Phase coherence in quasicondensate experiments: an ab initio analysis via the stochastic Gross-Pitaevskii equation

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We perform an ab initio analysis of the temperature dependence of the phase coherence length of finite temperature, quasi-one-dimensional Bose gases measured in the experiments of Richard et al. (Phys. Rev. Lett. 91, 010405 (2003)) and Hugbart et al. (Eur. Phys. J. D 35, 155-163 (2005)), finding very good agreement across the entire observed temperature range (0.8 < T/T\phi < 28).

Our analysis is based on the one-dimensional stochastic Gross-Pitaevskii equation, modified to self-consistently account for transverse, quasi-one-dimensional effects, thus making it a valid model in the regime \mu \sim \text{few} \hbar\omega_\perp. We also numerically implement an alternative identification of T\phi, based on direct analysis of the distribution of phases in a stochastic treatment.

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I. INTRODUCTION

Weakly interacting ultracold Bose gases are of great interest because they allow the ‘pure’ study of quantum phenomena on a macroscopic scale. Such systems are typically engineered in harmonic traps with different geometric configurations, whose dimensionality plays a crucial role in determining the properties of these gases. In three-dimensional harmonic traps, the system undergoes a phase transition which leads to the appearance of coherence across the entire sample, as demonstrated in [1-4]. By setting the trap frequency in one direction to be much larger than the others, the effective system dynamics is reduced to two dimensions [3-8], with many interesting phenomena occurring, such as the Berezinskii-Kosterlitz-Thouless transition [9-11]. Increasing the trap frequency in a further direction allows the realization of highly elongated traps, where the interesting physics occurs in the axial direction [3, 17, 22].

In such a regime two characteristic temperatures become relevant, associated with the onset of phase (T\phi) and density (T_d) fluctuations [23]. For temperatures T\phi < T < T_d, density fluctuations tend to be suppressed, and the system reduces to a condensate with fluctuating phase (quasicondensate) [22]. In such a geometry low-energy thermal excitations of the axial modes play a crucial role, as they tend to destroy the coherence in the sample [22, 40]; such excitations may have wavelengths greater than the transverse extent of the system, therefore acquiring a one-dimensional (1D) character [13, 22, 34-40]. An accurate analysis of the coherence properties in such systems is therefore necessary for potential applications, such as matter-wave interferometry [11, 50, 52], atom chips [54] and atom lasers [2, 57-60]. Although experiments can nowadays be engineered to produce gases which are both weakly interacting and practically 1D (\mu, k_B T \lesssim \hbar\omega_\perp, where \mu is the chemical potential, k_B T the thermal energy and \hbar\omega_\perp the transverse excitation energy) [22, 11, 41], the early experiments performed did not satisfy these conditions so well, and the system was instead in the 1D-3D crossover regime [17, 19, 22, 34-40].

In this work we propose and implement a modified stochastic model which enables us to perform a successful ab initio description in the ‘intermediate’ regime \mu, k_BT \sim \text{few} \hbar\omega_\perp where quasi-condensate physics dominates, but transverse effects still need to be appropriately accounted for. More specifically, we analyze two early experiments [22, 40] investigating the phase coherence properties of weakly interacting, quasi-1D Bose gases. These experiments measured the temperature dependence of the coherence length L_c, over which the gas maintains an appreciable coherence, both in the ‘strong’ (6 < T/T\phi < 28) [22] and ‘weak’ (0.8 < T/T\phi < 8) [40] phase fluctuation regimes. Although it is theoretically anticipated (for a homogeneous gas [61]), that the coherence length, scaled to the experimental half-length of the system L, should yield a universal curve when plotted against the reduced temperature T/T\phi, it would not actually be appropriate to incorporate their reported data into a single graph [62] spanning the entire regime 0.8 < T/T\phi < 28, because the two experiments used different techniques to extract the coherence.

The main achievements of this work are as follows: (i) we show that the measurements reported in both experiments are indeed consistent with such a unified curve; this is achieved by a completely ab initio analysis of the experimental data, using only the quoted experimental values for atom number, trap frequencies and temperature; (ii) our analysis, which is based on the quasi-1d stochastic Gross-Pitaevskii equation [63], appropriately modified here to extend its validity to the regime \mu \sim \text{few} \hbar\omega_\perp reveals excellent agreement in the ‘strong’ phase fluctuation regime; (iii) our attempt to more closely mimick the experimental procedure of [40] – whose findings have not been adequately interpreted to date – also yields good overall agreement (within error bars); in order to further improve this we implement an alternative approach of extracting a phase coherence temperature based on direct analysis of the phase distri...
butions from different realisations of our stochastic treatment.

II. METHODOLOGY

Our treatment is based on a modified one-dimensional form \[63\] of the stochastic Gross–Pitaevskii equation (SGPE) \[64, 66\]. The modification proposed in \[63\] – which was found to be essential to accurately simultaneously reproduce both \textit{in situ} density profiles and density fluctuations of recent quasi-one-dimensional experiments \[42, 45, 49\] – takes the form:

\[
\begin{align*}
\hbar \frac{\partial \psi(z,t)}{\partial t} & = [1 - i \gamma(z,t)] \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V(z) \right. \\
& \left. + \hbar \omega_\perp \left( \sqrt{1 + 4|\psi|^2 a_s - 1} - \mu \right) \psi(z,t) + \eta(z,t) \right). (1)
\end{align*}
\]

Here \(\psi\) describes the relevant (highly populated) low-energy modes of the system, \(V(z) = m \omega_\perp^2 z^2/2\) is the axial trapping potential and \(\eta\) is a complex Gaussian noise term, with correlations \(\langle \eta^*(z,t) \eta(z',t') \rangle = 2 \hbar \gamma(z,t) k_B T \delta(z-z') \delta(t-t')\), where \(\gamma(z,t)\) denotes the dissipation. (For further details on numerical implementations and related treatments see Refs. \[28, 63\].) The condition \(\mu \lesssim \hbar \omega_\perp\) can lead to a swelling of the condensate in the transverse direction, relative to the true 1D transverse ground state: this quasi-1D effect is due to repulsive interactions and is reproduced by using the modified nonlinear term \(\hbar \omega_\perp \left( \sqrt{1 + 4|\psi|^2 a_s - 1} \right)\) \(\) (where \(a_s\) is the s-wave scattering length), which reduces to the 1D result in the limit \(4|\psi|^2 a_s < 1\). For the ordinary Gross-Pitaevskii equation, this was shown in Refs. \[34, 51, 81\], and its validity was verified experimentally in \[48\].

In order to match to experimental atom numbers, it is crucial to also include in our treatment the contribution \((n_\perp)\) to the linear density profile of transverse thermal atoms with energy greater than \(\hbar \omega_\perp\). This is done via

\[
n(z; \mu, T) = \langle |\psi(z; \mu, T)|^2 \rangle + n_\perp(z; \mu, T) \quad (2)
\]

where \(\langle \cdots \rangle\) denotes ensemble averaging, obtained by averaging over many different realizations of the noise, and

\[
n_\perp(z; \mu, T) = \frac{1}{\lambda_{MB}} \sum_{j=1}^{\infty} (j+1) g_{1/2}(\mu_j(z)/k_B T) \quad (3)
\]

where \(g_{1/2}(\cdots)\) is the polylogarithm (or Bose function) of order \(1/2,\) and \(\lambda_{MB} = \hbar / \sqrt{2 \pi m k_B T}\) is the thermal de Broglie wavelength \[85\]. While in the limit \(\mu < \hbar \omega_\perp\) studied in our previous work \[63\], it is sufficiently accurate to use \(\mu_j(z) = \mu - V(z) - j \hbar \omega_\perp\) in Eq. \(4\) (see also \[43, 84\]), the effect of mean-field potential experienced by the transverse thermal atoms should also be taken into account in the regime \(\mu > \hbar \omega_\perp\), characteristic of the experiments of Refs. \[22, 40\] studied here. For this reason we use here the modified expression:

\[
\mu_j(z) = \mu - V(z) - j \hbar \omega_\perp - 2g(\langle |\psi|^2 \rangle + n_\perp) \quad (4)
\]

where \(g = 2a_s \hbar \omega_\perp\) is the effective interaction strength.

In the following sections we systematically use Eqs. \(1-4\) to match to the experimental atom number (noting also that the profiles generated by Eq. \(2\) correspond to transversely \textit{integrated} density profiles, typical of ultracold gas experiments), before performing analysis of the coherence properties.

For both experiments analysed here, the \textit{only} experimental inputs to our theory are the atomic species used \((^{87}\text{Rb} \text{in both cases}),\) the trapping configuration, the temperature and atom number corresponding to each \textit{reported} experimental data point \[87\]. In our \textit{ab initio} analysis we fix experimental trap configuration and temperature, and vary \(\mu\) until the desired number of atoms is reached in our simulations. Depending on the available experimental data, we either match to the \textit{total} or the \textit{quasicondensate} experimental atom number, as explained in each case. Representative error bars are also calculated for our simulations, based on variations in both number and temperature, in close analogy to the experimental analysis.

In order to discuss both experiments in a coordinated way, and demonstrate their consistency, we consider the dependence of the coherence length on temperature, with both parameters appropriately scaled to show the emergence of universal physics. In performing this analysis, we also address experiment-specific features, meaning the precise definitions of the coherence length and phase fluctuation temperature differ slightly within our subsequent analysis, however this difference is clearly labelled within figures.

III. COMPARISON TO THE EXPERIMENT OF RICHARD ET AL. \[22\]

The first experiment we consider is that of Richard et al. \[22\], in which a very elongated harmonic trap \((\omega_\perp/2\pi = 760 \text{ Hz} \text{ and } \omega_\parallel/2\pi = 5 \text{ Hz})\) was used to generate quasicondensates of atom numbers in the range \(0.25 \times 10^5 \text{ to } 0.65 \times 10^5\) and at temperatures \(90 \text{ nK} \lesssim T \lesssim 350 \text{ nK}\). These parameters enabled the ‘strong’ phase fluctuation regime \(6 < T/T_\varphi < 28\) to be probed.

In order to investigate the phase coherence properties of the gas, the axial momentum distribution was measured by means of Bragg spectroscopy and found to have a Lorentzian shape, consistent with an exponential decay of the correlation function in space anticipated in the regime of large phase fluctuations \[21, 22, 36, 88\]. This approach enabled the coherence length \(L_c\) to be extracted from the half width at half maximum of the profile, and its temperature dependence to be measured. Using the original data relevant for Ref. \[22\] (see also \[39\] and \[88\]), we can plot this in universal form by scaling the coherence length \(L_c\) to the half-length of the condensate \(L\), and plotting against the reduced temperature \(T/T_\varphi\). Here we use the general definition of \(T_\varphi\) in terms of the \(1\text{D}\) axial quasicondensate peak density, \(n_{qc}(0)\), i.e.
In order to simulate the experiment, we compute the first order correlation function in position space
\[ g^{(1)}(-z/2, z/2) = \langle \psi^*(z/2) \psi(-z/2) \rangle \]
(normalized to the averaged central density \( \langle |\psi(0)|^2 \rangle \)), finding an exponential behaviour against distance \( z \) as in the experiment (since \( T >> T_\phi \) here). We then compute the Fourier transform \( C^{(1)} = F[g^{(1)}] \) and extract the coherence length \( L_c \) by measuring \( \Delta k \), the half-width at half maximum of \( C^{(1)} \). We also use our simulated results to find \( T_\phi \), with the required inputs being the quasicondensate peak density and spatial extent of the gas, each of which we obtain \textit{ab initio}. Specifically, the quasicondensate density is extracted as

\[ n_{qc}(z) = \sqrt{2 \langle |\psi(z)|^2 \rangle^2 - \langle |\psi(z)|^4 \rangle} \quad (6) \]
and its peak value is used in the expression for \( T_\phi[n_{qc}(0)] \). Moreover, the half-length of the quasicondensate in our simulations is given by a temperature dependent Thomas-Fermi radius \( R_{TF}(T) \), extracted from Eq. 11 analogously to the procedure used in the modified Popov theory (see [27, 28, 91] and [89]).

Our \textit{ab initio} SGPE results presented in Figure 1 (upward filled black triangles) show excellent agreement to the experimental results (hollow red triangles) in this ‘strong’ phase fluctuation regime within their respective characteristic error bars (see caption for details). Note that the coherence length is smaller than the quasicondensate extent \( (L_c/L < 1) \) in the temperature range probed here, illustrating the fundamental role of phase fluctuations in such an elongated geometry. As \( T/T_\phi \) approaches zero, we expect the coherence length to increase: to show this, we have generated a further set of numerical points for \( T/T_\phi < 6 \) (downward filled black triangles), which indeed confirm this picture, and the ‘universal nature’ of such a scaled diagram.

In the following section, we turn to the investigation of the experiment of Hugbart et al. [40], in which the regime \( T/T_\phi < 8 \) was probed using different methods.

IV. COMPARISON TO THE EXPERIMENT OF HUGBART ET AL. [40]

The Bragg spectroscopy method used in the previous experiment limited accurate investigations of the coherence properties of the gas to \( T/T_\phi > 6 \), as at lower temperatures the width of the momentum distribution of the gas was no longer easily resolved [33]. In the experiment described in [40], an alternative interferometry technique (see also Ref. [21]) was used to measure the spatial correlation function in the regime \( 0.8 < T/T_\phi < 8 \).

In particular, after the condensate was released, two Bragg pulses were applied, playing the role of matter-wave beam splitters. The contrast of the resulting interference fringes was then obtained from the modulus of the Fourier transform of the interference pattern, and the coherence length was extracted from the decrease of the contrast as a function of the distance between the two interfering condensates. The experimental data (hollow red circles of Fig. 2), show that, in the regime \( T \approx T_\phi \), the coherence extends over more than half of the system size.

In order to access such low values of \( T/T_\phi \), it was technically easier to use slightly less elongated traps than in [22]. In fact, two different trap configurations were used (first: \( \omega_\perp/2\pi = 395 \) Hz and \( \omega_z/2\pi = 8.67 \) Hz; second: \( \omega_\perp/2\pi = 655 \) Hz and \( \omega_z/2\pi = 6.55 \) Hz); the data obtained with the second trap were subdivided into two different blocks, characterised by two different values of the evaporation parameter, which proved necessary in order to vary \( T/T_\phi \), while keeping the condensed fraction fairly constant within each data block. The total atom numbers measured in this experiment were found to lie...
Figure 2. (Color online) Scaled coherence length $L_c/L$ vs. reduced temperature $T/T_\phi$. Comparison of experimental data [40] (hollow red circles) to predictions of the SGPE model (filled black squares). The reported experimental best fit (dashed red line) and the theory used in [40] to interpret the experimental data (solid line) are also shown, alongside the SGPE fit from Fig. 1 (dot-dashed black line). Typical error bars for data points in both experiment and SGPE model are indicated, with the SGPE ones arising from a characteristic 20\% variation in total atom number and an additional 5\% variation in temperature; (note that the SGPE error bars for the point with $T/T_\phi \approx 1.2$ lie within the point size and are not visible). The SGPE results are scaled to the experimentally-reported ones. The reason for this discrepancy is twofold: firstly, in [40] it is an ‘effective’ correlation function that is evaluated, which leads to a different definition of the coherence length $L_c$ (see Section IV A); secondly, the theoretical values of $T/T_\phi$ obtained in our treatment do not span precisely the same range as in the experiment (see Section IV B).

With regard to the latter point, we note that the analysis of Ref. [40] was actually based on a slightly modified definition for $T_\phi$ compared to that of Eq. (5); in particular, in [40], $T_\phi$ was defined in terms of the number of quasicondensate [93] atoms $N_{qc}$ (and not on the peak quasicondensate density), via the expression

$$T_\phi[N_{qc}] = 15\hbar^2 N_{qc}/16 mk_BT^2.$$  (7)

This is in fact a simplified form of Eq. (5), valid for 3D condensates [98] (i.e. condensates where the transverse density profiles can be well approximated by a Thomas-Fermi profile).

In the following sections, we further investigate potential sources of discrepancy between our simulations and the experimental results of Ref. [40], focusing our analysis on a modified effective correlation function defined below, as relevant for this experiment. Throughout our analysis, the coherence length is always referred to as $L_c$, with the method of its extraction clearly identified in each figure.

A. Effective correlation function

The main source of the observed discrepancy should be related to the experimental measurement of an ‘effective’ correlation function (see Eq. (9) in [40]), instead of the correlation $C^{(1)}$ measured in Richard et al. [22] and discussed thus far. In the experiment of Ref. [40], this effective correlation was introduced in order to cancel the random phase caused by the shot to shot fluctuations of the global position of the contrast fringes; it was found that taking the absolute value of the Fourier transform of the fringe pattern before averaging achieved this aim (for a more detailed explanation see Section 4.2 in Ref. [40]), but modified the coherence length relative to that of $C^{(1)}$. For this reason, we should not expect the measurements from the two experiments to lie on the same curve, as they measure two different quantities [62].

In analogy to the experimental method used to extract the coherence length, we implement this feature by taking the absolute value of the Fourier transform of $g^{(1)}$ from each individual run, before averaging over the different realizations of the noise; our ‘effective’ correlation function, which we here call $C^{(1,\text{mod})}$, takes the form

$$C^{(1,\text{mod})} = \langle |F[g^{(1)}(-z/2, z/2)]| \rangle.$$  

The ‘effective’
correlation function is found to have similar behaviour to \( C^{(1)} \), but it decays faster (in the region of interest) in momentum space, thus resulting in larger values of the coherence length. This is shown in Fig. 3 which compares the theoretical results to the experimental measurements using \( C^{(1)} \) (brown filled squares) and \( C^{(1,\text{mod})} \) (blue filled circles). Although we consciously do not exactly reproduce the experimental measurement sequence, the results obtained from the SGPE analysis of the ‘effective’ correlation function clearly show a very similar trend to the experimental findings over the probed regime; in particular, they tend to lie on the reported line of best fit of the experimental data (dashed red line). We note that our calculation of \( C^{(1,\text{mod})} \) leads to a much improved agreement with the experimental data than the original theoretical analysis reported in Ref. [40] (solid red line) which largely overestimates the amount of coherence in the system.

![Figure 3](image-url)

**Figure 3.** (Color online) Scaled coherence length \( L_c/L \) vs. reduced temperature \( T/T_\phi \). Comparison of numerical SGPE findings extracted via \( C^{(1,\text{mod})} \) (filled blue circles) and via \( C^{(1)} \) (filled brown squares) against experimental data points of Ref. [40] (hollow red circles). The theory used in Ref. [40] is also shown (solid line) together with the experimental best fit (dashed red line). SGPE numerical points are scaled to \( T_\phi[N_{qc}] \), and respective error bars account for 20% variation in total atom number and a further 5% variation in temperature. Inset: comparison of SGPE results, in which temperatures are scaled to the experimentally quoted values of \( T_\phi \). SGPE results shown in the inset are obtained by matching either the total number of atoms (blue filled circles, main figure and inset), or the quasicondensate atom number (green crosses, inset only).

Although the experimental results and our simulated points (extracted via \( C^{(1,\text{mod})} \)) demonstrate very good agreement when accounting for their respective error bars, the experimental data still appear to systematically extend to slightly larger values of \( T/T_\phi \), as visible in Fig. 3. This could be attributed either to a systematic shift in the experimental determination of \( T \) (e.g. arising in expansion imaging), which however increases with increasing \( T/T_\phi \) ratio, or to the method by which the inputs to \( T_\phi \) are extracted in the analysis. In the rest of the paper we assume that this shift arises solely from the latter and attempt to further improve on the spanned range of \( T/T_\phi \).

**B. Identification of \( T_\phi \)**

A given set of trap frequencies, temperatures and total atom numbers should fix the characteristic temperature \( T_\phi \) to a particular value which is the same between theory and experiment; for this reason, in order to attempt to span precisely the same range of \( T/T_\phi \) as in the experiments, we plot in the inset of Fig. 3 our theoretically-observed values for the reduced coherence length \( L_c/L \) versus \( T/T_\phi \) using here the experimentally-obtained values for \( T_\phi \). This still reveals good agreement between our theoretical findings (blue filled circles) and the experimentally-extracted ones (hollow red circles), within the reported experimental error bars.

We have also performed a separate analysis, based again on the SGPE but in which we instead match our numerically extracted quasicondensate number to the corresponding experimentally-extracted ‘condensate’ number [57], and measure the coherence length via \( C^{(1,\text{mod})} \). These closely related measurements are shown in the inset of Fig. 4 with green crosses; we find good overall agreement both with the experimental findings (hollow red circles) and with the previous analysis, based on matching the experimental total atom number instead (filled blue circles).

In order to further resolve the remaining discrepancy between theory and experiment in the regimes of \( T/T_\phi \) analysed, in the next section we discuss an alternative method to extract \( T_\phi \) from the SGPE simulations.

### 1. \( T_\phi \) extracted from the phase distribution

In this section we investigate an alternative method of reproducing the experimental results of Ref. [40] from an SGPE analysis. Our approach is motivated from footnote 47 of Ref. [40], stating that \( T_\phi \) can be obtained from the relation \( L_\phi/L = T_\phi/T \) [17] with \( L_\phi \) identified as the characteristic separation over which the phase fluctuates by 1 radian at the trap centre.

In order to extract values for \( T_\phi \) according to the relation described above, we take here the approach of systematically analysing the phase distributions of the ensemble of stochastic fields \( \psi \) at several distances from the trap centre. Within the Thomas-Fermi radius, we find these distributions to be well fitted with Gaussian functions, whose standard deviation increases with increasing distance from the trap centre due to the enhanced role of
V. CONCLUSIONS

An \textit{ab initio} model capable of capturing the phase coherence properties of highly-elongated, weakly-interacting Bose gases is of fundamental importance for future applications such as atom-interferometers and atom lasers. In this work we have analysed one such model, the stochastic Gross-Pitaevskii equation (SGPE), with novel and self-consistent modifications in order to take account of quasi-one-dimensional effects. Comparing to the experiment of Richard \textit{et al}., Ref. 22, which considered the regime of relatively strong phase fluctuations $T/T_\phi > 6$, we found excellent agreement between experiment and the SGPE theory, based solely on using the experimental parameters (i.e. trap configurations, atom number, temperatures) as inputs for the theory.

We further compared the SGPE theory to the findings of Hugbart \textit{et al}., Ref. 40 for the opposite low temperature regime $T \lesssim \text{few } T_\phi$, which is more challenging to probe in experiments. While the SGPE analysis gave a temperature dependence of the coherence length in quantitative agreement with the experimental trend (within error bars), undertaking a point-by-point analysis of the experimental data was found to span a slightly narrower range of $T/T_\phi$ than the experimentally-reported curve, indicating a systematic deviation. We argued that this discrepancy may arise as a result of the different identifications of $T_\phi$ between theory and experiment, possibly due to the different means of processing the ‘raw’ experimental data and stochastic numerical results, e.g. due to differences in extracting the quasicondensate atom number which then feeds into the expression for $T_\phi$.

We have partially examined this issue by using instead a phase sensitive means of extracting $T_\phi$ from characterisation of the ensemble phase distribution in SGPE simulations. In particular, motivated by footnote 47 of Ref. 40, we identified $T_\phi$ through the relation $T_\phi = T(L_\phi/R_{TF}(T))$ where $L_\phi$ was chosen as the characteristic separation from the trap centre at which the phase distribution is fitted by a Gaussian with a particular value for the standard deviation. In our treatment however this value is a free parameter chosen here so as to match the experimental range of $T/T_\phi$.

Finally, as the measurements performed in 40 rely on an interferometric technique, this opens up a way to indirectly ‘extract’ the Penrose-Onsager mode in a quasi-one-dimensional experiment solely from knowledge of density and the two lowest equal-time correlation functions, with the Penrose-Onsager condensate mode density emerging as the phase- and density-fluctuation suppressed part of the density via $n_{PO}(z) \approx n(z) \sqrt{2 - g^{(2)}(z) g^{(1)}(0, z)}$ 28, 74.

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Appendix A: Analysis of the phase distribution

The numerical procedure implementing the SGPE model makes use of the Condor system [99], which is a mechanism to run parallel simulations, on different machines. Using this method we generate a large number of stochastic realizations of the wavefunction $\psi$ (typically $\sim 1000$ in a very short time); in this way we can then extract physical observables by performing the average over the ensemble of stochastic trajectories. In each numerical realization, which is somewhat analogous to an individual experimental run, the phase of the wavefunction $\psi$ takes on a certain value at each position, leading to a distribution across the ensemble; it is then interesting to investigate how the distribution of the several realizations of the phase changes with the distance from the trap centre, at a definite value of $T/T_F$.

In Fig. 5 we report phase histograms of the stochastic field $\psi(z)$, locked to the phase of the Penrose-Onsager (condensate) mode $\phi_0$, at three values of the distance from the trap centre, for the experimental data point with $T \approx T_\phi$ (left point with error bar in Figs. 2-4). To be more specific, we plot the phase of $a_c^* \psi(z)/|a_c|$ where $a_c$ is the amplitude of the Penrose-Onsager mode, given by $a_c = \Delta z \sum_i \phi_c^*(z_i) \psi(z_i)$, where $\Delta z$ is the grid spacing (see [74, 100] for further details and implementation).

The broadness of the generated distributions is an indication of the amount of coherence at a specific spatial point in the system: we expect the distribution to become broader with increasing distance from the trap centre, and to be almost flat at the edge, where the system becomes purely thermal (Fig. 5). We define $L_\phi$ to be the distance at which the phase histogram is well fitted by a Gaussian with a particular value of the standard deviation $\sigma$. Following such a procedure, we can then define a phase fluctuation temperature $T_\phi$ from the relation $T_\phi \approx T(L_\phi/R_{TF}(T))$ [17, 24]. The standard deviation is here a free parameter, and we find that the value of $\sigma = 0.65 \pm 0.05$ provides very good matching to the experimental data, as shown in Fig. 4 (with corresponding range $z = L_\phi \sim (0.15 - 0.6) R_{TF}(T)$ for this chosen value of $\sigma$).

Figure 5. Distribution of the phase of $a_c^* \psi(z)/|a_c|$ (where $a_c$ is the amplitude of the Penrose-Onsager mode), for a chosen temperature $T \approx T_\phi$ (left point with error bars in Figs. 2-4), at different locations from the trap centre. Shown are the distributions at the condensate edge (left plot), at the point $z = L_\phi$ (central plot) where the distribution is fitted by a Gaussian (illustrated by the red solid curve) of standard deviation $\sigma = 0.65$ (which for the particular numerical point considered occurs at $z \approx 0.5 R_{TF}(T)$) and at the trap centre $z = 0$ (right plot). The distributions are centered and normalised, and the phase $\phi$ is scaled to $\pi$.

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Note that the thermal atoms above, which are treated statically here, are not expected to significantly affect the axial modes (due to the relatively lower density of these high energy atoms), thus justifying the approximate form of the interaction term in Eq. 1 (in which $n_{\perp}$ is omitted).

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[88] Alternatively such a graph may be generated by extracting $L$ from Fig. 3 of [24] and the remaining data from Fig. 3 of [37].

[89] Specifically, for the modified nonlinearity considered here, we obtain at the Thomas-Fermi radius, $R_{TF}(T)$, the condition $\mu - V(R_{TF}(T)) = \hbar \omega_{\perp} (\sqrt{1 + 8a_s n_{TF} - 1})$ (see also Sec. V B of Ref. [28]). Approximating locally $\rho \approx |\psi|^2 = n_{qC}$ in the SGPE generates a Thomas-Fermi radius which is also in excellent agreement with the independent modified Popov prediction.

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[98] The two expressions are reconciled through the relation $\eta_{TF}\sim (\eta_{TF})_{norm} = 1$ (see also Sec. V B of Ref. [28]). Approximating locally $\rho \approx |\psi|^2 = n_{qC}^\perp$ in the SGPE generates a Thomas-Fermi radius which is also in excellent agreement with the independent modified Popov prediction.

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