Distributed Optimal Dispatching of Interconnected Electricity-Gas-Heating System

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ABSTRACT In the researching background of integrated energy system, a single electricity-gas-heating system (EGHS) can be regarded as an active producer. In order to solve the joint optimization of integrated energy systems under the condition of incomplete information, this paper proposes a distributed optimal scheduling framework of EGHS. First, establish a coupling model of the interconnected EGHS, and perform strict second-order cone convexity of the complex natural gas flow model. Next, use the bus splitting method to realize the decoupling between different regions of the interconnected system, and employ the alternating direction multiplier method (ADMM) to solve the model. Then, construct two-region energy system (78-node grid + 40-node gas grid + 40-node heat grid) and three-region energy system (117-node grid + 60 node gas grid + 60-node heat grid) as simulation examples to verify the effectiveness of the distributed optimization framework. In the end, the algorithm solution process, the effectiveness of scheduling results, and the comparison of optimization results under different interconnection methods are analyzed in detail.

INDEX TERMS Electricity-gas-heating system, alternating direction method of multiplies, distributed optimal scheduling, second-order cone programming.

I. INTRODUCTION
With the further advancement of the reform of energy industrialization, the energy internet has received increasing attention and researches [1]–[4]. As an important physical carrier of the energy internet, the integrated energy system is considered to be the main form of energy for the future society [5]–[7].

The continuous development of gas-fired power generation, power-to-gas (P2G) technology and combined heat-power (CHP) technology has transformed the construction of the integrated electricity-gas-heat energy system (EGHS) from theory to reality [8], [9]. At present, most researches on the optimal dispatch of integrated energy systems focus on internally regional optimization, that is, the coordination and complementation of electric energy, natural gas energy and thermal energy in a single zone [10]–[13]. Reference [10] proposes an improved sequential energy flow analysis method to solve the optimal energy flow of electrical interconnection system. Reference [11] studies the role of hourly economic demand response in the optimization of the stochastic day-ahead scheduling of electric power systems with natural gas transmission constraints. Reference [12] proposes a nonlinear optimal model of wind power accommodation for an electricity-heat-gas integrated microgrid with P2G, taking the minimum operating cost, the minimum wind curtailment, and the minimum comprehensive cost as objectives. Reference [13] studied the optimization of low-carbon operation of EGHS. The above studies all focus on the optimization of the single-zone integrated energy system inside the region, but the research on the collaborative optimization of the interconnected integrated energy system, especially the multi-region EGHS collaborative optimization, is still in scarcity, and many studies have not considered the topology of the thermal network, but only involve the type of thermal load [14]. As the heating area continues to expand and the heat demand continues to increase, in china, the National Energy Administration vigorously promotes the construction of “combined heat and power” projects, so it is necessary to accurately model the heat network in the integrated energy system [15].

In reality, the cross-regional interconnection of multiple integrated energy systems has become a typical feature of
the modern energy internet. The interconnected integrated energy system can be divided into multiple sub-regions based on geographic location, energy supplier attribution etc. Then through the inter-regional resource transmission, sub-regions can achieve further integration and optimization of resources [16]. In addition, in the interconnected EGHS, each EGHS can be regarded as an active prosumer, and the optimal scheduling can be achieved by obtaining or outputting energy from other EGHS. Therefore, it is necessary to study the collaborative optimization of multi-region integrated energy system.

In addition, the above studies mostly use centralized solutions, which assume that there is an upper control center that can transparently collect data information and process it centrally. It is foreseeable that as the integrated energy system continues to develop, the feasibility of this assumption will be challenged. On one hand, as the scale of the integrated energy system continues to expand, the information managed by the control center multiplies, challenging the computing and communication capabilities of the control center [17], [18]; on the other hand, as the number of energy suppliers continues to increase, each regional subsystem may choose its own energy supplier to optimize within the zone to form a single-zone decision-making body, and refuses to disclose its own operational information for privacy protection. Therefore, the optimal scheduling framework of the integrated energy system should change from a centralized framework to a distributed framework.

As a weakly-centralized computing technology, distributed computing can effectively solve the problem of information privacy. Distributed computing methods mainly include the principle method of auxiliary problems [19], [20], alternating direction method of multiplies algorithm [20]–[23], Benders decomposition algorithm [24], [25], etc. The ADMM algorithm is an important method for solving separable convex optimization problems. It has the advantages of wide adaptability and fast convergence speed, which is more in line with the requirements of distributed computing in integrated energy systems.

Therefore, the research focus of this paper is to construct interconnected EGHS distributed scheduling framework. First, a single-zone EGHS is used as the basic decision-making body, and the interconnected EGHS optimal scheduling model is established with the optimization goal of a minimum energy supply cost. Then, use the diagram of bus splitting method to realize the decoupling between regions of the multi-region interconnection system, and the topological decoupling of the inter-region heating network is realized for the first time. Finally, penalty convexity concave process (PCCP) is used to convexize the non-convex sources in the natural gas dynamic model, and ADMM is used to solve the model to obtain an accurate solution of the convergence, and the improvement strategy of PCCP initial point value is put forward. By solving the two-region interconnected EGHS system and three-region interconnected EGHS system models under different interconnection methods, the accuracy of the proposed model and the effectiveness of the distributed optimization framework are verified.

The remaining of this paper is organized as follows. Section II establishes the single-zone EGHS day-ahead scheduling model. Section III introduces interval decoupling mechanism for interconnected EGHS. Case studies are carried out in Section IV. Finally, Section V briefly summarizes the paper.

II. SINGLE-ZONE EGHS DAY-AHEAD SCHEDULING MODE

A. ELECTRIC- GAS- HEATING INTEGRATED ENERGY SYSTEM

The electric-gas-heating energy system studied in this paper is composed of electric power network, natural gas network and thermal power network. The gas network can convert natural gas to heat energy and electric energy through CHP equipment, and the gas network can convert natural gas to electric energy through gas turbine. The coupling between the systems is shown in Figure 1.

FIGURE 1. Structure of electricity-gas-heating system.

B. POWER NETWORK MODEL

In this paper, the DC power flow model is used to model the power network constraints:

\[
\begin{align*}
A_G P_{G,i} - P_{D,i} - A_{P2G} P_{P2G,i} - A_{CHP} P_{CHP,i} - B \theta_i &= 0 \\
\theta_{ij} - \theta_{ij}$, $i, j \in \Omega_{EN}$ \\
\theta_{i}^{\min} &\leq \theta_i \leq \theta_{i}^{\max} \\
P_{G,i}^{\min} &\leq P_{G,i} \leq P_{G,i}^{\max} \\
0 &\leq P_{P2G,i} \leq P_{P2G,i}^{\max} \\
0 &\leq P_{CHP,i} \leq P_{CHP,i}^{\max} \\
\theta_{i} &\leq \theta_{i}^{\max} \\
\theta_{\text{slack}} &= 0
\end{align*}
\]

\(\Omega_{EN}\) is the set of grid nodes; \(A_G, A_{P2G}, A_{CHP}\) are node-unit correlation matrix, node-P2G correlation matrix and node-CHP correlation matrix; \(P_{D,i}\) is node load matrix; \(B\) is virtual part of node admittance matrix; \(\theta_i\) is node voltage phase vector; \(P_{P2G,i}^{\max}\), \(x_{ij}\) are the maximum transmission power and
reactance of direct $ij$; $P_{G,i}$ is the active power output vector of unit; $P_{2G,i}$ is the active power vector of P2G unit; $a_d, a_u$ are the upper and lower bound vector of unit climbing rate constraints respectively; $\theta_{slack}$ is the phase angle of equilibrium node.

C. NATURAL GAS NETWORK MODEL

In order to accurately model the natural gas network, this paper describes the dynamic characteristics of the natural gas network using the gas flow partial differential equation of natural gas pipelines:

$$\frac{(b'_{ij,x+2} - b'_{ij,x})^2}{\Delta x_{ij}} + M_1 (f'_{ij,x})^2 = 0$$  \hspace{1cm} (2)

$$\frac{\partial (b'_{ij,x})^2}{\partial x} + M_2 (f'_{ij,x})^2 = 0$$  \hspace{1cm} (3)

where $b'_{ij,x}, f'_{ij,x}$ are the gas pressure and gas flow at the time of pipe $ij$ at the time $x$; $M_1, M_2$ are the pipe transfer constants. In reference [27], the Euler finite difference method is used to transform the partial differential equation (3), which is difficult to be solved directly into an algebraic equation. The pipe $ij$ is divided into $N$ segments, and the length of each segment $\Delta x_{ij} = l_j/N$, the equation is expressed as follows:

$$\frac{b'_{ij,x} - b'_{ij,x-1}}{\Delta t} + M_1 (f'_{ij,x+1} - f'_{ij,x}) = 0$$  \hspace{1cm} (4)

$$\frac{(b'_{ij,x+1})^2 - (b'_{ij,x})^2}{\Delta x_{ij}} + M_2 (f'_{ij,x})^2 = 0$$  \hspace{1cm} (5)

where $\Delta t$ is the time step, generally takes 15 min, $\Delta x_{ij}$ is the space step.

There is a delay effect in the transmission of natural gas in the pipeline, and some natural gas is retained in the pipeline, which is called “pipe storage”. After completing a scheduling period, the storage should meet the pipe storage constraint. In addition, the natural gas network should also meet the constraints of pressure station, node pressure constraints, gas source constraints, gas tank constraints and node flow balance constraints, as follows:

$$\sum_{t \in T} \sum_{(i,j) \in \Omega_p} (f'_{ij,0} - f'_{ij,N}) \Delta t = 0$$

where $\Omega_p$ is a collection of natural gas pipelines; $b'_{ij,0}$ is the air pressure before and after the pressurization station respectively; $k^{\min}_c, k^{\max}_c$ are the lower and upper limits of the pressurization station respectively; $f'_{in}$ is the pipeline flow, $f'_{in}^{\max}$ is the upper limit of the pipeline flow; $\Omega_g, \Omega_{GB}$ are gas source set and natural gas node set respectively; $b'_{ij}$ is gas pressure at node $i$ at time $t$; $f'_{ij,t}$ is gas source $j$ at time $t$; $\Omega_s$ is gas storage tank set; $Q'_{in,t}, Q'_{out,t}$ is the natural gas input and output of gas storage tank $n$ at time $t$ respectively; $\mu^{\cap}_{in}, \mu^{\cap}_{out}$ are the gas charging and discharging efficiency of gas storage tank $n$ at time $t$ respectively; $S'_{n,t}$ is the gas storage tank $n$ rated capacity; $B_{g, P2G, B_{in}, B_{CHP, B_{GT, A}}}$ are gas source output vector, P2G gas injection vector, CHP gas injection vector, gas flow injection vector and gas load vector.

D. HEATING NETWORK MODEL

In this paper, with the water flow as the heat medium, the dynamic model of hot water pipe network is constructed to describe the model of heating network accurately. The dynamic characteristics of heat-supply network are mainly reflected in the transmission delay and temperature loss of hot water. In the heat-supply network model constructed in this paper, the water flow is regarded as constant, and the dynamic characteristics of the heat-supply network are described as follows:

$$\frac{\partial T'_{k,t}}{\partial t} + \frac{G^g_{k,t}}{\pi \rho_w R^g_k} \frac{\partial T'_{k,t}}{\partial x} + \frac{2\mu_k}{c_w \rho_w R^g_k} (T'_{k,t} - T'_{k-1,t}) = 0$$  \hspace{1cm} (7)

where $T'_{k,t}, M^g_{k,t}$ are the water temperature and water flow at the distance $x$ from the pipe mouth of the $k$ pipeline at the time $t$; $\mu_k$ is the heat loss factor; $C_w$ is the specific heat capacity of the water; $\rho_w$ is the density of the water; $R_k$ is the radius of the pipe; $T'_{k,t}$ is the external environment temperature of $k$ pipeline at the time $t$.

From formula (11) the following can be obtained:

$$T'_{k,t+\Delta t_k} = T'_{k,t} + (T'_{in,t} - T'_{k,t}) \exp (\frac{-2\mu_k \Delta t_k}{c_w \rho_w R^g_k})$$  \hspace{1cm} (8)

where $T'_{in,t}, T'_{out,t}$ are the inflow temperature and outflow temperature of the $k$ pipeline at the time $t$, $t_k$ is the time that the water flows through the $k$ pipeline.

If the water flow is treated as a constant, the following can be obtained:

$$\Delta t_k = \frac{\pi \rho_w L_k R^g_k}{M_k}$$  \hspace{1cm} (9)

where $L_k$ is the length of the $k$ pipeline; $M_k$ is the water flow of the $k$ pipeline.

The electricity and heat output of CHP unit can be expressed as follows:

$$P'_{c,t} = k_c \cdot H'_{c,t}$$  \hspace{1cm} (10)

$$H'_{c,t} = c \cdot M'_{c,t} (T'_{c,t} - T_{c,t})$$  \hspace{1cm} (11)
where $T_{chp}^{c}$ is the electric heat ratio of the unit, the general working mode of CHP is to set the electricity based on the heat, taking $K_{chp}^{e}$ is the specific heat ratio of the unit; $H_{c, i}$ is the unit heat output; $M_{c, i}^{chp}$ is the water flow; $T_{s, i}^{c}$ is the water supply temperature; $c$ is the specific heat of water.

In addition, the heating network should also meet the heat transfer station constraints, node water temperature constraints, water temperature upper and lower limits constraints, as follows:

$$H_{b, i}^{hes} = c M_{b, i}^{hes} (T_{b, i}^{in} - T_{b, i}^{out}) = c w T_{mix, g}$$

where $H_{b, i}^{hes}$ is the heat load of the heat exchange station; $\Omega_{pipe, in, g}$, $\Omega_{pipe, out, g}$ are the pipe sets with node $g$ as outflow point and inflow point respectively; $T_{mix, g}$ is the mixed temperature of node $g$; $T_{g}^{max}$, $T_{g}^{min}$ are the upper and lower limit of the water temperature at node $g$ respectively.

E. COUPLING CONSTRAINT

The power network and the natural gas network are coupled through the gas turbine P2G unit, which must satisfy the following constraints:

$$f_{GT, i, t} = h_{t, i} P_{GT, i, t} + h_{o, i} P_{GT, i, t} + h_{0, i} i \in \Omega_{GT}$$

$$f_{P2G, i, t} = h_{2, i} P_{P2G, i, t} J \in \Omega_{P2G}$$

where $\Omega_{GT}$, $\Omega_{P2G}$ are gas turbine unit and P2G set respectively; $P_{GT, i, t}$, $f_{GT, i, t}$ are the gas turbine active power and corresponding natural gas consumption at the time $t$ respectively; $P_{P2G, i, t}$, $f_{P2G, i, t}$ are the P2G unit active power and corresponding natural gas conversion quantity at the time $t$ respectively; $h_{2, i}, h_{1, i}, h_{0, i}, h_{P2G, i}$ are the coefficients.

The natural gas network is coupled with the thermal network through the CHP device, and the power network is coupled simultaneously. The CHP mode of operation is “heat to power”, meeting the constraints:

$$P_{CHP, c, t} = K_{CHP, c} H_{CHP, c, t}$$

$$f_{CHP, c, t} = \left( P_{CHP, c, t} + H_{CHP, c, t} \right) \eta_{CHP}$$

where $\Omega_{CHP}$ is a set of CHP; $H_{CHP, c, t}$, $P_{CHP, c, t}$ are CHP conversion heat power and electric power; $K_{CHP, c}$ is the electric heat ratio of CHP unit; $f_{CHP, c, t}$ is CHP unit natural gas consumption; $\eta_{CHP}$ is CHP unit conversion efficiency; $H_{GV}$ is natural gas heat value, taken as 39kJ/m³.

F. OPTIMIZATION OBJECTIVE

Single zone EGHS day-ahead scheduling model aims to minimize the cost of system operation, that is, to minimize the cost of system power supply by optimizing scheduling:

$$f = \sum_{i \in \Omega_{CO}} \left( \sum_{j \in \Omega_{G}} (\sum_{i \in \Omega_{S}} (a_{i} P_{G, i, t}^2 + b_{i} P_{G, i, t} + c_{i}) + \sum_{j \in \Omega_{S}} f_{b, j, t} \right)$$

$$+ \sum_{k \in \Omega_{W}} \delta_{k} (P_{W, k, max} - P_{W, k, t}) + \sum_{n \in \Omega_{S}} C_{S, n, t} Q_{S, n, t}$$

$$+ \sum_{m \in \Omega_{h}} C_{h, m, t} h_{h, m, t}$$

where $T$ is the dispatch period; $\Omega_{G}$ is the set of coal-fired units; $\Omega_{W}$ is the set of gas sources; $\Omega_{S}$ is the set of wind turbines; $\Omega_{h}$ is the set of heat sources; $P_{G, i, t}$ is the contribution of the set of coal-fired units $i$ at time $t$; $a_{i}$, $b_{i}$, $c_{i}$ are the set cost coefficient; $f_{b, j, t}$ is the gas output of $j$ at time $t$; $\delta_{k}$ is the gas purchase cost coefficient; $P_{W, k, max}$ is the upper limit of the output of the wind turbine; $P_{W, k, t}$ is the output of the wind turbine at time $t$; $\delta_{k}$ is the penalty coefficient of discarding the wind; $T_{out}$, $T_{g, t}$ is the gas output of gas storage tank $n$ at time $t$; $C_{S, n, t}$ is the cost coefficient of the gas tank; $h_{h, m, t}$ is the heat source $m$ contribution at time $t$, $C_{h, m, t}$ is the purchase cost of heat.

III. INTERVAL DECOUPLING OF INTERCONNECTED EGHS

On the physical level, the adjacent single zones EGHS are coupled through tie lines and tie pipes. In the energy flow layer, the electric energy, gas energy and thermal energy of each region are interacted through the power connection line, natural gas connection pipeline and hot water pipeline respectively. In the distributed scheduling framework of interconnected EGHS proposed in this paper, the single zone EGHS is taken as the basic decision-making body. Each decision-making main body carries on the unified dispatch decision-making main body. In the existing research, the decoupling of interconnected EGHS generally selects the voltage phase angle and line power as the coupling boundary variables in interconnected power network, the gas flow of pipelines as the coupling boundary variables in interconnected natural gas network, and the temperature of pipeline flow as the coupling boundary variables in interconnected heat network. Use the bus-bar splitting method to decouple the interconnected interconnected EGHS, illustrated with figure 2 as an example.

A. DECOUPLING OF INTERCONNECTED POWER NETWORK

As shown in figure 2, the cross-region power line c-d is disconnected from the midpoint, and two grid nodes $a_d$ and $b_c$ are added respectively in the A and B regions. If the node voltage phase angle and active power are chosen as the boundary variables, then the value of boundary variables at
\[ a_d \text{ and } b_e \text{ should be the same, which satisfies the following constraint:} \]
\[
\begin{align*}
\theta_{a_d} &= X_{E1AB} \\
\theta_{b_e} &= X_{E2AB} \\
P_{c_{ad}} &= X_{E2AB} \\
P_{b_{d}} &= X_{E2AB}
\end{align*}
\]

where \( X_{E1AB}, X_{E2AB} \) are the voltage phase angle and active power of the boundary coordination variable respectively, which are used to coordinate the boundary variables of the two regions. \( \theta_{a_d}, \theta_{b_e} \) are the voltage phase angles of node \( a_d \) and \( b_e \) respectively; \( P_{c_{ad}}, P_{b_{d}} \) are the line power of \( c-a_d \) and \( b_e-d \) respectively.

**B. DECOUPLING OF INTERCONNECTED NATURAL GAS NETWORK**

Similarly, the e-f midpoint of the trans-regional gas connection pipeline is disconnected, and two gas network node \( a_f \) and \( b_e \) are added respectively in the A and B regions. If the gas flow rate is chosen as the boundary variable, then the values of the boundary variables at the \( a_f \) and \( b_e \) points should be the same, satisfying the following constraint:

\[
\begin{align*}
f_{c_{af}} &= X_{GAB} \\
f_{b_{ef}} &= X_{GAB}
\end{align*}
\]

where \( X_{GAB} \) is the boundary coordinating variable of the natural gas flow; \( f_{c_{af}}, f_{b_{ef}} \) are the natural gas flow of pipeline \( e-a_f \) and \( b_e-f \) respectively.

**C. DECOUPLING OF INTERCONNECTED HEAT NETWORK**

Similarly, the g-h midpoint of the cross-district heat pipeline is disconnected, and \( ah \) and \( bg \) nodes are added in the A and B regions respectively. Select the temperature of the pipeline water flow as the boundary variable:

\[
\begin{align*}
T_{a_f} &= X_{HAB} \\
T_{b_e} &= X_{HAB}
\end{align*}
\]

where \( X_{HAB} \) is the boundary coordinating variable of the water flow temperature; \( T_{a_f}, T_{b_e} \) are the node flow temperature of \( a_f \) and \( b_e \) respectively.

Through the proposed method, decoupling of interconnected zones of different sizes can be achieved.

**IV. ALGORITHM FOR SOLVING INTERCONNECTED EGHS MODEL**

The ADMM can be used to solve the model after the interval decoupling of interconnected EGHS is finished, but the ADMM algorithm is essentially a deformation of augmented Joseph-Louis Lagrange method, whose convergence can be guaranteed only when the original problem is convex. In the gas network model constructed in this paper, the expressions (5) and (13) are non-convex constraints, and the ADMM solution can’t guarantee the convergence, so it is necessary to convexize the model.

**A. PCCP ALGORITHM**

PCCP is a kind of penalty concave-convex algorithm.

Formula (13) can be directly transformed into the second-order cone form of formula (21). Since the optimization objective of the scheduling model constructed in this paper is to minimize the energy supply cost of the system, and the unnecessary consumption of natural gas will inevitably lead to the increase of the energy supply cost of the system, the transformation is reasonable and effective:

\[
f_{GT,i,t} \geq h_{2,i}P_{GT,i,t}^2 + h_{1,i}P_{GT,i,t} + h_{0,i}i \in \Omega_{GT}
\]

For formula (5), first complete the following equivalent transformation:

\[
\begin{align*}
(b_{i,j,d+1})^2 + \Delta x_{j}M_2 (f_{i,j,d})^2 - (b_{i,j,d})^2 \leq 0 \\
(b_{i,j,d+1})^2 - \Delta x_{j}M_2 (f_{i,j,d})^2 + (b_{i,j,d})^2 \leq 0
\end{align*}
\]

After the above transformation, formula (22) is convex constrained and (23) is still nonconvex. The convex relaxation of formula (23) is performed by using the sequential second-order cone programming method presented in the reference [27]. If the quadratic term is linearly expanded by the first order Taylor series, then expression (23) can be approximated as follows:

\[
\begin{align*}
(b_{i,j,d})^2 - 2b_{i,j,d+1}b_{i,j,d+1} - (b_{i,j,d+1})^2 \\
- \Delta x_{j}M_2 \left[ 2f_{i,j,d}f_{i,j,d} - (f_{i,j,d})^2 \right] \leq \delta_{i,j,d}
\end{align*}
\]

where \( b_{i,j,d+1}, b_{i,j,d+1} \) are the air pressure and air flow at the position \( d+1 \) of the tube at time \( t \) after the \( k \)-th iteration; \( \delta_{i,j,d} \) is a relaxation variable with a value greater than or equal to 0.

This relaxation will bring a certain relaxation gap, so a penalty convex-concave process is introduced to make the relaxation gap shrink by penalty function. Add the relaxation variable to the optimization objective function:

\[
W^{(k)} = f + \rho^{(k)} \sum_{i \in T} \sum_{(i,j) \in \Omega_{pipe}} \sum_{d \in \Omega_d} \delta_{i,j,d}
\]
where $W^{(k)}$ is the objective function value with penalty function obtained by the $k$th optimization; $\rho^{(k)}$ is the penalty factor of the outer PCCP for the $k$th iteration; $\Omega_d$ is the set of relaxation variables.

In each iteration, the existence of penalty term will make the relaxation gap shrink until the convergence condition is satisfied. The convergence conditions of the PCCP algorithm are:

$$
\max_k \left| W^{(k)} - W^{(k-1)} \right| < \epsilon_1 \left| W^{(k-1)} \right| \quad (26)
$$

where $\epsilon_1$ and $\epsilon_2$ are convergence admissibility; $\delta$ is the relaxation matrix.

If the convergence condition is not satisfied, the penalty factor is updated as follows:

$$
\rho^{(k+1)} = \min \left( v_c \rho^{(k)}, \rho^{\text{max}} \right) \quad (27)
$$

where $\rho^{\text{max}}$ is the maximum of penalty factor and $v_c$ is the dynamic adjustment factor.

The sequential second-order cone algorithm is essentially a heuristic algorithm, and the selection of the initial point has an important effect on the performance of the algorithm. In this paper, we propose a new strategy to improve the initial point, which is different from the conventional method of selecting zero point as the initial point. For the first iteration, we ignore formula (24) and construct the second-order cone programming (SOCP) model in order to find out the excellent initial point. After each iteration of ADMM, the output value of gas turbine and P2G is directly selected as the initial iteration point of the next iteration, which can ensure the speed and quality of the solution. At the same time, the penalty factor dynamic adjustment parameter $v_c$ is an open parameter, and the value of $v_c$ is directly related to the speed of PCCP.

### B. ADMM ALGORITHM

The ADMM algorithm can be applied to solve the interconnected EGHS after interval decoupling and model convexization. The core idea of ADMM algorithm is to add the coupling constraints (18), (20) and (21) into the objective function of each sub-problem by Joseph-Louis Lagrange relaxation. In the case of figure 2, the objective function in region A can be expressed as follows:

$$
\min W_A + \lambda_{E1AB}(\theta_b - \theta_d) + \frac{\beta_E}{2} (\theta_b - \theta_d)^2
+ \lambda_{E2AB}(P_{b,d} - P_{c,d}) + \frac{\beta_E}{2} (P_{b,d} - P_{c,d})^2
+ \lambda_{GAB}(f_{b,f} - f_{e,f}) + \frac{\beta_G}{2} (f_{b,f} - f_{e,f})^2
+ \lambda_{HAB}(T_{b_e} - T_{a}) + \frac{\beta_H}{2} (T_{b_e} - T_{a})^2 \quad (28)
$$

where $\lambda_{E1AB}, \lambda_{E2AB}, \lambda_{GAB}$ and $\lambda_{HAB}$ are Lagrange multiplier; $\beta_E, \beta_G$ and $\beta_H$ are penalty parameters.

Each sub-system realizes distributed optimization of interconnected EGHS by updating Lagrange multipliers.

The Lagrange multipliers are updated as follows:

$$
\begin{align}
\lambda_{E1AB}^{k+1} &= \lambda_{E1AB}^k + \beta_E (\theta_d^k - \theta_b^k) \\
\lambda_{E2AB}^{k+1} &= \lambda_{E2AB}^k + \beta_E (P_{d,b}^k - P_{b_d}^k) \\
\lambda_{GAB}^{k+1} &= \lambda_{GAB}^k + \beta_G (f_b^k - f_e^k) \\
\lambda_{HAB}^{k+1} &= \lambda_{HAB}^k + \beta_H (T_{b_e}^k - T_{a_b}^k)
\end{align} \quad (29)

\begin{align}
\epsilon_{E1}^{k+1} &= \max \left| \theta_d^k - \theta_b^k \right| \\
\epsilon_{E2}^{k+1} &= \max \left| P_{b_d}^k - P_{b_d}^k \right| \\
\epsilon_{G}^{k+1} &= \max \left| f_b^k - f_e^k \right| \\
\epsilon_{H}^{k+1} &= \max \left| T_{b_e}^k - T_{a_b}^k \right|
\end{align} \quad (30)

$$

where $\epsilon_{E1}^k, \epsilon_{E2}^k, \epsilon_{G}^k$ and $\epsilon_{H}^k$ are the absolute values of the maximum difference between the boundary variables of each subsystem in the $k$th iteration; $\epsilon^k$ is the maximum value of $\epsilon_{E1}^k, \epsilon_{E2}^k, \epsilon_{G}^k$ and $\epsilon_{H}^k$; $C_{all}$ is the total objective value of the overall multi-zone system for the $k$th iteration; $a$ and $b$ are the convergence parameters.

When the absolute value of the maximum difference between two adjacent boundary variables is less than the set value, and the change rate of the objective function value is less than the set value, the algorithm is considered convergent. The flow chart of the algorithm is shown in Appendix Figure 8.

### V. CASE STUDIES

In this paper, two regions and three regions are used as simulation examples respectively to verify the validity of the proposed distributed optimal scheduling framework for interconnected EGHS. The simulation program is written by Matlab2018a and Gams, and runs on a PC with Intel E5-2670 V3 CPU, 2.3 Ghz frequency and 64 GB memory. In this chapter, the process of algorithm solution, the validity of scheduling results and the comparison of optimization results under different interconnection modes are analyzed in detail.

The key parameters of PCCP and ADMM are given in Table 1. The parameters of the simulation system are shown in tables 5 and 6 of the appendix. The node coupling and the
energy supply price in each region of the example are shown in table 7 of the appendix.

A. CASE OF TWO-ZONE INTERCONNECTED EGHS
Two-zone interconnected EGHS is made up of two single-zone systems with the same topological structure. The single-zone system topology diagram is shown in figure 3.

Table 2 presents the results and computing time of the centralized solution method, the convex distributed algorithm and the independently operation of two zones. The analysis of Table 2 data shows that:

1) Among the three groups of operation results, the economy of the system is the worst when the system is run in isolation. Because the energy interaction between the two regions is 0 when running in isolation, it is impossible to coordinate the resources and complement the energy through the tie lines and the tie pipes. But in the centralized and distributed solution methods, because of the existence of tie lines and tie pipes, the whole operation economy is optimized by the coordination optimization between the two regions. It is verified that the proposed interconnected EGHS model is helpful to improve the system operating economy. At the same time, it is found that the difference between the results of the proposed convex distributed algorithm and the centralized operation is 0.2957, and the ratio of the difference is 0.029%, which validates the effectiveness of the proposed distributed optimization framework.

2) In the comparison of running time, it is found that the single running mode of parallel optimization is the shortest, and the centralized optimization is the second. The time of distributed optimization is longer than that of centralized optimization because: a. ADMM algorithm is a serial running structure, and A and B regions need to be solved in turn; b. in addition to the boundary variables iteration of ADMM algorithm, each iteration of ADMM requires PCCP iteration in its own sub-region to realize model convexization. Although the distributed framework takes a longer time to run, it is acceptable to sacrifice part of the running time for high-quality solution under the condition of low communication cost. In practical application, the convergence accuracy of ADMM and PCCP can be reduced properly, and the computing speed can be increased by multiple.

Fig. 4 shows the variation of the algorithm residuals in formula (33) of ADMM algorithm. The iterative process of the function value difference is shown in Fig. 9 of appendix. According to the graph, the absolute value of the maximum difference between the boundary variables satisfies the convergence condition after 22 iterations. If we ignore the PCCP process and directly apply the traditional ADMM solution, then after 35 iterations, the absolute value of the maximum difference between the boundary variables is still larger than the convergence criterion, because the sub-problem after
ignoring the PCCP process is non-convex, so the convergence can not be guaranteed when directly using the ADMM solution. The effectiveness of the proposed convexization method is verified.

**B. CASE OF THREE-ZONE INTERCONNECTED EGHS**

In order to verify the applicability and validity of the proposed framework, a three-zone interconnected EGHS example system is constructed. The example system is composed of three single-zone EGHS with the same topology. The single zone EGHS structure is shown in figure 3 and the whole structure is shown in figure 5.

Table 3 presents the operation of the centralized solution, the proposed convex distributed algorithm and the two-zone single operation, meanwhile demonstrating the results and computation time of the distributed algorithm under these three different operation modes. The results show that the difference between the proposed algorithm and the centralized optimization is 0.5050, and the proportion of difference is 0.034%, which validates the applicability and validity of the proposed distributed optimization framework. After comparing the optimization results under different operation modes, it can be seen that the algorithm is faster when only considering the single energy interaction of the interval, because the boundary variables are reduced in ADMM algorithm, so the variables that need to be determined for convergence decrease. However, the optimal results are all inferior to considering the interactive operation of electric, gas and thermal energy, indicating that the construction of a multi-energy complementary integrated system can fully tap the potential of the overall economic operation. At the same time, it is found that the optimization result of only connecting hot water pipelines is close to the single optimization result under three kinds of single interconnected EGHS operation condition, and is inferior to the optimal results of other two kinds of single connection way. This is because the energy consumption of heat-supply network is lower than that of power network and gas network, so heat-supply network is
TABLE 4. Comparison of results under different initial point settings.

| Initial point setting          | Objective function value /10^4$ | Number of iterations (average) | Calculating time /s |
|-------------------------------|---------------------------------|--------------------------------|---------------------|
| Zero Point                    | 1486.6710                       | 9                              | 9.3599              |
| Only consider SOCP to get the initial point | 1485.9371                       | 6.6232                         | 6.9017              |
| An improved strategy for initial point selection | 1485.9082                       | 4.3467                         | 4.4136              |

FIGURE 7. Typical PCCP convergence process.

often the storage part of the whole system in EGHS, which is used to improve the reliability of the whole system and the flexibility of dispatching.

Fig. 6 shows the operation of the power grid and the transmission power of the tie-lines in zone A under the distributed operation mode. In the power network, during the early morning hours, the power load is low, and the wind turbine output is high, so the P2G device power is high, converting the wind power into the natural gas storage to consume wind power. 7 hours later, when the daytime work begins, the power load increases and the grid can absorb all the wind power, so the power of P2G device is reduced to 0. In the evening, when the power load reaches its peak and the output of the coal-fired unit reaches its upper limit, some power is obtained from zone B, where the power supply cost is lower, to meet the operation constraints of the zone, and zone C is also at the peak load, supplying electricity to zone C, where energy costs are higher. When the power load is at its peak, the zone A will use the higher output cost gas turbine output maintains power balance, and at other times, the coal-fired unit’s own output can meet the load demand, so the gas turbine output has been maintained at the lowest level, therefore the output of gas turbine is kept to the minimum except the fluctuation during 17 ~ 19h. CHP works in the mode of “heat to power”, so CHP output power is 0 except 6~12h and 16~22h when the heat network demand power from CHP. Similarly, the operation of the gas network and heat network in zone A are shown in figures 10 and 11 of the appendix.

FIGURE 8. Flow chart of the algorithm.

FIGURE 9. Convergence of objective function value residuals in a two-zone system.
Figure 12 in the appendix records the converging process of the boundary variables in the distributed solution of three-zone interconnected EGHS. In order to explore the influence of the initial value on the process of PCCP, the initial value of PCCP is set as zero in three-zone interconnected EGHS case, only considering SOCP and the proposed method to obtain the initial point. The results are shown in Table 4. The analysis shows that it is simple to take the zero as the initial point, but the quality of the solution is the worst and the computation time is the longest and each region requires 9 PCCP iterations to achieve PCCP convergence conditions to convex the model. Although the optimization result of SOCP is similar to the proposed method, it is still time-consuming because it can not make use of the output information of gas units and P2G in each region after each ADMM iteration. By using the proposed strategy to select the initial point, the performance of the algorithm is greatly improved, and the number of iterations is significantly reduced while the quality of solution is guaranteed.

Fig. 7 shows a typical convergence process of PCCP iteration. When the zero point is taken as the initial point, in the first iteration, the relaxation variable constraint is very serious, and when only take the SOCP into account to get the initial point, the outcome is better than the method of choosing the zero point but inferior to the proposed strategy. By using the improved strategy of initial point selection, the
relaxation variable basically satisfies the convergence constraint in the first iteration, and with the increase of penalty factor in the iteration process, the over-limit of constraint decreases rapidly, and PCCP converges after 4 iterations. Because $v_c$ is an open parameter, the value of $v_c$ is directly related to the speed of solving PCCP. Figure 13 in the appendix is the average iteration times of PCCP and the objective function value obtained by the whole algorithm under different $v_c$ values. The graph shows that when the $v_c$ value is small, the average number of iterations of PCCP is more, because the penalty factor increases slowly. When the $v_c$ value is 5, the average number of iterations of PCCP is less, and when the $v_c$ value is over 5, the average number of iterations of PCCP decreases less, and the quality of the solution decreases slightly. Combining the speed and the quality of the solution, in the model proposed in this paper, the $v_c$ value is set to 5. Need to explain is that for different models, $v_c$ optimal value range is different, so it is necessary to set parameters according to specific models.

**VI. CONCLUSIONS**

Aiming at the optimal scheduling problem of interconnected EGHS, this paper first proposes a distributed scheduling framework for interconnected EGHS considering the topology of the heat network, which includes the ADMM-PCCP algorithm coordinated internally and externally, then the algorithm solving process, the effectiveness of scheduling results and the comparison of optimization results under

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**FIGURE 13.** The effect of different $v_c$ values on the algorithm.

**TABLE 5.** Parameters of generators.

| Gas unit | Electric network node | Gas Network Node | $P_{\text{max}}$/MW |
|----------|-----------------------|------------------|---------------------|
| G1       | 30                    | 4                | 520                 |
| G7       | 36                    | 10               | 290                 |
| G8       | 37                    | 12               | 280                 |

| Coal fired unit | Cost coefficient | $P_{\text{max}}$/MW |
|-----------------|-------------------|---------------------|
| G2              | 0.700 27 0       | 646                 |
| G3              | 0.682 22 0       | 725                 |
| G6              | 0.425 25 0       | 687                 |
| G9              | 0.458 26 0       | 865                 |
| G10             | 0.890 25 0       | 1100                |

**TABLE 6.** Parameters of gas sources.

| Gas source | Gas network node | Upper limit of capacity/m³ | Lower limit of capacity/m³ | Upper limit of output(m³/s) | Lower limit of output(m³/s) |
|------------|------------------|-----------------------------|-----------------------------|----------------------------|-----------------------------|
| S1         | 1                | —                           | —                           | 150.52                     | 10.27                       |
| S2         | 8                | —                           | —                           | 150.24                     | 11.97                       |
| C1         | 2                | 175.00                      | 29.17                       | 29.17                      | 29.17                       |
| C2         | 5                | 100.00                      | 16.67                       | 16.67                      | 16.67                       |
| C3         | 13               | 25.00                       | 4.17                        | 4.17                       | 4.17                        |
| C4         | 14               | 117.22                      | 19.54                       | 19.54                      | 19.54                       |
Different interconnection modes are analyzed in detail. The results show that the coordinated optimization scheduling of interconnected EGHS can improve the economy of the system, and the optimization potential of the whole system is greater under the multi-energy flow interaction between regions, so the effectiveness and necessity of interconnected EGHS and multi-energy-flow complementarity are proved; besides, the performance of PCCP can be improved greatly by setting reasonable penalty factor to adjust parameters and using the proposed initial-point selection strategy. It should be noted that distributed optimal scheduling is essentially still a cooperative game with incomplete information, and in an open free market environment, different types of game behaviors exist between different energy suppliers in a single region and between multi-regional dispatching centers. Therefore, in the follow-up work, the author will mainly study the related game strategies.

**APPENDIX**

See Figures 8–13 and Tables 5–7.

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**TABLE 7. Node coupling in each district and energy supply price relationship.**

| Zone Type     | Network Type | Connection relationship | Energy supply price relationship |
|---------------|--------------|-------------------------|---------------------------------|
| Two zones     | Power Network | Node 39 in zone A is connected to node 8 in zone B | A: B1 = 0.95 |
|               | Gas Network   | Node 1 in zone A is connected to node 20 in zone B | A: B1 = 1.1 |
|               | Heating Network | Node 15 in zone A is connected to node 1 in zone B | A: B1 = 1.1 |
| Three zones   | Power Network | Node 39 in zone A is connected to node 8 in zone B | A:B:C = 1.0:9524:1.0526 |
|               | Gas Network   | Node 1 in zone A is connected to node 20 in zone B | A:B:C = 1.0:1.1111:0.9091 |
|               | Heating Network | Node 15 in zone A is connected to node 1 in zone B | A:B:C = 1.0:1.1111:0.9091 |
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