The $\text{HeH}^+$ molecular ion in a magnetic field.

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Abstract

A detailed study of the low-lying electronic states $^1\Sigma, ^3\Sigma, ^3\Pi, ^3\Delta$ of the $\text{HeH}^+$ molecular ion in parallel to a magnetic field configuration (when $\alpha$-particle and proton are situated on the same magnetic line) is carried out for $B = 0 - 4.414 \times 10^{13}$ G in the Born-Oppenheimer approximation. The variational method is employed using a physically adequate trial function. It is shown that the parallel configuration is stable with respect to small deviations for $\Sigma$-states. The quantum numbers of the ground state depend on the magnetic field strength. The ground state evolves from the spin-singlet $^1\Sigma$ state for small magnetic fields $B \lesssim 0.5$ a.u. to the spin-triplet $^3\Sigma$ unbound state for intermediate fields and to the spin-triplet strongly bound $^3\Pi$ state for $B \gtrsim 15$ a.u. When the $\text{HeH}^+$ molecular ion exists, it is stable with respect to a dissociation.

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I. INTRODUCTION

It is well-known that a very strong magnetic field may appear on the surface of neutron stars, \( B \approx 10^{11} - 10^{13} \) G. It seems natural to expect that in a vicinity of the surface some atomic and molecular systems can occur. For many years this assumption has motivated a development of atomic-molecular physics in a strong magnetic field hoping that it might lead to understanding of the spectra of these compact stars and be important for a construction of a model of the atmosphere. Recent revolutionary observation of the absorption features in a spectra of the isolated neutron star 1E1207.4-5209 by Chandra X-ray observatory \[1\] brought an extra attention to the field.

So far, a major attention has been paid to a study of one-electron atomic and molecular systems (for a review, see \[2\]). During the last years the existence of several exotic, strongly bound one-electron molecular systems has been predicted. In particular, it was shown that for a sufficiently strong magnetic field \( B \gtrsim 5 \times 10^{12} \) G the exotic ion \((HeH)^{2+}\) can exist in parallel configuration (the magnetic field is directed along internuclear axis) as optimal as well as its excited states \(1\pi, 1\delta\). \[3\] A complete list of possible one-electron atomic and molecular systems is known for \( B \lesssim B_{\text{schwinger}} = 4.414 \times 10^{13} \) G. In turn, the exploration of two-electron systems has been mostly restricted to a study of atomic-type systems \(H^-\), \(He\) with the only exception of the \(H_2\) molecule (see \[4\], \[5\] and references therein). Recently, a first detailed study of the molecular ions \(H_3^+\) \[6\] and \(He_2^{2+}\) \[7\] has been carried out (for a brief review of a general situation see \[8\]). It was shown that for large magnetic fields if such an ion exists its optimal configuration is the linear parallel one - heavy charged particles are situated along a magnetic line. A goal of the present paper is to make a study of the Coulomb system \((\alpha\text{pee})\) in a magnetic field ranging in \(B = 0 - 4.414 \times 10^{13} \) G and thus to establish the possible existence of the simplest two-electron diatomic heteronuclear molecular ion \(HeH^+\) and to explore its lowest excited states. In the field-free case the theoretical existence of \(HeH^+\) was established long ago by Beach \[9\] (see also \[10\]) while its several excited states were found (see, e.g., \[11\]). Experimentally, this molecular ion was first observed in the mass spectra of helium-hydrogen mixtures (see, e.g., \[12\]). The \(HeH^+\) ion is thought to be among the first molecular species to appear in the Universe (see \[13\] and references therein). An attempt to explore the \(HeH^+\) ion in ultra-strong magnetic field was made in \[14\].

Atomic units are used throughout \((h=m_e=e=1)\), although energies are expressed in Ry-
dbergs (Ry). The magnetic field $B$ is given in a.u. with $B_0 = 2.35 \times 10^9 \text{G}$.

II. GENERALITIES

We study the system $(\alpha p ee)$ placed in a uniform constant magnetic field. It is assumed that the $\alpha$-particle and proton are infinitely massive (Born-Oppenheimer approximation of zero order). It seems natural from physical point of view that the optimal configuration is parallel when the $\alpha$-particle and proton are situated along the magnetic field line. Nevertheless a stability of this configuration with respect to small perturbations will be checked.

The Hamiltonian which describes the system $(\alpha p ee)$ when the magnetic field is oriented along the $z$ direction, $B = (0, 0, B)$ is

$$\mathcal{H} = \sum_{\ell=1}^{2} (\hat{p}_\ell + \mathcal{A}_\ell)^2 - \sum_{\ell=1,2,\kappa=\alpha,p} \frac{2Z_\kappa}{r_{\ell\kappa}} + \frac{2}{r_{12}} + \frac{4}{R} + 2B \cdot S,$$

(1)

(see Fig. I for the geometrical setting and notations), where $\hat{p}_\ell = -i\nabla_\ell$ is the momentum of the $\ell$-th-electron, the index $\kappa$ runs over the $\alpha$-particle and proton, $Z_\alpha = 2, Z_p = 1$ and $r_{12}$ is the interelectronic distance, $S = \hat{s}_1 + \hat{s}_2$ is the operator of the total spin. The vector potential $\mathcal{A}_\ell$ which corresponds to the constant uniform magnetic field $B$ is chosen in the symmetric gauge,

$$\mathcal{A}_\ell = \frac{1}{2} (B \times r_\ell) = \frac{B}{2}(-y_\ell, x_\ell, 0).$$

(2)

Finally, the Hamiltonian can be written as

$$\mathcal{H} = \sum_{\ell=1}^{2} \left(-\nabla_\ell^2 + \frac{B^2}{4} \rho_\ell^2\right) - \sum_{\ell=1,2,\kappa=\alpha,p} \frac{2Z_\kappa}{r_{\ell\kappa}} + \frac{2}{r_{12}} + \frac{4}{R} + B(\hat{L}_z + 2\hat{S}_z),$$

(3)

where $\hat{L}_z = \hat{l}_{z1} + \hat{l}_{z2}$ and $\hat{S}_z = \hat{s}_{z1} + \hat{s}_{z2}$ are the $z$-components of the total angular momentum and total spin, respectively, and $\rho_\ell^2 = x_\ell^2 + y_\ell^2$, $\ell = 1, 2$.

The problem is characterized by two conserved quantities: (i) the operator of the $z$-component of the total angular momentum (projection of the angular momentum on the magnetic field direction) giving rise to the magnetic quantum number $m$ and (ii) the operator of the $z$-component of the total spin (projection of the total spin on the magnetic field direction) giving rise to the spin quantum number $m_s$. Hence, any eigenstate has two quantum numbers assigned: the magnetic quantum number $m$ and the spin quantum number $m_s$. 
As the magnetic field increases a contribution from the linear Zeeman term (interaction of the spin with a magnetic field $B \cdot S$) becomes more and more important. Hence, it is natural to assume that for large magnetic fields the states with minimal $m_s$ (when both electron spins are antiparallel to the magnetic field) will represent the states with lowest total energies.

The space of eigenstates is split into subspaces (sectors) with each of them characterized by definite values of $m$ and $m_s$. Thus, to classify eigenstates we follow the convention widely accepted in molecular physics using the quantum numbers $m$ and $m_s$. The notation is $2S + 1M$, where $2S + 1$ is the spin multiplicity and corresponds to 1 for singlet $S = 0$ and 3 for triplet $S = 1$, as for the labeling $M$ it is used the Greek letters $\Sigma, \Pi, \Delta$ that correspond to the states with $|m| = 0, 1, 2,...$, respectively. We study only states with $m = 0, -1, -2$.

As a method to explore the problem we use the variational procedure. The recipe of choice of trial functions is based on physical arguments [15]. A trial function for states with orbital magnetic quantum number $m$ is chosen in the form

$\psi^{(\text{trial})} = (1 + \sigma e P_{12}) \rho_1^{m|} e^{i m \phi_1} e^{-\alpha_1 r_{1A} - \alpha_2 r_{1B} - \alpha_3 r_{2A} - \alpha_4 r_{2B} - B \beta_1 r_1^2 / 4 - B \beta_2 r_2^2 / 4 + \gamma r_{12} ,}$ (4)

(cf. [4]), where $\sigma_e = 1, -1$ stand for spin singlet ($S = 0$) and triplet states ($S = 1$), respectively. The $P_{12}$ is the operator which interchanges electrons ($1 \leftrightarrow 2$). The parameters $\alpha_{1-4}$,
β_{1-2} and γ as well as the internuclear distance \( R \) are eight variational parameters.

Calculations were performed using the minimization package MINUIT from CERN-LIB. Multidimensional integration was carried out using a dynamical partitioning procedure: a domain of integration was divided into subdomains following the integrand profile and then each subdomain was integrated separately (for details see, e.g., [2]). Numerical integration was done with a relative accuracy of \( \sim 10^{-6} - 10^{-7} \) by use of the adaptive D01FCF routine from NAG-LIB. Computations were performed on a dual DELL PC with two Xeon processors of 2.8 GHz each (ICN) and dual DELL PC with two Xeon processors of 3.06 GHz each (CINVESTAV). We implemented a parallelization in the routine for the calculation of the integrals. Minimization process for each particular magnetic field and for a given state was quite painful and lengthy, it took from 100 to 1000 hours of CPU time, however, when a minimum is found it was needed a few minutes to compute a minimal variational energy.

III. RESULTS

In absence of a magnetic field the HeH^{+} molecular ion exists and it is stable. Its ground state is the \( ^1\Sigma \) \((S = 0 \text{ and } m = 0)\) state [9]. In order to establish the existence of HeH^{+} in a magnetic field we carried out a detailed study of the Coulomb system (\( \alpha \text{pee} \)) in parallel configuration in the state \( ^1\Sigma \) for magnetic fields \( B = 0 - 10000 \) a.u. (see Table I). For this case the variational trial function \( \psi_\text{trial} \) (4) is used at \( \sigma_e = 1 \text{ and } m = 0 \).

The calculations indicate the optimal geometry is parallel (\( \alpha \)-particle and proton stay on the same magnetic line). For this configuration for all studied magnetic fields a minimum in the total energy curve \( E_T(R) \) is developed. Furthermore, we checked that this minimum is stable with respect to small inclinations of the internuclear axis from the magnetic field direction. It is found that with an increase of the magnetic field strength the total energy of the (\( \alpha \text{pee} \)) system in the \( ^1\Sigma \) state grows, it becomes more bound (in particular, the double ionization energy increases \( E_f = 2B - E_T \)) and the system becomes more compact (the internuclear equilibrium distance decreases), see Table I. For \( B \leq 100 \) a.u. the (\( \alpha \text{pee} \)) system in the \( ^1\Sigma \) state is stable with respect to the dissociation He^{+} + H or He + p. The vibrational and rotational energies are calculated for \( B = 0.5 \) a.u. The vibrational energy is larger than the rotational energy which is typical for field-free case.

We carried out a detailed study of the spin-triplet state \( ^3\Sigma \) \((S = 1 \text{ and } m = 0)\) of the
TABLE I: HeH\(^+\) ion in the state \(^1\Sigma\): Total \(E_T\), double ionization \(E_I\), vibrational \(E_{vib}\) and rotational \(E_{rot}\) energies in Ry, and equilibrium distance in a.u. of the \((\alpha pee)\) system in the state \(^1\Sigma\). Magnetic fields for which the \(^1\Sigma\) state is the ground state are marked by * (see text). \(^a\) Result from \([9]\) (rounded). Total energy \(E(*) = E(\text{He}^+(1s)) + E(\text{H}(1s))\) in Ry with electrons in spin-singlet state is from \([2]\). Total energy of He are from \([16]\), \(^x\) marks a result of linear interpolation.

| B (a.u) | \(E_T\) | \(E_I\) | \(R_{eq}\) | \(E_{vib}\) | \(E_{rot}\) | \(E(*)\) | \(E(\text{He}(1^10^+)\)) |
|--------|--------|--------|--------|--------|--------|--------|----------------|
| 0\(^*\) | -5.943 | 5.943  | 1.451  | 0.016  | -5.0   | -5.807 |
|        | -5.957 \(^a\) | 1.463  |
| 0.5\(^*\) | -5.843 | 6.843  | 1.430  | 0.018  | 0.005  | -4.8\(^x\) | -5.6\(^x\) |
| 1      | -5.626 | 7.626  | 1.404  | -4.544 | -5.459 |
| 10     | 5.337  | 14.663 | 0.789  | 7.725  | 6.129  |
| 100    | 166.513| 33.487 | 0.370  | 173.327| 169.837|
| 1000   | 1927.35| 72.65  | 0.212  | 1944.37|        |
| 10000  | 19848.67| 151.30 | 0.0928 | 19893.62|        |

(\(\alpha pee\)) system in parallel configuration in the domain of magnetic fields \(0 \leq B \leq 10000\) a.u. For this state the variational trial function \(\psi_{\text{trial}}(4)\) with \(\sigma_e = -1\) and \(m = 0\) is used.

Similar to \(^1\Sigma\) all total energy curves \(E_T(R)\) of the \((\alpha pee)\) system for magnetic fields \(B = 0 – 10000\) a.u. in the \(^3\Sigma\) state displays a minimum. However, to the contrary to the \(^1\Sigma\) state the minimum is always shallow, see Fig. 2-5 as an illustration. It is found that with an increase of the magnetic field strength the total energy decreases. The system gets more "compact" - the internuclear equilibrium distance of a shallow minimum decreases slowly, see Table II. We expect that the system in the \(^3\Sigma\) state "exists" in a form of two separated atomic-like systems \(\text{He}^+(1s) + \text{H}(1s)\) situated at large distances with electron spins directed against a magnetic field direction.
A comparison of the total energies of the \((\alphapee)\) system in \(^1\Sigma\) and \(^3\Sigma\) states in equilibrium indicates to the appearance of a level crossing (see Tables I and II). This crossing of the \(^1\Sigma\) and \(^3\Sigma\) states occurs at \(B \approx 0.5\) a.u. It implies that the \(^1\Sigma\) state is the lowest energy state at \(B \lesssim 0.5\) a.u., where the \((\alphapee)\) system is stable towards possible decays. Hence, one can conclude that the stable HeH\(^+\) molecular ion exists at \(B \lesssim 0.5\) a.u. with \(^1\Sigma\) state as the ground state. For \(B \gtrsim 0.5\) a.u. the situation is changed - the \(^3\Sigma\) state may become the ground state of the HeH\(^+\) molecular ion. In fact, the state \(^3\Sigma\) is at most metastable with very short lifetime or very likely is simply unbound - the HeH\(^+\) ion dissociates to He\(^+\)(1s) + H(1s) with the electrons in spin-triplet state. A general conclusion can be drawn that for \(0.5 < B < 15\) a.u. the system \((\alphapee)\) likely exists in a form of two separate atomic-type systems He\(^+\)(1s) + H(1s) with total electron spin equal to one.

As next we carried out a detailed study of the state \(^3\Pi\) \((S = 1\) and \(m = -1)\) of the \((\alphapee)\) system in parallel configuration for magnetic fields \(0 \leq B \leq 4.414 \times 10^{13}\) G (see Figs. 2-5). For this state the same variational trial function \(\psi^{trial}(4)\) is used at \(\sigma_e = -1\) and \(m = -1\).

The results of the calculations indicate unambiguously the existence of a deep minimum in the total energy curve \(E_T(R)\) of the \((\alphapee)\) system in parallel configuration for all studied magnetic fields \(0 \leq B \leq 4.414 \times 10^{13}\) G. Hence, it can be concluded that the HeH\(^+\) molecular ion may exist in the \(^3\Pi\) state. Table III contains the results for the total energy \(E_T\) and the internuclear equilibrium distance \(R_{eq}\) for the state \(^3\Pi\). Perhaps, it is worth mentioning that the previously obtained results for ultra-magnetic fields \(^{14}\) are less accurate for both studied two-electron systems: (i) the HeH\(^+\) ion in the present calculation and (ii) the He atom in the calculation made in \(^{17}\). With an increase of the magnetic field strength the total energy decreases, the system becomes more bound (double ionization energy increases being equal for triplet states to \(E_I = -E_T\)) and more compact (the internuclear equilibrium distance decreases). Table III shows the total energies of three dissociation channels of HeH\(^+\) in \(^3\Pi\) state: \(\rightarrow \text{He}(1^3(-1)^+) + p, \rightarrow \text{He}^+(1s) + \text{H}(2p_{-1})\) and \(\rightarrow \text{He}^+(2p_{-1}) + \text{H}(1s)\) for different magnetic fields. It can be seen that the state \(^3\Pi\) is stable towards any dissociation. It is worth noting that the dissociation energy needed increases monotonously in all range of studied magnetic fields. Therefore the stability of the system with a magnetic field increases.

One can see that the crossing of the total energies of the \(^3\Pi\) and \(^3\Sigma\) states at equilibrium occurs at \(B \approx 15\) a.u. For larger magnetic fields the total energy of the \(^3\Pi\) state is lower than the total energy of \(^3\Sigma\). Hence, the ground state for the HeH\(^+\) ion for \(B \gtrsim 15\) a.u. is
TABLE II: HeH$^+$ ion in the state $^3\Sigma$: Total $E_T$, double ionization $E_I$ energies in Ry and equilibrium distance in a.u. of the ($\alpha$pee) system in the state $^1\Sigma$. Magnetic fields for which the $^3\Sigma$ state might be the ground state are marked by $^\ast$. Total energy $E(*) = E(\text{He}^+(1s)) + E(\text{H}(1s))$ (in Ry) for the electrons in spin-triplet state is from [2]. Total energies for the He atom with total electron spin 1 are from [16]. $^\circ$ Ref [11].

| B(a.u) | $E_T$  | $R_{eq}$ | $E(*)$ | $E(\text{He}(1^3\Sigma^+))$ |
|-------|--------|----------|--------|-----------------------------|
| 0     | -5.0064 | 4.50     | -5.0   | -4.3504                     |
|       | -5.0087$^a$ | 4.35     |        |                             |
| 0.5$^\ast$ | -5.8712 | 4.32     | -5.8637| -5.3796$^{**}$              |
| 1$^\ast$ | -6.5524 | 4.04     | -6.5443| -5.3013                     |
| 10$^\ast$ | -12.209| 2.70     | -12.275| -9.255                      |
| 15$^\ast$ | -13.936| 2.47     | -14.043| -10.622$^{**}$              |
| 100   | -26.172 | 1.68     | -26.673| -19.686                     |
| 1000  | -54.020 | 1.20     | -55.628|                             |
| 10000 | -103.07 | 0.83     | -106.38|                             |

realized by the $^3\Pi$ state. Eventually, one can draw a conclusion that the $^3\Sigma$ state is the lowest energy state of the ($\alpha$pee) system for $0.5 < B \lesssim 15$ a.u. and the HeH$^+$ ion can exist.

In order to complete the study we consider the state $^3\Delta$ ($S = 1, m = -2$) of the ($\alpha$pee) system in the domain of magnetic fields $0 \leq B \leq 4.414 \times 10^{13}$ G. It is worth mentioning that the state $^3\Delta$ for the field-free case was studied in [11] and [18]. For this state the same variational trial function $\psi^{\text{trial}}(4)$ is used but with $\sigma_e = -1$ and $m = -2$. For all total energy curves $E_T(R)$ for different magnetic fields the minimum is found. Hence, the ion HeH$^+$ may exist in $^3\Delta$ state for all magnetic fields. Table [IV] shows the total $E_T$
TABLE III: HeH\textsuperscript{+} ion in the state $^3\Pi$: Total $E_T$ and vibrational $E_{\text{vib}}$ energies in Ry and equilibrium distance $R_{eq}$ in a.u. of the HeH\textsuperscript{+} ion in the $^3\Pi$ state. Magnetic fields for which the $^3\Pi$ state is the ground state are marked by *. Total energies $E(\ast\ast) = E(\text{He}^+(2p_{-1}) + \text{H}(1s))$ and $E(\ast\ast\ast) = E(\text{He}^+(1s) + \text{H}(2p_{-1}))$ (in Ry) for the electrons in spin-triplet state from \[2\]. Total energy for He atom in spin-triplet electronic state in magnetic field is from \[17\] (rounded). * Result of interpolation. \(a\) from \[11\]. \(b\) from \[16\]. \(c\) from \[14\].

| B(a.u) | $E_T$ | $R_{eq}$ | $E_{\text{vib}}$ | $E(\ast\ast)$ | $E(\ast\ast\ast)$ | $E(\text{He}(1^3(-1)^{+}))$ |
|--------|-------|----------|-----------------|----------------|-------------------|-----------------------------|
| 0      | -4.263| 7.61     | -2.0            | -4.25          | -4.266\(^b\)      |                             |
|        | -4.276\(^a\) | 7.7 |                 |                |                   |                             |
| 1      | -5.975| 3.297    | -3.829          | -5.795         | -5.919            |                             |
| 10     | -12.026| 1.135 | -8.732          | -11.030        | -11.659           |                             |
| 15\(*\) | -13.96| 0.925    | 0.04            | -12.66         | -13.58\(^*\)      |                             |
| 100\(*\) | -28.36| 0.440    | 0.15            | -20.30         | -24.37            | -26.15                      |
| 1000\(*\) | -64.24| 0.203    | 0.50            | -43.98         | -51.60            | -56.06                      |
|        | -63.07\(^c\) |     |                  |                |                   | -54.95\(^c\)               |
| 10000\(*\) | -133.49| 0.104   | 1.41            | -86.79         | -99.98            | -110.30                     |
|        | -129.7\(^c\) |     |                  |                |                   | -106.9\(^c\)              |
| 4.414 \times 10^{13} \text{G} \(*\) | -160.50 | 0.092  |                 |                |                   |                             |

With an increase of the magnetic field strength the total energies decrease, the system becomes more bound (double ionization energies increase, $E_t = -E_T$, for triplet states) and more compact (the internuclear equilibrium distance decreases). At $B \approx 300$ a.u. a crossing of the total energies of $^3\Sigma$ and $^3\Delta$ at equilibrium occurs. For larger magnetic fields the total energy of the $^3\Delta$ is smaller than the $^3\Sigma$ state. It implies that for $B \gtrsim 300$ a.u. the $^3\Delta$ state is the lowest excited state.
TABLE IV: Total energy in Ry and equilibrium distance in a.u. of the HeH$^+$ ion in the state $^3\Delta$.

| B (a.u.) | $E_T$   | $R_{eq}$ |
|---------|---------|----------|
| 0       | -4.108  | 16.97    |
| 1       | -5.664  | 4.717    |
| 10      | -11.02  | 1.581    |
| 100     | -25.78  | 0.486    |
| 1000    | -58.68  | 0.218    |
| 10000   | -123.62 | 0.112    |
| $4.414 \times 10^{13}$ G | -149.0  | 0.096    |

**FIG. 2**: Total energy curves for triplet states $^3\Sigma$ and $^3\Pi$ for a magnetic field $B = 10$ a.u.

**IV. CONCLUSION**

The low-lying electronic states $^1\Sigma$, $^3\Sigma$, $^3\Pi$, $^3\Delta$ of the Coulomb system $\alpha$pee in parallel configuration are studied for $B = 0 - 4.414 \times 10^{13}$ G using the variational method in the Born-Oppenheimer approximation of the zero order. It is shown that for all studied states and all magnetic fields the parallel configuration is always optimal being stable with respect to small inclinations. The state of the lowest total energy depends on the magnetic field strength. It evolves from the spin-singlet $^1\Sigma$ state for small magnetic fields $B \lesssim 0.5$ a.u. to spin-triplet $^3\Sigma$ (unbound state) for $B \gtrsim 0.5$ a.u. and, finally, for $B \gtrsim 15$ a.u. to spin-triplet
FIG. 3: Total energy curves for triplet states $^3\Sigma$ and $^3\Pi$ for a magnetic field $B = 100$ a.u.

FIG. 4: Total energy curves for triplet states $^3\Sigma$ and $^3\Pi$ for $B = 1000$ a.u.

$^3\Pi$ state (strongly bound state). This result shows that the HeH$^+$ molecular ion exists as a bound state for magnetic fields $B \lesssim 0.5$ a.u. and $B \gtrsim 15$ a.u. in a parallel configuration and it is stable. In a domain $15 \lesssim B \lesssim 300$ a.u. the state having the total energy next after the one of the ground state is the unbound (repulsive) $^3\Sigma$ state. For $B \gtrsim 300$ a.u. the $^3\Delta$ state is the lowest excited state of the HeH$^+$ molecular ion. As for $0.5 \lesssim B \lesssim 15$ a.u. the system ($\alpha$pee) likely exists in a form of two separate atomic-type systems He$^+(1s) + H(1s)$ with
total electron spin equal to one which are situated at very large distance from each other. It is worth emphasizing that for \( B \gtrsim 2000 \text{ a.u.} \) both one- and two-electron ions \((HeH)^{2+}\) and \((HeH)^{+}\) can exist. As for the ion \((HeH)^{2+}\) it becomes stable at \( B \gtrsim 10000 \text{ a.u.} \)

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