Recent Experiments with Bose-Condensed Gases at JILA

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ABSTRACT

We consider a binary mixture of two overlapping Bose-Einstein condensates in two different hyperfine states of $^{87}$Rb with nearly identical magnetic moments. Such a system has been simply realized through application of radiofrequency and microwave radiation which drives a two-photon transition between the two states. The nearly identical magnetic moments afford a high degree of spatial overlap, permitting a variety of new experiments. We discuss some of the conditions under which the magnetic moments are identical, with particular emphasis placed on the requirements for a time-averaged orbiting potential (TOP) magnetic trap.

Keywords: Bose-Einstein condensation, rubidium, rf and microwave spectroscopy, binary condensate mixtures, TOP magnetic trap

1. INTRODUCTION

The experimental realization of Bose-Einstein condensation (BEC) in the alkali metals has stimulated a flurry of scientific interest. In addition to single trapped condensates, double condensates (of two different spin states) have also been observed. In the experiments described below, another double condensate system is described, in which the two condensate spin states overlap to a higher degree than in the previous experiment. Moreover, transitions can be driven between the two states to couple them, produce superposition states, or measure their relative phase. This double condensate also makes possible a number of novel experiments which probe the interactions between two interpenetrating quantum fluids.

This paper describes the new double condensate system. We discuss in particular the conditions under which spin states are trapped and can be made to overlap. Since the experiments described here are performed in a trap with a rotating magnetic field, we also explore the peculiarities of spectroscopy in such a field.

2. CONDENSATE PRODUCTION

We use a new, third-generation JILA BEC apparatus, in which the time-averaged orbiting potential (TOP) magnetic trap of generation 2 is combined with the double magneto-optical trap (MOT) system of generation 3 to produce condensates containing up to one million atoms. $^{87}$Rb atoms are first collected in a vapor cell MOT and then magnetically guided through a transfer tube into a second ultrahigh vacuum (10$^{-12}$ Torr) MOT, where up to 10$^9$ atoms are collected in $\sim$ 15 seconds. After further cooling in optical molasses and optically pumping to the $|F = 1, m_f = -1\rangle$ hyperfine state, the trapped atoms are loaded into a magnetic trap. The atoms are further cooled by forced evaporation, in which an applied radiofrequency (rf) magnetic field drives transitions in the most energetic atoms to untrapped states (e.g., $|1, 0\rangle$). We adjust the final evaporation frequency to choose the fraction of atoms which are condensed into the ground state, which can range from 0%, a cold thermal cloud, to nearly 100%, a pure condensate. The atoms are held in the trap for up to several seconds and subsequently released and allowed to expand ballistically for imaging. The entire cycle typically takes under one minute, and the apparatus is reliable enough to operate unattended, producing nearly identical condensates without interruption for over an hour at a time.

The technique of resonant absorption imaging is used to probe the cloud. In this destructive process, a brief pulse of repump light (5$S_{1/2}, F = 1$ to 5$P_{3/2}, F' = 2$) transfers the $|1, -1\rangle$ atoms to the $F = 2$ hyperfine level. The atoms are subsequently illuminated by the probe beam, which drives the $5S_{1/2}, F = 2$ to $5P_{3/2}, F' = 3$ cycling

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†A recent preprint by the MIT group also alludes to a multiple-component BEC confined in an optical trap.
transition. Atoms scatter the light out of the probe beam and the resulting shadow is imaged upon a charge-coupled device (CCD) array. We process these data to extract the optical depth as a function of position, which permits us to determine the size of the cloud and number of atoms of which it is composed, as well as thermodynamic quantities such as its temperature $T$.

### 3. TWO OVERLAPPING CONDENSATES

In a recent experiment at JILA, Myatt and co-workers produced two overlapping condensates in the $|2, 2\rangle$ and $|1, -1\rangle$ hyperfine states of $^87$Rb. Due to their different magnetic moments, their mutual repulsion, and the effect of gravity, the condensates occupied slightly different regions in the trap and could not be made to overlap one another completely. A third spin state of $^87$Rb may also be trapped which has (to first order) the same magnetic moment as the $|1, -1\rangle$ state, and (as will be shown below) can be made to overlap more completely with it. This system permits the realization of a rudimentary Rb clock in a magnetic trap, and provides a vehicle for understanding the interplay and dynamics between two condensates trapped in different spin states without the complications introduced by the lack of condensate overlap.

#### 3.1. Magnetic Moments

In the low-field limit, the magnetic energy of a state characterized by magnetic moment $\mu$ and quantum number $m_f$ quantized along the direction of the field $B$ is

$$U = -\mu \cdot B = g_F m_f M_B B$$

where $g_F$ is the Landé $g$-factor and $\mu_B$ is the Bohr magneton. The magnetic moment precesses about the field at the Larmor frequency

$$\omega_L = \frac{-g_F \mu_B B}{\hbar}.$$  

States for which the product $g_F m_f$ is positive minimize their energy in low fields (“weak-field seekers”) and may be trapped in the minimum of a magnetic field. For $^87$Rb, this leads to three trappable states (Fig. 1): the $|2, 2\rangle$ and $|2, 1\rangle$ states in the $F = 2$ hyperfine manifold (where $g_F = +\frac{1}{2}$), and the $|1, -1\rangle$ state in the $F = 1$ manifold (where $g_F = -\frac{1}{2}$). Note that, because of the sign of $g_F$, the trapped $|1, -1\rangle$ state precesses in a sense opposite to that of the other two trapped states. This difference gives rise to some subtle behavior in the rotating magnetic field of the TOP trap, as we shall see below.

In the absence of gravity, all three of the trapped states sit atop one another, although the $|2, 2\rangle$ atoms are bound a factor of two more tightly by their larger magnetic moment. The centers of the atomic clouds are displaced (“sag”) in the presence of gravity by an amount which scales as $g/\mu$, where $g$ is the acceleration due to gravity. As a result, the $|2, 2\rangle$ atoms sag about half as much than their $|2, 1\rangle$ counterparts, and the degree of spatial overlap between the states is reduced. To a first approximation, however, the $|2, 1\rangle$ and $|1, -1\rangle$ atoms sag equally in the trap; consequently, we choose these two states for the fully overlapping condensate experiments.

Beyond first order, the magnetic moments of the $|2, 1\rangle$ and $|1, -1\rangle$ states depend (differently) upon the applied magnetic field, and a more exact analysis is required. The Hamiltonian for these atoms includes couplings between the nuclear spin $I = 3/2$, the electronic spin $J = 1/2$, and the externally applied magnetic field; this is expressed

$$\hat{H}_{BR} = \frac{\hbar \nu_{hfs}}{2} \mathbf{I} \cdot \mathbf{J} + g_J \mu_B \mathbf{J} \cdot \mathbf{B} - g_I \mu_n \mathbf{I} \cdot \mathbf{B},$$

where $\nu_{hfs}$ = 6834682612.8 Hz is the zero-field hyperfine splitting in $^87$Rb, $g_J$ and $g_I$ are the electronic and nuclear $g$-factors, and $\mu_n$ is the nuclear magneton. The energy of a particular state as a function of the magnitude of the magnetic field is found by diagonalizing Eq. (3), which yields the well-known Breit-Rabi formula

$$E_{BR}(B) = -\frac{\hbar \nu_{hfs}}{2(2J + 1)} - g_I \mu_n B m_f \pm \frac{\hbar \nu_{hfs}}{2} \sqrt{1 + \frac{4m_f x}{2J + 1} + x^2}$$

where the upper (lower) sign is taken for $F = 2$ ($F = 1$), and the parameter $x$ is defined as

$$x = \frac{(g_J \mu_B + g_I \mu_n) B}{\hbar \nu_{hfs}}.$$
Figure 1. A schematic of the $^{87}$Rb ground states. Atoms may be magnetically trapped in the states indicated by the solid polygons. We begin with atoms in the $|F = 1, m_f = -1 \rangle$ state; evaporative cooling proceeds by driving rf transitions to the untrapped $|1, 0 \rangle$ state. The two-photon transition to the trapped $|2, 1 \rangle$ state is accomplished as shown, with an intermediate state detuning of 1.24 MHz.

A plot of the splitting between the $|2, 1 \rangle$ and $|1, -1 \rangle$ states is shown in Fig. 2(b). The magnetic moments of the two states are identical when the slope of this difference becomes zero, i.e., at $B = 3.24$ G. At this field$^\dagger$ two noninteracting atomic clouds (or condensates) will overlap.

3.2. Creating the Double Condensate

The double condensate is produced by driving atoms from the $|1, -1 \rangle$ condensate to the $|2, 1 \rangle$ state with a two-photon excitation$^\dagger$ involving a microwave photon at $\sim 6.8$ GHz and an rf photon at $\sim 1$ MHz$^\ddagger$. The two-photon drive is detuned 1.24 MHz from the intermediate $|2, 0 \rangle$ state, as shown in Fig. 1. The rf pulse is generated by a Wavetek 395 synthesizer and coupled into the system in the same manner as the rf used for forced evaporative cooling. The microwave radiation is generated by an HP 8672A frequency synthesizer, amplified to approximately 1 W by a travelling wave tube amplifier, and coupled into the system through a truncated waveguide. The coupling is rather inefficient, and the resulting two-photon Rabi frequency is only 1.1 kHz. For stability and absolute hyperfine interval measurements (described below) both synthesizers are frequency-locked to the 10 MHz reference of an HP 53570A global positioning system (GPS)-stabilized time and frequency reference receiver.

By varying the rf detuning we can sweep the drive across the transition frequency and measure the number of atoms in the $|2, 1 \rangle$ condensate (Fig. 3). The repump light is blocked in the imaging sequence in order to prevent atoms in the $|1, -1 \rangle$ state from being imaged. Each point corresponds to a measurement with a pure condensate (i.e., evaporated to the point at which there was no visible thermal cloud). For this particular measurement the transfer pulse was applied after the atoms were released from the trap, but identical results are obtained with atoms transferred to the upper state while in the trap. The width of the line is consistent with the length of the applied pulse, which is approximately 500 µsec.

Strictly speaking, any transfer of atoms to the upper state which is incomplete results in a single condensate in a superposition of the two states. The probe light projects these states after the condensate is released from the trap. Unless the relative phase between the two condensates is to be measured (as in the Ramsey method described below) there is little experimental difference between the superposition state and two independent condensates.

$^\dagger$This requirement is only on the magnitude of the field seen by the atoms, and can arise from a combination of (rotating) bias fields and quadrupole components.

$^\ddagger$Note that this scheme differs from that used in the previous JILA experiment$^\ddagger$ in which the two condensates were produced simultaneously during the evaporative cooling cycle.
Figure 2. (a) The shifts of the $^{87}\text{Rb}$ energy levels as a function of the magnetic field, as calculated from the Breit-Rabi formula (Eq. 4). (b) The frequency difference between the $|2, 1\rangle$ and $|1, -1\rangle$ states (less the zero-field hyperfine splitting) is shown for weak magnetic fields. When the slope of the curve is zero ($B = 3.24$ G) the two states have the same effective magnetic moment.

Figure 3. Absorption line for the two-photon excitation between the $|1, -1\rangle$ and $|2, 1\rangle$ states, measured after the atoms were released from the trap. The width of the line is determined by the length of the pulse. Similar lines were also measured in the trap.
3.3. Effect of a Rotating Field

The preceding analysis is sufficient for a static magnetic trap, such as that used in the experiment of Myatt et al. Additional complications arise, however, when a time-dependent magnetic trap (such as the TOP trap) is used to provide the confinement. The TOP trap involves a magnetic field of magnitude $B_b$ rotating counterclockwise (as viewed from the positive $z$-axis) at angular frequency $\omega_t$ in the $xy$-plane,

$$B_b(t) = B_b(\cos \omega_t t \hat{x} + \sin \omega_t t \hat{y}). \tag{6}$$

The effective Hamiltonian in a frame co-rotating with the magnetic field transforms as

$$\hat{H}_{\text{eff}} = R(-\omega_t t) \hat{H}_{\text{BR}} R(-\omega_t t) - i \hbar R(-\omega_t t) \frac{\partial}{\partial t} R(-\omega_t t) \tag{7}$$

with the time-dependent rotation operator $R$ defined by

$$R(-\omega_t t) = \exp \left( i \frac{\hbar}{\hbar} F_z \omega_t t \right). \tag{8}$$

The operator $F_z$ is the $z$-component of the total (nuclear plus electronic) spin vector, and the sign of the argument of $R$ is chosen to rotate the coordinate axes in the same sense as the field. The Breit-Rabi Hamiltonian $\hat{H}_{\text{BR}}$ is invariant under the transformation. We therefore obtain

$$\hat{H}_{\text{eff}} = \hat{H}_{\text{BR}} - F_z \omega_t. \tag{9}$$

For weak fields (i.e., for $\omega_L \gg \omega_t$) this additional term may be approximated as an effective magnetic field $B_\omega$ pointing along the $z$-axis which causes Larmor precession of the spin at the trap rotation frequency, i.e.,

$$\omega_L = \frac{g_F \mu_B B_\omega}{\hbar}$$

$$B_\omega = -\frac{\hbar \omega_t}{g_F \mu_B} \tag{10}$$

The total spin vector for each state precesses about the magnetic field in a direction specified by the sign of its Landé $g$-factor $g_F$; as a consequence, the direction of the effective magnetic field $B_\omega$ is also determined by this sign. For our two trapped states we can take out the overall sign of $B_\omega$ explicitly and write

$$B_{\text{total}} = \sqrt{B_b^2 + (-2B_\omega z)^2} \tag{11}$$

where the upper (lower) sign is for the $|2, 1\rangle$ ($|1, -1\rangle$) state.

Up to this point we can see that the effective field is the same for both states. In TOP traps, however, there is also a magnetic quadrupole field

$$B_q = B'_q (x\hat{x} + y\hat{y} - 2z\hat{z}) \tag{12}$$

(when $B'_q = \partial B/\partial x$ and the sign of the quadrupole has been chosen such that the field points toward the center of the trap on the $z$-axis). Gravity causes the equilibrium displacement of the atoms to sag below the center of the trap as defined by the quadrupole gradient. If the force on the atoms due to gravity is in the $-z$-direction and the atoms are displaced some distance $z$ from the center of the trap, Eq. (11) must be modified to read

$$B_{\text{total}} = \sqrt{B_b^2 + (-2B'_q z \mp |B_\omega|)^2} \tag{13}$$

where the upper (lower) sign is taken for atoms in the $|2, 1\rangle$ ($|1, -1\rangle$) state. As a first result, we note that the measured transition frequency between the two states in the TOP trap will not be that shown in Fig. 2(b), which assumes that both states see the same field. In the TOP trap, each state sees a field different from that seen by the other since the rotating term $B_\omega$ adds to the field produced by the quadrupole gradient for one state and is subtracted from it for the other.

The equilibrium position of the atoms is where the total force on the atoms is zero, or

$$\frac{\partial}{\partial z} [E_{\text{BR}}(B_{\text{total}})] = M_{\text{Rb}} g. \tag{14}$$
Our second result is that the two equilibrium displacements will in general not be the same when $B_b = 3.24$ G. We may, however, find new conditions under which the two states once again share an equilibrium displacement in the TOP trap by calculating the relative sag as a function of the magnetic quadrupole gradient as well as the bias field magnitude, rotation frequency, and sense of rotation (all of which are TOP trap parameters which we can adjust within certain limits). For instance, Fig. 3 shows the calculated sag for a trap rotation frequency $\omega_t = 2\pi \cdot 7.202$ kHz and a quadrupole gradient $B'_q = 61.5$ G/cm, where the upper curve is for clockwise rotation ($\omega_t < 0$) and the lower curve for counterclockwise rotation ($\omega_t > 0$). In this case, the relative sag is calculated to be zero when $B = 9.4$ G and the field rotates in the counterclockwise sense. Note there is no value of $B_b$ for which the equilibrium displacements in the trap are the same for atoms in both states when the TOP field rotates in the clockwise sense.

In order to confirm the relative sag predicted by the theory for the rotating magnetic field, we drove the $m = 0$ center-of-mass (“slushing”) motion by discontinuously changing the trap fields (and, hence, oscillation frequencies) for pure $|1, -1\rangle$ and $|2, 1\rangle$ condensates. The resulting condensate axial motion is shown in Fig. 3. For a field of 3.2 G, the $|2, 1\rangle$ atoms are seen to oscillate about an equilibrium position somewhat higher than that of the $|1, -1\rangle$ atoms; the reverse is true for 11 G. These observations agree with the theoretical prediction of Fig. 3.

4. APPLICATIONS, PRELIMINARY RESULTS, AND CONCLUSIONS

The $|1, 0\rangle\rightarrow|2, 0\rangle$ “clock” transition in $^{87}$Rb is widely used as a frequency reference. In addition to systematic effects, the precision to which such an atomic transition can be measured is limited by the time over which the transition can be observed. By using condensates confined in a magnetic trap, the observation time can be on the order of the lifetime of the condensates, which may be up to several seconds. Unfortunately, the states traditionally used in the “clock” transition cannot be magnetically trapped since $m_f = 0$.

As noted in Sec. 3.2 (above), the $|2, 1\rangle$ to $|1, -1\rangle$ transition frequency is (like the clock transition) independent of $B$ to first order. We can measure this frequency by using Ramsey’s technique of separated oscillatory fields (SOF)). Unlike a beam experiment, the fields here occur as two pulses separated in time rather than two interaction regions separated in space. We begin with atoms in the $|1, -1\rangle$ state. A first $\pi/2$ pulse creates a condensate in a superposition of the two states, at which point its wavefunction freely evolves at the frequency of the hyperfine splitting $\omega_{\text{clock}} = \omega_{|2, 1\rangle} - \omega_{|1, -1\rangle}$. Our local oscillator (the sum of the microwave and rf frequencies) evolves at $\omega_{\text{LO}}$. The number of atoms observed in the upper state after some interaction time $t$ is given by

$$\frac{N_{2, 1}}{N_{\text{tot}}} = \sin^2 \left( \frac{\omega_{\text{clock}} - \omega_{\text{LO}}}{2} t + \phi_0 \right)$$

(15)

where $\phi_0$ is an unimportant relative phase originating from the details of the pulse timing. We detune the local oscillator roughly 1 kHz from the transition and measure the number of atoms in the $|2, 1\rangle$ condensate for various interaction times. From a fit to the data we obtain a measurement of the detuning, as shown in Fig. 3.

If the equilibrium positions of the two condensates are not identical then the wavefunctions for the two condensate states will begin to separate in space and will not effectively interfere with one another when the second $\pi/2$ pulse is applied. The data presented in this figure determine the hyperfine splitting between the two states to about 1 part in $10^9$, and were taken with atoms released from the trap and allowed to expand ballistically in only the rotating magnetic field. Ramsey fringes have recently been obtained with condensates confined in the magnetic trap (after having adjusted the trap parameters to make the condensate equilibrium positions identical) and will be considered in a future paper.

Another set of future experiments focuses upon the interactions between two condensates in two different hyperfine states, which constitutes one of a class of studies of mixed condensates. Choosing the equilibrium displacements to be identical simplifies the interpretation of the dynamical behavior of the two condensates, and maximizes the overlap which can be achieved. In particular, the system described here permits very good control over the relative number of atoms in the two states. Experiments under consideration include the measurement of the relative scattering length of the two states by examining the mode spectrum excited by transferring all of the atoms from the $|1, -1\rangle$ state to the $|2, 1\rangle$ state, and the phase separation dynamics of the two condensates.

This single measurement does not give us the sign of the detuning, and we are required to choose a different detuning and repeat the measurement to get at this quantity.
Figure 4. Calculated relative displacement between the two states as a function of magnetic field for \( \omega t = 2\pi \cdot 7202 \) Hz and a quadrupole gradient \( B'_q = 61.5 \) G/cm. For \( \omega t > 0 \) the TOP trap bias field rotates in the counterclockwise direction as viewed from the z-axis.

Figure 5. The relative centers of oscillation of the two states can be seen to differ in this set of plots which show the center of mass (“sloshing”) motion in the vertical direction. At 3.2 G (a), the \([2, 1]\) atoms (solid circles) are seen to sag less than the \([1, -1]\) atoms (hollow squares); the situation is reversed at 11 G (b). These observations agree with the theoretical prediction of Fig. 4. (The lines are fit to the data.)
Figure 6. Ramsey fringes, measured using two time-separated pulses of rf and microwave radiation on condensate atoms released from the trap and allowed to expand ballistically. The frequency measured is the difference between the GPS-stabilized local oscillator frequency and the frequency of the atomic hyperfine transition. Similar fringes have now been observed with magnetically trapped atoms.

In conclusion, we have produced a new two-condensate system out of the $|1, -1\rangle$ and $|2, 1\rangle$ hyperfine states of $^{87}$Rb. Due to their very similar magnetic moments these two condensates can be made to overlap to a very high degree in both static and rotating-field magnetic traps. This system, and others like it, may be used as a vehicle for studying the dynamics of interpenetrating quantum fluids and for precision metrology.

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