Generic f(R) theories and classicality of their scalarons

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In this Letter, we study quantum stability bound on the mass of scalaron in generic theories of $f(R)$ gravity. We show that in these scenarios, the scalaron mass increases faster with local density of the environment than one-loop quantum correction to it thereby leading to violation of quantum bound on the chameleon mass. The introduction of quadratic curvature corrections in the action are shown to stabilize the model.

I. INTRODUCTION

The late time cosmic acceleration [1, 2] has recently been accepted as one of the fundamental phenomena of nature whose underlying cause remains yet to be unfolded. The standard lore preaches that the late time acceleration is caused by the presence of a cosmic fluid with large negative pressure; the cosmological constant $\Lambda$ [3] presents a distinguished example of such a fluid.

As an alternative to cosmological constant, a variety of scalar field models were investigated with a hope to alleviate the fine tuning and coincidence problems associated with the model without assigning a fundamental reason to switch off $\Lambda$. Unfortunately, the scalar field dark energy models are not entirely problem free, assumptions about model parameters/tuning are tacitly made in these models.

There is an alternative school of thought in cosmology which advocates the need for paradigm shift and believes that cosmic acceleration results from large scale modification of gravity [4]. Such a proposal sounds healthy as general theory of relativity, which passes the solar test and has been accepted as one of the fundamental phenomena of gravity, is plugged with several difficult problems: the curvature singularity is easily accessible in the scenario and requires ugly fine tuning for its cure [5]. Being inspired by Starobinsky’s original proposal [6], the HSS model was extended by adding quadratic curvature correction [7] to address the said problems.

The quadratic correction provides in a sense quantum correction to gravity sector which turns out to be important in the scenario under consideration. It becomes equally important to investigate whether the quantum 1-loop correction to scalaron potential remains small as density of the environment increases.

In this Letter we shall study the quantum stability

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bound for scalar in Starobinsky $f(R)$ gravity model. We also address the same issue in the framework of an extended scenario by incorporating the quadratic curvature corrections in the Starobinsky model.

**Chameleon field**

Let us consider the following action in the Einstein frame

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + S_m \left[ A(\phi)^2 g_{\mu\nu}, \Psi_m \right]$$

(1)

The equation for the field $\phi$ which follows from the action can be written as

$$\Box \phi = \frac{dV}{d\phi} + (\rho - 3P)A^4 \frac{dA}{d\phi}$$

(2)

where $(\rho, P)$ are energy density and the pressure in the Jordan frame. We consider this frame as our physical frame in which the stress-energy tensor is conserved hence we assume that our quantities are independent of the scalar field $\phi$. We note that in the original paper [14], the authors defined a conserved density in the Einstein frame for a FLRW space-time. The definition that we adopt here gives a definition of the effective potential for any background (also in presence of pressure) and within this definition the effective mass of the chameleon field is the mass of the scalaron in $f(R)$. It is however clear that because in most of the cases $A \simeq 1$ the quantities in the two frames are very close.

The eq. (2) can be cast in the form

$$\Box \phi = \frac{dV_{eff}}{d\phi}$$

(3)

where $V_{eff} = V + \frac{\rho - 3P}{4} A^4$.

The existence of the chameleon mechanism depends on the form of the effective potential which in turn depends on the local density and pressure. When pressure is negligible and density is large, the scalar field may acquire a large mass for a suitably chosen potential leading to suppression of the fifth force locally. The scalaron mass is defined as usual

$$m_{eff}^2 = \frac{d^2 V_{eff}}{d\phi^2}$$

(4)

The scalar field is assumed to be settled in the minimum of the effective potential. It is therefore simple to recast the effective mass in the following form

$$m_{eff}^2 = V'' - V' \left( \frac{3A'}{A} + \frac{A''}{A'} \right)$$

(5)

To avoid a ghost instability, we require that $V''/V' > \frac{3}{4} \frac{A''}{A'}$.

In the simple case when $A$ is given by $A = e^{\beta \phi/M_p}$, we have

$$m_{eff}^2 = V'' - 4 \frac{\beta^2}{M_p} V'$$

(6)

In what follows we shall consider the chameleon mechanism in $f(R)$ theories of gravity where it occurs naturally.

**A. Chameleon theory and $f(R)$ gravity**

Let considering $f(R)$ action in the Jordan frame,

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}, \Psi_i]$$

(7)

We next use a conformal transformation

$$g_{\mu\nu} \rightarrow \ f_{,R} \ g_{\mu\nu} \ = \ e^{-2\beta \phi/M_p} g_{\mu\nu}$$

(8)

with $\beta = -1/\sqrt{6}$, to transform the action to the Einstein frame

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + S_m \left[ e^{2\beta \phi/M_p} g_{\mu\nu}, \Psi_i \right],$$

(9)

where

$$V(\phi) = M_{pl}^2 \frac{R f_{,R} - f}{2 f_{,R}^2}.$$  

(10)

The first and second derivatives of the potential $V(\phi)$ are given by

$$V_{,\phi} = \frac{M_{pl}^2}{\sqrt{6}} \frac{2 f - f_{,R}}{f_{,R}^2},$$  

(11)

$$V_{,\phi\phi} = \frac{1}{3 f_{,R}^2} \left( 1 + \frac{R f_{,RR} - 4 f f_{,RR}}{f_{,R}^2} \right).$$  

(12)

One can see that effective potential belongs to Chameleon theory as ($\phi$ directly couples to matter),

$$V_{eff} = V(\phi) + \frac{\rho - 3P}{4} A^4,$$  

(13)

where $\rho$ and $P$ are respectively the density and the pressure in Jordan frame and $A = 1/ \sqrt{f_{,R}}$. 
It is easy to find that the minimum of the effective potential from eq. (13),

\[ 2f - Rf_R = \frac{\rho - 3P}{M_p^2} \]  

(14)

It is interesting to notice that the minimum of the potential is invariant under the addition to the action of a \( R^2 \)-term. We shall use this aspect in the discussion to follow.

Also one can rewrite the effective mass \((6)\) with the help of \((11,12)\) as

\[ m_{\text{eff}}^2 = \frac{1}{3f_{RR}} \left( 1 - \frac{Rf_{RR}}{f_R} \right) \]  

(15)

which corresponds to the mass of the scalaron in the Einstein frame. In fact in the Jordan frame, we have \( M^2 = \frac{1}{3f_{RR}} \left( f_R - Rf_{RR} \right) \) and therefore \( M = \sqrt{f_R m_{\text{eff}}} = A^{-1} m_{\text{eff}} \) which is the standard factor which relates the mass of the field in Jordan to its counterpart in Einstein frame. We should emphasize that we recover the mass of the scalaron because we consider the effective potential instead of the potential and also because we consider the density \( \rho \) and the pressure in the Jordan frame as defined in \((13)\).

We often encounter local densities much larger than cosmological density \( \rho_{\text{lab}} \) such as \( \rho_{\text{lab}} \approx 10 \text{ g/cm}^3 \) and use the classical description for scalar degree of freedom in \( f(R) \) which assumes the quantum correction to scalaron potential to be small. Following Ref. \( [15] \), we shall now address the issue of quantum stability in generic theories of \( f(R) \) gravity.

### Quantum stability bound

The effective potential defined in \((13)\) depends on the energy density and therefore on the position of space-time. At the equilibrium, the field minimizes the potential \( V_{\text{eff},\phi} = 0 \), hence the chameleon appears as a massive field \( (V_{\text{eff}} \approx m_{\text{eff}}^2 \phi^2/2) \). In Einstein frame, we have General Relativity with a scalar field. In scenario under consideration, the curvature scalar is small as seen later and the effects of the expansion are negligible. Hence the model is close to quantum field theory in flat space-time where we can neglect the effects of gravity \(^1\) and quantize the scalar field sector in standard way. Also we should emphasize that we can always work in the Einstein frame, as long as the conformal transformation is not singular. Indeed this the case for the model studied as \( R \approx \rho/M_p^2 \), to be demonstrated shortly.

\[ 1 \text{ The gravity sector is the massless spin two particle without the scalaron.} \]

In \( [13] \), the authors considered the one-loop Coleman-Weinberg correction to the chameleon field potential.

\[ \Delta V_{\text{1-loop}}(\phi) = \frac{m_{\text{eff}}^4(\phi)}{64\pi^2} \ln \left( \frac{m_{\text{eff}}^2(\phi)}{\mu_0^2} \right), \]  

(16)

where \( \mu_0 \) is a cut off mass scale. It can be chosen equal to the mass of the field for a particular environment (density) which would kill the quantum correction but the correction would reappear at other values of the density.

At large values of density of interest to us or large mass of the field, we can set log term to unity

\[ \Delta V_{\text{1-loop}}(\phi) \approx \frac{m_{\text{eff}}^4}{64\pi^2}. \]  

(17)

Since we expect quantum corrections to be small, we should have \( [13] \) a small modification of the shape of the potential \( V \) which implies \( \Delta V_{\text{1-loop,}\phi}/V_{\phi} \) and \( \Delta^2 V_{\text{1-loop,}\phi}/V_{\phi} \) should be small.

Secondly, at the minimum of the effective potential \((13)\), we have\(^2\)

\[ V'(\phi) + \frac{\beta}{M_{\text{pl}}^2} \rho e^{4\beta\phi/M_{\text{pl}}} = 0 \]  

(18)

from which we obtain

\[ \frac{d\phi}{d\rho} = -\frac{\beta}{M_{\text{pl}}^2 m_{\text{eff}}^2(\phi)} e^{4\beta\phi/M_{\text{pl}}}, \]  

(19)

which gives

\[ \frac{\Delta V_{\text{1-loop,}\phi}}{V_{\phi}} \approx \frac{m_{\text{pl}}^2}{96\pi^2 \beta^2} \frac{dn_{\text{eff}}}{d\rho} < 1 \]  

(20)

\[ \frac{\Delta^2 V_{\text{1-loop,}\phi}}{V_{\phi}} \approx \frac{m_{\text{pl}}^2}{96\pi^2 \beta^2} \frac{d^2 m_{\text{eff}}}{d\rho^2} < 1 \]  

(21)

and after integration

\[ m_{\text{eff}} < \sqrt{\frac{48\pi^2 \beta^2 \rho^2}{M_{\text{pl}}^4}}^{1/6} = 0.0073 \left( \frac{|\beta| \rho}{10^g \text{ cm}^{-3}} \right)^{1/3} \text{ eV}. \]  

(22)

where, the constant of integration is fixed to zero; we can demand that the correction is zero for very low densities where the effective mass is zero. Let us briefly comment on the viability of Coleman-Weinberg one loop

\[ 2 \text{ For simplicity we neglect the pressure.} \]
correction used here for chameleon. It corresponds to the quantum mechanically corrected potential

\[ V_{\text{tot}} = V_{\text{eff}}(\phi) + \frac{i}{2} \ln \det \left[ \partial^2 + m^2_{\text{eff}} \right] \]  

(23)

where the first term represents classical part of the potential. The quantum correction is formally divergent and requires ultraviolet cut off. In case we use non-covariant scheme of regularization, say, Pauli-Willars regularization, with cut off \( M_{uv} \), the quantum correction is represented by three terms: (1) \( M^4_{uv} \), (2) \( m^2_{\text{eff}} M^2_{uv} \) and (3) \( m^4_{\text{eff}} \ln(m^2_{\text{eff}}) \). The first term can be absorbed in the definition of renormalization of cosmological constant, the third term represents the one-loop quantum correction to be used in the discussion to follow. However, the second term is much larger than the second and would invalidate usage of the quantum bound based upon the third term only.

It is well known that the term quadratic in cut off is specific to any regularization scheme which breaks Lorentz symmetry. In case of gauge theories, the regularization scheme which does not respect the underlying symmetry of the theory leads to wrong results [17]. Indeed, in the present context, the regularized value of the quantum correction using dimensional regularization gives rise to expression (16) without the dangerous term third term only.

\[ \rho = \frac{1}{2(2\pi)^3} \int d^3k \sqrt{k^2 + m^2_{\text{eff}}} \]  

(24)

As we previously said, a regularization that do not respect the symmetries of the problem is incorrect [18], and produce the terms detailed beforehand. Hence a dimensional regularization of the energy density gives in the \( \overline{MS} \) scheme

\[ \rho = \lim_{d \to 4} \frac{\mu_0^{4-d}}{2(2\pi)^{d-1}} \int d^{d-1}k \sqrt{k^2 + m^2_{\text{eff}}} \]

\[ \simeq \frac{m^2_{\text{eff}}}{64\pi^2} \ln \left( \frac{m^2_{\text{eff}}}{\mu_0^2} \right) + \cdots \]  

(25)

where \( \mu_0 \) is introduced to clean up the units.

We shall hereafter would specialize to \( f(R) \) gravity. We should emphasis that the scalaron potential in general is a complicated one and certainly does not belong to the class of renormalizable theory. However, in the neighborhood of its minimum, the latter can be approximated by a polynomial. Thus we can apply the quantum bound obtained using the Coleman-Weinberg formula for effective potential.

In \( f(R) \), \( \beta = -1/\sqrt{6} \), which implies that

\[ m_{\text{eff}} < 5.4 \times 10^{-3} \left( \frac{\rho}{10^9 \text{ cm}^{-3}} \right)^{1/3} \text{ eV}. \]  

(26)

Eq. (26) provides an upper bound on the mass of the field. As we mentioned before, we shall be interested in the scrutiny of generic \( f(R) \) theories, namely HSS and would specialize to Starobinsky parametrization for convenience.

### Parameters of Starobinsky model

We are interested to study the quantum stability of Starobinsky \( f(R) \) gravity [8].

\[ f(R) = R - \mu R_c \left[ 1 - (R^2 / R_c^2)^{-n} \right]. \]  

(27)

During the de-Sitter phase, the solution is described by [11] in an empty Universe. The curvature scalar \((R_{\text{dS}})\) is solution of

\[ \mu = \frac{1}{2} \frac{x(1 + x^2)^{n+1}}{(1 + x^2)^{n+1} - 1 - (1 + n)x^2}, \]  

(28)

where \( x = R_{\text{dS}} / R_c \). Considering \( R_c \) of the order the curvature scalar today we have \( \mu \simeq 1 \).

In the region of high density \((R \gg R_c)\), we have

\[ f(R) \simeq R - \mu R_c \left[ 1 - (R / R_c)^{-2n} \right]. \]  

(29)

It can easily be noticed from [13] that the minimum is \( R \simeq \rho / M^2_{pl} \) as in General Relativity. Hence the gravitational exact equivalence of the standard frame work of General Relativity. The scalaron which settles at the minimum of the effective potential has small variation around this point because of the space dependence of the density of matter [14].

This translates to the chameleon field via its definition [8] and gives the minimum of the effective potential

\[ \frac{\phi}{M_{pl}} = \sqrt{\frac{3}{2}} \ln f' \simeq \sqrt{\frac{3}{2}} \left[ f' \left( \frac{\rho}{M_{pl}} \right) - 1 \right] \]  

(30)

Let us now consider the experimental bound that comes from the solar system tests of the equivalence principle (LLR). Using the thin-shell parameter [14] for the Earth \( \epsilon_{\text{th}} \) we have

\[ \epsilon_{\text{th}} = \frac{\phi_{\infty} - \phi_{\oplus}}{6|\beta| M_{pl} \Phi_{\oplus}} < \frac{8.8 \times 10^{-7}}{|\beta|}. \]  

(31)
where \((\phi_\infty, \phi_0)\) are respectively the minimum of the effective potential at infinity and inside the planet and \(\Phi_0\) the Newton potential for the Earth.

Using the value \(\Phi_0 \simeq 7 \times 10^{-10}\), the previous bound translates into \(\phi_\infty/M_{pl} < 10^{-15}\), which after using Eq. (30) leads to

\[
|f'(\rho_{\infty}/M_{pl}^2) - 1| < 10^{-15} \tag{32}
\]

For the HSS model and with the density \(\rho_\infty \simeq 10^{-24}\ g\ cm^{-3}\) and \(R_c \simeq H_0^2\), we have \(10^{-5(2n+1)} < 10^{-15}\) tells us that \(n > 1\) \[10\].

We will show that for this set of parameters the quantum stability is violated in the Starobinsky model as the mass of scalaron in the model grows fast with density and can easily cross the quantum bound.

**Quantum stability of f(R) gravity**

According to [13], the bound on \(m_{\text{eff}}\) obtained using the 1-loop Coleman-Weinberg correction is given by

\[
m_{\text{eff}}(\rho) < 5.4 \times 10^{-3} \left(\frac{\rho}{10^{12} \ g\ cm^{-3}}\right)^{1/3} \text{eV}, \tag{33}\]

Also from Eq. (13), we can express the scalar field mass in Starobinsky model as a function of the density \(\rho\)

\[
m_{\text{eff}}(\rho) \simeq \frac{1}{\sqrt{6\mu n(2n+1)}} \sqrt{R_c} \left(\frac{\rho}{M_{pl}^2 R_c}\right)^{n+1}. \tag{34}\]

where we assumed that the density is large enough compared to the cosmological density \(M_{pl}^2 R_c \sim \rho_c \simeq 10^{-29}\ g\ cm^{-3}\)

From the previous discussion, we know that \(\mu \simeq 1\) and \(n > 1\), which gives

\[
m_{\text{eff}}(\rho) \simeq 3 \times 10^{-34} \left(\frac{\rho}{\rho_c}\right)^{n+1} \text{eV}. \tag{35}\]

At the cosmological density, \(\rho \sim \rho_c \sim 10^{-29}\ g/cm^3\), the quantum stability bound, \(5 \times 10^{-13}\) eV, is larger than the scalar field mass \(m_{\text{eff}} = 3 \times 10^{-34}\) eV. However, the quantum bound \(\propto \rho^{1/3}\) while \(m_{\text{eff}} \propto \rho^{n+1}\) (with \(n > 1\)). It is therefore clear that the \(m_{\text{eff}}\) will be excluded easily by this quantum stability bound at some high density.

Indeed, the scalar mass \(m_{\text{eff}}\) is quickly excluded by the quantum stability bound at \(\rho \approx 10^{-87n/(2+3n)}\ g/cm^3\).

That means according to this bound, \(f(R)\) gravity in the point of view of chameleon theory is excluded in any typical dense medium, i.e., in the air \(\left(\rho_{\text{air}} \sim 10^{-3}\ g/cm^3\right)\).

**Extended Starobinsky model: Adding \(\alpha R^2\) term**

In high density regime the quantum curvature corrections to Einstein-Hilbert action become important. These corrections might provide a cut off to the scalaron mass. In what follows, we shall consider the extension of Starobinsky model by adding \(\alpha R^2\) term to its action,

\[
f(R) = R - \mu R_c \left[1 - \left(\frac{R}{R_c}\right)^{2n}\right] + \alpha R^2, \tag{36}\]

This correction was briefly suggested in the original paper [5] to avoid the problem of scalaron mass from becoming too large and being inspired by the Starobinsky’s original idea, it was introduced in [12] as a solution to the Frolov singularity problem (see also Refs. [13] on the same theme). As noticed above, the addition of this term does not change the position of the minimum of the field but it will provide a natural upper bound to the mass of the chameleon field. Thus the gravitational sector is unchanged compared to general Relativity, we have the same curvature scalar but the scalar sector is modified because of this additional term. The shape of the effective potential is changed.

In the regime of large densities, we have

\[
m_{\text{eff}}^2 \simeq \frac{1}{6\alpha(1 + 12\rho/M_{pl}^2)} \tag{37}\]

We should emphasize that the scalaron is massless in the Einstein frame when the density diverge contrary to the Jordan frame where the effective mass goes to \(1/6\alpha\), this is because of the conformal factor \(f_R\) and it will have no effect on the following discussion. In fact, we consider hereafter \(\rho \approx \rho_{\text{lab}}\) which gives in both frames \(m_{\text{eff}} \approx 1/6\alpha\).

The classicality condition [55] gives a lower bound \(\alpha \geq 6 \times 10^3\) eV\(^{-2}\) when according to the bound from Big Bang nucleosynthesis and CMB physics \(\alpha \ll 10^{35}\) eV\(^{-2}\) [10]. But the tightest bound comes from the Eot-Wash experiments which implies that \(\alpha \ll 4 \times 10^4\) eV\(^{-2}\). We still have a range of viability of the model as soon as we add the quadratic curvature correction term in the action.

Also it should be noticed that \(R^2\) term is introduced here as a cure of late time cosmic dynamics. However, we know that \(R^2\) can gives rise to inflation at early epochs. And if the model is used to describe an early acceleration phase we would have [8, 21] \(\alpha \approx 10^{-45} \left(N/50\right)^2\) eV\(^{-2}\) where \(N\) is the number of e-folds. This is certainly not be compatible with the quantum bound. We should, however, note that at high energies in the early universe, the quantum correction may be quite different than the one given by Coleman-Weinberg potential and the simple analysis presented here may not be valid in that regime.
II. CONCLUSION

In this letter, we have investigated the issues of classicality of scalarons in the Starobinsky of $f(R)$ gravity model. The model is studied in the Einstein frame where we have General Relativity with a scalar field, the scalaron. At the densities studied, local analysis, we have shown that the curvature scalar is equivalent to the one in General Relativity. The scalar field appears as a Klein-Gordon massive field and can be quantized using the standard procedure. In this context, we have shown that the quantum bound on scalaron mass derived in [15] can be a tight constraint on $f(R)$ dark energy models. Within the range of viability of the parameters of the model, that we derived, the quantum corrections are large for low densities. The scalaron masses increases very fast with medium density than the quantum bound on it. The mass of the scalaron is unbounded and can exceed the Planck mass at a reasonable value of the density of the medium. Clearly, the model cannot be trusted in this case. We therefore need to introduce a cut off to the mass of scalaron as the expression of our ignorance and use the model below the cut off.

In view of the aforesaid, we used the extended Starobinsky model by adding quadratic curvature correction to the Lagrangian. We have shown that this term does not change the curvature scalar $R \approx \rho/M_p^2$ but only effects the form of the potential $V$ and therefore the mass of the scalaron. This term influences principally the scalar sector of the theory. It produces a natural bound on the mass of the scalaron that we constrained via the classicality condition and the E"ot-Wash experiments. We have demonstrated that extended scenario is consistent with the quantum bound on the scalaron mass.

It should also be noticed that in the regime of high densities or low densities but large scales, gravity is never weak and quantization of the scalaron might become complicated. As noticed by Starobinsky several years back, the quantum corrections in his model during inflation are small\cite{22} and hence there is no strong bound on the mass of scalaron from the classicality condition in this case. The analysis performed in this letter is done in a regime where the gravity is weak, density is low and scales are small. Hence the standard results of quantum field theory could be applied. Last but not least, we should clearly point out a subtle point of our analysis. To be fare, underlying argument regarding the quantum bound relies on the assumption of a semiclassical gravity in which chameleon field is quantum and gravitational field classical. But scalaron field has geometric origin which controls the curvature of space-time. To be systematic, a full analysis should be performed. Thus the scalaron field theory, which is obviously not renormalizable, can be judiciously used below some ultraviolet cut off $M_{uv}$. The quantum corrected effective Lagrangian contains a term proportional to $m_{pl}^4 (M_{uv}^2)^2$ which might effect the analysis presented here\cite{22}; in our opinion, the problem requires further investigation.

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