Analysis of Distributed Average Consensus Algorithms for Robust IoT networks

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Abstract—Internet of Things (IoT) is a heterogeneous network consists of various physical objects such as large number of sensors, actuators, RFID tags, smart devices, and servers connected to the internet. IoT networks have potential applications in healthcare, transportation, smart home, and automotive industries. To realize the IoT applications, all these devices need to be dynamically cooperated and utilize their resources effectively in a distributed fashion. Consensus algorithms have attracted much research attention in recent years due to their simple execution, robustness to topology changes, and distributed philosophy. These algorithms are extensively utilized for synchronization, resource allocation, and security in IoT networks. Performance of the distributed consensus algorithms can be effectively quantified by the Convergence Time, Network Coherence, Maximum Communication Time-Delay. In this work, we model the IoT network as a q-triangular r-regular ring network as q-triangular topologies exhibit both small-world and scale-free features. Scale-free and small-world topologies widely applied for modelling IoT as these topologies are effectively resilient to random attacks. In this paper, we derive explicit expressions for all eigenvalues of Laplacian matrix for q-triangular r-regular networks. We then apply the obtained eigenvalues to determine the convergence time, network coherence, and maximum communication time-delay. Our analytical results indicate that the effects of noise and communication delay on the consensus process are negligible for q-triangular r-regular networks. We argue that q-triangulation operation is responsible for the strong robustness with respect to noise and communication time-delay in the proposed network topologies.

Index Terms—Consensus Algorithms, Internet of Things, q-Triangular Networks, Small-World, Scale-Free Networks

I. INTRODUCTION

INTERNET of Things is formed by a large number of smart devices that dynamically cooperate and make their resources available to achieve a common goal. Smart devices consist of large number of sensors, actuators, RFID tags, smart devices, and servers connected to the internet. These devices should be able to take decisions, cooperative, and exchange information with the other devices and human beings. Consensus algorithms are simple distributed algorithms can be employed for the synchronization, security, and resource allocation problems in IoT Networks. These algorithms are suitable to resource constrained, dynamic, and fault tolerant networks. These algorithms performance is measured by the Convergence Time, Network Coherence, and Maximum Communication Time-Delay [1], [2]. Convergence Time is defined as the time required by the nodes to converge to the average of the initial node values. Convergence time is evaluated by the second smallest eigenvalue of Laplacian matrix. In consensus algorithms, in the absence of noise, nodes converge asymptotically to average of the initial state values. Due to noise, the node values fluctuates and they will not converge to the average, which can be studied by the network coherence.

Network coherence [3], [4] measures the deviation of node values from average of the initial state values. It is effectively captures the variance of the fluctuations in the first-order consensus system. In the second-order system, each node has two state variables, where the state of the entire system is thus captured by two vectors. Communication Time-Delay [5] measures ability of consensus algorithm resistant to communication delay between nodes, which is decided by the largest eigenvalue of Laplacian matrix. All these metrics depend on the eigenvalues of the Laplacian matrix, which represents the topology of any network. To avail the advantages of consensus algorithms, it is necessary to measure these performance metrics for IoT networks. To understand the effect of network parameters on convergence time of consensus algorithm, authors modeled the WSN as a one dimensional lattice network and derive the explicit expressions of convergence rate for periodic gossip algorithms. Lattice networks model the finite sized resource constrained networks and facilitate the closed form expressions for convergence time.

Small-world and Scale-free topologies are popular complex network models that can be applied to IoT networks to improve the robustness against random networks. Small-world model creates shorter average path length which reduces the maximum communication delay drastically in IoT. Moreover, adding shortcuts in communication will also improves the robustness. Scale-free model [8] is one of the classic models in which the node degree follows power-law distribution. These topologies give better performance in withstanding random attacks. Because of these salient features, small-world and scale-free topologies have been utilized in modeling IoT networks [9], [10], [11]. Triangulation is a popular graph operation [12], [13] in network science to obtain network models for studying the scale-free and small-world characteristics of networks. In our work, we modeled the IoT network as a q-triangular r-regular ring network and derived the closed-form expressions of convergence time, network coherence, and maximum communication time-delay. Our numerical results depict that our proposed network topology significantly improves the robustness of the consensus algorithms against communication delays and noise.
II. RELATED WORK

In literature, consensus algorithms have been widely studied for IoT networks. In particular, most of the researchers used these algorithms for distributed computation, security, and synchronization problems. In [14], authors employed the consensus algorithm for distributed monitoring of IoT networks. This work used f-consensus theory and derived convergence conditions that allows a nice trade-off between precision and resource consumption. A consensus based distributed algorithm for service detection and data processing is proposed in [15] for IoT networks. This algorithm calculates the consensus locally and combines in an iterative fashion to improve the robustness of the consensus process. Resource allocation, task allocations are critical issues to be focused in resource constrained networks. In [16], authors propose a consensus based optimization algorithm for resource allocation in heterogeneous IoT networks. They have shown that network converges to a solution where network resources are homogeneously distributed.

A distributed algorithm where objects cooperate to reach a consensus on resources allocation is proposed in [17]. This work implements optimization process to select the objects that would guarantee the minimum Quality of Information and improve the lifetime of objects. Authors in [18], evaluated the average consensus algorithm on IoT testbed. This work avoids the deadlock problems and deals with packet losses, delays, and multi-rate behavior in IoT networks. A distributed soft clustering algorithm based on average consensus algorithm for the IoT is presented in [19]. This work claimed it provides the stable clustering quality for IoT networks. Blockchain technology is well known for its potential use in security mechanisms and protect from different attacks. However, this technology is suitable to power constrained devices. Consensus algorithms have been extensively used in blockchains for IoT networks. In [20], authors presented the survey of the various blockchain based consensus methods that are suitable to resource constrained IoT networks.

In [21], authors proposed a blockchain system with credit-based consensus mechanism for IoT networks which ensures system security and transaction efficiency simultaneously. Trust models are very popular in ensuring security for peer to peer, WSN, and IoT networks. In [22], author proposes a trust model based on consensus to evaluate the trustworthiness of IoT nodes to detect malicious nodes. Authors propose a novel consensus algorithm called Proof-of-Authentication in [23] to introduce a cryptographic authentication mechanism for resource constrained devices. Authors proposed a lightweight proof of block and trade consensus algorithm for IoT blockchain in [24]. In [25], authors presented a distributed clustering algorithm for IoT networks where observations are distributed and data transmission is only allowed between one-hop neighbors. A blockchain based on consensus algorithm for IoT applications is proposed in [26]. In [27], authors presented a survey on blockchain-based consensus methods for resource-constrained IoT networks. Authors presented a lightweight consensus algorithm in [28] that can be implemented in the IoT environment. This algorithm evaluates the feasibility of medical supply and drug transportation to mitigate privacy issues. In our work, we provide the theoretical tools to study the consensus algorithms for IoT networks. The properties of small-world and scale-free networks ensure the network robustness against random attacks. This motivates us to develop the q-triangular r-regular network which incorporates the properties of both small-world and scale-free networks. We derive the closed form expressions of convergence time, network coherence, and maximum communication-time delay and study the effect of network parameters on these performance metrics. Our theoretical results provide important insights to design and control the convergence of consensus algorithms.

A. Main Contributions

1) Firstly, we model the IoT network as a q-triangular r-regular ring network and compute the Laplacian eigenvalues.
2) Secondly, we derive the explicit expressions for convergence time, first order network coherence, second order network coherence, and maximum communication-time delay of q-triangular r-regular ring networks for average consensus algorithms.
3) Finally, we present the numerical results and study the effect of node degree, network size, triangulation parameter on convergence time, network coherence, and maximum communication-time delay of consensus algorithms.

B. Organization

The remainder of the paper is organized as follows. We provide some preliminaries and review the consensus algorithms in Section III. In Section IV, we derive the explicit eigenvalues of consensus algorithm for q-triangular r-regular networks. In Section V, we derive the explicit expressions of convergence time, network coherence, and maximum communication time-delay. We present the numerical results and study the effect of triangulation parameter, network size, and node degree on the convergence time, network coherence, and maximum communication time-delay of consensus algorithms in Section VI. Finally, we discuss the conclusions in Section VII.

Notations: Table 1 presents the notations and corresponding definitions used in the paper.

III. BRIEF REVIEW OF AVERAGE CONSENSUS ALGORITHM

In this section, we introduce some basic concepts in spectral graph theory and review the consensus algorithm. We consider an n-vertex simple connected graph \( G = (V, E) \), where \( V = V(G) = \{1, 2, \ldots, n\} \) is the vertex set and \( E = E(G) = \{e_1, e_2, \ldots, e_m\} \) is the edge set of the graph. The adjacency matrix \( A(G) \) of \( G \) is a square matrix of order \( n \), whose \((i,j)^{th}\) entry is equal to 1(or 0) if the vertices \( i \) and \( j \) are adjacent(or not adjacent). The incidence matrix \( B \) of \( G \) is a matrix of order \( n \times m \), whose \((i,j)^{th}\) entry is equal to 1(or 0) if the vertex \( i \) and the edge \( e_j \) are incident(or not incident). Let \( D(G) \) be the diagonal matrix of vertex degree. Then the Laplacian matrix of \( G \) is...
**TABLE I**

| Notation | Definition |
|----------|------------|
| \( x \) | Vector of a state variables |
| \( n \) | Number of nodes |
| \( \lambda_2 \) | Second Smallest Eigenvalue |
| \( W \) | Weight matrix |
| \( T \) | Convergence Time |
| \( \lambda \) | Eigen Value |
| \( h \) | Consensus parameter |
| \( \gamma \) | Consensus parameter |
| \( B \) | Incidence Matrix |
| \( H^{(1)} \) | First Order Network Coherence |
| \( H^{(2)} \) | Second Order Network Coherence |
| \( T_{max} \) | Maximum Communication Time-Delay |
| \( r \) | Node Degree |
| \( q \) | Triangulation Parameter |
| \( n \) | Consensus Parameter |
| \( \lambda_{n-1} \) | Largest Eigenvalue |

\[
L(G) = D(G) - A(G). \text{ Also we have } L(G) = 2D(G) - BB^T.
\]

Let \( x_k(0) \) denotes the real scalar variable of node \( k \) at \( t = 0 \). Average consensus algorithm computes the average \( x_{avg} = \sum_{k=1}^{n} x_k(0) \) at every node through a distributed approach which does not require any centralized node. At time instant \( t + 1 \), the real scalar variable at node \( i \) is expressed as

\[
x_i(t+1) = x_i(t) + h \sum_{j \in N_k} (x_j(t) - x_k(t)), \quad k = 1, ..., n, ~ (1)
\]

where \( h \) is a consensus parameter and \( N_k \) denotes neighbor set of node \( k \). This can be also expressed as a simple linear iteration as

\[
x(t + 1) = Wx(t), \quad t = 0, 1, 2, ..., \quad (2)
\]

where \( W \) denotes weight matrix, and \( W_{kj} \) is a weight associated with the edge \((k, j)\). If we assign equal weight \( h \) to each link in the network, then optimal weight for a given topology is

\[
W_{kj} = \begin{cases} 
    h & \text{if} \quad (k, j) \in E, \\
    1 - h \deg(v_k) & \text{if} \quad k = j, \\
    0 & \text{otherwise}. 
\end{cases} \quad (3)
\]

and weight matrix is given by

\[
W = I - hL. \quad (4)
\]

where \( I \) is a \( n \times n \) identity matrix.

Definition 1: Convergence time \((T)\) \[2\] is defined as the time required for nodes to reach the consensus. And it is measured by the

\[
T = \frac{1}{\ln \left( \frac{1}{\gamma} \right)}, \quad (5)
\]

where \( \gamma \) is convergence parameter.

Physical objects in IoT network should be robust with respect to different parameters, including hardware failure, environmental uncertainty and communication failures. Robustness to uncertainty and noise can be effectively measured by network coherence \[30, 31\].

**Definition 2**: Network coherence is also defined as robustness to noise, and it can be measured by the deviation of each node’s state from the global average of all current states. In the first-order consensus problem, each node has a single state subject to noise. Network coherence of a first order system is measured by

\[
H^{(1)} = \frac{1}{2N} \sum_{k=2}^{N} \frac{1}{f(\lambda_k)} 
\]

Where \( f(\lambda_k) \) is the \( k^{th} \) eigenvalue of Laplacian matrix.

**Definition 3**: In the second-order consensus problem, each node has two states. Network coherence of a second order system is measured by

\[
H^{(2)} = \frac{1}{2N} \sum_{k=2}^{N} \frac{1}{f(\lambda_k)^2} 
\]

Definition 4: Maximum Communication Time-Delay \[31\]. \[5\] measures the ability of consensus algorithm resilient to maximum communication delay between nodes and it is expressed as

\[
T_{max} = \frac{\pi}{2f(\lambda_{n-1}(L))} 
\]

where \( f(\lambda_{n-1}) \) is the largest eigenvalue of Laplacian matrix.

In this work, we use the best constant weights algorithm to derive the closed-form expressions of convergence time as this algorithm gives the fastest convergence rate among the other uniform weight methods \[32, 33\].

**Algorithm 1**: Best Constant Weights Algorithm

1. Derive the expression for eigenvalues of Laplacian matrix.
2. Compute the second smallest eigenvalue of Laplacian matrix \((f(\lambda_1(L)))\) and largest eigenvalue of Laplacian matrix \((f(\lambda_{n-1}(L)))\).
3. Obtain \( \gamma \) using

\[
|1 - h f(\lambda_1(L))| = |1 - h f(\lambda_{n-1}(L))| \quad (9)
\]

4. Substitute the \( \gamma \) in \( |1 - h f(\lambda_1(L))| \) to obtain the convergence parameter \((\gamma)\).
5. Finally, convergence time \((T)\) can be calculated by

\[
T = \frac{1}{\ln \left( \frac{1}{\gamma} \right)}.
\]

**IV. Q-TRIANGULAR R-REGULAR RING NETWORKS AND RELATED MATRICES**

In this section, we derive the eigenvalues of Laplacian matrix for q-triangular r-regular ring networks. We can see r-regular ring network in Fig. 1 for \( r = 4 \) and q-triangular
and another is corresponding to a vertex in $V_2$. Therefore, we have

$$B^T \begin{bmatrix} J_1 \\ -J_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (12)$$

**Theorem 1.** Let $G$ be a simple, connected, $r$-regular graph with $n$ nodes and $m$ edges. Let $0 = \lambda_1 \leq \lambda_2 \cdots \leq \lambda_n$ be the eigenvalues of $L(G)$ and $x_1, x_2, \cdots, x_n$ be their corresponding eigenvectors. Let $\mu_k = (qr + \lambda_k - 2)^2 + 4q(2r - \lambda_k)$ and $y = \frac{qr + \lambda_k - 2 + \sqrt{\mu_k}}{2q(2r - \lambda_k)}$. Then if $G$ is non-bipartite: $\frac{1}{2}[qr + \lambda_k + 2 \pm \sqrt{\mu_k}]$, are eigenvalues of $L(R_q(G))$ corresponding to the eigenvector $\begin{bmatrix} x_i \\ yB^T x_i \end{bmatrix}$ for $i = 1, 2, \cdots, n$ and 2 is an eigenvalue of $L(R_q(G))$ with multiplicity $mq - n$, corresponding to the eigenvectors $\begin{bmatrix} 0 \\ V_j \end{bmatrix}$ for $j = 1, 2, \cdots, mq - n$, where $\{V_1, V_2, \cdots, V_{mq-n}\}$ is a basis of the null space of the matrix $C = (B B \cdots B)$.

If $G$ is bipartite: $\frac{1}{2}[qr + \lambda_k + 2 \pm \sqrt{\mu_k}]$, are eigenvalues of $L(R_q(G))$ corresponding to the eigenvector $\begin{bmatrix} x_i \\ yB^T x_i \end{bmatrix}$ for $i = 1, 2, \cdots, n - 1$, 2 is an eigenvalue of $L(R_q(G))$ with multiplicity $mq - n + 1$, corresponding to the eigenvectors $\begin{bmatrix} 0 \\ U_j \end{bmatrix}$ for $j = 1, 2, \cdots, mq - n + 1$, where $\{U_1, U_2, \cdots, U_{mq-n+1}\}$ is a basis of the null space of the matrix $C = (B B \cdots B)$ and $r(q + 2)$ is an eigenvalue of $L(R_q(G))$ corresponding to the eigenvector $\begin{bmatrix} x_n \\ 0 \end{bmatrix}$.

**Proof.** Assume that the construction of $R_q(G)$ from $G$ happens by $q$ consecutive steps. First step we construct $R_1(G)$ by adding $m$ number vertices in $G$, then in the $i$th step we construct $R_i(G)$ by adding $m$ number vertices in $R_{i-1}(G)$ for $i = 2, 3, \cdots, q$. Let $V^{(i)}$ is the set of newly added vertices in $i$th step. Then we have $V(R_q(G)) = V(G) \cup V^{(1)} \cup V^{(2)} \cdots \cup V^{(q)}$. With this ordering of vertices in $R_q(G)$ we get

$$L(R_q(G)) = \begin{bmatrix} qD(G) + L(G) & -B & -B & \cdots & -B \\ -B^T & 2I_m & 0 & \cdots & 0 \\ -B^T & 0 & 2I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -B^T & 0 & 0 & \cdots & 2I_m \end{bmatrix}.$$
When $G$ is non-bipartite, we have for each $i \in \{1, 2, \cdots, n\}$,

$$L(R_q(G)) = \begin{bmatrix} x_i \\ y B^T x_i \\ \vdots \\ y B^T x_i \end{bmatrix} = \begin{bmatrix} qr x_i + \lambda x_i - qy B^T x_i \\ -B^T x_i + 2y B^T x_i \\ \vdots \\ -B^T x_i + 2y B^T x_i \end{bmatrix}$$

Thus we get for $i = 1, 2, \cdots, n$, $f(\lambda_k)$ are the eigenvalues of $L(R_q(G))$. Now as the graph $G$ is bipartite, so $\text{rank}(B) = n$, then the matrix $C = (B \ B \ \cdots \ B)$ which is of order $n \times mq$, is also have rank $n$. Therefore we have

$$L(R_q(G)) = \begin{bmatrix} 0 \\ V_j \end{bmatrix} = 2 \begin{bmatrix} 0 \\ V_j \end{bmatrix}.$$  

Thus we get 2 is an eigenvalue of $L(R_q(G))$ with multiplicity $mq - n$. 

When $G$ is bipartite, we have $\lambda_n = 2r$. So the term $y$ in equation (13) is not defined for $\lambda = 2r$. But for all $i \in \{1, 2, \cdots, n-1\}$, equation (13) is satisfied. Thus $f(\lambda_k)$ are the eigenvalues of $L(R_q(G))$ for each $i \in \{1, 2, \cdots, n-1\}$. Also, as $G$ is bipartite, so $\text{rank}(B) = n - 1$, therefore $\text{dim}(\text{Ker}(C)) = mq - n + 1$. So if $U_1, U_2, \cdots, U_{mq-n+1}$ is a basis of $\text{Ker}(C)$, then for each $j \in \{1, 2, \cdots, mq-n+1\}$, we have

$$L(R_q(G)) \begin{bmatrix} 0 \\ U_j \end{bmatrix} = 2 \begin{bmatrix} 0 \\ U_j \end{bmatrix}.$$  

Thus we get 2 is an eigenvalue of $L(R_q(G))$ with multiplicity $mq - n + 1$. Also we have

$$L(R_q(G)) = \begin{bmatrix} x_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} qr + 2r \\ -B^T x_n \\ \vdots \\ -B^T x_n \end{bmatrix}.$$  

Therefore $qr + 2r$ is an eigenvalue of $L(R_q(G))$.  

V. CONVERGENCE ANALYSIS FOR Q-TRIANGULAR R-REGULAR RING NETWORK

In this section, we derive the explicit expressions of Convergence Time, First Order Network Coherence, Second Order Network Coherence, and Maximum Communication Time-Delay.

**Theorem 2.** Convergence Time of q-triangular r-regular ring network for average consensus algorithm is

$$T = \frac{1}{\ln \left( \frac{(q+1)r+3-l}{1-(q+1)r+l} \right)}$$

where

$$l_1 = \frac{\sin \frac{\pi}{n}(r+1)}{\sin \frac{\pi}{n}}.$$  

**Proof.** Eigenvalues of Laplacian matrix for r-regular ring network is

$$\lambda_k = r - 2 \sum_{j=1}^{\frac{n}{r}} \cos \left( \frac{2\pi kj}{n} \right).$$  

From Dirichilet Identity $1 + 2 \sum_{j=1}^{\frac{n}{r}} \cos (jx) = \frac{\sin (\frac{\pi x}{\frac{n}{r}})}{\sin \frac{\pi}{\frac{n}{r}}}$, we can further simplify the above expression as

$$\lambda_k = r + 1 - \frac{\sin (r + 1) x}{\sin \frac{2\pi}{n}}.$$  

where $n$ is number of nodes and $r$ is node degree. From Theorem 1, we can write the eigenvalues of Laplacian matrix for q-triangular regular network can be written as

$$f(\lambda_k) = \frac{1}{2} \left( qr + \lambda_k + 2 \pm \sqrt{(qr + \lambda_k - 2)^2 + 4q(2r - \lambda_k)} \right).$$  

We substitute the (17) in (18) to obtain the eigenvalues of Laplacian matrix for q-triangular r-regular ring network as

$$f(\lambda_k) = \frac{1}{2} \left( q^2 + 3 - l \pm \sqrt{(q_2 - 1 - l)^2 + 4q(r - 1 + l)} \right),$$  

where $l_2 = \frac{\sin \frac{\pi}{\frac{n}{r}}}{\sin \frac{\pi}{\frac{n}{r}}}$ and $(q+1)r = q_2$. In the $\lambda_0, \lambda_1, \ldots, \lambda_{n-1}$, we have observed $\lambda_1$ is the second smallest eigenvalue and $\lambda_{n-1}$ is the largest eigenvalue of Laplacian matrix. Hence, we can write the largest eigenvalue of Laplacian matrix of q-triangular r-regular ring network as

$$f(\lambda_{n-1}) = \frac{1}{2} \left( q_2 + 3 - l_3 + \sqrt{(q_2 - 1 - l_3)^2 + 4q(r - 1 + l_3)} \right),$$  

where $l_3 = \frac{\sin \frac{\pi}{\frac{n}{r}}}{\sin \frac{\pi}{\frac{n}{r}}}$ and similarly, we can write the smallest eigenvalue of Laplacian matrix of q-triangular r-regular ring network as

$$f(\lambda_1) = \frac{1}{2} \left( q_2 + 3 - l_3 - \sqrt{(q_2 - 1 - l_3)^2 + 4q(r - 1 + l_3)} \right).$$
From best constant algorithm, we can obtain the convergence parameter as
\[
\gamma = \sqrt{(q+1)r - 1 - \frac{\sin((r+1)\pi)}{\sin \frac{\pi}{n}}} + 4q\left(\frac{r - 1 + \frac{\sin((r+1)\pi)}{\sin \frac{\pi}{n}}}{(q+1)r + 3 - \frac{\sin((r+1)\pi)}{\sin \frac{\pi}{n}}}\right)
\] (22)
Substituting the \(\gamma\) value in (5) proves the theorem.

**Theorem 3.** Network Coherence of \(q\)-triangular \(r\)-regular ring network for first order system is
\[
H^{(1)} = \frac{1}{2N} \sum_{k=1}^{(N-1)} \frac{2}{(q+1)r + 3 - l + c_1},
\] (23)
and second order system is
\[
H^{(2)} = \frac{1}{2N} \sum_{k=1}^{(N-1)} \frac{2}{((q+1)r + 3 - l + c_1)^2},
\] (24)
where \(c_1 = \frac{\sqrt{(1-(q+1)r+l)^2 + 4q(r+l-1)}}{\sin \frac{\pi}{n}}\).

**Proof.** Substitute the (18) into (6) and (7) to prove the Theorem.

**Theorem 4.** Maximum Communication Time-Delay of \(q\)-triangular \(r\)-regular ring network for average consensus algorithm is
\[
R_{\text{max}} = \frac{\pi}{(q+1)r + 3 - l - c_1},
\] (25)
\(c_1 = \frac{\sqrt{(1-(q+1)r+l)^2 + 4q(r+l-1)}}{\sin \frac{\pi}{n}}\).

**Proof.** Substitute the (20) into (8) proves the theorem.

**VI. RESULTS AND DISCUSSION**

In this section, we present the numerical results to investigate the effect of node degree, network size, and triangulation parameter on convergence time, first order network coherence, second order network coherence, and maximum communication time-delay of average consensus algorithm. In Fig. 3, we plot the convergence time against triangulation parameter for \(n=100\). We have observed the convergence time linearly increases with the triangulation parameter \(q\) and convergence time reduces with the increase in \(r\) values. Increase in degree will leads to more participation among IoT nodes. This will naturally reduces the convergence time. To observe the effect of network size on convergence time, we plot the Fig. 4 for \(r=4\). It is noted that convergence time is drastically increasing with the network size. For \(r=50\), we plot the maximum communication time-delay against triangulation parameter \(q\) for different network sizes in Fig. 5, and observed that maximum communication time-delay exponentially reduces with the \(q\). This is because of nodes are able to reach the long distance nodes due to triangulation operation. To study the effect of number of node degree on maximum communication time-delay, we plot Fig. 6 for \(n=100\). It is observed that maximum communication time-delay decreases with \(r\) values. In Fig. 7, we plot the first order network coherence versus triangulation parameter for \(r=4\) and observed that network coherence exponentially decreases with the triangulation parameter and decreases with the node degree. To observe the effect of node degree on second order network coherence, we plot the Fig. 8. We have observed that there is a very little increase in second order network coherence with the increase in \(r\) values. In Fig. 9, we plot the first order consensus versus \(q\) for \(r=5\) and observed that first order network coherence increases with the network size. As shown in Fig. 10, the effect of network size on second order network coherence is drastically reduced.
Fig. 5. Maximum Communication Time-Delay versus $q$ for Average Gossip Algorithm ($r=50$).

Fig. 6. Maximum Communication Time-Delay versus $q$ for Average Gossip Algorithm ($n=100$).

Fig. 7. First Order Coherence versus $q$ for Average Gossip Algorithm ($r=4$).

Fig. 8. Second Order Coherence versus $q$ for Average Gossip Algorithm ($n=100$).

Fig. 9. First Order Coherence versus $q$ for Average Gossip Algorithm ($r=5$).

Fig. 10. Second Order Coherence versus $q$ for Average Gossip Algorithm ($r=5$).
VII. CONCLUSIONS

In this paper, we have modeled the IoT network as an q-triangular r-regular network and studied the consensus algorithms, with an emphasis on the convergence time, network coherence, and maximum communication time-delay. We first provided the eigenvalues of Laplacian matrix for q-triangular r-regular networks. We then derived the explicit expressions of convergence time, network coherence, and maximum communication time-delay for q-triangular r-regular ring networks. We studied numerically the convergence time, first order coherence, second-order coherence, and maximum communication time-delay with respect to network size, triangulation parameter, and node degree. Our results indicate that proposed network topology is resistant to noise and communication time-delay of average consensus algorithms for large-scale IoT networks. We argued that the scale-free and small-world structure of the q-triangulation networks is responsible for the robustness. Future work should include the unveiling the effects of energy consumption and position of nodes on the performance metrics of consensus algorithms for robust IoT networks.

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REFERENCES

[1] R. O. Saber and R. M. Murray, “Consensus protocols for networks of dynamic agents,” 2003.
[2] L. Xiao, S. Boyd, and S.-J. Kim, “Distributed average consensus with least-mean-square deviation,” Journal of parallel and distributed computing, vol. 67, no. 1, pp. 33–46, 2007.
[3] S. Patterson and B. Bamieh, “Consensus and coherence in fractal networks,” IEEE Transactions on Control of Network Systems, vol. 1, no. 4, pp. 338–348, 2014.
[4] B. Bamieh, M. R. Jovanovic, P. Mitra, and S. Patterson, “Coherence in large-scale networks: Dimension-dependent limitations of local feedback,” IEEE Transactions on Automatic Control, vol. 57, no. 9, pp. 2235–2249, 2012.
[5] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” IEEE Transactions on Automatic control, vol. 49, no. 9, pp. 1520–1533, 2004.
[6] S. Dhuli, K. Gaurav, and Y. N. Singh, “Convergence analysis for regular wireless consensus networks,” IEEE Sensors Journal, vol. 15, no. 8, pp. 4522–4531, 2015.
[7] S. Kouachi, S. Dhuli, and Y. N. Singh, “Convergence rate analysis of periodic gossip algorithms for one-dimensional lattice wsns,” IEEE Sensors Journal, vol. 20, no. 21, pp. 13 150–13 160, 2020.
[8] N. Chen, T. Qiu, X. Zhou, K. Li, and M. Atiquzzaman, “An intelligent robust networking mechanism for the internet of things,” IEEE Communications Magazine, vol. 57, no. 11, pp. 91–95, 2019.
[9] T. Qiu, B. Li, W. Qu, E. Ahmed, and X. Wang, “Tosg: A topology optimization scheme with global small world for industrial heterogeneous internet of things,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3174–3184, 2018.
[10] T. Qiu, J. Liu, W. Si, M. Han, H. Ning, and M. Atiquzzaman, “A data-driven robustness algorithm for the internet of things in smart cities,” IEEE Communications Magazine, vol. 55, no. 12, pp. 18–23, 2017.
[11] I. Sohn, “Small-world and scale-free network models for iot systems,” Mobile Information Systems, vol. 2017, 2017.
[12] Y. Yi, Z. Zhang, and S. Patterson, “Scale-free loop structure is resistant to noise in consensus dynamics in complex networks,” IEEE transactions on cybernetics, vol. 50, no. 1, pp. 190–200, 2018.
[13] Y. Zeng and Z. Zhang, “Hitting times and resistance distances of q-triangulation graphs: Accurate results and applications,” arXiv preprint arXiv:1808.01025, 2018.
[14] D. Carvin, P. Owezarski, and P. Berthou, “A generalized distributed consensus algorithm for monitoring and decision making in the iot,” in 2014 International Conference on Smart Communications in Network Technologies (SaCoNeT). IEEE, 2014, pp. 1–6.
[15] S. Li, G. Oikonomou, T. Tryfonas, T. M. Chen, and L. Da Xu, “A distributed consensus algorithm for decision making in service-oriented internet of things,” IEEE Transactions on Industrial Informatics, vol. 10, no. 2, pp. 1461–1468, 2014.
[16] G. Colistra, V. Pilloin, and L. Atzori, “The problem of task allocation in the internet of things and the consensus-based approach,” Computer Networks, vol. 73, pp. 98–111, 2014.
[17] V. Pilloin and L. Atzori, “Consensus-based resource allocation among objects in the internet of things,” Annals of Telecommunications, vol. 72, no. 7, pp. 415–429, 2017.
[18] B. Orostica and F. Núñez, “Robust gossiping for distributed average consensus in iot environments,” IEEE Access, vol. 7, pp. 994–1005, 2018.
[19] B. Oróstica and F. Núñez, “A multi-cast algorithm for robust average consensus over internet of things environments,” Computer Communications, vol. 140, pp. 15–22, 2019.
[20] H. Yu, H. Chen, S. Zhao, and Q. Shi, “Distributed soft clustering algorithm for iot based on finite time average consensus,” IEEE Internet of Things Journal, 2020.
[21] M. Salimitari, M. Chatterjee, and Y. P. Fallah, “A survey on consensus methods in blockchain for resource-constrained iot networks,” Internet of Things, p. 100212, 2020.
[22] J. Huang, L. Kong, G. Chen, M.-Y. Wu, X. Liu, and P. Zeng, “Towards secure industrial iot: Blockchain system with credit-based consensus mechanism,” IEEE Transactions on Industrial Informatics, vol. 15, no. 6, pp. 3680–3689, 2019.
[23] Z. Ma, L. Liu, and W. Meng, “Towards multiple-mix-attack detection via consensus-based trust management in iot networks,” Computers & Security, vol. 96, p. 101898, 2020.
[24] D. Puthal, S. P. Mohanty, V. P. Yanambaka, and E. Koungianos, “Pobt: A novel consensus algorithm for fast scalable private blockchain for large-scale iot frameworks,” arXiv preprint arXiv:2001.07297, 2020.
[25] S. Biswas, K. Sharif, F. Li, S. Maharjan, S. P. Mohanty, and Y. Wang, “Pobt: A lightweight consensus algorithm for scalable iot business blockchain,” IEEE Internet of Things Journal, vol. 7, no. 3, pp. 2343–2355, 2019.
[26] H. Chen, H. Yu, S. Zhao, and Q. Shi, “Consensus-based distributed clustering for iot,” in ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). IEEE, 2020, pp. 8324–8328.
[27] A. Dorri and R. Jurdak, “Tree-chain: A fast lightweight consensus algorithm for iot applications,” in 2020 IEEE 45th Conference on Local Computer Networks (LCN). IEEE, 2020, pp. 369–372.
[28] M. Salimitari and M. Chatterjee, “A survey on consensus protocols in blockchain for iot networks,” arXiv preprint arXiv:1809.05613, 2018.
[29] S. Maitra, V. P. Yanambaka, D. Puthal, A. Abdelgawad, and K. Yelamardhi, “Integration of internet of things and blockchain toward portability and low-energy consumption,” Transactions on Emerging Telecommunications Technologies, p. e1103, 2020.
[30] G. F. Young, L. Scardovi, and N. E. Leonard, “Robustness of noisy consensus dynamics with directed communication,” in Proceedings of the 2010 American Control Conference. IEEE, 2010, pp. 6312–6317.
[31] Y. Qi, Z. Zhang, Y. Yi, and H. Li, “Consensus in self-similar hierarchical graphs and sierpiński graphs: Convergence speed, delay robustness, and coherence,” IEEE transactions on cybernetics, vol. 49, no. 2, pp. 592–603, 2018.
[32] M. Toulouse, B. Q. Minh, and P. Curris, “A consensus based network intrusion detection system,” in IF Convergence and Security (ICITCS), 2015 5th International Conference on. IEEE, 2015, pp. 1–6.
[33] K. Avrachenkov, M. El Chamie, and G. Neglia, “A local average consensus algorithm for wireless sensor networks,” 2011.
[34] R. B. Bapat, Graphs and matrices. Springer, 2010, vol. 27.