Coulomb versus nuclear break-up of $^{11}$Be halo nucleus in a non perturbative framework.

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Abstract

The $^{11}$Be break-up is calculated at 41 MeV per nucleon incident energy on different targets using a non perturbative time-dependent quantum calculation. The evolution of the neutron halo wave function shows an emission of neutron at large angles for grazing impact parameters and at forward angles for large impact parameters. The neutron angular distribution is deduced for the different targets and compared to experimental data. We emphasize the diversity of reaction mechanisms, in particular we discuss the interplay of the nuclear effects such as the twining mode and the Coulomb break-up. A good agreement is found with experimental data.

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I. INTRODUCTION

A major discovery of the last decade in nuclear physics is the observation of halo nuclei \cite{1}. The presence of these extended systems has been uncovered by break-up studies. However, an unambiguous interpretation of these reactions requires a deep understanding of reaction
mecanisms such as diffractive effects [2]. In particular, the interplay between nuclear and Coulomb dissociation is of major importance.

On the experimental side, measurements of the neutron angular distribution have been performed for the one-neutron break-up of \(^{11}\)Be on Au, Ti and Be targets [3] at 41 MeV per nucleon. They clearly present two contributions. One peaked at small angles (below 10 degrees) which strongly varies with the target; for the Au target the cross section is as high as 50 barns per steradian at small angles whereas for the Be target it is around 1 barn per steradian. A second one located at large angles which shows a lesser dependence with the target; the cross section around 30 degrees is about 100 m barn per steradian for all studied targets. The small angle region was understood through models based on Coulomb-excitation theory to come from Coulomb dissociation [3] and the large angle emission was thought to be a consequence of nuclear break-up. Perturbative calculations [4] including both interactions or non perturbative calculations [3] which only account for the Coulomb field have been performed in the last years. Only recently the time dependent Schrödinger equation was solved [8] and applied to the break-up of \(^{11}\)Be using different numerical techniques than the one we have used [9]. In this later paper, the authors predict the presence of neutrons emitted with a high angular momentum in the \(^{11}\)Be frame which corresponds to the neutrons measured at large angles that were mentioned above. This prediction, which the authors do not observe when they use the eikonal approximation, is in agreement with our calculation as it will be discussed further in this paper. At the same time our paper was in submission, we noted the work of ref. [10] that also presents a method of solving the time dependent Schrödinger equation and applies it to the break-up of \(^{11}\)Be and \(^{19}\)C to calculate the relative energy between the emitted neutron and the core.

In our article, we present a non-perturbative model which accounts both for the Coulomb and the nuclear effects through the resolution of the full time-dependent Schrödinger equation onto a cartesian mesh. Our calculation shows that the emission of neutrons at large angles is due to the interaction of the particle with the nuclear potential. Such an emission has already been observed in reactions between stable nuclei [11] where the nucleon ori-
nated from the target and not the ejection as in the case of $^{11}\text{Be}$ break-up. This mechanism was called "towing mode" as the particle was pulled out from the target and towed by the projectile for a short while. Through the resolution of the time-dependent Schrödinger equation we reproduced this emission to the continuum with specific angles and energies in agreement with the measurements [3].

In the following, we will show through our calculation that both Coulomb and nuclear fields play an important role in the case of the neutron break-up of halo nuclei.

II. DESCRIPTION AND INPUTS OF THE MODEL

We use the calculation presented in a previous paper [3] and apply it to the reactions $^{197}\text{Au},^{48}\text{Ti},^{9}\text{Be} (^{11}\text{Be},^{10}\text{Be} + \text{n})$. This calculation describes the wave function distortion of an initially bound particle as it passes by the potential induced by the reaction partner. This is performed in the framework of independent particles.

The wave function of the neutron halo is deduced from the potential found by N. Vinh Mau [12] to reproduce the inversion of the $2s$ and the $1p$ states in the $^{11}\text{Be}$ nucleus for which a derivative of the Wood-Saxon potential is added at the surface. The potential between the neutron and the $^{10}\text{Be}$ core reads

$$V_{\text{Be}}(r) = \frac{V_0}{1 + \frac{a}{r}} + 16: \frac{e^{2\frac{1}{2} + \frac{b}{r}}}{1 + e^{\frac{r}{a}}}$$

where the di useness $a$ is 0.75 fm and the coefficient $b$ is equal to $-10.56$ MeV for the $2s$ state. The radius $R$ equals to $1.27$ A^{1-3} fm. Numerically the initial wave-packet is obtained by diagonalizing the one-body Hamiltonian in spherical coordinates in a sphere of 30 fm radius with a space-step of $r = 0.02$ fm. The depth $V_0$ was taken to be $-40$ MeV. The wave function calculated in spherical coordinates for $^{11}\text{Be}$ is then mapped onto Cartesian coordinates in order to calculate the dynamical evolution. Special attention has been given to the purity of the ground state $^{11}\text{Be}$ neutron halo by performing an additional imaginary time evolution on the Cartesian network, after which the $2s$ state is bound with $-0.503$ MeV.
The dynamical evolution of the system is given by the following single particle Schrödinger equation which reads in the $r$ space

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} \psi(r) + V_T(r, r_T(t)) + V_{Be}(r, r_{Be}(t)) \psi(r) = \frac{i}{\hbar} \frac{\partial \psi(r)}{\partial t}$$  \hspace{1cm} (2)$$

where $V_T$ and $V_{Be}$ are the time-dependent potentials between the neutron and the target and between the neutron and the projectile respectively. $r_T(t)$ and $r_{Be}(t)$ correspond to the target and the projectile positions respectively. The nuclear potential of the Au and Be target, $V_T(r)$, is taken to be of Woods-Saxon shape with a diffusion $a = 0.5 \text{ fm}$, a radius $R = 1.27A^{1/3} \text{ fm}$ ($A$ being the mass number of the $^{197}$Au or $^{9}$Be) and of depth adjusted to obtain the experimental binding energy of the last neutron. For the Au target, $V_0 = -49.3 \text{ MeV}$, giving a $3p$ state bound by about 8 MeV and for the Be target, $V_0 = -40. \text{ MeV}$, giving a $1p$ state bound by about 1.66 MeV. For the $^{48}$Ti target, we used the Becchetti and Greenlees prescriptions to obtain the real part of the nuclear potential between the neutron and the Ti, leading to a depth of $V_0 = -41.2 \text{ MeV}$, a diffusion of 0.75 fm and a $r_0$ value of 1.17 fm.

In order to take into account the target and the $^{10}$Be core displacement, the evolution is performed for each time step using the classical Coulomb trajectory for the center of mass motions ($r_T(t)$, $r_B(t)$, $r_{Be}(t)$, $r_{Be}(t)$) from a distance of $-400 \text{ fm}$ along the initial velocity axis between the projectile and the target and equal to the impact parameter $b$ on the perpendicular axis. This explicit treatment of the core recoil is responsible for the Coulomb excitations of the neutron around the $^{10}$Be core. The numerical method used for the trajectory calculation is the Runge-Kutta method.

We have chosen to use the split-operator method \textsuperscript{[15]} for the time evolution. This is a well known way to solve the time dependent Schrödinger equation (Eq.2) on a 3D cartesian lattice. It is faster and as accurate as the methods based on Taylor expansion of the evolution operator which are also routinely used to solve time dependent problems. Moreover it allows to treat large lattice performing a rather long time calculation.

The time step used is of $1.7 \text{ fm/c}$ on a mesh of $64x64x64 \text{ fm}^3$ with a step size of $0.5 \text{ fm}$.
have tested the numerical accuracy of the method with respect to the mesh parameters. The calculation is performed in the initial projectile frame and a Galilean frame transformation is performed to extract the observed quantities.

III. RESULTS

A. Density plots

The result of these calculations (Fig.[3]) is presented for the $^{11}\text{Be} + \text{Au}$ reaction as the probability density integrated over the z-axis, $2s(x;y) = \int (x;y;z)^2 dz$, for impact parameters $b = 10, 12, 15, 20$ and $40$ fm, after the evolution of the $2s$ wave function initially in the $^{10}\text{Be}$ potential.

After the evolution, we subtract the contributions of the wave-functions which are bound in the $^{10}\text{Be}$ potential in order to keep only the emitted part of the wave-function. This subtraction is performed by projecting out all the bound states $j >$ of the neutron in the $^{10}\text{Be}$, leading to the emitted wave function $j >$.

$$j > = j > \sum j << j >$$

(3)

The bound wave functions in the $^{10}\text{Be}$ core potential are the $2s$ state itself bound by $-0.503$ MeV, the $1s$ state bound by $-23$ MeV and the $1p$ state using the same nuclear potential and which is then bound by about $-11$ MeV. Note that we do not consider the contribution of the $1p$ wave function which has been experimentally evidenced at $-0.183$ MeV since it is not a bound wave function of our potential.
**FIG. 1.** Density plots of the 2s wave function in $^{11}$Be after scattering on a $^{197}$Au target for impact parameters of 10, 12, 15, 20 and 40 fm. The Coulomb trajectory is represented by the dashed line for each impact parameter. These plots are displayed in a logarithmic scale, black representing the most intense density. The ratio in density is 3 between each grey area.

**Fig. 2** shows the same evolutions as **Fig. 1** after the subtraction of these components. A sizeable fraction of the wave function is then removed around the $^{10}$Be core position. One can see that the small impact parameters ($b < 20$ fm) are responsible for neutrons emitted at large angles in the opposite direction of the core compared to the initial direction. For larger impact parameters, neutrons are forward focussed. This small angle emission can be understood as a Coulomb dissociation in which the $^{10}$Be core has been shaken by the Coulomb field of the target. In our calculation, for large impact parameters, we observe a displacement of the neutron wave function compared to the core which might be the first oscillation of a soft dipole resonance as it has been suggested in ref. [16]. Due to the weak binding of the last neutron, the halo has separated from the core, transferring little momentum to the neutron which is then emitted along the initial trajectory. The calculation shows a weak emission (see **Fig. 2** right) which must, however, be integrated over a large impact parameter domain to obtain the total cross section.

In order to compute cross-sections, we used the following method. For large impact parameters the fraction of wave function emitted is small and thus according to the perturbation theory, to gain time and reduce the error inherent to the method, we performed the calculation up to the minimum distance of approach and multiplied the result by two.
For large impact parameters we have tested that this procedure gives the same results as the complete calculation demonstrating that the impact parameters are large enough to guarantee a first order perturbation.

On the contrary, for small impact parameters (below 20 fm) the nuclear break-up cross section is large and the nuclear potential spreads the wave function on the whole mesh (see Fig. 2) we stopped the calculation at a distance of 20 fm after the projectile has passed by the target and checked that the extracted values do not change for a slightly longer evolution. Since the nuclear break-up is much larger than the Coulomb break-up cross section for the small impact parameters, as will be shown in section B, the fraction of Coulomb break-up missed there is negligible.

B. Interplay between nuclear and Coulomb break-up.

Since the $^{10}$Be core is detected in the experiment, we assumed a minimum impact parameter corresponding to a grazing trajectory. We followed the strong absorption model for which the minimum parameter is calculated as $1.4 (A_T^{1/3} + A_B^{1/3})$ fm yielding to $b_{min} = 11$ fm for the Au target, $b_{min} = 8$ fm for the Ti target and $b_{min} = 6$ fm for the Be target. To obtain a total cross section, this calculation was performed for impact parameters running from the grazing parameter, to an impact parameter where the fraction of wave function emitted no
longer changes with increasing impact parameter. This is shown in Fig[3] where the fraction of wave function emitted \( \text{Frac} = \frac{R}{d^3r} \) is plotted versus the impact parameter.

For the Au target this percentage starts to saturate around 200 fm whereas for the Ti it is around 100 fm and 40 fm for the Be target. At these impact parameters the fraction of wave function emitted is around 0.04%. At the saturation, only the noise inherent to the numerical method remains and we do not include larger impact parameters in the cross section estimate.

In this figure we clearly see a change in the slope of \( \text{Frac} \), between the small and the large impact parameters which could indicate the transition between the nuclear and the Coulomb perturbation.

\[ \text{FIG. 3. Fraction of wave function emitted (Frac) versus the impact parameter for the three targets (symbols). Dashed lines are those obtained with the formula } b \text{ for the large impact parameters (see text). Insert: scaling of } \text{Frac} \text{ with } A^{2/3}_f \text{ (symbols). The solid line is the } t \text{ of the scaling with } e^{-b} \text{ (see text).} \]

In order to get a deeper insight in the interplay between nuclear and Coulomb break-up, we have studied the mass and charge dependence of the fraction emitted (Frac) with the impact parameter. At large impact parameters, above 40 fm, the wave function does not see
the target nuclear potential and only the Coulomb field is felt by the core. We observe in Fig. 3 that at large impact parameters $\text{Frac}$ can be fitted with the formula $b$ for the Ti and Au targets. We found $= 2.4$ and which scales quadratically with the charge as $= 0.0088 Z^2$. Fits are shown on Fig. 3 as dashed lines (we assumed the same dependence for the Be target). Since the probability of emission is small we can conclude that the excitation is perturbative. This is confirmed by the $Z^2$ dependence of this probability.

We extrapolated the Coulomb effect to the small impact parameters and by subtracting the curve to the fraction emitted we obtained a function which we expect to only contain the nuclear effect. Indeed, after renormalizing these results with $A_{T}^{2-3}$, we obtained a quantity named $\text{Scaling} = (\text{Frac} \quad b)A_{T}^{2-3}$, which no longer depends on the target (see inset of Fig. 3). This leads to a cross section that scales with $A_{T}^{2-3}$ as expected for the nuclear emission [17]. This curve can be fitted in turn by an exponential ($\text{Scaling} / e^b$ with $= 0.40 \quad 0.01$ fm$^{-1}$).

We would like to emphasize that this empirical fitting method indicates a rather good separation between Coulomb and nuclear effects in our calculation. Such a separation, which comes as an hypothesis in perturbative framework, is however not natural in our framework where both effects are accounted for at the same time and can therefore interfere. By treating the Coulomb and nuclear part separately we have controlled that the possible interferences are small.

Using the parameters extracted from the fits and integrating over the impact parameter, we extract the variation of the break-up cross section with the charge of the target for both the Coulomb and the nuclear interactions. This evolution is presented in Fig. 3. We observe a Coulomb cross section that scales with $Z^{1.65}$ close to the value of 1.725 from ref. [18].

It should be noticed that in the case of the Au target we have checked this analysis by turning on and off the nuclear and the Coulomb field separately in the calculation.
FIG. 4. Coulomb (squares) and nuclear break-up (triangles) cross sections as a function of the target charge \(Z_T\) obtained from the fits to the calculated fraction of the wave function emitted and the to the Scaling parameter, integrating from the corresponding \(b_{\text{in}}\). The Coulomb contribution scales with \(Z^{1.65}\) (not shown in the figure). The sum of the Coulomb and the nuclear cross-section is also reported as circles joined by a plain line and compared to the experimental data (plain circles with error bars) of ref. [3].

C. Cross sections and angular distributions

Once we have the fraction of wave-function emitted we can access the differential cross-section by integrating the corresponding probability between \(b_{\text{in}}\) and \(b_{\text{max}}\) defined in previous section. For each impact parameter \(b_n\) (fm), the cross section is equal to \(2 \cdot b_n \cdot 10^{-2}\) (barn) times the modulus squared of the fraction of wave function which is emitted, where \(b_n\) is taken to be equal to \((b_{n+1} - b_{n-1})=2\). The step in impact parameter between two calculations varied from 1 fm (for small impact parameters) to 40 fm above \(b=150\) fm.

The angular distribution of the emitted neutron is extracted by applying the Fourier transform to obtain the remnant part of the wave function in the momentum space [3] (Fig.4).

\[
\langle p \rangle = \frac{Z}{2} \int \exp(\frac{i}{h} \cdot r) \cdot (r) dr
\]  

(4)

It should be noticed that to fully take into account the nuclear state interaction this calculation should be performed on the asymptotic wave function. We have controlled that \(\langle p \rangle\) does
not evolve anymore for longer evolution. Therefore, it can be considered as the final momentum distribution of the emitted particles. For small impact parameters, nuclear effects are important and the nuclear refraction of the neutron halo gives rise to an emission at large angles. This can be seen in the spectra extracted from the calculations performed at impact parameters of 10 to 20 fm which exhibit a component at angles above 30 degrees. This component is also seen in the density plots of Fig. 2 and present an anti-correlation with the core trajectory compared to the incident direction. This emission to the continuum is known as Towing Mode. In the case described with stable nuclei, a nucleon from the target is pulled out, towed along by the projectile and finally expelled at large angles and large velocities in the laboratory frame [11]. In the case of the $^{11}$Be experiments, the emitted neutron belongs to the projectile (the $^{11}$Be) which breaks up as it is perturbed by the target nuclear potential. In the $^{11}$Be frame, this nuclear break-up can be seen as the emitted neutron being accelerated by the target potential (see Fig. 2). We expect the same mechanism to be present for this halo nucleus as for a stable nucleus. However, in the case of a halo nucleus, the cross section of this nuclear break-up should be increased compared to a stable nucleus, due to the large extension of the neutron wave function and thus the larger impact parameter at which it sees the nuclear potential of the other nucleus. This will be discussed further when comparing with a more bound wave function (see section D).

We also display the angular distribution of the initial 2s wave function. In a sudden core removal model, it would correspond to the distribution of the emitted neutrons. However, it differs from the presented distributions as well as from the experimental distribution of ref. [3], showing the importance of the reaction mechanism in the break-up process.

At larger impact parameters, above 50 fm, only the component between 0 and 15 degrees remains (Fig. 4 right). There, the neutron wave function does not feel the nuclear potential anymore and the break-up comes from the deviation of the $^{10}$Be core in the Coulomb field. Indeed, because the halo neutron of $^{11}$Be is so weakly bound, a light shaking of the core is enough to induce the break-up. This, in turn, is not expected with a more strongly bound nucleus as it will be discussed further in section D.
FIG. 5. Angular distributions of the fraction of the wave function emitted for impact parameters from 10 to 20 fm (left) and from 50 to 190 fm (right). On the left, the Fourier transform of the initial 2s wave function is also shown as a plain line.

We have compared our calculation with a time dependent perturbative approach proposed by Bonaccorso, Brink and Margueron presented in ref. [5, 19] (called here perturbative) where they treat both the Coulomb and the nuclear break-up as a transfer to the continuum. The result we show have been compared with the same optical potential (presented here) and the wave function of the neutron halo is taken as the outer part of a 2s wave-function and normalized to the wave function used in our dynamical evolution. At small impact parameters we know that a large fraction of the wave function is emitted (Fig. 3 shows that more than 50% of the wave function is emitted for 10 fm of impact parameter) which indicates that the perturbative approach may not be adequate in the case of the Au target, however at large impact parameters this value becomes small (1% at 30 fm of impact parameter) and the perturbative approach is well justified. We thus expect the results of our calculations to be different from the perturbative approach for small impact parameters and to become more and more similar as we go at larger impact parameters. Fig. 5 shows this comparison for three regions of impact parameters for the break-up on a Au target.
FIG. 6. Angular distribution for the emission of the $^{11}\text{Be}$ halo neutron using a perturbative approach (dashed lines) and our calculation (plain lines) for three regions of impact parameter.

For the region between 10 and 12 fm the two calculations predict a quite different shape of the neutron angular distribution. At large impact parameter, the two calculations exhibit similar shapes of the neutron angular distributions but a larger cross section is observed around 5 degrees in the perturbative approach. This difference, present at all impact parameters, might be due to the remaining differences between the ingredients of the two methods. In particular it should be stressed that the wave functions used are not the same in both cases. In the perturbative calculation the inner part of the wave function is the Hankel function $h_1$ while in our approach we use the exact solution of the time dependent Schrodinger equation. We compared two time dependent theories which show some discrepancies. The
use of these theories as a spectroscopic tool for halo nuclei requires a better understanding of these differences. A deeper analysis is in progress and will be presented in a forthcoming article.

**D. Comparison between halo and non-halo neutron emissions**

Our calculation has also been performed to infer the evolution of a strongly bound wave function. We used a Wood-Saxon potential with $V_0 = -70.5$ MeV, $A = 11$, and obtained a 2s wave function bound by 7 MeV. The calculation has been performed for a Au target and for impact parameters running from 10 fm to 110 fm. The result is shown in Fig. 7 and is compared to the break-up of the $^{11}$Be halo for the same impact parameter range. The differential cross section of neutron emission for the bound nucleus is more than hundred times lower than for $^{11}$Be below 10 degrees. For large angles, around 40 degrees, the differential cross section is about 4 times lower. This latter difference can be understood by the extension of the wave function, which is much larger in case of a halo.

**Fig. 7.** Angular distribution for the emission of the $^{11}$Be halo neutron (plain line) and a non-halo neutron (dashed line) bound by 7 MeV after the scattering on a Au target summed over impact parameters between 10 fm and 110 fm.
In particular, it is often argued that Coulomb effects shadow the nuclear mechanism in heavy targets like Au. Although the relative proportion of these two effects is largely in favor of Coulomb dissociation for a heavy target such as Au, we show that nuclear break-up is of major importance for large angle emission with a cross section around 0.5 barn. Whereas Coulomb dissociation on a heavy target such as Au could be a direct measurement of the binding energy of the particle, the amount of towed particle could in turn bring information on the extension of the wave function, and hence answer the question whether we are dealing with a halo state or not. Furthermore, in stable nuclei, it has been shown that transfer to the continuum due to the Towing Mode might be a tool to infer information on shell structure as presented in ref. [11]. One might then be able to use this large angle emission to obtain additional information on nuclear halo wave-function properties.

E. Comparison with the $^{11}$Be data

To compare our calculation to the data of ref. [3], we extracted the differential cross section by summing the calculations from $b_{\text{min}}$ up to $b_{\text{max}}$. For the Au target we took $b_{\text{max}} = 210$ fm, 120 fm for Ti and 40 fm for Be. Calculations are shown in Fig. 8, multiplied by 0.84 to take into account the spectroscopic factor of the 2s state found experimentally and reported in ref. [14]. The calculation is compared to the experimental data of ref. [3] for all three targets taking into account the experimental threshold of 26 MeV for the neutron detection. Note that our calculation includes all the inelastic channels in which the target is excited but also those in which the $^{10}$Be remnant is in an excited state below its neutron separation energy. This is also included in the data that measured the $^{10}$Be and the neutron in coincidence. However, our calculation does not take into account the possible two-body dissipation since we use a single-particle framework. The calculations (plain lines) are in good agreement with the experimental data, both at small and large angles. In Fig. 8 we have reported the result of the calculations for three impact parameter regions showing the separation between the nuclear and the Coulomb break-up. The simultaneous reproduction
of the data for the Au, Ti, and Be targets demonstrates that the Coulomb and the nuclear interactions are well taken into account. In these calculations, the contribution of a neutron in a 1d state coupled to a 2+ excitation of the core as deduced from the experiment [14] has not been taken into account so far. However, since we already reproduce the whole cross section for the Au target with 84% of the 2s break-up, the contribution of the 1d state would seem to be small in this case.

**Figure 8.** Calculated angular distributions for impact parameter running from $b_{\text{min}}$ to $b_{\text{max}}$ (11 to 210 fm for the Au target (left figure)) (plain line) for neutron of energy higher than 26 MeV. Dots with error bars are data points from ref. [3]. Middle figure is for the Ti target and right figure for the $^9$Be target. Contribution of the calculation for three impact parameter regions, $b_{\text{min}}$ to 15 fm (short dashed lines), 15 to 40 fm (dotted lines) and 40 to $b_{\text{max}}$ (long dashed lines) are also presented.

**F. Relative energy**

We have calculated the relative energy between the emitted neutron and the $^{10}$Be using the momentum of the core after the Coulomb trajectory. This is presented in Fig. 9 for three different impact parameter regions for the Ti target. This spectrum shows that large relative energies correspond to small impact parameter hence the nuclear break-up whereas
large impact parameters lead to small values of the relative energy as expected for the Coulomb break-up.

**FIG. 9.** Relative energy between the neutron and the Be core after breakup on a Ti target for impact parameter running from $b_{\text{min}} = 8$ fm to $b_{\text{max}} = 120$ fm (plain line). Contribution of the calculation for three impact parameter regions, 8 to 15 fm (short dashed lines), 15 to 40 fm (dotted lines) and 40 to 120 fm (long dashed lines) are also presented.

**IV. CONCLUSION**

We have investigated the neutron break-up of a halo nucleus in the reactions $\text{Au, Ti, Be (}^{11}\text{Be}, {^{10}\text{Be+n}}$ at 41 MeV per nucleon, in the framework of a time dependent quantum model. Results were compared with the experimental neutron angular distributions of Ref. [3] and a good agreement was found. Our calculation, that includes both the Coulomb and the nuclear interactions, confirms that the forward peaked neutrons are due to the Coulomb break-up and that the neutrons emitted at large angles come from the interaction of the halo neutron with the target nuclear potential. This directive mechanism is called towing mode [11]. A strong angular correlation between the towed particle and the projectile was observed in reactions between stable nuclei and is also expected for the $^{11}\text{Be}$ break-up. However, the experiments performed so far did not measure the scattering angle of the...
remnant $^{10}$Be. Our calculations show that neutrons emitted below 15 degrees arise from the Coulomb dissociation and most of the cross section comes from large impact parameters as the neutron halo of $^{11}$Be is weakly bound. The shaking of the $^{10}$Be core by the Coulomb field leads to the dissociation of the halo and the emission of the neutron in the forward direction. This can also be understood as a Coulomb excitation of $^{11}$Be above the particle threshold, followed by neutron emission. This is the only mechanism contributing to the break-up when the impact parameter is such that the halo wave function does not overlap with the nuclear perturbative potential. The Coulomb break-up is very much hindered for strongly bound neutrons, whereas the nuclear break-up decreases by a factor of four due to the lesser extension of the wave function. Calculations for the 1d wave function should be performed for the $^{11}$Be break-up to infer its contribution and understand better the data. 

More generally, those calculations could be used to extract information on the wave function of the last bound neutron for unstable nuclei for which the properties are not well known yet, provided that a measurement of the neutron angular distribution cross section is performed both at large and small angles.
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