Research on the mapping grinding of dimple surface with ordered pattern based on topological theory

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Abstract

Since the structured dimple surface is one of the important functional drag-reducing surfaces, the research on its efficient fabrication technology is of great significance for the practical application of this surface. Therefore, in order to grind structured dimple surface, a topological mapping grinding method for structured dimple surface was proposed based on grinding kinematics principle and point set topology. Firstly, the topological spaces of the workpiece and the grinding wheel were established, and the topological features of the structured dimple surface and the structured grinding wheel were extracted based on the analysis of the topography features of the structured surface with ordered pattern. Then, the topology mapping equation of the grinding process was constructed based on the analysis of the generating mechanism of the dimple surface about grinding geometry, and the structured grinding wheel was designed according to the topology mapping equation. Finally, according to the grinding geometry simulation, the influence of grinding parameters on the generating of dimple surface topography was studied, and the grinding experiment was carried out. The results show that the structured grinding wheel designed based on the topological features of the structured dimple surface can achieve the grinding of the structured dimple surface. The ground dimple surface is a topological dimple surface, and its feature parameters can be changed with the change of grinding parameters, but the topological feature attributes remain unchanged under the condition of satisfying the proper grinding speed ratio.

Keywords  Grinding · Topological grinding · Structured dimple surface · Structured grinding wheel

1 Introduction

“Structured,” “textured,” or “engineered” surfaces are important for improving the performance of mechanical workpieces or products [1]. In recent years, with the continuous development of surface biomimetic technology, the theoretical research on structured surfaces has continued to deepen, and its surface morphology has also tended to be diverse [2–4]. In particular, the research on bionic structured drag reduction surfaces has attracted more and more attention from scholars and engineers, and has become one of the important research directions in the field of surface engineering design [5–8]. However, how to put the theoretical research results of structured surfaces into practical engineering applications, the research on the high-efficiency or high-precision and low-cost manufacturing technology of the surface has become a bridge between theoretical research results and practical engineering. At present, the manufacturing technologies of structured surfaces mainly include rolling, embossing, electrolytic etching, laser ablation, photolithography masks, cutting and grinding technologies, etc. [9–14]. Among them, grinding technology is one of the economical or efficient technologies to achieve structured surface fabrication of mechanical parts with difficult-to-machine material properties or mass production properties. In the field of grinding structured surface, its research is mainly reflected in the following aspects.

In the research of using spiral grooved grinding wheel to grind structured surface, Stepień [15–19] first proposed a method of dressing spiral grooves on the surface of the grinding wheel to grind the surface texture of grooves, convex hulls, and dimples. The corresponding structured surface was cut out, and the creation mechanism of the surface was studied in depth. On this basis, Kim and Ko [20–22] used the CAM simulation method to design a parametric grinding...
wheel, and analyzed the influences of the dressing geometric parameters and grinding parameters on the surface morphology of the workpiece. Cao et al. [23] developed the dressing theory of the grinding wheel with spiral groove and the control strategy of grinding cylindrical structured surface. Liu et al. [24] developed the influence law of the dressing spiral groove on the structured surface morphology and grinding force of the workpiece, and established the structured morphology and grinding force model. The model prediction results and the experimental results have good consistency. Mohamed et al. [25] carried out the research on predicting the surface texture of the workpiece by the grinding wheel with circumferential groove. Through the study of the influence of grinding parameters, grinding direction and groove coefficients on the structured height, spacing, and angle on the surface structure, a surface prediction model was established to realize the prediction of structured morphology.

In order to realize the grinding of multi-form structured surfaces, Oliveira and Silva et al. [26–29] proposed a dressing method for grinding wheel, and the grinding wheel dressed by this method was used to grind friction reduction surfaces with various structured morphologies such as triangle wave, groove, dimple, and convex hull, and also provides the kinematic conditions for realizing the grinding of structured surface. Moreno et al. [30] applied this research to the grinding of the friction reduction surface of the machine tool guide rail, and studied the friction performance of the ground surface to improve the wear resistance of the guide rail, which proved that the application of this grinding technology is feasible.

In the grinding of structured groove surface for drag reduction, Denkena et al. [31–34] carried out the research of the profile grinding for the riblet surface. With the help of the shaping and dressing of the grinding wheel, the riblet surface on the compressor blade was ground, and the generating mechanism of the burrs during grinding process and the influence of the grinding wheel wear on the riblet geometry was also deeply studied, and corresponding solutions was given. At the same time, a new grinding wheel with a nutria-like tooth structure was proposed. Xie et al. [35, 36] carried out a research on the grinding of the micro-groove array on the rake face of cutting tool, which improved the cutting performance of the tool. Wu et al. [37] used lasers to structure the grinding wheel wit coarse-grained diamond and realized the micro-structured grinding on SiC material surface.

In terms of theoretical research on the design of grinding wheels for grinding structured surface, although in order to improve the grinding performance of the grinding wheel, the research on structured grinding wheels has made good progress [38]. However, the theoretical research on the design of structured grinding wheels for grinding complex structured surfaces is relatively weak. Monie et al. [39] conducted research on this problem and gave meaningful design strategies.

The abovementioned research has promoted the development of structured surface grinding very well. However, it can also be seen that in addition to the profile grinding method that can design the grinding wheel according to the structured surface morphology, in most cases, the grinding wheel is dressed to a certain structured morphology, and then the structured surface morphology of the workpiece is obtained by selecting the grinding parameters appropriately. The research on designing a structured grinding wheel based on the morphological features of the structured surface and then grinding the corresponding structured surface is relatively weak, which makes the study of the grinding theory of structured surface have a certain degree of passivity and not conducive to the practical application for grinding structured surface. Therefore, based on the point set topology theory [40] and the principle of grinding kinematics, this paper proposes a topological mapping grinding strategy for structured dimple surface.

2 Basic strategy of the topological grinding of dimple surface

As shown in Fig. 1, suppose the coordinate system of the grinding wheel is $S (O_r X_r Y_r Z_r)$, the coordinate system of the workpiece is $W (O_w X_w Y_w Z_w)$, and the center of $S (O_r X_r Y_r Z_r)$ is at the rotation center of the grinding wheel end face, and the center of $W (O_w X_w Y_w Z_w)$ is at the center of the first dimple being ground. In the grinding process of dimple surface, when the grinding parameters (grinding wheel speed $n_r$, feed speed $v_w$, and grinding depth $a_p$) meet the appropriate functional relationship, the abrasive cluster subset $S_{ij}$ on the grinding wheel surface with the base radius $R_i$ can remap and ground out the dimple subset $W_{ij}$ on workpiece surface in a broad sense. In this process, the abrasives with randomly distributed features in the abrasive cluster subset cut the workpiece surface with their respective cycloid trajectories, in which only the trajectory of the effective dynamic cutting edge can finally envelop the dimple subset $W_{ij}$. Therefore, in Euclidean space, it is very complicated and difficult in engineering to obtain the equation of the abrasive cluster subset $S_{ij}$ and its arrangement function by coordinate transformation according to the coordinate equation of the dimple subset $W_{ij}$ of the workpiece and its arrangement function. At the same time, in the grinding process shown in Fig. 1, under the condition of a given grinding depth $a_p$, due to the contact arc length effect between the grinding wheel and the workpiece, which makes the ground dimple profiles at the grind-in and grind-out areas in the feed speed direction of the workpiece not steep. Therefore, it is difficult to obtain dimples that are precisely consistent.
with the design by the replay grinding method. However, from the analysis of theoretical research results on structured surface for reducing friction, it can be known that the drag reduction or friction reduction performance of dimple surface is related to the geometric shape of the dimple surface, and more importantly, it is related to some feature parameters of its geometry [5–8, 41–43], which has topological features. Therefore, based on the topology theory, the structured grinding wheel is designed according to the topological features of the dimple surface of the workpiece, and then the topological surface of the workpiece is ground for meeting the use function of the dimple surface. Through this grinding method, not only the grinding process of the dimple surface can be simplified, but also the potential of structured grinding wheels can be expanded.

3 Theory of the topological grinding for dimple surface

3.1 Establishment of topological space in grinding process

In the process of dimple grinding as shown in Fig. 1, the Euclidean space of the grinding wheel and the Euclidean space of the workpiece are both measurable spaces, which satisfy the relevant elements of the topological space [40]. Therefore, if the topological space of the grinding wheel is \( S_T \) and the topological space of the workpiece is \( W_T \), the grinding process of the dimple surface can be regarded as the homeomorphic mapping process of the grinding wheel space \( S_T \) to the workpiece space \( W_T \) [40], namely:

\[
f : S_T \rightarrow W_T
\]

where \( S_T = [\bigcup S_{ij}, i = 1, 2, 3, \ldots, I_s; j = 1, 2, 3, \ldots, J_t] \) is represented as the composition of abrasive cluster subset space, and \( I_s \) and \( J_t \) are the maximum ordinals of the abrasive cluster structured units in the axial direction and circumferential direction of the wheel. \( W_T = [\bigcup W_{ij}, i = 1, 2, 3, \ldots, I_w; j = 1, 2, 3, \ldots, J_w] \) is represented as the composition of dimple subset space on workpiece surface, and \( I_w \) and \( J_w \) are the maximum ordinals of the rows and columns on the workpiece surface corresponding to the arrangement of the abrasive cluster subset. \( f = f(n_s, v_w, a_p) \) is a topological mapping function determined by grinding parameters and grinding wheel geometric parameters. In the grinding process, \( S_{ij} \) with convex set features is mapped to an anticonvex set \( W_{ij} \) on the workpiece surface with the \( 1/n_s \) as the cycle. Of course, this is only an abstract expression in the sense of topology, and the actual grinding process is a physical process in which the dimple surface is obtained through the removing the material of ground workpiece.

3.2 Topology analysis and modeling of structured dimple surfaces

3.2.1 Modeling of the structured dimple surface

From the perspective of morphology, structured friction-reducing surface with dimples has a variety of geometric shapes, and each structured dimple surface is formed by dimple elements with specific shapes arranged on the workpiece surface in accordance with certain rules. The shape, size, and arrangement of the dimples affect the surface performance for friction-reducing [41–43]. For example, the shape of the dimple has a spherical crown, an ellipsoidal crown, a rectangle, a triangle, etc., and the arrangement of the dimples can be array, staggered, phyllotactic, and random or arrangement following a specific function. In order to facilitate discussion and analysis of issues, taking the array, staggered, and phyllotactic arrangement of spherical crown-shaped dimples as an example, the topological feature modeling and analysis are carried out. Figure 2 shows three arrangements of ellipsoidal crown-shaped dimples in Euclidean space.

In Fig. 2a, the mathematical description equation of Euclidean space is given by taking the elliptic dimple structure unit as an example. For the convenience of studying the
The contour is deformed topologically into ellipsoid crown or spherical crown shape, and the deformation does not overlap, does not produce sharp points, and does not change convex-concave of the contour. Therefore, taking the center of the dimple as the center of the elliptical dimple unit and keeping it consistent with the workpiece coordinate system, the mathematical model of the dimple is as follows:

\[
\begin{align*}
\ell^2 &= \frac{2h^2w^2}{c_w^2} - \frac{4c_w(2h - c_w)c_w^2}{(w_w)^2} - h_w + c_w \\
\frac{4c_w^2}{\ell_w^2} + \frac{4c_w^2}{\ell_w^2} &\leq 1
\end{align*}
\]

where \(c_w\) is the radius of the \(z_w\)-axis of the ellipse where the ellipsoid crown is located, and \(l_w\), \(w_w\), and \(h_w\) are the length, width, and depth of the dimple unit respectively.

When \(l_w > w_w\), the shape of the dimple unit is elliptic crown. When \(l_w = w_w\), the shape of the dimple unit is spherical crown.

For the three regular dimple arrangements shown in Fig. 2b, c, and d, there is a certain arrangement interval and position deviation between adjacent dimple units. Therefore, let the interval between dimple units in the same row or column form the arrangement cycle of the \(x_w\)-axis direction and the \(y_w\)-axis direction as \(T_{wx}\) and \(T_{wy}\) respectively. The deviation between the positions of different rows and columns results in the phase differences of \(\varphi_{wx}\) and \(\varphi_{wy}\) in the two directions. For the array pattern, the arrangement interval of the structural unit is controlled by the arrangement cycle \(T_{wx}\) and \(T_{wy}\). The rows and rows are arranged neatly, and the phase difference is \(\varphi_{wx} = \varphi_{wy} = 0\). The cycle of staggered pattern is \(T_{wx}\) and \(T_{wy}\). In the case of equal arrangement, the phase difference between even and odd rows in the staggered pattern is \(\varphi_{wx} = T_{wx}/2\), and the phase difference between even and odd columns is \(\varphi_{wy} = T_{wy}/2\). The phyllode arrangement here is the expansion arrangement of the Iterson G Van model [44] based on the cylindrical arrangement with radius \(R_b\) and growth order \(c\), which is an arrangement on a plane with an expansion length of \(2\pi R_b\) [45]. The cycle of expansion is \(T_{wx} = T_{wy} = 0\), the phase difference \(\varphi_{wx} = 2\pi R_b [(i - 1)\theta/360 - m + 1]\) in the \(x_w\) direction, and the phase difference \(\varphi_{wy} = c(i - 1)\) in the \(y_w\) direction, and each dimple unit is not in the same row and column. Then Eq. (1) can be regarded as the basic dimple, and the mathematical equation of the central position of any dimple can be established. The equation is as follows:

(a) Array pattern:

\[
\begin{align*}
x_{wo} &= (j - 1)T_{wx} \\
y_{wo} &= (i - 1)T_{wy} \\
z_{wo} &= 0
\end{align*}
\]

(b) Staggered pattern:

\[
\begin{align*}
x_{wo} &= (j - 1)T_{wx} \\
y_{wo} &= (i - 1)T_{wy} \\
z_{wo} &= 0
\end{align*}
\]

(c) Phyllotactic pattern:
\[
\begin{align*}
    x_{ij} &= \varnothing_{wx} = 2\pi R_h \left[ \frac{(j-1)\theta}{360} - m + 1 \right] \\
y_{ij} &= \varnothing_{wy} = c(i - 1) \\
z_{ij} &= 0
\end{align*}
\] (5)

In Eqs. (2), (3), and (4), \( i = 1, 2, 3, \ldots, I_w; j = 1, 2, 3, \ldots, J_w, \) \((x_{ij}, y_{ij}, z_{ij})\) is the coordinate of the center point of the dimple \( W_{ij}, \) \( i \) is the ordinal number of the row, and \( j \) is the ordinal number of the column. For staggered pattern, \( i \) and \( j \) must be both odd or both even. For the phyllotactic pattern, \( \) the phyllotactic angle \( \theta = 137.508^\circ, j = 1, 2, 3, \ldots, I_w, \) is the ordinal number of dimple units. \( m \) is the ordinal number that controls the \( j \)th structural unit in the \( x_w \)-axis direction, and \( m \leq (j - 1)\theta / 360 \leq m + 1, m = 1, 2, 3, \ldots, M. \)

3.2.2 Establishment of the topological feature vectors of the dimple surface

According to the topological grinding strategy in Sect. 1, it is necessary to ensure that the main features of the workpiece surface remain unchanged during the topological grinding process. In order to simplify the problem, for the three kinds of structured dimple surfaces described in Eqs. (2) to (5), only some main parameters are extracted as topological invariants, which are used as topological properties of dimple surfaces. For the point set \( W_{ij} \) of the dimple unit, the length vector \( L_w, \) width vector \( W_w, \) and depth vector \( H_w, \) of the dimple unit can be used to establish the matrix \( W_d \) that controls the shape and size of the dimple unit and its anticonvex set attributes. As for the arrangement rule between units, the arrangement cycle \( (T_{wx}, T_{wy}) \) and the phase difference \( (\varnothing_{wx}, \varnothing_{wy}) \) can be used as the arrangement attribute between control units and the neighborhood attribute of adjacent dimples. Thus, \( L_w, W_w, \) and \( H_w, \) are extracted as the topological feature parameters of the dimple unit. \( T_{wx}, T_{wy}, \) \( \varnothing_{wx}, \) and \( \varnothing_{wy} \) were taken as the arranged topological feature parameters. The topological feature vector of dimple unit and its arranged topological feature vector were established. Topological feature vector is as follows:

(1) Topological feature vector of the dimple unit:
\[
W_d = [L_w W_w H_w 1]^T
\] (6)

In Eq. (6), \( L_w \) \( W_w \) and \( H_w \) are the set of feature parameters that describe the length, width, and depth of the geometric morphology of the dimple, respectively, and are a set of feature parameters corresponding to the direction of the maximum parameters \( l_w, w_w, \) and \( h_w, \) which can reflect the using function. In actual operation, the \( X_w O_w Y_w \) coordinate plane and the \( X_w O_w Z_w \) coordinate plane are equally spaced and discrete to determine these parameters.

(2) Arranged topological feature vector:
Let the topological feature vector of the dimple arrangement be as follows:
\[
W_a = [T_{wx} T_{wy} \varnothing_{wx} \varnothing_{wy}]^T
\] (7)

Then the feature parameter vectors corresponding to three specific arrangements are as follows:

(a) Array pattern:
\[
W_{aa} = [T_{wx} T_{wy} 0 0]^T
\] (8)

(b) Staggered pattern:
\[
W_{as} = [T_{wx} T_{wy} T_{wx}/2 T_{wy}/2]^T
\] (9)

(c) Phyllotactic pattern:
\[
W_{ap} = [0 0 2\pi R_h \left[ \frac{(j-1)\theta}{360} - m + 1 \right] c(j - 1)]^T
\] (10)

3.3 Topology analysis and modeling of structured grinding wheel

3.3.1 Modeling of the structured surface of grinding wheel

Figure 3 is the morphology of the structured grinding wheel corresponding to the arrangement of three kinds of abrasive clusters adopted in Fig. 1. In the process of topological mapping grinding, the circumscribed geometric properties and layout properties of the abrasive cluster \( S_w \) on the wheel surface will be topologically mapped along the \( X_w \)-axis of the workpiece coordinate system according to the different grinding parameters. Its radial topological properties will be mapped along the \( Z_w \)-axis of the workpiece according to the different grinding parameters, while the axial properties will remain unchanged and map to the \( Y_w \)-axis direction of the workpiece. For the convenience of discussion, the circumferential arc length, axial width, and radial height of the abrasive cluster of the structured grinding wheel are set as \( l_w, w_w, \) and \( h_w, \) respectively corresponding to \( l_w, w_w, \) and \( h_w, \) of the dimple unit. Then the point set of the abrasive cluster \( S_{ij} \) will be obtained from the point set of the dimple \( W_{ij} \) on the workpiece surface through topological mapping. Therefore, the equation of abrasive cluster unit on the grinding wheel surface is described as:
\[
f_x(x, y, z) = 0(x, y, z) \in S_{ij}
\] (11)

Suppose that the arc length arrangement cycle \( T_{sl} \) and axial cycle \( T_{sy} \) of the abrasive clusters in the grinding wheel
correspond to the $\varnothing_{sl}$ and $\varnothing_{sy}$ of the dimples on the workpiece surface, respectively; then, for the array pattern of abrasive clusters shown in Fig. 3a, there will be $\varnothing_{sl} = 0$ and $\varnothing_{sy} = 0$; for the staggered pattern shown in Fig. 3b, there are $\varnothing_{sl} = T_{sl}/2$ and $\varnothing_{sy} = T_{sy}/2$; for the phyllotactic pattern shown in Fig. 3c, each circle on the circumference of the grinding wheel and each axial generatrix have only one abrasive cluster unit, and the position of the abrasive cluster on the circumference of the grinding wheel can be described by the phase difference between each other; therefore, according to the Iterson G Van model [44] of the phyllotactic pattern on cylinder surface, $\varnothing_{sl} = (j - 1)R_s\theta_s \pi /180$ and $\varnothing_{sy} = c(i-1)$. Therefore, the position equation of the abrasive cluster is expressed as follows:

(a) Array pattern:

\[
\begin{align*}
  x^{ij}_{so} &= R_s \cos \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right), \\
  y^{ij}_{so} &= (i-1)R_s \varnothing_{sy}, \\
  z^{ij}_{so} &= R_s \sin \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right)
\end{align*}
\]  

(b) Staggered pattern:

\[
\begin{align*}
  x^{ij}_{so} &= R_s \cos \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right), \\
  y^{ij}_{so} &= (i-1)R_s \varnothing_{sy}, \\
  z^{ij}_{so} &= R_s \sin \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right)
\end{align*}
\]  

(c) Phyllotactic pattern:

\[
\begin{align*}
  x^{ij}_{so} &= R_s \cos \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right) = R_s \cos \left( \left(\frac{j-1}{}180 \right) \pi \right), \\
  y^{ij}_{so} &= \varnothing_{sy} = c(i-1), \\
  z^{ij}_{so} &= R_s \sin \left( \left(\frac{j-1}{}R_s \right) \varnothing_{sl} \right) = R_s \sin \left( \left(\frac{j-1}{}180 \right) \pi \right)
\end{align*}
\]  

In Eqs. (12) to (14), $i = 1, 2, \ldots, I_s, j = 1, 2, \ldots, J_s$. $I_s$ is the axial ranking number of abrasive clusters on the wheel surface; $J_s$ is the number of circumferential rows of abrasive clusters. For staggered pattern, $i$ and $j$ must be odd or even, at the same time. For phyllotactic pattern, $i \neq j = 1, 2, \ldots, I_s, J_s = J_s$. $(x^{ij}_{so}, y^{ij}_{so}, z^{ij}_{so})$ is the position coordinates of the center of the abrasive cluster.

### 3.3.2 Establishment of the topological feature vectors of grinding wheel

During the grinding process, the topological features of the grinding wheel surface are correspondingly mapped to the workpiece surface. Based on the above analysis of the abrasive clusters of the grinding wheel and the established equations, the geometric and arrangement topological parameters...
of the abrasive clusters can be extracted, and the topological feature vector \( S_d \) of the abrasive clusters on the wheel surface and the topological feature vector \( S_a \) of its arrangement can be established, as follows:

1. **Topological feature vector of abrasive cluster unit**

\[
S_d = \begin{bmatrix} L_s & W_s & H_s & 1 \end{bmatrix}^T \tag{15}
\]

In Eq. (15), \( L_s \) is the set of feature arc lengths of the abrasive clusters along the cylindrical surface of the wheel substrate. \( W_s \) is the set of the feature widths of the abrasive clusters along the axial direction of the cylindrical surface, and \( H_s \) is the set of the feature heights of the abrasive clusters relative to the normal direction of the cylindrical surface. \( L_s, W_s, \) and \( H_s \) correspond to \( L_w, W_w, \) and \( H_w \) of the dimple unit on the workpiece surface.

2. **Topological feature vector of the arrangement of abrasive clusters**

Suppose the feature vector of the arrangement of abrasive clusters is:

\[
S_a = \begin{bmatrix} T_{sl} & T_{sy} & \varnothing_{sl} & \varnothing_{sy} \end{bmatrix}^T \tag{16}
\]

Then, the topological feature vectors of the three abrasive clusters corresponding to Eq. (16) are as follows:

(a) **Array pattern:**

\[
S_{aa} = \begin{bmatrix} T_{sl} & T_{sy} & 0 & 0 \end{bmatrix}^T \tag{17}
\]

(b) **Staggered pattern:**

\[
S_{as} = \begin{bmatrix} T_{sl} & T_{sy} & \frac{T_w}{2} & \frac{T_w}{2} \end{bmatrix}^T \tag{18}
\]

(c) **Phyllotactic pattern:**

\[
S_{ap} = \begin{bmatrix} T_{sl} & T_{sy} \frac{(i-1)\pi R \Theta_s}{180} & c(i-1) \end{bmatrix}^T \tag{19}
\]

### 3.4 Topological mapping matrix of grinding dimple process

#### 3.4.1 Analysis of the mapping process of grinding dimple surface

In the process of grinding the surface of the dimple, the structured grinding wheel grinds the surface of the workpiece by an up-grinding method. The abrasive particles of the abrasive cluster make an extended epicycloid motion relative to the workpiece, and the cutting sequence of each abrasive grain in the circumferential direction of the grinding wheel is different, and its radial protrusion height is also different, so it produces overlapping trajectories in the cross-section that interferes with the workpiece, removes material through the envelope [46], and generates a dimple geometry. Figure 4 shows the motion trajectory diagram of the enveloping dimples in the direction of a plurality of abrasive grains on two adjacent abrasive clusters. It is not difficult to know from Fig. 1 that the process of mapping dimples from the abrasive cluster topology, no relative movement occurs between the \( Y_s \) direction of the grinding wheel and the \( Y_w \) direction of the workpiece. Therefore, the geometric mapping between the topology feature width \( W_s \) of the abrasive cluster and the topology feature width \( W_w \) of the dimple maintain identically. At the same time, the geometric mapping between the arrangement features \( T_s \) and \( \varnothing_{sy} \) of the abrasive cluster and the arrangement features \( T_w \) and \( \varnothing_{sy} \) of the dimples will also remain identical. For the abrasive cluster with convex set feature on the grinding wheel surface, the radial height feature \( H_s \) of the abrasive cluster on the substrate surface is set to maintain an identical geometric mapping with the topological feature depth \( H_w \) of the anti-convex set dimple on the workpiece surface in the direction of \( Z_w \). Therefore, during the grinding process, the mapping change is between the feature length \( L_s \) of the abrasive cluster and the feature length \( L_w \) of the dimple in the \( X_w \)-axis direction under the conditions of different grinding parameters, and between the arrangement parameters \( T_{sl}, \varnothing_{sl}, \) and \( T_{lw}, \varnothing_{lw} \).

Figure 5 shows a schematic diagram of grinding the dimples in the direction of the center section of the abrasive cluster. In Fig. 5, when the grinding wheel moves from the \( O_{sl} \) position to the \( O_{s2} \) position, the abrasive cluster completed the generating of the first dimple, and when it moves to the \( O_{s4} \) position, the subsequent abrasive clusters will generate the second dimple. In this process, the abrasive cluster \( A_s \) point on the \( \overline{A_sB_s} (= l_s) \) segment of the abrasive cluster is cut in from the \( A_w \) point on the

![Fig. 4 The dimple section enveloped by abrasive grain trajectories](image-url)
surface of the workpiece, and the $B_i$ point is cut out from the $B_w$ point of the workpiece, and a dimple unit $A_wB_w$ ($= l_w$) is ground. The $B_wC_w$ section of the grinding wheel surface has no abrasive contact with the workpiece, and forms the $R_wC_w$ section corresponding to the workpiece surface, which becomes the platform-shaped separation interval between the dimple units, and the interval length is $l_wBC$. When $l_wBC \geq 0$, a dimpled surface is ground on the workpiece surface. In fact, it can be inferred from Fig. 5 that adjacent dimple elements intersect when $T_{wx} < l_w$; the dimple units are connected when $T_{wx} = l_w$; the dimple units are separated and a dimple-type structured surface can be generated when $T_{wx} > l_w$. Therefore, based on the grinding kinematics relationship, when the abrasive clusters of the grinding wheel rotate for one cycle time, that is, $\varphi_s/2\pi n_s = T_{sl}/2\pi n_s R_s$, the workpiece will be fed with $v_w$ for an arrangement cycle $T_{sl}$; the relationship between the arrangement cycles $T_{sl}$ and $T_{wx}$ is as follows:

$$T_{wx} = \frac{\varphi_s}{2\pi n_s} \times v_w = \frac{T_{sl}}{2\pi R_s} \times \frac{v_w}{n_s}$$  \hspace{1cm} (20)$$

In Eq. (20), $\varphi_s = 2\pi/N$ is the center angle of adjacent abrasive clusters, and $N$ is the largest ordinal number of abrasive clusters arranged on the same circumference of the grinding wheel. For array and staggered pattern, $N = J_s$; for phyllotactic pattern, $N = 1$.

Similarly, since the geometric position relationship between the arrangement cycle $T_{sl}$ and the phase difference $\varphi_{ls}$ is fixed, the following relationship also exists between the phase difference $\varphi_{ls}$ and $\varphi_{wx}$:

$$\varphi_{wx} = \frac{\varphi_s}{2\pi n_s} \times v_w = \frac{\varphi_{sl}}{2\pi R_s} \times \frac{v_w}{n_s}$$  \hspace{1cm} (21)$$

In the actual design of the radial geometry of the abrasive cluster on the surface of the grinding wheel base, it is necessary to ensure that the radial height growth rate of the inlet section and the attenuation rate of the cutting section of the abrasive cluster as shown in Fig. 5 can enable continuous progressive grinding when grinding the dimples on the workpiece surface, without rapidly increasing the grinding depth. In this process, the generating length $l_w$ of the dimple in the $X_w$ direction is the combined effect of the geometric interference chord length of the grinding wheel under the condition of the grinding depth $a_p$ of the abrasive cluster and the movement length of the workpiece during the cutting-in and cutting-out time of the abrasive cluster [46]. However, due to the different radii of the abrasives on the abrasive clusters relative to the center of the grinding wheel, the chord length of the interference changes, and the chord length increases with the change of feed time is also non-linear. Therefore, it can be approximately simplified as the generating length of the dimple is composed of
the interference chord length at the average grinding depth \( \overline{d}_p \) and the average feed motion length in the \( X_w \) direction when the grinding wheel rotates through the abrasive cluster area, namely:

\[
l_w \approx \frac{l_s}{2\pi R_s} \times \frac{v_w}{n_s} + 2\sqrt{2R_s \overline{d}_p} \tag{22}
\]

In Eq. (22), \( \overline{R}_s \) is the average value of the radial radius of the abrasive clusters corresponding to the \( l_s \) section which is the average value of the abrasive cluster radius. In the grinding mapping process, from the perspective of the grinding depth identity mapping, \( a_p = h_s = h_w \) can be determined. Under the condition that the feature parameter of the depth of the dimple is directly mapped along the radial direction of the grinding wheel, the values of \( \overline{R}_s \) and \( \overline{d}_p \) can be calculated.

### 3.4.2 Topological mapping matrix between workpiece and grinding wheel

According to the above analysis of the generating process of the maximum feature size and arrangement of the dimples, it can be seen that the topological corresponding relationship between the dimple and the abrasive cluster is equivalent transformation in the width and depth directions, while the topological deformation of stretching and bending occurs in the length direction due to the influences of the contour shape of the abrasive cluster and grinding parameters. Moreover, there is one-to-one mapping relationship between the set points of the dimple and the set points of the abrasive cluster as well as the arrangement relation, which forms a topological mapping relationship. Therefore, according to Eqs. (20), (21), and (22) and the analysis results of the mapping relationship of the dimples in different directions, the size topological mapping matrix \( F_d \) and the arrangement topological mapping matrix \( F_a \) in the grinding process can be established as follows:

**Topological mapping matrix of the size features:**

\[
F_d = \begin{bmatrix}
\frac{v_w}{2\pi R_s n_s} & 0 & 0 & 2\sqrt{2R_s \overline{d}_p} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{23}
\]

where \( \overline{R}_s \) is the average radius in the area corresponding to the length of the \( i \)th divided section of the abrasive cluster, and \( \overline{d}_p \) is the average grinding depth of the \( i \)th divided section. When the abrasive cluster is the maximum length \( l_s \), \( \overline{R}_s = R_s \) and \( \overline{d}_p = \overline{d}_p \).

**Topological mapping matrix of the arrangement features:**

\[
F_a = \begin{bmatrix}
\frac{v_w}{2\pi R_s n_s} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{v_w}{2\pi R_s n_s} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{24}
\]

Therefore, based on Eqs. (6), (7), (15), (16), (23), and (24), the topological mapping equations in the grinding process can be obtained as follows:

\[
\begin{align*}
W_d &= F_d S_d \\
W_a &= F_a S_a
\end{align*} \tag{25}
\]

Similarly, if the mapping of the grinding process is regarded as an identity mapping, for the convenience of designing the grinding wheel, the corresponding inverse mapping relationship from the workpiece to the grinding wheel can be given:

\[
\begin{align*}
S_d &= F_d^{-1} W_d \\
S_a &= F_a^{-1} W_a
\end{align*} \tag{26}
\]

Among them, \( F_d^{-1} \) and \( F_a^{-1} \) are as follows:

\[
F_d^{-1} = \begin{bmatrix}
\frac{2\pi R_s n_s}{v_w} & 0 & 0 & -\frac{4\pi R_s n_s}{v_w} \sqrt{2R_s \overline{d}_p} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{27}
\]

\[
F_a^{-1} = \begin{bmatrix}
\frac{2\pi R_s n_s}{v_w} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{2\pi R_s n_s}{v_w} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{28}
\]

### 4 Design and manufacturing of structured grinding wheel

#### 4.1 Design of the grinding wheel

In the actual design of the grinding wheel, the substrate radius \( R_s \) and the configuration form of the grinding wheel can be given first, as well as the number of circumscribed columns \( J_s \), so that the circumscribed \( T_{sd} \) and \( \Omega_{sd} \) of the grinding wheel can also be determined first. Then, according to \( T_{wx} \), \( \Omega_{wx} \), \( T_{wy} \), and \( \Omega_{wy} \) known to the workpiece, \( \Omega_{xy} \) and \( v_w/n_s \) can be determined by using the configuration equations in Eqs. (25) and (26). Then, according to the feature quantities \( L_{wx}, W_{wx}, \) and \( H_w \) of the dimples, \( L_{wy}, W_{wy} \), and \( H_y \) are obtained by using the size equations of the dimples in Eqs. (25) and (26).
In the design of the abrasive cluster of the grinding wheel, firstly, the dimple unit shown in Fig. 2a is discretely divided along the equal spacing of $X_w$ and $Y_w$ directions to determine the feature parameter set of length, and at the same time, the depth feature set of $Z_s$ direction is determined. Then, the cylinder surface of the grinding wheel is correspondingly divided in the topological space of the grinding wheel. When the abrasive cluster unit is divided, the axial discrete segmentation corresponds to the $Y_w$ direction division of the workpiece dimple, the circumferential segmentation with discrete equal angle corresponds to the $X_w$ direction division of the workpiece dimple, and then the peripheral size set $L_w$, axial size set $W_w$, and the radial size set $H_w$ can be obtained and the feature dimensions of $L_w$, $W_w$, and $H_w$ are calculated by using the size mapping equation of abrasive cluster in Eq. (26).

According to the above method, under the condition of the known dimple size as shown in Table 1, the arrangement and size parameters of the corresponding abrasive clusters are designed, and the digital grinding wheels with the array, staggered, and phyllotactic pattern of the abrasive clusters shown in Fig. 6 are designed by using software.

### 4.2 Manufacture of the grinding wheel

In the manufacturing process of the grinding wheel, a combination of masking, corrosion, and electroplating is used [47]. Firstly, the mask is designed and manufactured according to the structure of the grinding wheel shown in Fig. 6 and the parameters in Table 1. Then lithography and corrosion were carried out on the substrate of the grinding wheel to form a convex hull arranged in order on the substrate surface of the grinding wheel, and the convex hull was precisely shaped to achieve the designed geometric shape. When dressing the geometry of the convex hull, the amount of pre-dressing should be given based on the average diameter of the abrasive size. Finally, the consolidating abrasive grains and thickening coating on the convex hull by electroplating form a grinding wheel to ensure that the contour height of the abrasive clusters meets the design requirements. In order to improve the mechanical properties of the electroplated layer, the electroplated grinding wheel was placed in a thermostat for hydrogen evolution and stress relief. The substrate diameter of the electroplated grinding wheel is $60\text{mm}$, and the axial thickness is $15\text{mm}$. The abrasive used is CBN.

### Table 1 Topological design parameters of workpiece and grinding wheel

| Parameters          | Workerpiece (known) |          | Wheel: Diameter $\Phi60\text{mm}$, Thickness $15\text{mm}$ |
|---------------------|---------------------|----------|----------------------------------------------------------|
|                     | Array pattern       | Staggered pattern | Phyllotactic pattern | Array pattern | Staggered pattern | Phyllotactic pattern |
| Unit sizes, mm      | $I_w = 3$           | $l_w = 3$            | $I_w = 3$            | $I_w = 26$   | $l_w = 26$       | $l_w = 26$       |
|                     | $w_w = 2$           | $w_w = 2$            | $w_w = 2$            | $w_w = 2$   | $w_w = 2$       | $w_w = 2$       |
|                     | $h_w = 0.020$       | $h_w = 0.020$        | $h_w = 0.020$        | $h_w = 0.020$| $h_w = 0.020$  | $h_w = 0.020$  |
| Unit arrangement $= 20(\mu\text{m})$ | $T_w = 3$           | $T_w = 3$            | $T_w = 0$            | $T_w = 0$   | $T_w = 0$       | $T_w = 0$       |
|                     | $\varnothing_w = 3.25$ | $\varnothing_w = 3.25$ | $\varnothing_w = 3.25$ | $\varnothing_w = 3.25$ | $\varnothing_w = 3.25$ | $\varnothing_w = 3.25$ |
|                     | $\varnothing_w = 1.5$ | $\varnothing_w = 1.5$ | $\varnothing_w = 0.7(i - 1)$ | $\varnothing_w = 0.7(i - 1)$ | $\varnothing_w = 0.7(i - 1)$ | $\varnothing_w = 0.7(i - 1)$ |
| Rows and columns    | $I_s = 5$           | $I_s = 9$            | $I_s = J_s = 18$     | $I_s = 5$   | $I_s = 9$       | $I_s = J_s = 18$ |
|                     | $j = 1, 2, 3, \ldots, J_w$ | $j = 1, 2, 3, \ldots, J_w$ | $j = 1, 2, 3, \ldots, J_w$ | $j = 1, 2, 3, \ldots, J_w$ | $j = 1, 2, 3, \ldots, J_w$ | $j = 1, 2, 3, \ldots, J_w$ |

Fig. 6 Digital grinding wheels with different arrangement of abrasive clusters. a With array pattern, b with staggered pattern, c with phyllotactic pattern.
abrasive with 140/170 mesh. The electroplating solution is a bright nickel plating solution with NiSO₂ as the main salt [47]. Figure 7 shows an electroplated grinding wheel with an array, staggered, and phyllotactic pattern of the abrasive clusters.

5 Geometric simulation of grinding dimple surface

5.1 Simulation strategy and conditions

According to the topological feature parameters of Table 1 and the digital grinding wheel shown in Fig. 6, the geometric simulation of generating the dimple surface during the grinding process was carried out using computer software programming. The simulation flow chart is shown in Fig. 8. In the simulation, the distance between the grinding wheel and the workpiece surface as shown in Fig. 1 is set as \( e = R_s + h_s - a_p \). In the simulation, the position of the abrasive grain relative to the trajectory point of the workpiece surface is calculated according to the trajectory equation of the surface grinding, and then the grinding interference state between the abrasive particle and the workpiece is judged to determine the surface topography of the workpiece. The abrasive used was CBN-950 abrasive provided by Funik Ultra hard Material Co., Ltd., with a particle size of 140/170 mesh. The abrasive was measured with a digital microscope. The actual size dispersion ranged from 105 to 86 μm, and the average value was 90 μm. Therefore, since the average effect was mainly observed in the simulation, 90 μm was used as the calculation basis, and only the average effect of the macroscopic profile of the dimple is considered and the effect of the micro-roughness is ignored. At the same time, in order to simplify the explanation of the problem, the influence of grinding parameters on the maximum size feature parameters and arrangement parameters is mainly investigated.

5.2 Simulation results and analysis

Figure 9 shows the arranged dimple surfaces with array, staggered, and phyllotactic pattern obtained by simulation under the design parameters shown in Table 1. The simulation results show that the maximum characteristic lengths
of the pit elements in the $X_w$ direction are $l_w = 2.99$ mm, 2.96 mm, and 2.97 mm, and the error is not more than 1.3% compared with the design length. The maximum feature widths in the $Y_w$ direction are both $w_w = 2$ mm, and the maximum feature depths in the $Z_w$ direction are $h_w = 20 \mu$m, both of which are kept equal to the design size. The arrangement periods in the $X_w$ direction are $T_{wx} = 3.22$ mm, 6.5 mm, and 0, respectively, and the error is not more than 1.3% compared with the design period. The phase differences are $\varnothing_{wx} = 0, 3.25$ mm and 4.96 mm (initial unit), which are kept equal to the design parameters. In the $Y_w$ direction, the arrangement period is $T_{wy} = 3$ mm, 3 mm, and 0; the phase difference is $\varnothing_{wy} = 0, 1.5$ mm, and 0.7 mm; and the arrangement parameters are kept equal to the design parameters. In conclusion, the simulation results are consistent with the design dimensions, which indicate that the surface grinding of structured dimples with different arrangements can be achieved by using the topological grinding method.

In order to obtain the mapping effect of $v_w/n_s$ on the structured dimple surface of the workpiece, on the basis of the design geometric parameters shown in Table 1, when $a_p = 20 \mu$m, the speed ratio $v_w/n_s$ was changed for simulation. Figure 10 shows the pit-structured surface morphologies obtained at $v_w/n_s = 9, 12, 14$, and 15, respectively. When $v_w/n_s = 9$, the $l_w = T_{wx} = 2.26$ mm in the $X_w$ direction is smaller than the design parameters $l_w = 3$ mm and $T_{wx} = 3.25$ mm, and the adjacent structured dimple units intersect, which is no longer sufficient for grinding dimple condition. With the increase of $v_w/n_s$, when $v_w/n_s = 12, 14$, and 15, the maximum feature lengths of the dimple unit in the $X_w$ direction are $l_w = 2.96$ mm, 3.09 mm, and 3.20 mm respectively, showing an increasing trend and deviating from the design length which were $-1.3\%, 3\%$, and $6.7\%$, respectively. The width in the $Y_w$ direction is $w_w = 2$ mm, which is equal to the design width. The maximum depth in the $Z_w$ direction is $h_w = 20 \mu$m, which is equal to the design depth. The arrangement periods in the $X_w$ direction are $T_{wx} = 3$ mm, 3.5 mm, and 3.75 mm, showing an increasing trend, and the deviations from the design period are $-7.7\%, 7.7\%$, and 15.4%, respectively. The period in the $Y_w$ direction is also kept equal to the design period $T_{wy} = 3$ mm. The simulation results show that with the further increase of the rotational speed ratio, depth, and width of the structural unit remain unchanged, and the length $l_w$ and period $T_{wx}$ of the structural dimple unit increase, and it is further clarified that when the grinding parameters deviate from the design parameters, the guarantee is that the topology attributes are unchanged.

Figure 11 shows the surface topography of the workpiece obtained when the grinding depth $a_p$ is reduced to 10 $\mu$m and 15 $\mu$m, respectively, under the simulation conditions of Fig. 9a. When the grinding depths were 10 $\mu$m and 15 $\mu$m, the maximum feature lengths of the dimple units were $l_w = 2.25$ mm and 2.77 mm, respectively, and the deviations from the design lengths were $-25\%$ and $-7.7\%$, respectively. The widths are $w_w = 1.3$ mm and 1.7 mm, respectively, and the deviation from the design width is $-30\%$ and $-15\%$, respectively. The arrangement period of the units in the $X_w$ direction is $T_{wx} = 3$ mm, which is 1.3% less than the design period. The period in the $Y_w$ direction remains $T_{wy} = 3$ mm, which is equal to the design period.

The above Figs. 9, 10, and 11 are just a few examples to illustrate the problem. In fact, we have simulated different grinding parameters in the designing process of the grinding wheel. The results show that with the increase of...
the grinding depth, the dimple arrangement cycle does not change, the length and width of the dimple unit increase, and the growth rate is gradually reduced by the influence of the dimple shape. When the parameters of the grinding wheel are constant, the speed ratio \( \frac{v_w}{n_s} \) not only affects the length and arrangement period of the dimple units in the \( X_w \) direction, but also controls the separation, touch, and intersection between the dimple units, thus changing the topography of structured dimple surface.

6 Grinding experiment

6.1 Experimental setup

The grinding experiment was done on the CNC machining center as shown in Fig. 12. The grinding wheel used is shown in Fig. 7, and the structural parameters of the grinding wheel are shown in Table 1. The size of the workpiece is 50 mm × 25 mm × 15 mm, the workpiece material is 45 steel, its hardness is HRC36-42, and the initial surface roughness is Ra0.3. The workpiece is fixed on the worktable by a precision vise, and grinding method is up-grinding. No coolant is used during grinding, and the Taylor Hobson Form Talysurf i profiler is used to measure the three-dimensional topography of the ground surface of the workpiece.

6.2 Experimental results and analysis

Figure 13 shows the dimple surface topography obtained using a grinding wheel with arrayed abrasive clusters under the design parameters shown in Table 1. The average maximum feature sizes of the measured structured dimple units are \( l_w = 3.1 \) mm, \( w_w = 1.6 \) mm, and \( h_w = 12 \) \( \mu \)m. Compared with the theoretical design size under the condition of \( a_p = 20 \) \( \mu \)m, the resulting deviations are 0.33\%, −20\%, and −40\%. The arrangement parameters are \( T_{w_x} = 3.2 \) mm and \( T_{w_y} = 3 \) mm respectively, and the maximum error is not more than 1.5\%, which is basically consistent with the designed arrangement parameters. Superficially, this result
shows that there is a big difference between the feature sizes of the dimple units obtained by grinding and the theoretical design sizes. However, through the microscopic observation and measurement of the ground dimples, and the grinding process analysis, it can be found that the reason for this phenomenon is caused by the abrasive distribution characteristics of the surface of the electroplated CBN grinding wheel.

When designing the geometric shape of the abrasive clusters on the surface of the grinding wheel, the theoretical grinding depth is based on the average geometric size of the abrasive grains, and the protruding height of the abrasive grains is considered to be equal, while the actual abrasive protruding heights on the surface of the electroplated grinding wheel are unequal and normally distributed [48, 49]. During the grinding experiment, when the theoretical grinding depth $a_p = 20 \, \mu m$ was set based on the spark generation, only part of the abrasive particles actually participated in the grinding, and the actual grinding depth at this time did not reach the theoretical grinding depth. If compared with the simulation results in Fig. 11, and considering that the actual size distribution of the adopted 140/170 mesh CBN abrasive is in the range of 86 to 105 $\mu m$, the actual grinding depth of this experiment should be around $a_p = 12 \, \mu m$, which means that the experimental result is correct. Therefore, this experimental result also shows that improving the consistency of the protruding height of the abrasive grains on the surface of the electroplated grinding wheel is very important to realize this grinding method.

Figure 14 shows the surface topography of the structured dimples ground using a grinding wheel with staggered abrasive clusters. At this time, under the design parameters shown in Table 1, the speed ratio is reduced to $v_w/n_s = 10$ and the grinding depth is reduced to $a_p = 10 \, \mu m$. The average maximum feature sizes of the dimple unit obtained by actual measurement are $l_w = 2.2 \, mm$, $w_w = 1.6 \, mm$, and $h_w = 8 \, \mu m$, and the relative changes compared with the theoretical design sizes under the condition of $a_p = 20 \, \mu m$ are $-26.6\%$, $-20\%$, and $-60\%$. The measured arrangement periods are $T_{wx} = 5 \, mm$ and $T_{wy} = 3 \, mm$, respectively, and the variation compared with the design parameters is $23\%$ and $0\%$, respectively. The phase difference of the arrangement is $\varnothing_{wx} = 2.5 \, mm$ and $\varnothing_{wy} = 1.5 \, mm$, respectively, and the variation compared with the design parameters is $23\%$ and $0\%$, respectively. By measuring and observing the topography of the ground dimple surface, it can be seen that when the grinding parameters are smaller than the design parameters, and the feature parameters of the ground dimple surface will decrease, but the topological attributes of the surface remain unchanged.

Figure 15 shows the topography of the structured dimple surface ground using a grinding wheel with phyllotactic arranged abrasive clusters. At this time, the speed ratio is reduced to $v_w/n_s = 10$ under the design parameters in Table 1. The average maximum feature sizes of the measured dimple unit are $l_w = 2.7 \, mm$, $w_w = 1.6 \, mm$, and $h_w = 15 \, \mu m$, respectively. Compared with the theoretical design size under the condition of $a_p = 20 \, \mu m$, the relative changes...
are −10%, −20%, and −25%. The measured arrangement phase differences are $\phi_{wx} = 3.8$ mm and $\phi_{wy} = 0.7$ mm. Compared with the theoretical design value, the relative changes produced are −23.4% and 0%, respectively. It can be seen that the phase difference in the $Y_w$ direction remains consistent with the design parameter, and other feature parameters are reduced. The reason is mainly due to the reduction of the feed rate and the actual grinding depth is less than the designed grinding depth of $a_p = 20 \mu m$. At this time, the actual grinding depth is about 15 $\mu m$.

Figures 16 and 17 show the topography of the structured dimple surfaces ground using a grinding wheel with abrasive clusters arranged in an array pattern and in a phyllotactic pattern, respectively. At this time, the grinding speed ratio is

![Fig. 13](image13.png)

**Fig. 13** The ground surface topography by the grinding wheel with array pattern at $a_p = 20 \mu m$, $n_s = 1800r/min$, $v_w = 23400$ mm/min and $v_w/n_s = 13$. a Surface picture, b three-dimensional surface topography, c mid-section profile

![Fig. 14](image14.png)

**Fig. 14** The ground surface topography by the grinding wheel with staggered pattern at $a_p = 10 \mu m$, $n_s = 1800r/min$, $v_w = 18000$ mm/min, and $v_w/n_s = 10$. a Surface picture, b three-dimensional surface topography, c mid-section profile
reduced from \( v_w/n_s = 13 \) to \( v_w/n_s = 6 \). For the surface shown in Fig. 16, the dimple units intersect in the \( X_w \) direction, and the corresponding arrangement properties basically disappear, and a grooved topography is formed. The depth of the groove is \( h_w = 17 \, \mu m \), and its cross-sectional profile is undulating due to the overlapping grinding of adjacent abrasive clusters. The groove width in the \( Y_w \) direction is \( w_w = 1.5 \, mm \), and the arrangement period \( T_{wy} = 3 \, mm \). For the surface shown in Fig. 17, the pits intersect in the \( Y_w \) direction, and a certain arrangement topology can still be observed; the average maximum feature sizes of the dimple are \( l_w = 2.6 \, mm \), \( w_w = 1.6 \, mm \), and \( h_w = 16 \, \mu m \), respectively. The initial phase differences of the dimple arrangement are \( \phi_{wx} = 2.29 \, mm \) and \( \phi_{wy} = 0.7 \, mm \), respectively. These grinding results also show that the selected grinding parameters can no longer satisfy the kinematic conditions.
for creating the pit surface expressed by Eqs. (20) and (21). However, it can be seen that due to the reduction of the grinding speed ratio, the number of dynamic grinding edges involved in grinding increases [46, 49], and the density of cutting groove marks on the enveloped dimple or groove surface increases, and the bottom surfaces of the dimples and grooves become smoother.

When the surface topography of the structured dimples obtained by the experiment is compared with the topography obtained by the simulation, it can also be observed that the surface obtained by the experiment is relatively rough, the boundary contour is inconsistent, and the cross-sectional contour is undulating. At the same time, the problem that the actual grinding depth is smaller than the theoretical grinding depth is also found in the experiment process. In the simulation, in order to better obtain the influence of the relevant parameter changes on the grinding surface topography, the average abrasive size is adopted to obtain the generating law of the dimple surface. During the grinding experiment, due to the low density of abrasive grains in the abrasive cluster, the unequal protruding height of abrasive grains, the manufacturing error of abrasive grain clusters, the lack of dynamic grinding edges, and the principle error of tool setting, these problems make the experimental results deviate from the simulation results. Due to the limited number of abrasive grains on a single abrasive cluster, and in order to ensure the proper grinding speed ratio and meet the requirements of the feed speed, the selected speed of the grinding wheel is small, which reduces the number of dynamic cutting edges that actually participate in grinding. The dimple surface enveloped by the less dynamic cutting edge is rough. Especially when the actual grinding depth is small, this phenomenon is more obvious, as shown in Fig. 14. The unequal protruding height of the abrasive grains makes the cutting-in and cutting-out sequence of the abrasive grains different during the dimple grinding process, so that the created dimple boundary contour is irregular, as shown in Figs. 14a and 15a. The uneven protruding height of the abrasive grains also causes the undulation of the cross-sectional profile inside the dimple due to the residue of the removed material. The manufacturing error of the abrasive particle clusters leads to inconsistencies in the cross-section of the dimples ground on the workpiece surface. Of course, the undulation of the cross-sectional profile of Fig. 16c is mainly caused by the overlapping grinding of adjacent abrasive clusters. The inconsistency of the cross-sectional profiles in Figs. 15c and 17c is mainly due to the different cross-sectional positions of the dimples due to the arrangement effect. In the actual grinding process, the setting of the grinding depth is based on the grinding spark generated by the contact workpiece as the tool-setting reference; that is, the maximum protruding height of abrasive grains on the surface of the electroplated grinding wheel is also used as the tool-setting reference. The setting of the grinding depth in the simulation process is determined based on the average protruding height of abrasive grains. Therefore, there is an error between the actual grinding depth and the grinding depth set by the simulation. However, in general, these problems mentioned above do not affect the feasibility of this grinding strategy.

Fig. 17 The ground surface topography by the grinding wheel with phyllotactic pattern at \( \alpha_j = 20 \, \mu m, n_j = 1800 \, \text{r/min}, v_w = 10800 \, \text{mm/min}, \) and \( v_w/n_j = 6. \) a Surface picture, b three-dimensional surface topography, c mid-section profile
7 Conclusions and outlook

Based on the geometric topology theory, the mapping relationship matrix between the topological space of workpiece and the topological space of the grinding wheel is established. The structured grinding wheels with abrasive clusters are designed and the analysis and geometric simulation of the surface creation in the topological grinding process are performed. At the same time, the grinding wheel was manufactured and the grinding experiment was carried out. The results show that the topological grinding of the structured dimple surface can be realized by the designed grinding wheel with ordered abrasive clusters based on the topological features of the dimple surface. Although the ground dimple structured surface is an approximation of the theoretical surface, it still maintains the topological attributes unchanged. In order to realize the grinding of the structured dimple surface, the selection of the grinding parameters must satisfy the corresponding kinematic conditions. When the adopted grinding parameters are equal to the design parameters, the feature parameters required by the design can be ground. When the selected grinding parameters deviate from the design parameters, the feature parameters of the ground dimple surface will change, but the topological attributes will not change. These studies provide a new way for the grinding of the structured surface.

In the follow-up work, the manufacturing quality of the grinding wheel should be further improved, such as improving the manufacturing accuracy of the substrate, improving the consistency of the protruding height of the abrasive grains on the grinding wheel surface, and increasing the abrasive density. At the same time, the feasibility study of topological grinding of different structured surfaces is carried out, and the grinding wheel is designed according to the functional feature parameters of the structured surface to realize this grinding strategy.

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