Gravity, Lorentz violation, and effective field theory

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Abstract. This proceedings contribution summarizes the effective field theory approach to searches for violations of Lorentz invariance in gravity with a focus on propagation effects.

1. Introduction
The question of how General Relativity (GR) and the Standard Model (SM) of particle physics merge to form a single consistent theory is one of the most fundamental questions we currently face in physics. Over the past several decades, the realization that certain scenarios for this merge could generate violations of local Lorentz invariance (Lorentz violation) [1] has led to the development of a framework known as the gravitational Standard-Model Extension (SME) [2, 3, 4], which is designed to search for Lorentz violation comprehensively and systematically. The SME is an effective field-theory framework that characterizes arbitrary Lorentz-violating corrections to GR and the SM. These corrections can be sought in existing experiments in hopes of gaining insight about the nature of Planck-scale physics with existing technology, and dozens of SME-based experiments have been performed to date [5].

The SME consists of an expansion about the action of GR and the SM in operators of increasing mass dimension $d$. The idea is to search systematically for Lorentz violation throughout physics rather than individually testing a large number of special models (though specific models [6] also provide useful insight). The approach resonates philosophically with other familiar test frameworks including the Parameterized Post Newtonian (PPN) formalism [7], while differing in its goals, construction, and physical effects. In terms of goals, the SME parameterizes deviations from exact Lorentz symmetry throughout physics, some of which are modifications to GR, while the PPN parameterizes differences from GR, some of which are Lorentz violating. By way of construction, the SME parameterizes deviations from the action of GR and the SM, while the PPN parameterizes deviations from the post-newtonian metric of GR. Another approach that has some philosophical similarity with the SME is the Parameterized Post Einsteinian (PPE) framework [8]. The key points are that (i) the SME is a broad and general test framework having philosophical similarities with other test frameworks and (ii) the SME is unique in its comprehensive focus on Lorentz symmetry via an effective field-theory approach. While a number of approaches to Lorentz violation restrict attention to models that are isotropic in a special frame, the SME treats general anisotropic effects.
2. Linearized gravity

Much work has been done across a wide variety of gravitational systems in the SME. Lorentz violation in the pure-gravity sector at mass dimension four has been sought in a large number of systems including laboratory [9, 10, 11], solar-system [12, 13, 14], and astrophysical [15, 16, 17] tests. Much of this experimental and observational work is based on the theoretical and phenomenological developments of Ref. [18]. Searches for Lorentz violation in matter-gravity couplings have also been done [9, 11, 14, 19], initiated by the theoretical and phenomenological work of Refs. [20]. Studies of gravitational Lorentz violation beyond $d = 4$ have also begun. Here it has been shown that laboratory tests and gravitational-wave tests provide complementary coverage of the coefficient space [21]. Short-range gravity tests have made significant progress in this area [22], as have gravitational-wave tests [15, 17, 23, 24]. Exploration of the role of solar-system tests in searches for higher mass dimension operators has also begun [25], as have considerations of nonlinear gravity effects [26]. We also note in passing that Finsler geometry provides a geometric interpretation for a number of Lorentz-violating effects in the effective field-theory construction [27].

In this section we focus on the full Lorentz-violating expansion about linearized gravity and the associated effects on the propagation of gravitational waves. Here the generic form of the Lagrange density including both the Lorentz-violating and Lorentz-invariant contributions can be written [23],

\[ \mathcal{L}^{(d)} = \frac{1}{4} h_{\mu\nu} \hat{K}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}, \quad (1) \]

where the operator

\[ \hat{K}^{(d)\mu\nu\rho\sigma} = K^{(d)\mu\nu\rho\sigma\epsilon_1...\epsilon_{d-2}} \partial_{\epsilon_1} \partial_{\epsilon_2} \ldots \partial_{\epsilon_{d-2}} \]

has mass dimension $d \geq 2$. Here $h_{\mu\nu}$ is the usual metric perturbation and the coefficients $K^{(d)\mu\nu\rho\sigma\epsilon_1...\epsilon_{d-2}}$ are taken as small constants. Intuition for Eq. (1) can be gained by noting that it contains the standard linearized limit of the Einstein–Hilbert action, which is recovered in the limit

\[ \hat{K}^{(d)\mu\nu\rho\sigma} \rightarrow \epsilon^{\mu\rho\sigma\epsilon_1...\epsilon_{d-2}} \eta_{\epsilon_1\epsilon_2} \partial_{\epsilon_1} \partial_{\epsilon_2}. \quad (3) \]

In other cases, the coefficients $K^{(d)\mu\nu\rho\sigma\epsilon_1...\epsilon_{d-2}}$ parameterize the amount of Lorentz violation in the theory. A decomposition of the operator into irreducible parts generates 14 classes of operators. Three of these classes respect the usual gauge invariance of GR. These can be understood as being associated with spontaneous Lorentz violation [3, 28]. Initial exploration of these classes has been done [23].

Similar considerations in the photon sector lead to effects in vacuum that are analogous to the propagation of light in matter including energy-dependent birefringence, dispersion, and anisotropy. These effects have been sought astrophysically using a variety of observations including gamma-ray bursts [29, 30, 31] and CMB observations [29, 32]. The three classes of operators identified above are associated with the gravitational versions of the photon effects noted. These effects can be characterized via the dispersion relation

\[ \omega = \left(1 - \varsigma^0 \pm \sqrt{(\varsigma^1)^2 + (\varsigma^2)^2 + (\varsigma^3)^2}\right)|p|, \quad (4) \]

found from the relevant limit of (1) [23], where

\[ \varsigma^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) k^{(d)}_{(l)jm}, \]

\[ \varsigma^1 \mp i \varsigma^2 = \sum_{djm} \omega^{d-4} \pm i Y_{jm}(\hat{n}) \left(k^{(d)}_{(E)jm} \pm i k^{(d)}_{(B)jm}\right), \]

\[ \varsigma^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) k^{(d)}_{(V)jm}. \quad (5) \]
Here the relevant contributions to the coefficients $\mathcal{K}^{(d)}_{\mu\nu\rho\sigma\epsilon_1\ldots\epsilon_d-2}$ can be decomposed into the spherical coefficients $k^{(d)}_{(1)jm}$, $k^{(d)}_{(2)jm}$, $k^{(d)}_{(3)jm}$, and $k^{(d)}_{(4)jm}$ using spherical harmonics $Y_{jm}$, and spin-weighted spherical harmonics $sY_{jm}$ [29]. The features in the analogy with optics in matter can be seen directly from (4). Birefringence is associated with the plus/minus in (4), dispersion with (35). Gravitational waves were also used indirectly in early work placing limits via the lack of [34] gravitational-wave observations. These first analyses were performed via the chirp signals published by LIGO. The dispersive effects of Lorentz violation have subsequently been sought by the LIGO collaboration in the context of an isotropic phenomenological model [33]. Gravitational waves were also used indirectly in early work placing limits via the lack of gravitational Čerenkov radiation by scalar cosmic rays [36]. More recently, this idea has been expanded upon within the effective field-theory framework of the SME to include the full space of anisotropic effects, Lorentz violation associated with higher mass dimension operators, and gravitational Čerenkov radiation by other species including photons [15].

As an example of recent progress, the seminal observation of gravitational waves from a binary neutron-star merger by the LIGO-Virgo collaboration coincident with the observation of gamma-ray photons by the Fermi telescope provided an impressive new test of the effects contained in the gravitational dispersion relation [17]. The fact that the gamma-ray photons arrived about 1.7 seconds after the gravitational waves associated with the merger implies constraints on the relative speed of the gravitational waves and the photons [17] that are a significant improvement over prior direct speed-of-gravity tests [38]. Within effective field theory, the relative group velocity of the gravitational waves and the electromagnetic waves, $\Delta v = v_g - v_\gamma$, is controlled by differences in coefficients for Lorentz violation in the gravitational sector and the photon sector at each mass dimension. The explicit form of the group-velocity difference can be obtained from the dispersion relation in each sector [23, 29, 31] via standard methods. As birefringent coefficients can often be better sought with higher sensitivity using birefringence directly [23], we focus here on non-birefringent effects. Following this specialization, the resulting difference in group velocities for the two sectors can be written

$$\Delta v = -\sum_{djm} (d-3) Y_{jm}(\hat{n}) \left( E^d_{g} k^{(d)}_{(1)jm} - E^d_{\gamma} c^{(d)}_{(1)jm} \right). \quad (6)$$

Here $E_g$ is the energy corresponding to the observed gravitational-wave frequency, and $E_\gamma$ is the energy of the observed gamma rays. The result is presented in a spherical harmonic, $Y_{jm}$, basis, with $k^{(d)}_{(1)jm}$ and $c^{(d)}_{(1)jm}$ being the nonbirefringent spherical-basis coefficients for Lorentz violation in the gravity sector and electromagnetic sector, respectively. The sum is over even $d \geq 4$, and $j \leq d - 2$. The direction $\hat{n}$ refers to the direction of the incoming messengers from the event and is described by the standard spherical polar coordinates $\theta, \phi$ in the Sun-centered celestial-equatorial frame [37]. Use of this frame is standard in SME studies as it facilitates easy comparison among results of different experiments.

The focus of the analysis performed by the LIGO, Virgo, Fermi Gamma-ray Burst Monitor, and INTEGRAL collaborations was on the minimal Lorentz-violating effects associated with operators of mass dimension $d = 4$, which are nondispersive and hence inaccessible to other
gravitational wave-based approaches. Here, Eq. (6) can be written
\[ \Delta v = - \sum_{jm} Y_{jm}(\hat{n}) \left( \frac{1}{2}(-1)^{1+j}s_{jm}^{(4)} - c_{(i)jm}^{(d)} \right), \]  
(7) where \( k_{(4)jm} = \frac{1}{2}(-1)^{1+j}s_{jm}^{(4)} \) [21] is used to facilitate contact with earlier SME work at \( d = 4 \). The resulting speed constraints generated a ten order of magnitude improvement over existing sensitivities to \( s_{00}^{(4)} \) [16, 12] and order of magnitude improvements in a number of other \( d = 4 \) gravity-sector coefficients in the context of a maximum-reach analysis [9] in which each of the gravity-sector coefficients were considered one at a time.

Before ending discussion of this result, we provide a few remarks to help the reader develop an intuition for spherical coefficients. First, note that based on the range of the indices in Eq. (6), nine \( d = 4 \) spherical coefficients \( s_{jm}^{(4)} \) contribute. This is consistent with the counting in the Cartesian basis where \( s_{\mu\nu}^{(d)} \) is symmetric and traceless. Note also that the coefficients satisfy \( (s_{jm}^{(4)})^* = (-1)^m s_{j(-m)}^{(4)} \), where * is complex conjugation. The phase convention for the spherical harmonics is chosen such that they satisfy the same condition. Contact can then be made between the Cartesian and spherical coefficients by matching terms from each side of an expression such as
\[ s_{\mu\nu}^{(d)} \hat{p}_\mu \hat{p}_\nu = \sum_{jm} Y_{jm}(\hat{p}) s_{jm}^{(4)}. \]  
(8) For example, one finds \( 2s_{YZ}^{(4)} = \sqrt{\frac{15}{2\pi}} \text{Im} s_{21}^{(4)} \). Reference [39] provides explicit relations for all nine coefficients. For additional details, see Ref. [29].

4. Summary
Effective field theory methods provide a framework for searching for Lorentz violation. This proceedings contribution summarizes this approach in the context of linearized gravity with a focus on propagation effects. As an example of the approach, we highlight the recent progress via multimessenger-astronomy observations. The future is bright for such searches as additional gravitational-wave observations distributed across the sky can be expected to generate tests that are increasingly sensitive and robust.

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