Failure-Tolerant Connectivity Maintenance for Robot Swarms

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Abstract. Connectivity maintenance plays a key role in achieving a desired global behaviour among a swarm of robots. However, connectivity maintenance in realistic environments is hampered by lack of computation resources, low communication bandwidth, robot failures and unstable links. In this paper, we propose a novel decentralized connectivity-preserving algorithm that can be deployed on top of other behaviours to enforce connectivity constraints. The algorithm takes a set of targets to be reached while keeping a minimum number of redundant links between robots, with the goal of guaranteeing bandwidth and reliability. Robots then incrementally build and maintain a communication backbone with the specified number of links. We empirically study the performance of the algorithm, analyzing its time to convergence, as well as robustness to faults injected into the backbone robots. Our results statistically demonstrate the algorithm’s ability to preserve the desired connectivity constraints and to reach the targets with up to 70 percent of individual robot failures in the communication backbone.

Keywords: Swarm robotics · Connectivity Maintenance · Fault-Tolerance.

1 Introduction

Swarm robotics is a field of engineering that deals with relatively simple physically agents to achieve a global behaviour that emerges as a result of local interactions [23]. Swarm robotics has been widely investigated in the last decade [2] for a number of different applications, mainly due to its inherent benefits: robustness, scalability, and flexibility. With a large swarm, in general, the loss of a single agent does not jeopardize the overall mission and a failed agent could be replaced with another. Robotic swarms are deemed cost effective solutions when dealing with large, spatially distributed tasks like exploration [15], search and rescue [24], and area coverage [9].

In many of these applications the robots need communication between each other to coordinate. For the information to propagate, the swarm needs to be connected, i.e. there has to be a communication path between all the robots in the swarm. The problem of maintaining connectivity is widely discussed in literature, with a number of different recent approaches [17,18]. Some of these approaches design control strategies to enforce algebraic connectivity [6] among
a group of connected nodes \[8,16,22,7]\], while others address the connectivity problem by enforcing virtual forces on a pre-existent structure \[12,28,26]\.

During a real world mission robots can fail for a number of reasons (environmental factors, wear and tear, etc) and break connectivity, compromising the mission. In addition, maintaining connectivity is likely not the only requirement for mission success. Consider the Fukushima accident in 2011: robots were used to inspect the collapsed nuclear power plant (with a video feed), and they were subject to extremely high failure rates due to radiation. The work in \[13\] considers a robustness factor into the designed control law to tackle robot failures while enforcing connectivity. The convergence in \[13\] is slow due to the computation of the Fiedler vector (an algebraic connectivity measure) using a power iteration method, which requires multiple information exchanges throughout the entire swarm. In this paper, we propose a decentralized, failure-tolerant connectivity maintenance approach that can be added to existing control algorithms. Our approach is lightweight in both communication and computation requirements, freeing resources to achieve the mission goals.

In practice, we progressively and dynamically use the robots in the swarm to form a communication backbone from a root to a set of targets. We set the number of links between the robots as a configurable redundancy factor. The contributions of the work can be summarized as:

1. formalization of a chain-based backbone algorithm, that progressively places the robots towards the targets with a configurable number of links;
2. study of the algorithm performance using a physics-based simulator;
3. analysis on the performance of the algorithm with simulated robot failures.

The paper is structured as follows: a brief summary of the related work in literature is given in Section 2; we describe the mathematical model (kinematic and communication) in Section 3; our proposed algorithm is described in Section 4; Section 5 provides experimental results and analysis; finally, Section 6 draws some concluding remarks.

2 Related work

The problem of maintaining connectivity in a multi-robot system was addressed in several ways in the literature. One approach is to design reactive control laws while imposing connectivity as a constraint. For example, in \[10\] two control laws were developed for rendezvous and formation tasks, imposing an initially connected configuration to be an invariant set, effectively preserving connectivity. A similar approach was implemented in \[30\], achieving rendezvous among a group of agents and preserving an initial connected condition using a potential based controller. This class of approaches relies on global coordination, reducing their overall scalability, and making them more appropriate for small groups of robots.

Other works are more explicit, and use control laws that maximize algebraic connectivity (i.e. the second eigenvalue of the Laplacian of the robots’ connectivity graph). One example is \[21\], which describes a method for the distributed...
estimation of algebraic connectivity using a power iteration. Sabattini et al. [21] uses this estimation to drive agents in a way that maximizes connectivity. Kim et al. [11] proposes another approach that depends on solving an optimization problem on the relative locations of robots, maximizing the second eigenvalue of the Laplacian. Another example [4] uses a distributed approach to calculate the Fiedler vector (the eigenvector corresponding to the second eigenvalue of the Laplacian), then estimate other relevant elements of Laplacian for each agent in a decentralized manner, and finally use these to derive a gradient based control law that maximizes connectivity. The biggest advantage of these methods is that they work fairly well for any topology. However, the downside of using algebraic connectivity is that the distributed estimation of the adjacency matrix that is needed to compute the Laplacian requires multiple iterations of information flow through the graph to converge to a reasonable value. The time consumed to compute the algebraic connectivity makes it a very brittle estimate in case of noise, and limits its applicability to real world missions.

Another class of methods enforce the desired connectivity among robots by constructing a given communication topology, a spanning tree for instance. Schuresko et al. [25] describes a robust and mission-agnostic algorithm to generate a spanning tree. Aragues et al. [1] implements an area coverage mission while preserving a minimum spanning tree among robots. The approach keeps connectivity with minimal interference on the area coverage mission. However, this method requires a specific initial condition that cannot always be guaranteed during a real world deployment. Majcherczyk et al. [14] treats the problem of decentralized deployment of multiple robots to different target locations while preserving connectivity. The algorithm in [14] defines different roles for robots, such that when a target is specified, a branch of the robot network is deployed and additional robots are supplied to build a structure reaching the target while maintaining connectivity. Panerati et. al. [17] proposed a hybrid methodology with a navigation controller enforcing connectivity and a global scheduler to provide the navigation controller with optimal policies. Despite the use of a global scheduler to support the navigation controller, the approach is relatively slow in comparison with our approach.

Our work is comparable to [14], as we dynamically build structures to reach given targets. The main difference is that we are able to specify the number of redundant links required for a particular target, including fault-tolerance in our method. Moreover, our approach aims at minimizing the computation and communication load of each robot, allowing deployment of other behaviours on top of the connectivity maintenance algorithm.

3 Model

3.1 Communication model

We assume our robots to have situated communication [29]: senders broadcast messages within a limited range $R$, and receivers within this range estimate the relative position of the sender in their local coordinate frame.
Inter-agent communication can be modelled as an undirected graph $G = (\nu, \epsilon, A)$, with $\nu = \{r_1, ..., r_N\}$ the node set representing the robots, and $\epsilon$ being the edge set representing communication links. An edge between $r_i$ and $r_j$ exists in $\epsilon$, if and only if, the distance $\|p_i - p_j\| \leq R$, with $p_i$ and $p_j$ being the i and j node positions respectively, $R$ being the communication range.

Spectral graph theory [3] offers methods to estimate the algebraic connectivity of a given graph using the adjacency matrix $A$ and the Laplacian $L$. The Laplacian of a graph can be estimated using the adjacency matrix and the degree matrix $D$. An entry $a_{ij}$ in the adjacency matrix is 1 if the edge $(i,j) \in \epsilon$ exists, 0 otherwise. The adjacency matrix of an undirected graph is symmetric. The degree matrix $D$ is a diagonal matrix denoting the number of edges to a node. The Laplacian $L$ is defined as $L = D - A$. The algebraic connectivity or Fiedler value [6] is defined as being the smallest non-zero eigenvalue $\lambda_2$ of $L$.

Notably, if $\lambda_2 > 0$ the graph is connected. An undirected graph is said to be strongly connected if there exists a path between any two nodes in the graph. Later in the experimental section 5, we study the evolution of network topology using the algebraic connectivity.

### 3.2 Robot Kinematics

Let the state of the robot $i$ be its position $p_i \in \mathbb{R}^m$ and let its state at time $t$ be $p_i(t)$. The state of the swarm at time $t$ be the vector $P(t) = \{p_1(t), p_2(t), ..., p_N(t)\}$.

We assume the kinematics of the robots to be

\[
\begin{bmatrix}
\dot{x}_i(t) \\
\dot{y}_i(t) \\
\dot{\Theta}_i(t)
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i(t) & 0 \\
\sin \theta_i(t) & 0 \\
0 & 1
\end{bmatrix} 
\begin{bmatrix}
v_i(t) \\
\theta_i(t)
\end{bmatrix}
\] (1)

where the velocity of the robot can be directly controlled. We consider 2-dimensional non-holonomic robots with differential drive, hence $p_i(t) = [x_i(t), y_i(t), \theta_i(t)]^\top$.

Our connectivity maintenance algorithm issues high level commands in terms of desired velocity, so that when a robot joins a chain to reach a target, we apply simple PID control on the linear and angular velocities in Equation 1.

### 3.3 Objectives

The objective of this work is to incrementally construct a communication backbone using a minimal number of robots, considering the communication requirements of the targets. Let, $T = \{t_1, t_2, ..., t_x\}$ be the set of targets to be reached, $Z_i = \{l_1, l_2, ..., l_y\}$ be the set of chains in any given target $t_i$. A chain $l_i$ corresponds to the communication chain $i$ connecting a robot visiting a target $t_i$, to the swarm. $r^j_i = \{r_1, r_2, ..., r_z\}$ be the set of robot agents forming the chain $j$ to a target $i$. The global objective of the backbone construction is:

\[
\min \sum_{i \in V} \sum_{j \in V} |r^j_i| 
\] (2)
subjected to,

\[ nl(t_i, r_i) \geq N t_i, \forall t_i \in T \]  

\[ v(t_i, r_i) = 1, \forall t_i \in T \]  

\[ \lambda_2(dt) > 0, \forall dt \in T_t \]  

Equation 2 minimizes the number of robots in each link of all the targets to be reached, i.e., robots used to build the communication chains. Let \( r_i \) be the robot visiting the target \( t_i \), \( v(t_i, r_i) \) denotes whether the robot \( r_i \) reached the target \( t_i \).

\[ v(t_i, r_i) = \begin{cases} 
1, & \text{if link } r_i \text{ reached target } t_i \\
0, & \text{otherwise} 
\end{cases} \]  

Equation 4 ensures all the targets in set \( T \) are reached. Let \( nl(t_i, r_i) \) be the number of links between the swarm and robot \( r_i \) currently visiting target \( t_i \), and \( nl(t_i) \) be number of links required to target \( t_i \). The inequality in Equation 3 ensures that there exists at least \( N t_i \) links between the swarm and robot \( r_i \). Equation 5 ensures that the resulting graph of the robot network is always connected.

4 Approach

4.1 Top-down specification

In this work, the robots in the swarm are self organizing, with a global behaviour emerging as a result of local interactions among robots. We assume that the robots are randomly deployed, and we assume that the robot network is initially connected. We believe this initial condition to be reasonable, since the robots are deployed as a cluster from a deployment area in real world scenarios. The desired global behavior of the swarm is to construct a tree from a central reference robot to the robots visiting one or more target, using minimal number of robots as in 2.

For this purpose, we build a communication chain for the robots visiting each target \( t_i \) from the target set \( T \). We specify a target \( t_i \) by its position, orientation, and required number of links. We assume these requirements to be variable: in a real world mission they might depend on what is accomplished at the target. For instance, in an exploration mission the targets could be landmarks from which photos or videos are required. A video capture might require more bandwidth than the a photo capture, resulting in a different number of links to achieve a desired bandwidth.

This algorithm assigns four types of roles to the robots in a swarm, namely: 1. root, 2. free, 3. networker, and, 4. worker. The root is assumed to be the center of the communication chains. Robots in the swarm start to build a tree incrementally from the root robot. Worker robots are assumed to be the robots visiting a target at a distant location. The robots with the networker role maintain a certain distance from their neighbors to secure a communication link. These
Fig. 1: Screen capture of the simulation result: (a) 20 robots creating a single communication chain, (b) 40 robots creating 2 communication chains, (c) 60 robots creating 3 communication chains and (d) 80 robots creating 4 communication chains (the green inter-agent connections indicate the ability to communicate).
robots use the control law in Equation [7] Free robots form a cluster around the root, waiting for a networker or a worker to be selected to serve as a relay. At first, the worker and root robots are selected. It would be ideal to select a robot that is closest to the target as worker. We assume that robots with worker roles are assigned to all targets in $T$ in advance, using a task allocation algorithm for example [13]. A free robot can either switch to be a networker or a worker depending on the immediate need of the swarm.

Once the worker robots and the root robot are selected, the worker extends the communication chain starting from the root. When the worker robot determines it has reached a threshold distance $d_s$ from the root, the robot chooses a free robot as a networker to act as relay to the root. Subsequently, when a networker reaches a suitable distance, it selects a new free robot to serve as a networker, and so on until the worker reaches the desired target.

One way to model the interaction between the robots in a chain from a worker to the root is by using the notion of virtual springs and dampers [32], as shown in Figure 2. Each robot exerts a virtual force ($F_{C_{ij}}$) on the other to stay within the safe communication distance ($d_s$). The exerted force is determined by:

$$F_{C_{ij}} = F_{ij}^s + F_{ij}^d = k(d_{ij} - d_s) + c \frac{d}{dt}(d_{ij} - d_s)$$

(7)
where \( k \) and \( c \) are the spring stiffness constant and damping coefficient, respectively. \( d_{ij} \) is the distance between the robot \( i \) and robot \( j \), \( d_s \) is the length of the spring, which defines the safe communication distance between the agents.

We use Equation 8, a simplified version of Equation 7, to generate a velocity control law. The velocity of the robot is expressed in a similar way to a spring force, while discarding the damping effect, which leads to a simpler kinematic controller that is more compatible with the control strategy described earlier in the robot’s kinematic model. In fact, it is similar, yet simpler, to gradient based velocity control [10]:

\[
\mathbf{u}_i = k(d_{ij} - d_s)
\]

where \( u_i \) is the velocity vector of the robot, \( d_{ij} \) is distance between robots \( i \) and \( j \), and \( d_s \) is the desired communication distance between the agents. We define \( P_i \) to be the set of parents of robot \( i \) connecting \( i \) to the root robot, either directly or through other robots acting as communication relay.

4.2 Bottom-up specification

Robots in the swarm follow a simple set of rules using local information, depending on their roles. There exists a parent-child relationship between the robots in the chain.

**Algorithm 1** Worker robot control rules

```plaintext
1: procedure worker routine
2:   if DISMANTLING then
3:     move in parents heading
4:     if root distance \( \leq \) safe distance then
5:       broadcast root dismantle complete
6:   end if
7:   else if Parent unresponsive then
8:     move towards parent, find any robot in parents' link
9:     if root distance \( \leq \) safe distance then
10:    broadcast root dismantle complete
11:   end if
12:   else
13:     if Parents in range and distance \( \leq \) safe distance then
14:       Move towards target
15:     end if
16:     if Number of parents \( \leq \) num of links required and distance \( \geq \) safe distance then
17:       Select a parent
18:     end if
19:   end if
20: end procedure
```
Workers A worker robot in the swarm initiates the growth of the chain, when it has reached the safe communication distance with the root. In that case, it chooses new robots to act as relays. Once the selected robot/s are within a safe communication distance, the worker starts to move towards the target. The virtual spring and damper system ensures the integrity of the chain over time. However, when a worker determines that one of its parents is unresponsive using the method detailed in section 4.3, it retracts the chain by moving in the direction of the failed parent and reconnects the broken link. The pseudo code listing 1 outlines the rules followed by a worker to form the chain. The control input of the worker robot $u^w_i$ is formulated as a sum of virtual forces:

$$u^w_i = \begin{cases} u^{sd}_i + f(d^p_i)(u^{target}_i + u^{obstacle}_i), & \text{if } d^p_i < d_c, \forall p \in P_i \\ u^p_i, & \text{otherwise} \end{cases}$$

where $u^{sd}_i$ is described in Equation 8, with neighbours being all parents, as in Equation 10. $u^{target}_i$ defines the control input to attract a robot towards a target, and can be described as

$$u^{target}_i = k_t(p_i - p_t)$$

where $p_i$ is the position of agent $i$, $p_t$ is the position of the target and $k_t$ is a constant gain. $u^{obstacles}_i$ defines the control velocity that results from a repulsive potential created by obstacles, so as to let the robot move in a direction that avoids the obstacle, in a very similar way to what is described in [27]. $d_c$ is the critical communication distance above which communication becomes unreliable and results in a broken link. If the distance between the parent and the child increases above a critical communication distance $d_c$, the robot performs an emergency maneuver towards the parent using the virtual force created by $u^p_i$.

$$f(d^p_i) = \begin{cases} 1, & \text{if } f(d^p_i) < d_s \\ 0, & \text{otherwise} \end{cases}$$

Networker Networker robots act as communication relays, extending a chain in the communication backbone for the worker robots to reach the target. Algorithm 2 outlines the rules used by the networker robots: when a networker gets selected to join a building chain, the robot navigates to a safe distance from its child and maintains safe communication distance and acts as a parent for the selecting robot. Moreover, if the networker reaches a safe communication distance, if the chain needs to be further extended, it selects a new robot to join. If a link in the chain breaks in case of a robot failure, the robots at both ends of the broken links take half of the responsibility to regain connection. The parent robots move towards the children, and vice versa the children robot of failed robot move towards the parent. If any of the robots of the same chain are
Algorithm 2 Networker robot control rules

1: procedure networker routine
2: \[\text{if child in view then}\]
3: \[\text{if new parent required to extend the chain then}\]
4: \[\text{Select new parent}\]
5: \[\text{else}\]
6: \[\text{Maintain safe distance between child and parent}\]
7: \[\text{end if}\]
8: \[\text{else if new child not in view then}\]
9: \[\text{find child and move towards the child}\]
10: \[\text{else if old child not in view or unresponsive then}\]
11: \[\text{move towards child and find any robot in chain}\]
12: \[\text{else if Parent unresponsive then}\]
13: \[\text{move towards parent, find any robot in parents' link}\]
14: \[\text{else if DISMANTLING then}\]
15: \[\text{move with parents heading}\]
16: \[\text{end if}\]
17: \[\text{end procedure}\]

encountered, the connection is reestablished and the parent child relationship detailed above starts with the newly bridged robots. The control law maintaining the integrity of the networker position in the chain is:

\[ u_{in} = \begin{cases} u_{ip}, & \text{if } d_{pi} > d_c, \forall p \in P_i \\ u_{sd} + f(d_{pc}i)(u_{obstacle}), & \text{otherwise} \end{cases} \]  

(13)

\[ u_{sd} = u_{sp} + u_{sd} \]  

(14)

where \( u_{ip} \) is the force that attracts a networker towards its parent, if the distance is over the critical distance \( d_c \), as in Equation (13). \( u_{sd} \) is the control law that ensures the integrity of the networker position from its parent and child. If a networker reaches a critical distance from its parent, its child takes the responsibility of regaining safe communication distance. In other words, a chain retracts if the communication distance gets above critical distance. Equation (15) enables and disables obstacle avoidance if the distance between parent or child increases above \( d_s \).

\[ f(d_{pc}i) = \begin{cases} 1, & \text{if } d_{pi} < d_s \text{ or } d_{ci} < d_s \\ 0, & \text{otherwise} \end{cases} \]  

(15)

**Free and Root** The role of the root robot in the swarm is to serve as a reference point to build the communication backbone and monitor the growth of the chains. The root listens to the broadcasts from the networkers to monitor the expansion of the communication chains. If the root predict that there is an insufficient number of robots to build all the chains for all the targets, it
Algorithm 3 Root and free robot control rules

1: procedure Root routine
2: Listen to status broadcast from robots connecting a chain
3: if Insufficient robots predicted then
4: Find chain with least robots and broadcast dismantle
5: Find chains to expand
6: Broadcast expand message to chains to expand
7: end if
8: end procedure
9: procedure Free routine
10: if New request received then
11: Accept request
12: navigate to child
13: end if
14: Compute LJ potential with root and other free robot
15: Use accumulated value as movement command
16: end procedure

broadcasts messages to dismantle the chain with the least number of robots. The pseudo code describing the root robot rules can be found in Algorithm 3 at line 1.

Free robots are the robots that get selected by the networkers or the workers to act as relays for extending the chains. The free robots form a cluster around the root using the force created by a Lennard-Jones (LJ) potential \[31\], which uses the position of the root and the free robots to place the robots in a cluster around the root’s proximity. The control law defining the interaction between the free robots and the root robot is:

\[
\begin{align*}
\mathbf{u}_{\text{lj}}^i &= \frac{\epsilon}{d_{ij}^6} \left[ \delta_{d_{ij}}^4 \right] - \left[ \delta_{d_{ij}}^2 \right] \\
\mathbf{u}_i^f &= \mathbf{u}_{\text{lj}}^i + \mathbf{u}_{\text{obstacle}}^i
\end{align*}
\]

(16)

The control law in eq. 17 defines the control used by the free robots to maintain the cluster around the root over time.

4.3 Inter-agent information flow

The communication between the robots is gossip-based, with strictly local broadcasts making information flow like in many insect colonies \[5\]. We define four different broadcast topics: 1. status broadcast; 2. request and response; 3. parent strand info; and 4. child strand Info. Every robot in the swarm locally broadcasts its current status under the status topic, including its current role, previous role, parent need and target chain. Every robot listens to the status message of all its neighbours. The status information is serialized into a 4 byte value, with each sub-part of the status consuming 1 byte each. Each robot keeps lists for each of
the roles, updated when receiving a new status message. From the updated lists, the list corresponding to the free robots is used during the selection, process described in [4] by a worker or networker. The parent need of the status broadcast is used by parent robots to determine whether more extension to the chain is required. The target robot broadcasts a status message with a parent need of 1 if the target was not reached, and 0 otherwise. This information flows all the way down to the robot connecting the root, and this robot selects new parents if the target needs more expansion. Moreover, the status messages are also used to predict robot failure. When a robot in a chain determines the absence of a status message from a particular parent or a child over a period of time, this robot is declared inactive. Inactive robots result in broken links, a broken link is tackled using parent and child strand info broadcasts. Using the strand info, the robot sensing broken link moves in the direction of the failed robot and tries to find any robot in the strand info that was connecting the failed robot and bridges connection with this robot that was after the failed robot. The robots while moving to bridge the connections of a failed link, moves in a way that ensures link connectivity with the rest of the chain by enforcing the forces described above.

The parent strand info messages are a serialized string containing all the parent robots in a chain up to the robot receiving the broadcast. This broadcast starts from the root and flows through all the children in a chain. The Root broadcasts its own id, which is received by its children, who append their ID to the message and rebroadcast it, and so on, until it reaches the worker robot. Using the parent info broadcast, a robot can determine the chain of robots connecting it to the root. In case of an intermediate robot failure, the robots can determine which robots to look for to reconnect the chain. The child strand info messages are similar to the parent strand info broadcasts, except that the information of the chain flows from worker to root.

The request and response broadcast topic is used to send a request to a neighboring robot, for instance to ask a free robot to join a chain. This topic is also used by the robots to send a response to a request sent by a robot.

5 Experiments

5.1 Setup

The experimental evaluations are aimed at studying the performance of the algorithm under different conditions. The experiments were conducted using a realistic physics-based multi-robot simulator ARGoS [20]. The robot controllers are developed using Buzz [19], a programming language specific to swarm robotics. We performed two sets of large scale simulations, using four different link requirements. First we study the convergence properties of the algorithm. In particular, Figure 3 reports the time needed by the swarm to reach the targets while maintaining a connected network with the desired number of links. This figure also reports the optimal time that might be required by the robots to navigate to a predefined location to form a topology, identical to the one formed by the proposed algorithm. The second set of experiments observes different failure
probabilities, and how they affect our algorithm’s performance. These injected failures make robots with an assigned backbone role unresponsive (i.e., we remove their ability to communicate) with a random probability, and report the time to reach targets.

In the first set of experiments, four different configurations were used, with \( N \in \{20, 40, 60, 80\} \), where \( N \) is the number of robots used in the experiment. The second set of experiments involved robots with \( N \in \{40, 80\} \). We placed the robots in a square 10x10 meters arena during both experiments, the robots were given four targets, which are equally spaced by 90 degrees from each other. The experimental design parameters used during the evaluations are reported in table 1.

### 5.2 Results

Figure 1 reports the final configuration of the robot network, with different configurations used during the evaluations. The sub-figure, (a), shows the resulting formation of 20 robots reaching the targets with a single link, (b), reports the 40 robot configuration with 2 links, and (c) and (d) report the resulting communication chains with 60 (3 links) and 80 robots (4 links) respectively.

The box plot in figure 3 reports the number of time steps required by the robots to build the chain and reach the distant target locations over 35 trials,
Table 1: Experimental design parameters used during the evaluations.

| Symbol | rationale                  | value | unit   |
|--------|----------------------------|-------|--------|
| $C$    | communication range        | 2     | meters |
| $dt$   | simulation control step    | 0.1   | second |
| $d_\delta$ | movement threshold   | 0.3   | meters |
| $k$    | spring constant gain       | 0.8   | no unit|
| $d_s$  | safe communication distance| 1.4   | meters |
| $d_c$  | critical communication distance| 1.7   | meters |
| $\epsilon$ | Lennard-jones potential epsilon | 60 | no unit |
| $\delta$ | Lennard-jones potential target | 0.50 | meters |

Fig. 4: Time taken to build the chains with different percentage of fallible robots (the markers are slightly offset for visual clarity).
Fig. 5: Percentage of mission failure vs robot faults.

with each trial starting from random locations. During the simulations, a single control step was set to 0.1s. Robots building a single-link chain consumed the highest time to reach the targets, with a median of 35 trial correspond to 1400 control steps. The two-link and three-link configuration consumed about 1270 and 1223 control steps. The four-link network with 80 robots interestingly consumed the least control steps to reach the targets with 1214 median control steps.

The decreasing number of control steps with the increasing number of robots is interesting, because opposite effect would be expected, if this were a centralized or similar approach, the time consumed might increase as the number of robots scales. The effect of decrease in the amount of time consumed with increase in number links could be because of the increase in the summation of force that is exerted on the robots with more links. Moreover, the optimal time consumed by the robots in identical configurations is used as a baseline to indicate the performance of the algorithm. These optimal time to reach the targets is computed, assuming the robot are in a perfect world (no collisions, perfect knowledge of the control inputs to reach the target). In reality, neither these conditions is possible, nor the robots can navigate without a control law.

Figure 7 reports the maximum, minimum, and median bandwidth consumed by any given robot in the swarm during the simulations. The 10-robot configuration consumed the minimal bandwidth of 17 bytes/timestep and the 100-robot
case consumed the maximum bandwidth of 174 bytes/timestep, however the median bandwidth were around 100 bytes/timestep in all configurations. The major part of the bandwidth consumed is contributed by the requests and responses sent by the robots to form a chain. This class of messages depends on the average number of neighbours for each robot. The other messages contributing to the bandwidth consumption in increasing order are strand information broadcasts and status messages.

![Diagram](image.png)

Fig. 6: Evolution of algebraic connectivity over a trial of 5 different configurations.

Figure 6 reports algebraic connectivity ($\lambda_2$) of the resulting network graph. We observe that the result is a very good representation of the underlying network topology as introduced in section 3. During the initial stages of the simulation, the robots are in a cluster (larger $\lambda_2$); as the experiment progresses $\lambda_2$ gets close to zero, in particular with a single link network. This is expected, since breaking a link might result in a partition of the network.

Error bars in figure 4 report the minimum, maximum and median time consumed with different rates of failure over 35 trials. Intuitively, the time taken for the algorithm to converge increases as the amount of fallible robots increase. The time consumed by the robots were also quite fluctuating mainly due to the nature of the analysis. During this analysis the 80 robot case with two links to maintain, consumed more time to regain a chain break because of the robots
trying to maintain two links at once. The results follow a similar trend with up to 0.2 failure probability, with almost identical medians. The time tread starts to diverge from 0.2 failure probability and climbs to 2067 and 2125 time steps at 0.7 failure probability, for 40 and 80 robots respectively. Figure 5 illustrates the percentage of mission failures with increase in faulty robots.

![Fig. 7: Max, min and median bandwidth consumed by any robot with the backbone construction approach over different configurations.](image)

6 Conclusions

We proposed a decentralized approach to enforce connectivity constraints, capable of working alongside an existing algorithm given its low computational and communication requirements. The algorithm progressively builds a communication backbone for a set of robots visiting a distant target. Our approach is self-organizing and inherently robust to single agent failure. We tackle agent failures by propagating simple information through the communication backbone.

We studied the performance of the proposed algorithm through a set of simulation experiments that empirically demonstrate the properties of the proposed algorithm through time to convergence, robustness to failure and scalability to up to hundreds of agents. Our results show that the algorithm withstands up
to 70 percent agent failures and regains network connectivity in the presence of broken communication links. Our approach allows configuring the number of communication links either to increase redundancy for critical missions or to provide more bandwidth for communication hungry missions. Moreover, our approach provides methods to tackle resource constrained scenarios -with fewer robots, by dismantling un-grown chain’s; constructing a single full chain, and visiting one target at a time. By design, the approach could tackle moving targets, however we did not investigate this in this work, we plan to explore it in future works.

We envision to extend the approach in a number of ways, starting from demonstrating its ability to run alongside other behaviours, to investigating methods to tackle the availability of very few agents. We also plan to investigate and deploy the algorithm on board a KheperaIV ground robot and on a small swarm of flying UAV’s (like DJI m100s and 3DR Solo).

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