Transport of mixed ultracold atoms at the S and D band in a moving optical lattice

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The ultracold atoms in a moving optical lattice with its high controllability is a feasible platform to research the transport phenomenon. Here, we study the transport process of ultracold atoms at the D band in a one-dimensional optical lattice, and perform the manipulation of atomic transport by modulating the population proportion of S band and D band. In the experiment, we first load ultracold atoms into the optical lattice using shortcut method, and then accelerate the optical lattice by scanning the phase of lattice beams. The atomic transport in D band and S band is demonstrated respectively. We find the group velocity of atoms in D band is opposite to that in S band. By doping atoms of D band into that of S band, group velocity of the atomic superposition state is modulated, and the manipulation of atomic group velocity from positive to negative is realized. Furthermore, the influence of lattice depth and acceleration on the transport distance are studied. The calculations with multi-orbital simulation are coincident with the experimental results. Our work provides a useful method to manipulate the atomic transport in higher or mixed orbits.

I. INTRODUCTION

The transport phenomenon has been attracting tremendous efforts in recent years [1, 2], which occurs in a system of particles driven by external force and moving in a periodic potential [3, 4]. And transport process is studied in the areas of electronic materials [5, 6], trapping ions [7, 8], and ultracold atoms [9–11].

The ultracold atoms in optical lattices, due to its high controllability, is widely applied to simulating the physics of condensed matters [12–14]. For instance, novel physical phases [15–18] and dynamical mechanisms [19–22] of atomic superfluid in optical lattices are observed with different manipulation technologies [23, 24], and atomic circuits and atomic qubit are achieved in optical lattices [25, 26]. Among them, high orbital physics in optical lattices has attracted many attentions, like the reconfigurability of the P-band Bosons in hexagonal lattice [27, 28], the achievement of Ramsey interferometry between S band and D band [29], the observation of atomic scattering at D band [30], and the unconventional superfluid order at F band [31] et al. Furthermore, the optical lattice is also an effective platform to study transport phenomenon [32–35], such as the long-range transport [36–38], and the effect of Bosonic and Fermionic statistics on atomic transport [39]. However, there are few researches on the transport of atoms in the higher band and mixed band in an optical lattice.

Here, we report the observation of transport process in a system of ultracold atoms at D band in the moving one-dimensional optical lattice, and perform a transport manipulation of atomic superposition of S band and D band, which realizes the change of atomic group velocity from positive to negative. We use our previous shortcut method [23] to load atoms from the harmonic trap into the optical lattice with different atom population proportions of S band and D band within tens of microseconds. Then we perform the transport process of D band atoms in the moving one-dimensional optical lattice, and observe the opposite group velocity of S-band atoms and D-band atoms. Due to this different transport characteristic, doping D-band atoms into S-band atoms can modify the atomic group velocity obtained from moving optical lattice. After accelerating the optical lattice, atoms with different proportions of D-band atoms are observed to get different group velocities from positive to negative. Further, we study the influence of lattice depth and acceleration on the transport distance. Using the multi-orbital method [26], the atomic group velocities with different parameters are calculated, and the calculations agree with the experimental results. This work paves the way for the research of ultracold atom transport phenomena in the higher and mixed bands.

This paper is organized as follows. In Sec. II, we describe the transport process of S-band atoms and D-band atoms in the moving lattice, and then the method of numerical simulation is given. Our experimental setup and time sequence are represented in Sec. III A. In Sec. III B, we introduce the shortcut method used to load atoms into the mixed bands. The experimental results of transport process in a single band and the superposition of S-band and D-band are shown in Sec. IV A and Sec. IV B, respectively. And the influence of the lattice depth and lattice acceleration on the transport distance are studied in Sec. IV C. Finally, we give the discussion and conclusion in Sec. V.
II. TRANSPORT PROCESS IN MOVING OPTICAL LATTICE AND THEORETICAL SIMULATION

A. Transport process in moving optical lattice

We study the transport process of ultracold atoms in the moving one-dimensional optical lattice. As shown in Fig.1 (a), in the laboratory frame (Lab frame), the optical lattice is accelerated by acceleration $a_L$, and the atoms in the optical lattice are pushed by it. The Hamiltonian of the system is:

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} V_0 \cos(2k_L(x - \frac{1}{2}a_L t^2)), \quad (1)$$

where $\hat{p}$ is momentum operator, $m$ is the mass of the atom, $V_0$ is the lattice depth, $k_L = 2\pi/\lambda$ is the wave vector of the optical lattice, and $\lambda$ is wavelength of the optical lattice. The Hamiltonian can be considered as atoms applied by a fictitious force in the moving lattice frame. Hence, to analyze the motion of atoms, we first consider the atomic motion in the moving frame, and then transform the reference system to Lab frame to obtain the atomic group velocity.

In the moving frame, the optical lattice accelerated by $a_L$ creates a fictitious force $-ma_L$. When the acceleration $a_L$ is small enough, we can ignore Landau-Zener tunneling (LZT), and the atoms perform Bloch oscillation (BO). The Fig.1 (b) shows the motion of atoms at D band in quasi momentum space. Firstly, the atoms
distribute at center of the first Brillouin zone (BZ), and with the evolution time $t$ increasing, the quasi momentum $q$ is changed by the fictitious force. According to Bloch oscillation, when the atom is subjected to an external force $F$ (For the moving lattice $F = -ma_L t$), the quasi momentum $q(t)$ will increase as $[40, 41]$:

$$q(t) = F t = -ma_L t. \quad (2)$$

And the group velocity $v_g'(q(t))$ is decided by the dispersion relation $E_\alpha(q)$:

$$v_g'(q) = \frac{\hbar}{E_\alpha(q)} \frac{dE_\alpha(q)}{dq}. \quad (3)$$

where $\alpha$ represents the band and can be chosen as band $S, P, D, F, ...$. According to Eqs. (2) and (3), as shown in Fig. 1 (c), the change of group velocity over time $v_g'(t)$ for atoms in D band and S band is represented by the blue line and orange line, respectively.

When we return to the Lab frame, the velocity of optical lattice $v_L(t)$ needs to be added, and the group velocity of atoms $v_g(t)$ becomes:

$$v_g(t) = v_g'(t) + v_L(t). \quad (4)$$

Using Eq. (2), the velocity of lattice $v_L$ can be written as:

$$v_L = a_L t = -q/m. \quad (5)$$

In Fig. 1 (c), the dashed line marked the curve $-v_L = q/m$. From Fig. 1 (c), it can be found that the group velocity $v_g'(t)$ of S band and D band are on the opposite side of $q/m$. Hence, in the Lab frame, the atoms at S band and D band will acquire group velocity in the opposite direction. This result is different from the situation that atoms are driven by external forces in a static lattice, where the atoms at S band and D band will get group velocity in the same direction.

### B. Multi-orbital calculation method

To calculate the motion of atoms more accurately, the Landau-Zener tunneling could not be neglected [3, 42] which describes that the atoms jump to other bands during the Bloch oscillation, and we use the multi-orbital simulation method [26]. The multi-orbital simulation differentiates time to calculate the instantaneous evolution operator. By implementing all the instantaneous evolution operators to the initial state, the final state is calculated. For the moving lattice, we divide the time $t$ into $dt$, and project the instantaneous Hamiltonian into momentum state $|n\rangle = e^{2i\pi k_L x}$, where $x$ is the space coordinate position. Ignoring the interaction, the instantaneous Hamiltonian can be written as [26]:

$$H_{nn'} = \frac{\hbar^2 k_L^2}{2m} (2n)^2 \delta_{n,n'}$$

$$+ \frac{1}{4} V(t) \times (\delta_{n,n'+1} + \delta_{n,n'-1}),$$

where $n, n' = 0, \pm 1, \pm 2, ...$, and $V(t)$ is the potential energy changing with time. By solving eigen equation of the Hamiltonian, the instantaneous eigenvalue $\varepsilon_\alpha(t)$ is calculated, where $\alpha$ represents the Bloch band. By implementing all the instantaneous evolution operator $e^{-i\varepsilon_\alpha(t)dt/\hbar}$ to the initial state $|\alpha, n = 0\rangle$, the state after evolution time $t$ is gotten:

$$|\psi(t)\rangle = \prod_t e^{-i\varepsilon_\alpha(t)dt/\hbar} |\alpha, n = 0\rangle. \quad (7)$$

Using the state $|\psi(t)\rangle$, we calculate theoretical results of the group velocity $v_g(t)$ after time $t$:

$$v_g(t) = \langle \psi(t) | \hat{p} | \psi(t) \rangle, \quad (8)$$

where momentum operator could be written as $\hat{p} = \sum_n 2n\hbar k_L |n\rangle \langle n|$.}

### III. EXPERIMENTAL DESCRIPTION

#### A. Experimental setup and Sequence

Our experiment starts with a Bose Einstein condensate (BEC) of $^{87}$Rb with $2 \times 10^5$ atoms, which is prepared in a hybrid trap as our previous experiments described [27, 43]. After that, we use the shortcut sequence to load atoms into the one-dimensional optical lattice (the detailed description is in the following subsection), as shown in Fig. 2(a). The one-dimensional optical lattice is composed of an incident light beam and its reflected light, of which wavelength $\lambda$ is 1064 nm. To study the transport process, we make the lattice move with the acceleration $a_L$ for evolution time $t$. The acceleration is produced by an electro optic modulator (EOM, Thorlab EO-PM-NR-C2) between BEC and the reflector. Hence, the lattice potential $V_L(x)$ can be written as:

$$V_L(x) = \frac{1}{2} V_0 \cos(2k_L x - 2\varphi(t)), \quad (9)$$

Where the additional phase $\varphi(t)$ is added to the reflected beam using the EOM. By controlling the voltage of EOM, the phase $\varphi(t)$ can be scanned. Hence, to accelerate the optical lattice with $a_L$, as shown in Fig. 2(a), the phase $\varphi(t)$ should change as:

$$\varphi(t) = \frac{1}{2} a_L k_L t^2. \quad (10)$$

After the atoms evolve for time $t$ in the moving lattice, we turn off the lattice beam and take absorption imaging with the time of flight (TOF) $t_{TOF} = 32$ ms to detect the atoms.

Fig. 2(a) shows the experimental sequence, where time $t_1, t_2, t_3$ mark the stage of BEC, atoms loaded into the optical lattice, and after accelerating, respectively. Fig. 2(b) shows the typical images at these three moments. At $t_1$, atoms condensate at zero momentum state. After the
shortcut sequence at $t_2$, atoms symmetrically distribute at several momentum peaks. The distribution is decided by the proportion and phase of atoms at each band, but the integral of momentum at $t_2$ is always zero. After accelerating of optical lattice at $t_3$, the atoms move among momentum peaks, and the integral of momentum is a non-zero value. To deal with the absorption images, we fit the distance of two adjacent atom clouds to calibrate momentum $\hbar kL$ in the image, and by the calibration we can obtain momentum of each point in the images.

In the experiment, limited by EOM control voltage, the maximum reachable lattice phase is fixed. Hence, the low acceleration $a_L$ causes the change of quasi-momentum small, while the large acceleration leads to obvious Landau-Zener tunneling. Hence, we choose the acceleration $a_L$ around several times of gravitational acceleration, where the obvious transport process could be observed. One feasible method to solve the limit of EOM control voltage is to use additional EOMs.

![Image](image_url)

**FIG. 2. The experimental sequence and typical images.** (a) The experimental sequence. The horizontal axis shows the experimental time, and we divide it into several stages. The first stage is the preparation of BEC. In the second stage, the shortcut method is used to prepare the atoms into the aimed band of the optical lattice. The curve of lattice beam shows the shortcut pulses, and $t_{j}^{\text{on/ off}}$ represent the shortcut sequence. In the third stage, the optical lattice is accelerated by changing the optical lattice phase. Finally, after 32 ms time of flight, we take the absorption images to detect the atoms. $t_1, t_2, t_3$ mark the three typical moments: the BEC is prepared, the atoms are loaded into the optical lattice, and the atoms after accelerating. (b) shows the typical images according to the moment $t_1, t_2, t_3$. The above curves show the fitting of the images. The horizontal axis represents the momentum $p$.

**B. Shortcut method to load atoms into the aimed bands of optical lattice**

Now we describe how to load atoms into the different bands of an optical lattice. Here we use the shortcut method, which is a simple and efficient method to prepare atomic states [23]. The basic idea of the shortcut method is continuously turning on and off of the beam of lattice to modulate the atomic state, as shown in Fig. 2(a). In our experiment, the initial state $|\psi_i\rangle$ of BEC is the ground state of harmonic trap. After several shortcut pulses, the final state of atoms $|\psi_f\rangle$ can be written as [23, 30]:

$$|\psi_f\rangle = \prod_{j=1}^{n_p} U_{\text{off}}(t_{j}^{\text{off}})U_{\text{on}}(t_{j}^{\text{on}}) \times |\psi_i\rangle,$$

where $n_p$ is the number of shortcut pulses, $U_{\text{on/ off}}$ is the evolution operator of atomic state when the laser light is on (off), and $t_{j}^{\text{on/ off}}$ is the evolution time of the $j^{th}$ pulse.

Through well designing the shortcut sequence $t_{j}^{\text{on/ off}}$, we can prepare the BEC from the Harmonic Trap to our aimed state $|\psi_a\rangle$ with high fidelity. The fidelity $\eta$ describes the efficiency of state preparation and is defined as [23, 30]:

$$\eta = |\langle \psi_f | \psi_a \rangle|^2.$$

In the experiment, the aimed state $|\psi_a\rangle$ is the state at a single band or the superposition at band $|S\rangle$ and band $|D\rangle$ with zero quasi momentum:

$$|\psi_a\rangle = \gamma_1 |S\rangle + \gamma_2 |D\rangle,$$

where $|\gamma_1|^2 + |\gamma_2|^2 = 1$. Table. I shows the shortcut sequence to prepare the superposition used in the experiment with $V_0 = 10 E_r$, where $E_r = \hbar^2 k^2/2m$. The theoretical fidelity $\eta^T$ for each sequence is near 1, and $\eta^E$ is the experimental fidelity.

| $t_{1}^{\text{(us)}}$ | $t_{1}^{\text{off}}$ | $t_{2}^{\text{on}}$ | $t_{2}^{\text{off}}$ | $\gamma_1$ | $\gamma_2$ | $\eta^T$ | $\eta^E$ |
|------------------|-----------------|-----------------|-----------------|---------|---------|--------|--------|
| 10               | 32              | 26              | 10              | 1       | 1       | 0.9999 | 0.994 |
| 2                | 30              | 0               | 0               | $\sqrt{0.8}$ | $\sqrt{0.2}$ | 0.9994 | 0.974 |
| 14               | 47              | 11              | 34              | $\sqrt{0.6}$ | $\sqrt{0.4}$ | 0.9996 | 0.964 |
| 21               | 2               | 31              | 39              | $\sqrt{0.3}$ | $\sqrt{0.7}$ | 0.9994 | 0.956 |
| 22               | 43              | 61              | 39              | 0       | 1       | 0.9998 | 0.991 |

**IV. EXPERIMENTAL RESULTS**

**A. Transport process of atoms distributed at a single band**

Using the shortcut method and the moving lattice, the transport process of atoms at S band and D band with different evolution time $t$ are observed. Fig. 3 (a1) and (a2) show typical images of the results, where $V_0 = 10 E_r$, $a_L = 15 \text{ m/s}^2$. The longitudinal axis of Fig. 3 (a1) and (a2) represents the evolution time $t$ of atoms in the moving lattice. The horizontal axis represents the momentum $p$ of atoms, and the color represents atomic density. In the images, atoms distribute around quasi-momentum peaks $p = \pm 2\xi \hbar kL$ ($\xi = 0, \pm 1, \pm 2, ...$), and the number of atoms at each momentum state changes over time. It is because that in moving frame the velocity of each momentum
FIG. 3. The transport process of atoms in S band and D band. (a1) and (a2) are the absorption images of the transport process of S-band atoms and D-band atoms. The horizontal axis represents the momentum \( p \), and the longitudinal axis \( t \) is the evolution time of atoms accelerated in the optical lattice. The color marks the density of atoms. (b) shows the change of group velocity \( v_g \) of D-band atoms and S-band atoms over time. The circles are the experimental data, and the dashed lines are the theoretical simulation. The errorbar is the standard error of five measurements. The parameters in the figure: \( V_0 = 10 \, E_r \), \( a_L = 15 \, m/s^2 \).

state increases as \( q/m \), while in Lab frame the velocity becomes \( q/m + a_L t = 0 \). Hence, the center of each momentum peak remains unchanged, and the change of group velocity reflects in transfer of atom population at each momentum peak.

For the atoms at S band, initially, atoms mainly distribute in the central zero-order momentum peak, and a part of atoms are symmetrically distributed in the positive and negative first-order momentum peaks, as shown in Fig. 3 (a1). With the evolution time \( t \) increasing, the atoms at S band gradually move towards the positive momentum peak in time range of the experiment. The scattering halos between two momentum peaks may be from the collisions of different momentum components [44]. By calculating the average momentum of all the atoms, the group velocity of atoms at S band is gotten, as the orange data in Fig. 3(b) shows. The \( v_g \) of S-band atoms rises with small oscillations in the range of experimental time, which agrees with the theoretical dashed line.

For the atoms at D band, initially, atoms symmetrically distribute in the three central momentum peaks, as shown in Fig. 3 (a2). The atoms distributed in the positive and negative first-order momentum peaks are much more than that of S band, which is due to the higher energy of atoms in D band. With the evolution time \( t \) increasing, the atoms at D band gradually move towards the negative momentum peak. By the same method as that for S-band atoms, we calculate the group velocity of atoms at D band. The group velocity of D-band atoms changes from zero to negative and then gradually oscillates over time, as shown in the blue points in Fig. 3(b). That is consistent with the theoretical analysis that the group velocity of D-band atoms obtained from the moving lattice is opposite to that of S-band atoms.

B. Transport of atoms with the superposition states

Because the group velocity of S-band atoms and D-band atoms obtained in the moving lattice are opposite, by doping the D-band atoms into the S-band atoms, the group velocity of atoms obtained from the accelerating optical lattice would gradually change from positive to negative. The proportion of D-band atoms \( P_d \) is defined as:

\[
P_d = \frac{N_d}{N}, \tag{14}
\]

where \( N_d \) is the number of D-band atoms, and \( N \) is the sum of atom number.

Using the shortcut method, we could design the sequence to load atoms into the optical lattice with differ-
ent atom proportions. Fig. 4 shows the process of group velocity changing with different proportions of D-band atoms $P_d$ in the moving optical lattice, where dashed lines are the theoretical simulations by the multi-orbital simulation method. The top figure of Fig. 4 shows the transport process of $P_d = 0.2$. The group velocity gradually rises, and due to the doped D-band atoms, the increasing rate of group velocity over time is slower than atoms at S band. With $P_d$ increased, the group velocity of atoms gradually increases in the negative direction. As the points with $P_d = 0.4$ in Fig. 4 show, the group velocity of atoms gradually oscillates around zero. When $P_d$ reaches 0.7, as shown in the bottom figure of Fig. 4, the change of atomic group velocity, like the D-band atoms, first increases from zero to negative and then gradually oscillates over time. The oscillations of atomic superposition at S band and D band are more obvious than the oscillations of atomic group velocity, like the D-band atoms, which restrains the change of group velocity. Hence, the increasing rate of $S_a$ with $P_d$ reduces when $a_L$ increases, as shown in Fig. 4.

C. The change of transport distance with the different lattice depths and accelerations

Further, we study the influence of the lattice depth $V_0$ and lattice acceleration $a_L$ on the transport process. To display the result, we define the atomic transport distance $S_a$ with the group velocity $v_g(t)$:

$$S_a = \int_{t=0}^{T_B/2} v_g(t)dt,$$

where $T_B$ is the Bloch oscillation period, for $a = 15 \text{ m/s}^2$, $T_B/2 = 287 \mu s$. In the experiment, we sum the product of measured group velocity and measured time interval to calculate the transport distance. Fig. 5(a) shows the distance $S_a$ of atoms at S band and D band with different lattice depth $V_0$, where $a_L = 15 \text{ m/s}^2$. The selection of lattice depth $V_0$ is to keep the atoms in superfluid. The dashed lines represent theoretical calculation by the multi-orbital method, and $d_L = \lambda/2$ is lattice constant of the one-dimensional optical lattice. With the different lattice depth $V_0$, the transport distance $S_a$ of S-band atoms are almost unchanged. However, the transport process of atoms at D band is more sensitive to the lattice depth compared to that of atoms at S band. It is because that $v'_{g,S}$ of atoms at S band is much lower than $-v_{L,S}$, as shown in Fig. 1 (c). So the $v_g$ of S-band atoms is dominated by $v_L$, which is independent with lattice depth $V_0$, and the $S_a$ of S-band atoms are almost unchanged with $V_0$ increased. But, for atoms at D band, $v'_{g,D}$ and $v_L$ are comparable, as shown in Fig. 1 (c), so $v_g$ changes obviously with $V_0$ increased.

Fig. 5(b) shows the relationship between transport distance $S_a$ and proportion of D-band atoms $P_d$ with different lattice depth $V_0$ and acceleration $a_L$, where the dashed lines mark the simulation results. With $P_d$ increased, the $S_a$ linearly increases from positive to negative. The increasing rate of $10 \text{ E}_r$ is larger than $5 \text{ E}_r$ and $15 \text{ E}_r$, which is in accord with the trend of $S_a$ in Fig.5(a). With the acceleration $a_L$ increased, the Landau-Zener tunneling increases. The atoms tunneling to other bands would remain at the same momentum peak in Lab frame, which restrains the change of group velocity. Hence, the increasing rate of $S_a$ with $P_d$ reduces when $a_L$ increases, as shown in Fig. 5(b).

FIG. 5. Atomic transport distance $S_a$ with the different lattice depth $V_0$ and lattice acceleration $a_L$. In (a), the blue points and the orange points represent the transport distance $S_a$ for the atoms in the D band and S band, respectively, with the lattice depth $V_0=5,8,10,12,15 \text{ E}_r$. The dashed lines are the theoretical simulations. (b) The green, blue and yellow data show the transport distance $S_a$ with D-band atom proportion $P_d$ for the different $V_0 = 5, 10, 15 \text{ E}_r$ when $a_L = 15 \text{ m/s}^2$, respectively, and the purple data shows that for $V_0 = 10 \text{ E}_r$ and $a_L = 30 \text{ m/s}^2$. The dashed lines are the simulation results. The errorbar of each point is the standard error of five measurements.

V. DISCUSSION AND CONCLUSION

The discrepancy between experimental results and theoretical simulations comes from several aspects. Firstly, in the experiment, the error of the lattice depth calibration is less than 5%, and the error of acceleration calibration is about 2%. Secondly, the experimental fidelity of shortcut method could not reach 1, so there is an error in the D-band atom proportion. Moreover, in the multi-orbital simulation, we ignore the interaction and the momentum width of BEC, which also influence the precision of the simulation [3].

As for the high bands, we only study the transport process of atoms at D band, and achieve group velocity modulating process using the S-band and D-band atoms. Besides, the transport process of P band in the moving lattice is also different from the atoms in a static lattice. The group velocity $v'_g$ of P band in moving frame is on the same side of the $-v_L = q/m$ as the S band. So in the laboratory frame, the atoms at S band and P band can acquire the group velocity in the same direc-
in the experiment, the phase-shift shortcut method [43, 45] could be used to load atoms into the P band of one-dimensional optical lattice to demonstrate deduction. Furthermore, using the shortcut method [23], the sequence to load atoms into mixed bands of 2D or 3D optical lattices could be designed, which can extend this protocol to two or three dimensions.

In summary, we observe the opposite group velocity of D-band and S-band atoms in the one-dimensional accelerating optical lattice. Using the shortcut method, we design the sequence to load atoms into the optical lattice with different atom proportions of D-band and S-band, and perform a transport manipulating process of mixed atoms at S band and D band by changing the atom proportion. Furthermore, the influence of lattice depth and acceleration on the transport distance are studied. We use the multi-orbital simulation method to calculate the transport process, and this calculation agrees with the experimental results. The transport process of D-band atoms could contribute to the understanding of transport phenomenon in high bands, and the transport manipulation could be used to detect topological properties [46].

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