Detecting Bypassing Micro Black Holes

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Abstract. Contemporarily, convincing evidence is lacking for the existence of micro black holes due to their small masses. In this paper, a specific situation is discussed in which a Schwarzschild micro black hole, with a mass of up to $10^{13}$ kg and hence the lifetime longer than the universe's age flies towards the Earth with the typical speed of an asteroid, $O(10 \text{ km/s})$. The analysis of the two-body problem shows that Earth's surface will be tidally affected only if the initial configuration space, namely space of the impact parameter and initial velocity, satisfies a very strict relationship. Even if the micro black hole directly goes through the Earth core, its small mass restricts its accretion rate so much that its accreted mass is negligible. We also consider the possibility of detecting the black hole optically by its optical distortion. Analysis of the size of its Einstein ring indicates that one would need accurate instruments to detect such an effect. Finally, we analyze the emission spectrum of the micro black hole by assuming the validity of the Hawking radiation and conclude that its unique energy spectrum might be detected and used as evidence of a bypassing micro black hole.

1. Introduction

There is also a theory that proposes that PBHs can replace the role of dark matter to some extent [1]. First, they are the solution of general relativity, so they are the only ones that can avoid creating a new elementary particle. The second reason is that their nature is highly restrictive. Finally, the parameter space for PBHs is inherently bounded at both the high-mass and low-mass ends. An obvious upper limit of mass is set by the observed astrophysical objects made of dark matter. Meanwhile, the LIGO interferometer has found several merging events of massive black holes in recent years [2]. The mass of these black holes is larger than that expected by the supernova explosion and the remnants of normal stellar evolution. Therefore, it can be inferred that LIGO may have found a group of massive black holes formed in the early universe. These PBHs may be the main components of dark matter in the universe.

According to Penrose and Hawking's theorem, suppose we consider the black hole problem under the condition of general relativity. In that case, there is a physical singularity in the center of the black hole. Obviously, it is impossible to obtain the solution of a black hole without singularity. Therefore, by referring to the nonlinear electrodynamics to obtain the mass parameters or using the Van der Waals fluid phase structure and Gibbs free energy diagram to study the dynamics of these phase transitions, we can explore the deeper dynamics and thermodynamics of the black hole.

In this paper, we discuss the possibility of detecting a micro black hole, with mass $M \sim O(10^{13} \text{ kg})$, from its gravitational and optical effects and eliminating the potential concern that such a black hole may have a catastrophic effect on Earth.
2. Analysis
In the analysis, we made the following assumptions.

1) We assume the micro black hole mass to be at the order of $M \sim 10^{13}$ kg and take its initial velocity to be similar to asteroids ($v \sim O(10 \text{ km/s})$. The reason will be explained in section 2.1.

2) We assume Bondi accretion onto the micro black hole when it goes through the Earth and the atmosphere. The Bondi accretion sets a maximum accretion rate for the micro black hole. The Eddington accretion rate can also be used, but we want to set an upper bound on the accretion rate, so the Bondi accretion rate will be applied.

3) We assume the black hole to be a Schwarzschild black hole. However, we are still having a special two-body problem in generality, in which the black hole has a smaller mass. Damour shows that to the first order in $G$, a gravitationally interacting two-body system is equivalent to the relativistic dynamics of an effective test particle of reduced mass

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}$$

moving in a Schwarzschild geometry:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) + O(G^2),$$

where $M = m_1 + m_2$ [3].

4) We assume the rest frame of the Earth to be an inertial reference frame. Within the time of the interaction, the effect of centripetal force and Coriolis force will be negligible.

2.1. The Choice of Mass Range based on the Hawking Radiation
Taking into quantum effect into the analysis of black holes, Hawking theorized out that the creation of particle-antiparticle pair in the vicinity of the event horizon can cause the black hole to radiate energy out and, therefore, mass loss [4]. Under the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) + O(G^2)$$

a black hole with mass $M$ evaporates at a rate

$$\dot{M} = -\frac{\hbar c^4}{15360 \pi G^2 M^3},$$

where $\hbar$ is reduced Planck’s constant, $c$ is the speed of light, and $G$ is the gravitational constant. Given that the black hole has an initial mass $M_0$ at $t = 0$, Eq. 3 can be solved to give

$$M(t) = \left(M_0^3 - \frac{\hbar c^4}{5120 \pi G^2}\right)^{1/3}.$$

This means that the black hole with initial mass $M_0$ has a lifetime

$$\tau(M_0) = \frac{5120 \pi G^2}{\hbar c^4} M_0^3 \propto M_0^3.$$

Hence, there is a lower bound of the micro black hole mass in this analysis: micro black holes with a small mass will fully evaporate before interacting with the Earth. One can estimate the mass range. Suppose that a micro black hole has a velocity $v_\infty \sim 10 \text{ km/s}$ relative to Earth (the typical velocity of an asteroid) and that it has an impact parameter $\xi \sim R_\oplus$. To set a safe upper limit, suppose further that the significant gravitational interaction takes place at $r \sim 100R_\oplus$. Then, under the assumption of hyperbolic trajectory, the encounter time is

$$T \leq 2 \frac{M}{\mu} \int_{r_0}^{100R_\oplus} \frac{-dr}{\sqrt{E-U_{eff}(\xi,v_\infty,r)}} \sim 10^5 \text{ s.}$$
We want $\tau \gg T$, so $M \gg 10^7$ kg.

Meanwhile, Page and Hawking studied the isotropic $\gamma$-ray background around 100MeV and placed the upper limit of the number density of primordial black holes with an initial mass of $10^{12}$ kg at $10^4$ pc$^{-3}$ on average and $10^{10}$ pc$^{-3}$ in the halos of galaxies [5]. Micro black holes usually have mass $M < 10^{-19}M_\odot \sim 10^{11}$ kg and the formation mechanism of a micro black hole in the early universe restrict the number of micro black holes with larger mass. A micro black hole with the age of the universe has mass $M \sim 10^{11}$ kg. However, we will place a safe upper limit of $M \sim \mathcal{O}(10^{10}$ kg).

2.2. Gravitational Scattering
In this section, we consider how the micro black hole does not have direct contact with the Earth material. This means that the black is scattered off from the Earth and that the impact parameters $\xi$ is larger than the radius of the earth:

$$\xi > R_\oplus.$$ (6)

We will ignore the micro black holes’ interaction with the Earth atmosphere because the Bondi accretion rates set an upper limit at about

$$r = \frac{\dot{M}_{\text{BH}}}{M} \sim 10^{-21} \text{ s}^{-1},$$ (7)

and the trajectory of the black hole will not be affected by air molecules. Because of Eq. 3, the problem may now be reduced to a test particle of mass

$$\mu = \frac{M M_\oplus}{M + M_\oplus} \approx M$$

moving along the geodesics of the Schwarzschild geometry in Eq. 2 with $M = M + M_\oplus \approx M_\oplus$. An effective potential may then be found:

$$U_{\text{eff}}(r) = -\frac{GM\mu}{r} + \frac{L^2}{2\mu r^2} \left(1 - \frac{r_s}{r}\right).$$ (8)

In our case, $r_s \approx 8.9$ mm, so a Newtonian approximation is reasonable:

$$U_{\text{eff}}(r) \approx -\frac{GM\mu}{r} + \frac{L^2}{2\mu r^2} \approx -\frac{GM_\oplus M}{r} + \frac{L^2}{2M r^2}.$$ (9)

Now, the problem is reduced to a classical two-body problem with reduced mass $\mu \approx M$. The geometric solution to the Newtonian two-body problem is

$$r(\phi) = \frac{r_0}{1 - \varepsilon \cos(\phi - \phi_0)},$$ (10)

where the eccentricity

$$\varepsilon = 1 - \sqrt{1 + \frac{2EL^2}{g^2M_\oplus^2M^2\mu}}.$$ (11)

determines the type of trajectory. In our case, the angular momentum is $L \approx M_\xi v_0$ (where $\xi$ is the impact parameter), so $\varepsilon > 0$, and therefore the trajectory is hyperbolic (in the CM frame). Given $M \ll M_\oplus$, it is the micro black hole that moves, whereas the Earth stays at the origin of the CM frame.

In reality, the Earth is not a solid body; it could be affected by the tidal forces from the black hole. The tidal forces originate from the difference in the gravitational forces at different locations. In our case, we will consider the differences between the gravitational attraction on the mass element $\delta M$ at the center of the Earth and that at the closet point to the black hole:
The tidal disruption occurs when the tidal force is comparable in size with Earth’s self-gravitational pull, so we have the condition
\[ \delta M \approx \frac{GM}{(\rho - R_{\oplus})^2} \approx \delta M \frac{GM_{\oplus}}{R_{\oplus}^2}. \] (13)

This requires the closest approach \( \rho \) to satisfy
\[ \rho \approx R_{\oplus}. \] (14)

In other words, tidal disruption only occurs when the black hole is close to the Earth's surface. In the case of scattering, this has already shown that the gravitational attraction from a 10^{13}-kg micro black hole on the Earth is almost negligible.

The micro black hole dissipates energy to the Earth in the forms of sound waves and gravitational waves, in a total of about \( \Delta E \approx 4 \times 10^9 \) J [6]. This is negligible compared to the kinetic energy carried by the black hole at a typical speed of \( v_{\infty} \approx 10 \) km/s (so \( T \approx 10^{21} \)). In other words, to a very good approximation, mechanical energy is a constant of motion. One can show that the closest approach has given the impact parameter \( \xi \) and initial (asymptotic) velocity \( v_{\infty} \) is
\[ r_0 = \frac{\sqrt{G^2M_{\oplus} + \xi^2v_{\infty}^4 - GM_{\oplus}}}{v_{\infty}^2}. \] (15)

We just showed that tidal disruption occurs when \( r_0 < \rho \), which requires the velocity to satisfy
\[ v_{\infty} < \frac{\sqrt{2GM_{\oplus}R_{\oplus}}}{\xi^2 - R_{\oplus}^2} \equiv v_{\text{crit}}. \] (16)

The blue shaded area in fig. 3 is the space of initial conditions of the micro black hole such that the Earth will be tidally disrupted. It indicates that the chance of tidal disruption is low: for a micro black hole with the typical velocity of a meteoroid, \( v_{\infty} \approx O(10 \) km), tidal disruption occurs only when the impact parameters are in the region \( \xi \in [R_{\oplus}, 2R_{\oplus}] \).

It is better to loosen the constraint of what counts as the tidal effect since we do not need an object to be tidal disrupt the Earth to detect it gravitationally. We simply need it to have a noticeable tidal effect on Earth's surface. We now consider the critical initial velocity of the black hole at which the tidal effect is at least 1% of the moon’s tidal effect on Earth. The tidal force from the moon is at the order of \( F/M \approx 10^{-6} \) m/s^2. Eq. 12 gives that an upper bound of the closet approach of such a tidal effect:
\[ \rho_T \approx 1.04R_{\oplus} \] (17)

This gives a critical velocity.
\[ v_{\text{crit}} = \frac{2GM_{\oplus}\rho_T}{\sqrt{\xi^2 - \rho_T^2}}. \] (18)

The orange region in fig. 1 shows the critical initial velocity for the tidal effect to be as significant as the moon as a function of impact parameter. From the figure, one can see that even this condition would be too strict on the impact parameter.
Hence, we can conclude that the mass of the micro black hole sets a strict range for the initial velocity: to create a tidal effect as strong as the tidal effect from the moon, the black hole has to have a relative velocity as low as \( v \sim 1 \text{ km/s} \) or have an impact parameter in the region \( \xi \in [R_\oplus, 2R_\oplus] \). However, neither condition is so probable in practice. Hence, equipment with high sensitivity may be needed to detect its existence when a micro black hole passes by the Earth.

2.3. Directly Passing through the Earth and Accreting Mass
We now consider the case in which the impact parameter is smaller than the Earth radius, i.e., \( \xi < R_\oplus \). In this case, the micro black hole will go through the Earth, possibly accreting some significant amount of mass since the Earth’s average density \( \rho_\oplus \approx 5.51 \times 10^3 \text{ kg/s} \) is much larger than the atmosphere. Without considering the intermolecular forces of the Earth material, we idealize the material as pressureless gas to find an upper limit. With the assumptions, the equation of motion for the black hole is

\[
p_r = F_r
\]

where

\[
p_r = \dot{M}(r) j + \frac{4\pi G^2 M(t)^2}{(c_\infty + v_\infty)^{3/2}} r(t)
\]

and

\[
F_r = -\frac{GM_\oplus}{R_\oplus^2} r(t) \Theta(R_\oplus - r(t)) \Theta(r(t) + R_\oplus) - \frac{GM_\oplus}{r(t)^2} \Theta(r(t) - R_\oplus) \Theta(-r(t) - R_\oplus)
\]

where

\[
\Theta(x) = \int_{-\infty}^{x} \delta(x') dx' = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}
\]

is the Heaviside-theta function. Then, the total accreted mass is

\[
\Delta M = \int_0^\tau \dot{M} dt = \dot{M} \tau
\]

where \( \tau \) is the duration that the black hole is inside the Earth. If we assume \( r(0) = R_\oplus \) and \( r(-\infty) = v_\infty \), then \( \tau \) is defined to satisfy \( r(\tau) = -R_\oplus \).
We resort to numerical calculations to solve Eq. 19. Figure 2 shows the trajectory of the micro black hole with four different asymptotic velocities, namely $v_\infty = 0.01 \text{ m/s}, 10 \text{ km/s}, 60 \text{ km/s},$ and 100 km/s, as well as the accreted mass ratio $r \equiv \Delta M/M$. It can be seen from the figure that even if the asymptotic velocity is as low as $v_\infty = 0.01 \text{ m/s}$, the accreted mass from the Earth is negligible. For a micro black of mass $M = 10^{13} \text{ kg}$, this amounts to $\Delta M \sim 10^{-7} \text{ kg}$. For a more common asymptotic velocity like 10 km/s, this is even smaller.

Therefore, due to its small mass, the micro black hole is unable to attract much mass from the Earth. It follows that even if a micro black hole straightly goes through the Earth, significant gravitational effects cannot be observed, let alone having any catastrophic impact on the Earth. However, Khriplovich et al. point out that a micro black hole can leave a distinctive pattern in crystalline material when it goes through the earth, depositing a dose of about $10^5 \text{ Gy}$ [8]. Nonetheless, we might not be able to catch its gravitation signal the moment a micro black hole goes through the Earth.

| $v_\infty$ | $r$ |
|------------|-----|
| 0.01 \text{ m/s} | $4.54 \times 10^{-20}$ |
| 10 \text{ km/s} | $3.63 \times 10^{-20}$ |
| 60 \text{ km/s} | $9.95 \times 10^{-21}$ |
| 100 \text{ km/s} | $6.07 \times 10^{-21}$ |

Figure 2. The trajectory of the micro black hole with $v_\infty = 0.01 \text{ m/s}, 10 \text{ km/s}, 60 \text{ km/s},$ and 100 km/s. Here, $r = \Delta M/M$ is the fraction of total accreted mass assuming Bondi accretion.

2.4. Optical Distortion

Excluding the detecting of finding a micro black hole via its gravitational field when it passes near the Earth, we now consider an optical approach. That is, we consider whether the deflection of light by the micro black hole is detectable when it is close to Earth. In a Schwarzschild geometry given by Eq. 2, the orbital equation of light is given by

$$\frac{d\phi}{dr} = \pm \frac{1}{r^2} \left[ \frac{1}{\xi^2} - \frac{1}{r^2} \left( 1 - \frac{2GM}{c^2 r} \right) \right]^{-1/2}$$

where $\xi$ is the impact parameter. Therefore, the deflection angle is

$$\delta \phi = 2 \left\{ \int_{r_0}^{\infty} \frac{1}{r^2} \left[ \frac{1}{\xi^2} - \frac{1}{r^2} \left( 1 - \frac{2GM}{c^2 r} \right) \right]^{-1/2} dr \right\} - \pi.$$

where $r_0$ is the closest point, which satisfies

$$\frac{1}{r_0^2} \left( 1 - \frac{2GM}{c^2 r_0} \right) = \frac{1}{\xi^2}.$$


From a perturbation for small \( \frac{GM}{c^2 \xi} \approx 2r_\circ / \xi \) (which is true because in this case \( \xi \sim R_\odot \gg r_\circ \)), the deflection angle is
\[
\delta \phi \approx \frac{4GM}{c^2 \xi}.
\] (25)

The deflection will cause a ring to be formed around the black hole, called the Einstein ring, and the angular separation gives its size
\[
\theta = \sqrt{\frac{4GM}{c^2 d_s d_L}} = \frac{2r_\circ}{d_s d_L}
\] (26)

where \( d_L \) is the distance between the observer and the black hole, \( d_S \) is the distance between the black hole and object, and \( d_{LS} \) is the distance between the object and the observer. Since the mass of the micro black hole is small, its Schwarzschild radius is small. For a micro black hole with mass \( M = 10^{13} \text{ kg} \), \( 2r_\circ \approx 2.97 \times 10^{-14} \text{ m} \). Therefore, we can make an approximation that \( d_{LS} \approx d_S + d_L \), so that \( \theta = \theta(d_S, d_L) \). Then, if we parametrize the two variables as \( d_S = \rho \cos \phi \) and \( d_L = \rho \sin \phi \), where \( \rho \in [0, \infty) \) and \( \phi \in [0, \frac{\pi}{2}] \), we get
\[
\theta(\rho, \phi) = \sqrt{\frac{(2r_\circ \cos \phi + \sin \phi)}{\rho \sin \phi \cos \phi}}.
\] (27)

We can see that as \( \rho \to 0, \theta \to \infty \). However, this is not practical, since it requires that the observer, the black hole, and the object are infinitesimally close to one another.

We can estimate how close the three objects have to be together for the deflection ring to be detectable. Given \( \rho \), it can be found that \( \theta \) is maximized when \( \phi = \pi/4 \), which means \( d_S = d_L \equiv d \). For the deflection to be resolvable by optical telescopes, we require at least \( \theta \sim 1'' \sim 10^{-6} \text{ rad} \). This would require \( d \sim 0.01 \text{ m} = 1 \text{ cm} \). In other words, in this case, the three objects are centimeters apart, which is a strict constraint. This means that to make observation possible, we would need more sensitive equipment. If the three objects of interest happen to be kilometers apart, we would need an optical instrument with a resolution of at least \( O(10^{-10}) \sim O(10^{-5''}) \), and increasing a factor of 10 in the distance would require the resolution angle to be reduced by a factor of 100 from Eq. 27.

2.5. Hawking Radiation

The mass that the black hole evaporates is released to the surroundings as radiations. Thus, the evaporating black has an effective temperature, given by
\[
T = \frac{\hbar c^3}{8\pi G k_B T}.
\] (28)

Therefore, when a micro black hole passes by, we might be able to detect its emission spectrum. The intensity of radiation as a function of its frequency given by
\[
L(\omega) = \frac{\omega^4}{4\pi^3 c^2} \left( \frac{\omega}{\hbar c} \right)^2 e^{\frac{\hbar c}{2k_B T}} - 1
\]
so its peak can be found by solving \( l'(\omega) = 0 \), which yields the peak frequency as a function of mass
\[
\omega_{\text{max}}(M) = \frac{c^2}{8\pi GM} \left( 3 + W(-3e^{-3}) \right) \approx \frac{2.82 \times 144 c^3}{8\pi GM},
\] (30)
where \( W(x) \) is the Lambert W function. For the black hole in hour analysis, \( M \sim O(10^{13} \text{ kg}) \), so \( \omega_{\text{max}} \sim O(10^{21} \text{ Hz}) \) which corresponds to a gamma ray peaked at energy \( E_{\text{max}} \sim O(1 \text{ MeV}) \). Given the effective energy \( T \sim 10^{10} \text{ K} \), which is rare on the Earth's surface or its ambient space, Eq. 29 might give a unique radiation spectrum to differentiate it from the background.

\(^1\) This also means that the assumption \( d \gg r_\circ \) also works.
3. Discussions and conclusion
The We have performed analyses for the two scenarios of a $10^{13}$ kg micro black hole encounter with the Earth. The following assumptions have been made.

- The micro black hole has a mass $M \sim O(10^{13}$ kg) and initial velocity $v \sim O(10$ km/s).
- The micro black hole accretes mass at a rate bounded by the Bondi accretion.
- The micro black hole is a Schwarzschild black hole.
- The reference frame of the study is inertial in the time scale of interaction.

Even if we set a very high upper limit, we showed that the gravitational interaction between the Earth and the micro black hole is not significant and requires special equipment with high sensitivity to determine its existence. When a micro black hole passes through the earth, due to its small mass, the micro black hole is unable to attract much mass from the Earth; the micro black hole with mass $M \sim O(10^{13}$ kg) can only absorb up to $\Delta M = Mr \sim O(10^{-17}$ kg) of Earth material. Hence, even if a micro black hole straightly passes through the Earth, significant gravitational effects cannot be observed.

In the future, we may use GW interferometers on the ground and in space to observe PBH. We will get more information about their properties and distribution in the universe to provide clues for finding their origin and the early evolution of the universe. Even though there is a lot of evidence of PBH as DM, PBH itself may be the cause of the observed superluminiscence X-ray source and SMBH and imbh in the center of galaxies and globular clusters alleviates the sub-structure and too big to fail problem of the standard CDM paradigm. Moreover, the exploration of the origin and properties of PBH will lead to many new cosmological problems, such as the formation of early galaxies; The evolution of X-ray and gamma scattering background; The mass and spin distribution of PBH [7].

Besides, we consider the possibility of detecting a bypassing black hole optically. It is concluded, based on the size of the Einstein ring, that one would either need a piece of sensitive optical equipment that can resolve an opening angle of at least $O\left(10^{-5}\text{''}\right)$, or be lucky enough that the black hole has just gone in front of equipment with a background just in the back to detect its deflection. Either condition presents a difficulty for us to detect a bypassing micro black hole at the current moment. Nonetheless, the unique gamma radiation spectrum may allow us to detect its existence when one micro black hole bypasses.

If micro black holes are detected, we can have a better understanding of quantum gravity and solve the mysteries of physics on the Planck scale.

References
[1] Carr, B. (1975) The primordial black hole mass spectrum. The Astrophysical Journal, 201:1–19.
[2] Abott B. et al. (2016) Observation of gravitational waves from a binary black hole merger. Phys. Rev. Lett., 116:061102.
[3] Damour T. (2016) Gravitational scattering, post-minkowskian approximation, and effective-one-body theory. Phys. Rev. Lett. D, 94: 104015.
[4] Hawking S. (1975) Particle creation by black holes. Communications in Mathematical Physics, 43:199–220.
[5] Page D. and Hawking S. (1976) Gamma rays from primordial black holes. The Astrophysical Journal, 206:1–7.
[6] Khriplovich I., Pomeransky A., Produit N., and Ruban G. (2008) Can one detect passage of a small black hole through the earth? Phys. Rev. D, 77: 064017.
[7] García-Bellido J. (2019) Primordial black holes and the origin of the matter–antimatter asymmetry. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 377:20190091.