From the Cosmological Constant: Higgs Boson, Quantum Gravity, and Horizon Scales

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Abstract

We suggest discovery targets for the Higgs boson and quantum gravity mass scales in terms of a constraint between the electron and the cosmological constant. We go on to show that this constraint originates as a structural parameter of the electron, and leads naturally to a new cosmic horizon.

I. Introduction

The Higgs boson really needs no introduction, as it is the focus of world-wide attention by the particle physics community at FermiLab and CERN, where the Large Hadron Collider (LHC) was built primarily for its Higgs discovery potential. If found when the LHC begins science runs later this year, it will constitute the last missing link in the highly successful standard model of particle physics. Since its completion one-third of a century ago, the Glashow-Weinberg-Salam (GSW) theory of electroweak (EW) interactions has rendered precise predictions for the masses of the W and Z vector bosons [1]. In contrast, the GSW theory can only assert that the Higgs mass is given by its vacuum expectation value (vev \(\sim 246\) Gev), scaled by an unknown coupling constant, which is not computable from within the SM [2].

Outside the SM, versions such as the minimally supersymmetric standard model (MSSM) have produced many predictions over the last decade [3]. Indeed, some predictions of the Higgs mass lie outside the window over which current experimental searches focus on, such as conformal symmetry [4], with \(M_H \sim 10^{-33}\) ev, to extra-dimensional gauge fields [5], with \(M_H > 60\) Gev, and to 5-D versions of the SM [6] with \(M_H > 600\) Gev. The W and Z vector bosons were subsequently observed in the early 1980’s [7]. What has gone unobserved, despite intense efforts at CERN and Fermilab is the Higgs boson, the essential element to complete the GSW electroweak triad. As of 1996, continuing attempts to refine the value of the W boson and top quark masses has resulted in a shrinking of the theoretical search window for the Higgs [8], in concert with the experimental one released last year, from the CDF/D0 collaboration at Fermilab [9].

Experimental Discovery Window

In March of 2007, it appeared that a dramatic narrowing of the search window for the Higgs boson had arrived, when an indirect exclusion window was published [10], ranging from 114 - 144 Gev, at the 95% confidence level (CL). Two years later, in March 2009, the upper bound from the direct search window extended the range from the LEP2 lower bound of 114.4, up to and excluding the band from 160 - 170 Gev at 95% CL [11]. Recent analysis has reduced this exclusion band to 162 - 166 Gev [12]. Again in March of 2010, Erler [13] has produced a powerful global analysis, culled from electro-weak precision and Higgs search data which delineates a 90% CL search window for the Higgs boson, extending from 115 to 148 Gev, nearly identical to that of Blazey [10], as are seven predictions ranging from 117 to 146 Gev from extra-dimensional theories [3]. The probability distribution in [13] is highly skewed toward the lower end, due to prior LEP2 and Tevatron searches. We will see below that naturally occurring scales point strongly to this region as well, favoring a light Higgs.
Higgs boson-inflaton-electron connection

In a recent paper [14], Beck has presented a statistical argument that there is a constraint between the properties of the electron and the cosmological constant, which can be expressed more transparently through the relation,

\[ l_p^2 \Lambda = \left( \frac{l_p}{\alpha \lambda_e} \right)^6, \]

where \( l_p \) is the Planck length, \( \lambda_e \) is the electron Compton wavelength, \( \alpha \) is the fine structure constant, and \( \Lambda \) is the cosmological constant, corresponding to a value for the vacuum energy density which is in excellent agreement with the WMAP value, namely 3.9 Gev/m^3, vs. 4.1 Gev/m^3 from (1) [14]. The physical content of (1) is striking, in that the horizon entropy associated to the cosmological constant is just the inverse of the left-hand side, while the QED parameters \( \alpha \) and \( \lambda_e \) appear on the right. Indeed, the square of the Planck length can be viewed as the geometric mean of the zitterbewegung \( (\lambda_e/2) \) and Schwarzschild event horizon radii, essential to the black hole model of the Dirac electron [37-42]. This suggests that entropies associated to cosmological and electron scales are interdependent. Taking the square root of (1), we can write it in terms of the cosmological constant mass as:

\[ \left( \frac{m_e}{m_p} \right)^3 = 8\pi \left( \frac{m_{cc}}{m_p} \right)^2, \]

where \( m_p = 2.43 \times 10^{18} \) Gev is the reduced Planck mass. Thus (2) predicts,

\[ (3) \quad m_{cc} = 0.00237 \text{ ev} \]

So one could suggest that \( m_{cc}(m_e, \alpha) \), and indeed there are several models in which the cosmological constant is postulated to have a quantum electrodynamical (QED) origin [15-17]. Although it is not obvious whether the electron is the source of the cosmological constant or vice versa, it is clear that (2) constitutes an intriguing constraint between elementary particle and cosmological parameters where apriori, none is apparent in either standard model. Indeed for years now, it has been part of the folklore that the proton bears a remarkable relation to these parameters as well [18],

\[ (4) \log \left( \frac{m_e}{M_p} \right) = -11.6 \approx -\alpha^{-\frac{1}{2}} = -11.7 \]

As a result, one could argue that \( m_{cc}(M_p, m_e, \alpha) \), and is more naturally expressed in terms of just two dimensionless parameters,

\[ m_{cc}(\alpha, \mu), \text{ where } \mu = \frac{M_p}{m_e} = 1836, \text{ the proton-electron mass ratio.} \]

As Ozer points out [19], there is a remarkable scale equivalence between the Higgs, Inflaton, and, cosmological constant masses,

\[ (5) \frac{m_{cc}}{M_H} \approx \frac{M_H}{M_I} \sim 10^{-14} \]

Since the vacuum energies associated to the epochs of Hubble acceleration and inflation can be modeled by Higgs fields [20-21],we conjecture that this proportion may correspond to an equality:

\[ (6) \quad M_H^2 = m_{cc} M_I \]

Indeed, dimensional analysis suggests a natural, model-independent measure of the inflaton mass. It has long been known that the electron charge in combination with Newton’s gravitational constant, defines an electro-gravitational mass, which can be expressed in terms of the fine structure constant and the Planck mass, viz,

\[ (7) \quad M_{EG}^2 = \alpha m_p^2 \]
If we now construct a reduced electro-gravitational mass, which is the geometric mean of the reduced Planck mass and some intermediate mass scale, such that,

\[ M_{EG}^2 = m_x m_p = \alpha m_p^2, \]  

such a mass scale is then given by,

\[ m_x = \alpha m_p = 1.78 \times 10^{16} \text{Gev}. \]

This is in remarkable agreement with the MSSM-GUT scale of \( 2 \times 10^{16} \text{Gev} \) [22], as they are ostensibly identical. Taking the inflaton mass as the geometric mean of this GUT-scale mass, and the mass corresponding to the reheating temperature given in [20], of \( 2.0 \times 10^{15} \text{Gev} \), we compute an inflaton mass of,

\[ M_I = \sqrt{M_{GUT} M_{\text{reheat}}} = 6.0 \times 10^{15} \text{Gev}. \]

Inserting this value into (6) gives a Higgs mass of,

\[ M_H = 120 \text{Gev}. \]

In Erler [13], this value for the Higgs mass is in close proximity to a narrow maximum in the Higgs mass probability, peaked around 117 Gev; A mere 5 % lower reheating temperature in (10) reproduces the peak value, as does a recent computation of the two-photon Higgs decay process [23]. These calculations strongly suggest that a Higgs boson, with a mass near 120 Gev will be discovered in Tevatron or LHC data, and that the cosmological constant, electron, inflaton, and GUT scale masses are all mutually constrained. The implications for cosmological and particle physics are enormous if this interlinkage evolved from our vacuum state near the time of inflation up to the recent dark energy epoch, as a sort of ‘DNA’ coding of our vacuum, encompassing the 61 decades connecting the Planck and dark energy epochs. It also explains why the value of the cosmological constant is non-vanishing: all mass scales may ultimately derive from it.

II Tev-scale Quantum Gravity vs. Supersymmetry (SUSY)

Twelve years ago, an alternative resolution of the hierarchy problem was announced [24], in which large extra dimensions (LXD) effectively lowered the scale of quantum gravity from the Planck scale of \( 10^{19} \text{Gev} \), down to the Tev scale, accessible to particle accelerators such as the Tevatron or the LHC, which might observe them directly. Thus a reduced quantum gravity scale now vies with SUSY to resolve the hierarchy problem.

In a very recent paper [25], Gasperini has convincingly argued that the vacuum energy, first measured twelve years ago by astronomers, owes its existence to SUSY breaking, which should manifest at the Tev scale. A direct calculation of the upper bound gives,

\[ M_{SUSY} \leq \sqrt{m_{cc} m_p} = 2.4 \text{Tev} \]

He goes on to argue that failure to observe SUSY effects such as sparticles below this scale, would likely constitute experimental evidence against the existence of SUSY. This would leave the theory of LXD as the primary candidate for solving the hierarchy problem. Fortunately, a new test of LXD has just emerged [26], which produces a unique particle decay signature at a threshold near 6 Tev. Thus we enquire as to whether the relations in (2) could predict an energy scale consistent with this threshold. If there are six LXD, they constitute a 6-volume, \( V_{(6)} \) from which one can compute the reduced scale of quantum gravitational effects as,

\[ M_{QG}^4 = \sqrt{\frac{m_p^2}{V_{(6)}}} \]
From differential geometry [27], the volume of a 6-dimensional hypersphere is given by,

\[ V_6 = \frac{\pi^3}{6} R^6 \]

Equations (1) & (2) can be squared & combined to give

\[ \left( \frac{l_p}{\alpha \lambda_e} \right)^6 = 8\pi \left( \frac{m_p}{m_{cc}} \right)^4, \]

Where \( m_p \) is now identified as the Planck mass.

In (15), we recognize the left-hand denominator as the classical Lorentz radius of the electron,

\[ R_L = \alpha \lambda_e \]

If we now identify the Lorentz radius of the electron as the radius of the 6-d hypersphere, we can express it in terms of the cosmological constant mass as,

\[ R_L^6 = \left( \frac{m_p}{m_{cc}} \right)^4 \frac{l_p^6}{8\pi} \]

Thus from (14) we obtain the 6-volume,

\[ V_6 = \frac{\pi^2}{18} \left( \frac{m_p}{m_{cc}} \right)^4 l_p^6 \]

Inserting this into (13) gives, after some algebra,

\[ M_{QG}^2 = \left( \frac{48}{\pi^2} \right) \frac{1}{4} m_{cc} m_p \]

Thus we arrive at a reduced quantum gravity scale given by,

\[ M_{QG} = 6.5 \text{ TeV} \]

Which is in excellent agreement with the threshold value cited in [26].

III. The Dirac-Kerr-Newman black hole model of the electron

**Introduction**

In this, the golden age of cosmology, one cannot help but harken back to the golden age of black hole research, 1963-1977, during which most of today's current theory of the structure of classical black holes was finalized. Today this understanding is being applied and extended to diverse areas of study, ranging from extreme black holes in brane world theory, to billion-solar mass black holes powering quasars. One would hope that similar pioneering efforts to understand the nature of dark energy, both theoretically and experimentally, will pay similar dividends in as little time. Twelve years after its discovery, much progress has been made, and ambitious observational projects are in the works to further refine other models, such as quintessence [29].

Yet it is fair to say that we are far from a universal consensus on the nature and dynamics of the dark energy driving the acceleration of the Hubble expansion. Most studies have employed astronomical methods, with a continuing focus on supernovae [30,31], the ISW effect [32,33] and the CMB [34]. These have reached perhaps the penultimate refinement by the recent measurements of Vikhlinin et al.[36], of the dynamics of galactic clusters, in which three different measurement methodologies have set the value of the equation of state parameter to the value: \( w = - 0.991 \pm 0.045 \), which strongly suggests that a cosmological constant (\( w = -1 \)) affords the best fit to the accelerated expansion data of the universe.

Working under this assumption, we show that the clearest physical meaning of Beck's relation [14] is in the form of a cosmological length scale relative to the Compton wavelength and Schwarzschild radius of the electron. Adopting the Kerr-Newman model of the electron [36-40], as a spinning, charged black hole, we show that the modulus of the complex event horizon is identical with the radius of the naked ring singularity expressed in terms of these parameters, and that a geometric mean of the Schwarzschild...
radius of the electron and a new cosmological length scale is identical to the electron zitterbewegung radius. We speculate about the interpretation of this new scale.

Attempts to model the Dirac electron as a Kerr-Newman (DKN) black hole were shown by Carter to yield exactly Dirac's value for the gyromagnetic factor of $g = 2$ [36]. Ambitious attempts to develop this model further have demonstrated a finite value for the self-energy [37], charge and spin quantization [38], a beautiful isometry between the Kerr-Newman geometry and Dirac spinors [39,40], and a prediction that the positron may possess negative mass [41]. Nonetheless, acceptance of the DKN model has been difficult, due to the absence of detectable signatures in electron scattering experiments, Hawking radiation, and the appearance of a naked singularity, brought out most clearly by the expression for the event horizon radius,

$$R_{DKN} = \frac{R_S}{2} \left( 1 \pm \sqrt{1 - \frac{\alpha}{\alpha} \left( \frac{\lambda_e}{l_p} \right)^2 - \frac{1}{4} \left( \frac{\lambda_e}{l_p} \right)^4} \right),$$

where $R_S = 1.35 \times 10^{-57} m$ is the Schwarzschild radius of the electron.

This is clearly complex, since the last term is on the order of $10^{89}$. Using Beck's relation, (1), we can express (21) in terms of the cosmological constant,

$$R_{DKN} = \frac{R_S}{2} \left( 1 \pm \sqrt{1 - \left( \frac{1}{\alpha^2 \lambda_e \sqrt{\Lambda}} \right) - \left( \frac{1}{2 \alpha^4 \lambda_e \sqrt{\Lambda}} \right)^2} \right).$$

If we compute the modulus of this complex event horizon radius, we obtain the radius of the Kerr ring singularity itself,

$$R_{DKN} = \frac{R_S}{4 \alpha^3 \lambda_e \sqrt{\Lambda}} = 1.92 \times 10^{-13} m = R_z,$$

which is one-half the Compton wavelength of the electron, or Dirac's zitterbewegung radius. Thus it would appear that the cosmological constant is a structural parameter of the DKN electron-black hole. The cancellation of factors of two in the expression for the square of the Planck length,

$$l_p^2 = R_{DKN} \times R_S = \left[ \frac{\hbar}{2 \pi m c} \times \frac{2 G m c}{c^2} \right],$$

strongly suggests that black hole radii are the physical origin of the quantum gravity scale, and not merely dimensionful groupings of the fundamental constants, $\hbar$, $G$, and $c$. The algebraic structure of eq.(23) is highly suggestive of another geometric mean, which we write as,

$$\lambda_e^2 = R_{mv} \times R_S.$$

A quick calculation yields,

$$R_{mv} = 1.10 \times 10^{32} m = 3.6 \text{ Peta-parsecs (Ppc)}.$$  

The hierarchy of current length scales in cosmology [42] is as follows:

$R_H = 4.2$ Gpc is called the Hubble scale for obvious reasons, and is the length scale corresponding to our 13.7 Gy old universe.

$R_{particle} = 14.1$ Gpc is the more physically relevant particle horizon, and corresponds to light signals emitted at the big bang.

$R_\infty = 19.0$ Gpc. We shall refer to $R_\infty$ as the eternal horizon, and is the largest physical scale calculable in FRW cosmology.

If our universe has an infinite lifetime, this distance is the farthest we can ever directly observe. As such it is an information barrier to events happening beyond it.
\[ R_{mv} = 3.6 \text{ Ppc}, \] exceeds both the particle and eternal horizon scales of our universe by five orders of magnitude.

We conjecture that this may be the actual scale at which the presence of the multiverse is manifest and predictable, as a spatial separation between our eternal horizon, and that of a parallel universe. Indeed, any speculation about the variation of the fine structure constant over cosmological time scales would seem to be at odds with Beck’s relation. Thus it is quite plausible that any large variations in a huge multiverse scale would correspond to small variations in the cosmological constant, thereby rendering the fine structure term truly constant, as is consistent with the evidence to date.

One can only speculate how the cosmological constant influences the properties of the electron, which if it can be interpreted as a Kerr-Newman black hole, should possess only three good quantum numbers, namely mass, spin, and charge to completely specify its properties. However, it is clear that the electron charge and mass are degenerate with respect to \( \Lambda \), such that only \( e \) or \( m_e \), along with \( \Lambda \) and spin are required. Indeed, \( \Lambda \) appears to be electromagnetic in nature, despite the fact that dark energy exerts a negative pressure, and would seem to preclude this. Nonetheless, the proposal by Beck and collaborators [43] to search for a \( \Lambda \)-driven cutoff in the noise spectrum of a Josephson junction, would seem to depend critically upon an electro-gravitational coupling of dark energy.

**Conclusion**

It appears that through its unique linkage to the cosmological constant, the electron apportions scales which probably correspond to black holes and the Higgs boson. Coincidentally, it is also near the scale at which SUSY effects, such as sparticles, are expected to manifest, if at all. It would be extraordinary if nature, in defiance of Occam’s razor, accommodated two such physical phenomena capable of resolving the hierarchy problem. Hence one expects to see one or the other, but not both. Given the paucity of experimental evidence for SUSY in three decades of searching, as well as recent evidence against it, in which the measured \( B_s \) meson switching rate was much smaller than the SUSY prediction [28], this author feels that the odds favor observing a signature of quantum gravity effects, such as enhanced lepton production beyond the SM prediction, and/or black holes possibly later this year, now that the LHC has begun operating at the 7.0 Tev center-of-mass energy level.

With regard to the conjectured multiverse scale \( R_{mv} \), there is no widespread agreement on any theoretical or experimental value. After the recent observation of a large scale cosmic void [45], speculation arose as to whether this void was a multiverse signature [45]. Unfortunately, subsequent analysis of the data presented compelling evidence that the void was simply an artifact of the statistics used in its interpretation as such [46]. The multiverse scale presented here is unexpected, in that it relies upon two measures which one would think to be uniquely intrinsic to our universe, namely the cosmological and fine structure constants. Perhaps nature has made it possible to ascertain a scale for the multiverse in terms of constants that nonetheless might vary from universe to universe. If this is the case, it also follows that electrons possess a multiversal reality via their description as Kerr-Newman black holes, and may offer a microcosmic connection to the multiverse, complementary to the astronomical one.

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