VQ-AR: Vector Quantized Autoregressive Probabilistic Time Series Forecasting

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Abstract

Time series models aim for accurate predictions of the future given the past, where the forecasts are used for important downstream tasks like business decision making. In practice, deep learning based time series models come in many forms, but at a high level learn some continuous representation of the past and use it to output point or probabilistic forecasts. In this paper, we introduce a novel autoregressive architecture, VQ-AR, which instead learns a discrete set of representations that are used to predict the future. Extensive empirical comparison with other competitive deep learning models shows that surprisingly such a discrete set of representations gives state-of-the-art or equivalent results on a wide variety of time series datasets. We also highlight the shortcomings of this approach, explore its zero-shot generalization capabilities, and present an ablation study on the number of representations. The full source code of the method will be available at the time of publication with the hope that researchers can further investigate this important but overlooked inductive bias for the time series domain.

1 Introduction

Time series forecasting is an important business and scientific machine learning problem which is typically used to support decision making for downstream tasks. Classical methods like in [22] model each time series in a dataset individually using hand-crafted features and are used extensively as they providing strong baselines.

Recently deep learning based methods offer an alternative approach by utilizing a shared model trained over the whole dataset of time series, without the need for explicit feature engineering. These methods are competitive, especially when dealing with a large amount of data, against the classical methods, but can also suffer from overfitting or learn spurious correlations due to their flexibility.

Often the forecasts are point valued, however it is much more useful for the decision making process to also incorporate the inherent (aleatoric) uncertainty present in the observations via probabilistic forecasts. In the univariate setting this is usually done by learning the parameters of some chosen distribution, or via quantile regression or non-parametric and semi-parametric models which explicitly learn the conditional quantiles of the next time step given the past [39]. Other approaches include using Conformal Predictions [49, 54]. In the multivariate setting [50] however, one needs to resort to some approximation of the full multivariate distribution [47] due to computational tractability or learn the conditional distribution via Normalizing Flows [46, 10], Energy Based Methods [45], or Generative Adversarial Networks (GANs) [30].
As in most supervised deep learning methods, these models learn some representation of the history of a time series in order to best forecast, which can be thought of as a kind of self-supervised learning task (not unlike GPT [47] or Bert [11]) in the Natural Language Processing (NLP) setting. Thus in order to provide better historical representations we have a large collection of architectural choices we can make for the problem or dataset at hand.

The core insight of this work is the realization that there might only be a discrete collection of representations of time series histories which could potentially be used for the prediction task. The success of modern NLP on the back of discrete token representations leads us to consider if something similar would also work in the time series domain. Thus, in this paper we take inspiration from the Vector Quantized-Variational AutoEncoder (VQ-VAE) [51] model to learn a set of latent representations in order to best forecast in an autoregressive fashion.

Thus, the main contribution of this paper is a novel autoregressive time series forecasting method which is trained end-to-end and provides excellent inductive bias for the probabilistic forecasting task. We highlight the model’s performance and properties via extensive experimentation and show that it achieves comparable results against ensembles of tree-based methods without extensive feature engineering.

2 Background

We introduce some notation and go over some background material needed for the model first.

In this work we assume the univariate forecasting setup of DeepAR [48], however the method also applies to the multivariate setting. Formally we assume we are given a training dataset of \( D \geq 1 \) time series \( D_{\text{train}} = \{x_{i,1:T_i}\} \) where \( i \in \{1, \ldots, D\} \) and at each time \( t \) we have \( x_t \in \mathbb{R} \) or in \( \mathbb{N} \). The problem will require us to predict \( P \geq 1 \) steps into the future and thus will come with a back-testing test set of these \( D \) time series denoted by \( D_{\text{test}} = \{x_{i,T_i+1:T_i+P}\} \). Note that even though we are denoting the time index \( t \) by an incremental counter here, in reality each \( t \) has a date-time associated with it, which increments regularly based on the frequency of the dataset. Often we will have all the \( T_i \) be the same date-time for the time series of a particular dataset we use.

2.1 Probabilistic Forecasting

In the probabilistic forecasting problem, we wish to learn the unknown future distribution of the data given its past. Rather than considering the whole history of each time series \( i \) in a dataset (which might not be of the same size) we can instead consider some fixed context window of size \( C \geq 1 \) of our choosing and try to learn this potentially complex unknown distribution of the future values given the context window sized past denoted by

\[
p_X(x_{T_i+1:T_i+P}|x_{T_i+1-C:T_i}).
\]  

Thus, if we denote the parameters of our deep learning model by \( \theta \), we can approximate (1) by an autoregressive model which we can write via the chain-rule of probability as

\[
p_X(x_{T_i+1:T_i+P}|x_{T_i+1-C:T_i}; \theta) = \prod_{t=T_{i}+1}^{T_{i}+P} p_X(x_t|x_{T_{i}+1:C:t-1}; \theta).
\]

And as we can see at a high level the probabilistic forecasting problem reduces down to learning some representation of the context window past together with a distribution model of the next time step(s) given this representation. For example, the DeepAR model uses a Recurrent Neural Network (RNN) (like the LSTM [20] or GRU [8]) to represent the context window history together with associated covariates, denoted by real-valued \( \mathbb{R}^F \) sized vectors \( c_{i,T_i+P} \) (known for all times), as

\[h_t = \text{RNN}(\text{concat}(x_{i-1:T_i}, c_{i})), h_{i-1}; \theta),\]

with \( h_t \in \mathbb{R}^H \) and \( h_0 = \vec{0} \), to learn \( p_X(x_i|h_t; \theta) \). The distribution head in DeepAR learns the parameters of some chosen distribution, e.g. the mean and variance of a Gaussian and we can maximize the log-likelihood of the distribution with respect to the ground-truth \( x_t \) via SGD for all \( i \) and \( t \) in \( D_{\text{train}} \).

By incorporating different architectural inductive biases to learn the historic representation together with different distribution heads, researchers have come up with a zoo of models to solve the
probabilistic forecasting problem in the deep learning domain, for not only the univariate and multivariate or regular and irregular time series setting, but also for spatial-temporal problems as well.

2.2 VQ-VAE

Leaving time series forecasting aside for the moment we can consider the fact that latent representations can potentially also be discrete. Thus instead of learning a continuous valued distributional representation as in Variational AutoEncoders (VAE) [28], the VQ-VAE model learns a fixed size or discrete set of representations of the data via an encoder-decoder bottleneck.

Formally the model consists of an encoder that maps its input onto a fixed sized set of latent variables and a decoder that reconstructs the input from one of these fixed number of latents. Both the encoder and decoder use this shared codebook of vectors. Thus if we denote the encoding representation by $h_{\text{enc}} \in \mathbb{R}^E$, then this vector is quantized based on its distance to the prototype vectors in the codebook $\{z_1, \ldots, z_J\}$ such that the $h_{\text{enc}}$ is replaced by the closest prototype vector:

$$\text{Quantize}(h_{\text{enc}}) := z_n \text{ where } n = \arg \min_j \|h_{\text{enc}} - z_j\|_2,$$  

(2)

for a codebook size $J \geq 1$ of our choosing.

The mappings are learned by back-propagating the gradient of a reconstruction error via the decoder to the encoder using the Straight-Through gradient estimator [19, 4]. Apart from this loss the VQ-VAE model also has two terms that encourages the alignment of the vector space of the codebook with the output of the encoder, namely the codebook loss which pushes the selected codebook entry $z$ closer to the encoder representation and the commitment loss which encourages the output of the encoder to stay close to the chosen codebook vector to stop it switching too frequently from one code vector to another. The “stop-gradient” or “detach” operator $\text{sg}(\cdot)$ blocks the gradients from flowing into its argument and $\beta$ is the hyperparameter which penalizes the code vector fluctuating, then the two extra entries of the loss are given by

$$\|\text{sg}(h_{\text{enc}}) - z\|^2 + \beta \|\text{sg}(z) - h_{\text{enc}}\|^2.$$  

(3)

Since the optimal code would be the average of the representations, [51] presents an exponential moving average update scheme of the latents instead of the codebook loss (first term of (3)) during training. The SoundStream [55] paper proposes to initialize the codebook by the k-means centroids of the representations from the first batch. Additionally, as proposed in Jukebox [12], we deploy the heuristic where we replace codebook vectors that have an exponential moving average cluster size less than some threshold $Q$ by a random vector from the batch.

3 VQ-AR Method

We motivate this method with the observation that time series data in a dataset visually looks similar with repeated patterns, and thus we ask if instead of a continuous representation of the histories as learned by most deep learning models, could a discrete set of representations suffice? With this in mind, we introduce the VQ-AR model which incorporates an encoder-decoder RNN together with a Vector Quantizer (VQ) bottleneck.

Unlike in the DeepAR setting, where we learn the temporal representation of each time series’ context window to predict the distribution of the next step, we instead first encode it to a quantized representation $z_n$ given by (2) using an encoding-RNN’s state denoted by

$$h_{\text{enc}} = \text{RNN}_{\text{enc}}(\text{concat}(x_{t-1}, c_t), h_{\text{enc}}_{t-1}; \theta),$$

with $h_{\text{enc}} \in \mathbb{R}^E$ and $h_{0\text{enc}} = \vec{0}$. Subsequently we model the distribution of the next time step $p_X(x_t | z_n; \theta)$ using a decoding-RNN

$$h_{\text{dec}} = \text{RNN}_{\text{dec}}(\text{Quantize}(h_{\text{enc}}), h_{\text{dec}}_{t-1}; \theta),$$

which only takes the Vector-Quantized latent $z_n$ (2) as input, with $h_{\text{dec}} \in \mathbb{R}^H$ and $h_{0\text{dec}} = \vec{0}$. See Figure [a] for a schematic of the model.
Because the prior $p(z_j) = 1/J$ is a uniform categorical distribution and the posterior

$$q(z_n|x_{t-1}, c_i; \theta) = \begin{cases} 1 & \text{if } n = \arg\min_j \|h_{t}^{\text{enc}} - z_j\|_2 \\ 0 & \text{otherwise}, \end{cases}$$ (4)

is a categorical distribution parametrized by one-hot probability vectors, the KL-divergence term in the Evidence Lower Bound (ELBO) [6] is constant ($\log J$), and since the ELBO’s expectation term is over a deterministic distribution (4), we can simply train the whole model by maximizing the log-likelihood term of the next time step: $\log p_X(x_i^t | z_n, h_{t-1}^{\text{dec}}; \theta)$.

Thus, similar to the DeepAR model we minimize the negative log-likelihood of some chosen parametric distribution and train the whole model end-to-end together with the VQ loss (3). For example, to model Gaussian emissions the distribution head consist of a linear layer which outputs the mean $\mu_t \in \mathbb{R}$ and standard deviation $\sigma_t \in \mathbb{R}^+$, making sure that the standard deviation is always positive via the softplus ($\cdot$) non-linearity. These parameters are used to instantiate the distribution $N(\mu_t, \sigma_t)$ from which we calculate the negative log-likelihood: $-\log \mathcal{N}(x_i^t)$. The full loss of the model for time point $t - 1$ in the training window and series $i$ is thus denoted by

$$L_{i-1}(\theta) := -\log \mathcal{N}(x_i^t) + \|sg(h_{t}^{\text{enc}}) - z_n\|^2_2 + \beta \|sg(z_n) - h_{t}^{\text{enc}}\|^2_2.$$ (5)

### 3.1 Training

We construct batches $B$ of context and subsequent prediction window of total size $T + 1$, by sampling a random time series (and corresponding covariates) from $D_{\text{train}}$ and then sample this window randomly within. We will then minimize the mini-batch loss, which is the mean of all the individual losses (5) denoted by

$$\mathcal{L}(\theta) := \frac{1}{|B|T} \sum_{x_{1:T+1} \in B} \sum_{t=1}^T L_i(\theta),$$

via the Adam [27] SGD optimizer at each step of training with an appropriate batch size $|B|$ and learning rate.

### 3.2 Inference

At inference time we go over the last context sized window of each time series in the training dataset $D_{\text{train}}$ and then start forecasting for the $P$ prediction steps by sequentially sampling from the distribution head and autoregressively passing this value back into the model (together with the covariates which are known for all times). In fact, we can repeat this process $S$ number of times (e.g.
\( S = 100 \) to then report any empirical probability interval of interest as well as probabilistic metrics, like Continuous Ranked Probability Score (CRPS) \( [15, 37] \), with respect to the ground-truth values in \( D_{\text{test}} \). Point forecasting metrics can also be evaluated with respect to the empirical median or mean from the \( S \) samples at each time step of the prediction.

Note, unlike in some generative modeling use cases, \textit{we do not} sample with a reduced temperature distribution to obtain higher quality samples, nor do we Beam-search or do Nucleus sampling as in NLP settings during inference.

### 3.3 Covariates

As mentioned in section 2.1, we can incorporate covariates for each time point of the time series (as well as for all future times) by creating time features given the granularity of the time series in question. For example, for daily data we could consider the day of the week, week of the month, month of the year, etc., features. Although not strictly necessary, but certainly helpful, we can take inspiration from classical methods and construct time series covariates, such as lagged features, moving averages, difference to previous time points, exponential moving averages, and moving percentiles, etc., of \( x_t \) (and/or of temporal covariates) depending on the frequency of the time series in question.

Where applicable holidays or other recurring events can also be turned into temporal features. A running “age” counter can also serve as an indication of the length of a time series. Apart from these temporal features, one can embed the categorical identity \( i \in \{1, \ldots, D\} \) of each time series into a vector via a trainable embedding layer whose time independent output is copied over for all time points being considered. If the time series are grouped then their categorical tree structure can also be represented via similar embeddings.

Unless explicitly stated, for all our experiments we only use time, age, lagged features, and categorical identities as covariates.

### 3.4 Scaling

Entities in time series data can potentially have arbitrary magnitudes and so to be able to train a shared deep learning model with respect to inputs of any scale we can utilize the following heuristic: for each context window we can calculate the mean value of the data in the interval, specifically

\[
\nu^t = \frac{\sum_{t=1}^{T} x^t}{T} \quad \text{if it is not zero or 1 otherwise}
\]

and divide the time series by this before using it in the model. The outputs from the distribution heads are then multiplied by this \( \nu^t \) when sampling or we rescale in the parameter space of the distribution (e.g. when we require integer samples from a Poisson or Negative-Binomial head).

We omit stating this heuristic in all our derivations and treat it as an implementation detail.

### 4 Experiments

We test the performance of VQ-AR for the forecasting task in terms of performance and also highlight the limitations of the model and generalization properties of the representations in this section.

For our experiments we use Exchange \([31]\), Solar \([31]\), Elec. \([\text{Traffic}]\), Taxi \([\text{Wikipedia}]\) open datasets, preprocessed exactly as in \([47]\), as well as M5 \([35]\) and proprietary E-Commerce dataset, with their properties listed in Table 5 in the appendix B. As can be noted in the table, we do not need to normalize scales for the Traffic dataset. From the names of the datasets, we see that we cover a number of time series domains including finance, weather, energy, logistics, page-views, as well as E-commerce.

\[\text{https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014}\]
\[\text{https://archive.ics.uci.edu/ml/datasets/PEMS-SF}\]
\[\text{https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page}\]
\[\text{https://github.com/mbohlkeschneider/gluon-ts/tree/mv_release/datasets}\]
As indicated above, for evaluation we employ the CRPS metric which measures the compatibility of a cumulative distribution function (CDF) $F$ with the ground-truth observation $x$ as

$$\text{CRPS}(F, x) = \int_{\mathbb{R}} (F(y) - I\{x \leq y\})^2 \, dy,$$

where $I\{x \leq y\}$ is the indicator function which is one if $x \leq y$ and zero otherwise. CRPS is a proper scoring function, meaning CRPS attains its minimum when the predictive distribution $F$ and the data distribution are equal. Employing the empirical CDF of $F$, i.e. $\hat{F}(y) = \frac{1}{S} \sum_{s=1}^{S} I\{x^{(s)} \leq y\}$ using $S$ samples $x^{(s)} \sim F$ as a natural approximation of the predictive CDF, CRPS can be directly computed from sampled predictions from the model at each time point \[26\]. The final metric is averaged over all the prediction time steps and time series in a dataset.

### 4.1 Forecasting

We will compare VQ-AR with the following deep learning baseline probabilistic univariate models

- **DeepAR \[48\]:** an RNN based probabilistic model which learns the parameters of some chosen distribution for the next time point;
- **MQCNN \[53\]:** a Convolutional Neural Network model which outputs chosen quantiles of the forecast upon which we regress the ground truth via Quantile loss;
- **SQF-RNN \[14\]:** an RNN based non-parametric method which models the quantiles via linear splines and also regresses the Quantile loss;
- **IQN-RNN \[16\]:** combines an RNN model with an Implicit Quantile Network (IQN) \[9\] head to learn the distribution similar to SQF-RNN;
- **LSF \[18\]:** a method that transforms a point-estimator, coming from for example gradient boosting methods, into a probabilistic one;

as well as the classical ETS \[23\] which is an exponential smoothing method using weighted averages of past observations with exponentially decaying weights as the observations get older together with Gaussian additive errors (E) modeling trend (T) and seasonality (S) effects separately.

We evaluate the model on the datasets detailed above and follow the recommendations of the M4 competition \[34\] regarding forecasting performance metrics. Thus, we also report the mean scale interval score \[15\] (MSIS$^5$) for a 95% prediction interval, the 50-th and 90-th quantile percentile loss (QL50 and QL90 respectively), as well as the CRPS score. The point-forecasting performance of models is measured by the normalized root mean square error (NRMSE), the mean absolute scaled error (MASE) \[24\], and the symmetric mean absolute percentage error (sMAPE) \[33\]. For pointwise metrics, we use sampled medians with the exception of NRMSE, where we take the mean over our prediction samples. The results of our extensive experiments are detailed in Table \[1\]. Note that the VQ-AR is essentially compressing the time series histories to discrete tokens which could have applications in edge-computing, by trading off performance with space.

The M5 competition was a twin competition measuring both point forecasting accuracy \[35\] as well as probabilistic uncertainty \[36\] and the top solutions were dominated by tree-based methods. In fact, the winning solution to the accuracy benchmark is based on an ensemble of 220 gradient boosting tree (GBT) models while the winner of the uncertainty challenge is an ensemble of 126 GBTs with hundreds of hand-crafted features. In Table \[2\] we provide a comparison between tree-based methods and neural methods. As detailed in \[25\] it is still an open problem to improve deep-learning models with respect to tree-based methods and as can be seen, the VQ-VR method provides competitive metrics via a simple single model without any extensive feature engineering.

### 4.2 Robustness

We check the robustness of VQ-AR with respect to the noise added to the context window inputs (but not covariates) when forecasting. Our hypothesis is that due to the quantization nature of the method, the added noise (up to a certain level) will correspond to the same codebook vector and thus this
Table 1: Comparison metrics using different methods: SQF-RNN with 50 knots, DeepAR and VQ-AR with Student-T (-t), Negative Binomial (-nb) or IQN (-iqn) emission heads, ETS, MQCNN, and IQN-RNN on the datasets.

| Dataset     | Method       | CRPS  | QL50  | QL90  | MSIS   | NRMSE  | sMAPE  | MASE   |
|-------------|--------------|-------|-------|-------|--------|--------|--------|--------|
| Exchange    | SQF-RNN-50   | 0.010 | 0.013 | 0.006 | **14.15** | 0.020  | 0.013  | 1.800  |
|             | DeepAR-t     | 0.012 | 0.016 | 0.007 | 69.29  | 0.022  | 0.030  | 9.980  |
|             | ETS          | 0.008 | 0.010 | 0.005 | 15.89  | 0.015  | 0.011  | **1.517** |
|             | DeepAR-t     | 0.007 | 0.010 | 0.004 | 17.37  | 0.014  | 0.013  | 3.041  |
|             | ETS          | 0.015 | 0.016 | 0.011 | 60.04  | 0.026  | 0.045  | 5.440  |
|             | VQ-AR-t      | 0.010 | 0.013 | 0.007 | 18.10  | 0.019  | 0.015  | 2.658  |
| Solar       | SQF-RNN-50   | 0.330 | 0.431 | 0.175 | 5.65   | 0.929  | 1.342  | 1.004  |
|             | DeepAR-t     | 0.418 | 0.543 | 0.254 | 7.33   | 1.072  | 1.393  | 1.275  |
|             | ETS          | 0.646 | 0.661 | 0.383 | 18.55  | 1.112  | 1.546  | 1.938  |
|             | DeepAR-t     | 0.010 | 0.013 | 0.007 | 18.10  | 0.019  | 0.015  | 2.658  |
|             | ETS          | 0.010 | 0.013 | 0.007 | 18.10  | 0.019  | 0.015  | 2.658  |
| Electricity | SQF-RNN-50   | 0.078 | 0.097 | 0.044 | 8.66   | 0.632  | 0.144  | 1.051  |
|             | DeepAR-t     | 0.062 | 0.078 | 0.046 | 6.79   | 0.687  | 0.117  | 0.849  |
|             | ETS          | 0.076 | 0.100 | 0.050 | 9.99   | 0.838  | 0.156  | 1.247  |
|             | DeepAR-t     | 0.054 | 0.068 | 0.036 | 5.88   | 0.653  | 0.107  | 0.717  |
|             | ETS          | 0.054 | 0.068 | 0.036 | 5.88   | 0.653  | 0.107  | 0.717  |
| Traffic     | SQF-RNN-50   | 0.153 | 0.186 | 0.117 | 8.40   | **0.401** | 0.243  | 0.76   |
|             | DeepAR-t     | 0.172 | 0.216 | 0.117 | 8.02   | 0.472  | 0.244  | 0.89   |
|             | ETS          | 0.373 | 0.386 | 0.287 | 17.67  | 0.647  | 0.489  | 1.543  |
|             | DeepAR-t     | 0.139 | 0.168 | 0.117 | **7.11** | 0.433  | 0.171  | 0.656  |
|             | ETS          | 1.220 | 0.563 | 2.005 | 116.69 | 0.723  | 0.636  | 2.712  |
|             | DeepAR-t     | 0.138 | 0.164 | 0.113 | 7.79   | 0.409  | 0.185  | 0.641  |
|             | ETS          | 0.138 | 0.164 | 0.113 | 7.79   | 0.409  | 0.185  | 0.641  |
| Taxi        | SQF-RNN-50   | 0.296 | 0.362 | 0.188 | 5.53   | **0.570** | 0.609  | 0.741  |
|             | DeepAR-nb    | 0.299 | 0.379 | 0.203 | 5.44   | 0.610  | 0.582  | 0.771  |
|             | ETS          | 1.059 | 1.297 | 0.617 | 12.24  | 2.147  | 1.519  | 1.552  |
|             | DeepAR-nb    | 0.295 | 0.370 | 0.201 | 6.51   | 0.583  | 0.629  | 0.758  |
|             | ETS          | 1.220 | 1.451 | 0.488 | 48.61  | 2.645  | 0.912  | 3.041  |
|             | DeepAR-nb    | 0.286 | 0.362 | 0.193 | **5.43** | 0.572  | **0.570** | 0.741  |
|             | ETS          | 0.286 | 0.362 | 0.193 | **5.43** | 0.572  | **0.570** | 0.741  |
| Wikipedia   | SQF-RNN-50   | 0.558 | 0.708 | 0.466 | 8.26   | 1.634  | 1.556  | 0.912  |
|             | DeepAR-nb    | 0.539 | 0.679 | 0.469 | 8.01   | 1.547  | 1.550  | 0.915  |
|             | ETS          | 0.838 | 1.051 | 0.696 | 23.67  | 2.560  | 1.560  | 3.161  |
|             | DeepAR-nb    | 0.539 | 0.677 | 0.475 | 8.26   | **1.511** | 1.602  | 0.899  |
|             | ETS          | 0.574 | 0.725 | 0.497 | 8.04   | 1.775  | 1.616  | 0.921  |
|             | DeepAR-nb    | 0.527 | 0.694 | 0.457 | **7.20** | 1.628  | **1.535** | **0.895** |
|             | ETS          | 0.527 | 0.694 | 0.457 | **7.20** | 1.628  | **1.535** | **0.895** |
| M5          | SQF-RNN-50   | 0.454 | 0.627 | 0.648 | 24.82  | 8.492  | 1.653  | 1.246  |
|             | DeepAR-nb    | 0.531 | 0.612 | 0.638 | 38.12  | 8.160  | **1.409** | **1.071** |
|             | ETS          | 2.605 | 2.991 | 2.457 | 117.50 | 31.236 | 1.791  | 10.270 |
|             | DeepAR-nb    | 0.534 | 0.627 | 0.590 | 32.92  | 7.588  | 1.650  | 1.076  |
|             | ETS          | 0.649 | 0.751 | 0.712 | 27.56  | 9.336  | 1.677  | 1.325  |
| E-Commerce  | SQF-RNN-50   | 0.483 | 0.562 | 0.555 | **22.33** | 6.702  | 1.629  | 1.074  |
Table 2: Weighted Quantile Loss (lower is better) for M5 test set predictions using different methods for specified quantiles and their average.

| Quantile | M5 winner | M5 runner-up | DeepAR-t | DeepAR-nb | MQCNN | LSF | VQ-AR-nb |
|----------|-----------|--------------|-----------|-----------|-------|-----|---------|
| 0.005    | 0.010     | 0.042        | 0.023     | 0.016     | 0.010 | 0.012| 0.013   |
| 0.025    | 0.050     | 0.086        | 0.075     | 0.061     | 0.051 | 0.054| 0.057   |
| 0.165    | 0.312     | 0.337        | 0.319     | 0.319     | 0.320 | 0.316| 0.316   |
| 0.25     | 0.444     | 0.461        | 0.458     | 0.450     | 0.451 | 0.451| 0.445   |
| 0.50     | 0.698     | 0.690        | 0.766     | 0.705     | 0.712 | 0.724| **0.697**|
| 0.75     | 0.687     | 0.722        | 0.821     | 0.699     | 0.709 | 0.728| 0.690   |
| 0.835    | 0.585     | 0.598        | 0.754     | 0.600     | 0.609 | 0.607| 0.591   |
| 0.975    | 0.182     | 0.196        | 0.430     | 0.202     | 0.213 | 0.218| 0.189   |
| 0.995    | 0.055     | 0.076        | 0.310     | 0.074     | 0.084 | 0.086| 0.063   |
| Mean     | **0.336** | 0.357        | 0.440     | 0.347     | 0.351 | 0.355| 0.340   |

For some noise level $l = \{0.2, 0.4, \ldots, 1.0\}$ we calculate the standard deviation of the context window target $\sigma_{\text{context}}$ and add to each time point noise sampled from $\mathcal{N}(0, l \times \sigma_{\text{context}})$ and then pass this together with the covariates to the appropriate RNN.

Table 3 details the metrics for different levels of noise. As we can see with a small amount of noise the VQ-AR method is more robust than IQN-RNN, however as we increase the noise the selected codebook vectors become too different leading to worse performance than with an RNN based method with smooth representations. The MSIS metric which measures the upper 95% and lower 5% probability interval however is more robust.

Table 4 highlights the prediction metrics on unseen datasets and as can be noted, the representations learned as discrete entities tend to perform better in the zero-shot setting especially since the datasets come from dissimilar domains.

4.3 Generalization

The VQ-AR method is essentially an information bottleneck that serves as a form of regularization. Thus our hypothesis is that the learned codebook representations $\{z_j\}$ would need to be somewhat universal time series representations in comparison to methods that can potentially learn continuous representations without such a constraint. In order to test this we measure the zero-shot prediction performance of VQ-AR against DeepAR by first training these two models on the Electricity dataset (without any categorical time series covariates) and then testing the trained model’s performance on the test set of Solar and Traffic.

Table 4 highlights the prediction metrics on unseen datasets and as can be noted, the representations learned as discrete entities tend to perform better in the zero-shot setting especially since the datasets come from dissimilar domains.

4.4 Ablation

For our ablation study, we wish to test the performance of the model with respect to the number of codebook vectors $J$. In this experiment for the M5 and E-Commerce datasets we train and test the performance of VQ-AR using $J = \{2, 4, \ldots, 512\}$. We record the metrics in Figure 2 and to our
Figure 2: Test set CRPS of VQ-AR-iqn on (a) M5 and (b) E-Commerce datasets with different number of codebook vectors \((J)\). Surprisingly, the decoding RNN can still give valid predictions given some sequence of just two codebook vectors.

surprise, we see that we get decent forecasts using only two codebook vectors for all the datasets considered (and not only for the ones in the figure). More codebook vectors tend to give better performance. As \(J \rightarrow \infty\) we approach the DeepAR method albeit with more RNN layers.

5 Related Work

The use of Vector Quantization (VQ) in time series forecasting has not been explored, as far as we are aware. However, generative models like VQ-VAE have been used extensively for sequential modeling problems like for videos \([43]\) and audio/speech \([12, 55, 2]\) generation. Typically these models are trained in a two-stage process where a VQ-VAE model is first trained and then a sequential generative model is trained on top of the codebook vectors. The VQ-Wav2Vec \([2]\) uses a causal convolution to first learn a representation of an audio signal some steps into the future for further downstream task. This model is not autoregressive, takes bounded audio inputs only and is not able to be used as a forecaster which would need to account for the inductive bias of scale and additional covariates.

The SOM-VAE \([13]\) model is related in that it uses VQ to learn interpretable discrete representation of sequential data for clustering. Also in the paper \([42]\), the authors discretize the inputs to time series models via different binning transformations, as a way to mitigate the scaling issue discussed in section \([44]\) and investigate performance given this input transformation on different architectures.

Neural forecasting methods \([5]\) are related with the caveat that all of them, as far as we are aware, can be thought of as some component that learns a representation of the context window in a continuous fashion together with an emission component, be it point forecasting or probabilistic. CNN \([3]\) based methods can typically be replaced by RNNs or causal Transformers \([52]\) and most works try to introduce some time series or problem specific inductive bias into the architectural choices \([32]\) or emission models \([44]\) or inputs \([42]\) rather than explicitly on the representations themselves.

6 Summary and Discussion

We have presented VQ-AR a conceptually simple and novel yet powerful autoregressive time series forecasting method based on learning discrete historic representations of time series histories in order to forecast, trained in an end-to-end fashion. We have shown not only does it perform better or similar to the competitive methods considered, but is also able to achieve good performance in comparison to ensembles of hundreds of tree-based models which typically dominate forecasting benchmarks, all via a single model. The architectural principle applied in this method can be adapted for multivariate
forecasting and the underlying RNN based encoder-decoder can be replaced by a Transformer if so desired.

For future work, we would like to test the inductive bias introduced in this work for the multivariate setting and also incorporate it within the Transformer architecture especially for very long time series sequences. As mentioned in the appendix [A], the gradient estimator considered can also be replaced by its smooth-relaxation variant or by methods with less bias to help improve the VQ training.

Finally, we have also highlighted a critique of VQ-AR especially in terms of its robustness to noisy inputs, and in the future, we would like to investigate how to mitigate this potential issue.

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References

[1] Alexander Alexandrov, Konstantinos Benidis, Michael Bohlke-Schneider, Valentin Flunkert, Jan Gasthaus, Tim Januschowski, Danielle C. Maddix, Syama Rangapuram, David Salinas, Jasper Schulz, Lorenzo Stella, Ali Caner Türkmen, and Yuyang Wang. GluonTS: Probabilistic and Neural Time Series Modeling in Python. Journal of Machine Learning Research, 21(116):1–6, 2020.

[2] Alexei Baevski, Steffen Schneider, and Michael Auli. vq-wav2vec: Self-supervised learning of discrete speech representations. In International Conference on Learning Representations, 2020.

[3] Shaojie Bai, J. Zico Kolter, and Vladlen Koltun. An empirical evaluation of generic convolutional and recurrent networks for sequence modeling, 2018.

[4] Yoshua Bengio, Nicholas Léonard, and Aaron C. Courville. Estimating or propagating gradients through stochastic neurons for conditional computation. arXiv preprint arXiv:1308.3432, 2013.

[5] Konstantinos Benidis, Syama Sundar Rangapuram, Valentin Flunkert, Bernie Wang, Danielle Maddix, Caner Turkmen, Jan Gasthaus, Michael Bohlke-Schneider, David Salinas, Lorenzo Stella, Laurent Callot, and Tim Januschowski. Neural forecasting: Introduction and literature overview, 2020.

[6] Christopher M. Bishop. Pattern Recognition and Machine Learning. Springer, 2006.

[7] Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel Ziegler, Jeffrey Wu, Clemens Winter, Chris Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners. In H. Larochelle, M. Ranzato, R. Hadsell, M. F. Balcan, and H. Lin, editors, Advances in Neural Information Processing Systems, volume 33, pages 1877–1901. Curran Associates, Inc., 2020.

[8] Kyunghyun Cho, Bart van Merriënoorber, Dzmitry Bahdanau, and Yoshua Bengio. On the properties of neural machine translation: Encoder–decoder approaches. In Proceedings of SSST-8, Eighth Workshop on Syntax, Semantics and Structure in Statistical Translation, pages 103–111, Doha, Qatar, October 2014. Association for Computational Linguistics.

[9] Will Dabney, Georg Ostrovski, David Silver, and Remi Munos. Implicit Quantile Networks for Distributional Reinforcement Learning. In Jennifer Dy and Andreas Krause, editors, Proceedings of the 35th International Conference on Machine Learning, volume 80 of Proceedings of Machine Learning Research, pages 1096–1105, Stockholmsmässan, Stockholm Sweden, 2018. PMLR.
[10] Emmanuel de Bézenac, Syama Sundar Rangapuram, Konstantinos Benidis, Michael Bohlke-Schneider, Richard Kurle, Lorenzo Stella, Hilaf Hasson, Patrick Gallinari, and Tim Januschowski. Normalizing kalman filters for multivariate time series analysis. In H. Larochelle, M. Ranzato, R. Hadsell, M. F. Balcan, and H. Lin, editors, Advances in Neural Information Processing Systems, volume 33, pages 2995–3007. Curran Associates, Inc., 2020.

[11] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. BERT: Pre-training of deep bidirectional transformers for language understanding. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4171–4186, Minneapolis, Minnesota, June 2019. Association for Computational Linguistics.

[12] Prafulla Dhariwal, Heewoo Jun, Christine Payne, Jong Wook Kim, Alec Radford, and Ilya Sutskever. Jukebox: A generative model for music. CoRR, abs/2005.00341, 2020.

[13] Vincent Fortuin, Matthias Hüser, Francesco Locatello, Heiko Strathmann, and Gunnar Rätsch. Som-vae: Interpretable discrete representation learning on time series. In ICLR (Poster). OpenReview.net, 2019.

[14] Jan Gasthaus, Konstantinos Benidis, Yuyang Wang, Syama Sundar Rangapuram, David Salinas, Valentín Flunkert, and Tim Januschowski. Probabilistic Forecasting with Spline Quantile Function RNNs. In Kamalika Chaudhuri and Masashi Sugiya, editors, Proceedings of the 22nd International Conference on Artificial Intelligence and Statistics AISTATS 2019, Naha, Okinawa, Japan, volume 89 of Proceedings of Machine Learning Research, pages 1901–1910. PMLR, 2019.

[15] Tilmann Gneiting and Adrian E Raftery. Strictly proper scoring rules, prediction, and estimation. Journal of the American Statistical Association, 102(477):359–378, 2007.

[16] Adèle Gouttes, Kashif Rasul, Mateusz Koren, Johannes Stephan, and Tofigh Naghibi. Probabilistic Time Series Forecasting with Implicit Quantile Networks. In Time Series Workshop @ ICML 2021, 2021.

[17] Charles R. Harris, K. Jarrod Millman, St’efan J. van der Walt, Ralf Gommers, Pauli Virtanen, David Cournapeau, Eric Wieser, Julian Taylor, Sebastian Berg, Nathaniel J. Smith, Robert Kern, Matti Picus, Stephan Hoyer, Marten H. van Kerkwijk, Matthew Brett, Allan Haldane, Jaime Fern’andez del R’io, Mark Wiebe, Pearu Peterson, Pierre G’erard-Marchant, Kevin Sheppard, Tyler Reddy, Warren Weckesser, Hameer Abbasi, Christoph Gohlke, and Travis E. Oliphant. Array programming with NumPy. Nature, 585(7825):357–362, September 2020.

[18] Hilaf Hasson, Bernie Wang, Tim Januschowski, and Jan Gasthaus. Probabilistic forecasting: A level-set approach. In A. Beygelzimer, Y. Dauphin, P. Liang, and J. Wortman Vaughan, editors, Advances in Neural Information Processing Systems, 2021.

[19] Geoffrey Hinton, Nitish Srivastava, Kevin Swersky, Tijmen Tieleman, and Abdel-rahman Mohamed. Neural networks for machine learning: Lecture 15b deep autoencoders. https://www.cs.toronto.edu/~hinton/coursera_lectures.html, 2012.

[20] S. Hochreiter and J. Schmidhuber. Long Short-Term Memory. Neural Computation, 9(8):1735–1780, November 1997.

[21] J. D. Hunter. Matplotlib: A 2D graphics environment. Computing in Science & Engineering, 9(3):90–95, 2007.

[22] R.J. Hyndman and G. Athanasopoulos. Forecasting: Principles and practice. OTexts, 2021.

[23] Rob J. Hyndman and Yeasmin Khandakar. Automatic time series forecasting: The forecast package for R. J. Stat. Soft., 27(3):1–22, 2008.

[24] Rob J. Hyndman and Anne B. Koehler. Another look at measures of forecast accuracy. International Journal of Forecasting, 22(4):679–688, 2006.

[25] Tim Januschowski, Yuyang Wang, Kari Torkkola, Timo Erkkilä, Hilaf Hasson, and Jan Gasthaus. Forecasting with trees. International Journal of Forecasting, 2021.
[26] Alexander Jordan, Fabian Krüger, and Sebastian Lerch. Evaluating Probabilistic Forecasts with scoringRules. *Journal of Statistical Software, Articles*, 90(12):1–37, 2019.

[27] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In Yoshua Bengio and Yann LeCun, editors, *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*, 2015.

[28] Diederik P. Kingma and Max Welling. An Introduction to Variational Autoencoders. *Foundations and Trends in Machine Learning*, 12(4):307–392, 2019.

[29] Thomas Kluiver, Benjamin Ragan-Kelley, Fernando Pérez, Brian Granger, Matthias Bussonnier, Jonathan Frederic, Kyle Kelley, Jessica Hamrick, Jason Grout, Sylvain Corlay, Paul Ivanov, Damián Avila, Sofia Abdalla, and Carol Willing. Jupyter notebooks – a publishing format for reproducible computational workflows. In F. Loizides and B. Schmidt, editors, *Positioning and Power in Academic Publishing: Players, Agents and Agendas*, pages 87–90. IOS Press, 2016.

[30] Alireza Koochali, Andreas Dengel, and Sheraz Ahmed. If you like it, gan it—probabilistic multivariate times series forecast with gan. *Engineering Proceedings*, 5(1), 2021.

[31] Guokun Lai, Wei-Cheng Chang, Yiming Yang, and Hanxiao Liu. Modeling Long- and Short-Term Temporal Patterns with Deep Neural Networks. In *The 41st International ACM SIGIR Conference on Research & Development in Information Retrieval, SIGIR ’18*, pages 95–104, New York, NY, USA, 2018. ACM.

[32] Bryan Lim, Sercan Ö. Arık, Nicolas Loeff, and Tomas Pfister. Temporal fusion transformers for interpretable multi-horizon time series forecasting. *International Journal of Forecasting*, 37(4):1748–1764, 2021.

[33] Spyros Makridakis. Accuracy measures: theoretical and practical concerns. *International Journal of Forecasting*, 9(4):527–529, 1993.

[34] Spyros Makridakis, Evangelos Spiliotis, and Vassilios Assimakopoulos. The M4 Competition: 100,000 time series and 61 forecasting methods. *International Journal of Forecasting*, 36(1):54–74, 2020. M4 Competition.

[35] Spyros Makridakis, Evangelos Spiliotis, and Vassilios Assimakopoulos. The m5 competition: Background, organization, and implementation. *International Journal of Forecasting*, 2021.

[36] Spyros Makridakis, Evangelos Spiliotis, Vassilios Assimakopoulos, Zhi Chen, Anil Gaba, Ilia Tsetlin, and Robert L. Winkler. The m5 uncertainty competition: Results, findings and conclusions. *International Journal of Forecasting*, 2021.

[37] James E. Matheson and Robert L. Winkler. Scoring Rules for Continuous Probability Distributions. *Management Science*, 22(10):1087–1096, 1976.

[38] The Pandas development team. pandas-dev/pandas: Pandas, February 2020.

[39] Youngsuk Park, Danielle Maddix, François-Xavier Aubet, Kelvin Kan, Jan Gasthaus, and Yuyang Wang. Learning quantile functions without quantile crossing for distribution-free time series forecasting. In Gustau Camps-Valls, Francisco J. R. Ruiz, and Isabel Valera, editors, *Proceedings of The 25th International Conference on Artificial Intelligence and Statistics*, volume 151 of *Proceedings of Machine Learning Research*, pages 8127–8150. PMLR, 28–30 Mar 2022.

[40] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. PyTorch: An imperative style, high-performance deep learning library. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché Buc, E. Fox, and R. Garnett, editors, *Advances in Neural Information Processing Systems 32*, pages 8026–8037. Curran Associates, Inc., 2019.
[41] Max B. Paulus, Chris J. Maddison, and Andreas Krause. Rao-blackwellizing the straight-through gumbel-softmax gradient estimator. In *International Conference on Learning Representations*, 2021.

[42] Stephan Rabanser, Tim Januschowski, Valentin Flunkert, David Salinas, and Jan Gasthaus. The effectiveness of discretization in forecasting: An empirical study on neural time series models. In *6th Workshop on Mining and Learning from Time Series*, 2020.

[43] Ruslan Rakhimov, Denis Volkhonskiy, Alexey Artemov, Denis Zorin, and Evgeny Burnaev. Latent video transformer. In Giovanni Maria Farinella, Petia Radeva, José Braz, and Kadi Bouatouch, editors, *VISIGRAPP (5: VISAPP)*, pages 101–112. SCITEPRESS, 2021.

[44] Syama Sundar Rangapuram, Matthias W Seeger, Jan Gasthaus, Lorenzo Stella, Yuyang Wang, and Tim Januschowski. Deep state space models for time series forecasting. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, *Advances in Neural Information Processing Systems 31*, pages 7796–7805. Curran Associates, Inc., 2018.

[45] Kashif Rasul, Calvin Seward, Ingmar Schuster, and Roland Vollgraf. Autoregressive denoising diffusion models for multivariate probabilistic time series forecasting. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning*, volume 139 of *Proceedings of Machine Learning Research*, pages 8857–8868. PMLR, 18–24 Jul 2021.

[46] Kashif Rasul, Abdul-Saboor Sheikh, Ingmar Schuster, Urs Bergmann, and Roland Vollgraf. Multivariate Probabilistic Time Series Forecasting via Conditioned Normalizing Flows. In *International Conference on Learning Representations 2021 (Conference Track)*, 2021.

[47] David Salinas, Michael Bohlke-Schneider, Laurent Callot, Roberto Medico, and Jan Gasthaus. High-dimensional multivariate forecasting with low-rank Gaussian Copula Processes. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché Buc, E. Fox, and R. Garnett, editors, *Advances in Neural Information Processing Systems 32*, pages 6824–6834. Curran Associates, Inc., 2019.

[48] David Salinas, Valentin Flunkert, Jan Gasthaus, and Tim Januschowski. DeepAR: Probabilistic forecasting with autoregressive recurrent networks. *International Journal of Forecasting*, 2019.

[49] Kamił Stankevičiūtė, Ahmed Alaa, and Mihaela van der Schaar. Conformal time-series forecasting. In A. Beygelzimer, Y. Dauphin, P. Liang, and J. Wortman Vaughan, editors, *Advances in Neural Information Processing Systems*, 2021.

[50] Ruey S. Tsay. *Multivariate Time Series Analysis: With R and Financial Applications*. Wiley Series in Probability and Statistics. Wiley, 2014.

[51] Aaron van den Oord, Oriol Vinyals, and Koray Kavukcuoglu. Neural discrete representation learning. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, *Advances in Neural Information Processing Systems*, volume 30. Curran Associates, Inc., 2017.

[52] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is All you Need. In I. Guyon, U.V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, *Advances in Neural Information Processing Systems 30*, pages 5998–6008. Curran Associates, Inc., 2017.

[53] Ruofeng Wen, Kari Torkkola, and Balakrishnan Narayanaswamy. A multi-horizon quantile recurrent forecaster. In Vitaly Kuznetsov, Oren Anava, Scott Yang, and Azadeh Khaleghi, editors, *NIPS 2017 Time Series Workshop*, 2017.

[54] Chen Xu and Yao Xie. Conformal prediction interval for dynamic time-series. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning*, volume 139 of *Proceedings of Machine Learning Research*, pages 11559–11569. PMLR, 18–24 Jul 2021.

[55] Neil Zeghidour, Alejandro Luebs, Ahmed Omran, Jan Skoglund, and Marco Tagliasacchi. Soundstream: An end-to-end neural audio codec. *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 2021.
Table 5: Number of time series, domain, frequency, total training time steps and prediction length properties of the training datasets used in the experiments.

| Dataset   | $D$ | Dom. Freq. | Time step | Pred. len. |
|-----------|-----|------------|-----------|------------|
| Exchange  | 8   | $\mathbb{R}_{\geq 0}$ day | 6,071 | 30         |
| Solar     | 137 | $\mathbb{R}_{\geq 0}$ hour | 7,009 | 24         |
| Elec.     | 320 | $\mathbb{R}_{\geq 0}$ hour | 15,782 | 24         |
| Traffic   | 862 | $(0,1)$ hour | 14,036 | 24         |
| Taxi      | 1,214 | $\mathbb{N}_{\geq 0}$ 30-min | 1,488 | 24         |
| E-Commerce| 2,731| $\mathbb{N}_{\geq 0}$ day | 887 | 28         |
| Wikipedia | 9,535| $\mathbb{N}_{\geq 0}$ day | 762 | 30         |
| M5        | 30,490| $\mathbb{N}_{\geq 0}$ day | 1,941 | 28         |

A Gradient Estimators

Since the sampling and quantization in (4) are not differentiable the VQ-VAE paper applies a Straight-Through gradient estimator which propagates the gradients with respect to the codebook vectors through to the output of the encoding network. If the codebook vector and encoding representation are close then this is an adequate approximation but there is no bound on the bias of the gradients with this scheme and thus it falls under the biased-low variance taxonomy [41] of gradient estimators as shown in Figure 3.

B Experiment Details

To help reduce confusion we also collect the different hyperparameters in a single location with their description and default values listed in Table 6.
Table 6: Glossary of the main hyperparameters, their description and default values unless specified.

| Parameter | Description                               | Default |
|-----------|-------------------------------------------|---------|
| $D$       | Number of time series in a dataset        |         |
| $P$       | Prediction horizon length                 |         |
| $C$       | Context window length                     | $P \times 6$ |
| $F$       | Size of covariate vector                  |         |
| $E$       | Encoding RNN’s hidden size and size of codebook vectors | 64 |
| $H$       | Decoding RNN’s hidden size                | 40      |
| $J$       | Number of codebook vectors                | 128     |
| $\beta$  | Commitment cost                           | 0.25    |
| $Q$       | Threshold to replace codebook vector      | 2       |
| $S$       | Number of samples                         | 100     |

For our experiments we selected the default options for the models as implemented in the GluonTS library. In particular for VQ-AR, we modified the base implementation of DeepAR and thus kept the parameters of the models to be similar where applicable. We trained each model for `max_epochs=50` with the `batch_size=256` and a default `learning_rate=1e-3`. For the evaluation we used `quantiles=[0.1, 0.2, \ldots, 0.9]` to calculate the CRPS metric.