Identification of underdamped process dynamics

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This paper presents off-line and on-line identification of underdamped process dynamics in terms of transfer function models. Simple analytical expressions are derived based on the describing function technique. The nonlinear device relay with hysteresis is used to extract process information that help in estimating the unknown process model parameters. Another advantage of relay with hysteresis is that it reduces the effect of measurement noise, which is common in process industries. To further eliminate the ill effects of noise, the Fourier series-based curve fitting technique is employed. Unlike many identification methods that require to solve simultaneously a set of nonlinear equations, the advantage of the proposed method is that the process model parameters are estimated using explicit analytical expressions. The validity and robustness of the proposed method is illustrated with the help of six typical transfer function models from the literature. Results with and without noise are compared using Nyquist’s plots.

Keywords: describing function; identification; underdamped systems; curve fitting

1. Introduction

Recently, relay-based identification of process model parameters has received considerable attention. A process or system is controlled with the help of a controller, which can be designed once the process dynamics are identified. In the literature, various methods are proposed to identify different types of systems such as overdamped, integrating, critically damped, first-order plus dead time (FOPDT), second-order plus dead time (SOPDT) and higher order. A tutorial review (Liu, Wang, & Huang, 2013) on identification of various process models from relay or step test is presented including the significance of the describing function (DF) method. Atherton (2006) presented the importance of relay in autotuning of controllers and also highlighted about the DF method. Padhy and Majhi (2006, 2009) proposed relay-based on-line identification of stable and unstable FOPDT processes, autotuning of proportional integral derivative (PID) controllers for stable processes, employing the DF technique and also identified higher order systems in terms of first-order transfer function models. Recently, Desai and Prasad (2013) illustrated a new approach to reduce higher order models into lower order models. Liu and Gao (2008) developed separate algorithms for estimation of FOPDT model parameters, with single-biased and unbiased relay feedback tests. Lee, Sung, and Edgar (2007) used relay feedback with integrals of responses to get process information and also mentioned that suppression of effects of higher order harmonic terms is achieved with the help of these integrals. Kumanan and Nagaraj (2013) pointed out the significance of system identification for controller design and proposed tuning of PID controllers, employing firefly algorithm. Bajarangbali and Majhi (2012) proposed off-line and on-line estimation of FOPDT process model parameters utilizing relay with hysteresis and the DF method. Applying the single symmetric relay feedback test and Laplace transform method, Vivek and Chidambaram (2005) derived mathematical expressions to estimate FOPDT model parameters. Analytical expressions (De La Barra & Mossberg, 2007) are proposed to identify second-order underdamped systems, considering rectangular pulse inputs. Panda (2006) proposed the time-domain approach for identification of SOPDT underdamped processes using relay test. Majhi (2007) developed relay-based identification algorithms for the general model structure applying the state-space approach. Exact expressions (Liu, Gao, & Wang, 2008) are derived for estimation of critically damped, overdamped and underdamped model parameters from biased/unbiased relay feedback test. But these methods are time consuming and require comprehensive calculations. In the literature, system identification is done either off-line or on-line, utilizing ideal relay or biased relay and few authors have considered the presence of measurement noise. The greatest advantage of the presented method lies in that unlike many methods available in the literature it does not require one to solve a set of nonlinear equations to estimate process dynamic model parameters. Similar to the developments suggested in Bajarangbali and Majhi (2012), preliminary procedures are formulated for

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identifying SOPDT process dynamics in this paper. Hence, methods are proposed for off-line and on-line identification of underdamped process dynamics employing relay with hysteresis in closed loop with the process. For on-line identification, a PID controller is connected in parallel with relay, assuming initial controller parameters. The contents of this paper are systematically presented in the following sections, the identification method is detailed in Section 2, derivation of mathematical expressions is given in Section 3, results and verification are illustrated in Section 4 and conclusions are derived in Section 5.

2. Identification method
The identification scheme as shown in Figure 1 mainly consists of three components such as a relay with hysteresis, an underdamped process and a PID controller. In the off-line identification method, only relay and process are there, whereas in the on-line case, controller is there in parallel with relay connected to the process input, which can be equivalently written as shown in Figure 1. The reference input \( R \) is made zero so that the process generates sustained oscillations also called as the limit cycle as shown in Figure 2, with the help of relay input. This limit cycle characterizes the process, with parameters such as amplitude and time period. These limit cycle parameters are used in derived mathematical expressions to estimate unknown process model parameters. During on-line identification, PID controller parameters are initially chosen appropriately to get the limit cycle, once the process is modeled in transfer function form, controller can be designed for satisfactory performance of the system. To illustrate usefulness of the proposed method in the presence of measurement noise, a white Gaussian noise of some variance is added to the process output. This noisy output is processed using the Fourier series-based curve fitting technique (Matlab, 2007) to eliminate maximum noise content.

The transfer model of an underdamped SOPDT process is considered in the following form,

\[
G(s) = \frac{Ke^{-\theta s}}{as^2 + bs + 1}. \tag{1}
\]

Here the unknown parameters \( K \) (steady-state gain), \( \theta \) (time delay), \( a \) and \( b \) (process parameters) are to be estimated.

3. Derivation of mathematical expressions
In this section, mathematical expressions are derived in terms of relay parameters, limit cycle and controller parameters to identify the unknown process model dynamics. In the DF method, the nonlinear device relay with hysteresis is approximated by a gain of

\[
N = \frac{4h(\sqrt{A^2 - \varepsilon^2} - j\varepsilon)}{\pi A^2}, \tag{2}
\]

where \( N \) is the gain, \( h \) the relay height, \( \varepsilon \) the hysteresis width and \( A \) the amplitude of the limit cycle. Equation (1) can also be written in the following form

\[
G(s) = \frac{Ke^{-\theta s}}{(s - \alpha_1)(s - \alpha_2)}, \tag{3}
\]

or

\[
G(j\omega) = \frac{Ke^{-j\omega\theta} \alpha_1 \alpha_2}{(j\omega - \alpha_1)(j\omega - \alpha_2)}, \tag{4}
\]

where

\[
\alpha_1 = \frac{-b + \sqrt{b^2 - 4a}}{2a} \quad \text{and} \quad \alpha_2 = \frac{-b - \sqrt{b^2 - 4a}}{2a}. \tag{5}
\]

For underdamped systems, \( b^2 < 4a \), so, \( \alpha_1 \) and \( \alpha_2 \) are rewritten as

\[
\alpha_1 = \frac{-b + j\sqrt{4a - b^2}}{2a} \quad \text{and} \quad \alpha_2 = \frac{-b - j\sqrt{4a - b^2}}{2a}. \tag{6}
\]
3.1. Expressions for off-line identification

As mentioned earlier, the off-line identification method involves only relay and the process in the closed loop and therefore, the condition to obtain the stable limit cycle becomes

\[ NG(j\omega) + 1 = 0. \] (7)

Substitution of Equations (2) and (4) in Equation (7) results in

\[ 4h(\sqrt{A^2 - \varepsilon^2} - j\varepsilon)Ke^{-j\omega\theta}a_1a_2 \left( \frac{n^2 + \alpha^2}{\pi A} \right) + 1 = 0. \] (8)

Equation (8) can also be written as

\[ 4h(\sqrt{A^2 - \varepsilon^2} - j\varepsilon)Ke^{-j\omega\theta}a_1a_2 \left( \frac{n^2 + \alpha^2}{\pi A} \right) = -1. \] (9)

Now, considering the magnitudes of Equation (9) and solving, we obtain

\[ (\omega^2 + \alpha_1^2)(\omega^2 + \alpha_2^2) = \left( \frac{4hK\alpha_1\alpha_2}{\pi A} \right)^2. \] (10)

Further solving, Equation (10) is reduced to

\[ \alpha_1\alpha_2 = \omega\pi A \sqrt{\omega^2 + \alpha_1^2 + \alpha_2^2} \left( \frac{4hK\alpha_1\alpha_2}{4hK^2 - (\pi A)^2} \right). \] (11)

Now, equating the phase angles of Equation (9) and rearranging gives

\[ \tan^{-1}\left( \frac{\omega}{\alpha_1} \right) + \tan^{-1}\left( \frac{\omega}{\alpha_2} \right) - \tan^{-1}\left( \frac{\varepsilon}{\sqrt{A^2 - \varepsilon^2}} \right) = \pi + \omega\theta. \] (12)

Finally, the following expression is obtained on solving the above equation:

\[ \alpha_1 + \alpha_2 = \left( \frac{\alpha_1\alpha_2 - \omega^2}{\omega} \right) \tan \left( \pi + \omega\theta \right) + \tan^{-1}\left( \frac{\varepsilon}{\sqrt{A^2 - \varepsilon^2}} \right), \] (13)

where \( \omega = 2\pi/T \) and \( T \) is the time period of the limit cycle. Hence, the unknown parameters \( a \) and \( b \) are estimated from Equations (11) and (13), since \( a_1 \) and \( a_2 \) are in terms of these unknown parameters. Here the steady-state gain \( K \) is assumed to be known a priori or can be estimated from some other method and \( \theta \) is obtained from \( \theta = t_1 - t_0 \), proposed by Majhi (2007), where \( t_0 \) is the time instant at the place of relay switching with reference to the limit cycle and \( t_1 \) is the time instant at the place of abrupt change in second derivative of the limit cycle. The parameters \( t_0 \) and \( t_1 \) are measured either during positive or negative half limit cycle with reference to zero crossing line, the detailed theory can be referred from Majhi (2007).

3.2. Expressions for on-line identification

As shown in Figure 1, an on-line identification scheme consists of a PID controller along with relay and process. The PID controller is considered as

\[ G_c(s) = K_{pr}\left( 1 + \frac{1}{T_{in}(s)} + \frac{T_{dr}(s)}{\delta T_{dr}(s) + 1} \right), \] (14)

where \( K_{pr} \) is the proportional gain, \( T_{in} \) the integral time constant, \( T_{dr} \) the derivative time constant and \( \delta \) the derivative filter constant, which is generally very small and neglected here for convenience in deriving expressions. Hence, Equation (14) can be represented as

\[ G_c(s) = K_{pr}\left( 1 + \frac{1}{T_{in}(s)} + T_{dr}(s) \right) \] (15)

or

\[ G_c(j\omega) = K_{pr}\left( 1 + \frac{1}{j\omega T_{in}} + j\omega T_{dr} \right). \] (16)

Similar to the off-line identification method, the condition to obtain sustained oscillations is

\[ N\tilde{G}(j\omega) + 1 = 0, \] (17)

where

\[ \tilde{G}(j\omega) = \frac{G(j\omega)}{1 + GG_c(j\omega)}. \] (18)

So, Equation (17) is reduced to

\[ G(j\omega)(N + G_c(j\omega)) + 1 = 0. \] (19)

After substituting the corresponding expressions in Equation (19), the following equation is obtained:

\[ \frac{K e^{-j\omega\theta}a_1a_2}{(j\omega - a_1)(j\omega - a_2)} \left( \frac{4h}{\pi A^2} \sqrt{A^2 - \varepsilon^2} - j\varepsilon \right) + K_{pr}\left( 1 + \frac{1}{j\omega T_{in}} \right) + j\omega T_{dr} \] (20)

or

\[ \frac{K e^{-j\omega\theta}a_1a_2}{(j\omega - a_1)(j\omega - a_2)} \left( \frac{4h}{\pi A^2} \sqrt{A^2 - \varepsilon^2} + j\varepsilon \right) + K_{pr}\left( 1 + \frac{1}{j\omega T_{in}} \right) + j\omega T_{dr} \] (21)

Equation (21) is shortened to

\[ \frac{K e^{-j\omega\theta}a_1a_2}{(j\omega - a_1)(j\omega - a_2)}(p + jq) = -1, \] (22)

where

\[ p = \frac{4h\sqrt{A^2 - \varepsilon^2}}{\pi A^2} + K_{pr} \] and

\[ q = K_{pr}\omega T_{dr} - K_{pr}\frac{T_{dr}}{\omega T_{in}} - \frac{4h\varepsilon}{\pi A^2}. \] (23)
Equating the magnitudes of Equation (22), we obtain
\[ K\alpha_1\alpha_2\sqrt{p^2 + q^2} = \sqrt{(\omega^2 + \alpha_1^2)(\omega^2 + \alpha_2^2)} \] (24)

Further solving Equation (24), one obtains
\[ \omega^2(\omega^2 + \alpha_1^2 + \alpha_2^2) = (\alpha_1\alpha_2)^2(K^2(\omega^2 + q^2) - 1). \] (25)

Equation (25) is simplified to obtain
\[ \alpha_1\alpha_2 = \omega\sqrt{\frac{\omega^2 + \alpha_1^2 + \alpha_2^2}{K^2(\omega^2 + q^2) - 1}} \Rightarrow \]
\[ b = \sqrt{\frac{K^2(\omega^2 + q^2) - 1}{\omega^2}} - a(\omega^2 - 2). \] (26)

Now, considering the phase angles of Equation (22), the following expression can be written:
\[ \tan^{-1}\left(\frac{q}{p}\right) + \tan^{-1}\left(\frac{\omega}{\alpha_1}\right) + \tan^{-1}\left(\frac{\omega}{\alpha_2}\right) = \pi + \omega\theta. \] (27)

The above equation is further modified to
\[ \alpha_1 + \alpha_2 = \left(\frac{\alpha_1\alpha_2 - \omega^2}{\alpha_2}\right)\tan\left(\pi + \omega\theta - \tan^{-1}\left(\frac{q}{p}\right)\right) \Rightarrow a = \frac{1}{\omega^2} + \frac{b}{\omega}\cot\left(\pi + \omega\theta - \tan^{-1}\left(\frac{q}{p}\right)\right). \] (28)

Equations (26) and (28) are solved to estimate the unknown parameters \(a\) and \(b\) in Equation (1) and also in Equation (3) in the form of \(\alpha_1\) and \(\alpha_2\). As mentioned earlier, the PID controller parameters are initially selected to get the stable limit cycle, hence from many simulation results, the parameters selected are \(K_{pr} = 0.01, T_{in} = 0.5\) and \(T_{dr} = 0.125\). Remaining parameters, \(K\) and \(\theta\), are obtained by the procedure mentioned in Section 3.1.

4. Results and verification

In this section, the proposed method is verified using well-known examples from the literature. These examples signify the importance of processes in the form of the transfer function model. Simulation results are used to estimate the unknown process model parameters with the help of derived mathematical expressions. To verify the efficacy of the proposed method under realistic conditions, model parameters are also estimated in the presence of measurement noise. To get the noise effect, a white Gaussian noise is added to the process output and noisy limit cycle is processed through the curve fitting technique to obtain a clean signal. The proposed method is also tested for noise with low frequency. Simulations are not carried out with several noise patterns but in real-time scenarios, the noise can be of various forms. Practical issues such as noise and effect of

neglected dynamics play a major role in the model-based controller design. Our identification method provides a tool for the model-based controller design not aiming at achieving a very accurate model at any working condition but just at those frequencies that are interesting for control purposes. Results are compared using Nyquist’s plots and the multiplicative error (Err.) between model and process as given follows:

\[ \text{Err.} = \int_{0}^{\omega_{pf}} \left| \frac{G_m(j\omega) - G(j\omega)}{G_m(j\omega)} \right| d\omega, \] (29)

where \(\omega_{pf}\) is the phase crossover frequency of the process, \(G_m(j\omega)\) the model proposed and \(G(j\omega)\) the actual process.
Robustness of the method is studied in Examples 4 and 5 with different relay settings.

Example 1 For off-line identification, let us consider an underdamped system (Lavanya, Umamaheswari, & Panda, 2006) with transfer function $G(s) = e^{-3s}/(s^2 + 0.4s + 1)$. A relay with parameters $b = 1$ and $\epsilon = 0.1$ generates the limit cycle with $A = 2.5716$ and $T = 7.9202$, so $\omega = 0.7933$. Assume that $K = 1$ and $\theta$ is obtained from the measurements of $t = 3.0443$ and $q = 0.0443$, as $\theta = 3.0000$. Substituting $A$, $T$, $\omega$, and $K$ and $\theta$ in Equations (11) and (13) and solving these equations, the unknown parameters estimated are $a = 0.9991$ and $b = 0.4127$. The model parameters proposed by Lavanya et al. (2006) are $a = 1.326$, $b = 0.68$, $K$ = 1.0 and $\theta$ = 3.0; however, the authors have not shown the results in the presence of measurement noise. Now, a white Gaussian noise of variance 0.00329 and sampling time of 0.01 s is added to the process output to get a noisy signal of 0.001 NSR. Using the curve fitting technique, the denoised signal is obtained as shown in Figure 3. Repeating the above procedure, estimated process model parameters are $\theta = 2.9797$, $a = 1.0134$ and $b = 0.4158$. Similarly, the process dynamics identified in the presence of low frequency noise with 0.00047 NSR and 0.01 s sampling time are $\theta = 3.0209$, $a$ = 0.9871 and $b$ = 0.4012. Figure 4 shows the low frequency noisy signal and the denoised limit cycle outputs. The transfer function models with multiplicative errors are given in Table 1, and the results are further compared using Nyquist’s plots as shown in Figure 5. The closeness of these Nyquist’s plots

| Example | Proposed model without noise | Proposed model with noise (white Gaussian) | Proposed model with low frequency noise | Model proposed by Lavanya et al. (2006) |
|---------|------------------------------|-------------------------------------------|----------------------------------------|----------------------------------------|
| Example 1 | $1.0 e^{-3.0s}$/(0.9991s^2 + 0.4127s + 1) | $1.0 e^{-2.9797s}$/(1.0134s^2 + 0.4158s + 1) | $1.0 e^{-3.0209s}$/(0.9871s^2 + 0.4012s + 1) | $1.0 e^{-3.0s}$/(1.326s^2 + 0.68s + 1) |
| Example 2 | $1.0 e^{-3.0s}$/(0.9975s^2 + 0.4125s + 1) | $1.0 e^{-2.9864s}$/(1.0039s^2 + 0.4173s + 1) | $1.0 e^{-2.9721s}$/(1.0041s^2 + 0.4204s + 1) | $1.0 e^{-3.0s}$/(1.326s^2 + 0.68s + 1) |
| Example 3 | $1.50 e^{-1.0s}$/(4.0096s^2 + 0.4928s + 1) | $1.50 e^{-0.9814s}$/(4.0145s^2 + 0.4922s + 1) | $1.50 e^{-1.0s}$/(4.0096s^2 + 0.4928s + 1) | $1.50 e^{-1.0s}$/(4.0096s^2 + 0.4928s + 1) |
| Example 4 | $1.0 e^{-3.35s}$/(3.96s + 1) | $1.0 e^{-3.27s}$/(4.21s + 1) | $1.0 e^{-3.35s}$/(3.96s + 1) | $1.0 e^{-3.35s}$/(3.96s + 1) |
| Example 5 | $1.0 e^{-0.7495s}$/(9.5791s^2 + 1.8875s + 1) | $1.0 e^{-0.6831s}$/(9.6549s^2 + 1.8516s + 1) | $1.0 e^{-3.3598s}$/(3.9308s + 1) | $1.0 e^{-3.3598s}$/(3.9308s + 1) |

Err. = 0.0058  
Err. = 0.0082  
Err. = 0.0067  
Err. = 0.1584  
Err. = 0.0057  
Err. = 0.0039  
Err. = 0.0026  
Err. = 0.1584  
Err. = 0.0023  
Err. = 0.0028  
Err. = 0.0128  
Err. = 0.2948  
Err. = 0.3196  
Err. = 0.0498  
Err. = 0.0548  
Err. = 0.2984
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clearly indicates that the proposed method can be used for estimation of process model parameters.

Example 2 For on-line identification, the process of Example 1 is reconsidered here, i.e. \( G(s) = e^{-3s}/(s^2 + 0.4s + 1) \). The initial controller parameters mentioned earlier as \( K_{pr} = 0.01, T_{in} = 0.5 \) and \( T_{dr} = 0.125 \) are used with relay \((h = 1, \varepsilon = 0.1)\) to obtain the limit cycle with \( A = 2.59, T = 8.0019 \) and \( \omega = 0.7852 \). From measurements of \( t_1 = 3.0442 \) and \( t_0 = 0.0442 \), the time delay parameter, \( \theta = 3.0000 \) is obtained and \( K \) is assumed to be known earlier. Using corresponding values in Equations (26) and (28), the unknown parameters estimated are \( a = 0.9975 \) and \( b = 0.4125 \). The parameters estimated in the presence of measurement noise (white Gaussian) of 0.001 NSR and 0.01 s sampling time are \( a = 1.0039, b = 0.4173 \) and \( \theta = 2.9864 \). Similarly, for low frequency noise of 0.00047 NSR and 0.01 s sampling time, the process model parameters estimated are \( a = 1.0041, b = 0.4204 \) and \( \theta = 2.9721 \). As mentioned earlier, the process model parameters suggested by Lavanya et al. (2006) are \( a = 1.326, b = 0.68 \), \( K = 1.0 \) and \( \theta = 3.0 \) without noise. Using Nyquist’s plots, the results are compared as shown in Figure 6 and models are shown in the form of transfer functions in Table 1 with corresponding errors.

Example 3 This example considers on-line identification of an underdamped process with smaller time delay (Majhi, 2007), \( G(s) = 1.5e^{-s}/(4s^2 + 0.5s + 1) \). Using initial controller parameters as mentioned in Example 2 and relay having parameters \( h = 1 \) and \( \varepsilon = 0.2 \), the limit cycle is generated with \( A = 4.7744 \) and \( T = 10.9819 \), so \( \omega = 0.5721 \). Assume \( K = 1.5 \) and \( \theta \) is estimated from the measurements of \( t_1 = 1.0882 \) and \( t_0 = 0.0882 \) as \( \theta = 1.0000 \). Substituting

the parameters for \( A, T, \omega, \omega^2, K \) and \( \theta \) in Equations (26) and (28) and solved to estimate the unknown parameters, \( a = 4.0096 \) and \( b = 0.4928 \). In the presence of measurement noise (white Gaussian) of 0.001 NSR and 0.01 s sampling time, the process model parameters estimated are \( \theta = 0.9814, a = 4.0145 \) and \( b = 0.4922 \). However, for this process, on-line identification is not considered by the earlier author (Majhi, 2007). The proposed transfer function models are mentioned in Table 1 with multiplicative errors, and the results are compared using Nyquist’s plots as shown in Figure 7.

Example 4 Let us consider this example to study the robustness of the proposed method for off-line
identification with the process (Chen, 1989), \( G(s) = \frac{e^{-s}}{(9s^2 + 2.4s + 1)} \). The relay test with \( h = 1 \) and \( \varepsilon = 0.1 \) generates the limit cycle having the parameters \( A = 0.6642, T = 11.996, \omega = 0.5238 \). Assume \( K = 1.0 \), the time delay \( \theta = 1.0000 \) is estimated from the measurements of \( t_1 = 1.3025 \) and \( t_0 = 0.3025 \). The expressions (11) and (13) are solved using the known parameters to find out \( a = 9.0991 \) and \( b = 2.2857 \). For this system, (Chen, 1989), suggested two different FOPDT models as mentioned in Table 1 with corresponding errors, further the results are compared with the help of Nyquist’s plots as shown in Figure 8. To carry out the robustness test, process dynamics are identified for different relay settings as given in Table 2 and the percentage (%) error of model parameters estimated for different NSR values with 0.01 s sampling time are given in Table 3 along with limit cycle parameters.

Table 2. Proposed models with different relay settings for Example 4.

| \( \varepsilon \) | Model expression | Error |
|---|---|---|
| 0.1 | \( \frac{1.0e^{-0.1s}}{(9.0991s^2 + 2.2857s + 1)} \) | 0.0128 |
| 0.2 | \( \frac{1.0e^{-0.2s}}{(9.0991s^2 + 2.2793s + 1)} \) | 0.0135 |
| 0.3 | \( \frac{1.0e^{-0.3s}}{(9.1157s^2 + 2.2716s + 1)} \) | 0.0144 |

Table 3. Effects of measurement noise (white Gaussian) on parameters in % error, for Example 4.

| NSR | \( A \) | \( T \) | \( a \) | \( b \) | \( \theta \) |
|---|---|---|---|---|---|
| 0.03 | -1.3268 | 0.0172 | 1.5715 | -0.4958 | -4.19 |
| 0.01 | 0.3478 | 0.0141 | -0.1802 | -0.0219 | 0.62 |
| 0.003 | 0.1416 | -0.0016 | 0.0011 | 0.1228 | 0.26 |
| 0.001 | -0.0772 | -0.0016 | 0.0011 | 0.6405 | 0.62 |

Table 4. Proposed models with different relay settings for Example 5.

| \( \varepsilon \) | Model expression | Error |
|---|---|---|
| 0.1 | \( \frac{1.0e^{-0.1s}}{(9.0991s^2 + 2.2894s + 1)} \) | 0.0124 |
| 0.2 | \( \frac{1.0e^{-0.2s}}{(9.0991s^2 + 2.2829s + 1)} \) | 0.0131 |
| 0.3 | \( \frac{1.0e^{-0.3s}}{(9.1075s^2 + 2.2732s + 1)} \) | 0.0141 |

Example 5 Robustness of the presented method is also shown for on-line identification with this example, by reconsidering the process of Example 4. The relay parameters selected are \( h = 1, \varepsilon = 0.1 \) and initial controller settings are used as mentioned in Example 2 to generate the limit cycle with \( A = 0.6835, T = 12.096, \omega = 0.5194 \). With measurements of \( t_1 = 1.2956 \) and \( t_0 = 0.2956, \theta \) is found to be 1.0000 and assume \( K = 1.0 \). Substituting the corresponding parameters in expressions (26) and (28), the model parameters estimated are \( a = 9.0991 \) and \( b = 2.2894 \). The process dynamics identified for different relay settings given in Tables 4 and 5 indicate the % error of the limit cycle and model parameters calculated for different NSR values with 0.01 s sampling time. Hence, the proposed method gives good results for different values of relay and noise variance.

Example 6 An underdamped SOPDT plant with a zero is considered in this example as \( G(s) = (-0.25s + 1) e^{-0.75s}/(9s^2 + 2.4s + 1) \). The on-line relay \( (h = 1 \) and \( \varepsilon = 0.1 \) test is conducted to get the limit cycle parameters as \( A = 0.6896, T = 12.0984, \omega = 0.5193 \) using the same controller parameters as in Example 2. Assuming \( K = 1.0 \), the process time delay \( \theta = 0.7495 \) is obtained from \( t_1 = 1.0439 \) and \( t_0 = 0.2944 \). These corresponding parameters are substituted in expressions (26) and (28) to estimate the remaining process model parameters without measurement noise as, \( a = 9.5791 \) and \( b = 1.8875 \). Similarly, in the presence of low frequency measurement noise of 0.01 NSR and 0.01 s sampling time, the process model parameters estimated are \( \theta = 0.6831, a = 9.6549 \) and \( b = 1.8516 \). Applying (Chen, 1989), method, the given system is identified in terms of the FOPDT system. The identified models in the form of transfer function are given in Table 1 with multiplicative errors and these models are also compared using
5. Conclusions

Methods are proposed for off-line and on-line identification of underdamped process dynamics, employing the simple and effective DF technique. A single relay with hysteresis is used in closed loop to get process information. For on-line identification, controller parameters are initially chosen and once the process model is known, suitable controller can be designed. Since measurement noise is critical issue in process industries, validity of the proposed method is illustrated even under noisy environment. Hence, the advantage of the proposed method is that using the DF technique, simple analytical expressions are derived to estimate process model parameters explicitly and effect of measurement noise is reduced. Results are verified with the help of multiplicative errors, Nyquist’s plots and these plots indicate good match neighboring the critical point. To test the robustness of the method, process dynamics are identified for different relay settings.

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