Part-to-full shape matching of different human subjects

Panjawee Rakprayoon a, Miti Ruchanurucks a,**, Somying Thainimit a,* Ikuhisa Mitsugami b

a Electrical Engineering Department, Kasetsart University, Bangkok, Thailand
b Hiroshima City University, Hiroshima, Japan

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ABSTRACT
Shape matching is a fundamental operation in digital geometry processing and computer graphics. Challenges in shape matching include finding correspondences of partial shapes with deformations, as well as topological noise and ambiguities. This paper presents a partial shape correspondence algorithm based on the concept of the functional map. An iterative dense matching algorithm, incorporating sparse and guided dense matching, is proposed along with a new objective function including both descriptor matching error and transformation error. Rank estimation with the rank direction is proposed to achieve more accurate slope approximation of the functional map. The slope is beneficial because it directly influences the matching efficiency. The experimental results obtained using FAUST and SHREC’16 datasets demonstrate the effectiveness of our proposed algorithm for matching the shapes of different human subjects and shapes with large missing parts compared with state-of-the-art algorithms. The proposed algorithm provides an average geodesic distance of $<0.033$ even when the missing part is up to 80% of the area.

1. Introduction

Nonrigid shape matching has been widely studied in computer vision and computer graphics. It has been utilized as a technique for shape analysis, shape recognition, shape transformation, and shape recovery. The aim of shape matching or shape correspondence is to find a set of corresponding points between shapes, given point clouds, or mesh elements. Instead of using the original shape representation, shape matching often utilizes shape descriptors that characterize the properties of the shape. Various shape descriptors that are invariant to shape deformations have been proposed. Widely used descriptors include the geodesic distance [1, 2, 3, 4, 5, 6, 7, 8, 9], global point signature (GPS) [10, 11, 12], wave kernel signature (WKS) [13], heat kernel map [14], heat kernel signature (HKS) [15], extensions of heat kernel signature [16], and average mixing kernel signature (AMKS) [17]. Recently, research interest has moved toward learning-based approaches [18, 19, 20, 21, 22, 23, 24], which train existing descriptors from a dataset.

The correspondences between shapes are constructed by maximizing the similarity between the descriptors or between shapes obtained by transforming the descriptors. Additional constraints such as the preservation of the distance between points can be imposed upon the optimization. If only a subset of shape features is utilized, the matching is called ‘sparse matching’; otherwise, it is called ‘dense matching’. The sparse approach requires less time and memory than the dense approach [25, 26, 27, 28].

Alternatively, shape correspondence can be divided into complete correspondence and partial correspondence. Partial correspondence aims to find point correspondences between shapes having partially common surfaces. In real-world scenarios, partial shapes are often obtained. The partiality may be caused by missing views, occlusions, and large deformations of shapes that occur during three-dimensional (3D) acquisition. In practical applications, acquiring a full model may be nearly impossible, e.g., a full 3D model of a patient with severe burns. In such case, an accurate estimation of burn size is crucial for proper treatments. An extent of the burn injury, which is the percentage of the body surface damaged out of the total body surface of the patient is used to estimate the burn size. Our work is motivated by this real-world application, where only partial models of patients can be acquired [29]. Therefore, we proposed exploiting a complete model of another human subject to find correspondence between the two shapes in order to estimate the whole body surface area of patients. The more the accurate correspondences, the better the estimation of the surface area. Additionally, an analysis of human movements in sports and physical therapy could benefit if only

* Corresponding author.
** Corresponding author.
E-mail addresses: miti.r@ku.th (M. Ruchanurucks), sying.thai@gmail.com (S. Thainimit).

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one full model of any subject is required for tracking and studying movements of other subjects in near-real-time.

The functional maps presented by Ovsjanikov et al. [30] play an important role in the design and development of several shape-matching algorithms for solving non-rigid matching problems. The point-to-point correspondence between shapes can be mapped to the domain of the functions on its surfaces. Instead of point-to-point mapping, a linear operator called a “functional map” is used to map between the spaces of coefficients projected onto the Laplace-Beltrami eigenfunction basis. The shortcomings of point-to-point matching include the difficulty of incorporating global constraints, extensive computation, and limitations in matching coarse similarities or symmetry ambiguities. By considering mapping based on such a function, the algorithms with a functional map provide more flexibility and more manageable computation than conventional point-to-point correspondence approaches. Several methods [31, 32, 33, 34, 35, 36, 37, 38, 39] have been used to improve the functional map representation. For example, Kovnatsky et al. [38] adopted a soft error criterion to estimate the functional map efficiency. To enhance the matching accuracy and reduce the number of outliers, Rodolà et al. [39] introduced a probabilistic pointwise map for recovering point-to-point correspondences from existing functional map approaches.

For the partial matching problem, Rodolà et al. [40] analyzed the properties of Laplacian eigenfunctions that result in a particular slanted diagonal structure of the functional map matrix. They proposed an efficient partial functional map (pFM) framework to compute correspondences by optimizing the functional map in which eigenfunctions of the missing part of the full shape are truncated. Postolache et al. [41] proposed part-to-full shape matching method based on the discrete Hamiltonian eigenbasis. To reduce the complexity of the optimization, Litany et al. [42] proposed the fully spectral partial matching (FSPM) method to solve correspondences of partial shapes with topological noise by optimizing the transformation coefficient matrix using the functional map in the spectral domain. The optimization is simplified, resulting in a constant complexity, independent of the number of shape vertices. Arbel et al. [43] proposed a partial shape matching algorithm using the properties of nearest-neighbor fields. Wu et al. [44] proposed a method for integrating the eigenvalue equivalence Hamiltonian operator with the optimization formula of functional maps.

For the iterative map refinement techniques, Vestner et al. [45] included the proposed Product Manifold Filter for denoising the correspondence manifolds to increase the quality of input mapping. Melzi et al. [46] introduced the iterative spectral upsampling method by increasing a step size of the functional map, called Zoomout, to improve the quality of correspondences. Gasparetto et al. [47] proposed the minimization of the transportation of function with a fuzzy mapping between shapes to obtain the sub-vertex correspondence map between surfaces.

The aforementioned methods focus on matching studies using the same model. Our study addresses partial correspondence focusing on different human subjects. The proposed method is inspired by pFM framework and our observation that the rank of functional map matrix had a slanted diagonal structure. Consider a two-dimensional manifold \( \mathcal{X} \) and define the space of square-integrable functions on manifold \( \mathcal{X} \) as \( L^2(\mathcal{X}) = \{ f: \mathcal{X} \to \mathbb{R}, f_\infty < \infty \} \). The definition of the standard inner product is \( \langle f, g \rangle = \int \mathcal{X} f(u)g(u)du \), where \( f \) and \( g \) are two functions of a point \( u \) on the manifold \( \mathcal{X} \). To compute any function \( f \in L^2(\mathcal{X}) \), the Laplace-Beltrami operator \[ \Delta f(x) = \sum_{i,j} \lambda_i \phi_i \phi_j(x) \] is used to optimize the functional map. This results in functional map \( \phi \) and \( \psi \) that form an orthonormal basis for \( L^2(\mathcal{X}) \). Function \( f \) can be expressed using the Fourier series expansion as \( f(x) = \sum_{i,j} \hat{f}_{ij} \phi_i \psi_j(x) \).

The correspondence between nonrigid shapes \( \mathcal{X} \) and \( \tilde{\mathcal{X}} \) can be estimated using the linear operator \( T : L^2(\mathcal{X}) \rightarrow L^2(\tilde{\mathcal{X}}) \) on \( f \) defined by

\[
Tf = \sum_{i,j} \hat{f}_{ij} \psi_j \phi_i,
\]

where bases \( \{ \phi_i \}_{i \geq 1} \) and \( \{ \psi_j \}_{j \geq 1} \) are orthogonal on \( L^2(\mathcal{X}) \) and \( L^2(\tilde{\mathcal{X}}) \), respectively, and coefficients \( c_{ij} = T\phi_i \psi_j \). Because the transformation is encoded in coefficients \( c_{ij} \), correspondences between two shapes can be achieved by solving for the unknown matrix \( C \), i.e., \( c_{ij} \). Additionally, matrix \( C \) can be truncated because the Laplace-Beltrami eigenfunction provides a compact representation. Therefore, only \( k \) eigenfunctions corresponding to the \( k \) largest eigenvalues are needed to represent the full map. For the near-isometric shape, the matrix \( C_{k \times k} \) has a diagonally dominant structure. Furthermore, \( C^T \) includes the identity matrix to preserve the volume between the two shapes. The solution of matrix \( C \) can be obtained by utilizing the optimization problem presented as

\[
\min_{C} ||C\bar{F} - \tilde{G}||^2_F,
\]

where \( \bar{F} = (\phi_i,\psi_j) \) and \( \tilde{G} = (\psi_i,\phi_j) \) given \( N \) corresponding functions \( g_i \approx Tf_j, l = 1, \ldots, N \). For implementation, the shape descriptors, which are invariant to shape deformation, e.g., HKS or WKS, are used as the corresponding functions, and the iterative closest point (ICP) algorithm [49] is used to optimize the functional map.

2. Background

2.1. Functional correspondence

Consider a two-dimensional manifold \( \mathcal{X} \) and define the space of square-integrable functions on manifold \( \mathcal{X} \) as \( L^2(\mathcal{X}) = \{ f: \mathcal{X} \to \mathbb{R}, f_\infty < \infty \} \). The definition of the standard inner product is \( \langle f, g \rangle = \int \mathcal{X} f(u)g(u)du \), where \( f \) and \( g \) are two functions of a point \( u \) on the manifold \( \mathcal{X} \). To compute any function \( f \in L^2(\mathcal{X}) \), the Laplace-Beltrami operator \[ \Delta f(x) = \sum_{i,j} \lambda_i \phi_i \phi_j(x) \] is used to optimize the functional map. This results in functional map \( \phi \) and \( \psi \) that form an orthonormal basis for \( L^2(\mathcal{X}) \). Function \( f \) can be expressed using the Fourier series expansion as \( f(x) = \sum_{i,j} \hat{f}_{ij} \phi_i \psi_j(x) \).

The correspondence between nonrigid shapes \( \mathcal{X} \) and \( \tilde{\mathcal{X}} \) can be estimated using the linear operator \( T : L^2(\mathcal{X}) \rightarrow L^2(\tilde{\mathcal{X}}) \) on \( f \) defined by

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Tf = \sum_{i,j} \hat{f}_{ij} \psi_j \phi_i,
\]

where bases \( \{ \phi_i \}_{i \geq 1} \) and \( \{ \psi_j \}_{j \geq 1} \) are orthogonal on \( L^2(\mathcal{X}) \) and \( L^2(\tilde{\mathcal{X}}) \), respectively, and coefficients \( c_{ij} = T\phi_i \psi_j \). Because the transformation is encoded in coefficients \( c_{ij} \), correspondences between two shapes can be achieved by solving for the unknown matrix \( C \), i.e., \( c_{ij} \). Additionally, matrix \( C \) can be truncated because the Laplace-Beltrami eigenfunction provides a compact representation. Therefore, only \( k \) eigenfunctions corresponding to the \( k \) largest eigenvalues are needed to represent the full map. For the near-isometric shape, the matrix \( C_{k \times k} \) has a diagonally dominant structure. Furthermore, \( C^T \) includes the identity matrix to preserve the volume between the two shapes. The solution of matrix \( C \) can be obtained by utilizing the optimization problem presented as

\[
\min_{C} ||C\bar{F} - \tilde{G}||^2_F,
\]

where \( \bar{F} = (\phi_i,\psi_j) \) and \( \tilde{G} = (\psi_i,\phi_j) \) given \( N \) corresponding functions \( g_i \approx Tf_j, l = 1, \ldots, N \). For implementation, the shape descriptors, which are invariant to shape deformation, e.g., HKS or WKS, are used as the corresponding functions, and the iterative closest point (ICP) algorithm [49] is used to optimize the functional map.

2.2. Partial functional correspondence

For shapes with holes and cuts, Rodolà et al. [40] showed that map \( C \) has a slanted diagonal structure. Consider a partial functional map transformation \( T : L^2(\mathcal{X}_p) \rightarrow L^2(\tilde{\mathcal{X}}_p) \). The eigenfunction \( \phi_i \) of partial shape \( \mathcal{X}_p \) corresponds to the eigenfunction \( \psi_i \) of full shape \( \mathcal{X} \) for some \( i \leq j \). This results in functional map \( \mathcal{C} \) manifesting a slanted diagonal structure. The slope of this particular matrix structure depends on the area ratio between the partial and the full shapes. With the specified first \( k \) eigenvalues, the slope can be estimated using \( r/k \), where \( r = \text{rank}(C) \) and \( r < k \). To solve the partial matching problem, the objective function of correspondence optimization is set as
3. Proposed method

This section describes the proposed iterative partial shape matching method in detail. The proposed method is a semi-automatic approach that requires an initial set of correspondence landmarks between the partial shape \( X \) and the full shape \( Y \). The differences between our proposed method and pFM method are

1. We improve matching accuracy through better rank estimation obtained using the proposed rank direction.
2. For each iteration, the guided dense matches are also incorporated in the functional map refinement process. The guided dense matches are obtained based on both sets of point-wise distance error and transformation error (details are in section 3.5.2). In pFM, only the transformation error is used. The inclusion of the distance error in energy function helps to provide a more accurate functional map.

The proposed iterative method is described in Algorithm 1.

3.1. Correspondence landmark selection

The algorithm starts by manually selecting landmark matches. Common criteria for choosing landmark positions [8] is to choose the extreme points (points with high curvature) of an object such as fingertips of human body. Strait [50] also recommended choosing landmarks that represent features that are significant anatomically such as joints connecting a pair of bones and should be placed across a sample of shapes. In our work, the end points of human shapes (legs, arms, and the top of the head) and points located near the central symmetry line of the body (red line in Figure 1) are used as the initial landmarks. Figure 1 shows two example sets of initial pairwise landmarks.

3.2. Laplace-Beltrami eigenvector construction

The first hundred \((k = 100)\) Laplace-Beltrami eigenfunctions are a common choice for the basis function used in the functional map framework because the Laplace-Beltrami eigenfunctions reflect the geometry of the surface. In this study, the Laplacian on the point clouds of a manifold is generated using the discretized Laplace operator described in [48]. The Beltrami eigenfunction is then constructed using the generalized eigen-decomposition details in [40].

For manifolds \( \mathcal{X} \) and \( \mathcal{Y} \) containing the respective \( n \) and \( m \) vertices, the set of \( n \) and \( m \) eigenvalues corresponding to its ordered eigenvalues are contained as columns of the matrix. Finally, we have an \( n \times k \) matrix of \( \Phi = [\Phi_1 \ \Phi_2 \ ... \ \Phi_k] \) and an \( m \times k \) matrix of \( \Psi = [\Psi_1 \ \Psi_2 \ ... \ \Psi_m] \) as the bases of manifolds \( \mathcal{X} \) and \( \mathcal{Y} \), respectively.

3.3. Initial sparse and guided dense correspondences

The initial sparse and guided dense correspondences for shape matching are constructed by minimizing the deformation between the two shapes. In this step, shape deformation is characterized by the disparity of the normalized geodesic distance between pairs of points on the shape surface. A set of correspondences \( Q \) can then be obtained by minimizing the distance error, as

\[
E_d(Q) = \|d^Q_{\mathcal{X}} - d^Q_{\mathcal{Y}}\|,
\]

where \( d^Q_{\mathcal{X}} \) and \( d^Q_{\mathcal{Y}} \) are the normalized geodesic distance descriptors on a corresponding point \( x \in Q \) of shape \( \mathcal{X} \) and a corresponding point \( y \in Q \) of shape \( \mathcal{Y} \), respectively. For initial sparse correspondences, minimization is performed according to the set of landmark correspondences

\[
\min_{Q, \mathcal{Q}} \|C\cdot G - \hat{G}(v)\|_{2,1} + \rho_{cor}(C) + \rho_{perc}(v)
\]

where the operator \( \| \cdot \|_{2,1} \) is equal to the sum of the 2-norm of the matrix columns, i.e., \( \| \mathbf{x} \|_{2,1} = \sum_j |x_j| \), and \( \rho_{cor}(C) \) is correspondence regularization considering the characteristics of matrix \( C \). As the \( C \) of pFM does not have full rank, the slanted diagonal matrix \( C \) can be estimated using

\[
C^T C \approx \begin{bmatrix} I_{N-k} & 0 \\ 0 & 0 \end{bmatrix}_{k \times k},
\]

where the last \( k-r \) is zero. The function \( \rho_{perc}(v) \) is proposed for calculating the restricted area indicated by part \( v \) with the shortest boundary for finding the corresponding area between segmented shapes.

Algorithm 1

Part-to-full shape matching

Input: Point clouds of a partial shape \( \mathcal{X} \) and full shape \( \mathcal{Y} \)

Output: Set of correspondences \( Q \)

Initialization: Initial landmark matches \( Q_0 \)

1. Construct bases \( \{\phi_i\}_{i=1}^N \) and \( \{\psi_j\}_{j=1}^M \) for \( L^2(\mathcal{X}) \) and \( L^2(\mathcal{Y}) \), respectively.
2. Compute the initial sparse matches.
3. Construct the initial guided dense matches based on the initial sparse matches.
4. Compute two sets of descriptors \( \{f_i\} \) and \( \{g_j\} \) on \( \mathcal{X} \) and \( \mathcal{Y} \) and represent them as coefficient matrices \( \tilde{F}_i \) and \( \tilde{G}_j \), respectively, using the landmark matches.
5. Estimate the initial \( C \) according to the landmark matches.
6. for \( i = 1 \) to \( N \) (\( N = 30 \))
7. Search rank \( r_\mathcal{X} \) of the functional map \( C \). if \( i \leq 5 \)
8. Compute the optimal \( C \) using the coefficient matrices \( \tilde{F}_i \) and \( \tilde{G}_j \) and continually identify candidate correspondences.
9. Separately refine the solution \( C \) based on the candidate correspondences and the guided dense matches and update \( C \) with the one that provides the least transformation error calculated by (6). Then, identify the correspondences \( Q_k \).
10. Update the sparse matches and guided dense matches using both the point-wise distance error and transformation error denoted by (8).
11. Expand the landmark set by concatenating with pairwise points chosen from the best matching point of the sparse matches.
12. Update the coefficient matrices \( \tilde{F}_i \) and \( \tilde{G}_j \) according to the updated landmark set.
13. end for
14. Assign the correspondences \( Q = \{ \arg \min_{Q} E_i(Q) \} \) for \( i = 1, 2, ..., N \).
Figure 1. Initial landmark correspondences between two shapes represented by black dots connected by straight black lines. The unfilled dot is the corresponding point on the far side. Red lines indicate the central symmetry line of the body.

contained in Q. The initial guided dense matches are constructed similarly using the obtained initial sparse matches.

3.4. Initial functional map C

To obtain the solution of the functional map C, we exploit the optimization based on (3) without considering \( p_{\text{corr}}(\mathbf{w}) \) and using the coefficient matrices \( \mathbf{F}_i = \mathbf{F}_i^T \mathbf{F}_i \) and \( \mathbf{G}_i = \mathbf{G}_i^T \mathbf{G}_i \). The descriptors \( \mathbf{F}_i \) and \( \mathbf{G}_i \) are generated using the normalized GPS-D descriptor (see details in Appendix A) based on pairwise landmarks. Thus, our objective function is generated using the normalized GPS-D descriptor (see details in Appendix A) based on pairwise landmarks. Thus, our objective function is

\[
\min_{\mathbf{C}} \| \mathbf{C} \mathbf{F}_i - \mathbf{G}_i \|_{2,1} + p_{\text{corr}}(\mathbf{C}). \tag{5}
\]

Optimization over C is achieved using the ICP technique. The ICP scheme searches the eigenspace \( \mathbb{R}^k \) to find the shortest distance between a column of \( \mathbf{C} \mathbf{F}_i^T \) and the corresponding column of \( \mathbf{G}_i^T \). The correspondence regularization \( p_{\text{corr}}(\mathbf{C}) \) is determined using the method described in [40]. Thus, we solve for the map C that minimizes the transformation error defined by (6)

\[
E_i(\mathbf{Q}) = \| \mathbf{C} \mathbf{F}_i^T - \mathbf{G}_i^T \|_F. \tag{6}
\]

3.5. Iterative dense matching algorithm

The proposed iterative dense matching algorithm utilizes the initial sparse and initial guided dense matches to improve the overall matching accuracy. The iterative algorithm involves rank estimation with the rank direction and refinements of sparse, guided, dense, and dense matches.

3.5.1. Rank estimation with rank direction

For partial functional correspondence according to pFM method, the slope of the slanted diagonal structure of map C is calculated using the rank of map C. According to our observations, the ground truth of matrix C of the partial and the full shapes of different human subjects returns a different rank value from the rank \( r \) computed using pFM. The difference between the rank \( r \) and the returned rank of the corresponding shapes \( \mathcal{X}_p \) and \( \mathcal{Y}_p \) consists of both positive and negative values. To guide the search toward rank \( r \), the rank direction \( \overrightarrow{D}_r \) is estimated during the first iteration of Algorithm 1. Starting with the rank \( r \), the shape matching errors using the rank values near by the rank \( r \) are calculated. The rank value with the minimum shape matching error is set as the \( r_d \) and the rank direction, \( \overrightarrow{D}_r \), is determined using (7).

\[
\overrightarrow{D}_r = \begin{cases} 
-1, & r < r_d, \\
0, & r = r_d, \\
1, & r > r_d 
\end{cases} \tag{7}
\]

Constrained by the obtained rank direction, the search for refined \( r_d \) is continued in the next few iterations. By observations, the value of \( r_d \) converges at the fifth iteration. The obtained transformation errors remain stable for the remaining iterations. Therefore, we set the iteration for rank \( r_d \) estimation at the fifth iteration for all datasets in order to reduce the amount of time required.

3.5.2. Correspondence refinement

We propose incorporating sparse and guided dense matching with the algorithm to provide more accurate correspondences and efficient manageable computation. The correspondences encoded in map C are repeatedly refined using both the guided dense matches and landmark matches.

After the estimated rank \( r_d \) is obtained, map C is refined using the landmark matches. The optimization based on (5) yields the candidate correspondences. The other set of correspondences is computed similarly but by using the guided dense matches instead. The refined functional map C between the candidate correspondences and guided dense matches is selected by considering the smallest shape matching error defined by (6).

Next, we update the sparse matches using the landmark matches and the obtained refined map. The updated sparse matches are used to update the guided dense matches. The new corresponding points are chosen by minimizing the energy function, which includes both the transformation error \( E_i(\mathbf{Q}) \) defined by (6) and the distance error \( E_d(\mathbf{Q}) \) defined by (4). The energy function E of all corresponding points in \( \mathbf{Q} \) is denoted as

\[
E(\mathbf{Q}) = w_1 E_i(\mathbf{Q}) + w_2 E_d(\mathbf{Q}), \tag{8}
\]

where \( w_1 \) and \( w_2 \) are adjusted weights.

Consequently, the set of landmark matches is expanded by concatenating \( \mathbf{Q}_l \) with the new correspondence chosen from the best matching point of the sparse matches using (8).

Finally, the sets of the new corresponding functions obtained using the new pairwise landmark are computed and concatenated as a new column of each set \( \{f\} \) and set \( \{g\} \) to update the coefficient matrices \( \mathbf{F}_i \)
and $\tilde{G}_i$. In the last iteration, the set $Q_i$ that returns the smallest functional map error is assigned as the final correspondence.

4. Experimental results and discussions

We perform several experiments to demonstrate the performance and robustness of the proposed method. System validations and comparisons are performed using two datasets: FAUST [51] and SHREC’16 [52]. FAUST dataset contains real human shapes acquired using a 3D scanning device. The shapes contain topological artifacts and are partially occluded. SHREC’16 is a challenging partial correspondence benchmark. The dataset includes eight different classes of 120 shapes with large deformations, topological noise, and synthetic topological changes, such as shortcuts. To perform the experiments, we use the three classes of three different human subjects. Each class contains 15 pairwise shapes with various poses and cuts for an individual. A null shape with the standard pose for each class is used as a full template shape to match with the partial shape. In total, 45 pairwise shapes of human subjects from SHREC’16 dataset are used in our experiments.

FAUST database consists of 10 human shapes subdivided into 10 different poses for each person. All 100 shapes have 6,890 vertices prepared as ground-truth correspondences. The first pose of the first person is selected as the template. All poses of full shapes of the other nine persons are partially cut and grouped into four datasets, and 360 of the resulting partial shapes are selected as the model input.

The parameters of the energy function in (8) are set as $w_1 = 0.5$ and $w_2 = 0.5$ for FAUST and $w_1 = 0.1$ and $w_2 = 0.9$ for SHREC’16. The number of iterations $N$ is set to 30. The eigenvalues $\lambda = (\lambda_1 \ldots \lambda_k)$ corresponding to the eigenfunctions are computed with $k = 100$.

4.1. Quality of correspondences measurement

The quality of correspondence is expressed as the cumulative percentage of matches that rely on the normalized geodesic error defined in (9). The units of normalized geodesic distance between matching points $(x_c, y_c) \in Q$ and ground-truth correspondences $(x_c, \bar{y}_c)$ are approximated on the full template surface $Y_f$ represented by

$$
\epsilon(y_c) = \frac{D_G(y_c, \bar{y}_c)}{\sqrt{\text{area}(Y_f)}},
$$

where $D_G(y_c, \bar{y}_c)$ is the geodesic distance between points $y_c$ and $\bar{y}_c$. The mean geodesic distance error is the average of (9) for all matching points. Empirically, the solution of matrix $C$ and the dense correspondences can be obtained within the 30th iteration.

4.2. Proposed part-to-full shape matching performance

4.2.1. Performance of sparse matching

Our first experiment demonstrates the advantages of utilizing sparse matches and guided dense matches. The proposed iterative sparse matching algorithm iteratively updates the sparse matches according to the set of landmark matches. The initial sparse matching is included to accelerate the coarse shape alignment. The mean geodesic distance error induced by sparse matching converged quickly and remained small, as shown in Figure 2. The improved sparse matches are visualized, as shown on the right side of Figure 3. The left side of the figure displays the corresponding initial sparse matches. The experimental results indicate that the obtained sparse matches around the upper part of the body (head, face, and upper body) are substantially improved. Thus, the proposed sparse matching refinement can cope with symmetry ambiguity of the body surface.

Figure 2. Sparse matching assessment depicted as mean geodesic error for increasing iterations.

Figure 3. Initial sparse matches (left pair) and final sparse matches (right pair). Correspondences with geodesic error $\leq 0.02$ are plotted in green whereas other correspondences with error $>0.02$ are plotted in red.
4.2.2. Performance of guided dense matching

The second experiment investigates the effects of the guided dense matches. The average geodesic errors obtained when using and not using the set of guided dense matches \( Q_g \) to optimize the functional map and shape correspondence are shown in Figure 4. The guided dense matches succeed in two ways. First, they maintain a small and stable average distance error from the beginning of the iteration. Second, the average matching error is substantially reduced compared to that obtained without using the guided dense matches. In the last iteration, the error is reduced by 50.25%. Therefore, the combination of sparse matching and guided dense matching improves both the stability and accuracy of shape matching.

4.2.3. Performance of rank estimation with rank direction

The average geodesic errors of correspondences between shapes depend on the estimated rank of matrix \( C \), as shown in Figure 5. According to pFM, the estimated rank is \( r = 73 \), as indicated by the green bar in Figure 5. The rank value is improved by utilizing the rank direction to restrict the search over the set of rank intervals. The proposed rank estimation directs rank descent toward the optimal value. The obtained rank (\( r_d \)) is 82.

For partial matching, map \( C \) constitutes a slanted diagonal structure matrix whose slope is associated with the area ratio of the partial shape to the full shape. Consequently, the last \( (k-r_d) \) columns of the matrix are zero, as shown in Figure 6. With a better rank value, the slope of map \( C \) is estimated more accurately, resulting in a better accuracy.

The quantitative results for the %correspondences obtained using \( r \) and \( r_d \) are shown in Figure 7. In this test, we applied pFM using the same parameters and descriptors to optimize matrix \( C \) as the original work but with the intervention of the obtained rank \( r \) with \( r_d \) for various pairwise shapes. The results indicate that the more accurate rank obtained using the proposed rank estimation method enhances the matching performance.

4.3. Robustness against landmark initialization

This experiment demonstrates that our method is robust to possible situations for obtaining landmarks. We investigate the effects of a small number of initial landmarks, inaccurate landmark positions, and outliers on the system performance.

The matching accuracies of the proposed method when two to six initial landmarks are used are presented in Table 1 and Figure 8(a). In our experiments, cases with three to five landmarks exhibit comparable system performances. A larger number of initial landmarks corresponds
to a higher accuracy. The sharp accuracy increment starts at six initial landmarks. According to the results, the initial number of landmarks should be $\geq 3$, because the accuracy obtained using three landmarks increases by 34.09% from the result obtained using two landmarks, with an error of 0.02.

Next, we investigate the effects of initialization with inaccurate landmark positions. A distance deviation ($\Delta e_d$) ranging from 0 to 0.045 is introduced to the initial landmarks. In the dataset, deviations $>0.045$ are considered outliers. Figure 8(b) shows the corresponding accuracies. The results of the proposed method are well within the defined deviations. The deviation has a smaller effect when the required distance error is $>0.06$.

This experiment investigates the performance of our system against outliers, which are often found in automatic landmark position detection. One reference type and four types of outliers are considered in the experiments. Type A is used as the reference, containing the correct landmark correspondences. Types B and C contain common mismatches on the fingers and toes, where the little finger is mismatched with the thumb in Type B and the little toe is mismatched with the big toe in Type C. Types D and E contain symmetrical mismatches. Type D consists of landmark corresponding points located at the back of the body instead of the front (front-to-back landmark misplacement). Type E contains mismatched corresponding points between the left and right sides (left-to-right landmark misplacement), as shown in Figure 9.

The efficiency of the proposed method for handling different types of outliers is shown in Figure 8(c). According to our experimental results, the proposed method can reasonably handle small mismatches around the fingers and toes (Types B and C). With intrinsic symmetry of body shape, our approach can handle front-back symmetry (Type D) better than left-right symmetry (Type E). Type E mismatches substantially

Table 1. Mean geodesic error and matching accuracy of correspondences with a geodesic error of 0.02 obtained using different numbers of initial landmarks.

| Number of Initial landmarks | %Correspondences | Mean geodesic error |
|-----------------------------|------------------|---------------------|
| 2                           | 36.3331          | 0.0818              |
| 3                           | 55.1315          | 0.0322              |
| 4                           | 57.5489          | 0.0288              |
| 5                           | 58.9098          | 0.0256              |
| 6                           | 65.8107          | 0.0215              |

Figure 7. Comparison of matching accuracy between cases using rank $r_d$ and rank $r$ to find solution of matrix $C$ based on pFM algorithm.

Figure 8. Robustness of our method for three different cases of landmark correspondences: (a) matching accuracy with varying number of initial landmarks; (b) matching performance when the corresponding points of landmarks having small geodesic distance errors ($\Delta e_d$); (c) matching efficiency for pairwise landmarks having different types of outliers.
degrade the system performance. In automatic human shape matching, front-back symmetry is more common than left-right symmetry; none-
theless, we will develop a shape-matching task to resolve left-right symmetry confusion in the future. We also plan to make our scheme automatic.

4.4. Comparisons with state-of-the-art

To evaluate the matching performance, the results of our iterative method are compared with the results of the three state-of-the-art algorithms (pFM, FSPM, and Zoomout) obtained using the same dataset. We

Figure 9. Five different types of pairwise landmarks. The correct and incorrect matches are indicated by blue and red lines, respectively. Type A contains correct landmark matches. Type B contains mismatches between little finger and thumb only. Type C also includes mismatches between the little toe and big toe. The front-to-back landmark misplacement and left-to-right landmark misplacement are in Types D and E, respectively.

Figure 10. Correspondence performance of our method, pFM, FSPM and Zoomout measured using FAUST shapes: (a) dataset of same human subjects; (b) dataset of different human subjects.
quantitatively measure the matching performance based on FAUST and SHREC’16 datasets.

4.4.1. Matching performance comparisons using FAUST dataset

The percentages of correspondences obtained using the same (intra-class) and different (inter-class) human subjects are shown in Figs. 10(a) and (b), respectively. The obtained results of the proposed method and pFM, FSPM and Zoomout approaches are shown in black, blue, magenta, and orange, respectively.

For intra-class and inter-class comparisons, the proposed method outperforms the other methods. The results indicate that our method can handle different human subjects better than the other methods.

The other three algorithms are automatic methods, whereas our approach is semi-automatic. This large increment may be partially due to the more accurate landmark initialization. Hence, to observe the effects of landmark initialization, we initialize pFM method using the initial set of guided dense matches \( Q_g \). The obtained result is plotted as a blue dashed line in Figure 10(b). The more accurate landmark initialization substantially improves the % correspondence from the original pFM. However, it still cannot surpass the performance of the proposed system.

4.4.2. Matching performance comparisons using SHREC’16 dataset

We evaluate the accuracy of the matching algorithms using the challenging SHREC’16 database. This database contains shapes with different amounts and types of deformation and partiality. Because of the lack of ground truth for different subjects, evaluations and comparisons are performed on the same human subject only.

The partial matching accuracies of the four approaches are shown in Figure 11(a). The results for pFM and FSPM methods are comparable and within a small geodesic error of <0.045, the two approaches outperform our approach. However, our approach can achieve >90% accuracy within geodesic errors of 0.06 and 96.75% with an error of 0.1, whereas the two approaches achieve an accuracy of approximately 84%. We observe that the obtained distortion is large near locations with non-isometric deformations, such as kicking legs. Similar to many existing methods, our approach has limitations for handling non-isometric deformations.

Finally, we study the change in performance with an increasing level of partiality. The obtained average geodesic errors versus the percentage of the cut area of the surface (missing part) are plotted in Figure 11(b). Compared with the other methods, the average distance error obtained using our method remains small and becomes more stable with an increasing percentage of cut area. Zoomout is the second best, both in terms of small distant error and its stability. Furthermore, the average error of the outliers in our approach is < 0.1, as shown in Figure 11(c). Hence, our approach is more robust against shapes with large missing parts.

4.4.3. Runtime comparison

The average runtime per iteration is approximately 3.25 min for the proposed method, 11.39 min for pFM, 1.0142 min for FSPM, and 0.012 min for Zoomout. The algorithms are implemented in C++/MATLAB using an Intel(R) i7-6700 3.4 GHz CPU with four logical cores.

Even though the low-rank approximation used in pFM based on Laplace-Beltrami eigenfunctions can suppress dense matrices, it still consumes a large amount of time. The integration of the proposed sparse and guided dense matching can significantly reduce the runtime but it still falls behind Zoomout and FSPM. Zoomout method takes the shortest runtime because its iterative upsampling of the functional map starting from a \( 2 \times 2 \) matrix, while those of other approaches initiate the runtime comparison.

Figure 11. Comparisons using challenging SHREC’16 benchmark. (a) Percentages of correspondences (b) Mean geodesic error versus percentage of cut area of surface and (c) Mean geodesic error of outliers versus percentage of cut area of surface.
calculation with matrices of larger sizes. This substantially reduces the computation speed. The runtime of the FSPM method is the second best calculation with matrices of larger sizes. This substantially reduces the FSPM and Zoomout are suggested for initialization or coarse correspondence algorithms.

5. Conclusions

This paper proposes a semi-automatic part-to-full shape correspondence algorithm with a functional map-based approach that focuses on matching different humanoid models. We introduce a rank direction scheme to improve the rank estimation. The proposed rank direction leads to the estimation approaching the optimal rank value, resulting in better matching efficiency. We propose the integration of the descriptor matching error in iterative sparse and guided dense matching to improve refinement of the functional map. The integration substantially improves the stability of shape matching. We performed several experiments and comparisons in both intra-class and inter-class settings. Our proposed method outperforms the state-of-the-art method in an inter-class setting. Furthermore, the quantitative comparison results demonstrate that the system performance is remarkably stable throughout a wide range of partialities. We demonstrate that our proposed matching algorithm is robust with regard to inaccurate landmark initializations. Our approach can cope with landmark position deviations, a small number of landmarks, and front-to-back landmark misplacement.

One main limitation of our approach is the longer computational time than those of FSPM and Zoomout since our optimization complexity is efficient constant time complexity. Therefore, both FSPM and

Additional information

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