On the Covariant Hamilton-Jacobi Equation for the Teleparallel Equivalent of General Relativity

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Abstract

The covariant Hamilton-Jacobi equation for the Teleparallel Equivalent of General Relativity is derived based on the analysis of constraints within the De Donder-Weyl covariant Hamiltonian theory developed by Kanatchikov.

1 Introduction

The covariant Hamilton-Jacobi equation for General Relativity in metric variables has been found by Théophile De Donder in 1930 \cite{DeDonder1930} and rediscovered by Petr Hořava in 1991 \cite{Horava1991}. In 1935 Hermann Weyl has considered a Hamilton-Jacobi theory for multidimensional variational problems \cite{Weyl1935} based on earlier ideas of Volterra \cite{Volterra}, Carathéodory \cite{Caratheodory} and the Göttingen school of Hilbert \cite{Hilbert}. A good historical review with many references can be found in the book by Hanno Rund \cite{Rund}. In the 1980-es the interest to such theories has been revived by Hans Kastrup in his comprehensive review paper \cite{Kastrup1989} and Yoichiro Nambu \cite{Nambu1989} in the context of classical string dynamics. The connection between the geodesic fields \cite{Carinena1990} in the covariant Nambu-Hamilton-Jacobi theory of strings and classical fields described by simple bivectors has been discussed in \cite{Barbour1999,Barbour2000}. The geometric formulations using the language of differential geometry have been developed later e.g. in \cite{Barbour2006,Barbour2007,Barbour2008}. The relation to the standard Hamilton-Jacobi theory in field theory which is using an explicitly distinguished time dimension and canonical Hamiltonian formalism \cite{Dirac1950,Dirac1951} has been discussed only in three papers \cite{Barbour2009,Barbour2010,Barbour2011} and it still remains a subject worthy of further investigation. Let us note that, whereas the covariant Hamilton-Jacobi equations in modern field theory and general relativity are relying on an additional structure of a global foliation with space-like leaves \cite{Barbour2012,Barbour2013}, the covariant Hamilton-Jacobi theories in the calculus of variations do not introduce any additional structure on the space of independent space-time variables and treat them equally.

The covariant Hamilton-Jacobi theories of multidimensional variational problems which describe classical fields have naturally lead to the question of whether a formulations of quantum theory for fields may naturally reproduce the corresponding Hamilton-Jacobi theories in the quasiclassical limit. The modern answer to this question is given
by the programme of precanonical quantization which started in \[28–33\] using the earlier discovery of the bi-graded (Gerstenhaber) analogue of the Poisson bracket in the simplest De Donder-Weyl (DW) covariant Hamiltonian formalism \[34–39\]. This formalism is related to the covariant Hamilton-Jacobi theory discussed by Th. De Donder \[1\] and H. Weyl \[3\] in the same way as the standard Hamilton-Jacobi theory in mechanics is related to the Hamiltonian formalism \[19\]. Later on precanonical quantization has been applied to gauge fields \[33, 40–42\] and general relativity in the standard metric formulation \[43–46\] and the Palatini vielbein formulation \[47–51\]. The relation of this approach with the standard canonical quantization in the Schrödinger representation has been established in \[22, 40, 41, 52–56\] both for quantum scalar fields in flat and curved space-times and Yang-Mills gauge theory.

The classical limit of precanonical quantization has been analysed by means of a generalization of the Ehrenfest theorem \[30, 31, 33, 50\]: the De Donder-Weyl covariant Hamiltonian field equations are shown to be the equations satisfied by the expectation values of certain operators. However, the reduction of precanonical Schrödinger equation to the De Donder-Weyl covariant Hamilton-Jacobi equation so far has been demonstrated only for scalar fields in \[30, 57\].

In this paper we would like to extend the earlier work by De Donder \[1\] and Hořava \[2\] to the case of the teleparallel equivalent of general relativity (TEGR). The interest in teleparallel theories of gravity has been revived recently due to their theoretically attractive features such as a close connection with the Poincaré gauge theory of gravity, a possibility of defining the energy-momentum tensor of the field of gravity, and potential applications in the field of cosmology and gravitational waves. It has stimulated numerous generalizations and theoretical developments in different directions. Among those the attempts to analyze the canonical Hamiltonian formalism and the complicated structure of constraints have been made which are a prerequisite for canonical quantization. To our knowledge, no Hamilton-Jacobi theory based on the canonical Hamiltonian analysis has been formulated for teleparallel gravities so far despite it could be a bridge or a hint to a quantum formulation, as it happened with the canonical Hamilton-Jacobi formulation of General Relativity by Asher Peres in 1962 \[58\] serving as an inspiration for the Wheeler-De Witt equation in canonical quantum gravity. As the precanonical quantization of TEG has been recently presented in \[59, 60\], we are interested to formulate its covariant Hamilton-Jacobi counterpart as a potential testing ground of the ability of this quantum formulation of teleparallel gravity to reproduce the correct classical limit in the quasiclassical approximation in a similar way to the derivation of the classical equations of general relativity from the quantum geometrodynamics by Ulrich Gerlach \[61\]. Here we will limit our consideration to the case of the TEG and leave a consideration of more general teleparallel theories of gravity to a later occasion.

\[^{1}\text{We thank the author of these conference talks for sharing his slides prior to the publication of the Proceedings.}\]
2 Palatini formulation of TEGR

In spite of many different formulations of TEGR in metric and vielbein variables \cite{62,63} it is the Palatini formulation by Maluf \cite{64} which has been successfully applied for the covariant Hamiltonian analysis of TEGR in the sense of De Donder-Weyl formulation \cite{59,60}. In this formulation the vielbeins $e^a_\alpha$ and the variables $t_{abc} = -t_{acb}$ are the independent field variables. Using the Lagrangian density in the form

$$L = \frac{1}{16\pi G} \epsilon \Sigma(t)^{abc} (t_{abc} - 2T_{abc})$$  \hspace{1cm} (1)

where

$$T^c_{\alpha \beta} := \partial_\alpha e^c_\beta - \partial_\beta e^c_\alpha$$

$$T^c_{ab} := e^\alpha_a e^\beta_b T_{\alpha \beta}^c, \quad T_b := T_{ab}$$

$$\Sigma(t)_{abc} := \frac{1}{2}(\eta_{ac} t_b - \eta_{bc} t_a) + \frac{1}{4}(t_{abc} + t_{bac} - t_{cab})$$

and $\epsilon := \det(e^a_\alpha)$. The variation of $t_{abc}$ identifies the field $t_{abc}$ on classical solutions as the torsion field $T_{abc}$. The variation of vielbeins $e^a_\alpha$ reproduces the Einstein equations in vielbein variables \cite{64}.

3 De Donder-Weyl covariant Hamiltonian analysis of TEGR

3.1 De Donder-Weyl Hamiltonian formulation in flat space-time

A Lagrangian function $L(y, y_\alpha, x^\alpha)$ can be viewed as a function of space-time variables, $x^\alpha$, the field variable, $y$, and the first-jet coordinates, $y_\alpha$. On classical field configurations $y = y(x)$ and $y_\alpha = \partial_\alpha y(x)$, where $\partial_\alpha$ is a short-hand notation for $\frac{\partial}{\partial x^\alpha}$. The variation of the field configuration $y(x)$ yields the Euler-Lagrange field equations. The De Donder-Weyl Hamiltonian formulation of the latter is based on a new set of variables

$$p^\alpha_y := \frac{\partial L}{\partial y_\alpha}$$ \hspace{1cm} (3)

called polynomials corresponding to the field variable $y$ and

$$H := p^\alpha_y y_\alpha - L := H(y, p^\alpha_y, x^\alpha)$$ \hspace{1cm} (4)

called the De Donder-Weyl or DW Hamiltonian density. In terms of new variables the Euler-Lagrange field equations are cast in the first order Hamiltonian form

$$\partial_\alpha y = \frac{\partial H}{\partial p^\alpha_y}$$ \hspace{1cm} (5)

$$\partial_\alpha p^\alpha_y = -\frac{\partial H}{\partial y}$$ \hspace{1cm} (6)
This transformation is possible when the Lagrangian density is regular in the sense that the determinant of the matrix \( W^{\alpha\beta} = \frac{\partial^2 L}{\partial y^\alpha \partial y^\beta} \) is nonvanishing.

The Hamilton-Jacobi equation corresponding to the Hamiltonian equations (5), (6) can be obtained in the form of the first order partial differential equation for eikonal functions on the covariant field configuration space \( S^\alpha(y, x) \):

\[
\frac{\partial S^\alpha}{\partial x^\alpha} + H \left( y, \frac{\partial S^\alpha}{\partial y}, x^\alpha \right) = 0
\]

which describes the solutions of the Euler-Lagrange equation \( y = y(x) \) as geodesic fields in the space of field variable \( y \) and space-time variables \( x^\alpha \) which are given by the equations called the embedding conditions

\[
p^\alpha_y(y(x), y_\alpha(x)) = \frac{\partial S^\alpha}{\partial y}
\]

Here the left hand side is the explicit expression of polymomenta \( p^\alpha \) in terms of Lagrangian variables \((y, y_\alpha)\) taken along a classical configuration \( y(x) \).

### 3.2 The constraints

Let us apply the procedure of the covariant DW Hamiltonian theory to the Lagrangian density \( \Pi \). From the expressions of the polymomenta corresponding to the independent field variables \( e \) and \( t \) we immediately see that they are constrained. Namely,

\[
p^\mu_{abc} \approx 0, \\
p^\mu_e + \frac{1}{4\pi G} \epsilon \Sigma(t)_a \epsilon_{bc} e^\mu_c e^\beta_b \approx 0
\]

The first set of constraints will be denoted \( C^\mu_{tabc} \) and the second one \( C^\mu_e \), the symbol \( \approx \) stands for ”weakly equals to” according to the tradition set by P. Dirac in his theory of constraints [66]. The appearance of constraints is problematic for the straightforward application of DW Hamiltonian formalism. However, Kanatchikov [67] has generalised the Dirac’s constraints algorithm to the covariant DW Hamiltonian theory which was applied to the Palatini formulation of vielbein gravity [47–51] and to other modified theories of gravity [68–70].

In the follow-up we will often use a condensed notation that the indices \( e \) and \( t \) mean the whole set of indices carried by these fields and their repetition means summation over all those indices according to the Einstein summation rule. In this notation the primary constraints read

\[
C^\alpha_t \approx 0, \ C^\alpha_e \approx 0
\]

and the primary DW Hamiltonian density is obtained in the form

\[
\mathcal{H} := p^\mu_t \partial_\mu t + p^\mu_e \partial_\mu e - L \approx \frac{1}{4} p^\mu_{\alpha\beta} e^\beta_b e^\mu_c t^\alpha_{bc}
\]

Kanatchikov [67] has developed an algorithm of the constraints analysis in the covariant DW Hamiltonian formalism. Its central idea is to use the forms of constraints and their brackets. The forms of constraints are given by

\[
C_t := C^\alpha_t v_\mu, \ C_e := C^\mu_e v_\mu
\]
where \( v_\mu := \partial_\mu \mathcal{J}(dx^1 \wedge ... \wedge dx^n) \) if the space-time is \( n \)-dimensional. The brackets of forms are defined by the polysymplectic form on the unconstrained phase space of polymomentum and field variables

\[
\Omega = de \wedge dp_e^\mu \wedge v_\mu + dt \wedge dp_t^\mu \wedge v_\mu
\]  

(14)

\( \Omega \) maps a form \( C \) of degree \( (n-1) \) to a vector field \( \chi_C \)

\[
\chi_C \mathcal{J} \Omega = dC
\]

(15)

and the bracket of two \( (n-1) \)-forms is defined as

\[
\{ C_1, C_2 \} := \chi_{C_1} \mathcal{J} dC_2.
\]

(16)

Note that this definition produces brackets for a limited class of forms called Hamiltonian forms. The Lie algebra structure defined by this bracket is embedded into a bigger bi-graded structure defined on forms of all degrees below \( (n-1) \) which is known as the Gerstenhaber algebra [34–39].

A straightforward calculation yields the following brackets of forms of constraints:

\[
\{ C_{e^a_\alpha}, C_{e^d_\beta} \} := C_{e^a_\alpha} e^d_\beta = \frac{1}{4\pi G} \partial_{e^a_\alpha} (e \Sigma(t)_{d}^{bc} \epsilon^{[\alpha}_c e^{\beta]}_d) v_\nu
\]

(17)

\[
\{ C_{e^a_\alpha}, C_{t_{abc}} \} := C_{e^a_\alpha} t_{abc} = -\frac{1}{4\pi G} e^{[\nu}_k e^{\beta]} \partial_{t_{abc}} e^{\alpha]_{k}} t_{abc} v_\nu
\]

(18)

\[
\{ C_t, C_{t'} \} := C_{tt'} = 0
\]

(19)

It shows that we have a second-class constraints according to Dirac’s classification [66]; a proper generalization of Poisson bracket is needed to account for such constraints.

### 3.3 Generalized Dirac brackets

A generalization of Dirac bracket in the context of constrained DW Hamiltonian systems with 2-nd class constraint has been proposed by Kanatchikov in [67]. It has been used in his precanonical quantization of Einstein gravity in vielbein variables [47–51]. For \( (n-1)- \) and \( 0- \) forms \( A \) and \( B \)

\[
\{ A, B \}^D := \{ A, B \} - \Sigma U, V \{ A, C_U \} \bullet C_{V}^{\sim 1} \wedge \{ C_V, B \}
\]

(20)

The indices \( U, V \) enumerate the primary constraints,

\[
A \bullet B := *^{-1} (*A \wedge *B)
\]

(21)

and the pseudoinverse matrix \( C^{\sim 1} = C_{\mu}^{\sim 1} dx^\mu \) is defined by the relation

\[
C^{\sim 1} \bullet C \wedge C^{\sim 1} = C^{\sim 1}
\]

(22)
where the distributive law for $\wedge$ and $\bullet$ products is that the wedge product $\wedge$ acts first. The result of the calculation of generalized Dirac brackets \cite{60} reads

\[
\{ pt, t' \}^D = 0 \quad (23)
\]
\[
\{ pt, pt' \}^D = 0 \quad (24)
\]
\[
\{ pt, e \}^D = 0 \quad (25)
\]
\[
\{ pt, pe \}^D = 0 \quad (26)
\]
\[
\{ pe, pe' \}^D = 0 \quad (27)
\]
\[
\{ ev_{\mu}, e' \}^D = 0 \quad (28)
\]
\[
\{ t, pe \}^D = \delta_{ee'} \quad (29)
\]
\[
\{ t, e' \}^D = C_{\alpha e'}^{e} \quad (30)
\]
\[
\{ t, p_e \}^D = C_{\alpha e'}^{e} \quad (32)
\]

From (23) we conclude that variables $t$ have vanishing Dirac brackets with their conjugate polymomenta and from (23) we conclude that the Dirac brackets between different components of $t$ are not vanishing. It means that the unconstrained polymomentum phase space of variables $(e, t, p_e^\alpha, p_t^\alpha)$ is reduced to the space of vielbeins $e$ and their polymomenta $p_e$ where the variables $t$ are functions on this reduced polymomentum phase space $t = t(e, p_e)$ such as their Dirac brackets with the variables of the reduced polymomentum phase space are given by (30) and (32).

### 3.4 Functions $t(e, p_e)$ and $H(e, p_e)$ on the reduced polymomentum phase space

The analysis of constraints in the previous section has shown that the primary polymomentum phase space of variables $(e, t, p_e^\alpha, p_t^\alpha)$ is effectively reduced to the space of vielbeins $e$ and their polymomenta $p_e$ which we have introduced in the Lagrangian Palatini formulation \cite{11} are becoming functions of the reduced polymomentum phase space. Then the DW Hamiltonian density also becomes a function on the reduced polymomentum phase space. Here we present the explicit expressions of those functions.

From (2) we obtain

\[
 t_{abc} + \eta_{ac} t_b - \eta_{ab} t_c = 2 \Sigma_{bac} - 2 \Sigma_{cab}, \quad t_c = \frac{2}{2 - n} \Sigma_c
\]

Therefore,

\[
 t_{abc} = 2 \Sigma_{bac} - 2 \Sigma_{cab} - \frac{2}{n - 2} (\eta_{ab} \Sigma_c - \eta_{ac} \Sigma_b)
\]

Using the constraints \cite{10},

\[
 e \Sigma_{a}^{bc} \approx -4\pi G e_{[a}^{c} e_{\beta]}^{\alpha} p_{\beta}^{\alpha}
\]
\[
 e \Sigma^{c} \approx -4\pi G e_{[a}^{c} e_{\beta]}^{\alpha} p_{\beta}^{\alpha}
\]
Then
\[ t_{abc} \approx 8\pi G \left( e_a[\alpha e_b[\beta] P^\alpha_{e^\beta_\delta} - e_a[\alpha e_c[\beta] P^\alpha_{e^\beta_\delta} + \frac{1}{n-2} (\eta_{ab} e^d_{[\alpha e_c[\beta] P^\alpha_{e^\beta_\delta} - \eta_{ac} e^d_{[\alpha e_b[\beta] P^\alpha_{e^\beta_\delta}])} \right) \]  

(33)

The DW Hamiltonian density (12) can be expressed now as a function of the variables of the reduced polymomentum phase space as follows
\[ tH \approx \frac{1}{4} P^\alpha_{e^\beta_\delta} e^b_{[\alpha} e^c_{\beta]} P^\alpha_{e^\beta_\delta} \approx 4\pi G t^{-1} P^\alpha_{e^\beta_\delta} e^b_{[\alpha} e^c_{\beta]} \left( e^\alpha_{[\alpha e_b[\beta] P^\alpha_{e^\beta_\delta} + \frac{1}{n-2} \eta^a_{[\alpha e_d[\beta] P^\alpha_{e^\beta_\delta} \right) \]  

(34)

4 Covariant DW Hamilton-Jacobi equation

In general space-times the covariant DW Hamilton-Jacobi equation is formulated in terms of the eikonal densities \( S^\mu \) and the DW Hamiltonian function in (8) is replaced by the DW Hamiltonian density \( tH \). The Dirac bracket (29) shows that the polysymplectic structure on the reduced polymomentum phase space is the standard one
\[ \Omega^\text{red} = de \wedge dp^\alpha_e \wedge u^\alpha \]  

(35)

or, respectively, the related \( k \)-symplectic structure used in [15] is given by the family of forms \( de \wedge dp^\alpha_e \).

This observation allows us to use the geometric theory of HJ equations developed in [14–16] to write down the covariant DW Hamilton-Jacobi equation in the form
\[ \partial_\mu S^\mu + tH \left( e, p^\alpha_e = \frac{\partial S^\alpha}{\partial e}, x \right) = 0 \]  

(36)

in which the expression (34) of \( H \) on the reduced polymomentum phase space is used. Then the theorems proven in [14–16] guarantee that the embedding condition
\[ p^\alpha_e (e(x), \partial_\alpha e(x), x^\alpha) := \frac{\partial S^\alpha}{\partial e} \]  

(37)

describes the classical solutions of the teleparallel equivalent of the Einstein equations which are derived from the variational principle based on (1). Due to the constraint (10) and the fact that on classical solutions \( t_{abc} = T_{abc}(x) \) the embedding condition has the explicit form
\[ \frac{1}{2\pi G} \varepsilon^a_{(T) \alpha} \varepsilon^b_{e^\alpha_e} = \frac{\partial S^\alpha}{\partial e^a_{\beta}} - \frac{\partial S^\beta}{\partial e^a_{\alpha}} \]  

(38)

which is valid along the solutions \( e = e(x) \).

5 Conclusion

Using the Kanatchikov’s algorithm for the treatment of constraints within the covariant De Donder-Weyl Hamiltonian formulation we have constructed the DW Hamiltonian density of the Palatini formulation of the teleparallel equivalent of General Relativity on
the reduced polymomentum phase space and formulated the covariant DW Hamilton-Jacobi equation which is defined on the configuration space of vielbein variables and space-time variable. The problems to be considered in future work is a generalization of our consideration to non-Einsteinian teleparallel gravity theories and the study of the relation of our covariant Hamilton-Jacobi formulation with the standard Hamilton-Jacobi formulation based on 3+1 decomposition which is not yet formulated within the canonical Hamiltonian formalism for teleparallel gravities. It is also interesting to investigate if the DW Hamilton-Jacobi equation we have derived can be obtained in the quasiclassical limit of the precanonical quantization of TEGR described in [59,60].

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