Numerical Simulation of Exhaust Gas Cooling in Channels with Periodic Elbows for Application in Compact Heat Recovery Systems

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Abstract. Miniature and Micro devices represent the new frontier for advanced heat and mass transfer technology. Due to the small length scales, the use of CFD is very useful for designing and optimizing microluidic devices since experimentation and visualization at these scales can be difficult. In this work a high temperature air microluidic cooling strategy for applications such as compact waste heat recovery, exhaust gas recirculation and fuel cell thermal management is proposed. Initially, the application of a simple straight microchannel is considered. In an effort to partially compensate for the poor thermal properties of air, right-angle bends are introduced in order to induce Dean vortices which periodically restart the thermal boundary layer development, thus improving the heat transfer and fluid mixing. Numerical simulations in the range of $100 \leq Re_{Dh} \leq 1000$ have been carried out for channels of square cross-section. Channel wall lengths of 1.0 mm are investigated for elbow spacings of 5 mm, 10 mm and 15 mm. High temperature air ($300^\circ$C) at atmospheric inlet pressure is the working fluid. The results indicate that the elbows substantially improve the local and average heat transfer in the channels while increasing the pressure drop. Design considerations are discussed which take into account the heat transfer and pressure drop characteristics of the channels.

1. Introduction

The recent drive towards miniaturisation of heat transfer devices for a wide range of applications, including electronics cooling, solar heat collectors, compact waste heat recovery and exhaust gas recirculation heat exchanger designs, requires advanced heat transfer surface technologies. Heat transfer applications involving miniature and microchannels are appealing due to their large heat transfer surface area per unit volume. However, the flow within them is often in the laminar range due to pressure drop constraints. Numerous studies have considered different channel geometries (Lei and Trupp [1], Farhanieh and Sunden [2], Chen et al. [3], Lee and Lee [4]) but, due to the small channel size, the thermal and hydraulic boundary layers develop quickly, typically within 10 to 20 hydraulic diameters downstream of the inlet. Disturbing the thermal boundary layer within the channel by forcing it to follow a convoluted path through corrugated channels (Sawyers et al. [5]), around springs or twisted tapes (Kumar and Judd [6], Ray and Date [7, 8]) and through ribbed turbulators (Murata and Mochizuki [9]) have all been studied extensively, often resulting in a substantial pressure drop penalty. Further, due to fabrication limitations of such enhanced surface designs at small scales, they are often not relevant to microchannels.
As the size of the channel is reduced, the characteristic length scales are of the same order of magnitude as the hydrodynamic boundary layer and, as a consequence, momentum transfer perpendicular to the streamwise direction can increase substantially (Ma and Gerner [10]). One way of augmenting this is by introducing abrupt changes in the flow direction using serpentine channels. The result is the creation of flow instabilities commonly referred to as Dean Vortices (Kalb and Seader [11], Yang et al. [12], Sillekens et al. [13], Chintada et al. [14], Acharya et al. [15], Wang and Yang [16]) that periodically restart the growth of the hydrodynamic and thermal boundary layers, leading to substantially higher heat transfer coefficients. Compared to straight channel studies involving serpentine, or right angle turns as studied here, the Dean instabilities cause flow impingement on the walls, recirculation and flow separation in a rather structured fashion that brings the cold fluid in the centre of the channel to the hot walls (Rosaguti et al. [17], Geyer et al. [18]), or vice versa. The result is an increase in heat transfer with a relatively low pressure drop penalty compared to obstructive enhancement methods.

As a consequence of the small-scale channel and significant transverse convection and momentum transfer, temperature variations in the transport fluid can cause variations in the fluid properties that cannot be neglected as in many macro-scale systems. This observation in small-scale channels is difficult to quantify experimentally and has significant importance in waste heat recovery or exhaust gas recirculation heat exchanger devices undergoing temperature changes of hundreds of degrees. Further, detailed experimental studies have, until recently, been limited due to challenges in precision fabrication of miniature and micro channels and difficulties with data collection and analysis due to measurement devices affecting the flow dynamics of micro-scale systems (Papautshy et al. [19]). Consequently, in an effort to study the impact of Dean vortices on microchannels, with the goal of developing design guidelines and tools, a numerical investigation has been pursued prior to developing an experimental program. Numerical simulations in the range of $100 \leq \text{Re}_{Dh} \leq 1000$ have been carried out for channels of square cross-section. A channel wall length of 1.0 mm is investigated for elbow spacings of 5 mm, 10 mm and 15 mm. High temperature air (300°C) at atmospheric inlet pressure is the working fluid to emulate exhaust gas conditions.

### 2. Modeling Methodology

#### 2.1. Channel Specifications

Both a straight channel and channels containing right-angle bends are considered in this investigation. This is shown schematically in Figure 1. Each channel is square in cross section with a wall length of $a = 1.0 \text{ mm}$. The channels are considered for identical projected length of $c = 60 \text{ mm}$. To compare the influence of the right-angle bends on the flow and heat transfer, three run lengths, $b = 5 \text{ mm}, 10 \text{ mm}$ and $15 \text{ mm}$ were tested for Reynolds numbers of $\text{Re}_{a} = 100, 400, 600, 800$ and $1000$. Since large variations in fluid properties occur under these conditions, the Reynolds number has been defined for the inlet conditions for such that,

$$\text{Re}_{a} = \frac{\rho U a}{\mu}$$

where $\rho$ is the air density and $\mu$ is the dynamic viscosity evaluated at the air inlet temperature and atmospheric pressure. The wall length, $a$, is equivalent to the hydraulic diameter for the channel and $U$ is the characteristic velocity which is determined by rearranging Eq. 1 for a predetermined Reynolds number.

#### 2.2. Governing Equations

A three-dimensional model of the problem was formulated using the commercial software package FLUENT version 6.2.16. The governing equations for the problem are mass, momentum and energy conservation;
Figure 1. Straight channel (Top) and channel with right angled bends.

\[ \nabla \cdot (\rho \vec{u}) = 0 \]  

(2)

\[ \nabla \cdot (\rho \vec{u} \vec{u}) = \nabla p + \nabla \cdot (\mu [\nabla \vec{u} + \nabla \vec{u}^T]) \]  

(3)

\[ \nabla \cdot (\rho c_p T \vec{u}) = \nabla \cdot (k \nabla T) \]  

(4)

The coupled system of equations was solved for laminar, steady flow and heat transfer accounting for variations in fluid thermophysical properties [20]. A grid independence study was performed to ensure that the results are grid independent.

For the current investigation, the concept heat exchanger is envisaged for use in exhaust gas heat recovery where water is to be boiled in channels above and below the exhaust. To model this, the inlet temperature was assumed uniform at \( T(0, y, z) = 300^\circ C \) and the wall temperatures were constant at \( T_w = 100^\circ C \) due to the evaporative cooling condition. With regard to the flow, a uniform inlet velocity field, \( u(0, y, z) = U \) was assumed and a pressure outlet condition, \( P(x, y, c) = P_{out} \) was assumed as atmospheric. On each of the wall surfaces a no-slip boundary condition, \( u = v = w = 0 \), was used.

3. Results and Discussion

3.1. Influence of Dean Vortices

The principle under investigation in this study is that the abrupt changes in direction brought about by the right-angle bends cause the formation of Dean vortices within the fluid channel which is an effective means of heat and mass transfer enhancement. Figures 2 and 3 illustrate this effect for the case of \( Re_a = 1000 \) and a run length of \( b = 5 \) mm. In Figure 2 a view out of the first bend from a downstream location of the channel is shown which depicts the development of two counter-rotating Dean vortices.

The effect on the entire flow structure is better illustrated in Figure 3, where the mixing effect is evident in the first three bends. As will be discussed in more detail in Section 3.2, the run length of \( b = 5 \) mm is short enough that the flow does not recover fully between subsequent bends, thus establishing an irregular flow structure along the channel length. This is compared to the longer run lengths for which the flow is more periodic alternating between developing and developed.

The main influence of the creation of Dean vortices is intermittent re-mixing of the fluid which tends to redistribute the hot core fluid towards the cooled wall (or vice versa in heating applications) thus enhancing the heat transfer in this developing region. This is shown in Figure 4 as the hot core fluid subsequent to each bend (red) is better mixed with the cooler fluid near the wall (green).
The result of enhanced mass transfer is an improvement in the heat transfer; quantified in Figure 5. Here, the perimeter-averaged wall heat flux of the channel is plotted against axial distance for both a straight and serpentine channel ($Re_a = 1000$, $b = 5$ mm). It is evident that approaching the first bend of the channel the heat flux profile of the elbowed channel follows the straight channel profile closely, as would be expected since they both have the same entrance conditions. However, downstream of the first elbow the enhanced mixing caused by the Dean vortices incurs a very notable increase in the average local heat flux.

The impact of the enhanced mixing becomes diminished further along the channel length as the bulk liquid temperature approaches the wall temperature and the heat flux in both cases asymptotically approaches zero. This is also illustrated in Figure 6, where it is evident that the enhanced heat transfer due to the 90$^\circ$ elbows causes the averaged average fluid temperature to approach the wall temperature at a higher rate compared with that of the straight channel. Thus, for the serpentine channels, the enhanced heat transfer due to mixing is partially offset by a reduction in driving temperature differential, causing the heat flux profile to approach the straight channel. This may, however, allow for a reduction in required length of the heat exchanger.

### 3.2. Effect of Run Length and Reynolds number

Three different run lengths were investigated, $b = 5$ mm, 10 mm and 15 mm, for Reynolds numbers of $100 \leq Re_a \leq 1000$. Figures 7, 8 and 9 shows the effect of varying $b$ and $Re_a$ on the
heat flux distribution within the channel.

From Figure 7 it is evident that for the longest run length, b = 15 mm and $Re_a = 800$, the heat flux profile initially follows that of the straight channel until the first bend is approached. At approximately 10 mm an increase in the local heat flux over that of the straight channel occurs that peaks at the bend location and is then sustained for approximately 15 mm. As the second bend is approached, the hydraulic and thermal boundary layers develop to nearly the uniform axial flow condition. At each subsequent bend the magnitude of the enhanced heat transfer diminishes as the bulk liquid cools. For $b = 10$ mm and the same Reynolds number it is evident from Figure 7 that this situation behaves similarly to the $b = 15$ mm case with peak heat fluxes of similar magnitude which occur more frequently due to the reduced elbow spacing. However, for $b = 5$ mm a different behavior is observed. For this scenario the run length is not sufficient for the flow to develop fully and after the first bend the heat flux remains notably above the straight channel value. The flow field remains irregular and well mixed throughout the channel length.

Figure 8 shows this effect more clearly by considering the perimeter-averaged local wall heat transfer coefficient for the different run lengths and $Re_a = 800$. The perimeter-averaged local wall heat transfer coefficient $h$ is calculated using the expression,

$$h = \frac{\bar{q}'}{\bar{T}_b - \bar{T}_w}$$

(5)
where $q''$ is the average local heat flux around the parameter of the channel and $\bar{T}_b$ is the area averaged cross-sectional fluid temperature at that location. For longer run lengths the heat transfer coefficient profiles are generally periodic with similar amplitudes at each bend location dropping to near the benchmark straight channel value at regular intervals. However, for $b = 5$ mm the profile is more erratic and the heat transfer coefficient is sustained above the baseline case along the entire channel length but has a lower heat transfer coefficient at the peak elbow locations due to the irregular flow pattern.

**Figure 10.** Local average wall heat transfer coefficient for varying run length and $Re_a = 800$.

**Figure 11.** Pathlines colored by velocity magnitude (m/s) for $b = 15$ mm and $Re_a = 400$ (Top) and $Re_a = 100$ (Bottom).

Comparison of Figures 7, 8 and 9 illustrates the influence of decreasing Reynolds number on the wall heat flux along the channels. As expected, the length of the entry region decreases with decreased Reynolds number for the straight channel. For the augmented channels decreasing the Reynolds number has the effect of monotonically reducing the influence of the bends on the flow and heat transfer to the extent that, for $Re_a = 100$ the effect of the elbows on the flow is minimal. The reason for this is illustrated in Figure 11, where the pathlines through the first bend for the $b = 15$ mm channel are shown for $Re_a = 400$ and 100. For $Re_a = 400$ the formation of the Dean vortices establishes a mixing region that penetrates far downstream of the bend. Conversely, for $Re_a = 100$ the strength of the Dean vortices is reduced and the flow becomes fully developed only a short distance downstream of the bend, thus reducing the effect on the heat transfer.

Figure 12 shows the influence of Reynolds number on the heat transfer coefficient for the case of $b=15$ mm, though similar trends are observed for the $b = 10$ mm case. Initial reduction in $Re_a$ from 1000 to 800 shows little influence on the heat transfer coefficient. Further reduction in $Re$ shows a continual reduction in the magnitude of the heat transfer coefficient.

Considering only peak values, Figure 13 indicates that the peak heat transfer coefficient increases asymptotically with Reynolds number with an approximate dependence of $h_{max} \propto Re^{1/2}$. More rigorously, a regression analysis results in the following power law dependence:

$$h_{max} = 10.22 Re_a^{0.577}$$

(6)

From the above discussion it is difficult to select the ideal heat exchanger design since the channels behave differently. Furthermore, each channel will have an increase in the pressure
Figure 12. Local average wall heat transfer coefficient for Reynolds number and fixed run length of b = 15 mm.

Figure 13. Variation of peak heat transfer coefficient with Reynolds number for b = 10 mm & 15 mm.

drop associated with it, partially due to the longer overall length but also due to the mixing of the flow. If a heat exchanger is to be designed to balance the increased heat transfer coefficient with the associated pressure drop penalty, it is useful to define the level of enhancement due to the right-angle bends with respect to the values for a straight channel. The first of the quantities will be termed the Enhancement Factor (EF), as defined by,

$$EF = \frac{\bar{h}}{h_{sc}}$$

where $\bar{h}$ is the average heat transfer coefficient along the length, not including the initial entrance region, and $h_{sc}$ is the heat transfer coefficient for fully developed flow in the straight channel.

Similarly, a Penalty Factor (PF) is defined as,

$$PF = \frac{\Delta P}{\Delta P_{sc}}$$

where $\Delta P$ is the pressure drop associated with the augmented channel and $\Delta P_{sc}$ is the pressure drop associated with the straight channel.

The enhancement and penalty factors for all of the channel scenarios are plotted in Figure 14 and 15. In Figure 14 it can be seen that EF is only slightly effected by the run length, b. This would suggest that even though the periodic developing and redeveloping flow structure for b = 15 mm is fundamentally different in nature than the irregular and erratic flow structure associated with b = 5 mm, the increase in the peak magnitude of the heat transfer coefficient for the longer run length compensates for the fact that it is long enough to develop fully and reduces to near the value associated with the straight channel (i.e. Figure 10). Conversely, the PF increases notably with decreasing b. Since all of the augmented channels have the same overall length (85.5 mm) the increase in pressure drop must be due to the level of agitation in the flow associated with an increased number of flow redirections as b decreases. In particular, for b = 5 mm where the flow is well mixed along the entire channel due to the short run lengths (Figure 2), the pressure drop penalty is considerable.

The results of Figure 14 and 15 suggest that for practical design purposes there is minimal heat transfer advantage associated with short run lengths while there is a considerable pressure drop penalty. Thus, for the parameter range tested in this investigation, the longest run length tested gives the best thermal-hydraulic performance. Further studies should be carried out in order to expand the range of parameters.
Figure 14. Heat transfer coefficient Enhancement Factor (EF).

Figure 15. Pressure drop Penalty Factor (PF) versus Reynolds number.

4. Conclusion

Dean vortices within the fluid channel are shown to be an effective means of heat and mass transfer enhancement in miniature channels. The main influence of the creation of the Dean vortices is intermittent re-mixing of the fluid which tends to redistribute the hot core fluid towards the cooled wall, thus augmenting the heat transfer in this developing region. The enhancement and penalty factors for all of the channel scenarios are in the range of 2 to 5 for heat transfer and pressure drop. These are dependent on the Reynolds number, with the exception of shorter run lengths. At the shortest run length of b = 5 mm the flow does not recover fully between subsequent redirections, establishing an irregular flow structure along the channel length compared with the longer run lengths for which the flow is more periodic in nature. The result is a substantial increase in pressure drop for the short run case. Consequently, it is evident that an optimum configuration exists which is a function of hydraulic diameter and run length. Further studies are underway to extend the parameter ranges and develop design optimization to facilitate adoption in waste heat recovery processes.

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