Multimethod genetic algorithm for the three-dimensional orthogonal packing problem

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Abstract. The article is devoted to design and investigation of a set of heuristics intended to optimize the three-dimensional orthogonal packing problem with the multimethod genetic algorithm. The proposed heuristics are based on five rules for selecting objects and seven rules for selecting free spaces of containers. The quality and time effectiveness for all these rules were determined. The probabilities of including these rules into new heuristics are given. The article contains results of computational experiments carried out on the standard instances of the three-dimensional orthogonal bin packing problem.

1. Introduction
The three-dimensional packing problem is the problem of finding the optimal placement of a given set of three-dimensional objects into limited three-dimensional spaces (containers). The three-dimensional bin packing problem is a special case of the three-dimensional packing problem, in which all objects are to be placed in the minimum number of containers in the form of parallelepipeds [1]. The solution of a large number of resource allocation problems is reduced to solving the three-dimensional orthogonal packing problem. The area of practical interest of this problem includes applications related with the packing, loading, scheduling and routing processes [2]. One of the most common practical applications of this problem is a container transportation of goods in logistics [3].

The three-dimensional bin packing problem is set by a given set of \( N \) orthogonal containers (three-dimensional parallelepipeds) with the dimensions \( \{W_j^1, W_j^2, W_j^3\} \), \( j \in \{1,..,N\} \) as well as a set of \( n \) orthogonal objects (three-dimensional parallelepipeds) with the dimensions \( \{w_i^1, w_i^2, w_i^3\} \), \( i \in \{1,..,n\} \).

The goal of this NP-complete problem [4] is to find a placement of all objects into minimal number of containers under the following conditions of correct placement:

- all edges of objects and containers are parallel;
- all packed objects do not overlap with each other \( (\forall j \in \{1,..,N\}, \forall d \in \{1,..,3\}, \forall i,k \in \{1,..,n\}, i \neq k, (x_{ij}^d \geq x_{kj}^d + w_k^d) \vee (x_{kj}^d \geq x_{ij}^d + w_i^d)) \);
- all packed objects are within the bounds of the containers, i.e. \( \forall j \in \{1,..,N\}, \forall d \in \{1,..,3\}, \forall i \in \{1,..,n\} \ (x_{ij}^d \geq 0) \wedge (x_{ij}^d + w_i^d \leq W_j^d) \).

The statement of this problem includes specifying a load direction of containers as the priority selection list of the coordinate axes: \( L = \{l_1; l_2; l_3\} \), \( l_d \in \{1;3\} \forall d \in \{1,..,3\} \).
The developed model of potential containers described in detail in [5] is used to describe the contents of a three-dimensional container. The potential container (PC) is a free space in the form of a parallelepiped with the maximal possible dimensions which can be used for placing new objects into it. Overall dimensions of a PC \( j \) are described by a vector \( \{p_{1j}; p_{2j}; p_{3j}\} \).

2. Heuristics for the multimethod genetic algorithm

The classic genetic algorithm is one of the most common algorithms applied to solve packing problems. This algorithm works with chromosomes which code the sequences of objects to be selected for placing them into containers. The implemented multimethod genetic algorithm (MGA) [6] works with chromosomes which code the sequences of one-pass algorithms of solving the problem which are named by heuristics [7]. The authors propose new heuristics based on the 5 rules for selecting unpacked objects (table 1) and 7 rules for selecting potential containers (table 2) intended to solve the three-dimensional orthogonal packing problem with the MGA.

| Index | Rule | Description |
|-------|------|-------------|
| 1     | \( O_1 \) | Select object \( i \) with the maximal volume \( (w_{ij}^1 \times w_{ij}^2 \times w_{ij}^3) \). |
| 2     | \( O_2 \) | Select object \( i \) with the minimal volume \( (w_{ij}^1 \times w_{ij}^2 \times w_{ij}^3) \). |
| 3     | \( O_3 \) | Select the first object from the set of ordered in descending dimensions of objects in accordance with the priority selection list \( L \). |
| 4     | \( O_4 \) | Select the first object \( i \) with the maximal value \( w_{ij}^1 \times w_{ij}^2 \) and the minimal value \( w_{ij}^3 \). |
| 5     | \( O_5 \) | Select the first object \( i \) with the minimal value \( w_{ij}^1 \times w_{ij}^2 \) and the maximal value \( w_{ij}^3 \). |

| Index | Rule | Description |
|-------|------|-------------|
| 1     | \( P_1 \) | Select the PC \( j \) with the maximal volume \( (p_{1j}^1 \times p_{1j}^2 \times p_{1j}^3) \). |
| 2     | \( P_2 \) | Select the fit PC \( j \) with the minimal volume \( (p_{1j}^1 \times p_{1j}^2 \times p_{1j}^3) \). |
| 3     | \( P_3 \) | Select the first fit PC from the set of PCs arranged by their coordinates in descending order in accordance with the priority selection list \( L \). |
| 4     | \( P_4 \) | Select the fit PC \( j \) with the maximal value \( p_{1j}^1 \times p_{1j}^2 \) and minimal value \( p_{1j}^3 \). |
| 5     | \( P_5 \) | Select the fit PC \( j \) with the minimal value \( p_{1j}^1 \times p_{1j}^2 \) and maximal value \( p_{1j}^3 \). |
| 6     | \( P_6 \) | Select for a given object \( i \) the most fit PC with the minimal positive value \( p_{1j}^1 - w_{ij}^1 \). If there are several such PCs, select the fit PC with the minimal positive value \( p_{1j}^2 - w_{ij}^2 \). If there are several such PCs, select the fit PC with the minimal positive value \( p_{1j}^3 - w_{ij}^3 \). |
| 7     | \( P_7 \) | Select for a given object \( i \) the most fit PC with the maximal value \( p_{1j}^1 - w_{ij}^1 \). If there are several such PCs, select the fit PC with the maximal value \( p_{1j}^2 - w_{ij}^2 \). If there are several such PCs, select the fit PC with the maximal value \( p_{1j}^3 - w_{ij}^3 \). |
As a result, it is possible to use 35 heuristics based on different pairs of rules for selecting objects and potential containers in the MGA.

The efficiency of the proposed rules has been analyzed on the standard test instances proposed by S. Martello, D. Pisinger and D. Vigo [8]. All the test instances are divided into 8 classes. Each class contains 4 sets of instances with 50, 100, 150, and 200 objects and the same number of cubic containers. The authors considered 10 instances for each class of test instances. The averaged results are presented on figures 1–4.

**Figure 1.** Quality of packing for different rules for selecting objects.

**Figure 2.** Speed of different rules for selecting objects.

**Figure 3.** Quality of packing for different rules for selecting potential containers.

**Figure 4.** Speed of different rules for selecting potential containers.

The worst quality of packing is obtained while rules $O_2$ and $O_5$ are used. The fastest rules for selecting objects are $O_1$, $O_3$ and $O_4$. All the rules for selecting potential containers provide the same quality of packing on average. The fastest rule for selecting potential containers is $P_3$.

According to the results of the test experiments, the probabilities of usage the different rules in when creating new heuristics, were chosen (table 3, 4).
Table 3. Probabilities of usage rules for selecting objects.

| Rule | O_1 | O_2 | O_3 | O_4 | O_5 |
|------|-----|-----|-----|-----|-----|
| Probability, % | 90  | 1   | 3   | 5   | 1   |

Table 4. Probabilities of usage rules for selecting potential containers.

| Rule | P_1 | P_2 | P_3 | P_4 | P_5 | P_6 | P_7 |
|------|-----|-----|-----|-----|-----|-----|-----|
| Probability, % | 18  | 18  | 26  | 18  | 18  | 1   | 1   |

3. Multimethod genetic algorithm

The implemented multimethod genetic algorithm, intended to solve the three-dimensional orthogonal packing problem, has the following settings:

- the probability of the crossover operator is 0.80;
- the probability of the mutation operator is 0.10;
- the probability of the inversion operator is 0.50;
- the size of a chromosome is equal to number of objects to be packed;
- the size of population is 100 chromosomes;
- the initial population consists form 35 chromosomes composed using only one individual heuristic and 65 chromosomes composed with heuristics with accordance with probabilities of using the rules given in tables 3, 4;
- the sign of convergence of the algorithm is the obtaining of 20 generations with the same best found chromosome;
- the time of solution is limited to 300 s;
- the depth of search is limited to 1000 generations.

The computational experiments were conducted on the personal computer (Intel Core i5-3350P 3.10 GHz, RAM 6.00 GB).

Table 5 contains the average results of testing the developed MGA with the proposed heuristics in comparison with the following algorithms:

- C-EPBFD – the constructive heuristic algorithm proposed by T. Crainic, G. Perboli and R. Tadei [2];
- MPV-BS – the branch and bound method proposed by S. Martello, D. Pisinger and D. Vigo [8];
- HA – the heuristic algorithm proposed by A. Lodi, S. Martello and D. Vigo [9];
- LB – the lower bound of the problem calculated by M.A. Boscetti [10].

Since the instances form the test classes II and III are equal to the instances form the test class I [8], the results for these test classes are not presented in table 5.

Table 5. Results of testing the multimethod genetic algorithm.

| Class of test instances | MGA | C-EPBFD | MPV-BS | HA | LB |
|-------------------------|-----|---------|--------|----|----|
| Class I                 | 129,2 | 130,5   | 133,3  | 132,3 | 124,0 |
| Class IV                | 297,4 | 294,0   | 295,0  | 294,3 | 292,2 |
| Class V                 | 66,9  | 72,6    | 81,5   | 73,6  | 66,4 |
| Class VI                | 97,1  | 98,1    | 106,5  | 100,2 | 93,0 |
| Class VII               | 63,0  | 62,8    | 68,2   | 63,7  | 55,4 |
| Class VIII              | 83,5  | 85,4    | 90,9   | 87,1  | 77,8 |
| Total                   | 737,1 | 743,4   | 775,4  | 751,2 | 708,8 |
The results of computation experiments presented in table 5 show that the proposed algorithm (MGA) provides the densest placement of objects for four of six classes of test instances as well as it provides the most efficient placement of objects on average for all the classes of test instances of three-dimensional orthogonal packing.

4. Conclusion
The realized multimethod genetic algorithm with the chromosomes composed of heuristics compiled by the proposed rules for selecting objects and potential containers demonstrates the high results of solving the three-dimensional orthogonal packing problem obtained on most classes of standard test instances. This algorithm besides usage of efficient heuristics also uses inefficient heuristics with small probabilities which help to get out of the local optima during search of the optimal solution. Among all heuristics a heuristic which uses the rules $O_1$ and $P_3$ is the most effective both in speed and quality of resulting packing, hence only this heuristic can be used to solve to three-dimensional orthogonal packing problem when the requirement of speed is critical.

Future research can be related to the development of new heuristics which will collect new objects in groups in order to consider these groups as single composed objects in the formation of packing.

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