Phonon phenomenon in the interaction of guided ultrasonic waves with a surface grating

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Abstract. This paper deals with the interaction of guided ultrasonic waves with a surface grating. For particular frequencies, reflected converted waves are observed. At the entrance of the periodic grating, a phonon relation is written between the incident mode, the converted mode and the phonon related to the grating. An experimental verification of the phonon relation is carried out. Interpretation of the conversion phenomena is done by considering the dispersion curves of the propagating modes in a plate with an infinite periodical surface grating. The dispersion curves exhibit folding effect and several stop bands appear.

1. Introduction
Guided ultrasonic waves in plates (Lamb waves) are very sensitive to the surface state. If the surface is slightly rough, the amplitude of the propagating waves is attenuated. The particular case of periodic rough surface is quite different. For certain frequency the incident wave gives rise to converted backward modes. In a previous paper \cite{1} authors show that the conversion/reflection phenomenon is a consequence related to the opening of forbidden frequency bands. A forbidden band corresponds to the interaction of one or two given modes and does not affect the other propagating modes \cite{2, 3}. A simple relation between the wave number of incident wave, the wave number of the converted wave and the phonon of the grating exists. The case of a two dimensional rectangular grating is studied. Lamb waves are generated by the wedge method while laser vibrometer is used to measure the normal displacement of the plate surface opposed to the grating. Then signal processing allows us to plot the dispersion curves of the guided waves in the first Brillouin zone. The dispersion curves exhibit folding effect and several stop bands appear: classical band gaps at the boundary of the Brillouin zone and mini-stop-bands inside the Brillouin zone. Mini-stop-band leads to anti-crossing of dispersion curves and possible coupling occurs between the modes. It is shown that these “mini-stop bands” exist for the frequencies where converted waves are experimentally observed.

2. Dispersion curves in the first Brillouin zone
The studied plate is engraved with parallelepipedic grooves (fig.1). Periods are equal to 8mm in x direction and 6mm in the perpendicular direction. The thickness of the aluminium plates is 4.54 mm whereas the corrugation depth $p$ is 180 $\mu$m. The corrugation of the surface is weak and the guided waves are the well-known Lamb waves which are defined in an infinite smooth plate. The analytical
dispersion curves of Lamb waves in a smooth plate are plotted in figure 2.a. Within the studied frequency range the periods of the surface corrugation lengths are commensurate with the wavelength $\lambda_e$ of the Lamb waves.

Figure 1. Geometry of the studied sample.

In the experimental study, only generation of P-SV waves are allowed. Therefore the FEM study can be limited to a 2D analysis. The waveguide is considered as a repetition along the $x$ direction of an elementary cell of the grating (Fig. 1). On each boundary of the pattern, a Bloch-Floquet condition has to be fulfilled. The angular frequency $\omega$ is a periodic function of the wave vector $k$ and then the study is restricted to the first Brillouin zone. The dispersion curves of the waves propagating in the periodic waveguide are calculated with the FEM. The dispersion diagram is plotted by varying the wave vector in an half Brillouin zone $[0, \pi/A]$, and then the other half zone $[-\pi/A,0]$ is plotted symmetrically (fig 2.b). In figure 2.b, the periodicity of the guide implies that the Lamb waves dispersion curves fold back for $k = \pm \pi/A$ (limit of Brillouin zone) and $k=0$. Figure 3 presents the dispersion curves on periodical corrugated plate in the 300kHz to 450kHz frequency range. They exhibit two kinds of gaps: stop band at the limit of the Brillouin zone or “mini stop band” into the Brillouin zone.

Figure 2. (color online) (a) Lamb wave dispersion curves on smooth plate; (b) Lamb wave dispersion curves on periodical corrugated plate
Mini stop bands are due to the opening of gap at the crossing of two Lamb mode dispersion curves. A forbidden band between $F=415\text{kHz}$ and $F=419\text{kHz}$ exists for the S0 and A1 modes (fig. 3). This mini stop band leads to “anti-crossing” of the dispersion curves. There is a continuous curve from S0 to A1 modes. One another type of stop band exists at the boundary of the Brillouin zone. These stop bands are located in a frequency range where a dispersion curve fold back. An example of stop band is shown in Fig. 3 from $F=322\text{ kHz}$ to $F=326\text{ kHz}$. This stop band is located at the limit $k = \pi / \Lambda$ where the S0 dispersion curve is folding back. At this point there is an intersection of two dispersion curves: the incident S0 wave dispersion curve ($k>0$) and the reflected one ($k<0$) shifted by $2\pi / \Lambda$.

In a previous paper [1], authors show that at each band gap, a relation can be written between the wave vectors of the incident and reflected converted waves (respectively $\vec{k}_{\text{inc}}$ and $\vec{k}_{\text{conv}}$):

$$\vec{k}_{\text{inc}} - \vec{k}_{\text{conv}} = n \vec{G} \quad (1)$$

where $n$ is an integer. This relation implies one basis vector of the reciprocal lattice $\vec{G}$ defined by $|\vec{G}| = 2\pi / \Lambda$ and $\vec{G} = \vec{G} u_x$ if the propagation concerns only the $x$ direction which is the case in this paper. The integer $n$ means that incident and converted wavelengths are also coupled to the harmonics $(\Lambda / n)$ of the grating. For stop band located at the edge of the Brillouin zone, $\vec{k}_{\text{inc}}$ and $\vec{k}_{\text{conv}}$ are related to the same mode. For mini stop bands these wave vectors are related to two different modes. About the frequency $F=360\text{kHz}$ there is an anticrossing of A0 and A1 modes; relation (1) holds with a value of $n=1$. In this case, the two waves propagate in the same direction. There is no forbidden band contrarily to the band gap observed between S0 and A1 at about $F=417\text{kHz}$.

3. Experimental study

Lamb waves are generated by the wedge method using an emitting piezocomposite transducer (central frequency 1 MHz). The emitting transducer is positioned at a distance $x=20 \text{ mm}$ from the grating. The normal displacements of the surface are detected by a laser interferometer from $x=10 \text{ mm}$ (origin $x=0$ corresponds to emitting transducer position) to $x=90 \text{ mm}$ by $0.1 \text{ mm}$ step. For each spatial position, the amplitude is recorded. A 2D FFT is applied to the resulting $(x, t)$ image in order to obtain the experimental dispersion curves of the waves propagating in the plate in the $(k, f)$ space (figures 4 and 5). The superimposition of the theoretical dispersion curves of Lamb waves (plotted in fig 2.a) leads to the identification of the modes. On the frequency range $[500-800 \text{ kHz}]$, the S0 mode is generated whereas incident A1 mode is present in the frequency range $[1-1.25 \text{ MHz}]$. Figure 5 shows that different back reflected converted waves are present under and upstream the grating. Spots a, b, c are due to the conversion of the incident S0 Lamb wave on the grating whereas spots d, e and f are due to the incident A1 Lamb wave.

At $F=320\text{kHz}$ (spot (a)), the S0 incident wave is reflected. Around this frequency a gap exists in the dispersion curve of the S0 mode (figure 3). Indeed, at the edge of the Brillouin zone
−=−=\bar{k}_{inc} n \bar{G}

with \bar{k}_{inc} = \bar{k}_{S0}, \bar{k}_{conv} = \bar{k}_{S0} and n=1. At k=0 or \( k = \pm \pi / A \), the opening of a stop band implies that two counterpropagating modes are coupled. This coupling is also verified for spot (b) at \( F=775kHz \).

A strong conversion in A1 mode is also observed in figure 5 spot (c). At this frequency the wavevector of the A1 mode verifies the relation 1 with \( n=3 \):

\[ -k_{A1} = k_{S0} - \frac{2\pi}{A/3} \].

This conversion is linked to the existence of the third harmonic in the PSD of the grating profile. Conversion in S0 mode (spot (e)) and S1 mode (spot(f)) are due to the interaction of the A1 incident wave with the harmonic \( A/5 \) of the grating. Spot (d) is linked to the third harmonic. The square profile has only odd harmonics. However spot (b) corresponds to the fourth harmonic. Imperfections of the experimental grating can explain this fact. Stop bands seem to appear according to the grating period and this leads to an energy transfer between modes.

4. Conclusion

The interpretation of the interaction of Lamb waves propagating on a plate with one surface covered by periodic grating needs the use of well known concepts in solid physics or in photonics: phonon, Brillouin zone, mini stop band. The experimental evidence of these phenomena is achieved by means of optical measurement of the normal displacement of the plane interface associated to relevant signal processing.

References
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