HYDRODYNAMICAL SIMULATIONS OF THE Ly\textalpha\ FOREST: MODEL COMPARISONS

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ABSTRACT

We investigate the properties of the Ly\textalpha\ forest as predicted by numerical simulations for a range of currently viable cosmological models. This is done in order to understand the dependencies of the forest on cosmological parameters. Focusing on the redshift range from 2 to 4, we show that (1) most of the evolution in the distributions of optical depth, flux, and column density can be understood by simple scaling relations; (2) the shape of optical depth distribution is a sensitive probe of the amplitude of density fluctuations on scales of a few hundred kpc; and (3) the mean of the \( b \) distribution (a measure of the width of the absorption lines) is also very sensitive to fluctuations on these scales and decreases as they increase. We perform a preliminary comparison to observations, where available. A number of other properties are also examined, including the evolution in the number of lines, the two-point flux distribution, and the He \( \Pi \) opacity.

Subject headings: hydrodynamics — large-scale structure of universe — methods: numerical — quasars: absorption lines

1. INTRODUCTION

Several numerical simulations of the Ly\textalpha\ forest in cold dark matter (CDM)–dominated cosmologies have been performed in recent years and compared with observations (Cen et al. 1994; Zhang, Anninos, & Norman 1995; Hernquist et al. 1996; Miralda-Escudé et al. 1996; Zhang et al. 1997; Davé et al. 1997; Bond & Wadsley 1998; Zhang et al. 1998). Remarkably, all the simulations have been able to reproduce the measured neutral hydrogen column density distribution, the size of the absorbers (Charlton et al. 1997), and the line number evolution reasonably well, despite the differences in the cosmological models used: Cen et al. adopt a \( \Lambda \)CDM model; Zhang et al. investigate sCDM models with both an unbiased and a cluster scale normalization; and Hernquist et al. (1996) evolve an sCDM model with a cluster scale normalization. The distribution of Doppler parameters has fared somewhat less well: the predicted distribution peaks toward lower values than observed when the simulations are performed with adequate resolution (Bryan et al. 1999; Theuns, Leonard, & Efstathiou 1998). Nonetheless, the generally good agreement with observations of the Ly\textalpha\ forest suggests that the models are capturing the essential physical properties of the absorbers. This has prompted recent work by Croft et al. (1999) aimed at using flux statistics of the observational data to extract the fluctuation spectrum of the underlying cosmology. We are thus encouraged to investigate the possibility that differences in the statistical properties of the Ly\textalpha\ forest predicted by different cosmological models may provide a means of testing the models.

The objective of this paper is to compare the Ly\textalpha\ forest statistics derived from simulations in different cosmological models and to investigate what key properties of the cosmological models control a given statistic. The statistics predicted by simulations of the different models are calculated under idealized observational conditions in order to isolate what intrinsic differences may exist between the models themselves. This is a necessary first step toward a more definitive comparison with data that includes observational complications, such as noise and continuum fitting, specific to a given observational data set. Such a comparison using data from several observational groups will be presented in a forthcoming paper (A. Meiksin et al. 2000, in preparation). We do, however, make a preliminary comparison with observation here using the tabulated statistics of the Ly\textalpha\ forest as determined primarily by Kim et al. (1997) for several QSO lines of sight in order to test how well these models are doing.

We present results from 10 numerical simulations using five different background cosmological models, three of which are flat with no cosmological constant, one of which is open, and one of which is flat with a nonzero cosmological constant. For five of the simulations, which we will

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for the apparent high redshifts. The radiation field is normalized to the grid resolution and simulation box size for the statistics use these simulations to check numerical dependencies on optical depth. In §3 we investigate the model differences, power dependence, and redshift evolution in the raw opacity data as characterized by nonparametric statistics of the flux and optical depth. In §4 we present a line analysis of the spectra generated by the various simulations focusing on the column density distribution and line number evolution statistics. In §5 we discuss the Doppler b parameter distributions and related nonparametric statistics, and in §6 we present model predictions for He\textsubscript{II} absorption. We summarize our results in §7.

2. THE MODELS AND SIMULATIONS

All the model background spacetimes we consider are in the context of cold dark matter (CDM)-dominated cosmologies. We examine the following five models: a standard critical density flat CDM model (sCDM), a flat CDM model with a nonvanishing cosmological constant (ΛCDM), a topologically open CDM model (OCDM), the standard CDM model but with the power spectrum of the density perturbations tilted (tCDM) to match the normalization on large scales as determined from the COBE measurements of the cosmic microwave background (Bunn & White 1997), and a flat critical density mixed dark matter model with a hot component added to the CDM (CHDM). There are several important and well-established astrophysical measurements that constrain the various combinations of cosmological parameters. The parameters for each model, which we list in Table 1, have been determined to provide good consistency with these observations. For example, the combination \( \Omega_\Lambda h^2 \) is restricted by big bang nucleosynthesis constraints and the measured abundance of primordial deuterium to lie in the range 0.015–0.025 (Copi, Schramm, & Turner 1995; Burles & Tytler 1998). In addition, because the H\textsc{i} column density scales approximately as \( (\Omega_\Lambda h^2)^2 \) for a fixed UV radiation intensity, we choose \( \Omega_\Lambda \) and \( h \) so that \( \Omega_\Lambda h^2 \) is the same for three of the models: sCDM, ΛCDM and OCDM. The fluctuation normalization in a sphere of 8 \( h^{-1} \) Mpc is defined to match observations of the number density of galaxy clusters (White, Efstathiou, & Frenk 1993; Bond & Myers 1996) in all the models. In addition, a tilt has been applied to the CDM power spectrum in the tCDM model in order to match approximately the amplitude of the CMB quadrupole as measured by COBE (Bunn & White 1997). The cosmological constant in the ΛCDM case is consistent with the upper limit (\( \Omega_\Lambda < 0.7 \)) of Maoz & Rix (1993) and the best-fit parameters of Ostriker & Steinhardt (1995). One of the major problems with the sCDM model is its difficulty in matching observations of the large-scale structures in the universe. Since the standard CDM model is historically one of the most studied models, however, we use it as our canonical model to which the perhaps more viable additional models considered here may be compared and through which we investigate the dependence of the Ly\textsubscript{a} statistics on the fluctuation power spectrum. We refer the reader to Zhang et al. (1995, 1997, 1998) for further details and results from our previous sCDM simulations.

The initial data were generated using COSMICS (Bertschinger 1995) with the BBKS transfer function (Bardeen et al. 1986) to compute the starting redshifts and the initial particle positions and velocity perturbations appropriate for all models except CHDM. We used CMBFAST (Seljak & Zaldarriaga 1996) to solve the linearized Boltzmann equations to set the initial conditions for CHDM. For the comoving box size adopted (9.6 Mpc) and the corresponding comoving grid cell size (37.5 kpc in our high-resolution runs), the relevant wavenumber domain of the simulations at \( z = 0 \) is 168 > \( k > 0.65 \) Mpc\(^{-1} \), where \( k = 2\pi/\ell \) and \( \ell \) is the length scale. Over this domain, the sCDM, ΛCDM, and OCDM models all have a similar power distribution. The tCDM and CHDM models, on the other hand, have an overall lower normalization (see Table 1) in addition to a steeper slope that drops slightly more sharply than the other models over the smaller scales. In

| Model   | \( \Omega_0 \) | \( \Omega_\Lambda \) | \( \Omega_b \) | \( \Omega_\Lambda h^2 \) | \( \sigma_{8h} \) | \( \Delta x \) (kpc) | \( \Omega_\Lambda h^2 \) | \( \sigma_{8a} \) |
|---------|---------------|-----------------|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| sCDM    | 1             | 0               | 0.06          | 0.5             | 0.5             | 0.7             | 37.5            | 0.015           | 1.89            |
| ΛCDM    | 0.4           | 0.6             | -0.4          | 0.0355          | 0.65            | 1.0             | 37.5            | 0.015           | 2.03            |
| OCDM    | 0.4           | 0.4             | 0.2           | 0.0355          | 0.65            | 1.0             | 37.5            | 0.015           | 2.50            |
| tCDM    | 1             | 0.5             | 0.07          | 0.6             | 0.81            | 0.5             | 37.5            | 0.025           | 1.09            |
| CHDM    | 1             | 0               | 0.07          | 0.6             | 0.98            | 0.7             | 75              | 0.025           | 1.14            |

Note—\( \Omega_0 \) is the total density parameter, \( \Omega_\Lambda = \Lambda/3H_0^2 \) is the cosmological constant density parameter, \( \rho_0 = \Omega_0/2 - \Omega_\Lambda \) is the deceleration parameter, \( \Omega_b \) is the baryonic mass fraction, \( h \) is the Hubble parameter, \( n \) is the slope of the primordial density perturbation power spectrum, \( \sigma_{8h} \) is the fluctuation normalization in a sphere of radius 8 \( h^{-1} \) Mpc, \( \Delta x \) is the comoving simulation spatial resolution in units of kpc, \( \Omega_\Lambda h^2 \) is the baryon density in physical units (independent of \( H_0 \)), and the last column is proportional to Gnedin’s 1998 measure of power at small scales. For CHDM the fraction of the energy density carried in neutrinos is 0.2. The sCDM, ΛCDM, and OCDM models were also simulated with the Hercules codes at lower spatial resolution (\( \Delta x = 75 \) kpc).

referring to as the model comparison study, the parameters of the cosmological models have been selected by their ability to match the local or low-redshift observations, although all of these models except the standard cold dark matter (sCDM) model are also consistent with COBE measurements of the cosmic microwave background. A tilted CDM model is further designed to match COBE constraints on the normalization of the power spectrum on large scales. In the five remaining simulations, we keep the underlying cosmology fixed (sCDM) while varying the normalization of the fluctuation power spectrum in order to clarify the dependence of the Ly\textsubscript{a} statistics on this parameter. We also use these simulations to check numerical dependencies on grid resolution and simulation box size for the statistics presented here. The radiation field is normalized to the absorption properties of the Ly\textsubscript{a} forest as measured at high redshifts.
to simulate the gasdynamics. Since nonequilibrium chemistry and cooling processes can be important, six particle species (H I, H II, He I, He II, He III, and the electron density) are followed with a substepped backward finite-difference technique (Abel et al. 1997; Anninos et al. 1997). This is the same nonequilibrium chemistry and cooling model used in our previous studies of the Ly\text{\textalpha} forest (Zhang et al. 1995, 1997, 1998; Charlton et al. 1997; Bryan et al. 1999). For sCDM, ΛCDM, OCDM, and tCDM we use 256\textsuperscript{3} grid cells in the simulation box to follow the evolution of 128\textsuperscript{3} dark matter particles. This results in a comoving spatial resolution of Δx = 37.5 kpc. For CHDM 128\textsuperscript{3} grid cells are used with 64\textsuperscript{3}(128\textsuperscript{3}) cold (hot) dark matter particles, respectively, resulting in a lower spatial resolution, Δx = 75 kpc, for this model. For the sCDM simulations with varying σ\textsubscript{b} we present simulations with both 128\textsuperscript{3} and 256\textsuperscript{3} grid cells resulting in both low and high spatial resolutions of 75 and 37.5 kpc, respectively. Although we showed in our previous work (Bryan et al. 1999) that by a simulation box length of 9.6 Mpc (comoving) the effects of missing large-scale power on the column density and b parameter distributions for redshifts z ≥ 2 were very small, we include one low spatial resolution sCDM simulation with 256\textsuperscript{3} grid cells and double the simulation box length (19.2 Mpc) to check the stability of the new nonparametric statistics considered in this paper under changes in simulation box size.

We have also simulated three of our models (sCDM, ΛCDM, and OCDM) with a different numerical code, Hercules (Anninos, Norman, & Clarke 1994; Anninos et al. 1997). Hercules is a nested grid code that utilizes a multiscale PM method for the dark matter, artificial viscosity methods for the baryonic fluid and the same nonequilibrium chemistry and cooling model as above. The simulations produced from this code use 128\textsuperscript{3} particles and 128\textsuperscript{3} cells for both the nested and parent grids. However, in order to derive a more representative sample for statistics, the results discussed in this paper are extracted from the parent grid only. Thus, these simulations are of lower spatial resolution than most of the Kronos simulations, although the dark matter mass resolution is the same. For statistics that are insensitive to spatial resolution, a comparison of the results of the two codes is useful to insulate that simulation results are robust against changes in numerical technique.

Synthetic spectra are generated along 300 (900) random lines of sight through the Kronos (Hercules) simulated volume using the method of Zhang et al. (1997) including the effects of peculiar velocity and thermal broadening of the gas. (We have verified that decreasing the sample size from 900 to 300 for the Kronos data does not affect the results except for a slight increase in the scatter of the line properties for the highest, optically thick column density systems, a regime in which our results become unreliable anyway because of the absence of radiative transfer in the code.) Since we are primarily concerned in this paper with a comparison of model predictions, we have not included noise or continuum fitting in the analysis. Furthermore the resolution of the spectra, 1.2 km \text{\textpersecond}, is the same for all the simulations, a value that is smaller than current observations. However, we have shown elsewhere (Bryan et al. 1999) that, as long as we restrict ourselves to high-quality observational data, the impact of not including these observational difficulties is small. In addition to analyzing the raw optical depth and flux distributions, line lists are
extracted from the data using a Voigt profile-fitting procedure. This is described in more detail elsewhere (Zhang et al. 1997), but we outline it briefly here. First, maxima in the optical depth distribution are identified as line centers. Then Voigt profiles are fit, using a nonlinear minimization, to the part of the spectrum which is above \( \tau_{HI} = 0.05 \) and between neighboring minima. This results in the same spectral threshold \( F_t = e^{-\tau} = 0.95 \) as the high-resolution Keck HIRES spectrometer. Each line of sight chosen produces a sample spectrum with on the order of 10–100 lines per redshift interval \( \Delta z = 0.1 \) depending on the redshift and cosmological model. The statistics of these line lists are discussed in §§ 4 and 5.

The amplitudes of the distributions found in the models cannot be used as a basis for comparing the models since they may be arbitrarily rescaled for any individual model using the ionization bias factor \( b_{ion} = \Omega_B / \Gamma \), where \( \Omega_B \) is the fraction of the critical density carried in baryons and \( \Gamma \) is the Haardt-Madau (1996) parameterization of the metagalactic UV ionizing background extracted from the observed distribution of quasars. It is important to normalize all the models consistently before comparing the shapes of any of the distributions. This may be done in a variety of ways. We do so by matching the mean \( H_I \) opacity in each simulation to the measured intergalactic \( H_I \) opacity at \( z = 3 \). In Zhang et al. (1997), we found that the opacity measurements of Steidel & Sargent (1987) and Zuo & Lin (1993) gave a mean \( H_I \) opacity at \( z = 3 \) of \( \bar{\tau}_I = 0.27–0.35 \), although values as much as 30%–60% larger have been claimed (Press, Rybicki, & Schneider 1993; Rauch et al. 1997). Because of the uncertainty in this measurement, we also require consistency with the number density of lines observed above a threshold of \( \log N_H = 13.5 \). Using the three quasars in Hu et al. (1995) for which lines in the full redshift range \( 3 < z < 3.1 \) are listed, we find a total of 61 lines for the three lines of sight in this redshift interval with \( \log N_{HI} > 13.5 \), for which the line lists should be complete. (An estimate based on using the available lines for all four QSO line lists in Hu et al. in the redshift interval \( 2.9 < z < 3.1 \) gave essentially the same line density.) Normalized to \( \bar{\tau}_I = 0.30 \), the CHDM, sCDM, ACDM, OCDM, and tCDM models predict, respectively, 60.8, 62.1, 62.7, 63.7, and 59.5 lines, in close agreement with the observed number. Normalizing to \( \bar{\tau}_I = 0.35 \), the respective numbers of predicted lines are 73.7, 74.3, 73.9, 75.2, and 72.8. While these are not badly inconsistent with the observed number, they are all fairly high. We normalize the spectra according to \( \bar{\tau}_I = 0.30 \) throughout this paper, noting that this value is still not well agreed upon. In Figure 2 we plot a related statistic, \( \tau_{eff} \) (Zhang et al. 1997) for the normalized spectra of our models and compare to recent data by Kirkman & Tytler (1997). After normalization all of our models are consistent with the data over the redshift range \( 2 \leq z \leq 4 \) considered by this paper.

3. DIRECT OPTICAL DEPTH AND FLUX MEASUREMENTS

Historically, Lyz absorption spectra have been analyzed in terms of the statistics of spectral line features and as such have been plagued with difficulties of the line-fitting procedure such as line identification and blending. Many of these difficulties become increasingly severe at higher redshifts, making the results of the analysis uncertain. It is thus natural to ask whether statistics dependent directly on the observed flux and optical depth without recourse to line fitting might be of use in describing the forest and discriminating among competing models. Statistics of this kind have recently been proposed by several authors (Miralda-Escudé et al. 1998; Rauch et al. 1997; Cen 1997). Since these non-parametric measures are also easier to relate theoretically to the physical state of the absorbing gas, we begin our discussion with them.

3.1. Optical Depth Probability Distribution Function

The optical depth \( \tau \) is related to the transmitted flux \( F \) by \( F = \exp(-\tau) \). We define the optical depth probability distribution \( dP/d\tau \) as the probability that a pixel will have optical depth between \( \tau \) and \( \tau + d\tau \). In Figure 3 we use spectra generated from the sCDM high-resolution simulation to show \( \tau dP/d\tau \) versus \( \tau \) for redshifts \( z = 2, 3, \) and 4 (top panel). Although the peak of the distribution decreases
and the distribution broadens slightly with decreasing redshift, the principal contributor to the redshift evolution seen in Figure 3 is the evolution of the optical depth $\tau$. Hui, Gnedin, & Zhang (1997) discuss in detail the dependence of $\tau$ on the distribution and properties of neutral hydrogen along the line of sight in an expanding universe. Since we would like to understand the redshift evolution of the optical depth in terms of simple scaling laws, we repeat some of their discussion here in order to isolate the key factors controlling this redshift evolution and clarify the scaling law assumptions. The optical depth is defined as

$$\tau(\nu) = \int_{x_{\nu}}^{x_{\text{ns}}} n_{\text{HI}} \sigma_a dx \left(1 + \frac{x}{1 + z}\right), \tag{2}$$

where $\nu_a$ is the observed frequency, $n_{\text{HI}}$ is the number density of neutral hydrogen, $z$ is the redshift of the absorbing gas, $\sigma_a$ is the absorption cross section for Ly$\alpha$, and the integral is over the line of sight between the quasar ($x_a$) and the observer ($x_b$) in comoving coordinates. In practice the form of the Ly$\alpha$ absorption cross section limits the integration range per absorber to a small portion of the line of sight. It is thus useful to make a change of variable to velocity coordinates $u$ about some characteristic average redshift $\bar{z}$ in the problem. For example, for simulated data the redshift $\bar{z}$ might be a given output redshift for the simulation. The observed frequency $\nu_a$ and the frequency $\nu$ of the radiation in the absorber rest frame are then related by

$$\nu = \nu_a (1 + \bar{z}) (1 + u/c), \tag{3}$$

where

$$u = \frac{H(\bar{z}) (x - \bar{x})}{1 + \bar{z}} + v_{\text{pec}}(x), \tag{4}$$

$\bar{x}$ is the comoving position along the line of sight whose redshift is exactly $\bar{z}$, $v_{\text{pec}}$ is the physical velocity of the gas, and $H(\bar{z})$ is the Hubble parameter defined by

$$H(\bar{z}) = H_0 \sqrt{\Omega_m (1 - \bar{z})^3 + (1 - \Omega_m - \Omega_\Lambda) (1 + \bar{z})^2 + \Omega_\Lambda}. \tag{5}$$

The first term in equation (4) represents the contribution of the residual Hubble flow about the mean, while the second term is due to the physical bulk flow of the gas. We assume $u/c \ll 1$ and neglect contributions from turbulent flows since they would be unlikely in the low column density regions we are considering. Under this change of variable the Ly$\alpha$ cross section becomes

$$\sigma_a = \frac{\sigma_{a0} c}{b \sqrt{\pi}} e^{-u_0^2/b^2}, \tag{6}$$

where $\sigma_{a0} = 4.5 \times 10^{-18} \text{ cm}^2$ sets the scale of the absorption cross section in terms of fundamental constants, $u_0$ is the velocity $u$ for which the frequency $\nu$ in the rest frame of the absorbing gas is equal to the Ly$\alpha$ frequency $\nu_a$, and $b = (2k_B T/m_p)^{1/2}$ is the thermal width. For absorption lines of neutral hydrogen with column densities $N_{\text{HI}} < 10^{17} \text{ cm}^{-2}$, the thermal profile dominates the cross section, so we neglect the contribution of the natural line width to $\sigma_a$. The optical depth $\tau$ can now be written as

$$\tau = \frac{\sigma_{a0} c}{\sqrt{\pi}} \sum_{\text{streams}} \int_{x_{\nu}}^{x_{\text{ns}}} \frac{n_{\text{HI}}}{b(1 + \bar{z})} du \left| \frac{du}{dx} \right|^{-1} e^{-(u-u_0)^2/b^2} du. \tag{7}$$

The sum over streams represents the possibility that a given velocity $u$ corresponds to more than one position $x$. Although the integration formally runs over the full line of sight from quasar to observer, the Gaussian form for the cross section effectively limits the $u$ integration to a narrow range around $u_0$ (thus justifying our replacement of $z$ everywhere by $\bar{z}$). To simplify notation we drop the bar, letting $z$ represent $\bar{z}$ in what follows.

We assume that the number density of hydrogen traces the baryon gas density well. (There has been little metal production in these low-density regions, and there is no interaction that would cause the helium and hydrogen to separate.) Thus the number density of neutral hydrogen is $n_{\text{HI}} = \rho_b X_{\text{HI}}$, where $X_{\text{HI}}$ is the neutral fraction and $\rho_b$ the gas density. In ionization equilibrium (which is well satisfied except for the period of initial reionization) the neutral fraction of hydrogen is $X_{\text{HI}} \propto \rho_b T^{-0.7}$ such that the number density of neutral hydrogen (relevant to Ly$\alpha$ absorption) scales as

$$n_{\text{HI}} \propto (\Omega_b h^2)^2 \Gamma^{-1}(1 + z)(1 + \delta_b)^2 T^{-0.7}, \tag{8}$$

where $\delta_b$ is the baryon overdensity. Studies (Hui & Gnedin 1997; Weinberg et al. 1996) of the equation of state for the gas find that for unshocked gas at low to moderate baryon overdensities ($\delta_b \leq 5$) the equation of state is well fitted by a power law:

$$T \propto (1 + z)^{1.7}(1 + \delta_b)^{-1}. \tag{9}$$

Thus

$$n_{\text{HI}} \propto (\Omega_b h^2)^2 \Gamma^{-1}(1 + z)^{4.8}(1 + \delta_b)^{2.7 - 0.7 \gamma}. \tag{10}$$

For a uniform radiation field and reionization that occurs before $z = 5$, as is the case in our simulations, $\gamma \approx 1.4$. This is in agreement with the value $\gamma \approx 1.5$ found by Zhang et al. (1998) for clouds with column densities in the range $12.5 < \log N_{\text{HI}} < 14.5$. Furthermore, the assumption that most of the optical depth arises from low column density absorbers, large structures whose overdensities and peculiar velocities are slowly varying compared to the thermal profiles, means that multiple streaming is rare, the sum over streams in equation (7) can be dropped, and $|du/dx| \approx H/(1 + z)$. We then integrate over the thermal profile to obtain (Croft et al. 1997a)

$$\tau \propto \frac{c \sigma_{a0} (\Omega_b h^2)^2}{\Gamma(z) H} (1 + z)^{4.8}(1 + \delta_b)^{1.7}. \tag{11}$$

Note that in this limit $\tau$ need no longer have a thermal profile about its maximum (Hui et al. 1997). If $\delta_b$ is evolving slowly over this redshift range, $\tau$ should scale as

$$\tau \propto \frac{(1 + z)^{4.8}}{\Gamma(z) H}. \tag{12}$$

In the middle panel of Figure 3 we use this simple scaling law to rescale the $z = 2$ and $z = 4$ SCDM distributions from the top panel to $z = 3$, the redshift at which all the models are normalized. We do this in order to test how well the simulations obey this simple scaling relation: if they followed it exactly then all three curves would overlap. Most, but not all, of the redshift evolution of this distribution is accounted for by the scaling of $\tau$ given in equation (12). Since the evolution of the metagalactic UV radiation field $\Gamma$ is relatively slight over this redshift range, we are left with the remarkable conclusion that most of the evolution of the
Lyα forest is a direct consequence of the universal expansion.
The direct numerical results of Zhang et al. (1998) support
this conclusion. If we include the evolution of the baryon
overdensity, as shown in Figure 4 for the sCDM simulation,
and shift the overdensity distribution until the peaks
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resolution sCDM model with
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resolution and simulation box length. From this we can see
that the optical depth PDF at a given redshift is insensitive
to the spatial resolution of the simulation. Furthermore,
doubling the simulation box length causes only a slight
narrowing of the distribution at extreme values of \( \tau \) that
are difficult to observe. In the top panel of Figure 5 we show
\( \tau \, dP/d\tau \) for three sCDM simulations with varying power at
fixed spatial resolution \( \Delta x = 75 \) kpc. The shape of the
optical depth PDF is strongly dependent on the amount of
small-scale power present. Models with less power at these
scales produce narrow, sharply peaked distributions. As the
power increases, the distribution flattens and broadens. In the
middle panel of Figure 5 we show \( \tau \, dP/d\tau \) versus \( \tau \) at
redshift \( z = 3 \) for the simulations in the model comparison
study. These distributions again display a clear dependence
dependence of the optical depth, as shown in equation (11) for
the optical depth distributions predicts additional scaling factors of \( \approx 1.64 \) (0.77) at \( z = 2 \) (4), respectively, for \( \tau \) values that bring the distributions (shown in the bottom
panel of Fig. 3) into close agreement. The remaining small
differences, the slight broadening of the distribution and a
reduction in its peak amplitude with decreasing redshift,
most probably reflect the fact that the shape of the baryon
overdensity distribution is also evolving slowly with \( z \).

In the bottom panel of Figure 5 we show \( \tau \, dP/d\tau \) versus \( \tau \)
at \( z = 3 \) for the sCDM \( \sigma_8 = 0.7 \) model with varying spatial
resolution and simulation box length. From this we can see
that the optical depth PDF at a given redshift is insensitive
to the spatial resolution of the simulation. Furthermore,
doubling the simulation box length causes only a slight
narrowing of the distribution at extreme values of \( \tau \) that
are difficult to observe. In the top panel of Figure 5 we show
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most probably reflect the fact that the shape of the baryon
overdensity distribution is also evolving slowly with \( z \).

We quantify this relation between the shape of the \( \tau \) dis-
tribution and the amplitude of the power spectrum by

\[
\tau \frac{dP}{d\tau} \propto e^{-\left(\ln \tau - \ln \tau_0\right)^2/2\sigma_\tau^2}.
\]

(13)

Although this does not fit the profiles in Figure 5 in detail, it
does provide an adequate description as long as we restrict
the range of optical depths used in the fitting. Here we adopt
0.02 < \( \tau < 4 \), corresponding roughly to the observable
range. A different range or a different fitting function changes the
details, but not the nature of our result. In Figure 6, we show the correlation between \( \sigma \), a measure of the width of the distribution, and \( \sigma_{34} \), the amplitude of the
linear power spectrum on small scales as defined in equa-
tion (1). The strength of the correlation is striking. The low
scatter around the power-law relation shown in this figure
bolsters our claim that the shape of the \( \tau \) distribution func-
tion is insensitive to other cosmological parameters. To give
an idea of the uncertainty in each point, we fit both the high-
and low-resolution simulations for the sCDM \( \sigma_8 = 0.3 \) and \( \sigma_8 = 0.7 \) models. In both of these cases \( \sigma \) differs by
less than 10%.

3.2. Flux Probability Distribution

Although the optical depth PDF is easier to model theo-
retically, the flux PDF (where \( dP/dF \) is the probability that
a pixel will have transmitted flux between \( F \) and \( F + dF \)) is
closer to actual observation. The top panel of Figure 7 shows
the flux probability distribution functions for the high spatial
resolution sCDM model with \( \sigma_8 = 0.7 \) at \( z = 2, 3, \) and 4. The bottom panel of Figure 7 shows the prediction of the simple \( \tau \) scaling given in equation (12) applied to the
flux and these same flux probability distributions. Again we
attempt to rescale the $z = 4$ and $z = 2$ distributions to $z = 3$ in order to test the scaling. This results in a highly nonlinear mapping of the flux and the flux PDF from $z = j$ to $z = 3$ given by $dP_j/dF \rightarrow \eta F^{1-\eta} dP_j/dF$ and $F_j \rightarrow F_j^{1/\eta}$ for $\tau_j \rightarrow \tau_j/\eta$, where $\eta = 0.356$ (6.511) for $j = 2$ (4), respectively. While the shapes of the distributions in the top panel appear quite different, much of the $z$ evolution of the flux probability distributions is explained by this simple scaling, the remainder representing mostly the effect of the evolution of the baryon density in the cosmological model. We do not plot the scaled distribution for $z = 4$ below the scaled flux of 0.5 because this already corresponds to an unscaled flux of 0.015, close to saturation and most likely noise dominated in the observations.

The flux PDF depends only weakly on simulation grid resolution (Bryan et al. 1999) and box size. As shown in the bottom panel of Figure 8, the distributions for the sCDM $\sigma_8 = 0.7$ model at $z = 3$ are stable under changes in spatial resolution and simulation box length. At $z = 2$ small differences arise for low transmitted flux $F < 0.3$. Increasing the spatial resolution increases the flux PDF slightly for low transmitted flux, while increasing the simulation length depresses the distribution slightly in this range. However, even at $z = 2$ the distributions are nearly identical for transmitted flux $F > 0.4$, where model differences are greatest. The shape of the flux PDF is strongly dependent on the power spectrum of the underlying cosmology. In the top panel of Figure 8 we show the flux probability distributions in the sCDM model (spatial resolution $\Delta x = 37.5$ kpc) for cluster scale normalizations $\sigma_8 = 0.3$ and 0.7. The dependence on the normalization of the power spectrum is clear. The number of pixels found with flux in the central flux range $0.3 < F < 0.9$ is greater for models with lower power ($\sigma_8 = 0.3$), while the number of pixels with flux in the low- ($F < 0.3$) and high- ($F > 0.9$) flux ends of the distribution are less than for models with greater power ($\sigma_8 = 0.7$). This is in qualitative agreement with Croft et al. (1997b). We note, however, that our result (using the Kronos code) does not require any smoothing of the simulations as was the case for their TreeSPH simulations. In the middle panel of Figure 8 we present the $z = 3$ flux PDFs for the five models of the model comparison study. Models with lower power at small scales (tCDM and CHDM) have a larger flux PDF for $0.3 < F < 0.9$ than sCDM and LCDM, while the low-density model (OCDM) with the highest spectral power at these scales has the smallest flux PDF in this range, as
expected. Furthermore, the differences between models can be substantial. For example, at $F = 0.6$ the OCDM results lie 10% below the ΛCDM model result, while the CHDM result lies above the ΛCDM result by about a factor of 1.4. We remind the reader that the mean of the distribution has been fixed to match observations. Thus, this statistic should be useful to constrain competing models.

### 3.3. Fraction of High Lyα Opacity

Another statistic that has been suggested as possibly useful for discriminating between models is the fraction of a quasar spectrum with Lyα optical depth greater than a specified value $\tau_0$, i.e., the cumulative distribution in optical depth. Small differences in the amplitude normalization of the primordial power spectrum may be enhanced in the cumulative opacity data (Cen 1997).

Figure 9 shows the linear correlation between the opacity at line center and the column density of absorption features in the sCDM model, ranging from the optically thin to thick at the Lyman edge. In general, $N_{HI} \propto b \tau_0$, where $N_{HI}$ is the column density, $b$ is the Gaussian line width, and $\tau_0$ is the optical depth at line center (Spitzer 1978; Zhang et al. 1997). Since there is (at most) only a weak correlation between $N_{HI}$ and $b$, the above relation requires that $N_{HI} \propto \tau_0$. Figure 9 shows that this nearly unbroken relation $\tau_0 \propto N_{HI}$, exists down to the incompleteness density of $\sim 10^{12}$ cm$^{-2}$. The lower bound on the opacity ($\tau_0 > 0.05$) is set by the transmission or spectral threshold $F_\lambda = e^{-\tau} = 0.95$ used in the line identification procedure. Using Figure 9 as a guide, we investigate the cumulative opacity distribution with the following minimum opacity thresholds: $\tau > 0.1$, 1, and 7, which, if associated with the line centers, would correspond roughly to column densities of $\log N_{HI} = 12.5$, 13.5, and 14.5, respectively. The distributions $P(\tau > \tau_0)$ for the above minimum opacity thresholds are plotted in the top panel of Figure 10 at redshifts $z = 2$, 3, and 4 for the models in the model comparison study. In comparing groups with the same $\tau_0$, the smaller threshold curves are more highly clustered and less sensitive to the background cosmological model parameters. This is especially evident in the top panel of Figure 11, where we show the cumulative distributions of the optical depth at redshift $z = 3$ for these models.

Unfortunately the cumulative optical depth distributions suffer from two problems at the high optical depths where the statistics are most sensitive. First, these cumulative statistics enhance not only differences in the optical depth due to differences in the cosmological models but also small differences due to numerical effects. As shown in the bottom panel of Figure 10, increased simulation spatial resolution tends to increase the cumulative opacity distribution at optical depth thresholds $\tau_0 > 1$ and redshifts $z < 3$. In the bottom panel of Figure 11 we see that at $z = 3$ both simulations spatial resolution and box size effects become signifi-

---

**Fig. 9.** Scatter plot of the line center opacity as a function of column density for the sCDM model run with the Hercules code. Also shown (solid line) is the average line center opacity in the 24 density bins of width $\Delta \log N_{HI} = 0.24$ cm$^{-2}$. The cutoff in opacity at $\tau_0 = 0.05$ is due to the spectral threshold $e^{-\tau} = 0.95$ used in identifying the absorption features.

**Fig. 10.** Top: Fraction of pixels greater than three threshold optical depths, $\tau_0 = 0.1$ (circles), $\tau_0 = 1.0$ (squares), and $\tau_0 = 7.0$ (triangles), plotted against redshift for models in the model comparison study. The curves for tCDM and CHDM are indistinguishable for the lowest two thresholds. Bottom: The same statistic for sCDM $\sigma_8 = 0.7$ with varying simulation box size and spatial resolution.

**Fig. 11.** Top: Fraction of pixels exceeding a threshold optical depth $\tau_0$ at $z = 3$ for models in the model comparison study. The curves for tCDM and CHDM are indistinguishable. Bottom: The same statistic for sCDM $\sigma_8 = 0.7$ models with varying simulation box size and spatial resolution.
cant for $\tau_0 > 2$. Second, observations suffer from larger statistical errors at the high thresholds where the cumulative statistics become most sensitive because there are fewer pixels in the data at high optical depth. Thus, while regions in these distributions exist where the differences between models are still much greater than differences due to numerical effects, these statistics must be used with care.

4. LINE PARAMETER STATISTICS

In this section we present a line analysis of the spectra generated from the various model simulations. We compare and contrast the cosmological models based on the column density distribution and the evolution of line number.

4.1. H I Column Density Distribution

One of the most robust line statistics used in the analysis of the Ly$\alpha$ forest is the H I column density distribution, which is well converged by simulation box sizes of 9.6 Mpc and is insensitive to changes in the simulation grid resolution or treatment of gas hydrodynamics (Bryan et al. 1999; Zhang et al. 1997). The H I column density distribution, defined to be $N_{\text{HI}} = \int n_{\text{HI}}(1 + z) \, dz$, is closely related to the optical depth $\tau$ through the dependence of each on the number density of neutral hydrogen. Thus, using equation (10) and the approximations that led to equation (11), we expect the H I column density to scale as

$$N_{\text{HI}} \propto \frac{(\Omega_b h^2)^2}{(1 + z)^{4.8}} (1 + \delta_b)^{1.7} \, du.$$  

In the top panel of Figure 12 we show the raw (uncut) H I column density distribution for the high-resolution sCDM model for redshifts $z = 2$, 3, and 4. In the bottom panel of Figure 12 we see that the column density scaling

$$N_{\text{HI}} \propto \frac{(\Omega_b h^2)^2}{(1 + z)^{4.8}},$$  

the same relation as the naive scaling relation given in equation (12) for the optical depth $\tau$, accounts for the redshift evolution of the column density distribution amazingly well. This demonstrates that the column density, an integrated quantity, is much less sensitive than the optical depth distributions to the redshift evolution of the gas overdensity within an absorbing structure. The differences seen in the low column density end of the distributions, particularly for $z = 4$, may be a result of the simulation spatial resolution (Bryan et al. 1999). Although the simulations become less reliable for $N_{\text{HI}} > 10^{16}$ owing to the neglect of optical depth effects, the differences observed in the high column density end may still be a reflection of the evolution of the gas over density between redshifts $z = 3$ and $z = 2$ (see Fig. 4). In Figure 13 (top) we explore the dependence of this distribution at a given redshift ($z = 3$) on the power spectrum of the underlying cosmology. We find qualitative agreement with semianalytic arguments (Gnedin 1998; Hui et al. 1997) in that models with less power on small scales (such as sCDM with $\sigma_8 = 0.3$ and $\sigma_{34} = 0.812$) have H I column distributions with significantly steeper slopes than models (such as sCDM with $\sigma_8 = 0.7$ and $\sigma_{34} = 1.89$) with more power at these scales. However, as we discuss in more detail below and in Table 2, quantitative agreement between the simulations and the predictions of these semianalytic arguments seems more difficult to achieve.

In Figure 13 (bottom), we show the H I column density distribution at redshift $z = 3$ for (Kronos) simulated spectra in the model comparison study and compare the simulated data with data from Kirkman & Tytler (1997) and the fits provided by Kim et al. (1997). The distributions are conventionally quantified by fitting them to power laws, $dN/dN_{\text{HI}} \propto N_{\text{HI}}^p$. We use the same sets of column density cuts on the simulated data in the model comparison study as Kim et al. in order to expedite comparison with the data and use a direct unweighted least-squares fit (all quoted errors are $2\sigma$) to extract the slope $p$ from the simulated data. Our results are summarized in Table 2. We find again the expected dependence on the fluctuation power spectrum. For the column density range $13.7 < \log N_{\text{HI}} < 14.3$ (given by the column labeled $p_8$) the most shallow slope is

![Fig. 12.—Top: Uncut H I column density distributions for the high-resolution sCDM $\sigma_8 = 0.7$ model for redshifts $z = 2$, 3, and 4. Bottom: The same H I column density distributions with $N_{\text{HI}}$ scaled according to eq. (15) to $z = 3$.](image1)

![Fig. 13.—Top: H I column density distribution for high spatial resolution ($\Delta x = 37.5$ kpc) sCDM models with $\sigma_8 = 0.3$ (dashed) and $\sigma_8 = 0.7$ (solid). Bottom: H I column density distributions at redshift $z = 3$ for (Kronos) simulated spectra in the model comparison study. Observational data are from Kirkman & Tytler (1997) and Kim et al. (1997).](image2)
for OCDM, the low matter density model with $\sigma_{24} = 2.50$ while CHDM and tCDM with $\sigma_{24} = 1.14$ and 1.09, respectively, give the steepest distributions (see Table 1). The predicted column density distributions generally also steepen with time (decreasing redshift). Kim et al. find $\beta = 1.46 \pm 0.07$ (2 $\sigma$) for this column density range at $z = 2.85$. This is formally inconsistent with all the models at the $3 \sigma$ level except for OCDM, although it is marginally consistent with sCDM and $\Lambda$CDM.

Results for the column density range $12.8 < \log N_{HI} < 14.3$ in Table 2 are shown in the column labeled $\beta_{ij}$. The average distributions are generally shallower when extended to lower column densities, showing that the distributions are curved. Kim et al. similarly find a shallower distribution over this column density range with their results for lines at $\langle z \rangle = 2.31$, $\langle z \rangle = 2.85$, and $\langle z \rangle = 3.35$ shown in the last row of Table 2. The result at $\langle z \rangle = 2.31$ is inconsistent with all of the simulation results at $z = 2$, but note that the quoted uncertainty in the observation is 8 times smaller than at $\langle z \rangle = 2.85$, despite comparable numbers of absorbers. By contrast, the result at $\langle z \rangle = 2.85$ is formally consistent at the $3 \sigma$ level with all the models. At $\langle z \rangle = 3.35$, Kim et al. find $\beta = 1.59 \pm 0.13$ (2 $\sigma$). This result is consistent with the simulation results for the OCMD model and marginally consistent (at the $3 \sigma$ level) for the sCDM and $\Lambda$CDM models. The observational data also suggest a weak steepening of the distribution with increasing redshift, contrary to our findings. However, these discrepancies might be due to instrumental effects since lower redshift ($z < 2.7$) HIRES data are generally from somewhat lower signal-to-noise ratio spectra.

We may compare the simulation results with the semi-analytic predictions of Hui et al. (1997) to understand the trend of changing steepness with power spectrum. We provide the predicted values of $\beta$ according to the prescription of Hui et al. in Table 2. We assume $T \propto P_B^{0.5}$, as found by Zhang et al. (1998) for this column density range in an sCDM simulation. The uncertainty in the $T-P_B$ relation introduces only a 10% uncertainty in the prediction for $\beta - 1$, so it seems reasonable to retain it for the other models as well for this purpose. The predicted values of $\beta$ for the sCDM, tCDM, and CHDM models at $z = 3$ match the simulation values to within 1 $\sigma$, in agreement with the comparison in Hui et al. with one of our earlier sCDM models. However, the predictions for OCDM and $\Lambda$CDM at $z = 3$ are in disagreement with the semianalytic arguments giving too steep a slope. The predictions do less well for all models at $z = 2$. In particular, the simulation results show a steepening of the column density distribution toward decreasing redshifts for all the models, opposite to the predicted trend.

Over the wider column density range $10^{12.8} < N_{HI} < 10^{14}$ (summarized in the column labeled $\beta_{ij}$ in Table 2), we see that the average distributions continue to steepen toward higher column densities. Kim et al. obtain $\beta = 1.46$ for this column density range at $\langle z \rangle = 2.85$, with a steepening to $\beta = 1.55$ at $\langle z \rangle = 3.7$. The results for the tCDM and CHDM models ($1.92 \pm 0.06$ and $1.92 \pm 0.06$ at $z = 3$, respectively) are substantially steeper than these values. Because the column density distribution is curved, i.e., deviates from a pure power law at the low column density end, the distribution is better fitted by a double power law with a “break” at column densities $N_{HI} = 10^{14}$ (Carswell et al. 1987). This softening of the slope at $N_{HI} \sim 10^{14}$ probably reflects a change in the physical character of the absorbers from low-density structures evolving primarily with the universal expansion to structures undergoing gravitational collapse (Bryan et al. 1999). The results of a double-power-law fit to the column density distributions are given in the last two columns of Table 2, where $\beta_1$ is the slope of the column density distribution for lines at the low column density end ($10^{12.8} < N_{HI} < 10^{14}$) and $\beta_2$ is the slope of the column density distribution for lines at the high end ($10^{14} < N_{HI} < 10^{16}$). Giallongo et al. (1996) obtain $\beta = 1.8$ for systems with $N_{HI} > 10^{14}$ and $2.8 < z < 4.1$, that is consistent with earlier measurements by Petitjean et al. (1993).

Finally, we note that the analogous column density distributions derived from the lower resolution Hercules runs give similar results and slopes as the Kronos data. For example over the full column density range at $z = 3$, Hercules data give slopes for the column density distribution of

| Model       | $z$ | $\beta_1$ | $\beta_{HGZ}$ | $\beta_{z}$ | $\beta_1$ | $\beta_2$ | $\beta_2$ |
|-------------|----|-----------|----------------|-------------|-----------|-----------|-----------|
| sCDM        | 2  | 1.64 ± 0.03| 1.49           | 1.84 ± 0.23 | 1.87 ± 0.05| 1.63 ± 0.04| 1.98 ± 0.08|
|             | 3  | 1.56 ± 0.03| 1.55           | 1.77 ± 0.10 | 1.71 ± 0.03| 1.52 ± 0.04| 1.79 ± 0.05|
| OCM         | 2  | 1.61 ± 0.04| 1.50           | 1.80 ± 0.17 | 1.74 ± 0.04| 1.56 ± 0.05| 1.73 ± 0.08|
|             | 3  | 1.48 ± 0.04| 1.56           | 1.78 ± 0.12 | 1.66 ± 0.04| 1.42 ± 0.03| 1.74 ± 0.03|
| CHDM        | 2  | 1.56 ± 0.04| 1.47           | 1.78 ± 0.17 | 1.71 ± 0.04| 1.50 ± 0.03| 1.78 ± 0.08|
|             | 3  | 1.41 ± 0.03| 1.52           | 1.45 ± 0.10 | 1.58 ± 0.04| 1.41 ± 0.05| 1.71 ± 0.04|
| tCDM        | 2  | 1.89 ± 0.04| 1.63           | 1.93 ± 0.18 | 1.90 ± 0.05| 1.86 ± 0.06| 1.83 ± 0.13|
|             | 3  | 1.72 ± 0.08| 1.70           | 2.03 ± 0.12 | 1.95 ± 0.06| 1.64 ± 0.13| 2.03 ± 0.10|
|            | 2  | 1.87 ± 0.04| 1.66           | 1.88 ± 0.17 | 1.95 ± 0.05| 1.85 ± 0.05| 1.90 ± 0.14|
| Kim et al.  | 2  | 1.65 ± 0.08| 1.67           | 1.89 ± 0.10 | 1.92 ± 0.06| 1.57 ± 0.13| 2.01 ± 0.08|
|            | 3  | 1.39 ± 0.06| 1.46           | 1.46 ± 0.07 | 1.46 ± 0.06| 1.55 ± 0.07| 2.01 ± 0.08|
|            | 3.35| 1.59 ± 0.13| ...            | ...         | ...       | ...       | ...       |
|            | 3.7 | ...         | ...            | ...         | ...       | ...       | ...       |

Note—$\beta_1$ over the range $10^{12.8} < N_{HI} < 10^{14.3} \text{ cm}^{-2}$, $\beta_{HGZ}$ over the range $10^{13.7} < N_{HI} < 10^{14.3} \text{ cm}^{-2}$, $\beta_s$ over the range $10^{14.3} < N_{HI} < 10^{15.5} \text{ cm}^{-2}$, $\beta_1$ and $\beta_2$ represent the slopes found by splitting the distributions into two halves, $10^{12.8} < N_{HI} < 10^{14} \text{ cm}^{-2}$ and $10^{14} < N_{HI} < 10^{16} \text{ cm}^{-2}$, respectively. The column labeled $\beta_{HGZ}$ is the prediction of the semianalytic model of Hui et al. 1997. The last row provides the measured values reported by Kim et al. 1997. All quoted errors are 2 $\sigma$. 

TABLE 2
DETERMINATIONS OF THE H I COLUMN DENSITY DISTRIBUTION SLOPE OVER VARIOUS COLUMN DENSITY RANGES

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1.71, 1.66, and 1.62 for the sCDM, ΛCDM, and OCDM models, respectively, consistent within errors with the Kronos results. This suggests that the distribution function is a robust diagnostic, being relatively insensitive to grid resolution and numerical method. A preliminary comparison with the data favors models with more power at these scales than in our CHDM or tCDM cosmologies. However, there appears to be some discordance in the observations, so a more definitive comparison will require more work.

4.2. Line Number Evolution

The number of Lyα lines at a particular redshift reveals how many intergalactic absorbers exist at that time between the quasar and observer and, given certain assumptions on their geometry, the size and volume filling factor of the absorbers can also be deduced. Since the column density of Lyα lines corresponds to the mean overdensity and size of the clouds fairly well (Charlton et al. 1997; Zhang et al. 1997), it is useful to see how the number of lines evolves with different column density cutoffs, as this will track the evolution of morphologically distinct small scale structures in the universe.

Figure 14 shows the evolution of the number of lines with H I column densities greater than $10^{13}$ cm$^{-2}$, $10^{13.5}$ cm$^{-2}$, and $10^{14}$ cm$^{-2}$, respectively, comparing results for the models in the model comparison study with the observed data from Kulkarni et al. (1996) at $z \sim 2$, Hu et al. (1995) at $z \sim 3$, and Lu et al. (1997) at $z \sim 4$. For a fixed transmission cutoff (here $F_0 = 0.95$) and column density threshold, the total number of lines per unit redshift decreases with time because the opacity of the universe decreases from both the increasing flux of radiation and the expansion of the universe. With the exceptions of the tCDM and CHDM models for the lowest column density threshold $N_{H_\alpha} > 10^{13}$ where incompleteness due to line blending becomes significant at higher $z$, the deviation from a fixed power law behavior tracks predominantly the behavior of the radiation flux. Fitting the evolutions to the form $dN/dz \propto (1 + z)^{\gamma}$ over the range $2 < z < 4$, we find the exponents are fairly similar in the different models. We summarize these results in Table 3 (all errors are 2 $\sigma$). To compare these simulated results with observational data we fit the combined line lists from Kulkarni et al., Hu et al., Kirkman & Tytler, and Lu et al. to the same power-law behavior and display those results in the row labeled “combined” in Table 3, again with 2 $\sigma$ errors. (Lines near the QSO emission redshift were avoided because of the proximity effect, as were lines associated with metal systems.) Kim et al. obtain $\gamma = 2.78 \pm 1.42$ (2 $\sigma$), fitted over $2 < z < 3.5$ for systems with column densities $10^{13.77} < N_{H_\alpha} < 10^{16}$ cm$^{-2}$. Using the same column density cuts and simulation data for $z = 2$ and 3 only, we find power-law exponents (labeled $\gamma_c$ in Table 3) for the sCDM, ΛCDM, OCDM, tCDM, and CHDM models of the comparison study in good agreement with the observational result. For the lower column density range $10^{13.1} < N_{H_\alpha} < 10^{14}$ cm$^{-2}$, the power-law exponents are labeled $\gamma_p$ in Table 3. Kim et al. (1997) obtain $\gamma_p = 1.29 \pm 0.90$ (2 $\sigma$) fitted over $2 < z < 4$. Our sCDM, ΛCDM, and OCDM model predictions are in good agreement with this observational result, although tCDM and CHDM show a somewhat stronger evolution.

All the models yield comparable levels of evolution at each of the column density cutoffs, and, in fact, the evolution slopes in all the models agree for the most part within errors with the observed values. Two trends in the model predictions are apparent. The first is that at a given column density the shape of the number density function is $\propto (1 + z)^{\gamma}$ with $\gamma \approx 2$ for $z \approx 2$, and $\gamma < 1$ for $z \approx 4$.
density threshold the slope \( \gamma \) of the line number evolution is correlated to the slope \( \beta \) of the column density distribution with \( \gamma \) increasing for models with larger \( \beta \) (i.e., models with less power in the fluctuation spectrum at small scales). This is not unexpected since the evolution for a fixed transmission threshold is essentially determined by the radiation field (which is the same in all the models) and, to a lesser degree, the expansion of the universe. In our previous studies (Zhang et al. 1997) we have found little intrinsic cloud evolution over these redshift intervals. If we assume that the evolution of the gas overdensity does not contribute significantly to the evolution of line number or column density at these redshifts, we can use equation (15) and the power-law dependence of the column density distribution \( dN/N_{\text{HI}} dz \propto N_{\text{HI}}^{-\beta} \) to predict the number of lines above a fixed column density threshold as a function of redshift \( z \). We find

\[
\frac{dn}{dz} (N_{\text{HI}} > N) \propto \left( \frac{1+z}{\Gamma(z)\text{H}(z)} \right)^{(\beta-1)}.
\]

(16)

The second trend is that stronger column density lines for a given model exhibit a greater rate of redshift evolution. This again is primarily due to equation (16) and the fact that the column density distribution is curved with \( \beta \) steepening for higher column densities.

In Figure 15 we compare the scaling predictions of equation (16) to the simulation results using the high-resolution sCDM \( \sigma_8 = 0.7 \) model for column density thresholds \( N_{\text{HI}} > 10^{13}, 10^{13.5}, \) and \( 10^{14} \) cm\(^{-2} \), respectively. Since \( \beta \) also evolves weakly with \( z \), we use \( \beta \) at \( z = 3 \) as representative of the average for \( 2 < z < 4 \) in equation (16). For the lowest two column density thresholds, we use the single power-law fit to the column density distribution over the range \( 10^{12.8} < N_{\text{HI}} < 10^{16} \text{ cm}^{-2} \), while we use \( \beta_2 \) from the two power-law fit for the high \( (N_{\text{HI}} > 10^{14} \text{ cm}^{-2}) \) column density threshold. We normalize the scaling predictions to the simulated number of lines at \( z = 3 \) because that is where all the models in our study were normalized to the observational data. For the lower two column density thresholds, the scaling prediction tends to overestimate the number of lines at \( z = 4 \). For the lowest column density threshold \( (N_{\text{HI}} > 10^{13}) \), this again is partly due to incompleteness in the simulation line lists caused by line blending, an effect that becomes more severe for low column densities at high \( z \). Furthermore, the slope of the low column density end softens for \( N_{\text{HI}} < 10^{14} \), as discussed in § 4.1. This deviation of the column density distribution from the pure power law assumed in the scaling relation would also cause the scaling law to overproduce the low column density lines. For the column density threshold \( N_{\text{HI}} > 10^{13.5} \), the discrepancy between the scaling prediction and the simulation results at \( z = 4 \) is reduced. This is to be expected since the high spatial resolution sCDM line lists should be complete at this column density threshold. For lines with \( N_{\text{HI}} > 10^{14} \), where the absorbers share a common morphological type and the column density distribution is well fitted by a single power law, agreement between the scaling prediction and the simulations is good. In the lower panel of Figure 15 we compare the scaling predictions with the simulation results for the models in the model comparison study for the high column density threshold case. The scaling predictions for all models agree reasonably well with the simulations.

Encouraged by these results we solve equation (16) for the shape of the UV ionizing background \( \Gamma(z) \) in terms of the Hubble parameter \( H(z) \) modeling the universal expansion and the (in principle) measurable quantities \( dn/dz (N_{\text{HI}} > 10^{14}) \) and \( \beta_2 \), the slope of the column density distribution over this column density range:

\[
\Gamma(z) \propto \left( \frac{1+z}{H(z)} \right)^{1/3} \left( \frac{dn}{dz} \right)^{1/3}.
\]

(17)

We use equation (17) to compute \( \Gamma(z) \) with simulation data from the model comparison study and compare these predictions to the Haardt-Madau spectrum actually used in...
probably will not be able to do that for very long. We have
also found that the scaling relations, that all of the models reproduce the
assumed Haardt-Madau evolutionary trend.

5. DOPPLER b PARAMETER

Recent papers (Bryan et al. 1999; Theuns et al. 1998) have
shown that both the Doppler b parameter and a related
nonparametric statistic, the mean flux difference as a func-
tion of velocity, require high simulation spatial resolution
to model properly. In this section we investigate the depen-
dencies of these statistics on the properties of the cosmo-
logical model. Because these statistics are highly sensitive to
the spatial resolution of the simulation, we present results
only for those models (sCDM, ΛCDM, OCDM, and tCDM) simulated with our highest spatial resolution
Δx = 37.5 kpc. By this spatial resolution the b parameter
distribution and mean flux difference as a function of velocity are just approaching convergence (Bryan et al. 1999).
For example, a further increase in the simulation spatial
resolution by a factor of 2 to Δx = 18.75 kpc causes a
reduction in the median b parameter by 13%, 6%, and 3% at
z = 4, 3, and 2, respectively. Thus Δx = 37.5 kpc should be
considered the minimum resolution necessary to model
these statistics.

The Doppler b parameter measures the amount of line
broadening due to thermal broadening, physical velocities,
Hubble expansion broadening, and the shape of the
absorber density profile (Bryan et al. 1999). Both Hubble
and thermal broadening are significant for the lower
column density lines that arise from structures found in
voids that are still expanding in absolute coordinates. The
thermal contribution becomes dominant only for the higher
column density lines that have turned around and are gravit-
tationally collapsing. Furthermore, the b parameter is
highly sensitive to the simulation spatial resolution. Lower
resolution simulations numerically thicken the lines causing
the width of the lines, and thus b to be overestimated (Bryan
et al. 1999; Theuns et al. 1998). In our previous work (Bryan
et al. 1999) we argued that the shape of the b-distribution
was in rough agreement with observation and particularly
that the high-b power-law tail of the distributions arises
naturally in hierarchical models when quasar lines of sight
pass obliquely through the filamentary absorbing structures
(Rutledge 1998). However, the median of the simulated b
parameter distribution for the sCDM model, calculated from
simulations with high spatial resolution, was now sub-
stantially below the ~30 km s⁻¹ median seen in the obser-
vations. Thus the sCDM model, which previously had
appeared to be in agreement with observations, is now
discrepant. In Figure 17 we show the Doppler b parameter
distributions extracted from the high grid resolution
(Δx = 37.5 kpc) Kronos simulations at redshift z = 3 for the
sCDM, ΛCDM, tCDM, and OCDM models for lines with
column densities in the range 10¹⁺³.¹ cm⁻² < N_H< 10¹⁴
cm⁻². We present for comparison data from Kim et al.
(1997) for z = 3.35. The ΛCDM and OCDM models, like
sCDM, have their b distributions shifted too much to the
left (to low b values) to agree with observation. Only tCDM,
the model with the least fluctuation power at small scales
and thus broader density structures at this redshift, has a
median b approaching the observational values. We explore
this dependence on the fluctuation power spectrum with the
two highest resolution sCDM models (with σ₈ = 0.7 and
0.3, respectively) in the lower panel of Figure 17 and see that
indeed the model with lower spectral power produces a b
parameter distribution shifted toward higher b (as predicted
by Hui & Rutledge 1999). The increase in b for models with
less fluctuation power at small scales may be partly due to
line-blending effects at these low column densities.
However, as shown below, the shift to higher b values for
these models persists for lines with higher column densities
as well where line blending should not be as significant and
thus can not be explained by line blending alone.

To facilitate a better comparison of the models with
observations, we plot the median Doppler parameters as a
function of redshift in Figure 18 where we have imposed the
same column density cuts on the lines as those used by Kim

![Figure 16](image-url)

Although the prediction is highly sensitive to the
slope of the column density distribution used (whose errors
are still quite large), it is gratifying, given the simplicity of
the scaling relations, that all of the models reproduce the
assumed Haardt-Madau evolutionary trend.
et al. (1997). The median $b$ for lines with column densities $10^{13.8} \text{cm}^{-2} < N_{\text{H}_1} < 10^{16} \text{cm}^{-2}$ and $10^{13.2} \text{cm}^{-2} < N_{\text{H}_1} < 10^{14} \text{cm}^{-2}$ are shown in the top and bottom panels, respectively. While the $\Lambda$CDM, sCDM, and OCDM models predict roughly the observed evolutionary trend for both sets of column density cuts, the median $b$ values lie systematically more than 6 km s$^{-1}$ below the observational data. OCDM, the model with the most power at these scales, is the most discrepant. Since the $b$ distributions have not quite converged by $\Delta v = 37.5$ kpc and increased spatial resolution lowers the predicted median $b$, these values represent a lower bound on the actual discrepancy. The tCDM model predicts median $b$ parameters more consistent with observation. However, the redshift evolution predicted by this model appears to be in disagreement with the data. We can compare these results with other recent data sets. Confining to lines with $N_{\text{H}_1} > 10^{13} \text{cm}^{-2}$, we obtain from the published line lists, for $1.9 < z < 2.0$ (1 $\sigma$ errors), $(b_{\text{mean}}, b_{\text{median}}) = (32.1 \pm 2.6, 29.7 \pm 3.3)$ km s$^{-1}$ (1 $\sigma$) (Kulkarni et al.), $3 < z < 3.1$ $(b_{\text{mean}}, b_{\text{median}}) = (38.0 \pm 1.6, 33.6 \pm 2.0)$ km s$^{-1}$ (Hu et al.), $(b_{\text{mean}}, b_{\text{median}}) = (27.3 \pm 1.9, 25.9 \pm 1.2)$ km s$^{-1}$ (Kirkman & Tytler), and at $4 < z < 4.1$ $(b_{\text{mean}}, b_{\text{median}}) = (32.6 \pm 2.4, 25.9 \pm 3.1)$ km s$^{-1}$ (Lu et al.), again clearly discrepant with the model predictions. Thus, none of the models considered here can restore agreement with the observational data.

We also argued in Bryan et al. (1999) that this discrepancy is not a result of the particular choice of line-fitting algorithm but appears for sCDM in the nonparametric moments of the two-point flux distribution functions as well. The two-point function $P_{2}(F_1, F_2, \Delta v)$ gives the probability that 2 pixels with separation $\Delta v$ will have flux $F_1$ and $F_2$. We plot the normalized moments of this function averaged over the flux range $F_a$ to $F_b$ given by

$$\frac{\int_{F_a}^{F_b} dF_1 \int_{0}^{F_2} dF_2 P_2(F_1, F_2, \Delta v) F(F_1 - F_2)}{\int_{F_a}^{F_b} dF_1 \int_{0}^{F_2} dF_2 P_2(F_1, F_2, \Delta v)}$$

which represents the average flux difference as a function of velocity for pixels in the range $F_a$ to $F_b$. In Figure 19 we plot the above statistic at $z = 3$ as a function of velocity for various flux ranges. In the bottom panel of Figure 19 we use the sCDM $\sigma_8 = 0.7$ model to show that this statistic is stable under changes in simulation box size for $z \geq 3$ but is highly sensitive to simulation spatial resolution. By $z = 2$, simulation box size effects begin to appear in the lowest flux range, $0.0 < F < 0.1$, as we might expect from similar behavior observed in the low-flux region of the flux PDF. Differences in the other flux ranges at $z = 2$ due to box size effects remain small. In the middle panel of Figure 19 we plot the above statistic at $z = 3$ for the high-resolution models (sCDM, $\Lambda$CDM, OCDM, and tCDM) of the model comparison study, and in the top panel of Figure 19 we study its dependence on the small-scale fluctuation power using our high-resolution sCDM models with $\sigma_8 = 0.7$ and 0.3, respectively. There is little difference for low velocities, independent of flux level, owing to the high coherence of the lines. At very large velocity differences there is no coherence, and the value is just the difference between the mean value of the transmitted flux and the mean flux in a given flux interval (Bryan et al. 1999). It is at intermediate velocity separations where the statistic is heavily influenced by the structure of the lines. There $\Lambda$CDM, OCDM, and sCDM with $\sigma_8 = 0.7$, whose power spectra are very similar, produce very similar distributions, while the tCDM model is quite distinct. We may quantify these model differences by determining at what $\Delta v$ the model prediction passes through a given average flux difference. For the flux interval $0 < F < 0.1$, the simulation predictions pass through the mean flux difference of 0.3 at $\Delta v \approx 35$ km s$^{-1}$ for $\Lambda$CDM, OCDM, and sCDM $\sigma_8 = 0.7$ and at $\Delta v \approx 45$ km s$^{-1}$ for tCDM. Although observational data are limited, these are all lower than the $\Delta v \approx 55$ km s$^{-1}$ from Figure 3 of Miralde-Escudé et al. (1998).

It is important to ask what is needed to restore agreement between the simulations and observations. Although we can not completely rule out the possibility that the line-fitting algorithm contributes to differences in the simulated and observed $b$ parameter distributions, we argue that its effect should not be significant because the discrepancy is seen at a comparable level (Bryan et al. 1999) in the fit-independent two-point distribution of the flux as well. The mean optical depth of our models was scaled to agree with observations, but this normalization is in some dispute. However, changing this normalization has little effect on the median of the $b$-distribution. For example, using sCDM ($\sigma_8 = 0.7$), an increase in $\xi$ from 0.225 to 0.35 at $z = 3$ causes the median $b$ value to decrease from 20.8 km s$^{-1}$ to 20.1 km s$^{-1}$, a change of only $\sim 1$ km s$^{-1}$. One possibility might be to change the density structure through the power spectrum of the cosmological model. Other recent suggestions revolve around finding ways to increase the temperature of the gas (Theuns et al. 1999; Haehnelt & Steinmetz 1998; Madau & Efstathiou 1999; Abel & Haehnelt 1999; Nath, Sethi, & Shchekinov 1999). In Bryan & Machacek (2000) we explore several of these latter possibilities in more detail for the $\Lambda$CDM model. In particular we find that if radiative transfer effects increase the He II photoheating rate by a factor of

![Figure 19](image-url)
2 over that used in the optically thin limit (as suggested by Abel & Haehne 1999), agreement between the ΛCDM model and observation can be restored.

6. FLUX STATISTICS FOR He II

Previous work (Zhang et al. 1997, 1998; Croft et al. 1997a) indicates that He II Lyα (304 Å) absorption may be significant in regions where H I Lyα is not. Thus the study of He II Lyα absorption in quasar spectra provides a unique probe of structure in the lowest density regions of the universe. Comparison of both H I and He II absorption within the context of a given cosmological model may also yield important information about the spectral shape of the metagalactic UV radiation field and its redshift evolution. While current observations still struggle to obtain sufficient resolution to detect any but the broadest individual He II Lyα lines, it is still possible to determine mean statistics of the He II flux and optical depth that are not so sensitive to instrumental resolution. We define the mean optical depth \( \bar{\tau}_{\text{He II}} = -\ln \langle F \rangle \), where \( \langle F \rangle \) is the mean transmitted flux (\( F = 1 \) signifying complete transmission). In Figure 20 we present \( \bar{\tau}_{\text{He II}} \) as a function of redshift for the sCDM models with varying power normalizations (top) and for the models in the model comparison study (bottom). Several trends are apparent. First, all models produce a rapid rise in mean optical depth with increasing redshift (roughly a factor of 2 between \( z = 2 \) and \( z = 3 \)), with tCDM and CHDM rising slightly more steeply. This is consistent with previous work on a smaller number of hierarchical cosmologies (Zhang et al. 1997; Croft et al. 1997a) and with the interpretation that the observed optical depth is due primarily to absorption by gas in underdense regions. The redshift evolution of the optical depth is thus dominated by the change in the gas density due to universal expansion and (to a lesser degree) by the shape of the UV metagalactic ionizing background (here assumed to be the vertical line at 1996 with frequency dependence \( \propto v^{-1.8} \)). Second, for a given redshift \( z \), models with less power on small fluctuation scales have progressively larger optical depths. This is again consistent with the interpretation that the absorption is due to gas in predominantly underdense regions since less gas in these low-power models will have turned around and collapsed.

The first observation of a flux decrement at the wavelength where the He II Lyα absorption should occur was made by Jakobsen et al. (1994) using the HST Faint Object Camera to observe quasar Q0302−003. They obtained a 90% confidence lower limit of \( \bar{\tau}_{\text{He II}} > 1.7 \) at \( z = 3.286 \). Subsequently improved measurements using spectra from this same quasar were made by Hogan, Anderson, & Rugers (1997) with the Goddard High Resolution Spectrograph on HST and by Heap et al. (2000) using the Space Telescope Imaging Spectrometer (STIS). STIS provides better sensitivity and background determinations than previous measurements. Davidsen, Kriss, & Zheng (1996) used the Hopkins Ultraviolet Telescope to study the average He II opacity in the spectrum of quasar HS 1700+64 over the redshift interval 2.2 < z < 2.6 (lower than that available with HST). They find \( \bar{\tau}_{\text{He II}} = 1.0 \pm 0.07 \), although as shown in Figure 3 of Croft et al. (1997a) there is considerable scatter when the wavelength range is divided into 10 Å bins.

Measurements of He II absorption have also been made by Anderson et al. (1999) with STIS using the spectrum of quasar PKS 1935−692. Although the number of lines of sight studied so far is limited and thus a detailed comparison of observations with our model simulations (that average over 300 lines of sight) is premature, these data are also presented in Figure 20. On face value, these data favor higher optical depths and thus models with lower fluctuation power. However, none of the simulation models presented here can reproduce the apparent break at \( z = 3 \) in the optical depth observed by Heap et al. (1998). If this break persists, it would most likely signal a departure from the Haardt-Madau quasar reionization spectrum assumed here. Observations of an abrupt change in the Si IV/C IV line ratios in the Lyα forest at \( z \sim 3 \) (Songaila & Cowie 1996; Songaila 1998) also indicate a significant change in the metagalactic ionizing flux between redshifts 2.9 < z < 3.0. One possibility is that we are seeing the epoch at which He II reionizes. Further evidence supporting this possibility is the so-called "patchy" He II opacity near \( z = 3 \) observed by Reimers et al. (1997) that may indicate He II reionization is not yet complete by that redshift. Recent attempts to extract the temperature-density relation of the IGM from Lyα forest data using observational data above and below \( z = 3 \) (Bryan & Machacek 1999; Ricotti, Gnedin, & Shull 1999) also suggest that the gas temperature may actually rise below \( z = 3 \), signaling the onset of new heating consistent with this interpretation.

For completeness and comparison with previous work (Croft et al. 1997a), we show in Figure 21 (from top to bottom) the He II Lyα flux probability distribution functions at \( z = 4, 3, \) and 2, respectively, for the models in the model comparison study. The distribution functions are calculated from the flux smoothed with window functions corresponding to the same full width at half-maximum (FWHM) as STIS with high, 50 km s\(^{-1}\) resolution (left-hand column) and low, 500 km s\(^{-1}\) resolution (right-hand column). As Figure 21 shows, the shape of the flux PDF is highly dependent on the smoothing. We see, however, that models with less fluctuation power on small scales have far fewer truly transparent regions (pixels with \( F \) near 1). For the
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7. SUMMARY

We have performed several simulations of the Lyα forest using different background cosmological models, numerical codes, and grid resolutions. Five different cosmological models were considered here: the standard flat critical density cold dark matter model (sCDM), a flat CDM model with a nonzero cosmological constant (ΛCDM), an open CDM model (OCDM), a flat critical density CDM model with a tilted power spectrum matching both the COBE amplitude and small-scale clustering constraints (tCDM), and a flat critical density mixed dark matter model (CHDM). The high-resolution shock capturing code Kronos was used with comoving grid resolution Δx = 37.5 kpc (Δx = 75 kpc for CHDM) for the benchmark calculations presented in this paper. Three of the models (sCDM, ΛCDM, and OCDM with identical parameters) were also evolved with the artificial viscosity–based code Hercules at the lower grid resolution Δx = 75 kpc. Both simulation techniques give similar results for statistics, such as the slope of the column density distribution, that are insensitive to grid resolution.

We have presented results from several statistical analyses of absorption features present in the Lyα spectra, both from the unprocessed optical depth data and from the reduced line lists. Explicitly, we have considered the optical depth and transmitted flux probability distribution functions, the cumulative optical depth distributions, the H I column density distributions, line number evolution, Doppler b parameter distribution, the average flux difference as a function of velocity (first moment of the two-point flux distribution function), and the mean optical depth and flux probability distribution functions for He II absorption. We find the following:

1. Simple scaling laws describe the redshift evolution of the optical depth, flux PDF, the H I column density distribution, and, in conjunction with the slope of the column density distribution, the line number evolution remarkably well. This demonstrates that most of the evolution of the Lyα forest is a direct consequence of universal expansion.

2. The shape of the optical depth PDF is strongly correlated to the amplitude of the density fluctuation spectrum. Differences between models may be significant in the observationally accessible region 0.02 < τ < 4. Thus, this statistic may be a useful discriminator among models. Similar conclusions hold for the related flux PDF.

3. Cumulative opacity distributions for the models are strongly clustered at low optical depth thresholds and high z. Significant differences do occur for optical depth thresholds τ_0 > 1, but the usefulness of these high thresholds may be limited by numerical effects and observational statistics.

4. The column density distribution function is a robust statistic relatively insensitive to grid resolution and numerical method. Its redshift evolution is described well by the same naive scaling law that describes the evolution of the optical depth. The slope of the column density distribution
is sensitive to the amplitude of the power spectrum on scales roughly the size of the absorbers (∼ 100 kpc). Models with less power at these scales produce steeper distributions in qualitative agreement with semianalytic arguments (Hui et al. 1997). A preliminary comparison with data favors models with more power (sCDM, ACDM, and OCDM) over those with less power (tCDM and CHDM).

All models show comparable evolution for the number of lines above a given H I column density threshold in reasonable agreement with the data. Thus, this statistic is not a sensitive discriminator among models.

Although the shape of the Doppler b parameter distribution is well reproduced by all the models, the median of the distribution for sCDM, ACDM, and OCDM models is well below observed values. The median of the b parameter increases for models with less power on small scales. Thus, the observations favor low-power models, such as tCDM, making it difficult for any model considered in this study to simultaneously give good agreement with both the H I column density and b parameter data. This discrepancy is confirmed as well in the nonparametric first moment of the two-point flux distribution function and so is not solely the result of the line-fitting algorithm employed. The solution to this problem may require additional heating to raise the temperature of the IGM and produce more pressure broadening of the absorbing structures or a modification of the power spectrum of the underlying cosmology itself.

All of the models simulated in this study produce a rapid rise in He II mean optical depth with increasing redshift consistent with the interpretation by previous work (Zhang et al. 1997; Croft et al. 1997a) that the observed optical depth is due to absorption by gas in underdense regions where universal expansion dominates the evolution of the gas density. Models with less power on small scales (tCDM and CHDM) produce larger mean He II optical depths. Preliminary comparison with the data tends to favor these low-power models. However, none of the models can reproduce the break seen by Heap et al. near z = 3. If this break persists in the data, it would most likely reflect that the Haardt- Madau (1996) form for the metagalactic UV ionizing background, based on homogeneous reionization by quasars alone in a clumpy medium, must be modified.

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