Enabling Interactivity on Displays of Multivariate Time Series and Longitudinal Data

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Abstract

Temporal data is information measured in the context of time. This contextual structure provides components that need to be explored to understand the data and that can form the basis of interactions applied to the plots. In multivariate time series we expect to see temporal dependence, long term and seasonal trends and cross-correlations. In longitudinal data we also expect within and between subject dependence. Time series and longitudinal data, although analyzed differently, are often plotted using similar displays. We provide a taxonomy of interactions on plots that can enable exploring temporal components of these data types, and describe how to build these interactions using data transformations. Because temporal data is often accompanied other types of data we also describe how to link the temporal plots with other displays of data. The ideas are conceptualized into a data pipeline for temporal data, and implemented into the R package cranvas. This package provides many different types of interactive graphics that can be used together to explore data or diagnose a model fit.

Keywords: Interactive graphics; Multivariate time series; Longitudinal data; Multiple linked windows; Data visualization; Statistical graphics.

1 Introduction

Constructing interactive graphics for temporal data can be enabled by building upon static displays. Aspects of the graphical elements in the displays can be made accessible to modification by user actions, for the purpose of facilitating different exploration of the temporal components in the data. To explain how to do this we first need to understand how time might be structured, and the common types of temporal data displays.

1.1 Characterizing time

In data, time is coded in many different ways: as a date/time format ("Wed Oct 15 09:51:53 2014"), as discrete or continuous values, sporadic events or intervals. Recoding a time variable into other units like week in the year or days in the month, although convenient for some tasks,
brings imprecision, as do events like leap years, and seconds. There may not be a well-defined absolute time line, and periodicity can be hard to quantify. Variables measured over time may be measured on different scales, e.g. atmospheric particulate matter and mortality might be drawn from different sources to study the effect of pollution on human health and measured at different resolutions.

The most common description of time is as a continuous or discrete ordered numerical variable. All data is technically discrete, but if measurements are recorded often enough, and long enough they are effectively continuous. For example, currency exchange rates change on a microsecond basis, blood pressure measurements made with a wearable device records at every minute. For these examples time could essentially be considered continuous. However, it may be not helpful to evaluate trends on this microscale, and aggregating at an hourly, daily or monthly value may be sufficient. For simplicity, the methods developed in this paper assume the time variable is measured on a discrete scale.

On a discrete scale, it may be possible to have regular or irregular time spacing. Regular time spacing means that the measurement is collected on constant time intervals, e.g. average monthly temperature in climate records. Irregular time spacing typically arise from events like measurements taken during visits to the doctor’s office. Figure 1 illustrates data collected at regular, and irregular, intervals, respectively.

Figure 1: Time series plots for regular time spacing (left) and irregular time spacing (right). Tick marks at bottom indicate the time sampling.

When many measurements are made more complications ensue. In climate records we may have temperature measurements taken at many different locations. In longitudinal data, we may have records for many patients. Making comparisons between many time series is a challenge. This work addresses this for a moderate number of series.

1.2 Visualizing time

Longitudinal data and time series data, although analyzed very differently, have in common the context of time that is commonly plotted in similar ways. Here are examples of common types of temporal displays.

Time is most conventionally displayed on a horizontal axis of a plot. There are many different variations:
A line graph, the basic building block for temporal data, displays the measured variable on the vertical axis, time horizontally, and consecutive time points are connected with line segments. Figure 1 gives two examples of line graphs, for a single measured variable: (left) a classical time series plot, (right) a profile plot of longitudinal data. When there are many series, for example time series of different stocks or different geographic locations, or many patients, the series may be overlaid on the same plot (Figure 2 row 1 left), or faceted in several blocks (Figure 2 row 1 right).

Small multiples are used to display multiple series in separate plots (Figure 2 row 2 right). The terminology small multiples was introduced by Tufte (1983). A special modification was developed for multiple times series, called sparklines Tufte (2006). Small multiples can also be generated by subsetting based on categorical covariates (e.g. Cleveland (1993)).

Stacked graphs. Originated by Playfair in 1700’s and recently discussed by Byron and Wattenberg (2008); Javed et al. (2010); Heer et al. (2010), a stacked graph draws the time series sequentially, and uses the previous time series as the baseline for the current series (Figure 2 row 2 left). It is mostly used for the longitudinal data rather than multivariate time series since the individuals from the longitudinal data share the same scale.

Themriver and streamgraph. Themriver is created by Havre et al. (2000), which is a special case of the stacked graphs, since it moves the starting baseline from the bottom to the center, and makes the plot symmetric vertically (Figure 2 row 3 left). Streamgraph is developed later by Byron and Wattenberg (2008). It changed the algorithm to avoid the symmetry which increases the internal distortion.

Horizongraphs. The horizon graph is inspired by two-tone pseudocoloring (Saito et al. 2005) and formally developed at Panopticon Software (Reijner 2008). Two-tone pseudo coloring is a technique to visualize the details of multiple time series precisely and effectively. However, the horizon graph became more popular after mirroring the lower part of the series and simplifying the color scheme (Figure 2 row 3 right). The horizon graphs were designed for visualizing the stock prices and economic/financial data, so the features fit the requirements very well: (1) The data have a baseline, which is usually the value at the starting time point. Then the baseline can be used to mirror the negative part to the positive, in order to save the graph space, where ‘negative/positive’ means smaller/greater than the baseline. (2) The positive and negative performance should be distinguished, so the horizon graph provides two hues. (3) The number of the color bands should be small, usually three color bands for the positive values and three for the negative. Finding the band height is easy for the stock prices since they can use 10% of the initial value, and in most cases the price will not increase or decrease for more than 30%.

When time can be broken into two components it might be displayed on both horizontal and vertical axes:

A high frequency time series often has hierarchic or nested period levels, like year, day, minute, etc. Those levels can be placed on horizontal and vertical axes to reveal the periodic
Figure 2: Six variations of horizontal axis time plots for multivariate time series: (From top left to bottom right) overlaid line plots, faceted line plots, stacked graph, faceted area chart, themeriver, horizon graph. Plots in the right column are examples of small multiples.
dependency. For example, Keller and Keller (1993) used days and hours on two axes. In these graphs, the measurements of the time series are drawn in the grids via aesthetic settings like color or size.

- Calendar heat maps. Van Wijk and Van Selow (1999) proposed a colored calendar visualization (weeks and days on two axes). d3.js (Bostock et al., 2011) applies the calendar heat maps and makes it interactive.

Because time in some circumstances can be considered to be cyclical it is sometimes displayed in the polar coordinates:

- Nightingale’s coxcomb. Florence Nightingale might be the earliest author of a time series plot in polar coordinates. In the original plots, two unstacked barcharts were made in polar coordinates. Each diagram represents for one year. Later, people use the Nightingale’s coxcomb (Nightingale, 1858), also called circular histogram or rose diagram (Nemec, 1988), to plot the time series with a regular period like year or day.

- Spiral graphs. This approach is proposed by Weber et al. (2001). It can be seen as a temporal heatmap in polar coordinates. Figure 3 (right) shows an example. This approach is good for seeking the period, but the length for the same time unit changes over the loops. Besides, spiral graphs would be unhandy for the short period problems and multiple time series.

![Figure 3: Three types of time series plots in polar coordinates for the same data as used regularly spaced series shown in Figure 1: direct conversion (left), wrapped by the period (middle), and spiral graph with the colored grey scale representing the values (right).](image)

### 1.3 Interactive graphics

Interactive graphics emphasize the user manipulation of plot elements via input devices like the keyboard and mouse (Symanzik, 2012). Swayne and Klinke (1999) surveyed the use of the term “interactive graphics”, which revealed some differences in what people mean when they use the
term. They found that most commonly people perceived interactive graphics to mean that a new plot can be recreated quickly from the command line, like base R plots. They suggested using a different term, direct manipulation, to mean directly changing elements of the plot using input devices. However, this term did not gain traction in the community, and we still use interactive graphics. To be clear, we use it here to indicate direct manipulation of plot elements through input actions of mouse or key strokes.

The work described here builds from a history of statistics software systems that support interactive graphics: e.g. PRIM-9 (Fisherkeller et al., 1988), Data Desk ([Velleman and Velleman, 1988]), LISP-STAT (Tierney, 1990), XGobi (Swayne et al., 1998) and GGobi (Cook and Swayne, 2007), MANET (Unwin et al., 1996), Mondrian (Theus, 2002). The software Diamond Fast (Unwin and Wills, 1988), XQz (McDougall and Cook, 1994), and Fortune (Kotter and Theus, 1996) provided tools specifically for exploring time series data. With the current popularity of R language (R Core Team, 2014), ideally interactive graphics can integrate closely and flexibly with statistical modeling. Packages that support this to varying extents are rggobi (Wickham et al., 2008), iplots (Urbanek and Wichtrey, 2013), rgl (Adler et al., 2003), cranvas (Xie et al., 2013), ggvis (RStudio and Inc., 2014), and animint (Hocking et al., 2014).

Of these software, cranvas, which evolved substantially from GGobi, is the vehicle for the ideas described in this paper. At its foundation is a data pipeline that channels data to plot elements, and provides interaction through reactive data elements, using plumbr (Lawrence and Wickham, 2014). The graphics are constructed using Qt (Qt Project, 2014) that enables flexible plot design and fast rendering for smooth interaction. cranvas has many different types of plots and possible interactions. The design of cranvas, Xie et al. (2014) provides single display interactions and the linking between different displays. Single display interactions include brushing, zooming, panning, and querying. Linked brushing can be done between different displays. To integrate temporal displays in this system requires integrating with this setup.

The next section describes the building blocks for temporal displays, which is followed by a taxonomy of interactive tasks that is desirable to use for exploration (Section 3). How to realize the interactions is described in Section 4. Linking between temporal data displays and other plots is described in Section 5.

2 Layering to create a plot

For interactive graphics, layering up the plot to enable different interactions can be useful, and efficient for large data. The base layer is typically the plot of all of the data. An overlay of a brush layer, where only elements actively being colored are displayed, can provide the efficiency of faster rendering. A brush layer is common to all displays because brushing is a basic function for interactive graphics. Background layers like an axis layer or grid layer are common too, but are not necessary in displays like maps. Table 1 lists the layers of common plot types in cranvas.

For temporal data displays, there are three basic layers: point, line, and area. The coordinates of the points are initially calculated from the data, and interactions may change the locations of the points in the display. The line layer connects the current positions of the points, so it requires the order, or, path information to know how to make the connections. The area layer shades the
Table 1: Examples from **cranvas** of layers used in constructing plots. Some are common to all plots, and each plot has some layers that are unique. The “keys” layer listens for key strokes that change interaction modes. The “cue” layer on the histogram contains listeners and handles for dragging to change the binwidth interactively.

| display specific                  | scatterplot      | point                  |
|----------------------------------|------------------|------------------------|
| histogram                        | bar, cue         |                        |
| map                              | polygon, googlemaps, path, point |
| time plot                        | point, line, area, stats |
| common                           | required         | brush, identify, keys  |
|                                  | optional         | grid, x-axis, y-axis, x-label, y-label, title |

area under the line by constructing a baseline, matching the minimum data values, which enables closing the series to create a set of polygons. Each of these layers can take different interactions, and some care needs to be taken in realizing the effect on the different layers.

The point attributes, selected, color, size, or visibility, are generated for each observation when creating the plumbr mutaframe. The base element for the temporal plots are the points, and in **cranvas** brushing changes the attribute of each point in the mutaframe – essentially, points are brushed. The number of lines is one less than the number of points, and line color follows the first point in the defining pair. The number of polygons for the area display is the same as the number of lines, so color follows lines directly to polygons. The additional construction points of the area layer are only used in the area layer, and do not have independent attributes. This affects brushing behavior which is discussed later.

### 3 A taxonomy of interactions for temporal data displays

[**Wills** (2012)](Wills_2012) summarizes the interactivity for temporal displays as changes to parameters or data. Data is mapped into coordinates in the plot. Parameters can be considered to be attributes like color, labels, geometric elements, facet, or they can be considered to be aspects used to get the data into the plot like transformations, binning, dimension reductions or scales. Changes to the data or parameters provoke changes to the plots.

Parameters can often be attached to graphical user interface (GUI) items like sliders, that can generate the change in the plot. But more generally, interactions happen by direct action on the plot. For example, in brushing, the user selects elements like points in the plots. The software needs to locate these items in the data, update the attributes of these selected points, and broadcast these changes to other plots.

Some interactions, like brushing, selection, linking, zooming, panning, and querying, are universal for all plot types. Temporal and longitudinal data solicit special interactions to explore aspects of temporal dependence and trend. [**Buja et al.** (1996)](Buja_et_al_1996) describe a taxonomy of interactive tasks for multivariate data. Here we describe a taxonomy of tasks for temporal data, that enable exploration of different components of time series and longitudinal data:
Figure 4: Lynx trappings for 1821–1934: (a) Time series, (b – d) stages of $x$-wrapping, matching peaks, (e) faceted on the wrapped series, (f) area plot, (g) mirrored on the mean, blue indicating values above the mean, and yellow below the mean. (Video illustrating these interactions is available at [https://vimeo.com/112431547](https://vimeo.com/112431547) and [https://vimeo.com/112432400](https://vimeo.com/112432400).)
Figure 5: Quarterly pig production for 1967-1978 in UK measured by five variables: (a) faceted, (b) y-wrapped (see video at {https://vimeo.com/112435889}). Profit and gilts seem slightly lag-related. Herd size and production may be related in a lag relationship also, but neither is seasonal. In the y-wrapped version density indicates the magnitude of values, and long periods of higher values, like in herds size are more visible.

- Wrapping: In the x-direction explores seasonality and temporal dependence. In the y-direction it is done to compare magnitude of peaks and dips. It is easiest to explain x-wrapping: the series is cut at a fixed-length interval, the part of the series that extends beyond this interval is re-drawn from the initial point. This will change the x-coordinates of the data. The main purpose of x-wrapping is to explore the regularity of the periodicity. Some series that look to follow a regular period can be quickly revealed to have irregularities. The classical example is the lynx trappings data for 1821–1934 in the MacKenzie River District of North-West Canada (Campbell and Walker, 1977), which looks periodic (Figure 4). The wrapping shows that the period is not quite regular, matching one peak off-sets other peaks, and the period varies between 9-11 years. It is also possible to see that the increases are slower than the drops, that the population builds up and tends to plummet. To model this data well requires one also knows the snowshoe hare population. Figure 5 gives an example of the y-wrapping. It shows another classical example: quarterly pig production measured by four variables, herd size, production, profit and gilts, from 1967-1978 in United Kingdom (Andrews and Herzberg, 1985). The y-wrapping induces something that might be considered a temporal boxplot, where density produced from overlaying the wrapped peaks emphasizes long runs of ups, or periodic ups.

- Faceting: This creates small multiples to organize and examine across structural data components. These components might be a period such as year, or month, or variables when multiple are measured at the same time points, or in longitudinal data these might be individuals. In the interactive setting these components can be used to slowly pull overlaid series apart, a process that might be more revealing that disjointly laying out each in a static plot. Figure 6 illustrates sequential faceting on two variables and three individuals. The
Figure 6: Order of interaction matters when faceting with two variables and three individuals: (a) all series overlaid, color indicates variable, (b) facet first on variable (A, B), (c) facet first by individual (1, 2, 3), (d) facets (b) by individual, and (e) facets (c) by variable. Both final configurations are useful: (d) supports the primary comparison of individuals, with variable comparisons secondary and (e) supports comparison of variables within individuals. (Video footage illustration is available at [https://vimeo.com/112438919](https://vimeo.com/112438919)).
order of these operations changes the final result, and changes which comparison is primary and which secondary. Figure 7 gives two more examples, by a grid of spatial locations, and by covariates, respectively. Watch the videos to see how these are achieved interactively. There is also another video at https://vimeo.com/112505175 shows faceting on period for a time series with a regular period of every 12 observations.

- Mirroring: This splits the series vertically at a given value, and reflects the bottom half across this axis. With additional wrapping, the result is called a horizon graph, and it is used to compare the magnitude of peaks and troughs, particularly for binary phenomena like gains and losses. The \( y \)-coordinates of the data are modified by this interaction. The choice of split value are typically mean, median, midpoint of the range, or in economic data the initial series value. Figure 4 (f) and (g) shows mirroring of the lynx trappings data with the mean divider. We can see the peaks are sharp and irregular and the valleys are smooth and regular.

- Shifting: A series can be grabbed and shifted against another series. This is a more tangible operation than wrapping in order to compare periodicity and temporal dependence. Figure 8 shows three series that have been picked up and shifted together against the other three series to match peaks.

- Switching: At any time it should be possible to switch between line and area displays. Line plots are efficient but filling the area under the curve can give a stronger sense of the patterns in the series, especially when trying to compare multiple series. The video at https://vimeo.com/112530645 demonstrates switching.

Figure 7: Faceting can be conducted bi-directionally: (left) by spatial grid for spatiotemporal data (https://vimeo.com/112503285), (right) by two covariates, sex and age, in longitudinal data (https://vimeo.com/112509324).
Figure 8: Illustration of shifting: three bottom series (variable A) are shifted horizontally to the right, to match the peak time with the top three series (variable B). (See also the video at https://vimeo.com/112439923.)

4 Display Pipeline

Wilkinson et al. (2000, 2001, 2006) conceptualized and implemented a grammar of graphics that carefully details a mapping of data to plots. It was extended and implemented in ggplot2 by Wickham (2009). Data is parametrized into elements, and assigned to graphical elements, e.g. points, text, lines, polygons. Wills (2012) made a simple extension for interactive graphics: “allow the user to manipulate one of the two inputs, data or parameters, and show the changes in the chart”. Parameters, very generally, describe a very broad class of characteristics, e.g. display aesthetics like color or linetype, positional coordinates, statistics such as bins, scales like limits or color ladders, facets, and transformations. Much of what is needed to realize the taxonomy of tasks for interacting with time series plots can be considered to be data transformations. This section describes the transformations required to perform shifting, faceting, wrapping, and mirroring.

Let $(x, y)$ denote the positional coordinates for a temporal data set, where both are $n$-dimensional vectors. This notation is unconventional for time series, which typically uses $x_t$ to represent the value records at time $t$, but it is necessary for the graphical display because it allows us to think about horizontal and vertical positions and adjustments to these positions. Because, many sequential interactions can be made, and different types of interactions applied after each other, it is useful to incorporate notation specifying these into the equations. Let $I$ be the temporal sequence of interactions, e.g. \{facet, wrap, facet, zoom, \ldots\}, and $j \in J = \{1, 2, \ldots, J_i\}$ indicate the number of interactions made of type $i \in I$. Let $u_{ij} = (u_{ij1}, u_{ij2}, \ldots)$ denote the user’s input, e.g. key strokes, $l_{ij} = (l_{ij1}, l_{ij2}, \ldots, l_{ijn})$ be a line group indicator for each point, since some interactions might force new sets of lines, $p_i = (p_{i1}, p_{i2}, p_{i3}, \ldots)$ be a parameter vector, e.g. a wrapping stop value of 3 points in series, and $m_{ij} = (\Delta x_{ij}, \Delta y_{ij})$ denote the movements in $x$- and $y$- directions, where $i \in I$ is an interaction type, and $j$ is the number conducted. The new data coordinates are given by

$$(x, y)_{s+I_{ij}} = (x, y)_s + m_{ij},$$
where $s$ indicates the state before interaction. The movement $m_{ij}$ can be written as a function on $p_i$, $u_{ij}$, $l_{ij}$, $j$, and the initial coordinates $(x, y)_0$, 

$$m_{ij} = f_i(p_i, u_{ij}, l_{ij}, j, (x, y)_0).$$

These are the specific functional definitions used to generate the movement for the interactions available in cranvas.

4.1 Wrapping

Figure 4 (except bottom right plot) illustrates the horizontal wrapping of the lynx trapping data. The default wrapping interaction, by clicking a keystroke, induces the point at the end of the series, $x(n)$, to be cropped and moved to the very left side of the plot, at the same $x$-position as $x(1)$. With repeated keystrokes, the most recent elements of the series will be cropped and gradually wrapped onto the earliest elements. Only the $x$-coordinates are changed – the $y$-coordinates remain unchanged.

For simplicity, we assume that the difference between consecutive time values is 1. Let $x(1), \cdots, x(n)$ be the sorted $x$-coordinates of the $n$ points in the series, that is the points in time order. The $x$-limits after $j$ keystrokes for $x$-wrapping will be reset to $(x(1), x(n) - j)$, so the plot is rescaled accordingly. Let $\Delta_{n-j} = x(n-j) - x(1) + 1$, then the new $x$-coordinate, $x^*$ of $x$ is

$$x^* = \begin{cases} x(n-j) & \text{if } x - x(1) + 1 \mod \Delta_{n-j} = 0 \\
(x - x(1) + 1) \mod \Delta_{n-j} + x(1) - 1 & \text{o.w.} \end{cases}$$

$$= \begin{cases} x(n-j) & \text{if } (x - x(1) + 1) \mod \Delta_{n-j} = 0 \\
(x - x(1) + 1) - \left\lfloor \frac{x - x(1) + 1}{\Delta_{n-j}} \right\rfloor \times \Delta_{n-j} + x(1) - 1 & \text{o.w.} \end{cases}$$

$$= x - \left(\left\lfloor \frac{x - x(1) + 1}{\Delta_{n-j}} \right\rfloor - 1\right) \times \Delta_{n-j},$$

enabling the movements for $i = \text{wrap}$ to be described as

$$m_{ij} = \begin{cases} \left(-\left\lfloor \frac{x - x(1) + 1}{\Delta_{n-j}} \right\rfloor - 1\right) \times \Delta_{n-j}, 0) & 1 \leq j \leq n - 3 \\
\left(-\left\lfloor \frac{x - x(1) + 1}{\Delta_3} \right\rfloor - 1\right) \times \Delta_3, 0) & j \geq n - 2 \end{cases}$$

or equivalently in terms of line group indicators as well as points as

$$m_{ij} = \begin{cases} \left(-(l_{ij} - 1) \times \Delta_{n-j}, 0) & 1 \leq j \leq n - 3 \\
\left(-(l_{ij} - 1) \times \Delta_3, 0) & j \geq n - 2. \end{cases}$$

The line group indicator $l_{ij}$ will depend on the number of interactions $j$. The wrapping can be defined as an algorithm also:
1. Shift the data values up, usually by 1
2. Check the new $x$-limits, if a point has value large than upper limit, crop it using modulus arithmetic
3. Points that are cropped, have their line group indicator incremented
4. Connect the points that have the same line group indicator, in time order.

Sometimes it is useful to wrap the series faster. If you have a long time series, it might be useful to make full year jumps. This can be achieved with the above equations by setting the sequence of $j$ to respect this period. In other instances it may be useful to have a multiplicative wrapping so that it looks like the series wraps faster and faster with each step. That means every keystroke will send a different number of points from the right to the left. The number of points wrapped by the $j$th step, can be represented by the user input parameter $u_{ij}$. Then the $x$-range after $j$ steps is $(x(1), x(n-\sum_{a=1}^{j} u_{ia}))$, yielding $\Delta_{n-\sum_{a=1}^{j} u_{ia}} = x(n-\sum_{a=1}^{j} u_{ia}) - x(1) + 1$, and

$$x^* = x - \left(\left\lceil\frac{x - x(1) + 1}{\Delta_{n-\sum_{a=1}^{j} u_{ia}}}\right\rceil - 1\right) \times \Delta_{n-\sum_{a=1}^{j} u_{ia}};$$

$$m_{ij} = \begin{cases} (-1)(i - 1) \times \Delta_{n-\sum_{a=1}^{j} u_{ia}}, & 1 \leq \sum_{a=1}^{j} u_{ia} \leq n - 3 \\ (-1)(i - 1) \times \Delta_{3}, & \sum_{a=1}^{j} u_{ia} \geq n - 2. \end{cases}$$

If the user wants to skip all intermediate positions and use only one jump to the fully wrapped position, then the new $x$-range will be $(x(1), x(p_{i2}))$, where the parameter $p_{i2}$ is the length of period. Hence for $j \geq 1$,

$$x^* = x - \left(\left\lceil\frac{x - x(1) + 1}{\Delta_{p_{i2}}}\right\rceil - 1\right) \times \Delta_{p_{i2}};$$

$$m_{ij} = (-1)(i - 1) \times \Delta_{p_{i2}}, 0).$$

To generalize our case to the irregular time series, we should specify a wrapping speed parameter $p_{i3}$, i.e., with every key stroke, the $x$-range is shortened by at least $p_{i3}$. The wrapping speed parameter will determine how many points are shifted every time, because if the difference between largest two points is greater than $p_{i3}$, then only one point is shifted; if the difference is smaller than $p_{i3}$, then more than one points are shifted. After $j$ steps, the total number of points shifted is a function of $p_{i3}$ and $x_0$. Denote the function as $g_j$, so the new $x$-range is $(x(1), x(n-g_j(p_{i3}, x_0)))$, and the movements can be calculated then.

Movements from the $y$-wrapping on the $y$-direction, as shown in Figure 5, could be obtained by similar formulas. It is messier to realize because the $y$-values are typically not in a sequential order which means that more structural components need to be added to the data to actually draw the wrapped series. Some of the issues are discussed later in this paper.
Figure 9: Cartoon illustrating the $x$-wrapping. There are three consecutive wrapping steps. At step $j = 1$ the $x$ value of the last point changes from 21 to 16, and the line group indicator increments to 2. At step $j = 2$ the $x$ values of the last two points are changed, and both have line group indicators equal to 2. The wrapping stop parameter was set to $p_{ij} = 3$, which means that after one more step, $n - j = 3$, the wrapping would stop because each series has only 3 points. In the actual implementation the scale of the horizontal axis is changed at each step so that the full plot width is used.
4.2 Faceting

When \( i = \text{facet by individual} \), an initial setting of the parameter \( p_{i1} = 0.05 \), which means that every hit on the key will lift the \( l \)th standardized line by \( (l - 1) \times 0.05 \). Hence for \( j \in J \),

\[
m_{ij} = \begin{cases} 
(0, 0.05 (l_i - 1) j) & 1 \leq j < 20, \\
(0, l_i - 1) & j \geq 20.
\end{cases}
\]

We can also generalize the equation above by

\[
m_{ij} = \begin{cases} 
(0, p_{i1} (l_i - 1) j) & 1 \leq j < \frac{1}{p_{i1}}, \\
(0, l_i - 1) & j \geq \frac{1}{p_{i1}},
\end{cases}
\]

where \( p_{i1} \in (0, 1) \).

The example shows that \( m_{ij} \) is a function of \( p_i, j, \) and \( l_{ij} \), where \( l_{ij} = l_i \) in this example means that the line indicator for faceting is free from \( j \).

For \( i = \text{facet by variable/period} \), one click will fully split the variables, so \( j \) does not matter. All lines should be standardized between \([0, 1]\) first, then the movement is given by

\[
m_{ij} = (0, l_i - 1).
\]

Note that \( l_i \) in this case differs from \( l_i \) in faceting by individual.

4.3 Mirroring

To realize the interaction shown in Figure 4 (g), firstly we need to point the divider – mean in this example. Hence for \( i = \text{mirroring} \), the divider parameter \( p = \frac{1}{n} \sum_{d=1}^{n} y_d \). Then by \( j \in J \) hits on some triggering key, the movements are

\[
m_{ij} = \begin{cases} 
(0, p + \max(p - y, y - p) - y) & j = 1, 3, 5, \cdots \\
(0, 0) & j = 2, 4, 6, \cdots
\end{cases}
= \begin{cases} 
(0, \max(2p - 2y, 0)) & j = 1, 3, 5, \cdots \\
(0, 0) & j = 2, 4, 6, \cdots
\end{cases}
\]

Note that if the mirroring is revisited after some other interactions, then the count of \( j \) should not be reset.

4.4 Shifting

Figure 8 illustrates shifting the series, which is used to compare one series against another. The user input uses \( u_{ij} \), since the user can drag the series horizontally to any position. The starting point \( u_{ij1} \) and end point \( u_{ij2} \) of dragging on the \( x \)-axis, as well as the selected series \( u_{ij3} \) are the input from the user. The horizontally shifting will not change \( y \)-coordinates, so for \( i = \text{x-shifting} \) and \( j \in J \), we have

\[
m_{ij} = ((u_{ij2} - u_{ij1}) \times I \{l_{ij} = u_{ij3}\}, 0),
\]

where \( I \) is the indicator function.
4.5 Additivity of interactions

Most of the interactions could be considered to be additive. Figure 6 shows an example, where two different results are generated by different ordering of interactions. Faceting on individual is done after faceting on variable, with the process following panels (a) → (b) → (d). Faceting on variable after individual, as in the process (a) → (c) → (e), produces a different configuration of the time series. The additive application of interactions is not commutative. Both results are useful, because each facilitates a different type of comparison of the series, using proximity. Plot (d) enables the comparison of individuals, within variables, while plot (e) enables the comparison of series within individual. It is also interesting to note that wrapping vertically after mirroring, will result in a horizon graph, like Figure 2 (f).

The cumulative interactions could entirely change both $x$ and $y$ coordinates of data. For example, Figure 4 firstly directs 75 steps of $x$-wrapping, and then a faceting by period. That gives the eventual movement by

$$m = (0, (l_{facet by variable} - 1) \times \text{max}(l_{facet by individual}) + (l_{facet by individual} - 1)).$$

Table 2 takes the first point of each series as an example to show $y$ and the changes at the three stages.

| $l_{variable}$ | $l_{individual}$ | $y$ at (a) | $y$ at (b) | $y$ at (d) |
|----------------|-----------------|------------|------------|------------|
| 1              | 1               | 0.16       | 0.16+(1-1)=0.16 | 0.16+(1-1)×3+(1-1)=0.16 |
| 1              | 2               | 0.33       | 0.33+(1-1)=0.33 | 0.33+(1-1)×3+(2-1)=1.33 |
| 1              | 3               | 0.26       | 0.26+(1-1)=0.26 | 0.26+(1-1)×3+(3-1)=2.26 |
| 2              | 1               | 0.84       | 0.84+(2-1)=1.84 | 0.84+(2-1)×3+(1-1)=3.84 |
| 2              | 2               | 0.84       | 0.84+(2-1)=1.84 | 0.84+(2-1)×3+(2-1)=4.84 |
| 2              | 3               | 0.90       | 0.90+(2-1)=1.90 | 0.90+(2-1)×3+(3-1)=5.90 |

Table 2: $y$-coordinates of the first point on each line, at Figure 6 (a) no faceting, (b) faceting by variable, (d) faceting by variable then individual.

4.6 Incremental vs baseline operations

Calculations can be made incrementally or with respect to a stable state (baseline), which, respectively, stores multiple copies of data, or a single storage of the data with storage of movement.
1. Incremental: Let
\[ s_0 = \text{initial status}, \]
\[ s_{t+1} = s_t + u_{i1}. \]

Note that every new status is only one interaction after the previous status. The coordinates are given by
\[ (x, y)_{s_{t+1}} = (x, y)_{s_t} + m_{i1} \]
\[ m_{i1} = f_i(p_i, u_{i1}, l_{i1}, (x, y)_{s_0}) \]

This procedure always computes the next position directly from the current position. The change only depends on the corresponding interaction parameters and the input. The current status is stored in the memory and ready to use for the next step. This method is an intuitive design for interactive graphs with few special interaction types. For example, scatterplots in cranvas can change the size and transparency of dots. The new size (or transparency) is always calculated by the multiplication of the current size (or transparency) and a constant. The constant is greater than 1 if the aesthetic parameter is increasing, and less than 1 if the parameter is decreasing. The exponential growth of the parameter accelerates the change and reduces the times of repeated interaction.

The advantage of this procedure includes the straightforward design, the convenience of moving to the previous or next status, and the efficiency of avoiding the recomputation. However, when there are many special interactions that could transform the data, we need to record both the initial and current data positions of each interaction type in the stream \( I \), because when moving backwards, we need to know when the initial state is reached and then stop. Then for an interaction stream \( I \) of length \( k \), at least \( k + 1 \) phases \((x, y)_{s_0}, (x, y)_{s_1}, (x, y)_{s_2}, \ldots, (x, y)_{s_k}\) should be saved, where \( t_1, t_2, \ldots, t_k \) are the time of the end of interaction types \( 1, 2, \ldots, k \). When the data set is large, those copies will occupy too much memory. Also, the storage and management of \( u_{i1} \)'s and \( l_{i1} \)'s is messy. Another drawback is that numerical errors could be introduced after the same number of forward and backward interactions, due to the floating-point arithmetic calculation.

2. Baseline: To store the \( k + 1 \) phases, we do not make \( k + 1 \) copies of the data set, instead, the movement item is traceable. The coordinates of any status can be computed by
\[ (x, y)_{s_t} = (x, y)_{s_0} + \sum_{i,j} m_{ij} \]
\[ = (x, y)_{s_0} + \sum_{i,j} f_i(p_i, u_{ij}, l_{ij}, j, (x, y)_{s_0}). \]

Note that when a new position is required, the calculation starts from the initial position instead of the previous status. The movements from the original to the current position are computed instantly.
The idea of baseline operation is not new. In the tour movement of XGobi and GGobi, the target position is calculated by the initial position and the projection parameters that come from the auto-oriented settings or user-oriented interactions (Cook et al. 1995; Cook and Buja 1997). However, the interactivities that we discussed in this paper is more complex, because we need to consider the interaction stream I, but the tour movement does not need to. When there is only one type of modification, the formulas in Section 4 will provide the target position easily. But when different types of modifications are mixed, the baseline method structures the computation well.

With this procedure we do not need to save the intermediate positions of the data, but we have to save the inputs for movements. Now the problem turns to: how to store the inputs including $p_i$, $u_{ij}$, $l_{ij}$, and $j$? The answer is, to save $l_{iJ_i}$ with the data, where $J_i$ is the largest $j$ in each $i \in I$. This is because $p_i$ is fixed and $j$ is known, $u_{ij}$ is usually a short array, but $l_{ij}$ is of the same length as the data, and depends on other parameters like $u_{ij}$. So the data frame that we use to save the data includes not only the coordinates, point parameters like size and color, but also the line group indicators $l_{iJ_i}$.

The advantage of this procedure is apparent: we do not have to save multiple copies of data, and it is a better way to manage the data and parameters during the interactions. However, it is not a comprehensive solution. We assumed that the movements are additive, but this is not always desirable. When changes in type of interaction make calculations better performed on a mid-way state, then it is better to stop, use this state as the baseline and then continue adding movements to this state.

5 Linking

Linking between plots is a critical component of using multiple linked windows (Stuetzle 1987) to explore data. Xie et al. (2014) describes types of linking and how it is realized in cranvas. It is possible to both self-link, which is important for temporal data, and link on different data sources or aggregation levels, using categorical variables. For the temporal and longitudinal data, linking is complicated when there is the need for different forms of the dataset or additional data. Two situations are discussed in the following sections.

5.1 Self-linking

Self-linking is primarily used to highlight all of the points in a time series when any one is selected. It is the most common behavior that a user would use. When there are multiple time series, it may also be useful to link to all points representing values recorded at a particular time.

Data underlying multiple time series, as for most of the other plots available in cranvas, are usually in “wide data” format (Table 3). One row contains the values recorded for a particular time, and aesthetic parameters are associated with each row. In this form if the display shows the multiple series, then when a user selects one point by brushing, all the points (values for V1, V2, V3) for this time are highlighted. This form is not conducive to selecting either a single point or an entire time series.
A more flexible format is provided by melting the data into the “long data” format (Table 4). In this format it is easy to realize brushing and self-linking in different ways: the user can select a single point, and (1) only this point is highlighted, (2) all points for that line (e.g. V1) are highlighted, or (3) all points for that time are highlighted (Figure 10). The latter two are achieved by treating the line group indicator or “Time” as a categorical linking variable, respectively.

| Variables | Parameters |
|-----------|------------|
| Time      | brushed    | .color    |
| 1         | FALSE      | red       |
| 2         | FALSE      | blue      |
| ...       | ...        | ...       |

Table 3: Basic tabular form of data underlying plot (left), think of this as the “wide data” format. Each row contains values recorded at one time point. The aesthetic parameters are associated with one time point. Brushing on this form will highlight the points for a particular time (right), three points if all three series are drawn. It is probably more desirable for the behavior to be different: that selecting a single point will highlight all the values for that series, or only a single point, which can be achieved by a data re-structuring.

Figure 10: When a single point is brushed, there could be three modes of highlighting: (left) only a single point is highlighted; (center) all points for that series are highlighted, by treating the line group indicators as the categorical linking variables; (right) all points for that time are highlighted, by using the time as a linking variable.

The long data format is typically the basic format for longitudinal data, where there may be differing number time points per subject, and measured at different times. So this approach to implementing the brushing works here, too.

5.2 Linking between plots

Linking between plots builds a reactive brushing chain that when the data points on one plot are brushed, then they are highlighted on all the plots. In the normal cases of cranvas, data behind the plots is unique, so the brush interaction will modify the parameter attached to the data and trigger the listeners of plots to highlight the corresponding part. However, linking between a time series plot and other plots is different, because the data to create the time plot is in the long data format.
Table 4: The “long data” format, which is called the melted form in Wickham (2007). This format allows a lot of flexibility. The “Variable” column can be used as a categorical linking variable, so that all points corresponding to “V1” are highlighted, when any one is selected, or a single point can be highlighted in the simplest brushing style, by changing the parameters of just that row. It would also be possible to use this form to use “Time” as a categorical variable for linking, and highlight all values recorded at a particular time.

format, while the data to create the other plots, like scatterplots or histograms, are in the wide data format. Hence, a link between the two data formats must be constructed.

Xie et al. (2014) delineated how to link two data objects. First a linking variable must be pointed out, then two listeners are attached on the two objects. If the .brushed parameter switches in one dataset, then the listener is triggered. And if any observations from the second dataset have the same value in the linking variable as the first dataset, then the corresponding .brushed parameter in the second dataset will be changed.

Note that the linking between wide data and long data is not a one-to-one linking. “Time” is the linking variable between two formats. In the direction from wide to long data, each entry in the wide data can project to multiple entries in the long data. In the opposite direction, an entry in the long data will map to one entry in the wide data. The unbalanced linking could produce a problem, as shown in Table 5.

This problem can be solved by cutting off the backward linking. To facilitate the cutoff, two signals are added respectively in the listeners of the two data objects. When one listener is triggered, the signal will be turned on until the listener finishes its work. During this period, the other listener cannot work. As in Table 5, the arrow from (b) to (c) will be cut off.

Figure 11 shows the linking between a longitudinal time plot, a map, and a histogram. The data come from the Google Flu Trends (http://www.google.org/flutrends/).

5.3 Additional linking issues

Besides the issue of wide data and long data, there are other datasets created during the interactions that could produce some linking issues, such as the follows.
Table 5: Linking between the long and wide data format may produce a problem. Suppose we start from brushing a point (V1 at time 2) in the long data (a). Then the listener of the long data is triggered and changes the .brushed parameter for time 2 in the wide data (b). Then the listener of the wide data is triggered and switches the .brushed parameter for all the observations at time 2 in the long data (c), which will update (a) but make a conflict.
Figure 11: Linking between different plots for the Google Flu Trends data from November 24, 2013 – March 16, 2014. (Left) Time series faceted by state (see https://vimeo.com/112528131 for the transition to the facets), (top right) choropleth map grey scale indicating time of the peak in the series, light is earlier, (bottom right) histogram of the number of searches. Two states in the map are brushed, which highlights all flu searches from these states in the time series and histogram.
• The polygon data from the area layer. The area layer does not only need the values in the time series, but also the baseline to form many polygons. To draw the polygons, the data must be rearranged in an order of polygon vertexes. Each polygon is made of four vertexes: two from the time series and two from the baseline. Because the polygon layer should listen to the point and line layers, the link between the original data and the polygon data is one-way.

• Copies of the dataset during the incremental operation. When adopting the incremental procedure in Section 4.6, the variations of original dataset will be created with multiple stages of the interactions. However, these additional datasets are not required to get linked, because we only make the copies of the coordinates, not the properties. To use the coordinates, we combine them with the properties in the last minute, so there will not be any linking issues.

• Additional areas from the vertical faceting. The example of vertical faceting is shown in Figure 5 and Figure 12 explained how the interaction creates the additional areas. In Figure 12, the black dots are given by the time series, and the red dots are created during the cropping step of faceting, by the choice of cutting lines. The black and red dots are mixed in some order to form the shaded polygons. Whenever a point is brushed, one, two, or even more cropped polygons should be highlighted. Hence the two-direction linking between the cropped polygons and points should be constructed. Note that this is a one-to-n mapping, and the linking variable is the point ID, which should be assigned to the polygons when the red dots are generated.

6 Querying

In each of the examples, and plots shown the time axis is simply drawn using consecutive integers. This is necessary for convenience and generalizability. To know which actual time value requires querying, by mousing over the display, if labels have been set up in detail, the user can learn what day, month, year or individual identifier is under the cursor.
7 Conclusions and future work

This paper describes how interactions can be added to temporal displays by using sequences of affine transformations. This approach proves a rich variety of ways to slice and dice temporal data to explore seasonality, dependence, trends, anomalies and individual differences. These interactive temporal displays can be embedded in a large interactive graphics software system, enabling linking between plots to explore more general data where temporal components are just one aspect of many. Not everything is solved in terms of additivity, speed and interaction direction, but most common actions are possible for reasonably sized data.

Optimal aspect ratio for temporal displays is an unsolved problem, and is something that we have grappled with in the implementation. According to (Cleveland, 1993) time series should be “banked” to 45°, which means that on average the angle of the lines, in comparison to the x- or y-axes is 45°. This is computed and used in the initial plots in cranvas, but as wrapping and faceting are conducted it probably should be re-calculated. Ideal integration of aspect ratio re-drawing with plot interactions could be examined in new work.

Future work would extend the interactions to include transforming between euclidean and polar coordinates, and between time and frequency domains. Earlier work in Dataviewer (Buja et al., 1988) allowed users to interactively lag time series to generate lag plots to explore auto-correlation. This could be reasonably be accomplished similarly to the interactions described here.

Using data transformations to generate interactions is efficient but it assumes the components are ordinal. This is not necessarily true, for example, in the flu searches series (Figure 11) were faceted on the categorical variable state. This requires that states are first recoded to numerical values. In R, this is implicitly alphabetical order. However, because the implementation is created in R, the factor levels can be re-ordered easily, enabling the recoding to numerical value to be quite fluid. In the flu searches example, the states were re-ordered by earliest peak of searches.

This paper focused on interactive graphics for multivariate time series and longitudinal data, but the ideas should extend some to other temporal-context data.

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