A Method for Surveying Control Network Optimization Based on Reliability Properties

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Abstract  Surveying control network optimization design is related to standards, such as precision, reliability, sensitivity and the cost, and these standards are related closely to each other. A new method for surveying control network simulation optimization design is proposed. This method is based on the inner reliability index of the observation values.

Keywords  surveying control network; optimization design; reliability; redundant observation weight

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Introduction

There are two methods for surveying control network optimization designs, the analytic method and the simulative method, and both depend on computer programs with vast calculations.

The analytic method is a rigorous one, based on the optimization theory, and its mathematic model is generally as follows:

\[
\begin{align*}
    & f(X) \rightarrow \min \\
    & \varphi(X) \geq 0 \\
    & \Psi(X) = 0 \\
\end{align*}
\]

In Eq.(1), the first expression is the target function, and the last two are the restriction conditions. The target function may be precision, reliability, sensitivity or cost, and which also may be the index for the restriction conditions. The essence of this method is to seek the optimizing answer by finding the extremum of the target function based on the restriction conditions. In practice, we should adjust the observation scheme which needs be optimized, so the result of analytic method is not the optimum. It is difficult to develop a universal software for optimization with the analytic method. There is more research on analytic method, but the application of it is few.

The simulative method is a test-calculation one. Usually, an initial design scheme is set according to the optimization task and designer’s experience, and then adjustments and calculation analysis are made with the simulation observations. Calculations, modifications and such, are conducted until the requirement are met. The shortcoming of simulative method is that it depends on designer’s knowledge and experience, and the differences between schemes designed by different people are perhaps great.

This study presents an optimization design method for surveying control networks which is based on the inner reliability of observations. This method just needs one reliability index quantified, and the result of does not change with the designer’s knowledge and experience. It has the character of consistency and strictness. To use it, a module just needs to be added to the general software packed for data proc-

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essing of surveying networks.

It is valuable to illuminate that the method fits the ground side angle network, as well as the GPS network. GPS baseline vector can be regarded as one side projected to the Gaussian plane corresponding to the surface of a certain ellipsoid, and its length and direction can be obtained. Therefore, GPS network can be regarded as the plane network with sides and directions observed.

1 \( r_i \)' s calculation and its character

The reliability of network can be classified into collectivity reliability, inner reliability of observations (local reliability), and exterior reliability. Hereon, we just mainly discuss the inner reliability of observations.

For a surveying control network, the inner reliability \( r_i \) of observation \( l_i \) can be obtained by the parameter adjustment model \((l, Ax, \sigma^2, p^{-1})\):

\[
r_i = (Q_{ir}, P_{ir})
\]

and \( r_i \) satisfies:

\[
\sum_i r_i = r = n - t
\]

In Eq.(2), \( Q_{ir} \) is the covariance matrix of the observations’ correction value and \( P \) is the observations’ weight matrix. The down limit of gross error \((\nabla l_i)\) can be found for observation \( l_i \) using the following equation:

\[
\nabla l_i = \frac{\sigma_i \cdot \omega_i}{\sqrt{r_i}}
\]

where \( \omega_i \) is the non-center parameter, and its value, relating to marked level \( \alpha \) and test efficacy \( \gamma \), is always \( 2.79 (\alpha = 0.05, \gamma = 0.80) \) or \( 4.13 (\alpha = 0.001, \gamma = 0.80)^{[2]} \); \( \sigma_i \) is the mean square error (MSE) of observation \( l_i \); and \( r_i \) is also named as the redundant observation weight of \( l_i \). The larger \( r_i \), the less \( \nabla l_i \), and the capability for finding out gross error is stronger. While the observations are independent each other (this study just discusses this instance only), there is:

\[
r_i = 1 - \frac{\hat{\sigma}^2}{\sigma_i}
\]

where \( \hat{\sigma}^2 \) is the variance of the adjusted value of \( l_i \).

The inner reliability of observation \( r_i \) has the following characteristics.

1) \( 0 \leq r_i \leq 1 \). The less \( r_i \), the more important the observation in the network. If \( r_i = 0 \), the observation’s value is known in the network; and the larger \( r_i \), the less important the observation in the network. If \( r_i = 1 \), it indicates that the known value is observed, and that this observation should be deleted.

2) The less \( r_i \), the larger \( \nabla l_i \) that can be discovered in observations; the larger \( r_i \), the less \( \nabla l_i \) that can be discovered in observations; and when \( r_i \) becomes larger, the influence of \( \nabla l_i \) to the adjusted result gets less.

3) For a certain network or design scheme, which is to say that the figure of the network, the number of observations and the precision are all fixed, the higher the observation precision, the less \( r_i \); and the lower observation precision, the larger \( r_i \), i.e. the inner reliability of observation is inversely related to the its precision.

4) For a certain observation precision, the more redundant the observation, the larger \( r_i \), and the more cost for network constitution.

5) For a unattached network, \( r_i \) is irrespective with the position of the benchmark.

Suppose that the number of points in the network is \( m \), the number of known points is \( m_k \), the number of unknown points is \( m_u \), the number of known sides is \( k \), and the number of known azimuths is \( k_a \); the number of station with azimuth observation is \( m_a \); the number of station only with side observation is \( m_s \), hereinto, the number of stations using the known points with azimuth or side observation is \( m_t \); and the number of all observations is \( n \) (the number of azimuth observations is \( n_a \), the number of side observations is \( n_s \), and the number of necessary observations is \( n_t \)), then the redundant observations \( r \) of the network and the average redundant observation weight \( \bar{r} \) of observations can be calculated with the formulae as follows:

\[
r = n - t = n - \left(3m_a + 2m_s - 2m_k - k - k_a\right)
\]

\[
\bar{r} = \frac{L}{n}
\]

Table 1 lists the number of observation and average redundant weight for connecting traverse with different points. While the number of unknown points is
more than 4, \( \mathcal{R} \) is less than 0.2. According to reliability index, connecting traverse can not be the construction control network and deformation monitoring network.

### Table 1  Number of observation and average redundant weight for connecting traverse

| Number of unknown points | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( n \)                  | 8   | 11  | 14  | 17  | 20  | 23  | 26  | 29  | 32  |
| \( \mathcal{R} \)        | 0.375 | 0.27 | 0.21 | 0.18 | 0.15 | 0.13 | 0.115 | 0.10 | 0.09 |

In order for the network to have enough reliability, we put forward the concept of design value of average redundant weight while optimization design, and denoted it as \( \mathcal{R}^0 \). For different networks, the value of \( \mathcal{R}^0 \) can be 0.3-0.6. With \( \mathcal{R}^0 \), the design value \( n^0 \) of the number of observations can be calculated as follows:

\[
n^0 = \frac{I}{1 - \mathcal{R}^0}
\]  

\[(8)\]

**2 Optimization design based on \( r_i \)**

As we know, the quality standard of a network includes precision, reliability and the cost for network construction, and there are two more rules, sensitivity and distinguishable, for deformation monitoring network. It has been proved that inner reliability of observation is related to the precision of observation and the cost for future network construction. Moreover, the sensitivity of the deformation monitoring network is the precision of the points in specific directions, and the distinguishable quality of deformation and gross error is inevitably referred to precision. Therefore, the inner reliability of observation is related close to the precision of observation, cost of network constitution, sensitivity of deformation monitoring network and the distinguishable quality, some of which can be simplified as Eqs.(3)-(5) or the mathematical formulae presented in Reference [3], but some of which are difficult to express with mathematical formulae. With the character of inner reliability of observation and the discussion above, we can conclude the idea of optimization design based on reliability as follows.

1) There should be some redundant observations in a network, and the larger the number of observations \( r \), the better reliability of the network, and the more cost for network construction.

2) With the number of redundant observations fixed, the difference between the precision of observations should not be too large, and the precision of side and angle should be probably matching.

3) According to the request (the aim and precision) for the surveying control network design, instruments should be selected and be ascertained of their precision, graph design and exploration on the spot should be made, and the initial observation scheme should be set down. Observation precision should be at the highest that the instruments can obtain, which should make the precision to have space to reduce while optimization. In the initial observation scheme, all the potential observation sides and directions should be observed, and it should have the largest redundant number, and network’s initial observation network should be a “fat network” or a “dense network”.

4) Simulation calculations and adjustments for the initial observation scheme should be made, and the results should be analyzed. Above all, we should see if the precision is reasonable, if not, we should adjust the observation scheme. With a reasonable precision, the method based on inner reliability index of the observation value optimizes the network according to the strategy that made the network from “fat” to “thin”, “dense” to “sparse”. The steps are as follows: fan apt \( \mathcal{R}^0 \) should be fixed, \( n^0 \) should be calculated with Eq.(8) and \( n_j \) (the number of observation value that should be threw off), the redundant observation weights should be ordered from large to small, \( n_j \) (the observation values with larger redundant observation weight) should be thrown, and then simulation calculations and adjustments should again be made for the scheme. If the precision of the observation value is pretty selected, we can obtain an optimization design scheme with just one or two times calculation.

**3 Example and analysis**

A bridge control network is simulated with an absolute coordinate system, and is laid out with ground
side angle surveying technology. The length of the bridge is about 800 m. According to the graphic design and exploration on the spot, as Fig.1(a) shows, 9 points are disposed in the network, 3 points (QZ1, QZ2, QZ3) in the axes, 2(QN1, QN2) points in the south bridge approach, and 4 points in the south and north with each side having 2 points. The precision of the weakest point is ≤ 4.0 mm, the relative precision of weakest side is prior to 1/120 000, the MSE of the side of the largest two points is ≤ 5.0 mm, and its relative precision is prior to 1/300 000.

Two schemes, one with two fixed points (QZ1 and QZ2), and the other with one point and one side fixed, were adopted. The initial scheme is a complete side angle network, observing 70 directions and 34(35) sides with opposite directions, as Fig.1(a) shows. The observation precision of initial direction is 1.8′′, and the precision of the side is a = 2, b = 2 ppm. The calculation results of the initial scheme are as follows: the position precision of the weakest point is 1.7′′, the relative precision of weakest side is 1/160 000, and the MSE of the side of the largest two points is just 1.0 mm. Obviously, the precision is on the high side. In the light of experience, the observation precision of initial direction played down to 2.0′′.

With the method proposed in this study and corresponding software, making optimization design, the result is shown in Table 2.

Table 2  Optimization result of bridge control network

| Schemes                  | Average redundant observation weight | Number of direction and side | MSE of weakest point /mm | MSE of weakest side/mm and its relative precision | MSE of the side of the farthest two points/mm and its relative precision |
|--------------------------|--------------------------------------|------------------------------|----------------------------|--------------------------------------------------|------------------------------------------------------------------------|
| Initial scheme           | 0.78; 0.77                           | 70; 34,35                    | 1.7; 1.9                   | 0.9; 1:155 000                                  | 1.0; 1:260 000                                                          |
| Fix two points           | 0.60                                 | 38; 19                       | 2.8                       | 1.1; 1:135 000                                  | 2.0; 1:640 000                                                          |
| Fix two points           | 0.55                                 | 34; 17                       | 3.4                       | 1.1; 1:134 000                                  | 2.4; 1:530 000                                                          |
| Fix one point and one side | 0.60                               | 40; 20                       | 2.6                       | 1.1; 1:134 000                                  | 2.1; 1:595 000                                                          |
| Fix one point and one side | 0.55                               | 35; 18                       | 3.7                       | 1.1; 1:132 000                                  | 3.7; 1:377 000                                                          |

As Table 2 shows, the scheme with one point and one side fixed and 0.55 as the average redundant weight is a better optimization design, shown in Fig.1(c), which is close to the scheme with two points fixed and 0.55 as the average redundant weight, shown in Fig.1(b). However, if the two points were fixed, we should survey the side QZ1-QZ2 with higher precision. Analyzing Fig.1(c) carefully, we can discover that the optimization design scheme is a comparatively nice one, and the directions or sides joining the points are well-proportioned. Analyzing the directional values deleted, there are 3 points (QZ1,
QN1 and QZ3) with all the directions surveyed. The directions of short sides in point NN were all surveyed, and just 2-3 directions of short sides in the rest points were surveyed. For the sides, the short sides in the south and north were all surveyed, but just QZ2-ND and BD-ND, two sides among the 8 sides link the south and the north, were surveyed, and just the directions of the other 6 sides were surveyed. Furthermore, more were surveyed in just a single direction. This suggests that the observation values deleted were related more to the length of the sides.

With the method of gradual approaching, the average redundant weight is reduced more, such as 0.54-0.50. Then, the graph of the network will be malformed, but the precision does not lose greatly. With all balanced, the scheme shown in Fig.1(c) was adopted.

4 Conclusions

The optimization design for surveying network, especially for construction control network and deformation monitoring network, is still worthy for research and application. Based on thoroughly re-searching the inner reliability of observation, this study concludes the relations between redundant observation weight and quality standards, such as precision, cost of network construction, sensitivity and so on. The idea and algorithm of simulative methods for optimization design based on reliability of observation are put forward. Based on the general software packed for data processing of surveying network, we have developed an optimization design programmer, and have explained the process of network optimization design by example. The method fits ground side angle network, as well as the GPS network. With this method, we not only can process network optimization design, but also evaluate if the network design is good or not.

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