Nonlinear signal-based control for single-axis shake tables supporting nonlinear structural systems

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Summary
Nonlinear signal-based control (NSBC) is very powerful for controlling structural systems with parameter variations and has the advantage that the controllers can be designed using classical control theory and expressed by transfer functions. This report describes the first application of NSBC to shake table tests with nonlinear specimens and compares the performance of NSBC with that of a basic control approach that relies on the inverse transfer function of the controlled system. NSBC and the basic approach were numerically applied to shake table tests to excite a nonlinear single-degree-of-freedom system with earthquake acceleration motion, considering the nonlinear specimen characteristics and estimation errors associated with the table dynamics. NSBC achieved excellent control with near 100% accuracy, whereas the basic approach provided insufficient control. Although inaccurate estimation of the pure time delay in the controlled system causes instability at the practice of NSBC, proper design of the nonlinear signal feedback controller prevented instability. In experimental examinations, controllers for the two approaches were designed on the basis of the table dynamics with/without a specimen, which were preliminarily identified by performing tests using a random wave with small amplitude excitations. With no specimen present, both approaches yielded the expected acceleration motion on the table with high accuracy. However, with the specimen present, only NSBC successfully achieved excellent control of the shake table with near 100% accuracy, whereas the basic approach did not because of the specimen nonlinearity. These results numerically and experimentally demonstrate the efficiency and practicality of NSBC for shake tables supporting nonlinear structures.

KEYWORDS
nonlinear systems, Nyquist stability criterion, pure time delay, shake table, substructuring experiment, trilinear hysteretic spring
Shake tables are among the most common and key experimental devices in earthquake and structural engineering and are used in various experiments to examine the seismic performances of buildings or infrastructures subjected to earthquake excitation. In such experiments, a shake table is requested to excite a specimen until its structural condition reaches an ultimate level. However, a specimen in this condition displays nonlinear characteristics triggered by the structural damage, resulting in deterioration of the table control accuracy. This deterioration must be minimised to maintain experimental reliability and to examine the seismic performances of specimens as accurately as possible.

Substructuring shake table testing, an advanced experimental technique, is becoming possible as a means of examining the seismic performances of extremely large structures. Practical applications of this experimental technique have been reported recently by several research groups. The experimental reliability is directly associated with the shake table control performance, because the shake table acts as a transfer system that connects the physical and numerical domain. For further development of advanced experimental techniques, accurate control of shake tables supporting nonlinear structures is a fundamental issue.

E-Defense is currently the largest three-dimensional shake table in the world (size: 20 m × 15 m, maximum loading capacity: 12,000 kN). Over 80 full- or large-scale experiments have been implemented since its commencement in 2005. Its basic approach is a three-variable control method using the acceleration, velocity, and displacement of the table. When the table dynamics are affected by a specimen, traditional input compensation is applied to enhance the shake table control. This input compensation is an iterative off-line method and is applied to the table supporting the specimen. Because this modification is based on the specimen dynamics in the elastic range, its nonlinearities are not currently reflected in the compensation. Thus, control accuracy deterioration is observed when shake table tests are performed for large structures with strong nonlinear characteristics. This issue is not particular to E-Defense, as other facilities with shake tables have similar issues to a greater or lesser degree.

Consequently, much attention has been paid to this issue over the past decades in the structural and earthquake engineering communities. To overcome this problem, this paper introduces a new shake table control method, based on nonlinear signal-based control (NSBC), which has been developed specifically to control systems with parameter changes.

Various control approaches have been developed and incorporated to enhance shake table control, especially when the interaction between the table and specimen is not negligible and the specimen displays strong nonlinear characteristics. Iterative control input modification is commonly employed in shake table tests because it naturally guarantees stability owing to the off-line iteration process. Although such modification is effective for structural systems that repeatedly display some fixed nonlinearity, it is not efficient when “one-time-only” unknown dynamics or nonlinearities are present in the controlled system. To compensate for the errors caused by such dynamics, feedback action must also be employed.

$H_{\infty}$ control is known to reflect some of the uncertainty of the controlled system in the controller design in advance. Nonlinear $H_{\infty}$ control, which more explicitly deals with nonlinear systems, has also been studied. Although the basic control approach has been applied to experiments for earthquake engineering purposes, the nonlinear approach still has shortcomings that impede its application, especially in earthquake and structural engineering.

Model reference adaptive control, the most common adaptive control approach, is based on feedback linearization, in which the controller is designed such that its closed loop approaches a linear reference model. The performance of minimal control synthesis, which can tune the controller gains using less information about the controlled system than is required by model reference adaptive control, has also been demonstrated in shake table experiments. However, in adaptive control approaches, inefficient adaptation may occur, especially when rapid parameter variations occur in the controlled system, which directly jeopardises the control performance. To enhance the robustness and efficiency of adaptation, many techniques, such as Kalman filters, observers, real-time parameter estimation, fuzzy theory, and a backstepping technique, have been proposed.

NSBC has recently been developed for controlling nonlinear systems, without requiring detailed information about the nonlinearities in the systems. This control approach is literally based on the feedback action of the nonlinear signal $\sigma$, which is generated as the difference between the two outputs of the controlled system and its linear model under the same input signal. NSBC has the advantage that the controllers are designed on the basis of classical control and expressed by transfer functions. Because NSBC feeds only the output at the controlled point back to the controllers, it does not require many measurement devices or observers. In addition, the linear model, which plays a key role in this approach, can be simply built by using a priori information about the controlled system or dynamics identified by a random wave excitation with small amplitude.
In addition to the control performance, stability is also a major issue at shake table tests. Stability is normally affected by nonlinear structural dynamics in the specimen to be tested and a pure time delay that inevitably exists in the shake table control system. In general, compensation for the delay is very difficult, because it is the gap between the onsets of the input and output signals in the actuation system. Furthermore, this pure time delay is known to influence control performance and stability significantly in substructuring experiments. Thus, to perform shake table tests safely and with high accuracy, stability also needs to be considered during and before the tests.

Following the introduction of NSBC and its application to substructuring experiments using a hydraulic actuator, this report describes the first application of NSBC to shake table control. The innovative features of this study are as follows:

- NSBC controller design is addressed for shake tables supporting nonlinear multiple-degree-of-freedom (MDOF) systems. Its stability analysis against pure time delay is also studied by considering the effects of the delay estimation error.
- To examine the performance of NSBC numerically, control practices are simulated for a shake table with a nonlinear single-degree-of-freedom (SDOF) system. Various conditions such as errors in the table dynamics modelling or the pure time delay estimation are considered in the simulations.
- The performance of NSBC is experimentally examined in actual control of a single-axis shake table with/without a specimen that displays nonlinear characteristics over some deformation.

The remainder of this paper is organised as follows. In Section 2, the controller design of NSBC and its stability analysis are discussed. Sections 3 and 4 describe the numerical and experimental examinations, respectively, of the performance of NSBC in shake table tests with a nonlinear SDOF system. Finally, Section 5 summarises the conclusions.

## 2 | NSBC FOR SHAKE TABLE TESTS

In a shake table test, the interaction between the actuation system of the table and the specimen is significant, especially when the specimen is heavy and large. To consider this interaction directly, NSBC regards the shake table driven by the actuation system as part of the controlled system together with the specimen, as shown in Figure 1. Then, the pure time delay \( \tau \) solely becomes a factor of the transfer system in Figure 1. Thus, accurate estimation of the pure time delay, \( \tau \) in Figure 1, is essential to maximise the performance of NSBC. In addition, a linear model of the controlled system and the nonlinear signal, which is obtained from the outputs of the controlled system and its linear model under the same input signal \( u \), are necessary. Using these notations, the output of the linear model can be expressed as

\[
y_0(s) = y_0(s) - \sigma(s) = G_0(s)e^{-\tau s}u(s),
\]

(1)

where \( s \) is the Laplace operator, \( y_0 \) is the output of the controlled system \( G_0 \) containing the dynamics of the table, \( y_0 \) is the output of its linear model \( \bar{G}_0 \), and \( \sigma \) is the nonlinear signal. Based on Equation (1), the error signal \( e \) between the reference signal \( r \) and \( y_0 \) can be expressed as
\[ e(s) = r(s) - y_0(s) = r(s) - \bar{y}_0(s) - \sigma(s) = r(s) - \bar{G}_0(s)e^{-\tau s}u(s) - \sigma(s). \]  

(2)

The control input of NSBC is determined by

\[ u(s) = K_r(s)r(s) + K_e(s)e(s) + K_\sigma(s)\sigma(s), \]

(3)

where \( K_r, K_\sigma, \) and \( K_e \) are controllers acting on \( r, e, \) and \( \sigma, \) respectively. Note that, in general, the combination of \( K_r \) and \( K_\sigma \) is sufficient for controlling actuation systems and the addition of \( K_e \) is mainly employed for substructuring tests. Substituting Equation (3) into Equation (2), the error signal can be rewritten as

\[ e(s) = \frac{1 - \bar{G}_0(s)e^{-\tau s}K_r(s)}{1 + \bar{G}_0(s)e^{-\tau s}K_e(s)}r(s) - \frac{1 + \bar{G}_0(s)e^{-\tau s}K_\sigma(s)}{1 + \bar{G}_0(s)e^{-\tau s}K_e(s)}\sigma(s). \]

(4)

To achieve zero error in Equation (4), particularly for \( \tau = \bar{\tau} = 0, \) suitable controller transfer functions are

\[
\begin{align*}
K_r(s) &= \frac{1}{\bar{G}_0(s)}F_r(s), \\
K_\sigma(s) &= -\frac{1}{\bar{G}_0(s)}F_\sigma(s), \\
K_e(s) &= \frac{1}{\bar{G}_0(s)}F_e(s),
\end{align*}
\]

(5)

where \( F_r, F_\sigma, \) and \( F_e \) are the filters associated with the controllers acting on \( r, \sigma, \) and \( e, \) respectively.

### 2.1 Stability analysis

The linear model in NSBC must be designed so that it can represent the controlled system as much as possible. When the linear model of the shake table cannot perfectly represent the table, the unmodelled dynamics \( \Delta G_0 \) appear as modelling error:

\[ \Delta G_0(s) = G_0(s) - \bar{G}_0(s). \]

(6)

When the pure time delay is not accurately identified (i.e., \( \tau \neq \bar{\tau} \)), its estimation error appears as \( \Delta \tau = \tau - \bar{\tau}. \) Now, the nonlinear signal is given by

\[ \sigma(s) = y_0(s) - \bar{y}_0(s) = \Delta G_{0\Delta \tau}(s)\bar{G}_0(s)e^{-\tau s}u(s), \]

(7)

where \( \Delta G_{0\Delta \tau}(s) = \left( 1 + \frac{\Delta G_0(s)}{\bar{G}_0(s)} \right)e^{-\Delta \tau s} - 1. \) When \( \Delta \tau = 0, \) the unmodelled dynamics become \( \Delta G_{0\Delta \tau}(s) = \frac{\Delta G_0(s)}{\bar{G}_0(s)} \) and the nonlinear signal can be expressed as \( \sigma(s) = \Delta G_0(s)e^{-\tau s}u(s). \) Thus, when \( \bar{\tau} \) is identical to \( \tau, \) the nonlinear signal can directly provide the unmodelled dynamics of the controlled system. Therefore, the pure time delay estimation needs to be as accurate as possible, to utilise the nonlinear signal fully for nonlinear system control.

Based on Equations (4) and (7) having the control input \( u \) in Equation (3), the signals fed back in NSBC can be expressed as

\[
\begin{bmatrix}
e(s) \\
\sigma(s)
\end{bmatrix} = W(s)^{-1}W_0(s)r(s),
\]

(8)

where

\[
W(s) = \begin{bmatrix}
W_{11}(s) & W_{21}(s) \\
W_{12}(s) & W_{22}(s)
\end{bmatrix} = \begin{bmatrix}
\Delta G_{0\Delta \tau}(s)\bar{G}_0(s)e^{-\tau s}K_e(s) & -1 + \Delta G_{0\Delta \tau}(s)\bar{G}_0(s)e^{-\tau s}K_\sigma(s) \\
1 + \bar{G}_0(s)e^{-\tau s}K_e(s) & 1 + \bar{G}_0(s)e^{-\tau s}K_\sigma(s)
\end{bmatrix} \quad \text{and} \quad W_0(s) = \begin{bmatrix}
W_{01}(s) \\
W_{02}(s)
\end{bmatrix} = \begin{bmatrix}
-\Delta G_{0\Delta \tau}(s)\bar{G}_0(s)e^{-\tau s}K_r(s) \\
1 - \bar{G}_0(s)e^{-\tau s}K_e(s)
\end{bmatrix}.
\]

In Equation (8), the inversion of \( W(s) \) corresponds to the closed-loop characteristic equation for the signals of \( e \) and \( \sigma. \) Then, the closed-loop characteristic equation can be written as

\[ L(s) = \det W(s) = 1 + H(s)e^{-\tau s}, \]

(9)
where \( H(s) = (\Delta G_{0\Delta r}(s) + 1)G_0(s)K_e(s) - \Delta G_{0\Delta r}(s)G_0(s)K_e(s), \) which is the open-loop characteristic equation of Equation (9). When the suitable controller transfer functions in Equation (5) are employed in the open-loop characteristic equation, it can be rewritten as

\[
H(s) = \Delta G_{0\Delta r}(s)F_\sigma(s) + \left(1 + \frac{\Delta G_0(s)}{G_0(s)}\right)e^{-\Delta \tau F_e(s)}.
\]  

(10)

According to Equation (10), the stability of NSBC is affected by the unmodelled dynamics \( \Delta G_0(s) \), the pure time delay \( \Delta \tau \), and two filters \( F_e \) and \( F_\sigma \).

In this study, the stability of NSBC is analysed by using the Nyquist stability criterion, which is an appropriate choice for a system with a pure time delay. When the error feedback action is not activated (i.e., \( F_e = 0 \)) in Equation (10), \( H(s) \) becomes

\[
H(s) = \Delta G_{0\Delta r}(s)F_\sigma(s).
\]  

(11)

When \( \Delta \tau = 0 \) in Equation (11), it simply becomes

\[
H(s) = \frac{\Delta G_0(s)}{G_0(s)}F_\sigma(s).
\]  

(12)

Equation (12) indicates that the stability of NSBC with \( \{K_r, K_\sigma\} \) is determined by the unmodelled dynamics and the nonlinear signal feedback filter \( F_\sigma \) in practice. As shown later, the practice with \( \{K_r, K_\sigma\} \) is sufficient for controlling a shake table even with a nonlinear specimen. Thus, the stability analysis here is discussed for \( \{K_r, K_\sigma\} \). The stability analysis for \( \{K_r, K_\sigma, K_e\} \) can be found in the study.47

2.2 | Shake table supporting a nonlinear specimen

NSBC does not heavily rely on accurate information about the parameters in the controlled system. The minimum amount of information that enables the user to build a linear model of the system is sufficient. To support the design of the linear model, modelling of a shake table with/without a specimen is discussed in this section.

With no specimen on the table, as shown in Figure 2a, the table dynamics for displacement control are normally expressed as a second-order transfer function as follows:

\[
G_0(s) = \frac{\nu_0(s)}{u(s)} = \frac{\omega_0^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2} = \frac{k_0}{m_0 s^2 + c_0 s + k_0},
\]  

(13)

where \( \omega_0, \zeta_0, m_0, c_0, \) and \( k_0 \) are the natural circular frequency and the damping ratio, mass, damping coefficient, and stiffness of the table, respectively. In the time domain, Equation (13) can be equivalently written as

\[
m_0\ddot{y}_0(t) + c_0\dot{y}_0(t) + k_0y_0(t) = k_0u(t).
\]  

(14)

Equation (14) is identical to the common expression of a linear SDOF system. In addition, the displacement reference signal \( r \) directly becomes the control input \( u \) in this form.

When a shake table supports a specimen expressed by a nonlinear MDOF system as shown in Figure 2b, its equation of motion becomes

\[
M\ddot{Y}(t) + F_e(t) + F_k(t) = \eta(t)u(t),
\]  

(15)

where \( Y(t) = [y_0(t) \quad y_1(t) \quad \cdots \quad y_n(t)]^T, \eta(t) = [k_0(t) \quad 0 \quad \cdots \quad 0]^T \) and \( M = \text{diag}(m_0, m_1, \cdots, m_n) \). When the specimen is a linear system, the restoring force becomes
When the linear system is mounted on the table, the output $y_i$ on the $i$th storey and the input signal $u$ obey the relation

$$y_i(s) = G_i(s)y_{i-1}(s) = \left( \prod_{j=0}^{i-1} G_j(s) \right) u(s),$$

(16)

where $i = 0, 1, ..., n$ and $G_i(s)$ is the transfer function of outputs of $y_i$ and $y_{i-1}$ ($n$. $y_{-1} = u$). From Equations (15) and (16), the transfer function $G_i(s)$ can be expressed as

$$G_i(s) = \frac{y_i(s)}{y_{i-1}(s)} = \frac{V_{i+1}(s)}{H_{i+1}(s) - G_{i+1}(s)},$$

(17)

where $V_{i+1}(s) = \frac{c_i s + k_i}{c_i + s + k_{i+1}}$, $W_{i+1}(s) = \frac{m_i s^2 + (c_i + c_{i+1}) s + k_{i+1} + k_i}{c_i + s + k_{i+1}}$, $G_{n+1}(s) = 0$, $V_{n+1}(s) = c_n s + k_n$ and $W_{n+1}(s) = m_n s^2 + c_n s + k_n$.

Although Equations (15)–(17) are formulated for linear controlled systems, this formulation can be extensively used for the linear model, which is essential for the design of NSBC. The linear model of the controlled system in Equation (15) becomes

$$\ddot{\mathbf{M}} \ddot{\mathbf{Y}}(t) + \dddot{\mathbf{C}} \mathbf{Y}(t) + \mathbf{K} \mathbf{Y}(t) = \mathbf{f}(t),$$

(18)
where \( \bar{\eta} = [\bar{k}_0 \ 0 \cdots 0]^T \), \( K = \begin{bmatrix} k_0 + \bar{k}_1 & -\bar{k}_1 & 0 & \cdots & 0 \\ -\bar{k}_1 & k_1 + \bar{k}_2 & \ddots & \vdots & \vdots \\ 0 & \ddots & \ddots & -\bar{k}_n \\ 0 & 0 & \ddots & -\bar{k}_n \\ 0 & 0 & \cdots & -\bar{k}_n \end{bmatrix} \), \( C = \begin{bmatrix} c_0 + \bar{c}_1 & -\bar{c}_1 & 0 & \cdots & 0 \\ -\bar{c}_1 & \bar{c}_1 + \bar{c}_2 & \ddots & \vdots & \vdots \\ 0 & \ddots & \ddots & -\bar{c}_n \\ 0 & 0 & \ddots & -\bar{c}_n \\ 0 & 0 & \cdots & -\bar{c}_n \end{bmatrix} \), \( \bar{M} = M \), and \( Y(t) = [Y_0(t) \ Y_1(t) \cdots Y_n(t)]^T \). As for the linear model, the transfer function of \( y_i \) and \( y_i - y_{i-1} \) can be expressed as

\[
\bar{G}_i(s) = \frac{\bar{Y}_i(s)}{\bar{Y}_{i-1}(s)} = \frac{\bar{V}_{i+1}(s)}{\bar{W}_{i+1}(s) - \bar{G}_i(s)}.
\]

where \( \bar{V}_{i+1}(s) = \frac{\bar{c}_i s}{\bar{c}_{i+1} s + \bar{k}_{i+1}} \), \( \bar{W}_{i+1}(s) = \frac{\bar{m}_i s^2 + (\bar{k}_{i+1} + \bar{c}_i) s + \bar{k}_i}{\bar{c}_i s + \bar{k}_i} \), \( \bar{G}_{n+1}(s) = 0 \), \( \bar{V}_{n+1}(s) = \bar{c}_n s + \bar{k}_n \) and \( \bar{W}_{n+1}(s) = \bar{m}_n s^2 + \bar{c}_n s + \bar{k}_n \). NSBC controllers can be designed on the basis of Equation (19).

### 3 | NUMERICAL EXAMINATIONS

This section discusses the numerical examinations of NSBC performance for shake table control, including comparison with the performance of a basic control approach. The basic control approach employed is feedforward control using the inversion of the controlled system, which corresponds to NSBC without any feedback actions.

The controlled system is a shake table with a nonlinear SDOF system, as depicted in Figure 3a. The specimen has a nonlinear spring expressed by a trilinear hysteretic loop: the spring stiffness decreases to \( r_{11} \) of its initial value over the first elastic limit \( \Delta_{11} \) and to \( r_{12} \) over the second elastic limit \( \Delta_{12} \), as shown in Figure 3b. The reference signal to be realised on the table is an acceleration signal recorded by Japan Meteorological Agency (JMA) during the 1995 Hyogo-ken Nanbu/Kobe earthquake, which is referred to as JMA Kobe motion in this report.

The control performance will be assessed on the basis of the similarity between the reference signal and the table output realised in practice. The following indices are employed to quantify the similarities in the frequency and time domains:

\[
S_t = \left(1 + \frac{\sum e(t)^2}{\sum r(t)^2}\right)^{-1} \times 100\% \\
S_f = \left(1 + \frac{\sum A_r(f)^2}{\sum A_e(f)^2}\right)^{-1} \times 100\% 
\]

where \( A_r \) and \( A_e \) are the Fourier amplitude spectra of the reference signal \( r \) and the error signal \( e \), respectively. \( S_t \) and \( S_f \) are percentages based on the root mean square of the error and reference signal in the time and frequency domains, respectively. Both indices reach 100% when the error signal is infinitely small and 0.0% when it is infinitely large. \( S_t \) is affected by the noise observed in actual control practices, whereas \( S_f \) can focus on some frequency range of interest. In this study, \( S_f \) is evaluated within the range of 0.01–20.0 Hz.

![FIGURE 3](image-url)  Shake table control in numerical simulations: (a) shake table with a nonlinear specimen and (b) trilinear hysteretic loop.
A primary purpose of shake table tests in earthquake and structural engineering is to obtain the hysteretic loops of specimens. Thus, control performance should be assessed in terms of the hysteretic loops obtained using the control approach of interest. Based on the true hysteretic loop, the similarity of a hysteretic loop obtained using a control approach can be quantified as

\[ S_h = \left( 1 + \frac{\sum \sqrt{\left(1 - \hat{f}(t)\right)^2 + \left(1 - \hat{\delta}(t)\right)^2}}{\sum \sqrt{\tilde{f}_1(t)^2 + \tilde{\delta}_1(t)^2}} \right)^{-1} \times 100\%, \]

where \( \hat{f} \) and \( \hat{\delta} \) are force and inter-storey drift that are standardised by the maximum values of outputs \( f \) and \( \delta \), which are obtained at 100\% accurate control, respectively. In addition, \( \tilde{f}_1 \) and \( \tilde{\delta}_1 \) are the force \( f \) and the inter-storey drift \( \delta \) that are standardised by each maximum value. The evaluation using \( S_h \) is stricter than the visual evaluation in the figure of \( f-\delta \), because this index reflects the difference in the time series.

Note that assessing \( S_h \) requires a true hysteretic loop with 100\% accurate control, which is not obtainable in actual experimental control practices. Because \( S_h \) and \( S_0 \) are affected by the pure time delay, these indices in this study are calculated by intentionally applying the delay to the reference signal to minimise the increase of \( S_h \) caused only by the delay.

### 3.1 Shake table with a nonlinear SDOF system

The control performance of NSBC is numerically examined by using a shake table supporting an SDOF system with a trilinear hysteretic spring, as shown in Figure 3. Its equations of motion are

\[
\begin{align*}
\{ m_1 \ddot{y}_1(t) + f_{1c}(t) + f_{1k}(t) &= 0 \\
 m_0 \ddot{y}_0(t) + c_0 \dot{y}_0(t) + k_0 y_0(t) - f_{1c}(t) - f_{1k}(t) &= k_0 u(t)
\end{align*}
\]

where \( f_{1c} \) and \( f_{1k} \) are the forces deriving from the damper and spring in the specimen, respectively. This specimen has a constant damping term, which is given by \( f_{1c}(t) = c_1 (\dot{y}_1(t) - \dot{y}_0(t)) \). The restoring force in the trilinear hysteretic spring is

\[ f_{1k}(t) = r_1 k_1 \delta_1(t) + (1 - r_1) k_1 z_{1l}(t) + (r_1 - r_1^l) k_1^l z_{1l}(t), \]

where \( \delta_1(t) = y_1(t) - y_0(t), \ z_{1l}(t) = \delta_1(t) \chi(\Delta_{1l}(t) - z_{1l}(t)) + \chi(-\delta_1(t)) \chi(\Delta_{1l} + z_{1l}(t)) \), and \( l = 1, 2 \). The step unit function \( \chi \) becomes \( \chi(a) = 1 (\chi(a) = 0) \) at \( a \geq 0 (a < 0) \) and \( \Delta_{1l} \) is the \( l \)th \( (l = 1, 2) \) elastic limit of the spring in the specimen. On the right-hand side, the first term describes the elasticity and the second and third terms describe the elasto-perfectly plastic hysteretic loops. The set of these three terms generates the trilinear hysteretic loop shown in Figure 3b.

In NSBC, regardless of the types of nonlinearity in the damping and stiffness terms, the linear model of the controlled system is uniformly given by

\[
\begin{align*}
\{ m_1 \ddot{y}_1(t) + c_1 \dot{y}_1(t) + \bar{y}_0(t) - \bar{y}_0(t) &= 0 \\
 m_0 \ddot{y}_0(t) + c_0 \dot{y}_0(t) + \bar{k}_0 \bar{y}_0(t) - \bar{y}_0(t) - \bar{c}_1 (\dot{y}_1(t) - \dot{y}_0(t)) - \bar{k}_1 (y_1(t) - y_0(t)) &= \bar{k}_0 u(t)
\end{align*}
\]

Then, the following transfer functions can be obtained:

\[
\begin{align*}
\frac{\bar{y}_1(s)}{\bar{y}_0(s)} &= \frac{c_1 s + k_1}{m_1 s^2 + c_1 s + k_1} \\
\frac{\bar{y}_0(s)}{u(s)} &= \frac{k_0}{m_0 s^2 + (c_0 + c_1) s + (k_0 + k_1) - (c_1 s + k_1) \bar{c}_1(s)}
\end{align*}
\]

The controller design of NSBC and nonlinear signal feedback action rely on Equation (20).

A nonlinear controlled system cannot be accurately described in the Laplace domain. However, an equivalent system can be built on the basis of the parameters used in the linear model and is described by
\[
G_1^*(s) = \frac{\gamma_c \bar{c}_1 s + \gamma_{k1} \bar{k}_1}{m_1 s^2 + \gamma_c \bar{c}_1 s + \gamma_{k1} \bar{k}_1}
\]
\[
G_0^*(s) = \frac{\gamma_{k0} \bar{k}_0}{(m_0 s^2 + (\gamma_{c0} \bar{c}_0 + \gamma_c \bar{c}_1 s + \gamma_{k0} \bar{k}_0 + \gamma_{k1} \bar{k}_1) - (\gamma_c \bar{c}_1 s + \gamma_{k1} \bar{k}_1) G_1^*(s)}
\]

where \(\gamma_{c0} = \frac{c_0}{\bar{c}_0}\), \(\gamma_{k0} = \frac{k_0}{\bar{k}_0}\), \(\gamma_c = \frac{c_1}{\bar{c}_1}\), and \(\gamma_{k1} = \frac{k_1}{\bar{k}_1}\), which are determined or estimated from the parameter variations in the controlled system.

Based on the equivalent system in Equation (21) and the linear model in Equation (20), the unmodelled dynamics, which can be applied in stability analysis, can be estimated to be

\[
\Delta G_0(s) = G_0^*(s) - \bar{G}_0(s)
\]

Because maintaining stability becomes more difficult as the unmodelled dynamics become larger, the worst possible parameter variations of the controlled system should be reflected in the stability analysis.47

### 3.2 Example 1: Examination of nonlinear systems

Based on the controlled system shown in Figure 3a, the performances of NSBC and the basic control approaches for the case in which the table dynamics are exactly known and unchanged during the control practice were examined in Example 1. The table parameters are set to \(m_0 = 200\) kg, \(c_0 = 12.57\) kN/s/m, and \(k_0 = 197.3\) kN/m, yielding \( \omega_0 = 5.0 \times 2\pi \) rad/s and \( \zeta_0 = 1.0 \) with no specimen present. The specimen parameters are set to \(m_1 = 200\) kg, \(c_1 = 0.38\) kN/s/m, and \(k_1 = 71.1\) kN/m, yielding a natural frequency \( \omega_1 \) of \(3.0 \times 2\pi\) rad/s and a damping ratio \( \zeta_1 \) of 0.05 as a linear SDOF system. In this example, the parameters of the nonlinear spring are determined to be \(r_{11} = 0.5\), \( \Delta_{11} = 15\) mm, \(r_{12} = 0.1\), and \( \Delta_{12} = 30\) mm, indicating that the stiffness of the spring becomes half of its initial value over the first elastic limit and 1/10 of its initial value over the second elastic limit. In the simulation, the pure time delay and its estimation delay are set to \(\tau = \bar{\tau} = 4.0\) ms.

In this example, the linear model is directly built by using parameters of the shake table and specimen in the elastic range \(\frac{m_0}{m_0} = \frac{c_0}{c_0} = \frac{k_0}{k_0} = \frac{m_1}{m_1} = \frac{c_1}{c_1} = \frac{k_1}{k_1} = 1\). Then, the linear model is expressed as Equation (20) with these parameters. This example can be subsequently utilised to examine the control accuracy deterioration caused only by the nonlinear characteristics in the specimen.

In a preliminary simulation for the specimen and ground motion, the maximum deformation of the specimen became \(\delta_{1\text{max}} = 0.05\) m. Thus, the equivalent stiffness on the table would not be smaller than \(f_{1\text{max}}/(2\delta_{1\text{max}})\).

Next, the range of the stiffness variation in the specimen is estimated to be \(1 < \gamma_{k1} < f_{1\text{max}}/(2\delta_{1\text{max}})\). Based on this range, the parameters employed in the equivalent system are determined to be \(\gamma_{c0} = \gamma_{k0} = \gamma_c = 1\) and \(\gamma_{k1} = 0.245\). The unmodelled dynamics are expressed as Equation (22) with these parameters. According to the Nyquist diagram in Figure 4, NSBC maintains stability even when the anticipated parameter variation occurs in the specimen. The numerical results of both approaches are summarised in Table 1.

The basic control approach \(K_r\) shows inaccurate control in Figure 5a,b. The control accuracy is clearly deteriorated only by the nonlinearity in the specimen, because the table dynamics in this example are perfectly known and reflected in the controller. In Figure 5c, the obtained hysteretic loop has \(S_h = 80.1\%\) with the true loop that is obtained at 100% accurate control. This difference indicates that inaccurate control results in the production of inaccurate hysteretic loops.

Meanwhile, NSBC shows high control performance in Table 1 and Figure 6a,b. The similarities in the time and frequency domains have exceeded 99.5%. The hysteretic loop obtained using this control approach is visually identical to the true hysteretic loop, as shown in Figure 6c, and \(S_h = 95.4\%\).
Example 2: Examination of nonlinear characteristics and modelling error in the table dynamics

Example 1 has been based on ideal modelling of the table dynamics. In Example 2, the control performance when the table dynamics are not accurately known is examined. All of the parameters used in this example are exactly the same as in Example 1, apart from the damping and stiffness terms in the linear model. To demonstrate the table

| $E_{\text{max}}$ (m/s$^2$) | 3.2 | 0.5 |
|---------------------------|-----|-----|
| $S_f$ (%)                 | 93.3 | 99.97 |
| $S_s$ (%)                 | 89.4 | 99.6 |
| $S_h$ (%)                 | 80.1 | 95.4 |

FIGURE 4  Nyquist diagram for Example 1

FIGURE 5  Basic control approach for Example 1: (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop

FIGURE 6  Nonlinear signal-based control for Example 1: (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop

3.3 | Example 2: Examination of nonlinear characteristics and modelling error in the table dynamics

Example 1 has been based on ideal modelling of the table dynamics. In Example 2, the control performance when the table dynamics are not accurately known is examined. All of the parameters used in this example are exactly the same as in Example 1, apart from the damping and stiffness terms in the linear model. To demonstrate the table
damping and stiffness being inaccurately estimated as twice their actual values, the parameters in the linear model are set to \( \frac{m_0}{m_1} = \frac{c_1}{c_1} = \frac{k_1}{k_1} = 1.0 \) and \( \frac{\gamma_{c0}}{\gamma_{c1}} = \frac{\gamma_{k0}}{\gamma_{k1}} = 2.0 \). Then, the linear model in this example is expressed as Equation (20) with these parameters. This example is utilised to examine the control deterioration caused by inaccurate modelling of the table dynamics as well as nonlinear characteristics.

The maximum anticipated deformation of the specimen is the same as in Example 1 (i.e., \( \delta_{1\text{max}} = 0.05 \) m), as is the specimen parameter variation range. Thus, the parameters of the equivalent system are found to be \( \gamma_{c0} = 1/2 \), \( \gamma_{k0} = 1/2 \), \( \gamma_{c1} = 1 \), and \( \gamma_{k1} = 0.245 \). The unmodelled dynamics are expressed as Equation (22) with these parameters. According to the Nyquist diagram in Figure 7, adding modelling error in the table dynamics has changed the shapes of the circles, as can be seen by comparing Figures 4 and 7. However, the effect is not sufficiently large to cause instability. Thus, NSBC can maintain stability even when the anticipated specimen parameter variations will occur. The numerical results of both approaches are summarised in Table 2.

The basic control approach in this example has yielded even less similarity than in Example 1, as shown in Table 2, simply because of the addition of unmodelled table dynamics to the conditions in Example 1. Thus, the hysteresis loops in Figure 8c have become even less similar than those in Example 1 (Figure 5c).

Nonetheless, NSBC has achieved accurate control even in this case, yielding \( S_f = 99.8\% \) and \( S_f = 99.2\% \), which are as good as the results of Example 1. In addition, as shown in Figure 9c, the hysteresis loop obtained using this control

**FIGURE 7** Nyquist diagram for Example 2

**TABLE 2** Results of Example 2

|          | \( K_r \) | \( K_r, K_a \) |
|----------|-----------|---------------|
| \( E_{\text{max}} \) (m/s\(^2\)) | 2.8       | 0.7           |
| \( S_f \) (%)  | 93.6      | 99.8          |
| \( S_t \) (%)  | 86.2      | 99.2          |
| \( S_h \) (%)  | 72.9      | 94.0          |

**FIGURE 8** Basic control approach for Example 2: (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop
approach is nearly identical to the true hysteretic loop with 100% accurate control. The quantified similarity has become $S_h = 94.0\%$, which is nearly the same as in Example 1. Thus, even when the linear model is designed using inaccurate information about the table dynamics, NSBC can achieve excellent performance.

3.4 | Example 3: Examination of nonlinear characteristics, modelling error in the table dynamics, and pure time delay estimation error

NSBC requires accurate pure time delay estimation. However, the error between the actual and estimated delays is likely to occur at experimental sites. Thus, the pure time delay estimation error is considered in Example 3, in addition to the conditions of Example 2. The delays are set to $\tau = 4.0 \text{ ms}$ and $\tau = 1.0 \text{ ms}$, yielding $\Delta \tau = 3.0 \text{ ms}$. In this case, the basic control approach, using only the feedforward controller $K_r$, is not affected by the estimation error. Thus, this example is focused on NSBC using $K_r$ and $K_\sigma$.

The maximum anticipated deformation of the specimen is the same as in Examples 1 and 2 (i.e., $\delta_{1\text{max}} = 0.05 \text{ m}$), and the parameter variation ranges for the equivalent system are the same as in Example 2: $\gamma_{c0} = 1/2$, $\gamma_{k0} = 1/2$, $\gamma_{c1} = 1$ and $\gamma_{k1} = 0.245$. The unmodelled dynamics are expressed as Equation (22) with these parameters. According to the Nyquist diagram in Figure 10a, the controllers employed in Example 2 cannot maintain stability because of the presence of the pure time delay estimation error. $K_\sigma$ employing $F_\sigma = 1.0$ has presumably contributed the source of instability, because $K_r$ does not affect the stability.

Here, we propose an alternative controller design of $F_\sigma$. In this example, $F_\sigma$ is set to be a second-order Butterworth bandpass filter. Two bandpass ranges are chosen for this filter, $0.2$–$20.0$ and $0.2$–$50.0$ Hz, to examine its control performance and stability. These ranges are selected to eliminate high-frequency noise and the DC component, which contaminate experimental data. According to the Nyquist diagrams in Figure 10b,c, stability is found to be maintained with each bandpass range even with the pure time delay estimation error and parameter variations in the specimen. The results of the two simulations are summarised in Table 3.

The $0.2$–$20.0$ Hz filter has yielded $S_f = 97.8\%$ and $S_t = 96.6\%$, and its hysteretic loop has resulted in $S_h = 88.6\%$, which is sufficiently close to the true loop in Figure 11. Meanwhile, the $0.2$–$50.0$ Hz filter has achieved better control (i.e., $S_f = 99.6\%$ and $S_t = 99.3\%$) and a better hysteretic loop with $S_h = 95.5\%$, as shown in Figure 12. Because the cut-off frequency assigned in $F_\sigma$ affects the stability margin and control performance, it should be properly designed by following the control conditions. When the actual pure time delay is not exactly known and large parameter variations are anticipated, the filter should be designed to obtain sufficient stability margins.

4 | EXPERIMENTAL EXAMINATIONS

The control performance of NSBC was examined via a series of experiments using a single-axis electrodynamic shake table, shown in Figure 13. The table specifications were weight: 200 kg, table size: $1.2 \text{ m} \times 1.2 \text{ m}$, loading capacity: 500 kg, maximum stroke: $\pm 300 \text{ mm}$, maximum velocity: $1.0 \text{ m/s}$, and available frequency range: $0.5$–$15.0$ Hz.

The simple physical model depicted in Figure 13b was built on the table as a specimen imitating a typical building. The specimen had dimensions of $0.9 \text{ m} \times 0.9 \text{ m}$ with a height of $0.6 \text{ m}$ and was designed to have a mass of $200.0 \text{ kg}$, so that the ratio between the masses of the table and specimen was nearly 1.0. A set of plates connected with the masses of
**FIGURE 10** Nyquist diagrams for Example 3: (a) ideal bandpass range, (b) bandpass range of 0.2–20.0 Hz, and (c) bandpass range of 0.2–50.0 Hz

**TABLE 3** Results of Example 3

| Bandpass range          | $K_r$, $K_c$     |
|-------------------------|------------------|
|                         | 0.2–20.0 Hz      | 0.2–50.0 Hz      |
| $E_{\text{max}}$ (m/s²) | 1.6              | 0.8              |
| $S_f$ (%)               | 97.8             | 99.6             |
| $S_t$ (%)               | 96.6             | 99.3             |
| $S_h$ (%)               | 88.6             | 95.5             |

**FIGURE 11** Nonlinear signal-based control for the bandpass range (0.2–20.0 Hz): (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop
the table and specimen was used to reflect the flexibility of buildings and display nonlinear characteristics when the deformation exceeded the elastic range. These plates were designed to make the natural frequency of the specimen be 3.0 Hz.

A magnetostrictive displacement transducer was attached to the table to measure its displacement. Four accelerometers were utilised in total, with two placed on the table and another two on the specimen. Two wire displacement transducers were attached near the tops of the protection stands shown in Figure 13b to measure the deformation of the specimen (i.e., steel plates). In this study, the hysteretic loop will be acquired from the acceleration and deformation of the specimen.

4.1 Actual table dynamics

Proportional–integration–derivative control was employed as the base in the shake table test and acted as the inner controller for the table displacement, as shown in Figure 14. This controller was designed as \( C_{in}(s) = 5 + \frac{20}{s} + 0.2 \frac{s}{0.01s + 1} \)
to achieve reasonable control of the shake table without a specimen. The first-order low-pass filter described by \( F_{in}(s) = 50 \cdot 2\pi/(s+50 \cdot 2\pi) \) was employed to eliminate noise from the controlled signal (i.e., displacement measured at the shake table). The feedback loop in Figure 14 is represented by the closed loop transfer function \( G_0 \). When a specimen is placed on the table, \( G_0 \) will have a higher-order transfer function than when no specimen is present.
NSBC was employed as the outer controller for this shake table to enhance the tracking performance of table acceleration. In this study, the linear model, which has a key role in NSBC, was obtained by identification of the controlled system in the elastic range.

Band-limited white-noise excitation, which is commonly used in earthquake engineering experiments, was employed for shake table identification with/without the specimen and contained frequency components of 0.1–50.0 Hz. An identification test was performed on the table without the specimen with table displacements of up to 5.0 mm. Based on this test, the pure time delay was found to be 4.0 ms and the table dynamics were identified to be

\[
G_d(s) = \frac{y_0(s)}{u(s)} = \frac{987}{s^2 + 62.83 + 987},
\]

These results are illustrated in Figure 15a together with the test results in which the effect of the pure time delay is eliminated.

Another identification test was performed with the same excitation for the table supporting the specimen, with the maximum displacement of the table remaining within 5.0 mm. The pure time delay was again 4.0 ms, and the following transfer functions were obtained:

\[
G_d(s) = \frac{y_0(s)}{u(s)} = \frac{276.3s^2 + 165.8s + 5.941e04}{s^4 + 17.67s^3 + 673.3s^2 + 3732s + 5.941e04},
\]

The orders of the polynomial equations in Equation (24) were determined by Equation (20). The results are illustrated in Figure 15b together with the test results in which the effect of the pure time delay is eliminated.

The transfer function of the controlled system to be employed in the controller design depends on control purposes as follows: \( G_0(s) = G_d(s) \) for acceleration control and \( G_0(s) = G_d(s) \) for displacement control. The shake table control in this study was performed for acceleration control with \( G_0(s) = G_d(s) \).

4.2 | Shake table without a specimen

Experimental examinations were first performed for the shake table without a specimen, shown in Figure 13a. The reference signal was JMA Kobe motion, which was used in the numerical simulations. The controllers for the basic control approach and NSBC were based on \( G_d(s) \) in Equation (23).

4.2.1 | Basic control approach

The input of the basic control approach is determined by \( u(s) = K_r(s) \ddot{r}(s) \) with \( K_r(s) = G_d(s)^{-1} \). In this examination, the amplitude of the reference signal was considered as a varying parameter. The experimental results corresponding to amplitudes of 25%, 50%, and 100% are summarised in Table 4.
The basic control approach achieved excellent control for all amplitudes, as shown in Table 4. The transfer function obtained by system identification (i.e., Equation 23) was sufficient for this control method. According to these results, the shake table did not significantly change its dynamics with variations in the amplitude of the reference signal. In addition, as shown in Table 4, $S_i$ improves as the amplitude increases, simply because noise affects the index more significantly at small amplitudes than at large amplitudes.

4.2.2 | Nonlinear signal-based control

The design of NSBC was based on the transfer function in Equation (23), and the linear model was set to $\overline{G}_0(s) = G_a(s)$. Its control input is determined by $u(s) = K_r(s)\ddot{r}(s) + K_\sigma(s)\dot{\sigma}(s)$, where $\dot{\sigma}(s) = \ddot{y}_0(s) - \ddot{y}_0(s)$, $K_r(s) = \overline{G}_0(s)^{-1}$, and $K_\sigma(s) = \overline{G}_0(s)^{-1}F_\sigma(s)$. $F_\sigma$ was designed to be the second-order Butterworth filter with a bandpass range of 0.2-20.0 Hz. This range was selected to prepare for pure time delay estimation error and unmodelled dynamics in the shake table and specimen.

NSBC achieved excellent control for all amplitudes, as shown in Table 5. The NSBC results are slightly better than the basic control results in Table 4, indicating that NSBC provides control as effectively as the basic control approach when the dynamics of the controlled system are accurately known.

4.3 | Shake table with a specimen

The control performance was then experimentally examined using the shake table with the specimen shown in Figure 13b. The reference signal was the JMA Kobe motion. The controllers were based on $G_a(s)$ in Equation (24) for acceleration control.

4.3.1 | Basic control approach

The basic control approach was based on inversion of the controlled system. The control input is determined by $u(s) = K_r(s)\ddot{r}(s)$, where $K_r(s) = G_a(s)^{-1}$. The experimental results for amplitudes of 25% and 50% are summarised in Table 6. Note that amplitudes over 50% were not implemented due to safety management.

| Amplitude | 25% | 50% | 100% |
|-----------|-----|-----|------|
| $E_{\text{max}}$ (m/s$^2$) | 0.28 | 0.55 | 0.92 |
| $S_f$ (%) | 99.5 | 99.6 | 99.8 |
| $S_i$ (%) | 96.4 | 98.6 | 99.1 |

| Amplitude | 25% | 50% | 100% |
|-----------|-----|-----|------|
| $E_{\text{max}}$ (m/s$^2$) | 0.28 | 0.54 | 0.89 |
| $S_f$ (%) | 99.96 | 99.97 | 99.98 |
| $S_i$ (%) | 95.2 | 98.8 | 99.2 |

| Amplitude | 25% | 50% | 100% |
|-----------|-----|-----|------|
| $E_{\text{max}}$ (m/s$^2$) | 0.72 | 2.68 |  |
| $S_f$ (%) | 95.7 | 84.3 |  |
| $S_i$ (%) | 87.9 | 68.3 |  |
When the amplitude was 25%, $S_f$ and $S_t$ were not very high, as shown in Table 6. In Figure 16a,b, the results for the amplitude of 50% more clearly show the control performance deterioration, which was caused by the specimen nonlinearity, as observed in Figure 16c. The control accuracy is lower for the 50% amplitude than for the 25% amplitude, because the nonlinear characteristics become stronger as the amplitude increases.

### 4.3.2 Nonlinear signal-based control

The design of NSBC was based on the transfer function in Equation (24), and the linear model was set to $G_0(s) = G_0(s)$. The control input is again determined by $u(s) = K_r(s)\dot{r}(s) + K_\sigma(s)\dot{\sigma}(s)$, where $\dot{\sigma}(s) = \ddot{y}_0(s) - \ddot{y}_0(s)$, $K_r(s) = G_0(s)^{-1}$, and $K_\sigma(s) = G_0(s)^{-1}F_\sigma(s)$. $F_\sigma$ was the second-order Butterworth filter with a bandpass range of 0.2–20.0 Hz. This range was determined to prepare for the pure time delay estimation error and unmodelled dynamics of the shake table with the specimen. The experimental results for various amplitudes are summarised in Table 7.

According to Tables 6 and 7, the results of NSBC are obviously better than those of the basic approach for the amplitudes of 25% and 50%. At 50% amplitude, NSBC achieved $S_f = 99.7\%$ and $S_t = 96.4\%$, even though the specimen in this case displayed the nonlinearity in Figure 17c.

For further investigation, an experiment based on the same NSBC controllers was performed with an amplitude of 80%. In comparison with Figures 17c and 18c, the nonlinearity observed in this case is clearly stronger than that in the case with an amplitude of 50%. Even with the severe nonlinearity, NSBC achieved accurate control, as shown in

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**FIGURE 16** Basic control approach for JMA Kobe motion 50%: (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop

**TABLE 7** Experimental results obtained using NSBC

| Amplitude | 25% | 50% | 80% |
|-----------|-----|-----|-----|
| $E_{max}$ (m/s²) | 0.54 | 0.84 | 1.0 |
| $S_f$ (%) | 99.6 | 99.7 | 99.7 |
| $S_t$ (%) | 93.2 | 96.4 | 98.2 |

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**FIGURE 17** Nonlinear signal-based control for JMA Kobe motion 50%: (a) time history, (b) Fourier amplitude spectra, and (c) hysteretic loop
Figure 18, with $S_f = 99.7\%$ and $S_t = 98.2\%$. Judging from this highly accurate control, the hysteretic loop should be very close to that under 100% accurate control.

5 | CONCLUSIONS

This paper has introduced the design of NSBC for shake tables with nonlinear MDOF systems. The performance of NSBC has been examined by conducting numerical and experimental shake table tests.

In the numerical tests, NSBC achieved near 100% accurate control even though its controllers were designed on the basis of inaccurate table dynamics and the specimen displayed nonlinear characteristics. Under the same conditions, the basic control approach exhibited insufficient performance. NSBC basically requires an estimation of the pure time delay, but estimation error is likely to occur in actual applications. In numerical simulations reflecting this estimation error, proper design of the nonlinear signal feedback controller has prevented instability although slightly compromising the control accuracy.

For the experimental shake table control tests, the NSBC and basic approach controllers were designed on the basis of system identification results obtained by conducting small amplitude excitations. With no specimen on the shake table, both control approaches achieved excellent control. When the specimen was on the table, the basic control approach failed to achieve accurate control, because of the nonlinear characteristics in the specimen. However, NSBC yielded the expected acceleration signal on the table with near 100% accuracy, even though the specimen displayed more severe nonlinear characteristics with this control approach than with the basic approach.

In this study, the performance of NSBC has been examined by conducting shake table tests with a simple specimen imitating a building. Thus, examinations based on more realistic multistorey structures made of different materials (e.g., wood, concrete, or brick) are important further work. The application of NSBC to multiaxis shake tables and substructuring experiments are also fundamental subjects for further advancement of shake table experiments.

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