A New Analytical Model for SIM–OFDM Contradicts the Previously Claimed Features

Ahmed N. Jabbar, Samir J. Almuraab, and Abdulkareem A. Kadhim, Senior Member, IEEE

Abstract—The Subcarrier Index Modulation OFDM (SIM–OFDM) appeared in 2009 promising a −3 dB transmitted power reduction without affecting the system performance. Therefore, it became an attractive choice to upgrade the communication systems with researchers’ increasing interest. Despite the research efforts in SIM–OFDM field, there was no in-depth investigation for such transmitted power reduction or the system’s performance. The claimed power reduction relies on probabilistic assumptions that were not validated considering system operation concepts. This paper provides a new analytical model that characterizes the actual SIM–OFDM behavior. The contribution of this model is the inclusion of the majority condition in the derivation of 1’s pmf which modifies the 1’s pmf into a complex nonlinear function that is always higher than 1/2. The new pmf effect upon the power reduction, synchronization, and the overall Bit Error Rate (BER) is investigated. The new analytical model shows that the −3 dB power reduction cannot be achieved. Also, no successful synchronization can be established unless extra subcarrier is added that will create a frame like communication system. Such scheme increases BER if the carrier is falsely detected creating a Frame Error Rate (FER) which might lead to serious problem.

Index terms—OFDM, SIM–OFDM, Enhanced SIM–OFDM, BER OFDM, OFDM Synchronization.

I. INTRODUCTION

The modern communication systems are competing to provide the highest possible bit rates with the best Quality of Service (QoS) level. However, high data rates require advanced techniques, sophisticated equipment, and high transmission power to ensure high Signal to Noise Ratio (SNR) to meet the designated QoS. The Orthogonal Frequency Division Multiplexing (OFDM) played a major role in the modern communication systems. It provided the highest bandwidth efficiency leading to maximum transmission rates. The OFDM simply divides the high bit rate serial data stream into multi-parallel low-rate data streams. These streams are orthogonalized using Inverse Fast Fourier Transform (IFFT) before modulating the data using the RF subcarrier [1, 2].

The recent developments in communication networks promised data rates as high as few Giga bits for the users [3–6]. Such rates rely on favour channel environments and high SNR. Such considerations employ close distance base stations and high transmission power resulting in health risk and may violate government regulations. Such effects can be mitigated by adopting the so-called Green Communication [7, 8]. The green communication is a term that embraces all the attempts, developments, upgrades and new techniques aiming to reduce the consumed or transmitted power to an eco-friendly level. The eco-friendly means that the power should not impact the health of the fauna and/or the flora not to mention the human health.

The techniques that were adopted to reduce the power are either spatial or temporal modulation or, recently, the Subcarrier Index Modulation (SIM) [9]. The spatial or temporal modulation, such as the Multiple Input Multiple Output (MIMO), tries to divide the power into multi parallel paths or time slots in order to reduce the power intensity per path. The MIMO focuses on upgrading the RF physical layer of the network.

Subcarrier Index Modulation–Orthogonal Frequency Division Multiplexing (SIM–OFDM) is considered as two-dimensional modulation where part of the data is modulated as a conventional OFDM and the other part is used to switch ON and OFF the subcarriers like the ON/OFF Keying (OOK) [10]. The authors of [10] showed that this technique can ensure a −3 dB reduction in the transmitted power without significantly sacrificing the BER performance. However, implementing this technique faces some issues that were not referred to by the authors of [10] nor the researchers implemented SIM–OFDM in their papers. These issues can be summarized as follows: the −3 dB power reduction if it is true, the synchronization between the transmitter and receiver and the BER level. Such issues will dramatically affect the SIM–OFDM performance. This research is addressing and discussing these issues to build an exact model describing the SIM–OFDM behaviour using mathematical equations. The final model is presented in order that the researchers should be aware of how these issues may compromise the system performance while implementing SIM–OFDM.

This research provides a new analytical model for the behaviour of SIM–OFDM for the first time. The analytical model showed that the previously claimed features of the SIM–OFDM are not accurate due to the rough estimation for the 1’s and 0’s probability mass function (pmf ). Thus, this
new model shows that the claimed –3 dB power reduction cannot be accomplished practically only with partial success. Also, the previously published BER expression describing the SIM–OFDM performance did not consider the synchronization effect. When the synchronization is considered, a new error source will emerge and may dominate all other BER sources. Therefore, the new analytical model advices the researchers to carefully consider these problems before developing their SIM–OFDM based systems in the future for correct operation.

The remaining parts of the paper are organized as follows; the description of SIM–OFDM system is given in Section II covering both the conventional SIM–OFDM and its modified versions. Section III shows the complete analysis to determine the correct 0’s and 1’s, the synchronization problem and the Frame Error Rate (FER). The conclusion and remarks are presented in Section IV.

II. SUBCARRIER INDEX MODULATION OFDM SYSTEM

The conventional OFDM system is shown in Fig. 1. This system contains \( N \) mappers feeding the \( N \) points IFFT. Each mapper requires \( M \) bits to generate \( 2^M \) possible symbols. Then, at every instance the system is fed by \( M \cdot N \) bits block as shown in Fig. 1.

![Fig. 1. Conventional OFDM system.](image)

The proposed SIM-OFDM in [10] suggested that \( N \) bits are extracted to switch ON/OFF the subcarriers. These \( N \) bits are called \( B_{OOK} \). After extracting the \( N \) bits, the remaining block bits are \((M-1) \cdot N\). This can be interpreted in two ways, either that all the mappers’ symbols are halved, \( 2^M/2 \) keeping \( N \) constant, or the number of mappers is halved, \( N/2 \), keeping the mappers with \( 2^M \) symbols. The latter way, \( N/2 \), is implemented in the SIM–OFDM to reduce the transmitted power by a half [10]. This implies that the \( B_{OOK} \) should contain at least \( N/2 \) 1’s to switch ON the subcarriers. Hence the condition in (1) is checked at every modulation instance

\[
N_{maj} = \max \left\{ N_{book\,max} \cdot \left( N_{FFT} - N_{book\,max} \right) \right\} \quad (1)
\]

where \( N_{book\,max} \) is the number of 1’s and \( \left( N_{FFT} - N_{book\,max} \right) \) is the number of 0’s. The majority \( N_{maj} \) in (1) will count the number of 1’s in \( B_{OOK} \). If the number of 1’s is less than the number of 0’s then the \( B_{OOK} \) bits are inverted. Otherwise, the \( B_{OOK} \) is passed without any inversion to the OOK stage. This condition ensures that the system will switch ON the required number of subcarriers to satisfy the number of mappers. The SIM–OFDM system is shown in Fig. 2(a) and the \( N_{maj} \) flowchart is in Fig. 2(b).

The authors in [10] showed that the system can take advantage of the power difference to operate in two modes that are Power Re-allocation Policy (PRP) and Power Saving Policy (PSP). In PRP, the average symbol’s power can be doubled to increase the SNR. Using PRP SIM–OFDM, the transmitted power is equivalent to the conventional OFDM as given by (2) [10]

\[
\gamma_{PRP} = 10 \log_{10} \left( \frac{P_S}{E[N_{maj}]} \right) - 10 \log_{10} \left( \sigma^2 \right) \quad (2)
\]

where \( P_S \) is the average symbol power, and \( \sigma^2 \) is the Additive White Gaussian Noise (AWGN) variance per subcarrier. The value of \( E[N_{maj}] \) depends on the expected occurrences of the 1’s during the transmission. Since \( E[N_{maj}] = N/2 \) which, according to (2), clarifies that \( P_S \) can be doubled without exceeding the conventional OFDM transmitted power.

In PSP, the symbol power is kept without alteration to ensure the transmission power is halved. The average power per carrier is given by (3):
The analytical solution of the portability of error (P_e) for the PRP and PSP is given by (4) and (5) respectively for 4-QAM constellation [10]

\[ P_{e}^{\text{PRP}} = \frac{1}{2} \left[ 1 - \sqrt{1 - \frac{\gamma_s}{1 + N_{0}^{\text{SC}}} } \right] \]

\[ + \frac{1}{2} \left[ \frac{1}{8} \left( \sqrt{\frac{\gamma_s}{2 + 2\gamma_s}} + \frac{1}{2} \sqrt{\frac{2 + 2\gamma_s}{\gamma_s}} \tan^{-1} \left( \sqrt{\frac{2 + 2\gamma_s}{2\gamma_s}} \right) \right) \right] \]

\[ P_{e}^{\text{PSP}} = \frac{1}{2} \left[ 1 - \sqrt{1 - \frac{0.5\gamma_s}{1 + N_{0}^{\text{SC}}} } \right] \]

\[ + \frac{1}{2} \left[ -\frac{1}{8} \left( \sqrt{\frac{\gamma_s}{2 + 2\gamma_s}} + \frac{1}{2} \sqrt{\frac{2 + 2\gamma_s}{\gamma_s}} \tan^{-1} \left( \sqrt{\frac{2 + 2\gamma_s}{2\gamma_s}} \right) \right) \right] \]

where: \( \gamma_s \cup N_0^{\text{SC}} \) is the SNR and \( \alpha, E_s \) and \( N_0^{\text{SC}} \) are the Rayleigh fading magnitude coefficient, the average energy per symbol, and the noise power per symbol respectively.

The authors in [11] showed that the BER analysis was incomplete and that the SIM–OFDM suffers also from burst BER due to error propagation. The error propagation comes from the false detection of one bit or multi bits that form the 8OOK. Hence, an attempt by the authors in [11] to Enhance the SIM–OFDM performance by reducing this burst BER was introduced. The new method is called as Enhanced SIM–OFDM (ESIM–OFDM). They suggested that the 8OOK bits should not be used directly, instead each “1” is followed by a 0 and each “0” is followed by a “1”. This will eliminate the need for matched filters to reduce the burst error and prevent the confusion in the 8OOK bits detection. Figure 3 shows the suggested version of 8OOK bits in the ESIM–OFDM [11]

\[ \gamma_{\text{PSP}} = 10\log_{10} \left( \frac{P_e}{N} \right) - 10\log_{10} \left( \sigma^2 \right), \text{ dB} \] (3)

The new BER for ESIM–OFDM is shown in (6) [11]:

\[ \text{BER}_{\text{ESIM-OFDM}} = \text{BER}_{\text{book}} + \text{BER}_{\text{QAM}} \] (6)

Despite that ESIM–OFDM solved the problem of error propagation, its disadvantage is a slight reduction in the spectral efficiency as given in (7) [11]:

\[ \gamma_{\text{GENERAL}} = N_a \log_2 \left( \frac{M}{N} \right) + \frac{\log_2 \left( \frac{N!}{N_a !(N - N_a)!} \right)}{N} \] (7)

where \( N_a \) is the number of active carriers and \( N \) is the BOOK length.

III. BUILDING THE NEW SIM–OFDM ANALYTICAL MODEL

The following sections will describe the exact SIM–OFDM model. The issues covered by this model are the actual transmitted power reduction, the synchronization problem and the BER.

A. The BOOK Bits and the Exact –3 dB Power Reduction Model

According to [10 and 11], the transmitted power can be reduced to half (–3dB). However, this claim can only be ensured if the process is completely independent random variable. In probability theory, every discrete independent mutually exclusive random process will have a uniform probability distribution (equiprobable elements). Assume a random process \( X \) having \( K \) possible outcomes \( X = [x_1, x_2, \ldots, x_K] \) then the pmf of any element \( x_i \) will be as in (8):

\[ \text{pmf} (x_i) = 1/K \] (8)

In SIM–OFDM, each \( B_{\text{BOOK}} \) bit can be either “0” or “1” with equal probability (i.e., pmf (0) = pmf (1) = 1/2). Thus, for a long transmission period, each subcarrier will have a probability of 1/2 to be ON. This means that only half the subcarriers will be ON and the other half is OFF on average. This explains the –3 dB reduction in the transmitted power.

Careful investigation of Fig. 2 and (2) implies that the actual pmf of “0” and “1” in the \( B_{\text{BOOK}} \) bit pattern cannot be 1/2 because the process is altered by the \( N_{\text{maj}} \) condition. In other words, the binary values are now correlated and the they are no longer independent. The resultant “1” pmf depends on \( N \) which can be either even or odd resulting in a different 1’s distribution. For example, assuming \( N=4 \) then the possible states of the \( B_{\text{BOOK}} \) are equal to \( 2^2=16 \) as shown in Appendix A (Table A-I). The states in the table can be divided into two groups. The first group has the same number of 0’s and 1’s while the second group has different number of 0’s and 1’s. The first group will be left without any changes by the \( N_{\text{maj}} \) condition and has a recurrence of one. The second group with higher number of 0’s than 1’s will be inverted by the \( N_{\text{maj}} \) condition given by (1). These changed states will be identical to the unchanged states and that reduces the number of states of the second group to half. The inverted states are represented using red framed cell and they have twice recurrence. This conversion definitely increases the probability of “1” from 1/2 to 0.6875 for \( N=4 \) SIM–OFDM system.

When \( N \) is odd then, there will be no states with equal number of 1’s and 0’s. This means that half of the states will be inverted causing the number of total states to be reduced to
half. Table A-II in Appendix-A shows the possible states for \(N=5\) before and after applying the \(N_{maj}\) condition with a pmf (1) = 0.6875.

To put the example shown in the appendix A, Table A-I and A-II, in mathematical equations, assume that there is a \(B_{OOK}\) bit block containing \(N\) bits. The total number of the elements for \(N\) bits is \(N \cdot 2^{N}\). The probability of selecting the “1” equals to the ratio of the total number of 1’s to the total number of elements. The number of 1’s can be calculated using the combination theory multiplied by the number of 1’s at every state as given by (9)

\[
pmf_1(N) = \sum_{k=0}^{N} \frac{N!}{k!(N-k)!} \frac{2^k}{N^2} = \frac{1}{2}
\]

where \(k\), is the summation counter and also the number of 1’s in the current state. Equation (9) shows that for an independent mutually exclusive random process involving two outcomes process, like the bit, the pmf of both “1” and “0” is \(1/2\).

After applying the condition given by (1), the number of 1’s will be altered as shown by Tables A-I and A-II in the appendix. To find the new 1’s pmf of the \(B_{OOK}\) bits, \(N\) should be treated as either even valued or odd valued

For \(N\) even there are states where the number of 1’s equals the number of 0’s; hence, the condition will not be applied. While for \(N\) is odd, there are no states with equal number of 0’s and 1’s. This fact affects the repetition of the state after the condition. Starting with odd valued \(N\) then, the \(B_{OOK}\) pattern, after the \(N_{maj}\) condition in (1), will have 1’s from \((N+1)/2\) to \(N\). The 1’s ratio or the pmf (1) is shown in (10)

\[
\text{pmf}_{odd}(N) = \frac{2 \cdot \sum_{k=1}^{N} \frac{N!}{k!(N-k)!}}{2^N} = \frac{2 \cdot \text{hypergeom}\left(1, \frac{N+1}{2}, \frac{N}{2}, -1\right)}{2^N\left(N\right)}
\]

Equation 10 shows that the 1’s pmf depends on the value of \(N\) which reflects the effect of the condition unlike (9) which shows that the 1’s pmf is a constant. The pmf given in (10) is a complicated function which can be plotted with respect to \(N\) as shown in Fig. 4. Figure 4 shows that the 1’s pmf is always higher than 1/2 and the value of 1/2 is an asymptote which can be reached only when the value of \(N \rightarrow \infty\). For example, given \(N = 3, 5, 7\) and 15 then 1’s pmf are 0.75, 11/16 = 0.6875, 21/32 = 0.65625 and 2477/4096 = 0.60474 respectively.

Equation 10 can be converted to a power level in dB as in (11). This will give the real transmitted power reduction using SIM–OFDM technique

\[
\text{pmf}_{odd}(N) = 10\log_{10}\left(pmff_{odd}(N)\right)\quad (11)
\]

The plot of (11) is shown in Fig. 5 also shows that the \(-3\) dB reduction is reach at \(N \rightarrow \infty\). The power above \(-3\) dB is a wasted power.
Equation 12 shows clearly that the 1’s pmf is higher than 1/2 and converges to 1/2 when $N \to \infty$. As a numerical example, the pmf values for $N$ equals 2, 8, 16 and 20 are $3/4 = 0.75$, $163/256 = 0.6367$, $39203/65536 = 0.5982$ and $308333/524288 = 0.5881$ respectively. The even valued $N$ pmf for the 1’s is plotted in Fig. 6.

Figure 6 also shows that the pmf = 1/2 can only be reached when $N \to \infty$. The amount of actual transmitted power reduction in dB is given by (13)

$$pmf_{\text{even}}(1) = 10 \log \left( pmf_{\text{even}}(1) \right)$$

(13)

Plotting (13) will illustrate the actual transmitted power reduction as in Fig. 7

Figure 8 shows the pmf of the 1’s for even and odd valued $N$. Figure 8 shows that there is a slight difference between the even and odd valued $N$ at small values of $N$ but they almost coincide at large $N$.

The above analysis shows that the transmitted power cannot be reduced to half nor guaranteeing half of the carriers are ON. In consequence, the algorithm proposed for the ESIM–OFDM [11] will fail too as it requires half of the carriers at maximum to be ON to operate correctly.

### B. The Synchronization Between the Transmitter and the Receiver

The synchronization between the transmitter and the receiver is a pivotal concept that ensures correct data delivery. The synchronization is a wide term but this research will only be concerned with how the receiver knows that the condition in (1) is applied or not. When the transmitter finds the number of 1’s is less than the 0’s then the $B_{\text{OOK}}$ bits will be passed through NOT gates. When this process is executed, the receiver should be informed to reverse the process to retrieve the original data. Otherwise, the data will be lost and the BER will increase immensely.

In [10] and [11] there was no definite technique to inform the receiver that the NOT is applied or not. In [10] followed by [11] and other researchers [12–19], the suggested technique is to use the free carriers with no modulating data, or in other words the free carriers, to notify the receiver whether the condition is used or not. However, all the suggested techniques are based on the fact that the pmf is 1/2 for the 1’s. The previous section showed that this fact is no longer applicable when the $B_{\text{OOK}}$ is passed through (1). Even so, all the researches did not show how to embed the notification when all the $B_{\text{OOK}}$ bits are 1’s. Not to mention that any carrier used for this purpose will create a bit pattern that is part of the $2^N$ states. Hence, the receiver will not differentiate between a notification and information. Nevertheless, the notification carrier should be modulated with a special value to avoid the confusion with the data. Choosing this special value for highly dense constellations is not an easy task. A practical way for notifying the receiver that the transmitter activated the condition or not is to add an extra carrier for this purpose. This
A. N. JABBAR et al.: A NEW ANALYTICAL MODEL FOR SIM-OFDM

arrangement will require the bandwidth \((B)\) to be increased by an amount shown in (14) and illustrated by Fig. 9

\[
B = (N + 1) \cdot B_c \quad \text{Hz} \tag{14}
\]

Fig. 9. The SIM–OFDM spectrum with added flag signal.

where: \(B_c\) is the carrier bandwidth and \(N\) is the number of carriers. The expansion ratio \((E)\) in \(B\) will converge to “1” with the increase of \(N\) as seen in (15)

\[
E = \frac{(N + 1)B_c}{N \cdot B_c} = \frac{N + 1}{N} \quad \lim_{N \to \infty} \left( E \right) = \lim_{N \to \infty} \left( \frac{N + 1}{N} \right) = 1
\tag{15}

The author of [20] showed that the SIM–OFDM and its derivatives will have a very low throughput that is \(\leq 2\) bit/sec/Hz. With the added flag carrier, the throughput will be reduced further more unless the number of carriers is large enough to maintain the 2 bit/sec/Hz.

C. The BER Model

The BER is an important measurement criterion that is used to evaluate the communication system performance. Equations 4, 5 and 6 in [10 and 11] shows the BER over Rayleigh fading AWGN channel for the SIM–OFDM and ESIM–OFDM respectively. These equations show that the BER is the sum of two independent processes which are the modulation technique, if QAM, and the \(B_{OOK}\) BER which is equivalent to ASK system. However, these equations are based on the assumption that the probability of “1” equals to 1/2.

Depending on the analysis in section III.A that showed the pmf \((1)\) is dynamic depends on the value of \(N\); then, the BER needs more review and reformation. Following the approach in [10 and 11] and including the synchronization problem solution suggested in section III.B, it can be seen that the BER comes from three sources instead of two which are QAM, \(B_{OOK}\) and the added flag carrier.

The false detection of the added flag carrier \(C_f\), shown in Fig. 9, can be calculated as in (16) assuming the flag can take either “0” or “1” states

\[
\text{FER} = p \left( C_f^1 \right) p \left( C_f^0 \right) = \frac{1}{2} \quad \lim_{N \to \infty} \left( \text{FER} \right) = \frac{1}{2}
\tag{16}
\]

where \(F_{ER}\) is the Frame Error Rate, the superscript \(T\) refers to transmitted, \(D\) to detected and \(p(\cdot)\) is the probability. The flag, as shown before, is directly related to the number of the \(B_{OOK}\) bits \(N\) which can be either even or odd. Starting with the odd \(N\) then the probability of activating the flag or not is equiprobable because half of the states are inverted and the others are kept without inversion. Therefore, the probability of the flag to be “1” or “0” equals to 1/2. This means that (16) is equivalent to the probability of error in Binary Phase Shift Keying (BPSK) as given in (17) for \(F_{ER}\)

\[
F_{ER} = Q \left( \frac{E_b}{N_0} \right) \tag{17}
\]

where \(E_b\) is the flag bit energy, \(N_0\) is the AWGN power spectral density and \((\cdot)\) is the tail distribution function of the standard normal distribution.

Recalling the SIM–OFDM system as described by [10 and 11] where an average \(N/2\) of the subcarriers are active with a symbol’s length \(M\) bits then, the average of error bits \(F_b\) in a transmitted instance due to \(F_{ER}\) is given by (18)

\[
F_b = M \cdot N \cdot \text{pmf} \left( 1 \right) \cdot \text{FER} = \frac{M \cdot N}{2} \cdot \text{FER} \tag{18}
\]

To find the actual number of bits that contribute in \(F_b\), the pmf \((1)\) is substituted by the actual 1’s probability given by (10) and (12) in (18). Taking \(N \to \infty\) then pmf \((1)\) for both even and odd \(N\) will coincide on each other, see Fig. 8, and reaching 1/2. Therefore, (18) can be written as in (19) even when (10) and (12) are substituted in (18)

\[
\lim_{N \to \infty} \left( F_b^{\text{even, odd}} \right) = \frac{M \cdot N}{2} \cdot \text{FER} \tag{19}
\]

Equation (19) shows that the FER effect can dominate the QAM and the \(B_{OOK}\) BER. This is due to the fact that \(F_b\) controls the complete frame bits unlike QAM that controls only \(M\) bits and the \(B_{OOK}\) controls only \(N\) bits. Consequently, FER should be treated with caution during the design. It is worth to mention that when FER occurs, the error in \(B_{OOK}\) and QAM will be meaningless as they will be included in the erroneous frame.

IV. CONCLUDING REMARKS

The SIM–OFDM is an elegant technique to reduce the transmitted power by incorporating the data to switch ON the carrier if “1” or OFF if the data is “0”. This technique promised a \(-3\) dB transmitted power reduction based on the probability of “1” which is 1/2. However, by carefully examining the SIM–OFDM and applying the probability theories, it has been shown that the real situation differs significantly from what was promoted. This new model gives the actual description for the SIM–OFDM behaviour and performance. The analytical model showed that the 1’s pmf depends on the length of IFFT \(N\). Also, for all values of \(N\), the reduction in the transmitted power is above \(-3\) dB with
appreciable power waste especially for low $N$. This waste reduces the bandwidth utilization and the system’s efficiency. Another observed issue is the need for a flag carrier to signal the receiver when to apply the bits inversion to the $B_{ook}$. This increases the needed bandwidth and further squanders the transmitted power without useful modulated data. This problem can be reduced and the system’s efficiency can be increased by increasing $N$ to approach its limit when $N \to \infty$. Although this might be a practical solution but increasing $N$ will increase the BER due to the increment in frame’s bits size. The increment in system’s BER comes from the misinterpretation of the flag. This can have a catastrophic effect because the flag controls a complete frame rather than $N$ bits by the $B_{ook}$ or $M$ bits by the QAM. Hence, any error in detecting the flag will generate a burst error that overwhelms the other error sources. Therefore, careful management should be followed to balance the system’s efficiency and the BER by choosing the appropriate IFFT length $N$. This balance can be achieved by applying the optimization algorithms for optimum performance to the analytical model presented in this research. The other point that should be referred to is that, the SIM–OFDM can work with best results in a high Signal to Noise Ratio (SNR) and not in hostile environments. This high SNR ensures the minimum error in detecting both the carrier and the $B_{ook}$ leading to a stable communication system. Another solution is to use a powerful encoding scheme to minimize the FER.

APPENDIX A

A numerical example illustrating the effect of $N$ on the binary states before and after applying the $N_{maj}$ condition.

### Table A-I

| Value | $B_{ook}$ bit pattern before $N_{maj}$ | $B_{ook}$ pattern after $N_{maj}$ |
|-------|--------------------------------------|----------------------------------|
| 0     | 0 0 0 0                               | 1 1 1 1                          |
| 1     | 0 0 0 1                               | 1 1 1 0                          |
| 2     | 0 0 1 0                               | 1 1 0 1                          |
| 3     | 0 0 1 1                               | 0 1 1 1                          |
| 4     | 0 1 0 0                               | 1 0 1 1                          |
| 5     | 0 1 0 1                               | 0 1 0 1                          |
| 6     | 0 1 1 0                               | 0 1 1 0                          |
| 7     | 1 0 0 0                               | 1 1 1 1                          |
| 8     | 1 0 0 1                               | 1 1 0 1                          |
| 9     | 1 0 1 0                               | 1 0 1 0                          |
| 10    | 1 0 1 1                               | 1 0 1 1                          |
| 11    | 1 1 0 0                               | 1 1 1 0                          |
| 12    | 1 1 0 1                               | 1 1 0 1                          |
| 13    | 1 1 1 0                               | 1 1 0 0                          |
| 14    | 1 1 1 1                               | 1 1 1 1                          |

pmf (1) 1/2 0.6875

### Table A-II

| Value | $B_{ook}$ bit pattern before $N_{maj}$ | $B_{ook}$ pattern after $N_{maj}$ | pmf (1) |
|-------|--------------------------------------|----------------------------------|---------|
| 0     | 0 0 0 0                               | 1 1 1 1                          |         |
| 1     | 0 0 0 1                               | 1 1 1 0                          |         |
| 2     | 0 0 1 0                               | 1 1 0 1                          |         |
| 3     | 0 0 1 1                               | 0 1 1 1                          |         |
| 4     | 0 1 0 0                               | 1 0 1 1                          |         |
| 5     | 0 1 0 1                               | 0 1 0 1                          |         |
| 6     | 0 1 1 0                               | 0 1 1 0                          |         |
| 7     | 1 0 0 0                               | 1 1 1 1                          |         |
| 8     | 1 0 0 1                               | 1 1 0 1                          |         |
| 9     | 1 0 1 0                               | 1 0 1 0                          |         |
| 10    | 1 0 1 1                               | 1 0 1 1                          |         |
| 11    | 1 1 0 0                               | 1 1 1 0                          |         |
| 12    | 1 1 0 1                               | 1 1 0 1                          |         |
| 13    | 1 1 1 0                               | 1 1 0 0                          |         |
| 14    | 1 1 1 1                               | 1 1 1 1                          |         |

pmf (1) 1/2 0.6875

### References

1. K. D Rao, *Channel Coding Techniques for Wireless Communications*. India: Springer, 2015.
2. R. Prasad, *OFDM for Wireless Communications Systems*, London: Artech House Inc., U.K., 2004.
3. M. Wen, C. Niang and L. Yang, *Index Modulation for 5G Wireless Communications*, Springer, 2017.
4. A. Zaidi et. al., *5G Physical Layer Principles, Models and Technology Components*, Academic Press, U.K., 2018.
5. S. Z Asif, *5G Mobile Communications Concepts and Technologies*, CRC Press, U.S.A., 2019.
6. A. M. Langer, *Analysis and Design of Next-Generation Software Architectures 5G, IoT, Blockchain, and Quantum Computing*, Springer Nature Switzerland, 2020.
7. S. Khan, and J. L. Mauri, *Green Networking and Communications ICT for Sustainability*, CRC Press, U.S.A., 2014.
8. Edited by H. A. Saraweeera, J. Yang, A. Zappone and J. S. Thompson, *Green Communications for Energy-Efficient Wireless Systems and Networks*, The Institution of Engineering and Technology, U.K. 2021.
9. G. Kaddoum and E. Soujeri, “On the Comparison Between Code-Index Modulation and Spatial Modulation Techniques”, *International Conference on Information and Communication Technology Research (ICTRC2015)*, 2015.
[10] R. Abu-Alhiga and H. Haas, “Subcarrier-Index Modulation OFDM”, IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications, Tokyo, Japan, 2009.

[11] D. Tsonev, S. Sinanovic and H. Haas, “Enhanced Subcarrier Index Modulation (SIM) OFDM”, IEEE GLOBECOM Workshops (GC Wkshps), Houston, TX, USA, 2011.

[12] E. Basar, U. Aygolu, and E. Panayirci, “Orthogonal Frequency Division Multiplexing with Index Modulation in the Presence of High Mobility”, First International Black Sea Conference on Communications and Networking (BlackSeaCom), 2013.

[13] Y. Xiao et. al., “OFDM With Interleaved Subcarrier-Index Modulation”, IEEE Communications Letters, Vol. 18, No. 8, 2014.

[14] E. Basar, “OFDM with Index Modulation Using Coordinate Interleaving”, IEEE Wireless Communications Letters, Vol. 4, Issue 4, 2015.

[15] T. Van Luong and Y. Ko., “A Closed-Form Symbol Error Probability for MCIC-OFDM with Frequency Diversity”, IEEE 18th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2017.

[16] S. Dogan, A. Tusha and H. Arslan, “OFDM with Index Modulation for Asynchronous mMTC Networks”, Sensors, Vol. 18, No. 4, 2018.

[17] Y. Shi, “Genetic Algorithm aided OFDM with All Index Modulation”, IEEE Communications Letters, Vol. 23, Issue 12, 2019.

[18] M. Zhou et. al., “A Novel DNN Based Channel Estimator for Underwater Acoustic Communications with IMOFDM”, IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC), 2020.

[19] Z. Qasem, H. A. Leftah, H. Sun, J. Qi and H. Esmaiel, “X-Transform Time-Domain Synchronous IM-OFDM-SS for Underwater Acoustic Communication”, IEEE Systems Journal (Early Access), 2021.

[20] N. Ishikawa, S. Sugura, and L. Hanzo, “Subcarrier-Index Modulation Aided OFDM – Will It Work?”, IEEE Access, Vol. 4, 2016.

Ahmed N. Jabbar received the B. Sc. in Electronic and Communication Eng. in 1994 and M. Sc. In 1997 from Nahrain University, Baghdad, Iraq. He received the title of lecturer in 2009, associate professor in 2012 and professor title in 2018 from the University of Babylon. Currently, he is a professor and Ph. D. student in University of Babylon, College of Eng., Electronic and Communication Eng. Babylon city, Iraq since 2019. The field of interest is High Speed Communication Systems, Microwave System Design and Analysis, Data Mining, Parallel Hardware System Design using FPGA and DSP for AI in Communication Systems.

Samir J. Almuraab was born in Hilla, Babylon, Iraq, in 9 Oct. 1959. He received the B.Sc. degree in Electrical Engineering/Electronics and Communications in 1981 from University of Sulaymaniyah, and the M.Sc. degree in Electrical Engineering/Electronics and Communications in 1986 from University of Baghdad and Ph.D. in communication engineering at the Department of technical Education- Electrical Engineering, University of Technology. Currently He works as a professor and Head of the Scientific Composition at the Electrical Department at the Faculty of Engineering, University of Babylon. His main interest is Wireless Communication, Spread spectrum systems, Digital video broadcasting (T.S.&C), Coding, Wireless Sensor Network Applications, Bioinformatics, Signal Processing, Healthcare System.

Abdulkareem Abdulrahman Kadhim was born in Baghdad, Iraq, in 1958. He received his B.Sc. degree in Electrical and Electronics Engineering in 1981 from MEC, Iraq, and M.Sc. and Ph.D. degrees from Loughborough University of Technology, UK, in 1984 and 1989, respectively, in Digital Communication Systems. He is an IEEE Senior Member and Member of ACM. Currently, he is a professor of Digital Communications in the College of Information Engineering, Al-Nahrain University, Iraq. He has published 66 papers in international and national journals and scientific conferences. He successfully supervised 11 Ph.D. dissertations and 62 M.Sc. theses. His research interests include Modern Error Correction Codes for Next Generation Networks, Detection of Coded and Modulated Signals, Low Complexity Decoders, Millimetre Wave Channel Modelling, Network Coding, Software Defined Environments, and Efficient Routing for Wireless Sensor Networks (WSNs) and IoT Networks.