Progress Towards Determining the Density Dependence of the Nuclear Symmetry Energy Using Heavy-Ion Reactions

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Abstract. The latest development in determining the density dependence of the nuclear symmetry energy using heavy-ion collisions is reviewed. Within the IBUU04 version of an isospin- and momentum-dependent transport model using a modified Gogny effective interaction, recent experimental data from NSCL/MSU on isospin diffusion are found to be consistent with a nuclear symmetry energy of $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$ at subnormal densities. Predictions on several observables sensitive to the density dependence of the symmetry energy at supranormal densities accessible at GSI and the planned Rare Isotope Accelerator (RIA) are also made.

Keywords: Equation of State, Neutron-Rich Matter, Nuclear Symmetry Energy, Transport Models, Heavy-Ion Reactions, Neutron Stars

PACS: 25.70.-z, 21.30.Fe, 21.65.+f, 24.10.Lx

1. Introduction

The Equation of State (EOS) of isospin asymmetric nuclear matter can be written within the well-known parabolic approximation, which has been verified by all many-body theories, as

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4),$$

(1)
where $\delta \equiv (\rho_n - \rho_p) / (\rho_p + \rho_n)$ is the isospin asymmetry and $E_{\text{sym}}(\rho)$ is the density-dependent nuclear symmetry energy. The latter is very important for many interesting astrophysical problems\cite{1}, the structure of radioactive nuclei\cite{2,3} and heavy-ion reactions\cite{4,5,6,7}. Unfortunately, the density dependence of symmetry energy $E_{\text{sym}}(\rho)$, especially at supranormal densities, is still poorly known. Predictions based on various many-body theories diverge widely at both low and high densities. In fact, even the sign of the symmetry energy above $3\rho_0$ remains uncertain\cite{8}. Fortunately, heavy-ion reactions, especially those induced by radioactive beams, provide a unique opportunity to pin down the density dependence of nuclear symmetry energy in terrestrial laboratories. Significant progress in determining the symmetry energy at subnormal densities has been made recently both experimentally and theoretically\cite{9,10}. High energy radioactive beams to be available at GSI and RIA will allow us to determine the symmetry energy at supranormal densities. In this talk, we highlight the recent most exciting progress in determining the symmetry energy at subnormal densities and present our predictions on several most sensitive probes of the symmetry energy at supranormal densities.

2. An isospin- and momentum-dependent transport model for nuclear reactions induced by radioactive beams

Crucial to the extraction of critical information about the $E_{\text{sym}}(\rho)$ is to compare experimental data with transport model calculations. We outline here the major ingredients of the version IBUU04 of an isospin- and momentum-dependent transport model for nuclear reactions induced by radioactive beams\cite{11}. The single nucleon potential is one of the most important inputs to all transport models. Both the isovector (symmetry potential) and isoscalar parts of this potential should be momentum dependent due to the non-locality of strong interactions and the Pauli exchange effects in many-fermion systems. In the IBUU04, we use a single nucleon potential derived from the Hartree-Fock approximation using a modified Gogny effective interaction (MDI)\cite{12}, i.e.,

$$\begin{align*}
U(\rho, \delta, \vec{p}, \tau, x) &= A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} \\
&+ B \left( \frac{\rho}{\rho_0} \right)^\sigma (1 - x\delta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma - 1}}{\rho_0^\sigma} \delta \rho_{\tau'} \\
&+ \frac{2C_{\tau,\tau'}}{\rho_0} \int d^3p' \frac{f_{\tau}(\vec{r}, \vec{p})}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} \\
&+ \frac{2C_{\tau,\tau'}}{\rho_0} \int d^3p' \frac{f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}.
\end{align*}$$

(2)

In the above $\tau = 1/2 \ (-1/2)$ for neutrons (protons) and $\tau \neq \tau'$; $\sigma = 4/3$; $f_{\tau}(\vec{r}, \vec{p})$ is the phase space distribution function at coordinate $\vec{r}$ and momentum $\vec{p}$. The parameters $A_u(x), A_l(x), B, C_{\tau,\tau'}, C_{\tau,\tau''}$ and $\Lambda$ were obtained by fitting the momentum-dependence of the $U(\rho, \delta, \vec{p}, \tau, x)$ to that predicted by the Gogny Hartree-Fock
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and/or the Brueckner-Hartree-Fock calculations, the saturation properties of symmetric nuclear matter and the symmetry energy of 30 MeV at normal nuclear matter density $\rho_0 = 0.16 \text{ fm}^{-3}$ \cite{12}. The incompressibility $K_0$ of symmetric nuclear matter at $\rho_0$ is set to be 211 MeV. The parameters $A_u(x)$ and $A_l(x)$ depend on the $x$ parameter according to $A_u(x) = -95.98 - x \frac{2P}{\rho_0}$ and $A_l(x) = -120.57 + x \frac{2P}{\rho_0}$. The parameter $x$ can be adjusted to mimic predictions on the $E_{\text{sym}}(\rho)$ by microscopic and/or phenomenological many-body theories. The last two terms contain the momentum-

dependence of the single-particle potential. The momentum dependence of the symmetry potential stems from the different interaction strength parameters $C_{\tau,\tau'}$ and $C_{\tau,\tau}$ for a nucleon of isospin $\tau$ interacting, respectively, with unlike and like nucleons in the background fields. More specifically, we use $C_{\text{unlike}} = -103.4 \text{ MeV}$ and $C_{\text{like}} = -11.7 \text{ MeV}$. As an example, shown in Fig. 1 is the density dependence of the symmetry energy for $x = -2, -1, 0$ and 1.

Systematic analyses of a large number of nucleon-nucleus and $(p,n)$ charge exchange scattering experiments at beam energies below about 100 MeV indicate undoubtedly that the symmetry potential at $\rho_0$, i.e., the Lane potential, decreases approximately linearly with increasing beam energy $E_{\text{kin}}$ according to $U_{\text{Lane}} = a - bE_{\text{kin}}$ where $a \simeq 22 - 34 \text{ MeV}$ and $b \simeq 0.1 - 0.2$ \cite{13, 14}. This provides a stringent constraint on the symmetry potential. The potential in eq.2 meets this requirement very well as seen in Fig. 2 where the symmetry potential $(U_n - U_p)/2\delta$ as a function of momentum and density for the parameter $x = -1$ is displayed.

One characteristic feature of the momentum dependence of the symmetry potential is the different effective masses for neutrons and protons in isospin asymmetric

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Density dependence of the symmetry energy for four $x$ parameters.}
\end{figure}
nuclear matter, i.e.,

$$\frac{m^*_r}{m_r} = \left\{ 1 + \frac{m_r}{k_F^2} \frac{dU_z}{dk} \right\}^{-1}_{k=k_F^p},$$  \hspace{1cm} (3)$$

where $k_F^{p}$ is the nucleon Fermi wave number. With the potential in eq. 2, we found that the neutron effective mass is higher than the proton effective mass and the splitting between them increases with both the density and isospin asymmetry of the medium[11].

Since both the incoming current in the initial state and the level density of the final state in nucleon-nucleon (NN) scatterings depend on the effective masses of colliding nucleons in medium, the in-medium nucleon-nucleon cross sections are expected to be reduced by a factor $\sigma_{NN}^{medium}/\sigma_{NN} = (\mu_{NN}^*/\mu_{NN})^2$, where $\mu_{NN}$ and $\mu_{NN}^*$ are the reduced mass of the colliding nucleon pairs in free-space and in medium, respectively[15]. Thus, because of the reduced in-medium nucleon effective masses and their dependence on the density and isospin asymmetry of the medium, the in-medium NN cross sections are not only reduced compared to their free-space values, the $nn$ and $pp$ cross sections are split and the difference between them grows in more asymmetric matter. We expect the isospin-dependence of the in-medium NN cross sections to play an important role in nuclear reactions induced by neutron-rich nuclei[16].

Fig. 2. Symmetry potential as a function of momentum and density for MDI interaction with $x = -1$. 
3. Probing the symmetry energy at subnormal densities with isospin diffusion

Tsang et al. [9] recently studied the degree of isospin diffusion in the reaction $^{124}$Sn + $^{112}$Sn by measuring [17]

$$R_t = \frac{2X_{^{124}Sn+^{112}Sn} - X_{^{124}Sn+^{112}Sn} - X_{^{112}Sn+^{124}Sn}}{X_{^{124}Sn+^{112}Sn} - X_{^{112}Sn+^{124}Sn}}$$

(4)

where $X$ is the average isospin asymmetry $\langle \delta \rangle$ of the $^{124}$Sn-like residue. The data is indicated in Fig. 3 together with the time evolutions of $R_t$ and the average central densities calculated with $x = -1$ using both the MDI and the soft Bertsch-Das Gupta-Kruse (SBKD) interactions are also shown. It is seen that the isospin diffusion process occurs mainly from about 30 fm/c to 80 fm/c corresponding to an average central density from about $1.2\rho_0$ to $0.3\rho_0$. The experimental data from MSU are seen to be reproduced nicely by the MDI interaction with $x = -1$, while the SBKD interaction with $x = -1$ leads to a significantly lower $R_t$ value [10].

Fig. 3. The degree of isospin diffusion as a function of time with the MDI and SBKD interactions. The corresponding evolutions of central density are also shown.

Effects of the symmetry energy on isospin diffusion were also studied by varying the parameter $x$ [10]. Only with the parameter $x = -1$ the data can be well reproduced. The corresponding symmetry energy can be parameterized as $E_{sym}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$. In the present study on isospin diffusion, only the free-space NN cross sections are used and thus effects completely due to the different
density dependence of symmetry energy are investigated. As the next step we are currently investigating effects of the in-medium NN cross sections on the isospin diffusion.

4. Probing the symmetry energy at supranormal densities

Several probes that are sensitive to the high density behavior of the symmetry energy have been proposed. As an illustration, we present here three most sensitive observables that can be measured in future experiments at RIA and GSI.

4.1. Pions yields and $\pi^-/\pi^+$ ratio at RIA and GSI

At the highest beam energy at RIA, pion production is significant. Pions may thus carry interesting information about the EOS of dense neutron-rich matter[18, 19]. Shown in Fig.4 are the $\pi^-$ and $\pi^+$ yields as a function of the $x$ parameter. It is interesting to see that the $\pi^-$ multiplicity depends more sensitively on the symmetry energy. The $\pi^-$ multiplicity increases by about 20% while the $\pi^+$ multiplicity remains about the same when the $x$ parameter is changed from -2 to 1. The multiplicity of $\pi^-$ is about 2 to 3 times that of $\pi^+$. This is because the $\pi^-$ mesons are mostly produced from neutron-neutron collisions. Moreover, with the softer symmetry energy the high density region is more neutron-rich due to isospin fractionation[19]. The $\pi^-$ mesons are thus more sensitive to the isospin asymmetry of the reaction system and the symmetry energy. However, one should notice that it is well known that pion yields are also sensitive to the symmetric part of the nuclear
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It is thus hard to get reliable information about the symmetry energy from \( \pi^- \) yields alone. Fortunately, the \( \pi^-/\pi^+ \) ratio is a better probe since statistically this ratio is only sensitive to the difference in the chemical potentials for neutrons and protons\(^{20}\). This expectation is well demonstrated in Fig. 5. It is seen that the pion ratio is quite sensitive to the symmetry energy, especially at low transverse momenta. Thus, it is promising that the high density behavior of nuclear symmetry energy \( E_{\text{sym}}(\rho) \) can be probed using the \( \pi^-/\pi^+ \) ratio.

![Fig. 5. The \( \pi^-/\pi^+ \) ratio as a function of transverse momentum.](image)

**4.2. Isospin fractionation and n-p differential flow at RIA and GSI**

The degree of isospin equilibration or translucency can be measured by the rapidity distribution of nucleon isospin asymmetry \( \delta_{\text{free}} = (N_n - N_p)/(N_n + N_p) \) where \( N_n (N_p) \) is the multiplicity of free neutrons (protons)\(^{21}\). Although it might be difficult to measure directly \( \delta_{\text{free}} \) because it requires the detection of neutrons, similar information can be extracted from ratios of light clusters, such as, \(^3\)H/\(^3\)He, as demonstrated recently within a coalescence model\(^{22, 23}\). Shown in Fig. 6 are the rapidity distributions of \( \delta_{\text{free}} \) with (upper window) and without (lower window) the Coulomb potential. It is interesting to see that the \( \delta_{\text{free}} \) at midrapidity is particularly sensitive to the symmetry energy. As the parameter \( x \) increases from \(-2\) to \(1\) the \( \delta_{\text{free}} \) at midrapidity decreases by about a factor of \(2\). Moreover, the forward-backward asymmetric rapidity distributions of \( \delta_{\text{free}} \) with all four \( x \) parameters indicates the apparent nuclear translucency during the reaction\(^{24}\).

Another observable that is sensitive to the high density behavior of symmetry
energy is the neutron-proton differential flow \[ F^{x}_{n-p}(y) \equiv \frac{\sum_{i=1}^{N(y)} (p_{x}^{i} w_{i})}{N(y)}, \] where \( w_{i} = 1(-1) \) for neutrons (protons) and \( N(y) \) is the total number of free nucleons at rapidity \( y \). The differential flow combines constructively effects of the symmetry potential on the isospin fractionation and the collective flow. It has the advantage of maximizing the effects of the symmetry potential while minimizing those of the isoscalar potential. Shown in Fig. 7 is the n-p differential flow for the reaction of \(^{132}Sn + ^{124}Sn\) at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm. Effects of the symmetry energy are clearly revealed by changing the parameter \( x \).

5. Conclusions

The density dependence of the symmetry energy is very important for both nuclear physics and astrophysics. Significent progress has been made recently by the heavy-ion community in determining the density dependence of the nuclear symmetry energy. Based on transport model calculations, a number of sensitive probes of the symmetry energy have been found. The momentum dependence in both the isoscalar and isovector parts of the nuclear potential was found to play an important role in extracting accurately the density dependence of the symmetry energy. Comparing with recent experimental data on isospin diffusion from NSCL/MSU,
we have extracted a symmetry energy of $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$ at subnormal densities. It would be interesting to compare this conclusion with those extracted from studying other observables. More experimental data including neutrons with neutron-rich beams in a broad energy range are needed. Looking forward to experiments at RIA and GSI with high energy radioactive beams, we hope to pin down the symmetry energy at supranormal densities in the near future.

This work was supported in part by the US National Science Foundation of the under grant No. PHY 0098805, PHYS-0243571 and PHYS0354572, Welch Foundation grant No. A-1358, and the NASA-Arkansas Space Grants Consortium award ASU15154. L.W. Chen was supported by the National Natural Science Foundation of China grant No. 10105008. The work of G.C. Yong and W. Zuo was supported in part by the Chinese Academy of Science Knowledge Innovation Project (KECK2-SW-N02), Major State Basic Research Development Program (G2000077400), the National Natural Science Foundation of China (10235030) and the Important Pare-Research Project (2002CAB00200) of the Chinese Ministry of Science and Technology.

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