Bilinear observer-based robust adaptive fault estimation for multizone building VAV terminal units

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ABSTRACT
In this paper, we propose a novel bilinear observer-based robust fault detection, isolation and adaptive fault estimation methodology to precisely estimate a class of actuator faults, namely bias in damper position and lock-in-place faults, in Variable-Air-Volume (VAV) terminal units of Heating Ventilation and Air-Conditioning (HVAC) systems. The proposed adaptive fault estimator is robust in the sense that the fault estimates are not affected by the unmeasured disturbance variable and that the effects of measurement noises on fault estimates are attenuated. The fault estimation algorithm with the integrated building control system improves occupants comfort and reduces the operation, maintenance, and utility cost, thereby reducing the impact on the environment. The effectiveness of the methodology for adaptive estimation of multiple or single VAV damper faults is successfully demonstrated through different simulation scenarios with SIMBAD (SIMulator of Building And Devices), which is being used in industries for testing and validation of building energy management systems.

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1. Introduction

Heating Ventilation and Air-Conditioning (HVAC) systems employing Variable Air Volume (VAV) terminal boxes offer an energy-efficient solution to maintain the thermal comfort of occupants inside the building. A VAV box is an end-user terminal unit of the zone, which actuates the volume amount of processed air inside each zone. In order to maintain the desired thermal comfort inside the zone, a temperature controller in the outer loop generates reference signals to these boxes equipped with electromechanical dampers and internal flow controllers (Yamé, Jain, and Sauter 2015). Several model-based linear (Coffey 2013) and nonlinear (Castilla et al. 2014), and data-driven control (Qin, Lysecky, and Sprinkle 2014) techniques are designed for HVAC building systems to ensure the thermal comfort of occupants. However, during the service life of HVAC-VAV units, the sensors and dampers may develop some failures, namely lock-in-place and bias failures. Typically, the detection and identification of the above anomalies involve subunit-by-subunit inspection in a building automation system (BAS) or building energy management system (BEMS), which is challenging to execute frequently. The human operator may not notice this problem when the system is operated in the closed-loop configuration due to a passive control scheme tailored to such faults, where the prime objective is to ensure the tracking of setpoints possibly at the expense of consuming higher energy (O’Neill et al. 2014). As a result, the energy-efficient operation is severely affected under the event of damper lock-in-place and bias faults in VAV boxes. To remove the above manpower and maintenance cost-intensive manual tasks, automated fault detection and diagnosis (FDD) methodologies should be integrated within the building HVAC-VAV systems. This integrated system enables the maintenance crews to identify and predict likely occurring failure modes and to conduct fault impact analysis, determine their criticality on the BAS and be directed in the quick repair process ensuring the system availability. An experimental study on the energy cost impacts for a range of common system faults in various components of the HVAC-VAV systems is conducted in Lee and Yik (2010).

In the literature, several model-based, data-driven (Jacob et al. 2010), and grey-box model-based FDD schemes (Geoffroy et al. 2022) for HVAC-VAV units have been developed (Gunay, Shen, and Yang 2017). The data-driven or machine learning methods for automated FDD of sensor and actuator faults are developed using neural...
networks (Montazeri and Kargar 2020), Kalman filters (Sun et al. 2014), expert knowledge (Li et al. 2018), and support vector machines (SVM) (Namburu et al. 2007). Using linear state-observers, model-based FDD methods for detecting and isolating actuator faults have also been developed (Glass et al. 1995). Thumati et al. (2011) presented model-based fault detection and isolation scheme for HVAC-VAV systems for diagnosing degradation faults in the cooling and thermal insulation coils. A linear causal model-based framework for the detection and isolation of sensor and actuator multiple faults in HVAC-VAV systems, namely stuck dampers, stuck valves, and biased sensor measurements, is presented in Shahnazari et al. (2018). Yoshida, Kumar, and Morita (2001) presented an online FDD in the VAV air handling unit to diagnose sensor errors arising out of faulty bias during long-term use. The method is based on the dynamic and recursive model building of a VAV control subsystem from the measured data, which ensured that robustness against sensor error is self-attained. Darure, Yamé, and Hamelin (2016) presented a fault estimation algorithm, for damper sticks in VAV boxes where a dedicated bank of unknown input residual generators was employed for fault detection and isolation (FDI). In the above data-driven and model-based FDD schemes, the robustness of the fault diagnosis methods is not studied. The robustness problem of an FDD method deals with the accuracy of detection, isolation, and estimation of the magnitude of faults under several types of uncertainties, such as modelling errors, sensor measurement noise, transients due to switching of operating mode and unknown disturbances. Wang et al. (2011) presented a robust FDD strategy for the isolation of faults in pressure-independent VAV dampers using a hybrid approach based on cumulative sum control charts and a rule-based fault classifier. The hybrid approach is robust to false alarms caused by impacts of normal transient changes within the system. Bonvini et al. (2014) presented a robust FDD algorithm based on Unscented Kalman Filter techniques for detecting common chiller plant faults in an HVAC system, which is robust to sensor errors and data paucity. Mulumba et al. (2015) proposed a robust model-based fault diagnosis method based on SVM techniques for 11 different faults in the AHU categorized under the broad class of stuck and leakages faults in various valves, dampers, and duct respectively, which is robust to model parameter uncertainty. The effectiveness of another hybrid approach based on the SVM technique for fault detection and isolation in a single zone HVAC system is demonstrated through numerical simulation in Liang and Du (2007), where three major faults, including the recirculation damper stuck, cooling coil fouling/block and supply fan speed decreasing, are investigated. Zhao et al. (2019) presented a comprehensive survey on artificial intelligence-based FDD methods concluding that the effectiveness of data-driven and knowledge-based schemes rely heavily on training data and expert knowledge respectively, without which robustness cannot be achieved.

With regards to the vast development of fault diagnosis approaches for HVAC-VAV-based buildings, many of them are built upon linear and data-driven statistical models of the building thermal dynamics. Since a fault diagnosis module is an integral part of a control scheme, addressing the aftereffects of faults, statistical or purely data-based techniques may not be useful in implementing an active fault-tolerant control (AFTC) scheme (Jain, Yamé, and Sauter 2018). With the dynamically changing zone occupancy profile and outside weather conditions, the linearized models capturing the zone thermal dynamics need to be updated at regular time intervals in linear model-based approaches. To address this issue, an observer-based fault diagnosis approach is developed in Subramaniam, Jain, and Yamé (2020) by dealing directly with the physics-informed thermal dynamics which turns out to be a bilinear control system (Elliot 2009) derived from the very behaviour of VAV actuators. The fault diagnosis problem for control affine nonlinear process systems has also been addressed in Shahnazari and Mhaskar (2018) and in the presence of uncertainty in Du, Scott, and Mhaskar (2013). However, in these works, only the fault detection and isolation modules are developed using high-gain observers for at most two simultaneously occurring faults under the absence of measurement noise. The robustness issues with regards to unmeasurable disturbances and measurement noise remain an open problem. In the literature, several control schemes are reported assuming the availability of either both the disturbances (Oldewurtel et al. 2012) or one of them while estimating the other (Kim et al. 2016). In Dong and Lam (2014), moving average models are proposed to estimate both the disturbances in real-time for developing the model-predictive control (MPC) scheme. In Dobbs and Henchy (2014), the measurement of outside weather conditions is assumed to be known, while the information of occupancy and subsequently, the internal heat gain is estimated online in the MPC scheme. A practical implementation of the MPC scheme assuming the availability of both disturbances is developed in Bengoa et al. (2014) and Goyal, Barooah, and Middelkoop (2015). In Coffman and Barooah (2018), a simultaneous identification method is proposed to identify both linear thermal model (of resistance-capacitance network type) and the internal load from measured input-output data assuming that the latter is constant and therefore modelled.
as a process with a zero derivative. While the problem of estimating time-varying occupancy and ambient air flow signals is addressed using the noisy carbon dioxide and flow sensor measurements in Zavala (2014). In Subramaniam, Jain, and Yamé (2020), the information of both disturbances was assumed to be known in the solution of the fault estimation problem for HVAC building systems. For its practical implementation, this may additionally require high computational modules to precisely estimate these disturbances in real-time (Lazos, Sproul, and Kay 2014). To address this issue, the solution to the fault estimation problem should be robust with regards to these disturbances. It is well known in the literature that such robust solutions are constrained by some matrix existence conditions, which may not be satisfied in some scenarios (Gao, Liu, and Chen 2016). This work takes further steps towards a realistic approach by assuming that the disturbance is only known partially.

The main contributions of this paper are:

- a detailed design of a novel fault diagnosis methodology integrated with robust adaptive estimation of faults, which is inherited from the physics-informed nonlinear (bilinear) dynamics of controlled thermal zones by VAV terminal units is derived, thereby making the methodology applicable to a broad operating range of the system;
- the robustness of the algorithm with regards to unmeasured disturbances and noisy sensor measurements is rigorously established such that the fault estimates are insensitive to unmeasured disturbances, and the effect of measurement noise on these estimates is attenuated;
- a class of actuator faults, namely lock-in-place faults and bias faults in VAV dampers, is considered in this paper;
- a simulation study of a one-storey building with three-zones is presented within the Matlab-based SIMBAD (SIMulator of Building And Devices) software environment to demonstrate the effectiveness of the derived algorithms, where two scenarios consisting of four different single and multiple VAV damper fault cases are considered.

The paper is organized as follows: The system description, fault modelling, and the problem statement are given in Section 2. In Section 3, the robust adaptive fault estimation algorithm based on a nonlinear observer is derived. The two different simulation scenarios composed of four different cases of single and multiple faults occurring in HVAC-VAV dampers of a one-storey three-zone building are presented in Section 4. Finally, in Section 5, some concluding remarks are provided.

2. System description and problem formulation

In this section, firstly, we briefly present the bilinear model of thermal dynamics in a multizone building. Subsequently, two types of VAV damper faults are modelled, followed by the problem statement addressed in this paper.

2.1. Thermal dynamics of a multizone building

The air conditioning technical plan of a typical HVAC-VAV system in a multizone building with its traditional control loops is sketched in Figure 1. Each zone of the building is actuated by a VAV unit receiving the processed air from the central air-handling unit (AHU) at a constant supply air temperature, $T_s$, whose setpoint is reset based on outdoor air temperature (Montgomery and McDowall 2008, Chapter 9). The AHU being the centralized subunit, includes a mixer, cooling/heating coils, and a variable-speed supply fan. The air is processed within the AHU subunit through the mixer damper, mixing the outside air with the return air from all the zones, and cooling/heating coils, controlling the supply airflow temperature, which is separately controlled by conventional/non-conventional energy sources. Each VAV box is equipped with a controlled damper that regulates the volume of hot/cold primary air depending on the outside weather conditions to be delivered inside the zones according to thermal requirements. Applying the first law of thermodynamics to each zone, the temperature dynamics of the i-th zone can be obtained, which is directly proportional to the total heat transfer rate from different sources in that zone, namely the HVAC system, internal heat gain which is the cumulative heat flux due to occupants and electronic devices in zone $i$, convective heat transfer rate due to natural ventilation, outside temperature, external and internal wall (or shared wall with adjacent zones) temperatures. Under the standard modelling assumptions, i.e.

(1) the natural ventilation is negligible;
(2) the thermal load generated by lights and other electronic equipment depends on the occupancy profile;
(3) the blinds are closed, and the solar heat gain in the i-th zone contributed from the k-th external wall is negligible;
(4) the zone temperature is obtained from well-mixed air of the i-th zone, the temperature profile of the walls is uniformly distributed across their thickness;

the following bilinear state-space model describes the thermal dynamics of multizone buildings

$$
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} = 
\begin{bmatrix}
A & B & E \\
C & 0_{n \times n} & 0_{n \times (n+1)}
\end{bmatrix}
\begin{bmatrix}
x^T \\
u^T \\
d^T
\end{bmatrix}
$$
where \( \otimes \) denotes the Kronecker product, \( 0 \) denotes the zero matrix of appropriate dimension, \( x, u, y, \) and \( d \) denote the state, input, output and the disturbance variable given as

\[
x(t) = \begin{bmatrix} T_z \end{bmatrix}, \quad y(t) = T_z,
\]

\[
u(t) = \begin{bmatrix} m_1 \cdots m_n \end{bmatrix}, \quad d(t) = \begin{bmatrix} Q_{int} \mid T_{oa} \end{bmatrix},
\]

\[
T_z = \begin{bmatrix} T_{z1} \cdots T_{zn} \end{bmatrix}, \quad T_w = \begin{bmatrix} T_{intw} \mid T_{intw} \end{bmatrix},
\]

\[
Q_{int} = \begin{bmatrix} Q_{int,1} \cdots Q_{int,n} \end{bmatrix},
\]

\[
T_{extw} = \begin{bmatrix} T_{extw,1} \cdots T_{extw,p} \end{bmatrix},
\]

\[
T_{intw} = \begin{bmatrix} T_{intw,12} \cdots T_{intw,n} \end{bmatrix}
\]

for \( i, j = 1, 2, \ldots, n \), \( i \neq j \), model parameters \( D, A, B, C, E \) are matrices of compatible dimensions, and \( p_1 \) is the number of external walls. These matrices are composed of thermodynamical parameters, which can be modelled as electrical quantities in terms of capacitances and thermal resistances of the zones and that of the external and internal walls (Subramaniam, Jain, and Yamé 2020).

The description of the variables introduced in (2)–(3) is provided in Table 1. Let \( p = p_1 + p_2 \) be the total number of walls, with \( T_{extw} \in \mathbb{R}^{p_1} \) and \( T_{intw} \in \mathbb{R}^{p_2} \), then \( x \in \mathbb{R}^{n+p}, u \in \mathbb{R}^n, y \in \mathbb{R}^n, \) and \( d \in \mathbb{R}^{n+1} \). The output matrix \( C = [I_n \; \mathbf{0}_{n \times p}] \), where \( I_n \) is the \( n \)-dimensional identity matrix, indicates that the zone temperatures are measured directly but the wall temperatures are not measured. The bilinear term is expressed as

\[
D(u \otimes x) = \sum_{i=1}^{n} D_i \hat{m}_i(t) x(t) = D_u(t)x
\]

with \( D = \begin{bmatrix} D_1 & \cdots & D_n \end{bmatrix} \), where \( D_i \)'s are square matrices of order \((n + p)\) and \( D_u(t) \) is a time-varying matrix of appropriate dimension. Typically, sensors are deployed to measure the zone temperatures, which inherently induce noise in the measured output signals. The sensor measurement noise can be modelled by a uniformly distributed additive noise signal denoted by \( \eta(t) \). Then, the governing overall state-space model is represented by

\[
\begin{bmatrix} \dot{x} \\ z \end{bmatrix} = \begin{bmatrix} A & B \\ C & E \end{bmatrix} \begin{bmatrix} x^T \\ u^T \end{bmatrix} + \begin{bmatrix} \eta(t) \end{bmatrix}
\]

**Figure 1.** Technical layout of the HVAC-VAV system in a multizone building and its traditional cascaded control strategy.

**Table 1.** Building thermal variables and parameters.

| Variables | Description |
|-----------|-------------|
| \( T_p \) | Temperature of the \( i \)-th zone |
| \( T_z \) | Zone temperature vector |
| \( A, B, C, D, E \) | Building thermal parameters |
| \( F(t) \) | VAV damper fault distribution matrix |
| \( m_i \) | Airflow rate setpoints to the \( i \)-th zone |
| \( Q_{int,i} \) | Internal heat gain of the \( i \)-th zone |
| \( Q_{int} \) | Internal gain vector |
| \( T_{oa} \) | Outside ambient temperature |
| \( T_{extw,i} \) | Wall temperature of the \( i \)-th zone |
| \( T_{intw,i} \) | Wall temperature of the internal wall between the \( i \)-th and \( j \)-th zone |
| \( T_{extw} \) | External wall temperature vector |
| \( T_{intw} \) | Internal wall temperature vector |
| \( T_s \) | Supply air temperature |
| \( \eta \) | Measurement noise signal vector |
| \( e_i \) | Fault in the \( i \)-th zone |
| \( \epsilon_i \) | Fault vector containing faults of all zones |
| \( u_i \) | Actual airflow rate to the zones |
| \( L_i, R \) | Fault estimator design matrices |
| \( k_Q \) | Constant coefficient parameters of the \( i \)-th zone building thermal dynamics |
| \( \phi(t) \) | Impulse response of the causal filter |
| \( \epsilon_i \) | \( i \)-th column vector of the standard basis of \( \mathbb{R}^n \) |
where signal $z(t)$ is the measured output, i.e., the zone’s measured temperatures vector.

### 2.2. Fault modelling in VAV terminal units

To ensure the energy-efficiency of the building HVAC system while satisfying the thermal comfort requirement within the zones, the fault-free operation of VAV dampers plays a significant role because based on the thermal load, these dampers adjust the volume flow rate of processed air being dispatched into the zone. The outer-loop temperature controller ensures that the temperature of each zone is regulated to its setpoint by providing the variable airflow rate setpoint to the inner-loop flow controller. Whenever the VAV damper of the $i$-th zone becomes faulty, the energy-efficient operation of the HVAC system and the thermal comfort of the occupants of the building are drastically hampered. Two types of frequently occurring VAV damper faults, namely a constant bias in the damper position and damper lock-in-place failure, are considered in this work. A lock-in-place failure occurs when the VAV damper is stuck anywhere between the operating range of the VAV damper given by $[0, 1]$, where 0 represents a damper fully closed, and 1 when it is fully open. A bias fault refers to the case when the commanded position by the controller is offset by a bias in the actual damper position. Let $f_a(t)$ denote the actuator fault signal.

- **Lock-in-place of VAV dampers:**
  Under the event of this fault occurring at time $t_f$, the actual input signal to the plant (5) becomes the faulty signal, $u_f(t)$, modelled compactly as
  \[
  u_f(t) = [1_2 \otimes I_n] [u^T(t) \mid f_a^T(t)]^T
  \]
  with $u_f(t)$ being a constant signal for all $t \geq t_f$, and where $1_2$ is the 2-dimensional vector of entries equal to 1.

- **Bias in VAV damper position:**
  Here, the fault can also be modelled as given in (6), but $u_f(t)$ may not be a constant signal with $f_a(t)$ denoting a constant bias (Jain, Yamé, and Sauter 2018).

Substituting any of the above fault types in (5) yields the following state-space model of the faulty system as

\[
\begin{bmatrix}
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
A & B & E & 0_n & F(t)
\end{bmatrix}
\begin{bmatrix}
x
u^T
u^T
f_a^T
\eta^T
\end{bmatrix}
+ \begin{bmatrix}
D(u \otimes x) & 0
\end{bmatrix}
\]

where $f_a = [f_1 \cdots f_n]^T$ denotes the fault vector and $F(t)$ a time-varying fault distribution matrix. Let $\kappa = diag(\kappa_0^1, \ldots, \kappa_0^n)$, see Table 1, with $diag(\bullet)$ denoting the diagonal matrix of $\bullet$ element, and $e_i$ be the $i$-th column vector of the standard basis of $\mathbb{R}^n$, then it is straightforward to compute $F(t)$ as $F(t) = [F_1(t) \ 0_{p \times n}]$, with sub-matrix $F_1(t)$ given by $F_1(t) = \kappa (\sum_{i=1}^n e_i e_i^T (I_n T_s - y(t)) \otimes e_i^T)$ where $1_n$ is the $n$-dimensional vector with all entries equal to 1 and recalling that $T_s$ is the supply air temperature from the AHU.

### 2.3. Problem formulation

The problem we aim to address in this paper is to design a bilinear observer for the robust adaptive estimation of the magnitude of VAV damper faults under the effect of partially decouplable disturbances such that the energy-efficient operation of the overall closed-loop system can be maintained. To this end, the disturbance signal, $d(t)$, in (5) is partitioned into measured and unmeasured disturbance signals denoted by $d_m(t)$ and $d_{um}(t)$ respectively, i.e., $d = [d_m^T \mid d_{um}^T]^T$. The associated matrix $E$ can also be partitioned as $E = [E_m \mid E_{um}]$. Taking into account the above partition, the objective is to ensure the robustness of the fault estimator with regards to the effect of the unmeasured disturbance $d_{um}(t)$ and that the effects of $\eta(t)$ are attenuated as much as possible. Particularly, the following explicit partition of the disturbance $d_m = Q_{int}$, and $d_{um} = T_{oa}$ is used throughout. Note that the former disturbance can be computed online using either stochastic models (Dong and Lam 2014) or a technique based on passive infrared sensors (Goyal, Barooah, and Middelkoop 2015). The latter disturbance is a weather variable that significantly impacts the building dynamics as an unknown large disturbance input. Nevertheless, the temperature forecast can be predicted to some extent, the uncertainty of forecast error is a major challenge that requires proper treatment in the design of FDD algorithms for building HVAC systems. This uncertainty is also critical in greenhouse indoor climate control and monitoring problem, which is harmful to plants and fruits (Chen and You 2021). The main non-trivial issues to be solved for the fault estimation problem are, therefore:

- How to detect and isolate the faults?
- How to estimate the fault magnitude?
- How to remove the effect of unmeasured disturbance on fault estimation?
- How to attenuate the effect of sensor measurement noise on fault estimation?

### 3. Robust adaptive fault estimator

The following definition and lemma are required to develop the fault estimation algorithm.
Definition 3.1 (Krener and Isidori 1983; Subramaniam, Jain, and Yamé 2019): The dynamical system
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}_a
\end{bmatrix} = \begin{bmatrix}
A - LC & B & E_m & L & F(t) \\
C & 0 & 0 & 0 & 0 \\
-(\vartheta(t) * C) & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\times \begin{bmatrix}
x^T \\
u^T \\
d^T_m \\
z^T \\
f_a^T \\
\end{bmatrix}
+ \begin{bmatrix}
\phi(u, z) \\
0 \\
0 \\
\end{bmatrix}
\]
with \(\phi(u, y)\) representing a matrix-valued nonlinear function of input and output variables is defined as an output injected nonlinear system.

Proposition 3.1: The dynamical system (5) is an output injected nonlinear system.

Proof: See Appendix 1.

3.1. Development of the VAV damper fault estimation algorithm

The bilinear observer-based robust adaptive fault estimator (RAFE) is proposed as
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}_a
\end{bmatrix} = \begin{bmatrix}
A - LC & B & E_m & L & F(t) \\
C & 0 & 0 & 0 & 0 \\
-(\vartheta(t) * C) & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\times \begin{bmatrix}
x^T \\
u^T \\
d^T_m \\
z^T \\
f_a^T \\
\end{bmatrix}
+ \begin{bmatrix}
\phi(u, z) \\
0 \\
0 \\
\end{bmatrix}
\]
where ‘s’ represents the convolution operator, \(\hat{x}(t)\) denotes the state estimate, \(\hat{y}(t)\) the output estimate. Here, it is worth noticing that the unknown fault \(f_a(t)\) is viewed as a time-varying parameter whose estimate \(\hat{f}_a(t)\) is obtained by the adjusting law given by \(\hat{f}_a(t)\) in (8), where the to-be-designed parameters are constant matrices \(L, G, \) and \(\vartheta(t)\) the impulse response of a causal filter. Let the state estimation error be defined as \(e(t) = x(t) - \hat{x}(t),\) then from (7) and (8) the error dynamics reads
\[
\begin{align*}
\dot{e} &= [(A - LC) \ E_m \ -L \ F(t)] \\
&\times [\begin{bmatrix}
e^T \\
d^T_m \\
\eta^T \\
(f_a - \hat{f}_a)^T
\end{bmatrix}] \\
&+ (\phi(u, y) - \phi(u, z))
\end{align*}
\]
Proposition 3.2: Define \(\tilde{\eta}(t)\) as \(\tilde{\eta} = [\eta_u^T \ \eta^T]\) with \(\eta_u(t) = (\sum_{i=1}^{n} e_i^T u(t) \otimes e_i^T) \eta(t).\) Then, the nonlinear error dynamics (9) is equivalent to the linear dynamics
\[
\begin{align*}
\dot{e} &= [\tilde{A} \ B \ E_m \ F(t)] \\
&\times [\begin{bmatrix}
e^T \\
d^T_m \\
(f_a - \hat{f}_a)^T
\end{bmatrix}]
\end{align*}
\]
where \(\tilde{A} = (A - L C),\) and \(\tilde{B} = \begin{bmatrix} B & -L \end{bmatrix}\).

Proof: See Appendix 2.

For fault estimation, we use the error dynamics resulting from a similarity transformation \(\Gamma\) applied on (10), with \(\varepsilon = \Gamma\zeta,\) and where matrix \(\Gamma\) is the modal matrix of \(\bar{A},\) i.e. \(\Gamma^{-1}\bar{A}\Gamma = \bar{A}\) is a diagonal matrix (Chen 1995). Consequently, (10) is transformed to
\[
\begin{align*}
\begin{bmatrix}
\dot{\zeta} \\
\dot{\eta}_y
\end{bmatrix} &= \begin{bmatrix}
\tilde{A} & \tilde{B} & E_{um} & \bar{F}(t) & 0 \\
C & 0 & 0 & 0 & 0 \\
(\vartheta(t) * G) & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\times \begin{bmatrix}
\varepsilon^T \\
\eta_y^T \\
\eta^T \\
(f_a - \hat{f}_a)^T
\end{bmatrix}
\end{align*}
\]
where \(\zeta = z - \bar{y}, \tilde{A} = \Gamma^{-1}\bar{A}\Gamma, \tilde{B} = \Gamma^{-1}\bar{B}, E_{um} = \Gamma^{-1}E_{um}, \bar{F}(t) = \Gamma^{-1}F(t)\) and \(\bar{C} = \Gamma\bar{C}.\) Next, we shall introduce a matrix \(G \in \mathbb{R}^{n \times n}\) satisfying certain conditions, which would be helpful in designing the fault estimation filter in Proposition 3.4. The following proposition provides a sufficient condition for the existence of the matrix \(G.\)

Proposition 3.3: If \(\tilde{E}_{um}\) is a vector with non-zero entries, i.e. \(\tilde{E}_{um} = \begin{bmatrix} h_1 & \cdots & h_n \end{bmatrix}^T\) with \(h_i \neq 0, i = 1, 2, \ldots, n,\) then there exists a gain matrix \(G\) such that

(i) the vector \(\tilde{E}_{um} d_{um} \in \ker(G),\) and
(ii) \(\tilde{C}\bar{F}(t)(f_a - \hat{f}_a) \notin \ker(G).\)

Proof: See Appendix 3.

Remark 3.1: The rank of the matrix \(G\) and consequently, that of \(G\tilde{C}\bar{F}(t)\) is \(n-1\) for all time \(t,\) which implies that at least one of the eigenvalues of these matrices is zero. From the structure of the matrix \(G,\) there is only one eigenvalue at zero. Since the rank deficient matrix \(G\) is used in the adjusting law \(\hat{f}_a(t),\) only \(n-1\) faults can be estimated from (8). Nevertheless, multiple VAV damper faults can also be estimated using a multiple fault estimator scheme based on (8), which shall be presented in the next section.

Remark 3.2: The design matrix \(G\) in Proposition 3.3 is well defined whenever \(h_i \neq 0,\) for all \(i = 1, 2, \ldots, n.\) It is worth noting that \(\tilde{C}\tilde{E}_{um} = CE_{um}.\) Since \(C = \begin{bmatrix} I_n & 0_{n \times p} \end{bmatrix},\) and the outside ambient temperature always affects the zone dynamics, every element of the associated distribution vector is nonzero.

To prove the next proposition, we need the following lemma.

Lemma 3.1 (Subramaniam, Jain, and Yamé 2020): Let \(\bar{A}\) be a Hurwitz matrix and \(\hat{A}\) a perturbation time-varying...
matrix of the same dimension as $\tilde{A}$. Let $N$ and $M$ be positive definite symmetric matrices satisfying

$$\tilde{A}^T M + M \tilde{A} = -N. \quad (12)$$

Then, the autonomous system $\dot{\hat{e}} = (\tilde{A} + \tilde{\hat{A}}(t))\hat{e}$ is asymptotically stable if $\|\hat{e}(t)\| < \frac{\lambda_{\text{max}}(N)}{\lambda_{\text{min}}(M)}$, where $\lambda_{\text{min}}(\cdot), \lambda_{\text{max}}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix respectively.

We are now in a position to state the main result of the paper.

**Proposition 3.4:** Let $\vartheta(t)$ be the causal filter given by

$$\vartheta(t) = \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} \hat{e}(t) & I(t) \end{bmatrix}^T$$

where $\beta_1, \beta_2 \in \mathbb{R}^{n \times n}$ are the filter design matrices with $\hat{e}(t)$ and $I(t)$ denoting the Dirac delta function and the Heaviside function respectively. Let

$$\tilde{A} = \begin{bmatrix} \Gamma^{-1} & 0_n \end{bmatrix} \begin{bmatrix} (A - LC) - B \beta_1 GC & B \\ -B_2 GC & 0_n \end{bmatrix} \begin{bmatrix} \Gamma \\ 0_n \end{bmatrix}$$

and

$$\tilde{\hat{A}}(t) = \begin{bmatrix} \Gamma^{-1} & 0_n \end{bmatrix} \begin{bmatrix} -H_2(t) \beta_1 GC & H_2(t) \\ 0_n & 0_n \end{bmatrix} \begin{bmatrix} \Gamma \\ 0_n \end{bmatrix},$$

where $H_2(t) = \begin{bmatrix} H_2(t)^T \\ 0_n^T \end{bmatrix}^T$ with $H_2(t) = -\kappa (\sum_{i=1}^n \frac{e_i e_i^T y(t) \otimes e_i^T}{\sigma_{\text{min}}(\cdot)})$. If there exist matrices $\beta_1, \beta_2, \text{ and } L$ such that

(i) $\tilde{A}$ is Hurwitz,

(ii) $\tilde{\hat{A}}(t) \vartheta(t) \in \mathbb{R}^{(n+1) \times n}$, where $\tilde{\vartheta}_a = \tilde{\vartheta} - \tilde{\vartheta}_a$ denotes the fault estimation error,

(iii) $\|\tilde{\hat{A}}(t)\| < \frac{\lambda_{\text{max}}(N)}{2\lambda_{\text{min}}(M)}$, for some positive definite symmetric matrices $N, M$ satisfying (12),

then the fault estimation error is independent of the unmeasured disturbance signal $d_{\text{um}}$ and the effect of measurement noise $\eta$ on $\tilde{\vartheta}_a$ is attenuated.

**Proof:** See Appendix 4.

**Remark 3.3:** The main focus of this paper is on the issue of FDD robustness against uncertainties arising from noise and unmeasured disturbances in buildings equipped with VAV terminal units subject to a class of actuator faults. It is clear that, as the FDD algorithm is based on the building dynamical model, there will always be a mismatch between the actual building and its mathematical model, even in the fault-free case. Such discrepancies can be sources of false and missed alarms which can compromise the FDD system performance. To overcome such difficulties, the FDD algorithm must be enhanced with robustness against uncertainties arising from the building modelling. Note that for lock-in-place failures of the VAVs, the diagnosis performed by the algorithm is virtually unaffected by the building model uncertainties because these failures have larger effects on the designed FDD system than those of the modelling uncertainties. However, bias faults may have small effects on the FDD system and can be hidden as a consequence of modelling uncertainties. We will not dwell further into this aspect as the details depends strongly on the uncertainty modelling assumptions. This is left for a future research. In this paper, the robustness against uncertainties arising from the mismatch between the model and the real building has been dealt in a practical way at the fault detection stage by setting judiciously the detection threshold.

**3.2. Implementation of the robust fault diagnosis methodology**

In this section, the implementation of the overall robust fault diagnosis methodology consisting of fault detection, isolation, and adaptive estimation of VAV damper faults is presented. The design matrices $L, G, \beta_1,$ and $\beta_2$ of the RAFE (8) are synthesized using Propositions 3.3 and 3.4. From Remark 3.1, faults in up to $n - 1$ VAV dampers can be estimated using the RAFE (8). Therefore, prior to the estimation of a fault, it is required to identify which zones have a faulty VAV damper. Let us denote by $R$ the overall residual generator. This generator is composed of $n$ residual generators $R_i, i = 1, 2, \ldots, n$, where each $R_i$ outputs a signal $r_i(t)$ which is a vector belonging to $\mathbb{R}^n$ and computed using the formula

$$r(t) = \gamma_i G \zeta_y(t, \tilde{\vartheta}_a = 0) \quad (14)$$

with $\gamma_i \in \mathbb{R}^{n \times n}$ denoting a full rank matrix associated with the $i$-th residual signal of the bank. The signal $\zeta_y(t, \tilde{\vartheta}_a = 0)$ in (14) is computed from (11) by substituting $\tilde{\vartheta}_a = 0$. So the overall residual signal of $R$ is

$$r(t) = \begin{bmatrix} r_1(t) & r_2(t) & \cdots & r_n(t) \end{bmatrix}^T$$

where

$$\begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_n(t) \end{bmatrix} = \begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{1n} \\ r_{21} \\ \vdots \\ r_{2n} \\ \vdots \\ r_{nn} \end{bmatrix}.$$
and the square of the Euclidean norm of \( r(t) \) is given by \( \| r(t) \|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij}^2 \). The matrix \( \Upsilon_i \) in (14) is designed such that whenever there is a damper fault in the VAV terminal unit of the \( i \)-th zone, then only the \( i \)-th element of the vector \( r_i(t) \) is zero and the \( i \)-th elements of the other residual vectors \( r_k(t), \forall k \neq i \), are greater than some threshold \( \rho \), that is,

\[
\left\{ \begin{array}{ll}
\left( r_i(t) = 0 \right) \land & \text{true, fault is in } i \text{-th zone} \\
\left( |r_{ki}(t)| \geq \rho \right) & \text{false, } i \text{-th zone is fault - free}
\end{array} \right\}
\]

(15)

where \( r_{ki}(t) \) denotes the \( i \)-th element of vector \( r_k(t) \), \( | \cdot | \) denotes the modulus and the symbol ‘\( \land \)’ denotes the conjunction operation. Using (15), single and multiple VAV damper faults can be successfully isolated. Fault isolation is preceded by a fault detection stage carried out as

\[
\| r(t) \| < \varrho, \text{ for no faults}
\]

\[
\| r(t) \| \geq \varrho, \text{ for VAV damper faults}
\]

where \( \varrho \) is a given threshold value with units in °C. The implementation of the complete robust fault detection and diagnosis methodology is shown in the block diagram of Figure 2, where based on the measured signals, \((z, u, d_m)\), a fault occurrence is detected using the detection logic (16) followed by an isolation of the faulty VAV dampers using the rule (15), and finally the adaptive fault estimator is triggered for estimating precisely the magnitude of the fault using (8) with parameter matrices computed from Propositions 3.3 and 3.4. Note that two RAFEs are required for the implementation of the FDD methodology. In the event that VAV damper faults occur simultaneously in all the zones, then both the RAFEs are used for the estimation of the faults, with RAFe1 used to estimate the damper faults occurred in \( n \) – 1 zones, while RAFe2 is required for the estimation of the fault occurred in the \( n \)-th zone. If there occurs a single fault or multiple faults in up to \( n \) – 1 zones, then the faults estimates are obtained using only RAFe1.

### 3.3. Discussion

A pseudocode describing various steps for implementing the proposed fault detection and estimation methodology is provided in Algorithm 1. The algorithm is built upon the information of the plant matrices, where the information of the unmeasurable disturbance variable is not required. The matrix \( G \) is well-defined (see Remark 3.2). For any building envelope, the pair \((A, C)\) is detectable. From a practical viewpoint, \( L \in \mathbb{R}^{n} \) can be designed such that all the eigenvalues of the matrix \((A - LC)\) lie on the left half of the complex plane. Also, compute the modal matrix \( \Gamma \) of \((A - LC)\). To design the filter matrices, firstly, the parameter \( \beta_1 \) is designed such that \( \tilde{C}\tilde{F}(t)\tilde{d} \notin \ker \beta_1 \Gamma \), and \|\tilde{A}(t)\| < \frac{\min(N(M))}{2\max(M)}\), for some positive definite symmetric matrices \( N, M \). Secondly, the parameter \( \beta_2 \) is designed such that

\[
\begin{bmatrix}
(A - LC) - B\beta_1GC \\
-\beta_2GC
\end{bmatrix} \quad B \quad 0
\]

is positive definite.

#### Algorithm 1 Robust Adaptive Fault Estimator

1: \text{procedure RAFe}(A, B, C, D, \tilde{E}, \tilde{m})
2: Design \( G \) according to Proposition 3.3
3: Compute \( \beta_1, \beta_2, \bar{L} \) according to Proposition 3.4
4: Compute \( \bar{\Gamma} \) such that \( \bar{\tilde{A}} = \Gamma^{-1}(A - LC)\Gamma \) is diagonal
5: Compute \( \tilde{B}, \tilde{C}, \tilde{E}_{um}, \tilde{F}(t) \) according to (11)
6: Select \( \Upsilon_i, i = 1, 2, \ldots, n \) according to (14)
7: \text{while } \| r(t) \| \geq \varrho \text{ do}
8: Identify fault actuator(s) according to (15)
9: Estimate the fault magnitude using (8)
10: \text{end while}
11: \text{end procedure}

**Figure 2.** Robust fault detection and diagnosis methodology for multizone building VAV terminal units.
Hurwitz. In order to detect and isolate the faulty dampers, the matrix $\Upsilon_i, i = 1, \ldots, n$ can be computed using the generalized eigenvectors of $G$ as shown in the simulation section. Finally, the threshold parameter $\varphi$ can be determined through extensive simulations or, if possible, experimentally through real tests on the plant, which is usually done as a mandatory prerequisite step in the failure mode and effects analysis (FMEA) stage (Yang et al. 2018).

4. Simulation results

In order to demonstrate the effectiveness of the complete fault diagnosis methodology, a case study of a one-storey building containing three zones is simulated using the physics-based simulator SIMBAD, which is a Matlab/Simulink toolbox developed by CSTB (SIMBAD 2005) for prototyping and analysing control performance and monitoring methods in building HVAC systems (Togashi and Miyata 2019). The SIMBAD simulator, being able to virtually reconstruct real buildings behaviours based on their actual configurations and different weather and occupancy conditions as well as use patterns, has been extensively used for testing and validating control and monitoring algorithms by industry engineers in preliminary steps before coding and deploying these algorithms in real building automation systems (Husaunndee, Vaezi-Nejad, and Visier 2001).

4.1. Setup of the controlled HVAC-VAV system

The building considered is a non-residential building consisting of three office spaces (zones), as illustrated in Figure 3. The working hours are from 08h00 to 12h00 in the morning and from 14h00 to 18h00 in the afternoon for five days a week. The occupancy profile in Zone-3, Zone-2 and Zone-1 is set to five, four and six respectively, which contribute to the thermal load of the building. The minimum and maximum airflow rate possible by the VAV boxes of each zone corresponding to the damper position at 0 and 1 are $u_{\text{min}} = 0.49 \times 10^{-3}$ kg/s and $u_{\text{max}} = 0.3163$ kg/s, respectively. The three zones in this building have three external walls, denoted by $T_{\text{extw}}, i = 1, 2, 3$, which interacts with the ambient weather, and three internal walls, denoted with $T_{\text{intw}}, T_{\text{intw,12}}$ and $T_{\text{intw,13}}$, which interacts with neighbouring zones. The supply temperature $T_s$ for conditioning the air is kept at $27^\circ$C. Based on the occupancy profile and weather data, the thermal load in each zone, which acts as a disturbance in (5), is computed within SIMBAD. The disturbance signals are shown in Figure 4. The sensor noise in the measurement of zone temperature is simulated with zero mean and variance of $20 \times 10^{-5}$.

With reference to the closed-loop control of the multizone system shown in Figure 1, the variable speed fan in the air-handling unit drives appropriate volume air mass to the input of VAV terminal units of the individual zones. The VAV boxes are pressure-independent terminal units (Levermore 2000), which are generally controlled by a cascaded control strategy consisting of a master controller, which is the temperature controller, and a slave controller, which controls the airflow rate. The outer-loop temperature control is a PI (Proportional + Integral) control law $u(t) = K_P (z(t) - y_{\text{set}}(t)) + K_I \int_0^t (z(\tau) - y_{\text{set}}(\tau)) \, d\tau$, which regulates each zone temperature to its reference value of $T_{z,\text{set}} = 20 \times 13$ $^\circ$C, with the controller gains selected as $K_p = I_3$ and $K_I = 0.0005I_3$. Under the event of VAV dampers faults, the desired volume of air is not delivered to the zone, thereby affecting the thermal comfort of occupants and the energy-efficient operation of the overall HVAC system.
4.2. Fault diagnosis

We consider four different fault cases categorized in two different scenarios for demonstrating the effectiveness of the proposed fault diagnosis methodology, where the measured disturbance variable is $Q_{int}$ and the unmeasured disturbance variable is $T_{oa}$. Nevertheless, there could be uncertainties in the measured disturbance variable also, since the mathematical bilinear model used in simulation scenarios captures the approximate dynamics of the overall HVAC building system simulated with SIMBAD toolbox, which is typically suitable for control and monitoring purposes. It is now apparent that uncertainties in the sense of the mismatch between the plant and the mathematical model are inherently present in simulations. It is shown through numerical simulations in subsequent sections that the developed fault estimator is not only robust to unmeasured disturbances as rigorously proved in the previous sections but also with regards to inherently unavoidable modelling uncertainties.

In Scenario-1, multiple VAV damper faults (lock-in-place and bias) occur in a mutually exclusive manner, and in Scenario-2, simultaneous faults occur in all zones. All considered fault cases are given in Table 2. For both the scenarios, the design matrices for the robust adaptive control and monitoring purposes. It is now apparent that

The lock-in-places and the corresponding constant airflow rates to each zone are given in Table 3. Under this fault, the zones temperature are shown in Figure 5(a). It is worth noticing that the ‘intense’ variations in zone temperature measurements appear due to the departure and arrival of occupants. The norm of the residual signal is recorded in Figure 5(b). With the detection threshold values as $\rho = 2 \degree C$ and $\varrho = 5 \degree C$, the fault is successfully detected at the end of day 1, 3, and 5 according to (16). Moreover, when the damper fault disappears, then the norm of the residual signal is again less than the threshold value. For fault isolation, the vector signals $r_i(t), i = 1, 2, 3$, illustrated in Figure 5(c), are used to determine the faulty damper. Recall that if the fault has occurred in the $i$-th zone, then the $i$-th signal of the vector $r_i(t)$ is zero while that of the other vectors, i.e. $r_{i\neq i}(t)$, satisfy the condition given in (15). This can be visualized in Figure 5(c) that during day 2, the signal $r_1(t)$ is zero, and the signals $r_2(t), r_3(t)$ satisfy $|r_2(t)| > 2$ and $|r_3(t)| > 2$. Also, during the same time, other signals in Figure 5(c), satisfy $|r_{ik}| < 2, Vi = 1, 2, 3$ and $k = 2, 3$, which implies that the fault has occurred only in zone-1. Therefore, using RAFE$_3$, the fault in zone-1 is estimated, which is illustrated in Figure 6. Now after day 2, the fault in zone-1 disappears and a fault in zone-2 appears at the end of day 3. From Figure 5(b), the norm of the residual signal satisfies the condition given in (16). On checking the residual signals in Figure 5(c), we have $r_{22}(t) = 0$ and $|r_{12}(t)| \geq 2, |r_{32}(t)| \geq 2$, which indicates that there is a fault in zone-2. Similarly, the lock-in-place failure of the VAV damper of zone-3, zone-2, and zone-1 at the end of day 5, 3, and 1 respectively and disappears or is corrected in the respective zones by the end of the next day.

VAV dampers of zone-3, zone-2, and zone-1 at the end of day 5, 3, and 1 respectively and disappears or is corrected in the respective zones by the end of the next day.

Table 2. Fault scenarios for evaluation of the fault diagnosis methodology.

| Scenario | Case   | Fault type  | Faulty zones |
|----------|--------|-------------|--------------|
| 1 (mutually exclusive) | 1 | Lock-in-place | 1, 2, 3 |
|          | 2 | Bias        |              |
| 2 (simultaneous)  | 1 | Lock-in-place | 1, 2, 3 |
|          | 2 | Bias        |              |

Table 3. Fault scenario-1.

| Case | Fault type | Damper-1 | Damper-2 | Damper-3 |
|------|------------|----------|----------|----------|
| 1    | Lock-in-place | (Airflow rate) 0.061 kg/s | (Airflow rate) 0.064 kg/s | (Airflow rate) 0.071 kg/s |
| 2    | Bias       | -0.33    | -0.30    | -0.32    |

VAV dampers of zone-3, zone-2, and zone-1 at the end of day 5, 3, and 1 respectively and disappears or is corrected in the respective zones by the end of the next day.

4.2.1. Scenario 1: mutually exclusive occurring multiple faults

In this scenario, multiple VAV damper faults occur in the building HVAC system. Two fault cases are considered, where in the first case, a lock-in-place failure appears in
the adaptive fault estimation filter driven by noisy sensor measurement and without having any information of $d_w$ successfully reconstructs the airflow rates at the input of the zones in the fault-free mode as well as in the faulty condition.

For bias faults, a bias is considered in all VAV dampers occurring and disappearing as above. A constant bias in different damper positions is also given in Table 3. Since the building HVAC system operates under feedback control, the controller output also saturates under the bias faults, due to which the airflow rates to each zones will be constant, mentioned in the bottom row of Table 3. The zones temperature, the norm of the residual, and the signals generated by the bank of residual generators at the output of the VAV boxes are listed in Table 4. Under the lock-in-place failure, the temperature of the zones is shown in Figure 9(a). The norm of the residual signal generated by the bank of residual generators

4.2.2. Scenario 2: simultaneously occurring multiple faults
In this scenario, similar types of faults as above occur in zone-1 and zone-2 simultaneously at the end of day 2, and in zone-3 at the end of day 4. Unlike above, both type of faults once occurred do not disappear. The damper positions and the corresponding airflow rates

Figure 5. Scenario-1, Case-1. (a) Zones temperature, (b) Residual Norm and (c) Residual bank signals.

Figure 6. Scenario-1, Case-1: output signals of the PI controller and that of the VAV boxes.
Figure 7. Scenario-1, Case-2. (a) Zones temperature, (b) Residual Norm and (c) Residual bank signals.

Figure 8. Scenario-1, Case-2: output signals of the PI controller and that of the VAV boxes.

Table 4. Fault scenario-2.

| Case | Fault type | Damper-1 | Damper-2 | Damper-3 |
|------|------------|----------|----------|----------|
| 1    | Lock-in-place (Airflow rate) | 0.7 (0.071 kg/s) | 0.6 (0.041 kg/s) | 0.6 (0.041 kg/s) |
| 2    | Bias (Airflow rate) | -0.33 (0.071 kg/s) | -0.32 (0.064 kg/s) | -0.35 (0.055 kg/s) |

is shown in Figure 9(b), and the individual residual signals are recorded in Figure 9(c). From Figure 9(b), the normed signal exceeds the threshold value of 5°C as soon as the lock-in-place failure occurs in VAV dampers at the end of the day. According to the fault isolation logic (15), the signal $r_{1}(t)$ in Figure 9(c) is zero, while the signals $r_{21}(t), r_{31}(t)$ satisfy $|r_{21}(t)| \geq 2$ and $|r_{31}(t)| \geq 2$, which implies that a fault has occurred in zone-1. It is also noticed that after day 2, the signal $r_{22}(t)$ is zero, while the signals $r_{12}(t), r_{32}(t)$ satisfy $|r_{12}(t)| \geq 2$ and $|r_{32}(t)| \geq 2$, which implies that a fault has also occurred in zone-2. At the same time, the signals $r_{33}(t), i = 1, 2, 3$ satisfy $|r_{33}(t)| < 2$ until the end of day 4. However, the signals $r_{13}(t) \geq 2$ and $r_{23}(t) \geq 2$ with $r_{33}(t) = 0$ after that day. This implies that now a fault has also occurred in zone-3 after day 4. Finally, since the VAV damper fault is now present in all the zones after day 4, only RAFE1 is activated up to day 4, while the estimator RAFE2 is activated afterwards. The robust adaptive estimation of the magnitude of the VAV damper lock-in-place failures is illustrated in Figure 10. Similarly, the signals corresponding to bias faults occurring in all zones are recorded in Figures 11 and 12.
5. Conclusion

In this work, a new robust fault diagnosis methodology is developed for multizone building VAV terminal units, which ensures some robustness properties with regards to unmeasured disturbances and sensor noises. The methodology consists of detecting, isolating, and adaptively estimating any single or multiple VAV damper failures. In the considered context, the unmeasured disturbance is the outdoor temperature which causes the thermal dynamics of the building to vary significantly over a large operating region, thus making accurate fault diagnosis a challenging task due to the non-linearity underlying this wide operating region. Unlike the usual building dynamics linearization-based approaches, the robust FDD algorithm is built upon a bilinear observer derived from the physics-informed thermal dynamics of the building over its entire operating region essentially determined by the outdoor climate conditions and, in particular, the outside temperature. The key feature of this observer is that it is augmented by a judiciously designed filter-based adjustment law for adaptive fault estimation that achieves a reduced sensitivity to measurement noises and a decoupling of the estimates from the unmeasured disturbance. Simulation results using the MATLAB-based simulator SIMBAD are reported to demonstrate the effectiveness of the robust FDD methodology under different VAV damper fault scenarios in a one-story building with three zones. Though the present work was mainly motivated by dealing with the outdoor temperature as an unmeasured disturbance, in the future
applicability of the development methodology shall be investigated to estimate the faults under the unavailability of internal heat gains from a zone of the building as well as under modelling uncertainties.

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**Data availability statement**

The authors confirm that the data supporting the findings of this study are available within the article.

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**References**

A., M. S., T. Jain, and J. J. Yamé. 2019. "Output Injected Nonlinear Observer for Diagnosing Faults in Multi-Zone Building."
In 2019 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 660–665. Hong Kong: IEEE/ASME.

A., M. S., T. Jain, and J. J. Yamé. 2020. “Bilinear Model-Based Diagnosis of Lock-in-Place Failures of Variable-Air-Volume HVAC Systems of Multizone Buildings.” Journal of Building Engineering 28: 101023. doi:10.1016/j.jobe.2019.10.1023.

Bengea, S. C., A. D. Kelman, F. Borrelli, R. Taylor, and S. Narayanan. 2014. “Implementation of Model Predictive Control for an HVAC System in a Mid-Size Commercial Building.” HVAC&R Research 20 (1): 121–135. doi:10.1080/10789696.2013.834781.

Bonvini, M., M. D. Sohn, J. Granderson, M. Wetter, and M. A. Piette. 2014. “Robust On-Line Fault Detection Diagnosis for HVAC Components Based on Nonlinear State Estimation Techniques.” Applied Energy 124: 156–166. doi:10.1016/j.apenergy.2014.03.009.

Castilla, M., J. Álvarez, J. Normey-Rico, and F. Rodríguez. 2014. “Thermal Comfort Control Using a Non-Linear MPC Strategy: A Real Case of Study in a Bioclimatic Building.” Journal of Process Control 24 (6): 703–713. doi:10.1016/j.jprocont.2013.08.009.

Chen, C. 1995. Linear System Theory and Design. New York: Oxford University Press, Inc.

Chen, W.-H., and F. You. 2021. “Semiclosed Greenhouse Climate Control under Uncertainty Via Machine Learning and Data-Driven Robust Model Predictive Control.” IEEE Transactions on Control Systems Technology 30 (3): 1186–1197. doi:10.1109/TCST.2021.3094999.

Coffman, A. R., and P. Barooah. 2018. “Simultaneous Identification of Dynamic Model and Occupant-Induced Disturbance for Commercial Buildings.” Building and Environment 128: 153–160. doi:10.1016/j.buildenv.2017.10.020.

Darure, T., J. J. Yamé, and F. Hamelin. 2016. “Fault-Adaptive Control of VAV Damper Stuck in a Multizone Building.” In 3rd Conference on Fault and Tolerant Systems (SysTol), 170–176. Barcelona: IEEE.

Dobbs, J. R., and B. M. Hencsey. 2014. “Model Predictive HVAC Control with Online Occupancy Model.” Energy and Buildings 82: 675–684. doi:10.1016/j.enbuild.2014.07.051.

Dong, B., and K. P. Lam. 2014. “A Real-Time Model Predictive Control for Building Heating and Cooling Systems Based on the Occupancy Behavior Pattern Detection and Local Weather Forecasting.” Building Simulation 7: 89–106. doi:10.1007/s12273-013-0142-7.

Du, M., J. Scott, and P. Mhaskar. 2013. “Actuator and Sensor Fault Isolation of Nonlinear Process Systems.” Chemical Engineering Science 104: 294–303. doi:10.1016/j.ces.2013.08.009.

Elliot, D. L. 2009. Bilinear Control System: Matrices in Action. Dordrecht: Springer Science & Business Media.

Gao, Z., X. Liu, and M. Chen. 2016. “Unknown Input Observer-Based Robust Fault Estimation for Systems Corrupted by Partially Decoupled Disturbances.” IEEE Transactions on Industrial Electronics 63 (4): 2537–2547. doi:10.1109/TIE.2015.2497201.

Geoffroy, H., J. Berger, B. Colange, S. Lespinats, and D. Duttykh. 2022. “The Use of Dimensionality Reduction Techniques for Fault Detection and Diagnosis in a Ahu Unit: Critical Assessment of Its Reliability.” Journal of Building Performance Simulation: 1–19. doi:10.1080/19401493.2022.2080864.

Glass, A., P. Gruber, M. Roos, and J. Todtli. 1995. “Qualitative Model-Based Fault Detection in Air-Handling Units.” IEEE Control Systems Magazine 15 (4): 11–22. doi:10.1109/37.408465.

Goyal, S., P. Barooah, and T. Middelkoop. 2015. “Experimental Study of Occupancy-Based Control of HVAC Zones.” Applied Energy 140: 75–84. doi:10.1016/j.apenergy.2014.11.064.

Gunay, B., W. Shen, and C. Yang. 2017. “Characterization of a Building’s Operation Using Automation Data: A Review and Case Study.” Building and Environment 118: 196–210. doi:10.1016/j.buildenv.2017.03.035.

Husaunnudee, A., H. Vaezi-Nejad, and J. Visier. 2001. “Use of Simulation Tools for Design and Test of Control and Energy Management Systems in Intelligent Buildings.” International Journal of Solar Energy 21 (2-3): 111–130. doi:10.1080/0145910108914367.

Jacob, D., S. Dietz, S. Komhard, C. Neumann, and S. Herkel. 2010. “Black-Box Models for Fault Detection and Performance Monitoring of Buildings.” Journal of Building Performance Simulation 3 (1): 53–62. doi:10.4190/jbps.2010.01.1388.

Jain, T., J. Yamé, and D. Sauter. 2018. Active Fault-Tolerant Control Systems: A Behavioral System Theoretic Perspective. 1st ed., Studies in Systems, Decision and Control. New York: Springer International Publishing.

Kim, D., J. Cai, K. B. Ariyur, and J. E. Braun. 2016. “System Identification for Building Thermal Systems under the Presence of Unmeasured Disturbances in Closed Loop Operation: Lumped Disturbance Modeling Approach.” Building and Environment 107: 169–180. doi:10.1016/j.buildenv.2016.07.007.

Krener, A. J., and A. Isidori. 1983. “Linearization by Output Injection and Nonlinear Observers.” Systems & Control Letters 3 (1): 47–52. doi:10.1016/0167-6911(83)90037-3.

Lazos, D., A. B. Sproul, and M. Kay. 2014. “Optimisation of Energy Management in Commercial Buildings with Weather Forecasting Inputs: A Review.” Renewable and Sustainable Energy Reviews 39: 587–603. doi:10.1016/j.rser.2014.07.053.

Lee, S., and F. Yik. 2010. “A Study on the Energy Penalty of Various Air-Side System Faults in Buildings.” Energy and Buildings 42 (1): 2–10. doi:10.1016/j.enbuild.2009.07.004.

Levermore, G. 2000. Building Energy Management Systems. 2nd ed., Applications to Low-Energy HVAC and Natural Ventilation Control. London: E & FN Spon, Taylor & Francis Group.

Li, D., Y. Zhou, G. Hu, and C. J. Spanos. 2018. “Identifying Unseen Faults for Smart Buildings by Incorporating Expert Knowledge with Data.” IEEE Transactions on Automation Science and Engineering 16 (3): 1412–1425. doi:10.1109/TASE.8856.

Liang, J., and R. Du. 2007. “Model-Based Fault Detection and Diagnosis of HVAC Systems Using Support Vector Machine Method.” International Journal of Refrigeration 30: 1104–1114. doi:10.1016/j.ijrefrig.2006.12.012.

Montazeri, A., and S. M. Kargar. 2020. “Fault Detection and Diagnosis in Air Handling Using Data-Driven Methods.” Journal of Building Engineering 31: 101388. doi:10.1016/j.jobe.2020.01.01388.

Montgomery, R., and R. McDowall. 2008. Fundamentals of HVAC Control Systems. Burlington, MA: Elsevier.

Mulumba, T., A. Afshari, K. Yan, W. Shen, and L. K. Norford. 2015. “Robust Model-Based Fault Diagnosis for Air Handling Units.”
Energy and Buildings 86: 698–707. doi: 10.1016/j.enbuild.2014.10.069.

Namburu, S. M., M. S. Azam, J. Luo, K. Choi, and K. R. Patipati. 2007. “Data-Driven Modeling, Fault Diagnosis and Optimal Sensor Selection for HVAC Chillers.” *IEEE Transactions on Automation Science and Engineering* 4 (3): 469–473. doi: 10.1109/TASE.2006.888053.

Oldewurtel, F., A. Parisio, C. N. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari. 2012. “Use of Model Predictive Control and Weather Forecasts for Energy Efficient Building Climate Control.” *Energy and Buildings* 45: 15–27. doi: 10.1016/j.enbuild.2011.09.022.

O’Neill, Z., X. Pang, M. Shashanka, P. Haves, and T. Bailey. 2014. “Model-Based Real-Time Whole Building Energy Performance Monitoring and Diagnostics.” *Journal of Building Performance Simulation* 7 (2): 83–99. doi: 10.1080/19401493.2013.777118.

Qin, X., S. Lysecky, and J. Sprinkle. 2014. “A Data-Driven Linear Approximation of HVAC Utilization for Predictive Control and Optimization.” *IEEE Transactions on Control Systems Technology* 23 (2): 778–786. doi: 10.1109/TCST.2014.2332873.

Shahnazari, H., and P. Mhaskar. 2018. “Actuator and Sensor Fault Detection and Isolation for Nonlinear Systems Subject to Uncertainty.” *International Journal of Robust and Nonlinear Control* 28 (6): 1996–2013. doi: 10.1002/rnc.3996.

Shahnazari, H., P. Mhaskar, J. M. House, and T. I. Salsbury. 2018. “Heating, Ventilation and Air Conditioning Systems: Fault Detection and Isolation and Safe Parking.” *Computers and Chemical Engineering* 108: 139–151. doi: 10.1016/j.compchemeng.2017.08.012.

SIMBAD. 2005. “SIMulator of Building and Devices, Centre Scientifique et Technique du Batiment.” France.

Sun, B., P. B. Luh, Q. Jia, Z. O’Neill, and F. Song. 2014. “Building Energy Doctors: An SPC and Kalman Filter-Based Method for System-Level Fault Detection in HVAC Systems.” *IEEE Transactions on Automation Science and Engineering* 11 (1): 215–229. doi: 10.1109/TASE.2012.2226155.

Thumati, B. T., M. A. Feinstein, J. W. Fonda, A. Turnbull, F. J. Weaver, M. E. Calkins, and S. Jagannathan. 2011. “An Online Model-Based Fault Diagnosis Scheme for HVAC Systems.” In *IEEE International Conference on Control Applications (CCA)*, 70–75. Denver, CO: IEEE.

Togashi, E., and M. Miyata. 2019. “Development of Building Thermal Environment Emulator to Evaluate the Performance of the HVAC System Operation.” *Journal of Building Performance Simulation* 12 (5): 663–684. doi: 10.1080/19401493.2019.1601259.

Wang, H., Y. Chen, C. W. Chan, and J. Qin. 2011. “A Robust Fault Detection and Diagnosis Strategy for Pressure-Independent VAV Terminals of Real Office Buildings.” *Energy and Buildings* 43 (7): 1774–1783. doi: 10.1016/j.enbuild.2011.03.018.

Yamé, J., T. Jain, and D. Sauter. 2015. “An Online Controller Redesign Based Fault-Tolerant Strategy for Thermal Comfort in a Multi-Zone Building.” In *IEEE Conference on Control Applications*, 1901–1906. Sydney, NSW, Australia: IEEE.

Yang, C., W. Shen, Q. Chen, and B. Gunay. 2018. “A Practical Solution for HVAC Prognostics: Failure Mode and Effects Analysis in Building Maintenance.” *Journal of Building Engineering* 15: 26–32. doi: 10.1016/j.jobe.2017.10.013.

Yoshida, H., S. Kumar, and Y. Morita. 2001. “Online Fault Detection and Diagnosis in VAV Air Handling Unit by RARX Modeling.” *Energy and Buildings* 33 (4): 391–401. doi: 10.1016/S0378-7788(00)00121-3.

Appendices

**Appendix 1**

**Proof:** The bilinear term in (5) reads as

\[
D(u \otimes x) = \phi(u, x) = \begin{bmatrix}
D_{11}(t) & D_{12}(t) \\
D_{21}(t) & D_{22}(t)
\end{bmatrix}
\begin{bmatrix} T_x \\ Tw
\end{bmatrix}
\]

where \(D_{11}, D_{12}, D_{21}, \) and \(D_{22}\) are block matrices of \(Du(t)\), see (4).

It turns out that for a multizone building system controlled by VAV terminal units, matrices \(D_{12}(t), D_{21}(t), D_{22}(t)\) are all zero matrices of appropriate dimensions. This results in

\[
D(u \otimes x) = \phi(u, x) = \begin{bmatrix}
D_{11}(t) & 0_{p \times n} \\
0_{n \times p} & 0_{p \times 1}
\end{bmatrix}
\begin{bmatrix} T_x \\ 0_{p \times 1}
\end{bmatrix}
= \phi(u, y)
\]

(A1)

with \(D_{11}(t) = -\kappa \sum_{i=1}^{n} e_{i} e_{i}^{T} u(t) \otimes e_{i}^{T}\). Clearly, (5) is an output injected nonlinear (bilinear) system.

**Appendix 2**

**Proof:** Using Proposition 3.1, the matrix-valued nonlinear function \(\phi(u, z)\) in (9) can be decomposed as the sum of \(\phi(u, y)\) and a matrix consisting of the product of \(D_{11}(t)\) and \(\eta\), i.e.

\[
\phi(u, z) = \phi(u, y + \eta) = \begin{bmatrix}
D_{11}(t) & 0_{p \times 1}
\end{bmatrix} \begin{bmatrix} y + \eta \\ 0_{p \times 1}
\end{bmatrix}
= \phi(u, y) + \begin{bmatrix}
D_{11}(t) & 0_{p \times 1}
\end{bmatrix} \eta
\]

(A2)

where matrix \(D_{11}(t)\eta\) can be written as \(D_{11}(t)\eta = -\kappa \sum_{i=1}^{n} e_{i} e_{i}^{T} u(t) \otimes e_{i}^{T} \times \eta(t) = -\kappa \eta(t)\). Putting (A2) into (9) removes the nonlinear term from the error dynamics. Thanks to the model structure of a multizone building, the matrix \(B\) is given as \(B = [ B_{1}^{T} \mid 0_{n \times n} ]^{T}\) where \(B_{1} \in \mathbb{R}^{n \times n}\) with \(B_{1} = k Tw\). As a consequence, the second term in (A2) can be expressed as

\[
\begin{bmatrix}
D_{11}(t) \eta \\ 0_{p \times 1}
\end{bmatrix} = -\frac{1}{k} B \eta u, \text{ which on substituting in (9) yields the linear system (10).}
\]
Appendix 3

**Proof:** The proof of the proposition is constructive. Indeed, construct matrix $G \in \mathbb{R}^{n \times n}$ as

$$G = \begin{bmatrix} 1 & \frac{h_1}{h_2} & \cdots & 0 & 0 \\ \vdots & \frac{h_2}{h_3} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & -\frac{h_{n-1}}{h_n} \\ -\frac{h_n}{h_1} & 0 & \cdots & 0 & 1 \end{bmatrix}.$$ 

(i) then it is straightforward to check that $\mathcal{C}E_{um}d_{um} \in \ker(G)$.

(ii) From (7), since $F(t) = \left[ x(\sum_{i=1}^{n} e_i e_i^T (\Pi_{T_z} - T_z) \otimes e_i) \right] \in \mathbb{R}_+^1$, the matrix $G\hat{C}F(t)$ is expressed as $G\hat{C}F(t) = G \times \text{diag}(F_{11}(t), \ldots, F_{nn}(t))$ where $F_{ii}(t)$ denotes the $i$-th diagonal element of $F(t)$. Writing the above matrix explicitly, we get

$$G\hat{C}F(t) = \begin{bmatrix} F_{11} & \frac{h_1}{h_2}F_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{h_n}{h_1}F_{11} & 0 & \cdots & F_{nn} \end{bmatrix}. \quad (A3)$$

Now the vector $\hat{C}F(t)(\hat{f}_a - \hat{f}_a) \in \ker(G)$ if and only if $T_zT_n - T_z = 0_n$. Thanks to the building HVAC dynamics, the last condition is never satisfied.

$$\blacksquare$$

Appendix 4

**Proof:** The proof contains two parts. In the first part, we show $d_{um}$ does not affect the fault estimates. In the second part, the boundedness of the fault estimation error is shown.

**Part (a):** Substituting $x(t)$ from (7) in the last block-row of (8) and using transformation (11) yields the adjusting law as

$$\hat{f}_a(t) = (\beta_1 \hat{G}\hat{C}A + \beta_2 \hat{G}\hat{C} \tilde{B}) \xi + \beta_1 \hat{G}\hat{C}B\eta + \hat{\theta}(t) + \tilde{G} \eta.$$ 

From Proposition 3.3, the fourth term in (A4) is zero. Subsequently, simplifying it further using the solution of the linear system (11), Equation (A4) reads as

$$\hat{f}_a(t) = (\beta_1 \hat{G}\hat{C}A + \beta_2 \hat{G}\hat{C} \tilde{B}) \int_{t_0}^{t} e^{A(t-s)} (\tilde{E}_{um}d_{um}(t) + \tilde{B}\eta(t)) \, ds + \beta_1 \hat{G}\hat{C} \tilde{B}\eta + \hat{\theta}(t) + \tilde{G} \eta + \alpha, \quad (A5)$$

where in other terms (o.t.) there is no signature of $d_{um}(t)$. Since the matrix $\tilde{A}$ is diagonal, the associated matrix exponential is also diagonal. Clearly from (A5) and Proposition 3.3, $\mathcal{C}E_{um}d_{um} \in \ker(G)$ implies $\mathcal{C}E_{um}d_{um} \in \ker(\beta_1 \hat{G} + \beta_2 \tilde{B})$, which concludes this part.

**Part (b):** Substituting $\tilde{B}$ and $\tilde{\eta}$ from Proposition 3.2 in Equation (A4), the derivative of the fault estimation error $\hat{f}_a$ reads

$$\dot{\hat{f}}_a = -(\beta_1 \hat{G}\hat{C} + \beta_2 \hat{G}\hat{C} \tilde{B}) \xi - \frac{1}{T_z} \beta_1 \hat{G}\hat{C} \tilde{B} \eta + \dot{\hat{f}}_a + \beta_1 \hat{G}\hat{C} \tilde{B} \eta - \hat{\theta}(t) + \tilde{G} \eta - \beta_1 \hat{G}\hat{C} \tilde{B} \eta,$$ 

and the state equation of (11) reduces to

$$\dot{\xi} = \tilde{A}\xi + \frac{1}{T_z} \tilde{B} \eta + \tilde{E}_{um}d_{um} + \tilde{f}(t)\tilde{f}_a.$$ 

(7)

If there is a damper lock-in-place failure in VAV boxes, then $u(t)$ is a constant signal. In addition, under closed-loop control, the airflow setpoint generated by the controller, $u(t)$, saturates due to hard constraints on the VAV dampers. From (6), if $u(t)$ and $u(t)$ are constant signals, then $f_2(t)$ is also a constant signal and consequently, $f_2(t) = 0$. For bias faults, $f_2(t)$ is also equal to zero. Concatenating vectors $\xi$ and $\hat{f}_a$, the derivative of the concatenated vector reads (see Equations (A6) and (A7))

$$\begin{bmatrix} \dot{\xi} \\ \dot{\hat{f}}_a \end{bmatrix} = \frac{1}{T_z} \begin{bmatrix} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} -\frac{1}{T_z} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} -1 \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} \tilde{B}(t) \xi + \tilde{E}_{um}d_{um} \end{bmatrix}. \quad (A8)$$

Define $\tilde{x} = \begin{bmatrix} \xi^T \\ \hat{f}_a^T \end{bmatrix}$, then (A8) can be compactly expressed as $\dot{\tilde{x}} = \tilde{A}(t)\tilde{x} + \tilde{B}(t)\tilde{u}$. Applying the transformation $\tilde{x} = \Lambda^{-1}\tilde{x}$ with the unimodular transformation matrix $\Lambda = \left[ \begin{matrix} \tilde{h}_{1+p} & 0_{(n+p) \times n} \\ \beta_1 \tilde{G} & I_n \end{matrix} \right]$, yields $\dot{\tilde{x}} = \Lambda \tilde{A}(t)\Lambda^{-1}\tilde{x} + \Lambda \tilde{B}(t)\tilde{u}$. Thanks to the model structure of the building thermal dynamics, the time-varying fault distribution matrix $F(t)$ can be expressed as $F(t) = \tilde{B} + H_2(t)$, then $\tilde{F}(t) = \tilde{B} + H_2(t)$, and $\tilde{B}(t)$.

Define $\tilde{A}(t) = \Lambda \tilde{A}(t)\Lambda^{-1}$ and $\tilde{B}(t)$ in (A9), the matrix $\Lambda \tilde{A}(t)\Lambda^{-1}$ can be decomposed as the sum of a constant matrix $\Lambda \tilde{A}(t)$ and a time-varying matrix $\Lambda \tilde{B}(t)$, i.e.

$$\Lambda \tilde{A}(t)\Lambda^{-1} = \Lambda + \tilde{A}(t)$$

with

$$\begin{bmatrix} \Lambda^{-1} & 0_n \\ \Lambda^{-1} & \Lambda^{-1} \end{bmatrix} \left[ \begin{matrix} -\tilde{A} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} -\tilde{A} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} \tilde{B}(t) \xi + \tilde{E}_{um}d_{um} \end{bmatrix}. \quad (A9)$$

From the above, if there exist matrices $\beta_1, \beta_2$ and $L$ such that $\Lambda \tilde{A}(t)$ or equivalently $\left[ \begin{matrix} -\tilde{A} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} -\tilde{A} \tilde{B} \xi + \tilde{E}_{um}d_{um} \\ \eta \end{bmatrix} + \begin{bmatrix} \tilde{B}(t) \xi + \tilde{E}_{um}d_{um} \end{bmatrix}$ is Hurwitz, and from Lemma 3.1 if $|\| \tilde{A}(t) \| | < \frac{\tilde{h}_{1+p}}{\tilde{h}_{1+p}}$, then the zero-input dynamics of (A8) is asymptotically stable. Moreover, as the signals $(\eta, \eta)$ in (A8) are bounded, the fault estimation error is also bounded. 

$$\blacksquare$$