Generalized protocol for distribution and concentration of Quantum information

Partha Mukhopadhyay
Cryptololgy Research Group
Applied Statistics Unit
Indian Statistical Institute
Kolkata-700108
India

22.08.2003

Abstract

Alice can distribute a quantum state $|\phi\rangle$ to $N$ spatially separated parties (say Bobs) by telecloning. It is possible for Charlie to reconstruct the quantum state to him if he shares same kind of telecloning quantum channel with Bobs using only LOCC. For $N = 3$ reconstruction can be done faithfully using Smolin’s 4 party unlockable bound entangled state as shared channel. In this note we investigate, in multiparty setting, the general structure of quantum channel and protocol by which faithful distribution and concentration of quantum information can be done.

1 Introduction

In the usual quantum teleportation scheme [1] an unknown quantum state can be faithfully transmitted to a remote receiver via an initially shared maximally entangled state between sender and receiver. This can be considered as basic unit of quantum communication network. But in the case of distributed multiparty communication information is transferred usually in the following way. Firstly the unknown state is distributed among many distant spatially separated intermediate receivers via an initially shared channel between them. Then the original state is remotely concentrated back to the actual receiver by applying Local Operations and Classical Communication (LOCC) by the intermediate receivers via another channel. In both distribution and concentration we require initially shared entangled state as channel state among the parties in the network. In a recent work [2] it was shown that an unknown state of a 2-level system (called qubit) can be distributed among $N$ distant qubits such that each qubit is either optimally cloned or anti-cloned. This process is called telecloning. As an example a sender (Alice, say) first distributes an unknown qubit $|\phi\rangle = \alpha |0\rangle + \beta |1\rangle$ among three spatially separated distant qubits ($M_a$, $M_o$, $M_c$) and after distribution the state becomes

$$|\psi_D\rangle_{M_aM_oM_c} = \alpha \sqrt{\frac{2}{3}} \left\{ |0\rangle |00\rangle + \frac{1}{\sqrt{2}} |1\rangle (|01\rangle + |10\rangle) \right\} +$$

$$\beta \sqrt{\frac{2}{3}} \left\{ |1\rangle |11\rangle + \frac{1}{\sqrt{2}} |0\rangle (|01\rangle + |10\rangle) \right\}$$

(1)
where the first qubit is the anticloned and the last two are cloned ones. Now this distributed state can again be concentrated to another single qubit (holding by Bob, say) using Smolin’s four qubit unlockable bound entangled state given by $\frac{1}{4}(P[\phi^+] + P[\phi^-] + P[\psi^+] + P[\psi^-])$ shared between parties holding $(M_a, M_o, M_e)$ and Bob (Smolin state [4]) using only LOCC [4]. $\phi^+, \phi^-, \psi^+, \psi^-$ are Bell’s state.

In this note we investigate a general class of $N$ party pure and mixed state which can be used for faithful distribution and concentration of qubit using only LOCC. We also propose the protocols used for distribution and concentration. We observe that telecloning state and Smolin’s state belong to the generalized class.

2 Distribution and concentration of qubit

In this section, we will show how an unknown state can be distributed among several intermediate spatially separated parties and then again remotely reconstructed to another distant party without any global operation.

Suppose we have $(N+1)$ parties where $N$ is odd, and Alice, one among of them, has a unknown qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where, $|\alpha|^2 + |\beta|^2 = 1$. Alice firstly distributes her qubit among the $N$ Bobs, and Bobs then reconstruct the original state to another remote party Charlie, without any global operation.

So the protocol has two phase. We call the first phase as distribution phase and the second phase as concentration phase.

In the first phase, Alice shares the following state with $N$ Bobs

$$\chi^{(1)}_{A,B_1,\ldots,B_N} = \left(\frac{1}{\sqrt{2}}\right) \left([0]_A \otimes \sum_i \alpha_i |S_i\rangle_{B_1,\ldots,B_N}ight) + \left([1]_A \otimes \sum_i \alpha_i |S_i\rangle_{B_1,\ldots,B_N}\right). \quad (2)$$

Where $|S_i\rangle_{B_1,\ldots,B_N}$ is the qubit wise complement state of $|S_i\rangle_{B_1,\ldots,B_N}$. The state is properly normalized, i.e $\sum_i \alpha_i \bar{\alpha}_i = 1$. Where $\bar{a}$ is the complex conjugate of $a$. In each $|S_i\rangle_{B_1,\ldots,B_N}$ the no of $|0\rangle_{B_i}$’s is odd. Now Alice performs Bell state measurement on her two qubits, the unknown qubit and one qubit of the shared channel, where Bell states are

$$|Bell_1\rangle = |\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}},$$

$$|Bell_2\rangle = |\psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}},$$

$$|Bell_3\rangle = |\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

and

$$|Bell_4\rangle = |\phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}.$$  

Depending on her outcome, which is one of the following four states $\phi^+, \phi^-, \psi^+, \psi^-$, Alice makes phone call to $N$ Bobs to perform usual unitary operation $I, \sigma_z, \sigma_x, \sigma_y$ respectively on their own qubits i.e each Bob will individually apply $I$ for measurement result $\phi^+$, $\sigma_z$ for $\phi^-$, $\sigma_x$ for $\psi^+$ and $\sigma_y$ for $\psi^-$. After this the state of the $N$ Bobs’ become $|\xi^{(1)}_i\rangle = \frac{1}{k}(\alpha \sum_i a_i |S_i\rangle + \beta \sum_i a_i |\bar{S}_i\rangle)$, where $k$ is the normalizing factor. We note that the telecloning channel is a member of this general class channel shared between Alice and Bobs.
Now instead of using $|\chi^{(1)}\rangle$, as a channel state, if we take the channel state as a mixed one, which is the mixture of any no. of states of the form $|\chi^{(j)}\rangle$, i.e. $\rho = \sum_{j} p_j P[|\chi^{(j)}\rangle]$, with $\sum_j p_j = 1$

where

$$
|\chi^{(1)}\rangle_{A,B_1,...,B_N} = \left(1/\sqrt{2}\right)[|0\rangle_A \otimes \sum_i a_i |S_i\rangle_{B_1,...,B_N} + |1\rangle_A \otimes \sum_i a_i |\bar{S}_i\rangle_{B_1,...,B_N}],
$$

$$
|\chi^{(2)}\rangle_{A,B_1,...,B_N} = \left(1/\sqrt{2}\right)[|0\rangle_A \otimes \sum_i b_i |S_i\rangle_{B_1,...,B_N} + |1\rangle_A \otimes \sum_i b_i |\bar{S}_i\rangle_{B_1,...,B_N}],
$$

and so on, then after using the same distribution protocol the Bobs will share a state $\sigma = \sum_j q_j P[|\xi^{(j)}\rangle]$ where $\sum_j q_j = 1$.

We note that Smolins 4 party bound entangled state belongs to the mixed state channel for $N = 3$, $p_i$’s are equal and the coefficients of $|S_i\rangle$ (and so for $|\bar{S}_i\rangle$) are same for $|\chi^{(1)}\rangle$, $|\chi^{(2)}\rangle$ and $|\chi^{(3)}\rangle$.

In the concentration phase, Charlie wants to reconstruct the qubit to him. Charlie shares a properly normalized state

$$
|\chi^{(2)}_{B_1',...,B_N',C}\rangle = \left(1/\sqrt{2}\right)[(\sum_i b_i |S_i\rangle) \otimes |0\rangle + (\sum_i \bar{b}_i |\bar{S}_i\rangle) \otimes |1\rangle]
$$

with $N$ Bobs among whom the original state was distributed, where $\sum_i b_i \bar{b}_i = 1$ and $\bar{b}_i$ is complex conjugate of $b_i$. Now each of the Bob performs Bell measurement on his two qubits and let Bob$_i$ gets $|\text{Bell}^{(i)}\rangle$. Bobs inform their measurement results to Charlie by phone call. Charlie then performs unitary operation on his qubit given by $i \in \{1,N\} \Pi(\sigma_k^{(i)})$, where $\sigma_k^{(i)}$ is $I$, $\sigma_x$, $\sigma_y$ or $\sigma_z$, depending on whether Bob$_i$’s outcome is $\phi^+$, $\psi^+$ $\psi^-$ or $\phi^-$. For example if Bob$_1$ gets $|\phi^+\rangle$, Bob$_2$ gets $|\psi^-\rangle$ ... Bob$_N$ gets $|\psi^+\rangle$ then Charlie will perform $I, \sigma_y, ... , \sigma_x$. After the operation done by Charlie the state $|\psi\rangle$ is formed at Charlie’s end exactly. This protocol holds good for the mixed state channel also.

Interestingly this protocol does not work if $N$ is even. We are able to propose a general protocol which works for any $N$.

As before, Alice holding a qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where, $|\alpha|^2 + |\beta|^2 = 1$, wants to distributes the qubit among $N$ Bobs.

The entangled channel shared among Alice and $N$ Bobs are given by

$$
|\chi^{(1)}\rangle_{A,B_1,...,B_N} = \left(1/\sqrt{2}\right)[|0\rangle_A \otimes \sum_{i=0}^{n-1} a_i |0^{(n-i)}1^i\rangle_{B_1,...,B_N} + |1\rangle_A \otimes \sum_{i=0}^{n-1} a_i |1^{(n-i)}0^i\rangle_{B_1,...,B_N}].
$$

The state is properly normalized. Now Alice performs Bell state measurement on her two particles and makes phone calls to $N$ Bobs informing the measurement outcomes.

Now if Alice’s outcome is $\phi^+$, each Bob operates $I$ on their corresponding qubit. If Alice’s outcome is $\phi^-$, first of the intermediate Bobs performs $\sigma_z$ on his qubit and other performs $I$ on their corresponding qubit.
Now if Alice’s outcome is $\psi^+$, each Bob operates $\sigma_z$ on their corresponding qubit. If Alice’s outcome is $\psi^-$, first of the intermediate Bobs performs $\sigma_y$ on his qubit and other performs $\sigma_x$ on their corresponding qubit.

So after the distribution the states between $N$ Bobs will be given by $|\xi(1)\rangle = \frac{1}{k}(\alpha \sum_{i=0}^{n-1} a_i |0^{(n-i)}1^i\rangle + \beta \sum_{i=0}^{n-1} a_i |1^{(n-i)}0^i\rangle)$, where $k$ is the normalizing factor.

Instead of using pure state channel if we use mixed state channel then the result will be of same nature as in the last protocol.

Let us now consider the distribution phase. Charlie, who wants to reconstruct the qubit to him, shares properly normalized entangled channel $|\chi(2)\rangle_{B_1,...,B_N,C} = \left(\frac{1}{\sqrt{2}}\right)|\sum_{i=0}^{n-1} b_i |0^{(n-i)}1^i\rangle_{B_1,...,B_N} \otimes |0\rangle_C + \sum_{i=0}^{n-1} b_i |1^{(n-i)}0^i\rangle_{B_1,...,B_N} \otimes |1\rangle_C\right]$. (7)

with $N$ Bobs’.

Now each Bob performs bell state measurement on his two qubit and informs Charlie the outcome of their respective measurement.

Now Charlie can reconstruct the qubit if he follows the following Algorithm:

1. Charlie initialize a counter $S$ to zero.

2. Whenever Charlie finds the measurement outcome of any Bob $\phi^-$ or $\psi^-$, he increases $S$ by one.

3. Charlie stores the measurement outcome of $B_1$.

Now, after knowing measurement outcomes from each Bob Charlie follows the following set of rules to reconstruct the qubit:

1. If $S$ is even then Charlie can reconstruct the distributed qubit to him by operating $I$ or $\sigma_z$ or $\sigma_x$ or $\sigma_y$ on his qubit whenever the corresponding measurement outcome of $B_1$ is $\phi^+$ or $\phi^-$ or $\psi^+$ or $\psi^-$ respectively.

2. If $S$ is odd then Charlie can reconstruct the distributed qubit to him by operating $\sigma_z$ or $I$ or $\sigma_y$ or $\sigma_x$ on his qubit whenever the corresponding measurement outcome of $B_1$ is $\phi^+$ or $\phi^-$ or $\psi^+$ or $\psi^-$ respectively.

If we consider any mixture of the pure state as channel (as in the 1$t$ protocol) and follow the same protocol then also faithful reconstruction of the qubit can be done.

### 3 Conclusion

We have proposed a large class of pure and mixed state multiparty quantum channel which can be faithfully used for distribution and concentration of quantum information. In the case of first protocol, we observed that telecloning channel and Smolin’s state channel are the special cases of our pure and mixed state channel respectively. But surprisingly we could not extend the protocol for $N$ even.

In the second case we are able to present a class of channels and corresponding protocol which can be used for any $N$. It remains an interesting problem to find, what will be the most general
channel and protocol that can be used for any $N$.

Author acknowledges Guruprasad Kar, Sibasish Ghosh, Anirban Roy of Indian Statistical Institute (India) and Debasis Sarkar of Calcutta University (India) for many useful suggestion and helpful discussion during this work.

E-mail: partha@isi@yahoo.com

References

[1] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W.K Wotters, *Phys. Rev. Lett.* **70** 1895 (1993).

[2] M. Murao, D. Jonathan, M. B. Plenio and V. Vedral *Phys. Rev. A* **59** 156 (1999); [quant-ph/9806082](quant-ph/9806082)

[3] J.A. Smolin, *Phys. Rev. A* **63** 032306 (2001); [quant-ph/0001001](quant-ph/0001001)

[4] M. Murao and V. Vedral *Phys. Rev. Lett.* **86** 352 (2001); [quant-ph/0008078](quant-ph/0008078)

[5] Todd A. Brun, [quant-ph/0102046](quant-ph/0102046)