OPACITY BROADENING OF $^{13}$CO LINEWIDTHS AND ITS EFFECT ON THE VARIANCE–SONIC MACH NUMBER RELATION

C. Correia$^1$, B. Burkhart$^2$, A. Lazarian$^3$, V. Ossenkopf$^4$, J. Stutzki$^3$, J. Kainulainen$^4$, G. Kowal$^5$, and J. R. de Medeiros$^1$

$^1$ Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, 59072-970, Brazil; caioftc@dfte.ufrn.br
$^2$ Astronomy Department, University of Wisconsin, Madison, 475 North Charter Street, WI 53711, USA
$^3$ Physikalisches Institut der Universität zu Köln, Zülpicher Strasse 77, D-50937 Köln, Germany
$^4$ Max-Planck-Institute for Astronomy, Königstuhl 17, D-69117 Heidelberg, Germany
$^5$ Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, 05508-090, Brazil

Received 2013 August 16; accepted 2014 February 12; published 2014 March 21

ABSTRACT

We study how the estimation of the sonic Mach number ($M_s$) from $^{13}$CO linewidths relates to the actual three-dimensional sonic Mach number. For this purpose we analyze MHD simulations that include post-processing to take radiative transfer effects into account. As expected, we find very good agreement between the linewidth estimated sonic Mach number and the actual sonic Mach number of the simulations for optically thin tracers. However, we find that opacity broadening causes $M_s$ to be overestimated by a factor of $\approx 1.16$–1.3 when calculated from optically thick $^{13}$CO lines. We also find that there is a dependence on the magnetic field: super-Alfvenic turbulence shows increased line broadening compared with sub-Alfvenic turbulence for all values of optical depth for supersonic turbulence. Our results have implications for the observationally derived sonic Mach number–density standard deviation ($\sigma_{\rho/\langle\rho\rangle}$) relationship, $\sigma_{\rho/\langle\rho\rangle}^2 = b^2 M_s^2$, and the related column density standard deviation ($\sigma_{N/\langle N\rangle}$) sonic Mach number relationship. In particular, we find that the parameter $b$, as an indicator of solenoidal versus compressive driving, will be underestimated as a result of opacity broadening. We compare the $\sigma_{N/\langle N\rangle}$–$M_s$ relation derived from synthetic dust extinction maps and $^{13}$CO linewidths with recent observational studies and find that solenoidally driven MHD turbulence simulations have values of $\sigma_{N/\langle N\rangle}$ which are lower than real molecular clouds. This may be due to the influence of self-gravity which should be included in simulations of molecular cloud dynamics.

Key words: ISM: structure – magnetohydrodynamics (MHD) – methods: numerical

Online-only material: color figures

1. INTRODUCTION

Supersonic magnetized turbulence is observed in multiple tracers across several different interstellar medium phases. This includes the neutral medium, traced by H$^1$ (see Chepurnov et al. 2010; Burkhart et al. 2010; Peek et al. 2011); the warm ionized medium, traced by H$\alpha$ (see Hill et al. 2008); electron density fluctuations (Armstrong et al. 1995; Chepurnov & Lazarian 2010); synchrotron polarization (Gaensler et al. 2011; Burkhart et al. 2012); and the molecular medium (see Heyer et al. 1998; Goldsmith et al. 2008), which includes a variety of molecular tracers including the often used carbon monoxide (CO) line.

Turbulence and magnetic fields in these environments are interrelated with a variety of physical processes, including cosmic ray transport (Yan & Lazarian 2004; Beresnyak et al. 2011), magnetic reconnection (Lazarov & Vishniac 1999; Kowal et al. 2009), and star formation (McKee & Ostriker 2007 and references therein). Furthermore, it is clear that, in order to understand MHD turbulence and related physical mechanisms, one needs to be able to measure basic plasma parameters such as the sonic and Alfvenic Mach numbers ($M_s \equiv V_{turb}/c_s$, where $c_s$ is the sound speed, and $M_A \equiv V_{turb}/V_{Alfven}$, respectively).

It is clear from simulations, observations, and theoretical works that compressible turbulence, which generates shocks, is important for creating filaments and local regions of high density contrast (Kowal & Lazarian 2007). Shocks broaden the gas/dust density and column density probability distribution function (PDF) as well as increase the peaks and drag out the distribution’s tails toward higher gas densities (Burkhart et al. 2009). Based on this observation, several authors (e.g., Vazquez-Semadeni 1994; Padoan et al. 1997; Federrath et al. 2010; Burkhart & Lazarian 2012), to name a few, have developed relationships between the PDF moments, such as the variance or standard deviation of density ($\sigma_{\rho/\langle\rho\rangle}$), and the sonic Mach number, for example,

$$\sigma_{\rho/\langle\rho\rangle}^2 = b^2 M_s^2,$$

where $b$ is a parameter that depends on the type of turbulence forcing with $b = 1/3$ for pure solenoid forcing and $b = 1$ for pure compressive forcing (Federrath et al. 2008). Once gas becomes dense enough, collapse can occur via self-gravity and the PDF forms power-law tails toward higher density regions (Klessen et al. 2000; Collins et al. 2012). Additional variants of Equation (1) have also been developed, e.g., including plasma $\beta$ (see Molina et al. 2012).

Equation (1) is hard to constrain observationally as volume densities and the three-dimensional (3D) velocity structure are not available from observations. Important information on turbulent supersonic motions in molecular clouds comes from the observed non-thermal broadening of the linewidths of different spectral lines, e.g., from carbon monoxide (CO) emission. However, CO, in particular $^{13}$CO, is often partially or fully optically thick and traces only a limited dynamic range of column densities (see Goodman et al. 2009; Burkhart et al. 2013a, 2013b).

Fortunately, dust extinction column density maps of infrared dark clouds (IRDCs), including mid infrared (MIR) and near infrared (NIR) wavelengths, can be used to trace a much larger dynamic range of densities in order to probe the PDF ($A_v = 1–25$ for NIR and $A_v = 10–100$ for MIR, see Lombardi...
& Alves 2001; Kainulainen et al. 2011; Kainulainen & Tan 2013, henceforth KT13). However, extinction maps carry no dynamical information regarding velocities and for this, molecular line profiles are needed to measure the dynamics of clouds, including the sonic Mach number for a given cloud temperature.

In this Letter, we investigate the robustness of measuring the sonic Mach number from maps of $^{13}$CO using synthetic observations derived from 3D MHD turbulence simulations. We further create synthetic dust maps in order to compare the PDF standard deviation with the measured sonic Mach number, following the approach in the observational work of KT13.

2. NUMERICAL DATA

We generate a database of 12 3D numerical simulations of isothermal compressible (MHD) turbulence with resolution $512^3$ (all models are listed in Table 1). For models with $M_s < 20$ we use the MHD code detailed in Cho & Lazarian (2003) with large scale solenoidally driven turbulence. Our two models with $M_s > 20$ are from the AMUN code (Kowal et al. 2011). The magnetic field consists of the uniform background field and a time-dependent fluctuating field: $B = B_{\text{ext}} + b(t)$; $b(0) = 0$. For more details on the simulations scheme see Burkhart & Lazarian (2012, henceforth BL12) and Burkhart et al. (2013b).

Our models are divided into two groups corresponding to sub-Alfvénic ($B_{\text{ext}} = 1.0$) and super-Alfvénic ($B_{\text{ext}} = 0.1$) turbulence. For each group we compute five models with different gas pressures falling into subsonic and supersonic regimes with $M_s$ ranging from 0.7 to 26 (see Table 1).

We post-process the simulations to include radiative transfer effects from the $^{13}$CO $J = 2–1$ line. For more information on this code and the assumptions involved see Osenkopf (2002) and Burkhart et al. (2013a, 2013b). Our original simulation set the sonic Mach number using the sound speed. We rescale our simulation’s velocity field in such a way that each has the same value of temperature and retain the original sonic Mach number. When applying radiative transfer post-processing we vary the density scaling factor by increasing and decreasing it by a factor of 30 from the standard numerical density of 275 cm$^{-3}$ (n henceforth), which represents a typical value for a giant molecular cloud. We also include a very high density case with 82,500 n. Each line of sight (LoS) has a spectral resolution of 0.5 km s$^{-1}$ and is taken perpendicular to the mean magnetic field. The cube size of our clouds is 5 pc, the gas temperature is 10 K, and the CO abundance $^{13}$CO/H = 1.5 $\times$ 10$^{-6}$.

2.1. Analysis of the Synthetic Observations

Once we generate the synthetic $^{13}$CO line profile maps we measure the dispersion of the velocity profile using a Gaussian fit. Using a gas temperature of $T = 10$ K we can calculate the sonic Mach number as

$$M_s = \frac{\sigma^{\text{3D}}_{\text{CO}}}{c_s} = \sqrt{3\sigma^{\text{3D}}_{\text{CO}} / c_s},$$  \hspace{1cm} (2)

where $\sigma^{\text{3D}}_{\text{CO}}$ is the velocity dispersion in one dimension and $c_s$ is the sound speed:

$$c_s = \sqrt{\frac{K_B T}{\mu M_H}}.$$  \hspace{1cm} (3)

We then disregarded the column densities with $A_v < 7$ mag, following the procedure of KT13. In the left panel Figure 1, we show an example plot of the $^{13}$CO line profiles for same super-sonic super-Alfvénic simulation with different density values, and hence different optical depths. The opacity broadening effect slightly widens the observed profile. In the right panel of

### Table 1

| Density | Super-Alfvénic ($M_s > 7.0$) | Sub-Alfvénic ($M_s < 0.7$) |
|---------|------------------------------|----------------------------|
|         | $\sigma^{\text{3D}}_{\text{CO}}$ | $M_s$ | $\tau$ | $\sigma^{\text{3D}}_{\text{CO}}$ | $M_s$ | $\tau$ |
|         | (km s$^{-1}$) | | | (km s$^{-1}$) | | |
| $n_0$   | 6.75 | 34.4 | 0.0017 | 4.21 | 21.5 | 0.0022 |
| 275$n_0$| 6.29 | 32.1 | 0.03 | 3.85 | 19.6 | 0.092 |
| 8250$n_0$| 6.25 | 31.9 | 1.2 | 4.77 | 24.4 | 2.5 |
| 82,500$n_0$ | 6.42 | 32.8 | 2.0 | 5.64 | 28.8 | 29.6 |
| $M_s \approx 6.25$ | | | | $M_s \approx 25.3$ |
| $M_s \approx 9.0$ | | | | $M_s \approx 7.9$ |
| $n_0$ | 1.83 | 9.3 | 0.0006 | 1.26 | 6.4 | 0.0003 |
| 275$n_0$ | 1.92 | 9.8 | 0.3 | 1.41 | 7.2 | 0.3 |
| 8250$n_0$ | 2.20 | 11.2 | 3.0 | 1.76 | 9.0 | 3.2 |
| 82,500$n_0$ | 2.32 | 11.8 | 54.7 | 1.94 | 9.9 | 74.4 |
| $M_s \approx 7.1$ | | | | $M_s \approx 6.8$ |
| $n_0$ | 1.34 | 6.8 | 0.008 | 1.00 | 5.1 | 0.0037 |
| 275$n_0$ | 1.47 | 7.5 | 0.1 | 1.12 | 5.7 | 0.4 |
| 8250$n_0$ | 1.71 | 8.7 | 3.0 | 1.54 | 7.9 | 10.0 |
| 82,500$n_0$ | 1.81 | 9.2 | 68.9 | 1.68 | 8.6 | 96.1 |
| $M_s \approx 4.3$ | | | | $M_s \approx 4.5$ |
| $n_0$ | 0.84 | 4.3 | 0.0067 | 0.68 | 3.5 | 0.015 |
| 275$n_0$ | 0.84 | 4.3 | 0.3 | 0.85 | 4.4 | 0.4 |
| 8250$n_0$ | 1.00 | 5.1 | 13.0 | 1.09 | 5.6 | 13.0 |
| 82,500$n_0$ | 1.06 | 5.4 | 56.6 | 1.18 | 6.0 | 128.0 |
| $M_s \approx 3.1$ | | | | $M_s \approx 3.2$ |
| $n_0$ | 0.49 | 2.5 | 0.016 | 0.57 | 2.9 | 0.0064 |
| 275$n_0$ | 0.57 | 2.9 | 0.5 | 0.61 | 3.1 | 0.1 |
| 8250$n_0$ | 0.73 | 3.7 | 20.0 | 0.80 | 4.1 | 14.0 |
| 82,500$n_0$ | 0.78 | 4.0 | 140.0 | 0.86 | 4.4 | 20.3 |
| $M_s \approx 0.7$ | | | | $M_s \approx 0.7$ |
| $n_0$ | 0.16 | 0.8 | 0.082 | 0.16 | 0.8 | 0.094 |
| 275$n_0$ | 0.18 | 0.9 | 3.8 | 0.17 | 0.9 | 4.0 |
| 8250$n_0$ | 0.24 | 1.2 | 71.0 | 0.21 | 1.1 | 81.0 |
| 82,500$n_0$ | 0.27 | 1.4 | 480.0 | 0.23 | 1.2 | 806.0 |

Notes. Each simulation has four different density values (given in Column 1) in order to probe varying optical depth. Columns 2–4 give cloud parameters for super-Alfvénic simulations while Columns 5–7 give parameters for sub-Alfvénic simulations. Those parameters are the one-dimensional (1D) velocity dispersion, estimated sonic Mach number (see Section 2) and the average optical depth of the cloud given by SimLine3D (see Osenkopf 2002). The actual $M_s$ as measured from the original MHD simulations with no radiative transfer is shown above each group of density scaling factors.

$^a$ Initial Alfvén Mach Number.

$^b$ Estimated $M_s$ from the measured linewidths.

$^c$ Actual $M_s$ from original MHD simulations.

In addition to the $^{13}$CO simulations, we create synthetic dust maps using the column density maps from our MHD simulations with the LoS taken perpendicular to the mean magnetic field. We scale our column density mean value of unity to $2 \times 10^{22}$ cm$^{-2}$ and then take a simple scaling law (see Bohlin et al. 1978) from column density to extinction as

$$N_H = 1.9 \times 10^{21} \text{cm}^{-2} A_v,$$  \hspace{1cm} (4)

where $A_v$ is the visual extinction.
Mach numbers from 13CO line maps and compare these with column density PDF distribution. An increase in sonic Mach number broadens the dust relationship as if it were calculated from the observations. We also calculate the PDFs of the synthetic dust extinction the actual sonic Mach numbers as measured in the simulations. (A color version of this figure is available in the online journal.)

Figure 1. Left panel: $^{13}$CO line profiles for the same supersonic super-Alfvénic simulation with different density values. The solid line is the actual $^{13}$CO line profile of densities $9n$ (black) and $2850n$ (blue) while the thinner dashed line is a three-term Gaussian fit using the native IDL routine GAUSSFIT. Right panel: synthetic dust column density PDF for simulations with $M_s \approx 8$ (black) and $M_s \approx 0.7$ (blue); thinner dashed line is the corresponding Gaussian fit for the PDF.

(A color version of this figure is available in the online journal.)

Figure 2. Measured $M_t$ from $^{13}$CO vs. actual $M_s$. Black solid line is a linear fit to the super-Alfvénic clouds; blue dashed line is for sub-Alfvénic clouds; red dot-dashed line is for no radiative transfer clouds. (A color version of this figure is available in the online journal.)

We utilize both our synthetic dust maps and $^{13}$CO line dispersion data to estimate the standard deviation–sonic Mach number relation as it would be calculated from the observations and compare this with data from KT13. We calculate the standard deviation using two different methods. One is direct calculation of the column density standard deviation ($\sigma_{\text{direct}}$) the sub-Alfvénic (blue lines) and super-Alfvénic (black lines) simulations separately with the $y$-intercept held to pass through the origin. We fit slopes to each optical depth regime and list the values in Table 2.

We find that the optically thin cases (triangle and diamond symbols) reproduce well the actual sonic Mach number from the measured one. However, for our highest optical depth cases the ratio between the observationally measured $M_t$ and the actual $M_s$ increases to between $1.16$–$1.3$ (fitted slopes are reported in Table 2). Additional line broadening due to opacity enlarges the discrepancy between the measured and actual $M_s$ as the optical depth increases. This occurs because the center of line emission becomes saturated and photons escape the cloud at the wings of the emission line. Indeed, CO opacity broadening can be explained by the curve of growth relationship as well as by numerous past studies (Lepine 1991; Leung & Brown 1977; Leung & Liszt 1976, see Table 2).

Additionally, we observe a magnetic field dependency: super-Alfvénic turbulence shows increased line broadening compared with sub-Alfvénic turbulence for all values of optical depth for supersonic turbulence. This effect may be due to the velocity dispersion of super-Alfvénic turbulence having a greater dependence on density fluctuations for supersonic turbulence (see Burkhart et al. 2009, Figures 9 and 10), which causes an additional broadening. This occurs regardless of the optical depth of the line (see Table 2). We investigated the effect of opacity broadening on a LoS parallel to the mean magnetic field and found results similar to those reported in Figure 2: the slopes of the measured versus actual $M_s$ covering the range of optical depth values were $1.06$ for sub-Alfvénic simulations and $1.28$ for super-Alfvénic simulations. This indicates that the anisotropy present in MHD turbulence is not responsible for the line broadening and instead the strength of the magnetic field is more important.

4. STANDARD DEVIATION–SONIC MACH NUMBER RELATION

We calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations.

3. SONIC MACH NUMBER FROM $^{13}$CO LINE PROFILES

Table 2

| Slope (Denoted as $c$) for the Measured $M_t$–Actual $M_s$ Data Fit |
|----------------------|---|---|
|                     | $c^a$ | $c^b$ |
| Super-Alfvénic      |      |      |
| $9n$                 | 1.25  | 0.99 |
| $275n$               | 1.19  | 1.06 |
| $8250n$              | 1.22  | 1.23 |
| $82,500n$            | 1.26  | 1.30 |
| Sub-Alfvénic        |      |      |
| $9n$                 | 0.84  | 0.80 |
| $275n$               | 0.80  | 0.90 |
| $8250n$              | 1.00  | 1.17 |
| $82,500n$            | 1.16  | 1.28 |

Notes. Column 2 is the slope for a fit including $M_s > 20$. Column 3 is for a fit with $M_s < 20$.

$^a$ Including $M_s > 20$.

$^b$ Only for $M_s < 20$.

We calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations. In the following sections, we calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations.

Figure 1, we show an example of the synthetic dust PDFs (with $A_v$ cut off of 7) of simulations with subsonic (supersonic) Mach number. An increase in sonic Mach number broadens the dust column density PDF distribution.

In the following sections, we calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations.

Notes. Column 2 is the slope for a fit including $M_s > 20$. Column 3 is for a fit with $M_s < 20$.

$^a$ Including $M_s > 20$.

$^b$ Only for $M_s < 20$.

Figure 2 plots $M_t$ as measured from the $^{13}$CO line profiles (measured $M_t$) versus actual $M_s$ of the simulations. We plot four different values of density, which effectively change the optical depth (see Table 1). We also overplot linear fits to the discrepancy between the measured and actual $M_s$ in Table 2). Additional line broadening due to opacity enlarges the discrepancy between the measured and actual $M_s$ as the optical depth increases. This occurs because the center of line emission becomes saturated and photons escape the cloud at the wings of the emission line. Indeed, CO opacity broadening can be explained by the curve of growth relationship as well as by numerous past studies (Lepine 1991; Leung & Brown 1977; Leung & Liszt 1976, see Table 2).

Additionally, we observe a magnetic field dependency: super-Alfvénic turbulence shows increased line broadening compared with sub-Alfvénic turbulence for all values of optical depth for supersonic turbulence. This effect may be due to the velocity dispersion of super-Alfvénic turbulence having a greater dependence on density fluctuations for supersonic turbulence (see Burkhart et al. 2009, Figures 9 and 10), which causes an additional broadening. This occurs regardless of the optical depth of the line (see Table 2). We investigated the effect of opacity broadening on a LoS parallel to the mean magnetic field and found results similar to those reported in Figure 2: the slopes of the measured versus actual $M_s$ covering the range of optical depth values were $1.06$ for sub-Alfvénic simulations and $1.28$ for super-Alfvénic simulations. This indicates that the anisotropy present in MHD turbulence is not responsible for the line broadening and instead the strength of the magnetic field is more important.

4. STANDARD DEVIATION–SONIC MACH NUMBER RELATION

We calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations. In the following sections, we calculate the observed sonic Mach numbers from $^{13}$CO line maps and compare these with the actual sonic Mach numbers as measured in the simulations. We also calculate the PDFs of the synthetic dust extinction maps and examine the standard deviation–sonic Mach number relationship as if it were calculated from the observations.

Notes. Column 2 is the slope for a fit including $M_s > 20$. Column 3 is for a fit with $M_s < 20$.

$^a$ Including $M_s > 20$.

$^b$ Only for $M_s < 20$.
divided by the mean value as

$$\sigma_{N/(N)} = \sqrt{\frac{1}{N} \sum_{i=1}^{n} \left( \frac{N_i}{N} - \langle N/N \rangle \right)^2}.$$  \hspace{1cm} (5)

The other method assumes a log-normal distribution and calculates the variance ($\sigma_{\ln N}$) and mean ($\mu$) values of the logarithm of the extinction maps by fitting a Gaussian distribution, illustrated in Figure 1, right panel. This method does not assume that we know a priori the actual mean value of the extinction distribution, which is indeed the case in the observations; however, the assumption of log normality is not always appropriate for all molecular clouds as gravity creates deviations from the log-normal distribution (see Collins et al. 2012). From the fit we are able to determine the parameters $\mu$ and $\sigma_{\ln N}^2$, which are then used to calculate the mean ($\langle N \rangle$) and standard deviation ($\sigma_N$) of the corresponding log-normal distribution as

$$\langle N \rangle = e^{\mu + \sigma_{\ln N}^2/2}$$  \hspace{1cm} (6)

and

$$
\sigma_N = \left[\left(e^{\sigma_{\ln N}^2} - 1\right)e^{2\mu + \sigma_{\ln N}^2}\right]^{1/2}.
$$  \hspace{1cm} (7)

For both methods, we calculate the standard deviation for an extinction distribution with an $A_v$ cut off of $\approx 7$ for compatibility with the method used in KT13 (hence we denote these as $\sigma_{\ln N}$ and $\sigma_{\ln N, \text{cut}}$). We also investigate the standard deviation calculations for the complete $A_v$ distribution. This is not available from observations; however, it is interesting to see how large of a difference in the standard deviation is observed between this idealized case and a more realistic distribution with a low $A_v$ cut off.

We find that there is not a substantial difference between the values of the standard deviation calculated either with Equation (5) or Equations (6) and (7). This is expected for our data since we are dealing with log-normal distributions in the case of pure isothermal MHD turbulence. The differences between $\sigma_{\ln N}$ and $\sigma_{\text{direct}}$ extend up to values of 0.15. We additionally find that there is not a large difference between the standard deviation of the full column density distribution and the column density distribution which employs a cut off value of $A_v = 7$. The differences between $\sigma_{\ln N}$ and $\sigma_{\text{direct}}$ with a cut off value of $A_v = 7$ extend up to values of 0.13. However, for a subset of real clouds, it may be difficult to discern if one is dealing with a true log-normal distribution or the beginnings of a high density power-law tail and thus the difference in using direct calculation versus a log-normal fit to calculate the standard deviation of the column density in observational clouds might be high.

Figure 3 shows the column density dispersion for four different methods of calculating the standard deviation: $\sigma_{\text{direct}}$, $\sigma_{\text{dir, cut}}$, $\sigma_{\ln N}$, and $\sigma_{\ln N, \text{cut}}$ (direct calculation, direct calculation with an $A_v = 7$ cut off, log-normal fit, and log-normal fit with an $A_v = 7$ cut off, respectively) versus estimated $M_f$ for sub-Alfvénic (top panel) and super-Alfvénic cases (bottom panel). The values of $M_f$ plotted are the average $M_f$ calculated from the linewards of the four density cases with error bars representing the standard deviation between the values. We perform a linear fit to the data, as used in KT13,

$$\sigma_{N/(N)} = a_1 \times M_f + a_2,$$

where $a_1$ is the slope between the sonic Mach number and column density dispersion and $a_2$ is the intercept.

Along with the synthetic observations presented in this Letter, Figure 3 overplots data from the IRDCs presented in KT13 (their Figure 7, left frame, $M_f$ from a Gaussian fit, and reported in KT13’s Table 1) which are shown with red diamonds. The KT13 values of $\sigma_{N/(N)}$ plotted here are calculated by direct calculation, and thus are most compatible with our method of direct calculation with an $A_v = 7$ cut off (blue triangles). Additionally, we include the predicted relation for the column density standard deviation–sonic Mach number relation from BL12, Equation (4) with $b = 1/3$ for solenoidal mixing and $A = 0.11$:

$$\sigma_{N/(N)} = \sqrt{\left(b^2M_s^2 + 1\right)^A - 1}.$$  \hspace{1cm} (9)

The observational points of KT13 consistently show higher values of $\sigma$ than the MHD simulations, regardless of the method of calculation for the standard deviation or the Alfvén Mach number. Upon further inspection, the KT13 points also have a spread in the values of $\sigma$ for a given sonic Mach number, suggesting that other physics might come into play in the interpretation of the variance of the data. Processes that could create larger values of variance than expected from solenoidally driven turbulence include gravitational contraction (Collins et al. 2012) and compressive forcing (Federrath et al. 2010; Kainulainen et al. 2013).

Comparing the mean value of the slopes for the cases of direct calculation of the standard deviation with the $A_v$ cut off, $a_1 = 0.0103 \pm 0.0047 (0.0136 \pm 0.0050$ for sub-Alfvénic; $0.0069 \pm 0.0044$ for super-Alfvénic), with KT13’s Figure 7, left frame, which reports $a_1 = 0.0095 \pm 0.0066$, we see that there is a good agreement. We see significantly less agreement of our $a_1$ values with the values of standard deviation derived from the fitted log-normals, $a_1 = 0.0110 \pm 0.0067 (0.0133 \pm 0.0089$ for sub-Alfvénic; $0.0086 \pm 0.0046$ for super-Alfvénic), compared to those of KT13 (their Figure 8, left frame), $a_1 = 0.051 \pm 0.018$. The discrepancy may be due to PDF broadening by gravity in the observations (i.e., an unresolved power-law tail), thus distorting the standard deviation’s relationship with the sonic Mach number.

The other possible explanation for a steeper relation could be that the driving in the KT13 data is compressive forcing.
however, they derived an expression for $b$,\
\[ b = \frac{\sigma_{\nu}(\nu)}{M_s} = \frac{\sigma_N|\nu|}{M_s} R^{-1/2} = a_1 \times R^{-1/2}, \] (10)

where $R$ is the 3D–2D variance ratio and was found to be between $R = [0.03, 0.15]$ (Brunt 2010). KT13 estimated a $b$ value with a $3\sigma$ uncertainty of $b = 0.20^{+0.37}_{-0.22}$, which indicates that the driving is generally solenoidal to mixed $(b = 1/3$ for solenoidal and $b = 1$ for compressive driving; Federrath et al. 2008).

However, it is clear that applying a sonic Mach number opacity correction in the case of high optical depths will increase the measured value of $b$, since the slope $a_1$ will steepen by the correction factor. In the case of high optical depth this factor is as high as $\approx 1.3$. Thus the slope, $a_1$, steepens by this factor and, assuming the observations are optically thick, the value of $b$ calculated by KT13 will increase to $b = 0.25^{+0.25}_{-0.15}$. These values still indicate solenoidal to mixed driving. Hence the most likely explanation for the higher variance values in KT13 as compared with simulations is the influence of gravity. This is not unexpected, as Collins et al. (2012, Table 1) shows that the variance measured from the PDFs in 3D density increases as gravity acts on the cloud.

5. DISCUSSION

The models of star formation must invoke stirring by turbulence, including collecting matter by compressible turbulent motions (see McKee & Ostriker 2007 and references therein) and the removal of magnetic flux from the collapsing region by the process of reconnection diffusion, which is much faster than the traditionally considered process of ambipolar diffusion (Lazarian et al. 2012). This motivates observational quantitative studies of turbulence and this Letter is a part of such studies.

This Letter studies the effect of self-absorption on the observational measurement of the sonic Mach number as measured from $^{13}$CO linewidths with a range of densities and optical depths. $^{13}$CO line broadening and extinction PDFs employed in this Letter to study the sonic Mach number are in no way the only methods available to researchers to study turbulence in molecular clouds. Sonic Mach numbers can be also successfully obtained with the kurtosis and skewness of the PDF distribution (Kowal et al. 2007;Burkhart et al. 2009, 2010). The power spectra of density and velocity can be obtained with velocity channel analysis and velocity coordinate spectrum techniques (Lazarian 1999, 2004, 2006) which were successfully tested numerically (see Chepurnov et al. 2008) and applied to HI and CO data sets (see Padoan et al. 2009; Chepurnov et al. 2010; Lazarian 2009, for a review). In view of the complexity of astrophysical turbulence, simultaneous use of the combination of these techniques presents an unquestionable advantage.

Our empirical finding is that the evaluation of the Mach number from the linewidth is possible even for strongly self-absorbing species, but a correction factor should be applied for opacity broadening. For the range of absorption depths that we studied we found that this correction factor is at most 1.3. It also corresponds to the earlier one-dimensional studies of spectral line broadening in the presence of absorption in, e.g., Leung & Brown (1977) and Leung & Liszt (1976). An additional consideration is that IRDCs are cold ($T = 10–40$ K) and are likely to be close to isothermal, but if instead of 10 K the cloud had a different temperature there would be a factor of $1/\sqrt{T}$ change in $M_s$.

6. CONCLUSIONS

We create synthetic $^{13}$CO emission maps, with varying optical depth, and dust column density maps from a set of 3D MHD simulations. We derive a standard deviation–sonic Mach number relation, as found from the observations, and compare this with recent results from real clouds discussed in KT13. We find the following.

1. Calculations of $M_s$ from linewidths of $^{13}$CO are robust for optically thin $^{13}$CO but are overestimated by a factor of up to $\approx 1.3$ for optically thick clouds. This is due to the well-known, but often overlooked, effect of opacity broadening.
2. This overestimation of the sonic Mach number as derived from $^{13}$CO linewidths will cause the slope of the $\sigma_N|\nu|/M_s$ relation to become shallower and this will result in lower values of the measured $b$ parameter.
3. The $\sigma_N|\nu|$ values of clouds reported in KT13 are larger than values found from ideal solenoidally driven simulations of turbulence. This could be due to the fact that in real molecular clouds gravitational influences exist, which will increase the measured column density standard deviation.

We thank the anonymous referee for helpful comments. This work is supported by continuous grants from INEsPaço/FAPERN/CNPq/MCT. C.C. acknowledges a graduate PDSE/CAPES grant Process n°3932/13–0. B.B. acknowledges support from the Wisconsin Space grant. A.L. and B.B. acknowledge the Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas and the IIP/UFRN (Natal) for hospitality. V.O. and J.S. acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) project n°30S ~ 177/2 – 1. V.O., J.S., and J.K. were supported by the central funds of the DFG-priority program 1573 (ISM-SPP).

REFERENCES

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Beresnyak, A., Yan, H., & Lazarian, A. 2011, ApJ, 728, 60
Balogh, R. C., Savage, B. D., & Drake, J. F. 1978, ApJ, 224, 132
Brunt, C. M. 2006, A&A, 445, 281
Chepurnov, A., Lazarian, A., Stanimirovi´c, S., Heiles, C., & Peek, J. E. G. 2010, ApJ, 714, 1398
Cho, J., & Lazarian, A. 2003, MNRAS, 345, 325
Collins, C. D., Kritsuk, A. G., Padoan, P., et al. 2012, ApJ, 750, 13
Esquivel, A., & Lazarian, A. 2010, ApJ, 710, 125
Federrath, C., Klessen, R. S., & Schmidt, W. 2008, ApJ, 688, L79
Federrath, C., Roman-Duval, J., Klessen, R. S., Schmidt, W., & Mac Low, M.-M. 2010, A&A, 513, A67
Gazis, B. M., Haverkorn, M., Burkhardt, B., et al. 2011, Natur, 478, 214
Goldsmith, P. F., Heyer, M., Narayanan, G., et al. 2008, ApJ, 680, 428

1 One can also determine the Alfvénic Mach number $M_a$ using different contours of isocorrelation obtained with velocity centroids (see Esquivel & Lazarian 2010 and references therein), Tsallis statistics (Lazarian et al. 2012; Tofflemire et al. 2011), and bispectrum (Burkhart et al. 2010, 2013b).
Goodman, A. A., Rosolowsky, E. W., Borkin, M. A., et al. 2009, Natur, 457, 63
Heyer, M. H., Brunt, C., Snell, R. L., et al. 1998, ApJS, 115, 241
Hill, A. S., Benjamin, R. A., Kowal, G., et al. 2008, ApJL, 686, 363
Kainulainen, J., Beuther, H., Banerjee, R., Federrath, C., & Henning, T. 2011, A&A, 530, A64
Kainulainen, J., Federrath, C., & Henning, T. 2013, A&A, 553, L8
Kainulainen, J., & Tan, J. C. 2013, A&A, 549, A53 (KT13)
Klessen, R. S., Heitsch, F., & Mac Low, M.-M. 2000, ApJ, 535, 887
Kowal, G., Falceta-Gonçalves, D. A., & Lazarian, A. 2011, NJPh, 13, 053001
Kowal, G., Lazarian, A., & Beresnyak, A. 2009, ApJ, 658, 423
Kowal, G., Lazarian, A., Vishniac, E. T., & Otmianowska-Mazur, K. 2009, ApJ, 700, 63
Lazarian, A. 2009, SSRv, 143, 357
Lazarian, A., Esquivel, A., & Crutcher, R. 2012, ApJ, 757, 154
Lazarian, A., & Pogosyan, D. 1999, BAAS, 31, 1449
Lazarian, A., & Pogosyan, D. 2004, ApJ, 614, 943
Lazarian, A., & Pogosyan, D. 2006, ApJ, 652, 1348
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Lepine, J. R. D. 1991, in IAU Symp. 147, Fragmentation of Molecular Clouds and Star Formation, ed. E. Falgarone, F. Boulanger, & G. Duvert (Dordrecht: Kluwer), 451
Leung, C. M., & Brown, R. L. 1977, ApJL, 214, L73
Leung, C.-M., & Liszt, H. S. 1976, ApJ, 208, 732
Lombardi, M., & Alves, J. 2001, A&A, 377, 1023
Mckee, C. F., & Ostriker, E. C. 2007, ARA&A, 45, 565
Molina, F. Z., Glover, S. C. O., Federrath, C., & Klessen, R. S. 2012, MNRAS, 423, 2680
Ossenkopf, V. 2002, A&A, 391, 295
Padoan, P., Jones, B. J. T., & Nordlund, A. P. 1997, ApJ, 474, 730
Padoan, P., Juvela, M., Kritsuk, A., & Norman, M. L. 2009, ApJL, 707, L153
Peek, J. E. G., Heiles, C., Peek, K. M. G., Meyer, D. M., & Lauroesch, J. T. 2011, ApJL, 735, 129
Tollefjør, B. M., Burkhardt, B., & Lazarian, A. 2011, ApJ, 736, 60
Vazquez-Semadeni, E. 1994, ApJ, 423, 681
Yan, H., & Lazarian, A. 2004, ApJ, 614, 757