Determination of Azimuthal Anisotropy of $\pi^0$ from the Measured Anisotropy of Photons in Ultra-Relativistic Nuclear Collisions

Rashmi Raniwala$^1$, Sudhir Raniwala$^1$ and Yogendra Pathak Viyogi$^2$

$^1$ Physics Department, University of Rajasthan, Jaipur 302004, India  
$^2$ Variable Energy Cyclotron Centre, Calcutta 700064, India

Abstract

A method is suggested to deduce the anisotropy in neutral pions by measuring the azimuthal anisotropy of photons in ultra-relativistic nuclear collisions. The ratio of the estimated anisotropy in photons to the anisotropy in neutral pions is seen to scale with a parameter which depends on photon multiplicity and anisotropy. This parameter can be determined from experimental data.

1 Introduction

In ultra-relativistic nuclear collisions the colliding nuclear matter emerges with a pattern that has its origin in the incompressibility of nuclear matter and the mean field effects. The observation of collective behaviour in the system validates the use of a hydrodynamic description of the evolution of the collision. Within the hydrodynamic model the collective motion results from pressure gradient in the matter, which depends upon the compressibility of the underlying equation of state [1]. In the case of a phase transition from hadronic matter to quark gluon plasma, increased number of degrees of freedom may lead to a softening of the equation of state [2, 3, 4, 5]. Collective flow manifests in altered event shapes which can be studied by measuring the momentum distributions of produced particles. The azimuthal distribution of these event shapes are studied using coefficients in their Fourier expansion. One of the
major current aims of the study of ultra-relativistic heavy ion experiments is to determine the magnitude of anisotropy present in the final state momentum distributions of various particle species, in different regions of phase space. The systematic dependence of this anisotropy on energy, centrality and other collision parameters is predicted to be sensitive to the equation of state [2].

Most of the experimental studies on azimuthal anisotropy have been restricted to charged particles (a large number of results for charged particle measurements can be found in the review article [2]). The measured anisotropy of charged particles includes effect due to final state Coulomb interactions and may inhibit a direct comparison of results with the predictions of strong interaction physics.

Measurements of anisotropy in the azimuthal distribution of photons have been reported recently [6, 7]. Since almost 90% of photons produced in ultra-relativistic nuclear collisions originate from the decay of $\pi^0$, the anisotropy measured in photons should have their origin in the anisotropy of $\pi^0$ produced in the reaction. However, the measured anisotropy in photons will also include some effect due to decay. The decay introduces non-flow correlations amongst kinematic variables of photons due to four-momentum conservation and also dilutes the existing correlation in azimuthal angles. Determination of the effect of decay should enable deduction of the anisotropy in $\pi^0$. This would complement the study of flow using charged particles and would be free from Coulomb effects. Such a determination will provide much needed additional information to characterise the final state momentum distributions [8]. A comparison of the anisotropies deduced from charged and neutral pions would help in estimating the effect of final state Coulomb interaction on the charged particles.

In the present work we study the effect of $\pi^0$ decay on the observed anisotropy of photons for various assumed flow parameters and multiplicities using a fast simulation technique. Preliminary results can be found in [8, 9, 10]. We show how the anisotropy parameter for the parent $\pi^0$ distribution can be deduced using the measured anisotropy in the azimuthal distribution of photons.

2 Simulation

A complete determination of the flow parameters in different regions would require a measurement of the three-dimensional momentum distribution of all the particles. Several experiments determine the anisotropy parameters in azimuthal angle distributions ($dN/d\phi$) of produced particles. [8, 9, 11, 12]. In the present work we restrict ourselves to the study of photon multiplicity in a de-
tector having full azimuthal acceptance in one unit of pseudo-rapidity suitably chosen with practical detectors in mind [7, 8, 9]. We generate events having azimuthally anisotropic shapes and analyse them using methods employed for the analysis of experimental data.

2.1 Anisotropy

Let us assume that the event shape in the azimuthal plane can be described by an arbitrary ellipse. The most general equation of an ellipse in polar coordinates is

\[ r(\phi) = \frac{1}{2\pi} \left[ 1 + 2v_1 \cos(\phi - \psi_1) + 2v_2 \cos 2(\phi - \psi_2) \right] \]  \hspace{1cm} (1)

where \( \psi_1 \) and \( \psi_2 \) define the direction of the shift of the centroid of the ellipse from the origin and the orientation of the major axis of the ellipse respectively. \( v_1 \) is the magnitude of the shift of the centroid and \( v_2 \) is a measure of the difference in the major and the minor axes of the ellipse. \( v_1 \) and \( v_2 \) denote the first order and the second order flow components, termed as directed and elliptic flow.

2.2 Event generation

For the fast simulation, a set of events consisting of \( \pi^+, \pi^- \) and \( \pi^0 \) is generated. The particle multiplicity in the event is selected in such a way that the multiplicities of charged particles \( (\pi^+, \pi^-) \) and decay photons within the one unit of pseudo-rapidity acceptance of the given detector are in the range of 100 to 4000. Such a wide range of multiplicity corresponds to varying centralities encompassing the SPS, RHIC and LHC energies and serves to determine the effect of fluctuations due to finite number of particles. The multiplicity distribution of particles is assumed to be Gaussian with a width \( \sigma = 2\sqrt{M} \) for a mean multiplicity \( M \).

We assume the azimuthal distributions to have initial flow components \( v^m_i \), where \( m (=1,2) \) denotes the order of flow. Using these values, the azimuthal angles of the charged and neutral pions are assigned according to the distribution given by eqn. (1). \( \psi_1 \) is chosen randomly. For the present work we have chosen \( \psi_2 = \psi_1 \), which corresponds to the major axis of the ellipse along the direction of the shift of the centroid. Physically this represents in-plane flow. The data are also generated for different values of \( v^1_i \) and \( v^2_i \), corresponding to different initial flow.

The pseudo-rapidity and \( p_T \)-distributions are taken from a parametrised form of the HIJING event generator [13] at the relevant energy. Neutral pions
are generated in the pseudo-rapidity interval extending to 1.5 units on both sides of the assumed acceptance of the detector. These are allowed to decay and the η, p_T and φ of the photons are obtained. The photons falling within the acceptance region of the detector are used for further analysis.

2.3 Analysis

The set of events thus generated is analysed to extract the anisotropy parameters v_m using the Fourier expansion technique in a manner identical to the analysis of experimental data. The present analysis is based on 10000 events for each combination of multiplicity and anisotropy.

The generated data have been analysed to obtain the following:

1. ψ₁^{est} : the estimated direction of the first order event plane. Because of finite particle multiplicity the estimated event plane ψ₁^{est} in the generated data fluctuates about the actual event plane, ψ₁, with a spread that depends on the initial anisotropy and the multiplicity.

2. v_1' = \langle \cos(φ - ψ₁^{est}) \rangle : a measure of the directed flow in the direction of ψ₁^{est}, where the average is over all particles of all events.

3. ψ₂^{est} : the estimated direction of second order event plane. This also fluctuates about the actual direction ψ₂.

4. v_2' = \langle \cos 2(φ - ψ₂^{est}) \rangle : a measure of the ellipticity about ψ₂^{est} where the average is over all particles of all the events. This is a measure of the difference of the major axis and the minor axis of the event shape.

Since the estimated event plane differs from the actual event plane, the values of v_m' are systematically lower by a factor \langle \cos m(ψ_m - ψ_m^{est}) \rangle, where the average is over all events for a particular sample. This factor is the resolution correction factor (RCF). The values v_m that represent the magnitude of directed (m = 1) and elliptic (m = 2) flow, for the sample of events, are obtained by v_m'/RCF.

Experimentally, RCF is obtained using the subevent method described in [14]. Here every event is divided randomly into two subevents of equal multiplicity and the angle ψ_m is determined for each subevent. This enables a determination of a parameter χ_m directly from the experimental data using the relation [14, 15]:

\[
\frac{N_{events}(m|ψ_m^{a} - ψ_m^{b}| > \pi/2)}{N_{total}} = e^{-\frac{\chi^2_m}{2}}
\]  

(2)
where $N_{\text{total}}$ denotes the total number of events, $\psi_m^a$, $\psi_m^b$ are the estimated angles of the two subevents (labelled $a$ and $b$) and the numerator on the left denotes the number of events having the angle between subevents greater than $\pi/2m$. The parameter $\chi_m$ is then used to determine RCF according to the relation given in the Ref. [14].

The generated azimuthal distribution of the charged pions are also analysed in a manner similar to the azimuthal distribution of photons. Comparing the results of charged particles and photons enables a determination of the effect of decay.

![Figure 1](image.png)

Figure 1: Estimated value $v_1$ of directed flow for charged particles and photons as a function of multiplicity. Symbols denote different initial anisotropies varying from 0 to 0.1. For photons the initial anisotropy was introduced in the $\pi^0$ distribution. The error bars are statistical.

## 3 Results

### 3.1 Effect of decay

The results for the estimated anisotropy $v_1$ for directed flow are shown in Fig. 1(a) as a function of multiplicity for charged particles and Fig. 1(b) for photons for varying values of $v_1^m$. Similar results for elliptic flow are shown.
in Fig. 2. For charged particles the analysis technique correctly reproduces
the initial anisotropy for all multiplicities for both directed and elliptic flow,
demonstrating the validity of the technique used. For lower multiplicities the
errors on the estimated values are larger due to greater fluctuations.

Figure 2: Estimated value $v_2$ of elliptic flow for charged particles and pho-
tons as a function of multiplicity. Symbols denote different initial anisotropies
varying from 0 to 0.1. For photons the initial anisotropy was introduced in the
$p^0$-distribution. The error bars are statistical.

The same analysis technique, when applied to photons, yields values of
$v_m$ that are generally different from the initial anisotropy present in the $p^0$
distribution. This difference in the anisotropy values for photons and $p^0$ is
attributed to two effects of decay: (i) it dilutes the initial flow by randomising
the existing azimuthal correlation, and (ii) it mimics flow, the correlations
due to momentum conservation producing an anisotropy in the final state
distributions.

To estimate the effect of momentum conservation correlation we ge-
erated azimuthally symmetric distributions, corresponding to $v_m^{\pi}=0$. The decay
photons falling within the acceptance of the detector were analysed, and the
values of $v_m$ obtained for them are also shown in Fig. 1(b) and in Fig. 2(b). One
observes that the deduced anisotropy in photons is always non-zero. At low
multiplicities the values can be as high as those observed in charged particle
measurements [12]. The values for elliptic flow are systematically lower than
for directed flow, implying lesser effect of momentum conservation correlation on the elliptic event shape.

For all values of initial anisotropy, the estimated anisotropy in photons decreases with increasing multiplicity. At low multiplicity the correlation due to momentum conservation competes with dilution due to decay. The relative importance depends on the initial anisotropy in neutral pions. For low anisotropy in $\pi^0$, this leads to enhanced anisotropy in photons.

The above results hold for both orders of flow. It is seen that the decay dilutes the elliptic flow more than it dilutes directed flow.

### 3.2 Estimation of $\pi^0$ flow

While the effect of decay is seen to depend on the values of multiplicity and the amount of initial flow, the ratio of anisotropy observed in photon distributions, $v_m(\gamma)$, to that in the parent $\pi^0$ distribution, $v_m^{in}(\pi^0)$, is seen to scale with the parameter $\chi_m$. Fig. 3 shows this ratio as a function of $\chi_m$ for a large combination of values of anisotropy and multiplicity.

![Figure 3: Ratio of estimated value of anisotropy in photons to the initial value in $\pi^0$ as a function of measured anisotropy parameter $\chi$, for different values of anisotropy and multiplicity (a) for $m = 1$, directed flow and (b) for $m = 2$, elliptic flow. Symbols denote different initial $\pi^0$ anisotropies. Logarithmic $x$-axis is used for better visualisation of the results at lower $\chi_m$ values. The error bars are statistical.](image-url)
We have parametrised the observed scaling behaviour by a relation:

\[ \frac{v(\gamma)}{v^m(\pi^0)} = \frac{a}{(\chi - b)^2} + c \]  

(3)

The relation was motivated by the fact that the effect of two particle correlations scales as \(1/M\) \([14]\) and \(\chi_m\) is proportional to \(\sqrt{M}\). If the exponent is chosen a free parameter, the best-fit value varies between 1.9 and 2.1, depending upon the number of points used in the fit.

The continuous curves in Fig. 3 show the fitted functions for both directed and elliptic flow. The values of the constants for different values of \(m\) are given in Table 1. The values of \(b\) reflect the lower limit of \(\chi\) upto which the anisotropy in \(\pi^0\) can be deduced by this method. The lower value of \(b\) for \(m=2\) implies that elliptic flow in neutral pions can be studied for lower anisotropy and multiplicity. \(c\) is the limiting value of the fractional anisotropy remaining in photons.

Table 1: Fitted values of the constants of the scaling relation.

| Order (\(m\)) | \(a\)  | \(b\)  | \(c\)  |
|---------------|--------|--------|--------|
| 1             | 0.075  | 0.473  | 0.801  |
| 2             | 0.031  | 0.286  | 0.619  |

It is noteworthy that a simple parameterisation can be used up to quite low values of \(\chi\). The parameter \(\chi_m\) depends on anisotropy and multiplicity \((\chi_m = v_m \sqrt{2M})\) and can be determined from experimental data using eqn. (3). Combining this with the anisotropy values \(v_m\) determined from the experimental data on photon distributions, one can deduce the anisotropy present in the parent \(\pi^0\) distribution using the above scaling relation.

To check the efficacy of the method, we simulated another data set with \(v^m_m = 0.02\) and \(v^m_m = 0.04\) and for a range of multiplicities varying between 250 and 3000. This data set was analysed as earlier and the value of anisotropy in neutral pions was estimated using the scaling relation. The estimated values are close to the input values, the ratio of the two deviating from unity only for the cases when both the multiplicity and the initial flow are small.

The error on the deduced \(\pi^0\) anisotropy will be small for large values of \(\chi\), as is evident from the local slope of the curve in Fig. 3. The percentage errors on the deduced anisotropy values are shown as a function of multiplicity in Fig. 4 for both orders of flow. These have been calculated by taking the statistical errors in \(v_m(\gamma)\) and \(\chi_m\), both experimentally measurable quantities. The errors are generally within 10%, except for elliptic flow at low multiplicities.
Figure 4: The percentage error on the value of $\pi^0$ anisotropy, deduced using the scaling relation, as a function of multiplicity of photons for (a) directed flow and (b) elliptic flow. The circles represent initial anisotropy $v_m(\pi^0) = 0.02$ and squares represent $v_m(\pi^0) = 0.04$.

4 Summary

A set of events having built-in azimuthal anisotropies has been analysed using techniques employed for the analysis of experimental data. The anisotropies are introduced in both charged and neutral pion distributions. Photon distributions are obtained from the $\pi^0$ distributions. The pion distributions are generated to have the desired multiplicities in the range 100 to 4000 in one unit of pseudo-rapidity.

Analysis of charged particle distribution leads to estimates of anisotropy values in close agreement with the initial anisotropy present in the event. Due to the effect of decay, the values obtained from photon distributions are different, being lower, or higher, than the initial anisotropy depending on the value of the initial anisotropy and multiplicity.

The ratio of estimated anisotropy in photon distributions to the initial anisotropy in $\pi^0$ distributions is seen to scale with a parameter $\chi$ which can be determined from experimental data and depends on both anisotropy and multiplicity. This scaling behaviour provides us with a simple relation to deduce the initial anisotropy in $\pi^0$ distributions by the experimental study of photon distributions. The anisotropy is obtained with varying precision which improves when either flow or multiplicity increases.
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