Phase-space methods in nuclear reactions around the Fermi energy

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Some prescriptions for in-medium complex particle production in nuclear reactions are proposed. They have been implemented in two models to simulate nucleon-nucleus (nIPSE) and nucleus-nucleus (HIPSE) reactions around the Fermi energy [1, 2]. Our work emphasizes the effect of randomness in cluster formation, the importance of the nucleonic Fermi motion as well as the role of conservation laws. The key role of the phase-space exploration before and after secondary decay is underlined. This is illustrated in the case of two debated issues: the memory loss of the entrance channel in central collisions and the \((N, Z)\) partitions after the pre-equilibrium stage.

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In nuclear reactions around the Fermi energy, nuclei can break into several pieces of various sizes: the so-called multifragmentation process [3]. A striking experimental feature is the large number of accessible charge and energy partitions. In order to understand the statistical aspects of the explored phase-space, several scenarios have been proposed. However, in view of the complexity of nuclear reactions due for instance to impact parameter mixing, pre-equilibrium emission and thermal decay, it is hard to trace back the process of cluster formation. This issue remains a highly debated question. During the past three years, we have tried to answer the following question: "What are the minimal hypothesis required to reproduce the characteristics of nuclear fragmentation?". Based on a detailed and a successful comparison with experimental data, we have ended up with surprising conclusions. Simple prescriptions have been found for the formation and the emission of complex particles [1, 2, 4]. First, these rules are introduced and illustrated. In a second part, we discuss two aspects of current interest related to the phase-space exploration after pre-equilibrium stage.

I. RULES FOR THE FORMATION AND THE EMISSION OF CLUSTERS

In this section, rules for the cluster formation and the production of fragmentation partitions are defined. These rules have been extensively discussed in ref. [4] and we only give here a short summary. Let us consider a proton colliding a heavy target with an energy \(E_B\) close to the Fermi energy. A schematic three steps scenario is considered (see figure 1). First, the incident nucleon is absorbed. The second step corresponds to the in-medium formation of the cluster. We assume that clusters are made of nucleons randomly picked inside a Thomas-Fermi distribution. Cluster properties, position \(r_c\) and the kinetic energy \(\varepsilon_k\) are calculated on the basis of the characteristics of the nucleons in phase-space. Using a Monte-Carlo sampling, an ensemble of configurations is generated (called "total accessible phase-space") is obtained. As an illustrative example, the phase-space of an \(\alpha\) particle is given in top right of figure 2. The last step corresponds to cluster emission. All configurations obtained in the second step will not necessarily lead to the emission of a cluster. Two constraints on the emission can be identified. The first one, which is independent of the entrance channel, is due to the mutual interaction between the cluster and the other nucleons. Given an interaction potential between the cluster and the emitter, denoted by \(V_{A+A_c}(r_c)\), the emitted particle must overcome the barrier, denoted by \(V_B\). This gives a lower limit on the cluster kinetic energy. The second constraint is directly dependent on the reaction type and is due to the energy balance. The constraint of a positive excitation energy for the configuration leads to

\[
E_B - Q - V_{A+A_c}(r_c) \geq \varepsilon_k(r_c)
\] (1)

where \(Q\) is the Q-value of the configuration. Therefore, only a fraction of the total phase-space accessible for the cluster will indeed lead to emission in the continuum.
FIG. 2: Top-left: Two-body potential between the $\alpha$ and the emitter. Above the line (I), the cluster cannot be emitted. This upper limit is directly given by the energy balance of the reaction. Below the line (II), the cluster cannot overcome the barrier (since here, quantum tunnelling is not considered). In between the two lines, there is a small "escape window" for the emission of the cluster. Top-right: Total available phase-space of the cluster. This latter is significantly reduced due to the energy constraint. The two curves correspond respectively to the lower and upper limit in the kinetic energy. Bottom: Calculated kinetic energy distribution (open squares) of the $\alpha$ particle obtained by propagating each configuration in the "escape window" up to infinity. The calculated spectrum is compared with the experimental data (black circles).

This fraction corresponds to the "explored phase-space". These two constraints are shown in Fig. 2 (top left) for a proton-induced reaction at $E_B = 39$ MeV. There, an $\alpha$ particle can only be emitted in a small interval of kinetic energy (called "escape window"

II. PHASE-SPACE EXPLORATION: TWO ILLUSTRATIONS

A detailed comparison of the models with the experimental data can be found in [1] for nucleus-nucleus and [2] for nucleon-nucleus reactions. Besides the fact that comparisons are made on the phase-space after the secondary decay, HIPS and nIPSE models give also access to configurations before secondary emission. This intermediate stage is of particular interest for making the link with statistical physics. In the following, we give two examples where the models provide important information in the nuclear context.

A. Loss of memory of the entrance channel in central events

A possible way to trace back the complete loss of memory of the entrance channel is to consider the degree of shape equilibration. Following ref. [8], we define the isotropic ratio as $\langle R_{iso} \rangle = E_\perp / 2E_\parallel$ where $E_\perp$ and $E_\parallel$ correspond respectively to the average total transverse and parallel energy. For a spherical symmetric partition, we do expect $\langle R_{iso} \rangle = 1$, this limit corresponding to a complete shape equilibration. In figure (3), we have plotted the evolution of $\langle R_{iso} \rangle$ as a function of multiplicity for the reaction Xe+Sn at 50 MeV/A. We observe an increase of this quantity and a saturation around a value of 0.75. A similar behavior has been found in experiments.

1 n-IPSE: nucleon-Ion Phase-Space Exploration
2 HIPSE: Heavy-Ion Phase-Space Exploration
and assigned as an incomplete loss of the entrance channel memory. In that case, the multiplicity has been assumed to be strongly correlated with the centrality of the reaction. In HIPSE, the average multiplicity indeed increases with the centrality. However, large fluctuations around the mean value are also observed. As a consequence, a given multiplicity corresponds to a large set of impact parameters (denoted by $b$). The great advantage of HIPSE is that it gives direct access to $b$. In the left side of figure (3), the evolution of $\langle R_{iso} \rangle$ is presented as a function of $b$. We clearly observed a saturation around 1 for smallest impact parameters. This observation leads to two conclusions for the model. First, the multiplicity is not a perfect selector of the centrality and the saturation below 1 is a spurious saturation due to impact parameter mixing. Second, most central events in HIPSE are completely equilibrated in shapes and a complete loss of memory of the entrance channel is observed.

![Figure 3: Evolution of the isotropic ratio as a function of the total multiplicity (left) and impact parameter (right). The black circles indicate the average evolution.](image)

**B. Isospin equilibration**

With new radioactive beams facilities, we do expect to explore a large set of nuclei in the nuclear chart. In HIPSE, a particular care has been taken to treat consistently the isospin degrees of freedom. For the moment, HIPSE has only been applied to reactions with stable nuclei but can similarly be used for radioactive beams. This aspect is illustrated in the reaction $^{48}$Ca+$^{40}$Ca at beam energy $E_B = 25$ MeV/A. In figure (4), the $(N,Z)$ distributions before the decay respectively for the quasi-projectile (QP) and the quasi-target (QT) in the left side and for the mid-rapidity clusters and compound nucleus in the right side are shown. This figure clearly shows the partial isospin equilibration between the QP and QT, while clusters in the mid-rapidity region or close to compound nucleus are equilibrated in average. It also illustrates the large fluctuations in the nuclear chart.

![Figure 4: Illustration of the Neutron-Proton phase-space explored for the Quasi-Target (QT), Quasi-Projectile (QP) in the left side and for the Mid-rapidity clusters and Compound nucleus in the right side for the reaction $^{48}$Ca+$^{40}$Ca at beam energy $E_B = 25$ MeV/A (all impact parameters are mixed). In both cases, the largest area indicates the total phase-space available (in the mass table used in HIPSE), points indicates nuclei in the valley of stability, dashed lines correspond to the Target and Projectile $N/Z$ and the central solid line to the equilibrium $N/Z$.](image)

### III. CONCLUSION

We propose simple prescriptions that may be used to describe the pre-equilibrium emission of clusters in the course of nuclear reactions. These rules are based on a random sampling of the nucleons taking into account the Fermi motion and a proper account of nuclear effects as well as the conservation laws. Using these rules, two models dedicated respectively to nucleon nucleus (nIPSE) and nucleus-nucleus reactions (HIPSE) around the Fermi energy have been developed.

Besides the good reproduction of a large variety of experimental observations on a large set of different reactions and energies, these models underlines the importance of phase-space explored before and after the secondary decay. This aspect is illustrated in two examples of present interest, namely the memory loss in most
central collisions and the \((N, Z)\) partitions after the pre-equilibrium stage.

[1] D. Lacroix, A. Van Lauwe and D. Durand,\textit{ Phys. Rev. C} 69, 054604 (2004). A. Van Lauwe, D. Lacroix, and D. Durand, Proceedings of the IWM 2003 conference, Caen, France, 2003. A. Van Lauwe, Thèse de l’Université de Caen, France (2003).
[2] D. Lacroix, V. Blideanu, and D. Durand,\textit{ Phys. Rev. C} 71, 024601 (2005).
[3] D. Durand, E. Suraud, B. Tami, “Nuclear Dynamics in the Nucleonic Regime”, IOP Publishing, (2001).
[4] D. Lacroix and D. Durand, Proceedings of the 2nd Argonne/MSU/JINA/INT RIA Workshop on “Reaction mechanisms for rare isotope beams”, East Lansing, MSU, 9-12 March 2005, USA.
[5] F. E. Bertrand and R. W. Peelle,\textit{ Phys. Rev. C} 8, 1045 (1973).
[6] V. Blideanu et al., \textit{Phys. Rev. C} 70, 014607 (2004).
[7] G.A. Souliotis et al, Phys. Lett. \textbf{588B}, 35 (2004).
[8] C. Escano-Rodriguez et al, Los-alamos preprint, \texttt{arXiv:nucl-ex/0503007} (2005).