Study of dynamic properties of measuring equipment at the design stage

I I Sytko¹, V E Makhov²

¹ Saint-Petersburg Mining University, 21 Line, St. Petersburg, Russia, 199106
² Mozhaisky Military Space Academy, St. Petersburg, Russia

E-mail: ivan-sytko@yandex.ru

Abstract. The modeling of measuring tools was carried out based on modern computer technologies in VisSim and Mathcad software. The transfer function of a measuring instrument in a closed state with negative feedback is determined. The range of changes in the feedback coefficient at which the dynamic system is stable is determined. The optimal feedback coefficient is determined by the criterion of the transition process minimum duration. The quality indicators of the measuring instrument are determined by the transition function: the time of setting (regulation), over-regulation, the number of fluctuations and the degree of attenuation. The transfer function of the measuring instrument is determined by error. The error rates for steady state position, speed and acceleration are obtained by decomposing the Taylor series of the transfer function by mistake in the Mathcad environment, using the series function. The stability reserves in amplitude and phase are determined before and after introduction of corrective links to the structure of the measuring instrument. The sequential regulator corrective link was synthesized according to the criterion – minimization of stabilization time. The serial controller parameters were calculated in Mathcad by gain coefficient interactive selection and time constant, with the ability to control its parameters by logarithmic frequency characteristics, as well as by static and dynamic properties in transient and steady-state modes. The dynamic safety is determined when one of the measuring tool parameters deviates from the nominal value.

1. Introduction
The relevance of designing measuring tools with given dynamic properties is caused by the fact that it is necessary to ensure speed, stability and stability margin for phase and amplitude; static and dynamic accuracy; estimate dynamic error caused by deviation of parameters from nominal values.

The object of the study includes dynamic characteristics of the designed measuring tools, stability, speed.

Field of application: the methodological device and simulation models can be used to evaluate the dynamic properties and static accuracy of measuring tools at the design stage.

The methods for determining dynamic properties are based on the study of an experimental measuring tool, which leads to an increase in time and material costs during the design stage.

The practical solution of on-line determination of dynamic properties of measuring tools at the design stage, taking into account technical and metrological requirements, provides the prerequisites for wider use of the methodological device and imitation models in the study of various technical systems.

It should be considered that the on-line determination of dynamic properties of measuring tools at the design stage, taking into account technical and metrological requirements, is an urgent task.
2. Purpose of the study
The purpose of the study was to develop a methodological device and simulation models based on computer technologies that provide for error rates in static mode, stability analysis, correction of the transfer function in accordance with given quality indicators in static and dynamic mode, as well as the assessment of dynamic error when the parameters deviate from nominal values.

3. Materials and methods
The existing transfer functions of measuring tools were studied using VisSim and Mathcad software.

VisSim and Mathcad were chosen for experimental studies. A capacitive fuel gage was chosen as a measuring tool, which transfer function in a closed state with negative feedback is described as follows:

\[ W_g(s) = \frac{13.64}{1 + (0.5s + 1)(0.2s + 1)(0.4s + 1) + 13.64K_{oc}} \]  

(1)

where \( K_f \) – feedback coefficient of closed dynamic system with negative feedback.

To determine the range of change of the feedback coefficient at which the dynamic system is stable, the Hurwitz algebraic criterion was chosen. The characteristic polynomial of the closed system [1] looks as follows:

\[ C_y(s, K_{oc}) = 0.04s^3 + 0.38s^2 + 1.1s + (1 + 13.64K_{oc}) \]  

(2)

Mathcad software was used to define the system stability domain depending on the feedback coefficient. The system is stable if the feedback coefficient varies from –0.073 to +0.69.

The optimal feedback coefficient was determined according to the criterion – minimization of stabilization time [2] and amounts to \( K_f = 0.2 \).

VisSim software was used to study the closed dynamic system model (Fig. 1) described by the transfer function (1). Quality indicators in the initial mode such as setting time (regulation), readjustment, number of cycles and degree of attenuation, as well as the position error in the steady-state mode are defined, which are summarized in Table 1.

![Figure 1. Defining quality indicators in closed transition dynamic system before correction](image)

In order to determine the quality of the dynamic system in the steady-state mode, an error-based transfer function was determined according to the transfer function (1) and the feedback coefficient \( K_f = 0.2 \):
The quality of the dynamic system in the steady-state mode is characterized by error coefficients in terms of position ($c_0$), speed ($c_1$) and acceleration ($c_2$), which connect the input effect with the system parameters. These coefficients can be determined if the transfer function (3) is expanded in a Taylor series \[W_e(s)\]  

\[
W_e(s) = 1 - W_3(s) = 1 - \frac{13.64}{1 + (0.5 \ s + 1) \times (0.2 \ s + 1) \times (0.4 \ s + 1) + 2.73} \tag{3}
\]

The quality of the dynamic system in the steady-state mode is characterized by error coefficients in terms of position ($c_0$), speed ($c_1$) and acceleration ($c_2$), which connect the input effect with the system parameters. These coefficients can be determined if the transfer function (3) is expanded in a Taylor series \[W_e(s)\]. Therefore, let us expand the Taylor series of the transfer function by error $W_e(s)$ in Mathcad using the `series` function

\[
W_e(s) \text{ series}, s, 3 \rightarrow \{c_0, c_1, c_2\}
\]

The position error ($c_0$) obtained in VisSim and Mathcad differs by not more than 0.01%. The obtained error coefficients by position ($c_0$), speed ($c_1$) and acceleration ($c_2$) are summarized in Table 1.

| Quality indicators in the transient mode | Quality indicators in the steady-state mode (error coefficients) |
|-----------------------------------------|---------------------------------------------------------------|
| Stabilization time ($t_s$), sec | By position ($c_0$)  
3.4 | 0.268 |
| Readjustment ($\sigma$), % | By speed ($c_1$)  
37 | 1.08 |
| Number of oscillations ($N_o$) | By acceleration ($c_2$)  
2.6 | 0.24 |
| Attenuation rate ($\zeta$) | |

As it is known [1, 2], for reliable operation of the dynamic system, it shall be kept at a certain distance from the stability boundary.

The degree of proximity of a closed system to the stability boundary [1, 7] is measured using various criteria: Routh-Hurwitz, Mikhailov, Nyquist.

The Nyquist criterion is mainly used in practice to estimate the stability margin of a closed system by its log magnitude (LM) and phase-frequency characteristic (PFC).

Figure 2 shows LM and PFC of the dynamic system with the transfer function (1), the stability margin by amplitude and phase is also determined. It is known [1, 2] that the stability margin during the design of dynamic systems should make by amplitude – $(6 \div 20)$ dB and by phase – $(20 \div 60)$°.

At the final stage of analysis of the dynamic system at the design stage, a dynamic error is determined, which occurs when the parameters deviate from nominal values. The feedback coefficient was chosen as the deviation parameter, which decreased by 10% and made 0.18. The calculation of this error was carried out by methods [8] using the transfer function of a measuring tool by error

\[
W_e(s, \alpha) = \frac{13.64}{1 + (0.5 \ s + 1) \times (0.2 \ s + 1) \times (0.4 \ s + 1) + 13.64 \times (K_{oc} - \alpha)} \tag{4}
\]

and the inverse Laplace transform. In Mathcad [8], this error is determined using the `series` and `invlaplace` function and can be presented as follows:

\[
y(t, \alpha) = L^{-1}(W_e(s, \alpha) \cdot x(s)) \tag{5}
\]
4. Experimental results and discussion

The studies were conducted using VisSim and Mathcad software and the existing transfer functions of measuring tools [1, 2, 8, 9].

The next stage to improve the quality of the dynamic system, which provides the necessary properties – stability and required quality in the transition mode – is the introduction of compensating elements (regulators) into the structure [1, 7, 10].

The degree of proximity of a closed system to the stability boundary [1, 7] is measured using various criteria: Routh-Hurwitz, Mikhailov, Nyquist.

In the medium frequency range, a serial controller with a transfer function is considered the best type

\[ W_c(s) = K_c (1 + T_c s) \]  

Then the desired transfer function in a closed state with a feedback coefficient (Kf=0.2) will be as follows:

\[ W_{\infty}(s) = \frac{W(s) \times W_c(s)}{1 + W(s) \times W_c(s) \times W_{\infty}(s)} = \frac{2,728 \times K_c \times (1 + T_c s)}{1 + 1,1 s + 0,38 s^2 + 0,04 s^3 + 2,728 \times K_c \times (1 + T_c s)} \]  

The calculation of the acceleration coefficient (Kc) and the time constant (Tc) of the regulator was carried out in Mathcad by interactive selection of regulator parameters with the possibility to control its parameters by log magnitude, as well as by static and dynamic properties using the transient characteristic. The calculated parameters of the regulator were as follows: acceleration coefficient (Kc=4.5), time constant (Tc=0.2 s).

For the corrected closed dynamic system described by the transfer function (4) all other parameters in transient and steady-state modes are obtained in VisSim and Mathcad [8, 9, 11], which are summarized in Table 2 and Figure 3.
Table 2. Quality indicators of the dynamic system after correction

| Quality indicators in the transient mode after correction |   |
|----------------------------------------------------------|---|
| Stabilization time ($t_s$), sec | 1.4 |
| Readjustment ($\sigma$), % | 25 |
| Number of oscillations ($N_o$) | 2.2 |
| Attenuation rate ($\zeta$) | 0.18 |

| Quality indicators in the steady-state mode (error coefficients) after correction |   |
|-------------------------------------------------------------------|---|
| By position ($c_0$) | 0.162 |
| By speed ($c_1$) | 0.313 |
| By acceleration ($c_2$) | 0.024 |

Figure 3. Hodograph, LM and PFC of the dynamic system after correction

5. Conclusion

It was shown that the introduction of the serial regulator into the structure of the dynamic system, which parameters were determined by the criterion – minimization of the stabilization time, ensured the required indicators of the dynamic system in the transition and steady-state modes. Besides, the dynamic system now has the required stability margin by amplitude and by phase, which will ensure its efficiency during operation and influence of various destabilizing factors.

The obtained results can be used to design measuring tools in order to obtain the required static and dynamic properties through simulation methods.

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