Investigating the Existence of Chaos in Inflation Data in relation to Chaotic Foreign Exchange Rate

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Foreign exchange (ForEx) rates are amongst the most important economic indices in the international monetary markets. ForEx rate represents the value of one currency in another and it fluctuates over time. It is related to indicators like inflation, interest rate, gross domestic product, and so forth. In a series of works, we investigated and confirmed the chaotic property of ForEx rates by finding positive largest Lyapunov exponent (LLE). As inflation influences ForEx, in this work we would like to address the specific question, Is inflation data also chaotic? We collected data for time period of 2000 to 2013 and tested for nonlinearity in data by surrogate method. Calculating LLE, we find existence of chaos in inflation data for some countries.

1. Introduction

An exchange rate represents the value of one currency in another. An exchange rate between two currencies fluctuates over time. The foreign exchange market is the largest and most liquid of the financial markets. Foreign exchange rates are amongst the most important economic indices in the international monetary markets. As floating exchange rate system was implemented by industrialized countries in 1973, they are no more determined almost solely by balance of payments as in the fixed rate regime. Foreign exchange rates are affected by many highly correlated economic, political, and even psychological factors. The interaction of these factors is in a very complex fashion [1].

Ever since the breakdown of (linear) structural exchange model, the dominant view of exchange rate dynamics has been used on the “news” model. It was observed that changes in the exchange rate are related to news in the fundamentals. Set of fundamentals covers (i) the inflation for the country concerned, (ii) the money supply for the country under scrutiny, (iii) the Money Market Rate, which is used as a measure of the short term interest rate, (iv) the lending rate and the long-term government bond yield which are both proxies of the long-term interest rate, and the latter was, however, only available for the low inflation countries, (v) industrial production, and (vi) the trade balance relative to the GDP. Characterizing the nature of the relationship between exchange rate changes and the news in its underlying fundamentals has long been an objective of empirical international macroeconomics [2]. In an analysis of high frequency exchange data, Goodhart [3] documents that very often the exchange rate does not respond to observable news and that many exchange rate movements cannot be associated with the news. This empirical research suggests that, in addition to random shocks, there are other driving forces in the exchange market that are important to understanding its dynamics [4]. Most of the countries are buying the US dollar in order to curb the appreciation of their currencies [5].

There is no clear relationship among the factors as well as their collective relationship to determine ForEx rate of a country. From the complex viewpoint of its dependencies on different factors and adding the most difficult factor of speculation in this huge market, ForEx is almost impossible to predict. Nonlinearity of speculative dynamics leads to chaotic motion of the exchange rate as mentioned before. Bask considered Swedish Kroner versus Deutsche Mark, ECU, US$, and Yen in his study using data of daily observation and found indication of deterministic chaos in all exchange rate
series [6, 7]. In a series of works (2007, 2012, and 2013), we investigated and confirmed the chaotic property of foreign exchange rates of several countries [8–11]. Chaotic processes are characterized by positive Lyapunov Exponents (LEs) and we calculated LEs from ForEx data.

Economic indicators are snippets of financial and economic data published regularly by governmental agencies and the private sector. Traders can measure the economic health of a given country (and its currency) through its economic indicators. These statistics help market observers monitor the economy’s pulse—which are followed in the financial markets. We briefly introduce working definitions of them below. In this work, we collected data of gross domestic product (GDP), Consumer Price Index (CPI) and inflation, and ForEx rate of different countries. In our earlier work [10] we investigated balance of trade (BoT) with respect to US for some countries in relation to their ForEx rate [11].

As discussed earlier, the relationships of these and other indicators in defining ForEx are yet to be established. In this work we have collected data of some of these indicators from year 2000 to 2013 of the above few important indicators. Figures plotted from this data indicate that ForEx is closely related to other indicators. Our main focus is to explore the nature of dependence of Consumer Price Index (CPI) on ForEx rate. We would like to address the specific question, Is CPI data is also chaotic as along with other indicators, it influences ForEx whose fluctuation is already found to be chaotic?

This question has become more pertinent in present years because, in European Union (EU) countries, the common currency Euro have a common exchange rate to US Dollar for member countries who have different economic conditions and different CPI. If domestic economic indicators define partly or wholly ForEx rate, what happens for these EU countries? Needless to say problems some of the Eurozone countries are facing are hard to control probably for this reason. As Gilles [12] put it, over the last decade, countries in the Eurozone have quietly but stubbornly diverged in terms of inflation, growth, fiscal performance, and competitiveness. Some have had 2% inflation on average, others have 4%, some have built up trade surpluses, others are increasingly indebted with respect to the rest of the world, some have kept their government budget in check, and others have let debt grow. This divergence comes from different policy choices, different institutions, and different cultures. But the common currency has in fact added to the divergence. Now what would have happened to such a country if it had not been a member of the Eurozone? Perhaps its central bank would have been worried about inflation and would have raised the interest rate. This would have cooled the economy down and especially hit the oversized construction industry and perhaps even deflated the housing bubble, or foreign investors would have taken into account the critical developments of the external accounts and attacked the currency.

Little work was attempted in this direction. Guastello (1995) showed low-dimensional chaos for US inflation rates during the 1948–1995 era with Lyapunov dimensionality 1.5. He also confirmed that although there was short time linear prediction of inflation rates, the global picture was, nonetheless, chaotic [13]. In another study (2001) he stressed the presence of chaotic attractors for inflation rate in the US [14]. Results from linear and nonlinear analyses provide overwhelming evidence in support of the nonstationarity of the inflation rate in Africa [15]. We would like to study this situation for some EU countries, as well as some other countries having their own ForEx rate. Obviously, we are not attempting any analysis of justification of Euro. Our analysis will be confined to understanding the economic indicators, particularly inflation data in relation to ForEx rate through data analysis. Details of data collection are given in Section 2, while in Section 3 we estimate qualitatively the performance of three economic indicators, as well as ForEx data. In Section 4, we briefly explain nonlinear analysis tools applied here, that is, surrogate test and Lyapunov exponent. Results of analysis are given in Section 5 which are discussed in Section 6 as conclusion. We make some remarks under Discussion in Section 7.

2. Data Collection and Description

To get an idea of gross domestic product (GDP) annual growth rate (expressed as per cent change over previous year), we have collected data for the following countries: France, Germany, Greece, Italy, India, Malaysia, Spain, Singapore, Sri Lanka, United Kingdom, and average for Euro area.

We have also collected the data of same countries on annual basis of inflation, consumer prices (annual %), obtained from World Databank, maintained by the World Bank for the period 2003 to 2012 on annual basis [16].

Detailed inflation or CPI data are as follows: the inflation rate is based upon the Consumer Price Index (CPI). The CPI inflation rates used are on a yearly basis (compared to the same month the year before). For example, CPI for January 2013 is difference in inflation over that in January 2012 expressed as per cent. Inflation.eu [17] maintains historical data for many countries which have been used in this paper. We have collected data on monthly basis from January 2000 to September 2013 for the following countries: France, Italy, Germany, Spain, Greece, India, Singapore, Sri Lanka, and UK. So each country has a dataset consisting of 165 data, one for each month. For Sri Lanka, data from January 2001 to April 2008 are available, so datapoints are 88 in number from Department of Census and Statistics, government of Sri Lanka [18]. Singapore CPI data was taken from data maintained by the government of Singapore [19].

Foreign Exchange data are taken from OANDA which maintains historical data, from January 200 to November 2013. So each set has 3490 points [20].

3. Performance of Indicators

Here we shall report the analysis of data collected for previously defined indicators. Earlier we have shown (2013) that GDP has overall important role in influencing ForEx rate going up or down in the long hand. Here the GDP concerns the nominal value, that is, without inflation adjustment over
Table 1: Showing qualitatively performance of indicators for different countries.

| Country  | ForEx rate (to US$) | GDP growth annual % | Inflation annual % |
|----------|---------------------|---------------------|--------------------|
| France   | 0.8 to 1.3          | Decreasing          | Stable around 2    |
| Germany  | Same                | Less than 2001-02 value | Stable around 2 |
| Greece   | Same                | Negative since 2009 | Decreasing since 2011 |
| Italy    | Same                | Decreasing since 2010 | Increasing |
| Spain    | Same                | Decreasing, negative | High, about 4 |
| Australia | Stable around 1 for last 2 years | Stable around 3 | Decreasing |
| India    | Increased from 44 to 69 | Decreasing, less than 2001–03 value | Very high |
| Malaysia | Decreased from 3.8 to 3.2 | Same as 2004–08 | Higher than 2001–05 |
| Singapore | Decreased from 1.7 to 1.3 | Lower than 2003–08 value | Higher than 2001–08 value |
| UK       | Same, near 2003–04 value | Lower than 2001–08 value | High, more than 2001–07 value |

period of study. Over time, the growth in GDP causes inflation, and inflation begets hyperinflation. Once this process is in place, it can quickly become a self-reinforcing feedback loop. This is because, in a world where inflation is increasing, people will spend more money because they know that it will be less valuable in the future. This causes further increases in GDP in the short term, bringing about further price increases.

Here, in Figure 1, we have plotted the GDP data for ten countries and we observe the following.

(1) For all countries without any exception, GDP growth rate is negative in year 2009. There is a clear dip in the curve. This is clearly due to fall in GDP value due to global recession.

(2) In subsequent year, GDP growth has increased for many countries. But for last two years (2011 and 2012), GDP is again sliding down. This is true for all three categories. For Greece, GDP growth rate never turned back to positive values, almost the same for Spain.

(3) During 2001-2002, GDP growth rate was sharply down for almost every country though in lesser scale compared to 2009 fall.

We have also plotted in Figure 1 inflation of consumer prices as annual per cent change and ForEx rate for these countries against time period of 2000 to 2013. At the top of each block of Figure 1, we plot ForEx rate daily data. For EU countries, this is not applicable as they have common ForEx rate of Euro given in separate block. From the plots of Figure 1 the indicators changed as shown in Table 1.

4. Nonlinear Analysis of Inflation Data

Here we shall concentrate on detailed nonlinear data analysis of inflation data collected to get more insight into it. The basic point we like to investigate is whether CPI data analysis shows chaos or not. First we check the nonlinearity of data using surrogate method. For characterizing chaos both qualitatively and quantitatively, we find LLE using TSTOOL package. Due to scarcity of data, we use another algorithm to check the results.

4.1. Test for Nonlinearity Using Surrogate Data Method. We follow the approach of Theiler et al. [21]. The surrogate signal is produced by phase-randomizing the given data. It has spectral properties similar to the given data; that is, the surrogate data sequence has the same mean, the same variance, the same autocorrelation function, and therefore the same power spectrum as the original sequence, but (nonlinear) phase relations are destroyed. Details of the method for the countries considered have been given in the previous work [8] or as used with additional noise reduction [10]. We used the TSTOOL package by Parlitz et al. [22], under MATLAB [23] software to create surrogate data for a scalar time series. From this analysis, we got some idea about the degree of nonlinearity associated with the time series of foreign exchange data up to year 2008. We are not repeating the same analysis because we are considering the same countries and compared to our previous data, we now have 450 more points, which are only 5% of total (from January 2008 to October 2009). But we certainly have to use the results.

4.2. Finding Lyapunov Exponent Using TSTOOL Package. Chaotic processes are characterized by positive Lyapunov exponent (LE) calculated following the approach of Wolf et al. [24] as detailed in Das et al. [8, 9]. Again, we used the TSTOOL [22] to find the LLE. The function used is largelyap which is an algorithm based on work by Wolf et al.; it computes the average exponential growth of the distance of neighboring orbits via the prediction error. The increase of the prediction error versus the prediction time allows an estimation of the LLE. In the particular MATLAB code, largelyap, the average exponential growth of the distance of neighboring orbits is studied in a logarithmic scale, this time via prediction error p(k). Dependence of p(k) on the number of time steps may be divided into three phases. Phase I is the transient where the neighboring orbits converge to the direction corresponding to the LLE (λ). During phase II, the distance grows exponentially with exp(λt) until it exceeds the range of validity of the linear approximation of the flow. Then phase III begins where the distance increases slower than exponentially until it decreases again due to folding in
the state space. If phase II is sufficiently long, a linear segment with slope $\lambda$ appears in the $p(k)$ versus $k$ diagram [22]. While calculating the LLE, we have obtained the prediction error $p(k)$ versus $k$ diagrams as output and are given in Figure 2.

4.3. Finding Lyapunov Exponent Using Rosenstein Algorithm. The algorithm is also attractive from a practical standpoint because it does not require large datasets [25]. The first step of the approach involves reconstructing the attractor dynamics from a single time series. The reconstruction depends on the lag or reconstruction delay and the embedding dimension [25]. This method works well when datapoints are small, 100 to 1000 for the systems examined. The greatest difficulty occurs with the Rossler attractor where a 20–25% negative bias in the results for even number of points is equal to 3000–5000.

In our case, inflation datapoints are small (nearly 165). For complex data like inflation, the embedding dimension is high, but the above method fails with higher dimension but smaller datasets. We used MATLAB program [26] to find LLE for some countries, keeping in mind the error range. We find LLE from inflation data of France as 0.212, India as 0.664, Spain as 0.212, and Germany as 0.231. The output of the program is given in Figure 3. It is to be noted that absolute value of LLE varies depending on the algorithm used, but its sign should be the same as positive LLE indicates chaos. In our case, the results obtained using both TSTOOL and Rosenstein algorithm indicate chaos.
5. Results

To ascertain the nonlinearity of data, we have applied surrogate methods as described in Section 4.1. We have used Theiler algorithm [23] to produce three surrogate datasets of each series. We have calculated LLE from original and three surrogate sets and compared the values to see how much they change in per cent. This is given in Table 1. For a sufficiently chaotic dataset, LLE for a surrogate set would differ considerably for reasons described in Section 4.1. In this paradigm, per cent deviation of LLE values in corresponding surrogate sets will indicate chaotic nature of the original data. From the results given in Table 2, we find the following.

(A) For inflation data, chaotic nature of data is as follows:
- low (change < 20%): France, Germany, and Spain;
- high (change < 40%): Greece and India;
- very high ((change > 40%)): Italy, Singapore, Sri Lanka, and UK.

(B) For ForEx data, chaotic nature of data is as follows:
- low: EU and Australia;
- high: Malaysia;
- very high: Singapore, Sri Lanka, and India.

6. Conclusion

From above results, we can draw the following conclusions.

(1) We can say that, from above calculation, LLE values indicate that inflation data exhibits chaos. They are highly chaotic for at least four countries listed above.

(2) Inflation data points are very low in number. Consumer Price Index is calculated monthly, so during the period under present study (year 2000 to 2013), we have about 160 points only. But there is evidence that for many countries (say, e.g., India and Sri Lanka) prices fluctuate daily like in ForEx markets, but only monthly values are recorded. Considering algorithm to calculate LLE, such small number of datapoints is not sufficient. Compared to this, each ForEx dataset contains nearly 3500 points.

(3) Another important aspect of inflation data is that as this factor immediately affects people’s life they are more closely monitored by respective governments. Even the intervention during present global recession is so acute that some reverse phenomena like deflation have taken place in many European countries. Deflation which refers to fall in money prices of commodities also constitutes a serious threat to the monetary system. Inflation falling low as seen...
Table 2: Calculation of largest Lyapunov exponent (LLE) using TSTOOL in MATLAB.

| Country | Original data | Surrogate set 1 % change in surrogate over original | Surrogate set 2 % change in surrogate over original | Surrogate set 3 % change in surrogate over original |
|---------|---------------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| Inflation (monthly data, January 2000 to September 2013) |
| France  | 1.8           | 2.1                                                 | 16.67                                               | 1.9                                                 | 5.56                                               | 2.2                                                 | 22.22                                               |
| Germany | 1.7           | 2.1                                                 | 23.53                                               | 1.9                                                 | 11.76                                              | 2                                                   | 17.65                                               |
| Greece  | 1.7           | 2.2                                                 | 29.41                                               | 1.7                                                 | 0.00                                               | 2.2                                                 | 29.41                                               |
| Italy   | 1.2           | 1.9                                                 | 58.33                                               | 1.6                                                 | 33.33                                              | 1.7                                                 | 41.67                                               |
| Spain   | 1.7           | 1.8                                                 | 5.88                                                 | 1.7                                                 | 0.00                                               | 1.9                                                 | 11.76                                               |
| UK      | 1.3           | 1.9                                                 | 46.15                                               | 1.5                                                 | 15.38                                              | 1.4                                                 | 7.69                                                |
| Sri Lanka | 0.6         | 0.8                                                 | 33.33                                               | 0.4                                                 | −33.33                                             | 0.85                                                | 41.67                                               |
| India   | 1.1           | 1.4                                                 | 27.27                                               | 1.3                                                 | 18.18                                              | 1.5                                                 | 36.36                                               |
| Singapore | 0.7          | 1.2                                                 | 71.43                                               | 1.2                                                 | 71.43                                              | 0.8                                                 | 14.29                                               |
| Foreign exchange rate (daily data, January 2000 to October 2013) |
| EU      | 3.8           | 4.2                                                 | 10.53                                               | 4.1                                                 | 7.89                                               | 3.9                                                 | 2.63                                                |
| Australia | 3.1          | 3.4                                                 | 9.68                                               | 3.5                                                 | 12.90                                              | 3.6                                                 | 16.13                                               |
| India   | 2.4           | 3.3                                                 | 37.50                                               | 4.2                                                 | 75.00                                              | 3.8                                                 | 58.33                                               |
| Singapore | 2.3          | 3.5                                                 | 52.17                                               | 3.4                                                 | 47.83                                              | 3.3                                                 | 43.48                                               |
| Malaysia | 2.7          | 3.1                                                 | 14.81                                               | 3.6                                                 | 33.33                                              | 3.5                                                 | 29.63                                               |
| Sri Lanka | 1.6          | 3.3                                                 | 106.25                                              | 3.8                                                 | 137.50                                             | 4                                                   | 150.00                                              |
in Figure 1 for countries (say, e.g., Greece) threatens Europe [27]. In the present study, we are interested in fluctuations of the CPI, not its absolute values. So, whether inflation is high or low is not of direct importance here, but such interventions sometimes in unspoken terms make data analysis more difficult. Particularly, these become important when we try to see CPI in relation to other economic indicators mentioned in this paper.

(4) If we compare the LLE values of inflation data and ForEx rate data, we find ForEx rate to be more chaotic in nature. For India, Singapore, and Sri Lanka we have calculated LLEs from both inflation and ForEx data. We find for these countries, LLE of ForEx data is more than double of LLE of inflation data. So, even if inflation is chaotic, it is much lesser chaotic than ForEx rate.

(5) In our earlier analysis (2007), we showed ForEx market to exhibit chaos [8]. For some countries (like India, China, and Sri Lanka) chaos is much higher compared to countries like Japan, Singapore, and Sweden when countries like Malaysia and Thailand stand in between. The results were calculated for period (year 1975 to 2005) before recession started. Here we find that the same results are reflected for data during year 2000 to 2013. Chaotic nature of ForEx countries is retained even during the global recession as Singapore, Sri Lanka, and India show high chaotic ForEx market.

(6) Interestingly, we find countries with chaotic ForEx market show higher chaotic inflation rate (Singapore, Sri Lanka, and India). So there may be good relation between the two indicators. But countries within EU show different levels of chaos in inflation data (France and Germany low but Italy high). Historically, to participate in the currency, member states are meant to meet strict criteria, such as a budget deficit of less than three per cent of their GDP, a debt ratio of less than sixty per cent of GDP (both of which were ultimately widely flouted after introduction), low inflation, and interest rates close to the EU average [28]. But current analysis shows different picture in this regard. So the relationship is more complex in nature. The important point that arises is how and why countries with same ForEx (that is Euro) have different inflation rate. So probably, in relating inflation and ForEx rates, other domestic and international factors have to be accounted for.

7. Discussion

We found chaotic nature of the inflation rate. But as pointed out earlier, experimentation with higher amount of data has to be made. For that reason, more rigorous data collection practice has to be adopted. If we can establish at least some empirical mathematical relation of indicators towards setting up a model of a system consisting of these indicators, we can investigate important parameters controlling the system. These issues deserve more thoughtful studies which will have far reaching effect on handling the system for the benefit of citizens.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] M. A. Torkamani, S. Mahmodzadeh, S. Pourroostaei, and C. Lucas, “Chaos theory and application in foreign exchange rates vs. IRR (Iranian rial),” International Journal of Human and Social Sciences, vol. 2, article 7, 2007.

[2] P. Grauwe de and I. Vansteenkiste, “Exchange rates and fundamentals: A nonlinear relationship?” Working Paper 577, Center for Economic Studies & Ifo Institute for Economic Research, 2001, http://www.CESifo.de.

[3] C. Goodhart, “News and the foreign exchange market,” LSE Financial Market Group Discussion Paper 71, 1990.

[4] P. de Grauwe and H. Dewachter, “A chaotic model of the exchange rate: the role of fundamentalists and chartists,” Open Economies Review, vol. 4, no. 4, pp. 351–379, 1993.

[5] P. de Grauwe and M. Grimaldi, Intervention in the Foreign Exchange Market in a Model with Noise Traders, University of Leuven, Leuven, Belgium, 2003.

[6] M. Bask, “Dimensions and Lyapunov exponents from exchange rate series,” Chaos, Solitons and Fractals, vol. 7, no. 12, pp. 2199–2214, 1996.

[7] M. Bask, “A positive Lyapunov exponent in Swedish exchange rates?” Chaos, Solitons and Fractals, vol. 14, no. 8, pp. 1295–1304, 2002.

[8] A. Das and P. Das, “Chaotic analysis of the foreign exchange rates,” Applied Mathematics and Computation, vol. 185, no. 1, pp. 388–396, 2007.

[9] G. Çoban, A. H. Büyüklü, and A. Das, “A linearization based non-iterative approach to measure the gaussian noise level for chaotic time series,” Chaos, Solitons and Fractals, vol. 45, no. 3, pp. 266–278, 2012.

[10] A. Das, P. Das, and G. Çoban, “Chaotic analysis of the foreign exchange rates during 2008 to 2009 recession,” African Journal of Business Management, vol. 6, no. 15, pp. 5226–5233, 2012.

[11] P. Das and A. Das, “Comparison of recession during 2008–2009 and 2009–2013 through chaotic analysis of the foreign exchange rates,” Journal of Contemporary Management, vol. 2, pp. 100–112, 2013.

[12] S.-P. Gilles, “A crisis mechanism for the Euro: the European stability mechanism,” VOX, March 2011, http://www.voxeu.org/article/euro-failure.

[13] S. J. Guastello, “The search for a natural rate of price inflation: US 1948–1995,” Chaos Network, vol. 7, no. 3, pp. 16–23, 1995.

[14] S. J. Guastello, “Attractor stability in unemployment and inflation rates,” in Evolutionary Controversies in Economics, pp. 89–99, 2001.

[15] C. A. Augustine, “Are inflation rates really nonstationary? New evidence from non-linear STAR framework and African data,” International Journal of Economics and Finance, vol. 3, no. 3, 2011.

[16] World Databank, World Bank, 2013, http://databank.worldbank.org/data/home.aspx.
[17] Inflation.eu—worldwide inflation data, December 2013, http://www.inflation.eu.

[18] Movements of Sri Lanka CCPI 2001, Department of Census and Statistics, Government of Sri Lanka, 2013, http://www.statistics.gov.lk/.

[19] Singapore inflation data, Government of Singapore, 2013, http://data.gov.sg/.

[20] OANDA—a trusted source for currency data, November 2013, http://www.oanda.com/.

[21] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, and J. Doyne Farmer, “Testing for nonlinearity in time series: the method of surrogate data,” Physica D: Nonlinear Phenomena, vol. 58, no. 1-4, pp. 77–94, 1992.

[22] U. Parlitz, I. Wedekind, W. Lauterborn, and C. Merkwirth, TSTOOL & User Manual, Ver. 1.11, DPI Göttingen, 2001, http://www.physik3.gwdg.de/tstool/gpl.txt.

[23] MATLAB Release13, The MathWorks, Natick, Mass, USA, 2002.

[24] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, “Determining Lyapunov exponents from a time series,” Physica D. Nonlinear Phenomena, vol. 16, no. 3, pp. 285–317, 1985.

[25] M. T. Rosenstein, J. J. Collins, and C. J. de Luca, “A practical method for calculating largest Lyapunov exponents from small data sets,” Physica D: Nonlinear Phenomena, vol. 65, no. 1-2, pp. 117–134, 1993.

[26] S. Mohammadi, Lyaprosen.m—A Matlab Implementation of Rosenstein Algorithm, University of Tehran, 2009.

[27] Wall Street J, November 2013, http://online.wsj.com/home-page.

[28] Wikipedia, EURO, 2013, http://en.wikipedia.org/wiki/Euro.
