The design of an improved matched filter in DSSS-GMSK system

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Abstract. This paper introduces the principle of DSSS-GMSK system, analyses the superiority of GMSK modulation over MSK modulation. Accord that the method of de-spread before demodulation can effectively improve the capability of the system with spread spectrum gain, this paper researches an improved method with matched filter to de-spread and demodulate the DSSS signals. The local PN code is modulated with GMSK modulation before being correlated with received signal, then we can get the synchronous PN code, de-spread and demodulate the signal. MATLAB simulation shows that this method is more efficient than the method of demodulation before de-spread in low SNR environment.

1. Introduction
Spread-spectrum system has been widely used with the advantage of low probability of being captured, strong ability on anti-jamming, high resolving power of distance and the power of random stochastic location by multi-user. As a working mechanism of spread-spectrum system, direct sequence spread-spectrum (DSSS) uses BPSK/QPSK/MSK as its common mode of modulation. GMSK modulation, which is an improvement on MSK modulation, uses a Gaussian low pass filter before MSK. GMSK modulated signals have more tightly band limit than other constant envelope signals, so that they are more suitable.

The advantage of DSSS system is the spread spectrum gain to the signal. To get the gain, the signals must be de-spread before demodulation. DSSS-BPSK/QPSK/MSK systems all use the method of de-spread before demodulation to manage the signals. The local PN code is made correlation calculation with received signals by matched filter, then we can get the correlation peak, which could help us get the synchro PN code and demodulate the signals. But in GMSK modulated system, we could not get the correlation peak by using the same method. To solve this problem, literature [4] and literature [5] both use the method of demodulation before de-spread to process the received signals. But this method could not efficiently use the spread-spectrum gain. The system could not work well in low SNR environment. In order to efficiently use the spread-spectrum gain, this paper researches an improved method with zero IF direct correlator to demodulate the DSSS signals, analyses and emulates the principle and capability of the system.

2. Principle and capability of DSSS-GMSK modulation
2.1. Principle of DSSS-GMSK modulation

GMSK (Gaussian-filtered minimum-shift keying) is a kind of modulation scheme with constant envelope, which develops from MSK. The principle of GMSK modulation is that the signals go through a Gaussian low-pass filter before MSK modulation. The impulse response of the Gaussian filter is defined as follows:

\[ h(t) = \sqrt{\frac{\pi}{\alpha}} \exp\left(-\frac{\pi}{\alpha} t^2\right) \]  

(1)

Where \( \alpha = \sqrt{\ln 2/2}/B \), \( B \) is the 3dB bandwidth of the Gaussian filter. If the width of a pulse is \( T \), then \( BT \) is the normalized 3dB bandwidth. We can obtain the frequency impulse signal of the Gaussian filter:

\[ g(t) = Q\left(\frac{2\pi B_T}{\sqrt{\ln 2}} \left(t - \frac{T}{2}\right)\right) - Q\left(\frac{2\pi B_T}{\sqrt{\ln 2}} \left(t + \frac{T}{2}\right)\right) \]

(2)

Where \( Q(x) = \int_{\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-x^2/2\right) dx \).

Suppose that the transmitting data multiplied by PN sequence is \( a_k (a_k \in \{-1,1\}) \), then the phase of the modulated signals is shown as follows:

\[ \phi(t) = \frac{\pi}{2T} \left(\sum_{k} a_k g(\tau - nT - \frac{T}{2})\right) \]

(3)

Where \( kT \leq \tau \leq (k+1)T \). The frequency of the carrier wave is \( f_c \), then the GMSK modulated signal is produced after IF modulation:

\[ s(t) = \cos\left(2\pi f_c t + \phi(t)\right) = \cos\left(2\pi f_c t_k\right) \cos\phi(t_k) - \sin\left(2\pi f_c t_k\right) \sin\phi(t_k) \]

(4)

2.2. Capability analysis of GMSK modulated signals

GMSK is an improvement on MSK. Figure 1 gives the BER performances and frequency spectrum for GMSK and MSK modulated signals. Although the error rate performance of GMSK modulated signal is less than MSK modulated signal, but the gap is quite small. Figure 2 shows that with respect to MSK modulated signals, GMSK modulated signals have a more compact spectrum and faster band attenuation. So GMSK has a broader development prospect.

![Figure 1. BER comparison of GMSK and MSK.](image)

Normalized 3dB bandwidth is a very important parameter of GMSK modulated signals. From figure 2 we can see that the spectrum structure of GMSK modulated signals is closely related to BT value. The smaller the BT value, the more compact the frequency spectrum. When \( BT \to +\infty \), the GMSK modulated signals change into MSK modulated signals. BT value will also have an impact on the error rate of GMSK modulated signals. Figure 1 also shows that the BER performance will be getting worse as BT value decreases. So, BT value usually is 0.3 in GSM system.
3. Improved method with matched filter to de-spread and demodulate the DSSS signals

Two methods are used for the signal processing at the receiving terminal of the DSSS system: de-spread before demodulation and demodulation before de-spread. In high SNR environment, the two methods both have good BER performances. But in low SNR environment, the method of de-spread before demodulation has a better BER performance because this method could effectively use the spread spectrum gain. At the receiving terminal, we must get the the synchronous PN code before despread the received signals. Common methods to get synchronous PN code are sliding correlation and matched filter. The principle of matched filter is that we use the PN code as the coefficients of the filter to filter the received signals. The result will be sent to the threshold decider. If the result is larger than the threshold value, the PN code is synchronous.

In DSSS-BPSK system, signals at the receiving terminal is shown as follows:

\[ s(t) = \sqrt{2P} c(t - \tau) \cos(\omega'_b t + \phi) \]  

(5)

Where \( c(t) \) is the PN code multiplied by transmitted data. \( \tau \) is the time delay. \( \omega'_b \) is the frequency of the received signal, \( \phi \) is the initial phase which can be set to zero. The impulse response of the matched filter is: \( h(t) = c(NT_c - t) \), where \( 0 \leq t \leq NT_c \), \( NT_c \) is the period of PN code. After DDC, the signals will be sent to the matched filter. The result is that:

\[ y(t) = s(t) * h(t) \]

\[ = \int_{-\infty}^{t} s(\lambda) h(t - \lambda) d\lambda \]

\[ = \int_{-\infty}^{t} \sqrt{P} c(\lambda - \tau) c(NT_c - t + \lambda) \cos(\Delta\omega \lambda) d\lambda \]

(6)

Where \( \Delta\omega \) is the variation of the carrier frequency. The correlation peak could be found from the result. However, the GMSK modulated signals are shown as formula(4), the result of the signals correlated with PN code is that:

\[ z(t) = \cos(\Delta\omega t + \phi) h(t) \]

\[ = \int_{-\infty}^{t} \cos(\Delta\omega \lambda + \phi(\lambda)) h(t - \lambda) d\lambda \]

\[ = \int_{-NT_c}^{t} \cos(\Delta\omega \lambda + \phi(\lambda)) \cdot c(NT_c - t + \lambda) d\lambda \]

(7)

We can not find the correlation peak from formula(8), then we can not get the synchronous PN code and...
despread the received signals. To solve this problem, this paper researches an improved method with matched filter to de-spread and demodulate the DSSS signals. Figure 3 shows the process of the method. The local PN code, which modulated by GMSK mode first, is set as the coefficient of the matched filter. Then the matched filter is used to filter the received signals.

\[
\begin{align*}
\text{Data I} & \quad \int \quad + \quad t \quad \text{threshold} \\
\text{Data Q} & \quad \int \quad + \quad t \quad \text{clock}
\end{align*}
\]

**Figure 3.** Process of the improved method.

Assume that the carrier is synchronous, after DDC, the GMSK In-phase data and Quadrature data of the signals are that:

\[
\begin{align*}
s_i(t) &= A \cos(\phi(t)) \\
s_q(t) &= A \sin(\phi(t))
\end{align*}
\]

Where \( \phi(t) = \frac{\pi}{2T} \int_{-\infty}^{t} \sum a_n g(\tau - nT - \frac{T}{2}) d\tau \). Assume the period of the PN code is \( k \), then the local GMSK modulated signals are that:

\[
r(t) = \cos(\phi(t)), 0 \leq t \leq kT
\]

The output of the matched filter is that:

\[
\begin{align*}
y_i(t) &= s(t) * r_i(t) = \int_{-\infty}^{t} s(\lambda) r_i(t - \lambda) d\lambda \\
&= A \int_{t-kT}^{t} \cos(\phi(\lambda)) \cos(\phi(t-\lambda)) d\lambda \\
y_q(t) &= s(t) * r_q(t) = \int_{-\infty}^{t} s(\lambda) r_q(t - \lambda) d\lambda \\
&= A \int_{t-kT}^{t} \cos(\phi(\lambda)) \sin(\phi(t-\lambda)) d\lambda
\end{align*}
\]

Then the peak value is that:

\[
z(t) = y_i(t)^2 + y_q(t)^2
\]

If \( z(t) \) is larger than threshold value, then the local PN code is synchronous. According to the correlation peak values in each symbol period, we can demodulate the received signals.

4. Simulation results
Assume that signal transmission rate \( R_s = 80kHz \), PN code period \( k = 128 \), then PN code transmission rate is 10.24MHz. The sampling rate is 12 times the code transmission rate. Time delay is set to be 48 sampling duration. Carrier is synchronous at the receiving terminal. We can get the simulation results with the input SNR of \(-6dB\):

1. The output of the matched filter is shown as Figure 4.

From Figure 4 we can see the correlation peak clearly. Then we can get the synchronous PN code by setting a suitable threshold value.
Figure 4. Output of the matched filter.

Figure 5 shows the correlation peak after de-spread. Since every symbol has 1536 sampling points, the correlation peak will appear every 1536 sampling points. We can get the sending signals through the positive and negative of every peak value.

Figure 5. Correlation peak after de-spread.

Figure 6 shows the BER performances of the two methods. The method this paper researched has a better BER performance than the method of demodulation before de-spread in low SNR environment.

Figure 6. BER performances of the two methods.
5. Summary
This paper introduces the principle of DSSS-GMSK system, analyses the GMSK modulated signals. According to the special form of the GMSK modulated signals, with the thought of de-spread before demodulation, this paper improves the matched filter. Simulation results show that we can get the correlation peak from the filter output and despread the signals. When input SNR is $-6dB$, the BER can be $10^{-5}$.

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