Synchronous activity among individual neurons or their ensembles is a robust phenomenon observed in many regions of the brain, in sensory systems and in other neural networks. With constant synaptic connections the regions of neural synchronization are quite narrow in parameter space and the origin of the observed robustness of synchronization is not clear. It is known that many neurons in the cortex, in the cerebellum and in other regions of the brain, in sensory systems and in other cortices are typical in sensory systems [4, 5] and neurons in the electrosensory lobe of fish [3]. The connections between neurons, and its inverse, observed, for example, in neurons with unidirectional, activity-dependent excitatory coupling. The map allows us to speculate about the general applicability of learning-enhanced synchronization.

We consider here the simplest neural network: two neurons with unidirectional, activity-dependent excitation by activity-dependent coupling. The map allows us to speculate about the general applicability of learning-enhanced synchronization.

There have been recent experimental advances in the understanding of such plasticity, and, in particular, of the critical dependence on timing in presynaptic and postsynaptic signaling. Two manifestations of this kind of synaptic plasticity are the Spike-Timing Dependent Plasticity (STDP) [1, 2] seen in excitatory connections between neurons, and its inverse, observed, for example, in the connections between excitatory and inhibitory neurons in the electrosensory lobe of fish [2]. The connections between excitatory neurons through inhibitory interneurons are typical in sensory systems [4, 5] and cerebral cortex [3]. These also express synaptic plasticity [1] and play an important role in the control and synchronization of neural ensembles in hippocampus.

We report here on the synchronization of two model neurons coupled through STDP or inverse STDP synapses. We demonstrate that such coupling leads to neural synchronization which is more rapid, more flexible and much more robust against noise than synchronization mediated by constant strength connections. (For reviews, see [6, 7, 10]). We also build a simple discrete map that illustrates the enhancement of synchronization by activity-dependent coupling. The map allows us to speculate about the general applicability of learning-enhanced synchronization.

The parameters in these equations are given in [2].

Each neuron receives a constant input $I_{stim}$ forcing it to spike with a constant, $I_{stim}$-dependent frequency. The second neuron is synaptically driven by the first via an excitatory current dependent on the postsynaptic $V_2(t)$ and presynaptic $V_1(t)$ membrane voltages:

$$I_{syn}(t) = g(t)S(t)V_2(t).$$

$S(t)$ is the fraction of open synaptic channels. It satisfies first-order kinetics:

$$\frac{dS(t)}{dt} = \alpha(1 - S(t))H(V_1(t)) - \beta S(t),$$

with $H(V_1(t)) = (1 + \tanh(10V_1(t)))/4$.

The time dependent synaptic coupling strength $g(t)$ is conditioned by the dynamics of the pre- and postsynaptic
neurons. We consider two types of activity-dependent couplings: (1) an excitatory synapse with STDP, and (2) an excitatory synapse with inverse STDP. Through STDP $g(t)$ changes by $\Delta g(t)$ which is a function of the time difference $\Delta t = t_{post} - t_{pre}$ between the times of post- and presynaptic spikes. We use the additive update rule

$$\Delta g(t) = G(\Delta t) = A \text{sgn}(\Delta t) \exp(-\gamma|\Delta t|)$$

for STDP, and $\Delta g(t) = -G(\Delta t)$ for inverse STDP. We used $A = 0.004 \mu S$ and $\gamma = 0.15 \text{ms}^{-1}$.

We studied the synchronization properties of this coupled system by setting the autonomous period of the postsynaptic neuron to 15 ms, then evaluating the actual period of its oscillation $T_2$ as a function of the imposed autonomous oscillation period $T_1$ of the presynaptic neuron. In Fig. 1 we show $T_1/T_2$ as a function of $T_1$ in two cases: (a) a synaptic coupling with constant strength 0.008 $\mu S$ and (b) a synaptic coupling with inverse STDP. In the later case the steady-state coupling strength depends on the ratio of neuronal frequencies ($c$). Its average over all $T_1$ values is 0.002 $\mu S$, which is much lower than the strength in the case of constant coupling.

In Fig. 2 we see the familiar ‘Devil’s Staircase’ associated with frequency locking domains of a driven nonlinear oscillator. Only frequency locking with ratios 1:1, 2:1, 3:1, and 4:1 leads to synchronization plateaus with significant width. In Fig. 3, we see that the synchronization domains are substantially broadened due to activity-dependent coupling, especially for $T_1/T_2 = 1$. Some synchronization plateaus exhibit multistability, which we confirmed by observing the associated hysteresis. These results show that even a weak, but adaptive connection with strength that is determined dynamically is able to greatly enhance and enrich synchronization.

We also studied the robustness of this enhanced synchronization in the presence of noise by adding zero mean, Gaussian, white noise to the membrane currents of each neuron. We examined the behaviour of the system with RMS noise amplitudes $\sigma = 0.01, 0.05, 0.1, 0.5$ nA.

For $\sigma = 0.01$ nA no phase-locking plateaus were destroyed. At $\sigma = 0.05$ nA the 4:1 plateau became distorted. Larger $\sigma$ sequentially eliminated synchronization plateaus until only the 1:1 plateau remained. The 1:1 plateau was seen for all $\sigma$. In Fig. 4 we illustrate the effect of the noise on synchronization when $\sigma = 0.1$ nA with (a) constant and (b) inverse STDP coupling. While in (a) most of the plateaus have disappeared, in (b) the 1:1, the 2:1 and even the 3:1 frequency locking regimes remained. In sharp distinction to classical synchronization, frequency locking through activity-dependent coupling is significantly more robust in the presence of noise.

To understand the mechanisms behind such a remarkable robustness we studied the diffusion of oscillation phase caused by noise. For $\sigma = 0.5$ nA in Fig. 5, we show that in the case of 1:1 synchronization and coupling with constant strength 0.008 $\mu S$ noise-induced phase diff-

![FIG. 1: Devil’s Staircase for (a) constant synaptic strength and (b) synaptic strength varying according to inverse STDP coupling. $T_1$ and $T_2$ are the observed periods of the presynaptic (driving) neuron and postsynaptic (driven) neuron respectively. In (c) the final value of synaptic strength is displayed.](image-url)

fusion results in $2\pi$ phase slips that destroy synchronized state. Quite contrary Fig. 5 shows that in the case of activity-dependent coupling phase slips are absent and the phase difference does not increase. In this particular case the strength of coupling varied around the mean of 0.0064 $\mu S$ with standard deviation of 0.0026 $\mu S$.

In Fig. 6 we plot the average rate of phase slips for different amplitudes of the noise. In line with the above observation we see that in the case of activity-dependent coupling (dashed line) phase slips are suppressed in a wide range of noise amplitudes. We argue here that this suppression of phase slips is the primary mechanism responsible for robustness of synchronization mediated by activity-dependent coupling. After the introduction of a discrete map model we will discuss this mechanism in more detail.

We also considered synchronization through an activity-dependent synapse in the interesting case when the presynaptic neuron produces bursts of spikes and the postsynaptic neuron spikes irregularly. We found that synchronization through an STDP synapse is very fast; even a few spikes are enough for the frequency locking to establish itself. Neurons in the same set up with constant coupling synchronize much more slowly and only if the strength of the connection is appropriate for the given ratio of their frequencies. Hence, activity-dependent synapses allow adaptation ‘on the run,’ synching a postsynaptic neuron to the firing properties of its presynaptic partner.
FIG. 1: The difference of oscillation phases of two neurons as a function of time in the cases of (a) constant and (b) activity-dependent coupling.

FIG. 2: Same as Fig. [1], but with zero mean, Gaussian, white noise with $\sigma = 0.1$ nA added to the membrane currents.

FIG. 3: The difference of oscillation phases of two neurons as a function of time in the cases of (a) constant and (b) activity-dependent coupling.

FIG. 4: Average rate of phase slips as a function of RMS noise amplitude for the case of 1:1 synchronization and constant (solid line) or activity-dependent (dashed line) coupling.

To understand the above results in a general way we have constructed a discrete time map model of periodic generators with STDP-like coupling. This map accounts for the dependence of the coupling strength on the activity of generators. Take $T_0$ and $T_0$ as the autonomous periods of the first and second generators. As a result of unidirectional coupling, the period of the second generator will change by some amount $\Delta T$ each time it receives a spike from the first generator. Assuming initial phases to be 0, the time of the $n+1$-st spike of the first generator and $m+1$-st spike of the second generator are taken to satisfy

\[ t_{n+1}^{(1)} = t_n^{(1)} + T_1^0 \]
\[ t_{m+1}^{(2)} = t_m^{(2)} + T_2^0 - \Delta T_{m,n}, \]

where $n$ and $m$ are such that $t_{m+1}^{(2)} \leq t_n^{(1)} \leq t_{m+1}^{(2)}$. In general, $\Delta T_{m,n}$ would be a function of $T_1^0$, $T_2^0$, $t_n^{(1)}$, and $t_m^{(2)}$, and the coupling strength $g_{m,n}$. We argue that the two main variables here are $t_n^{(1)}$ and $t_m^{(2)}$, and $g_{m,n}$. In the simplest case $\Delta T_{m,n}$ can be approximated by

\[ \Delta T_{m,n} = g_{m,n} F(t_n^{(1)} - t_m^{(2)}). \]

where the function $F(x)$ is the analog of a phase response curve for our model. To obtain results quantitatively comparable with our neuronal model, we fit it by non-negative quadratic function that describes phase response of our model neurons: $F(x) = 835 + 63x - 9x^2$ for $0 \leq x \leq T_2^0$ and 0 otherwise. $g_{m,n}$ obeys the inverse STDP update rules:

\[ g_{m+1,n} = g_{m,n} - G(t_n^{(1)} - t_m^{(1)}) \]
\[ g_{m,n} = g_{m,n-1} - G(t_n^{(2)} - t_m^{(2)}). \]

In Fig. 3 we show the Arnol’d Tongues calculated for the map (1) in the cases of (a) constant and (b) inverse STDP coupling. As with the model neurons, we see that activity-dependent coupling greatly enlarges the zones of synchronization.

This discrete map can be further analyzed to find its fixed points corresponding to $n : m$ synchronization and to examine their stability. We present here only the case of 1:1 synchronization. Then $m = n$, and the system of equations (4) can be written in the following simple form:

\[ \tau_{n+1} = \tau_n + T_1^0 - T_2^0 + g_n F(\tau_n) \]
\[ g_{n+1} = g_n - G(T_1^0 - \tau_{n+1}) - G(-\tau_{n+1}). \]

where $\tau_n = t_n^{(1)} - t_n^{(2)}$. The fixed points of (1) are given by $g_n^* = (T_2^0 - T_1^0)/F(\tau_n^*)$ and $\tau_n = T_1^0/2$. Stability calculations show that for such $F(\tau)$ and $G(\tau)$ these fixed points are stable. The second fixed point illustrates that activity-dependent coupling introduces a new limitation on the relationship between the phases of two oscillators. It is this limitation that causes the suppression of phase
slips under the influence of noise. Detailed analysis shows that in the course of noise-affected synchronization the strength of activity-dependent coupling adjusts dynamically to keep this phase relationship close to satisfaction and, hence, suppresses phase slips.

In conclusion, we have analyzed the effects of activity-dependent coupling on synchronization properties of coupled neurons. We showed that such coupling results in a substantial extension of the temporal synchronization zones, leads to more rapid synchronization and makes it much more robust against noise. The enlargement of synchronization zones means that with STDP-like learning rules the number of synchronized neurons in a large heterogeneous population must increase. In fact, this is an aspect of the popular idea due to Hebb [14]. It is supported by the results in [13, 10] which indicate that the coherence of fast EEG activity in the gamma band increases in a process of associative learning.

Based on our discrete map model results, we argue that the particular details of the signal-generating devices (e.g. neurons) and their connections (e.g. synapses) are not essential and the obtained results have general applicability. In fact, we observed similar phenomena of robust and enhanced synchronization in computer simulations of other types of periodic generators (such as Van-der-Pol and \( \theta \)-oscillators) with STDP-like activity-dependent coupling.

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\(-95 \, mV \), \( \alpha = 10 \, ms^{-1} \), \( \beta = 0.2 \, ms^{-1} \), \( C = 1.43 \times 10^{-4} \, \mu F \), \( \alpha_n = 0.032(50 - V)/(exp((-50 - V)/5) - 1) \), \( \beta_n = 0.5 \times exp((-55 - V)/40) \), \( \alpha_m = 0.32(-52 - V)/(exp((-52 - V)/4) - 1) \), \( \beta_m = 0.28(25 + V)/(exp((25 + V)/5) - 1) \), \( \alpha_h = 0.128 \times exp((-48 - V)/18) \) and \( \beta_h = 4/(exp((-25 - V)/5) + 1) \).
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