Screening Masses in Dimensionally Reduced (2+1)D Gauge Theory

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We discuss the screening masses and residue factorisation of the SU(3) (2 + 1)D theory in the dimensional reduction formalism. The phase structure of the reduced model is also investigated.

1. Introduction

We present a numerical study of a 2D SU(3) gauge theory with adjoint scalar fields, defined by dimensional reduction of pure gauge QCD in (2+1)D at high temperature. We show that the reduced model not only reproduces Polyakov Loops and their correlations but also gives a good approximation for the spatial string tension \[ T \]. We also show that the correlations between Polyakov loops are saturated by two colourless bound states. Their contributions in correlation functions of local composite operators \( \phi_n \) respectively of degree \( n = 2p \) and \( 2p + 1 \) in the scalar fields \( (p = 1, 2) \) fulfill factorization. The contributions of two particle states are detected. Their size agrees with the estimates based on a meanfield-like decomposition of the \( p = 2 \) operators into polynomials in \( p = 1 \) operators. In contrast to the naive picture of Debye screening, no sizable signal in any \( \phi_n \) correlation can be attributed to \( 1/n \) times a Debye screening length associated with \( n \) elementary fields. These results are quantitatively consistent with the picture of scalar “matter” fields confined within colourless boundstates whose residual “strong” interactions are very weak [4].

2. Dimensional Reduction and its Validity Region

The reduced lattice action \( S_{\text{eff}} \) results from integrating out the infrared convergent non-static modes of the gauge fields of the (2 + 1)D theory at temperature \( T \) and coupling \( g_3 \). \( S_{\text{eff}} \) depends on 2D gauge fields and on a “Higgs” field (noted \( \phi(x) \)), the static components respectively of the spatial and time 3D gauge fields. In the large \( T \) limit and for momenta \( p \ll T \) (large distance physics), \( S_{\text{eff}} \) can be written (see [1] for details)

\[
S_{\text{eff}} = S_U + S_{U,\phi} + V(\phi).
\]

Here \( S_U \) is pure gauge, \( S_{U,\phi} \) is the gauge invariant \( \phi \) kinetic term, and \( V(\phi) \) is a local potential involving the quadratic and quartic self-couplings \( h_2, h_4 \), which are computed in [1] as functions of \( g_3 \) anf \( T \) or, equivalently - of the (2 + 1)D lattice parameters: coupling \( \beta_3 \), time-direction extension \( L_0 \) and spacing \( a \)

\[
\beta_3 = \frac{6}{ag_3^2}; \quad L_0 = \frac{1}{aT}; \quad \tau = \frac{T}{g_3^2} = \frac{\beta_3}{6L_0}.
\]

To determine the region of validity of the dimensional reduction, we measured correlations of the Polyakov Loops, which are static operators

\[
L(\vec{x}) = \frac{1}{3} Tr \exp(i\phi(\vec{x})/\sqrt{\tau}),
\]
Figure 1. $M_S$ is a mass in physical units and the string tension, extracted from the spatial Wilson Loops. We performed the 2D simulations on a $32^2$ lattice with $L_0 = 4$, using Multi-hit Metropolis as well as Hybrid Metropolis-Heatbath algorithms. The method turned out to be valid for temperatures down to $T = 1.5T_c$, $T_c$ being the deconfinement temperature of the $(2+1)D$ model [3] (Fig.1). In order to make sure that we are close to the scaling region, we also performed some measurements with a lattice spacing twice smaller. That justifies keeping $L_0 = 4$ and comparing our 2D data with the $(2+1)D$ data of [3].

3. General properties of the reduced model

The system has a strong first order phase transition with respect to $R_\tau$, the imaginary time inversion symmetry of the original model, i.e. the $\phi \rightarrow -\phi$ symmetry, for which $tr\phi^3$ is an order parameter. We find that in the thermodynamical limit the parameters of the reduced model correspond to a point in the broken $R_\tau$ phase (but very close to the transition line). To avoid getting into the broken phase we start with weak field configurations, and check that the system stays in the (metastable) phase monitored by $<tr\phi^3> = 0$.

The phase diagram in the $h_2, aT$ plane is sketched in Fig.2: at two temperatures the value of the quadratic self coupling $h_2$ at the transition is compared with that for the reduced model, and the phases are indicated.

Careful study of the correlations $\phi_{n,m} = <Tr\phi^n(x)Tr\phi^m(y)>$ reveals two channels called $S$ and $P$ (Fig.3), associated with operators respectively even and odd under $R_\tau$, i.e. in $\phi$. The mass $M_S$ measured in the even channel agrees with that measured from Polyakov loops correlations (Fig. 1), in which the lowest power in $\phi$, namely 2, dominates. The ratio of the corresponding screening masses does not show any clear tendency, being 1.8, 2.0, 1.7 and 1.6 in order of increasing temperature. It may still go to the value of 1.5 at a very high temperature as expected from a naive composite gluon picture.
It should go to one at large $r$. Deviations at shorter distances are to a large extent compatible
with the propagation of two particles, namely two $S$ or $P$ respectively in $S$ and $P$ channels. A Wick-like treatment in the mean field approximation leads to, for example, the following result for the $\phi_{4,4}$

$$\phi_{4,4}(r) = \left( \frac{5}{4} \right)^2 \left[ \phi_2^2 \phi_{2,2}(r) + \frac{1}{2} \phi_{2,2}^3(r) \right]$$

The measured values of $X_2$ and those $\tilde{X}_2$ which include the two $S$ correction to $\phi_{4,4}$ are successfully compared in Fig.4.

**REFERENCES**

1. P. Bialas, A. Morel, B. Petersson, K. Petrov and T. Reisz, “High Temperature 3D QCD: Dimensional Reduction at Work”, *Nucl. Phys.* **B581** (2000) 477.
2. P. Bialas, A. Morel, B. Petersson, K. Petrov and T. Reisz, “QCD with Adjoint Scalars in 2D: Properties of the Colorless Scalar Sector”, *Nucl. Phys.* **B603** (2001) 369.
3. C. Legeland, Ph.D. Thesis, “Aspects of (2+1) Dimensional Lattice Gauge Theory” (University of Bielefeld, Germany, September 1998).