The nature of Galactic Center could be probed by lensing experiments capable of testing the spatial and velocity distributions of stars nearby and beyond it. Several hypotheses are possible (e.g. massive neutrino condensation, boson star) which avoid the shortcomings of the supermassive black hole model.

1 Introduction

Several observational campaigns have identified the center of our Galaxy with the supermassive compact dark object Sagittarius A* (Sgr A*) which is an extremely loud radio source. Detailed information comes from dynamics of stars moving in the gravitational field of such a central object. The statistical properties of spatial and kinematical distributions are of particular interest: Using them, it is possible to establish the mass and the size of the object which are $(2.61 \pm 0.76) \times 10^6 M_\odot$ concentrated within a radius of $0.016 \text{ pc}$ (about 30 lds). From this data, it is possible to state that a supermassive compact dark object is present at the Galactic Center and, furthermore, it is revealed by the motion of stars moving within a projected distance of less than 0.01 pc from the radio source Sgr A* at projected velocities in excess of 1000 km/s. Furthermore, a large and coherent counter-rotation, especially of the early-type stars, is revealed. Observations of stellar winds nearby Sgr A* give a mass accretion rate of $dM/dt = 6 \times 10^{-6} M_\odot\text{yr}^{-1}$. Hence, the dark mass must have a density $\sim 10^9 M_\odot\text{pc}^{-3}$ or greater and a mass-to-luminosity ratio of at least $100 M_\odot/L_\odot$. The result is that the central dark mass is statistically very significant ($\sim 6 - 8\sigma$) and cannot be removed even if a highly anisotropic stellar velocity dispersion is assumed. As a first conclusion, several authors state that, in the Galactic Center, there is either a single supermassive black hole or a very compact cluster of stellar-size black holes. Due to the above mentioned mass accretion rate, if Sgr A* is a supermassive black hole, its luminosity should be more than $10^{40} \text{erg s}^{-1}$. On the contrary, observations give a bolometric luminosity of $10^{37} \text{erg s}^{-1}$. This discrepancy is the so-called “blackness problem” which has led to the notion of a “black hole on starvation” at the Galactic Center. Besides, the most recent observations probe the gravitational potential at a radius larger than $4 \times 10^4$ Schwarzschild radii of a black hole of mass $2.6 \times 10^6 M_\odot$ so that the supermassive black hole hypothesis is far from being conclusive. On the other hand, stability criteria rule out the hypothesis of a very compact stellar cluster in Sgr A*. In fact,
detailed experiments of evaporation and collision mechanisms give maximal lifetimes of the order of \(10^8\) years which are much shorter than the estimated age of the Galaxy. Recently, other viable alternative models for the Galactic Center (and the center of several other galaxies) has been proposed. Essentially, the authors wonder if nonbaryonic condensations, given by massive neutrinos (or other fermions as gravitinos), or massive bosons could account for dynamics and size of Sgr A*, without considering the supermassive black hole. The main ingredient of such proposals is that nonbaryonic matter interacts gravitationally forming a supermassive ball in which the degeneracy pressure of fermions or Heisenberg uncertainty principle for bosons balance their self–gravity. Both mechanisms, also if in a completely different way, prevent from gravitational collapse. Such nonbaryonic condensations could have formed in the early epochs during a first–order gravitational phase transition.

2 Sgr A* as a neutrino star

Various experiments are today running to search for neutrino oscillations. It is very likely that exact predictions for \(\nu_e - \nu_{\tau u}\) and \(\nu_e - \nu_{x}\) oscillations will be soon available. From all this bulk of data, it is possible to infer reasonable values of mass for \(\nu_e\), \(\nu_\mu\), and \(\nu_\tau\). For our purposes, we are particularly interested in fermions which masses range between 10 and 25 keV/c\(^2\). This choice allows the formation of supermassive degenerate objects (from \(10^6M_\odot\) to \(10^9M_\odot\)) with a large amount of radio emission. The theory of heavy neutrino condensates, bound by gravity, can be easily sketched by a Thomas–Fermi model for fermions. We can set the Fermi energy \(E_F\) equal to the gravitational potential which binds the system, that is \(\frac{\hbar^2k_F^2(r)}{2m_\nu} - m_\nu\Phi(r) = E_F = -m_\nu\Phi(r_0)\), where \(\Phi(r)\) is the gravitational potential, \(k_F\) is the Fermi wave number and \(\Phi(r_0)\) is a constant chosen to cancel the gravitational potential for vanishing neutrino density. The length \(r_0\) is the estimated size of the condensation. If we take into account a degenerate Fermi gas, we get \(k_F(r) = (6\pi^2n_\nu(r)/g_\nu)^{1/3}\), where \(n_\nu(r)\) is the neutrino number density and we are assuming that it is the same for neutrinos and antineutrinos within the condensation. The number \(g_\nu\) is the spin degeneracy factor. Immediately we see that the number density is a function of the gravitational potential, i.e. \(n_\nu = f(\Phi)\), and the model is specified by it. The gravitational potential will obey a Poisson equation where neutrinos (and antineutrinos) are the source term: \(\Delta \Phi = -4\pi Gm_\nu n_\nu\). We can assume the spherical symmetry and define the variable \(u = r[\Phi(r) - \Phi(r_0)]\) then the Poisson equation reduces to the radial Lanté–Emden differential equation

\[
\frac{d^2u}{dr^2} = -\left(\frac{4\sqrt{2}m_\nu^4Gg_\nu}{3\pi\hbar^3}\right)\frac{u^{3/2}}{r},
\]

with polytropic index \(n = 3/2\). This equation is equivalent to the Thomas–Fermi differential equation of atomic physics, except for the minus sign that is due to the gravitational attraction of the neutrinos as opposed to the electrostatic repulsion between the electrons. If \(M_B\) is the mass of the baryonic star internal to the condensation, the natural boundary conditions are \(u(0) = GM_B\), \(u(r_0) = 0\). The general solution of (1) has scaling properties and it is able to reproduce the observations. It well fits the observations toward the Galactic Center which estimate a massive object of \(M = (2.6 \pm 0.7) \times 10^6M_\odot\) which dominates the gravitational potential in the inner (\(\leq 0.5\)pc) region of the bulge. In summary, a degenerate neutrino star of mass \(M = 2.6 \times 10^6M_\odot\), consisting of neutrinos with masses \(m \geq 12.0\) keV/c\(^2\) for \(g_\nu = 4\), or \(m \geq 14.3\) keV/c\(^2\) for \(g_\nu = 2\), does not contradict the observations. Considering a standard accretion disk, the data are in agreement with the model if Sgr A* is a neutrino star with radius \(R = 30.3\) ld (\(\sim 10^5\) Schwarzschild radii) and mass \(M = 2.6 \times 10^6M_\odot\) with a luminosity \(L \sim 10^{37}\) erg sec\(^{-1}\). Similar results hold also for the dark object (\(M \sim 3 \times 10^9M_\odot\)) inside the
center of M87. Assuming the existence of such a neutrino condensate in the Galactic Center, it could act as a spherical lens for the stars behind so that their apparent velocities will be larger than in reality. Comparing this effects with the proper motion of the stars of the cluster near Sgr A*, exact determinations of the physical parameters of the neutrino ball could be possible. In this case, gravitational lensing, always used to investigate baryonic objects, could result useful in order to detect a nonbaryonic compact object. Furthermore, since the astrophysical features of the object in Sgr A* are quite well known, accurate observations by lensing could contribute to the exact determination of particle constituents which could be, for example, neutrinos or gravitinos. Our heavy neutrino ball, being massive, extended and transparent, can be actually considered as a magnifying glass for stars moving behind it. If an observer is on Earth and he is looking at the Galactic Center, he should appreciate a difference in the motion of stars since lensed stars and non-lensed stars should have different projected velocity distributions. In other words, depending on the line of sight (toward the ball or outside the ball) it should be possible to correct or not the projected velocities by a gravitational lensing contribution and try to explain the bimodal distribution actually observed. Detailed calculations in this sense are given in Capozziello & Iovane 1999, where the spatial and kinematical distributions of the stars nearby the Galactic Center are reproduced by using the neutrino condensate as a thick lens (a sort of magnifying glass).

3 Sgr A* as a boson star

Also a gravitationally–bound boson condensate could explain the supermassive object in Sgr A*. In general, the theory of a boson star can be constructed starting from the Lagrangian density of a massive complex self-gravitating scalar field (taking $\hbar = c = 1$)

$$\mathcal{L} = \frac{1}{2} \sqrt{|g|} \left( \frac{m_{Pl}^2 \pi}{8} R + \partial_\mu \psi^* \partial^\mu \psi - U(|\psi|^2) \right),$$

where $R$ is the scalar of curvature, $|g|$ the modulus of the determinant of the metric $g_{\mu\nu}$, and $\psi$ is a complex scalar field with potential $U$. Since the potential is a function of the squared modulus of the field, we obtain a global $U(1)$ symmetry. This symmetry is related with the conserved number of particles. The form of the potential gives a mini-boson, a boson, or a soliton stars. Conventionally, it is given by $U = m^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4$ where $m$ is the scalar mass and $\lambda$ a dimensionless coupling constant. Mini-boson stars are spherically symmetric equilibrium configurations with $\lambda = 0$. Boson stars, on the contrary, have a non-null value of $\lambda$. Soliton stars are non-topological solutions with a finite mass, confined in a region of space, and non-dispersive. They are given by a potential of the form $U = m^2 |\psi|^2 \left( 1 - \frac{|\psi|^2}{\Phi_0^2} \right)^2$, where $\Phi_0$ is a constant. Let us see, now, which are the parameters of the different scalar objects which may reproduce the features of the central object in our Galaxy. We are looking for a mass of $(2.6 \pm 0.76) \times 10^6 M_\odot$, a radius of the accretion disk of 0.016 pc~30 light days, and a luminosity of $10^{37}$ erg s$^{-1}$. An interesting fact is that, for all the above scalar objects, the radius is always related with the mass in the same way: $M = m_{Pl}^2 R$, where $m_{Pl}$ is the Planck mass. In the case of Galactic Center, it is clear that the main parameter is the mass and not the radius. In the scalar star models, from the given central mass, the radius we obtain for the star is comparable to that of the horizon ($R = m_{Pl}^2 \times 2.61 \times 10^6 M_\odot = 3.9 \times 10^{11}$ cm). In other words, we expect an object which extends about 10 solar radius from the center and which is singularity free. Due to this intrinsic feature, it is impossible to use gravitational lensing as above. There the presence of an “extended” supermassive neutrino condensation in Sgr A* allows to distinguish the stars ‘nearby’ and ‘behind’ the object which effectively acts as a thick spherical lens. The
star spatial positions and projected kinematics have a bimodal distribution depending on the line of sight (toward the ball or outside the ball). In the boson condensation case, we have a pointlike lens (with respect to an observer on Earth) and, also if the boson star is “transparent”, we cannot expect any bimodal distribution of stars behind it. However, due to the extremely large mass of the object, a standard gravitational lens analysis fails, since strong lensing effects have to be considered. The observed bimodal distributions in space and velocities have to be ascribed to intrinsic, in some sense ‘genetic’, effects. The question is now for which values of the parameters, we can obtain a scalar object of such a mass. For the case of mini-boson star, we need an extremely light boson $m = 5.08 \times 10^{-17} \text{eV/c}^2$. This is the only parameter of this model and so it is unequivocally fixed. For boson stars, we get $m[\text{GeV}] = 7.9 \times 10^{-4}(\lambda/4\pi)^{1/4}$. It is possible to fulfill the previous relationship, for instance, with a boson of about 1 MeV and $\lambda = 1$. In the case of a non-topological soliton, we obtain $m[\text{GeV}] = 7.6 \times 10^{12}\text{GeV}^3/(\Phi_0^2[\text{GeV}]^2)$. If the parameter $\Phi_0$ is of the order of boson mass, we need very heavy bosons: $m = 1.2 \times 10^4\text{GeV/c}^2$. Based only on the constraints imposed by the above mass–radius relationship, we may conclude that: \(i\) if the boson mass is comparable to the expected Higgs mass (hundreds of GeV), then the Galactic Center could be a non-topological soliton star; \(ii\) an intermediate mass boson could be enough to produce a heavy object in the form of a boson star; \(iii\) a mini-boson star needs the existence of an ultra-light boson. If boson stars really exist, they should be the remnants of first-order gravitational phase transitions and their mass should be ruled by the epochs when they decoupled from the cosmological background. The Higgs particle, besides its leading role in inflationary models, should be the best and natural candidate as constituent of a boson condensation, if the phase transition occurred in early epochs. A boson condensate should be considered as a sort of topological defect. In this case, Sgr A* should be a soliton star. If soft phase-transitions took place during cosmological evolution (e.g. soft inflationary events), the leading particles could have been intermediate mass bosons and so our supermassive object should be a genuine boson star. If the phase transitions are very recent, the ultra-light bosons could belong to the Goldstone sector giving rise to mini-boson stars. In literature, we can find several examples of particles capable to fit the issues of boson stars but the ultimate answer is left to the cosmological observations and particle physics experiments.

As it is widely discussed in literature, gravitational lensing observations or very large baseline interferometry (VLBI) could give the “signature” to discriminate among the models of Galactic Center present in literature. For example, the investigation of the large “shadow” of the event horizon of the central object by an observer (we on Earth) should give information on dynamics and intrinsic structures. Interesting proposals and simulations in this sense are given in. Besides, the project ARISE (Advanced Radio Interferometry between Space and Earth) is going to use the technique of Space VLBI to increase our understanding of black holes and their environments, by imaging the extreme physical configurations produced in their proximities by strong gravitational fields. From a theoretical point of view, developments and results in gravitational lensing in very strong field regimes will be of extreme importance.

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