Vortex Motion Noise in Micrometre-Sized Thin Films of the Amorphous Nb$_{0.7}$Ge$_{0.3}$ Weak-Pinning Superconductor

D. Babić,¹ T. Nussbaumer,¹ C. Strunk,¹ C. Schönenberger,¹ and C. Sürgers²

¹Institute of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland
²Physikalisches Institut, Universität Karlsruhe, D-76128 Karlsruhe, Germany

We report high-resolution measurements of voltage ($V$) noise in the mixed state of micrometre-sized thin films of amorphous Nb$_{0.7}$Ge$_{0.3}$, which is a good representative of weak-pinning superconductors. There is a remarkable difference between the noise below and above the irreversibility field $B_{irr}$. Below $B_{irr}$, in the presence of measurable pinning, the noise at small applied currents resembles shot noise, and in the regime of flux flow at larger currents decreases with increasing voltage due to a progressive ordering of the vortex motion. At magnetic fields $B$ between $B_{irr}$ and the upper critical field $B_c$, flux flow is present already at vanishingly small currents. In this regime the noise scales with $(1 - B/B_c)^2 V^2$ and has a frequency ($f$) spectrum of $1/f$ type. We interpret this noise in terms of the properties of strongly driven depinned vortex systems at high vortex density.

PACS numbers: 74.76.Db, 74.40.+k, 74.60.Ge

I. INTRODUCTION

When set in motion by a current $I$, vortices in superconductors generate a voltage $V$. The resulting $V(I)$ curve may be either non-linear, implying depinning phenomena, or linear, indicating flux flow (FF). Such $V(I)$ characteristics do not provide complete information on the nature of vortex motion, especially if the pinning is weak. This is a point where information available from the voltage noise becomes a powerful indicator of the underlying physics. The finding that vortices moving as bundles composed of $N$ magnetic-flux quanta $\phi_0$ may produce shot noise attracted considerable attention, and resulted in extensive subsequent work which was eventually extended beyond a simple shot-noise approach. Samples used in these studies were mainly polycrystalline conventional superconductors with appreciable pinning and non-linear $V(I)$ characteristics up to very close to $B_c$. Noise experiments have also been carried out on high-T$_c$ superconductors, which are in a "liquid" state of negligible pinning over a large portion of the magnetic field vs. temperature plane, displaying linear $V(I)$ curves. However, the intrinsic anisotropic character of vortex matter in these compounds is a serious obstacle to the understanding of the mechanisms that contribute to voltage noise related to motion of vortices against a weak pinning potential.

Thus, a number of phenomena in the weak-pinning regime have remained largely unexplored from the point of view of vortex motion noise. The same holds for the noise properties in the depinned state, i.e. for $B > B_{irr}$. For instance, the interplay of bulk pinning and surface barriers which are both obstacles for vortex motion (and can be of similar strengths when pinning is weak), has been studied mostly by analyzing the $V(I)$ curves and the magnetoresistance $R(B, T)$. Similarly, dynamic ordering of vortex motion has also been explored by measuring the average transport properties. Noise measurements can reveal effects which are beyond reach of measurements of the average voltage. For example, if pinning is absent the $V(I)$ is linear, but one could ask does this mean that vortices really move completely "silently" or are there some dynamic effects which introduce fluctuations in their velocity? Moreover, it is known that shot noise probes the properties of "granular magnetic-flux charge", $N\phi_0$, but the details of this process are still subject to discussion - especially if $N$ is small (characteristic of weak pinning).

In this paper we present high-resolution noise measurements which address the above topics. We have chosen a system particularly suitable for such research, namely Nb$_{0.7}$Ge$_{0.3}$ amorphous thin films of thickness $d$ comparable to the coherence length $\xi$. These films are conventional, isotropic, weak-coupling s-wave BCS superconductors in the dirty limit, and for $\xi \sim d$ they exhibit an extended $(B, T)$ range of easily-movable vortices. In contrast to the complicated situation in high-T$_c$ compounds, here vortices can be considered as undeformed "cylinders" of a volume $\xi^2 d$ and the Ginzburg-Landau (GL) parameters can be found straightforwardly. We also note that our shaping the samples in the form of narrow wires turned out to be crucial for observing the overall properties of the noise, i.e. for both $B < B_{irr}$ and $B > B_{irr}$.

In the regime where $V(I)$ and $R(B, T)$ still indicate the presence of pinning we find a noise similar to that in Ref.², i.e. which for small applied currents resembles shot noise, being linear in $V$ and frequency independent at low frequencies, and decreases for more ordered vortex motion at high currents. Closer to $B_{c2}$, over a rather extended range, we find no evidence for pinning in neither $V(I)$ or $R(B, T)$. The lower boundary of this region is therefore taken as the irreversibility field $B_{irr}$. The noise for $B_{irr} < B < B_{c2}$ is qualitatively different from that in the presence of pinning. It exhibits a $1/f$ frequency spectrum and is quadratic in $V$. Moreover, it scales with $(1 - B/B_{c2})^2 V^2$. The monotonic increase with increasing $V$, and in particular the scaling which involves no pinning...
dependent parameters, motivates us to propose that the noise of this kind is a peculiar property of strongly driven vortices at high vortex density.

II. EXPERIMENT

Our samples (20 nm thick) were produced by magnetron sputtering of Nb and Ge on to oxidised silicon wafers through masks prepared by electron-beam lithography, using a double-layer resist (PMMA/PMMA-MA). The measurements were carried out in a 4He cryostat, above the λ-point of liquid helium. Voltage noise, \( V(I) \) and \( R(B,T) \) were measured extensively on a \( W = 5 \) µm wide and \( L = 50 \) µm long wire connected to two wide contact pads (sample S5). In order to investigate size effects in the noise we performed a less comprehensive set of measurements on a \( W = 1 \) µm and \( L = 10 \) µm sample (sample S1). By analysing the low-current (10 nA; 10 Acm\(^{-2}\)) \( R(B,T) \) measurements within the framework of a model appropriate for dirty weak-coupling superconductors, \( T_c \) we characterised sample S5 in detail.

The transition temperature \( T_c \) is determined as the midpoint of the \( 10\% \) - \( 90\% \) (0.1 K) zero-field transition curve. The transition curve is smooth and free of "kinks" that would indicate the presence of inhomogeneities, and we ascribe the rather wide transition (in units of \( T/T_c \)) to a pronounced two-dimensional character of the sample. A similar conclusion was drawn in Ref.\(^{13}\) for an YBa\(_2\)Cu\(_3\)O\(_{\gamma-\delta}\) single crystal investigated systematically with respect to different values and consequently different anisotropies. Very weak temperature dependence of the normal state resistivity \( \rho_N \) above \( T_c \) permits the estimation of \( \rho_N(T = 0) = (2.3 \pm 0.2) \mu \Omega m \). Using this value and \(-\langle dB_{22}/dT \rangle \) measured, determined from the \( R(B = \text{const.}, T) \) measurements (not shown), we calculate\(^1\) the GL parameters: \( \xi(0) = 7.4 \) nm, \( \kappa = 77 \) and \( \lambda(0) = 1.63\kappa(0) = 930 \) nm. The parameters of sample S5 are in good agreement with published work.\(^{14}\) Sample S1 had a slightly lower \( T_c \) (2.55 K) and larger \( \rho_N \), but otherwise showed fairly the same properties as sample S5. The method of noise measurements is described in detail in Ref.\(^{14}\). In short, the signal from a sample is processed through two low-noise amplifiers the outputs of which are cross-correlated in a spectrum analyser. The noise setup is calibrated against the equilibrium Nyquist noise \( 4k_BT\rho_N \) in the normal state (\( \rho_N \) is the normal state resistance). By this approach we have obtained a resolution of \( \lesssim 10^{-20} \) V\(^2\)/s, necessary for measurements of small noise signals appearing in the case of weak pinning. For both samples the frequency window for the noise measurements was 106.5 - 114 kHz, except for the measurements of the frequency dependence of the noise power spectrum \( S_V \), performed at several frequencies between 20 kHz and 250 kHz.

All the noise measurements were carried out at fixed temperatures, \( T = 2.4 \) K (\( T/T_c = 0.82 \)) for sample S5 and \( T = 2.25 \) K (\( T/T_c = 0.88 \)) for sample S1. Since sample S1 had lower \( T_c \), we had to choose a larger value of \( T/T_c \) in order to avoid temperature instabilities that appear in the vicinity of the \( \lambda \)-point.

III. MAGNETORESISTANCE AND CURRENT-VOLTAGE CHARACTERISTICS

First we analyse the \( R(B,T) \) and \( V(I) \) results. In the lower inset to Fig.1 we show \( R(B, 2.4 \) K) for sample S5. Above \( \sim 0.65 \) T we found good agreement with the FF theory of Larkin and Ovchinnikov (LO)\(^{15}\) The LO FF conductivity is given by

\[
\sigma_{FF} = \frac{1}{\rho_N} \left[ 1 + \frac{1}{(1 - T/T_c)^{1/2}} \left( \frac{B_{c2}}{B} \right) g(B/B_{c2}) \right],
\]

where \( g(z) = (1 - z)^{3/2} \). The solid line, representing the LO FF resistance \( R_{FF} = R_N/\sigma_{FF}\rho_N \), is drawn by taking \( T_c \) = 2.91 K, \( B_{c2} \) = 1.18 T and \( R(B_{c2}(2.4 \) K)) = 1375 \( \Omega \) (\( \rho_N = 2.75 \mu \Omega m \)). The mentioned uncertainty in \( \rho_N \) implies a certain range of the \( B_{c2} \) values that do not deteriorate the fit. This range is \( \sim 1.14 - 1.22 \) T and agrees fairly well with \( B_{c2} \sim 1.09 - 1.12 \) T obtained by the extrapolation method of Ref.\(^{14}\). Henceforth we use \( B_{c2} = 1.18 \) T. Taking different values of \( T_c \) (within the transition width) has little effect on the quality of the fit. We conclude that for the fields above \( \sim 0.65 \) T the vortices flow freely even at very small applied currents, and thus \( B_{\nu\nu}(2.4 \) K) \( \sim 0.65 \) T, which is, as we show below, in agreement with the \( V(I) \) results.

For magnetic fields below \( \sim 0.65 \) T the LO theory does not explain the magnetoresistance data, and \( R \) is smaller than \( R_{FF} \). This indicates that the vortices are slowed down by experiencing a pinning potential. However, \( R \) is finite even at magnetic fields as low as \( \sim 0.5B_{c2} \), which implies a very weak pinning. In the upper inset to Fig.1 we show a log-log plot of a typical \( V(I) \) in this region, for \( 0.27 \) T. Over two decades in \( I \) the \( V(I) \) is ohmic (\( R = 25 \) \( \Omega \) ) before it turns upwards. This suggests a hopping vortex motion (HVM), most probably thermally activated. In the model of thermally activated HVM, vortex velocity is given by \( v_\phi = l(n_+ - n_-) \), where \( l \) is the hop length and \( n_\pm \propto \exp[-(U + U_F)/k_BT] \) the hopping rates over a potential \( U \) in the direction (+) and opposite (-) to the driving force \( F = -\nabla U \). Since \( F \propto IB \) and \( V = BLv_\phi \), for \( I \approx 0 \) the \( V(I) \) is linear. From our measurements of \( R(B = \text{const.}, T) \) we can estimate the values of \( U/k_BT \), which are remarkably small. At \( 2.4 \) K, \( U/k_B \) is lower than 10 K and is a decreasing function of \( B \). At higher currents the \( V(I) \) gradually changes to a \( V(I) \propto (I - I_c) \) dependence, as we show for 0.27 T in Fig.1 by open circles. This suggests a force-induced transition to flux flow, i.e. an ordering of the vortex motion with increasing driving force. This assumption will be supported further by the noise results presented in Section IV A. Finally, at even higher currents \( V \) jumps to a value of the order of \( V_N \approx R_N I \) (Fig.1, upper inset) due to the appearance of
IV. NOISE RESULTS

In the rest of the paper we present and discuss the results of our noise measurements, which if not specified otherwise refer to sample S5. We introduce ΣV to denote the excess noise, which is the difference between the total measured noise SV and the thermal (Nyquist) noise 4kBT(dV/dI). The currents used in the noise measurements were always kept below those corresponding to the appearance of the high-current non-linearities mentioned in Section III, since we are interested in situations where the average transport properties are still unaffected by the high-current dynamical processes described in the LO theory. In Section IV A we analyse the noise in the regime of non-linear V(I) curves, i.e. for \( B < B_{irr} \), and in Section IV B we turn to the noise for \( B_{irr} < B < B_{c2} \), where the V(I) is linear and \( R(B, 2.4 \text{ K}) \) agrees well with the LO FF theory.

A. Noise in the regime of non-linear \( V(I) \)

In Fig.2 we show a typical \( \Sigma_V(V) \) curve in the regime of non-linear \( V(I) \), i.e. for \( 0.27 \text{ T} \) (corresponding to the \( V(I) \) curve in Fig.1). The maximum background Nyquist noise is \( \sim 2.5 \times 10^{-20} V^2/s \). \( \Sigma_V(V) \) first increases linearly up to \( V \approx 0.2 \text{ mV} \) which is close to the upper limit of HVM in \( V(I) \). At higher voltages, where \( V(I) \) becomes proportional to \( (I - I_c) \), \( \Sigma_V \) gradually decreases with increasing \( V \). From this decrease of \( \Sigma_V(V) \) we infer that the vortex motion becomes more and more ordered when the driving force progressively dominates over the pinning potential. At large driving force the pinning potential causes not only a finite offset \( I_c \) in \( V(I) \) but also random fluctuations of the vortex velocity, which is most probably the origin of the small residual noise above \( V \sim 0.5 \text{ mV} \). This residual noise is expected to vanish together with \( I_c \) at \( B_{irr} \), which is indeed observed in our experiment. It is worthwhile to note that the onset of collective vortex motion has stronger effect on \( \Sigma_V \) than on \( V(I) \). In \( \Sigma_V \) the depinning threshold \( I_c \) is indicated by a pronounced maximum above which an ordering of the vortex motion occurs. On the other hand, \( V(I) \) shows no sharp feature at \( I_c \), implying that \( I_c \) has to be determined by extrapolation of the linear part of \( V(I) \) down to \( V = 0 \). Since the linear regime extends only over a small current range between the HVM regime and the high-current non-linearities, the determination of \( I_c \) is more ambiguous than in \( \Sigma_V \). The non-monotonic character of \( \Sigma_V(V) \) supports our interpretation more strongly, and also supplements research on dynamic vortex ordering studied by analysing the average transport properties.

Similarly to shot noise (in current) of electrons, which is a linear function of \( I \), shot noise (in voltage) of vortices is a linear function of \( V \). To check whether the linear increase of \( \Sigma_V(V) \) in the low-voltage regime can be interpreted as shot noise we investigated the frequency and sample-width dependences of \( \Sigma_V \). The studies of vortex-motion shot noise offer different models for the slopes \( \Gamma \) of linear \( \Sigma_V(V) \) plots, as we discuss later, but they agree in predicting a frequency-independent \( \Gamma \) up to a frequency \( f_c \sim v_0/W = V/BLW \). Because the wire width \( W \) is small, in the present case the (calculated) \( f_c \) is large, more than 500 kHz for all the measured points, except for a few ones very close to \( V = 0 \). We measured \( \Sigma_V(f) \) at a characteristic point \( (V = 65 \mu V) \) of a linear \( \Sigma_V(V) \) curve, and found that \( \Sigma_V(f) \) is essentially flat between 20 kHz and 250 kHz, as shown in the lower inset to Fig.2. This result meets the above-mentioned expectation for shot noise.

In the first work on vortex shot noise the factor \( \Gamma \) was related to the "charge" of a vortex bundle, i.e. \( \Gamma = 2N\phi_0 \). The low level of noise found in this study for Corbino disc geometry implied that the noise in the samples of bar geometry was produced essentially at their edges. This finding can be understood in terms of the surface barriers (of Bean-Livingston type) for a vortex entering and leaving a sample. In short, a de-
Pinned vortex bundle "shoots" across a bar-geometry sample, interacts only weakly with the pinning centres and the rest of (pinned or slowly moving) vortices, and the noise is created by the bundle overcoming the barriers at the entry and exit from the sample. In this case \( \Gamma \) does not depend on sample width. However, in later studies it was found that if vortex bundles travel a distance \( x < W \) before their motion is interrupted by the pinning centres, \( \Gamma \) should be inversely proportional to \( W \). The reason for this can be inferred from the Josephson relation \( V = \phi_0 d_i / d_{\text{eff}} / 2\pi \), where \( \phi \) is the phase of the superconducting order parameter. A moving vortex causes the phase shift of \( 2\pi \) only if it moves over the whole distance \( W \). If the actual distance \( x \) is shorter than \( W \), the phase change associated with one voltage pulse is a factor \( x / W \) less than \( 2\pi \), and the consequence is \( \Gamma = 2\phi_0 N x / W \). Note that in this case the noise is produced in the bulk, i.e. at the pinning centres. The reduction factor \( x / W \) explains the result of Ref.17 that the noise produced in the bulk (by slow vortices moving over small distances, or by local bundle-velocity fluctuations) was much smaller than that due to the "shooting" bundles overcoming the surface barriers. A more complicated expression for \( \Gamma \) was derived in Ref.8, where it was found that if there is a distribution of the strengths and positions of pinning centres the above expression becomes \( \Gamma = 2\phi_0 \langle N^2 \rangle / \langle x^2 \rangle / \langle N \rangle \langle x \rangle W \), where the brackets denote averages over the distribution function.

In the upper inset to Fig.2 we plot \( \Gamma(B) \) for sample S5 and \( \Gamma(B) / 8 \) for sample S1. Over the whole field range where we found well defined linear \( \Sigma_V(V) \) curves the slopes \( \Gamma(B) \) for both samples decrease with increasing field in the same manner, and \( \Gamma \) for sample S1 is approximately eight times larger. If we take into account slightly different experimental conditions for the two samples this is in fair agreement with \( \Gamma \propto 1/W \). At magnetic fields lower than \( \sim 0.20 \) T the resistances of the samples, the measured voltage and the corresponding voltage noise are small, which leads to a larger error in \( \Gamma \).

We now address the question of whether the noise is produced by the pinning or by the surface barriers. The surface barriers are important at applied magnetic fields of the order of, or lower than, the thermodynamic critical field \( H_c = B_{c2}/\mu_0 k\sqrt{T} \). In our case, \( \mu_0 H_c \sim 11 \) mT is much lower than the fields at which we found the noise of a measurable magnitude. In addition, the approximate scaling of \( \Gamma \) with sample width suggests that the bulk pinning, and not the sample edges, dominates the noise. In turn, measurements of the width dependence of \( \Gamma \) may be an alternative to other experiments for determining whether the surface barriers influence the measured transport properties.

The fact that our measurements allow us to exclude the surface barriers as the main origin of the noise in our samples also sheds more light on the nature of the \( B_{\text{irr}} \) and the meaning of the potential \( U \) of HVM. It is known that for some samples (e.g. single crystals of the Bi_2Sr_2CaCu_2O_{8+\delta}) high-Tc superconductor) surface barriers may have considerable effect on both the irreversibility field \( B_{\text{irr}} \) and the thermally activated transport. This is not the case in the present situation, the \( B_{\text{irr}} \) can be attributed to a transition to a depinned vortex state and the \( U \) is related to bulk pinning, as we have anticipated in Section III.

We attribute the decrease of \( \Gamma \) with increasing \( B \) to the weakening of pinning as \( B \) approaches \( B_{\text{irr}} \), since for \( B > B_{\text{irr}} \) we found no linear \( \Sigma_V(V) \) curves and, moreover, the overall noise magnitude decreases as \( B \) increases towards \( B_{\text{irr}} \). The decrease of \( \Gamma(B) \) for \( B \) well below \( B_{\text{irr}} \) could be explained within the framework of the models of Refs.1,17 if the unknown parameters \( x, N \), and, respectively, \( \langle x \rangle, \langle x^2 \rangle, \langle N \rangle \) and \( \langle N^2 \rangle \) depend on magnetic field in the right way. Since \( \Gamma \) comprises these parameters as products and ratios (see above), they cannot be extracted independently from our data. However, both models break down in the limit \( B \to B_{\text{irr}} \). This can be understood as follows. The effects of pinning are (1) formation of vortex bundles in order to increase the driving force and thus facilitate their motion against the pinning potential, (2) reduction of the hopping distance below the sample width \( W \). The pinning force vanishes at \( B_{\text{irr}} \), implying \( \langle N^2 \rangle^{1/2} \to \langle N \rangle \to 1 \) and \( \langle x^2 \rangle^{1/2} \to \langle x \rangle \to W \), i.e. \( \Gamma \to 2\phi_0 \), which is in contrast to the experimental observation shown in Fig.2.

A reason for this breakdown of the classical models can possibly be inferred from the comparison of the transports of (normal) electrons and vortices close to the limit of perfect transmission. Our experimental realisation - where vortices are created at the entry into a sample and...
vanish at the exit, is equivalent to a two-terminal mesoscopic conductor - where electrons have their source and drain in the reservoirs. Whenever the transmission coefficient $\Theta$ for electron transport through such a mesoscopic conductor is close to unity, shot noise is suppressed by a factor $(1 - \Theta)^2$. In the ballistic limit ($\Theta = 1$) there is no noise associated with electron transport. If vortices are not slowed down by bulk pinning and/or surface barriers, their motion is determined by the viscous drag only. This situation represents perfect vortex motion, conceptually similar to ballistic transport of electrons. Therefore, if there are no dynamic effects present (see Section IV B), in the limit of perfect vortex transmission across a sample the noise should vanish. A more quantitative treatment of vortex motion shot noise at high transmittance requires further research.

### B. Noise in the regime of linear $V(I)$

Above $B_{irr} \sim 0.65$ T, where the vortex density is large and $V \approx R_{FF}I$ for all our noise measurements, no noise described in Section IV A was found. Instead, as we show in Fig.3a, $\Sigma_V$ is a monotonic function of $V$, increasing as $V^2$, and as a function of magnetic field it decreases as $B$ approaches $B_{c2}$. Moreover, as shown in Fig.3b, there is a scaling $\Sigma_V \propto (1 - B/B_{c2})^2 V^2$ which holds over $B_{irr} < B < B_{c2}$ and is insensitive to variations of $B_{c2}$ in the range $1.14 - 1.22$ T. The frequency dependence of $\Sigma_V$ in this regime is of $1/f$ type, more precisely $1/f^\alpha$ with $\alpha = 1.5 \pm 0.1$ (Fig.3b, upper inset). In the normal state, above $B_{c2}$, $\Sigma_V = 0$ and $S_V$ is simply the voltage-independent Nyquist noise.

The existence of any noise in the regime where the vortices are most likely to be completely depinned, as seen from the $R(B,T)$ and $V(I)$, is rather surprising, since in the pinned state the magnitude of the noise described in Section IV A is becoming progressively smaller as $B \rightarrow B_{irr}$. Furthermore, if the background pinning would still influence the noise significantly one would not expect an increase of $\Sigma_V$ with increasing $V$, because at larger driving force the role of pinning is less important. Therefore, the origin of the noise shown in Fig.3 has to be sought in dynamic properties of depinned vortices far from equilibrium, with a guideline along the LO theory of non-equilibrium phenomena in flux-flow dissipation. In addition, a possible partial or complete melting of the vortex lattice, which could occur at $B_{irr}$, should also be taken into account.

There is experimental evidence in support of our assumption that the peculiar noise observed is not related to depinning processes. In Fig.4 we show $\Sigma_V(V)$ for 0.61 T, i.e. just at the crossover from HVM to LO FF in $R(B,T)$. For low voltages, $\Sigma_V(V) \propto V$ (as indicated by the dashed line), suggesting that the vortices undergo the HVM. At $V \sim 0.8$ mV the noise starts to deviate from the linear dependence, showing in a small voltage range a tendency to decrease, typically for the vortex motion becoming more ordered with increasing driving force. However, at higher $V$ the decrease of $\Sigma_V(V)$ does not continue but instead $\Sigma_V$ approaches the same $\Sigma_V(V) \propto V^2$ behaviour as for the higher fields (the solid line in Fig.4 indicates the scaling in Fig.3b). Although the vortex motion is becoming more and more uniform the noise increases, which can hardly be explained in terms of vortex interaction with a pinning potential.

Quadratic voltage dependence and $1/f$ power spectrum are generally known to be the properties of resistance fluctuations. Hence, a possibility that our finding represents resistance fluctuations, i.e. vortex velocity fluctuations, requires attention. At a fixed $(B,T,I)$ point, two parameters influence vortex velocity and consequently FF resistance: $\rho_N$ and vortex core area $A_c$. Thus, if there are fluctuations in either $\rho_N$ or $A_c$, the

![Fig. 3: (a) Vortex motion noise $\Sigma_V(V)$ for $0.67 \leq B \leq 1.06$ T. The dashed lines are fits to $\Sigma_V \propto V^2$ dependence. (b) The curves from (a) plotted against $(1 - B/B_{c2})V$. Solid line: $\Sigma_V = \gamma (1 - B/B_{c2})^2 V^2$ with $\gamma (110 \text{ kHz}) = 2.1 \times 10^{-13} \text{ s}$. Upper inset: Frequency dependence of this noise, measured at $0.67$ T and $1.5$ mV, showing $\Sigma_V(f) \propto f^{-\alpha}$ with $\alpha = 1.5 \pm 0.1$, as indicated by the solid line.](image)
FF resistance fluctuates as well. The fact that the measured noise above $B_{c2}$ is just the Nyquist noise rules out fluctuations of $\rho_N$, leaving us with a possibility that $A_c$ fluctuates. We argue below that such fluctuations may occur if the vortex velocity is large and the vortex density high.

The non-equilibrium properties of vortex cores and the related influence on flux-flow dissipation were studied theoretically by LO [1] and in Ref. [2]. If the electric field generated in moving vortex cores is sufficiently strong, quasiparticles in the cores can gain enough energy to overcome the potential barriers at vortex edges and to escape into the surrounding superfluid. This leads to a reduction of the core size, the vortex viscosity decreases, and the vortex velocity increases, resulting in the non-linearities in $V(I)$ at large currents and finally the jump shown in the upper inset to Fig.1. At low vortex density the electron-phonon relaxation processes are sufficiently efficient to cool the hot quasiparticles to the bath temperature, as the heating occurs in the cores only and the cooling over the whole volume.

However, the situation changes at large vortex density. With increasing vortex density the cooling efficiency decreases and the quasiparticles are heated-up to an elevated temperature [13, 14]. This may cause an increase of thermal fluctuations of the quasiparticle density. As a consequence, the quasiparticle pressure on the vortex "walls" may fluctuate, which would then result in the fluctuations of $A_c$. The related fluctuations of $\nu_\phi$ are measured as voltage fluctuations.

Since the average transport properties can be for $B > B_{irr}$ consistently described by the LO theory, it is tempting to check whether the LO expression for $\sigma_{FF}$ (see Section III) allows to relate the possible core-size fluctuations and the measured fluctuations in voltage. Because the observed noise occurs where $V(I)$ is linear, the $\Sigma_V \propto V^2$ dependence can be explained by assuming fluctuations of the conductivity, i.e. $\Sigma_V \Delta f = (\delta V)^2 = (\delta \sigma_{FF})^2 / \sigma_{FF}^2 V^2$. $\Delta f$ is the frequency interval over which the noise spectrum is averaged. To relate the fluctuations $\delta \sigma_{FF}$ and $\delta A_c$ we can rewrite $\sigma_{FF}$ in terms of the vortex core area $A_c \sim \xi^2 / \phi_0 / B_{c2}$ and the intervortex distance $l_B \sim \sqrt{\phi_0 / B}$, so that $z = B / B_{c2} = A_c / l_B^2$.

Then we calculate $\delta \sigma_{FF} = (1/l_B^2) (\partial \sigma_{FF} / \partial \delta A_c) \delta A_c$ from Eq. [1] and obtain

$$\frac{\langle \delta \sigma_{FF} \rangle^2}{\sigma_{FF}^2} = \frac{G(B / B_{c2}, T / T_c)}{(\langle \delta A_c \rangle^2 / A_c^2),} \quad (2)$$

where $G(z, T / T_c) = [dg(z)/dz - g(z)z^2] / [(1 - T / T_c)^{1/2} + g(z)z^2]$. The form of $\delta A_c$ is not known a priori. However, it can be deduced by combining Eq.[2] and the experimentally observed behaviour $(\delta \sigma_{FF})^2 / \sigma_{FF}^2 = (\delta V)^2 / V^2 = \gamma (1 - B / B_{c2})^2 \Delta f$ (see Fig.3b). This results in

$$\frac{\langle \delta A_c \rangle^2}{A_c^2} = \frac{\gamma (1 - B / B_{c2})^2 \Delta f}{G(B / B_{c2}, T / T_c)}, \quad (3)$$

In Fig.5 we plot this expression against $B / B_{c2}$ in order to check whether there is any approximation that would lead to a simple picture of the fluctuations. It is seen that $\langle \delta A_c \rangle^2 / A_c^2$ can be well approximated for $B / B_{c2} \lesssim 0.92$ by a power law, i.e., $\langle \delta A_c \rangle^2 / A_c^2 \sim (B / B_{c2})^{-n}$ with $n \approx 2$. The simulations for other values of $T / T_c$ show that the power-law approximation holds well for essentially any value of $T / T_c$. The power n weakly depends on $T / T_c$ but is reasonably close to 2 in the region $0.7 < T / T_c < 0.95$.

The apparent $(B / B_{c2})^{-2}$-decrease of $\langle \delta A_c \rangle^2 / A_c^2$ has a simple visualisation: such a functional dependence corresponds to a plausible assumption that the fluctuations $\delta A_c$ of the vortex area are proportional to the space $\sim l_B^2$ available, so that $\langle \delta A_c \rangle / A_c \sim (l_B^2 / \xi^2) \sim (B / B_{c2})^{-2}$. The above modelling based on the LO conductivity hence shows that the assumption of core-size fluctuations may reproduce the measured voltage and magnetic field dependences of the voltage noise.

With the experimentally determined value of the prefactor $\gamma = 2.1 \times 10^{-13}$ s for $f = 110$ kHz and $\Delta f = 7.5$ kHz we obtain the relative fluctuation amplitude $\delta A_c / A_c$ of the order of $10^{-5}$. However, as we discuss below, the $1/f$ spectrum implies that the fluctuations are distributed over a range of relaxation times. As a consequence, the small value of $\delta A_c / A_c$ only represents the contribution of those core-size fluctuations which occur in this frequency window around the given frequency.

The observed $1/f$ spectrum cannot be explained if all vortex cores fluctuate in exactly the same manner. The fluctuation of the size of a vortex core is assumed to be a random process with a characteristic time $\tau$. If $\tau$ would be the same for all cores, this would result in a Debye-Lorentzian spectrum of the fluctuations, white up to the cutoff frequency $1/\tau$. On the other hand, a distribution of $\tau$ and a superposition of Debye-Lorentzian spectra may result in a $1/f$ spectrum. Properties of the distribution
then also determine how much the fluctuations with a
given τ contribute to δAe/Δc, measured at (f, Δf). Such
a distribution may arise, for example, as a consequence of
different local correlations.

That vortex motion can strongly depend on local condi-
tions was demonstrated in Ref.23, where it was found
that, in the presence of pinning, vortices move in a
form of intermittent "rivers" between the pinned islands.
In our case one can hardly discuss a motion around the
pinned islands, since any important pinned fraction
would affect the average transport properties sig-
ificantly, which is not observed (see the discussion of
Fig.1). This however does not necessarily imply that
there are no "floating islands", i.e. vortex lattice do-
 mains moving together with the "liquid" phase. The
average flux-flow dissipation in such a (depinned) system
would be still well described by the LO theory, since the
ratio B/Bc influences the magnetoresistance much more
strongly than the exact geometry of a system of moving vortices. However, the local vortex correlations could be
different for vortices deeply in the islands, in the "li-
quid", close to the island boundaries, etc., which could
lead to different relaxation times for the core fluctuations.
These different relaxation times would then give a
1/f noise spectrum.

We are aware that our arguments offer only a qualita-
tive picture, and that further clarification of the above
ideas is required. However, at the moment we do not
know of any quantitative theoretical model which would
account for the observed peculiarities of vortex motion
noise above Brr, nor are we aware of any related sys-
tematic experimental work dealing with a range Brr < B < Bc
as large as ~ 50 % of Bc. Thus, we believe that the results and discussion of this Section could be
used as a possible starting point for further experimental
and theoretical work.

V. SUMMARY AND CONCLUSIONS

We have measured voltage noise in the mixed state
of micrometre-sized wires of amorphous Nb0.7Ge0.3 thin
films. The samples are well described by conventional
theories for dirty weak-coupling superconductors, have
weak pinning, relatively low irreversibility field Brr, and
the vortex structure is much simpler than in high-Tc
superconductors. These properties make the samples suit-
able for exploring the vortex motion noise in the weak-
pinning regime.

At low magnetic fields, i.e. for B < Brr, and small ap-
plied currents the voltage-current curves exhibit proper-
ties characteristic of thermally activated hopping of vor-
tices. The related noise is a linear function of voltage,
with the slopes Γ of noise vs. voltage curves inversely propor-
tional to the sample width, and is basically fre-
quency independent up to 250 kHz. This behaviour is
in agreement with the shot noise model and the assump-
tion that the noise is generated by bulk pinning and not
by surface barriers. Γ decreases with increasing B over
the whole magnetic field range of the shot-noise-like be-
avour, which does not contradict the presently available
models of vortex motion shot noise. These models how-
ever fail to explain the disappearance of the shot noise as
B → Brr. For B < Brr but at larger currents the vortex
motion becomes more uniform and the noise decreases.
The decrease and the low level of the noise is ascribed
to the ordering of vortex motion with increasing driving
force.

In a narrow range of B slightly below Brr, at low V
one still observes the above-mentioned two types of noise
but at large V the noise becomes quadratic in V. This
signifies the appearance of the dynamic effects inherent
to large vortex density, a behaviour fully developed for
B > Brr. For B > Brr the V(I) curves are linear over
the whole range of our measurements and the magnetore-
sistance agrees well with the flux-flow theory of Larkin
and Ovchinnikov. The noise in this regime is completely
different from that for B < Brr. Over the whole volt-
age range it increases quadratically with increasing volt-
age, its frequency spectrum is of 1/f type, and it scales
with (1 − B/Bc)2V2. The origin of this noise is not
entirely clear. We present a qualitative explanation in
terms of the non-equilibrium properties of moving vortex
cores which are subjected to fluctuations of their radius.

We are grateful to J. Aarts for contributing to this
work in its initial stage. Valuable discussions with K. E.
Nagaev, V. B. Geshkenbein, G. B. Lesovik, G. Blatter, H. v.
Löhneysen, F. Nori, E. H. Brandt and J. R. Cooper are greatly acknowledged. This work was
supported by the Swiss National Science Foundation.

* Corresponding author. Present address: Department of
Physics, Faculty of Science, University of Zagreb, Croa-
tia; Electronic address: dbabic@phy.hr
† Present address: Institute for Experimental and Applied Physics, University of Regensburg, Germany

1 D. J. van Ooijen, and G. J. van Gurp, Phys. Lett. 17, 239 (1965); G. J. van Gurp, Phys. Rev. 166, 436 (1968).

2 For an overview see: J. R. Clem, Phys. Rep. 75, 1 (1981). This paper also contains details of theoretical approach to vortex motion noise.

3 P. J. M. Wöltgens, C. Dekker, S. W. A. Gielkens and H. W. de Wijn, Physica C 247, 67 (1995).

4 G. D’Anna, P. L. Gammel, H. Safar, G. B. Alers and D. J. Bishop, J. Giapintzakis and D. M. Ginsberg, Phys. Rev. Lett. 75, 3521 (1995).

5 D. H. Kim, K. E. Gray, N. Jukam, D. J. Miller, Y. H. Kim, J. M. Lee, J. H. Park and T. S. Hahn, Phys. Rev. B 60, 3551 (1999).

6 C. P. Bean and J. D. Livingston, Phys. Rev. Lett. 12, 14 (1964).

7 E. Zeldov, A. I. Larkin, V. B. Geshkenbein, M. Konczykowski, D. Majer, B. Khaykovich, V. M. Vinokur and H. Shtrikman, Phys. Rev. Lett. 73, 1428 (1994).

8 D. T. Fuchs, R. A. Doyle, E. Zeldov, S. F. W. R. Rycroft, T. Tamegai, S. Ooi, M. L. Rappaport and Y. Myasoedov, Phys. Rev. Lett. 81, 3944 (1998); S. F. W. R. Rycroft, R. A. Doyle, D. T. Fuchs, E. Zeldov, R. J. Drost, F. H. Kes, T. Tamegai, S. Ooi and D. T. Foord, Phys. Rev. B 60, R757 (1999).

9 J. M. E. Geers, C. Attanasio, M. B. S. Hesselberth, J. Aarts and P. H. Kes, Phys. Rev. B 63, 094511 (2001).

10 M. H. Theunissen and P. H. Kes, Phys. Rev. B 55, 15183 (1997); For a comprehensive study see: M. H. Theunissen, PhD. Thesis, University of Leiden, 1997.

11 F. Habbal and W. C. H. Joiner, J. Low Temp. Phys. 28, 83 (1977); J. D. Thompson and W. C. H. Joiner, Phys. Rev. B 20, 91 (1979).

12 P. H. Kes and C. C. Tsuei, Phys. Rev. B 28, 5126 (1983).

13 D. Babić, J. R. Cooper, J. W. Hodby and Chen Changkang, Phys. Rev. B 60, 698 (1999).

14 M. Henny, S. Oberholzer, C. Strunk and C. Schönenberger, Phys. Rev. B 59, 2871 (1999).

15 A. I. Larkin and Yu. N. Ovchinnikov, in Nonequilibrium Superconductivity, edited by D. N. Lengenberg and A. I. Larkin (North Holland, Amsterdam, 1986).

16 F. Lefloch, C. Hoffmann and O. Demolliens, Physica C 319, 258 (1999).

17 C. M. Knoedler and R. F. Voss, Phys. Rev. B 26, 449 (1982).

18 E. Zeldov, D. Majer, M. Konczykowski, A. I. Larkin, V. M. Vinokur, V. B. Geshkenbein, N. Chikumoto and H. Shtrikman, Europhys. Lett. 30, 367 (1995).

19 G. B. Lesovik, Pis’ma Zh. Eksp. Teor. Fiz. 49, 515 (1989) [JETP Lett. 49, 594 (1989)]; M. Büttiker, Phys. Rev. Lett. 65, 2901 (1990).

20 For an overview of the various aspects of $1/f$ noise and resistance fluctuations see e.g. M. B. Weissman, Rev. Mod. Phys. 60, 537 (1988).

21 A. I. Bezuglyj and V. A. Shklovskij, Physica C 202, 234 (1992).

22 From the LO theory it follows that out of equilibrium the vortex viscosity is velocity dependent and given by $\eta(v_\phi) = \eta_0 [1 + (v_\phi/v^*_{\phi})^2]^{-1}$, where $\eta_0$ is the viscosity for small vortex velocities and $v^*_{\phi}$ a characteristic vortex velocity. Also, $A_e(v_\phi) = A_e(0) [1 + (v_\phi/v^*_{\phi})^2]^{-1}$, which leads to $\eta(v_\phi) = \eta_0 A_e(v_\phi)/A_e(0)$. Therefore, one should not confuse the vortex-size dependences of $\eta_0$ and $\eta(v_\phi)$: while $\eta_0$ is larger for smaller $A_e(0)$, $\eta(v_\phi)$ is smaller for smaller $A_e(v_\phi)$.

23 T. Matsuda, K. Harada, H. Kasai, O. Kanamura and A. Tomonura, Science 271, 1393 (1996).