Electroweak Corrections to the Decay $H^+ \to W^+ h$

in the Minimal Supersymmetric Model

Ya Sheng Yang*and Chong Sheng Li†

Department of Physics, Peking University, Beijing 100871, P.R. China.

March 25, 2022

ABSTRACT

We calculate the $O(\alpha_{\text{ew}} m_t^2 / m_W^2)$ and $O(\alpha_{\text{ew}} m_b^4 / m_W^4)$ supersymmetric electroweak corrections to the process $H^+ \to W^+ h$ in the Minimal Supersymmetric Model. These corrections arise from the virtual effects of the third family (top and bottom) quarks and squarks (top-squark and bottom-squark). We find that for $m_{H^+} > 200\text{GeV}$ at low $\tan \beta (\leq 3)$, the corrections can increase the tree-level decay widths and the branching ratios more than 20% and 40%, respectively.

PACS number: 14.80.Cp, 14.80.Ly, 12.38.Bx

Keywords: Radiative correction, Charged Higgs decay, Supersymmetry

*E-mail: ylsdd@ibm320h.phy.pku.edu.cn
†E-mail: csli@ibm320h.phy.pku.edu.cn
1. Introduction

The minimal supersymmetric standard model (MSSM) takes the minimal Higgs structure of two doubles [1], which predicts the existence of three neutral and two charged Higgs bosons $h, H, A,$ and $H^\pm$. When the Higgs boson of the Standard Model (SM) has a mass below 130-140 Gev and the $h$ boson of the MSSM is in the decoupling limit (which means that $H^+$ is too heavy anyway to be possibly produced), the lightest neutral Higgs boson may be difficult to distinguish from the neutral Higgs boson of the standard model (SM). But charged Higgs bosons carry a distinctive signature of the Higgs sector in the MSSM. Therefore, the search for charged Higgs bosons is very important for probing the Higgs sector of the MSSM and, therefore, will be one of the prime objectives of the CERN Large Hadron Collider (LHC). In fact, in much of the parameter space preferred by MSSM, namely $M_{H^\pm} > M_{W^\pm}$ and $1 < \tan \beta < m_t/m_b$ [2,3], the LHC will provide the greatest opportunity for the discovery of $H^\pm$ particles. Previous studies [4,5] have shown that the dominant decay modes of the charged Higgs boson are $H^+ \to t + b$ with $m_{H^\pm} > m_t + m_b$ and $H^+ \to \tau^+ \nu$. However, recent analyses [6,7] indicate that the alternative decay channel $H^+ \to W^+ h^0$ could be very important. In fact, its branching ratio can be rather large, competing with the top-bottom decay mode and overwhelming the tau-neutrino one for $M_{H^\pm} \geq m_t$ at low $\tan \beta$ [7]. Thus an more accurate calculation of the decay mechanisms is also necessary to provide a solid basis for experimental analysis of observing $H^+ \to W^+ h^0$ at the LHC. In Ref. [6], R. Santos et al. calculated the top quark loops corrections to the decay width of the process $H^+ \to W^+ h^0$ in the framework of the two-Higgs doublet model for some reasonable choice of the free parameters and found that such corrections can be as large as 40%. In this letter, we present the calculation of the $O(\alpha_{ew} m_t^2/m_W^2)$ and $O(\alpha_{ew} m_t^4/m_W^4)$ supersymmetric (SUSY) electroweak (EW) corrections to this process in the MSSM. These corrections arise from the virtual effects of the third family (top and bottom) quarks and squarks (top-squark and bottom-squark). We will give attention manily to these corrections in parameter range $\tan \beta \leq 3$ and $m_{H^+} > 200$ GeV, where the $H^+ \to W^+ h^0$ decay
rate is significant.

2. Calculations

Feynman diagrams contributing to supersymmetric electroweak corrections to $H^+ \rightarrow W^+ h$ are shown in Fig.1 and 2. We carried out the calculation in the t’Hooft-Feynman gauge and used dimensional reduction, which preserves supersymmetry, for regularization of the ultraviolet divergences in the virtual loop corrections using the on-mass-shell renormalization scheme[8], in which the fine-structure constant $\alpha_{ew}$ and physical masses are chosen to be the renormalized parameters, and finite parts of the counterterms are fixed by the renormalization conditions. The coupling constant $g$ is related to the input parameters $e$, $m_W$, and $m_Z$ via $g^2 = e^2 / s_w^2$ and $s_w^2 = 1 - m_W^2 / m_Z^2$.

As far as the parameters $\beta$ and $\alpha$, for the MSSM we are considering, they have to be renormalized, too. In the MSSM they are not independent. Nevertheless, we follow the approach of Mendez and Pomarol[9] in which they consider them as independent renormalized parameters and fixed the corresponding renormalization constants by a renormalization condition that the on-mass-shell $H^+ \bar{l}\nu_l$ and $h\bar{l}l$ couplings keep the forms of Eq.(3) of Ref.[9] to all order of perturbation theory.

The relevant renormalization constants are defined as

$$m_{W0}^2 = m_W^2 + \delta m_W^2, \quad m_{Z0}^2 = m_Z^2 + \delta m_Z^2,$$

$$\tan \beta_0 = (1 + \delta Z_\beta) \tan \beta,$$

$$\sin \alpha_0 = (1 + \delta Z_\alpha) \sin \alpha,$$

$$W_0^\pm = (1 + \delta Z_{W})^{1/2} W^\pm + i Z_{H^\pm W^\pm} \partial^\mu H^\mp,$$

$$H_0^\pm = (1 + \delta Z_{H^\pm})^{1/2} H^\pm,$$

$$H_0 = (1 + \delta Z_H)^{1/2} H + Z_{Hh}^{1/2} h,$$

$$h_0 = (1 + \delta Z_h)^{1/2} h + Z_{hH}^{1/2} H.$$

(1)

Taking into account the $O(\alpha_{ew} m_{t(b)}^2 / m_W^2)$ and $O(\alpha_{ew} m_{t(b)}^4 / m_W^4)$ SUSY EW corrections, the renormalized amplitude for $H^+ \rightarrow W^+ h$ can be written as

$$M_{ren} = M_0 + \delta M,$$

(2)
where $M_0$ is the tree-level amplitude arising from Fig.1(a), which is given by
\[ M_0 = -\frac{ig}{2} \cos(\beta - \alpha) p^\mu \varepsilon_\mu(k). \] (3)

Here $p$ is the momentum of the incoming charged higgs Boson, and $k$ is the momentum of the outgoing $W^+$ Boson. $\delta M$ represents the SUSY EW corrections, which is given by
\[
\delta M = -\frac{ig}{2} \cos(\beta - \alpha) p^\mu \varepsilon_\mu(k) \left[ \frac{\delta g}{g} - \tan(\beta - \alpha) (\cos \beta \sin \beta \delta Z_\beta - \tan \alpha \delta Z_\alpha) \right. \\
+ \left. \frac{1}{2} (\delta Z_{H^+} + \delta Z_h + \delta Z_W) - \tan(\beta - \alpha) \delta Z_{H^h}^{1/2} \right] \\
+ igm_W \sin(\beta - \alpha) Z_{H^W}^{1/2} p^\mu + i \sum_{j=1}^{4} \Lambda_j p^\mu \varepsilon_\mu(k),
\] (4)

with
\[
\frac{\delta g}{g} = \frac{\delta e}{e} + \frac{1}{2} \frac{\delta m_Z^2}{m_Z^2} - \frac{1}{2} \frac{\delta m_W^2}{m_W^2}, \\
\delta Z_\beta = -\delta g + \frac{1}{2} \frac{\delta m_W^2}{m_W^2} - \frac{1}{2} \frac{\delta Z_{H^+}}{\tan \beta} Z_{H^W}^{1/2}, \\
\delta Z_\alpha = -\frac{\delta g}{g} + \frac{1}{2} \frac{\delta m_W^2}{m_W^2} - \frac{1}{2} \frac{\delta Z_h}{\cot \alpha} Z_{H^h}^{1/2} - \sin^2 \beta \delta Z_\beta.
\] (5)

Here $\Lambda_j (j = 1-4)$ are the vertex form factors coming from Fig.1(b)-(e). The $\delta e/e$ appearing in Eq.(5) does not contain the $O(\alpha_{ew} m_{t(b)}^2 / m_W^2)$ corrections and needs not be considered in our calculations.

Calculating the self-energy diagrams in Fig.2, we can get the explicit expressions of all the renormalization constants as following:
\[
\delta m_W^2 = \frac{g^2}{16\pi^2} \left\{ (m_b^2 - m_t^2) (1 + \frac{m_b^2 - m_t^2}{m_W^2} B_0^{\text{tot}}) - 2m_t^2 B_0^{\text{tot}} \right\} \\
- \frac{1}{2m_W^2} \left[ (m_b^2 - m_t^2) + (m_b^2 + m_t^2) m_W^2 B_0^{W^{\text{tot}}} \right],
\]
\[
\delta Z_W = \frac{g^2}{32\pi^2 m_W^2} \left\{ \frac{(m_b^2 - m_t^2)^2}{m_W^2} (B_0^{\text{tot}} - B_0^{W^{\text{tot}}}) + [(m_b^2 - m_t^2)^2 \\
+ (m_b^2 + m_t^2) m_W^2 B_0^{W^{\text{tot}}} \right\},
\]
\[
\delta m_Z^2 = \frac{g^2 s_W^2}{18c_W^2 \pi^2} \left[ \frac{m_b^2 (3 - 2s_W^2) (B_0^{\text{tot}} - B_0^{W^{\text{tot}}}) - m_t^2 (3 - 4s_W^2) (B_0^{\text{tot}} - B_0^{W^{\text{tot}}} \right] \\
+ \frac{g^2}{32\pi^2} \left[ m_b^2 (B_0^{Z^{\text{tot}}} - 2B_0^{W^{\text{tot}}}) - m_t^2 (B_0^{Z^{\text{tot}}} + 2B_0^{W^{\text{tot}}} \right],
\]
\[
\delta Z_{H^+} = \frac{3}{16\pi^2} [2(h_t^2 m_t^2 + h_b^2 m_b^2)(B_1^{H^+tb} + m_b^2 B_0^{H^+tb} + m_H^2 B_1^{H^+tb})
- 4h_t h_tm_b m_t \beta_{11} \beta_{21} B_0^{H^+tb} + \sum_{i,j',j''} (\theta_{ii'}^b)^2 (\Theta_{ij'j''}^t)^2 (h_t \Theta_{ii'j''}^5 + h_t \Theta_{ij'j''}^6)^2 B_0^{H^+b_{ij''}}],
\]
\[
\delta Z_{h} = \frac{3}{16\pi^2} \left\{ 2h_t^2 \beta_{11} (B_1^{ttb} + m_b^2 B_0^{ttb} + m_h^2 B_1^{ttb}) + 2h_b^2 \alpha_{21} (B_1^{bb}) + 2m_b^2 B_0^{b_{bbb}} + m_h^2 B_1^{b_{bbb}} + \sum_{i,j',j''} [(h_t \theta_{ii'}^b \theta_{jj'j''}^t \Theta_{ij'j''})^2 B_0^{H^+b_{ij''}}
+ (h_t \theta_{ii'}^b \theta_{jj'j''}^b \Theta_{ij'j''})^2 B_0^{B_{ij''}}],
\]
\[
Z_{H^+W} = \frac{3}{16\sqrt{2}\pi^2 m_H^2 m_W^2} [(h_t m_t \beta_{11} + h_t m_b \beta_{12}) ((m_b^2 - m_H^2)(B_0^{ttb} - B_0^{H^+tb}) - m_H^2 B_0^{H^+tb})
+ \sum_{i,j',j''} \theta_{ii'}^b \theta_{jj'j''}^t \Theta_{ij'j''}^t (m_b^2 - m_H^2)(B_0^{b_{ij''}} - B_0^{H^+b_{ij''}})],
\]
\[
Z_{H^+H}^{1/2} = \frac{3\alpha_{11} \alpha_{12}}{16\pi^2 (m_H^2 - m_h^2)^2} [2m_b^2 (1 + B_0^{b_{bbb}} + B_0^{b_{bb}}) - 2m_b^2 (1 + B_0^{b_{ttb}} + B_0^{b_{tt}})
- m_b^2 (B_0^{b_{b_{bb}}})]
+ \frac{3}{16\pi^2 (m_H^2 - m_h^2)} \sum_{i,j',j''} [(h_t \theta_{ii'}^b \theta_{jj'j''}^t)^2 \Theta_{ij'j''}^2 B_0^{b_{ij''}} + (h_t \theta_{ii'}^t \theta_{jj'j''}^t)^2 \Theta_{ij'j''}^t B_0^{b_{ij''}}]
- \frac{3\alpha_{11} \alpha_{12}}{16\pi^2 (m_H^2 - m_h^2)} \sum_i [(h_b^2 m_b^2 (1 + B_0^{b_{bb}}) + h_t^2 m_b^2 (1 + B_0^{b_{tt}})],
\]
(6)

with
\[
B_{n}^{ijk} = (-1)^n \left\{ \frac{\Delta}{n+1} - \int_0^1 dy y^n \ln \left[ \frac{m_H^2 y (y - 1) + m_b^2 (1 - y) + m_h^2 y}{\mu^2} \right] \right\},
\]
(7)
\[
B_{n'}^{ijk} = (-1)^n \int_0^1 dy y^{n+1} (1 - y) \frac{m_h^2 y (y - 1) + m_b^2 (1 - y) + m_h^2 y}{\mu^2}.
\]
(8)

Here \( h_b \equiv g m_b / \sqrt{2} m_W \cos \beta \) and \( h_t \equiv g m_t / \sqrt{2} m_W \sin \beta \) are the Yukawa couplings from the bottom and top quarks, respectively. The notations \( \theta_{ij}^t, \theta_{ij}^b, \alpha_{ij}, \beta_{ij}, \varphi_{ij} \) and \( \Theta_{ij}^n \) used in the above expressions are defined in Appendix A.

Calculating the diagrams in Fig.1(b)-(c), we can get the explicit expressions of the vertex form factors \( \Lambda_j (j = 1-4) \) as following:

\[
\Lambda_1 = -\frac{3g h_b}{16\pi^2} \sin \alpha \{ h_b \sin \beta (B_0^{H^+tb} - B_1^{H^+tb}) + [-h_t m_b m_t \cos \beta (C_0 + 2C_2) 
- 2h_b \sin \beta C_{00} + h_b m_b^2 \sin \beta (C_0 - 2C_2) - h_b m_h^2 \sin \beta (C_{12} + 4C_{22})
+ h_t m_h^2 \sin \beta (C_{12} + 2C_{22}) + h_b m_W^2 \sin \beta (C_1 - 3C_{12})
\]
\]

5
The renormalized decay width is given by

\[ \Gamma(Ren) = \frac{3g_{ht}}{16\pi^2} \cos \alpha \{h_t \cos \beta(-B_0 H^{tb} + B_1 H^{tb}) + |h_b m_b m_t \sin \beta(C_0 + 2C_2) + 2h_t \cos \beta C_{00} - h_t m_t^2 \cos \beta(C_0 - 2C_2) + h_t m_{H^+}^2 - h_t m_W^2 \cos \beta(C_{12} + 4C_{22}) + h_t m_h^2 \cos \beta(C_{12} + 2C_{22}) - h_t m_W^2 \cos \beta(C_1 - 3C_{12}) - 4C_{22}\}(m_{H^+}^2, m_{H^+}^2, m_{h}^2, m_{h}^2, m_{\tau}^2, m_{\tau}^2) \} \]

\[ \Lambda_2 = \frac{3gh_t}{16\pi^2} \cos \alpha \{h_t \cos \beta(-B_0 H^{tb} + B_1 H^{tb}) + |h_b m_b m_t \sin \beta(C_0 + 2C_2) + 2h_t \cos \beta C_{00} - h_t m_t^2 \cos \beta(C_0 - 2C_2) + h_t m_{H^+}^2 - h_t m_W^2 \cos \beta(C_{12} + 4C_{22}) + h_t m_h^2 \cos \beta(C_{12} + 2C_{22}) - h_t m_W^2 \cos \beta(C_1 - 3C_{12}) - 4C_{22}\}(m_{H^+}^2, m_{H^+}^2, m_{h}^2, m_{h}^2, m_{\tau}^2, m_{\tau}^2) \} \]

\[ \Lambda_3 = \frac{3gh_b}{8\sqrt{2}\pi^2} \sum_{ijk} \sum_{ij'k'} \phi_{ii'}(\theta_{jj'}^t)^2 \theta_{kk'} \Theta_{ii'j'j} \left(h_b \Theta^5_{j'k'} + h_t \Theta^6_{j'k'} \right) \]

\[ \times C_2(m_{H^+}^2, m_{H^+}^2, m_{h}^2, m_{h}^2, m_{\tau}^2, m_{\tau}^2) \]

\[ \Lambda_4 = -\frac{3gh_t}{8\sqrt{2}\pi^2} \sum_{ijk} \sum_{ij'k'} \phi_{kk'} \Theta_{ij'j'}^2 \Theta_{ii'j'j} \left(h_b \Theta^5_{j'k'} + h_t \Theta^6_{j'k'} \right) \]

\[ \times C_2(m_{H^+}^2, m_{H^+}^2, m_{h}^2, m_{h}^2, m_{\tau}^2, m_{\tau}^2) \]  \hspace{1cm} (9)

where \(C_i\) and \(C_{ij}\) are the three-point Feynman integrals[10].

The corresponding amplitude squared is

\[ |M_{\text{ren}}|^2 = |M_0|^2 + 2Re(\delta M M_0^\dagger). \]  \hspace{1cm} (10)

The renormalized decay width is given by

\[ \Gamma(\text{H}^+ \rightarrow \text{W}^+ h) = \Gamma_0(\text{H}^+ \rightarrow \text{W}^+ h) + \delta \Gamma(\text{H}^+ \rightarrow \text{W}^+ h) \]

\[ = \frac{1}{16\pi m_{H^+}^3} \sqrt{(m_{h}^2 - m_{H^+}^2 - m_{W}^2)^2 - 4m_{H^+}^2 m_{W}^2 |M_{\text{ren}}|^2}. \]  \hspace{1cm} (11)

The tree-level branching ratio of \(\text{H}^+ \rightarrow \text{W}^+ h\) decay is

\[ B_0(\text{H}^+ \rightarrow \text{W}^+ h) = \Gamma_0(\text{H}^+ \rightarrow \text{W}^+ h) / \Gamma_0(\text{H}^+), \]  \hspace{1cm} (12)

where the tree-level total decay width \(\Gamma_0(\text{H}^+)\) of the charged Higgs boson is approximated by

\[ \Gamma_0(\text{H}^+) = \Gamma_0(\text{H}^+ \rightarrow \text{W}^+ h) + \Gamma_0(\text{H}^+ \rightarrow tb) + \Gamma_0(\text{H}^+ \rightarrow cs) + \Gamma_0(\text{H}^+ \rightarrow \tau \nu). \]  \hspace{1cm} (13)

While calculating the one-loop branching ratio \(B(\text{H}^+ \rightarrow \text{W}^+ h)\), the QCD corrections[11] to the \(\text{H}^+ \rightarrow tb\) width, \(\delta \Gamma(\text{H}^+ \rightarrow \text{tb})\), are included into the total width \(\Gamma(\text{H}^+)\), but its leading EW corrections[12] are neglected since they are much smaller than the QCD corrections:

\[ B(\text{H}^+ \rightarrow \text{W}^+ h) = \Gamma(\text{H}^+ \rightarrow \text{W}^+ h) / \Gamma(\text{H}^+) \]  \hspace{1cm} (14)
with
\[
\Gamma(H^+) = \Gamma_0(H^+) + \delta \Gamma(H^+ \to W^+ h) + \delta \Gamma(H^+ \to tb) \tag{15}
\]

3. Numerical results and conclusion

We now present some numerical results for the SUSY EW corrections to the decay \(H^+ \to W^+ h\). The SM input parameters in our calculations were taken to be \(\alpha_{ew}(m_Z) = 1/128.8\), \(m_W = 80.375\text{GeV}\) and \(m_Z = 91.1867\text{GeV}\)[13], and \(m_t = 175.6\text{GeV}\) and \(m_b = 4.7\text{GeV}\). Other parameters are determined as follows

(i) The one-loop relations[14] between the Higgs boson masses \(M_{h,H,A,H^-}\) and the parameters \(\alpha\) and \(\beta\) in the MSSM were used, and \(m_{H^+}\) and \(\beta\) were chosen as the two independent input parameters. As explained in Ref[9], there is a small inconsistency in doing so since the parameters \(\alpha\) and \(\beta\) of Ref[14] are not the ones defined by the conditions given by Eq.(Eq(3)) of Ref[9]. Nevertheless, this difference would only induce a higher order change[9].

(ii) For the parameters \(m_{Q,\tilde{U},\tilde{D}}^2\) and \(A_{t,b}\) in squark mass matrices
\[
M_{\tilde{q}}^2 = \begin{pmatrix}
M_{\tilde{q}L}^2 & m_{\tilde{q}}M_{LR} \\
m_{\tilde{q}}M_{RL} & M_{\tilde{q}R}^2
\end{pmatrix}
\tag{16}
\]

with
\[
M_{\tilde{q}L}^2 = m_{\tilde{Q}}^2 + m_{\tilde{q}}^2 + m_Z^2 \cos 2\beta (I_{\tilde{q}L} + e_q \sin^2 \theta_W), \\
M_{\tilde{q}R}^2 = m_{\tilde{U},\tilde{D}}^2 + m_{\tilde{q}}^2 + m_Z^2 \cos 2\beta e_q \sin^2 \theta_W, \\
M_{LR} = M_{RL} = \begin{pmatrix}
A_t - \mu \cot \beta \ (\bar{q} = \bar{t}) \\
A_b - \mu \tan \beta \ (\bar{q} = \bar{b})
\end{pmatrix}
\tag{17}
\]

to simplify the calculation we assumed \(M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}}\) and \(A_t = A_b\), and we used \(M_{\tilde{Q}}, A_t\) and \(\mu\) as input parameters except the numerical calculations as shown in Fig.5(a) and (b), where we took \(m_{\tilde{t}_1}, m_{\tilde{b}_1}, A_t = A_b\) and \(\mu\) as the input parameters.

Figure 3 shows the tree-level partial width are relatively large for low values of \(\tan \beta (= 1.5, 2)\) when \(m_{H^+} > 200\text{GeV}\).
In Figs. 4(a) and (b) we present the SUSY EW corrections to the tree-level decay widths and the branching ratios as functions of $m_{H^+}$ for different values of $\tan \beta$, respectively, assuming $M_{\tilde{Q}} = M_{\tilde{t}} = M_{\tilde{b}} = 300\text{GeV}$, $A_t = A_b = 300\text{GeV}$, and $\mu = -100\text{GeV}$. Figure 4(a) shows that the relative corrections increase the partial width significantly at low $\tan \beta$, which can exceed 40\% for $\tan \beta = 1.5$ and 30\% for $\tan \beta = 2$, respectively. Fig. 4(b) gives the tree-level branching ratios and the branching ratios after including $O(\alpha_{\text{ew}} m_{t(b)}^2 / m_W^2)$ and $O(\alpha_{\text{ew}} m_{t(b)}^4 / m_W^4)$ SUSY EW corrections for $\tan \beta = 1.5$ and 3, respectively. From Fig. 4(b), we see that the branching ratios are enhanced by the corrections, especially for $m_{H^+} > 300$ GeV, they can be increased by 50\%. But the corrections to the branching ratios decrease with an decrease of $m_{H^+}$, especially for $m_{H^+}$ below 200 GeV, they become negligibly small.

In Fig. 5(a) and (b) we assumed $M_{\tilde{Q}} = M_{\tilde{t}}$, $A_t = A_b = 500\text{GeV}$, $\mu = 100\text{GeV}$, $m_{\tilde{t}_1} = 170\text{GeV}$ and $m_{\tilde{b}_1} = 200\text{GeV}$. Fig. 5(a) shows the relative corrections to the tree-level widths as a function of $m_{H^+}$ for $\tan \beta = 1.5$, 2, 6, 10 and 30. Comparing Fig. 5(a) with Fig. 4(a), we see that the relative corrections are enhanced by the relatively light squark masses in the case of heavy charged Higgs masses at low or high $\tan \beta$. Fig. 5(a) also shows at low $\tan \beta$ the corrections always increase the decay width, which can exceed 50\%, while at high $\tan \beta$ the corrections decrease the decay width significantly. There are dips at $m_{H^+} = m_{\tilde{t}_1} + m_{\tilde{b}_1} = 370\text{GeV}$ due to the threshold effects. As shown in Fig. 4(b), Fig. 5(b) shows the branching ratios are enhanced by the SUSY EW corrections for $\tan \beta = 1.5$ and 3, especially when $m_{H^+} > 300\text{GeV}$.

In conclusion, we have calculated the $O(\alpha_{\text{ew}} m_{t(b)}^2 / m_W^2)$ and $O(\alpha_{\text{ew}} m_{t(b)}^4 / m_W^4)$ SUSY EW corrections to the process $H^+ \to W^+ h$ in the MSSM. In general, these corrections increase the decay widths and the branching ratios and are sensitive to both of $\tan \beta$ and the mass of the charged Higgs boson. For $m_{H^+} > 200\text{GeV}$ at low $\tan \beta (\leq 3)$, the corrections can increase the decay widths and the branching ratios more than 20\% and 40\%, respectively.

This work was supported in part by the National Natural Science Foundation of
China, the Doctoral Program Foundation of Higher Education of China, and a grant from the State Commission of Science and Technology of China.

Appendix A

We present some notations used in this paper here. We introduce an angle \( \varphi = \beta - \alpha \), and for each angle \( \alpha, \beta, \varphi, \theta^t \) or \( \theta^b \), we define

\[
\alpha_{ij} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}, \quad \beta_{ij} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix}, \quad \varphi_{ij} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix},
\]

\[
\theta^t_{ij} = \begin{pmatrix} \cos \theta^t & \sin \theta^t \\ -\sin \theta^t & \cos \theta^t \end{pmatrix}, \quad \theta^b_{ij} = \begin{pmatrix} \cos \theta^b & \sin \theta^b \\ -\sin \theta^b & \cos \theta^b \end{pmatrix}.
\]

We define six matrix \( \Theta_{ijkl}^i (i = 1 - 6) \) for the couplings between squarks and Higgses:

\[
\Theta_{11j}^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 2m_t \cos \alpha & A_t \cos \alpha + \mu \sin \alpha \\ A_t \cos \alpha + \mu \sin \alpha & 2m_t \cos \alpha \end{pmatrix},
\]

\[
\Theta_{12j}^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 2m_t \sin \alpha & A_t \sin \alpha - \mu \cos \alpha \\ A_t \sin \alpha - \mu \cos \alpha & 2m_t \sin \alpha \end{pmatrix},
\]

\[
\Theta_{21j}^2 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 2m_b \sin \alpha & A_b \sin \alpha + \mu \cos \alpha \\ A_b \sin \alpha + \mu \cos \alpha & 2m_b \sin \alpha \end{pmatrix},
\]

\[
\Theta_{22j}^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 2m_b \cos \alpha & A_b \cos \alpha - \mu \sin \alpha \\ A_b \cos \alpha - \mu \sin \alpha & 2m_b \cos \alpha \end{pmatrix},
\]

\[
\Theta_{51j}^5 = \begin{pmatrix} m_b \sin \beta & 0 \\ A_b \sin \beta + \mu \cos \beta & m_t \sin \beta \end{pmatrix},
\]

\[
\Theta_{52j}^5 = \begin{pmatrix} -m_b \cos \beta & 0 \\ -A_b \cos \beta + \mu \sin \beta & 0 \end{pmatrix},
\]

\[
\Theta_{6j1}^6 = \begin{pmatrix} m_t \cos \beta & A_t \cos \beta + \mu \sin \beta \\ 0 & m_b \cos \beta \end{pmatrix}.
\]
\[ \Theta^6_{ij^2} = \begin{pmatrix} m_t \sin \beta & A_t \sin \beta - \mu \cos \beta \\ 0 & 0 \end{pmatrix} \]
References

[1] H.E. Haber and G.L. Kane, Phys. Rep. 117, 75(1985); J.F. Gunion and H.E. Haber, Nucl. Phys. B272, 1(1986).

[2] CMS Technical Proposal. CERN/LHC94-43 LHCC/P1, December 1994.

[3] CDF Collaboration, Phys. Rev. Lett. 79, 35(1997); D0 Collaboration, Phys. Rev.Lett. 82, 4975(1999).

[4] V.Barger, R.J.N. Phillips and D.P. Roy, Phys.Lett. B324, 236(1994); J.F. Gunion and S.Geer, preprint UCD-93-32, September 1993, hep-ph/9310333; J.F. Gunion, Phys.lett. B322, 125(1994); D.J. Miller, S.Moretti, D.P. Roy and W.J. Stirling, Phys.Rev. D61, 055011(2000); S. Moretti and D.P. Roy, B470 209(1999).

[5] K. Odagiri, preprint RAL-TR-1999-012, February 1999, hep-ph/9901432; S. Raychaudhuri and D.P. Roy, Phys.Rev. D53, 4902(1996).

[6] R.Santos, A.Barroso, and L. Brucher, Phys. Lett. B391, 429(1997).

[7] Stefano Moretti, Phys.Lett. B481, 49(2000).

[8] S. Sirlin, Phys. Rev. D22, 971 (1980); W. J. Marciano and A. Sirlin,ibid. 22, 2695(1980); 31, 213(E) (1985); A. Sirlin and W.J. Marciano, Nucl. Phys. B189, 442(1981); K.I. Aoki et.al., Prog. Theor. Phys. Suppl. 73, 1(1982).

[9] A. Mendez and A. Pomarol, Phys.Lett. B279, 98(1992).

[10] G.Passarino and M.Veltman, Nucl. Phys. B160, 151(1979); A.Axelrod, ibid. B209, 349 (1982); M.Clements et al., Phys. Rev. D27, 570 (1983); A.Denner, Fortschr. Phys. 41, 4 (1993); R. Mertig et al., Comput. Phys. Commun. 64, 345 (1991).

[11] C.S. Li and R.J. Oakes, Phys. Rev. D43, 855(1991); A. Mendez and A. Pomarol, Phys.Lett. B252, 461(1990); C.S. Li and J. M. Yang, Phys.Lett. B315, 367(1993); Heinz Konig, Modern Phys. Lett. A10, 1113(1995); A. Bartl, H. Eberl, K. Hidaka, T. Kon, W. Majerotto and Y. Yamada, Phys. Lett. B378, 167(1996); A. Djoual, M. Spira and P.M. Zerwas, Z. Phys. C70, 427(1996).
[12] J. M. Yang and C.S. Li, Phys. Rev. D47, 2872(1993); M. A. Diaz, Phys.Rev. D48, 2152(1993)

[13] Particle Data Group, C.Caso et al, Eur.Phys.J. C3, 1(1998).

[14] J.Gunion, A.Turski, Phys. Rev. D39, 2701(1989); D40, 2333(1990); J.R.Espinosa, M.Quiros, Phys. Lett. B266, 389(1991); M.Carena, M.Quiros, C.E.M.Wagner, Nucl. Phys. B461, 407(1996).
**Figure Captions**

**FIG. 1** Feynman diagrams contributing to supersymmetric electroweak corrections to $H^+ \rightarrow W^+ h$: (a) is the tree-level diagram; (b)–(e) are the one-loop diagrams.

**FIG. 2** Feynman diagrams contributing to the renormalization constants.

**FIG. 3** The tree-level $H^+ \rightarrow W^+ h$ decay widths.

**FIG. 4** (a): The SUSY EW relative corrections to the $H^+ \rightarrow W^+ h$ decay width; (b): The branching ratio of the $H^+ \rightarrow W^+ h$ decay, assuming $M_{\tilde{Q}} = M_{\tilde{t}} = M_{\tilde{b}} = 300\text{GeV}$, $A_t = A_b = 300\text{GeV}$, and $\mu = -100\text{GeV}$. Logarithmic coordinate is used in (b).

**FIG. 5** (a): The SUSY EW relative corrections to the $H^+ \rightarrow W^+ h$ decay width; (b): The branching ratio of the $H^+ \rightarrow W^+ h$ decay, assuming $M_{\tilde{Q}} = M_{\tilde{t}}$, $A_t = A_b = 500\text{GeV}$, $\mu = 100\text{GeV}$, $m_{\tilde{t}_1} = 170\text{GeV}$ and $m_{\tilde{b}_1} = 200\text{GeV}$. Logarithmic coordinate is used in (b).
Fig 1

(a) $H^+ \rightarrow W^+ h$

(b) $H^+ \rightarrow W^+ b h$

(c) $H^+ \rightarrow W^+ t h$

(d) $H^+ \rightarrow W^+ \tilde{b} h$

(e) $H^+ \rightarrow W^+ \tilde{b} t h$
Fig 2
Fig. 3

\[ \Gamma_0 \text{(GeV)} \]

\[ m_{H^+} \text{(GeV)} \]
Fig. 3 (a)

\[ \delta \Gamma / \Gamma_0 \text{(\%)} \]

- \( \tan \beta = 1.5 \)
- \( \tan \beta = 2 \)
- \( \tan \beta = 6 \)
- \( \tan \beta = 10 \)
- \( \tan \beta = 30 \)

\[ m_{H^+} \text{(GeV)} \]

- 150
- 200
- 250
- 300
- 350
- 400
- 450
- 500
Fig. 3 (b)

- $B_0(\tan \beta = 1.5)$
- $B(\tan \beta = 1.5)$
- $B_0(\tan \beta = 3)$
- $B(\tan \beta = 3)$

Branching Ratio vs. $m_{H^+}$ (GeV)
Fig. 4(a)

\[ \delta \Gamma / \Gamma_0 (\%) \]

\[ m_{H^*} (\text{GeV}) \]

- \( \tan \beta = 1.5 \)
- \( \tan \beta = 2 \)
- \( \tan \beta = 6 \)
- \( \tan \beta = 10 \)
- \( \tan \beta = 30 \)
Fig. 4 (b)

Branching Ratio

$m_{H^+}$ (GeV)

$B_0 (\tan \beta = 1.5)$

$B (\tan \beta = 1.5)$

$B_0 (\tan \beta = 3)$

$B (\tan \beta = 3)$