Study on Concrete Workability Based on Comparison between the Minimum Paste Demand and the Closest Packing Density

Gaolong Zhang, Zhaoyu Yan, Fuqiang He, Ruipan Wang, and Changping Chen

1 School of Civil Engineering and Architecture, Xiamen University of Technology, Xiamen, Fujian 361024, China
2 College of Materials, Xiamen University, Xiamen, Fujian 361005, China

Correspondence should be addressed to Fuqiang He; 77163594@qq.com and Changping Chen; cpchen@126.com

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The mixture design method (MDM) has been used to design the concrete workability with the three-graded stone. The results indicated that the proportion of the stone corresponding to the minimum paste demand ($P_{\text{min}}$) can meet that corresponding to the approximate closest packing density ($ACPD_{\text{max}}$). When the workability reaches its maximum value, the paste thickness ($T_{\text{paste}}$) and mortar thickness ($T_{\text{mortar}}$) do not reach their respective extreme values. Therefore, the combined effect of the $T_{\text{paste}}$ and $T_{\text{mortar}}$ needs to be considered when evaluating concrete workability. Compared with concrete designed by the maximum packing density, the concrete designed by the $P_{\text{min}} + ACPD_{\text{max}}$ has a lower cost; however, stone surface area ($S_{\text{stone}}$) and the average particle size of the stone ($D_{A\text{-stone}}$) are increased by 8.6% and reduced by 5.7%, respectively. Therefore, the contradictory weakening effect of an increase in the $S_{\text{stone}}$ and a decrease in the $D_{A\text{-stone}}$ on the interface transition zone in both cases requires further investigation.

1. Introduction

Since Powers [1] recognized that aggregate gradation has a great effect on the properties of concrete, the method of achieving the maximum packing density by changing aggregate gradation has become the most important design concept during the preparation of modern concrete. Powers et al. [1, 2] considered that in the case of the paste amount remaining the same, the larger the packing density is, the smaller the paste demand will be, the more excessive paste there will be, and the thicker the formed paste coating on the aggregate will be. Therefore, they thought that an increase in the packing density could increase the concrete workability while maintaining the same paste amount and water-to-binder ratio [3]. Additionally, the volume stability of concrete can also be increased with an increase in the packing density. Therefore, an aggregate system needs to achieve its maximum packing density. This may explain why many researchers use the closest packing aggregate system to design concrete, especially high-strength, high-performance concrete, and self-compacting concrete [4].

Many models have been proposed to predict the packing density of aggregate, including the discrete model [2–5], the binary mixture model [1, 5, 6], the ternary mixture model [2, 7], the multicomponent mixture model [8, 9], the continuous model [2, 5, 6, 10], and so forth. In addition, some researchers even used the 3D computer simulation model [11–13], the digital image processing model [14, 15], and the discrete element model [16–19] to predict the packing density. Considering the interaction between aggregate particles, Dewar et al. [20] developed a model for predicting the packing density of multisize aggregate. De Larrard et al. [21] considered the interaction among aggregate particles and the compaction effect of the aggregate simultaneously and proposed a compressible packing model to predict the packing density. Kwan et al. [5] considered the effect of the shape factor and the convexity ratio of the aggregate and established a model for predicting the packing density of single-size aggregate.
The above models are of great value for predicting and optimizing the packing density. However, the factors affecting the packing density including particle size range, gradation, particle shape, surface roughness, and various effects during the mixing and compaction (e.g., the wall effect and the loosing effect) [5, 13, 22]. Therefore, it is very difficult to predict the packing density by using a certain model to comprehensively evaluate the effect of all these factors. Even if these factors can be involved in a model, they are difficult to be quantified accurately, which may cause the prediction results to deviate greatly from the actual situation. In order to achieve the maximum packing density, it is necessary to further increase the content of finer particles. In this situation, the specific surface area of aggregate will increase, resulting in a paste thickness reduction, and thus concrete workability will decrease [23]. Therefore, it is necessary to consider whether a concrete will meet its minimum paste demand in the case of the maximum packing density of an aggregate system [24]. This means that when evaluating concrete workability, combined effect of the aggregate packing and paste amount on the workability should be considered. The volume of the paste filled among aggregate particles has been calculated by the void fraction of aggregate [1, 2, 25]. Some researchers attempted to calculate the volume of the filling paste by assuming the paste thickness and the shape of the aggregate [25]. However, the fluctuation in the paste thickness may make the calculated results vary over a relatively large range [25]. Ji et al. [26, 27] experimentally determined the packing density and the specific surface area of the aggregate and the filling paste amount to achieve the minimum paste demand [26, 27]. However, their method for testing the specific surface area is complicated [26], and they did not consider the closest packing density in their study. Wang et al. [28] presented an optimized method for designing ultrahigh performance concrete (UHPC) with high wet packing density, in which the multiple effects of solid and liquid phases on UHPC packing mode are considered, the designed UHPC contributes high packing density, leading to optimized pore structure and extraordinary compressive strength. This method may be a potential one for concrete workability design based on the consideration of the combined effects of solid particle packing and liquid.

The use of two- or three-graded stone is a common selection in many concrete engineering. The closest packing density of two-graded stone can be determined by testing the mixed packing density of the two-graded aggregate with different proportions [26, 27]. Although Shi et al. [25] suggested that three-graded stone can be optimized by the simplex centroid design method (SCDM), there is no literature on how to get the closest packing density of a three-graded aggregate system. In fact, some researchers have used the SCDM for predicting the properties of a ternary system of cement-based materials [29–33]. Jiao et al. [29] studied the rheological properties of a coarse aggregate-sand-paste system by the SCDM. Douglas et al. [29–31] accurately predicted the strength of mortar and concrete in a ternary cementitious system using the SCDM. In addition, the SCDM can accurately predict the optimum composition of a ternary cementitious system of ladle slag-lime-quartz powder under autoclaved curing conditions [32] and the optimum composition of any ternary cementitious system when considering the expansion effect of the alkali aggregate reaction [33]. As a statistical method, the SCDM is one of the types of mixture design method (MDM). Another type of the MDM, extreme vertex mixture design method (EVMDM) in the MDM, is suitable for situations with constraints in which the extreme value differences in each component proportion are not completely the same, such as the coarse aggregate-sand-paste system in concrete [34].

As reviewed above, how to experimentally realize the maximum packing density and minimum paste demand still has no good solution. Therefore, the difference between the concrete prepared based on the two situations cannot yet be effectively compared. This study fills a gap in current research by experimentally realizing the maximum packing density and the minimum paste demand based on EVMDM and SCDM, which use a limited number of trials and have no parameters assumptions and any models. A comparison between the maximum packing density and the minimum paste demand systems is presented. Additionally, unlike other studies in which only the effect of the paste coating on the workability is considered, the combined effect of thicknesses of the paste coating and mortar coating on the concrete workability is discussed in this study. Finally, a concrete mix design method based on the minimum paste demand and the approximate closest packing density is suggested.

2. Raw Materials and Experimental Method

2.1. Raw Materials. P.O. 42.5 cement, which conforms to the Chinese Standard GB175-2007 [35], grade II fly ash, which conforms to the Chinese Standard GB/T 1596–2017 [36], and grade 595 slag powder, which conforms to the Chinese Standard GB/T 18046–2008 [37], were used in this study. Polycarboxylate superplasticizer, which conforms to the Chinese Standard JB/T 223–2007 [38], was used in this experiment. The chemical compositions of the cement, fly ash, and slag powder are shown in Table 1.

The coarse aggregate used in the experiments is three-graded limestone crushed stone with single particle sizes of 5–10 mm (the small size stone), 10–20 mm (the middle size stone), and 16–31.5 mm (the large size stone). The particle size distributions were tested three times according to the Chinese National Standard GB14685-2011 [39], and the average values of the test results are taken as the final values. The particle size distributions of the three-graded stone are shown in Table 2 and the grading curves are shown in Figure 1.

The fine aggregate used here was river sand with the fineness modulus of 2.6. The compressed packing density, loose packing density, and the apparent density values of the sand are 1662 kg/m³, 1531 kg/m³ and 2622 kg/m³, respectively, tested according to the Chinese National Standard GB14684-2011 [40]. The particle size distribution of the sand was tested three times according to the Chinese National Standard GB14684-2011 [40], and the average value was taken as the final value. The particle size distribution of the sand is shown in Table 3, and the grading curve of the sand is shown in Figure 1.
2.2. Mixing Design of the Coarse Aggregate System. In order to obtain better contour plots, the pivot point enhanced simplex centroid design method (ppe-SCDM) was used to design the three-graded stone system (L-M-S system), and 10 groups of different proportions of the coarse aggregate can be obtained, as shown in Table 4.

2.3. Concrete Mix Proportion. To determine the proportions of the three-graded stone in the L-M-S system based on workability (slump and slump flow), 10 groups of coarse aggregate with different proportions were designed by the ppe-SCDM. The binder dosage in the concrete was fixed at 420 kg/m³ of which 60% was cement, 30% was fly ash, and 10% was slag powder, according to the durability design requirements for mass concrete, which are not within the scope of this study. The water-binder ratio (W/B) was calculated according to the National Standard of China JGJ55-2011 [41], the W/B and sand-to-aggregate ratio (S/A) were fixed at 0.36 and 0.39, respectively. The dosage of the polycarboxylate superplasticizer (type I, solids content 20%) was 0.8% of the binder, by mass. The proportions of the concrete mixtures are shown in Table 5.

For the purpose of determining the binder dosage and S/A used in concrete, the EVMDM was adopted to design the paste-sand-stone ternary system (P-S-S system). The binder dosage was in the range of 380–420 kg/m³, the S/A varied from 0.35 to 0.45, and thus, 13 groups of concrete mixtures were obtained, as presented in Table 6. The W/B and the proportion of the three-graded stone were both fixed at 0.36, and the proportion of the large size stone: middle size stone: small size stone was equal to 0.32:0.34:0.34 (corresponding to the minimum paste demand, see Section 3.1.2 for the details). The proportions of the cement, fly ash, and slag powder in the binder were kept the same as those used in the L-M-S system. The dosage of polycarboxylate superplasticizer (type II, solids content 15%) was 1.6% of the binder, by mass.

2.4. Experimental Method. According to the China National Standard GB14685-2011 [39], the loose packing density, compressed packing density, and apparent density of the above 10 groups of three-graded stone were tested. The tested results are shown in Table 4. The concrete slump and slump flow tests were carried out in accordance with the National Standard of China GB/T50080-2016 [42].

3. Results and Discussion

3.1. Determination of the Mixing Ratio of the Three-Graded Stone

3.1.1. Mixing Ratio of the Three-Graded Stone Based on the Maximum Packing Density. The packing density contour plots of the L-M-S system are shown in Figure 2. It can be

Table 2: Particle size distributions of the three-graded stone.

| Sieve size (mm) | Large size stone | Middle size stone | Small size stone |
|-----------------|------------------|-------------------|-----------------|
|                 | Average value (g) | Coefficient of variation (%) | Cumulative percentage retained (%) | Average value (g) | Coefficient of variation (%) | Cumulative percentage retained (%) | Average value (g) | Coefficient of variation (%) | Cumulative percentage retained (%) |
| 31.5            | 269              | 0.17              | 4.2              | 0               | 0                | 0                            | 0               | 0                | 0                            |
| 26.5            | 2030             | 0.19              | 35.5             | 0               | 0                | 0                            | 0               | 0                | 0                            |
| 19              | 3956             | 0.08              | 96.6             | 547             | 0.07             | 10.8                        | 0               | 0                | 0                            |
| 16              | 129              | 0.05              | 98.6             | 2725            | 0.03             | 64.6                        | 0               | 0                | 0                            |
| 9.5             | 56               | 0.11              | 99.4             | 1759            | 0.18             | 99.4                        | 166             | 0.05             | 6.8                          |
| 4.75            | 5                | 0.10              | 99.5             | 6               | 0.14             | 99.5                        | 2010            | 0.06             | 89.4                         |
| 2.36            | 1                | 1.18              | 99.5             | 1               | 0.29             | 99.5                        | 240             | 0.21             | 99.3                         |
| Remaining       | 32               | 0.35              | 100.0            | 25              | 0.14             | 100.0                       | 18              | 0.10             | 100.0                        |
| Total           | 6478             | 0.01              | —                | 5063            | 0.04             | —                           | 2434            | 0.08             | —                            |

Figure 1: Particle size distribution curves of the aggregate.

Table 1: Chemical compositions of the cement, fly ash, and slag power (by mass, %).

| Material   | SiO₂  | Al₂O₃ | Fe₂O₃ | CaO   | MgO   | SO₃  | K₂O | CaO  | Loss |
|------------|-------|-------|-------|-------|-------|------|-----|------|------|
| Cement     | 25.54 | 8.13  | 5.59  | 52.27 | 2.3   | 2.16 | 0.64| 1.22 | 1.75 |
| Fly ash    | 70.59 | 15.16 | 2.10  | 2.01  | 0.65  | 0.43 | 4.17| 2.24 | 5.92 |
| Slag power | 30.62 | 15.04 | 0.47  | 42.29 | 7.04  | 2.28 | 0.52| 0.94 | 0.36 |

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### Table 3: Proportions and properties of the designed coarse aggregate system.

| No. | Stone proportions, %, by mass | Apparent density (kg/m³) | Loose packing density (kg/m³) | Compressed packing density (kg/m³) |
|-----|--------------------------------|--------------------------|-------------------------------|----------------------------------|
|     | Large size stone (16–31.5 mm) | Middle size stone (10–20 mm) | Small size stone (5–10 mm) |                                    |
| 1   | 50.0                           | 0.0                       | 50.0                          | 2698.3                           |
| 2   | 16.7                           | 16.7                      | 66.7                          | 2746.1                           |
| 3   | 0.0                            | 50.0                      | 50.0                          | 2794.0                           |
| 4   | 0.0                            | 100.0                     | 0.0                           | 2792.0                           |
| 5   | 100.0                          | 0.0                       | 0.0                           | 2650.0                           |
| 6   | 33.3                           | 33.3                      | 33.3                          | 2723.7                           |
| 7   | 0.0                            | 0.0                       | 100.0                         | 2764.3                           |
| 8   | 66.7                           | 16.7                      | 16.7                          | 2709.6                           |
| 9   | 16.7                           | 67.7                      | 16.7                          | 2761.9                           |
| 10  | 50.0                           | 50.0                      | 0.0                           | 2705.9                           |

### Table 4: Particle size distribution of the sand.

| Sieve size (mm) | Average value (g) | Coefficient of variation (%) | Percentage retained (%) | Cumulative percentage retained (%) |
|-----------------|------------------|-------------------------------|-------------------------|-----------------------------------|
| 4.75            | 1.0              | 0.073                         | 0.2                     | 0.2                               |
| 2.36            | 40.0             | 0.034                         | 8.0                     | 8.2                               |
| 1.18            | 67.0             | 0.042                         | 13.4                    | 21.6                              |
| 0.60            | 148.5            | 0.013                         | 29.7                    | 51.3                              |
| 0.30            | 149.0            | 0.019                         | 29.8                    | 81.1                              |
| 0.15            | 75.0             | 0.037                         | 15.0                    | 96.1                              |
| Residue         | 19.5             | 0.091                         | 3.9                     | 100.0                             |
| Total           | 500.0            | 0.001                         | 100.0                   | —                                 |

### Table 5: Proportions of the three-graded stone and concrete mix proportions in the L-M-S system.

| No. | Large size stone (16–31.5 mm) | Middle size stone (10–20 mm) | Small size stone (5–10 mm) | Apparent density (kg/m³) | Loose packing density (kg/m³) | Compressed packing density (kg/m³) |
|-----|--------------------------------|--------------------------------|-----------------------------|--------------------------|-------------------------------|-----------------------------------|
| 1   | 40.0                           | 26.7                           | 20.0                        | 33.3                     | 40.0                          | 26.7                              |
| 2   | 22.9                           | 46.7                           | 20.0                        | 33.3                     | 40.0                          | 26.7                              |
| 3   | 66.7                           | 33.3                           | 33.3                        | 33.3                     | 40.0                          | 26.7                              |
| 4   | 22.7                           | 16.7                           | 20.0                        | 33.3                     | 40.0                          | 26.7                              |
| 5   | 50.0                           | 0.0                            | 100.0                       | 2764.3                   | 1579.2                        | 1696.2                            |
| 6   | 67.7                           | 67.7                           | 16.7                        | 2769.6                   | 1579.2                        | 1696.2                            |
| 7   | 0.0                            | 0.0                            | 100.0                       | 2769.6                   | 1579.2                        | 1696.2                            |

### Table 6: Mixture design of the P-S-S system.

| No. | Binder, %, by mass | Sand, kg/m³ | Stone, %, by mass | S/A (%) |
|-----|-------------------|-------------|------------------|---------|
| 1   | 22.1              | 390.4       | 32.9             | 45.0    | 1080.3 | 0.42 |
| 2   | 23.8              | 420.0       | 26.9             | 49.3    | 1182.9 | 0.35 |
| 3   | 21.5              | 380.0       | 34.8             | 43.6    | 1047.6 | 0.44 |
| 4   | 23.3              | 410.4       | 32.3             | 44.5    | 1068.0 | 0.42 |
| 5   | 22.9              | 403.6       | 28.9             | 48.2    | 1157.4 | 0.37 |
| 6   | 23.0              | 406.3       | 26.9             | 48.2    | 1157.4 | 0.37 |
| 7   | 23.8              | 420.0       | 33.6             | 48.2    | 1157.4 | 0.37 |
| 8   | 22.7              | 400.8       | 30.9             | 48.2    | 1157.4 | 0.37 |
| 9   | 21.5              | 380.0       | 28.4             | 48.2    | 1157.4 | 0.37 |
| 10  | 22.6              | 398.3       | 34.8             | 48.2    | 1157.4 | 0.37 |
| 11  | 23.3              | 410.4       | 28.9             | 48.2    | 1157.4 | 0.37 |
| 12  | 22.6              | 399.5       | 32.9             | 48.2    | 1157.4 | 0.37 |
| 13  | 22.1              | 390.4       | 29.7             | 48.2    | 1157.4 | 0.37 |
seen in Figure 2 that the compressed and loose packing density values of the L-M-S system reached 1736 kg/m³ and 1599 kg/m³, respectively. The proportions of the three-graded stone corresponding to the maximum compressed packing density \((CPD_{\text{max}})\), the approximate maximum compressed packing density \((ACPD_{\text{max}})\), the maximum loose packing density \((LPD_{\text{max}})\), and the approximate maximum loose packing density \((ALPD_{\text{max}})\) of the L-M-S system are given in Table 7. It can be seen in Table 7 that proportion regions of the three-graded stone of \(ALPD_{\text{max}}\) and \(ACPD_{\text{max}}\) do not have any overlapping area, and the proportion regions of the three-graded stone of \(LPD_{\text{max}}\) are not within that of \(ACPD_{\text{max}}\). This means that the compressed packing density can reach its maximum only if these three types of stones are incorporated in a considerable proportion. That is, the smaller-size stones are gradually filled into the voids between the larger-size stones, so as to achieve the closest packing state. When the stones were loosely packed, large size stones cannot be incorporated to achieve the maximum loose packing density. This is because the inclusion of large size stones will form larger voids, and these larger voids cannot be filled very well without the compression.

3.1.2. Mixing Ratio of the Three-Graded Stone Based on the Concrete Workability. The contour plots of the concrete slump and slump flow are plotted in Figure 3. From Figure 3, the maximum slump \((S_{\text{max}})\), the approximate maximum slump \((AS_{\text{max}})\), the maximum slump flow \((D_{\text{max}})\), and the approximate maximum slump flow \((AD_{\text{max}})\) of concrete prepared using the L-M-S system can be obtained, as shown in Table 8. It can be seen from Tables 7 and 8 that the three-graded stone proportions of \(ACPD_{\text{max}}\) of the L-M-S system overlap with that of \(AS_{\text{max}} + AD_{\text{max}}\). The proportions of the three-graded stone corresponding to \(S_{\text{max}}\) and \(D_{\text{max}}\) of the L-M-S system are almost the same, and the proportion ranges of the three-graded stone corresponding to \(AS_{\text{max}}\) and \(AD_{\text{max}}\) in the L-M-S system are also almost the same. The proportions of the three-graded stone corresponding to \(S_{\text{max}}\) and \(D_{\text{max}}\) in the L-M-S system fall within the overlapping region of the three-graded stone proportions among \(ACPD_{\text{max}}\), \(AS_{\text{max}}\), and \(AD_{\text{max}}\) of the L-M-S system. However, the proportions of the three-graded stone in the case of \(CPD_{\text{max}}\) and \(LPD_{\text{max}}\) are not within this range. This means that the proportions of the three-graded stone corresponding to \(S_{\text{max}}\), \(D_{\text{max}}\), and \(CPD_{\text{max}}\) satisfy that of \(ACPD_{\text{max}}\) in the L-M-S system and the compressed packing density is more suitable for characterizing the packing state of the coarse aggregate in concrete than the loose packing density.

It can be read in Figure 2 that the compressed packing density in the case of \(S_{\text{max}}\) is 1730.6 kg/m³ in the L-M-S system, which is very close to that of \(CPD_{\text{max}}\) (1736 kg/m³). The surface area corresponding to \(S_{\text{max}}\) is 245.7 m², and the surface area corresponding to \(CPD_{\text{max}}\) is 266.9 m². This means that paste volume of the three-graded stone corresponding to \(S_{\text{max}}\) is very close to the minimum paste filling volume, but the surface area corresponding to \(S_{\text{max}}\) is smaller than that corresponding to \(CPD_{\text{max}}\) in the L-M-S system. Therefore, the paste demand for the same paste thickness is correspondingly smaller. It can be seen from Figure 3 that the slump and slump flow corresponding to \(CPD_{\text{max}}\) are 173 mm and 321 mm, respectively, which are relatively smaller than \(S_{\text{max}}\) (196 mm) and \(D_{\text{max}}\) (390 mm). This shows that with the same amount of paste, the aggregate system according to \(CPD_{\text{max}}\) needs more filling and coating paste than that corresponding to \(S_{\text{max}}\), and thus there is a less excessive paste. When obtaining a certain targeted slump, the three-graded stone system corresponding to \(S_{\text{max}}\) has the minimum paste demand \((PD_{\text{min}})\) aggregate system in the L-M-S system.

The effect of the particle sizes and surface areas of the three-graded stone respectively corresponding to \(PD_{\text{min}}\) and \(CPD_{\text{max}}\) on the interface transition zone (ITZ) is a key consideration. The ITZ of concrete has a great effect on the strength and durability of concrete [43–48]. When mixing fresh concrete, a water film is formed around the large aggregate particles, resulting in a much higher W/B around the large particles than that of the remaining portion. When other conditions are constant, the larger the aggregate size, the greater the local W/B in the ITZ, and the lower the strength and durability of the concrete [43, 49]. The surface area \((S_{\text{stone}})\) and the average particle size \((D_{\text{A-stone}})\) of the three-graded stone corresponding to \(PD_{\text{min}}\) and \(CPD_{\text{max}}\) in the L-M-S system can be calculated according to equation (A.9) and equation (A.14) (see Appendix A), which are 245.7 m², 15.33 mm, and 266.9 m², 14.45 mm, respectively. Compared with \(PD_{\text{min}}\), \(S_{\text{stone}}\) corresponding to the \(CPD_{\text{max}}\) is increased by 8.6%, and the \(D_{\text{A-stone}}\) is reduced by 5.7%. The increase in the \(D_{\text{A-stone}}\) leads to an increase in the weakening degree to the ITZ. However, the decrease in \(S_{\text{stone}}\) leads to a decrease in the ITZ quantity and thus reduces the weakening degree to ITZ. The weakening degree to ITZ depends on the combined effects of \(S_{\text{stone}}\) and \(D_{\text{A-stone}}\). Therefore, the effect on ITZ in the case of \(CPD_{\text{max}}\) and \(PD_{\text{min}}\) was used to prepare concrete in this study.

It is clear from the above discussion that the proportion of the three-graded stone in the cases of \(PD_{\text{min}}\) also satisfies \(ACPD_{\text{max}}\); therefore, the proportion of the three-graded stone corresponding to \(PD_{\text{min}}\) was used to prepare concrete in this study.

3.2. Determination of the Binder Dosage and S/A Based on EVMDM

3.2.1. The Contour Plots of Concrete Slump and Slump Flow of the P-S-S System. The contour plots of the slump and slump flow of the P-S-S system are shown in Figure 4. It can be seen in Figure 4 that the varying trends of concrete slump and slump flow are mostly the same. With an increase in the paste volume, the slump and slump flow gradually increase, and with a decrease in the S/A, the slump and slump flow first increase and then decrease.

Based on the concrete encapsulation model [50], the paste thickness \((d_{\text{paste}})\) coated on the surface of the aggregate, and the mortar thickness \((d_{\text{mortar}})\) coated on the surface of the coarse aggregate can be calculated according to
equations (B.12) and (B.13) (see Appendix B), respectively, whose results are shown in Figure 5. It can be seen in Figure 5 that $T_{\text{paste}}$ increases gradually with an increase in the binder dosage and the decrease in $S/A$ and varies between 30 and 59 μm, which is consistent with the range of $T_{\text{paste}}$ obtained by other researchers [51]. This is because the total
surface area of aggregate can be significantly reduced by reducing S/A, and $T_{\text{paste}}$ increases with a decrease in S/A when the paste volume is constant. When the binder dosage is at the maximum value (420 kg/m$^3$) and S/A is at the minimum value (0.35), $T_{\text{paste}}$ reaches its maximum value (approximately 59 $\mu$m). $T_{\text{mortar}}$ increases with an increase in the binder dosage and S/A. When the binder dosage is at the maximum value (420 kg/m$^3$) and S/A is 0.43 (near its maximum value, 0.45), $T_{\text{mortar}}$ reaches its maximum value (approximately 1.7 mm).

Associated with Figures 4 and 5, it can be found that when the slump and slump flow reach their maximum values, $T_{\text{paste}}$ and $T_{\text{mortar}}$ vary in the range of 50–55 $\mu$m and 1.3–1.4 mm, respectively, which is in the middle region of $T_{\text{paste}}$ and $T_{\text{mortar}}$ respectively. This indicates that $T_{\text{paste}}$ and $T_{\text{mortar}}$ are not at their maximum or minimum values in the case of $PD_{\text{min}}$; however, $T_{\text{paste}}$ is closer to its maximum value (59 $\mu$m), and $T_{\text{mortar}}$ is closer to its minimum value (1.0 mm). Jiao et al. [52] regarded that when S/A is fixed, the slump and slump flow are mainly affected by $T_{\text{paste}}$. However, their study cannot reflect the combined effects of the paste volume and S/A on the concrete workability by changing the paste volume and S/A simultaneously. The results obtained from Figures 4 and 5 demonstrate that when considering the effect of $T_{\text{paste}}$ on the workability of concrete, the effect of $T_{\text{mortar}}$ must also be considered at the same time, and EVMDM can determine the most reasonable range of $T_{\text{paste}}$ and $T_{\text{mortar}}$ in a convenient way.

3.2.2. Determination of the Binder Dosage and the S/A Based on $PD_{\text{min}}$ of the P-S-S System. The paste-sand-stone proportions corresponding to $S_{\text{max}}$, $AS_{\text{max}}$, $D_{\text{max}}$, and $AD_{\text{max}}$ of the P-S-S system can be read in Figure 4, as shown in Table 9. It can be seen in Table 9 that the proportions of paste-sand-stone corresponding to $S_{\text{max}}$ and $D_{\text{max}}$ are almost the same; the proportions of paste-sand-stone corresponding to $AS_{\text{max}}$ and $AD_{\text{max}}$ are very similar. When the binder dosage varies in the range of 411–420 kg/m$^3$ and S/A varies from 0.37 to 0.39, the concrete workability reaches its approximate maximum value. When the binder dosage is 420 kg/m$^3$ and S/A is 0.38, the slump and slump flow synchronously reach their maximum values, which are 239 mm and 560 mm, respectively.

3.2.3. Determination of the Binder Dosage and the S/A Based on the Targeted Workability of the P-S-S System. A typical contour plot of the targeted slump ($S_{\text{tar}}$, assumed to be 180–200 mm) and targeted slump flow ($D_{\text{tar}}$, assumed to be 350–450 mm) is shown in Figure 6. It can be seen in Figure 6 that when $S_{\text{tar}}$ and $D_{\text{tar}}$ are synchronously met (the red part in Figure 6), the proportion of the minimum paste amount is 0.215, the proportion of the sand and the stone drops within the regions of 0.284–0.304 and 0.481–0.501, respectively. Namely, the binder dosage is 380 kg/m$^3$, and S/A is from 0.36 to 0.39. It can also be read from Figure 6 that when the proportions of the paste, sand, and stone are in the range of 0.226–0.238, 0.317–0.338, and 0.436–0.448, respectively, the concrete workability can also meet the targeted requirements. Correspondingly, the binder dosage varies between 399 and 420 kg/m$^3$, and S/A varies within the range of 0.42–0.44.

The compressed packing density of the sand-stone system with different S/A values is shown in Figure 7. It can be seen in Figure 7 that the sand-stone system has the maximum packing density when S/A is 0.41. Therefore, S/A corresponding to $CPD_{\text{max}}$ is not the same as that corresponding to $PD_{\text{min}}$. Corresponding to $PD_{\text{min}}$, the binder dosage is 420 kg/m$^3$ and S/A is 0.38, which are not the same as those of the maximum packing density of the sand-stone system. In this situation, $T_{\text{paste}}$ is approximately 49.5 $\mu$m and $T_{\text{mortar}}$ is approximately 1.33 mm. When taking S/A (0.41) corresponding to the maximum packing density of sand-stone system and when the binder dosage is 420 kg/m$^3$, $T_{\text{paste}}$ and $T_{\text{mortar}}$ are approximately 44 $\mu$m and 1.49 mm, respectively, while the slump and slump flow are 210 mm and 405 mm, respectively, which are far from those corresponding to $PD_{\text{min}}$ in the P-S-S system. This shows that the combined effect of an increase in $T_{\text{mortar}}$ and a decrease in
Packing density (kg/m³)

|      | 1720 | 1760 | 1800 | 1840 | 1880 | 1920 | 1960 | 2000 | 2040 | 2080 |
|------|------|------|------|------|------|------|------|------|------|------|
| Stone| 0.461| 0.473 | 0.469 | 0.475 | 0.469 | 0.480 | 0.481 | 0.482 | 0.483 | 0.484 |
| Sand | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.295 | 0.215 |
| Paste| 0.34 | 0.36 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.47 | 0.48 | 0.49 |

Table 9: Binder dosage and the S/A in the case of $S_{\text{max}}$ and $AS_{\text{max}}$ of the P-S-S system (by mass).

| Items  | $AS_{\text{max}}$ | $S_{\text{max}}$ | $AD_{\text{max}}$ | $D_{\text{max}}$ | $AS_{\text{max}} + AD_{\text{max}}$ | Binder (kg/m³) | S/A   |
|--------|------------------|------------------|------------------|------------------|-----------------------------------|----------------|-------|
| Paste  | 0.230–0.238      | 0.238            | 0.233–0.238      | 0.238            | 0.233–0.238                       | 411–420        | 0.37–0.39 |
| Sand   | 0.279–0.301      | 0.289            | 0.280–0.293      | 0.287            | 0.280–0.293                       |                |       |
| Stone  | 0.461–0.483      | 0.473            | 0.469–0.481      | 0.475            | 0.469–0.481                       |                |       |

Note: $S_{\text{max}}$ and $D_{\text{max}}$ are the maximum slump and maximum slump flow values of the P-S-S system, respectively; $AS_{\text{max}}$ and $AD_{\text{max}}$ are the approximate maximum slump values within a range of 10 mm from the $S_{\text{max}}$ and the approximate maximum slump flow values within a range of 30 mm from the $D_{\text{max}}$ in the P-S-S system, respectively.

Figure 5: Contour plots of $T_{\text{paste}}$ (a) and $T_{\text{mortar}}$ (b) of the P-S-S system.

Figure 6: Contour plot of the concrete slump and slump flow (mm, slump = 180–200 mm, slump flow = 350–450 mm).

Figure 7: Relationship between S/A and the compressed packing density of the sand-stone system. Note: the proportion of the three-graded stone corresponding to $CPD_{\text{max}}$ of the L-M-S system was used.

$T_{\text{paste}}$ significantly reduces the concrete workability. Therefore, it is not enough to study the concrete workability only based on $T_{\text{paste}}$. The combined effect of $T_{\text{paste}}$ and $T_{\text{mortar}}$ on the workability should be considered. Cementitious materials will cause early thermal expansion and shrinkage in the hydration process, which will also produce a drying shrinkage in a long-term drying environment [53]. When W/B is constant, reducing the content of cementitious materials can effectively reduce the effect of the paste on the volume stability of concrete [23]. Therefore, one can control $T_{\text{paste}}$ at a lower level by keeping $T_{\text{mortar}}$ within a reasonable range.

$S_{\text{stone}}$ of the three-graded stone corresponding to $S_{\text{tar}}$ and $S_{\text{max}}$ values of the P-S-S with different binder dosages and S/

A and $CPD_{\text{max}}$ values with S/A = 0.41 are shown in Table 10. It can be seen in Table 10 that $S_{\text{stone}}$ value corresponding to $CPD_{\text{max}}$ is larger than that corresponding to $S_{\text{tar}}$ and $S_{\text{max}}$. In the case of $S_{\text{tar}}$, $S_{\text{stone}}$ with smaller binder dosage is larger than that with large binder dosage. Therefore, it also has a larger ITZ. However, in the case of the maximum packing density of sand-stone system, $D_{\text{A-stone}}$ of the three-graded stone decreases by 5.7%, which makes it more difficult to form a water film compared with $S_{\text{max}}$ situation and thus
reduces the effect of the particle size on W/B in ITZ. When W/B is constant, reducing the content of cementitious materials can effectively reduce the effect of the paste on volume stability of concrete [23]. To achieve the targeted workability requirements of a 180–200 mm slump and a 350–450 mm slump flow, the binder dosage = 380 kg/m$^3$ and S/A = 0.36–0.39 can be selected to design the concrete with the lowest cost. It is worth noting whether the P-S-S system based on the PD$_{\text{min}}$ is consistent with the P-S-S system according to the best durability needs further research. It can be recommended that EVMDM is also suitable for durability design based on adjustments of the P-S-S system.

### 4. Conclusions

In this study, ppe-SCDM and EVMDM have been used to design the workability of concrete with the three-graded stone, which can experimentally realize the minimum paste demand and the maximum packing density of the three-graded stone in a convenient way. S$_{\text{stone}}$, S$_{\text{sand}}$, D$_{\text{A-stone}}$, $T_{\text{paste}}$, and $T_{\text{mortal}}$ were calculated to discuss their effect on concrete workability. Some of the main conclusions obtained are as follows:

1. The proportions of the three-graded stone in the case of CPD$_{\text{max}}$ and LPD$_{\text{max}}$ are quite different. The compressed packing density is more suitable for characterizing the packing state of the three-graded stone in concrete than the loose packing density. The proportions of the three-graded stone in the case of S$_{\text{max}}$ and D$_{\text{max}}$ of the L-M-S system fall within the overlapping region of the three-graded stone proportions among the ACPD$_{\text{max}}$, AS$_{\text{max}}$, and AD$_{\text{max}}$ of the L-M-S system. However, the proportion of the three-graded stone corresponding to CPD$_{\text{max}}$ is not within this overlapping region. This indicates that the proportion of the three-graded stone corresponding to S$_{\text{max}}$ and D$_{\text{max}}$ can meet that corresponding to ACPD$_{\text{max}}$.

2. When the binder dosage and S/A are in the range of 380–420 kg/m$^3$ and 0.35–0.45, respectively, $T_{\text{paste}}$ and $T_{\text{mortal}}$ vary between 30 and 59 $\mu$m and 1.2–1.7 mm, respectively. However, $T_{\text{paste}}$ and $T_{\text{mortal}}$ vary in the range of 50–55 $\mu$m and 1.3–1.4 mm, respectively, when the slump and slump flow reach their maximum values. This means that when the workability reaches a maximum value, $T_{\text{paste}}$ and $T_{\text{mortal}}$ do not reach their extreme values; however, $T_{\text{paste}}$ is closer to its maximum value, and $T_{\text{mortal}}$ is closer to its minimum value. Therefore, it is very necessary to consider the combined effects of $T_{\text{paste}}$ and $T_{\text{mortal}}$ on concrete workability, not just the effect of $T_{\text{paste}}$.

3. The MDM can determine PD$_{\text{min}}$ and the optimal S/A under any target workability requirements. At the same time, the concrete designed by the MDM approximately reaches the ACPD$_{\text{max}}$ value. Therefore, it can be expected that the concrete prepared using this method has a lower cost. However, compared with PD$_{\text{min}}$ (meeting the ACPD$_{\text{max}}$), S$_{\text{stone}}$ of the three-graded stone system corresponding to CPD$_{\text{max}}$ is increased by 8.6%, and D$_{\text{A-stone}}$ is reduced by 5.7%. The increase of the particle size will cause an increase in the weakening degree of ITZ. However, the decrease in S$_{\text{stone}}$ will reduce the weakening degree of ITZ. The weakening degree of ITZ depends on the combined effects of S$_{\text{stone}}$ and D$_{\text{A-stone}}$. Therefore, the contradictory weakening effect of an increase in S$_{\text{stone}}$ and a decrease in D$_{\text{A-stone}}$ to ITZ in both cases requires further evaluation.

### Abbreviations

- A$_I$-stone': specific surface area of stones with diameter $D_i$ in per cubic metre concrete, m$^2$/kg
- A$_{\text{stone}}'$: specific surface area of stone in a certain particle size range, m$^2$/kg
- ACPD$_{\text{max}}$: approximate maximum compressed packing density of the three-graded stone, kg/m$^3$
- AD$_{\text{max}}$: approximate maximum slump flow of concrete, mm
- ALPD$_{\text{max}}$: approximate maximum loose packing density of the three-graded stone, kg/m$^3$
- A$_{\text{sand}}$: specific surface area of sand, m$^2$/kg
- AS$_{\text{max}}$: approximate maximum slump of concrete, mm
- A$_{\text{stone}}$: specific surface area of stone, m$^2$/kg
- CPD$_{\text{max}}$: maximum compressed packing density of the three-graded stone, kg/m$^3$
- D$_{\text{A-stone}}$: average particle size of stone in per cubic metre of concrete, mm
- D$_{\text{I-sand}}$: diameter of the I-th sand particle size, mm
- D$_I$-stone': average diameter of the I-th stone particle size, mm
- D$_{\text{max}}$: maximum slump flow of concrete, mm
- D$_{\text{tar}}$: targeted concrete slump flow
- ITZ: interface transfer zone

| Binder content (kg/m$^3$) | S/A | S$_{\text{stone}}$ (m$^2$) | Comment |
|---------------------------|-----|--------------------------|---------|
| 420                       | 0.37| 253.6                    |         |
| 420                       | 0.38| 249.6                    | PD$_{\text{min}}$ |
| 420                       | 0.39| 245.5                    |         |
| 420                       | 0.41| 257.9                    | CPD$_{\text{max}}$ |
| 411                       | 0.37| 255.3                    |         |
| 411                       | 0.38| 251.2                    | PD$_{\text{min}}$ |
| 411                       | 0.39| 247.2                    |         |
| 411                       | 0.41| 259.6                    | CPD$_{\text{max}}$ |
| 380                       | 0.36| 265.3                    |         |
| 380                       | 0.38| 257.0                    | PD$_{\text{min}}$ |
| 380                       | 0.39| 252.8                    |         |
| 380                       | 0.41| 265.6                    | CPD$_{\text{max}}$ |
| 399                       | 0.42| 237.1                    |         |
| 399                       | 0.43| 233.0                    | PD$_{\text{min}}$ |
| 399                       | 0.44| 228.9                    |         |
| 399                       | 0.41| 261.9                    | CPD$_{\text{max}}$ |
Appendix

A. Calculation of $S_{\text{stone}}$ and $S_{\text{sand}}$ and $D_{A-\text{stone}}$

Assuming that all of the stone particles are spherical, the apparent density of the stones is $\rho_{\text{stone}}$, the mass of a stone particle with a diameter of $D_{i-\text{stone}}$ is $m_{i-\text{sin}}$, the surface area of this stone particle is $S_{i-\text{sin}}$, and the volume of this stone is $V_{i-\text{sin}}$ which can be expressed as follows [48]:

$$V_{i-\text{sin}} = \frac{m_{i-\text{sin}}}{\rho_{\text{stone}}} = \frac{4}{3} \pi \left( \frac{D_{i-\text{stone}}}{2} \right)^3 = \frac{1}{6} D_{i-\text{stone}} \cdot \pi (D_{i-\text{stone}})^2 = \frac{1}{6} D_{i-\text{stone}} \cdot S_{i-\text{sin}}.$$  \hspace{1cm} (A.1)

The $m_{i-\text{sin}}$ can be obtained from equation (A.1), expressed as follows (A.2):

$$m_{i-\text{sin}} = \frac{1}{6} D_{i-\text{stone}} \cdot S_{i-\text{sin}} \cdot \rho_{\text{stone}}.$$  \hspace{1cm} (A.2)

Assuming that the number of the stone particles with a diameter of $D_{i-\text{stone}}$ is $n$ in a cubic metre of concrete, the total mass of the stone particles with diameter of $D_{i-\text{stone}}$ is $m_{i-\text{stone}}$, and the surface area the stone particles is of $S_{i-\text{stone}}$, the specific surface area, $A_{i-\text{stone}}$ of the stone particles can be obtained from the following equation:

$$A_{i-\text{stone}} = \frac{S_{i-\text{stone}}}{m_{i-\text{stone}}}.$$  \hspace{1cm} (A.3)

Substituting equation (A.2) into equation (A.3), equation (A.4) is obtained:
\[ A'_{\text{stone}} = \frac{S_{\text{stone}}}{m_{\text{stone}}} = \frac{n \cdot S_{\text{stone}}}{n \cdot m_{\text{stone}}} = \frac{6 \cdot n \cdot S_{\text{stone}}}{D_{\text{stone}} \cdot \rho_{\text{stone}}} \]

Assuming the total mass of stone particles in a certain size range is \( M'_{\text{stone}} \), the average stone diameter of the \( I \)-th particle size is \( D_{\text{I-stone}} \), the mass is \( M_{\text{I-stone}} \), and the mass ratio to the total stone in the certain size range is \( K_I \). The specific surface area, \( A'_{\text{stone}} \), of the stone in a certain particle size range can be calculated by the following equation:

\[ A'_{\text{stone}} = \frac{\sum_I ((6000/D_{\text{I-stone}}) \rho_{\text{stone}}) M_{\text{I-stone}}}{M'_{\text{stone}}} = \frac{6000}{\rho_{\text{stone}}} \sum_I K_I \]  

For the three-graded stone, we assume that the mass ratios of large, middle, and small size stones to the total stones are \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \), respectively. Because the stone particle sizes may overlap in different stone size ranges, we assume that the mass ratios of stones with \( D_{\text{I-stone}} \) to the large size stone, middle size stone, and small size stone are \( K_{I_1} \), \( K_{I_2} \) and \( K_{I_3} \), respectively. The specific surface area, \( A'_{\text{stone}} \), of the stone in the three-stone grinding system can be calculated by the following equation:

\[ A'_{\text{stone}} = \frac{6000}{\rho_{\text{stone}}} \sum_I \left( \frac{\alpha_1 \cdot K_{I_1} + \alpha_2 \cdot K_{I_2} + \alpha_3 \cdot K_{I_3}}{D_{\text{I-stone}}} \right) \]  

The specific surface area correction coefficient, \( \beta \), is used to correct the error caused by irregularly shaped stones (needle-like, etc.) that are present in the stone system. The specific surface area correction coefficient of the aggregate is recommended to be 1.00 – 1.15 [43], and \( \beta = 1.10 \) for the three-graded stone that was used in this study. The specific surface area, \( A'_{\text{stone}} \), of the stone can be expressed by the following equation:

\[ A_{\text{stone}} = \beta \cdot A'_{\text{stone}} \]  

Assuming the mass of stone per cubic metre of concrete is \( M_{\text{stone}} \), the surface area of stone per cubic metre of concrete can be expressed by the following equation:

\[ S_{\text{stone}} = A_{\text{stone}} \cdot M_{\text{stone}} \]  

Substituting equation (A.5) and equation (A.6) into equation (A.7), equation (A.9) can be obtained:

\[ S_{\text{stone}} = \beta \frac{6000}{\rho_{\text{stone}}} \sum_I \left( \frac{\alpha_1 \cdot K_{I_1} + \alpha_2 \cdot K_{I_2} + \alpha_3 \cdot K_{I_3}}{D_{\text{I-stone}}} \right) \cdot M_{\text{stone}} \]  

Similarly, assuming that the sand is sphere, for the sand within a certain particle size range, the apparent density is \( \rho_{\text{sand}} \) and the mass ratio of the sand with \( D_{\text{I-sand}} \) to total sand mass is \( K_I \). Then, the specific surface area of sand, \( A_{\text{sand}} \), can be expressed by the following equation:

\[ A_{\text{sand}} = 6000 \frac{\sum_I K_I \cdot M_{\text{I-sand}}}{\rho_{\text{sand}} \sum_I D_{\text{I-sand}}} \]  

Assuming the mass of sand per cubic metre of concrete is \( M_{\text{sand}} \), the surface area of sand per cubic metre of concrete can be expressed by the following equation:

\[ S_{\text{sand}} = A_{\text{sand}} \cdot M_{\text{sand}} \]  

Substituting equation (A.10) into equation (A.11), equation (A.12) can be obtained:

\[ S_{\text{sand}} = \frac{6000}{\rho_{\text{sand}}} \sum_I K_I \cdot M_{\text{I-sand}} \]  

At the same time, the average diameters of the stone \( (D_{\text{A-stone}}) \) in a certain size range of the three-graded stone can also be obtained from the above assumptions, which can be expressed as:

\[ A'_{\text{stone}} = \pi \cdot D_{\text{A-stone}}^3 \]  

Substituting equation (A.6) into equation (A.13) and transforming it, \( D_{\text{A-stone}} \) can be obtained, as shown in the following equation:

\[ D_{\text{A-stone}} = \frac{6000}{\pi \rho_{\text{stone}}} \sum_I \left( \frac{\alpha_1 \cdot K_{I_1} + \alpha_2 \cdot K_{I_2} + \alpha_3 \cdot K_{I_3}}{D_{\text{I-stone}}} \right) \]  

From equations (A.9) and (A.12), the surface area of the three-graded stone \( (S_{\text{stone}}) \) and the surface area of the sand \( (S_{\text{sand}}) \) per cubic metre of concrete can be determined, respectively. The average particle size of the three-graded stone \( (D_{\text{A-stone}}) \) per cubic metre of concrete can be obtained by equation (A.14).

### B. Calculation of \( T_{\text{paste}} \) and \( T_{\text{mortar}} \)

Assuming the binder dosage in the concrete is \( M_{\text{binder}} \), and the water-to-binder ratio is \( W/B \), then the paste mass \( (M_{\text{paste}}) \) can be expressed by the following equation:

\[ M_{\text{paste}} = (1 + W/B) \times M_{\text{binder}} \]  

Assuming the apparent density of the paste is \( \rho_{\text{paste}} \) the paste volume \( (V_{\text{paste}}) \) in the concrete can be expressed by the following equation:

\[ V_{\text{paste}} = \frac{M_{\text{paste}}}{\rho_{\text{paste}}} \]  

Assuming the concrete bulk density is \( \rho_{\text{concrete}} \) the sand-to-aggregate ratio is \( S/A \), and the compressed packing density of the sand and stone are \( \rho'_{\text{sand}} \) and \( \rho'_{\text{stone}} \), respectively. The compressed packing volume of the sand and
stone in per cubic metre of concrete can be calculated by equations (B.3) and (B.4), respectively:

\[
V_{\text{paste}} = \frac{\left( \rho_{\text{concrete}} - M_{\text{paste}} \right) \times S/A}{\rho_{\text{sand}}}, \quad (B.3)
\]

\[
V_{\text{paste}} = \frac{\left( \rho_{\text{concrete}} - M_{\text{paste}} \right) \times 1 - S/A}{\rho_{\text{stone}}}. \quad (B.4)
\]

Assuming the void ratio of the sand is \( P_{\text{sand}} \), then the volume of mortar in per cubic metre of concrete \( (V_{\text{mortar}}) \) can be expressed by the following equation:

\[
V_{\text{mortar}} = V_{\text{sand}} + V_{\text{paste}} - V_{\text{sand}} \times P_{\text{sand}}. \quad (B.5)
\]

Assuming the compressed packing void ratio of stone is \( P_{\text{stone}} \), then the volumes of excess paste \( (V_{\text{epaste}}) \) and excess mortar \( (V_{\text{emortar}}) \) can be expressed by equation (B.6) and equation (B.7), respectively:

\[
V_{\text{epaste}} = V_{\text{paste}} - V_{\text{sand}} \times P_{\text{sand}}. \quad (B.6)
\]

\[
V_{\text{emortar}} = V_{\text{mortar}} - V_{\text{stone}} \times P_{\text{sand}}. \quad (B.7)
\]

Assuming the specific surface area of the sand and stone are \( A_{\text{sand}} \) and \( A_{\text{stone}} \), respectively, then \( A_{\text{sand}} \) and \( A_{\text{stone}} \) can be calculated based on equations (A.9) and (A.12), (see Appendix A for the details). The specific surface area correction coefficients of aggregate are 1.10 and 1.00 for stone and sand used in this study, respectively. The surface area of the sand \( (S_{\text{sand}}) \) and stone \( (S_{\text{stone}}) \) per cubic metre of concrete can be calculated by equations (B.8) and (B.9), respectively:

\[
V_{\text{paste}} = \left( \rho_{\text{concrete}} - M_{\text{paste}} \right) \times S/A \times A_{\text{sand}}. \quad (B.8)
\]

\[
V_{\text{paste}} = \left( \rho_{\text{concrete}} - M_{\text{paste}} \right) \times (1 - S/A) \times A_{\text{stone}}. \quad (B.9)
\]

The paste thickness \( (T_{\text{paste}}) \) and mortar thickness \( (T_{\text{mortar}}) \) can be expressed by equations (B.10) and (B.11), respectively:

\[
T_{\text{paste}} = \frac{V_{\text{epaste}}}{S_{\text{sand}} - S_{\text{stone}}}. \quad (B.10)
\]

\[
T_{\text{mortar}} = \frac{V_{\text{emortar}}}{S_{\text{stone}}}. \quad (B.11)
\]

Substituting equations (B.6) and (B.8) into equation (B.10), equation (B.12) can be obtained:

\[
T_{\text{paste}} = \frac{\left( (1 + W/B) \times M_{\text{binder}} / \rho_{\text{stone}} \right) - \left( \left( \rho_{\text{concrete}} - (1 + W/B) \right) \times S/A / \rho_{\text{sand}} \right) \times P_{\text{sand}}}{\left( \rho_{\text{concrete}} - (1 + W/B) \right) \times M_{\text{binder}} \times \left[ S/A \times A_{\text{sand}} + (1 - S/A) \times A_{\text{stone}} \right]}. \quad (B.12)
\]

Substituting equations (B.7) and (B.9) into equation (B.11), equation (B.13) can be obtained:

\[
T_{\text{mortar}} = \frac{\left( \rho_{\text{concrete}} - (1 + W/B) \times M_{\text{binder}} \right) \times \left[ (S/A \times (1 - P_{\text{sand}}) / \rho_{\text{sand}}) \times (1 - S/A) \times P_{\text{stone}} / P_{\text{paste}} \right] + \left( (1 + W/B) \times M_{\text{binder}} / \rho_{\text{paste}} \right)}{\left( \rho_{\text{concrete}} - (1 + W/B) \times M_{\text{binder}} \right) \times (1 - (S/A) \times A_{\text{stone}})}. \quad (B.13)
\]

Based on the above derivation process, \( T_{\text{paste}} \) and \( T_{\text{mortar}} \) can be obtained by equations (B.12) and (B.13), respectively.

**Data Availability**

All data are available in the manuscript.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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