Physical and Constructive (Limiting) Criteria of Gear Wheels Wear

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Abstract: We suggest using a generalized model of friction - the model of elastic-plastic deformation of the body element, which is located on the surface of the friction pairs. This model is based on our new engineering approach to the problem of friction - triboergodynamics. Friction is examined as transformative and dissipative process. Structural-energetic interpretation of friction as a process of elasto-plastic deformation and fracture contact volumes is proposed. The model of Hertzian (heavy-loaded) friction contact evolution is considered. The least wear particle principle is formulated. It is mechanical (nano) quantum. Mechanical quantum represents the least structural form of solid material body in conditions of friction. It is dynamic oscillator of dissipative friction structure and it can be examined as the elementary nanostructure of metal’s solid body. At friction in state of most complete evolution of elementary tribosystem (tribocontact) all mechanical quanta (subtribosystems) with the exception of one, elasticity and reversibly transform energy of outer impact (mechanic movement). In these terms only one mechanical quantum is the lost – standard of wear. From this position we can consider the physical criterion of wear and the constructive (limiting) criterion of gear teeth and other practical examples of tribosystems efficiency with new tribology notion – mechanical (nano) quantum.

Keywords: friction, energy, evolution, dissipation, nanostructure, wear standard.

1. Introduction

We suggest using a generalized model of friction the model of elastic-plastic deformation of the body element, which is located on the surface of the friction pairs. This model is based on our new engineering approach to the problem of friction - triboergodynamics. Triboergodynamics [1] is an extension (one of its parts) of general Ergodynamics of deformable bodies [2-4]. Ergodynamics is a synthesis to the problem of deformation most general laws of thermodynamics for non-reversible processes, molecular kinetics and dislocation theory in their mutual, dialectical tie on the basis of a most general law of nature – the law of energy conservation at its transformations. Triboergodynamics is based on modern knowledge of friction too: 1. Friction is a phenomenon of resistance to relative motion between two bodies, originating at their surfaces contact area; 2. Friction is the process of transformation and dissipation of energy of external movement into other kinds of energy; 3. Friction is the process of elasto-plastic deformation localized in thin surface layers of rubbing materials.
Thus, within the framework of triboergodynamics the model of elastic-plastic deformation of contact volumes is examined as a generalized mechanism of transformation and dissipation energy and determines essence of resistance to surfaces displacement. The major distinction of triboergodynamics from general Ergodynamics of deformed solids is «scale factor» which exhibits itself in existence of critical friction volume. This volume determines the limit friction parameters and separate, in essence, the surface deformation from the traditional volume deformation.

2. Short fundamentals of ergodynamics of deformed solids

2.1. Structural model

Deformable body is considered as an open, multicomponent, substantially nonhomo-geneous and nonequilibrium system with hierarchy of different levels (from submicro- to macrolevel) of metastable structural elements (defects and damages) which are statistically uniformly distributed in the volume. Some of these elements are virtual sources and sinks of elementary defects (vacancies, dislocations, etc.), the others are a barrier to their motion. The structure state is defined by the basic parameters [4]:

\[ \gamma_\sigma \] is overstress factor of interatomic bonds which evaluates nonuniformity of external stresses \( \sigma \) distribution in the bonds \( \sigma^0 / \sigma \geq 1 \); \( u_e \) is the density of latent (free) energy of defects and damages; \( v \) is the coefficient of nonuniformity of latent energy distribution in volume which is equal to ratio between latent energy density in local volume \( u_e^0 \) and average value of \( u_e \). A complex structural parameter \( k = \gamma_\sigma / v^{0.5} = \sigma_* / S_* \) specifies a relationship between theoretical \( \sigma_* \) and actual \( S_* \) strength of a solid body.

2.2. Physical model and structural-energetic interpretation of the process

Macroscopic phenomenon - plastic deformation and fracture of the body element is considered as a cooperation of a huge number of microscopic elementary acts of atomic-molecular regroupings under external force field (mechanical, thermal, electrical, etc.) which are activated by the thermal energy fluctuations. From the thermodynamic point of view, all the mechanisms and structural levels of the process are divided into two most characteristic groups of adaptive and dissipative (relaxation) types. They differ in physical nature and kinetic behavior. The simple acts controlling generation and accumulation of unit defects in deformed body element (damage) are classified as the first group. The specific (referred to unit volume) pumping power of excessive (latent) energy \( \dot{u}_e \) is an overall characteristic of the processes rate

\[ \dot{u}_e = \frac{du_e}{dt} = A \sinh\left[\left(\alpha \sigma_i^2 - \nu u_e\right)/2kT\right]. \]  (1)

The mechanisms and simple acts controlling relaxation (dissipative) processes of plastic deformation are classified as the second group. The specific power of thermal effect \( \dot{q} \) of plastic deformation is overall characteristic of the processes

\[ \dot{q} = \frac{dq}{dt} = B \sinh\left[\left(\alpha \sigma_i^2 + \nu u_e\right)/2kT\right]. \]  (2)

Here \( A \) and \( B \) are the kinetic coefficient

\[ A = \frac{2kT}{hV_0} \sum_{i=1}^{n} U'_i(\sigma_0, T) \exp\left[-\frac{U'_i(\sigma_0, T)}{kT}\right], \]  (3)

\[ B = \frac{2kT}{hV_0} \sum_{i=1}^{n} U'_i(\sigma_0, T) \exp\left[-\frac{U'_i(\sigma_0, T)}{kT}\right], \]  (4)

\[ U'_i(\sigma_0, T) = U'_{0i} + \Delta U'(T) \pm \beta \sigma_0^2, \quad U'_i(\sigma_0, T) = U^*_{0i} + \Delta U^*(T) \pm \beta \sigma_0^2, \]  (5)
\[ \alpha = \frac{\gamma_s V_0}{6G}, \quad \beta = \frac{\gamma_s V_0}{2K}, \]

\( U_{oi}, U_{oi}^* \) — activation energy of formation and diffusion of the \( i \)-th defect; \( \sigma_0, \sigma_i \) — hydrostatic stress and stress intensity; \( V_0 \) — atomic volume; \( k \) — Boltzmann constant; \( h \) — Planck constant; \( T \) — absolute temperature; \( G, K \) — shear and bulk modules.

### 2.3. Thermodynamic analysis of interrelation between deformation and fracture

From thermodynamic point of view, the plastic deformation and the fracture are defined by a competition of two opposite interrelated and simultaneous trends (figure 1): growth of latent energy density \( u_e \) of various defects and damages which are generated and accumulated in the material due to work done by the external forces \( \omega_p \) and reduction (release) of the density as a result of relaxation processes in deformed body element. The first trend is concerned with strain hardening and damage of material, the second — with dynamic recovery and dissipation of the strain energy which govern the thermal effect of plastic deformation \( q \).

**Figure 1.** Scheme of the energy balance for the plastic deformation of a solid body [2-4].

A significant portion of the dissipative energy \( q \) is not retained in the deformed element, but passes through it and is dissipated in the environment due to heat exchange \( \dot{q} \). Only insignificant portion of the energy \( q \) is accumulated in deformed element as a heat component of internal energy \( \Delta u_T = q - \dot{q} \), increasing its temperature (self-heating effect). According to conservation law:

\[ \omega_p = \Delta u_e + q \quad \text{and} \quad \dot{\omega}_p = \dot{u}_e + \dot{q}. \]  

In mechanics of deformable solids the irreversible work \( \omega_p \) and power \( \dot{\omega}_p \) of deformation are related to stress-strain state of the element by
From (7) and (8) a one-to-one relation follows between stress-strain and thermodynamic states of the element

\[ \dot{\varepsilon}_i^p = \frac{\dot{\omega}_p}{\sigma_i} = \frac{1}{\sigma_i} (\dot{\varepsilon}_i^e + \dot{\varepsilon}_i^q) = \dot{\varepsilon}_i^e + \dot{\varepsilon}_i^q. \]  (9)

Consequently, from thermodynamic point of view, the total values of work \( \omega_p \) and irreversible strain \( \varepsilon_i^p \) as well as their rates \( (\dot{\omega}_p, \dot{\varepsilon}_i^p) \) may be presented as a sum of two components related to strain hardening and damage \( (\dot{\varepsilon}_i^e = \dot{\varepsilon}_i^e / \sigma_i) \), and dynamic recovery \( (\dot{\varepsilon}_i^q = \dot{\varepsilon}_i^q / \sigma_i) \) controlling quasi-viscous flow of the body element, respectively. This deduction is of important value in analyzing interrelation between deformation and fracture processes. Only a portion of plastic (irreversible) strain \( \varepsilon_i^e \) which is controlled by microscopic processes related to strain hardening and accumulation of latent energy of defects and damages is responsible for damage and fracture of the body element. The significant portion of the irreversible strain \( \varepsilon_i^q \) controlled by relaxation (dissipative) processes does not effect the damage and fracture of the body element and only causes quasi-viscous flow (steady state creep). The relationship between work and extent of irreversible deformation and their components varies in a very wide range and depends on the structure and deformation conditions of the material [2].

### 2.4. Thermodynamic condition of local fracture

As a parameter of damage (scattered fracture) we shall take the density of internal energy stored in the deformed volume. The energy is defined as a sum of two components: potential (latent) energy \( u_e \) and kinetic (thermal) energy \( u_T \) that is,

\[ \Delta u = \Delta u_e + \Delta u_T, \quad \dot{u} = \dot{u}_e + \dot{u}_T. \]  (10)

The energy is related to static (\( \Delta u_e \)) and dynamic (\( \Delta u_T \)) damages and distortions of crystal lattice in deformed body. Consequently, it is responsible for scattered fracture (damage). The body element is looked upon as fractured if at least in one local volume responsible for fracture the internal energy density reaches the critical (ultimate) value \( u_* \). This value corresponds to the loss of stability «in great» by crystal lattice. At this instant the cracks of critical size (after Griffith-Orowan-Irwin) and sharp localization of the process at the crack tip occur in a local volume. The thermodynamic condition of local fracture is written as

\[ u(\vec{r}_a, t_* ) = u(\vec{r}_a, 0) + \int_0^{t_*} \dot{u}(\vec{r}_a, t) dt = u_* = \text{const}. \]  (11)

Here \( u(\vec{r}_a, 0) \) — density of internal energy of the material in initial (before deformation, \( t = 0 \)) state; \( \dot{u}(\vec{r}_a, t) \) — specific power of internal energy sources in local macrovolume of the material responsible for fracture; \( \vec{r}_a \) — parameter characterizing coordinates \( (x_*, y_*, z_*) \) of the local volume responsible for fracture.

### 2.5. Thermodynamic criterion of fracture

According to structural-energetic analogy between mechanical fracture and melting of metals and alloys [5] and experimental data [2], the critical value of internal energy \( u_* \) in the local volume responsible for fracture agrees with known thermodynamic characteristic of material \( \Delta H_\varphi \) (enthalpy of melting)
\[ u_* = \Delta H_S = \int_0^T c_p dT + L_S. \]  \hfill (12)

Here \( T_S \) — melting temperature; \( c_p \) - specific heat; \( L_S \) - latent melting heat.

### 2.6. Relationship between force and energy criteria of local fracture

The analysis of kinetic equation of state (damage) (1) indicates that the real solid body approaches the stationary (stable) state under constant action of external fields (\( \sigma_0 = \text{const}, \ \sigma_i = \text{const}, \ T = \text{const} \)) if

\[ \Delta u_e = \text{const} \quad \text{and} \quad \dot{u}_e = 0. \]  \hfill (13)

From the kinetic Equation (1) under condition (13) an important consequence follows

\[ \sigma_i = \sigma_s, \quad \sigma_s = \left( \frac{v u_e^*}{\alpha} \right)^{1/2} = \frac{1}{K_a} (6G U_e^*)^{1/2}, \]  \hfill (14)

according to which the structure state of material \( \sigma_s (\alpha, u_e, v) \) adapts (shakes down) in a stable stage to external conditions, which are defined unambiguously by deviatory component of the stress tensor \( \sigma_i (\sigma_s = \sigma_i) \). Relationship (14) generalizes the known proposition of dislocation theory on mutual relation between the flow stress \( \sigma_s \) and the density of latent (stored) energy \( u_e \) [6] for the case of combined stress state. The material damage \( u_e \) in the local volume responsible for fracture becomes critical, therefore, relationship (14) makes it possible to estimate the actual strength \( S \) of the material.

\[ S = \left( \frac{v u_e^*}{\alpha} \right)^{1/2} = \frac{1}{K_a} (6G U_e^*)^{1/2}. \]  \hfill (15)

Here

\[ u_e^* = u_e - u_{e0} = \Delta H_S - \int_0^T c_p dT. \]  \hfill (16)

From (15) and (16) under \( k_{\sigma} = 1.0 \) and \( T = 0 \) the theoretical shear strength is:

\[ \sigma_s = (6G \Delta H_S)^{1/2} = \left( \frac{3E \Delta H_S}{1+\mu} \right)^{1/2}. \]  \hfill (17)

Here \( E, \mu \) — elasticity modulus and Poisson's ratio.

### 3. Tribodynamics method

#### 3.1. Structural-energetic interpretation of friction process

It is known friction is characterized a product of frictional forces \( F \) by friction distance \( \ell \), that is, the work \( \omega_f \), expended on overcoming frictional force:

\[ \omega_f = F \ell, \]  \hfill (18)
\[ \omega_f = \Delta u_e + q, \]  \hfill (19)
\[ \omega_f = \dot{u}_e + \dot{q}. \]  \hfill (20)

Here \( \omega_f = d\omega_f / dt \) is a power of friction dissipation of energy; \( \dot{u}_e = du_e / dt \) is the rate of storing latent energy in deformed (contact) volumes; \( \dot{q} = dq / dt \) the power of thermal effect of plastic deformation (friction).

Since the contact volumes of both materials of the friction pair (figure 1) are deformed, Equations (19) and (20) should be rewritten as:

\[ \omega_f = \Delta u_{e1} + \Delta u_{e2} + q_1 + q_2, \]  \hfill (21)
These equations show, that from thermodynamic point of view, the work $\omega_f$ of friction forces, (friction power $\dot{\omega}_f$) is related to plastic deformation of the contact volumes. The work $\omega_f$ may be divided conventionally into two specific parts. The first part is related to variation of the latent (potential) energy ($\Delta u_{e1}$ and $\Delta u_{e2}$) in deformed (contact) volumes. This is the energy of various simple defects and damages which are generated and accumulated in the bulk. This energy is unique and the total characteristic of submicro- and microstructural variations occurring in plastically deformed volumes [2, 3]. This is a measure of strain hardening and damage of material.

The relations between $\Delta u_{e1}$ and $\Delta u_{e2}$, as well as $q_1$ and $q_2$ are defined by physico-chemical properties of the materials of the friction pair, their structure and friction conditions. Since the contact volumes (not unit sizes) of the materials forming a friction couple become strained by friction (figure 2), equations (1) and (2) can be rewritten in the form

\[
W_f = \Delta U_e + Q = \Delta U_{e1} + \Delta U_{e2} + Q_1 + Q_2, \quad (23)
\]

\[
W_f = \dot{U}_e + \dot{Q} = \dot{U}_{e1} + \dot{U}_{e2} + \dot{Q}_1 + \dot{Q}_2, \quad (24)
\]

where $\Delta U_e = V_f \Delta u_e$; $\dot{U}_e = V_f \dot{u}_e$; $V_f$ - is the deformaible (friction) volume.

Solving equations (23) and (24) for the frictional force $F$, one obtains:

\[
F_l = \frac{\Delta U_{e1} + \Delta U_{e2}}{l} + \frac{Q_1 + Q_2}{l}, \quad (25)
\]

\[
F_v = \frac{\dot{U}_{e1} + \dot{U}_{e2}}{v} + \frac{\dot{Q}_1 + \dot{Q}_2}{v}, \quad (26)
\]

where $l$ and $v$ are the friction path and the slip velocity. Dividing equations (25) and (26) by the normal force $N$ gives generalized equations for the friction coefficient:

\[
\mu_l = \frac{\Delta U_{e1} + \Delta U_{e2}}{Nl} + \frac{Q_1 + Q_2}{Nl}, \quad (27)
\]

\[
\mu_v = \frac{\dot{U}_{e1} + \dot{U}_{e2}}{Nv} + \frac{\dot{Q}_1 + \dot{Q}_2}{Nv}. \quad (28)
\]
Therefore, the friction is generally described by the energy balance equation and with thermodynamical point of view [1-3] is the process of two interrelated, oppositely directed and concurrent trends operating in a strained contact. According to the energy balance scheme (figure 1) for plastic deformation and fracture [2] presented above, equations for friction work $W_f$, frictional force $F$ and friction coefficient $\mu$ (without lubrication) has view:

$$W_f = \Delta U_e + Q = \Delta U_{e_1} + \Delta U_{e_2} + \Delta U_{T_1} + \Delta U_{T_2} + \dot{Q}_1 + \dot{Q}_2,$$

$$\dot{W}_f = \dot{U}_e + \dot{Q} = \dot{U}_{e_1} + \dot{U}_{e_2} + \dot{U}_{T_1} + \dot{U}_{T_2} + \dot{\dot{Q}}_1 + \dot{\dot{Q}}_2,$$

$$F_f = \frac{\Delta U_{e_1} + \Delta U_{e_2}}{l} + \frac{Q}{l} = \frac{\Delta U_{e_1}}{l} + \frac{\Delta U_{e_2}}{l} + \frac{Q_1 + Q_2}{l},$$

$$F_v = \frac{\dot{U}_{e_1} + \dot{U}_{e_2}}{v} + \frac{\dot{Q}_1 + \dot{Q}_2}{v} = F_{\text{mechanical}} + F_{\text{molecular}},$$

$$\mu_l = \frac{\Delta U_{e_1} + \Delta U_{e_2}}{N_l} + \frac{Q_1 + Q_2}{N_l} = \mu_{\text{adapt}} + \mu_{\text{dis}} = \mu_{\text{adapt}} + \mu_{T(\text{dis})} + \mu_{\dot{Q}(\text{dis})},$$

$$\mu_v = \frac{\dot{U}_{e_1} + \dot{U}_{e_2}}{N_v} + \frac{\dot{Q}_1 + \dot{Q}_2}{N_v} = \mu_{\text{deformation}} + \mu_{\text{adhesion}},$$

where $\Delta U_e = V_f \Delta u_e$; $Q = V_f q$; $\dot{Q} = V_f \dot{q}$; $\dot{U}_e = V_f \ddot{u}_e$; $\dot{u}_e = d u_e / d t$ - is the rate of latent energy density change in the contact volumes; $V_f$ - is the deformable (friction) volume; $\mu$ - friction coefficient; $\mu_{\text{adapt}}$ - adaptive friction coefficient; $\mu_{T(\text{dis})}$ and $\mu_{\dot{Q}(\text{dis})}$ - statical and dynamical components of dissipative friction coefficient; $\Delta U_T$ - thermal component of internal energy; $N$ - normal load; $l$ - distance of friction; $v$ - sliding velocity. The latent energy density $\Delta u_e$ is an integral parameter of tribostate and damageability (failure ($\Delta u_e^\ast$)).

Thus, viewed thermodynamically, the work done by friction forces $W_f$ (the friction power $\dot{W}_f$), the friction force $F$ and the friction coefficient $\mu$ may be classified conventionally into two specific components with different kinetic behavior [3]. The first component is associated with microscopic mechanisms of adaptive type and relates to the change of latent (potential) energy ($\Delta u_{e_1}, \Delta u_{e_2}$) of various elementary defects and damages that are generated and accumulate in the deformable volumes of materials friction pair (figure 1). This energy is a unique and integral characteristic of the submicro- and microstructural transformations that occur in plastically strained materials [2-4]. This energy is a measure of strain hardening and damageability of materials. The second component is associated with microscopic mechanisms of dissipative type and relates to dynamic recovery processes in which latent energy is released and heat effect of friction ($q_1, q_2$) take place. This energy originates in the motion and destruction of various elementary defects of opposite signs, the egress of these defects to the surface, the healing of reversible submicroscopic discontinuities, etc. The ratios of the components $\Delta u_{e_1}$ and $\Delta u_{e_2}$ as well as $q_1, q_2$ of the balance vary over a wide range, depending on the physical, chemical, and structural properties of the materials that comprise the friction couple and the friction process conditions.

Thus, the thermodynamic analysis of friction (plastic deformation and fracture) has led to generalized (two-term) relations for the force $F$ and coefficient of friction $\mu$, which agrees with current concepts of the nature of friction [7,8] – molecular-mechanical theory (32) and deformable-adhesion theory (34). But it is more correct to speak about the adaptive-dissipative nature (model) of friction (33). Relationships (21)-(28) which generalize the mechanism of energy dissipation at friction allow to classify the tribosystem states. According to ergodynamics of deformed solids (relationships $\Delta u = \Delta u_e + \Delta u_f$ and $q = \Delta u_f + \dot{\dot{q}}$) and equations (23)-(24) may be transformed to:
\[ W_f = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{T1} + \Delta U_{T2} + \tilde{Q}_1 + \tilde{Q}_2, \quad (35) \]
\[ \dot{W}_f = \dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{T1} + \dot{U}_{T2} + \ddot{Q}_1 + \ddot{Q}_2. \quad (36) \]

As follows from equations of energy balance (35), (36), all exhibitions of friction and wear may be reduced conventionally at least to two basically different states: the first state defines all types of damage and wear, the second — the so-called "wearless" condition.

The state of damage and wear is characterized by the components of energy balance (35), (36), which are responsible for accumulation of internal energy in deformed volumes \( \Delta u = \Delta u_{e1} + \Delta u_{e2} + \Delta u_{T1} + \Delta u_{T2} \), i.e. the process is irreversible. The "wearless" state is characterized by the components responsible for dynamic dissipation (reversibility) of strain energy into elastic and structural dissipated energy of friction contact \( \tilde{q} = \tilde{q}_1 + \tilde{q}_2 \).

In its turn, the first state may be classified depending on the relation between potential \( \Delta u_e \) and kinetic \( \Delta u_T \) components of internal energy. It is subdivided conventionally into mechanical damage and wear (due to so-called structure activation) and thermal damage and wear (due to thermal activation). For instance, let the thermal component of internal energy \( \Delta u_T \) be equal to zero (\( \Delta u_T = 0 \)) and the internal energy variation at damage and wear be defined only by variation of the potential component \( \Delta u_e (\Delta u = \Delta u_e) \). Then, the mechanical damage and wear with brittle fracture of surfaces take place. On the contrary, if we have \( \Delta u_e = 0 \) (\( \Delta u = \Delta u_T \)), then the thermal damage and wear with ductile fracture of surfaces take place. All the intermediate values of the components are associated with quasi-brittle or quasi-ductile fracture of solids.

In the most general case, the energy balance at dry friction (23) should be written as:
\[ W_f = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{e3} + Q_1 + Q_2 + Q_3. \quad (37) \]

In the special case, where the friction is localized into volume of the "third body" (figure 2) equation (37) develops into:
\[ W_f = \Delta U_{e3} + \tilde{Q}_3. \quad (38) \]

Here \( \Delta U_{e3} = V_3 \Delta u_{e3} \).

3.2. Energy interpretation of Leonardo da Vinci (Amonton’s) friction coefficient

According to thermodynamic theory of strength [2], the structure parameter should be related to the portion of the accumulated plastic deformation that is responsible for strain hardening. This portion is uniquely and integrally defined by the density of the potential component of internal energy (that is, the latent energy density \( \Delta u_e \)) of various defects and damages that accumulate in a plastically strained material. With this in mind, if we neglect the heat effect \( Q \) of friction, one will infer from the thermodynamic analysis of friction of equations (27)-(28) that the Amonton (Leonardo da Vinci) friction coefficient is
\[ \mu = \frac{\Delta u_e}{\mu^* N_l} = \frac{F}{N}; \quad F = \frac{\Delta U_{e}}{I}; \quad Q \approx 0, \quad \mu^* = 1. \quad (39) \]

Consequently, the coefficient of friction has a very deep physical sense. On the one hand, it is the parameter which generally characterizes the resistance of relative displacement (movement) of surfaces, for it reflects the portion of energy, which «is done by friction away» as accumulated latent energy \( \Delta U_e \), by relation to parameter of external forces work \( \mu^* N_l \) (energy of external relative movement). On the other hand, it is the generalized characteristic of damage, for it is defined of the latent energy density \( \Delta u_e \) as integral characteristic of the structure defectiveness measure, because this energy is the generalized parameter of damage. Here too, coefficient of friction generally reflects the structural order (disorder) of deforming contact volume, since the parameter \( \Delta u_e = \Delta u_e V_f \) is
defined of the energy of defects and damages of different types, that are accumulated into contact volumes $V_f$ solids.

Therefore, coefficient of friction is a true and generalized parameter of tribosystem state. From this conclusion we can say that the analysis of the evolution of the states of a tribosystem is primarily an analysis of the latent deformation energy accumulated within the contact friction volumes.

3.3. Generalized experimental friction curves

The dependences obtained for the friction coefficient $\mu$ are in agreement with experimental curves $\mu = \mu(N, v)$ (figure 3-5).

A subsequent analyses of modern experimental data using equations (23)-(34) has shown that the experimental friction curves (figure 3-5) of type $\mu = \mu(N, v)$ are generalized friction curves that reflect the evolution (the change in the friction coefficient) of tribosystem.

![Figure 3. P.Conti’s experimental results [9].](image-url)
3.4. Structural-energy regularities of rubbing surfaces evolution

We propose an energetic interpretation of the experimental friction curves $\mu = \mu(N,v)$ (figure 6). According to our concept [1, 11], the ascending portion of the friction coefficient curve $\mu$ is mainly controlled by processes associated with the accumulation of latent energy $\Delta U_e$ in various structural defects and damages. Here the increase in $\mu$ is due to the increasing density of latent (potential) energy $\Delta u_e$ and the increasing adaptive friction volume $V_f$. The descending portion of the friction curve is mainly controlled by processes associated with the release and dissipation of energy $Q = \Delta U_T + \dot{Q}$. Here the decrease in $\mu$ is due to the decrease in latent energy density within the friction volume $V_f$ or (which is virtually the same) to the decrease of the adaptive friction volume $V_{adapt}$ ($u_e = u_e^*$) and to the increase of the dissipative volume $V_{dis}$ ($\dot{q}^* = u_e^*$).

Evolution of tribosystem, presented as a diagram view (figure 6), has an adaptive-dissipative character (29)-(34) and reflects the competitive (dialectical) nature of friction. Evolution curve has the row of principal points (1-5) of transitional tribosystem states, which strictly obeys the balance principle of friction; there are more characteristic areas of tribosystem behavior between these points. These areas reflect the common properties of nonlinear dynamic of evolution.

So, in figure 6 it is possible to see the following conventionally designated points and stages: 0-1 – a stage of static friction and deformational strengthening; 1 – a point of limit for deformational strengthening; 1-2 – a stage of pumping of excess energy; 2 – a point of gripping (adhesion) and transition of outer friction into internal (critical non-stability); 2-3 – a stage of forming dissipation structures (formation of heat fluctuation in friction volume); 3 – a point of minimum compatibility (maximum frictionness); 1-2-3 – a stage of selforganization; 3-4 – a stage of compatibility; 4 – a point of wearlessness (anormal-low friction); 5 – a point of thermal adhesion.
An ideal evolution of tribosystem is symmetrical. The process starts and finishes within areas of elastic behavior. A plastic maximum (a superactivated condition) exists between them as a condition of selforganisation and adaptation.

In the most general case evolution (adaptation) regularities of tribosystems may be presented as a 2-stage (figure 6). At the first stage (0-2) of adaptation the evolution of friction contact rushes to form some critical volume of friction $V_f^*$ (point 2). It is elementary tribosystem that is the elementary and self-sufficient energy transformer. The first stage - latent energy density growth $\Delta u_e$ to a limited magnitude $\Delta u_e$ within critical friction volume $V_f^*$.

This friction volume $V_f^*$ is constant at the second stage of evolution, but here it is evolutionary developed owing to structural transformation; by this one may realize wide spectrum of compatibility friction structures (figure 6). The second stage (2-4) – structural transformation of critical friction volume (elementary tribosystem) $V_f^*$ into adaptive $V_{adapt}$ and dissipative $V_{dis}$ volumes (figure 7). The limit (point 4) of this stage is characterized by a full transformation of adaptive critical volume $V_{adapt}^*$ into $V_{dis}^*$ dissipative.

The volumes mentioned above characterize different regularities of transforming energy of outer mechanical movement at friction. Adaptive volume $V_{adapt}$ is connected with non-reversible absorption of deformation energy. It is in this volume where latent deformation energy $\Delta u_e$ accumulates and where the centres of destruction initially emerge (birth). Dissipative volume $V_{dis}$ is capable of reversible transformation (dissipate) of outer movement energy. It doesn’t accumulate latent deformation energy owing to reversible elastic-viscous-plastic deformation.

Suggested theoretical and calculation evaluation [1, 11] showed that dissipative friction volume performs reversible elastic energy transformation of outer mechanical movement with density $\tilde{q}^*$ equal to critical density of latent energy $u_e^*$.

**Figure 6.** Structural-energy diagram for evolution of rubbing surfaces [1, 14].
Culmination of tribosystem evolution is its final and limited condition of point 4 – a state of anomalously low friction and wearlessness (maximum efficient). A schematic evolution of the contact volume of friction in diagram’s points 1-5 is presented in figure 7.

![Figure 7](image)

**Figure 7.** A schematic evolution of the contact volume of friction in diagram’s points 1-5 [1].

Calculation shows [1] that at an ideal tribosystem evolution an adaptive (Amontons) friction coefficient \( \mu_{adapt} \) in a point 2 of a diagram falls abruptly down, reaching in a point 4 the value of elastic friction coefficient \( \mu_{elast} \). For point 4 of compatibility area 3-4 an equation of energy balance (33) showed be put in the following way:

\[
\mu_{adapt} = \mu^* - \mu_{dis} = 1 - \mu_{dis} = \mu_{plast} = 0 = \mu_{elast}; \mu^* = 1.0. \tag{40}
\]

Thus, point 4 stands for an ideal evolution of contact friction volume a condition of ideal elastic-viscous-plastic deformation. Equation (40) shows as a matter of fact exactly it, i.e. Amontons friction coefficient \( \mu_{adapt} \) being in its essence plastic friction coefficient \( \mu_{plast} \) has a minimum value equal to zero. It follows then, that plastic friction became elastic with friction coefficient \( \mu_{elast} \). It means that plastic deformation of contact volume friction is implemented with the maximum dynamic dissipation (\( \dot{Q} = \max \)) of accumulated latent energy. That is why the value of accumulated energy in point 4 is equal to zero (\( \Delta U_e = 0 \)). This fact proves an ideal condition at full evolution of contact volume. From the physics point of view this condition may be explained by the full dissipation of accumulated energy \( \Delta U_e^* \) in point 2 and by newly emerged structures of point 4 in the form of elastic energy of interaction between them (dynamic dissipation energy \( \dot{Q}^* \)). Here \( \mu_{dis} = 1.0 \). The structural elements themselves are defectlessness - \( \mu_{adapt} = 0 \), and friction is elastic - \( \mu = \mu_{elast} \).

It has been demonstrated [1] that value of minimum adaptive friction volume \( V_{min} \) corresponding to the zero meaning of plastic friction component \( \mu_{adapt} \) is not equal to zero, but is equal to some minimum structural element of deformed solid body.

3.5. About mechanical (nano) quantum of dissipative friction structures
The result of ideal elementary tribosystem (contact) evolution is forming of unique nanostructure – a mechanical (nano) quantum. Strict notions about mechanical quantum have been obtained [1] considering equation of quasiideal solid body for point 4 of diagram of friction evolution –

\[ \bar{Q}^* = \bar{S}_Q T = \mu_{\text{dis}}^* Nl_f = V_f^* u_e^* = V_f^* \bar{q}_* , \]  

which is particular case of solving equation of energy friction balance (29) at \( \mu_{\text{adapt}} = 0 \) and \( \mu_{\text{dis}} = 1 = \mu_{\text{dis}}^* \). Here \( \bar{S}_Q \) – inertia entropy of compatible friction volume; \( T \) - characteristic temperature of contact friction volume; \( l_f \) - linear size of elementary contact.

Correspondingly, in conditions of maximum compatibility (point 4) when tribosystem implements full evolution cycle of adaptation with formation of most perfect dissipative structure, the behaviour of structure is subject to equation of quasiideal solid body condition. So, it is to be presumed that, interaction between elements of this structure, are minimized – a condition of ideal elasticity in dynamics. Equation (31) with taking into account Plank-Boltzmann formula

\[ W k T S = \ln \left( \frac{Nl_f}{\mu_{\text{dis}}^*} \right) , \]



where \( k \) - Boltzmann constant; \( W \) - condition probability; \( S_U \) - configuration entropy of friction, contact volume.

Tribosystem always tends to some optimal condition, characterized, i.e. to a most probable condition \( W' = N_f \ln W \) for the given friction conditions.

Analysis and solution of these equations [1] allows to demonstrate the principle of constant probability value (parameter of tribosystem condition (order)) \( W \) for the whole range of compatible friction precisely \( \ln W = 3 \) and \( W = e^3 = 20,08553696... \).

The value of thermodynamic probability \( W \) equal to 20,08553696... was interpreted [1, 11-13] as a minimum value of linear, atomic oscillators in one of three directions of minimum adaptive friction volume \( V_{\text{adapt}}^{\text{min}} \) corresponding to condition of practically absolute elastic friction – anomalously-low friction (safe deformation threshold). Then the number of atomic oscillators in this volume equals \( V_{\text{adapt}}^{\text{min}} \) and as a result:

\[ V_Q = (20,08553695...)^3 = 8103,083969 \text{ atom's oscillators} \]

It is the universal size (volume) of mechanical quantum [1, 11-13].

On the other hand, adopting the meaning of Boltzmann entropy \( S \), a universal friction constant \( R_f = k N_f \) [1,11-13] is obtained, which characterizes in physical meaning «energetical size» of elementary tribosystem (TS), containing in ideal conditions the same number of atomic oscillators \( N_f \) (mechanical quanta \( N_Q \)):

\[ R_f = k \cdot N_f = k \cdot W^3 \cdot N_Q = R_{MQ} \cdot N_Q , \]  

\[ R_{MQ} = k \cdot W^3 , \]

where \( R_{MQ} \) - universal constant of deformation at friction.
As it follows from calculations [1] the size of minimum adaptive friction volume $V_{\text{min}}^{\text{adapt}}$ coincides in its value with the size of submicroscopic area in crevice mouth, which is equal for metals $(4...9) \times 10^{-6}$ mm, i.e. of critical volume size responsible to fracture. Thus the size of minimum adaptive friction volume $V_{\text{min}}^{\text{adapt}} = V_{\text{elast}}$, can be presented as the size of some mechanical quantum.

| Metals and steels | $E \times 10^{-3}$, MPa | $u^*_e$ | $\Delta H_S \times 10^{-3}$, MJ/m$^3$ | $E / 3E_r$ |
|-------------------|-------------------------|--------|-------------------------------|------------|
| Cr                | 235.4                   | 8.5    | 27.69                         |            |
| Mg                | 44.4                    | 1.9    | 23.37                         |            |
| Ag                | 79.0                    | 3.7    | 21.35                         |            |
| Au                | 78.7                    | 4.0    | 19.67                         |            |
| Co                | 200.1                   | 10.6   | 18.88                         |            |
| Fe                | 211.4                   | 9.9    | 21.35                         |            |
| Ta                | 184.4                   | 10.6   | 17.39                         |            |
| Ti                | 105.9                   | 6.7    | 15.8                          |            |
| Nb                | 104.0                   | 9.2    | 11.3                          |            |
| Zr                | 95.6                    | 5.7    | 16.77                         |            |
| Mo                | 316.9                   | 12.0   | 26.4                          |            |
| W                 | 392.4                   | 14.4   | 27.25                         |            |
| Ni                | 201.1                   | 9.4    | 21.39                         |            |
| Iron              | 210.9                   | 10.1   | 20.88                         |            |
| 20                | 200.1                   | 9.5    | 21.06                         |            |
| 1Kh13             | 206.0                   | 8.9    | 23.14                         |            |
| 3Kh13             | 218.8                   | 9.2    | 23.78                         |            |
| Kh18N9T           | 199.1                   | 9.4    | 21.19                         |            |
| Kh18M9            | 199.1                   | 9.6    | 20.74                         |            |
| 30Kh              | 214.1                   | 10.2   | 20.99                         |            |
| 30N3              | 207.5                   | 10.3   | 20.11                         |            |
| 40                | 209.4                   | 9.7    | 21.58                         |            |
| 30G2              | 207.2                   | 10.0   | 20.72                         |            |
| 30KhGN3           | 208.0                   | 10.2   | 20.4                          |            |
| G13               | 204.0                   | 10.0   | 20.4                          |            |
| 50S2G             | 196.2                   | 10.3   | 19.05                         |            |
| U8                | 198.0                   | 10.3   | 19.22                         |            |
| U12               | 198.0                   | 10.4   | 19.04                         |            |

$\Delta H_S = 3E_r, \quad E / 3E_r = 20.77, \quad E / 3E_r = 20.77.$

This mechanical quantum constitutes a minimum number of atoms capable to provide such a configurational distribution (structure) which obtains the property of reversibly taking and dissipating (recovering) energy of outer mechanical movement. It also constitutes minimum structure in conditions of plastic deformation and it is formed at tribosystem transition (deformed volume) through an ultimately activated (critical) condition (see figure 6) due to development of selforganisational tribosystem adaptation processes. Mutual rotation-oscillation movement of these mechanical quanta in respect of each other within elementary tribosystem (contact) determines condition of most perfect dissipative friction structure. Properly speaking, such condition is described by equation of quasiideal solid body condition (41), a condition when interaction between structural elements (mechanical
quanta) is minimized – a condition of ideal elasticity of quasiviscous flow. Calculation friction coefficient between quanta equals about $10^{-8}$ [1, 11, 13].

A conclusion that mechanical quantum constitutes a minimum structural form at plastic deformation (friction) is supported by calculation. If values of elasticity modules $E$ correspond to atomic (true) elasticities $E_r$ then values equal to 60 are obtained, where $60 = 3W$ can be interpreted as a characteristic of volume elasticity of one mechanical quantum – minimum adaptive friction volume $V_{\text{adapt}}^\text{min}$. Calculation assessment of parameter $W = 20 = E / 3E_r$, done for various metals and steels gives an average value 20.77 ((Table)); $\Delta H_S = 3E_r$ - enthalpy of melting.

A conclusion is made [1] that the number of atoms (mechanical quantum (MQ)) within volume of one elementary tribosystem (TS) in conditions of ideal tribosystem evolution is a constant value. Thus, it is possible to speak about the quantity of substance equal by mass to one elementary tribosystems and to one mechanical quantum.

3.6. Synergism of tribosystem and optimum states

Mechanical quantum is dynamic oscillator of dissipative friction structure. An ideal quasielastic contact condition at its full evolution constitutes effect of most fully dissipated energy of outer mechanical movement throughout newly formed (by mechanism of selforganization) structural elements – mechanical quantums (dynamic oscillators) which most fully realize their rotationary – oscillatory behavior in relation to each other within elementary tribosystem volume. Their resistance to relative interaction here is minimally elastic and corresponds to elasticity of ideal atomic (thermodynamically balanced) interactions at the level of electron orbits.

Universal constants of mechanical quantum and elementary tribosystem (material point) determine quantum model of surface damping:

$$\mu_{\text{dis}} = \frac{3R_{MQ}Tn_i}{NfU_{Q^n_i}} = \frac{U_{Q^n_i}}{U_{Q^n*}} = \frac{n_i}{n_*} = 1 - \mu_{\text{adapt}};$$

(46)

taking into account destruction quantums $n_{\text{dest}}$ (non-reversible process component) and damping quantums $n_{i}$ (reversible, elastic component – fatigue number), and also probability evolution tribosystem model to a most ordered condition:

$$\mu_{\text{adapt}} = 1 - \mu_{\text{dis}} = 1 - \frac{R_f T \ln W_i}{Nf} = 1 - \frac{\ln W_i}{\ln W_*},$$

(47)

where $3R_{MQ}T = U_{Q^n} -$ energy of one mechanical quantum; $W_i$ and $W_*$ - current and ultimate probabilities of tribosystems compatibility conditions.

According to a model of quantum surface damping at friction in state of most complete evolution (adaptation) of elementary tribosystem all mechanical quantums with the exeption of one elasticity and reversibly transform energy of outer impact (mechanic movement). One mechanical quantum of radiation ($\approx 8103$ atoms) – is a minimum loss (essence of wearlessness or other wear primary standard).

Linear size of quantum is equal to diameter of spherical ideal crystal with atomic roughness:

$$D_{MQ} = 2 \cdot W \cdot \bar{d}_a \cdot (3/4 \cdot \pi)^{1/3} = 7.177 \text{ nm}.$$  

(49)

Here $\bar{d}_a$ - mean atomic diameter for metals; $W = e^3$ - parameter of state for mechanical quantum [1].

Mechanical quantum (figure 8) can be examined as the elementary nanostructure of metal’s solid body.
Calculations have shown [1] the number $N_Q$ of such mechanical «quanta» (subtribosystems) within the elementary tribosystem’s volume $V_f^* = V_{dis}^*$ to be $0.63 \times 10^8$, which is close to the safe number $n_*$ of fatigue cycles.

According to the quantum damping model of surfaces under friction, when we have the state of more full evolution of elementary tribosystem, the all mechanical quanta to elastic and reversible transform the energy of external mechanical motion. Only one mechanical quantum (8103 atoms) is the minimum loss (the essence of «wearlessness»).

In these terms (point 4) only one mechanical quantum [1,11-14] is the lost – standard wear. The tribosystem (friction contact) has the ideal damping properties – «wearlessness».

The principle of mechanical quantum determines nanoquantum levels of all friction parameters of compatible (optimal) tribosystems and other.

4. Nanoquantum models of tribosystem maximum capacity for work

4.1. Gear wear calculation principle

The all parameters of compatibility (optimal) friction have to be in quanta levels - commensurable with the parameters of the one mechanical quantum – standard of wear.

So, all heavy-loaded tribosystems it is necessary to examine with position of tribosystem ideal evolution. This ideal state of tribocatct is true indicator of tribosystem state for practical examples of tribology. It is the standard of maximum tribosystems efficiency - anomalously low friction and wearlessness.

The state of friction contact under its most full evolution is the characteristic with exploitation of hard loaded Hertzian contact, for example, on the surfaces of gear wheels teeth and systems of wheel-rail and other. We can examine the active surface of gear wheel, which consist of equilibrium spherical form asperities after run-in. During one revolution of gear wheel each asperity of gear wheel teeth is loaded one time too. Under it the loss of one friction contact is equal to one mechanical (nano) quantum. Therefore, the whole contact volume is fatigue failed during about 63 millions cycles. The linear wear $h_*$ of gear wheel is equal to diameter size $Q_T^S = 2.85 \times 10^{-6}$ m of an equilibrium friction volume $V_f^*$ (figure 9) [15]. It is the physical criterion of wear. One may understood that the constructive (limiting) criterion of gear teeth is equal to the limit clearance between tooth surfaces. For example, it is about $0.3 \div 0.4$ modulus of gear wheel tooth wear when we have the loss of bend strength of teeth.
Thus, an elementary nano-structure of deformed solids may examine as the standard of wear and to apply with optimization the life time of real hard pressed Hertzian contact systems.

4.2 Estimation of bearing capacity for work of internal combustion engines

Take the engine with a frequency of \( n = 1500 \text{ min}^{-1} \) shaft rotation. Take the limit wear (linear) of bearing is equal to \( h^* = 0.1 \text{ mm} \). We know the line size elementary tribosystem: \( D_{TS} = 2.85 \text{ mkm} = 2.85 \times 10^{-6} \text{ m} \). For every revolution shaft one elementary tribosystem (equilibrium, run-in contact) loses one mechanical quantum. The number of turns required to wear one elementary tribosystem equals the number of mechanical quantum in this tribosystem.

So, there are \( n_{MQ} = 0.63 \times 10^8 \) revolutions.

Now you can define the time of wear for one elementary tribosystem:

\[
T_{TS} = \frac{n_{MQ}}{n} = \frac{0.63 \times 10^8}{1500} = \frac{42000 \text{ min}}{60} = \frac{42000}{60} = 700 \text{ hour} = \frac{700}{24} = \frac{29166}{365} = 0.0799 \text{ year}
\]

Now let's define the number of layers of elementary tribosystems into linear wear - 0,1 mm:

\[
a_{h^*} = \frac{h^*}{D_{TS}} = \frac{1 \times 10^{-4}}{2.85 \times 10^{-6}} = 0.35 \times 10^2 = 35.
\]

Now, let's define the time of wear of shaft-bearing system with the ultimate linear given wear - \( h^* = 0.1 \text{ mm} \), namely:

\[
t_{\text{motor}} = T_{TS} \cdot a_{h^*} = 0.0799 \times 35 = 2.7968 \text{ year}.
\]

Total we have 2,7968 years of continuous operation on limit load.

If you work 8 hour per day, then we get the following result:

\[
2.7968 \times 3 = 8.39 \text{ year}.
\]

This is a very real result for modern cars. And if the ride is not 8 hours per day and less durability increases considerably-really to decades.

For this result we have the wear rate - \( i = 4 \text{ nm/h} \). It is good correspondence with the data for wear rate - \( i = 5 \text{ nm/h} \) by Prof. F. Franek [15].

4.3 The principle of critical velocity of rolling wheels
This speed limit is determined by the principle of filling the entire nominal friction sliding system area with elementary tribosystems, damping process. Above this speed happens full unloading tribosystem, detachment of wheels from the surface of the rail as distorts the principle of minimum resistance to movement (the principle of one elementary tribosystem or irreversibility). In this case, all mechanical quantums of elementary tribosystem will repel the wheel. There will be no quantum which activates a process to maintain the system in an excited state.

The calculation will be performed in the following order. Denote elementary nominal contact area. By definition [1], on elementary, nominal area of tribosystem can accommodate and work

\[ n_{TS} = 0.63 \times 10^8 \] 

elementary tribosystems. Each elementary tribosystem (for model of spherical roughness) has a size of \( D_{TS} = 2.85 \times 10^{-6} \) m and is able to implement rolling wheel path in the elementary act of rolling at length of this tribosystem.

Thus, if per unit of time all elementary tribosystems work on elementary nominal area of friction time, then the path passable wheel per unit of time, equal to

\[ L_{ETS} = D_{TS} \cdot n_{TS} = 2.85 \times 10^{-6} \cdot 0.63 \times 10^8 = 179.55 \text{ m} . \]

Consequently, the critical speed of wheel rolling is equal

\[ v_\ast = L_{ETS} \cdot 3600 = 646.38 \text{ km/h} . \]

This result is close to modern speed of 574.8 km/h (TGV, France).

5. Conclusions

5.1. Structural-energy analysis of the friction process allows us to examine the friction process as the evolution process;
5.2. From the energy balance equations of friction follows that the evolution of tribosystem (contact) has an adaptive-dissipative character.
5.3. Experimental friction curves of \( \mu = \mu(N,v) \) type may be examined as generalized friction experimental curves;
5.4. The fuller evolution of tribosystem has symmetrical view - the friction process is started and finished within elastic area.
5.5. Under fuller evolution of friction contact (elementary tribosystem) the unique nanostructure is formed; the basis of this structure is the mechanical (nano) quantum and the contact (material point of mechanics) consists of about \( 0.63 \times 10^8 \) such quantums.
5.6. We can examine the mechanical quantum as the least structural form of solid material body and the standard of wear.
5.7. All parameters of compatibility (optimal) friction have to be in quanta levels - commensurable with the parameters of the one mechanical quantum.
5.8. Interaction between nanoquantums is nature the net elasticity. The value of the coefficient of friction between mechanical quantums has order \( \mu_{MQ} = 1.587 \times 10^{-8} \).
5.9. Exploitation of gear wheels and other heavy-loaded tribosystems (Hertzian contact) are subjected to model of nanoquanta damping, when one mechanical quantum is the standard of wear (subtribosystem).

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