Modified method of identification of mutual fractional-order inductance

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Abstract. The paper presents a method for identifying the parameters $M, \gamma$ of a fractional-order transformer, which parameters $L, \beta_1, \beta_2$ have been previously determined. This method is based on the measurement of the phase resonance frequency in a few systems containing: the investigated fractional-order transformer and two standard capacitors. The measurements need to be performed only for one series opposite-aiding connection of the fractional-order transformer. The dependencies allowing the determination of the fractional-order mutual inductance parameters have been given.

1 Introduction

There are many works devoted to the analysis of systems with fractional-order elements $L, C$, their realization and parameter identification, e.g. [1-3].

For several years, there has been a rapid growth of interest in fractional differential-integral calculus application in describing fractional-order magnetically-coupled coils systems [4-6]. The work [4] describes the concept and properties of such fractional-order coupled inductances. In [5], the electromagnetic Maxwell equations of the fractional-order mutual inductance are analyzed. The wireless power transmission system has been modeled as a fractional-order coupled coils system in [6]. The existence of fractional-order coupled coils (fractional-order transformer) implies the need to determine the parameters of the fractional-order elements. In [7], a method has been proposed for parameters identification of the fractional-order coils with an iron core, which is based on the approximation of the transient response to the unit-step voltage using the least squares method.

The paper is an extension and continuation of [8], where the new method for the identification of all the parameters $L, \beta_1, \beta_2, \beta, M, \gamma$ of the fractional-order coupled inductances, has been proposed. The paper presents a proposal for a modified method of the identification of the fractional-order parameters $M, \gamma$ of the mutual inductance, based on the phase resonance phenomenon in the series circuit of the class $RLC\omega$ compared to [8], without the need of the input impedance measurement in the combination of series and opposite-aiding connection of the transformer system.

2 Modification proposal

The equivalent circuit of the system for the parameters $\gamma, M$, determination of the fractional-order mutual inductance, is shown in Fig. 1.

The circuit from Fig. 1 is supplied by the sinusoidal voltage source of adjustable frequency.

The circuit impedance, seen from the source terminals, is given by a formula:

$$Z(j\omega) = R + (j\omega)^{\beta_1} L_{\beta_1} + (j\omega)^{\beta_2} L_{\beta_2} - 2(j\omega)^{\gamma} M_{\gamma} - \frac{1}{\omega C} \quad (1)$$

where: $R$ - the equivalent resistance of the series connection of the coil resistances.

Transforming, the real and imaginary part of the impedance is:
However, the parameter \( \gamma \) can be determined as:

\[
\gamma = \log \left( \frac{\omega_1^{\beta_1} L_{\beta_1} \sin \left( \frac{\beta_1 \pi}{2} \right) + \omega_2^{\beta_2} L_{\beta_2} \sin \left( \frac{\beta_2 \pi}{2} \right)}{\omega_1^{\beta_1} L_{\beta_1} \sin \left( \frac{\beta_1 \pi}{2} \right) + \omega_2^{\beta_2} L_{\beta_2} \sin \left( \frac{\beta_2 \pi}{2} \right)} - \frac{1}{\omega_1 C_1} \right).
\]

Then the value of the parameter \( \gamma \) can be determined as:

\[
\gamma = \log \left( \frac{\omega_1^{\beta_1} L_{\beta_1} \sin \left( \frac{\beta_1 \pi}{2} \right) + \omega_2^{\beta_2} L_{\beta_2} \sin \left( \frac{\beta_2 \pi}{2} \right)}{\omega_1^{\beta_1} L_{\beta_1} \sin \left( \frac{\beta_1 \pi}{2} \right) + \omega_2^{\beta_2} L_{\beta_2} \sin \left( \frac{\beta_2 \pi}{2} \right)} - \frac{1}{\omega_1 C_1} \right).
\]

However, the parameter \( M_\gamma \) can be determined by substituting the obtained value of the coefficient \( \gamma \) with the formula (3) for one of the performed measurements, for example for:

\[
M_\gamma = \frac{\omega_1^{\beta_1} L_{\beta_1} \sin \left( \frac{\beta_1 \pi}{2} \right) + \omega_2^{\beta_2} L_{\beta_2} \sin \left( \frac{\beta_2 \pi}{2} \right)}{2 \omega^{\gamma} \sin \left( \frac{\gamma \pi}{2} \right)} - \frac{1}{\omega^{\gamma} C_1}.
\]

The described algorithm has been illustrated with a simulation example.

### 3 Example

The circuit from Fig. 1 has been supplied from a source with an adjustable frequency value, for which the input voltage value has been assumed \( U(j\omega) = 1 \text{ V} \).

Parameters of the primary and secondary side of the transformer have been determined according to the procedure described in [8] and were respectively: \( \beta_1 = 0.503, L_{\beta_1} = 8.813 \text{ mH} \cdot s^{(1-\gamma)}, \beta_2 = 0.502, L_{\beta_2} = 3.113 \text{ mH} \cdot s^{(1-\gamma)} \).

For two capacitors with known capacitances \( C_1 = 10 \text{ mF}, C_2 = 3.53 \text{ mF} \) in the investigated circuit, as in Fig. 1, two values of resonance frequencies \( f_1 = 100 \text{ Hz}, f_2 = 200 \text{ Hz} \) have been recorded. From the dependencies (4) and (5), the searched values of the fractional-order parameters of the mutual inductance have been determined:

\[
\gamma = 0.503
\]

and:

\[
M_\gamma = 1.554 \text{ mH} \cdot s^{(1-\gamma)}
\]

### 4 Summary

The paper proposes a modified method, compared to the method presented in [8], for identifying \( M_\gamma, \gamma \) parameters of a fractional-order transformer. This method is based on the measurement of the phase resonance frequency in a circuit containing the analyzed transformer and two switchable standard capacitors. The dependencies allowing the determination of the fractional-order mutual inductance parameters have been given, on the basis of the described measurements.

The advantage of the modified method for determining the fractional-order parameters is the fact, that only one series opposite-aiding connection of the fractional-order coupled coils is enough to determine the searched parameters. The need to measure the input impedance of the circuit from Fig. 1 is also avoided.

### References

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