The Beginning of Chiral Symmetry

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Abstract
For readers interested in the history of chiral symmetry we present the translation of two papers from 1955 and 1959 from German to English, in which chiral symmetry properties for weak and strong interactions have been postulated and discussed.
Chiral symmetry is one of the fundamental properties of the theory of particle physics. It requires invariance of the Lagrangian under independent global transformations of left and right handed fermion fields. For a single particle generation, the standard model Lagrangian of particle physics, with its electroweak and strong gauge and Yukawa interactions, has this symmetry. This holds also in natural extensions of the standard model, the grand unified theories, such as \( SO(10) \) and \( E_6 \). For several generations the Yukawa interaction is no more invariant, but chiral symmetry can be restored, if a generation symmetry is built into the model. The fermion masses then arise from the spontaneous breaking of chiral and generation symmetry.

For some readers it may therefore be of interest to look at the very first suggestions of chiral symmetry published many years ago. For their convenience we translate here from German to English the two original papers dealing with chiral symmetry for weak and strong interactions, published in Zeitschrift für Physik in 1955 \(^1\) and 1959 \(^2\) respectively. The fact that they have been written in German may be the reason why they are less known than later papers on this subject. In the translation we tried to be as close as possible to the originals. In the first paper, chiral symmetry of the weak interaction Lagrangian was postulated in the form of a \( \gamma_5 \) symmetry. Because this paper was written before Lee and Yang suggested the existence of parity changing processes, the symmetry operation consisted of the simultaneous replacement of two fermion field operators by \( \gamma_5 \) times these fields. After the discovery of parity changing transitions, the extension of the symmetry by simply transforming each fermion field separately with \( \gamma_5 \) became possible. The corresponding invariance requirement led immediately to the unique and well-known chirality invariant form of the weak interaction.

The suggestion that chiral symmetry could also be a symmetry of the strong interaction, even though this interaction is parity conserving, was made in the second paper we translate here. To our knowledge it was the first paper proving that chiral invariance could also be a property of the strong interaction Lagrangian. A non-linear square root type meson-nucleon coupling served for this purpose. Today, one uses for the effective interaction with the pion field a more convenient expression with the meson field in the exponent. In both papers presented here chiral symmetry is valid for the interactions, but is still violated by the mass terms in the free part of the Lagrangian. It is a symmetry "as if the masses were zero". Only later this deficiency could be removed with an improved form of the effective Lagrangian for mesons and baryons and, for the basic theory, by generating the fermion masses using the Higgs mechanism.

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\(^1\) B. Stech and J. H. D. Jensen, Z. Phys. 141, 175 (1955)
\(^2\) G. Kramer, H. Rolnik and B. Stech, Z. Phys. 154, 564 (1959)
The coupling constants in the theory of $\beta$ decay\textsuperscript{1}.

By

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By only using data about $\beta$ decay (not $\mu$ meson decay) the restrictions in the choice of the coupling constants well secured by experiment are discussed. After that two “a priori” arguments will be described where one of them yields the alternative: $(STP)$ or $(VA)$. The other “a priori” argument determines the exact value of the coefficients, namely: $S - T + P$ and $V - A$ respectively.

Introduction.

The theory of $\beta$ decay is still suffering from the ambiguity of the possible ansätze for the interaction operator, even if one restricts oneself to a point interaction of the four fermions participating in the process. The most obvious ansatz would be a simple scalar one for the interaction operator

$$W = U + U^* \quad \text{with} \quad U = g \cdot S$$

(1)

where $S$ stands for the abbreviation

$$S = (\bar{\psi}_p \cdot \psi_n)(\bar{\phi}_e \cdot \phi_\nu).$$

(2)

Here $g$ is a constant measuring the coupling of the four fermion fields; $\bar{\psi} = \psi^\dagger \gamma_1$, where $\psi^\dagger$ is the hermitean conjugate spinor and $\psi_p, \psi_n, \psi_e, \psi_\nu$ are the field functions (operators) of proton, neutron, electron and neutrino, respectively. Such an ansatz is not sufficient to describe all $\beta$ processes since this interaction ansatz implies that the emerging proton is of the same spinor type as the disappearing neutron. Thus, the spin of the nucleon could not flip in the $\beta$ process. However, the experimental data reveal that $\beta$ processes with spin flip occur with approximately the same probability as those in which the nucleon keeps its spin orientation\textsuperscript{2}. Therefore it is necessary to take into account the possibility of such spin flip processes in the ansatz for the interaction. For allowed transitions and in a non-relativistic description of the nucleons, this happens in the generalization suggested by G\textsc{amow} and T\textsc{eller}:

$$U = g_P S + g_G G$$

(3)

with $S$ as above, and $G = (\bar{\psi}_p \sigma \psi_n)(\bar{\phi}_e \sigma \phi_\nu)$ where $\sigma$ is PAULI’s resp. DI\textsc{rac}’s spin vector operator.

A somewhat simpler description of this case is obtained using an ansatz for the interaction in which in each scalar spinor product a heavy and a light particle are grouped together, e.g. $$(\bar{\psi}_p \cdot \phi_\nu)(\bar{\phi}_e \cdot \psi_n).$$

(4)

\textsuperscript{1}To the 60\textsuperscript{th} birthday of Professor Dr. H. KOPPERMANN.

\textsuperscript{2}For a discussion of the experimental material see the report by K\textsc{onopinski} and L\textsc{anger} [Ann. Rev. of Nucl. Sci. 2, 261 (1953)] and the literature cited there, and also J. B. G\textsc{erhart}: Phys. Rev. 95, 288 (1954); B. M. R\textsc{ustad} and S. L. R\textsc{uby}: Phys. Rev. 89, 880 (1953); J. S. A\textsc{llen} and W. K. J\textsc{entschke}: Phys. Rev. 89, 902 (1953) and D. C. P\textsc{easlee}: Phys. Rev. 91, 1447 (1953).
Here the spin of the emerging nucleon is not coupled any more to the spin of the disappearing nucleon, and already a simple scalar ansatz of the type (4) contains both Fermi and Gamow-Teller transitions. We will come back to this later in §3. For the time being we shall use the traditional terminology.

For the general discussion including “forbidden” transitions it is necessary to treat also the nucleons relativistically since relativistic terms of the nucleons contribute to the “forbidden” transitions to the same order of magnitude as the finite ratio of the spatial extension of the nucleon fields to the wave lengths of the electron and neutrino states. The most general relativistic invariant ansatz for a point interaction is, cf. Konopinski, l.c. – (in the following we will write for the field functions of the particles simply the particle symbol: n for \( \psi_n \) and accordingly p for \( \psi_p \), e for \( \psi_e \), and \( \nu \) for \( \psi_\nu \)) — :

\[
U = g_S S + g_V V + g_T T + g_A A + g_P P
\]

(5)

with

\[
\begin{align*}
S &= \langle \bar{p}n \rangle \langle \bar{e}\nu \rangle \\
V &= \sum_\lambda \langle \bar{p}\gamma_\lambda n \rangle \langle \bar{\epsilon}\gamma_\lambda \nu \rangle \\
T &= \sum_{\lambda' < \lambda} \langle \bar{p}\gamma_\lambda' \gamma_\lambda n \rangle \langle \bar{\epsilon}\gamma_\lambda' \gamma_\lambda \nu \rangle \\
A &= \sum_{\lambda' < \lambda' < \lambda} \langle \bar{p}\gamma_\lambda' \gamma_\lambda' \gamma_\lambda n \rangle \langle \bar{\epsilon}\gamma_\lambda' \gamma_\lambda' \gamma_\lambda \nu \rangle \\
P &= \langle \bar{p}\gamma_5 n \rangle \langle \bar{\epsilon}\gamma_5 \nu \rangle
\end{align*}
\]

(5a)

Here \( \gamma_\lambda \) are the hermitean Dirac matrices with the commutation relations

\[
\gamma_\lambda \gamma_{\lambda'} + \gamma_{\lambda'} \gamma_\lambda = 2\delta_{\lambda\lambda'} \quad \text{for} \quad \lambda = 1, 2, 3, 4.
\]

For the product \( \gamma_1 \gamma_2 \gamma_3 \gamma_4 \) we write as usual \( \gamma_5 \). The single round brackets transform as a scalar in \( S \), as a four vector in \( V \), as an antisymmetric tensor in \( T \), as a pseudo vector in \( A \) and as a pseudo scalar in \( P \).

The fundamental problem of the theory of \( \beta \) decay is the finding of experimental rules or of theoretical arguments restricting the choice of the five coupling constants \( g_S, g_V, g_T, g_A, g_P \) and determining them unambiguously.

The reality of the coupling constants is guaranteed by the requirement that the interaction ansatz should be invariant by simultaneously rewriting all four fermions into their anti particles\(^3\).

\(^3\) Cf. e.g. De Groot, H. A., and S. R. Tolhoek Physica, Haag 16, 456 (1950). — Phys. Rev. 84, 150 (1951). — Biedenharn, L. C., and M. E. Rose: Phys. Rev. 83, 459 (1951).

\(^4\) Cf. also Pais and Jost: Phys. Rev. 87, 871 (1952).

\(^5\) Peaslee, l. c., cf. footnote 8 on the next page.

\(^6\) Cf. also E. J. Konopinski and H. M. Mahmoud: On the Universal Fermi Interaction, Phys. Rev. 92, 1045 (1953). We thank Mr. Konopinski for sending the manuscript before publication.
and $\mu$ meson decay which fixes the spectrum of the decay electrons of the $\mu$ meson – (the spectrum goes to zero at the maximum electron energy). But according to the most precise measurements\(^7\) the spectrum has a finite value at the energy boundary.

In the present article we like to summarize the consequences for the interaction ansatz resulting from experiments restricted to $\beta$ processes and to search for and to discuss the simplest “theoretical” principles which are in accordance with the empirical results and tighten them.

§1. A simple restriction for the coefficients in the interaction ansatz

The alternative statement with the best experimental confirmation reads, cf. Konopinski and Langer, 1. c.

*either* one has

\[ g_V = g_A = 0 \quad (7a) \]

*or*

\[ g_S = g_T = g_P = 0. \quad (7b) \]

In other words: there can either be a (STP) combination or a (VA) combination\(^8\). This statement is affirmed by precise measurements of electron and positron spectra of allowed and forbidden transitions\(^9\). Of course, the exact vanishing of the relevant groups of $g$ coefficients cannot be extracted from experiments. However, one would certainly come to a contradiction with the measured spectra if the absolute value of one of the $g$ factors having been set to zero would be bigger than $1/10$ of the value of the biggest $g$ coefficient of those having not been set to zero. This is a careful estimation, probably one can conclude that the corresponding $g$ values must be even closer to zero.

There exists a very simple theoretical argument that provides exactly the alternative expressed in (7). First it may be formulated quite formally:

One requires that the interaction (5) – except for a sign – will transform into itself if one performs simultaneously the replacement ($e$ and $\nu$ denote, as above, the field functions):

\[ e \rightarrow \gamma^5 e \quad \nu \rightarrow \gamma^5 \nu. \quad (9) \]

One finds from a quite elementary calculation that under the given replacement $V$ and $A$ transform into themselves while $S$, $T$ and $P$ change their sign:

\[
\begin{align*}
(\overline{\gamma^5 e} \gamma^5 \gamma^5 \nu) & = (\overline{\nu} \gamma_5 \nu) \quad \text{and accordingly for the terms in } A, \\
(\overline{\gamma^5 e} \gamma^5 \gamma^5 \nu) & = - (\overline{\nu} \gamma_5 \nu) \quad \text{and accordingly for the terms in } T \text{ and } P.
\end{align*}
\]

\(^7\)Bramson, H., A. Seifert and W. Heavens: Phys. Rev. 88, 304 (1952). – Sagane, R. and collaborators: Phys. Rev. 95, 863 (1954). – Harrison, F. B. and collaborators: Phys. Rev. 96, 1159 (1954). – Vilain, J. H. and W. Williams: Phys. Rev. 94, 1011 (1954).

\(^8\)The notation (AV) etc. means that at least one of the coefficients $g$ of the terms in brackets are not equal to zero while the $g$ factors of the terms not appearing in the brackets vanish.

\(^9\)This is equivalent with the absence of all so-called “Fierz terms” in the spectrum, Konopinski and Langer, 1. c.
The requirement of invariance under this transformation excludes therefore that \((VA)\) can appear together with \((STP)\) in the interaction operator.

With respect to the physical interpretation of the transformation (9) it may be pointed out that the replacement of \(\psi\) by \(\gamma_5\psi\) is equivalent to the replacement of \(m\) by \(-m\) in the Dirac equation, for from

\[
\left( \sum_\lambda \gamma_\lambda \frac{\partial}{\partial x_\lambda} + \frac{mc}{\hbar} \right) \psi = 0
\]  

follows

\[
\left( \sum_\lambda \gamma_\lambda \frac{\partial}{\partial x_\lambda} - \frac{mc}{\hbar} \right) \gamma_5 \psi = 0.
\]

If the electron would have, like the neutrino, a vanishing rest mass then, with \(e\) being a solution of the Dirac equation, \(\gamma_5e\) would be a solution as well. The finite value of the rest mass of the electron is probably irrelevant for the general form of the interaction operator. Thus it can make good sense to require: “The interaction operator should have the same general form as if the rest mass of the electron would be zero.” But then the invariance of the interaction operator under the transformation (9) is an almost compelling requirement.

One still could tighten this selection principle in the following sense. If one presumes that the \(\beta\) interaction operator has the same form as for four different fermions which all have a vanishing rest mass, then one could require the invariance of the interaction operator under the simultaneous replacement of every arbitrary pair of field functions \(\psi\) by \(\gamma_5\psi\); so e. g. also for \(e \rightarrow \gamma_5e\) and \(p \rightarrow \gamma_5p\). With this strict requirement the only combinations possible would be:

- either \(g_S = g_P = g_P = 0\); \(g_A = \pm g_V\), i.e.: \(V + A\) or \(V - A\)
- or \(g_V = g_A = 0\); \(g_P = + g_S\); \(g_T\) arbitrary, i.e.: \(S + \text{const } T + P\)
- or \(g_V = g_A = 0\); \(g_P = - g_S\); \(g_T = 0\), i.e.: \(S - P\).

We suppress the explicit proof that this selection follows from the invariance requirement mentioned, since it requires some calculations, and also because in the next paragraph we will discuss an invariance requirement more plausible to us having consequences which are in accord with the selections mentioned just now, but tighten this selection even further.

For the following we will only stress on the existence of the experimentally well established alternative (7) and the fact that one can obtain it from a very little incisive theoretical requirement.

\[\text{§2. Fixing the coefficients in the interaction ansatz}\]

The statement (7) is still very general since it states nothing about the relative magnitude of the coefficients in the combination \((STP)\) and in \((VA)\) respectively. Before coming to this point we will first quote additional experimental findings.

From the measurements of \(f \cdot t\) values of some specific allowed transitions it can be concluded, cf. especially 11 and 12, that \(|g_G/g_F|\) is close to 1. This

\[10\] the invariance of the interaction operator under the \(\gamma_5\) transformation of the electron and neutrino states has the consequence that all terms linear in the electron mass (“Fierz terms”) in the spectrum vanish. In Peaslee 1. c. a \(\gamma_5\) invariance for two undistinguishable neutrinos in \(\mu\) decay has been required. Only in this case the requirement of invariance including the sign suggests itself.

\[11\] Gerhart, J. B.: l.c.

\[12\] Kofoed-Hansen, O.: Phys. Rev. 92, 1075 (1953). — Staehelin, P.: Phys. Rev. 92, 1076 (1953).
implies for the case of a (STP) interaction that \(|g_T/g_S| \approx 1\) and for the case of a (VA) interaction \(|g_A/g_V| \approx 1\). On the other hand, the ratio \(g_P/g_S\) is within a large range undetermined.

Now we would like to point out that the special combinations

\[
\begin{align*}
\text{either} & \quad g_S = -g_T = g_P & \text{and} & \quad g_A = g_V = 0, \quad \text{i. e.:} & \quad S - T + P \\
\text{or} & \quad g_A = -g_V & \text{and} & \quad g_S = g_T = g_P = 0, \quad \text{i. e.:} & \quad A - V,
\end{align*}
\]

follow from a very simple isotropy requirement. From the known general formulae obtained for the angular correlation of \(\beta\)-decay\(^{13}\) it follows that for \(|g_S| = |g_T|\) and \(g_V = g_A = 0\) or for \(|g_V| = |g_A|\) and \(g_S = g_T = g_P = 0\) there is no angular correlation between electron and the neutrino in the decay of the free neutron\(^{14}\). This can be understood from a plausible general isotropy requirement, if one describes the \(\beta\) decay, as usual, by the annihilation of two particles (neutron and neutrino) and the creation of two others (proton and electron)\(^{15}\). In a reference frame, in which the neutron and the vanishing neutrino (which is in a state of negative energy) have opposite momenta and thus total momentum zero, no direction is preferred and the following requirement seems plausible: “In this reference frame the distribution of the direction of the emitted protons should be isotropic”. Since the electrons have the opposite momenta of the protons in this reference frame, also the distribution of the directions of the electrons will then be isotropic.

§3. Consequences of the isotropy requirement

One can overlook the consequences of this isotropy requirement best in a representation of the interaction operator differing from the usual one, namely, in which in analogy with the expression (4), each nucleon is coupled with one of the light particles. In this case the two annihilated, and the two created fermions as well, can be combined to form Lorentz covariant expressions. For this purpose it is necessary to introduce the so-called conjugate spinors, cf. e. g. Pais and Jost 1. c. These are defined as follows: To every spinor \(\psi\) a conjugate spinor \(\psi^C\) is defined by the following relation:

\[
\psi^C = C\tilde{\psi}^T. \tag{11}
\]

The meaning of \(\tilde{\psi}\) is given above near eq. (11), \(T\) shall indicate the transposition, i. e. the interchange of columns and rows. The unitary transformation matrix \(C\) is fixed by the following properties\(^{16}\)

\(^{13}\)Cf. e. g. GROOT, S. R. DE, and H. A. TOLHOEK: 1.c.

\(^{14}\)The angular correlation in non-relativistic approximation is proportional to

\[
\frac{1}{3}[\bar{\rho}\sigma n]^2(g_T^2 - g_A^2) - |(\bar{\rho}n)|^2(g_S^2 - g_V^2);
\]

but for the decay of the free neutron and the square of the Gamow-Teller matrix element \((\bar{\rho}n)\) is three times the square of the Fermi matrix element \((\bar{\rho}n)\).

\(^{15}\)In the representation \(n \rightarrow p + e + \nu^C\), \(\nu^C\) denotes an “antineutrino”, cf. also MAHMoud and KOnopinski 1.c.

\(^{16}\)In the Dirac representation of the \(\gamma\) s the matrix \(C\) has, up to an arbitrary factor \(\pm \sqrt{\frac{\alpha}{3}}\), the form

\[
C = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}
\]
\[ C^c C = 1, \quad C^T C^{-1} = -1, \quad C^{-1} \gamma_\lambda C = -\gamma^T_\lambda \]

\( \psi^c \) and \( \psi \) transform under Lorentz transformations in the same way; they satisfy the Dirac equation for free particles but differ by having eigenvalues with opposite signs for energy, momentum and spin components.

With this notation the five invariants \( S, V, T, A, P \) of eq. (5a) are linear combinations of the five new invariants

\[
\begin{align*}
\tilde{S} &= \sum (\nu C_n)(\bar{p} e^C) \\
\tilde{V} &= \sum (\nu C \gamma_\lambda n)(\bar{p} \gamma_\lambda e^C) \\
\tilde{T} &= \sum (\nu C \gamma^\lambda \gamma_\lambda n)(\bar{p} \gamma^\lambda \gamma_\lambda e^C) \\
\tilde{A} &= \sum (\nu C \gamma^5 \gamma_\lambda n)(\bar{p} \gamma^5 \gamma_\lambda e^C) \\
\tilde{P} &= (\nu C \gamma^5 n)(\bar{p} \gamma^5 e^C)
\end{align*}
\]

The linear relation between the invariants (5a) and (12) is\[17\]:

\[
\begin{align*}
-4 \tilde{S} &= (S - T + P) - (V - A) \\
-2 \tilde{V} &= 2(S - P) + (V + A) \\
-2 \tilde{T} &= (3S + T + 3P) \\
-2 \tilde{A} &= 2(S - P) - (V + A) \\
-4 \tilde{P} &= (S - T + P) + (V - A)
\end{align*}
\]

The isotropy requirement discussed at the end of §2 now says that the interaction potential

\[ U = a_S \tilde{S} + a_V \tilde{V} + a_T \tilde{T} + a_A \tilde{A} + a_P \tilde{P} \quad (14) \]

– applied to a process in which neutron and neutrino have opposite momenta – is invariant under an arbitrary rotation of the electron and proton directions.

Thus, the replacement

\[ \bar{p} \rightarrow \bar{p} \Lambda^{-1}, \quad e^C \rightarrow \Lambda e^C, \quad (15) \]

where \( \Lambda \) denotes the transformation matrix\[18\] acting on the spinor components should leave \( U \) unchanged. (The momentum conservation insures that the exponential functions of the plane waves do not have to be considered). Because of the known transformation properties of the vector, tensor etc. combinations the invariance holds only for

\[ a_V = a_T = a_A = 0 \quad (16) \]

e.i. only the “scalar” and “pseudoscalar” expressions \( \tilde{S} \) and \( \tilde{P} \) can contribute to the interaction. From \[15\] one has now the strict requirement that \( U \) can

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\[17\]A similar transformation has been performed by M. Fierz \[Z. Physik 104, 553 (1937)\]. Cf. also Caianiello, E. R.: Nuovo Cim. 8, 534, 749 (1941); 9, 336 (1952), and Michel, L.: Proc. Phys. Soc. A 63, 514 (1950).

\[18\]See e.g.: Pauli, W.: Handbuch der Physik XXIV, 1, p. 221. If the rotation is given by the Lorentz transformation of the space coordinates \( x'_\lambda = \sum \alpha \mu x_\mu \), then the corresponding transformation matrix \( \Lambda \) of the spinors is determined by \( \Lambda^{-1} \gamma_\lambda \Lambda = \sum \alpha \mu \gamma_\mu \).
only be a linear combination of \(-2(\tilde{S} + \tilde{P}) = S - T + P\) and \(2(\tilde{S} - \tilde{P}) = V - A\). Taking the alternative (7) into account then

\[
\begin{align*}
\text{either} & \quad U = -g(S - T + P) = 2g(\tilde{S} + \tilde{P}) \quad (17a) \\
onumber \\
\text{or} & \quad U = -g(V - A) = 2g(\tilde{S} - \tilde{P}) \quad (17b)
\end{align*}
\]

are the only possible forms for the interaction.

It shall be pointed out, that the alternative (7) is only needed in the weak form \(g_S g_V = g_T g_A = 0\) (i.e. that \(V\) cannot appear together with \(S\) and that \(A\) cannot appear together with \(T\)). This, however, is well established experimentally, because it can already be concluded from the precisely measured spectra of allowed transitions.

In the formulation of the isotropy principle a reference system was used, in which under rotations only the spinor components are transformed. Therefore, one can interpret the “isotropy” as a decoupling of the spin orientations of the created particles from the spin orientation of the annihilated particles. This point of view can also be formulated quantitatively: For free particles the Hamilton operator commutes with \((\vec{\sigma} \cdot \vec{p})\) and thus commutes for particles with a fixed momentum direction with \((\vec{\sigma} \cdot \vec{e})\), if the unit vector \(\vec{e}\) points in the momentum direction. This operator is the operator of an infinitesimal spin rotation around the momentum vector. If one now requires the invariance of the interaction operator \(U\) under such spin rotations of the created particles, keeping the spins of the annihilated particles fixed,

\[
\vec{p} \rightarrow \vec{p}(\vec{\sigma}\vec{e}) \quad \text{and} \quad \vec{e} \rightarrow -(\vec{\sigma}\vec{e})\vec{e}, \quad \text{i.e.} \quad e^C \rightarrow (\vec{\sigma}\vec{e})e^C, \quad (18)
\]

then only such relativistic invariants can contribute to the interaction ansatz \([14]\), which contains \(\gamma\) combinations commuting with \((\vec{\sigma}\vec{e})\). Commuting with \((\vec{\sigma}\vec{e})\), however, are only 1 and \(\gamma_5\). Also in this way one finds the result \(16\). The latter leads in non-relativistic approximation (for the heavy particles) to \(|g_V| = |g_G|\) and reveals here again and clearly the decoupling of neutron and proton spin: spin flip and non-spin flip processes have the same probability.

The result \(16\) \((a_V = a_T = a_A = 0)\) which has been obtained with the help of the isotropy requirement is also in accord with the requirement of invariance (up to the sign) of the part of the interaction operator containing the created (or the annihilated) particles alone under the particle-antiparticle conjugation \(21\). One finds using the definition \(22\)

\[
\begin{align*}
P^C(\psi_1 \Omega \psi_2^C) &= (\psi_2^C \Omega \psi_1^C) \\
P^C(\vec{p} e^C) &= -(\vec{p} e^C) \\
P^C(\vec{p}_\gamma e^C) &= -(\vec{p}_\gamma e^C)
\end{align*}
\]

However, since the corresponding requirements as well as the one for time reversal separately for created and annihilated particles seems not very plausible, we dispense from deriving our result \(16\) from such conditions.

**Note added in proof.** Both interaction ansätze \((17a) (S - T + P)\) and \((17b) (A - V)\) are antisymmetric by interchanging either the created particles among

\[19\] Cf. Konopinski and Langer: 1. c.
\[20\] \(\vec{p} = \frac{\hbar}{2}\vec{V}\); in the representation of the states by plane waves the operator \(\vec{p}\) can be replaced by its eigen value.
\[21\] For an earlier application of the particle-antiparticle conjugation of two particles in \(\beta\) decay that, however, has led to results not compatible with the experimental results, see S. R. De Groot and H. A. Tolhoek, 1. c.
\[22\] Cf. e. g.: Peaslee, D. C.: 1. c. and Pais and Jost, 1. c.
themselves or the annihilated particles among themselves as a simple calculation shows, cf. also Peaslee 1. c., Caianiello 1. c. and Pursey\textsuperscript{23}. This restricted part of the general antisymmetry requirement discussed by Critchfield and Wigner\textsuperscript{24} is therefore compatible with our isotropy requirement discussed above. The isotropy requirement, however, is more restrictive. The requirement of antisymmetry under the interchange of the created or the disappearing particles would also allow the combination\textsuperscript{25} $S - A - P$ which is not compatible with the isotropy requirement and the experimental findings.

The antisymmetry requirement for the “created particles” together with the invariance under the substitution \textsuperscript{26} is, however, equivalent with the isotropy requirement together with the invariance postulate \textsuperscript{26}.

\section*{§4. The $S - T + P$ Interaction}

A theoretical decision in favor of or against one of the two possibilities presented in (17), differing in the new representation only by a sign, has not yet been accomplished. Thus, at this place only a discussion of the experimental results is possible. Especially important to us are the recoil measurements in $\beta$ decay of He\textsuperscript{6} that have been performed by Rustard and Ruby, 1. c. and by Allen and Jentschke, 1. c. These measurements show that $T$ is an essential part of the $\beta$ interaction operator. They exclude the combination (17b). This finding is in agreement with earlier evidences by Nordheim\textsuperscript{26} and Peaslee, 1. c. that $P$ is likely a part of the interaction.

If one accepts this alternative and the theoretical selection principles put forward, then $\beta$ decay is a $S - T + P$ interaction which, in a new notation, takes the simple form

$$U = 2g (\tilde{S} + \tilde{P}) = 2g \{ (\nu^C n)(\gamma^C e^C) + \{ (\nu^C \gamma_5 n)(\mathbf{p} \gamma^C e^C) \}. \quad (19)$$

One can see that the tensor product $T$ appears in the interaction expression quasi only by an “inapt choice of coordinates”. To summarize: by taking “created” and “disappearing” particles separately in one spinor product, only scalar and pseudoscalar “interactions” appear. Apart from the theoretical aesthetic point of simplicity recommending the ansatz \textsuperscript{19}, the usage of this simple form means also a calculational simplification in quantitative computations of complicated – e.g. multiply forbidden – transitions, of the exact consideration of Coulomb corrections etc. We hope to be able to present this in a later article.

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$\gamma_5$-Invariance and Parity Conservation in Strong Interactions

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Abstract

In this paper it is shown, that it is possible to construct parity conserving baryon-pion interactions, which are $\gamma_5$ invariant in exactly the same manner as electromagnetic and weak interactions.

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In previous publications [1, 2] the possibility was pointed out to construct couplings between baryons and π-mesons, which are invariant against space reflections and are in addition explicitly $\gamma_5$-invariant in the same way as the weak and electromagnetic interactions. Here we understood as $\gamma_5$ invariance the invariance of the interaction operator under the substitution $\psi_K \rightarrow \gamma_5 \psi_K$ for any spin particle $K$, which appears in the coupling. For example, neutron and proton are considered as different particles. Just this kind of $\gamma_5$-invariance seems to be present in the weak interactions [3, 4, 5, 6]. The proof in [1, 2] for the space-reflection invariance of certain $\gamma_5$ invariant π-meson-baryon couplings is not correct. In this paper a rigorous proof shall be given and the conditions explained, under which an explicitly $\gamma_5$-invariant π-meson-baryon interaction is simultaneously parity-invariant.

At first, for simplicity we consider only the π-meson-nucleon interaction. The $\gamma_5$-invariant interaction part of the Lagrange density is of the following form

$$L_W = \frac{f}{m} \bar{\psi} \gamma_\mu (1 + \gamma_5) \vec{T} \psi \frac{\partial}{\partial x_\mu} \vec{\Phi}. \quad (1)$$

In this equation $\vec{T}$ stands for an isospin-vector matrix, which in general depends still on the π-meson field. Now, we try to determine this isospin matrix $\vec{T}$ such, that the coupling (1) is space-reflection invariant. Since the reflection invariance of a $\gamma_5$-invariant form of coupling, as it is present in (1), is not easy to see, transformations of the field operators should be performed. If we succeed to determine $\vec{T}$ in such a way, that a transformation of the field operators can be given, which on one side leads to an obvious parity invariant coupling and on the other side does not change the physical consequences of the theory (S-matrix), then we have reached our goal.

A simple example is the coupling of neutral mesons, where $T$ naturally has only one component. In this case (1) is already with $T = 1$ parity invariant. Here, the simple substitution (directly in the Lagrange-function or in the field equations)

$$\psi(x) = c \cdot e^{i \frac{f}{m} \vec{\Phi}(x)} \psi'(x), \quad \vec{\Phi}(x) = \vec{\Phi}'(x) \quad (2)$$

leads to field equations in $\psi'(x)$ and $\vec{\Phi}'(x)$ with a pure axial coupling [7], $c$ is a renormalization constant (see Appendix). The possibility, to eliminate the vector current $\gamma_\mu \psi$ from the field equations, follows from the vanishing of the divergence of this current. Whereas in the $\gamma_5$-invariant form of the Lagrange density and in the equations of motion the parity operator has a complicated form, for the primed field operators the usual form can be used and the reflection invariance can be seen directly. The physical equivalence of the transformed and untransformed field equations follows from the fact, that the ingoing and outgoing asymptotic fields of $\psi'(x)$, $\Phi'(x)$ agree with the corresponding fields of $\psi(x)$, $\vec{\Phi}(x)$ and therefore lead to the same S-matrix. The proof of this assertion - also for the following more general case - will be given in the appendix.

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1 We thank Dr. Symanzik and Dr. W. Theis for a hint in this respect
In the symmetrical meson theory the situation is more complicated, because of the presence of the non-commuting isospin matrices. In this case we use instead of (2) the following substitution \(^2\)

\[ \psi(x) = c \cdot U \psi'(x), \quad \Phi(x) = \Phi'(x). \quad (3) \]

Here \(U\) should represent a matrix invariant under rotations in isospace of the following form

\[ U = \frac{1 + i \vec{\tau} \vec{w}}{\sqrt{1 + \vec{\Phi}^2 w^2}}, \quad (4) \]

where for the present \(w\) is an arbitrary function of \(\vec{\Phi}^2\) and \(\vec{\tau}\) denote the usual isospin matrices. In order that the ingoing and outgoing fields for \(\psi'(x)\) and \(\Phi'(x)\) agree with those for \(\psi(x)\) and \(\Phi(x)\), we require that \(U\) allows a power series expansion in \(f/m\).

Performing the substitution (3) in the Lagrange operator or in the field equations the following new coupling arises:

\[ L'_W = |c|^2 \bar{\psi} i \gamma_\mu \left\{ i U^\dagger \frac{\partial}{\partial \Phi'} U + \frac{f}{m} U^\dagger \vec{T} U (1 + \gamma_5) \right\} \psi' \frac{\partial}{\partial x^\mu} \vec{\Phi}' \quad (5) \]

The first term of this expression results from the interaction-free part of the Lagrange function.

Eq.(5) is certainly a reflection-invariant coupling, if the \(\gamma_5\)-free term contains even and the term multiplied with \(\gamma_5\) contains odd powers of the field \(\Phi'(x)\). (This requirement implies that the meson field \(\Phi'(x)\) is a pseudoscalar field.) Choosing in Eq.(1)

\[ \vec{T} = \frac{1}{2} \frac{1}{1 + (f/m)^2 \vec{\Phi}^2} \left( \vec{\tau} + \frac{f}{m} [\vec{\Phi} \times \vec{\tau}] \right) \quad (6) \]

the requirements are fulfilled, if for the transformation (3) and (4) the function

\[ w(\vec{\Phi}^2) = \frac{f/m}{1 + \sqrt{1 + (f/m)^2 \vec{\Phi}^2}} \quad (7) \]

is used. \(w(\vec{\Phi}^2)\) satisfies the requirement on the transformation \(U\) made in connection with the asymptotic condition, namely the possibility of an expansion of it in powers of \(f/m\).

Eq.(1) with (6) can be written in the following form:

\[ L_W = \frac{f}{m} (j^\mu + j^A_\mu) \left( \frac{\partial}{\partial x^\mu} \vec{\Phi} - \frac{f^2}{m^2} \frac{\vec{\Phi}}{1 + (f/m)^2 \vec{\Phi}^2} (\vec{\Phi} \cdot \frac{\partial}{\partial x^\mu} \vec{\Phi}) \right). \quad (8) \]

In this equation \(j^\mu\) denotes the total isovector current following from (1) with (6), which obeys a continuity equation, whereas \(j^A_\mu\) stands for the axial vector current

\[ j^A_\mu = \bar{\psi} i \gamma_\mu \gamma_5 \vec{\tau} \psi. \quad (9) \]

\(^2\)In a somewhat different formulation the transformation (3) has been used several times for the proof of equivalence theorems in the meson theory [8, 9, 10, 11].
This kind of presentation of $L_W$ corresponds to the form aimed at in [1, 2]. The equation (4) in [2] differs from (8) in the last term, which however is necessary for the space-reflection invariance.

Written in the transformed field the coupling has the following form, because of $L(\psi) = L'(\psi')$:

\[
L'_W = |c|^2 \left\{ \frac{\Gamma}{m} \bar{\psi}' i \gamma_\mu \gamma_5 \bar{\psi}' \left( \frac{\partial}{\partial x_\mu} \bar{\Phi}' - \frac{f^2}{\sqrt{2} (1 + a^2)} \bar{\Phi}' \left( \bar{\Phi}' \frac{\partial}{\partial x_\mu} \bar{\Phi}' \right) \right) \right\} - \left( \frac{f}{m} \right)^2 \bar{\psi}' i \gamma_\mu \gamma_5 \bar{\psi}' \left[ \bar{\Phi}' \times \frac{\partial}{\partial x_\mu} \bar{\Phi}' \right],
\]

where

\[
a = \sqrt{1 + \left( \frac{f}{m} \right)^2 \bar{\Phi}'^2}
\]

In Eq.(10) the reflection invariance is obvious, whereas the $\gamma_5$- invariance for every particle, in contrast to (1), does not appear explicitly anymore.

Of course, the form of Eq.(10) for $L'_W$ is only an example and is determined by the specially chosen ansatz for $\bar{T}$ in Eq.(6). This $\bar{T}$ seems to be the simplest one, which makes the coupling $\gamma_5$-invariant and reflection invariant simultaneously.

The coupling to the electromagnetic field produces no new difficulties. It turns out that the gauge invariant coupling of the photon to the Lagrange function containing the interaction Eq.(1) leads to the same result as the explicitly reflection-invariant coupling to the Lagrange function with the interaction Eq.(5). This is due to the rotation invariance of the matrix $U$ in isospace, which leads to the following relation:

\[
U^\dagger \frac{\vec{\gamma}}{2} U = \frac{1}{2} \vec{\gamma} + i U^\dagger \left( \vec{\Phi} \times \frac{\partial U}{\partial \vec{\Phi}} \right).
\]

It is suggestive to require the simultaneous parity and $\gamma_5$- invariance also for all $\pi$-meson-baryon couplings. As long as one can neglect the mass difference of the $\Lambda$- and $\Sigma$ particle, the pairs $\Sigma^+$, $\sqrt{2}(\Lambda - \Sigma^0)$ and $\sqrt{2}(\Lambda + \Sigma^0)$, $\Sigma^-$ can be introduced [12]. Then the arguments given above for the $\pi$-meson-nucleon interaction can be carried over to all isospin doublets literally.

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**Appendix**

In this appendix the asymptotic equality of the fields $\psi(x)$ and $\psi'(x)$ shall be shown. We will require, that the function $U$ from Eqs.(3) and (4) can be inverted and that the field $\psi'(x) = (cU)^{-1} \psi(x)$ can be expanded into powers of

\[3\text{Because of the required } \gamma_5 \text{ invariance the electromagnetic coupling is not allowed to contain Pauli terms [1, 2].}\]
The adiabatic hypothesis \((f/m \to 0 \text{ for } t \to \pm \infty)\), used in the past, lets it appear being plausible that the two fields \(\psi'(x)\) and \(\psi(x)\) agree in the limit of large time.

To proceed more rigorously, the results of the work of Zimmermann \[13\] can be used. For this purpose we define ingoing and outgoing fields by the relation

\[
\begin{align*}
\psi_{\text{in/out}}(x) &= \psi(x) + \int S_{\text{ret/adv}}(x - x') (\gamma_\mu \frac{\partial}{\partial x_\mu} + M) \psi(x') d^4x' \\
\psi'_{\text{in/out}}(x) &= \psi'(x) + \int S_{\text{ret/adv}}(x - x') (\gamma_\mu \frac{\partial}{\partial x_\mu} + M) \psi'(x') d^4x'.
\end{align*}
\]  

(A1)

The proof of the equality of the two asymptotic fields is accomplished, if the two following points can be proven:

I. A one-nucleon state \(\Psi_\alpha\) of the Hilbert space can be constructed with the help of the field \(\psi_{\text{in/out}}'(x)\) as well as with the help of the field \(\psi_{\text{in/out}}(x)\) from the vacuum \(\Omega\):

\[
\Psi_\alpha = \int \psi_{\text{in/out}}^*(x) f_\alpha(x) d^3x |\Omega\rangle = \int \psi_{\text{in/out}}'(x) f_\alpha(x) d^3x |\Omega\rangle
\]  

(A2)

In these expressions \(f_\alpha\) is a solution of the free Dirac equation.

II. The field \(\psi_{\text{in/out}}'(x)\) possesses with the fields \(\psi_{\text{in/out}}(x)\) and \(\Phi_{\text{in/out}}(x)\) the same commutation relations as the field \(\psi_{\text{in/out}}(x)\) has with these field operators.

From points I and II it follows for all Hilbert-space states \(\Psi_1\) and \(\Psi_2\):

\[
\int f_\alpha^*(x) \langle \Psi_1 | \psi_{\text{in/out}}'(x) | \Psi_2 \rangle d^3x = \int f_\alpha^*(x) \langle \Psi_1 | \psi_{\text{in/out}}(x) | \Psi_2 \rangle d^3x
\]

which shows the complete equivalence of the two asymptotic fields.

Point I can be obtained from the Lorentz invariance of the field operators \(\psi(x)\) and \(\psi'(x)\). Denoting by \(\Psi_p\) a one-nucleon state with momentum \(p\) and \(-p^2 = M^2\), it is

\[
\langle \Omega | \psi_{\text{in}}(x) | \Psi_p \rangle = \frac{1}{(2\pi)^{\frac{3}{2}}} e^{ipx} u_p
\]  

(A3)

and

\[
\langle \Omega | \psi_{\text{in}}'(x) | \Psi_p \rangle = \langle \Omega | \psi'(x) | \Psi_p \rangle = \frac{1}{(2\pi)^{\frac{3}{2}}} e^{ipx} u_p.
\]  

(A4)

In order that on the right side of Eq.(A.4) the plane wave has the same factor as in Eq.(A.3) the renormalization constant \(c\) from Eq.(4), respectively Eq.(2), must obey the equation

\[
c = (2\pi)^{\frac{3}{2}} u_p \langle \Omega | U^{-1}(0) \psi(0) | \Psi_p \rangle.
\]  

(A5)

From the fact, that according to (A.1) the ingoing and outgoing fields obey the free Dirac equation, it moreover follows

\[
\langle \Omega | \psi_{\text{in/out}}'(x) | \Psi \rangle = \langle \Omega | \psi_{\text{in/out}}(x) | \Psi \rangle = 0
\]  

(A6)
for all states of the Hilbert space with $-p^2 \neq M^2$. From (A.3), (A.4) and (A.6) the statement I follows.

Furthermore (A.3), (A.4) and (A.6), together with the corresponding relations for the field $\Phi_{in/out}(x)$, allow the calculation of the vacuum expectation values of the wanted commutation relations \[13\]. Therefore the latter ones are identical for $\psi'_{in/out}(x)$ and for $\psi_{in/out}(x)$ fields. It is more difficult to prove the full statement II. In order that the equality of the commutation relations is valid in general and not only for the vacuum expectation values, it is necessary to show, that all commutation relations are c numbers. Expanding $\psi'(x)$ in a power series in $f/m$ it seems possible to perform such a proof in analogy to Zimmermann \[13\] for every approximation in $f/m$, if the appearing operator products are suitably defined.

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