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The discrete charm of flavour and CP violation

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Abstract. We point out that in the Standard Model (SM) there is no explanation why $|V_{23}|^2 + |V_{13}|^2$ is of order $10^{-3}$. A framework is described for explaining this small mixing, involving the introduction of vector-like quarks. A symmetry is introduced so that at a first stage $V_{CKM} = 1$ and only the third generation of quarks acquires a mass. It is shown that when interactions of vector like quarks are taken into account a realistic quark mass spectrum is generated together with a correct $V_{CKM}$ matrix.

1. Introduction

The closely related questions of Flavour and CP Violation are two of the major open problems in Particle Physics. Discrete symmetries have played a major role in the development of Particle Physics. The symmetries C, P, CP and T are all violated in Nature. Yet, they are very important, since they are conserved by electromagnetic and strong interactions. Only CPT is not violated, as far as we know today. Parity is explicitly violated in the Standard Model (SM), but in the SM CP can be violated explicitly in the Yukawa sector provided that there are three or more fermion generations. Spontaneous CP violation, can only occur if the scalar sector of the SM is extended. The first model of spontaneous CP violation was proposed by Lee [1] in the context of the SM extended with one additional Higgs doublet, in order to accommodate the experimental evidence for CP violation in the quark sector at a time when it was not yet known that there were more than two generations of quarks. Models with two Higgs doublets have a very rich phenomenology [2, 3] and they provide very interesting candidates for physics beyond the SM, which have the potential of being tested at the LHC. There is, by now, clear experimental evidence that the Cabibbo-Kobayashi-Maskawa (CKM) matrix is complex [4]. Therefore, in any model of spontaneous CP violation, with the breaking of CP arising from a CP violating vacuum, it must be possible to generate a complex CKM matrix from the vacuum phase, in order to conform to experiment. This makes it non-trivial to have viable models of spontaneous CP violation. The simplest realistic model of spontaneous CP violation, involves the addition to the SM of a vector-like isosinglet quark and a complex singlet scalar [5, 6, 7]. Models with several Higgs doublets have potentially large Higgs mediated flavour changing neutral currents. Experimental data show that the latter are extremely suppressed thus implying the need for a mechanism to avoid, in a natural way, sizeable effects. One option is to use a symmetry that eliminates these currents at tree level, this is the natural flavour conservation paradigm [8, 9]. In alternative it is possible to use a symmetry that leads to tree level effects suppressed by small entries of $V_{CKM}$ [10]. These are the so-called BGL models which have been extended later-on.
[11, 12] and where the suppression conforms with the experimental evidence.

At present, all experimental results on quark mixing and CP violation in the quark sector, are in essential agreement with experiment. However, there is plenty of room for New Physics. CP violation has profound implications for cosmology, since CP breaking is one of the required ingredients [13] in order to generate the observed Baryon Asymmetry of the Universe (BAU). By now, it is clear that in the SM it is not possible to generate sufficient BAU. One of the reasons for this is the smallness of CP violation in the SM. This is a clear indication that there should be additional sources of CP violation, beyond the one present in the SM. The great challenge is to find out where are the new sources of CP violation and how they could be detected. In the quark sector, an interesting possibility may arise if vector-like quarks are added to the spectrum of the SM [6, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. These quarks can have bare masses in the Lagrangian, since their mass terms are invariant under the gauge symmetry. They may also receive their mass from the couplings to SU(2) singlet scalars, once these acquire a vacuum expectation value (vev). In any case, they mix with the standard quarks, and the $3 \times 3$ unitarity of the CKM matrix can be violated. These violations of $3 \times 3$ unitarity lead in general to Z-mediated Flavour-Changing-Neutral-Currents (FCNC) at tree level, which are naturally suppressed. In spite of this suppression, these FCNC, like $B_d - \bar{B}_d$ mixing, $B_s - \bar{B}_s$ mixing and $D^0 - \bar{D}^0$ mixing [32], can contribute to processes which in the SM only occur at loop level.

Another possible source of CP breaking is leptonic CP violation. It is well known that in the SM neutrinos are strictly massless, so the discovery of neutrino oscillations, implying that at least two of the neutrinos have mass, is a clear experimental evidence of Physics Beyond the SM. Let us assume that one adds three right-handed neutrinos to the SM. Then the most general renormalizable Lagrangian must include Majorana-type bare mass terms for the right-handed neutrinos, together with Dirac neutrino masses, leading to the seesaw mechanism [33, 34, 35, 36, 37] of type one, providing an elegant explanation for the smallness of the neutrino masses. In this scenario, there are three types of CP violation which may arise: CP violation at low energies of Dirac type, CP violation at low energies of Majorana type and CP violation at high energy which provides a very attractive scenario of Baryogenesis through Leptogenesis [38]. At low energies one may detect leptonic CP violation of Dirac type through neutrino oscillations, which is one of the major tasks for future experiments. Majorana CP phases are difficult to detect at low energies but they affect double-beta decay. An important question is whether there is a relationship between leptonic CP violation at low energy and CP violation entering in Leptogenesis. It has been shown that in general such a relationship does not exist [39, 40] unless one introduces leptonic flavour symmetries.

2. Comparing the Quark and Leptonic Sectors

In the SM, the flavour structure of Yukawa couplings is not constrained by the gauge symmetry which implies that the Yukawa matrices are $3 \times 3$ general complex matrices. This leads to the existence of 36 parameters in the quark flavour sector. It is clear that there is a large redundancy, since in the quark flavour sector we have only ten physical parameters, namely the six quark masses plus the four physical parameters of the CKM matrix. In the lepton sector, one encounters an entirely analogous situation in the case of massive Dirac neutrinos. In the case of Majorana neutrinos, one has two extra CP violating phases which reflect the Majorana character of neutrinos. This large flavour redundancy is due to the freedom that one has to make weak basis (WB) transformations which, for the quark sector, can be written:

$$d_R^0 \rightarrow V'^R d_R^0, \quad u_L^0 \rightarrow U'^L u_L^0, \quad d_R^0 \rightarrow V'^R d_R^0, \quad u_R^0 \rightarrow W' u_R^0$$

(1)
As previously mentioned, the Yukawa matrices are arbitrary complex matrices. However, one can make a weak basis transformation so that both $Y_u, Y_d$ become Hermitian matrices. In the context of specific anzätze with texture zeros it is common to consider Hermitian matrices (see, for example, [41]) yet, this need not be the case [42]. The large redundancy in Yukawa couplings renders it specially difficult to extract from the data some flavour symmetry, in a bottom up approach. Even if such symmetry exists, in what WB would the symmetry be apparent? There is a specially convenient WB where one of the quark mass matrices ($M_d$ or $M_u$) is diagonal real and the other matrix is Hermitian. It can be readily seen that in this basis the number of free parameters in the quark mass matrices equals the number of physical measurable parameters, namely the six quark masses and the four physical parameters in $V_{CKM}$.

The discovery of neutrino oscillations providing experimental evidence for non-vanishing neutrino masses, brought further intriguing aspects to the Flavour Problem. Why are the pattern of fermion mixings so different in the quark and lepton sectors? Do we understand why lepton mixing is large? Do we understand why quark mixing is small? An important difference between the quark and lepton sectors has to do with experiment. In the quark sector the $V_{CKM}$ matrix is overdetermined, in the sense that once we measure $V_{ub}$, $V_{cb}$ and $V_{cb}$ through strange particle and B-meson decays, there is a large amount of physical quantities like $B_d - \overline{B_d}$, $B_s - \overline{B_s}$ mixings, the strength of CP violation in the kaon sector as well as CP violation in the B sector, with the measurement of the angles $\beta$ and $\gamma$, which within the SM, have to be fitted through a single parameter, namely the CP violating phase entering in the parametrization of $V_{CKM}$. This is to be contrasted to the situation encountered in the lepton sector.

Let us assume that neutrinos are Majorana particles and consider the effective neutrino mass matrix at low energies. The leptonic mass terms are then of the form:

$$L_{\text{mass}} = -\frac{1}{2} \nu^{0T}_L C^{-1} m_{\nu_L} \nu^0_L - \overline{\nu}^0_L m_{\ell R} \nu^0_R + \text{h.c.} \quad (2)$$

The neutrino mass matrix is a complex symmetric matrix, with six real parameters and three phases. If we work in the WB where the charged lepton mass matrix is diagonal and real, the number of independent parameters of the neutrino mass matrix is equal to the number of physical quantities derived from it, to wit, three neutrino masses, and six parameters - three mixing angles and three CP violating phases - appearing in the leptonic mixing matrix. It has been pointed out that it is not possible to completely determine the neutrino mass matrix from viable experimental measurements [43]. This motivated the authors of Ref. [43] to consider a restricted class of neutrino mass matrices with zero textures consistent with all experimental data and to explore their phenomenological implications. The imposition of these textures is based on the underlying idea that they are linked to a flavour symmetry. The effective neutrino mass matrix written in Eq. (2) must be generated through physics beyond the SM. The seesaw framework is a simple and elegant possibility. The authors of Ref. [44] studied seesaw realisations of the texture zeros proposed in [43]. Implications for CP violation in the lepton sector at high energies, and in particular for leptogenesis, of these seesaw realizations were analysed in [45]. With two right-handed neutrinos only, zero textures allow to establish a connecting link between the sign of the baryon asymmetry of the universe and the sign of the CP violating parameter which will be measured in low energy neutrino oscillations [46]. With three right-handed neutrinos these signs cease to be correlated [47] but a connection still exists. In this respect the Casas and Ibarra parametrisation [48] is very useful to show that zeros in the Dirac type neutrino mass matrix, in the basis where the charged lepton mass matrix is diagonal, lead to orthogonality relations such that for some of the textures the variables relevant for leptogenesis can be fully determined in terms of low energy parameters and heavy neutrino masses [49, 50, 51]. An alternative way of reducing the number of free parameters in the Lagrangian is by means of discrete symmetries, such as for example an $A_4$ symmetry [52, 53] (for a review see [54]), leading, once
again, to definite predictions. In general without a flavour model the three CP violating phases appearing at low energies and the three phases relevant for high energy leptonic CP violation, and in particular Leptogenesis, are not related [39, 40].

3. The Origin of CP Violation

In the SM with three or more generations, CP violation may originate at the Lagrangian level with complex Yukawa couplings, as it has been pointed out by Kobayashi and Maskawa [55]. The most elegant way to show this consists of separating the Lagrangian of the SM in two parts:

\[ \mathcal{L} = \mathcal{L}_{CP} + \mathcal{L}_{\text{remaining}} \]  

(3)

\( \mathcal{L}_{CP} \) consists of the gauge part of the Lagrangian which necessarily conserves CP. One can then consider the most general CP transformation which leaves \( \mathcal{L}_{CP} \) invariant. Applying this CP transformation to \( \mathcal{L}_{\text{remaining}} \) one can easily derive that the necessary condition for CP invariance for any number of fermion generations is [56]:

\[ \text{Tr} [H_u, H_d]^3 = 0 \]  

(4)

The condition of Eq. (4) is automatically satisfied for one or two generations. For three generations Eq. (4) is a necessary and sufficient condition for CP invariance and it can be expressed in terms of quark masses and mixing:

\[ \text{tr} [H_u, H_d]^3 = 6i \left( m_t^2 - m_c^2 \right) \left( m_c^2 - m_u^2 \right) \left( m_c^2 - m_d^2 \right) \times \]

\[ \times \left( m_b^2 - m_s^2 \right) \left( m_b^2 - m_d^2 \right) \left( m_s^2 - m_d^2 \right) \text{Im} Q_{uscb} \]  

(5)

where \( Q \) stands for a rephasing invariant quartet of \( V_{CKM} \), defined by \( Q_{\alpha i \beta j} \equiv V_{\alpha i}V_{\beta j}V_{\alpha j}^*V_{\beta i}^* \) \( (\alpha \neq \beta, i \neq j) \).

Another scenario for the generation of CP violation is the one put forward by Lee [1] about the same time that the KM mechanism [55] was proposed. The idea is that CP is a good symmetry of the Lagrangian but the vacuum is not CP invariant.

In general a realistic model of Spontaneous CP violation has to fulfil the following conditions.

i) The model should have a natural suppression mechanism for FCNC both in the scalar and vector sectors.

ii) The vacuum CP violating phase phase should be able to generate a complex CKM matrix. This requirement arises from the fact that experimentally there is clear evidence that the CKM matrix is non-trivially complex even if one allows for the presence of New Physics.

The minimal viable model of spontaneous CP violation involves the following extension of the SM:

i) Introduction of at least one vector-like quark with charge either -1/3 or +2/3.

ii) Introduction of one complex scalar field, singlet under the gauge group, which we denote S.

With the above mentioned addition to the SM, one achieves spontaneous CP violation through a non-trivial phase in the vev of the scalar isosinglet S. For definiteness, let us assume that we have introduced a down type vector-like quark. This leads to a 4 × 4 quark mass matrix in the down sector and the quark mixing matrix becomes a 3 × 4 matrix, leading to deviations of 3 × 3 unitarity in the CKM matrix connecting standard quarks and also to Z-mediated FCNC in the down sector. One of the nice features of this class of models is the fact that both deviations of 3 × 3 unitarity and tree level FCNC are naturally suppressed by the ratio of masses \( m^2/M^2 \) of standard quarks and of the heavy isosinglet quark. Recall that since the mass term of the vector-like quark is gauge invariant, its value can naturally be much larger than the electroweak scale.
There is mixing between standard quarks and the vector-like quark which in turn generates at low energies a complex $3 \times 3$ effective down quark mixing. Upon diagonalisation of the quark mass matrices, this leads to a complex $3 \times 3$ CKM matrix, in agreement with experiment. As mentioned above there are naturally suppressed FCNC at tree level which in spite of this suppression can contribute to $B_d - \overline{B_d}$ and $B_s - \overline{B_s}$ mixings.

4. Generating a Viable Model of Quark Mixing with Vector-Like Quarks

In the SM the Yukawa couplings $Y_u, Y_d$, responsible for generating the up and down quark mass matrices, are two independent complex matrices. As a result, we show next that there is no reason for having the CKM matrix close to the identity, even if one takes into account the strong hierarchy of quark masses. In order to prove this result we shall consider the extreme chiral limit, where only the top and the bottom quarks have a mass.

4.1. The Extreme Chiral Limit

In the EC limit the up and down quark mass matrices are two rank one matrices which can be written as:

$$M_d = U^d_L \text{diag}(0, 0, m_b) U^d_R, \quad M_u = U^u_L \text{diag}(0, 0, m_t) U^u_R \quad (6)$$

Note that no generality is lost in writing in Eq. (6) the quark masses in a given ordering, taking into account that a permutation which would change these positions could always be incorporated in the matrices $U^d_{L,R}$.

Keeping in mind that at this stage the first two generations are massless, one can make an arbitrary redefinition of the massless quarks through a unitary transformation with the following form:

$$W_{u,d} = \begin{bmatrix} X_{u,d} & 0 \\ 0 & 1 \end{bmatrix} \quad (7)$$

where $X_{u,d}$ are $2 \times 2$ unitary matrices. One can then show that the quark mixing matrix $V_{CKM}$ is an orthogonal matrix mixing only the third and second quark generation, parametrised by an angle $\alpha$ which is entirely arbitrary in the EC limit. It is clear that in the SM the smallness of $|V_{23}|^2 + |V_{13}|^2$ has no relation with the fact that the quark mass ratios are small. The experimentally observed smallness of $|V_{23}|^2 + |V_{13}|^2$ may be an indication of the need of a flavour symmetry.

5. Obtaining Small Mixing Through a Symmetry

Let us introduce the following $Z_6$ flavour symmetry in the SM:

$$Q^0_{L1} \rightarrow e^{i\tau} Q^0_{L1}, \quad Q^0_{L2} \rightarrow e^{-2i\tau} Q^0_{L2}, \quad Q^0_{L3} \rightarrow e^{-i\tau} Q^0_{L3}$$

$$d^0_{R1} \rightarrow e^{-i\tau} d^0_{R1}, \quad d^0_{R2} \rightarrow e^{-i\tau} d^0_{R2}, \quad d^0_{R3} \rightarrow e^{-2i\tau} d^0_{R3} \quad (8)$$

$$u^0_{R1} \rightarrow e^{i\tau} u^0_{R1}, \quad u^0_{R2} \rightarrow e^{i\tau} u^0_{R2}, \quad u^0_{R3} \rightarrow u^0_{R3}; \quad \Phi \rightarrow e^{i\tau} \Phi; \quad \tau = \frac{2\pi}{6}$$

where the $Q^0_{Lj}$ stand for the left-handed quark doublets, $d^0_{Rj}$ and $u^0_{Rj}$ denote right-handed quark singlets and $\Phi$ is the scalar doublet. The Yukawa interactions can be written:

$$\mathcal{L}_Y = \left[ -Q^0_{Lj} \Phi Y_d d^0_{Rj} - Q^0_{Lj} \tilde{\Phi} Y_u u^0_{Rj} \right] + \text{h.c.}, \quad (9)$$
As a result of the $Z_6$ symmetry, the Yukawa couplings have the following flavour structure:

$$Y_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}, \quad Y_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

which leads to $V_{CKM} = \mathbb{1}$, at this stage.

5.1. Generation of Quark Masses for the First Two Families

We show next that one may generate masses for the first two generations, through the introduction of vector-like quarks. For definiteness, we introduce three down ($D^0_{Li}$, $D^0_{Ri}$) and three up ($U^0_{Li}$, $U^0_{Ri}$) vector-like quarks which are singlets under the gauge group. The Yukawa interactions can now be written:

$$\mathcal{L}_Y = \left[ -\overline{Q}^0_{Li} \Phi (Y_d)_{i\alpha} d^0_{Ra} - \overline{Q}^0_{Li} \Phi (Y_u)_{i\beta} u^0_{R\beta} + h.c. \right]$$

where the index $i$ runs from 1 to 3 while the indices $\alpha$ and $\beta$ run from 1 to 6. We introduce the following generic bare mass terms

$$\mathcal{L}_{b.m.} = \left[ -\overline{D}^0_{Lj}(\eta_d)_{j\alpha} d^0_{Ra} - \overline{U}^0_{Lk}(\eta_u)_{k\beta} u^0_{R\beta} + h.c. \right]$$

The $Z_6$ symmetry is extended to the full Lagrangian, and we further introduce a complex scalar singlet denoted $S$. Soft-breaking terms are introduced in the scalar sector to avoid a massless Goldstone boson and also to solve the domain-wall problem. The couplings of the scalar singlet to the quark singlets can be written:

$$\mathcal{L}_S = \left[ -\overline{D}^0_{Lj}(\eta_d)_{j\alpha} S + (g_d')_{j\alpha} S^* \right] d^0_{Ra} - \overline{U}^0_{Lk}(\eta_u)_{k\beta} S + (g_u')_{k\beta} S^* u^0_{R\beta} + h.c.$$ (13)

After gauge symmetry breaking one obtains:

$$\mathcal{L}_M = \left[ -\frac{v}{\sqrt{2}} \overline{d}^0_{Li}(Y_d)_{i\alpha} d^0_{Ra} - \frac{v}{\sqrt{2}} \overline{u}^0_{Li}(Y_u)_{i\alpha} u^0_{R\beta} - \overline{D}^0_{Lj}(\mu_d)_{j\alpha} d^0_{Ra} - \overline{U}^0_{Lk}(\mu_u)_{k\beta} u^0_{R\beta} \right] + h.c.$$ (14)

In a more compact form one can write:

$$\mathcal{L}_M = - \left( \overline{d}^0_L D^0_L \right) \mathcal{M}_d \left( \begin{array}{c} d^0_R \\ p^0 \end{array} \right) - \left( \overline{u}^0_L U^0_L \right) \mathcal{M}_u \left( \begin{array}{c} u^0_R \\ p^0 \end{array} \right)$$ (15)

where $\mathcal{M}_d$, $\mathcal{M}_u$ are $6 \times 6$ matrices

$$\mathcal{M}_d = \begin{pmatrix} m_d & \omega_d \\ X_d & M_d \end{pmatrix}, \quad \mathcal{M}_u = \begin{pmatrix} m_u & \omega_u \\ X_u & M_u \end{pmatrix}$$ (16)

These mass matrices can be diagonalised through unitary transformations:

$$\left( \begin{array}{c} d^0_L \\ D^0_L \end{array} \right) = \left( \begin{array}{c} A_{dL} \\ B_{dL} \end{array} \right) \left( \begin{array}{c} d_L \\ p \end{array} \right) \equiv U^0_L d_L, \quad \left( \begin{array}{c} u^0_L \\ U^0_L \end{array} \right) = \left( \begin{array}{c} A_{uL} \\ B_{uL} \end{array} \right) \left( \begin{array}{c} u_L \\ p \end{array} \right) \equiv U^0_L u_L$$

$$\left( \begin{array}{c} d^0_R \\ D^0_R \end{array} \right) \equiv U^0_R d_R; \quad \left( \begin{array}{c} u^0_R \\ U^0_R \end{array} \right) \equiv U^0_R u_R$$ (17)

One can then write the charged currents as:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \left( \overline{\pi}^0_L \gamma^\mu d^0_L \right) W^\mu_\mu + h.c. = -\frac{g}{\sqrt{2}} \left( \overline{\pi}^\mu_L V \gamma^\nu \gamma^\mu d_L \right) W^\mu_\mu + h.c.$$ (18)
with $V = A_u^\dagger A_d L$. The neutral gauge couplings are:

$$\mathcal{L}_Z = \frac{g}{\cos \theta_W} Z_\mu \left[ \frac{1}{2} (\bar{u}_L W^{u \gamma_\mu} u_L - \bar{d}_L W^{d \gamma_\mu} u_L) - \sin^2 \theta_W \left( \frac{2}{3} \bar{u}_L \gamma_\mu u_L - \frac{1}{3} \bar{d}_L \gamma_\mu d_L \right) \right]$$

(19)

with $W^d = V^\dagger V$ and $W^u = V V^\dagger$. In the quark mass eigenstate basis the SM-like Higgs couplings, and the couplings of the would-be Goldstone bosons $G^+, G^0$ can be written:

$$\mathcal{L}_{H h} = - \frac{\sqrt{2} g^2}{v} \left[ \bar{u}_L V D_d d_R - \bar{u}_R D_u V d_L \right] - i \frac{g^0}{v} \left[ \bar{d}_L W^d D_d d_R + \bar{u}_L W^u D_u u_R \right] + \text{h.c.}$$

(20)

The vector-like quarks transform in the following way under $Z_6$:

$$D_{L1}^0 \rightarrow e^{-3i_\tau} D_{L1}^0 \quad D_{L2}^0 \rightarrow e^{-2i_\tau} D_{L2}^0 \quad D_{L3}^0 \rightarrow e^{-i_\tau} D_{L3}^0$$

$$D_{R1}^0 \rightarrow e^{-2i_\tau} D_{R1}^0 \quad D_{R2}^0 \rightarrow e^{-3i_\tau} D_{R2}^0 \quad D_{R3}^0 \rightarrow D_{R3}^0$$

$$U_{L1}^0 \rightarrow e^{-i_\tau} U_{L1}^0 \quad U_{L2}^0 \rightarrow U_{L2}^0 \quad U_{L3}^0 \rightarrow e^{i_\tau} U_{L3}^0$$

$$U_{R1}^0 \rightarrow U_{R1}^0 \quad U_{R2}^0 \rightarrow e^{-i_\tau} U_{R2}^0 \quad U_{R3}^0 \rightarrow e^{2i_\tau} U_{R3}^0 \quad S \rightarrow e^{i_\tau} S \quad \tau = \frac{2\pi}{6}$$

(21)

Diagonalisation of the quark mass matrices is performed through:

$$U_L^d \mathcal{M}_d U_R^d = \mathcal{D}_d \equiv \text{diag}(d_d, d_u)$$

(22)

with:

$$U_L = \begin{pmatrix} K & R \\ S & T \end{pmatrix}$$

(23)

Unitarity of $U_L$ implies $KK^\dagger = \mathbb{1} - RR^\dagger$ as well as $K^\dagger K = \mathbb{1} - S^\dagger S$. Deviations of $3 \times 3$ unitarity are small since $R$ and $S$ are suppressed by the ratio $m/M$. The matrices $K_d, K_u$ can be computed from an effective Hermitian squared matrix given by:

$$\mathcal{H}_{eff} = (mm^\dagger + \omega \omega^\dagger) - (mX^\dagger + \omega M^\dagger)(XX^\dagger + MM^\dagger)^{-1}(Xm^\dagger + M\omega^\dagger)$$

(24)

To an excellent approximation $K$ is the matrix that diagonalises $\mathcal{H}_{eff}$:

$$K^{-1} \mathcal{H}_{eff} K = d^2$$

(25)

We assume that the $6 \times 6$ matrix $\mathcal{M}_d$ has a Froggatt-Nielsen structure [57]:

$$\mathcal{M}_d = \mu \begin{pmatrix} \frac{\lambda_1}{\lambda} & \frac{\lambda_2}{\lambda} & \frac{\lambda_3}{\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda y & 0 & 0 \\ 0 & 0 & 1 & \lambda x & 0 & 0 \\ 0 & 0 & C_1 & \frac{D_1}{\lambda} & 0 & 0 \\ C_2 & \frac{D_2}{\lambda} & 0 & \lambda x & 0 & 0 \\ \frac{A_1}{\lambda} & \frac{A_2}{\lambda} & 0 & 0 & \frac{A_3}{\lambda} & 0 \end{pmatrix}$$

(26)
with $\mu \approx m_b$. This leads to the following structure for $H_{\text{eff}}$:

$$H_{\text{eff}} \sim \begin{pmatrix} \lambda^6 r_2 z^2 & \lambda^5 u y z & -\lambda^3 c_3 z \\ \lambda^5 u^* y z & \lambda^4 r'y^2 & -\lambda^2 c_2 y \\ -\lambda^3 c_3^* z & -\lambda^2 c_2^* y & 1 - \lambda^2 r' \end{pmatrix} \quad (27)$$

with:

$$r = |a_3|^2 + |c_3|^2$$

$$r' = |a_2|^2 + |b_2|^2 + |c_2|^2$$

$$\hat{r} = |c_1|^2 + |c_2|^2 + |c_3|^2 + (c_1 + c_1^*) x$$

$$u = a_2^* a_3 + c_2^* c_3$$

$$a_i = A_i / D_i, \quad b_i = B_i / D_i, \quad c_i = C_i / D_i$$

It can be shown that a realistic $V_{\text{CKM}}$ is generated with:

$$|V_{cb}| \approx K_{23} \lambda^2 \quad |V_{ub}| \approx K_{13} \lambda^3 \quad (29)$$

where the $K_{ij}$ are of order one, namely $K_{23} = c_2 y$ and $K_{13} = c_3 z$.

The general FCNC structure was analysed in detail [58] for this model, including loop-induced FCNC. The decay channels of the vector-like quarks were also studied, together with the possibility of discovering them at the LHC.

6. Conclusions

We emphasise a flavour fine-tuning problem present in the SM, which results from the fact that there is no reason to have a $V_{\text{CKM}}$ matrix close to the identity, even taking into account the strong hierarchy of quark masses. We describe a solution to this fine-tuning problem which involves the introduction of a flavour symmetry and the introduction of vector-like quarks of charges (-1/3) and (2/3), together with a complex singlet scalar. Prior to the introduction of the vector-like quarks $V_{\text{CKM}}$ equals the identity, but in the presence of vector-like quarks, a correct quark mass spectrum and a realistic CKM mixing matrix is obtained. We also point out that in realistic models, some of the vector-like quarks are at the reach of second run of LHC.

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