New method for evaluation of bendability based on three-point-bending and the evolution of the cross-section moment

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Abstract. Friction-free 3-point bending has become a common test-method since the VDA 238-100 plate-bending test [1] was introduced. According to this test the criterion for failure is when the force suddenly drops. It was found by the author that the evolution of the cross-section moment is a more preferable measure regarding the real material response instead of the force. Beneficially, the cross-section moment gets more or less a constant maximum steady-state level when the cross-section becomes fully plastified. An expression for the moment M is presented that fulfils the criteria for energy of conservation at bending. Also an expression calculating the unit-free moment, M/Mₘₑ, i.e. current moment to elastic-moment ratio, is demonstrated specifically proposed for detection of failures. The mathematical expressions are simple making it easy to transpose measured force F and stroke position S to the corresponding cross-section moment M. From that point of view it’s even possible to implement, e.g. into a conventional measurement system software, studying the cross-section moment in real-time during a test. It’s even possible to calculate other parameters such as flow-stress and shape of curvature at every stage. It has been tested on different thicknesses and grades within the range from 1.0 to 10 mm with very good results. In this paper the present model is applied on a 6.1 mm hot-rolled high strength steel from the same batch at three different conditions, i.e. directly quenched, quenched and tempered, and a third variant quench and tempered with levelling. It will be shown that very small differences in material-response can be predicted by this method.

Figure 1. The bending-equipment used with rotatable friction-free die-rollers measuring the force vs. knife-position converted by present model to cross-section moment. The model is applied on a hot-rolled high strength steel from the same batch with the three different conditions. Beside the moment the evolution of the flow-stress and the curvature are estimated.
1. Introduction
Testing by bending has lots of advantages. By bending it is more like making a screening as the strain is linearly distributed through the thickness compared to a tensile-test that is uniform. At bending the specimen can also be turned, i.e. top- and lower-side down, able to test each side separately investigating any tendencies for asymmetry. From that point of view at tensile-testing one gets only an average of the material response. Other deformation-mechanisms are also involved in bending compared to tensile-testing that makes the test important, and thus validating the bendability performance. Commonly the bending-force is used as a measure detecting the stage for the appearance of bend failure. In this paper it will be shown that the moment is a more beneficial parameter for this purpose as the cross-section moment becomes more or less constant. Whenever it starts to decrease it’s a sign on a failure. The characteristic for the force does not have a steady state behaviour as it by nature starts to decrease as the test progresses. This is a drawback especially for more ductile materials and high performance high strength steels. While testing such materials when the force drops naturally, the test will be interrupted indicating a failure that may not be the case. From that point of view, in such a case, the VDA238-100 test specification has been expanded giving instructions for making additional preparation of the specimens with pre-straining in advance before bending, i.e. forcing the material to fail at an earlier stage. However this statement investigating the cross-section moment during bending is not new as several papers within this area have been published. In [2] a similar study was performed aiming to obtain material characteristics from bending-tests, based on a model solved by numerical iterations. Other investigations, e.g. [3-6], are mainly based on constitutive related material-models, achieving a theoretical moment. Another very complex method mentioned in [7], calculating such moment-characteristic by making several tensile-tests. However this paper will demonstrate an alternative method estimating the cross-section moment experimentally and directly. It is based on the energy absorbed by the specimen and it is also demonstrated how to achieve other properties such as the unit-free moment, flow-stress and strains. Beside this, it is demonstrated how to predict failures by studying sudden changes of the moment. In this paper the model is applied to a 6.1 mm thick hot-rolled high strength steel. The source material was selected from the same batch and treated into three different conditions, i.e. directly quenched, quenched and tempered, plus quench and tempered with levelling. The die used in the test, shown in Figure 1, is an up-scaled variant of the equipment applied for the VDA 238-100 test, with friction-free die-rollers so the material doesn’t slide or drag over the rollers. The parameters for the setup are shown in Table 1.

| Material thickness, $t$ (mm) | Die-radius, $R_d$ (mm) | Knife-radius, $R_k$ (mm) | Half die-width, $L_o$ (mm) | Width of specimen, $B$ (mm) |
|-----------------------------|------------------------|--------------------------|---------------------------|-----------------------------|
| 6.1                         | 40                     | 4.0                      | 47.75                     | 70.0                        |

2. The estimation of the cross-section moment $M$ and bending-angle $\beta_2$
At small bending-angles, the moment can be estimated by: $\frac{F_y L_m}{2}$, where $F_y$ is the vertical load applied and $L_m$ the moment-arm, see Figure 2 and 3, with the geometrical expression:

$$L_m(\beta_3) = L_0 - (R_d + R_k) \cdot \sin \beta_3$$

representing the horizontal distance between the two tangent-points, i.e. at the knife and die-support considering a straight flange. $R_d$ and $R_k$ are the radii for the die and the knife. By assuming a straight flange the bending-angle $\beta_1$, i.e. half total bending-angle, can easily be defined as it only relates to the geometry of the setup and the stroke-position $S$, i.e. presented in a similar form in [8]:

$$\beta_1 = \sin^{-1} \left( \frac{Q}{L_0^2 + (S - Q)^2 - Q^2} \cdot \sqrt{L_0^2 + (S - Q)^2} \right) \cdot \frac{180}{\pi} \text{[degrees]}$$

Where:

$$Q = R_d + R_k + t, \quad \text{and, } t, \text{ is the material thickness.}$$
To full-fill the energy of conservation, equation (5), the present expression for the cross-section moment \( M \) becomes as:

\[
M(F_y, \beta_1) = \frac{F_y}{2} \frac{L_m(\beta_1)}{\cos^2 \beta_1} = \frac{F_y}{2} \frac{(L_0 - (R_d + R_k) \sin \beta_1)}{\cos^2 \beta_1}
\]

(2)

with the corresponding bending-angle:

\[
\beta_2 = \beta_1 - \int \frac{t \sin \beta_1}{L_0 - (R_d + R_k) \sin \beta_1} \, d\beta_1
\]

(3)

As can be seen in equation (3) \( \beta_2 \) becomes a bit less than the commonly used angle in bending \( \beta_1 \). An explicit solution, equation (A1), of the integral solving \( \beta_2 \) is presented in Appendix A. The moment-curves are calculated, based on sampled force-data (Figure 4) obtained from the bending tests performed are shown in Figure 5. By applying the constant steady-state moment \( M_{\text{max}} \) to equation (2) and deriving \( F \), a theoretical force-curve can be estimated, see dashed lines in Figure 4. The theoretical estimated force-curve is the path that the force should follow if the material does not fail.

3. Estimation shape- and contact-angle, \( \beta_S \) and \( \beta_C \)

Regarding the present model, it can be theoretically shown that it applies even for curved flanges. From that point of view it is of interest to investigate the evolution of the curvature and the movement of the points of contact between knife and material introducing the shape of curvature-angle \( \beta_S \) and the contact-angle \( \beta_C \). The sum these two angles makes the total bending angle \( \beta_2 \), see Figures 3.

\[
\beta_2 = \beta_S + \beta_C
\]

(4)

The energy input \( U \) at bending obtained by the integrals:

\[
U = \int F_y \, ds = \int 2M \, d\beta_2
\]

(5)

Considering the distribution of the cross-section moment (see Figure 2) related to each individual bending-angle \( \beta_C \) and \( \beta_S \), an alternate form of energy-equation can be expressed by:

\[
U = 2M \left( \beta_C + \frac{\beta_S}{2} \right)
\]

(6)

Then equation (4) and equation (6) give:

\[
\beta_S = 2\beta_2 - \frac{U}{M} \quad \text{and} \quad \beta_C = \frac{U}{M} - \beta_2
\]

(7)

The individual angles \( \beta_S \) and \( \beta_C \) obtained from the three experiments are plotted in Figure 6.

Figure 2. Showing the parameters involved in the model and moment-distribution considered.

Figure 3. Showing the angle of contact \( \beta_C \) and the shape of curvature-angle \( \beta_S \) as well as the tangent point of contact considering a straight flange (dashed lines).
Figure 4. Measured forces and estimated theoretical forces, considering a constant moment; $M = M_{\text{max}}$.

Figure 5. Calculated moment-curves based on the present model showing constant steady-state behaviours until failure.

Figure 6. Showing the evolution of the contact-angle at the knife $\beta_C$ and the shape of curvature angle of the flange $\beta_S$. At the stage when $\beta_C$ starts to increase, the bending goes from 3-to 4-points of bending. The sum of both angles gives the entire bending-angle $\beta_2$. Also the difference between $\beta_1$ and $\beta_2$ is shown.

4. Estimation of curvature $1/R$
As the expressions for both $\beta_S$ and $\beta_C$ are known the curvature can be derived applying the well-documented formula: $\frac{1}{R} = |y''|\left[1 + (y')^2\right]^{-3/2}$, where $y' = \frac{dy}{dx} = \tan \beta_C$. The present formula becomes as simple as:

$$\frac{1}{R} = \frac{2U}{M \cdot L_N} \cdot \cos(\beta_C)$$

(8)

where $L_N$ is the horizontal distance between the real contact-points, see Figure 2, obtained by:

$$L_N(\beta_1, \beta_C) = L_0 - R_k \sin \beta_C - R_d \sin \beta_1$$

(9)

By assuming the neutral-layer positioned at center thickness at the contact-point, the plane strain, $\varepsilon_1$, becomes:

$$\varepsilon_1 = \frac{1}{R} \cdot \frac{t}{2} = \frac{t \cdot U}{M \cdot L_N} \cdot \cos \beta_C$$

(10)

The distribution of the curvature $1/R$ and/or the strain $\varepsilon_1$ can easily be plotted against the coordinate $X$, see Figure 2, at a certain bending-angle called $\beta_2^*$. At the current stage the moment is $M^*$ and moment-arm $L_N^*$. As the moment $M$ is linearly distributed between the two contact points, knife and die (Figure 2), the coordinate $X$ can be formulated as: $X = \frac{M(\beta_2^*)}{M^*} \cdot L_N^*$

(11)
In Figure 7, the estimated curvatures $1/R$ obtained from the three tests performed are plotted against $X$. The results represent the status at $\beta_2=15$ degrees (still bending in progress and at a total bending angle of 30 degrees).

5. The flow-stress, $\sigma_1$

The moment, $M$, can be calculated assuming symmetric stress-distribution in tension and compression across the thickness:

$$M = \frac{B t^2}{\varepsilon_1^2} \int_0^{\epsilon_{\text{max}}} \sigma_1 \cdot \varepsilon_1 \; d\varepsilon_1,$$

where $B$ is the width of the specimen and $t$ the material thickness.

By deriving $\sigma_1$ from equation above, gives:

$$\sigma_1 = \frac{2}{B t^2} \frac{1}{\varepsilon_1} \cdot \frac{d}{d\varepsilon_1} (M \cdot \varepsilon_1^2) \quad (12)$$

Calculated flow-stress curves from the tests performed are shown in Figure 8.

6. Unit-free moment, $M/Me$

The unit-free moment is the ratio between current moment, $M$, to the elastic moment, $M_e$ and its maximum value represents a fully plastified cross-section, equal to 1.5 and its minimum value is 1.0 during pure elastic condition. These ratios can be obtained by the expressions for the elastic- and fully plastic moment, i.e.: $M_{el} = \frac{B t^2 R_{m\prime}}{6}$ and $M_{max} = \frac{B t^2 R_{m\prime}}{4}$ where $R_{m\prime}$ is the ultimate strength for plane strain condition. That makes the ratio:

$$\frac{M_{max}}{M_{el}} = 1.5.$$

Then the total interval becomes:

$$1 \leq \frac{M}{M_e} \leq 1.5.$$

A function evident for the entire interval can be derived, applying equation (12) that gives:

$$\frac{M}{M_e} = 3 \cdot \left( \frac{dM/d\beta_1}{M/\beta_1} + 2 \right)^{-1} \quad (13)$$

The expression can easily be confirmed by the criteria’s for its min- and max-value, i.e. the elastic-, and the fully plastified state. Calculated M/Me-curves from the tests are presented in Figure 9. As can be seen when a failure occurs, the M/Me ratio increase above its theoretical maximum value.

A simplified version based on equation (13) aimed for detection of failures is:

$$\frac{M}{M_e} \approx 3 \cdot \left( \frac{dM/d\beta_1}{M/\beta_1} + 2 \right)^{-1} \quad (14)$$

![Figure 7. Results of estimated curvatures after 30 degrees bending (i.e. total bending-angle) plotted along the horizontal distance between the two contact-points, i.e. die support and knife.](image)

![Figure 8. The flow-stresses obtained show different behaviours, especially the YPE discontinuous yielding-effect after tempering (Q+T) and its change after levelling (Q+T+L).](image)
7. Determination of position for failures
In this study a comparison is performed between detecting of failures by force and by the present method based on the moment or more specifically by the unit-free moment, i.e. simplified version in equation (14). As soon as the unit-free moment does increase above its maximum limit of 1.5, see Figure 9, a failure can be suspected. When material starts to crack during bending it will initially indicate a very small reduction in force. Regarding the force-based method and according [1] a failure is determined when force has dropped 60N. In this case the material thickness is 6.1 mm compared to [1] that is based on thinner materials, i.e. approximately 2.0 mm, giving an equivalent value of 540 N. For comparisons and visualizing all functions in one single graph values were normed. Numerically by present method initial cracks were indicated as soon as the unit-free moment reached a chosen value of 1,525. Corresponding normed values for these two limits are 0.99 and 1.05, respectively. In Figures 10-12 the results are shown from the three tests performed. In Figure 13, the diagram in Figure 12 is magnified confirming a very small decrease in the force detected. Additionally, also the normed theoretical force is plotted showing the stage at which the measured force is dropping. In Table 2, the numerical results are shown for positions and corresponding bending-angles for failures detected by the two methods.

![Figure 9](image_url)

Figure 9. The unit-free moment M/Me obtained for the three tests performed. At failure the M/Me-ratio increases rapidly making it applicable for detection of failures.

![Figure 10](image_url)

Figure 10. Identifying the position at failure for Test Q.

![Figure 11](image_url)

Figure 11. Identifying the position at failure for Test Q+T.
Identifying the position at failure for Test Q+T+L.

Diagram in Figure 12 magnified showing the very small drop in force and moment confirming the response in M/Me.

Table 2. Comparison between force- and moment detection of failures

| Method | F | M/Me | Error |
|--------|---|------|-------|
| Q      | 14.35 | 13.19 | 1.16  | 8.8% |
| Q+T    | 17.50 | 16.53 | 0.97  | 5.9% |
| Q+T+L  | 21.77 | 20.74 | 1.03  | 4.9% |

| F | M/Me | Error |
|---|------|-------|
| 20.31 | 18.40 | 1.91  | 10.4% |
| 25.72 | 24.00 | 1.72  | 7.2%  |
| 33.79 | 31.77 | 2.02  | 6.3%  |

8. Discussion

Beside detection of failures, the present method gives the opportunity for determining the material behaviour from different aspects. First of all by simply making the transpose from force to moment by equation (1 and 2), one get a more clear picture of the material response. However as bending is more like “free bending” the material itself decides its curvature. From that point of view and from a material-development perspective it can be of interest able to investigate small differences in properties that have an influence on the bendability. In Figure 6 the evolution of the estimated individual angles, $\beta_S$ and $\beta_C$ are presented showing small differences comparing the tempered materials, i.e. Q+T and Q+T+L. The true- and the commonly used bending angle $\beta_2$ and $\beta_1$ are shown, starting to diverge and the gap becomes bigger with larger bending-angle. By studying the curvature $\frac{1}{R}$, in Figure 7, the unlevelled material (Q+T) has a rapid and local increase in the shape of curvature close to the knife contact point. According to [7] this tendency can be related to the flow-stress and especially if the material doesn’t follow a perfect elasto-plastic behaviour. In Figure 8, the calculated flow-stress curves are shown and at the very beginning there are big differences in between the tests. For the tempered material (Q+T) a discontinuous yielding-effect is detected showing an upper and lower yield-point. This kind of discontinuity of the flow-stress is a typical reason behind the tendency for so-called kinking [7], i.e. localization of strains in the bend. One can also see that after the levelling procedure (Q+T+L) this effect is taken away probably why this increase in bendability. The quenched material (Q) failed at an early stage reasonable due to its very high strength and lower ductility. Regarding detection of failures, and according to the VDA238-100 test, the criterion for detecting a failure is when the main drop in force occurs resulting in a larger measure compared to corresponding value related to the initial state of the failure. Regarding the unit-free moment M/Me, shown in Figure 9, that is very sensitive to very small changes of the cross-section moment, referring to the equation (13) including the derivative of the moment. A material with some type of deformation-hardening behaviour, will reach a steady state value below 1.5. However the theoretical maximum ratio a material can reach is 1.5. At failure this derivative becomes negative and thereby the ratio M/Me will increase above this maximum limit. From that point of view the unit-free moment
becomes as a very good measure for detecting failures. In this paper a simplified version has been derived specifically aimed for the detection of failures, given in equation (14). This function has been applied on the three bending tests performed and compared with the common method, i.e. by the force, see Figures 10-13. In Table 2, one can see that there is a big difference between the results depending on method applied.

By experience even though there is no loss of energy, like friction involved, there is a tendency for the estimated moment, M, to increase with large bending-angles. The internal moment from the material bent and the external moment estimated have to be the same, the internal (material) increase can be explained by the fact that the neutral-layer at the bend moves inwards. The increase of the external moment is due to the equilibrium of the horizontal forces involved makings an additional force acting at the bend close to the knife. When the material tends to move outwards from the knife it results in this additional moment. Regarding the failure detection, this kind of extra additional moment is not as crucial.

9. Conclusion
It has been shown by present method that it is possible to estimate the evolution of the cross-section moment, curvatures and flow pattern during bending and even an opportunity for displaying these data in real time during a test. The moment-based method has been applied on real cases where very small differences in material behaviours have been detected. It has also been demonstrated a new technique for prediction of the occurrence of initial failures, applied on the three tests performed and compared with the common technique according to [1]. The results obtained confirming the new method even more accurate. Moreover present method is also applicable for softer grades that become a problem when studying the changes in force.

Beside the present investigation, this method has been applied on a wide range of steel-grades, thicknesses, and dimensional sizes of specimens with very good results. One of these applications was to make bending-tests on micro-specimens. These small plates were prepared from gleeble-specimens and ground into thin sheet-samples and bent by a micro-tool for bending.

Appendix A
Explicit expression for the true bending-angle $\beta_2$ [rad]:

$$\beta_2 = \beta_1 - \int \frac{t \sin \beta_1}{L_0 - (R_k + R_d) \sin \beta_1} =$$

$$= \beta_1 (R_T + t)/R_T - 2tP_2 \left[\tan^{-1}(P_1) - \tan^{-1}(P_1 - P_2 \tan \left(\frac{\beta_1}{2}\right))\right]/R_T$$

(A1)

Where:

$R_T = R_k + R_d$, $P_1 = R_T \cdot \left[L_0^2 - R_T^2\right]^{-1/2}$, $P_2 = L_0 \cdot \left[L_0^2 - R_T^2\right]^{-1/2}$

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