Effect of multiple charge traps on dephasing rates of a Josephson charge qubit system

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We examine the dephasing rate of a Josephson charge qubit system due to background charge fluctuations. We consider single qubit and two-charge traps. The transition probability was controlled to a state where two traps were occupied. The transition probability was affected by the Coulomb blockade effect that occurs between two charge traps. To obtain the dephasing rate, we computed the spectra of random frequency modulation signals. Our results show that the interaction between charge traps suppresses dephasing.

KEYWORDS: Josephson charge qubit, quantum computation, dephasing, background charge fluctuations

Among various proposals, quantum bits (qubits) in solid state materials, such as, superconducting Josephson junctions\(^1\) and quantum dots,\(^2\)\textsuperscript{--}\(^4\) have the advantage of scalability. Proposals to implement a quantum computer using superconducting nanocircuits are proving to be very promising\(^5\),\(^6\) and several experiments have already highlighted the quantum properties of these devices.\(^7\) Such a coherent-two-level system constitutes a qubit and the quantum computation can be carried out as the unitary operation functioning on the multiple qubit system. Essentially, this quantum coherence must be maintained during computation. However, dephasing is hard to avoid due to the system’s interaction with the environment. In terms of a bonding-antibonding bases, the decay of off-diagonal elements of the qubit density matrix signals that dephasing is occurring. This dephasing is characterized by the dephasing time \(T_2\). Various environments can cause dephasing. In superconducting nanocircuits various sources of decoherence are present,\(^5\) such as fluctuations originating from the surrounding circuit, quasiparticle tunneling, background charge fluctuation (BCF), and flux noise. For a charge qubit system, BCF is one of the most critical dephasing channels.\(^8\)\textsuperscript{--}\(^11\)

BCFs have been observed in various kinds of systems.\(^12\)\textsuperscript{--}\(^15\) In nanoscale systems, they are the electrostatic potential fluctuations due to the dynamics of electrons, or holes on a charge trap. In particular, the charge at a charge trap fluctuates with the Lorentzian spectrum form, which is called random telegraph noise in the time domain.\(^14\),\(^16\) The random distribution of
the positions of such dynamical charge traps and their time constants lead to BCFs or 1/f noise.\(^{17}\) In solid-state charge qubits, these BCFs result in a dynamical electrostatic disturbance and hence, dephasing. The theoretical effect of 1/f noise on a charge Josephson qubit has been examined previously.\(^{8–11}\)

We investigated how the electrostatic disturbance coming from two or more dynamical charge traps affects the quantum coherence of a qubit. In past studies, an environment composed of free charge traps had been considered.\(^{8,10}\) When such an environment is interacting with itself, its characteristic nature would be expected to affect the relaxation phenomena. In present study, we especially concentrated on the correlation effect between the charges in the traps. We consider pure dephasing as an event which occurs when the dynamical charge traps induce fluctuation in extra bistable bias. It should be noted that this dephasing process does not mean the qubit is entangled with the environment, but rather, that the stochastical evolution of an external classical field, is suppressing the off-diagonal density matrix elements of the qubit after being averaged out over statistically distributed samples.

The system under consideration is Cooper pair box.\(^{5}\) Under appropriate conditions (charging energy \(E_C\) much larger than the Josephson coupling \(E_J\) and temperatures \(k_B T \ll E_J\)) only two charge states are important, and the Hamiltonian of the qubit \(H_{qb}\) reads

\[
H_{qb} = \frac{\delta E_C}{2} \sigma_z + \frac{E_J}{2} \sigma_x
\]

where the charge bases \(\{|0>, |1>\}\) is expressed using the Pauli matrices, and the bias \(\delta E_C \equiv E_C(1-C_x V_x/e)\) can be turned by varying the applied gate voltage \(V_x\). The environment is a set of BCF electrostatistically coupled to the qubit,\(^{8,10,18,19}\)

\[
H_{qb-imp} = \sum_{i=1}^{N} \frac{\hbar J_{C_i}}{2} \sigma_z (d^\dagger_i d_i - \frac{1}{2})
\]

where \(d^\dagger_i\) and \(d_i\) are the electron creation and annihilation operators of a charge trap, \(i\) is the index of \(N\) charge traps, and the coupling with the qubit is such that each BCF produces a bistable extra bias \(\hbar J_{C_i}\). Because qubit Hamiltonian consists of \(E_J\) and \(\delta E_C\), the dephasing consists of that with dissipation and pure dephasing. In general, the dephasing with dissipation can be neglected as follows. For physical setups, \(\delta E_C \simeq 122\ \mu\text{eV}\), and \(E_J \simeq 34\ \mu\text{eV}\);\(^{9}\) By perturbation method,\(^{5}\) the ratio of the dephasing rate with dissipation to pure dephasing rate is roughly given by \(\frac{E_J^2}{\delta E_C^2 (\delta E_C^2 + E_J^2)/\hbar^2 + \lambda^2}\) in the presence of the bistable extra bias, where \(\lambda\) is the transition rate of the dynamical charge trap. For the above experimental setups with the dynamical charge trap with low frequency, we can neglect the effect of \(E_J\) because \(E_J < \delta E_C\) and \(\frac{\sqrt{\delta E_C^2 + E_J^2}}{\hbar} \gg \lambda\). Then the pure dephasing event is critical. In final results of present study, we discuss about the many charge traps which are interacting with each other. For this case, the dominant process is different, we discuss about this behavior latter. We neglect the back action from the qubit to charge traps, namely, the transition rates of charge traps do not
depend on the qubit state. This assumption is justified because the qubit induces static shift of the chemical potential of charge traps. And the change in the chemical potential of a charge trap does not renormalize the transition rate of itself.

Using the environment variable \( X_i(t) = \langle d_i^\dagger(t)d_i(t) \rangle - 1/2 \), where \( \langle A(t) \rangle \) is the expectation value of the operator \( A(t) \) about the electron reservoir of the charge trap, we rewrite the Hamiltonian in terms of the Pauli matrix as

\[
\mathcal{H} = \frac{\delta E_C}{2} \sigma_z + \sum_i^N \frac{hJ_{Ci}}{2} \sigma_z X_i(t),
\]

we assume that the charge traps are strongly coupled with their charge reservoirs and the time evolution of \( X_i(t) \) is a Poisson process.

Following the time evolution of density matrix of qubit, we obtain the following off-diagonal element \( \rho_{12}(t) = \rho_{12}(t_0) e^{i\delta E_C/\hbar(t-t_0)} e^{i \int_0^t dx(t)} \), where \( t_0 \) is the initial time, and \( x(t) = \sum_i^N J_{Ci} X_i(t) \) takes \( 2^N \) with possible different values of \( a_1, \ldots, a_2^N \). The fluctuation in tunneling coupling constant is pure dephasing and does not accompany relaxation of the population. Therefore, diagonal elements of the qubit density matrix do not change.

In the following, we estimate the ensemble average of off diagonal element \( E[\rho_{12}(t)] = \rho_{12}(t_0) e^{i\delta E_C/\hbar(t-t_0)} e^{i \int_0^t dx(t)} \). For this quantity, we can apply the characteristic functional method, \(^{21,22}\) namely, \( R(t) \equiv \langle e^{-i \int_0^t dx(t)} \rangle = \sum_{i,k=1}^{2N} p_i R_{ik}(t) \), where \( p_i \) is the occupation probability of the state \( i \), which can be determined by the stationary condition, \( 0 = -\mu_i p_i(t) + \sum_{j=1,\neq i}^{2N} \lambda_{ij} p_j(t) \), where \( \lambda_{ij} \) is the transition probability defined for very short time \( \Delta t \) with \( i \neq j \), and \( \mu_i \equiv \sum_{j\neq i} \lambda_{ij} \) is the emission rate during this time. The \( p_i \) has the properties \( \sum_i p_i = 1 \) and \( \mu_i p_i = \sum_{j \neq i} \lambda_{ij} p_j \). The average of \( x, \eta \), is given by \( \sum_i p_i a_i \) and the variance \( \sigma \) is given by \( \sqrt{\sum_i p_i a_i^2 - \eta^2} \). The function \( R_{ik} \) satisfies following the first order differential equation,

\[
\frac{dR_{ik}(t)}{dt} = (ia_k - \mu_k)R_{ik}(t) + \sum_{m \neq k} \lambda_{mk} R_{km}(t)
\]

\[
\equiv \sum_m R_{im} \Lambda_{mk},
\]

with the initial condition \( R_{ij}(t=0) = \delta_{ij} \). \( T^{-1}_2 \) characterizes the exponential tail of long-time dephasing behavior. This quantity is obtained by \( \text{Min}(-\text{Re}(\epsilon_i)) \), where \( \epsilon_i \) are the eigenvalues of \( \Lambda \). While for very short \( t \), the \( R(t) \) shows Gaussian behavior. \(^8\)

First, we examine the single charge trap case \( (N=1) \) where we have

\[
\Lambda = \begin{pmatrix}
ia_1 - \lambda_u & \lambda_u \\
\lambda_d & ia_2 - \lambda_d
\end{pmatrix}.
\]

where the \( \lambda_u \) (\( \lambda_d \)) is the transition rate from the 1st state to the 2nd state (2nd state to the 1st state). The resultant \( T^{-1}_2 \) is given by \( T^{-1}_2 = \frac{1}{2}(\lambda_u + \lambda_d - \text{Re}\sqrt{A}) \) where \( A = 4a_1a_2 + 4i(a_1\lambda_d + a_2\lambda_u) + (\lambda_u + \lambda_d - ia_1 - ia_2)^2 \), and \( J_C = a_2 - a_1 \). For weak coupling
Fig. 1. Scheme of two charge trap system. The wells represent the charge traps and the arrows represent the transition between each of four states. $\lambda_{ij}$'s indicate the transition probabilities from state $i$ to state $j$.

$|J_C| \ll \max(\lambda_u, \lambda_d)$, $J_C = d = -2a_1 = 2a_2$, and $d$ characterize strength of bistable extra bias, $T_2^{-1}$ is given by

$$T_2^{-1} = \lambda_u \lambda_d d^2 / (\lambda_u + \lambda_d)^3. \quad (6)$$

For strong coupling, $(|d| \gg \lambda_u, \lambda_d)$, $T_2^{-1} = (\lambda_u + \lambda_d)/2$. These results coincide with those found by Itakura and Tokura,\textsuperscript{8} where the dephasing time was derived using a different method.

Next, we examined the two traps, ($N=2$) including the Coulomb blockade effect occurring between the traps. When two traps are located close to one another, there should be capacitance coupling between two occupied traps. However, we neglect tunneling between the charge traps.

There are four states: both charge traps empty, left charge trap occupied, right charge trap occupied, and both charge traps occupied (Fig. 1). $\lambda'_{ij}$'s are transition rates from $i$ state to $j$ state, and we neglect the transition processes between 1 state and 4 state, and 2 state and 3 state. In general, we notice $\lambda_{12} \geq \lambda_{34}$ and $\lambda_{13} \geq \lambda_{24}$, where the equations are for the absence of Coulomb blockade effect.

For actual calculation, we restricted the parameters for the transition rate which are symmetric for two charge traps, $\lambda_u = \lambda_{12} = \lambda_{13}, \lambda_d = \lambda_{21} = \lambda_{31}, \lambda'_u = \lambda_{24} = \lambda_{34}, \lambda'_d = \lambda_{12} = \lambda_{13}$. The occupation probabilities are $p_1 = \frac{\lambda_d \lambda'_u}{D}, p_2 = p_3 = \frac{\lambda_u \lambda'_d}{D}, p_4 = \frac{\lambda_u \lambda'_u}{D}$, where $D = \lambda_d \lambda'_d + (2\lambda'_d + \lambda'_u) \lambda_u$.

First we discuss the high temperature behavior, where energy differences of each state are lower than temperatures, while $\delta E_C$ and gap energy of cooper pair of qubit are much higher than temperatures. To demonstrate the effect of Coulomb interaction transparently, we chose parameters, $\lambda_u = \lambda_d = \lambda'_d \equiv \lambda$ and $\lambda'_u \equiv \lambda'$, and calculate $T_2$ while changing $\lambda'$ in the range $0 \leq \lambda' \leq \lambda$. Here, the occupations are $p_1 = p_2 = p_3 = \frac{\lambda}{2\lambda + \lambda'}$ and $p_4 = \frac{\lambda'}{2\lambda + \lambda'}$, and the amplitudes are, $a_1 = -d, a_2 = a_3 = 0$ and $a_4 = d$. The dephasing rates for weak coupling
\[ T_{2}^{-1} = \begin{cases} \frac{\lambda^2}{4\pi}, & \text{for } \lambda' = \lambda \\ \frac{d^2}{2\pi^2}, & \text{for } \lambda' = 0 \end{cases} \] (7)

In Fig. 2, the results by numerically solving Eq. (4) are plotted. This plot shows that two limits \( T_{2}^{-1} = \frac{\lambda^2}{4\pi} \) and \( \frac{d^2}{2\pi^2} \) are smoothly connected in the intermediate parameter region for \( d/\lambda = 0.1 \) (weak coupling case). In the limit of no interaction \( (\lambda = \lambda') \), the dephasing rate becomes twice of that for the single charge trap. For a strong interacting limit \( (\lambda' = 0) \), the time evolution reduces to that of a single charge trap with asymmetric transition rate of \( \lambda_{u}^{\text{single}} = \frac{1}{3}\lambda_1 \) and \( \lambda_{d}^{\text{single}} = \frac{2}{3}\lambda_1 \) with \( \lambda_1 = \frac{2}{3}\lambda \) and the dephasing rate is smaller than that of a single charge trap. For \( d/\lambda = 2 \), there is a rapid increase in the dephasing rate when \( \lambda \sim \lambda' \). There are four eigenvalues for characteristic equation of \( R_{\text{im}} \). Therefore there are four characteristic dephasing rates for this case. This singularity appears because two of them become same, thus the transition from weak coupling to strong coupling occurs there. Such a singularity also appears for the dephasing due to single charge trap. All plots show that the dephasing rate increases with \( \lambda'/\lambda \), which indicates that the effect of interaction, or, the screening effect, suppresses the dephasing compared with that of non-interacting charge traps. In this analysis, the average, \( \eta \), changes with the ratio \( \lambda'/\lambda \). If we choose \( a_i \) such that the average of \( \eta \) is invariant, the results are the same. The reason is that for both cases, \( a_2 - a_1 = a_3 - a_1 = d \) and \( a_4 - a_2 = a_4 - a_3 = d \), independent of \( \lambda'/\lambda \), and the difference between the former case and the latter case only leads to the modulation of Rabi oscillation frequency.

We also examined the Gaussian behavior, which is the short-time regime for \( t < \min(\frac{1}{d}, \frac{3}{\max(\lambda, \lambda')}) \). For dephasing due to a single charge trap, the off-diagonal element of density matrix decay is represented as, \( R(t) \simeq \exp(-\frac{1}{2}(\frac{t}{T_{2g}})^2) \), where \( T_{2g}^{-2} \) is given by \( \frac{\lambda^2}{4\pi} \). For two non-interacting charge traps, \( T_{2g}^{-2} = \frac{d^2}{\pi} \), and for strong interaction \( (\lambda' = 0) \), \( T_{2g}^{-2} = \frac{d^2}{4} \). This behavior shows that the dephasing is suppressed as interaction increases, even for the Gaussian behavior. It should be noted that the decay rate of Gaussian behavior depends on the total charge of charge traps, where we chose zero as the mean of amplitudes. In present examinations, the effect of interaction between charge traps is discussed, while the numerical estimation of dephasing rate due to non-interacting BCF had been done in refs. 8–10. Please note that \( T_{2g}^{-2} \) depends only on distribution of \( d^2 \) and number of charge traps, although \( T_2^{-1} \) depends on distributions of \( d^2/\lambda \) and number of charge traps; Eq. (7). The former result coincides with that of ref. 10 when the initial state is in thermal equilibrium.

At lower temperatures than the energy differences of each state, we have asymmetric transition rates. Therefore, we must consider the effect of temperature. In order to satisfy the detailed balance condition, temperature and electron correlation leads to the following forms of the transition rate:\[ \lambda_d = \lambda', \lambda_u = \lambda e^{-\Delta \over \kappa B T} \text{ and } \lambda_d' = \lambda e^{-\Delta+E_{charge} \over \kappa B T} \]. The definitions of
energy difference are: \( \Delta = E_2 - E_1 = E_3 - E_1 \), \( \Delta + E_{\text{charge}} = E_4 - E_2 = E_4 - E_3 \). The \( E_{\text{charge}} \) is the capacitive energy between two charge traps. In this case, the probabilities of population obey classical Boltzmann distribution, where \( p_i = \frac{e^{-E_i/k_BT}}{\sum_j e^{-E_j/k_BT}} \). At high temperatures \( (T \gg E_4/k_B) \), the occupation probabilities become \( p_1 = p_2 = p_3 = p_4 = 1/4 \). In Fig. 3, we show the dephasing rate obtained numerically with the amplitudes set to \( a_1 = -d, a_2 = a_3 = 0, a_4 = d \).

We chose the numerical parameter of \( d/\lambda = 0.1 \) (weak coupling). Using analytical expressions of dephasing rate for single charge trap, Eq. (6), the equation of the normalized dephasing rate for weak coupling case \( (d \ll \lambda) \) is given by \( T_{2,\text{single}}^{-1} = \frac{d^2}{\lambda} \frac{e^{-\Delta/k_BT}}{(1 + e^{-\Delta/k_BT})^3} \). The behavior of the traps for \( N=2 \) requires detailed examination. For weak interaction \( (E_{\text{charge}} \ll \Delta) \), the dephasing rate due to the two charge traps is twice that of the dephasing rate due to the single charge traps. From Eq. (6), the dephasing rate becomes suppressed as the asymmetry of the transition rates increases. At low temperatures \( (k_BT \ll \Delta) \), the dephasing rate is suppressed exponentially, because the asymmetry of the transition rates increases with a decrease in temperature. At high temperatures \( (k_BT \gg \Delta) \), the dephasing rate is again suppressed. The reason is that, the characteristic transition rate \( (\lambda_u + \lambda_d) \), increases as the temperature increases. When coupling between the qubit and charge traps is weak \( (d \ll \max(\lambda_u, \lambda_d)) \), the magnitude of the fluctuations in the trace of a state on the Bloch sphere decreases with increasing, \( \lambda_u + \lambda_d \), hence the dephasing rate decreases as well.
Next, we must consider the behavior of the traps when Coulomb interaction is strong ($E_{\text{charge}} \gg \Delta$). Except for very high temperatures ($k_B T \gg E_{\text{charge}} + \Delta$), the dephasing rate due to two dynamical charge traps is, $T_2^{-1} \lambda / d^2 = 2e^{-\Delta / k_B T} / (1 + 2e^{-\Delta / k_B T})^3$. Then, comparing with Eq. (6), two charge traps are equivalent to a single charge trap with asymmetric transition rate, $\lambda_d = \lambda$, $\lambda_u = 2e^{-\Delta / k_B T} \lambda$. At intermediate temperatures ($E_{\text{charge}} \gg k_B T \gg \Delta$), the dephasing rate is smaller than that of a single charge trap. The reason for this behavior is that two traps behave as a single charge trap with a larger characteristic transition rate, $\lambda (1 + 2e^{-\Delta / k_B T})$, compared with that of the single charge trap $\lambda (1 + e^{-\Delta / k_B T})$. At low temperatures, ($\Delta, E_{\text{charge}} \ll k_B T$), the dephasing rate decreases exponentially as the temperature decreases irrespective of $E_{\text{charge}}$.

Finally, we examine $N$ identical charge traps which are located close to one another. To simplify the discussion, we consider a system of $N$ charge traps symmetrically coupled with a qubit. There are: one empty state ($i = 0$), $N$ single occupied states ($i = 1$), $(N-1)N/2$ two occupied states ($i = 2$), $\cdots$, one fully occupied state ($i = N$). For strong and long-range Coulomb interaction, the empty state and single occupied states are relevant. When there is weak coupling, we have $T_2^{-1} = \frac{d^2 N e^{-\Delta / k_B T}}{\lambda (1 + Ne^{-\Delta / k_B T})}$, where $d$ and $\lambda$ are coupling constants.
between the qubit and the background charges, and characteristic transition rate of charge traps, respectively, which are identical for all charge traps. When there is non-interaction, \( N \) charge traps behave independently, and \( T_{1}^{-1} = \frac{\varepsilon^{2}}{N} \frac{N e^{-\Delta/k_{B} T}}{(1+e^{-\Delta/k_{B} T})^{3}} \), hence the interaction between charge traps suppresses the dephasing rate. At high temperatures, the analytical solution of \( T_{1}^{-1} \) and \( T_{2}^{-2} \) are given by \( T_{1}^{-1}(\text{strong interaction}) = \frac{8}{(N+1)^{2}} \), \( T_{2}^{-2}(\text{strong interaction}) = \frac{1}{N} \), where we chose zero as the mean of amplitudes. Therefore, the dephasing rate becomes suppressed more effectively as the number of charge traps increases. It should be noted that when charge traps are interacting strongly each other, the dephasing rate with dissipation in the large \( N \) limit is given by \( \frac{E_{2}^{2}}{\delta E_{C}^{2} + E_{2}^{2}} \frac{\varepsilon^{2}}{N} \). Then the dephasing rate with dissipation becomes also suppressed with increasing \( N \). While the dephasing rate with dissipation becomes gradually dominant over the pure dephasing rate as increasing \( N \), we do not argue this effect since both rate vanish with \( N \).

In conclusion, we examined the dephasing rate of a two-level system, coupled with a classical environment made of \( N \) charge traps. The environment changes its bistable extra bias, which results in pure dephasing. When the charge traps fluctuate independently, the total dephasing rate is the simple summation of the dephasing rate of each charge trap. If multiple charge traps are interacting with each other, the dephasing rate is slowed, when \( T \) is not much smaller than \( \Delta/k_{B} \). At high temperatures, \( T \gtrsim \Delta/k_{B} \), more than one charge traps with large Coulomb interaction results in a smaller dephasing rate than that of the single charge trap. It should be noted that the other channels of dephasing exist, such a dephasing rate should be added to present dephasing rate. And present estimation of dephasing rates corresponds to that of free induction decay,\(^9\) not that during gate operation in such a case the charge degeneracy state (\( \delta E_{C} = 0 \)) should be manipulated. The numerical evaluation of dephasing rate for such a situation has been done in refs. 8, 9.

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