Majorana-mediated spin transport without spin polarization in Kitaev quantum spin liquids

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We study the spin transport through the quantum spin liquid (QSL) by investigating the real-time and real-space dynamics of the Kitaev spin system with a zigzag structure in terms of the time-dependent Majorana mean-field theory. After the magnetic field pulse is introduced to one of the edges, the spin moments are excited in the opposite edge region although no spin moments are induced in the Kitaev QSL region. This unusual spin transport originates from the fact that the $S = 1/2$ spins are fractionalized into the itinerant and localized Majorana fermions in the Kitaev system. Although both Majorana fermions are excited by the magnetic pulse, only the itinerant Majorana fermions flow through the bulk regime without the spin excitation, resulting in the spin transport in the Kitaev system. We also demonstrate that this phenomenon can be observed even in the system with the Heisenberg interactions using the exact diagonalization.

Spin transport without an electric current has attracted not only practical interest in spintronics but also considerable attention in modern condensed matter physics. In insulating magnets, the carriers of the spin current are conventionally considered to be magnons, which are elementary excitations of the spin transport in the quantum spin liquid (QSL) in a magnetically ordered state [1–4]. By contrast, the possibility of the spin transport in the nonmagnetic system could flow through the Kitaev QSL region without spin polarization. Thus, it is highly desired to examine the spin transport in the nonequilibrium dynamics, which should be important to observe the itinerant nature of the Majorana fermions in the bulk.

In this Letter, to address the spin transport through the Kitaev QSL, we investigate the real-time dynamics triggered by an impulse magnetic field on one of the edges. Using the time-dependent mean-field (MF) theory, we examine the time evolution of the magnetization and dynamics of the fractionalized Majorana quasiparticles. We demonstrate that a spin-polarized wavepacket created at the edge propagates to the other edge even when the two edges are separated by the QSL region without spin polarization. We also address how robust this anomalous phenomenon is against the Heisenberg interactions by means of the exact diagonalization (ED). Finally, we propose the ways to extract the results intrinsic to the Kitaev QSL with the fractionalized quasiparticles in experiments.

We consider the Kitaev model in the $L_x \times L_y$ cluster of the honeycomb lattice with zigzag edges, which is schematically shown in Fig. 1. The norms of the primitive translational vectors $\alpha$ and $\beta$ are assumed to be unity. The periodic boundary condition is imposed along the $\beta$-direction. The system is considered here is composed of three regions. In the middle (M) region, no magnetic field is applied and the Kitaev QSL is realized without spin polarization. In the right (R) region, the static magnetic field $h_R$ is applied. We introduce $L_R$, which is defined as the number of $\alpha$ bonds included in this region with respect to the $\alpha$ direction (see Fig. 1). Moreover, we term the $L$ region composed of the left-edge sites. In this region, we introduce the time-dependent magnetic field $h_L(t)$. The corresponding Hamiltonian is

$$\mathcal{H}(t) = -J_K \sum_{\gamma=x,y,z} \sum_{\langle i,j \rangle_{\gamma}} S^\gamma_i S^\gamma_j - h_R \sum_{\gamma=\alpha} S^\gamma_i - h_L(t) \sum_{\gamma=\beta} S^\gamma_i,$$

where $S^\gamma_i$ is the $\gamma(=x,y,z)$ component of an $S = 1/2$ spin operator at the $i$th site. The ferromagnetic exchange $J_K(>0)$ is defined on three different types of the nearest-neighbor bonds, $x$ (red), $y$ (blue), and $z$ (green) bonds (see Fig. 1). It is known that, in the uniform lattice, the magnetic field in-
introduces the phase transition to the spin-polarized state around \( h_1/J_R \approx 0.042 \) within the MF theory. Therefore, we restrict ourselves to the case with \( h_R < h_1 \) to discuss the spin transport inherent in the Kitaev QSL.

We study the time evolution of the system upon stimuli of the magnetic pulse in the L region (see Fig. 1) \cite{44, 45}. To this end, we introduce the time-dependent Majorana MF theory. The details of the formulations are shown in Ref. \cite{46}. The Hamiltonian Eq. (1) is obtained as a fermion model by applying the Jordan-Wigner transformation to the spin operators \[47-49\]. Furthermore, by introducing two kinds of Majorana fermions, \( y \) and \( \bar{y} \), from a complex fermion at each site, Eq. (1) is rewritten as

\[\mathcal{H}(t) = -\frac{J_K}{4} \sum_{r} (y_{r-a}^{A} y_{r}^{B} + 
abla_{r} y_{r}^{A} y_{r}^{B}) - \frac{J_K}{4} \sum_{r} (y_{r}^{A} y_{r}^{B} + \nabla_{r} y_{r}^{A} y_{r}^{B}) - \frac{h_R}{2} \sum_{r} (y_{r}^{A} y_{r}^{B} - \nabla_{r} y_{r}^{A} y_{r}^{B}) + \frac{h_L}{2} \sum_{r} \nabla_{r} y_{r}^{A} y_{r}^{B}, \quad (2)\]

where \( r \) indicates the position of the \( z \) bond (the center of the \( z \) bond; see Fig. 1). \( y_{r}^{A} \) and \( \bar{y}_{r}^{A} \) (\( y_{r}^{B} \) and \( \bar{y}_{r}^{B} \)) are the Majorana fermion operators connected with the \( z \) bond in the sublattice \( A \) (\( B \)), as shown in Fig. 1. When \( h_R = h_L(t) = 0 \), \( \mathcal{H}, \eta_r \] = 0 at each \( r \), and \( \eta_r = (\bar{y}_{r}^{A} y_{r}^{B}) \) is the \( Z_2 \) local conserved quantity. In the case, the model is solvable as the Hamiltonian is bilinear in terms of \( y \), and its low-energy dispersion is given as \( E_r = v|k| - h_K \) around the K point with the velocity \( v = \sqrt{3}J_K/4 \). This indicates that \( y \) and \( \bar{y} \) are regarded as the itinerant and localized Majorana fermions, respectively.

Since the magnetic field hybridizes two kinds of the Majorana fermions, the Hamiltonian is no longer exactly solvable. Here, we apply the Hartree-Fock type decoupling to the interaction on the \( z \) bond as \( i y_{r}^{A} y_{r}^{B} \nabla_{r} y_{r}^{A} y_{r}^{B} \sim i y_{r}^{A} y_{r}^{B} \Theta_1(x,t) + \Theta_2(x,t) \nabla_{r} y_{r}^{A} y_{r}^{B} - \Theta_2(x,t) \Theta_3(x,t) + \Theta_4(x,t) \nabla_{r} y_{r}^{A} y_{r}^{B} \Theta_3(x,t) + \Theta_4(x,t) \Theta_5(x,t) - \Theta_2(x,t) \Theta_5(x,t) \nabla_{r} y_{r}^{A} y_{r}^{B} \Theta_2(x,t) - \Theta_4(x,t) \Theta_5(x,t) \Theta_2(x,t) + \Theta_4(x,t) \Theta_2(x,t) \Theta_5(x,t) \Theta_2(x,t) \Theta_2(x,t) \Theta_2(x,t) \Theta_2(x,t) \), where we have introduced the six kinds of \( x \)- and \( t \)-dependent MFs as \( \Theta_1(x,t) = \langle i y_{r}^{A} y_{r}^{B} \rangle \equiv \langle \eta(x,t) \rangle, \Theta_2(x,t) = \langle i y_{r}^{A} y_{r}^{B} \rangle \equiv \langle \xi(x,t) \rangle, \Theta_3(x,t) = \langle i y_{r}^{A} y_{r}^{B} \rangle \equiv -2\langle S_{r}^{z}(t) \rangle = -2\langle S_{r}^{z}(x - x_d/2,t) \rangle, \Theta_4(x,t) = \langle i y_{r}^{A} y_{r}^{B} \rangle \equiv -2\langle \xi(x,t) \rangle = -2\langle \xi(x - x_d/2,t) \rangle, \Theta_5(x,t) = \langle i y_{r}^{A} y_{r}^{B} \rangle \equiv -2\langle \eta(x,t) \rangle = -2\langle \eta(x - x_d/2,t) \rangle \rangle \), (3)

This indicates that the propagation is attributed to the gapless Majorana excitation in the bulk within a small static field.

FIG. 1. Kitaev model on a honeycomb lattice with zigzag edges. Red, blue, and green lines represent \( x \), \( y \), and \( z \) bonds, respectively. Solid (open) circles represent spin-1/2 in the \( A \) (\( B \)) sublattice. In this figure, four \( z \) (green) bonds exist along the \( \alpha \) direction in the \( R \) region, namely, \( L_R = 4 \).

FIG. 2. Real-time dynamics of the Kitaev spin system with \( L_a = 50 \) and \( L_b = 300 \) after the magnetic-field pulse at \( t = 0 \). The contour plot of \( \Delta S_{r}^{z}(x,t) \) on the plane of the time \( t \) and the space \( x \) in the system (a) without and (b) with the \( M \) (white) region where the QSL is realized with no spin polarization between the \( L \) (left gray) and \( R \) (right gray) regions (see the top of the panels). The dashed lines represent \( x = v t \) with the Majorana velocity \( v \) (see text).
Now, we consider the real-time dynamics of the non-magnetic Kitaev spin system triggered by the magnetic-field pulse at the left edge to discuss how the wavepacket flows through the M region (the Kitaev QSL without spin polarization). In this region, the local conserved quantity is present in each hexagon, and spin correlations are extremely short-ranged \[50\]. Figure \[2\](b) shows the time evolution of \( \Delta S^z(x,t) \) in the system with \( L_R = 10 \). We find that the magnetic moment is always zero in the M region and no proximity effect is found around the interface between L and M regions. Nevertheless, in the R region, \( \Delta S^z(x,t) \) is induced and the wavepacket flows with the Majorana velocity \( v \). This result indicates that the spin excitations propagate in the nonmagnetic region via the itinerant Majorana fermions, which cannot be explained by classical pictures such as the spin wave theory.

To discuss the propagation of the spin excitation through the QSL region in more detail, we examine the time evolutions of \( \Delta \xi \) and \( \Delta \eta \), which correspond to the dynamics of itinerant and localized Majorana fermions, as shown in Figs. \[3\](a) and \[3\](b). We find in Fig. \[3\](a) that the excitation created in the left edge at \( t = 0 \) propagates in the whole region, which results from the motion of the itinerant Majorana fermions. By contrast, Fig. \[3\](b) shows that \( \Delta \eta \) vanishes in the M region owing to the existence of the local \( Z_2 \) symmetry, while it appears in the R region. This is the similar behavior as in \( \Delta S^z \). These suggest that, after the excitation at the left edge, only the itinerant Majorana fermions propagate in the bulk, where no oscillation appears in the magnetization, and finally reach the R region. The weak magnetic field in the R region yields the hybridization between the itinerant Majorana fermions and the localized fermions, resulting in time-dependent nonzero spin moments there. Thus, in the Kitaev QSL, the spin transport is mediated by the Majorana fermions although the spin moments never appear. Moreover, we have confirmed that the magnitude of the spin moment induced in the R region exhibits a power-low decay as a function of the length of the M region. This is ascribed to the gapless dispersion of the Majorana fermions, in contrast to the existence of the gap in the spin excitation in the Kitaev model.

The pulse-amplitude dependence in this phenomenon is also remarkable. In the R region, \( \Delta S^z \) turns out to be proportional to \( A^2 \). This can be explained by considering the local symmetry at the left edge \[57\]. This non-linear feature is intrinsic in the Kitaev model, in contrast to the conventional systems with \( \Delta S^z \propto A \). To study how visible anomalous behavior is in the system with the Heisenberg interaction, we apply the ED method to the Hamiltonian \( \mathcal{H}(t) + J_H \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \) with the antiferromagnetic Heisenberg coupling \( J_H > 0 \). It is known that, when \( h_R = h_L(t) = 0 \), the Kitaev QSL is stable against small \( J_H \) \[9\][12][19][25]. In our calculations, the initial ground state is obtained with the Lanczos method and the time evolution is simply evaluated by the Runge-Kutta method.

The obtained results for the 24-site cluster with \( L_a = 4 \), \( L_b = 3 \), and \( L_R = 1 \) are shown in Fig. \[3\]b. In the calculations, we have confirmed that the induced moment is always parallel to the \( z \) direction. First, we show the results for the genuine Kitaev model with \( J_H = 0 \) in Fig. \[4\]a). One can find the propagation of the magnetic excitation from one edge to the other through the QSL region without spin polarization, which is consistent with the Majorana MF result discussed above. In the presence of the Heisenberg term \( (J_H/J_K = 0.03) \), \( \Delta S^z(x,t) \) takes nonzero values in the M region, as shown in Fig. \[4\]b), suggesting that the Heisenberg interaction affects the flow of the spin excitation. In particular, the spin modulation in the M region is more prominent compared to that in the R region. This difference from the genuine Kitaev model originates from the fact that the Heisenberg interaction yields the interaction between itinerant and local-
targeted Majorana fermions. Therefore, the spin moments appear in the M region near the interface to the L region as a proximity effect, as shown in Fig. 3(b).

We note that $\Delta S^z$ in the R region is similar to the case without the Heisenberg interactions. This implies that the spin transport inherent in the Kitaev model still survives. It is naively expected that in the M and R region, the Heisenberg and Kitaev interactions mainly give $A$ and $A^2$ contributions in the spin oscillation, as discussed above. Therefore, the unique feature for the Kitaev system is extracted by examining the average of the magnetic responses after the magnetic pulses with $A$ and $-A$. In Fig. 3(c), this quantity is hardly seen in the M region but clearly observed in the R region, which is a consequence of the Kitaev QSL with itinerant Majorana fermions.

When the pulse amplitude $A$ is relatively large, the Kitaev interaction plays a dominant role for the spin propagation and the spin transport without spin polarization becomes practically prominent. Figure 4(d) presents the results with the large $A$. The spin moments induced in the M region are relatively small, but the spin excitation propagates to the right edge, at which the spin moments induced are much larger than those in the M region. This phenomenon is essentially the same as that in the genuine Kitaev case shown in Fig. 4(a). The above two results suggest that the spin transport mediated by the fractionalized itinerant quasiparticles without spin excitations can be observed even in the presence of additional interactions.

Finally, we discuss the relevance of the present results to real materials. The setup of our study could be implemented by considering a Kitaev candidate material sandwiched by ferromagnetic insulators. The candidate materials have been proposed as $A$IrO$_3$ ($A=$Na, Li) [58,63] and $\alpha$-RuCl$_3$ [35,38]. The stimuli of the magnetic field pulse can be injected from a ferromagnetic insulator by the spin pumping [64,67] or circular polarized light irradiation [68,69]. Our results suggest that the spin-excited flow propagates to the other edge even if the magnetic polarization is absent in the Kitaev magnet, and therefore, we expect that the time-dependent magnetic moment is observed in the ferromagnetic insulator connected to the other side of the Kitaev magnet with a small overlapping. This time evolution can be experimentally measured by the Kerr or Faraday rotations [68,70], which will provide convincing evidence of the fractionalized itinerant quasiparticles in the bulk of the Kitaev magnet.

Note that in the real system, a magnetic order hinders the appearance of the Kitaev QSL [71,76]. This effect can be avoided by the finite temperature measurement above the Néel temperature, where the itinerant quasiparticles are active, and/or the recent progress of the thin film [13,77,83], which suppresses the magnetic ordering due to the suppression of the interlayer coupling. Moreover, by changing the intensity of the injection of the spin excitation, one could estimate the magnitude of the additional interactions such as the Heisenberg one. The effect of the off-diagonal interactions, so called $\Gamma$ term, is not addressed in the present study but we expect that this gives a similar effect to the Heisenberg one [86,88].

In summary, we have demonstrated that, after the magnetic excitation at one of the edges in the Kitaev spin system, the spin moments never appear in the bulk, but are fluctuated in the opposite edge. We have revealed that this unusual spin transport is governed by the fractionalized itinerant Majorana fermions. The spin transport without spin polarization should be feasible, even in the system with the Heisenberg coupling by using the pulse field dependence in $\Delta S^z$.

We also note that it might be possible to control the motion of the localized Majorana fermions (vison) in the bulk, by switching on/off the magnetic field. This should be important for realizing the vison transport in the experiments. It is also interesting to study the spin transport in the generalized Kitaev models [89,91], where the existence of spin fractionalization has also been suggested [90,92,93]. The real-time spin dynamics should be one of the possible candidates to clarify the presence of the quasiparticles.

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This can be explained as follows. Since the system has the local symmetry before and after the introduction of the pulsed field, each eigenstate is specified by a set of the eigenvalues of $\eta$ in the L region. Let us start with the initial state given as $|\Psi;\eta_1,\eta_2,\cdots\rangle$. After the pulse at the left-edge, the final state should be given within the first-order perturbation as $|\Psi(t)\rangle = |\Psi;\eta_1,\eta_2,\cdots\rangle + \sum_i c_i |\Phi_1,\eta_1,\eta_2,\cdots\rangle + \cdots$, where $c_i$ is some constant and $|\Psi\rangle$ and $|\Phi\rangle$ belong to the distinct subspaces specified by different sets of eigenvalues of $\eta$ at the left edge. On the other hand, when the operator $O$ commutes with $\eta$ at the left edge, $O|\Psi\rangle$ belongs to the same subspace as $|\Psi\rangle$. (For example, $O$ is $\ldots,\xi,\eta$ in the M and R regimes.) Therefore, the first order component of $A$ in $|\Psi(t)\rangle$ vanishes.
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