Measuring Luttinger Liquid Correlations from Charge Fluctuations in a Nanoscale Structure

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We suggest an experiment to study Luttinger liquid behavior in a one-dimensional nanostructure, avoiding the usual complications associated with transport measurements. The proposed setup consists of a quantum box, biased by a gate voltage, and side-coupled to a quantum wire by a point contact. Close to the degeneracy points of the Coulomb blockaded box, and in the presence of a magnetic field sufficiently strong to spin polarize the electrons, the setup can be described as a Luttinger liquid interacting with an effective Kondo impurity. Using exact nonperturbative techniques we predict that the differential capacitance of the box will exhibit distinctive Luttinger liquid scaling with temperature and gate voltage.

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It is theoretically well established that interacting electrons in one dimension (1D) do not form a Fermi liquid, but rather a composite — a Luttinger liquid [1] — where all low-lying excitations are collective, and separately carry charge and spin. Despite intense efforts, however, there are very few experiments that unambiguously point to Luttinger liquid behavior in a real 1D electron system. Quantum wires [2] and single-walled carbon nanotubes [3] are prime examples of systems where the electron dynamics is effectively one-dimensional. Still, interpretations of relevant experimental data based on Luttinger liquid theory remain controversial. In most experiments until now, one has measured transport properties, and it has been notoriously difficult to assess the extent to which external sources, contacts, impurities, etc., influence the results.

In this Letter, we propose a non-transport experiment on a 1D nanoscale structure which avoids the problems mentioned above. The system is composed by a 1D quantum box side-coupled to a single-mode quantum wire via a point contact (Fig. 1), and could be built from a gated GaAs semiconductor or cleaved edge overgrowth structure [4]. A magnetic field is applied such that the electrons become spin-polarized. The charging of the box is then monitored as a function of an applied gate voltage (Fig. 1), and could be built from a gated GaAs semiconductor or cleaved edge overgrowth structure [4].

We take the quantum box to be sufficiently small to exhibit Coulomb blockade [5], but large enough for the electrons in the box to be modeled by a (confined) Luttinger liquid. More precisely, we study the limit \( \delta E \ll k_B T_K \ll c^2/2C_V \), where \( \delta E \) is the average level spacing of the box close to the Fermi level, \( T_K \) is the temperature scale at which correlation effects set in (to be defined below), and \( c^2/2C_V \) is the charging energy of the box (with \( C_V \) the full capacitance of the box). \( \delta E \) thus serves as a low-energy cutoff restricting the validity of our analysis [6].

The system can be modeled by a Hamiltonian

\[
H = H_{el} + H_c + H_{tun},
\]

where

\[
H_{el} = \sum_{k,\alpha} \epsilon_k a_{k,\alpha}^\dagger a_{k,\alpha} + \sum_{q,\alpha,\beta} \hat{U}_{\alpha\beta}(q) \rho_{q,\alpha} \rho_{-q,\beta},
\]

\[
H_c = \frac{Q^2}{2C_V} + \zeta V Q_1,
\]

\[
H_{tun} = \frac{t}{\ell} \sum_{k,p} (a_{k,0}^\dagger a_{p,1} + h.c.).
\]

Here \( a_{k,\alpha} \) are the electron destruction operators in the wire (\( \alpha = 0 \)) and the box (\( \alpha = 1 \)), with the energy \( \epsilon_k \) measured from the Fermi level \( \epsilon_F \). In the interac-

\[
\text{FIG. 1: Schematic picture of the proposed setup. A 1D quantum box side-coupled to a quantum wire via a point contact. V is a gate voltage.}
\]
tion term \( p_{q,\alpha} \) are the Fourier components of the corresponding density operators in the wire and the box, and \( \hat{U}_{\alpha\alpha}(q) \) is the Fourier transform of the screened interaction potential in the wire and the box [between wire and box] (with the screening supplied by carriers in nearby gates). Since the wire and the box are defined on the same substrate, we shall take \( \hat{U}_{00}(q) = \hat{U}_{11}(q) \), assuming that their transverse widths are the same. The charging energy of the box is described by \( H_c \) with \( Q_1 \) measuring the surplus charge in the box w.r.t. the (zero bias) Fermi level, \( \zeta \) being a dimensionless parameter which depends on the layout of the sample, and \( V \) the gate voltage. The last term, \( H_{\text{tun}} \), governs the tunneling between the wire and the box, with \( t \) the tunneling rate through the point contact. Note that all effects from the finite size of the 1D box (incl. Coulomb blockade) are carried by \( H_c \), and that in \( H_{el} \) and \( H_{\text{tun}} \) the tunneling \( \ell \) of the box for simplicity is taken to be the same as that of the extended wire (here assumed to be sufficiently large for additional charging effects to be ignored). Also note that, while \( H_c \) encapsulates only the mean-field Coulomb interaction among electrons in the box, the electron-electron interaction in \( H_{el} \) is dynamic and influences the spectrum already for a fixed number of electrons in wire and box.

To make progress, we decompose the electron fields \( \psi_{\alpha}(x) \sim \int dk e^{ikx} d_{k,\alpha} \) in left (\( \psi_{R,\alpha}(x) \)) and right (\( \psi_{L,\alpha}(x) \)) components (with \( x \) the coordinate along the wire), expanded about the two Fermi points \( \pm k_F \) of the linearized spectrum. Keeping only the "local" piece \( U_{\alpha\beta}(x) = \hat{U}_{\alpha\beta}(0) \delta(x) \) of the potential, and setting \( \hat{U}_{01}(0) = \hat{U}_{00}(0) \equiv g \), \( H_{el} \) can be expressed on diagonal Sugawara form \[ \text{(5)} \] as

\[
H_{el} \approx \frac{1}{2\pi} \int dx \left[ \frac{v_c}{4} (J_R J_R^\dagger + : J_L J_L : ) + \frac{v_F}{3} (J_R^\dagger J_R + : J_L^\dagger J_L : ) \right],
\]

where "\( \approx \)" is a reminder that \[ \text{(5)} \] contains the local part of the interaction only. Here \( v_c = v_F (1 + 4g/v_F) \), with \( v_F \) the Fermi velocity, and the normal ordering is taken w.r.t. the filled Dirac sea. The currents are defined by

\[
J_{R/L} = \sinh \theta : \psi_{R/L,\alpha}^\dagger \psi_{R/L,\alpha} + \cosh \theta : \psi_{L/R,\alpha}^\dagger \psi_{L/R,\alpha} ;
\]

\[
J_{R/L} = \frac{1}{2} : \psi_{R/L,\alpha}^\dagger (x) \sigma_{\alpha\alpha'} \psi_{L/R,\alpha'}(x) : ,
\]

with \( 2\theta = \arctanh(2g/(v_F + 2g)) \), \( \sigma \) being the vector of Pauli matrices, and the indices \( \alpha, \alpha' = 0,1 \) summed over.

One immediately recognizes \( H_{el} \) in \[ \text{(5)} \] as a Luttinger liquid Hamiltonian, with dynamically separated charge and "pseudospin" currents \( J_{R/L} \) and \( J_{L/R} \) respectively \[ \text{(5)}. \] Taking into account the boundaries of the box \[ \text{(5)}, \] as well as the finite range of the screened Coulomb interaction \[ \text{(5)}, \] will add more structure to Eq. \[ \text{(5)}. \] Also, in a more realistic theory one expects that \( \hat{U}_{00} < \hat{U}_{01} \), implying that the manifest SU(2) pseudospin symmetry of \( H_{el} \) in \[ \text{(5)} \] gets broken. However, for transparency and ease of notation, we here choose to work with the simple theory where \( H_{el} \) is represented by \[ \text{(5)}, \] and return below to discuss the more general case.

Having built in the Luttinger liquid correlations into the model via \[ \text{(5)}, \] we now explore how these influence the charging of the box. Let us first recall that in a "classical" picture the charge in a quantum box biased by a gate voltage \( V \) can change only when \( V \) is tuned to the discrete values \(-ne/(2C_e)\) (with \( n \) an odd integer) for which the Coulomb blockade is lifted \[ \text{(1)}. \] This leads to the celebrated "Coulomb staircase" with steps at the degeneracy points at which the charging energy for \((n/2)\pm1/2\) electrons is the same. This simple picture is modified by quantum charge fluctuations, enhanced by the coupling of the box to the quantum wire.

To study the fluctuation effects, we probe the system with a gate voltage close to a degeneracy point, for example \( V = -e/(2C_e) + u \), with \( u \ll e/C_e \) (i.e., a small voltage bias away from the chosen degeneracy point). In the limit of small \( t \), we can then truncate the Hilbert space to the \( Q_1 = 0 \) and \( Q_1 = e \) states (since in this limit transitions to virtual states of higher energy are suppressed). Following an exact formulation of Matveev \[ \text{(11)}, \] the resulting two-level system \( H_c + H_{\text{tun}} \) in \[ \text{(1)}, \] can be mapped onto an anisotropic Kondo interaction

\[
H_K = \frac{J_\perp}{2} \psi_{\mu,\alpha}^\dagger(0) \sigma_{\alpha\alpha'} \psi_{\mu',\alpha'}(0) S^\perp - h S^z , \tag{6}
\]

where \( J_\perp = 2t \) and \( h = eu \), and where \( S \) is an additional "pseudospin" of magnitude \( 1/2 \) that implements the constraint on the allowed states (with \( S \) localized at the position \( x = 0 \) of the point contact). Note that all indices in \[ \text{(6)} \] (\( \mu, \mu' = L, R; \alpha, \alpha' = 0,1; j = x, y \)) are summed over. It is here important to realize that the presence of backscattering terms in \( H_K \) is due to the fact that the quantum box is side-coupled to the wire via the point contact. This is different from the case of an end-coupled box, which supports only forward Kondo scattering \[ \text{(11)-13)}, \] As it turns out, it is precisely the backscattering in \[ \text{(6)} \] that imprints Luttinger liquid characteristics on the charging of the box, measured by the average \( \langle Q_1 \rangle \). Its dependence on the gate voltage is given by the differential capacitance \( c(u, T) = -(1/(\zeta e^2))[(\partial \langle Q_1 \rangle)/\partial V] \), which, via the Matveev mapping \[ \text{(11)}, \] gets modeled by an impurity susceptibility \( \chi_{imp}(h, T) = \partial (S^z)/\partial h \equiv c(u, T) \), describing the response of the local pseudospin to a "magnetic field" \( h \equiv eu \) at \( x = 0 \).

The original problem has thus been replaced by that of calculating the susceptibility of a (pseudo)spin-1/2 impurity coupled to a Luttinger liquid \( H_{el} \) \[ \text{(6)} \] by an anisotropic Kondo interaction \( H_K \) \[ \text{(6)}, \] The presence of backscattering in \( H_K \) still makes this a hard problem, however. A perturbative RG analysis \[ \text{(14)}, \] reveals that the backscattering terms become relevant for interacting electrons, taking the theory to a nontrivial
Fixed point. Here we approach the problem via a non-
perturbative route, exploiting boundary conformal field
theory (BCFT) \[14\] to trade the Kondo interaction \(H_K\)
for a scale invariant boundary condition on the bulk the-
ory \(H_{\text{cl}}\) in \[15\]. One can then use BCFT to extract the
critical exponents that govern the scaling of \(\chi_{\text{imp}}\) (alias
the differential capacitance) for small values of \(T\) and \(u\)
(i.e. close to the fixed point).

The fixed point describing the \textit{isotropic} spin-1/2
Kondo effect in a Luttinger liquid \[14\] \[16\] has been shown to correspond to a particular selection rule for quantum numbers of the BCFT embedding
\(U(1) \otimes U(1) \otimes SU(2)_2 \otimes Ising\) \[17\]. Here the two \(U(1)\)
factors represent the spectra of left- and right-moving charge
excitations, while the \(SU(2)_2 \otimes Ising\) block derives from a
coset construction of the \(SU(2)_1 \otimes SU(2)_1\) left- and right-
moving pseudospin excitation spectra (with the indices labeling the \textit{levels} of the corresponding Kac-Moody al-
bras \[7\]). Given this structure, it is straightforward to verify that the anisotropy in \[6\] introduces irrelevant operators only (in exact analogy to the Kondo effect for noninteracting electrons \[13\]). Thus, the fixed point for the present problem is the same as for the isotropic model, and we can exploit the BCFT scheme developed in Ref. \[17\].

Knowing the fixed point allows us to identify the lead-
ing boundary operators that drive the finite-\(T\) scaling of \(\chi_{\text{imp}}\). Note that, in contrast to the isotropic case in Ref. \[17\], operators that break (pseudo)spin-rotational
invariance are now allowed (by the anisotropy of \(H_K\) in \[6\]). A systematic search \[12\] yields two leading opera-
tors \(O^{(1)} = T_\epsilon \otimes 1_{\text{Ising}} \otimes 1_{\text{c}}\) and \(O^{(2)} = J^z \otimes \epsilon \otimes O_c\), with scaling dimensions \(\Delta^{(1)} = 2\) and \(\Delta^{(2)} = 3/2 + 1/2K_c\),
respectively. \(K_c\) is the usual Luttinger liquid "charge parameter" with perturbative expression
\[1 + 4g/\nu \epsilon_c\]^{-1/2} (here allowed to take values in the interval \(1/2 \leq K_c \leq 1\), \(T_b\) is the \(SU(2)_2\) energy-momentum ten-
sor, \(\ll\) is the identity operator in the indexed sector, \(J^z\) is the \(z\)-component of the \(SU(2)_2\) pseudospin current, \(\epsilon\) is the Ising energy density, and \(O_c\) is a symmetrized prod-
ct of \(U(1)\) vertex operators (for details, see Ref. \[17\]).

Given the operators \(O^{(1)}\) and \(O^{(2)}\), the scaling behavior of \(\chi_{\text{imp}}(T, h=0)\) can be calculated via an expansion in their conjugate scaling fields \(\lambda_1\) and \(\lambda_2\). Passing to a Lagrangian formalism, we write the partition function
as a path integral, treating the (inverse) temperature as an
imaginary time. To simplify the calculation we also
lead to the calculation of the imperfect suscepti-
ability \[21\], but since we shall be interested in the scaling
exponents only, this change is immaterial. Using a linked
cluster expansion, we can then write
\begin{equation}
\chi_{\text{imp}}(T, 0) = \lambda_1 I[O_3^{(1)}] + \frac{1}{2} \sum_{i=1, 2} \lambda_i^2 I[O_3^{(i)}, O_4^{(i)}] + \ldots, \tag{7}
\end{equation}
where
\begin{equation}
I[O_3, \ldots, O_j] = \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{4\pi^2 \beta} \int_{-\beta/2}^{\beta/2} d\tau_1 \ldots d\tau_j \langle j^1_j \hat{J}_2 \hat{O}_3 \ldots \hat{O}_j \rangle_c,
\end{equation}
with \(\langle \ldots \rangle_c\) a connected \(n\)-point function, and \(J^1_k = J^zh(x_k), k = 1, 2\). Given the boundary operators
\(O_j^{(1, 2)} = O^{(1, 2)}(\delta_j), j = 3, 4\), that enter \[7\], we use the appropriate operator product expansions (OPEs) \[8\] to
collapse the integrands to products of two-point func-
tions. This allows us to easily calculate the integrals and we
obtain (using that \(c(T, u=0) = \chi_{\text{imp}}(T, h=0)\)):
\begin{equation}
c(T, u=0) = A + B[K_c]T^{1/K_c} + CT^2 + \ldots, \tag{8}
\end{equation}
with \(A\), \(B[K_c]\) and \(C\) constants (where \(B[K_c] = \text{const} \{1/K_c - 1\}\), and where "\ldots" indicate sub-
leading corrections. The short-range electron-electron
interaction, encoded by the parameter \(K_c\), is thus seen to
induce a \textit{nonanalytic term in the differential capacitance},
scaling as \(T^{1/K_c}\), while vanishing in the noninteracting
limit \((K_c = 1)\).

Our result in \[8\] predicts a distinct signal of Luttinger
liquid correlations in the proposed setup. For what tem-
peratures should one expect to see it? Taking the 1D quantum box to have a length \(\ell' \sim 1 \mu m\) and choosing
parameters assuming an experiment using a GaAs heterostructure \[2\], the energy space \(\delta E\) close to the Fermi level
coincides to roughly 0.5K. The temperature
that sets the upper limit for the validity of our theory
is the effective Kondo temperature \(T_K\), with expression
\(T_K = E_C^* \exp(-1/2\nu\ell')\) in the limit \(g\ell' < 2\ell' \[14\].
Here \(E_C^* = E_C(1 - 4\nu\ell') + \ldots\) is the renormalized charging
energy \[21\], and \(\nu\) is the density of states at the Fermi level. With \(\ell' \sim 0.2\mu m\) and \(E_C \sim e^2/2CS_c\), where \(CS_c \sim 30aF\) in a typical device, we obtain \(T_K \sim 2K\).
With these estimates, our prediction in \[8\] applies for temperatures in the interval \(0.5K < T < 2K\).

Considering the narrowness of the estimated temperature interval, it may experimentally be easier to study the scaling of the capacitance with gate voltage at a fixed
temperature. Approximating the window \(0.5K < T < 2K\) by the \(T \rightarrow 0\) limit, the scaling can be obtained via a \textit{Weigner expansion} \[22\] of the effective ("Kondo lan-
guage") impurity free energy. Close to the critical point
\(T = 0, h = 0\) we thus write
\begin{equation}
F_{\text{imp}} = \text{const.} + T f(\frac{\hbar}{T^\Delta}) + gT^{1-\Delta'}f'(\frac{\hbar}{T^\Delta}) + \ldots \tag{9}
\end{equation}
Here \(f\) is a scaling function, \(\Delta = 1/2\) is the boundary
dimension acquired by the \textit{local} magnetic field \(h\), and \(f'\) is the gradient of \(f\) w.r.t. the leading irrelevant scaling field
\(g'\). The corresponding operator \(c \otimes O_c\) is generated from the OPE of \(J^z\) with \(J^z \otimes \epsilon \otimes O_c\) and \(g'\) is thus proportional to \(h\) and carries RG eigenvalue
\(\Delta' = -(1/K_c - 1)/2 < 0\). In the limit \(s \rightarrow \infty\), \(f(s) \sim s^{1/\Delta}\). Thus, when \(T \rightarrow 0\),
the second term in Eq. (9) gives an analytic contribution \( \sim h^2 \). Inspection of the third term in (10) reveals that it can contribute a finite correction \( \delta F_{imp} \) only via a term \( \sim g(1-\Delta')/\Delta \) in the expansion of \( f' \), implying that \( \delta F_{imp} \sim h^{1+1/\Delta_c} \). Contributions from higher order terms in Eq. (9) are of\( O[h^{4}] \). Summarizing, we obtain

\[
c(T=0, u) = D + E[K_c]u^{1/K_c} + Fu^2 + ... \quad (10)
\]

Here \( D, E[K_c] \) and \( F \) are constants, with \( E[K_c] \to 0 \) as \( K_c \to 1 \).

Before concluding, we must address the question how the boundaries of the box, as well as the finite range and the anisotropy of the screened Coulomb interaction, influence the physics. Although these features must be accounted for in a faithful modeling of an experimental sample, they will not qualitatively change the charge fluctuation effects derived in Eqs. (8) and (10): As to the boundary effects from the quantum box, these will suppress the spectral weight at the Fermi level, at low energies reducing the effective value of \( K_c \). The finite range \( R \) of the screened Coulomb interaction further depresses \( K_c \) by a factor \( (\ln(R/d))^{-1/2} \), where \( d \) is the (common) transverse width of wire and box (10) (with \( 3 < R/d < 15 \) in typical experiments on gated GaAs heterostructures (2)). Both effects are moderate, though, and as long as the renormalized \( K_c \) is larger than \( 1/2 \) the nonanalytic terms in (8) and (10) will remain the leading ones. Turning to the expected anisotropy \( \hat{U}_{00}(0) \equiv g' < \hat{U}_{00}(0) = g \), this will generate an exactly marginal term proportional to \( (g-g')J_{R}^{(1)} \), in addition to shift the velocities in (5). While the boundary operators identified above will still be present (with \( K_c \) renormalized upwards, with a new perturbative expression \( K_c = (1+2g+g')/2 \), it is conceivable that the spin sector may now contribute additional boundary operators with noninteger dimensions. However, if these results an exponent smaller than \( 1/K_c \), this implies only that the nonanalytic scaling of the capacitance gets enhanced. Conversely, the \( 1/K_c \) scaling remains the leading one. In either case, the picture that we have uncovered by using an SU(2) invariant description in (5) will remain valid.

In summary, we predict, under conditions specified above, that the differential capacitance of a quantum box side-coupled to a quantum wire exhibits a nonanalytic scaling in temperature and gate voltage, with the same scaling exponent in both cases. We have traced the effect to the strong electron correlations inherent in one-dimensional systems, and we expect that high-precision charge measurements should be able to detect it. An experimental verification may shed new light on the elusive Luttinger liquid behavior of electrons in one dimension.

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