1. Introduction

A protoneutron star (PNS) is a very hot \( T \sim 10^{11} \) K, rapidly rotating, lepton rich object that has been formed from the collapse of a massive stellar progenitor. It is believed that lepton and negative entropy gradients generates hydrodynamical instabilities which can play a significant dynamical role in the early stage of the PNS evolution. (Grossman et al. 1993, Bruenn & Dineva 1996, Miralles et al. 2000). While convective instability is presumably connected to the entropy gradient, the so-called neutron-finger instability is instead generated by a negative lepton gradient. The latter is due to dissipative processes which are rather fast in PNSs and it grows on a timescale \( \sim 30 – 100 \) ms, that is one or two orders of magnitude longer than the growth time of convection (Miralles et al. 2000). Turbulent motions caused by hydrodynamic instabilities in combination with rotation make turbulent dynamo one of the most plausible mechanism of the pulsar magnetism.

The character of turbulent dynamo depends on the Rossby number, \( Ro = P/\tau \), where \( P \) is the PNS spin period and \( \tau \) the turnover time of turbulence. If \( Ro \gg 1 \), the effect of rotation on turbulent motions is weak and the mean-field dynamo is inefficient. The small-scale dynamo can be operative, however, even at very large
Rossby numbers. If $Ro \leq 1$ and the turbulence is strongly influenced by the rotation, then the PNS can be subject to a large-scale mean-field dynamo action. Note that the small-scale dynamo can still operate in this case. The Rossby number is typically large in the convective region, $Ro \sim 10 - 100$, and the mean-field dynamo is not likely to work in this region. On the contrary, except very slowly rotating PNSs, the Rossby number is of the order of unity, $Ro \sim 1$, in the neutron-finger unstable region (Bonanno et al. 2003, 2005) where turbulent motions are slower, and the turbulence is strongly modified by rotation. This favors the efficiency of mean-field dynamos in the neutron-finger unstable region. This dynamo mechanism is then very different from the one proposed by Thompson & Duncan (1993) who argued that only small-scale dynamos can operate in most PNSs.

2. The model

The problem for dynamo modelling in PNSs has been described by Bonanno et al. (2003, 2005). We model the PNS as a sphere of radius $R$ with two different turbulent zones separated at $R_c < R$. The inner part ($r < R_c$) corresponds to the convective region, while the outer one ($R_c < r < R$) to the neutron-finger unstable region. The mean-field equation reads

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B} + \alpha \vec{B}) - \nabla \times (\eta \nabla \times \vec{B}),$$

(1)

where $\eta$ and $\alpha$ are the turbulent magnetic diffusivity and $\alpha$-parameter of the dynamo theory, respectively (For details see Bonanno et al. 2006). We assume that the rotation is the only large-scale motion and $\vec{v} = \vec{\Omega}(\vec{r}) \times \vec{r}$. We can basically distinguish two different dynamo regime: in the $\alpha - \Omega$ regime the dynamo action is mainly due to the differential rotation, while in the $\alpha^2$ regime the dynamo action is mainly due to the turbulent helicity. We found that most of the PNS do generate a large scale mean field dynamo provided the initial period is shorter than a critical period.

Fig. 1. On the left panel is depicted a typical field configuration for $\alpha^2$-dynamo while in the right panel is depicted a field configuration in the presence of differential rotation. Color levels are for the toroidal field, while dashed lines represents poloidal field lines.
\(P_c\) which is of the order of \(0.1 \sim 1\) s (See Bonanno et al. 2003). The typical field configuration in presence of differential rotation is depicted in Fig.1 where we have supposed that the boundary of the NF unstable region is located at 0.6 stellar radii. As it is possible to note the region of the maximum strength of the generated field is inside the region of instability.

3. Results

The critical period that determines the onset of the mean-field dynamo action is rather long, and dynamos should be effective in most PNSs. The unstable stage lasts \(\sim 40\) s, and this is sufficient for the dynamo to reach saturation. We can estimate a saturation field assuming the simplest \(\alpha\)-quenching with non-linear \(\alpha\) given by \(\alpha(\tilde{B}) = \alpha_{nf}(1 + B^2/B_{eq}^2)^{-1}\), where \(\tilde{B}\) is the characteristic value of the generated field, and \(B_{eq}\) is the equipartition small-scale magnetic field. The generated field reduces the \(\alpha\)-parameter and slows down the generation. The saturation is reached when non-linear \(\alpha\) becomes equal to the critical value \(\alpha_0\) corresponding to the marginal dynamo stability. Then, the saturation field is \(B_s \approx B_{eq}\sqrt{P_c/P - 1}\). A subsequent activity of the PNS as a radiopulsar is determined by the poloidal component of the saturation field, \(B_{ps}\) and by the ratio \(\xi = |B_p/B_s|\). Using the estimate \(B_s \sim B_{ps}(1 + \xi)\), for \(B_{ps}\) we obtain \(B_{ps} \approx B_{eq}(1 + \xi)^{-1}\sqrt{P_c/P - 1}\). The equipartition field, \(B_{eq} \approx 4\pi\rho v_T^2\), varies during the unstable stage, rising rapidly soon after collapse, reaching a quasi-steady regime, and then going down when the temperature and lepton gradients are smoothed. We can estimate \(B_{eq}\) at a peak of instabilities as \(\sim 10^{16}\) G in the convective zone, and \(\sim (1 - 3) \times 10^{14}\) G in the neutron-finger unstable zone (Urpin & Gil 2004). However, the temperature and lepton gradients are progressively reduced as the PNS cools down, hence, \(v_T\) and \(B_{eq}\) decrease whereas the turnover time \(\tau\) increases. The mean field dynamo description can be applied as the quasi-steady condition \(\tau_{cool} \gg \tau\) is fulfilled, \(\tau_{cool}\) being the cooling timescale. The final equipartition field is the same for the both the unstable zones. We thus have \(B_{eq} \sim (1 - 3) \times 10^{13}\) G for the largest turbulent scale \((\ell_T = L \sim 1 - 3\) km) if \(\tau_{cool} \sim few\) seconds.

We can distinguish few types of neutron stars which possess different magnetic characteristics: (i) Strongly magnetized neutron stars. If the initial period satisfies the condition \(P \leq P_m \equiv P_c[1 + (1 + \xi)^2]^{-1}\) then the dynamo action leads to the formation of a strongly magnetized PNS with \(B_{ps} > B_{eq} \sim 3 \times 10^{13}\)G. Since \(R_0 \leq 1\) for fast rotators, we can expect that the rotation of such stars is almost rigid. For a rigid rotation, the \(\alpha^2\)-dynamo is operative, hence, \(\xi \sim 1 - 2\) in the neutron-finger unstable region. Therefore, strongly magnetized stars can be formed if the initial period is shorter than \(\sim 0.1P_c\). Then for the surface field of such PNSs we have \(B_{ps} \sim 0.3B_{eq}\sqrt{P_c/P - 1} \sim 10^{13}\sqrt{P_c/P - 1}\) G. The shortest possible period is likely \(\sim 1\) ms, hence, the strongest mean magnetic field that can be generated by the dynamo action is \(\sim 3 \times 10^{14}\) G. Since the small-scale magnetic field is weaker than the large-scale field at the surface of such stars, one can expect that
radiopulsations from them may have a more regular structure than those from low-field pulsars. (ii) **Neutron stars with moderate magnetic fields.** If \( P_c > P > P_m \) then the generated mean poloidal field (for example, dipolar) is weaker than the small-scale field, \( B_{ps} < B_{eq} \sim 3 \times 10^{13} \text{G} \). The Rossby number is \( \sim 1 \) or slightly larger for such PNSs, and their rotation can be differential. Departures from a rigid rotation are weak if \( P \) is close to \( P_m \) but can be noticeable for \( P \) close to \( P_c \). As a result, the parameter \( \xi \) can vary within a wider range for this group of PNSs, \( \xi \sim 5-60 \), and the toroidal field should be essentially stronger than the poloidal one in the neutron-finger unstable region. The toroidal field is stronger than the small-scale field if \( P_c/2 > P > P_m \), and weaker if \( P_c > P > P_c/2 \). Note that both differential rotation and strong toroidal field can influence the thermal evolution of such neutron stars. Heating caused by the dissipation of the differential rotation is important during the early evolutionary stage because viscosity operates on a relatively short timescale \( \sim 10^2 - 10^3 \) yrs. On the contrary, ohmic dissipation of the toroidal field is a slow process and can maintain the surface temperature \( \sim (1 - 5) \times 10^5 \text{K} \) during \( \sim 10^8 \) yrs (Miralles et al. 1998). (iii) **Neutron stars with no large-scale field.** If the initial period is longer than \( P_c \) then a large scale mean-field dynamo does not operate in the PNS but the small-scale dynamo can still be efficient. We expect that such neutron stars have only small-scale fields with the strength \( B_{eq} \sim 3 \times 10^{13} \text{G} \) and no dipole field. Likely, such slow rotation is rather difficult to achieve if the angular momentum is conserved during the collapse, and the number of such exotic PNSs is small.

Likely, the most remarkable property of these neutron stars is a discrepancy between the magnetic field that can be inferred from spin-down measurements and the field strength obtained from spectral observations. Features in X-ray spectra may indicate the presence of rather a strong magnetic field \( \sim 3 \times 10^{13} \text{G} \) (or a bit weaker because of ohmic decay during the early evolution) associated to sunspot-like structures at the surface of these objects. The field inferred from spin-down data should be essentially lower.

In particular the recent discovery of young slowly spinning radio pulsar 1E 1207.4-5209 (Gotthelf & Halpern 2007) with a very weak magnetic field seems to support our theory.

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