Determination and Application of Microscopic Index $R_{ca}$ in Evaluating the Mechanical Properties of Soils

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Abstract. This paper presents an elaborated discussion on a soil microstructural index: the contact area ratio $R_{ca}$, including its determination method and potential applications. Starting from the calculation of 3D porosity, a reliable threshold is obtained by introducing the connection between macro and micro properties of soils, and then the contact area ratio $R_{ca}$ could be well determined. Next triaxial compression test and SEM test are conducted and combined to investigate the evolution law of the microscopic contact area ratio $R_{ca}$ during the compression process. Finally, the application value of this micro index in the evaluation of soil mechanical properties is demonstrated by introducing two potential application, which lays the foundation for further research in this field.

1. Introduction

Natural soils are porous materials formed by bonding or connection between particles. A great number of studies have shown that the mechanical properties of soils from macro perspective such as strength, deformation and permeability are closely affected or even determined by its microstructure [1-4]. Benefit from the modern advanced technology in the field of microstructure measurement and image processing, it is more convenient to collect and analyze the micro information of soil structure. At present, techniques applied for measuring the soil microstructure mainly include:

- MIP (Mercury Intrusion Porosimetry). It is used as measurement of the pore size distribution in a soil [5].
- CT (Computed Tomography) scanning. It’s a nondestructive testing method commonly used for the observation of crack inside the soils [6].
- SEM (Scanning Electron Microscopy)/ESEM (Environmental Scanning Electron Microscopy) technique. This method can provide cross sectional images which are used for analyzing the morphology and arrangement characteristics of pores or particles [7].

Once the micrographs are obtained, useful structural information should be extracted with the help of image processing program (e.g. MATLAB and image-pro plus), so that accurate and reliable comment on soil properties can be drawn. For a long period, qualitative analysis about the soil microstructure are most popular and can be easily found in the literature. Later, as the deepening of researches, more and more scholars pay their attentions to the quantitative analysis of soil microstructure. Some assessment indices in the micro-level are developed from quantitative evaluation.
of microstructure. Then connections between the macro and micro properties of soil can be established. As a result, multi-scale modeling of soil properties is being carried out [8-9].

There are various kinds of micro quantitative assessment indices proposed by now, each index focuses distinctively from the other, such as fractal dimensions [10] which describe the irregularity of soil particles, and alignment entropy [11] which reflect the orientation of aggregates. The index \( R_{ca} \) (average contact area ratio) proposed by author is another microscopic assessment which is originally developed to investigate the transfer mechanism of effective stress in the soil [12]. In this paper, the determination and some further applications of index \( R_{ca} \) will be fully elaborated combined with macro and micro tests.

2. Definition and Determination of \( R_{ca} \)

2.1. 3D Porosity

Before discussing the index \( R_{ca} \), a new concept defined as 3D porosity should be introduced firstly. Figure 1 shows a typical SEM image of soft clay microstructure, which is a grayscale graph with integer grayscale value between 0 and 255. Different pixel regions in the SEM image correspond to different grayscale value, which reflects the distance between sample surface and the projecting plane.

![Figure 1. SEM image of soft clay microstructure.](image)

In figure 1, regions \( A_1 \) (boundary \( B_1 \), denoted by red line) and \( A_2 \) (boundary \( B_2 \), denoted by yellow line) are picked by different grayscale value \( Y_1 \) and \( Y_2 \). When the values of \( Y_1 \) and \( Y_2 \) are relatively close, the volume of pores surrounded by \( B_1 \) and \( B_2 \) can be indirectly expressed as:

\[
V_i = \left( \frac{S_i + S_{i+1}}{2} \right) \times (Y_i - Y_2) \tag{1}
\]

where \( S_i \) and \( S_{i+1} \) represent the pixel area of \( A_1 \) and \( A_2 \), respectively. Then the method for calculating 3D porosity is established based on the SEM image as:

\[
n_{3D} = \frac{\sum_{i=1}^{w} \left( \frac{S_i + S_{i+1}}{2} \right) \times (Y_{i+1} - Y_i)}{Y_m \times S_A} \tag{2}
\]

where \( S_A \) is the pixel area of SEM image picked for analysis; \( Y_m \) is the threshold for pore structure.

It can be seen from equation (1) that the value of \( n_{3D} \) varies with threshold \( Y_m \). However, studies on the threshold have never came to a unified conclusion thus many different threshold determination algorithms have been proposed over the past four decades [13]. A newly proposed method for threshold determination based on 3D porosity will be explained in the following.
2.2. Contact Area and $R_{ca}$ for Soft Clay
At microscopic level, 3D porosity can be obtained by statistically averaging calculated values from SEM images of different parts of soil sample. The relationship between $n_{3D}$ and threshold is shown in figure 2, which shows they are positively correlated. On the other hand, the real porosity $n$ of soil can be measured macroscopically using geotechnical testing method. A reasonable threshold is obtained by setting $n_{3D} = n$ in figure 2 according to the basic principle that micro and macro porosity should be equal.

In turn, the determined threshold is utilized to binarize SEM image (see figure 3). The white parts in figure 3 are defined as the contact area of soil aggregates. And the ratio of contact area to the total area of SEM images are defined as the contact area ratio $R_{ca}$.

![Figure 2. Relationship between $n_{3D}$ and threshold.](image2.png)  
![Figure 3. Black-white map of SEM image.](image3.png)

To verify the reliability and stability of determination for $R_{ca}$, SEM images taken from 10 different positions in a soil sample have been used to calculate the contact area ratio $R_{ca}$. The result shows that the obtained $R_{ca}$ is stable with a standard deviation no more than 2% [12]. In addition, another noteworthy issue is the influence of magnification on the calculation results. Tang et al [14] discussed the factors that affecting analysis of soil microstructure using SEM and suggested the reasonable magnification for quantitative analysis is at round 1200 times. Therefore, magnification adopted in the subsequent analysis is ×1000 and ×1500.

3. Variation of Microscopic Contact Area during Triaxial Compression with Drainage
To quantitatively investigate the variation of microscopic contact area during triaxial compression, macro and micro tests are designed and the evolution law for $R_{ca}$ is discussed in this section.

3.1. Triaxial Compression Test
Soil samples are soft clay taken from a site in Taizhou, whose basic physical properties are shown in table 1.

| Table 1. Physical properties of soil samples. |
|---------------------------------------------|
| Unit Weight (kN/m³) | Water Content (%) | Specific Gravity $G_s$ | Liquid Limit (%) | Plastic Limit (%) | Plastic Index $I_p$ |
|---------------------|------------------|------------------------|-----------------|-----------------|-------------------|
| 18.6                | 42.7             | 2.73                   | 45.6            | 24.7            | 20.9              |

In this test, remodeled specimens were adopted. the original samples were firstly dried and crushed into fine powder that could pass through a 2-mm sieve. Then the screened samples were sealed and preserved. All the tests were done on 39.1-mm-diameter and 80-mm-high specimens which were
carefully compacted in four layers. Initial void ratio of all specimens was controlled at about 0.91. Triaxial tests were carried out with the GDS triaxial system (GDSTTS) at Zhejiang University. Two series of drained triaxial compression tests were carried out. Four specimens with different degrees of compression (denoted by axial strain $\varepsilon_1$) were designed for each group. Test scheme in detail can refer to table 2. The well-prepared specimens were placed in a vacuum device for saturation and then a back pressure of 300kPa was employed for back saturation. After that B-check was performed to make sure that $B>0.98$. Then consolidation can be carried out, and the compression is controlled by axial displacement whose loading rate is 0.0075 mm /min.

Table 2. Test scheme.

| Test Number | Cell Pressure (kPa) | Axial Strain $\varepsilon_1$ (%) |
|-------------|---------------------|---------------------------------|
| 1-1         |                     | 0                               |
| 1-2         | 100                 | 4.995                           |
| 1-3         |                     | 11.126                          |
| 1-4         |                     | 17.406                          |
| 2-1         |                     | 4.729                           |
| 2-2         | 200                 | 8.862                           |
| 2-3         |                     | 14.998                          |
| 2-4         |                     | 19.955                          |

3.2. SEM Test and Image Processing
After the triaxial tests, soil slices of approximately $10\text{mm} \times 10\text{mm} \times 5\text{mm}$ were cut off from the middle part of specimens. After air drying the disturbed particles on the surface were blown off to obtain a representative structural cross-section. Then the samples were taken into the vacuum evaporation coating instrument for gold-plated film to enhance the conductivity of clay. After that SEM test were carried out with Quanta FEG 650. Figure 4 shows some representative SEM images of microstructure for samples from first group. It can be seen clearly that the soils become denser during the compression.

Figure 4. SEM image of samples with different degrees of compression: (a) 1-1 (b) 1-2 (c) 1-3.

Before the quantitative analysis and evaluation of SEM images, some procedures for preprocessing should be done: image enhancement and image denoising. In this study, these processes are finished using MATLAB software.

3.3. Variation of $R_{ca}$ During Compression
The real porosity for each specimen at any stage of triaxial compression could be obtained via:
\[
\frac{n}{n_0} = \frac{\varepsilon_v}{1 - \varepsilon_v}
\]

where \(n_0\) is the initial porosity (equals to 0.476 in this study) and \(\varepsilon_v\) is the volumetric strain that can be directly obtained from triaxial test.

Therefore, the determination method for \(R_{ca}\) illustrated in section 2 is employed calculate the average contact area ratio \(R_{ca}\) under different compression degrees and the results are given in figure 5.

![Figure 5. Variation of \(R_{ca}\) with axial strain \(\varepsilon_1\).](image)

Figure 5 shows the variation of the average contact area ratio \(R_{ca}\) with the axial strain \(\varepsilon_1\) during compression. It can be seen that as \(\varepsilon_1\) develops, \(R_{ca}\) gradually increases firstly and then decreases when the axial strain reaches about 20%. This is due to: when the axial strain reaches 20%, inhomogeneous deformation became obvious, middle part of specimens swelled, and the cross-section expanded, resulting in the decrease of \(R_{ca}\).

4. Applications of Index \(R_{ca}\) in Evaluating the Soil Properties

4.1. Influence on Soil Stiffness

Relationship between the tangent modulus \(E_t\) and average contact area ratio \(R_{ca}\) is shown in Figure 6 (taking specimens 1-4 and 2-4 as example). An approximate linear dependence can be found. In other words, in the process of elastoplastic deformation, soil stiffness increases linearly with the increase of contact area ratio \(R_{ca}\). Furthermore, the \(E_t\)-\(R_{ca}\) curves under two different confining pressures in figure 6 are approximately parallel, which indicates that the correlation between \(E_t\) and \(R_{ca}\) is not affected by the confining pressure to some extent, but an intrinsic nature of soil.

![Figure 6. Relationship between \(R_{ca}\) and \(E_t\).](image)
4.2. Influence on Transfer of Effective Stress
Judging from the results of macro and micro tests in this paper, the contact area ratio for soft clay generally varies in the range of 10% ~ 40%. It may imply that the contact area between soil particles isn’t so small that can be ignored, especially during the compression process, as the compaction of soil, contact area will significantly change.

However, the derivation of effective stress principle in classical soil mechanics is often established on the assumption that the contact area between soil particles is negligible. Although many scholars have questioned on this, there is still no reliable evidence. This topic was also discussed by Rong et al [15], who reformulated the expression of effective stress principle and introduced coefficient $\alpha$ (coefficient of pore pressure transfer) which represents the proportion of the particle surface to which the pore pressure is applied. According to this definition, there should be an approximate relationship between $\alpha$ and $R_{ca}$, i.e.

$$R_{ca} = 1 - \alpha$$

(4)

Comparing the potential range of coefficient $\alpha$ in reference [15] and the statistical value of $R_{ca}$ in this study, it can be inferred that equation (4) is approximately effective.

5. Conclusions
This paper presents the methodology for determining the microscopic index $R_{ca}$ and its evolution law in the process of compression. Meanwhile, two potential applications of $R_{ca}$ in describing mechanical properties soil are discussed. The main findings of this research can be summarized as:

(1) Starting from the principle of SEM imaging, 3D porosity for soil microstructure is established, which is employed to determine the threshold based on the connection between the macro and micro properties of soils. The threshold is then used to binarize SEM grayscale images, as a result, the contact area ratio $R_{ca}$ are defined.

(2) Triaxial compression test in macro level and SEM test in micro level are combined to explore the evolution law of contact area ratio $R_{ca}$ during the compression process. The results show that as the deformation increases, $R_{ca}$ increases at first and then decreases. And the increase is due to the compaction of soil; while the decrease is induced by inhomogeneous deformation which will lead to the expansion of cross section.

(3) By comparing the relationship between tangent modulus $E_i$ and contact area ratio $R_{ca}$ during the compression process, a negative correlation between them can be observed. And the increase of $R_{ca}$ will cause the linear increase of the soil stiffness, this correlation is independent on the confining pressure, which could be an intrinsic attribute of soil. Besides, it can also be inferred that the contact area ratio $R_{ca}$ has a certain influence on the transfer mechanism of effective stress in the soil.

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