Spatially q-deformed radial momentum of d-dimensional Klein-Gordon equation for Harmonic Oscillator plus Inverse Quadratic potential investigated using hypergeometric method

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Abstract. Spatially quadratic deformation radial momentum of D-dimensional Klein-Gordon equation with three-dimensional Harmonic Oscillator and Inverse Quadratic potential was investigated using Hypergeometric Method. By using appropriate variable transformation, D-Dimensional Klein-Gordon equation for q deformed momentum was reduced into one order dimensional Schrodinger like equation with Pöschl-Teller like potential. The relativistic energy and wave function equations were obtained from the solution of Schrodinger like equation. Then, the relativistic energy and wave function were calculated and visualized using Matlab R2013 software, respectively. The q deformed parameter had the effect to the relativistic energy and wave function. The increase of q deformed parameter causes the increase of energy spectra and the decrease of amplitude.

1. Introduction
In the 20th century, several scientists debated about the concept of classical physics that refers to Newton's law and the concept only explain some macroscopic materials behavior. Some experiments showed that Newton's physics could not explain some microscopic materials behavior. Therefore, a new theory was founded which was called the theory of quantum mechanics. It could explain about microscopic objects behavior with interaction of energy on the scale of atoms and subatomic particles [1,2].

In quantum mechanics, the relativistic effect will be considered when a particle is moving in a strong potential field [3,4]. The Klein-Gordon equation is used to describe a zero-spin particle [3,4,5], pions and kaons, in large potentials [6]. In relativistic equation consists of four vector potential V(r) and space-time scalar potential S(r) [3]. By using non-trivial nonrelativistic limit S(r) = V(r), we obtain the potential function 2V(r), not V(r). Besides that, by using nonrelativistic limit, we obtain the potential interaction V(r), not 2V(r) [7].

Hypergeometric method was one of the methods which was used to solve the Klein-Gordon equation. This method was obtained from Hypergeometric function by substituting new variable to get the second-order differential equation [2,5].

Some potentials can be used to obtain the exact solution of Klein-Gordon equation, especially shape invariant potentials, such as Kratzer, Pöschl-Teller, and Manning Rosen potentials [8,9]. Indeed, the
Harmonic Oscillator plus Inverse Quadratic potential can be used to solve the Klein-Gordon equation too [11]. In this research, we investigate and calculate energy spectrum and wave function for D-dimensional Klein-Gordon equation with spatially q-deformed of radial momentum for Harmonic Oscillator plus Inverse Quadratic potential using Matlab R2013 Software.

There are several sections of this paper such as the D-dimensional Klein-Gordon equation, the Hypergeometric method, result and discussion, and conclusion.

2. Experimental

2.1. D-dimensional klein-gordon equation

The motion of particle with mass \( M \) and relativistic energy \( E \) on scalar \( S(r) \) and vector \( V(r) \) potentials in natural unit \( (\hbar = 1, c = 1) \) is given as [7,12,13]:

\[
\left\{ \nabla_D^2 + \left[(V(r) - E)^2 - (S(r) + M)^2 \right] \right\} \psi_{\ell}^{(\ell)}(r) = 0
\]  

(1)

where \( \nabla_D^2 \) is Laplacian operator which is expressed by [13]

\[
\frac{1}{r^{D-1}} \frac{\partial}{\partial r} \left( r^{D-1} \frac{\partial}{\partial r} \right) - \frac{L_{D-1}^2}{r^2}
\]  

(2)

The angular momentum \( L_{D-1}^2 \) in equation (2) can be written as

\[
L_{D-1}^2 = - \sum_{j=1}^{D-2} \frac{1}{\sin^n \theta_{j+1}} \left( \frac{1}{\sin^n \theta_j} \left( \frac{\partial}{\partial \theta} \sin^{n+1} \theta_j \frac{\partial}{\partial \theta} \right) \right) + \frac{1}{\sin^n \theta_{D-1}} \left( \frac{\partial}{\partial \theta} \sin^{n+1} \theta_{D-1} \frac{\partial}{\partial \theta} \right)
\]  

(3)

The D-dimensional Klein-Gordon equation for \( S(r) = V(r) \) can be written as [7,12]

\[
\left\{ \nabla_D^2 + \left[(E^2 - M^2 - (E + M)V(r)) \right] \right\} \psi_{\ell}^{(\ell)}(r) = 0
\]  

(4)

The total of wave function equation is [3,7,12]

\[
\psi_{\ell_{1},...,\ell_{D-2}}^{(\ell)} = R_{\ell}(r)Y_{\ell_{1},...,\ell_{D-2}}^{(\ell)}(\theta_{1},\theta_{2},...,\theta_{D-1}) \quad \text{with} \quad R_{\ell}(r) = \frac{F}{r^{(\ell-1)}}
\]  

(5)

By inserting equations (2) and (5) into equation (4), we obtain

\[
\frac{\partial^2 F}{\partial r^2} \left( \frac{D-1}{2} \right) \left( \frac{D-3}{2} \right) F + \left( E^2 - M^2 - (E + M)V(r) \right) F = \frac{L_{D-1}^2}{r^2} F
\]  

(6)

Equation (6) is the radial part of wave function equation with central potential \( V(r) \).

2.2. Spatially q deformed of quadratic radial momentum

In this section, we will discuss about the deformation in quantum mechanics. The q deformation of momentum can be expressed by [16]

\[
\frac{\partial^2}{\partial r^2} = D^q_r
\]  

(7)

The \( q \)-deformed of quantum mechanics equation was [14]

\[
D^q_r = \left(1 + qr^2\right) \frac{d}{dr}
\]  

(8)

The \( q \) deformed of quadratic radial momentum \( D^q_r \) can be rewritten by

\[
D^q_r = (1 + qr^2)^2 \frac{d^2}{dr^2} + 2qr(1 + qr^2) \frac{d}{dr}
\]  

(9)

By using new variable defined as
\[ y = \sqrt{qr} \rightarrow dy = \sqrt{q} dr \rightarrow dr = \frac{dy}{\sqrt{q}} \]  \hspace{1cm} (10)

and by inserting equation (10) into equation (9), we have

\[ D^2_y = (1 + y^2)^2 q \frac{d^2}{dy^2} + 2qy(1 + y^2) \frac{d}{dy} = \frac{\partial^2}{\partial r^2} \]  \hspace{1cm} (11)

Equation (11) was the general form of deformation momentum of radial part.

2.3. Harmonic Oscillator plus Inverse Quadratic potential

Harmonic Oscillator plus Inverse Quadratic can be expressed by [4]

\[ V(r) = kr^2 + \frac{g}{r^2} \]  \hspace{1cm} (12)

with \( k \) is constant of arbitrary and \( g \) is power of inverse quadratic potential.

By changing the \( r \) variable in equation (10) into equation (12) and by substituting equation (12) into equation (6), we get

\[ (1 + y^2)^2 \frac{d^2 F}{dy^2} + 2y(1 + y^2) \frac{dF}{dy} = \left( \frac{D - 1}{2} \right) \left( \frac{D - 3}{2} \right) + \lambda_{D-1} \frac{\partial^2}{\partial r^2} \]  \hspace{1cm} (13)

Equation (13) can be solved by Hypergeometric method to gain energy and wave function equations.

2.4. Hypergeometric methods

The second order differential equation of Hypergeometric function can be written in form [5,15]

\[ z(1 - z) \frac{\partial^2 \phi}{\partial z^2} + (c - (a + b + 1)z) \frac{\partial \phi}{\partial z} - ab \phi = 0 \]  \hspace{1cm} (14)

from equation (14), we have singular points at \( z = 0 \) which was written in series form [8]

\[ a_n = \frac{(a)(a+1)(b)(b+1)\ldots(a+(n-1))(b+(n-1))}{(c)(1+c)\ldots((n-1)+c)n!} \]  \hspace{1cm} (15)

and the solution form of Hypergeometric differential equation from equation (14) was given as [8,15]

\[ \sum_{n=0}^\infty \frac{(a)_n(b)_n}{n!(c)_n} z^n \]  \hspace{1cm} (17)

If \( a = -n \) or \( b = -n \), the solution of equation (17) to get the energy eigen value of the system in polynomial rank \( n \).

3. Results and Discussions

The solution for equation (13) can be solved by using variable approximation with

\[ y = \tan x \rightarrow dy = \sec^2 x \ dx \]  \hspace{1cm} (18)

By inserting equation (18) into equation (13), we have

\[ \frac{d^2 F}{dx^2} = \left( \frac{D - 1}{2} \right) \left( \frac{D - 3}{2} \right) + \lambda_{D-1} \]  \hspace{1cm} (19)

or

\[ \sum_{n=0}^\infty \frac{(a)_n(b)_n}{n!(c)_n} z^n \]
\[
\frac{d^2 F}{dx^2} + \left( D - \frac{1}{2} \right) \left( D - \frac{3}{2} \right) \lambda_{D-1} + (E + M)g + (E + M) \frac{k}{q^2} + \left( \frac{E^2 - M^2}{q} \right) F
\]

by setting

\[
A_s = \left( D - \frac{1}{2} \right) \left( D - \frac{3}{2} \right) + (E + M)g + \lambda_{D-1} \\
B_s = (E + M) \frac{k}{q^2} \\
E_s = \left( D - \frac{1}{2} \right) \left( D - \frac{3}{2} \right) + \lambda_{D-1} + (E + M)g + (E + M) \frac{k}{q^2} + \left( \frac{E^2 - M^2}{q} \right)
\]

Equation (20) can be rewritten by

\[
\frac{d^2 F}{dx^2} - \frac{A_s}{\sin^2 x} F - \frac{B_s}{\cos^2 x} F = -E_s F
\]

or

\[
\frac{d^2 F}{dx^2} - \frac{\eta(\eta - 1)}{\sin^2 x} F - \frac{\mu(\mu - 1)}{\cos^2 x} F = -E_s F
\]

By using the variable approximation, we get

\[
A_s = \eta(\eta - 1) \rightarrow \eta = \frac{1 + \sqrt{1 + 4A_s}}{2} \quad \text{and} \quad \mu(\mu - 1) = B_s \rightarrow \mu = \frac{1 + \sqrt{1 + 4B_s}}{2}
\]

The form of equation (25) was Schrödinger like equation with Pöschl-Teller potential. Therefore, the form of solution equation (26) resemble with solution of Schrodinger like equation with Pöschl-Teller potential by Hypergeometric method.

We set the variable approximation in equation (24) \[8,15\]

\[
\cos^2 x = z \rightarrow dz = -2 \sin x \cos x dx \rightarrow \frac{dz}{dx} = -2 \sqrt{z(1-z)}
\]

By inserting equation (27) into equation (24), the form of equation (24) becomes

\[
z(1-z) \frac{d^2 F}{dz^2} + (1-2z) \frac{dF}{dz} + \left( \frac{E_s'}{4} - \frac{A_s}{4(z-1)} - \frac{B_s}{4z} \right) F = 0
\]

From equation (28), we gain the general solution of wave function equation

\[
F = z^\alpha (1-z)^\beta f(z)
\]

By substituting equation (29) into equation (28), we obtain the second order of Hypergeometric function equation which was

\[
z(1-z)f'' + \left( 2\alpha + \frac{1}{2} \right) - (2\alpha + 2\beta + 1)z \right) f' + \left( \frac{E_s'}{4} - (\alpha + \beta) \right) f = 0
\]

Equation (30) equivalent with equation (14). Consequently, we have \( c = 2\alpha + \frac{1}{2} \) and \( a + b = 2\alpha + 2\beta \) and

\[
\alpha = \frac{\mu}{2} \quad \text{and} \quad \beta = \frac{\eta}{2}
\]

From equation (30), we can obtain the relativistic energy equation
\[ E_n = (\mu + \eta + 2n)^2 \]  

By setting \( E'_n = E'_s \) and by substituting equation (23) into equation (32), we obtain the relativistic energy equation

\[
(E^2 - M^2) = q\left(\sqrt{\frac{1+4B_s}{2}} + \sqrt{\frac{1+4A_s}{2}} + 2n\right)^2 - q\left(D - \frac{1}{2}\right)\left(D - \frac{3}{2}\right) - q\lambda_{D-1} - q(E + M)g - (E + M)\frac{k}{q}
\]

with \( A_s \) and \( B_s \) in equations (21) and (22), respectively.

The relativistic energy in equation (33) can be calculated by Matlab R2013 software with several variation parameters that the result of relativistic was showed in Table 1.

Moreover, solution of wave function equation can be gained by substitution equation (27) into equation (29) that can be expressed by

\[ F = \left(\cos^2 x\right)^\alpha \left(\sin^2 x\right)^\beta \cdot R_i(a, b, c; z) \]  

with \( \alpha \) and \( \beta \) in equation (31), and based on the explanation above that \( c = 2\alpha + \frac{1}{2} \) and \( a + b = 2\alpha + 2\beta \), equation (34) can be written in form

\[ F = \left(\cos^2 x\right)^{\frac{1+\sqrt{4+B_s}}{4}} \left(\sin^2 x\right)^{\frac{1+\sqrt{4+A_s}}{4}} C'(-1)^n \left(2\alpha + \frac{1}{2}\right)_2 R_i(-n, 2\alpha + 2\beta + 1, 2\alpha + \frac{1}{2}; \cos^2 x) \]

or

\[
F_n(x) = \left(\cos^2 x\right)^{\frac{1+\sqrt{4+\left(E+M\right)\frac{k}{q}}}{4}} \left(\sin^2 x\right)^{\frac{1+\sqrt{4+\left(E+M\right)g+\lambda_{D-1}}}{4}} C'(-1)^n \left(2\alpha + \frac{1}{2}\right)_2 R_i(-n, 2\alpha + 2\beta + 1, 2\alpha + \frac{1}{2}; \cos^2 x)
\]

Equation (36) was the wave function equation for D-dimensional Klein-Gordon equation with spatially q deformed of quadratic radial momentum which can be expressed with quantum number variation that the result of wave function equation was showed in Table 2. After that, the wave function of radial part equation can be visualized by Matlab R2013 software and the result of visualization shows in Figure 1 and Figure 2.
### Table 1. The Relativistic Energy for D-Dimensional Klein-Gordon Equation with Spatially q-Deformed of Quadratic Radial Momentum in Natural Unit $g = 5, k = 5, M = 0.5$

| D | q | n | l | $E_n$ | D | q | n | l | $E_n$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | -0.499971168 | 0.01 | -0.499964827 | 0.01 | -0.499959665 | 0.01 | -0.499956137 |
| 0.04 | -0.49536735 | 0.04 | -0.498435396 | 0.04 | -0.499368076 | 0.04 | -0.499296456 |
| 0.07 | -0.49573384 | 0.07 | -0.496264398 | 0.07 | -0.498054961 | 0.07 | -0.497839721 |
| 0.1 | -0.497068669 | 0.1 | -0.496442892 | 0.1 | -0.495958588 | 0.1 | -0.495579007 |
| 0.4 | -0.487638977 | 0.4 | -0.439255889 | 0.4 | -0.432415071 | 0.4 | -0.426815068 |
| 0.7 | -0.471016256 | 0.7 | -0.297536024 | 0.7 | -0.28147968 | 0.7 | -0.267228517 |
| 1 | -0.461346221 | 1 | -0.260742085 | 1 | -0.033211057 | 1 | -0.010785701 |
| 0.1 | 0.0-0.49991117 | 0.1 | 0.0-0.499884828 | 0.1 | 0.0-0.499870616 | 0.1 | 0.0-0.499838166 |
| 1 | 0.49976016 | 1 | 0.498585166 | 1 | 0.499872202 | 1 | 0.499837491 |
| 0.04 | 0.49408918 | 0.04 | 0.49149714 | 0.04 | 0.497922615 | 0.04 | 0.49772351 |
| 0.07 | 0.49722615 | 0.07 | 0.49723231 | 0.07 | 0.497548132 | 0.07 | 0.49792802 |
| 1 | 0.49301763 | 1 | 0.49300722 | 1 | 0.492466691 | 1 | 0.491991777 |
| 0.1 | -0.48954215 | 0.1 | -0.48929430 | 0.1 | -0.486950665 | 0.1 | -0.48564484 |
| 0.4 | -0.424850746 | 0.4 | -0.289756944 | 0.4 | -0.27290057 | 0.4 | -0.256513525 |
| 1 | -0.27290057 | 1 | -0.256513525 | 1 | -0.243126335 | 1 | -0.227497937 |
| 0.7 | -0.303125824 | 0.7 | 0.223567345 | 0.7 | 0.254687619 | 0.7 | 0.26384619 |
| 1 | 0.254687619 | 1 | 0.28866721 | 1 | 0.323110119 | 1 | 0.356134655 |
| 0.1 | -0.24554695 | 1 | 0.86653603 | 1 | 1.08323585 | 1 | 1.12763641 |
| 0.4 | -0.46895064 | 0.4 | -0.485660665 | 0.4 | -0.485470768 | 0.4 | -0.48364448 |
| 1 | -0.485425368 | 1 | -0.47597391 | 1 | -0.47366318 | 1 | -0.471946969 |
| 0.01 | 0.0-0.498511138 | 0.01 | 0.0-0.497974712 | 0.01 | 0.0-0.497940707 | 0.01 | 0.0-0.497918762 |
| 1 | 0.49740701 | 1 | 0.49718762 | 1 | 0.496986763 | 1 | 0.49680244 |
| 0.04 | 0.496968673 | 0.04 | 0.496800644 | 0.04 | 0.49663000 | 0.04 | 0.49647683 |
| 0.07 | 0.497622508 | 0.07 | 0.496217198 | 0.07 | 0.495834479 | 0.07 | 0.495583896 |
| 1 | 0.49583447 | 1 | 0.495483895 | 1 | 0.495162865 | 1 | 0.494858739 |
| 0.1 | -0.495162685 | 0.1 | -0.494867937 | 0.1 | -0.494956903 | 0.1 | -0.494397095 |
| 0.7 | -0.495649537 | 0.7 | -0.488332747 | 0.7 | -0.48713736 | 0.7 | -0.486106896 |
| 1 | -0.487174367 | 1 | -0.48106898 | 1 | -0.481526765 | 1 | -0.484226456 |
| 0.3 | -0.485126765 | 0.3 | -0.484224656 | 0.3 | -0.48338134 | 0.3 | -0.482634454 |
| 0.01 | 0.49507743 | 0.01 | 0.467697444 | 0.01 | 0.465995681 | 0.01 | 0.464424394 |
| 0.4 | -0.469507743 | 0.4 | -0.467697444 | 0.4 | -0.465995681 | 0.4 | -0.464424394 |
| 1 | -0.031850124 | 1 | -0.035492971 | 1 | -0.03160153 | 1 | -0.03160153 |
| 0.7 | 1.040845878 | 0.7 | 1.084575103 | 0.7 | 1.13317679 | 0.7 | 1.183253783 |
| 1 | 1.25314282 | 1 | 2.61946377 | 1 | 2.679551264 | 1 | 2.745316912 |
| 2 | 2.679551264 | 2 | 2.745316912 | 2 | 2.84208808 | 2 | 2.88459555 |
Table 1. shows that the energy values were affected by D-dimension, quantum number, orbital number and deformation parameters. The increase of quantum number, q deformation and orbital number cause the increase of energy values. The energy values were positive and negative that those indicates free particle and bound energy of a particle in atom’s shell, respectively. Meanwhile, the q deformation shows about the energy gap in atom’s shell.

Table 2. The Wave Function for D-Dimensional Klein-Gordon Equation with Spatially q-Deformed of Quadratic Radial Momentum

| n  | $F_n(x)$                                      |
|----|-----------------------------------------------|
| 0  | $F_0(x) = \left( \cos^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} \left( \sin^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} C'$ |
| 1  | $F_1(x) = - \left( \cos^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} \left( \sin^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} C' \left( 2\alpha + \frac{1}{2} \right)^2 \left( 1 - \frac{(2\alpha + 2\beta + 2)}{(2\alpha + 1)^2} \right) (\cos^2 x) $ |
| 2  | $F_2(x) = \left( \cos^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} \left( \sin^2 x \frac{1 + \frac{3}{4}(E + M)^2}{\sqrt{q^2}} \right)^{\frac{1}{4}} C' \left( 2\alpha + \frac{1}{2} \right) \left( 2\alpha + \frac{3}{2} \right) \left( 1 - \frac{2(2\alpha + 2\beta + 3)}{(2\alpha + 1)^2} (\cos^2 x) + \frac{(2\alpha + 2\beta + 3)(2\alpha + 2\beta + 4)}{(2\alpha + 1)^2}(\cos^2 x)^2 \right) $ |
Figure 1. The wave function unnormalized for Oscillator Harmonic plus Inverse Quadratic Potential in $D = 3$ with (a) $q = 0.01$ and (b) $q = 1$

Figure 2. The wave function unnormalized for Oscillator Harmonic plus Inverse Quadratic Potential in $D = 6$ with (a) $q = 0.01$ and (b) $q = 1$
The radial part of wave function in Figure 1 shows that vibration of particle which was influenced by Oscillator Harmonic plus Inverse Quadratic potential. In Figure 1 and Figure 2 show about the model of wave function with amplitude as radius of particle with Oscillator Harmonic plus Inverse Quadratic potential. As a result, a particle in strong potential field has the chance of finding particle is greater when the radius of a particle is closer to core that shows in the increase of quantum number causes the increase of amplitude. Then, the increase of q deformation quadratic parameters causes the decrease of amplitude from wave function. For instance, n = 2 in Figure 1 with (a) q = 0.01 has maximum amplitude $\pm 217 \text{ fm}$ and (b) q = 1 has maximum amplitude $\pm 65 \text{ fm}$. Besides that, in Figure 2 with (a) q = 0.01 has maximum amplitude $\pm 310 \text{ fm}$ and (b) q = 1 has maximum amplitude $\pm 160 \text{ fm}$. Meanwhile, the increase of D-dimension and quantum number cause the increase of amplitude from wave function. For instance, for q = 0.01 in Figure 1 and Figure 2 have maximum amplitude around 0.85 (n = 0), 1.3 (n = 1), 217 (n = 2) and around 0.9 (n = 0), 1.4 (n = 1), 310 (n = 2), respectively.

4. Conclusions
The D-dimensional Klein-Gordon equation with spatially q-deformed of quadratic radial momentum for Harmonic Oscillator plus Inverse Quadratic potential can be solved by Hypergeometric Method. The energy values were positive and negative that those indicates free particle and bound energy of a particle in atom’s shell, respectively. The increase of D-dimension, quantum number, orbital number and deformation parameters causes the increase of energy values. Then, the increase of q deformation quadratic parameters causes the decrease of amplitude from wave function, while the increase of quantum number and D-dimension parameters cause the increase of amplitude from wave function.

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