Modeling of dielectric elastomer oscillators for soft biomimetic applications

E-F M Henke\textsuperscript{1,2,\dag}, Katherine E Wilson\textsuperscript{\ddag} and I A Anderson\textsuperscript{3,4,\dag}

1 Institute of Solid State Electronics, TU Dresden, 01062 Dresden, Germany
2 Biomechanics Lab, Auckland Bioengineering Institute, The University of Auckland, 70 Symonds Street, Auckland 1010, New Zealand
3 Institute of Semiconductors and Microsystems, TU Dresden, 01062 Dresden, Germany
4 Department of Engineering Science, The University of Auckland, Auckland, New Zealand

E-mail: m.henke@auckland.ac.nz and i.anderson@auckland.ac.nz

Keywords: dielectric elastomers, artificial muscles, artificial central pattern generator, soft robotics

Abstract

Biomimetic, entirely soft robots with animal-like behavior and integrated artificial nervous systems will open up totally new perspectives and applications. However, until now, most presented studies on soft robots were limited to only partly soft designs, since all solutions at least needed conventional, stiff electronics to sense, process signals and activate actuators. We present a novel approach for a set up and the experimental validation of an artificial pace maker that is able to drive basic robotic structures and act as artificial central pattern generator. The structure is based on multi-functional dielectric elastomers (DEs). DE actuators, DE switches and DE resistors are combined to create complex DE oscillators (DEOs). Supplied with only one external DC voltage, the DEO autonomously generates oscillating signals that can be used to clock a robotic structure, control the cyclic motion of artificial muscles in bionic robots or make a whole robotic structure move. We present the basic functionality, derive a mathematical model for predicting the generated signal waveform and verify the model experimentally.

1. Introduction

You never have to think about digestion. The enteric nervous system’s continuous ganglionicated nerves located within the wall of the gut respond to local muscle strain, switching living muscles on or off so that food is pushed forward along the digestive track; without control from the central nervous system [1]. The approach of distributed control is present in most living systems and relieves central control units of the task of controlling continuously recurring processes and reflexes. This architecture is efficient and reduces complexity in communication between the central control and distant fundamental processes. In common robotics the design still focuses on centralized control architectures, relaying on powerful controllers and complicated communication between sensors, actuators and controllers. Peristalsis in the gut as well as other regular rhythmic motions such as walking, swimming, flying, and respiration are locally generated by autonomous neural ganglia known as central pattern generators (CPGs) [2].

We present a concept and a mathematical model of an entirely soft, autonomously running signal generator suitable to drive artificial muscles for soft, biomimetic robots—an artificial central pattern generator (aCPG). So-called dielectric elastomer oscillators are a contribution to the rapidly growing soft robotics tool-kit. Dielectric elastomer oscillators (DEOs) enable distributed, autonomous signal generation, that can be controlled in a wide range by external signals or mechanical stimuli. Figure 1 points out a possible architecture for soft robots possessing distributed soft control.

Presently, the vast majority of soft robots use pneumatic [3–5] or hydraulic [6] actuators. They can be made to operate in multi-degree of freedom motion by using multiple chambers. A four-legged crawling robot by Shepherd et al [5] can move by separate pneumatic actuation of each leg. In another instance an octopus-arm actuator can be steered using tendon-like cords [7]. The bulk of their active elements are low-modulus, compliant materials like silicone or natural rubber, possessing Young’s moduli of the order of
$10^5$ Pa; very low compared with conventional robot motors, gears and fixtures built from materials with moduli in the range $10^9$–$10^{12}$ Pa [8, 9]. But whether hard or soft, they require external power and control units that are typically composed of dense and stiff materials. Thus, soft actuators can never be entirely soft in a completed assembly of actuator and controller. The integration of sensory functionalities into soft structures is even more challenging. Although there are currently promising developments in flexible and stretchable electronics [10], most conventional sensory technologies are not suited for integration into soft structures. To emulate the example of the gut, a technology is needed that is soft and fully autonomous.

Within the range of smart materials, there is one class with mechanical characteristics very similar to biological muscles [11]: the dielectric elastomer (DE) transducer. Its mechanism is electro-mechanic and this offers the opportunity for rapid and fully integrated control of the actuation behavior.

Dielectric elastomers can simultaneously act as actuators with large actuation [11, 12]. They can be made to match and exceed the performance of natural muscles, earning the moniker of artificial muscles [13]. Besides that, they can perform as sensors for large strain [14–16] and as power generators [17]. Some of these functions can be utilized simultaneously, enabling multi-functional DEs [18, 19].

The muscle actuation of the gut, cited above, provides a clue on how to avoid bulky and heavy external control units in a soft actuator: by using strain itself as the control mechanism. Beruto et al [20] give an exploration how to implement that control mechanism into soft robots. Silicon-graphite mixtures show piezoresistive behavior with large changes in piezoresistivity—order of five orders of magnitude around the percolation threshold—and thus, can be used for strain and pressure sensing [21].

A similar effect can be achieved by a dielectric elastomer switch (DES); a piezoresistive membrane area that is mechanically pre-stretched above its percolation threshold and combined with a dielectric elastomer actuator (DEA). The conductivity of the switch can then be changed by several orders of magnitude upon a voltage-induced elongation of the DEA, compressing the switch and changing its resistance. DEAs and DESs can be assembled into strain-dependent electrical signal inverters. When an odd number of these inverters is assembled in a closed loop cycle, one obtains a soft electro-mechanical oscillator that is entirely and uniquely controlled by elastomer strain [22]; without any conventional electronic components being used. Those DEOs can be used to autonomously drive robots. Figure 2 depicts a first untethered robot driven by a DEO, only requiring a single DC voltage—Trevor the Caterpillar [23].

Although DEOs have been presented in several applications [22–24], there is little theoretical background about the functionality of DEOs. We present a model that is able to simulate the functionality of DEOs based on their individual electro-mechanical components. Every component is analyzed and transformed into a Simulink model that finally makes up a full, functional DEO model. A DEO model is necessary to analyze and fully understand the electromechanical processes within the individual components and to design specific DEOs adapted to a certain application.

2. Functionality of dielectric elastomer oscillators—DEOs

It has been shown that DE devices are able to represent all basic boolean gates [25] and basic memory [26],

Figure 1. Fully soft robotic architecture inspired by the caterpillar, possessing distributed soft sensor, actuator and signal processing nodes throughout its entire structure, rather than controlling its functionality by a centralized control unit.

Figure 2. Trevor the Caterpillar—a autonomous biomimetic crawling robot using DEOs for signal generation and crawling motion. Reprinted with permission from SOFT ROBOTICS 4/4, 2017, pp 533–66, published by Mary Ann Liebert, Inc., New Rochelle, NY [23].

6 Mechanically stretching the membrane above a critical stretch value at which there are no conductive paths anymore.
without the need of conventional semiconductor-based electronics. Using those logic gates in a closed loop enables spontaneous, self-primed oscillators that can run cyclic processes, such as crawling or wing flapping, or simply transform DC high voltage into AC high voltage signals. To explain this signal generation, it is necessary to identify and characterize all individual electro-mechanical dielectric components of DEOs.

2.1. Set-up of a DEO

A DEO consists of three or more odd numbered DE inverters in a closed loop that spontaneously start an electro-mechanical oscillation upon a certain value of DC input voltage [22, 23]. This oscillation generates 120° phase-shifted voltage signals of exponential shape. Each inverter consists of one DEA of width \( w_{DEA} \) and length \( l_{DEA} \) and one DES. If there is a high voltage applied to the input of one DEA, it elongates and compresses the DES, coupled to it. This compression yields a drop of the DES's resistance \( R_{DES} \) and the output voltage drops simultaneously (for a detailed description see [23]). All individual parts of the DEO can be represented and modeled by electrical, or electro-mechanical elements. DEAs can be considered variable capacitors with leakage, parallel, voltage-dependent resistors; DESs as highly variable mechanosensitive resistors and \( R_S \) as standard resistors. An equivalent electrical network is depicted in figures 4(A) and (B). (A) depicts a simplified model, where the DESs are represented as simple inverters coupled to the remaining circuitry. This first model did not sufficiently represent the DEO’s behavior and failed in predicting the experimentally measured signal waveforms.

An improved DEO model circuit is shown in figure 4(B). In this representation the DESs are modeled by highly non-linear voltage-dependent resistors. Depending on the actual value of the individually generated voltage signals \( V_i \), the DEAs are actuated to a certain amount of actuation \( u_i \). Depending on \( u_i \), the voltage \( V_i \) indirectly controls the resistance \( R_i \) of the attached DES via a characteristic function: 

\[
R_{DES \ i} = R_{DES \ i}(u_i(V_i))
\]

The functional chain of an entire DEO is shown in figure 4(C). Each inverter \( i \) receives the generated voltage value \( V_{i-1} \) of the inverter before. This generates an actuation \( u_i \), which controls the resistor value \( R_i \) (representing \( R_{DES \ i} = R_{DES \ i}(u_i(V_i)) \) here) and, thus, the individual output voltage \( V_i \) that is passed to the next inverter stage \( i + 1 \).

2.2. Materials and methods

All DEOs described here have been assembled from equi-biaxially pre-stretched VHB 4905 double-sided tape. First, the tape was stretched in a self-made iris-like stretcher to an initial pre-stretch of \( \lambda_{pre} = 1.75 \) and the piezoresistive DES tracks were imprinted using a silicone stamp and a 5:1 mixture of noconductive
grease (Molykote 44 Medium) and Cabot Vulcan XC72 carbon black. In a second step, the membrane was stretched to the final pre-stretch of $\lambda_{\text{pre}} = 3.5$ and the remaining DE structures were applied using carbon grease (Nye’s Nyogel 756G). Afterwards, the DEO membrane was stuck to a laser-cut PMMA frame. The detailed procedure can be retraced in [27, 28].

In the here presented experiments, the serial resistors $R_{S}$ have been of constant values and were changed in every experiment physically. A future more integrated layout of DEOs will possess variable $R_{S}$, represented by mechanosensitive DESs themselves.

3. A general model for DEOs

As figure 4 shows, a full DEO possesses nine components that have to be represented in a model: three DEAs, three DESs and three $R_{S}$. All elements are electrically connected as depicted in the network in figure 4. The charging resistors $R_{S}$ are considered to be constant, all other components vary their electrical characteristics upon the voltage $V_{i}$ applied to them and influence each other by mechanical constraints. The most important and simultaneously most complex parts of a DEO are its three DES, which are described below.

3.1. Electromechanical model of a DES

DESs possess a complex, electro-mechanical behavior, transforming an electrical input voltage $V_{\text{in}}$ into a DES resistor value $R_{\text{DES}}$. It is governed by two processes: the hyper-viscoelastic, voltage-induced actuation $u(V_{\text{in}}(t))$ of the DEA and the highly nonlinear resistor change $R_{\text{DES}}(u(V_{\text{in}}(t)))$ due to piezoresistivity. The model for DESs (figure 5) was derived from measurement results. For several inverters—assembled from a DEA connected to a DES—the step-responses in actuation $u(t)$ and resistance $R_{\text{DES}}(t)$ was recorded. In our model $u(t)$ is the actuation of a DEA in $x_{1}$-direction. Experimental values $u(t)$ were measured using a high accuracy laser displacement sensor (Keyence LC-2400). Since the actuation happens in plane, a small PMMA piece was glued on top of the DEA membrane, just before the DES, to reflect the laser beam and measure the actuation. Based on those measurements the two building blocks of the inverter model were derived: a generalized Kelvin model to represent the actuation $u(V(t))$ and a characteristic resistance function $R_{\text{DES}}(u)$. A combination of both blocks resulted in a model chain to represent $R_{\text{DES}}(V_{\text{in}}, t)$. In figures 5(A) and (B) an inverter is shown in its most extreme states: In (A) there is no input voltage $V_{\text{in}}$ thereby $R_{\text{DES}}$ is high and the output voltage $V_{\text{out}}$ is high; in (B) $V_{\text{in}}$ is high, thus $R_{\text{DES}}$ is low and $V_{\text{out}}$ is low either. To secure a reliable switching, the inverter frame possess a so called strain amplifier [24], a special edge design in $x_{1}$-direction. This structure limits the membrane length at the DES’s position to minimum required length, while leaving a larger membrane area

![Figure 4. Equivalent network of a DEO depicted in 3, with DEA capacitors $C_{i}$, variable DESs resistors $R_{i}$ and serial resistors $R_{S}$, generating three oscillating voltage signals $V_{i}$ from one single input voltage $V_{\text{in}}$. (A) Simplified circuit—DEs represented by an inverter module. (B) More realistic DEO model, representing the DES as voltage-controlled variable resistors $R_{i}$. (C) Functional chain of the entire DEO.](image)
besides the DES. The longer membrane areas besides
the DES enable an actuation mainly in direction \(x_1\)
while there is little actuation in direction \(x_2\).

Figure 5(C) shows typical transition functions \(R_{DES}(V_{in})\) versus the input voltage. As can be clearly
seen, the individual transitions differ in their characteristics. This is mainly because of the manual produc-
tion of all components and one of the main causes of
the model deviating from experiments. We used the
Simulink model depicted in figure 5(E) to represent the
viscoelastic actuation behavior of the DEAs driving the
DESs. The system function \(u(t)\), represented by a system
transfer function block \(\frac{num(s)}{den(s)}\) (figure 5(E)) was derived
from an example inverter experiment using a rectangu-
lar signal of \(V_{in} = 3000\) V, a period of \(T = 10\) min and
the duty cycle of \(D = 0.5\). \(u(t)\) was derived from the
measured actuation responses by a MATLAB script,
resulting in the generalized 5th order viscoelastic model:

\[
u(t) = a_0 + \sum_{i=1}^{5} a_i \left(1 - e^{-t/b_i}\right).
\]

(1)

The necessary parameters \(a_i\) and \(b_i\) are listed in table 1.

Table 1. Parameters describing the actuation of an inverter according to equations (1) and (3).

|   | 1   | 2   | 3   | 4   | 5   |
|---|-----|-----|-----|-----|-----|
| \(a_i\) in mm | 0.75 | 0.5416 | 0.2307 | 0.1691 | 0.5038 | 0.4094 |
| \(b_i\) in \(s\) | 0    | 0.0041 | 0.0330 | 0.1436 | 0.8449 | 6.6667 |
| \(N_i\) | \(9.72 \times 10^{14}\) | \(1.16 \times 10^{15}\) | \(1.45 \times 10^{15}\) | \(2.52 \times 10^{14}\) | \(8.43 \times 10^{12}\) | \(3.67 \times 10^{10}\) |
| \(D_i\) | \(1.15 \times 10^{11}\) | \(8.83 \times 10^{11}\) | \(8.03 \times 10^{11}\) | \(1.22 \times 10^{11}\) | \(5.43 \times 10^{10}\) | \(1.25 \times 10^{10}\) |

For an easier implementation into the model, the
transfer function was identified by applying the
access transfer function data function tfdata(U). This
function returns numerator and denominator of the
system’s transfer function:

\[
U(s) = \frac{1}{E_0} + \sum_{i=1}^{5} \frac{1}{E_i + \mu_i s}, \quad E_i = \frac{V_{in}}{a_i}, \quad \mu_i = \frac{E_i}{b_i}, \quad \forall i.
\]

(2)

Table 1 lists the values \(N_i\) and \(D_i\). The model suffi-
ciently represents example experiments. Figure 6
compares experiment and corresponding simulation.
As can be seen, the model follows the viscoelastic
actuation measured in the experiment. After applying
the voltage step \(V_{in}\) the DEA actuates. There is an
immediate actuation, followed by a decreasing
actuation speed. This step-response follows a typical
viscoelastic characteristic. During the next cycles, the
DEA does not relax to its initial length and reaches
higher maximum actuation each time a voltage step
occurs. This behavior represents the long term over-
all drift of the highly viscoelastic membrane material
and settles after several minutes. This drift most
likely explains the measured signal drift of DEOs shown in figure 6(A). If the DEA is allowed to rest for more than 5 min it returns to its initial shape. As depicted in figure 5(E) the calculated actuation $u$ is transferred into a look-up table block, calculating the DES’s resistor values $R_{DES}(u)$, upon corresponding actuation values $u$. The data stored in this look-up table was derived from experiments, ran with the same DEA as above, connected to a DES.

To validate an acceptable model accuracy of the entire DES transition, several experiments, using different signal waveforms, were conducted. Figure 7 depicts an example experiment, applying a triangular voltage signal of $V_{in}$, depicted versus time $t$ in (B). The data stored in this look-up table was derived from experiments, ran with the same DEA as above, connected to a DES.

Regarding the final DEO model, the transfer function $R_{DES}(V_{in}(t))$ dominates the system response. This function defines which input voltage level is necessary over what time period to switch a DES to its conductive state and, hence, defines the over-all oscillation characteristic and signal waveforms. Figures 7(D) and (E) show the characteristics for $u(V_{in}(t))$ and $R_{DES}(V_{in}(t))$, for the model and the experiment. Both charts display an acceptable concordance between measurement and simulation. However, they also show that there is still a deviation between simulation and experiment, pointing to the difficulty of modeling the complex, electro-mechanical behavior of DEOs, using highly viscoelastic materials, such as VHB. A future application of silicone membranes possessing less viscosity [29] and reliable production technologies will enable more accurate models.

### 3.2. Model of individual inverter transitions

Modeling an entire DEO does not just require an accurate model for the individual DEOs, it is also important to understand the charging and discharging behavior of the individual electrical components. Therefore, it is necessary to describe and understand how the individual inverters within the oscillator influence each other and how to represent their behavior mathematically. Figure 8(A) depicts a model network for a single transition between two inverters of a DEO represented in figure 8(B).

Basically, the network can be described as resistor–capacitor network with variable time constants for the charging and discharging cycle. The discharging process via the DES at the output of the inverter, represented by $R_{DES}(V_{i−1})$, is triggered by the voltage $V_{i−1}$ at the input of the inverter before. The most basic model of a DEO inverter transition is made under the assumption that there are two possible stages during a full signal period: a charging and a discharging process, via $R_S$, or $R_{DES}(V_{i−1})$, respectively. During the charging cycle, the voltage $V_{i−1}$ is low, the DEA is not actuated and $R_{DES}(V_{i−1})$ is not conducting. The current driven by the supply voltage $V_{in}$ charges the capacitor $C_{i+1}$ leading to a rise of the voltage $V_i$. If there is a suita-
ble voltage level $V_{i-1}$ at the input side of $C_i$, DEA acts, compresses DES, and $R_{DES_i}(V_{i-1})$ reaches a low resistance and starts conducting current. This state is mainly governed by the discharging of $C_{i+1}$ via $R_{DES_i}$. In parallel, there is a leakage resistor $R_{mem_i+1}$, representing the current leaking through the membrane.

This model is too simple to adequately describe a whole DEO, but enables a basic understanding of the functionality of the full electro-mechanical network. Supplying an input voltage $V_{in}$ to this network (figure 8(A)) yields the algebra-differential equation set:

$$V_{in} = R_S i + V_i,$$

$$i = C_{i+1} \frac{dV_i}{dt} + \frac{V_i}{R_{mem_i+1}},$$

$$\frac{dV_i}{dt} = \frac{V_{in}}{\tau_S} - V_i \left( \frac{1}{\tau_{mem}} + \frac{1}{\tau_{DES}} + \frac{1}{\tau_i} \right),$$

with $\tau_S = R_S C_{i+1}$, $\tau_{mem} = R_{mem_i+1} C_{i+1}$, $\tau_{DES} = R_{DES} C_{i+1}$. Transforming the above equation into Laplace domain enables the creation of a Simulink model for an individual inverter transition (figure 9).

![Figure 7. Viscoelastic actuation of a DEA, coupled to a DES, representing an individual DES transition—comparison of experiment (solid lines) and simulation (dashed lines): (A)—DEA actuation $u(t)$ response versus time $t$ to the triangular voltage course depicted in (C). (B)—DES resistance $R_{DES}(t)$ response versus time $t$ to the triangular voltage course depicted in (C). (C)—applied voltage $V_{in}$ versus time $t$. (D)—Actuation response $u(V_{in})$ versus the applied input voltage $V_{in}$. (E)—Resistance response $R_{DES}(V_{in})$ versus applied input voltage $V_{in}$.](image-url)
The time constant $\tau_{\mathrm{DES}}$ highly depends on the resistance of the DES. The capacitor $C_{i+1}$ is either charging or discharging, depending on the ratio between $\frac{1}{R}$ and $k = \frac{1}{\tau_S} + \frac{1}{\tau_{\mathrm{DES}}} + \frac{1}{\tau_{\mathrm{mem}}}$.

The actual resistance of the DESs can be modeled using the DE inverter model, described in section 3.1 (figure 5). Combining both Simulink networks and putting them in a closed loop yields a first-order model of an entire DEO. A single stage of such a closed loop is depicted in figure 10. It represents the marked DE components highlighted in figure 8(B). In this model, the feedback gain $k = \frac{1}{R}$ is replaced by the DES network (figure 5(E)); it is represented by the math function block $R_{\mathrm{DES}}$. This math function calculates the actual value of $R_{\mathrm{DES}}$ depending on the course of $V_S(t)$. A DEO modeled with the above Simulink network would work and start a spontaneous oscillation. However, due to the simplified approach, the results do not match reality close enough. In order to improve the model, we take variable electric components into account in section 3.3.

3.3. Model of varying electric components

In reality most of the individual components of DEOs have voltage-dependent values, which have to be represented in a realistic model. The electric network from figure 8(A) possesses four electric components: the serial charging resistor $R_S$ is considered to be constant at all time, the DES’s resistor $R_{\mathrm{DES}}$ whose voltage dependence was already discussed in section 3.1, a voltage-dependent leakage resistor through the membrane $R_{\mathrm{mem}}$ [30] and the voltage-dependent capacitor $C$ of the DEA. In addition there is another parallel resistor $R_{\mathrm{meas}}$ that represents the measurement equipment, what was neglected so far. Although high resistance voltage dividers with $R_{\mathrm{meas}} = 5005 \, \Omega$ were used to reduce the high-voltage signals to low voltages, there still is a large influence on the DEOs, since its resistor values can reach similar values. In total, there are five components, from which one can be modeled as constant ($R_S$) and four depend on the actual voltage. All components reside in the feedback gain:

$$k = \frac{1}{R_S C} + \frac{1}{R_{\mathrm{DES}} C} + \frac{1}{R_{\mathrm{mem}} C} + \frac{1}{R_{\mathrm{meas}} C}, \quad (7)$$

and can be represented as variable time constants $\tau_i$. All individual time constants are modeled by individual Simulink subsystems and will be described in the following.

3.3.1. Voltage-dependent DEA capacitors

Actuation of DEAs always affect their physical shape and, thus, their capacitance. A voltage induced elongation changes the electrode’s area and the membrane’s thickness. For simplicity reasons we assume that the DEA’s area change is mainly caused by an elongation $u_i$ in length direction: $A = w_0(l_0 + u_i)$. This assumption is valid for DEOs where $l < w$ and the actuation $u$ has a larger effect onto $l$ then on $w$ (see figure 5(B)). Furthermore, there is less free membrane pulling in $w$-direction, since the frame edges are closer than in $l$-direction. The so-called strain amplifier [24] increases the actuation in $l$-direction (figure 5(A)). The generated stretch is $\lambda_I = \frac{l}{l_0}$. Due to the material’s
incompressibility, the volume has to stay constant: 
\[ \lambda_1 \lambda_2 \lambda_3 = 1 \] leading to 
\[ \lambda_2 = 1, \lambda_3 = \frac{l_0 + u_i}{d_0 + \lambda_3} \] [31]. Considering a DEA being an ideal plate capacitor, the 
capacity \( C_i(u_i) \) yields:

\[ C_i(u_i) = \varepsilon_0 \varepsilon_r \frac{(l_0 + u_i) w_0}{d_0 \lambda_3}. \] (9)

This equation is implemented in a subsystem in Simulink. The necessary actual value \( u_i \) is calculated 
by the DES subsystem (figure 5(E)) and transferred to 
the capacitor subsystem, depicted in figure 11. Based
on the initial geometry \( w_0 \) and \( l_0 \) of the DEA and the 
elongation \( u_i \) it calculates the actual capacity \( C_i(u_i) \) 
for each simulation step. This value is then used to 
calculate the time constants \( \tau_i \) in equation (8).

3.3.2. Voltage-dependent leakage current

Conductivity in DE membranes is a complex 
process. It depends on several material properties, 
such as material, type of electrodes and temperature.
However, dielectric elastomer membranes always 
possess a certain amount of current leaking through 
them [32]. This leakage is voltage-dependent and can 
be estimated as mathematical function [30]. We again 
consider the area change being only dependent on 
\( I = l_0 + u_i \) and get:

\[ i = \sigma_0 E e^{x \tau_i A}, \] (10)

\[ i = \sigma_0 w_0 \frac{V_i}{d_0 \lambda_3} e^{V_i/d_0 \lambda_3}, \] (11)

with \( \sigma_0 \) being the initial conductivity, \( E_b \) an empirically 
derived constant with electrical field dimension, \( E \) 
the actual electric field and \( A = w l \) the area of the

DEA. Recalling equation (8), we find that the model 
has to calculate the term \( \frac{1}{\tau_{\text{mem}} + \frac{1}{\tau_{\text{DES}}}} \). A combination with 
equation (11) yields:

\[ \frac{1}{\tau_{\text{mem}} i} = \frac{1}{C_i R_{\text{mem}} i}, \] (12)

\[ \frac{1}{\tau_{\text{mem}} i} = \left( C_i \sigma_0 w_0 \frac{l_0 + u_i}{d_0 \lambda_3} e^{V_i/d_0 \lambda_3} \right)^{-1}. \] (13)

Figure 12(A) depicts the constructed network, 
representing equation (13). The network requires 
three inputs, the actual output voltage \( V_{i-1} \) of the 
inverter before, the capacity \( C_i \) and the actual actuation 
\( u_i \) of DEA. The output is the calculated term \( \frac{1}{\tau_{\text{mem}} i} \) 
according to equation (13). The voltage dependent 
values of the membrane resistance \( R_{\text{mem}} \) and the 
resulting current \( i_{\text{mem}} \) leaking through a single DEA 
membrane are depicted in figure 12(B). The size of 
the example membrane is \( l_{\text{DEA}} = 60 \) mm, \( l_{\text{DEA}} = 100 \) mm. This is the maximum size of DEAs used in the 
experiments and, thus, the maximum expected 
leakage current. For a more realistic evaluation of 
the individual leakage currents, all three \( i_{\text{mem}} \) are 
depicted in figure 12(C) during a typical oscillation. 
The maximum leakage current values are typically in 
a range of several \( \mu A \) per membrane. At the measured 
voltage levels this current does not limit the operation 
of the DEO. The currents drawn through the serial 
resistors \( R_S \) and the DESs at their minimum resistance 
are typically one order of magnitude higher. However, 
as can be seen in figure 12(B), the membrane resistance 
reaches values below the DES’s high resistances values 
at higher voltages. That limits the maximum output 
voltage of the individual inverter stages.
3.4. A full model of a DEO

Recalling sections 3.1 through 3.3, it becomes clear that most active parts within the DEO influence each other by a certain amount. Therefore, we divided the entire model into several subsystems. In this section we now give an overview how to assemble the mentioned subsystems to a full functional DEO model. For a better overview we divided the DEO into three stages as depicted in figure 4. Connecting three of those stages in a closed loop (figure 13) results in a functional Simulink model that behaves like a physical DEO. Every stage is supplied with the constant input voltage $V_{in}$ and calculates its output voltage $V_i$. This signal is transmitted to the next stage. In there, the DES subsystem calculates the actuation $u_i$ which is needed in stage $i$ to calculate the value of the DEA capacitor and the membrane’s resistance. The loop is closed by voltage and actuation feedback loops $V_3$ and $u_1$. The three generated oscillating voltages $V_i$ can be tapped at the outputs of the individual subsystems.

Within the subsystems of figure 13 all electric and electro-mechanical components are represented by the before-described models. Figure 14 depicts the internal network for such a stage. This concerns all the components $R_S$, DES 1, $C_2$ and $R_{mem}$, depicted in figure 4(B) and the measurement resistor $R_{mem}$ in parallel. The network continuously solves the differential equation:

$$\frac{dV_i}{dt} = \frac{V_{in}}{\tau_S} - V_i \left( \frac{1}{\tau_S + \frac{1}{\tau_{DES}}} + \frac{1}{\tau_{mem}} + \frac{1}{\tau_{meas}} \right).$$

(14)

The input signals from the neighboring stages are supplied to the corresponding subsystems. The resulting time constants are summed up and used to calculate the second term of equation (14).

At the current state of the art, especially due to manual manufacturing processes and related uncertainties in electrical and mechanical properties of the individual components, it is not yet possible to perfectly match all investigated DEOs. However, the presented model is a useful tool to predict the waveforms of signals generated by DEOs. Using such a simulation supports the design process of soft dielectric elastomer robots using DEO aCPGs for driving their actuators.

4. Results and discussion

For the evaluation of our DEO model 28 DEOs with varying DEA geometries have been manufactured by hand. Each one was run with 11 different serial resistors $R_S = 33\, \text{M} \Omega$–$1000\, \text{M} \Omega$ at up to four voltages $V_{in}$ between 3250 and 4000 V. This results in a maximum number of 1232 individual experiments. Of that number not all DEOs were oscillating at each
voltage and resistor combination. Especially the lowest voltage \( V_{in} = 3250 \, \text{V} \) did not always result in a stable oscillation. Furthermore, serial resistors of \( R_S = 1000 \, \text{M} \, \Omega \) did either not result in an oscillation or did not influence the waveform compared to an oscillation at \( R_S = 800 \, \text{M} \, \Omega \). Also the model only oscillates with low amplitudes or settles in a quasi-static state at \( R_S \) values of 1000 M\( \Omega \) and above. Table A1 in appendix lists the parameters used in the Simulink model.

4.1. General signal waveform

Figure 15 compares three measured and simulated signal waveform patterns for one DEO possessing three different serial resistors. The embedded DEAs possess an electrode width of \( w_{DEA} = 53.5 \, \text{mm} \) and a length of \( l_{DEA} = 27 \, \text{mm} \). As can be seen the simulated waveforms accord to the measured signals quite well for all experiments. The signals follow the predicted charging and discharging courses and are \( 120^\circ \) phase-shifted. The minimum, simulated signal values fit very well to the experiments. The maximum simulated signal levels are slightly higher than the experimentally measured values. There are two possible causes for this. On the one hand, the model does not take the passive resistors of the conductive carbon tracks into account. This may result in higher output signal levels than measured experimentally. On the other hand, the viscoelastic response of the DEA does not represent the real response perfectly. In figure 6(A) the simulated actuation \( u_{sim} \) does not relax to the minimum measured actuation for lower input voltages after several cycles. Since already slight differences in actuation can cause large resistor changes in \( R_{DES} \), this deviation in the model can cause differences between simulation and experiment. To achieve a better agreement, the model of the viscoelastic DEA response has to be improved. However, recalling the uncertainties in material properties for VHB tapes, the applied pre-stretch and the unreliability of manual production, the Simulink model predicts the generated waveforms for the used oscillator sufficiently well. The supplementary video (DEO_video2.m2v) shows a working DEO, running at different frequencies. The depicted DEO differs from the one shown in figure 3(C). This is due to a simplification of experiments, where we used exchangeable conventional serial resistors \( R_{Si} \), to investigate the effects of variable \( R_S \).

The introduction of the varying components and the measurement resistors, described in section 3.3, was necessary, despite complicating the model. Neglecting the measurement resistors \( R_{meas} \) would result in significantly higher oscillation frequencies by reducing the total time constant of the system. The varying capacitances and the voltage-dependent leak-
age current, mainly influence the wave-forms of the simulated signals. Disregarding those processes would increase the slope of the rising signal flanks and lead to higher maximum signal levels. The flattening of the signal wave-forms for above 2000 V is mainly governed by the rising current leaking through the membrane. This process limits the maximum voltage levels that can be generated using VHB membranes for DEOs.

4.2. Influences of DEA geometry and input voltage

The main purpose for the development of the presented model is the prediction of oscillation frequencies in order to design DEOs upon certain applications. Considering the three DES transitions in the DEOs with their constant behavior, leaves the DEA geometry as relevant design parameters and the input voltage as variable model input. In order to evaluate the model behavior and predict the possible range of the oscillation frequency \(f_0\), the model was run for 28 different DEOs at three different input voltages \(V_{in} = 3500 \text{ V}, 3750 \text{ V}, 4000 \text{ V}\). The model parameters derived from the experiments of one DES (figure 6 and table 1) have been used to simulate the signal waveforms for all different DEOs. Figure 16 compares the predicted with the experimentally determined frequency values for several experiments. Figures 16(A) and (B) depict the general influence of the DEA geometry and the value of \(R_S\) on \(f_0\). As already shown in figure 15, there is a large dependency of \(f_0\) on the resistors \(R_S\). Furthermore, changes in \(w_{DEA}\) and \(l_{DEA}\) shift the \(f_0(R_S)\) curve upwards or downwards. Additionally, the applied voltage influences \(f_0\) (figure 16(C)). A higher voltage causes a higher oscillation frequency.

Figure 16(D) summarizes the results, by depicting the calculated oscillation frequency for all \(w_{DEA}-l_{DEA}\) combinations and different serial resistor values \(R_S\). Each plane represents one resistor value—from bottom to top: \(R_S = 800 \text{ M} \Omega, 70 \text{ M} \Omega, 33 \text{ M} \Omega\). Considering the calculated results within the entire design space, DEOs can be designed within a frequency range of \(f_0 = 0.06 \text{ Hz}–10.38 \text{ Hz}\).

Furthermore, the model can be used to estimate the maximum size of an DEO or the number of inverter stages that can be run by a single source. The current intake depends mainly on the currents running through the DES in their low resistance state. The leakage current through the membranes is typically more than one order of magnitude lower. In our study the source was able to supply 125 \(\mu\text{A}\) per channel at four channels, resulting in \(i_{max} = 500 \text{ \(\mu\text{A}\}}\). Using the model, we could calculate a maximum current intake of the largest used DEO of \(i_{in} = 144 \text{ \(\mu\text{A}\}}\). Our source would be able run larger DEOs.

4.3. Limitations of the model

The presented model shall be seen as a study to determine the fundamental characteristics of DEOs and how to control their waveforms and oscillation
frequencies. The uncertainty of the properties of the used materials, such as VHB and carbon grease, as well as their calculation from experiments, result in large divergences of the electro-mechanical properties and uncertainties. Furthermore, no time dependent drift in the electric properties of any of the dielectric components has been taken into account. For that reason, the model does not claim to perfectly predict quantitatively the behavior of all DEOs but shall be understood as a toolkit to understand and design DEO prototypes in point of fact.

However, the presented model behaves like typical DEOs do, represents signal waveforms qualitatively correctly and predicts frequencies sufficiently. To improve the model the influence of geometry change on the DEs itself has to be studied more detailed and the long-term drift should be introduced.

4.4. Outlook
Besides the model improvement also the DEOs itself can be advanced. The replacement of VHB by silicone membranes will reduce the viscoelastic losses within the membrane and, thus, reduce the time constants and increase the oscillation frequencies in general. We further expect a significant reduction of leakage resistors and voltage leaking through the DEA’s membrane. The replacement of carbon-grease by carbon doped silicone electrodes, both for DEAs and DESs, will increase the reliability and predictability of the electro-mechanical properties of all components. This will enable the derivation of an improved DEO model and a detailed experimental verification.

At this point, we do not possess a production technology to create DES on silicone membranes and, hence, neither silicone based inverters nor DEOs could be produced. Therefore, we are limited to the presented data and model at the moment. However, first model allows better understanding of the governing behaviors in the DEO and its subsystems. The DEO behaves well as a CPG — stable oscillating signals are generated from a single DC voltage input. Based on the parameters discussed above, custom DEOs for specific aCPGs or other applications can be designed. Furthermore, the same approach can be followed to produce models of other logical arrangements of DE components, such as digital gates assembled of several DEAs and DESs [25].

Acknowledgment
This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 706754 and by a US Army Research, Development & Engineering Command Grant (FA5209-15-P-0214).
Appendix. System parameters used in the Simulink model

Table A1. Model parameters used to simulate multiple DEOs using the Simulink model depicted in figure 14: \( w_{\text{DEA}} \), \( l_{\text{DEA}} \)—initial DEA geometry, \( R_s \)—serial charging resistors, \( R_{\text{meas}} \)—measurement resistor, \( V_{\text{in}} \)—DC input voltage, \( E_F \)—empirical field constant for current leakage model through the membranes, \( \sigma_F \)—low field conductivity of the DEA membranes.

| \( w_{\text{DEA}} \) in mm | \( l_{\text{DEA}} \) in mm | \( R_s \) in M\( \Omega \) | \( R_{\text{meas}} \) in M\( \Omega \) | \( V_{\text{in}} \) in V | \( E_F \) in \( \frac{V}{\text{m}} \) | \( \sigma_F \) in \( \frac{\text{A}}{\text{m}} \) |
|-------------------------|-------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 33.5                    | 20                      | 33              | 5005            | 3250            | \( 37.5 \times 10^6 \) | \( 1.05 \times 10^{12} \) |
| 43.5                    | 27                      | 50              |                 | 3500            |                 |                 |
| 53.5                    | 37                      | 70              |                 | 3750            |                 |                 |
| 67.5                    | 47                      | 100             |                 | 4000            |                 |                 |
| 73.5                    |                         |                 |                 |                 |                 |                 |
| 83.5                    | 200                     |                 |                 |                 |                 |                 |
| 93.5                    | 250                     |                 |                 |                 |                 |                 |
|                         | 300                     |                 |                 |                 |                 |                 |
|                         | 500                     |                 |                 |                 |                 |                 |
|                         | 800                     |                 |                 |                 |                 |                 |
|                         | 1000                    |                 |                 |                 |                 |                 |

ORCID iDs

E-F M Henke  
https://orcid.org/0000-0002-0540-9150

Katherine E Wilson  
https://orcid.org/0000-0001-7246-4208

I A Anderson  
https://orcid.org/0000-0003-4698-7317

References

[1] Mazzuoli-Web and Schemann M 2015 Front. Cell. Neurosci. 9 408
[2] Marder E and Bucher D 2001 Curr. Biol. 11 R986–96
[3] Suzumori K, Ikura S and Tanaka H 1992 IEEE Control Syst. 12 21–7
[4] Chou C P and Hannaford B 1996 IEEE Trans. Robot. Autom. 12 90–102
[5] Shepherd R F, Ilievski F, Choi W, Morin S A, Stokes A A, Mazzeo A D, Chen X, Wang M and Whitesides G M 2011 Proc. Natl Acad. Sci. 108 20400–3
[6] Katschmann R K, Marchese A D and Rus D 2016 Hydrophilic autonomous soft robotic fish for 3D swimming Experimental Robotics (Berlin: Springer) pp 405–20
[7] Calisti M, Giorelli M, Levy G, Mazzolai B, Hochner B, Laschi C and Dario P 2016 Biophysics of Robotics 7 306002
[8] Majidi C 2014 Soft Robot. 1 5–11
[9] Trivedi D, Rahn C D, Kier W M and Walker D 2008 Appl. Bionics Biomech. 5 99–117
[10] Rogers J A, Someya T and Huang Y 2010 Science 327 1603–7
[11] Carpi F, De Rossi D, Kornbluh R, Pelrine R E and Sommer-Larsen P 2011 Dielectric Elastomers as Electromechanical Transducers: Fundamentals, Materials, Devices, Models and Applications of an Emerging Electroactive Polymer Technology (Amsterdam: Elsevier)
[12] Liu T, Keplinger C, Baumgartner R, Bauer S, Yang W and Suo Z 2013 J. Mech. Phys. Solids 61 611–28
[13] Bar-Cohen Y 2002 J. Spacecr. Rockets 39 822–7
[14] Gisby T, Xie S, Calius E and Anderson I 2009 Integrated sensing and actuation of muscle-like actuators Proc. SPIE 7287 728707
[15] O’Brien B, Gisby T, Xie S Q, Calius E and Anderson I 2010 Biomimetic control for dea arrays Proc. SPIE 7642 764220
[16] O’Brien B, Gisby T and Anderson I A 2014 Stretch sensors for human body motion Proc. SPIE 9056 905618
[17] McKay T, O’Brien B, Calius E and Anderson I 2010 Smart Mater. Struct. 19 055025
[18] Kornbluh R D, Pelrine R, Pei Q, Heydt R, Stanford S, Oh S and Eckerle J 2002 Electroelastomers: applications of dielectric elastomer transducers for actuation, generation, and smart structures Proc. SPIE 4698 254–70
[19] Anderson I A, Gisby T A, McKay T G, O’Brien B M and Calius E P 2012 J. Appl. Phys. 112 041101
[20] Beruto D, Capurro M and Marro G 2005 Sensors Actuators A 117 301–8
[21] O’Brien B M, Calius E P, Inamura T, Xie S Q and Anderson I A 2010 Appl. Phys. A 100 385–9
[22] O’Brien B M and Anderson I A 2012 IEEE/ASME Trans. Mechatronics 17 197–200
[23] Henke E F M, Schlatter S and Anderson I A 2017 Soft Robot. 4 552–66
[24] O’Brien B M, Rosset S, Shea H R and Anderson I A 2012 Cutting the fat: artificial muscle oscillators for lighter, cheaper, and slimmer devices SPIE Smart Structures and Materials + Nondestructive Evaluation and Health Monitoring (International Society for Optics and Photonics) p 834008
[25] Wilson K E, Henke E F M, Slipper G A and Anderson I A 2016 Extreme Mech. Lett. 9 188–94
[26] Wilson K E, Henke E F M, Slipper G A and Anderson I A 2017 SPIE Smart Structures and Materials + Nondestructive Evaluation and Health Monitoring (International Society for Optics and Photonics) p 101632H
[27] Henke E F M, Schlatter S and Anderson I A 2016 (arXiv:1603.05599)
[28] Henke E F M, Wilson K E and Anderson I A 2017 Entirely soft dielectric elastomer robots Electroactive Polymer Actuators and Devices (EAPAD) vol 10163 (International Society for Optics and Photonics) p 101631N
[29] Molberg M, Leterrier Y, Plummer C J, Walder C, Löwe C, Opris D M, Nüesch F A, Bauer S and Månson J A E 2009 J. Appl. Phys. 106 054112
[30] Chiang Foo C, Cai S, Jin Adrian Koh S, Bauer S and Suo Z 2012 J. Appl. Phys. 111 034102
[31] Holzapfel G A 2010 Nonlinear Solid Mechanics: a Continuum Approach for Engineering ed G A Holzapfel (Chichester: Wiley)
[32] Gisby T, Xie S, Calius E, Anderson I and Bar-Cohen Y 2010 Leakage current as a predictor of failure in dielectric elastomer actuators Proc. SPIE 7642 764213