A Nonsingular Kerr-Sen Black Hole

Byon N. Jayawiguna

Departemen Fisika, FMIPA, Universitas Indonesia, Depok, 16424, Indonesia.

Abstract

We present a novel solution describing four dimensional rotating regular charged black hole in the low energy heterotic string theory. This present solution is characterized by its mass, charged, as well as angular momentum. Some aspect including event horizon, ergosphere, and the angular velocity are discussed. It is shown that later, our expression for event horizon and ergosphere cannot be obtained analytically. On the other hand, to ensure whether our solution describe a nonsingular rotating charged black hole, we use two type of scalar invariant; contracted Ricci tensor and Kretschmann scalar. From our result, the inclusion of charge does not alter the regularity of a black hole. We also proposed a natural prescription to have rotating charged black hole solution with a minimal violation of weak energy conditions.

*Electronic address: byon.nugrah@ui.ac.id
I. INTRODUCTION

String theory, until now, is one of the best candidate to explain quantum gravity and their related phenomenology in extra dimensions. Fortunately, it is always possible to study the nature of string theory from our dimensions point of view. If we have large mass parameter compared to its Planck mass and curvature below the Planck scale, then static charged black hole in low energy string theory exist [1]. Rotating version in the low energy obtained by Sen [2] in 1992 with an extra term that differ from GHS one. He obtained an exact solution solution of rotating charged black hole in four dimensional spacetime by using generating technique [2, 3] and employ Kerr [4] as a vacuum solution. Hence the solution in Einstein frame characterized by its mass, spin, and electric charge. Switched off the charge parameter makes the solution goes back to the well known Kerr, whereas for vanishing spin combined with some rescale, the solution reduces to the Einstein-Maxwell-dilaton (GHS) black hole. We can infer that the vacuum solution from string theory is the same as ordinary Einstein gravity. In charged static black hole, coupling between the nontrivial dilaton and Maxwell tensor makes the solution differ from Einstein gravity. The only difference is that the appearance of the surface singularity. For rotating case, although we have the same family on charged black hole, Kerr-Sen and Kerr-Newman solution appeared to be different since it has, again, a nontrivial dilaton. There are several feature that distinct between the two solution. One of the example is that the hidden conformal properties that has been studied in [5] do not have Q-picture hidden conformal symmetry compared to the rotating Einstein-Maxwell [6]. Most of them can be done from the black hole (with ring singularity) under the main assumption from validity of the energy condition. Since collapsing matter forms a spacetime singularity at the origin [7].

Inspired by Bardeen [8], regular (nonsingular) black hole have been considered that has a spherically symmetric, static, asymptotically flat spacetime, and regular at the center. While there has been no comprehensive solid theory on quantum gravity, several toy models like black hole without singularity has been reported. The physical source of a regular black hole with magnetic monopole and nonlinear electrodynamics has been obtained by Ayon-Beato and Garcia [10–12]. These spacetime has an event horizon and no pathological singularity, and the magnetic monopole in reference [12] has been verified with the conserved charge evaluated in the boundary. Of course this present spacetime is not a vacuum solution to
GR but they can be introduced as exotic field or modified gravity. They could spared the singularity because they meet the weak energy condition, but not the strong one [14]. From this extension literature, one can find a rotating version corresponds to our previous reference on static spacetime via Newman-janis algorithm [13]. A rotating regular black hole was obtained by Bambi-Modesto [14]. They use two type of static regular black hole; Hayward [15] and Ayon-Beato magnetic monopole [12]. Another rotating regular black hole with N-J algorithm has been reported in [16]. In the presence of cosmological constant, Neves-Sa [17] obtained the solution with syng e g-method. From the rotating regular literature (both in vacuum and charged case), they all showed that the rotation makes the weak energy condition to be violated, but the violation can be very small depends on how we treat the parameter. The shared feature also applied to another vacuum nonsingular rotating black hole [18].

While there has been extensive literature on vacuum rotating regular black hole, there are no comprehensive study on regular black hole in the low energy string theory. Hence, in this paper, we examine charged rotating regular black hole in the low energy heterotic string theory. We will not follow the Newman-Janis algorithm and syng e g-method in this context. Instead, we use generating technique as the reference [2], and using Ghosh regular black hole as a seed solution [18] since it obeys the vacuum solution in large region. The output of this solution is in string frame and we will use the conformal transformation to obtain the metric in Einstein frame. We utilize the conserved quantity in order to obey the three physical parameters ($M, Q, J$).

This paper is structured as follows. Section II, we review charged rotating black hole in the low energy heterotic string theory both in string and Einstein frame combined with the several aspect such as angular velocity and the Hawking temperature. Section III contains of our novel solution by adding the regular factor in the Kerr metric and using generating technique into regular (nonsingular) Kerr-Sen black hole. The vacuum regular solution are already obtained by [18]. After obtaining the solution in the Einstein frame, we analyze several aspect including horizon, ergosphere, as well as angular velocity. Moreover, to test the existence of regular charged black hole, we use two type of scalar invariant that is contracted Ricci tensor as well as the Kretschmann scalar. Section IV, we investigate the weak energy condition and compare it to the other reference. Section V, we conclude of our statement.
II. KERR-SEN BLACK HOLE

Sen [2] in 1992 obtained the exact solution of rotating charged black hole in low energy heterotic string theory. Since he used Kerr black hole as a seed, the solution later commonly referred to as a Kerr-Sen. The corresponding effective action in string frame for this context reads

$$S = \int d^4x \sqrt{-\tilde{g}} e^{-\Phi} \left[ \mathcal{R} + (\nabla \Phi)^2 - \frac{1}{8} F^2 - \frac{1}{12} H^2 \right],$$  \hspace{1cm} (1)

where $\tilde{g}$ is a determinant of metric tensor $g_{\mu\nu}$, $\mathcal{R}$ is a Ricci scalar, $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ is a Maxwell field, $\Phi$ as a scalar dilaton, and $H_{\kappa\mu\nu} = \partial_\kappa B_{\mu\nu} + \partial_{\mu} B_{\kappa\nu} + \partial_{\nu} B_{\kappa\mu} - \frac{1}{4} (A_\kappa F_{\mu\nu} + A_\nu F_{\kappa\mu} + A_\mu F_{\nu\kappa}).$ \hspace{1cm} (2)

is a 3-form tensor field which contain the antisymmetric 2-form tensor field and Chern-Simons term. By using the conformal transformation

$$ds^2_E = e^{-\Phi} ds^2,$$  \hspace{1cm} (3)

we can get the action in Einstein frame

$$S = \int d^4x \sqrt{-g} \left[ R(g) - \frac{1}{2} (\nabla \Phi)^2 - \frac{e^{-\Phi}}{8} F^2 - \frac{e^{-2\Phi}}{12} H^2 \right].$$  \hspace{1cm} (4)

When all the non gravitational field is zero, the action reduces to the well known Einstein-Hilbert action. Let us discuss the metric solution in Einstein frame. In Boyer-Lindquist coordinate, Kerr-Sen solution can be written as

$$ds^2_E = - \left( 1 - \frac{2Mr}{\rho^2_{KS}} \right) dt^2 - \frac{4aMr \sin^2 \theta}{\rho^2_{KS}} dt d\phi + \rho^2_{KS} \left( \frac{dr^2}{\Delta_{KS}} + d\theta^2 \right)$$

$$+ \left[ r^2 + a^2 + 2br \frac{2Mr a^2 \sin^2 \theta}{\rho^2_{KS}} \right] \sin^2 \theta \ d\phi^2.$$  \hspace{1cm} (5)

The complete solution [5] characterized by mass $M$, angular momentum $J$, and charge $Q$. Rotational parameter $a$ is defined as a ratio between angular momentum and its mass. Their related non gravitational fields are

$$A_\mu dx^\mu = \frac{Qr}{\rho^2_{KS}} (dt^2 - a \sin^2 \theta d\phi^2),$$  \hspace{1cm} (6)

$$B_{\nu\phi} = \frac{2bra \sin^2 \theta}{\rho^2_{KS}},$$  \hspace{1cm} (7)
and
\[ e^{-2\phi} = \frac{\rho_{KS}^2}{r^2 + a^2 \cos^2 \theta}, \]  
(8)

Where \( \rho_{KS}^2 = r^2 + 2br + a^2 \cos^2 \theta \), \( \Delta_{KS} = r^2 + 2br - 2Mr + a^2 \), and \( b = Q^2/2M \). To get the solution, one can obtained Hassan-Sen transformation [3] resulting a string frame metric solution. Using conformal transformation combined with the conserved mass, angular momentum, and charge, we obtained the solution in [5]. We will discuss the conserved quantity in the next section. However, the black hole’s event horizon is obtained when \( \Delta(r_\pm) = 0 \) satisfied. From this condition, we conclude that the Kerr-Sen black hole have at least two radius of horizon \( r_\pm = M - b \pm \sqrt{(M - b)^2 + a^2} \), where the \( r_\pm \) is the outer and inner horizon respectively. Moreover, we can obtained the angular velocity of a Kerr-Sen black hole by setting the Killing vector \( \partial_t + \Omega \partial_\phi \), null at the horizon. Thus, we get
\[ \Omega_H = \frac{J/M}{2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2}}. \]  
(9)

The Hawking temperature at the horizon also reads
\[ T_H = \frac{\sqrt{(2M^2 - Q^2)^2 - 4J^2}}{4\pi M \left(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2}\right)}. \]  
(10)

When \( b \) parameter is vanish, the Einstein frame metric reduces to Kerr black hole and simultaneously we have vanishing non gravitational field. Furthermore, vanishing rotational parameter combined with rescaled \( r \rightarrow r - 2b \), gives us the GMGHS black hole describing charged black hole in string theory [1].

III. A NONSINGULAR KERR-SEN BLACK HOLE

In the previous section, we already discuss Kerr-Sen black hole. The solution in Einstein frame is obtained from Hassan-Sen transformation resulting the string frame and employed conformal technique to transform into Einstein frame metric solution. In this section, we present a novel solution describing regular rotating charged black hole in the low energy string theory. We employ H-S transformation for getting the result. Since the transformation requires us to have stationary and axialsymmetry solution as a seed, we use vacuum rotating regular black hole solution from [18]. To get into it, first we discuss the H-S transformation. Hassan and Sen show that the set field \( \{g_{\mu
u}, \phi, A_\mu, B_{\mu
u}\} \) will satisfies the equation of
motion if the relationship of a new and old fields are

\[ \mathcal{M}' = \mathcal{M} \mathcal{Y}^T, \quad \Phi' = \Phi + \ln \left| \sqrt{g'/g} \right|, \quad (11) \]

where the accent sign (') is stands for the new field. The definition for matrix \( \mathcal{M} \) and \( \Omega \) in the (11) is as follows

\[ \mathcal{M} = \begin{pmatrix}
(K^T - \eta)g^{-1}(K - \eta) & (K^T - \eta)g^{-1}(K + \eta) & -(K^T - \eta)g^{-1}A \\
(K^T + \eta)g^{-1}(K - \eta) & (K^T + \eta)g^{-1}(K + \eta) & -(K^T + \eta)g^{-1}A \\
-A^T g^{-1}(K - \eta) & -A^T g^{-1}(K + \eta) & A^T g^{-1}A
\end{pmatrix}, \quad (12) \]

and

\[ \mathcal{Y} = \begin{pmatrix}
I_{I \times 7} & \ldots & \ldots \\
\ldots & \cosh \alpha & \sinh \alpha \\
\ldots & \sinh \alpha & \cosh \alpha
\end{pmatrix}, \quad (13) \]

where the \( K, g^{-1}, A, \) and \( \eta \) is the matrix of \( K_{\mu
u}, g_{\mu
u}, A_\mu, \) and \( \eta_{\mu\nu}. \) The dot sign in the matrix \( \mathcal{Y} \) indicates the zero matrix whereas \( I_{I \times 7} \) is an identity matrix 7x7. Important to note that the matrix \( K_{\mu\nu} \) is given by \( K_{\mu\nu} = -B_{\mu\nu} - g_{\mu\nu} - \frac{1}{4}A_\mu A_\nu, \) whereas the flat metric convention is \( \eta_{\mu\nu} = \text{diag}(1, 1, 1, -1). \) From the definition, we can conclude that the matrix \( \mathcal{M} \) contains all of the set field. The idea of this step is that the action (11) will be reduced to the Einstein-Hilbert action when all fields \( (\Phi, A_\mu, B_{\mu\nu}) \) vanish (vacuum case as a seed solution). So by using Hassan-Sen transformation (11), we will obtain a new solution with non vanishing fields \( \{ B'_{\mu\nu}, A'_\mu, \phi' \}. \) The metric tensor expression that obey these conditions is specifically written as

\[ ds^2 = - \left( 1 - \frac{2mre^{-k/r}}{\rho^2} \right) dt^2 - \frac{4amre^{-k/r} \sin^2 \theta}{\rho^2} dtd\phi + \rho^2 \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \left( r^2 + a^2 + \frac{2a^2 mre^{-k/r} \sin^2 \theta}{\rho^2} \right) \sin^2 \theta \ d\phi^2, \quad (14) \]

where \( \rho^2 = r^2 + a^2 \cos^2 \theta, \Delta = r^2 + a^2 - 2Mr e^{-k/r}, \) and \( k \) is a free parameter that measure the deviation from the classical rotating Kerr metric and assume to be positive [18]. When \( k = 0, \) the solution should reduces to the Kerr black hole solution. However, after finding
the non zero matrix $\mathcal{M}$ and $\mathcal{M}'$, we obtained the solution in string frame

$$
\tilde{g}_{\mu} = -\frac{(r^2 + a^2 \cos^2 \theta - 2mre^{-k/r})(r^2 + a^2 \cos^2 \theta)}{[r^2 + a^2 \cos^2 \theta + 2mre^{-k/r} \sin^2(\alpha/2)]^2}, \quad \tilde{g}_{rr} = \frac{(r^2 + a^2 \cos^2 \theta)}{r^2 + a^2 \cos^2 \theta - 2mre^{-k/r}}
$$

$$
\tilde{g}_{\theta \theta} = r^2 + a^2 \cos^2 \theta, \quad \tilde{g}_{\phi \phi} = -\frac{4me^{-k/r}r a \cosh^2(\alpha/2)(r^2 + a^2 \cos^2 \theta) \sin^2 \theta}{[r^2 + a^2 \cos^2 \theta + 2mre^{-k/r} \sin^2(\alpha/2)]^2}
$$

$$
+ 2mra^2 e^{-k/r} \sin^2 \theta + 4mre^{-k/r}(r^2 + a^2) \sin^2(\alpha/2) + 4m^2r^2 e^{-2k/r} \sin^4(\alpha/2)
\right).
$$

The corresponding non gravitational field reads

$$
A = A_\mu dx^\mu = \frac{2me^{-k/r}r \sinh(\alpha)}{r^2 + a^2 \cos^2 \theta + 2mre^{-k/r} \sin^2(\alpha/2)}(dt^2 - a \sin^2 \theta d\phi^2),
$$

$$
B_{t\phi} = \frac{2mre^{-k/r} \sin^2(\alpha/2) \sin^2 \theta}{r^2 + a^2 \cos^2 \theta + 2mre^{-k/r} \sin^2(\alpha/2)},
$$

and

$$
\tilde{\Phi} = -\ln \left| \frac{r^2 + a^2 \cos^2 \theta + 2mre^{-k/r} \sin^2(\alpha/2)}{r^2 + a^2 \cos^2 \theta} \right|.
$$

These metric and their related matter fields constitute a complete solution for non singular black hole in string frame. By using the conformal transformation (3), we obtain the metric solution in Einstein frame

$$
d_{E}^2 = -\frac{\rho^2}{\tilde{\rho}^2} \left( 1 - \frac{2me^{-k/r}}{\rho^2} \right) dt^2 - \frac{4amre^{-k/r} \cosh^2\alpha/2 \sin^2 \theta}{\tilde{\rho}^2} dtd\phi + \tilde{\rho}^2 \left( \frac{dr^2}{r^2 + a^2 - 2mre^{-k/r}} + d\theta^2 \right) + \frac{\sin^2 \theta}{\tilde{\rho}^2} \left[ (r^2 + a^2) \rho^2 + 2mre^{-k/r}ra^2 \sin^2 \theta 
\right.

\left. + 4mre^{-k/r}(r^2 + a^2) \sin^2(\alpha/2) + 4m^2r^2 e^{-2k/r} \sin^4(\alpha/2) \right) d\phi^2,
$$

where $\tilde{\rho}^2 = \rho^2 + 2mre^{-k/r} \sin^2(\alpha/2)$. Although the metric solution in Einstein frame has been successfully obtained, the black hole must also admit the no-hair theorem. The theorem is stated that the black hole solution will be always externally observable by three classical parameters: mass $M$, charge $Q$, and angular momentum $J$. In fact, obtaining those three parameter is not as simple as its stated. One can verify that the three parameters $(M, Q, J)$ living in the asymptotically flat black hole by evaluating Komar integral or
ADM formulation. In this paper, we verify the no hair theorem variable with the ADM one by using the gravitational Hamiltonian [19]. With appropriate choices of lapse and shift function, we obtained the mass and angular momentum

\[ M = -\frac{1}{8\pi} \lim_{S_t \to \infty} \oint_{S_t} (k - k_0) \sqrt{\sigma} d^2 \theta, \]  

(20)

\[ J = -\frac{1}{8\pi} \lim_{S_t \to \infty} \oint_{S_t} (K_{ab} - K h_{ab}) \varphi^a r^b \sqrt{\sigma} d^2 \theta, \]  

(21)

where \( S_t \) two surface at constant \( t \), \( k = \sigma^{AB} k_{AB} \) is the extrinsic curvature of \( S_t \) embedded in hypersurface \( \Sigma_t \), \( k_0 \) is the extrinsic curvature embedded in flat space, \( K_{ab} \) is the extrinsic curvature in a boundary. The minus sign in angular momentum was added to recover the right hand rule. However, we can obtained the explicit form for mass and angular momentum by setting the Eintein frame solution (19) in the asymptotic region (\( r \gg m \)). Straightforward calculation yields

\[ M = m \cosh^2(\alpha/2), \quad \text{and} \quad J = a m \cosh^2(\alpha/2). \]  

(22)

it is shown that the corresponding ADM mass and angular momentum is the same as Kerr-Sen black hole [2]. For total charge \( Q \), one can obtained by using Maxwell equation and employ Gauss theorem to make the total charge evaluated in a boundary. The formalism yields

\[ Q = \oint F^{\mu\nu} dS_{\mu\nu} = \frac{m}{\sqrt{2}} \sinh(\alpha), \]  

(23)

where \( dS_{\mu\nu} = \frac{1}{4} \varepsilon_{\mu\nu\alpha\beta} dx^\alpha \wedge dx^\beta \), which gives the same electric charge to that of Kerr-Sen solution.

After obtaining all of these conserved quantity, it will be more convenient to express \( m \), and \( \alpha \) in terms of physical parameters \( M \), \( J \), and \( Q \) obtained from equation (22) and (23). After some algebra, we get

\[ m = M - b, \quad \text{and} \quad \sinh(\alpha) = \frac{Q \sqrt{2}}{M - b}, \]  

(24)

where \( b = Q^2/2M \). The rotational parameter \( a \) have the same result with KS black hole; ration between angular momentum and its mass. By inserting relation (24) into equation (19) combined with the other hyperbolic identities, we find a new class of solution describing
non singular rotating charged black hole in the low energy heterotic string theory:

\[
ds_E^2 = - \left(1 - \frac{2Mr e^{-k/r}}{\varphi^2}\right) dt^2 - \frac{4aMr e^{-k/r} \sin^2 \theta}{\varphi^2} dt d\phi + \varphi^2 \left[\frac{dr^2}{\Xi} + d\theta^2\right] + \left[\varphi^2 + \frac{2Mr^2 e^{-k/r} \sin^2 \theta}{\varphi^2}\right] \sin^2 \theta d\phi^2,
\]

and the matter fields using (24)

\[
A = Qre^{-k/r}, \quad B = 2bra e^{-k/r} \sin^2 \theta, \quad \text{and} \quad e^{-2\Phi} = \frac{\varphi^2}{\rho^2},
\]

where \( \varphi^2 = r(r + 2be^{-k/r}) + a^2 \cos^2 \theta \), and \( \Xi = r(r + 2be^{-k/r}) + a^2 - 2Mr e^{-k/r} \). The field both metric and matter solution we obtained constitute a complete solution of the non singular Kerr-Sen black hole system in string and Einstein frame. Thus, the ordinary Kerr-Sen black hole can be obtained by switched off the free parameter \( k \), where \( k > 0 \). When we switched off the \( b \) parameter, the solution should reduce to vacuum case [18]. It is also shown that from the metric part, the black hole is asymptotically-flat. From the matter part, the field are vanish in large \( r \). In this paper, we only analyze the aspect only in the Einstein frame. Now, let us calculate the event horizon of the black hole. The radius can be obtained by the condition \( \Xi(r_{\pm}) = 0 \). The equation is given by

\[
r_{\pm}^2 + 2br_{\pm} e^{-k/r_{\pm}} + a^2 - 2Mr_{\pm} e^{-k/r_{\pm}} = 0.
\]

As we can see that the equation above contains transcendent functions. This expression, unfortunately, cannot solve \( r \) analytically with general parameter \( (b, M, k, a) \). Thus, numerical methods is the best way we can achieve by plotting the equation [27] with some fix parameter. First we try to varying the free parameter \( k \) and charge parameter \( b \). According to Fig. 1, the numerical of the transcendental functions reveals that it is possible to construct a regular black hole with non vanishing \( a, b, k \) parameters, and it admits two positive roots \( r_{\pm} \). In Fig. 1 we shows a typical plot depicting an event horizon of black hole with varying free parameter (in the left panel), and charge (in the right panel). It is shown that the black hole can have two horizon(s) and shrinked as the parameter \( (k \) and \( b \) gets larger. When the parameter reach the critical value \( k_c = 0.385 \) and \( b_c = 0.4 \), the two horizon merges (extreme conditions) resulting degenerate horizon \( (r_{+} = r_{-}) \). Thus when the parameter \( k < k_c \) and \( b < b_c \), we have two positive root \( r_{\pm} \) and has no horizon for \( k > k_c \) and \( b > b_c \).

However, in general rotating object, there is a radius that distinguish with the static one; static limit surface. For the case of nonsingular Kerr-Sen spacetime, there also exist the

\[
\frac{\partial}{\partial r}\left[\frac{\varphi^2}{\rho^2}ight] = 0,
\]

where \( r = \sqrt{r^2 + a^2 \cos^2 \theta} \). The equation is given by

\[
r_{\pm}^2 + 2br_{\pm} e^{-k/r_{\pm}} + a^2 - 2Mr_{\pm} e^{-k/r_{\pm}} = 0.
\]
FIG. 1: A typical plot that represents the event horizon of a black hole with varying the crucial parameters. (Left) the free parameter $k$, and (Right) is for charge. At a first sight, we can see that the black hole can have two horizon(s). In this plot, we have set $M = 1$.

The static limit surface which $g_{tt}(r_{sl}) = 0$

$$r_{sl}^2 + 2br_{sl}e^{-k/r_{sl}} + a^2 \cos^2 \theta - 2Mr_{sl}e^{-k/r_{sl}} = 0,$$  

(28)  

where the issue is the same as equation (27) so we employ the same method by investigating the natural characteristic of $g_{tt}$ metric with plot. Einstein frame metric $g_{tt}$ versus radius is shown in Fig. 2 with $M = 1$ in both panel. The radius that satisfied the equation (28) is called $r_{sl}$, and becomes smaller as the parameter gets larger. The extreme condition also occur if $k_c = 0.55$ and $b_c = 0.64$. From this conditions, we can infer that the existence of black hole with two horizon occurs when the free parameter and charge has smaller value than its critical. Moreover, this metric reduces into Minkowski spacetime in large $r$ and everywhere regular when the radius shift into origin.

The region between the $r_{sl}$ and the horizon $r_+$ is called ergosphere. Penrose [20] stated that energy can be extracted from black hole’s ergoregion. The shape of this region is strongly depends on free parameter, charge, and rotational parameter. The exact horizon and static limit radius are shown in Table. I. The first table is for ergosphere with fix rotational and charge parameter, and the second table represent the ergosphere with rotational and free parameter as a fix value. The shared feature with ref. [18] is that the ergosphere region grows as free parameter $k$ gets larger. By adding some charge into equation (14) (which is our solution in equation (25)), we still have a same properties (see Table. I). To analyze
FIG. 2: Graph for $-g_{tt}$ metric versus radius $r$ with $M = 1$. This metric represents the static limit radius. Both graph shows that the radius is shrinked as the parameter gets larger.

TABLE I: (Left) Radius of event horizon, static limit and ergosphere ($\delta = r^e_{sl} - r^e_{eh}$) with $M = 1$, $a = 0.4$, and $b = 0.2$. (Right) Radius of event horizon, static limit and ergosphere ($\delta = r^e_{sl} - r^e_{eh}$) with $M = 1$, $a = 0.4$, and $k = 0.2$.

further, consider a nonsingular Kerr-Sen spacetime with constant space coordinate $(r, \theta, \phi)$

$$ds^2 = - \left( 1 - \frac{2Mr\epsilon^{-k/r}}{\varphi^2} \right) dt^2, \quad (29)$$

and the ergosphere is located at $r < r^e_{sl}$. It is shown that the metric become positive ($ds^2 > 0$) when $r^2 + 2br\epsilon^{-k/r} + a^2 \cos \theta - 2Mr\epsilon^{-k/r} < 0$. Hence, the observer follow an orbit of asymptotic time translation Killing field $\xi^a = \left( \frac{\partial}{\partial t} \right)^a$ resulting the angular velocity that
can be written as
\[ \Omega(r, \theta) = \frac{2M a e^{-k/r}}{(\Xi + 2M a e^{-k/r})^2 - \Xi^2 a^2 \sin^2 \theta}, \]
where the angular velocity of a black hole is \( \Omega_H(r_+) \) when \( \Xi(r_+) = 0 \). As an observer shift further \((r \to \infty)\), the angular velocity goes to zero, which means an observer does not feel the frame dragging effect from the vicinity of a black hole. When \( k \) vanish, the expression (30) reduces to the well known Kerr-Sen black hole’s angular velocity [2].

FIG. 3: The curvature invariant versus radius. (Top) \( \mathcal{R}^E = R^{\mu\nu} R_{\mu\nu} \), and (Bottom) Kretschmann scalar \( \mathcal{K} = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \) with different value of free parameter \( k \) and charge \( b \). It is shown that the spacetime is regular everywhere.

On the other hand, what we mention earlier is that the space time is regular everywhere when the radius shift into the origin of a black hole in Boyer-Lindquist coordinate. In order
to ensure whether the metric [25] has a regularity or not, we approach this problem by looking at the behavior of the scalar invariant; \( R^2 = R^{\mu\nu}R_{\mu\nu} \) as well as the Kretschmann scalar \( \mathcal{K} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \). This present scalar invariant can be obtained with a non-vanishing Ricci and Riemann tensor. Unfortunately, their both expression is quite complicated and hence we would not show it here. Both scalar quantity \( R^2 \) and \( \mathcal{K} \) are shown in Fig. 3. We can infer that the addition of charge does not alter the regularity of a black hole. Our scalar invariant plot for vanishing charge is the same as [18].

**IV. WEAK ENERGY CONDITION**

In this subsection, we study weak energy condition for nonsingular Kerr-Sen spacetime. In order to investigate the matter associated with (25), we employ an orthonormal tetrads in which the energy momentum tensor is diagonal [14, 16–18, 21]

\[
e^{(a)}_{\mu} = \begin{pmatrix} g_{tt} - \Omega g_{t\phi} & 0 & 0 & 0 \\ 0 & \sqrt{\pm g_{rr}} & 0 & 0 \\ 0 & 0 & \sqrt{g_{xx}} & 0 \\ g_{t\phi}/\sqrt{g_{\phi\phi}} & 0 & 0 & \sqrt{g_{\phi\phi}} \end{pmatrix},
\]

(31)

where this present tetrads formalism corresponds to the locally non-rotating frame

\[
e^{(0)} = \sqrt{\pm (g_{tt} - \Omega g_{t\phi})} \, dt, \quad e^{(1)} = \sqrt{\pm g_{rr}} \, dr,
\]

\[
e^{(2)} = \sqrt{g_{xx}} \frac{dx}{1 - x^2}, \quad e^{(3)} = \frac{g_{t\phi}}{\sqrt{g_{\phi\phi}}} \, dt + \sqrt{g_{\phi\phi}} \, d\phi
\]

(32)

According to the matrix, we must assure that the nonzero components of our tetrads formalism has a real root. Notice that \( g_{xx} > 0 \) and

\[
g_{\phi\phi} = \frac{(r^2 + a^2 + 2b r e^{k/r})(r^2 + 2b r e^{-k/r} + a^2 x^2) + 2M r a^2 e^{-k/r} (1 - x^2)}{r^2 + a^2 x^2 + 2b r e^{-k/r}},
\]

(33)

also ensures a regular and positive metric. However, from

\[
g_{tt} - \Omega g_{t\phi} = -\frac{(r^2 + 2b r e^{-k/r} + a^2 x^2)(r^2 + a^2 + 2b r e^{-k/r} - 2M r e^{-k/r})}{(r^2 + a^2 + 2b r e^{-k/r})(r^2 + 2b r e^{-k/r} + a^2 x^2) + 2M r a^2 e^{-k/r} (1 - x^2)},
\]

(34)

it is shown that the equation above is negative. Thus, the sign from matrix (31) must be selected depending on the considered region.
FIG. 4: Plots showing the behavior of weak energy condition. (Top Left) for different free parameter (from 0.3 to 0.36) with $b = 0.3$, and $x = 0.5$. (Top Right) is for different value of $x$ (run from -1 to 1) with $k = 0.4$ and $b = 0.3$. (Bottom Left) $\rho + P_2$ with $b = 0.5$ and $x = 0.5$. (Bottom Right) $\rho + P_2$ with $b = 0.5$ and $x = 0.5$. In this plot, we have set $x = \cos \theta$.

Components for the energy-momentum tensor in the orthonormal frame can be written as

$$T^{(a)(b)} = e^{(a)}_{\mu} e^{(b)}_{\nu} G^{\mu\nu},$$

where $G^{\mu\nu}$ is the Einstein tensor. As is well known that the weak energy condition requires us to obey $\rho \geq 0$, and $\rho + P_i \geq 0$, where $i = 1, 2, 3$. By considering Einstein frame metric [25], we have found that

$$\rho = \frac{1}{r^2 \left( e^{k/r} \left( a^2 x^2 + r^2 \right) + 2br \right)^3} \left\{ be^{k/r} \left[ a^2 x^2 (b - 2M)(k + r)^2 + br^2 \left( k^2 + 6kr + r^2 \right) 
-2Mr^2(k + r)^2 \right] + 2kr^2(b - M) \left( a^2 x^2 + r^2 \right) e^{2k/r} + 2b^2r(b - M)(k + r)^2 \right\} = -P_1,$$
\[
P_2 = \frac{1}{r^3 (e^{k/r} (a^2 x^2 + r^2) + 2br)^3} \left\{ -br e^{k/r} \left[ a^2 b x^2 (r - k)(3k + r) + br^2 (-3k^2 + 6kr + r^2) 
+ 2M \left( k^2 - 2kr - r^2 \right) (a^2 x^2 + r^2) \right] + k(b - M)e^{\frac{2k}{r}} (a^2 x^2 + r^2) \left[ a^2 k x^2 + r^2 (k - 2r) \right] 
- 2b^2 r^2 (b - M) (-k^2 + 2kr + r^2) r^3 \left[ e^{k/r} (a^2 x^2 + r^2) + 2br \right]^3 \right\},
\]

and
\[
\rho + P_2 = \frac{1}{r^3 (e^{k/r} (a^2 x^2 + r^2) + 2br)^3} \left\{ 2br e^{k/r} \left[ a^2 b x^2 \left( k^2 - 2kr - r^2 \right) + br^2 \left( k^2 - 6kr - r^2 \right) 
+ 2Mr (2k + r) (a^2 x^2 + r^2) \right] + k(b - M)e^{\frac{2k}{r}} \left[ a^2 k x^2 + r^2 (k - 4r) \right] 
- 4b^2 r^3 (b - M)(2k + r) \right\}.
\]

Where we switched off the charge parameter \( b \), we have the same expression as vacuum nonsingular rotating black hole obtained by Ghosh [18]. However, it is not quite simple to investigate whether our solution satisfied the energy condition or not by only considering three equation above. So we have plotted weak energy condition in Fig. 4 with respect to radius and several parameter \((k \text{ and } x = \cos \theta)\). What we can conclude of our result and figure is that the violation of weak energy condition cannot be prevented if we have nonzero rotational and free parameter, since with \( b = 0 \), the \( \rho + P_2 \) appeared to be violated as in ref. [18] (also see [17], for the solution in ref. [14] with a presence of cosmological constant).

The inclusion of charge in the low energy string theory does not makes the energy condition satisfied, but the violation can be very small depending on how we treat the parameter [14, 16–18].

V. CONCLUSION

In this paper, we examine a rotating charged regular black hole in the low energy heterotic string theory in four dimensional spacetime. We using generating technique inspired by [2] and [3] in vacuum nonsingular rotating black hole [18], where \( k \) in the literature referred as free parameter that determine the deviation from Kerr metric. As a result, we obtained a string frame metric tensor as well as nongravitational field solution. Rather than considering in this frame, we use conformal transformation in order to obtain the black hole solution in Einstein frame. Since black hole solution will be characterized by three classical parameter,
we employ the corresponding conserved quantity by solving the gravitational Hamiltonian in a boundary $\partial \Sigma$ with appropriate choices of lapse $N$ and shift $N^a$ function. For charge, we can obtain the total charge in the boundary using Gauss theorem. Surprisingly, we have found that the conserved quantity is the same as Kerr-Sen black hole. Thus we have a complete solution containing Einstein frame metric and nongravitational field (vector potential, 2-form antisymmetric field, and nontrivial dilaton) which characterized by its mass, charge, angular momentum, and free parameter. When $k = 0$ we have Kerr-Sen solution. The solution (25) also reduces to the well known Ghosh rotating black hole [18] when we switched off the charge parameter.

Moreover, our novel solution reduces to Minkowski spacetime in large $r$ and seems regular at the origin in Boyer-Lindquist coordinate. To assure this behavior, we employ two type of scalar invariant; contracted Ricci tensor ($R^2$) and Kretschmann scalar ($K$) that build from contracted Riemann tensor. Since the expression for both scalar invariant are complicated, we instead using numerical plot to study the behavior of invariant quantity. We have plot it with two different parameter ($k$ and $b$). It has been reported that the addition of charge does not alter the regularity of a black hole. Thus we still have a charged regular black hole in the low energy string theory. The event horizon of this object can be obtained by solving $\Xi(r_+) = 0$. Since we have a transcendental function, the radius cannot solved analytically and unfortunately we have the same story for static limit radius. So we shift our way by obtaining the radius from plot. As we can see the horizon shrinks faster than the static limit radius as free and charge parameter gets larger. At the same time, a black hole has larger area for energy extraction in the ergosphere as $k$ and $b$ gets larger. The observer that stand still in the ergosphere becomes spacelike and hence should follow an orbit of asymptotic time translation Killing vector resulting the angular velocity as a function of radius and polar angle. On the other hand, to analyze the matter content of our solution (25), we investigate the weak energy condition. We introduce the usual orthonormal tetrads where corresponds to the locally non-rotating frame inspired by [14, 17, 18, 21]. We already shown that the rotation lead to violation of WEC, as in ref. [14, 17, 18]. Thus we can infer that by adding some charge in Ghosh solution, does not make the WEC satisfied, but the violation can be very small depending on how we set the charge or free parameter.

With our solution, it is tempting to investigate some phenomenological aspect like geodesics, deflection, and thermodynamics quantity. These several points are now under
considerations in the next several manuscript.

Acknowledgments

I thank I. Prasetyo, Haryanto Siahaan, and H. S. Ramadhan for the useful discussions and insight for the weak energy condition. Last but not least, I also thank her for the constant support.

[1] D. Garfinkle, G. T. Horowitz and A. Strominger, “Charged black holes in string theory,” Phys. Rev. D 43 (1991), 3140
[2] A. Sen, “Rotating charged black hole solution in heterotic string theory,” Phys. Rev. Lett. 69 (1992), 1006-1009 [arXiv:hep-th/9204046 [hep-th]].
[3] S. Hassan and A. Sen, “Twisting classical solutions in heterotic string theory,” Nucl. Phys. B 375 (1992), 103-118
[4] R. P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics,” Phys. Rev. Lett. 11 (1963), 237-238
[5] A. M. Ghezelbash and H. M. Siahaan, “Hidden and Generalized Conformal Symmetry of Kerr-Sen Spacetimes,” Class. Quant. Grav. 30 (2013), 135005 [arXiv:1206.0714 [hep-th]].
[6] C. M. Chen, Y. M. Huang, J. R. Sun, M. F. Wu and S. J. Zou, Phys. Rev. D 82 (2010), 066004 doi:10.1103/PhysRevD.82.066004 [arXiv:1006.4097 [hep-th]].
[7] S. W. Hawking and G. F. R. Ellis, “The Large Scale Structure of Space-Time,” doi:10.1017/CBO9780511524646
[8] J.M. Bardeen, in: Conference Proceedings of GR5, Tbilisi, USSR, 1968, p. 174.
[9] I. Dymnikova, “Vacuum nonsingular black hole,” Gen. Rel. Grav. 24, 235-242 (1992)
[10] E. Ayon-Beato and A. Garcia, “Regular black hole in general relativity coupled to nonlinear electrodynamics,” Phys. Rev. Lett. 80, 5056-5059 (1998) [arXiv:gr-qc/9911046 [gr-qc]].
[11] E. Ayon-Beato and A. Garcia, “Nonsingular charged black hole solution for nonlinear source,” Gen. Rel. Grav. 31, 629-633 (1999) [arXiv:gr-qc/9911084 [gr-qc]].
[12] E. Ayon-Beato and A. Garcia, “The Bardeen model as a nonlinear magnetic monopole,” Phys. Lett. B 493, 149-152 (2000) [arXiv:gr-qc/0009077 [gr-qc]].

[13] E. T. Newman and A. I. Janis, J. Math. Phys. 6, 915-917 (1965) doi:10.1063/1.1704350

[14] C. Bambi and L. Modesto, “Rotating regular black holes,” Phys. Lett. B 721, 329-334 (2013) [arXiv:1302.6075 [gr-qc]].

[15] S. A. Hayward, “Formation and evaporation of regular black holes,” Phys. Rev. Lett. 96, 031103 (2006) [arXiv:gr-qc/0506126 [gr-qc]].

[16] B. Toshmatov, B. Ahmedov, A. Abdujabbarov and Z. Stuchlik, “Rotating Regular Black Hole Solution,” Phys. Rev. D 89, no.10, 104017 (2014) [arXiv:1404.6443 [gr-qc]].

[17] J. C. S. Neves and A. Saa, “Regular rotating black holes and the weak energy condition,” Phys. Lett. B 734, 44-48 (2014)

[18] S. G. Ghosh, “A nonsingular rotating black hole,” Eur. Phys. J. C 75 (2015) no.11, 532

[19] E. Poisson, “A Relativist’s Toolkit: The Mathematics of Black-Hole Mechanics,” doi:10.1017/CBO9780511606601

[20] R. Penrose, “Gravitational collapse: The role of general relativity,” Riv. Nuovo Cim. 1, 252-276 (1969)

[21] J. M. Bardeen, W. H. Press and S. A. Teukolsky, “Rotating black holes: Locally nonrotating frames, energy extraction, and scalar synchrotron radiation,” Astrophys. J. 178, 347 (1972)