The Features of the Flexomagnetoelectric Effect in an External Magnetic Field

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Abstract—The effect of a magnetic field on the behavior of 180-degree domain walls in a uniaxial ferromagnetic film with an inhomogeneous magnetoelectric interaction has been studied. It has been shown that, depending on the magnitude and direction of the field, it is possible to enhance or weaken the flexomagnetoelectric effect in the sample. In addition, it was found that in the reverse field the switching of the interaction between the electric field source and the domain wall from attraction to repulsion is possible.

Keywords: uniaxial ferromagnetic film, flexomagnetoelectric effect, 180-degree domain wall, nonuniform electric field, magnetic field

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1. INTRODUCTION

The study of magnetoelectric effects observed in a certain class of magnets called multiferroics is of great interest [1, 2]. These are characterized by two or more order parameters and have a number of unusual properties that can be used in new generation spintronics and magnetic memory devices. As is well known, ferrite-garnet films, in which the giant magnetoelectric effect (linear) was found at room temperature, also are multiferroics [3]. Some time later, a new effect of this type was discovered in them, which consists of the displacement of domain walls (DWs) under the action of a uniform electric field [4]. In analyzing the experimental data, the authors suggested that it can be explained by the manifestation of the flexomagnetoelectric effect (FMEE) [1], i.e., the presence of an inhomogeneous magnetoelectric interaction (IMEI) in the materials under study, as first considered in [5]. The results obtained in [4] initiated new studies in this direction [6–11], which made it possible to study the effect of an electric field on the structure and properties of magnetic inhomogeneities of various topologies in magnetic films with IMEI more thoroughly.

At the same time, in [12, 13], another interpretation of the experimental data [3] was proposed, which is not related to “charged” DWs. It is based on a possible change in the anisotropy constant of a material due to the displacement of ions of the same type relative to the equilibrium position under the action of a nonuniform electric field. It should be noted that in [14], based on the fluorescence spectroscopy of single molecules, the flexomagnetoelectric nature of the induced electric polarization in iron garnet films was confirmed. Nevertheless, a comparative analysis of the above mechanisms showed [15] that both of them at a qualitative level fully explain the behavior of DWs in a nonuniform electric field. Hence, each of the mechanisms contributes to the phenomenon under study. However, which of them is dominant remains to be clarified in further research. In addition, it is of practical interest to study various factors (external or internal) that significantly influence the degree of manifestation of this effect. In particular, it was shown in [15–19] that some properties of DWs (displacement, velocity, etc.), as well as its transformation in a nonuniform electric field, are significantly affected by an external magnetic field, in particular, its planar component [15, 19]. For this purpose, in this work, a theoretical analysis of the influence of an external magnetic field on the manifestation of the FMEE in the magnets under study has been conducted.

2. PROBLEM STATEMENT

We consider a uniaxial ferromagnet in the form of a film of thickness \( D \). It is assumed that the easy magnetization axis of perpendicular anisotropy is directed along the normal to the film and is parallel to the \( O_z \) axis (Fig. 1), while the \( O_y \) axis coincides with the direction along which the sample is inhomogeneous, i.e., the magnetic moments rotate along it. The magnetization vector \( \mathbf{M} = M_s \mathbf{m} \) (\( M_s \) is the saturation magnetization) is expressed in terms of the unit vector...
m, defined via the variables θ and φ: \( m = (\sin \theta \cdot \cos \phi, \sin \phi, \cos \theta \cdot \cos \phi) \).

The energy of the magnet, reduced to the area of the cross section of the film by the \( xOz \) plane, is taken in the form

\[
E = \int \left\{ A \left[ \left( \frac{d\phi}{dy} \right)^2 + \cos^2 \phi \left( \frac{d\theta}{dy} \right)^2 \right] + K_u \right. \times \left( \sin^2 \theta \cos^2 \phi + \sin^2 \phi \right) + \varepsilon_{\text{int}} + \varepsilon_H + 2\pi M^2_s \sin^2 \phi \right\} dy.
\] (1)

Here, \( A \) is the exchange parameter, \( K_u \) is the uniaxial anisotropy constant, and \( \varepsilon_{\text{int}} \) and \( \varepsilon_H \) are the IMEI and Zeeman interaction energy densities, respectively. The last term represents the energy density of magnetizing fields from space charges \([20, 21]\). In this case, it is assumed that the film is thick (\( \Delta_0 \ll D < \Lambda_0 \)), where \( \Delta_0 = \sqrt{A/K_u} \) is the characteristic size of the DW and \( \Lambda_0 = \sqrt{A/2\pi M_s} \) is the Bloch line width \([22]\) and the contribution of demagnetizing and scattering fields is neglected. Accordingly, the formula for \( \varepsilon_H \) is

\[
\varepsilon_H = -M_s (mH), \quad (2)
\]

and \( \varepsilon_{\text{int}} \) is expressed as \([23]\)

\[
\varepsilon_{\text{int}} = M_i E (b_i m \text{div} m + b_2 m \text{rot} m), \quad (3)
\]

where \( b_1 \) and \( b_2 \) are the magnetoelectric constants and \( E \) and \( H \) are the electric and magnetic field strengths, respectively. In the given case, these fields are assumed to be nonuniform and act in bounded regions of space,

\[
E = E_0/cosh^{-1}(y/L), \quad H = H_0/cosh^{-1}(y/L), \quad (4)
\]

where \( E_0 = E(0) \) and \( H_0 = H(0) \) are the values of the corresponding fields in the center of the band of their action and \( L_1 \) and \( L_2 \) are the characteristic sizes of the corresponding bands along the \( Oy \) axis. It is assumed that the field \( E \) is directed along the \( Oy \) axis (\( E || Oy \)) and the field \( H \) is directed along the \( Oy \) axis (\( H \parallel Oy \)).

Then, the expression for \( \varepsilon_{\text{int}} \), written in angular variables, will take the form

\[
\varepsilon_{\text{int}} = EM_i^2 \left\{ \left( b_1 \cos^2 \phi + b_2 \sin^2 \phi \right) \cos \theta \frac{d\phi}{dy} + b_2 \sin \theta \sin \phi \cos \phi \frac{d\theta}{dy} \right\}.
\] (5)

Fig. 1. The geometry of the problem.

The structure and properties of magnetic inhomogeneities are determined from the Euler–Lagrange equations:

\[
\frac{d}{d\xi} \left( \cos^2 \phi \frac{d\theta}{d\xi} \right) - \sin \theta \cos \theta \cos^2 \phi + \left( \lambda_1 + \lambda_2 \right) f(\xi) \sin \theta \cos^2 \phi \frac{d\phi}{d\xi} + \sin \theta \cos \theta \cos^2 \phi \frac{d\phi}{d\xi} = 0;
\]

\[
\frac{d^2 \phi}{d\xi^2} - \sin \phi \cos \phi \left[ \cos^2 \theta - \left( \frac{d\theta}{d\xi} \right)^2 \right] - \left( \lambda_1 + \lambda_2 \right) f(\xi) \sin \theta \cos^2 \phi \frac{d\theta}{d\xi} + \left( \lambda_1 \cos^2 \phi + \lambda_2 \sin^2 \phi \right) \cos \theta \frac{d\phi}{d\xi} + \frac{1}{M_s H_u} \frac{\partial \varepsilon_H}{\partial \phi} - Q^{-1} \sin \phi \cos \phi = 0.
\] (6)

Here, \( \lambda_i = E_i/E_i = E_i M_i^2 b_i / 2K_u \Delta_0, \quad E_i = 2K_u \Delta_0 / M_s^2 \), \( i = 1, 2, \xi = y/\Delta_0, \quad \lambda_1 = L_1 / \Delta_0, f(\xi) = \cosh^{-1}(\xi/L_1), \) and \( Q = K_u/2\pi M_s^2 \). The quantities \( \lambda_1 \) and \( E_i \) are the reduced and characteristic electric fields, respectively, \( \xi \) is the reduced coordinate, \( Q \) is the quality factor of the material, and \( H_u = 2K_u / M_s \) is the field of uniaxial anisotropy. In what follows, we will use another dimensionless parameter, \( h = H_0 / H_u \) (reduced magnetic field).

Numerical analysis of these equations, taking into account the IMEI, showed \([21]\) that in uniaxial ferromagnets at \( h = 0 \), depending on the chosen boundary conditions imposed on \( \theta \) and \( \phi \) at \( \xi \to \infty \), the existence of three types of micromagnetic structures is possible. These are \( 180^\circ \) DWs with a noncircular trajectory of the magnetization vector \([24]\), \( 0^\circ \) DWs with a quasi-Bloch structure \([24, 25]\), and \( 0^\circ \) DWs of the Néel type \([25]\). In this work, the main attention will be paid to the behavior of the \( 180^\circ \) DW in the ferromagnet under study in an external magnetic field, which is associated with similar experimental studies of the FMEE \([15, 19]\), in which only this type of boundaries was observed.
3. TRANSFORMATION OF A 180° DW IN A MAGNETIC FIELD, \( H = 0 \)

Obviously, a 180° Bloch-type DW will transform in an external magnetic field \( H \), but the nature of these changes will depend both on the magnitude and orientation of the field \( H \) relative to the DW plane. In this case, it makes no sense to consider the case where \( H \parallel Ox \) for a 180° DW, since such a field will only lead to a displacement of the DW as a whole.

Let us first consider the case of \( h = 0 \). A numerical study of Eqs. (6) (here we consider the case of \( \lambda_1 = \lambda_2 = \lambda \)) shows [21] that a 180° Bloch-type DW exposed to a nonuniform electric field undergoes a number of transformations of its texture with an increase in \( \lambda \). 180° Bloch-type DW → 180° DW with a quasi-Bloch structure → 180° DW with a quasi-Néel structure → 180° Néel-type DW. Magnetic inhomogeneities that are in intermediate positions in this chain of transformations refer to DWs with a noncircular trajectory of the magnetization vector [10, 21, 24]. This means that the magnetic moments in both types of DWs have both Bloch \((m_s \neq 0)\) and Néel \((m_s \neq 0)\) components. However, the difference between them is that a 180° DW with a quasi-Bloch structure does not have regions with a purely Néel law of rotation of magnetic moments \((m_s = 0)\), while there are such sections in the second type.

It should be noted that the cascade of transformations of the 180° DW structure, which occur with an increase in the electric field, is initially accompanied by the induction of bound charges in the vicinity of the 180° DW and the subsequent increase in the electric polarization (both its differential value \( p = v p_0 \) and integral value \( P = N p_0 \), where \( v \) and \( N \) are the reduced differential and integral polarizations and

\[
\rho_0 = M_s^2 b_0 / \Delta_0 \text{ is the characteristic value of polarization [21]. When the field reaches the value } \lambda = \lambda_s, \text{ at which the 180° DW becomes completely non-Néelian, the dependence } N = N(\lambda) \text{ (Fig. 2, black curve) has a kink: a sharp increase is replaced by a region of a slow (adiabatic) increase in } N. \]

4. TRANSFORMATION OF A 180° DW IN A MAGNETIC FIELD, \( \Lambda = 0 \)

Let us consider the effect of an external magnetic field on the structure and properties of a 180° DW. We assume that \( H \parallel Ox \) and coincides with the direction of the magnetic moments in the DW plane at \( y = 0 \). In this case, the magnetic moments form with the field an angle \( \psi \) lying in the interval \( 0 \leq \psi \leq \theta_0 \), where \( \theta_0 = \arcsin (h) \). Analysis of Eqs. (6) for this case shows that in the absence of an electric field \( (\lambda = 0) \), the magnetization \( M_0 \) in the domains makes with the \( Ox \) axis an angle \( \theta_0 \). Accordingly, a 180° Bloch-type DW under the action of a magnetic field \( h \) becomes narrower \((180° - 2\theta_0)\) with the law of rotation of the vector \( \mathbf{m} \) in the wall, determined by the expressions (at \( l_2 \rightarrow \infty \))

\[
\theta = 2\arctan \left[ \sqrt{1 - h^2 \tanh \left( \frac{\sqrt{1 - h^2} \xi}{2} \right)} \right] / h, \quad (7)
\]

\[
\varphi = 0.
\]

Hence, with an increasing field \( h \), the maximum magnetization turn angle \( \theta_m \) in such a DW, equal to \( \theta_m = (180° - 2\theta_0) \), will continuously decrease, while its width \( \Delta \) will increase (Fig. 3).

When the field \( h \) reaches the critical value \( h = 1 \) \((H = H_d)\), the limiting magnetization orientations in
the domains, $\mathbf{m}_1$ and $\mathbf{m}_2$ ($\mathbf{m}_1 = \mathbf{m}(-\infty)$, $\mathbf{m}_2 = \mathbf{m}(\infty)$) become parallel ($\mathbf{m}_1 | \mathbf{m}_2$) and the width of such a DW increases without limit. Accordingly, $\theta_m \to 0$ and the wall disappears. However, if the magnetic field is non-uniform and acts in a bounded region comprising a band of width $l_2$ (along the $Oy$ axis), in this case, according to calculations, with an increase in $h$, the DW width $\Delta$ will also increase, but with a smaller slope of the corresponding curve (Fig. 3). At the same time, the turn angle $\theta_m$ will decrease, but the limiting value $\theta_m = 0$ will be reached in much stronger fields ($h > 1$).

If the magnetic field is directed oppositely to the $Ox$ axis, then the 180° DW will transform according to a different scenario. In this case, the magnetic moments in the domains will also begin to deviate from the $Oz$ axis towards the direction of the field $\mathbf{H}$, but the turn of the vector $\mathbf{m}$ will be narrower, $\theta_m \geq \pi$. In this case, the structure of the 180° DW will be described by another magnetization distribution, which has the form (at $l_2 \to \infty$)

$$
\theta = -2\arctan \left[ \frac{1 + \sqrt{1 - h^2 \coth^2 \left( \sqrt{1 - h^2 \xi^2} / 2 \right)}}{h} \right], \quad (8)
$$

Accordingly, the hodograph of the magnetization vector $\mathbf{m}$ will describe a “longer” trajectory on the surface of a sphere of unit radius ($\theta_m = \pi + 2\theta_0$) than in the first case of the orientation of $\mathbf{H}$. Thus, this wall comprises a (180° + 20°) DW. With increasing $h$, the angle $\theta_m$ will also increase and, in the limit at $h = 1$, $\theta_m = 2\pi$, i.e., the (180° + 20°) DW will become a 360° DW. Accordingly, the magnetic moments located at the center of the wall (near $y = 0$) will be directed oppositely to the field $\mathbf{H}$. As is known, such a wall becomes unstable with respect to fluctuations of the Néel-type magnetization vector and, at a certain value of the field $h$ [26], it collapses and disappears.

If the magnetic field $\mathbf{H}$ is directed along the $Oz$ axis, a qualitative change in the structure of the 180° DW in the magnetic field occurs. In this case, the wall, remaining as a 180° DW, transforms from the Bloch type into a quasi-Bloch wall, because the magnetization $\mathbf{M}_0$ leaves the DW plane ($\varphi \neq 0$). In addition, the magnetization $\mathbf{M}_0$ in the domains deviates from the $xOz$ plane (coinciding with the DW plane) by an angle $\varphi_0 = \varphi(\infty) \neq 0$ (Fig. 4, green dashed line (T)).

With an increase in $h$, the maximum angle of emergence $\varphi_m$ increases and, at some $h = h_1$, reaches the value of $\varphi_m = \pi / 2$. With a further increase in the field up to $h = h_2$ (at $Q = 3$, $l_2 = 1000$, and $h_2 = 0.4$), the Néelian contribution to the DW structure increases ($m_0$ increases), while the Blochian contribution decreases ($m_1 \to 0$). Finally, at $h = h_3$, the wall becomes completely Néel-type. With a further increase in $h$, the wall becomes unstable and collapses.

With a decrease in the size of the band of nonuniformity of the magnetic field $l_2$, this critical field increases. In the reverse field, the process of transformation of a 180° DW will repeat, but the angle $\varphi_m$ in this case will take values of the opposite sign.

5. TRANSFORMATION OF A 180° DW IN A MAGNETIC FIELD ($A \neq 0$)

Let us now study the influence of an external magnetic field on the flexomagnetoelectric effect. We will assume that $\mathbf{H} \parallel Oy$ and the chirality of the DW is such that the direction of the magnetic moments (at $y = 0$) coincides with $\mathbf{H}$. Then, upon switching on the field, a similar transformation, considered in the previous section, will take place: a 180° DW with a quasi-Bloch structure is transformed into a (180° − 20°) DW also with $\mathbf{m}$ leaving the plane of rotation of the magnetic moments (Fig. 5a). However, with an increase in $h$, which tends to turn the magnetic moments along the field (i.e., return them back to the DW plane at a constant value of the parameter $\lambda$), the maximum angle of emergence $\varphi_m$ decreases (Fig. 5b).

In addition, the maximum value of the differential polarization $p_m$ also decreases (Fig. 5), which leads to a decrease in the integral polarization $N$. However, with an increase in the electric field strength $E_0$ ($\lambda$ increases), the relative decrease $\Delta N / N (\Delta N = N(h_i) - N(h_j))$, $N(h_i)$ are the values of the integral polarization at different $h_i (i = 1.2)$ but at the same $\lambda$) will decrease until reaching zero in the limit. In this case, all curves $N = N(\lambda)$ converge in the limit ($\lambda \to \infty$) to the same asymptote (Fig. 2), which corresponds to the dependence curve for a 180° Néel-type DW ($h = 0$). The same behavior
is demonstrated by the dependences $\varphi_m = \varphi_m(\lambda)$ and $\nu_m = \nu_m(\lambda)$. Hence, the action of a magnetic field with $H \parallel Ox$ weakens the FMEE. In addition, the presence of a magnetic field leads to the smoothing of the transition from a $180^\circ$ quasi-Néel DW to a $180^\circ$ Néel-type DW (the plots of the dependence $N = N(\lambda)$ in Fig. 2 have no kink) and also to a lowering of the critical field $\lambda_c$ of this transition.

If the magnetic field is directed oppositely to the $Ox$ axis, then the magnetic moments in the domains will deviate from the $Oz$ axis in the opposite direction and a $180^\circ$ DW will also transform into a $(180^\circ - 2\theta_0)$ DW. In this case, the angle of emergence of the magnetization from the wall plane increases significantly (Fig. 6); accordingly, the differential polarization $\nu$ also increases, which leads to an increase in the integral polarization $N$ (Fig. 2). Thus, in the reverse field, the FMEE in the sample is significantly enhanced.

Let us now consider the situation where the initial magnet is exposed to a magnetic field $H \parallel Oy$. In this case, a $180^\circ$ Bloch-type DW transforms under the action of the magnetic field into a quasi-Bloch wall already at $\lambda = 0$. In this case, the magnetization $M_0$ in the domains deviates from the $xOz$ plane by an angle $\varphi_0$ (Fig. 4). If $\lambda \neq 0$, the process of changing the topology of the wall is enhanced; with increasing $h$, both the angle $\varphi_0$ and the maximum angle of deviation from the homogeneous state ($\varphi_m - \varphi_0$) increase, as well as the maximum value of the differential polarization $\nu_m$. Accordingly, the value of the integral polarization $N$ also increases (Fig. 7).

In this case, an interesting regularity is observed: the higher the value of $h$ is, the lower the electric fields are at which the transition of a quasi-Bloch $180^\circ$ DW to a Néel-type wall is achieved, while the maximum value of the integral polarization decreases (Fig. 8).

When the electric field reaches its critical value $\lambda = \lambda_c$, the structure of a $180^\circ$ DW becomes Néel-type. In this case, the graph of the dependence of the
The maximum angle of deviation from the homogeneous state $\varphi_m$ of a 180° DW vs. parameter $\lambda$ in a magnetic field $H \parallel Oy$: (4, black curve) $h = 0$, (3, red curve) $h = 0.1$, (2, blue curve) $h = 0.2$, (1, yellow curve) $h = 0.4$, (3, red dashed curve) $h = -0.1$, and (6, blue dashed curve) $h = -0.2$.

Fig. 8. The maximum angle of deviation from the homogeneous state $\varphi_m$ of a 180° DW vs. parameter $\lambda$ in a magnetic field $H \parallel Oy$: (4, black curve) $h = 0$, (3, red curve) $h = 0.1$, (2, blue curve) $h = 0.2$, (1, yellow curve) $h = 0.4$, (3, red dashed curve) $h = -0.1$, and (6, blue dashed curve) $h = -0.2$.

integral polarization $N$ on $\lambda$ has a kink similar to that at $h = 0$. Hence, under the action of a magnetic field $H$ along the $Oy$ axis the FMEE is enhanced, but this happens in low fields $h$, while, in high fields, $h$, the effect weakens.

In the case where the direction of $H$ is opposite to the $Oy$ axis, magnetic moments, in turning towards the field, eventually form an angle $\varphi_0 = \varphi(\infty)$, which becomes negative and reduces the maximum angle $\varphi_m$ of emergence of the magnetization from the DW plane (Fig. 4). As a result, the values of $v_m$ and $N$ also decrease. With a further increase in $h$, the value of $N$ decreases and at a certain value $h = h_0$, it becomes zero and at $h > h_0$, negative (Fig. 8). This means that a 180° DW will have to be repelled from the source of the nonuniform electric field. Thus, by switching the direction of the magnetic field, it is possible to change the sign of the polarization and thereby change the character of the interaction of the 180° DW with an external electric field. This result agrees with the experimental data well [4]. This allows using electric and magnetic fields to regulate the motion of a DW, which is of practical interest.

CONCLUSIONS

Thus, the above results show that the presence of an external magnetic field has a significant influence on the flexomagnetoelectric effect observed in films of ferrite garnets with the IMEE. The degree of this influence depends on both the magnitude and orientation of the magnetic field relative to the plane of a 180° DW. In particular, in this work, the structure of a 180° DW was studied in two mutually perpendicular directions: $H \parallel Ox$ and $H \parallel Oy$. According to calculations, a significant (multiple) enhancement of the effect will take place under the action of electric and magnetic fields on a 180° DW such that $E \parallel Oz$ and $H \parallel Oy$, and the greatest enhancement can be achieved even in low magnetic fields. This agrees with the experimental data [15, 19], according to which, the greatest displacement of a DW in a nonuniform electric field occurs under the action of a magnetic field perpendicular to the wall plane. In this case, the enhancement of the integral polarization $N$ is achieved by increasing the angle of emergence of the magnetization vector from the DW plane. Accordingly, the magnitude of the volume magnetic charges, as determined by the expression $\rho_v = -M_s \text{div} m$ [18, 26], increases, which ultimately leads to an increase in the parameters and in $N$.

From the results we obtained it also follows that by changing the orientation of the magnetic field to the opposite one it is possible to change the character of the manifestation of the flexomagnetoelectric effect: either to enhance it (in the case of $H \parallel x$) or weaken. However, by switching the direction of $H$, one can also achieve a change in the character of the interaction of the DW with the electric field from attraction of the DW to its repulsion and vice versa. This property can be important in applied research. On the other hand, this property indicates that the flexomagnetoelectric mechanism is also dominant when the DW is subjected to a nonuniform electric field. The point is that a perpendicular magnetic field can change the width of a DW and its toplogy, but not to move it.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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