Manufacturable Gradient-Variable Cellular Structures Design

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Abstract. Topology optimization (TO) has been widely used in the field of additive manufacturing (AM), but the structure designed by the density-based optimization method has obvious material gradient and complex internal structures, which are difficult to be directly manufactured with specific process. This paper proposes a multi-scale design method for designing cellular structure with topology optimization. Firstly, micro units with variable density are designed to support smooth interconnections. Then, we provide a assembling method to generate models with density distributions similar to topology optimization method, while guarantee the manufacturability of the structures. The experimental results show that the generated model does not need support in fabrication, and retains the optimized mechanical properties, which can be used as a TO based fill pattern for 3D printing.

1. Introduction

In the past decade, TO methods for AM technology have become an active research filed. AM is able to create complex functional structures without increasing costs, so it can take advantage of topology-optimized designs to create innovative structures. This combination has an increasingly widespread impact in various industrial fields in recent years. However, applying TO methods in AM can also cause many problems that are difficult to solve.

First, TO methods usually generate internal structures with pores or large holes. Although AM is an efficient approach to fabricate object with complex structures, there are also many limitations. Specifically, when the design object is an internal structure with a complete outer casing. For some AM processes, such as the Fused Deposition Modeling (FDM), Stereolithography (SLA), and Selective Laser Melting (SLM), internal support structures are generated, which is difficult to remove in the post-processing.

Secondly, many applications have different requirements on the microstructures. For example, the pore size and connectivity of the microstructure have a very important influence on the cell attachment and growth in bioscaffold design. Controlling microstructure and macroscopic material distribution at the same time is a challenged task.

By combining the TO and the cellular structures design, this paper proposes a multiscale structural design method TO porous features. We can select and design the right microstructure according to the application needs. At the same time, the global distribution of materials is also considered to ensure that the design of the structure in the macro to meet the requirements of mechanics, lightweight and...
2. Related works

2.1. Topology optimization

TO method, also referred as layout optimization method, seeks to find the optimal load path for a certain boundary condition, which enables to find the best material distribution in the part that fulfills certain constraints, such as compliance constraints and displacement constraints. Since 1988, Bendse and Kikuchi introduce the homogenization method into the optimization process [1], big progress is have achieved in TO methods. Traditional structural optimization has been extended to solve some new optimization problems such as cellular structure design [2], mass transfer [3], metamaterial design [4], multifunctional material design [5] and optimization of performance and appearance fixed structures [6]. A lot of manufacturing-oriented TO algorithms are presented and the most representative one is density-based algorithm [7]. This method establishes the relationship between the relative density of voxel elements and the physical properties of materials. The relative density of elements is often used as the design variable. However, the most obvious disadvantage of this optimization method is that it produces intermediate density voxel elements that are difficult to manufacture.

The discrete structure-based solid isotropic material penalty function (SIMP) method [8] is a classic density based TO method, which considers that the intermediate density is not manufacturable. The penalty function is used to generate cavity and solid structures. The rational approximation of material properties (RAMP) method [9] is the other commonly used method to solve the problems that do not converge with SIMP [10]. In addition to using the above penalty function to solve the density optimization problem, the bi-directional evolutionary structural optimization (BESO) method [11] is developed to find the optimal value of the density distribution of the voxel material in the structure by using the dichotomy, and gradually discretize the intermediate density to a binary density structure of 0 and 1.

In order to ensure the manufacturability of the design results, more constraints are integrated into the local filter [12] [13]. While the design process is not intuitive and tends to fall into local optimization.

2.2. Cellular structure

Cellular structures such as foam, honeycomb, and lattice structure are used in applications due to their special mechanical properties. There are several existing methods to generate cellular structures, including model boolean operation (MBO), image-based, implicit surfaces, and TO method can be used to design cellular structures. The MBO method assembles new cellular structures by boolean operation on the basic cellular structures [14] [15]. The common used basic structures include rod units, plate units and body units.

Image-based method is to obtain cellular structures by reverse engineering through the micro-CT image. Cellular structures generated by this method are most consistent with the original model, and they are most used in the field of biological implants [16]. Implicit surface modeling method is a method of forming porously connected structures by minimal surfaces such as triply periodic minimal surfaces(TPMS). Using TPMS as the filling structure can improve the robustness of the structure designed by the optimization method [17].

Another direction to design cellular structure units is to use a pure TO method, which uses basic TO methods to design cellular structures that meet specific properties requirements. Although some articles suggest that an optimized self-supporting structure can be designed, it is not a priori and likely to be unstable or even failed [12].

The density-based optimization method can achieve optimal distribution of materials within the structure, but cannot guarantee the manufacturability. In addition, special microstructures are required in many applications. In order to obtain manufactured cellular structures with excellent mechanical
properties, this paper proposes a unit assembly method combining density-based TO method and MBO structure design method.

3. Design framework

The design framework is shown in figure 1, which consists of three steps. Firstly, the voxel density distribution map of the macrostructure is obtained by the TO. Then, the designed microstructure unit families are selected by control parameters. And finally, the microstructures are assembled into the macrostructure through the mapping function, which generates cellular structure with certain distribution of material density.

3.1. Topology optimization
We take a compliance minimization problem as an example, the objective function is represented as

\[
\arg_x \min C(X) = F^T U = U^T KU
\]

\[
s.t. \quad V^* = \sum_{i=1}^{n} X_i
\]

In equation (1), \( F \) and \( U \) represent the load and displacement of the structure, \( K \) represents the overall stiffness of the structure, \( V^* \) represents the residual material volume of the optimization target, and \( X_i \) represents the voxel element density of the macrostructure.

The TO algorithm used in this paper is the SIMP. As mentioned above, in order to make the design structure easier to manufacture the original SIMP algorithm uses penalty strategy. The intermediate density is penalized so that the intermediate density unit approaches both ends to generate the solid and the cavity. In assembly, we use a unit structure with different densities to fit the local density distribution. Therefore, the density distribution of the gradient is more appropriate. In the experiments, different penalty factors are used and tested.

3.2. Unit design
Once we get the density map by SIMP, each element of the map can be replaced with a unit structure with the same density. The key problem is to provide such type of unit, which has the ability to express a wide density range. There are different designs for different application requirements, but some of the must be met. These rules consist of effective structures, stable structures, manufacturable structures, and smooth connectivity. Then these units can be added into a unit database to support fast search and matching.

3.3. Assembling
Assembly is a process that includes one-to-one unit replacement and structure reconstruction. Given an element from the density map, the voxel with relative density less than a certain value \( X_{\text{min}} \) is
determined to be the microscopic structural unit with the smallest material density. The position information of the units with different material densities can also be obtained from the density map. When selecting the same family of microstructural units, it is necessary to consider the problems of connection between different density microstructural units controlled by control parameters. If choose different families of microstructure units, it is still need to consider the connection problems between the units.

4. Unit design
As mentioned in section 3.2, a viable design unit must meet the following requirements: effective structures, stable structures, manufacturable structures, and smooth connectivity. The density range of a structural unit with a cavity is between $[X_{\text{min}}, 1]$. Where $X_{\text{min}}$ is a lower limit to its density because the unit must have sufficient material density to ensure its own manufacturing and strength.

4.1. Effective structures
In principle, the material in a certain density can be randomly distributed, but it causes effective structures. Taking a two-dimensional structure as an example, as shown in figure 2. (a) shows the material inside the unit is directly broken off. (b) and (c) show the material inside unit have no contribution to the structural bearing capacity.

![Figure 2. Two-dimensional ineffective unit structures.](image)

There are several three-dimensional ineffective unit structures in figure 3. Figure 3(a) shows hexahedral voxel inner isolated sphere solid model is directly broken off. Figure3 (b) - (d) respectively show that the material in unit does not contribute to the structural bearing capacity.

![Figure 3. Three-dimensional ineffective unit structures.](image)

4.2. Stable structures
The instability of unit structure includes the minimum size of the structural section and the largest structural stress area of unit. These structures have structural weaknesses as shown by the arrows in figure 4. The structural weakness will not only increase the macrostructural instability, but also reduce structural service life.

![Figure 4. Two-dimensional unit with weakness.](image)

There is also structural weakness in three-dimensional structural unit, as shown in figure 5. The unit consists of a spherical structure that is hollowed out in the cubic unit. Along with the material in the unit decrease, the structure unit becomes more and more unstable.
4.3. Manufacturable structures

The manufacturability of the unit is related to the specific processing constraints and the minimum size that can be machined. There is no concept of support structure in the two-dimensional structures. When there is a cantilever structure in the three-dimensional structure, support structures will be required if the angle between the cantilever structure and the printing plane is less than a specific support angle. Take FCC unit in figure 6 as an example. Because of there are suspended beam structures in the upper and lower boundaries, this unit structure needs support structures.

![Figure 6. FCC unit.](image)

It is necessary to determine the minimum structural size of the structure and its position. We can judge the minimum size by the intersection of the structure and printing plane, as shown in figure 7. The material distribution of the FCC unit on print plane $a$ and plane $b$, the minimum size structures on the planes are divided into rectangles, ellipses and quarter circles. Arrows 1, 2 and 3 refer to the minimum size of this structure.

![Figure 7. Different material distribution of FCC unit on plane $a$ and plane $b$.](image)

4.4. Smooth connectivity

The designed units need to be assembled into the macrostructure. Therefore, it is necessary to consider the connection problem between adjacent units. Figure 8(a) shows the mismatch between the same family, and figure 8(b) shows the mismatch situation of different family units. The mismatching phenomenon will not only cause stress concentration even cause structural failure. Similarly, in the three-dimensional structure, there is also a mismatch situation between the adjacent units. In order to avoid the mismatch situation of adjacent units, the units with similar boundary features will be designed or selected.

![Figure 8. Units connection.](image)
4.5. Design strategy
Combining the above-mentioned unit design rules, a two-dimensional unit structure is designed. As shown in figure 9, the idea of the unit design is to design a hollow rectangular structure. This structure not only meets the manufacturability requirements but also ensures that adjacent units can be smoothly connected.

For two-dimensional situation, the material density of such a unit is controlled by the inner hollow material quadrilateral side length \( D \). Assuming that the side length of the quadrilateral unit is \( a \), the unit material density can be calculated by equation (2).

\[
\rho = \left( a^2 - D^2 \right) / a^2, \quad (D \leq a - 2d_{\text{min}}, \; 2d_{\text{min}} < a)
\]

Where \( d_{\text{min}} \) represents the minimum manufacturing size of a manufacturing process.

According to the unit design rules, a three-dimensional structure is designed as shown in figure 10. The basic structural feature is the top and bottom plates in a hexahedral voxel unit, a central cross plate structure, and an inner hexahedral prism structure. The main control parameters of the three-dimensional unit are the top and bottom plate thickness \( d_1 \), the cross plate thickness \( d_2 \), the cross-sectional square of inner prism side length \( D \). In order to make adjacent units smoothly connected, the dimensions of plate thickness \( d_1 \) and \( d_2 \) are fixed.

Assuming the minimum processing size is \( d_{\text{min}}=0.5\text{mm} \) and parameters \( d_1=0.2\text{mm} \), \( d_2=0.5\text{mm} \), the density \( \rho \) can be directly calculated by equation (3).

\[
\rho = (-D - \sqrt{2} / 2)^2 \times (a - 0.4) + 1.4a^2 - 0.65a + 0.1, \quad (0 \leq D \leq \sqrt{2}a / 2 + \sqrt{2} / 4)
\]

The range of the unit density can be obtained by adjusting the structural control parameter \( D \). The range is \([\frac{(1.4a^2 - 0.6a + 0.1)}{a^3}, \frac{(0.5a^3 - 0.7a^2 - 0.325a + 0.05)}{a^3}]\) derived by equation (3), which is too narrow to map voxel relative density. It is necessary to extend the basic structural units of this family. In figure 11, upper row is shown as an overall view of the extend basic unit structures, and lower row is shown as the cross-sectional pattern corresponding to the upper row of structures.
5. Assembling
Assembling is a process that converts density map from TO to a cellular structure. Given each voxel in the density map, the assembling process finds a microstructure from the unit database with the same material density. The density range of the selected unit family and the scale of unit must be determined, which have a large impact on the accuracy of the finite element analysis (FEA) calculation.

5.1. Determine the scale of unit
Unit assembly is complete based on the material density mapping relationship. Calculating the density range of two-dimensional unit is relatively easy to obtain directly by equation (2). The density range calculation is quite complicated of three-dimensional unit, as shown in equation (4). The density range consists of five subranges of three types of basic units.

$$\rho = \begin{cases} 
\left[1.4a^2 - 0.65a + 1 - (a - 0.4)D_E^2\right]/a^3, (0 \leq D_E \leq 0.5) \\
\left[1.8a^2 - 0.4a - (a - 0.4)D_E\right]/a^3, (0.5 < D_E \leq a - 0.8) \\
(1.4a^2 - 0.65a + 0.1)/a^3, (0 \leq D_{SC} \leq \sqrt{2}/2) \\
\left[1.4a^2 - 0.65a + 1 - (a - 0.4)(D_{SC} - \sqrt{2}/2)^2\right]/a^3, (\sqrt{2}/2 < D_{SC} \leq \sqrt{2}/2a + \sqrt{2}/4) \\
0.5a^3 + 0.7a^2 - 0.725a + 0.05 + \sqrt{2}(a^2 - 0.9a + 0.2)D_{SP}, (\sqrt{2}/2a + \sqrt{2}/4 \leq D_{SP}) 
\end{cases}$$

(4)

The structural density control parameters of this family of microstructure units are the inner prism side length $D_E$, $D_{SP}$, $D_{SC}$ and the unit scale $a$, which can provide a wide range of density. For a unit with complex structures, the density is difficult to calculate directly using equation. In this case, the interpolation method is used to calculate the density range.

The parameter $a$ is proportional to the density range, but inversely proportional to the result accuracy of the FEA calculation. Taking a classic three-dimensional MBB structure as an example, the macrostructure size is $150 \times 30 \times 20 \text{mm}$, and the smallest manufacturable unit size is set to $0.5 \text{mm}$. Relationship of the deformation of macrostructure and unit family density range with unit scale are shown in figure 12.

![Figure 12](image)

Figure 12. Relationship of deformation of macrostructure and material density with unit scale.

5.2. Structure reconstruction
Taking the microstructures shown in figure 11 as an example, using the limit thought we can know that $\rho_{\text{min}}$ gradually approaches 0 with the increase of the scale of unit. The voxel with relative density between interval 0 and $X_{\text{min}}$ has no corresponding microscopic structural unit. If the voxels in this region are mapped to empty material unit, a large number of support structures generate in macrostructure. This kind of cellular structure is not manufactured, so the strategy to solve above
problem is to map voxels with relative density less than \( X_{\text{min}} \) to minimum density unit. Although this strategy leads to the material volume fraction of the overall structure larger than the optimized target volume, it can avoid generating support structures.

Given each voxel in the density map, we can get the density, the parameter of the corresponding unit can be calculated from equation (4). Then we can select the unit from the unit database. The replacement process is applied to each voxel of the density map. After these operations, a cellular structure model is constructed. Finally, the cellular model is converted into a mesh model by marching cube algorithm.

6. Experiment and discussion
The experiment is divided into two parts. Firstly, the SIMP algorithm obtains the three-dimensional density map. Then the unit family and the scale of the unit are selected and determined. The cellular structure with optimal material density distribution is constructed by the process mentioned above.

In the experiment, we use a classic MBB beam model with \((150 \times 30 \times 20 mm)\). The load and boundary conditions are shown in figure 13 (a). In order to compare the stiffness of our method and SIMP, different penalty values are used. The target volume fraction is 20%.

![Figure 13. (a) MBB beam structure diagram; (b) Density map with \( P = 1 \); (c) Density map with \( P = 3 \).](image)

We use the three-dimensional unit structures shown in figure 11 that can satisfy the requirements of manufacturability. The appropriate unit scale of such unit families has been solved according to figure 12, and it is 4.7mm.

![Figure 14. (a) Designed by SIMP; (b) Designed by our method; (c) Designed by our method; (d) Designed by our method; (e) Designed by our method; (f) MBB structure under three-point bending test.](image)

Figure 14 gives the fabricated results. Figure 14(a) is designed by SIMP. Its support structure is generated by Cura 3.6.0. Figure 14 (b) is designed by our method. The penalty value used in SIMP algorithm is 1. In the assembling process, the voxel density that is less than \( X_{\text{min}} \) are mapped to empty material unit. The final model is processed by Cura 3.6.0. Support structures are generated because the cavities cannot be fabricated directly. Figure 14(c) also demonstrates a cellular structure designed by our method. In this model the voxel density that is less than \( X_{\text{min}} \) are mapped to the unit with the minimum density.

Figure 14(d) shows a fabricated mode with a shell structure. In this model the cellular structure we designed can be seen as a three-dimensional infill pattern with optimized material distribution. And figure 14(e) shows a shell structure that using a grid support structure generated in Cura 3.6.0. The infill material density is equal to the model shown figure 14(d). Finally, figure 14(f) shows the MBB structure under three-point bending test.

The three-point bending test results are shown in figure 15 and figure 16, respectively. The slopes of the different line graphs show the stiffness of different structures. The middle cliffs in the line graph indicate local failures in these structures. And the end cliffs indicate that the overall structures have failed.

From figure 15, the beam1 has the best overall stiffness. This structure can withstand a maximum load of 757 N and a corresponding maximum displacement of 2.4 mm. The stiffness of the beam3
obtained by our design method has a similar stiffness, which can withstand the maximum load of 733N. The corresponding maximum displacement is 2.87mm. The stiffness of beam2 is the worst among the three structures.

Figure 16 shows the stiffness comparison of beam4 with the structures designed as internal supports and beam5 with grid structures generated by Cura as internal supports. The maximum load that the beam4 can withstand is 196 N, and the maximum displacement is 3.5mm. Beam5 are 435N and 1.97mm. In fact, beam4 is not as good as beam5. The low-density unit structures designed in this paper is not as strong as a continuous grid structures. Because the volume fraction is too small that causes a lot isolated rod structures inside the generated model.

For models generated by SIMP, we regulated with Cura to guarantee that all fabricated models have the same volume fraction. In summary, the stiffness of the structure designed by our method has the similar stiffness with the original SIMP optimized structure, but no additional support structures are needed.
7. Conclusions
The cellular structure design method proposed in this paper can generate global optimized structures with specified microstructures. The model designed is easier to manufacture. No additional supports are needed. The designed cellular structure can also be seen as a new fill pattern of the 3D printing structure to make the internal structure have the optimal material distribution. The next goals are to design more usable units and extend the internal support filling method to the traditional slicing software.

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