Polarized forward-backward asymmetries of lepton pair in $B \to K_1 \ell^+ \ell^-$ decay in the presence of New physics

Faisal Munir$^*$, Saadi Ishtiaq$^{1,3}$, Ishtiaq Ahmed$^{12,4}$

$^1$Center for Future High Energy Physics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
$^2$National Centre for Physics, Quaid-i-Azam University Campus, Islamabad, 45320 Pakistan
$^3$Department of Physics, University of Gujrat, Hafiz Hayat Campus, Gujrat, Pakistan
$^4$Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, Brazil

(Dated: November 24, 2015)

Double polarized forward-backward asymmetries in $B \to K_1(1270,1400)\ell^+ \ell^-$ with $\ell = \mu, \tau$ decays are studied, using most general non-standard local four-fermi interactions, where the mass eigenstates $K_1(1270)$ and $K_1(1400)$ are the mixture of $^1P_1$ and $^3P_1$ states with the mixing angle $\theta_K$. We have calculated the expressions of nine doubly polarized forward-backward asymmetries and it is presented that the polarized lepton pair forward-backward asymmetries are greatly influenced by the new physics. Therefore, these asymmetries are interesting tool to explore the status of new physics in near future, specially at LHC.

I. INTRODUCTION

Rare $B$ decays mediated through the flavor changing neutral current (FCNC) $b \to s (d) \ell^+ \ell^-$ transitions not only provide a testing ground for the gauge structure of standard model (SM) but are also an effective way to look for the physics beyond the SM. As we know that in SM the Wilson coefficients $C_7, C_9$ and $C_{10}$ of the operators $O_7, O_9$ and $O_{10}$ at $\mu = m_b$ are used to describe the $b \to s \ell^+ \ell^-$ transition. Therefore, in these transitions, NP effects can be incorporated in two different ways: one is through new contributions to Wilson coefficients and the other is via the introduction of new operators in effective Hamiltonian which are absent in the SM.

Though the decay distribution of inclusive decays such as $B \to X_s d \ell^+ \ell^-$ is theoretically better understood but hard to be measured experimentally. In opposite, the exclusive decays such as $B \to (K, K^*, K_1, \rho)\ell^+ \ell^-$ are easy to detect experimentally but are tough to calculate theoretically as the difficulty lies in describing the hadronic structure, which are the main source of uncertainties in the predictions of exclusive rare decays.

The exploration of physics beyond the SM through various inclusive $B$ meson decays such as $B \to X_s d \ell^+ \ell^-$ and their corresponding exclusive processes, $B \to M \ell^+ \ell^-$ with $M = K, K^*, K_1, \rho$ etc., have already been studied [1–5]. These studies showed that the above mentioned inclusive and exclusive decays of $B$ meson are very sensitive to the flavor structure of the SM and provide an effective way to explore NP effects.

Regarding this precise measurements of different experimental observables for $b \to s \ell^+ \ell^-$ decay such as branching ratio, forward-backward asymmetry, various polarization asymmetries of the final state leptons, etc could be useful in establishing the status of new physics (NP) in near future, specially at LHC. For this reason, many exclusive $B$ meson processes based on $b \to s (d) \ell^+ \ell^-$ such as $B \to K (K^*) \ell^+ \ell^-$ [6–12], $B \to \rho \ell^+ \ell^-$ [13], $B \to \gamma \ell^+ \ell^-$[14–16] and $B \to \ell^\pm \ell^-$ [17] have already been studied.

It has been mentioned in [18] that measurement of many additional observables, would be possible by studying the simultaneous polarizations of both leptons in the final state, which in turn would be useful in testing the SM and highlighting new physics beyond the SM. It should be mentioned here that double lepton polarization asymmetries in $B \to K^* \tau^+ \tau^-$ [19], $B \to K \ell^+ \ell^-$ [20], $B \to \rho \ell^+ \ell^-$ [21] and $B \to K_1 \ell^+ \ell^-$ [22, 23] have already been studied. Along with other observables, forward backward asymmetry is also an efficient observable to explore NP beyond the SM. In this regard, double lepton polarization forward-backward asymmetries in $B \to K^* \ell^+ \ell^-$ [24, 25], $B \to K \ell^+ \ell^-$ [26], $B \to \rho \ell^+ \ell^-$ [27] and in $B_s \to \gamma \ell^+ \ell^-$ [28] have already been explored. We would like to emphasise here that the situation which makes $B \to K_1 \ell^+ \ell^-$ decay more interesting than $B \to K^* \ell^+ \ell^-$ is the mixing of axial vector states $K_{1A}$ and $K_{1B}$ which are the $^3P_1$ and $^1P_1$ states respectively. Therefore, it is also interesting to see that how polarized forward-backward asymmetries of $B \to K_1 \ell^+ \ell^-$ are influenced in the presence of new physics. So in the present work polarized forward-backward asymmetry in the exclusive decay $B \to K_1 \ell^+ \ell^-$ are addressed using most general...
effective Hamiltonian, including all forms of possible interactions, similar to the case of \( B \to K^* \ell^+ \ell^- \) decay. The physical states \( K_1(1270) \) and \( K_1(1400) \) are superposition of the P-wave states in the following way

\[
|K_1(1270)\rangle = |K_{1A}\rangle \sin \theta_K + |K_{1B}\rangle \cos \theta_K
\]

\[
|K_1(1400)\rangle = |K_{1A}\rangle \cos \theta_K - |K_{1B}\rangle \sin \theta_K
\]

If we define, \( y = \sin \theta_K \) then above Eqs. become

\[
|K_1(1270)\rangle = |K_{1A}\rangle y + |K_{1B}\rangle \sqrt{1 - y^2}
\]

\[
|K_1(1400)\rangle = |K_{1A}\rangle \sqrt{1 - y^2} - |K_{1B}\rangle y
\]

where the magnitude of the mixing angle \( \theta_K \) has been estimated \([29]\) to be \( 34^\circ \leq |\theta_K| \leq 58^\circ \) and the study of \( B \to K_1(1270)\gamma \) impose the limit \([30]\) on the mixing angle as

\[
\theta_K = -(34 \pm 13)^\circ
\]

where minus sign of \( \theta_K \) is related to the chosen phase of \( K_{1A} \) and \( K_{1B} \) \([30]\).

The manuscript is presented as follows. In sec. II, we devise our required theoretical framework which is followed by two Subsections, \( \text{II A and II B} \) relating to mixing of \( K_1(1270) \) and \( K_1(1400) \), form factors and constraints on the coefficients of NP operators used in this study. Sec. III, is devoted to analytical calculations and the explicit expressions of doubly polarized forward-backward asymmetries. In Sec. IV, we give the numerical analysis with discussion about the observables under considerations. We end our work by giving concluding remarks in Sec. V.

II. THEORETICAL FORMALISM

At the quark level \( B \to K_1(1270, 1400)\ell^+ \ell^- \) decays are induced by the transition \( b \to s \ell^+ \ell^- \), which in the SM, is described by the following effective Hamiltonian \([31]\)

\[
H_{\text{eff}}^{SM}(b \to s \ell^+ \ell^-) = -\frac{G_F}{{\sqrt{2} \pi}} V_{tb} V_{ts}^* \left\{ C_9^{\text{effSM}}(\bar{s} \gamma_\mu L b)(\bar{\ell} \gamma^\mu \ell) + C_{10}(\bar{s} \gamma_\mu L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell) - 2m_b C_7^{\text{effSM}}(\bar{s} i \sigma_{\mu \nu} q'' \bar{R} b)(\bar{\ell} \gamma^\mu \ell) \right\}
\]

where \( R, L = (1 \pm \gamma_5)/2 \) are the projector operators and \( q^2 \) is the square of momentum transfer while \( C's \) are Wilson coefficients. The effective Wilson coefficient \( C_9^{\text{effSM}}(\mu) \), can be decomposed into the following three parts \([3, 5]\)

\[
C_9^{\text{effSM}}(\mu) = C_9(\mu) + Y_{SD}(z, \bar{s}) + Y_{LD}(z, \bar{s}),
\]

where the parameters \( z \) and \( \bar{s} \) are defined as \( z = m_c/m_b, \bar{s} = q^2/m_b^2 \). It is important to mention here that in our numerical calculations of asymmetries and their average values, we do not include \( Y_{LD}(z, \bar{s}) \), otherwise the asymmetries would be largely effected by the contributions of \( J/\psi \) and \( \psi(2s) \) resonance around \( s = 10 GeV^2 \) and \( s = 14 GeV^2 \) respectively. The explicit expressions for short-distance contributions \( Y_{SD}(z, \bar{s}) \) and long distance contributions \( Y_{LD}(z, \bar{s}) \) are given in \([4, 6]\).

New physics effects are explored for \( B \to K_1\ell^+\ell^- \) channel by considering the most general local four-fermi interactions. In this regard the total effective Hamiltonian is given by

\[
H_{\text{eff}} = H_{\text{eff}}^{SM} + H_{\text{eff}}^{\text{VA}} + H_{\text{eff}}^{\text{SP}} + H_{\text{eff}}^{T}
\]
where

\[ \mathcal{H}_{eff}^{VA} = \frac{G_F \alpha}{\sqrt{2\pi}} V^*_t V_b \left\{ C_{LL} \bar{s} L \gamma^\mu b_L \gamma^\mu l_L \\
+ C_{LR} \bar{s} R \gamma^\mu b_L l_R \gamma^\mu l_L \\
+ C_{RR} \bar{s} R \gamma^\mu b_R \gamma^\mu l_R \right\} \]

\[ \mathcal{H}_{eff}^{SP} = \frac{G_F \alpha}{\sqrt{2\pi}} V^*_t V_b \left\{ C_{LRLL} \bar{s} L l_R b_R l_L b_L l_L r_R + C_{RLRR} \bar{s} R b_R \gamma^\mu l_R \right\} \]

\[ \mathcal{H}_{eff}^{T} = \frac{G_F \alpha}{\sqrt{2\pi}} V^*_t V_b \left\{ C_T \bar{s} \sigma_{\mu \nu} b \sigma_{\mu \nu} l \\
+ i C_T \epsilon_{\mu \nu \alpha \beta} \bar{l} \sigma_{\mu \nu} \gamma_{\alpha \beta} b \right\} \]

(6)

while \( \mathcal{H}_{eff}^{SM} \) is given in Eq. (4) and \( C_X \) are the coefficients of the four-Fermi interactions. Defining the combinations

\[ R_V = \frac{1}{2}(C_{LL} + C_{LR}), \quad R_A = \frac{1}{2}(C_{LR} + C_{LL}) \]
\[ R'_V = \frac{1}{2}(C_{RR} + C_{RL}), \quad R'_A = \frac{1}{2}(C_{RR} - C_{RL}) \]
\[ R_S = \frac{1}{2}(C_{LRLL} + C_{RLRR}), \quad R_P = \frac{1}{2}(C_{LRLL} - C_{RLRR}) \]
\[ R'_S = \frac{1}{2}(C_{RLRL} + C_{RLRR}), \quad R'_P = \frac{1}{2}(C_{RLRL} - C_{RLRR}) \]

where \( R_A, R_V, R'_A, R'_V, R_S, R_P, R'_S, R'_P, C_T \) and \( C_{TE} \) represents the NP couplings. Using the expression of the effective Hamiltonian Eq. (5) the decay amplitude for \( B \to K_1 l^+ l^- \) is given by

\[ \mathcal{M}(B \to K_1 l^+ l^-) = \frac{\alpha G_F}{2\sqrt{2\pi}} V^*_t V_b \]
\[ \times \left\{ \langle K_1(p_K, e) | \bar{s} \gamma^\mu (1 - \gamma_5) b | B(p_B) \rangle \left\{ (C_9^{eff} + R_V) \bar{l} \gamma^\mu l \right\} \right. \]
\[ + \left. (C_{10} + R_A) \bar{l} \gamma^\mu l \right\} + \langle K_1(p_K, e) | \bar{s} \gamma^\mu (1 + \gamma_5) b | B(p_B) \rangle \left\{ (C_9^{eff} + R_V) \bar{l} \gamma^\mu l \right\} \]

\[ \times \left\{ R'_V \bar{l} \gamma^\mu l + R'_A \bar{l} \gamma^\mu l \right\} \]
\[ - 2 \frac{C_7^{eff}}{s} m_b \langle K_1(p_K, e) | \bar{s} i \sigma_{\mu \nu} g^\nu (1 + \gamma_5) b | B(p_B) \rangle \bar{l} \gamma^\mu l \]
\[ + \langle K_1(p_K, e) | \bar{s} (1 + \gamma_5) b | B(p_B) \rangle \left\{ R_S \bar{l} l + R_P \bar{l} \gamma_5 l \right\} \]
\[ + \langle K_1(p_K, e) | \bar{s} \gamma^\mu (1 - \gamma_5) b | B(p_B) \rangle \left\{ R'_S \bar{l} l + R'_P \bar{l} \gamma_5 l \right\} \]
\[ + 2 C_T \langle K_1(p_K, e) | \bar{s} \sigma_{\mu \nu} b | B(p_B) \rangle \bar{l} \sigma_{\mu \nu} l \]
\[ + 2 i C_{TE} \epsilon_{\mu \nu \alpha \beta} \langle K_1(p_K, e) | \bar{s} \sigma_{\mu \nu} b | B(p_B) \rangle \bar{l} \sigma_{\alpha \beta} l \] \( \) (7)

Note: One can also consider the new physics contribution coming from the operator \( O_7^c = C_7^c \bar{s} \sigma_{\mu \nu} b LF_{\mu \nu} \). However, in the present study we do not include these effects.

A. Form Factors and Mixing of \( K_1(1270) - K_1(1400) \)

The hadronic matrix elements of quark operators appearing in Eq. (1) over the meson states, for the exclusive \( B \to K_1(1270, 1400) \ell^+ \ell^- \) decays can be parameterized in terms of the form factors as:
Additionally with defined in Eqs. (1-2). The \( B \) where the mass of strange quark has been neglected. K, where \( p_B(p_k) \) are the momenta of the \( B(K) \) mesons and \( \varepsilon_\mu \) correspond to the polarization of the final state axial vector \( K_1 \) meson. In Eq. (8) we have

\[
V_3(q^2) = \frac{m_B + m_{K_1}}{2m_{K_1}} V_1(q^2) - \frac{m_B - m_{K_1}}{2m_{K_1}} V_2(q^2)
\]

with

\[
V_5(0) = V_6(0)
\]

Additionally

\[
\langle K_1 | \bar{s}_i \gamma_\mu b | B(p_B) \rangle = \left[ \varepsilon_\mu^* (p_B + p_k)_\nu - \varepsilon_\nu^* (p_B + p_k)_\mu \right] F_1(q^2)
\]

\[
= \left( \frac{m_B^2 - m_{K_1}^2}{q^2} \right) F_2(q^2) \left( \varepsilon_\mu^* q_\nu - \varepsilon_\nu^* q_\mu \right)
\]

\[
+ \left( \frac{F_1(q^2)q_\mu - F_3(q^2)q_\mu}{m_B^2 - m_{K_1}^2} \right) \varepsilon^* \cdot q
\]

\[
\times \left[ (p_B + p_k)_\nu q_\mu - (p_B + p_k)_\mu q_\nu \right]
\]

(11)

(12a)

(12b)

with \( F_1(0) = 2F_2(0) \). Where Eqs. (12a, 12b) are obtained by contracting Eq. (11) with \( q^\mu \). Moreover, the matrix element \( \langle K_1(p_k, \varepsilon) | \bar{s}(1 \pm \gamma_5) b | B(p_B) \rangle \) can be calculated by contracting Eq. (8) with \( q^\mu \) and by making use of the equation of motions along with Eq. (10), we have

\[
\langle K_1(p_k, \varepsilon) | \bar{s}(1 \pm \gamma_5) b | B(p_B) \rangle = \frac{1}{m_b} \left\{ \pm m_{K_1} (\varepsilon^* \cdot q) V_6(q^2) \right\}
\]

(13)

where the mass of strange quark has been neglected.

As the physical states \( K_1(1270) \) and \( K_1(1400) \) are mixed states of the \( K_{1A} \) and \( K_{1B} \) with mixing angle \( \theta_K \) as defined in Eqs. (12). The \( B \to K_1 \) form factors can be parameterized as [22]
where the mixing matrix $M$ is

$$M = \begin{pmatrix} \sin \theta_K & \cos \theta_K \\ \cos \theta_K & -\sin \theta_K \end{pmatrix}. \quad (16)$$

So the form factors $A_{K_1}^{K_1}$, $V_{0,1,2}^{K_1}$ and $F_{0,1,2}^{K_1}$ satisfy the following relations

$$\begin{align*}
\begin{pmatrix} A_{K_1}^{K_1(1270)} \\ V_{2}^{K_1(1270)} \\ \frac{m_B + m_{K_1}(1270)}{m_B + m_{K_1}(1400)} \end{pmatrix} &= M \begin{pmatrix} A_{K_1A}^{K_1A} \\ m_B + m_{K_1A} \\ \frac{m_B + m_{K_1A}}{m_B + m_{K_1B}} \end{pmatrix},
(17)
\end{align*}$$

$$\begin{align*}
\begin{pmatrix} (m_B + m_{K_1(1270)}) V_{K_1(1270)} \\ V_{K_1(1400)} \\ \frac{m_B + m_{K_1(1270)}}{m_B + m_{K_1(1400)}} \end{pmatrix} &= M \begin{pmatrix} (m_B + m_{K_1A}) V_{K_1A}^{K_1A} \\ (m_B + m_{K_1B}) V_{K_1B}^{K_1B} \\ \frac{m_B + m_{K_1A}}{m_B + m_{K_1B}} \end{pmatrix},
(18)
\end{align*}$$

$$\begin{align*}
\begin{pmatrix} F_{K_1(1270)} \\ F_{K_1(1400)} \\ \frac{m_{K_1(1270)}}{m_{K_1(1400)}} \end{pmatrix} &= M \begin{pmatrix} F_{K_1A}^{K_1A} \\ F_{K_1B}^{K_1B} \\ m_{K_1A} \end{pmatrix},
(19)
\end{align*}$$

$$\begin{align*}
\begin{pmatrix} (m^2 - m_{K_1(1270)}^2) F_{2}^{K_1(1270)} \\ (m^2 + m_{K_1(1400)}^2) F_{2}^{K_1(1400)} \\ \frac{m_{K_1(1270)}}{m_{K_1(1400)}} \end{pmatrix} &= M \begin{pmatrix} (m^2 - m_{K_1A(1270)}^2) F_{2}^{K_1A} \\ (m^2 + m_{K_1B(1400)}^2) F_{2}^{K_1B} \\ \frac{m_{K_1A(1270)}}{m_{K_1B(1400)}} \end{pmatrix},
(20)
\end{align*}$$

where we have supposed that $p_{K_1(1270),K_1(1400)}^{\mu} \simeq p_{K_1A,K_1B}^{\mu}$. Using the above matrix elements, the decay amplitude for $B \to K_1 l^+ l^-$ can be written as

$$\begin{align*}
M(B \to K_1 l^+ l^-) &= \frac{\alpha G_F}{4 \sqrt{2} \pi} V_{tb} V_{ts}^{\ast} \left\{ (\bar{\ell} \gamma_\mu l) \\
\times \left\{ -2 \mathcal{A}_t \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu\alpha} p_{K_1}^{a} q^{\beta} - i B_1 \epsilon_\mu^{\ast} + i B_2 \epsilon^{\ast} \cdot q (p_B + p_{K_1})_\mu \right\} \\
+(\bar{\ell} \gamma_\mu \gamma_5 l) \times \left\{ -2 \mathcal{A}_t \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu\alpha} p_{K_1}^{a} q^{\beta} - i D_1 \epsilon^{\ast}_\mu \\
+i D_2 \epsilon^{\ast} \cdot q (p_B + p_{K_1})_\mu + i D_3 \epsilon^{\ast} \cdot q q_\mu \right\} \\
+i G_1 (\bar{l} l) \epsilon^{\ast} \cdot q + i G_2 (\bar{l} \gamma_5 l) \epsilon^{\ast} \cdot q \\
+4 i C T \epsilon_{\mu\nu\alpha\beta} (\bar{l} \sigma^{\mu
u}) l \left\{ \mathcal{G}_5 \varepsilon^{\alpha (p_B + p_{K_1})} \right. \\
- \varepsilon^{\ast (p_B + p_{K_1})} + \mathcal{G}_3 (\varepsilon^{\ast q} q^{\beta} - \varepsilon^{\ast q} q^{\alpha}) \\
+ \mathcal{G}_4 \varepsilon^{\ast} \cdot q [(p_B + p_{K_1})^\beta q^\alpha - (p_B + p_{K_1})^\alpha q^\beta] \left. \right\} \\
+4 C_T (\bar{l} \sigma_{\mu\nu} l) \left\{ \mathcal{G}_5 \varepsilon^{\mu (p_B + p_{K_1})} \\
- \varepsilon^{\ast \mu} (p_B + p_{K_1})_\mu \right. \\
+ \mathcal{G}_3 (\varepsilon^{\mu q} q^{\nu} - \varepsilon^{\ast q} q^{\nu}) \\
+ \mathcal{G}_4 \varepsilon^{\ast} \cdot q [(p_B + p_{K_1})^\nu q^\mu - (p_B + p_{K_1})^\mu q^\nu] \right\}
\right\} \right\}
(24)
\end{align*}$$

The auxiliary functions appearing in [24] can be written as follows:
where $q$ however these bounds are weakened when we include $R$ for self consistency these bounds are given below:

$A = 2(C_9^{eff} + R_V + R'_V) - \frac{A(q^2)}{m_B(1+k)} + \frac{4m_b C_7^{eff} F_1(q^2)}{q^2}$

$B_1 = 2(C_9^{eff} + R_V - R'_V) m_B(1 + \hat{k}) V_1(q^2)$

$+ 4m_b C_7^{eff} (1 - \hat{k}) \left( \frac{F_2(q^2)}{(q^2/m_B^2)} \right)$

$B_2 = 2(C_9^{eff} + R_V - R'_V) m_B(1 + \hat{k})$

$+ 4m_b C_7^{eff} \left( \frac{F_2(q^2) + (q^2/m_B^2)}{(1 - \hat{k}^2)} \right) F_3(q^2)$

$C_1 = 2(C_{10} + A + R'_A) A(q^2) \frac{m_B(1+k)}{m_B}$

$D_1 = 2(C_{10} + A - R'_A)m_B(1 + \hat{k}) V_1(q^2)$

$D_2 = 2(C_{10} + A - R'_A) V_2(q^2) \frac{m_B(1+k)}{m_B}$

$D_0 = \frac{4\hat{k}}{m_B} (C_{10} + A - R'_A) V_3(q^2) - V_0(q^2) \left( \frac{q^2/m_B^2}{} \right)$

$G_1 = -4(R_S - R'_S) \frac{k}{(m_b/m_B)} V_0(q^2)$

$G_2 = -4(R_P - R'_P) \frac{k}{(m_b/m_B)} V_0(q^2)$

$G_3 = \frac{m_B^2 (1 - \hat{k})^2}{q^2} F_2(q^2)$

$G_4 = \frac{F_2(q^2)}{q^2} + \frac{F_3(q^2)}{m_B^2 (1 - \hat{k})^2}$

$G_5 = F_1(q^2)$

where $q = (p_+ + p_-) = (p_B - p_{K_1})$ and $\hat{k} \equiv m_{K_1}/m_B$.

**B. Phenomenological bounds on NP couplings**

In the present paper, we use the constraints on the NP couplings parameters from A. Kumar *et al* [32]. However, for self consistency these bounds are given below:

In the absence of $R_{V, A}$ the bounds are

$$|R'_V|^2 + |R'_A|^2 \leq 16.8,$$  \hspace{1cm} (25)

however these bounds are weakened when we include $R_{V, A}$

$$|R'_V|^2 + |R'_A|^2 \leq 39.7,$$  \hspace{1cm} (26)

On the other hand the constraints on tensor coupling entirely come from $B(\bar{B} \rightarrow X_s \mu^+ \mu^-)$ which are

$$|C_T|^2 + 4|C_{TE}|^2 \leq 1.3,$$  \hspace{1cm} (27)

The limits on scalar and pseudo scalar couplings are extracted from $B(\bar{B}_s^0 \rightarrow \mu^+ \mu^-)$

$$|R_S - R'_S|^2 + |R_P - R'_P|^2 \leq 0.44,$$  \hspace{1cm} (28)

and from $B(\bar{B} \rightarrow X_s \mu^+ \mu^-)$ [33] [34] are

$$|R_S|^2 + |R_P|^2 < 45, \quad R'_s = R_S, \quad R'_p = R_P.$$  \hspace{1cm} (29)
III. ANALYTICAL CALCULATIONS OF DOUBLY POLARIZED FORWARD BACKWARD ASYMMETRIES

Now we have all the ingredients to calculate the physical observables. The double differential decay rate is given as

\[
\frac{d^2\Gamma(B \to K_1 l^+ l^-)}{d\cos\theta ds} = \frac{1}{2m_B} \rho \sqrt{\lambda} |\mathcal{M}|^2
\]

(30)

where \( \rho \equiv \sqrt{1 - \frac{4m_t^2}{s}} \) and \( \lambda \equiv m_B^2 + m_{K_1}^2 + s^2 - 2m_B^2 m_{K_1}^2 - 2m_B^2 s - 2m_{K_1}^2 s \). By using the expression of the decay amplitude given in Eq. \[24\] one can get the expression of the dilepton invariant mass spectrum as

\[
\frac{d\Gamma(B \to K_1 l^+ l^-)}{ds} = \frac{G_F^2 \alpha^2 m_B}{2^{14} \pi^5} |V_{tb}V_{ts}^*|^2 \rho \sqrt{\lambda} (8\pi)^3 \delta
\]

(31)

where

\[
\delta = 4(2m^2 + s)\left\{ \frac{8\lambda}{3} |A|^2 + \frac{12m_{K_1}^2 s + \lambda}{3m_{K_1}^2} |B_1|^2 + \frac{\lambda}{3m_{K_1}^2} \text{Re}(B_1 B_2^*) + \frac{\lambda^2}{3m_{K_1}^2} |B_2|^2 \right\}
\]

\[
+ \frac{32\lambda}{3}(s - 4m^2) |C_1|^2 + \left[ \frac{4\lambda(2m^2 + s)}{3m_{K_1}^2} + 16(s - 4m^2) \right] |D_1|^2 - \frac{8m^2 \lambda}{m_{K_1}^2} \text{Re}(D_1 D_2^*) - \frac{4\lambda}{3m_{K_1}^2} [(2m^2 + s)]
\]

\[
\times (m_B^2 - m_{K_1}^2) + s(s - 4m^2) |\mathcal{E}|^2 + 0s(2m_B^2 + 2m_{K_1}^2 - s) + \lambda(2m^2 + s) |D_2|^2
\]

\[
+ \frac{8m^2 \lambda}{m_{K_1}^2} (m_B^2 - m_{K_1}^2) \text{Re}(D_2 D_0^*) + \frac{8m^2 \lambda}{m_{K_1}^2} |D_0|^2 - 256m^2 \lambda \text{Re}(\mathcal{C}_E^* \mathcal{C}_E) + \frac{32m_{K_1}^2}{\lambda} \{ \mathcal{G}_4 \lambda t + \mathcal{G}_5 (\lambda t - \lambda) \}
\]

\[
+ 12m_{K_1}^2 (m_B^2 - m_{K_1}^2)) \} \text{Re}(B_1 C_1^*) + \frac{8m^2 \lambda}{m_{K_1}^2} (\mathcal{G}_5 j + \mathcal{G}_3 t + \mathcal{G}_4 \lambda \text{Re}(B_2 C_2^*)
\]

\[
- \frac{4m^2 \lambda}{m_{K_1}^2} \text{Re}(D_1 G_2^*) + \frac{4m^2 \lambda (m_B^2 - m_{K_1}^2)}{m_{K_1}^2} \text{Re}(D_2 G_2^*) + \frac{4m^2 \lambda}{m_{K_1}^2} \text{Re}(D_0 G_2^*) + \frac{(s - 4m^2) \lambda}{m_{K_1}^2} |G_1|^2
\]

\[
+ \frac{s \lambda}{m_{K_1}^2} |G_2|^2 - \frac{256}{3m_{K_1}^2} \left[ (12m_{K_1}^2 (8m^2 ((m_B^2 + m_{K_1}^2) - 4s) - 2\lambda - 2s (m_B^2 + m_{K_1}^2) + s^2)
\right.
\]

\[
+ \lambda (4m^2 - s)(s - 8m_{K_1}^2) |G_5|^2 + (4m^2 - s) s \{ 2\lambda j \mathcal{G}_4 \mathcal{G}_5 + \lambda^2 |G_4|^2 + (24m_{K_1}^2 (m_B^2 - m_{K_1}^2))
\]

\[
+ 13\lambda \mathcal{G}_6 \mathcal{G}_5 + 13\lambda \mathcal{G}_4 \mathcal{G}_5 + 12(\lambda + 8m_{K_1}^2) |G_3|^2 \} |C_T|^2 + \left[ \frac{64}{3m_{K_1}^2} [2 \mathcal{G}_4 \mathcal{G}_5 s j + 2 \mathcal{G}_3 \mathcal{G}_4 st
\]

\[
+s \lambda |G_4|^2 \} \mathcal{G}(8m^2 + s) + \mathcal{G}_3 \mathcal{G}_4 \mathcal{s} (3m^2 (2\lambda + 24m_{K_1}^2 (m_B^2 - m_{K_1}^2)) + 4s (12m_{K_1}^2 (m_B^2 - m_{K_1}^2) + \lambda))
\]

\[
+ |G_3|^2 4s (3m^2 (2\lambda + 8m_{K_1}^2) + s (12m_{K_1}^2 + \lambda)) + |G_5|^2 \} \mathcal{G}(3s (\lambda - 8m_{K_1}^2 (s - 2m_B^2 + m_{K_1}^2)))
\]

\[
+ (8m_{K_1}^2 - s) \lambda + 12m_{K_1}^2 (\lambda + m_B^2 - m_{K_1}^2)^2 + \lambda s (8m_{K_1}^2)) \} |C_T|^2
\]

where \( t \equiv -m_B^2 + m_{K_1}^2 + s \) and \( j \equiv -m_B^2 - 3m_{K_1}^2 + s \).

we first define the six orthogonal vectors belonging to the polarizations of \( l^- \) and \( l^+ \) which we denote here by \( S_i \) and \( W_i \) respectively where \( i = L, N \) and \( T \) corresponding to the longitudinally, Normally and transversally polarized lepton \( l^\pm \) respectively. \[18 \[35 \[36 \]
where \( p^+, p^- \) and \( p_{K_1} \) denote the three momenta vectors of the final particles \( l^+, l^- \) and \( K_1 \) respectively. These polarization vectors \( S_\mu^i(W_\mu^i) \) in Eqs. \( \text{(32)} \) and \( \text{(33)} \) are defined in the rest frame of \( l^- (l^+) \). When we apply lorentz boost to bring these polarization vectors from rest frame of \( l^- (l^+) \) to the centre of mass frame of \( l^+ \) and \( l^- \), only the longitudinal polarization four vector get boosted while the other two polarization vectors remain unchanged. After this operation the longitudinal four vector read as

\[
S_\mu^L = (0, \frac{E_l p_-}{m}, \frac{-E_l p_-}{m}, \frac{-m p_-}{m}) \\
W_\mu^L = (0, \frac{E_l p_+}{m}, \frac{-E_l p_+}{m}, \frac{-m p_+}{m})
\]

(34)

To achieve the polarization asymmetries one can use the spin projector \( \frac{1}{2} (1 + \gamma_5 S) \) for \( l^- \) and for the \( l^+ \) spin projector is \( \frac{1}{2} (1 + \gamma_5 W) \). Normalized, unpolarized differential forward-backward asymmetry is defined as

\[
A_{FB} = \frac{\int_{-1}^{0} \frac{d^2 \Gamma}{dsd \cos \theta} d \cos \theta - \int_{0}^{1} \frac{d^2 \Gamma}{dsd \cos \theta} d \cos \theta}{\int_{-1}^{1} \frac{d^2 \Gamma}{dsd \cos \theta} d \cos \theta}
\]

(35)

When the spins of both leptons are taken into account, the \( A_{FB} \) will be a function of the spins of final leptons, and is defined as

\[
A_{FB}^{ij} = \left( \frac{d \Gamma}{ds} \right)^{-1} \left\{ \int_{0}^{1} d \cos \theta - \int_{-1}^{0} d \cos \theta \right\} \\
\times \left\{ \frac{d^2 \Gamma(s^- = i, s^+ = j)}{dsd \cos \theta} - \frac{d^2 \Gamma(s^- = i, s^+ = -j)}{dsd \cos \theta} \right\} \\
- \left\{ \frac{d^2 \Gamma(s^- = -i, s^+ = j)}{dsd \cos \theta} - \frac{d^2 \Gamma(s^- = -i, s^+ = -j)}{dsd \cos \theta} \right\}
\]

(36)

\[
A_{FB}^{ij} = A_{FB}(s^- = i, s^+ = j) - A_{FB}(s^- = i, s^+ = -j) \\
- A_{FB}(s^- = -i, s^+ = j) + A_{FB}(s^- = -i, s^+ = -j)
\]

(37)

Using these definitions for the double polarized FB asymmetries, we have found the expressions of numerators as follows:
\[ A_{FB}^{LL} = \sqrt{s\lambda(s-4m^2)} \left[ 4\{\Re(A_D^* + \Re(B_1^*) \} ight. \\
+ \frac{2m}{m_{K_1}^2 s} \left\{ t\Re(B_1 G_1^*) + \lambda\Re(B_2 G_1^*) \right\} + \frac{64}{s} \\
\times \left( (m_B^2 - m_{K_1}^2)G_3 + sG_3 \right) \Re(C_1 C_T^*) + \frac{32m}{m_{K_1}^2 s} \\
\times \left( (m_B^2 - 5m_{K_1}^2 - s)G_5 - G_3t + \lambda G_4 \right) \Re(D_1 C_{TE}^*) \\
+ \frac{8m}{m_{K_1}^2} (G_5j + G_3t + \lambda G_4) \left\{ \frac{4(m_B^2 - m_{K_1}^2)}{s} \right\} \\
\times \Re(D_2 C_{TE}^*) + 4\Re(D_0 C_{TE}^*) \\
\left. + \frac{1}{m} \left( \Re(G_1 C_T^*) + 2\Re(G_2 C_{TE}^*) \right) \right\} \] \\

\[ A_{FB}^{NN} = \frac{m}{m_{K_1}^2 s} \sqrt{\lambda(s-4m^2)} \left[ -2(t\Re(B_1 G_1^*) + \lambda\Re(B_2 G_1^*)) \\
- 32(G_5j + G_3t + \lambda G_4) \Re(D_1 C_{TE}^*) \\
- (m_B^2 - m_{K_1}^2)\Re(D_2 C_{TE}^*) - s\Re(D_0 C_{TE}^*) \\
+ \frac{s}{4m} \left( \Re(G_1 C_T^*) + 2\Re(G_2 C_{TE}^*) \right) \right\] \\

\[ A_{FB}^{TT} = \frac{2m}{m_{K_1}^2 s} \sqrt{\lambda(s-4m^2)} \left[ t\Re(B_1 G_1^*) + \lambda\Re(B_2 G_1^*) \\
+ 16(G_5j + G_3t + \lambda G_4) \Re(D_1 C_{TE}^*) \\
- (m_B^2 - m_{K_1}^2)\Re(D_2 C_{TE}^*) - s\Re(D_0 C_{TE}^*) \\
+ \frac{s}{4m} \left( \Re(G_1 C_T^*) - 2\Re(G_2 C_{TE}^*) \right) \right\] \\

\[ A_{FB}^{LT} = \frac{8\lambda}{3\sqrt{s}} \left[ 8(s + 4m^2)G_5 \Re(A_C^* C_{TE}^*) - ms|A|^2 \right. \\
- 1024m|G_5|^2|C_{TE}|^2 - 4(s - 4m^2)G_5 \Re(C_1 C_T^*) \\
\left. + \frac{1}{m_{K_1}^2} \left\{ m \left( |B_1|^2 + t\Re(B_1 B_2^*) + \lambda|B_2|^2 \right) \right\} \\
+ (G_3 + G_5 + G_4t) \left\{ 2(s + 4m^2)\Re(B_1 C_T^*) \\
- 4(s - 4m^2)\Re(D_1 C_{TE}^*) \right\} \\
+ (G_5j + G_3t + G_4\lambda) \left\{ 2(s + 4m^2)\Re(B_2 C_T^*) \\
- 4(s - 4m^2)\Re(D_2 C_{TE}^*) \right\} \\
+ 64m \left[ |G_5|^2(s - 4m_{K_1}^2) + 2s(G_5 G_3 + G_5 G_4 j) \\
+ G_3 G_4 t) + s|G_3|^2 + s\lambda|G_4|^2 |C_T|^2 \right\} \]
\[ A_{FB}^{TT} = \frac{8\lambda}{3\sqrt{s}} \left[ ms|A|^2 - 8(s + 4m^2)\mathcal{G}_5 \text{Re}(AC_{TE}^*) \right. \\
+ 1024m|\mathcal{G}_5|^2|C_{TE}|^2 - 4(s - 4m^2)\mathcal{G}_5 \text{Re}(C_1C_7^*) \\
\left. + \frac{1}{m_{K_1}} \left\{ - m(B_1|B_1|^2 + tR|B_1B_1^*| + \lambda|B_2|^2) \\
- (\mathcal{G}_3 + \mathcal{G}_5 + \mathcal{G}_4)t \left\{ 2(s + 4m^2)\text{Re}(B_1C_7^*) \\
+ 4(s - 4m^2)\text{Re}(D_1C_{TE}^*) \right\} \\
- (\mathcal{G}_3 + \mathcal{G}_5 + \mathcal{G}_4)\lambda \left\{ 2(s + 4m^2)\text{Re}(B_2C_7^*) \\
+ 4(s - 4m^2)\text{Re}(D_2C_{TE}^*) \right\} \\
- 64m |\mathcal{G}_5|^2(s - 4m_{K_1}^2) + 2s(\mathcal{G}_5\mathcal{G}_3 + \mathcal{G}_5\mathcal{G}_4j) \\
+ \mathcal{G}_3\mathcal{G}_4t + s|\mathcal{G}_3|^2 + s\lambda|\mathcal{G}_4|^2 |C_T|^2 \right\} \right] \\
A_{FB}^{NT} = -A_{FB}^{TN} \] (38)

\[ A_{FB}^{LN} = \frac{4\lambda\sqrt{s - 4m^2}}{3} \left[ 8\mathcal{G}_5 \text{Im}(AC_{TE}^*) - m\text{Im}(AC_1^*) \right. \\
\left. + \frac{1}{m_{K_1}} \left\{ m(\text{Im}(B_1D_1^*) + t\text{Im}(B_1D_2^*) + \text{Im}(B_2D_1^*)) \\
+ \lambda\text{Im}(B_2D_2^*) \right\} + 4s(\mathcal{G}_3 + \mathcal{G}_4) \left( 2\text{Im}(B_1C_{TE}^*) + 2t\text{Im}(B_2C_{TE}^*) \right) \right] \]

\[ A_{FB}^{NL} = \frac{4\lambda\sqrt{s - 4m^2}}{3} \left[ - 8\mathcal{G}_5 \text{Im}(AC_{TE}^*) - m\text{Im}(AC_1^*) \right. \\
\left. + \frac{1}{m_{K_1}} \left\{ m(\text{Im}(B_1D_1^*) + t\text{Im}(B_1D_2^*) + \text{Im}(B_2D_1^*)) + \lambda\text{Im}(B_2D_2^*) \right\} \\
- \frac{8}{m_{K_1}} \left\{ (\mathcal{G}_3 + \mathcal{G}_5 + t\mathcal{G}_4)\text{Im}(B_1C_{TE}^*) + (\mathcal{G}_3 + \mathcal{G}_5 + \mathcal{G}_4)\lambda\text{Im}(B_2C_{TE}^*) \right\} \right] \]

Note: It is worthful to mention here we have included short distance part, \( Y_{SD}(z, \hat{s}) \), of \( C_0^{effSM} \) in our numerical calculation which contains also the imaginary part, therefore, in \( A_{FB}^{NT}(A_{FB}^{TN}) \) and \( A_{FB}^{NL}(A_{FB}^{NL}) \) only those terms contribute which contain auxiliary functions \( A, B_1 \) and \( B_2 \).

IV. NUMERICAL RESULTS AND DISCUSSION

In this section we examine the effects of different new physics operators on polarized lepton pair forward-backward asymmetries. For this purpose, we analyze the behaviour of polarized \( FB \) asymmetries and their average values in the presence of constraints on NP couplings that are given in section III. Regarding this, different scenarios for NP Lorentz structure are displayed in Table IV-VI. Numerical values of different input parameters are given in Table I, while the SM Wilson coefficients at \( \mu = m_b \) are given in Table II. In addition to calculate the numerical values of observables under consideration, we have used the light-cone QCD sum rules form factors \( 38 \), summarized in Table III. The momentum dependence dipole parametrization for these form factors is:

\[ T_i^X(q^2) = \frac{T_i^X(0)}{1 - a_i^X(q^2/m_B^2) + b_i^X(q^2/m_B^2)^2} \] (39)
where \( T \) denotes the \( A, V \) or \( F \) form factors and the subscript \( i \) can take the value 0, 1, 2 or 3. The superscript \( X \) belongs to \( K_{1A} \) or \( K_{1B} \) state.

**TABLE I: Default values of input parameters used in the calculations** [36]

| \( m_B = 5.28 \) GeV, \( m_b = 4.28 \) GeV, \( m_\mu = 0.105 \) GeV, \( m_\tau = 1.77 \) GeV, \( f_B = 0.25 \) GeV, \( |V_{tb}V_{ts}^*| = 45 \times 10^{-3}, \) \( \alpha^{-1} = 137, \) \( G_F = 1.17 \times 10^{-5} \) GeV\(^{-2}, \) \( \tau_B = 1.54 \times 10^{-12} \) sec, \( m_{K_{1}(1270)} = 1.270 \) GeV, \( m_{K_{1}(1400)} = 1.403 \) GeV, \( m_{K_{1B}} = 1.34 \) GeV. |

Before proceeding to analyze the NP, first we would like to mention here that the authors of ref [31,32,39] concluded that all observables such as branching ratio, forward backward and single lepton polarization asymmetries, etc for \( B \to K_{1}(1430)\mu^+\mu^- \) are sensitive to mixing angle \( \theta_K \). In this context, it is interesting to see the dependence of the values of double lepton polarizations forward-backward asymmetries on mixing angle \( \theta_K \). In this study, we have found that \( A_T^{LL,B}, A_L^{TL,B} \) and \( A_L^{LT,B} \) are sensitive to \( \theta_K \) for the decay \( B \to K_1(1430)\mu^+\mu^- \) as shown in fig 11(a-c) but not much sensitive for \( B \to K_1(1270)\mu^+\mu^- \). Therefore, besides to other observables, the precise measurements of these asymmetries (for former decay channel) at LHC may also provide help to put some stringent constraint on the mixing angle \( \theta_K \) in near future. However, as it is mentioned in ref. [40] that the branching ratio for \( K_1(1430) \) is two order suppressed i.e. \( Br(B \to K_1(1270)\mu^+\mu^- (\tau^+\tau^-)) \) are of the order of \( 10^{-4} \) (10\(^{-5}\)) while \( Br(B \to K_1(1430)\mu^+\mu^- (\tau^+\tau^-)) \) are of the order of \( 10^{-8} \) (10\(^{-10}\)). For this reason we are not interested in the results of \( B \to K_1(1430)\mu^+\mu^- (\tau^+\tau^-) \).

Now to see the behaviour of double polarized \( FB \) asymmetries under the influence of new physics couplings, we have drawn the \( s \)-dependence of these asymmetries in figs. 1-10. In all these graphs the grey shaded band corresponds to the region of the SM values of these asymmetries due to uncertainties in mixing angle \( \theta_K \) while dashed line corresponds to the SM value when the central values of the form factors are taken. In fig. 1 (6) we present the dependence of \( A_T^{LL,B} \) on \( s \) for the decay \( B \to K_1(1270)\mu^+\mu^- (B \to K_1(1270)\tau^+\tau^-) \) when only vector type couplings are switched on. figs. 1a-1c depict the effects of different NP scenarios presented in tables (IV,V) on \( s \) dependence of \( A_T^{LL,B} \). These figures show that the zero position of \( A_T^{LL,B} \) shifts towards left and right-side of the corresponding SM value within allowed values of different NP coefficients. For example fig. 1a depicts scenario S1 (see Table IV), where by fixing the value of \( R_A \), three different curves of \( A_T^{LL,B} \) are drawn within the allowed range of \( R_V \). It shows that zero position of \( A_T^{LL,B} \) shifts towards left and right-side of the corresponding SM value for all allowed values of \( R_V \) in S1 scenario. Similarly figs. 1b and 1c depict scenarios S4 and S6 given in table V. figs. 2a-2c depict the effects of tensor interactions (table VI) on \( s \) dependence of \( A_T^{LL,B} \). For instance fig. 2c shows the case of S9 when both tensor couplings \( C_T \) and \( C_{TE} \) are present with opposite polarity. It is important to mention here that only those scenarios of all NP couplings are shown in figures for which the zero position of \( A_T^{LL,B} \) is shifted distinctly in comparison to that of the zero position in SM. In contrast to \( B \to K_1(1270)\mu^+\mu^- \), \( A_T^{LL,B} \) does not have zero crossing for \( B \to K_1(1270)\tau^+\tau^- \). figs. 6(a-c) depict scenarios S1, S4, S6 and fig. 7(a-c) depict scenarios S7, S8 and S9 which show, respectively, the possible effects

**TABLE II: The Wilson coefficients \( C_i^\mu \) at the scale \( \mu \sim m_b \) in the SM** [37].

| \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) | \( C_7 \) | \( C_9 \) | \( C_{10} \) |
|---|---|---|---|---|---|---|---|---|
| 1.107 | -0.248 | -0.011 | -0.026 | -0.007 | -0.031 | -0.313 | 4.344 | -4.669 |

**TABLE III: \( B \to K_{1A,1B} \) form factors** [38], where \( a \) and \( b \) are the parameters of the form factors in dipole parametrization.

| \( T_i^X (q^2) \) | \( T(0) \) | \( a \) | \( b \) | \( T_i^X (q^2) \) | \( T(0) \) | \( a \) | \( b \) |
|---|---|---|---|---|---|---|---|
| \( V_{1A} \) | 0.34 | 0.635 | 0.211 | \( V_{1B} \) | -0.29 | 0.729 | 0.074 |
| \( V_{K_{1A}} \) | 0.41 | 1.51 | 1.18 | \( V_{K_{1B}} \) | -0.17 | 0.919 | 0.855 |
| \( V_{0A} \) | 0.22 | 2.40 | 1.78 | \( V_{0B} \) | -0.45 | 1.34 | 0.690 |
| \( A_{K_{1A}} \) | 0.45 | 1.60 | 0.974 | \( A_{K_{1B}} \) | -0.37 | 1.72 | 0.912 |
| \( F_{K_{1A}} \) | 0.31 | 2.01 | 1.50 | \( F_{K_{1B}} \) | -0.25 | 1.59 | 0.790 |
| \( F_{L_{1A}} \) | 0.31 | 0.629 | 0.387 | \( F_{L_{1B}} \) | -0.25 | 0.378 | -0.755 |
| \( F_{T_{1A}} \) | 0.28 | 1.36 | 0.720 | \( F_{T_{1B}} \) | -0.11 | 1.61 | 10.2 |
TABLE IV: Scenarios for different possible fixed values of $R_A$, when only $R_A$ and $R_V$ couplings are present

| Scenario | $R_A$ | $R_V$ |
|----------|-------|-------|
| S1       | $-1.10$ | $-6.5 \leq R_V \leq 1$ |
| S2       | $9$    | $-6.5 \leq R_V \leq 1$ |

TABLE V: Scenarios for different possible fixed values of $R'_A$, when only $R'_A$ and $R'_V$ couplings are present

| Scenario | $R'_A$ | $R'_V$ |
|----------|-------|-------|
| S3       | $-3 \leq R'_A \leq 3$ | $0.1$ |
| S4       | $-3 \leq R'_A \leq 3$ | $2.75$ |
| S5       | $0.1$  | $-3 \leq R'_V \leq 3$ |
| S6       | $2.75$ | $-3 \leq R'_V \leq 3$ |

when only vector and tensor type couplings are present in $A'_{FB}$ for $B \rightarrow K_1(1270)\tau^+\tau^-$. In all these scenarios the value of $A'_{FB}$ remains positive in high $s$ region as predicted by SM value except S7. fig. 7a shows that when tensor coupling $C_T$ is present only (Scenario S7), $A'_{FB}$ can get the negative values in opposite to SM prediction. Therefore, if negative values of $A'_{FB}$ are measured in future experiments for $B \rightarrow K_1(1270)\tau^+\tau^-$, these results will be unambiguous indication of existence of new physics beyond the SM (i.e. existence of tensor type interactions).

In figs. 3(a-d) and 4(a-c), we present the dependence of $A'_{FB}$ on $s$ for muons as final state leptons while figs. 8(a-c) and 9(a-c) show the dependence of $A'_{FB}$ on $s$ for tausons as final state leptons. fig 3a (8a) presents S1 (i.e when only $R_A$ and $R_V$ couplings are present), where three different curves for $A'_{FB}$ are plotted by fixing $R_A = -1.10$ and by taking three different values of $R_V$ within the allowed range (i.e. $-6.5 \leq R_V \leq 1$) for the case of muons (tausons) as final state doubly polarized leptons. This figure tells us that zero position of $A'_{FB}$ gets shifted towards left and right with respect to SM zero position for all the different selected values of $R_V$ with the allowed range while fig. 8a shows the NP effects when tausons are the final state leptons. One can also see from this figure that NP effects are significant. Similarly figs. 3b-3d present the NP effects on $A'_{FB}$ when scenarios S2, S5 and S6 are considered for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$. One can also notice from the expressions given in $A'_{FB}(A'_{FB})$ that $A'_{FB} = -A'_{FB}$ when we consider only vector type couplings. Therefore the effects of vector type couplings on $A'_{FB}$ are same as $A'_{FB}$. Moreover, figs. 4a-4c (9a-9c) present scenarios S7, S8 and S9 for the case of muons (tausons) as final state leptons when tensor type couplings, $C_T$ and $C_{TE}$, are considered. For instance, fig. 4c represents scenario S9 (i.e. when both tensor interactions are present) in which we consider the case when both $C_T$ and $C_{TE}$ are present with opposite polarity. All these figures show that $A'_{FB}$ is greatly effected by NP couplings in particular to tensor interactions. Furthermore, for the case of tausons, when different new physics couplings are switched on, for some of the cases $A'_{FB}$ gets opposite value in entire high $s$ region as compared to its SM values predictions.

Similar to $A'_{FB}$, $s$ dependence of $A'_{FB}$ for different scenarios is shown in figs. 5(a-c) for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$ while figs. 10(a-c) present for the case of tausons as final state leptons. figs. 5(a-c) show the effects of tensor type interactions (S7, S8 and S9). These figures show that all these new physics scenarios effect the $s$ dependence value of $A'_{FB}$ significantly. Additionally, figs. 10(a-c) manifest scenarios S7, S8 and S9 for the case of tausons. Again from these figures we conclude that different NP couplings modify the value of $A'_{FB}$ significantly in the high $s$ region.

It is emphasized here that in our analysis only $A'_{FB}$, $A'_{FB}$ and $A'_{FB}$ are observed to be considerably effected by NP couplings of different types. Therefore the other remaining polarized lepton pair forward-backward asymmetries are not discussed.

Moreover, we eliminate the dependence of forward-backward polarized asymmetries on $s$ by performing integration over $s$ and find the average values of above mentioned asymmetries which are also experimentally useful tools to explore the new physics. We calculate the averaged double lepton polarization forward-backward asymmetries by

TABLE VI: Different Scenarios when tensor couplings are present

| Scenario | $C_T$ | $C_{TE}$ |
|----------|-------|---------|
| S7       | $-1.14 \leq C_T \leq 1.14$ | $0$  |
| S8       | $0$   | $-0.57 \leq C_{TE} \leq 0.57$ |
| S9       | $\pm 0.54$ | $-0.5 \leq C_{TE} \leq 0.5$ |
using the following formula

$$\langle A_{FB}^{ij} \rangle = \frac{\int d^2 s A_{FB}^{ij}(s) ds}{\int d^2 s (m_{K_i}^2 - m_{K_j}^2)^2 (m_{K_i}^2 - m_{K_j}^2)^2 ds}$$  \hspace{1cm} (40)$$

As mentioned in Sec. II that in the calculation of average values we do not include long distance contribution, $Y_{LD}(z, \delta)$. Now we discuss the effects of NP on $\langle A_{FB}^{ij} \rangle$, in the following sections.

A. Tensor type interactions present only

In this section, we discuss the explicit dependence of tensor type couplings on the average values of different polarized forward-backward asymmetries. For this purpose 12e and 12f show the effects of NP tensor and axial tensor operators, respectively, on $\langle A_{FB}^{ij} \rangle$ for the case of muons. Fig. 12e depicts the scenario S7 (see Table-VI. i.e. when $C_T$ present only), in which $\langle A_{FB}^{LL} \rangle$ significantly varies from its SM value. The value of $\langle A_{FB}^{LL} \rangle$ increases and reaches to a maximum value of $\approx 0.21$ and then again decreases within the allowed range ($-1.14 \leq C_T \leq 1.14$). It is also clear that $\langle A_{FB}^{LL} \rangle$ does not change its sign while $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ both change their sign in the allowed range. Moreover, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ show opposite trend such that $\langle A_{FB}^{LT} \rangle \ (|A_{FB}^{TL}|)$ remains positive (negative) for ($-1.14 \leq C_T \leq -0.05$) while it becomes negative (positive) for ($-0.05 \leq C_T \leq 1.14$). All other polarized forward-backward asymmetries are insignificant for scenario S7. When only second type of tensor interaction $C_{TE}$ is switched on (Scenario S8), fig. 12f manifest its possible effects on $\langle A_{FB}^{ij} \rangle$. It can be easily noted here that only $\langle A_{FB}^{LL} \rangle$ does not change its sign while all other change their sign, when $C_{TE}$ is varied from -0.57 to 0.57. One can also observe that only $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are effect significantly similar to the case when only $C_T$ type interaction is switched on. Similar to figs. 12e and 12f, we plot averaged double lepton polarization forward-backward asymmetries in figs. 12c and 12d for the case of tausons, when only tensor type interactions are present. Fig. 13e shows S7 scenario, where $\langle A_{FB}^{ij} \rangle$ is plotted for the allowed range of $C_T$. From this plot we see that $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$, are greatly influenced by NP tensor operator $C_T$ as compared to their SM values, where by signs of some of these polarized forward-backward asymmetries are flipped as well. In comparison to this fig. 13f shows even more distinct effects on the values of all $\langle A_{FB}^{ij} \rangle$ except $\langle A_{FB}^{ij} \rangle$, $\langle A_{FB}^{ij} \rangle$, $\langle A_{FB}^{ij} \rangle$ and $\langle A_{FB}^{ij} \rangle$ (not included), observed for S8, when only $C_{TE}$ operator is switched on.

B. $R_V$ and $R_A$ couplings present only

When only $R_V$ and $R_A$ couplings are present, for the case of muons, figs. 12a and 12b represent scenarios S1 and S2 respectively. In fig. 12a, When the value of $R_A = -1.10$ is fixed and $R_V$ is varied in allowed range from -6.5 to 1, $\langle A_{FB}^{LL} \rangle$ is drastically changed from its SM value, while $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are also modified appreciably from their SM values. The value of $\langle A_{FB}^{LL} \rangle$ remains negative for the values of $R_V$ from -6 to -3 and it acquires positive values for ($-3 \leq R_V \leq 1$), where the maximum value $\langle A_{FB}^{LL} \rangle = 0.24$ is observed at $R_V = 1$. It is also clear from this plot that $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ follow the opposite pattern, such that $\langle A_{FB}^{LT} \rangle \ (\langle A_{FB}^{TL} \rangle)$ remains positive (negative) for ($-6 \leq R_V \leq -2.5$) and negative (positive) from -2.5 to 1. Similarly when S2 is considered (fig. 12b), all three double polarizations of $FB$ asymmetries, $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ not only vary in magnitude for the allowed region of $R_V$ but also change their polarities, where $\langle A_{FB}^{LL} \rangle$ becomes positive to negative at $R_V = -3$ whereas $\langle A_{FB}^{LT} \rangle \ (\langle A_{FB}^{TL} \rangle)$ changes its sign from positive (negative) to negative (positive) at $R_V \approx -1.2$. All other averaged polarized $FB$ asymmetries which are left out show negligible NP effects. When the case of tausons is considered, figs. 13a and 13b, it is observed that presence of couplings $R_V$ and $R_A$ effect $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$, significantly. One can observe from fig. 13a that the magnitude of $\langle A_{FB}^{LL} \rangle$, varies significantly within the allowed range ($-6 \leq R_V \leq 1$) along with the change in polarity of $\langle A_{FB}^{LL} \rangle$. While when we consider S2 (fig. 13b), it shows the opposite behaviour for $\langle A_{FB}^{LL} \rangle$, while similar behaviour for $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ as compared to S1.

C. $R'_V$ and $R'_A$ couplings present only

When only $R'_V$ and $R'_A$ couplings are present, figs. 12c and 12d depict scenarios s4 and s6 respectively for muons, while figs. 13c and 13d represent the case of tausons. Again from these figures, one can observe that only $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are considerably effected for the case of muons as well as for the case of tausons in the presence of $R'_V$ and $R'_A$ couplings. In fig. 12c $\langle A_{FB}^{LL} \rangle$ and $\langle A_{FB}^{LT} \rangle$ acquire only positive sign where by $\langle A_{FB}^{TL} \rangle$ acquire only negative
sign for all allowed values of $R_A$. For $S6$ (fig. 12d), $\langle A_{FB}^{LL} \rangle$ increases from 0.03 at $R_V = -3$ and reaches to a maximum value of $\approx 0.23$ at $R_V = 3$ whereas $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ follow the opposite fashion, compared to $S4$. For the case of tauons, fig. 13c show that when we switch on only $R_A$, only $\langle A_{FB}^{LL} \rangle > \langle A_{FB}^{LT} \rangle > \langle A_{FB}^{TL} \rangle > \langle A_{FB}^{NN} \rangle > \langle A_{FB}^{TT} \rangle > \langle A_{FB}^{TN} \rangle > \langle A_{FB}^{NT} \rangle > \langle A_{FB}^{LL} \rangle$ for entire allowed range ($-3 \leq R_A \leq 3$). Similar conclusion can be drawn from fig. 13d (S6) such as $\langle A_{FB}^{LL} \rangle > \langle A_{FB}^{LT} \rangle > \langle A_{FB}^{TL} \rangle > \langle A_{FB}^{NN} \rangle > \langle A_{FB}^{TT} \rangle > \langle A_{FB}^{TN} \rangle > \langle A_{FB}^{NT} \rangle > \langle A_{FB}^{LL} \rangle$ for ($-3 \leq R_V \leq 0.7$). Also polarities of $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are flipped.

V. CONCLUSION

In conclusion, we calculate double polarized $FB$ asymmetries using most general model independent form of the effective Hamiltonian including all possible non-standard local four-fermi interactions. Our analysis shows that similar to the other observables, polarized $FB$ asymmetries are also sensitive to the mixing angle $\theta_K$. While considering the different NP scenarios our analysis exhibit that the averaged double lepton polarization forward-backward asymmetries are very sensitive to NP couplings. The key points are as under.

When vector axial-vector couplings are considered for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$, only averaged polarized forward-backward asymmetries, $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are affected significantly whereas all other averaged polarized $FB$ asymmetries are suppressed. Similarly when only tensor interaction $C_T$ is present again $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are modified considerably as compared to their SM values while $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$, $\langle A_{FB}^{TL} \rangle$ and $\langle A_{FB}^{NN} \rangle$ are influenced greatly when only $C_{TE}$ coupling is present. In similarity to the decay $B \rightarrow K_1(1270)\mu^+\mu^-$, when the case of tauons is considered it is found again that $\langle A_{FB}^{LL} \rangle$, $\langle A_{FB}^{LT} \rangle$ and $\langle A_{FB}^{TL} \rangle$ are modified as compared to their SM values, when either vector axial-vector operators or tensor interactions of type $C_T$ are present only. Moreover all types of averaged doubly polarized $FB$ asymmetries except $\langle A_{FB}^{NN} \rangle$, $\langle A_{FB}^{TT} \rangle$ and $\langle A_{FB}^{TN} \rangle$ and $\langle A_{FB}^{NT} \rangle$ are influenced greatly when only $C_{TE}$ tensor couplings are switched on.

Additionally, the dependence of polarized lepton pair forward-backward asymmetries $A_{FB}^{LL}$, $A_{FB}^{LT}$ and $A_{FB}^{TL}$ on $s$ for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$ depict the left and right side shifting of zero crossing positions of these forward-backward polarized asymmetries from their corresponding SM values, when vector axial-vector and tensor type NP operators are considered. Moreover, signs of some of these polarized $FB$ asymmetries are also flipped for few allowed values of different NP couplings. Similar conclusion is drawn for the case of tauons as final state leptons.

Acknowledgments

The authors would like to thank Prof. Fayyazuddin for their valuable guidance and useful discussions. One of the author I. Ahmed. would like to acknowledge the grant (2013/23177-3) from FAPESP.

[1] T. Goto, Y. Okada, Y. Shimizu and M. Tanaha, Phys. Rev. D 55, 4273 (1997) [arXiv:hep-ph/9609512].
[2] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B 353, 591 (1991).
[3] C. S. Lim, T. Morozumi and A. I. Sanda, Phys. Lett. B 218, 343 (1989); X.-G. He, T. D. Nguyen and R. R. Volkas, Phys. Rev. D 38, 814 (1988); B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. B 319, 271 (1989); Y. G. Kim, P. Ko and J. S. Lee, Nucl. Phys. B 544, 64 (1999) [arXiv:hep-ph/9810336]; C.-S. Huang, W.-J. Huo and Y.-L. Wu, Mod. Phys. Lett. A 14, 2453 (1999) [arXiv:hep-ph/9911203].
[4] N. G. Deshpande, J. Trampetic and K. Panose, Phys. Rev. D 39, 1461 (1989); P. J. O'Donnell and H. K. T. Kung, Phys. Rev. D 43, R2067 (1991); N. Paver and Riazzuddin, Phys. Rev. D 45, 978 (1992); W.-S. Hou, R.S. Willey and A. Soni, Phys. Rev. Lett. 58, 1608 (1987) [Erratum-ibid. 60, 2337 (1987)].
[5] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. B 273, 505 (1991).
[6] A. Ali, E. Lunghi, C. Greub and G. Hiller, Phys. Rev. D 66, 034002 (2002) [arXiv:hep-ph/0112300].
[7] T. M. Aliev, M. K. Cakmak and M. Savci, Nucl. Phys. B 607, 305 (2001) [arXiv:hep-ph/0009313]; T. M. Aliev, A. Ozpineci, M. Savci and C. Yuce, Phys. Rev. D 66, 115006 (2002) [arXiv:hep-ph/0208128]; T. M. Aliev, A. Ozpineci and M. Savci, Phys. Lett. B 511, 49 (2001) [arXiv:hep-ph/0103261]; T. M. Aliev and M. Savci, Phys. Lett. B 481, 275 (2000) [arXiv:hep-ph/0003188]; T. M. Aliev, D. A. Demir and M. Savci, Phys. Rev. D 62, 074016 (2000) [arXiv:hep-ph/9912525]; T. M. Aliev, C. S. Kim and Y. G. Kim, Phys. Rev. D 62, 014026 (2000) [arXiv:hep-ph/9910501]; T. M. Aliev, E.O. Iltan, Phys. Lett. B 451, 175 (1999) [arXiv:hep-ph/9804458].
[8] C.-H. Chen, C. Q. Geng, Phys. Rev. D 66, 034006 (2002) [arXiv:hep-ph/0207038]; C.-H. Chen, C. Q. Geng, Phys. Rev. D 66, 014007 (2002) [arXiv:hep-ph/0205306].
[9] G. Erkol, G. Turan, Nucl. Phys. B 635, 286 (2002) [arXiv:hep-ph/0204129]; E. O. Iltan, G. Turan and I. Turan, J. Phys. G 28, 307 (2002) [arXiv:hep-ph/0106136].
[10] W.-J. Li, Y.-B. Dai and C.-S. Huang, Eur. Phys. J. C 40, 565 (2005) [arXiv:hep-ph/0410317].
[11] Q.-S. Yan, C.-S. Huang, W. Liao and S.-H. Zhu, Phys. Rev. D 62, 094023 (2000) [arXiv:hep-ph/0004262].
[12] F. Kruger, E. Lunghi, Phys. Rev. D 63, 014013 (2001) [arXiv:hep-ph/0008210].
[13] R. Mohanta, A. K. Giri, Phys. Rev. D 75, 035008 (2007) [arXiv:hep-ph/0611068].
[14] S. R. Choudhury, N. Gaur and N. Mahajan, Phys. Rev. D 66, 054003 (2002) [arXiv:hep-ph/0203041]; S. R. Choudhury, N. Gaur, [arXiv:hep-ph/0205076]; S. R. Choudhury, N. Gaur, [arXiv:hep-ph/0207353].
[15] T. M. Aliev, V. Bashiry and M. Savci, Phys. Rev. D 71, 035013 (2005) [arXiv:hep-ph/0411327].
[16] U. O. Yilmaz, B. B. Sirvanli and G. Turan, Phys. Rev. D 62, 094023 (2000) [arXiv:hep-ph/0004262].
[17] F. Kruger, E. Lunghi, Phys. Rev. D 63, 014013 (2001) [arXiv:hep-ph/0008210].
[18] R. Mohanta, A. K. Giri, Phys. Rev. D 75, 035008 (2007) [arXiv:hep-ph/0611068].
[19] S. R. Choudhury, N. Gaur, [arXiv:hep-ph/0205076]; S. R. Choudhury, N. Gaur, [arXiv:hep-ph/0207353].
[20] T. M. Aliev, V. Bashiry and M. Savci, Phys. Rev. D 72, 034031 (2005) [arXiv:hep-ph/0506259].
[21] V. Bashiry, JHEP 0906, 062 (2009) [arXiv:0902.2578 [hep-ph]].
[22] S. R. Choudhury, A. S. Cornell, N. Gaur and G. C. Joshi, Phys. Rev. D 69, 054018 (2004) [arXiv:hep-ph/0307276].
[23] T. M. Aliev, V. Bashiry and M. Savci, Eur. Phys. J. C 35, 197 (2004) [arXiv:hep-ph/0311294].
[24] H. Hatanaka, K. C. Yang, Phys. Rev. D 77, 034023 (2003) [arXiv:hep-ph/0301198 [hep-ph]].
[25] A. J. Buras and M. Muni, Phys. Rev. D 52, 186 (1995) [arXiv:hep-ph/9501281]; N. G. Deshpande and J. Trampetic, Phys. Rev. D 60, 2583 (1999); M. Misirak, Nucl. Phys. B 393, 23 (1993) [Erratum-ibid. 439, 461 (1995)].
[26] A. K. Alok, A. Dighe, D. Ghosh, D. London, J. Matias, M. Nagashima and A. Szynkman, JHEP 1002, 053 (2010) [arXiv:0912.1382 [hep-ph]].
[27] S. Ishaq, F. Munir and I. Ahmed, JHEP 07, 006 (2013).
FIG. 1: The dependence of $A_{LL}^{B_{FB}}$ on $s$ for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$, where the dashed-dotted, solid and dashed curves in each figure correspond to $A_{LL}^{B_{FB}}$ for three different allowed values of vector type couplings.

FIG. 2: The same as in figure 1, but for tensor type couplings.
FIG. 3: The same as in figure 1, but for $A^T_T$.

FIG. 4: The same as in figure 3, but for tensor type interactions.
FIG. 5: The dependence of $A^T_{FB}$ on $s$ for the decay $B \rightarrow K_1(1270)\mu^+\mu^-$, where the dashed-dotted, solid and dashed curves in each figure correspond to $A^T_{FB}$ for three different allowed values of tensor type couplings.
FIG. 6: The same as in figure 1, but for $B \to K_1(1270)\tau^+\tau^-$. 

FIG. 7: The same as in figure 2, but for $B \to K_1(1270)\tau^+\tau^-$. 
FIG. 8: The same as in figure 3, but for $B \rightarrow K \tau^+ \tau^-$. 

FIG. 9: The same as in figure 4, but for $B \rightarrow K \tau^+ \tau^-$. 
FIG. 10: The same as in figure 5, but for $B \rightarrow K_{1}(1270)\tau^{+}\tau^{-}$.
FIG. 11: Dependence of double polarized forward-backward asymmetries for the decay $B \rightarrow K_1(1430)\mu^+\mu^-$ on mixing angle $\theta_K$, where $y = \sin \theta_K$. The dashed-dotted, solid and dashed angle dependent curves correspond to $s = 3\text{GeV}^2, 5\text{GeV}^2$ and $7\text{GeV}^2$, respectively.
FIG. 12: Averaged double lepton Polarized forward backward asymmetries \( \langle A_{FB}^{ij} \rangle \) for the decay \( B \to K_{1}(1270)\mu^+\mu^- \) in different scenarios.
FIG. 13: Averaged double lepton Polarized forward backward asymmetries \( \langle A_{FB}^{ij} \rangle \) for the decay \( B \to K_1(1270)\tau^+\tau^- \) in different scenarios.