The Spontaneous Breaking of Chiral Symmetry without Goldstone Bosons

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Abstract

Considering a self-interaction only of mirror fermions in the context of a lattice-regularized fermion field theory, we show that the system undergoes spontaneous breaking of chiral symmetry and mirror-fermion masses are generated. However, it is explicitly shown that there are no Goldstone bosons appearing together with this spontaneous symmetry breaking phenomenon, since Lorentz invariance, one of very general prerequisites of the Goldstone theorem, is violated. The result and its possible application are briefly discussed.

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1. The “Goldstone theorem”\cite{1,2} has played a very important role in modern physics, in particular, theoretical particle physics. The theorem states: “there appear spinless particles of zero mass whenever a continuous symmetry group leaves the Lagrangian but not the vacuum invariant\cite{3}.” Its application to the Standard Model (SM), i.e., the spontaneous symmetry breaking phenomenon\cite{4}, succeed in cooperating the gauge symmetry $SU_{L}(2)\otimes U_{Y}(1)$ of the (SM) with the massive gauge bosons $W^{\pm},Z^{0}$ observed phenomenologically. Three Goldstone bosons become the longitudinal modes of the massive gauge bosons. However, we have never observed any extra massless modes of Goldstone type in the elementary spectrum although this spectrum shows a total violation of the chiral (flavour) symmetry of the SM. Thus, possibilities of violating the chiral symmetry of the SM without Goldstone modes deserve study. In order to avoid the appearance of extra massless modes, one might consider an explicit breaking of chiral symmetries through either the fermionic sector\cite{5} or anomalies in the gauge sector\cite{6}. In this note, we attempt to showing the possibility that a lattice-regularized fermion field theory, which is chirally symmetric, undergoes a spontaneous symmetry breaking without Goldstone modes appearing. However, the absence of massless bosons is a consequence of the inapplicability of the “Goldstone theorem” rather than a contradiction of it, since the spontaneous breaking of chiral symmetry to be considered does not obey one of the very general prerequisites of the “Goldstone theorem”, i.e., Lorentz invariance, which is a necessary condition in demonstrating the theorem\cite{3,7}.

2. We thus consider a chirally symmetric lattice-regularized fermion field theory. There must exist ordinary and mirror fermions in the low-energy spectrum of the theory\cite{8}. As for the Wilson fermion\cite{9}, a dimension-5 operator (the Wilson term) in the continuum limit is introduced ($S_{d}$ is a massless Dirac action)

$$S = S_{d}(\bar{\psi}, \psi) + \frac{r}{a} \sum_{x,\mu} \bar{\psi}(x) \partial_{\mu}^{2} \psi(x), \quad (1)$$

where $a$ is the lattice spacing and the lattice “laplacian” $\partial_{\mu}^{2}$ is defined as

$$\partial_{\mu}^{2} \psi(x) = \psi(x + a_{\mu}) + \psi(x - a_{\mu}) - 2\psi(x). \quad (2)$$

This high-dimension operator becomes an irrelevant operator in the infrared and does not affect the continuum limit at all. However, chiral symmetry in (1) is explicitly broken. One might introduce a self-interaction of mirror fermions only, and generalize (1) to

$$S = S_{d}(\bar{\psi}, \psi) - \frac{G_{2}}{2} \sum_{x,\mu} (\bar{\psi}_{L}(x) \partial_{\mu}^{2} \psi_{R}(x))(\bar{\psi}_{R}(x) \partial_{\mu}^{2} \psi_{L}(x)). \quad (3)$$
Analogous to the Wilson term (1), this dimension-10 operator in the continuum limit is not relevant for ordinary fermions with $ka \sim 0$. However, for mirror fermions with $ka \sim \pi$, this operator is not irrelevant and it presents a self-interaction of these mirror fermions at the cutoff scale. Obviously, eq.(3) is chiral invariant. Note that in general, there are several possibilities of chirally invariant four fermion interactions on a lattice space time, e.g.,

$$\beta_1 \bar{\psi}(x)\psi(x)\bar{\psi}(x)\psi(x) + \beta_1 \sum_\mu \bar{\psi}(x \pm \mu)\psi(x)\bar{\psi}(x)\psi(x) + \cdots,$$

which represent complicated interactions between ordinary fermions and mirror fermions. The origin of these interactions (which might stem from the quantum gravity [10, 11] or a high-frequency contribution of the theory [12]) will not be a focus of this paper. We have tuned the four-fermion couplings in eq.(4) such that only mirror fermions interact with themselves (3) and the interactions between ordinary fermions and mirror fermions vanish.

On the other hand, in order to generate effective masses $M$ of order $O(\frac{1}{a})$ for mirror fermions, which split mirror fermion from ordinary fermions as occurs in (1), one expects that the Fermi-type coupling $G_2$ be large enough so that theory (3) undergoes spontaneous symmetry breaking with non-vanishing vacuum expectation value ($r = \bar{r}a$)

$$\bar{r} = \frac{G_2}{4} \sum_{\mu,x} \left( \frac{1}{4} \right) \langle \bar{\psi}_L(x)\partial^2_\mu \psi_R(x) + h.c. \rangle.$$

This belief can be justified based on computation of the effective potential and gap-equation of the system by adopting the Landau mean-field method and the large-$N_f$ approach ($G_2 N_f$ fixed, $N_f \gg 1$ is the number of flavour in eq.(3)). The effective potential is given by ($f_l = \int_{-\pi}^\pi \frac{d^4 l}{(2\pi)^4}$)

$$V(\bar{r}) = \frac{4\bar{r}^2}{G_2} - N_f \text{tr} \int_l \ell n \left[ \gamma_\mu \sin l_\mu a + \bar{r} w(l) \right] + \cdots,$$

where $l_\mu = p_\mu a; w(l) = \sum_\mu (1 - \cos l_\mu)$, the first term is the Landau mean-field approximations (tree-level), the second term is a one-loop approximation (with mean-fields) in the large-$N_f$ approach, both are $O(N_f)$ terms, and the dots denote $O(N_f^0)$ terms. In eq. (5) the imaginary part, $iG_2^2 \sum_{\mu,x} \left( \frac{1}{4} \right) \langle \bar{\psi}_L(x)\partial^2_\mu \psi_R(x) - h.c. \rangle$, can always be gauged away by an appropriate transformation because of the chiral symmetry of the action (3). By minimizing the lowest-order effective potential $V(\bar{r})$ eq. (5), we readily obtain a self-consistent “gap equation”:

$$r = \frac{g_2}{2} \int_l w(l) \frac{r w(l)}{\text{den}(l)},$$
where $g_2 a^2 = N_f G_2$ and $\text{den}(l) = \sin^2 l_\mu + (rw(l))^2$. This gap equation is exact for $N_f \to \infty$. Numerical calculation shows, for $g_2 > 0.2$, there exist a non-trivial solution $r > 0$. One clearly finds that the solution $r > 0$ stems from the contribution of mirror fermions, owing to the factor $w(l)$ in eq. (7). The trivial solution $r = 0$ cannot be realized since it does not represent the true ground state ($\Delta E = V(r) - V(0)$)

$$\Delta E = -\frac{2N_f}{a^4} \int \sum_{k=1}^\infty \frac{1}{k+1} \left[ \frac{(rw(l))^2}{s^2(l) + (rw(l))^2} \right]^{k+1} + O(N_f^0),$$

which is obtained by substituting (4) into (5). Thus, the system does undergo spontaneous symmetry breaking. Mirror-fermion masses are generated through their self-interactions (Nambu-Jona Lasinio mechanism [2]). The effective action above the true ground state ($r \neq 0$) is just the Wilson fermion (1).

3. Are there Goldstone modes appearing together with the spontaneous symmetry breaking discussed above? We discuss this question by performing an explicit calculation of the scattering amplitude of mirror fermions and anti mirror-fermions. The self-interaction of mirror fermions in action (4) can be written as

$$-\frac{G_2}{a} \sum_{x,\mu} \left( \frac{1}{4} \right) \left[ (\bar{\psi}(x) \partial_\mu \psi(x))(\bar{\psi}(x) \partial_\mu \psi(x)) - (\bar{\psi}(x) \gamma_5 \partial_\mu \psi(x))(\bar{\psi}(x) \gamma_5 \partial_\mu \psi(x)) \right],$$

which clearly gives us the scattering vertices of mirror fermions in the scalar channel ($s$) and the pseudo-scalar channel ($p$) (see Fig.1)

$$V_i \sim (-)^i \frac{G_2}{2} \sum_{\mu \nu} \delta_{\mu \nu} T_{\mu \nu}(k, q), \quad i = p, s \quad (10)$$

$$T_{\mu \nu} = (1 - \cos(k + \frac{q}{2}) \mu)(1 - \cos(k + \frac{q}{2}) \nu),$$

where $(-)^p = -1$, $(-)^s = 1$ and the momentum $q$ for composite particles is much smaller than the momentum $k$ of mirror fermions $q \ll k \sim \pi$. Then, we consider a one-loop bubble diagram (see Fig.2)

$$\sum_{\mu \nu} \left( (-)^i \frac{G_2}{2} \right)^2 T_{\mu \nu}(k, q) B^{(i)}_{\mu \nu}(q)$$

with

$$B^{(i)}_{\mu \nu}(q) = -\frac{1}{a^2} \int \text{tr} \left[ \Gamma_i S(l + \frac{q}{2}) \Gamma_i S(l - \frac{q}{2}) \right] T_{\mu \nu}(l, q),$$

where $\Gamma_p = \gamma_5$, $\Gamma_s = 1$ and the mirror-fermion propagator $S$ satisfying gap-equation (2) is given by

$$S(l \pm \frac{q}{2}) = \frac{-\gamma_\mu s_\mu (l \pm \frac{q}{2}) + rw(l \pm \frac{q}{2})}{\text{den}(l \pm \frac{q}{2})},$$

*The critical value is not very large, approximation is good even for small $N_f$. 3
where $\sin l_{\mu}$. Straightforward calculation leads us to

$$B^{(i)}(q) = (-)^i 4N_f \int \frac{s^2(l) - (-)^i(rw(l))^2}{[\text{den}(l)]^2} T_{\mu\nu}(l) + Q(q)E_{\mu\nu}^{(i)} + O[(q^2)^2],$$

(15)

where $Q(q) = \frac{4}{a^2} \sum_{\mu} \sin^2 \frac{q_{\mu}}{2}$ and

$$E_{\mu\nu}^{(i)} = (-)^i \frac{N_f}{4} \left[ - \int \frac{c^2(l)}{[\text{den}(l)]^2} T_{\mu\nu}(l) + \delta_{\mu\nu} \int \frac{s^2(l) - (-)^i(rw(l))^2}{[\text{den}(l)]^2} s^2(l) \right].$$

(16)

Note that throughout our calculation, the operators $\partial_{\mu}^2$, $w(l)$ and $T_{\mu\nu}(l)$ can be understood as the projective operators to pick out mirror components of fermion field. Now we are in a position to consider the chain approximation (leading order in large-$N_f$ expansion) to the scattering amplitude of mirror fermions and anti mirror fermions, $A^{(i)} \sim \sum_{\mu} \langle \bar{\psi}(x) \partial_{\mu}^2 \Gamma_i \psi(x) \bar{\psi}(0) \partial_{\mu}^2 \Gamma_i \psi(0) \rangle$ (see Fig.3)

$$A^{(i)}(q^2) = (-)^i \frac{G_2}{2} \sum_{\mu\nu} \delta_{\mu\nu} T_{\mu\nu}(k, q) + \left( (-)^i \frac{G_2}{2} \right)^2 \sum_{\mu\nu} T_{\mu\nu}(k, q) B^{(i)}_{\mu\nu}(q) + \cdots$$

(17)

$$= \frac{G_2}{2} \sum_{\mu\nu} T_{\mu\nu}(k, q) \left( \frac{1}{\delta_{\mu\nu} - (-)^i \frac{G_2}{2} B^{(i)}_{\mu\nu}(q)} \right),$$

(18)

where $(B^{(i)}_{\mu\nu}(q))^n = B^{(i)}_{\mu\nu}(q) B^{(i)}_{\sigma\rho}(q) \cdots B^{(i)}_{\lambda\omega}(q)$. Substituting the $\delta_{\mu\nu}$ in denominator of (18) by the gap-equation (7)

$$\delta_{\mu\nu} = \delta_{\mu\nu} \frac{g_2}{2} \int_l w(l) \frac{w(l)}{\text{den}(l)},$$

(19)

and $B^{(i)}_{\mu\nu}(q)$ by eq. (15), we examine the poles of (18). These poles are located at

$$\frac{G_2 N_f}{a^2} \left( \delta_{\mu\nu} \frac{1}{2} \int_l \frac{(w(l))^2}{\text{den}(l)} - 2 \int_l \frac{T_{\mu\nu}(l)}{\text{den}(l)} \right).$$

(20)

for the pseudo-scalar channel and

$$\frac{G_2 N_f}{a^2} \left( \delta_{\mu\nu} \frac{1}{2} \int_l \frac{(w(l))^2}{\text{den}(l)} - 2 \int_l \frac{s^2(l) - (rw(l))^2}{[\text{den}(l)]^2} T_{\mu\nu}(l) \right).$$

(21)

for the scalar channel. One finds that (20) does not vanish and eq. (18) does not possess a zero pole. Thus, we do not find any massless modes in the pseudo-scalar channel of the scattering amplitudes of mirror fermions. Actually, the non-vanishing of eqs. (20) and (21) tells us that the composite modes of mirror fermions have their masses at the order of the cutoff and are then unobservable in the low-energy spectrum. However, instead of the self-interaction of mirror fermions introduced, we consider four-fermion interaction $G_1 \bar{\psi}(x) \psi(x) \bar{\psi}(x) \psi(x)$ with chiral symmetry breaking v.e.v. $\langle \bar{\psi}(x) \psi(x) \rangle \neq 0$. Then, $\partial_{\mu}^2 \sim 1$, $T_{\mu\nu}(l) \sim \delta_{\mu\nu}$ and $w(l) = \sum_{\mu\nu} T_{\mu\nu} \sim \sum_{\mu\nu} \delta_{\mu\nu}$
are no longer projective operators for mirror fermions, eq. (20) vanishes and one obtains a Goldstone boson in the pseudo-scalar channel. For the scalar channel, the composite scalar is still massive at the order of the cutoff due to the contribution of mirror fermions.

4. We turn to discuss the reasons for the absence of Goldstone modes. Lattice regularization of a fermion field theory \( S_d(\bar{\psi}, \psi) \) in \( (1) \) by itself is not explicitly Lorentz invariant, in particular, the high-dimension operators in eqs. \( (1,3) \) are Lorentz-symmetry violating. Lorentz symmetry is expected to be restored in the continuum limit, where these high-dimension operators are irrelevant. If we consider the scattering vertex four-fermion interaction \( G_1 \bar{\psi}(x) \psi(x) \psi(x) \psi(x) \) with chiral symmetry breaking \( \langle \bar{\psi}(x) \psi(x) \rangle \neq 0 \) and \( A^{(\mu)} \sim \sum_{\mu} \langle \bar{\psi}(x) \gamma^\mu \psi(x) \bar{\psi}(0) \gamma^\mu \psi(0) \rangle \), all of them are Lorentz invariant and remain relevant operators in the continuum limit, where Lorentz symmetry is fully restored and we have Goldstone modes as those in the continuum Nambu-Jona Lasinio model. However, in the context we discuss, the scattering vertex \( (3) \), chiral symmetry breaking \( (5) \) and scattering amplitude \( (18) \) of mirror fermions, whose momenta probe the detail of lattice structure, violate Lorentz invariance and these operators turn out to be irrelevant in the continuum limit. Thus, the approach of demonstrating the Goldstone theorem, where the Lorentz invariance of the theory is a necessary prerequisite, cannot be applied. Although the self-coupling \( G_2 \) of mirror fermions can be strong enough to break chiral symmetry, a Goldstone mode is not bound to be produced.

Starting from the Goldstone theorem, let us suppose that there is a Goldstone mode appearing, corresponding to the spontaneous breaking of chiral symmetry \( r \neq 0 \) \( (7) \), which leads to the effective Wilson action \( (1) \) through asymmetric vacuum \( (8) \). However, the chiral-symmetry breaking term, i.e., Wilson term \( (1) \), turns out to be irrelevant and chiral symmetry and Lorentz symmetry are restored as the continuum limit is approached, the Goldstone boson, which is supposed to be produced, would not disappear in this limit owing to its long-range property. Then, in the continuum limit, we would have obtained the Goldstone boson without any symmetry breaking. This turns out to be in contradiction with the Goldstone theorem itself, which should be true in the continuum limit. Thus, this logical argument leads us to the absence of the Goldstone modes in the spontaneous symmetry breaking phenomenon we discussed, although we have not presented a rigorous demonstration.

Once the Wilson term is generated without Goldstone bosons, nothing can prevent the dimension-3 operator \( m \bar{\psi}(x) \psi(x) \) \( (12,13) \) and the anomaly \( (14) \) from being produced if we turn on gauge interactions. Also, we might find a way out of the problem of the non-conservation of fermion numbers \( (3) \) in a lattice-regularized version
of the Standard Model.

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Figure Captions

Figure 1: The scattering vertices of mirror fermions.
Figure 2: One-loop bubble diagram.
Figure 3: The scattering amplitude of mirror fermions in the chain approximation.
This figure "fig1-1.png" is available in "png" format from:

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