Observational implications of precessing protostellar discs and jets

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ABSTRACT

We consider the dynamics of a protostellar disc in a binary system where the disc is misaligned with the orbital plane of the binary, with the aim of determining the observational consequences for such systems. The disc wobbles with a period approximately equal to half the binary’s orbital period and precesses on a longer timescale. We determine the characteristic timescale for realignment of the disc with the orbital plane due to dissipation. If the dissipation is determined by a simple isotropic viscosity then we find, in line with previous studies, that the alignment timescale is of order the viscous evolution timescale. However, for typical protostellar disc parameters, if the disc tilt exceeds the opening angle of the disc, then tidally induced shearing within the disc is transonic. In general, hydrodynamic instabilities associated with the internally driven shear result in extra dissipation which is expected to drastically reduce the alignment timescale. For large disc tilts the alignment timescale is then comparable to the precession timescale, while for smaller tilt angles δ, the alignment timescale varies as (sin δ)−1. We discuss the consequences of the wobbling, precession and rapid realignment for observations of protostellar jets and the implications for binary star formation mechanisms.

Key words: accretion, accretion discs – binaries: general – ISM: jets and outflows – stars: formation – stars: pre-main-sequence

1 INTRODUCTION

Recent observations of young stellar objects (YSOs) provide evidence for the existence of binary systems with circumstellar discs that are misaligned with the binary’s orbital plane. The Hubble Space Telescope (HST) and adaptive optics images of the HK Tau pre-main-sequence system provide the most striking and direct evidence, assuming that it is indeed a binary system (Stapelfeldt et al. 1998; Koresko 1998). In addition, there are several examples of pre-main-sequence binaries or unresolved YSOs from which two protostellar jets are seen to emanate in different directions (e.g. Davis, Mundt & Eisloffel 1994). It is assumed that these twin jets originate from binary systems with two circumstellar discs that are misaligned with each other. Thus, one or both of the discs must be misaligned with the binary’s orbital plane. Furthermore, observations of solar-type main-sequence binary systems indicate that the stellar rotational equatorial planes in wide binaries (≥ 40 a.u.) are frequently misaligned with the orbital plane, while for closer systems there is a tendency for alignment (Hale 1994). Assuming the equatorial plane of a star is determined by the plane of its original circumstellar disc, this indicates that circumstellar discs are frequently misaligned with the orbital plane in wide binaries.

In binary systems with a circumstellar disc whose plane is misaligned with the orbital plane, the circumstellar disc is expected to precess due to the tidal interaction of the companion (e.g. Papaloizou & Terquem 1995). Disc precession has been used to explain long-period variations in the light curves of a number of X-ray emitting binary systems (Gerend & Boynton 1976; Katz et al. 1982; Wijers & Pringle 1999). For YSOs, the major application of disc precession has been to predict the precession of protostellar jets, since many Class 0 and I objects are observed to emit jets (Bontemps et al. 1996). Several hydrodynamic investigations into the appearance of precessing jets have been...
performed (Raga, Cantó & Biro 1993; Biro, Raga & Cantó 1995; Cliffe et al. 1996; Völker et al. 1999) and precession has been used to explain the changes in the flow directions of several jets (Eislöffel et al. 1996; Davis et al. 1997; Mundt & Eislöffel 1998), although Eislöffel & Mundt (1997) stress that alternative explanations exist. Recently, Terquem et al. (1999) discussed the orbital periods that are required to give visibly precessing jets and the expected precession periods for systems that are observed to have misaligned jets.

In this paper, we present a simple review of the behaviour of a circumstellar disc which is misaligned with the orbital plane in a binary system, and discuss the implications of this behaviour for observations of discs and jets in binary protostellar systems. In Section 2, along with simple precession, we also consider the effect of the oscillating torque produced by the companion, which could cause the disc and jet to 'wobble' with a period of half the orbital period. In Section 3, we consider the effect of dissipation in the disc and calculate the timescale for a disc to realign itself with the binary's orbital plane due to dissipation. The general results of this study, along with the expected observational consequences and the implications for models of binary star formation are discussed in Section 4. Finally, we give our conclusions in Section 5. The reader more interested in the results and conclusions of the paper than the details of the analysis may care to move directly to Section 4 from this point.

2 PRECESSION INDUCED BY THE BINARY COMPANION

We consider here in simple terms the effect of a binary companion (the secondary) on a gaseous disc around the primary, whose plane is not aligned with the plane of the binary orbit.

2.1 Ring precession

We first consider the effect of the secondary on a circular ring of material orbiting the primary at radius $a$. We consider the binary star system with component masses $M_p$ (primary) and $M_s$ (secondary), with a circular orbit, and with stellar separation $D$. Now also consider a ring of material, of negligible mass, $m_r$, in orbit about the primary star. The ring has mean radius $a < D$, and is tilted to the plane of the binary orbit at an angle $\delta$. We can regard the effect of the secondary star on the dynamics of the ring as a perturbation. We use a set of non-rotating coordinates centred on the primary star, with the OZ axis parallel to the rotation axis of the binary. In these coordinates, the acceleration caused by the secondary star at position vector $r$ is given by

$$\mathbf{F}_s = -\left(\frac{GM_s}{D^3}\right) r + \left(3\frac{GM_s}{D^3}\right) (r \cdot \mathbf{D}) \mathbf{D},$$

where $\mathbf{D}$ is the position vector of the secondary relative to the primary. We should remark that the fictitious force arising from the acceleration of the coordinate system has been included, cancelling the $(GM_s/D^3) \mathbf{D}$ term, and that terms of relative order $(a/D)$ have been neglected. Here the first term simply represents an augmentation of the effect of the gravity of the primary due to the presence of the secondary. It affects the centrifugal balance of the ring, but otherwise has no effect on its dynamics. The second term is the tidal term and represents a force in a direction parallel (or anti-parallel) to the instantaneous vector joining the two stars, and with magnitude proportional to the distance of the position $r$ from the plane passing through the primary perpendicular to the line joining the two stars. From a physical point of view, this term can be thought of as comprising two components. The first is a force which acts similarly to a centrifugal force in that it is everywhere directed perpendicular to and away from the OZ axis, and in that its magnitude is proportional to the distance from that axis. In terms of azimuthal Fourier components $\exp(i \phi)$, where $\phi$ is the azimuth about the OZ axis, this term is derived from an axially symmetric potential, and thus has $m = 0$. The second is similar except that the magnitude of the force is proportional to distance from the OZ axis multiplied by $\cos(2\phi)$, and thus has $m = 2$.

2.1.1 The $m = 0$ term

Breaking the tidal term into these two parts enables us to consider their effects on the dynamics of the ring in a more straightforward manner. Suppose that the coordinates of the secondary are given by

$$\mathbf{D} = (-D \sin(\Omega t), D \cos(\Omega t), 0),$$

where $2\pi/\Omega_b$ is the binary period, and thus

$$\Omega_b = (G(M_s + M_p)/D^3)^{1/2}.$$ (3)

Suppose that the ring is tilted in our system of coordinates about the OX axis. Thus the axis of the ring is given by

$$\mathbf{k} = (0, -\sin \delta, \cos \delta).$$ (4)

Then the effect of the $m = 0$ part of the force is to exert a torque $\mathbf{G}_0 = (G_{0x}, 0, 0)$ on the ring about the negative OX axis, of magnitude

$$G_{0x} = -\frac{3}{4} (GM_s m_r a^2 / D^3) \sin \delta \cos \delta,$$ (5)

where $m_r$ is the mass of the ring. The direction of the $m = 0$ torque, $\mathbf{G}_0$, lies along the line of nodes of the ring (the line along which the ring intersects the OXY-plane). If $\Omega = \Omega_k$ is the angular velocity of the ring, then $\mathbf{G}_0 \cdot \Omega = 0$. Thus the effect of the torque on the ring is to cause the plane of the ring to precess retrogradely about the OZ axis with a precession rate given by $-\omega_p$, where

$$\omega_p = |G_{0x}|/(\sin \delta m_r a^2 \Omega).$$ (6)

Note that $\Omega$ is given by

$$\Omega = (GM_p/a^3)^{1/2},$$ (7)

and $m_r a^2 \Omega$ represents the angular momentum of the ring.

Thus we find that the mean precession rate is

$$\omega_p = \frac{3}{4} \cos \delta (GM_s / D^3 \Omega),$$ (8)

or, equivalently, that

$$\omega_p / \Omega = \frac{3}{4} \cos \delta (M_s / M_p) (a / D)^3.$$ (9)

We should also note that the above treatment, which essentially assumes that the ring maintains its shape as if it were a solid body, will be valid provided that $\Omega \gg \Omega_b \gg \omega_p$, which is satisfied in this case (see below).
2.1.2 The $m = 2$ term

The effect of the $m = 2$ term is to give rise to an oscillating torque

\[ G_2 = (G_{2x}, G_{2y}, G_{2z}) \]  

(10)
on the ring which has a frequency of $2\Omega_b$. The $m = 2$ torque is of the form

\[ G_{2x} = -(3/4) (GM_s m a^2 D^3) \sin \delta \cos \delta \cos(2\Omega_b t), \]

(11)

\[ G_{2y} = -(3/4) (GM_s m a^2 D^3) \sin \delta \cos \delta \sin(2\Omega_b t), \]

(12)

and

\[ G_{2z} = -(3/4) (GM_s m a^2 D^3) \sin^2 \delta \sin(2\Omega_b t). \]

(13)

We note also that $G_2 \cdot \mathbf{v} = 0$. Indeed, the total torque on the ring may be expressed simply as

\[ \mathbf{G} = (3/2)(GM_s m a^2 D^3) (\mathbf{D} \times \mathbf{k}). \]

(14)

Because of the oscillating nature of the $m = 2$ torque, the mean precession rate is unaffected by the $m = 2$ term. However, the instantaneous precession rate oscillates with a period of (approximately) one half of the orbital period (Katz et al. 1982). Since the amplitude of the torque due to the $m = 2$ term is equal to the torque generated by the $m = 0$ term the modulation is not a negligible effect.

Perhaps the simplest way to envisage what is happening is to consider the motion of the symmetry axis $\mathbf{k}$ of the ring. The $m = 0$ term causes the axis to move in a cone, semi-angle $\delta$, around the $OZ$ axis at a rate $-\omega_p$. Superimposed on this, the $m = 2$ term causes a wobble of the symmetry axis in the direction of precession, and a nodding motion perpendicular to this direction. The amplitude of the wobble (in radians) is roughly $\omega_p/(2\Omega_b)$, and the amplitude of the nodding motion is the same but multiplied by a factor of $\tan \delta$ (see Katz et al. 1982). We should remark that this does not lead to a divergence as $\delta \to \pi/2$ because $\omega_p$ is proportional to $\cos \delta$. We should note that, strictly, because the ring is precessing in a retrograde direction because of the $m = 0$ term, the period of the wobble is in fact $2\pi/(2\Omega_b + \omega_p)$.

2.2 Disc precession

We have seen above that the precession rate for a ring of radius $a$ is proportional to $a^{3/2}$. Thus if we regard a disc as being made up of a succession of concentric rings, the effect of the $m = 0$ potential is to cause the disc to precess differentially. However, if the disc is able to hold itself together in some way, either by means of wave-like communication (Larwood et al. 1996), or by viscous communication (Wijers & Pringle 1999), then it may be able to respond coherently to the $m = 0$ term in the potential by precessing at the same rate at all radii, that is, as if it were a solid body. The precession rate can then be calculated straightforwardly from the above, by rewriting the ring mass, $m_r$, as the mass of an annulus in the disc, $2\pi \Sigma a da$, where $\Sigma$ is the disc surface density at radius $a$.

The net precession rate is then given by

\[ \omega_p = K \cos \delta (GM_s/D^3 \Omega_d), \]

(15)

where (Terquem 1998)

\[ K = \frac{3}{4} R^{-3/2} \left[ \int_0^R \Sigma a^3 da \right] / \left[ \int_0^R \Sigma a^{3/2} da \right]. \]

Here $\Omega_d$ is the angular velocity of disc material at the outer edge of the disc which is at radius $R$, and the lower limit in the integrals can be simply a small radius rather than zero.

Thus, for a disc,

\[ \omega_p/\Omega_d = K \cos \delta (M_s/M_p)(R/D)^3. \]

(17)

For a constant surface density disc $K = 15/32$ (Larwood et al. 1996); for a surface density proportional to $R^{-3/2}$, $K = 3/10$.

2.3 Application to protostellar discs

In protostellar discs the ratio of disc semi-thickness to radius, $H/R$, is of order 0.1 (Burrows et al. 1996; Stapelfeldt et al. 1998), which is larger than the typical dimensionless viscosity $\alpha \sim 0.01$ (Hartmann et al. 1998). Under these circumstances bending waves can propagate through the disc (assumed Keplerian) (Papaloizou & Lin 1995, Pringle 1997), and thus the disc can communicate with itself in a wave-like manner. The propagation speed for such bending waves is of order $c_s a$, where $c_s$ is the disc sound speed (Papaloizou & Lin 1995, Pringle 1999). Thus the condition that the disc be able to precess as a solid body subject to forced differential precession might be expected to be that the wave crossing timescale be less than the precession timescale, that is

\[ R/c_s \lesssim \omega_p^{-1}. \]

(18)

This is the criterion given by Larwood et al. (1996), and by Papaloizou & Terquem (1995). Using the standard result for accretion discs (e.g. Pringle 1981) that $c_s/(R \Omega_d) \sim H/R$, we find that the condition may be written

\[ \omega_p/\Omega_d \lesssim H/R. \]

(19)

For a disc in a binary system which is truncated by tidal forces, the outside disc edge is typically a substantial fraction of the binary separation (e.g Papaloizou & Pringle 1977; Paczyński 1977; Artymowicz & Lubow 1994; Larwood et al. 1996). As a typical value, for reasonable mass ratios, we shall take $R/D \approx 0.3$. Thus, taking $K \approx 0.4$, $q = M_s/M_p \approx 1$, and $\cos \delta \approx 1$, we find that

\[ \omega_p/\Omega_d \approx 0.011 (K/0.4)(R/0.3D)^3 q \cos \delta. \]

(20)

We note that condition (19) is readily satisfied for protostellar discs. However, as we shall note below, this is not the whole story.

Using the above estimates, we note that

\[ \Omega_d/\Omega_b = [M_p/(M_p + M_s)]^{1/2} (D/R)^{3/2} \]

\approx 4.3 [2/(1 + q)]^{1/2} (R/0.3D)^{-3/2}. \]

(21)

Thus typically

\[ \omega_p/\Omega_b \approx 0.05 (K/0.4) q [2/(1 + q)]^{1/2} (R/0.3D)^{3/2} \cos \delta. \]

(22)

This implies that the amplitude of the wobble/nodding motion about steady precession (discussed in Section 2.1.2) is of order a degree or so at the outer edge of the disc although, as we note in Section 2.3, the amplitude inside the disc depends on the properties of the disc.
Some protostellar discs may be sufficiently massive that a noticeable deviation from Keplerian rotation may occur. This effect, and also the self-gravitation of the disc, could change the nature of the propagating bending waves, making them dispersive (Papaloizou & Lin 1995). However, assuming that self-gravity is not dominant (i.e. the Toomre parameter $Q \gtrsim 1$), these effects are expected to be small because the fractional deviation from Keplerian rotation is at most of order $H/R$, and our estimates here are probably still valid. The character of wave propagation also differs between vertically isothermal and thermally stratified discs (Lubow & Ogilvie 1998), however this is relevant only when the radial wavelength is comparable to the disc thickness, and therefore does not affect the propagation of long-wavelength bending waves as considered here.

3 THE EFFECT OF DISSIPATION

As we have seen, the tidal effect of the secondary star can be regarded being due to the $m = 0$ and $m = 2$ parts of the tidal potential independently. Thus the effect of dissipation within the disc on the motions caused by these two parts of the potential can also be calculated separately.

3.1 The $m = 0$ term

The effect of the $m = 0$ term on the misaligned disc is to produce a torque $G_0$ on the disc which is in the direction of aligning it with the binary orbit. This comes about because there is a potential energy, $\Phi$, associated with the misalignment of the disc which is given by

$$\Phi = (3GM_*/8D^3) \sin^2 \delta \int_0^R a^2 \Sigma 2\pi \sin \delta. \quad (23)$$

This has minima at $\delta = 0$ (alignment) and $\delta = \pi$ (anti-alignment), and a maximum at $\delta = \pi/2$. Thus we would normally expect loss of energy associated with the motions induced by the $m = 0$ part of the tidal potential to lead to the disc becoming aligned with the orbital plane.

However, because the $m = 0$ term is symmetric about the $OZ$-axis, $\hat{z}$, any torque produced by the $m = 0$ term must lie in the OXY-plane. We have seen that in the absence of dissipation the torque is such that $G_0$ is parallel to $k \times \hat{z}$, where $k$ is the unit vector in the direction of the angular momentum $J$ of the disc. The effect of viscosity is to give rise to a phase delay in the torque about the $OZ$ axis, which has the effect of producing an additional (viscous) torque $G_{tw}$, which is perpendicular to $\hat{z}$, which lies in the plane defined by $J$ and $\hat{z}$, and which is directed such that $G_{tw} \cdot k < 0$, so that its effect is dissipative. Thus the net effect of this torque is to align the disc with the $OZ$-axis, but in such a way that $J \cdot \hat{z}$ is conserved. This means that in the process of alignment, angular momentum is removed from the disc, giving rise to an enhanced accretion rate (see Papaloizou & Terquem 1995). We compute the timescale on which this torque leads to disc alignment below.

3.2 The $m = 2$ term

In the absence of dissipation, the effect of the $m = 2$ term is to induce oscillations in the disc which have a frequency of $2\Omega$, and which therefore cause the angular momentum vector $J$ to oscillate about some mean value. Thus we would naively expect that the effect of dissipation on the motions induced by the $m = 2$ part of the tidal potential would be to reduce the amplitude of the oscillations, but to have no net effect on the mean disc plane, that is, to have no long-term time-averaged effect on the value of $\delta$.

Papaloizou & Terquem (1995) and Terquem (1998) have argued that the effect of dissipation on the $m = 2$ induced motions is to give rise to a torque which may increase the inclination of the disc. This possibility has been investigated in detail by Lubow & Ogilvie (2001), who formulated the problem in terms of the linear stability of an initially coplanar disc in the presence of the tidal field. It was confirmed that the $m = 2$ component of the tidal potential causes the inclination to grow, but the effect is usually negligible and is outweighed by the effect of the $m = 0$ potential, so the net outcome is that the inclination decays in time. An exception occurs if there is a coincidence between the frequency of the oscillating torque, $2\Omega_0$, and the natural frequency of a global bending mode of the disc. However, this resonance can occur only if the disc is very thick ($H/R \gtrsim 0.4$) or much smaller than the standard tidal truncation radius ($R/D \ll 0.3$). We therefore neglect the effect of the $m = 2$ term on the long-term time-averaged value of $\delta$ and restrict our attention to the alignment effects of the $m = 0$ term.

3.3 Application to protostellar discs

In the analysis so far we have assumed that the disc is able to communicate internally sufficiently fast that the disc can proceed as a rigid body. This communication takes the form of bending waves which propagate through the disc at a speed of order $\frac{1}{2}c_s$. However, these bending waves are associated with nearly resonant horizontal epicyclic motions that are strongly shearing, being proportional to the distance above the mid-plane (Papaloizou & Pringle 1983). Viscosity in the disc can then act on these shearing motions, leading to dissipation of energy in the precessional motion, and alignment of the disc with the orbital plane.

We have seen that the torques exerted at different radii in the disc by the $m = 0$ component of the tidal potential would naturally result in differential precession. In order for the disc to resist this, hydrodynamic stresses must be established within the disc so that the net torque on each ring is such as to maintain a uniform, global precession rate. The required internal torque, although it varies with radius and vanishes at the edges of the disc, is therefore generally comparable to the total tidal torque on the disc, and is given approximately by

$$G_{int} \sim 2\pi R^4 \Omega \omega_0 \sin \delta. \quad (24)$$

These hydrodynamic stresses are associated with the horizontal epicyclic motions mentioned above, which take the form

$$\nu_\phi \sim 2\nu_\phi \sim A R, \quad (25)$$

referred to cylindrical polar coordinates $(r, \phi, z)$ based on the mean disc plane, and where $A$ is independent of $z$. As a result of the hydrodynamic stress $\rho \nu_\phi \cdot r \Omega$ there is a net horizontal angular momentum flux
2\pi \int \rho v_0 \Omega z\, dz \sim 2\pi \Delta \Sigma R^2 H^2 \Omega_d. \quad (26)

Equating this with \( \Gamma_{\text{int}} \) gives
\[
A \sim (R/H)^2 \omega p \sin \delta, \quad (27)
\]
and so
\[
v_0'/c_0 \sim 2\omega_p'/\omega_p \sim (R/H)^2 (\omega_p/\Omega_d) \sin(\delta/H). \quad (28)
\]
Note that for these velocities to be subsonic we require that
\[
\omega_p/\Omega_d \lesssim (H/R)^2 \sin \delta. \quad (29)
\]

For \( \sin \delta > H/R \), this is a stronger condition than the one
derived by Papaloizou & Terquem (1995), and if \( \delta \sim 1 \), it is
only marginally satisfied in protostellar discs. The fact that
the internal disc velocities are likely to be close to sonic,
means that the dissipation may well be strongly enhanced.
We discuss this further below.

For an internal disc viscosity \( \nu \), these velocity perturbations
lead to a rate of energy dissipation in the disc of order
\[
dE/dt \sim m_d \alpha^2 (\nu/H^2) (R/H)^4 (\omega_p/\Omega_d)^2 \sin^2 \delta, \quad (30)
\]
where \( m_d \) is the mass of the disc.

As we have seen above, the effect of this dissipation is
to lead to an alignment of the disc with the orbital plane.
The means by which this is accomplished is through the
viscous torque \( G_{\text{int}} \), which leads to alignment by removing
the component of disc rotation which lies in the orbital plane
(recall that \( G_{\text{int}} \cdot \hat{z} = 0 \)). Thus the amount of energy to be
dissipated in order to bring about alignment is
\[
E_{\text{kin}} \sim m_d R^2 \Omega_d^2 \sin^2 \delta. \quad (31)
\]

From these estimates we deduce the alignment timescale
for the disc \( t_{\text{align}} \), which is given by
\[
t_{\text{align}} \sim E_{\text{kin}}/(dE/dt) \sim (R^2/\nu) (H/R)^4 (\Omega_d/\omega_p)^2. \quad (32)
\]
We note further that the viscous evolution timescale for a
disc \( t_{\nu} \) is given by
\[
t_{\nu} \sim R^2 / \nu. \quad (33)
\]
Thus, from this analysis we would conclude that for precession
by the induced velocities which are subsonic, the alignment timescales for
a misaligned protostellar disc is typically of order, or somewhat longer than, the viscous evolution timescales, and that the additional accretion rate
caused by the process of alignment is typically at most comparable
to the accretion rate already present in the disc.

The viscous evolution timescale can be written in terms of
the dimensionless measure of viscosity, \( \alpha \), as
\[
t_{\nu} \sim \Omega_d^{-1} (R/H)^2 \alpha^{-1}. \quad (34)
\]
Using this we find that
\[
t_{\text{align}} \sim \omega_p^{-1} (H/R)^2 (\Omega_d/\omega_p). \quad (35)
\]
This expression for the alignment timescale and the expression
for the precession rate (equation 22) have been verified by the
numerical calculations of Lubow & Ogilvie (2000).

3.3.1 The effect of sonic induced velocities

However, since the velocities associated with the induced
shearing in these discs are close to sonic, the parametric
instabilities discussed by Gammie, Goodman & Ogilvie
(2000) are likely occur. These authors concluded that the
internal shear motions induced by the disc bending are, for
sonic shearing motions, unstable on a local (disc) dynamical
timescale, \( \Omega^{-1} \). The local turbulence induced by these
instabilities increases the damping rate for the shearing motions.
For example, if \( v' \sim c_s \), then we may estimate the
dissipation rate in the disc due to these instabilities as
\[
dE/dt \sim m_d c_s^2 \Omega_d, \quad (36)
\]
which implies an alignment timescale of
\[
t_{\text{align}} \sim \omega_p^{-1}/\sin \delta. \quad (37)
\]
This implies that alignment occurs almost as fast as precession.

Furthermore, even when the induced velocities, \( v' \) are
subsonic, the instabilities grow on a timescale \( \sim H/v' \) (Gammie et al. 2000),
that the growth time is less than the
viscous damping timescale \( \sim H^2/v \), i.e. provided that
\[
\alpha \lesssim v'/c_s, \quad (38)
\]
or, alternatively, that
\[
\sin \delta \gtrsim \Omega_d/\omega_p (H/R)^2 \alpha \quad (39)
\]
i.e. that
\[
\sin \delta \gtrsim 0.01 \frac{(\alpha/0.01) (H/0.1R)^2 (K/0.4) (R/0.3D)}{1} q \cos \delta. \quad (40)
\]
In this case the parametric instabilities again have enhanced
dissipation which is likely to lead to an enhanced effective viscosity, \( \alpha_{\text{eff}} \), such that the enhanced damping rate is of
order the instability growth rate, viz.
\[
\alpha_{\text{eff}} \sim v'/c_s. \quad (41)
\]
This enhanced damping implies an alignment timescale of
\[
t_{\text{align}} \sim \omega_p^{-1} (H/R)^2 (\Omega_d/\omega_p)^2 / \sin \delta \sim \omega_p^{-1} (H/0.1R)^2 (K/0.4)^{-2} (R/0.3D)^{-6} q^{-2} \cos \delta \quad (42)
\]

We should stress that this estimate is probably a lower
limit to the alignment timescale since the efficiency of the
damping due to the parametric instability may not be as
perfect as suggested by equation 11 (Gammie et al. 2000).

4  DISCUSSION

4.1 General results

We have presented a simple discussion of the dynamics of
a misaligned disc in a binary star system, and have applied
the results to the parameters of discs and binary parameters
which are relevant to young stellar objects. Our general
conclusions are as follows:

\[\text{Implications of precessing discs and jets} \quad 5\]
For typical protostellar disc parameters, and for circular binary orbits, tidal forces cause a misaligned disc to precess like a solid body about the angular momentum vector of the binary orbit, with a precession period of order \( P_p \sim 20 P_{orb} \), where \( P_{orb} \) is the orbital period (equation 23 in Papaloizou & Terquem 1995; Larwood et al. 1996; Terquem 1998; Terquem et al. 1999). In addition, the outer disc plane is forced to wobble with a period of \( 5 P_{orb} \) (Katz et al. 1982).

We have presented order of magnitude estimates for the timescale on which dissipation within the disc leads to alignment with the orbital plane. (For the reasons described above, we overlook the possibility raised by Papaloizou & Terquem (1995) that dissipation might lead to disc misalignment). If the disc evolution is determined by a simple isotropic viscosity, then we find, in line with the estimates given by Papaloizou & Terquem (1995) and by Terquem et al. (1999) that the alignment timescale is, for protostellar disc parameters, of order the normal viscous evolution timescale.

However, we have also pointed out that the velocities induced within the disc by the action of tidal forces on the tilt are, for typical protostellar disc parameters, transonic, and that the criterion used by Papaloizou & Terquem (1995) to justify the use of their linearization procedure is incorrect if the disc tilt exceeds the opening angle of the disc (equation 23). The induced velocities take the form of a horizontal epicyclic motion proportional to the distance above the mid-plane within the disc which oscillates in a frame rotating with the fluid with a period approximately equal to that of the orbital period of the disc material. It has long been suspected that such a shear flow is unstable (Katz & Coleman 1993), and recent work by Gammie et al. (2000) has demonstrated that the flow is indeed unstable to a parametric hydrodynamic instability which has a growth rate of order the shearing timescale and leads to rapid dissipation. We have estimated the effect of such instabilities. For large disc tilts, and for typical protostellar disc parameters, we find that the disc alignment timescale is comparable to the precession timescale. However, for smaller tilt angles \( \delta \), we find that the alignment timescale varies as \((\sin \delta)^{-1}\). It is worth noting that the enhanced dissipation will also result in a greater disc luminosity and a larger mass accretion rate than those provided by standard viscous evolution. These effects should be considered when modelling protostellar discs in binary systems. However, given the large range of observed accretion rates from protostellar discs (e.g. Hartmann et al. 1998) and the uncertainties in current models of protostellar discs, it would be difficult to detect an unambiguous signature of the enhanced luminosity or accretion rate.

We should draw attention to the fact that, in common with previous authors, all the results presented above are for binary stars with circular orbits. If, as is the case for many binary stars, the orbit is non-circular, then the analysis is similar but rather more complicated. However, the main effects of orbital eccentricity on the results reported above are to decrease the ratio of disc radius to orbital semi-major axis and to modify the time-averaged potential due to the companion. Tidal truncation of the disc now takes place at periastron, so that \( R \approx 0.3a(1-e) \), while the modified time-averaged potential can be approximated by replacing the orbital separation \( D \), by \( a(1-e^2)^{1/2} \) (Holman, Touma & Tremaine 1997), where \( a \) is the semi-major axis and \( e \) is the eccentricity. Thus, to a first approximation, the major effects of orbital eccentricity can be taken into account by replacing \( R/0.3D \) in the above formulae by the quantity

\[
\sqrt{\frac{1-e}{1+e}} \left( \frac{R}{0.3a(1-e)} \right). \tag{43}
\]

### 4.2 Precession and wobbling of protostellar jets

Disc precession has been used as an ingredient in the explanation of long-period variations in the light curves of a number of X-ray emitting binary systems (Gerend & Boynton 1976; Katz et al. 1982; Wijers & Pringle 1999). However, for protostellar systems the major application for tidally induced disc precession has been to provide an explanation for changes in flow direction of protostellar jets (e.g. Eislöffel & Mundt 1997; Terquem et al. 1999). Although changes in flow direction are often discussed in terms of precession, there are, as discussed by Eislöffel & Mundt (1997), various alternative explanations for such phenomena, and there is as yet no convincing case of a jet which has been steadily precessing for many precession periods. In the light of this we discuss, in general terms, the kind of effects which tidally induced disc precession might be expected to lead to from a theoretical point of view.

Protostellar jets appear to be produced during the major accretion phase in the star-formation process, that is during the Class 0 and Class I phases of the life of a protostellar core/young star (Bontemps et al. 1996). This phase of the star formation process is thought to last about 10⁵ years (Lada 1999). In addition, given that the jet velocities are of order the escape velocities from the central stars, and that the mass outflow rates are a non-negligible fraction of the likely accretion rates, it seems a reasonable assumption that the jets are formed close to the centre of the disc (e.g. Pringle 1993), and therefore that the jet direction is governed by the disc axis in the central regions of the disc. Thus, since for a strongly misaligned disc, the alignment timescale is of order the precession timescale, which is of order \( \sim 20 \) orbital periods, we expect that, regardless of how the misalignment might have come about, strongly misaligned discs and jets are only likely to occur in binaries with orbital periods longer than \( \sim 5000 \) years, that is with separations larger than about \( \sim 100 \) a.u. Examples of systems which appear to have misaligned jets include: Cep E (Eislöffel et al. 1996); T Tau (Bohm & Solf 1994); HH 1,2,144 VLA 1/2 (Reipurth et al. 1993); and HH 111/121 (Gredel & Reipurth 1993). These systems are either known to be wide binaries (\( > 100 \) a.u.), in agreement with expectations, or have not yet been resolved.

Since the alignment and precession timescales are comparable for strongly misaligned discs, one would not expect to observe the multiple large scale wiggles which might be the result of a jet undergoing many precession cycles at a large angle to the orbital axis. One might, however, see the results of such a jet whose direction starts at large angle to the binary axis, and then changes direction on the sky as it simultaneously precesses and aligns (with the binary axis) on a timescale of \( \sim 20 \) orbital periods. For such a jet, with jet axis at a large angle to the binary axis, provided that the inner and outer parts of the disc are in good communication, the jet direction is forced to wobble by the \( m = 2 \).
tides with a period of half the orbital period. The amplitude of the wobble at the outside of the disc is only about a degree or so, but since the communication to the disc centre is wave-like, the amplitude of the wobble at the disc centre must depend on the wave amplitude induced there, which in turn depends on the properties of the disc, and in particular on the dependences with radius of the angular momentum density ($\propto \Sigma r^{1/2}$) and the group velocity ($\propto c_\Sigma$) (e.g. Terquem 1998). Such a regular wobbling of the jet direction may already have been observed in one of the jets emanating from Cep E (Eislöffel et al. 1996). The jet appears to undergo a regular wobble with an amplitude of 4° and a period of $\sim 400$ years (both dependent on the inclination). The mechanism driving this oscillation may be able to be tested observationally using the VLA or adaptive optics in the infrared. If the oscillation of the jet is due to a wobble of the disc, the period of the binary should be $\sim 800$ years (separation $\sim 80$ a.u. or 0.12 pc), whereas if the oscillation is due to precession of the disc the binary’s period should be much shorter ($\lesssim 20$ years) and the binary would not be resolvable ($\lesssim 0.01''$). Many YSOs with single jets also show evidence for direction changes, and a high-resolution survey to determine the binarity of such sources would be invaluable for testing the theory of wobbling and precessing jets.

For jets that are weakly misaligned, we note that the alignment timescale is inversely proportional to $\delta$, (implying that the misalignment angle decreases with time as $t^{-1}$) and scales, for typical protostellar disc parameters, approximately with the orbital period. Furthermore, the ratio of the precession timescale to the alignment timescale scales with $\delta$. In order to see multiple wiggles caused by jet precession in a jet whose length corresponds typically to a dynamical timescale of $\sim 10^5$ years, we require a precession period of less than or of order a few thousand years, and thus binary periods of less than or of order a few hundred years, and binary separations less than or of order a few tens of a.u.

For example, an initially strongly misaligned binary with a separation of a few tens of a.u. will rapidly evolve into a weakly misaligned system, but even after $\sim 10^5$ years will still show a misalignment angle of $\sim 0.03$ radians, that is, a few degrees, and thus, with a precession period of a few thousand years, such a system would be marginally observable. Wider binaries than this would have precession periods too long to show wiggling of a jet whose dynamical age is $\sim 10^4$ years, whilst closer binaries with shorter precession periods would have alignment angles too small to be observed. We note that for small misalignment angles, the amplitude of the wobble of the outer disc caused by the $m = 2$ tidal term (which is approximately $\delta \omega_0 / (2 \Omega_b) \approx \delta / 40$ as $\delta \to 0$) is probably too small to be detected.

In summary, one expects jets from strongly misaligned discs to be a rarity amongst binaries closer than $\sim 100$ a.u. In wider binaries, jets in strongly misaligned systems are wiggled at twice the binary orbital frequency with an amplitude which depends on the details of the disc structure. Weakly misaligned systems are longer-lived, and therefore potentially observable in closer binaries. As $\delta \to 0$, precession should give rise to observable jet wiggling (with amplitude of angle $\delta$ decreasing as $\propto \delta^{-1}$) in binaries with separation of order a few tens of a.u.

### 4.3 Implications for binary star formation

We now consider briefly the implications that evidence for disc misalignment might have for the various theories of binary star formation, noting that in order for a misaligned system to be observed, the system as a whole must be assembled over a timescale which is shorter than the alignment timescale, $t_{\text{align}}$.

#### 4.3.1 Binary fragmentation

Most published fragmentation calculations result in discs which are coplanar with the resulting fragments. Of these, there are typically two cases: fragmentation due to initial density perturbations (e.g. Boss & Bodenheimer 1979; Boss 1986), or centrifugally-supported fragmentation (e.g. fragmentation of a massive protostellar ring or disc; Norman & Wilson 1978; Bonnell 1994; Bonnell & Bate 1994; Burkert & Bodenheimer 1996; Burkert, Bate & Bodenheimer 1997) with some calculations exhibiting both types of fragmentation (e.g. Bonnell et al. 1991; Bate, Bonnell & Price 1995). The fragmentation of centrifugally-supported material leads to the orbit(s) and discs of all fragments occupying the same plane. In most calculations where the fragmentation occurs due to initial density perturbations, the initial conditions have a single axis of rotation and there is typically an $m = 2$ density perturbation which is perpendicular to the rotation axis. With these simple initial conditions, a binary forms from the initial $m = 2$ density perturbation in a plane perpendicular to the rotation axis and passing through the centre of the cloud, while the discs that form around these fragments have rotation axes that are parallel to the rotation axis of the cloud. Thus, the protostellar discs lie in the same plane as the orbit.

In order to produce discs that are misaligned with the orbital plane, it is natural to expect that the angular momentum distribution of the molecular gas must have strong spatial variations to enable the discs to have different rotation axes from the larger-scale orbit. However, this is not necessary. All that is required is to force the binary to form in a plane that is not perpendicular to the rotation axis. This can be achieved simply by rotating the $m = 2$ density perturbation slightly so that it is no longer perpendicular to the rotation axis. (e.g. Boss & Bodenheimer 1979; Boss et al. 1980; Burkert & Bodenheimer 1996; Burckert, Bate & Bodenheimer 1997) with some calculations exhibiting both types of fragmentation (e.g. Bonnell et al. 1991; Bate, Bonnell & Price 1995). Such a lack of correlation between the axis of rotation and the initial density perturbation(s) is expected if the collapse of a molecular gas is triggered by an external source (e.g. a shock wave or gravitational interaction with a passing object), or if the pre-collapse clump of gas is formed dynamically within a turbulent molecular cloud. In fact, given that the rotational energy of observed molecular clumps is generally insignificant compared with their gravitational energy (Goodman et al. 1993), it is hard to find a reason why there should be any correlation. In this case, the two fragments form above and below the plane that is perpendicular to the rotation axis and passes through the cloud’s centre and, hence, the orbital plane is no longer perpendicular to the rotation axis of the initial cloud. The rotation axes of the discs that form around the fragments, on the other hand, are still parallel to the rotation axis of the initial cloud. Thus, even though the pre-collapse gas is all rotating around a single axis, possibly in solid-body rotation, the discs are misaligned with the...
orbital plane of the binary. Such fragmentation calculations have been performed by Bonnell et al. (1992) and Bonnell & Bastien (1992). In their particular case, the \( m = 2 \) density perturbation was in the form of the initial molecular cloud core being prolate. We note that in such calculations, although the discs are misaligned with the orbital plane, they are still aligned with each other. To produce the added complexity of discs which are initially misaligned with each other would require spatial variations in the angular momentum distribution. However, even if the discs are initially aligned, they are almost certain to undergo precession at different rates (equation 17) and, therefore, will very rapidly become misaligned with each other.

4.3.2 Misaligned systems from dynamical interactions

The formation of wide binaries with misaligned discs via fragmentation that was described above is the simplest example of ‘prompt initial fragmentation’ (Pringle 1989) where each stellar component and its concomitant disc forms from a spatially-distinct region of the collapsing cloud, the accretion of large amounts of material on to the binary as a whole is avoided (see below), and, thus, the discs need not be initially aligned.

More complicated initial conditions than those described above can lead to the formation of a small, three-dimensional, cluster of stars (e.g. Larson 1978; Chapman et al. 1993; Klessen et al. 1998). The stars in such a group are expected to undergo interactions with one another, such as dissipative star-disc encounters (Larson 1990; Clarke & Pringle 1991a, 1991b; Clarke & Pringle 1993; McDonald & Clarke 1993, 1995).

Highly-dissipative encounters may lead to the formation of binary (or multiple) systems, via capture of the passing object, typically with misaligned discs. However, it remains to be determined to what extent these dissipative interactions also lead to disc alignment through strong tidal interactions (Heller 1993; Hall, Clarke & Pringle 1996; Hall 1997), especially given that in reality the major infall phase onto the separate protostellar nuclei occurs contemporaneously with (Bonnell et al. 1997; Klessen et al. 1998), rather than prior to, the dissipative binary formation process (see the next section).

Another possibility is that the gravitational interaction of a passing object, although not leading to capture, may tilt the disc(s) within a binary system or around a single star. If such an interaction occurred during the main accretion phase, the tilting of the disc would likely lead to a change in the direction of an emanating jet which could be mistaken for precession or realignment of a misaligned disc in a binary system. Thus, to unambiguously identify a precessing jet we emphasize that the jet should exhibit several oscillations.

4.3.3 Subsequent accretion

Whether a binary is formed directly by fragmentation or more complicated interactions between several protostars, the above discussion ignores the effect of accretion of material after the binary has formed.

If a protobinary forms with discs that are initially misaligned with the orbital plane, but subsequently accretes the majority of its mass, the misalignment will generally be diminished. For binaries formed directly via fragmentation, the mass of the protobinary immediately after its formation is less for binaries with smaller initial separations (Boss 1986). Therefore, to obtain the same final total mass, closer binaries must accrete more, relative to their initial mass, than wider binaries and, hence, close binaries (\(< 100 \) a.u.) are less likely to exhibit misaligned discs than wider binaries. Binaries formed via dissipative interactions in small clusters are also likely to accrete material subsequently (Bonnell et al. 1997; Klessen et al. 1998). Finally, along with these direct effects of infalling material on misalignment, as mentioned earlier, binaries with separations \(< 100 \) a.u. are also unlikely to have strongly misaligned discs because the alignment timescale for close binaries (\(< 100 \) a.u.) is short in comparison to typical protostellar accretion timescales.

Conversely, if a binary is formed with discs that lie in the orbital plane, there remains the possibility that later infall of material with a different angular momentum vector to the binary’s orbit might lead to a degree of misalignment. In particular, it is simple for the spin of a disc to be affected by late infall of a small amount of material once the main accretion phase is over, and the disc masses have been substantially reduced. On the other hand, the magnitude of the misalignment which could occur during the main accretion phase of the stars in the process of formation, when the jet-like outflows are at their strongest, and when tidal forces too are at their greatest, has yet to be determined. Further numerical investigation of accretion and fragmentation scenarios are required to test this further.

Finally, as with the above mentioned effects of an encounter on the disc and/or jet of a single star, a change in the angular momentum vector of material being accreted by a single star could also lead to a change in the orientation of its disc and, thus, a wandering of an emanating jet. Once again, therefore, to unambiguously identify a precession, several oscillations must be observed.

5 CONCLUSIONS

We have investigated the dynamics of a protostellar disc in a binary system where the disc is misaligned with the orbital plane of the binary. The disc is found to wobble with a period approximately equal to half the binary’s orbital period and to precess with a period of order 20 binary periods. We also determine the characteristic timescale for realignment of the disc with the orbital plane due to dissipation. If the dissipation is determined by a simple isotropic viscosity then we find, in line with previous studies, that the alignment timescale is of order the viscous evolution timescale (of order 100 precession periods). However, for typical protostellar disc parameters, if the disc tilt exceeds the opening angle of the disc, then tidally induced shearing within the disc is transonic. In general, hydrodynamic instabilities associated with the internally driven shear result in extra dissipation which is expected to drastically reduce the alignment timescale. For large disc tilts the alignment timescale is then comparable to the precession timescale, while for smaller tilt angles \( \delta \), the alignment timescale increases by a factor as \( (\sin \delta)^{-1} \).

These general results lead to several observational con-
sequences. Since the alignment timescale in strongly misaligned discs is so short, such discs are only likely to occur in binaries with periods $\gtrsim 5000$ years (i.e. separations $\gtrsim 100$ a.u.). This expectation is in good agreement with the separations of binary systems which are observed to have misaligned jets. In addition, because the alignment timescale is of order the precession period, multiple large wiggles are not expected to be seen. At best one might observe a ‘bending’ of the jet as the disc simultaneously precesses and aligns to the orbital plane. However, such bending could also result from other processes (e.g. a single star whose disc, and thus jet, orientation is altered by interaction with a passing object or by the accretion of material with a different angular momentum vector than that of the original disc). For this and other reasons, we emphasize that to unambiguously identify a precessing jet several oscillations must be observed. Finally, although multiple large oscillations are not expected to be seen from systems with strongly misaligned discs, the jet direction may be forced to wobble by a few degrees on a timescale of half the orbital period, with an amplitude that depends on the details of the disc structure. Such a wobble may have been observed in one of the jets emanating from the Cep E system and we strongly encourage efforts to determine the binarity of sources which emit double jets or display evidence of wobbling jets so that the theoretical expectations can be tested.

For discs which are misaligned by small angles the alignment timescale is much longer. Therefore, precession of jets from such systems may be detectable for systems with orbital periods of order one hundred years (separations of a few tens of a.u.) in order to produce multiple wiggles in the $\sim 10^4$ year dynamical timescale of a jet. Wobbling of the jet on a timescale of half the orbital period is unlikely to be large enough to be detected in this case.

Finally, we discuss the implications of the existence of discs that are misaligned with the orbital plane of a binary for mechanisms for binary star formation. Although most published fragmentation calculations have not resulted in the formation of discs which are misaligned with the orbital plane of the binary, it is trivial to produce initial conditions where this is the case (e.g. Bonnell et al. 1992; Bonnell & Bastien 1992). Furthermore, these calculations do not require spatial variation of the direction of the angular momentum vector in the initial clouds; there is a single axis of rotation and even solid-body rotation is permitted. Binary systems with misaligned discs may also be formed directly via dissipative interactions in small clusters of protostars formed via ‘prompt initial fragmentation’. However, in either case, formation of the binary is likely to be contemporaneous with the accretion phase and, thus, strongly misaligned discs are unlikely for binaries with separations $\lesssim 100$ a.u. both because of the rapid realignment timescale and because the subsequent accretion of a large amount of material by the binary would tend to align the orbital and disc planes. Alternatively, binary systems with misaligned discs could be formed through the gravitational interaction of a passing object with a previously aligned disc, or by the infall of a small amount of material with a different angular momentum vector to that of the binary’s orbit near the end of accretion phase. Any of these mechanisms could explain the misaligned disc which may be present in HK Tau (Stapelfeldt et al. 1998; Koresko 1998) provided that the time that has elapsed since the misaligned disc was formed is less than the current alignment timescale. Thus, if HK Tau is a binary with a misaligned disc, this does not necessarily mean that it was formed this way initially or even that the discs were misaligned during the main accretion phase when the system would be expected to have produced jets. Larger surveys of the alignment of discs in binary systems are required for us to draw conclusions on formation mechanisms.

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