Analysis of phase noise influence on micro-Doppler feature extraction of vibrating target

Zihao Liu, Bo Peng, Xiang Li

Abstract: It is generally considered that increasing the carrier frequency of radar is an important way to improve the precision of micro-motion measurement. However, the increase of the radar frequency may raise the phase noise intensity of the radar transmitting signal and make the extraction more difficult, therefore it is particularly necessary to study the influence of phase noise on the extraction of micro-motion characteristics. In this study, specific studying about the effect of phase noise on the extraction of micro-Doppler (m-D) features is carried out. The effect of phase noise on the extraction performance of the m-D features is evaluated based on a parameter micro-motion signal-to-clutter ratio and an empirical formula is put forward based on the experiments. The results of simulation experiments indicate that increasing the carrier frequency cannot improve the extraction performance of micro-motion features in the case of using both time–frequency analysis method and the new developed sinusoidal frequency modulation Fourier transform (SFMFT) method. Meanwhile, increasing the frequency of target's micro-motion has an improvement effect through time–frequency method due to the increase of Doppler frequency.

1 Introduction

The extraction of micro-Doppler (m-D) features is an important means to discover the characteristics of the targets [1, 2]. Micro-Doppler features have been widely used to recognise the targets and there have been decades of work on the extraction of micro-Doppler features. Many studies are aimed at the m-D effect of ballistic targets [3–6]. Most of the analysis methods are based on the time–frequency distribution method [4–9], some of them use the generalised Hough transform [6], Hilbert–Huang transform [7] and the inverse Radon transform [8]. Micro-motion induces phase modulation in radar echo signals and is reflected in the change of the instantaneous frequency. Moreover, phase noise is the characterisation of the random phase jitter of the signal and a random phase modulation which will cause the change of the instantaneous frequency of radar signal. Phase noise directly affects the extraction performance of m-D features and currently research on this is not very abundance.

Jose have studied the moving human micro-Doppler signature in forest environments, two continuous-wave radars at 1 and 5 GHz are used to detect human motion and a time–frequency transform is applied to analyse the echo signal. The paper finally draws the conclusion that the responses from the different body parts are not easily distinguishable for the 1 GHz case, and higher frequencies are preferable to identify human motion in a Doppler signature. However, with higher carrier frequency, it does not achieve the corresponding improvement effect [9]. Gu [10] carried out an experiment with two Doppler radars which carrier frequency is separately 2.4 and 24 GHz, the results showed that when using the Doppler radar to measure the vibration target, the radar input signal-to-noise ratio (SNR) did not rise with the increase of carrier frequency. The study explains the phenomenon in theory; however, the significant difference in phase noise of two carrier motion frequencies should also be taken into account. Yan and Wang [11] have reported on the frequency stability constraints of radar target on m-D feature extraction, in the study the inverse Radon transform is used to process the time–frequency curve and preliminary conclusions are obtained. Zhou and Wang studied the effect of phase noise on the extraction of micro-motion characteristics and drawn the conclusion that the near-end phase noise within 1 KHz offset carrier frequency had greater effects on the extraction of m-D features [12].

The last two studies have been carried out from the point of view of the measured data and simulation results, but are lack of the quantitative relationship between the model and intensity of phase noise and the micro-motion measurement, and are also lack of a detailed comparison of the typical micro-motion estimation methods in the respect of the sensitivity to phase noise.

In this article, the oscillator phase noise model and the single target micro-motion model is studied first. Then in order to analyse the effects of factors like vibration amplitude and sampling rate on the extraction performance of micro-motion features, the simulation of micro-motion feature extraction is carried out using both time–frequency analysis method and the SFMFT method [13, 14]. Finally, the experimental results are summarised and the conclusion is concluded.

2 Methodology

In the following text, a radar detection model which consists of a single vibrating point as a target and a single frequency continuous-wave radar will be set up to study the relationship between the extraction of micro-motion features and phase noises. As shown in Fig. 1, the radar observation model consists of several steps: parameters setting of the transmitted signals, interaction between the transmitted signals and vibrating targets, sampling and processing of the received radar signals. Suppose that the radar target is a single scattering point which is acting sinusoidal

Fig. 1 Radar observation model of a single vibrating point target
vibration with vibration frequency $f_v$, amplitude $D_V$, origin phase $\phi_v$, and distance between the vibration centre and the Radar $R_c$. Then the real-time distance between the target and radar $R(t)$ is

$$R(t) = R_c + D_V \sin(2\pi f_v t + \phi_v)$$ (1)

If not noted specially, the default simulation parameters in the following text would be as listed in Table 1.

It is known from the literature [11, 15] that in the frequency domain, the phase noise of the highly stable crystal oscillator can be described by the single-side band (SSB) power spectrum density model:

$$S_p(f) = k_0 + k_i f_m^2 + k_s f_m^3$$ (2)

where $f_m$ indicates the frequency offset from the carrier frequency, $k_0$ indicates phase modulation white noise, $k_i f_m^2$ indicates frequency modulation white noise, and $k_s f_m^3$ indicates frequency modulation glint noise. As shown in Fig. 2, the SSB power spectrum density curve consists of three section curves with different slopes. These characteristics can be used as reference when designing the phase noise model in the remainder of this article.

In addition, the slope of the phase noise in the range $100\,\text{Hz} < f < 1000\,\text{Hz}$ is $-20\,\text{dB/dec}$. It can be concluded that as to a certain frequency source, if there are two frequency points offset from carrier frequency $f_m$ and $f_{nc}$, then the power of phase noise $P_{pd/m}$ at these two frequency points $f_m$ and $f_{nc}$ has the following relationship:

$$P_{pd/m} = P_{pd}(f_m) - 20 \left( f_{nc} = 10 \cdot f_m \right)$$

$$P_{pd/m} = P_{pd}(f_m) - 6 \quad (f_{nc} = 2 \cdot f_m)$$ (3)

As shown in Table 2, according to the rules above, one can define the characteristics of phase noise by defining several phase noise intensity values which are decaying exponentially at several specific frequencies offset from carrier frequency.

2.1 Micro-Doppler analysis methods for the study of phase noise influence

There are two main methods to analyse the received signal: time–frequency analysis method and SFMFT method. The time–frequency analysis has been widely used, but also has some insuperable defects. SFMFT method can retrieve the phase modulation process well and has better estimation accuracy.

A common way to practice time–frequency analysis is peak detection algorithm, which extracts the time–frequency curve and estimates the parameters on the basis of the curve. This algorithm fails to achieve the desired effect when the amplitude of vibration is too small which may make the extracted curve be a straight line.

The SFMFT method uses summation of a series of sinusoidal frequency modulation orthogonal basis to represent sinusoidal frequency modulation signal. Digital signal $x(t)$ can be analysed using SFMFT method to get the frequency spectrum of its phase modulation:

$$X(k) = \text{SFMFT}[x(n)] = \frac{1}{N} \sum_{n=0}^{N-1} \ln|\sin(\cdot)| \cdot (\exp(\text{j}(\frac{2\pi n}{N} \cdot k)))$$ (4)

where the twiddle factor can be calculated by $\exp(\text{j}(\frac{2\pi n}{N}))$. Using SFMFT method, one can accurately measure the parameters of the vibration target even if the vibration amplitude is very small.

An experiment is carried out to investigate the performance of the two methods mentioned above. The experimental conditions are listed in Tables 1 and 2; the results of the time–frequency analysis and SFMFT method are shown in Fig. 3.

| Table 1 | Default simulation parameters |
| --- | --- |
| $f_v$, Hz | 20 |
| $\phi_v$, rad | 0 |
| $D_V$, mm | 5 |
| $R_c$, m | 20 |
| carrier frequency, GHz | 1 |
| sampling frequency, kHz | 20 |

2.2 Explication of the simulation experiments

As shown in Figs. 3a and b, the vibration of the target adds frequency modulation to the radar signal. Figs. 3c and d show the result of the time–frequency curve extraction, it can be seen that there is an error in curve extraction result when noise exists. Figs. 3e and f show the fast Fourier transform (FFT) results of Figs. 3c and d, respectively, an obvious clutter can be seen in Figs. 3f. Figs. 3g and h show the results when utilising SFMFT method to analyse the received radar signal. A distinct peak can be seen where the abscissa value is equal to $20\,\text{Hz}$ in Figs. 3e and g. Figs. 3f and h are obtained under the condition of phase noise, a peak can still be seen, but there is a fading clutter which may make the extraction of the vibration frequency more difficult.

2.3 Parameter MSCR

In order to weighing how easily it is to extract the vibration feature successfully, a parameter is defined to weigh how prominent the peak is compared with clutter in the vibration frequency spectrum. The parameter can be calculated based on the formula

$$\text{MSCR} = 10 \cdot \ln(H_1) - \ln(H_2)$$

$$= 10 \cdot \ln \frac{H_1}{H_2}$$ (5)

where MSCR is micro-motion signal-to-clutter ratio (MSCR). $H_1$ represents the vertical ordinate value where horizontal ordinate is vibration frequency, and $H_2$ represents the maximum peak value except $H_1$ in the spectrum of analysis results of FFT and SFMFT methods. Therefore, it is obviously that:

If parameter MSCR $> 0$, it means that the vertical ordinate value at the vibration frequency is larger than any other peak values. For instance, the MSCR can be calculated to be equal to $2.29$ in Fig. 3h. In this situation the vibration frequency can be extracted from the frequency spectrum diagram without difficulty. On the contrary, if parameter MSCR $< 0$, it means that the micro-motion features cannot be extracted.

As the phase noise is a kind of random noise which takes a random effect on the estimation of micro-motion parameters, it should be a reasonable idea to use the Monte Carlo method to analyse the effect of phase noise.

3 Results

3.1 Different intensities of ordinary phase noise

In this experiment, five different intensities of phase noise are listed in Table 3. Under each condition, the experiment is executed 1000 times, the MSCR's mean value in the case of using SFMFT
method is shown in Fig. 4a. The time–frequency analysis of the received signal is also carried out and the trend of parameter MSCR is displayed in Fig. 4b. In addition, the default experimental parameters are as listed in Table 1. As shown in Figs. 4a and b, a decrease of the parameter MSCR along with the increase of phase noise intensity can be observed, which means that the greater the phase noise's intensity is, the more difficult the estimation of vibration parameters will be.

### 3.2 Different carrier frequencies

According to the theories mentioned above, if a low noise frequency multiplier is used to double the frequency of the signal, the signal after doubling operation will take some changes in the performance of phase noise compared to the original signal. Specifically, the phase noise at the same frequency point \( f_m \) will be increased by 6 dBc/Hz.

Two independent experiments were carried out, both of them has four different carrier frequencies: 1, 2, 4 and 8 GHz. Parameters of these two experiments are exactly the same except the phase noise performance, the phase noise performance in each experiment are listed in Tables 4 and 5. The vibration amplitude is 0.5 mm in experiment 1 and 2 and 5 mm in experiments 3 and 4.

After using SFMFT method and time–frequency analysis method to analyse the radar echo signal and execute the calculation of parameter MSCR, the experimental results of experiments are listed in Fig. 5.

In Figs. 5a and b, red dashed line represents the changing trend of parameter MSCR in the case of constant phase noise and the blue dashed line with asterisks represents the changing trend of MSCR in the case of an increased phase noise. It could be seen from Fig. 5 that if the carrier frequency of the signal is doubled and the phase noise characteristic is constant, the extraction performance of micro-motion parameters can be improved. It also
Table 5  Phase noise performance of experiments 2 and 4

| Offset from carrier frequency, Hz | Performance of 1 GHz carrier frequencies, dBc/Hz | Performance of 2 GHz carrier frequencies, dBc/Hz | Performance of 4 GHz carrier frequencies, dBc/Hz | Performance of 8 GHz carrier frequencies, dBc/Hz |
|----------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
|                                  | 10      | 50      | 100     | 500     | 1000    | 5000    | 9999    |
|                                  | −40     | −64     | −70     | −84     | −90     | −104    | −110    |
|                                  | −34     | −58     | −64     | −78     | −84     | −91     | −104    |
|                                  | −28     | −52     | −58     | −72     | −78     | −85     | −98     |
|                                  | −22     | −46     | −52     | −66     | −72     | −79     | −92     |

Fig. 5  Average value of MSCR under different radar carrier frequencies
(a) Parameter MSCR of experiments 1 and 2 using SFMFT method, (b) Parameter MSCR of experiments 3 and 4 using time–frequency analysis method

Fig. 6  Average value of MSCR under different amplitudes of vibration
(a) Phase noise intensity – parameter MSCR in each vibration amplitude using SFMFT method, (b) Phase noise intensity – parameter MSCR in each vibration amplitude using time–frequency analysis method

Fig. 7  Average value of MSCR under different frequencies of target's vibration
(a) Phase noise intensity – parameter MSCR under different vibration frequencies using SFMFT method, (b) Phase noise intensity – parameter MSCR under different vibration frequencies using time–frequency analysis method

3.4 Different frequencies of vibration

To verify the assumption that increasing the frequency of vibration can also improve the extraction performance of micro-motion, a comparison simulation experiment with different vibration frequencies is carried out. The intensity of phase noise is in the form of rectangle. The boundary frequency is set to be 100 Hz, and the sampling frequency is 40 kHz to ensure there is no phase ambiguity. The result of the experiment is shown in Figs. 7a and b.

It can be seen from Fig. 7b that when the vibration frequency increases, the parameter MSCR increases. According to the formula of Doppler frequency:

$$f_d = A \cdot \frac{4\pi f_m f_a \cos(\alpha)}{c}$$

where $f_d$ indicates the Doppler frequency, $A$ indicates the amplitude of vibration, $f_m$ indicates the vibration frequency, $f_a$ indicates the radar carrier frequency. The Doppler frequency increases along with the increase of vibration frequency, and Doppler frequency can be detected in the time–frequency distribution results.

Therefore, the increase of vibration frequency can help to improve the extraction performance of micro-motion parameters. Instead, using the SFMFT method one cannot figure out the difference between different vibration frequencies because the peak value in the spectrum will not increase with the increase of vibration frequency.

3.5 Different numbers of sampling points, sampling rates and time lengths

As the time length of observation is equal to the number of sampling points multiplied by the sampling rate, one can control...
one of the variables to be constant and change the other two variables. Three controlled experiments were set up to clarify if different sampling points, sampling rates or time lengths have influence on the extraction performance of vibration parameters.

Notably, the intensity of phase noise is in the form of rectangle, the boundary frequency is 100 Hz, and the power of phase noise is the amplitude of vibration is 5 mm.

Experiment 1: The sampling rate is set constantly to be 20 kHz, and the number of sampling points is 1000, 2000,..., 25,000, respectively.

Experiment 2: The time length is set to be 1 s, and the sampling rate is 1, 2, ..., 25 kHz, respectively.

Experiment 3: The number of sampling points is set to be 10,000, and the sampling rate is 1, 2, 4, 8, 10, 20, 25, 100, 200 kHz, respectively.

The simulation results are displayed in Fig. 8. A conclusion can be obtained after analysing Fig. 8a that in the case of a constant sampling rate, increasing the number of sampling points can effectively improve the effect of the extraction of micro-motion parameters. Fig. 8b illustrates that in the case of constant length of time, improving the sampling rate can effectively improve the extraction performance. Fig. 8c indicates that when the number of sampling points is fixed, increasing the sampling rate has no contribution to the extraction performance and may even cause decrease of the parameter MSCR because of the shortening of time. The same conclusion can be obtained after the observation of Figs. 8d–f.

In a word, increasing the sampling rate when the time length is constant and increasing the time length when the sampling rate is constant could be effective methods to improve the extraction performance of micro-motion parameters.

3.6 Empirical formula

In the following text, empirical formula will be put forward. On the basis of this empirical formula, the exact value of the estimation accuracy under any given experimental condition can be calculated. Since the experimental results of the time–frequency analysis and the SFMFT method differ greatly, two empirical formulas are proposed, respectively.

3.6.1 Empirical formula in the condition of using SFMFT method: When utilising SFMFT method to analyse the estimation accuracy, the empirical formula is shown as following:

\[
\text{MSCR} = -0.8258P + 8.3361\ln(f_c) + 2.9203\ln(A)
\]

\[
-0.0288f_s + 7.6361\ln(p_s) - 53.83489
\]

where \(P\) represents the intensity of rectangular phase noise which unit is dBc/Hz and \(f_c\) represents the carrier frequency which unit is GHz and \(A\) represents the vibration amplitude which unit is millimetre and \(f_s\) represents the sampling frequency which unit is kHz and \(p_s\) represents the sampling points which unit is one thousand points. As the vibration has no effect on the analysis results of the SFMFT method, the vibration frequency \(f_m\) does not appear in the formula. Due to the logarithmic operation in the calculating of MSCR, the parameters that have a linear promoting effect on the estimation effect have a natural logarithmic symbol, such as \(f_c\), \(A\) and \(p_s\).

When the phase noise intensity is in the ordinary form, assuming that the demarcation point of the descent rate is 100 Hz and the rate of descent is 30 dB/dec within the point and 20 dB/dec outside the point, the empirical formula is shown as following:

\[
\text{MSCR} = -1.0548P + 9.8742\ln(f_c) + 9.8603\ln(A)
\]

\[-0.0262f_s - 10.6114\ln(p_s) + 12.7354\]

3.6.2 Empirical formula in the condition of using time–frequency method: The empirical formula is similar to the formula of SFMFT method except that the vibration frequency \(f_m\) has a significant influence on the estimation results which unit is Hz.

\[
\text{MSCR} = -0.5819P + 11.7627\ln(f_c) + 12.9514\ln(A) + 1.2285f_m
\]

\[-0.4456f_s + 10.0939\ln(p_s) - 76.243\]

\[
\text{MSCR} = -0.509P + 9.246f_c + 7.263\ln(A) + 0.294f_m
\]

\[-0.549f_s + 4.324\ln(p_s) - 3.196\]

Formulae (9) and (10) are empirical formulas for rectangular and ordinary phase noise intensity, respectively. After the condition parameters are calculated, the values of the parameter MSCR can be estimated by the above formula.

4 Conclusion

In this paper, the radar echo signal is analysed by means of time–frequency analysis method and SFMFT method, and the m-D features are extracted. The effect of phase noise on the extraction of m-D features is evaluated by calculating the parameter MSCR. The results of simulation experiment indicate that increasing the carrier frequency cannot improve the extraction performance of micro-motion features; large amplitude of vibration can help increase the tolerance to phase noise; increasing the sampling points and the sampling rate can improve the effect of the estimation of the m-D parameters. However, contrary to the
conclusion of time–frequency analysis, increasing the frequency of the vibration will not help to improve the extraction performance of the m-D features in the case of using the SFMFT method. Further investigation is currently in progress in our laboratory.

5 References

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