Some consequences of shear on galactic dynamos with helicity fluxes

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ABSTRACT

Galactic dynamo models sustained by supernova (SN) driven turbulence and differential rotation have revealed that the sustenance of large-scale fields requires a flux of small-scale magnetic helicity to be viable. Here we generalize a minimalist analytic version of such galactic dynamos to explore some heretofore unincluded contributions from shear on the total turbulent energy and turbulent correlation time, with the helicity fluxes maintained by either winds, diffusion or magnetic buoyancy. We construct an analytic framework for modelling the turbulent energy and correlation time as a function of SN rate and shear. We compare our prescription with previous approaches that include only rotation. The solutions depend separately on the rotation period and the eddy turnover time and not just on their ratio (the Rossby number). We consider models in which these two time-scales are allowed to be independent and also a case in which they are mutually dependent on radius when a radial-dependent SN rate model is invoked. For the case of a fixed rotation period (or a fixed radius), we show that the influence of shear is dramatic for low Rossby numbers, reducing the correlation time of the turbulence, which, in turn, strongly reduces the saturation value of the dynamo compared to the case when the shear is ignored. We also show that even in the absence of winds or diffusive fluxes, magnetic buoyancy may be able to sustain sufficient helicity fluxes to avoid quenching.

Key words: dynamo – MHD – turbulence – galaxies: ISM – galaxies: magnetic fields.

1 INTRODUCTION

1.1 Background

In situ galactic dynamo theory has long been a leading paradigm to explain the ordered large-scale magnetic fields of galaxies (Ruzmaikin, Sokolov & Shukurov 1988). In this paradigm, a weak seed field, perhaps supplied primordially, is amplified via the action of turbulence and differential rotation in the galactic interstellar medium. How such dynamos work in detail has been a longstanding research enterprise (Ruzmaikin et al. 1988; Brandenburg & Subramanian 2005; Shukurov et al. 2006; Hansaz et al. 2009; Chamandy et al. 2014; Blackman 2015; Kulsrud 2015).

Standard (20th century) mean field α–Ω galactic dynamos typically have at least three key ingredients: (1) supernova-driven (SN-driven) turbulence, which in the presence of galactic rotation and stratification produces a kinetic-helicity-driven ‘α’ effect that converts toroidal to poloidal field; (2) differential rotation that shears the poloidal field into the toroidal direction; and (3) some kind of turbulent diffusion or loss term of the mean field in a thin disc that ensures that the net toroidal flux in the disc reflects the observed field geometry (e.g. quadrupole).

A challenge for 20th century galactic dynamo theory has been the absence of a physical understanding of how the dynamo saturates. That basic theory is kinematic, considering only the growth of the large-scale field without including the dynamics of the field on the driving flow. Intertwined with this deficiency has been the realization that standard mean field textbook α–Ω dynamos also do not conserve magnetic helicity (Blackman & Field 2000; Vishniac & Cho 2001, for reviews, see Brandenburg & Subramanian 2005; Blackman 2015).

Principles of dynamically including magnetic helicity conservation in magnetohydrodynamics (MHD) turbulence from Pouquet, Frisch & Léorat (1976) and modified lessons from steady-state mean-field considerations of Gruzinov & Diamond (1994) and Bhattacharjee & Yuan (1995) were synthesized into time-dependent mean field dynamical toy models (Blackman & Field 2002) using a simple closure (now referred to as ‘minimal τ’). In these models, the growth of a helical component of the large-scale field is accompanied by the growth of the oppositely signed small-scale helical...
field, which, in turn, represents a backreaction that saturates the dynamo. For dynamos without shear, this leads to a steady state, but for dynamos with shear, this can lead to catastrophic quenching, alleviated only when helicity fluxes carry away the excess small-scale field. Ultimately, this requires a dynamo sustained by helicity fluxes (Blackman & Field 2000). Depending on which terms in the electromotive force (EMF) actually dominate, a complementary perspective is that the large-scale dynamo is sustained directly via helicity fluxes even in the absence of any kinetic helicity (e.g. Vishniac & Cho 2001; Vishniac & Shapovalov 2014). Helicity-flux-driven dynamos are conceptually related to the sustenance of large-scale fields in the different context of laboratory magnetically dominated plasmas (Strauss 1985; Bhattacharjee & Hameiri 1986).

Incorporation of some of these principles has led the numerical demonstration of the helpful role of magnetic helicity in numerical simulations of dynamos in stellar contexts (Brandenburg & Sandin 2004; Chatterjee, Guerrero & Brandenburg 2011) as well as practical galactic dynamo models with helicity fluxes (Shukurov et al. 2006; Sur, Shukurov & Subramanian 2007; Chamandy 2016).

A second challenge of galactic and mean field dynamo theory is to incorporate the influences of rotation and shear on the turbulence, dynamo coefficients and EMF. One approach is to expand the turbulent quantities into a base state that is independent of shear and rotation plus corrections that depend on them. The resultant mean turbulent EMF (whose curl enters the growth if the mean magnetic field) can then be expanded into a sum of all possible terms that are linear in the mean magnetic field and linear in the mean rotation or shear (Brandenburg & Subramanian 2005; Rädler & Stepanov 2006). The relevance and interpretation of each of these terms must be assessed independently for a given circumstance. However, this approach does not capture all of the effects of rotation and shear to all orders. Doing so formally is impractical, but physical approaches can provide insight and shortcuts.

1.2 Strategy and outline

The influence of rotation can be partly gauged by the ratio of the non-linear term in the Navier–Stokes equation to the Coriolis term in the rotating frame. This dimensionless ratio, the Rossby number, is given by

$$Ro(\tau_{\text{ed}}) = \tilde{Ro}(\Omega) = \frac{1}{\Omega \tau_{\text{ed}}},$$

where $\Omega$ is the rotation speed and $\tau_{\text{ed}}$ is the eddy turnover time, presently defined in terms of the turbulence supplied specifically by SNe. The latter is important to keep in mind as we will also utilize a separate correlation time $\tau_{\text{cor}}$ not necessarily equal to $\tau_{\text{ed}}$. The above equation introduces our convention of writing $Ro$ for the Rossby number for fixed $\Omega$, allowing $\tau_{\text{ed}}$ to vary, and $\tilde{Ro}$ for the Rossby number at fixed $\tau_{\text{ed}}$, allowing $\Omega$ to vary. How dynamos depend separately on $\tau_{\text{ed}}$, $\Omega$ and on differential rotation is not completely understood. Even for the basic $\alpha$–$\Omega$ type dynamo, the question of how the kinetic component of the helicity coefficient contribution $\alpha_0$ depends on rotation and shear warrants revisiting for strong shear.

There are a few precursors in this context. Ruzmaikin et al. (1988) calculated an effect of rotational quenching on $\alpha$. Ruzmaikin et al. (1988) considered the effect of the Coriolis force without shear, and their prescription for the effect of rotation on $\alpha_0$ can be recast by replacing the correlation time of the turbulence $\tau_c = \tau_{\text{ed}}Ro^{-1/2}$ when $Ro \geq 1$, and $\tau_c = \tau_{\text{ed}}$ otherwise. In Chamandy, Shukurov & Taylor (2016), the same resulting piecewise-defined $\alpha$ was used. Blackman & Thomas (2015) and Blackman & Owen (2016) included an effect of shear on the correlation time by arguing that $\tau_{\text{cor}}$ equals $\tau_{\text{ed}}$ times a factor that depends on $Ro$ and shear.

In this paper, we explore and generalize a physical model for the influence of shear and rotation on both $\tau_{\text{ed}}$ and the turbulent energy density for galactic dynamos. We will see that when $Ro$, $\tilde{Ro} \gg 1$, the SN turbulence dominates both the turbulent energy density and its correlation time. In the regime $Ro$, $\tilde{Ro} \ll 1$, shear can dominate both of these quantities. We build our model in three separate ways, first fixing $\Omega$ and changing $\tau_{\text{ed}}$, and then fixing $\tau_{\text{ed}}$ and changing $\Omega$. Then, we consider a model in which they are mutually dependent on radius, via their connection to the star formation rate (SFR). The need for this arises because the dynamo depends separately on those two parameters not just in their dimensionless combination of the Rossby number. We explicitly derive $\tau_{\text{ed}}$ in terms of the SR rate and show how $\tau_{\text{cor}}$ changes as a function of rotation and shear. Both the effect of shear on the turbulent correlation time and as a supplemental source of turbulent energy have not been included in galactic dynamo models, although in the absence of SNe, shear is expected to be a source of galactic turbulence (Sellwood & Balbus 1999).

We also incorporate a magnetic buoyancy (MB) term (Parker 1966) in the helicity flux term of the dynamo equations, generalizing the corresponding term of Sur et al. (2007), which included only an advective wind flux term. The buoyant speed itself depends on the magnetic field, which increases the non-linearity of the dynamo equations.

This paper is arranged as follows. In Section 2, we relate the turbulent velocity and correlation time to the Rossby number in both fixed-$\tau_{\text{ed}}$ and fixed-$\Omega$ cases, developing expressions for both the turbulent energy density and correlation time as a function of shear, rotation and SN rate. For a given shear profile, we consider three cases: (i) fixed $\Omega$, varying $\tau_{\text{ed}}$; (ii) fixed $\tau_{\text{ed}}$, varying $\Omega$; and (iii) mutually dependent variation of $\Omega$ and $\tau_{\text{ed}}$. We apply these relations to the dynamo equations in Section 3. The solutions are found numerically in Section 4, where we show both steady-state solutions and time evolution of the magnetic fields. We identify where the results from our calculations that include the new ingredients differ from previous approaches. We also discuss the influence of MB and the consequences of our calculations for observed pitch angle. We conclude in Section 5.

2 EFFECT OF SHEAR ON CORRELATION TIME AND TURBULENT ENERGY

The Rossby number is a function of two variables ($\tau_{\text{ed}}$, $\Omega$). The value of $\tau_{\text{ed}}$ can vary for different SN rates, and $\Omega$ depends on the details of galaxy formation and the mass therein. In practice, these two quantities could be correlated because a fixed initial mass function for stars, and a baryon mass correlated with total mass, would increase both the rate of SN and the rotation rate at a fixed radius. Below, we consider separately cases where we allow these two quantities to be independent and then consider a case where they are mutually dependent. When they are independent, the dynamo then depends on these two variables independently, not just on their ratio.

We first construct a physical model for the influence of shear on the turbulent energy and correlation time by fixing $\Omega$ and allowing $\tau_{\text{ed}}$ to change. We then construct the analogue where we keep $\tau_{\text{ed}}$, allowing $\Omega$ to change. We show in Appendix that these two approaches can be unified. In the last part of this section, we consider the case where the two quantities are mutually dependent.
In what follows, quantities with a subscript 0 (e.g. \( \tau_{\text{c0}}, v_0, l_{\text{c0}} \) and so on) are evaluated at their fiducial values such that \( R_{00} = R_0(\tau_{\text{c0}}) = (\tau_{\text{c0}}/\Omega_0)^{-1} = 1. \)

### 2.1 Correlation time and turbulent energy for fixed \( \Omega \), fixed shear, but different SN rates

#### 2.1.1 Effect of the shear on the correlation time

We distinguish between the turbulent correlation time \( \tau_{\text{cor}} \) and the naked eddy turnover time \( \tau_{\text{ed}} \) determined by SN in the absence of shear, and \( \tau_{\text{c0}} \) as the fiducial value of the latter. We define the ratio of the former to latter as

\[
y(\Omega) \equiv \frac{\tau_{\text{cor}}}{\tau_{\text{c0}}},
\]

where \( \Omega = \Omega(\tau_{\text{ed}}) \) for fixed \( \Omega \) in this section. The quantity \( \tau_{\text{cor}} \) must satisfy the physically expected behaviours in the low and high \( \Omega \) limits, namely \( \tau_{\text{cor}} \rightarrow \tau_{\text{ed}} \) as \( \Omega \rightarrow \infty \) and \( \tau_{\text{cor}} \rightarrow \tau_\ast \) as \( \Omega \rightarrow 0 \), where \( \tau_\ast \) is defined as

\[
\tau_\ast = \frac{\Delta r}{r \Delta \Omega} = \frac{\Delta r}{r \Delta \tau_\ast} \equiv \frac{1}{q} \frac{\tau_{\text{c0}}}{q} = \frac{\tau_{\text{ed}}}{q}
\]

along with the rotation profile \( \Omega \propto r^{-q} \). The physical meaning of \( \tau_\ast \) is evident if we consider radially separated points on two concentric rings orbiting in the galaxy with radii \( r - \Delta r/2 \) and \( r + \Delta r/2 \), respectively. Their relative velocity will be \( r \Delta \Omega = r \Delta \tau_\ast \), and \( \tau_\ast \) characterizes the time-scale for these points to further separate by \( \Delta r \) in the azimuthal direction. In terms of \( y \), the aforementioned asymptotic limits imply

\[
y = \begin{cases} 
1/q \quad & \text{as } \Omega \rightarrow 0 \\
1/\Omega \quad & \text{as } \Omega \rightarrow \infty 
\end{cases}
\]

Deriving \( y \) from first principles is a challenging endeavour, but we can make good progress with a physically motivated approach. We posit that quadratic time correlations of turbulent quantities decay exponentially in time over a correlation time that has separate independent exponential factors from shear and SN turbulence. Then,

\[
y^{-1} = \tau_{\text{ed}}^{-1} + \tau_\ast^{-1},
\]

or, equivalently,

\[
y = \frac{1}{\Omega + q}
\]

Equation (6) satisfies the constraint (4). In the fast rotation limit \( \Omega \rightarrow 0 \), we have \( y = 1/q \) so that equation (28) predicts a correlation time that asymptotically approaches a constant for \( q > 0 \), as we will see below.

#### 2.1.2 Effect of the shear on the turbulent energy

Next, we consider the effect of the shear on the turbulent energy. Technically, the turbulent energy consists of both energy from SNe and differential rotation since rotating MHD shear flows with \( q > 0 \) are unstable (Velikhov 1959; Balbus & Hawley 1991). In terms of energy density input rate, this implies

\[
\frac{\rho v^2}{\tau_{\text{cor}}} = \frac{E}{\tau_{\text{cor}}/\tau_\ast},
\]

where \( \rho \) is the average density of interstellar medium (ISM), \( v \) is the mean square root velocity of the turbulence, \( E \) is the energy input to the ISM per SN, \( l_\ast = v \tau_\ast \) is the eddy scale, and \( \varepsilon \) is the energy density input by shear and is taken to be a fraction of the fiducial shear energy density \( \rho v^2_\ast = \rho (l_{\text{c0}}/\tau_\ast)^2 \). More specifically, we then have

\[
\varepsilon = \xi \rho (l_{\text{c0}}/\tau_\ast)^2 = \xi \rho \tau_{\text{ed}}^2 q^2 \tau_{\text{ed}}^2 \equiv \xi q^2 \rho v^2_\ast,
\]

where we take \( \xi = 0.1 \). To provide physical meaning for the second term in equation (7), we note that the energy density supplied by SNe per unit time can be expressed as \( E V V^{-1} \), where \( V \) is the volume of the galaxy and \( V \) is the rate at which SNe are produced in \( V \). Crudely assuming that SNe occur isotropically, we have

\[
\frac{\Gamma}{\tau_{\text{ed}}} \approx \frac{V}{\tau_{\text{ed}}},
\]

where \( \Gamma_{\text{ed}} \) indicates the turbulent correlation scale from SN. Therefore,

\[
E_\varepsilon = \frac{E}{\tau_{\text{c0}}/\tau_\ast}.
\]

We further assume that \( E \) is a constant, and that the variation of \( \rho v^2 \) with \( \Omega \) is small compared to \( v \), so that \( \rho \) can be taken as approximately constant as well. For the fiducial point values, the ratio of the second term on the right-hand side to the left-hand side of equation (7) is

\[
0.1 \rho v^2_{\varepsilon}/\tau_{\text{c0}} \equiv 0.1 \rho v^2_\varepsilon/(\tau_{\text{c0}}/q) = \xi q^2 \rho v^2_\varepsilon = \frac{1}{20}
\]

in using (2) and (6). For a flat rotation profile (as in typical spiral galaxies), \( q \simeq 1 \) so that this ratio is small, and at the fiducial point values, we can neglect the second term on the right-hand side of equation (7) to obtain

\[
E_\varepsilon / \rho \approx v^3_{\varepsilon}/\tau_{\text{ed}}^4,
\]

which can then be used to simplify equation (7) to

\[
f = \frac{E}{\rho} = \frac{R^4}{f^3} + \frac{q^3}{10},
\]

where

\[
f(\Omega) \equiv v/v_0.
\]

Equation (13) determines the non-linear relation between the turbulent speed \( v \) and \( \Omega \). However, its solution has simple asymptotic behaviours. In the large \( \Omega \gg 1 \) regime, \( \tau_{\text{ed}} \rightarrow 0 \) and SN energy input rate dominates over that of the shear, so we can drop the second term in equation (13), which leads to \( f = (R^4 y^3)^{1/5} \). In the \( \Omega \ll 1 \) regime, the second term of equation (13) dominates so we drop the first term on the right-hand side to obtain \( f = (q y^3/10)^{1/2} \). The two terms contribute equally at \( \Omega = 0.22 \), so we approximate \( f \) as

\[
f = \begin{cases} 
(R^4 y^3)^{1/5} & \text{as } \Omega \geq 0.22 \\
(q y^3/10)^{1/2} & \text{as } \Omega \leq 0.22
\end{cases}
\]

These relations capture the fact that as SNe become scarce, the average turbulent speed of the ISM would decrease, but since shear provides a fixed baseline of turbulent energy, \( v \) approaches a constant.

Given equation (15), we are poised to check one more plausibility condition for \( y \), namely that the magnitude of \( \alpha_0 \approx \tau_{\text{cor}}(v \cdot \nabla \times v) \) cannot be larger than \( v \), since the helical fraction cannot exceed unity. For quasi-isotropic turbulence (Durney & Robinson 1982; Ruzmaikin et al. 1988),

\[
\alpha_0 \approx \frac{\xi}{v_\ast} \tau_{\text{ed}}^2 v^2 \Omega/h.
\]

MNras 469, 1466–1475 (2017)
where $\xi$ is a factor smaller than 1 and $h \propto r^{-1/2}$ is half of the scaleheight of the galaxy in an isothermal self-gravitating slab model (Spitzer 2008). The required inequality is then $\tau_{\text{cor}}^2 v^2 \Omega / h \leq v$, or, equivalently,

$$y^2 f^3/5 \leq \frac{h_0}{v_0 \tau_{\text{cor}}} = 5$$

(17)

upon using fiducial values $h_0 = 0.5$ kpc, $\tau_{\text{ed}} = 10^{15}$ s and $v_0 = 10^6$ cm s$^{-1}$, the validity of which can be checked using equations (6) and (15).

Note also that after turbulent energy is taken over by shear as $Ro$ decreasing, $l_{\text{cor}} = v \tau_{\text{cor}}$ will never exceed the scaleheight $2h$, since

$$\frac{2h}{l_{\text{cor}}} = \frac{2h_0 f^{-1/2}}{l_{\text{ed}} f y} = 10 f^{-3/2} y^{-1},$$

(18)

and it can be verified that the quantity above is always greater than unity using (15).

2.2 Correlation time and turbulent energy for fixed SN rates but different rotation rates

Complementing the previous subsection, here we instead fix the SN rate (and thus $\tau_{\text{ed}}$) but allow for different rotation rates. By direct analogy to (6), we define $\tilde{y}$ by

$$\tilde{y} = \frac{\tau_{\text{cor}}}{\tau_{\text{ed}}}.$$

(19)

Note that we have $\tau_{\text{ed}} = \tau_{\text{ed0}}$ here. The asymptotic limits are now

$$\tilde{y} = \left\{ \begin{array}{ll} \tilde{\rho} / q & \tilde{\rho} \to 0 \\ 1 & \tilde{\rho} \to \infty \end{array} \right.,$$

(20)

By analogy to equation (4), we take

$$\tilde{y} = \frac{\tilde{\rho}}{Ro + q}. $$

(21)

For the turbulent energy, we now generalize the energy input from shear to allow varying angular velocity, assuming that a fixed fraction is available. Thus, equation (8) is replaced by (with $\xi = 0.1$)

$$e = \xi q^2 \rho v_0^2 \tilde{\rho}^{-2}, $$

(22)

which gives the correct value of $e$ at the fiducial point where $\tilde{\rho} = Ro = Ro_0 = 1$. Now, $f = y / v_0$ is given by

$$\frac{\tilde{f}^2}{\tilde{y}} = \frac{1}{f^3} + \frac{q^3}{10Ro^4}, $$

(23)

of which the solution is approximately

$$\tilde{f} = \left\{ \begin{array}{ll} (\tilde{y})^{1/5} & \tilde{\rho} \geq 0.355 \\ (\tilde{y}^2/10\tilde{\rho})^{1/2} & \tilde{\rho} \leq 0.355 \end{array} \right..$$

(24)

The plausibility analogue to equation (17) becomes

$$y^2 f^3/5 \tilde{\rho}/\tilde{\rho} \leq 5,$$

(25)

which is also satisfied if we use equations (21) and (24), and same fiducial values $h_0 = 0.5$ kpc, $\tau_{\text{ed0}} = 10^{15}$ s and $v_0 = 10^6$ cm s$^{-1}$ as in the last subsection.

2.3 Generalizing the correlation time to include the case of rotation without shear

For rigid rotation, $q \to 0$, and $y = Ro^{-1}$ yields $\tau_{\text{cor}} = \tau_{\text{ed}}$ as expected in the absence of shear. The effect of rigid rotation without shear on $\tau_{\text{cor}}$ can be estimated to be

$$\tilde{\rho} = \frac{(\tilde{y})^{1/5} \tilde{\rho}}{Ro} \geq 0.355$$

(26)

Combining this with the case of Section 2.2 (fixed $\tau_{\text{ed}}$), we can then write

$$\tau_{\text{cor}} = \min\left[ \tilde{\rho}^{1/2}, \tilde{y}(Ro) \right].$$

(27)

For the case of Section 2.1 (fixed $\Omega$), but with $q = 0$, we can write $T_{\text{c}} = \tau_{\text{ed}} \Omega^{-1/2}$, and then

$$\tau_{\text{cor}} = \min\left[ Ro^{-1/2}, y(Ro) \right].$$

(28)

Note that equations (27) and (28) incorporate the separate influences on the correlation time from pure rotation and shear. Fig. 1 shows the correlation time in our approach.

We may express $Ro$ as a function of the radial coordinate $r$, given the rotation profile, i.e.

$$Ro = \frac{1}{\tau_{\text{ed0}} \Omega} = \frac{1}{\tau_{\text{ed0}} \Omega_0 (r/r_0)^{-q}} = \left( \frac{r}{r_0} \right)^q,$$

(29)

where we have used $\tau_{\text{ed0}} \Omega_0 = 1$. Replacing $Ro$ by $r$ using the relation above and assuming that all other variables are independent of $r$ provides us with one of the simplest ways to write down an $r$-dependent model.

2.4 Case when correlation time and $\Omega$ both depend on $r$

In the cases considered above, we have assumed that the eddy time and the rotation periods are independent, but both likely depend
on radius for galaxies. We now suppose that $\tau_{cd}$ varies with the radial coordinate $r$ according to the prescription adopted by Prasad & Mangalam (2016). Specifically, we adopt the relation

$$\tau_{cd} = \frac{r}{r_0} \propto r,$$  \hspace{1cm} (30)

where $r_0 = 8$ kpc and it is determined from the following argument: If $\tau_{cd}^0$ is proportional to the SN rate and the SN rate is proportional to the surface density of the SFR, $\Sigma_{SFR}$ (Shukurov 2004; Rodrigues et al. 2015), we have $\tau_{cd}^0 \propto \Sigma_{SFR}$. Further, we assume a Schmidt–Kennicutt-like power-law relation $\Sigma_{SFR} \propto \Sigma^3$ (Schmidt 1959; Kennicutt 1989; Heiderman et al. 2010), where $\Sigma$ is the gas surface density and typically $1 \leq \xi \leq 1.4$. For simplicity, we take $\xi = 1$. The mean galactic gas surface density $\Sigma_g \propto 1/r$ if the gas surface density hovers around a fixed fraction of order unity near the critical Toomre density for gravitational stability (Toomre 1964; Cowie 1981). Then, combining these above relations, we arrive at equation (30). If the helicity flux is driven by a galactic fountain, which, in turn, is driven by SNe (Shapiro & Field 1976; Tenorio-Tagle & Bodenheimer 1988; Shukurov 2004; Rodrigues et al. 2015), we might consider that the outflow speed also satisfies

$$U \propto 1/\tau_{cd} \propto 1/r, \quad \text{or,} \quad U = \frac{r_0}{r} U_0.$$  \hspace{1cm} (31)

In addition, for a flat rotation curve, $\Omega \propto 1/r$. Now since both $\tau_{cd}$ and $\Omega$ vary with $r$, we need the unified relations derived in Appendix, which results in

$$y(r) = r/2$$  \hspace{1cm} (32)

and

$$F(r) = \max[(1/2r^3)^{1/5}, (1/20r^3)^{1/3}].$$  \hspace{1cm} (33)

### 3 DYNAMO MODEL FOR LOW AND HIGH ROSSBY NUMBERS

The induction equation for the mean field is given by (for reviews, see Brandenburg & Subramanian 2005; Blackman 2015)

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \mathbf{E} - \beta \mathbf{J},$$  \hspace{1cm} (34)

where $\mathbf{B}$ and $\mathbf{U}$ are the (ensemble or spatial averaged) mean magnetic field and velocity field, respectively; $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the mean current (taking $\mu_0 = 1$); $\beta$ is the Ohmic resistive diffusion coefficient; $\mathbf{E} = \alpha \mathbf{B} - \beta \mathbf{J}$ is the mean turbulent EMF, where $\beta$ is the turbulent magnetic diffusivity; and $\alpha \equiv \alpha_0 + \alpha_m$ is the pseudoscalar helicity coefficient separated into kinetic and magnetic contributions, $\alpha_0$ and $\alpha_m$, respectively.

We adopt cylindrical coordinates and apply the ‘no-z’ approximation (Subramanian & Mestel 1993; Moss 1995; Phillips 2001; Sur et al. 2007) to reduce the partial differential equation to a simpler set of ordinary differential equation; the reduced dynamo equations read\(^1\)

$$\partial_t B_\phi = -\frac{2}{\pi} R_o (1 + \alpha_m) B_\phi - \left( R_U + \frac{\pi^2}{4} \right) B_\phi.$$  \hspace{1cm} (35)

\(^1\) Here we are working in the $\alpha-\Omega$ dynamo approximation. For the more general $\alpha-\Omega$ dynamo, an extra term $-\mathbf{R}_T/\tau_{cd} \propto 1/\tau_{cd}$ would appear on the right-hand side of (36). This term is negligible compared to the term $\mathbf{R}_T \mathbf{B}_\phi$, since $|\mathbf{R}_T/\tau_{cd}| \sim r^2 R_\lambda^2/15 \ll 1$ using (39), and $|\mathbf{R}_T/\tau_{cd}| \sim r^2/15 \ll 1$ using (40), for all values of interest of $R_0$.

$$\partial_t B_\phi = R_o B_\phi - \left( R_U + \frac{\pi^2}{4} \right) B_\phi.$$  \hspace{1cm} (36)

$$\partial_t \alpha_m = -R_U \alpha_m - \frac{\beta_3 \pi}{\beta_2} \alpha_m - C \left[ (1 + \alpha_m) \left( B_\phi^2 + B_\beta^2 \right) \right]$$

$$+ \frac{3}{8} \left( -\pi (1 + \alpha_m) R_o B_\phi + \alpha_m \frac{R_o}{R_m} \right) + \lambda \frac{R_o}{R_m} \left( B_\phi^2 - B_\beta^2 \right),$$  \hspace{1cm} (37)

where $B_\phi$ and $B_\beta$ are, respectively, the radial and azimuthal components of the total magnetic field. The $z$ component is assumed to be much less than these two and is neglected.

The second term on the right-hand side of (37) governs the effect of diffusive fluxes $\beta_3 \nabla^2 \alpha_m$ (Brandenburg, Candelaresi & Chatterjee 2009; Hubbard & Brandenburg 2010; Mitra et al. 2010), where $\beta_3$ is the diffusion coefficient. For most of the discussion of the solutions in Section 4, we take $\beta_3 = 0$ (the case of Sur et al. 2007), except for Section 4.5, where we adopt $\beta_3/\beta_2 = 1$ in a model using the radial coordinate $r$ as a free parameter and find that this diffusive helicity flux term raises the saturated magnetic energy as it exceeds the wind flux term $R_0$ for the fiducial parameters chosen, over much of the disc. The last term in equation (37) is the Vishniac–Cho flux (Vishniac & Cho 2001) with dimensionless coefficient $\lambda$. We find that in accordance with Sur et al. (2007) this flux has an influence only after the field already grows substantially, and has its strongest influence at low Rossby numbers. Even then, the buoyancy flux tempers the influence of the Vishniac–Cho flux. In the solutions presented in the sections below, we focus primarily on the case of $\lambda = 0$.

The magnetic fields are normalized by the equipartition field strength $R_{eq} = \sqrt{\nabla \cdot \nabla / \nabla}$, so that $B_\phi = v_A \beta_1$, and so on, with $v_A$ being the Alfvén speed. Note that $R_{eq}$ is a function of $v$ and thus varies with both the eddy turnover time and the galactic rotation speed. We normalize the time by the diffusion time-scale $\tau_2^2/\beta_2$, which again depends on the Rossby number. The dimensionless parameters in the above dynamo equations are

$$R_o = \frac{\alpha_0 h}{\beta_2}, \quad R_U = \frac{U h}{\beta_2}, \quad R_m = \frac{h^2 \Omega}{\beta_2}, \quad C = 2 \left( \frac{h}{T} \right)^2,$$  \hspace{1cm} (38)

where $U$ is the buoyant speed in the $z$ direction containing both a convective flow part $U_0$ and an MB part $U_0$; $\alpha$ is normalized by $\alpha_0$; and $l = v\tau_{cd}$ is the correlation length-scale of the turbulence.

For the fixed-$\Omega$ case of Section 2.1, substituting (2) and (14) into those dimensionless parameters gives

$$R_o = \gamma R_{c0}, \quad R_U = \gamma^{-1} f^{-5/2} R_{c0} + R_{V0},$$

$$R_m = \gamma^{-1} f^{-3} q R_{c0}, \quad C = \gamma^{-2} f^{-3} C_0,$$  \hspace{1cm} (39)

where $R_{c0}$ is the MB term, which will be clarified later.

For the fixed-$\tau_{cd}$ case of Section 2.2, we use (19) and $f = u/v_0$ to obtain

$$R_o = \gamma \tilde{R}^{-1} R_{c0}, \quad R_U = \gamma^{-1} f^{-5/2} R_{c0} + R_{V0},$$

$$R_m = \gamma \tilde{f}^{-3} \tilde{R}^{-1} R_{c0}, \quad C = \gamma^{-2} \tilde{f}^{-3} C_0.$$  \hspace{1cm} (40)

For the $r$-dependent model in Section 2.4, we use (A7) along with $U \propto 1/r$ to obtain

$$R_o = R_{c0}/2, \quad R_U = 2 R_{U0}/f^2 F^{-3/2},$$

$$R_m = 2 R_{c0}/f^2 F^3, \quad C = 4 C_0/f^2 F^3.$$  \hspace{1cm} (41)

\[ 
\text{MNRA} 469, 1466–1475 (2017) \]
where \( \vec r = r/r_0 \) with \( r_0 = 8 \) kpc, and we use the following typical data for our Galaxy to calculate the fiducial values (same as in Sur et al. 2007, for the comparison later):

\[
\begin{align*}
\tau_\omega &= 10^{15} \text{ s}, \\
v &= 10 \text{ km s}^{-1}, \\
r\Omega &= 200 \text{ km s}^{-1}, \\
l &= 0.1 \text{ kpc}, \\
h &= 0.5 \text{ kpc}, \\
U_0 &= 1 \text{ km s}^{-1},
\end{align*}
\]

which give (with \( q = 1 \))

\[
R_{u0} \approx 1, \quad R_{U0} \approx 0.3, \quad R_{ed} \approx -15, \quad C_0 \approx 50, \quad R_\infty \approx 10^4,
\]

and the corresponding fiducial Rossby number \( R_{\Omega0} \approx 1 \).

The instantaneous dynamo number in the kinematic regime can be defined as the square of the ratio of the coefficients of the amplifying rate terms \( \gamma_\delta \) and the decay rate terms \( \gamma_d \):

\[
D_\text{ins} \equiv \frac{\gamma_\delta^2}{\gamma_d^2}, \quad (42)
\]

with

\[
\gamma_\delta^2 = \frac{2}{\pi (1 + \alpha_m) R_\alpha |R_u|} \quad (43)
\]

and

\[
\gamma_d^2 = \left( R_U + \frac{\pi^2}{4} \right)^2 \quad (44)
\]

being, respectively, the product of growth and decay terms in (35) and (36). We can define the dynamo growth time, divided by the diffusion time \( \tau_\text{diff} = h^2/\beta_1 \), as

\[
\frac{\tau_\text{dyn}}{\tau_\text{diff}} = \frac{1}{\gamma_\delta - \gamma_d}. \quad (45)
\]

The bottom panel of Fig. 2 shows \( \tau_\text{dyn} \) (thick blue line) in comparison with the age of the universe \( \tau_u \approx 10^3 \tau_\text{ed}, \tau_\text{ed} \) and \( \tau_x \) for our fixed-\( \Omega \) case, while the dashed purple line (\( \tau_\text{dyn} \)) indicates the dynamo growth time for our fixed-\( \tau_\text{ed} \) case. All times in the plot are normalized by \( \tau_\text{diff} \). The vertical dot-dashed lines at \( R_\Omega = 0.22 \) and 0.355, respectively, correspond to the transition values of equations (15) and (24), respectively, and marking for each of these cases the transition from shear-dominated to SN-dominated turbulent velocities as \( R_\Omega \) increases. The top panel of Fig. 2 shows the 3D space that unifies the cases of Sections 2.1 and 2.2 via equation (A5).

Several interesting features are evident in the bottom panel of Fig. 2. First, in both cases, for either the fixed \( \Omega \) or fixed \( \tau_\text{ed} \), \( \tau_\text{diff}^{-1} \rightarrow 0 \) when \( R_\Omega \) approaches \( \sim 1.2 \). As a consequence, for \( R_\Omega \lesssim 1.2 \), the initial growth of the magnetic field will be too slow to produce a significant large-scale field. Secondly, \( \tau_\text{dy}n \) becomes independent of \( \tau_\text{ed} \) when \( R_\Omega \lesssim 0.1 \) for fixed \( \Omega \) (blue curve) because of the complete dominance of the shear as a supplier of turbulence. In contrast, for the case of fixed \( \tau_\text{ed} \) (dashed purple line), the growth time blows up for \( R_\Omega \lesssim 0.02 \), highlighting that field growth becomes insignificant at these values in this case. The top panel of Fig. 2 shows how these two different cases are mutually compatible in 3D. The solutions further demonstrating these points will be discussed in the next section.

For the dynamo to have a significant influence on the large-scale field, its growth time must be less than the age of the universe \( \tau_u \). The associated condition \( \tau_\text{dy}n \leq \tau_u \) leads to an upper bound on \( R_\Omega \) above which the dynamo solution cannot produce significant observable large-scale fields. In addition, we impose a lower bound on \( R_\Omega \) for fixed \( \tau_\text{ed} \) by the condition \( \Omega_{\text{max}} c = \epsilon/10 \), with \( c \) being the speed of light, simply so that we focus on the cases where the rotation speed is non-relativistic. Combining these two constraints, we can express the physically meaningful range as

\[
R_\Omega \leq 1.171 \quad (\log R_\Omega \leq 0.069) \quad (46)
\]

for fixed \( \Omega \), and

\[
0.024 \leq R_\Omega \leq 1.161 \quad (-1.62 \leq \log R_\Omega \leq 0.065) \quad (47)
\]

for fixed \( \tau_\text{ed} \).

4 SOLUTIONS

For the first three subsections below, we focus on the fixed-\( \Omega \) case, before addressing a few important features of the fixed-\( \tau_\text{ed} \) case in the penultimate subsection. In the last subsection, we consider solutions for the case when \( \Omega \) and \( \tau_\text{ed} \) mutually depend on \( r \).

4.1 Steady state without MB

We first consider the case without MB. By solving equations (35)–(37) for a steady state (\( \partial_t = 0 \)), we obtain the darkest blue dotted line in Fig. 3. The \( y \)-axis, representing the magnetic field strength, is scaled with the equipartition field strength \( B_0^2 = 4\pi \rho v^2 \), which depends on \( R_\Omega \). To the left-hand side of the vertical dot-dashed line...
the turbulence and correlation time (though shear is still maintained for which is the expression for \( \text{relative strength of the two components of the magnetic field. The increase in } |p| \text{ when MB is included results from a greater loss in the toroidal field.} \)

Figure 3. Magnetic energy \( B^2 = B^2_t + B^2_p \) as a solution of equations (35–37). (a) Full 2D solution surface showing the dependence on both \( \tau_\text{cor} \) and \( \tau_\text{ad} \) without MB (blue surface) compared to the 2D solution surface in the absence of the influence of shear on the turbulence. The solid (dashed) black curves are obtained by taking a slice at the fiducial value \( \tau_\text{cor}/\tau_0 = 1 \) (\( \tau_\text{ad}/\tau_0 = 1 \)), corresponding to the case with fixed \( \Omega = \Omega_0 \) (\( \tau_\text{ad} = \tau_\text{ad0} \)) and varying \( \tau_\text{ad} \). (b) The 1D solution for fixed \( \Omega \) but varying \( \tau_\text{cor} \) corresponding to the solid red curves in (a). The two darkest dotted curves represent the results with and without MB, respectively. The competition between enhanced large-scale growth due to buoyant ejection of small-scale magnetic helicity and the buoyant loss of large-scale fields can be assessed by comparing the two curves. The top dotted curve is obtained by neglecting the contribution from shear in both the turbulent energy and the correlation time. Dynamos supported purely by shear and MB become possible for small \( \text{Ro} \), indicated by the lowest dotted curve.

line at \( \text{Ro} = 0.22 \), the turbulence is mostly driven by shear, and to the right-hand side, it is driven by SNe. The cusp irregularity at \( \text{Ro} = 0.22 \) occurs because of our piecewise-defined (15), which, in principle, can be removed by rigorously solving \( f \) but not essential for the level of detail explored here. We used (15), which is sufficient to capture the asymptotic behaviour for large and small \( \text{Ro} \). The darkest blue dotted line solution includes the influence from differential rotation of both \( \tau_\text{cor} \) and \( \nu \), and can be compared with the top dotted line, obtained by taking for the full range of \( \text{Ro} \):

\[
f = (\text{Ro}^\lambda y)^{1/2}; \quad y = \min(\text{Ro}^{-1}, \text{Ro}^{-1/2}),
\]

which is the expression for \( f \) that neglects the effect of shear in the turbulence and correlation time (though shear is still maintained for the \( \Omega \) effect in the \( T_\phi \) equation).

In Fig. 4, we show how different components of the magnetic fields depend on the Rossby number. We define the pitch angle by

\[
p = \arctan \frac{B_t}{B_p} = \arctan \frac{R_{\text{cor}}}{R_U + \nu^2/4},
\]

where we have used (36) for the last equality. The magnitude of \( p \) decreases with decreasing \( \text{Ro} \) when the turbulent energy is mostly provided by SNe (region to the right-hand side of the vertical line), in agreement with the numerical solution in Chamandy et al. (2016) (see their fig. 2, where they used the Coriolis number \( C_0 = 1/\text{Ro} \)). As expected, the pitch angle goes to a constant as \( \text{Ro} \to 0 \), since without SNe the turbulent energy and the correlation time depend only on the rotation profile. Then, \( \text{Ro} \) drops out of the equations and the dynamo saturates to a state purely driven by shear at fixed \( \nu \). The smallness of the pitch angle is consistent with the basic observation that galactic magnetic fields are predominantly azimuthal (Beck & Wielebinski 2013).

### 4.2 Role of MB versus outflow and diffusive flux

We now investigate the inclusion of MB. Although Foglizzo & Tagger (1994) suggest that differential rotation will stabilize the Parker mode, we neglect this effect in our rough calculations here. We use the buoyancy speed as calculated in Parker (1979). For a weak magnetic field of sub-equipartition (with the turbulence) strength, \( U_\text{b} \approx v_\lambda^2/v = v(B_t^2 + B_p^2) \). For a magnetic field comparable to equipartition strength, \( U_\text{b} \approx v_\lambda = v\sqrt{B_t^2 + B_p^2} \). The field-related buoyancy coefficient (assuming \( |B_p| \gg |B_t| \)) is then

\[
R_{\text{bb}} = \frac{U_{\text{b}} h}{v^2 \tau_\text{cor}} = \left( \frac{C_0}{2} \right)^{1/2} y^{-3/2} \min \{ B_p, B_p^2 \}
\]

for fixed \( \Omega \). (For the case of fixed \( \tau_\text{ad} \), we would just replace \( y \) by \( \tilde{y} \) and \( f \) by \( \tilde{f} \) in the above expression.)

MB extracts small-scale magnetic helicity, but also large-scale fields. As a consequence, there is a competition between the loss
of large-scale field and benefit to amplification from small-scale magnetic helicity removal. The bottom dotted line of Fig. 3 shows the solution for the fixed-Ω case. Here, the presence of MB lowers the overall field strength compared to the case when \( R_{\text{th}} = 0 \). For fixed \( \Omega \), we also note the possibility of dynamo purely supported by only MB, where \( U_0 = R_{\text{th}} = 0 \). This solution is represented by the lightest blue curve in Fig. 3.

The curves represented by green diamonds and red triangles of Fig. 4 show the different behaviours of the toroidal and poloidal magnetic fields. The growth of toroidal field (\( B_\theta \), blue circles and green diamonds) is suppressed by MB, whereas the poloidal field (\( B_r \), yellow squares and red triangles) is amplified by MB. This is understandable by noting the competing roles of MB mentioned above, and the fact that in equations (35) and (36), MB is more significant for the toroidal field loss because \(|B_\theta| \gg |B_r|\).

The importance of the diffusive helicity flux (second term on the right-hand side of equation 37) can be assessed by its separate ratios to the wind term (first term on the right-hand side of equation 37) and the MB (third term on the right-hand side of equation 37). For \( \beta_1 = \beta_2 \), these are, respectively,

\[
\frac{\text{diff}}{\text{wind}} = \frac{\pi}{2R_0} y f^{5/2}
\]

and

\[
\frac{\text{diff}}{\text{MB}} = \left( \frac{\tau_0^2}{2C_0} \right)^{1/2} y f^{3/2} \min \{ B_\theta, B_\phi^2 \}
\]

in the case of fixed \( \Omega \). Since both ratios are smaller than 1 when \( R_\Omega < 1 \), keeping or neglecting the diffusive helicity flux term will not change the results significantly.

The pitch angle profile under the influence of MB is shown in Fig. 4. This curve explicitly reveals that MB more strongly suppresses azimuthal fields.

For this model, we can predict the tangent of the pitch angle as a function of \( Ro \). The result is shown in Fig. 5, where we compare our numerical prediction with that of Chamandy et al. (2016) [who found \( \tan p \sim \tau_0 (v/h)^2 / (q \Omega) \)]. The red part shows a power law \( \ln (-\tan p) = 1.13 \ln (Ro) + \text{constant} \). The limited data in Van Eck et al. (2015) from their fig. 8 suggest a slope of 0.4–0.5, if we assume that the surface SFR density is proportional to the surface supernova rate density \( \alpha_1 / \tau_\text{ed} \propto Ro \). This is closer to the predicted value of Chamandy (2016) than ours, but more data and work are ultimately needed to pin down the tightness of these trends and predictions.

4.3 Time-dependent solutions

We now compare the time evolution of magnetic fields from the dynamo solutions for different values of \( Ro \) in Fig. 6. The time is normalized by the constant \( \tau = 2\pi / \Omega \), and the magnetic fields are normalized by the \( (Ro\text{-dependent}) \) equipartition field strength.

The two lower curves show the transition from decaying solutions to those with an asymptotic sustenance of a steady state as \( Ro \) is dialled below \(~1.2\). As \( Ro \) is decreased downwards from 1.25, the dynamo growth time decreases. The growth time reaches a minimum (the dotted curve, \( Ro = 0.6 \)) and then increases, finally saturating (the solid curve), in agreement with Fig. 2. The dashed black curve indicates the fiducial point \( Ro = 1 \).

4.4 Fixing \( \tau_\text{ed} \) and changing \( \Omega \)

Fig. 7 shows the dynamo solutions using the relation (21) and the corresponding non-dimensional parameters in the dynamo equations for the case of fixed \( \tau_\text{ed} \) and varying \( \Omega \). The vertical dot-dashed line marks the transition value \( \tilde{Ro} = 0.355 \) between shear-dominated and SN-dominated turbulence. The maximum steady-state field strength \(~0.02 B_\text{eq}^2 \) occurs at intermediate \( Ro \sim 0.2 \), and decays with \( \tilde{Ro} \) for both lower and higher \( Ro \). This contrasts the
saturated steady states of Figs 3 and 4 for small Ro where we fixed
\(\Omega\) and allowed \(\tau_{\text{ed}}\) to vary.

4.5 Dynamo solutions as a function of radius when SN rate
depends on \(\Omega\)

Fig. 8 shows the result in using the model discussed in Section 2.4
and equation (41). The horizontal axis is normalized by \(r_0 = 8\) kpc.
Here, we define \(e_{\text{turb}} = \rho u^2 = \rho_0 v_0^2 r^2 / \tau_{\text{ed}}\) and \(e_B = B^2\) as the
turbulent energy density and magnetic energy density, respectively, and
show them in blue curves. The ISM mass density is assumed to have the
same dependence on \(r\) as the galactic surface density, i.e. \(\rho \propto 1/r\).
The red curve represents the model with (48) being used, i.e. it
neglects the effect of shear on both correlation time and
turbulent energy density. Beyond the galactic central region \(r / r_0 < 0.2\)
where a more sophisticated dynamo model is needed, we obtain
a nearly flat profile for both turbulent and magnetic energy density
correlation time of the turbulence (Blackman & Thomas 2015).

Curiously, if we compare the two dashed lines, i.e. the turbulent
energy densities with and without considering the shear, the latter
is above the former, yet the former one includes energy sources
from both SNe and shear. This is not surprising if we realize that
even though cooperating shear into the model increases the turbulent
energy input rate, the correlation time is decreased at the same time,
leading to a net effect of lowering \(e_{\text{turb}}\) (see equation 7).

The black curve of Fig. 8 represents the magnetic energy density
if we take the diffusive helicity flux term into consideration. Here,
the diffusion coefficient \(\beta_d\) is assumed to be equal to the turbulent
diffusivity \(\beta_t\), which may be an overestimate because usually the
ratio \(\beta_d / \beta_t\) is taken to be \(\ll 1\); for example, in Brandenburg et al.
(2009), it is 0.05, and in Mitra et al. (2010), a value of \(\sim 0.3\) is found
(although relatively low compared to what would be appropriate for
galaxies). Using (32), (33) and (51), we find that the contribution
from the wind term (characterized by \(R_\gamma\)) is comparable to that
from the diffusive term (characterized by \(\pi/2\)) when \(r < 0.3\), and \(\pi/2 \gg R_\gamma\)
when \(r > 0.3\), showing a dominance of this diffusive helicity flux
in almost the whole disc. The inclusion of \(\beta_d\) increases the saturated
value of magnetic energy by nearly an order of magnitude, given
our fiducial parameter choices.

5 CONCLUSIONS

We have generalized a 2D ‘no-\(z\)’ galactic dynamo model with helicity
fluxes to include two effects of differential rotation beyond its
role in the \(\Omega\)-effect that have not been previously combined
galactic dynamo models. First, differential rotation provides an
additional energy source for ISM turbulence, beyond that of SNe.
Secondly, differential rotation can shred turbulent eddies, reducing
the correlation time of the turbulence (Blackman & Thomas 2015).

We have incorporated these effects and relaxed the commonly
assumed equality between the correlation time and the SN-driven
eddies turnover time. We show that the effect of shear on the correla-
tion time can be important even when shear does not dominate
the turbulent energy. For low SN rates and strong shear, both effects
are important. We separately studied the influence of differential
rotation on the mean field dynamo solutions as a function of the SN
input rate and the rotation period when these quantities are taken
to be independent and also when they are proportional to each other.
The latter would be expected from correlations of the SN rate with
the SFR and, in turn, the galactic surface density and rotation rate
(Prasad & Mangalam 2016).

Our solutions show that the observable steady-state mean field
dynamo field strengths at low Rossby numbers are significantly
lower than those when the correlation time is independent of shear.
The model also predicts the pitch angle of the mean field, a measure
of radial to toroidal field magnitude, and a clean quantity to compare
with observations (Chamandy & Taylor 2015). Unlike previous
work, we have also included MB as a contributor to the helicity
fluxes, which becomes most important when Ro, \(\tilde{R}_o < 1\). We find
that dynamos for which the helicity fluxes are entirely determined
by buoyancy are possible even in the absence of advective or diffusive
fluxes.

We also considered a model (Section 2.4) where both \(\tau_{\text{ed}}\) and
\(\Omega\) are functions of \(r\). All dimensionless parameters were then reinter-
preted as functions of \(r\) only, as in (41). When our model is
used in this way to explore the radial dependence of quantities
within a galaxy, we derived that the magnetic energy density profile
is relatively flat in radius, consistent with observations (Beck &
Wielebinski 2011), and it is a result that serves as a test/consistency
check for the model. The shape of the curve is sensitive to the
Schmidt–Kennicutt index of Section 2.4. If we switch it from 1 to
1.4, the radius at which the steady-state magnetic energy density
drops to zero will move from \(r \sim 0.2\) to \(0.7\).

Earlier prescriptions for galactic dynamos with \(\tilde{R}_o < 1\) included
only the effect of \(\Omega\) on the reduction of correlation time in \(a_0\)
(Ruzmaikin et al. 1988), and without explicitly including the role
of shear as a source of energy for the turbulence. We showed herein
that shear causes a further reduction in the correlation time not captured
by the previous treatments and an \(\alpha \propto \Omega^{-3/4}\) for fast rotation
in the fixed-\(\tau_{\text{ed}}\) case (Section 2.2). This is a weaker reduction for fast
rotators than rotational quenching in the absence of the shear effects,
which predicts \(\alpha \propto \Omega^{-1}\) (Rudiger 1978).

Our calculations herein focused only on two specific influences
of the role of shear, and we do not purport to have captured all of
the effects of shear on the turbulence, and also we have not included
all terms in the EMF that depend on rotation. There are also other
approaches to helicity-flux-driven mean field dynamos that bypass
the \(\alpha\) coefficient altogether. Our point in this paper, however, is
to focus on specific effects on shear that have been understudied.
Future work should incorporate and assess the relevance of lessons
learned here in the derivation of other dynamo coefficients not
previously considered.
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APPENDIX: UNIFIED TREATMENT OF SECTIONS 2.1 AND 2.2

We can formally combine the two cases (Sections 2.1 and 2.2) in a single formalism by defining

\[ \tilde{\tau}_{ed} = \frac{\tau_{ed}}{\tau_{ed0}}, \quad \tilde{\tau}_r = \frac{\tau_r}{\tau_{r0}} = \frac{\Omega_0}{\Omega}, \]  

so that \( \alpha = \tilde{\tau}_r/\tilde{\tau}_{ed} \). Then, we can write

\[ y = \frac{\tau_{cor}/\tau_{ed0}}{1 + \frac{1}{\tilde{\tau}_{ed}^{-1} + q^2 \tilde{\tau}_{r}^{-1}}}. \]  

The energy rate balance equation is

\[ \frac{\rho u^2}{\tau_{cor}} = \frac{E}{\tau_{ed0}^4} + \frac{0.1q^2 \rho v_{0}^2 \Omega_0^2}{\tau_{s}}. \]  

which, using \( F = v/v_0 \), can then be expressed as

\[ \frac{F^2}{y} = \frac{1}{\tau_{ed0}^4 F^3} + \frac{q^3}{10 \tau_r}. \]  

The solution is approximately

\[ F(\tilde{\tau}_{ed}, \tilde{\tau}_r) \approx \max \left\{ \left( \frac{y}{\tilde{\tau}_{ed}^2} \right)^{1/5}, \left( q^3 y/10 \tilde{\tau}_r \right)^{1/2} \right\}. \]  

and it is related to \( f \) (equation 15) and \( \tilde{f} \) (equation 24) in Section 2 through

\[ f = F(\tilde{\tau}_{ed}, 1), \quad \tilde{f} = F(1, \tilde{\tau}_r). \]  

Non-dimensional parameters (defined in Section 3) then exhibit the following scalings:

\[ \alpha \propto y/\tilde{\tau}_r, \quad R_o \propto 1/F^{3/2} y, \quad R_o \propto 1/F^3 y^{3/2}, \quad C \propto 1/F^3 y^2. \]  

Fix-\( \Omega \) and fix-\( \tau_{ed} \) cases correspond to, respectively, taking \( \tilde{\tau}_r = 1 \), \( \tilde{\tau}_{ed} = R \sigma^{-1} \) and taking \( \tau_{ed} = 1 \), \( \tilde{\tau}_r = R o \) in the above relations.

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