Search for the semileptonic decay $D^{0(+) \rightarrow b_1(1235)} \rightarrow (0)^+_{c} e + \nu_e$
Using 2.93 fb$^{-1}$ of $e^+e^-$ annihilation data collected at a center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector operating at the BEPCII collider, we search for the semileptonic $D_0^{(*)}$ decays into a $b_1(1235)^{-}$ axial-vector meson for the first time. No significant signal is observed for either charge combination. The upper limits on the product branching fractions are $B_{D_0^{(*)} \rightarrow b_1(1235)^{-}} e^+\nu_e < 1.12 \times 10^{-4}$ and $B_{D_0^{(*)} \rightarrow b_1(1235)^{0}} e^+\nu_e < 8.12 \times 10^{-4}$. 

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Semileptonic decays of the $D^{0(+)\pm}$ provide an outstanding platform to explore the dynamics of both weak and strong interactions in the charm sector. The semileptonic $D^{0(+)}$ decays into pseudoscalar and vector mesons have been widely studied in both experiment [1] and theory. Extensive studies of the semileptonic $D^{0(+)}$ decays into axial-vector mesons $K_1(1270)$ and $b_1(1235)$ play an important role in the understanding of nonperturbative strong-interaction dynamics in weak decays [2–8]. Nevertheless, knowledge of these decays is limited. The observation of the Cabibbo-favored decay $D^+ \to K_1(1270)^0 e^+\nu_e$ has been reported by the BESIII experiment [9], and evidence for $D^0 \to K_1(1270)^- e^+\nu_e$ has been found at CLEO [10]. The measured branching fractions are consistent with theoretical predictions based on the Isgur-Scora-Grinstein-Wise (ISGW) quark model [2] and its upgrade (ISGW2) [3], as well as those based on the covariant light-front quark model [6]. As for the singly Cabibbo-suppressed decays $D^{0(+)} \to b_1(1235)^-(0) e^+\nu_e$, no experimental study has yet been carried out. Experimental measurements of the semileptonic decays $D^{0(+)} \to b_1(1235)^-(0) e^+\nu_e$ are important to test theoretical calculations and to understand nonperturbative effects in heavy meson decays [2, 3, 6]. Moreover, the observation of the $b_1(1235)^-(0)$ meson in semileptonic decays would provide a clean environment to study its nature [11]. In this paper, we report the first search for the semileptonic decays $D^0 \to b_1(1235)^-(0) e^+\nu_e$ and $D^+ \to b_1(1235)^0 e^+\nu_e$. The data used in this analysis, corresponding to an integrated luminosity of 2.93 fb$^{-1}$ [12], was accumulated at a center-of-mass energy of 3.773 GeV with the BESIII detector. Throughout this paper, charge conjugate channels are always implied.

### II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector is a magnetic spectrometer [13] located at the Beijing Electron Positron Collider (BEPCHII) [14]. The cylindrical core of the BESIII detector consists of a helium-based multi-layer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules interleaved with steel. The acceptance of charged particles and photons is 93% over $4\pi$ solid angle. The charged-particle momentum resolution at 1 GeV/$c$ is 0.5%, and the $dE/dx$ resolution is 6% for the electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end cap part is 110 ps.

Simulated samples produced with the GEANT4-based [15] Monte Carlo (MC) package which includes the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes the beam energy spread and initial state radiation (ISR) in the $e^+e^-$ annihilations modeled with the generator KKMC [16]. The inclusive MC samples consist of the production of $D\bar{D}$ pairs with consideration of quantum coherence for all neutral $D$ modes, the non-$D\bar{D}$ decays of the $\psi(3770)$, the ISR production of the $J/\psi$ and $\psi(3686)$ states, and the continuum processes. The known decay modes are modeled with EVTGEN [17] using the branching fractions taken from the Particle Data Group [1], and the remaining unknown decays from the charmonium states with LUNDCHARM [18]. The final state radiations from charged final state particles are incorporated with the PHOTOS package [19]. The signal process $D^{0(+)} \to b_1(1235)^-(0) e^+\nu_e$ is simulated with $b_1(1235)^-(0)$ decaying into $\omega\pi^-(0)$, using the ISGW2 model [3]. A relativistic Breit-Wigner function is used to parameterize the resonance $b_1(1235)^-(0)$, the mass and width of which are fixed to the world-average values of $1229.5 \pm 3.2$ MeV/$c^2$ and $142 \pm 9$ MeV, respectively [1].

### III. DATA ANALYSIS

The process $e^+e^- \to \psi(3770) \to D\bar{D}$ provides an ideal opportunity to study semileptonic $D^{0(+)}$ decays with the double-tag (DT) method, because there are no additional particles that accompany the $D$ mesons in the final states [20]. Throughout the paper, $D$ denotes $D^0$ or $D^+$. At first, single-tag (ST) $D^0$ mesons are reconstructed by using the hadronic decay modes of $D^0 \to K^+\pi^-$, $K^+\pi^-\pi^0$, and $K^+\pi^-\pi^+\pi^-$; while ST $D^-$ mesons are reconstructed via the decays $D^- \to K^+\pi^-\pi^-$, $K^0_S\pi^-$, $K^0_S\pi^-\pi^0$, $K^0_S\pi^-\pi^0$, $K^0_S\pi^+\pi^-\pi^-$, and $K^+K^-\pi^-$. Then the semileptonic $D$ candidates are reconstructed with the remaining tracks and showers. The candidate event in which $D$ decays into $b_1(1235)^0 e^+\nu_e$ and $D$ decays into a tag mode is called a DT event. Since the branching fraction of the subsequent decay $b_1(1235)^0 \to \omega\pi$ is not well measured, the product of the branching fractions of the decay $D \to b_1(1235)^0 e^+\nu_e$ ($\mathcal{B}_{SL}$) and its subsequent decay $b_1(1235)^0 \to \omega\pi$ ($\mathcal{B}_{b1}$) is
determined using
\[ B_{\text{SL}} \cdot B_{\text{DT}} = \frac{N_{\text{DT}}}{N_{\text{ST}}} \cdot \varepsilon_{\text{SL}} \cdot B_{\text{SL}} \cdot (B_{\nu e})^k, \]
where \( N_{\text{ST}} \) and \( N_{\text{DT}} \) are the yields of the ST \( D \) mesons and the DT signal events in data, respectively; \( B_{\text{SL}} \) and \( B_{\nu e} \) are the branching fractions of \( \omega \to \pi^+\pi^-\pi^0 \) and \( \pi^0 \to \gamma\gamma \), respectively; \( k \) is the component, which corresponds to the number of \( \pi^0 \) mesons in the final states and \( \varepsilon_{\text{SL}} \) is the average efficiency of reconstructing \( D \to b_1(1235)e^+\nu_e \). The average signal efficiency, weighted over the tag modes, is calculated by
\[ \varepsilon_{\text{SL}} = \frac{\Sigma_i [(\varepsilon_{i\text{DT}} \cdot N_{i\text{ST}})^{1/2}]}{N_{\text{ST}}} \]
where \( N_{i\text{ST}} \) is the ST yield of \( D \to i \), \( \varepsilon_{i\text{DT}} \) is the detection efficiency of reconstructing \( D \to i \), and \( \varepsilon_{i\text{DT}} \) is the detection efficiency of reconstructing \( D \to b_1(1235)e^+\nu_e \) at the same time.

The ST \( D \) candidates are selected with the same criteria employed in our previous works [9, 21–29]. For each charged track (except for those used for reconstructing \( K_S^0 \) meson decays), the polar angle with respect to the MDC axis (\( \theta \)) is required to satisfy \( |\cos \theta| < 0.93 \), and the point of closest approach to the interaction point (IP) must be within 1 cm in the plan perpendicular to the MDC axis and within \( \pm 10 \) cm along the MDC axis. Charged tracks are identified by using the \( dE/dx \) and TOF information, with which the combined confidence levels under the pion and kaon hypotheses are computed separately. A charged track is assigned as the particle type which has a larger probability.

Candidate \( K_S^0 \) mesons are formed from pairs of oppositely charged tracks. For these two tracks, the distance of closest approach to the IP is required to be less than 20 cm along the MDC axis. No requirements on the distance of closest approach in the transverse plane or on particle identification (PID) criteria are applied to these tracks. The two charged tracks are constrained to originate from a common vertex, which is required to be away from the IP by a flight distance of at least twice the vertex resolution. The invariant mass of the \( \pi^+\pi^- \) pair is required to be within (0.486, 0.510) GeV/\( c^2 \).

Neutral pion candidates are reconstructed via the \( \pi^0 \to \gamma\gamma \) decays. Photon candidates are chosen from the EMC showers. The EMC time deviation from the event start time is required to be within [0, 700] ns. The energy deposited in the EMC is required to be greater than 25 (50) MeV if the crystal with the maximum deposited energy in that cluster is in the barrel (end cap) region [30]. The opening angle between the photon candidate and the nearest charged track is required to be greater than 10°. For any \( \pi^0 \) candidate, the invariant mass of the photon pair is required to be within (0.115, 0.150) GeV/\( c^2 \). To improve the momentum resolution, a mass-constrained (1-C) fit to the nominal \( \pi^0 \) mass [1] is imposed on the photon pair. The four-momentum of the \( \pi^0 \) candidate returned by this kinematic fit is used for further analysis.

In the selection of \( D^0 \to K^+\pi^- \) events, the background from cosmic rays and Bhabha events are rejected by using the same requirements described in Ref. [31]. To separate the ST \( D \) mesons from combinatorial backgrounds, we define the energy difference \( \Delta E \equiv E^+_D - E_{\text{beam}} \) and the beam-constrained mass \( M_{\text{BC}} \equiv \sqrt{E^2_{\text{beam}}/c^4 - p_D^2/c^2} \), where \( E_{\text{beam}} \) is the beam energy, and \( E_D \) and \( p_D \) are the total energy and momentum of the ST \( D \) meson in the \( e^+e^- \) center-of-mass frame. If there is more than one \( D \) candidate in a specific ST mode, the one with the least \( |\Delta E| \) is kept for further analysis.

To suppress combinatorial backgrounds, the ST \( D \) candidates, which are reconstructed by using the modes with and without \( \pi^0 \) in the final states, are imposed with the requirements of \( \Delta E \in (-0.055, 0.045) \) GeV and \( \Delta E \in (-0.025, 0.025) \) GeV, respectively. For each ST mode, the yield of ST \( D \) mesons is extracted by fitting the corresponding \( M_{\text{BC}} \) distribution. The signal is described by an MC-simulated shape convolved with a double-Gaussian function which compensates the resolution difference between data and MC simulation. The background is parameterized by the ARGUS function [32]. All fit parameters are left free in the fits. Figure 1 shows the fits to the \( M_{\text{BC}} \) distributions for individual ST modes. The candidates with \( M_{\text{BC}} \) lying in (1.859, 1.873) GeV/\( c^2 \) for \( D^0 \) tags and (1.863, 1.877) GeV/\( c^2 \) for \( D^- \) tags are kept for further analysis. Summing over the tag modes, the total yields of ST \( D^0 \) and \( D^- \) mesons are obtained to be 2321009 ± 1875_{\text{stat}} \) and 1522474 ± 2215_{\text{stat}} \), respectively [23].

![Fig. 1. Fits to the M_{BC} distributions of the ST D candidates.](image)

We require that there are four and three charged tracks reconstructed in \( D^0 \to b_1(1235)e^+\nu_e \) and \( D^+ \to b_1(1235)e^+\nu_e \) candidates, respectively. These tracks exclude those used to form the ST \( D \) candidates. For each candidate, one charged track is identified as a positron and the others are required to be identified.
as pions. The selection criteria of charged and neutral pions are the same as those used in selecting the ST $D$ candidates. To suppress fake $\pi^0$ candidates, the decay angle of $\pi^0$, defined as
\[
\cos \theta_{\pi^0} = \frac{|E_{\gamma 1} - E_{\gamma 2}|}{|\vec{p}_{\pi^0} \cdot \vec{c}|},
\]
is required to be less than 0.9. The requirement has been optimized using the inclusive MC sample. $E_{\gamma 1}$ and $E_{\gamma 2}$ are the energies of the two daughter photons of the $\pi^0$, and $\vec{p}_{\pi^0}$ is the reconstructed momentum of the $\pi^0$. For the selected $D^0 \rightarrow \pi^+\pi^-\pi^0 e^+\nu_e$ and $D^+ \rightarrow \pi^+\pi^-\pi^0 e^+\nu_e$ candidates, there are always two possible $\pi^+\pi^-\pi^0$ combinations to form the $\omega$. The invariant masses of both combinations are required to be greater than 0.6 GeV/$c^2$ to suppress the backgrounds from $D \rightarrow a_0(980)e^+\nu_e$. One candidate is kept for further analysis if either of the combinations has an invariant mass falling in the $\omega$ mass signal region of $(0.757, 0.807)$ GeV/$c^2$. To form a $b_1(1235)$ candidate, the $\omega\pi$ invariant mass is required to be within $(1.080, 1.380)$ GeV/$c^2$. The background from $D^{0(*)} \rightarrow K_1^{*}(1270)[K^0_S\pi^+(0)\pi^-(-0)]e^+\nu_e$ is rejected by requiring the invariant masses of any $\pi^+\pi^-\pi^0$ combinations to be outside $(0.486, 0.510)$ GeV/$c^2$ ($(0.460, 0.510)$ GeV/$c^2$). These requirements correspond to three times the invariant mass resolution about the nominal $K^0_S$ mass [1].

The $e^+$ candidate is required to have a charge of opposite sign to that of the charm quark in the ST $D$ meson. The $e^+$ candidate is identified by using the combined $dE/dx$, TOF, and EMC information. The combined confidence levels for the positron, pion, and kaon hypotheses ($CL_e$, $CL_\pi$, and $CL_K$) are computed. The positron candidate is required to satisfy $CL_e > 0.001$ and $CL_\pi/(CL_e+CL_\pi+CL_K) > 0.8$. Its deposited energy in the EMC is required to be greater than 0.8 times its momentum reconstructed by the MDC, to further suppress the background from misidentified hadrons and muons.

The peaking backgrounds from hadronic $D$ decays with multiple pions in the final states are rejected by requiring that the invariant mass of $b_1(1235)e^+ (M_{b_1e^+})$ is less than 1.80 GeV/$c^2$. To suppress backgrounds with extra photon(s), we require that the energy of any extra photon $(E_{\gamma \text{extra}}^0)$ is less than 0.30 GeV and there is no extra $\pi^0$ ($N_{\text{extra}}^0$) in the candidate event.

The neutrino is not detectable in the BESIII detector. To distinguish semileptonic signal events from backgrounds, we define $U_{\text{miss}} \equiv E_{\text{miss}} - |\vec{p}_{\text{miss}}| \cdot c$, where $E_{\text{miss}}$ and $\vec{p}_{\text{miss}}$ are the missing energy and momentum of the DT event in the $e^+e^-$ center-of-mass frame, respectively. They are calculated as $E_{\text{miss}} \equiv E_{\text{beam}} - E_0 - E_{\pi^+} - E_{\pi^-} - E_{\pi^0}$ and $\vec{p}_{\text{miss}} \equiv \vec{p}_D - \vec{p}_0 - \vec{p}_\pi + \vec{p}_e$, where $E_{0,\pi^+}$ are the measured energy and momentum of the $b_1(1235)(e^+)$ candidates, respectively, and $\vec{p}_D \equiv -\vec{p}_D \cdot \sqrt{E_{\text{beam}}^2/c^2 - m_D^2}$, where $\vec{p}_D$ is the unit vector in the momentum direction of the ST $D$ meson and $m_D$ is the nominal $D$ mass [1]. The use of the beam energy and the nominal $D$ mass for the magnitude of the ST $D$ mesons improves the $U_{\text{miss}}$ resolution. For the correctly reconstructed signal events, $U_{\text{miss}}$ peaks at zero.

Figure 2 shows the $U_{\text{miss}}$ distributions of the accepted candidate events. Unbinned maximum likelihood fits are performed on these distributions. In the fits, the signal and background are modeled by the simulated shapes obtained from the signal MC events and the inclusive MC sample, respectively, and the yields of the signal and background are left free. Since no significant signal is observed, conservative upper limits will be set by assuming all the fitted signals are from $b_1(1235)$.

The detection efficiencies $\varepsilon_{\text{SL}}$ are estimated to be $0.0704 \pm 0.0006$ and $0.0412 \pm 0.0002$ for the $D^0 \rightarrow b_1(1235)e^+\nu_e$ and $D^+ \rightarrow b_1(1235)e^+\nu_e$ decays, respectively. The blue dotted curves in Fig. 3 show the raw likelihood distributions versus the corresponding product of branching fractions.

IV. SYSTEMATIC UNCERTAINTY

With the DT method, many systematic uncertainties on the ST side mostly cancel. The sources of the systematic uncertainties in the measurements of the product of branching fractions are classified into two cases. The first one is from the uncertainties relying on effective efficiencies and are assigned relative to the measured branching fractions. The uncertainty associated with the ST yield $N_{ST}^{tot}$ is estimated to be 0.5% [21–23]. The uncertainty from the quoted branching fraction of the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay is 0.8%. The uncertainties from the tracking and PID of $e^\pm$ are studied with a control sample of $e^+e^- \rightarrow \gamma e^+e^-$. The uncertainties from the tracking and PID of $\pi^0$ reconstruction are obtained by studying a DT control sample $\psi(3770) \rightarrow D\bar{D}$ with hadronic $D$ decays [21, 22]. The systematic uncertainties from the tracking (PID) efficiencies are assigned as 1.0% (1.0%) per $e^\pm$ and 1.0% (1.0%) per $\pi^\pm$, respectively. The $\pi^0$ reconstruction efficiencies include photon finding, the $\pi^0$ mass window, and the 1-C kinematic fit, the systematic uncertainty of which is taken to be 2.0% per $\pi^0$. The systematic uncertainty from the $\pi^0$ decay angle requirement is determined to be 2.0% per $\pi^0$ by studying the DT events of $D^0 \rightarrow K^-\pi^+\pi^0$ versus $D^0 \rightarrow K^+\pi^-\pi^0$ and $K^+\pi^-\pi^0$. The systematic uncertainty associated with the $\omega$ mass window is assigned to be 1.2% using a control sample of $D^0 \rightarrow K^0_{SL}\omega$ reconstructed versus the same $D^0$ tags as those used in the nominal analysis. The systematic uncertainties from the $E_{\text{extra}}^\text{max, SL}$ and $N_{\text{extra, \pi^0}}$ requirements are estimated to be 1.4% and 2.0% for $D^0 \rightarrow b_1(1235)e^+\nu_e$ and $D^+ \rightarrow b_1(1235)e^+\nu_e$, respectively, which are estimated using DT samples of $D^0 \rightarrow K^-e^+\nu_e$ and $D^+ \rightarrow K^0_{SL}e^+\nu_e$ decays reconstructed versus the same tags as the nominal analysis. The systematic uncertainty related to the MC generator is
estimated using alternative signal MC samples, which are produced by varying the mass and width of the $b_1(1235)$ by ±1σ. The maximum changes of the signal efficiencies, 5.1% and 2.7%, are assigned as the systematic uncertainties for $D^0 \rightarrow b_1(1235)^- e^+\nu_e$ and $D^+ \rightarrow b_1(1235)^0 e^+\nu_e$, respectively. The uncertainties from limited MC statistics, propagated from those of the ST and DT efficiencies, are 0.7% and 0.9% for $D^0 \rightarrow b_1(1235)^- e^+\nu_e$ and $D^+ \rightarrow b_1(1235)^0 e^+\nu_e$, respectively. By adding these uncertainties in quadrature, the total systematic errors associated with the signal efficiencies ($\sigma_s$) are obtained to be 8.2% and 7.3% for $D^0 \rightarrow b_1(1235)^- e^+\nu_e$ and $D^+ \rightarrow b_1(1235)^0 e^+\nu_e$, respectively. The second kind of systematic uncertainty originates from the fit to the $U_{\text{miss}}$ distribution of the semileptonic $D$ decay candidates. It is dominated by the uncertainty from imperfect knowledge of the background shape. The uncertainty associated with the signal shape is negligible. The background shape is obtained from the inclusive MC sample using a kernel estimation method [33] implemented in RooFit [34]. Unlike the other sources of uncertainties, the background shape directly affects the likelihood function. The smoothing parameter of RooKeysPdf is varied within a reasonable range to obtain alternative background shapes. The absolute change of the signal yield, which gives the largest upper limit on the branching fraction, is taken as the systematic uncertainty ($\sigma_s$). It is found to be 1.7 for $D^0 \rightarrow b_1(1235)^- e^+\nu_e$ and 1.1 for $D^+ \rightarrow b_1(1235)^0 e^+\nu_e$.

V. RESULTS

To incorporate the second kind of systematic uncertainty, the updated likelihood is then convolved with another Gaussian function with mean of 0 and a width equal to $\sigma_s$ similarly. Here $\sigma_s$ is an uncertainty of the product of the branching fractions calculated with Eq. (1) by replacing $N_{\text{DT}}$ with $\sigma_s$.

The red solid curves in Fig. 3 show the resulting likelihood distributions for the two decays. The upper limits on the product of branching fractions at the 90% confidence level (C.L.), obtained by integrating $L(B)$ from zero to 90% of the total curve, are

$$B_{D^0 \rightarrow b_1(1235)^- e^+\nu_e} \cdot B_{b_1(1235)^- \rightarrow \omega\pi^-} < 1.12 \times 10^{-4}$$

and

$$B_{D^+ \rightarrow b_1(1235)^0 e^+\nu_e} \cdot B_{b_1(1235)^0 \rightarrow \omega\pi^0} < 1.75 \times 10^{-4}.$$
Fig. 3. Likelihood distributions versus the corresponding product of branching fractions for (left) $D^0 \rightarrow b_1(1235)^0 e^+ \nu_e$ and (right) $D^+ \rightarrow b_1(1235)^- e^+ \nu_e$, with (red solid curves) and without (blue dotted curves) smearing the systematic uncertainties. The black arrows correspond to the upper limits at the 90% confidence level.

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