Interpretation of High Energy String Scattering in terms of String Configurations

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ABSTRACT

High energy string scattering at fixed momentum transfer, known to be dominated by Regge trajectory exchange, is interpreted by identifying families of string states which induce each type of trajectory exchange. These include the usual leading trajectory $\alpha(t) = \alpha't + 1$ and its daughters as well as the "sister" trajectories $\alpha_m(t) = \alpha(t)/m - (m - 1)/2$ and their daughters. The contribution of the sister $\alpha_m$ to high energy scattering is dominated by string excitations in the $m^{th}$ mode. Thus, at large $-t$, string scattering is dominated by wee partons, consistently with a picture of string as an infinitely composite system of "constituents" which carry zero energy and momentum.

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Superstring theory is apparently not an ordinary quantum field theory, and yet it seems to satisfy all of the axioms of $S$-matrix theory. It is also the only existing physical theory of quantum gravity which has a chance of avoiding the notorious ultraviolet divergences of quantized Einstein gravity, a disease which also afflicts all of the supersymmetric extensions of that theory. Unfortunately, we do not yet know how to properly formulate string theory. String field theory is an option that is still being pursued, but it must be said that the concept of a string field is, to say the least, cumbersome. All of perturbation theory can be so simply understood using the first-quantized world sheet approach, that one feels there must be a better way.

The string field concept is derived directly from the perturbative particle content of string theory: one assigns an independent string field component to each state of the string. It has long been recognized that the field content of quantum field theory has no necessary correspondence to the particle content. Just think of QCD where no fundamental field is associated with any particle state. Why should it be any different for string theory? If we knew only the particle states (i.e. the hadrons) of QCD, we could apply a familiar strategy for uncovering the fundamental degrees of freedom: probing the short distance structure of hadrons reveals the fundamental degrees of freedom, namely quarks and gluons. In this article we apply the same strategy to string theory. The string scattering amplitudes are known in perturbation theory. We shall try to use them to illuminate the short distance structure of string theory.

To probe short distances, it is necessary to study scattering amplitudes at very large momentum. It would be desirable to consider taking all components of all momenta large, but this is impossible for $S$-matrix elements since the external lines are all on mass-shell. For example, consider a four particle scattering amplitude. There are only two independent invariants, $s = -(p_1 + p_2)^2$ and $t = -(p_4 + p_1)^2$. All others are related to these via momentum conservation and the mass-shell conditions, e.g. $u = -(p_1 + p_3)^2 = \Sigma - s - t$ where $\Sigma = \sum_{k=1}^{4} m_k^2$. Gross and Mende\textsuperscript{1} have already studied the maximally high momentum limit where all three
invariants $s, t, u$ become large in magnitude. This is achieved by taking high energy at fixed scattering angle. The striking conclusion is that the amplitudes decay exponentially in all these invariants, in sharp contrast to the power law fall off of all known quantum field theories. If this exponential decay is associated with the vertex form factors, the result could be interpreted as the scattering of two softly bound composite systems. The result, by itself, does not necessarily imply that any hypothetical constituents of the composites are themselves soft. This latter conclusion is expected from the naive form factor estimate for a charge attached to a point on the string

$$F(Q^2) \sim e^{-\alpha' Q^2 \ln n_C}$$

and the electroproduction structure function

$$W(x_{Bj}) \sim e^{-x_{Bj} n_C},$$

where we have arbitrarily suppressed the contribution of string oscillation modes $n > n_C$. The coefficient of $x_{Bj}$ in the exponent is infinite when the cutoff is removed ($n_C \to \infty$), indicating zero probability for finding the point charge with finite momentum. (Recall that $x_{Bj}$ equals the momentum fraction carried by the struck constituent.) Since we really don’t know how local fields should couple to a string, one of our aims in this article is to expose a true $S$-matrix signature of this wee parton dominance.

Instead of the extreme short distance limit studied by Gross and Mendel, in this article we consider large momentum limits in which a maximal number of invariants are large, but some are much larger than others. Such limits probe the Regge trajectories of string theory and hence might shed light on the more stringy aspects of the short distance limit. Consider the contribution of the leading Regge trajectory to the four open string scattering amplitude (with no Chan-Paton factors):

$$A_4(s, t) \xrightarrow{s \to \infty} \frac{1 + e^{-i\pi \alpha(t)}}{2 \Gamma(-\alpha(t)) s^{\alpha(t)}}.$$
where

\[ \alpha(t) = \alpha' t + 1. \]

As \( t \) varies from 0 to \(-\infty\) these Regge contributions interpolate between the stringy particle spectrum and the high energy fixed angle limit.

Long ago Hoyer, Tornquist and Webber \cite{2,3} discovered that in addition to the familiar linear Regge trajectories, there is an infinite sequence of sister trajectories (see Fig. 1)

\[ \alpha_n(t) = \frac{1}{n} \alpha(t) - \frac{n-1}{2} \]

with ever decreasing slope which contribute to high energy string processes. These new trajectories couple only to higher point functions (typically \( \alpha_n \) contributes only to amplitudes involving at least \( 2(n + 1) \) strings), but their existence nonetheless suggests an interesting modification of the approach to fixed angle scattering described above. In particular, although each individual trajectory decreases linearly with \( t \) as \( t \to -\infty \), the envelope of the complete family of trajectories decreases only as \(-\sqrt{-t}\) in the same limit. If the Cerulus-Martin lower bound \cite{4} on fixed angle scattering cross sections

\[ |A(s, t)| > e^{-f(\theta)\sqrt{s}\ln s} \]

holds, it follows that the scattering amplitude can not be dominated by a Regge trajectory \( \alpha(t) \) for all \( t < 0 \) unless that trajectory satisfies \( \alpha(t) > -C \sqrt{-t} \) with \( C > 0 \) as \( t \to -\infty \). Thus if the envelope of sisters is taken as the effective leading trajectory for the tree approximation to string theory, we can say that the tree approximation is in this sense compatible with the bound. In multi-loop string amplitudes an effect similar to that of the sister trajectories is produced by multi-Regge cuts, so in their work, Gross and Mende anticipate that multiloop corrections could restore the bound. We are raising here the possibility that the “classical”
string theory (i.e. tree approximation) contains a mechanism for restoring the bound, e.g. by exploiting a different background.

There are two approaches toward extracting Regge trajectories from scattering amplitudes. The first is to analyse directly the high energy limit at fixed $t$ and to extract the trajectory as the power of $s$ that characterizes the asymptotics. This is the only way the sisters have been obtained so far. Unfortunately this analysis obscures the string interpretation. The second method is to look for angular momentum singularities in the partial wave scattering amplitudes. Since these angular momentum singularities are directly associated with the excitation spectrum of the string, the string interpretation should be more transparent in this second approach. In this note we shall explain the origin of the sister trajectories in terms of the sum over string excited states contributing as intermediate resonances in the scattering amplitudes. This analysis allows a more direct interpretation of them in terms of string configurations. It also provides a link with an earlier analysis of the spin content of the physical states of string theory\textsuperscript{[5]} in which this pattern of sister trajectories naturally appears. Our work confirms the relevance of the spin content analysis to the high energy behavior of string scattering amplitudes. Since the whole family of sister trajectories encodes information about the short distance structure of string theory, we hope that this interpretation will yield hints about the relevant degrees of freedom that might be used in a reformulation of string theory.

Instead of the usual procedure of trying to define explicit partial wave projections on multiparticle scattering amplitudes, we can directly exploit the string oscillator formalism to display the poles in any selected channel as a sum over string states according to the decomposition of the identity

\[
I = \sum_{n_1=0}^{\infty} \frac{1}{n_1!} \sum_{n_2=0}^{\infty} \frac{1}{2n_2!} \cdots \left( a_{-1}^{n_1} a_{-2}^{n_2} \cdots | 0 \right) \cdot \left( 0 | \cdots a_2^{n_2} a_1^{n_1} \right). \tag{1}
\]
The normalizations in (1) are fixed by the projector condition $I^2 = I$, and the commutation relations

$$[a^\mu_m, a^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}. \quad (2)$$

We have also suppressed the $\mu_k$ indices: all indices in the ket are understood to be contracted with corresponding indices in the bra. Pick a channel in $M$ open string scattering with total momentum $P$ and consider the dependence on the invariant $t = -P^2$. The poles in the scattering amplitude can then be exhibited as

$$A_M = \langle L | \prod_{r=1}^{\infty} \sum_{n_r=0}^{\infty} \frac{1}{r^{n_r} n_r!} a^{n_r}_{-r} | 0 \rangle \cdot \langle 0 | a^{n_r}_{R} \sum_{j=1}^{\infty} \frac{1}{j n_j - \alpha(t)} | R \rangle. \quad (3)$$

Our aim is to identify families of states which can be associated with Regge trajectories of a definite type and then to examine how each such family generates singularities in the complex angular momentum plane.

Let us begin by asking which family of states generates the leading Regge trajectory $\alpha(t) = \alpha't + 1$. These should be the states that maximize the angular momentum at fixed rest mass. For the classical string these would be the motions in which a straight string rotates about its center in a plane. For the first quantized string, we fix a $(mass)^2$ level and notice that we can maximize the angular momentum by maximizing the number of oscillator raising operators, which we do by using only the $a^{\mu}_{-1}$ operators in constructing the state:

$$a^{\mu_1}_{-1}a^{\mu_2}_{-1} \cdots a^{\mu_N}_{-1} | 0, P \rangle \quad (4)$$

where $-\alpha'P^2 = N - 1$. In the string’s rest system the angular momentum of these states can be identified by the fact that the space components of each oscillator transform as an $O(D - 1)$ vector and the time component as a scalar. The spatial components always enter as a completely symmetric $O(D - 1)$ tensor, so the irreducible angular momentum states are formed by removing all the traces. The maximal angular momentum is clearly $J = N$ and corresponds to the states with
all $\mu_k$ spatial and projected to be traceless in all spatial indices. We must associate
the leading trajectory with the entire family of these maximal angular momentum
states with $N = 0, 1, 2, \cdots$. Clearly for this family, $J = N = \alpha' t + 1$.

The next step is to examine how this family of states contributes to the scat-
ering amplitude. To end up with only the leading trajectory we would have to
carry out all of the restrictions on the $\mu_k$ and the trace projections described in the
preceding paragraph. This would be tedious and, for our purposes, not very illumi-
nating. If we take the whole family (4) instead, it is clear that we should generate
the leading trajectory accompanied by some lower lying daughter trajectories par-
allel to the leading one and spaced by integer values in angular momentum. In the
relativistic context it is indeed more natural to consider such groups of trajectories.
So let us restrict the sum over states in (3) to (4).

$$A^{(1)}_M = \sum_{n=0}^{\infty} \frac{1}{n!} \langle L | a_{n-1} | 0 \rangle \cdot \langle 0 | a^n_1 | R \rangle \frac{1}{n - \alpha(t)}.$$  (5)

The sum over $n$ in this formula is closely related to a partial wave expansion.
Instead of being a pure angular momentum, $n$ is a label denoting a group of angular
momentum values whose maximal value is $n$. But for purposes of interpreting
high energy behavior it is just as effective as the pure angular momentum. Thus
instead of the conventional Sommerfeld-Watson transformation on a true partial
wave expansion, we will apply the same transformation to (a slightly rearranged
version of) the sum over $n$ in (5). To do this one has to continue the summand to
continuous complex values of $n$. The explicit $n$ dependence shown in (5) has a clear
continuation. Moreover the “Regge” pole $n = \alpha(t)$ is quite explicit. However the
continuation of the $n$ dependence in the product of matrix elements is not obvious
so we now turn to this question.

Call the momenta of the $I < M$ strings, described by the state $|R\rangle$, $p_i$ and
those $J = M - I$, described by the state $\langle L |$, $q_j$. Each vertex factor is in the form
of a Koba-Nielsen integral over variables which we call $y_i$ for the right factor and
\[ x_j \] for the left factor. We can choose, for example, \( y_1 = x_1 = 0 \) and \( y_I = x_J = 1 \) with all other variables taking values between 0 and 1. Define \( P_n(y) = \sum_i p_i y_i^n \) and \( Q_n = \sum_j q_j x_j^n \). Then the products of matrix elements in the \( n^{th} \) term of (5) can be written

\[
F_n \equiv \langle L | a_{n-1}^n | 0 \rangle \cdot \langle 0 | a_1^n | R \rangle = \int \prod_{i=2}^{I-1} dy_i \prod_{j=2}^{J-1} dx_j L(x,q)R(y,p)[Q_1(x) \cdot P_1(y)]^n.
\]

(6)

Clearly the \( n \) dependence is isolated in the last factor. This formula shows that for fixed integer \( n \) the invariants \( p_i \cdot q_j \) enter through a homogeneous polynomial of order \( n \). These invariants are the multi-particle analogue of the crossed channel variable \( s \) in 2-2 scattering. They are linear in the cosines of the various scattering angles, and this explains why the sum over \( n \) is analogous to the partial wave expansion. It is the \( p_i \cdot q_j \) which are taken large in Regge limits. For \( M > 4 \) there is some degree of choice in specifying a high energy limit, and an associated choice of the suitable summation variable to continue into the complex plane. A simple limit would be to take all these invariants to infinity at fixed ratio, and if this were possible the variable \( n \) of (5) is the suitable one. However, for a fixed dimension of space-time, this limit is impossible except for low values of \( M \). In this paper, we consider high energy limits which are possible in 4 space-time dimensions. That is, if our theory is defined in a higher number of dimensions, we shall only allow large momentum components in a four dimensional subspace. If we let \( s \) be one of the large invariants, then because of the 4-dimensionality constraints on the large momentum components entering the \( q_i \cdot p_j \), \( Q \cdot P \) will not be homogeneous in \( s \) but we can write \( Q \cdot P = As + B \) with \( A \) independent of \( s \) and \( B = O(1) \) as \( s \to \infty \). Then

\[
[Q_1(x) \cdot P_1(y)]^n = \sum_{k=0}^{n} \binom{n}{k} (As)^{n-k} B^k,
\]

And a rearrangement of the sum in (5) yields

\[
A_M^{(1)} = \sum_{n=0}^{\infty} \frac{1}{n!} s^n \sum_{k=0}^{\infty} \frac{1}{k! n+k-\alpha(t)} F_n^k
\]

(7)
where $F_n^k$ is obtained from $F_n$ by replacing $[Q_1 \cdot P_1]^n$ by $A^n B^k$ in (6). We stress that the choice of $B$ is determined by the high energy limit we wish to analyze. Although the limit determining the choice $B = 0$ might be feasible for low $M$, it would only be possible for all $M$ in an infinite number of space-time dimensions. Note also that the term containing the leading Regge pole is the one with $k = 0$.

Turning at last to the problem of continuing $n$, we first note that for integer $n$, $A$ is just a polynomial in $y$ and $x$ and has a benign effect on the integrals; in particular the vanishing of $A$ in the integration range causes no difficulty. However when we attempt to continue $n$ to continuous complex values, the vanishing of $A$ in the integration range will induce singularities in the $n$ dependence at negative values of $n$. Before discussing these singularities we must first spell out how the continuation in $n$ is to be done. Carlson’s theorem shows that such a continuation is unique up to ambiguities like $\sin n\pi$. If $A$ changes sign in the integrand, our expression is plagued by a dependence $(-)^n$ which is just of this ambiguous type. This difficulty can be handled by defining separate continuations for even and odd $n$:

$$A^n = \begin{cases} |A|^n & n \text{ even} \\ A|A|^{n-1} & n \text{ odd.} \end{cases}$$

This is completely analogous to the usual definition of signatured amplitudes in Regge theory.

Call the continuation of $F_n^k$ for even $n$, $F_n^{k(+)}(n)$, and that for odd $n$, $F_n^{k(-)}(n)$. Then the Sommerfeld-Watson representation for the r.h.s. of (7) reads

$$A_M^{(1)} = \frac{1}{2\pi i} \int_{C_0} dz \frac{s^z}{\sin \pi z \Gamma (z + 1)} \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\xi_+(z) F^{k(+)}(z) + \xi_-(z) F^{k(-)}(z)}{z + k - \alpha(t)}$$

(8)

Here the contour $C_0$ is a set of infinitesimal circles around all the nonnegative integers. The signature factors $\xi_\pm = (e^{-i\pi z} \pm 1)/2$ guarantee that the signatured amplitudes contribute only where they should: $F^{(\pm)}$ for even(odd) $n$ respectively. The Sommerfeld-Watson transformation consists in deforming the contour $C_0$ as
far as possible to the left in the complex $z$ plane. Singularities exposed by this deformation then contribute to the large $s$ behavior, the rightmost one dominating. Inspection of (8) shows that there is always the “normal” Regge behavior coming from picking up the explicit poles at $z = \alpha(t) - k$. In a situation where $A$ never vanishes (for example, when all the $p_i \cdot q_j$ have the same sign) there are no further singularities in $z$ and one has only normal Regge behavior.

However, generally the $p_i \cdot q_j$ have both signs and $A$ vanishes inside the integration region. Then integration near such a zero induces further singularities that are not of the “normal” Regge type. In the truncated amplitude $A^{(1)}$ the new singularities are fixed (independent of $t$) and can give rise to fixed power behavior. Thus they can modify the expected rate at which the amplitude vanishes with $s$ at large $t$. The occurrence of these fixed singularities in $z$ depends on the process (e.g. for $M = 4$ no such singularity occurs), and on which high energy limit is taken. We restrict attention to high energy limits in which only momentum components within a four-dimensional subspace of space-time are allowed to become large. The main consequence of this restriction is that there are relations among the invariants $p_i \cdot q_j$ which are becoming large, and, in particular $B \neq 0$ for $M \geq 6$.

Let us confirm for $M = 6$ that a double pole at $z = -1$ appears. In this case $(I = J = 3, x_2 \equiv x, y_2 \equiv y)$, putting $s = p_2 \cdot q_2$, $\eta_1 = p_3 \cdot q_2 / s$, and $\eta_2 = p_2 \cdot q_3 / s$,

$$P \cdot Q = p_3 \cdot q_3 + xp_3 \cdot q_2 + yp_2 \cdot q_3 + yxp_2 \cdot q_2 \equiv s(y - \eta_1)(x - \eta_2) + \left\{\frac{p_3 \cdot q_3 p_2 \cdot q_2 - q_3 \cdot p_2 p_3 \cdot q_2}{s}\right\}$$

In a 4 dimensional high energy limit there is a relation between the invariants that forces $\kappa$, the quantity in braces, to be $O(1)$ for large $s$ so it plays the role of $B$ in our earlier discussion. The role of $A$ is played by the factored expression

* Singularities arising from an interior region of integration typically occur at negative (“nonsense”) “wrong signature” integers. Thus the signature factor in the Sommerfeld-Watson representation supplies a zero so at least a double pole singularity is required to give rise to fixed power behavior.


\[(y - \eta_1)(x - \eta_2)\]. Thus we have for \(M = 6\)

\[
F^{k(+)}(z) = \kappa^k \int_0^1 dy R(y, p)|y - \eta_1|^z \int_0^1 dx L(x, q)|x - \eta_2|^z
\]

If \(0 < \eta_1, \eta_2 < 1\) we see that integration near \(y \approx \eta_1, x \approx \eta_2\) produces a double pole at \(z = -1\), a nonsense wrong-signature point,

\[
F^{k(+)} \sim \frac{4\kappa^k R(\eta_1, p)L(\eta_2, q)}{(z + 1)^2},
\]

which contributes the high \(s\) behavior

\[
\frac{2i}{s} R(\eta_1, p)L(\eta_2, q) \sum_{k=0}^{\infty} \frac{\kappa^k}{k!} \frac{1}{k - 1 - \alpha(t)}
\]

(9)

to \(A_6^{(1)}\). There is no singularity at \(z = -1\) in \(F^{k(-)}(z)\).

The contribution (9) to the high energy behavior of \(A^{(1)}\) is intimately linked to the first sister trajectory \(\alpha_2(t)\). At the moment it appears to be a fixed singularity in the angular momentum plane, but that is because we have only included a subset of the intermediate states (4). The complete set of states would be obtained by including all states of the same form as (4) but with the ket \(|0, P\rangle\) replaced by any state obtained by applying an arbitrary monomial of the \(a_{-n}\) with \(n \neq 1\) to \(|0, P\rangle\):

\[
a_{-2}^{n_2} a_{-3}^{n_3} \cdots |0, P\rangle
\]

(10),

where the integers \(n_k\) represent powers of the \(k^{th}\) oscillators and Lorentz indices are suppressed. Thus the total contribution from the double pole at \(z = -1\) will itself be a sum over intermediate states of the form (10). The pole factor for each such contribution will be \(1/(-1 + 2n_2 + 3n_3 \cdots - \alpha(t))\) and the leading power of \(s\) will be \(s^{-1+n_2+n_3\cdots}\). The states with maximal angular momentum amongst the states (10) are those with only \(a_{-2}\) oscillators, \(i.e. n_3 = n_4 = \cdots = 0\). If we consider only
their contribution and perform a new Sommerfeld-Watson transformation on the sum \( n_2 \to z_2 \), the pole at \( z_2 = \frac{1}{2}(\alpha(t) + 1) \) modifies the power \( s^{-1} \) contribution we found in \( A^{(1)} \) to \( s^{-1+\alpha(t)+1/2} = s^{\alpha(t)/2-1/2} \). This power is exactly the second sister trajectory \( \alpha_2(t) \). Clearly the story now repeats itself \textit{ad infinitum}. As we deform the \( z_2 \) contour we can expect to encounter fixed singularities at \( z_2 = -1 \). Their contribution will be a sum over states of the form

\[
a_{-3}^{n_3}a_{-4}^{n_4} \cdots |0, P\rangle
\]

associated with a pole factor \( 1/(-1 - 2 + 3n_3 + 4n_4 + \cdots - \alpha(t)) \) and power of \( s \), \(-1-1+n_3+n_4+\cdots\). The states in (11) with maximal angular momentum are those with only \( n_3 \neq 0 \). Inserting this set of states and continuing \( n_3 \to z_3 \) yields a Regge pole at \( z_3 = \frac{1}{3}(\alpha(t) + 1 + 2) \) and the power of \( s \) becomes \(-1 - 1 + n_3 = \frac{1}{3}\alpha(t) - 1 \) which is exactly the third sister \( \alpha_3(t) \). At the \( k \)th step of this process we generate a singularity

\[
z_k = \frac{1}{k}(\alpha(t) + 1 + \cdots + (k - 1)) = \frac{1}{k}(\alpha(t) + k(k - 1)/2)
\]

yielding a power of \( s \),

\[-(k - 1) + z_k = \frac{1}{k}\alpha(t) - \frac{k - 1}{2} = \alpha_k(t)\]

Our analysis has shown how the sister trajectories are generated successively. Each new sister is required to cancel an unphysical singularity in the residue function of the previous sister. In this iterative scheme, one could consider an approximation consisting in deleting the contribution of all string excitations with mode number larger than some cutoff \( n_C \). Then the first \( n_C \) sisters would appear as trajectories with finite slope, with all the higher ones replaced by a fixed (double) pole which prevents the included sister trajectories from dominating at indefinitely large \(-t\) (see Fig. 2). One can plausibly associate such a fixed power behavior with
a “hard” point-like constituent present in the approximate model. For example, taking $n_C = 1$ gives a crude caricature of the behavior expected from $QCD$ which does predict hard point-like constituents. But in string theory, closer examination ($i.e.$ increasing $n_C$) shows that this “hard” constituent itself has structure associated with the second modes of excitation. And so it goes, at higher and higher $-t$ the excitations in the higher modes dominate the scattering. This correlation of high $-t$ probes with the high frequency string excitations, shows that one never reveals truly hard partons. The string is a composite of wee partons only. It is very interesting that these high mode contributions enter in just such a way as to fulfill the Cerulus-Martin bound. That bound is expected in conventional quantum field theories as a consequence of unitarity and power boundedness. Its validity in string theory lends some support to the idea that the absence of hard partons in string theory is not inconsistent with physical principles. It is also suggestive that, in its fundamental formulation, string theory may well be a new kind of quantum field theory.

**Fig. 2.** Envelope of the first few sister trajectories. The dotted line indicates the effective trajectory in a string model with a mode number cutoff $n_C = 6$.

In conclusion we return to our comparison of string theory and QCD. Both theories possess a spectrum of particles lying on Regge trajectories $\alpha(t)$ rising linearly as $t \to +\infty$. To avoid conflict with the Cerulus-Martin bound, the leading trajectory cannot decrease linearly as $t \to -\infty$. In QCD this is achieved because that limit is dominated by hard parton scattering, and consequently the leading trajectory approaches a constant in this limit. There are no hard partons in string theory, but the family of sister trajectories envelops an effective trajectory that saturates the C-M bound.
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