Comment on “The low lying modes of triplet-condensed neutron matter and their effective theory”

L. B. Leinson

Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation RAS (IZMIRAN), 142190 Troitsk, Moscow, Russia

Abstract

Contrary to what is claimed in the article by P. F. Bedaque and A. N. Nicholson [Phys. Rev. C 87, 055807 (2013)], their result do not contradict but rather complement the conclusion of the paper by L. B. Leinson [Phys. Rev. C 85, 0655021 (2012)] with respect to the low lying nonunitary excitations in the anisotropic neutron superfluid. In fact, the corresponding Goldstone modes (angulons), existing at zero temperature, can be found from Eq. (59) of the work by Leinson. Being massless at absolute zero these modes acquire a mass gap at finite temperature.
The authors of a recent paper \[1\] derived a low-energy effective theory describing the massless Goldstone bosons (angulons) associated with broken rotational symmetry in a \(^3P_2(m_j = 0)\) condensed neutron superfluid at zero temperature. The authors claim that their result is in contradiction with the work by Leinson \[2\]. Below, we argue that this is not the case and the massless Goldstone modes are the solutions of the general dispersion equation at \(T = 0\).

In Ref. \[2\], the author studied the collective excitations of the \(^3P_2(m_j = 0)\) condensate at arbitrary temperatures. It was found that the eigenmodes of the pseudo-vector current should satisfy Eq. (59) of this work. The dispersion equation has been solved for the case of vanishing wave vector, \(q = 0\), and the conclusion was made that, at finite temperature, there are no gapless modes associated with broken rotational symmetry. Instead, there have been found the nonunitary excitations corresponding to periodic flapping of the total angular momentum \(\mathbf{j}\) (the same angular oscillations) which are gapless at zero temperature but having a mass gap, \(\tilde{\omega}_1\), at finite temperatures below the critical temperature \(T_c\).

This however is not in a contradiction to the results of Ref. \[1\] because, in the case of anisotropic pairing, the Nambu-Goldstone theorem, valid at absolute zero, is violated at finite temperatures. To be convinced in the violation of the Goldstone theorem one needs to solve the equation (59) of Ref. \[2\] for finite values of the temperature and wave vector. We solve this equation for the case when the temperature \(T\) is very small as compared to the superfluid energy gap \(\Delta\), and \(\omega, qV_F \ll 2\Delta\) (System of units \(\hbar = c = 1\), and the Boltzmann constant \(k_B = 1\)). The Fermi velocity is small in the nonrelativistic system of neutrons, \(V_F \ll 1\).

At low temperature, when \(y \equiv \Delta(T)/T \gg 1\), in Eq. (59) of Ref. \[2\], we obtain
\[
A \simeq \ln \pi - \ln (\gamma b y) - 2 \kappa \exp (-y b),
\]
and
\[
\mathcal{I}_{FF} \simeq \frac{1}{2 b^2} (1 + \delta_{FF}), \quad \delta_{FF} \equiv \frac{1}{s^2} (\cos^2 \theta_{n q} - s^2) \sqrt{\frac{2\pi}{b y}} e^{-y b},
\]
where \(s = \omega / (q V_F)\), and \(\theta_{n q}\) is the angle between the wave vector and the direction of the quasiparticle momentum.

In the case \(\sqrt{y} \gg 1\) one can neglect \(\delta_{FF}\), thus obtaining the following dispersion equation for the oscillations with \(m_j = \pm 1\):
\[
\left\langle s^2 - \cos^2 \theta_{n q} \right\rangle \frac{b_1^* b_1}{b^2} - \frac{2 \kappa \Delta^2}{q^2 V_F^2} b_1^* b_1 e^{-y b} \right\rangle^2 - \left\langle \frac{b_1^* b_1}{b^2} \cos^2 \theta_{n q} \right\rangle \right\rangle \left\langle \frac{b_1^* b_1}{b^2} \cos^2 \theta_{n q} \right\rangle = 0,
\]
(2)
where the angle brackets denote the averaging over directions of the quasiparticle momentum,
\[ \kappa \equiv \frac{1}{2} e^{1/4} K_0 (1/4) \simeq 0.989667, \tag{3} \]
and \( K_0 (z) \) being the Bessel function. The vectors \( \mathbf{b}_1 \) and \( \mathbf{\bar{b}}_2 \) are defined in Ref. [2]:
\[ \bar{b}^2 = \frac{1}{2} (1 + 3 \cos^2 \theta) \, , \quad \mathbf{b}_1^* \mathbf{b}_1 = \frac{3}{4} (1 + \cos^2 \theta) \, . \]
Here \( \theta \) is the polar angle on the Fermi surface.

Performing the angular integration one can find the solutions in the form:
\[ \omega^{(1)} = \sqrt{\frac{9}{4\pi \sqrt{3} + 18} q^2 V_F^2 + \frac{27 - 2\pi \sqrt{3}}{6\pi \sqrt{3} + 27} q^2 V_F^2 + \bar{\omega}_1^2 (T \ll \Delta)}, \tag{4} \]
\[ \omega^{(2)} = \sqrt{\frac{16\pi \sqrt{3} - 27}{12\pi \sqrt{3} + 54} q^2 V_F^2 + \frac{(27 - 2\pi \sqrt{3})}{6\pi \sqrt{3} + 27} q^2 V_F^2 + \bar{\omega}_1^2 (T \ll \Delta)}, \tag{5} \]
where
\[ \bar{\omega}_1 (T \ll \Delta) = \frac{6 \kappa^{1/2}}{2\pi \sqrt{3} + 9} \Delta (T) \left( \mathbf{b}_1^* \mathbf{b}_1 e^{-\mathbf{\bar{b}}_2} \right)^{1/2} . \tag{6} \]

In the limit \( T = 0 \), from Eqs. (4) and (5) we obtain the massless solutions which numerically recover the dispersion law for the angulons in Eq. (25) of Ref. [1]. In particular case where the propagation is along the \( z \)-axis and in the orthogonal direction the corresponding velocities \( s_i \equiv \omega_i / (q V_F) \) (for two modes 1 and 2) are
\[ s^{(1,2)}_z = \sqrt{\frac{27 - 2\pi \sqrt{3}}{6\pi \sqrt{3} + 27}} = 0.5198, \tag{7} \]
\[ s^{(1)}_\perp = \sqrt{\frac{9}{4\pi \sqrt{3} + 18}} = 0.4757, \tag{8} \]
\[ s^{(2)}_\perp = \sqrt{\frac{16\pi \sqrt{3} - 27}{12\pi \sqrt{3} + 54}} = 0.7096. \tag{9} \]
Thus, for zero temperature we find two Goldstone bosons which are associated with the breaking of rotational invariance in two planes, in accordance with Ref. [1].

At finite temperature the nonunitary Goldstone bosons acquire a mass gap, \( \bar{\omega}_1 \), which is exponentially small at low temperatures but considerably grows when the temperature increases. Its temperature dependence is presented in Fig. 1 which demonstrates the result obtained in Ref. [2]. The mass gap \( \bar{\omega}_1 \) passes through a maximum and tends to zero both for \( T \to 0 \) and for \( T \to T_c \).
FIG. 1. The mass gap of the low lying nonunitary excitations versus reduced temperature in the neutron $^3P_2(m_j = 0)$ condensate. The gap energy is given in units of $\Delta_0 \equiv \Delta (T = 0)$.

Thus the Goldstone theorem is violated in the case of spontaneous breaking of the rotation symmetry at finite temperatures. A natural explanation of this phenomenon is as follows. In our model, the low lying oscillations are associated with a flapping of the total angular momentum. This can be imagined as a departure of the symmetry axis of the bound pair about the symmetry axis of the equilibrated condensate, what is equivalent to oscillations of the preferred direction of the Cooper pair relative to the axis of the energy gap in the quasiparticle energy. The quasiparticle readjustment can not follow this rapid motion. As a result any rotation of the total angular momentum from its original orientation will cost energy which grows with the number of broken pairs and with the magnitude of the superfluid energy gap.

Immediately below the critical temperature the moment of inertia, associated with the quasiparticles, increases along with increase of the superfluid energy gap at lowering of the temperature. However the moment of inertia declines owing to a decrease of number of thermal quasiparticles along with the further lowering of the temperature. At zero temperature the restoring force is expected to vanish along with the number of thermal quasiparticles. Accordingly, we obtain the temperature behavior of the function $\tilde{\omega}_1$ shown in the above plot.

The mass gap in the angulon spectra leads to important consequences because, unlike
the massless modes predicted in Ref. [1], the gapped waves with \( \omega > qV_F \) do not undergo the Landau damping and can propagate at finite temperature.

It should be noted that a possible violation of the Nambu-Goldstone theorem at finite temperatures is known (See e.g. [3]).

REFERENCES

[1] P. F. Bedaque, and A.N. Nicholson, Phys. Rev. C 87, 055807 (2013)
[2] L. B. Leinson, Phys. Rev. C 85, 0655021 (2012)
[3] P. Wölfe, Phys. Rev. Lett. 37, 1279 (1976).