Gray models of convection in core collapse supernovae

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Abstract. A major difficulty facing modelers of core collapse supernovae lies in the need for an accurate description of the flow of neutrinos that leak out of the post-collapse core. In the interior of the collapsed core the neutrinos have a short mean free path (MFP) and are diffusive while in the exterior the MFP is much larger than the star and the neutrinos free stream radially. The greatest difficulty in modeling the transport of neutrinos is to correctly describe the flow of neutrinos in the intermediate regions where the neutrino distribution function transitions between the two extremes. In this region the neutrino distribution function transitions from a Local Thermodynamic Equilibrium (LTE) distribution to a non-LTE (NLTE) distribution. The difficulties of numerically modeling the neutrino transport in this transition region have been compounded through the use of the gray approximation to radiation transport. The gray approximation assumes that the neutrino distribution can be described by a distribution function that is parameterized in terms of a neutrino temperature, $T_\nu$, and a neutrino chemical potential, $\mu_\nu$. However, these parameters must be assumed. Furthermore, the parameters will also differ between the LTE and NLTE regions. Additionally, within the gray approximation the location at which the neutrino distribution function transitions from LTE to NLTE must be assumed. By considering a series of models where the LTE/NLTE decoupling point is varied we show that the outcome of the numerical models is critically sensitive to the choice of the decoupling point when the gray approximation is employed. We also examine the effects of the neutrino-electron scattering (NES) rate which is difficult to correctly formulate within the gray approximation. We show that NES has a dramatic affect on the the overall neutrino heating rate and the dynamics of the model. This result conflicts with the results of high resolution multi-group models which model neutrino transport without employing the gray approximation.

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1. Introduction

For years researchers have realized that portions of the region of a post-bounce core collapse supernova interior to a stalled prompt shock are convectively unstable. The origin of this convective instability is fairly well understood. After the “bounce” of the collapsed stellar core the prompt shock wave is formed near the sonic point which separates the inner homologous core from the supersonically infalling outer core. As the shock begins to propagate outward from this point (at an enclosed baryon mass of about $0.5 - 0.7 M_\odot$) it weakens as energy is expended in dissociating the heavy nuclei into free nucleons. This weakening of the shock produces a negative entropy gradient that is convectively unstable.

Until recently, researchers had numerically modeled the prompt phase of a core collapse supernova via 1-dimensional (1-D) radiation hydrodynamic models. While these models, in many cases, included fairly sophisticated microphysics and radiation transport techniques they were unable to accurately model the convection which is inherently multi-dimensional in nature. This situation changed radically with the pioneering work of [1] who were the first to conduct multi-dimensional simulations of this convection. Within the past few years there have been a spate of models [2, 3, 4, 5, 6] that have begun to carry out 2-D simulations of the evolution of this convectively unstable region in the post-bounce epoch. Most of these models have shown convection to take place in the region between the neutrinosphere and the stalled prompt shock.

However, the advancement to multi-dimensional numerical models of post-bounce supernovae has forced some compromises in the way that neutrino transport is modeled. In this paper we report on some consequences of the use of the gray approximation to radiation transport in 2-D supernova models. In the context of supernova models the gray approximation has taken on a unique form. The neutrinos in the interior high density regions of the supernova are assumed to be in LTE with matter while in outer regions of the core the neutrinos are assumed to be thermally and chemically decoupled from the matter [7] (hereafter CVB). In the high density regions the assumption of LTE is adequate. However, the fact that the neutrinos are decoupled in the exterior regions of the core require an assumption of spectral properties for each species of neutrino.

The use of the gray approximation to describe the evolution of the neutrinos in a core collapse supernova should be contrasted with the multi-group treatment that has been employed in a plethora of 1-D calculations [8, 9, 10, 11, 12, 13]. The multi-group approach does not assume a spectral energy distribution for the neutrinos, rather the spectrum is explicitly modeled. This is accomplished by solving a monochromatic transport equation [14] for a set of discrete energy “groups” which span the spectral range of interest.

From 1-D simulations it has been known for some time that for the problem of modeling neutrino transport in supernovae the gray methods and the multi-group methods of neutrino transport can yield substantially different results. In particular we have discovered that the results obtained by the use of the gray approximation are extremely sensitive to the ad hoc choice of parameters needed to describe the decoupling of neutrinos and matter and to the choices made in the implementation of the weak interaction rates that describe the coupling of neutrinos to matter. Our purpose in this paper is to describe these sensitivities and discuss how they effect the multi-dimensional models in the convective epoch of supernovae.
2. Numerical Models

The numerical hydrodynamic algorithm employed in these calculations is a modified version of the ZEUS-2D code (Stone & Norman 1992) in spherical polar coordinates. The ZEUS-2D algorithm explicitly solves the Euler equations of Newtonian hydrodynamics:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}
\]

\[
\frac{\partial (e \rho)}{\partial t} + \nabla \cdot (e \rho \mathbf{v}) + (P + Q)\nabla \cdot \mathbf{v} = \left( \frac{De}{Dt} \right)_{\text{source}} \tag{2}
\]

\[
\frac{\partial (v_i \rho)}{\partial t} + \nabla \cdot (v_i \rho \mathbf{v}) + \nabla (P + Q) + \rho \nabla \phi = 0 \tag{3}
\]

This algorithm offers several advantages for this particular problem (for details see [15]). We have made several major changes to this algorithm to accommodate the unique nature of dense matter hydrodynamics. First, we have added a separate continuity equation to treat the advection of electrons. Second, we have modified the ZEUS-2D algorithm to incorporate an arbitrary equation of state (EOS) instead of the ideal gas assumed in [16]. In doing so, we have utilized the matter temperature $T$ as the fundamental variable in solving the Lagrangean part of the gas energy equation instead of using the internal energy.

A third major modification has been made in order to circumvent the strong restriction on the timesteps imposed by the CFL stability limit in polar coordinates near the origin. We assume that the flow is radial inside a certain radius, $r_r$ which eliminates the restriction posed by the angular part of the CFL criterion near the origin. For the calculations described in this paper we have employed $r_r = 22.5$ km which is inside the proto-neutron star formed by the core bounce. In the calculations discussed in this paper we have assumed that gravity is described by a spherically symmetric potential which is easily calculated by integration over the domain of computation.

Calculations were carried out in spherical polar coordinates with two different mesh sizes. The 1-D models were actually carried out using the 2-D code with very low angular grid resolution, i.e. 3 angular grid zones spanning the polar angular range of $(\pi/4, 3\pi/4)$. This low angular grid resolution is sufficient to ensure that non-radial flow features never develop. The radial grid resolution spans three separate ranges. There are 60 radial zones covering the innermost 25 km of the grid. Between 25 km and 400 km there are 132 zones, and from 400 km to 900 km there are 25 zones that geometrically increase in size with radius. The 2-D models maintain the same radial zoning but employ 90 angular zones over the same angular domain for an angular zone width of $\Delta \theta = 1^\circ$.

In the calculations described in this paper we have modeled the evolution of three neutrino species: $\nu_e$, $\bar{\nu}_e$, and a generic species $\nu_x$ which represents the $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$, and $\bar{\nu}_\tau$ neutrinos. The evolution of the neutrino energy density in this model is described by the mixed-frame flux-limited diffusion equation:

\[
\frac{DE_\nu}{Dt} + \nabla \cdot (D \nabla E_\nu) + E_\nu \left( P_2 \frac{\partial \bar{\nu}_e}{\partial r} + Q_2 \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right) = -\left( \frac{De}{Dt} \right)_{\text{source}} \tag{4}
\]

where $E_\nu$ in this case refers to a generic neutrino energy density that could be $E(\nu_e), E(\bar{\nu}_e)$, or $E(\nu_x)$. The $P_2$ and $Q_2$ coefficients involve the Eddington factor $\chi$ and
are described in [8]. The equation is implicitly finite differenced on the same staggered mesh as the hydrodynamic code. The source term is linearized so that the entire implicit system of transport equations is solved once per timestep. Finally, in the calculation described in this paper we have assumed that the neutrinos diffuse only radially which decouples the solution of (1) into a separate tridiagonal system for each angular zone of the problem. Our intent is not to mimic the variety of schemes used in other works, but to employ a single scheme to illustrate how the results of the simulations depend on the choice of decoupling location. We describe the details of the finite differencing solution of equation (4) in [15].

The diffusion coefficient $D$ is calculated by the Levermore–Pomraning [17] prescription. In order to calculate the source terms and diffusion coefficient of equation (4) the gray approximation requires the assumption of a spectral distribution for each neutrino species. We discuss this choice in section 3.

For the equation of state we employ the finite temperature EOS of [18] using the Sk180 nuclear force parameter set [13]. The EOS was tabularized using a thermodynamically consistent interpolation scheme [19] and is described in [15].

The weak interaction rates were implemented in a fashion similar to the two-fluid rates of CVB. One exception to this are the electron capture and neutrino capture rates. These rates as implemented by CVB, HBHFC, and BHF do not preserve detailed balance in the regions in LTE. We have not made the approximations employed in CVB in order to simplify the phase space integrations but instead preserve detailed balance by calculating the rates exactly by numerical quadrature. The other difference between our rates and the rates of the aforementioned calculations is that we restrict the NES heating rates to reflect only the down-scattering of neutrinos seen in multi-group calculations. If this is not done the rates as implemented in the aforementioned calculations can lead to an unphysical cooling of matter by NES in high density regions. Further details of the NES rate are discussed in section 3.3.

For the initial conditions we collapsed a progenitor core using our BRYTSTAR spherically symmetric multi-group radiation hydrodynamics code [13] which was run in Newtonian mode. For the models discussed here we employed the S15S7b core of [20]. This particular progenitor model has a relatively small iron core. The use of a multi-group code during collapse and bounce allowed us to obtain post-bounce cores with accurate lepton and entropy profiles. We extract the initial data from the BRYTSTAR runs at the point where the prompt shock begins to stall. For the 2-D models we apply a small ($\sim 1\%$) sinusoidal perturbation to the electron fraction in the convectively unstable region.

3. Results

3.1. Choice of Decoupling Point

In the interior of the collapsed stellar core the neutrino distributions are in LTE because of the short MFP of the neutrinos. The neutrinos in this region are in chemical and thermal equilibrium. This permits a simple and accurate characterization of the neutrino distribution as a Fermi–Dirac distribution where $T_\nu = T$ (the matter temperature) and $\mu_\nu$ is given by the condition for beta equilibrium. The neutrino number density and
energy density are determined by $T_\nu$ and $\mu_\nu$. In this situation the gray approximation yields an excellent description of the evolution of the neutrinos.

In contrast, in the exterior regions of the collapsed core the MFP exceeds the radius of the star and the neutrinos do not interact with matter. Here the distribution function for the neutrinos reflects the spectrum from where the neutrinos decouple from matter, i.e. the spectrum “freezes out” in regions outside of the radius of the decoupling point. The neutrino distribution at this point is not in thermal or chemical equilibrium with the matter. However, within the gray approximation a distribution function must be assumed. In the case of the current generation of 2-D gray supernova models the assumption has been that the distribution function in this NLTE region can continue to be characterized by a Fermi-Dirac distribution function. However, the neutrino temperature and chemical potential in this region are not given by the conditions for chemical and thermal equilibrium with matter. Rather, one must either assume that $T_\nu$ and $\mu_\nu$ maintain the values that characterize the distribution function at the decoupling point or make an ad hoc assumption of their values. In either case the neutrino number density and energy density in the NLTE region are unrelated to $T_\nu$ and $\mu_\nu$. These parameters serve only to characterize the spectrum of the neutrinos and the matter-neutrino interaction rates. With an ad hoc choice of parameters the sensitivities of the models to the choice of parameters are clear: one can “tune” the neutrino reheating rate in the gain region to yield an explosion (or not) for a given model. For this reason we consider only the case where the spectrum is assumed to be that of the decoupling point.

Unfortunately, the gray approximation does not constrain the location of the point where the decoupling of neutrinos and matter occur. Common lore is that decoupling occurs near the neutrinosphere, where the neutrino optical depth is unity. However, this notion is somewhat ill defined for several reasons. First, the neutrino opacity is a strong function of neutrino energy, causing the physical location of the neutrinosphere to vary with neutrino energy. Thus the location of the neutrinosphere is somewhat nebulous. Secondly, it is not clear what sources of neutrino opacity should be considered in defining the neutrinosphere. The iso-energetic neutrino–nucleon scatterings which contribute strongly to the overall opacity isotropize the neutrino distribution but do not thermalize it. Thus the surface of last-scattering is not the point of thermal decoupling for the neutrinos. The neutrino emission-absorption reactions are not efficient at thermalizing the distribution because of the large difference between the average neutrino energies and the nucleon rest mass–energy. Therefore, it is unlikely that the neutrino distribution is thermalized exactly at the point where the optical depth due to emission–absorption is unity. Neutrino–electron scattering is a more efficient means of thermalizing the neutrino distribution but the overall contribution of NES to the opacity is small compared to the other interactions. For these reasons there is no clear physical criterion to determine where decoupling should occur.

As a practical matter one can still employ criteria based on an energy averaged optical depth or MFP to select a decoupling point in a gray model [21, 3]. Such criteria determine a “gray neutrinosphere” based on the total opacity as calculated using the grey spectrum. But the question remains: To what extent does the choice of the criterion affect the outcome of the model? If the choice of decoupling point, and thus the spectrum, in the NLTE regions were robust the outcome of the numerical simulations should not crucially depend on the particular choice. As we have previously mentioned there is no clear physical reason why any of the aforementioned criteria should yield a precise
location of the decoupling point. Accordingly, we examine in this paper how varying these choices can affect the radiation–hydrodynamic models of the convective region.

3.2. Effects of Variations in the Decoupling Point

In order to ascertain the sensitivity of the models to the choice of decoupling point we carried out several 1-D and 2-D models where the decoupling points were varied slightly in order to examine how the outcome of the simulation would be affected. The method we employ to vary the location of the neutrino decoupling point is to choose the decoupling point for each neutrino species based on the MFP, $\lambda$ for that species. We select the decoupling point based on the parameter $\xi = \Delta r / \lambda$. By trial and error we have found that varying $\xi$ between 0.5 and 2 we can cause the neutrino decoupling point and spectrum to vary such that the location of the gray neutrinosphere for the $\nu_e$ neutrinos varies over the range of approximately 20 – 40 Km. The neutrinospheres for the $\bar{\nu}_e$ and $\nu_\mu$ neutrinos vary accordingly. We briefly describe some of our findings regarding the effects of this variation on the models. For more detail the reader is referred to [15].

For simplicity, we first consider three 1-D models where we choose the decoupling point based on $\xi = 1/2$, 1, and 2. Since $\xi$ is defined in terms of the zone size there is no clear reason why one of these choices should be preferred over another. The neutrino luminosities for these three models are displayed in Fig. 1. Note that the electron neutrino luminosities initially decrease by a factor of approximately as $\xi$ is increased from 1/2 to 2. This difference gets larger later in the calculation as the $\xi = 2$ model is overwhelmed by infalling matter driving the opacity up. Note that the difference in the electron neutrino luminosities between the $\xi = 1/2$ and the $\xi = 1$ cases is not nearly as pronounced as it is between $\xi = 1$ and $\xi = 2$ cases.

The large change in neutrino luminosities between the various models can be easily understood in terms of the rapid variation of the temperature as a function of radius near the neutrinosphere. A typical example of this variation is shown in Fig. 2. Since little neutrino cooling has occurred in the first few hundred milliseconds the temperature profile is still largely determined by the hydrodynamic evolution of the collapsed core during the prompt phase. As $\xi$ is decreased the decoupling occurs at a larger radius. For the timescales we are considering in this paper the decoupling radius for the the electron neutrinos is approximately 23 km for $\xi = 2$ and approximately 39 km for $\xi = 1/2$. The corresponding electron neutrino temperatures at the decoupling points are $T_{\nu_e} \approx 9.2$ MeV for $\xi = 2$ and $T_{\nu_e} \approx 6.0$ MeV for $\xi = 1/2$. The decoupling of the neutrinos at larger radii results in a decreased electron neutrino temperature and a softer neutrino spectrum. This decrease in the electron neutrino temperature corresponds to a lower average neutrino energy and in turn a lower opacity at a given radius. Consequently, a lower opacity results in a higher electron neutrino luminosity. A corresponding change is also reflected in the electron anti-neutrino luminosities. The radius of the neutrinosphere for the electron anti-neutrinos decreases as $\xi$ is increased. This is most pronounced for the electron anti-neutrinos as the decoupling cutoff is varied from $\xi = 1/2$ to $\xi = 1$ where the radius of the neutrinosphere shifts from 39 km to 22 km. The electron anti-neutrino luminosity drops substantially with this shift. However, as the cutoff is further shifted to $\xi = 2$ there is little change in the radius of the neutrinosphere for the electron anti-neutrinos. This is again reflected in the luminosities where the $\xi = 2$ case actually gives slightly higher luminosities then the $\xi = 1$ case for approximately 60 milliseconds after
core bounce.

In contrast to the electron neutrinos and electron anti-neutrinos the \( \mu \) and \( \tau \) neutrinos (and anti-neutrinos) show smaller variations in luminosity as the cutoff parameter is varied. Again, this can be attributed to the small variation in the \( \nu_\mu \) neutrinosphere radii from 19.5 km for \( \xi = 1/2 \) to 17 km for \( \xi = 2 \). As expected the decoupling for the \( \nu_\mu \)'s occurs deeper in the star. The neutrino temperatures for the \( \nu_\mu \) neutrinos in the NLTE region originate near the peak of the temperature profile depicted in Fig. 2. Small variations in the decoupling point about this peak do not produce large variations in the \( \nu_\mu \) spectrum. The variation in the luminosities and average energies of the three neutrino species also can have a substantial effect on the dynamics and the chemical composition of the material in the region exterior to the decoupling point. This same effect is present in 2-D models where the effects on the dynamics are more pronounced because of the convection that occurs in these models.

The variations in the dynamical behavior of the 2-D models can be understood in
terms of the cumulative neutrino heating differences among the three models. The total amount of heating in the gain region is shown in Fig. 3 at a time of approximately 45 milliseconds after the beginning of the simulation when the convection is well developed. Note that the gain radius moves radially outward as $\xi$ decreases. The luminosities increase as $\xi$ decreases, as in the 1-D case, and the cumulative amount of heating in the gain region increases. Since the region from $r = 60 - 200\text{Km}$ is the base of the convective region the increased heating in the gain region drives the convection more vigorously.

While the peak heating rate in the outer regions for the $\xi = 0.5$ case is comparable to that of the $\xi = 1.0$ case the width of the gain region is narrower than either the $\xi = 1$ or $\xi = 2$ cases, thus resulting in the least total heating. Consequently the $\xi = 0.5$ model is the least vigorous of the three cases. This can clearly be seen in Fig. 4 where the entropy profiles are shown for the three models. Here the differences in the strength of the convection are clearly revealed. Given the cumulative heating profiles shown in Fig. 3 it is not surprising that the $\xi = 1$ case is showing the most vigorous behavior. By $t = 90$ msec this model has exploded to the edge of the grid. In contrast, the narrow gain region of the $\xi = 0.5$ case results in the the weakest convection of the three models. By $t = 90$ msec the shock in the $\xi = 0.5$ case actually recedes to a radius of approximately 290Km and shows signs of growing weaker. It is unclear whether this model will explode or not. However, while the $\xi = 2$ model has a lower cumulative heating profile it has a broad gain region and the model is slowly exploding. Because of space limitations we are unable to discuss on the effects of the decoupling point choice on the chemical compositions.

We briefly mention that the variations in the spectra have a substantial effect on the electron fraction in the convective region. The composition of the material is set by the competing effects of electron neutrino and anti-neutrino absorption in establishing kinetic equilibrium in this material. Thus the change in the neutrino spectra and luminosities, as $\xi$ is varied, in turn yields large differences in the electron fraction. The reader is referred to [15] for a detailed discussion of this effect.

The wide variation among the three models clearly demonstrates the sensitivity of the models to the choice of location of the LTE/NLTE decoupling point. Such variation clearly points to the need to eliminate the use of the gray approximation if we are to have predictive models that do not employ free parameters.

### 3.3. Role of the Gray NES Rate

The formulation of the NES rate in the gray approximation is also a source of substantial error in numerical RHD models of supernovae. As we previously mentioned NES is not a significant source of neutrino opacity in the collapsed core but it has a strong effect on spectral softening during core collapse. However, through high resolution 1-D multigroup calculations it is known that the NES heating rate in the gain region during the post bounce epoch is dominated by the heating rate due to absorption of neutrinos on nucleons (see Fig. 6 of [24]). From this previous work we know that the inclusion of NES heating in post-bounce simulations should have little effect on the dynamics of the model. It is also known that the main effect of NES is the spectral down-scattering of neutrinos thus softening the spectrum and enhancing the overall luminosity.

The phenomenon of spectral softening is at odds with the a priori choice of spectral parameters that must be made to employ the gray approximation. Once a spectrum has
been assumed there is no way to self-consistently account for the spectral rearrangement that NES will cause. Nevertheless, a number of attempts have been made to estimate the heating rate due to NES within the gray approximation \[7, 21, 3\]. The reader is referred to these papers for details of the formulation of these rates in the gray approximation. In some cases the formulations clearly have problems as has been pointed out by CVB in \[7\].

In order to try to estimate the effect of these rates on the convective models we carried out a comparison of the models with and without the NES heating. The results are dramatic. The entropy profiles for a $\xi = 1$ model (the most robustly exploding 2-D model) with and without NES are depicted in Fig. 5 at $t \approx 47$ msec. In the case without NES heating the explosion seems to “fizzle” for this particular choice of decoupling point while it is extremely vigorous for the case where NES heating is included using the CVB rates. The reason for this is clear from the heating rate profile shown in Fig. 6. The NES heating rate clearly gives a significant contribution over a very large region exterior to the gain radius. Clearly the excess heating in the region exterior to $r = 150$Km caused by the inclusion of NES heating is sufficient to completely alter the outcome of the model. At $t \approx 60$ msec when the model with NES is well on the way to exploding, the shock in the model without NES has receded slightly, as it becomes overwhelmed by infalling matter, and seems to weaken. This significant neutrino reheating of matter due to NES conflicts with the results obtained by several multi-group flux-limited diffusion \[10, 8, 22\] and multi-group Boltzmann \[23\] calculations which have shown only minimal effects due to NES.

The effects of the NES rate arise from the formulation of the rate in terms of Fermi integrals over the neutrino and electron distributions which does not correctly account for momentum exchange. Furthermore, gray formulations of this rate do not vanish in thermal equilibrium as they should. CVB have made note of these difficulties and we refer the reader to that work \[7\] for a thorough discussion of the problem. We wish to point out that the bizarre behavior caused by NES in the gray case indicates that it should not be included in these simulations in its existing form. These difficulties further indicate the need to carry out future supernova models using multi-group transport which allows for the neutrino–matter interaction rates to be formulated more accurately.
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