Proximity Approaches and Design Strategies for Non-Cooperative Rendezvous* 
V-bar Hopping vs. Spiral Approach

Takahiro SASAKI,† Yu NAKAJIMA, and Toru YAMAMOTO
Research and Development Directorate, JAXA, Tsukuba, Ibaraki 305–8505, Japan

As the amount of debris in orbit increases, so does the risk of collisions and their seriousness. All nations involved with space operations acknowledge this growing threat. One solution receiving increased attention is active debris removal. The first step in a debris removal mission would be to approach the debris. In this phase, it is important to ensure passive abort safety and to guarantee the robustness against collisions in the case of off-nominal thruster burns, that may be caused by spacecraft anomalies such as navigation sensor or actuator failures. This paper compares two types of passive abort safe trajectories—the V-bar hopping and spiral approaches—considering the ΔV budget, the duration of operations, and variation in the line-of-sight vector to the target. This paper also proposes design strategies for determining the parameters in the two candidate trajectories, considering passive abort safety. The robustness of the trajectories against collisions due to off-nominal thruster burns is also demonstrated through Monte Carlo simulations. The paper investigates which trajectories are suitable for an active debris removal mission to a non-cooperative target.

Key Words: Active Debris Removal, Formation Flying, Proximity Operations, Spacecraft

Nomenclature

\[ a: \text{semi-major axis, m} \]
\[ e: \text{eccentricity} \]
\[ e: \text{eccentricity vector} \]
\[ i: \text{inclination, rad} \]
\[ i: \text{inclination vector} \]
\[ N_{\text{hop}}: \text{the number of hops on the V-bar} \]
\[ N_{\text{spiral}}: \text{the number of spirals along the V-bar} \]
\[ u: \text{mean argument of latitude, rad} \]
\[ v: \text{velocity vector w.r.t. hill coordinates, m/s} \]
\[ x: \text{position vector w.r.t. hill coordinates, m} \]
\[ \alpha: \text{relative orbit elements} \]
\[ \gamma: \text{off-nominal rate} \]
\[ \chi: \text{hopping rate} \]
\[ \psi: \text{spiral rate} \]
\[ \Delta v: \text{impulsive delta-v, m/s} \]
\[ \theta: \text{relative ascending node, rad} \]
\[ \lambda: \text{mean longitude, rad} \]
\[ \phi: \text{relative perigee, rad} \]
\[ n: \text{orbital mean motion, rad/s} \]
\[ \Omega: \text{right ascension of ascending node, rad} \]

Subscripts

\[ 0: \text{initial value} \]
\[ d: \text{destination value} \]
\[ N: \text{normal direction} \]
\[ R: \text{radial direction} \]
\[ T: \text{tangential direction} \]

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†Corresponding author, sasaki.takahiro@jaxa.jp

1. Introduction

Orbital debris has become an increasingly severe problem and a challenge to all nations. To solve this problem, active debris removal (ADR) missions are gaining progressively more attention. The first step in an ADR mission would be to approach the debris, which is usually a non-cooperative target. In a non-cooperative rendezvous, obtaining continuous sensor measurements for navigation is quite difficult due to loss of sensor lock or the effect of solar reflection on the optical camera. It is important to ensure passive abort (PA) safety and to guarantee robustness against collisions in the case of off-nominal thruster burns, that may be caused by spacecraft anomalies such as navigation sensor or actuator failures.

Considerable knowledge of safe PA trajectories has been gained from previous missions that entailed both cooperative and non-cooperative rendezvous. Rendezvous with the International Space Station (ISS), which is a cooperative target, uses one of the safe PA trajectories, the R-bar and V-bar approaches. They have been extensively and safely implemented, especially when a precise relative navigation solution is required. The Space Shuttle used both R-bar and V-bar approaches as Fig. 1 shows. The ETS-VII is an engineering test satellite developed by the National Space Development Agency of Japan (NASDA). It approached the target using V-bar hopping from 1100 m to 150 m based on relative GPS navigation, and then approached along the V-bar using rendezvous laser radar (RVR) navigation in the final phase. The Orbital Express was a flight demonstration program established by the Defense Advanced Research Projects Agency (DARPA) to develop and validate key technologies required for cost-effective servicing of next-generation satellites. Some rendezvous scenarios have been successfully
is an Earth observation mission that uses interferometry to deliver a digital-elevation map of the Earth. It exploits cooperative FF implemented by e/i-vector separation. GRACE was an Earth observation mission to map the gravity field of the Earth. It exploited cooperative FF in leader-follower configuration, and it used e/i-vector separation only for swapping the leader satellite at one point in the mission. PRISMA demonstrated the practicality of GPS-based, radio-frequency based, and vision-based technologies using fiducial markers for cooperative FF. PRISMA was also used to mimic some scenarios of non-cooperative rendezvous, but the intersatellite link was never deactivated. The other mission that includes a not fully-cooperative rendezvous is the Restore-L mission\(^5\) planned by the National Aeronautics and Space Administration (NASA). It is equipped with a robot-controlled service and capture satellite. Since the target satellite (i.e., Landsat-7) is still operational, it can provide complete attitude control and communicate with the ground.

There are very few missions that perform ADR using a non-cooperative rendezvous. The AVANTI experiment\(^{16–18}\) in the FireBird missions of the German Aerospace Center (DLR) focused on a non-cooperative target. In this mission, the rendezvous was performed with a relative GNC designed to exploit e/i PA safety. The ESA is planning the e.Deorbit mission,\(^9\) the objective of which is to remove Envisat (a 8-ton inactive satellite) with a challenging rendezvous, capture, and de-orbit process. The plan is to perform e/i-vector separation. The ESA is also studying a hybrid trajectory method consisting of a V-bar hopping approach followed by e/i-vector separation prior to deorbit as a part of the e.Inspector mission,\(^{20}\) which is a rendezvous-only mission with a cubesat. Additionally, Astroscale is now developing the ADR satellite ELSA-d,\(^{21}\) which consists of two spacecraft, a chaser (180 kg) and a target (20 kg). They will be launched stacked together and will demonstrate technologies for rendezvous and proximity operations, including e/i-vector separation trajectories, but the target is not fully non-cooperative because it has a docking plate. JAXA is joining the Commercial Removal of Debris Demonstration (CRD2) project,\(^{22,23}\) the aim of which is developing space debris removal as a new, sustainable space business. The objective of the CRD2 project is to remove large space debris (e.g., the upper stage of Japan’s H-IIA rocket) at low cost using a small ADR satellite. These missions are summarized in Table 2. Tables 1 and 2 show that V-bar hopping should not be used for a non-cooperative rendezvous. The AVANTI experiment is the only mission for which the spiral trajectory is demonstrated on orbit for non-cooperative ADR. Furthermore, quantitative trade-off research to investigate non-cooperative rendezvous has not been done yet. This paper presents a fairly objective comparison of the V-bar hopping approach and the spiral approach for non-cooperative rendezvous, which would be encountered during ADR.

Some parameters are needed to design the candidate trajectories. The number of hops on the V-bar approach and the hopping interval/duration are needed to design the V-bar hopping trajectory. Designing a spiral trajectory requires
the number of spirals along the V-bar, which depends on the difference in altitudes, and the magnitude of the e/i-vector. Furthermore, the hopping and spiral rates are introduced. As the ADR satellite approaches the target, the hopping interval becomes shorter at the constant hopping rate in the V-bar hopping approach and the size of the relative orbits becomes smaller at the constant spiral rate every orbital revolution in the spiral approach. Basically, the accuracy and reliability of the navigation system are improved as the ADR satellite becomes closer to the target debris. This paper presents analytical solutions for these parameters while considering PA safety and duration of operation, and proposes design strategies for non-cooperative rendezvous using the candidate trajectories. Using the proposed strategies, determining the parameters for safe trajectory design becomes easier, and quantitative trade-offs between the candidates can be made objectively. As a result, the characteristics of each trajectory can be examined and their applicability to non-cooperative rendezvous missions can be clarified.

This paper first discusses the approach policy of two candidate trajectories for non-cooperative targets of ADR missions and shows the design of example nominal trajectories. This paper then considers the ΔV budget, duration of operations, and variation in the line-of-sight (LoS) vector through numerical simulations. The robustness of candidate trajectories against collisions is also demonstrated through Monte Carlo simulations, and the characteristics of the two candidate trajectories for ADR missions to non-cooperative targets are examined and clarified.

2. Problem Statements

This section summarizes three problem statements.

Mission scenario: This paper focuses on the proximity approaches (the V-bar hopping and spiral approaches) of an ADR satellite from 1 km to 100 m, while minimizing the risk of collision when off-nominal thruster burn occurs. Since a delay in detecting mechanical failure is critical in a proximity operation, PA safety is an important factor during the rendezvous phase. In this paper, the goal of ADR satellites is defined by the nominal destination at 100 m, which is different between trajectories as shown in Fig. 2, where ADR satellites precisely estimate the motion of the target debris during the final approach along the circular path around the debris and capture.

Definition of coordinates and safe regions: LVLH, R-bar/V-bar, and RTN coordinates are defined with their origins set to the target’s center of mass, as shown in Fig. 2. An LVLH coordinate is frequently used for the reference frame of the Earth observation satellite’s attitude. The LVLH and RTN coordinates are often used in V-bar hopping and spiral approaches, respectively. In defining the volume through which a satellite can safely approach, the Keep-Out-Sphere (KOS) should be designated as a forbidden volume that the satellite cannot enter on approach, as shown by the red sphere in Fig. 2. However, the proximity approach of a satellite generally entails GNC errors, including a guidance error (i.e., due to modeling error in guidance algorithm), navigation error (i.e., in estimating orbital and attitude determinations) and control error (i.e., excessive/insufficient thruster burn or control input delay). This means that the nominal destinations for V-bar hopping and spiral approach should be set while determining a safe distance, as shown in Fig. 2.

Variation in the LoS vector: ADR satellites need to keep debris in the sensor field-of-view (FOV) to derive relative position and attitude. It is preferable to maintain LVLH attitude as long as the debris is kept in sensor FOV since attitude-pointing control usually imposes strict constraints on a satellite’s systems, such as navigation sensors (e.g., optical cameras and 3D LiDARs), power control, thermal maintenance, and communications. Therefore, small variation in the LoS vector leads to simplifying satellite system design. In this paper, the LoS vector is used as one of the comparative items for trade-offs.

3. Trajectory Design

This section discusses two types of trajectories (V-bar hopping and spiral approach) considering trajectory safety.

3.1. V-bar hopping approach

V-bar hopping would seem to be the most balanced approach for a debris removal satellite in terms of PA safety
and fuel consumption considering the V-bar hold for observing the target and shooting many images. After enough data is collected, the ADR satellite will fly around to the capture surface of the debris. In this approach, the satellite covers a predefined distance in small steps through ΔV impulses in the radial direction or radial/tangential directions at the V-bar crossing.

**Hill-Clohessy-Wiltshire (HCW) equation:** If the orbit of the target debris is circular, a HCW equation is a useful representation for calculating the required ΔV. Since a V-bar hopping approach has no out-of-plane (±y) motion, the in-plane components in the HCW equations are only used as follows:

\[
\begin{bmatrix}
x_{c}(\Delta t) \\
v_{c}(\Delta t)
\end{bmatrix} =
\begin{bmatrix}
\Theta_{11}(\Delta t) & \Theta_{12}(\Delta t) \\
\Theta_{21}(\Delta t) & \Theta_{22}(\Delta t)
\end{bmatrix}
\begin{bmatrix}
x_{c}(0) \\
v_{c}(0)
\end{bmatrix}
\]

(1)

where

\[
\begin{align*}
\Theta_{11}(\Delta t) &= \begin{bmatrix} 1 & 6(n\Delta t - \sin n\Delta t) \\ 0 & 4 - 3\cos n\Delta t \end{bmatrix}, \\
\Theta_{12}(\Delta t) &= \begin{bmatrix} \frac{1}{n}(4\sin n\Delta t - 3n\Delta t) & \frac{2}{n}(1 - \cos n\Delta t) \\ -\frac{2}{n}(1 - \cos n\Delta t) & \sin n\Delta t \end{bmatrix}, \\
\Theta_{21}(\Delta t) &= \begin{bmatrix} 0 & 6n(1 - \cos n\Delta t) \\ 0 & 3n\sin n\Delta t \end{bmatrix}, \\
\Theta_{22}(\Delta t) &= \begin{bmatrix} 4\cos n\Delta t - 3 & 2\sin n\Delta t \\ -2\sin n\Delta t & \cos n\Delta t \end{bmatrix}
\end{align*}
\]

(2-5)

Together with \(x_{c} = [x, z]^{T}\) and \(v_{c} = [v_{x}, v_{z}]^{T}\) with respect to hill coordinates, and Θ is the state transition matrix of the HCW equations. The symbol \(n\) is the orbital mean motion of the target. Using Eq. (1), the required ΔV \( (= v_{c}(0)) \) which achieves the desired position \(x_{c}(\Delta t)\) in Δt s is obtained by

\[
v_{c}(0) = \Theta_{12}^{-1}(\Delta t)(x_{c}(\Delta t) - \Theta_{11}(\Delta t)x_{c}(0)).
\]

(6)

Note that V-bar hopping usually assumes \(z(0) = z(\Delta t) = 0\) (at the point on the V-bar). This equation provides a numerical solution set of the required ΔV. If \(\text{rank}(\Theta_{12}(\Delta t)) = 2\), the inverse matrix \(\Theta_{12}^{-1}(\Delta t)\) is always obtained. However, if \(\text{det}(\Theta_{12}(\Delta t)) \neq 0\) then this cannot be solved. It occurs when \(\text{det}(\Theta_{12}(\Delta t)) = 0\) or equivalently

\[
8(1 - \cos nt) - 3nt\sin nt = 0.
\]

(7)

Next, an analytical solution set of the required ΔV is introduced using the HCW equation in Eq. (1). Using the assumption of a V-bar hopping

\[
z(0) = z(\Delta t) = 0,
\]

(8)

the following solution can be obtained:

\[
v_{x}(0) = \frac{n\sin n\Delta t}{8(1 - \cos n\Delta t) - 3n\Delta t\sin n\Delta t} \cdot \Delta x
\]

(9)

\[
v_{z}(0) = \frac{4n\sin^{2}(n\Delta t/2)}{8(1 - \cos n\Delta t) - 3n\Delta t\sin n\Delta t} \cdot \Delta x
\]

(10)

\[
v_{x}(\Delta t) = v_{x}(0)
\]

(11)

\[
v_{z}(\Delta t) = -v_{z}(0).
\]

(12)

Note that \(\Delta x = |x(\Delta t) - x(0)|\) is a step on the V-bar. From Eqs. (9) and (10), the range for choosing \(\Delta t\) is restricted as follows:

\[
8(1 - \cos n\Delta t) - 3n\Delta t\sin n\Delta t > 0
\]

(13)

\[
0 < \Delta t < \frac{\pi}{n}.
\]

(14)

Equation (13) satisfies the condition of singularity avoidance in Eq. (7). This is a PA constraint of the V-bar hopping approach, which guarantees a satellite is going away from the debris after maneuver failure. Figure 3 compares the numerical and analytical solutions of V-bar hopping trajectory in Eqs. (6) and (9)–(10), which were found to match within expected error in the numerical computations. Note that the step on the V-bar and the time are \(\Delta x = 0.1\) m and \(\Delta t = 2000\) s, respectively, as given in the simulation.

**Parameter design policy:** The constraint of burn only in the radial direction requires firing at specific times of the orbit (i.e., the second burn is exactly half an orbital period \(\pi/n\) after the first one). Since such a maneuver plan does not take into account the margin for PA safety from Eq. (14), a thruster misalignment or navigation error could disrupt PA safety. This paper allows for both radial and tangential maneuvers to ensure an arbitrary margin. Based on the constraint for PA safety in Eq. (14) and the reference, assumptions for designing the parameters are given as follows:

- Total hopping time \(t_{\text{total}}\) can be designed according to the mission.
- One hopping interval is set to \(\Delta t\), obtained from the number of hops \(N_{\text{hop}}\) and Eq. (14).
- Both radial and tangential maneuvers are allowed.
- Each hopping duration is set proportional to distance from the target, thus the hopping interval becomes shorter as the chaser gets closer to the target at the constant rate \(\gamma_{\text{hop}}\).

Note that \(\gamma_{\text{hop}}\) is the hopping rate, which is the proportional constant, and the number of hops \(N_{\text{hop}}\) is determined by

![Comparison of numerical and analytical solutions](image-url)

Fig. 3. Comparison of numerical and analytical solutions (\(\Delta t = 1/3\) rev, \(t_{\text{total}} = 1\) rev).
Using these design parameters, a desired position of the chaser is likely to enter the KOS. Therefore, the minimum number of hops \( N_{\text{hop}, \min} \) is defined by the GNC error and thruster maneuver error.

**Example:** From these assumptions, the parameters of a V-bar hopping trajectory are \( y_{\text{hop}} = 0.8 \), \( t_{\text{total}} = 6000 \) s (which is described as 1 orbital period 2\( \pi / (5n) \)), and \( \Delta t = 1200 \) s (which is described as 1/5 orbital period 2\( \pi / (5n) \)), and \( N_{\text{hop}, \min} = 5 \). From Eq. (15), \( N_{\text{hop}} = 5 \). Note that \( \Delta t \) is described as a constant and must be shorter than 1/2 orbital period \( \pi / n \) in Eq. (14). Using these design parameters, a desired position set \( x_d \) is calculated as follows:

\[
x_d = \begin{bmatrix} x_{d0} \\ x_{d1} \\ x_{d2} \\ x_{d3} \\ x_{d4} \\ x_{d5} \end{bmatrix}
\]

\[= [-1000, -732, -518, -347, -210, -100] \text{ m} \]

Figure 4 shows a nominal trajectory of the V-bar approach. The difference between the mean orbital altitudes of a satellite and a debris determine PA safety in the V-bar hopping approach. Introducing the hopping rate \( y_{\text{hop}} \), the hopping interval becomes shorter as the chaser gets closer to the target at the constant rate. Basically, the accuracy and reliability of the navigation system are improved as the ADR satellite gets closer to the target debris. Figure 5 shows an example of free-drift trajectories after maneuver failure (i.e., missed thruster burn) in a nominal trajectory of the V-bar hopping approach. The blue line shows the nominal trajectory of V-bar hopping and the green lines show the off-nominal trajectories in the case of maneuver failures. In this figure, PA is successfully achieved.

**3.2. Spiral approach**

The spiral approach, which usually considers relative orbit elements (ROEs) between the chaser and the target debris, is related to the phasing of the relative eccentricity/inclination (e/i)-vector and the trajectory runs spirally along the V-bar. The spiral trajectory can be used to design proximity operation geometries characterized by passive safety and stability.

**Relative orbit elements:** ROEs \( \delta \alpha \) are a useful representation to describe the relative orbit and are defined as follows:

\[
\delta \alpha = \begin{bmatrix} \Delta a/a \\ \Delta \lambda \\ \Delta e_x \\ \Delta e_y \\ \Delta i_x \\ \Delta i_y \end{bmatrix} = \begin{bmatrix} \delta a \\ \delta \lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{bmatrix}
\]

with relative eccentricity vector \( \delta e \) and relative inclination vector \( \delta i \) being defined as follows:

\[
\delta e = \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}, \quad \delta i = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}
\]

where \( a \) is the semi-major axis, and \( e \) and \( i \) are eccentricity and inclination, respectively. Parameters \( \Omega, \alpha, \) and \( \lambda \) are right ascension of the ascending node, mean argument of latitude, and mean longitude, respectively. Note that the phases of the relative e/i-vectors in Eq. (19) are termed argument of periapher \\( \phi \) and ascending node \( \theta \) of the relative orbit. The mean argument of latitude \( \alpha \) is sometimes used as a parameter in ROEs instead of \( \lambda \) but using \( \lambda \) allows decoupling the in-plane from the out-of-plane components when introducing impulsive maneuvers. ROEs can be geometrically characterized as shown in Fig. 6 and are valid only for bounded relative orbits (i.e., \( \delta \alpha = 0 \)). In the R-T plane, the ratio of the major and minor axes in the ellipse is maintained at 2:1. The R-N plane is a key factor when considering PA safety. For \( \delta e = \delta i \), the trajectory in the R-N plane is described as a circle.

**Gauss equation:** The Gauss equation provides the conse-
sequent change of ROEs from an impulse maneuver as follows:

$$\Delta(a\dot{\alpha}) = \begin{bmatrix} a\delta a \\ a\delta \lambda \\ a\delta e_x \\ a\delta e_y \\ a\delta i_x \\ a\delta i_y \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ \sin u & 2 \cos u & 0 \\ -\cos u & 2 \sin u & 0 \\ 0 & 0 & \cos u \\ 0 & 0 & \sin u \end{bmatrix} \begin{bmatrix} \Delta v_R \\ \Delta v_T \\ \Delta v_N \end{bmatrix}.$$  (20)

This equation decouples the in-plane and out-of-plane relative motions. Maneuvers in the radial or tangential direction affect the eccentricity vector. Although propellant consumption with tangential maneuvers is twice as efficient as that with radial maneuvers, a radial maneuver has no effect on the relative semi-major axis. Maneuvers in a normal direction only affect the inclination vector, which controls the out-of-plane motion. For bounded relative motion ($\delta a = 0$), the minimum collision risk is provided by parallel or anti-parallel relative $e$/$i$-vectors. In this case, the relative perigee $\phi$ and relative ascending node $\theta$ in Eqs. (19) satisfy $\phi = \theta$ or $\phi = \theta + \pi$.  

**Relative orbit control maneuvers**: Orbital control maneuvers in the normal direction for out-of-plane reconfiguration are given by

$$\Delta v_N = v \parallel \Delta i \parallel$$  (21)

where

$$u = \arctan(\Delta \delta i_x / \Delta \delta i_d).$$  (22)

Orbital control maneuvers in the tangential direction for in-plane reconfiguration with spiral motion are given by

$$\Delta v_T = \frac{v}{4} (\| \Delta e \| + \| \Delta a/a \|)$$  (23)

$$\Delta v_T = \frac{v}{4} (\| \Delta e \| - \| \Delta a/a \|)$$  (24)

where

$$u^1 = \arctan(\Delta \delta e_y / \Delta \delta e_x)$$  (25)

$$u^2 = u^1 + \pi$$  (26)

for the maneuver locations, respectively. Orbital control maneuvers in the radial direction for in-plane reconfiguration without spiral motion are given by

$$\Delta v_R = \frac{v}{4} (2 \| \Delta e \| + \| \Delta \delta \lambda \|)$$  (27)

$$\Delta v_R = \frac{v}{4} (2 \| \Delta e \| - \| \Delta \delta \lambda \|)$$  (28)

where

$$u^1 = \arctan(\Delta \delta e_y / \Delta \delta e_x) + \pi/2$$  (29)

$$u^2 = u^1 + \pi$$  (30)

for the maneuver locations, respectively. Note that a three-tangential maneuver scheme provides an analytical closed-form solution for the $\Delta V$ optimal reconfiguration changing whatever of the four in-plane ROEs instead of radial and tangential (RT) maneuvers in Eqs. (23)–(24) and (27)–(28). Such triple maneuvers are effective when the minimum impulse bit of the thrusters on board the ADR satellite is small enough. Otherwise, the control error become large due to thrust error. Although the thruster capability is determined by the satellite system design, in this paper, the RT maneuver set is adopted considering the worst case of fuel consumption.

**Parameter design policy**: To design the parameters, the following assumptions apply:

- The trajectory in the R-N plane is described as a circle ($\delta e = \delta i$).
- $\delta e$ and $\delta i$ in the initial relative orbit is twice as large as the nominal destination.
- The spiraling relative trajectory becomes smaller by the constant value $\delta e_0$ and $\delta i_0$ as a spiral rate $\gamma_{spiral}$.
- The number of spiral rotations $N_{spiral}$ depends on the duration of operation.

Note that the spiral rate is defined as $\gamma_{spiral} = [\delta e_0 \; \delta i_0]^T$, and the components ($\delta e$ and $\delta i$) are obtained by

$$\delta e = \frac{\delta e_0 - \delta e_d}{N_{spiral}}$$  (31)

$$\delta i = \frac{\delta i_0 - \delta i_d}{N_{spiral}}$$  (32)

where the subscripts 0 and d are the initial and destination values, respectively.

For spiral motion, a larger spiral bias $\delta a$ provides a larger approach rate to the debris. Satellites approach by $3\pi \delta a$ per orbital period. Setting the distance of approach $\Delta x$ is given as follows:

$$\delta a = \frac{\Delta x}{3\pi N_{spiral}}$$  (33)

where $N_{spiral}$ is the number of spirals within $\Delta x$. Note that in this paper $\Delta x$ is equal to 900 m (~1000 – 100). Figure 7 shows the relationship between initial/destination relative orbits and spiral motion. From this figure, the initial $e/i$ values $\delta e_0/\delta i_0$ and the destination $e/i$ values $\delta e_d/\delta i_d$ are determined by

![Image](image_url)
Note that the relationship between initial relative e/i values and destination one is usually satisfied by \( \delta e_0 > \delta e_d = \delta i_d \geq \delta a + (r_{KOS} + r_{marg}) \). \hspace{1cm} (34)

Example: Figure 8 shows the nominal trajectory of the spiral approach. Note that the figures are described as the LVLH frame instead of the RTN notation. Figure 9 shows an example of free-drift trajectories after maneuver failure in a nominal trajectory of the spiral approach. The blue line shows the nominal trajectory of a spiral approach and the green lines show the off-nominal trajectories after maneuver failures. Spiral motion passes by the target debris with a trajectory around the origin as a point on the V-bar. When the e/i-vector is separated (parallel or anti-parallel), the PA safety of the spiral approach is guaranteed. Operationally, the size of the relative orbits may depend on the reliability of the relative navigation solutions and additional constraints on the FOV of the sensors. However, this paper fixes \( a\delta a \) with Eq. (33) and the respect of PA safety is obtained by setting the initial and final sizes of the e/i-vectors to comply with Eq. (34) since, in this paper, no specific sensor configuration is assumed in order to implement a fair comparison.

4. Numerical Simulations

This section compares the \( \Delta V \) budgets and variation in the LoS vector of two trajectories through numerical simulations, while fairly considering the duration of operations (e.g., the number of hops or spirals). The robustness of two trajectories against collisions due to off-nominal thruster burn is also demonstrated through Monte Carlo simulations.

4.1. Comparison of AV budget

This subsection compares the \( \Delta V \) budget in the case of V-bar hopping and that in a spiral approach. The initial/final positions of the V-bar hopping approach are set to \( x_0^{VHLH} = \)
Tables 3 and 4 show the ΔV budget for each maneuver in the V-bar hopping and spiral approaches, respectively. Note that the spiral approach only considers ΔV budget for each interval and shows that the ΔV budget of the spiral approach takes much less than that of the V-bar hopping approach. Sensor FOV requirements can be given by

\[ \Phi_{\text{FOV}} = \max \{ \Phi_{\text{in-plane}}, \Phi_{\text{out-of-plane}} \}. \]  

The maximum direction angle of the V-bar hopping approach is much less than that of the spiral approach. Sensor FOV requirements can be given by

\[ \Phi_{\text{FOV}} = \max \{ \Phi_{\text{in-plane}}, \Phi_{\text{out-of-plane}} \}. \]  

Figure 10(a) presents a target direction angle varying within 10 deg, which implies that, by using sensors with 10 deg FOV or larger, the attitude can be kept steadily pointing towards the V-bar direction during rendezvous.  

4.3. Monte Carlo simulations

Monte Carlo (MC) simulations are executed to clarify the robustness against collisions in the case of off-nominal thrust. This paper considers both excessive and insufficient thruster burns. To model the resulting uncertainties, a ran-
dom variable $\gamma$ with a standard uniform distribution $U(0, 2)$ is added for each maneuver point as follows:

$$\Delta V_{thr} = \gamma \cdot \Delta V_{true}$$

(38)

where

$$\gamma = \begin{cases} 
0 & : \text{passive abort} \\
\hat{\gamma}, 0 < \hat{\gamma} < 1 & : \text{insufficient maneuver} \\
1 & : \text{desired maneuver} \\
\hat{\gamma}, \hat{\gamma} > 1 & : \text{excessive maneuver} 
\end{cases}$$

(39)

Since the guidance profile is interrupted as soon as an off-nominal maneuver occurs, thruster burn is no longer executed. Thus, the PA safety of the trajectories can be estimated in order to confirm the passive trajectory after an off-nominal maneuver. If a trajectory of an ADR satellite invades the KOS because of an off-nominal thruster burn, the result is deemed to be unsafe.

Figures 11 and 12 show the MC simulation results and Fig. 13 shows the relation between off-nominal rate and the number of incursions into the KOS. From these figures, in the V-bar hopping approach, excessive maneuvers in the vicinity of the target may lead to invasion of the KOS. This result implies that V-bar hopping is more likely to enter KOS when the navigation error is not sufficiently small. On the other hand, there is no case in which a satellite in a spiral approach invades the KOS. The number of satellite invasions into the KOS are shown in Table 6. Therefore, due to off-nominal thruster burn, the spiral approach has a lower risk of collisions than the V-bar hopping approach.

4.4. Discussions

Summary of the V-bar hopping approach: The V-bar hopping approach would seem to offer the best balance between PA safety and fuel consumption for non-cooperative rendezvous missions for the R-bar and V-bar approaches. In this paper, the hopping approach moves the satellite a pre-defined distance in small steps through $\Delta V$ impulses in the radial and tangential directions at the V-bar crossing. This would be performed when the close-range navigation system has verified a reliable and sufficiently precise solution.

Summary of the spiral approach: The spiral approach defines a trajectory with eccentricity/inclination $(e/i)$-vector separation and evaluates the PA safety of the ADR satellite geometrically. This approach has the advantages of requiring small $\Delta V$ corrections and robustness against collisions due to the off-nominal thruster burns to which V-bar hopping
is subject. However, the variation in the LoS vector is large, which implies strict constraints on satellite design.

**General discussion:** Trajectory safety, partially guaranteed by robustness, has an impact on non-cooperative rendezvous such as many ADR missions. However, a large variation in the LoS vector leads to strict constraints on a satellite’s systems, such as GNC sensors, power control, thermal management, and communications. Table 7 compares the two candidate trajectories. From the criteria, it is apparent which trajectory should be adopted for a particular mission. The V-bar hopping approach is suitable for missions under the following conditions:

- The amount of $\Delta V$ is not strictly constrained;
- The onboard relative navigation system has already converged to a reliable and robust solution; and
- The requirements of satellite design are strictly determined by mission specifications per the configuration of mission instruments.

On the other hand, spiral approaches are suited to missions where:

- The amount of $\Delta V$ is strictly constrained due to the size of the chaser (e.g., ADR satellite);
- The navigation system has not been verified to provide a reliable, sufficiently precise solution in advance; and
- An amount of operational time is available to collect enough data from the sensors and verify the image processing and navigation algorithms in the various in-orbit conditions.

Therefore, the type of approach trajectory should be selected according to the mission and related conditions.

5. **Conclusion**

This paper shows the quantitative trade-offs between the V-bar hopping and spiral approach trajectories while considering PA safety in non-cooperative rendezvous missions. The orbital dynamics, control maneuvers and abort characteristics of two candidate trajectories are introduced and developed. Furthermore, design strategies for approach trajectories for ARD missions are proposed. It then compared the $\Delta V$ budget, duration of operation, and variation in the LoS vector through numerical simulations. The robustness against collisions due to off-nominal thruster burn was also demonstrated through Monte Carlo simulations. The proposed policy for designing trajectories considering PA safety facilitated the design of safe approach trajectories in non-cooperative rendezvous missions and allowed for quantitative trade-offs for candidate trajectories.

As a result, it is shown that the V-bar hopping approach has the advantage of clearly observing the target and enabling simpler satellite system design thanks to the small variation in the LoS vector when the onboard relative navigation system has already converged to a reliable and robust navigation solution. On the other hand, the spiral approach has the advantages of a small $\Delta V$ (i.e., total $\Delta V$ budget with the spiral approach is smaller than that with V-bar hopping by one order of magnitude) to achieve a proximity approach and robustness against collisions that could occur because of off-nominal thruster burn. This paper clarifies the characteristics of two PA-safe trajectories and makes it possible to select the better trajectory considering safety and the mission.

This paper focused on evaluation by thrust error in the Monte Carlo simulations and assumed that navigation is reliable since the quality of navigation depends on the application (i.e., what sensors are used and which navigation filters are implemented). Therefore, determining the proper duration of operation in non-cooperative rendezvous missions remains an open question that entails correcting navigation errors in orbit and attitude determination which, in turn, depends on the noise inherent in navigation sensors, the navigation filter algorithm, and tuning.

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Matteo Ceriotti

*Associate Editor*