Influence of the wave generator type on the stress distribution in the flexible wheel of double harmonic transmissions

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Abstract. The main factor that influences the lifecycle of the double harmonic transmission is the durability of the flexible toothed wheel, which is the most strongly stressed element of the transmission. The complexity of the stress distribution in the flexible toothed wheel depends on many factors, such as: the type of wave generator, the flexible wheel construction shape, the coupling mode of the flexible wheel and the transmission load. In the paper it was studied the stress distribution in the flexible wheel body, as well as the variation of the displacements of the nodes located on certain generatrix of the flexible wheel, by considering three types of mechanical generators with two deformation waves, materialized by: 2 rolls, 2 eccentric discs or cam. The numerical simulation of the flexible wheel was made in the elastic domain using a Finite Element Method with SolidWorks software.

1. Introduction

The harmonic gear transmissions occupy an important place in the field of drives (power up to 10 kW) due to their significant advantages (compact construction, high gear ratio, silent operation, low weight, reduced gauge, etc.) compared to conventional gears (cylindrical, conical and worm), at which the fundamental law of engagement is respected [1-8].

For harmonic gear transmissions, the functioning is based on a new principle of transmission of the rotation motion and a new gears engagement character, which imposed an adequate constructional modeling of the flexible toothed wheel shape. Their operating principle assures the transmission of the rotation motion through elastic deformation of one wheel of the mechanism, which is the flexible toothed wheel [9-17].

The increasing interest in harmonic gear transmissions led to the emergence of a growing number of their constructive variants in industry, in order to satisfy a wide range of applications [18-21]. By constructive-functional diversification of the harmonic gear transmissions a new constructive variant was obtained, which is called the double harmonic transmission (DHT). The particularity of this transmission lies in the construction of the flexible toothed wheel, which has an annular shape and is provided at each extremity with toothing. These two toothings can be located both on the outer or inner surface of the flexible wheel, or one may be external and the other inside [22].

The analysis of the influence of the wave generator type on the stress distribution and variation of the resulting displacement of the characteristic points of the flexible toothed wheel wall was achieved by numerical simulation of the flexible wheel behavior for various loads of the transmission. For the elastic deformation of the flexible wheel were used the cases of three types of mechanical generators with two deformation waves (made with the help of 2 rolls, 2 eccentric discs or cam).
2. **The double harmonic transmission construction**

The double harmonic transmission variant that has been researched is one with annular flexible wheel, which has at one end the external teeth and at the other end the internal teeth.

The double harmonic transmission construction is presented in Figure 1, which content the following elements: 1 - the wave generator as driving element, 2 - the flexible toothed wheel as intermediate element, 3 - the rigid wheel as fixed element and 4 - the rigid mobile wheel as driven element.

![Figure 1. The double harmonic transmission construction: a) DHT image and b) 3D model.](image)

The double harmonic transmission operation principle is similar to the simple harmonic transmission. By forcefully mounting the wave generator (1) inside the flexible wheel (2), a deformation of it is produced, which will take an elliptical shape in the cross-section. Thus, the deformed flexible wheel will have four engagement zones positioned equidistant at an angle of 90°.

In the frontal plane of action of the wave generator, in the diametrically opposed zones corresponding to the major axis of the ellipse, the external teeth of the flexible wheel will be engaged with the internal teeth of the rigid fixed wheel (3). In the other front plane of the flexible wheel, in the diametrically opposed zones corresponding to the small axis of the ellipse, the inner teeth of the flexible wheel will be engaged with external teeth of the mobile rigid wheel (4).

In order to ensure the conditions of engagement between the flexible wheel and the rigid fixed wheel, respectively between the flexible wheel and the rigid mobile wheel, it is necessary to correlate the teeth numbers of the conjugated wheels so that the following relations are fulfilled:

\[
\begin{align*}
&z_3 - z_2 = k \cdot n_u \\
&z'_3 - z'_4 = k \cdot n_u
\end{align*}
\]

where: \( k \) - is a constant (\( k = 1, 2, \ldots \)), \( n_u \) - the number of waves deformation (\( n_u = 2 \)), \( z \) - the number of teeth of the wheels.

Figure 2 shows three constructive types of mechanical generators with two deformation waves, materialized by means of cam numbered (1), two eccentric discs (2) and two rolls (3), respectively in Figure 3 are shown some flexible toothed wheel models of the double harmonic transmission studied.

![Figure 2. Mechanical wave types generators.](image)

![Figure 3. Flexible toothed wheel models.](image)
3. Stress distribution of the flexible toothed wheel simulation study

The determination of the stress distribution of the flexible toothed wheel was achieved by numerical simulation of the flexible wheel with the help of the Simulation module in the SolidWorks software, using the Finite Element Method.

The numerical simulation of the flexible toothed wheel involved the following stages: adopting the geometry of the wave generator model and the flexible wheel, creating the simulation study, selecting of the flexible wheel material, applying the restraint, applying the forces, meshing the model, running the simulation study, viewing of the results, analyzing of a new study [23].

The geometric models of the 3 wave generators are shown in Figure 4, where the numerical simulation of the flexible wheel was performed distinctly for three cases of wheel deformation, caused by three types of wave generators, namely: with 2 rolls (Figure 4, a), with 2 eccentric discs (Figure 4, b) and by cam (Figure 4, c).

The model of the flexible toothed wheel has the shape of a short cylinder that is open at both extremities and has two toothed crowns (one outer and one inner). The characteristic geometric dimensions of the adopted model are as follow: inner radius, \( r = 29.3 \) mm; wheel length, \( l = 30 \) mm; width of the teeth, \( b = 12 \) mm and wall thickness, \( s = 0.6 \) mm. The material selected for the flexible wheel is an alloy steel which has the following mechanical characteristics: Young’s modulus, \( E = 2.1 \times 10^{11} \) N/m\(^2\); Poisson’s ratio, \( \nu = 0.28 \); mass density, \( \rho = 7700 \) kg/m\(^3\), tensile strength, \( \sigma_{at} = 723.83 \) MPa and yield strength, \( \sigma_{y} = 620.42 \) MPa.

Figure 5 shows the applied restraints to the flexible toothed wheel for the three types of wave generators: with 2 rolls (Figure 5, a), with two eccentric discs (Figure 5, b), respectively with a cam (Figure 5, c).

![Figure 4. The geometric models wave generators.](image)

![Figure 5. Applied restraints of the flexible toothed wheel.](image)
The geometric model of the flexible wheel has the Oz axis as symmetry axis. The xOy plane is oriented on the NSVE surface of the flexible wheel, where the Ox axis is positively oriented from V to E and the Oy axis is positively oriented from S to N.

The restraints were applied in the points N1 and S3 in the contact areas of the 2 rolls with the flexible wheel, respectively in the points N1 and S4 in the contact areas of the 2 eccentric discs with the flexible wheel.

To both types of wave generators, four restrictions were applied: 2 restrictions that cancel the displacement of the contact points in the Ox direction (\(x_{N1} = 0\) and \(x_{S3} = 0\)), respectively (\(x_{N1} = 0\) and \(x_{S4} = 0\)) and other 2 restrictions that materialize the deformation of the flexible wheel in the direction of the Oy axis (\(y_{N1} = 0.3\) mm and \(y_{S3} = 0.3\) mm), respectively (\(y_{N1} = 0.3\) mm and \(y_{S4} = 0.3\) mm).

In the case of the deformation of the flexible wheel with a cam generator, four points N1, S3, V5 and E7 were considered in the contact areas and eight restrictions were applied: 4 restrictions that cancel the displacement of those points in the Ox direction (\(x_{N1} = 0; x_{S3} = 0; x_{V5} = 0\) and \(x_{E7} = 0\)) and other 4 restrictions materializing the deformation of the flexible wheel in the direction of the Oy axis (\(y_{N1} = 0.3\) mm and \(y_{S3} = 0.3\) mm - oriented towards the outside of the flexible wheel, respectively \(y_{V5} = 0.3\) mm and \(y_{E7} = 0.3\) mm - oriented towards the inside of the flexible wheel).

On all 3 types of wave generators, a Roller/Slider restriction was applied to the parallel and opposite side of the NSVE surface. This restraint allows the points on this plane to move freely in their plane without being able to move perpendicularly to this plane.

Figure 6 shows the applied loads on the flexible toothed wheel, and Figure 7 shows the mesh of the flexible wheel model, which generates a number of 63,687 finite elements and a number of 123,511 nodes.

\[
F_{r_{\text{max}}} = q_{t_{\text{max}}} \cdot b = \pi \cdot M_{a4} \cdot p \cdot (2 \varphi_2 \cdot d_2^2) / (2 \varphi_2 \cdot d_2) \quad (3)
\]

\[
F_{r_{\text{max}}} = F_{r_{\text{max}}} \cdot \tan \alpha \quad (4)
\]

where: \(q_{t_{\text{max}}}\) - is the tangential force per unit length of the teeth; \(b\) - the length of the teeth; \(p\) - the circular pitch; \(d_2\) - the diameter pitch circle of the flexible wheel; \(\varphi_2\) - the positioning angle of the engagement zone; \(\alpha\) - the profile angle of the tooth and \(M_{a4}\) - the output torque.
The equivalent stresses von Mises in the flexible wheel body were calculated through linear static analysis, for 5 cases of solicitation of the double harmonic transmission: $M_{t4} = (0, 25, 50, 75, 100)$ Nm. The calculation of the equivalent stress in the case of compounded loads was made after von Mises (using the 5th theory of resistance) with the relation [2]:

$$
\sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \cdot \sigma_2 - \sigma_2 \cdot \sigma_3 - \sigma_3 \cdot \sigma_1}
$$

where: $\sigma_1$, $\sigma_2$ and $\sigma_3$ - are the normal stresses after main directions.

In the numerical analysis carried out in the SolidWorks software, the variations of the resultant displacement and von Mises stress in the flexible wheel body depend on the transmission output torque. The results obtained can be viewed by graphical display (diagrams and color maps) or in analytical form (by numerical values for von Mises stress and displacements) were studied.

3.1. Case of the wave generator with 2 rolls

The analysis model (Figure 4, a) consists of a flexible toothed wheel and a wave generator with 2 rolls. The 2 rolls were modeled by two identical straight cylinders, characterized by the roll diameter, $d_r = 22$ mm and the roll length of $b_r = 8$ mm.

Figure 8 shows the von Mises stress ($\sigma_{vMmax}$) and the resultant displacement ($\Delta$) for the wave generator with 2 rolls, for a transmission output torque of $M_{t4} = 100$ Nm.

![Figure 8. The von Mises stress and the resultant displacement for the wave generator with 2 rolls.](image)

3.2. Case of the wave generator with 2 eccentric discs

The analysis model (Figure 4, b) consists of a flexible toothed wheel and a wave generator with 2 discs mounted eccentrically. The 2 discs were modeled by two identical straight cylinders, characterized by the disc diameter, $d_d = 56$ mm and the disc length, $b_d = 8$ mm.

Figure 9 shows the von Mises stress ($\sigma_{vMmax}$) and the resultant displacement ($\Delta$) for the wave generator with 2 eccentric discs, for a transmission output torque of $M_{t4} = 100$ Nm.

![Figure 9. The von Mises stress and the resultant displacement for the wave generator with 2 eccentric discs.](image)
3.3. Case of the wave generator with cam
The analysis model (Figure 4, c) consists of a flexible toothed wheel and a wave generator with a plate cam, characterized by a constant width, \( b_c = 8 \text{ mm} \).

Figure 10 shows the von Mises stress (\( \sigma_{\text{vMmax}} \)) and the resultant displacement (\( \Delta \)) for the wave generator with cam, for a transmission output torque of \( M_{\text{td}} = 100 \text{ Nm} \).

![Figure 10](image)

Figure 10. The von Mises stress and the resultant displacement for the wave generator with cam.

4. Numerical simulation results of the flexible toothed wheel
In the numerical simulations of the flexible toothed wheel, it has been studied the influence of the type of the wave generator on the displacements of the nodes positioned on some wheel generatrix, as well as on the von Mises stresses appearing in the flexible wheel body, by considering the different output torque at the double harmonic transmission.

Thus, the numerical simulations of the flexible wheel were made both for the case of the unloaded double harmonic transmission (\( M_{\text{td}} = 0 \)) and for the cases of its 4 loading stages (\( M_{\text{td}} = 25, 50, 75, 100 \text{ Nm} \)). The succession of the numerical simulations of the flexible wheel has been preserved for all three cases of deformations, using the three types of analysed wave generator.

The obtained results from the numerical simulations of the flexible wheel are presented in Table 1 - for the resultant displacement (\( \Delta \)) of the characteristic nodes located on the N and E generatrix direction of the flexible wheel, respectively in Table 2 - for the maximum von Mises stress (\( \sigma_{\text{vMmax}} \)).
**Table 1.** Resultant displacements of the characteristic nodes.

| Node no. | Positioning coordinate $z_{nod}$ (mm) | Resultant displacement $\Delta$ (mm) | rolls - N | discs - N | cam - N | cam - E |
|----------|--------------------------------------|--------------------------------------|-----------|-----------|---------|---------|
| 1        | 0                                    | 0.3                                  | 0.3       | 0.3       | 0.3     | 0.3     |
| 2        | -3.72                                | 0.3                                  | 0.3       | 0.3       | 0.3     | 0.3     |
| 3        | -8.27                                | 0.299                                | 0.299     | 0.3       | 0.3     | 0.3     |
| 4        | -12.2                                | 0.297                                | 0.297     | 0.299     | 0.299   | 0.299   |
| 5        | -16.75                               | 0.295                                | 0.294     | 0.299     | 0.299   | 0.299   |
| 6        | -20.65                               | 0.292                                | 0.293     | 0.299     | 0.298   | 0.298   |
| 7        | -25.15                               | 0.291                                | 0.292     | 0.298     | 0.298   | 0.298   |
| 8        | -30                                  | 0.291                                | 0.291     | 0.298     | 0.297   | 0.297   |

It is notice from Table 1 that the positions of the characteristic nodes on the considered generatrix of the flexible wheel are given by the negative values of the coordinate $z$, because the Oz axis has a positive orientation in the opposite direction to the mentioned generatrix.

**Table 2.** Maximum von Mises stress.

| Output torque $M_{44}$ (Nm) | Tangential force $F_t$ (N) | Radial force $F_r$ (N) | Stress $\sigma_{\text{v Mises max}}$ (MPa) |
|---------------------------|---------------------------|-----------------------|------------------------------------------|
|                           |                           |                       | rolls | discs | cam      |
| 0                         | 0                         | 0                     | 362.1 | 308.3 | 287.1    |
| 25                        | 13.08                     | 4.76                  | 368.5 | 312.6 | 291.4    |
| 50                        | 26.16                     | 9.52                  | 370.1 | 314.8 | 292.5    |
| 75                        | 39.24                     | 14.28                 | 370.9 | 315.4 | 293.8    |
| 100                       | 52.32                     | 19.04                 | 371.8 | 317.3 | 294.5    |

Figure 12 shows the resultant displacement variation diagrams of the characteristic nodes on the finite elements located in the direction of the N and E generatrix.

**Figure 12.** Variation of the resultant displacement $\Delta = \Delta(z)$.

From the analysis of the resultant displacement diagrams ($\Delta = \Delta(z)$), it can be observed that for the characteristic node positioned on the generatrix N of the flexible wheel, at the coordinate $z = 0$, for the resultant displacement has been obtain exactly the value of the maximum radial deformation of the flexible wheel ($\Delta(0) = w_0 = 0.3$ mm), for all 3 types of analysed wave generators.

Figure 13 shows the von Mises stress variation diagrams for the deformation of the flexible wheel by the analysed wave generators.
Figure 13. Maximum von Mises stress diagram $\sigma_{Vmax} = \sigma_{Vm}(M_t)$.

It can be noticed that in the case of a cam wave generator, appear the smallest stresses in the body of the flexible wheel, because its deformation is a controlled one, realized by winding the entire inner contour of the wheel on the cam profile.

The generators with rolls or eccentric discs have the disadvantage that the flexible toothed wheel can be deformed freely outside the contact areas with rolls or discs, which increases the stress in the wheel body.

5. Conclusions

The paper is a contribution of the authors in the study of the dynamic of the flexible toothed wheel of a double harmonic transmission, for three distinct deformation cases using three types of wave generators (2 rolls, 2 eccentric discs and by cam).

The numerical simulations of the flexible toothed wheel for the three types of wave generators have confirmed the dependence of the stress distribution on the flexible wheel wall of the transmission output torque ($M_\alpha$). The maximum von Mises stress ($\sigma_{Vmax}$) increases when the transmission output torque increases, but the maximum von Mises stresses always remain below the yield strength ($\sigma_c$). The maximum stress value occurs in the vicinity of the two points of application of the elastic deformation force of the flexible toothed wheel from the active components of the wave generator.

From the analysis of the resultant displacement diagram of the nodes located on the N generatrix of the flexible wheel, it can be observed that the resulting displacement has a decreasing character with the increasing of the distance of the NSVE surface nodes of the flexible wheel. The maximum resultant displacement was obtained for the node (given by coordinate $z = 0$) positioned in the action area of the wave generator on the flexible wheel and it is equal to the maximum value of the radial deformation ($\Delta(0) = w_0 = 0.3$ mm).

From the analysis of the obtained results, it can be concluded that the cam wave generator is superior to the 2 rolls and 2 eccentric discs generators, regarding the assurance of a uniform and controlled deformation of the flexible wheel and regarding the stress distribution developed in the flexible wheel wall.

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