light at 580 nm were dropped into the silicone oil. In photographing the fluorescent emission at 580 nm, a long-pass filter (SCF-505-560, Sigmakoki, Saitama, Japan) was set on the camera lens (Micro-Nikkor 105-F2.8, Nikon, Tokyo, Japan).

The second method was 2D3C time-resolved PIV, that is, 2D3C PIV for the velocimetry of the nozzle internal flow on the \( XY \) plane. Adopting 2D3C PIV, the three velocity components in the \( X \), \( Y \), and \( Z \) directions were obtained. In the 2D3C PIV, an additional set of a high-speed camera, lens, and long-pass filter that were the same as those used in the 2D2C PIV was added. Two high-speed cameras were set as shown in Fig. 5. The oil flow system and the other parts of the PIV system were the same as those in the 2D2C PIV setup. The thickness of the laser sheet was set to 800 μm using a slit. Table 2 gives the measurement settings for 2D2C and 2D3C PIV.

In these experiments, the camera shutter speed was set at 20,000 fps and the double-pulse laser illuminated particles for each half frequency of the camera shutting (10,000 Hz). The time interval between laser illuminations was adjusted between 10 and 40 μs depending on the Reynolds number such that tracer particles moved two pixels or more in the case of minimal movement and 15 pixels or less in the case of maximum movement within the paired imaging. In analyzing
The PIV results, we adopted PIV analysis software (FtrPIV, Flowtech Research, Kanagawa, Japan), which makes use of direct cross-correlation. A window of $33 \times 33$ pixels with a 50% overlap was set for interrogation and 10 pixels were added to the interrogation window for the search range.

The 2D3C PIV results are compared with the 2D2C PIV results for verification of the accuracy of the former results. The 2D3C PIV was conducted at $Z = -6.0$ mm. Figure 6 compares values of $U_{\text{mag},xz}$ (defined by Eq. (2)) obtained at this position using the two PIV methods. Error bars indicating twice the standard deviation, $2\sigma_{U_{\text{mag}}}$, were calculated using Eq. (3). The 2D3C PIV results are in good agreement with the 2D2C PIV results.

![Diagram](image1)

**Fig. 4** Experimental setup for 2D2C PIV

**Fig. 5** Experimental setup for 2D3C PIV

| Table 2 2D2C and 2D3C PIV settings |
|------------------------------------|
| **2D2C-PIV** | **2D3C-PIV** |
| Reynolds number | 1000 - 3000 | 1000 - 3000 |
| Observation range of Z axis [mm] | -20 - 0 | -6.0 |
| Sampling rate [Hz] | 10000 | 10000 |
| Light interval time [μsec] | 10 - 60 | 20 - 30 |
| LASER sheet thickness [mm] | 0.5 | 0.8 |

\[
Re = \frac{\bar{W}d}{\nu} \tag{1}
\]

\[
U_{\text{mag},xz} = \sqrt{U_x^2 + U_z^2} \tag{2}
\]

\[
2\sigma_{U_{\text{mag}}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (U_{\text{mag},i} - U_{\text{mag}})^2} \tag{3}
\]

![Graph](image2)

**Fig. 6** Comparison of 2D2C PIV and 2D3C PIV results for different Reynolds numbers: (a) $R = 15$ mm and (b) $R = 60$ mm. Solid lines show the 2D2C PIV results whereas broken lines show the 2D3C PIV results.
2.3 POD analysis using the time-resolved PIV results

Snapshot POD analysis was conducted to investigate the flow characteristics. 2C POD analysis was performed for the 2D2C PIV results whereas 3C POD analysis was performed for the 2D3C PIV results. We used the POD method presented by Taira (2017). The 2C turbulent velocity field \( u' = (u_x', u_z') \) in 2D2C PIV or 3C turbulent velocity field \( u' = (u_x', u_y', u_z') \) in 2D3C PIV recorded at each time instant \( t_1, t_2, t_3, \ldots, t_N \) (i.e., \( u'(t_1), u'(t_2), u'(t_3), \ldots, u'(t_N) \)) are arranged in matrix form in Eq. (4). We used \( N = 2500 \) sample PIV images for POD. The turbulent velocity component \( u' \) was obtained by subtracting the time-averaged value from each flow velocity component \( \overline{u} \) as expressed by Eq. (5). The snapshot POD method solves the eigenvalue problem of the velocity correlation matrix \( U^T U \) according to Eq. (6). Eigenvalues \( \lambda_n \) are sorted in descending order and the \( n \)th-order POD mode is expressed by Eq. (7) using the corresponding eigenvectors \( B_n \). When \( u' \) is used in PIV, the dimension of \( \lambda_n \) is the same as that of the kinetic turbulent energy. The order of the POD mode thus represents the order of magnitude of the kinetic turbulent energy. The velocity data decomposed as described above can be reconstructed as expressed by Eq. (8), using any \( M \)th POD mode. \( a_n \) is a time-dependent coefficient of the \( n \)th mode \( (1 \leq n \leq N) \), the so-called POD coefficient, which represents the magnitude of each POD mode. The POD coefficient is given by Eq. (9).

\[
U = [u'(t_1) \ u'(t_2) \ u'(t_3) \ \cdots \ u'(t_N)].
\]

\[
u'(t) = u(t) - \overline{u}
\]

\[
U^T U B_n = \lambda_n B_n
\]

\[
\phi_n = \frac{UB_n}{\|UB_n\|}
\]

\[
u_n(t) = \overline{u} + \sum_{n=1}^{M} a_n(t) \phi_n
\]

\[
a_n(t) = u'(t) \phi_n^T
\]

3. Results and discussion

3.1 POD analysis for 2D2C PIV

This section investigates the number of modes needed to reproduce the Lagrangian speed shown by Kawaguchi, M. et al. (2021). Kawaguchi, M. et al. (2021) have also shown that the Lagrangian speed is consistent with the wave speed of the ejected oil jet. Figure 7 shows 0.25-s time-averaged flow speed distributions that were also obtained by Kawaguchi M. et al. (2021). These distributions are used as POD mode 0 in 2C POD and referenced in the discussion of the POD reconstruction data below. It is seen that the region of high-speed flow is biased toward the outside of the curved nozzle under all conditions. Figure 8 shows the behavior of the oil jet in the two nozzles for different Reynolds numbers. The interface between the jet and air has strong waviness for the small radius of curvature (i.e., \( R = 15 \)) and large Reynolds numbers (i.e., \( Re = 2000 \) and \( Re = 3000 \)). Compared to the KH instability caused by the ambient air and jet velocity, the turbulence of nozzle internal flow has larger effect on the jet behavior immediately after the nozzle exit. We investigated the turbulence of nozzle internal flow in our previous work (Kawaguchi, M. et al. 2021), using \( U_{ratio,x} \). The ratio (i.e. \( U_{ratio,x} = u_x / U_{mag} \)) refers to the ratio of the absolute value of the flow velocity \( U_{mag} \) to the velocity component \( u_x \) in the pipe diameter direction. The results of \( U_{ratio,x} \) investigation showed that under the condition of smaller \( R \) and larger \( Re \), \( R15 \) and \( Re3000 \), the flow from inner to outer occurs at the nozzle outside, and the flow from outer to inner occurs at the nozzle inside. As a result, the jet interface spreads in both directions and symmetrical jet behavior is obtained.

Figures 9 and 12 show the effects of the Reynolds number on the basis vector pattern and POD coefficients of modes 1 to 4, which have high contribution rates, for \( R15 \) and \( R60 \) respectively. Twin modes are confirmed for \( R15 \), that is, modes
3 and 4 at Re2000 in Fig. 9 (b) and modes 1 and 2 and modes 3 and 4 at Re3000 in Fig. 9 (c). The twin modes have similar basis vector patterns and similar POD coefficients. Meanwhile, the characteristic twin structure in the basis vector pattern and POD coefficients seen for R15 is not seen for R60 in Fig. 12. Figures 10 and 13 show the cross-correlation coefficients of POD coefficients $S_{\text{POD,coef}}$ of modes 1 and 2 and modes 3 and 4 in Figs. 9 and 12 in addition to modes 5 and 6 and modes 7 and 8 for the two radii of curvature of the nozzle and each Reynolds number, calculated using Eq. (10). The horizontal axis shows the offset time $\Delta T$ whereas the vertical axis shows the cross-correlation coefficient of the POD coefficients $S_{\text{POD,coef}}$. When one POD coefficient waveform of a twin mode is shifted at a particular $\Delta T$ and matches with another waveform of the mode, the positive value of the cross-correlation coefficient becomes large. Negative cross-correlation coefficients indicate that the waveforms are off by half a cycle. When there are twin modes of basis vectors and the waveforms of the POD coefficients of each mode are strongly correlated, propagation can appear, see Figs. 9 and 10. Therefore, we investigated whether the maximum absolute value of the correlation coefficient was large enough to indicate a strong correlation. Two-thousand four-hundred of the 2500 PIV images were used in calculating the cross-correlation coefficient of the POD coefficients. In the R15 case, modes 3 and 4 at Re2000 and modes 1 and 2 and modes 3 and 4 at Re3000 have strong correlation, with the absolute value of the correlation coefficient being 0.8 or greater. These twin modes are considered to indicate the propagation of turbulence.

Figures 11 and 14 present the cumulative rate of the contribution of modes to the POD for R15 and R60 respectively. The figures confirm that the cumulative contribution rate, calculated using Eq. (11), is dominated by a small number of modes with small mode numbers in all cases. Figures 15–20 show the transient original 2D2C PIV result and the POD reconstruction results obtained using the lowest-number modes (i.e., starting from mode 0) that make cumulative contributions of 30%, 50%, 70% and 100% for the two radii of curvature of the nozzle and each Reynolds number. Figures 15–17 confirm that the outline of the flow speed on the outside of the bend can be reproduced by superimposing a relatively small number of modes under any Reynolds number condition in R15. This reproduction is possible because the basis vector of each mode in R15 has a large flow speed distribution on the outside of the bend. These characteristic modes are thought to represent the propagation of turbulence. Under the presented conditions, twin modes are seen, and the oil jet fluctuation at the interface between the air and oil is stronger in R15 than in R60 as shown in Fig. 8. Meanwhile, Figs. 18 to 20 for R60 suggest that the outline of the flow speed on the outside cannot be reproduced even if modes having a cumulative contribution of 70% are used for reconstruction. Using the POD results, the Lagrangian speed was calculated by reconstructing modes 0 to 2 and the modes from mode 0 to mode $M_{30}$, $M_{50}$, $M_{70}$, and $M_{100}$ that make cumulative contributions $C_i = 30\%$, $50\%$, $70\%$, and $100\%$ (Figs. 15 to 20) respectively, as shown in Fig. 21. Similar to what was found in the visualization of the reconstruction of POD results, the original Lagrangian speed could be reproduced using modes 0 to $M_0$ (or more modes) at each Reynolds number for R15, whereas the original Lagrangian speed could not be reproduced with even modes 0 to $M_0$ for R60. These results reveal that a smaller radius of curvature of the nozzle and a larger Reynolds number increase the waviness of the oil jet. This behavior is due to characteristic twin modes that have a large energy basis vector at the outside of the bend and a high flow speed inside the nozzle.

\[
S_{\text{POD,coef}} = \frac{\sum_{i=0}^{299} (\text{POD}_i, t = t - \text{POD}_i, t = t + \Delta T - \text{POD}_i, t = t + \Delta T)}{\sum_{i=0}^{299} (\text{POD}_i, t = t - \text{POD}_i, t = t + \Delta T - \text{POD}_i, t = t + \Delta T)^2} \sqrt{\sum_{i=0}^{299} (\text{POD}_i, t = t - \text{POD}_i, t = t + \Delta T - \text{POD}_i, t = t + \Delta T)^2}
\]

\[
C_{\lambda M} = \frac{\sum_{i=1}^{M} \lambda_i}{\sum_{i=1}^{2500} \lambda_i} \times 100
\]
Fig. 7 PIV results of the 0.25-s time-averaged distribution of the flow speed (mode 0): (a) R15 and (b) R60. The X axis gives the radial position of the nozzle, with positive values inside the nozzle and negative values outside the nozzle. (Kawaguchi, M. et al., 2021)

Fig. 8 Oil jet behaviors for different Reynolds numbers: (left) R15 and (right) R60 (Kawaguchi, M. et al. 2021). The fluctuation of the oil jet at the interface between the air and oil is stronger in R15 than in R60 for the same Reynolds number.
Fig. 9 Basis vector patterns and POD coefficients ($R_{15}$)

(a) $R_{1000}$
(b) $R_{2000}$
(c) $R_{3000}$

Fig. 10 Cross-correlation of POD coefficients for modes 1 and 2, modes 3 and 4, modes 5 and 6, and modes 7 and 8

(a) $Re_{1000}$
(b) $Re_{2000}$
(c) $Re_{3000}$

Fig. 11 Cumulative rate of the POD contribution of modes in 2D2C PIV for $R_{15}$
Fig. 12 Basis vector patterns and POD coefficients ($R_{60}$)

(a) $R_{60}$, $Re_{1000}$

(b) $R_{60}$, $Re_{2000}$

(c) $R_{60}$, $Re_{3000}$

Fig. 13 Cross-correlation of POD coefficients for modes 1 and 2, modes 3 and 4, modes 5 and 6, and modes 7 and 8

(a) $Re_{1000}$

(b) $Re_{2000}$

(c) $Re_{3000}$

Fig. 14 Cumulative rates of the POD contribution of modes in 2D2C PIV for $R_{60}$
(a) Original case of $R15$, $Re1000$

(b) Reconstruction result (modes 0 to $M_{50}$)

(c) Reconstruction result (modes 0 to $M_{50}$)

(d) Reconstruction result (modes 0 to $M_{70}$)

Fig. 16 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $R15$ and $Re1000$

(a) Original case of $R15$, $Re2000$

(b) Reconstruction result (modes 0 to $M_{50}$)

(c) Reconstructed result (mode 0 to $M_{50}$)

(d) Reconstructed result (mode 0 to $M_{70}$)

Fig. 15 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $R15$ and $Re1000$
Fig. 17 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for R15 and Re3000

Fig. 18 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for R60 and Re1000
Fig. 19 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $Re_{60}$ and $Re_{2000}$

Fig. 20 2D2C PIV original result and 2C POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $Re_{60}$ and $Re_{3000}$
3.2 3C POD analysis for 2D3C PIV

The flow on the $XY$ cross section was measured using 2D3C PIV to investigate the occurrence of turbulence confirmed in the $XZ$ cross section by 2D2C PIV in Section 3.1. 2D3C PIV obtains three components on the $XY$ plane and in the $Z$ direction. The measurement position was set at $Z = -6.0$ mm, which is 6.0 mm upstream from the nozzle exit. Figure 22 (a) and (b) respectively shows the time-averaged 2D3C PIV results for R15 and R60 at Reynolds numbers of 1000, 2000, and 3000. These time-averaged flow speed distributions are used as POD mode 0 in 3C POD. It is seen that the region of high-speed flow is biased toward the outside of the bend under all conditions. At all Reynolds numbers in the R60 case, there is an area of high-speed flow at the outside of the bend at $X \approx -2$ and $Y \approx 0$. However, such an area of high-speed flow at the outside of R15 is not seen at Reynolds numbers of 2000 and 3000; instead, there is strong waviness of the oil jet as shown in Fig. 8.

We used 3C POD to investigate the motion of flow. Figure 23 (a) shows that, for R15, there are enclave characteristic modes with a high flow speed at a position deviating in the $Y$ direction from the region of the maximum flow speed in Fig. 22. These characteristic POD modes may be responsible for the main stream of the internal flow of the R15 nozzle. Figure 23 (b) shows that the enclave characteristic modes cannot be confirmed at $Re2000$ and $Re3000$ for R60. However, it is seen that there are regulation modes that may generate fluctuations at the inside of the bend where the $X$ axis is positive when the time-averaged flow speed is low at $Re3000$. Figure 24 presents the cumulative contribution rates of modes for the 3C POD results. The reconstruction results presented in Fig. 25 for R15 reveal that the area of high-speed flow at the outside of the bend reproduces the fluctuation in the $Y$ direction as the Reynolds number increases owing to the superposition of a relatively small number of modes, whereas the area of high-speed flow hardly fluctuates in R60 as shown in Fig. 26. We consider that the fluctuation of the area of high-speed flow at the outside of the bend results from the large amount of energy propagating downstream under the conditions that the radius of curvature of the nozzle is small (i.e., R15) and the Reynolds number is large. The fluctuation is further affected by the number of modes required to reproduce the Lagrangian speed on the outside of the bend in 2D2C PIV. We consider that this spatial and transient fluctuation of the flow speed generates a wave at the oil–jet interface immediately after the nozzle outlet.

Fig. 22 Time-averaged 2D3C PIV results for R15 (left) and R60 (right) and Reynolds numbers of 1000, 2000, and 3000
Fig. 23 Basis vector patterns and contribution ratios of modes calculated by POD

Fig. 24 Cumulative contribution rates of the 3C POD modes: R15 (left) and R60 (right)
Fig. 25 2D3C PIV original result and POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $R15$
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Fig. 26 2D3C PIV original result and POD reconstruction results obtained using modes 0 to $M_{30}$, $M_{50}$, and $M_{70}$ for $R60$
4. Conclusions

This study investigated the relationship between the 2C POD and the Lagrangian speed obtained in 2D2C PIV and analyzed the flow field obtained by 3C POD using 2D3C PIV. The following findings are taken from the results of the study.

1. 2D2C PIV and 2C POD

We found that several modes having high contribution rates affect the reproduction of the Lagrangian velocity under conditions for which the oil jet has strong waviness, that is, a small radius of curvature of the nozzle and a large Reynolds number.

Under the Re2000 and Re3000 conditions for the R15 nozzle, there are twin modes that have a high-contribution ratio characterized by the spatial offset of the basis vector and the temporal offset of the POD coefficient at the outside of the bend where there is a high-flow-speed bias. Under these conditions, the Lagrangian speed is reproduced by reconstructing a relatively small number of high-contribution modes. Under the Re60 condition, the Lagrangian speed with a high flow speed at the outside of the bend is not reproduced even if the modes having a cumulative contribution rate of up to 70% are used under all the Reynolds number conditions considered. This is because the modes that make a large energy contribution control the turbulence of the low-speed flow at the inside of the bend in Re60. Therefore, the jet interface does not show strong waviness in Re60 under these conditions.

2. 2D3C PIV and 3C POD

Modes having characteristic basis vectors that sway the area of high-speed flow in the Y direction were confirmed for the small radius of curvature of the nozzle and a large Reynolds number.

The number of modes required to reproduce the Lagrangian speed at the outside of the bend in 2D2C PIV was measured. We consider that the spatial and transient fluctuation of the flow speed generates a wave at the oil jet interface immediately after the nozzle outlet.

The results clarify the effects of the internal flow of a circular 90-degree curved nozzle on the behavior of the ejecting oil jet.

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