Boson-Fermion Model of High-$T_C$ Superconductivity
—-a Progress Report*

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1. The Motivation

A common feature of the cuprate superconductors is that the coherence length is comparable with the lattice spacing [1]. Therefore the Cooper pairs are rather localized in the coordinate space and consequently can be regarded as boson degrees of freedom [2]. The superconductivity is thereby associated with the kinematical Bose-Einstein condensation and there must be uncondensed pairs in the normal phase.

For a Bose-Einstein condensation to happen, the thermal wavelength of the bosons with mass $m_b$ at the transition temperature $T_C$, $\lambda_C = \sqrt{\frac{2\pi}{m_b k_B T_C}}$ should be comparable with the inter-boson distance, $l$, so that quantum mechanical coherence takes place. Indeed, the ratio $\frac{\lambda_C}{l}$ is 1.38 for an ideal Bose gas and 1.65 for $HeII$ at the $\lambda$-transition. For high $T_C$ materials, this ratio can be extracted from the results of the $\mu SR$ experiment [3] of Uemura et. al. According to them, the transition temperature $T_C$ of the under-doped

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up to optimal-doped cuprates is inversely proportional to the square of the magnetic penetration depth at $T = 0$, $\lambda_0$ with a constant of proportionality universal for all cuprates,

$$T_C(K) = 0.25 \times 10^5 \frac{m_e}{e^2 \lambda_0^2},$$

(1.2)

where $m_e$ is the electron mass in vacuum and the unit of $m_e/(e^2 \lambda_0^2)$ is Å$^{-3}$. Taking into account of possible contribution to the superfluid density from the fermionic component, we have

$$\frac{1}{\lambda_0^2} \geq \frac{4e^2}{m_e c l^2},$$

(1.3)

where $c$ denotes the average distance between CuO$_2$ layers and $l$ stands for the average inter-boson distance within each layer. Combining (1.3) and (1.2), we end up with

$$\frac{\lambda_C}{l} \leq 2.8,$$

(1.4)

which is consistent with the picture of Bose-Einstein condensation.

With simply an ideal electron(hole) gas in chemical equilibrium with an ideal gas of Cooper pairs, we are able to explain Uemura’s universal dependence of $T_C$ on the superfluid density [4].

2. The Boson-Fermion Model

The boson-fermion model was proposed independently in Ref. 5 and in Ref. 3. The grand Hamiltonian of the system reads

$$H = \sum_{\vec{p}, s} \epsilon_{\vec{p}} a_{\vec{p}s}^\dagger a_{\vec{p}s} + \sum_{\vec{p}} \omega_{\vec{p}} b_{\vec{p}}^\dagger b_{\vec{p}}$$

$$+ \frac{1}{\sqrt{\Omega}} \sum_{\vec{p}, \vec{q}} g_{\vec{p}, \vec{q}} (b_{\vec{p}} a_{\vec{p}+\vec{q}s}^\dagger a_{\vec{p}-\vec{q}s}^\dagger + b_{\vec{p}}^\dagger c_{\vec{p}} a_{\vec{p}+\vec{q}s} a_{\vec{p}-\vec{q}s}^\dagger),$$

(2.1)

where $a_{\vec{p}s}$, $a_{\vec{p}s}^\dagger$, $b_{\vec{p}}$ and $b_{\vec{p}}^\dagger$ are annihilation and creation operators of electrons(holes) and bosons, the subscript $s$ denotes the spin orientation, and $\Omega$ is the total volume of the system. The momentum dependence of the coupling $g_{\vec{p}, \vec{q}}$ will determine the pairing symmetry, e.g., $g = \text{const} \rightarrow s$-wave
and \( g \propto q_z^2 - q_y^2 \rightarrow d\)-wave. To illustrate the main physics, we assume a 3D isotropic jellium in which

\[
\epsilon_{\vec{p}} = \frac{p^2}{2m_f} - \mu
\]  

(2.2)

and

\[
\omega_{\vec{p}} = \frac{p^2}{2m_b} + 2(\nu - \mu)
\]  

(2.3)

and a constant \( g \), where \( m_f(m_b) \) is the effective mass of fermions (bosons), \( \mu \) is the chemical potential of the system, \( 2\nu \) is the energy of a static boson relative to two static fermions \( (2\nu = \omega_{\vec{p}}|_{\vec{p}=0} - 2\epsilon_{\vec{p}}|_{\vec{p}=0}) \).

The conserved electric charge number is given by

\[
Q = \sum_{\vec{p},s} a^\dagger_{\vec{p}s} a_{\vec{p}s} + 2 \sum_{\vec{p}} b^\dagger_{\vec{p}} b_{\vec{p}}.
\]  

(2.4)

Because of the strong Coulomb repulsion, we assume that \( \nu > 0 \) and the bosons are resonances. A dimensionless coupling can be defined as the ratio of the boson half width in vacuum, \( \Gamma/2 \), to the energy \( 2\nu \), i.e.

\[
g^2 = \Gamma^2 = \frac{g^2}{2\nu} = \frac{g^2}{\pi} \left( \frac{m_f}{2} \right)^{\frac{3}{2}} \frac{1}{\sqrt{\nu}},
\]  

(2.5)

which serves as an effective expansion parameter.

The physics of the model at weak coupling \( \hat{g} \ll 1 \) and \( T = 0 \) is determined by the total number density of charges,

\[
n = n_f + 2n_b
\]  

(2.6)

in relation to the characteristic density

\[
n_{\nu} = \frac{(2m_f\nu)^{\frac{3}{2}}}{3\pi^2},
\]  

(2.7)

where \( n_f \) is the number density of fermions and \( n_b \) that of bosons. \( n_{\nu} \) corresponds to a filled Fermi sea with Fermi energy \( \nu \). If \( n < n_{\nu} \), the electrons (holes) will fill in the Fermi levels below \( \nu \) and there can only be virtual bosons through interaction condensing at the zero momentum bosonic level. The long range order in this case is of BCS type. If \( n > n_{\nu} \), the excess electrons (holes) after filling up the Fermi level \( \nu \) prefer to combine into bosons which condensing at the zero momentum level and the long range order is of Bose-Einstein type.
The grand partition function of the model (2.1) is

\[ Q = \text{Tr} e^{-\beta H} \]  

(2.8)

and the thermal average of an operator \( O \) is defined to be

\[ < O > = \frac{\text{Tr} O e^{-\beta H}}{\text{Tr} e^{-\beta H}}. \]

(2.9)

With a Bose condensate

\[ B = \frac{1}{\sqrt{\Omega}} < b_{\vec{p}=0} >, \]

(2.10)

we may substitute

\[ b_{\vec{p}} = \sqrt{\Omega} B \delta_{\vec{p},0} + \beta_{\vec{p}} \]

(2.11)

into the Hamiltonian (2.1) and obtain \( H = H_0 + H_1 \), with

\[
H_0 = 2\Omega(\nu - \mu)B^2 + \sum_{\vec{p},s} \epsilon_{\vec{p}} a_{\vec{p}s}^{\dagger} a_{\vec{p}s} + \sum_{\vec{p}} \omega_{\vec{p}} \beta_{\vec{p}}^{\dagger} \beta_{\vec{p}} + gB \sum_{\vec{q}} (a_{\vec{q}\uparrow}^{\dagger} a_{-\vec{q}\uparrow} + a_{-\vec{q}\downarrow} a_{\vec{q}\uparrow})
\]

(2.12)

and \( H_1 \) the rest of the terms of (2.1) which are of higher orders at weak coupling. \( B \) has been chosen real. The Hamiltonian (2.12) can be easily diagonalized through a Bogoliubov transformation and yields a fermionic spectrum

\[ E_{\vec{p}} = \sqrt{\epsilon_{\vec{p}}^2 + \Delta^2} \]

(2.13)

with a gap energy \( \Delta = gB \). The thermodynamic potential within this approximation reads

\[ \ln Q = 2\beta(\mu - \nu)B^2\Omega + 2 \sum_{\vec{p}} \ln(1 + e^{-\beta E_{\vec{p}}}) - \sum_{\vec{p}} \ln(1 - e^{-\beta(\omega_{\vec{p}} + 2\nu - 2\mu)}). \]

(2.14)

The thermodynamical equilibrium corresponds to the maximum of \( \ln Q \) at fixed \( \beta \) and \( \mu \), i.e.

\[ \left( \frac{\partial \ln Q}{\partial B^2} \right)_{\beta,\mu} = 0 \]

(2.15)
\[
\left( \frac{\partial^2 \ln Q}{(\partial B^2)^2} \right)_{\beta,\mu} \leq 0. \tag{2.16}
\]

Combining (2.15) and the relation
\[
n = \frac{1}{\Omega} \left( \frac{\partial \ln Q}{\partial \mu} \right)_{\beta,B}, \tag{2.17}
\]
we can solve for the gap energy \( \Delta(T) \) and the transition temperature \( T_C \) in terms of the density \( n \).

At low density, \( n < n_\nu \), we find that
\[
\Delta(0) = 8\mu \exp \left( -2 - \frac{\nu - \mu}{\hat{g}^2 \sqrt{\nu \mu}} \right) \tag{2.18}
\]
with \( \mu \simeq \frac{(3\pi^2 n)^{3/2}}{2m} \) and that
\[
\frac{\Delta(0)}{k_B T_C} = \pi e^\gamma \tag{2.19}
\]
with \( \gamma = 0.5772... \) the Euler constant. At higher density, \( n > n_\nu \), we obtain that
\[
k_B T_C = 2\pi m_b \left( \frac{n - n_\nu}{2\zeta(3/2)} \right)^{2/3} \tag{2.20}
\]
with \( \zeta(3/2) = 2.612... \) and that
\[
\Delta(T) = \Delta(0) \sqrt{1 - \left( \frac{T}{T_C} \right)^{2/3}} \tag{2.21}
\]
with \( \Delta(0) = g \sqrt{(n - n_\nu)/2} \). Therefore, the phenomenological model (2.1) provides a simple interpolation between a BCS condensation and a Bose-Einstein condensation.

3. The Pseudo-gap

A remarkable phenomenon of under-doped cuprates is the pseudo gap in their electron(hole) spectrum above \( T_C \) [6]. It has been found numerically in the boson-fermion model [7]. Here I shall present an analytical calculation [8], which highlight the importance of the dimensionality of the system.
Starting with the retarded electron propagator

\[ S_R(p_0, \vec{p}) = -i \int_0^{\infty} dt e^{ip_0t} \langle \{ a_{\vec{p}}(t), a_{\vec{p}}^\dagger(0) \} \rangle, \]  

(3.1)

where the time development of \( a_{\vec{p}}(t) \) and its conjugate follows from the total Hamiltonian (2.1), the spectral function \( A(p_0, \vec{p}) \) is given by its imaginary, i.e.

\[ A(p_0, \vec{p}) = \frac{1}{\pi} \text{Im} S_R(p_0, \vec{p}), \]  

(3.2)

The function \( A(p_0, \vec{p}) \) gives rise to the probability density of a single electron (hole) excitation at energy \( p_0 \) and momentum \( \vec{p} \) and can be measured directly by ARPES. For a BCS superconductor, \( A(p_0, \vec{p}) \) is peaked at the quasi particle pole \( p_0 = \epsilon_\vec{p} \) in the normal phase and at \( p_0 = \pm \sqrt{\epsilon_\vec{p}^2 + \Delta^2} \) below \( T_C \).

1). Two-dimensions: This case simulates the actual cuprate materials above \( T_C \), which consist of a set of parallel \( CuO_2 \) layers. As long as \( T \) is not too close to \( T_C \), the interlayer hopping can be neglected. With a jellium approximation, \( \epsilon_\vec{p} \) and \( \omega_\vec{p} \) remain given by (2.2) and (2.3) but with a 2D momentum \( \vec{p} \). On writing

\[ S_R(p_0, \vec{p}) = \frac{1}{p_0 - \epsilon_\vec{p} - \Sigma_R(p_0, \vec{p})}, \]  

(3.3)

the retarded self-energy function, \( \Sigma_R(p_0, \vec{p}) \), is given to the one-loop order by

\[ \Sigma_R(p_0, \vec{p}) = g^2 \int \frac{d^2 \vec{q}}{(2\pi)^2} \frac{N_b(\vec{q}) + N_f(\vec{q} - \vec{p})}{p_0 - \omega_\vec{q} + \epsilon_{\vec{q} - \vec{p}} + i0^+} \]  

(3.4)

with

\[ N_b(\vec{p}) = \frac{1}{e^{\beta \omega_\vec{p}} - 1} \]  

(3.5)

and

\[ N_f(\vec{p}) = \frac{1}{e^{\beta \epsilon_\vec{p}} + 1} \]  

(3.6)

respectively. There is no Bose-Einstein condensation in a truly 2D system, the bosonic chemical potential \( 2(\mu - \nu) \equiv -\delta \) vanishes at \( T = 0 \) and the integral (3.4) diverges logarithmically. For an approximate 2D system and with a fixed carrier density, \( \delta \ll k_B T \) within a considerable range of \( T \ll \nu/k_B \) above \( T_C \) [8], and the integral (3.4) is dominated at small \( \vec{q} \). This gives rise to a crude estimate that

\[ \Sigma_R(p_0, \vec{p}) \sim \frac{\bar{\Delta}^2}{p_0 + \epsilon_\vec{p}} \]  

(3.7)
with
\[
\bar{\Delta}^2 = \frac{2r}{\pi} \tilde{g}^2 \nu k_B T \ln \frac{k_B T}{\delta}.
\] (3.8)
and \(r = m_b/m_f\), which leads to a perfect gap. A more refined calculation of the integration (3.4) was performed in [8] for \(p_0 \sim k_B T\) with the result at \(p = p_F\) (Fermi momentum):
\[
A(p_0, \vec{p})|_{p=p_F} = \frac{1}{\pi} \text{Im} \left( \frac{1}{p_0 - u(p_0) - iu(p_0) + u(\delta)} \right),
\] (3.9)
where
\[
u(p_0) = \frac{2r}{\pi} \tilde{g}^2 \nu \left[ \frac{k_B T \text{sign}(p_0 - \delta)}{\sqrt{4r \mu \delta + (p_0 - \delta)^2}} \ln \frac{\sqrt{4r \mu \delta + (p_0 - \delta)^2} + |p_0 - \delta|}{\sqrt{4r \mu \delta + (p_0 - \delta)^2} - |p_0 - \delta|} \right]
- \frac{\ln r}{r - 1} + \frac{1}{2} f(e^{-\beta(p_0-\delta)}) \sqrt{\frac{\pi k_B T}{r \mu}}
\] (3.10)
and
\[
\nu(p_0) = -\frac{2r}{\pi} \tilde{g}^2 \nu \left[ \frac{\pi k_B T}{\sqrt{4r \mu \delta + (p_0 - \delta)^2}} + \frac{1}{2} \left( f(e^{\beta(p_0-\delta)}) + \zeta(\frac{1}{2}) \right) \sqrt{\frac{\pi k_B T}{r \mu}} \right]
\] (3.11)
with
\[
f(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} dx \sqrt{x} \frac{ze^{-x}}{(1 + ze^{-x})^2}
\] (3.12)
and \(\zeta(1/2) = -1.4604\ldots\). The function (3.9) is plotted in Fig. 1.

The argument based on the logarithmic divergence can be generalized to higher orders in \(g\) and a Borel summation of the most divergent diagrams yields [8]:
\[
A(p_0, \vec{p})|_{p=p_F} = \frac{|p_0|}{\Delta^2} e^{-\frac{p_0^2}{\Delta^2}}.
\] (3.13)
It consists of two peaks at \(p_0 = \pm \bar{\Delta}/\sqrt{2}\) and a depletion of states at the Fermi level \(p_0 = 0\).

2). Three dimensions: With three dimensional \(\vec{p}\) and \(\vec{q}\), the simple argument based on the logarithmic divergence does not work since the integral (3.4) converges as \(\delta \to 0\). The evaluation of the integral for 3D case leads to
\[
\Sigma(p_0, \vec{p})|_{p=p_F} = \frac{\pi r}{4} \tilde{g}^2 k_B T + \frac{r}{\pi} \tilde{g}^2 \mu \int_{0}^{\infty} dx \frac{x}{e^{\beta(x^2-1)} + 1}
\]
The spectral function at $p = p_F$, with $\beta \mu = 10$, $r = 2$, $\ln k_B T / \delta = 8$ and $\hat{g}^2 = \frac{\pi}{80}$ for $D = 2$ (solid line) and $D = 3$ (dashed line), where $x = p_0 / k_B T$ and $y = A(p_0, \vec{p})|_{p=pF} k_B T$.

$$-rac{i r \hat{g}^2 k_B T}{4} \left[ \ln \frac{k_B T}{\delta + \frac{(p_0 - \delta)^2}{4 \mu}} + 2 \ln (1 + e^{\beta (p_0 - \delta)}) \right].$$

The corresponding $A(p_0, \vec{p})$ function is also plotted in Fig. 1 for comparison and the pseudo-gap disappears. Since the 3D calculation is only to the one-loop order, this is not in contradiction with a weak pseudo-gap, as inferred from the numerical solutions of truncated Dyson-Schwinger equations [9].

The dependence of the pseudo-gap can be tested by changing the separation between CuO$_2$ layers, or by comparison with 3D strongly correlated superconductors, say fullerenes perhaps.

4. DC Transport Coefficients

On the normal phase DC transport coefficients (resistivity, Hall number and magnetoresistance), lies perhaps, the most severe disagreement between the observation and the conventional Fermi liquid theory. A large number of measurements converges to a universal temperature dependence above $T_C$ for all cuprates [10]. The resistivity $\rho$ depends linearly on $T$ in contrast with the $T^2$ behavior of a good metal. The Hall number $n_H$, instead of being a constant which equals to the actual carrier density in case of a good metal, increases linearly with $T$ and reaches a value about a factor two of the chemically determined carrier density. The magnetoresistance, $\Delta \rho / \rho$ being
proportional to $T^{-n}$ ($n = 3.5 - 4$) is at entirely variance with the Kholer’s law (which says that the ratio $\frac{\delta\rho}{\rho} / B^2$ is temperature independent) for good metals.

In the boson-fermion model, there are two kinds of charge carriers above $T_C$: electrons or holes (fermions) and uncondensed Cooper pairs (bosons). If the lifetime of the latter is longer than the relaxation time of the electric current, both of them contribute to the DC conductivity. A Green’s function calculation is, however, very difficult since the simplifications based on incoherent superposition of quasi elastic collisions fail for bosons. Instead, we ask the following question [11]: Given a chemical equilibrium of fermions and bosons ($g = 0$ in (2.1)) and a simple power dependence on $T$ for the relaxation time of each kind of carriers, Is it possible to produce a reasonable fit of available experimental data?

Consider the situation with an electric field $\vec{E}$ parallel to the $CuO_2$ plane and a magnetic field $\vec{B} = B\hat{z}$ perpendicular to it, the electric current density is given by [11]

$$J_a = \sigma_{ab}(\vec{B})E_b$$

(4.1)

with $a, b = x, y$. The Taylor expansion in $B$ of the conductivity tensor reads

$$\sigma_{ab}(\vec{B}) = (\sigma + \Delta\sigma)\delta_{ab} + \sigma_H\epsilon_{ab} + O(B^3),$$

(4.2)

where

$$\sigma = \left(\frac{\tau_f n_f}{m_f} + 4\frac{\tau_b n_b}{m_b}\right)e^2,$$

(4.3)

is the zero field conductivity,

$$\sigma_H = \left(n\frac{\tau_f^2 n_f}{m_f^2} + 8\frac{\tau_b^2 n_b}{m_b^2}\right)e^3B$$

(4.4)

is the Hall conductivity and

$$\Delta\sigma = \left(n\frac{\tau_f^3 n_f}{m_f^3} + 8\frac{\tau_b^3 n_b}{m_b^3}\right)e^4B^2,$$

(4.5)

with $n_f$ and $n_b$ the number densities of each type of carriers, $\tau_f$ and $\tau_b$ the corresponding relaxation times, and $m_f$ and $m_b$ the corresponding effective masses. We have assumed a parabolic band for bosons and a nonparabolic band for fermions. In case of the nonparabolic band, the effective mass $m_f$ is defined by

$$\frac{1}{m_f} = \frac{1}{2n_f} \int \frac{d^2\vec{k}}{(2\pi)^2} v_k^2 \delta(\epsilon_k)$$

(4.6)
and the band correction factors $\eta$ and $\eta'$ of (4.4) and (4.5) are given by

$$
\eta = \frac{m_f^2}{2n_f} \epsilon_{ab} \epsilon_{ij} \int \frac{d^2 \vec{k}}{(2\pi)^2} v_a v_i \nabla_j v_b \delta(\epsilon_{\vec{k}})
$$

(4.7)

and

$$
\eta' = \frac{m_b^3}{2n_f} \epsilon_{ab} \epsilon_{ij} \int \frac{d^2 \vec{k}}{(2\pi)^2} v_a \nabla_b v_i \nabla_j v_l \delta(\epsilon_{\vec{k}}).
$$

(4.8)

It follows from (4.3)-(4.5) that the DC resistivity

$$
\rho = \frac{1}{e^2} \left( \frac{\tau_f n_f}{m_f} + 4 \frac{\tau_b n_b}{m_b} \right)^{-1},
$$

(4.9)

the Hall number

$$
n_H = \frac{\sigma^2}{e \sigma_H} B = \frac{\left( \frac{\tau_f n_f}{m_f} + 4 \frac{\tau_b n_b}{m_b} \right)^2}{\eta' \frac{\tau_f n_f}{m_f} + 8 \frac{\tau_b^2 n_b}{m_b}}
$$

(4.10)

and the Kholer’s ratio

$$
K = -\rho^3 \frac{\Delta \sigma + \rho \sigma_H^2}{B^2} = \frac{e^4 \tau_f n_f \rho^3}{m_f} \left[ (\eta' - \eta^2) \frac{\tau_f^2}{m_f} + 4 e^2 \tau_b n_b \left( \eta \frac{\tau_f}{m_f} - \frac{2 \tau_b}{m_b} \right)^2 \rho \right].
$$

(4.11)

The approximate agreement with Luttinger theorem as observed by photoemission ruled out any issue based on a large fraction of bosons and we expect that $n_b << n_f$. The temperature dependence of $n_f$ and $n_b$ can be neglected. The DC resistivity is dominated by fermions and we have

$$
\tau_f \propto T^{-1}.
$$

(4.12)

An explanation of this non-Fermi liquid behavior is suggested in [12]. If the factor $\eta$ is sufficiently small such that the denominator of (4.10) is dominated by the second term, a Hall number considerably larger than the actual carrier density emerges and the linear temperature dependence can be obtained if we assume

$$
\tau_b \propto T^{-1.5}.
$$

(4.13)

With both (4.12) and (4.13), the Kholer’s ratio would be proportional to $T^{-1.5}$ if the factor $\eta'$ is also sufficiently small. For a nonparabolic band, it is possible to have small $\eta$ since it switches sign from a particle Fermi sea to a hole Fermi sea. But a small $\eta'$ may not be easy to tune since its integrand is positive definite. We are still not in the position to claim the triumph of the boson-fermion model in explaining the DC transport coefficients.
5. Two Theoretical Issues

To justify the boson-fermion model as an adequate low density phenomenological model of certain strongly correlated electronic systems, the following issues have to be settled.

5.1 Fermi distribution function

Consider a purely electronic system on a lattice with electronic operators \( \alpha_{\vec{p},s} \) and \( \alpha_{\vec{p},s}^\dagger \) in the momentum representation. The Fermi distribution function is defined as

\[
n_{\vec{p}} = \sum_s < |\alpha_{\vec{p},s}^\dagger \alpha_{\vec{p},s}| >
\]  

(5.1)

with \( | > \) the ground state of the system. With the Bogoliubov type of trial state [13]

\[
| > = \exp \left[ \sum_{\vec{p}} \theta_{\vec{p}} (\alpha_{-\vec{p},\downarrow} \alpha_{\vec{p},\uparrow} - \alpha_{\vec{p},\uparrow}^\dagger \alpha_{-\vec{p},\downarrow}^\dagger) \right] |0 >,
\]

(5.2)

we find that

\[
n_{\vec{p}} = 2 \sin^2 \theta_{\vec{p}},
\]

(5.3)

For a BCS ground state, we have

\[
\cos 2\theta_{\vec{p}} = \frac{\epsilon_{\vec{p}}}{\sqrt{\epsilon_{\vec{p}}^2 + \Delta^2}}
\]

(5.4)

and

\[
\sin 2\theta_{\vec{p}} = \frac{\Delta}{\sqrt{\epsilon_{\vec{p}}^2 + \Delta^2}}
\]

(5.5)

with \( \Delta << \epsilon_F \) and the corresponding \( n_{\vec{p}} \) is slightly smeared by \( \Delta \) from that of an ideal Fermi gas. The location of the kink (Fermi surface when \( \Delta = 0 \)) is determined by Luttinger theorem.

For a Hubbard model with a strong on-site attraction, the ground state is the Bose condensate of local pairs. This corresponds to a \( \vec{p} \) independent \( \theta_{\vec{p}} \) and the corresponding Fermi distribution function is a plateau within the Brillouin zone.

In case of the boson-fermion model, the distribution function corresponds to an different interpolation from that in [13] between the former two cases. The \( n_{\vec{p}} \) profile is the superposition of the BCS case with the Fermi level retreated towards the bottom of the band and with a plateau outside the Fermi sea.

The Fermi distribution function for the three different cases is shown schematically in Fig. 2. The profile in Fig. 2c may be generated with
5.2 The pole trajectory underneath the physical sheet at $T > T_C$

In the BCS model, the pairing instability would be triggered at the absence of a long range order by a pair of complex conjugate poles of the two fermion scattering amplitude below $T_C$ on the energy $E$-plane. For $T > T_C$, they slip through the cut at $E = 2\epsilon_F$ to the unphysical sheets below and leave the real axis vertically with an increasing $T$. Their effect becomes insignificant outside the critical region. On the other hand, for the local pair system the bound state pole stays always on the real $E$ axis for $T > T_C$. The pole trajectory for the boson-fermion model (2.1) (for which the pole of the scattering amplitude of two fermions coincides with the pole of the boson propagator ) has been calculated to the one-loop order. Without a Bose condensate, there would be a pair of complex conjugate poles triggering the instability for $T < T_C$. As $T$ being increased away from $T = T_C$, they emigrant to the unphysical sheet but leave the real axis obliquely with a tunable slope proportional to $g^2$ (Fig. 3). For small $g$, they stay close to the real $E$ for a wide range of temperature above $T_C$ and could make significant contributions to transport processes. Starting with a purely electronic Hamiltonian, such resonance boson poles can be implemented for a few body system [14]. But to reproduce the pole trajectory of the boson-fermion model for a many-body system remains an open problem.

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Figure 3  The pole trajectory on the complex energy plane of the two electron (hole) scattering amplitude for $T > T_C$. The dashed line is for the boson-fermion model and the dotted line is for BCS model. The arrows point to the direction corresponding to an increasing $T$. The heavy solid line represents the branch cut and the intersection point of all trajectories corresponds to $E = 2\epsilon_F$.

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