Dependence of maximum static friction on waiting time using dynamics of vortices in superconductors

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Abstract. Vortices in high-\(T_c\) superconductors were investigated in terms of the microscopic friction. The \(I-V\) characteristics and the transient response of driven vortices were measured in \(La_{2-x}Sr_xCuO_4\) thin films with different Sr concentrations and samples with columnar defects. We found the remarkable dependence of the maximum static friction force on the waiting time at low temperatures. The result suggests that the competition between the flux creep by the thermal fluctuation and the pinning yields a characteristic time scale to stabilize vortices. This relaxation phenomenon is very similar to the so-called boundary lubrication, which occurs under the existence of a thin lubricant film between interfaces. Such a strong dependence changed into a weak logarithmic dependence at higher temperatures. These results imply that the dynamics of vortices can be used for the model not only of the dry friction but also of the lubricated friction. Also, the dependence of the kinetic friction force on the sliding velocity showed the non-Amontons-Coulomb-like behavior even for the lubricated friction. Thus, we confirmed that the Amontons-Coulomb’s law does not realize when the thermal fluctuation is sufficiently large.

1. Introduction

Recently, scientific investigations of tribology from the microscopic viewpoint have found novel phenomena, which cannot be explained by macroscopic theories of friction[1, 2]. As is widely known, an empirical rule for the dry friction is the Amontons-Coulomb’s law. However, many deviations from this empirical law were found in experimental results. For instance, a kinetic friction force, \(F_k\), slightly depends on the velocity, \(v\), of a sliding object at low velocities. Also, a maximum static friction force, \(F_{s_{\text{max}}}\), depends on the waiting time, \(t_w\), which is the intermission time of a repetitively applied driving force. This \(F_s(t_w)\) and the related aging effects have been investigated for the solid-solid interface[3], the interface with lubricant[4], the agar gel[5], the charge density wave[6], and the vortices in superconductors[7-9]. The origin of these deviations from the Amontons-Coulomb’s law has been clarified partly by considering the deformation and the destruction of the microscopic asperity[10]. However, no consensus has been achieved for the detailed mechanism of the energy dissipation at a microscopic interface.

In addition to the dry friction as was described just above, the sliding friction under the existence of a liquid lubricant at the interface is also important for an industrial application. There are different regimes distinguished by the thickness of the lubricant. When it is thick,
the elastohydrodynamics (EHD) of the lubricant mainly contributes to the friction, which is called as the EHD regime. On the other hand, if the lubricant layer is very thin, the molecular characteristics of the lubricant turns out to be essential, and exhibits peculiar dynamics such as the solidification of the lubricant. This regime is called as the boundary lubrication (BL) regime, whose understanding is essential for the construction of micrometer- or nanometer-scale machines. Such lubricated friction is more difficult to be analyzed, because the precise manipulation of the interface becomes more complicated. Recently, some kinds of superior experimental equipments such as surface-force apparatus have been developed, and the detailed lubricant dynamics depending on molecules has been investigated[4]. Furthermore, computer simulations of the microscopic friction have been carried out on a large scale[11].

In type-II superconductors under a finite magnetic field, the magnetic field partly penetrates in the form of tiny flux lines, and its unit is called as the flux quantum, Φ0[12]. Around the flux lines supercurrent circulates to screen the magnetic field. Therefore, the flux line is called as a vortex. These vortices repel with each other, and are pinned by defects in the sample. When we apply enough amount of the driving electric current above a critical value, vortices can move by the Lorentz force, and induce a finite voltage. This Lorentz force is compensated by the pinning force, the viscous force due to the dissipation of quasiparticles in the central part of the vortex (core), where the superconductivity is weakened[13], and a thermal-fluctuation force. These are represented by the following equation for the displacement of a vortex, \( \vec{u}(\vec{r}, t) \),

\[
\eta \frac{d\vec{u}(\vec{r}, t)}{dt} + \kappa_p \vec{u}(\vec{r}, t) = \vec{j}(\vec{r}, t) \times \Phi_0 \vec{z} + \vec{j}_{\text{fluct}},
\]

(1)

where \( \eta \) is the viscous coefficient, \( \kappa_p \) is the pinning coefficient, \( \vec{j} \) is a current density, \( \vec{z} \) is the unit vector along a magnetic field, and \( \vec{j}_{\text{fluct}} \) is the thermal fluctuation force. Here we ignored the inertial term, which is considered to be very small for the low frequency motion. If we assume that there is no pinning effect, an ohmic resistivity (flux flow resistivity, \( \rho_{FF} \)), which is proportional to the external magnetic field, appears due to the viscous force. In terms of the correspondence to the physics of friction, the Lorentz force corresponding to the critical current density, \( j_c \), is equal to \( F^\text{max}_s \). On the other hand, for \( F_k \), the correspondence is more complicated. For the dry friction, the interaction between pinning centers and a movable object is important, and this is the only force that contributes to \( F_k \)[14]. However, it is important to note that the average viscosity should be subtracted since the friction is the energy dissipation from a representative degree of freedom to others. Thus, we can obtain \( F_k \) by measuring the \( I-V \) characteristics of driven vortices as follows[15].

\[
\vec{F}_k = \vec{j}(\vec{r}, t) \times \Phi_0 \vec{z} - \eta \left\langle \frac{d\vec{u}(\vec{r}, t)}{dt} \right\rangle = \vec{j}(\vec{r}, t) \times \Phi_0 \vec{z} \left( 1 - \frac{\rho}{\rho_{FF}} \right),
\]

(2)

where \( \rho \) is the resistivity and \( \left\langle \cdot \right\rangle \) represents the time average. According to equation (2), we need the value of \( \rho_{FF} \) to obtain \( F_k \). Equation (1) shows that the motion of vortices becomes almost purely dissipative (flux flow) when vortices move with a high velocity or at a high frequency. Thus, the transport measurement up to a sufficiently large driving force can be used to obtain \( \rho_{FF} \). Alternatively, \( \eta \), which is defined by \( \eta \equiv \Phi_0 B / \rho_{FF} \), can also be deduced from the microwave-surface-impedance measurement[16, 17].

Our approach is that we regard the dynamics of driven vortices as a model system of the physics of friction. This is significant because there are neither deteriorations nor damages of the sample caused by the wear. Thus, we can repeat experiments under the same environment[15]. Previously, studying the relationship between \( F_s(t_w) \) and \( F_k(v) \) in high-\( T_c \) superconductors La1.85Sr0.15CuO4, we showed that the driven-vortex system exhibited a non-Amontons-Coulomb's behavior owing to the existence of a large thermal fluctuation[18]. In this paper, we further investigate both of \( F_s(t_w) \) and \( F_k(v) \) in a high-\( T_c \) superconductor,
Table 1. Parameters of the samples used in this study; the Sr concentration $x$ of La$_{2-x}$Sr$_x$CuO$_4$, the size between the voltage electrodes (width×length), $T_c$ (50%), $\Delta T_c$ (10% - 90%), and the ion irradiation dose, $B_{\Phi}$, of the columnar defects.

| sample | $x$ | size (mm×mm) | $T_c$ (K) | $\Delta T_c$ (K) | $B_{\Phi}$ (T) |
|--------|-----|--------------|-----------|-----------------|----------------|
| # A    | 0.12| 1.47×0.66    | 35.20     | 2.05            | –              |
| # B    | 0.15| 1.18×0.21    | 35.06     | 3.40            | –              |
| # C1   | 0.15| 0.05×0.07    | 38.53     | 2.58            | 1.0            |
| # C2   | 0.15| 0.04×0.09    | 35.11     | 2.91            | 2.0            |

La$_{2-x}$Sr$_x$CuO$_4$ for various kinds of pinning force, and try to look for the correspondence of the driven vortices to the lubricated friction.

2. Experiment
We prepared 3000 Å -thickness films of the optimally doped ($x = 0.15$) and the underdoped ($x = 0.12$) La$_{2-x}$Sr$_x$CuO$_4$ by the pulsed laser deposition (PLD) technique. Details on the preparation of the films were described elsewhere[19]. An 0.5 mm thick LaSrAlO$_4$(001) was used for the substrate because the mismatch of the lattice constant was very small. Some of the samples were irradiated by 5.8 GeV Pb ions shower parallel to the c-axis using the Grand Accélérateur National d’Ions Lourds(GANIL) in Caen, France. For samples with columnar defects, we further fabricated the samples into the bridge-type form by the photolithography- and the chemical-etching techniques. We investigated 4 samples. Table 1 shows the characteristic parameters of these samples.

For the $I$-$V$ characteristics measurement, the sample was attached to an oxygen-free-copper sample holder in a vacuum environment. Magnetic fields were applied by a superconducting magnet, and all the measurements were carried out after field cooled conditions to avoid the non-uniformity of vortices due to the pinning effect. During the measurements, the fluctuation of the temperature and the magnetic field were less than 2 mK and 1 mT, respectively. To avoid the heating effect, we applied a rectangular pulsed electrical current, and measured the $I$-$V$ characteristics. We changed the duration time and the intermission time of the pulse in order to check the joule-heating effect. If the $I$-$V$ characteristics were found to be independent of these parameters, we judged that the heating effect can be ignored. Typical duration time and intermission time were 0.125 ms and 62.5 ms, respectively.

Furthermore, we applied a sawtooth-like pulsed current to obtain the transient response of the vortices. By comparing the critical current densities for different waiting times, we obtained the waiting-time dependence of $F_s^{\text{max}}$, $F_s(t_w)$, in the dynamics of vortices. Empirically, heating effect is almost negligible below 0.5 W. In our measurements the maximum power generated in the sample was about 1 $\mu$W. Thus, we safely conclude that the obtained $F_s(t_w)$ was not caused by the heating effect of electrodes.

3. Results and discussion
Figure 1(a) is the temperature dependence of the dc resistivity of the sample #C1 near the superconducting transition under various magnetic fields. Driving current density was 10 A/cm$^2$. This resistive broadening was widely known as an indication of the large superconducting fluctuation[20]. This phenomenon has been interpreted also in terms of the glass transition[21]. We estimated the vortex glass-liquid transition line, $T_g(H)$, from the data points where a finite resistance ($> 3 \mu\Omega$) appeared by changing temperature under constant magnetic field. In figure
Figure 1. (a) Resistivity of sample #C1 near the superconducting transition under several magnetic fields. (b) $H$-$T$ phase diagram and the vortex glass-liquid transition line, $T_g(H)$, was depicted with a criteria where $\rho$ becomes $3 \mu \Omega$ in (a). Solid line, which is $(\text{const.}) \times (1 - T/T_c)^2$, is a guide for the eye.

Figure 2. The waiting-time dependence of the maximum static friction force. (a) Near $T_g(H)$, $F_s$ depends on $t_w$ logarithmically in samples #A and #B. (b) All data show the appearance of characteristic time scale at rather lower temperatures than $T_g(H)$ in samples #A and #B. Solid line is the fitting curve of equation (3). Inset: Representative transient responses of vortices. Solid line is the waveform of a driving current. Arrows indicate the current values where finite voltages appeared. (c) In sample #C2, there is no large systematic time dependence.

1(b), we show $T_g(H)$ obtained in this way. To investigate the dynamics of vortices near the critical current density, we mainly measured the $I$-$V$ characteristics under $T_g(H)$.

First, we discuss the results of figure 2, that is the dependence of $F_s^{\text{max}}$ on the waiting time, $t_w$, $F_s(t_w)$, measured at several points which depicted in figures 3(a)~3(c). The inset of figure 2(b) shows a representative transient response of vortices to a sawtooth-like pulsed driving current, which is also shown in the same figure. We found two different kinds of the dependences of $F_s^{\text{max}}$ on $t_w$. As is shown in figure 2(a), near $T_g(H)$, $F_s$ depends on $t_w$ very weakly. If we try to express the explicit $t_w$ dependence, it depends logarithmically on $t_w$. On the other hand, in figure 2(b), a characteristic time scale appears at lower temperatures. Below this time scale, the variation of $F_s^{\text{max}}$ becomes prominent. These features were observed both in samples #A and #B, whose pinning strengths are different from each other. However, the sample with columnar defects (#C2) did not show such drastic variations of $F_s(t_w)$, as is seen in figure 2(c). We consider the reason of this phenomenon as follows; columnar defects pin the vortices more strongly than ordinary intrinsic defects of the sample. Thus, the relaxation by the
We note that the dependence shown in figure 2(b) can be fitted by a model equation for the BL regime, which is shown by solid lines. A recent computer simulation found that the order parameter obeying the Ginzburg-Landau equation could describe the dynamics in the BL regime[22]. So it is expected that the lubricated friction can be also described by the dynamics of vortices, because the vortex motion is also represented by the time-dependent Ginzburg-Landau equation. In the BL, because of the thermal redistribution and the solidification of the lubricant film, $F_s(t_w)$ obeys the following empirical equation[23],

$$F_s(t_w) = A + \frac{B\theta_0}{\theta_0 + (1 - \theta_0) \exp\left(-\left(\frac{t_w}{\tau}\right)^\beta\right)}$$  \hspace{1cm} (3)

where $\beta = 1$ in Ref. [23], and $\theta_0$ is the order parameter denoting the state of the lubricant film. $\theta_0 = 0$ means a solid state, whereas $\theta_0 = 1$ means a liquid state. We found that the fitting of the strong $t_w$ dependence data to equation (3) was good, and obtained the fitting parameters of $\theta_0=0.93\sim0.99$ and $\tau=0.5\sim15$ ms. As for $\beta$, we used $\beta = 0.5$ in figure 2(b) for the better fitting. Large numbers of the order parameter $\theta_0$ means that the thermal fluctuation is fairly large in the vortex system, which is consistent with our previous conclusion[15]. With increasing temperature, $\tau$ increases in general[4]. Thus, we expect that the drastic change of $F_s^{\text{max}}$ on $t_w$ becomes more prominent at higher temperatures. However, as was already shown, $F_s(t_w)$ changes logarithmically near $T_g(H)$. This suggests that the vortex dynamics changes qualitatively near $T_g(H)$. We speculate that the $t_w$ dependence around here can be expressed in terms of the EHD. Indeed, logarithmic time dependence means the absence of any characteristic time scale, which is consistent with the fact that in the EHD regime, there is no characteristic time scale for the solidification of the lubricant. To sum up, the data in figure 2 suggest that we can obtain various different models of the friction for the different thickness of the lubricant by raising temperature, as is schematically shown in figure 3(d).

Next, we discuss the kinetic friction force, $F_k$. Figure 4 shows the velocity dependence of $F_k$ obtained from the measured $I$-$V$ characteristics. $F_k$ strongly depends on the velocity in a
Figure 4. The velocity dependence of the kinetic friction force, obtained from the I-V characteristics for (a) sample #A and (b) sample #C2. At each temperature, the dotted line is $F_k = j \Phi_0 - \eta v$, and the solid line is $F_k = j \Phi_0$. $\eta$ is deduced from Ref. [24] in figure (a), and from the I-V characteristics in figure (b).

Table 2. Proposed correspondence between the mechanical friction at the solid interface and the driven vortices as the model system of friction.

| Mechanical friction | Dry friction of vortices | Lubricated friction of vortices |
|---------------------|--------------------------|-------------------------------|
| $F_{s_{\text{max}}}$ | $j_c \Phi_0$             | $j_c \Phi_0$                  |
| $F_k(v)$            | $j \Phi_0 - \eta v$      | $j \Phi_0$                    |
| Increasing $T$      | -                        | BL→EHD                       |

very different manner from the Amontons-Coulomb’s behavior. This type of behavior occurs if the interface is very pristine[14]. For the dry friction, as was already seen in equation (2), we obtained $F_k$ by subtracting the average viscous force from the Lorentz force. This means that only the interaction between vortices and pinning centers contributes to $F_k$. However, the viscous force plays a crucial role for the friction with lubricant. Thus, we believe that the expression of $F_k$ for the lubricated friction should be modified from equation (2), as $F_k = j \Phi_0$. Table 2 shows the whole correspondence between the various types of friction and the model using the dynamics of vortices.

Previously, the scaling relation between $F_s(t_w)$ and $F_k(v)$ was proposed for the dry friction of thick papers[25], as

$$F_s(t_w) = F_k(v = \frac{D_0}{t_w}),$$

(4)

where $D_0$ is a characteristic length scale for the stick-slip motion. On the other hand, we showed that there is no such scaling relation between $F_s(t_w)$ and $F_k(v)$ for driven vortices[18]. In Ref. [18], we analyzed the vortex data in terms of the dry friction. However, in the dry friction experiments at the solid interface, the wear often takes place at the interface, and the wear debris works as a lubricant. Thus, there is a possibility that the existence of the wear debris is essential for the scaling relation (equation (4)) to be valid. However, as is shown in figure 4, $F_k$...
increases with $v$ both for $F_k = j\Phi_0 - \eta v$ and $F_k = j\Phi_0$. This means that the absence of the scaling relation is always valid in the vortex dynamics, irrespective with whether we subtract the viscous force from the Lorentz force or not. Therefore, the origin of the absence of the scaling relation is not the absence of the lubricant. This rather supports our previous conclusion that $F_k(v)$ of driven vortices, which was largely different from the Amontons-Coulomb-like behavior, was originated from the large thermal broadening of a dynamic phase transition[15]. If the measurement of $F_k(v)$ at much lower temperatures becomes possible, we believe that it shows the Amontons-Coulomb-like behavior.

4. Conclusion
We investigated the dynamics of driven vortices in a high-$T_c$ superconductors, $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$, in terms of the physics of friction. The waiting-time dependence of the maximum static friction force was observed, and it showed the strong dependence at rather low temperatures below the glass-liquid transition line. From the fitting to a model equation of the boundary lubrication, we found that the dynamics of driven vortices can be also used as a model of the lubricated friction. Furthermore, by changing temperature or magnetic field we can tune various conditions of lubricated interface for the friction problem, from the elastohydrodynamic lubrication to the boundary lubrication. As for the kinetic friction force, we suggest that we can choose different friction model, including the dry friction and the lubricated friction, by excluding or including the viscous force from the model equation. In our experimental data, $F_k(v)$ and $F_s(t_w)$ do not scale with each other in any regions. This strongly suggests that the Amontons-Coulomb’s law is not realized when the thermal fluctuation is sufficiently large.

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