Design of inter-stations $H_\infty$ decoupling controller in VSC-HVDC system

Yu Yu$^{1,3}$,Hongliang Li$^1$,Qijiang Chen$^2$,Ye Tian$^1$ and Lichao Xu$^1$

$^1$Hubei Key Laboratory for High-efficiency Utilization of Solar Energy and Operation Control of Energy Storage System, Hubei University of Technology, Wuhan, 430068
$^2$China City Environment Protection Engineering Limited Company, Wuhan, China

$^3$E-mail: minetooybox@163.com

Abstract. The purpose of this work is to reduce the mutual interference between converter stations when the AC power or DC voltage fluctuates in VSC-HVDC system. The global small signal mathematical model of VSC-HVDC system is established firstly. Then, according to the principle of mixed sensitivity $H_\infty$ robust control, the appropriate weight function is selected and the decoupling controller between stations is deduced. In addition, the high-order controller deduced is reduced to an appropriate order which is beneficial to engineering application by using the balanced truncation method of coprime factors. Finally, the electromagnetic transient simulation is carried out to verify that the $H_\infty$ decoupling controller between stations can be effectively weakened the interaction between two converter stations and improves the dynamic performance and robust stability of the system.

1. Introduction

VSC-HVDC system can be effectively applied in renewable power integration, city DC distribution and passive load supplying [1-3]. However, due to the weak inertia of DC lines, a complex cross coupling exists between converter stations in VSC-HVDC system. The independence and reliability of system will be reduced if the interference between stations is neglected in the design of controller [4,5].

Aiming at the decoupling of VSC-HVDC system, literature [6] proposed a decoupling algorithm based on dynamic feedforward compensation which has better decoupling effect than traditional current vector control strategy. Literature [7-8] proposed an improved feedforward control strategy for the current loop in traditional PI double closed-loop structure, enhanced the anti-disturbance ability of the system. Literature [9-10] proposed a control strategy based on internal model control, which realized the decoupling control of active and reactive power. Literature [11] combined the advantages of integral sliding mode control and direct power control, and proposes a dual integral sliding mode direct power control strategy. Literature [12] deduced the Euler-Lagrange model of VSC-HVDC system in dq coordinate system, and designed a current inner loop passive decoupling controller. In the above references, the decoupling control only solves the internal active and reactive power decoupling of single converter station, ignoring the coupling relationship between converter stations. Literature [13] used state variable feedback to realize decoupling between converter stations, however, the calculation of feedback coefficient is complexity, and more physical variable need to be measured.

In this paper, the global small signal mathematical model of VSC-HVDC system is deduced. Based on the mixed sensitivity $H_\infty$ control theory, the inter-stations decoupling controller is calculated. The controller is convenient to design and does not need to measure additional physical variables.
Feedback decoupling between stations is achieved by introducing the controlled variable of the other station. Finally, the decoupling effect is verified by MATLAB and PSCAD simulation.

2. Topology and small signal model of VSC-HVDC system

The circuit topology of the VSC-HVDC system is shown in Figure 1. The AC grid1 connects to the rectifier station and the grid 2 is connected to the inverter station. The rectifier adopts the constant active power \( P_1 \) and the constant reactive power \( Q_1 \) control strategy, and the inverter adopts the constant DC voltage \( U_{di} \) and the constant reactive power \( Q_2 \) control strategy.

![Figure 1. Topology of VSC-HVDC system.](image)

The small signal model is the basis of the controller design [14-15]. The global small signal model of the VSC-HVDC system includes the converter model, AC power source model, transformer and reactor model, DC regulated capacitor model, and the DC line model.

2.1. VSC Converter Equivalent Mathematical Model

The VSC converter is simplified to the inertia link, and the mathematical model of the VSC converter is shown as equation (1). When \( i = 1 \), the physical variables describe the rectifier station. When \( i = 2 \), the physical variables describe the inverter station. \( u_{cdi} \) and \( u_{cqi} \) are the d and q-axis components of AC voltage of converter, \( u_{cdi}^* \) and \( u_{cqi}^* \) are the d and q-axis components reference values of the SPWM modulated wave. The switching delay \( \tau = 1.5T \), \( T \) is the carrier period. \( k \) is the converter gain, \( U_{di} \) is the DC voltage of the converter, \( S \) is the amplitude of the SPWM carrier.

\[
\begin{bmatrix}
  u_{cdi} \\
  u_{cqi}
\end{bmatrix} = \frac{k}{\tau s + 1} \begin{bmatrix}
  u_{cdi}^* \\
  u_{cqi}^*
\end{bmatrix}
\]

\[k = \frac{U_{di}}{2S}\]

2.2. Mathematical Model of AC Side of Converter

The simplified model of the converter is shown in Figure 2, \( u_{ai} \) is the voltage at the AC bus, \( u_{ci} \) is the AC voltage of the VSC converter, \( i_{ai} \) is the AC current of the converter, \( R_i + j\omega L_i \) is the equivalent impedance of the transformer and reactor of AC side. \( E_i \) is the voltage of the AC grid \( i \), \( R_{ai} \) and \( L_{ai} \) are the internal impedance of the power supply. The active power of the rectifier station flows from the AC side to the DC side. The structure of the VSC inverter station is the same as rectifier station, but power flow direction is opposite. The mathematical model of the transformer and reactor in the dq axis is shown as equation (2). In the equation, \( u_{adli} \) and \( u_{aqli} \) are the AC bus voltage d and q-axis components, \( i_{adli} \) and \( i_{aqli} \) represent the AC line current d and q-axis components, and \( \omega \) is dq axis rotation angle frequency. Similarly, the mathematical model of the AC power source is shown as equation (3). Since the supply voltage vector \( E_i \) is selected as the d-axis of dq rotating coordinate, the d-axis component of the supply voltage is \( E_i \) and the q-axis component is 0.

![Figure 2. Equivalent circuit of the converter.](image)
2.3. Mathematical Model of DC regulated capacitor

In Figure 2, \( i_{dc} \) is the converter DC current, \( i_{dc} \) is the DC transmission line current, \( C_i \) is the shunt capacitance, and \( i_{cji} \) is the capacitor current, \( U_{di} \) is the DC voltage of the VSC converter. The active power and reactive power of the converter in the dq coordinate system are shown as equation (4).

\[
\begin{align*}
\mathbb{P}_c &= \frac{3}{2} \left( u_{cd_i} i_{cd_i} + u_{cq_i} i_{cq_i} \right) \\ \mathbb{Q}_c &= \frac{3}{2} \left( u_{cd_i} i_{cq_i} - u_{cq_i} i_{cd_i} \right)
\end{align*}
\]

The DC active power \( P_{dc} = i_{dc} U_{dc} \). \( i_{dc} \) is shown as equation (5) neglecting the power loss on the VSC converter. The voltage and current of the DC regulated capacitor can deduce as equation (6).

\[
\frac{C_i d U_{di}}{2} = i_{di} - i_{dc}
\]

2.4. Mathematical Model of DC Line

The cascaded series-connected pi-type model of DC line model is shown as Figure 3. \( i_i \) represents the DC line current. The DC line impedance is simplified to \( R_d + j\omega L_d \), where \( R_d = R_{d1} + R_{d2} + \cdots + R_{dn} \).

\[
L_{di} \frac{d i_{di}}{dt} = U_{d1}/2 - U_{d2}/2 - R_{cd_i} i_{di}
\]

2.5. Global Small Signal Model of VSC-HVDC System

Combining the equation (1) ~ (7), the small-signal model of the VSC-HVDC system could be written in the form of state equation as equation (8) shows. State variables \( x = [\Delta i_{d1} \Delta i_{sq1} \Delta u_{d1} \Delta u_{sq1} \Delta u_{d2} \Delta u_{sq2} \Delta u_{d3} \Delta u_{sq3} \Delta u_{d4} \Delta u_{sq4}]^T \). The variables with triangular prefixes are the small signal parameters. Input variables \( u = [\Delta u_{d1} \Delta u_{d2} \Delta u_{dq1} \Delta u_{dq2} \Delta u_{dq3} \Delta u_{dq4}]^T \), where \( \Delta u_{dq1} = \Delta u_{sq1} \), \( \Delta u_{dq2} = \Delta u_{sq2} \), \( \Delta u_{dq3} = \Delta u_{sq3} \), \( \Delta u_{dq4} = \Delta u_{sq4} \).

\[
\begin{align*}
L_{di} \frac{d i_{di}}{dt} &= U_{d1}/2 - U_{d2}/2 - R_{cd_i} i_{di} \\
&= \mathbb{P}_c - \mathbb{Q}_c - R_{cd_i} i_{di}
\end{align*}
\]
ω₀ is the rotation angular frequency of grid voltage. The system matrix \( A \) is a 15×15 square matrix, and the input matrix \( B \) is a 15×4 matrix. The output variables are \( y = [\Delta P_1, \Delta Q_1, \Delta u_2, \Delta Q_2] \). The output matrix \( C \) is a 4×15 matrix, and \( D \) is a 4×4 square matrix. The transfer function matrix of controlled object \( G(s) \) can be obtained by the state space equation, which is a 4×4 order transfer function matrix.

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]  

(8)

3. Design of inter-stations \( H\infty \) decoupling controller of VSC-HVDC system

Control target of S/T/KS mixed sensitivity \( H_s \) control [16] is shown as equation (9), where \( S=(I+G)\overline{I} \) is the sensitivity function, \( T=I-S \) is the complementary sensitivity function and \( KS \) is the input sensitivity function. \( W_w, W_T \) and \( W_p \) are weight functions. \( G \) is the controlled object, \( K \) is the \( H_s \) controller. The typical weight function setting method [17-18] is shown as equations (10).

\[
\min \| N(K) \|_\infty , N = \begin{bmatrix} W_{KS} \\ W_T \\ W_{pS} \end{bmatrix}
\]

(9)

\[
W_p = \frac{s/M_s + \omega_b}{s + \omega_b \omega_l}, W_T = \frac{s + \omega_l/M_l}{A_s + \omega_l}
\]

(10)

\( M_s \) and \( M_l \) are stability boundaries related to system robustness and dynamic performance. \( M_s<2, M_l<1.25 \) is usually required. \( \omega_b \) and \( \omega_l \) are the closed-loop bandwidth of \( S \) and \( T \) respectively. \( W(s) \) has low-pass characteristics and \( W_T(s) \) has high-pass characteristics. The closed-loop bandwidth of the system is between the two crossover frequencies. \( A_s \) and \( A_l \) are the maximum tracking error values, which are usually set to zero. According to the control requirements, the parameters of the weight function are set as: \( \omega_b=400, \omega_l=800, A_s=1/10000, A_l=1/1000, M_s=1.9, M_l=1. \)

\[
K(s) = \frac{u(s)}{y(s)} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}
\]

(11)

According to the mixed sensitivity weight function, the decoupling controller is obtained and reduced to an appropriate order by the balanced truncation method [19]. The controller \( K(s) \) of the rectifier and inverter station are shown in equations (11).

4. Simulation

The parameters of the VSC-HVDC system are shown in Table 1. The small-signal mathematical model and electromagnetic transient model are established in MATLAB and PSCAD respectively.

4.1. Decoupling Effect of Inter-Stations \( H\infty \) Decoupling Controller on MATLAB

The compares of the step response of \( U_{d2} \) between the \( H_s \) control system and the PI control system are shown in Figure 4. Controlled variables of \( H_s \) control system can respond quickly to the reference value from Figure 4(a). Compared with PI control system, the overshoot of the response is smaller and the adjustment time to achieve stability is shorter in \( H_s \) control system. As Figure 4(b)–(d) show, the disturbances of other three controlled variables influenced by \( U_{d2} \) in \( H_s \) control system is significantly little than that of PI control system. The results verify that the \( H_s \) controller proposed in this paper has satisfactory inter-stations decoupling effect.

4.2. Inter-Stations \( H\infty \) Decoupling Control System Electromagnetic Transient Simulation

The active power command of the rectifier station changes from 13MW to -13MW at 15s. The simulation results are shown in Figures 5. As Figure 5(a) shows, the reaction of \( P_l \) of \( H_s \) control system is quicker and smoother than that of PI control system. As Figure 5(b)–(d) show, the other
three control variables are significantly deviated from the steady state in the PI control system and the disturbances in the $H_\infty$ control system are smaller. In conclusion, the $H_\infty$ controller calculated in this paper has better decoupling effect than PI controller.

**Table 1.** The Parameters of the VSC-HVDC System.

| Parameter                          | Value           |
|------------------------------------|-----------------|
| DC line Equivalent resistance $R_d/\Omega$ | 1               |
| DC line Equivalent inductance $L_d/H$   | 0.01            |
| Grid-connected transformer $T_1$ ratio kV/kV | 115/25          |
| AC grid 1 initial phase angle $\phi$ | 0               |
| Connecting reactor resistance $R_1/\Omega$ | 0.8             |
| Connecting reactor inductance $L_1/H$ | 0.05            |
| AC grid 1 equivalent internal resistance $R_m/\Omega$ | 0               |
| AC grid 1 equivalent inductance $L_m/H$ | 0.01            |
| Active power control target $P_1/MW$   | 12              |
| Reactive power control target $Q_1/Mvar$ | -5              |
| Grid-connected transformer $T_2$ ratio kV/kV | 115/25          |
| AC grid 2 initial phase angle $\phi$ | 0               |
| Connecting reactor resistance $R_2/\Omega$ | 0.8             |
| Connecting reactor inductance $L_2/H$ | 0.05            |
| AC grid 2 equivalent internal resistance $R_m/\Omega$ | 0               |
| AC grid 2 equivalent inductance $L_m/H$ | 0.01            |
| DC voltage control target $U_d/kV$   | 60              |
| Reactive power control target $Q_2/Mvar$ | 3               |

**Figure 4.** Step Response of DC Voltage of Inverter Station.
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Figure 5. Comparison of $H_{\infty}$ Controller and PI Controller under Active Power Fluctuation.

5. Conclusions
The weak inertia of DC line result in the complex coupling relationship between converter stations in VSC-HVDC system when the AC power or DC voltage fluctuates. This paper designed the inter-stations S/T/KS mixed sensitivity $H_{\infty}$ decoupling controller based on the global small signal mathematical model of VSC-HVDC system. When DC voltage float, such as the short-term fault of DC line, the disturbances of active and reactive power variables of rectifier station influenced by DC voltage in $H_{\infty}$ control system is significantly little than that of PI control system. Then, the $H_{\infty}$ decoupling control system has stronger fault recovery capability and can avoid accident expansion. When large power fluctuates, such as active power reversal in rectifier station, the disturbance of the DC voltage and the reactive control variables of inverter station in $H_{\infty}$ control system are smaller than that of PI control system. It’s proved that the $H_{\infty}$ decoupling controller proposed in this paper realizes decoupling control and weakens the interference between the two converters.

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References
[1] HE Zhiyuan, LIU Dong, PANG Hui 2018 Research of Simulation Technologies of VSC-HVDC and DC Grids Power System Technology 42(01): 1-12
[2] MA Weimin, WU Fangjie, YANG Yimingo 2014 Flexible HVDC Transmission Technology’s Today and Tomorrow High Voltage Engineering 40(08): 2429-2439
[3] XU Zheng, XUE Yinglin, Zhang Zheren 2014 VSC-HVDC Technology Suitable for Bulk Power Overhead Line Transmission Proceedings of the CSEE 34(29): 5051-5062
[4] S. Tan, X. Du and G. Wang 2018 Research on Interaction Between AC side and DC side in VSC-HVDC 2018 IEEE International Power Electronics and Application Conference and Exposition, Shenzhen pp. 1-5
[5] Jiahui Chen, M. Han, Lingfei Xiong 2016 Electromechanical transient modeling without coupling terms for VSC-HVDC 2016 IEEE PES Asia-Pacific Power and Energy Engineering Conference, Xi’an pp. 1029-1033
[6] SHAO Bingbing, ZHANG Shuqiang 2019 Power Coupling Characteris-tics and Decoupling Strategy of VSC-HVDC System Connected to the Weak AC Power Grid Power System Technology: 1-11[2019-03-22]. https://doi.org/10.13335/j.1000-3673.pst.2018.2105.
[7] DENG Qi, ZHANG yingmin, LI Xingyuan 2017 An improved feed-forward control strategy for interconnected VSC-HVDC transmission system Electrica Measurement & Instrumentation 54(22): 48-53+110
[8] CHEN Qian, LI Chong, JIN Yuqing 2014 Optimization of Grid-connected VSC Controller by Decoupling Models High Voltage Engineering 40(08): 2478-2484
[9] CHU Zhengchao, Zhang Yingmin, LI Xingyuan 2016 Research on Internal Modal Decoupling Control of a Flexible VSC-HVDC Sichuan Electric Power Technology 39(01): 12-17
[10] GUO Lei, ZHANG Yingmin, LI Xingyuan 2016 A kind of control strategy for VSC-HVDC system decoupling and harmonics supression Electrical Measurement & Instrumentation 53(02): 34-39+44
[11] ZHENG Lianqing, ZHU Meng, LU Zhiguo 2013 Design of Double Integral Sliding Mode Decoupling Controller for VSC-HVDC System Journal of Hunan University(Natural Sciences) 40(02): 52-58
[12] FAN Ximing, GUAN Lin, HE Jianming 2013 Decoupling Passivity Control of VSC-HVDC Connected Wind Power Transactions of China Electrotech-nical Society 28(10): 311-319
[13] YANG Jie 2015 Small Signal Model and Decouple control of Voltage Source Converter based Multi-Terminal Direct Current Wu-han University
[14] YANG Jie, LIU Kaipei, YU Yu 2015 Small Signal Modeling for VSC-HVDC Used in AC Grid Interconnection Proceedings of the CSEE 35(09): 2177-2184
[15] C. Guo, W. Liu, J. Zhao and C. Zhao 2018 Impact of control system on small-signal stability of hybrid multi-infeed HVDC system IET Generation, Transmission & Distribution 12(19) pp. 4233-4239
[16] Al Sil, T K Gangopadhyay, S Paul and A K 2009 Maitra, Design of robust Power System Stabilizer using H∞ mixed sensitivity technique 2009 International Conference on Power Systems, Kharagpur pp. 1-4
[17] M Y Hammoudi, O Kraa, R Saadi, M Y Boukhlouf 2018 Non linear control of a Fuel Cell Interleaved Boost Converter using Weighted Mixed Sensitivity H∞ 2018 International Conference on Electrical Sciences and Technologies in Maghreb, Algiers pp. 1-5
[18] A. Banerjee and N R Chaudhuri 2016 Robust Damping of Inter-Area Oscillations in AC-MTDC grids using H∞ Mixed-sensitivity ap-proach 2016 IEEE Power and Energy Society General Meet-ing, Boston, MA pp. 4-5
[19] S Pandey, R S Yadav, S K Chaudhary, K G 2018 Upadhyay and S. P. Singh. Hankel Norm Approximation of A Highly Unstable System 2018 5th IEEE Uttar Pradesh Section International Conference on Electrical, Electronics and Computer Engineering (UPCON), Gorakhpur pp. 1-5