Controlled Electromagnetically Induced Transparency and Fano Resonances in Hybrid BEC-Optomechanics

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Cavity-optomechanics, a tool to manipulate mechanical effects of light to couple optical field with other physical objects, is the subject of increasing investigations, especially with regards to electromagnetically induced transparency (EIT). EIT, a result of Fano interference among different atomic transition levels, has acquired a significant importance in many areas of physics, such as atomic physics and quantum optics. However, controllability of such multi-dimensional systems has remained a crucial issue. In this report, we investigate the controllability of EIT and Fano resonances in hybrid optomechanical system composed of cigar-shaped Bose-Einstein condensate (BEC), trapped inside high-finesse Fabry-Pérot cavity with one vibrational mirror, driven by a single mode optical field and a transverse pump field. The transverse field is used to control the phenomenon of EIT. It is detected that the strength of transverse field is not only efficiently amplifying or attenuating out-going optical mode but also providing an opportunity to enhance the strength of Fano-interactions which leads to the amplification of EIT-window. To observe these phenomena in laboratory, we suggest a certain set of experimental parameters. The results provide a route for tunable manipulation of optical phenomena, like EIT, which could be a significant step in quantum engineering.

Cavity-optomechanics that couples coherent electromagnetic field with other mechanical objects via radiation pressure has become a significant area of research over the few years. The demonstration of cavity-optomechanics with ultra-cold particles, like Bose-Einstein condensate (BEC), provides stunning manipulation of solid-state physics along with optical physics. Through several investigations, different aspects of BEC have made spectacular contributions in understanding complex systems. In optomechanical systems, coupling is obtained by radiation pressure and indirectly via quantum dot and ions. Optomechanics helps mechanical effects of light to cool movable mirror to its quantum mechanical ground state and provides a platform to study strong coupling effects in hybrid systems. Recent magnificent discussions and simulations on bistable behavior of BEC-optomechanical system, high fidelity state transfer, entanglement in cavity-optomechanics, dynamical localization in field of cavity-optomechanics, and coupled arrays of micro-cavities provide clear understanding for cavity-optomechanics.

Recently, electromagnetically induced transparency (EIT), a phenomenon of direct manifestation of quantum coherence, has been extensively investigated and has provided a lot of remarkable applications. In atomic system, EIT occurs due to Fano-interactions or quantum interference effects induced by coherently driving atomic wavepacket with an external pump laser field. EIT effect has been theoretically investigated in optomechanical system and later experimentally verified in both optical and microwave domains. Fano resonances, caused by Fano-interactions at different system configurations, have played an important role in understanding photo-electron spectra in atomic physics and have made a magnificent contribution in the latest field of plasmonics. Fano resonances have also been investigated in hybrid cavity-optomechanics by using different configurations.

In this report, we investigate the controlled behavior of electromagnetically induced transparency (EIT) and Fano Resonances in hybrid BEC-optomechanical system, by choosing similar method of control as proposed...
in ref. 52 where the tunable bistable dynamics have been discussed. The optomechanical system is composed of a cigar-shaped Bose-Einstein condensate (BEC) trapped inside high-finesse Fabry-Pérot cavity, with one fixed mirror and one moving-end mirror oscillating with frequency ω_{cc} and maximum amplitude q_0, driven by a single mode optical field and a transverse pump field η_{P}. Transverse optical field is used to control the phenomenon of electromagnetically induced transparency (EIT) in the output field spectra. By observing output probe field spectra, we show that the probe laser field can efficiently be amplified or attenuated depending on the strength of transverse optical field η_{P}. Furthermore, we demonstrate emergence of Fano resonances using transverse optical field. To observe these phenomena in laboratory, we have suggested a certain set of experimental parameters very near to the present experimental quests.

**BEC-Optomechanics.** We consider a Fabry-Pérot cavity of length L with a fixed mirror and a moving-end mirror driven by a single mode optical field with frequency ω_{p}, as shown in Fig. 1. Inside the high-Q cavity, optical fields generate a single dimensional optical lattice in which Cigar-Shaped BEC, containing N-two level atoms, is trapped. Moreover, a transverse optical field, with strength η_{P} and frequency ω_{P}, is used to drive BEC perpendicularly, which scatters transverse photons to the intra-cavity field and maximum amplitude q_0, however, in the absence of intra-cavity radiation pressure, it performs Brownian motion due to external heat-bath.

The complete Hamiltonian of system can be divided into two parts, \( \hat{H} = \hat{H}_m + \hat{H}_p \), where \( \hat{H}_m \) is related to the motion of side mirror inside the cavity and optical fields interacting with system, \( \hat{H}_p \) describes the atomic degree of freedom inside the system. The Hamiltonian \( \hat{H}_m \) is given as

\[
\hat{H}_m = \hbar \Delta_c \hat{c}^\dagger \hat{c} + \frac{\hbar \omega_{cc}}{2} (\hat{q}^2 + \hat{p}^2) - \xi \hbar \hat{c}^\dagger \hat{c} \hat{q} - i \hbar \eta_{P}(\hat{c}^\dagger e^{i \Delta_{P} t} - \hat{c} e^{-i \Delta_{P} t}),
\]

where \( \hbar \Delta_c \hat{c}^\dagger \hat{c} \) is energy of the intra-cavity field, with detuning \( \Delta_c = \omega_{c} - \omega_{cc} \) and frequency \( \omega_{c} \), \( \hat{q} \) and \( \hat{p} \) are dimensionless position and momentum operators, respectively, for vibrational end mirror having canonical relation \( [\hat{q}, \hat{p}] = i \), revealing scaled Planck’s constant \( \hbar = \frac{\sqrt{\omega_{cc} L}}{\chi} \) is the strength of coupling between vibrating end mirror and intra-cavity optical field, where \( \chi = \sqrt{\hbar / (2 \mu \omega_{cc})} \) is zero point motion mass of the mirror, \( i \hbar \eta_{P}(\hat{c}^\dagger - \hat{c}) \) is the relation between intra-cavity field and external pump field, \( [\eta_{P}] = \frac{\hbar \omega_{cc}}{\chi \omega_{P}} \) with external pump field power \( P \) and intra-cavity field decay \( \kappa \). Finally, last term describes external probe field and \( [E_{P}] = \frac{\hbar \omega_{P}}{\chi \omega_{P}} \) is related to the power of the probe field. (Note: The system is nearly same as studied by us in ref. 52 except the addition of an external probe field which is crucial to investigate EIT). \( \Delta_{P} = \omega_{P} - \omega_{cc} \) is detuning between probe laser field frequency \( \omega_{P} \) and external pump field \( \omega_{P} \).

We consider the motion of BEC quantized along with the cavity-axis in strong detuning regime to adiabatically eliminate internal excited states of atoms and dilute enough so that atom-atom interaction effects can be ignored. In addition, the Hamiltonian describing BEC and its association with cavity mode is written under momentum side-mode approximation (detailed derivation are shown in ref. 52) as,
\[ \hat{H} = \frac{\hbar U_0 N}{2} \hat{c}^\dagger \hat{c} + \frac{\hbar \Omega}{2} \left( \hat{p}^2 + \hat{Q}^2 \right) + \xi_{sm} \hbar \hat{c} \hat{c} \hat{Q} + \hbar \eta_{\text{eff}} \hat{Q} \hat{c}^\dagger + \hat{\gamma}, \]

where \( U_0 = g^2 / \Delta_s \) is the far off-resonant Rabi frequency with detuning \( \Delta_s \) and \( N \) is the total number of bosonic particles in atomic mode. \( \hat{p} \) and \( \hat{Q} \) are dimensionless momentum and position quadratures, respectively, for atomic mode, with canonical relation \([\hat{Q}, \hat{p}] = i\), describing motion of atomic mode under mechanical effects of light with recoil frequency \( \Omega = 4 \omega_s = 2 \hbar k^2 / m_v \). \( \xi_{sm} = \frac{\gamma_s}{\hbar} \) is the coupling of atomic mode with intra-cavity optical field, where \( m_v = \hbar \omega_s^2 (L^2 N U_0^2 \omega_s) \) is side mode mass of condensate. Further, \( \eta_{\text{eff}} = \sqrt{m} \eta_s \) describes coupling of atomic degree of freedom with perpendicularly interacting optical field, where \( |\eta_s|^2 = \frac{\gamma_s}{\hbar \omega_s} \) is intensity of transverse field. From Eq. (2), it can be noted that in the absence of transverse field (i.e. when there is no excitation for \( \cos(kx) \)) we recover same expression for atomic mode of cavity-optomechanics as in refs 3 and 25.

To incorporate the dissipation effects of intra-cavity field due to photon shot noises and thermo-mechanical noises effects associated with the motion of moving-end mirror and BEC via standard quantum noise operators, we derive Langevin equations from Hamiltonian \( \hat{H} \).

\[ \frac{d \hat{c}}{dt} = \hat{\epsilon} = (i \hat{\Delta} + i \xi \hat{Q} - i \xi_{sm} \hat{Q} - \kappa) \hat{c} + i \eta_{\text{eff}} \hat{Q} + \eta + E \rho e^{-i \Delta_s t} + \sqrt{2 \kappa} \xi_{sm}, \]

\[ \frac{d \hat{p}}{dt} = \hat{\dot{p}} = -\omega_m \hat{q} + \xi \hat{c}^\dagger \hat{c} - \gamma_s \hat{p} + \hat{f}_m', \]

\[ \frac{d \hat{q}}{dt} = \hat{\dot{q}} = \omega_m \hat{p}, \]

\[ \frac{d \hat{\dot{p}}}{dt} = \hat{\ddot{p}} = -4 \omega_s \hat{Q} - \xi_{sm} \hat{c}^\dagger \hat{c} - \gamma_s \hat{p} + \hat{f}_a', \]

\[ \frac{d \hat{\dot{Q}}}{dt} = \hat{\ddot{Q}} = 4 \omega_s \hat{p} - \gamma_s \hat{Q} + \hat{f}_a'. \]

The term \( \hat{f}_m \) describes the Markovian input shot noise associated with intra-cavity field and \( \hat{\Delta} = \Delta_s - NU_0 / 2 \) represents effective detuning of the system. \( \gamma_s \) is the decay rate of the moving-end mirror motion while \( \hat{f}_m' \) describes thermo-mechanical noise operator associated with Brownian motion of mechanical mode. Further, \( \gamma_s \) and \( \hat{f}_a' \) are damping of atomic mode due to harmonic trapping potential and \( \hat{f}_a' \) is Markovian noise operator connected with motion of atomic side modes, respectively. It should be noted that the Eqs (3–7) are same as derived in ref. 52 except the addition of external probe field in Eq. (3) to investigate EIT. We consider positions and momentum quadratures as classical variables to obtain steady state solutions of the Langevin equations. Further, we take optical field decay at its fastest rate so that time derivative can be set to zero in Eq. (3). The static solutions are given as,

\[ c_s = \frac{\eta + i \eta_{\text{eff}} \hat{Q} + E \rho e^{-i \Delta_s t}}{\kappa + i (\hat{\Delta} - \hat{\xi} Q + \xi_{sm} \hat{Q})}, \quad q_s = \frac{\xi \hat{c}^\dagger \hat{c}}{\omega_m}, P_s = 0, \]

\[ Q_s = \frac{-\xi_{sm} \hat{c}^\dagger \hat{c}}{4 \omega_s [1 - \gamma_s / 4 \omega_s]}, \quad P_s = \frac{\gamma_s}{4 \omega_s} Q_s, \]

where \( c_s, q_s \) and \( Q_s \) represent steady-state solution of intra-cavity field, the mechanical mirror displacement and the position of the BEC mode, respectively. As expected, the steady-state solutions given in Eq. (8) are same as derived in ref. 52 except the addition of external probe field relation in first equation.

To observe output field spectra, we deal with mean response of the system to probe field in presence of external pump field (control laser) and transverse field. First, we linearized quantum Langevin equations by inserting ansatz \( \hat{\epsilon}(t) = c_0 + \delta c(t), \hat{\dot{\epsilon}}(t) = q_0 + \delta q(t), \hat{\ddot{\epsilon}}(t) = p_0 + \delta p(t), \hat{\dot{q}}(t) = Q_0 + \delta Q(t) \) and \( \hat{\ddot{Q}}(t) = P_0 + \delta P(t) \) in Langevin equations and taking care of only first-order terms in fluctuating operators \( \delta \hat{\epsilon}(t), \delta \dot{q}(t), \delta \dot{p}(t), \delta Q(t) \) and \( \delta P(t) \). The linearized quantum Langevin equations are now given as,

\[ \delta \hat{\ddot{c}}(t) = - (\kappa + i \hat{\Delta}) \delta c(t) + G_m \delta q(t) - G_s \delta Q(t) + i \eta_{\text{eff}} \delta Q(t) + E \rho e^{-i \Delta_s t} + \sqrt{2 \kappa} \xi_{sm}, \]

\[ \delta \hat{\ddot{q}}(t) = - (\kappa - i \hat{\Delta}) \delta c(t) + G_m \delta q(t) - G_s \delta Q(t) - i \eta_{\text{eff}} \delta Q(t) + E \rho e^{-i \Delta_s t} + \sqrt{2 \kappa} \xi_{sm}, \]

\[ \delta \dot{q}(t) = \omega_m \delta p(t), \]
$$\delta p(t) = -\omega_p \delta q(t) + G_m(\delta c(t) + \delta c^\dagger(t)) - \gamma_m \delta p(t) + \tilde{f}_m,$$

$$\delta \dot{q}(t) = \omega_r \delta p(t) + \tilde{f}_r,$$

$$\delta \dot{P}(t) = -\omega_m \delta Q(t) + G_m(\delta c(t) + \delta c^\dagger(t)) - \gamma_P \delta P(t) + \tilde{f}_P,$$

where $\Delta = \delta_q + \xi_{\text{eff}} Q$ is the effective detuning of the system and $G_m = \xi \epsilon |G_r| G_a = \xi \epsilon |G_r|^2 |G_a|$ are the effective coupling of optical field with the moving-end mirror and the condensate mode, respectively. To solve mean value equation of the system, we write expectation value of operators in form expressed as,

$$<\hat{c}(t)> = \frac{\kappa - i(\Delta + \Delta_p) + X(\Delta_p)}{Y(\Delta_p)},$$

$$<\hat{q}(t)> = \frac{E_p}{X(\Delta_p)Y(\Delta_p)} \left[ Y(\Delta_p) + \left( \kappa + i(\Delta + \Delta_p) \right) + X(\Delta_p) \right] \left( \kappa - i(\Delta - \Delta_p) + X(\Delta_p) \right),$$

where

$$X(\Delta_p) = -\frac{G_m^2 \omega_r^2}{\omega_r^2 - \Delta_p^2 + i\gamma_m \Delta_p} - \frac{G_m^2 \omega_m^2}{\omega_m^2 - \Delta_p^2 + i\gamma_m \Delta_p},$$

$$Y(\Delta_p) = -\Delta^2 - \kappa^2 - 2i\kappa \Delta_p + \Delta_p^2 + \frac{2\omega_r \omega_m^2 (\kappa + i\Delta_p)}{i\gamma_m \Delta_p - \Delta_p^2 + \omega_m^2} + \frac{2\kappa \omega_m^2}{i\gamma_m \Delta_p - \Delta_p^2 + \omega_m^2}. $$

Eqs (15) and (16) clearly describe dependence of output field spectra on the coupling of different degrees of freedom. In particular, we can observe the rule of transverse optical field coupling with BEC mode in output field. Furthermore, to investigate EIT-like behavior, we write output field spectra by using input-output relation $c_{\text{out}} = \sqrt{2\kappa} c - c_{\text{in}}$, where $c_{\text{in}}$ and $c_{\text{out}}$ represent input and output field, respectively. Moreover, we ignore quantum noises associated with $c_{\text{out}}$ and $c_{\text{in}}$ as discussed earlier. The out-going optical field can be expressed as,

$$<c(t)> = c_0 + c_+ E_p e^{-i\Delta_p t} + c_- E_p e^{i\Delta_p t}. $$

By using above relation and input-output field theory, we describe the components of output field spectra at probe field frequency as,

$$c_0 = \sqrt{2\kappa} c_ - \eta,$$

$$c_+ = \sqrt{2\kappa} \tilde{c}_+ - 1,$$

$$c_- = \sqrt{2\kappa} \tilde{c}_-.$$

In order to examine EIT in output field, we define total out-going optical mode $E_T$, at probe frequency $\omega_p$, as $E_T = c_+ + 1 = \sqrt{2\kappa} \tilde{c}_+$, where $E_T$ not only describes optical field leaking-out from cavity but also accommodates noise effects associated with the system. In absence of optical coupling with moving-end mirror and BEC mode, the output field spectra $E_T$ at probe frequency will be reduced to,

$$E_T = \frac{2\kappa}{\kappa + i(\Delta - \Delta_p)}. $$

**Controllable EIT in Output Field.** The hybrid BEC-optomechanical system shown in Fig. 1 is simultane-ously driven by external pump field with frequency $\omega_p$ and probe field with frequency $\omega_P$, generating radiation pressure force which oscillates at frequency difference $\Delta_p = \omega_P - \omega_p$. When this resultant force resonates with
atomic mode trapped inside cavity with damping rate and detuning are discussed in Fig. 2, in the absence of transverse field coupling power follows similar stability conditions as developed and discussed in refs 52,55,57. We consider the absence of transverse optical field coupling pump field is taken as.

Figure 2. Single electromagnetically induced transparency in output field. The absorption (Re[\(E_T\)]) and dispersion (Im[\(E_T\)]) quadratures of the output field spectra \(E_T\) as a function of probe detuning \(\Delta_p/\omega_m\) and in the absence of of transverse field coupling \(\eta_{gf}/\kappa = 0\). (a) Describes the behavior of single-EIT in real quadrature of output probe field with different atom-field and mirror field coupling strengths \(G_d/\omega_m = 0, G_m/\omega_m = 0\) (blue curve), \(G_d/\omega_m = 0.02, G_m/\omega_m = 0\) (red curve), and \(G_d/\omega_m = 0.03, G_m/\omega_m = 0\) (green curve). (b) Contains similar behavior of imaginary quadrature of output probe field with different couplings \(G_d/\omega_m = 0, G_m/\omega_m = 0\) (blue curve), \(G_d/\omega_m = 0.02, G_m/\omega_m = 0\) (red curve), and \(G_d/\omega_m = 0.03, G_m/\omega_m = 0\) (green curve). One can observe the emergence of single-EIT window with atom-field coupling due to the quantum interference among atomic transition pathways. The frequency of moving-end mirror is considered as, \(\omega_m = 1.02 \times 2\pi MHz\). The atomic mode and mechanical mode damping rates are fixed to \(\gamma = 0.21 \times 2\pi kHz\) and \(\gamma = 1.1 \times 2\pi kHz\), respectively. The cavity optical decay is taken as, \(\kappa = 1.3 \times 2\pi kHz\).

frequency close to the frequency of mechanical mode \(\omega_m\) or atomic mode (BEC) \(4\omega_p\), it gives rise to Stokes and anti-Stokes scatterings of light from the strong intra-cavity standing field. But Stokes scattering is strongly suppressed because, conventionally, optomechanical systems are operated in resolved-sideband regime \(\kappa \ll \omega_m\), which is off-resonant with Stokes scattering and so only anti-Stokes scattering survives inside the cavity. Therefore, due to the presence of probe field and pump field inside system, Fano-interactions occur in mirror and atomic transition pathways which cause appearance of electromagnetically induced transparency (EIT) like behavior in output field spectra.

To make this study of tunable EIT and Fano resonances in hybrid BEC-Optomechanics experimentally feasible, we choose a regime of particular parameters very close to the recent experimental studies. The EIT behavior in output field can only be observed when intra-cavity optical field is in stable regime. For this purpose, we follow similar stability conditions as developed and discussed in refs 52,55,57. We consider \(N = 2.3 \times 10^4 Rb\) atoms trapped inside Fabry-Pérot cavity with length \(L = 1.25 \times 10^{-4}\), driven by single mode external field with power \(P_0 = 0.0164 mW\), frequency \(\omega_p = 3.8 \times 2\pi \times 10^{14} Hz\) and wavelength \(\lambda_p = 780 nm\). The strength of external pump field is taken as \(\eta/\kappa = 1.8\). The intra-cavity optical mode oscillates with frequency \(\omega_c = 15.3 \times 2\pi \times 10^{14} Hz\), with decay rate \(\kappa = 1.3 \times 2\pi kHz\). Further, vacuum Rabi frequency of atomic mode is considered \(U_0 = 3.1 \times 2\pi MHz\) with detuning \(\Delta = 0.52 \times 2\pi MHz\). Intra-cavity field produces recoil of \(\omega_p = 3.8 \times 2\pi kHz\) in atomic mode trapped inside cavity with damping rate \(\gamma = 0.21 \times 2\pi kHz\). The moving-end mirror of cavity is considered as a perfect reflector oscillating with frequency \(\omega_m = 1.02 \times 2\pi MHz\) with damping \(\gamma = 1.1 \times 2\pi kHz\). From given parameters, one can observe that the system is in resolved-sideband regime because \(\omega_m \gg \kappa\), this condition is also referred to good-cavity limit.

The real (Re[\(E_T\)]) and imaginary (Im[\(E_T\)]) quadratures of out-going probe field as a function of probe-cavity detuning are discussed in Fig. 2, in the absence of transverse field coupling \(\eta_{gf}/\kappa = 0\). Here, \(Re[\(E_T\)]\) and \(Im[\(E_T\)]\) accounts for in-phase and out-phase, respectively, quadratures of the output field spectra and also referred to absorption and dispersion behavior of out-going optical mode. Figure 2(a,b) demonstrate the single-EIT behavior of such absorption and dispersion quadratures, respectively, of output field spectra for different coupling strengths \(G_d/\omega_m = 0, G_m/\omega_m = 0\) (blue curve), \(G_d/\omega_m = 0.02, G_m/\omega_m = 0\) (red curve), and \(G_d/\omega_m = 0.03, G_m/\omega_m = 0\) (green
Figure 3. Controlled electromagnetically induced transparency with transverse field. The real (Re$[E_T]$) and imaginary (Im$[E_T]$) quadratures of the output probe field $E_T$ as a function of probe detuning $\Delta_p/\omega_m$ and transverse optical field $\eta_{Gm}/\kappa$. (a,d) Accommodate double-EIT behavior in real and imaginary quadratures, respectively, in the absence of transverse field $\eta_{Gm}/\kappa = 0$ and with various coupling strengths $G_m/\omega_m = 0$, $G_m/\omega_m = 0.08$, $G_m/\omega_m = 0.05$ (red curve), and $G_m/\omega_m = 0.1$, $G_m/\omega_m = 0.08$ (green curve). Similarly like atomic mirror, quantum interference in mechanical mirror transition paths cause the emergence of another transparency window in output field spectra. (b,e) Demonstrate single-EIT windows with coupling strengths $G_m/\omega_m = 0.03$, $G_m/\omega_m = 0$, as a function of transverse field strengths $\eta_{Gm}/\kappa = 0$ (blue curve), $\eta_{Gm}/\kappa = 0.02$ (red curve) and $\eta_{Gm}/\kappa = 0.03$ (green curve). Similarly, (c,f) show double-EIT behavior of output field spectra with coupling $G_m/\omega_m = 0.1$, $G_m/\omega_m = 0.08$, as a function of transverse field strengths $\eta_{Gm}/\kappa = 0$ (blue curve), $\eta_{Gm}/\kappa = 0.02$ (red curve) and $\eta_{Gm}/\kappa = 0.03$ (green curve). The transverse field scatters photons inside the cavity causing modification in both atomic as well as mechanical mirror transitions. The projection of these modifications on optical field leaking-out from cavity can be seen in given results. The remaining parameters used in numerical calculations are same as in Fig. 2.

To observe single-EIT behavior, we have considered case when system is only coupled with intra-cavity atomic mode or BEC and the coupling of moving-end mirror with intra-cavity optical mode is zero because the single-EIT behavior in cavity-optomechanics in presence of mirror coupling has already been discussed in previous works like 43,45,46,50. We consider that the optomechanical system is being operated in strong coupling regime which means intra-cavity optical mode is strongly coupled to atomic mode of the system. When collective density excitation of atomic mode becomes resonant with intra-cavity optical mode, strong coupling between atomic and intra-cavity standing wave is generated. It is only possible when the strength of coupling between single atom and single photon of the cavity $g_0 = 10.9 \times 2\pi \text{MHz}$ is larger than both decay rate of the atomic excited state $\gamma_a = 0.21 \times 2\pi \text{kHz}$ as well as intra-cavity field decay $\kappa = 1.3 \times 2\pi \text{kHz}$ ($g_0 \gg \gamma_a, \kappa$). We can also note from mathematical expression of atomic coupling that the strength of atomic mode coupling is directly proportional to the vacuum Rabi frequency ($G_m \propto U_m = g_0^2/\Delta_p$).

In Fig. 2, one can easily observe that there are no signatures of EIT in absorption and dispersion spectra of output field (blue curves) when intra-cavity field is not coupled with mechanical mode and condensate mode ($G_m/\omega_m = 0$, $G_m/\omega_m = 0$). While in red curve, the single-EIT window appears in output probe field due to intra-cavity optical mode coupling with atomic mode (BEC) ($G_m/\omega_m = 0.02$). However, coupling of intra-cavity field with mechanical mode is kept zero $G_m/\omega_m = 0$. Green curve shows the enhancement in single-EIT window in output probe field by increasing the coupling strength to $G_m/\omega_m = 0.03$. These results clearly prove the existence of single-EIT window in output probe field when intra-cavity optical mode is only coupled to atomic mode of cavity-optomechanics.

Figure 3 describes the EIT behavior in the output field spectra of the optomechanical system in presence of probe detuning $\Delta_p/\omega_m$ and transverse optical field $\eta_{Gm}/\kappa$. Figure 3(a,d) represent absorption (real) and dispersion (imaginary) quadratures, respectively, of output probe field in the absence of transverse field $\eta_{Gm}/\kappa = 0$ with various coupling strengths $G_m/\omega_m = 0$, $G_m/\omega_m = 0$ (blue curve), $G_m/\omega_m = 0.08$, $G_m/\omega_m = 0.05$ (red curve), and $G_m/\omega_m = 0.1$, $G_m/\omega_m = 0.08$ (green curve). These results show that there are no signs of EIT-like behavior in absorption and dispersion spectra of output field (blue curves) when system is isolated form mechanical mode and condensate mode ($G_m/\omega_m = 0$, $G_m/\omega_m = 0$). On the other hand, in red curve, two EIT windows appear in output probe field because optical mode of the system is now coupled to both mechanical mode (moving-end mirror) ($G_m/\omega_m = 0.05$) as well as to condensate mode with coupling strength $G_m/\omega_m = 0.08$. Such behavior is also known as double-EIT response of output field45,51. When system is coupled to atomic mode and mechanical mode at the same time and optical mode of the system becomes resonant to both these...
modes, it gives rise to anti-Stokes scattering inside the system causing appearance of another EIT window in output field. Green curves in Fig. 3(a,d) demonstrate similar behavior double-EIT when the coupling strengths are increased to $G_{\omega_m} = 0.1$, $G_{\rho}/\omega_m = 0.08$. We observe that the quadratures of double-EIT behaviors are increased by increasing coupling strengths. The given results in Fig. 3(a,d) show such double-EIT behaviors in output field when optomechanical system is coupled to moving-end mirror of the system and BEC trapped inside the system.

Figure 3(b,e) demonstrate single-EIT behavior in absorption and dispersion quadratures, respectively, of output probe field under the influence of transverse optical field $\eta_{\rho}\Omega$ when intra-cavity optical mode is coupled to condensate mode with coupling strength $G_{\rho}/\omega_m = 0.03$ while the coupling of optical mode with moving-end mirror is zero $G_{\omega_m}/\omega_m = 0$. It should be noted that we cannot observe transverse optical field effects on single-EIT when intra-cavity optical degree of freedom is only coupled to the moving-end mirror, as shown in single-EIT results in previous works like45,45,46,50,51 because transverse optical field is only interacting with BEC trapped inside the cavity. Therefore, we only consider condensate mode coupling while studying effects of transverse field on single-EIT. Blue curves show single-EIT windows in output probe field in the absence of transverse optical field $\eta_{\rho}\Omega = 0$. On the other hand, red and green curves demonstrate the effects of transverse field strengths $\eta_{\rho}\Omega = 0.02$ and $\eta_{\rho}\Omega = 0.03$, respectively, on the single-EIT behavior. When transverse field photon interacts with atomic mode of the system, it gives rise to the total photon number $n$ inside the cavity by scattering transverse photons into the system, which give rise to the quantum interferences and directly enhance the EIT behavior in output field. We can observe such effects of transverse field in the results that the strength of single-EIT is efficiently amplified by increasing the strength of transverse optical field.

Similarly, Fig. 3(c,f) represent double-EIT behavior in absorption and dispersion quadratures respectively, of output probe field as a function of transverse optical field $\eta_{\rho}\Omega$. Intra-cavity optical mode is coupled to both condensate mode with coupling strength $G_{\rho}/\omega_m = 0.1$ and to the moving-end mirror is $G_{\omega_m}/\omega_m = 0.08$. Blue curves in both these figures describe double-EIT behavior in the absence of transverse field $\eta_{\rho}\Omega = 0$. Besides, red and green curves represent double-EIT with transverse field strengths $\eta_{\rho}\Omega = 0.02$ and $\eta_{\rho}\Omega = 0.03$, respectively. We can observe, like single-EIT results, double-EIT windows are enhanced by increasing the transverse optical coupling. Therefore, in accordance to these results, we can confidently state that by increasing transverse optical field coupling, we can control the phenomenon of EIT in hybrid BEC-optomechanics.

**Tunable Fano resonances.** The formation of Fano resonance in the output optical mode of hybrid optomechanical system is a fascinating phenomenon caused by quantum mechanical interaction between different degrees of freedom of the system50,51. The constructive and destructive quantum interferences among narrow discrete intra-cavity optical resonances are the foundations for Fano resonances in output field of such complex systems. The transverse field effects on EIT presented in Figs 2 and 3 are similar to the single and double-Fano resonances but tuned by transverse optical field. We conventionally observe Fano line shapes in EIT windows by tuning effective detuning of the system. The variation in effective detuning of the system brings modifications to the Fano-interactions, among atomic and mirror transition pathways, which causes shift in EIT window. Afterwards, we demonstrate Fano behavior of system output field with respect to different parameters.

Figure 4 shows Fano resonances in the absorption (real) and dispersion (imaginary) profile of output probe field in the absence of transverse field coupling $\eta_{\rho}\Omega = 0$, as a function of normalized probe field detuning $\Delta_{\omega_m}/\omega_m$ and normalized effective detuning of the system $\Delta/\omega_m$. The coupling of intra-cavity optical mode with atomic mode is $G_{\rho}/\omega_m = 0.03$ while, the coupling of optical mode with mechanical mode is kept zero ($G_{\omega_m}/\omega_m = 0$) which means, optomechanical system is only coupled to the condensate mode trapped inside the cavity. Figure 4(a,b) describe absorption and dispersion profile, respectively, in output probe field as a function of normalized probe detuning. Blue curve in absorption shows Fano line with effective system detuning $\Delta/\omega_m = 0.84$. While, red and green curves in real quadrature represent fano behavior under the influence of effective detuning $\Delta/\omega_m = 0.87$ and 0.9, respectively. Similarly, blue curve in dispersion profile shows the existence of Fano line with effective system detuning $\Delta/\omega_m = 0.99$. Besides, red and green curves in imaginary quadrature of output field represent fano behavior under influence of effective detuning $\Delta/\omega_m = 1.02$ and 1.06, respectively. We can observe, each curve with different height follows a same dip in absorption and dispersion response which causes the formation of resonance in out-going optical mode.

We further investigate the existence of double-Fano resonances in output probe field by introducing another coupling in the optomechanical system and modifying effective detuning51. As the phenomenon of EIT is very sensitive to the coupling with different degrees of freedom in the system, therefore, by introducing another coupling, we can convert single-Fano resonance to double-Fano resonance, as shown in Figs 4(c) and 5(d). The coupling of intra-cavity optical mode with moving-end mirror is $G_{\rho}/\omega_m = 0.1$ and the coupling of optical mode with condensate mode is $G_{\rho}/\omega_m = 0.08$. Figure 5(c) describes absorption and Fig. 5(d) describes dispersion profile in output probe field as a function of normalized probe detuning. Blue curves, in Fig. 5(c,d), show the double-Fano line with effective detuning values $\Delta/\omega_m = 0.85$ and 1.1, respectively. Similarly, red, curves, in absorption and dispersion, accommodate the double-Fano response under the influence of effective detuning $\Delta/\omega_m = 0.94$ and 0.96, respectively and green curves account for the influence of effective detuning $\Delta/\omega_m = 0.87$ and 1.15, respectively. By analyzing these results, we come to know that the single-Fano resonance can be transformed to double-Fano resonances by coupling intra-cavity field with both atomic as well as mechanical degrees of freedom50,51.

In previous Fano resonance results, we have ignored the effects of transverse optical field coupling. However, it will be important to keep these effects and analyze the behavior of Fano resonances. Figure 5 illustrates such effects on Fano resonances emerging in output field spectra in the presence of transverse field coupling $\eta_{\rho}\Omega = 0.03$. Figure 5(a) shows real quadrature of out-going mode, containing single-Fano resonance, where blue, red and green curves correspond to the effective detuning strengths $\Delta/\omega_m = 0.85$, 0.87 and 0.9, respectively.
On the other hand, Fig. 5(b) represents imaginary quadrature of output field, where blue, red and green curves correspond to the influence of system detuning strengths $\Delta/\omega_m = 0.99, 1.02$ and $1.06$, respectively. One can observe, how quadratures of singe-Fano lines are increased due to the presence of transverse field. Transverse optical field causes scattering of photons inside the cavity which gives rise to intra-cavity photon number and this nonlinear factor brings modification to the out-going optical mode of cavity. It is understood that if we further increase the strength of transverse coupling, it will definitely further modify Fano behavior in output field. Figure 5(c,d) show similar behavior for double-Fano resonances in real and imaginary profile of out-going probe field under the influence of transverse field strength. Figure 5(c) shows absorption and Fig. 5(d) shows dispersion behavior at transverse optical field strength $\eta_\text{eff}/\kappa = 0.03$. Blue curves, in Fig. 5(c,d), show the effects of $\eta_\text{eff}/\kappa$ on double-Fano curves appearing in out-going mode with effective detuning values $\Delta/\omega_m = 0.85$ (blue curve), $\Delta/\omega_m = 1.1$ (red curve) and $\Delta/\omega_m = 1.15$ (green curve). The Fano-interactions between atomic and mechanical mirror transition levels are generating resonances with different system detuning strengths, which can be seen in the results. Remaining parameters are same as in Fig. 2.

By comparing results of Figs 4 with 5, we can easily note the effects of transverse optical field on the double-Fano resonance of the optomechanical system. The absorption and dispersion quadratures of single-Fano as well as double-Fano resonances are notably modified by increasing transverse field strength.

**Discussion**

In conclusion, we discuss the controllability of electromagnetically induced transparency (EIT) and Fano Resonances in hybrid optomechanical system, by following same method as investigated in ref. 52. The system contains cigar-shaped Bose-Einstein condensate (BEC) trapped inside high-finesse optical cavity with one moving-end mirror and driven by a single mode optical field along the cavity axis and a transverse pump field. As the transverse optical field directly interacts with condensate mode which causes the scattering of transverse photon inside the cavity so, by varying transverse field, we can modify the dynamics of system. We have shown the controlled behavior of EIT in output probe field by using transverse field. We discuss existence of single-EIT window, in output field of cavity, in the absence of moving-end mirror which means intra-cavity optical mode was...
only coupled to atomic mode (BEC) of the system. The single-EIT as well as double-EIT windows, in out-going probe field, are efficiently amplified by increasing the strength of transverse optical field. Furthermore, single and double-Fano resonances are discussed in out-going probe field of the system. The transverse optical field shows similar effects on the emergence of Fano resonances as it does to EIT. We have also suggested a certain set of experimental parameters to observe these phenomena in laboratory.

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Author Contributions
K.A.Y. and W.-M.L. proposed the ideas. K.A.Y. interpreted physics, performed the theoretical as well as the numerical calculations and wrote the main manuscript. W.-M.L. checked the calculations and the results. Both of the authors reviewed the manuscript.

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