Multi-user Cognitive Interference Channels:  
A Survey and New Capacity Results

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Abstract—This paper provides a survey of the state-of-the-art information theoretic analysis for overlay multi-user (more than two pairs) cognitive networks and reports new capacity results. In an overlay scenario, cognitive / secondary users share the same frequency band with licensed / primary users to efficiently exploit the spectrum. They do so without degrading the performance of the incumbent users, and may possibly even aid in transmitting their messages as cognitive users are assumed to possess the message(s) of primary user(s) and possibly other cognitive user(s). The survey begins with a short overview of the two-user overlay cognitive interference channel. The evolution from two-user to three-user overlay cognitive interference channels is described next, followed by generalizations to multi-user (arbitrary number of users) cognitive networks. The rest of the paper considers K-user cognitive interference channels with different message knowledge structures at the transmitters.

Novel capacity inner and outer bounds are proposed. Channel conditions under which the bounds meet, thus characterizing the information theoretic capacity of the channel, for both Linear Deterministic and Gaussian channel models, are derived.

The results show that for certain channel conditions distributed cognition, or having a cumulative message knowledge structure at the nodes, may not be worth the overhead as (approximately) the same capacity can be achieved by having only one global cognitive user whose role is to manage all the interference in the network. The paper concludes with future research directions.

I. INTRODUCTION

More efficient usage of the spectrum is needed given the ever increasing demand for wireless broadband services. Cognitive radio combined with spectrum sharing have been proposed as a solution to this apparent spectrum crunch. This would involve smart new “cognitive” wireless devices intelligently coexisting with users with priority access to the spectrum, either minimally impacting them, or not impacting them at all.

The usage of cognitive radios allows for “cognition” in wireless networks, which broadly speaking implies that the wireless devices are able to adapt in real-time to the wireless environment. This may translate to a number of assumptions and/or schemes technically. We list three common sets of assumptions and their corresponding nomenclature below [1].

1) The approach where cognitive radios sense white spaces (time, space or frequency voids) and adjust their transmissions to fill the sensed voids has been referred to as interweave and avoids interference altogether, at the (possible) expense of spectral efficiency.

2) Contrary to keeping its transmission orthogonal to the primary user’s transmissions, in the interweave paradigm, the underlay paradigm allows a cognitive radio to simultaneously transmit with primary user(s), if the interference caused at the primary receiver(s) is kept below a certain threshold that is commonly referred to as the “interference temperature constraint.” In this case, a cognitive radio adjusts its transmission power in order to satisfy the interference temperature. The maximal tolerable interference for the surrounding users, as well as channel state information of the interfering channel gains, is assumed available at the cognitive transmitter.

3) The case where the cognitive devices have additional information of codebooks, messages or channel gains of other user(s), and they simultaneously transmit with primary license holders, is referred to as an overlay paradigm, or a Cognitive Interference Channel (CIFO) [1].

In this paper we focus on an overlay form of cognition, where secondary users have a-priori non-causal message knowledge of primary license holder(s). All nodes furthermore are assumed to have global codebook and channel state information, as is common in an information theoretic analysis of complex network models. Intuitively, this idealized assumption allows the cognitive radios to cooperate in sending the primary user’s message, while at the same time transmitting their own message by using an interference mitigation technique.

In this paper, we survey the fundamental limits of communication for a multi-user overlay cognitive networks with an arbitrary number of secondary / cognitive user(s) having non-causal message knowledge of primary user(s). The users transmit in the same frequency band and thus in general interfere with one another. The performance metric considered is the information theoretic notion of channel capacity. In other words, we are interested in the maximum rate of communication for which arbitrarily small probability of error can be achieved by every user, which may be seen as a benchmark when building practical systems. We will focus on results for general memoryless and practically relevant additive white Gaussian noise (AWGN) channel models, as well as high signal to noise ratio (SNR) approximations of Gaussian
channels called linear deterministic channels (LDA) \([3]\).

A. Motivation for Non-Causal Message Knowledge

The asymmetric form of cooperation of an overlay cognitive multi-user network, in which cognitive users possess primary users’ messages prior to transmission but not vice versa, may be motivated in a number of ways. For example, if certain receivers were not able to decode their own messages (due to for example packets being lost or damaged), they may request a re-transmission. If other transmitters were able to hear and decode the original transmission, during the re-transmission phase, then knowledge of messages at other transmitters is justified. It may also be justified as an upper bound to what a real cognitive transmitter may be able to do, under the assumption that it possesses the primary’s codebook, and hence listens and tries to decode the primary users’ messages.

The problem of Coordinated Multipoint (CoMP) joint transmission, also known as base station cooperation, has been considered at length over the decades. Such models usually consider a network with base stations connected via unrestricted backhaul links (error-free and unlimited capacity) over which messages or subsets of messages can be shared. The analytically tractable Wyner model \([4]\) (where nodes are placed on a line and suffer from interference only from a limited number of left and right neighbors) has been a widely adopted model for studying the advantages of base station cooperation in the downlink of cellular networks. In \([5]\) the authors show that achievable rates with cooperation are upper bounded by a theoretic limit that is independent of the transmit power; thereby proving that arbitrarily high gains can not be achieved through cooperation. The work in \([6]\) provides an overview of the information theoretic results and techniques to study multi-cell MIMO cooperation. An overlay multi-user cognitive interference network is a form of CoMP or network with base station cooperation in which there are only backhaul links in certain directions, leading to an asymmetry in the message knowledge structure.

B. Contributions

In this paper we give an overview of the state-of-the-art results for overlay cognitive networks dating back to the two-user cognitive interference channel (2-CIFC) and its reported capacity results. The survey continues with the introduction of the three-user extension of the 2-CIFC. For the 3-CIFC different models of cognition, or different message knowledge structure at the transmitters, are possible. After overviewing known results for the 3-CIFC, the results when the number of transmitters in the network is an arbitrary integer greater than three are then presented. Our major contributions include:

1) The derivation of new outer bounds on the capacity regions of \(K\)-CIFC.
2) Sum-capacity achieving schemes for both the LDA and the Gaussian noise symmetric \(K\)-CIFC are presented.
3) Interestingly, we show that under certain symmetric channel conditions, the throughput / sum-capacity of a multi-user cognitive interference with a cumulative message knowledge structure (or distributed cognition) can be achieved with a much simpler message knowledge structure. In particular, we show that at high SNR, having an interference channel with a cognitive relay is throughput / sum-capacity achieving; the same holds at finite SNR but to within a finite constant additive gap.

4) The asymmetric \(K\)-CIFC is considered and capacity results for the LDA are presented. Translation of these results at finite SNR for the Gaussian noise model is part of on-going work.

The general information theoretic study of cognitive networks is extensive and we do not attempt to survey it all; we focus on genie-aided multi-user cooperative networks in the sense that we assume messages are known to secondary users prior to transmission. The interference channel with partial transmitter cooperation, the causal cognitive interference channel (channels with cooperation between transmitters) considered in \([7]\) and \([8]\), the cognitive interference channel with fading \([9]\) are beyond the scope of this paper.

C. Paper Organization

The paper is organized as follows. Section \([1]\) introduces notation, definitions, and channel models; Section \([II]\) briefly surveys known results for multi-user cognitive interference channels; Section \([IV]\) derives novel outer bounds, which are then matched to novel achievable schemes for the linear deterministic channel in Section \([V]\) and for the Gaussian noise channel in Section \([VI]\). Section \([VII]\) concludes the paper. Some proofs may be found in Appendix.

This survey is meant to offer an entry point on the literature on the subject of cognitive networks to readers who are familiar with communication theory and point-to-point information theory \([10]\), but not necessarily with the latest advances in network information theory \([11]\). In order to make its content accessible to a wide audience, we have provided comments, insights and references to the most technical aspects of the discussion.

II. CHANNEL MODELS

Before discussing known and new results, we introduce the formal information theoretic definition of the problem. We start with describing our notation convention.

A. Notation

In the following we shall follow the notation convention of \([11]\). Lower case variables are instances of upper case random variables which take on values in calligraphic alphabets. The set of integers from \(n_1\) to \(n_2\) is denoted by \([n_1 : n_2]\). \(Y^j\) is a vector of length \(j\) with components \((Y_1, \ldots, Y_j)\). For a vector \(X\) and an index set \(I\), \(X_I = (X_i : i \in I)\). \(N(\mu, \sigma^2)\) denotes the density of a complex-valued circularly symmetric Gaussian random variable with mean \(\mu\) and variance \(\sigma^2\). \(P(\cdot)\) denotes a probability distribution function (a subscript indicating the random variable may be included to avoid confusion), \(P[\cdot]\) the probability measure (for the probability of an event), and \(E[\cdot]\) the expectation. The mutual
information between random variables $X$ and $Y$ is denoted by
$I(X,Y) = \mathbb{E} \left[ \log \frac{P(X,Y)}{P(X)P(Y)} \right]$ where $P(X)$ and $P(Y)$ are the marginal distributions of $P(X,Y)$.

**B. The General Memoryless CIFC**

**CIFC-CoMS:** The general memoryless $K$-user Cognitive InterFerence Channel with Cognitive only Message Sharing (K-CIFC-CoMS) is shown in Fig. 1(E). It consists of $K$ source-destination pairs sharing the same physical channel, where one transmitter has non-causal knowledge of the messages of all the other transmitters. Here transmitters $[1 : K-1]$ are referred to as primary users and are assumed to have no cognitive abilities. Transmitter $K$ is non-causally cognizant of the messages of the primary users. More formally, the K-CIFC-CoMS channel consists of:
- channel inputs $X_i \in \mathcal{X}_i$, $i \in [1 : K]$,
- channel outputs $Y_i \in \mathcal{Y}_i$, $i \in [1 : K]$, and
- a memoryless channel with joint transition probability distribution (or conditional channel distribution) $P(Y_1, \ldots, Y_K | X_1, \ldots, X_K)$.

A code with non-negative rate vector $(R_1, \ldots, R_K)$ is defined by:
- Mutually independent messages $W_i$, one for each transmitter $i \in [1 : K]$, that are uniformly distributed over $[1 : 2^{N R_i}]$, where $N$ denotes the block length and $R_i$ the rate in bits per channel use.
- Encoding functions $f_i^{(N)} : [1 : 2^{N R_i}] \rightarrow \mathcal{X}_i^N$ such that $X_i^N := f_i^{(N)}(W_i)$, for primary user $i \in [1 : K-1]$, while $f_K^{(N)} : [1 : 2^{N R_K}] \times \ldots \times [1 : 2^{N R_K}] \rightarrow \mathcal{X}_K^N$ such that $X_K^N := f_K^{(N)}(W_1, \ldots, W_K)$ for the cognitive user.
- Decoding functions $g_i^{(N)} : \mathcal{Y}_i^N \rightarrow [1 : 2^{N R_i}]$ such that $\hat{W}_i = g_i^{(N)}(Y_i^N)$, $i \in [1 : K]$.
- The (average over all messages) probability of error for user $i \in [1 : K]$ is denoted as $P[\hat{W}_i \neq W_i]$.

The capacity region of the K-CIFC-CoMS consists of all rate tuples $(R_1, \ldots, R_K)$ for which there exists a sequence of codes indexed by the block length $N$ such that the probability of error of every user can be made arbitrary small, formally, such that $P_e^{(N)} := \max_{i \in [1 : K]} P[\hat{W}_i \neq W_i] \rightarrow 0$ as $N \rightarrow \infty$.

Note that the capacity under average probability of error criteria $P[\hat{W}_i \neq W_i] = \sum_k P[W_i = k]P[\hat{W}_i \neq k | W_i = k]$, $i \in [1 : K]$, may be larger than the capacity under maximal probability of error criteria $\max_k P[\hat{W}_i \neq k | W_i = k]$, $i \in [1 : K]$, [11] Chapter 4.

**CIFC-CMS:** The general memoryless $K$-user Cognitive InterFerence Channel with Cumulative Message Sharing (K-CIFC-CMS) is shown in Fig. 1(F). It consists of $K$ source-destination pairs sharing the same physical channel, where there are $K-1$ cognitive transmitters and one primary user. Here transmitters $\ell \in [2 : K]$ are referred to as cognitive users and are assumed to have non-causal message knowledge of the users’ messages with lesser index. The K-CIFC-CMS and the K-CIFC-CoMS differ thus in the encoding at the cognitive transmitters. In particular, the K-CIFC-CMS consists of encoding functions $f_i^{(N)} : [1 : 2^{N R_i}] \times \ldots \times [1 : 2^{N R_K}] \rightarrow \mathcal{X}_i^N$ such that $X_i^N := f_i^{(N)}(W_1, \ldots, W_K)$, for $i \in [1 : K]$, while all the rest is as for the K-CIFC-CoMS.

**C. The Gaussian Channel**

The single-antenna complex-valued $K$-CIFC with Additive White Gaussian noise (AWGN) has input-output relationship
\[
Y_\ell = \sum_{i \in [1 : K]} h_{\ell i} X_i + Z_\ell, \quad \ell \in [1 : K], \tag{1a}
\]
where, without loss of generality, the inputs are subject to the power constraint
\[
\mathbb{E}[|X_i|^2] \leq 1, \quad i \in [1 : K], \tag{1b}
\]
and the noises are marginally proper-complex Gaussian random variables with parameters
\[
Z_\ell \sim \mathcal{N}(0, 1), \quad \ell \in [1 : K]. \tag{1c}
\]

1This specific way of writing the overall system error probability $P_e^{(N)}$ (as the maximum of individual error probabilities) highlights the capacity of an IFC without node cooperation, similarly to the broadcast channel [11] Chapters 5 and 8, does not depend on the joint channel conditional distribution $P(Y_i | X_1, \ldots, X_K)$ but only on the marginal distributions $P(Y_i | X_1, \ldots, X_K)$, $i \in [1 : K]$—a fact that may be leveraged in deriving outer bounds.
The channel gains \( h_{ij}, (i, j) \in [1 : K] \times [1 : K] \), are assumed constant for the whole codeword duration and therefore known to all terminals. This is equivalent to assuming a non-fading / static channel (for which all channel gains can be learnt by every node to any degree of accuracy without impacting the transmission rates as the blocklength tends to infinity [11 Chapter 3]), or to a fading channel with perfect instantaneous channel state information at all nodes [11 Chapter 23].

Determining the exact capacity region (a convex set in \( \mathbb{R}_{+}^{K} \)) as a function of \( K \) as complex-valued parameters (the channel gains) is a formidable task. Moreover, it is well known that in general different achievable schemes are needed depending on the relative strength of the desired signal at a receiver and the strength of the interfering terms [11 Chapter 6]; determining such regimes is still an art in many cases. To circumvent these problems, in the past decade it has become apparent that it is easier to approximate [12]. The approach is as follows. One first studies a deterministic / noiseless approximation of the Gaussian channel at high SNR, in which the noise is neglected to focus solely on the interference problem. The key is to choose a deterministic model for which capacity can be easily determined but that still retains the distinguishing features of the Gaussian channel. Intuitions from such a well chosen noiseless model can be translated into outer and inner bounds for the Gaussian case at any finite SNR, with the property that the worst case gap / difference between the outer and the inner bounds, taken over all possible channel gains, is a (small hopefully) constant.

D. The Linear Deterministic Approximation (LDA) of the Gaussian Channel at High SNR

The Linear Deterministic Approximation (LDA) of Gaussian noise \( K \)-CIFC has input-output relationship [3]

\[
Y_{\ell} = \sum_{i \in [1 : K]} S^{m-n_{\ell i}} X_{i}, \; \ell \in [1 : K], \tag{2}
\]

where \( m := \max\{n_{i,j}\} \), \( S \) is the binary shift matrix of dimension \( m \) (made of all zeros except for the first lower diagonal), all inputs and outputs are binary column vectors dimension \( m \), the summation is bit-wise over the binary field, and the channel gains \( n_{\ell i} \) for \( (\ell, i) \in [1 : K]^{2} \), are non-negative integers. The channel in (2) can be thought of as the high SNR approximation of the channel in (1) with their parameters related as \( n_{ij} = \lceil \log(1 + |h_{ij}|^{2}) \rceil, (i, j) \in [1 : K]^{2} [3]. \)

The model in (2) can be ‘played with’ without much network information theory knowledge, by simply reasoning in terms of recovering bits at each receiver from the received linear combinations of the transmitted bits. For the LDA, linear schemes often turn out to be optimal [3 Chapter 3].

E. Performance Metrics: Sum-Capacity, Generalized Degrees of Freedom, and Constant Gap Approximation

In this work we shall focus primarily on achieving the sum-capacity, or throughput, for the Gaussian channel model by leveraging results for the LDA \( K \)-CIFC-CMS.

The throughput is often used as a performance metric of interest from a network operator point of view, where the revenue is assumed to be proportional to the total delivered traffic irrespective of which receiver actually obtains / pays for the bits. This is of course just one performance measure, that unfortunately neglects important issues such as fairness among users. The sum-capacity should thus simply be thought of here as a ‘summary’ of the capacity region, the latter which gives us the ultimate complete network performance characterization with all the involved tradeoffs among competing users.

As finding the exact capacity of a Gaussian network is challenging, it is helpful to first study asymptotic approximations of the sum-capacity from the LDA. Towards this goal, it is convenient to introduce the notion of Generalized Degrees of Freedom (gDoF) of a symmetric network [19].

The gDoF is meant to capture the behavior of the capacity for different relative rates of growth of the interference links compared to the direct links, which is typical of the wireless channel [19], when the network is not noise-limited. The symmetric assumption is made so as to reduce the number of parameters in the network model.

Let SNR and INR be non-negative numbers, which are intended to characterize the signal and interference to noise ratios, respectively. Let us parameterize the magnitude of the \( K^{2} \) channel gains in (1a) as

\[
|h_{\ell i}|^{2} := \text{SNR}, \; i \in [1 : K], \tag{3a}
\]

\[
|h_{\ell i}|^{2} := \text{INR} = \text{SNR}^{\alpha}, \; (\ell, i) \in [1 : K]^{2}, \ell \neq i, \tag{3b}
\]

for some non-negative real-valued \( \alpha \). The phases of the channel gains are assumed to be such that any submatrix of the channel matrix \( H = [h_{\ell i}] \) is full-rank [20]. The gDoF is

\[
d(\alpha) := \lim_{\text{SNR} \to +\infty} \frac{C_{\Sigma}}{\log(1 + \text{SNR})}, \tag{4}
\]

where \( C_{\Sigma} := \max\{R_{1} + \ldots + R_{K}\} \) is the sum-capacity optimized over all achievable rate tuples.

The special case \( \alpha = 1 \) is referred to as Degrees of Freedom (DoF); the DoF provides a high SNR (or interference limited) approximation of the sum-capacity of the network when all channel gains scale at the same rate, or alternatively, when there is an average power constraint \( P \) at all nodes, the channel gains are kept fixed and \( P \) is increased to infinity.

The gDoF of the Gaussian channel and the sum-capacity of the LDA may be related as follows. Although not proved in general, so far it has been the case (except possibly for \( \alpha = 1 \)) that \( C_{\Sigma - \text{LDA}}(\alpha)/n_{d} = d(\alpha) \) where \( C_{\Sigma - \text{LDA}}(\alpha) \) is the sum-capacity of the symmetric LDA with \( n_{d} = n_{d} \) and \( n_{ij} = \alpha n_{d}, \; \forall j \neq i \). Intuitively, the gDoF in (4) counts how many equivalent independent interference-free streams can be reliably sent across the network simultaneously. For example, in the classical 2-IFC without cognition (see Fig. 1(A) and [11 Chapter 6]), \( d(\alpha) \in [1/2, 1] \), \( d(\alpha) = 1/2 \) means that each user can send one interference-free stream for half of the time (i.e., time division is optimal at high SNR), while \( d(\alpha) = 1 \) means that each user at each point in time can send one stream as if it was alone on the network [19].

Knowing the gDoF of the channel amounts to correctly establishing the optimal scaling of the sum-capacity at high SNR, that is, for \( \text{SNR} \gg 1 \) we have \( C_{\Sigma} \approx d(\alpha) \cdot \log(\text{SNR}) \); for this reason the gDoF is also known as “pre-log factor”
or “multiplexing gain.” So far it has been the case that the characterization of the gDoF allows one to make an educated guess on good outer an inner bounds for the original Gaussian channel. Specifically, from the study of the LDA one infers how to enhance the original Gaussian channel (by giving the nodes ‘genie side information’ for example) so that the capacity of this enhanced channel, $C_{\Sigma}(H)$, can be easily determined and thus provides an upper bound to the sum-capacity of the original channel with channel gains $H = [h_{i,j}]$. At the same time, one mimics the LDA capacity achieving scheme and derives a lower bound, $C_{\Sigma}(H)$, to the sum-capacity. The goodness of the bounds is measured by their additive gap. Let $\text{gap} := \sup_{H} (C_{\Sigma}(H) - C_{\Sigma}(H))$. If $\text{gap} < +\infty$ we say that the capacity is approximately known to within gap bits per channel use. Notice that the gap holds for all channel gain matrices $H$ and represents the difference between an outer and inner bounds for the worst channel in the Gaussian family.

### III. Survey

After having introduced the formal channel model definition and the performance metric of interest, we are ready to summarize known results for the CIFC. A summary of the main capacity results mentioned in this survey for Gaussian, LDA and general memoryless channels with the corresponding references are presented for the reader convenience in Table 1.

| Channel Model          | Capacity Results                                                                 | Remarks                                                                 | References |
|------------------------|---------------------------------------------------------------------------------|------------------------------------------------------------------------|------------|
| K-CIFC-CMS Gaussian    | Characterization of gDoF and sum-capacity of fully symmetric $K$-user Gaussian to with constant gap | MIMO broadcast DPC scheme with one encoding order (K-1) FMS + one global cognitive user sufficient to obtain outer bound | Thm. 10    |
| K-CIFC-CMS LDA         | Sum-capacity of fully symmetric $K$-user Scheme requires messages equivalent to CMS channel | Thm. 8       |
| K-CIFC-CMS Gaussian and General Memoryless | Sum-capacity general (non-symmetric) channel under strong channel gain conditions | Joint Decoding Scheme | Thm. 9     |
| K-CIFC-GMS Gaussian and General Memoryless | Sum-capacity general (non-symmetric) channel under strong channel gain conditions | Joint Decoding Scheme | Thm. 9     |
| J-CIFC-GMS Gaussian and General Memoryless | Capacity region for a 3 user channel under strong channel gain conditions | Compound MAC scheme | Thm. 4     |
| IPC+CR LDA             | Characterization of capacity region regions in yellow, red and green in Fig 6 | Capacity still open in moderately weak and weak from cognitive relay (region includes the blue region in Fig 6) | Thm. 17    |
| IPC+CR Gaussian        | Characterization of capacity region Capacity still open in moderately weak and weak from cognitive relay | Thm. 17    |
| 3-CIFC-CMS Gaussian    | Characterization of sum-capacity regions in yellow, red and green in Fig 6 | IPC+CR message knowledge structure is sufficient to obtain CMS outer bound | Thm. 18    |
| 3-CIFC-CMS Gaussian    | Characterization of sum-capacity under strong conditions | IPC+CR message knowledge structure is sufficient to obtain CMS outer bound | Thm. 18    |
| K-CIFC-CMS Gaussian    | Characterization of sum-capacity to within constant gap | IPC+CR or CMS message knowledge structure are sufficient to obtain CMS outer bound | Thm. 18    |

Table 1: Capacity Results for Different Cognitive Interference Channel Models.

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2DPC, or Gelfand-Pinsker binning for channels with states [24, 11 Chapter 3], is a coding technique to convey data through a channel subjected to an interference known non-causally to the transmitter but unknown to the receiver. The technique, sometimes called precoding, in the Gaussian noise channel with additive interference is such that the achievable rate is as if the receiver knew the interference as well and were able to perfectly subtract it from the received signal, thus resulting in the same capacity as if the interference was not present [25]. DPC can be seen as a building block of Marton’s achievable region for the broadcast channel [11 Chapter 6.3] and is capacity achieving for the $K$-user MIMO Gaussian broadcast channel [26].

Rate splitting [11 Section 6.5] refers to a technique where a user divides its message into independent substreams and possibly codes them with different techniques. Superposition coding [11 Section 5.3] is a technique originally devised for the 2-user broadcast channel where a ‘cloud center’ codeword is intended to be decoded by the weakest receiver, while both the ‘cloud center’ codeword and the ‘satellite’ codeword are decoded by the strongest receiver; these techniques allows users with different channel qualities to decode as much information as they can without becoming the network bottleneck. Simultaneous non unique-decoding [11 Section 6.5] can be informally thought of as giving a receiver the possibility to decode a non-integer $I$ / ‘don’t care’ message if that helps to achieve a larger rate for the intended message [28].

The advantage of simultaneous non unique-decoding in a superposition coding scheme with rate splitting is that the receivers can decode only part of the interference and thus the resulting achievable rate region is described by fewer rate constraints than more traditional coding techniques, as showed in [29].
aspect for determining the capacity to within a constant gap for the Gaussian 2-CIFC is to compare the unifying outer bound in [27, Theorem 6] to the “Scheme (C)” (inspired by Marton’s achievable region for the broadcast channel) in [33, eq.(21)] with the auxiliary random variable assignment in [33, eq.(22)], which was inspired by the “New Capacity Result for the Semi-Deterministic Channel” (i.e., the signal at the cognitive receiver is an arbitrary deterministic function of the channel inputs) in [27, Theorem 11]. With this, it is a matter of simple entropy computations to find out that the gap for a user is equal to its number of receive antennas.

The performance of the multi-input and multi-output (MIMO) 2-CIFC was examined in [34], and results were further refined in [35], where the capacity of the MIMO 2-CIFC was derived to within an additive and multiplicative gap proportional to the number of antennas at the secondary receiver. While a finite additive gap (i.e., difference between outer and inner bounds) is meaningful at high SNR, a finite multiplicative gap (i.e., ratio between outer and inner bounds) is meaningful at low SNR—see also the related notion of ‘wideband slope’ [36], [37].

In [38], the DoF for the MIMO 2-IFC with different combinations of non-causal message knowledge at the transmitters and the receivers was obtained. We note that contributions on the 2-CIFC extend beyond those mentioned in this survey. The work in [35] provides a more comprehensive overview of the different outer bounds, achievability schemes and capacity results for different channel models of the 2-CIFC, including the Gaussian noise model.

B. The Three-User Cognitive Interference Channel (3-CIFC)

Several 3-user extensions of the 2-CIFC model have been studied in the literature that are the main focus of this paper. For the 3-user case, which we term the 3-CIFC, the work [39] proposed the following models for cognition/message sharing:

- with cumulative message sharing (3-CIFC-CMS)
- with primary message sharing (3-CIFC-PMS)
- with cognitive-only message sharing (3-CIFC-CoMS).

The 3-CIFC-CMS, shown in Fig. 1(C), models a network of two cognitive users: Tx2 knows the message of primary user Tx1, and Tx3 knows the messages of Tx1 and Tx2. The 3-CIFC-PMS is shown in Fig. 1(B), in this network there are 2 cognitive users: Tx2 and Tx3, who only know the message of the primary Tx1. In the 3-CIFC-CoMS, shown in Fig. 1(E), there are two primary users who do not know each others’ message and a single cognitive user who knows both primary messages. In Fig. 1(D) we have also plotted an Interference Channel with a Cognitive Relay (IFC+CR), a channel model studied in [17], [18], [40], in which there are two primary Tx1 and Tx2 and one cognitive Tx3 which has knowledge of the two primary messages and aids in their transmission. This third node is a relay only, as it does not have a message of its own to transmit. Clearly, because the channel with CMS has more message knowledge at its transmitters, anything the channel with PMS can do, the channel with CMS can as well. Hence, in Fig. 1 we have used the \( \subseteq \) to denote that the capacity region of the PMS is contained in that with the CMS message knowledge structure.

Limited prior work has emerged on the 3-CIFC: in [41], the authors considered the 3-CIFC-PMS and CMS, achievable rate regions based on rate splitting and binning were derived. The 3-CIFC-CoMS scenario was later introduced in [42]. Achievable rate regions for the discrete memoryless channel were obtained and were numerically evaluated for the Gaussian noise channel; a numerical comparison of the achievable rates in Gaussian noise was made in [39]. Inner and outer bounds for the special case of the 3-CIFC-CoMS in which the cognitive user is assumed not to interfere with the primary users were obtained in [43]. We shall see later on that there exist channel gain conditions for which the sum-capacities of the IFC+CR and of the 3-CIFC-CoMS are to within a constant gap, which implies that sometimes the most throughput-wise efficient use of cognitive abilities is to manage interference.

However, general relationships between the capacity regions of the three channel models has not yet been examined. In this work we make progress by showing some relationships between the capacity regions of the different models. This has implications in understanding which message knowledge structures are most desirable in practice.

For example, it turns out that having message knowledge at the transmitters—corresponding to a PMS or a CoMS—is sufficient for achieving (under certain channel gain conditions, and to within constant gap) the sum-capacity upper bound derived for the CIFC-CMS. This has obvious practical advantages and may reduce the amount of signaling needed in practice on the backhaul in order to achieve the desired message knowledge structure.

C. The K-user Cognitive Interference Channel (K-CIFC)

Much less work has been done on cognitive interference channels with an arbitrary (K) number of transmit and receive pairs. Given the large number of channel gain parameters involved, and the many ways messages may be shared at the transmitters, studies have so far usually been restricted to symmetric scenarios, or have even assumed some links to be zero altogether. For example, in [44] the authors considered a channel model that consists of one primary user and \( K - 1 \) cognitive users, a K-user extension of the PMS scenario: each cognitive user only knows the primary message in addition to

\footnote{A MAC models the uplink of a cellular system with multiple transmitters with mutually independent messages and a single receiver interested in decoding all messages [11, Chapter 4]. A compound MAC has multiple receivers, all interested in decoding all messages. The compound channel is often used to model channel state information uncertainty at the transmitters.}
their own message. However, they restrict the channel so that
the cognitive users do not cause interference to one another
but only to the primary receiver and are interfered only by
the primary transmitter; for this channel model the capacity in
the very strong interference regime (i.e., INR > SNR^2 in
the symmetric case) is obtained by using lattice codes[7].

In [13], we further considered the K-CIFC-CMS. A sum-
capacity upper bound was derived by giving nested genie side-
information to receivers. The side information given to the
secondary receivers consists of messages and output sequences
of receivers with lesser index. It was shown that this upper
bound can be achieved exactly for the symmetric linear
deterministic K-CIFC.

For the Gaussian K-CIFC the capacity was derived to
within an additive and multiplicative gap independent of the
channel gains. Interestingly, for the symmetric K-CIFC-CMS,
the achievable scheme required only cognitive message
knowledge at one user, a global cognitive user. In particular,
the achievability scheme required only cognitive message
K channel gains. Interestingly, for the symmetric
within an additive and multiplicative gap independent of the
deterministic K bound can be achieved exactly for the symmetric linear
of receivers with lesser index. It was shown that this upper
capacity results for such networks.

The achievable scheme utilizes DPC at the cognitive trans-
dering cannot increase the DoF above
that of the classical K-IFC [50], which motivates the study of
non-causal / cognitive networks. For the 2-user case, the DoF
with cognition is given in [51], Eq.1. We note also that the
DoF of (non-generic) networks with a sparse topology, i.e.,
networks for which some channel gains are zero, cannot be
smaller than that of the classical K-IFC.

In [52], the authors characterized the DoF of a K-IFC in
which each transmitter, in addition to its own message, has
access to a subset of the other users’ messages; in particular, it
was shown that the maximum possible DoF = K is attainable
if the sum of the number of jointly cooperating transmitters
and the number of jointly decoding receivers is greater than
or equal to K + 1. In general, cognition cannot decrease the
DoF over that of the classical K-IFC. The question of interest
is thus how much cognition is needed to ‘beat’ the classical
K-IFC, or alternatively, for a given DoF above that of the
classical K-IFC what is the minimum amount of cognition
needed to achieve it. The novel results in this paper try to
answer these questions.

The DoF (α = 1) of the K-IFC in which receiver k suffers
from interference due to the transmission of users indexed
i ≤ k only, and where transmitter k knows the messages of J
preceding transmitters, was characterized in [53] as

\[ \text{DoF}(K, J) = 1 - \frac{1}{K} \left( \frac{K}{J + 2} \right). \]

The achievable scheme utilizes DPC at the cognitive trans-
ers indexed i ∈ [1 : J+1] while silencing transmitter J+2
(repeated at all users).

In [13], the gDoF of the K-CIFC-CMS was shown to be

\[ d^{K-CIFC-CMS}(\alpha) = K \max\{1, \alpha\} - \alpha. \]

In Fig. [2] the gDoF of the K-CIFC-CMS for K ∈ [2 : 4] is
plotted along with the gDoF of the MISO broadcast channel,
which provides an upper bound by giving cognitive message
knowledge of all messages at all transmitters, given by [54]

\[ d^{K-BC}(\alpha) = K \max\{1, \alpha\}, \]

and the gDoF of the K-IFC, which provides a lower bound
where transmitters have no cognitive abilities, given by [54]

\[ d^{K-IFC}(\alpha) = K \min \left\{ 1, \max \left( \frac{\alpha}{2}, \frac{1 - \alpha}{2}, \max (\alpha, 1 - \alpha) \right) \right\}. \]

Two interesting observations can be made on the normalized
(by the number of users) gDoF [13]: the first is that unlike
the K-IFC and the MISO broadcast channels, the normalized
gDoF of the K-CIFC-CMS is a function of the number of
users in the networks K, and the second is that the normalized
gDoF loses \( \alpha/K \) with respect to the \( d^{K-BC}(\alpha)/K \) (a vanishing loss as K increases).
Moreover, each rate bound in the joint channel conditional distribution as long as the upper bound derived for the K-KK memoryless channel. Theorem 1 may be tightened with respect to the joint channel conditional distribution as long as the marginal channel conditional distributions are preserved.

IV. NOVEL OUTER BOUNDS

We start by reporting an outer bound region for the general memoryless K-CIFC-CoMS as defined in Section [II-B]. Based on the K-CIFC-CoMS result, we then derive an outer bound for the 3-CIFC-CMS. Note that an outer bound on the general memoryless K-CIFC-CMS is also an outer bound for the K-CIFC-CoMS as the CMS has more message knowledge at the transmitters. One of our goals is to show that for distributed cognition (K-CIFC-CMS) is not always needed from a sum-capacity perspective. In particular, the sum-capacity upper bound derived for the K-CIFC-CMS may sometimes be achieved (or achieved to within a constant gap) with a message structure corresponding to that of the K-CIFC-CoMS, indicating that extra message knowledge sometimes only leads to bounded gains.

**Theorem 1 (K-CIFC-CMS Outer Bound)**

The capacity region of the general memoryless K-CIFC-CMS is contained in the region

\[
R_i \leq I(Y_i; X_i, X_K|X_{[1,i−1]}),
\]

\[
\sum_{j=1}^{K} R_j \leq \sum_{j=1}^{K} I(Y_j; X_{[j,K]}|X_{[1,j−1]}, Y_{[1,j−1]}),
\]

for \(i \in [1 : K]\) for some joint input distribution \(P_{X_1, ..., X_K}\).

Moreover, each rate bound in (5) may be tightened with respect to the joint channel conditional distribution as long as the marginal channel conditional distributions are preserved.

Since we will be examining the 3-CIFC-CoMS at length, we explicitly provide the statement and proof for the case \(K = 3\) below, which is based on Theorem 1 for \(K = 3\).

**Theorem 2 (3-CIFC-CoMS Outer Bound)**

The capacity region of the general memoryless 3-CIFC-CoMS is contained in the region defined by

\[
R_1 \leq I(Y_1; X_1, X_3|X_2),
\]

\[
R_2 \leq I(Y_2; X_2, X_3|X_1),
\]

\[
R_3 \leq I(Y_3; X_3|X_1, X_2),
\]

\[
R_1 + R_3 \leq I(Y_1; X_1, X_3|X_2) + I(Y_3; X_3|X_2, X_1, Y_1),
\]

\[
R_2 + R_3 \leq I(Y_2; X_2, X_3|X_1) + I(Y_3; X_3|X_1, X_2, Y_2),
\]

\[
R_1 + R_2 + R_3 \leq I(Y_1; X_1, X_2, X_3) + I(Y_2; X_2, X_3|X_1, Y_1)
+ I(Y_3; X_3|X_1, Y_1, X_2, Y_2),
\]

\[
R_2 + R_1 + R_3 \leq I(Y_2; X_2, X_1, X_3) + I(Y_1; X_1, X_3|X_2, Y_2)
+ I(Y_3; X_3|X_2, Y_2, X_1, Y_1),
\]

for some joint input distribution \(P_{X_1, X_2, X_3}\).

**Proof:** An outer bound for the 3-CIFC-CoMS, where transmitter \(i \in [1 : 2]\) only knows its own message can be obtained by giving side information to the two primary users so as to transform the CoMS message structure in Fig. [II-E] into the CMS one in Fig. [II-F]. For each possible permutation of the primary users’ indices we obtain a region as in (6b); by intersecting these regions we obtain the outer bound for the 3-CIFC-CMS in (6).

Since we are mainly interested in the sum-capacity, we explicitly derive a sum-capacity upper bound for the 3-CIFC-CoMS from Theorem 2 as follows.

**Corollary 3.** The sum-capacity of the general memoryless 3-CIFC-CoMS is upper bounded by

\[
R_1 + R_2 + R_3 \leq \min\{a, b\},
\]

\[
a := \min \left\{ I(Y_5; X_3|X_1, X_2, Y_1), I(Y_3; X_3|X_1, X_2, Y_2) \right\}
+ I(Y_1; X_1, X_3, X_2) + I(Y_2; X_2, X_3|X_1),
\]

\[
b := \min \left\{ I(Y_1; X_1, X_2, X_3) + I(Y_2; X_2, X_3|X_1, Y_1),
I(Y_1; X_1, X_3|X_2, Y_2) + I(Y_2; X_1, X_3, Y_1) \right\},
\]

for some input distribution \(P_{X_2, X_1, X_3}\).

**Proof:** The sum-rate upper bound in (7) is obtained as \(\min\{6d, 6e, 6a, 6b\}\).

We conclude this section by highlighting some of the ‘desirable characteristics’ one seeks in an outer bound that can be found in Theorems 1 and 2 and Corollary 3. Our outer bounds apply to any memoryless channel (because of no assumptions on the channel structure, as opposed to structure specific bounds such as for ‘injective semi-deterministic’ channels [11] Section 6.7]). As such, they can be used as building blocks for other networks (through a cooperation or a genie side information argument as we did with Theorem 1 to obtain...
Theorem 2. Last but not the least, they are easily computable (because they do not involve random variables that are not part of the problem definition, but only channel inputs and outputs). For example, our bounds as exhausted by independent and equally likely input bits for the LDA, and by jointly Gaussian inputs for the Gaussian IFC.

In the following we shall derive capacity results that will however not cover all possible parameter regimes or the whole capacity region. We offer next our thoughts as to why this may be the case. The LDA and the Gaussian noise channels are examples of Injective Semi-Deterministic (ISD) channels. ISD IFCs are such that the channel output of each user is a deterministic function of the intended input and a noisy function of the interferers with the property that, knowing the channel output and the intended input (which is the case after decoding) it is possible to recover the noisy function of the interferers [53]. The ISD characteristics can be leveraged to obtain outer bounds that may not be derived for the general memoryless case. For example, in [53] it was shown that for the ISD 2-IFC the Han and Kobayashi achievable region [11, Section 6.5] is optimal to within a constant gap by deriving ISD-specific sum-rate upper bounds and bounds of the type 2R1 + R2 and R1 + 2R2. Interestingly, it turns out that such ISD-specific bounds are not needed in order to characterize to within a constant gap the capacity of the 2-IFC CoMS or that of the 2-IFC with output feedback [56]; but they are needed for the IFC+CR [17] and for the IFC with causal cognition [57]. At this point it is not clear whether our partial novel capacity results for the LDA and the Gaussian noise channels are due to a weakness in the achievable scheme or a lack of ISD-specific bounds, or both. Answering this question is a subject of current investigation.

V. NOVEL CAPACITY RESULTS FOR THE LDA

In this section we shift our focus to the 3-user LDA, before returning to arbitrary K-user Gaussian CIFCs. As mentioned before, the LDA models the Gaussian channel at high SNR and provides insights into interference-limited behaviors of the network, i.e., how the capacity is limited by the interference created and caused by other users rather than by noise. We next study both the symmetric LDA in Section V-A and the non-symmetric LDA in Section V-B with three users. We show that conclusions for symmetric channels may or may not extend to more general asymmetric settings.

A. Sum-Capacity of the Symmetric LDA 3-CIFC-CoMS

In our prior work, we showed that the sum-capacity of the 3-CIFC-CoMS is the same as that for the 3-CIFC-CoMS for a symmetric (all cross-over links are the same strength, all direct links the same strength) LDA [13, Theorem 4]. We now strengthen and generalize this result by obtaining channel conditions under which the sum-capacity of the 3-CIFC-CoMS is actually achieved by a scheme which only requires the message knowledge of an IFC+CR [17], [18], [40], that is, setting R3 = 0 and ignoring the knowledge of message W1 at Tx2 is sum-capacity optimal.

We show results for the following somewhat symmetric parametrization of the channel gains for the LDA with channel gains as described in [2] and as depicted in Fig. 3:

\[
\begin{align*}
n_{22} &= n_{11} = n_d & (8a) \\
n_{12} &= n_{21} = n_i = \alpha n_d & (8b) \\
n_{13} &= n_{23} = n_c = \beta n_d. & (8c)
\end{align*}
\]

Notice that we have not placed any conditions yet on the links coming into the cognitive receiver, i.e., on n_{31}, n_{32}, n_{33}.

Our main result is as follows.

Theorem 4. The message knowledge structure of the IFC+CR is sufficient to achieve the sum-capacity of the LDA 3-CIFC-CoMS and LDA 3-CIFC-CMS channels under the following channel gain conditions

\[
\{n_{33} \leq \beta n_d\} \cup \{\alpha \geq 1\} \cup \{\beta \geq \alpha\}. \tag{9}
\]

In addition, the outer bounds for the IFC+CR and those for the 3-CIFC-CoMS (and hence 3-CIFC-CoMS) coincide for the following channel gains conditions, though it is not generally known whether these outer bounds are achievable

\[
\{n_{33} \leq \beta n_d\} \cup \{\beta < \alpha < 1\} \cap \{2 \leq 3\alpha + \beta\}. \tag{10}
\]

Proof: The upper bound on the sum-capacity for the LDA 3-CIFC-CMS in Theorem 1 was evaluated in [13, eq.(8)] and is given by

\[
R_1 + R_2 + R_3 \leq \max\{n_{11}, n_{12}, n_{13}\} + f(n_{22}, n_{23}|n_{12}, n_{13}) + [n_{33} - \max\{n_{31}, n_{23}\}]^+ , \tag{11}
\]

where the function \( f(c, d|a, b) \) in (11) is defined as \( \max\{c + b, a + d\} - \max\{a, b\} \) if \( c - d = a - b \) and \( \max\{a, b, c, d\} - \max\{a, b\} \) if \( c - d = a - b \). In the following we aim to find channel gain conditions under which it is sum-rate optimal for the global cognitive transmitter in a 3-CIFC-CoMS to behave as a cognitive relay. If

\[
n_{31} \leq \max\{n_{13}, n_{23}\}, \tag{12}
\]

the sum-capacity outer bound in (11)
result, please recall our discussion at the end of Section IV about ISD-type outer bounds.

Finally, we note that having $R_3 > 0$ does not qualitatively alter the capacity results. In fact the condition $n_{33} > \max\{n_{13}, n_{23}\}$ (i.e., the complement of the condition in (12)) suggests the following communication strategy. Recall that $n_{33} > \max\{n_{13}, n_{23}\}$ means that some of the $n_{33}$ bits received at Rx$_3$ from Tx$_3$ are neither received at Rx$_1$ nor at Rx$_2$, i.e., they do not create interference (commonly referred to as “bits are received below the noise floor of the non-intended receivers”). Therefore, the most cognitive user can use its bits not received at Rx$_1$ and at Rx$_2$ to convey its own message to Rx$_3$ while keeping using an IFC+CR type scheme to manage interference for those bits that are received at Rx$_1$ and at Rx$_2$.

We now briefly consider the role of symmetry in our LDA results, and consider examples of 3-CIFC-CMS with asymmetric channel gains. We ask whether our statements on message knowledge and cognitive relay behavior continue to hold then. It turns out that under certain asymmetric channel gains, the statements continue to hold. In other cases, however, having cognitive transmitters as in a CIFC-CMS is critical to achieve the outer bound in (11). We provide examples that illustrate both observations. The characterization of exactly when—in terms of channel gain relationship—not both the former mentioned schemes is optimal is still an open problem.

### B. Sum-Capacity for some Asymmetric LDA 3-CIFC-CMS

In order to explore whether the conclusions made regarding cognition and message knowledge are due to our restriction to symmetric channel gain conditions, we present some examples which achieve the sum-capacity outer bound in (11) (valid for all channel gains) for the LDA asymmetric 3-CIFC-CMS. We note that attempting to characterize the sum-capacity for all asymmetric channel gain conditions is an open problem; the number of channel gains that needs to be considered is large and many different relative orderings of these channel gains may need to be considered in general.

A note on reading the following figures. In the following examples, bits within each transmit signal vector are represented by different shades of the same color. Bits that arrive at the same level (equal shifts) from the different users will neutralize each other. Bits arriving at the same level but with different color interfere. Having knowledge of primary users messages, cognitive users can transmit bits with different colors (corresponding to primary users) and its own bits.

**a) Example 1, Fig. 5** Parameters: $n_{11} = 5, n_{12} = 3, n_{13} = 3, n_{21} = 3, n_{22} = 2, n_{23} = 3, n_{31} = 5, n_{32} = 3, n_{33} = 2$ bits.

Key idea: even in asymmetric cases it may be sum-capacity optimal to set $R_3 = 0$ when $n_{33} \leq \max\{n_{13}, n_{23}\}$. The shifted transmit signals $X_1, X_2, X_3$ arrive at the three receivers Rx$_1, $Rx$_2, $Rx$_3$ with shifts equal to $n_{ij}, i, j \in \{1, 2, 3\}$. We assume that Rx$_1, $Rx$_2, $Rx$_3$ are interested in decoding green, red and yellow bits respectively. With $n_{33} \leq \max\{n_{13}, n_{23}\}$, one might suspect that $X_3$ may convey more information to
the primary receivers than to its intended receiver. In this case, setting $R_3 = 0$ is optimal, and the best use of the cognitive capabilities of user 3 is to “broadcast” to the non-intended receivers, even in asymmetric scenarios. In Fig. 5, we give an achievable strategy for this example. The cognitive transmitter that has in addition to its own message, message $W_1$ and $W_2$, sends a linear combination of these messages thus behaving much like a cognitive relay. Recall that the addition is bit wise over the binary field; therefore, the interference at receivers $R_{x1}$ and $R_{x2}$ are zero forced simultaneously (by addition modulo 2). In this example, there are no yellow bits successfully decoded at $R_{x3}$ as $R_3 = 0$. The sum-capacity in this case is 8 bits as given in (11).

b) Example 2, Fig. 6 Parameters: $n_{11} = 1, n_{12} = 2, n_{13} = 3, n_{21} = 1, n_{22} = 3, n_{23} = 2, n_{31} = 1, n_{32} = 3, n_{33} = 4$ bits. Key idea: when the cognitive user has $n_{33} > \max\{n_{13}, n_{21}\}$, it may sneak in bits to achieve $R_3 > 0$, while using the cognitive relaying strategy for the other users. The condition $n_{33} > \max\{n_{13}, n_{21}\}$ suggests that the intended signal at $R_{x3}$ is sufficiently strong to be able to support a non-zero rate. The form of the sum-capacity also suggests that most cognitive user can “sneak in” extra bits for user 3 in such a way that they do not appear at the other receivers, in other words they appear below the noise level. In Fig. 6, schemes used in Example 1 (where user 3 acts as a cognitive relay only) are again used here, but in addition, user 3 is able to sneak in yellow bits which are below the noise level at the primary receivers by sending a combination of the interfering and desired messages, a linear deterministic version of the Gaussian dirty paper coding. The number of bits for $R_3$ in this case is $n_{33} - n_{13}$. The sum-capacity in this case is 4 bits as given in (11).

c) Example 3, Fig. 7 Parameters: $n_{11} = 3, n_{12} = 2, n_{13} = 4, n_{21} = 1, n_{22} = 2, n_{23} = 2, n_{31} = 3, n_{32} = 3, n_{33} = 3$ bits. Key idea: Distributed message knowledge (not just one fully cognitive transmitter) is needed to achieve the sum-capacity outer bound (in general). As in Example 1, we set $R_3 = 0$. However, notice that in this case user 3 is not able to zero force the interference caused by primary user 1 at receiver 2.

In this case cognitive user 2 can precode against the interference caused by user 1 and thus the message knowledge structure in this case corresponds to $W_1$ at $Tx_1$, $W_1$, $W_2$ at $Tx_2$ and again $W_1$, $W_2$ at $Tx_3$ . The sum-capacity in this case is 6 bits as given in (11). Note that the upper bound derived for the IFC+CR in [17] Theorem 2, Theorem 3, eq.(5a), eq.(8)] when evaluated for the LDA with the channel gains of this example yields a sum-capacity of 4 bits. Therefore, message knowledge at $Tx_2$ is needed to achieve the upper bound of 6 bits as given in (11).

VI. NOVEL CAPACITY RESULTS FOR THE AWGN

The insights gained for the LDA have often been translated into Gaussian capacity results to within a constant gap for any finite SNR. In light of these success stories, we will leverage the results obtained for the 3-user LDA in the previous section into approximate capacity results for the 3-user Gaussian noise channel in Section VI-A. In particular, we will consider channels which satisfy channel gain relationships equivalent to those in yellow and red in Fig. 5 at finite SNR. In Section VI-B we will discuss extension to the general $K$-user case.

The general Gaussian model in (1) is described by $K^2$ parameters; in the following we reduce the number of parameters involved by focusing on the symmetric case defined by: for $j \in [1 : K - 1]$,

$$h_{jj} = |h_{ij}|, \text{ (primary direct links)},$$

$$h_{jk} h_{ij}, \text{ (secondary→primary links)},$$

$$h_{jk} = h_{ij}, k \not\in \{j, K\} \text{ (primary interfering links)}. \quad (16c)$$

The symmetric setting in (16) (equivalent for the 3-user case to the parameterization in (3) for the LDA) makes users $j \in [1 : K - 1]$ completely equivalent in terms of channel gains, but it does not impose any restriction on the channel gains of the cognitive receiver (i.e., $h_{K}, i \in [1 : K]$) which are kept general. Moreover, the direct channel gains can be taken to...
be real-valued without loss of generality because the receivers can compensate for the phase of one of its channel gains.

A. Gaussian 3-CIFC-CMS in Strong Interference

In this section we aim to examine the relationship, in terms of sum-capacity, between the different Gaussian CIFC models under strong interference. In particular, we will show an achievability scheme with message knowledge equivalent to that of an IFC+CR that achieves (to within a constant gap) the sum-capacity outer bound of the 3-CIFC-CMS channel under strong interference conditions. Since in the LDA, a scheme with the message knowledge structure of the IFC+CR achieves an outer bound for the 3-CIFC-CMS under strong interference conditions, highlighted in red ($\alpha \geq 1$) in Fig. 4, intuition may suggest that a constant gap result is also possible for Gaussian noise channels at all SNR. In particular, we try to match the red high SNR regimes by a channel gain relationship of the form $|h_1|^2 \geq |h_2|^2$ for the Gaussian 3-CIFC-CMS.

Before going into the details of the derivation, we mention that a sum-capacity achievability scheme under a strong interference condition for the LDA (see Example 2 in Fig. 6) where Rx$_1$ is interfered by a strong interferer Tx$_3$ (transmit signal $X_3$ has more bits than $X_1$) required cooperation from the strong interferer with Tx$_3$ in the transmission of the latter’s bits (green bits intended for Rx$_1$ are relayed by Tx$_3$ at the highest signal power level of the transmit signal). Therefore, we leverage these insights for the Gaussian channel. More precisely, we allow the cognitive transmitter to coherently beam form with the primary transmitter by allowing it to transmit the same (scaled) Gaussian transmit signal as the primary user. In the following, we give an achievability scheme for the Gaussian channel which is based on cooperation.

**Theorem 5.** When the following holds

$$|h_{33}|^2 \leq |h_3|^2 \quad (17a)$$

$$\log(1 + h_2^H \Sigma h_2) \leq \log(1 + h_2^H \Sigma h_1) \quad (17b)$$

$$h_5 := \left[ \begin{array}{c} \frac{|h_d|}{h_c} \\ \frac{1}{h_c} \end{array} \right], \quad h_1 := \left[ \begin{array}{c} \frac{1 - |\rho_1|^2}{\rho_2^* - \rho_1 \rho_2^*} \\ \frac{1 - |\rho_2|^2}{\rho_2^* - \rho_1 \rho_2^*} \end{array} \right], \quad \Sigma := \left[ \begin{array}{cc} |\rho_1|^2 & \rho_2 - \rho_1 \rho_2^* \\ \rho_2 - \rho_1 \rho_2^* & |\rho_2|^2 \end{array} \right],$$

for all $|\rho_1| \leq 1, i \in [1 : 3]$, the sum-capacity outer bound of the 3-CIFC-CMS in Theorem 4 is achieved to within 3 bits by an achievable scheme with a message structure of IFC+CR.

Remark 1. The regime highlighted in red in Fig. 4 in the LDA is characterized by $\alpha \geq 1$, which we try to match with a channel gain relationship of the form $|h_1|^2 \geq |h_3|^2$ for the Gaussian 3-CIFC-CMS. Let $|h_d| = \text{SNR}^{1/2}$, $h_3 = \text{SNR}^{\alpha/2} e^{i\theta}$, and $h_c = \text{SNR}^{\beta/2} e^{i\phi}$; then, solving for (17b) gives

$$\text{SNR} (1 - |\rho_1|^2) + 2 \text{Re}(\rho_3 - \rho_1 \rho_2^*) \text{SNR}^{\alpha/2} \text{SNR}^{\beta/2} e^{i\phi} \leq \text{SNR}^{\alpha} (1 - |\rho_1|^2) + 2 \text{Re}(\rho_3 - \rho_1 \rho_2^*) \text{SNR}^{\beta/2} \text{SNR}^{\alpha/2} e^{i\phi};$$

by taking the limit as SNR approaches infinity in (18) the condition $\max\{1, 1/2 + \beta/2\} \leq \max\{\alpha, \alpha/2 + \beta/2\}$ is obtained, which is equivalent to $\alpha \geq 1$.

Remark 2. Theorem 5 shows that for the 3-user case, full cognition only gives a constant gap improvement for the symmetric sum-capacity under strong interference conditions compared to an achievability scheme with a message structure as for the IFC+CR. We note however that the sum-capacity for the K-CIFC-CMS under strong interference conditions has been characterized in [14] completely (for arbitrary $K$ and exact sum-capacity). The achievability scheme in that case amounts to having all cognitive users beam form to the primary receiver (as in a MISO channel). Extending our Theorem 5 to arbitrary $K$, i.e., characterizing the sum-capacity to within a constant gap for the $K$-CIFC-CMS by using an achievability scheme with message knowledge of $(K - 1)$-IFC+CR, is still an open problem.

**Proof:** As a sum-capacity outer bound we use the bound initially derived for the K-CIFC-CMS under strong interference conditions in [14], Theorem 1 and proved in Appendix B which specialized for the case of $K = 3$ users reads: when the following channel conditions hold

$$I(X_3; Y_3 | X_1, X_2) \leq I(X_3; Y_2 | X_1, X_2), \quad (19a)$$

$$I(X_2, X_3; Y_2 | X_1) \leq I(X_2, X_3; Y_1 | X_1) \quad (19b)$$

then the sum-capacity is upper bounded by

$$R_1 + R_2 + R_3 \leq I(X_1, X_2, X_3; Y_1), \quad (20)$$

where, by the ‘Gaussian maximizes entropy’ principle [11, Section 2.1], it suffices to use jointly Gaussian inputs both in (19) and in (20). Therefore, with Gaussian inputs the sum-capacity upper bound in (20) becomes

$$R_1 + R_2 + R_3 \leq \log \left( 1 + (|h_1| + |h_d| + |h_c|)^2 \right) =: C_{\text{sum,up.}} \quad (21)$$

and the channel gain conditions in (19) reduce to (17), as shown in [14], eq.(16) and (17).

For achievability, we use a scheme for the IFC+CR, where messages $W_1$ and $W_2$ are known at Tx$_3$.

Tx$_3$ and Tx$_2$ use independent Gaussian random codes, and Tx$_3$ sends a superposition of the codewords generated by the primary users, and nothing for Rx$_3$. Rx$_1$ and Rx$_2$ are required to decode both messages non-uniquely. The achievable sum-capacity is thus as for a compound MAC and is given by

$$R_1 + R_2 + R_3 \leq \min_j I(X_1, X_2, X_3; Y_j) \quad (22)$$

$$= \log \left( 1 + (|h_1| + \sqrt{|h_d|^2 + |h_i|^2})^2 \right) =: C_{\text{sum,low.}} \quad (23)$$

The gap between the sum-capacity outer bound in (21) and the inner bound in (23) is thus

$$C_{\text{sum,up}} - C_{\text{sum,low}} = \log \left( \frac{1 + (|h_1| + |h_d| + |h_i|^2)}{1 + (|h_1| + \sqrt{|h_d|^2 + |h_i|^2})^2} \right) \leq \log \left( \frac{2 |h_1| + |h_d| + |h_i|}{|h_1| + \sqrt{|h_d|^2 + |h_i|^2}} \right)^2 \leq \log(8).$$

6 The sum-capacity upper bounds in Theorem 1 and 20 coincide under the condition in [19] as shown in Remark 3 in [14].
where the inequalities follow form: (a) \(|h_c| + \sqrt{|h_d|^2 + |h_i|^2} \geq 1\), then \(|h_c| + |h_d| + |h_i| \geq 1\) and 1 + \(|h_c| + |h_d| + |h_i|^2 \leq 2(|h_c| + |h_d| + |h_i|^2)^2\), and (b) \(|h_d| + |h_i| \leq 2 \max\{|h_d|, |h_i|\}\) while \(|h_d|^2 + |h_i|^2 \geq \max\{|h_d|, |h_i|\}\), we see (setting \(m = \max\{|h_d|, |h_i|\}\) that

\[
\frac{|h_c| + |h_d| + |h_i|}{|h_c| + \sqrt{|h_d|^2 + |h_i|^2}} \leq \frac{|h_c| + 2m}{|h_c| + m} \leq 2\frac{|h_c| + m}{|h_c| + m} = 2.
\]

This completes the proof.

So far we extended the LDA red region of Fig. 4 to the Gaussian noise case. At present, we do not have results for the LDA green region or for the asymmetric setting. The LDA yellow region of Fig. 7 will be discussed next.

We next show that Theorem 3 which was inspired by Theorem 4 can be extended to any number of users.

B. Gaussian K-user Cognitive Interference Channel

In this section, we present more general results for the K-user cognitive interference channels for \(K \geq 3\) users. Very few results exist in general for such channels. We first recall our previous result from [13] which states that a MIMO broadcast Dirty Paper Coding scheme [26] but with one encoding order (due to the cumulative message sharing) is sum-capacity optimal to within a constant gap for the symmetric Gaussian channel. With a particular choice of power splits (in an attempt to match the upper bound), the message knowledge sufficient to obtain the outer bound is that \((K - 1)\)-CIFC-PMS and one global cognitive user. Such a scheme achieves the sum-capacity of the K-CIFC-CMS to within the following gap, despite the reduced message knowledge.

Theorem 6 ([13, Theorem 4]). The sum-capacity upper bound in (5) is achievable for the symmetric Gaussian K-CIFC-CMS to within 6 bits for \(K = 3\) and to within \((K - 2)\log(2) + 3.88\) bits for \(K \geq 4\).

We now show that under specific channel gain conditions, an achievability scheme with message knowledge corresponding to that of a K-CIFC-CoMS and a \((K - 1)\)-IFC+CR achieves the K-CIFC-CMS outer bound (to within a constant gap). Both these results show that more message knowledge is not necessarily needed for higher gDoF (i.e., can only improve the gap). Before going into the details of the derivation we have the following remark regarding the particular structure of the channel gain condition considered in this section.

The regime highlighted in yellow in Fig. 4 in the LDA is characterized by \(\alpha \leq \beta \leq 1\), which we try to match with a channel gain relationship of the form \(|h_1|^2 \leq |h_c|^2 \leq |h_d|^2\) for the Gaussian 3-CIFC-CMS. In particular, whenever a cognitive Tx transmits the same bits as those transmitted by the primary transmitter (see for example Example 1 in Fig. 5 where \(X_3\) is a combination of transmitted bits from \(T_{x_1}\) and \(T_{x_2}\)), we match this to a zero forcing scheme for the Gaussian noise channel. Moreover, if a cognitive transmitter sneaks in bits below the noise level (see for example Example 2 in Fig. 6 where yellow bits appeared only at \(R_{x_3}\)), we scale the Gaussian transmit signal by the interfering channel gain such that when reaches the non-intended receiver, it is received below the noise level. In the following, we consider an equivalent K-user extension of the three user network and our main result is summarized in the next theorem.

Theorem 7. When the following channel conditions hold

\[|h_{KK}| \leq |h_c|^2, \quad (K - 1)|h_i|^2 \leq |h_c|^2 \leq |h_d|^2, \quad (24)\]
a scheme with message knowledge structure corresponding to that of a \((K - 1)\)-IFC+CR achieves the sum-capacity outer bound of a K-CIFC-CMS to within

\[
gap \leq \log \left( \frac{(2\sqrt{K-1} + K - 2)^2}{(\sqrt{K-1} - 1)^2} \right) + (K - 1) \log(2),
\]

while when the following channel conditions hold

\[|h_{KK}| > |h_c|^2, \quad (K - 1)|h_i|^2 \leq |h_c|^2 \leq |h_d|^2, \quad (25)\]
a scheme with message knowledge structure corresponding to that of a K-CIFC-CoMS achieves the sum-capacity outer bound of a K-CIFC-CMS to within

\[
gap \leq \log \left( \frac{(K^2 - 2) (2\sqrt{K-1} + K - 2)^2}{(\sqrt{K-1} - 1)^2} \right) + (K - 2) \log(2).
\]

Proof: The sum-capacity outer bound in (5) for the channel model described in (16) evaluated over jointly Gaussian inputs is given by

\[
\sum_{k=1}^{K} R_k \leq \log \left( 1 + (|h_d| + (K - 2)|h_i| + |h_c|)^2 \right)
\]

\[+ (K - 2) \log \left( 1 + \frac{|h_d| - |h_i|^2}{2} \right) + (K - 2) \log(2)
\]

\[+ \log \left( 1 + \frac{|h_{KK}|^2}{1 + (K - 1)|h_c|^2} \right). \quad (26)
\]

For achievability, consider the following transmit signals

\[X_i = \alpha_i T_{izf} + \gamma_i T_{ip}, \quad i \in [1 : K - 1], \quad (27a)\]

\[X_K = -\beta_K \sum_{i=1}^{K-1} T_{izf} + \gamma_K T_{KP}, \quad (27b)\]

where \(T_{izf}\) and \(T_{ip}\) are mutually independent \(N(0, 1)\), for all \(i \in [1 : K]\), and where, for the average power constraint to be satisfy, we require

\[|\alpha_i|^2 + |\gamma_i|^2 \leq 1, \quad i \in [1 : K - 1] \quad (28a)\]

\[|\gamma_K|^2 + (K - 1)|\beta_K|^2 \leq 1 \quad i \in [1 : K - 1]. \quad (28b)\]

For the case in (4) we set

\[\gamma_i = 0, \quad i \in [1 : K], \quad (29a)\]

\[\alpha_i = \beta_K = \frac{h_i}{h_c}. \quad (29b)\]
With this choice of power splits and after lower bounding the achievable rate $R_1$ using the condition in (24), we have that the following rates are achievable

$$R_1 = \log \left( 1 + \left( 1 - \frac{1}{\sqrt{K}} \right)^2 |h_1|^2 \right)$$  \hspace{1cm} (30)$$
$$R_i = \log \left( 1 + \left| |h_{al}|-h_i|^2 \right| \right) \quad i \in [2 : K - 1].$$  \hspace{1cm} (31)

The sum-capacity outer bound in (26) can be upper bounded so that user $i$ may not be worth the overhead as (approximately, or to within definition, or having a cumulative message knowledge structure, terms of capacity for different cognitive networks that differ having only one "globally cognitive" user whose role is to manage all the interference in the network. Whether this is true for asymmetric scenarios and for the capacity region rather than sum-capacity is an open question.

For multi-user cognitive networks with more than three users, many problems are still open including: characterization of the sum-capacity for asymmetric scenarios for both the linear deterministic and Gaussian networks, characterization of parameter regimes for the asymmetric Gaussian and linear deterministic Gaussian networks under which the observations made for the symmetric networks hold, and characterization of the DoF and gDoF of MIMO multi-user cognitive interference channels.

**APPENDIX A**

**PROOF OF TH. 3**

When $\alpha = 1$, the sum-capacity in (14b) and (13c) are the same; therefore, we focus on the case when $\alpha \neq 1$, we find the channel gain conditions that satisfy the following condition: (14a) - (15b) $\leq 0$. We consider the six different relative orderings of $(1, \alpha, \beta)$, evaluating the bounds in (14) and seeing under what conditions these are no larger than those in (15). For example if $\alpha > \beta > 1$, then (14a) becomes $\alpha + \beta$, (15a) becomes $\alpha + \beta$ and (15b) becomes $2 \alpha + 2 \beta$. Hence, for this entire regime the CIFC-CMS outer bound is equal to the ICF+CR outer bound, which we know from (17) is achievable. Similar arguments hold for the orderings $\alpha > 1 > \beta$, $\beta > \alpha > 1$, $1 > \beta > \alpha$ and $1 > \alpha > \beta$. In the regime $1 > \alpha > \beta$ \begin{align*}
(14a) &= 2 + \beta - \alpha \\
(15a) &= 2 + \beta - \alpha \\
(15b) &= 2 \max(1 - \alpha, \alpha) + 2 \beta.
\end{align*}

The conditions under which $2 + \beta - \alpha \leq 2 \max(1 - \alpha, \alpha) + 2 \beta$ may be simplified to $2 \leq 3 \alpha + \beta$, which is drawn in blue in Fig. 3. In this regime, the outer bounds for the CIFC-CMS and the IFC+CR coincide. However, it is not known whether these bounds are achievable under either scheme in general.

**APPENDIX B**

**PROOF OF (20)**

Let $\epsilon_N > 0 : \epsilon_N \to 0$ as $N \to +\infty$. We have

$$N \sum_{j=1}^{K} (R_j - \epsilon_N) \leq \sum_{j=1}^{K} I(W_j; Y_j^{N})$$  \hspace{1cm} (a)$$
$$\leq \sum_{j=1}^{K} I(W_j; Y_j^{N}|W_{[1:j-1]}) \leq \sum_{j=1}^{K} I(X_j^{N}; Y_j^{N}|X_{[1:j-1]})$$  \hspace{1cm} (b)$$
$$= \sum_{j=1}^{K-1} I(X_j^{N}; Y_j^{N}|X_{[1:j-1]}) + I(X_{K}^{N}; Y_{K}^{N}|X_{[1:K-1]})$$  \hspace{1cm} (c)$$
$$\leq \sum_{j=1}^{K-1} I(Y_j^{N}|X_{[1:j-1]}) + I(X_{K}^{N}; Y_{K-1}^{N}|X_{[1:K-1]})$$  \hspace{1cm} (d)$$
$$= \sum_{j=1}^{K-2} I(Y_j^{N}|X_{[1:j-1]}) + I(X_{K-1;K}^{N}; Y_{K-1}^{N}|X_{[1:K-2]})$$  \hspace{1cm} (e)$$

VII. CONCLUSION

In this paper, we surveyed and studied the relationship in terms of capacity for different cognitive networks that differ by the amount of cognition at the transmitters.

One general trend that emerged is that "distributed cognition," or having a cumulative message knowledge structure, may not be worth the overhead as (approximately, or to within a bounded gap) the same sum-capacity can be achieved by having only one "globally cognitive" user whose role is to
where: (a) follows from Fano’s inequalities $H(W_j | Y_{-j}) \leq N \epsilon_j, \forall j \in [1 : K]$; (b) from the independence of messages, (c) by definition of encoding functions (for all $M_j \subseteq [1 : j], j \in [1 : K]$) and by the data processing inequality, (d), (e), (f), (g) and (h) from the condition in \((19)\) for $j = K, j = K - 1, \ldots, j = 2$ and \((14)\) Lemma 2), (h) from the chain rule of entropy, conditioning reduces entropy and memoryless property of the channel, (i) by introducing a time-sharing random variable $Q$ independent and uniformly distributed on $[1 : N]$, (j) by conditioning reduces entropy and since $Q$ is independent of the channel.

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