Nuclear clustering in the Energy Density Functional Approach

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Abstract. Nuclear Energy Density Functionals (EDFs) are a microscopic tool of choice extensively used over the whole chart to successfully describe the properties of atomic nuclei ensuing from their quantum liquid nature. In the last decade, they also have proved their ability to deal with the cluster phenomenon, shedding a new light on its fundamental understanding by treating on an equal footing both quantum liquid and cluster aspects of nuclei. Such a unified microscopic description based on nucleonic degrees of freedom enables to tackle the question pertaining to the origin of the cluster phenomenon and emphasizes intrinsic mechanisms leading to the emergence of clusters in nuclei.

1. Introduction
Correlations can lead profound changes in a system’s bulk properties. Already present in ideal quantum gases, where, at low temperature, quantum statistics manifest in entirely different behaviour of bosons and fermions, pending the irreducible representation of the permutation group they belong to, the situation is even more involved when correlations stem from the constituents’ interactions: they dictate the phase of matter preferred by the system. This is particularly true in nuclear systems whose rich behaviour lies in the quantum arrangement of four kinds of fermions (spin-isospin different combinations) interacting, \textit{inter alia}, by the complicated reminiscent strong force acting between their underlying quarks and gluons. Atomic nuclei usually display a quantum liquid like nature even in their ground state, in other words a phase with delocalized structure and elementary excitations with long mean free path whose eigenvalues therefore reflect the shape and space dependence of the confining potential over the whole nuclear volume \cite{1}. The corollary of this feature is a quasi homogeneous repartition of the
nucleons in the nuclear volume. In some cases, this picture breaks down. Nuclear clustering, \textit{i.e.} the arrangement of nucleons in additional bound sub-structures in the nucleus itself, provides a typical example.

Clustering is an essential feature of many-nucleon dynamics that coexists with the nuclear mean-field. Therefore, although in most cluster models the existence of such structures is assumed a priori and the corresponding effective interactions are adjusted to the binding energies and scattering phase shifts of these configurations, a fully microscopic understanding of cluster formation necessitates a more general description that encompasses both cluster and quantum liquid aspects in light as well as in heavier nuclei. At present the only comprehensive approach to nuclear structure is based on the framework of energy density functionals (EDFs). Nuclear EDFs enable a complete and accurate description of ground-state properties and collective excitations over the whole nuclide chart \cite{2, 3, 4} and therefore define an appropriate tool to describe the coexistence of cluster and quantum-liquid aspects of light nuclei (Fig 1). EDFs do not assume any specific form for the nuclear A-body wave function apart to be a product wave function. Within this framework, we can identify three main conditions for $\alpha$-clusterisation:

\begin{itemize}
  \item[i)] the effect of the depth of the mean potential
  \item[ii)] the effect of nuclear deformation
  \item[iii)] the effect of baryonic density
\end{itemize}

2. Effect of the depth of the confining potential
As studied in Refs. \cite{5, 6}, the depth of the confining potential impacts $\alpha$-clusterisation through the localisation properties of the nucleonic single-particle orbitals. If in solid state physics, one is able to experimentally fine tune the depth of a potential trapping fermionic atoms to study its effect on the formation of dimers \cite{7}, any experimental study on the depth of the nuclear
confining potential turns out in a delicate issue. Equivalent informations can be obtained in the theoretical side, by comparing various realisations of nuclear EDFs yielding the same observables but distinguishable by different depths of their mean potentials. Relativistic and the non-relativistic Gogny and Skyrme functionnals are such realisations. For example, $^{36}$Ar observables are equivalently described by the three aforementioned functionals (Fig. 2). On the other hand the relativistic functional involves a deeper confining potential than the non-relativistic ones. As explained in Refs. [5, 6], and also by E. Khan in the present volume, a deeper confining potential causes the nucleonic single-orbitals to be more localised leading to a clustering phase. Consequently, a more and more pronounced degree of localisation of nucleons can be noticed in the total density inserts of Fig. 2 going from Skyrme SLy4, then Gogny D1S, to DD-ME2.

![Figure 2](image)

Figure 2: Binding energy of $^{36}$Ar versus the quadrupole deformation parameter with inserts displaying the total density at corresponding deformations.

### 3. Effect of the deformation

The relationship between $\alpha$ clusters and single particle states in deformed nuclei is a well known effect [8, 9, 10]. A harmonic oscillator confining potential, when deformed, loses at first the degeneracy of the single particle states presented at sphericity, however recreates it again when the ratio of its frequencies becomes the ratio of integer numbers. The new magic numbers generated can be expressed as combinations of the spherical ones, so that the deformed harmonic oscillator potential admits an interpretation in terms of a series of shifted overlapping spherical potentials. Clustering can then be viewed as a mass exchange process between these spherical
potentials. We shall focus here on $\alpha$ clusterisation. As noted by Aberg [10], an isolated single-particle state in a deformed $N=Z$ nuclei corresponds to an alpha clusterisation, because of both the Kramers (time-invariance) degeneracy and the isospin symmetry: 2 protons and 2 neutrons shall have similar wave functions and therefore drive their common localisation leading to alpha clusterisation. Hence, the degeneracy raising among levels seems to be the master condition for alpha clusterisation. Let us quantitatively analyse this idea using the microscopic EDF framework on the example of $^{12}$C. Increasing its quadrupole deformation on the prolate side make it pass through different clusterised states until eventually reach the linear $\alpha$-chain configuration. We then define the average energy difference between consecutive single particle levels in those systems as a function of the deformation parameter. When the consecutive level spacing is larger than 5 MeV, the value is kept at 5 MeV which seems sufficient to consider a degeneracy raising among levels. Fig. 3 shows a correlation with a maximum average degeneracy raising and the arising of clusters: the more raised the single-particle degeneracy, the more alpha-localised the density.

Figure 3: Mean gap value between consecutive occupied neutron levels versus the quadrupole deformation parameter. The inserts display the total density at corresponding deformations

4. Effect of the density
Infinite nuclear matter at subsaturation density finds energetically favorable to undergo a Mott-like phase transition breaking its homogeneity by forming sub-units at saturation density. The occurrence of such a phenomenon in finite nuclei can be studied in the framework of EDF [11, 12]. In order to isolate the influence of density, i.e. to suppress the impact of deformation on clusterisation, calculations are performed imposing a vanishing quadrupole moment. Constraining on the total r.m.s. radius of the finite nucleus allows to reach low density phases. For example, inflating $^{16}$O makes it go from a homogeneous spherical shape to a 4 $\alpha$ clusterized phase with tetragonal shape. The effect of the phase transition on the total binding energy is displayed in Fig. 4.

5. Conclusion
EDFs are a powerful tool to tackle both quantum liquid and cluster properties from fundamental grounds. They allow to investigate on the origin of the clustering phenomenon, linked not only to
Figure 4: Total binding energy versus the r.m.s radius of $^{16}$O with DD-ME2.

the depth of the confining potential as well as the baryon density, but also, in a finite size system, to the deformation. The study of cluster structures in neutron rich systems is straightforward [13](for instance Fig. 5), and the inclusion of beyond mean field correlations is in progress.

Figure 5: Left panel : total (a), proton (b) and neutron (c) intrinsic densities of $^{14}$Be at equilibrium deformation. Right panel, from bottom to top: 3D density of the $\alpha + \alpha$ core; contour plots of the core density and the density of the valence neutrons in the (Oxz) plane; 3D density of the valence neutrons..

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