Topological Excitations in Double-Layer Quantum Hall systems

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Abstract

Double-layer quantum Hall systems with spontaneous broken symmetry can exhibit a novel manybody quantum Hall effect due to the strong interlayer coherence. When the layer separation becomes close to the critical value, quantum fluctuations can destroy the interlayer coherence and the quantum Hall effect will disappear. We calculate the renormalized isospin stiffness $\rho_s$ due to quantum fluctuations within the Hartree-Fock-RPA formalism. The activation energy of the topological excitations thus obtained demonstrates a nice qualitative agreement with recent experiment.

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Double-layer quantum Hall systems (DLQHS) have drawn much theoretical and experimental attention recently [1–8]. When the distance $d$ between the layers is comparable to the mean particle spacing, strong interlayer correlations induce a novel manybody quantum Hall effect at $\nu_T = 1/m$ ($m$ is an odd integer), where $\nu_T$ is the total filling factor [3,8,10]. Since each layer at $\nu = 1/(2m)$ alone does not support quantum Hall effect, it manifests a genuine manybody correlation effect. When the layer separation becomes large and close to the critical value, quantum fluctuations can destroy the interlayer coherence and accordingly quantum Hall effect will disappear [3,10,9]. By mapping the layer degrees of freedom onto the $S = \frac{1}{2}$ isospin variables, the interlayer correlations can be quantified as isospin stiffness $\rho_s$. Since most of the recent experimental systems [3,10] belong to the quantum fluctuation dominated regime, it is crucial to obtain the reliable estimate of the renormalized isospin stiffness $\rho_s$ which incorporates the effect of quantum fluctuations.

In this Letter, we calculate the renormalized isospin stiffness $\rho_s$ and the effective tunneling amplitude $t_R$ within the Hartree-Fock-RPA formalism. By taking into account the important vertex corrections [7,9,12], we explicitly show that $\rho_s$ vanishes at $d/\ell \approx 1$, where $\ell = \sqrt{\hbar c/|e|B}$ is the magnetic length. The activation energy of the lowest charge excitations (e.g. Meron pairs or domain-wall string solitons) is estimated and shows a nice qualitative agreement with the recent experiment [10]. We consider the DLQHS with finite tunneling $t$ and parallel magnetic field $B_\parallel \hat{y}$. We concentrate on the $\nu_T = 1$ quantum Hall effect and assume that the electrons are lying on the lowest Landau level due to the strong perpendicular magnetic field $B_\perp \hat{z}$ and the real spins are fully spin-polarized due to the finite Zeeman gap. With the choice of the following gauge $A = (0, B_\perp x, -B_\parallel x)$, the Hamiltonian of the system can be written by

$$
\mathcal{H} = -te^{-Q^2/4} \sum_X \left( e^{iQX} c_{X,\uparrow}^\dagger c_{X+} + e^{-iQX} c_{X\downarrow}^\dagger c_{X-}\right)
+ \frac{1}{2} \sum_{\sigma, \sigma', X_1, X_2, X', \neq 0} V_{\sigma \sigma'}(X', X_1 - X_2)c_{X_1 - X', \sigma}^\dagger c_{X_2 - X', \sigma'}^\dagger c_{X_1 \sigma}
$$

where $\sigma = +1(-1)$ represents an electron in the upper(lower) layer, $X$ the guiding center coordinate, and $Q = dB_\parallel/\ell^2 B_\perp$. With the following convention of the units $e = \ell = \hbar = 1,$

$$
\begin{align}
\mathcal{H} = & -te^{-Q^2/4} \sum_X \left( e^{iQX} c_{X,\uparrow}^\dagger c_{X+} + e^{-iQX} c_{X\downarrow}^\dagger c_{X-}\right)
+ \frac{1}{2} \sum_{\sigma, \sigma', X_1, X_2, X', \neq 0} V_{\sigma \sigma'}(X', X_1 - X_2)c_{X_1 - X', \sigma}^\dagger c_{X_2 - X', \sigma'}^\dagger c_{X_1 \sigma}
\end{align}
$$
\(V_{\sigma\sigma'}\) is an intralayer interaction \(V_A\) for \(\sigma = \sigma'\) and an interlayer interaction \(V_E\) for \(\sigma \neq \sigma'\):

\[
\tilde{V}_A = 2\pi/\epsilon q, \quad \tilde{V}_E = 2\pi e^{-qd}/\epsilon q, \quad \text{and}
\]

\[
V_{\sigma\sigma'}(X, X_1 - X_2) = \frac{1}{L^2} \sum_{q_y} e^{-q^2/2} \tilde{V}_{\sigma\sigma'}(X, q_y) e^{i q_y (X_1 - X_2)}
\]

(2)

where \(q = \sqrt{X^2 + q_0^2}\). We notice that \(B_\parallel\) induces an Aharonov-Bohm phase \(e^{\pm iQX}\) depending on the sense of interlayer tunneling.

The relatively weak interlayer interaction comparing to the intralayer interaction suppresses local density fluctuations. Since this corresponds to the \(m_z^2\)-term in the magnetic analogy, the isospins are forced to lie on the \(\hat{x} - \hat{y}\) isospin space [6]. Hence the DLQHS can be viewed as quantum XY-ferromagnets. We obtain the following energy functional

\[
E[\theta] = \int d^2r \left\{ \frac{1}{2} \rho_s (\nabla \theta)^2 - \frac{t}{2\pi} \cos [\theta(r) - Qx] \right\}
\]

(3)

where \(\theta(r)\) represents the isospin orientation. This is precisely the Pokrovsky-Talapov (PT) model [15], which exhibits a commensurate-incommensurate transition. For small \(Q\) and hence \(B_\parallel\), the phase obeys \(\theta(r) = Qx\). As \(B_\parallel\) increases, the local field tumbles too rapidly and a continuous phase transition to an incommensurate state with broken translation symmetry occurs [2,6]. The ground state energy of the commensurate state is given by

\[
\epsilon(Q, t) \equiv \frac{E[Q, t]}{A} = \frac{1}{2} \rho_s Q^2 - \frac{t}{2\pi} e^{-Q^2/4}
\]

(4)

where \(\rho_s\) measures the interlayer coherence. By imposing the following condition \(\sqrt{t}/\rho_s \gg Q > 0\), one can guarantee that the ground state of the system is the commensurate one and \(\rho_s\) can be obtained by the following procedure

\[
\rho_s = \lim_{t \to 0} \lim_{Q \to 0} \frac{d^2 \epsilon(Q, t)}{dQ^2}.
\]

(5)

An appealing feature of the commensurate state is that the self-consistent Hartree-Fock ground state is also an eigenstate of the tunneling Hamiltonian alone. One can easily diagonalize the tunneling Hamiltonian in terms of the \(\alpha_X, \beta_X\)-operators

\[
H_t = -te^{-Q^2/4} \sum_X (\alpha_X^\dagger \alpha_X - \beta_X^\dagger \beta_X)
\]

(6)
where $\alpha_X, \beta_X$ correspond to the symmetric and antisymmetric state with respect to the spatially varying tumbling field: $\alpha_X = (e^{-iQX/2c_X\uparrow} + e^{iQX/2c_X\downarrow})/\sqrt{2}$, $\beta_X = (e^{-iQX/2c_X\uparrow} - e^{iQX/2c_X\downarrow})/\sqrt{2}$. The ground state $|\Phi\rangle$ is a completely filled state of the $\alpha_X$-particles. Since $|\Phi\rangle$ is an eigenstate of the full Hartree-Fock Hamiltonian too, it is straightforward to calculate the Hartree-Fock ground state energy yielding 

$$
\frac{E_{HF}}{A} = \frac{1}{A} \langle\Phi |H| \Phi \rangle 
= -\frac{t}{2\pi} e^{-Q^2/4} - \frac{1}{4\pi} \int dX dq_y \frac{e^{-q^2/2} V_o^Q(X, q_y)}{(2\pi)^2} 
$$

(7)

where $V_o^Q(X, q_y) = (\tilde{V}_A + \tilde{V}_E \cos QX)/2$. Using Eq.(4), we obtain the Hartree-Fock estimate of isospin stiffness

$$
\rho_0^s = \frac{1}{32\pi^2} \int_0^{\infty} dq \tilde{V}_E(q) e^{-q^2/2} q^3. 
$$

(8)

Now we want to calculate the fluctuation corrections to $\rho_0^s$ using the Hartree-Fock-RPA scheme. The Green function $G_{X'X}^{\sigma\sigma'}(\tau - \tau')$ is defined as follows

$$
G_{X'X}^{\sigma\sigma'}(\tau - \tau') \equiv \langle T_\tau C_{\sigma X}(\tau) C^\dagger_{\sigma' X'}(\tau') \rangle. 
$$

(9)

The retarded Green function $G_{X'X}^{\sigma\sigma'}(\omega)$ of the Hartree-Fock mean field Hamiltonian is given by

$$
G_{X'X}^{\sigma\sigma'}(\omega) = \delta_{X,X'} \frac{1}{2} e^{\frac{i}{2}(\sigma - \sigma')QX} \left\{ \frac{1}{\omega - \epsilon_{\sigma} - i\eta} + \sigma\sigma' \frac{1}{\omega - \epsilon_{\sigma'} + i\eta} \right\} 
$$

(10)

where $\epsilon_{\sigma} = -te^{-Q^2/4} - \sum_X \langle 0X | \tilde{V}_0^Q | X \rangle$, $\epsilon_{\sigma'} = te^{-Q^2/4} - \sum_X \langle 0X | \tilde{V}_x^Q | X \rangle$, and $\tilde{V}_x^Q(X, q_y) = (\tilde{V}_A - \tilde{V}_E \cos QX)/2$. $\delta_{X,X'}$ indicates that the system is translationally invariant along the $\hat{y}$-direction in our chosen gauge and we only keep a single index $X$ from now on. The RPA ground state energy can be calculated as follows

$$
E_{RPA} = \sum_{n=2}^{\infty} \frac{i}{2n} \langle \Phi |(DV)^n | \Phi \rangle 
$$

(11)

where $D$ is the Hartree-Fock particle-hole propagator defined by
tunneling process which picks up an Aharonov-Bohm phase of \( \sigma \omega \). The integration over frequency yields

\[
D_{XX'}^{(n)}(\omega) = \int \frac{d\omega'}{2\pi} G_X^{(n)}(\omega + \omega') G_{X'}^{(n)}(\omega') = \frac{i}{4} \sigma \sigma' e^{i\frac{1}{2}(\sigma - \sigma')Q(XX')} A(\omega)
\]

(12)

where \( \Delta_x(Q) = \Delta_{SAS} e^{-Q^2/4} + \sum_X \langle 0X|\tilde{V}_E \cos QX|X0 \rangle \) represents the \( Q \)-dependent exchange-enhanced gap with \( \Delta_{SAS} = 2t \) and \( A(\omega) = (\frac{1}{\omega - \Delta_x(Q) + i\eta} - \frac{1}{\omega + \Delta_x(Q) - i\eta}) \). The strong enhancement of the tunneling gap due to the Coulomb exchange energy stabilizes our perturbative analysis. When \( \sigma \) and \( \sigma' \) are opposite to each other, \( D_{XX'}^{(n)} \) represents an interlayer tunneling process which picks up an Aharonov-Bohm phase of \( e^{\pm iQ(XX')} \). We first consider the \( n \)-th order bubble diagram (refer to Fig.(1))

\[
E^{(n)}_{RPA} = \frac{(-i)^{n-1}}{2n} \sum_{\{\sigma, X_{i}\}} \int \frac{d\omega}{2\pi} D_{X_{i}X_{i+1}}^{\sigma\sigma_{i}}(\omega) \cdots D_{X_{n-1}X_{n}}^{\sigma_{n-2}\sigma_{n-1}}(\omega) \langle X_1X_4|V_{\sigma_1\sigma_2}|X_2X_3\rangle \cdots \langle X_{2n-1}X_2|V_{\sigma_{2n-2}\sigma_{2n-1}}|X_{2n}X_1\rangle.
\]

(13)

We notice that the momentum transfer \( X = X_{2i-1} - X_{2i} \) is conserved. The sum over isospins can be performed using the following trick. Since \( X \) stays the same for all the bubbles, the phase factor in \( D_{X_{i}X_{i+1}}^{\sigma\sigma_{i}} \) can be rearranged as follows

\[
D_{X_{i}X_{i+1}}^{\sigma\sigma_{i}}(\omega) \cdots D_{X_{n-1}X_{n}}^{\sigma_{n-2}\sigma_{n-1}}(\omega) = [iA(\omega)]^n \prod_{i=1}^{n} \left[ \frac{\sigma_{2i-1}\sigma_{2i}}{4} e^{-\frac{i}{2}(\sigma_{2i-1} - \sigma_{2i})QX} \right].
\]

(14)

Using Eq.(14), the isospin-sum can be factorized and we obtain the following useful relations

\[
\frac{1}{4} \sum_{\sigma_1, \sigma_2} \sigma_1 \sigma_2 e^{-\frac{i}{2}(\sigma_1 - \sigma_2)QX} \langle X_1X_4|V_{\sigma_1\sigma_2}|X_2X_3\rangle = V^Q_x(X, X_1 - X_3).
\]

(15)

Since our gauge choice conserves the momentum along the \( \hat{y} \)-direction, we finally obtain

\[
\frac{E^{(n)}_{RPA}}{A} = \frac{i}{2} \frac{1}{L^2} \sum_{X_{i}X_{i+1}} \int \frac{d\omega}{2\pi} \left[ \frac{1}{2\pi} e^{-\frac{q^2}{2}Q_x^2} \tilde{V}^Q_x(X, q_y) A(\omega) \right]^n.
\]

(16)

The sum of \( E^{(n)}_{RPA} \) with respect to \( n \) yields

\[
\frac{E_{RPA}}{A} = \frac{i}{2} \frac{1}{L^2} \sum_{X} \int \frac{d\omega}{2\pi} \left\{ -\ln \left[ 1 - \frac{1}{2\pi} e^{-\frac{q^2}{2}Q_x^2} \tilde{V}^Q_x(X, q_y) A(\omega) \right] - \frac{1}{2\pi} e^{-\frac{q^2}{2}Q_x^2} \tilde{V}^Q_x(X, q_y) A(\omega) \right\}.
\]

(17)

The integration over frequency \( \omega \) can be explicitly performed and \( E_{RPA} \) can be written by
\[ \frac{E_{\text{RPA}}}{A} = \frac{1}{2L^2} \sum_{X,q_y} \left\{ \sqrt{\Delta_x^2(Q) + \frac{1}{\pi} e^{-q^2/2} \tilde{V}_x^Q(X,q_y) \Delta_x(Q) - \Delta_x(Q)} \right\} \]

\[ - \frac{1}{2L^2} \sum_{X,q_y} \frac{1}{2\pi} e^{-q^2/2} \tilde{V}_x^Q(X,q_y). \]

(18)

The last term in Eq.(18) plus \( E_{HF} \) does not have a \( Q \)-dependence. By expanding both \( \Delta_x(Q) \) and \( \tilde{V}_x^Q \) up to the quadratic order in \( Q \), we obtain

\[ \rho_s = \int \frac{d^2 q}{(2\pi)^2} \left\{ \frac{16\pi e^{-q^2/2} \tilde{V}_E \Delta_{x0} - 2e^{-q^2/2} \rho_0^n \tilde{V}_x}{\sqrt{\Delta_{x0}^2 + \frac{1}{\pi} e^{-q^2/2} \tilde{V}_x \Delta_{x0}}} \right. \]

\[ + \left. \frac{16\pi e^{-q^2/2} \tilde{V}_E \Delta_{x0} - \Delta_{x0}}{\sqrt{\Delta_{x0}^2 + \frac{1}{\pi} e^{-q^2/2} \tilde{V}_x \Delta_{x0}}} \right\} \]

(19)

where \( \tilde{V}_x = (\tilde{V}_A - \tilde{V}_E)/2 \) and \( \Delta_{x0} = \Delta_x(Q = 0) \).

It is well-known that the coherent double-layer quantum Hall state is unstable due to the presence of a charge density wave (CDW) instability at a finite wave-vector \( k\ell \equiv 1 \) for \( d/\ell \geq 1.2 \). In order to correctly capture this important quantum fluctuations, we take into account the vertex corrections, i.e., a sum of ladders in the polarization insertions in Fig.(1). It amounts to replacing \( \Delta_x(Q) \) with the correct dispersion relation \( \omega(q,Q) \) of the collective mode in Eq.(18)

\[ \omega(q,Q) = \sqrt{D_z(q,Q)D_y(q,Q)} \]

(20)

where \( D_z(q,Q) \) and \( D_y(q,Q) \) are given by

\[ D_z(q,Q) = \Delta_{SAS} + \frac{1}{\pi} \tilde{V}_x^Q e^{-q^2/2} + \int \frac{d^2 k}{(2\pi)^2} \tilde{V}_E(k) \cos Qk_y e^{-k^2/2} - \int \frac{d^2 k}{(2\pi)^2} \tilde{V}_A(k) e^{-k^2/2} e^{ik \times q \hat{z}} \]

(21)

\[ D_y(q,Q) = \Delta_{SAS} + \int \frac{d^2 k}{(2\pi)^2} \tilde{V}_E(k) \cos Qk_y e^{-k^2/2} - \int \frac{d^2 k}{(2\pi)^2} \tilde{V}_E(k) \cos Qk_y e^{-k^2/2} e^{ik \times q \hat{z}}. \]

(22)

Fig.(2) demonstrates that \( \rho_s \) obtained by RPA with vertex corrections indeed exhibits a dramatic reduction from the Hartree-Fock or RPA estimates. One can also notice that \( \rho_s \) vanishes at \( d/\ell \cong 1 \) due to the proximity to the strong CDW instability. Quantum
fluctuations can also reduce the normalized magnetization $< m_x >$—the order parameter of the DLQHS. One can calculate $< m_x >$ using the following procedure \[6\]

$$< m_x > = -\lim_{t \to 0} \lim_{Q \to 0} 2\pi \frac{d\epsilon(Q, t)}{dt}$$

$$= 1 - \int \frac{d^2q}{2\pi} \left\{ \frac{\Delta_{x0} + \frac{1}{2\pi} e^{-q^2/2\bar{V}_x} \Delta_{x0}}{\sqrt{\Delta_{x0}^2 + \frac{1}{2\pi} e^{-q^2/2\bar{V}_x} \Delta_{x0}}} - 1 \right\}. \quad (23)$$

The renormalization of $< m_x >$ implies that the effective tunneling amplitude $t_R$ is reduced by a factor of $< m_x >$: $t_R = t < m_x >$. The inset of Fig.(2) shows that $< m_x >$ via RPA with vertex corrections is strongly reduced from the Hartree-Fock or RPA estimates. Based on the general field-theoretical point of view, we speculate that in the absence of tunneling, $< m_x >$ will vanish at the same value of $d$ as $\rho_s$ \[6\]. Here $< m_x >$ vanishes at $d/\ell \sim 0.7$, which is comparable to $d/\ell \sim 1$ from $\rho_s$.

The activation energy of the lowest charge excitations can be calculated from $\rho_s$ and $t_R$ obtained above. The subscript $R$ is omitted from now on. When the tunneling is very weak, i.e., $\bar{t} \leq 2.4 \times 10^3 \bar{\rho}_s^3$, the lowest charge excitations are meron-pairs, where $\bar{t}, \bar{\rho}_s$ are measured in units of $e^2/\epsilon \ell$ \[6,13\]. Since the merons carry charge $\pm \frac{1}{2}e$ depending on the vorticity and the core-spin configurations, the energy $E_{MP}$ of meron pairs with charge $\pm e$ can be determined by balancing the Coulomb repulsion and the logarithmic attraction: $E_{MP} \sim 4\pi \rho_s$ \[6\]. As $t$ increases or $\rho_s$ decreases, it becomes too costly to flip the isospins over a broad area due to the large tunneling energy cost. Hence the meron-pairs are confined by a linear string tension $T_0 = 8(t\rho_s/2\pi)^{1/2}$ and the lowest charge excitations become domain-wall string solitons(DWS) \[6,16\]. The activation energy of the DWS can be estimated by balancing the Coulomb repulsion and linear string tension: $E_{DWS} \sim (e^2T_0/\epsilon)^{1/2}$ \[6,16\]. We have chosen $t/(e^2/\epsilon \ell) \cong 4 \times 10^{-3}$ \[17\]. Fig.(3) shows the $d/\ell$-dependence of the activation energy. Since one needs to create a charge neutral excitation at $\nu_T = 1$, $E_A$ is multiplied by a factor of two. Below $d/\ell \sim 0.7$, the meron pairs are the lowest charge excitations and above are the DWS. The theoretical result thus obtained demonstrates a nice qualitative agreement with the experiment \[10\]. When $d/\ell$ vanishes, one recovers a skyrmion-antiskyrmion pair energy $8\pi \rho_s$ \[18\]. In real experimental systems, when the layer separation becomes small,
the interlayer tunneling becomes much easier and the system acts like a single wide quantum well. As the Zeeman gap is much smaller than $\Delta_{\text{SAS}}$, the real spin degrees of freedom will become important and the lowest charge excitations are real spin-textured quasiparticles.

To summarize, we have calculated the renormalized isospin stiffness $\rho_s$ and the effective tunneling amplitude $t_R$ within the Hartree-Fock-RPA formalism. We have explicitly shown that $\rho_s$ vanishes at $d/\ell \approx 1$. We have also estimated the $d/\ell$-dependent activation energy of the topological excitations.

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FIGURES

FIG. 1. The $n$-th order RPA bubble diagram.

FIG. 2. The renormalized isospin stiffness $\rho_s$: the solid line stands for the Hartree-Fock result $\rho_s^0$, the dashed line for the RPA, and the long-dashed line for the RPA with the vertex corrections. The inset shows the corresponding renormalized magnetizations $< m_x >$.

FIG. 3. The activation energy of the lowest charge excitations.
K. Moon Fig. 3
K. Moon Fig. 2
K. Moon Fig. 1