I. Introduction

The need for Dark Matter (DM), confirmed by current astronomical observations, is a sufficient reason to look beyond the Standard Model (SM). A reliable way to include DM into a model of physics beyond the Standard Model (SM) is to introduce a discrete symmetry under which some new particles are odd, while all Standard Model (SM) particles are even. The lightest odd particle is stable and is a candidate for DM. In supersymmetry (SUSY) this discrete symmetry is R-parity, in Universal Extra Dimension (UED) models it is KK parity, and for Little Higgs models this discrete symmetry is called T parity \[1, 2, 3\].

In this paper we will study the DM candidates of a \(T\) parity invariant version of the Simplest Little Higgs \[4, 5\] model. Unlike other \(T\) parity invariant Little Higgs models \[6, 7\], it is not possible to make all new particles odd under \(T\) parity. The gauge boson \(Z'\) remains even. Since tree level \(Z'\) exchange is allowed, the \(Z'\) needs to be heavy to avoid conflict with precision electroweak experiments. This means we must take the scale \(f\) to be relatively high. The price we pay for this is fine tuning. Despite this fine tuning, the lightest \(T\) parity odd field, which is either a scalar \(\eta\) or a heavy neutrino remains as an interesting DM candidate. The scalar \(\eta\) is similar to the electroweak singlet limit of the LH scalar DM found in \[8\]. It annihilates predominantly into higgs boson and \(t\bar{t}\) pairs, but the cross section is small unless \(\eta\) is quite heavy. The heavy neutrino is a new addition to Little Higgs model DM. It is also distinct from SUSY neutralino and UED heavy neutrino DM since it is nearly inert under the SM weak gauge group. The SLH heavy neutrino does couple to the heavy gauge bosons and to the higgs.

We investigate the viability of both of these DM candidates using the standard relic abundance calculation. As an additional check, we also explore the constraints direct detection experiments place on SLH DM. In an effort to make this analysis more general, we do not limit ourselves to a particular value of the symmetry breaking scale \(f\). Hopefully this will allow our analysis to be applicable to other models similar to SLH.

We find that \(\eta\) can be an acceptable DM candidate only when coannihilating with a slightly more massive fermion. The \(\eta\) mass that yields the proper abundance will depend on the scale \(f\) and the type of fermion it coannihilates with, but typically \(m_\eta \sim 1.0\) TeV. We find \(\eta - \text{nucleon}\) elastic cross section to be well below the current experimental sensitivity for the \(\eta\) mass range of interest. The SLH heavy neutrino is an acceptable DM particle over a wider region of parameter space. However, direct detection constraints are also much stricter. Current direct detection experiments rule out scenarios with \(f \lesssim 3.0\) TeV, and are predicted to be sensitive to \(f \sim 4.5\) TeV by the time they finish running. To be consistent with the above bounds, heavy neutrino DM must have \(m_N \sim 600\) GeV for \(f = 3.0\) TeV and \(m_N \sim 1.0\) TeV for \(f = 4.5\) TeV.

This paper is organized as follows: In section II we introduce the SLH model and show how it must be modified to make it \(Z_2\) invariant. We describe some of the existing constraints on this model from precision electroweak observables in section III. After a brief overview of the standard relic abundance calculation in section IV, we examine the DM candidates \(\eta\) (section V) and the heavy neutrino (section VI) in more detail. In section VII we include the effects of coannihilation, which we find to be substantial. Section VIII is devoted to the experimental constraints on SLH model DM from direct detection experiments.

II. Fields and Representations

In the Simplest Little Higgs model, the SM weak gauge group is extended to \(SU(3)_w \times U(1)\). All SM electroweak doublets are therefore accompanied by a new partner to form \(SU(3)_w\) triplets. We follow the second set of representations given in SLH since it has no \(SU(3)_w^3\) anomaly. In this set of representations, the first and second generation quarks are promoted to a 3 of \(SU(3)_w\), while the third generation quarks and the leptons are a 3. All fields
and their representations under \((SU(3)_c, SU(3)_w)\) are listed below.

\[
\Psi_{Q3} = (3, 3) \quad \Psi_{Q1}, \Psi_{Q2} = (3, 0) \quad \Psi_{Li} = (1, 3) \quad d^c = (\bar{3}, 1) \quad d^c_1, d^c_2, d^c_3 = (\bar{3}, 1) \quad e^c_i = (1, 1)_i
\]

\[
u^c_1, \nu^c_2 = (\bar{3}, 1) \quad u^c_1, u^c_2 = (\bar{3}, 1) \quad n^c_i = (1, 1)_i
\]

Under this assignment, the partner of the third generation SM particles are even and new particles are odd. The PGBs feel a potential generated radiatively by the higgs triplets, the transformation is

\[
\Phi_1 \rightarrow +\hat{\Omega}\Phi_2.
\]

which requires \(f_2 = f_1 = f/\sqrt{2}\). Under this transformation, the higgs doublets are T parity even, while the remaining PGB, \(\eta\), is T parity odd. The vevs of the \(\Phi\) are also T parity even.

Having defined the action of T parity on \(SU(3)\) triplets, we immediately know how the octet of gauge fields \(A_\mu\) transform.

\[
A_\mu \rightarrow \hat{\Omega}A_\mu\hat{\Omega}
\]

This transformation leaves the diagonal gauge bosons and the gauge bosons in the upper left 2 by 2 block even, while all other gauge bosons are odd. This is immediately a problem since the \(Z'\), a new particle, remains even. For now we press on with our discrete transformation, but we will discuss the consequences of an even \(Z'\) in more detail in the next section.

**Enforce T parity on the Yukawa sector:**

To determine how the rest of the fields transform, we need to enforce T parity invariance on the Yukawa sector. To be consistent with the T parity transformation of the higgs, this means that both \(\Phi_1\) and \(\Phi_2\) must couple with equal strength.

For the first two generations of up quarks, the bottom quark, and the leptons, enforcing T parity is straightforward and requires no additional operators. The fields \(d^c, u^c_1, u^c_2, \text{ and } e^c_i\) are all T parity even. Enforcing T parity on the down quark Yukawas is slightly more difficult, for reasons that become clear by examining the top Yukawa in greater detail.

The operator that leads to the top mass is

\[
\lambda_u \Psi_{Q3}(\Phi_1^d u^c_1 + \Phi_2^d u^c_3).
\]

The transformation properties of the \(u^c_1\) are determined by requiring that the linear combination of the \(u^c_1\) that gets a mass with the heavy top quark is odd. Expanding out the higgs fields to lowest order, we find

\[
-i\frac{\lambda_u h}{2}(u^c_1 - u^c_3)Q_3 + \lambda_u \frac{f}{\sqrt{2}}(u^c_1 + u^c_3)T + \cdots. \tag{6}
\]

The correct T parity transformation for the \(u^c_1\) is therefore

\[
u^c_1 \rightarrow -u^c_3. \tag{7}
\]

Comparing \(6\) to the corresponding equation in SLH, we see that we have halved the number of parameters:
\[ f_1, f_2, \lambda_{u1}, \lambda_{u2} \rightarrow \lambda_u, f. \]  
The reduced number of parameters leads us to a simple relation between the mass of the top quark and the mass of new heavy quark \( T \),  
\[ \frac{m_T}{m_t} = \left( \frac{\sqrt{2} f}{v} \right). \]  
For \( f = 2.0 \text{ TeV} \) this ratio gives \( m_T = 2.0 \text{ TeV} \), almost twice its value at the 'golden point' of SLH.

The problem with the first and second generation down type Yukawas is now clear. The operator  
\[ \lambda_d \Psi Q_1 (\Phi_1 d^c_1 + \Phi_2 d^c_2) \]  
has exactly the same form as \[ 4 \], and would imply a relationship similar to \[ 5 \] between the SM down quark mass and the heavy \( D \) quark mass. For \( m_d \approx 10 \text{ MeV} \) this relation predicts a heavy down quark with a mass less than a GeV! Clearly we need to use a different operator to give mass to the heavy down type quarks.

To avoid this catastrophe, we introduce operators that give mass to the \( T \) parity even and \( T \) parity odd portions separately  
\[ \lambda_d \Psi Q_1 (\Phi_1 + \Phi_2) d^c_H + \epsilon_d \Psi Q_1 (\Phi_1 - \Phi_2) d^c_L, \]  
where \( d^c_H = (d^c_1 + d^c_2), d^c_L = (d^c_1 - d^c_2) \) are respectively the odd and even combinations \[ 32 \] of \( d^c_1, d^c_2 \). The new particles and SM quark masses are no longer related for \( Q_1,2 \).

The only remaining term is the mass term for the heavy neutrinos \( N_i, n_i^c \), both of which should be \( T \) parity odd. The operator  
\[ \lambda_n n^c (\Phi_1 + \Phi_2) L \]  
is sufficient.

The couplings \[ 10, 11 \] violate the little higgs mechanism. By this we mean that they generate the operator \( \Phi_1^2 + \Phi_2^2 + h.c. \) at one loop, and therefore lead to quadratically divergent radiative higgs mass terms  
\[ \delta m^2 \sim \lambda^2 f^2. \]  
The larger the mass of the heavy quark or neutrino, the larger the contribution to the higgs mass. Since we are not aiming to remove fine tuning in this model, we will not restrict particle masses on account of \[ 10 \]. However it does encourage us to look for light \( \lesssim 1 \text{ TeV} \) DM.

Now that we have made the SLH model consistent with \( T \) parity we can identify DM candidates. The DM particle will be the lightest \( T \) parity odd particle (LTOP). The neutral \( T \) parity odd fields available are the neutrinos \( (N_i, n_i^c) \), the scalar \( \eta \), and the heavy gauge bosons \( W_0^{\pm, 0} \). The mass of the heavy neutrino can easily be a few hundred GeV for a small value of \( \lambda_n \), and we expect the mass of \( \eta (\sim \mu) \) to be approximately the weak scale. The \( W_0^0 \) obtain a mass \( m_{W_0} = \frac{\sqrt{2} f}{v} \), when \( SU(3)_c \otimes U(1)_X \) breaks to \( SU(2)_W \otimes U(1)_Y \). The \( W^0 \) mass and all of its interactions are fixed for a given scale \( f \). Because of this lack of flexibility and because the \( W^0 \) is usually heavier than the \( \eta \) and heavy neutrino, we will not consider the \( W^0 \) as a DM candidate here.

III. Parameters and Tunings

As we are unable to make the \( Z' \) odd under \( T \) parity, we need it to be very heavy in order to avoid conflict with precision electroweak observables (EWPO). The only way to get a heavy \( Z' \) in the SLH model is to increase the symmetry breaking scale \( f \). The lower bound on \( f \) was originally estimated to be \( f \gtrsim 2.0 \text{ TeV} \) \[ 4 \], but later analysis point to a stricter bound of \( f \gtrsim 4.5 \text{ TeV} \) \[ 10, 11, 12 \]. Rather than abide by one of these bounds, we will investigate SLH DM for a variable scale \( f \gtrsim 2.0 \text{ TeV} \). This allows our analysis to be more general, and applicable to other similar models. Whenever results for a fixed \( f \) are desired, we will use the value \( f = 4.0 \text{ TeV} \).

For a given \( f \), the masses of the heavy gauge bosons and heavy top are automatically determined.

\[ m_T = m_t \left( \frac{\sqrt{2} f}{v} \right), \quad M_{Z'} = \frac{\sqrt{2} g f}{\sqrt{3 - \tan \theta_{W'}^2}}, \quad M_{W'} = \frac{g f}{\sqrt{2}}. \]  
The radiative contributions of these heavy particles to the higgs mass can then be determined by the usual technique as shown in \[ 4, 5 \]. As a consequence of a high value of \( f \), the masses in \[ 13 \] can be quite large, causing the theory to be somewhat finely tuned. However, since it is not our goal in this paper to remove fine tuning, we will accept it at its current level.

For relatively light DM \( (\lesssim 1 \text{ TeV}) \), we must also check to make sure that any effective four fermion operators are appropriately suppressed. Four fermions operators come about in this scheme through box diagrams with SM external legs and \( T \) parity odd particles on the internal loop. The constraints on new contributions to four electron operators \( (eeee) \) are the most stringent, followed by constraints on four quark operators \( (qqqq) \). We have estimated the contributions of heavy particle loops to \( (eeee), (qqqq) \) in the model we have presented, and find that these contributions are less than \( 1/(5-10 \text{ TeV})^2 \) in the DM mass region of interest. The \( (qqqq) \) contributions from \( D - \eta \) boxes are suppressed by factors of \( (v/f) \) at each vertex (basically we have decoupled the heavy quarks from their SM partners by introducing the extra operators). Charge conservation forbids an \( e N \eta \) vertex, so only \( W' - N \) box diagrams contribute to \( (eeee) \). These contributions are safely suppressed by the larger mass of the \( W' \).

IV. Relic DM Abundance Calculation

We now give a brief overview of the relic DM calculation before examining the heavy neutrino and \( \eta \) further. We follow the standard procedure outlined in \[ 13, 14, 15 \].

The number density \( n \) of a cold dark matter particle \( \chi \) obeys the Boltzmann equation  
\[ \frac{dn}{dt} + 3Hn = -\left\langle \sigma v_{rel} \right\rangle (n^2 - n^2_{eq}). \]  
(14)
Here \( H \) is the expansion rate of the universe, \( n_{\text{eq}} \) is the equilibrium number density, and \( \sigma v_{\text{rel}} \) is the annihilation cross section of the particle times the relative velocity. The \( \langle \rangle \) around the annihilation cross section indicates that we take a thermal average. We will use the standard value for the equilibrium density of a cold, nonrelativistic particle of mass \( m \) with \( g \) degrees of freedom:

\[
n_{\text{eq}} = g \left( \frac{m T}{2 \pi^2} \right)^{3/2} \exp(-m T). \tag{15}
\]

Since we expect the cold DM particles to be slow, we first take the non-relativistic limit of the cross section. To obtain the nonrelativistic cross section, we substitute \( s = 4M^2 + M^2 v^2 \) into the cross section and keep only first order terms in \( v^2 \). Thermal averaging then gives

\[
\sigma v_{\text{rel}} \approx_{\text{NR}} a + b v^2 \rightarrow (\sigma v_{\text{rel}}) \approx a + \frac{6b}{x}. \tag{16}
\]

where \( x = \frac{M}{T} \).

From the thermally averaged cross section we can determine the contribution of particle \( \chi \) to the total energy density of the universe. The density of particle \( \chi \) divided by the critical density corresponding to a flat universe, denoted as \( \Omega_{\chi} h^2 \), is determined to be

\[
\Omega_{\chi} h^2 \approx \frac{1.04 \times 10^9}{M_{\text{pl}}} \frac{x_F}{\sqrt{g^* (a + \frac{6b}{x_F})}}. \tag{17}
\]

Here \( x_F \) is the critical temperature below which the expansion term alone determines the evolution of the number density. It is referred to as the freezeout temperature and it can be expressed analytically as

\[
x_F = \log \left( \frac{0.047 g M_{\chi} M_{\text{pl}} (a + \frac{6b}{x_F})}{\sqrt{g^* x_F}} \right). \tag{18}
\]

The parameter \( g^* \) is the total (spin, color etc.) number of relativistic degrees of freedom at temperature \( x_F \). Typically, \( x_F \approx 20 \).

The results \((17\text{-}18)\) are derived by solving the Boltzmann equation assuming constant entropy per comoving volume in the limits \( x \ll x_F \) and \( x \gg x_F \), then matching the two solutions together. A more complete discussion of this procedure can be found in \((12\text{-}16)\). We will consider DM to be cosmologically acceptable if \( \Omega_{\chi} h^2 \) falls within the 2 \( \sigma \) limits from WMAP, \( 0.094 \leq \Omega_{\chi} h^2 \leq 0.129 \) \((16\text{-}17)\).

Coannihilation

If there is a particle \( (\chi_2) \) just slightly heavier than the LTOP \( (\chi_1) \), then coannihilation processes \( \chi_1 \chi_2 \rightarrow XX' \), etc. become important in determining the LTOP number density \((18)\). To study the effects of coannihilation we must modify our relic abundance formalism slightly. The formalism of \((17\text{-}18)\) assumes that all T parity odd particles have decayed into the LTOP by \( x_F \). In order to consider particles with approximately the same mass as the LTOP we must drop this assumption and examine the evolution of the total number density of T parity odd particles \( n_x = n_{\chi_1} + n_{\chi_2} + \cdots \). The evolution equation for the total number density takes the familiar form

\[
\frac{dn}{dt} + 3 H n = - (\sigma_{\text{eff}} v_{\text{rel}}) (n^2 - n_{\text{eq}}^2). \tag{19}
\]

Here \( \sigma_{\text{eff}} \) is the sum of all cross sections \( \sigma_{ij} = \sigma (\chi_i \chi_j \rightarrow XX') \) weighted by the degrees of freedom \( g_i \) in \( \chi_i \) and the mass difference \( \Delta_i = (m_{\chi_i} - m_{\chi_1})/m_{\chi_1} \). For \( N \) coannihilating particles, we have

\[
\sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} \frac{g_i g_j}{g_{\text{eff}}} (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} e^{-x(\Delta_i + \Delta_j)}, \tag{20}
\]

where

\[
g_{\text{eff}} = \sum_{i} g_i (1 + \Delta_i)^{3/2} e^{-x \Delta_i}. \tag{21}
\]

Note that the mass appearing in \( x = \frac{m_{\chi_1}}{T} \) above is the LTOP mass. The effects of particles much heavier than the LTOP are exponentially suppressed, so only particles very close in mass to the LTOP are relevant in the sum \((20)\).

The solution to \((19)\) is the same as \((17\text{-}18)\) but with modified \( a \) and \( b \).

\[
\Omega_{\chi} h^2 \approx \frac{1.04 \times 10^9}{M_{\text{pl}}} \frac{x_F}{\sqrt{g^*(I_a + \frac{6b}{x_F})}}, \tag{22}
\]

where

\[
I_a = x_F \int_{x_F}^{\infty} \frac{a_{\text{eff}}}{x^2} \, dx, \quad I_b = 2x_F^2 \int_{x_F}^{\infty} \frac{b_{\text{eff}}}{x^3} \, dx.
\]

The averaged coefficients \( a_{\text{eff}}, b_{\text{eff}} \) are defined by substituting \( a_{ij}, b_{ij} \) for \( \sigma_{ij} \) in \((20)\).

We now examine the relic abundance calculations for \( \eta \) and \( N \) in more detail to locate the regions in parameter space where they are suitable DM candidates. For a given value of \( f \), we will vary \( \mu, \lambda_a \), and \( \lambda_d \) (or equivalently \( m_\eta, m_N, m_D \)). For simplicity we will first consider the scenario where the LTOP is much lighter than the other new particles. Once we understand the abundances in the \( m_{\text{LTOP}} \ll m_{\text{NP}} \) limit, we will explore the effects of coannihilation.

V. Scalar \( \eta \) as the LTOP

The first dark matter candidate that we consider is \( \eta \), the remaining PGB from \( SU(3)_c \otimes U(1)_X \rightarrow SU(2)_W \otimes U(1)_Y \) breaking. It receives a mass \( m_\eta^2 \sim \mu^2 \) from the scalar potential.

To determine the viability of \( \eta \) dark matter we need to calculate the annihilation cross section. As \( \eta \) is an
EW singlet, it does not couple directly to the $W^\pm$ or $Z^0$ gauge bosons. The $\eta\eta Z'$ couplings is also zero because $\eta$ is a real scalar field. The dominant annihilation mode for $m_\eta \lesssim f$ is into a pair of higgs bosons $\eta\eta \rightarrow h_0h_0$. This annihilation proceeds through the $\eta^2 h h$ term in the scalar potential, and also through the $t$-channel exchange of a heavy $W'$ gauge boson.

The other important annihilation mode for $\eta$ is $\eta\eta \rightarrow \bar{t}t$. As the $\eta\bar{t}t'$ coupling is large, we might expect $\eta\eta \rightarrow \bar{t}t$ through $T$ exchange to be the dominant annihilation mode. However, this process is helicity suppressed, meaning that this process has $a \approx 0$ (see Eq. 10). The $b$ term for $\eta\eta \rightarrow \bar{t}t$ can be large, but its effect on the overall cross section is suppressed by an additional factor of $x_F \sim \mathcal{O}(20)$.

There is also a contribution to $\eta\eta \rightarrow \bar{t}t$ from higgs exchange. The interaction responsible for this annihilation comes from the scalar potential and has strength $\sim m_\eta^2/\eta^2$. For heavy $\eta$ this becomes $\mathcal{O}(1)$. Annihilation modes of $\eta$ to light fermions or SM gauge bosons are all negligible, suppressed by small fermion masses or by additional powers of $(v/f)$.

In Figure 1, we plot the regions where $\Omega_\eta h^2$ is allowed by cosmology as a function of the mass of $\eta$ and $F$. We assume a higgs mass of 140 GeV. For a given $f$, we see that $\Omega_\eta h^2$ does not fall into the allowed range until the $\eta$ mass is $\mathcal{O}(f)$. Because of the higgs exchange contribution, $\eta$ annihilation into $\bar{t}t$ continues to grow as $m_\eta$ is increased. When $m_\eta \sim f$, the cross section is elevated to the point that we get the right relic abundance. Once $m_\eta$ is much larger than $f$ the cross section is too big and $\eta$ DM is insufficient. Although there is a $m_\eta - f$ region where $\Omega_\eta h^2$ is within the WMAP experimental limits, we cannot accept it as a viable DM candidate. The $\eta$ mass which gives the right relic abundance is always heavier than the mass of the $W'$, and thus the $\eta$ in this scenario would never be the LTOP. As we will see in section VII., the acceptable $m_\eta - f$ region changes significantly when we allow coannihilation with nearly degenerate fermions.

VI. Heavy Neutrino(s) as the LTOP

The heavy neutrino is a slightly more complicated LTOP since it is a dirac fermion. To calculate its annihilation cross section we must calculate the annihilation cross sections of each of it’s Weyl components ($N, n^c$) with themselves and with each other.

$$
\sigma_{tot} = \sigma(NN \rightarrow XX') + \sigma(n^cn^c \rightarrow XX') + \\
\sigma(N\bar{N} \rightarrow XX') + \sigma(n^c\bar{n}^c \rightarrow XX') + \\
\sigma(Nn^c \rightarrow XX') + \sigma(N\bar{n}^c \rightarrow XX') + h.c.,
$$

(23)

where $X, X'$ are some SM particles. Though this looks tedious, many of the sub-cross sections are either zero or very small. The only significant terms are $\sigma(NN \rightarrow XX')$ and $\sigma(Nn^c \rightarrow XX')$.

Unlike many heavy neutrino DM candidates, the SLH heavy neutrino interacts very weakly with SM gauge bosons. It does not couple to the $W^\pm$, and its coupling to the $Z^0$ is suppressed by a factor of $\frac{\eta}{\sqrt{v}} (1 - \tan^2 \theta_W)$. All annihilations proceed through the exchange of a heavy gauge boson, a heavy fermion, or the higgs.

The $N\bar{N}$ annihilation cross section contains the mode $N\bar{N} \rightarrow \nu\bar{\nu}, \ell^+\ell^-$ through $t$-channel $W'$ exchange. For light $m_N$ this is the dominant mode as the coupling of the $N$ to the $W'$ is large. For heavier $m_N$, $N\bar{N}$ annihilation to fermions via $s$-channel $Z'$ quickly becomes the most important process. The reason for this dominance is simply the presence of a pole at $m_N = M_{Z'}/2$ in the $s$-channel propagator. In vicinity of $m_N = M_{Z'}/2$ the cross section is so large that its nonrelativistic limit must be treated specially. The usual Taylor expansion about $v = 0$ (Eq. 17) is very inaccurate and can result in negative cross sections. To avoid this inaccuracy, we follow one of the prescriptions suggested in [15] and set $v = 0$ in all $s$ channel $Z'$ propagators.

For $m_N \gg M_{Z'}/2$, $Nn^c$ annihilation into heavy SM quarks via higgs exchange may also become relevant as the $hNn^c$ coupling is proportional to $\frac{m_N}{f}$. It is interesting to note that, contrary to many DM candidates, the annihilation into $W^+W^-$ is not very significant for either DM candidate considered here.

We have calculated the relevant terms in [18] and the resulting $\Omega_N h^2$ is plotted in Figure 2. for both fixed $f$ and variable $f$.

In the $m_N - f$ plane, we see two strips where the $\Omega_N h^2$ is consistent with experiment. The $m_N$ values of the both strips increases with $f$, yet they remain roughly...
Right at the $Z'$ pole, the annihilation cross section blows up causing the relic abundance to plummet. However, as $m_N$ moves away from the resonant value $M_{Z'}/2$ by $\sim \pm 200$ GeV, $\Omega_N h^2$ crosses the allowed WMAP region. The acceptable $m_N$ regions always border the $Z'$ pole. Thus as $f$ is increased, raising $M_{Z'}$, the allowed neutrino DM mass also increases.

VII. Including Coannihilation

As we saw in section IV, particles slightly heavier than the LTOP can have a significant effect on the relic DM abundance. The nature of this effect depends on the self-annihilation cross section and the number of degrees of freedom of the heavy particle, as well the annihilation cross section of the heavier particle with the LTOP. Coannihilation of the LTOP with a particle that has the same (or fewer) degrees of freedom and participates in the same interactions usually results in a smaller cross section, while coannihilation with a strongly interacting particle with many degrees of freedom can increase the cross section by an order of magnitude or more. In the Simplest Little Higgs model there can be coannihilation among more than one generation of heavy neutrino, between the heavy neutrinos and the heavy quarks, and also between $\eta$ and a heavy fermion $H^\prime$.

Heavy neutrino LTOP including coannihilation

We first consider coannihilation among three degenerate heavy neutrino flavors. Two neutrinos of the same flavor can annihilate with each other as described previously, and two neutrinos of different flavor can annihilate through the $t$-channel exchange of a $W'$. The flavor changing processes $N_iN_j \rightarrow \nu_i\bar{\nu}_j$, $\ell_i\ell_j^\dagger$ are less efficient than the self-annihilation processes, and as a result the average cross section at a given mass $m_N$ for three neutrino flavors is smaller than the cross section for a single neutrino flavor. This results in a larger $\Omega_N h^2$, as can be seen in Figure 3.

In order to achieve the correct relic abundance with three neutrinos, we need a larger self annihilation cross section to compensate for the extra degrees of freedom. This requires $m_N$ to be even closer to $M_{Z'}/2$, and thus the two acceptable DM regions are closer together.

Coannihilation of a heavy neutrino with a heavy quark has a very different effect. Since they are colored, heavy quarks can annihilate to SM quarks through gluon exchange, and thus $\sigma(D\bar{D} \rightarrow XX')$ and $\sigma(d_i\bar{d}_i \rightarrow XX')$ include $O(\alpha_s^2)$ terms. The quark self-annihilation cross sections are consequently orders of magnitude larger than $\sigma(N\bar{N} \rightarrow XX')$, especially for small $m_N,m_D$. As the heavy quarks couple to the $Z'$, their cross section is also enhanced near the pole at $m_D = M_{Z'}/2$.

The large heavy quark self annihilation cross sections dominate the average cross section $\sigma_{<fJ}$ since they are weighted by the large number of degrees of freedom in

![Graph showing the regions where $\Omega_N h^2$ is within the 2 $\sigma$ WMAP limits for a range of $f$ and $m_N$. In the bottom plot: $\Omega_N h^2$ for a heavy neutrino LTOP versus the mass of the heavy neutrino in GeV for fixed $f = 4.0$ TeV. In both plots we have assumed that all other T parity odd particles have mass $\gg m_N$.](image)
All neutrinos are degenerate. Right: \( \Omega = 4 f \) (line) and three (dash-dot line) degenerate neutrino flavors for the colored heavy quarks. The mixed annihilation channel \( \sigma(N \bar{N} \to XX') \) through \( W^\prime \) exchange is enhanced by color factors relative to the mixed neutrino annihilation cross section, but it is still much smaller than the self annihilation cross sections \( \sigma(N N \to XX'), \sigma(D \bar{D} \to XX') \). However, because \( \sigma_{\text{eff}} \) is so strongly controlled by \( \sigma(D \bar{D} \to XX') \) and \( \sigma(d \bar{d} \bar{f} \rightarrow XX') \), the inefficiency of the mixed \( ND \) processes has a negligible effect on the average cross section except for when \( m_D, m_N \) are very close to \( m_{Z'}/2 \).

In Figure 4, we plot the acceptable regions of heavy neutrino DM including coannihilation with a heavy quark as a function of \( m_N, f \). If the heavy quark is just slightly heavier than the heavy neutrino, we see that there can be an additional neutrino mass region where heavy neutrino DM is cosmologically allowed. The range of \( m_D/m_N \) where this second region occurs is very small. For \( m_D/m_N > 1.05 \), this additional allowed mass region either does not exist or exists where \( m_D \) is experimentally forbidden. Even if \( 1.0 < m_D/m_N < 1.05 \), the additional region is only acceptable when \( f \gtrsim 2.5 \) TeV. For lower \( f \), heavy quark annihilation through gluons is so dominant that \( \sigma_{\text{eff}} \) is increased to the point where there is insufficient DM.

**Coannihilation of \( \eta \) with a heavy fermion:**

Following the same procedure as above, we can calculate \( \Omega_\eta h^2 \) including coannihilation with a heavy fermion (either heavy neutrino or heavy quark). The coannihilation channels \( \eta N \to XX' \) and \( \eta D \to XX' \) are mediated by heavy neutral \( W^\prime \) exchange. The heavy fermions have larger annihilation cross sections and more degrees of freedom than \( \eta \), so their self-annihilation controls \( \sigma_{\text{eff}} \).

To show the effect of a degenerate heavy fermion on \( \Omega_\eta h^2 \), we plot \( \Omega_\eta h^2 \) including coannihilation with a heavy neutrino in Figure 5.

The most important consequence of coannihilation in this scenario is that the \( s \) channel \( Z' \) exchange terms originally in the neutrino self annihilation cross section are folded into the effective cross section for \( \eta \). As a result there is now a large dip in \( \Omega_\eta h^2 \) near \( M_{Z'}/2 \). For \( m_N \) within 10% of \( m_\eta \) this dip is substantial enough that we achieve acceptable \( \Omega_\eta h^2 \) for much lighter \( m_\eta \). Comparing figures (1) and (4), we can see that coannihilation decreases the allowed \( m_\eta \) by almost a factor of 10. If \( m_N \) is even closer to \( m_\eta \) the dip becomes larger and we get the correct relic abundance at lighter \( m_\eta \).

We now summarize the results of our relic abundance calculations before investigating possible experimental signatures. In order for \( \Omega_\eta h^2 \) to be consistent with the region allowed by WMAP for an \( \eta \) that is actually the LTOP, we must have coannihilation with a nearly degenerate fermion. Coannihilating with a heavy neutrino with \( m_N/m_\eta = 1.1 \) for \( f = 4.0 \) TeV, we achieved the correct relic abundance for \( m_\eta \sim 1.0 \) TeV. If the LTOP is a heavy neutrino, \( \Omega_N h^2 \) falls within the experimental limits for a much wider range of \( m_N \) and \( f \). Taking \( f \) fixed and varying \( m_N \), there are two regions of acceptable \( \Omega_N h^2 \) - one on each side of the pole in the cross section at \( m_N = M_{Z'}/2 \). For \( f = 4.0 \) TeV the two allowed neutrino mass regions are \( m_N \sim 900 \) GeV and \( m_N \sim 1300 \) GeV. If the other heavy neutrinos are nearly degenerate with the LTOP, the allowed mass regions at a given \( f \) are

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**Figure 3:** Top: Acceptable \( \Omega_N h^2 \) for a single flavor of heavy neutrino (black) and including coannihilation among three flavors of heavy neutrino (gray, green online) versus \( m_N, f \). All neutrinos are degenerate. Right: \( \Omega_N h^2 \) for one (solid line) and three (dash-dot line) degenerate neutrino flavors for \( f = 4.0 \) TeV. Dashed lines are WMAP limits.
closer to $M_{Z'}/2$. Including coannihilation with a heavy quark, we can achieve adequate values for $\Omega_N h^2$ for $m_N$ as low as 200 GeV. To achieve a tolerable $\Omega_N h^2$ at such a low $m_N$, $m_D/m_N$ must fall in a very specific range.

FIG. 4: Left: Regions of cosmologically allowed $\Omega_N h^2$ including coannihilation with a heavy down type quark. Three different quark masses are shown; $m_D/m_N = 1.1$ (gray, green online), 1.05 (light gray, yellow online), degenerate (solid black). See figure 2 for $\Omega_N h^2$ without coannihilation. As before the allowed range for $\Omega_N h^2$ is $0.094 \leq \Omega_N h^2 \leq 0.129$. Right: $\Omega_N h^2$ versus $m_N$ including coannihilation with a degenerate heavy quark for $f = 4.0$ TeV. The dashed lines indicate the WMAP limits.

FIG. 5: Regions of acceptable $\Omega_\eta h^2$ including coannihilation with a heavy down neutrino, versus $m_\eta$. Three different $N - \eta$ mass differences are shown; $\Delta m = m_N - m_\eta = 10\% m_N$ (black), 5$\%$ $m_N$ (gray, green online), degenerate (light gray, yellow online). There is an additional region not shown here because it requires $m_\eta > M_{W'}$. The allowed range for $\Omega_N h^2$ is $0.094 \leq \Omega_N h^2 \leq 0.129$.

VIII. Possible Signatures of SLH Dark Matter

Now that we have identified a few candidate scenarios for DM in the SLH model with $T$ parity, we look to see what signatures each predicts.

By looking for anomalous atomic recoil in systems of very cold atoms, an upper limit can be placed on the DM-nucleon elastic scattering cross section. Currently the most stringent limits come using $^{73}\text{Ge}$ in the CDMS experiment located in the Soudan mine in Minnesota. CDMS places separate limits on the spin independent (SI) and spin dependent (SD) interactions of DM with nuclei. Spin independent interactions are generically proportional to the reduced mass of the DM - nuclei system, while spin dependent interactions are proportional to the spin of the nuclei. For larger nuclei ($A \geq 40$) and non-Majorana DM, spin independent interactions are usually orders of magnitude larger than the spin dependent interactions.

Based off of the total DM-nucleon elastic cross section, we can also make an estimate of the rate for indirect detection. DM particles are collected over time in a massive stellar object like the sun and they annihilate with each other. The energetic neutrino remnants from
these DM annihilations reach the earth and can be seen in high energy neutrino detectors. This rate does depend somewhat on the dominant annihilation mode of the DM and other model dependent parameters [19, 24].

Detection of heavy neutrino DM

Since the heavy neutrino is a dirac fermion its coupling to the $Z'$ has $(V-A)$ structure. The vector portion, when combined with the vector part of a $q{ar q}Z'$ interaction, contributes to the spin independent heavy neutrino-nucleon scattering cross section. One might expect this contribution to be small given the weakness of the coupling and the large mass of the $Z'$. A calculation of the $N$--nucleon effective cross section shows that this is not the case. The $Z'$ couples to both components of a weak $SU(2)$ doublet with equal strength. The effective proton/neutron-$Z'$ couplings are then approximately three times the size of the quark-$Z'$ coupling. The coherent interaction of a $Z'$ with the protons and neutrons in a large nucleus can thus be sizable [30]. Following [14, 19, 22, 24] we calculate the neutrino-nucleus scattering cross section (per nucleon) in $^{73}$Ge to be

$$\sigma_{N-\text{nuc}} \cong 3.7 \times 10^{-42} \left(\frac{2 \text{ TeV}}{f}\right)^4 \text{ cm}^2. \quad (24)$$

Normalized according to the convention in [28, 29], the cross section is nearly independent of the heavy neutrino mass. The only $m_N$ dependence is in the nuclear form factors, which we will ignore here. The cross section [24] therefore applies to both neutrino mass regions for a given $f$. For $f \lesssim 3.0$ TeV, the $\sigma_{N-\text{nuc}}$ from [24] is excluded by the current CDMS data [22, 28]. Between $f = 3.0$ TeV and $f = 4.5$ TeV, $\sigma_{N-\text{nuc}}$ is beneath the current CDMS limits, but within the predicted final sensitivities of the current run, CDMSII. Heavy neutrino DM in this region would certainly be visible in later stages of CDMS, if not earlier. In Fig.6 we show how the $N$-nucleon scattering cross section per nucleon compares with the current and predicted CDMS sensitivities [30] as a function of $f$.

There is also a SD neutrino-nucleus cross section from the axial-vector portion of $Z'$ exchange. We estimate it to be several orders of magnitude smaller than the SI cross section and will therefore ignore its effects. For a total ($SI + SD$) nuclear cross section of $\mathcal{O}(10^{-6} \text{ pb})$ (per nucleon), we expect $\mathcal{O}(1)$ event/year per km$^2$ [24] in a neutrino telescope.

Detection of $\eta$ DM

By the same procedure, we can calculate the effective $\eta$--nucleon cross section. Since $\eta$ is a scalar, there is no spin dependent interaction. The full $\eta$-nucleon cross section is just the spin-independent $\eta$-nucleon cross section.

An $\eta$ scatters off of a light quark ($u, d, s$) predominantly through $t$ channel higgs exchange. Other $\eta q \rightarrow \eta q$ processes, either from $s$ channel heavy quark exchange or from higher order Yukawa terms like $\eta^2(q\bar q)$, are small. The higgs exchange interactions are proportional $m^2_{\eta q}$ and to the quark masses, thus the strange quark contribution is the largest. To get the full SI $\eta$-nucleon cross section, we must also include $\eta$-gluon scattering $\sigma(\eta g \rightarrow \eta g)$. This occurs through higgs exchange with a quark loop which emits two gluons. Loops containing heavy SM quarks ($c, b, t$) or new heavy quarks ($S, D, T$) can make substantial contributions to $\sigma(\eta g \rightarrow \eta g)$ since both $\alpha_s$ and the heavy quark Yukawa couplings are large [14, 25, 26]. Using a standard approximation for the heavy quark (both SM and T-odd) loops [31] we calculate the $\eta$--nucleon cross section in the limit $m_T, m_D \gg m_\eta$ to be

$$\sigma_{\eta-\text{nuc.}} \sim 10^{-45} \left(\frac{m_\eta}{500 \text{ GeV}}\right)^2 \left(\frac{2.0 \text{ TeV}}{f}\right)^4 \text{ cm}^2. \quad (25)$$

This cross section is well below the current and projected CDMS limits for the entire $m_\eta - f$ range of interest. Given $\sigma_{\eta-\text{nuc.}} \lesssim 10^{-9}$ pb and the lack of any spin dependent interactions with the nucleus, the potential for indirect detection of $\eta$ in a neutrino telescope is also very dim [24].

IX. Conclusion

One consequence of enforcing a $\mathbb{Z}_2$ symmetry onto a Little Higgs model is that the lightest odd particle is a
potential DM candidate. We have investigated this here in the context of the Simplest Little Higgs model. We are unable to make all new particles odd, and therefore we are forced to work within a somewhat finely tuned model. Accepting this fine tuning, we examined the two DM candidates in this model; a heavy neutrino $N$ and a scalar $\eta$. Through the standard relic abundance calculations we have found the circumstances under which these DM candidates are allowed by cosmology. We have also checked to make sure these circumstances are consistent with current direct detection limits.

The first DM candidate we investigated was the scalar $\eta$. We found that $\eta$ cannot be the LTOP unless there is a nearly degenerate (within 10%) fermion. Coannihilating with a nearly degenerate heavy fermion, there is parameter space where $\Omega N h^2$ is within the WMAP limits and $m_\eta$ is in the TeV range. For $f = 4.0$ TeV, this occurs for $m_\eta \sim 1.0$ TeV, although the actual number will depend on the type of fermion and the degree of degeneracy. We estimate the $\eta - \text{nucleon}$ cross section to be $\lesssim O(10^{-9})$ pb in the entire region of interest. This cross section is well below the predicted sensitivity bounds of both the current CDMS run and the first SuperCDMS phase. $\eta$ DM in this scenario would be difficult to find.

The other DM candidate we considered was a heavy neutrino. When the mass of the neutrino is close $M_{Z^\prime}/2$, the annihilation cross section is enhanced and lowers $\Omega N h^2$ into an acceptable range. Typically this happens for $|m_N - M_{Z^\prime}/2| \sim 200$ GeV. Provided that the neutrino mass is in the right range, $\Omega N h^2$ stays in the allowed region for a large range of the other T-odd particle masses. However, heavy neutrino DM is ruled out by direct detection unless $f \gtrsim 3.0$ TeV. Based on the projected final CDMS II sensitivity (see fig. [10]), the bound on $f$ could become as high as 4.5 TeV if no heavy neutrino DM is seen by the end of the current CDMS run. These bounds are approximately $m_N$ independent, although CDMS constraints on more massive particles are somewhat weaker. The bound $f \gtrsim 4.5$ is consistent with the strictest EWPO bound [14]. Heavy neutrino DM for $f = 4.5$ TeV has mass $\sim 1$ TeV, although it can be much lighter if a heavy quark is nearly degenerate with the heavy neutrino. As neither the heavy neutrino nor $\eta$ has significant SD interactions with nuclei, we expect indirect detection signals of SLH DM to be very small.

Some other distinct features of this model are the T-even $Z^\prime$, a very heavy $m_{T} \sim f$ top quark partner, and the lack of heavy T parity even quarks [2, 7]. In addition to the direct detection signals we discussed, this model and its DM candidates may yield interesting collider phenomenology which remains to be investigated.

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[33] Calculations with $m_h = 115$ GeV and $m_h = 200$ GeV yield almost identical results.

[34] Strictly speaking, this inaccuracy is problematic only for $m_N > \Gamma_{Z'}$. We calculate $\Gamma_{Z'} \approx 70$ GeV $\ll m_N$.

[35] There can also be coannihilation of either DM candidate with the $W'$ but we have neglected these effects here.

[36] There is also a small contribution from $Z$ exchange, which we include in all calculations.