Spin Information from Vector-Meson Decay in Photoproduction

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Abstract

For the photoproduction of vector mesons, all single and double spin observables involving vector meson two-body decays are defined consistently in the \(\gamma N\) center of mass. These definitions yield a procedure for extracting physically meaningful single and double spin observables that are subject to known rules concerning their angle and energy evolution. As part of this analysis, we show that measuring the two-meson decay of a photoproduced \(\rho\) or \(\phi\) does not determine the vector meson’s vector polarization, but only its tensor polarization. The vector meson decay into lepton pairs is also insensitive to the vector meson’s vector polarization, unless one measures the spin of one of the leptons. Similar results are found for all double spin observables which involve observation of vector meson decay. To access the vector meson’s vector polarization, one therefore needs to either measure the spin of the decay leptons, make an analysis of the background interference effects or relate the vector meson’s vector polarization to other accessible spin observables.

24.70.+s, 25.20Lj, 13.60Le, 13.88.+e
I. INTRODUCTION

There is increased interest in measuring the photo- and electro-production of vector mesons from nucleon targets. Precision experiments, including coincidence measurements of the vector meson’s decay products, have been made possible by new high-flux, continuous beams of polarized electrons and photons with polarized targets, along with large angle spectrometers and recoil polarimeters. In this flowering of interest in vector mesons and spin physics, most experimental proposals follow the earlier analysis in which the vector meson’s decay is described in its rest frame. The production amplitude for $\vec{\gamma} + \vec{N} \rightarrow \vec{V} + \vec{N}'$, i.e. the photoproduction of vector mesons, is used to produce a density matrix which describes the spin state of the produced vector meson. The rest frame of the vector meson differs from the production frame and since standard spin observables are defined most naturally in the $\gamma N$ CM, it is necessary to treat the decay in that same CM frame. In much of the literature the decay and density matrix of the vector meson are described in the rest frame of the vector meson and standard spin observables are not invoked. This difference in approach is partly a question of differing motivation. Extraction of the vector meson’s density matrix in its rest frame, using either the Gottfried-Jackson, Adair or Helicity axes, can provide a framework for determining the character of $t-$channel exchange mechanisms. On the other hand, efforts to define the angular and energy evolution of standard spin observables, which could be driven by specific resonances and other dynamical mechanisms, does require that spin observables be defined in a consistent manner, especially for cases involving measuring the vector meson decay along with having polarized photons and/or nucleons (e.g. for double or triple spin observables). Our purpose is to define single and double spin observables in a manner consistent with the treatment of the decay process. Therefore, we describe all spin observables and decays in the same overall $\gamma N$ CM frame.

We begin by discussing the number and character of the basic amplitudes needed to describe spin observables. The photoproduction of vector mesons is described by twelve independent, complex helicity amplitudes. The determination of these 12 amplitudes requires the measurement of twenty-three independent observables at each energy and angle. It is unlikely that such a complete set of vector meson photoproduction experiments will ever be accomplished. Nevertheless, it is natural that one should start by measuring all single spin observables, plus the differential cross section. Let us consider the number of single spin observables, e.g. observables for which either the photon, the target nucleon, the recoil nucleon or the vector meson are polarized. With the $\hat{z}$ axis along the photon beam momentum, the $x$-$z$ plane is the scattering plane, see Fig 1. The vector meson is produced in the final state $\hat{z}'$ direction, with the normal to the scattering plane remaining as $\hat{y}' = \hat{y}$, see Fig. 1. For the photon, the single spin observable is the photon asymmetry $\vec{\Sigma}^\gamma$, (also called $\vec{A}^\gamma$), which has only a non-zero $\hat{x}$ component. The target nucleon single spin observable is called the target polarization $\vec{T}^N$ (also called the nucleon asymmetry $\vec{A}^N$ ), which has only a non-zero $\hat{y}$ component. For the recoil nucleon there is the polarization $\vec{P}^{N'}$, which has only a non-zero $\hat{y}$ component. The vector meson has a vector polarization $\vec{P}^V = \hat{y}P^V_y$ and a

\footnote{The vector meson is designated by $\vec{V}$, which denotes either a $\rho$ or a $\phi$ meson, both of which have two meson decay modes. The $\rho$ decays to two pions, the $\phi$ to two kaons.}
tensor polarization $T_{ij}^V$ (in a cartesian basis). Thus, with the differential cross-section $\sigma(\theta)$, there are eight observables to consider before going on to double, triple, and even quadruple, spin observables, which require experiments with simultaneous information about two, three, and four particle spins. Experiments where one uses polarized photon beams and measures the vector meson decays, fall into this category of double spin observables. In experiments where one has an unpolarized photon beam, an unpolarized target, and also one does not measure the recoil nucleon’s polarization, the angular distribution of the two-body decay of the vector meson provides information about the single spin observables of the vector meson. For example, one could measure the angular distribution of the two decay pions for the case of $\rho$ meson photoproduction, or measure the two decay kaons for $\phi$ photoproduction. Such experiments are planned using the CLAS detector at TJNAF. What information about the vector meson is obtained from measuring the two-body decays?

In this paper, we first discuss the vector meson’s single spin observables, obtainable with an unpolarized photon beam. Then we consider the case of a polarized photon beam along with measurement of the vector meson’s two-body decay modes; for this case one deals not with a single spin observable, but a double spin observable or a spin correlation. The full set of spin observables with beam, target and/or recoil particles polarized was addressed in Ref. [1]. Note that all spin observables and the associated amplitudes and density matrices are defined in the $\gamma N$ CM frame.

The eight independent single spin observables for this reaction are: the differential cross-section $\sigma(\theta)$, the target asymmetry $T_N^y$, the photon beam asymmetry $\Sigma_{\gamma x}^\rho$, the recoil polarization $P_N^y$, the vector meson’s vector polarization $P_V^y$, and the vector meson’s tensor polarization in a spherical tensor basis $T_{V_{20}}^V$, $T_{V_{21}}^V$, and $T_{V_{22}}^V$. Because of parity conservation, one has $P_{V_x}^\rho = 0$, $P_{V_z}^\rho = 0$, $T_{V_{2-2}}^V = T_{V_2}^V$, and $T_{V_{2-1}}^V = -T_{V_{21}}^V$. In the language of density matrices, measuring the four single spin observables of the vector meson ($P_V^y$, $T_{V_{20}}^V$, $T_{V_{21}}^V$, and $T_{V_{22}}^V$) is equivalent to knowing all nine elements of the hermitean $3 \times 3$ spin density matrix $\langle 1 \lambda | \rho_{\gamma N}^V | 1 \lambda' \rangle$, for the case of an unpolarized photon beam. To be specific these are the elements of $\rho_{\gamma N}^0$, defined in Ref. [4], which formalism is often used in analysis of experimental data.

With the CM amplitude for $\gamma + N \rightarrow V + N'$ denoted by $T$, the final CM density matrix is $\rho_f = T_\rho T^\dagger = \frac{1}{4} TT^\dagger$, where the $\frac{1}{4}$ arises from averaging over the initial unpolarized photon and unpolarized target nucleon. This $\rho_f$ reduces to $\frac{1}{2} \rho^V$, where the final baryon’s spin is to be summed over since it is not measured. The final density matrix then involves just the spin state of the vector meson and is described by (see Ref. [1]):

$$\rho^V = \frac{1}{3}[I + \frac{3}{2} \vec{S} \cdot \vec{P}^V + \tau \cdot T^V].$$

(1.1)

Note the above differs from the definition used in Pichowsky et al. [1], since we now require that in the limit of a pure state, the polarization be limited to be at most one in magnitude and the tensor polarization $T_{V_{20}}^V$ varies between a maximum of $\frac{1}{\sqrt{2}}$ and a minimum of $-\sqrt{2}$. 2

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2 Other changes from Ref. [1] are: $\vec{P}^V = \text{Tr}(TT^\dagger \vec{S})/\text{Tr}(TT^\dagger)$, $T_{2\mu}^V = \text{Tr}(TT^\dagger \tau_{2\mu})/\text{Tr}(TT^\dagger)$, and $\tau_{2\mu} = \sqrt{3}[S_1 \times S_1]_{2\mu}$. These changes were made to comply with the Madison conventions.
Since $\rho^V$ is a hermitian $3 \times 3$ matrix, Eq.(1.1) is a general form, where $\bar{S}$ is the spin-1 operator and $\tau$ is the symmetric traceless rank-2 operator with cartesian components $\tau_{ij} = \frac{3}{2}(S_i S_j + S_j S_i) - 2 \delta_{ij}$. On a spherical basis $\tau \cdot T^V$ is defined as $\sum \mu (-1)^\mu \tau_{2-\mu}\ T^V_{2\mu} = \frac{1}{3} \sum_{i,j} \tau_{ij} T_{ij}$. The function $\bar{P}^V$ defines the vector meson’s vector polarization and $T^V_{2\mu}$ the tensor polarization of the vector meson, when both $T^V$ and the density matrix are defined in the CM frame. We note that other authors discuss the elements of the frame dependent density matrix in the rest frame of the vector meson (see for example Refs. 4,5 and also Ref. 3 for $K^*$ production and decay). In our paper, all single spin observables $\bar{P}^V$ and $T^V_{2\mu}$ as well as double, triple, and quadruple spin correlations are defined only in the $\gamma N$ CM frame; we do not invoke rest frames for individual particles. The use of the overall $\gamma N$ CM frame also allows to exploit algebraic relations between various sets of spin observables which can be derived for vector meson photoproduction. Such rules can be obtained as discussed in Ref. 4 and are extremely important in order to define complete sets of independent observables.

All of the spin observables depend on the squared total invariant energy $s$ and the $\gamma - V$ squared momentum transfer $t$ or, equivalently, on the incident photon energy and the CM scattering angles $\Theta, \Phi$ for the outgoing vector meson. That dependence is suppressed in using the notation $T^V_{2\mu}$ instead of the full expression $T^V_{2\mu}(E, \Theta, \Phi)$, but should not be forgotten. Rules for the angular and energy dependence were discussed in Ref. 4 and could be used to analyze important underlying dynamics, such as resonances, cusps, or missing state effects. The density matrix in the vector meson’s rest frame has been defined earlier by others using either the Gottfried-Jackson, Adair, or Helicity axes as shown in Fig. 2 ($\rho^{GJ}, \rho^A, \rho^H$). Extraction from data yields different values for $\rho^{GJ}, \rho^A, \rho^H$ (see for example Ref. 3), which are related by simple rotation matrices. These different vector meson rest frame axes are useful for the study of exchange mechanisms and for study of helicity conservation, but are not useful for the definition of spin observables, which requires consistent use of the $\gamma N$ CM system.

II. SINGLE SPIN OBSERVABLES FOR THE VECTOR MESON

To understand the connection between the single spin observables of the vector meson and the density matrix elements in the $\gamma N$ CM system, we express the matrix elements of the density matrix $\rho^V$ in terms of the meson’s vector and tensor polarization by:

$$
\rho^V = \frac{1}{9} \left( \begin{array}{ccc}
1 + \frac{3}{2} P^V_{P_z} + \frac{\sqrt{3}}{2} T^V_{T_{20}} & \frac{3}{2} P^V_{P_{1-1}} & \frac{3}{2} T^V_{T_{2-1}} \\
-\frac{3}{2} P^V_{P_{11}} - \frac{\sqrt{3}}{2} T^V_{T_{21}} & 1 - \sqrt{2} T^V_{T_{20}} & \frac{3}{2} P^V_{P_{1-1}} - \frac{\sqrt{3}}{2} T^V_{T_{2-1}} \\
\sqrt{3} T^V_{T_{22}} & -\frac{3}{2} P^V_{P_{11}} + \frac{\sqrt{3}}{2} T^V_{T_{21}} & 1 - \frac{3}{2} P^V_{P_z} + \frac{\sqrt{3}}{2} T^V_{T_{20}}
\end{array} \right). 
$$

(2.1)

Here, $P^V_{P_{\pm 1}} = \mp \sqrt{2} (P^V_{P_x} \pm iP^V_{P_y})$. Because of the hermiticity of $\rho^V$ and parity conservation in the reaction, the matrix elements in Eq.(2.1) satisfy the conditions

$^3$We take $\Phi \equiv 0$ for the scattering plane and introduce azimuthal angles for the two-meson decay plane, $\phi_1, \phi_2 = \pi + \phi_1$. 

4
\[ \rho_{\lambda_1 \lambda_2} = \rho_{\lambda_1 \lambda_2}^*, \quad \text{and} \quad \rho_{\lambda_1 \lambda_2}^V = (-1)^{\lambda - \lambda'} \rho_{-\lambda \lambda}, \tag{2.2} \]

where \( \lambda, \lambda' \) are helicity labels and take the values +1, 0, -1. Therefore, \( P_y^V = P_y^V = 0, T_{10}^V, T_{21}^V, T_{22}^V \) are all real, and \( T_{2-1} = -T_{21}, T_{2-2} = T_{22} \). The density matrix simplifies to

\[ \rho^V = \frac{1}{3} \left( \begin{array}{ccc} 1 + \sqrt{2} T_{21} & \frac{3}{2} \sqrt{2}(-i P_y^V) - \sqrt{2} T_{21} & \frac{3}{2} \sqrt{2}(-i P_y^V) + \sqrt{2} T_{21} \\ \frac{3}{2} \sqrt{2} T_{10} \sqrt{3 T_{22}} & 1 - \sqrt{2} T_{20} & 1 + \sqrt{2} T_{20} \end{array} \right). \tag{2.3} \]

Note that \( \text{Tr} \rho^V = 1 \). The \( 3 \times 3 \) hermitian spin density matrix \( \rho^V \) is described fully by four independent real quantities, \( \rho_{00}, \rho_{1-1}, \text{Re}(\rho_{10}) \), and \( \text{Im}(\rho_{10}) \). These four quantities in the \( \gamma N \) CM frame are simply related to the four vector-meson double spin observables \( P_y^V, T_{20}^V, T_{21}^V, \) and \( T_{22}^V \) by:

\[ \begin{align*}
P_y^V &= -2 \sqrt{2} \text{Im} \rho_{10}^V \\
T_{20}^V &= \frac{1}{\sqrt{2}} (1 - 3 \rho_{00}^V) \\
T_{21}^V &= -\sqrt{6} \text{Re} \rho_{10}^V \\
T_{22}^V &= \sqrt{3} \rho_{1-1}^V. 
\end{align*} \tag{2.4} \]

In sections IV and V, we discuss which of these four quantities can be determined by measuring the angular distribution of the two mesons or leptons that arise from vector meson decay.

### III. DOUBLE SPIN OBSERVABLES FOR THE VECTOR MESON WITH POLARIZED PHOTON BEAM

We now consider those double spin observables involving a polarized photon beam and observation of the vector meson decay. These are part of a broader class of double spin observables with two particles polarized; namely: beam-target \( C^{\gamma N} \), beam-recoil \( C^{\gamma N'} \), target-recoil \( C^{NN'} \), target-vector meson \( C^{VN} \), and recoil-vector meson observables \( C^{N'V} \). Here, we restrict ourselves to the beam-vector meson double spin observables \( C^{\gamma V} \).

The final density matrix elements describing the vector meson is now related to the initial density matrix for the incident photon beam by: \( \rho_f = \frac{1}{2} T \rho \gamma T^\dagger \), where the photon is described by

\[ \rho^\gamma = \frac{I}{2} [1 + \vec{P}^S \cdot \vec{\sigma}]. \tag{3.1} \]

Here, \( \vec{P}^S \) is the Stokes “vector,” as discussed in Ref. [3]. The final density matrix can be written as: \( \rho_f = \rho^V + P_x^S \rho^x + P_y^S \rho^y + P_z^S \rho^z \). The vector meson density matrices \( \rho^x, \rho^y, \) and \( \rho^z \), which arise from the \( \sigma_x^\gamma, \sigma_y^\gamma, \) and \( \sigma_z^\gamma \) terms in \( \rho^\gamma \) in Eq. (3.1), are expressed in the \( \gamma N \) CM frame in terms of standard beam-vector meson double spin observables as: [4]

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4 We have already invoked the constraints due to parity and the hermiticity of the density matrix.
\[
\rho^x = \frac{1}{3} [\Sigma_x^\gamma + \frac{3}{2} \sum_j S_j C_{xj}^{\gamma V} + \sum_\mu (-1)^\mu \tau_{2-\mu} C_{x2\mu}^{\gamma V}],
\]

which parallels Eq. (1.1).

We now consider the case of \( P_x^S = P_y^S = 0, P_x^S = \pm 1 \), which corresponds to a photon linearly polarized perpendicular \((\hat{y})\) for \( P_x^S = +1 \), or parallel \((\hat{x})\) for \( P_x^S = -1 \) to the scattering plane. When the decay products of the final vector meson are also measured, we have a final state density matrix \( \rho^x \). Because of the hermiticity of \( \rho^x \) and parity conservation, the matrix elements of \( \rho^x \) satisfy the conditions

\[
\rho_{\lambda\lambda'}^x = \rho_{\lambda'\lambda}^x, \quad \text{and} \quad \rho_{\lambda\lambda'}^x = (-1)^{\lambda-\lambda'} \rho_{-\lambda,-\lambda'}^x.
\] (3.3)

As a consequence, one finds that \( C_{x\gamma}^{\gamma V} = C_{x\mu}^{\gamma V} = 0 \), \( C_{x20}^{\gamma V}, C_{x21}^{\gamma V}, C_{x22}^{\gamma V} \) are all real, and \( C_{x22}^{\gamma V} = -C_{x21}^{\gamma V}, C_{x21}^{\gamma V} = C_{x22}^{\gamma V} \). The density matrix \( \rho^x \) simplifies to

\[
\rho^x = \frac{1}{3} \left( \begin{array}{ccc}
\Sigma_x^\gamma + \sqrt{2} C_{x20}^{\gamma V} & \frac{3}{\sqrt{2}} (-i C_{x21}^{\gamma V}) - \frac{3}{2} C_{x20}^{\gamma V} & \frac{3}{\sqrt{2}} C_{x22}^{\gamma V} \\
\frac{3}{\sqrt{2}} (i C_{x21}^{\gamma V}) - \frac{3}{2} C_{x20}^{\gamma V} & \Sigma_x^\gamma - \frac{3}{2} C_{x21}^{\gamma V} & \frac{3}{\sqrt{2}} (-i C_{x20}^{\gamma V}) + \frac{3}{2} C_{x21}^{\gamma V} \\
\frac{3}{\sqrt{2}} (i C_{x21}^{\gamma V}) + \frac{3}{2} C_{x20}^{\gamma V} & \frac{3}{\sqrt{2}} (-i C_{x20}^{\gamma V}) + \frac{3}{2} C_{x21}^{\gamma V} & \Sigma_x^\gamma + \sqrt{2} C_{x22}^{\gamma V}
\end{array} \right).
\] (3.4)

Note that \( \text{Tr}[\rho^x] = \text{Tr}[\frac{1}{2} \sigma_3^x T^x] = \Sigma_3^x \). This \( 3 \times 3 \) hermitian spin density matrix \( \rho^x \) is described fully by five independent real quantities, \( \rho_{11}^x, \rho_{00}^x, \rho_{1-1}^x, \text{Re}(\rho_{10}^x), \text{Im}(\rho_{10}^x) \). These five quantities are related in the \( \gamma N \) CM frame to the four vector-meson double spin observables \( C_{x20}^{\gamma V}, C_{x21}^{\gamma V}, C_{x22}^{\gamma V} \), and \( C_{x22}^{\gamma V} \) and the single spin observable \( \Sigma_3^x \) \(^5\) by:

\[
\begin{align*}
C_{x20}^{\gamma V} &= -2\sqrt{2} \text{Im} \rho_{10}^x \\
C_{x21}^{\gamma V} &= \frac{3}{2} (\Sigma_x^\gamma - 3\rho_{00}^x) \\
C_{x22}^{\gamma V} &= -6 \text{Re} \rho_{10}^x \\
C_{x21}^{\gamma V} &= \sqrt{3} \rho_{00}^x \\
\Sigma_x^\gamma &= \rho_{00}^x + 2\rho_{11}^x.
\end{align*}
\] (3.5)

Next we consider the case of \( P_x^S = P_y^S = 0, P_y^S = \pm 1 \), which corresponds to a photon linearly polarized at an angle of \( \pm 45^\circ \) with respect to the \( \hat{x} \) axis. When the decay products of the final vector meson are also measured, the final state density matrix \( \rho^y \) is:

\[
\rho^y = \frac{1}{3} [\Sigma_y^\gamma + \frac{3}{2} \sum_j S_j C_{yj}^{\gamma V} + \sum_\mu (-1)^\mu \tau_{2-\mu} C_{y2\mu}^{\gamma V}],
\]

Because of the hermiticity of \( \rho^y \) and parity conservation, the matrix elements in Eq. (3.6) satisfy the conditions

\(^5\) The polarized photon asymmetry, \( \Sigma_3^\gamma \), is a single spin observable, which enters in Eq. (3.4) via the \( P_x^S \) term of Eq. (3.1.)
\[ \rho^y_{\lambda'\lambda} = \rho^{y*}_{\lambda'\lambda}, \quad \text{and} \quad \rho^y_{\lambda\lambda'} = -(-1)^{\lambda'-\lambda} \rho^y_{\lambda'\lambda}. \]  

Note that condition (3.7) for \( \rho^y \) has the opposite sign from the similar condition (3.3) for \( \rho^x \). As a consequence different observables are now constrained. One obtains, \( \Sigma^y = 0 \), \( C^V_{yy} = 0 \), \( C^V_{yy'y'} = 0 \), \( C^V_{yxy'} - C^V_{y'y} = 0 \), \( \sqrt{3} \ C^V_{y21} = -i C^V_{y'y} \), and \( \sqrt{3} \ C^V_{y22} = i C^V_{y'y} \).

Now \( C^V_{y21} \) and \( C^V_{y22} \) are both purely imaginary, and \( C^V_{y2-1} = C^V_{y21}, \ C^V_{y2-2} = -C^V_{y22} \).

The density matrix simplifies to

\[
\rho^y = \frac{1}{3} \begin{pmatrix}
\frac{3}{2} C^V_{yy'} & \frac{3}{2} \sqrt{2} C^V_{xy} - \frac{1}{2} \sqrt{2} C^V_{xy'} & -\sqrt{3} C^V_{y21} \\
\frac{3}{2} \sqrt{2} C^V_{xy} + \frac{1}{2} \sqrt{2} C^V_{xy'} & 0 & -\frac{3}{2} C^V_{y22} \\
\sqrt{3} C^V_{y22} & \frac{3}{2} C^V_{y22} & 0
\end{pmatrix}. \tag{3.8}
\]

Note that \( \text{Tr}[\rho^y] = 0 \), because \( \Sigma^y = 0 \). The \( 3 \times 3 \) hermitian density matrix \( \rho^y \) is fully described by four independent real quantities, \( \rho^y_{11}, \text{Im}(\rho^y_{1-1}), \text{Re}(\rho^y_{10}), \) and \( \text{Im}(\rho^y_{10}) \). These four quantities are related in the \( \gamma N \) CM frame to the four vector-meson double spin observables \( C^V_{y2'}, \ C^V_{y2'}, \ C^V_{y21}, \) and \( C^V_{y22} \) by:

\[
C^V_{y2'} = 2 \sqrt{2} \text{Re}(\rho^y_{10}) \\
C^V_{y2'} = 2 \rho^y_{11} \\
C^V_{y21} = \sqrt{6} i \text{Im}(\rho^y_{10}) \\
C^V_{y22} = -\sqrt{3} \rho^y_{1-1}. \tag{3.9}
\]

Finally, we consider the case of \( P^S_x = P^S_y = 0 \), \( P^S_z = \pm 1 \), which corresponds to a circularly polarized photon, with helicity \( \pm 1 \), e.g. right or left circular polarization. With measurement of the decay products of the final vector meson, the final state density matrix \( \rho^z \) is given by:

\[
\rho^z = \frac{1}{3} \begin{pmatrix}
\frac{3}{2} C^V_{zz'} & \frac{3}{2} \sqrt{2} C^V_{zz'} + \sqrt{3} C^V_{zz''} & -\sqrt{3} C^V_{zz21} \\
\frac{3}{2} \sqrt{2} C^V_{zz'} - \sqrt{3} C^V_{zz''} & 0 & -\frac{3}{2} C^V_{zz22} \\
\sqrt{3} C^V_{zz22} & \frac{3}{2} \sqrt{2} C^V_{zz'} - \sqrt{3} C^V_{zz''} & 0
\end{pmatrix}. \tag{3.10}
\]

Similarly, \( \Sigma^z = 0 \), \( C^V_{zy} = 0 \), \( C^V_{zy0} = 0 \), \( C^V_{zyx'} = 0 \), \( C^V_{zy'y} = 0 \), \( \sqrt{3} C^V_{zy21} = -i C^V_{zy2'} \), and \( \sqrt{3} C^V_{zy22} = i C^V_{zy2'y} \). Again \( C^V_{zy21} \) and \( C^V_{zy22} \) are purely imaginary and \( C^V_{zy2-1} = C^V_{zy21}, C^V_{zy2-2} = -C^V_{zy22} \). The density matrix simplifies to

\[
\rho^z = \frac{1}{3} \begin{pmatrix}
\frac{3}{2} C^V_{zz'} & \frac{3}{2} \sqrt{2} C^V_{zz'} + \sqrt{3} C^V_{zz21} & -\sqrt{3} C^V_{zz22} \\
\frac{3}{2} \sqrt{2} C^V_{zz'} - \sqrt{3} C^V_{zz21} & 0 & -\frac{3}{2} C^V_{zz22} \\
\sqrt{3} C^V_{zz22} & \frac{3}{2} \sqrt{2} C^V_{zz'} - \sqrt{3} C^V_{zz21} & 0
\end{pmatrix}. \tag{3.11}
\]

The \( 3 \times 3 \) hermitian spin density matrix \( \rho^z \) is fully described by four independent real quantities, \( \rho^z_{11}, \text{Im}(\rho^z_{1-1}), \text{Re}(\rho^z_{10}), \) and \( \text{Im}(\rho^z_{10}) \). These four quantities are related in the \( \gamma N \) CM frame to the four vector-meson double spin observables \( C^V_{zz'}, \ C^V_{zz'}, \ C^V_{zz21}, \) and \( C^V_{zz22} \) by:
\[ C_{zz'} = 2 \sqrt{2} \text{Re} (\rho_{10}^z) \]
\[ C_{z\bar{z}'} = 2 \rho_{11}^z \]
\[ C_{z\bar{z}} = \sqrt{6} i \text{Im} (\rho_{10}^z) \]
\[ C_{\bar{z}\bar{z}} = -\sqrt{3} \rho_{11}^z . \]

(3.12)

Which single and double spin observables can be measured using the decay pions or leptons?

IV. PION PAIR IN FINAL STATE

The \( \rho \) meson is observed via its decay products. For \( \rho \) decay into a pion pair, one determines the angular distribution of the two pions in the reaction

\[ \gamma + N \rightarrow \pi + \pi + N' , \]
(4.1)

for example in the overall \( \gamma N \) (or \( \rho N' \)) CM system (see Fig. 1). Here, we do not deal with the nontrivial separation of the direct meson decay mode from other mechanisms for producing the two final pions. To define spin observables that involve the \( \rho \) meson, such a separation is required. In Appendix A, we outline how such background terms would enter into our discussion and the simple form of the decay that results when one is allowed to ignore mechanisms for two-meson production other than via the vector meson production.

In this section, we present the \( \rho \rightarrow \pi \pi \) decay, but the discussion applies also to the case of \( \phi \rightarrow K \bar{K} \).

A. Decay Distributions for Single Spin Observables

Assuming that the pions are produced solely via vector meson decay, the two pion angular distribution in the overall \( \gamma N \) (or \( VN' \)) CM frame is to be measured. As shown in Appendix A, the two pion angular distribution in the \( VN' \) CM frame can be described using the angles \( \bar{\theta} \) and \( \bar{\phi} \) between the direction of the vector meson’s momentum \( \vec{q} = q\hat{z}' \) and the vector \( \vec{v}_1 - \vec{v}_2 \), where \( \vec{v}_1 = \vec{p}_1/E_1 \) and \( \vec{v}_2 = \vec{p}_2/E_2 \) are the velocity vectors of the two decay mesons (e.g. \( \pi_1 \) and \( \pi_2 \)). The angles \( \bar{\theta}, \bar{\phi} \) reduce, in the vector meson rest frame, to the angles of the decay pion \( \pi_1 \), e.g. \( \bar{\theta}, \bar{\phi} \rightarrow \theta_1, \phi_1 \). (We note that the reference frame for these angles is \( \hat{x}', \hat{y}, \hat{z}' \), where the \( \hat{z}' \)-axis is along the direction of the vector meson \( V \).)

For an unpolarized photon beam, the two-pion angular distribution is given by

\[ dN^V = W^V(\bar{\theta}, \bar{\phi}) \ d\cos \bar{\theta} \ d\bar{\phi} . \]
(4.2)

One counts the number of mesons at \( \bar{\theta}, \bar{\phi} \), which entails measuring the momentum and energy of each meson, forming the two-meson four vector \( \vec{p}_1 + \vec{p}_2 = \vec{q} \), and only select those pion pairs that satisfy \( q^2 = m_{\rho}^2 \), allowing for the width of the vector meson. \( \Box \)

\(^6\)Boosting along the \( \rho \) direction from the \( \rho N' \) overall CM frame to the \( \rho \) rest frame, one can show
The decay distribution $W(\tilde{\theta}, \tilde{\phi})$ can be expressed in terms of the matrix elements of the $\gamma N$ CM density matrix $\rho^V$, and the CM decay amplitude $M$ (see Appendix A). The relation is $W^V(\tilde{\theta}, \tilde{\phi}) = \text{Tr}[M\rho_f M^\dagger]$, where the trace appears because of summation over the vector meson helicities. Since we evaluate $\rho_f = \mathbf{T}_\rho\mathbf{T}_f$ in the $\gamma N$ CM frame, we need to evaluate the $\rho$ decay amplitude $M$ in the same overall CM frame (see Appendix A). Therefore, we are dealing with the traditional frame in which spin observables are defined.

Before parity conservation is imposed, one has [2]

$$W^V(\tilde{\theta}, \tilde{\phi}) = \frac{1}{4\pi} \xi_V(\tilde{\theta}) (1 - \sqrt{\frac{1}{2}} T^V_{20}(3\cos^2\tilde{\theta} - 1)$$

$$+ \sqrt{3} \text{Re} (T^V_{21}) \sin 2\tilde{\theta} \cos \tilde{\phi} + \sqrt{3} \text{Im} (T^V_{21}) \sin 2\tilde{\theta} \sin \tilde{\phi}$$

$$- \sqrt{3} \text{Re} (T^V_{22}) \sin^2\tilde{\theta} \cos 2\tilde{\phi} - \sqrt{3} \text{Im} (T^V_{22}) \sin^2\tilde{\theta} \sin 2\tilde{\phi}).$$  \hspace{1cm} (4.3)

The function $\xi_V(\tilde{\theta})$ is a kinematical factor that arises from considering the vector meson decay in the $\gamma N$ CM system. It is expressed in terms of the decay angle $\tilde{\theta}$ as

$$\xi_V(\tilde{\theta}) = \frac{1}{(\sin^2\tilde{\theta} + (\frac{E_\rho}{m_\rho})^2 \cos^2\tilde{\theta})^{5/2}}, \hspace{1cm} (4.4)$$

where $m_\rho$ is the vector meson’s mass and $E_\rho$ is the vector meson’s energy, which depends on the production angles $\Theta, \Phi$. For $E_\rho \approx m_\rho$ clearly $\xi_V(\tilde{\theta}) \rightarrow 1$, and the above expressions reduce to those obtained when the rest frame of the vector meson is used along with the amplitude in the $\gamma N$ CM frame. Therefore, $\xi_V(\tilde{\theta})$ is the major effect of a consistent treatment of the decay and production, along with the use of the decay angle $\tilde{\theta}$. We normalize the above angular distribution by dividing by the known factor $\xi_V$ and setting

$$\int d\Omega \frac{W^V(\tilde{\theta}, \tilde{\phi})}{\xi_V(\tilde{\theta})} \equiv 1.$$

Already at this stage the angular dependence $W^V(\tilde{\theta}, \tilde{\phi})$ does not depend on the vector polarization $P^V_y$. Neither does the function $W^V$ depend on $P^V_x$ or $P^V_z$. After parity conservation has been imposed, one has

$$W^V(\tilde{\theta}, \tilde{\phi}) = \frac{1}{4\pi} \xi_V(\tilde{\theta}) \left[ 1 - \sqrt{2} T^V_{2\mu}(\Theta, \Phi) C^\ast_{2\mu}(\tilde{\theta}, \tilde{\phi}) \right]$$

$$= \frac{1}{4\pi} \xi_V(\tilde{\theta}) \left[ 1 - \sqrt{\frac{1}{2}} T^V_{20}(3\cos^2\tilde{\theta} - 1) + \sqrt{3} T^V_{21} \sin 2\tilde{\theta} \cos \tilde{\phi} - \sqrt{3} T^V_{22} \sin^2\tilde{\theta} \cos 2\tilde{\phi} \right].$$  \hspace{1cm} (4.5)

Here $C^\ast_{2\mu} \equiv \sqrt{\frac{21}{8}} Y^\ast_{2\mu}$ is a spherical harmonic function. A general proof that $W^V(\tilde{\theta}, \tilde{\phi})$ for decay to two pseudoscalar mesons is independent of the vector meson’s polarization $\vec{P}^V$ is presented in Appendix B.

Both $P^V_{x'}$ and $P^V_{z'}$ vanish because of parity conservation, but $P^V_y$ is non-zero, and contains all the information about the vector polarization of the vector meson. As seen from Eq. (4.5), the observable $P^V_y$ remains unmeasurable from the two meson decay mode.

that the angle $\tilde{\theta}$ is related to the $\rho$ rest frame angle $\theta_1$ of the decay pion $\pi_1$ by $\tan \tilde{\theta} = \frac{E_\pi}{m_\pi} \tan \theta_1$, while the azimuthal orientation of the scattering plane with respect to the $\hat{z'}$ axis remains the same, $\tilde{\phi} = \phi_1$.  

9
B. Decay Distributions for Double Spin Observables

For initial photon polarization, the angular distribution of the decay mesons becomes
\[ W(\bar{\theta}, \phi) = W^V(\bar{\theta}, \phi) + P^S W^x(\bar{\theta}, \phi) + P^S W^y(\bar{\theta}, \phi) + P^S W^z(\bar{\theta}, \phi), \] (4.6)
where \( W^i(\bar{\theta}, \phi) = \text{Tr}[M^i M^T] \). Again both \( \rho \) and \( M \) are defined in the \( \gamma N \) CM system. The explicit forms of \( W^i(\bar{\theta}, \phi) \) are
\[
\begin{align*}
W^x(\bar{\theta}, \phi) &= \frac{1}{4\pi} \xi_V(\bar{\theta}) \left[ \Sigma^\gamma_x - \sqrt{\frac{1}{2}} C^\gamma_{x20}(3 \cos^2 \bar{\theta} - 1) + \sqrt{3} C^\gamma_{x21} \sin 2 \bar{\theta} \cos \phi - \sqrt{3} C^\gamma_{x22} \sin^2 \bar{\theta} \cos 2 \phi \right], \\
W^y(\bar{\theta}, \phi) &= \frac{1}{4\pi} \xi_V(\bar{\theta}) \left[ \sqrt{3} \text{Im}(C^\gamma_{y21}) \sin 2 \bar{\theta} \sin \phi - \sqrt{3} \text{Im}(C^\gamma_{y22}) \sin^2 \bar{\theta} \sin 2 \phi \right], \\
W^z(\bar{\theta}, \phi) &= \frac{1}{4\pi} \xi_V(\bar{\theta}) \left[ \sqrt{3} \text{Im}(C^\gamma_{z21}) \sin 2 \bar{\theta} \sin \phi - \sqrt{3} \text{Im}(C^\gamma_{z22}) \sin^2 \bar{\theta} \sin 2 \phi \right].
\end{align*}
\] (4.7)
Here, \( \Sigma^\gamma_x \) is a single spin observable (the polarized photon asymmetry); all other observables in Eq. (4.7), \( C^\gamma_{x20}, C^\gamma_{x21}, C^\gamma_{x22}, C^\gamma_{y21}, C^\gamma_{y22}, C^\gamma_{z21}, C^\gamma_{z22} \), involve polarized photons and the tensor polarization of the vector meson and therefore are double spin observables.

Again one concludes that the double spin observables \( C^\gamma_{x20}, C^\gamma_{y21}, C^\gamma_{y22}, C^\gamma_{z21}, C^\gamma_{z22} \), which involve a polarized photon and the vector polarization of the vector meson, do not contribute and cannot be measured by meson decay distributions.

V. LEPTON PAIR IN FINAL STATE

The \( \rho \) meson can also decay into a pair of leptons. For \( \rho \) decay into an electron-positron pair one has the reaction
\[ \gamma + N \rightarrow e^+ + e^- + N', \] (5.1)
for which one might determine the angular distribution of the two decay leptons in the \( \gamma N \) CM system. The vector meson’s decay into a muon pair
\[ \gamma + N \rightarrow \mu^+ + \mu^- + N', \] (5.2)
has a similar angular distribution. The angular distribution of the two leptons can be expressed in terms of the matrix elements of the vector meson’s density matrix \( \rho^V \).

A. Decay Distributions for Single Spin Observables

The angular distribution of the decay lepton pair for the case that the lepton spins are not measured and the photon is not polarized is given by (see Appendix B):
\[
W^V(\bar{\theta}, \phi) = \frac{1}{8\pi} \left[ 3 \xi^3_{V}(\bar{\theta}) + \xi_V(\bar{\theta}) \{ -1 + \sqrt{2} T^V_{2\mu}(\Theta, \Phi) C^\mu_{2\mu}(\bar{\theta}, \phi) \} \right]
= \frac{1}{8\pi} \left[ 3 \xi^3_{V}(\bar{\theta}) + \xi_V(\bar{\theta}) \{ -1 + \frac{1}{2} T^V_{2\mu}(3 \cos^2 \bar{\theta} - 1) \right.
- \sqrt{3} T^V_{21} \sin 2 \bar{\theta} \cos \phi + \sqrt{3} T^V_{22} \sin^2 \bar{\theta} \cos 2 \phi \}].
\] (5.3)
Note that the angular dependence $W^V(\bar{\theta}, \bar{\phi})$ is independent of the vector meson’s vector polarization $\bar{B}^V$, but does depend on the vector meson’s tensor polarization $T^V_{2\mu}$. Proof of this assertion is given in Appendix B.

This should be contrasted to the fact that complete spin information (vector polarization as well as tensor polarization) about $W$ and $Z$ vector bosons can be extracted from their leptonic decay angular distributions. For example, Ref. [3] deals with vector meson weak decay of the type $W^- \rightarrow \ell^- \bar{\nu}_\ell$ or $Z \rightarrow \ell^- \ell^+$ in which the parity violation term $1 \pm \gamma_5$ yields $S$, $P$ and D-wave interference terms that depend on the vector meson’s vector polarization. Therefore, angular distributions of weak decays of vector mesons, with the lepton spin unmeasured, do depend on the vector polarization. The difference with the processes discussed in this paper is that in vector meson photoproduction the $\rho$ and $\phi$ meson decays are strong or electro-magnetic interactions that conserve parity. However, one should note that the operator $\frac{1}{2}(1 \pm \gamma_5)$ also serves as a spin projection operator. This indicates that if we can measure the spin of one of the decay leptons, in the parity conserving $\rho$ or $\phi$ leptonic decay, the dependence on the vector polarization will appear also. Indeed, that is the result for the case of two-lepton decay with one lepton spin measured.

1. Final Leptonic Spin

If one can measure the spin of one of the final leptons, the helicity projection operator $\frac{1}{2}(1 \pm \gamma_5)$ is introduced, where for example the $+$ sign means projecting out a right-handed electron or a left-handed positron. The angular distribution of the decay leptons now does depend on the vector meson’s vector polarization $P^V_y$:

$$W^V(\bar{\theta}, \bar{\phi}) = W^V(\bar{\theta}, \bar{\phi}) \mp \xi_V(\bar{\theta}) \frac{1}{4\pi} P^V_y \sin \bar{\theta} \sin \bar{\phi},$$

(5.4)

where the $\mp$ sign corresponds to the $\pm$ sign in the spin projection operator. Here, $W^V(\bar{\theta}, \bar{\phi})$ is given in Eq. (5.3) for the case where no lepton spin is measured.

The above results are for the case of no photon polarization and therefore involve single spin observables. Now consider the case of beam-vector meson double spin observables.

B. Decay Distributions for Double Spin Observables

For initial photon polarization, the angular distribution of the decay leptons, without measuring their spin, becomes

$$W(\theta, \phi) = W^V(\bar{\theta}, \bar{\phi}) + P^x_x W^z(\bar{\theta}, \bar{\phi}) + P^y_y W^y(\bar{\theta}, \bar{\phi}) + P^z_z W^z(\bar{\theta}, \bar{\phi}),$$

(5.5)

where $W^i(\bar{\theta}, \bar{\phi}) = \text{Tr}[M^i \rho M^i \rho]$. The explicit forms of $W^i(\bar{\theta}, \bar{\phi})$ are

$$W^x(\bar{\theta}, \bar{\phi}) = \frac{1}{8\pi} [3 \xi_V^3(\bar{\theta}) \Sigma_{\bar{\theta}}^\gamma + \xi_V(\bar{\theta}) \{ - \Sigma_{\bar{\theta}}^\gamma + \frac{1}{2} C_{20}^V (3 \cos^2 \bar{\theta} - 1)$$

$$- \sqrt{3} C_{21}^V \sin 2\bar{\theta} \cos \bar{\phi} + \sqrt{3} C_{22}^V \sin^2 \bar{\theta} \cos 2\bar{\phi} \}],$$

$$W^y(\bar{\theta}, \bar{\phi}) = \frac{1}{8\pi} \xi_V(\bar{\theta}) [ - \sqrt{3} \text{Im}(C_{21}^V) \sin 2\bar{\theta} \sin \bar{\phi} + \sqrt{3} \text{Im}(C_{22}^V) \sin^2 \bar{\theta} \sin 2\bar{\phi} ],$$

$$W^z(\bar{\theta}, \bar{\phi}) = \frac{1}{8\pi} \xi_V(\bar{\theta}) [ - \sqrt{3} \text{Im}(C_{21}^V) \sin 2\bar{\theta} \sin \bar{\phi} + \sqrt{3} \text{Im}(C_{22}^V) \sin^2 \bar{\theta} \sin 2\bar{\phi} ].$$

(5.6)
Here, $\Sigma^x_\gamma$ is a single spin observable (the polarized photon asymmetry); the other observables in Eq. (5.4), $C^{\gamma V}_{xy}, C^{\gamma V}_{yx'}, C^{\gamma V}_{yz'}, C^{\gamma V}_{zx'}, C^{\gamma V}_{zz'}$, involve polarized photons and tensor polarization of the vector meson and are double spin observables.

1. Final Leptonic Spin

The 5 double spin observables $C^{\gamma V}_{xy}, C^{\gamma V}_{yx'}, C^{\gamma V}_{yz'}, C^{\gamma V}_{zx'}, C^{\gamma V}_{zz'}$, which involve the vector polarization of the vector meson, do not contribute to the lepton decay distribution (see Eq. (5.6)) unless one measures the spin of one of the decay leptons or has a weak interaction.

If a lepton spin is determined, all $W^i(\bar{\theta}, \bar{\phi})$ in Eq. (5.5) are replaced by $\tilde{W}^i(\bar{\theta}, \bar{\phi})$, where

$$
\tilde{W}^x(\bar{\theta}, \bar{\phi}) = W^x(\bar{\theta}, \bar{\phi}) \mp \xi_V(\bar{\theta}) \frac{1}{4\pi} C^{\gamma V}_{xy} \sin \bar{\theta} \sin \bar{\phi},
$$

$$
\tilde{W}^y(\bar{\theta}, \bar{\phi}) = W^y(\bar{\theta}, \bar{\phi}) \mp \xi_V(\bar{\theta}) \frac{1}{4\pi} \left[ C^{\gamma V}_{yz'} \cos \bar{\theta} + C^{\gamma V}_{yx'} \sin \bar{\theta} \cos \bar{\phi} \right],
$$

$$
\tilde{W}^z(\bar{\theta}, \bar{\phi}) = W^z(\bar{\theta}, \bar{\phi}) \mp \xi_V(\bar{\theta}) \frac{1}{4\pi} \left[ C^{\gamma V}_{zx'} \cos \bar{\theta} + C^{\gamma V}_{zz'} \sin \bar{\theta} \cos \bar{\phi} \right].
$$

(5.7)

Therefore, from the angular distribution one can now extract double spin observables that involve the vector meson’s vector polarization $C^{\gamma V}_{xy}, C^{\gamma V}_{yx'}, C^{\gamma V}_{yz'}, C^{\gamma V}_{zx'}, C^{\gamma V}_{zz'}$. However, the measurement of two-lepton decays, which is already difficult by virtue of the $10^{-4} - 10^{-5}$ decay branching ratio to the dominant two meson mode, is made essentially impossible by the need to measure the spin of a decay lepton. Hence, for practical purposes the vector meson’s vector polarization remains unmeasurable, even for the case of two lepton decay.

The general discussion of why the vector polarization does not enter into the $W^V(\bar{\theta}, \bar{\phi})$ angular distribution, when no final lepton spin is measured, is given in Appendix B.

VI. CONCLUSION

In pionic and leptonic parity-conserving two-body decays of vector mesons, we have shown that measurement of the final decay angular distributions yields only the tensor polarization of the vector meson. Access to the vector meson’s vector polarization is possible if one measures the final state leptonic spin (or if one has access to a weak decay), but both are presently not feasible. Hence, one of the simpler single spin observables, $P^V_y$ is difficult to measure and we must learn to live without it. The basic dynamics must be extracted from other observables. One possibility for finding $P^V_y$ is to ascertain an interference mechanism with the background that could isolate vector polarization effects. Another possibility, albeit remote, is to measure the final leptonic spin. Finally, using the algebra of measurement contained in Ref. [1,7] and associated Fierz transformations, it might be possible to obtain the vector polarization from sets of other single, double, triple, etc., spin observables, which can be measured. That step requires a solution of the problem of determining a complete set of measurements for vector meson photoproduction.

It has been shown in Appendices A and B that single spin observables involving measuring the decay pions or leptons (without lepton spin measurements) do not yield information
about the vector meson’s vector polarization, but only depend on the tensor polarization. Vector mesons with two-body decays that conserve parity, are therefore self-analyzing with respect to their tensor polarization.

The proof in Appendices A and B also shows that all double, triple, and quadruple spin observables involving the vector polarization of the vector meson cannot be obtained from processes where the final vector meson decays into two mesons. Therefore, within the set of single and double spin observables there are 16 spin observables that are inaccessible, unless one measures the lepton spin in a leptonic decay mode. These inaccessible observables are: one single spin observable: $P^V_y$; five beam-vector meson double spin observables: $C_{x_2}^{TV}, C_{y_2}^{TV}, C_{y_2'}^{TV}, C_{z_2'}^{TV}$; five target-vector meson double spin observables: $C_{x_z}^{NV}, C_{x_z'}^{NV}, C_{y_z}^{NV}, C_{y_z'}^{NV}, C_{z_z}^{NV}, C_{z_z'}^{NV}$; and five recoil nucleon-vector meson double spin observables: $C_{x_z}^{NV}, C_{x_z'}^{NV}, C_{y_z}^{NV}, C_{y_z'}^{NV}, C_{z_z}^{NV}, C_{z_z'}^{NV}$.

Which observables are accessible with present experimental techniques? For an unpolarized photon beam, just by measuring the angular distribution of the decay mesons or the decay leptons, one does find the values of the 3 single tensor polarization observables of the vector meson ($T_2^0, T_2^1, T_2^2$). For a polarized photon beam, one finds 7 double (photon-vector meson) spin observables $C_{x_2}^{TV}, C_{y_2}^{TV}, C_{y_2'}^{TV}, C_{z_2'}^{TV}, C_{x_z}^{TV}, C_{y_z}^{TV}, C_{y_z'}^{TV}$, involving the polarization of the photon and the tensor polarization (but again not the vector polarization) of the vector meson. For a polarized target, the decay angular distribution determines 7 double (target-vector meson) spin observables $C_{x_2}^{NV}, C_{y_2}^{NV}, C_{y_2'}^{NV}, C_{z_2'}^{NV}, C_{x_z}^{NV}, C_{y_z}^{NV}, C_{y_z'}^{NV}$, and by detecting the recoil polarization and the angular decay distribution one can obtain 7 double (recoil-vector meson) spin observables $C_{x_z}^{NV}, C_{x_z'}^{NV}, C_{y_z}^{NV}, C_{y_z'}^{NV}, C_{z_z}^{NV}, C_{z_z'}^{NV}$.

Recall that all these observables are related to the spin density matrices in the $\gamma N$ CM frame, and are functions of the incident beam energy and of the scattering angles $\Theta, \Phi$ of the vector meson. There is therefore still a very rich source of information to be obtained from the angular distribution of the vector meson’s decay products, if one does a careful analysis of the data.

As part of that careful analysis, if one wishes to extract spin observables, then both the vector meson production amplitude and its decay must be expressed in the $\gamma N$ CM system. As a consequence, a kinematic factor $\xi_V(\bar{\theta})$ must be included.

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7 The procedures discussed in Appendices A and B also imply that spin-2 mesons with parity conserving two-body decays can be self-analyzing with respect to their rank-2 and rank-4 tensor polarization, but do not reveal their vector or octupole polarizations via the angular distributions of their decay products.
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APPENDIX A: ROLE OF BACKGROUND TERMS

In this appendix we outline the role of background terms, such as arise from direct production of two mesons without intermediate vector meson production. By ignoring this background, we obtain the simplified version for the decay angular distribution used in this paper and in Ref. [2].

The amplitude for the photoproduction of two pseudoscalar mesons has the general form

\[ T = M T^V + T^B, \]  

(A1)

where \( T^V \) describes the photoproduction of a vector meson and \( M \) describes the subsequent decay of the vector meson into two pseudoscalar mesons. The term \( T^B \) describes other “background” mechanisms for the direct production of, for example, two pions. \[8\] \[9\] We assume that it will be possible to select only those pion pairs with the correct kinematics of having \( \rho \)-like attributes. Namely, in the CM frame of the two pions, they should be in a pure relative P-wave (the quantum number of the \( \rho \)) and the energy of both pions should satisfy \( E_{\text{pion}1} = E_{\text{pion}2} = m_\rho/2 \). Satisfying these two constraints does not guarantee that only the \( M T^V \) term contributes; one could get direct production contributions from \( T^B \) without an intermediate \( \rho \).

In the \( \rho \) rest frame the decay amplitude \( M_\lambda \propto D^{1}_{\lambda 0}(\theta_1, \phi_1)^* \). Here \( \theta_1, \phi_1 \) are the angles of one of the decay pions in the \( \rho \) rest frame (where the momentum of the recoil nucleon is along the \(-\hat{z}'\) axis).

The two-body decay amplitude \( M_\lambda \) of \( \rho \) into \( \pi_1 \) given by \((E_1, \vec{p}_1)\) and \( \pi_2 \) given by \((E_2, \vec{p}_2)\) has the relativistic invariant form

\[ M_\lambda = -i g_{\rho \pi \pi} \varepsilon^\mu(q, \lambda)(p_1 - p_2)_\mu, \]  

(A2)

where \( \varepsilon^\mu(q, \lambda) \) is the polarization vector of the \( \rho \) meson. We can therefore express the decay in the \( \gamma N \) CM frame using Eq.(A2). Introducing the relative velocity of the decay products \( \vec{v} = \vec{v}_1 - \vec{v}_2 \), one can write \( M_\lambda \) as a 3-vector dot product and a helicity independent kinematic factor

\[ M_\lambda = \frac{2E_1 E_2}{E_1 + E_2} \varepsilon^\mu(q, \lambda)(\vec{v}_1 - \vec{v}_2) = \frac{2E_1 E_2}{E_1 + E_2}|\vec{v}_1 - \vec{v}_2|D^1_{\lambda 0}(\tilde{\theta}, \tilde{\phi})^*. \]  

(A3)

The above form of \( M_\lambda \) holds in the \( \rho \) rest frame and in the \( \gamma N \) CM frame, and \( \tilde{\theta} \) and \( \tilde{\phi} \) are the angles (see Fig. 2) between \( \hat{z}' \) and the relative velocity \( \vec{v} = \vec{v}_1 - \vec{v}_2 \). In the \( \rho \) rest frame Eq.(A3) reduces to

\[ M_\lambda = i g_{\rho \pi \pi} 2p_1 D^1_{\lambda 0}(\theta_1, \phi_1)^*. \]  

(A4)

For an unpolarized beam, an unpolarized target and with no measurement made of the recoil nucleon’s polarization, the reaction \( \gamma + N \rightarrow \pi_1 + \pi_2 + N' \) is described by the transition probability

\[ TT^\dagger = (MT^V + T^B)(MT^V + T^B)^\dagger = MTT^V T^V M^\dagger + MTT^V T^{B\dagger} + TT^B T^V M^\dagger + TT^B T^{B\dagger}. \]  

(A5)
In our discussion, we have assumed $\mathbf{T}^B = 0$. Using that assumption, we can describe the angular distribution of the decay pions in the overall CM frame by

$$W^V(\bar{\theta}, \bar{\phi}) = \text{Tr}[M \rho^V M^\dagger], \quad (A6)$$

where

$$\rho^V = \mathbf{T}^V \mathbf{T}^{V\dagger}. \quad (A7)$$

Note that $M_\lambda$ is given in the $\gamma N$ CM frame by Eq. (A3). The kinematic factor $\xi_V(\bar{\theta})$ in the text arises by combining:

$$\left(\frac{2E_1E_2}{E_1+E_2} |\vec{v}_1 - \vec{v}_2|\right)^2 = \frac{4p_1^2}{\sin^2 \bar{\theta} + (\frac{m_\pi}{m_{\gamma}})^2 \cos^2 \bar{\theta}}, \quad (A8)$$

with the invariant density of state factors

$$\frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \delta^4(q - p_1 - p_2).$$

Here $\bar{\theta}$ is the angle between the velocity difference vector $\vec{v}_1 - \vec{v}_2$ and the vector meson’s momentum $\vec{q} = q \hat{z}'$ in the $\gamma N$ CM system.

**APPENDIX B: MESON AND LEPTON DECAY ANGULAR DISTRIBUTION**

Under the assumption that background terms can be neglected, we now demonstrate that the vector polarization, both for pionic and leptonic decays, does not appear in the decay angular distribution. This has already been shown in the text. Here, we show that the result is due to general symmetry properties.

The expression (A3), yields for the pionic decay case:

$$W^V(\bar{\theta}, \bar{\phi}) = \text{C}_V(\bar{\theta}) \sum_{\lambda, \lambda'} D^1_{\lambda 0}(\bar{\theta}, \bar{\phi}) D^1_{\lambda' 0}(\bar{\theta}, \bar{\phi}) \times (-1)^{\lambda'} < 1 \lambda | [I + \frac{3}{2} \vec{S} \cdot \vec{P}^V + \tau \cdot T^V] | 1 - \lambda' >, \quad (B1)$$

where $C$ incorporates the decay coupling and kinematic factors, which are independent of the helicities and the angles $\bar{\theta}, \bar{\phi}$. Note that $D^1_{\lambda 0}(\bar{\theta}, \bar{\phi}) D^1_{\lambda' 0}(\bar{\theta}, \bar{\phi})$ is a symmetric function of the $\rho$ helicity indices $\lambda, \lambda'$; whereas, $(-1)^{\lambda'} < 1 \lambda | \vec{S} | (1 - \lambda') >$ is an antisymmetric function of $\lambda, \lambda'$, using the Wigner-Eckart theorem. Thus, the term involving the vector polarization $\vec{P}^V$ vanishes in the description of the purely two pion decay mode. If the background term $\mathbf{T}^B$ is re-introduced, then there would be terms in $W(\bar{\theta}, \bar{\phi})$ for which $\vec{P}^V$ plays a role.

The above conclusion also applies to the lepton pair decay when the spins of the decay leptons are not measured. The decay amplitude $M_{\lambda, \lambda_+ \lambda_-}$ for the $1^-$ decay of the leptons, $^8$

---

8Two leptons, $\mu^+\mu^-$, in a $1^-$ state (the quantum numbers of the $\rho$), must be in either a triplet S- or triplet D-wave. A P-wave (singlet or triplet) would form positive parity states $0^+, 1^+$, or $2^+$, and therefore is not allowed.
using a \( \bar{\psi}\gamma^{\mu}\psi\rho_{\mu} \) coupling \(^9\) for lepton-pair to vector meson equals

\[
M_{\lambda,\lambda',\lambda} = -ig_{\rho_1\rho_2}\varepsilon^{\mu}(q, \lambda)\bar{u}(p_-, \lambda_-)\gamma_\mu v(p_+, \lambda_+),
\]

(B2)

where the constant \( g_{\rho_1\rho_2} \) includes all factors and is fixed by the decay width of the \( \rho \) meson into a lepton pair \((l_1^+ l_2^-)\). The lepton’s helicities are \( \lambda_+ \) and \( \lambda_- \), and \( \lambda \) is the vector meson’s helicity. In forming the decay distribution, we need the combination \( M_{\lambda,\lambda',\lambda} M_{\lambda',\lambda,\lambda}^\ast \) where \( M \) is the decay amplitude in the overall CM frame.

Summing over the unobserved final lepton helicities and using the usual projection operator rules, the above reduces to a trace; the result is:

\[
\sum_{\lambda_+\lambda_-} M_{\lambda,\lambda',\lambda_-} M_{\lambda',\lambda_+,\lambda}^\ast = g^2 m^2/2m^2 \delta_{\lambda\lambda'} - 2g^2 m^2 \varepsilon^\mu(q, \lambda)\varepsilon^{\ast\nu}(q, \lambda') \text{Tr}[(\not p_2 + m)\gamma_\mu(\not p_1 - m)\gamma_\nu]
\]

(B3)

In evaluating Eq.(B3) terms of order \((m/m_\rho)^2\) are ignored. As for the meson decay \( \bar{\theta} \) and \( \bar{\phi} \) are the angles between \( \vec{s}' \) and the relative velocity of the lepton pair \( \vec{v} = \vec{v}_1 - \vec{v}_2 \). The angular distribution of the lepton pair is now described by:

\[
W^V(\bar{\theta}, \bar{\phi}) = \sum_{\lambda} \sum_{\lambda_+\lambda_-} M_{\lambda,\lambda',\lambda_-} M_{\lambda',\lambda_+,\lambda}^\ast < 1 - \lambda' |[I + 3/2 \vec{S} \cdot \vec{P}^V + \tau \cdot T^V]|1\lambda' >
\]

(B4)

where \( \sum_{\lambda_+\lambda_-} M_{\lambda,\lambda',\lambda_-} M_{\lambda',\lambda_+,\lambda}^\ast (-1)^\lambda \) again is a symmetric function of \( \lambda, \lambda' \). Thus the vector polarization also does not contribute to the angular distribution of the leptons, when the lepton spins are summed over, in the absence of a background term.

If one of the leptonic spins is measured then the above reasoning does not apply and terms involving the vector meson’s vector polarization appear.

**APPENDIX C: SPIN-DEPENDENT LEPTON DECAY ANGULAR DISTRIBUTION**

We mention an interesting similarity in the description of the parity conserving decay \( V \to \ell^+ \ell^- \) relevant in this paper and the parity non-conserving decays \( Z \to \ell^+ \ell^- \) or \( W^- \to \ell^- \bar{\nu}_\ell \). This similarity demonstrates why in the weak decay the angular distribution alone gives complete information on the polarization of the vector meson, while in the electromagnetic decay one needs an additional measurement of the lepton helicity to obtain the same complete spin information.

We start from a Lorentz covariant form of the density matrix \( \rho^{\mu\nu} \) related to the previously discussed density matrix \( \rho^{\lambda\lambda'} \) by

\[9\] The decay angular distribution with addition of a tensor coupling also does not depend on the vector polarization of the vector meson.
\[
\rho^{\mu\nu} = \sum_{\lambda\lambda'} \varepsilon^{\mu}(q, \lambda) \rho^{\lambda\lambda'} \varepsilon^* \nu(q, \lambda'),
\]
(C1)

or

\[
\rho^{\lambda\lambda'} = \sum_{\mu\nu} \varepsilon^*_{\mu}(q, \lambda) \rho^{\mu\nu} \varepsilon_{\nu}(q, \lambda').
\]
(C2)

If we define the decay amplitude into leptons with respective momenta \(\vec{p}_1\) and \(\vec{p}_2\) and spin projections \(m_1\) and \(m_2\) as

\[
M^{m_1,m_2}_\lambda = \sum_\mu I^{m_1,m_2}_\mu \varepsilon^{\mu}(q, \lambda),
\]
(C3)

where

\[
I^{m_1,m_2}_\mu = g \bar{u}_{m_1}(p_1) \gamma_\mu (a - b \gamma_5) v_{m_2}(p_2),
\]
(C4)

where the parameters \(a\) and \(b\) will be discussed below. The angular distribution for definite \(m_1\) and \(m_2\) is found from

\[
W^{m_1,m_2}(\bar{\theta}, \bar{\phi}) = \sum_{\lambda\lambda'} M^{m_1,m_2}_\lambda \rho^{\lambda\lambda'} M^{\dagger \top \; m_1,m_2}_{\lambda'}

= \sum_{\lambda\lambda'} \sum_{\mu\nu} L^{m_1,m_2}_\mu \varepsilon^{\mu}(q, \lambda) \rho^{\lambda\lambda'} \varepsilon^* \nu(q, \lambda') L^{m_1,m_2*}_\nu

= \sum_{\mu\nu} \rho^{\mu\nu} L^{m_1,m_2}_\mu L^{m_1,m_2*}_\nu.
\]
(C5)

We can now define the Lorentz tensor

\[
L^{m_1,m_2}_{\mu\nu} \equiv I^{m_1,m_2}_\mu L^{m_1,m_2*}_{\nu}

= g^2 \bar{u}_{m_1}(p_1) \gamma_\mu (a - b \gamma_5) v_{m_2}(p_2) \bar{v}_{m_2}(p_2) \gamma_\nu (a - b \gamma_5) u_{m_1}(p_1).
\]
(C6)

Summing over spin projections \(m_1\) and \(m_2\) leads to \(W(\bar{\theta}, \bar{\phi}) = \sum_{m_1,m_2} W^{m_1,m_2}(\bar{\theta}, \bar{\phi})\) and \(L_{\mu\nu} = \sum_{m_1,m_2} L^{m_1,m_2}_{\mu\nu}\) and

\[
L_{\mu\nu} = g^2 \text{Tr}[\bar{u}(p_1) \gamma_\mu (a - b \gamma_5) v(p_2) \bar{v}(p_2) \gamma_\nu (a - b \gamma_5) u(p_1) ]

= \frac{g^2}{4m^2} \text{Tr}[(\not{p}_1 + m) \gamma_\mu (a - b \gamma_5)(\not{p}_2 - m) \gamma_\nu (a - b \gamma_5)].
\]
(C7)

The Lorentz tensor \(L_{\mu\nu}\) defined in Eq. (C7) describes all three cases that are relevant for the discussion in this Appendix. For \(a = 1\) and \(b = 0\), \(L_{\mu\nu}\) describes the angular distribution \(W(\bar{\theta}, \bar{\phi})\) of parity-conserving leptonic decay (for example \(\rho \to \ell^+\ell^-\)), where one sums over the lepton helicities. Because the spin projection operator has the form \(\frac{1}{2}(1 \pm \gamma_5)\), the choice \(a = \frac{1}{2}, b = \pm \frac{1}{2}\), in \(L_{\mu\nu}\) corresponds with parity conserving leptonic decay where \(\ell^-\) is respectively purely right handed or purely left handed. For \(a = b = 1\), the same \(L_{\mu\nu}\) describes the angular distribution of a parity non-conserving weak decay, where the lepton helicities are not observed (for example in a process where \(Z \to \ell^+\ell^-\)). From the above expressions for \(L_{\mu\nu}\) in Eq. (C7) and \(W(\bar{\theta}, \bar{\phi})\) of Eq. (C5) one can obtain the angular distribution of the decay leptons in each of the three cases. Only in the last two cases the angular distribution \(W(\bar{\theta}, \bar{\phi})\) contains information about the vector polarization of the vector meson due to the presence in Eq. (C7) of terms linear in \(\gamma_5\).
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FIG. 1. Kinematics for the $\gamma N \rightarrow VN'$ reaction, where the incident photon is in the $\hat{z}$ direction, the vector meson ($V = \rho$) is produced in the $\hat{z}'$ direction at the scattering angle $\Theta, \Phi$ and the normal to the scattering plane is in the $\hat{y}$ direction. The decay plane of either the two mesons $\pi_1, \pi_2$ or two leptons is shown in the $\gamma N$ CM system, where the decay products are in the $\theta_1, \phi_1$ and $\theta_2, \phi_2$ directions.
FIG. 2. The Gottfried-Jackson (GJ), Adair(A), and Helicity(H) choice of axes are shown. All three choices are defined in the vector meson’s rest frame and are related to each other by simple rotations (by angles $\alpha$ or $\Theta$) about their common normal to the scattering plane, e.g. about $\hat{y}'$. The H choice is defined by the direction of the vector meson’s momentum $\vec{q}$ and the normal $\hat{y}'$ to the scattering plane (both of which remain the same for the vector meson rest and the $\gamma N$ CM frames). The GJ choice also uses the normal $\hat{y}'$ to the scattering plane, but the $\hat{z}'_{GJ}$ direction in the vector meson’s rest frame is defined by the incident photon’s direction—as seen from the vector meson’s rest frame. That last step entails a Lorentz transformation so that the photon’s direction is seen along the vector $\gamma'$ at the angle $\alpha$ as shown in the figure. The A choice again uses the common normal to the plane, plus the original (untransformed) photon direction. Each choice has its advantages, but they are not useful for defining single and double spin observables, which involve the photoproduction amplitude in the $\gamma N$ CM system.
FIG. 3. A vector meson $(V = \rho)$ is produced in the $\hat{z}'$, direction at the scattering angle $\Theta, \Phi$, where we take the scattering plane to have $\Phi \equiv 0$. The velocities $\vec{v}_1 = \vec{p}_1/E_1, \vec{v}_2 = \vec{p}_2/E_2$, of the two mesons (or two leptons) are shown in the $\gamma N$ CM system, where they are in the $\theta_1, \phi_1$ and $\theta_2, \phi_2 = \pi + \phi_1$ directions. Note $\bar{\theta}, \bar{\phi}$ are defined as the angles between the velocity difference vector $\vec{v}_1 - \vec{v}_2$ and the direction of the vector meson $\vec{q} = q\hat{z}'$, all in the $\gamma N$ CM system. The decay plane is out of the scattering plane by an azimuthal angle $\phi$, which equals $\phi_1$. 