Lepton Flavor Violating $\tau^- \rightarrow \mu^-PP$ Decays in the Two Higgs Doublet Model III

Wenjun Li

Department of Physics, Henan Normal University, XinXiang, Henan, 453007, P.R.China
Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China

YingYing Fan, Gongwei Liu

Department of Physics, Henan Normal University, XinXiang, Henan, 453007, P.R.China

In this paper, the lepton flavor violating $\tau^- \rightarrow \mu^-PP$ ($PP = K^+K^-, K^0\bar{K}^0, \pi^+\pi^-, \pi^0\pi^0$) decays are studied in the framework of the two Higgs doublet model (2HDM) III. We calculate these decays branching ratios and get the bounds of model parameter $|\lambda_{\tau\mu}|$ from the experimental upper limits. Our results show that, the neutral Higgs bosons have tree-level contributions to these decays. Among these decays, the $\tau^- \rightarrow \mu^-K^+K^-$ decay is most sensitive to $|\lambda_{\tau\mu}|$. In the existing parameters space, these decays could reach the measure capability of B factory. These processes can provide some valuable information to future research and furthermore present the reliable evidence to test the 2HDM III model.

PACS numbers: 13.35.Dx, 12.15.Mm, 12.60.-i

I. INTRODUCTION

Flavor physics have made rapid development in these decades. In addition to B physics, $\tau$ physics, including determination of $\alpha_s$ from the inclusive hadronic width, charged-current universality tests, and lepton flavor violation decays etc., also belongs these days to one branch of flavor physics. Among these topics, lepton flavor violation (LFV) decays raises in importance after the discovery of neutrino flavor oscillations and related non-zero neutrino masses[1]. In the SM these processes are forbidden or suppressed strongly, therefore, LFV decays could be a sharp tool to seek for some new scenarios with new LFV source and/or new particles. The theoretical investigations of $\tau \rightarrow 3l, \tau \rightarrow l\gamma, \tau \rightarrow lP(V^0)$ have sprung up in different contexts[2–4]. With only two meson in final states, $\tau^- \rightarrow \mu^-PP$ decays are so clean as to provide some information of QCD. And the experimental upper limit of $\tau \rightarrow K^+K^-(K^0\bar{K}^0, \pi^+\pi^-, \pi^0\pi^0)$ are [6]:

$$B(\tau^- \rightarrow \mu^-K^+K^-) < 6.8 \times 10^{-8}, \ 90\% CL$$
\[
\mathcal{B}(\tau^- \rightarrow \mu^- \pi^+ \pi^-) < 3.3 \times 10^{-8}, \ 90\% \text{CL}
\]
\[
\mathcal{B}(\tau^- \rightarrow \mu^- K^0\bar{K}^0) < 3.4 \times 10^{-6}, \ 90\% \text{CL}
\]
\[
\mathcal{B}(\tau^- \rightarrow \mu^- \pi^0 \pi^0) < 1.4 \times 10^{-5}, \ 90\% \text{CL}
\]

where the former two values have improved the previous upper bounds by almost an order of magnitude.

There are also lots of theoretical researches on \(\tau \rightarrow lPP\) decays in many possible extensions of the SM\cite{7-10}. For \(\tau \rightarrow lPP\) decays with hadrons in final states, their amplitudes could be separated into leptonic vertexes and hadronic parts. One approach to handling the latter is usually parameterized as hadrons mass and their decay constants, which could be determined by experimental values. Some authors have made analysis of these processes from views of vector meson dominance, chiral symmetry breaking, Breit-Wigner propagators, etc.\cite{7}. For the hadronisation of final state in \(\tau\) decays, its scale is at the order of 1GeV which lies in the non-perturbative region. Hence, we need consider the non-perturbative methods one of which is the Chiral Perturbative Theory (\(\chi PT\))\cite{11}. Different from one pseudoscalar meson in final state, the resonances are participated in the processes of \(\tau^- \rightarrow \mu^- PP\). Stemmed from \(\chi PT\), the Resonance Chiral Theory (\(R\chi T\)) has developed\cite{12}. Using \(R\chi T\), E. Arganda et al. have investigated these processes in two constrained MSSM-seesaw scenarios\cite{8}. M. Herrero et al. also have make an discussion on the sensitivity of LFV tau decays the Higgs sector of SUSY-seesaw models\cite{9}. The new particles effects to \(\tau^- \rightarrow \mu^- PP\) decays in the TC model and the LHT model are calculated by Yue chongxing’s group\cite{10}.

In 2HDM model III, it naturally introduces flavor-changing neutral currents (FCNCs) at tree level. In order to satisfy the current experiment constrainst, the tree-level FCNCs are suppressed in low-energy experiments for the first two generation fermions. While processes concerning with the third generation fermions would be larger. These FCNCs with neutral Higgs bosons mediated may produce sizable effects to the \(\tau - \mu\) transition. The authors in \cite{13} have discussed the \(\tau \rightarrow 3\mu\) decay and the Higgs sector’s contributions. \(\tau \rightarrow \mu P(V^0)\) decays have been studied under this scenario in our previous work\cite{5} where the hadronisation in final state is merely expressed in terms of the meson decay constants and meson masses. The \(\tau \rightarrow \mu P\) decays could yield one pseudoscalar meson from the vacuum state through the scalar and pseudoscalar currents. Hence, this type decay could occur at the tree level through the neutral Higgs bosons exchange. In this paper, we extend our discussion to the case of two pseudoscalar mesons in the hadronic final state and deal with these four decays by \(R\chi T\). Our results suggest that, the neutral Higgs bosons contribute at the tree level in 2HDM model III. The model parameter \(\lambda_{\tau \mu}\) is restrained at \(O(10 \sim 10^3)\) and the decay branching ratios could as large as the current upper limits.

The paper is organized as follows: In section II, we make a brief introduction of the theoretical framework for the two-Higgs-doublet model III. In section III, we briefly introduce the Resonance Chiral Theory. In the next section, we deliberate the calculation of the decay amplitudes with Resonance Chiral Theory and our numerical
predictions. Our conclusions are listed in the last section.

II. THE TWO-HIGGS-DOUBLET MODEL III

As the simplest extension of the SM, the Two-Higgs-Doublet Model has an additional Higgs doublet. In order to ensure the forbidden FCNCs at tree level, it requires either the same doublet couple to the $u$-type and $d$-type quarks (2HDM I) or one scalar doublet couple to the $u$-type quarks and the other to $d$-type quarks (2HDM II). While in the 2HDM III [14, 15], two Higgs doublets could couple to the $u$-type and $d$-type quarks simultaneously. Particularly, without an ad hoc discrete symmetry exerted, this model permits flavor changing neutral currents occur at the tree level.

The Yukawa Lagrangian is generally expressed as the following form:

$$L_Y = \eta_{ij}^U \bar{Q}_{i,L} H_1 U_{j,R} + \eta_{ij}^D \bar{Q}_{i,L} D_{j,R} + \xi_{ij}^U \bar{Q}_{i,L} H_2 U_{j,R} + \xi_{ij}^D \bar{Q}_{i,L} D_{j,R} + h.c.,$$

where $H_i (i = 1, 2)$ are the two Higgs doublets. $Q_{i,L}$ is the left-handed fermion doublet, $U_{j,R}$ and $D_{j,R}$ are the right-handed singlets, respectively. These $Q_{i,L}$, $U_{j,R}$ and $D_{j,R}$ are weak eigenstates, which can be rotated into mass eigenstates. While $\eta^{U,D}$ and $\xi^{U,D}$ are the non-diagonal matrices of the Yukawa couplings.

We can conveniently choose a suitable basis to denote $H_1$ and $H_2$ as:

$$H_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + \phi_1^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2} G^+ \\ iG^0 \end{pmatrix} \right], \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ \phi_2^0 + iA^0 \end{pmatrix},$$

where $G^{0, \pm}$ are the Goldstone bosons, $H^{\pm}$ and $A^0$ are the physical charged-Higgs boson and CP-odd neutral Higgs boson, respectively. Its virtue is the first doublet $H_1$ corresponds to the scalar doublet of the SM while the new Higgs fields arise from the second doublet $H_2$.

The CP-even neutral Higgs boson mass eigenstates $H^0$ and $h^0$ are linear combinations of $\phi_1^0$ and $\phi_2^0$ in Eq. (4),

$$H^0 = \phi_1^0 \cos \alpha + \phi_2^0 \sin \alpha, \quad h^0 = -\phi_1^0 \sin \alpha + \phi_2^0 \cos \alpha,$$

where $\alpha$ is the mixing angle.

After diagonalizing the mass matrix of the fermion fields, the Yukawa Lagrangian becomes [16]
\displaymath\begin{align*}
-H^0 U \left\{ \sqrt{2} \frac{M_U}{v} \cos \alpha + \left[ \xi^U R + \xi^{U\dagger} L \right] \sin \alpha \right\} U - \frac{H^0}{\sqrt{2}} D \left\{ \sqrt{2} \frac{M_D}{v} \cos \alpha + \left[ \xi^D R + \xi^{D\dagger} L \right] \sin \alpha \right\} D \\
- \frac{h^0}{\sqrt{2}} U \left\{ -\sqrt{2} \frac{M_U}{v} \sin \alpha + \left[ \xi^U R + \xi^{U\dagger} L \right] \cos \alpha \right\} U - \frac{h^0}{\sqrt{2}} D \left\{ \sqrt{2} \frac{M_D}{v} \sin \alpha + \left[ \xi^D R + \xi^{D\dagger} L \right] \cos \alpha \right\} D \\
- H^+ U \left[ V_{CKM}^{\dagger} \xi^D R - \xi^U V_{CKM} L \right] D - H^- D \left[ \xi^D V_{CKM} L - V_{CKM}^{\dagger} \xi^U R \right] U
\end{align*}
\tag{6}
\end{displaymath}

where U and D now are the fermion mass eigenstates and

\displaymath\begin{align*}
\hat{\eta}^{U,D} &= (V_{U,D}^{L,R})^{-1} \cdot \eta^{U,D} \cdot V_{U,D}^{L,R} = \sqrt{2} \frac{M_{U,D}}{v} (M_{U,D}^{ij} = \delta_{ij} m_{U,D}^{ij}), \\
\xi^{U,D} &= (V_{U,D}^{L,R})^{-1} \cdot \xi^{U,D} \cdot V_{U,D}^{L,R},
\end{align*}
\tag{7}
\end{displaymath}

where \( V_{U,D}^{L,R} \) are the rotation matrices acting on up and down-type quarks, with left and right chiralities respectively. Thus \( V_{CKM} = (V_{U}^{L})^{\dagger} V_{D}^{L} \) is the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix. In general, the matrices \( \hat{\eta}^{U,D} \) of Eq.(7) are diagonal, while the matrices \( \xi^{U,D} \) are non-diagonal which could induce scalar-mediated FCNC. Seen from Eq.(6), the coupling of neutral Higgs bosons to the fermions could generate FCNC parts. For the arbitrariness of definition for \( \xi^{U,D}_{ij} \) couplings, we can adopt the rotated couplings expressed \( \xi^{U,D} \) in stead of \( \hat{\xi}^{U,D} \) hereafter.

In this work, we use the Cheng-Sher ansatz \cite{14}

\displaymath\begin{align*}
\xi^{U,D}_{ij} &= \lambda_{ij} \frac{\sqrt{2} \sqrt{m_i m_j}}{v}
\end{align*}
\tag{9}
\end{displaymath}

which ensures that the FCNCs within the first two generations are naturally suppressed by small fermions masses. This ansatz suggests that LFV couplings involving the electron are suppressed, while LFV transitions involving muon and tau are much less suppressed and may lead to some loop effects which are promising to be tested by the future B factory experiments. In Eq.(9), the parameter \( \lambda_{ij} \) is complex and \( i, j \) are the generation indexes. In this study, we shall discuss the phenomenological applications of the type III 2HDM.

## III. THE RESONANCE CHIRAL THEORY

For the intermediate and low energy parts of hadronic spectrum, they locate at the non-perturbative region where ordinary perturbative QCD methods does not work. Hence, many non-perturbative approaches have been developed, such as \( \chi PT \) \cite{11}, QCD sum rules \cite{17}, lattice gauge theory \cite{18} and so on. The large-\( N_C \) expansion of \( SU(N_C) \) QCD \cite{19} is a suitable idea.

\( \chi PT \) is a appropriate method with \( 1/N_C \) expansion which is very successful in the energy reign of \( \simeq 1 GeV \). It could deal with \( \tau^- \rightarrow \mu^- P \) decays which will be discussed in our later paper. While for \( \tau^- \rightarrow \mu^- PP \) processes, the resonances paly a dynamical role so that we should by the aid of \( R\chi T \) \cite{12}. Motivated by partially \( \chi PT \) and large \( N_c \) QCD, \( R\chi T \) could describe the immediate energy region. Its advantages are not only to realize the non-linear
of spontaneous chiral symmetry breaking, but also to study meson structure without imposing any structure in advance. Some application on \( R\chi T \) have been elaborated in \cite{8,9,20}. For energies \( 1 \sim 2\text{GeV} \), we restrict it to only the lightest resonance in each channel. The detailed deliberation can be found in Ref.\cite{8}.

The symmetry of QCD breaks spontaneously from \( SU(3)_R \otimes SU(3)_L \) to \( SU(3)_V \) and produces eight Goldstone bosons in the spectrum. We regard these Goldstone bosons as the lightest hadrons. The \( \chi PT \) Lagrangian can be constructed from pseudoscalar fields and external source \( v^i(x), a^i(x), s_i(x), p^i(x) \):

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1,
\]

\[
\mathcal{L}_0 = -\frac{1}{2g^2} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \overline{q} i \gamma^\mu (\partial_\mu - iG_\mu) q, \quad \mathcal{L}_1 = \overline{q} [\gamma_\mu (v_\mu + \gamma_5 a^\mu) - (s - ip\gamma_5)] q.
\]

where the \( \mathcal{L}_1 \) is the massless QCD Lagrangian. \( G_{\mu\nu} \) denotes the gluon fields, \( v^\mu, a^\mu, s, p \) are matrices in the flavor fields and \( \lambda_i \)s are the Gell-Mann matrices. The QCD generating functional \( Z_{QCD}[v, a, s, p] \) could be written:

\[
e^{iZ_{QCD}[v, a, s, p]} = \int [DG_{\mu}] [Dq] [D\bar{q}] e^{\int d^4x \mathcal{L}_{QCD}[q, \bar{q}, G, v, a, s, p]}
\]

We introduce the lightest \( U(3) \) nonet of pseudoscalar mesons:

\[
\phi(x) = \sum_{a=0}^{8} \frac{\lambda_a}{\sqrt{2} \rho_a} \phi_a
\]

\[
= \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta^8 + \frac{1}{\sqrt{3}} \eta^0 \\
\pi^- \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta^8 + \frac{1}{\sqrt{3}} \eta^0 \\
\chi^+
\end{pmatrix}
\]

\[
\pi^+ \\
\chi^+
\]

\[
\chi^+ = u^\dagger \chi u^\dagger + u \chi^\dagger u, \quad \chi = 2B_0(s + ip).
\]

The leading \( O(p^2) \chi PT SU(3)_L \times SU(3)_R \) chiral Lagrangian is

\[
\mathcal{L}^{(2)} = \frac{F^2}{4} (u_\mu u^\mu + \chi_+) \\
u_\mu = i [u^\dagger (\partial_\mu - i\gamma_5) u - u (\partial_\mu - i\gamma_5) u^\dagger], \quad \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u.
\]

The leading \( O(p^2) \chi PT SU(3)_L \times SU(3)_R \) chiral Lagrangian is

\[
\mathcal{L}^{(2)} = \frac{F^2}{4} (u_\mu u^\mu + \chi_+) \\
u_\mu = i [u^\dagger (\partial_\mu - i\gamma_5) u - u (\partial_\mu - i\gamma_5) u^\dagger], \quad \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u.
\]
Indeed in the isospin limit we have:

\[
\chi = 2B_0 M + \cdots = \left( \begin{array}{c}
m_\pi^2 \\
m_\pi^2 \\
2m_K^2 - m_\pi^2 \
\end{array} \right) + \cdots
\]

\[
B_0 m_u = B_0 m_d = \frac{1}{2} m_\pi^2, \quad B_0 m_s = m_K^2 - \frac{1}{2} m_\pi^2
\]

The mass eigenstates \( \eta \) and \( \eta' \) are related to the octet \( \eta_8 \) and singlet \( \eta_0 \) states through the rotation matrix:

\[
\left( \begin{array}{c}
\eta \\
\eta'
\end{array} \right) = \left( \begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array} \right) \left( \begin{array}{c}
\eta_8 \\
\eta_0
\end{array} \right)
\]

where \( \theta = 18^\circ \).

In the processes of \( \tau \) decaying to two pseudo-scalars, vector and scalar resonances generally play a propelling role. However, due to the higher masses of scalar resonances, their effects are ignored\[8\]. Then we carry out antisymmetric tensor fields to introduce vector resonances. The nonet of resonance fields \( V_{\mu\nu} \) reads:

\[
\begin{pmatrix}
\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_0 \\
\rho^- \\
K^*-
\end{pmatrix} = \begin{pmatrix}
\rho^0 \\
-\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_3 \\
K^*0
\end{pmatrix}
\]

Then the \( R\chi T \) Lagrangian puts:

\[
\mathcal{L}_{R\chi T} = \mathcal{L}_\chi^{(2)} + \mathcal{L}_V^{(2)}, \quad \mathcal{L}_V = \mathcal{L}_{kin}^V + \mathcal{L}_{(2)}^V,
\]

\[
\mathcal{L}_{kin}^V = \frac{1}{2} \langle \nabla \lambda V_{\mu\nu} \nabla^\lambda V_{\mu\nu} \rangle + \frac{M_\pi^2}{4} \langle V_{\mu
u} V_{\mu
u} \rangle, \quad \mathcal{L}_{(2)}^V = \frac{F_V}{2 \sqrt{2}} \langle V_{\mu\nu} f_{\mu\nu}^I \rangle + i \frac{G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle
\]

where \( \mathcal{L}_V \) is the resonance Lagrangian. The relevant definitions can be found in Ref.\[8\]. The corresponding QCD generating functional is written as

\[
e^{iZ_{QCD}[v,a,s,p]} = \int [Du][DV] e^{i \int d^4x \mathcal{L}_{R\chi T}[u,V,v,a,s,p]}
\]

Through making the proper partial derivatives of the functional action, we can get the hadronisation of bilinear quark currents:

\[
V_{\mu}^i = \bar{q} \gamma_{\mu} \frac{\lambda^i}{2} q = \left. \frac{\partial \mathcal{L}_{R\chi T}}{\partial \bar{V}_{\mu}^i} \right|_{j=0}, \quad A_{\mu}^i = \bar{q} \gamma_{\mu} \gamma_5 \frac{\lambda^i}{2} q = \left. \frac{\partial \mathcal{L}_{R\chi T}}{\partial a_{\mu}^i} \right|_{j=0},
\]

\[
S^i = -\bar{q} \lambda^i q = \left. \frac{\partial \mathcal{L}_{R\chi T}}{\partial S_i} \right|_{j=0}, \quad P^i = \bar{q} \gamma_5 \lambda^i q = \left. \frac{\partial \mathcal{L}_{R\chi T}}{\partial P^i} \right|_{j=0}
\]

The final results from Eq.(20) are

\[
V_{\mu}^i = \frac{F_V^2}{4} < \lambda^i (uu^\dagger u^\dagger u^\mu u) > - \frac{F_V}{2 \sqrt{2}} < \lambda^i \partial^\mu (u^\dagger V_{\mu\nu} u + u V_{\mu\nu} u^\dagger) >
\]
\begin{align}
A^i_\mu &= \frac{F^2}{4} < \lambda^i(uu_\mu u^\dagger + u^\dagger u_\mu u) > \quad (22) \\
S^i &= \frac{1}{2} B_0 F^2 < \lambda^i(u^\dagger u^\dagger - uu) > \quad (23) \\
P^i &= \frac{i}{2} B_0 F^2 < \lambda^i(u^\dagger u^\dagger + uu) > \quad (24)
\end{align}

IV. THE DISCUSSION FOR $\tau^- \to \mu^- PP$ DECAYS

In 2HDM model III, the neutral Higgs bosons mediated tree diagrams have contributions to $\tau^- \to \mu^- PP$ processes. The amplitudes could be factorized into leptonic vertex corrections and hadronic parts described with hadronic matrix elements, which express as:

\[
(\mu^- PP|\mathcal{M}|\tau^-) = \frac{i G_F}{2\sqrt{2}} \cdot m_q \sqrt{m_\tau m_{\mu^*}} \cdot \left\{ \begin{array}{l}
H^q \cdot \lambda_\tau \cdot (\bar{\mu} \tau)_{S+P} + H^q \cdot \lambda_\tau^* \cdot (\bar{\mu} \tau)_{S-P} \\
N^q \cdot \lambda_\tau \cdot (\bar{\mu} \tau)_{S+P} - N^q \cdot \lambda_\tau^* \cdot (\bar{\mu} \tau)_{S-P}
\end{array} \right\} < PP|(\bar{q} q)|0 > + < PP|(\bar{q}^* q)|0 >
\]

where $H^q, N^q$ are auxiliary functions can be find in Appendix. From the Eq.(25), we can see that three neutral Higgs bosons perform roles to two pseudo-scalar mesons through $(\bar{q} q)_{S \pm P}$ operators. It differs from the case of MSSM models where the $\gamma$ contributions is the dominate one and only $H^0$ and $h^0$ take effects at the large $\tan \beta$.

For the heavy Higgs bosons, the hadronic final state is not sensitive to resonances and known little. It should be noted that the pseudo-scalars currents have contributions to only one pseudo-scalar meson in final states. So we have dealt with the hadronic matrix elements by virtue of Eq.(23), (24) and the following currents:

\[
\begin{align*}
- \tilde{u} u &= \frac{1}{2} S^3 + \frac{1}{2 \sqrt{3}} S^8 + \frac{1}{\sqrt{6}} S^0, \\
- \tilde{d} d &= - \frac{1}{2} S^3 + \frac{1}{2 \sqrt{3}} S^8 + \frac{1}{\sqrt{6}} S^0, \\
- \tilde{s} s &= - \frac{1}{\sqrt{3}} S^8 + \frac{1}{\sqrt{6}} S^0
\end{align*}
\]

The obtained amplitudes read as:

\[
\mathcal{M}(\tau^- \to \mu^- PP) = \frac{i G_F}{2\sqrt{2}} \cdot \sqrt{m_\tau m_{\mu^*}} \cdot \left\{ T(PP) \cdot \lambda_\tau \cdot (\bar{\mu} \tau)_{S+P} + T^*(PP) \cdot \lambda_\tau^* \cdot (\bar{\mu} \tau)_{S-P} \right\} \quad (27)
\]

where

\[
T(K^+ K^-) = A \cdot [Re(\lambda_{uu}) \cdot m^2_\pi + Re(\lambda_{ss}) \cdot (2m^2_K - m^2_\pi)] + Bi \cdot [Im(\lambda_{ss}) \cdot (2m^2_K - m^2_\pi) - Im(\lambda_{uu}) \cdot m^2_\pi]
+ F \cdot m^2_K
\]

\[
T(K^0 \bar{K}^0) = A \cdot [Re(\lambda_{dd}) \cdot m^2_\pi + Re(\lambda_{ss}) \cdot (2m^2_K - m^2_\pi)] + Bi \cdot [Im(\lambda_{ss}) \cdot (2m^2_K - m^2_\pi) + Im(\lambda_{dd}) \cdot m^2_\pi]
+ F \cdot m^2_K
\]

\[
T(\pi^+ \pi^-) = m^2_\pi \cdot \{ A[Re(\lambda_{uu}) + Re(\lambda_{dd})] + Bi \cdot [Im(\lambda_{dd}) - Im(\lambda_{uu})] + F \}
\]
\[
T(\pi^0\pi^0) = \frac{m_\pi^2}{2\sqrt{2}} \cdot \{A[Re(\lambda_{uu}) - Re(\lambda_{dd})] - Bi \cdot [Im(\lambda_{uu} + Im(\lambda_{dd}))]\}
\]
\[
A = \frac{\sin^2\alpha}{m_{H^0}^2} + \frac{\cos^2\alpha}{m_{h^0}^2}, \quad B = \frac{1}{m_{A^0}^2}, \quad F = 2\sin\alpha\cos(\frac{1}{m_{H^0}^2} - \frac{1}{m_{h^0}^2})
\]

(31)

**TABLE I: Constraints on the \( \lambda_{ij} \) in quark and lepton sector.**

| Bound and restrictions               | Process and Restriction                        | References |
|--------------------------------------|-----------------------------------------------|------------|
| \(|\lambda_{uu}|, |\lambda_{dd}| \sim O(1)\)               | \(F^0 - \bar{F}^0\) mixing \((F = K, B_d, D), R_b, \rho, B \to X, \gamma\) | 21         |
| \(|\lambda_{tt}| = 0.02, |\lambda_{bb}| = 50\)                    | \(B_d - \bar{B}_d\) mixing, \(b \to s\gamma\) | 22         |
| \(|\lambda_{tt}| < 0.5, |\lambda_{bb}| < 70, |\lambda_{tt}\lambda_{bb}| \sim 3, |\lambda_{ss}| \in [80, 120]\) | \(B_d - \bar{B}_d\) mixing, \(b \to s\gamma, \rho_0, R_b, NEDM\) | 23         |
| \(|\lambda_{tt}| = |\lambda_{tc}| = 0.1, |\lambda_{bb}| = |\lambda_{bb}| = 50\) | \(h^0 \to f\bar{f}\) | 24         |
| \(|\lambda_{tt}| = 0.3, |\lambda_{bb}| = 35, \lambda_{ij} = 0\) | \(B \to PP, PV, \nu\nu\) | 25         |
| \(|\lambda_{tt}| = |\lambda_{tt}| = 5.50,\) | \(B_{d,s} \to l^+l^-\) | 22         |
| \(|\lambda_{tt}| = |\lambda_{tt}| = |\lambda_{tt}| = |\lambda_{tt}| = 10\) | \(h^0 \to f\bar{f}\) | 24         |
| \(\lambda_{\tau\mu} \sim O(10) - O(10^2)\) | \((g - 2)_\mu, m_{A^0} \to \infty\) | 26         |
| \(\lambda_{\tau\mu} \sim O(10^2) - O(10^3)\) | \(\tau \to 3\mu, \tau \to \mu\gamma\) | 27         |

1 Note that the constraints in [26] and [27] are denoted by our notation.

**V. NUMERICAL RESULTS**

In our calculation, the input parameters are the Higgs masses, mixing angle \(\alpha\), \(|\lambda_{ij}|\) and their phase angles \(\theta_{ij}\). We using the values of neutral Higgs boson masses in literature [22, 23, 26], where the experimental constraints of \(B - \bar{B}\) mixing, \(b \to s\gamma, \rho^0, R_b\) considered.

\[
m_{H^0} = 160\text{GeV}, \quad m_{h^0} = 115\text{GeV}, \quad m_{A^0} = 120\text{GeV}, \quad \alpha = \pi/4, \quad \theta = \pi/4
\]

(32)

According the mesons quark constants in final states, the involved factors of quark sector are \(\lambda_{uu}, \lambda_{dd}\) and \(\lambda_{ss}\). The bounds of \(|\lambda_{\tau\mu}|\) from different phenomenological considerations [23, 26, 27] are demonstrated in Tab.I, too. For the first generation FC couplings are suppressed, the values of \(\lambda_{uu}\) and \(\lambda_{dd}\) are less than 1 [21]. And the \(B_0 - \bar{B}_0\) mixing constrains approximately \(\lambda_{ss}\) in \(80 - 120\) [23]. These bounds are considered in our calculation. In the following paragraphs, we will analysis the relations of these decays branching ratios and relevant parameters. First, we take \(|\lambda_{\tau\mu}| = 5\) and study the relations of branching ratio and other parameters.

The Fig.1 shows \(Br\) for \(\tau^- \to \mu^-K\bar{K}\) decays versus model parameters, where the left figure is \(Br(\tau^- \to \mu^-K^+K^-)\) versus \(|\lambda_{uu}|\) and \(|\lambda_{ss}|\), and the right is \(Br(\tau^- \to \mu^-K^0\bar{K}^0)\) versus \(|\lambda_{dd}|\) and \(|\lambda_{ss}|\). One can see from Fig.1(a) that \(Br(\tau^- \to \mu^-K^+K^-)\) raises with the increase of \(|\lambda_{ss}|\) but does not vary with \(|\lambda_{uu}|\) growing. For \(\tau^- \to \mu^-K^0\bar{K}^0\) decay, the same as that of \(\tau^- \to \mu^-K^+K^-\) decay except that \(|\lambda_{dd}|\) replaces \(|\lambda_{uu}|\). In the
FIG. 1: Left: $Br$ for $\tau^- \rightarrow \mu^- K^+ K^-$ versus $|\lambda_{uu}|$ and $|\lambda_{ss}|$; Right: $Br$ for $\tau^- \rightarrow \mu^- K^0 \bar{K}^0$ versus $|\lambda_{dd}|$ and $|\lambda_{ss}|$ with $|\lambda_{\tau\mu}| = 5$.

FIG. 2: Left: $Br$ for $\tau^- \rightarrow \mu^- \pi^+ \pi^-$ versus $|\lambda_{uu}|$ and $|\lambda_{dd}|$; Right: $Br$ for $\tau^- \rightarrow \mu^- \pi^0 \pi^0$ versus $|\lambda_{uu}|$ and $|\lambda_{dd}|$ with $|\lambda_{\tau\mu}| = 5$.

FIG. 3: Left: $Br$ for $\tau^- \rightarrow \mu^- K \bar{K}$ versus $|\lambda_{\tau\mu}|$ for $|\lambda_{uu}| = 0.5, |\lambda_{ss}| = 100$. The solid line stands for $\tau^- \rightarrow \mu^- K^+ K^-$, the dashing line for $\tau^- \rightarrow \mu^- K^0 \bar{K}^0$, the horizon line for current experimental upper limit for $\tau^- \rightarrow \mu^- K^+ K^-$. Right: $Br$ for $\tau^- \rightarrow \mu^- \pi\pi$ versus $|\lambda_{\tau\mu}|$ for $|\lambda_{uu}| = |\lambda_{dd}| = 0.5$. The solid line stands for $\tau^- \rightarrow \mu^- \pi^+ \pi^-$, the dashing line for $\tau^- \rightarrow \mu^- \pi^0 \pi^0$, the horizon line for current experimental upper limit for $\tau^- \rightarrow \mu^- \pi^+ \pi^-$. 
mentioned parameter spaces, both two decays ratios could reach the order of \(10^6 - 8\). Comparing Eq.(24) and (25), we could find that the structures of these decay amplitudes are similar. Hence \(Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)\) also rises with the increase of \(|\lambda_{ss}|\) but is almost not affected by the modification of \(|\lambda_{dd}|\). It resulted from the suppressed \(|\lambda_{uu}|, |\lambda_{dd}|\) and that \(|\lambda_{ss}|\) is larger than \(|\lambda_{uu}|, |\lambda_{dd}|\) two order of magnitudes.

The functions of \(\tau^- \rightarrow \mu^- \pi\pi\) decays versus model parameters are presented in Fig.2, where the left figure is \(Br(\tau^- \rightarrow \mu^- \pi^+\pi^-)\) versus \(|\lambda_{uu}|\) and \(|\lambda_{dd}|\), and the right is \(Br(\tau^- \rightarrow \mu^- \pi^0\pi^0)\) versus \(|\lambda_{uu}|\) and \(|\lambda_{dd}|\). Both decay amplitudes are relevant to \(|\lambda_{uu}|\) and \(|\lambda_{dd}|\). When \(|\lambda_{\tau\mu}| = 5\), their decay ratios move upward with \(|\lambda_{uu}|\) and \(|\lambda_{dd}|\) and both extend the order of \(10^{(-14)}\). We find that \(Br(\tau^- \rightarrow \mu^- \pi^+\pi^-)\) climbs more rapidly than \(Br(\tau^- \rightarrow \mu^- \pi^0\pi^0)\). It is because the contributions of \(|\lambda_{uu}|\) and \(|\lambda_{dd}|\) to the former amplitude are larger than those to the latter amplitude.

Then, we take \(|\lambda_{uu}| = |\lambda_{dd}| = 0.5, |\lambda_{ss}| = 100\) and analysis the relation of branching ratios versus \(|\lambda_{\tau\mu}|\). Fig.3 gives four decays branching ratios versus \(|\lambda_{\tau\mu}|\) for other fixed parameters. The left figure is branching ratios for \(\tau^- \rightarrow \mu^- K \bar{K}\), where the solid line stands for \(\tau^- \rightarrow \mu^- K^+ K^-\), the dashing line for \(\tau^- \rightarrow \mu^- K^0 \bar{K}^0\). The experimental data for \(\tau^- \rightarrow \mu^- K^+ K^-\) and \(\tau^- \rightarrow \mu^- K^0 \bar{K}^0\) are denoted by the lower horizon line and upper horizon dash line, respectively. From left figure, we could see the lower horizon line constraints \(|\lambda_{\tau\mu}|\) at \(\simeq 4.5\) for \(Br(\tau^- \rightarrow \mu^- K^+ K^-)\), and the upper limit for \(Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)\) constraints \(|\lambda_{\tau\mu}|\) at the order of \(O(1)\). It should be noted that the latest experimental value of \(Br(\tau^- \rightarrow \mu^- K^+ K^-)\) is based on 671 nb\(^{-1}\) data. If the data of \(Br(\tau^- \rightarrow \mu^- K^0 \bar{K}^0)\) have been updated, the bound obtained will be more respective. The right figure is \(Br\) for \(\tau^- \rightarrow \mu^- \pi\pi\) where the solid line stands for \(\tau^- \rightarrow \mu^- \pi^+\pi^-\) and the dashing line for \(\tau^- \rightarrow \mu^- \pi^0\pi^0\). The experimental data for \(\tau^- \rightarrow \mu^- \pi^+\pi^-\) and \(\tau^- \rightarrow \mu^- \pi^0\pi^0\) are denoted by the lower horizon line and upper horizon dash line, respectively. From the right figure, we could find \(Br(\tau^- \rightarrow \mu^- \pi^+\pi^-)\) is more sensitive to \(|\lambda_{\tau\mu}|\) than \(Br(\tau^- \rightarrow \mu^- \pi^0\pi^0)\). In all, the \(\tau^- \rightarrow \mu^- K^+ K^-\) decay would make tighter constraints on the Higgs couplings than those from decays.

**VI. CONCLUSION**

Sum up, we have calculated the branching ratios of \(\tau^- \rightarrow \mu^- PP (PP = K^+ K^-, K^0 \bar{K}^0, \pi^+ \pi^-, \pi^0\pi^0)\) decays in the model III 2HDM. The neutral Higgs bosons contribute to theses decays at the tree-level. Comparing to the \(\tau^- \rightarrow \mu^- P\) decays, the resonances play a part in \(\tau^- \rightarrow \mu^- PP\) processes and to which the massive Higgs bosons have insensitivity. Only the scalar currents contribute to these decays. Our work suggests that the parameter \(|\lambda_{\tau\mu}|\) is constrained at the order of \(O(1 \sim 10^3)\) by the experimental data. And in the rational parameters space, their branching ratios can reach the experimental values. The \(\tau^- \rightarrow \mu^- K^+ K^-\) decay is most sensitive to \(|\lambda_{\tau\mu}|\). Our
study is hoped to supply good information for the future experiment and explore the structure of the 2HDM III model.

Acknowledgments

The work is supported by National Science Foundation under contract No.10547110, Henan Educational Committee Innovative Research Team Foundation under contract No.2010IRTSTHN002, Henan Educational Committee Foundation under contract No.2007140007.

[1] Y. Fukuda, et al., Super-Kamiokande Collaboration, Phys. Lett. B433, 9(1998); ibid. Phys. Lett. B436, 33(1998); ibid. Phys. Rev. Lett. 81, 1562(1998).

[2] J.G. Hayes, S.F. King, I.N.R. Peddie, Nucl.Phys. B739:106-119(2006); M. Gómez-Bock, et al., Phys. Rev. D80, 055017 (2009).

[3] J. Hisano, et al., Phys. Rev. D53, 2442(1996); Yasuhiro Okada, et al., Phys. Rev. D61, 094001(2000); Seungwon Baek, et al., Nucl. Instrum. Meth. A 503, 244(2001); W. Grimus, L. Lavoura, Phys. Rev. D66, 014016(2002); G. Cvetic, et al., Phys. Rev. D66, 034008(2002); Erratum-ibid. D68, 059901(2003); Andrea Brignole and Anna Rossi, Phys. Lett. B566, 217(2003); E. Arganda, Phys. Rev. D71, 035011(2005); Sebastian Jager, PITHA-05/07 [arXiv:hep-ph/0505243]; Shinya Kanemura, et al., OU-HET 525, SISSA 54/2005/EP [arXiv:hep-ph/0507264]; M. Cannoni, O. Panella, Phys. Rev. D81:036009(2010).

[4] M. Frank, Phys. Rev. D62:015006(2000); A. de Couvea, S. Lola, K. Tobe, Phys. Rev. D63:035004(2001); M. Sher, Phys. Rev. D66:057301(2002); T. Fukuyama, T. Kikuchi, N. Okada, Phys. Rev., D68:033012(2003); A. Brignole and A. Rossi, Nucl. Phys. B701, 3(2004); Yue Chong-Xing, Wang Li-Hong, Ma Wei, Phys. Rev., D74:115018(1-15)(2006); P. Paradisi, JHEP, 0602:050(2006); M. Blanke, et al., JHEP, 0705:013(2007).

[5] Wenjun Li, Yadong Yang and Xiangdan Zhang, Phys. Rev. D73, 073005(2006); Wenjun Li, et al., [arXiv:hep-ph/0812.0727].

[6] C.Amsier, et al., Phys. Lett. B667:1(2008); Y.Miyazaki, et al., (The Belle Collaboration), Phys. Lett. B682:355-362(2010)[arXiv:hep-ph/0908.3156].

[7] A. Ilakovac, Phys. Rev., D54:5653-5673(1996); Chuan-Hung Chen, Chao-Qiang Geng, Phys. Rev. D74:035010(2006).

[8] E. Arganda, M. J. Herrero and J. Portolés, JHEP 0806, 079(2008), [arXiv:hep-ph/0803.2639v3].

[9] M.Herrero, J.Portoles and A.Rodriguez-Sanchez , [arXiv:hep-ph/0909.0724].

[10] Wei Liu, Chong-Xing Yue, Jiao Zhang, [arXiv:hep-ph/0910.2514].

[11] S. Weinberg, Physica A 96 (1979) 327; J. Gasser and H. Leutwyler, Annals Phys. 158 (1984) 142; J. Gasser and H. Leutwyler, Nucl. Phys. B 250, (1985)465; R.Dashen and M. Weinstein, Phys. Rev. 183(1969)1261;L.-F. Li and...
[12] G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, (1989)311; G. Ecker, et al., Phys. Lett. B223(1989)425.
[13] Akihiro Matsuzaki, Hidekazu Tanaka, Phys. Rev. D79:015006(2009).
[14] T. P. Cheng and M. Sher, Phys. Rev. D35, 3484(1987).
[15] W.S. Hou, Phys. Lett. B296, 179(1992); A. Antaramian, L.J. Hall, and A. Rasin, Phys. Rev. Lett. 69, 1871(1992); L. Hall and S. Weinberg, Phys. Rev. D48, R979 (1993); M.J. Savage, Phys. Lett. B266, 135(1991); L. Wolfenstein and Y.L. Wu, Phys. Rev. Lett. 73, 2809(1994).
[16] David Bowser-Chao, King-man Cheung, Wai-Yee Keung, Phys. Rev. D59, 115006(1999), arXiv:hep-ph/9811235.
[17] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385-448, (1979).
[18] Wilson K G, Phys. Rev. D10, 2445(1974).
[19] G.¢ Hooft, Nucl. Phys. B 72, (1974)461; Nucl. Phys. B 75, (1974)461; E. Witten, Nucl. Phys. B 160, (1979) 57.
[20] G. Colangelo, et al., Phys.Rev..D54:4403-4418(1996); Karol Kampf, Jiri Novotny and Jaroslav Trnka, Eur.Phys.J.C50: 385-403(2007); ActaPhys.Polon.B38:2961-2966(2007); D. Gomez Dumm, et al., [arXiv:hep-ph/0911.2640]; S. Iwashyn, A. Korchin, Nucl. Phys. B (Proc.Suppl.) 181-182:189-193(2008); Guo zhihui, Phys. Rev., D78: 033004(2008).
[21] D. Atwood, L. Reina, and A. Soni, Phys. Rev. Lett. 75, 3800(1995); Phys. Rev. D53, 1199(1996); Phys. Rev. D55, 3156(1997).
[22] Yuan-Ben Dai, Chao-Shang Huang, Jian-Tao Li, Wen-Jun Li, Phys.Rev. D67, 096007(2003).
[23] Chao-Shang Huang, Jian-Tao Li, Int. J. Mod. Phys. A20, 161(2005). arXiv:hep-ph/0405294.
[24] R. Martinez, J.A. Rodriguez, D.A. Milanes, Phys. Rev. D72, (2005)035017, arXiv:hep-ph/0502087.
[25] Zhenjun Xiao, et al., Phys.Rev. D63 (2001) 074005.
[26] R.A. Diaz, R. Martinez, J.Alexis Rodriguez, Phys. Rev. D67, 075011(2003).
[27] U. Cotti, M. Pineda, and G. Tavares-Velasco, [arXiv:hep-ph/ 0501162].