Non singular M theory Universe in
Loop Quantum Cosmology – inspired Models

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ABSTRACT

We study an M theory universe in the Loop Quantum Cosmology – inspired models which involve a function, the choice of which leads to a variety of evolutions. The M theory universe is dominated by four stacks of intersecting brane–antibranes and, in general relativity, it becomes effectively four dimensional in future while its seven dimensional internal space reaches a constant size. We analyse the conditions required for non singular evolutions and obtain explicit solutions in the simplified case of a bi–anisotropic universe and a piece–wise linear function for which the evolutions are non singular. One may now ask whether the physics in the Planckian regime can enhance the internal volume to phenomenologically interesting values. In the simplified case considered here, there is no non trivial enhancement. We make some comments on it.
1. Introduction

The \((9 + 1)\) dimensional superstring theory, equivalently the \((10 + 1)\) dimensional M theory, is considered to be a quantum theory of gravity. Any candidate for a quantum theory of gravity may be expected to provide, among other things, a detailed description of black hole physics and also of the beginning of the universe. For example, such a theory should explain black hole entropy and Hawking radiation, and should resolve the black hole and the big bang singularities which occur in general relativity descriptions.

String/M theory has provided detailed descriptions of black hole entropies and Hawking radiations for certain classes of extremal and near extremal black holes. The black holes are described by appropriate stacks of intersecting brane–antibranes, their entropies arise from the degrees of freedom living on these branes, and the Hawking radiation arise from interactions between these degrees of freedom. See, for example, \([1] – [7]\). As for the black hole or the big bang singularities, there are no similarly detailed string/M theoretic descriptions although there have been a variety of ideas. See \([8] – [23]\) for a sample of them.

The \((3 + 1)\) dimensional Loop quantum gravity (LQG) based on Ashtekar variables is considered to be another candidate for a quantum theory of gravity \([24] – [30]\). The areas and volumes are quantised in LQG and the black hole entropies are described in terms of the quanta of area \([31] – [35]\). Quantising the homogeneous sector of LQG leads to Loop quantum cosmology (LQC) and it provides a resolution of big bang singularity: instead of ending in a big bang singularity, the universe undergoes a bounce when its density is Planckian. As one goes back in time, a large universe contracts as in general relativity, then reaches a minimum size where its density is Planckian, bounces back from this minimum, and starts expanding again as in general relativity as one goes further into its past \([36] – [44]\).

The quantum evolution of a \((3 + 1)\) dimensional universe in LQC can be described well by effective equations which reduce to general relativity equations in the classical limit \([44]\). Recently, we have constructed LQC – inspired models by empirically generalising these effective equations to \((d + 1)\) dimensions and studied several aspects of these models \([45, 46, 47]\). These models are characterised by two functions but we will fix one of them by working in what is referred to as \(\tilde{\mu}\)–scheme. The remaining function can
be chosen so as to lead to general relativity, or to LQC, or to a variety of evolutions, singular as well as non singular. For example, one can model a bouncing universe or an universe which enters and stays in the 'Hagedorn phase' where the density and temperatures are constant [46].

In string or M theory universes, the spacetime is ten or eleven dimensional. They all have big bang singularities when evolved using general relativity equations. These singularities may now be resolved in the LQC – inspired models. The string/M theory universes may then have a bounce instead of a big bang singularity, or a variety of more general non singular evolutions.

In this paper, we study the evolution of a \((10 + 1)\) dimensional M theory universe in the LQC – inspired models. We consider the universe studied in [18] – [23] which, for entropic reasons, is dominated by four stacks of intersecting brane–antibranes. In general relativity, due to the U–duality relations among the densities and the pressures, this universe becomes effectively \((3 + 1)\) dimensional in future while the seven dimensional internal space reaches a constant size [21, 22, 23].

In the present study, we first analyse qualitatively the conditions required for non singular evolution in the LQC – inspired models. Then we simplify our set up in order to obtain explicit solutions: Instead of considering a general anisotropic universe, we consider a bi–anisotropic universe where the space is \(d = (\tilde{n} + n)\) dimensional and where the quantities corresponding to the \(\tilde{n}\) and the \(n\) dimensional spaces are seperately isotropic; and, consider a simplified, piece–wise linear function for which the evolutions are non singular.

We obtain explicit solutions for a bi–anisotropic universe and then consider the M theory universe for which the evolution is non singular, and which becomes effectively \((3 + 1)\) dimensional in future while its internal space reaches a constant size. One may now ask whether the physics in the non singular Planckian regime can enhance the future constant size of the internal space. Such a large internal space, obtained with no fine tuning, may be useful in phenomenological model building, see for example [48, 49]. Answering this question using the explicit solutions obtained in this paper, we find no non trivial enhancement of the internal size.

Although this answer may be disappointing and is perhaps not unexpected, we like to emphasise that it is now possible to ask such a question and to seek its answer for an M theory universe in the LQC – inspired mod-
els. This is because the question itself is meaningful, and its answers may then be sought, only if a higher dimensional universe evolves non-singularly in the Planckian regime, and only if it dimensionally compactifies in future. However, more analysis is needed to determine whether or not a large volume compactification is possible in the LQC–inspired models but this is beyond the scope of the present paper.

This paper is organised as follows. In section 2, we present the equations of motion in general relativity and in the LQC–inspired models. In section 3, we present the density and the pressures for the most entropic constituents of an M theory universe, incorporating U–duality relations. In section 4, we analyse qualitatively the general evolution and make some simplifying assumptions. In section 5, we obtain explicit solutions. In section 6, using these solutions, we analyse the size of the internal space. In section 7, we summarise the paper and conclude by mentioning a few topics for further studies. In Appendix A, we present the anisotropic solutions to general relativity equations. In Appendix B, we present the isotropic solutions to the equations in the LQC–inspired models. In Appendix C, we present solutions for a case left out in section 5.

2. LQC–inspired models: Equations of motion

In this section, we write down the equations of motion first in general relativity and then in the Loop Quantum Cosmology (LQC)–inspired models. Let the space be $d$ dimensional and toroidal with $d \geq 3$ and with coordinates $x^i$, $i = 1, 2, \cdots, d$. Consider a homogeneous and anisotropic universe whose $(d + 1)$ dimensional line element $ds$ is given by

$$ds^2 = -dt^2 + \sum_i e^{2\lambda_i} (dx^i)^2$$

where the scale factors $e^{\lambda_i}$ are functions of $t$ only. Here and in the following, we will explicitly write the indices to be summed over. The general relativity equations are given, in the standard notation with $\kappa^2 = 8\pi G_{d+1}$, by

$$R_{AB} - \frac{1}{2} g_{AB} R = \kappa^2 T_{AB} \quad , \quad \sum_A \nabla^A T_{AB} = 0$$

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where \( A, B = (0, i) \) and \( T_{AB} \) is the energy momentum tensor. Let \( T_{AB} \) be diagonal and be given by \( T_{AB} = \text{diag} \ (\rho, \ p_i) \) where \( \rho \) is the density and \( p_i \) is the pressure in the \( i^{th} \) direction. Then, after a straightforward algebra, equations (2) give

\[
\sum_{ij} G_{ij} \lambda^i_t \lambda^j_t = 2\kappa^2 \rho \tag{3}
\]

\[
\lambda^i_{tt} + \Lambda \lambda^i_t = \kappa^2 r^i \tag{4}
\]

\[
\rho_t + \sum_i (\rho + p_i) \lambda^i_t = 0 \tag{5}
\]

where the \( t-\)subscripts denote derivatives with respect to \( t \) and

\[
G_{ij} = 1 - \delta_{ij} \quad , \quad G^{ij} = \frac{1}{d-1} - \delta^{ij}
\]

\[
\Lambda = \sum_i \lambda^i_t \quad , \quad r^i = \sum_j G^{ij} (\rho - p_j) . \tag{6}
\]

Note that \( \sum_j G^{ij} G_{jk} = \delta^i_k \) and \( r^i = p_i + \frac{\rho - \sum_j p_j}{d-1} \). Also define \( Y_i \) by

\[
Y_i = \sum_j G_{ij} \lambda^j_t = \Lambda_t - \lambda^i_t \tag{7}
\]

so that, using equation (3), equation (4) for \( \lambda^i_{tt} \) may be written as

\[
\lambda^i_{tt} + \sum_j \frac{(\lambda^j_t - \lambda^j_t) Y_i}{d-1} = \kappa^2 \left( r^i - \frac{2\rho}{d-1} \right) . \tag{8}
\]

Equations (3) and (8) will resemble closely the equations (10) and (11) in the LQC – inspired model, to be given below.

We now consider the evolution of a \((d + 1)\) dimensional homogeneous anisotropic universe in the LQC – inspired models. These models were constructed in our earlier works by a natural, straightforward, and empirical generalisation of the effective equations which describe the quantum evolution of an anisotropic universe in LQC. The model we consider here is specified by an arbitrary function \( f(x) \) with the only requirement that \( f(x) \to x \)
in the limit $x \to 0$. The general relativity equations follow for $f(x) = x$ and the LQC effective equations follow for $f(x) = \sin x$ and $d = 3$.

In the $(3 + 1)$ dimensional Loop Quantum Gravity (LQG) formalism, the canonical pairs of phase space variables consist of an $SU(2)$ connection $A_a^i$ and a triad $E_a^i$ of density weight one where $i, a = 1, 2, 3$. For LQC, in the notation used here, the triad variable $E_a^i \propto e^{\lambda_i} e^{\Lambda_i}$ and the connection variable $A_a^i \propto \hat{c}^i$ which will turn out to be related to $(e^{\Lambda_i})_t$. Also, let $m^i = \bar{\mu}^i \hat{c}^i$ where $\bar{\mu}^i \propto e^{-\lambda_i}$ in what is referred to as the $\bar{\mu}$-scheme. The exact expressions for $A_a^i$, $E_a^i$, and $\bar{\mu}^i$ and their derivations are somewhat involved and are not needed here. See the review [44] for a detailed description.

Starting with the LQC variables in $(3 + 1)$ dimensions, generalising them empirically to $(d + 1)$ dimensions, and after a long algebra, the equations for the LQC – inspired models may be written concisely in terms of the variables $m^i$, $i = 1, 2, \cdots, d$. In these models, the conservation equation (5) for $\rho_t$ remains the same but equations (3) and (8), which is equivalent to (4), are modified. In terms of the functions $f^i$, $g_i$, and $X_i$ defined by

$$f^i = f(m^i), \quad g_i = \frac{d f^i}{dm^i}, \quad X_i = g_i \sum_j G_{ij} f^j,$$

these modified equations in our LQC – inspired models are given by

$$\sum_{ij} G_{ij} f^i f^j = 2 \gamma^2 \lambda_{qm}^2 \kappa^2 \rho = \frac{\rho}{\rho_{qm}} \tag{10}$$

$$(m^i)_t + \sum_j \frac{(m^i - m^j) X_j}{(d - 1) \gamma \lambda_{qm}} = \gamma \lambda_{qm} \kappa^2 \left( r^i - \frac{2 \rho}{d - 1} \right) \tag{11}$$

$$\frac{X_i}{\gamma \lambda_{qm}} = Y_i = \sum_j G_{ij} \lambda_j^i$$

$$\leftrightarrow \quad \lambda_i^j = \frac{\sum_j G^{ij} X_j}{\gamma \lambda_{qm}} \tag{12}$$

where $\rho_{qm} = \frac{1}{2 \gamma \lambda_{qm} \kappa}$, the constant $\gamma$ is analogous to the Barbero – Immirzi parameter in LQC, and $\lambda_{qm}$ is a length parameter which characterises the quantum of the $(d - 1)$ dimensional area : $\lambda_{qm}^{d-1} \sim \gamma^2 \kappa^2$. Note that, upon using (12) for $\lambda_i^j$ and equations (6) for $r^i$, the conservation equation (5) may be written in terms of $X_i$ as
\[(\gamma \lambda_{qm}) \rho_t + 2\rho \frac{\sum_i X_i}{d-1} = \sum_i r^i X_i \, . \quad (13)\]

Equation (13) also follows upon calculating $\rho_t$ from equation (10) and then using equation (9) for $X_i$ and (11) for $(m^i)_t$ . Equivalently, equation (10) may be derived as an integral of equations (11) and (13). Also note that for any linear function $f(x) = cx + c_0$ where $c$ and $c_0$ are constants, one has
\[f^i = cm^i + c_0 \, , \quad g_i = c \, , \quad (\gamma \lambda_{qm}) \lambda_i^i = cf^i \, . \quad (14)\]

Equations (10) and (11) then give the general relativity equations (3) and (8) with $\kappa^2$ now replaced by $c^2 \kappa^2$ .

We note here that Helling has pointed out in [50] that functions of the form $f(x) = \sum_n a_n \sin (b_n x)$ should be admissible within the LQC formalism itself. He further shows by giving an example that some of these functions with infinite sums can lead to a more singular evolution than in general relativity. Also, Bodendorfer et al have constructed a higher dimensional LQG by generalising Ashtekar variables [51, 52, 53]. Upon quantising its homogeneous sector, one can obtain $(d + 1)$ dimensional LQC where $f(x) = \sin x$ [54, 55]. It is likely that functions of the form $f(x) = \sum_n a_n \sin (b_n x)$ should be admissible here also. Admitting such functions in $(d + 1)$ dimensional LQC may provide a firm foundation for the LQC – inspired models.

### 3. M theory universe

One may now study the $(10 + 1)$ dimensional M theory universe in the LQC – inspired models by incorporating in equations (10) – (12) the density $\rho$ and the pressures $p_i$ for its constituents.

We consider the M theory universe studied in [18] – [23] which is dominated by constituents that are most entropic. Such constituents are given by four stacks of M theory brane–antibranes which intersect according to the Bogomol’nyi – Prasad – Sommerfield (BPS) rules \footnote{According to the BPS rules, two stacks of 5 branes intersect along three common spatial directions; two stacks of 2 branes intersect along zero common spatial directions; a stack of 2 branes intersect a stack of 5 branes along one common spatial direction; and each stack of branes is smeared uniformly along the other brane directions. There can be a wave along common intersection direction. See [5, 6, 7] for more details and for other such M theory configurations.} and wrap the seven di-
rections, labelled 1, 2, ⋅⋅⋅, 7: namely, two stacks each of $M^2$ and $M^5$ brane–antibranes wrap respectively the directions 12, 34, 13567, and 24567. Such $N$ stacks of $M^2$ and $M^5$ brane–antibranes intersecting according to the BPS rules may be described by a total energy momentum tensor $T_{AB}$ which is made up of $N$ mutually noninteracting and separately conserved components. These energy momentum tensors may be taken to be diagonal. Thus, with $I = 1, 2, ⋅⋅⋅, N$, they may be written as

$$T_{AB} = \sum_I T_{AB}(I) \ , \ \sum_A \nabla^A T_{AB}(I) = 0 \quad (15)$$

where $T_{AB} = \text{diag} (\rho_i, p_i)$ and $T_{AB}(I) = \text{diag} (\rho_I, p_{iI})$. The total density $\rho$, the total pressure $p_i$ in the $i^{th}$ direction, and the total $r^i$ are then given by

$$\rho = \sum_I \rho_I \ , \ p_i = \sum_I p_{iI} \ , \ r^i = \sum_I r^i_I \quad (16)$$

where

$$r^i_I = \sum_j G^{ij} (\rho_I - p_{jI}) = p_{iI} + \frac{\rho_I - \sum_j p_{jI}}{d-1} \ . \quad (17)$$

Furthermore, for the line element $ds$ given in equation (1), the conservation equation (15) for $T_{AB}(I)$ leads to

$$(\rho_I)_t + \sum_i (\rho_I + p_{iI}) \lambda^i_I = 0 \ . \quad (18)$$

In the LQC – inspired models, using equations (12) for $\lambda^i_I$ and (17) for $r^i_I$, the conservation equation (18) may be written in terms of $X_i$ as

$$(\gamma \lambda_{qm}) (\rho_I)_t + 2 \rho_I \frac{\sum_i X_i}{d-1} = \sum_i r^i_I X_i \ . \quad (19)$$

To proceed further, one needs equations of state which determine the pressures $p_{iI}$ in terms of $\rho_I$. For $N$ stacks of $M^2$ and $M^5$ brane–antibranes intersecting according to the BPS rules, the U–duality symmetries of M theory may be shown [21, 22, 23] to require that the density $\rho_{(I)}$ of the $I^{th}$ stack and its pressures $p_{∥(I)}$ and $p_{⊥(I)}$ along the parallel and transverse directions must be related as follows:

$$p_{∥(I)} = -\rho_{(I)} + 2 p_{⊥(I)} \leftrightarrow (\rho - p_{∥})_{(I)} = 2 (\rho - p_{⊥})_{(I)} \ . \quad (20)$$
Specifying \( p_\perp(t) \) as a function of \( \rho(t) \) will determine the equations of state for \( p_\parallel(t) \) and thereby for all the pressures \( p_{iI} \). The U–duality symmetries further require this function to be the same for all \( I \). Hence, specifying a single function \( p_\perp(\rho) \) determines all \( p_{iI} \) in terms of \( \rho_I \) where \( i = 1, 2, \cdots, 10 \) and \( I = 1, 2, \cdots, N \). \(^2\)

Consider now the most entropic constituents mentioned earlier which are given by two stacks each of \( M2 \) and \( M5 \) brane–antibranes. \( N = 4 \) for this configuration and, for simplicity, we refer to it as \((2, 2', 5, 5')\) branes. Using equation (20), we now write the pressures in the \( i^{th} \) directions for the \((2, 2', 5, 5')\) branes in an obvious notation as follows:

\[
\{(\rho - p_i)_{(2)}\} : (2, 2, 1, 1, 1, 1, 1, 1, 1, 1) (\rho - p_\perp)_{(2)}
\]

\[
\{(\rho - p_i)_{(2')}\} : (1, 1, 2, 1, 1, 1, 1, 1, 1, 1) (\rho - p_\perp)_{(2')}
\]

\[
\{(\rho - p_i)_{(5)}\} : (2, 1, 2, 2, 2, 2, 1, 1, 1, 1) (\rho - p_\perp)_{(5)}
\]

\[
\{(\rho - p_i)_{(5')}\} : (1, 2, 1, 2, 2, 2, 1, 1, 1, 1) (\rho - p_\perp)_{(5')}
\]

The corresponding \( r_i^j = \sum_j G^{ij} (\rho - p_j)_{(*)} \) where \( * = 2, 2', 5, 5' \) are given, after a little algebra, by

\[
\{r_i^j\}_{(2)} : (-2, -2, 1, 1, 1, 1, 1, 1, 1, 1) \frac{(\rho - p_\perp)_{(2)}}{3}
\]

\[
\{r_i^j\}_{(2')} : (1, 1, -2, -2, 1, 1, 1, 1, 1, 1) \frac{(\rho - p_\perp)_{(2')}}{3}
\]

\[
\{r_i^j\}_{(5)} : (-1, 2, -1, 2, -1, -1, -1, 2, 2, 2) \frac{(\rho - p_\perp)_{(5)}}{3}
\]

\[
\{r_i^j\}_{(5')} : (2, -1, 2, -1, -1, -1, 2, 2, 2) \frac{(\rho - p_\perp)_{(5')}}{3}
\]  \( \cdots \)

Thus, given an equation of state function \( p_\perp(\rho) \), equations (10) – (12), (16) – (19), and (22) will describe the cosmological evolution of a \((10 + 1)\) dimensional M theory universe in our LQC – inspired models.

\(^2\)In a certain approximation, Chowdhury and Mathur have derived from first principles the energy momentum tensors for the intersecting branes [18, 19]. The pressures, thus derived, satisfy the U–duality relation (20) and follow from the present expressions as a special case when \( p_\perp = 0 \).
Note that if the densities $\rho_{(\ast)}$ are the same for all $\ast = 2, 2', 5, 5'$ then so will be the pressures $p_{\perp(\ast)}$ and, hence, $(\rho - p_{\perp})_{(\ast)}$. Consequently, it follows from the above expressions for $r_{i(\ast)}^1$ that the total $r_i^1 = r_{i(2)}^1 + r_{i(2')}^1 + r_{i(5)}^1 + r_{i(5')}^1 = 0$ for $i = 1, 2, \cdots, 7$. The ten dimensional space will then become effectively three dimensional in the limit $e^\Lambda \to \infty$ : the seven directions, labelled 1,2,\cdots,7, will neither expand nor contract and will reach constant sizes; and, the remaining three directions will continue to expand. In this paper, we assume that the densities $\rho_{(\ast)}$ are the same for all $\ast$ and that the equation of state is linear. Thus, we write

$$\rho_{(\ast)} = \frac{\rho}{4}, \quad p_{\perp(\ast)} = (1 - u) \rho_{(\ast)}$$

(23)

for $\ast = 2, 2', 5, 5'$ where $\rho$ is the total density and $u < 2$ is a constant. It then follows from equations (21) and (22) that the total $p_i = \sum_I p_{iI}$ and $r_i = \sum_I r_{iI}$ are given by

$$\{\rho - p_i\} : (6, 6, 6, 6, 6, 6, 6, 4, 4, 4) \frac{u \rho}{4}$$

(24)

$$\{r_i\} : (0, 0, 0, 0, 0, 0, 0, 6, 6, 6) \frac{u \rho}{12}.$$  (25)

4. General evolution and a bi–anisotropic universe

Consider now the general evolution resulting from equations (10)–(12) and (16)–(19). Equation (10) may be derived as an integral of the remaining equations. Hence, if it is satisfied at an initial time $t_0$ then it is satisfied for all $t$.

The equations of state, which may be derived from the underlying physics or may be assumed, will give the pressures $p_{iI}$ and the quantities $r_{iI}^1$ in terms of $\rho_I$. Then equations (11), (12), and (19) give the first time derivatives $m_i^1, \lambda_i^1$, and $(\rho_I)_t$ as polynomials in terms of $(\rho_I, m_i, f_i)$ and $g_i = \frac{df_i}{dm_i}$.  

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Even if the densities $\rho_{(\ast)}$ are unequal initially, the dynamics of the general relativity equations (4) resulting from the $r_{i(\ast)}^1$ given in equations (22) is such that these densities become equal in the limit $e^\Lambda \to \infty$ [21, 22, 23]. Such an M theory universe may therefore provide a detailed realisation of the maximum entropic principle that we had proposed in [17] to determine the number $3 + 1$ of large spacetime dimensions.

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where \( f^i = f(m^i) \). Differentiating these expressions repeatedly will then give all the higher time derivatives of \((m^i, \lambda^i, \rho_I)\) as polynomials in terms of \((\rho_I, m^i, f^i)\) and the higher derivatives of \(f^i\) with respect to \(m^i\). Therefore, it follows that if the function \(f(x)\) and all its derivatives are finite then all the time derivatives of \(\lambda^i\) will also remain finite and thus the evolution will be non singular. See [46] for a variety of such evolutions. Also, note that the function \(f(x) = x\) is not finite although all its derivatives are, and it leads to the big bang singularities of general relativity.

Consider obtaining solutions numerically for \((m^i, \lambda^i, \rho_I)\). Let the equations of state be given and let the initial values of \((m^i, \lambda^i, \rho_I)\) at \(t_0\) satisfying equation (10) be also given. Then, in principle, \(m^i(t), \lambda^i(t), \text{ and } \rho_I(t)\) follow from equations (11), (12), and (19) : The values of \(m^i(t_0)\) determine the values of \((f^i, g_i, X_i)\) at \(t_0\); equation (12) then determines \(\lambda^i\) at \(t_0\); and equations (11) and (19), together with the equations of state, then determine \(m^i\) and \(\rho_I\) at \(t_0\). These will then determine the values of \((m^i, \lambda^i, \rho_I)\) at \(t_0 \pm \delta t\). Repeating this procedure will give \((m^i, \lambda^i, \rho_I)\) for all \(t\). Thus, it is always possible to obtain solutions numerically.

However, solving equations (10) – (12) and (16) – (19) analytically and obtaining \(m^i(t), \lambda^i(t), \text{ and } \rho_I(t)\) explicitly is not always possible. We are able to obtain explicit solutions only in a few simple cases when \(N = 1\) and when the equation of state is linear : in the anisotropic case with \(f(x) = cx + c_0\) which gives general relativity, see Appendix A; and, in the isotropic case with \(f(x) = cx + c_0\) or \(f(x) = \sin x\), see [45, 46] and Appendix B.

Hence, in order to obtain explicit solutions which may provide insights into non singular evolution of an M theory universe, we now simplify our set up : Instead of considering a general anisotropic universe, we consider a bi–anisotropic universe where the space is \(d = (\tilde{n} + n)\) dimensional, and where the quantities, such as \(m^i, f^i, \lambda^i, p_i, r^i\), corresponding to the \(\tilde{n}\) and the \(n\) dimensional spaces are seperately isotropic. Thus, we write

\[
\left( m^i, f^i, g_i, X_i, \lambda^i, p_i, r^i \right) = \left( \tilde{m}, \tilde{f}, \tilde{g}, \tilde{X}, \tilde{\lambda}, \tilde{p}, \tilde{r} \right) \text{ for } i = 1, 2, \ldots, \tilde{n}
\]

\[
= \left( m, f, g, X, \lambda, p, r \right) \text{ for } i = \tilde{n} + 1, \ldots, \tilde{n} + n .
\]
Then the line element $ds$ in equation (1) is given by

$$ds^2 = - dt^2 + e^{2\lambda} \sum_{i=1}^{\tilde{n}} (dx^i)^2 + e^{2\tilde{\lambda}} \sum_{i=\tilde{n}+1}^{\tilde{n}+n} (dx^i)^2,$$  \hspace{1cm} (27)

we have $\Lambda = \tilde{n}\tilde{\lambda} + n\lambda$, and equations (10) – (12) become

$$\left(nf + \tilde{n}\tilde{f}\right)^2 - \left(nf^2 + \tilde{n}\tilde{f}^2\right) = 2\gamma^2 \lambda_{qm}^2 \rho = \frac{\rho}{\rho_{qm}}$$  \hspace{1cm} (28)

$$\tilde{m}_t + \frac{(\tilde{m} - m) nX}{(d-1) \gamma\lambda_{qm}} = \gamma\lambda_{qm} \kappa^2 \left(\tilde{r} - \frac{2\rho}{d-1}\right)$$

$$m_t + \frac{(m - \tilde{m}) \tilde{n}\tilde{X}}{(d-1) \gamma\lambda_{qm}} = \gamma\lambda_{qm} \kappa^2 \left(r - \frac{2\rho}{d-1}\right)$$  \hspace{1cm} (29)

$$\frac{\tilde{X}}{\gamma\lambda_{qm}} = \Lambda_t - \tilde{\lambda}_t, \quad \frac{X}{\gamma\lambda_{qm}} = \Lambda_t - \lambda_t$$

$$\Longleftrightarrow \tilde{\lambda}_t = \frac{nX - (n-1)\tilde{X}}{(d-1) (\gamma\lambda_{qm})}, \quad \lambda_t = \frac{\tilde{n}\tilde{X} - (\tilde{n} - 1)X}{(d-1) (\gamma\lambda_{qm})}$$  \hspace{1cm} (30)

where

$$\tilde{X} = \tilde{g} \left(nf + (\tilde{n} - 1)\tilde{f}\right), \quad X = g \left((n-1)f + \tilde{n}\tilde{f}\right)$$

$$\tilde{r} = \frac{\rho - np + (n-1)\tilde{p}}{d-1}, \quad r = \frac{\rho - \tilde{n}\tilde{p} + (\tilde{n} - 1)p}{d-1}.$$  \hspace{1cm} (31)

Let the equations of state be linear and be given by $\tilde{p} = (1 - \tilde{u}) \rho$ and $p = (1 - u) \rho$. Then, writing $\tilde{r} = \tilde{v}\rho$ and $r = v\rho$, one has

$$\tilde{v} = \frac{nu - (n-1)\tilde{u}}{d-1}, \quad v = \frac{\tilde{n}\tilde{u} - (\tilde{n} - 1)u}{d-1}.$$  \hspace{1cm} (32)

For the $(\tilde{n} + n)$ dimensional space to become effectively $n$ dimensional in the limit $e^\Lambda \rightarrow \infty$, it is necessary that $\tilde{v} = 0$ which then gives

$$\tilde{u} = \frac{n u}{n-1}, \quad v = \frac{u}{n-1}.$$  \hspace{1cm} (33)
Also, for the linear equations of state, the conservation equation (5) gives

\[ \rho = \rho_0 e^{(\tilde{u}-2) \tilde{n}(\tilde{\lambda} - \tilde{\lambda}_0) + (u-2) n(\lambda - \lambda_0)} . \]  

(34)

For an M theory universe, \( d = \tilde{n} + n = 10 \). The above equations for the bi–anisotropic universe are consistent with and become applicable to M theory universe dominated by (2, 2’, 5, 5’) branes if \( \tilde{n} = 7 \), \( n = 3 \), and the densities \( \rho_{(*)} \) are the same for all \( * = 2, 2', 5, 5' \). Therefore, we take \( \rho_{(*)} \) and the equation of state to be given by equations (23). Hence \( p = (1 - u) \rho \). Then, with \( \tilde{p} = (1 - \tilde{u}) \rho \), \( \tilde{r} = \tilde{v} \rho \), and \( r = v \rho \), one has \( \tilde{u} = \frac{3n}{2} \), \( \tilde{v} = 0 \), and \( v = \frac{n}{2} \), see equations (24), (25), and (33).

**A convenient choice for \( f(x) \)**

In the LQC–inspired models, it follows from equations (10) – (12) and (16) – (19) that the cosmological evolution will be non singular if the function \( f(x) \) and all its derivatives are finite. We do not know the fundamental origin, if any, of such a class of functions. Nevertheless, by modelling the non singular evolution of an universe in several ways by several choices of \( f(x) \), one may gain new insights into the Planckian regime of the evolution.

One question that may be asked in the present set up is the following. In an M theory universe where the constituent pressures are given by equation (21), the seven spatial directions wrapped by branes reach constant sizes and the remaining three continue to expand in the limit \( e^A \to \infty \). As we found in [22, 23] using general relativity equations, these constant sizes are generically of \( O(l_{11}) \) where \( l_{11} \) is the eleven dimensional Planck length. They may be made arbitrarily large, for example \( O(10^{15} l_{11}) \) which may be of phenomenological interest [48, 49], but it requires a simialrly large fine tuning to about 15 decimal places near the Planckian regime. One may now ask in an LQC – inspired model for an M theory universe whether it is possible to obtain a large internal space with no fine tuning.

Such a question may be addressed in the LQC – inspired models because now the evolution can be made non singular by choosing the function \( f(x) \) appropriately. Naturally, one may also hope to achieve a large internal space but with no fine tuning by choosing a suitable class of such functions. With
this question in mind, we consider functions which may cause the universe to be in the Planckian regime for a long time and study whether a long stay in the Planckian regime will result in a large internal space.

Accordingly, we consider a class of functions which are odd under $x \rightarrow -x$, have a period $4m_\ast$, are labelled by an integer $\nu \geq 1$, and are given in the interval $0 \leq x \leq 2m_\ast$ by

$$f(x;\nu) = A_0 \left(1 - \left(1 - \frac{x}{m_\ast}\right)^{2\nu}\right)$$

where $m_\ast = 2\nu A$ so that $f(x;\nu) \rightarrow x$ in the limit $x \rightarrow 0$. One may set $A = 1$ with no loss of generality but it is convenient not to do so. Note that $f(x;\nu) = 0$ at $x = 0$ and $2m_\ast$, that $f(x;\nu) = f_{\text{max}} = A$ at $x = m_\ast$, and that the integer $\nu$ controls the flatness of $f(x;\nu)$ near its maximum. Also note that when one or more $f^i$s are of $O(1)$ and near $f_{\text{max}}$, equations (10) and (12) imply that, generically, the values of $\rho$ and $\lambda_1^i$ are Planckian. Thus, larger values of $\nu$ will make the function flatter near the maximum and, hence, may cause the universe to be in the Planckian regime for a longer time.

Now, in order to obtain explicit solutions, we make a piece-wise linear approximation to this function as follows. Let $f(-x) = -f(x)$, let $f(x + 4m_\ast) = f(x)$, and let $f(x)$ be given in the interval $0 \leq x \leq 2m_\ast$ by

$$f(x) = x \quad \text{for} \quad 0 \leq x \leq A$$

$$= A \quad \text{for} \quad A \leq x \leq A + 2\Delta$$

$$= (2m_\ast - x) \quad \text{for} \quad A + 2\Delta \leq x \leq 2m_\ast$$

where $m_\ast = A + \Delta$. The parameter $\Delta$ controls the width of the flat part of $f(x)$ and, in that sense, is a proxy for $\nu$. Note that the functions given in equations (35) and (36) have discontinuities in their derivatives which are but artefacts of our modelling. We will ignore such discontinuities because they may all be smoothened out as much as required. Then, since the function remains finite and all its derivatives may be smoothened to finite values, the resulting evolution will be non singular.
5. Solutions for a bi–anisotropic universe

Consider the solutions to equations (10) – (12) when \( f(x) \) is the simplified, piece-wise linear function given in equation (36). Isotropic solutions are straightforward to obtain and they are given in Appendix B. Consider the solutions for a bi-anisotropic universe where \( d = \tilde{n} + n \) and the quantities corresponding to the \( \tilde{n} \) and the \( n \) dimensional spaces are seperately isotropic as given in equation (26).

Equations (28) – (30) describe the evolution of such an universe. Let the equations of state be given by \( \tilde{\rho} = (1 - \tilde{u})\rho \) and \( p = (1 - u)\rho \). Then \( \tilde{\rho} = \tilde{u}\rho \) and \( r = v\rho \) where \( \tilde{v} \) and \( v \) are given by equations (32), and equation (34) gives \( \rho \) in terms of \( \tilde{\lambda} \) and \( \lambda \). If \( \tilde{m} \) lies in the interval \((0, A)\) and \( m \) in \((A + 2\Delta, 2m_\ast)\) or vice versa, then we cannot solve equations (28) and (29) analytically. Hence we assume that \( \Delta \gg A \) so that, generically, this possibility will not arise.

When \( \tilde{m} \) and \( m \) both lie in the interval \((0, A)\), the evolution will be as in general relativity for which the solutions are given in Appendix A. Let \( t_0 \) be an initial time and let the initial values \( \tilde{m}_0 \) and \( m_0 \) both lie in the interval \((0, A)\). Then the initial values \( \tilde{\lambda}_0 \) and \( \lambda_0 \) are both positive, see equations (14). Hence, going forward in time, \( \tilde{m} \propto \tilde{\lambda}_t \) and \( m \propto \lambda_t \) will decrease monotonically for \( t > t_0 \) and will vanish in the limit \( t \to \infty \).

Going back in time, \( \tilde{m} \) and \( m \) will increase monotonically for \( t < t_0 \). They will enter the interval \((A, A + 2\Delta)\) one after the other, evolve further, and exit from it into the interval \((A + 2\Delta, 2m_\ast)\). Let these entries and exits occur at times \((t_1, t_1, t_2, t_2)\). Taking \( t_0 > t_1 > t_1 > t_2 > t_2 \) for the sake of definiteness, we denote the monotonically increasing values of \( \tilde{m} \) and \( m \) at these times by

\[
(\tilde{m}_0, \tilde{m}_1, m_1, m_2), \quad (m_0, m_1, m_1, m_2) \tag{37}
\]

where

\[
0 < \tilde{m}_0 < A, \quad 0 < m_0 < A
\]

\[
\tilde{m}_1 = A, \quad m_1 < A
\]
\[ A < \tilde{m}_1 < A + 2\Delta, \quad m_1 = A \]

\[ \tilde{m}_2 = A + 2\Delta, \quad A < m_2 < A + 2\Delta \]

\[ \tilde{m}_2 > A + 2\Delta, \quad m_2 = A + 2\Delta. \] (38)

Also, let the values of \( \tilde{\lambda} \) and \( \lambda \) at the times \((t_0, t_1, t_2, t_2)\) be denoted by

\[ (\tilde{\lambda}_0, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3), \quad (\lambda_0, \lambda_1, \lambda_1, \lambda_2, \lambda_2). \] (39)

In expressions (38), the equalities define the times \((t_1, t_2, t_2, t_2)\) and the inequalities mean that, as one goes back in time from \( t_0 \), the field \( \tilde{m} \) first enters the interval \((A, A + 2\Delta)\) at \( \tilde{t}_1 \), then \( m \) enters it at \( t_1 \), then \( \tilde{m} \) first exits from it into the interval \((A + 2\Delta, 2m_e)\) at \( \tilde{t}_2 \), and then \( m \) does the same at \( t_2 \). We now analyse the solutions as \( t \) varies from \( \infty \) to \( t_0 \) to \( t_1 \) to \( t_2 \) to \( -\infty \).

\[ t > t_1 \]

The fields \( \tilde{m} \) and \( m \) both lie in the interval \((0, A)\) for \( t > t_1 \) and, hence, \( f(x) = x \). Therefore, their evolution during these times will be as in general relativity. The initial values of the fields given at \( t_0 > t_1 \) and the general relativity solutions given in Appendix A will determine all the fields for \( t > t_1 \). In particular, the values \( \lambda_1 \) and \( \lambda_1 \) in expressions (39) will follow from these solutions.

\[ t_1 > t > t_1 \]

During \( t_1 > t > t_1 \), the field \( \tilde{m} \) lies in the interval \((A, A + 2\Delta)\) and varies from \( \tilde{m}_1 = A \) to \( \tilde{m}_1 > A \) whereas \( m \) lies in the interval \((0, A)\) and varies from \( m_1 < A \) to \( m_1 = A \). Therefore, during this evolution, \( \tilde{f} = A \), \( \tilde{g} = \tilde{X} = 0 \), \( f = m \), \( g = 1 \), and \( X = (n - 1)f + \tilde{n}A \). Define \( y \), \( z \), and \( a \) by

\[ y = (n - 1)f + \tilde{n}A, \quad z = (m - m), \quad a = \sqrt{\frac{\tilde{n}(d - 1)}{n}}A. \] (40)

Then, after a straightforward algebra, it follows from equations (28) and (29) that

\[ \frac{\rho}{\rho_{qm}} = \frac{n}{n - 1}(y^2 - a^2) \] (41)
\[ y_t = -c_y (y^2 - a^2) \]  
\[ z_t + b y z = -c_z (y^2 - a^2) \]

where
\[ c_y = \frac{n \left( \frac{a^2}{d-1} - v \right)}{2 \gamma \lambda_{qm}} \],  
\[ c_z = \frac{n \left( v - \tilde{v} \right)}{2 (n-1) \gamma \lambda_{qm}} \],  
\[ b = \frac{n}{(d-1) \gamma \lambda_{qm}} \].

Since \( \tilde{X} = 0 \), it follows that \( \Lambda_t - \tilde{\lambda}_t = 0 \) and hence, from equations (32), (34), and (41), that
\[ (\lambda - \lambda_1) = - \left( \frac{\tilde{n} - 1}{n} \right) (\tilde{\lambda} - \tilde{\lambda}_1) \]
\[ e^{(2 - (d-1) v) (\tilde{\lambda}_1 - \tilde{\lambda})} = \frac{\rho}{\rho_1} = \frac{y^2 - a^2}{y_1^2 - a^2}. \]  
(44)

Defining \( t_\infty \) by \( y(t_\infty) = \infty \), the solution \( y(t) \) for the equation (42) may be written as
\[ \left( \frac{y - a}{y + a} \right) e^{2ac_y t} = \left( \frac{y_1 - a}{y_1 + a} \right) e^{2ac_y t_1} = \left( \frac{y_1 - a}{y_1 + a} \right) e^{2ac_y t_1} = e^{2ac_y t_\infty} \]  
(45)

where \( y \geq a > 0, \ y_1 = (d-1)A, \) and the last two equalities give \( t_1 \) and \( t_\infty \) in terms of \( A \) and the initial values \( t_1 \) and \( y_1 \). Thus, if \( c_y \) is positive then \( y_t < 0, \ t_\infty < t_1 \), and \( y \) increases monotonically from \( a \) to \( y_1 \) to \( \infty \) as \( t \) decreases from \( \infty \) to \( t_1 \) to \( t_1 \) to \( t_\infty \). Defining \( s = \frac{b}{2c_y} \) and solving for \( z \) in terms of \( y \), it is easy to see that the solution for \( z(y) \) is given by
\[ z = (y^2 - a^2)^s \left( \frac{z_1}{(y^2_1 - a^2)^s} + \frac{c_z}{c_y} \int_{y_1}^y \frac{dy}{(y^2 - a^2)^s} \right) \].  
(46)

\[ t_1 > t > t_2 \]

The fields \( \tilde{m} \) and \( m \) both lie in the interval \( (A, A+2\Delta) \) when \( t \) decreases from \( t_1 \) to \( t_2 \). It then follows that
\[ \tilde{f} = f = A, \quad \tilde{g} = g = \tilde{X} = X = 0. \]
Equations (30) then give
\[ \tilde{\lambda}_t = \lambda_t = 0 \implies \tilde{\lambda}_2 = \lambda_1, \quad \tilde{\lambda}_2 = \lambda_1. \]  
(47)

Equations (28) and (29) give
\[ \frac{\rho}{\rho_{qm}} = d(d - 1)A^2 \]  
(48)
\[ \tilde{m} - \tilde{m}_1 = c_{\tilde{m}} (t_1 - t) \]  
(49)
\[ m - m_1 = c_m (t_1 - t) \]  
where \( c_{\tilde{m}} = \frac{d(d-1)A^2}{2\gamma\lambda_{qm}} \left( \frac{2}{d-1} - \tilde{v} \right) \), \( c_m = \frac{d(d-1)A^2}{2\gamma\lambda_{qm}} \left( \frac{2}{d-1} - v \right) \), \( \tilde{m}_1 > A \), and \( m_1 = A \). We will assume that \( \tilde{v} \) and \( v \) are both \( < \frac{2}{d-1} \), hence \( c_{\tilde{m}} \) and \( c_m \) are both positive. There is no loss of generality here since this is Planckian regime and the constituents with lowest \( \tilde{u} \) and \( u \) will dominate. Also, for an M theory universe, \( \tilde{v} = 0 \) which is clearly \( < \frac{2}{d-1} \). Evolving as in equation (49), \( \tilde{m} \) and \( m \) will reach the value \( (A + 2\Delta) \) respectively at \( t_2 \) and \( t_2 \) given by
\[ t_1 - t_2 = \frac{A + 2\Delta - \tilde{m}_1}{c_{\tilde{m}}}, \quad t_1 - t_2 = \frac{2\Delta}{c_m}. \]  
(50)

If \( \tilde{v} = v \) then \( c_{\tilde{m}} = c_m \) and, since \( \tilde{m}_1 > A \), it follows that \( t_2 > t_2 \). If \( \Delta \) is large so that \( 2\Delta \gg \tilde{m}_1 - A \) then \( A + 2\Delta - \tilde{m}_1 \approx 2\Delta \) and
\[ \frac{t_1 - t_2}{t_1 - t_2} \approx \frac{c_m}{c_{\tilde{m}}} = \frac{2 - (d-1)v}{2 - (d-1)\tilde{v}}. \]  
(51)

Hence, it follows that \( t_2 > t_2 \) if \( v > \tilde{v} \) and that \( t_2 > t_2 \) if \( \tilde{v} > v \). Since we have assumed that \( t_2 > t_2 \), we must have \( v \geq \tilde{v} \).

\[ t_2 > t > t_2 \]

During \( t_2 > t > t_2 \), the field \( m \) lies in the interval \( (A, A + 2\Delta) \) and varies from \( m_2 < A + 2\Delta \) to \( m_2 = A + 2\Delta \) whereas \( \tilde{m} \) lies in the interval \( (A + 2\Delta, 2m_2) \) and varies from \( \tilde{m}_2 = A + 2\Delta \) to \( \tilde{m}_2 > A + 2\Delta \). The corresponding solutions are similar to the ones when \( t_1 > t > t_1 \). They are given in Appendix C.
The fields $\tilde{m}$ and $m$ both lie in the interval $(A+2\Delta, 2m_*)$ for $t < t_2$ and, hence, $f(x) = 2m_* - x$. Therefore, their evolution during these times will be as in general relativity. The values of the fields at $t_2$, equation (14), and the general relativity solutions given in Appendix A will determine all the fields for $t < t_2$.

6. Evolution of $e^{\tilde{\lambda}}$ during the Planckian regime

During the time interval $t_1 > t > t_2$, the value of at least one of the functions $\tilde{f}$ and $f$ remains maximum $= A$. Hence, the universe may be considered to be in the Planckian regime during this interval. Moreover, during the sub-interval $t_1 > t > t_2$, one has $\tilde{f} = f = A$ and $\tilde{\lambda}_t = \lambda_t = 0$, hence $\tilde{\lambda} = \tilde{\lambda}_1$ and $\lambda = \lambda_1$. Thus, the density $\rho$ remains maximum and the scale factors $e^{\tilde{\lambda}}$ and $e^\lambda$ remain constant during this Planckian subperiod. With no loss of generality, we take these constant values of the scale factors to be $O(1)$, namely take $e^{\tilde{\lambda}_1} \simeq e^{\lambda_1} \simeq O(1)$.

Going forward in time from the interval $t_1 > t > t_2$, the universe may be considered to be in the classical regime of general relativity for $t > t_\tilde{\lambda}$ when $\tilde{m}$ and $m$ both lie the interval $(0, A)$ and, hence, $f(x) = x$. We will focus on $\tilde{\lambda}$ which, in an M theory universe considered here, will reach a constant value in the limit $e^{A} \to \infty$ in future, causing the ten dimensional space to become effectively three dimensional in this limit. During the Planckian regime, as $t$ increases from $t_1$ to $t_\tilde{\lambda}$, the field $\tilde{\lambda}$ will evolve and, for the conditions assumed in equation (38), will increase from $\tilde{\lambda}_1$ to $\tilde{\lambda}_\tilde{\lambda}$ whose value may be obtained by setting $t = t_1$ and $m(t_1) = m_1 = A$ in equation (44). We have $y_1 = (d-1)A$ and $y_\tilde{\lambda} = (n-1)m_\tilde{\lambda} + nA$ where $m_\tilde{\lambda} < A$, see equation (38).

Hence, it follows from equations (40) and (41), or from equation (48), that $\rho(t_1) = \rho_1 = d(d-1)A^2 \rho_{qm}$, and then from equation (44) that

$$e^{(2-(d-1)v)(\tilde{\lambda}_\tilde{\lambda} - \tilde{\lambda}_1)} = \frac{\rho_1}{\rho_{\tilde{\lambda}}} = \frac{y_1^2 - a^2}{y_{\tilde{\lambda}}^2 - a^2}.$$  \hfill (52)

The value of the scale factor given above is the result of Planckian dynamics in our LQC – inspired model with the function $f(x)$ given as in equation...
In the bi-anisotropic case, the volume of the $\tilde{n}$ dimensional internal space at time $t_1$ is given by $V_1 = e^{\tilde{n} \tilde{\lambda}_1}$. The evolution for $t > t_1$ will be as in general relativity and the internal volume will grow to a constant value $V_\infty$ as $e^\Lambda \to \infty$, which will occur as $t \to \infty$. In general relativity evolution, with no fine tuning, $V_\infty \simeq V_1$ within a couple of orders of magnitude [23]. Hence, a larger value of $V_1$ will result in a larger value of $V_\infty$.

We will now estimate the value of the scale factor $e^{\tilde{\lambda}_1}$. First consider the factor $(2 - (d - 1)v)$. Let $\tilde{v} = 0$. It then follows from equation (33) that $v = \frac{1}{n-1}$. Note that, in the Planckian regime, the constituents with lowest $u$ will dominate. Hence, setting $u \simeq 0$ is natural. Setting $u \simeq \frac{2(n-1)}{d-1}$ can easily result in a large value for $e^{\lambda_1}$ but it may be unphysical in the Planckian regime, see below.

Now consider the ratio $\rho_1$ for an M theory universe where $\tilde{n} = 7$, $n = 3$, and $d = 10$. Note that $\tilde{f} = f_1 = m_1 = A$ and that

$$\rho_1 = d(d-1) A^2 \rho_{qm} = 90 A^2 \rho_{qm}.$$ 

It follows from equation (28) that, for $f_1 = m_1 \geq 0$, 

$$\rho_1 \geq \tilde{n} (\tilde{n} - 1) A^2 \rho_{qm} = 42 A^2 \rho_{qm}$$ 

whereas, even for $f_1 = m_1 \geq -A$, one only has 

$$\rho_1 \geq (\tilde{n} (\tilde{n} - 1) + n(n - 1) - 2n\tilde{n}) A^2 \rho_{qm} = 6 A^2 \rho_{qm}.$$ 

Thus, the ratio $\frac{\rho_1}{\rho_1} \leq 15$ and, hence, the scale factor $e^{\lambda_1}$ increases only by a factor of $O(1)$ even though the universe stays for a long time in the Planckian regime.

Although obtained using a simplified, piece-wise linear function, it may be that the above results are generic and indicate that the scale factors may be enhanced by only a factor of $O(1)$ during the Planckian regime in the LQC – inspired models. However, it is possible that there are other avenues which may yield larger enhancements. For example : (1) Setting $u \simeq \frac{2(n-1)}{d-1}$ in the Planckian regime instead of $u \simeq 0$ may be physically acceptable for some reason, of which we are currently unaware. In a sense, this would be analogous to an inflation. Note that when $n = d$, one has $u \simeq 2$ and hence $p = (1 - u) \rho \simeq -\rho$. Naively, one would have expected the early universe
to be dominated by radiation for which \( u = 1 - \frac{1}{d} \) or by matter for which \( u \) is even smaller but we now know that \( u \approx 2 \) is physically acceptable under inflationary conditions. (2) In the M theory universe considered here, we assumed that the densities \( \rho(*) \) are the same for all \(* = 2, 2', 5, 5'\) in order to obtain explicit solutions. Generically, however, these densities will be different. It is then possible that the total density may be Planckian, but the constituent densities may differ sufficiently which may lead to large values for internal scale factors. Hence a more systematic analysis is needed before concluding that internal scale factors may be enhanced by only a factor of \( \mathcal{O}(1) \) during the Planckian regime in the LQC – inspired models. Such an analysis, however, is beyond the scope of the present paper.

7. Conclusion

We now summarise the paper. We studied the evolution of an M theory universe in the LQC – inspired models. This universe is dominated by four stacks of intersecting brane–antibranes and, in general relativity, it becomes effectively four dimensional in future while its seven dimensional internal space reaches a constant size.

In the LQC – inspired models, we first analysed the conditions required for non singular evolutions. Then we obtained explicit solutions by considering a \((\tilde{n} + n)\) dimensional bi–anisotropic universe where the quantities corresponding to the \( \tilde{n} \) and the \( n \) dimensional spaces are separately isotropic, and by considering a simplified, piece–wise linear function for which the evolutions are non singular.

We applied these solutions to the M theory universe and considered the question of whether the physics in the non singular Planckian regime can enhance the future constant size of its seven dimensional internal space. Using the explicit solutions, we found no non trivial enhancement of this size. This may be a generic feature of the LQC – inspired models but it is also possible that there are other avenues which may yield larger enhancements.

We now conclude by mentioning a few topics for further studies. The LQC – inspired models involve a function, the choice of which leads to a variety of evolutions. It is desirable to understand the origin of this function and to explore the physical principles which may restrict it as uniquely as possible.
In string/M theory, effective higher derivative actions can be constructed systematically. It is worthwhile to explore whether the equations of motion resulting from these higher derivative actions bear any relation to the effective equations in LQC or to the equations in the LQC – inspired models. The M theory considered here becomes effectively four dimensional in future while its seven dimensional internal space reaches a constant size. Its evolution can be made non singular in the LQC – inspired models. In such a set up, one can now explore various mechanisms which may lead to large internal volumes which are of phenomenological interest [48, 49]. It will be equally interesting if one can prove instead, either in general or within the LQC – inspired models, that such large internal volumes are not possible.

Appendix A : Anisotropic solutions in general relativity

Consider the general relativity equations (3) – (5) for the anisotropic case. When the equations of state are linear, it is straightforward to solve these equations and obtain analytic solutions [23]. It follows from equations (14) that, upon replacing $\kappa^2$ by $c^2 \kappa^2$, these solutions are applicable to the LQC – inspired models when $f(x) = cx + c_0$.

We now present these solutions. First, define a new variable $\tau$ by

$$dt = e^\Lambda d\tau \quad \longleftrightarrow \quad t - t_0 = \int_{\tau_0}^{\tau} d\tau e^\Lambda$$

where $t_0$ and $\tau_0$ are initial times. Then, for any function $\psi(t(\tau))$, we have

$$\psi_\tau = e^\Lambda \psi_t \quad , \quad \psi_{\tau \tau} = e^{2\Lambda} (\psi_{tt} + \Lambda_t \psi_t)$$

Defining $(\hat{*}) = e^{2\Lambda} (\ast)$ for $(\ast) = (\rho, p_i, r^i)$, equations (3) – (5) become

$$\sum_{ij} G_{ij} \lambda^i_\tau \lambda^j_\tau = 2\kappa^2 \dot{\rho}$$

$$\lambda^i_{\tau \tau} = \kappa^2 \dot{r}^i$$

$$(\dot{\rho})_\tau = \sum_i (\dot{\rho} - \dot{p}_i) \lambda^i_\tau$$
Let the equations of state be linear and be given by

\[ p_i = (1 - u_i) \rho \]  

(57)

where \( u_i \) are constants. Define \( l, v^i, \) and \( G \) by

\[ l = \sum_i u_i \lambda^i, \quad v^i = \sum_j G^{ij} u_j, \quad G = \sum_i v^i u_i = \sum_{ij} G^{ij} u_i u_j \]  

(58)

and let the initial values of various quantities at \( t = t_0 \) be given by

\[(\lambda^i,  \lambda^i_0, \rho; \Lambda, l, l_t; \tau, \lambda^i_\tau, l_\tau, \hat{\rho})_{t=t_0} = \left(\lambda^i_0, k^i, \rho_0; \Lambda_0, l_0, l_{t_0}; \tau_0, \lambda^i_\tau_0, l_\tau_0, \hat{\rho}_0\right) \]  

(59)

where

\[ \rho_0 > 0, \quad \sum_{ij} G_{ij} k^i k^j = 2\kappa^2 \rho_0 \]

\[ \Lambda_0 = \sum_i \lambda^i_0, \quad l_0 = \sum_i u_i \lambda^i_0, \quad l_{t_0} = \sum_i u_i k^i \]

\[ \lambda^i_\tau_0 = e^{\Lambda_0} k^i, \quad l_\tau_0 = e^{\Lambda_0} l_{t_0}, \quad \hat{\rho}_0 = e^{2\Lambda_0} \rho_0. \]  

(60)

Then equations (55) and (56) give

\[ \lambda^i_{\tau\tau} = \kappa^2 v^i \hat{\rho} \]  

(61)

\[ l_{\tau\tau} = \kappa^2 G \hat{\rho} \]  

(62)

\[ \hat{\rho} = \hat{\rho}_0 e^{l-l_0} \]  

(63)

and it follows from equations (61) and (62) that

\[ \lambda^i - \lambda^i_0 = \frac{v^i}{G} (l - l_0) + \sum_i u_i L^i (\tau - \tau_0). \]  

(64)

Since \( l = \sum_i u_i \lambda^i \), it follows that the integration constants \( L^i \) must satisfy the constraint \( \sum_i u_i L^i = 0 \). This constraint is identically satisfied if \( L^i = \)
\[ e^{\Lambda_0} \left( k^i - \frac{u^i}{G} \right) \] where \( l_{t0} = \sum_i u_i k^i \), see equations (60). Thus, the set of \( d \) number of initial values \( \{k^i\} \) is equivalent to the set of \( (1 + d) \) number of initial values \( \{l_{t0}, L^i\} \) together with one constraints on \( L^i \). Upon using \( \sum_i u_i L^i = 0 \), equation (54) gives

\[
(l_{\tau})^2 = 2 G \left( E + \kappa^2 \hat{\rho} \right), \quad 2 E = - \sum_{ij} G_{ij} L^i L^j .
\]

Now, in principle, equations (62), (63), and (65) give \( l(\tau) \) and equations (64) and (53) give \( \lambda^i(\tau) \) and \( t(\tau) \) from which \( \tau(t), l(t), \) and \( \lambda^i(t) \) follow. Also, it can be shown that if \( \sum_i u_i L^i = 0 \) and \( G = \sum_{ij} G_{ij} u_i u_j > 0 \) then \( E \geq 0 \) and \( E \) will vanish if and only if all \( L^i \) vanish [23]. Henceforth, we assume that \( G > 0 \) and \( E > 0 \).

For the case of bi–anisotropic universe considered in this paper, see equations (26) and (27), it follows straightforwardly that

\[
\tilde{p} = (1 - \tilde{u}) \rho, \quad p = (1 - u) \rho
\]

\[
\tilde{v} = \frac{nu - (n - 1)\tilde{u}}{\tilde{n} + n - 1}, \quad v = \frac{\tilde{n}\tilde{u} - (\tilde{n} - 1)u}{\tilde{n} + n - 1}
\]

\[
l = \tilde{n}\tilde{u}\tilde{\lambda} + nu\lambda, \quad G = \tilde{n}\tilde{u}\tilde{v} + nuv
\]

\[
\tilde{n}\tilde{u}\tilde{L} + nuL = 0, \quad 2E = \frac{n(n + \tilde{n} - 1)GL^2}{\tilde{n}\tilde{u}^2}
\]

where the expression for \( E \) follows after some algebra. If \( \tilde{v} = 0 \), which is necessary for the \((\tilde{n} + n)\) dimensional space to become effectively \( n \) dimensional in the limit \( e^{\Lambda} \to \infty \), then one has

\[
\tilde{u} = \frac{nu}{n - 1}, \quad v = \frac{u}{n - 1}
\]

\[
l = \tilde{u} \left( \tilde{n}\tilde{\lambda} + (n - 1)\lambda \right), \quad G = \frac{nu^2}{n - 1}
\]

\[
\tilde{n}\tilde{L} + (n - 1)L = 0, \quad 2E = \frac{(n - 1)(n + \tilde{n} - 1)L^2}{\tilde{n}}.
\]
Consider now the solution \( l(\tau) \) for equations (62), (63), and (65). As can be verified easily, it is given by

\[ \kappa^2 \hat{\rho} = \kappa^2 \hat{\rho}_0 e^{l-l_0} = \frac{E}{\sinh^2 \sigma (\tau_\infty - \tau)} \]  

(68)

where \( 2\sigma^2 = GE \). Note that the sign of \( \sigma \) is immaterial; that

\[ l_\tau = 2 \sigma \coth \sigma (\tau_\infty - \tau) ; \]  

(69)

and that setting \( l = l_0 \) and \( \tau = \tau_0 \) in equation (68) gives \( \tau_\infty \) in terms of \( E \) and \( \hat{\rho}_0 \). Equations (64) and (53) will now give \( \lambda^i(\tau) \) and \( t(\tau) \) from which \( \tau(\tau) \), \( l(\tau) \), and \( \lambda^i(t) \) follow. Taking \( \sigma > 0 \) and \( l_\tau > 0 \) for the sake of definiteness, we now mention some features of these solutions.

- Since \( l_\tau > 0 \), it follows from equation (69) that \( \tau_\infty > \tau_0 \). It follows from equation (68) that \( l(\tau) \) varies monotonically between \( -\infty \) and \( +\infty \), that \( l \to -\infty \) as \( \tau \to -\infty \), and that \( l \to \infty \) as \( \tau \to \tau_\infty \).

- In the limit \( \tau \to \tau_\infty \) from below, one has \( l(\tau) \sim -2 \ln (\tau_\infty - \tau) \to \infty \). Equations (64) and (53) then give, up to unimportant constants,

\[ t \sim (\tau_\infty - \tau)^{-\frac{2B-G}{\nu}} , \quad B = \sum_j v^j \]  

(70)

\[ e^{\lambda^i} \sim (\tau_\infty - \tau)^{-\frac{2B}{\nu}} \sim t^{\frac{2B}{2B-G}} \]  

(71)

\[ e^{\Lambda} \sim (\tau_\infty - \tau)^{-\frac{2B}{\nu}} \sim t^{\frac{2B}{2B-G}} . \]  

(72)

For the bi–anisotropic universe, it follows from equations (66) that

\[ 2B - G = \bar{\nu} \bar{v} (2 - \bar{u}) + n v (2 - u) . \]  

(73)

Hence, for \( \bar{v} = 0 \), one has \( e^{\lambda} \sim \text{const} \) and \( e^\Lambda \sim t^{\frac{2}{\nu(2-u)}} \) which is the standard \( n \) dimensional result.

- In the limit \( \tau \to -\infty \), one has \( l(\tau) \sim 2 \sigma \tau \to -\infty \). Equations (64) then imply that \( \lambda^i(\tau) \) are all linear in \( \tau \). Let \( \tau \to -\infty \) and \( e^\Lambda \to 0 \) in this limit and, up to unimportant constants, let

\[ \lambda^i \sim q^i \tau , \quad \Lambda \sim q \tau , \quad q = \sum_i q^i > 0 . \]
Then, after some algebra, it follows from equation (53) that

\[ e^\Lambda \sim e^{\theta \tau} \sim q t \to 0 \quad , \quad e^{\lambda^i} \sim e^{q^i \tau} \sim (q t)^{q^i} \] (74)

which are the Kasner–type solutions.

**Appendix B : Isotropic solutions in LQC – inspired models**

Consider the fully isotropic case where

\[ (m^i, f^i, g_i, X_i, \lambda^i, p_i, r^i) = (m, f, g, X, \lambda, p, r) \]

for \( i = 1, 2, \ldots, d \). Then

\[ g = \frac{df}{dm}, \quad X = (d - 1) gf, \quad r = \frac{\rho - p}{d - 1} \]

and equations (10) – (12) give

\[ f^2 = \frac{2 \gamma^2 \lambda^2 q^2 \rho}{d (d - 1)} \] (75)

\[ m_t = -\frac{\gamma \lambda q^2}{d - 1} (\rho + p) \] (76)

\[ \lambda_t = \frac{g f}{\gamma \lambda q^m} \implies \quad (\lambda_t)^2 = \frac{2 \kappa^2 (\rho g^2)}{d(d - 1)}. \] (77)

Let the equation of state be linear and be given by \( p = (1 - u) \rho \) where \( u < 2 \) is a constant. Then equations (5) and (75) – (77) may be solved explicitly if certain integrations and functional inversions can be performed. Equations (5) and (75) give

\[ \frac{\rho}{\rho_0} = \frac{f^2}{f_0^2} = e^{-(2-u) d (\lambda - \lambda_0)} \] (78)

which leads to \( \lambda(m) \). Equations (75) and (76) then lead to \( t(m) \) given by

\[ c_{qm} (t - t_0) = - \int_{m_0}^{m} \frac{dm}{f^2} \] (79)
where \( c_{qm} = \frac{(2-u)d}{2\gamma q_{wm}} \). Inverting \( t(m) \) then gives \( m(t) \) and \( \lambda(t) \). The integrations and functional inversions required here can be performed explicitly for \( f(x) = cx + c_0 \) and also for \( f(x) = \sin x \) but not for a generic \( f(x) \). The resulting solutions are given in [45, 46].

Consider now the isotropic solutions for the simplified, piece-wise linear function \( f(x) \) given in equation (36). Equation (78) gives the density \( \rho(m) \) and the scale factor \( e^{\lambda(m)} \). Let the initial value \( m_0 \) at time \( t_0 \) lie in the range \( 0 < m_0 < A \). It then follows that as \( m \) increases from 0 to \( m_0 \) to \( A \) to \( A + 2\Delta \) to \( 2m_0 \), the function \( f \) increases from 0 to \( m_0 \) to \( A \), remaining at \( A \), and then decreasing to 0. Hence, correspondingly, the scale factor \( e^{\lambda(m)} \) decreases from \( \infty \) to \( e^{\lambda_0} \), decreases further, then remains constant, and then increases again to \( \infty \).

The time \( t(m) \) follows straightforwardly upon performing the integration in equation (79), and is given by

\[
c_{qm} (t - t_0) = -\frac{1}{m_0} + \frac{1}{m} \quad \text{for} \quad 0 \leq m \leq A \\
= \frac{2}{A} - \frac{1}{m_0} - \frac{m}{A^2} \quad \text{for} \quad A \leq m \leq A + 2\Delta \\
= \frac{2}{A} - \frac{1}{m_0} - \frac{2\Delta}{A^2} + \frac{1}{m - 2m_\ast} \quad \text{for} \quad A + 2\Delta \leq m \leq 2m_\ast.
\]

(80)

Hence, as \( m \) increases from 0 to \( m_0 \) to \( A \) to \( A + 2\Delta \) to \( 2m_0 \), the time \( t \) decreases monotonically from \( \infty \) to \( t_0 \) to \( -\infty \), first as \( \frac{1}{m} \), then linearly as \( -\frac{m}{A^2} \), and then as \( \frac{1}{m - 2m_\ast} \).

**Appendix C : Bi–anisotropic solutions**

when only \( m \in (A, A + 2\Delta) \)

During \( t_b > t > t_c \), let \( m(t) \) lie in the interval \((A, A + 2\Delta)\) and let \( \tilde{m}(t) \) lie in \((0, A)\) or in \((A + 2\Delta, 2m_\ast)\). Then \( f = A, \ g = X = 0, \ \tilde{f} = c\tilde{m} + c_0 \) where \((c, c_0) = (1, 0)\) or \((-1, 2m_\ast)\), \( \tilde{g} = c \), and \( \tilde{X} = c\left(\tilde{m} - 1\right)\tilde{f} + nA \).
The times $t_b$ and $t_e$ are defined by the equalities in the following expressions for the values of $\tilde{m}$ and $m$ at $t_b$ and $t_e$:

$$\tilde{m}_b < A \ , \ m_b = A$$

$$\tilde{m}_e = A \ , \ A < m_e < A + 2\Delta$$

or

$$\tilde{m}_b = A + 2\Delta \ , \ A < m_b < A + 2\Delta$$

$$\tilde{m}_e > A + 2\Delta \ , \ m_e = A + 2\Delta . \quad (81)$$

Several equations are different if $\tilde{n} > 1$ or $\tilde{n} = 1$. Hence we consider these two cases separately.

$$\tilde{n} > 1$$

Define $y$, $z$, and $a$ by

$$y = (\tilde{n} - 1) \bar{f} + nA , \ z = (m - \tilde{m}) , \ a = \sqrt{\frac{n (d - 1)}{\tilde{n}}} A . \quad (82)$$

Note that $\tilde{X} = cy$ and that $nA < y < (d - 1)A$. After some algebra, it follows from equations (28) and (29) that

$$\frac{\rho}{\rhoqm} = \frac{\tilde{n}}{\tilde{n} - 1} \left( y^2 - a^2 \right) \quad (83)$$

$$y_t = -c_y \left( y^2 - a^2 \right) \quad (84)$$

$$z_t + b \ y \ z = -c_z \left( y^2 - a^2 \right) \quad (85)$$

where

$$c_y = \frac{\tilde{n} \ c \left( \frac{2}{d - 1} - \bar{v} \right)}{2 \ \gamma \lambda_{qm}} , \ c_z = \frac{\tilde{n} \ (\bar{v} - v)}{2 (\tilde{n} - 1) \ \gamma \lambda_{qm}} , \ b = \frac{\tilde{n} \ c}{(d - 1) \ \gamma \lambda_{qm}} .$$

The solutions $y(t)$ and $z(y)$ are given in equations (45) and (46). Since $X = 0$, it follows that $\Lambda_t - \lambda_t = 0$ and hence, from equations (32), (34), and
(83), that

\[
(\bar{\lambda} - \lambda_0) = - \left( \frac{n - 1}{\tilde{n}} \right) (\lambda - \lambda_0)
\]

\[
e^{2 - (d-1) \bar{v}} (\lambda_0 - \lambda) = \frac{\rho}{\rho_b} = \frac{y^2 - a^2}{y_b^2 - a^2} .
\] (86)

\[
\tilde{n} = 1
\]

Now \( d = n + 1 \). Define \( y \) and \( z \) by

\[
y = 2 \tilde{f} + (n - 1) A \quad , \quad z = (m - \tilde{m}) .
\] (87)

Note that \( \tilde{X} = ncA \) and that \((n-1)A < y < (n+1)A\). After some algebra, it follows from equations (28) – (29) that

\[
\frac{\rho}{\rho_{qm}} = nA y
\] (88)

\[
y_t = 2c \gamma \lambda_{qm} \kappa^2 \left( \tilde{v} - \frac{2}{d - 1} \right) \rho
\] (89)

\[
z_t + \frac{nc A z}{(d - 1) \gamma \lambda_{qm}} = \gamma \lambda_{qm} \kappa^2 (v - \tilde{v}) \rho .
\] (90)

Equations (88) – (90) lead to the solutions \( y(t) \) and \( z(y) \) given by

\[
y = y_b e^{-\frac{nc A}{\gamma \lambda_{qm}} \left( \frac{2}{d - 1} \tilde{v} \right) (t-t_b)}
\] (91)

and

\[
z = y^s \left( \frac{z_b}{y_b^{s}} + \sigma \int_{y_b}^{y} \frac{dy}{y^s} \right).
\] (92)

where \( s = \frac{1}{2 - (d-1) \bar{v}} \) and \( \sigma = \frac{(d-1) (\bar{v} - \bar{v}_0)}{2c (2 - (d-1) \bar{v})} \). Thus, if \( \bar{v} < \frac{2}{d-1} \) then \( y_t < 0 \) and \( y \) increases monotonically from \( y_b \) to \( \infty \) as \( t \) decreases from \( t_b \) to \( -\infty \). Also, equations (86) give \( \bar{\lambda} \) and \( \lambda \) in terms of \( y \).
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