Stable scalar tetraquark \( T_{bb\bar{u}\bar{d}}^- \)

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The mass and coupling of the scalar tetraquark \( T_{bb\bar{u}\bar{d}}^- \) (hereafter \( T_{bb}^- \)) are calculated in the context of the QCD two-point sum rule method. In computations we take into account effects of various quark, gluon and mixed condensates up to dimension ten. The result obtained for the mass of this state \( m = (10389 \pm 240) \) MeV demonstrates that it is stable against the strong and electromagnetic decays. We also explore the dominant semileptonic \( T_{bb}^- \to Z_{bc;\bar{u}\bar{d}}^0 \nu \) and nonleptonic decays \( T_{bb}^- \to Z_{bc;\bar{u}\bar{d}}^0 M \), where \( Z_{bc;\bar{u}\bar{d}}^0 \) is the color-sextet scalar tetraquark, and \( M \) is one of the final-state pseudoscalar mesons \( \pi^-, K^-, D^- \) and \( D_s^- \), respectively. The partial widths of these processes are calculated in terms of the weak form factors \( G_1(q^2) \), which are determined from the QCD three-point sum rules. Predictions for the mass, full width \( \Gamma_{\text{full}} = (10.88 \pm 1.88) \times 10^{-10} \) MeV, and mean lifetime \( \tau = 0.61^{+0.13}_{-0.09} \) ps of the \( T_{bb}^- \) obtained in the present work can be used in theoretical and experimental studies of this exotic state.

I. INTRODUCTION

Four-quark states composed of heavy diquarks \( QQ' \) and light \( q\bar{q} \) antiquidquarks are real candidates to exotic mesons existence of which does not contradict first principles of QCD. During last few years an interest to these tetraquarks is renewed, although this topic attracted interest of physicists many years ago \(^{1–5}\). The main qualitative results concerning a stability of the tetraquarks \( QQ'q\bar{q} \) against strong decays were also obtained in these articles. Thus, it was shown that such four-quark compounds may be stable if the ratio \( m_Q/m_q \) is sufficiently large. Investigations in the framework of different models confirmed that the axial-vector state \( T_{bb\pi^0}^- \) (in what follows \( T_{bb}^- \)) is a strong-interaction stable particle with the mass below the \( B\bar{B} \) threshold \(^{4,5}\).

Discovery of double-charmed \( \Xi_{cc}^{+} \) baryon stimulated a new interest to these heavy tetraquarks because parameters of \( \Xi_{cc}^{+} \) were used in phenomenological models to estimate the mass of \( T_{bb}^- \) \(^{6,7}\). The prediction for the mass of \( T_{bb}^- \) obtained in Ref. \(^6\) equals to \( m = (10389 \pm 12) \) MeV being 215 MeV below \( B\bar{B} \) and 170 MeV below \( B^-\bar{B}^0 \gamma \) thresholds, respectively. This means that the tetraquark \( T_{bb}^- \) is stable against the strong and electromagnetic decays and should dissociate to conventional mesons only weakly. The similar conclusion on strong-interaction stable nature of the tetraquark \( T_{bb}^- \) was made in Ref. \(^7\) on the basis of the heavy-quark symmetry analysis. Its mass was found there \( m = 10482 \) MeV which is 121 MeV below the open-bottom threshold.

In the context of the QCD sum rule method the axial-vector particle \( T_{bb}^- \) was recently studied in our paper \(^8\). In accordance with Ref. \(^8\) the mass of \( T_{bb}^- \) amounts to \( m = (10035 \pm 260) \) MeV that confirms once more its stability against the strong and electromagnetic decays. We calculated not only the mass and current coupling of the tetraquark \( T_{bb}^- \), but also explored the semileptonic decays \( T_{bb}^- \to Z_{bc;\bar{u}\bar{d}}^0 \nu \). In these processes we treated the final-state tetraquark \( Z_{bc;\bar{u}\bar{d}}^0 = \{bc|\bar{u}\bar{d}\} \) as a scalar particle with the color-triplet structure \( \{\bar{c}\bar{e}\}bc \otimes [\bar{c}\bar{e}]_3 \). The predictions for the full width \( \Gamma = (7.17 \pm 1.23) \times 10^{-8} \) MeV and mean lifetime \( \tau = 9.18^{+1.93}_{-1.34} \) ps of this axial-vector tetraquark are information useful for experimental investigation of a family of double-heavy exotic mesons. The parameters of the \( T_{bb}^- \) and its weak decays were considered also in Ref. \(^8\).

In the present article we are going to extend our analysis of exotic mesons \( bb\bar{u}\bar{d} \) and explore the scalar partner of the \( T_{bb}^- \) with the same quark content. We denote this particle \( T_{bb}^- \) and compute its spectroscopic parameters, full width and mean lifetime. The mass \( m \) and coupling \( f \) of the \( T_{bb}^- \) are evaluated by means of the QCD two-point sum rule method, where we take into account various quark, gluon, and mixed vacuum condensates up to dimension ten. A result for the mass of the state \( T_{bb}^- \) is important for our following investigations. Indeed, \( m \) determines whether \( T_{bb}^- \) is strong-interaction stable particle or not. A simple consideration allows one to see that the scalar tetraquark \( T_{bb}^- \) in \( S\)-wave can strongly fall-apart to a pair of conventional mesons \( B^-\bar{B}^0 \) provided its mass is higher than the threshold 10560 MeV. But, our calculations demonstrate that mass of this tetraquark is \( m = (10135 \pm 240) \) MeV, and therefore \( T_{bb}^- \) is a strong-interaction stable particle. It is also stable against an electromagnetic dissociation \( T_{bb}^- \to B^-\bar{B}^0(5721)^0 \gamma \), because this process may run only if the mass of the initial particle exceeds 11003 MeV which is not a case. As a result, to determine the full width and mean lifetime of \( T_{bb}^- \) we have to study its weak decays.

The dominant weak decays of the \( T_{bb}^- \) are generated...
by a subprocess $b \to W^- c$ which lead to its semileptonic and nonleptonic transformation to the exotic scalar meson $Z_{bc}^0$ (a brief form of $Z_{bc}^0 \pi$). The scalar tetraquark $Z_{bc}^0$ should belong to sextet representation of the color group $[6^e]_{bc} \otimes [6^e]_{bc}$. This restriction is connected with features of different tetraquarks’ weak decays, where the following selection rules are valid: for transitions with $\Delta S = 0$, when initial and final tetraquarks have the same spin $S$, weak transitions occur between states of the same color structure, i.e., only triplet-triplet and sextet-sextet transitions are allowed \cite{11,13}. For processes $|\Delta S| = 1$ initial and final tetraquarks that belong to different representation of the color group give rise to sextet-triplet and triplet-sextet weak decays \cite{8}. Because in our case the scalar particle $T_{bc}^-$ contains the diquark $bb$ and has the color-sextet organization, then the scalar tetraquark $Z_{bc}^0$ should also have the color-sextet structure.

Thus, weak processes to be explored are the semileptonic $T_{bc}^- \to Z_{bc}^0 \pi_l$, and nonleptonic $T_{bc}^- \to Z_{bc}^0 M$ decays of $T_{bc}^-$. In the present work, we consider the processes, where $M$ is one of the conventional pseudoscalar mesons $\pi^-, K^-, D^-$ and $D_s^-$. It is clear that nonleptonic processes can be kinematically realized if $m - m_Z > m_M$ with $m_Z$ and $m_M$ being the masses of the tetraquark $Z_{bc}^0$ and $M$ meson, respectively. The spectroscopic parameters $m_Z$ and $\tilde{J}_Z$ of the scalar tetraquark $Z_{bc}^0$ are necessary to calculate partial widths of all weak decays under consideration, and they will be found as well.

The full width of the tetraquark $T_{bc}^-$ are calculated by taking into account aforementioned semileptonic and nonleptonic decay modes. For these purposes, we employ the QCD three-point sum rule approach and compute weak form factors $G_1(q^2)$ and $G_2(q^2)$ required for our studies. Namely these form factors enter into expressions for the differential rate $\frac{d\Gamma}{dq^2}$ of semileptonic and partial width of nonleptonic processes. The sum rule computations, unfortunately, lead to reliable predictions for $G_{1(2)}(q^2)$ only at limited values of the momentum transfers $q^2$. To integrate $\frac{d\Gamma}{dq^2}$ over $m_t^2 \leq q^2 \leq (m - m_Z)^2$ and find the partial widths of the semileptonic decays, we need to extrapolate sum rule predictions to this domain. The latter is achieved by introducing fit functions $G_{1(2)}(q^2)$ that coincide with the sum rule results when they are accessible, and can be easily extrapolated to the whole integration region.

This article is organized in the following manner: In Section \textbf{II} we calculate the mass and coupling of the scalar tetraquarks $T_{bc}^-$ and $Z_{bc}^0$. To this end, we derive sum rules from analysis of the relevant two-point correlation functions: in numerical computations we take into account quark, gluon and mixed condensates up to dimension ten. In Section \textbf{III} using spectroscopic parameters of the initial and final tetraquarks and three-point sum rules, we compute the weak form factors $G_{1(2)}(q^2)$ in regions of the momentum transfers $q^2$, where the method leads to reliable predictions. In this section we also determine the model functions $G_{1(2)}(q^2)$ and find the partial widths of the semileptonic decays $T_{bc}^- \to Z_{bc}^0 \pi_l$. In Section \textbf{IV} we explore the nonleptonic decays $T_{bc}^- \to Z_{bc}^0 M$ of the tetraquark $T_{bc}^-$. Here, we write down our final predictions for the full width and lifetime of the $T_{bc}^-$. Section \textbf{V} is devoted to analysis of obtained results, and contains our concluding remarks.

\section{Spectroscopic Parameters of the Scalar Tetraquarks $T_{bc}^-$ and $Z_{bc}^0$}

The spectroscopic parameters $m$, and $f$ of the tetraquark $T_{bc}^-$ are required to reveal its nature and answer a question whether this state is stable against strong and electromagnetic decays or not. The mass and coupling of $Z_{bc}^0$ are important to explore the weak decays of the master particle $T_{bc}^-$. Besides, the tetraquark $Z_{bc}^0$, as its color-triplet partner $Z_{bc}^0$, may be strong- and/or electromagnetic-interaction stable particle, which is of independent interest for us.

The parameters of these states can be extracted from the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0| T \{ J(x) J^\dagger(0) \} |0 \rangle, \quad (1)$$

where $J(x)$ is the interpolating current for a scalar particle. In the case of $T_{bc}^-$ it has the following form

$$J(x) = \left[ b^T_a(x) C \gamma_5 b_b(x) \right] \left[ \bar{\pi}_a(x) \gamma_5 C \bar{d}^T_b(x) \right]. \quad (2)$$

For the tetraquark $Z_{bc}^0$ we use

$$\tilde{J}_Z(x) = \left[ b^T_a(x) C \gamma_5 a(x) \right] \left[ \bar{\pi}_a(x) \gamma_5 C \bar{d}^T_b(x) \right] + \bar{\pi}_a(x) \gamma_5 C \bar{d}^T_b(x). \quad (3)$$

In these currents $a$, and $b$ are color indices and $C$ is the charge-conjugation operator.

As it has been emphasized above, $J(x)$ and $\tilde{J}_Z(x)$ are symmetric in color indices and have the color-sextet organizations. They correspond to tetraquarks composed of the scalar diquark and antidiquark, which are most attractive and stable two-quark structures \cite{14}, consequently these currents describe the lowest lying ground-state sextet tetraquarks.

Spectroscopic parameters of different scalar tetraquarks were objects of detailed sum rule analysis, therefore we provide below only essential stages of calculations in the case of the tetraquark $T_{bc}^-$ and give final results for the $Z_{bc}^0$.

The sum rules to evaluate $m$ and $f$ can be obtained by matching two expressions of the correlation function $\Pi(p)$: the first expression is calculated using the physical parameters of $T_{bc}^-$ whereas the second one is written
The shorthand notation quark propagators, respectively. Above we also introduce where \( \times \) these operations for \( \Pi \) and contract relevant heavy and light quark fields. After rather lengthy, we do not write down it here.

The explicit expressions of the heavy and light quark propagators can be found, for instance, in Ref. [10]. The nonperturbative parts of the propagators contain various quark, gluon, and mixed condensates which are sources of nonperturbative terms in \( \Pi^{\text{OPE}}(p) \).

The first equality necessary to derive the sum rules are obtained by equating the amplitudes \( \Pi^{\text{Phys}}(p^2) \) and \( \Pi^{\text{OPE}}(p^2) \), and applying to both sides of this expression the Borel transformation: By this way we suppress contributions to the sum rules of higher resonances and continuum states. But even after the Borel transformation suppressed terms appear as a contamination in the physical side of the equality. Fortunately, they can be subtracted by invoking assumption about quark-hadron duality. The second equality required for our purposes is derived by applying the operator \( d/dl(−1/M^2) \) to the first one. These two expressions are enough to get the sum rules for \( m \)

\[
m^2 = \frac{\int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}}{\int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}},
\]

and for \( f \)

\[
f^2 = \frac{1}{m^2} \int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{(m^2 − s)/M^2}.
\]

The two-point spectral density \( \rho^{\text{OPE}}(s) \) is computed as an imaginary part of the correlation function \( \Pi^{\text{OPE}}(p) \). We include into analysis vacuum condensates up to dimension 10: because the final expression of \( \rho^{\text{OPE}}(s) \) is rather lengthy, we do not write down it here.

The sum rules [9] and [10] contain the universal vacuum condensates and masses of \( b \) and \( c \) quarks:

\[
\begin{align*}
\langle \bar{q}q \rangle &= -(0.24 ± 0.01)^3 \text{GeV}^3, \quad \langle s\bar{s} \rangle = 0.8 \langle \bar{q}q \rangle, \\
\langle \bar{g}_s\sigma Gq \rangle &= m_0^2 \langle \bar{q}q \rangle, \quad \langle \bar{g}_b\sigma Gs \rangle = m_0^2 \langle s\bar{s} \rangle, \\
m_b^2 &= (0.8 ± 0.1) \text{GeV}^2, \\
\left\langle \frac{\sigma_s G^2}{\pi} \right\rangle &= (0.012 ± 0.004) \text{GeV}^4, \\
\left\langle g_s^3 C^3 \right\rangle &= (0.57 ± 0.29) \text{GeV}^6, \\
m_c &= 1.27 ± 0.2 \text{GeV}, \quad m_b = 4.18_{−0.02}^{+0.03} \text{GeV}.
\end{align*}
\]

Besides, \( m \) and \( f \) depend on the Borel \( M^2 \) and continuum threshold \( s_0 \) parameters appeared in Eqs. [9] and [10] after the Borel transformation and continuum subtraction procedures, respectively. The \( M^2 \) and \( s_0 \) are the auxiliary parameters of the problem under discussion, a correct choice of which is an important task of computations. But proper regions for \( M^2 \) and \( s_0 \) should meet some restrictions imposed on the pole contribution (PC) and convergence of the operator product expansion (OPE). In fact, at maximum of \( M^2 \) the PC should obey the constraint

\[
\text{PC}= \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)} > 0.2,
\]

\[\text{FIG. 1: The mass } m \text{ of the tetraquark } T_{bc\overline{cd}} \text{ as a function of the Borel } M^2 \text{ and continuum threshold } s_0 \text{ parameters.} \]
where $\Pi(M^2, s_0)$ is the Borel-transformed and subtracted invariant amplitude $\Pi^{\text{OPE}}(p^2)$. The minimum of $M^2$ is fixed from analysis of the ratio

$$R(M^2) = \frac{\Pi^\text{DimN}(M^2, s_0)}{\Pi(M^2, s_0)} \leq 0.01. \quad (13)$$

In Eq. (13) $\Pi^\text{DimN}(M^2, s_0)$ denotes a contribution of the last term (or a sum of last few terms) to the correlation function. In the present calculations we use a sum of last three terms, and hence DimN $\equiv$ Dim$(8 + 9 + 10)$.

Our analysis demonstrates that the working windows for the parameters $M^2$ and $s_0$ are

$$M^2 \in [8.5, 12] \text{ GeV}^2, \quad s_0 \in [113, 118] \text{ GeV}^2, \quad (14)$$

and they satisfy all aforementioned constraints on $M^2$ and $s_0$. Indeed, at $M^2 = 12 \text{ GeV}^2$ the pole contribution is 0.21, whereas at $M^2 = 8.5 \text{ GeV}^2$ it amounts to 0.61. These two values of $M^2$ fix the boundaries of a region where the Borel parameter can be varied. At the minimum of $M^2 = 8.5 \text{ GeV}^2$ we get $R \approx 0.006$. Apart from that, at minimum of the Borel parameter the perturbative contribution forms 79% of the whole result overshooting significantly the nonperturbative terms.

Our results for $m$ and $f$ are

$$m = (10135 \pm 240) \text{ MeV}, \quad f = (2.26 \pm 0.57) \times 10^{-2} \text{ GeV}^4, \quad (15)$$

where uncertainties of computations are shown as well. Theoretical uncertainties in the case of $m$ equal to $\pm 2.4\%$, whereas for the coupling $f$ they amount to $\pm 25\%$ remaining, at the same time, within limits accepted in sum rule computations. It is worth noting that these uncertainties appear mainly due to variations of the parameters $M^2$ and $s_0$. In Fig. 1 we display the sum rule’s prediction for $m$ as a function of $M^2$ and $s_0$, where one can see residual dependence of the mass on these parameters.

The mass and coupling of the scalar tetraquark $\bar{Z}_{bc}^0$ are calculated by the same way. The phenomenological side of the corresponding sum rules is determined by Eq. (9) with evident replacement $(m, f) \to (\bar{m}_Z, \bar{f}_Z)$. Their QCD side is given by the following formula

$$\bar{\Pi}^{\text{OPE}}(p) = i \int d^4xe^{ipx} \text{Tr} \left[ \gamma_5 \bar{S}_b(x)\gamma_5 \bar{S}_c^b(x) \right] \times \left\{ \text{Tr} \left[ \gamma_j \bar{S}_d^{\nu b}(x)\gamma_5 s_u^\nu(-x) \right] + \text{Tr} \left[ \gamma_j \bar{S}_d^{a b}(x)\gamma_5 s_u^a(-x) \right] \right\}. \quad (16)$$

The mass $\bar{m}_Z$ and coupling $\bar{f}_Z$ of the tetraquark $\bar{Z}_{bc}^0$ can be found from Eqs. (10) and (11) by replacing $\rho^{\text{OPE}}(s) \to \bar{\rho}^{\text{OPE}}(s)$, where the spectral density $\bar{\rho}^{\text{OPE}}(s)$ is found using the correlation function $\Pi^{\text{OPE}}(p)$, and substituting $(m_b + m_c)^2$ instead of $4m_b^2$. As working windows for $M^2$ and $s_0$ we utilize

$$M^2 \in [5.5, 6.5] \text{ GeV}^2, \quad s_0 \in [53, 54] \text{ GeV}^2. \quad (17)$$

The regions (17) meet standard constraints of the sum rule computations. Thus, at $M^2 = 5.5 \text{ GeV}^2$ the ratio $R$ is 0.01, hence the convergence of the sum rules is satisfied. The pole contribution PC at $M^2 = 6.5 \text{ GeV}^2$ and $M^2 = 5.5 \text{ GeV}^2$ equals to 0.24 and 0.71, respectively. At minimum of $M^2$ the perturbative contribution constitutes 72% of the whole result exceeding considerably nonperturbative terms.

For $\bar{m}_Z$ and $\bar{f}_Z$ our computations yield

$$\bar{m}_Z = (6730 \pm 150) \text{ MeV}, \quad \bar{f}_Z = (6.2 \pm 1.4) \times 10^{-3} \text{ GeV}^4. \quad (18)$$

In Fig. 2 we plot the prediction obtained for the mass of the tetraquark $\bar{Z}_{bc}^0$ and show its dependence on $M^2$ and $s_0$.

### III. SEMILEPTONIC DECAYS $T_{b\bar{d}}^- \to \bar{Z}_{bc}^0 \nu_l$

The result for the mass of the tetraquark $T_{b\bar{d}}^-$ proves its stability against the strong and electromagnetic decays. In fact, the central value of the mass $m = 10135 \text{ MeV}$ is 425 MeV lower than the threshold 10560 MeV for strong decay to mesons $B^- \bar{B}^0$. Its maximal allowed value $m_{\text{max}} = 10375 \text{ MeV}$ is 185 MeV below this limit as well. In other words, the $T_{b\bar{d}}^-$ is a strong-interaction stable particle. The threshold 11003 MeV for the decay $T_{b\bar{d}}^- \to B^- \bar{B}^0(5271)^0 \gamma$ is higher than $m_{\text{max}}$ which forbids this electromagnetic process. Therefore, the full width and mean lifetime of the $T_{b\bar{d}}^-$ are determined by its weak decays.

This section is devoted to analysis of the dominant semileptonic decay $T_{b\bar{d}}^- \to \bar{Z}_{bc}^0 \nu_l$ triggered by the weak
transition of the heavy $b$-quark $b \to W^− c \to c \bar{c}$. It is evident, that the mass difference $3405$ MeV of the states $T_{bc}^−$ and $Z_{bc}^0$ makes all decays $T_{bc}^− \to Z_{bc}^0 \ell_1$, where $l = e, \mu$ and $\tau$ kinematically allowed ones. Here, we neglect processes generated by a subprocess $b \to W^− u$, because they are suppressed relative to dominant decays by a factor $|V_{ub}|^2/|V_{cb}|^2 \approx 0.01$ with $V_{q_i q_j}$ being the Cabibbo-Kobayasi-Maskawa (CKM) matrix elements.

At the tree-level the subprocess $b \to W^− c$ can be described using the effective Hamiltonian
\[
\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \gamma_\mu (1 - \gamma_5) \bar{d} \gamma_\mu (1 - \gamma_5) b. \tag{19}
\]

Here, $G_F$ and $V_{bc}$ are the Fermi coupling constant and CKM matrix element, respectively
\[
G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad |V_{bc}| = (42.2 \pm 0.08) \times 10^{-3}. \tag{20}
\]

A matrix element of $\mathcal{H}^{\text{eff}}$ between the initial and final tetraquarks
\[
\langle \bar{Z}_{bc}^0(p') | \mathcal{H}^{\text{eff}} | T_{bc}^- (p) \rangle = L_\mu H^\mu,
\]

consists of leptonic and hadronic factors. A leptonic part of the matrix element $L_\mu$ is universal for all semileptonic decays and does not contain information on features of tetraquarks. Therefore, we are interested in calculation of $H^\mu$ which is nothing more than the matrix element of the current
\[
J^{\mu \tau}_\mu = \bar{\tau} \gamma_\mu (1 - \gamma_5) b. \tag{22}
\]

It can be detailed using form factors $G_{1(2)}(q^2)$ that parametrize the long-distance dynamics of the weak transition. In terms of $G_{1(2)}(q^2)$ the matrix element $H^\mu$ has the form
\[
\langle \bar{Z}_{bc}^0(p') | J^{\mu \tau}_\mu | T_{bc}^- (p) \rangle = G_1(q^2) P_\mu + G_2(q^2) q_\mu, \tag{23}
\]

where $p$ and $p'$ are the momenta of the initial and final tetraquarks, respectively. Above, we also use notations $P_\mu = p'_\mu + p_\mu$ and $q_\mu = p_\mu - p'_\mu$. The $q_\mu$ is the momentum transferred to the leptons, hence $q^2$ changes in the region $m_l^2 \leq q^2 \leq (m - m_Z)^2$, where $m_l$ is the mass of a lepton $l$.

The sum rules for the form factors $G_{1(2)}(q^2)$ can be extracted from the three-point correlation function
\[
\Pi_\mu(p, p') = i^2 \int d^4x d^4y \epsilon^{ip'y - px} \langle 0 | T \{ \bar{J}_Z(y) J^{\tau \mu}_\mu (0) J^\tau (x) \} | 0 \rangle. \tag{24}
\]

As usual, we write down $\Pi_\mu(p, p')$ using the spectroscopic parameters of the tetraquarks, and get the physical side of the sum rule $\Pi^{\text{phys}}_\mu(p, p')$. The function $\Pi^{\text{phys}}_\mu(p, p')$ has the following form
\[
\Pi^{\text{phys}}_\mu(p, p') = \frac{(0) \bar{J}_Z(p') \langle \bar{Z}_{bc}^0(p') | J^{\mu \tau}_\mu | T_{bc}^- (p) \rangle}{(p^2 - m^2)(p'^2 - m_Z^2)} \times (T_{bc}^- (p) | J^\tau | 0) + \ldots, \tag{25}
\]

where the term in Eq. (24) is contribution of the ground-state particles: contributions of excited resonances and continuum states are denoted by dots.

The phenomenological side of the sum rules can be simplified by substituting in Eq. (25) expressions of matrix elements in terms of the tetraquarks’ masses and couplings, and weak transition form factors. To this end, we employ Eqs. (5) and (23), and additionally invoke the matrix element of the state $Z_{bc}^0$
\[
\langle 0 | \bar{J}_Z | \bar{Z}_{bc}^0(p') \rangle = f_Z m_Z. \tag{26}
\]

Then one gets
\[
\Pi^{\text{phys}}_\mu(p, p') = \frac{f m \bar{J}_Z m_Z}{(p^2 - m^2)(p'^2 - m_Z^2)} \times [G_1(q^2) P_\mu + G_2(q^2) q_\mu] + \ldots \tag{27}
\]

We find $\Pi_\mu(p, p')$ also using explicitly the interpolating currents in the correlator, and expressing (24) in terms of quark propagators, which lead to the QCD side of the sum rules
\[
\Pi^{\text{OPE}}_\mu(p, p') = i^2 \int d^4x d^4y \epsilon^{ip'y - px} \left\{ \left( \bar{\gamma}_5 S_{\bar{d}}^b(x - y) \right) \times \bar{\gamma}_5 S_{
\]
where \( \rho_{12}(s, s', q^2) \) are the spectral densities calculated with dimension-7 accuracy. In Eq. (29) the pair of parameters \((M_q^2, s_0)\) describes the initial tetraquark \( T_{bc}^- \), whereas the set \((\tilde{m}_Z^2, s_0)\) corresponds to the final state \( \tilde{Z}_{bc}^0 \).

In computations the working regions for \( M^2 \) and \( s_0 \) are chosen as in analyses of the masses \( m \) and \( m_Z \). Input information necessary for numerical calculations of \( G_{1(2)}(q^2) \) that includes the vacuum condensates, spectroscopic parameters of the tetraquarks \( T_{bc}^- \) and \( \tilde{Z}_{bc}^0 \) are presented in Eqs. (11), (15) and (18), respectively. In Fig. 3 we show obtained predictions for the form factors \( G_1(q^2) \) and \( G_2(q^2) \).

The sum rules give reliable results for \( G_{1(2)}(q^2) \) in the region \( m^2 \leq q^2 \leq 8 \text{ GeV}^2 \), which is not enough to calculate the partial width of the process \( T_{bc}^- \to \tilde{Z}_{bc}^0 \tau \bar{\tau} \) under analysis. Thus, the form factors \( G_{1(2)}(q^2) \) determine the differential decay rate \( d\Gamma/dq^2 \) of this process

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{bc}|^2}{64\pi^3 m^3} \lambda \left( m^2 - m_Z^2, q^2 \right) \left( \frac{q^2 - m_Z^2}{q^2} \right)^2 \times \\
(2q^2 + m_Z^2) \left[ G_2^2(q^2) \left( \frac{q^2}{2} - m^2 - m_Z^2 \right) - G_2^2(q^2) \left( \frac{q^2}{2} + (m_Z^2 - m^2)G_1(q^2)G_2(q^2) \right) \\
+ \frac{q^2 + m_Z^2}{q^2} \left[ G_1(q^2)(m^2 - m_Z^2) + G_2(q^2)q^2 \right] \right] \cdot \frac{1}{q^2},
\]

(30)

where

\[
\lambda \left( m^2, m_Z^2, q^2 \right) = \left[ m^4 + m_Z^4 + q^4 \right] - 2 \left( m^2 m_Z^2 + m^2 q^2 + m_Z^2 q^2 \right)^{1/2}.
\]

(31)

To find the width of a semileptonic decay, \( d\Gamma/dq^2 \) should be integrated over \( q^2 \) in the limits \( m^2 \leq q^2 \leq (m - m_Z)^2 \). But \( m^2 \leq q^2 \leq 11.59 \text{ GeV}^2 \) is wider than the region where the sum rules lead to strong results. This problem can be evaded by introducing fit functions \( \mathcal{G}_i(q^2) \) \((i = 1, 2)\): at the momentum transfers \( q^2 \) accessible for the sum rule computations they have to coincide with \( G_i(q^2) \), but have analytic forms suitable to carry out integrations over \( q^2 \).

For these purposes, we use the functions of the form

\[
\mathcal{G}_i(q^2) = \mathcal{G}_i^0 \exp \left[ g_1^I \frac{q^2}{m^2} + g_2^I \left( \frac{q^2}{m^2} \right)^2 \right],
\]

(32)

where \( \mathcal{G}_i^0, g_1^I, \) and \( g_2^I \) are constants which have to be fixed by comparing \( \mathcal{G}_i(q^2) \) and \( G_i(q^2) \) at common domains of validity. Performed numerical analysis gives

\[
\mathcal{G}_1^0 = -0.26, \ g_1^I = 10.14, \ g_2^I = -10.36
\]

\[
\mathcal{G}_2^0 = 0.35, \ g_1^I = 8.87, \ g_2^I = -7.91.
\]

(33)

The functions \( \mathcal{G}_i(q^2) \) are plotted in Fig. 3 one can see an agreement between the sum rule predictions and fit functions.

The masses of the leptons \( m_e = 0.511 \text{ MeV} \), \( m_\mu = 105.658 \text{ MeV} \), and \( m_\tau = (1776.82 \pm 0.16) \text{ MeV} \) used to find \( \Gamma(T_{bc}^- \to \tilde{Z}_{bc}^0 \tau \bar{\tau}) \) are taken from Ref. [15]. The results obtained for the partial width of the semileptonic decays \( T_{bc}^- \to \tilde{Z}_{bc}^0 \) are collected in Table I.

**TABLE I**: Partial width of the \( T_{bc}^- \) tetraquark’s weak decay channels.

| Channel | Partial width |
|---------|---------------|
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 e^- \nu_e \) | \((4.45 \pm 1.28) \times 10^{-10} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 \mu^- \nu_\mu \) | \((4.44 \pm 1.26) \times 10^{-10} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 \tau^- \nu_\tau \) | \((1.99 \pm 0.56) \times 10^{-10} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 \pi^- \) | \((5.13 \pm 1.42) \times 10^{-13} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 K^- \) | \((3.93 \pm 1.12) \times 10^{-14} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 D^- \) | \((8.49 \pm 2.41) \times 10^{-14} \text{ MeV} \) |
| \( T_{bc}^- \to \tilde{Z}_{bc}^0 D_s^- \) | \((2.92 \pm 0.82) \times 10^{-12} \text{ MeV} \) |

The nonleptonic decays \( T_{bc}^- \to \tilde{Z}_{bc}^0 M \)

Here, we study the nonleptonic weak decays \( T_{bc}^- \to \tilde{Z}_{bc}^0 M \) of the tetraquark \( T_{bc}^- \) in the framework of the QCD factorization method. This approach was applied to investigate nonleptonic decays of the conventional mesons \([16, 17]\), but can be used to investigate decays of the tetraquarks as well. Thus, nonleptonic decays of the scalar exotic mesons \( \tilde{Z}_{bc}^0 \), \( T_{bc}^{-}\to\tilde{Z}_{bc}^0 \nu \)) and \( T_{bc}^{-}\to\tilde{Z}_{bc}^0 \mu \)) (in a short form \( T_{bc}^- \)) were analyzed by this way in Refs. [13], respectively.

We consider processes, where \( M \) is one of the pseu-
doscalar mesons $\pi^-$, $K^-$, $D^-$, and $D_S^-$. We provide details of analysis for the decay $T_{b\pi}^- \to \bar{Z}_{bc}^0\pi^-$, and write down final predictions for other channels. At the quark level, the effective Hamiltonian for this decay is given by the expression

$$H_{\text{eff}}^{\text{lep}} = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* \left[ c_1(\mu) Q_1 + c_2(\mu) Q_2 \right],$$

where

$$Q_1 = \langle \bar{q}_u q_d \rangle_{V-A} \langle \bar{q}_d q_u \rangle_{V-A},$$

$$Q_2 = \langle \bar{q}_u q_d \rangle_{V-A} \langle \bar{q}_d q_u \rangle_{V-A},$$

and $i$, $j$ are the color indices, and $\langle \bar{q}_i q_j \rangle_{V-A}$ means

$$\langle \bar{q}_i q_j \rangle_{V-A} = \frac{1}{1 - \gamma_5} q_i q_j.$$

The only unknown matrix element $\langle \pi^- (q) | (\bar{q}_u q_d) | V_{bc} V_{ud}^* | 0 \rangle$ in $\mathcal{A}$ can be defined in the following form

$$\langle \pi^- (q) | (\bar{q}_u q_d) | V_{bc} V_{ud}^* | 0 \rangle = i f_{\pi} q_{\mu},$$

Then, it is not difficult to see that $\mathcal{A}$ is:

$$\mathcal{A} = i \frac{G_F}{\sqrt{2}} f_{\pi} V_{bc} V_{ud}^* a_1(\mu)$$

$$\times [ G_1(q^2) P_q + G_2(q^2) q^2 ],$$

For completeness we provide below the partial width of this process

$$\Gamma(T_{b\pi}^- \to \bar{Z}_{bc}^0\pi^-) = \frac{G_F^2 f_{\pi}^2 |V_{bc}|^2 |V_{ud}|^2}{32\pi m_{\pi}^3}$$

$$\lambda \left( m_{\pi}^2, m_{Z}^2, m_{\pi}^2 \right) \left[ G_1(m_{\pi}^2 - m_{Z}^2) + G_2 m_{\pi}^2 \right],$$

where the weak form factors $G_{1(2)}(q^2)$ are computed at $q^2 = m_{\pi}^2$. The decay modes $T_{b\pi}^- \to \bar{Z}_{bc}^0 K^- (D^-, D_S^-)$ can be analyzed in a similar manner. To this end, one has to replace in Eq. (41) $(m_{\pi}, f_{\pi})$ by the masses and decay constants of the mesons $K^-, D^-, and D_S^-$, and make the substitutions $V_{ud} \to V_{us}, V_{cd},$ and $V_{cs}.$

### TABLE II: Masses and decay constants of the final-state pseudoscalar mesons.

| Quantity | Value |
|----------|-------|
| $m_{\pi}$ | 139.570 MeV |
| $m_K$ | $(493.677 \pm 0.016)$ MeV |
| $m_D$ | $(1869.61 \pm 0.10)$ MeV |
| $m_{D_s}$ | $(1968.30 \pm 0.11)$ MeV |
| $f_{\pi}$ | 131 MeV |
| $f_K$ | $(155.72 \pm 0.51)$ MeV |
| $f_D$ | $(203.7 \pm 4.7)$ MeV |
| $f_{D_s}$ | $(257.8 \pm 4.1)$ MeV |
| $|V_{us}|$ | 0.97420 ± 0.00021 |
| $|V_{cd}|$ | 0.2243 ± 0.0005 |
| $|V_{cs}|$ | 0.218 ± 0.004 |
| $|V_{cs}|$ | 0.997 ± 0.017 |

All input information necessary for numerical analysis are collected in Table III it contains spectroscopic parameters of the final-state mesons, and CKM matrix elements. The coefficients $c_1(m_b)$, and $c_2(m_b)$ with next-to-leading order QCD corrections are borrowed from Refs. [18, 20]

$$c_1(m_b) = 1.117, \quad c_2(m_b) = -0.257.$$  

For the decay $T_{b\pi}^- \to \bar{Z}_{bc}^0\pi^-$, calculations yield

$$\Gamma(T_{b\pi}^- \to \bar{Z}_{bc}^0\pi^-) = (5.13 \pm 1.42) \times 10^{-13} \text{ MeV}.$$  

Partial widths of this and other nonleptonic decays of the tetraquark $T_{b\pi}^-$ are moved to Table III. It is evident that widths of these processes are very small, and can be safely neglected in computation of the full width of the $T_{b\pi}^-$. As a result, we get

$$\Gamma_{\text{full}} = (10.88 \pm 1.88) \times 10^{-10} \text{ MeV},$$

$$\tau = 6.05^{+1.26}_{-0.89} \times 10^{-13} \text{ s},$$

which are among main predictions of the present work.

### V. ANALYSIS AND CONCLUDING REMARKS

In the present work we have calculated the mass, width and lifetime of the stable scalar tetraquark $T_{b\pi}^-$ with the content $bb\pi$. This particle can be considered as a $ud$ member of the scalar multiplet $bb\pi$. Another particle from this multiplet $T_{b\pi}^-$ was studied in our article [13]. The tetraquark $T_{b\pi}^-$ is composed of $bb\pi$ quarks, has the mass

$$m = (10250 \pm 270) \text{ MeV},$$

(45)
and is stable against the strong and electromagnetic decays. By comparing the parameters of the tetraquarks $T_{bc}^{−}$ and $T_{bc}^{+}$ one can easily reveal a mass gap $115$ MeV in this multiplet, which is consistent with analysis of the open charm-bottom axial-vector states $Z_{a} = [cs][ar{b}q]$ and $Z_{a} = [cq][ar{b}q]$ [21]. In fact, the mass splitting between $Z_{a}$ and $Z_{b}$ equals approximately to $240$ MeV, which is caused by two $s$ quarks in the $Z_{a}$, hence a single $s$ generates the mass splitting $120$ MeV.

The second particle considered in this work is the tetraquark $Z_{bc}^{0}$ built of $bc$ meson with the symmetric color structure, and calculated its spectroscopic parameters $\bar{m}_{Z}$ and $\bar{f}_{Z}$. The scalar particle $Z_{bc}^{0}$ is stable against $S$-wave decays to mesons $B^{−}D^{+}$ and $\bar{B}^{0}D^{0}$ because thresholds for these processes $7149/7144$ MeV are higher than mass of the $Z_{bc}^{0}$. For the same reasons $Z_{bc}^{0}$ can not transform to conventional mesons through electromagnetic decays. In fact, threshold for a such process $Z_{bc}^{0} \rightarrow B^{−}D_{s1}(2460)^{+}\gamma$ is equal to $7739$ MeV and considerably exceeds the mass of the tetraquark $Z_{bc}^{0}$.

There are two other scalar exotic mesons with the same or close quark contents. First of them is the color-triplet particle $Z_{bc}^{0}$ built of $bc$ quarks with the mass $m_{Z} = (6660 \pm 150)$ MeV [3]. The second scalar tetraquark is $\tilde{s}$ partner of $Z_{bc}^{0}$, i.e., an exotic meson $Z_{bc}^{0}$ with $bc$ content and color-sextet organization. This particle was investigated in Ref. [13], in which its mass was estimated within the range

$$\bar{m} = (6830 \pm 140) \text{ MeV}. \quad (46)$$

The mass splitting inside of the multiplet of scalar color-sextet particles $bc$ is

$$\bar{m} - \bar{m}_{Z} = 100 \text{ MeV}, \quad (47)$$

is compatible with our above-stated discussion. Comparing the masses of the color-sextet and -triplet states $Z_{bc}^{0}$ and $Z_{bc}^{0}$, we get

$$\Delta m = \bar{m}_{Z} - m_{Z} = 70 \text{ MeV}. \quad (48)$$

The mass gap between axial-vector four-quark mesons $[cs][ar{s}q]$ with different color structures was studied in Ref. [22]. The color-triplet and -sextet states were interpreted there as candidates to resonances $X(4140)$ and $X(4274)$, respectively. Theoretical estimate for a difference of their masses amounts to $\Delta m \approx 180$ MeV. The triplet-sextet splitting in the scalar system $(Z_{bc}^{0}, Z_{bc}^{0})$ is numerically smaller than the same parameter in the case of axial-vector tetraquarks. But one should take into account that axial-vector particles $[cs][ar{s}q]$ are composed of a heavy diquark and antidiquark, whereas tetraquarks $bc\bar{q}q$ are built of heavy diquark and light antidiquark. Whether the triplet-sextet splitting depends only on spin-parities of these particles or bears also an information on their structures worths additional studies.

We have computed partial widths of the semileptonic $T_{bc}^{−} \rightarrow Z_{bc}^{0}\ell\nu$ and nonleptonic $T_{bc}^{−} \rightarrow Z_{bc}^{0}M$ processes, where $M$ is one of the pseudoscalar mesons $\pi^{-}, K^{-}, D^{-},$ and $D_{s}^{-}$. It turned out that partial widths of semileptonic decays are considerably higher than ones of these nonleptonic modes. Namely the semileptonic decay channels have been used to evaluate the full width and lifetime of $T_{bc}^{−}$.

Tetraquarks containing diquarks $bb$ and $bc$ are interesting objects for both theoretical and experimental investigations. There are experiments in which they can be observed: Discovery potential of some of these experiments was discussed in Refs. [23, 24]. Spectroscopic parameters of the scalar tetraquarks $T_{bc}^{−}$ and $Z_{bc}^{0}$, as well as weak decays of $T_{bc}^{−}$ studied in the present work provide new and useful information on features of double-heavy exotic mesons $bb\bar{q}q$ and $bc\bar{q}q$.

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