Torque-Matched Aerodynamic Shape Optimization of HAWT Rotor

Ali Al-Abadi\textsuperscript{1,2}, Özgür Ertunç\textsuperscript{1}, Florian Beyer\textsuperscript{1}, & Antonio Delgado\textsuperscript{1}

\textsuperscript{1}Institute of Fluid Mechanics, FAU Erlangen-Nuremberg, Germany
\textsuperscript{2}AlKhwarizmi College of Engineering, University of Baghdad, Iraq

E-mail: ali.al-abadi@fau.de, Ozguer.Ertunc@lstm.uni-erlangen.de

Abstract. Schmitz and Blade Element Momentum (BEM) theories are integrated to a gradient based optimization algorithm to optimize the blade shape of a horizontal axis wind turbine (HAWT). The Schmitz theory is used to generate an initial blade design. BEM theory is used to calculate the forces, torque and power extracted by the turbine. The airfoil shape (NREL S809) is kept the same, so that the shape optimization comprises only the chord and the pitch angle distribution. The gradient based optimization of the blade shape is constrained to the torque-rotational speed characteristic of the generator, which is going to be a part of the experimental set-up used to validate the results of the optimization study. Hence, the objective of the optimization is the maximization of the turbine’s power coefficient $C_p$ while keeping the torque matched to that of the generator. The wind velocities and the rotational speeds are limited to those achievable in the wind tunnel and by the generator, respectively. After finding the optimum blade shape with the maximum $C_p$ within the given range of parameters, the $C_p$ of the turbine is evaluated at wind-speeds deviating from the optimum operating condition. For this purpose, a second optimization algorithm is used to find out the correct rotational speed for a given wind-speed, which is again constrained to the generator’s torque rotational speed characteristic. The design and optimization procedures are later validated by high-fidelity numerical simulations. The agreement between the design and the numerical simulations is very satisfactory.

keywords: Horizontal Axis Wind Turbine, Optimization, Schmitz theory, BEM, rotor blade design

1. Introduction
In different design steps of a HAWT there are different optimization objectives, such as the higher aerodynamic efficiency, lighter structures, lower fatigue loads, noise and cost [1]. These objectives are normally subjected to different constraints and trade-off among them to get the optimum one has to be considered. The efficiency of the wind turbines depends on many subsystems such as rotor shape, gear box, electrical generator and control of the turbine. The aerodynamics of the rotor blades shape playing a decisive role in maximizing the efficiency. Increasing energy demand requires more and more optimized rotors for efficient exploitation of the available wind energy.

Jureczko [2] devised a gradient based aerodynamic and structural shape optimization for the wind turbine blades. The optimization problem was subjected to constraints of the blade chord length, the tip speed and the optimal blade weight. Evolutionary optimization algorithms with embedded aerodynamic simulators were also utilized for the multi-objective blade shape optimization [3, 4, 5, 6].
The control of the commercial wind turbines can be stall regulated-fixed pitch (SR-FP) which is coupled to the generator via gearbox or direct-drive. The second control approach is to control the magnitude of the rotor current by adjustable external load. For example, Muljadi et al. [7] evaluated the variable speed fixed-pitch control (VS-FP) strategy to maximize the efficiency under varying wind speed conditions. The third approach is the active pitch control which can provide a wide range of rotational speeds [8]. Pitch-control strategy was also integrated in a multi-objective rotor shape optimization procedures [9].

The coupling technology of the rotor with the generator has to be considered in the design. The use of the direct-drive increases in comparison to gear-box one as the wind industry needs scaling up, cutting costs, and improving reliability [10]. Replacing the traditional gearboxes and high-speed generators, which suffer tremendous stress because of wind turbulence, especially in offshore turbines, which faces extreme wind speeds and gusts, with bigger low-speed generators can reduce the complexity owing to easier operations and maintenance [11, 12].

Whatever the selected drive and control technology is, the best performance of the wind turbine can be achieved when the aerodynamic shape of the turbine is matched to the torque rotational speed characteristic of the driving unit. This matching is more essential for direct-drive technology, since the rotor constrained strictly with the torque rotational speed characteristic of the generator. The present study aims at developing a blade shape design, analysis and optimization method constrained to the torque rotational speed characteristic of the generator. An initial rotor design is performed with combination of Schmitz and BEM theories as shown in the second section. The gradient based optimization method, which is explained in section 3, is applied for maximizing the power coefficient while satisfying the torque constraint of the generator. The results of the developed design and optimization method are verified by numerical simulations of the flow over the optimized rotor geometry whose details are provided in section four. The paper ends with a conclusion section.

2. Design procedure

The design procedure is based on the combination of Schmitz and BEM theories. The blade profile is designed with Schmitz theory, while the torque and power are calculated as is done in BEM theory [13, 14, 15, 16].

BEM is not selected for the design since it iteratively founds the axial and tangential induction factors a and a’ for ring elements along the blade span. The iterations becomes then the bottle neck during optimization. Schmitz theory is favored for the design over the Betz, because it considers the rotation of the wake.

Schmitz theory relates the velocity triangles in figure 1 and finds the power in terms of the relative angle \( \varphi \). The optimal relative angle \( \varphi = \varphi_x = 2/3\varphi_1 \) gives the maximum power, which is found by differentiation of the power equation of the ring element along the span of the blade [16, 17]. Hence, at this optimum state, the tangential velocity \( u \) reads:

\[
-u = r\omega + \frac{1}{2}\Delta u
\]

where \( r \) is the radius, \( \omega \) is the angular speed and \( \Delta u \) is the speed difference due to the wake rotation.

Schmitz assumes a constant angle of attack \( \alpha \) along the span of the blade and consequently the lift coefficient \( C_l \) remains constant along the blade, and that in turn reduces the complexity of the equations. Moreover, the drag coefficient \( C_d \) is taken as zero. While keeping the relationship of \( \alpha + \beta = \varphi \) as in the theory of Betz, a new pitch angle \( \beta(r)_{\text{Schmitz}} \) distribution is found. Furthermore, the design angle of attack \( \alpha \) is selected to give maximum lift to drag ratio. In the present study it is set to \( 7^\circ \) and the airfoil of (NREL S809) profile is kept the same along the span. Equating the axial force obtained from the conservation of momentum equation and the
same force derived from the airfoil theory, the chord \( c(r) \) distribution is obtained. Thus, Schmitz theory provides an optimum blade profile which includes the rotation effect.

When the rotating wake effect is considered, the tangential induction factor is always greater than 0, i.e. \( a' > 0 \) and the axial induction factor is always less than its optimum value of Betz, i.e. \( a < 1/3 \). The tangential induction factor \( a' \) which causes the wake rotation is defined as:

\[
a' = \frac{u - r\omega}{r\omega}
\]

(2)

Figure 1. Velocities, angles and forces in the rotor plane in stall regulator-fixed pitch Theory

In the theory of Schmitz \( a \) and \( a' \) are not constant and change as a function of \( \lambda_r \). Equating the equations of the thrust \( dF \) on an annular element [15] yields a direct relation of \( a, a' \) and \( \lambda_r \) as follows:

\[
a \left( 1 - a \right) \frac{a'}{a'(1 + a')} = \lambda_r^2
\]

(3)

An alternative expression for \( dP = dT\omega \), which is dependent on \( a, a' \) and \( \lambda_r \), is needed. The power can be expressed as a function of the conservation of angular momentum:

\[
P = \frac{1}{2} \rho A v_1^3 \left[ \frac{8}{\lambda^2} \int_0^\lambda a'(1 - a) \lambda_r^3 d\lambda_r \right]
\]

(4)

The maximum power can be achieved when the term \( a'(1 - a) \) is the highest. An expression for \( a' \) can be found from (3) and substituting into (4), then setting the derivative with respect to \( a \) to 0 yields to:

\[
\lambda_r^2 = \frac{(1 - 5a + 4a^2)^2}{(1 - 4a + 3a^2)}
\]

(5)

To get a direct relationship between \( a \) and \( a' \) one has to equate (5) with (3) which results in:

\[
a' = \frac{1 - 3a}{4a - 1}
\]

(6)

This equation is valid for an ideal HAWT \( (C_d = 0) \) with wake rotation. From the relative velocities triangle of the Schmitz (figure 1), one also gets

\[
\tan \phi_x = \frac{1 - a}{1 + a' \lambda_r}
\]

(7)
Combining equations (6) and (7) gives

\[ a = \frac{(5 - \lambda_r \tan \varphi_x) - \sqrt{\lambda_r \tan \varphi_x - 9}}{8} \]  

(8)

Substituting \( \varphi_x \) into (8) gives the final distribution of \( a \) depending only on \( \lambda_r \), since \( \varphi_1 = \varphi_1(\lambda_r) \). Finally, \( a \) and \( a' \) becomes dependent solely on \( \lambda_r \) or \( \varphi_1 \). A comparison with BEM induction factors showed that they are almost identical for radius ratio higher than 0.2, as expected for the same assumptions.

After finding the induction factors for the designed blade, the torque \( T \), the axial force \( F \) and the power \( P \) can be calculated by integrating the tangential and axial force (per unit meter of the blade) along the span by using the BEM theory. Thus the torque produced by the wind on the rotor blades reads:

\[ T_{\text{rotor}} = B \int_0^R dT = B \int_0^R \left( \frac{1}{2} \rho w^2 c C_x \right) dr \]  

(9)

where \( B \) is the number of blades and \( C_x = C_l \sin(\varphi) \) with zero drag coefficient.

In order to include the profile loss (drag loss) and tip loss, the following efficiency factors should be multiplied by the local torque on a differential ring element along the span \( dT \), which is calculated in the absence of profile and tip losses [16]:

\[ \eta_{\text{profile}}(r) = \frac{C_l \sin(\varphi) - C_d \cos(\varphi)}{C_l \sin(\varphi)} = 1 - \frac{3r\lambda}{2R(C_l/C_d)} \]  

(10)

\[ \eta_{\text{tip}} = (1 - \frac{0.92}{B \sqrt{\lambda^2 + 4/9}})^2 \]  

(11)

This formulation is applied to design the blade geometry under given wind and rotational speed and to calculate the generated torque by the rotor.

### 3. Formulation of the Optimization

#### 3.1. Design Optimization

In reality the turbine adjusts itself to the rotational speed characteristic of the drive unit and finds the proper rotation speed for each wind velocity. The drive unit can be either direct one or comprises a gearbox and generator. An optimization method is then needed to change the blade shape in order to capture the maximum power from the wind under the torque rotational speed constraint of the drive unit. In this work, this method is called as torque matched aerodynamic shape optimization (TMASO).

The objective is to find the maximum power coefficient, which is

\[ C_p = \frac{\omega T_{\text{rotor}}}{\frac{1}{2} \rho v^3 \pi r^2} \]  

(12)

Hence, the objective function is formulated as:

\[ \text{Minimize} \ (-C_p) \]  

(13)

As the rotor coupled to the drive, \( T_{\text{rotor}} \) should match the torque of the drive unit \( T_{\text{drive}}(\omega) \). The difference between them should be minimized so that torque matching can be achieved. For this purpose, the following constraint is used during the optimization.

\[ \sqrt{(T_{\text{rotor}} - T_{\text{drive}})^2} = 0 \]  

(14)
Torque rotational speed characteristic of a generator, which will be used in the experimental validation phase of our studies, is selected for the present optimization study. The procedure optimizes the wind speed and rotational speed together with the shape of the rotor blade. The shape of the rotor blade is defined by the local pitch angle \( \beta(r) \) and the chord length \( c(r) \) along the blade [5, 15]. Bounds for the wind velocity \( v_1 \) and the rotor’s angular speed \( \omega \) are needed in the procedure. The range of \( v_1 \) and \( \omega \) was set between 8 and 20 m/s and between 200 and 400 rad/s, respectively. The radius of the rotor was set to be \( R = 0.25m \). The \( fmincon \) optimization function in MATLAB with the active set algorithm were used for the optimization.

Before starting with optimization, an initial blade design is made by using the Schmitz method at TSR of 7. In order to show the influence of the type of the losses, three different torque matched blade shape optimizations were performed: First optimization was done without considering the losses. In the second optimization, profile loss was added in the performance analysis and in the third optimization tip loss was added. The optimizations ended with chord and pitch angle distributions along the blade span as shown in figure 2. This study shows that the blade has to cover the power reduction caused by the losses with additional increase of both \( c \) and \( \beta \). Moreover, the effect of tip loss on the shape is higher than that of profile loss.

The result of the optimization with profile and tip losses are considered to be the optimum design. The optimum operation conditions are a wind velocity of about 15 m/s and rotor’s angular speed of about 300 rad/s. Hence, the optimized rotor delivers a maximum \( C_p \) of 0.44 at a TSR of about 5 (figure 3), which is different than the initially selected one.

![Figure 2. Initial design and the optimized pitch angle and chord distribution along the rotor radius ratio](image)

The torque matched optimized blade is considered as a point design, which gives the maximum power at one wind velocity with its associated rotational speed of the rotor. During the optimization, the relative angle is at its optimum value of \( \varphi_x \). In order to monitor the rotor performance under variable incoming wind velocity, torque matched \( C_p \) calculations were performed by using another optimization procedure. At a given wind velocity, the relative angle \( \varphi \) and the rotational speed were found and \( C_p \) values shown in figure 3 were calculated. This study reproduces the optimum point found in the previous step, namely a \( C_p \) of 0.44 at a TSR of approximately 5. In addition, one can recognize three important regions. The first one is the design range (maximum power range), where \( \alpha \) is near to its design value. Second region is the stall range where the TSR is in its lower values, which means that \( \alpha \) will reach its stall value, which in turns causes a dramatic drop in the power. The third region, is the low \( \alpha \) region which appear at high TSR range causes a slightly reduction of the power with increasing TSR.
4. Numerical Simulation

A commercial CFD code (Star-CCM+) was employed for the numerical analysis. The strategy used in simulation is based on creating two domains. Inner cylinder, which includes the wind turbine and the near-surrounding air, rotates with respect to the laboratory coordinate. Outer cylinder is kept stationary with respect to the laboratory coordinate system. The outer cylinder represents the ambient air around the wind turbine. The interface between the two domains are connected by using the frozen rotor model. The two domains and their dimensions in relation to the rotor radius R are shown in figure 4.

The dimensions for the outer domain were chosen to be away from the wind turbine to insure that the boundary condition at inlet, outlet and far-field surfaces has minimum influence on the aerodynamics forces exerted on the turbine rotor blades. The inner cylinder dimensions were chosen to cover tightly the rotor geometry.

Due to the complex shape of the rotor, the geometry has to be resolved in a good manner. Therefore, the polyhedral meshing model is chosen (see figure 5). This includes an unstructured mesh that has about five times less elements than the other unstructured alternative with
tetrahedral elements. Unstructured meshes have the advantage that they can mesh complex surfaces more easily than the structured meshes.

Spalart-Allmaras turbulence model is used. This model is especially designed for RANS solutions of aerodynamic flows [18]. Furthermore, it solves only one transport equation that determines the turbulent viscosity $\tilde{\nu}$ [19]. In its standard expression it is meant to be applied without a wall function [20]. On the other hand, the turbulent boundary layer, including the viscous sublayer, has to be resolved in a highly accurate way. This can be achieved with a good boundary layer mesh, which led to low $y^+$ values. Thus, at the turbine blades the prism layer mesher is taken as shown in figure 5. The prism layer is selected with a number of layers and an absolute thickness with a suitable stretch factor to insure the height of the prism layer next to the wall is less than the sublayer thickness, and this will guarantee that the values of $y^+$ during operation are kept always less than 5.

The total number of cells is about 7 million, where only 2 million cells are in the outer domain.

![Figure 6. Comparison between theoretically optimized and simulated power coefficients](image)

$C_p$ obtained from the theoretical calculations and the numerical simulations are compared in figure 6. Numerical simulations underestimates the theoretical ones about 7%. This deviation might be owing to the losses not considered in the theoretical part, which are possibly caused by the hub and the hub-blade connections. Where, the theoretical optimization analysis based on one dimensional theories with some losses corrections as explained before, whereas the numerical analysis include the full turbine, thus, it is expected to have additional losses from the hub and the blade root regions, beside the ability of including the three-dimensionality flow interactions in the numerical one. The mesh quality, which suppose to be resolved more near the surface to capture the pressure different and the viscous effect in the near blade boundary-layer region, could be another reason for the theoretical and numerical mismatching.

5. Conclusions
The study reveals a developed method for blade shape design, analysis and optimization constrained to the torque rotational speed characteristic of the generator.

The combination of Schmitz and BEM theories for an initial design of HAWT has advantages of getting new profile that includes the rotational wake. In addition, the forces, torque and power can be calculated with a relativity low computational effort. Hence, the selected design and analysis tool becomes appropriate for very high number of optimization iterations. Including the losses in the optimization code increases the $c$ and $\beta$ distributions values, which in turn
shifted the $C_p$ curves to lower TSR than the initially selected value, because the rotation speed drops for the same incoming wind velocity. Moreover, it is shown that tip loss causes the most effect on the blade shape.

Numerical simulations confirm the validity of the theoretical design, analysis and optimization method employed in this study.

The torque matched aerodynamic shape optimization improves the HAWT performance over a selected range of incoming wind velocity. The advantage of adopting this method coming from its ability to design an efficient turbine rotor that can be set and match the torque-rotational speed characteristics of any existing drive unit. This matching is more essential for direct-drive technology, since the rotor constrained strictly with the torque rotational speed characteristic of the generator. That in turn, can add a novelty to the conventional aerodynamics design of wind turbine.

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