Keeping it Together: Interleaved Kirigami Extension Assembly

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Traditional origami structures can be continuously deformed back to a flat sheet of paper, while traditional kirigami requires glue or seams in order to maintain its rigidity. In the former, non-trivial geometry can be created through overfolding paper while, in the latter, the paper topology is modified. Here we propose a hybrid approach that relies upon overlapped flaps that create in-plane compression resulting in the formation of “virtual” elastic shells. Not only are these structures self-supporting, but they have colossal load-to-weight ratios of order 10^4.

I. INTRODUCTION

The role of self-folding and unfolding has become ever more used as a framework to understand natural structure [1, 2]. Since the tunability of geometry is scale-invariant, origami and kirigami inspired structures translate from large to small scales as can be seen from the packaging materials in daily life [3] to deployable solar panels for space missions [4, 5]. Moreover, their transformability serves as a powerful tool to program shape-induced properties: these architected structures have been utilized in flexible electronics [6], mechanical metamaterials [7–9], and soft robots [10, 11]. In practice, origami and kirigami start with geometric design. However, paper, time, and human effort are all required in order to design origami structures with exquisitely detailed Gaussian curvature [12]. Recently, kirigami methods have been developed that allow, in addition to folding, cutting and rejoining that drastically reduce the complexity of the inverse design problem [13–15]. By maintaining a lattice structure and inducing Gaussian curvature via buckling of (two-dimensional) disclinations into the third dimension, these lattice kirigami methods provide an algorithmic approach to design that maintains edge lengths on the lattice and dual lattice.

Although the kirigami rules provide much simpler ways to introduce curvature, we still need glue, seams, or a zipper to rejoin the shape (similarly in the case of origami if we want to hold the shape at specific folding angles). Different from locking by geometry [9, 10], here we demonstrate a novel way of self-locking without any external forces – we get the utmost out of the material. Instead of excising material from a dislocation [13, 14], we make cuts without removing any material. Upon folding, a dislocation-antidislocation pair has a pair of interleaved overlapping areas, which serve as intrinsic locks (Fig. 1). This “interleaved kirigami extension assembly” is lightweight, simple (can be easily designed and manipulated by hand), and strong (can keep its shape and possesses great bearing capacity). Our design lends itself to deployable structures like portable shelters, architectural canopies, and furniture. In analogy with origami-inspired energy absorption structures [17], it should also be possible to stack the proposed pattern into multiple layers and harness the buckling behaviors at sidewalls to absorb energy.

We first demonstrate the extended assembly method. We modify the topology of ground lattice and arrange dislocation pairs to form a whole pattern. Then we explore its working mechanism by strength tests. By observing pre-buckling and post-buckling behaviors of different models, we reveal rescaled curves that collapse experimental data and demonstrate the influence of “flaps” and neighbors. We find that our construction is unusually strong and hypothesize that the geometry and mechanics conspire to create stable, “virtual” shells, held together by in-plane compression and friction. We test this hypothesis in numerous ways by studying related cutting motifs and modifications to our original design.

II. EXTENDED ASSEMBLY

We start with the honeycomb lattice and its dual lattice, the triangular lattice. As in lattice kirigami [13], which follows the prescription so elegantly revealed in [18], we create a dislocation in the lattice by introducing a disclination pair. Cutting π/3 from one hexagon...
and combining two hexagons, each with $5\pi/6$ removed creates a pentagon-heptagon pair, a 5-7 dipole (here and throughout, the tilde refers to the fact that these are defects on the dual lattice). The parallel mountain and valley folds that create topography can be oriented at any angle with respect to the cuts. When creating a plateau, as in Fig. 1, we restrict ourselves for the moment, to regular, convex polygons. The discrete version of the Gauss-Bonnet theorem (i.e., geometry) requires that the cone angles add to $5\pi/3$. Calling the interior angle of the polygon $\alpha$ and the angle between the cut and the fold $\beta$, we have $\alpha + 2\beta = 5\pi/3$. For a single plateau, this does not constrain us. However, since we eventually consider periodic arrays, we require that the polygons respect the underlying lattice symmetry to preserve the intrinsic geometry of the lattice and dual lattice. It follows that $\alpha = \pi/3$ or $2\pi/3$ and so $\beta = 2\pi/3$ or $\pi/2$, respectively. The latter gives us the “classic” vertical walls and the former corresponds to new, tilted walls, with angle $\sin^{-1}(1/3) \approx 19.47^\circ$ from the vertical. In general, for an isolated “$\tilde{5}$-butte” with an internal angle $\alpha$, the sidewalls make an angle $\theta = \sin^{-1}[\tan(\pi/3 - \alpha/2) \tan(\alpha/2)]$ with the vertical and so, in principle, we can consider buttes with interior angle up to $5\pi/6$ with inward tilting sidewalls. If we wanted angles larger than $\alpha = \pi$ we can accomodate them by inverting the structure – that is, by putting a $\tilde{7}$ defect on the top and a $\tilde{5}$ on the bottom of the sidewall corners.

In classic kirigami paper is removed and rejoined. Here, however, we make the cuts but do not remove any paper. Instead, we leave the excess paper and overlap it after folding. These extensions, as shown in Fig. 1, neatly fit into the valley fold of the adjoining plateaus. Although a single glide (like Fig. 1) can hold its shape under small stresses, it can be unfolded easily along its dislocation direction. However, in a triangular lattice of plateaus, unfolding requires that each unit relax in three dislocation directions. By surrounding each unit with three other units as in Figs. 2 and 3, we can weave together the extensions to create a locked configuration – unfolding in one direction is frustrated by compression in the other two. As we will demonstrate, this mechanical linkage leads to collossal specific strengths. In the following, we will refer to each tetrahedral frustum as “the basic unit.”

III. STRENGTH TESTING

We fabricated this motif using four kinds of paper with different thicknesses (0.14 mm, 0.20 mm, 0.28 mm and 0.38 mm) [19]. We start with a letter-size sheet (215.9 mm by 279.4 mm) then cut and score it with a Graphtec CE6000 Series Cutting Plotter. Folding was performed by hand along the score lines. To assess the strength of each assembly, we performed tests on an Instron 5564 with a 2kN load cell to get force-displacement curves. Throughout, we set a loading rate of 2mm/min. We sandwiched each sample between thick, acrylic plates (the top plate mass is 61.9g) to spread the stress evenly. The loading setup is shown in Fig. 4. We begin our discussion with the 0.28mm-thick paper and will return to paper differences later. The load-displacement curves are shown in Fig. 5a with the gray region representing the error bars over three samples. We see that the loading curve has a number of regions: i) the initial linear slope; ii) a shoulder – it is at this point that the sidewalls begin to buckle; iii) a new linear regime where the buckled structure is responding elastically; iv) the peak and beyond where the details of wall-buckling give huge variations in response – note that the final rise in force comes from the crushing of the crumpled paper. Regime iii) is sensitive to the roughness of the paper: we have tried other materials such as transparency sheets and standard...
printer paper and the low friction changes the behavior of this region. We also note that the variation from sample to sample becomes large after the buckling. We attribute this to hard to control variations in folding and cutting leading to “random” buckling of the sidewalls, either in or out. This sensitivity arises from the nearly degenerate deformation modes of the structure: we used the general implicit finite element (FEM) code ABAQUS/Standard to calculate the linear buckling modes of thin plates organized into the original model. As shown in Fig. 7, the inward and outward buckling modes have nearly degenerate eigenvalues and this is why small variations (imperceptible to us) change the nature of buckling (in or out). Nonetheless, the load curves are reproducible up until the first peak and we can collapse the data simply by rescaling by the peak force and the displacement at peak force as shown in Fig. 5b.

FIG. 3. From Left to Right: step by step assembly of the periodic plateau array. Note that this assembly requires bending of the sheets and so this structure is not flat-foldable.

We refer to this structure as the “original model”. To fully explore how neighbors contribute to the overall strength of the structure, we compared the original model with both pinned models and disconnected models. In the pinned models, we remove the influence of the excess flaps by pinning the flaps with tape as on the left in Fig. 8. The tape does not modify the units, each trapezoidal edge remains free; the tape only keeps the flaps from moving and prevents the conversion of downward forces to in-plane compression. Likewise, on the right of Fig. 8 we physically disconnected the plateaus to remove all interactions. All the models here were made with the 0.28mm paper.

As expected, the disconnected model is the weakest. However, surprisingly, up until the initial peak, the original model is stronger than the pinned model by roughly a factor of two. What accounts for this? We note that the transfer of vertical stresses on the plateaus into in-plane stresses on the structure can only occur when the flaps are both present and are free to move. This creation of compression locks together the the plateaus along their open edges transforming them from three separate Euler elastic beams into a rigid frustum-shaped shell. The strength of the original structure is created by these effective shells. The roughness of and friction between the slits confers more or less strength to the design. Indeed, smooth paper is inferior to rougher paper. Further, note that in the pinned and disconnected models the dominant buckling mode is out but the virtual shell model requires inward buckling. Fortuitously, the inward buckling is promoted by the displacement of the flaps. We can see from the data that the absolute force at which buckling begins, the shoulder, is roughly the same for all three models. It is only then the subsequent interaction between the sidewalls that add to the strength.

To test this hypothesis we constructed a square analog of the original model. As shown in Fig. 9 this model has square, overlapping flaps but has vertical sidewalls. Without any in-plane stresses, vertical sidewalls should be stronger than canted sidewalls since the sloped walls of the original model must bear larger compression for the same load. However, we do not find this. The square model is weaker, presumably because there is no transfer of load to in-plane compression and thus the square plateaus do not get compressed and behave as a solid, cubical shell.

The FEM results explain why the onset of buckling is the same in the original, pinned, and disconnected model. The compression tests were simulated by imposing vertical displacements at the top triangle of a single basic unit while only allowing rotation along the six fold lines. The first three modes were all bending modes with very close eigenvalues (Fig. 7). The fourth mode was a twisting mode with an eigenvalue gap. Note also, that in the FEM analysis, different buckling in the individual sidewalls does not seem to have an effect on neighboring walls – they buckle independently. Of course, when adjacent

FIG. 4. The loading setup. a) To avoid boundary effects, we only push on fully coordinated plateaus. b) Upon collapse, some walls buckle in while others buckle out. We denote the outward buckled walls with red lines. Three different samples show that the collapse has a random component, likely due to folding inhomogeneity.
sidewalls buckle in, they will contact and support each other, but the finite element analysis does not account for contact. Indeed, in experiment, units with two or three inward-buckled sidewalls had slightly higher buckling peaks.

To further test the validity of the virtual shell model, we also created “gapped” models where we removed some of the paper along the edges of the sidewalls (also shown in Fig. 8). The gapped model showed nearly the same loading curve as the pinned model. In the former case, the sidewalls could not contact and form shells, in the latter, there was no in-plane compression to press the walls together. Therefore both of these loading curves are giving us information about the Euler-like buckling of the walls which are identical.

In addition, we made measurements with different unit sizes, scaled by the linear factor $\lambda$. We always pushed on six, fully-coordinating units. Since the contact area shrank as $\lambda^2$, but, as Fig. 6 shows, the buckling force remained constant, the buckling pressure scales as $\lambda^{-2}$. This is precisely what one expects for shells: for spherical shells the collapse pressure scales as $(h/R)^2$ where $h$ is the thickness of the shell and $R$ is its radius \[20\]. Since we use identical paper, $h$ is unchanged as are all the moduli, corroborating the virtual shell picture. A full sheet of 1/2 scale units has a mass of 8.1g and buckles at around 1100 kN, a load-to-weight ratio around 14000!

Finally, we turn to how the paper type changes the behavior of the assemblies. It is important to note that paper is a complicated material. It is not isotropic since there is a machine rolling direction and the transverse direction \[21\]. Fibers are primarily aligned along the machine direction, leading to a mechanical difference in different directions. However, the structures we consider have three fold symmetry, washing out the anisotropy of the paper – we checked that the buckling thresholds are independent of orientation (within experimental error). In addition, the stiffness of cardboard (the 0.20 mm, 0.28 mm and 0.38 mm paper are all cardboard) is controlled
FIG. 7. Numerical images of the first four eigenmodes and the corresponding eigenvalues. The deformation level in images is enlarged by a factor of four for clarity.

FIG. 8. Pinned, gapped, and disconnected models. a) Diagrams of cuts and folds. For pinned models, we made cuts along solid lines and added tape on the hatched areas to hold the shape. For the gapped models, we removed the edges of the sidewalls to prevent them from touching adjacent walls upon compression. For disconnected models, we folded six units separately and stuck them at the positions they would sit at were they on the original model. b) Final states of all three models. Red short lines denote walls buckled outside. Note that outward buckling was the dominant mode in the pinned and disconnected cases.

by the top and bottom surface layers, made of chemical pulp while the mechanical pulp – the filler that makes up the rest of the material – is sandwiched between [22].

As shown in Fig. 10, all curves had almost the same buckling plateaus except the softest paper (0.14 mm). Because the 0.14 mm paper is thin its behavior is in the membrane regime where compression is difficult to support, i.e. \( a/h \approx 15/0.14 > 80 \) where \( a \) is the dimension along which we bend [23]. Note also that were the three heaviest papers uniform materials, Ref. [20] suggests that the peak strengths should be in the ratios \((28/20)^2 \approx 2\), and \((38/28)^2 \approx 1.8\). However, we find ratios around 3 and 2, respectively only confirming that paper is a complex multilayered material. Future work with more uniform and considerably more expensive material should resolve this discrepancy.

IV. SUMMARY AND CONCLUSIONS

In summary, we have designed an interleaved kirigami extension assembly and characterized its mechanical
properties. Not only does our pattern hold its shape through overlapping frictional flaps but they conduct forces effectively between adjacent units through in-plane compression. We have suggested that, in turn, the compression seals the individual units which then act as two-dimensional shells. A variety of tests confirm this hypothesis.

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