Developing The M6 Learning Model to Improve Mathematic Critical Thinking Skills

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Annotation. This research & development aims to develop and produce quality learning models. The main product in this study is the M6 learning model which has the following syntax: (1) focusing on initial skills; (2) justifying concepts; (3) investigating problems; (4) presenting ideas; (5) evaluating; and (6) concluding. The results show that the M6 learning model developed meets the validity requirements. This study concludes that the M6 learning model has met the requirements by the assessment of experts so that it can improve students’ critical thinking skills.

Keywords: development M6 learning model, mathematical critical thinking skills.

Introduction

Critical thinking skills are needed in the 21st century (Beers, 2011; Roekel 2012; Alismail & Mcguire, 2015; Abed et al., 2015; Lamp, 2017). Critical thinking skills can also be used as a very important topic in modern education which is needed to continue studies at a higher level (As’ari, 2014). Therefore, critical thinking skills are needed to be applied in the learning process as one of the goals of the education system (Massa, 2014). In addition, critical thinking skills are very important in many aspects, including improving student performance in learning (Marcut, 2005); stimulating intellectual abilities and making students actively involved in class activities (Fisher, 2003); can improve students’ understanding of mathematical concepts (Alcantara & Bacs, 2017); and can improve
student achievement in mathematics. In the field of natural science, critical thinking skills can be used as a basis for creating solutions in learning (Yacoubian, 2015).

Learning that can improve critical thinking skills is very important. According to Karakoc (2016), when critical thinking skills are developed, students will be encouraged to think for themselves; questioning hypotheses; analyze and synthesize events; developing new hypotheses; testing hypotheses based on facts; and evaluate information on the right basis. In addition, through continued emphasis on developing critical thinking skills, students have the opportunity to build knowledge and experience to learn how to solve problems (Norris, 1985; Snyder & Snyder, 2008; Sulistiani & Masrukan, 2016). Critical thinking skills are needed by all students to understand the information presented in learning (Lambert & Cuper, 2008). Students who fail to develop critical thinking skills will usually get lower academic scores (Quitadamo et al., 2008). Therefore, critical thinking skills need to be developed in mathematics learning because critical thinking skills allow students to analyze their own thinking to make choices and draw conclusions (Sulistiani & Masrukan, 2016).

Students’ critical thinking skills in mathematics are also not well developed. This is evidenced by the student not being able to solve the questions well when the researcher gives the test questions. After being researched, almost all students could not understand the meaning of the questions given so that students did not know the steps taken to produce the correct answer. This is in accordance with the results of Akgun & Duruk’s (2016) research which shows that students do not have good competencies that involve critical thinking skills. Furthermore, according to Hadi et al (2018), the indicator with the lowest score increase is analyzing and making conclusions because students are still not maximal in composing sentences in their own language.

Another evidence that shows that students’ critical thinking skills in mathematics are still low is that students have not been able to solve problems related to their ability to analyze, evaluate, and conclude. From the analysis of students’ answers, it was found that the percentage of students who could answer questions about the ability to analyze was 25%. The percentage of students who could answer questions about their ability to evaluate was 18.75%. The percentage of students who can answer questions about their ability to conclude is 9.375%. This shows that the student’s ability to solve math critical thinking skills is still very low. Therefore, there needs to be a way to overcome these problems so that students’ critical thinking skills can improve properly.

Several studies have developed learning models to improve critical thinking skills including Yeh (2009), Wannapiroon (2014), Buhaerah (2016), Muhlisin et al. (2016), and Samo et al. (2017). Yeh (2009) developed a direct learning model consisting of preliminary activities (reviewing previous material, conveying learning objectives, focusing students’ attention on the topic to be discussed), presentations, and the teacher guiding in making conclusions. Based on the findings, the shortcoming in this model is that at the time of presentation, the teacher still conveys material to students so that students are less able
to develop their critical thinking. Wannapiroon (2014), developed a Research-Based Blended Learning (RBBL) learning model which consists of the following steps: compiling and analyzing a problem; designing and planning investigations; interpret and evaluate problems; and presentation of research results. The disadvantage of this learning model is that students conduct problem solving investigations individually so that other students are less than optimal in critical thinking. Buhaerah (2015) developed a PMBK learning model with the steps of this learning model, including: identifying concepts and justifying concepts; solve the problem; and generalize; and analyze algorithms. Based on the findings, it can be stated that the shortcomings of this model are that the teacher only divides groups without paying attention to students’ abilities so that students’ critical thinking skills do not increase evenly. Muhlisinet et al. (2016), designed an RMS (Reading, Mind Mapping and sharing) learning model consisting of the following steps: reading; create concept maps; and share ideas. The weakness in this learning model lies in the sharing step, the teacher immediately provides feedback when students present in front of the class so that students are less able to develop critical thinking skills. Samo et al. (2017), developed a contextual thinking learning model consisting of the following steps: presenting contextual problems; ask critical and analytical questions; individual and group investigations; presentations and discussions; reflection; and test. The weakness in this learning model is that when asking critical questions and analysis, the teacher’s role is too dominating so that it will hinder the ideas possessed by students. From the study of these learning models there are several shortcomings so that the increase in students’ critical thinking skills is not optimal. Therefore, it is still necessary to improve the steps of the existing learning model so that the objectives of learning can be maximized. This is done in order to produce a new quality learning model that meets the validity criteria to improve critical thinking skills in mathematics.

Focusing on initial abilities can develop students’ critical thinking skills. This is reinforced by the opinion of Yeh (2009) and Marashi & Noochiwani (2011), that initial abilities can develop students’ critical thinking skills. Initial abilities are also very necessary for students to connect the achievements of the next material so that they get good results (Safdar et al., 2012; Clement, 2013; Salam et al., 2019). In addition, initial abilities are the raw material needed in learning to teach students new materials to be studied so that they can achieve learning objectives (Sa’dijah, 2007; Dehghani et al., 2011; Karbalaei, 2014; Cooper et al., 2018). The low initial ability of students will result in difficulty in learning mathematics so that learning outcomes will be low (Acharya, 2017). Therefore, the teacher must link the initial abilities students have with the material to be studied so that they can train students to develop students’ critical thinking skills in mathematics.

Justifying the concept is the first step in building knowledge in critical thinking. This is reinforced by Bajracharya (2010), that to build knowledge in critical thinking requires activities to justify concepts. Justifying the concept consists of reading and questioning the concepts being studied (Ennis, 2008; Bajracharya, 2010). Reading activities can build
meaning in learning and build relationships between concepts so that they can empower students’ critical thinking skills (Yu-hui et al., 2010; Wang, 2012; Akin et al., 2015; Mahanal et al., 2019). In addition, the existence of concept building activities can make it easier for students to understand what is being learned so that teachers must give students the opportunity to understand concepts in learning mathematics in class (Ahuna et al., 2014; Kusaeri et al., 2019; Mahanal et al., 2019). Choosing the right concept from students will have a good impact in developing students’ critical thinking skills (Hussain & Munshi, 2011). Mastery of mathematical concepts will have a relationship with the development of students’ critical thinking skills (Chukwuenum, 2013; Radulovic & Stancic, 2017).

Learning processes that can improve critical thinking skills include identifying problems, analyzing problems, group discussions, asking questions that stimulate critical thinking skills, making conclusions from various sources, evaluating findings, providing feedback, and learning assessments that stimulate students critical thinking skills (Walker, 2003; Duron et al., 2006; Qatipi, 2011; Lee et al., 2012; Peter, 2012; Aktas&Unlu, 2013; Zhao et al., 2016; Vong & Kaewurai, 2017). Investigating problems in critical thinking requires analyzing, evaluating, and making conclusions (Sendag & Ferhan, 2009; Haghparast et al., 2013; Paul & Elder, 2014; Gholami et al., 2016; Vong & Kaewurai, 2017). Other activities that support critical thinking skills include: identifying problems and presenting supporting evidence (Innabi, 2003). Students can also develop mathematical critical thinking skills when facing mathematical problems, identify possible solutions, and justify reasons for an outcome (Huang et al., 2016). The critical thinking skills also involve a reason and logical consideration in deciding or evaluating a problem at hand (Marin & Halpern, 2011; Thomas, 2011; Fahim & Pazesk, 2012).

According to Peter (2012), one way that teachers can develop critical thinking skills in mathematics at the high school and post-secondary levels is to use learning strategies that actively involve students in the learning process. Learning strategies that can be carried out include group discussions and debates about the ideas expressed (Avsec & Kocijancic, 2014). Furthermore, according to Vong & Kaewurai (2017), Hadi et al (2018), and Prayogi et al (2018), the steps of a learning model that can improve critical thinking skills include: 1) identifying the problem given; 2) investigating related problems; 3) discussing the problems found; 4) evaluating the problems found; 5) creating solutions to the problems given, and 6) presenting solutions. Providing the right questions in group discussions, gathering feedback from students, and providing feedback to students that will make students active in identifying problems, analyzing problems, concluding problems, and evaluating problems so that they can develop students’ critical thinking skills (Duron et al., 2006; Peter, 2012; Zhao et al., 2016). Furthermore, according to Peter (2012), critical thinking must be based on activities that include unstructured problems so that students can practice evaluating problems and concluding. Giving problems, gathering sufficient evidence, making hypotheses, making comments from the results of presentations are activities that can train students to identify problems, analyze problems,
conclude problems, and evaluate problems so that they can develop critical thinking skills (Duran & Dokme, 2016).

Based on the description, which is accompanied by some evidence of research conducted previously, further researchers want to conduct research on the development of the M6 learning model that meets the valid criteria by experts so that it can be implemented in improving students’ critical thinking skills.

Method

General Background

The type in this research is Research and Development (R & D). The purpose of this study is to produce an M6 learning model that meets the validity criteria in improving critical thinking skills. The M6 learning model is taken from the acronym which consists of: (1) focusing on initial abilities (Memfokuskan); (2) justifying the concept (menjustifikasi); (3) investigate the problem (Menyelidiki); (4) presenting ideas (Mempresentasikan); (5) evaluate (Mengevaluasi); and (6) conclude (menyimpulkan). The product produced in this research is in the form of a M6 learning model book. This model book consists of (a) rational development of the M6 learning model; (b) the theoretical basis of the learning model; (c) M6 learning model; (d) instructions for implementing the M6 learning model; and (e) bibliography. The model book that has been produced will assess the quality of the M6 learning model using the instruments that have been developed. The developed instrument was tested for validity and reliability. The focus of this research is the validity of the M6 learning model which meets the requirements by the assessment of experts which can improve students’ critical thinking skills in mathematics.

Instruments and Research Procedures

The development process in this study follows the steps of the development model proposed by Plomp (2013), then supported by including the components of the learning model proposed by Joyce and Weil (2009), and the product quality criteria proposed by Nieveen (2013). The process of developing the M6 learning model according to Susandi et al. (2020), can be seen in Picture 1.
Preliminary research was conducted to collect related data: (1) problem identification and needs analysis regarding existing learning models; (2) problem identification and needs analysis on learning theory; (3) problem identification and needs analysis on critical thinking theory; and (4) problem identification and needs analysis regarding the current conditions of mathematics learning. The result of this preliminary study is a model design in the form of a draft M6 Learning model. The results obtained in the literature study and preliminary studies are used as materials for developing learning device products as an operational form of the M6 Learning model. The steps in developing the M6 learning model are: a) the development of the M6 learning model; b) the realization of the M6 learning model; c) the preparation of the M6 learning model; and (d) validation of the M6 learning model device by the assessment of experts. The instrument for assessing the quality of the M6 learning model was filled by three education experts who examined and assessed the learning model developed by the researcher. One validator is a professor in the field of learning model development; one validator is an expert in the field of mathematics, and one validator is a senior mathematics teacher at a school. This was done to obtain data about the validity of the M6 learning model. The quality of the M6 learning
model is assessed by the content validity and construct validity (Nieveen et al., 2013). The results of the assessment are used as a reference for revising the M6 learning model developed. For content validation, it is assessed by the researchers themselves whether the M6 learning model has met Plomp’s (2013) procedure and the characteristics of the learning model developed or not. Validation of constructs used a validation sheet. Construct validation is seen based on the relationship and suitability of each component in the M6 learning model with the learning theory used as a basis for model development.

**Data Analysis**

The data on the validity of the M6 learning model are obtained from the validation sheet that has been filled in by the validator. Before analyzing the validation results, the validation sheet reliability test was conducted. To determine the percentage of instrument reliability in this study, the percentage of agreement developed by Emmer and Millett (in Borich, 1994) was used with the terms of the agreement if the difference between the validators’ assessments of item i was matched. In other situations, it is said to be disagree. Emmer and Millett’s percentage of agreement formula:

\[
R = \left[1 - \frac{A - B}{A + B}\right] \times 100\%
\]

Information:
- R: coefficient (degree) of instrument reliability.
- A: the maximum rating of the indicator is observed by the validator.
- B: the minimum assessment of the indicators observed by the validator.

According to Borich (1994), the research instrument is said to be reliable if the R value for all indicators is . After checking the reliability of the validation sheet, the next step is to analyze the validity data of the M6 learning model. The steps in analyzing the model validity data are as follows: (1) recap the scores of the three validators; (2) calculating the average score of the three validators on each indicator ; (3) calculating the average score of indicators in each aspect ); (4) calculating the average score of all aspects ); and (5) make conclusions about the validity of the M6 learning model, learning tools, and instruments. The criteria for the validity of the M6 learning model are shown in Table 1.

| Score   | Criteria                | Meaning                        |
|---------|-------------------------|--------------------------------|
| Valid   |                         | Products can be tested         |
| Enough Valid |                      | There are still small parts that are revised |
| Invalid |                         | Total product revision         |
Based on Table 1, if the conclusions obtained are valid, the resulting product can be tested. If the conclusion is valid enough, it indicates that there are still small parts that need revision. If an invalid conclusion is obtained, a total revision is needed. This revision may be on the model or on the device. If a model revision results in a revision of the device, then the device is immediately revised so that the model and device remain consistent. In addition, it is necessary to pay attention to whether there are revisions in the form of notes, suggestions, or comments from the validator. If a total revision must be made, the revised model and equipment must be re-validated again.

**Results and Discussion**

**Rational Development of Learning Model M6**

The rationality of each sequence of steps in the M6 learning model is based on the researcher’s argument, theoretical studies, and empirical studies. The first step is to focus on initial skills. At the beginning of learning the teacher first explains the topics, objectives, learning outcomes to be achieved, and motivates students to the topics to be discussed. This is because motivation affects the success of each student’s problem solving (Keller, 2010). Furthermore, the teacher formed a heterogeneous group. Heterogeneous group formation is based on Vygostsky’s social constructivism in those students share ideas with other students to build mutual understanding that is impossible to build individually so as to help students foster critical thinking skills (Avsec & Kocijancic, 2014; Shieh & Chang, 2014). In addition, peer interaction in groups with high potential can develop students’ critical thinking skills (Lim et al., 2011; Sofroniou & Poutos, 2016). Furthermore, according to Maricica & Spijunovich (2015), the same role in group work will affect the development of students’ critical thinking skills. Then the teacher asks questions in order to explore students’ initial abilities related to the material to be studied. This is done because initial ability is a prerequisite for mastering further learning in order to get better results (Salam et al., 2019). In addition, initial knowledge can build students’ critical thinking skills (Yeh, 2009). Based on the rationality study, the first step in the M6 learning model is to focus on initial abilities.

The second step is to justify the concept. The activity of justifying the concept is the first step to building knowledge in critical thinking (Bajracharya, 2010). In this step, the teacher assigns students to read and understand the contents of the book and determine concepts related to the problem topic being discussed. This is done because the activities of observing, memorizing, and building concepts can make it easier for students to understand what is being learned (Mahanal et al., 2019). Then the teacher appoints representatives of each group to question concepts that are not yet understood. This is because students are required to have the ability to choose the right concept so that it has a good impact on developing students’ critical thinking skills (Hussain & Munshi, 2011).
In addition, formulating questions and asking questions are activities that can develop critical thinking skills (Ennis, 2011). Based on the rationality study, the second step in the M6 learning model is to justify the concept.

The third step is to investigate the problem. In this step the teacher presents problems about analyzing, evaluating, and concluding; students create hypotheses, collect data, and test hypotheses in group discussions. This is done because critical thinking skills requires activities to analyze, evaluate, and make conclusions in investigating problems (Sendag & Ferhan, 2009; Paul & Elder, 2014). Furthermore, students immediately identify problems, formulate hypotheses, record observations and ideas, and test hypotheses in group discussions. Active participation and organizing work with small groups in students can develop critical thinking skills (Florea & Hurjui, 2015; Maricica & Spjunovicb, 2015). In this step, face-to-face social interaction also occurs which allows students to share alternative views or ideas and helps students see ideas differently so that critical thinking skills can develop (Rusmansyah et al., 2019; Saputra et al., 2019). In this step, the teacher also provides scaffolding to students who have difficulty solving the questions given. The scaffolding given is feedback in the form of questions that require students to find their own answers so that students can be trained to improve critical thinking skills. This is in accordance with the opinion of Wartono et al. (2018) that the critical thinking skills can be empowered by teachers by giving questions or problems that challenge students’ thinking. Based on the rationality study, the third step in the M6 learning model is to investigate the problem.

The fourth step is to present the idea. In this step, a presentation of the ideas that have been obtained during group discussions in front of the class is carried out by group representatives appointed by the teacher. During the presentation, students explain the ideas from the results of group discussions. In addition, students who did not present could judge ideas from other groups and then come up with new ideas based on the right reasons. This is because activities conveying ideas from group work can develop critical thinking skills (Florea & Hurjui, 2015). Then in this step, the teacher only provides scaffolding in the form of questions so that students return to the problems being discussed and find the correct answers when there is a debate that is far from the context of the problems being discussed. The purpose of scaffolding is given to students so that students can solve the problems they are facing (Maharani & Subanji, 2018). This is reinforced by the opinion of Dewaelsche (2015), that questions can be an effective tool in developing critical thinking skills. Based on the rationality study, the fourth step in the M6 learning model is to present ideas.

The fifth step is to evaluate. In this step, the teacher instructs students to correct the answers obtained during the presentation. After students find wrong answers, then the teacher invites students to reveal the reasons for the wrong answers. This is because evaluating findings is one of the goals of critical thinking (Vong & Kaewurai, 2017). In addition, evaluating activities in critical thinking involves activities to assess various
kinds of ideas by looking for clear reasons (Duron et al., 2006; Taleb & Chadwick, 2016). Furthermore, according to Irwanto et al (2018), students should be encouraged to express and protest opinions, identify and clarify problems, and respond to and present various solutions to the problems at hand. Based on the rationality study, the fourth step in the M6 learning model is to evaluate.

The sixth step is concluding. In this step, students are required to formulate conclusions from what was obtained in the evaluation step. In this step, the teacher also appoints several students to express the conclusions obtained from the results of the evaluation that has been carried out. Then the teacher corrects the conclusions of the students who have been appointed and the teacher concludes in general from the results of the student’s answers. After that, students make conclusions in their own language that is easy to understand. This is because one of the abilities students must have in order to have good critical thinking skills is that students have the ability to draw conclusions well (Ghazivgawai et al., 2014; Boa et al., 2018). Based on the rationality study, the sixth step in the M6 learning model is to conclude.

**Assessment of the Quality of Learning Model M6**

Before analyzing the results of the model book validation, first the reliability is calculated for the model validation sheet. Based on the results of the calculations, it was found that the price of R was greater than 75%, namely R = 86.26%. According to the established instrument reliability criteria, the model book validation sheet is said to be reliable. The data analysis of the learning model book was carried out based on four aspects on the validation sheet, namely the rational development of the M6 learning model, the theoretical basis, the model components, and the model implementation instructions. The summary of the results of the model book validity can be seen in Table 2.

**Table 2**

*Model Book Validation Results*

| Rated aspect                        | The average score of the validator | Average |
|------------------------------------|-----------------------------------|---------|
|                                    | I  | II | III |               |
| Rational development of the M6 model |   |    |     |               |
|                                    | 3.33 | 3.00 | 4.00 | 3.44          |
| Theoretical basis                  |   |    |     |               |
|                                    | 3.80 | 3.00 | 4.00 | 3.60          |
| Model components                   |   |    |     |               |
|                                    | 3.87 | 3.00 | 3.60 | 3.49          |
| Model implementation instructions  |   |    |     |               |
|                                    | 3.33 | 3.00 | 3.67 | 3.33          |
| Average                            |   |    |     |               |
|                                    | 3.59 | 3.00 | 3.82 | 3.47          |
Based on the data analysis of the results of the model book validation in Table 2. M6 learning model book has met the validity criteria with an average value of 3.47. This means that the M6 learning model has met the valid criteria so that the model can be tested.

The product objective of the M6 learning model is to improve students’ critical thinking skills in mathematics. The product of the M6 learning model was developed in accordance with the stages of the Plomp development model (2013). The M6 learning model was developed based on a preliminary study, namely literature study and field observations which showed that the learning model applied in schools and the books used for student learning had not explored more deeply about students’ critical thinking skills in mathematics. Whereas the critical thinking skills in mathematics is important for students to have in learning mathematics. This was also expressed by several previous researchers, including As’ari (2016); Susandi et al. (2018); Susandi et al. (2019a); and Susandi et al. (2019b).

The prototypes produced in this study were in the form of a model book, a lesson plan (RPP), a student book, a teacher’s book, and a mathematics critical thinking skill test. The preparation of this prototype considers the characteristics of students to practice critical thinking skills in mathematics. In the ongoing learning, it can be seen that students who have the critical thinking skills in mathematics are good when faced with a problem, then these students collect various information and analyze information, and use their thinking to find linkages between one information and another. In the end, students will get the right conclusions about the problems they face. The behavior that is often seen when students face a problem is that students look for several alternative solutions, evaluate them, and finally decide on the choice of solution. When looking for solutions, students’ critical mathematical thinking skills play a role in producing correct problem solving. Furthermore, these activities are linked into a complete structure in syntax, social system, reaction principle, support system, instructional impact, and accompanying impact. These components are in the M6 learning model that the researcher developed.

The M6 learning model has six syntaxes. In the first syntax begins by focusing on initial capabilities. In this step the teacher first divides students into several groups heterogeneously. This is done because teaching students in a group can improve critical thinking skills (Elder & Paul, 2010; Kelemen, 2014). The formation of heterogeneous groups is based on social constructivism by Vygotsky, that students will share ideas with other students to build mutual understanding that is impossible to build individually (Moreno, 2010). In groups there will be interactions that make students exchange ideas so that students’ critical thinking skills can be trained. This is in accordance with the opinion of Costley (2016), that interaction in groups can improve critical thinking skills because the learning environment becomes more controlled. Interaction between students can also train students to be involved in learning and can improve critical thinking skills (Hussin et al., 2019). Then after the group has been formed, the teacher asks questions related to the students’ initial abilities related to the material to be discussed. Early
abilities are important in learning because initial abilities are the raw materials needed in learning to direct students to new information to be learned (Dehghani et al., 2011; Cooper et al., 2018). The questions posed by the teacher about students’ initial abilities can train students to improve critical thinking skills (Pithers & Soden, 2000). Questions that lead students to practice critical thinking using Bloom’s Taxonomy indicators. This is reinforced by the opinion of Pappas, et al. (2013), that questions for training critical thinking can be measured using the Bloom Taxonomy indicator.

The second syntax is to justify the concept. In this step, the teacher assigns students to read and understand the contents of the book and determine concepts related to the problem topic being discussed. This is done because reading is not only about expressing what is in the text but also includes the process of building meaning in learning (Akin et al., 2015; Yu-hui et al., 2010). Reading also involves the coordination of affective and cognitive components such as observing, focusing, having perceptions, memorizing, building relationships, analyzing, and interpreting which can empower students’ critical thinking skills (Mahanal et al., 2019). Then the teacher appoints representatives of each group to question concepts that are not yet understood. This is done so that students are right in choosing the concepts used so that they have a positive impact on the development of students’ critical thinking skills (Karagol & Bekmezci, 2015). The ability to master concepts also has a relationship with the level of students’ critical thinking skills in mathematics (Chukwuyenum, 2013; Radulovic & Stancic, 2017). Formulating questions and asking questions can also train students to have critical thinking skills (Vieira et al., 2011). Furthermore, according to Lubliner (2004), asking questions is a tool that can be used to improve students’ thinking skills.

The third syntax is to investigate the problem. In this step the teacher presents problems about analyzing, evaluating, and concluding; students create hypotheses, collect data, and test hypotheses in group discussions. The presentation of the problem is carried out with the aim of stimulating students to solve problems from the information obtained (Dwyer et al., 2014; Fajrianthi et al., 2016). Furthermore, according to Rahaju, et al (2019), in mathematics learning, critical thinking can be developed by means of mathematical problem solving activities. Problems used to improve critical thinking skills are analyzing, evaluating, and concluding (Haghparast et al., 2014; Vong & Kaewurai, 2017). Furthermore, students immediately identify problems, formulate hypotheses, record observations and ideas, and test hypotheses in group discussions. Group discussions conducted by students in solving problems can improve critical thinking skills (King et al., 2013; Schoenberger & Spiller, 2014; Sofroniou & Poutos, 2016). In this group discussion activity students are required to understand the problem so that their critical thinking skills can increase. This is done because understanding problems can lead students to critical thinking processes (Aljaberi, 2015). In this group discussion there was social interaction between students to share different views or ideas so that students were active in developing critical thinking skills in mathematics (Saputri et al., 2019). In this step,
the teacher also provides scaffolding to students who have difficulty solving the questions given. The scaffolding given is feedback in the form of questions that require students to find their own answers so that students can be trained to improve critical thinking skills.

The fourth syntax is presenting ideas. In this step, a presentation of the ideas that have been obtained during group discussions in front of the class is carried out from group representatives appointed by the teacher. This is done because students can improve their critical thinking skills by expressing student opinions and challenging other students’ ideas in order to obtain solutions to the problems given (Brindley et al., 2009). During the presentation, students explain the ideas from the results of group discussions. In addition, students who did not present could judge ideas from other groups and then come up with new ideas based on the right reasons. This is done because it assesses ideas with reasons and provides alternative solutions for activities that can improve critical thinking skills (Florea & Hurjui, 2015; Monteleone et al., 2018). Furthermore, according to As’sari (2014) teaching critical thinking can be done by asking students to consider alternative ideas then asking students to think about the reasons. In this step, students are actively involved in learning so that they can improve their critical thinking skills in mathematics (McCormick et al., 2015; Killian & Bastas, 2015). Then in this step the teacher only provides scaffolding so that students return to the problems being discussed and find the correct answers when there is a debate that is far from the context of the problems being discussed.

The fifth syntax is to evaluate. In this step, the teacher instructs students to correct the answers obtained during the presentation. This is done because students are actively involved in the learning process to agree or disagree with information, assess to determine the truth, change wrong information to create new ideas, and compare various information to get conclusions (Florea & Hurjui, 2015; Husnaeni, 2016; Susanti & Hartono, 2019). Through such learning activities, students can learn to think critically (Michalsky & Kramarski, 2015). After students find wrong answers, then the teacher invites students to reveal the reasons for the wrong answers. This is done because assessing the reasons correctly or identifying the wrong opinion of a particular problem can train students critical thinking skills (Mason, 2008; Fahim & Ahmadi, 2012).

The sixth syntax is to conclude. In this step, students are required to formulate conclusions from what was obtained in the previous step. This is because making the right conclusions is part of the critical thinking skills (Moon, 2008; Eggen & Kauchak, 2012; Dwyer et al, 2014). In this step, the teacher also appoints several students to express the conclusions obtained from the results of the evaluation that has been carried out. The purpose of this step is for students to focus, find reasons, and carry out a comprehensive analysis to make conclusions correctly (Karakoc, 2016; Ulger, 2018). Then the teacher corrects the conclusions of the students who have been appointed and the teacher concludes in general from the results of the student’s answers. After that students make conclusions in their own language that is easy to understand.
The validity of the M6 learning model product is tested through the expert validation stage, the practitioner’s assessment, and students as a small group of product users. The product of developing the M6 learning model before it is validated is called prototype-I. Then the prototype-I was validated by experts to assess its validity. The results of expert validation on the product of the M6 learning model (prototype-I) show that there are still many weaknesses and shortcomings. It is indicated that the product of the M6 learning model is not yet feasible to be tested. Therefore, it must be revised beforehand until it really is said to be feasible to be tested in a small group. According to Nieveen (2013), the learning device developed is said to be of quality if it meets the validity requirements, which are suitable for use in the learning process. The product of the first revised M6 learning model produces prototype-II which is then re-validated by the experts. Based on the validation results, it was found that the development of model books and learning tools carried out in the lesson plans, teacher books, and student books were in the validity criteria with the “valid” category and could be applied. Therefore, based on a review of all aspects of the quality assessment of the M6 learning model, it shows that the M6 learning model meets valid requirements so that it can be used as a solution to improve critical thinking skills in mathematics.

The prototype-II, which has been valid, then simulates small group trials of the model teacher and observers. The purpose of this simulation is for the model teacher and observers to understand the steps that must be taken when carrying out the learning model process in the classroom by applying the M6 learning model. In addition to simulating the model teacher and observer, a student book readability test was also carried out. The purpose of the readability test is to get input and suggestions that the student book can clearly be read, understood, and can be implemented in a class that is the subject of a large group trial. The results of the assessment of practitioners and students in the small group trial showed that the product of the first revision of the M6 learning model, namely prototype-II, was in accordance with the developed objectives because it was included in the “feasible” category. However, there are some inputs from practitioners (teachers) on the M6 learning model product, resulting in revision. In student assessments based on student response questionnaires to small group trials, it shows that student books are effectively applied to learning in small classes so that they can be continued to large group testing (large scale). The product of the M6 learning model which was revised from the readability of the small group trial produced prototype-III which was then tested in large groups. A valid learning model can be used to see the next aspect, namely the practicality and effectiveness of the model being developed (Limatahu et al., 2018; Madeali & Prahani, 2018; Plomp, 2013).
Conclusion

Learning Model M6 is a learning model designed to improve critical thinking skills in mathematics. The M6 learning model consists of six syntax, namely: (1) focusing on initial abilities, (2) justifying concepts, (3) investigating problems (4) presenting ideas, (5) evaluating, and (6) concluding. The results showed that the M6 learning model has a reliability value of 86.26% which means it has a high reliability value. While the average value of the validity is 3.47, which means that the M6 learning model has met the validity criteria of the model. It can be concluded that the M6 learning model meets the requirements (valid in content and construct, and reliable) by experts. The implication of this research is that the quality M6 learning model can be used to improve students’ critical thinking skills in mathematics. This study has limitations, namely the M6 learning model has not been tested for its practicality and effectiveness. The focus of further research is needed to test the practicality and effectiveness of the M6 learning model to improve students’ critical thinking skills in mathematics.

References

Abed, S., Davoudi, A. M. H., & Hoseinzadeh, D. (2015). The effect of synectics pattern on increasing the level of problem solving and critical thinking skills in students of Alborz province. WALIA Journal, 31(1), 110–118.

Acharya, B. R. (2017). Factors affecting difficulties in learning mathematics by mathematics learners. International Journal of Elementary Education, 6(2), 8–15. https://doi.org/10.11648/j.ijeedu.20170602.11

Ahuna, K. H., Buffalo, C. G. T., & Kiener, M. (2014). A new era of critical thinking in professional programs. Transformative Dialogues: Teaching & Learning Journal, 7(3), 1–9.

Akgun, A., & Duruk, U. (2016). The investigation of preservice science teachers’ critical thinking dispositions in the context of personal and social factors. Science Education International, 27(1), 3–15.

Akın, F., Koray, O., & Tavukçu, K. (2015). How effective is critical reading in the understanding of scientific texts. Procedia – Social and Behavioral Sciences, 174, 2444–2451.

Aktas, G. S., & Unlu, M. (2013). Critical thinking skills of teacher candidates of elementary mathematics. Procedia-Social and Behavioral Sciences, 93, 831–835.

Alcantara, E. C., & Bacsa, J. M. P. (2017). Critical thinking and problem solving skills in mathematics of grade-7 public secondary students. Asia Pacific Journal of Multidisciplinary Research, 5(4), 21–27.

Alismail, H. A., & Mcguire, P. (2015). 21st Century standards and curriculum: current research and practice. Journal of Education and Practice, 6(6), 150–155.
Aljaberi, R. (2015). Creative and its relation to academic achievement and teaching performance of pre-service female teachers in Ajman University in UAE. *Procedia – Social and Behavioral Sciences, 174*, 560–567.

As’ari, A. R. (2014, April 12–13). *Ideas for developing critical thinking at primary school level* [Seminar presentation]. An International Seminar Addressing High Order Thinking at Muhammadiyah Makasar University.

As’ari, A. R. (2016, August 13). *Tantangan pengembangan dimensi keterampilan standar kompetensi lulusan kurikulum 2013 edisi revisi ditinjau dari rumusan kompetensi dasar matematika jenjang sekolah menengah pertama* [The challenge of developing the dimensions of competency standards for graduates of the revised 2013 curriculum in terms of the formulation of basic mathematics competencies for junior high school level] [Seminar presentation]. National Seminar on Mathematics and Its Learning. Department of Mathematics, FMIPA UM.

Avsec, S., & Kocijancic, S. (2014). Effectiveness of inquiry-based learning: how do middle school students learn to maximise the efficacy of a water turbine? *International Journal of Engineering Education, 30*(6), 1436–1449.

Bajracharya, I. K. B. (2010). Teaching mathematics through abc model of critical thinking. *Mathematics Education Forum, 2*(28), 13–17.

Beers, S. Z. (2011). 21st Century skills: preparing students for their future. *STEM. 1–6. http://cosee.umaine.edu/files/coseeos/21st_century_skills.pdf*

Boa, E. A., Wattanatorn, A., & Tagong, K. (2018). The development and validation of the blended socratic method of teaching (bsmt): an instructional model to enhance critical thinking skills of undergraduate business students. *Kasetsart Journal of Social Sciences, 39*(1), 81–89. Advance Online Publication. [https://doi.org/10.1016/j.kjss.2018.01.001](https://doi.org/10.1016/j.kjss.2018.01.001)

Borich, G. D. (1994). *Observational skills for effective teaching* (4th ed.). NJ: MacMillan.

Brindley, J., Blaschke, L. M., & Walti, C. (2009). Creating effective collaborative learning groups in an online environment. *The International Review of Research in Open and Distributed Learning, 10*(3), 1–18. [https://doi.org/10.19173/irrodl.v10i3.675](https://doi.org/10.19173/irrodl.v10i3.675)

Buhaerah. (2016). Pengembangan model pembelajaran matematika yang mengembangkan kemampuan berpikir kritis [Development of mathematical learning models that develop critical thinking skills] *Jurnal Pendidikan Matematika, 1*(1), 17–36.

Chukwuyenum, A. N. (2013). Impact of critical thinking on performance in mathematics among secondary school students in lagos state. *IOSR Journal of Research & Method in Education (IOSR-JRME), 3*(5), 18–25. [http://www.iosrjournals.org/iosr-jrme/papers/Vol-3Issue-5/D03518_25.pdf](http://www.iosrjournals.org/iosr-jrme/papers/Vol-3Issue-5/D03518_25.pdf)

Clements, D. (2013). The progress of education reform. *education commission of the states, 14*(5), 1–7. [http://www.du.edu](http://www.du.edu)

Cooper, J. D., Robinson, M. D., Slansky, J. A., & Kiger, N. D. (2018). *Literacy helping students construct meaning 10th*. Boston: Cengage Learning.
Costley, J. (2016). The effects of instructor control on critical thinking and social presence: variations within three online asynchronous learning environments. *The Journal of Educators Online-JEO*, 13(1), 109–171.

Dehghani, M., Mirdoraghi, F., & Pakmehr, H. (2011). The role of graduate students’ achievement goals in their critical thinking disposition. *Procedia Social and Behavioral Sciences*, 15, 2426–2430.

Dewalsche, S. A. (2015). Critical thinking, questioning, and student engagement in korean university English courses. *Linguistics and Education*, 32, 131–147.

Duran, M., & Dokme, I. (2016). The effect of the inquiry-based learning approach on student’s critical thinking skills. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(12), 2887–2908.

Duron, R., Limbach, B., & Waugh, W. (2006). Critical thinking framework for any discipline. *International Journal of Teaching and Learning in Higher Education*, 17(2), 160–166.

Dwyer, C. P., Hogan, M. J., & Stewart, I. (2014). An integrated critical thinking framework for the 21st century. *Thinking Skills and Creativity*, 12, 43–52. https://doi.org/10.1016/j.tsc.2013.12.004

Eggen, P., & Kauchak, D. (2012). *Learning strategies and models teach content and thinking skills*. Jakarta: PT Index Permata Putri Media.

Elder, L. & Paul, R. (2010). *Critical thinking development: a stage theory with implications for instruction*. Tomales, CA: Foundation for Critical Thinking.

Ennis, R. H. (2008). Nationwide Testing of critical thinking for higher education. *Teaching Philosophy*, 1–26. Vigilance: University of Illinois UC. (Online), http://www.criticalthinking.net/NatCTTest11_18_07U.pdf

Ennis, R. H. (2011). The nature of critical thinking: an outline of critical thinking dispositions and abilities. *International Conference on Thinking at MIT, Cambridge*. http://faculty.ed.uiuc.edu/rhennis

Fahim, M., & Ahmadi, H. (2012). Critical thinking, content schemata, and EFL readers’ comprehension and recall. *Journal of Comparative Literature & Culture*, 1(1), 23–28.

Fahim, M., & Pezeshk, M. (2012). Manipulating critical thinking skills in test taking. *International Journal of Education*, 4(1), 153–160.

Fajrianty, Hendriani, W., & Septarini, B. G. (2016). Development of critical thinking test with item response theory approach. *Jurnal Penelitian Dan Evaluasi Pendidikan*, 20(1), 45–55. https://doi.org/10.21831/pep.v20i1.6304

Fisher, R. (2003). Teaching thinking: philosophical enquiry in the classroom. *London, Continuum*, 4(1), 153–160.

Florea, N. M. & Hurjui, E. (2015). Critical thinking in elementary school children. *Procedia – Social and Behavioral Sciences*, 180, 565–572. https://doi.org/10.1016/j.sbspro.2015.02.161

Ghazivakili, Z., Norouzi Nia, R., Panahi, F., Karimi, M., Gholsorkhi, H., & Ahmadi, Z. (2014). The role of critical thinking skills and learning styles of university students in their academic performance. *Journal of Advances in Medical Education & Professionalism*, 2(3), 95–102. Retrieved from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4235550/
Gholami, M., Moghadam, P. K., Mohammadipoor, F., Tarahi, M. J., Sak, M., Toulabi, T., & Pour, A. H. H. (2016). Comparing the effects of problem-based learning and the traditional lecture method on critical thinking skills and metacognitive awareness in nursing students in a critical care nursing course. *Nurse Education Today, 45*, 16–21. https://doi.org/10.1016/j.nedt.2016.06.007

Hadi, S. A, Susantini, E., & Agustini, R. (2018). Training of students’ critical thinking skills through the implementation of a modified free inquiry model. *IOP Conference Series: Journal of Physics: Conference Series, 947*(1), 012063. https://doi.org/10.1088/1742-6596/947/1/012063

Haghpastarast, M., Nasaruddin, F. H., & Abdulllah, N. (2014). Cultivating critical thinking through e-learning environment and tools: a review. *Procedia – Social and Behavioral Sciences, 129*, 527–535.

Huang, H. F., Ricci, F. A., & Mnatsakanian, M. (2016). Mathematical teaching strategies: pathways to critical thinking and metacognition. *International Journal of Research in Education and Science (IJRES), 2*(1), 190–200.

Husnaeni (2016). The enhancement of mathematical critical thinking ability of Aliyah Madrasas student model using Gorontalo by interactive learning setting cooperative model. *Journal of Education and Practice, 7*(8), 159–164.

Hussain, I. & Munshi, P. (2011). Identifying reading preferences of secondary school students. *Creative Education, 2*(5), 429–434.

Hussin, W. N. T. W., Harun, J., & Shukor, N. A. (2019). Online interaction in social learning environment towards critical thinking skill: a framework. *Journal of Technology and Science Education, 9*(1), 4–12. https://doi.org/10.3926/jotse.544

Innabi, H. (2003). Aspects of critical thinking in classroom instruction of secondary school mathematics teachers in Jordan. The Mathematics Education into the 21st Century Project Proceedings of the International Conference The Decidable and the Undecidable in Mathematics Education. Brno, Czech Republic, 124–129.

Irwanto, Saputro, A. D., Rohaeti, E., & Prodjosantoso, A. K. (2018). Promoting critical thinking and problem solving skills of preservice elementary teachers through process-oriented guided-inquiry learning (POGIL). *International Journal of Instruction, 11*(4), 777–794. https://doi.org/10.12973/iji.2018.11449a

Joyce, B., Weill, M., & Calhoun, E. (2009). *Models of teaching* (8th ed.). Boston: Allyn & Bacon.

Karagöl, I., & Bekmezci, S. (2015). Investigating academic abilities and critical thinking dispositions of teacher candidates. *Journal of Education and Training Studies, 3* (4). doi: https://doi.org/10.1111/jets.v3i4.834

Karakoc, M. (2016). The significance of critical thinking ability in terms of education. *International Journal of Humanities and Social Science, 6*(7), 81–84.

Karbalei, A. (2012). Critical thinking and academic achievement. *Íkala, Revista de Lenguaje y Cultura, 17*(2), 121–128.

Kelemen, G. (2014). Specific methods for increasing learning abilities in students. *Procedia-Social and Behavioral Sciences, 116*, 4505–4510.
Keller, M. J. (2010). *Motivational design for learning and performance the arcs model approach*. USA: Springer.

Killian, M., & Bastas, H. (2015). The effects of an active learning strategy on students’ attitudes and students’ performances in introductory sociology classes. *Journal of the Scholarship of Teaching and Learning, 15*(3), 53–67.

King, F. J., Goodson, L., & Rohani, F. (2013). *Higher order thinking skills. Center for Advancement of Learning and Assessment*, 1–177. [http://www.cala.fsu.edu/files/higher_order_thinking_skills.pdf](http://www.cala.fsu.edu/files/higher_order_thinking_skills.pdf)

Kusaeri, & Aditomo, A. (2019). Pedagogical beliefs about critical thinking among Indonesian mathematics pre-service teachers. *International Journal of Instruction, 12*(1), 573–590.

Lamb, S. (2017). *Key skills for the 21st century: an evidence-based review*. Melbourne, Australia: State of New South Wales.

Lambert, J. & Cuper, P. (2008). Multimedia technologies and familiar spaces: 21st century teaching for 21st century learners. *Contemporary Issues in Technology and Teacher Education, 8*(3), 264–276.

Lee, W., Chiang, C. H., Liao, I. C., Lee, M. L., Chen, S. L., & Liang, T. (2012). The longitudinal effect of concept map teaching on critical thinking of nursing students. *Nurse Education Today, 33*(10), 1219–1223.

Lim, S. C., Cheung, W. S., & Hew, K. F. (2011). Critical thinking in asynchronous online discussion: an investigation of student facilitation techniques. *New Horizons in Education, 59*(1), 52–65.

Limatahu, I., Suyatno, Wasis, & Prahani, B. K. (2018). The effectiveness of CCDSR learning model to improve skills of creating lesson plan and worksheet science process skills (SPS) for pre-service physics teacher. *Journal Physics: Conference Series, 997*(1), 012032.

Lubliner, S. (2004). Help for struggling upper-grade elementary readers. *The Reading Teacher, 57*(5), 430–438.

Madeali, H., & Prahani, B. K. (2018). Development of multimedia learning-based inquiry on vibration and wave material. *Journal Physics: Conference Series, 997*(1), 012029.

Mahanal, S., Zubaidah, S., Sumiati, I. D., Sari, T. M., & Ismirawati, N. (2019). RICOSRE: A learning model to develop critical thinking skills for students with different academic abilities. *International Journal of Instruction, 12*(2), 417–434. [https://doi.org/10.29333/iji.2019.12227a](https://doi.org/10.29333/iji.2019.12227a)

Maharani, I. P., & Subanji (2018). Scaffolding based on cognitive conflict in correcting the students’ algebra errors. *International Electronic Journal of Mathematics Education, 13*(2), 67–74. [https://doi.org/10.12973/iejme/2697](https://doi.org/10.12973/iejme/2697)

Marashi, H. & Nouchirwani, S. (2011). The comparative impact of content-based and task based teaching in a critical thinking setting on EFL learners’ reading comprehension. *Journal of English studies, 1*(4), 27–39.

Marcut, L. (2005). Critical thinking applied to the methodology of teaching mathematics. *Educatia Matematica, 1*(1), 57–66.
Maricica, S., & Spijunovicb, K. (2015). Developing critical thinking in elementary mathematics education through a suitable selection of content and overall student performance. *Procedia – Social and Behavioral Sciences, 180*, 653–659.

Marin, L., & Halpern, D. (2011). Pedagogy for developing critical thinking in adolescents: Explicit instruction produces greatest gains. *Thinking Skills and Creativity, 6*, 1–13.

Mason, M. (2008). *Critical thinking and learning*. Australia: Blackwell Publishing.

Massa, S. (2014). The development of critical thinking in primary school: the role of teachers’ beliefs. *Procedia – Social and Behavioral Sciences, 141*, 387–392.

McCormick, N. J., Clark, L. M., & Raines, J. M. (2015). Engaging students in critical thinking and problem solving: a brief review of the literature. *Journal of Studies in Education, 5*(4), 100–113.

Michalsky, T., & Kramarski, B. (2015). Prompting reflections for integrating self-regulation into teacher technology education. *Teachers College Record, 117*(5), 1–38.

Monteleone, C., White, P., & Geiger, V. (2018). Defining the characteristics of critical mathematical thinking. *Proceedings of the 41st Annual Conference of the Mathematics Education Research Group of Australasia*, 559–566. Auckland: MERGA.

Moon, J. (2008). *Critical thinking an exploration of theory and practice*. USA: Routledge.

Moreno, R. (2010). *Educational psychology*. New York: Jhon Wiley & Sonc, Inc.

Muhlisin, A., Susilo, H., Amin, M., & Rohman, F. (2016). Improving critical thinking skills of college students through RMS model for learning basic concepts in science. *Asia-Pacific Forum on Science Learning and Teaching, 17*(1), 1–24.

Nieveen, N. (2013). *Educational design research*. Netherlands: Netherlands Institute for Curriculum Development (SLO).

Norris, S. P. (1985). Synthesis of research on critical thinking. *Educational Leadership, 42*(8), 40–45.

Pappas, E., Pierrakos, O., & Nagel, R. (2013). Using bloom’s taxonomy to teach sustainability in multiple contexts. *Journal of Cleaner Production, 48*, 54–64. https://doi.org/10.1016/j.jclepro.2012.09.039

Paul, R., & Elder, L. (2014). *Critical thinking: tools for taking charge of your professional and personal life*. Pearson FT Press. http://ptgmedia.pearsoncmg.com/images/9780133115284/samplepages/0133115283.pdf

Peter, E. E. (2012). Critical thinking: essence for teaching mathematics and mathematics problem solving skills. *African Journal of Mathematics and Computer Science Research, 5*(3), 39–43.

Pithers, R. T & Soden, R. (2000). Critical thinking in education: a review. *Educational Research, 45* (3), 237–249. https://doi.org/10.1080/001318800440579

Plomp, T. (2013). *Educational design research*. Netherlands: Netherlands Institute for Curriculum Development (SLO).

Prayogi, S., Yuanita, L., & Wasis (2018). Critical-Inquiry-Based-Learning: model of learning to promote critical thinking ability of Pre-service Teachers. *IOP Conference Series: Journal of Physics: Conference Series, 947*, 012013 https://doi.org/10.1088/1742-6596/947/1/012013

Qatipi, S. (2011). Questioning and its true values in the process of learning and teaching to promote critical thinking. *Problems of Education in the 21st Century, 38*, 71–81.
Quitadamo, I. J., Faiola, C. L., Johnson, J. E., & Kurtz, M. J. (2008). Community-Based Inquiry improves critical thinking in general education biology. *CBE Life Science Education, 7*(3), 327–337.

Radulović, L., & Stančić, M. (2017). What is needed to develop critical thinking in schools. *CEPS Journal, 7*(3), 9–25. [https://files.eric.ed.gov/fulltext/EJ1156618.pdf](https://files.eric.ed.gov/fulltext/EJ1156618.pdf)

Rahaju, R., Purwanto, P., Parta, I. N., & Rahardjo, S. (2019). Students’ critical thinking skills in making mathematical problems. *IOP Conference Series: Journal of Physics: Conference Series IOP Conf. Series: Journal of Physics: Conference Series*1318. doi:10.1088/1742-6596/1318/1/012094

Roekel, D. V (2012). *Preparing 21st century students for a global society: an educator guide to the “four cs”*. USA: National Education Association.

Rusmansyah, Yuanita, L., Ibrahim, M., & Isnawati, Prahani, B. K. (2019). Innovative chemistry learning model: improving the critical thinking skill and self-efficacy of pre-service chemistry teachers. *Journal of Technology and Science Education, 9*(1), 59–76. [https://doi.org/10.3926/jotse.555](https://doi.org/10.3926/jotse.555)

Sadijah, C. (2007). Critical attitude and problem-solving abilities of female students by using constructivism mathematics learning. *MIPA, 36*(2), 133–146.

Safdar, M., Hussain, A., Shah, I., & Rifai, Q. (2012). Concept maps: an instructional tool to facilitate meaningful learning, *European Journal of Educational Research, 1*(1), 55–64.

Salam, M., Ibrahim, N., & Sukardjo, M. (2019). Effects of instructional models and spatial intelligence on the mathematics learning outcomes after controlling for students’ initial competency. *International Journal of Instruction, 12*(3), 699–716. [https://doi.org/10.29333/iji.2019.12342a](https://doi.org/10.29333/iji.2019.12342a)

Samo, D. D., Darhim, & Kartasasmita, B. (2017). Developing contextual mathematical thinking learning model to enhance higher-order thinking ability for middle school students. *International Education Studies, 10*(12), 17–29.

Saputra, M. D., Joyoatmojo, S., Wardani, D. K., & Sangka, K. B. (2019). Developing critical-thinking skills through the collaboration of jigsaw model with problem-based learning model. *International Journal of Instruction, 12*(1), 1077–1094.

Saputri, A. C., Sajidan, Rinanto, Y., Afandi, & Prasetyanti, N. M. (2019). Improving students’ critical thinking skills in cell-metabolism learning using stimulating higher order thinking skills model. *International Journal of Instruction, 12*(1), 327–342.

Schoenberger, Orgad. M., & Spiller, D. (2014). Critical thinkers and capable practitioners. *Journal of Communication Management, 18*(3), 210–221. [https://doi.org/10.1108/JCOM-11-2012-0085](https://doi.org/10.1108/JCOM-11-2012-0085)

Sendag, S., & Ferhan, O. H. (2009). Effects of an online problem based learning course on content knowledge acquisition and critical thinking skills. *Computers and Education, 53*, 132–141. [https://doi.org/10.1016/j.compedu.2009.01.008](https://doi.org/10.1016/j.compedu.2009.01.008)

Shieh, R. S., & Chang, W. (2014). Fostering student’s creative and problem-solving skills through a hands-on activity. *Journal of Baltic Science Education, 13*(4), 650–661.

Snyder, L. G., & Snyder, M. J. (2008). Teaching critical thinking and problem solving skills. *The Delta Pi Epsilon Journal, 50*(2), 90–99.

Sofroniou, A., & Poutos, K. (2016). Investigating the effectiveness of groupworking mathematics. *Educ. Sci, 6*(30), 1–15.

Susandi, A. D., Sadijah, C., As‘ari, A. R. & Susiswo (2018). Error analysis on prospective teacher in solving the problem of critical thinking mathematics with apos theory. *Advances in Social Science, Education and Humanities Research, 218*, 71–75, First International Conference on Science, Mathematics, and Education, (ICoMSE 2017), Atlantis Press.
Susandi, A. D., Sadijah, C., As'ari, A. R. & Susiswo (2019a). What error happened to inferences of senior high school students using mathematical critical thinking ability? *International Journal of Scientific & Technology Research, 8*(9), 507–511.

Susandi, A. D., Sadijah, C., As'ari, A.R. & Susiswo (2019b). Students’ critical ability of mathematics based on cognitive styles. *Journal of Physics: Conference Series*, 1315 012018.

Susandi, A. D., Sadijah, C., As'ari, A. R., & Susiswo (2020). M6 learning model: the framework to design a learning model that improves students’ critical thinking skill. *Systematic Reviews in Pharmacy, 11* (6), 1245–1254. [https://doi.org/10.31838/srp.2020.6.182](https://doi.org/10.31838/srp.2020.6.182)

Susanti, E., & Hartono (2019). Mathematical critical thinking and creative thinking skills: how does their relationship influence mathematical achievement? ICMSTTL 2019: *Proceedings of the 2019 International Conference on Mathematics, Science and Technology Teaching and Learning*, 63–66. [https://doi.org/10.1145/3348400.3348408](https://doi.org/10.1145/3348400.3348408)

Sulistiani, E., & Masrukan (2016). The importance of critical thinking in learning mathematics to face the challenges of the MEA. In PRISMA, *Proceedings of the National Mathematics Seminar*, 605–612.

Taleb, H. M., & Chadwick, C. (2016). Enhancing student critical and analytical thinking skills at a higher. *Journal of Educational and Instructional Studies, 6*(1), 66–77.

Thomas, T. (2011). Developing First year students' critical thinking skills. *Asian Social Science, 7*(4), 26–35.

Ulger, K. (2018). The effect of problem-based learning on the creative thinking and critical thinking disposition of students in visual arts education. *Interdisciplinary Journal of Problem-Based Learning, 12*(1). doi: [https://doi.org/10.7771/1541-5015.1649](https://doi.org/10.7771/1541-5015.1649)

Vieira, R. M., Tenreiro, V. C., & Martins, I. P. (2011). Critical thinking: conceptual clarification and its importance in science education. *Science Education International, 22*(1), 43–54.

Vong, S. A., & Kaewurai, W. (2017). Instructional model development to enhance critical thinking and critical thinking teaching ability of trainee students at regional teaching training center in Takeo province, cambodia. *Kasetsart Journal of Social Sciences, 38*, 88–95.

Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cambridge: Harvard University Press.

Walker, S. (2003). Active learning strategies to promote critical thinking. *Journal of Athletic Training, 38*(8), 263–267.

Wang, A. Y. (2012). Exploring the relationship of creative thinking to reading and writing. *Thinking Skills and Creativity, 7*, 38–47.

Wannapiroon, P. (2014). Development of research-based blended learning model to enhance graduate students’ research competency and critical thinking skills. *Procedia – Social and Behavioral Sciences, 136*, 486–490.

Wartono, W., Hudha, M. N., & Batlolona, J. R. (2018). How are the physics critical thinking skills of the students taught by using inquiry-discovery through empirical and theoretical overview? *EURASIA Journal of Mathematics, Science and Technology Education, 14*(2), 691–697.

Yacoubian, H. A. (2015). A framework for guiding future citizens to think critically about nature of science and socioscientific issues. *Canadian Journal of Science, Mathematics and Technology Education, 15*(3), 248–260. [https://doi.org/10.1080/14926156.2015.1051671](https://doi.org/10.1080/14926156.2015.1051671)

Yeh, Y. C. (2009). Integrating e-learning into the direct-instruction model to enhance the effectiveness of critical thinking instruction. *InstrSci, 37*, 185–203.

Yu-hui, L., Li-rong, Z., Yue, N. (2010). Application of schema theory in teaching college English reading. *Canadian Social Science, 6*(1), 59–65.
Zhao, C., Pandian, A., & Singh, M. K. M. (2016). Instructional strategies for developing critical thinking in EFL classrooms. *English Language Teaching, 9*(10), 14–21.

M6 mokymosi modelio kūrimas matematinio kritinio mąstymo įgūdžiams tobulinti

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## Santrauka

Šio tyrimo tipas yra moksliniai tyrimai ir plėtra (angl. R&D). Šio tyrimo tikslas – sukurti M6 mokymosi modelį, atitinkantį matematinio kritinio mąstymo įgūdžių tobulinimo tinkamumo kriterijus. M6 mokymosi modelis yra sukurtas iš akronimo, kurį sudaro: 1) dėmesys pradiniams gebėjimams (indonez. *Memfokuskan*); 2) sampratos pagrindimas (indonez. *Menjustifikasi*); 3) problemas ištyrimas (indonez. *Menyelidiki*); 4) idėjų pristatymas (indonez. *Mempresentasikan*); 5) įvertinimas (indonez. *Mengevaluasi*) ir (6) išvadų darymas (indonez. *Menyimpulkan*). M6 mokymosi modelio kokybės vertinimo instrumentą išnagrinėjo ir įvertino trys švietimo ekspertai: mokymosi modelio kūrimo srities profesorius, matematikos srities ekspertas ir vyresnysis matematikos mokytojas mokykloje. Mokymosi modelio kokybei vertinti pirmiausia buvo apskaičiuota patikumo reikšmė, o po to buvo apskaičiuojamas vidutinis validumo balas. Rezultatai rodo, kad M6 mokymosi modelio patikimumo reikšmė yra 86,26 proc., tai reiškia, kad jo patikimumo reikšmė yra didelė. Vidutinė validumo reikšmė yra 3,47, tai reiškia, kad M6 mokymosi modelė atitiko modelio validumo kriterijus. Galima daryti išvadą, kad M6 mokymosi modelės atitinka ekspertų keliamus reikalavimus (validus turinio ir konstrukto atžvilgiu bei patikimus).

## Esminiai žodžiai: M6 mokymosi modelis, matematinio kritinio mąstymo įgūdžiai.

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