$C$, $P$, and $CP$ asymmetry observables based on triple product asymmetries

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The discrete symmetries $C$, $P$ and $CP$ are known to be violated by the weak interaction. It is possible to probe the breaking of these symmetries using asymmetries constructed from triple products based on the decay of some particle $M$ to a four body final state. These proceedings discuss the full set of possible asymmetries that can be probed and applications to various measurement scenarios, focusing mostly on charm mesons and baryons. The ramifications of what can be learned from such measurements are also discussed.

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1 Introduction

The discrete symmetries of parity ($P$), charge conjugation ($C$), time (or motion) reversal ($T$), and the combination $CP$ are known to be violated in weak interactions, but are conserved in both electromagnetic and strong interactions. The overall combination $CPT$ is experimentally conserved in locally gauge invariant quantum field theories such as the Standard Model of particle physics (SM), and a corollary of this is that Lorentz symmetry is also conserved in such theories. At some scale it is expected that $CPT$ will be violated as our understanding of subatomic physics adapts to account for quantum gravity. Parity violation was discovered by Wu et al. in 1957 [1], and $CP$ was found to be violated in 1964 by Christenson et al. [2]. For the past 50 years tests of $CP$ violation in quark interactions have provided a consistent picture: all known violations are consistent with the 1964 discovery. Most discrete symmetry violation measurements have concentrated on testing $CP$ given the implications noted by Sakharov that both $C$ and $CP$ violation are required for the Universe to evolve from the Big Bang to a matter dominated state as observed today [3]. However, the observed level of $CP$ violation is insufficient to be able to explain this observed matter dominance of the Universe. These proceedings discuss tests of these symmetries using asymmetries obtained using scalar triple products constructed from spins or momenta of the final state particles. We can consider the decay of some particle $M$ to a four body final state $abcd$ and the $C$ conjugate process $M \to \bar{abcd}$, where $ab$ and $cd$ can be used to construct decay planes in the centre of mass of the decaying particle. Considering the three momenta of the final state particles it is straightforward to see that a scalar triple product $\vec{p}_a \cdot (\vec{p}_b \times \vec{p}_c)$ is even under $C$ and $CPT$ and odd under $P, T$ and $CP$. To test a symmetry we need to identify a process $\psi \equiv M \to abcd$ and the $C$ conjugate under the symmetry. If $A$ is non-zero these probabilities differ and the symmetry is violated. We can write down the rate for decays of $M$ with a positive triple product (upward going decay) as $\Gamma_+$ and that with a negative triple product (downward going) as $\Gamma_-$. The corresponding anti-particle rates are denoted by $\bar{\Gamma}_\pm$. By considering $C, P$ and

\[ A = \frac{P(\psi') - P(\psi)}{P(\psi') + P(\psi)}. \]  

A second asymmetry can be constructed by considering the $C$ conjugate decay under the symmetry. If $A$ is non-zero these probabilities differ and the symmetry is violated. We can write down the rate for decays of $M$ with a positive triple product (upward going decay) as $\Gamma_+$ and that with a negative triple product (downward going) as $\Gamma_-$. The corresponding anti-particle rates are denoted by $\bar{\Gamma}_\pm$. By considering $C, P$ and

*Charge conjugation is implied throughout unless otherwise specified, and the pairings defined depend on the possible physical states that are reconstructable. A more general amplitude analysis based on a model summing over all relevant interfering amplitudes is desirable, however construction of a relevant model to fit to data requires experimental input.*
$CP$ acting on the four $\Gamma$s we can construct six distinct asymmetries; these are

$$A_P = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}, \quad \overline{A}_P = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}, \quad (2)$$

$$A_C = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+}, \quad \overline{A}_C = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+}, \quad (3)$$

$$A_{CP} = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}, \quad \overline{A}_{CP} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+}. \quad (4)$$

The subscript on $A$ denotes the symmetry used to construct the conjugate pair.

We can construct another six asymmetries by considering the remaining symmetry transformations on $A$ and $\overline{A}$ in turn. These are given by

$$a_P^C = \frac{1}{2} (A_P - \overline{A}_P),$$
$$a_{CP}^P = \frac{1}{2} (A_P + \overline{A}_P),$$
$$a_C^P = \frac{1}{2} (A_C - \overline{A}_C),$$
$$a_{CP}^C = \frac{1}{2} (A_C + \overline{A}_C),$$
$$a_P^{CP} = \frac{1}{2} (A_{CP} - \overline{A}_{CP}),$$
$$a_C^{CP} = \frac{1}{2} (A_{CP} + \overline{A}_{CP}). \quad (5)$$

Here these secondary asymmetries are denoted by a lower case $a$, the superscript corresponds to the first symmetry used and the subscript corresponds to the second one. We can determine which symmetry is under scrutiny by multiplying the superscripts and subscripts together. For example $a_C^P$ is a test of $CP$, $a_{CP}^P$ a test of $CP^2 = C$ etc.

In general these asymmetries receive contributions from all interactions, and the interest is in isolating effects that are dominated by weak interactions (i.e. theoretically clean), or where that may not be possible, to isolate effects that can either be understood in the longer term, or signify a non-trivial weak interaction effect even in the presence of pollution from strong force induced final state interactions (i.e. soft QCD and re-scattering). A concrete example of this issue is discussed below in the context of the measurement of $\alpha_b$ from $\Lambda_b$ decays. Hence interpretation of these asymmetries depends on the decay under study (and hence the model required to interpret data), and an example where one considers two interfering amplitudes is also discussed.

An important aside is to consider the language used in the literature today for discrete symmetry violation tests. In the past some of the literature has referred to one of these $CP$ violating triple product asymmetries as a manifestation of $T$
violation. Consider the process $abcd \rightarrow M$; this never happens hence it follows that it is not possible to construct the asymmetry of Eq. (1). One can invoke CPT as an argument to call a particular CP violation effect a $T$ violation effect; however that is somewhat futile as it trivialises the problem to the level where all CP violation effects are equivalently $T$ violation effects. For a long time people have been proposing (and performing) valid tests of $T$ violation and searching for CPT violation. In particular $T$ violation has been studied in kaons for several decades, culminating in the measurement of Kabir’s asymmetry by CPLEAR [5, 6]. More recently Banuls and Bernabeu [7] extended Kabir’s approach to use flavour and CP filter pairs to distinguish between discrete symmetries for entangled pairs of neutral mesons. This provides four additional tests of $T$, which unlike Kabir’s asymmetry are only tests of $T^\dagger$. There are additional experiments focused on testing CPT via edm measurements as well as using particle decay, mass differences and so on. The remainder of these proceedings refer to triple product asymmetries according to the symmetry under test and does not consider the trivialisation induced by invoking CPT as worthy of further note. We can summarise the preceding discussion with the following mis-appropriated quote from Douglas Adams’ Hitchiker’s Guide to the Galaxy “One of the things Ford Prefect had always found hardest to understand about humans was their habit of continually stating and repeating the very very obvious”, such as triple product asymmetries are not $T$ violating by themselves. It has been noted that with the extra ingredient of entanglement we can use triple products to probe $T$ violation (and hence CPT) [9].

2 Are these asymmetries of any use?

The twelve asymmetries listed above can be measured in a given decay, which prompts the question, what if anything can we learn from them. A fairly general model, following for example Valencia [10], could be used where one sums over arbitrary $S$, $P$ and $D$ wave amplitudes to identify which of the asymmetries can be driven by the existence of non-zero weak phase differences even in the presence of non-zero strong phase differences. However it is sufficient to illustrate the point with just two interfering amplitudes given by

$$A_+ = a_1 e^{i(\phi_1 + \delta_{1,1})} + a_2 e^{i(\phi_2 + \delta_{2,1})},$$

$$A_- = a_1 e^{i(\phi_1 + \delta_{1,-})} + a_2 e^{i(\phi_2 + \delta_{2,-})},$$

$$\overline{A}_+ = a_1 e^{i(-\phi_1 + \delta_{1,1})} + a_2 e^{i(-\phi_2 + \delta_{2,1})},$$

$$\overline{A}_- = a_1 e^{i(-\phi_1 + \delta_{1,-})} + a_2 e^{i(-\phi_2 + \delta_{2,-})}.$$ (6) (7) (8) (9)

It should be noted that one has to be careful with operator definitions when allowing CPT to be violated, and there is a second possible interpretation that could indicate that the non-zero $T$ violation measurement from $BABAR$ could be a CPT violation manifest as a fake $T$ violation [8].

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measurements by FOCUS reported values of $A$ vs fake asymmetries as in Eq. (12). If one were to extend the restrictive labeling used in some of the literature true vs fake asymmetries as in Eq. (12). If one were to extend the restrictive labeling used in some of the literature, one can only be non-zero for a non-zero weak phase difference; fake should be read as

\begin{align}
A_P &\propto r \sin \Delta \phi (\sin \Delta \delta_+ - \sin \Delta \delta_-) + r \cos \Delta \phi (\cos \Delta \delta_+ - \cos \Delta \delta_-) \quad (10) \\
\overline{A}_P &\propto r \sin \Delta \phi (\sin \Delta \delta_+ - \sin \Delta \delta_-) + r \cos \Delta \phi (\cos \Delta \delta_+ - \cos \Delta \delta_-) \quad (11) \\
A_C^P &\propto \frac{[2r^2 \cos \Delta \phi \sin(\Delta \delta_+ - \Delta \delta_-)] + r(1 + r^2)(\sin \Delta \delta_+ - \sin \Delta \delta_-)}{\sin \Delta \phi} \quad (12) \\
A_{CP}^P &\propto (\cos \Delta \delta_+ - \cos \Delta \delta_-)(r^2(\cos \Delta \delta_+ + \cos \Delta \delta_-) + r(1 + r^2) \cos \Delta \phi) \quad (13) \\
A_C &\propto 2r \sin[\Delta \delta_-] \sin[\Delta \phi] \quad (14) \\
\overline{A}_C &\propto 2r \sin[\Delta \delta_+] \sin[\Delta \phi] \quad (15) \\
A_C^P &\propto r \left[ (1 + r^2)(\sin \Delta \delta_+ - \sin \Delta \delta_-) + 2r \cos \Delta \phi \sin[\Delta \delta_+ - \Delta \delta_-] \right] \sin \Delta \phi \quad (16) \\
A_{CP}^P &\propto r \left[ (1 + r^2)(\sin \Delta \delta_+ + \sin \Delta \delta_-) + 2r \cos \Delta \phi \sin[\Delta \delta_+ + \Delta \delta_-] \right] \sin \Delta \phi \quad (17) \\
A_{CP} &\propto r \cos \Delta \phi(\cos \Delta \delta_+ - \cos \Delta \delta_-) + r \sin \Delta \phi(\sin \Delta \delta_+ + \sin \delta_-) \quad (18) \\
\overline{A}_{CP} &\propto r \cos \Delta \phi(\cos \Delta \delta_- - \cos \Delta \delta_+) + r \sin \Delta \phi(\sin \Delta \delta_- + \sin \delta_+) \quad (19) \\
A_{CP}^P &\propto r \left[ (1 + r^2)(\sin \Delta \delta_+ + \sin \Delta \delta_-) + 2r \cos \Delta \phi \sin(\Delta \delta_+ + \Delta \delta_-) \right] \sin \Delta \phi \quad (20) \\
A_{CP}^P &\propto r(\cos \Delta \delta_+ - \cos \Delta \delta_-)[r(\cos \Delta \delta_+ + \cos \Delta \delta_-) + (1 + r^2) \cos \Delta \phi]. \quad (21)
\end{align}

where $\Delta \phi = \phi_1 - \phi_2$, $\Delta \delta_{\pm} = \delta_{1,\pm} - \delta_{2,\pm}$ and $r = a_{1,2}/a_2$. One can see from this that six asymmetries can only be non zero for $\sin \Delta \phi \neq 0$. These are $A_C^P$, $A_C$, $\overline{A}_C$, $A_P^P$, $A_{CP}^P$, and $A_{CP}^P$. The asymmetries $A_C$, $\overline{A}_C$ have the familiar form of a time-integrated (a.k.a. direct) CP asymmetry. The remaining asymmetries can be non zero under more relaxed conditions that include non-zero strong phase differences even if weak phase differences are zero. The expected result that $A_T = A_C^P \propto \sin \Delta \phi$ can be seen in Eq. (12). If one were to extend the restrictive labeling used in some of the literature true vs fake asymmetries, these quantities then the six asymmetries $A_C^P$, $A_C$, $\overline{A}_C$, $A_P^P$, $A_{CP}^P$, and $A_{CP}^P$ would be called true, and the others fake. For this labeling to be placed correctly in context one has to decode the shorthand; true should be read as “can only be non-zero for a non-zero weak phase difference”; fake should be read as “can be non zero for non-zero weak and/or strong phase differences”.

3 Existing measurements in charm

The following four body final states have been studied in charm decays: $D^0 \rightarrow K^+K^-\pi^+\pi^-$, $D^\pm \rightarrow K^\pm K^0\pi^+\pi^-$, and $D_s^\pm \rightarrow K^\pm K^0_S\pi^+\pi^-$. Early measurements by FOCUS reported values of $A_P$, $\overline{A}_P$, and $a_{1,2}^P$ along with the direct CP asymmetry. The results obtained were consistent with zero for all asymmetries. Following on from this BABAR were able to establish non-zero values of $A_P$ and $\overline{A}_P$.
for the $D^0$ and $D_s$ mode. The $D^0$ result has been confirmed by LHCb. The results obtained are listed in Table 1. Recently $\text{BaBar}$ reanalysed their data in the context of the full set of asymmetries noted here [15]. An interesting issue is highlighted by the LHCb work; previous measurements had integrated over the di-meson invariant mass distribution due to the lack of sufficient statistics to perform an amplitude analysis. With the advent of LHCb this limitation goes away and they have produced results binned in $KK$ and $\pi\pi$ invariant masses in anticipation of performing such an amplitude analysis. However, in addition to the $KK$ and $\pi\pi$ masses it is also interesting for the experiments to study $K\pi$ mass distributions. These distributions could inform us if there are $S$ or $P$ wave $K\pi$ contributions in the data including, but not limited to, the $K^*(892)$.

Table 1: Experimental results published for triple product asymmetries in charm decays

| Expt.    | $A_P$             | $\overline{A}_P$ | $a_C^P$         |
|----------|-------------------|------------------|-----------------|
| $D^0 \to K^+K^-\pi^+\pi^-$ |                  |                  | $0.010 \pm 0.057 \pm 0.037$ |
| FOCUS    | $-$0.069 ± 0.007 ± 0.006 | $-$0.071 ± 0.007 ± 0.004 | 0.001 ± 0.005 ± 0.004 |
| $\text{BaBar}$ |                  |                  |                  |
| LHCb     | $-$0.0718 ± 0.0041 ± 0.0013 | $-$0.0755 ± 0.0041 ± 0.0012 | 0.0018 ± 0.0029 ± 0.0004 |

| $D^\pm \to K^\pm K^0_S\pi^+\pi^-$ |                  |                  |                  |
| FOCUS    |                  |                  | $0.023 \pm 0.062 \pm 0.022$ |
| $\text{BaBar}$ |                  |                  |                  |
| $+$0.011 ± 0.014 ± 0.006 | $+$0.035 ± 0.014 ± 0.007 | $-$0.012 ± 0.010 ± 0.005 |

| $D_s^\pm \to K^\pm K^0_S\pi^+\pi^-$ |                  |                  |                  |
| FOCUS    |                  |                  | $-$0.036 ± 0.067 ± 0.023 |
| $\text{BaBar}$ |                  |                  |                  |
| $-$0.099 ± 0.011 ± 0.008 | $-$0.072 ± 0.011 ± 0.011 | $-$0.014 ± 0.008 ± 0.003 |

Several sensitivity studies have been performed for $D \to VV$ and charm baryon decays at BES III. These are summarised in the papers by Kang and Li [16, 17]. In many modes one expects that a $\tau$-charm factory with $20\text{fb}^{-1}$ of data would be able to reach (sub)percent level measurements for a number of modes. LHCb should be able to measure a number of the modes studied in those references.

4 Other (potential) measurements

Interest in the use of triple product asymmetries to probe $CP$ violation has been around since the 1960s. In 1993 Heiliger and Seghal predicted that the decay $K_L \to \pi^+\pi^-e^+e^-$ would have a large $\mathcal{O}(14\%)$ effect manifest [18]. This large effect is the result of an interference between four amplitudes from: $K_L \to \pi^+\pi^-\gamma$ photon con-
version; bremsstrahlung from the $CP$ violating decay $K_L \to \pi^+\pi^-$; a $CP$ conserving magnetic dipole component; and finally a short distance component related to $s\bar{d} \to e^+e^-$. Shortly afterward this prediction was confirmed by KTeV and subsequently NA48 [19, 20]. Thus instead of the typical $O(10^{-3})$ or $O(10^{-6})$ level of $CP$ violation associated with $\varepsilon$ and $\varepsilon'$ in kaons we observe an $O(1)$ effect akin to the magnitudes of $CP$ violation effect observed in $B$ decays. In contrast the corresponding asymmetry measured in $K_S \to \pi^+\pi^-e^+e^-$ is found to be compatible with zero [20]. This logically follows from the fact that the $K_S \to \pi^+\pi^-$ bremsstrahlung term is $CP$ conserving along with the other three contributions in the $K_S$ decay. Other systems where one can perform triple product asymmetry measurements include $B$ decays, the decays of $b$ and $c$ baryons, and bosonic ($Z$, $H$, or associated production of pairs of bosons) decays to four body final states. It may also be possible to learn something about $\tau$ decays, where $\tau$ pairs produced at threshold have the advantage that one constrain the laboratory frame to be the same as centre of mass frame for the decaying lepton. Such measurements are discussed in [3].

5 Summary

In summary there are twelve triple product asymmetries that can be used to probe $C$, $P$ and $CP$. Eight of these have recently been introduced. Of these asymmetries, for a simple interfering amplitude model, six can only be non-zero if there is a non-zero weak phase difference. The remaining six asymmetries can be non-zero even if the weak phase difference is zero. Thus there are five new triple product asymmetries that can provide unambiguous tests of weak interactions. Collectively the six asymmetries $A_P^C$, $A_C$, $A_C^\prime$, $A_P^C$, $A_C^C$, and $A_C^{CP}$ provide us with the ability to perform unambiguous tests of $C$, $P$ and $CP$ in the search for non zero weak phase differences.

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