Energy and Color Flow in Dijet Rapidity Gaps

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Abstract

When rapidity gaps in high-$p_T$ dijet events are identified by energy flow in the central region, they may be calculated from factorized cross sections in perturbative QCD, up to corrections that behave as inverse powers of the central region energy. Although power-suppressed corrections may be important, a perturbative calculation of dijet rapidity gaps in $p\bar{p}$ scattering successfully reproduces the overall features observed at the Tevatron. In this formulation, the average color content of the hard scattering is well-defined. We find that hard dijet rapidity gaps in quark-antiquark scattering are not due to singlet exchange alone.

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Among the most intriguing recent experimental results in quantum chromodynamics is the observation of dijet rapidity-gap events, with anomalously low radiation in a wide interjet rapidity region [1, 2, 3]. These events are typically identified by low or zero hadron multiplicity in the central region, despite the high momentum transfer necessary to produce the jets.

The existence of such events was originally suggested on the basis of color flow considerations in QCD [4, 5]. If forward jets are produced by exchanging a pair of gluons in a color singlet state, color can be recombined independently in each forward region. Then much less radiation is expected between the jets than when the exchange is a color octet gluon, which requires recombining color between particles moving in nearly opposite directions. Rapidity gap events have special interest as clear illustrations of color coherence and its interplay with hadronization. In addition, because their observation requires large rapidity intervals, they offer a new window into a perturbative, yet Regge-like limit of QCD. Nonetheless, despite their intuitive appeal, the theoretical understanding of rapidity gaps has been somewhat hampered by two problems. One of these is the issue of “survival” [5, 6]. In any high-energy scattering, multiple soft interactions between spectators of the hard interaction may fill the gap by processes unrelated to the color content of the hard interaction. The second is that, since even the softest gluon carries color in the octet representation, it is not immediately obvious how the color of the hard scattering is to be defined.

In this paper, we observe that it is possible to overcome these problems, at least in part, by identifying rapidity gaps in terms of energy flow, rather than multiplicity. The energy flow $Q_c$ into the central rapidity interval between a pair of jets is an infrared safe observable. That is, $d\sigma/dQ_c$ can be written as a convolution of parton distributions with a perturbative hard-scattering function, which depends on $Q_c$. The issue of color flow may then be formulated self-consistently in the hard-scattering function. Corrections to the factorized cross section are proportional to powers of $\Lambda/Q_c$, with $\Lambda$ the scale of the QCD coupling, and may become large for small $Q_c$. As we shall see, however, the purely perturbative cross section remains well-defined, and energy flow gaps appear in this limit, once soft radiation is resummed including color effects [7, 8, 9]. In the conventional formulation for rapidity gaps, one writes $f_{\text{gap}} = f_{\text{singlet}} P_S$, with $f_{\text{gap}}$ the fraction of gap events, $f_{\text{singlet}}$ the fraction of “hard singlet” exchanges, and $P_S$ the survival probability. Compared to this, we generalize $f_{\text{singlet}}$, which then necessarily incorporates a perturbative survival probability. Non-perturbative survival considerations may reappear as we approach zero energy in the gap, but their importance should be reduced in a calorimetric measurement. Our results below support this possibility.

To be specific, we will study the process $p(p_A) + \bar{p}(p_B) \rightarrow J_1(p_1) + J_2(p_2) + X_{\text{gap}}$, where we sum inclusively over final states, while measuring the energy that flows into the intermediate region between two forward jets. For simplicity, we restrict ourselves to valence quarks and antiquarks, $q(k_A) + \bar{q}(k_B) \rightarrow q(k_1) + \bar{q}(k_2) + X$. We will begin by deriving a cross section for this process, specific to the geometry described by the D0 and CDF collaborations [1, 2, 10]. We go on to evaluate the cross section as a function of $Q_c$, and to view the results in the light of what we have learned from experiment. We will close with a few comments on the relation of our approach to
previous work, and on prospects for further development in this problem.

Following CDF and D0, we require the two jets, and therefore the outgoing partons coming from the hard scattering, \( q(k_1), \bar{q}(k_2) \), to be directed into fixed forward and backward (collectively denoted “forward”) regions of the calorimeter, defined by \(|y| > y_0\), where \( y \) is the (pseudo)rapidity \( y = (1/2) \ln \cot(\theta/2) \), with \( \theta \) the polar angle. In addition we require the jets to have transverse energies above an experimental threshold, \( E_T \). We will discuss cross sections for measured energy in a symmetric central region, spanning rapidity \( \Delta y = 2y_0 \). This geometry is presented schematically in Fig. 1. The inclusive dijet cross section for all events with energy in the central region equal to \( Q_c \) is a typical factorizable jet cross section, which may be written as

\[
\frac{d\sigma}{dQ_c} (S, E_T, \Delta y) = \sum_{f_A, f_B = u, d} \int d\cos \hat{\theta} \\
\times \int_0^1 dx_A \int_0^1 dx_B \phi_{f_A/p}(x_A, -\hat{t}) \phi_{f_B/p}(x_B, -\hat{t}) \\
\times \sum_{f_1, f_2 = u, d} \frac{d\hat{\sigma}^{(f)}}{dQ_c d\cos \hat{\theta}} \left( \hat{t}, \hat{s}, y_{JJ}, \Delta y, \alpha_s(\hat{t}) \right),
\]

(1)

where \( \phi_{f_A/p}, \phi_{f_B/p} \) are valence parton distributions, evaluated at scale \(-\hat{t}\), the dijet momentum transfer. \( d\hat{\sigma}^{(f)}/dQ_c d\cos \hat{\theta} \) is a hard scattering function, starting with the Born cross section at lowest order. The index \( f \) denotes \( f_A + f_B \rightarrow f_1 + f_2 \). The detector geometry determines the phase space for the dijet total rapidity, \( y_{JJ} \), the partonic center-of-mass (c.m.) energy squared, \( \hat{s} \), and the partonic c.m. scattering angle \( \hat{\theta} \), with \(-\hat{s}/2 \left( 1 - \cos \hat{\theta} \right) = \hat{t} \). For simplicity of presentation, we take \( Q_c \) to be the energy in the dijet c.m.

In the spirit of Refs. [4, 5, 6, 7], we now observe that we may perform a further factorization on the partonic hard-scattering function \( d\hat{\sigma}^{(f)}/dQ_c d\cos \hat{\theta} \). The underlying observation is that for \( Q_c \ll \sqrt{-\hat{t}} \), the soft gluon radiation that appears in the central

![Figure 1: Geometry of the calorimeter detector. \( Q_c \) is the energy flow into the central rapidity interval](image)
region decouples from the dynamics of the hard interaction that produces the dijet event. In technical terms, soft gluon emission may be approximated by an effective cross section, in which the hard scattering is replaced by a product of recoilless color sources (specifically, Wilson lines [8, 9]) in the directions of the incoming partons and the outgoing jets. The refactorized hard-scattering function then takes the form [11]

$$q_{c} \frac{d\hat{\sigma}^{(l)}}{dq_{c}} \left( \hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_{s}(-\hat{t}) \right) = H_{IL} \left( \frac{\sqrt{-\hat{t}}}{\mu}, \sqrt{\hat{s}}, \sqrt{-\hat{t}}, \alpha_{s}(\mu^{2}) \right) \times S_{LI} \left( \frac{q_{c}}{\mu}, y_{JJ}, \Delta y \right). \quad (2)$$

The functions $S_{LI}$ and $H_{IL}$ contain the dynamics of soft radiation from Wilson lines (at measured $q_{c}$), and the hard interaction, respectively, with a new factorization scale. The product itself must be independent of how we choose the scale, so long as $\mu > q_{c}$. The indices $I$ and $L$ label the possible color structures of the hard interaction, which correspond in $S_{LI}$ to the color matrices that couple the four Wilson lines representing the $2 \rightarrow 2$ hard subprocess. One index refers to the hard scattering in the amplitude, the other to the hard scattering in the complex conjugate. For quark-antiquark scattering, the Wilson lines are in the 3 (quark) and $3^*$ (antiquark) representations of $SU(3)$, respectively, and their product may be characterized by either singlet or octet color exchange in the $t$- or $s$-channel. For the physical reasons outlined above, we will choose a $t$-channel color basis. Corrections to Eq. (2) are expected from three-jet final states, for which the analysis below must, in principle, be repeated. We expect these corrections to be relatively small.

Because the left-hand side of Eq. (2) is independent of the precise choice of factorization scale $\mu$, the matrices $H_{IL}$ and $S_{LI}$ must satisfy evolution equations, in which their variations with $\mu$ cancel each other. The only variable that $H$ and $S$ hold in common is $\alpha_{s}(\mu^{2})$, and, as a result, the evolution equation for $S$ is

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) S_{LI} = - (\Gamma_{S}^{L})_{LB} S_{BI} - S_{LA}(\Gamma_{S}^{I}), \quad (3)$$

and similarly for $H$, with $\Gamma_{S}(\alpha_{s})$ an anomalous dimension matrix. Consider a $t$-channel singlet-octet basis, with color vertices schematically given by

$$c_{1} = I \times I, \quad c_{2} = \sum_{a} T^{a} \times T^{a}, \quad (4)$$

with $I$ the identity and $T^{a}$ the generators of $SU(3)$ in the quark representation. A one-loop calculation in this basis gives [11]

$$\Gamma_{S} \left( y_{JJ}, \Delta y, \hat{\theta} \right) = \frac{\alpha_{s}}{4\pi} \begin{pmatrix} \rho + \xi & -4\frac{C_{F}}{N_{c}} i\pi \\ -8i\pi & \rho - \xi \end{pmatrix}, \quad (5)$$

where $N_{c}$ is the number of colors. The functions $\xi$ and $\rho$ are

$$\xi(\Delta y) = -2N_{c}\Delta y + 2i\pi \frac{N_{c}^{2} - 2}{N_{c}}, \quad (6)$$
\[ \rho(y_{JJ}, \Delta y, \hat{\theta}) = \frac{N_c^2 - 1}{N_c} \times \left[ \ln \left( \frac{\cos(\hat{\theta}) + \tanh \left( \frac{\Delta y}{2} - y_{JJ} \right)}{\cos(\hat{\theta}) - \tanh \left( \frac{\Delta y}{2} - y_{JJ} \right)} \right) + \ln \left( \frac{\cos(\hat{\theta}) - \tanh \left( -\frac{\Delta y}{2} - y_{JJ} \right)}{\cos(\hat{\theta}) + \tanh \left( -\frac{\Delta y}{2} - y_{JJ} \right)} \right) + \frac{2}{N_c} \Delta y - 2i\pi \frac{N_c^2 - 2}{N_c} \right]. \]  

(7)

While \( \rho \) depends on the jet rapidities and on the partonic scattering angle, \( \xi \) depends on the geometry only, through \( \Delta y \). We note that the off-diagonal components of \( \Gamma_S \) are purely imaginary. This interesting feature is due to strong coherence effects in the one-loop calculation, related to angular ordering \[13\].

To study the \( Q_c \)-dependence of \( S \), it is convenient to diagonalize \( \Gamma_S \). In the basis in which \( \Gamma_S \) is diagonal, Eq. (3) implies that the components of \( S \) evolve independently in \( \mu \). In this basis we may calculate unambiguously the dependence on the central energy flow \( Q_c \). This is the technique that we summarize in the following.

The eigenvectors of \( \Gamma_S \) in Eq. (5) may be chosen as

\[ e_1 = \left( \frac{1}{8\pi} \left( \xi - \frac{1}{\sqrt{N_c}} \eta \right)^{-1} \right), \)

\[ e_2 = \left( \frac{i}{8\pi} \left( \xi + \frac{1}{\sqrt{N_c}} \eta \right) \right), \]

(8)

where we define

\[ \eta(\Delta y) \equiv \sqrt{N_c [\xi(\Delta y)]^2 - 32C_F\pi^2}. \]

(9)

A very useful feature is that these eigenvectors are independent of the jet rapidities, and depend only on \( \Delta y \). The corresponding eigenvalues of \( \Gamma_S \) are in general complex,

\[ \lambda_1 = \frac{\alpha_s}{2\pi} \left[ \frac{1}{2} \rho - \frac{1}{2\sqrt{N_c}} \eta \right], \]

\[ \lambda_2 = \frac{\alpha_s}{2\pi} \left[ \frac{1}{2} \rho + \frac{1}{2\sqrt{N_c}} \eta \right]. \]

(10)

In the limit of a large central region, \( \Delta y \gg 1 \), the function \( \xi \) has a large negative real part, while the real part of \( \eta \) is positive, and grows with \( \Delta y \). From Eq. (8), we see that, as \( \Delta y \to \infty \), \( e_1 \) reduces to a color “quasi-singlet”, and \( e_2 \) to a color “quasi-octet”. As a realistic example, we take the value \( \Delta y = 4 \), and find

\[ e_1 = \left( 0.455 e^{2.161i} \right), \]

\[ e_2 = \left( 0.101 e^{-0.981i} \right). \]

(11)
For this configuration, the second eigenvector is close to a color octet, but the first is still a mixture of octet and singlet, with the latter only slightly predominant. In the following, however, we find it suggestive to retain the names “quasi-singlet” and “quasi-octet” for the elements of the diagonal basis. In the limit of large $\Delta y$, the eigenvalue for the quasi-octet grows with $\Delta y$, while the eigenvalue of the quasi-singlet does not. This will produce the expected enhancement of the latter relative to the former in the resummed cross section at small $Q_c$. We shall use Greek indices to identify the basis in which $\Gamma_S$ is diagonal.

We can now write down a resummed cross section, working to lowest order in $\alpha_s(-\hat{t})$, but resumming all leading logarithms in $Q_c$. We transform Eq. (2) to the diagonal basis, and solve the evolution equation for $S$, to get

$$\frac{d\hat{\sigma}^{(f)}}{dQ_c d\cos \hat{\theta}} \left( \hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(-\hat{t}) \right) = H_{\beta\gamma}^{(1)} \left( \Delta y, \sqrt{\hat{s}}, \sqrt{-\hat{t}}, \alpha_s(-\hat{t}) \right) S_{\gamma\beta}^{(0)}(\Delta y)$$

$$\times \frac{E_{\gamma\beta}}{Q_c} \left[ \ln \left( \frac{Q_c}{\Lambda} \right) \right] E_{\gamma\beta}^{-1} \left[ \ln \left( \frac{\sqrt{-\hat{t}}}{\Lambda} \right) \right]^{-E_{\gamma\beta}}.$$  

(12)

The coefficients $E_{\gamma\beta}$ are given by

$$E_{\gamma\beta} \left( y_{JJ}, \hat{\theta}, \Delta y \right) = \frac{2\pi}{\beta_1} \left[ \hat{\lambda}_\gamma \left( y_{JJ}, \hat{\theta}, \Delta y \right) + \hat{\lambda}_\beta \left( y_{JJ}, \hat{\theta}, \Delta y \right) \right],$$

(13)

where $\beta_1$ is the first coefficient in the expansion of the QCD $\beta$-function, $\beta_1 = \frac{11}{3}N_c - \frac{2}{3}n_f$, and where we define $\hat{\lambda}_\beta$ by $\lambda_\beta = \alpha_s \hat{\lambda}_\beta + \cdots$.

In accordance with our approximation, the matrix $S_{\gamma\beta}^{(0)}$ is obtained by transforming the zeroth order $S_{LI}^{(0)}$ of Eq. (2) to the new basis. The matrix $S_{LI}^{(0)}$ is just a set of color traces,

$$S_{LI}^{(0)} = \begin{pmatrix} N_c^2 & 0 \\ 0 & \frac{1}{4}(N_c^2 - 1) \end{pmatrix},$$

(14)

and is transformed to the diagonal basis by the matrix $(R^{-1})_{K\beta} \equiv (e_\beta)_K$ [9],

$$S_{\gamma\beta}^{(0)} \equiv \left[ (R^{-1})_{\gamma M} \right] S_{MN}^{(0)} (R^{-1})_{N\beta}.$$  

(15)

Analogously, we take for $H_{IL}^{(1)}$ the square of the single-gluon exchange amplitude, represented in the color basis. Considering the dominant $t$-channel Born-level amplitude alone, which is purely octet, we have $H_{IL}^{(1)} = \delta_{I2} \delta_{L2} \hat{\sigma}_t$, where $\hat{\sigma}_t$ is the $t$-channel partonic cross section, including the coupling $\alpha_s(-\hat{t})$. The contribution of $s$-channel diagrams has a relatively small effect, and will be described elsewhere [11]. In the diagonal basis the hard matrix $H_{IL}^{(1)}$ becomes $H_{\beta\gamma}^{(1)}$, defined as

$$H_{\beta\gamma}^{(1)} = (R)_{\beta L} H_{IL}^{(1)} (R^t)_{K\gamma}. $$

(16)
Observe that $S^{(0)}$ and $H^{(1)}$ both acquire a $\Delta y$-dependence through the change of basis.

From Eqs. (12)-(13), using the results described above, it is possible to evaluate Eq. (1). For the valence partons we have taken the leading order CTEQ4L distributions [12]. In Fig. 2 we plot the shapes of the cross sections obtained in this way, as a function of the radiation into the central region, $Q_c$, for two different sets of conditions, $\sqrt{S} = 630$ GeV, $\Delta y = 3.2$, and $\sqrt{S} = 1800$ GeV, $\Delta y = 4.0$. We also show the contributions of quasi-octet and quasi-singlet terms. There is in addition a negative interference term, not exhibited separately. As anticipated above, we find a strong suppression of the quasi-octet component for very small values of $Q_c$, contrasted to a peak for the quasi-singlet in this limit. The reason for this difference is easily found. For most kinematic configurations, the coefficient $E_{11}$ from Eq. (13) is less than one, so that the quasi-singlet cross section in Eq. (12) decreases monotonically with increasing $Q_c$. For the quasi-octet, on the other hand, $E_{22}$ is always greater than unity, so that its contribution grows with $Q_c$, until the power of the logarithm is overcome by the dimensional factor of $1/Q_c$.

These results can be compared with the experimental data in Fig. 1 of Ref. [10], showing the measured number of events as a function of the number of towers counted in the central region of the calorimeter, clearly related to $Q_c$. We can understand the similarity of shapes in terms of the $Q_c$ dependence in Eq. (12), discussed above. This similarity is suggestive; indeed, from our simulation we have evaluated the minimum-maximum ratio of the cross section, finding about 30% at $\sqrt{S} = 630$ GeV and about 15% at $\sqrt{S} = 1800$ GeV, close to the analogous ratios in Fig. 1 of Ref. [10]. We have also determined an analog of a “hard singlet fraction” [1, 2], as the ratio of the area under the quasi-singlet curve to the area under the overall curve. It is about 5% at...
$\sqrt{S} = 630 \text{ GeV}$ and about 3% at $\sqrt{S} = 1800 \text{ GeV}$. The order of magnitude of the result is reasonable, although higher than the roughly 1% found at the Tevatron using track or tower multiplicities. How much of this difference is due to our new definition of the gap and how much to the lack of a nonperturbative survival probability remains to be explored. The sharp upturn that we observe below 1 GeV is due to the divergence of the perturbative running coupling at $Q_c = \Lambda$; nonperturbative effects will attenuate this rise.

Previous analysis of rapidity gaps in dijet events has tended to emphasize either the short-distance [5, 14] or long-distance [15, 16, 17] aspects of the problem. (The role of Sudakov logarithms in double-rapidity gap events has been discussed in [18].) Here, we have argued that by factorizing short- and long-distance effects, we may treat both dependences systematically. In our formalism, the mixing of color states begins at short distances precisely with two-gluon exchange [5, 14], summarized through the anomalous dimension $\Gamma_S$, while long-distance color ("bleaching") effects [15, 16, 17] follow the evolution of the different color components between the short-distance scale $\sqrt{-t}$ and the long-distance scale $Q_c$.

As we have observed above, our formalism does not include a nonperturbative survival probability [3, 4] associated with the interaction of spectator partons. Clearly, a full phenomenological analysis will also require the inclusion of processes involving gluons (including $q\bar{q} \to gg$) and sea quarks. The treatment of gluon-gluon scattering [16, 17] should be particularly interesting [4]. Nevertheless, we believe that the basic features shown in the valence-quark analysis outlined above will appear as well in a more complete discussion. A calorimetric analysis of dijet rapidity gap events, if possible experimentally, could shed valuable light on the dynamics of QCD.

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