Improving comb aliasing rejection using filters with stepped triangular impulse response

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Abstract. Decreasing the sampling rate by an integer is called decimation. Decimation finds applications in audio signal processing, Analog/Digital (A/D) converters, digital receivers, among others. The signal must be filtered by a digital filter, before decimation, to prevent aliasing, which may deteriorate decimated signal. This filter is called anti-aliasing or decimation filter. The simplest decimation filter is a comb filter which has all coefficients equal to unity, and consequently does not require multipliers for its implementation. This filter is usually used in the first stage of decimation. The aliasing occur in the bands around comb zeros, called folding bands. Consequently, comb filter naturally provides the aliasing rejection in folding bands. However, this attenuation is not enough in many applications, and must be increased. The principal goal is keeping comb low complexity, while improving its aliasing rejection. In this paper we propose novel method based on filter with stepped triangular impulse response (STIR). It is shown that the STIR filters are multiplier less filters and have all zeros on unit circle. It is elaborated how to choose the STIR filter parameters to get an improved comb aliasing rejection and have the same parameters of design, as the original comb filter. To this end, comb decimation factor is presented as the product of two integers, which are also the parameters of the STIR filter. The magnitude response of the proposed decimator is compared with that of the original comb to show the benefit of the proposed method.

1. Introduction

Decimation is a process of decreasing sampling rate by an integer, called decimation factor [1]. This operation has applications in audio signal processing, communications, analog/digital converters, among others. This process introduces aliasing which may deteriorate the decimated signal. In order to eliminate aliasing, the signal must be filtered, before decimation, by so called decimation filter. The most simple decimation filter is a comb filter, which does not require multipliers because all its coefficients are equal to unity. Comb filter must have high attenuation in the bands around its zeros, also called folding bands, where aliasing occur. However, comb filter does not provide enough attenuation in the folding bands, which may deteriorate the decimated signal [1].

Different methods were proposed to improve comb alias rejection [2-7]. Best results are obtained by rotation of comb zeros from their original positions, first introduced in [2], using so called RS (Rotated Sinc) filter. However, this method needs two multipliers and may result in instability due to the lost of pole zero cancellation on the unit circle, as a result of finite precision of the RS filter coefficients. The principal goal is keeping comb low complexity, while improving its aliasing rejection.

In this paper we propose novel method to improve comb alias rejection using a special symmetrical
multiplier less filter with a stepped triangular impulse response (STIR). This filter has three parameters of design. We analyze the choice of parameters to get an improvement of comb alias rejection, at low cost and get the same parameter of design as in comb filter itself. To this end, we consider comb with the decimation factor $M$, which can be presented as a product of two integers.

The rest of the paper is organized in the following way. Next section presents comb filter and a special class of filters with the stepped triangular response, and the choice of parameters to get benefit of improving comb aliasing rejection. Section 3 presents proposed comb decimation filter based on filter STIR. The proposed method is compared with the original comb filter.

2. Comb filter and stepped triangular response (STIR) filter

In this section we describe comb filter and introduce STIR filter.

2.1. Comb filter

Transfer function of comb filter is given as,

$$H(z) = \frac{1 - z^{-M}}{M} = \frac{1}{M} \sum_{k=0}^{M-1} z^{-k},$$

(1)

where $M$ is decimation factor.

Magnitude response of comb filter is in $\sin(x)/x$ form:

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(M\omega/2)}{\sin(\omega/2)}.$$  

(2)

As an example, Figure 1 shows pole-zero plot, and magnitude response of comb filter for $M=6$.

![Pole-zero plot and magnitude response for $M=6$.](image)

Figure 1. Comb filter, $M=6$.

2.2. STIR filter

The STIR filter with symmetrical coefficients, considered here has three parameters:

- The number of samples at each step, except the highest one, denoted as $M_1$.
- The number of different steps, denoted as $M_2$.
- The number of samples at highest step, denoted as $R$, $R>M_1$.

As an example Figure 2 shows pole-zero plots and impulse response for STIR filters for $M_1=2$, $M_2=3$, for two values of $R$: $R=3$, and $R=4$.

2.3. Properties of STIR filters

2.3.1. **STIR filter is the cascade of two combs of different lengths.** We write the transfer function $G(z)$ of STIR filter with the parameters: $M_1$, $M_2$ and $R$:
Figure 2. STIR filters for $M_1=2$, $M_2=3$, and two different values of $R$.

\[
G(z) = \sum_{k=0}^{M_1-1} z^{-k} + 2 \sum_{k=M_1}^{M_1(M_2-2)+M_1-1} z^{-k} + \ldots + (M_2-1) \sum_{k=M_1(M_2-2)+M_1}^{M_1(M_2-2)+M_1+R-1} z^{-k} + \sum_{k=M_1(M_2-2)+M_1+R}^{N-1} z^{-k},
\]

where $N$ is the length of the filter:

\[
N = (M_2 - 1) \times 2 \times M_1 + R.
\]

Denoting $R=M_1+r$, from (3) we get:

\[
G(z) = G_1(z)G_2(z^{M_1}),
\]

where:

\[
G_1(z) = \sum_{k=0}^{M_1M_2+r-1} z^{-k} = \frac{1-z^{-M_1M_2+r}}{1-z^{-1}}.
\]

\[
G_2(z) = \sum_{k=0}^{M_1-1} z^{-k} ; \quad G_2(z^{M_1}) = \sum_{k=0}^{M_1-1} z^{-kM_1} = \frac{1-z^{-M_1M_2}}{1-z^{-M_1}}.
\]

Note that $G_1(z)$ and $G_2(z)$ are transfer functions of comb filters with omitted normalization factor.

As an example, figure 3 shows impulse responses of filters (6) and (7) for the values $M_1=2$, $M_2=3$ and $R=4$ ($r=2$). According to (6) impulse response of the filter $G_1(z)$, denoted as $g_1(n)$, has length of $M_1 \times M_2 + r=8$. Similarly, from (7), the length of the impulse response of the filter $G_2(z)$, denoted as $g_2(n)$, is equal to $M_2$. The expanded $g_3(n)$ filter is shown in the second row of figure 3. The convolution
of the filters $g_1(n)$ and the expanded filter $g_2(n)$ is also presented in the second row of figure 3, thus presenting the STIR filter with parameters $M_1=2$, $M_2=3$, and $R=4$.

Figure 3. STIR as the convolution of two comb filters.

2.3.2. All zeros of STIR filter are on unit circle. Knowing that all zeros of comb filter, including expanded comb filter, are on the unit circle, and that STIR filter is cascade of two comb filters, it follows that all zeros of STIR filter are also on the unit circle. As an example figure 4 shows zeros of filters from figure 3.

Figure 4. Pole-zero plots of filters from figure 3.
3. Proposed STIR filter-based comb decimator

STIR filter may be used to improve comb alias rejection due to the following properties:

- STIR filter is a multiplier less filter
- The filter has all zeros on the unit circle
- Choosing the appropriate values of the filter parameters, its zeros may be placed into the comb folding bands.

The transfer function of the proposed STIR filter–based comb decimator is given as:

\[ H_p(z) = [H(z)]^K G_1(z) G_2(z^{M_1}) / A, \]  

where \( H(z) \) is a comb filter given in (1), \( K \) is the number of cascaded combs, \( G_1(z) \) and \( G_2(z) \) are given in (6) and (7), respectively, and \( A \) is the normalization factor.

In the following we consider the choice of parameters to ensure that the zeros of STIR filter fall into comb folding bands.

First, we assume that the decimation factor \( M \) can be presented as:

\[ M = M_1 M_2, \]  

where \( M_1 < M_2 \).

Using (9) the transfer function of the comb filter can be rewritten as:

\[ [H(z)]^K = \left[ \frac{1}{M_1} z^{-M_1} \right]^K \left[ \frac{1}{M_2} z^{-M_2} \right]^K = [H_1(z)]^K [H_2(z^{M_1})]^K, \]  

where

\[ H_1(z) = \left[ \frac{1}{M_1} z^{-M_1} \right]^K \quad \text{and} \quad H_2(z) = \left[ \frac{1}{M_2} z^{-M_2} \right]^K, \]  

Taking \( r = M_1 \), from (6) we get:

\[ G_1(z) = \sum_{k=0}^{M_1 M_2 + M_1 - 1} z^{-k} = \frac{1 - z^{-M_1 (M_2 + 1)}}{1 - z^{-1}}. \]  

The principal features of the proposed methods are the following:

- The proposed decimator is a multiplier less filter.
- The parameters of design of the overall filter are the same as in comb filter itself.
- The filter has two stages with decimation factors \( M_1 \) and \( M_2 \).
- The filters \( H_1(z) \) and \( G_1(z) \), are in the first stage, while in the second stage are the filters \( H_2(z) \) and \( G_2(z) \). (Note that the filter \( G_2(z) \) is equal to the filter \( H_2(z) \)).

The method is illustrated in following example.

Example 1:

We consider decimation factor \( M = 15 \), and \( M_1 = 3, M_2 = 5, K = 3 \). The magnitude responses, of the proposed filter and the original comb filter, are contrasted in figure 5. Note that the proposed filter provides an improved alias rejection in comparison with the original comb filter.
Figure 5. Magnitude responses of proposed and comb filter in Example 1.

4. Conclusion
This paper presents a novel method to improve comb aliasing rejection by using a special multiplier less symmetrical filter with the stepped triangular (STIR) impulse response. The parameters of STIR filter are the same as in a two-stage comb filter, i.e. the decimation factors of the first and second stages. The proposed filter has better aliasing rejection than the original comb filter at price of slight increasing of complexity.

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