Quantized state feedback-based $\mathcal{H}_\infty$ control for nonlinear parabolic PDE systems via finite-time interval

Teng-Fei Li · Xiao-Heng Chang · Ju H. Park

Abstract In this paper, the finite-time $\mathcal{H}_\infty$ control problem of nonlinear parabolic partial differential equation (PDE) systems with parametric uncertainties is studied. Firstly, based on the definition of the quantiser, static state feedback controller (SC) and dynamic state feedback controller (DC) with quantization are presented, respectively. The finite-time $\mathcal{H}_\infty$ control design strategies are subsequently proposed to analyze the nonlinear parabolic PDE systems with respect to the effect of quantization. And by constructing appropriate Lyapunov functionals for the studied systems, sufficient conditions for the existence of the feedback control gains and the quantizer’s adjusting parameters which guarantee the prescribed attenuation level of $\mathcal{H}_\infty$ performance are expressed as nonlinear matrix inequalities. Then, by using some inequalities and decomposition technic, the nonlinear matrix inequalities are transformed to standard linear matrix inequalities (LMIs). Furthermore, the optimal $\mathcal{H}_\infty$ control performances are pursued by solving optimization problems subject to the LMIs. Finally, to illustrate the feasibility and effectiveness of the finite-time $\mathcal{H}_\infty$ control design strategies, an application to the catalytic rod in a reactor is explored and a numerical example is provided.

Keywords Parabolic PDE · Quantization · Finite-time · LMIs · $\mathcal{H}_\infty$ control

1 Introduction

Many processes in engineering fields and multidimensional dynamical systems depend not only on time variable but also on spatial variable inherently, which means that the system performances are decided by both spatial position and time variable, such as fluid heat exchangers, thermal diffusion processes, and dissipative dynamical systems, et al. [1,2]. And these distributed parameter systems or processes can be frequently described by partial differential equations (PDEs). During the past several decades, an immense amount of interesting findings on the PDE systems have been made by scholars, for instance, the control synthesis and stability analysis of parabolic PDE systems [3–6], as well as the stability analysis of reaction-diffusion neural networks [7–10]. It’s worth noting that, by using the semigroup theory to map the spatial derivative terms of the PDEs in Euclidean space to a linear operator in Hilbert space, the PDE models in Euclidean space can be transformed into an abstract ordinary differential form in Hilbert space [6–10]. This provides a powerful
and effective tool for the investigation of PDEs. These research achievements are meaningful for the applications and further studies.

As the constructions of industrial distributed parameter systems governed by PDEs are more and more complicated, higher demands on safety and reliability are usually confronted. However, unexpected deviations or uncertainties generated by external and internal disturbances always occur inherently. These uncertainties may lead to system instability and performance degradation if they are not appropriately taken account in the controller design. Thus, the issue of control design for PDE systems with uncertain parameters is of practical and theoretical value. Fortunately, valuable researches of controller design and stability analysis for parabolic PDE systems with parametric uncertainties have been made by many scholars. For example, with the consideration of uncertain variables presented in the system structure, robust output-based controller design methods for quasi-linear parabolic PDE systems and parabolic PDE systems with time-dependent spatial domains can be seen in [11] and [12], respectively. In order to deal with both the parameter variations and boundary uncertainties, Cheng et al. [13] presented a boundary control law for a class of parabolic PDE system by using the Volterra integral transformation technique. On the basis of the Galerkin method, the nonlinear parabolic PDE system was described by finite dimensional ordinary differential equation (ODE) model, then predictive control strategy was provided [14] by employing a multilayer neural network to parameterize the unknown nonlinearities. And two formulations were developed [15] to investigate the sparse solutions for the optimal control problems subject to PDEs with uncertain coefficients. More recently, an adaptive feedback control design strategy was proposed [16] to stabilize a class of coefficient matrix uncertain ODE system which is casted with uncertain constant and coefficient matrix parabolic PDE. More researches of uncertain systems can be referred to [17–22] and references therein. However, these methods are mainly by Lyapunov’s direct method except [17], thus the LMI approach which acts as a powerful tool to deal with the control design problem of systems is worth further study.

On the other hand, associated with the control issues, the properties of these complicated systems are usually required to be considered in a finite-time interval. Thus, the finite-time bounded control is also worth investigating. Fruitful achievements of finite-time bounded control have been made [23–28] for the network system. Precisely, the bounded control strategies of semilinear PDE systems in the finite-time interval were also investigated by Song et al. [29,30]. Control problem in $H_{\infty}$ sense is also usually put forward under the uncertainties and disturbances [31–34]. These achievements also motivate the present study.

Recently, networked control systems have attracted more and more attentions of scholars due to the development of computer technology and control theory [35]. The application of a multipurpose networked control system usually brings flexible architectures and reduces the budget and installation. Thus, networked control systems have been widely applied in many practical systems [36–41]. To assimilate the advantages of networked control technology, some interesting results have been proposed for the stability analysis and control synthesis of parabolic PDE systems. Such as, Demetriou [42] considered a mobile network colocated sensors and actuators for a class of diffusion PDE systems. By utilizing the distributed in space measurements, a network-based $H_{\infty}$ filter for parabolic PDEs systems with the objective of enlarging the sampling time intervals to reduce the amount of communications while preserving a prescribed error system performance was designed in [43]. Other more recent researches can be referred in [44,45]. To benefit the development of computer technology, the research on networked control PDE systems is of practical significance.

However, due to the limited capability of the communication channel, the signal information (e.g., system state, measurement output, and control signal) is usually required to be quantized before it is transmitted into the network. Signal quantization has been widely adopted and studied in recent studies [46–51]. Nevertheless, accompanied with the signal quantization, quantization errors, which may yield system instability and performance degradation, and also bring difficulties for the system analysis and control, inevitably occur. Thus, the control synthesis for nonlinear systems with the effect of quantization is significant and necessary. Generally, the quantizer applied in the existing studies can be divided into two categories: static quantizer and dynamic one. For static signal quantization, there are some interesting results that have been reported. Such as, event triggered $H_{\infty}$ control problem of parabolic systems with Markovian jumping sensor
faults was studied under the effect of time delays in [52]. By employing a static logarithmic quantizer for the transmitted measurement output, distributed event-triggered networked control strategy for parabolic systems subjected to diffusion PDEs was proposed in [53]. Considering that the control signal was quantized by a static quantizer, state observer-based finite-time fuzzy bounded state feedback control strategy for semilinear PDE systems with Markov jump actuator failures [29], and \( \mathcal{H}_\infty \) asynchronous output feedback control strategy for nonlinear Markov jump distributed parameter systems [30], were provided, respectively. Nevertheless, dynamic signal quantization which is more general and more advantageous for parabolic PDE systems is still few reported in the existing studies.

The above researches are of abyssal significance in the field of control synthesis and attract more and more scholars to further studies. Although there are already various achievements for the controller design of parabolic PDE systems in the existing studies, there are still few researches on the quantized state feedback bounded control for parameter uncertain nonlinear parabolic PDE systems via finite-time interval, which is worthy of further study and application. Moreover, taking an overview of the existing results, it is realized that these results of the \( \mathcal{H}_\infty \) controller design for parabolic PDE systems are mostly relied on static feedback control, the finite-time \( \mathcal{H}_\infty \) control of parameter uncertain nonlinear parabolic PDE systems under DC has not been studied yet, which motivates the present study. Thus, the objective of this paper is to study the finite-time \( \mathcal{H}_\infty \) control problem for the parameter uncertain nonlinear parabolic PDE systems via quantized state feedback. Meanwhile, to ensure the prescribed \( \mathcal{H}_\infty \) performance index of the systems, sufficient conditions of the designed controller and quantizer’s adjusting parameters are developed in forms of matrix inequalities. The contributions and novelties can be summarized as follows:

1. Quantized static/dynamic state feedback control strategies for parameter uncertain nonlinear parabolic PDE systems via finite-time interval are studied. Sufficient conditions to obtain the prescribed \( \mathcal{H}_\infty \) performance are provided in terms of nonlinear matrix inequalities.

2. With the consideration of the system state is transmitted via the digital communication channel, dynamic quantizer which is regarded to be more general and more advantageous compared with a static one is adopted in this paper. Moreover, the quantization errors generated by the quantizer are well treated.

3. By using some inequalities and decomposition technic, the nonlinear matrix inequalities are transformed to standard LMIs. Furthermore, the optimal \( \mathcal{H}_\infty \) performances are pursued by solving optimization problems.

The rest sections of this paper are organized as follows: Sect. 2 addresses the studied parabolic PDE systems and some lemmas which are necessary for the results presented in this paper; Sect. 3 gives the static and dynamic state feedback controllers with quantization, and provides the sufficient conditions for the finite-time \( \mathcal{H}_\infty \) controller design; application to the catalytic rod in a reactor is explored and a numerical example is provided to illustrate the feasibility and efficiency of the designed methods in Sect. 4; Sect. 5 draws some conclusions.

Some notations are used in this paper, which are shown in Table 1.

### 2 Problem formulation and preliminaries

With the system notations expressed in Table 2, the mathematical model of nonlinear parabolic PDE systems with parametric uncertainties can be expressed as [6]:

\[
\begin{align*}
\frac{\partial x(s, t)}{\partial t} &= (A + \Delta A) \frac{\partial^2 x(s, t)}{\partial s^2} + f(x(s, t)) \\
&+ (B_u + \Delta B_u) u(t) + (B_\omega + \Delta B_\omega) \omega(t), \\
y(t) &= \int_\Omega C x(s, t) ds,
\end{align*}
\]

The following initial condition and boundary conditions are considered for the system (1):

\[
\begin{align*}
x(s, 0) &= x_0(s), \\
x(0, t) &= x(l, t) = 0.
\end{align*}
\]

Assumption 1 For all \( x \in \mathcal{R}^n \neq 0 \). Then an operator \( f : \mathcal{R}^n \rightarrow \mathcal{R}^n \) is said to be Lipschitz constraint if there is a scalar \( \kappa > 0 \) satisfying:

\[
\|f(x(s, t))\| \leq \kappa \|x(s, t)\|.
\]
Definition 1 (Finite-time bounded) Given a positive definite matrix $R$ and three positive constants $c_1$, $c_2$, $T$, with $c_1 < c_2$, if there exists a controller $u(t)$ for system (1), $\forall t \in [0, T]$ such that:

$$\int_0^T x^T(s, 0) R x(s, 0) ds < c_1 \Rightarrow \int_0^T x^T(s, 0) R x(s, 0) ds < c_2,$$  \hspace{1cm} (4)

then the closed-loop system (1) is said to be finite-time bounded with respect to $(c_1, c_2, T)$.

Lemma 1 [54] If there is a well-defined matrix $0 < \exists \in \mathbb{R}^{n \times n}$ and a vector function $z(\cdot)$ which is differentiable with $z(0) = 0$ or $z(l) = 0$, then:

$$\int_0^l \dot{z}^T(s) \exists \dot{z}(s) ds \geq \frac{\pi^2}{4l^2} \int_0^l z^T(s) \exists z(s) ds.$$  \hspace{1cm} (5)

Remark 1 The constant $\kappa$ in Assumption 1 is called the Lipschitz constant of $f$. And this assumption is widely used in the studies (see [6] and the references therein).
Lemma 2 [55] For real scalar \( t \in \mathbb{R}^+ \), vector function \( z : [0, t] \to \mathbb{R}^n \), if a well-defined matrix \( F \) satisfy \( 0 < F \in \mathbb{R}^{n \times n} \), then it can be derived that:

\[
\frac{1}{t} \left[ \int_0^t z(s) ds \right]^T F \left[ \int_0^t z(s) ds \right] \leq \int_0^t z^T(s) F z(s) ds.
\]

(6)

Lemma 3 [56] If \( N + [ST + \ast] < 0 \) holds with appropriate dimensions matrices \( N, S \) and \( T \), then for a given scalar \( \epsilon \), there exists a matrix \( U \) such that:

\[
\begin{bmatrix}
N & \ast \\
\epsilon ST + UT - \epsilon [U + \ast] & < 0.
\end{bmatrix}
\]

Lemma 4 [56] For symmetric matrix \( M \), real matrices \( X, Y \) with proper dimensions, and \( \Delta \) satisfies \( \Delta \Delta^T \leq I \), then:

\[
M + [X \Delta Y + \ast] < 0
\]

holds if and only if there is a positive scalar \( \phi \) such that:

\[
M + \frac{1}{\phi} XX^T + \phi YY^T Y < 0.
\]

(9)

The definition of quantizer \( q(t) \) is employed in [57]. If there is real numbers \( H > 0 \) and \( \Delta_q > 0 \) that satisfying:

\[
\|q(t) - t\| \leq \Delta_q, \text{if } \|t\| \leq H,
\]

\[
\|q(t)\| > H - \Delta_q, \text{if } \|t\| > H,
\]

where the range and error bound of the quantizer are represented by \( H \) and \( \Delta_q \), respectively. And a kind of dynamic quantizer is considered as:

\[
q_{\mu}(t) = \mu q \left( \frac{t}{\mu} \right),
\]

with the dynamic parameter \( \mu \).

3 Main results

In the section, static/dynamic state feedback control with quantization for the parabolic PDE system (1) will be presented. Figure 1 shows the diagram of state feedback control strategy. Under the influence of the disturbance \( \omega(t) \), the system state \( x(s, t) \) is transmitted to the Quantizer through the Sensors; after quantization, the system state is transmitted to the controller side through the Network; then the Controller products an input control signal to the Actuators side; finally, with the actuators, the prescribed attenuation level of \( \mathcal{H}_\infty \) performance of the Uncertain System can be guaranteed. The output is the integration of states over space.

Disturbance \( \omega(t) \) \rightarrow \text{Uncertain System} \rightarrow \text{Sensors} \rightarrow \text{Quantizer} \rightarrow \text{Controller} \rightarrow \text{Network} \rightarrow \text{Actuators} \rightarrow \text{System} \rightarrow \text{Output} \rightarrow \text{State} \rightarrow \text{State} \rightarrow \text{Output}

Fig. 1 Block diagram of state feedback control strategy

3.1 SC with quantization

The SC with quantization is considered as:

\[
u(t) = \int_\Omega K q_{\mu}(x(s, t)) ds,
\]

(12)

with the undetermined gain matrix \( K \). By the definition of quantizer described in (10) and (11), the above controller can be written as:

\[u(s, t) = \int_\Omega K \mu(s, t) q \left( \frac{x(s, t)}{\mu(s, t)} \right) ds = \int_\Omega K [\vartheta(s, t) + x(s, t)] ds,
\]

(13)

where \( \vartheta(s, t) = \mu(s, t) \left[ q \left( \frac{x(s, t)}{\mu(s, t)} \right) - \frac{x(s, t)}{\mu(s, t)} \right].\)

By substituting the controller into (1) in the presence of parametric uncertainties \( \Delta A, \Delta B_u, \Delta B_w \), the quantized state feedback control system is derived:

\[
\begin{aligned}
\frac{\partial x(s, t)}{\partial t} &= (A + \Delta A) \frac{\partial^2 x(s, t)}{\partial s^2} + f(x(s, t)) \\
&\quad + (B_u + \Delta B_u) K \int_\Omega [\vartheta(s, t) + x(s, t)] ds \\
&\quad + (B_w + \Delta B_w) \omega(t), \\
y(t) &= \int_\Omega C x(s, t) ds.
\end{aligned}
\]

(14)

The following theorem provides a sufficient design condition for controller (12).

Theorem 1 For the state feedback system (14), if there exist matrices \( P > 0 \), \( K \) with proper dimensions, scalars \( r_1 > 0, r_2 > 0 \), such that the following conditions are fulfilled under the prescribed scalar \( \gamma > 0 \), given quantizer’s range \( H \) and error \( \Delta_q \), given positive matrix \( R \) and positive constants \( \delta, c_1, c_2, T \):

\[
[P(A + \Delta A) + \ast] > 0.
\]

(15)
\[
\Pi_1 = \begin{bmatrix}
\theta_1 + \ast & c^T c \\
* & \ast & P & B_1 & 0 \\
* & * & -r_2 I & 0 & 0 \\
* & * & * & -r_1 I & 0 \\
* & * & * & * & -r_1 I
\end{bmatrix}
\] < 0,
\]
\[
c_2 > \frac{\lambda_2 c_1 + \hat{\omega} \gamma^2}{\lambda_1} e^{\lambda T}.
\]

where \( \theta_1 = \frac{1}{2} P(B_u + \Delta B_u)K, \) \( B_1 = P(B_{ow} + \Delta B_{ow}). \)
\( \lambda_1 = -\frac{\gamma^2}{4l^2} [P(A + \Delta A) + \ast] + (2r_2 + \frac{4r_1^2 \Delta \rho_2}{H^2}) I - \delta P, \) \( \lambda_2 = \lambda_{\min}(R^{-1} P R^{-1}), \)
and the dynamic parameter \( \mu(s, t) \) is adjusted on-line as:
\[
\frac{\rho_u}{H} \| x(s, t) \| \leq \mu(s, t) \leq \frac{2\rho_u}{H} \| x(s, t) \|,
\]
where \( \rho_u \geq 1. \) Then the quantized SC (12) subject to the adjusting rule (18), guarantees the prescribed \( \mathcal{H}_\infty \) performance \( \gamma \) for system (14) in the sense of finite-time bounded with respect to \( (c_1, c_2, T). \)

**Proof** For the state feedback system (14), select the Lyapunov function as:
\[
V(t) = \int_\Omega x^T(s, t) P x(s, t) ds,
\]
where \( P \) is a positive matrix to be determined. By the time derivative of \( V(t), \)
\[
\dot{V}(t) = \left[ \int_\Omega x^T(s, t) P (A + \Delta A) d\frac{\partial x(s, t)}{\partial s} \right] + *
+ \left[ \int_\Omega x^T(s, t) P f(x(s, t)) ds + * \right]
+ \left[ \int_\Omega x^T(s, t) ds P(B_u + \Delta B_u)K \right]
\int_\Omega \vartheta(s, t) ds + *
+ \left[ \int_\Omega x^T(s, t) ds P(B_{ow} + \Delta B_{ow}) \right]
\int_\Omega x(s, t) ds + *
+ \left[ \int_\Omega x^T(s, t) P(B_{ow} + \Delta B_{ow}) \omega(t) ds + * \right].
\]

Integrating by part with the consideration of the boundary conditions (2) gives:
\[
\int_\Omega x^T(s, t) P (A + \Delta A) d\frac{\partial x(s, t)}{\partial s} = x^T(s, t) P (A + \Delta A) \frac{\partial x(s, t)}{\partial s} |_{s=0}^{s=T}
- \int_\Omega \frac{\partial x^T(s, t)}{\partial s} P(A + \Delta A) \frac{\partial x(s, t)}{\partial s} ds
= - \int_\Omega \frac{\partial x^T(s, t)}{\partial s} P(A + \Delta A) \frac{\partial x(s, t)}{\partial s} ds.
\]

Then, by (16) that \( [P(A + \Delta A) + \ast] > 0, \) with Lemma 1, substituting (21) into (20), one can obtain:
\[
\dot{V}(t) \leq -\frac{\pi^2}{4l^2} \int_\Omega x^T(s, t) [P(A + \Delta A) + \ast] x(s, t) ds
+ \left[ \int_\Omega x^T(s, t) P f(x(s, t)) ds + * \right]
+ \left[ \int_\Omega x^T(s, t) P(B_u + \Delta B_u)K ds \right]
\int_\Omega \vartheta(s, t) ds + *
+ \left[ \int_\Omega x^T(s, t) P(B_{ow} + \Delta B_{ow})K ds \right]
\int_\Omega x(s, t) ds + *
+ \left[ \int_\Omega x^T(s, t) P(B_{ow} + \Delta B_{ow}) \omega(t) ds + * \right].
\]

By the on-line adjusting rule (18), we have:
\[
\| x(s, t) \| \leq \mu(s, t) \leq \frac{2\rho_u}{H} \| x(s, t) \|,
\]
according to (10),
\[
q \left( \frac{x(s, t)}{\mu(s, t)} \right) - \frac{x(s, t)}{\mu(s, t)} \| \leq \Delta_q.
\]

Consider the homogeneity property of Euclidean norm to \( \vartheta(s, t), \) we arrive at:
\[
\| \vartheta(s, t) \| \leq \frac{2\Delta_q}{H} \| x(s, t) \|.
\]

Let the Assumption 1 satisfied, then the following inequalities stand:
\[
\vartheta^T(s, t) \vartheta(s, t) \leq \frac{4\Delta_q^2 \rho^2 u}{H^2} x^T(s, t) x(s, t).
\]

By Lemma 2, it’s obvious that,
\[
\int_\Omega \vartheta^T(s, t) \vartheta(s, t) ds \int_\Omega \vartheta(s, t) ds
\leq l \int_\Omega \vartheta^T(s, t) \vartheta(s, t) ds.
\]
By adding and subtracting:
\[
\begin{align*}
& r_1 \int_{\Omega} \vartheta^T(s,t)ds \int_{\Omega} \vartheta(s,t)ds \\
& + r_2 \int_{\Omega} f^T(x(s,t)) f(x(s,t))ds \\
& \text{to the right side of (22),}
\end{align*}
\]
\[
\dot{V}(t) \leq \int_{\Omega} \left\{ x^T(s,t) A_1 x(s,t) ds \\
+ \int_{\Omega} x^T(s,t) \left[ \frac{P(B_u + \Delta B_u)K + *} l \right] ds \\
+ [ \int_{\Omega} x^T(s,t) Pf(x(s,t)) + * ] ds \\
+ \left[ \int_{\Omega} x^T(s,t) P(B_u + \Delta B_u)K l ds \right] \\
+ \int_{\Omega} \vartheta^T(s,t)ds + * \\
+ \left[ \int_{\Omega} x^T(s,t) P(B_u + \Delta B_u) K \right] \omega(t)ds + * \\
- r_1 \int_{\Omega} \vartheta^T(s,t)ds \int_{\Omega} \vartheta(s,t)ds - r_2 f^T(x(s,t)) f(x(s,t)) \right\} ds.
\]

Consider the system (14) with $\mathcal{H}_\infty$ performance index $\gamma$, and choose:
\[
J_T = \int_0^T \| y(t) \|^2 dt - \gamma^2 \int_0^T \| w(t) \|^2 dt. 
\] (29)

Combine (28) to (29) gives:
\[
\dot{V}(t) - \delta V(t) + \| y(t) \|^2 - \gamma^2 \| w(t) \|^2 \\
= \int_0^T \int_{\Omega} X^T(s,t) \Pi_1 X(s,t) ds dt,
\] (30)

where
\[
X(s,t) = [ \int_{\Omega} x^T(s,t) ds \ x^T(s,t) f^T(x(s,t)) \omega(t) ] \left[ \int_{\Omega} \vartheta^T(t)ds \right]^T.
\]

By (16), we know that $\Pi_1 < 0$, which means:
\[
e^{-\delta t} [ \dot{V}(t) - \delta V(t) ] < e^{-\delta t} [ \| y(t) \|^2 - \gamma^2 \| w(t) \|^2 ].
\] (31)

Then under the initial value condition, integrating (31) from 0 to $T$ arrives:
\[
0 < e^{-\delta T} V(T) - V(0) < \int_0^T e^{-\delta t} [ \| y(t) \|^2 - \gamma^2 \| w(t) \|^2 ] dt.
\] (32)

Inequality (32) further gives:
\[
\| y(t) \|^2 < \gamma^2 \| w(t) \|^2,
\] (33)

and
\[
V(T) < e^{\delta T} \left[ V(0) + \int_0^T e^{-\delta t} [ \| y(t) \|^2 ] dt \right] < e^{\delta T} \left[ \int_0^T x^T(s,0) P x(s,0) ds + \tilde{\omega} \gamma^2 \right] = e^{\delta T} \left[ \int_0^T x^T(s,0) R^{\frac{1}{2}} (R^{-\frac{1}{2}} P R^{-\frac{1}{2}}) R^{\frac{1}{2}} x(s,0) ds + \tilde{\omega} \gamma^2 \right] < e^{\delta T} \lambda_2 \int_0^T x^T(s,0) R x(s,0) ds + e^{\delta T} \tilde{\omega} \gamma^2 < e^{\delta T} \lambda_2 c_1 + e^{\delta T} \tilde{\omega} \gamma^2.
\] (34)

On the other hand,
\[
V(T) = \int_0^T x^T(s,0) P x(s,0) ds = \int_0^T x^T(s,0) R^{\frac{1}{2}} (R^{-\frac{1}{2}} P R^{-\frac{1}{2}}) R^{\frac{1}{2}} x(s,0) ds > \lambda_1 \int_0^T x^T(s,0) R x(s,0) ds.
\] (35)

Thus,
\[
\int_0^T x^T(s,0) R x(s,0) ds < \frac{\lambda_2 c_1 + \tilde{\omega} \gamma^2}{\lambda_1} e^{\delta T}, \forall t \in [0, T].
\] (36)

By (17), one can derive $\int_0^T x^T(s,0) R x(s,0) ds < c_2$.

Then, according to Definition 1, the prescribed $\mathcal{H}_\infty$ performance in the finite time interval $[0, T]$ for system (14) under the quantized SC (12) could be obtained. This ends the proof. □

Remark 2 To guarantee the prescribed $\mathcal{H}_\infty$ performance for system (1), the terms which reflect the effect of quantization errors and uncertainties as $\hat{\Theta}_1, \hat{\Delta}_1$, and $\Delta B_\omega$ exist in Theorem 1. The quantization errors for the network system also appeared in [57]. Here, in Theorem 1, both the quantization errors and system uncertainties that may yield the deterioration of system performances and even system instability are considered.

Due to the uncertain terms $\Delta A, \Delta B_u, \Delta B_\omega$ and coupled terms $P B_u K$, the conditions presented in Theorem 1 cannot be applied directly to design the quantized state feedback controller (12). In order to cope with this problem, the following method is provided to eliminate these terms, such that the finite-time $\mathcal{H}_\infty$ control design.
conditions are given in terms of standard LMIs which can be solved by MATLAB matrix toolbox.

Since \([\Delta A \Delta B_u \Delta B_w]\) = \(MS[\Omega_1 \Omega_2 \Omega_3 \Theta]\), then the matrix \(\Pi_{11}\) can be rewritten as:

\[
\Pi_{11} = \Pi_2 + \begin{bmatrix} \lambda_{11} \lambda_{12} \lambda_{21} \lambda_{22} \lambda_{31} \lambda_{32} \lambda_{33} \lambda_{34} \end{bmatrix},
\]

where

\[
\Pi_2 = \begin{bmatrix}
\Theta_2 + \frac{C^T C}{\lambda} & 0 & 0 & 0 & \Theta_2 \\
* & \lambda_2 & P & PB_w & 0 \\
* & * & -r_2 I & 0 & 0 \\
* & * & * & -r^2 I & 0 \\
* & * & * & * & -r_1 I \\
\end{bmatrix},
\]

\[
\lambda_{11} = \left[ -\frac{\pi^2}{4l^2} (PM)^T 0 0 0 \right]^T,
\]

\[
\lambda_{12} = \left[ 0 N_1 0 0 \right],
\]

\[
\lambda_{21} = \left[ \frac{1}{\lambda}(PM)^T 0 0 0 \right]^T,
\]

\[
\lambda_{22} = \left[ N_2 K 0 0 0 N_2 K \right],
\]

\[
\lambda_{31} = \left[ 0 (PM)^T 0 0 0 \right]^T,
\]

\[
\lambda_{32} = \left[ 0 0 0 N_3 0 \right],
\]

\[
\Theta_2 = \frac{1}{l} \begin{bmatrix} PB_u K + * \end{bmatrix},
\]

\[
\lambda_{33} = \left[ PB_u K + * \right],
\]

By Lemma 4 and Schur complement, for \(\epsilon_A, \epsilon_H, \epsilon_w > 0\), (37) is equal to:

\[
\Pi_3 = \begin{bmatrix} \Pi_2 \begin{bmatrix} \lambda_{11} \epsilon_A \lambda_{12} \lambda_{21} \epsilon_A \lambda_{22} \lambda_{31} \epsilon_w \lambda_{32} \lambda_{33} \epsilon_w \lambda_{34} \end{bmatrix} & -\text{diag} \left\{ \epsilon_A I, \epsilon_A I, \epsilon_H I, \epsilon_H I, \epsilon_w I, \epsilon_w I \right\} \end{bmatrix}
\]

(38)

Let \(K = U^{-1} V\), then the matrix (38) can be expressed as:

\[
\Pi_3 = \Pi_4 + \begin{bmatrix} \lambda_{41} U^{-1} V \lambda_{42} + * \end{bmatrix},
\]

where

\[
\Pi_4 = \left[ \begin{bmatrix} \lambda_{11} \epsilon_A \lambda_{12} \lambda_{21} \epsilon_A \lambda_{22} \lambda_{31} \epsilon_w \lambda_{32} \lambda_{33} \epsilon_w \lambda_{34} \end{bmatrix} & -\text{diag} \left\{ \epsilon_A I, \epsilon_A I, \epsilon_H I, \epsilon_H I, \epsilon_w I, \epsilon_w I \right\} \end{bmatrix},
\]

(39)

\[
\lambda_{41} = \begin{bmatrix} \frac{1}{l} (PB_u - B_u V)^T 0 0 0 0 0 \epsilon_u N_2^T 0 0 \end{bmatrix},
\]

\[
\lambda_{42} = \begin{bmatrix} I 0 0 0 I 0 0 0 0 \end{bmatrix}.
\]

Since \([P(A + \Delta A) + \epsilon_A] = [PA + \epsilon_A] + [PM N_1 + \epsilon_A]\), by Lemma 4, for a positive scalar \(\epsilon_1\), the following inequality stands:

\[
\begin{bmatrix} -[PA + \epsilon_A] + \epsilon_1 N_1^T N_1 & PM \\ * & -\epsilon_1 I \end{bmatrix} < 0.
\]

(40)

By using Lemma 3 and Schur complement to (39), if \(\Pi_3 < 0\), then the following result can be derived:

**Theorem 2** For the state feedback system (14), if there exist matrices \(P > 0\), \(U\) and \(V\) with proper dimensions, undetermined scalars \(r_1, r_2, \epsilon_A, \epsilon_H, \epsilon_w, \epsilon_1\), and \(\rho_{u1}\), such that the following conditions are fulfilled under the prescribed scalar \(\gamma > 0\), given quantizer’s range \(H\) and error \(\Delta_q\), a given scalar \(\epsilon_k\), given positive matrix \(R\) and positive constants \(\delta, c_1, c_2, T\):

\[
\begin{bmatrix} -[PA + \epsilon_A] + \epsilon_1 N_1^T N_1 & PM \\ * & -\epsilon_1 I \end{bmatrix} < 0.
\]

(41)

\[
\begin{bmatrix} \Omega_{11} \Omega_{12} \Omega_{13} \Omega_{14} \Omega_{15} \Omega_{16} \\ * \Omega_{22} 0 0 \Omega_{25} \Omega_{26} \\ * * \Omega_{33} 0 0 \Omega_{44} \Omega_{46} \\ * * * \Omega_{55} 0 \\ * * * * \Omega_{66} \end{bmatrix} < 0.
\]

(42)

\[
c_2 > \frac{\lambda_2 c_1 + \bar{\omega} \gamma^2}{\lambda_1} e^{\delta T},
\]

(43)

where

\[
\Omega_{11} = \text{diag} \left\{ \frac{1}{l} (PB_u + \epsilon_A) + \frac{C^T C}{\lambda}, r_2 \epsilon_1 \right\},
\]

\[
\Omega_{12} = \begin{bmatrix} 0 & 0 & \frac{1}{l} (PB_u V + \epsilon) & C^T C \end{bmatrix},
\]

\[
\Omega_{13} = \begin{bmatrix} 0 & 0 & \frac{\pi^2}{4l^2} (PA + \epsilon_A) - \delta P \end{bmatrix},
\]

\[
\Omega_{14} = \begin{bmatrix} \frac{1}{l} PM N_1 \\ 0 & 0 \end{bmatrix},
\]

\[
\Omega_{15} = \begin{bmatrix} 0 & 0 \end{bmatrix},
\]

\[
\Omega_{16} = \text{diag} \left\{ \frac{\epsilon_k}{l} (PB_u - B_u U), \frac{\rho_{u1} \Delta_q}{H} \right\},
\]

\[
\Omega_{22} = -\text{diag} \left\{ r_1, \gamma^2 \frac{I}{l}, r_1 I \right\},
\]

\[
\Omega_{25} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.
\]
\[\Omega_{26} = \begin{bmatrix} 0 & 0 & V \\ 0 & 0 & 0 \end{bmatrix}^T,\]
\[\Omega_{33} = -\text{diag}[\varepsilon_A I, \varepsilon_A I],\]
\[\Omega_{44} = -\text{diag}[\varepsilon_A I, \varepsilon_A I],\]
\[\Omega_{46} = \begin{bmatrix} 0 & 0 \\ \varepsilon_k \varepsilon_w & N_2^T \end{bmatrix} 0,\]
\[\Omega_{55} = -\text{diag}[\varepsilon_A I, \varepsilon_A I],\]
\[\Omega_{66} = -\text{diag}[\varepsilon_k \{U + \varepsilon\}, \frac{r_1}{4\Omega_I}], \rho_{u1} = \rho_u r_1, \lambda_1 = \lambda_{\text{min}}(R^{-\frac{1}{2}} \bar{P} R^{-\frac{1}{2}}), \lambda_2 = \lambda_{\text{max}}(R^{-\frac{1}{2}} \bar{P} R^{-\frac{1}{2}}), \]

and (11), the above controller can be written as:
\[u(t) = \int_\Omega \{C_d \phi(s, t) + D_d q_\mu(x(s, t))\} ds,\]

with the undetermined gain matrices \(A_d, B_d, C_d\) and \(D_d\). By the definition of quantizer described in (10) and (11), the above controller can be written as:
\[\frac{\partial \phi(s, t)}{\partial t} = A_d \phi(s, t) + B_d \phi(x(s, t)) + \vartheta(s, t),\]

\[u(t) = \int_\Omega \{C_d \phi(s, t) + D_d [x(s, t) + \vartheta(s, t)]\} ds.\]

By substituting the controller into (1), the quantized state feedback control system is derived:
\[\frac{\partial x(s, t)}{\partial t} = (A + \Delta A) \frac{\partial^2 x(s, t)}{\partial s^2} + f(x(s, t)) + (B_u + \Delta B_u) \int_\Omega C_d \phi(s, t) ds + (B_w + \Delta B_w) \omega(t)\]
\[+ (B_u + \Delta B_u) \int_\Omega D_d [x(s, t) + \vartheta(s, t)] ds,\]
\[y(t) = \int_\Omega C x(s, t) ds.\]

Assume that the initial state of the dynamic state feedback controller (44) is zero, i.e., \(\phi(s, 0) = 0\), then the following result provides the \(H_\infty\) control design for the closed-loop system (46) in the finite time interval \([0, T]\).

**Theorem 3** For the dynamic state feedback system (46), if there exist matrices \(P_1, P_2 > 0, A_d, B_d, C_d\) and \(D_d\) with proper dimensions, undetermined positive scalars \(r_1, r_2, r_3, r_4\), such that the following conditions are fulfilled under the prescribed scalar \(\gamma > 0\), given quantizer’s range \(H\) and error \(\Delta q\), a given positive matrix \(R\), and given positive constants \(\delta, c_1, c_2, T\): 
\[P_1 (A + \Delta A) + \varepsilon > 0,\]
\[\Psi_{11} = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ * & E_1 & P_1 & [P_2 B_d]^T \\ * & * & -r_1 I & 0 \\ * & * & * & -r_1 I \end{bmatrix} < 0,\]
\[c_2 > \frac{\gamma^2}{\lambda_1} \varepsilon_1 + \gamma \varepsilon_2^2 e^{\delta T}.\]

where \(C_1 = \frac{P_1 (B_u + \Delta B_u) D_d + \varepsilon}{P_1 (B_u + \Delta B_u) D_d}, E_1 = -\frac{\gamma^2}{4T} [P_1 (A + \Delta A) + \varepsilon] + (r_1 I + r_2) \frac{4 A^2_{\varepsilon} \varepsilon_1^2}{H^2} I + r_2 k^2 I - \delta P_1, F_1 = P_1 (B_w + \Delta B_w), W_1 = [P_2 A_d + \varepsilon] + r_3 I - \delta P_2, \lambda_1 = \lambda_{\text{min}}(R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}}), \]
\[\lambda_2 = \lambda_{\text{max}}(R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}}), \]

and the dynamic parameter \(\mu(s, t)\) is adjusted on-line as (18). Then under the DC (44), closed-loop system (46) is finite-time bounded with respect to \((c_1, c_2, T)\), and satisfies the \(H_\infty\) performance with index \(\gamma\).

**Proof** Select the following Lyapunov functional for the state feedback system (46):
\[\dot{V}(t) = \int_\Omega x^T(s, t) P_1 x(s, t) ds + \int_\Omega \phi^T(s, t) P_2 \phi(s, t) ds,\]
\[\dot{V}(t) \leq -\frac{\pi^2}{4T} \int_\Omega x^T(s, t) [P_1 (A + \Delta A) + \varepsilon] x(s, t) ds + \int_\Omega x^T(s, t) P_1 f(x(s, t)) ds + * + \int_\Omega x^T(s, t) P_1 (B_u + \Delta B_u) ds\]
\[
\begin{align*}
&\int_\Omega C_d \phi(s, t) ds + * \\
&+ \left[ \int_\Omega x^T(s, t) P_1(B_u + \Delta B_u) ds \right] \\
&+ \int_\Omega D_d \theta(s, t) ds + * \\
&+ \left[ \int_\Omega x^T(s, t) P_1(B_u + \Delta B_u) ds \right] \\
&+ \int_\Omega D_d \theta(s, t) ds + * \\
&+ \left[ \int_\Omega x^T(s, t) P_1(B_u + \Delta B_u) ds \right] \\
&+ \int_\Omega \phi^T(s, t) [P_2 A_d + *] \phi(s, t) ds \\
&+ \left[ \int_\Omega \phi^T(s, t) P_2 B_d \theta(s, t) ds \right] \\
&+ \int_\Omega \phi^T(s, t) P_2 B_d x(s, t) ds + * \\
&+ \left[ \int_\Omega x^T(s, t) P_1(B_o + \Delta B_o) \omega(t) ds + * \right] \\
&+ \int_\Omega \phi^T(s, t) [P_2 A_d + *] \phi(s, t) ds \\
&+ \left[ \int_\Omega \phi^T(s, t) P_2 B_d \theta(s, t) ds + * \right] \\
&+ \int_\Omega \phi^T(s, t) P_2 B_d x(s, t) ds + * \\
&- r_1 \int_\Omega \theta^T(s, t) \theta(s, t) ds \\
&- r_2 \int_\Omega f^T(x(s, t)) f(x(s, t)) ds \\
&- r_3 \int_\Omega \phi^T(s, t) ds \int_\Omega \phi(s, t) ds \\
&- r_4 \int_\Omega \theta^T(s, t) \theta(s, t) ds \bigg] ds. \quad (53)
\end{align*}
\]

Consider the system (46) with $\mathcal{H}_\infty$ performance index $\gamma$ as (29), Combine (28) to (29) gives:
\[
\begin{align*}
\dot{V}(t) - \delta V(t) + \|y(t)\|^2 - \gamma^2 \|w(t)\|^2 &= \int_0^T \int_0^t K(T) \Psi_1 \dot{X}(s, t) ds dt, \quad (54)
\end{align*}
\]
where $\dot{X}(s, t) = \int_\Omega x^T(s, t) ds \ x^T(s, t) f^T(x(s, t)) \ \phi^T(s, t) \int_\Omega \theta^T(s, t) ds \int_\Omega \phi^T(s, t) ds \ omega^2(t) \theta^T(s, t)|^T$.
By (48) that $\Psi_1 < 0$, which means:
\[
e^{-\delta t}[\dot{V}(t) - \delta V(t)] < e^{-\delta t} [\gamma^2 \|w(t)\|^2 - \|y(t)\|^2]. \quad (55)
\]

Then under the initial value condition, integrating (55) from 0 to $T$ arrives:
\[
0 < e^{-\delta T} V(T) - V(0) < \int_0^T e^{-\delta t} [\gamma^2 \|w(t)\|^2 - \|y(t)\|^2] dt. \quad (56)
\]
Since the initial state of the controller (44) is zero, the inequality (56) further means:
\[
\|y(t)\|^2 < \gamma^2 \|w(t)\|^2, \quad (57)
\]
and
\[
V(T) < e^{\delta T} \left[ V(0) + \int_0^T e^{-\delta t} [\gamma^2 \|w(t)\|^2] dt \right]
< e^{\delta T} \left[ \int_\Omega x^T(s, 0) P_1 x(s, 0) ds \\
+ \int_\Omega \phi^T(s, 0) P_2 \phi(s, 0) ds + \omega \gamma^2 \right]
< e^{\delta T} \left[ \int_\Omega x^T(s, 0) R^2 (R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}}) R^2 x(s, 0) ds \right]
\]
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\[ + \bar{\omega} \gamma^2 \]
\[ < e^{\delta T} \bar{\lambda}_2 \int_{\Omega} x^T(s, t) R x(s, t) ds + e^{\delta T} \bar{\omega} \gamma^2 \]
\[ < e^{\delta T} \bar{\lambda}_2 c_1 + e^{\delta T} \bar{\omega} \gamma^2. \]  

(58)

On the other hand,

\[ V(T) = \int_{\Omega} x^T(s, t) P_1 x(s, t) ds \]
\[ + \int_{\Omega} \phi^T(s, t) P_2 \phi(s, t) ds \]
\[ > \int_{\Omega} x^T(s, t) R x(s, t) ds \]
\[ > \bar{\lambda}_1 \int_{\Omega} x^T(s, t) R x(s, t) ds. \]

Thus,

\[ \int_{\Omega} x^T(s, t) R x(s, t) ds < \frac{\bar{\lambda}_2 c_1 + \bar{\omega} \gamma^2}{\bar{\lambda}_1} e^{\delta T}, \]

\[ \forall t \in [0, T]. \]  

By (49), one can derive \[ \int_{\Omega} x^T(s, t) R x(s, t) ds < c_2. \]

Then, according to Definition 1, the system (46) under the DC (44) with quantization is finite-time stable in the interval $[0, T]$, and the prescribed $\mathcal{H}_\infty$ performance could be satisfied. This ends the proof. \qed

Since $[\Delta A \ \Delta B_u \ \Delta B_w] = MSN_1 N_2 N_3$, similar to the proof of Theorem 2, the following theorem provides conditions in terms of standard LMIs.

**Theorem 4** If there exist positive matrices $P_1, P_2$, undetermined $P_{2A}, P_{2B}, A_d, B_d, U_1, U_2, V_1$ and $V_2$ with proper dimensions, undetermined positive scalars $\varepsilon_\Delta, \varepsilon_1, r_1, r_2, r_3, r_4, \rho_{ur}$, such that the following conditions are fulfilled under the prescribed scalar $\gamma > 0$, given quantizer’s range $H$ and error $\Delta_q$ a given positive matrix $R$ and positive constants $\varepsilon_k, \delta, c_1, c_2, T$:

\[
\begin{bmatrix}
- [P_1 A + \ast] + \varepsilon_1 N_1^T N_1 P_1 M \\
\ast
\end{bmatrix}
< 0,
\]

\[
\begin{bmatrix}
\tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \tilde{\Omega}_{15} & \tilde{\Omega}_{16} & \tilde{\Omega}_{17} \\
\ast & \tilde{\Omega}_{22} & 0 & \tilde{\Omega}_{24} & 0 & 0 & 0 \\
\ast & \ast & \tilde{\Omega}_{33} & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \tilde{\Omega}_{44} & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \tilde{\Omega}_{55} & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \tilde{\Omega}_{66} & \tilde{\Omega}_{67} \\
\ast & \ast & \ast & \ast & \ast & \ast & \tilde{\Omega}_{77}
\end{bmatrix}
< 0,
\]

\[ c_2 > \frac{\bar{\lambda}_2 c_1 + \bar{\omega} \gamma^2}{\bar{\lambda}_1} e^{\delta T}, \]

where

\[ \tilde{\Omega}_{11} = diag \left[ \frac{[B_u V_2 + \ast] + CT^C}{l} \right], \]

\[ -\frac{\pi^2}{4l^2} [P_1 A + \ast] + r_2 x^2 I - \delta P_1, \]

\[ \tilde{\Omega}_{12} = \begin{bmatrix} 0 & 0 \\ P_1 & P_{2R}^T \end{bmatrix}, \]

\[ \tilde{\Omega}_{13} = \begin{bmatrix} \frac{B_u V_2}{\delta} & 0 \\ 0 & 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{14} = \begin{bmatrix} 0 & 0 \\ P_1 B_w & 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{15} = diag \left[ \frac{P_1 M}{l} , P_1 M \right], \]

\[ \tilde{\Omega}_{16} = \begin{bmatrix} 0 & 0 \\ 0 & -\pi^2 N_1 \end{bmatrix}, \]

\[ \tilde{\Omega}_{17} = \begin{bmatrix} \frac{r_2}{l} [P_1 B_u - B_u U_2] + V_2^T \frac{[P_1 B_w - B_u U_1] \Delta_{\rho_w}}{H} 0 \\ 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{22} = -diag \left[ r_2 I, [P_{2A} + \ast] + r_3 x^2 I - \delta P_2 \right], \]

\[ \tilde{\Omega}_{24} = -diag \left[ 0, P_{2B} \right], \]

\[ \tilde{\Omega}_{23} = -diag \left[ r_1 I, r_3 I \right], \]

\[ \tilde{\Omega}_{37} = \begin{bmatrix} V_1^T & 0 & 0 \\ 0 & V_1 & 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{44} = -diag \left[ \frac{\gamma^2}{l} I, r_4 I \right], \]

\[ \tilde{\Omega}_{46} = \begin{bmatrix} 0 & N_1^T \\ 0 & 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{55} = -diag \left[ \varepsilon_1 I, \varepsilon_1 I \right], \]

\[ \tilde{\Omega}_{66} = -diag \left[ \varepsilon_1 I, \varepsilon_1 I \right], \]

\[ \tilde{\Omega}_{67} = \begin{bmatrix} \varepsilon_k \varepsilon_\Delta N_2 & \varepsilon_k \varepsilon_\Delta N_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]

\[ \tilde{\Omega}_{77} = -diag \left[ \varepsilon_k [U_2 + \ast], \varepsilon_k [U_1 + \ast], \frac{r_1^2 + \lambda_d}{4} I \right], \rho_{ur} = \rho_u (r_1^2 + r_3), \tilde{\lambda}_1 = \lambda_{\text{min}} (R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}}), \tilde{\lambda}_2 = \lambda_{\text{max}} (R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}}), \] the dynamic parameter $\mu(s, t)$ is adjusted on-line as (18), then the closed-loop system (46) is finite-time bounded, and the prescribed $\mathcal{H}_\infty$ performance of (29) with index $\gamma$ could be obtained under the DC (44). Moreover, the control gain matrices are given by $A_d = P_{2A}^{-1} P_{2A}, B_d = P_{2B}^{-1} P_{2B}, C_d = U_1^{-1} V_1, D_d = U_2^{-1} V_2$.

**Remark 3** The conditions in Theorem 2 are standard LMIs which can be solved by MATLAB LMI toolbox. Thus, the controllers can be frequently derived by solving (41)–(42) in Theorem 2 and (61)–(62) in Theorem 4. To obtain the best robust control effect, the following
optimization problem is employed:

$$\min \gamma^2,$$

subject to LMIs in Theorem 2 (or Theorem 4).

Remark 4 When the candidate parameters $\varepsilon_1, \varepsilon_A, \varepsilon_u, \varepsilon_\omega, \varepsilon_k$ and $r_2$ in theorems are associated with the spatial variable $s$, then the results also stand and the conditions could be relaxed.

Remark 5 If the Dirichlet boundary conditions (2) are replaced with the mixed boundary conditions $x(s, t)|_{s=0} = \frac{\partial x(s, t)}{\partial s}|_{s=l} = 0$, Theorems 1–4 can also be derived. Moreover, when the system structure $B_\omega$ and uncertain parameter $\Delta B_\omega$ are related to a spatial variable $s$, then the results also stand and the conditions could be relaxed.

### Table 3 Notations of the catalytic reaction system for \((66)\)

| Notations | Denotes |
|-----------|---------|
| $s$       | Spatial position variable, $s \in [0, l]$ |
| $t$       | Time variable, $t \in [0, T]$ with $T = 10$ |
| $l$       | $l = 2$, length of the catalytic rod, |
| $x(s, t)$ | Dimensionless temperature of the catalytic rod, $x(s, t) \in \mathcal{R}$ |
| $u(t)$    | Control input |
| $\beta_1$ | $= 6.5$, heat in the reaction |
| $\alpha$  | $= 2.0$, activation energy |
| $\beta_2$ | $= 1.3$, transfer coefficient of heat |
| $\omega(t)$ | $= 0.3e^{-0.2t} + e^{-0.2t} \cos(2\pi t) + e^{-0.2t} \sin(3\pi t)$, disturbance |
| $B_u$     | $= 22.7975$, the distribution coefficient of control actuators |
| $B_\omega$ | $= 16.3175$, constant coefficient respect to the disturbance |
| $C$       | $= -0.1469$, measurement output coefficient |
| $[\Delta A \ \Delta B_u \ \Delta B_\omega]$ | $= MS[N_1 \ N_2 \ N_3]$, parametric uncertainties, with $M = 0.5406$, $S = 0.5$, $N_1 = 0.0101$, $N_2 = -0.1569$, $N_3 = 0.2778$ |
| $H$       | $= 100$, range of quantizer |
| $\Delta$  | $= 0.1$, error of quantizer |
| $c_1, c_2$ | Positive constants, $c_1 = 0.4$, $c_2 = 300$ |
| $\delta$  | $= 0.001$, decay rate of system |
| $R$       | $= I$, given positive definite matrix used in Definition 1 |
| $\varepsilon_k$ | $= 2.5$, proper scalar satisfy Lemma 3 |
| $y(t)$    | Measured output |
variable, all the results proposed in this paper also stand with parameters $B_\omega(s)$ and $N_3(s)$.

**Remark 6** In the results presented in the theorems, the control gain matrices $K$, $C_d$ and $D_d$ are determined by the disturbance matrix $B_\omega$, matrices of modeling errors and uncertainties $\Delta A$, $\Delta B_\omega$, $\Delta B_\omega$. In return, the feedback controllers with the control gain matrices will restrain these disturbances, modeling errors, and various uncertainties.

**Remark 7** In this paper, the dynamic parameter $\mu(s, t)$ which is transmitted to the system are adjusting by the rule presented in [57].

**The adjusting rule:**

$$
\mu(s, t) = \begin{cases}
\text{floor}\left(\frac{2\rho_u}{H}|x(s, t)| \times 10^\ell\right) \times 10^{-\ell}, & 0 \leq \frac{\rho_u}{H}|x(s, t)| < \frac{1}{2}, \\
1, & \frac{1}{2} \leq \frac{\rho_u}{H}|x(s, t)| < 1, \\
\text{floor}\left(\frac{2\rho_u}{H}|x(s, t)|\right), & 1 < \frac{\rho_u}{H}|x(s, t)|,
\end{cases}
$$

(65)

where $\ell = \min\{\ell \in \mathbb{N}^+: \frac{2\rho_u}{H}|x(s, t)| \times 10^\ell > 1\}$ and the function $\text{floor}(\zeta)$ denotes the largest integer of $\zeta$, but less than $\zeta$.

### 4 Simulation studies

#### 4.1 Application to catalytic rod in a reactor

Consider a furnace which is filled with specie $A$ and a catalytic reaction of the form $A \rightarrow B$ takes place on a long thin rod shown in Fig. 2.

This rod acts as a cooling medium since the reaction in the furnace is exothermic. The process model of temperature on the rod which is also studied in [6] can be illustrated by:

$$
\begin{bmatrix}
\frac{\partial x(s, t)}{\partial t} \\
\frac{\partial^2 x(s, t)}{\partial s^2}
\end{bmatrix} = (1.0 + \Delta A) \begin{bmatrix}
\frac{\partial^2 x(s, t)}{\partial s^2} \\
\frac{\partial x(s, t)}{\partial s}
\end{bmatrix}
$$

$$
+ \beta_1(e^{-\frac{\rho_a}{H}|x(s, t)|} - e^{-\frac{\rho_a}{H}})
$$

$$
- \beta_2 x(s, t) + (B_a + \Delta B_a)u(t)
$$

$$
+ (B_\omega + \Delta B_\omega)\omega(t),
$$

$$
y(t) = \int_0^t Cx(s, t)ds,
$$

(66)

where the symbols are described in Table 3.

For simulation purposes, the initial temperature and the surface temperature of the catalytic rod are assumed to be relative level, which means the following conditions are met:

$$
x(s, t)|_{s=0} = x(s, t)|_{s=L} = 0,
$$

$$
x_0(s) = 0.2 \sin(\pi s).
$$

Then, the state of the open-loop system (66) can be obtained and shown in Fig. 3 with $\tilde{\omega} = 7.5$. By the simulation results, the Assumption 1 can be fulfilled with $\kappa = 0.55$. With the LMI toolbox in MATLAB 2014a on a laptop computer (Ideapad Y480, Intel(R) Core(TM) i5 2.5GHz; Memory:4.0 GB; OS:Windows 7, 64 bit), then by solving the optimization problem (64), the feasible results presented in Table 4 can be found within 10 s, which means the feasible solutions of the presented conditions can be easily found.

The state profile of closed-loop system under SC (12) and DC (44) are depicted in Figs. 4 and 5. The difference $e(s, t)$ between the state of the system under SC and DC is depicted in Fig. 6.

The simulations of SC and DC are described in Fig. 7 with blue dashed line and green dashed line respectively, and the red line denotes the difference between the SC and DC. The evolution of the norm $\|x(s, t)\|_2$ under the SC and DC are depicted in Fig. 8. The consideration of different level of uncertainties are presented in Table 5 and Fig. 9.

#### Table 4 Parameter derived in example 4.1

| Parameter | SC   | DC   |
|-----------|------|------|
| $\gamma$  | 0.6006 | 7.7378 |
| Control gains $K$ | $-8.5980$ | $-8.5980$ |
|             | $B_d = 7.3277 \times 10^{-11}$ | $C_d = -3.2142 \times 10^4$ |
|             | $D_d = -2.4922$ | $\rho_u$ | 1.0001 | 1.0982 |
| Variables $r_1$ | 0.0276 | 0.2371 |
| $r_2$ | 1.4692 $\times 10^{-4}$ | 0.0139 |
| $\varepsilon_A$ | 0.0027 | 8.2321 $\times 10^7$ |
| $\varepsilon_u$ | 0.0021 | 0.3647 |
| $\varepsilon_\omega$ | 0.0213 | 75.1742 |
| $r_1$ | 0.7922 | $\varepsilon_\Delta$ | 0.1245 |
Fig. 4 Close-loop state profile with SC

Fig. 5 Close-loop state profile with DC

Fig. 6 Difference between the state profile under SC and DC

4.2 A numerical simulation

Consider system (66) with

\[
A = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}, \quad B_u = \begin{bmatrix} 30.0334 & 1.5121 \\ 3.5533 & 30.3268 \end{bmatrix}, \\
B_{\omega} = \begin{bmatrix} 2.3872 & 3.0051 \\ 1.9722 & 1.2812 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 0.0826 & 0.1519 \\ -0.1974 & -0.1314 \end{bmatrix}, \quad \beta_1 = 10.0, \beta_2 = 1.0, \\
\alpha = 1.0, l = 2,
\]
and initial states: $M = \begin{bmatrix} 0.6039 & -0.3075 \\ 0.1769 & -0.1318 \end{bmatrix}$, $\gamma = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$.

The parametric uncertainties:

$[\Delta A \Delta B_u \Delta B_\omega] = MS[N_1 N_2 N_3]$, where $M = \begin{bmatrix} -0.6039 & -0.3075 \\ 0.1769 & -0.1318 \end{bmatrix}$, $S = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$.

$N_1 = \begin{bmatrix} 0.0101 & 0.0100 \\ 0.0101 & 0.0100 \end{bmatrix}$, $N_2 = \begin{bmatrix} -0.2383 & 0.4400 \\ 0.2296 & -0.6169 \end{bmatrix}$.

$N_3 = \begin{bmatrix} 0.2748 & 0.0923 \\ 0.6011 & 1.7298 \end{bmatrix}$, and the boundary conditions and initial states:

$x(s, t)|_{s=0} = x(s, t)|_{s=t} = 0$,

$x_0(s) = \begin{bmatrix} 0.2 \sin(\pi s) \\ 0.5 - 0.5 \cos(2\pi s) \end{bmatrix}$.

Then the state of open-loop system can be obtained and shown in Fig. 10 with the disturbance $\omega(t) = \begin{bmatrix} 0.2e^{-0.5t} + e^{-0.5t} \cos(2\pi t) + e^{-0.5t} \sin(3\pi t) \\ 0.1e^{-0.5t} + e^{-0.5t} \sin(2\pi t) + e^{-0.5t} \cos(3\pi t) \end{bmatrix}$ and $\bar{\omega} = 10$.

By the simulation results, the Assumption 1 can be fulfilled with $\kappa = 0.55$. Set the range $H = 100$ and error $\Delta = 0.1$ of the quantizer. And give the matrix and scalars as $R = I$, $\delta_k = 0.1$, $c_1 = 0.4$, $c_2 = 150$, $T = 10$, and $\delta = 0.001$, by solving the optimization problem (64) with the LMI toolbox in MATLAB 7.0 on a laptop computer, the feasible results presented in Table 6 can be found within 10 s.

The state profile of closed-loop system under SC (12) and DC (44) are depicted in Figs. 11 and 12. The difference $e(s, t)$ between the state of the system under SC and DC is depicted in Fig. 13. The simulations of SC and DC are described in Fig. 14. The evolution of the norm $\|x_1(s, t)\|_2$ and $\|x_2(s, t)\|_2$ under SC and DC with different level of uncertainties are depicted in Figs. 15 and 16.

Through simulation results presented above, it is obvious that the design methods for the nonlinear parabolic PDE systems with parametric uncertainties can guarantee the prescribed $\mathcal{H}_\infty$ performance in the finite-time interval. Moreover, under the same disturbance level, the optimal $\mathcal{H}_\infty$ control performance with SC (12) is smaller than the optimal $\mathcal{H}_\infty$ control performance with DC (44). According to the performance index $\gamma$ defined by (29), it’s obvious that a smaller value of $\gamma$ means a better control performance for the studied system.

**Remark 8** Compared with the existing solutions presented in [17], this paper provides quantized SC and DC strategies to deal with the control problem of nonlinear parabolic PDE systems. And by the simulation comparison to [17], with the consideration of the system state is transmitted via the digital communication channel, the prescribed attenuation level of $\mathcal{H}_\infty$ performance can be guaranteed within a finite-time interval.

**Comparative Explanations:** The developed quantized state feedback control design strategies in this paper provide efficient methods for the finite-time $\mathcal{H}_\infty$ control of the nonlinear parabolic PDE system with parametric uncertainties. Compared with the existing solutions in the studies, some contributions and advantages of the presented feedback control strategies could be identified in the following aspects:

![Fig. 9](image-url)

**Table 5** Different level of uncertainties with $\gamma = 0.5$

| Item | Uncertainties | $\gamma$ under SC | $\gamma$ under DC |
|------|---------------|-------------------|-------------------|
| 1    | $M = 0.1, N_1 = 0.01 \quad N_2 = -0.1, N_3 = 0.1$ | 0.5112 | 5.7188 |
| 2    | $M = -0.2, N_1 = 0.01 \quad N_2 = 0.2, N_3 = 0.2$ | 0.6365 | 12.1909 |
| 3    | $M = 0.3, N_1 = 0.03 \quad N_2 = -0.3, N_3 = 0.3$ | 0.7403 | 19.9751 |
| 4    | $M = -0.4, N_1 = 0.04 \quad N_2 = -0.4, N_3 = 0.4$ | 0.8434 | 43.4166 |
Table 6  Parameter derived in example 4.2

| Parameter | SC                  | DC       |
|-----------|---------------------|----------|
| $\gamma$  | 0.6081              | 4.0783   |
| Control gains | $K = \begin{bmatrix} -1.9280 & -0.8887 \\ -0.7982 & -1.4266 \end{bmatrix}$ | $A_d = \begin{bmatrix} -1.7046 & 0.0078 \\ 0.0078 & -1.7351 \end{bmatrix}$ |
|           |                     | $B_d = \begin{bmatrix} 0.2392 & -0.1003 \\ -0.1098 & 0.5966 \end{bmatrix}$ |
|           |                     | $C_d = \begin{bmatrix} -7.6521 & 1.4969 \\ 1.8059 & -9.9682 \end{bmatrix}$ |
|           |                     | $D_d = \begin{bmatrix} -1.5017 & -0.1777 \\ -0.0757 & -1.5162 \end{bmatrix}$ |
| $\rho_u$  | 1.0000              | 1.0588   |
| Variables | $r_1 = 0.1997$      | $r_1 = 7.4090 \times 10^3$ |
|           | $r_2 = 0.0046$      | $r_2 = 0.0450$ |
|           | $\varepsilon_A = 0.0137$ | $r_3 = 1.3206 \times 10^8$ |
|           | $\varepsilon = 0.0455$ | $r_4 = 1.4818 \times 10^4$ |
|           | $\varepsilon_\omega = 0.0121$ | $\varepsilon_1 = 75.9319$ |
|           | $\varepsilon_1 = 2.7065$ | $\varepsilon_\Delta = 0.5529$ |

Fig. 10  Open-loop state profile of system

Fig. 11  Close-loop state profile of system with SC
Quantized state feedback-based $H_\infty$ control

(1) Different from [11–14, 17–19], the system state information is considered to be transmitted via the digital communication channel which is used frequently on the sensor side. Then SC is studied for the parameter uncertain nonlinear parabolic PDE system via finite-time interval. And quantized DC is also investigated, which has not been studied yet in the existing studies. In addition, by the presented simulation results, it can be observed that the SC (12) has a better control effect and a lower optimal index of $H_\infty$ performance compared with the DC (44).

(2) In comparison with [29, 30, 52, 53], a kind of dynamic quantizer which is regarded to be more general and more advantageous than that of a static one for the control input signal is adopted in this paper.

(3) Compared with [13, 17], new finite-time $H_\infty$ control design conditions for nonlinear parabolic PDE system under the SC and DC are provided in terms of LMIs. And the $H_\infty$ performances are optimized by solving the optimization problems subject to the LMIs.

5 Conclusion

This paper has studied the finite-time $H_\infty$ control problem of parametric uncertain nonlinear parabolic PDE systems via SC and DC. Considering that the system state information is transmitted through a digital communication channel, a dynamic quantizer has
been adopted to deal with the limited capability of the communication channel. Moreover, the quantization errors generated by the quantizer also have been considered. By constructing appropriate Lyapunov functionals, finite-time bounded conditions of the existence of the designed controllers and adjusting parameters have been presented in terms of nonlinear matrix inequalities. Then, standard LMIs have been derived by using some inequalities and decomposition technic. In addition, the $H_\infty$ performances can be optimized by solving optimization problems subject to these standard LMIs. At last, an exploration to a catalytic rod in a reactor and a numerical example has been presented to verify the effectiveness of the proposed strategies. However, the limitations of the proposed methods in practical applications lie in two aspects: (1) How to find a proper scalar $\varepsilon_k$ to satisfy the conditions (41)–(43) and (61)–(63); (2) How to find the Lipschitz constraint $\kappa$ to satisfy Assumption 1. In the following research, to deal with the above limitations, the quantized control problem for nonlinear parabolic PDE systems with space-varying parametric uncertainties, and the boundary control problem for nonlinear PDE systems with time-varying delays via T-S fuzzy model will be investigated.

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**Data availability** The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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