Relativistic Covariant Equal-Time Equation
For Quark-Diquark System

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ABSTRACT. Relativistic three-dimensional quasipotential (equal-time) equations are considered, which describe bound states of fermion and boson of spin \( S = 0 \) or \( S = 1 \). The spin structure of the interaction quasipotentials in such systems is studied, and corresponding partial-wave equation for the simplest case is obtained. Such equations can be used in calculations of energy spectra, decay rates and structure functions of quark-diquark systems (nucleons and their resonances) as well as for description of \((\pi\mu)\)-atom.

KEYWORDS: quantum chromodynamics, diquark, bound state, equal-time quasipotential approach
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1 Introduction

The concept of constituent diquarks has been introduced in 1966, Ref. [1]. In a three-quark system spin-spin interaction can lead to an existence of the short-range correlations in two-quark subsystems [2], which are comparable in strength to the $\bar{q}q$-attraction inside mesons. There are experimental evidences for the diquark correlations in baryons [3]. Scalar diquarks are mentioned in [5] to be energetically favored. Moreover, in a series of recent papers [6] it was shown that the concept of diquarks as effective degrees of freedom arising as a result of such correlations has important meaning for descriptions of nonleptonic weak decays at low energies. Scalar diquarks also arise in superstring-inspired models [7]. Therefore, nucleon can be interpreted as the quark-diquark bound state and described on the base of well-known methods for a solution of two-body problem. In connection with that we can mention the papers of Lichtenberg with collaborators [8] and Efimov’s group [9]. There are also many speculations concerning ”diquonia” (diquark-antidiquark bound system) and ”dibaryons”, which are rather based on the radical point of view of considering diquarks as elementary constituents.

Secondly, we would like to note that now the new trend in physics of elementary particles connected with the researches of properties of so-called exotic atoms has been developed. Such systems represent the atoms in which one of electrons is replaced by an elementary particle [12]. The first works devoted to the consideration of these systems appeared in the forties [13]. In the middle of the 70-s the bound state of $\pi$-meson and muon [14], which also can be interpreted as the exotic atom, has been experimentally observed. It deserves to emphasize that the main properties of ($\pi\mu$)-atom have been studied theoretically in Ref. [15] even before its experimental discovery. In these papers the attention has been paid at the possibilities of exploring features of $\pi\mu$-atom by means of experimental investigation of the composite system of meson and lepton (one could find the following works in Ref. [16]).

In the works [17] the influence of the relativistic effects in the description of ($\pi\mu$)-atom was under consideration. For this purpose the equal-time quasipotential approach suggested by Logunov and Tavkhelidze in Ref. [18] has been used for the description of these bound states on the base of quantum field methods.

In the presented paper we employ the quasipotential approach, the Kadyshevsky’s version [19], to the model in which the nucleon is considered to be a bound state of a quark of spin 1/2 and a diquark, whose spin is $S = 0$ or $S = 1$. We are interested in the spin structure of the quasipotentials for interaction between fermion (e.g., quark or $\mu$-meson) and (pseudo) scalar particle (e.g., diquark or $\pi$-meson) as well as between fermion and vector particle which is described by Joos-Weinberg’s formalism [20]. We also find out the form of the quasipotential in the partial-wave equation for ($\pi\mu$)-atom [3] and propose the ways of numerical solutions of the above-mentioned equations.

1Let us still mention that another point of view is presented in some papers [1].
2See also the recent reviews [14, 11].
3The analogous problem for two spinor particle system has been solved in [21].
2 Equation For the Wave Function of the Composite System Formed by Fermion and $S = 0$ Boson

The quasipotential equation for the wave function of the composite system consisting of fermion and spinless boson has been obtained in [22]

$$2\Delta_{p,m_2\lambda_\rho}(M - \Delta_{p,m_1\lambda_\rho} - \Delta_{p,m_2\lambda_\rho})\Phi(\vec{\Delta}_{p,\lambda_\rho}) =$$

$$= \frac{1}{(2\pi)^3} \sum_\nu \int \frac{d^3\vec{\Delta}_{k,\lambda_\rho}}{2\Delta_{k,m_1\lambda_\rho}^0} V_{\sigma')(\vec{\Delta}_{p,\lambda_\rho}; \vec{\Delta}_{k,\lambda_\rho})\Phi(\vec{\Delta}_{k,\lambda_\rho}).$$

(1)

The quasipotential $\hat{V}$ coincides with the scattering amplitude of muon on pion in the first approximation in the coupling constant. Covariantly define $d$ in the c.m.s. 4-momentum of particles is presented by the following formulas [22]–[24]

$$\Delta_{p,\lambda_\rho} = (\Delta_{p,\lambda_\rho} p_1) = \vec{p}_1 - \frac{\vec{p}}{M}(p_0^1 - \frac{\vec{p}_1^2}{P_0 + M}) = -\vec{\Delta}_{p,m_2\lambda_\rho} \equiv \tilde{\vec{p}};$$

(2)

$$\Delta_{p,m_1\lambda_\rho} = \sqrt{\Delta_{p,\lambda_\rho}^2 + m_j^2} \equiv \tilde{p}_1^0; j = 1, 2$$

(3)

Here, $M$ is the mass of bound system, $m_1$ is muon mass, $m_2$ is pion mass, $L_{\lambda_\rho}^{-1}$ is the matrix of the Lorentz boost from the system with 4-momentum $P_\mu$ and 4-velocity $\lambda_\rho P_\mu \equiv P_\mu / \sqrt{P^2}$ to the rest system, $L_{\lambda_\rho}^{-1} P = (\vec{M}, 0)$. The covariant 4-momentum of the particle after interaction $(\Delta_{p,m_1\lambda_\rho}^0$ and $\vec{\Delta}_{k,\lambda_\rho})$ is defined similarly.

In Ref. [23] expression for the quasipotential is chosen in the form:

$$\hat{V}^{(2)}(\vec{\Delta}_{p,\lambda_\rho}, \vec{\Delta}_{k,\lambda_\rho}) = \sum_{pol.inds.} D^{(S=1/2)}_{\sigma_\rho} (V^{-1}(\Lambda_{\rho}, p_1)) \times$$

$$\times V_0(\vec{\Delta}_{k,\lambda_\rho} - \vec{\Delta}_{p,\lambda_\rho}) \tilde{\pi}(\vec{\Delta}_{p,\lambda_\rho}; \sigma_\rho \gamma_{\mu} u(\vec{\Delta}_{k,\lambda_\rho}; \nu_\rho)(\Delta_{p,\lambda_\rho} + \vec{\Delta}_{k,\lambda_\rho})^\mu \times$$

$$\times D^{(S=1/2)}_{\nu_\rho \nu_\lambda_\rho} \left\{ V^{-1}(\Lambda_{p_1}, k_1) \right\} D^{(S=1/2)}_{\nu_\lambda_\rho \nu_\lambda_\rho} \right\};$$

(4)

where $V_0$ is the local part of the quasipotential corresponding to the one-boson exchange, $\Delta_{p,\lambda_\rho}^\mu = (\Delta_{p,m_1\lambda_\rho}^0, \vec{\Delta}_{p,\lambda_\rho})$; $\tilde{\Delta}_{p,\lambda_\rho} = (\Delta_{p,m_2\lambda_\rho}^0, -\vec{\Delta}_{p,\lambda_\rho})$, $D^{(S=1/2)}$ are Wigner functions.

Let us rewrite (4) in more details using the results of Ref. [24]. We employ the expression of Ref. [24] for the 4-current

$$j_{\sigma_\rho \nu_\rho}(\vec{p}, \vec{k}) = \tilde{\pi}(\vec{p}, \sigma_\rho) \gamma_{\mu} u(\vec{k}, \nu_\rho) = \frac{2}{\sqrt{2m(\Delta_0 + m)}} \xi_{\sigma_\rho}^* \left[ p^\mu(\Delta_0 + m) + 2W^\mu(\vec{p})(\vec{\Delta}) \right] \xi_{\nu_\rho},$$

(5)

with

$$\vec{\Delta} \equiv \vec{k}(-\vec{p}) = (L_{\vec{p}}^{-1} \vec{k}); \quad \Delta_0 \equiv \sqrt{\Delta^2 + m^2} = \frac{p^\mu k_\mu}{m}$$

(6)

is momentum transfer in the Lobachevsky space. $W^\mu(\vec{p})$ is the vector of relativistic spin (Pauli–Lyubansky–Shirokov vector, $p_\mu W^\mu(\vec{p}) = 0$; $k_\mu W^\mu(\vec{p}) = \frac{m}{2} (\vec{\Delta})$). We “reseted” 

\[\text{\footnotesize We omit the circles above \(\vec{p}\) and \(\vec{k}\) in the following, implying still the covariant generalizations of the usual momenta.} \]
the polarization indices to a single momentum, e.g., $\vec{p}$ as earlier [21],[24]–[26]. As a result we obtain

$$\hat{V}^{(2)}(\vec{k}, \vec{p}) = \frac{2}{\sqrt{2m_1(\Delta^0_1 + m_1)}} \left\{ [p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2] (\Delta^0_1 + m_1) + \right.$$ 

$$\left. + (\vec{p}\Delta_1)(p_1^0 + p_2^0 + k_1^0 + k_2^0) + i\sigma[\vec{p}\Delta_1](p_1^0 + p_2^0 + k_1^0 + k_2^0) \right\} V_0(\vec{k}(-\vec{p})). \tag{7}$$

After transition to the nonrelativistic limit\(^5\), one can see that the quasipotential (\(7\)) transforms to the following form ($\vec{\Delta}_e = \vec{k} - \vec{p}$):

$$V^{(2)}_{\text{nonrel.}}(\vec{k}, \vec{p}) = -g^2 \frac{4m_1m_2}{\Delta^2_1} + g^2 \frac{1}{c^2} \left( 1 + \frac{m_2}{2m_1} \right) - g^2 \frac{1}{c^2} \left( 2 + \frac{m_1}{m_2} + \frac{m_2}{m_1} \right) \frac{\vec{p}^2 + \vec{k}^2}{\Delta^2_1} - 

- \frac{g^2}{c^2m_1} \frac{(\vec{k}^2 - \vec{p}^2)^2}{\Delta^2_1} - g^2 \frac{1}{c^2} \left( 1 + \frac{m_2}{m_1} \right) 2i\sigma[\vec{p}\Delta^2_1], \tag{8}$$

provided that the local part of the quasipotential is chosen in the form

$$V_0(\vec{k}(-\vec{p})) = \frac{g^2}{(p_1 - k_1)^2} = -\frac{g^2}{2m_1(\Delta^0_1 - m_1)}, \tag{9}$$

g\(_V\) is coupling constant for the quark - vector boson and diquark - vector boson interactions. After some calculations we obtain the matrix elements of the quasipotential (\(7\)), $V^{(2)}_\sigma(\vec{k}, \vec{p})$. They can be written in the following form:

$$V_{1/2}^{-1} = 2V_0(\vec{k}(-\vec{p})) (p_1^0 + p_2^0 + k_1^0 + k_2^0)(i\eta_1 + \eta_2), \tag{10}$$

$$V_{1/2}^{1} = 2V_0(\vec{k}(-\vec{p})) (p_1^0 + p_2^0 + k_1^0 + k_2^0)(i\eta_1 - \eta_2), \tag{11}$$

$$V_{1/2}^{1} = 2V_0(\vec{k}(-\vec{p})) \frac{(p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2)(\Delta^0_1 + m_1) +}{2m_1(\Delta^0_1 + m_1)} \left\{ [p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2] (\Delta^0_1 + m_1) + \right.$$ 

$$\left. + (p_1^0 + p_2^0 + k_1^0 + k_2^0)(\vec{p}\Delta_1) + i\eta_3(p_1^0 + p_2^0 + k_1^0 + k_2^0) \right\}, \tag{12}$$

$$V_{1/2}^{1} = 2V_0(\vec{k}(-\vec{p})) \frac{(p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2)(\Delta^0_1 + m_1) +}{2m_1(\Delta^0_1 + m_1)} \left\{ [p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2] (\Delta^0_1 + m_1) + \right.$$ 

$$\left. + (p_1^0 + p_2^0 + k_1^0 + k_2^0)(\vec{p}\Delta_1) - i\eta_3(p_1^0 + p_2^0 + k_1^0 + k_2^0) \right\}, \tag{13}$$

where $\vec{\eta} = [\vec{p}\Delta]$.

\(^5\)Let us mention that the another version of the quasipotential approach, based on two-time Green function formalism, has been used in [27]. This way leads to the quasipotentials depending on the total energy of bound system.

\(^6\)More exactly, to the quasi-relativistic limit similarly to the use of $1/c^2$ expansion in the Breit equation for two spinor particle interaction, see [28].
Expanding the wave function and the quasipotential in partial waves, we obtain the system of partial equations \[23\]

\[
2P_2^0(M - p_1^0 - p_2^0)\frac{1}{p}\Psi_{J}(p) = \frac{1}{2\pi} \int_{0}^{\infty} \frac{kdk}{k_0^1} \sum_{\nu} V_{\nu l}^l(k, p)\Psi_{J\nu}(k),
\]

where \(J = |l - 1/2|, l + 1/2, k = \Delta_{k,\lambda}; p = \Delta_{p,\lambda}\). The coefficients \(V_{\nu l}^l(k, p)\) can be found by the formula:

\[
V_{\nu l}^l(k, p) = \sum_{M, \sigma, \nu} \int_{0}^{\pi} \sin \theta_p d\theta_p \int_{0}^{2\pi} d\phi_p \int_{0}^{\pi} \sin \theta_k d\theta_k \int_{0}^{2\pi} d\phi_k \times
\]

\[
\times \left[ \Omega_{J,M}^{(1/2)}(\vec{n}_p) \right]^{\sigma} V_{\sigma}^\nu(\vec{k}(-)\vec{p}; \vec{p}) \left[ \Omega_{J',M}^{1/2}(\vec{n}_k) \right]_{\nu}. \tag{15}
\]

Here, \(\vec{n}_p = \frac{\vec{p}}{|\vec{p}|}, \vec{n}_k = \frac{\vec{k}}{|\vec{k}|}\), \(\theta_p, \phi_p\) are angular coordinates of the vector \(\vec{n}_p\); \(\theta_k, \phi_k\) are angular coordinates of the vector \(\vec{n}_k\); \(\Omega_{J'M}^{1/2}(\vec{n})\) are spherical spinors.

Let us choose the coordinate system in such a way that the vector \(\vec{n}_p\) is aligned to the \(OZ\) axis and the vector \(\vec{n}_k\) lies in the \(XZ\) plane. The results of calculations can be expressed in the integrals \(I_1^{(l)}\) and \(I_2^{(l)}\)

\[
V_{\nu l}^{\pm\frac{1}{2}}(p, k) = -(J + \frac{1}{2}) \frac{g_p^2}{m_1\sqrt{2pk}} \left\{ [p_1^0(p^0_1 + p^0_2 + k_1^0 + k_2^0)]^{\gamma^+} - 2m_1^2(\gamma^+ - \gamma^-) - \frac{p_k^0}{k} (p^0_1 + p^0_2 + k_1^0 + k_2^0) I_1^{(l)} - 2m_1^2 I_2^{(l)} \right\} +
\]

\[
= (J + \frac{1}{2}) \frac{g_p^2}{\sqrt{2pk}} (p^0_1 + p^0_2 + k_1^0 + k_2^0) \frac{l(l + 1)}{2l + 1} (I_1^{(l-1)} - I_1^{(l+1)})(1 - \delta_{l0}). \tag{16}
\]

The matrix element for the \(\Delta l = \pm 1\) transition drops out

\[
V_{\nu l \pm 1}^{\pm\frac{1}{2}}(p, k) = 0. \tag{17}
\]

Here, \(\gamma^+ = \frac{p_k^0 + m_1^2}{pk}\), \(\gamma^- = \frac{p_k^0 - m_1^2}{pk}\) and

\[
I_1^{(l)} = \int_{-1}^{1} dz \frac{P_l(z)}{(\gamma^- - z)\sqrt{\gamma^+ - z}}, \tag{18}
\]

\[
I_2^{(l)} = \int_{-1}^{1} dz \frac{P_l(z)}{\sqrt{\gamma^+ - z}}. \tag{19}
\]

\(P_l(z)\) is Legendre polynomial of the first kind.

The value of the second integral can be taken from Ref. [29, p. 822]

\[
I_2^{(l)} = \frac{2^{1-l}}{2l + 1} (\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1})^{2l+1}. \tag{20}
\]

In cases of the low angular momenta \((l = 0, 1, 2)\), the first integral can be directly calculated from [13]

\[
I_1^{(0)} = \frac{1}{\sqrt{\gamma^+ - \gamma^-}} \ln \left[ \frac{(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - \gamma^-})(\sqrt{\gamma^+ + 1} + \sqrt{\gamma^+ - \gamma^-})}{(\sqrt{\gamma^+ + 1} + \sqrt{\gamma^+ - \gamma^-})(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - \gamma^-})} \right], \tag{21}
\]

\[
I_1^{(1)} = \gamma^- I_1^{(0)} - 2(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1}), \tag{22}
\]

\[
I_1^{(2)} = (\frac{3}{2}\gamma^- + \frac{1}{2})I_1^{(0)} - (\gamma^+ - \sqrt{\gamma^+ + 1} + 3\gamma^-)(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1}). \tag{23}
\]
However, calculation of the first integral for an arbitrary \( l \), the orbital quantum number, is highly complicated and the result seems not to be expressed in the known special functions. See Appendix for some speculations in connection with this subject.

3 Equation For the Wave Function of the Composite System Formed by Fermion and \( S = 1 \) Boson

The equation for the equal-time WF of the composite system of fermion and \( S = 1 \) boson is analogous to the one presented in the Section II.

\[
2\Delta_{p,m2\lambda p}^{0}(M - \Delta_{p,m1\lambda p}^{0} - \Delta_{p,m2\lambda p}^{0})\Phi_{\sigma_{1}\sigma_{2}}(\Delta_{p,\lambda p}) = \\
\frac{1}{(2\pi)^3} \sum_{\nu_{1}\nu_{2}} \int \frac{d^{3}\Delta_{k,\lambda p}}{2\Delta_{k,m1\lambda p}} V_{\nu_{1}\nu_{2}}^{\nu_{1}\nu_{2}}(\Delta_{p,\lambda p}; \Delta_{k,\lambda p})\Phi_{\nu_{1}\nu_{2}}(\Delta_{k,\lambda p})
\]

(24)

It is obvious that in this case the quasipotential in the momentum representation does have the additional terms (which are responsible for the spin-spin interaction, the tensor interaction and the squared spin-orbit interaction) comparing to the case of fermion - \( S = 0 \) boson [30].

Following to the technique of "resetting" the polarization indices, we get analogously to the Section II \[7\]

\[
< p_{1}, p_{2}; \sigma_{1}, \sigma_{2}| \hat{V}^{(2)}| k_{1}, k_{2}; \nu_{1}, \nu_{2} > = \\
= \sum_{\text{pol.ind.}} D_{\sigma_{1}\sigma_{1}p}^{+(S=1/2)} \{ V^{-1}(\Lambda_{p}, p_{1}) \} D_{\sigma_{2}\sigma_{2}p}^{+(S=1)} \{ V^{-1}(\Lambda_{p}, p_{2}) \} \times \\
\times V_{\nu_{1}\nu_{2}p_{2}}^{\nu_{1}\nu_{2}p_{2}}(\tilde{k}(\nu, \bar{p}) \bar{p}) D_{\nu_{2}\nu_{2}k_{2}}^{(S=1/2)} \{ V^{-1}(\Lambda_{p}, k_{1}) \} \times \\
\times D_{\nu_{2}\nu_{2}k_{2}}^{(S=1)} \{ V^{-1}(\Lambda_{p}, k_{2}) \}
\]

(25)

\[
V_{\nu_{1}\nu_{2}p_{2}}^{\nu_{1}\nu_{2}p_{2}}(\tilde{k}(\nu, \bar{p}) \bar{p}) = \xi_{\nu_{1}p}^{*} \xi_{\nu_{2}p}^{*} \hat{V}^{(2)}(\tilde{k}(\nu, \bar{p}) \bar{p}) \xi_{\nu_{1}p} \xi_{\nu_{2}p},
\]

(26)

Let us use the equations for the 4- current of spinor particle defined by the formula (4) and Eq. (27) for the 4- current of vector particle in the Joos-Weinberg’s formalism

\[
j_{\nu_{2}p}^{\mu}(\bar{p}, \tilde{k}) = \xi_{\nu_{2}p}^{*} \left[ (p_{2} + k_{2})^{\mu} + \frac{1}{m_{2}} W^{\mu}(\bar{p}_{2})(\tilde{S}_{2}\tilde{N}_{2}) - \frac{1}{m_{2}} (\tilde{S}_{2}\tilde{N}_{2}) W^{\mu}(\bar{p}_{2}) \right] \xi_{\nu_{2}p}.
\]

(27)

Following to the rules of construction of the quasipotential over the on-shell scattering amplitude [18, 19] we obtain

\[
< \bar{p}_{1}, \bar{p}_{2}; \nu_{1}p_{1}, \sigma_{2}p_{2} | V^{(2)} | k_{1}, k_{2}; \nu_{1}p_{2}, \nu_{2}p_{2} >= \\
= < \bar{p}_{1}, \bar{p}_{2}; \nu_{1}p_{1}, \sigma_{2}p_{2} | T^{(2)} | k_{1}, k_{2}; \nu_{1}p_{2}, \nu_{2}p_{2} > = -g_{\nu} j_{\nu_{2}p_{2}}^{\nu_{1}p_{1}}(\bar{p}_{1}, \tilde{k}_{1}) g_{\nu_{1}p_{1}}^{\nu_{2}p_{2}}(\bar{p}_{2}, \tilde{k}_{2}) (p_{1} - k_{1})^{2}
\]

(28)

\[7\] Let us mention that \( \xi_{\nu_{1}p}^{*}, \xi_{\nu_{2}p} \) are usual Pauli two-component spinors normalized by equation \( \xi_{\nu_{1}p}^{*} \xi_{\nu_{1}p} = \delta_{\nu_{1}}^{\nu} \) and \( \xi_{\nu_{2}p}, \xi_{\nu_{2}p} \) are 3- component analogues of Pauli spinors for \( S = 1 \) particle.
with, as earlier, $\vec{p} = \vec{p}_1 = -\vec{p}_2$ and $\vec{k} = \vec{k}_1 = -\vec{k}_2$ are covariant generalizations of momenta.

As a result one can write the quasipotential operator as follows

$$\hat{V}^{(2)}(\vec{p}, \vec{k}) = -g_V^2 \sqrt{\frac{\Delta_1^0 + m_1}{2m_1}} \frac{p_1^0 (p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2}{m_1(\Delta_1^0 - m_1)}$$

$$- g_V^2 \frac{(p_1^0 + p_2^0 + k_1^0 + k_2^0)(\vec{p}\vec{\Delta}_1)}{m_1(\Delta_1^0 - m_1)} \sqrt{2m_1(\Delta_1^0 + m_1)} + g_V^2 \frac{i\vec{\sigma}_1[p\vec{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)}{m_1(\Delta_1^0 - m_1)}$$

$$+ g_V^2 \sqrt{\frac{\Delta_1^0 + m_1}{m_1 m_2(\Delta_1^0 - m_1)}} i\vec{S}_2[\vec{p}\vec{\Delta}_2](p_1^0 + p_2^0) +$$

$$+ g_V^2 \sqrt{\frac{m_1}{2(\Delta_1^0 + m_1)}} \frac{i\vec{S}_2[\vec{p}\vec{\Delta}_2](p_1^0 + p_2^0 + m_1 + m_2)^2}{m_1(\Delta_1^0 - m_1)}$$

$$+ g_V^2 \sqrt{\frac{m_1}{2(\Delta_1^0 + m_1)}} \frac{(\vec{\sigma}_1\vec{\Delta}_2)(\vec{S}_2\vec{\Delta}_1) - (\vec{\sigma}_1\vec{S}_2)(\vec{\Delta}_1\vec{\Delta}_2) + i\vec{S}_2[\vec{\Delta}_1\vec{\Delta}_2]}{m_1(\Delta_1^0 - m_1)}$$

$$- g_V^2 \frac{1}{\sqrt{m_1(\Delta_1^0 + m_1)}} \frac{\vec{\sigma}_1[p\vec{\Delta}_1]S_2\vec{p}\vec{\Delta}_2](p_1^0 + p_2^0 + m_1 + m_2)^2}{m_1 m_2(\Delta_1^0 - m_1)[(p_1^0 + m_1)(p_2^0 + m_2)].}$$

(29)

Here,

$$\vec{\Delta}_1 = \vec{k} - \frac{\vec{p}}{m_1} \left(k_1^0 - \frac{\vec{k}\vec{p}}{p_1^0 + m_1}\right), \quad \Delta_1 = \sqrt{k_1^2 + m_1^2},$$

$$\vec{\Delta}_2 = \vec{k} - \frac{\vec{p}}{m_2} \left(k_2^0 - \frac{\vec{k}\vec{p}}{p_2^0 + m_2}\right), \quad \Delta_2 = \sqrt{k_2^2 + m_2^2}. \quad \text{(30)}$$

and $p_1^0 = \sqrt{\vec{p}_1^2 + m_1^2}$, $k_1^0 = \sqrt{\vec{k}_1^2 + m_1^2}$, $p_2^0 = \sqrt{\vec{p}_2^2 + m_2^2}$, $k_1^0 = \sqrt{\vec{k}_2^2 + m_2^2}$.

In the quasirelativistic approximation (account of terms up to the order $1/c^2$) Eq. (29) yields

$$V^{(2)}(\vec{k}, \vec{p}) = -g_V^2 \frac{4m_1m_2}{\Delta_\xi^2} + g_V^2 \frac{1}{c^2} \left(1 + \frac{m_2}{2m_1}\right) - g_V^2 \frac{1}{c^2} \left(2 + \frac{m_1}{m_2} + \frac{m_2}{m_1}\right)$$

$$\vec{p}_1^2 + \vec{k}_1^2$$

$$- g_V^2 \frac{1}{c^2} \left(1 + \frac{m_2}{m_1}\right) \frac{2i\vec{\sigma}_1[p\vec{\Delta}_\xi^2]}{\Delta_\xi^2} - g_V^2 \frac{1}{c^2} \left(1 + \frac{m_1}{m_2}\right) \frac{i\vec{S}_2[\vec{p}\vec{\Delta}_\xi^2]}{\Delta_\xi^2}$$

$$- g_V^2 \frac{1}{c^2} \frac{(\vec{\sigma}_1\vec{\Delta}_\xi^2)(\vec{S}_2\vec{\Delta}_\xi^2) - (\vec{\sigma}_1\vec{S}_2)\Delta_\xi^2}{\Delta_\xi^2},$$

(31)

where again $\Delta_\xi^2 = \vec{k} - \vec{p}_1$ is the momentum transfer in the Euclidian space. As opposed to the Eq. (8) we have two additional terms corresponding to the tensor forces and the spin-orbit interaction of the second particle.

One can see that this case is more complicated comparing to the case of the Section II and it does not admit the analytical solution. Therefore, we intend to solve the equation with the quasipotential (29) in the following publications numerically. A good accuracy
in numerical solution of such a type of problems is provided by the spline method \[31\] or by the method for solving the spectral problems, developed in Ref. \[32\], and which is founded on the Galerkin’s procedure of discretization of integral operators. They have been used for the description of two spinor system in \[33\].

4 Conclusions

In the presented paper we have applied the covariant three-dimensional quasipotential approach to the description of quark-diquark bound states, which can be interpreted as nucleons and their resonances. We have derived the partial relativistic equal-time equation for \((\pi\mu)-\) atom and other bound systems (e.g. proton) composed from the particles with spin 1/2 (quark) and spin 0 (diquark). The spin structure of the quasipotential for the system of fermion and \(S = 1\) boson has been also under consideration.

The presence of huge terms in these equations induces us to employ the numerical methods for their solution.

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Appendix

First of all, let us mention that the integral (18)

\[
I_1^{(t)} = \int_{-1}^{1} \frac{P_l(z)dz}{(\gamma^- - z)\sqrt{\gamma^+ - z}}
\]

(32)
can be reduced by means of simple algebraic transformations to

\[
I_1^{(t)} = \frac{1}{\gamma^- - \gamma^+}I_2^{(t)} + \frac{2}{\sqrt{\gamma^+ - \gamma^-}}Q_l(\gamma^-) + \frac{1}{\gamma^+ - \gamma^-} \int_{-1}^{1} \frac{P_l(z)dz}{\sqrt{\gamma^+ - z} + \sqrt{\gamma^+ - \gamma^-}},
\]

(33)

where \(Q_l(x)\) is the Legendre function of the second kind. However, the calculation of the integral in (33) is as complicated as the previous one (32).

Next, we can use the multiple Mellin transform and the tables of formulas from Ref. [34,35] in order to calculate the integral (18). The multiple Mellin transform has the form:

\[
K^*(s_1, \ldots, s_n) = \int_{0}^{\infty} \cdots \int_{0}^{\infty} K(x_1, \ldots, x_n)x_1^{s_1-1} \cdots x_n^{s_n-1}dx_1 \cdots dx_n.
\]

(34)
If the function $K(c_1, \ldots, c_n)$ can be represented in such a form:

$$K(c_1, \ldots, c_n) = \int_0^\infty K_1(x)K_2\left(\frac{c_1}{x}\right)K_{n+1}\left(\frac{c_n}{x}\right)\frac{dx}{x}$$  \hspace{1cm} (35)

then the transform $K(s_1, \ldots, s_n)$ is calculated by the formula

$$K^*(s_1, \ldots, s_n) = K_1^*(s_1 + \ldots + s_n)K_2^*(s_1)\ldots K_{n+1}^*(s_n).$$  \hspace{1cm} (36)

After the substitution $z = 2/x - 1$, the needed integral is rewritten as follows:

$$I_1^{(l)} = \frac{(-1)^l \sqrt{\gamma^+ - 1}}{\gamma - 1} \int_0^\infty \frac{dx}{x} P_l\left(\frac{2}{x} - 1\right)H(x - 1) \cdot \frac{c/x}{\sqrt{1 + c/x(1 + \gamma^+ - 1/c)}},$$  \hspace{1cm} (37)

where $c = 2/(\gamma^+ - 1)$, $H(y) = \begin{cases} 1, & y \geq 0; \\ 0, & y < 0 \end{cases}$ is the Heaviside function.

Taking the transforms from the Tables [35, 36] we can use the reverse Mellin transformation to find out the value of the integral (37)

$$K(c_1, \ldots, c_n) = \frac{1}{(2\pi i)^n} \int_{\gamma_1 - i\infty}^{\gamma_1 + i\infty} \cdots \int_{\gamma_n - i\infty}^{\gamma_n + i\infty} K^*(s_1, \ldots, s_n)c_1^{-s_1} \ldots c_n^{-s_n}ds_1 \ldots ds_n,$$  \hspace{1cm} (38)

where $\gamma_k = \Re s_k, k = 1, \ldots, n$.

Thus,

$$I_1^{(l)} = (-1)^l \sqrt{\gamma^+ - 1} \sum_{k=0}^\infty \frac{1}{\Gamma\left(\frac{3}{2} + k\right)} \left(\frac{\gamma^- - \gamma^+}{\gamma^- - 1}\right)^k \times$$

$$\times G_{1,3}^{1,3} \left(\begin{array}{c} 2 \\ \gamma^+ - 1 \end{array}\right| \begin{array}{ccc} 1, & 1, & 1 \\ 1 + k, & -l, & 1 + l \end{array}\right),$$  \hspace{1cm} (39)

where $G_{H+C,A+D}^{A,B}$ is the Meijer $G$-function.

The another way is also possible: to consider every term in the integral (32) separately and to employ triple Mellin transforms. Using this technique we obtain (34)

$$K_1(x) = P_l\left(\frac{2}{x} - 1\right)H(x - 1) \Rightarrow K_1^*(s) = \Gamma\left[\frac{-s}{l + 1} - s,\frac{-s}{l} - \frac{l}{s} - s\right],$$  \hspace{1cm} (40)

$$K_2\left(\frac{c_1}{x}\right) = \frac{1}{\frac{c_1}{x} - 1} \Rightarrow K_2^*(s_1) = -\pi \Gamma\left[\frac{-s_1}{1} - s_1,\frac{1}{2} - s_1,\frac{1}{2} + s_1\right],$$  \hspace{1cm} (41)

$$K_3\left(\frac{c_2}{x}\right) = \frac{1}{\sqrt{1 - \frac{c_2}{x}}} \Rightarrow K_3^*(s_2) = \frac{\pi}{\Gamma\left(\frac{1}{2}\right)\cos\frac{s_2}{2}} \Gamma\left[\frac{1}{4} + s_2,\frac{3}{4} - s_2\right],$$  \hspace{1cm} (42)

where $c_1 = \frac{2}{\gamma^+ + 1}$, $c_2 = \frac{2}{\gamma^+ + 1}$ and $s = s_1 + s_2, \Gamma(s)$ is the Eiler’s $\Gamma$-function and $\Gamma\left[\begin{array}{c} a_1 \ldots a_k \\ b_1 \ldots b_m \end{array}\right]$ denotes

$$\Gamma\left[\begin{array}{c} a_1 \ldots a_k \\ b_1 \ldots b_m \end{array}\right] = \frac{\Gamma(a_1)\ldots\Gamma(a_k)}{\Gamma(b_1)\ldots\Gamma(b_m)}.$$  \hspace{1cm} (33)

The number of multiple terms in the integral (32) is three.  

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Then,

\[ K^*(s_1, s_2) = -\frac{\sqrt{2\pi^2}}{\Gamma\left(\frac{1}{2}\right)} \Gamma \left[ l + 1 - s_1 - s_2, -s_1 - s_2, -s_1, 1 + s_1, s_2, \frac{1}{2} - s_2 \right]. \] (43)

Similarly to the preceding calculation, employing the reverse Mellin transformation

\[ K(c_1, c_2) = \frac{1}{(2\pi i)^2} \int_{\gamma_1-i\infty}^{\gamma_1+i\infty} \int_{\gamma_2-i\infty}^{\gamma_2+i\infty} K^*(s_1, s_2) c_1^{-s_1} c_2^{-s_2} ds_1 ds_2 \] (44)

to our integral we come to

\[ K(c_1, c_2) = -\frac{\sqrt{2\pi^2}}{\Gamma\left(\frac{1}{2}\right)} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{k+n}}{k! n!} \times \Gamma \left[ \begin{array}{c} k + n + 1, k + n + 1, k + 1, \frac{1}{2} + n \\ l + k + n + 2, -l + k + n + 1, \frac{3}{2} + k, -\frac{1}{2} - k, \frac{1}{4} - n, \frac{3}{4} + n \end{array} \right] c_1^{k+1} c_2^n, \] (45)

which can be slightly simplified after use of well-known expressions for \( \Gamma \)-function,

\[ \Gamma(p)\Gamma(1 - p) = \frac{\pi}{\sin p\pi}, \quad \text{and} \quad \Gamma(k + 1) = k! \quad (k = 0, 1, \ldots) \]

Finally, the value of the integral \( (32) \) can be represented in the form of the complicated double sum of \( \Gamma \)-functions:

\[ I_1^{(l)} = \frac{1}{\sqrt{\pi(\gamma^+ + 1)}} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \Gamma \left[ \begin{array}{c} k + n + 1, k + n + 1, \frac{1}{2} + n \\ l + k + n + 2, -l + k + n + 1, n + 1 \end{array} \right] c_1^{k+1} c_2^n. \] (46)

It is not clear, what representation of the integral under consideration is more convenient; all of them have enormous form and are rather inconvenient for some applications. In our opinion, the further simplifications appear to be impossible. Therefore, the use of computer seems to be necessary.
References

1. M. Ida and R. Kobayashi, *Prog. Theor. Phys.* 36 (1966) 846; D. B. Lichtenberg and L. J. Tassie, *Phys. Rev.* 155 (1967) 1601.
2. S. Fredriksson, *Phys. Rev. Lett.* 52 (1984) 724; Z. Dziembowski and J. Franklin, *Phys. Rev.* D42 (1990) 905; M. Narodetskii, Yu. A. Simonov and V. P. Yurov, *Z. Phys.* C55 (1992) 695.
3. B. Stech, *Phys. Rev.* D36 (1987) 975; B. Pire, *Nucl. Phys.* A497 (1989) 223c; P. Kroll, M. Schürmann and W. Schweiger, *Z. Phys.* A338 (1991) 339; M. Anselmino and E. Predazzi, *Phys. Lett.* B254 (1991) 203; A. Buck, R. Alkofer and H. Reinhardt, *Phys. Lett.* B286 (1992) 29; M. Szczekowski, Preprint CERN-PPE-92-150 (1992) Geneva.
4. C. P. Forsyth and R. E. Cutkosky, *Nucl. Phys.* B178 (1981) 35; B. Ram and V. Kriss, *Phys. Rev.* D35 (1987) 400; S. Fleck, B. Silvestre-Brac and J.-M. Richard, *Phys. Rev.* D38 (1988) 1519; D. B. Leinweber, *Phys. Rev.* D47 (1993) 5096.
5. S. Fredriksson, M. Jändel and T. Larsson, *Z. Phys.* C14 (1982) 35; S. Fredriksson and T. Larsson, *Phys. Rev.* D28 (1987) 975; B. Pire, *Nucl. Phys.* A497 (1989) 223c; P. Kroll, M. Schürmann and W. Schweiger, *Z. Phys.* A338 (1991) 339; M. Anselmino and E. Predazzi, *Phys. Lett.* B254 (1991) 203; A. Buck, R. Alkofer and H. Reinhardt, *Phys. Lett.* B286 (1992) 29; M. Szczekowski, Preprint CERN-PPE-92-150 (1992) Geneva.
6. C. P. Forsyth and R. E. Cutkosky, *Nucl. Phys.* B178 (1981) 35; B. Ram and V. Kriss, *Phys. Rev.* D35 (1987) 400; S. Fleck, B. Silvestre-Brac and J.-M. Richard, *Phys. Rev.* D38 (1988) 1519; D. B. Leinweber, *Phys. Rev.* D47 (1993) 5096.
7. S. Fredriksson, M. Jändel and T. Larsson, *Z. Phys.* C14 (1982) 35; S. Fredriksson and T. Larsson, *Phys. Rev.* D28 (1983) 255.
8. M. Neubert and B. Stech, *Phys. Lett.* B231 (1989) 477; *Phys. Rev.* D44 (1991) 775; H. Dosch, M. Jamin and B. Stech, *Z. Phys.* C42 (1989) 167; B. Stech and Q. P. Xu, *Z. Phys.* C49 (1991) 491; M. Neubert, *Z. Phys.* C50 (1991) 243.
9. A. P. Contogouris et al., *Phys. Lett.* B287 (1992) 203.
10. G. V. Efimov, M. A. Ivanov and V. E. Lyubovitskij, *Z. Phys.* C47 (1990) 583.
11. M. Szczekowski, *Int. J. Mod. Phys.* A4 (1989) 3985.
12. J.-M. Richard, *Phys. Repts.* 212 (1992) 1.
13. V. G. Kirillov-Ugryumov, Yu. P. Nikitin and F. M. Sergeev, *Atoms and Mesons.* Atomizdat, Moscow, 1980 (in Russian); C. J. Batty, *Fiz. Elem. Chast. At. Yadra* 13 (1982) 164 [Sov. J. Part. and Nucl., 71].
14. J. A. Wheeler, *Phys. Rev.* 71 (1947) 320; E. Fermi and E. Teller, *Phys. Rev.* 72 (1947) 399.
15. M. Schwartz, In Proc. of XVIII Int. Conf. on HEP. Tbilisi, 1976. JINR, Dubna D12-14000, vol. I (1977) A7; R. Combes et al., *Phys. Rev. Lett.* 37 (1976) 249.
16. L. L. Nemenov, In Proc. of Int. School of Young Scientists on HEP. Gomel', 1971. JINR, Dubna 2-6371 (1972) 87; *Yad. Fiz.* 15 (1972) 1047 [Sov. J. Nucl. Phys., 582]; *Yad. Fiz.* 16 (1972) 125 [Sov. J. Nucl. Phys., 67].
17. U. Bar-Gadda and C. F. Cho, *Phys. Lett.* 46B (1973) 95; C. F. Cho, *Nuovo Cim.* 23A (1974) 557.
18. A. Karimhodzhaev and R. N. Faustov, Preprint JINR P2-11746 (1978) Dubna; *Yad. Fiz.* 29 (1979) 463 [Sov. J. Nucl. Phys., 232].
19. A. A. Logunov and A. N. Tavkhelidze, *Nuovo Cim.* 29 (1963) 380.
20. H. Joos, *Forts. Phys.* 10 (1962) 65; S. Weinberg, *Phys. Rev.* B133 (1964) 1318; ibid B134 (1964) 882; ibid 181 (1969) 1893.
21. V. V. Dvoeglazov and N. B. Skachkov, *JINR Communications* P2-84-200 (1984) Dubna; V. V. Dvoeglazov et al., *Yad. Fiz.* 54 (1991) 685 [Sov. J. Nucl. Phys., 398].
22. A. D. Linkevich and v. I. Savrin and N. B. Skachkov, *Yad. Fiz.* 37 (1983) 391 [Sov. J. Nucl. Phys., 398].
23. A. D. Linkevich et al., *JINR Communications* P2-82-563 (1982) Dubna.
24. N. B. Skachkov, *JINR Communications* E2-7159, E2-7333 (1973) Dubna; *TMF* 22 (1975) 213 [Theor. Math. Phys., 149]; Preprints *JINR* E2-81-294, E2-81-308, E2-81-399 (1981) Dubna; N. B. Skachkov and I. L. Solovtsov, *Fiz. Elem. Chast. At. Yadra* 9 (1978) 5 [Sov. J. Part. Nucl., 1].

25. Yu. M. Shirokov, *ZhETF* 34 (1951) 1005; A. A. Cheshkova and Yu. M. Shirokov, *ZhETF* 44 (1963) 1982 [Sov. Phys. JETP 17 (1963) 1333].

26. V. V. Dvoeglazov and N. B. Skachkov, *JINR Communications* P2-84-383 (1984) Dubna; V. V. Dvoeglazov, Ph. D. Thesis, *JINR* 2-91-331 (1991) Dubna.

27. V. N. Kapshai and G. Yu. Tyumenkov, *Izvest. VUZ: Fizika* 32 (1989) 57 [Sov. Phys. J., 123].

28. V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Relativistic Quantum Theory. Vol. 4 of Course of Theoretical Physics. Part I.* Moscow, Nauka, 1968 [English translation: Oxford, Pergamon Press, 1979, p. 280].

29. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Series and Products.* Moscow, Nauka, 1971 [English translation: New York and London, Academic Press, 1965. Ed. A. Jeffrey].

30. V. V. Dvoeglazov and N. B. Skachkov, *JINR Communications* P2-84-199 (1984) Dubna.

31. S. B. Stechkin and Yu. N. Subbotin, *Splines in Computational Mathematics.* Moscow: Nauka, 1976; L. Alexandrov and D. Karadzhov, *Zh. Vych. Mat. & Mat. Fiz.* 20 (1980) 923; A. V. Sidorov and N. B. Skachkov, *JINR Communications* P2-84-502 (1984) Dubna (in Russian).

32. E. P. Zhdakov et al., *JINR Communications* P11-85-465, P11-87-261, E11-88-494 (1985-88) Dubna.

33. B. A. Arbuzov et al., *TMF* 83 (1990) 175 [Theor. Math. Phys., 457]; *Mod. Phys. Lett.* A5 (1990) 1441; *Phys. Lett.* B240 (1990) 477.

34. O. I. Marichev, *Handbook of Integral Transforms of Higher Transcendental Functions: Theory and Algorithmic Tables.* Minsk, Nauka, 1978 [English translation: New York, Halsted Press, 1983. Transl. by L. W. Longdon].

35. A. P. Prudnikov, Ya. A. Brychkov and O. I. Marichev, *Integrals and Series. Vol. 3.* Moscow, Nauka, 1986 [English translation: London, Gordon and Breach, 1990. Transl. by G. G. Gould, p.54]

36. A. Erdélyi (ed.), *Tables of Integral Transforms. Vol. 1 (Bateman Manuscript Project).* New York, McGrow-Hill, 1954, p.312.