Energy is probably the most frequently used quantity to characterize a system in physics. Despite this, for very good reasons, in certain types of (high energy physics) experiments energy is not often used to present the data. In these pages I will review why this is the case and what can be gained in these experiments using energy as a primary quantity to look at the data.

For a system of mass $m$ and momentum $p$ the total energy is $E = \sqrt{p^2 + m^2}$. In high energy physics is often the case that the mass of the system, typically a particle, can be neglected with respect to the momentum of this particles, that is to say in high energy physics we often deal with objects close to their ultra-relativistic regime. In these circumstances energy and momentum become synonymous as $E = p$. This is a first reason why theoretical as well experimental discussions in high energy physics are often carried out in terms of momentum. Further to it, in collider experiments such as those at the Large Hadron Collider at CERN the energy is so high that the colliding protons do not interact as single particles - they reveal their constituent particles each carrying an unknown fraction of the proton energy. As a consequence, collisions between proton constituents happen at unknown energies, making difficult to use energy to characterize these collisions. As a matter of fact in high energy physics experiments it is often exploited the fact that the colliding particles and their constituents travel at high speed along...
a well specified direction, so that it is safe to assume that any motion of the collision products in the perpendicular directions is the result of the interactions happened in the collision. This line of thinking lead in the past decades to development of a large body of research [1] about how to study the details of particles collisions looking only at the momentum in the direction perpendicular to the direction of the colliding protons. The observables that use only information from the momentum in the perpendicular direction are called transverse, as opposed to longitudinal. For momentum this division results in the definition $\vec{p} = \vec{p}_T + \vec{p}_L$, where traditionally $p_L = p_z$ and $p_T = \sqrt{p_x^2 + p_y^2}$ in cartesian coordinates. In these coordinates energy is given by $E = \sqrt{p_x^2 + p_y^2 + p_z^2}$, which is invariant under spatial rotations of the reference axes, but transforms under Lorentz transformations as 

$$E' = \gamma E + \gamma \sqrt{1 - \gamma^2 \cos^2 \theta} |\vec{p}|,$$

for a transformation of velocity parameter $\vec{\beta}$ directed with an angle $\theta$ with respect to the direction of the momentum $\vec{p}$ and resulting in a Lorentz factor $\gamma = 1/\sqrt{1 - |\vec{\beta}|^2}$. Under the same transformation momentum components $p_i$ transform as 

$$p_i' = \gamma p_i - \gamma \beta_i E,$$

therefore if the velocity of the Lorentz transformation is orthogonal to $\vec{p}$ the transformation has no effect on $\vec{p}$. This implies that a boost directed along the $z$ axis (that is the beam axis) will leave unchanged the transverse momentum $p_T$. Clearly the invariance of this quantity is another good reason for using transverse momentum for many discussion of physics at particle colliders.

Nevertheless, restricting experiments to just use transverse observables effectively throws away part of the information on the collisions that the experiments set out to study. Furthermore there are cases in which combining transverse and longitudinal observables allows to recover important kinematical properties that we are going to review in the following Sections for the case the energy.

These properties are somewhat standard knowledge of the cosmic ray physics practitioners and we will see how they have been rediscovered and extended in recent years in high energy particle physics and what prospect they still have for future applications.

2 History and cosmic rays peaks

The observation of particles hitting Earth from outer space has traditionally been a carrier of discoveries and surprise in particle physics. Indeed many particles have been discovered in the cosmic radiation before they could be produced in laboratories with accelerators. One question that was in debate in the early 1950s in the particle physics community was the composition of the natural radiation that can be measured in the atmosphere with growing intensity at higher altitudes, what we call today cosmic rays. This radiation is mostly the result of interactions of primary protons hitting the upper atmosphere layer giving rise to a “shower” of cosmic ray secondary products that proceed further towards ground level and beyond. In the early 1950s one experiment was carried out to identify the origin of photons that appeared in the cosmic radiation and concluded that these photons were the trace of the presence of $\pi^0$ in the cosmic radiation. The $\pi^0$, in modern view, are expected to be abundantly produced when the atmosphere is hit by protons, but their presence was not experimentally proven until the measurement of Carlson [2]. This experiment identified $\pi^0 \rightarrow \gamma \gamma$ decay by measuring the spectrum of $\gamma$ and observing that it had a peak at $m_{\pi^0}/2$, which was expected from relativistic kinematics for the distribution of the energy of massless decay products from a heavy particle decay.

The argument is very simple and short and can be found in Ref. [3] as well as in standard textbooks [4] of cosmic rays physics. It essentially states that if we observe all the decay products of a scalar particle that decays in to massless particles, a process we denote $M \rightarrow ab$, the distribution of the energy of either of the decay products, say $a$, has a peak at $m_M/2$ irrespective of the distribution of boosts of the decaying
particle $M$ in the frame in which we carry out the measurement. Given its simplicity it is not surprising that this observation has re-surfaced the literature on cosmic rays in more than one occasion, sometimes with extensions of the original argument and new applications to new domains [5][6].

In modern problems of cosmic rays physics, however, it seems that this property of energy distributions is of limited use. For the study of complicated air-showers resulting from protons hitting the atmosphere this type of characterization of the decay product of a particle are insufficient to achieve interesting results and more complex modeling of these phenomena is necessary. Still, the of $\pi^0$ characteristic peak in the photon spectrum has allowed the FERMI collaboration to claim the identification of $\pi^0$ in the gamma rays from sources known to be remnants of supernova explosions [7] - quite an achievement to find out particles identity observing their decay products from few thousand light-year distance! This example well represents the possible domain of application of energy peaks, as it shows how, despite the large distance, despite the uncertain conditions of production of the $\pi^0$ in the supernova remnant environment, and despite the propagation of the $\gamma$ from the source to us, the energy spectrum of these photons still carries enough information to allow to identify its parent particle. Further to it, we are identifying the parent particle observing just one of the photons decay product at a time. This is remarkable because if we had measured energy and direction of both photons for each $\pi^0 \rightarrow \gamma\gamma$ decay we would know from four-momentum conservation that the parent particle was a pion, we would simply observe that

$$ (p_{\gamma_1} + p_{\gamma_2})^2 = m_{\pi^0}^2. \quad (1) $$

On the contrary, looking the energy peak distribution enables us to make a statement on the origin of the photons despite our complete ignorance on the properties of the other photon produced in the decay.

3 Recent Applications in high energy physics

The property of the peaks of energy distributions well known for spinless particles such as the neutral pion has been recently extended to the case of particles with spin [3]. The key observation that allows to extend the result to particles with spin is that when decays are observed from a collection of particles that populates evenly all the polarization states, this collection of particles is statistically equivalent to a scalar. Therefore if one observes decays of a massive fermions belonging to a collection made equally of left-handed and right-handed massive fermions the energy distribution of the decay products is guaranteed to have the same peak as if the decaying fermions were scalars. Therefore for a fermion parent particle $F$ decaying into two massless particles

$$ F \rightarrow ab $$

the energy spectrum of either decay products has a peak at $m_F/2$ irrespective of the distribution of boosts of the decaying particle $F$ in the frame in which we carry out the measurement.

This observation enables the use of energy peak distributions in a completely new set of problems, especially in high energy particle physics experiments, where fundamental particles with spin, e.g. the quarks and leptons of the Standard Model, are studied.

In the following we will deal with decays in which both the final state particles have a non-vanishing mass, in such case the energy peak for the particle $b$ is predicted to arise at

$$ E_b^* = \frac{m_F^2 - m_a^2 + m_b^2}{2m_F}. \quad (2) $$

This is the master formula that we will use for all the applications to two-body decays.
4 New physics mass measurements

As highlighted in the previous section 2 about the historical uses of peaks of energy distributions, the presence of a peak in the energy distribution of a decay product is a consequence of general kinematical arguments. We have also highlighted how striking it is to be able to say anything about cosmic gamma rays that (i) have been produced in an environment of which we know so little details, and (ii) have traveled very long distances in a medium of rather unknown properties. The condition of uncertainty in which energy peaks properties have found successful application described above make them a very interesting tool for the study of new physics at particle colliders. In fact when approaching a newly discovered particle very little is known about it and is very useful to have tools to study its properties making very minimal assumptions. For this reason the first set of applications that have been studied in high energy physics for the application of properties of energy peaks have to do with the characterization of new physics, in particular putative new particles to be discovered in collider experiments.

A classic problem one faces when a new particle is discovered is to measure the mass of the new state. In most cases of particles discovered in the past decades this problem was almost the same as that of discovering the new particle itself. For example, observing a new peak in the ratio \( R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) \) when the \( e^+ \) and \( e^- \) beams reaches a new record high energy automatically means a new particle has been produced and its mass be around the center of mass energy of the beams, \( \sqrt{s} = 2E_{e^+} \) if the collision is a classic symmetric beam-beam collision. However, in many new physics scenarios studied in nowadays experiments at particle colliders such simple strategies are not applicable. A first example of particle discovery in which the mass of the new particle is not so obviously known are the discovery of the \( W \) boson and that of the top quark. These particles have been discovered observing decays into final states containing invisible particles, e.g. \( W^\pm \rightarrow \ell^\pm \nu \) the decay of a \( W \) boson into a charged lepton and a neutrino. The neutrino, being a weakly interacting particle, is not detectable in the high energy experiments. Therefore we cannot use conservation of four-momentum as in eq. (1) because one of the two particles that emerges from the decay of the \( W \) boson is not experimentally accessible. The measurement of masses of particles that decay into final states containing invisible particles (e.g. neutrinos) is one of the classic problems in mass measurement practice that can receive new inputs from the use of energy peaks properties and we will discuss examples of this kind at length.

When invisible particles are involved in the decay, as in the decay of the \( W \) boson, we are forced to find “ad-hoc” solutions to measure the mass of the newly discovered particle. For the \( W \) boson the commonly adopted mass measurement method exploits the fact that \( W \) bosons are produced at hadron colliders in a very simple reaction

\[
pp \rightarrow W + X, \tag{3}
\]

where \( X \) denotes remnants of the protons that collided. In this situation it is possible to apply the properties of transverse momentum distribution know as the jacobian peak and further improve on these results using a special kinematical variables called transverse mass \([8]\). Very good results have been obtained pursuing methods based on jacobian peaks and transverse mass measurements. Nevertheless it is important to remark that these methods rely on the knowledge of the details of the production reaction eq. (3) and are indeed starting to show limitations on the ultimate precision attainable in these measurements. These limitations can be ultimately be traced back to the fact that dynamics of the Standard Model of particles physics, on top of simple kinematics, enters in the formulation of the methods to measure the \( W \) boson mass. For instance we have to know whether or not there is a subdominant production mechanism which produces pairs of \( W \) bosons, or which quarks inside the proton collide to produce a \( W \) boson, as well as their momentum distribution. This example should convince the reader that good strategies can be devised on case-by-case basis when we know
well enough the properties of the particle subject of mass measurement, provided that we know all the necessary inputs with sufficient accuracy. However without an accurate knowledge of the necessary inputs, or in absence of a prevailing and motivated underlying characterization of the particle, e.g. as a state foreseen in a given model, it is hard to make progress in measuring particle masses when invisible final state particles are involved.

Given the vast landscape of possibilities for models of new physics that have been conceived in the past decades it is of greatest importance to be able to approach the problem of mass measurement with a minimal amount of assumptions. The job of the high energy phenomenologist is therefore to find strategies with an optimal balance between the achievable precision on the mass measurement and the amount of assumptions to be made to carry out the measurement. Clearly, as more data flows in on the newly discovered particle, more and more assumption may become justified and the point of balance in the determination of a particle mass can switch more towards methods that use more dynamics, but it remains a very important task to measure masses when these assumptions are not yet fully justified - in this task is where we expect energy peaks techniques to be very useful.

In order to exemplify possible uses of energy distribution and peaks identification in these distribution we recall a few applications that have been studied in detail.

In Ref. [9] the properties of energy peaks have been applied to two body decays of supersymmetric particles, putative new particles that arise in well motivated scenarios of new physics. The case studied was the production of gluinos, denoted as $\tilde{g}$ as they are partners of the gluon particle $g$, that are produced by strong interactions in

$$pp \rightarrow \tilde{g}\tilde{g}$$ (4)

and further decay first in a two-body decay

$$\tilde{g} \rightarrow b\bar{b}$$ (5)

and then the sbottom particle $\tilde{b}$, the partner of the bottom quark particle $b$, decays into two bodies

$$\tilde{b} \rightarrow b\chi^0$$ (6)

yielding overall a cascade decay

$$\tilde{g} \rightarrow b\tilde{b} \rightarrow bb\chi^0,$$ (7)

that is the decay of a single gluino into two bottom quarks and a neutral particle, denoted by $\chi^0$, that is known as neutralino. The neutralino is, like a neutrino, charged only under weak interactions, therefore it does not leave any direct trace in detectors. For this reason we face, as for the case of the $W$ boson illustrated above, a decay into a semi-invisible final state, which poses serious challenges for a mass measurement.

Considering that two gluinos are produced in each reaction eq.(4), overall we have

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow bbbb\chi^0\chi^0,$$

that is a quite busy reaction with two invisible particles, the two $\chi^0$, and four identical particles, the four $b$ quarks, each of the $b$ quarks giving rise to essentially indistinguishable signals in the part of the detector that they will hit. Clearly, measuring the gluino mass in such a reaction is quite challenging in many respects. In fact, even if the $\chi^0$ particles were directly observables in the detector, such a busy final state would pose a challenge in understanding which particles come from one gluino and which form the other gluino. In other words if one wants to use simple four-momentum conservation as in the case of pion decay in eq.(1) one has first to single out the particles arising from each individual gluino. On top of this, there is the extra difficulty $\chi^0$ are not measurable at all in our detectors. Therefore if one wants to try to measure the mass with any strategy similar spirit to the use of four-momentum conservation there are two big challenges to be faced: i) four-momentum is carried away by invisible particles ii) visible particles can be divided in subset
belonging to each gluino in a factorial number of ways. Clearly a new strategy for the gluino mass measurement might alleviate significantly these obstructions - energy peaks measurements take care of both these issues.

The key idea of using energy peaks properties is to realize that information about the mass difference between particles involved in each two-body decay is carried by the peak position of the energy distribution. In the example of gluino decay above we have

\[ E_b^b = \frac{m_b^2 - m_b^2 + m_b^2}{2m_b} \approx \frac{m_b^2 - m_b^2}{2m_b} \]  

for the decay in eq.(5) and

\[ E_b^b = \frac{m_b^2 - m_b^2 + m_b^2}{2m_b} \approx \frac{m_b^2 - m_b^2}{2m_b} \]  

for the second step of the decay chain given in eq.(6), where in both cases one may neglect the $b$ quark mass. Measuring the two values of $E_b^b$ in energy spectrum of $b$ quarks can therefore yield information on two masses involved in the problem, e.g. $m_b$ and $m_b$ if one assumes a value for $m_b$ or evidence suggests it is negligible. Strikingly the information about the two masses comes from the measurement of a quantity that is directly accessible without any assumption or knowledge about the invisible particle $\chi$. Furthermore these relations hold for any process in which gluino is produced by strong interactions, so if we had a mixture of $pp \rightarrow g\tilde{b}$ and $pp \rightarrow g\tilde{g}$, which due to the complex collision environment at the Large Hadron Collider might be look-alike processes, we can use these relations to gather information on these two masses even in presence of such a nuisance.

Last but not least, attempting to measure masses with these single particles relations has the advantage to not require us to identify which $b$ quark is coming from each step of the decay. In general each step of the decay process will give rise to a characteristic spectrum and we will observe the sum of all the spectra coming from all the decay steps. For our two-step decay chain we expect to have spectra similar to one of the two distinctive cases shown in Figure 1. The right panel shows the spectrum of $b$ quark energies that one expects when the two mass differences involved in are similar, while the left panel shows the spectrum that we expect when the two mass differences are significantly different. In the latter case it is possible to attempt to extract from the spectrum the two values of the energy peaks relation on the left-hand side of eq.(8) and (9) analyzing separately the ranges of energy, one for each peak. An example of this type is described at length in Ref. [9].

Here we report the result of the extraction of the two peaks as shown in Figure 2. The energy spectrum peak values are identified through a fit in which the data is modeled by suitably chosen function. This function has two parameters, one for the peak position and one for the width of the peak shape. As explained in Ref. [3], from prime principle argument
one can see that this function has to depend on the combination $E^*/E + E/E^*$, rather than being a simple function of $E$. In this example, and many others, it has been found that data around the peak region can be fit by

$$A \cdot \exp \left\{ w \left( \frac{E^*}{E} + \frac{E}{E^*} \right) \right\},$$

(10)

where $E^*$ is the peak position, i.e. the parameter of interest for the mass measurement, $w$ fixes the width of the peak shape, and $A$ is a normalization factor. The translation of the peak fit results in actual mass measurement depends on the degree of knowledge one assumes on $m_\chi$ as well as if one wants to use further information on the masses that can be gathered in events in which new physics particles undergo a cascade decay. As described in detail in Ref. [9] for the example at hand one could exploit some features of the end-point of the distribution of the invariant mass $m_{bb}$ [12]. All in all Ref. [9] finds that an extraction of $m_{\tilde{g}}$ and $m_\chi$ with precision better than 10% is possible with the energy peak method and can be improved to few percent precision if additional information is used.

For the case of mass differences of comparable entity the energy peak shapes could be largely overlapping, as sketched in the left panel of Figure 1. In this case the fitting exercise is slightly more complicated because two peak shapes need to be identified at once, however there is no fundamental difference compared with the case described above. Of course the fit results may degrade significantly because of the overlapping peaks shapes. Ref. [9] reports results for this type of “merged” peaks spectrum that we report in Figure 3. The identification of two peak positions is clearly still possible despite they look like a single bump spectral shape. In fact it is possible to use a $\chi^2$ fit to the spectrum with either one or two peaks, modeled by eq.(10) or the sum of two such functions, and choose what type of fit to perform based on the $\chi^2$ value. In the cases studied by Ref. [9] the masses can be determined with accuracy slightly worse than those attainable in the separated bumps case, but in any case the attainable precision is better than 10%.

All the above results have been derived in the context of two-body decays. These can happen in cascades, as in the example discussed above for gluino production and decay. However it is also possible in new physics models that new particles do not have any allowed two-body decay. In these cases the new particles might still decay via three-body decays such as

$$\tilde{g} \rightarrow bb\chi^0.$$

While this decay has the same final state as the above eq.(7) the underlying physics is different, because the no resonance exists in the mass
spectrum of any pair of final states. Despite this absence of structure in the decay kinematics it is still possible to use energy peaks analysis to extract information on the particles masses. In fact any multi-body decay can be visualized as a (weighted) sum of decays fewer bodies decays. For instance we could imagine the three-body decay above to be described as a two body decay 

$$\tilde{g} \rightarrow B\chi^0$$

where the mass of the body $B$ changes in each recorded event within the allowed limits $m_B \in [2m_b, m_\tilde{g} - m_\chi]$. Decomposing the decay process in this way we can deal independently with each subset of the events with approximatively same value of $m_B$. In practice we can observe the range of values than $m_B$ attains in the data and divide it in sub-ranges $[m_B^{(i)} - \Delta, m_B^{(i)} + \Delta]$ in which we can consider $m_B$ to be constant and equal to $m_B^{(i)}$. For each of the values $m_B^{(i)}$ an equation of the kind of eq.(2) will hold, because, within the error due to $\Delta$ being small but not zero, the events effectively obey kinematical relations of two-body decays. Therefore if we measure the energy peak of the subset of events belonging to each of the $m_B^{(i)}$ values we can study the relation 

$$E^*_{B^{(i)}} = \frac{m_\tilde{g}^2 - m_\chi^2 + (m_B^{(i)})^2}{2m_\tilde{g}}$$

(11)

and fit the value of $m_\tilde{g}$ and $m_\chi$ from the several values obtained for different $m_B^{(i)}$ choices. This strategy to deal with three-body and multi-body decays has been investigated in Ref. [13]. An example result for the determination from data of the relation eq.(11) is shown in Figure 4. The figure also shows the comparison of two different choices for the fitting function. The best performing one is a modification of the parametrization introduced in eq.(10) that Ref. [13] motivates to better take into account the fact that $m_B^{(i)}$ can be large compared to $m_\tilde{g} - m_\chi$. We refer the reader to Ref. [13] for further details on the fit procedure adopted to extract $E^*_{B^{(i)}}$ for large values of $m_B^{(i)}$. What we would like to highlight here, rather than the details of the fit procedure, is the fact that phase-space slicing allows to use ideas born for two-body decays in the context of multi-body decays.

As it is true in most cases, the information from energy peaks analysis can be supplemented with the one coming form other measurements, such as the end-point of the $m_{bb}$ spectrum. Combining all these information Ref. [13] finds that the masses $m_\tilde{g}$ and $m_\chi$ can be determined with precision well below 10%. Given the non trivial interplay of fits and the role played by the choice of the fitting function used to extract the peak values, in the case of multi-body decays is possible to introduce noticeable biases in the mass extraction. Ref. [13] finds that, especially after a combination of the energy peaks results with the results from invariant mass $m_{bb}$ analysis, these biases are much mitigated.

The case of three-body decays, as we saw above, forces to deal with the extraction of energy peaks from spectra of massive particles energies. Once this becomes feasible a much larger range of decay processes become tractable with energy peaks technique. An example of such decays is the decay 

$$\tilde{t}_2 \rightarrow \tilde{t}_1 Z^0$$

which has been studied in Ref. [14]. In this work greater details are given on how to model the shape of the energy peak for massive particles. In particular it was shown that the peak of the $Z$ boson energy distribution can be determined with precision around or below 10% and information of comparable accuracy on the mass difference between the two particles $\tilde{t}_2$ and $\tilde{t}_1$ can be recovered looking solely at the $Z$ boson energy spectrum.

In addition to the cases of pair production of new physics states, energy peaks methods have been applied to single production of new resonances whose mass can be measured in semi-invisible final states, e.g. $pp \rightarrow G_1 \rightarrow ZZ \rightarrow \nu\nu\mu^+\mu^-$ studied in Ref. [15] where the energy spectrum of the two muons, $E(\mu\mu)$ is used to measure the mass of the intermediate resonance $G_1$.

Before closing this section it is worth recalling a few applications of
Theory: $m_{\gamma} = 1200$ GeV, $m_{c} = 100$ GeV

Fit with massive data ($m_{bb} \in [200, 650]$):

$F = 1042 \pm 65$ GeV

Fit with massless data ($m_{bb} \in [200, 600]$):

$E_{bb}^{*} = 964 \pm 56$ GeV

$E_{bb}^{*} = -24000 \pm 41000$ GeV

Figure 4: Energy peak extracted for different slices of the three-body phase-space with approximatively constant mass of the sub-system made of two $b$ quarks. Two lines represents two different fit functions used to extract the peak value from the data.

5 Precision top quark mass measurement

In the discussion above we have highlighted the several application of energy spectrum peak measurements in the context of measurements on putative new particles to be discovered at the Large Hadron Collider. While these application are interesting per se, it would be even more interesting if these ideas could be applied on real data. At the time of writing the Large Hadron Collider has not seen any hint of new physics, therefore the energy peak method cannot be applied yet on new physics mass measurements as outlined above. Nevertheless there is an important piece of data on which it is interesting to apply the energy peak method, that is the top quark data where energy peaks techniques can help in the measurement of the mass of the top quark.

It is important to stress that after more than 25 years of data collected on top quarks at two different accelerators (TeVatron and the Large Hadron Collider) we already know plenty about the top quark. Therefore the applications discussed above of properties of energy peaks related ideas on the use of energy distributions to measure particles masses. In Ref. [16] it has been considered how a weighted average of the energy spectrum of a decay product can be used to infer the mass of the decaying particle. The method outlined differs from all the applications above for the fact that the peak position is not central to the extraction of the mass of the decaying particle, but rather the shape of the entire spectrum. Due to the necessity of averaging on the whole spectrum, this kind of mass measurement requires to either have fully inclusive data on the decay products or, if part of the data is missing, to supplement the real data with simulated one. This is usually an issue in high energy physics experiments where, in a way or another, events are required to pass certain selections in order to be useable for a mass measurement or simply to reject backgrounds. The method has been studied at leading order in perturbation theory for Higgs boson [17] and top quark [18] mass measurement with encouraging results.
to obtain mass measurements at 10% precision are not necessarily interesting when a single experiment can measure the top quark mass with precision below 1% using other techniques. However, we have discussed in the beginning section how one of the properties that make energy peaks mass measurements interesting is that they make possible to carry out a measurement while keeping the number of assumptions at a minimum. For instance, we have highlighted how energy peak relations such as eq. (2) hold irrespectively of the mechanism or reaction used to produce the particle subject of mass measurement, provided that the sample of particles that we observe is made of equal populations of all possible particle polarizations. This fact is a key to apply energy peaks techniques to a precision measurement such as the measurement of the top quark mass.

In fact, even if we know the reaction for the production of the particle of interest, in concrete cases we might know how to carry out detailed calculations about this reaction only up to a certain order of perturbation theory or neglecting certain effects which might go beyond the presently developed modeling of fundamental interactions (e.g., non-perturbative or non-factorizable QCD effects). As a consequence of this unavoidable presence of theory uncertainties, the energy peaks properties acquire new interest in the context of mass measurements of Standard Model particles. In fact, no matter what is the cause of mis-modeling of production reactions, be it mis-modeling due to missing orders in perturbation theory or absence of description of certain phenomena, energy peaks predictions are expected to enjoy much milder theory uncertainties than those of methods based on other observables. The key of this resilience is the validity of results such as eq. (2) for any two-body decay from a scalar particle or a sample of particles with spin that populate evenly all polarization states.

In the specific case of top quark mass measurement all modern precise mass measurements are obtained comparing some data with theoretical calculations [19, 20, 21, 22]. The data is collected so far only in hadronic colliders and comes from reactions such as \( pp \rightarrow t \bar{t} \), which, due to hadronic nature of the collision, should be more properly written \( pp \rightarrow t \bar{t} + \text{hadrons} \). In nowadays high energy physics practice the extra hadrons produced together with the top quarks are clustered in sets of collimated particles called jets, therefore we will call this reaction \( pp \rightarrow t \bar{t} + \text{jets} \).

The number of jets that is formed together with the top quarks roughly corresponds to the number of orders in perturbation theory beyond the lowest one at which one should have to carry out calculations to describe this process with theoretical calculations. In addition, processes that give rise to any number of jets contribute to the simplest process \( pp \rightarrow t \bar{t} \) when the jets do not carry large energies or point in directions in which it is not possible to detect them. These complications from the hadronic nature of the reaction that produces the top quarks make difficult to obtain precision predictions for any quantity measurable in top quark physics. In spite of all this, calculations up to two orders beyond the leading one in QCD have been carried out and it is nowadays possible to compute total rates for top quark production with a theory uncertainty in the 5% ballpark [23, 24, 24, 25, 26]. Furthermore, it is possible to carry out calculations in which fixed orders in perturbation theory are matched to calculations in which large numbers of soft and collinear emissions of quanta are dealt with in the more suitable parton shower picture, even with the inclusion of non-resonant processes as well as resonant top quark pair production [27]. After all these tremendous efforts to compute in ever more fine details the production of top quarks at hadron colliders, theory errors on the top quark mass determination are still not negligible and it has become a tough problem to i) quantify these uncertainties in a reliable way ii) combine measurements in a meaningful manner when affected by such errors.

In view of the crucial role played by theoretical uncertainties in the determination of the top quark mass from hadronic colliders, the use of energy peaks techniques may offer a new and complementary way to look at these issues. In fact, the insensitivity of the energy peak relation...
eq. (2) offers a chance for smaller sensitivity to theory mis-modeling in extracting the top quark mass from data.

It is important to stress that in the context of a precision measurement some of the assumptions that lead to the derivation of eq. (2) may fail. For instance the necessity to impose selections on the data to isolate top quark events from background events may ruin, even if only slightly, the property of QCD to create samples of top quarks that populate evenly all polarization states, which was a key fact to extend the "old" results on decays of scalars to the case of particles with spin. Furthermore the energy peak techniques are ideally applied to two-body decays, so if one has radiative corrections to the decay process, e.g.

\[ t \rightarrow bW + X, \quad (12) \]

then one has to review the validity of eq. (2) to determine the top quark mass. These issues clearly need a dedicated study, in which the dynamics of the Standard Model and in particular of the QCD sector is used to model these effects and to assess their effect on the determination of the top quark mass. To deal in a consistent way with all these effects one can formulate a method to extract the top quark mass by comparing data with different templates for energy spectra corresponding to different values of the top quark mass. This method, using spectra computed assuming the Standard Model dynamics and including the first QCD corrections to production and decay of the top quark [28, 29], have been studied in [30]. It was found that theory errors due to missing orders in the description of both the production and decay of top quarks are rather small compared to those obtained for templates of other observables (evaluated for instance in Ref. [31]). Quite strikingly, this favorable comparison of theory uncertainties is true even when one compares with observables that are rather close to the energy, e.g. the sum of the two energies of the b quarks in each event studied in Ref. [31]. These results confirm the possible utility of mass measurements, even precision ones, based on single energy peaks techniques.

Furthermore it should be remarked that the CMS experiment has carried out a preliminary measurement of the top quark mass using energy peaks methods on early Large Hadron Collider data. The main result is reported in Figure 5, that is the first ever public measurement of an energy spectrum reported by CMS top quark physics group to carry out a measurement. In Ref. [32] the CMS experiment has used theory predictions at the lowest order in perturbation theory to extract the top quark mass from the measured b-jet energy spectrum and has obtained encouraging results. This measurement has confirmed the estimate of the statistical error presented in the original proposal of Ref. [3]. Furthermore the theory error estimated by the CMS experiment is in line with expectations for a result obtained from theory at the lowest order in perturbation theory and leaves room for improvement once fixed-order and parton shower improvements that have been developed in the meantime [27] will be included in a future reiteration of this measurement.

**Figure 5:** CMS measurement [32] of the b jet energy measurement for the top quark mass measurement.
6 Dark Matter

We have discussed how energy peak relations could be useful thanks to the fact that they do not require to observe more than one decay product for each decay. If such decay is a two-body decay we expect eq. (2) to hold up corrections that could arise for instance from radiative corrections such as those we discussed for the top quark decay eq. (12). These corrections are usually small, surely below 10%, as they do not reach that level even for a strongly interacting particle such as the top quark. Indeed for a weakly interacting particles we expect eq. (2) to hold very precisely.

One type of weakly interacting particles is very interesting in high energy physics: the Dark Matter candidate particles. These are putative particles that should explain a number of phenomena observed in the Universe such as: the observed rotation speed of stars versus their distance from the center of their galaxy, peaks in the power spectrum of the Cosmic Microwave Background, gravitational lensing effects of distant light sources and more (see Ref. [33, 34] for more details on the astrophysical and cosmological evidence for Dark Matter). Such weakly interacting particles are often a piece of a larger theory of physics beyond the Standard Model and are supposed to fill present day Universe and have had a large impact on its evolution ever since its formation.

In order for Dark Matter particles to be present in our Universe from its early times till today they are often charged under some special symmetry that prevents them from decaying. Depending on the model of new physics in which the Dark Matter particle is embedded this symmetry might be a simple parity symmetry under which all particles are either odd or even or some more complicated symmetry. Since “even” and “odd” are just two possible charges particles can have in the simplest symmetry these are called $Z_2$ discrete symmetries. Exactly like in arithmetics, two even charges make an even charge, as well as two odd charges do, but one odd and one even charge make an odd charge. Any interaction that is “odd”, because it involves an odd number of “odd” particles is forbidden.

In most model of new physics a $Z_2$ symmetry is used to distinguish the Standard Model particles, that are assigned to be “even”, and the new particles, that are assigned to be “odd”. In such a way any collision at particles colliders always has to produce a pair of new physics particles and never one single particle. The case seen above of gluino pair production eq. (4) is an example of this very common feature in new physics phenomenology. Similarly, when we considered the decay of the gluino in eq. (5), it was a decay into one Standard Model particle, the $b$ quark, and one new particle, the supersymmetric $\tilde{b}$ particle. For exactly the same reason, the $Z_2$ symmetry, when we considered the decay of the $\tilde{b}$ in eq. (6) we had one Standard model particle, a $b$ quark, and a new physics particle, the $\chi$. In supersymmetric models that attempt to provide a Dark Matter particle the $\chi$ particle is the lightest particle “odd” under the $Z_2$ symmetry. Therefore it is forbidden for $\chi$ to decay and, being electrically neutral, it can very well be a Dark Matter candidate.

The phenomenology of models for Dark Matter in which the Dark Matter is stable because of a $Z_2$ symmetry can be all be captured by the fact that the Dark Matter, when produced from a new particle decay, is produced singly, that is to say a new particle $B$ decays

$$B \rightarrow b\chi,$$

where $b$ denotes a Standard Model particle and $\chi$ the Dark Matter particle. In alternative models, in which a more complex symmetry is used to prevent the Dark Matter from decaying, one might have more than one Dark Matter per decay, for instance one could have

$$B \rightarrow b\chi\chi,$$

for suitable new physics particle $B$ and Standard Model particle $b$. Since the $\chi$ particle is not directly observable, the visible decay products of eq. (13) and eq. (14) at first look are the same, which poses the problem of finding methods to distinguish these two decay processes. The fact that energy peaks formulae can be applied just observing one particle
per decay suggests that energy peaks methods can be helpful to answer this question.

Given the importance of the underlying symmetry that stabilizes the Dark Matter, it is very important to explore methods to distinguish the two classes of models that give rise to different decays eq. (13) and eq. (14). Ref. [35] explored how to distinguish the two type of reactions leveraging the properties of energy peaks. The idea is to observe the energy of $b$, the visible decay product that appears in both kind of decays. If the peak of the energy distribution matches with the prediction for a two body decay eq. (2) one could state with some confidence level that the data suggests a $Z_2$ stabilization symmetry rather than a more complicated symmetry.

It is important to stress that, in order to compute the correct value for eq. (2) one would need to know the masses of the Dark Matter $\chi$ and of the heavier new physics particle $B$, both of which might be poorly known. To surpass this difficulty in Ref. [35] it has been pointed out that the value of eq. (2) relevant for a two-body decay can be obtained from other distributions in particular from the end-point of the variable $m_{T_2}$ introduced in [10, 11]. The reference value from an $m_{T_2}$ analysis can then be used to check if the energy spectrum of the $b$ particle has a peak at that value or at some lower values. An example of $b$ particle energy spectra for the case of production of heavy fermions $B$ particles and decay

$$pp \rightarrow BB \rightarrow bb\chi\chi \text{ or } bb\chi\chi\chi$$

is shown in Figure 6. In both panels the dashed vertical line is the reference value obtained from an $m_{T_2}$ analysis which matches quite well on the left hand plot with the peak of the distribution, while the right hand plot has a peak clearly shifted with respect to the dashed line.

From the results of Ref. [35] it appears that if new physics is discovered at colliders and the signals of new physics contain invisible particles the energy peak method can be used to count the number of invisible particles produced in each event, hence probing very fundamental properties of the new physics theory such as its symmetries and in particular the symmetry responsible for making the Dark Matter stable.

We would like to remark that the properties of energy distributions considered in these pages have also been recently applied in the context of gamma rays astronomy for the characterization of Dark Matter signals. Tell-tale tests of photon energy spectra from non-minimal Dark Matter scenarios have been studied in Ref. [36]. In this work it has been established that the features of the energy spectrum of photons that arise from multi-particle Dark Matter sector, either decaying or annihilating into photons or photons sources, can be revealed with future gamma rays observatories. Furthermore in Ref. [37] it was studied how features of the energy spectrum, such as the dependence on $E/E' + E'/E$ that we pointed out to introduce eq.(10), can be used to test the origin of present experiments excesses.

Finally it is worth recalling that the development of eq.(10) to describe the energy spectrum of massless particles close to their peak lead to the formulation of explanations of gamma rays excesses in present experiment that significantly differ from the usually adopted models of...
Dark Matter annihilation or decay. In the more standard scenarios it is usually assumed that Dark Matter annihilates or decays directly into SM states, e.g. $\chi\chi \rightarrow \psi_{SM} \bar{\psi}_{SM}$, and the photons observed in the experiments are among the decay products of the heavy SM states $\psi_{SM}$, that could be for instance SM quarks, into stable SM states such as $e^\pm$, $p$, $\bar{p}$, $\nu$, and $\gamma$. Clearly, the connection between the observed gamma rays spectrum and the properties of the Dark Matter sector is a complicated one, because it may involve hadronization of quarks, details of the formation of different species of QCD hadrons, bremsstrahlung of all involved charged particles, and other aspects of the dynamics of the Standard Model. At variance with the scenario of direct Dark Matter annihilation or decay into SM states, in Refs. [38, 39] it has been proposed that bump-like features of the excess gamma rays spectra can originate from multi-particle Dark Matter sectors through decays $\chi' \rightarrow \phi \chi$ followed by $\phi \rightarrow \gamma\gamma$ for mass $m_{\phi}$ around twice the energy of the peak region of the excess. The spectral shape of this type of signal is in direct connection with properties of the Dark Matter sector, hence allows to test more directly the models of Dark Matter and in some cases it has been shown to give better fits to the reported excess data [38, 39].

7 Energy peaks: a “one-prong” Breit-Wigner?

These results show that energy peaks are very useful tools to characterize newly discovered particles, especially useful when we have experimentally access to only one particle, or a subset, of the decay products of a particle. We have seen how the energy distribution can be used to probe the number of invisible particles in decays, e.g. to tell what symmetry makes Dark Matter a stable particle, or to measure masses when some part of the kinematical information is carried away by invisible particles and is not possible to draw a classic Breit-Wigner distribution to identify a particle mass.

Given the results obtained one might ask if an energy distribution can be considered as some sort of “one-prong” Breit-Wigner distribution. With this expression we mean to characterize energy peaks methods as a generic substitute for invariant mass analysis in the cases in which an invariant mass analysis is not possible for intrinsic properties of the problem at hand, for instance the presence of invisible particles as decay products which prevent from using four-momentum conservation. At first this may look a too far fetched comparison. However it turns out that is not at all inappropriate to put energy peaks next to well known Breit-Wigner peaks. In fact both the existence of a peak in a Breit-Wigner distribution and in an energy peak distribution rely on the fact that an on-shell resonance can be thought as an intermediate state of our reaction. For instance if we go back to top quark pair production and we consider the off-shell contributions

$$pp \rightarrow Wb\bar{t}$$

neither the distribution of invariant mass $m_{Wb}$ nor that of the energy of the $b$ quark will have a peak at $m_t$ or at the value predicted by eq. (2). Furthermore if one considers extra radiation in the decay process, such as the correction eq. (12) to the lowest order decay of the top quark, the mass $m_{Wb}$ and the energy of the $b$ quark will again fail to have peaks at $m_t$ and at the value predicted by eq. (2), respectively.

The lesson learned here is that the same kind of hypotheses lay behind the use of Breit-Wigner and energy peaks. It should be recalled however that for the energy peak relation to hold for a particle with spin one needs to make sure that the observed sample of decays has the same population of all the possible polarization states of the particle. This may be trivially the case when we look at particles without spin, but in general is something we have to check case by case for each application of energy peaks methods. Production of particles through strong and electromagnetic interaction, that conserve spatial parity, can guarantee such even population of polarization states. However one has to be careful to check that i) experimental effects such as event selection and ii) subdominant reactions mediated by weak interactions (that are violate spacial parity) do not ruin the applicability of energy peaks results. For
a fair comparison it should be said that event selection can in principle ruin a Breit-Wigner peak, but it is much harder to spoil such a peak than a peak in an energy distribution.

Of course an energy peak analysis is always a second choice when a Breit-Wigner analysis is possible, but we have seen that there are plenty of interesting cases in which Breit-Wigner analysis simply is unfeasible for absence of enough measurable particles (e.g. in cosmic rays studies, Dark Matter characterization and mass measurement at colliders). Furthermore in cases in which a Breit-Wigner analysis can be done there might still be interest in pursuing an energy peak analysis to corroborate results and double-check uncertainty estimates from the more powerful Breit-Wigner analysis, as we have seen for the case of the precision top quark mass measurement.

8 Outlook

From the above discussion and the several results presented we have learned that, despite the simplicity of the energy peak relation and the hypotheses behind it, the use of energy peaks methods is very powerful for certain problems in high energy particle physics. We have seen that when particles decay into some combination of visible and invisible particles it is possible to gain useful information on the decay looking at peaks in energy distributions of the visible particles.

In general if one has more than one visible particle per decay and some number of invisible ones it is interesting to combine energy peaks techniques with more traditional ones based on invariant masses, such as the end-point techniques, or methods based on transverse momenta such as the $m_{T2}$ variable. In fact in most of the applications we have discussed in this review the results obtained from energy peaks have always been combined with other techniques. All in all we have seen a nice degree of complementarity between energy peaks methods and other techniques more established in the high energy particle physics practice.

We have reviewed the development of energy peaks techniques beyond the historical results for spinless particles and we have extended to the case of particles with spin. This leads to a new hypothesis for the basic equation of energy peaks method to hold: the sample of particles that is observed decay must populate evenly all possible polarization states for eq.(2) to hold exactly. Armed with this knowledge we have applied the energy peak method to the measurement of masses of new physics particles that give rise to complex final states containing invisible particles. We have studied the determination of the mass of the gluino, the sbottom and the stop supersymmetric particles in different contexts[9]. Along the way we have developed refinements of the original result for decays into light or massless particle to cover the case of decays into particles of non-negligible mass [14]. We have also generalized the idea of energy peaks to multi-body decays exploiting phase-space slicing [13] and the capability to deal with massive decay products. All in all we have demonstrated the capability of energy peaks methods to yield mass measurements with precision at or below 10% in a variety of contexts for new physics particles.

Furthermore we have applied the ideas of energy peak methods to distinguish multi-body from two-body decays in the challenging case in which each decay yields only one visible particle. This is precisely what would be needed to tell which symmetry stabilizes the Dark Matter candidates in new physics models and we have proven that energy peaks methods can be used successfully to tell apart the simplest models of Dark Matter particles from models with more complex symmetries.

We have also investigated the application of the energy peak method to the precision measurement of particle masses. In the application of energy peaks methods to precision mass measurements we have highlighted the importance of radiative corrections to eq.(2) that should be taken into account when one seeks precision beyond the lowest order in perturbation theory. We have discussed how, using explicitly the dynamics of the Standard Model, these corrections can be computed and indeed a method for the measurement of the top quark mass that include these corrections has been devised in Ref. [30]. The results in this work
showed that precision measurements based on energy peaks experience small theoretical uncertainties. From Ref. [30] we can firmly conclude that that energy peak methods can be used for a precision mass measurement once eq.(2) is corrected according to the specific dynamics of the particles under study.

Once we enter in a domain of precision physics, as it becomes inevitable to use the dynamics of the Standard Model to carry out measurements, it is possible to extend the use of energy peaks methods to other measurements, such as that of the W boson mass. In this case, being the production of W bosons inherently of electroweak nature, we are not guaranteed at all that the W bosons will be produced with an even population of polarization states. However, using the dynamics of the Standard Model, we can predict the population of each polarization state. In general we expect small deviations from the predictions of eq.(2) because of the roughly symmetric collision environment at the Large Hadron Collider and of the small velocity that most W bosons have in the Drell-Yan reaction eq.(3). The possibility to extract the W boson mass from energy peaks has not been studied in details yet, but it remains an interesting topic for future work.

Within the contest of collider experiments we can envisage further applications for energy peaks methods. For instance one could compare the b jet energy spectrum, and in particular the peak region, measured in pp collisions and in p – ion and ion – ion collisions recored in the heavy ions physics runs of the Large Hadron Collider. From these comparisons one can learn the interactions that heavy flavor quarks, b quarks in particular, have with the medium that is created in the three types of collisions. This type of analysis would constitute an interesting second look at effects due to the formation of quark-gluon plasma such as the so-called “jet quenching” [40, 41]. Jet quenching is the dissipation of jets energy in collisions in which two heavy ions collide and give rise two jets, II → jj. In principle these jets have equal and opposite momenta, and so they do when measured “in vacuum”, that is to say in relatively not busy pp collisions. However, in a heavy ion collision the nuclear matter involved in the collision may give rise to a medium which tends to alter the jet momentum by interactions with the quarks and gluons that give rise to the jets. These interactions with the medium usually result in an energy loss by the jet, hence the name “quenching”.

Thanks to experience gained on b quarks energy spectra acquired in top quark mass measurements it is in principle possible to study this distribution in heavy ions collisions as well. Comparing the pp results to those for heavy ions, or studying how the distribution changes when the heavy ions event activity changes, one can single out the effects of the medium. Using the b quarks from top decay as hard probes of the medium one has the advantage of knowing quite precisely the expected overall energy scale of the hard event and in particular the expected energy peak position for b quarks in absence of medium effects. This may alleviate the uncertainties in the study of dijet events in heavy ions collisions and constitute a nice complememt of the studies of jet quenching in events in which a jet and a weak boson are produced, for instance II → Zj, where the leptonic decay of the Z boson offes a unit of measure to check medium effects on the recoiling jet [42].

Leveraging the experience gained in the study of energy peaks in laboratory experiments may also lead to novel uses of energy peaks techniques outside high energy peaks laboratories. Looking in this direction it is important to remark that in all the applications discussed above for high energy physics it was critical to be able to extract the peak position from a rather broad spectral shape (see for instance the peak shape in the real data taken by the CMS experiment reported in Figure 5). The capability to extract this peak arises from a very good modeling of the peak shape and to a certain extent of the tails of spectrum. The fact that this modeling has been validated on data from a laboratory experiment with rather small uncertainties might enable new applications of energy peaks. For instance it appears feasible to look for traces of mesons heavier than the π0 in cosmic rays, e.g. in satellite-born experiments such as FERMI or the future ComPair [43] and e-ASTROGAM [44] experiments that could be sensitive to η meson production in supernova remnants.
The signal from these \( \eta \) mesons would be hidden underneath that of the more abundant \( \pi^0 \) claimed by FERMI in Ref. [7], therefore it is very important to be able to model the whole shape of the \( \pi^0 \) peak up to photon energy values where the \( \eta \) peak might be visible. Observing such a \( \eta \) meson signal would put on firmer ground the hadronic origin of a major part of the cosmic radiation and would be a nice come-back of energy peaks methods to cosmic rays physics.

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