Mimetic Curvaton

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In this paper, we investigate the primordial perturbations of inflation model induced from the multi-field mimetic gravity, where there are two fields during inflation, and both adiabatic and isocurvature perturbation modes are generated. We show that although the original adiabatic perturbation mode indeed loses the kinetic term due to the constraint equation, by applying the curvaton mechanism where one of the fields is viewed as curvaton field, the adiabatic perturbation can be transferred from the isocurvature one at the end of inflation. Detailed calculations are performed for both inflationary and the consequent radiation-dominant and matter-dominant epochs. Therefore, the so-called “non-propagating problem” of the adiabatic mode will not harm the multi-field mimetic inflation models.

I. INTRODUCTION

The recently proposed mimetic gravity provides us with a novel mechanism of dark matter generation [1–3]. Such kind of gravity introduces an auxiliary metric $\tilde{g}_{\mu\nu}$, which connects with our physical metric $g_{\mu\nu}$ via the conformal transformation

$$g_{\mu\nu} = (\tilde{g}^{\alpha\beta}\partial_\alpha \phi \partial_\beta \phi)\tilde{g}_{\mu\nu}$$  

(1)

where $\phi$ is the auxiliary field. By varying the Hilbert-Einstein action with respect to the auxiliary metric $\tilde{g}^{\mu\nu}$, one can have an additional term in $T^\mu_{\nu\rho\sigma}$ that is proportional to $(G - T)$ (the trace difference between $G_{\mu\nu}$ and $T_{\mu\nu}$), mimicking the cold dark matter according to the fact that the effective pressure caused by this term is zero. This means that due to such a transformation, the longitudinal mode of gravity now become dynamical. Moreover, from (1) one can find that the scalar field $\phi$ obeys the constraint of

$$g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1$$  

(2)

thus a Lagrangian multiplier term can be added into Hilbert-Einstein action so that the physical metric $g^{\mu\nu}$ is still considered as variable instead of auxiliary metric $\tilde{g}^{\mu\nu}$ with no change on equations of motion [4].

The mimetic gravity can also be extended to describe inflation where a potential $V(\phi)$ of the scalar is introduced as the usual single field inflation model. However, as in [4] the authors have pointed out, considering the constraint equation (2), one actually cannot get a wavelength-dependent solution. In other words, we cannot define the quantum state of the perturbations in usual way. Therefore we need to modify the theory by adding new degree of freedoms. One way of doing this is to add the higher derivative terms such as $\Box \phi$ to the action, but pure $\Box \phi$ will generally lead to pathology such as gradient instabilities [4–7] (see also [8] for more complicated case). On the other hand, in order to get a stable mimetic gravity theory, various ways of modifying the higher derivative terms have been discussed, see e.g., [9–17]. Another way is to add extra fields, by which one then gets a multi-scalar mimetic gravity [18, 19]. While Ref. [18] considered two fields with a flat metric $\delta_{ab}$ in field space, in [19] the authors extended it into an arbitrary metric $G_{ab}$, violating the shift symmetry of the fields. By decomposing perturbations into adiabatic and entropy modes as the usual approach performed on the multi-field inflation models, one can calculate their evolution behaviors respectively. However, Ref. [19] claimed that, due to the constraint equation, the velocity of adiabatic mode can be described as a function of both two modes, eliminating the kinetic term of adiabatic mode in the total action, which means that the adiabatic mode does not propagate. Moreover, this conclusion holds independence on gauge choice. In this case, it will bring problems because the adiabatic perturbation of each space-time point will evolve independently, and the global homogeneity of our Universe required by the Cosmic Microwave Background (CMB) observations cannot be guaranteed. Therefore there must be a mechanism to avoid the case.

One feasible mechanism is the so-called curvaton mechanism [22, 23], in which it states that there is an extra light scalar field (comparing with inflaton) responding to the curvature perturbation. Since the curvaton field is much lighter than the inflaton field, its effects on background trajectory can be neglected, meanwhile, the curvaton will generate the most of the perturbations which are isocurvature perturbations. At the end of inflation, the curvaton field will decay into radiation or matter, by which it will transfer the isocurvature perturbation into the curvature perturbation. Moreover, we can realize the curvaton mechanism from the inflaton decay that can be dubbed as one field inflationary theory [20]. In this paper, we consider the curvaton mechanism under the context of the mimetic gravity.
framework of multi-field mimetic inflation, where one of the fields acts as a curvaton, thus one can still get the adiabatic perturbations after inflation [21]. Inspired by this framework, the adiabatic perturbation is sourced by the isocurvature one which is propagating, therefore the “non-propagating” problem demonstrated above can be avoided.

This paper is organized as follows: in Sec. II we briefly review the mimetic inflation model (both single- and multi-field), and show the problems for both cases; in Sec. III we study the background evolution for our mimetic curvaton model; in Sec. IV we investigate the perturbations in our model, by using the curvaton mechanism, for both inflationary and matter-dominant epochs. The final curvature perturbation in matter-dominant epoch is also obtained. Sec. V gives the main conclusion and final discussion.

II. MIMETIC GRAVITY FOR INFLATION

The original single-field inflation model that arises from mimetic gravity is described by the action [4]:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \lambda \left( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + M^4 \right) - V(\phi) \right],
\]

where the constraint is put inside the action with a multiplier \( \lambda \) and the potential \( V(\phi) \) is also added to the action. Considering the dimension, the constraint term is written as \( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + M^4 \) where \( M \) corresponds to the typical energy scale of \( \phi \). One can always do the rescaling of \( \phi \) as \( \phi \rightarrow \phi/M \) to reduce the constraint to the form of (2).

From action (3), it is straightforward to obtain the Friedmann equations in flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

\[
\begin{align*}
3M_P^2 H^2 &= V - 2\lambda M^4, \\
M_P^2 \dot{H} &= \lambda M^4,
\end{align*}
\]

where \( \partial^\alpha \phi \partial_\alpha \phi \) is eliminated by the constraint thus does not appear as in the usual case. As can be seen, \( \lambda \) appears in Friedmann equations as a part of energy density and totally determines the time evolution of Hubble parameter. From the Friedmann equations, one gets the equation of motion of \( \lambda \) as

\[
\lambda + 3\dot{H} \lambda - \frac{\dot{V}}{2M^4} = 0,
\]

and its solution is

\[
\lambda = \frac{1}{2M^4} a^{-3} \int_{t_1}^{t} dt' \dot{V} a^3 + C_1 a^{-3},
\]

where \( C_1 \) is an integral constant. Thus the parameter \( \lambda \) is determined by potential \( V \) and boundary condition.

From the above solution one can obtain remarks as follows:

1) during slow-roll inflation \( (t_1 < t < t_e) \) where slow-roll parameter \( \varepsilon \equiv -\dot{H}/H^2 \approx 0 \), one has \( \lambda = M_P^2 H/M^4 \approx 0 \), therefore \( V = 3M_P^2 H^2 \) which is almost independent on time, namely \( \int_{t_1}^{t} dt' \dot{V} a^3 \approx 0 \) till \( t = t_e \). These in turn implies that \( C_1 \) vanishes;

2) from the end of inflation to the beginning of matter-dominance \( (t_e < t < t_\text{mi}) \) where needs \( \varepsilon \approx 1 \) to terminate the inflation, from (4) we have \( V = -\lambda M^4 \). Taking \( C_1 = 0 \), this leads to \( V \sim a^{-2} \);

3) from matter-dominance to now \( (t_\text{mi} < t < t_0) \) where mimetic field \( \phi \) has almost totally decayed, yielding \( V \approx 0 \) and \( \dot{V} \approx 0 \), we have \( \int_{t_\text{mi}}^{t} dt' \dot{V} a^3 = 0 \) till \( t = t_0 \), so \( \int_{t_1}^{t} dt' \dot{V} a^3 = \int_{t_\text{mi}}^{t} dt' \dot{V} a^3 \) is a constant (definite integration), making \( \rho = V = -2\lambda M^4 \propto a^{-3} \) as unrelativistic matter. This is why we can consider it as a candidate of dark matter. Setting \( C_2 \equiv \int_{t_\text{mi}}^{t} dt' \dot{V} a^3 \), we will find that \( C_2 \) is nothing but the dark matter energy density today,

\[
\rho_{m0} = \rho_m a^3 = -\int_{t_1}^{t_0} \dot{V} a^3 dt = \int_{t_{\text{mi}}}^{t_{0}} dt' \dot{V} a^3 = -C_2 ,
\]

therefore the constant can be treated as a connection between the early epoch (right after inflation) and the late epoch of our Universe. It indicates that the cold dark matter fraction in mimetic gravity is produced in preheating epoch.

However, the single-field mimetic gravity cannot describe inflation properly. Due to the constraint equation, the universal equation of motion for the scalar perturbation \( \delta \phi \) is obtained in Ref. [4]:

\[
\delta \ddot{\phi} + H \dot{\delta \phi} + \ldots = 0 ,
\]

where there is no spatial derivative, thus \( \delta \phi \) is valid irrelevant of the wavelength. Note that we have omitted parts of Eq. (8) which is not important for our discussion. Therefore we have problem of defining the quantum origin of the perturbations. To be precise, from Eq. (8) we cannot have a plane-wave like solution as usual, and the momentum of the quantum fluctuations is not well-defined, either. In order to avoid this problem, in Ref. [19] the model is extended by adding another mimetic field to the system. If the mimetic gravity is described by more than one field with transformation \( g_{\mu\nu} = (G_{ab} \delta^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \phi) \delta_{\mu\nu} \), the action becomes

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \lambda \left( G_{ab} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + M^4 \right) - V(\phi^a) \right],
\]

where \( G_{ab} \) is metric of field space. As discussed in [19] as well as in [24], the model indeed has two scalar degrees of freedom as the same as in normal double-field
inflation models. For background evolution, Eqs. (4)-(6) are still applicable. The authors have calculated the perturbations generated by this model using the well-known adiabatic-entropy decomposition, finding that different from normal double-field inflation models, the adiabatic mode perturbation does not propagate in multi-field mimetic gravity! This problem occurs due to the fact that the perturbed mimetic constraint combines the adiabatic mode perturbation and the entropy mode perturbation, which gives the relation

\[ \dot{u}_T = \varepsilon H u_T + \dot{\theta} u_N, \]  

(10)

where \( u_T \) and \( u_N \) are adiabatic mode and entropy mode perturbations respectively, \( \varepsilon \) is slow-roll parameter, \( \theta \) is the angle of the tangent to the background trajectory with respect to one of the axis in the field space (see \([19, 25])\). As a result, the perturbed action becomes \([19]\)

\[ \delta^2 S = \int d^4x a^3 M_P^2 \varepsilon H^2 \left[ \mathcal{L}_{u_N} + \mathcal{L}_{u_T} + 2 \text{sgn}(\pm 1) \dot{\theta} u_T \right. \]

\[ \times \dot{u}_T - 2 \dot{\theta} u_N \dot{u}_T - 2 \left( \frac{M_{NT}^2}{M_P^2 \varepsilon H^2} - \varepsilon H \dot{\theta} \right) u_T u_N + \frac{2}{M_P^2} H \]

\[ \times \delta \lambda \left( u_T - \frac{\dot{u}_T}{\varepsilon H} + \frac{1}{\varepsilon H} \dot{\theta} u_N \right) \]  

(11)

with

\[ \mathcal{L}_{u_N} = \text{sgn}(\pm 1) \left( \dot{u}_N^2 - \frac{1}{a^2} u_N^2 \right) + \left( \dot{\theta}^2 - \frac{M_{NT}^2}{M_P^2 \varepsilon H^2} \right) u_N^2 \]

(12)

and

\[ \mathcal{L}_{u_T} = \dot{u}_T^2 - \frac{1}{a^2} (\partial u_T)^2 - 2 \varepsilon H u_T \dot{u}_T \]

\[ + \left( \text{sgn}(\pm 1) \dot{\theta}^2 - \frac{M_{NT}^2}{M_P^2 \varepsilon H^2} \right) \dot{u}_T^2. \]

(13)

where

\[ M_{NT}^2 \equiv \frac{1}{2} (N^a N^b V_{ab} - \text{sgn}(\pm 1) \varepsilon H^2 \mathcal{R}), \]

\[ M_{TT}^2 \equiv \frac{1}{2} T^a T^b V_{ab} + \varepsilon H (T^a V_a + M_P^2 \varepsilon H^3 (3 - \varepsilon)), \]

\[ M_T^2 \equiv \frac{1}{2} (N^a N^b V_{ab} + \varepsilon H N^a V_a), \]

(14)

where \( T^a \) and \( N^a \) are the tangent and normal unit vectors with respect to the background trajectory, while \( \mathcal{R} \) is the Ricci scalar of field space. Substituting Eq. (10) into \( \mathcal{L}_{u_T} \) in (13), it appears that \( \dot{u}_T^2 \) can be re-expressed using \( u_T \) and \( u_N \), thus the kinetic term of \( u_T \) get lost. Moreover, this problem cannot be avoided by gauge transformations. In the next section, we will involve curvaton mechanism to solve this problem.

### III. CURVATON MECHANISM IN MULTI-FIELD MIMETIC GRAVITY

In this section, we will investigate the curvaton mechanism in multi-field mimetic gravity. In order to show the realization of curvaton mechanism, with the freedom of choosing the metric of field space \( G_{ab} \), we choose the following metric:

\[ G_{ab} = \text{diag} \left\{ 1, 6 \sinh^2 \left( \frac{\sigma}{\sqrt{6} M} \right) \right\}. \]

(15)

This metric is also applied to the so-called \( \alpha \)-attractor inflation models \([29-33]\), which can preserve the local conformal symmetry with the transformation \( \tilde{g}_{\mu \nu} = \exp[-2\sigma(x)]g_{\mu \nu}, \tilde{\chi} = \exp[\sigma(x)]\chi \) and \( \tilde{\phi}^i = \exp[\sigma(x)]\phi^a \) with the original inflaton fields \( \chi \) and \( \phi^a \). Moreover, it can avoid the initial condition problem for inflationary potential as well. Applying this metric, action (9) becomes

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \lambda \left( -\frac{1}{2} \dot{\phi}^i + \frac{1}{2a^2} \partial_i \phi \partial^i \right) \partial_i \phi \partial^i \phi^a + 3 \sinh^2 \left( \frac{\sigma}{\sqrt{6} M} \right) \left( -\dot{\theta}^2 + \frac{1}{a^2} (\partial_i \phi \partial^i \theta) + M^4 \right) \right] \]

\[ - V(\varphi, \theta) \right\}. \]

(16)

The equations of motion of this two mimetic fields are

\[ \frac{\partial}{\partial \varphi} \left( \frac{\sigma}{\sqrt{6} M} \right) \left( -\dot{\theta}^2 + \frac{1}{a^2} (\partial_i \phi \partial^i \theta) + M^4 \right) \frac{\partial V(\varphi, \theta)}{\partial \varphi} = 0, \]

\[ \frac{\partial}{\partial \theta} \left( \frac{\sigma}{\sqrt{6} M} \right) \left( -\dot{\theta}^2 + \frac{1}{a^2} (\partial_i \phi \partial^i \theta) + M^4 \right) \frac{\partial V(\varphi, \theta)}{\partial \theta} = 0, \]

while the mimetic constraint is

\[ \dot{\phi}^2 - \frac{(\partial \phi)^2}{a^2} + 6 \sinh^2 \left( \frac{\sigma}{\sqrt{6} M} \right) \left( -\dot{\theta}^2 + \frac{1}{a^2} (\partial \theta)^2 \right) = 2M^4. \]

(18)

In the following, we will analyze the background evolution at two epochs separately, namely the inflation epoch and the following matter-dominant epoch. For the sake of simplicity, we will ignore the reheating and radiation-dominant epoch, assuming that these epochs took place instantaneously.

### A. the inflation epoch

In this subsection, we would like to show the realization of a inflation era in our model. We regard \( \varphi \) as
inflaton which drives inflation, while θ as curvaton, who plays little role in the background level, but plays main role in creating the curvaton perturbation. As demonstrated in the original curvaton paper [22], we require that the potential along with the trajectory of θ should be sufficiently flat, namely \(|\partial^2 V/\partial \theta^2| \ll H^2\). One of the convenient choice is to consider a potential that is nearly independent of θ, namely \(V(\tanh(\varphi/\sqrt{6}M), \theta) \approx V(\varphi)\). Since the background behavior at this epoch is mainly determined by the inflaton \(\varphi\), the equations of \(\varphi_0\) and \(\theta_0\) in Eq. (17) become:

\[
\ddot{\varphi}_0 + 3H \dot{\varphi}_0 + \frac{\lambda}{\lambda_0} \varphi_0 - \sqrt{\frac{1}{6}M^2} (2M^4 - \dot{\varphi}_0^2) + \frac{1}{\lambda} \frac{dV}{d\varphi_0} \approx 0 \, ,
\]

\[
\ddot{\theta}_0 + 3H \dot{\theta}_0 + \frac{\lambda}{\lambda_0} \theta_0 + \frac{2}{3M^2} \coth \left( \frac{\varphi_0}{\sqrt{6}M} \right) \varphi_0 \theta_0 \approx 0 \, .
\]

As the Refs. [29–33] has suggested, the specific form of \(V(\varphi)\) can be chosen as \(V(\varphi) \propto \tanh(2\varphi/\sqrt{6}M)\) at large \(\varphi\) plotted in Fig. 1. In Fig. 1 one can see that, the potential is rather flat in the large field stage, which is reasonable to cause a period of slow-roll inflation era.

From the Friedmann equation (4) as well as the equation for \(\lambda (5)\), the Hubble parameter can be directly expressed as the potential functional \(V(\varphi)\). However, to know its behavior with respect to time variable such as \(t\) or \(a\), we have to know the form of \(\varphi(t)\) as well.

It is not difficult to get one solution as \(\theta_0 \approx 0\) with the sufficiently flatness of potential along with the trajectory of \(\theta\). Moreover, in the current case we can also utilize the constraint equation (18) without solving the equation of motion of \(\varphi\) as usual inflation model. After performing slow-roll conditions and neglecting spatial derivative term as well as the solution of \(\theta_0\) field, Eq. (18) gives

\[
\dot{\varphi} \approx \pm \sqrt{2}M^2 \, .
\]

As a consistency check, we also numerically solve the equation of motion (19), and plot the behavior of \(\dot{\varphi}(t)\) in Fig. 2, while the slope gives the value of \(\dot{\varphi}\). One can see from the plot that Eq. (21) is indeed the solution of the equation of motion (19) at large field value region.

Making use of the above result, one can get the numerical plot of the square of Hubble parameter \(H^2\) as in Fig. 3. We show that the Hubble parameter is flat at the beginning and then decreases, which are correspondingly the inflation era and its end. We also plot the evolution behavior of each component (say, \(V\) and \(\lambda M^4\)) of \(H^2\). Following the analysis in the previous section, the inflation ends when \(V = -\lambda M^4\), namely \(\ln(-2\lambda M^4) = \ln(V) + \ln 2\). As showed in the Fig. 3, it actually happens at the time when the e-folding number \(N \equiv \ln(a/a_i) \approx 73.68\), which is consistent with the current constraints on inflation.

### B. matter-dominated epoch

In the matter-dominated epoch, the inflaton field \(\varphi\) has rolled along the potential from the large field region to the small field region. Therefore we have

\[
\frac{\varphi_0}{M} \ll 1\, , \sinh \left( \frac{\varphi_0}{\sqrt{6}M} \right) \approx \frac{\varphi_0}{\sqrt{6}M}\, , \cosh \left( \frac{\varphi_0}{\sqrt{6}M} \right) \approx 1 \, .
\]

Moreover, according to Eq. (6) and the analysis in Sec. II, one has

\[
\lambda \propto a^{-3} \, .
\]

The equation of \(\varphi_0\) in Eq. (17) becomes

\[
\dot{\varphi}_0 - \frac{1}{\varphi_0} (2M^4 - \dot{\varphi}_0^2) + \frac{1}{\lambda} V' = 0 \, .
\]

Note that in this case, the \(3H\) friction term that is to appear in the equation of motion has been cancelled by \(\dot{\varphi}/\lambda\) term, so the equation looks like that in Minkowski
constraining on the effective mass of $\varphi$ (modulo the amplitude $A$, which can be obtained from previous process of the inflaton's evolution.) Moreover, if we furtherly constrain the $m_{\text{eff}}$ to be $m_{\text{eff}}t \simeq m_{\text{eff}}/H \ll 1$, we can then pick up the leading order of the Taylor expansion of (25), which gives

$$\varphi_0^2 \simeq A_{\text{eff}} t .$$

Moreover, under the assumptions (22) and solution (26), the (17) becomes

$$\dot{\theta}_0 + 2\frac{\dot{\varphi}_0}{\varphi_0} \theta_0 = 0 ,$$

and it is easy to get the solution

$$\dot{\theta}_0 = \frac{C_\theta}{\varphi_0^2} \simeq \frac{C_\theta}{A_{\text{eff}} t} .$$

where $C_\theta$ is a integral constant. On the other hand, the mimetic constraint (18) becomes

$$\frac{A_{\text{eff}} t}{4t} + \frac{C_\theta^2}{M^2 A_{\text{eff}} t} = 2M^4 .$$

The constraint seems to be inconsistent with the solution (25) and (28). The inconsistency arises because as the inflation has ended, the inflaton field (maybe as well as the curvaton field) will decay into other products. This will make the constraint equation no longer the form of (18), but should also include the decay products. We denote the decaying products as another field $\psi$, and the constraint equation should be $\dot{\psi}^2 + 6 \sinh^2(\varphi/\sqrt{6}) \theta^2 + \dot{\psi}^2 = 2M^4$. The detailed calculation will be modified accordingly as can be seen in the Appendix A, and our main result will not get changed.

**IV. GENERATING CURVATURE PERTURBATIONS**

In this section, we will investigate the perturbations generated from the curvaton model, especially how adiabatic perturbations can be transferred from the isocurvature ones. We perturb the two scalar fields as $\varphi = \varphi_0 + \delta \varphi$ and $\theta = \theta_0 + \delta \theta$. Therefore according to the total equations of motion (17), the explicit perturbed equations of
motion are
\[
\begin{align*}
\delta \ddot{\varphi} + 3H \delta \dot{\varphi} + \frac{k^2}{a^2} \delta \phi - \frac{1}{M^2} \left[ \sinh^2 \left( \frac{\varphi_0}{\sqrt{6} M} \right) \delta \phi \right] - \cosh^2 \left( \frac{\varphi_0}{\sqrt{6} M} \right) \left[ \delta \theta_0 - \frac{1}{a} \partial_i \delta \theta_0 \partial^i \varphi_0 \right] \delta \phi & = 0, \\
- \sqrt{\frac{2}{3} M^2} \sinh \left( \frac{\varphi_0}{\sqrt{6} M} \right) \cosh \left( \frac{\varphi_0}{\sqrt{6} M} \right) \left( \delta \theta_0 \delta \dot{\theta} \right) & = 0, \\
- \frac{1}{3\sigma^2} \partial_i \delta \theta_0 \partial^i \delta \theta & + \frac{1}{M} \frac{d^2 V(V)}{d\varphi^2} \varphi_0 \delta \varphi = 0, \\
\delta \ddot{\theta} + 3H \delta \dot{\theta} + \frac{k^2}{a^2} \delta \phi - \frac{1}{a} \partial_i \delta \theta_0 \partial^i \varphi_0 + \sqrt{\frac{2}{3} M^2} \coth \left( \frac{\varphi_0}{\sqrt{6} M} \right) \left( \delta \varphi_0 \delta \theta_0 - \partial_i \delta \varphi_0 \partial^i \varphi_0 \right) & = 0, \\
- \frac{1}{3M^2} \text{csch}^2 \left( \frac{\varphi_0}{\sqrt{6} M} \right) \delta \varphi \left( \delta \varphi_0 - \delta \varphi \right) & = 0,
\end{align*}
\]

(30)

where we ignore the perturbation of $\lambda$. From the constraint equation (18), the perturbed mimetic constraint is

\[
\varphi_0 \delta \phi + 6 \sinh^2 \left( \frac{\varphi_0}{\sqrt{6} M} \right) \dot{\theta}_0 \dot{\theta} + \sqrt{\frac{6}{M}} \sinh \left( \frac{\varphi_0}{\sqrt{6} M} \right) \cosh \left( \frac{\varphi_0}{\sqrt{6} M} \right) \delta \varphi \delta \varphi_0 = 0.
\]

(31)

where we considered the homogeneity and isotropy of the background fields.

A. the inflation epoch

At the inflation epoch, from the slow-roll condition (22) one can obtain the equation of motion for $\delta \theta$:

\[
\dot{\delta} \theta + 3H \delta \theta + \frac{k^2}{a^2} \delta \phi - \sqrt{\frac{2}{3}} \frac{\dot{\varphi}}{M} \delta \theta = 0.
\]

(32)

Note that the last term comes from the constraint equation. In our case where $\dot{\varphi} \approx -\sqrt{2} M^2$ (see Eq. (21)), so $(\dot{\varphi}/M) \delta \theta \approx \delta \theta$. As long as we require $M \ll H$, namely the energy scale of the scalar field is much less than the Hubble parameter, the last term can be ignored. Thus Eq. (32) will be the same as the perturbation equation of usual curvaton field. On the other hand, due to the fact that the mimetic constraint requires the perturbations of both field should be non-vanishing, we also consider the perturbations of $\varphi$, unlike the usual case where we simply neglect the perturbations of inflaton. The perturbed equation of motion of $\varphi$ has a similar equation as $\delta \theta$ at super-horizon scale where $k^2 \ll H^2$ again,

\[
\delta \ddot{\varphi} + 3H \delta \dot{\varphi} + \frac{V''(\varphi)}{\lambda} \delta \varphi = 0,
\]

(33)

where $V'' \equiv d^2 V(V)/d\varphi^2$. Similarly, under the slow-roll condition $V''/H^2 \ll 1$, the last term can be ignored. Therefore, we obtain the solutions of $\delta \varphi$ and $\delta \theta$ as:

\[
\delta \varphi \approx \delta \theta \approx \frac{H}{2\pi}.
\]

(34)

B. the matter-dominated epoch

In matter dominated universe, the perturbed equations of motion of $\delta \varphi$ and $\delta \theta$ are both wave equations where $3H$ term is cancelled by $\lambda/\lambda$ term, as demonstrated in Sec. III B. Moreover, taking into account the assumption (22), we can obtain the perturbed equations of motion in matter dominated universe from eq (30) as

\[
\delta \ddot{\varphi} + \frac{k^2}{a^2} \delta \varphi - \frac{2M^4}{\varphi_0^2} \delta \varphi + 2 \varphi_0 \delta \dot{\varphi} + \frac{1}{\lambda} V'' \delta \varphi = 0,
\]

(35)

\[
\delta \ddot{\theta} + \frac{k^2}{a^2} \delta \phi + \frac{2\varphi_0}{\varphi_0^2} \delta \phi = 0.
\]

(36)

Furthermore, taking the mimetic constraint (18), perturbed constraint (31), and the solutions (25) and (28) into the equations of motion of $\delta \varphi$ (35), it becomes

\[
\frac{d^2}{dt^2} \left( \varphi_0 \delta \varphi \right) + \left( \frac{k^2}{a^2} + \frac{3}{2} m_{eff}^2 \right) \varphi_0 \delta \varphi = 0.
\]

(37)

Since $m_{eff} \ll H \ll k^2/a^2$, the effective mass term in the above equation can be omitted. Therefore solution of $\delta \varphi$ is

\[
\delta \varphi = \frac{1}{\varphi_0} \left( D_+ e^{iKt} + D_- e^{-iKt} \right),
\]

(38)

where $\varphi_0 \propto t^{1/2}$. Since the perturbations in this stage are inherited from those in the inflation stage, the coefficients $D_\pm$ is determined by the previous solutions. From Eq. (36) the solution of $\delta \theta$ with $C_0 = A m_{eff}/2\sqrt{3}$ in (28) is (see the Appendix A for detailed calculations)

\[
\delta \theta = \frac{\sqrt{3}}{3 A m_{eff} t} \left( D_+ e^{iKt} + D_- e^{-iKt} \right).
\]

(39)

The perturbations can be transferred to curvature perturbation, when either the curvaton field dominates the universe or the curvaton decays into the background, whatever is earlier [22, 23]. The curvature perturbation is related to the density perturbations of each component as

\[
\zeta = -H \frac{\delta \rho}{\rho} \simeq \frac{1}{3} \left( \frac{\rho_\varphi}{\rho_{tot}} \delta \varphi + \frac{\rho_\theta}{\rho_{tot}} \delta \theta \right),
\]

(40)

where we define the density contrast of $\varphi$ and $\theta$ as:

\[
\delta \varphi \equiv \frac{\delta \rho_\varphi}{\rho_\varphi}, \quad \delta \theta \equiv \frac{\delta \rho_\theta}{\rho_\theta}.
\]

(41)

Note that for $\varphi$ field, the energy density is mainly contributed by its potential energy, therefore we have $\rho_\varphi \propto m_{eff}^2 \varphi_0^2$ and $\delta \rho_\varphi \propto m_{eff}^2 (\varphi_0^{4}\delta \varphi)$, which gives $\delta \varphi \propto 1/t$ (Gaussian part) or $1/t^4$ (non-Gaussian part). While for $\theta$ field with no potential, the energy density is mainly contributed by its kinetic energy. Therefore one has
\[ \rho_0 \propto \varphi_0^2 \dot{\theta}_0^2 \sim C^2_0 / \varphi_0^2 \] and \( \delta \rho_0 \propto \varphi_0^2 (\dot{\theta}_0 \delta \theta) = \varphi_0^2 \sqrt{\dot{\theta}_0^2 (\delta \theta)^2} \), making \( \delta \theta \) time-independent. Since the \( \delta \varphi \) will dilute as time goes while \( \delta \theta \) does not, and \( \varphi_0^2 \sim m_{\text{eff}} t \ll 1 \) giving \( \rho_\varphi \ll \rho_0 \), the term containing \( \delta \theta \) will dominate over the other.

To be more specific, with \( P = k^3 |\delta \theta|^2 / 2\pi^2 \), we have the re-entering power spectrum with its initial value correspondence with the value at the end of inflation

\[
P_{\theta,\text{md}} = \frac{t_i^2}{t_{\text{decay}}^2}, \tag{42}
\]

where \( t_{\text{decay}} \) is the physical time of the almost total decay of \( \varphi \). Following [22], before working out the density contrast \( \delta \), we should calculate \( \langle \delta \theta^2 \rangle \) firstly and compare it with \( \dot{\theta}^2 \) to figure out the feature of power spectrum:

\[
\langle \delta \theta^2 \rangle = \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{P_{\theta,\text{md}}(k,t) \, dk}{k} = \frac{H_i^2}{10\pi^2 M_p^2 \pi c k_{\text{min}}} \left( \frac{k_{\text{max}}}{k_{\text{min}}} \right). \tag{43}
\]

Thus the Gaussian and non-Gaussian power spectrum are both time independent. The Gaussian power spectrum of curvature perturbation is

\[
P_\zeta = r^2 \frac{H_i^2}{2\pi^2 M_p^2 c \dot{\theta}_i^2} \tag{44}
\]

where \( r \equiv \rho_0 / \rho_{\text{tot}} \) is the energy fraction of curvaton-like field \( \theta \) and \( \dot{\theta}_i \) is the initial value of \( \theta \) after preheating as a constant.

The converted curvature perturbation from our curvaton-like field \( \theta \) is required to conform to Planck 2018. Considering the assumption that the value of effective mass of \( \varphi \) is quite small, as \( m_{\text{eff}} \ll H \), we point out that the inflationary energy scale is large enough to enable the hot big-bang universe. On the other hand, although the contribution in CMB from \( \delta \varphi \) is negligible, what’s interesting is that due to the mimetic constraint as well as the coupling, we do have fluctuation on inflaton-like field \( \varphi \) at inflationary era, which is expected to be tested with the future accuracy of observations.

\section{Conclusion}

In this paper, we study a kind of modified gravity inflation model with inflaton driven by the mimetic gravity field. In such kind of models, due to the constraint condition, the evolution of energy density is solely determined by the potential of inflaton field \( \varphi \). Due to the design of the potential, our model can connect the early-time inflation with the late-time matter domination. However, since the single-field mimetic inflation model has problem with quantum fluctuations, we refer to multi-field mimetic inflation model, and in order to explain how adiabatic (curvature) perturbation is generated in multi-field case, we apply the curvaton mechanism, where one of the fields is interpreted as a curvaton field.

To be more precise, we choose the metric of field space as the T-model of \( \alpha \)-attractor inflation, which keeps the conformal invariance. We have calculated the background and perturbation solutions for both inflation and the following matter-dominant era, where we assume that the reheating process occurs instantaneously. In the inflation era, the inflaton field evolves with a nearly constant velocity \( \dot{\varphi} \sim -\sqrt{2} M_2 \), while the curvaton field nearly remains static due to the mimetic constraint. In the matter-dominant era, the inflaton field oscillates around the minimum of its potential with small effective mass, while the massless curvaton field evolves as \( \dot{\theta} \sim \varphi^{-2} \). To satisfy the constraint equation as well, we also include another field that works as the decay product of both fields. For the perturbation part, since there is a coupling as well as the constraint equation that links the two fields (\( \varphi \) and \( \theta \)), we have to consider perturbations from both inflaton and curvaton. By analytical calculations, we show that these two fields behave similarly as shown in eq. (34). However, one cannot use the slow-roll condition after inflation, thus the differences between inflaton field and curvaton field will evolve differently in MD, for which our calculation shows that the curvature perturbation induced by the isocurvature perturbation is obtained.

Here, we emphasize the importance of our theoretical construction combined with curvaton mechanism and mimetic gravity. Our work suggests that although in mimetic gravity the constraint equation hides the kinetic term of the adiabatic perturbation mode in the action, so as to make it seemingly non-propagate, it actually can be produced from the perturbation of curvaton field, which is mainly isocurvature in the curvaton mechanism, so that one might not need to worry about this non-propagating issue. Moreover, if one introduces the coupling between the two fields, the perturbation of the inflaton field will also appear, which can contribute to the adiabatic perturbations as well. Although in this paper we take a specific example for the illustration, one is free to have it extended to more general cases, such as in the cucuton cosmological framework [36–38].

There still remains lots of questions that needs to be done. Firstly, we have ignored the reheating process, which may have effects on the evolution of perturbations after inflation. Furthermore, from the observational perspective, we could constrain the gravitational waves as well as parameters \( r \) which tests its validity for the non-Gaussianities. Finally, we could apply our method to investigate the primordial black hole generation, effects

\footnote{Here we used the approximation \( \langle \delta \dot{\theta}^2 \rangle \sim \langle \delta \theta^2 \rangle \) for oscillating solution of \( \delta \theta \).}
on Big Bang Nucleosynthesis, etc. We postpone these investigations to an upcoming work.

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Appendix A: Curvature perturbation in matter dominated universe

Here we consider a little bit more on the case where the decaying product $\psi$ is also included in the constraint equation in matter-dominated epoch, as was discussed in Sec. III B. From Eq. (17) The equation of motion for $\varphi$ is

$$\ddot{\varphi} - \varphi \dot{\varphi}^2 / M^2 + V'_\varphi / \lambda = 0 \ , \quad (A1)$$

where the mimetic constraint equation then gives

$$\dot{\theta}^2 = \frac{M^2}{\varphi_0^3} \left( 2M^4 - \psi^2 - \varphi_0^2 \right) \ . \quad (A2)$$

As has been demonstrated, thanks to the $\psi$ field, there will be one more degree of freedom in the constraint equation. Therefore it can be made consistent with the solution of equation of motions of $\varphi$ and $\theta$. If we construct the potential $V(\varphi)$ as

$$V/\lambda = m_{eff}^2 \varphi_0^2 / 4 + (2M^4 - \psi^2) \ln(\varphi_0 / \varphi_1) \ , \quad (A3)$$

then the $2M^4 - \psi^2$ part in equation of motion of $\varphi$ in Eq. (A1) is cancelled. We still can obtain the equation

$$\frac{d^2}{dt^2} (\varphi_0^2) + m_{eff}^2 \varphi_0^2 = 0 \ , \quad (A4)$$

and get the solution $\varphi_0 = \sqrt{A} \sin^{1/2}(m_{eff} t) \simeq \sqrt{A} m_{eff}$. Therefore the background solution of $\varphi$ and $\theta$ is not affected. On the other hand, assuming $\delta \lambda = 0$ and $\delta \psi = 0$ while applying Eq. (22), we can obtain the perturbed equations of motion

$$\begin{cases} \delta \ddot{\varphi} + \frac{k^2}{a^2} \delta \varphi - \left( \frac{\varphi_0^2}{M^2} + 1 \right) \frac{\varphi_0^2}{M^2} \delta \varphi - \frac{2 m_{eff} \theta_0}{M^2} \delta \theta \\ + \frac{1}{\lambda} V,_{\varphi \varphi} \delta \varphi = 0 \end{cases} \quad (A5)$$

$$\begin{cases} \delta \ddot{\theta} + \left( \frac{k^2}{a^2} - \frac{2}{3} \right) \delta \theta + \frac{1}{3} \left( M^2 - \frac{2}{\sqrt{\psi_0^2}} \right) \varphi_0 \theta_0 \delta \varphi + 2 \frac{\varphi_0^2}{\sqrt{\psi_0^2}} \delta \theta + 2 \varphi_0 \delta \varphi = 0 \end{cases} \quad (A5)$$

Since $\varphi_0^2 / M^2 \ll 1$, using Eqs. (A2) and (A3), we can derive the equation of $\varphi \delta \varphi$

$$\frac{d^2}{dt^2} (\varphi_0 \delta \varphi) + \left( \frac{k^2}{a^2} + \frac{1}{2} m_{eff}^2 - \frac{\varphi_0^2}{\varphi_0^2} - \frac{\dot{\varphi}_0}{\varphi_0} \right) \varphi_0 \delta \varphi = 0 \ , \quad (A6)$$

where $\varphi_0^2 / \varphi_0^2 = - \varphi_0 / \varphi_0 = 1 / 4t^2$. Since $m_{eff}^2 \ll H^2$, the $m_{eff}^2$ term can be omitted, thus the solution of $\delta \varphi$ is again Eq. (38). Moreover, considering the perturbed mimetic constraint

$$- \delta \ddot{\theta} \simeq \frac{M^2}{\varphi_0^2} \left( \dot{\varphi}_0 \delta \varphi + \varphi_0 \delta \varphi \right) \ , \quad (A7)$$

and the solution of $\dot{\theta}_0 = C_\theta / \varphi_0^2$, the last three term in equation of $\dot{\theta}$ in Eq. (A5) becomes

$$- \frac{2 C_\theta \dot{\varphi}_0}{\varphi_0^4} \delta \varphi + 2 \frac{C_\theta}{\varphi_0^3} \delta \dot{\varphi} - \frac{2 \dot{\varphi}_0^2}{C_\theta \varphi_0^2} \varphi_0 \delta \varphi + \varphi_0 \theta^2 \delta \varphi \ , \quad (A8)$$

where

$$\begin{align} - \frac{2 C_\theta \dot{\varphi}_0}{\varphi_0^4} & \simeq \frac{C_\theta}{(Am_{eff})^{3/2} t^{5/2}} , \\ + \frac{2 C_\theta}{\varphi_0^3} \delta \dot{\varphi} & \simeq \frac{C_\theta}{(Am_{eff})^{3/2}} \left( \pm i \frac{2k}{a} - \frac{1}{t} \right) \delta \varphi , \\ - \frac{2 \dot{\varphi}_0^2}{C_\theta \varphi_0^2} \varphi_0 \delta \varphi & \simeq - \sqrt{Am_{eff}} \frac{C_\theta}{4 C_\theta t^{3/2}} \left( \pm i \frac{2k}{a} - \frac{1}{t} \right) \delta \varphi , \quad (A9) \end{align}$$

Without the third line, $\delta \theta$ will evolves time-independently after it gets close to the horizon, which is not an acceptable solution for curvature perturbation. This point requires $12 C_\theta^2 = A^2 m_{eff}^2$, so that the perturbed equation (A5) becomes

$$\delta \ddot{\theta} + \left( \frac{k^2}{a^2} - \frac{2}{3} \right) \delta \theta \frac{2 \sqrt{3}}{3 Am_{eff}} \left[ \frac{1}{t^3} \mp i \frac{k}{a t^2} \right] D_{\pm e^{ik\tau}} = 0 \ , \quad (A10)$$

with solution

$$\delta \theta = \frac{\sqrt{3}}{3 Am_{eff} t} D_{\pm e^{ik\tau}} \simeq \frac{\sqrt{3}}{3 \varphi_0^2} D_{\pm e^{ik\tau}} \ . \quad (A13)$$

which has an oscillating behavior with the amplitude proportional to time inverse.
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