Supernovae, Hypernovae and Color Superconductivity

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Abstract

We argue that Color Superconductivity (CSC, Cooper pairing in quark matter leading to the breaking of SU(3) color symmetry) may play a role in triggering the explosive endpoint of stellar evolution in massive stars (M > 8M⊙). We show that the binding energy release in the transition of a sub-core region to the CSC phase can be of the same order of magnitude as the gravitational binding energy release from core collapse. The core temperature during collapse is likely below the critical temperature for CSC, and the transition is first order, proceeding on Fermi timescales when the pressure reaches a critical value of several times nuclear density. We also discuss the implications for hypernova events with total ejecta energy of 10-100 times that of type II supernova.

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1 Introduction and Review of CSC

Recent results have demonstrated rigorously that the ground state of quark matter at sufficiently high density exhibits Color Superconductivity (CSC), resulting from the Cooper pairing of quark quasiparticles near the Fermi surface \[1\] – \[15\]. At asymptotic density, the nature of the order parameter, the binding energy density and the critical temperature are all precisely calculable. Less is known about intermediate densities of several times nuclear density, however there are strong indications of a Cooper pairing instability, and estimates of the resulting gap are of order \(\Delta \sim 40\text{-}120 \text{ MeV}\) at a quark chemical potential \(\mu\) of \(\approx 400 \text{ MeV}\). Direct links between CSC and the astrophysics of compact objects have been suggested in \[16\].

In this letter we discuss the possible implications of CSC for the collapse and explosion of massive stars \((M > 8M_\odot)\). We argue that it is quite likely that at the moment of maximum compression of the collapsing Fe core, the densest part of the star crosses the critical density into the phase where CSC is energetically favored. The release of latent energy has the potential to generate an explosive shockwave which powers the resulting supernova (SN). Current simulations of supernovae are generally unable to reproduce the explosive behavior observed in nature: the shockwave generated by the mechanical bounce of the nuclear core stalls before reaching the surface, unless an appeal is made to additional phenomena such as neutrino reheating combined with non-spherical phenomena such as rotation or convection \[17, 18\]. We also note that the energy liberated in a CSC phase transition is potentially sufficient to power hypernovae (HN) \[19\], which have been linked to gamma ray burst events \[20\].

First, let us summarize some results on CSC from the recent literature. Precise results are only valid at asymptotic densities where the effective QCD coupling is small, however they should still be useful guide when dealing with intermediate densities. In any case, our qualitative results will be insensitive to factors of 2 in these formulas:

- Gap size: \(\Delta \sim 40\text{-}120 \text{ MeV}\)
- Critical temperature: \(T_c \simeq .57\Delta\)
- At asymptotic density the binding energy density is \(E_{\text{CSC}} = \frac{58}{3}\Delta^2\mu^2\). Recent studies \[18\] of the interface between the nuclear and CSC phase using specific models seem consistent with this result. Simple dimensional analysis (given the absence of any small parameter) also suggests a value in this range. Note that we are interested here in the latent energy associated with the first order transition to the CSC phase (or any other transition that occurs as the baryon density is increased beyond several times nuclear density). The baryon density...
changes on astrophysical timescales, or very slowly on the timescale of QCD dynamics, and the vacuum state at zero temperature (or at $T \ll T_c$) is found by minimizing the energy in the sector of the Hilbert space with fixed baryon number. Studies such as [15], which involve the free energy $\Omega$ at finite chemical potential (but not fixed density) are appropriate for determining pressure equilibrium between nuclear and CSC matter in circumstances in which baryon number can flow across a boundary (e.g. in a neutron star), but do not address a possible SN transition.

- Phase diagram: the normal nuclear phase is separated from the CSC phase (most likely the 2SC two flavor condensate phase, although possibly the CFL phase [15]) by a first order boundary.

## 2 Astrophysics of Core Collapse

Now let us review the standard scenario of Fe core collapse which is believed to lead to type II supernovae [17]. Nuclear burning during the $10^7$ year lifetime of the star leads to a shell structure, with the inner core eventually consisting of Fe ash. Because iron cannot participate in further exothermic nuclear reactions, there is an eventual cooling and collapse of the Fe core, whose mass is likely to be $(1 - 2) M_\odot$ (or, roughly the Chandresekhar mass). The collapse of this core is only halted by neutron degeneracy, which leads to a stiffening of the equation of state. The resulting bounce produces a shockwave, whose energy of $\sim 10^{51}$ ergs is a small fraction of the total available gravitational binding energy released by the collapse:

$$E_b \sim 3 \cdot 10^{53} \text{ergs} \left( \frac{M_{\text{core}}}{M_\odot} \right)^2 \left( \frac{R}{10\text{km}} \right)^{-1}. \quad (1)$$

Most of this energy escapes in the form of neutrinos during the supernova, as was observed in the case of SN1987a.

The pressure in the collapsed core at the instant of the bounce is most likely greater than the corresponding pressure in any remnant neutron star. In order to cause a bounce, the kinetic energy of the infalling material (which is a sizeable fraction of a solar mass) must be momentarily stored as compressional potential energy in the (sub)nuclear matter. This additional mechanical squeezing at the bounce suggests that if the critical density for CSC is ever reached in a neutron star, it will be reached at this instant.

Simulations of the core bounce result in densities of at least several times nuclear density ($5 - 10 \cdot 10^{14}\text{g/cm}^3$), and temperatures of roughly 10-20 MeV [17]. This temperature is likely
less than $T_c$ for CSC, possibly much less\footnote{It is conceptually easier to think about the case where $T$ is much less than $T_c$, since in this case the Free energy ($F = U - TS$) liberated by the transition is predominantly energy, with only a small component related to entropy. The relevant dynamics is governed by energetics rather than Free energetics.}, and hence the core of the star traverses the phase diagram horizontally in the density-temperature plane, crossing the critical density boundary into the CSC phase. It is important to note that the core region at bounce is probably cooler than post-bounce, since degenerate neutrinos tend to heat the proto-neutron star as they diffuse out \cite{21}. Studies quoting larger SN temperatures such as $T \sim 30$ MeV generally refer to this later stage \cite{22}.

Once the core crosses into the CSC part of the phase diagram, the transition proceeds rapidly, on hadronic timescales. Because the transition is first order, it proceeds by nucleation of bubbles of the CSC phase in the normal nuclear background. The rate of bubble nucleation will be of order $(\text{fm})^4$ (fm = $10^{-13}$ cm), due to strong coupling. (In a system governed by a dimensional scale $\Lambda$, the nucleation rate is given by $\Gamma \sim \Lambda^4 e^{-S}$, where $S$ is the action of the Euclidean bounce solution interpolating between the false (normal nuclear) vacuum and a bubble of critical size. At strong coupling, $S$ is of order one, so there is no large exponential suppression of the nucleation rate. The scale $\Lambda$ is of the order of $\Delta$ or $\mu = 400$ MeV.)

Causality requires that the mechanical bounce of the core happen over timescales larger than the light crossing time of the core, or at least $10^{-4}$ s. Hence, the phase transition occurs instantaneously on astrophysical timescales. A nucleated bubble of CSC phase expands relativistically – liberated latent heat is converted into its kinetic energy – until it collides with other bubbles. Because the system is strongly coupled, these collisions lead to the rapid production of all of the low energy excitations in the CSC phase, including (pseudo)Goldstone bosons and other hadrons. The resulting release of energy resembles an explosion of hadronic matter.

To estimate the total CSC energy released in the bounce, we use the result that the ratio of CSC binding energy density to quark energy density is of order $\left(\frac{\Delta}{\mu}\right)^2$. For $\mu \sim 400$ MeV, and $\Delta \sim 40$-120 MeV, this ratio is between .01 and .08, or probably a few percent.

$$E \sim \left(\frac{\Delta}{\mu}\right)^2 M_{\text{core}}. \quad (2)$$

In other words, the total energy release could be a few percent of a solar mass, or $10^{52}$ ergs! This is significantly larger than the energy usually attributed to the core shockwave, and possibly of the order of the gravitational collapse energy $E_b$. The implications for SN simulations are obviously quite intriguing.

In \cite{23} it was suggested that strange matter formation might overcome the energetic difficulties in producing type-II supernova explosions. While there are strong arguments...
that the CSC transition should be first order, and reasonable order of magnitude estimates of the latent heat [1, 15], it is not clear to us why there would be supercooling in a strange matter transition. The conversion of up quarks to strange quarks must proceed by the weak interactions, but the rate is still much faster than any astrophysical timescale. Thus, the population of strange quarks is likely to track its chemical equilibrium value as the pressure of the core increases. There may be an important effect on the nuclear equation of state from strangeness (e.g. a softening of the pressure-density relationship), but we do not see why there should be explosive behavior.

Our results are also relevant to hypernovae [19], which are observed to have ejecta kinetic energies 10-100 times larger (of order $10^{52-53}$ ergs) than those of ordinary type II supernovae. Accounting for this extra kinetic is extremely challenging in standard scenarios. However, for exceptionally massive stars with $M < 35M_\odot$ (for $M > 35M_\odot$ the hydrogen envelope is lost during H-shell burning, and the core size actually decreases [24]) there is a large core mass which leads to a larger release of CSC binding energy. In fact, the released energy might depend nonlinearly and sensitively on the star’s mass at the upper range. For example, the fraction of $M_{\text{core}}$ which achieves critical density might be a sensitive function of the mass of the star.

Another alternative is that hypernovae are the result of neutron star mergers rather than the explosion of an individual star. This possibility has been examined in relation to hypernovae as the engines of gamma ray bursts (GRBs) [20]. It seems quite plausible that in the merger of two cold neutron stars a significant fraction of the stars’ mass undergoes the CSC transition (i.e. crosses the critical pressure boundary for the first time; in this case the temperature is probably negligible relative to the CSC scale $\Delta$). This provides a substantial new source of energy beyond gravitational binding, and may solve the “energy crisis” problem for this model of GRBs [20].

Finally, we note that the trajectory of the SN core in the temperature-density phase diagram might be rather complicated. The parameters suggest a density transition (at $T < T_c$), but subsequent reheating of the core due to the explosion, or to neutrino diffusion [21] might raise the temperature above $T_c$, and lead to additional transitions across the temperature boundary [16, 22]. When $T \sim T_c$, the Free energies ($F = U - TS$) of the normal and CSC phases are comparable, due to the larger entropy of the normal phase. The evolution of bubbles in this regime is governed by relative Free energies rather than energetics alone, and the transition is presumably less explosive than the pressure transition at $T << T_c$. 
3 Discussion

Our understanding of QCD at high density has evolved dramatically over the past two years, leading to remarkable progress in understanding the QCD phase diagram and the color superconducting state of quark matter. We have argued here that the well-established picture of core collapse in massive stellar evolution suggests quite strongly that CSC may play an important role in type II supernovae, and possibly hypernovae (GRBs).

Our assumptions concerning key parameters are conservative, and taken from distinct (and heretofore independent) regimes of inquiry: stellar astrophysics and dense quark matter. Yet, they point to the interesting possibility that supernova explosions are powered by CSC binding energy. It is well established that the shockwave energy from core collapse is insufficient to produce an explosion, and recent results incorporating Boltzman transport of neutrinos show that neutrino reheating is also insufficient unless non-spherical phenomena such as rotation or convection are taken into account [18]. We are optimistic that future progress in simulations will tell us much about whether and how latent energy from color superconductivity plays a role in stellar explosions.

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