Topology Optimization Design of 3D Continuum Structure with Reserved Hole Based on Variable Density Method

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Abstract

An objective function defined by minimum compliance of topology optimization for 3D continuum structure was established to search optimal material distribution constrained by the predetermined volume restriction. Based on the improved SIMP (solid isotropic microstructures with penalization) model and the new sensitivity filtering technique, basic iteration equations of 3D finite element analysis were deduced and solved by optimization criterion method. All the above procedures were written in MATLAB programming language, and the topology optimization design examples of 3D continuum structure with reserved hole were examined repeatedly by observing various indexes, including compliance, maximum displacement, and density index. The influence of mesh, penalty factors, and filter radius on the topology results was analyzed. Computational results showed that the finer or coarser the mesh number was, the larger the compliance, maximum displacement, and density index would be. When the filtering radius was larger than 1.0, the topology shape no longer appeared as a chessboard problem, thus suggesting that the presented sensitivity filtering method was valid. The penalty factor should be an integer because iteration steps increased greatly when it is a non-integer. The above modified variable density method could provide technical routes for topology optimization design of more complex 3D continuum structures in the future.

Keywords: Topology Optimization, Reserved Hole, Multiple Loading Conditions, Variable Density Method, MATLAB

1. Introduction

Topology optimization design is a computational method for achieving optimal material distribution without knowing the special shape of the structure in advance. Therefore, topology optimization can be used to develop great potential of new structure with high performance. The history of the topology optimization dates back to the truss theory proposed by Michell in 1904. At that time, the theory can only be applied to single working condition that depended on the strain filed and was not applicable in practical engineering [1]. In 1964, Dorn et al. proposed ground structure approach [2] that was applied into topology optimization. From then on, topology optimization had become a more active research field. In recent years, topology optimization theory of continuum structure [3] has developed rapidly. Many topology optimization methods including variable thickness method, variable density method, asymptotic structure optimization, independent continuum mapping method (ICM), level set, and nodal density method [4] have been proposed. Topology optimization methods have been applied in many fields.

Especially the variable density method has been successfully applied in many practical engineering projects because of its simple programming procedure and high efficiency in calculation competence [5]. Yang et al. transformed topology optimization problems into linear programming problems and then used the variable density method to design engine components [6]. With the development of many CAE software based on the variable density method, the engineering examples of topology optimization that are solved by numerical simulation methods have increased daily [7], [8], [9], [10], [11], [12].

2. State of the Art

Numerical instability problems including chessboard and mesh dependence phenomenon are ubiquitous in topology optimization design based on variable density method [13]. Sigmund proposed a filtering radius method to solve this problem [14]. Zuo modified the previous filtering method [15]. Chen used an adjacent entropy filtering method based on graph theory to eliminate the chessboard and mesh dependence problems [16].

The essence of topology optimization is to solve the extrema problem. Therefore, the development of topology optimization is inseparable from optimal mathematical algorithms. The advantage of the optimization criterion method (OC method) is fast convergence speed, however, this method might be difficult to deal with the complex structure under the conditions of different constraints [17]. In 1960, Schmit adopted mathematical programming theory to solve optimization problems of elastic structure under the condition of multiple load [18]. His research pushed the development and application of topology optimization algorithms. Traditional optimization methods are not applicable for these problems because mathematical models are complicated, nonlinear, random, and blurry in real optimization design problems. Therefore, many new algorithms, including simulated annealing method [19],...
genetic algorithm [20], evolutionary algorithm [21], and neural network algorithm [22], [23], were presented. Based on the above modern methods, the global or approximately global solutions can generally be obtained. However, this will cost huge amount of calculation time. The minimum compliance problem in nonlinear programming can be dealt with by sequential quadratic programming (SQP) [24], [25] and moving asymptotes method (MMA) [26], [27]. Compared with SQP and MMA methods, the OC method has many advantages including good computation convergence and fast computation speed in topology optimization design. Hence, in this study, the OC method is used to solve the minimum compliance problem of the topology optimization model.

The remainder of this study is organized as follows. Section 3 establishes a 3D topology optimization model of continuum structure with the object function of minimum compliance and deduces the 3D finite element formulations by OC method. Section 4 studied the topology optimization design examples of 3D continuum structure with reserved hole and discusses the influence of mesh numbers, penalty factor, and filtering radius on results of topology optimization. Section 5 presents conclusions.

3. Methodology

3.1 Minimum Compliance Problem

The minimum compliance problem of topology optimization design of the continuum structure constrained by volume fraction is expressed as follows:

$$\min \ C(x)$$

subject to

$$\int_{\Omega} \alpha \Omega \leq V^*$$

$$x_i = 0 \text{ or } 1$$

where $C(x)$ is compliance of structure. $x$ is material pseudo-density as the design variable. $\Omega$ is the given design domain. $V^*$ is the required optimal design volume. $V_0$ is the original design volume.$\alpha$ is the volume fraction.

3.2 Newly Improved SIMP Model

The basic approach used to handle the problem of discrete variables in numerical calculation is to substitute continuous function for discrete function. The continuous function can generate many elements, and the continuous density of which is between 0 and 1. However, making this kind of structure material practical is difficult. To solve this problem, the penalty factor is usually introduced to suppress the appearance of intermediate-density element. By using variable density method and introducing penalty factor, the relationship between variable density $x_i$ and elasticity modulus $E_i$ based on SIMP model is expressed as follows:

$$E_i = E_s x_i = x_i^p E_0, \quad x_i \in [0,1]$$

where $E_s$ is elasticity modulus of solid materials. $p$ is the penalty factor. The evolved solid isotropic microstructures with penalization (SIMP) model is expressed as follows:

$$E_i = E_{min} + x_i^p (E_0 - E_{min}), \quad x_i \in [0,1]$$

where $E_{min}$ is the elasticity modulus of void material. To avoid the singularity of the stiffness matrix, the value is not zero as usual. The value is set to “0.001.” The improved SIMP model makes the penalty factor and elasticity modulus of void material mutually independent. Compared with the former model, the improved SIMP model has better convergence in computation [28].

3.3 Sensitivity Filtering

Topology optimization model based on variable density method is always accompanied with numerical problems, such as mesh-dependence, checkerboard pattern, local extremum, and so on. To solve these problems, the common way is to introduce density-filtering method shown as follows:

$$\bar{x}_i = \frac{\sum_{j \in N_N} H_{ij} v_j x_j}{\sum_{j \in N_N} H_{ij} v_j}$$

where $v_j$ is the volume of element. $H_{ij}$ is the weighting factor. $N_i$ is the element set adjacent to element $i$ and can be defined as

$$N_i = \{ j : \text{dist}(i, j) \leq R \}$$

where the operator $\text{dist}(i, j)$ is the center distance between element $i$ and element $j$, and $R$ is the filtering radius. The weighting factor $H_{ij}$ is:

$$H_{ij} = R - \text{dist}(i, j)$$

Filtering density $\bar{x}_i$ is modified density. This factor is introduced into the SIMP model, and the formula can be deduced as follows:

$$E_i(\bar{x}) = E_{min} + \bar{x}_i^p (E_0 - E_{min}), \quad \bar{x}_i \in [0,1]$$

3.4 Element Stiffness Matrix and Formulation of Finite Element

Based on improved SIMP model, as indicated in Formula (7), sensitivity filtering method, and Hooke’s Law, the 3D stress matrix of isotropic material element $i$ is expressed as:

$$D_i(\bar{x}) = E_i(\bar{x}) D_{y}^i, \quad \bar{x}_i \in [0,1]$$

where $D_{y}^i$ is the stress matrix composed of unit Young’s modulus and can be expressed as:

$$D_{y}^i = \frac{1}{(1 + \psi(\frac{\bar{x}_i}{2}))^3} \begin{bmatrix} 1 & \psi & \psi & 0 & 0 & 0 \\ \psi & 1 & \psi & 0 & 0 & 0 \\ \psi & \psi & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1 - 2\psi)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1 - 2\psi)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1 - 2\psi)}{2} \end{bmatrix}$$

where $\psi$ is Poisson ratio of isotropic material. Based on finite element theory, the element stiffness matrix of elastic
solid is the volume integral of stress matrix $D(\mathbf{x})$ and strain matrix $B$, which can be expressed as:

$$k_i(\mathbf{x}) = \int_{\Omega_i} B^T D(\mathbf{x}) B d\mathbf{x}$$

where $\xi_i (\varepsilon=1,2,3)$ is the natural coordinate system of hexahedron element. Strain matrix $B$ describes the relationship between the node displacement of elements and the strain. Based on SIMP model, the element stiffness matrix can be expressed as:

$$k_i(\mathbf{x}) = E_i(\mathbf{x}) k_i^0$$

where

$$k_i^0 = \int_{\Omega_i} B^T D^0 B d\mathbf{x}$$

Substituting Formula (9) into Formula (12), it can be further organized as:

$$k_i^0 = \frac{1}{(1+n)(1-2n)} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} & k_{34} \\ k_{14} & k_{24} & k_{34} & k_{44} \end{bmatrix}$$

where

$$k_{ij} = \begin{cases} -(6\varepsilon - 4)/9, & k_{1i} = 1/12, \\ -1/9, & k_{2i} = -(4\varepsilon - 1)/12, \\ -(4\varepsilon - 1)/12, & k_{3i} = 1/18, \\ 1/24, & k_{4i} = -1/12, \\ (6\varepsilon - 5)/36, & k_{5i} = -(4\varepsilon - 1)/24, \\ -1/24, & k_{6i} = (4\varepsilon - 1)/24, \\ (3\varepsilon - 1)/18, & k_{7i} = (3\varepsilon - 2)/18, \end{cases}$$

The global stiffness matrix is the collection of element stiffness matrix, which can be expressed as:

$$K(\mathbf{x}) = \sum_{i=1}^{n} K_i(\mathbf{x}) = \sum_{i=1}^{n} E_i(\mathbf{x}) k_i^0$$

where $n$ is the amount of elements. Based on the definition of the global stiffness matrix, Formula (16) can be further expressed as:

$$K(\mathbf{x}) = \sum_{i=1}^{n} K_i(\mathbf{x}) = \sum_{i=1}^{n} E_i(\mathbf{x}) k_i^0$$

where $K_i^0$ is the global constant stiffness matrix that is composed of the element stiffness matrix. According to Formula (7), the matrix can be expressed as:

$$K(\mathbf{x}) = \sum_{i=1}^{n} [E_i(\mathbf{x}) - E_i(\mathbf{x})] K_i^0$$

By solving Formula (19), we can obtain the displacement vector of nodes $U(\mathbf{x})$:

$$K(\mathbf{x}) U(\mathbf{x}) = F$$

where $F$ is the force vector of nodes.

### 3.5 Topology Optimization Model Based on Improved SIMP method

The solution of minimum compliance problem is to find the distribution form of material density, which makes structural deformation minimum under the action of the specified load and constraint. Therefore, compliance of structure can be defined as:

$$c(\mathbf{x}) = F^T U(\mathbf{x})$$

By introducing the volume restriction, the minimum compliance problem can be further expressed as follows:

Find $x = [x_1, x_2, ..., x_n]^T$

Minimize $c(\mathbf{x}) = F^T U(\mathbf{x})$

Subject to:

$$U(\mathbf{x}) = \int [E_i(\mathbf{x}) - E_i(\mathbf{x})] K_i^0$$

where density $\mathbf{x}$ is determined by Formula (4). $v = [v_1, ..., v_n]^T$ is the volume vector of elements.

### 3.6 The Sensitivity Analysis of Structure

Based on the improved SIMP model, sensitivity analysis should be indispensable in obtaining the solution to objective function.

In Formula (21), the derivative of volume constraint function $V(\mathbf{x})$ with respect to design variable is:

$$\frac{\partial V(\mathbf{x})}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial E_j(\mathbf{x})}{\partial x_i} \frac{\partial x_i}{x_i}$$

where $\frac{\partial E_j(\mathbf{x})}{\partial x_i} = v_j \frac{\partial x_i}{x_i} = \sum_{j=1}^{n} H_j v_j$. The mesh element used in the study is the cube element with unit volume. That is, $v_j = v_j = 1$.

In Formula (21), the derivative of compliance with respect to design variable $x_i$ is:
\[
\frac{\partial c(x)}{\partial x} = \sum_{i=1}^{n} \frac{\partial c(x)}{\partial x_i} \frac{\partial x_i}{\partial x} = 0
\]  

where

\[
\frac{\partial c(x)}{\partial x_i} = F^T \frac{\partial U(x)}{\partial x_i} + U(\xi) \frac{\partial U(x)}{\partial x_i}
\]

In Formula (19), the derivative of total stiffness with respect to design variable \( x_i \) is:

\[
\frac{\partial K(x)}{\partial x_i} U(\xi) + K(x) \frac{\partial U(x)}{\partial x_i} = 0
\]

Formula (25) can be further organized as:

\[
\frac{\partial U(x)}{\partial x_i} = -K(x)^{-1} \frac{\partial K(x)}{\partial x_i} U(\xi)
\]

Based on Formula (18), the equation can be expressed as:

\[
\frac{\partial K(x)}{\partial x_i} = \frac{\partial}{\partial x_i} \sum_{j=1}^{n} \left[ E_{\text{loc}} + \bar{\nu} \left( E_0 - E_{\text{min}} \right) \right] K_j^0
\]

Combined with Formula (24), Formula (26) and Formula (27), it can be expressed as:

\[
\bar{\tau} \bar{\xi} = U(\xi) \left[ \bar{\rho} \bar{x}_i \left( E_0 - E_{\text{min}} \right) K_j^0 \right] U(\xi)
\]

Given that \( K_j^0 \) is the collection of element stiffness matrix, Formula (28) can be expressed as:

\[
\bar{\tau} \bar{\xi} = a_i(\bar{\xi}) \left[ \bar{\rho} \bar{x}_i \left( E_0 - E_{\text{min}} \right) k_i^0 \right] U(\xi)
\]

where \( U(\xi) \) is the displacement vector of element node.

Because \( k_i^0 \) is positive definite, \( \frac{\partial c(x)}{\partial x_i} < 0 \).

### 3.7 Optimization algorithm

OC method is an indirect optimization method because it does not optimize the object function directly. The method makes K-T condition, which the optimal solution should meet in math as the guideline the most optimal structure should satisfy. The outstanding characteristic is with fast convergence speed and less iteration number. The K-T condition of optimization criteria method should satisfy

\[
\frac{\partial c(x)}{\partial x} + \lambda \frac{\partial \phi(x)}{\partial x} = 0
\]

where \( \lambda \) is Lagrangian multiplier. Formula (30) can be further expressed as:

\[
B_\eta = \frac{\partial c(x)}{\partial x} \left( \lambda \frac{\partial \phi(x)}{\partial x} \right)
\]

The large change of relative density from void to solid is not allowed. Therefore, moving limit \( \eta \) should be introduced into the design variable \( x \). The iterative density can be further expressed as follows:

\[
\eta = \left\{ \begin{array}{ll}
\max(0, x_i - \eta), & \text{if } x_i B_\eta^0 \leq \max(0, x_i - \eta), \\
\min(1, x_i + \eta), & \text{if } x_i B_\eta^0 \geq \min(1, x_i - \eta), \\
\eta & \text{otherwise}
\end{array} \right.
\]

where \( \eta \) is the moving limit. \( \eta \) is the damping coefficient ranging from 0 to 1. Introducing the damping coefficient and the moving limit aims to improve iteration convergence.

### 3.8 Procedure of Obtaining Solutions

Based on MATLAB programming, we depict the whole solving flow chart shown in Figure 1 and provide specific explanations for each step.

1. Input of original data: maximum iteration number, material parameters (elastic modulus and Poisson ratio), coordinate of force acting point, coordinates of constraint node and freedom numbers
2. Definition of elemental stiffness matrix and integration of total stiffness matrix
3. Finite element analysis and calculation of the element nodal displacement
4. According to the above calculated displacement, calculating sensitivity, and objection function (compliance)
5. Sensitivity filtering method
6. Convergence test is performed by Formula (34). If satisfied, the results (compliance, displacement, and nephogram of density distribution) will be the output; otherwise the solving step returns to Step 3, in which the density variable is updated to perform the finite element analysis.
4. Result Analysis and Discussion

4.1 Example Verification

As is shown in Figure 2, the design domain is a 3D continuum cantilever beam with reserved hole. The length in x direction is 40 mm; the width in y direction is 20 mm; and the thickness in z direction is 10 mm. A cylinder hole with radius of 7 mm is in the center of the entire structure. The discrete structure is partitioned into \(32 \times 16 \times 8\) hexahedral elements. The left side boundary condition is fully fixed and the right side at the bottom of the beam is subjected to a concentrated line load 1 kN/m. The elastic modulus of material \(E = 1\) GPa, the volume fraction is 0.3, the penalty factor \(p=3.0\), and the filtering radius \(r=1.5\). The objective function is to minimize the compliance of the entire structure. The iteration results are shown in Table 1.

Table 1 presents the topology optimization results in the iteration process. From the table, we can see that when the iteration step reaches step 5, the shape of the topology structure is unformed. When the iteration process reaches step 50, the structure is fully shaped into many supporting bars in the local region. With the increase of iteration numbers, the structure form has only a subtle change.

To reflect the change trend of compliance, maximum displacement, and density index with the increase of the iteration numbers, Figures 3(a), (b), (c) are drawn respectively. Herein, the density index is defined as the ratio of element numbers of which density is less than 0.01 and greater than 0.99 to the total element numbers. This index reflects the extent that the whole element density tends to be 0 or 1. When the iteration number begins to reach 20, the compliance and the maximum displacement decline sharply, whereas the density index climbs rapidly. As the iteration number continues to increase, all the results (compliance and maximum displacement) have only subtle changes. From the above density nephogram change and iteration results of various index, we can conclude that the presented numerical method is stable, reasonable, and with good convergence.
4.2 Mesh Dependency Analysis

To analyze the influence of element numbers on optimization results and computation time, five cases with mesh numbers 16×8×4, 20×10×5, 32×16×8, 40×20×10, 48×24×12 are discussed. The penalty factor is $p=3.0$ and filtering radius is $r=1.5$.

From Table 2, we can see that if the mesh numbers are too coarse or too fine, the calculated compliance and the maximum displacement are both too large. In special cases, when the mesh is 32×16×8, the optimal compliance is the minimum in five cases. As the mesh numbers increase, iteration steps increase and computation time becomes longer. Meanwhile, the density index becomes larger, that is, the element number ratio of the intermediate density decreases. In addition, the mesh numbers do not affect the structure form significantly. Based on overall consideration of the optimization results and calculating cost, the reasonable mesh numbers in this study are 32×16×8.

4.3 Penalty Factor Analysis

To analyze the influence of different penalty factors on the optimization results and computation time, we studied five cases with penalty factors 2.0, 2.5, 3.0, 3.5, and 4.0. Herein, the mesh number is 32×16×8 and filtering radius $r=1.5$ in five cases.

As is shown in Table 3, with the increase of penalty factor, the calculated compliance, maximum displacement, and density index become larger. Furthermore, when the penalty factor is non-integer, iteration steps grow apparently and computation cost increases. When the penalty factor is an integer, the iteration number is relatively small, and the larger the penalty factor is, the faster the convergence time will be. However, the optimal compliance and the maximum displacement both increase, which suggests that the optimization effect declines. Based on an overall consideration of optimization results and calculating cost, the reasonable penalty factor in this example should be 3.0.

| Mesh partition | 16×8×4 | 20×10×5 | 32×16×8 | 40×20×10 | 48×24×12 |
|----------------|--------|---------|---------|----------|----------|
| Iteration step  | 49     | 159     | 175     | 170      | 475      |
| Compliance (N·m) | $1836.01 \times 10^{-3}$ | $1787.01 \times 10^{-3}$ | $1774.25 \times 10^{-3}$ | $1931.22 \times 10^{-3}$ | $2077.22 \times 10^{-3}$ |
| Maximum displacement (m) | $0.2345 \times 10^{-3}$ | $0.2267 \times 10^{-3}$ | $0.1998 \times 10^{-3}$ | $0.2400 \times 10^{-3}$ | $0.2530 \times 10^{-3}$ |
| Density index | 0.9219 | 0.9450 | 0.9819 | 0.9915 | 0.9942 |

**Fig. 3.** Variation of compliance, maximum displacement, and density index with iteration numbers

**Table 2.** Final topology optimization results of different numbers of mesh elements
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