Giant extrinsic spin Hall effect due to rare-earth impurities

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Abstract. We investigate the extrinsic spin Hall effect (SHE) in the electron gas model due to magnetic impurities, by focusing on Ce- and Yb-impurities. In the dilute limit, the skew scattering term dominates the side jump term. For Ce-impurities, the spin Hall angle \(\alpha_{\text{SH}}\) due to skew scattering is given by \((-8\pi\delta_2)/7\), where \(\delta_2(\ll1)\) is the phase shift for the \(d\) \((l=2)\) partial wave. Since \(\alpha_{\text{SH}}\) reaches \(O(10^{-1})\) if \(\delta_2 \gtrsim 0.03\), considerably large SHE is expected to emerge in metals with rare-earth impurities. The present study provides a highly efficient way to generate a spin current.

Contents

1. Introduction 2
2. Model and Hamiltonian 3
3. Calculations 4
   3.1. \(T\)-matrix due to c–f resonant scattering 4
   3.2. Skew scattering term 5
   3.3. Side jump term 7
4. Discussions 7
   4.1. Origin of the skew scattering mechanism 7
   4.2. Estimations of the spin Hall angle and the skew scattering term 8
5. Summary 9
Acknowledgment 9
References 9

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1. Introduction

The spin Hall effect (SHE) has been attracting a great deal of interest due to its fundamental as well as technological importance. The intrinsic SHE originates from the Berry phase of the multiband Bloch function, so that the spin Hall conductivity (SHC) takes a material-specific value that is independent of impurity scattering in the low resistivity regime [1, 2]. In transition metals, the intrinsic SHE takes a considerably large value [3, 4], because of the ‘effective Aharonov–Bohm (AB) phase’ induced by d-orbital degrees of freedom and the atomic spin–orbit interaction (SOI) [5]–[8]. Intrinsic SHE in Sr$_2$RuO$_4$ has been calculated in [5] based on the t$_{2g}$ tight-binding model. By the same mechanism, SHE in transition metals had been studied based on the realistic multiorbital tight-binding model in [7] and the first-principles calculations [9]. The obtained SHCs agree well with recent experimental observations, in both the magnitude and sign [8].

In addition to the intrinsic mechanism, SHE is also induced by impurity scattering with the aid of SOI; this phenomenon is known as the extrinsic SHE [10]–[12]. It consists of a skew scattering term ($\sigma_{ss}^{\text{SH}} \propto \rho^{-1}$) and a side jump term ($\sigma_{sj}^{\text{SH}} \propto \rho^0$); this fact suggests that considerable large SHC can be realized due to the skew scattering term in the very low resistivity metals, beyond the material-specific intrinsic SHC. For this reason, study of the extrinsic SHE is significant for realizing excellent spintronics devices to produce or detect the spin current.

The extrinsic SHE due to nonmagnetic impurities or vacancies had been studied in [10]–[12], by developing the theory of the extrinsic anomalous Hall effect (AHE) [13]–[15]. The obtained SHC is, however, very small unless the enhancement factor for the SOI due to the multiband effect is taken into consideration. For this reason, it is not easy to estimate the magnitudes of $\sigma_{ss}^{\text{SH}}$ and $\sigma_{sj}^{\text{SH}}$ due to nonmagnetic impurities. In transition metals, there is no apparent experimental evidence for a large extrinsic SHE due to natural randomness.

Dilute magnetic impurities such as Fe and Ce have been found to induce a large extrinsic AHE in simple metals [16, 17]. The Hall angles in AuFe [17] and rare-earth doped metals [16] reach 0.01 $\sim$ 0.001 under high magnetic fields. Since the SHE has a similar mechanism to the AHE, one might expect that a giant SHE can be realized by magnetic impurities. However, theoretical analysis of the extrinsic SHE in a simple s-electron metal due to magnetic impurities has not been performed previously.

In this paper, we study the extrinsic SHE based on a single-impurity Anderson model for Ce and Yb atoms, both of which are typical magnetic impurities. Nonperturbative effect of the large SOI for f-electrons ($\sim$3000 K) is correctly accounted for. Due to skew scattering, the obtained spin Hall angle $\alpha_{\text{SH}} = \frac{\alpha_{2s}}{\alpha_{2l}} \frac{2\hbar}{\hbar}$, which is the ratio between the transverse spin Hall current and the longitudinal electric current, exceeds 0.1 for $\delta_2 \geq 0.03$, where $\delta_2$ is the phase shift for $d(l = 2)$ partial wave; it is more than ten times greater than that in nonmagnetic metals such as Pt [4]. Since the spin Hall angle represents the efficiency of the creating spin current, giant spin Hall current is expected to emerge in simple metals by introducing dilute magnetic impurities. Moreover, the extrinsic SHC can take a large value well above the Kondo temperature $T_K$, in contrast to the AHC. The origin of the giant SHE is found to be the large SOI and angular momentum in the rare-earth impurities, which had not been treated appropriately in previous studies of the extrinsic SHE.
2. Model and Hamiltonian

In this study, we use the single-impurity Anderson model for Ce and Yb atoms. In a Ce$^{3+}$ ion with a 4f$^1$ configuration, the $J = 7/2$ level is about 3000 K higher than the $J = 5/2$ level due to the strong atomic SOI [18]. Therefore, we consider only $J = 5/2$ states for the Ce-impurity. In the same way, we consider only $J = 7/2$ states in Yb$^{3+}$ ion with 4f$^3$ configuration. Here, we introduce the following single-impurity Anderson model for Ce/Yb atoms with both d- and f-orbitals [16]:

$$
H = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,\sigma m} E_{2\alpha}^d d_{2\alpha m}^d d_{\sigma m}^\dagger + \sum_{kM} E_{M}^f f_M^\dagger f_M
+ \sum_{k,\sigma m} \left\{ V_{km}^d d_{2\alpha m}^\dagger c_{k\sigma} + \sum_{kM} \left\{ V_{km}^f c_{k\sigma} f_M^\dagger + hc \right\} + \frac{U_{2\alpha}}{2} \sum_{M \neq M'} n_M^f n_{M'}^f \right\}, \tag{1}
$$

where we have employed the electron (hole) picture for the Ce (Yb) impurity. $c_{k\sigma}^\dagger$ is the creation operator of a conduction electron with spin $\sigma = \pm 1$. $f_M^\dagger$ is the creation operator of an f-electron with total angular momentum $J = 5/2$ (7/2) and $z$-component $M = (\pm J \leq M \leq J)$ for Ce$^{3+}$ (Yb$^{3+}$). $d_{2\alpha m}^\dagger$ is the creation operator of a d-electron with angular momentum $m = (\pm 2 \leq m \leq 2)$. $\varepsilon_k = k^2/2m$ is the energy for the conduction-electrons, and $E_{2\alpha}^f$ ($E_{2\alpha}^d$) is the localized f(d)-level energy. $V_{kM}^f$ and $V_{km}^d$ are the mixing potentials, which are given by

$$
V_{kM\sigma}^f = \sqrt{4\pi} V_{\sigma} \sum_{m} a_{m\sigma}^M Y_3^m (\hat{k}), \tag{2}
$$

$$
V_{km}^d = \sqrt{4\pi} V_{\sigma} Y_2^m (\hat{k}), \tag{3}
$$

where $a_{m\sigma}^M$ is the Clebsch–Gordan (C–G) coefficient and $Y_3^m (\hat{k})$ is the spherical harmonic function. We will show that the phase factor in $Y_3^m (\hat{k})$ and $a_{m\sigma}^M$ are indispensable to realizing the SHE. For $J = 5/2$, $a_{m\sigma}^M = -\sigma ((7/2 - M\sigma)/7)^1/2 \delta_{m,M-\sigma}/2$ [19, 20]. Note that the SOI was neglected in the study of the AHE in [16]. In the present study, we neglect the crystalline electric field of the f-orbitals, since it is small due to the small radius of the f-orbital wave function [16]. We set $\hbar = 1$ hereafter.

To discuss the scattering problem, it is useful to derive an effective Hamiltonian for the conduction electrons by integrating out the f and d electrons in equation (1). The obtained Hamiltonian is given by [16]

$$
H_c = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k',\sigma} J_{k,k',\sigma}^d c_{k\sigma}^\dagger c_{k'\sigma} + \sum_{kk',\sigma\sigma'} J_{k,k',\sigma\sigma'}^f c_{k\sigma}^\dagger c_{k'\sigma'}, \tag{4}
$$

where

$$
J_{k,k',\sigma}^d = \frac{1}{\mu - E^d} \sum_{km} V_{km}^d V_{k'm}^d
= \frac{1}{\mu - E^d} \sum_{m} Y_2^m (\hat{k}) \left[ Y_2^m (\hat{k'}) \right]^*, \tag{5}
$$

We omit the atomic SOI for d-electrons since it is much smaller than $|\mu - E^d|$.

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Figure 1. (a) Diagrammatic expression for the T-matrix due to c–f resonant scattering. (b) The diagram that vanishes after $k_1$-summation.

\[ T_{f_k \sigma, k'_{\sigma'} \sigma'} = J_{f_k \sigma, k'_{\sigma'} \sigma'} + \frac{1}{N} \sum_{k_1, \sigma_1} J_{f_{k_1 \sigma_1 \sigma_1}} G_{k_1, k'_{\sigma_1 \sigma'}} T_{f_{k_1, k'_{\sigma_1 \sigma_1}}}, \]

where its diagrammatic expression is shown in figure 1(a), $N$ is the number of $k$-points, and $G_{k}^0 = (\varepsilon - \varepsilon_{k})^{-1}$. We note that the term containing both $J_{k_0, k'_{\sigma_1 \sigma_1}}$ and $J_{d_{k, k'_{\sigma_1 \sigma_1}}}$ given in figure 1(b) vanishes identically due to the orthogonality of spherical harmonic functions. The solution of equation (7) for $k = k'$ and $\sigma = \sigma'$ is simply given by

\[ T_{f_k, k_0}^{f_{k, k_0}} (\varepsilon) = \frac{3 J_{f_k \sigma, k_0 \sigma}^{f_{k, k_0}}}{1 - J_{f_k \sigma, k_0 \sigma}^{f_{k, k_0}}}, \]
Figure 2. Diagrammatic expressions for SHC induced by skew scattering (a) within the lowest-order (extended Born approximation) contribution, and (b) in the T-matrix approximation with full-order diagrams.

where we have used the relations \( \sum_M |V^f_{kM\sigma}|^2 = 3|V^f_{k}\|^2 \) and \( \frac{1}{N} \sum_{k\sigma} V^f_{kM\sigma} G^0_k(\epsilon)V^f_{kM'\sigma} = |V^f_{k}|^2 g(\epsilon) \delta_{MM'} \). \( g(\epsilon) = \frac{1}{N} \sum_k G^0_k(\epsilon) \) is the local Green function. Assuming approximate particle–hole symmetry near \( \mu \), we put \( g(\epsilon) \equiv g(+i\delta) = -i\pi N(0) \). Then, the quasiparticle damping rate in the T-matrix approximation is given by

\[
\gamma = -n_{\text{imp}} \text{Im} T^{\text{R}}_{k\sigma,k'\sigma}(0) = 3n_{\text{imp}} \frac{\pi N(0) J^2_f}{1 + (\pi N(0) J_f)^{-1}},
\]

where \( n_{\text{imp}} \) is the impurity concentration. Note that equation (9) is exact for \( n_{\text{imp}} \ll 1 \).

3.2. Skew scattering term

In this section, we study the skew scattering term using linear response theory. Initially, we consider the case \( N(0) J_f \ll 1 \) and \( J_d \ll J_f \), where the Born approximation is valid.

In analogy to [15, 16], the skew scattering term is given by

\[
\sigma_{\text{ss}}^{\text{SH}} = -\frac{e}{2\pi} n_{\text{imp}} \frac{1}{N^2} \sum_{k,k',\sigma} \frac{\partial \epsilon_k}{\partial k_x} \frac{\partial \epsilon_{k'}}{\partial k'_x} |G^R_k(0)|^2 |G^R_{k'}(0)|^2 \left\{ T^{(2)\text{R}}_{k\sigma,k'\sigma}(0) J^d_{k',k} + \text{cc} \right\},
\]

where \(-e\) (\( e > 0 \)) is the electron charge. Its diagrammatic expression is shown in figure 2(a).

Here,

\[
T^{(2)\text{R}}_{k\sigma,k'\sigma}(0) = \frac{1}{N} \sum_{k_1,\sigma_1} J^f_{k\sigma,k_1\sigma_1} G^R_{k_1}(0) J^i_{k_1\sigma_1,k'\sigma}
= g^R(0) J^f_{k\sigma,k'\sigma} J^i,
\]
is the second-order term of the T-matrix; the first-order term in \( J_{k\sigma,k'\sigma}^f \) does not contribute to \( \sigma_{\text{SH}}^{ss} \) up to the first-order term in \( J_d \) [16]. Note that any diagram that contains the part shown in figure 1(b) vanishes identically. The retarded Green function is given by \( G_k^R(0) = (\mu - \varepsilon_k + i\gamma)^{-1} \), where \( \gamma \) represents the quasiparticle damping rate. Here, we put

\[
\gamma = \gamma_1 + \gamma_0,
\]

where \( \gamma_0 \) is the damping rate due to nonmagnetic scattering, such as c–d scattering \((\gamma_0 = 5\pi n_{\text{imp}} N(0) J_d^2)\) and the scattering due to disorders. The charge current is given by \( j^c = -e\partial\varepsilon_k/\partial k_v \), where \( v = x, y \). The spin current is then given by \( j^s = (\sigma/2)\partial\varepsilon_k/\partial k_v \).

First, we consider the angular integration in equation (10), which is given by

\[
\sum_{\sigma} \frac{\sigma}{2} \left( J_{k\sigma,k'\sigma}^d f \left( \hat{k}, \hat{k}' \right) \right)^\Omega,
\]

where \( f(\hat{k}, \hat{k}') \equiv \frac{m^2}{k^4} \frac{\partial\varepsilon_k}{\partial k} \frac{\partial\varepsilon_{k'}}{\partial k'} = \sin \theta_k \cos \phi_k \sin \theta_{k'} \sin \phi_{k'} \), and \( \langle \cdots \rangle \Omega \) denotes the average over the Fermi level, which is defined as \( \langle A(\hat{k}_1, \cdots, \hat{k}_n) \rangle \Omega \equiv \int \cdots \int \frac{d\hat{k}_1 \cdots d\hat{k}_n}{(4\pi)^3} A(\hat{k}_1 \cdots \hat{k}_n) \).

Since \( f(\hat{k}, \hat{k}') = 2\pi \left( Y_1^1(\hat{k}) - Y_1^{-1}(\hat{k}) \right) \left( Y_1^{-1}(\hat{k}') + Y_1^1(\hat{k}') \right) / 3i \), angular integration such as \( \int d\Omega_\omega Y_3^{M-\sigma/2}(\hat{k}) Y_1^{\pm \lambda}(\hat{k}) Y_1^{\pm \lambda}(\hat{k}) \) appears in equation (13). This integral is finite only when \( l = 2, 4 \). Therefore, the interference of the \( f(l = 3) \) and \( d(l = 2) \) partial waves is essential for skew scattering [16]. After performing the angular integrations, equation (13) is given as

\[
\sum_{\sigma} \frac{\sigma}{2} \left( J_{k\sigma,k'\sigma}^d f \left( \hat{k}, \hat{k}' \right) \right)^\Omega = \frac{4}{7} J_d J_f.
\]

Using the relations \( |G_k^R(0)|^2 \approx \frac{e}{\gamma} \delta(\mu - \varepsilon_k) \) for small \( \gamma \), and \( \frac{1}{N} \sum_{k} G_k^R(0) = g^R(0) = -i\pi N(0) \), equation (10) is transformed into

\[
\sigma_{\text{SH}}^{ss} = -\frac{2\pi}{7} n_{\text{imp}} J_d J_f^2 N(0) \sum_{k,k'} \frac{\partial\varepsilon_k}{\partial k} \frac{\partial\varepsilon_{k'}}{\partial k'} \frac{1}{\gamma^2} \delta(\mu - \varepsilon_k) \delta(\mu - \varepsilon_{k'}).
\]

Since \( \gamma_1 = 3\pi n_{\text{imp}} N(0) J_f^2 \) in the Born approximation, \( \sigma_{\text{SH}}^{ss} \) is given by

\[
\sigma_{\text{SH}}^{ss} = -\frac{2\pi}{7} \frac{1}{21\pi^2} J_d J_f^2 \frac{\gamma_f}{\gamma^*}.
\]

Now, we derive the skew scattering term using the T-matrix approximation, which gives the exact result for \( n_{\text{imp}} \ll 1 \). In this case, \( T_{k\sigma,k\sigma}^{(2)R}(0) \) in equation (10) is replaced with the full T-matrix \( T_{k\sigma,k\sigma}^{(2)}(0) = T_{k\sigma,k\sigma}^{(2)R}(0)(1 - G_k^R(0))^{-1} \), where the first-order term in \( J_{k\sigma,k\sigma}^f \) has been dropped. The diagrammatic expression for \( \sigma_{\text{SH}}^{ss} \) is shown in figure 2(b). The angular integration in the T-matrix approximation can be performed as

\[
\text{Re} \sum_{\sigma} \frac{\sigma}{2} \left( T_{k\sigma,k\sigma}^{(2)R}(0) J_{k\sigma,k\sigma}^f f \left( \hat{k}, \hat{k}' \right) \right)^\Omega = \frac{4}{7} J_d J_f^2 \frac{\pi N(0)}{1 + ((\pi N(0) J_f)^2)^{-1}}.
\]

Thus, \( \sigma_{\text{SH}}^{ss} \) in the T-matrix approximation is given by \( (1 + (\pi N(0) J_f)^2)^{-1} \) times equation (14). Therefore, the expression for the skew scattering term in equation (15) is valid beyond the Born approximation, by considering \( \gamma_1 \) given by equation (9).
Here, we discuss the temperature dependence of $\sigma_{\text{SH}}^{ss}$. From equation (15), $\sigma_{\text{SH}}^{ss}$ is proportional to $1/\gamma_t$ when $\gamma_t \gg \gamma_0$. Since $\gamma_t \propto -\ln(T/W_{\text{band}})$ for $T \gg T_K$ [18], $\sigma_{\text{SH}}^{ss}$ decreases as the temperature decreases. In contrast, $\sigma_{\text{SH}}^{ss} \propto \gamma_f$ for $\gamma_f \ll \gamma_0$, which increases at low temperatures if $\gamma_0$ is constant.

3.3. Side jump term

In this section, we briefly discuss the side jump term $\sigma_{\text{SH}}^{sj}$. In the $T$-matrix approximation, $\sigma_{\text{SH}}^{sj}$ is given by

$$\sigma_{\text{SH}}^{sj} = -\frac{e}{2\pi} \frac{1}{N^2} \sum_{k_\alpha, k'_{\alpha'}} \frac{\sigma - \partial J_{k_\alpha, k'_{\alpha'}}}{\delta k_x} \frac{\partial \varepsilon_{k'_{\alpha'}}}{\delta k_y} |G_{k_\alpha, k'_{\alpha'}}^{R}(0)|^2 \left[ T_{R k_\alpha, k'_{\alpha'}}^{fR} G_{k_{\alpha'}}^{R}(0) + \langle R \leftrightarrow A \rangle \right],$$

(17)

whose diagrammatic expression is shown in figure 3. In contrast to the skew scattering term, $J_d$ is not necessary for the side jump term. As reported in [20], the large anomalous velocity $v_{kM\sigma}$, which is not perpendicular to the Fermi surface, arises from the $k$-derivative of the phase factor in the c–f mixing potential as follows:

$$\frac{\partial V_f^r}{\partial k_x} \equiv v_{kM\sigma}^a = -i \left( M - \frac{\sigma}{2} \right) \frac{k_y}{k_x^2 + k_y^2} V_f^r.$$  

(18)

The final expression for the side jump term in the $T$-matrix approximation is given by

$$\sigma_{\text{SH}}^{sj} = -\frac{e \gamma_t}{2\pi} \frac{4 k_F}{3 \pi} \frac{\gamma'_f}{\gamma_f},$$

(19)

where we have used the relations $\sum_{M\sigma} M\sigma |V_{kM\sigma}^f|^2 = 3|V_f|^2(1 - \sin^2 \theta)$ and $\sum_{M\sigma} \sigma^2 |V_{kM\sigma}^f|^2 = 6|V_f|^2$. By using equations (15) and (19), the ratio $\sigma_{\text{SH}}^{sj}/\sigma_{\text{SH}}^{ss}$ is given by $\frac{28}{28} \frac{\gamma_t}{\gamma_f \gamma_{\text{imp}}}$. Therefore, $\sigma_{\text{SH}}^{sj}$ will exceed $\sigma_{\text{SH}}^{ss}$ in dirty metals. However, the skew scattering contribution will be dominant in simple metals with magnetic impurities for $n_{\text{imp}} \lesssim 0.01$ ($\rho_0 \lesssim 10 \mu \Omega \text{cm}$) [16].

4. Discussions

4.1. Origin of the skew scattering mechanism

In section 3.2, we have studied the SHC due to skew scattering by using the Green function method. In this section, we discuss the origin of the skew scattering mechanism.
based on the Boltzmann transport theory. For this purpose, we study a simplified two-orbital model with \( M = \pm 5/2 \), assuming the strong crystalline electric field. In this model, \( V_{f,i}^{\dagger} = -\sigma \sqrt{4\pi/7} V_f \sqrt{6} Y_3^{2\sigma} (\hat{k}) \delta_{M,-2\sigma} + Y_3^{2\sigma} (\hat{k}) \delta_{M,2\sigma} \). Since \( V_f^{\dagger} \propto Y_3^{2\sigma} (\hat{k}) \propto e^{-2i\phi_k} \) approximately, the second-order term of \( T_f \) is simply given as \( T_f^{(2)\text{R}} \sim -iN(0)e^{-2i(\phi_k-\phi_f)} \). In the Boltzmann transport theory, the spin Hall resistivity due to skew scattering is approximately, the second-order term of \( T_f \) is simply given as \( T_f^{(2)\text{R}} \sim -iN(0)e^{-2i(\phi_k-\phi_f)} \). In the Boltzmann transport theory, the spin Hall resistivity due to skew scattering is \( \rho_{\Delta \text{SH}} = \sum_{\sigma} \langle k, k' | w(k \sigma \to k' \sigma) \rangle_{\text{FS}} \), where \( w \) represents the scattering probability, which is proportional to \( n_{\text{imp}} |T_{k\sigma,k'\sigma}^{(2)\text{R}} + J_{k,k'}^{\dagger}|^2 \) due to Fermi’s golden rule in the present model. According to [16], skew scattering occurs when the scattering probability includes an asymmetric component \( w^{\text{as}}(k \to k') \propto \Im e^{i(\phi_k-\phi_f)} \propto (\hat{k} \times \hat{k}')_z \). In the present model, \( w^{\text{as}} \) arises from the interference of the \( f \) and \( d \) scattering channel, \( w^{\text{as}}(k \sigma \to k' \sigma) \propto \Im e^{i(\phi_k-\phi_f)} \) in this model since \( J_{k,k'}^{\dagger} \) contains the term \( Y_2^{\pm 2}(\hat{k}) \left[ Y_2^{\pm 2}(\hat{k}') \right]^* \propto e^{\pm 2i(\phi_k-\phi_f)} \). In summary, a conduction electron with \( \sigma \) hybridizes with \( l_{z} = -3\sigma \) state due to the strong SOI, and therefore the spin-dependent skew scattering probability \( w^{\text{as}}(k \sigma \to k' \sigma) \propto \Im e^{i(\phi_k-\phi_f)} \) arises from the interference of the \( f \) and \( d \) angular momenta. Thus, the origin of the SHE due to skew scattering mechanism is well understood based on the simplified two-orbital model.

4.2. Estimations of the spin Hall angle and the skew scattering term

Firstly, we estimate the magnitude of the spin Hall angle \( \tan \alpha_{\text{SH}} \equiv \frac{\sigma_{\Delta \text{SH}}}{\sigma_{xx}} \). The longitudinal conductivity is given by \( \sigma_{xx} = \frac{e^2 n}{2m\pi} \), where \( n = k_F^2/3\pi^2 \) is the density of the conduction electrons. Then, the spin Hall angle is given by

\[
\tan \alpha_{\text{SH}} = \frac{8\pi^2}{7} J_d N(0) \frac{\gamma_f}{\gamma_f},
\]

where \( \gamma_f \) is the Fermi energy. If \( J_d N(0) \ll 1 \), equation (20) can be rewritten as \( \tan \alpha_{\text{SH}} = -\frac{8\pi^2}{7} \gamma_f \). In the case where \( \gamma_f \gg \gamma_0 \), \( \alpha_{\text{SH}} \) reaches \( \sim 0.3 \) for \( \delta_2 \sim 0.1 \). Therefore, giant SHE will be realized in metals with rare-earth impurities.

Recently, giant SHE was observed in FePt/Au devices [22]. It was reported that the large spin Hall angle in Au is \( \sim 0 \). Since the intrinsic SHE in Au is small due to the Fermi surface being mainly composed of \( s \)-electrons [7], the observed giant SHE may be explained by the skew scattering mechanism. Very recently, the extrinsic SHE in AuFe metal has been studied with the first-principles band structure calculations independently of our study [23].

Secondly, we estimate the magnitude of the skew scattering term for \( \delta_2 = 0.1 \), where the spin Hall angle is 0.3. Then, \( \sigma_{\Delta \text{SH}} = (\hbar/2e) \Gamma (0.3) \times \sigma_{xx} \) from the above discussion. In the case where the resistivity is 10 \( \mu \Omega \) cm, we obtain \( \sigma_{\Delta \text{SH}} = 15000 (\hbar/e) \Omega^{-1} \text{ cm}^{-1} \) if we set the length of unit cell \( a = 4 \) \( \text{Å} \), which is the case for Pt and Au. The obtained skew scattering term is about 60 times larger than the SCH observed in Pt: 240 (\( \hbar/e \) \( \Omega^{-1} \text{ cm}^{-1} \). Since the resistivity in Pt and Au is \( \rho \sim 10 \mu \Omega \text{ cm} \) and \( \rho \sim 1 \mu \Omega \text{ cm} \), respectively, the skew scattering term due to magnetic impurities should dominate the intrinsic term. In contrast, the skew scattering due to ‘natural randomness’ has not been observed in the AHE in transition metals, even in \( \rho \sim 10 \mu \Omega \text{ cm} \) [24]. Therefore, the orbital degrees of freedom of the impurity is indispensable for the giant extrinsic SHE.

Finally, we compare the magnitude of \( \sigma_{\Delta \text{SH}}^{\text{as}} \) and \( \sigma_{\Delta \text{SH}}^{\text{sj}} \). If we put \( k_F = \pi/a \) and \( \gamma \approx \gamma_f \) in equation (19), \( \sigma_{\Delta \text{SH}}^{\text{sj}} = \frac{e^4}{2\pi a} \sim -4(3) \times 10^3 (h/e) \Omega^{-1} \text{ cm}^{-1} \) for \( a = 4 \) \( \text{Å} \). Therefore, the skew...
scattering term will be comparable in magnitude to the side jump term when \( \rho \sim 100 \mu \Omega \text{ cm} \). Thus, the skew scattering term is dominant over the side jump term when \( \rho \ll 100 \mu \Omega \text{ cm} \).

5. Summary

In summary, we have studied the extrinsic SHE due to magnetic impurities, where Ce and Yb impurities have been discussed as a typical case. The obtained expression for the skew scattering term shows that the spin Hall angle reaches \( O(10^{-1}) \), which is more than ten times larger than that in Pt. Therefore, we propose that the extrinsic SHE due to rare-earth impurities is useful as a method for creating large spin current efficiently. The present study also suggests that the giant SHE will arise from other rare-earth atoms as well as 4d and 5d atoms that possesses strong SOI. As in the case for the intrinsic SHE, orbital degrees of freedom are essential for generating a huge extrinsic SHE. According to equation (20), giant spin Hall angle always appears when \( \gamma_I \gg \gamma_0 \). Thus, the Kondo resonance, \( N(0)J_f \gg 1 \), is not a necessary condition for the giant SHE. In contrast, the AHC (= \( \sigma_{xy}/H_z \)) is enhanced by the Kondo effect since it is proportional to the uniform susceptibility.

In a similar way to the present study, we can obtain \( \sigma_{SH}^{SS} \) and \( \sigma_{SH}^{SJ} \) for \( J = 7/2 \), which is the case for Yb-impurities. In this case, \( a_{m}^{M} = (7/2 + M \sigma)/7 \right)^{1/2} \delta_{m,M-\sigma/2} \), and \( \gamma_I \) is given by \((4/3)\) times equation (9). The obtained \( \sigma_{SH}^{SS} \) and \( \sigma_{SH}^{SJ} \) are given by \((-3/4)\) times equation (15) and equation (19), respectively.

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References

[1] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 Phys. Rev. Lett. 92 126603
[2] Murakami S, Nagaosa N and Zhang S C 2004 Phys. Rev. B 69 235206
[3] Saitoh E, Ueda M, Miyaji H and Tatara G 2006 Appl. Phys. Lett. 88 182509
[4] Kimura T, Otani Y, Sato T, Takahashi S and Maekawa S 2007 Phys. Rev. Lett. 98 156601
Vila L, Kimura T and Otani Y 2007 Phys. Rev. Lett. 99 226604
Otani Y et al (unpublished)
[5] Kontani H, Tanaka T, Hirashima D S, Yamada K and Inoue J 2008 Phys. Rev. Lett. 100 096601
[6] Kontani H, Naito M, Hirashima D S, Yamada K and Inoue J 2007 J. Phys. Soc. Japan 76 103702
[7] Tanaka T, Kontani H, Naito M, Naito T, Hirashima D S, Yamada K and Inoue J 2008 Phys. Rev. B 77 165117
[8] Kontani H, Tanaka T, Hirashima D S, Yamada K and Inoue J 2008 arXiv:0806.0210
[9] Guo G Y, Murakami S, Chen T-W and Nagaosa N 2008 Phys. Rev. Lett. 100 096401
[10] Dyakonov M I and Perel V I 1971 JETP Lett. 13 467
[11] Hirsch J E 1999 Phys. Rev. Lett. 83 1834
[12] Takahashi S and Maekawa S 2008 J. Phys. Soc. Japan 77 031009
[13] Smit J 1958 Physica 24 39
[14] Berger L 1970 Phys. Rev. B 2 4559
[15] Crepieux A and Bruno P 2001 Phys. Rev. B 64 014416
[16] Fert A 1973 J. Phys. F: Mat. Phys. 3 2126
Fert A and Friederich A 1976 Phys. Rev. B 13 397
[17] Fert A and Jaoul O 1972 Phys. Rev. Lett. 28 303

New Journal of Physics 11 (2009) 013023 (http://www.njp.org/)
[18] Hewson A C 1993 *The Kondo Problem to Heavy Fermions* Cambridge Studies in Magnetism (Cambridge: Cambridge University Press)  
Yosida K 1996 *Theory of Magnetism* (Berlin: Springer)  
[19] Yamada K and Yosida K 1975 *Prog. Theor. Phys.* 53 970  
[20] Kontani H and Yamada K 1994 *J. Phys. Soc. Japan* 63 2627  
[21] Haldane F D M 1978 *Phys. Rev. Lett.* 40 416  
[22] Seki T, Hasegawa Y, Mitani S, Takahashi S, Imamura H, Maekawa S, Nitta J and Takanashi K 2008 *Nature* 7 125  
[23] Guo G Y, Maekawa S and Nagaosa N 2008 arXiv:0809.2921  
[24] Miyasato T, Abe N, Fujii T, Asamitsu A, Onoda S, Onose Y, Nagaosa N and Tokura Y 2007 *Phys. Rev. Lett.* 99 086602