Karl Popper and the Copenhagen Interpretation

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Abstract

Popper conceived an experiment whose analysis led to a result that he deemed absurd. Popper wrote that his reasoning was based on the Copenhagen interpretation and therefore invalidated the latter. Actually, Popper’s argument involves counterfactual reasoning and violates Bohr’s complementarity principle. The absurdity of Popper’s result only confirms Bohr’s approach.

I called thee to curse mine enemies, and, behold, thou hast altogether blessed them.

Numbers 24:10

The emergence of quantum mechanics led to considerable progress in our understanding of physical phenomena. However, it also led to serious misconceptions. In my current work as a theoretical physicist, I recently examined a conceptual experiment that was proposed some time ago by Karl Popper (1982). Its feasibility was challenged by Collett and Loudon (1987) who claimed that such an experiment would be inconclusive. Nevertheless, an actual experiment is currently under way (Kim and Shih, 1999). The rigorous theoretical analysis of these experiments is quite intricate and I shall only briefly outline it here. Most of the present article is an attempt to analyze the meaning of what Popper wrote and to understand his way of reasoning. I found it most surprising when I read the original argument in his book.

Popper’s experiment is a variant of the one considered long ago by Einstein, Podolsky, and Rosen (1935): a source S emits pairs of particles having a broad angular distribution but precisely opposite momenta,

\[ \mathbf{p}_1 + \mathbf{p}_2 = 0. \]
The example given by Popper is that of pairs of photons emitted by the decay of positronium at rest. Actually, the wavelength of gamma rays emitted by positronium is much too short for realizing Popper’s experiment, but pairs of photons resulting from parametric down-conversion in a nonlinear crystal (Kim and Shih, 1999) are suitable for that purpose: these photons have precisely correlated (though not opposite) momenta, and this is all we need. If we wish, we can refer our calculations to a Lorentz frame moving with a constant velocity \( c (p_1 + p_2)/(E_1 + E_2) \), so that Eq. (1) holds in that frame.

Note that Eq. (1) seems to conflict with the quantum “uncertainty principle.” Popper writes “we consider pairs of particles that move in opposite directions along the positive and negative \( x \)-axis.” If these were classical particles, opposite momenta would indeed lead to opposite positions. However, the quantum dynamical variables in Eq. (1) do not commute with \((q_1 + q_2)\). For the components along any axis, we have uncertainty relations

\[
\Delta(p_1 + p_2) \Delta(q_1 + q_2) \geq \hbar,
\]

which set a limit on how precisely opposite the positions of the particles will be observed. This issue was analyzed by Collett and Loudon (1987) who came to the conclusion that Popper’s experiment (described below) could not give conclusive results. This is just one example of how hazardous it is to use classical reasoning when we discuss quantum phenomena. I shall return to this point later.

However, it is no less hazardous to make heuristic use of the “uncertainty principle” in order to draw quantitative conclusions. What must be done in case of doubt is to write the Schrödinger equation that describes the physical situation, and to derive rigorously unambiguous results. As will be shown below, the analysis of Popper’s experiment is much subtler than either Popper, or Collett and Loudon, were inclined to think.

Popper’s proposed experiment proceeds as follows: two observers, whom I shall call Alice and Bob in accordance with current practice in quantum information theory, are located on opposite sides of the source, with arrays of detectors as shown in Figure 1. Alice can place an opaque screen with a narrow slit of width \( a \) in the way of her photons, so that those passing through the slit are diffracted by an angle of the order of \( \lambda/a \), where \( \lambda \) is the wavelength of the photons. The narrower the slit, the wider is the scattering angle.

On this point, Popper writes that “the wider scattering angles go with a narrower slit, according to the Heisenberg relations.” Actually, the diffraction angle \( \lambda/a \) is a well known
result of classical optics. The wavelength of the photons, which is the quantity that we can actually measure, is related to their momentum by the relation $\lambda = \frac{h}{p}$, which readily follows from Einstein’s equation for the photoelectric effect, $E = h\nu$. The latter predates Heisenberg’s uncertainty principle by more than 20 years. Still before Heisenberg, it was de Broglie’s bold intuition to extend the relation $\lambda = \frac{h}{p}$ to massive particles, and in that case $\lambda$ is called the de Broglie wavelength. However, the issue is not just one of misappropriation of credit. Here, Popper wanted to invoke Heisenberg’s “uncertainty” because he had in mind that the detection of a particle that had passed through Alice’s slit was a measurement of the $y$-coordinate of that particle at it passed through the slit, and therefore also a virtual measurement of the position of the other particle, since the two had precisely opposite directions. Let us examine Popper’s text:

According to the EPR argument, we have measured $q_y$ for both particles . . . with the precision $\Delta q_y \equiv a$ . . . We can now calculate the $y$-coordinate of the [other] particle with approximately the same precision . . . We thus obtain fairly precise ‘knowledge’ about the $q_y$ position of this particle—we have ‘measured’ its position indirectly. And since it is, according to the Copenhagen interpretation, our knowledge which is described by the theory . . . we should expect that the momentum of the [second] beam scatters as much as that of the beam that passes through the slit . . .

To sum up: if the Copenhagen interpretation is correct, then any increase of our mere knowledge of the position . . . of the particles . . . should increase their scatter . . .

The italics that appear in the above excerpt are those in the book. Popper refrains from openly saying that the above prediction is absurd (as it obviously is). He only says that he is “inclined to predict” that the test will decide against the Copenhagen interpretation. On this, I have several comments.

First, is not at all clear why Popper associates this absurd prediction (particle scatter due to potential knowledge by an observer) with the Copenhagen interpretation. This is another example of credit misappropriation, much worse than having quoted Heisenberg instead of Einstein or de Broglie. Whatever the “Copenhagen interpretation” is (a point that I shall discuss later), it is reasonable to expect that it is somehow related to the views
expressed by Niels Bohr. However, Popper himself wrote explicitly that his proposed experiment was an extension of the argument of Einstein, Podolsky, and Rosen (1935). It is well known that their argument was promptly criticized by Bohr (1935). I find it quite remarkable that an opinion which is diametrically opposite to Bohr’s be called the “Copenhagen interpretation.”

I also have other, more serious objections to the terminology used in the passage quoted above. In particular, I take exception to the phrase “we have measured $q_y$” of some particle. Here however, my criticism is not aimed at Popper because we are all guilty of occasionally talking like that. This is a misleading language, as explained long ago by Kemble (1937):

> We have no satisfactory reason for ascribing objective existence to physical quantities as distinguished from the numbers obtained when we make the measurements which we correlate with them. There is no real reason for supposing that a particle has at every moment a definite, but unknown, position which may be revealed by a measurement of the right kind, or a definite momentum which can be revealed by a different measurement. On the contrary, we get into a maze of contradictions as soon as we inject into quantum mechanics such concepts carried over from the language and philosophy of our ancestors... It would be more exact if we spoke of “making measurements” of this, that, or the other type instead of saying that we measure this, that, or the other “physical quantity.”

Terms that Popper used, such as “knowledge of the $y$-coordinate... or the $q_y$ position of this particle” are flagrant (and admittedly quite common) abuses of an improper language. When we are discussing quantum theory, we should refrain from using classical terminology—or at least be aware that we do so at our own risk.

In classical mechanics, a particle has (ideally) a precise position and a precise momentum. We can in principle measure them with arbitrary accuracy and thereby determine their numerical values. In quantum mechanics, a particle also has a precise position and a precise momentum. However, the latter are mathematically represented by self-adjoint operators in a Hilbert space, not by ordinary numbers. Their nature is quite different from that of the classical position and momentum. In the early quantum literature, operators
were called $q$-numbers, while plain numbers were $c$-numbers (Dirac, 1926). Likewise, to avoid confusion, we should have used in quantum theory names such as $q$-position and $q$-momentum, while the corresponding classical dynamical variables would have been called $c$-position and $c$-momentum. If such a distinction had been made, it would have helped to prevent much of the present confusion about quantum theory. It is the imperfect translation from the $q$-language to the $c$-language that led to the unfortunate introduction of the term “uncertainty” in that context.

We may note, incidentally, that the theory of relativity did not cause as much misunderstanding and controversy as quantum theory, because people were careful to avoid using the same nomenclature as in nonrelativistic physics. For example, elementary textbooks on relativity theory distinguish “rest mass” from “relativistic mass” (hard core relativists call them simply “mass” and “energy”).

The criticism above was aimed at the terminology used by Popper in proposing his experiment. Now, it is time to analyze the substance. First, we have to find out how precisely the two particles of each pair will be aligned opposite to each other, in spite of the uncertainty relation in Eq. (2). Note that, contrary to the so-called “uncertainty principle” which is an ill defined concept and has only a heuristic meaning, Eq. (2) is a rigorous mathematical consequence of the quantum formalism. It puts a lower bound on the product of the standard deviations of the results of a large number of measurements performed on identically prepared systems. Each one of these measurements is assumed to have perfect accuracy (any experimental inaccuracy would have to be added to the quantum dispersion). There is no “uncertainty” connotation here, unless this uncertainty merely refers to future outcomes of potential, perfectly accurate measurements that may be performed on such systems (Ballentine, 1970).

A long calculation (to be published separately) is needed to estimate how precise is the angular alignment of two particles emitted with opposite momenta. Actually, what Eq. (2) says is that if an ensemble of pairs of particles is prepared in such a way that $(p_1 + p_2)$ is sharp, then the positions of the points halfway between the particles are very broadly distributed. It says nothing on the angular alignment of distant particles. On that issue, a detailed calculation shows that if one particle is found in the direction given by polar and azimuthal angles $\theta$ and $\phi$, then the other will be found very nearly in the opposite direction, with angles $\pi - \theta$ and $\phi \pm \pi$, respectively. The allowed deviation from
perfect alignment is too small to be of any consequence in the present discussion.

It is therefore correct to assume, as Popper did, that if a particle is detected behind Alice’s slit, and if an identical slit were placed by Bob in a symmetric position, then Bob would definitely detect the other particle of that pair there. However, this does not mean that Bob’s knowledge creates a “virtual slit” through which his particles are diffracted by the same angle $\lambda/a$. Bob’s knowledge has no physical consequence because it is manifestly counterfactual. This can easily be seen by considering other counterfactual experiments. For example, Bob also knows, after he was informed by Alice of what she found, that if he had placed a slit of width $a/2$ at a position whose distance from the source is one half of the distance of Alice’s slit, then he would have detected his particle within that slit with certainty. In that case, his “virtual slit” is narrower, and therefore the diffraction angle is wider by a factor 2. In brief, we can imagine infinitely many such counterfactual experiments (which are mutually exclusive, of course), and each one of these conceptual slits leads to a different observable diffraction angle, which is absurd.

There is no doubt that Popper was right when he was “inclined to predict” that the test would give a negative result. However, Popper concluded that “the test decides against the Copenhagen interpretation” and this assertion requires further scrutiny. What is, indeed, the Copenhagen interpretation? There seems to be at least as many different Copenhagen interpretations as people who use that term, probably there are more. For example, in two classic articles on the foundations of quantum mechanics, Ballentine (1970) and Stapp (1972) give diametrically opposite definitions of “Copenhagen.” There is no real conflict between Ballentine and Stapp on how to understand quantum mechanics, except that one of them calls Copenhagen interpretation what the other considers as the exact opposite of the Copenhagen interpretation. I shall now explain my own Copenhagen interpretation. It relies on articles written by Niels Bohr. Whether or not you agree with Bohr, he is the definitive authority for deciding what is genuine Copenhagen.

Quantum mechanics provides statistical predictions for the results of measurements performed on physical systems that have been prepared in specified ways (Peres, 1995). (I hope that everyone agrees at least with that statement. The only question here is whether there is more than that to say about quantum mechanics.) The preparation of quantum systems and their measurement are performed by using laboratory hardware which is described in classical terms. If you have doubts about that, just have a look at
any paper on experimental physics. The necessity of using a classical terminology was emphasized by Bohr (1949) whose insistence on this point was very strict:

However far the [quantum] phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word ‘experiment’ we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.

The keywords in that excerpt are: classical terms ... unambiguous language ... terminology of classical physics. Bohr did not say that there are in nature classical systems and quantum systems. There are physical systems for which we may use a classical description or a quantum description, according to circumstances, and with various degrees of approximation. It is according to our assessment of the physical circumstances that we decide whether the $q$-language or the $c$-language is appropriate. Physics is not an exact science, it is a science of approximations. Unfortunately, Bohr was misunderstood by some (perhaps most) physicists who were unable to make the distinction between language and substance, and he was also misunderstood by philosophers who disliked his positivism.

It is remarkable that Bohr never considered the measuring process as a dynamical interaction between an apparatus and the system under observation. Measurement had to be understood as a primitive notion. Bohr thereby eluded questions which caused considerable controversy among other authors (Wheeler and Zurek, 1983). Bohr willingly admitted that any intermediate systems used in the measuring process could be treated quantum mechanically, but the final instrument always had a purely classical description (Bohr, 1939):

In the system to which the quantum mechanical formalism is applied, it is of course possible to include any intermediate auxiliary agency employed in the measuring process [but] some ultimate measuring instruments must always be described entirely on classical lines, and consequently kept outside the system subject to quantum mechanical treatment.
Yet, a quantum measurement is not a supernatural process. Measuring apparatuses are made of the same kind of matter as everything else and they obey the same physical laws. It therefore seems natural to use quantum theory in order to investigate their behavior during a measurement. This was first attempted by von Neumann (1932) in his treatise on the mathematical foundations of quantum theory. In the last section of that book, as in an afterthought, von Neumann represented the apparatus by a single degree of freedom whose value was correlated to that of the dynamical variable being measured. Such an apparatus is not, in general, left in a definite pure state, and does not admit a classical description. Therefore, von Neumann introduced a second apparatus which observes the first one, and possibly a third apparatus, and so on, until there is a final measurement, which is not described by quantum dynamics and has a definite result (for which quantum mechanics can only give statistical predictions). The essential point that was suggested, but not proved by von Neumann, is that the introduction of this sequence of apparatuses is irrelevant: the final result is the same, irrespective of the location of the “cut” between classical and quantum physics. (At this point, von Neumann also speculated that a final step would involve the consciousness of the observer—a rather bizarre statement in a mathematically rigorous monograph.)

These different approaches of Bohr and von Neumann were reconciled by Hay and Peres (1998), who introduced a dual description for the measuring apparatus. It obeys quantum mechanics while it interacts with the system under observation, and then it is “dequantized” and is described by a classical Liouville density, which provides the probability distribution for the results of the measurement. Alternatively, the apparatus may always be treated by quantum mechanics, and be measured by a second apparatus which has such a dual description. Hay and Peres showed that these two different methods of calculation give the same result, provided that the measuring apparatus satisfies appropriate conditions (otherwise, it is not a valid measuring apparatus).

The other fundamental feature of Bohr’s presentation of quantum theory is the principle of complementarity, which asserts that when some types of predictions are possible, others are not, because they are related to mutually incompatible experiments. For example, in the situation described by Einstein, Podolsky, and Rosen (1935), the choice of the experiment performed on the first system determines the type of prediction that can be made for the results of experiments performed on the second system (Bohr, 1935).
In Popper’s experiment, Bob can predict what would have happened if he had placed slits of various sizes at various positions, or no slit at all. However, all these possible setups are mutually incompatible. In particular, if Bob puts no slit at all, the result he obtains is not the one he would have obtained if he had put a slit. Counterfactual experiments need not have consistent results (Peres, 1978).

Note that Bohr did not contest the validity of counterfactual reasoning. He wrote (Bohr, 1935):

Our freedom of handling the measuring instruments is characteristic of the very idea of experiment . . . we have a completely free choice whether we want to determine the one or the other of these quantities . . .

Thus, Bohr found it perfectly legitimate to consider counterfactual alternatives: observers have free will and can arbitrarily choose their experiments. However, each experimental setup must be considered separately. In particular, no valid conclusion can be drawn from the comparison of possible results of mutually incompatible experiments. Bohr was sometimes accused of being elusive, because his approach does not provide answers to questions in which people may be interested. There are indeed questions that seem reasonable but do not correspond to any conceivable experiment: quantum theory has no obligation to answer meaningless questions.

To conclude this article, let me report the result of a rigorous analysis of Popper’s experimental setup, where only Schrödinger’s equation is used, without invoking any controversial interpretation. The irony of the answer is that Bob does observe a diffraction broadening, as if he had a virtual slit! However, that slit is not located between him and the source, but is precisely located where Alice’s real slit is, and is indeed identical to it. An experiment similar to Popper’s proposal was actually performed by Strekalov et al. (1995), who used a double slit, so that Bob had a virtual double slit, producing a neat interference pattern, not only a diffraction broadening. Figure 2 is a simplified sketch of that experiment. Its complete theoretical analysis involves advanced concepts of quantum optics and is quite intricate. I shall now give a brief outline of the theory, based on Schrödinger’s equation.

The only “knowledge” needed in the analysis of the experiment is the factual one, on the preparation and observation procedures. That knowledge is formally encapsulated in the
Hilbert-space vectors $|\Psi_0\rangle$ and $|\Psi_d\rangle$, whose coordinate-space representation is localized in the source of particles and in the detectors that were excited, respectively. (These vectors are also known as “quantum states.”) Schrödinger’s equation asserts that the initial vector $|\Psi_0\rangle$ evolves in time, as long as there is no detection event, according to a unitary transformation

$$|\Psi_0\rangle \rightarrow |\Psi_t\rangle = U_t |\Psi_0\rangle,$$

where $U_t = e^{-iHt/\hbar}$ for a time-independent Hamiltonian $H$. In the present case, the double slit can be represented by an infinite potential in $H$, or by an equivalent boundary condition.

Born’s rule (which makes the connection between the quantum formalism and observed probabilities of macroscopic events) asserts that the probability that a particular pair of detectors will “click” at time $t$ is $P = |\langle \Psi_d, \Psi_t \rangle|^2$, where the symbol $\langle u, v \rangle$ denotes the scalar product of two vectors, $|u\rangle$ and $|v\rangle$. We thus have (Peres, 1995)

$$P = |\langle \Psi_d, U_t \Psi_0 \rangle|^2 = |\langle U_t^\dagger \Psi_d, \Psi_0 \rangle|^2,$$

where $U_t^\dagger = U_{-t}$ is the unitary operator for the time-reversed dynamics. It may be practically impossible to realize experimentally that reversed dynamics, but it is legitimate to perform the calculation of the ordinary dynamics by proceeding backwards, starting at the detectors and ending at the source. In the present case, this is indeed much easier, because $|\Psi_0\rangle$ is entangled and has to satisfy Eq. (1), while

$$|\Psi_d\rangle = |\psi_1\rangle \otimes |\psi_2\rangle,$$

is a tensor product of two vectors, whose coordinate-space representations are well separated, since they are localized in the two detectors. Moreover, the Hamiltonian is the sum of those of the two particles, since the latter do not interact after they leave the source. Therefore the unitary evolution also factorizes: $U_{-t} = U_1 \otimes U_2$. We thus propagate $|\psi_1\rangle$ and $|\psi_2\rangle$ from the detectors toward the source. We have to compute

$$P = |\langle \Psi_0, (U_1 \otimes U_2, \Psi_2 \rangle |^2.$$

Now, since $|\Psi_0\rangle$ satisfies Eq. (1), the only contribution to $P$ comes from components of $|U_1 \psi_1 \otimes U_2 \psi_2\rangle$ with opposite momenta that also satisfy Eq. (1). This is illustrated in
Figure 2. For example, if we record all the detections on Bob’s side that are in coincidence with one particular detector of Alice, then Bob will observe an ordinary double-slit interference pattern, generated by a “virtual” double-slit, that actually is Alice’s real slit.

Note that it is necessary, for such an observation to be possible, that the region of the nonlinear crystal from where the rays emerge be very broad (Hong and Mandel, 1985) and the emergence point be undetermined. Likewise, if the experiment were done with positronium as Popper originally suggested, the positronium ought to be prepared with $\Delta y$ much larger than the distance between the slits. Expressed in an informal language, the requirement is that each one of the two photons that pass through both slits must also originate in both regions of the source. This demand is similar to the conditions required for the Pfleegor and Mandel (1967) experiment, where a single photon originates from two different lasers and gives rise to first order interference. A similar analysis also applies to Popper’s original experiment with a single slit (however, it would be more difficult to draw for it a figure like Figure 2).

In summary, according to the Copenhagen interpretation, as Bohr apparently understood it, quantum theory is not a description of physical reality. It also does not deal with anthropomorphic notions such as knowledge or consciousness. All it does is to provide correct answers to meaningful questions about experiments done with physical systems.

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**FIGURE 1.** Popper’s conceptual experiment. A pair of photons with opposite momenta is emitted by the source S. Alice’s detectors are on the left, those of Bob on the right.

**FIGURE 2.** Simplified sketch of the experiment of Strekalov *et al.* (1995). The figure shows a single pair of photons with opposite momenta, emitted by the source S. When many such pairs are detected in coincidence, interference patterns appear on both sides.
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Figure 1

Figure 2