Artificial Wormhole

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Abstract

It is shown that recently reported result by the OPERA Collaboration [1] of an early arrival time of muon neutrinos with respect to the speed of light in vacuum does not violate standard physical laws. We show that vacuum polarization effects in intensive external fields may form a wormhole-like object. The simplest theory of such an effect is presented and basic principles of formation of an artificial wormhole are also considered.

1. An actual wormhole requires the violation of averaged null energy condition (ANEC) Ref. [2] and is commonly supposed to be forbidden in classical physics. However, the situation changes when we consider a virtual wormhole. It represents a quantum topology fluctuation which takes place at very small (Plankian) scales and lasts for a very short period of time [3][4]. It does not obey the Einstein equations and, therefore, ANEC cannot forbid the origin of such an object. By other words, virtual wormholes violate readily ANEC. Moreover, virtual wormholes play an important role in particle physics. First, they may introduce in a natural way the cutoff at very small scales and remove divergencies in quantum field theory [5]. Secondly, they predict new phenomena and open new perspectives in applied physics. In particular, by applying an external field one may govern the intensity of such fluctuations and thus organize an artificial wormhole. It is curious that actual traversable wormholes may be supported by arbitrarily small quantities of “exotic matter” Ref. [6], while in our case the latter is represented by virtual wormholes.

We note that such an artificial wormhole is an extremely complex object (from microphysics standpoint) which requires numerical study. In the present paper we consider the simplest model which introduces an anisotropy in the speed of light which allows to give the principle explanation of the anomaly observed by the OPERA Collaboration [1] (faster than light travel of muon neutrinos). We also point out to Ref. [7] where another approach has been suggested to explain the OPERA anomaly.

2. In what follows we widely use results of [5] and will not repeat them here. A virtual wormhole can be described as follows. It is convenient from the very beginning to use the Euclidean approach (e.g., see Refs. [4], [8]-[10] and references therein). Then the simplest virtual wormhole is described by the
metric \((\alpha = 1, 2, 3, 4)\)

\[ ds^2 = h^2 (r) \delta_{\alpha\beta} dx^\alpha dx^\beta, \]

where

\[ h (r) = 1 + \theta (a - r) \left( \frac{a^2}{r^2} - 1 \right) \]

and \(\theta (x)\) is the step function. Such a wormhole has vanishing throat length. Indeed, in the region \(r > a\), \(h = 1\) and the metric is flat, while the region \(r < a\), with the obvious transformation \(y^\alpha = \frac{a^2}{r^2} x^\alpha\), is also flat for \(y > b\). Therefore, the regions \(r > a\) and \(r < a\) represent two Euclidean spaces glued at the surface of a sphere \(S^3\) with the center at the origin \(r = 0\) and radius \(r = b\). Such a space can be described with the ordinary double-valued flat metric in the region \(r > a\) by

\[ ds^2 = \delta_{\alpha\beta} dx^\alpha dx^\beta, \]

where the coordinates \(x^\alpha_\pm\) describe two different sheets of space. Now, identifying the inner and outer regions of the sphere \(S^3\) allows the construction of a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by gluing the two spaces in \((3)\) by motions of the Euclidean space (the Poincare motions). If \(R_\pm\) is the position of the sphere in coordinates \(x^\mu_\pm\), then the gluing is the rule

\[ x^\mu_+ = R^\mu_+ + \Lambda^\mu_\nu (x^\nu_\prime - R^\nu_+), \]

where \(\Lambda^\mu_\nu \in O(4)\), which represents the composition of a translation and a rotation of the Euclidean space (Lorentz transformation). In terms of common coordinates such a wormhole represents the standard flat space in which the two spheres \(S^3_\pm\) (with centers at positions \(R_\pm\)) are glued by the rule \((4)\). We point out that the physical region is the outer region of the two spheres. Thus, in general, the wormhole is described by a set of parameters: the throat radius \(b\), positions of throats \(R_\pm\), and rotation matrix \(\Lambda^\mu_\nu \in O(4)\).

3. Consider now the simplest massless scalar field and construct the Green function in the presence of a gas of virtual wormholes.

The Green function obeys the Laplace equation

\[ -\Delta G (x, x') = 4\pi^2 \delta (x - x') \]

with proper boundary conditions at throats (we require \(G\) and \(\partial G/\partial n\) to be continual at throats). The Green function for the Euclidean space is merely \(G_0 (x, x') = \frac{1}{(x-x')^2}\) (and \(G_0 (k) = 4\pi^2/k^2\) for the Fourier transform). In the presence of a single wormhole which connects two Euclidean spaces this equation admits the exact solution. For outer region of the throat \(S^3\) the source \(\delta (x - x')\) generates a set of multipoles placed in the center of sphere which gives the corrections to the Green function \(G_0\) in the form (we suppose the center of the sphere at the origin)

\[ \delta G = -\frac{1}{x^2} \sum_{n=1}^{\infty} \frac{1}{n + 1} \left( \frac{a}{x^2} \right)^{2n} \left( \frac{x'}{x} \right)^{n-1} Q_n, \]
where \( Q_n = \frac{4\pi}{2n} \sum_{l=0}^{n-1} \sum_{m=-l}^{l} Q'_{nlm} Q_{nlm} \) and \( Q_{nlm}(\Omega) \) are four-dimensional spherical harmonics e.g., see Ref. [11]. In the present paper we shall consider a dilute gas approximation and, therefore, it is sufficient to retain the lowest (monopole) term only. A single wormhole which connects two regions in the same space is a couple of conjugated spheres \( S^3_\pm \) of the radius \( a \) with a distance \( \vec{X} = \vec{R}_+ - \vec{R}_- \) between centers of spheres. So the parameters of the wormhole are \( \xi = (a, R_+, R_-) \). The interior of the spheres is removed and surfaces are glued together. Then the proper boundary conditions (the actual topology) can be accounted for by adding the bias of the source

\[
\delta(x - x') \to N(x, x') = \delta(x - x') + b(x, x').
\]

In the approximation \( a/X \ll 1 \) (e.g., see for details [12]) the bias takes the form

\[
b_0(x, x', \xi) = \frac{a^2}{2} \left( \frac{1}{(R_- - x')^2} - \frac{1}{(R_+ - x')^2} \right) \left[ \delta^4(x - R_+) - \delta^4(x - R_-) \right].
\]

We expect that virtual wormholes have throats \( a \sim \ell_{\text{pl}} \) of the Plankian size, while in the present paper we are interested in much larger scales. Therefore, the form (7) is sufficient for our aims. However this form is not acceptable in considering the short-wave behavior and vacuum polarization effects (stress energy tensor). In the last case one should account for the finite value of the throat size and replace in (7) the point-like source with the surface density (induced on the throat) i.e., see for details [5],

\[
\delta^4(x - R_\pm) \to \frac{i}{2\pi\sigma} \delta(|x - R_\pm| - a).
\]

In the rarefied gas approximation the bias function for the gas of wormholes is additive, i.e.,

\[
b_{\text{total}}(x, x') = \sum b_0(x, x', \xi_i) = N \int b_0(x, x', \xi) F(\xi) d\xi,
\]

where

\[
F(\xi) = \frac{1}{N} \sum_{i=1}^{N} \delta(\xi - \xi_i).
\]

We assume a homogeneous but anisotropic distribution \( F(\xi) = F(a, X) \), then for the bias we find

\[
b_{\text{total}}(x - x') = \int a^2 \left( \frac{1}{R_-^2} - \frac{1}{R_+^2} \right) \delta^4(x - x' - R_+) NF(a, X) d\xi
\]

Consider the Fourier transform \( F(a, X) = \int F(a, k) e^{-i k X} \frac{d^4 k}{(2\pi)^4} \) and using the integral \( \frac{1}{2\pi} = \int \frac{4\pi^2}{k^2} e^{-i k x} \frac{d^4 k}{(2\pi)^4} \) we find for \( b(k) = \int b(x) e^{i k x} d^4 x \) the expression

\[
b_{\text{total}}(k) = N \int a^2 \frac{4\pi^2}{k^2} (F(a, k) - F(a, 0)) da.
\]

\(^1\)The additional parameter (rotation matrix \( U \)) is important only for multipoles of higher orders.
Consider now a particular form for $F(a, X)$, e.g.,

$$NF(a, X) = n\delta (a - a_0) \frac{1}{2} \left( \delta^4 (X - r_0) + \delta^4 (X + r_0) \right),$$

(12)

where $n = N/V$ is the density of wormholes. Such a distribution corresponds to a coherent set of wormholes with the throat $a_0$, oriented along the same direction $r_0$ and with the distance between throats $r_0 = |R_+ - R_-|$. We assume that $r_0 = (0, \vec{r}_0)$ has only spatial direction. Then $NF(a, k) = \int NF(a, X) e^{ikx} d^4x$ reduces to $NF(a, k) = n\delta (a - a_0) \cos \left( \vec{k} \cdot \vec{r}_0 \right)$. Thus from (11) we find

$$b(k) = -na^2 \frac{4\pi^2}{k^2} \left( 1 - \cos \left( \vec{k} \cdot \vec{r}_0 \right) \right).$$

(13)

In the vacuum case the background fluctuations have an isotropic and homogeneous distribution and form the background cutoff function $\overline{N}(k)$, so that the regularized vacuum Green function $G_{\text{reg}}(k)$ has the form

$$G_{\text{reg}}(k) = \overline{N}(k) G_0(k) = \frac{4\pi^2}{k^2} \overline{N}(k).$$

(14)

General properties of the cutoff is that $\overline{N}(k) \to 0$ as $k \gg k_{\text{pl}}$, and $\overline{N}(k) \to \text{const} \ll 1$ on the mass shell (as $k \ll k_{\text{pl}}$). We shall use Plankian units, i.e., $k_{\text{pl}} = 2\pi$.

4. Consider the structure of the bias of the unit source in the coordinate representation. Substituting (12) into (10) we find

$$b(x) = -na^2 \frac{2}{x^2} \left( \frac{1}{(x + r_0)^2} - \frac{1}{(x - r_0)^2} \right).$$

(15)

We recall that here $\frac{1}{(x-x')^2} = G_0(x, x')$ is the standard Euclidean Green function which, upon the continuation to the Minkowsky space, transforms to the Feynman propagator which is important in quantum field theory. However when considering the propagation of signals we should use the retardation Green function $G_0 \to G_{\text{ret}} (x, x') = \frac{1}{R} \delta(t' - t + \frac{1}{c} R)$, while the bias has the same structure (e.g., see Ref. [13]). Thus we see that the additional source represents three outgoing spherical waves which originate at positions $x = 0$ and $x = \pm r_0$. Since $r_0$ has only spatial direction the additional source $b(x)$ forms the wavefront which overruns the standard wave in the direction $\vec{r}_0$ which should lead to the observed anomaly $\Delta t = r_0/c$. The intensity of such an additional signal is described by the portion of the primary signal scattered on virtual wormholes which is given by $b = -\int b(x) d^4x$

$$b = 2\pi^2 na^2 r_0^2 \ll 1.$$  

(16)

We recall that the dilute gas approximation requires $2\pi^2 na^4 \ll 1$ (i.e., the portion of the volume cut by virtual wormholes should be sufficiently small), while the ratio $r_0/a$ may be an arbitrary parameter and therefore in general $b$ may reach the order of unity.
5. Consider now the generating functional (the partition function) which is used to generate all possible correlation functions in quantum field theory (and the perturbation scheme when we include interactions)

\[ Z_{\text{total}} (J) = \sum_{\tau} \sum_{\varphi} e^{-S_E} \]  

where the sum is taken over field configurations \( \varphi \) and topologies \( \tau \) (wormholes), the Euclidean action is

\[ S_E = -\frac{1}{2} (\varphi \Delta \varphi) + 4\pi^2 (J \varphi) , \]

and we use the notions \( (J \varphi) = \int J(x) \varphi(x) d^4 x \). Here \( J \) denotes an external current. The sum over field configurations \( \varphi \) can be replaced by the integral

\[ Z^* (J) = \int [D\varphi] e^{\frac{1}{2} (\varphi \Delta \varphi) - (J \varphi)} . \]

Upon the simple transformations

\[ \frac{1}{2} (\varphi \Delta \varphi) - (J \varphi) = \frac{1}{2} (\bar{\varphi} \Delta \bar{\varphi}) - \frac{1}{2} (J G J) , \]

where \( \bar{\varphi} = \varphi - G J \) and \( G \) is the background Green function \( \{14\} \), we cast the partition function to the form

\[ Z^* = \int [D\bar{\varphi}] e^{\frac{1}{2} (\bar{\varphi} \Delta \bar{\varphi}) - \frac{1}{2} (J G J)} = Z_0(G) e^{-\frac{1}{2} (J G J)} , \]

where \( Z_0(G) = \int [D\varphi] e^{\frac{1}{2} (\varphi \Delta \varphi)} \) is the standard expression and \( G = G(\xi_1, \ldots, \xi_N) \) is the Green function for a fixed topology, i.e., for a fixed set of wormholes \( \xi_1, \ldots, \xi_N \).

Consider now the sum over topologies \( \tau \). To this end we restrict with the sum over the number of wormholes and integrals over parameters of wormholes:

\[ \sum_{\tau} \to \sum_N \int \prod_{i=1}^N d\xi_i = \int [DF] \]

where \( F \) is given by \( \{19\} \). We point out that in general the integration over parameters is not free (e.g., it obeys the obvious restriction \( |\vec{R}^+ - \vec{R}^-| \geq 2a_i) \).

This defines the generating function as

\[ Z_{\text{total}} (J) = \int [DF] Z_0(G) e^{-\frac{1}{2} (J G J)} . \]

Since in the vacuum case virtual wormholes have a homogeneous distribution, in the Fourier representation the bias \( N(x, x', \xi) \to N(k, k', \xi) \) which gives \( N(k, k') = N(k, \xi) \delta(k - k') \), then we find compare with \( \{14\} \)

\[ G(k) = G_0(k) N(k, \xi). \]
Then for the total partition function we find

\[ Z_{\text{total}}(J) = \int [DN(k)] e^{-I(N(k))} e^{-\frac{i}{2} \sum_k \frac{4\pi^2}{k^2} |J_k|^2 N(k)}, \tag{24} \]

where \( \sum_k = \frac{L^4}{(2\pi)^4} \int d^4k \) and \([DN] = \prod_k dN_k\). The functional \( I(N) \) comes from the integration measure (which includes the Jacobian of transformation from \( F(\xi) \) to \( N(k) \))

\[ e^{-I(N)} = \int [DF] Z_0(N(k, \xi)) \delta(N(k) - N(k, \xi)) \]

and has the sense of the action for the bias function \( N(k) \). In the true vacuum case \( J = 0 \) and by means of using the expression (24) we find the two-point Green function in the form

\[ G(k) = \frac{4\pi^2}{k^2} N(k) \tag{25} \]

where \( N(k) \) is the cutoff function (the mean bias) which is given by

\[ \overline{N}(k) = \frac{1}{Z_{\text{total}}(0)} \int [DN] e^{-I(N)} N(k). \]

The action \( I(N) \) can be expanded as

\[ I(N) = \overline{I}(N) + \frac{1}{2} \sum_k \frac{(N(k) - \overline{N}(k))^2}{\sigma_k^2} + \ldots \tag{26} \]

where \( \sigma_k^2 \) defines the dispersion of vacuum topology fluctuations. Since the bias \( N(k) \) plays the role of a projection operator which for a dense gas (e.g., see (13)) has the asymptotic \( \overline{N}(k) \to \text{const} \ll 1 \) as \( k \ll k_{pl} \) one may expect \( \sigma_k^2 = \overline{N}(k)(1 - \overline{N}(k)) \cong \overline{N}(k) \) as \( k \ll k_{pl} \).

6. Consider now topology fluctuations in the presence of an external current. In the presence of an external current \( J^{ext} \) the intensity of topology fluctuations changes. Indeed using (24), (26) we find

\[ I(N, J^{ext}) = \overline{I}(N) + \frac{1}{2} \sum_k \frac{(N(k) - \overline{N}(k))^2}{\sigma_k^2} + \frac{1}{2} \sum_k \frac{4\pi^2}{k^2} |J_k^{ext}|^2 N(k) + \ldots \tag{27} \]

which gives

\[ I(N, J^{ext}) = \overline{I}(N, J^{ext}) + \frac{1}{2} \sum_k \frac{(N(k) - \overline{N}(k, J^{ext}))^2}{\sigma_k^2} \tag{28} \]

\[ \text{In this expansion the cutoff } \overline{N}(k) \text{ is merely the solution of } \frac{\partial I(N)}{\partial N(k)} = 0. \]

\[ \text{We point out that the inverse dispersion } \sigma_k^{-2} \text{ is analogous to the Laplace operator } \Delta \text{ in } \text{(13)}. \]
where

\[ I(\overline{N}, J^{\text{ext}}) = I(\overline{N}) - \frac{1}{2} \sum_k \sigma_k^2 \left( \frac{4\pi^2}{2k^2} |J_k^{\text{ext}}|^2 \right)^2 \]  

(29)

and

\[ \overline{N}(k, J^{\text{ext}}) - \overline{N}(k) = b(J) = -\sigma_k^2 \frac{4\pi^2}{2k^2} |J_k^{\text{ext}}|^2 \]  

(30)

Since we expect that the external current has scales \( k \ll k_{\text{pl}} \), (where \( \sigma_k^2 N(k) \approx 1 \)) this expression can be cast into the form

\[ b(J) = -\frac{1}{2} \sigma_k^2 \frac{4\pi^2}{k^2} \overline{N}(k) |J_k^{\text{ext}}|^2 \simeq -\frac{1}{2} G_{\text{reg}}(k) |J_k^{\text{ext}}|^2 \]  

(31)

where \( G = G_{\text{reg}} \) is the physically measured (observed at laboratory scales) Green function. At very large scales upon renormalization of charge values we get \( G_{\text{reg}} = G_0 = \frac{4\pi^2}{k^2} \). Now comparing this function with (13) we relate the additional distribution of virtual wormholes and the current as

\[ N \int a^2 \frac{4\pi^2}{k^2} (F(a, 0) - F(a, k)) da \simeq \frac{1}{2} G_{\text{reg}}(k) |J_k^{\text{ext}}|^2. \]  

(32)

which gives at scales \( k \ll k_{\text{pl}} \)

\[ |J_k^{\text{ext}}|^2 \simeq 2na_0^2 (f(0) - f(k)). \]  

(33)

where \( na_0^2 f(k) = N \int a^2 F(a, k) da \), or for a particular distribution \( [13] \) the necessary current to produce the desired additional fluctuations takes the form

\[ |J_k^{\text{ext}}|^2 \simeq 2na_0^2 \left( 1 - \cos \left( \vec{k} \cdot \vec{r}_0 \right) \right) = \frac{b}{\pi^2 r_0^2} \left( 1 - \cos \left( \vec{k} \cdot \vec{r}_0 \right) \right) \]  

(34)

Here \( a_0 \) is the typical size of the throat of virtual wormholes and \( b \) is the portion of the signal scattered on topology \( (b < 1) \).

7. Thus, we see that sufficiently intensive external current (which probably was reached in the experiment \([1]\) ) is apt to produce the anomaly observed (faster than light travel). In conclusion we also note that we considered here the simplest situation which does not destroy the homogeneity of space. However it is clear that our consideration allows for the straightforward generalization on a more complex (e.g., spherically symmetric) case which opens the new perspective to create such an object in laboratory physics. Moreover, external intensive fields are widely met in astrophysics, and therefore, one may expect that such complex objects (actual wormhole-like objects) are indeed responsible for the dark matter phenomenon, e.g., see \([14]\) which however requires for the further studying.
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