Application Driven Joint Uplink-Downlink Optimization in Wireless Communications

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Abstract—This paper introduces a new mathematical framework which is used to derive joint uplink/downlink achievable rate regions for multi-user spatial multiplexing between one base station and multiple terminals. The framework consists of two models: the first one is a simple transmission model for uplink (UL) and downlink (DL), which is capable to give a lower bound on the capacity for the case that the transmission is subject to imperfect channel state information (CSI). A detailed model for concrete channel estimation and feedback schemes provides the parameter input to the former model and covers the most important aspects such as pilot design optimization, linear channel estimation, feedback delay, and feedback quantization.

We apply this framework to determine optimal pilot densities and CSI feedback quantity, given that a weighted sum of UL and DL throughput is to be maximized for a certain user velocity. We show that for low speed, and if DL throughput is of particular importance, a significant portion of the UL should be invested into CSI feedback. At higher velocity, however, DL performance becomes mainly affected by CSI feedback delay, and hence CSI feedback brings little gain considering the inherent sacrifice of UL capacity. We further show that for high velocities, it becomes beneficial to use no CSI feedback at all, but apply random beamforming in the DL and operate in time-division multiplex.

I. INTRODUCTION

A. Motivation

Mobile communication systems provide a diversity of high-quality mobile applications and services, which require a multitude of operation modes. Each mode is characterized among others by its requirements on latency, packet error rate, and supported rate. To satisfy these demands, currently deployed systems use different transport protocols, coding schemes, and modulation schemes. Furthermore, radio resource management (RRM) algorithms usually optimize the UL and DL data rate independently.

Consider a typical file download, which requires an UL-DL throughput ratio of $R_{UL}/R_{DL} \ll 1$. In the context of multi-user MIMO, it is known that a strong DL requires CSI feedback from the user terminal (UT) side to the base station (BS) side, where precise multi-user precoding for spatial multiplexing can then be performed. By contrast, the upload of files and real-time video-streaming (for instance for remote surveillance) require a stronger UL than DL, which results in $R_{UL}/R_{DL} \gg 1$. While the former examples represent asymmetric services, voice-over-IP or video-conferencing have symmetric rate demands, reflected by $R_{UL}/R_{DL} = 1$.

In today's mobile communication systems, all three service classes use the same physical layer mode, although they have very contrary demands, and satisfy these by individual resource scheduling in UL and DL. In this work, we alternatively consider an application-driven multi-cross-layer approach, which jointly optimizes both UL and DL not only on the upper layers but also on physical layer.

B. Outline of Main Contribution

This paper presents

1) a simplified model of an UL and DL transmission over a frequency-flat channel for capacity calculation under CSI imperfectness at BS and UT side,

2) a detailed model that yields the extent of CSI imperfectness for a concrete OFDM-based channel estimation and CSI feedback scheme, applied to a channel with a certain dispersiveness in time and frequency, and

3) an UL/DL tradeoff discussion, which analyzes the best choice of pilot density and CSI feedback amount, given that a weighted sum-rate of UL and DL is to be maximized.

\[ \text{Fig. 1. Information flow between uplink and downlink.} \]
In general, we consider the communication between a BS with \( N_{BS} \) receive and transmit antennas, and \( K \) UTs with one receive and transmit antenna each, as depicted in Fig. 1. The set of UTs is defined by \( \mathcal{K} = \{ 1..K \} \). Within this setup, we are particularly interested in optimizing the following parameters:

- the pilot density \( \rho_{UL} \in \mathbb{R}^+ \) in the UL,
- the pilot density \( \rho_{DL} \in \mathbb{R}^+ \) in the DL, and
- the amount of CSI feedback \( N_d \in \mathbb{R}^+ \) in bits per channel coefficient and physical resource block (PRB).

We generally assume a scenario-dependent optimized layout of UL and DL pilots in time and frequency, so that the pilot densities \( \rho_{UL} \) and \( \rho_{DL} \) are sufficient as optimization parameters. Fig. 1 shows the information flow in the considered system. The right side illustrates the UL and the left side the DL. Via MMSE channel estimation, each UT generates both a channel estimate \( \hat{h}_{DL}^{[t]} \) for data detection as well as a channel prediction \( \hat{h}_{UL}^{[t+N_d]} \) for CSI feedback. The latter is quantized to \( \hat{h}_{UL}^{[t+N_d]} \), introducing quantization noise \( n_q \), before it is sent by the UL to the BS (introducing a delay of \( N_d \) transmission time interval (TTI)). At the BS side, an MMSE estimator yields the channel estimate \( \hat{h}_{UL}^{[t]} \), which is used to decode the UL transmission. This includes the CSI feedback from the UT side, which is then used to perform linear precoding to improve the DL throughput.

C. Previous Work

To the best of the authors’ knowledge, this is the first work considering a model which incorporates all major aspects of imperfect channel state information in a bi-directional, multi-user wireless communication system. The problem of imperfect CSI at the transmitter as a function of the CSI at the receiver has been first considered by Caire and Shamai in [1]. In [2], Santipach and Honig considered both imperfect channel estimation and quantized channel feedback. More recently, Kobayashi et al. [3] analyze a system where the DL throughput depends on the channel estimation at the receiver and the amount of feedback, which is constrained by a given UL capacity (independent of the channel estimation at the BS). While [3] assumes a channel static during both UL and DL, the same authors consider FDD models in [4], which are also in the focus of this work. More specifically, [4] analyzes the performance of digital and analog feedback, evaluates also in the focus of this work. More specifically, [4] analyzes the performance of digital and analog feedback, evaluates the jointly achievable UL/DL rate region, and considers user scheduling in addition to a feedback optimization.

D. Outline of Paper

In Section II we introduce a simplified transmission model for UL and DL which yields lower bounds on the capacity for transmission under imperfect CSI. A detailed model for channel estimation and CSI feedback will be introduced in Section III and provides the parameter values for the simpler model in Section II. We use both models in Section IV to discuss the joint optimization of UL and DL throughput and in Section V to evaluate the individual parameter values, which achieve optimal sum-rate under a given UL/DL throughput ratio. The paper is concluded in Section VI.

II. SIMPLIFIED MODEL FOR CAPACITY CALCULATION

In our simplified model for capacity calculation, we assume that all entities are perfectly synchronized in time and frequency and that transmission takes place over a frequency-flat channel. We generally assume that all involved signals are realizations of Gaussian processes.

A. Uplink

In the UL, we face a multiple access channel (MAC), and model the transmission of each symbol as

\[
\mathbf{y} = \mathbf{H}^{UL} \mathbf{x} + \mathbf{n}, \tag{1}
\]

where \( \mathbf{y} \in \mathbb{C}^{[N_{BS} \times 1]} \) are the signals received at the BS antennas, \( \mathbf{H}^{UL} \in \mathbb{C}^{[N_{BS} \times K]} \) is the channel matrix, \( \mathbf{x} \in \mathbb{C}^{[K \times 1]} \) are the signals transmitted from the K terminals with \( \mathbf{P} = E \{ \mathbf{x} \mathbf{x}^H \} \), and \( \mathbf{n} \sim \mathcal{N}(0, \sigma^2_{UL} \mathbf{I}) \) is receiver-side noise. The UL is subject to a per-UT power constraint which we state as \( \forall k \in \mathcal{K} : E \{ x_k^2 \} \leq P_{UL}^\max \). Let the channel knowledge at the BS side be

\[
\hat{\mathbf{H}}^{UL} = \mathbf{H}^{UL} + \mathbf{E}^{UL}, \tag{2}
\]

which corresponds to the actual channel plus a channel estimation error \( \mathbf{E}^{UL} \in \mathbb{C}^{[N_{BS} \times K]} \). We further assume that all entries in \( \mathbf{E}^{UL} \) are uncorrelated Gaussian variables with \( E \{ \text{vec} (\mathbf{E}^{UL}) \text{vec} (\mathbf{E}^{UL})^H \} = \sigma^2_{UL,BS} \mathbf{I} \). The latter variance can be obtained through the Kramer-Rao lower bound [5] as \( \sigma^2_{UL,BS} = \sigma^2_{UL,BS} / (N_{pilots} \cdot P_{pilots}) \), if channel estimation has been performed based on the transmission of \( N_{pilots} \) pilots of power \( P_{pilots} \). It has been shown in [6] that we can find an inner bound on the capacity region connected to the transmission in (1) by observing a modified transmission

\[
\mathbf{y} = \mathbf{H}^{UL,eff} \mathbf{x} + \mathbf{n}, \tag{3}
\]

where the channel is reduced in power to an effective channel

\[
\forall i, j: \hat{h}_{UL,eff}^{i,j} = \frac{h_{UL}^{i,j}}{\sqrt{1 + \sigma^2_{UL,BS} / E \{ \| \mathbf{h}_{UL}^{i,j} \|^2 \}}} \tag{4}
\]

and with an additional channel estimation related noise term

\[
E \{ \mathbf{v} \mathbf{v}^H \} = \text{diag} \left( \text{diag} (\mathbf{E}^{UL,eff} \mathbf{P} (\mathbf{E}^{UL,eff})^H) \right),
\]

where

\[
\forall i, j: \hat{\mathbf{e}}^{UL,eff}_{i,j} = \sqrt{\frac{E \{ \| \mathbf{h}_{UL}^{i,j} \|^2 \} - \sigma^2_{UL,BS}}{E \{ \| \mathbf{h}_{UL}^{i,j} \|^2 \} + \sigma^2_{UL,BS}}}. \tag{5}
\]

Briefly, the derivation of (3) and (5) is based on the fact that the channel estimation noise is treated as a Gaussian variable, leading to an overestimation of its detrimental impact [7].

The sum-rate of all UTs can now be lower-bounded as [6]

\[
R_{UL} \leq \max_{\mathbf{P}} \log_2 \left( 1 + \Phi^{-1} \mathbf{H}^{UL,eff} \mathbf{P} (\mathbf{H}^{UL,eff})^H \right) \tag{6}
\]

with

\[
\Phi = \sigma^2_{UL} \mathbf{I} + \text{diag} \left( \text{diag} (\mathbf{E}^{UL,eff} \mathbf{P} (\mathbf{E}^{UL,eff})^H) \right). \tag{7}
\]

Note that (6) requires optimization over all power allocations \( \mathbf{P} \) that fulfill the individual power constraints. Under
perfect CSIR, the sum-rate maximizing strategy is to let all UTs transmit at maximum power, which, however, is not necessarily the case under imperfect CSIR, as the channel estimation related noise term in (7) depends on $P$. 

**B. Downlink**

The DL corresponds to a broadcast channel (BC), where the transmission of each symbol can be stated as

$$y = (H_{DL}^H) s = (H_{DL}^H) W x + n,$$  

(8)

where $y \in \mathbb{C}^{[K \times 1]}$ are now the signals received by the UTs, $H_{DL} \in \mathbb{C}^{[N_{BS} \times K]}$ is the DL channel matrix, $W \in \mathbb{C}^{[N_{BS} \times K]}$ is a precoding matrix, $x \in \mathbb{C}^{[K \times 1]}$ are signals to be transmitted to the $K$ UTs, and $n \sim N(0, \sigma^2_{DL,UT})$ is UT-side noise. The DL is usually different from the UL channel $H_{UL}$ due to different frequencies and hence different path loss and scattering. We consider a sum-power constraint $\text{tr} \{E\{ss^H\}\} \leq P_{DL}^\text{tot}$, and assume that the UTs have the (distributed) channel estimate

$$\hat{H}_{DL} = H_{DL} + E_{DL,UT},$$  

(9)

with $E\{\text{vec}(E_{DL,UT})\text{vec}(E_{DL,UT})^H\} = \sigma^2_{DL,UT}$, and that the BS side has an even noisier channel estimate

$$\hat{H}_{DL} = \sqrt{\alpha} (H_{DL} + E_{DL,UT}) + E_{DL,BS},$$  

(10)

with $E\{\text{vec}(E_{DL,BS})\text{vec}(E_{DL,BS})^H\} = \sigma^2_{DL,BS}$. Scaling factor $\alpha$ ensures that the power of the channel estimate $\hat{H}_{DL}$ at the BS side corresponds to that of $H_{DL}$ again [8]. The model is motivated through the fact that in an FDD system, CSI at the transmitter side can only be obtained through feedback from the receiver side. Hence, it is always strictly less accurate (due to quantization and delay) than at the receiver side. Again it is possible to model the impact of imperfect CSI through the observation of a modified transmission equation [9].

Controlable term

$$y = \hat{H}_{DL,\text{eff}}^\text{DL} W x + v_{\text{UE}} + v_{\text{BS}} + n,$$  

with $\forall \ i, j : h_{L,i,j}^{\text{DL,eff}} = h_{L,i,j}^{\text{DL}},$

$$v_{\text{UE}} \sim N_\mathcal{C}(0, \Delta \text{E}(DL_{\text{UE}}F_{\text{UE}}(E_{DL,UT})^H)),$$

$$v_{\text{BS}} \sim N_\mathcal{C}(0, \Delta \text{E}(DL_{\text{BS}}F_{\text{BS}}(E_{DL,BS})^H)), $$

$$\forall i, j : \epsilon_{\text{DL,BS}}^{\text{DL,BS}} = \sqrt{\frac{\sigma^2_{DL,BS} \{ \text{E} \{ |h_{L,i,j}^{\text{DL}}|^2 \} \} \}} + \sigma^2_{DL,UT},$$

$$\forall i, j : \epsilon_{\text{DL,UT}}^{\text{DL,UT}} = \sqrt{\frac{\text{E} \{ |h_{L,i,j}^{\text{DL}}|^2 \} \} \sigma^2_{DL,UT}} + \sigma^2_{DL,UT}.$$  

(11)

Here, $v_{\text{UE}}$ is a noise term related to imperfect CSI at BS and UT side, while $v_{\text{BS}}$ is connected to additional CSI imperfection at the BS side [9]. Note that, as in the UL, the modified transmission equation in (11) implies that statistical knowledge on the channel and estimation error is given at both BS and UT side. While direct capacity region calculation in a BC is tedious, UL/DL duality [10] can strongly facilitate this, and is also applicable in the context of imperfect CSI [9]. We are then facing a dual UL transmission under a sum power constraint, where the sum rate is given as in (12), optimized over the dual UL power $p \in \mathbb{R}_+^{1 \times K}$ s.t. $\tilde{p}^\top 1 \leq P_{UL}^\text{tot}$. In the denominator of the large fraction in (12), the first term is due to inter-UT interference, the second due to imperfect CSI at receiver and transmitter side, and the third due to additional CSI imperfection at the BS side.

**C. TDM as an Alternative in the DL**

In (11), the power of the effective channel goes to zero as the CSI at the transmitter side diminishes. However, our model does not capture the fact that the system can always operate in time division multiplexing (TDM) and perform random beamforming to each UT successively. The average sum-rate achievable with TDM is given as

$$R_{DL,TD} \leq \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\text{E} \{ |h_{k}^{DL,\text{eff},TD}|^2 \} \} \sigma^2_{TD,k} + \sigma^2_{DL} \right).$$  

(13)

where $h_{k}^{DL,\text{eff},TD}$ is again a power-reduced effective channel, and $\sigma^2_{TD,k}$ is a noise term connected to imperfect receiver-side channel knowledge at UT $k$, given as $\forall k \in K$ :

$$h_{k}^{DL,\text{eff},TD} = \sqrt{\frac{P_{DL}^\text{tot}}{N_{BS}}} \frac{\text{E} \{ |h_{k}^{DL}|^2 \} \} \text{E} \{ |h_{k}^{DL}|^2 \} \} + \sigma^2_{DL,UT},$$

$$\sigma^2_{TD,k} = \frac{\text{E} \{ |h_{k}^{DL}|^2 \} \} \text{E} \{ |h_{k}^{DL}|^2 \} \} + \sigma^2_{DL,UT}.$$  

(14)

A special aspect of TDM is that only one channel coefficient has to be estimated by each UT, namely the coefficient connected to the effective channel after random precoding at the BS side, reducing the pilot overhead in the DL. In the remainder of this work, we will always consider both instantaneous spatial multiplexing in the DL as well as TDM, and choose the better of both for any given scenario. Clearly, the value of sum-rate terms obtained through (6), (12) and (13) depends strongly on the choice of terms $\sigma^2_{UL,BS}$, $\sigma^2_{DL,UT}$ and $\sigma^2_{DL,BS}$, which in a practical system depend on the exact channel estimation and CSI feedback scheme as well as on the terminal speed $v$ and maximum delay spread $\tau_{max}$. We hence require a lookup table providing

$$[\sigma^2_{UL,BS}, \sigma^2_{DL,UT}, \sigma^2_{DL,BS}] = f (\rho_{UL}, \rho_{DL}, N_b, v, \tau_{max}).$$  

(15)

**III. DETAILED MODEL FOR CHANNEL ESTIMATION AND CSI FEEDBACK**

To obtain (15), let us consider a particular OFDMA system as it is used in the DL of LTE Release 8 [11], with a symbol rate $f_s = 14$ kHz and a sub-carrier spacing $\Delta F = 15$ kHz. For simplicity, we assume that OFDMA is also used in the UL (rather than SC-FDMA). Channel estimation is performed individually for each PRB spanning $N_s = 14$ OFDM symbols
times $N_c = 12$ sub-carriers, hence 168 channel accesses. The channel estimation performance could be improved using a larger observation window, which raises complexity issues. CSI feedback is also performed on a PRB basis, but with the option of using successive schemes that exploit the channel correlation over multiple TTIs.

See the detailed channel estimation and CSI feedback model in Fig. 2. Different from before, we now consider the vector $\hat{h}[t] \in \mathbb{C}^{[N_t,N_c] \times 1}$, which stacks all channel realizations connected to the link between one BS antenna $a$ and one UT $k$ for all channel accesses within a PRB at time $t$ into one vector. As we assume all links to be uncorrelated, channel estimation and CSI feedback have to be performed individually for each channel coefficient, where we omit the indices $a$ and $k$ in our notation for brevity. Matrix $S \in \{0,1\}^{[N_t,N_c \times N_{pos}]}$ indicates the $N_{pos}$ pilot positions within the PRB. Channel estimation is assumed to be subject to uncorrelated Gaussian noise $n \sim \mathcal{N}_C(0,\sigma_n^2I)$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Detailed channel estimation and CSI feedback model.}
\end{figure}

A. Channel Estimation

At the receiver side, a channel estimate $\hat{h}[t] \in \mathbb{C}^{[N_t,N_c] \times 1}$ for each OFDM symbol in a PRB at time $t$ is obtained by applying an MMSE filter $G$ to the received pilot symbols:

$$\hat{h}[t] = G \left( Sh[t] + n \right)$$

where the filter matrix is given as [12], [13]

$$G = \Phi_{hh} (0) S^H (\Phi_{hh} (0) S^H + \sigma_p^2 I)^{-1}.$$  

Under the assumption of a wide-sense stationary uncorrelated scattering (WSSUS) channel fading model [14], the filter $G$ exploits the correlation of $h$ in time and frequency, and is given as [12], [13]

$$\Phi_{hh} (N_d) = E \left\{ h[t-N_d] h[t] \right\} = E \left\{ |h|^2 \right\} \cdot (\Pi_T (N_d) \otimes \Pi_F),$$

$$\Phi_{hh} (0) S^H (\Phi_{hh} (0) S^H + \sigma_p^2 I)^{-1}.$$  

where $E\{|h|^2\}$ is the variance of the channel coefficients, $\otimes$ denotes the Kronecker product, and $\Pi_T (N_d)$ and $\Pi_F$ are given in [20] and [21] on the next page. Here, $J_0(\cdot)$ is the zero-th order Bessel function of the first kind, $\sin(x)$ denotes $\sin(x)/x$, and the maximum Doppler frequency is given as $f_D = f_c \cdot v/c$ [14], where $f_c$ is the carrier frequency, $v$ the UT speed, and $c$ the speed of light. $N_{q}$ denotes the delay (in TTIs) between the PRB, which is used to estimate the channel, and the PRB to which the estimate is applied at the transmitter. This delay will be of particular interest in the context of CSI feedback later, but is set to zero for the moment. The OFDM-symbol-wise mean-square error (MSE) of the obtained channel estimates can now be stated as [12], [13], [15], [16]

$$\text{MSE}_{\text{CSIR}} = \text{diag} \left( E \left\{ (\hat{h}[t] - h[t]) (\hat{h}[t] - h[t])^H \right\} \right)$$

$$= E \left\{ |h|^2 \right\} - \text{diag} \left( \Phi_{hh} (0) S^H (\Phi_{hh} (0) S^H + \sigma_p^2 I)^{-1} \Phi_{hh} (0) \right).$$  

Typically, the MSE of the outer OFDM symbols is worse than that between pilot positions. Since the badly estimated OFDM symbols have a dominant effect on the overall transmission, we calculate a representative value for $\sigma_{DL,UT}^2$ as

$$\sigma_{DL,UT}^2 = \max \left\{ \text{MSE}_{\text{CSIR}} \right\} - \max \left( \text{MSE}_{\text{CSIR}} \right).$$  

B. Channel Prediction and CSI Feedback

As depicted in Fig. 2 the pilots received at the UT side are also used to obtain a channel estimate $\hat{h}[t+N_d]$ that predicts the channel $N_d$ TTIs into the future, assuming the feedback process itself consumes $N_d$ TTIs. The channel prediction is achieved by the modified MMSE filter

$$G_P = \Phi_{hh} (N_d) S^H (\Phi_{hh} (0) S^H + \sigma_p^2 I)^{-1}. \Phi_{hh} (N_d).$$  

The (again OFDM-symbol-wise) MSE between the predicted channel estimate and the actual channel in the corresponding TTI of interest is given as

$$\text{MSE}_{\text{CSIR},P} = \text{diag} \left( E \left\{ (\hat{h}[t] - h[t]) (\hat{h}[t] - h[t])^H \right\} \right)$$

$$= E \left\{ |h|^2 \right\} - \text{diag} \left( \Phi_{hh} (N_d) S^H (\Phi_{hh} (0) S^H + \sigma_p^2 I)^{-1} \Phi_{hh} (N_d) \right),$$

which is equivalent to (22), except that CSI feedback delay $N_d$ is now taken into account.
\[ \Pi_T(N_d) = \begin{bmatrix} J_0 \left( 2 \pi \frac{f_0 N_s N_c}{f_t} \right) & \cdots & J_0 \left( 2 \pi \frac{f_0 (N_s N_c + N_c - 1)}{f_t} \right) \\ J_0 \left( 2 \pi \frac{f_0 (N_s N_c - 1)}{f_t} \right) & \cdots & J_0 \left( 2 \pi \frac{f_0 (N_s N_c + N_c - 2)}{f_t} \right) \\ \vdots & \ddots & \vdots \\ J_0 \left( 2 \pi \frac{f_0 (N_s N_c - N_c + 1)}{f_t} \right) & \cdots & J_0 \left( 2 \pi \frac{f_0 N_s N_c}{f_t} \right) \end{bmatrix} \]

\[ \Pi_F = \begin{bmatrix} \mathbf{1} & \cdots & \mathbf{1} \\ \mathbf{1} & \cdots & \mathbf{1} \\ \vdots & \ddots & \vdots \\ \mathbf{1} & \cdots & \mathbf{1} \end{bmatrix} \]

C. Redundant CSI Quantization in each TTI

Our work considers two CSI quantization approaches, which give a lower and upper bound for the performance of a practical system. In the first case, we assume that in each TTI \( t \), the channel prediction \( \hat{h}[t + N_c] \) is quantized independently of previous estimates and fed back to the BS side. As in [15, 16], we assume that a decorrelation filter \( \mathbf{V}^H \) is applied to \( \hat{h}[t + N_c] \), such that we obtain a vector of \( N_{\text{rank}} \) uncorrelated Gaussian quantities. Filter matrix \( \mathbf{V} \in \mathbb{C}^{[N_s N_c \times N_{\text{rank}}]} \) is obtained through an Eigenvalue decomposition of the signal covariance at the output of the MMSE predictor, i.e.

\[
\Phi_{hh} = E \left\{ \hat{h}[t]^H \hat{h}[t] \right\} = \Phi_{hh}(N_d) S^H (S \Phi_{hh}(0) S^H + \sigma^2 I)^{-1} S \Phi_{hh}(N_d) \quad \text{(27)}
\]

\[ = U \Sigma U^H, \quad \text{(28)} \]

after which \( \mathbf{V} \) is chosen such that it contains the \( N_{\text{rank}} \) column vectors from \( U \) that correspond to the strongest Eigenvalues on the diagonal of \( \Sigma \). The rank-reduced channel estimates are quantized, leading to an introduction of quantization noise \( \mathbf{n}_q \sim \mathcal{N}(0, \Phi_{qq}) \). Then, they are fed back to the transmitter through an error-free link, where a multiplication with \( \mathbf{V} \) yields the vector of channel estimates \( \hat{h} \). We consider a practical quantization approach [17], where an overall number of \( N_b \) bits is equally invested into each of the \( N_{\text{rank}} \) decorrelated channel estimates, and one bit per real dimension is lost w.r.t. the rate-distortion bound [18]. The quantization noise inherent in the feedback can now be stated as

\[
\Phi_{qq} = 2^{-\max(0, \frac{N_b}{N_{\text{rank}}}-2)} \mathbf{V}^H \Phi_{hh}^H \mathbf{V}. \quad \text{(29)}
\]

Finally, the MSE of the predicted channel at the transmitter side is given as [15]

\[
\text{MSE}^{\text{CSIT}} = \text{diag} \left( \mathbf{E} \left\{ \left( \hat{h}[t] - h[t] \right) \left( \hat{h}[t] - h[t] \right)^H \right\} \right) = \mathbf{E} \left\{ |h|^2 \right\} + \text{diag} \left( \Phi_{qq} - \mathbf{V} \mathbf{V}^H \Phi_{hh} \mathbf{V} \mathbf{V}^H \right). \quad \text{(30)}
\]

D. Successive CSI Feedback

The amount of CSI feedback can be significantly reduced if its correlation in time is exploited. This can be modeled by letting the UTs quantize the channel estimate \( \hat{h}[t + N_c] \) conditioned on the previous channel estimate \( \hat{h}[t + N_c - 1] \) sent to the BS. Hence, we are interested in the conditional covariance

\[
\Phi_{\hat{h}[t]|\hat{h}[t-1]} = \Phi_{hh} - \mathbf{E} \left\{ \hat{h}[t] \hat{h}[t-1]^H \right\} \left( \mathbf{E} \left\{ \hat{h} \hat{h}^H \right\} \right)^{-1} \mathbf{E} \left\{ \hat{h}[t-1] \hat{h}[t]^H \right\} = U \Sigma U^H, \quad \text{(31)}
\]

with

\[
\mathbf{E} \left\{ \hat{h}[t] \hat{h}[t-1]^H \right\} = \beta \mathbf{G}_p (S \Phi_{hh}(0) S^H + \sigma^2 I) \mathbf{G}_p^H \mathbf{V} \mathbf{V}^H
\]

where \( N'_b \) is the number of CSI feedback bits used in the previous TTI (assuming that unconditioned CSI was fed back at that time). We perform a rank reduction of the conditional covariance given in (31) as before and calculate the quantization noise under the assumption of the same quantizer as in (29). From (30) we can calculate \( \sigma^2_{\text{DL,BS}} \), again based on the assumption that the overall performance is dominated by the OFDM symbols for which channel estimation is worst:

\[
\sigma^2_{\text{DL,BS}} = \frac{\max(\text{MSE}_{\text{CSIT}}) - \max(\text{MSE}_{\text{CSIR}})}{\mathbf{E} \left\{ |h|^2 \right\} - \max(\text{MSE}_{\text{CSIR}})}. \quad \text{(33)}
\]

For successive CSI feedback, we can now consider one TTI of unconditioned CSI feedback using \( N'_b \) bits, followed by one TTI of successive CSI feedback with \( N_b \) bits. If we now adjust \( N'_b \) such that the resulting MSE is the same, we obtain the steady-state performance of a scheme continuously using successive feedback with \( N_b \) bits after an initialization phase.

The noise term \( \sigma^2_{\text{UL,UT}} \), which reflects the impact of channel estimation error at the receiver in the UL, can be calculated using the same methodology presented in this section, but considering only the channel estimation part.

IV. Joint UL/DL Optimization

Currently, the asymmetric operation of mobile communication systems is only considered by the radio link control, e.g., RRM and quality of service. Nonetheless, the joint view of UL and DL can also be applied to the physical layer and...
reflects the increasing demand for cross-layer optimization. An example is the adjustment of physical layer parameters in order to adapt to the demands implied by higher layers.

To evaluate the trade-off between UL and DL, we need to take into account the overhead connected to pilots and CSI feedback, and hence find expressions for the net sum rate in UL and DL. For the UL, this is given as

\[ R_{UL}^{net} = R_{UL} \cdot \frac{N_b \cdot N_c \cdot (1 - K \cdot \rho_{UL})}{N_c}, \]

Equation (34) considers the overall rate in a PRB (without pilot symbols), and subtracts the rate required for CSI feedback. The overall pilot effort is the product of pilot density \( N_b \) and number of UTs \( K \), since the BS has to be able to distinguish all terminals based on orthogonal pilot sequences. For the CSI feedback effort, on the other hand, we have to consider that \( N_b \) bits are required for all \( N_b \cdot K \) spatial coefficients of the MIMO channel.

The net sum rate in the DL depends on whether spatial multiplexing is performed or TDM with random precoding vectors. In the former case, we can state

\[ R_{DL}^{net} = R_{DL} \cdot (1 - (N_{BS} + K) \cdot \rho_{DL}), \]

as we need one pilot for each of the \( N_{BS} \) BS antennas (required for channel estimation connected to CSI feedback at the UT side), as well as one pilot for each UT-specific stream (required for channel estimation connected to data decoding at the UT side) [19]. In the case of TDM with random precoding, this increases to

\[ R_{DL}^{net} = R_{DL} \cdot (1 - \rho_{DL}), \]

as the transmission is only performed to one UT at a time, and this UT only needs to estimate the effective channel as a result of random precoding.

In our work, we perform a brute-force search over various concrete pilot sequences \( S \) (yielding different densities \( \rho_{UL}, \rho_{DL} \)) and different CSI feedback extents \( N_b \), which allows us to compute convex joint UL/DL rate regions as given for an example channel realization in Fig. 3. Each point on the surface of such a rate region is connected to a Pareto-optimal set of parameters \( \rho_{UL}, \rho_{DL} \) and \( N_b \), and constitutes the optimum w.r.t. a certain weighted UL/DL sum-rate optimization. In the example, the case of \( v = 1 \) km/h and a strong focus on the UL leads to a choice of \( N_b = 14 \), while \( N_b = 6 \) is preferable in the case of weighting UL and DL 1 : 6. For \( v = 100 \) km/h, we can see that regardless of UL/DL weights it is optimal to set \( N_b = 0 \) and operate the DL in TDM mode with an increased pilot density \( \rho_{DL} = 0.1 \).

V. RESULTS

In this section, we present the best choice of pilot densities and CSI feedback quantity as a function of target UL/DL rate ratio and terminal velocity. In general, we observe a scenario with \( N_{BS} = K = 4 \) for \( f_c = 2.6 \) GHz, and perform Monte-Carlo simulations with a large number of independent channel realizations in UL and DL, where all channel coefficients are uncorrelated in space, have zero-mean and are of unit variance \( E\{ |h|^2 \} = 1 \). All noise terms are set to \( \sigma_{UL}^2 = \sigma_{DL}^2 = \sigma_p^2 = 0.1 \), the sum transmit power at both BS and UT side is set to 1, and the maximum delay spread is \( \tau_{max} = 1 \mu s \), which can be seen as a worst-case delay in an urban scenario with rich scattering. For CSI feedback, \( N_{rank} = 2 \) is chosen empirically.

Fig. 4 shows the dependency of the terminal velocity and the optimal number of CSI feedback bits \( N_b \) per spatial channel coefficient and PRB (plot 4(a)) as well as the optimal pilot densities in UL and DL (plot 4(b)). For low speeds, and especially if the DL is considered important, it is beneficial to invest large extents of UL capacity into CSI feedback. The difference between non-succ. and succ. CSI feedback is rather small, as the weighted sum-rate optimization makes the system invest the gained capacity into boosting the DL, rather than decreasing CSI feedback. For moderate speeds, there is little difference between non-succ. and succ. CSI feedback, until for large speeds, DL performance is so strongly impaired through CSI feedback delay that the optimum extent of CSI feedback decreases until the system uses TDM in the DL. If strong priority is given to the UL, the system only invests into succ. CSI feedback and operates in TDM otherwise. Plot 4(b) shows that the UL pilot density remains constant except for large UT speeds. Depending on the desired UL/DL ratio, the DL pilot density switches between two modes: A low density for spatial multiplexing operation and a higher density for TDM operation. For large UT speeds, CSI feedback delay becomes the dominant issue, such that pilot density in the DL is reduced again.

Fig. 5 shows the same parameters, but as a function of target UL/DL ratio. We can see a similar trend as before, hence when the focus is shifted more towards the UL, and for larger
Fig. 4. Optimal choice of parameters as a function of UT speed.

Fig. 5. Optimal choice of parameters as a function of the desired UL/DL ratio.

Fig. 6. Weighted joint UL/DL sum-rate gains through adaptation (as opposed to fixed parameters).
UT speeds, less UL is invested into CSI feedback. As succ. CSI feedback improves the performance/feedback ratio, it leads to the fact that even for a strong focus on the UL it is still beneficial to operate the DL in spatial multiplexing mode. If the DL is operated in TDM, it is then beneficial to increase the DL pilot density, as only one pilot sequence is needed instead of $N_{\text{BS}} + K$ as for spatial multiplexing.

Fig. 6 finally shows the weighted UL/DL sum rate gain that can be achieved through an adaptive usage of pilot densities and CSI feedback quantity, as opposed to fixed parameters $N_{\text{BS}} = 12$, $p_{\text{UL}} = p_{\text{DL}} = 0.017$. These parameters have shown to be optimal on average for a terminal speed of $v = 10 \text{ km/h}$ and a desired UL/DL-ratio of $1:1$. In both cases, we have the option of switching between spatial multiplexing and TDM in the DL. Gains are shown as a function of UL speed in plot 6(a) and of the desired UL/DL ratio in plot 6(b) respectively. In regimes of low speed, a gain in the order of a few percent is visible, as an increase in CSI feedback quantity beyond $N_{\text{BS}} = 12$ can still improve rates. A large gain of adaptation is visible for large UT velocities, as here both UL and DL ask for more dense pilot structures, in particular in conjunction with DL TDM. Plot 6(b) shows adaptation gains as soon as a different target UL/DL ratio is desired. In general, adaptation gains are reduced if succ. CSI feedback is employed, as this requires less sacrifice of UL rates. In practical systems, we expect the gains through adaptation to be larger, as both channel estimation and CSI feedback schemes will perform significantly worse than the information theoretical bounds observed here, such that it will be even more essential to optimize the trade-off between UL and DL rates. We expect an additional gain if in the DL, pilot densities specific for BS antennas and stream-specific can be adjusted individually.

VI. CONCLUSIONS

Our analysis revealed two apparent trends, which are summarized in Table I. CSI feedback becomes more beneficial for decreasing UT speed (and less scattering) as well as for an increasing weight on the DL rate. Hence, a system might use a multi-cross layer approach in order to adaptively control the physical layer (pilot structure and CSI feedback) depending on the application (UL/DL ratio) as well as depending on the channel (velocity, scattering). In addition, our analysis demonstrated the potential of succ. CSI feedback to scale down the signaling overhead in regimes of low to moderate terminal velocity. Our multi-cross-layer approach could be further extended to include QoS constraints such as latency and packet error rate as well as more degrees of freedom, e.g., the change of physical layer parameters such as the block size in time and frequency, which would imply significant changes for the architecture of mobile communication systems.

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TABLE I

| UL/DL Ratio | Velocity / Large Scale Scattering |
|-------------|----------------------------------|
| $< 1$       | high $N_{\text{BS}}$ – a major part of the UL is used to improve the DL throughput, as the UL is less important and the high coherence facilitates precise CSIT. |
| $> 1$       | moderate $N_{\text{BS}}$ – the UL obtains more priority and therefore cannot trade the same number of feedback bits to improve the DL as in case of a lower UL/DL ratio. |
|             | no CSI feedb. – the prioritized UL cannot give up enough resources for feedback and the high velocity implies a dominant feedback-delay noise. It is best to operate the DL in TDM mode and abandon BS precoding. |

N_{\text{BS}}

| N_{\text{BS}} | Velocity / Large Scale Scattering |
|--------------|----------------------------------|
| low          | Moderate $N_{\text{BS}}$ – only a moderate part of the UL is spent for CSI feedback as the low coherence implies a feedback-delay-noise dominating channel estimation noise. |
| high         | Increasing $N_{\text{BS}}$. |

qualitative summary of the optimal number of feedback bits depending on the UL/DL ratio and the channel coherence.