CONSTRANTS ON $|U_{e3}|^2$ FROM A THREE-NEUTRINO OSCILLATION ANALYSIS OF THE CHOOZ DATA

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Abstract

Results of a new analysis of the CHOOZ data, performed in the framework of three-neutrino mixing, are presented. Both the cases of normal and inverted neutrino mass hierarchy are considered. The parameters characterizing the solar neutrino oscillations, $\Delta m^2_\odot$ and $\tan^2 \theta_\odot$, are assumed to lie in the region of the large mixing angle (LMA) MSW solution of the solar neutrino problem, which is favored by the current solar neutrino data. At $\Delta m^2_\odot \lesssim 10^{-4}$eV$^2$ the new CHOOZ exclusion curve in the $\Delta m^2_{31} - |U_{e3}|^2$ plane practically coincides with the exclusion curve obtained in the two-neutrino mixing analysis. For $\Delta m^2_\odot \gtrsim 2 \cdot 10^{-4}$ eV$^2$, the constraints of the CHOOZ data on $|U_{e3}|^2$ are more stringent. For, e.g., $\sin^2 \theta_\odot = 0.50$ and $\Delta m^2_\odot = 6 \cdot 10^{-4}$ eV$^2$, and at $\Delta m^2_{31} = 2.5 \cdot 10^{-3}$ eV$^2$ (the Super-Kamiokande best fit-point) we find that (at 90% C.L.) $|U_{e3}|^2 < 1.7 \cdot 10^{-2}$, which is by more than a factor of 2 smaller than the upper bound obtained in the original two-neutrino mixing analysis of the CHOOZ data ($|U_{e3}|^2 < 3.7 \cdot 10^{-2}$).
1 Introduction

There exist at present strong evidences in favor of neutrino oscillations obtained in the atmospheric [4] and solar neutrino experiments [2, 3]. The results of the recent SNO experiment [7], combined with the data from the Super-Kamiokande experiment [6], which clearly demonstrated the presence of $\nu_\mu$ ( $\nu_\tau$) in the flux of the solar neutrinos reaching the Earth [4, 5], are additional compelling evidence in favor of neutrino oscillations. All these data suggest the existence of mixing of at least three massive neutrinos:

$$\nu_{\alpha L} = \sum_{j=1}^{3} U_{\alpha j} \nu_{j L}, \quad \alpha = e, \mu, \tau. \quad (1)$$

Here $\nu_{\alpha L}$ is the field of the flavour neutrino $\nu_{\alpha}$, $\nu_{j}$, $j = 1, 2, 3$, is the field of neutrino with mass $m_j$ and $U$ is a 3x3 unitary matrix - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [8, 9].

Indications for $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ oscillations were found in the Los Alamos short baseline accelerator experiment LSND [10]. In order to describe the data of the atmospheric, solar and LSND experiments in terms of neutrino mixing and oscillations one needs to assume the existence of sterile neutrinos and mixing of (at least) four massive neutrinos (see, e.g., [11]). The LSND result will be checked by the MiniBooNE experiment [12] scheduled to start in 2002. We will consider in what follows only the case of three-neutrino mixing and oscillations.

The analyses of the Super-Kamiokande atmospheric neutrino data suggest that the atmospheric $\nu_\mu$ ($\overline{\nu}_\mu$) oscillate predominantly into $\nu_\tau$ ($\overline{\nu}_\tau$) and that these oscillations are induced by a neutrino mass squared difference $\Delta m^2_{atm}$ having a value in the range $[1]$ (99% C.L.)

$$1.3 \cdot 10^{-3} \text{ eV}^2 \lesssim \Delta m^2_{atm} \lesssim 5.0 \cdot 10^{-3} \text{ eV}^2. \quad (2)$$

Global analyses of the solar neutrino data including the SNO results [7] were performed, e.g., in [13, 14, 15, 16, 17]. The data were shown to favor the large mixing angle (LMA) MSW, the LOW and the quasi-vacuum oscillation (QVO) solutions of the solar neutrino problem. In the case of the LMA solution, the range of values of the corresponding neutrino mass-squared difference was found in [13] and [14] to extend (at 99% C.L.) up to values of $\Delta m^2_{\odot}$ as large as $\sim 5.0 \cdot 10^{-4} \text{ eV}^2$ and $\sim 8.0 \cdot 10^{-4} \text{ eV}^2$, respectively:

$$\text{LMA MSW [13, 14]} : \quad 2.0 \cdot 10^{-5} \text{ eV}^2 \lesssim \Delta m^2_\odot \lesssim (5.0 - 8.0) \cdot 10^{-4} \text{ eV}^2. \quad (3)$$

The most stringent constraints on the oscillations of electron (anti-)neutrinos were obtained in the CHOOZ [18] and Palo Verde [19] long baseline disappearance experiments with reactor $\overline{\nu}_e$. These constraints play a significant role, in particular, in our current understanding of the possible patterns of oscillations of the three flavour neutrinos and anti-neutrinos, $\nu_l$ and $\overline{\nu}_l$, $l = e, \mu, \tau$. The CHOOZ and Palo Verde experiments are sensitive to values of neutrino mass squared difference $\Delta m^2 \gtrsim 10^{-3} \text{ eV}^2$, which includes the corresponding atmospheric neutrino region, eq. (2). No disappearance of the reactor $\overline{\nu}_e$ was observed. This rules out the possibility of significant $\nu_\mu \leftrightarrow \nu_e$ ($\overline{\nu}_\mu \leftrightarrow \overline{\nu}_e$) oscillations of the atmospheric $\nu_\mu$ ($\overline{\nu}_\mu$), and $\nu_e$ ($\overline{\nu}_e$), which is consistent with the Super-Kamiokande atmospheric neutrino data [6]. Performing a two-neutrino oscillation analysis, the following upper bound on the value of the corresponding mixing angle, $\theta$, was obtained by the CHOOZ collaboration [4, 18] at 90% C.L. for $\Delta m^2 \geq 1.5 \times 10^{-3}\text{eV}^2$:

$$\sin^2 \theta < 0.09. \quad (4)$$

\footnote{The non-electron neutrino component in the flux of solar neutrinos can also include, or correspond to, $\overline{\nu}_\mu$ and/or $\overline{\nu}_\tau$.}

\footnote{The second possibility - of large $\sin^2 \theta > 0.9$, which is admitted by the CHOOZ data alone, is incompatible with the neutrino oscillation interpretation of the solar neutrino deficit (see, e.g., [20, 11]).}
The precise upper limit in eq. (4) is $\Delta m^2$-dependent. It is a decreasing function of $\Delta m^2$ as $\Delta m^2$ increases up to $\Delta m^2 \simeq 6 \cdot 10^{-3}$ eV$^2$ with a minimum value $\sin^2 \theta \simeq 10^{-2}$. The upper limit becomes an increasing function of $\Delta m^2$ when the latter increases further up to $\Delta m^2 \simeq 8 \cdot 10^{-3}$ eV$^2$, where $\sin^2 \theta < 2 \cdot 10^{-2}$ (see Figs. 1 - 3). Somewhat weaker constraints on $\sin^2 \theta$ have been obtained by the Palo Verde collaboration [19].

The results of the two-neutrino oscillation analysis of the CHOOZ data remain essentially valid in the case of 3-neutrino mixing and oscillations, provided the two independent neutrino mass-squared differences characterizing the oscillations in this case, $\Delta m^2_{21} > 0$ and $\Delta m^2_{31} > 0$ (see further), obey the hierarchical relation:

$$\Delta m^2_{21} \ll \Delta m^2_{31}. \quad (5)$$

In this case one can make the identification:

$$\Delta m^2_{21} \simeq \Delta m^2_\odot, \quad \Delta m^2_{31} \simeq \Delta m^2_{\text{atm}}. \quad (6)$$

Under the conditions (5) and (6) and the constraint (2), the oscillations of the reactor $\nu_e$ due to $\Delta m^2_{31}$ cannot develop on the baselines of the CHOOZ and Palo Verde experiments. The corresponding 3-neutrino $\nu_e$ survival probability reduces to a two-neutrino survival probability with $\Delta m^2 = \Delta m^2_{31}$ and $\sin^2 \theta = |U_{e3}|^2$, where $U_{e3}$ is the element of the PMNS mixing matrix which couples the electron field to the heaviest neutrino field in the mixing relation eq. (1). Thus, we have in the case under discussion:

$$\sin^2 \theta = |U_{e3}|^2 = \sin^2 \theta_{13}, \quad (7)$$

$\theta_{13}$ being the one of the three mixing angles in the standard parametrization of the PMNS matrix (see, e.g., [21]). The other two angles, $\theta_{12}$ and $\theta_{23}$, defined through the relations

$$|U_{e1}| = \cos \theta_{12} \sqrt{1 - |U_{e3}|^2}, \quad |U_{e2}| = \sin \theta_{12} \sqrt{1 - |U_{e3}|^2}, \quad (8)$$

and

$$|U_{\mu 3}| = \sin \theta_{23} \sqrt{1 - |U_{e3}|^2}, \quad |U_{\tau 3}| = \cos \theta_{23} \sqrt{1 - |U_{e3}|^2}, \quad (9)$$

control the oscillations of the solar $\nu_e$ and of the atmospheric $\nu_\mu$ ($\nu_\tau$), respectively. The upper limit (4) corresponds to:

$$|U_{e3}|^2 = \sin^2 \theta_{13} < 0.09. \quad (10)$$

Let us note that under the condition (3), the solar neutrino survival probability, $P_{\odot}^{3\nu}(\nu_\epsilon \to \nu_\epsilon)$, which is relevant for the interpretation of the solar neutrino data in terms of neutrino oscillations, depends on $\Delta m^2_{21}$, $\theta_{12}$ and $|U_{e3}|^2$ (or $\theta_{13}$) (see, e.g., [22]):

$$P_{\odot}^{3\nu}(\nu_\epsilon \to \nu_\epsilon) \equiv (1 - |U_{e3}|^2)^2 P_{\odot}^{2\nu}(\nu_\epsilon \to \nu_\epsilon) + |U_{e3}|^4, \quad (11)$$

where $P^{(2\nu)}(\nu_\epsilon \to \nu_\epsilon)$ is the solar $\nu_\epsilon$ survival probability in the case of two-neutrino $\nu_\epsilon \to \nu_\mu(\tau)$ transitions due to $\Delta m^2_{21}$ and $\theta_{12}$ [24], in the expression for which the solar matter potential in the standard two-neutrino case, $V_\odot = \sqrt{2}G_F N_\odot$, $N_\odot$ being the solar electron number density, is replaced by $(1 - |U_{e3}|^2)V_\odot$. The dependence of $P_{\odot}^{2\nu}(\nu_\epsilon \to \nu_\epsilon)$ on $\theta_{13}$, however, is rather weak and cannot be used to derive more stringent upper limit on $\sin^2 \theta_{13}$ than that obtained in the CHOOZ experiment, e.g., eq. (10). Similar conclusion is valid for the upper bounds on $|U_{e3}|^2$ which can be derived from the Super-Kamiokande atmospheric neutrino data (see, e.g., [24]).

The element of the PMNS matrix $U_{e3}$, more precisely, its absolute value $|U_{e3}| = \sin \theta_{13}$, plays a very important role in the phenomenology of the 3-neutrino oscillations. It drives the sub-dominant
\[ \nu_\mu \leftrightarrow \nu_e \ (\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e) \] oscillations of the atmospheric \( \nu_\mu \) (\( \bar{\nu}_\mu \)) and \( \nu_e \) (\( \bar{\nu}_e \)) \cite{23}. The value of \( |U_{e3}| \) (or \( \theta_{13} \)) controls also the \( \nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e, \nu_\bar{e} \rightarrow \nu_\mu \) and \( \bar{\nu}_\bar{e} \rightarrow \bar{\nu}_\mu \) transitions in the long baseline neutrino oscillation experiments (MINOS, CNGS), and in the widely discussed very long baseline neutrino oscillation experiments at neutrino factories (see, e.g., \cite{29}). The magnitude of the T-violation and CP-violation effects in neutrino oscillations is directly proportional to \( |U_{e3}| = \sin \theta_{13} \) (see, e.g., \cite{29}). Thus, the fundamental question about the T- and CP-violation in the lepton sector can be investigated in neutrino oscillation experiments only if \( |U_{e3}| \) is sufficiently large. If the neutrinos with definite mass are Majorana particles (see, e.g., \cite{23}), the predictions for the effective Majorana mass parameter in neutrinoless double \( \beta \)-decay depend in the case of hierarchical neutrino mass spectrum on the value of \( |U_{e3}|^2 \) (see, e.g., \cite{29}). The knowledge of the value of \( |U_{e3}| \) is crucial for the searches for the correct theory of neutrino masses and mixing as well.

In the present article we present results of a new analysis of the CHOOZ data which limit \( |U_{e3}|^2 \). The analysis is made in the framework of the three-neutrino mixing hypothesis. The parameters characterizing the solar neutrino oscillations, \( \Delta m^2_\odot \), and the neutrino mixing angle \( \theta_\odot \), are assumed to lie in the region of the LMA MSW solution of the solar neutrino problem. In the case of neutrino mass spectrum with normal hierarchy, for instance, we have, \( \Delta \rightarrow \) lies in the region of the LMA MSW solution of the solar neutrino problem. In the case of \( \Delta \)

\[ \Delta m^2_\odot \] characterizing the solar neutrino oscillations, \( \Delta \)

As we have already briefly discussed, the LMA MSW solution admits values of \( \Delta m^2_\odot \) as large as \( \sim (5-8) \cdot 10^{-4} \text{ eV}^2 \) \cite{13, 14}. For such values of \( \Delta m^2_\odot \) the oscillations of the reactor \( \bar{\nu}_e \) due to \( \Delta m^2_\odot \)

can develop on the baselines of the CHOOZ and Palo Verde experiments and the contributions of the “solar” (\( \Delta m^2_\odot \) and \( \theta_\odot \)) terms in the reactor \( \bar{\nu}_e \) survival probability can be sizeable. The hierarchical relation (3) is not valid in a large part of the interval of allowed values of \( \Delta m^2_{\text{atm}} \) (e.g., eq. (2)) as well. As a consequence, the limits on \( |U_{e3}|^2 \), obtained in the two-neutrino oscillation analyses of the CHOOZ and Palo Verde data are not valid in the case of 3-neutrino oscillations if \( \Delta m^2_\odot \) lies in a rather large sub-region of the LMA MSW solution region. Using the CHOOZ data, we obtain the 3-neutrino oscillation limits on \( |U_{e3}|^2 \) with values of \( \Delta m^2_\odot \) and \( \sin^2 \theta_\odot \) in the LMA MSW allowed region. We find that for \( \Delta m^2_\odot \gtrsim 2 \cdot 10^{-4} \text{ eV}^2 \) these limits are more stringent than the limits obtained from the original analysis of the CHOOZ data made in the framework of the two-neutrino mixing.

2 Three-Neutrino Oscillations of Reactor \( \bar{\nu}_e \)

We shall number (without loss of generality) the neutrinos with definite mass in vacuum \( \nu_j \), \( j = 1, 2, 3 \), in such a way that their masses obey \( m_1 < m_2 < m_3 \). With this choice one has \( \Delta m^2_{jk} > 0 \) for \( j > k \). We do not assume that the hierarchy relation (3) is valid in what follows.

Consider first the case of “normal hierarchy” between the neutrino masses, in which

\[ \Delta m^2_{21} \simeq \Delta m^2_\odot. \] (12)

The neutrino mixing angle which controls the solar neutrino oscillations and is determined in the analyses of the solar neutrino data, \( \theta_\odot \), coincides in this case with \( \theta_{12} \):

\[ \theta_\odot \simeq \theta_{12}. \] (13)

For \( \Delta m^2_{\text{atm}} \) there exist two possibilities:

\[ \Delta m^2_{\text{atm}} \simeq \Delta m^2_{31}, \] (14)

or

\[ \Delta m^2_{\text{atm}} \simeq \Delta m^2_{32}. \] (15)

Let us note that even for \( \Delta m^2_\odot \simeq \Delta m^2_{31} \sim (4-8) \cdot 10^{-4} \text{ eV}^2 \), the corrections to the solar \( \nu_e \) survival probability, eq. (14), due to \( \Delta m^2_{\text{atm}} \) when the hierarchical relation \( \Delta m^2_\odot \ll \Delta m^2_{\text{atm}} \) does
not hold, are practically negligible and do not change the results of the analyses of the solar neutrino data based on the “hierarchical” expression for the probability.

The exact expression for the survival probability of interest in the case of the three neutrino mixing can be written in the form

$$P(\nu_e \rightarrow \nu_e) = 1 - 2 |U_{e3}|^2 (1 - |U_{e3}|^2) \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E} \right)$$

$$- \frac{1}{2} (1 - |U_{e3}|^2)^2 \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E} \right)$$

$$+ 2 |U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2 \theta_\odot \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E} - \frac{\Delta m_{21}^2 L}{2 E} \right) - \cos \frac{\Delta m_{21}^2 L}{2 E} \right)$$

(16)

In deriving the expression for $P(\nu_e \rightarrow \nu_e)$ we have utilized eqs. (8) and (13).

The second term in the right-hand side of the expression for $P(\nu_e \rightarrow \nu_e)$, eq. (16), correspond to oscillations due to $\Delta m_{32}^2$ atm, the third one is a “solar neutrino oscillation” term, and the fourth one is an interference term between the amplitudes corresponding to the first three. Let us notice that the coefficient in front of the bracket of the “solar neutrino oscillation” term in eq. (16) is relatively large in the LMA MSW solution region, while the coefficient in front of the bracket of the fourth term is of the same order as coefficient in front of the bracket in the main second term.

In what follows we will present constraints on $|U_{e3}|^2$ as a function of $\Delta m_{31}^2$ for a number of fixed values of $\Delta m_{21}^2 \approx \Delta m_{\odot}^2$ and $\sin^2 \theta_{12} \approx \sin^2 \theta_\odot$.

In the case of “inverted hierarchy” between the neutrino masses one has:

$$\Delta m_{32}^2 \approx \Delta m_{\odot}^2.$$  

(17)

Now $|U_{e2}|$ and $|U_{e3}|$ are related to the mixing angle which controls the solar neutrino oscillations $\theta_\odot$:

$$|U_{e2}| = \cos \theta_\odot \sqrt{1 - |U_{e1}|^2}, \quad |U_{e3}| = \sin \theta_\odot \sqrt{1 - |U_{e1}|^2}. $$

(18)

There are again two possibilities for $\Delta m_{atm}^2$: the first coincides with that in eq. (14), while the second is given by:

$$\Delta m_{atm}^2 \approx \Delta m_{21}^2.$$  

(19)

The expression for the survival probability of interest can be written in the following form:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 2 |U_{e1}|^2 (1 - |U_{e1}|^2) \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E} \right)$$

$$- \frac{1}{2} (1 - |U_{e1}|^2)^2 \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E} \right)$$

$$+ 2 |U_{e1}|^2 (1 - |U_{e1}|^2) \cos^2 \theta_\odot \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E} - \frac{\Delta m_{21}^2 L}{2 E} \right) - \cos \frac{\Delta m_{21}^2 L}{2 E} \right),$$

(20)

where we have utilized eq. (18).

One comment is in order. Note that, apart from the fact that the role of $|U_{e3}|^2$ in eq. (16) is played by $|U_{e1}|^2$ in eq. (20), the coefficients in front of the last interference terms in eqs. (16) and (20) differ: in the case of eq. (16) it contains the factor $\sin^2 \theta_\odot$, while the analogous factor in
eq. (20) is $\cos^2 \theta$ ⊙. This implies that for $\sin^2 \theta$ ⊙ = $\cos^2 \theta$ ⊙, the constraints from the CHOOZ and Palo Verde data on $|U_{e3}|^2$ in the case of neutrino mass spectrum with “normal hierarchy” will be equivalent to the constraints on $|U_{e1}|^2$ in the case of spectrum with “inverted hierarchy”. However, these constraints will differ if $\sin^2 \theta$ ⊙ ≠ $\cos^2 \theta$ ⊙. The best fit point of the solar neutrino data in the LMA MSW solution region, for instance, corresponds to $\sin^2 \theta$ ⊙ = 0.27 and $\cos^2 \theta$ ⊙ = 0.73. Consequently, the upper bound on $|U_{e3}|^2$ in the “normal hierarchy” case can differ from the upper bound on $|U_{e1}|^2$ in the case of spectrum with “inverted hierarchy”.

3 New Constraints on $|U_{e3}|^2$ from the CHOOZ Data

Taking into account the possibility of relatively large values of $\Delta m^2_\odot$, which are allowed in the case of the most favorable by the data LMA solution of solar neutrino problem, we have re-analyzed the CHOOZ data and have obtained exclusion curves that took into account the contribution of the “solar terms” in the reactor $\bar{\nu}_e$ survival probability (eq. (16) and eq. (20)). We will present results derived for two values of the solar neutrino mixing angle, $\sin^2 \theta$ ⊙ = 0.5; 0.27, and for $\Delta m^2_\odot$ = 0; 2 · $10^{-4}$; 4 · $10^{-4}$; 6 · $10^{-4}$ eV$^2$. All these values of $\sin^2 \theta$ ⊙ and $\Delta m^2_\odot$ lie in the region of the LMA MSW solution of the solar neutrino problem [13, 14, 15, 16, 17].

3.1 The CHOOZ Data and their Statistical Analysis

The CHOOZ experiment, just like the other reactor experiments, detected the $\bar{\nu}_e$’s produced by each reactor of the homonym power plant through the reaction

$$\bar{\nu}_e + p \rightarrow n + e^+ \quad (21)$$

The $e^+$ energy deposit in the detector is strongly correlated with the incoming $\bar{\nu}_e$ energy (the correlation being linear as long as the proton recoil can be neglected). An evidence for neutrino oscillations in CHOOZ could thus result from:

1. a deficit of the $\bar{\nu}_e$ detection rate with respect to expectations;
2. distortion in the ratio of the measured positron spectrum versus predictions.

The most powerful test (analysis “A” [18]) combine both pieces of information. The event sample is divided into seven $e^+$ energy bins ranging from 0.8 to 6.4 MeV; for each energy bin, the ramp-up and down of reactors during the data taking period allowed to subtract the background as well as to extract the contribution of each reactor to the signal rate [31]. As a result, we have two $e^+$ experimental spectra at our disposal, to be compared with the predictions. The expected positron spectrum is obtained by Monte Carlo simulation of the detector response, after folding the no-oscillation $\bar{\nu}_e$ flux with the neutrino survival probability (eq. (14) or (20) depending on whether normal or inverted hierarchy is considered); so the positron yield for the k-th reactor and the j-th energy bin can be parametrized as follows:

$$\bar{\nu}(E_j, L_k, \theta, \Delta m^2_{31}) = \bar{Y}(E_j)\bar{P}(E_j, L_k, |U_{e3}|^2, \Delta m^2_{31}), \quad (j = 1, ..., 7, \ k = 1, 2) \quad (22)$$

where $\bar{Y}(E_j)$ is the distance-independent positron yield in the absence of neutrino oscillations, $L_k$ is the reactor-detector distance and the last factor represents the survival probability averaged over the energy bin and the finite detector and reactor core size [31]. Both measured and predicted yields can then be arranged into 14-element arrays and, in order to test the compatibility of a certain
oscillation hypothesis (|U_{e3}|^2, \Delta m_{31}^2) with measurements, we can build the following \( \chi^2 \) function
\[
\chi^2(|U_{e3}|^2, \Delta m_{31}^2, \alpha, g) = \sum_{i,j=1}^{14} \left( Y_i - \alpha \overline{Y}(gE_i, L_i, |U_{e3}|^2, \delta m_{31}^2) \right) V_{ij}^{-1} \left( Y_j - \alpha \overline{Y}(gE_j, L_j, |U_{e3}|^2, \delta m_{31}^2) \right) + \left( \frac{\alpha - 1}{\sigma_\alpha} \right)^2 + \left( \frac{g - 1}{\sigma_g} \right)^2 ,
\]
where \( V_{ij} \) is the error matrix, \( \alpha \) is the absolute normalization constant (\( \sigma_\alpha = 2.7\% \)), \( g \) is the energy-scale calibration factor (\( \sigma_g = 1.1\% \)), \( L_i = L_1 \) for \( i \leq 7 \) and \( L_i = L_2 \) for \( i > 7 \). The \( \chi^2 \) value for a certain parameter set is obtained by minimizing \( \chi^2 \) with respect to \( \alpha \) and \( g \).

Confidence intervals at 90% C.L. can be obtained by using a frequentist approach, according to the Feldman and Cousins prescription \( [5] \). The “ordering” principle is based on the logarithm of the ratio of the likelihood functions for the two cases:
\[
\lambda(|U_{e3}|^2, \Delta m_{31}^2) = \chi^2(|U_{e3}|^2, \Delta m_{31}^2) - \chi^2_{\text{min}}
\]
where the minimum \( \chi^2 \) value must be searched for within the physical domain (0 < |\( U_{e3} \)| < 1, \( \Delta m_{31}^2 > 0 \)). Smaller \( \lambda \) values imply a better agreement of the hypothesis with the data. The \( \lambda \) distribution for the given parameter set was evaluated by performing a Monte Carlo simulation of a large number (5000) of experimental positron spectra whose values are scattered around the predicted positron yields \( \overline{Y}(E_i, L_i, |U_{e3}|^2, \Delta m_{31}^2) \) with Gaussian-assumed variances. For each set we extracted the quantity \( \lambda_c(|U_{e3}|^2, \Delta m_{31}^2) \) such that 90% of the simulated experiments have \( \lambda < \lambda_c \).

The 90% confidence domain then includes all points in the \(|U_{e3}|^2, \Delta m_{31}^2\) plane such that
\[
\lambda_{\text{exp}}(|U_{e3}|^2, \Delta m_{31}^2) < \lambda_c(|U_{e3}|^2, \Delta m_{31}^2),
\]
where \( \lambda_{\text{exp}} \) is evaluated for the experimental data for each point in the physical domain.

The above procedure is then repeated to extract confidence intervals for different values of the solar neutrino oscillation parameters entering eqs. \( (16) \) and \( (20) \). The results are shown in Fig. 1 (normal hierarchy, \( \sin^2 \theta_\odot = 0.5 \)), Fig. 2 (same as before, with \( \sin^2 \theta_\odot = 0.27 \)) and Fig. 3 (inverted hierarchy, \( \sin^2 \theta_\odot = 0.27 \)). All parameters lying to the right of the contours are excluded by the CHOOZ data at 90% C.L., while the parameter regions to the left of the contours are compatible with the data.

### 3.2 The Results

As Figs. 1 - 3 show, the new exclusion curves are practically indistinguishable from the curves found in the two-neutrino oscillation analysis of the CHOOZ data if \( \Delta m_\odot^2 \lesssim 10^{-4}\text{eV}^2 \). Thus, the CHOOZ bound on \(|U_{e3}|^2\) is stable with respect to corrections due to the “solar” terms as long as \( \Delta m_\odot^2 \) does not exceed \( \sim 10^{-4}\text{eV}^2 \).

The 3-neutrino oscillation analysis leads, however, to more stringent constraints than the two-neutrino oscillation analysis if \( \Delta m_\odot^2 \gtrsim 2 \times 10^{-4} \text{eV}^2 \), as it is seen in Figs. 1 - 3. For \( \Delta m_\odot^2 \approx (4-6) \cdot 10^{-4} \text{eV}^2 \) we find that the upper bounds on \(|U_{e3}|^2\) (\(|U_{e1}|^2\)) are considerably more stringent than those derived in the two-neutrino oscillation analysis of the CHOOZ data. The upper bounds of interest depend on \( \sin^2 \theta_\odot \). At \( \Delta m_{31}^2 = 2.5 \cdot 10^{-2} \text{eV}^2 \), which is the best fit value of the Super-Kamiokande atmospheric neutrino data, and for \( \Delta m_\odot^2 = 4.0 \cdot 10^{-4} \text{eV}^2 \), we get for \( \sin^2 \theta_\odot = 0.50 (0.27) \):
\[
|U_{e3}|^2 \leq 2.9 \cdot 10^{-2} (3.0 \cdot 10^{-2}).
\]
For \( \Delta m_\odot^2 = 6.0 \cdot 10^{-4} \text{eV}^2 \), and \( \sin^2 \theta_\odot = 0.50 \) the same limit reads:
\[
|U_{e3}|^2 \leq 1.7 \cdot 10^{-2},
\]
while if $\sin^2 \theta_\odot = 0.27$, we get $|U_{e3}|^2 \leq 2.0 \cdot 10^{-2}$. The limits on $|U_{e3}|^2$ ($|U_{e1}|^2$ in the case of the inverted hierarchy) for different values of solar neutrino oscillation parameters and different values of $\Delta m^2_{31}$ are presented in Table 1.

Table 1: Limits on the $\nu_e$ mixing parameter $|U_{e3}|^2$ ($|U_{e1}|^2$ in the case of inverted hierarchy) for three values of $\Delta m^2_{31}$ and for different values of solar neutrino oscillation parameters.

| $\Delta m^2_{31}$ (eV$^2$) | $\Delta m^2_{\odot}$ (eV$^2$) | $|U_{e3}|^2$ ($\sin^2 \theta_\odot = 0.5$) | $|U_{e3}|^2$ ($\sin^2 \theta_\odot = 0.27$) | $|U_{e1}|^2$ ($\sin^2 \theta_\odot = 0.27$) |
|-------------------------|-------------------|-----------------|-----------------|-----------------|
| $2.5 \cdot 10^{-3}$    | 0                 | 3.7 $\cdot 10^{-2}$ | 3.6 $\cdot 10^{-2}$ | 3.8 $\cdot 10^{-2}$ |
|                         | 2 $\cdot 10^{-4}$ | 3.6 $\cdot 10^{-2}$ | 3.6 $\cdot 10^{-2}$ | 3.8 $\cdot 10^{-2}$ |
|                         | 4 $\cdot 10^{-4}$ | 2.9 $\cdot 10^{-2}$ | 3.0 $\cdot 10^{-2}$ | 3.5 $\cdot 10^{-2}$ |
|                         | 6 $\cdot 10^{-4}$ | 1.7 $\cdot 10^{-2}$ | 2.0 $\cdot 10^{-2}$ | 2.6 $\cdot 10^{-2}$ |
| $10^{-2}$               | 0                 | 3.6 $\cdot 10^{-2}$ | 3.4 $\cdot 10^{-2}$ | 3.4 $\cdot 10^{-2}$ |
|                         | 2 $\cdot 10^{-4}$ | 3.4 $\cdot 10^{-2}$ | 3.4 $\cdot 10^{-2}$ | 3.4 $\cdot 10^{-2}$ |
|                         | 4 $\cdot 10^{-4}$ | 2.8 $\cdot 10^{-2}$ | 2.9 $\cdot 10^{-2}$ | 2.9 $\cdot 10^{-2}$ |
|                         | 6 $\cdot 10^{-4}$ | 1.7 $\cdot 10^{-2}$ | 2.1 $\cdot 10^{-2}$ | 2.0 $\cdot 10^{-2}$ |
| $10^{-1}$               | 0                 | 2.5 $\cdot 10^{-2}$ | 2.3 $\cdot 10^{-2}$ | 2.3 $\cdot 10^{-2}$ |
|                         | 2 $\cdot 10^{-4}$ | 2.3 $\cdot 10^{-2}$ | 2.3 $\cdot 10^{-2}$ | 2.3 $\cdot 10^{-2}$ |
|                         | 4 $\cdot 10^{-4}$ | 2.0 $\cdot 10^{-2}$ | 2.1 $\cdot 10^{-2}$ | 2.1 $\cdot 10^{-2}$ |
|                         | 6 $\cdot 10^{-4}$ | 1.2 $\cdot 10^{-2}$ | 1.6 $\cdot 10^{-2}$ | 1.6 $\cdot 10^{-2}$ |

The results from the ongoing experiments with solar neutrinos Super-Kamiokande, SNO, SAGE and GNO, as well as from the future experiment BOREXINO, will lead to a considerable progress in the searches for the correct and unique solution of the solar neutrino problem. The LMA MSW solution will be tested in detail by the KamLAND experiment. The results of these experiments will allow to make a more definite conclusion concerning the possibility of large “solar term” effects on the limits of the important parameter $|U_{e3}|^2$, that can be inferred from the data of the CHOOZ and Palo Verde experiments.

4 Conclusions

In conclusion, we have presented here the results of a new three-neutrino oscillation analysis of the CHOOZ data, which imposes stringent constraints on the oscillations of electron (anti-)neutrinos. The earlier obtained CHOOZ constraints are valid in the case of two-neutrino oscillations, as well as in the case of 3-neutrino oscillations if the neutrino mass-squared differences characterizing the oscillations of the solar and atmospheric neutrinos, $\Delta m^2_{\odot}$ and $\Delta m^2_{\text{atm}}$, obey the hierarchical relation $\Delta m^2_{\odot} \ll \Delta m^2_{\text{atm}}$. In such a way, stringent upper limits on the $|U_{e3}|^2$ element of the PMNS matrix have been obtained.

In our analysis the parameters characterizing the solar neutrino oscillations, $\Delta m^2_{\odot}$ and the neutrino mixing angle $\theta_\odot$, were assumed to lie in the region of the LMA MSW solution of the solar neutrino problem, which is favored by the current solar neutrino data, including the SNO results. This solutions admits values of $\Delta m^2_{\odot}$ as large as $\sim (5-8) \cdot 10^{-4}$ eV$^2$. For $\Delta m^2_{\odot} \gtrsim 2 \cdot 10^{-4}$ eV$^2$ the oscillations of the rector $\mathbf{U}_\odot$ due to $\Delta m^2_{\odot}$ can develop on the baselines of the CHOOZ and Palo Verde experiments and the contributions of the “solar” terms in the reactor $\mathbf{U}_\odot$ survival probability can be sizeable. In addition, for the indicated values of $\Delta m^2_{\odot}$ the hierarchical relation between the “solar” $\Delta m^2_{\odot}$ and the “atmospheric” $\Delta m^2_{\text{atm}}$, does not hold in a large part of the interval of allowed values of $\Delta m^2_{\text{atm}}$. As a consequence, the limits on $|U_{e3}|^2$, obtained in the previous neutrino oscillation analyses of the CHOOZ and Palo Verde data are not valid for $\Delta m^2_{\odot}$ lying in a rather
large sub-region of the LMA MSW solution region. We have obtained upper limits on $|U_{e3}|^2$ in the indicated sub-region of values of $\Delta m^2_{\odot}$.

We have considered also the case of “inverted hierarchy” between neutrino masses, which differs under the conditions of interest from the case of “normal hierarchy”. Limits on the corresponding element of the PMNS matrix, $|U_{e1}|^2$, in the case of neutrino mass spectrum with inverted mass hierarchy were obtained as well.

Our analysis was based on the comparison of the positron spectra measured in the CHOOZ experiment with that expected in the case of 3-neutrino neutrino oscillations. We have found, in particular, that at $\Delta m^2_{\odot} \lesssim 10^{-4}$ eV$^2$, the effect of the “solar” $\Delta m^2_{\odot}$ and $\theta_{13}$ on the $\nu_e$ survival probability is very small and the CHOOZ upper bound on $|U_{e3}|^2$ practically coincides with that obtained in the original two-neutrino mixing analysis. For $\Delta m^2_{\odot} \gtrsim 2 \cdot 10^{-4}$ eV$^2$, the constraints of the CHOOZ data on $|U_{e3}|^2$ are more stringent than in the case of $\Delta m^2_{\odot} \lesssim 10^{-4}$ eV$^2$. They depend on the value of $\sin^2 \theta_{13}$. For, e.g., $\sin^2 \theta_{13} = 0.50$ and $\Delta m^2_{\odot} \simeq 4 \cdot 10^{-4}$ (6 $\cdot$ 10$^{-4}$)eV$^2$, we find that at $\Delta m^2_{31} = 2.5 \cdot 10^{-3}$ eV$^2$ (the best fit-point of the Super-Kamiokande atmospheric neutrino data), one has $|U_{e3}|^2 < 2.9 \cdot 10^{-2}$ ($|U_{e3}|^2 < 1.7 \cdot 10^{-2}$), while for “large” $\Delta m^2_{31}$ we found $|U_{e3}|^2 < 2 \cdot 10^{-2}$ ($|U_{e3}|^2 < 1.2 \cdot 10^{-2}$). These upper bounds are approximately by a factor of 1.3 (2.2) smaller than the upper bound obtained in the two-neutrino mixing analysis of the CHOOZ data. Let us stress that the value of the parameter $|U_{e3}|^2$ ($|U_{e1}|^2$ ) is extremely important for the future Super Beam and Neutrino Factory programs. In particular, the possibility to investigate CP violation in the lepton sector at these facilities depends on the value of this parameter.

We expect that a three-neutrino oscillation analysis of the Palo Verde data, performed under the conditions of the analysis made in the present article, will lead at $\Delta m^2_{\odot} \gtrsim 2 \cdot 10^{-4}$ eV$^2$ to more stringent constraints on $|U_{e3}|^2$ (or $|U_{e1}|^2$) than the two-neutrino oscillation analysis of the Palo Verde data [1].

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\[
\sin^2(\theta_{12}) = 0.5, \text{ normal hierarchy}
\]

Figure 1: Exclusion plot (90\% C.L.) obtained in the case of normal neutrino mass hierarchy for maximum “solar” mixing angle \(\sin^2\Theta = 0.5\) and for values of \(\Delta m^2\) from the LMA solution region. The CHOOZ result obtained in the case of two-neutrino mixing is also shown (doubly thick solid line).
\[ \sin^2(\theta_{12}) = 0.27, \text{ normal hierarchy} \]

**Figure 2:** The same as in Fig. 1 for \( \sin^2 \theta_\odot = 0.27 \), corresponding to the best fit value of the “solar” mixing angle \( \theta_\odot = \theta_{12} \) in the LMA region.
\( \sin^2(\theta_{12}) = 0.27 \), inverted hierarchy

Figure 3: The same as in Fig. 2 in the case of inverted hierarchy.