Dark energy and the Rutherford-Soddy radiative decay law

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I. INTRODUCTION

Recently reported evidence of a putative time variation of the electromagnetic coupling on cosmological time scales from the observation of absorption spectra of quasars [1] has led to a revival of interest on ideas that actually go back to Dirac [2] about the variation of fundamental couplings (see Ref. 3 for a review). Such variations are fairly natural in unification models with extra dimensions or expectation values of scalar fields. It is likely that if a variation of the electromagnetic coupling occurs one should also expect variations of other gauge or Yukawa couplings. Confirming these observations and proposing realistic particle physics models consistent with a cosmological variation of the fundamental coupling constants is therefore of great interest. In this work we shall consider the effect that a variation of the strong and weak couplings may have on the nuclear decay and the available bounds on the variation of those couplings.

A relevant feature of the observed variation of the electromagnetic coupling is that it is presumably a late event in the history of the universe and it is thus natural to associate it to the observed late accelerated expansion of the universe [6]. This late time acceleration of the expansion can be driven by a cosmological constant (see [7] and references therein), by a scalar field with a suitable potential, quintessence [8], or by a fluid with an exotic equation of state like the generalized Chaplygin gas (GCG) [9]. We mention that for the evolution of exotic equation of state like the generalized Chaplygin gas, quintessence [8], or by a fluid with an energy field [12, 13, 15, 16]. In this work we shall consider the variation of the strong and weak couplings in the context of a exponential quintessence model and of the GCG model.

II. NUCLEON DECAY AND THE EVOLUTION OF THE STRONG AND WEAK COUPLINGS

It is a quite well established experimental fact that the nuclear decay rate of any nuclei is described by the statistical Rutherford-Soddy law, first stated back in 1902:

$$\frac{dN}{dt} = -\lambda N$$

where \(\lambda = \frac{A_i}{\tau_{0}}\) and \(\tau_{0}\) being the nuclei’s lifetime. For a constant \(\lambda\), which reflects, as later stated by Rutherford, Chadwick and Ellis, the fact that the rate of transformation of an element has been found to be constant under all conditions [17], one obtains the well known exponential decay law:

$$N(t) = N_i e^{-\lambda(t-t_i)}$$

where \(N_i\) is the nuclei’s number counting at \(t_i\). This statistical law reflects radiative processes of electromagnetic, strong and weak nature that take place within the nuclei. Thus, \(\tau_{0}\) is related to the amplitude of the relevant decay process being therefore a function of order \(\gamma\) of the coupling constant \(\alpha_g\):

$$\tau_{0} = A \alpha_g^\gamma$$

with \(\alpha_s(E) = g_s^2(E)/\hbar c\) for the strong interactions, \(g_s\) being the strong coupling, and \(\alpha_W = G_pm_e^2c/\hbar^3\) for the weak interactions, \(G_p\) being Fermi’s constant and \(m_p\) the proton mass. Parameter \(A\) is a constant related to a specific process, suitable integration over the phase space, binding energy, quark masses, etc.

On the other hand, one expects that if dark energy couples with the whole gauge sector of the Standard Model, this coupling can be modelled by a generalization of the so-called Bekenstein model [3]

$$\mathcal{L}_{gauge} = -\frac{1}{16\pi} f(\phi) F_{\mu\nu}^a F^{\mu\nu a}$$

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where \( f \) is an arbitrary function of the dark energy field, \( \phi \), and \( F_{\mu\nu} \), the gauge field strength. Given that the variation of the gauge couplings is presumably small (c.f. below), we expand this function to first order

\[
f(\phi) = \frac{1}{\alpha_i} \left[ 1 + \xi \left( \frac{\phi - \phi_i}{M} \right) \right],
\]

where \( \alpha_i \) is the initial value of the gauge structure constant, \( \xi \) is a constant, and \( \phi_i \) is the initial value of the quintessence field and \( M \) a characteristic mass scale of the dark energy model. A model with a quadratic variation is discussed in Ref. [18]. It follows that

\[
[f(\phi)]^{-1} \approx \alpha_i \left[ 1 - \xi \left( \frac{\phi - \phi_i}{M} \right) \right].
\]

In this case, the gauge coupling, \( \alpha_g \), evolution is given by

\[
\alpha_g = [f(\phi)]^{-1} \approx \alpha_i \left[ 1 - \xi \left( \frac{\phi - \phi_i}{M} \right) \right],
\]

hence, for its variation, we obtain

\[
\frac{\Delta \alpha_g}{\alpha_g} = \xi \left( \frac{\phi - \phi_i}{M} \right).
\]

For the electromagnetic interaction one should take into account the Equivalence Principle limits, which implies

\[
\xi_{EM} \leq 7 \times 10^{-4}.
\]

Of course, this bound does not apply to short range interactions. However, as we shall see, this parameter must be constrained in order to ensure that strong and weak gauge coupling do not change significantly so to ensure the experimental standing of the Rutherford-Soddy law.

In what follows we consider only strong and weak decays given that the effects of a putative variation of the electromagnetic coupling have been the subject of various studies (see e.g. Ref. [15] and references therein). Thus, for strong and weak decays, keeping quark masses unchanged and disregarding the running of the strong interactions, one can write for \( \tau \):

\[
\tau_{DE} = A f(\phi)^{-\gamma},
\]

and since \( \xi < 1 \)

\[
\tau_{DE} \approx \tau_0 \left[ 1 - \gamma \xi_{S,W} \left( \frac{\phi_0 - \phi_i}{M} \right) \right],
\]

depending on whether the process is ruled by strong or weak interactions, where \( \phi_0 \) refers to the value of the dark energy scalar field at present. Introducing this expression into the rate of change of atoms

\[
\frac{dN}{dt} \approx -N \frac{1}{\tau_0} \left[ 1 + \gamma \xi_{S,W} \left( \frac{\phi_0 - \phi_i}{M} \right) \right],
\]

from which follows that

\[
N(t) \approx N_i \exp \left[-\lambda(\Delta t + \gamma \xi_{S,W} I)\right],
\]

where

\[
I = \int_{t_i}^{t_0} \left( \frac{\phi_0 - \phi_i}{M} \right) dt,
\]

and \( \Delta t = t_0 - t_i \).

We now consider the bounds on the variation of the gauge couplings (latest bounds on the variation of the electromagnetic coupling can be found for instance in [13] and references therein). Regarding the strong interaction, considerations on the stability of two-nucleon systems yield the bound

\[
\frac{\Delta \alpha_S}{\alpha_S} < 4 \times 10^{-2},
\]

for the deuteron, and

\[
\frac{\Delta \alpha_S}{\alpha_S} < 6 \times 10^{-1},
\]

for the diproton.

For the weak interaction, one finds, from Oklo data

\[
\frac{\Delta \alpha_W}{\alpha_W} < 4 \times 10^{-3}.
\]

On the other hand, from the weak interaction contribution to the nuclear ground state energy of \(^{147}\text{Sm}\), one gets

\[
\frac{\Delta \alpha_W}{\alpha_W} < 0.02.
\]

From considerations on the \( \beta \)-decay rates of \(^{187}\text{Re}\) lead to

\[
\frac{\Delta \alpha_W}{\alpha_W} < 3 \times 10^{-7}.
\]

III. DARK ENERGY MODELS

In order to quantify our predictions concerning the level of validity of the statistical Rutherford-Soddy decay law, we consider two observationally viable dark energy models.

A. A quintessence model with an exponential potential

For simplicity let us start considering the exponential potential \( V(\phi) = V_0 e^{-\beta \phi / M_p} \), in the case of scalar field
domination, there is a family of solutions \[19\]
\[
\phi(t) = \phi_0 + \frac{2}{\beta} \ln(t M_P) \tag{20}
\]
\[
\phi_0 = \frac{2}{\beta} \ln \left( \frac{V_0 \beta^2}{2 M_P^4 (6 - \beta^2)} \right) \tag{21}
\]
\[
\rho_\phi \propto \frac{1}{a^{3+\beta}} \quad a \propto t^{2/\beta^2} \tag{22}
\]
where \(a\) is the scale factor of the universe. For \(\beta < \sqrt{6}\), and \(\phi_0\) can always be chosen to be zero by redefining the origin of \(\phi\), in which case \(V_0 = \frac{2}{\beta^2} \left( \frac{\beta^2}{6} - 1 \right) M_P^4\). The parameters of this model can be set to satisfy all phenomenological constraints (see e.g. last reference in \[8\]).

**B. The Chaplygin gas model**

The GCG model considers an exotic perfect fluid described by the equation of state \[9\]
\[
p = -\frac{A}{\rho_\phi} \tag{23}
\]
where \(A\) is a positive constant and \(\alpha\) is a constant in the range \(0 \leq \alpha \leq 1\). For \(\alpha = 1\), the equation of state is reduced to the Chaplygin gas scenario. The covariant conservation of the energy-momentum tensor for an homogeneous and isotropic spacetime implies that
\[
\rho_\phi = \rho_{\phi 0} \left[ A_s + \frac{(1 - A_s)}{\alpha^{3(1+\alpha)}} \right]^\frac{1}{\alpha}, \tag{24}
\]
where \(A_s = A / \rho_{\phi 0}^{(1+\alpha)}\) and \(\rho_{\phi 0}\) is the present energy density of the GCG. Hence, we see that at early times the energy density behaves as matter while at late times it behaves like a cosmological constant. This dual role is at the heart of the surprising properties of the GCG model. Moreover, this dependence with the scale factor indicates that the GCG model can be interpreted as an entangled admixture of dark matter and dark energy.

This model has been thoroughly scrutinized from the observational point of view and is shown to be compatible with the Cosmic Microwave Background Radiation (CMBR) peak location and amplitudes \[20, 21\], with SNe Ia data \[22, 23, 24\], gravitational lensing statistics \[25\], cosmic topology \[26\], gamma-ray bursts \[27\] and variation of the electromagnetic coupling \[28\]. The issue of structure formation and its difficulties \[29\] have been recently addressed \[30\]. Most recent analysis based on CMBR data indicates that \(\alpha < 0.25\) and \(A_s > 0.93 \) \[21\].

Following Ref. \[23, 31\], we describe the GCG through a real scalar field. We start with the Lagrangian density
\[
\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi), \tag{25}
\]
where the potential for a flat, homogeneous and isotropic universe has the following form \[31\]
\[
V = \frac{1}{2} A_s^{\frac{1}{1+\alpha}} \rho_0 \left[ \cosh(\beta \phi) \frac{\alpha}{\alpha^{3(1+\alpha)}} + \cosh(\beta \phi) \frac{\alpha}{\alpha^{3(1+\alpha)}} \right], \tag{26}
\]
where \(\beta = 3(\alpha + 1)/2\).

For the energy density of the field we have
\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} = \rho_{ch} \tag{27}
\]
The Friedmann equation can be written as
\[
H^2 = \frac{8\pi G}{3} \rho_\phi \tag{28}
\]
and thus
\[
H(t) = H_0 \Omega_{\phi}^{1/2} \tag{29}
\]
where \(\Omega_{\phi}\) can be written as
\[
\Omega_{\phi} = \left[ A_s + \frac{(1 - A_s)}{\alpha^{3(1+\alpha)}} \right]^{3(1+\alpha)}. \tag{30}
\]

**IV. RESULTS**

We have computed the integral \(I\), Eq. \[17\], for the quintessence model with an exponential potential (Model I) and for the GCG model (Model II) and considered the implications for the variation of the Rutherford-Soddy decay law for strong and weak interactions. For Model I we have set \(\beta^2 = 2\) and that at present the scalar field contribution to the energy density is 70% of the critical density. For Model II, we have considered the values \(\alpha = 0.2, A_s = 0.95\), and used the relationship \(1 + z = 1/a\) which allows one to express the integral \(I\) in terms of \(z\).

For the strong interaction decay we have considered the \(^{238}U\) nucleus for which \(\tau_0 = 4.468 \times 10^9\) years. For the weak interaction decay we have considered the \(\beta^-\) decay of \(^{187}Re\) for which \(\tau_0 = 4.558 \times 10^9\) years. Using bounds \[15, 17\], and the integration interval equal to \(\tau_0\) we compute \(\zeta_{S,W}\) and the respective deviation from the Rutherford-Soddy law \(N/N_{RS}\) for the case of weak and strong interactions with no coupling to dark energy. Our results are depicted in Table 1 for \(\gamma = 2\).

**V. DISCUSSION AND CONCLUSIONS**

In this work we have considered the impact that the coupling of dark energy to gauge fields may have on the nuclear decay Rutherford-Soddy law. From the available bounds on the variation of the strong and weak gauge couplings we have estimated, in a linear model for the couplings, the deviations from the Rutherford-Soddy law. We find that thanks to the stringent bound
on the variation of the weak coupling arising from \(^{187}Re\) imply in significant constraints on any variation of the Rutherford-Soddy law for weak interactions, actually at the level \(3 \times 10^{-7}\) for the quintessence field with an exponential potential and at \(8 \times 10^{-7}\) level for the GCG model. For the case of strong interactions the bounds are much more modest and are about \(4 \times 10^{-2}\) for the quintessence model with exponential potential and \(10^{-1}\) for the GCG model. More stringent bounds on the deviation of the nuclear decay law for strong interactions could be obtained if tighter bounds on the variation of couplings were known. Furthermore, it is interesting that rather different deviations are found for distinct dark energy models. This may be relevant to distinguish them given that most often they yield (or can be fiddled) to give rise to rather similar behaviour in what concerns the cosmological observable parameters.

Our results indicate that, at least for strong interactions, bounds on the coupling constant are not constrained enough or that dark energy is not coupled to gauge fields, at least not through a linear variation model.

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**Table:**

| \(I\) (yrs) | \(|\zeta_s|\) | \(|\zeta_w|\) | \(\frac{\Delta N_{th}}{N_{th}}|s - 1|\) | \(\frac{\Delta N_{th}}{N_{th}}|w - 1|\) |
|----------------|-----------------|-----------------|-----------------|-----------------|
| **Model I**    | 1.22 \times 10^9 | 7.1 \times 10^{-2} | 5.24 \times 10^{-7} | 3.8 \times 10^{-2} | 2.79 \times 10^{-7} |
| **Model II**   | 1.08 \times 10^9 | 2.27 \times 10^{-1} | 1.7 \times 10^{-6} | 1.04 \times 10^{-1} | 8.1 \times 10^{-7} |

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