Nuclear structure in parity doublet model

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Abstract Using an extended parity doublet model with the hidden local symmetry, we study some properties of nuclei in the mean field approximation to see if the parity doublet model could reproduce nuclear properties and also to estimate the value of the chiral invariant nucleon mass $m_0$ preferred by nuclear structure. We first determine our model parameters using the inputs from free space and from nuclear matter properties. Then, we study some basic nuclear properties such as the nuclear binding energy with several different choices of the chiral invariant mass. We observe that our model could reproduce nuclear properties and also to in the mean field approximation to see if the parity doublet model could reproduce nuclear properties and also to estimate the value of the chiral invariant nucleon mass $m_0$ preferred by nuclear structure. We first determine our model parameters using the inputs from free space and from nuclear matter properties. Then, we study some basic nuclear properties such as the nuclear binding energy with several different choices of the chiral invariant mass. We observe that our model estimates slightly lower than the experimental value, and the value of the chiral invariant nucleon mass $m_0$ preferred by nuclear properties. We then calculate some properties of several selected nuclei with $m_0 = 700$ MeV and compare them with experiments. Finally, we study the neutron-proton mass difference in some nuclei.

1 Introduction

Nuclei are interesting and important quantum finite many-body systems, providing a solid testing ground for our understanding of the strong interactions and many-body techniques. In principle, we should be able to understand nuclei in terms of quarks and gluons in the framework of quantum chromodynamics (QCD). For instance, it will be especially interesting to investigate how QCD vacuum properties are reflected in the properties of nuclear matter and finite nuclei. This is, however, a formidable task to achieve due to the non-perturbative nature of the strong interaction at low energies. Since the nucleus is commonly believed to be composed of the protons and neutrons and it is highly nontrivial to understand nuclei in terms of quarks and gluons in the framework of QCD, it is natural to view the nucleus as a collection of interacting protons and neutrons. In addition, in the light of effective theory, thanks to the QCD separation scale due to confinement or spontaneous chiral symmetry breaking, using protons and neutrons as relevant degrees of freedom for nuclei would be still desirable.

Studying the origin of hadron masses is one of important problems in nuclear physics. As it is well known, the current quark mass could explain roughly 2% of the nucleon mass. A picture in nuclear physics is that the nucleon mass in the chiral limit could be explained by quark-antiquark condensates in QCD vacuum, i.e., spontaneous chiral symmetry breaking. In the parity doublet model [1], however, the nucleon mass has a piece called the chiral invariant mass $m_0$ that may have something to do with QCD trace anomaly. In Ref. [1], from the decay of $N^*, N^*(1535) \rightarrow N + \pi$, the value of the chiral invariant mass $m_0$ was determined as $m_0 \approx 700$ MeV.

Dense matter, which is intimately related to heavy ion collisions, nuclear structure and neutron stars, has been extensively investigated in the parity doublet models [2–10], while nuclear structure study in parity doublet models has not been made. In Ref. [3], the chiral invariant mass was estimated as $m_0 \sim 800$ MeV from nuclear matter properties, especially incompressibility. In Ref. [6], the chiral invariant mass $m_0$ was re-expressed as the sum of the contributions from tetraquark and gluon condensates. An extended parity doublet model [10] reasonably reproduces the properties of nuclear matter with the chiral invariant nucleon mass in the range from 500 to 900 MeV. As discussed in Ref. [11], it is expected that a larger $m_0$ implies a smaller Yukawa coupling of the $\sigma$ field to nucleons, since a part of the nucleon mass...
from the chiral symmetry breaking is smaller. The attractive force by the $\sigma$ should be balanced by the repulsive force mediated by $\omega$ at the saturation density, so that the $\omega$ contribution is smaller for large $m_0$. In the higher density region, $\omega$ contribution is expected to grow while $\sigma$ contribution decreases. As a result of the repulsive nature of the $\omega$, the equation of state is softer for large $m_0$ [11].

A promising microscopic theoretical tool for nuclear matter and (medium-mass and heavy) nuclei is energy density functional (EDF) theory [12]. With a few parameters, density functional theory provides a successful description of ground-state properties of spherical and deformed nuclei. Self-consistent relativistic mean field theory is a useful method to obtain the covariant energy density functionals. The covariant EDF includes the nucleon spin degree of freedom naturally and, therefore, can consistently explain the nuclear spin-orbit potential. For the details of covariant EDF, we refer to Refs. [13–16].

In this work, using the parity doublet model developed in Ref. [10], we study the properties of nuclei in self-consistent relativistic mean field theory to see if a parity doublet model works for nuclear properties and to find out the value of the chiral invariant mass preferred by nuclear structures. As a first attempt, we will focus on the properties of stable nuclei in the present study.

The extended parity doublet model [10], where the chiral invariant nucleon mass is in the range from 500 to 900 MeV, is briefly described in Sect. 2 and the results are given in Sect. 3. We then summarize the present work in Sect. 4.

2 Parity doublet model with hidden local symmetry

In this section we study finite nuclei in the context of the parity doublet model [1,17–19]. To investigate nuclear matter or finite nuclei using a parity doublet model, either isospin symmetric or asymmetric, one needs to introduce vector mesons in the model. A convenient way to do that, which respects chiral symmetry, is to use the Hidden Local Symmetry (HLS) [20,21]. In Ref. [10], a parity doublet model with the HLS was constructed for an asymmetric nuclear matter study. It was shown in Ref. [10] that the phase structure of cold dense matter depends on the value of the chiral invariant mass and also on isospin asymmetry. In this section, we use the extended parity doublet model constructed in Ref. [10].

We start with the Lagrangian,

$$\mathcal{L} = \bar{\psi}_1 i \partial_\mu \psi_1 + \bar{\psi}_2 i \partial_\mu \psi_2 + m_0(\bar{\psi}_2 \gamma_5 \psi_2 - \bar{\psi}_1 \gamma_5 \psi_2) + g_1 \bar{\psi}_1(\sigma + i \gamma_5 \tau \cdot \vec{\tau}) \psi_1 + g_2 \bar{\psi}_2(\sigma - i \gamma_5 \tau \cdot \vec{\tau}) \psi_2 - g_\omega NN \bar{\psi}_1 \gamma_\mu \omega^\mu \psi_1 - g_\omega NN \bar{\psi}_2 \gamma_\mu \omega^\mu \psi_2 - g_\rho NN \bar{\psi}_1 \gamma_\mu \vec{\rho}^\mu \cdot \vec{\tau} \psi_1 - g_\rho NN \bar{\psi}_2 \gamma_\mu \vec{\rho}^\mu \cdot \vec{\tau} \psi_2 - e \bar{\psi}_1 \gamma_\mu A_\mu \frac{1 - \tau_3}{2} \psi_1 - e \bar{\psi}_2 \gamma_\mu A_\mu \frac{1 - \tau_3}{2} \psi_2 + \mathcal{L}_M,$$

(1)

where the baryon fields $\psi_1$ and $\psi_2$ transform as

$$\psi_1 R \rightarrow R \psi_1, \quad \psi_2 L \rightarrow L \psi_1,$$
$$\psi_2 R \rightarrow L \psi_2, \quad \psi_2 L \rightarrow R \psi_2,$$

(2)

with $L$ and $R$ being the elements of $SU(2)_L$ and $SU(2)_R$ chiral symmetry group, respectively. The meson part of Lagrangian is given by

$$\mathcal{L}_M = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \vec{\tau} \cdot \partial^\mu \vec{\tau}$$
$$- \frac{1}{4} \Omega_{\mu\nu \rho} \Omega^{\mu\nu \rho} - \frac{1}{4} \tilde{R}_{\mu\nu \rho} \tilde{R}^{\mu\nu \rho} - \frac{1}{4} F_{\mu
u \rho} F^{\mu \nu \rho}$$
$$+ \frac{\vec{\mu}^2}{2} (\sigma^2 + \vec{\tau}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\tau}^2)^2 + \frac{\lambda_6}{6} (\sigma^2 + \vec{\tau}^2)^3 + \epsilon \sigma$$
$$+ \frac{1}{2} m_\omega^2 \rho_{\mu \rho} \rho^\rho - \frac{1}{2} m_\rho^2 \rho_{\mu \rho} \rho^\rho$$

(3)

with

$$\Omega_{\mu \nu \rho} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,$$
$$\tilde{R}_{\mu \nu \rho} = \partial_\mu \bar{\rho}_\rho - \partial_\rho \bar{\rho}_\mu,$$
$$F_{\mu \nu \rho} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

(4)

In this work, we adopt a mean field approximation and consider the following mean fields for mesons: $\sigma \rightarrow \langle \sigma \rangle$, $\omega_\mu \rightarrow \delta_{\mu 0} \langle \omega_0 \rangle$, and $\rho_{\mu} \rightarrow \delta_{\mu 3} \langle \rho_3 \rangle$. The pion mass $m_\pi$, $\sigma$ meson mass $m_\sigma$, and pion decay constant $f_\pi$ are given by

$$m_\pi^2 = \lambda \langle \sigma \rangle^2 - \mu^2 - 6 \lambda \langle \sigma \rangle^4,$$
$$m_\sigma^2 = 3 \lambda \langle \sigma \rangle^2 - \mu^2 - 5 \lambda \langle \sigma \rangle^4,$$
$$f_\pi = \langle \sigma \rangle.$$

(5)

After diagonalization of the baryon mass terms, we get the mass of the nucleon field $N$, which corresponds to one of the mass eigenstates, as

$$m_N = \frac{1}{2} \left( \sqrt{(g_1 + g_2)^2 \langle \sigma \rangle^2 + 4m_0^2} - (g_1 - g_2) \langle \sigma \rangle \right).$$

(6)

The equations of motion (EoM) for the stationary mean fields $\check{\sigma}$, $\check{\omega}_0$, $\check{\rho}_3$ and $A_0$ read

$$\left( - \check{\nabla}^2 + m_\sigma^2 \right) \langle \check{\sigma} \rangle = - \check{N} \langle \check{\sigma} \rangle N \langle \check{\sigma} \rangle \frac{\partial m_\sigma(\check{\sigma})}{\partial \check{\sigma}} \bigg|_{\check{\sigma} = \langle \check{\sigma} \rangle}$$
$$+ (-3f_\pi \lambda + 10 f_\pi^3 \lambda_6) \langle \check{\sigma} \rangle^2$$
$$+ (\lambda - 1 + 10 f_\pi^3 \lambda_6) \langle \check{\sigma} \rangle^3$$
$$+ 5f_\pi \lambda_6 \langle \check{\sigma} \rangle^4 + \lambda_6 \langle \check{\sigma} \rangle^5,$$

(7)

$$\left( - \check{\nabla}^2 + m_\rho_3^2 \right) \langle \check{\rho}_3 \rangle = \check{g}_{\rho NN} \check{N}^4 \langle \check{\rho}_3 \rangle N \langle \check{\rho}_3 \rangle,$$

(8)

$$\left( - \check{\nabla}^2 + m_\omega^2 \right) \langle \check{\omega}_0 \rangle = \check{g}_{\omega NN} \check{N}^4 \langle \check{\omega}_0 \rangle N \langle \check{\omega}_0 \rangle,$$

(9)

$$- \check{\nabla}^2 \langle A_0 \rangle = e \check{N}^4 \langle \check{\sigma} \rangle - \frac{\tau_3}{2} N \langle \check{\sigma} \rangle.$$
Note here that for calculational handiness we take the shift \( \sigma = f_\pi + \bar{\sigma} \). Since we are interested in finite nuclei, we will not consider the EoM for the parity partner of the nucleon, \( N^*(1535) \), which does not form its Fermi sea near the saturation density. In addition, since our primary goal here is to see if the parity doublet model can explain some basic nuclear properties such as the binding energy, we will not consider pairing correlations which are essential for odd-even staggering in nuclear properties. For instance, according to the semi-empirical mass formula, the contribution from the pairing term to the binding energy per nucleon of \(^{58}\text{Ni}\) is only about 0.03 MeV.

The EoM for the nucleon is given by

\[
\left[ \tilde{\alpha} \cdot \tilde{p} + \beta m_N \left( (\tilde{\sigma} (\tilde{x})) + V(\tilde{x}) \right) \right] N_i (\tilde{x}) = \epsilon_i N_i (\tilde{x}), \quad (11)
\]

where \( N_i \) is the single-particle wave function and

\[
V (\tilde{x}) = g_{\omega NN} (\omega_0 (\tilde{x})) + g_{\rho NN} (\rho_0^3 (\tilde{x})) \tau^3 + e \left( 1 - \frac{\tau_3}{2} \right) (A_0 (\tilde{x})). \quad (12)
\]

With assuming the spherical shape of the nucleus, we can solve the Eqs. (7)–(10) and Eq. (11) simultaneously to obtain the energy

\[
E = \int d^3x \mathcal{H} (\tilde{x}). \quad (13)
\]

After subtracting out the vacuum contribution, we write the Hamiltonian density \( \mathcal{H} (\tilde{x}) \) in the mean field approximation as

\[
\mathcal{H} = \tilde{N} \left( -i \gamma^i \partial_i + m_N \right) N + g_{\omega NN} (\omega_0) N \mathcal{N} + g_{\rho NN} (\rho_0^3) \tau^3 N + e (A_0) N \mathcal{N} \left[ 1 - \frac{\tau_3}{2} \right] N
\]

\[
- \frac{1}{2} \gamma^i (\tilde{\sigma}) \partial_i (\tilde{\sigma}) + \frac{1}{2} \gamma^i (\omega_0) \partial_i (\omega_0)
\]

\[
+ \frac{1}{2} \gamma^i (\rho_0^3) \partial_i (\rho_0^3) + \frac{1}{2} \gamma^i (A_0) \partial_i (A_0)
\]

\[
- \frac{\mu^2}{2} \left[ (f_\pi + \langle \tilde{\sigma} \rangle)^2 - f_\pi^2 \right] + \frac{\lambda}{4} \left[ (f_\pi + \langle \tilde{\sigma} \rangle)^4 - f_\pi^4 \right]
\]

\[
- \frac{\lambda_6}{6} \left[ (f_\pi + \langle \tilde{\sigma} \rangle)^6 - f_\pi^6 \right] - 4 \langle \tilde{\sigma} \rangle
\]

\[
- \frac{1}{2} m_\omega^2 (\omega_0)^2 - 1 + m_\rho^2 (\rho_0^3)^2. \quad (14)
\]

Then, the binding energy (BE) per nucleon is given by

\[
\text{BE} / A = - \frac{E}{A} + m_N. \quad (15)
\]

### Table 1 The inputs from free space (in MeV)

| \( m_N \) | \( m_{N^*} \) | \( m_{\omega} \) | \( m_{\rho} \) | \( f_\pi \) | \( m_\pi \) |
|-----------|-------------|-------------|-------------|----------|----------|
| 939       | 1535        | 783         | 776         | 93       | 138      |

### Table 2 Parameter set 1: \( K = 240 \) MeV

| \( m_0 \) (MeV) | 600      | 700      | 800      | 900     |
|-----------------|----------|----------|----------|---------|
| \( g_1 \)      | 14.836   | 14.171   | 13.349   | 12.329  |
| \( g_2 \)      | 8.427    | 7.762    | 6.941    | 5.921   |
| \( g_{\omega NN} \) | 9.132    | 7.305    | 5.660    | 3.522   |
| \( g_{\rho NN} \) | 3.927    | 4.065    | 4.149    | 4.218   |
| \( \mu^2 / f_\pi^2 \) | 21.821   | 18.842   | 11.693   | 1.537   |
| \( \lambda \)   | 39.367   | 34.584   | 22.578   | 4.388   |
| \( \lambda_6 f_\pi^2 \) | 15.344   | 13.540   | 8.683    | 0.649   |
| \( m_\sigma \) (MeV) | 411.299  | 385.805  | 330.440  | 269.255 |

### Table 3 Parameter set 2: \( K = 215 \) MeV

| \( m_0 \) (MeV) | 600      | 700      | 800      | 900     |
|-----------------|----------|----------|----------|---------|
| \( g_1 \)      | 14.836   | 14.171   | 13.349   | 12.329  |
| \( g_2 \)      | 8.427    | 7.762    | 6.941    | 5.921   |
| \( g_{\omega NN} \) | 9.132    | 7.305    | 5.660    | 3.522   |
| \( g_{\rho NN} \) | 3.948    | 4.080    | 4.157    | 4.221   |
| \( \mu^2 / f_\pi^2 \) | 23.377   | 20.980   | 13.346   | 2.502   |
| \( \lambda \)   | 42.369   | 38.921   | 26.128   | 6.673   |
| \( \lambda_6 f_\pi^2 \) | 16.709   | 15.739   | 10.580   | 1.969   |
| \( m_\sigma \) (MeV) | 413.612  | 384.428  | 324.007  | 257.583 |

### 3 Results

Following Ref. [10], we determine the free parameters in our model using the inputs in free space listed in Table 1 and nuclear matter properties.

The nuclear matter properties used to fix the parameters are given by

\[
\frac{E}{A} - m_N = -16 \text{ MeV}, \quad n_0 = 0.16 \text{ fm}^{-3},
\]

\[
K = 240 \pm 40 \text{ MeV}, \quad E_{\text{sym}} = 31 \text{ MeV}. \quad (16)
\]

We choose the value of \( m_0 \) in the range of 500–900 MeV [10]. Since the value of the incompressibility \( K \) is relatively not well fixed compared to the other nuclear matter properties, we use two different values of \( K \) as inputs. The determined parameters are shown in Table 2, where \( K = 240 \) MeV, and Table 3, where \( K = 215 \) MeV.

We remark here that the coefficient of the six-point interaction of the \( \sigma \) meson is positive, which implies that the potential is not bounded below and the system is not stable for infinite scalar mean field. However, since the \( \sigma \) field in our model is the chiral partner of the pion field, the mean...
value of the $\sigma$ field in dense matter is in general smaller than the one in free space due to (partial) chiral symmetry restoration. Therefore, our system will be stable within the mean field approximation in dense matter. It can be seen from Tables 2 and 3 that the couplings of the $\sigma$ field and $\omega$ field to nucleons decrease as $m_0$ increases, which is consistent with the observation made in Ref. [11].

In Fig. 1 we show nucleon density distribution $n(r)$ in $^{40}$Ca and $^{48}$Ca calculated with the parameter set 2. From Fig. 1 we observe that the central density tends to increase with $m_0$.

Now, we calculate the binding energy and the charge radius of some selected nuclei using the parameter sets. We present our results in Tables 4 and 5 together with the corresponding root-mean-square (RMS) deviations. We have used the experimental values compiled in Refs. [22, 23]. We remark here that we cannot obtain converged numbers in our calculations for several nuclei when $m_0 = 500$ MeV, and therefore we have ruled out the case with $m_0 = 500$ MeV. As it can be seen from Tables 4 and 5, our results approach the experimental values as $m_0$ is increased till $m_0 = 700$ MeV and start to deviate more afterwards. From Tables 4 and 5, we conclude that $m_0 = 700$ MeV is favored by nuclear properties such as the nuclear binding energies and charge radii.

We can understand our conclusion from the following two observations. First, from Tables 2 and 3, we can see that the values of $\tilde{\mu}_2/f_\pi^2$, $\lambda$, and $\lambda_2 f_\pi^2$ decrease slightly as $m_0$ changes from 600 MeV to 700 MeV, while the values change drastically when $m_0$ is larger than 700 MeV. Second, from Fig. 2 it can be seen that $\langle \omega \rangle$ decreases gradually as $m_0$ increases, while $\langle \sigma \rangle$ shows a peculiar behavior. As $m_0$ increases from 600 to 700 MeV, $\langle \sigma \rangle$ increases, while the value changes lit-

![Fig. 1](Color online) Nucleon density profile in $^{40}$Ca and $^{48}$Ca calculated with the parameter set 2

| $m_0$ (MeV) | 600 | 700 | 800 | 900 | Exp. |
|-------------|-----|-----|-----|-----|-----|
| BE/A (MeV)  |     |     |     |     |     |
| $^{16}$O    | 7.087 | 7.280 | 6.792 | 5.093 | 7.976 |
| $^{40}$Ca   | 7.736 | 7.906 | 7.538 | 6.191 | 8.551 |
| $^{48}$Ca   | 7.676 | 7.768 | 7.378 | 6.061 | 8.667 |
| $^{58}$Ni   | 7.391 | 7.486 | 7.108 | 5.849 | 8.732 |
| $^{70}$Ge   | 7.761 | 7.900 | 7.584 | 6.429 | 8.722 |
| $^{82}$Se   | 7.799 | 7.899 | 7.580 | 6.462 | 8.693 |
| $^{92}$Mo   | 7.741 | 7.821 | 7.507 | 6.424 | 8.658 |
| $^{112}$Sn  | 7.668 | 7.760 | 7.474 | 6.460 | 8.514 |
| $^{126}$Sn  | 7.575 | 7.801 | 7.516 | 6.536 | 8.443 |
| $^{138}$Ba  | 7.695 | 7.758 | 7.482 | 6.526 | 8.393 |
| $^{154}$Sm  | 7.596 | 7.691 | 7.447 | 6.540 | 8.227 |
| $^{170}$Er  | 7.526 | 7.587 | 7.354 | 6.484 | 8.112 |
| $^{182}$W   | 7.418 | 7.466 | 7.237 | 6.387 | 8.018 |
| $^{202}$Pb  | 7.277 | 7.303 | 7.062 | 6.221 | 7.882 |
| $^{208}$Pb  | 7.306 | 7.322 | 7.075 | 6.232 | 7.867 |

| RMS deviation |     |     |     |     |     |
|---------------|-----|-----|-----|-----|-----|
| $^{16}$O      | 0.827 | 0.737 | 1.047 | 2.147 | –   |
| $^{40}$Ca     | 2.877 | 2.792 | 2.790 | 2.803 | 2.699 |
| $^{48}$Ca     | 3.572 | 3.491 | 3.485 | 3.479 | 3.478 |
| $^{58}$Ni     | 3.605 | 3.537 | 3.532 | 3.522 | 3.478 |
| $^{70}$Ge     | 3.932 | 3.863 | 3.861 | 3.855 | 3.776 |
| $^{82}$Se     | 4.104 | 4.028 | 4.018 | 4.001 | 4.041 |
| $^{92}$Mo     | 4.223 | 4.154 | 4.145 | 4.125 | 4.140 |
| $^{112}$Sn    | 4.684 | 4.616 | 4.608 | 4.591 | 4.594 |
| $^{126}$Sn    | 4.764 | 4.703 | 4.695 | 4.675 | 4.685 |
| $^{138}$Ba    | 4.928 | 4.865 | 4.856 | 4.834 | 4.838 |
| $^{154}$Sm    | 5.117 | 5.045 | 5.031 | 5.004 | 5.105 |
| $^{170}$Er    | 5.250 | 5.181 | 5.169 | 5.144 | 5.279 |
| $^{182}$W     | 5.374 | 5.305 | 5.294 | 5.270 | 5.356 |
| $^{202}$Pb    | 5.555 | 5.493 | 5.485 | 5.462 | 5.471 |
| $^{208}$Pb    | 5.588 | 5.529 | 5.521 | 5.499 | 5.501 |
Table 5 The binding energy per nucleon and the charge radius \( (R_C) \) with the parameter set 2

| \( m_0 \) (MeV) | 600 | 700 | 800 | 900 | Exp. |
|-----------------|-----|-----|-----|-----|------|
| BE/A (MeV)      |     |     |     |     |      |
| \(^{16}\text{O}\) | 7.489 | 7.781 | 7.298 | 5.698 | 7.976 |
| \(^{40}\text{Ca}\) | 8.063 | 8.301 | 7.942 | 6.693 | 8.551 |
| \(^{48}\text{Ca}\) | 7.978 | 8.134 | 7.757 | 6.541 | 8.667 |
| \(^{58}\text{Ni}\) | 7.685 | 7.841 | 7.473 | 6.308 | 8.732 |
| \(^{70}\text{Ge}\) | 8.044 | 8.239 | 7.932 | 6.866 | 8.722 |
| \(^{82}\text{Se}\) | 8.066 | 8.219 | 7.910 | 6.881 | 8.693 |
| \(^{90}\text{Mo}\) | 7.993 | 8.123 | 7.822 | 6.828 | 8.658 |
| \(^{112}\text{Sn}\) | 7.911 | 8.050 | 7.774 | 6.844 | 8.514 |
| \(^{126}\text{Sn}\) | 7.980 | 8.070 | 7.802 | 6.909 | 8.443 |
| \(^{138}\text{Ba}\) | 7.920 | 8.028 | 7.764 | 6.890 | 8.393 |
| \(^{154}\text{Sm}\) | 7.821 | 7.958 | 7.724 | 6.894 | 8.227 |
| \(^{170}\text{Er}\) | 7.733 | 7.837 | 7.618 | 6.830 | 8.112 |
| \(^{182}\text{W}\) | 7.616 | 7.707 | 7.494 | 6.726 | 8.018 |
| \(^{202}\text{Pb}\) | 7.468 | 7.535 | 7.310 | 6.549 | 7.882 |
| \(^{208}\text{Pb}\) | 7.496 | 7.552 | 7.321 | 6.558 | 7.867 |
| RMS deviation   | 0.573 | 0.438 | 0.727 | 1.734 | –     |
| \( R_C \) (fm)  |     |     |     |     |      |
| \(^{16}\text{O}\) | 2.845 | 2.763 | 2.772 | 2.796 | 2.699 |
| \(^{40}\text{Ca}\) | 3.546 | 3.469 | 3.473 | 3.479 | 3.478 |
| \(^{48}\text{Ca}\) | 3.585 | 3.521 | 3.525 | 3.527 | 3.478 |
| \(^{58}\text{Ni}\) | 3.912 | 3.848 | 3.856 | 3.863 | 3.776 |
| \(^{70}\text{Ge}\) | 4.085 | 4.013 | 3.913 | 4.008 | 4.041 |
| \(^{82}\text{Se}\) | 4.209 | 4.145 | 4.144 | 4.135 | 4.140 |
| \(^{90}\text{Mo}\) | 4.401 | 4.339 | 4.344 | 4.344 | 4.315 |
| \(^{112}\text{Sn}\) | 4.671 | 4.608 | 4.609 | 4.602 | 4.594 |
| \(^{126}\text{Sn}\) | 4.754 | 4.697 | 4.698 | 4.688 | 4.685 |
| \(^{138}\text{Ba}\) | 4.920 | 4.862 | 4.861 | 4.849 | 4.838 |
| \(^{154}\text{Sm}\) | 5.111 | 5.045 | 5.039 | 5.022 | 5.105 |
| \(^{170}\text{Er}\) | 5.242 | 5.178 | 5.175 | 5.160 | 5.279 |
| \(^{182}\text{W}\) | 5.364 | 5.301 | 5.298 | 5.286 | 5.356 |
| \(^{202}\text{Pb}\) | 5.549 | 5.493 | 5.493 | 5.481 | 5.471 |
| \(^{208}\text{Pb}\) | 5.584 | 5.531 | 5.532 | 5.519 | 5.501 |
| RMS deviation   | 0.082 | 0.046 | 0.049 | 0.056 | –     |

Experiments in Table 7, where the RMS deviation of our results is 0.204 for the binding energy and 0.045 for the charge radius. As in Table 7, our results are in quantitative agreement with experiments. To see how our results compare with the ones from a Walecka-type mean field model, we also show the results from, for example, the spherical relativistic continuum Hartree-Bogoliubov (RCHB) theory with the relativistic density functional PC-PK1 [23, 24].

Using the parameter set 3, we finally calculate the \( r \)-dependence of neutron and proton masses which are given by Eq. (6) with the \( r \)-dependent \( \langle \sigma \rangle \), and present our results in Fig. 3. Our results show that, as expected, the neutron-proton mass difference is larger in the nuclear system with a larger isospin asymmetry.

4 Summary

In this work, using the extended parity doublet model [10], we calculated the properties of some stable nuclei in the mean field approximation to estimate the value of the chiral invariant mass preferred by nuclear properties. Since our primary goal in this work is to see if the parity doublet model can explain some basic nuclear properties such as the binding energy, we didn’t consider pairing correlations which are essential for odd-even staggering in nuclear properties. We observed that our results are closest to the experiments when we take \( m_0 = 700 \) MeV. Therefore, we have concluded that the chiral invariant mass contribution to the nucleon mass is around 700 MeV, which can be understood from the peculiar
behavior of $\tilde{\mu}^2 / f_{\pi}^2$, $\lambda$, $\lambda_6$, $f_\pi$, $\langle \sigma \rangle$, and $\langle \omega_0 \rangle$ as the value of $m_0$ changes. We also calculated the neutron and proton masses in a nucleus and observed that the neutron-proton mass difference becomes larger in an isospin asymmetric nucleus.

In the future we will extend our present study by including pairing correlations and deformations using more complete approaches such as (deformed) RCHB theory which can provide an appropriate treatment of the pairing correlation in the presence of the continuum through the Bogoliubov transformation in a microscopic and self-consistent way [25].

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### Data Availability Statement
This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The relevant data to show are all shown in the main text].

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