Intermediate baseline appearance experiments and three-neutrino mixing schemes

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Abstract

Three-neutrino mixing schemes suggested by Cardall & Fuller and Acker & Pakvasa are compared and contrasted. Both of these schemes seek to solve the solar and atmospheric neutrino problems and to account for the possible neutrino oscillation signal in the LSND experiment. These neutrino oscillation schemes have different atmospheric and solar neutrino signatures that will be discriminated by Super-Kamiokande and SNO. They will also have different signatures in proposed long-baseline accelerator and reactor experiments. In particular, both of these schemes would give dramatic (and dramatically different) signals in an “intermediate baseline” experiment, such as the proposed ICARUS detector in the Jura mountains 17 km from CERN.

1 Three-mixing schemes

At the present time there are three hints of neutrino mixing: the solar (e.g. [1]) and atmospheric (e.g. [2]) neutrino problems, and the signal in the LSND experiment [3]. Each of these can be solved by or interpreted as neutrino flavor oscillations. If an independent neutrino mass difference is associated with each of these three hints, four neutrino flavors are necessary. Since it is known that only three neutrino flavors participate in the weak interaction [4], the fourth neutrino flavor must be “sterile” [an SU(2) singlet]. Accounting for all three hints with only three active neutrino flavors would require that at least two of the phenomena must “share” one of the two available independent mass differences.
The possibility of a three-neutrino mixing scheme along these lines was noted by Cardall & Fuller [5]. Their scheme uses the smaller neutrino mass difference to account for the solar neutrino problem, and the larger mass difference to account for the atmospheric neutrino problem and the LSND signal. They proposed the following mass/mixing parameters:

\( \delta m_{21}^2 \approx 7 \times 10^{-6} \text{ eV}^2, \)
\( \delta m_{31}^2 \approx \delta m_{32}^2 \approx 0.3 \text{ eV}^2, \)

\[
U_{\text{CF,sma}} \approx \begin{pmatrix}
0.994 & 0.044 & 0.100 \\
-0.108 & 0.530 & 0.841 \\
-0.015 & -0.847 & 0.532
\end{pmatrix},
\]

Here \( \delta m_{ji}^2 = m_j^2 - m_i^2 \), where \( m_i^2 \) are the \( i \)th and \( j \)th squared neutrino mass eigenvalues, and we take \( m_j^2 > m_i^2 \). The unitary matrix \( U_{\text{CF,sma}} \) (where “CF” stands for “Cardall & Fuller” and “sma” stands for “small mixing angle”) connects the neutrino flavor and mass eigenstates: \( \nu_\alpha = \sum_i U_{\alpha i} \nu_i \), where \( \alpha = e, \mu, \) or \( \tau \) and \( i = 1, 2, 3 \). These mass/mixing parameters were designed to employ the small mixing angle MSW solution to the solar neutrino problem (see below). However, as can readily be seen from the analysis of Ref. [6], the general framework of the Cardall & Fuller scheme can also accommodate the solar neutrino large mixing angle MSW solution:

\( \delta m_{21}^2 \approx 2 \times 10^{-5} \text{ eV}^2, \)
\( \delta m_{31}^2 \approx \delta m_{32}^2 \approx 0.3 \text{ eV}^2, \)

\[
U_{\text{CF,lma}} \approx \begin{pmatrix}
0.873 & 0.478 & 0.100 \\
-0.330 & 0.428 & 0.841 \\
0.359 & -0.767 & 0.532
\end{pmatrix},
\]

and the solar neutrino “just-so” vacuum oscillation solution:

\( \delta m_{21}^2 \approx 6 \times 10^{-11} \text{ eV}^2, \)
\( \delta m_{31}^2 \approx \delta m_{32}^2 \approx 0.3 \text{ eV}^2, \)

\[
U_{\text{CF,vac}} \approx \begin{pmatrix}
0.807 & 0.582 & 0.100 \\
-0.381 & 0.384 & 0.841 \\
0.451 & -0.717 & 0.532
\end{pmatrix},
\]

Another variation on the Cardall & Fuller scenario has very recently been proposed in Ref. [7].
A rather different scheme has been suggested recently by Acker & Pakvasa [8], in which the smaller neutrino mass difference accounts for both the solar and atmospheric neutrino problems, and the larger mass difference accounts for the LSND signal. They propose the following mass/mixing parameters:

\[ \delta m_{21}^2 \approx 10^{-2} \text{ eV}^2, \]
\[ \delta m_{31}^2 \approx \delta m_{32}^2 \approx 1 - 2 \text{ eV}^2, \]
\[ U_{AP} \approx \begin{pmatrix} 0.700 & 0.700 & 0.140 \\ -0.714 & 0.689 & 0.124 \\ -0.010 & -0.187 & 0.982 \end{pmatrix}. \]

Relatively small alterations of this mixing matrix could also allow this scheme to accommodate slightly larger or smaller mass differences to account for a significant fraction of the LSND signal range.

2 Consequences for solar neutrino experiments

The solar neutrino problem arises from the observation that the measured solar \( \nu_e \) fluxes in experiments with chlorine [9], water [10], and gallium [11] detectors are only a fraction of the fluxes expected from models of the sun. Furthermore, the fact that the detectors employing the above substances have different thresholds and that they observe different \( \nu_e \) flux deficits points to an energy dependence of the \( \nu_e \) suppression. This energy dependence picks out values of \( \delta m^2 \) that may provide a neutrino mixing resolution to the puzzle (e.g. [6,12]): \( \delta m^2 \approx 10^{-5} \text{ eV}^2 \) for an MSW effect (matter-enhanced neutrino flavor conversion) solution, with both small and large mixing angle solutions as possibilities; or \( \delta m^2 \approx 10^{-10} \text{ eV}^2 \) for a “just-so” vacuum oscillation solution, with large mixing angle.

While the Cardall & Fuller schemes employ these standard solar neutrino solutions, the Acker & Pakvasa scheme employs an “energy independent” solar neutrino solution with \( \delta m^2 \approx 10^{-2} \) [13,14]. Their motivation is to use the same mass difference as that suggested by the claimed zenith angle dependence of the Kamiokande multi-GeV atmospheric neutrino data (see below). Such an energy independent solution is strongly disfavored if all solar neutrino experiments and the latest standard solar models are used [15], but is marginally possible if the \( ^8\text{B} \) neutrino flux is significantly (factor of \( \sim 0.6 \)) smaller than that predicted by the standard solar model. The possibility of a \( ^8\text{B} \) neutrino flux smaller than that of the SSM has received some motivation from a recent measurement [16] of the reaction \( \gamma + ^8\text{B} \rightarrow ^7\text{Be} + p \), the inverse of a reaction important to the determination of that flux. However, issues surrounding the
ability to extract the forward cross section from measurements of the inverse reaction have been controversial [17]. Another possibility for allowing a larger neutrino mass difference than those in the “standard” solar neutrino solutions is to ignore either the chlorine or gallium experiments [14,15].

The current evidence for neutrino oscillations associated with the sun rests simply on an observed neutrino flux deficit as compared with solar models. Two next-generation solar neutrino experiments, SNO [18] and Super-Kamiokande [19], will allow tests that can provide more conclusive evidence of neutrino mixing. These tests include time variable solar neutrino signals (seasonal for vacuum oscillations [20,21], day-night for the MSW solutions [22,23]) and distortions of the recoil electrons from neutrino interactions in the detector [21,23,24] (relevant for both the MSW solutions and the “just so” vacuum oscillation solution). These effects will be visible for the Cardall & Fuller schemes since they adopt the standard neutrino oscillation solutions. However, these effects will be absent in the “energy independent” solar neutrino solution of Acker & Pakvasa. This is a major observational difference between the Cardall & Fuller and Acker & Pakvasa mixing schemes.

3 Consequences for atmospheric neutrino experiments

Fluxes of $\nu_e$ and $\nu_\mu$ arise from the interactions of cosmic rays with the earth’s atmosphere. In order to minimize uncertainties associated with neutrino cross sections and a lack of knowledge of the absolute magnitude of the neutrino fluxes, the “ratio of ratios” $R$ is a commonly reported observable:

$$R = \frac{(\nu_\mu/\nu_e)_{\text{data}}}{(\nu_\mu/\nu_e)_{\text{Monte Carlo}}}.$$  \hspace{1cm} (13)

Studies involving large water detectors (Kamiokande [25] and IMB [26]) have reported values of $R \sim 0.6$. Experiments using iron calorimeter detectors have had mixed results: Soudan 2 [27] has observed a deficit in $R$ comparable to that seen in the water detectors, while NUSEX [28] and Fréjus [29] have not seen evidence for values of $R$ significantly different from unity.

An interesting aspect of the atmospheric neutrino puzzle is the zenith-angle dependence of $R$ reported by the Kamiokande group for multi-GeV neutrinos [30]. According to their analysis, $R$ decreases from zero zenith angle (neutrinos from the atmosphere immediately above the detector) to maximum zenith

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1 SNO, with its use of heavy water, will allow the very important measurement of the ratio of charged current/neutral current events for neutrino interactions with deuterium.
angle (neutrinos from the atmosphere on the other side of the earth). This dependence suggests that neutrinos from overhead do not travel far enough to oscillate significantly, while those coming from across the earth do travel sufficiently far to oscillate. Such a scenario restricts neutrino mass differences to the range $10^{-3} - 10^{-1}$ eV$^2$.

A neutrino mass difference of about $10^{-2}$ eV$^2$ has therefore come to be associated with the atmospheric neutrino problem. This is the mass difference employed in the Acker & Pakvasa neutrino oscillation scheme. The mass difference of $\sim 0.3$ eV$^2$ in the Cardall & Fuller scheme requires one to discount the zenith angle dependence of the Kamiokande multi-GeV data. This position is not entirely implausible, as the statistical significance of this effect has been questioned [31], and the IMB group saw no zenith angle dependence in their data [32].

Super-Kamiokande will be able to settle the question of zenith angle dependence definitively. Preliminary analyses already confirm a deficit in $R$ averaged over all directions, but nothing definitive can yet be said about the zenith angle dependence [33]. This is perhaps interesting in view of the fact that Super-Kamiokande already has more events than were obtained with the previous Kamiokande detector.

Another important point is that in the Acker & Pakvasa solution the atmospheric neutrino oscillations are primarily $\nu_\mu \leftrightarrow \nu_e$ to allow a simultaneous solution to the solar neutrino problem, while in the Cardall & Fuller scheme the atmospheric neutrino oscillation channel is primarily $\nu_\mu \leftrightarrow \nu_\tau$ to avoid conflict with $\nu_e$ disappearance experiments. Due to uncertainties in the $\nu_e$ and $\nu_\mu$ absolute fluxes, it is not currently possible to determine the oscillation channel of atmospheric neutrinos. However, higher precision cosmic ray measurements may help reduce the ambiguity.

4 Consequences for accelerator and reactor experiments

While solar and atmospheric neutrinos have provided tantalizing suggestions of neutrino mixing, it is highly desirable to directly observe neutrino oscillations in controlled terrestrial experiments. We point out that the neutrino mass difference associated with solar neutrinos is so small that all three of the Cardall & Fuller solutions, corresponding to different solar neutrino solutions, will have the same effects in the terrestrial experiments discussed here.
4.1 Short baseline experiments

The LSND experiment, which has reported an excess of $\bar{\nu}_e$ in a $\bar{\nu}_\mu$ beam from pion decays at rest, continues to run. KARMEN [34], a similar experiment, has recently received an upgrade that should eventually be able to confirm this LSND signal. While the LSND signal is compatible with large neutrino mass differences, negative results from CCFR [35], CHORUS [36], and NOMAD [37] restrict the viable range of of neutrino mass difference to $\delta m^2 \leq 10$ eV$^2$. In addition, LSND has recently completed a study of excess $\nu_e$ events in a $\nu_\mu$ beam from pion decays in flight [38], a channel whose backgrounds and systematics are different from those of the $\bar{\nu}_e$ beam from pion decays at rest. This excess can also be accounted for by neutrino oscillations, with mixing parameters that overlap those suggested by the decay at rest data. Finally, COSMOS [39], the proposed short baseline counterpart to the MINOS experiment, should just be able to see $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in both the Acker & Pakvasa and Cardall & Fuller schemes.

4.2 Long baseline experiments

Long baseline experiments are designed to explore the region of neutrino mixing parameter space suggested by the atmospheric neutrino problem, neutrino mass differences of $\sim 10^{-2}$ eV$^2$ in particular. Long baseline accelerator experiments such as MINOS (Fermilab to the Soudan mine) [40], KEK to Super-Kamiokande [41], and CERN to Gran Sasso [42] involve a $\nu_\mu$ beam and should be able to distinguish between $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_e$ oscillations. Figure 1 shows the oscillation (flavor conversion) probabilities for these channels implied by the Cardall & Fuller and Acker & Pakvasa schemes for a baseline of 735 km, which coincidentally is the approximate baseline length for both the CERN to Gran Sasso and MINOS experiments. The differences are evident. The Acker & Pakvasa scheme has large amplitude $\nu_\mu \leftrightarrow \nu_e$ mixing, with oscillations in energy space that could probably be observed experimentally. The Cardall and Fuller scheme has large amplitude $\nu_\mu \leftrightarrow \nu_\tau$ mixing, with the oscillations in energy space being so rapid that in practice an average would probably be measured.

There are also forthcoming long baseline reactor experiments, CHOOZ [43] and Palo Verde [44]. These are $\bar{\nu}_e$ disappearance experiments which will be sensitive to neutrino mass differences as low as $\sim 10^{-3}$ eV$^2$, but which will not provide increased sensitivity to the oscillation amplitude. These experiments are therefore sensitive to a $\nu_\mu \leftrightarrow \nu_e$ solution to the atmospheric neutrino problem (which has a very large effective mixing angle). They would see neutrino oscillations predicted by the Acker & Pakvasa scheme, but not those predicted
Fig. 1. Probabilities for (a) $\nu_e$ and (b) $\nu_\tau$ appearance in a $\nu_\mu$ beam, as a function of neutrino energy, for a baseline of 735 km (CERN to Gran Sasso and Fermilab to the Soudan mine). “CF” and “AP” stand for the Cardall & Fuller and Acker & Pakvasa oscillation schemes respectively.

by the Cardall & Fuller solution.

4.3 The Jura mountain neutrino detector

The ICARUS collaboration, which is producing neutrino detectors for the CERN to Gran Sasso long baseline experiment, has recently proposed placing
Fig. 2. Probabilities for (a) $\nu_e$ and (b) $\nu_\tau$ appearance in a $\nu_\mu$ beam, as a function of neutrino energy, for a baseline of 17 km (CERN to the Jura mountains). “CF” and “AP” stand for the Cardall & Fuller and Acker & Pakvasa oscillation schemes respectively.

one of their detector modules behind the Jura mountains 17 km from CERN [45]. This “intermediate baseline” experiment is of great interest in light of the three-neutrino mixing schemes discussed here, since the neutrino flight distance divided by the neutrino oscillation length is such that neutrino oscillations should be directly observable in energy space, as illustrated in Figure 2 [the estimated energy resolution for ICARUS is (5-15)\%]. This experiment was proposed with the Acker & Pakvasa scheme in mind, but Figure 2 shows
that the Cardall & Fuller scheme predicts an even more spectacular signal in the \( \nu_\tau \) appearance channel. We emphasize that an intermediate baseline experiment such as this is important to distinguish between the Acker & Pakvasa and Cardall & Fuller schemes, and to directly observe two features of those schemes which are presently only inferred from phenomenological considerations: (1) a neutrino mass difference in the range \( 0.1 \leq \delta m^2 \leq 1 \text{ eV}^2 \), and (2) significant \( \nu_\mu \leftrightarrow \nu_\tau \) mixing, which is (at least in part) indirectly responsible for the \( \nu_\mu \leftrightarrow \nu_e \) signal observed at LSND.

### 5 Conclusion

There exist at least two three-neutrino mixing schemes designed to satisfy three hints of neutrino mixing (the solar and atmospheric neutrino problems, and the signal at LSND). Some of their signatures in future experiments are given in Table 1. Accomodating all three of these hints would normally require four neutrino flavors; use of a three generation framework requires ignoring some aspect of the data. Cardall & Fuller have chosen to ignore the zenith angle dependence of the atmospheric neutrino data, while Acker &
Pakvasa have chosen to ignore the energy dependence of the solar neutrino data. The former choice has recently been identified in a thorough analysis as the choice of “minimum sacrifice” [46], and the validity of the above assumptions regarding atmospheric and solar neutrinos will eventually be tested by Super-Kamiokande and SNO. Nevertheless, direct experimental verification of the predictions of these schemes in a controlled terrestrial experiment is highly desirable. In particular, an “intermediate baseline” experiment such as the proposed ICARUS detector to be placed behind the Jura mountains 17 km from CERN is important to be able to see, in energy space, unaaveraged $\nu_\mu \leftrightarrow \nu_\tau$ oscillations driven by a neutrino mass difference of $0.1 \leq \delta m^2 \leq 1$ eV$^2$.

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