Exotic Fermions\(^1\)

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1 Introduction

As has often been said, the standard model, although extremely successful in explaining virtually all known experimental data, cannot be the whole story – there are too many questions still left unanswered. By themselves, exotic fermions do not solve any of these problems, but they often appear in models which do address some of these remaining questions. In this chapter I will discuss the constraints which can be put upon exotic fermions, particularly as regards their mixings with the ordinary fermions.

In the standard model, all left-handed ($L$) fermions transform as doublets under weak $SU(2)_W$, while all right-handed ($R$) fermions are singlets:

\[
\begin{pmatrix}
& \nu_e \\
\nu_e & \\
\mu^- & \\
\tau^- & \\
u^+ & \\
u^+ & \\
u^+ & \\
u^+ &
\end{pmatrix}_L
\begin{pmatrix}
& \mu^- \\
\mu^- & \\
\tau^- & \\
u^+ & \\
u^+ & \\
u^+ & \\
u^+ &
\end{pmatrix}_L
\begin{pmatrix}
& u \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e &
\end{pmatrix}_L
\begin{pmatrix}
& d \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e &
\end{pmatrix}_L
\begin{pmatrix}
& s \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e &
\end{pmatrix}_L
\begin{pmatrix}
& t \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e & \\
u_e &
\end{pmatrix}_L,
\]

Many models which go beyond the standard model predict the existence of new fermions which transform in a non-standard way under $SU(2)_W$. In $E_6$ models, for example, in the 27-plet one finds, in addition to the ordinary particles, vector singlet quarks and vector doublet leptons. Vector singlet (doublet) fermions refer to particles whose $L$ and $R$ components both transform as singlets (doublets) under $SU(2)_W$. One also finds new $SU(2)_W$-singlet Weyl neutrinos in the 27-plet. Mirror fermions are another type of exotic fermion, whose transformation properties under $SU(2)_W$ are opposite those of ordinary fermions, i.e. left-handed singlets and right-handed doublets. These appear, for instance, in grand unified theories which include family unification [1].

The possibilities for new fermions are listed in Table 1. In the following analysis, all particles whose $L$ and $R$ components obey the same transformation properties as those in Eqs. 1 and 2 (i.e. $L$-handed doublets, $R$-handed singlets) will be called ordinary. These include the standard fermions, as well as any new sequential (e.g. fourth family) fermions. Those particles whose $L$ and/or $R$ components transform differently than those of ordinary fermions are called exotic. Note that particles with noncanonical electric or colour charges are not considered here. This restricts the ‘exotic’ label to mirror fermions, vector doublets and singlets, and Weyl neutrinos.

There are two ways to look for signals of exotic fermions – directly and indirectly. The best limits on direct production of such particles come from LEP [2],

\[
M_N, M_{E-}, M_U, M_D > 45 \text{ GeV},
\]

although the bound on $M_N$ depends on the type of exotic neutrino. For example, the mass limit on exotic singlets can be considerably weaker. As to indirect signals, one possibility is to look for loop-induced effects in rare processes. This is a model-dependent enterprise, depending on the mass of the exotic fermions, their couplings to ordinary gauge bosons, the possible existence of other gauge bosons, etc., and will not be discussed here.
Sequential Fermions
\[
\left(\begin{array}{c}
N_e^- \\
E_R
\end{array}\right) \quad \left(\begin{array}{c}
U_L \\
D_R
\end{array}\right)
\]

Non-Canonical \(SU(2)_W \times U(1)\) Assignments

| a) Mirror Fermions | b) Vector Doublets | c) Vector Singlets | d) Weyl Neutrinos |
|--------------------|-------------------|-------------------|------------------|
| \(E_L^- \quad \left(\begin{array}{c}
N_e^- \\
E_R
\end{array}\right) \quad U_L \quad \left(\begin{array}{c}
U \\
D_R
\end{array}\right)\) | \(\left(\begin{array}{c}
N_e^- \\
E_R
\end{array}\right) \quad \left(\begin{array}{c}
U \\
D_R
\end{array}\right)\) | \(E_L^- \quad E_R^- \quad U_L \quad U_R \quad D_L \quad D_R\) | \(N_L \quad N_R\) |

Table 1: Possible \(SU(2)_W \times U(1)\) assignments for new fermions. Pairs of particles enclosed in parentheses indicate \(SU(2)_W\)-doublets; otherwise they are \(SU(2)_W\)-singlets. \(N\) and \(E\) refer to leptons of charge 0 and \(-1\), respectively; \(U\) and \(D\) are quarks of charge \(2/3\) and \(-1/3\).

The other indirect signal, which is the focus of this chapter, is to look for signs of exotic fermions through their mixings with ordinary fermions. These mixings can be analysed in a model independent way. In general, mixing between ordinary and exotic fermions will induce flavour-changing neutral currents (FCNC). The experimental absence of FCNC places extremely stringent limits on fermion mixing. However, there are directions in parameter space where it is possible to fine-tune away FCNC. Nevertheless, even these regions can be constrained by looking at data involving charged currents and flavour-conserving neutral currents. I will review here the constraints which current experimental data place upon the mixings of ordinary and exotic fermions. The material contained in this chapter comes principally from work by Langacker and London (Ref. [3]), and Nardi, Roulet and Tommasini (Ref. [4]). Where there are holes in the exposition, I refer the reader to these two articles for details.

This chapter is organized as follows. In section 2, I introduce the formalism needed to describe the mixing between ordinary and exotic fermions. For charged fermions, in order to avoid FCNC, it is necessary to consider fine-tuned directions in parameter space in which each ordinary charged fermion mixes with its own exotic fermion. In this way mixing is parametrized by one angle per ordinary \((L-\) or \(R\)-handed) charged fermion. Neutrinos are more complicated, both due to the possibility of Dirac and Majorana masses, and because there is no empirical evidence requiring the absence of FCNC between neutrino species. However, due to the fact that neutrinos are unobserved in experiment, it is possible to parametrize mixing by one effective angle, plus one auxiliary parameter, per neutrino species. In section 3, I review the experimental data which is used to constrain mixing. Here I will include the theoretical expressions, including mixing, which are to be fitted to the experimental results. The fact that certain results are normalized to other data must be
carefully taken into account to ensure that all mixing effects are included. The fits are given in section 4. There are enough constraints to limit all mixings of $L$- and $R$-handed ordinary fermions. Two types of fits are presented. In the first, only one particle at a time is allowed to mix. This yields the most stringent limits on fermion mixing. In the second fit, all particles can mix simultaneously, which weakens the constraints due to the possibility of fine-tuned cancellations of the effects of different particle mixings. I conclude in section 5.

2 Mixing Formalism

In this section, I present the formalism for describing the mixing of ordinary and exotic fermions. As mentioned in the introduction, charged fermions and neutrinos must be treated separately. The material in this section is taken almost completely from Ref. [3].

2.1 Charged Fermions

Since electromagnetic gauge invariance is unbroken, fermions with different charges cannot mix. Charged fermions can therefore be divided into three categories – those with $Q_{em} = 2/3$ ($u$-type), $Q_{em} = -1/3$ ($d$-type) and $Q_{em} = -1$ ($e$-type). For each of these types it is convenient to put the $L$ and $R$ gauge eigenstates of both ordinary and exotic fermions into a single vector,

$$
\psi^0_{L(R)} = \begin{pmatrix}
\psi^0_O \\
\psi^0_E
\end{pmatrix}_L(R),
$$

(4)

in which the subscripts $O$ and $E$ stand for ordinary and exotic, respectively. Here and below, the superscript 0 indicates the weak-interaction basis; the mass basis is denoted by the absence of superscripts. In the above equation there are $n_L$ ($n_R$) ordinary $L$-handed ($R$-handed) fields and $m_L$ ($m_R$) exotic $L$-handed ($R$-handed) fields.

The light ($l$) and heavy ($h$) mass eigenstates can be written similarly,

$$
\psi_{L(R)} = \begin{pmatrix}
\psi^l \\
\psi^h
\end{pmatrix}_L(R).
$$

(5)

The dimensionality of these vectors is as above, that is, there are $n_L$ ($n_R$) light $L$ ($R$) states and $m_L$ ($m_R$) heavy $L$ ($R$) states. Of course, the labels ‘light’ and ‘heavy’ should not be taken literally – for example, fourth generation particles (if they exist) are known to be heavy, yet they are included among the ‘light’ particles. This decomposition is useful as a reminder that in general we expect the light states to consist mainly of ordinary particles and the heavy states to be principally exotic.

The weak and mass eigenstates are related by a unitary transformation

$$
\psi^0_a = U_a \psi^0_a,
$$

(6)

in which $a = L, R$. It is useful to write the matrix $U$ in block form as

$$
U_a = \begin{pmatrix}
A_a & E_a \\
F_a & G_a
\end{pmatrix}.
$$

(7)
Since all our experimental data concerns only the light eigenstates, the important elements of \( U_a \) are the \( n_a \times n_a \) submatrix \( A_a \), which relates the light mass states and the ordinary weak states, and \( F_a \), which is \( m_a \times n_a \) and describes the overlap of the light eigenstates with the exotic fermions. The unitarity of \( U_a \) requires

\[
A_a^\dagger A_a + F_a^\dagger F_a = A_a A_a^\dagger + E_a E_a^\dagger = I,
\]

which shows that \( A_a \) is not by itself unitary. However, since we expect the light (heavy) particles to be mainly ordinary (exotic), we see that the deviation of \( A_a \) from unitarity is of second order in the mixing.

Let us now examine the effects of mixing on the neutral currents of the light fermions. In the weak basis, the coupling of the \( Z^0 \) to charged fermions can be written

\[
\frac{1}{2} J_\mu^Z = \overline{\psi}_O \gamma^\mu \psi^0_L + \overline{\psi}_E \gamma^\mu \psi^0_E - \sin^2 \theta_W J_{em}^\mu,
\]

in which \( I_{3W} = +1/2 \) for \( u \)-type fermions, and \( I_{3W} = -1/2 \) for \( d \)- and \( e \)-type fermions. Using Eqs. 6 and 7, the weak neutral current can now be expressed in terms of mass eigenstates. Keeping only those terms which involve just the light states, this gives

\[
\frac{1}{2} J_\mu^Z = \overline{\psi}_i L \gamma^\mu I_{3W} A_{L}^\dagger A_L \psi_i L + \overline{\psi}_i R \gamma^\mu I_{3W} F_{R}^\dagger F_R \psi_i R - \overline{\psi}_i \gamma^\mu Q_{em} \sin^2 \theta_W \psi_i.
\]

The important point to recognize here is that, since neither \( A_L \) nor \( F_R \) is unitary (Eq. 8), \( A_{L}^\dagger A_L \) and \( F_{R}^\dagger F_R \) are not necessarily diagonal. In other words, FCNC will in general be induced among the light particles.

It is useful to parametrize the FCNC between the light particles \( i \) and \( j \) as

\[
\lambda_{ij}^L = (A_L^\dagger A_L)_{ij} = - (F_L^\dagger F_L)_{ij}, \quad \lambda_{ij}^R = (F_R^\dagger F_R)_{ij}, \quad i \neq j.
\]

Note that these are of second order in light-heavy mixing. As can be seen from Table 2, the constraints on the \( \lambda_{ij}^{L,R} \), \( i \neq j \) are quite stringent, which strongly limits the mixing of ordinary and exotic fermions. However, it is possible to evade these bounds by considering the fine-tuned cases in which both \( A_L^\dagger A_L \) and \( F_R^\dagger F_R \) are diagonal. These correspond to those directions in mixing parameter space in which each ordinary fermion mixes with its own exotic fermion. For the rest of this chapter, I will assume \( \lambda_{ij}^{L,R} = 0 \) for \( i \neq j \).

With this (strong) assumption, using Eq. 8 one can write

\[
(A_a^\dagger A_a)_{ij} = (e_a^i)^2 \delta_{ij}, \quad (F_a^\dagger F_a)_{ij} = (s_a^i)^2 \delta_{ij}, \quad a = L, R,
\]

in which \( (s_a^i)^2 \equiv 1 - (e_a^i)^2 \equiv \sin^2 \theta_a^i \), where \( \theta_{L,R}^i \) is the mixing angle of the \( i \)th \( L \)-handed (\( R \)-handed) ordinary fermion and its exotic partner. With this notation, the neutral current in Eq. 10 becomes

\[
\frac{1}{2} J_\mu^Z = \sum_i \left[ \overline{\psi}_i L \gamma^\mu \bar{\epsilon}_L(i) \psi_i L + \overline{\psi}_i R \gamma^\mu \bar{\epsilon}_R(i) \psi_i R \right],
\]
Table 2: Limits on the flavour changing neutral current parameter $\lambda_{ij}$ (Eq. 11). The bounds on leptonic FCNC are taken or adapted from Ref. [6], while the limits on hadronic FCNC are taken, updated or adapted from Ref. [7]. There is no bound on $|\lambda_{bd}|$ from $B_d \to \bar{B}_d$ mixing because this mixing can in principle be explained by a nonzero $\lambda_{bd}$ [7].

| Quantity | Upper Limit | Source |
|-----------|-------------|--------|
| $|\lambda_{\mu e}|$ | $1 \times 10^{-6}$ | $\mu \not\to 3e$ [3] |
| $|\lambda_{\mu \tau}|, |\lambda_{e \tau}|$ | $7 \times 10^{-3}$ | $\tau \not\to 3\ell$ [2] |
| $|\lambda_{ds}|$ | $6 \times 10^{-4}$ | $\Delta m_{KL,KS} [2]$ |
| $|\lambda_{cu}|$ | $1 \times 10^{-3}$ | $K_L \to \mu^+\mu^-$ [2] |
| $|\lambda_{bd}|, |\lambda_{bs}|$ | $2 \times 10^{-3}$ | $D^0-\bar{D}^0$ mixing [2] |

where the sum is over the light particles and

$$\tilde{\epsilon}_L(i) = I_{3W}^i \left(c_L^i\right)^2 - Q_{em}^i \sin^2 \theta_W ,$$

$$\tilde{\epsilon}_R(i) = I_{3W}^i \left(s_R^i\right)^2 - Q_{em}^i \sin^2 \theta_W.$$  \hspace{1cm} (14)

From Eqs. 13 and 14, the effects of mixing are clear. First, the mixing of ordinary $L$ doublets with exotic $L$ singlets results in a nonuniversal reduction ($(c_L^i)^2$) of the isospin current. Second, mixing in the $R$-handed sector induces a $R$-handed current ($(s_R^i)^2$). The electromagnetic current is unchanged, reflecting the simple fact that only particles of the same charge can mix. In the presence of mixing, the vector and axial couplings for fermion $i$ are

$$v_i \equiv \tilde{\epsilon}_L(i) + \tilde{\epsilon}_R(i) = I_{3W}^i \left[\left(c_L^i\right)^2 + \left(s_R^i\right)^2\right] - 2Q_{em}^i \sin^2 \theta_W ,$$

$$a_i \equiv \tilde{\epsilon}_L(i) - \tilde{\epsilon}_R(i) = I_{3W}^i \left[\left(c_L^i\right)^2 - \left(s_R^i\right)^2\right].$$  \hspace{1cm} (15)

The hadronic charged current involving the light quarks is

$$\frac{1}{2} J_{W}^{\mu} = \bar{\psi}_{uL} \gamma^\mu V_L \psi_{dL} + \bar{\psi}_{uR} \gamma^\mu V_R \psi_{dR},$$  \hspace{1cm} (16)

in which $\psi_{uL}$ and $\psi_{dL}$ are column vectors of the light $L$ $u$-type and $d$-type quarks, respectively. Recall that ‘light’ $L$-handed particles include possible extra sequential or vector doublet quarks. Thus, the first 3 components of $\psi_{uL}$ and $\psi_{dL}$ are the standard quarks, while the remaining $n_l - 3$ quarks are nonstandard. The column vectors $\psi_{uR}$ and $\psi_{dR}$ are defined completely analogously. In Eq. 16, $V_L = A_L^{u}A_L^{d}$ is the generalized Cabibbo-Kobayashi-Maskawa (CKM) matrix. The point to observe here, however, is that $V_L$ is non-unitary in the presence of mixing between the ordinary and exotic fermions. It can be decomposed as

$$V_{Lij} = c_{L}^{ui} c_{L}^{dj} \tilde{V}_{Lij},$$  \hspace{1cm} (17)
where $\hat{V}_L$ is the true (unitary) CKM matrix. Here and below I use the term ‘true’ to refer to a quantity in the absence of mixing, and I denote this by a symbol with a caret. ‘Apparent’ quantities, which are represented by symbols with no caret, are those which are actually measured. In Eq. 17 we see that apparent CKM matrix elements are reduced from their true values by the nonuniversal factor $c_u^i c_d^j$. If $\psi_uL$ and/or $\psi_dL$ contain nonstandard ‘light’ quarks, this will manifest itself through the apparent nonunitarity of $\hat{V}_L$. The second term in Eq. 16 is a $R$-handed charged current, induced when both $R$-handed $u_i$ and $d_j$ quarks mix with exotic $SU(2)_W$-doublets. Like $V_L$, the apparent $R$-handed CKM matrix $V_R$ is non-unitary, but can be written

$$V_{Rij} = s_R^{u_i} s_R^{d_j} \hat{V}_{Rij},$$

(18)

where $\hat{V}_R$ is unitary.

### 2.2 Neutrinos

As mentioned in the introduction, neutrinos must be treated separately for several reasons. First of all, there are three types of $L$-handed neutrino weak eigenstates:

$$\begin{pmatrix} n^0_{OL} \\ e^0_L \\ n^0_{EL} \end{pmatrix}, \quad \begin{pmatrix} e^0_R \\ n^0_{EL} \end{pmatrix}, \quad n^0_{SL}.$$

(19)

Here, the $n^0_{OL}$ are ordinary $SU(2)_W$-doublets with $I_{3W} = 1/2$, the $n^0_{EL}$ are exotic $SU(2)_W$-doublets with $I_{3W} = -1/2$, and the $n^0_{SL}$ are exotic $SU(2)_W$-singlets. Note that the $n^0_{EL}$ are usually referred to as antineutrinos. However, Majorana masses are possible for neutrinos, in which case there is no real distinction between particle and antiparticle. This is the second difference between neutrinos and charged particles. In the general Majorana case all three types of $\nu$ can mix. Finally, there are no experimental constraints on FCNC involving neutrinos. Despite these differences, mixing between ordinary and exotic neutrinos can be analyzed using a formalism similar to that introduced in Sec. 2.1.

Since in the presence of Majorana masses one does not distinguish between particle and antiparticle, in dealing with neutrinos it is convenient to denote all $L$ states as $n^c_L$ and all $R$ states as $n^c_R$. These are related by $n^c_R = C(n^T_L)$, where $C$ is the charge conjugation matrix. Thus, in analogy to the charged fermion case, all $L$-handed weak eigenstate neutrinos are put together into a vector

$$n^0_L = \begin{pmatrix} n^0_{OL} \\ n^0_{EL} \\ n^0_{SL} \end{pmatrix}.$$

(20)

As above, the neutrino mass eigenstates are divided into two classes, ‘light’ (i.e. essentially massless) and ‘heavy’:

$$n_L = \begin{pmatrix} n_{lL} \\ n_{hL} \end{pmatrix}.$$

(21)
The weak and mass bases are related by a unitary transformation \( n^0_L = U_L n_L \), in which \( U \) can be decomposed as

\[
U_L = \begin{pmatrix} A & E \\ F & G \\ H & J \end{pmatrix}_L.
\] (22)

Similarly, \( n^0_R = U_R n^\dagger_R \), with \( U_R = U_L^\dagger \). In Eq. 22, the matrices \( A_L, F_L \) and \( H_L \) describe the overlap of the massless neutrinos with ordinary doublets \( (n^0_{OL}) \), exotic doublets \( (n^0_{EL}) \), and exotic singlets \( (n^0_{SL}) \), respectively. The LEP data has constrained the number of light \( SU(2)_W \)-doublets to be 3. Thus, exotic doublet \( \nu \)'s must have a mass greater than \( M_Z/2 \). This implies that the components of \( F_L \) are small. As to \( H_L \), I will assume that the light neutrinos are mainly \( n^0_{OL} \). If they are massless or have Majorana masses, then there are no light singlets, and all components of \( H_L \) are small. If the \( n_{iL} \) have small Dirac masses, then it is necessary to include 3 light singlets in the spectrum. In this case, the components of \( H_L \) corresponding to these singlets may be large, but the remaining components must be small. As far as the formalism is concerned, there is little difference between these two possibilities.

Dropping the subscript \( l \), the weak neutral current for the light neutrino states can now be written

\[
\frac{1}{2} J^\mu_Z = \frac{1}{2} \pi_L \gamma^\mu \left( A^\dagger_L A_L - F^\dagger_L F_L \right) n_L.
\] (23)

The \( A^\dagger_L A_L \) and \( F^\dagger_L F_L \) terms come from the neutral currents of the \( n^0_{OL} \) and \( n^0_{EL} \), respectively. As in Sec. 2.2, neither \( A_L \) nor \( F_L \) is unitary. On the other hand, unlike the charged fermion case, there is no experimental evidence to suggest that \( A^\dagger_L A_L \) and \( F^\dagger_L F_L \) are diagonal. However, as we will see, essentially the same effect is produced when one sums over the unobserved final state \( \nu \)'s in weak processes.

The leptonic charged current is

\[
\frac{1}{2} J^\mu_W = \pi_L \gamma^\mu A^\dagger_L c_L e_L + \pi_R \gamma^\mu F^\dagger_R s_R e_R
\]

\[
= \sum_{ia} \left[ \pi_L \gamma^\mu \left( A^\dagger_L \right)_{ia} c_L e_{aL} + \pi_R \gamma^\mu \left( F^\dagger_R \right)_{ia} s_R e_{aR} \right].
\] (24)

Note that since \( F^\dagger_R = F^\dagger_R \) and the second term in Eq. 24, which is the induced right-handed current, is of second order in light-heavy mixing. This term is produced when both the light neutrino and charged lepton mix with a member of an exotic doublet. The left-handed charged current is reduced in strength by the factor \( \left( A^\dagger_L \right)_{ia} c_L^a \) due to ordinary-exotic mixing.

We can now see the effect of summing over the final state \( \nu \)'s in a weak process. In the presence of mixing, the rate for the charged current transition \( e_a \rightarrow n_i \) relative to its value \( (\Gamma_0) \) in the absence of mixing is

\[
\frac{1}{\Gamma_0} \Gamma(e_a \rightarrow n_i) = (c_L^a)^2 (A^\dagger_L)_{ai} \left( A_L^\dagger \right)_{ia} + (s_R^a)^2 (F^\dagger_R)_{ai} \left( F_R^\dagger \right)_{ia}.
\] (25)

However, since the final \( \nu \)'s are unobserved, we must sum over them. The effect of this is to reduce the many parameters describing neutrino mixing to a single mixing angle per
neutrino flavour:

\[
\frac{1}{\Gamma_0} \sum_i \Gamma(e_a \rightarrow n_i) = (c_L^{e_a})^2 (c_L^{n_i})^2 + (s_R^{e_a})^2 (s_R^{n_i})^2 ,
\]

(26)

where the effective neutrino mixing angles \((c_L^{n_i})^2 = (A_L^{n_i} A_L^{n_i})_{aa}\) and \((s_R^{n_i})^2 = (F_R^{n_i} F_R^{n_i})_{aa}\) have been introduced. The second term in Eq. (26), which comes from the induced right-handed charged current, is of \(O(s^4)\). From now on we will be working to second order in light-heavy mixing, so that this term can be dropped.

The final state neutrino produced in Eq. (26) is

\[
|n_{aL}\rangle \equiv \sum_i \frac{(A_L^{n_i})_{ia}}{(c_L^{n_i})} |n_{iL}\rangle ,
\]

(27)

so that the cross section for scattering into the "right" charged lepton \((e_{aL})\) is

\[
\frac{1}{\sigma_0} \sigma(n_{aL} \rightarrow e_{aL}) = (c_L^{e_a})^2 (c_L^{n_i})^2 .
\]

(28)

(There is also the possibility of scattering into the "wrong" lepton (Ref. [8]), but this will not be discussed here.) One can also calculate the neutral current cross section for the rescattering of the neutrino in Eq. (27). Summing again over the final unobserved neutrinos, this can be found from Eqs. (23) and (27) to give

\[
\frac{1}{\sigma_0} \sum_i \sigma(n_{aL} \rightarrow n_{iL}) = 1 - \Lambda_a (s_L^{e_a})^2 ,
\]

(29)

where, in the second line, I have used the unitarity of \(U_L\) (Eq. (22) and the fact that the components of \(F_L\) and \(H_L\) are all of \(O(s)\)). Thus it is evident that this cross section depends not only on the mixing angle, but also on the type of neutrino(s) with which the ordinary neutrino mixes. Eq. (29) simplifies even further when one realizes that, for \(a\epsilon 1, 2, \ldots, p\) (\(p\) is the number of light \(\nu\)'s), \(A_L^{\nu}\) differs from the identity by terms of \(O(s)\). In this case we obtain

\[
\frac{1}{\sigma_0} \sum_i \sigma(n_{aL} \rightarrow n_{iL}) = 1 - \Lambda_a (s_L^{e_a})^2 ,
\]

(30)

where the parameter \(\Lambda_a\) is defined to be \(\Lambda_a = 4\lambda_F^a + 2\lambda_H^a\), with \((F_L^{\nu} F_L^{\nu})_{aa} \equiv \lambda_F^a (s_L^{\nu_a})^2\) and \((H_L^{\nu} H_L^{\nu})_{aa} \equiv \lambda_H^a (s_L^{\nu_a})^2\). The \(\lambda\)'s are constrained to lie between 0 and 1, so that \(\Lambda_a\) takes values between 0 and 4, depending on the mixing involved.

Finally, using Eq. (23) and the same approximations as above, it is possible to calculate the rate for the decay of the \(Z^0\) into undetected neutrinos. Assuming the existence of 3 light\(^{21}\) have assumed that the light \(\nu\)'s are either massless or Majorana; the case of light Dirac \(\nu\)'s does not change the formalism significantly.
neutrinos in the absence of mixing, and normalizing to the decay rate of the $Z^0$ into one neutrino, this gives

$$\frac{1}{\Gamma_1^{\nu}} \Gamma(Z^0 \to invisible) = 3 - \sum_a \Lambda_a (s_{L}^{\nu_a})^2.$$  \quad (31)

Having presented the formalism for the mixing of ordinary and exotic fermions, I will now turn to the experimental data which is used to constrain such mixings.

### 3 Experimental Data

In this section, I will present the experimental data which are used to constrain the mixing of ordinary and exotic fermions. I must emphasize at the outset that these results, taken from Ref. [4], are somewhat outdated, since the analysis was done in the summer of 1991. However, except for the $\nu^e_L$, the constraints obtained here would not be much improved if present data were used. I will comment further on this in Section 4.

In using the data to constrain fermion mixing, it must be remembered that mixing can cause a discrepancy between the experimental result and the theoretical expression in two ways. Not only can mixing directly affect the process being examined, but it can also appear indirectly. This can happen, for example, when the extraction of a particular result requires normalization to another piece of experimental data. Thus, in putting constraints on mixing, one must be very careful to include all mixing effects.

Most of the experimental results are precise enough that it is necessary to include radiative corrections in order that there be agreement with the standard model. In the present analysis, radiative corrections will be included, but only those due to ordinary particles without mixing. (The inclusion of mixing is a second order effect.) Radiative corrections involving exotic fermions are typically much smaller, although it must be acknowledged that in the case of exotic nondegenerate $SU(2)_W$ doublets, the corrections could be large [9].

In order to calculate radiative corrections, it is necessary to choose a set of input parameters. These are typically taken to be the electromagnetic coupling $\alpha$, measured at $q^2 = 0$, the Fermi constant $G_\mu$, and the $Z$-mass $M_Z$, fixed to be $M_Z = 91.175$ GeV [10]. The values of $\alpha$ and $M_Z$ as extracted from experiment are not affected by mixing. On the other hand, since $G_\mu$ is obtained directly from $\mu$-decay, there is an effect due to mixing. The measured value of $G_\mu = 1.16637(2) \times 10^{-5}$ GeV$^{-2}$ is related to its true value $\hat{G}_\mu$ by

$$G_\mu = \hat{G}_\mu e^{\nu_e^{\nu_e}} e^{\nu_{\mu}^{\nu_{\mu}}}$$  \quad (32)

due to the possible mixing of the leptons with exotic fermions. Since many experimental results are normalized to $\mu$-decay, indirect effects of mixing can appear in this way. Finally, it is necessary to include $t$-quark mass and the Higgs mass in the radiative corrections. These are fixed to be $m_t = 120$ GeV and $m_H = 100$ GeV, respectively.
3.1 \( M_W \)

Including radiative corrections, the theoretical expression for \( M_W \) as a function of \( \alpha, M_Z \) and \( G_\mu \) is given by \cite{11,12}

\[
M_W^2 = \frac{\rho M_Z^2}{2} \left[ 1 + \sqrt{1 - \frac{4A}{G_\mu \rho M_Z^2} \left( \frac{1}{1 - \Delta \alpha} + \Delta \rho_{\text{rem}} \right)} \right],
\]

(33)

where \( A = \pi \alpha/\sqrt{2} G_\mu \). Here, \( \rho \simeq 1 + 3G_\mu m_t^2/8\sqrt{2} \pi^2 \) contains the leading \( t \)-quark effects \cite{9}. \( 1/(1 - \Delta \alpha) \) renormalizes the QED coupling to the \( M_Z \) scale, including the large logs, and \( \Delta \rho_{\text{rem}} \) includes all remaining small corrections. The sole (indirect) dependence of \( M_W \) on fermion mixings is found in the ratio \( G_\mu/G_\mu' \) (Eq. 32).

The average value of \( M_W \) as measured by CDF and UA2 is \cite{13}

\[
M_W = 80.13 \pm 0.31 \text{ GeV},
\]

(34)

where the LEP result for \( M_Z \) has been used to convert the UA2 measurement of \( M_W/M_Z \) into a value for \( M_W \).

3.2 Charged Currents

There are a number of experiments involving charged currents which can be used to constrain fermion mixing. In the interest of brevity, I will present only those experimental results which are most important for bounding the mixing.

3.2.1 Lepton Universality

In the standard model, the coupling of the \( W \) to each of the lepton doublets \( (\nu_e \, e^-)_L \), \( (\nu_\mu \, \mu^-)_L \) and \( (\nu_\tau \, \tau^-)_L \) is universal, that is, \( g_e = g_\mu = g_\tau \). In the presence of mixing, this equality can be altered:

\[
\left( \frac{g_i}{g_e} \right)^2 = \frac{(c_{L\nu}^e)^2 (c_{L\nu}^\mu)^2 + (s_{R\nu}^e)^2 (s_{R\nu}^\mu)^2}{(c_{L\nu}^e)^2 (c_{L\nu}^\mu)^2 + (s_{R\nu}^e)^2 (s_{R\nu}^\mu)^2} \simeq 1 + (s_{L\nu}^e)^2 + (s_{L\nu}^\mu)^2 - (s_{L\nu}^e)^2 - (s_{L\nu}^\mu)^2, \quad i = \mu, \tau,
\]

(35)

where only terms of \( O(s^2) \) have been kept in the second line.

These ratios have been measured in several experiments. The most precise are

- Pion and Kaon decay:
  \[ \frac{\Gamma(\pi \to \mu\nu)}{\Gamma(\pi \to e\nu)} = \frac{\Gamma(K \to \mu\nu)}{\Gamma(K \to e\nu)}, \]
  (36)

- Tau and Muon decay:
  \[ \frac{\Gamma(\tau \to \mu\bar{\nu}\nu)}{\Gamma(\tau \to e\bar{\nu}\nu)} = \frac{\Gamma(\tau \to \mu\bar{\nu}\nu)}{\Gamma(\mu \to e\bar{\nu}\nu)}, \]
  (37)
Table 3: Experimental constraints on lepton universality (Eq. 35). There is a correlation between the data marked with a †, which has been taken into account in the fits [4].

The experimental data are shown in Table 3. These experiments constrain the left-handed mixing angles of the leptons. However, muon and tau decay have been measured accurately enough to put limits on the right-handed mixing angles of these leptons. The observables relevant to right-handed leptonic currents are all of $O(s^4)$. I will not discuss these here, but rather refer the reader to Refs. [3] and [14] for details.

### 3.2.2 Quark-Lepton Universality

In order to test quark-lepton universality, one typically compares the rate for the decay of a hadron with that of muon decay. In the standard model, in the absence of mixing, these should be equal, up to factors of CKM matrix elements. However, these matrix elements obey another constraint, namely that of the unitarity of the CKM matrix. In this sense, a test of quark-lepton universality is equivalent to a test of CKM matrix unitarity.

$V_{ud}$ is measured by comparing the rates for $\beta$-decay (vector current only) and $\mu$-decay. $V_{us}$ is obtained similarly, except that $K_{e3}$ and hyperon decay are used. In the presence of mixing, the true values of the CKM matrix elements differ from the measured values by [3]

$$V_{ui} = \frac{c_L^u c_L^i V_{Lui} + s_R^u s_R^i V_{Rui}}{c_L^u c_R^i} \quad i = d, s. \quad (38)$$

$V_{ub}$ is also related to its true value in this way, but in any case its size is too small to be of interest for this analysis.

Using the fact that $\sum_{i=1}^n |\tilde{V}_{Lui}|^2 = 1$, expanding Eq. (38) to $O(s^2)$, and defining

$$\kappa_{ij} = s_R^{ui} s_R^{d_j} \frac{\tilde{V}_{Rij}}{\tilde{V}_{Lij}}, \quad (39)$$

one obtains

$$\sum_{i=1}^3 |V_{ui}|^2 = 1 + (s_L^\mu)^2 + (s_L^\nu)^2 - (s_L^u)^2 - \sum_{i=4}^n |\tilde{V}_{Lui}|^2 + |V_{ud}|^2 \left(2\text{Re}(\kappa_{ud}) - (s_L^d)^2\right) + |V_{us}|^2 \left(2\text{Re}(\kappa_{us}) - (s_L^s)^2\right). \quad (40)$$
The experimental value for this quantity is \[15\]

\[
\sum_{i=1}^{3} |V_{ui}|^2 = 0.9981 \pm 0.0021. \tag{41}
\]

For those CKM matrix elements involving the $c$-quark, the analysis is similar to the above, except that the mixing of the first-generation particles can be neglected since such mixings are constrained considerably better from other processes than from the relatively imprecise measurements of $V_{cd}$ and $V_{cs}$. Thus we have

\[
V_{cd} = c_L^c \hat{V}_{Lcd},
\]

\[
V_{cs} = c_L^c c_L^s \hat{V}_{Lcs} + s_R^c s_R^s \hat{V}_{Rcs}, \tag{42}
\]

and

\[
\sum_{i=1}^{3} |V_{ci}|^2 = 1 - (s_L^c)^2 - \sum_{i=4}^{n} |\hat{V}_{Lci}|^2 + |V_{cs}|^2 \left(2 \text{Re}(\kappa_{cs}) - (s_L^s)^2\right), \tag{43}
\]

where $|V_{cb}|^2$ has been neglected. The experimental value is \[15\]

\[
\sum_{i=1}^{3} |V_{ci}|^2 = 1.08 \pm 0.37. \tag{44}
\]

The hadronic right-handed currents $\kappa_{ud}$ and $\kappa_{us}$ are constrained through the unitarity of the CKM matrix. There are additional, very stringent constraints coming from the predictions of PCAC for nonleptonic $K_{\pi 3}$ amplitudes relative to $K_{\pi 2}$ amplitudes \[17\]. Interpreting these limits as $1\sigma$ errors \[3\], one has

\[
\kappa_{ud}, \kappa_{us} = 0 \pm 0.0037. \tag{45}
\]

There are also additional (weak) constraints on $\kappa_{cd}$ and $\kappa_{cs}$, but they will not be discussed here (see Refs. \[3\] and \[14\]).

### 3.3 Neutral Currents (Low Energy)

At low energy, neutral current interactions can be parametrized through effective lagrangians in which the $Z^0$ has been integrated out. In this subsection, I will discuss three types of scattering processes – $\nu q$, $\nu e$ and $e q$. In all three cases, radiative corrections are important \[18\]. For simplicity, these corrections are not shown explicitly, but are included in the fits.

#### 3.3.1 Deep-Inelastic Neutrino Scattering

The effective lagrangian describing the scattering of neutrinos from quarks can be written as

\[
-\mathcal{L}^{\nu q} = \frac{4G_F}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \nu_L \sum_{i=u,d,...} \left[ \epsilon_L(i) \bar{q}_L^i \gamma^\mu q_L^i + \epsilon_R(i) \bar{q}_R^i \gamma^\mu q_R^i \right]. \tag{46}
\]
| Quantity | Experimental Value | Source |
|----------|--------------------|--------|
| $g^2_L$  | 0.2977 ± 0.0042    |        |
| $g^2_R$  | 0.0317 ± 0.0034    |        |
| $\theta_L$ | 2.50 ± 0.03 | Deep inelastic [15] |
| $\theta_R$ | 4.59 ± 0.44 | |
| $g^2_V$  | -0.10 ± 0.05       | Low-energy $\nu_\mu e$: BNL [19] |
| $g^2_A$  | -0.50 ± 0.04       |        |
| $g^2_V$  | -0.06 ± 0.07       | High-energy $\nu_\mu e$: CHARM I [20] |
| $g^2_A$  | -0.57 ± 0.07       |        |
| $g^2_V/g^2_A$ | 0.047 ± 0.046 | CHARM II [21] |
| $C_{1u}$ | -0.249 ± 0.066$^\dagger$ |        |
| $C_{1d}$ | 0.391 ± 0.059$^\dagger$ | Atomic parity [22] |
| $C_{2u} - \frac{1}{2}C_{2d}$ | 0.21 ± 0.37 | SLAC e-D [23] |

Table 4: Low-energy neutral current data. There are non-negligible correlations between the measurements marked with a $^\dagger$. These have been taken into account in the fits [4].

In order to extract the values of $\epsilon_L(i)$ and $\epsilon_R(i)$, the neutral current process are normalized to the corresponding charged current processes, that is, the ratios

$$R_\nu = \frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to \mu^- X)}, \quad R_\bar{\nu} = \frac{\sigma(\bar{\nu} N \to \bar{\nu} X)}{\sigma(\bar{\nu} N \to \mu^+ X)}$$ (47)

are used. Thus, mixing effects enter both in the numerator and in the denominator. Taking all effects into account, the values of $\epsilon_L(i)$ and $\epsilon_R(i)$ obtained from deep-inelastic neutrino scattering are

$$\epsilon_{L,R}(i) = F_1(s^2, \kappa) \tilde{\epsilon}_{L,R}(i),$$ (48)

where the $\tilde{\epsilon}_{L,R}(i)$ are defined in Eq. [14] and [3]

$$F_1(s^2, \kappa) = \frac{1 - \frac{1}{2} \Lambda_{\mu} \left(s_{\nu}^\mu\right)^2}{1 - (s_{\mu}^\mu)^2 - (s_{\mu}^\mu)^2 - \text{Re}(\kappa_{ud})}$$ (49)

incorporates the mixing effects in the neutrinos as well as in the normalization. The experimental values of $g^2_a \equiv \epsilon_a(u)^2 + \epsilon_a(d)^2$ and $\theta_a \equiv \tan^{-1}[\epsilon_a(u)/\epsilon_a(d)]$, $a = L, R$ are given in Table 4.

### 3.3.2 Neutrino-Electron Scattering

The neutral current interaction of $\nu_\mu$ and $e$ can be described by

$$-\mathcal{L}^{\nu_\mu e} = \frac{2G_F}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \nu_L \bar{e} \gamma_\mu (g^e_V - g^e_A \gamma_5) e.$$ (50)
As in deep-inelastic neutrino scattering, the vector- and axial-couplings of the electron are obtained by normalizing the neutral current process (in this case $\nu_\mu-e$ scattering) to a charged current process ($\nu_\mu$-hadron scattering). Again, mixing effects appear in both places. The low-energy experiments from BNL normalize to the quasielastic process $\nu_\mu n \rightarrow \mu^- p$, leading to

$$
g_V^e = F_2(s^2)v_e = F_2(s^2)\left[-\frac{1}{2}(c_L^e)^2 - \frac{1}{2}(s_R^e)^2 + 2\sin^2\theta_W\right],
g_A^e = F_2(s^2)a_e = F_2(s^2)\left[-\frac{1}{2}(c_L^e)^2 + \frac{1}{2}(s_R^e)^2\right],
$$

(51)

where

$$
F_2(s^2) = \frac{1 - \frac{1}{2}A_\mu \left(s_{\nu e}^\mu\right)^2}{1 - (s_{\nu e}^\mu)^2 - (s_{\nu e}^\mu)^2}. 
$$

(52)

The high-energy experiments at CERN and Fermilab normalize to $\nu N \rightarrow \mu^- X$ as in deep-inelastic scattering, so that in this case $g_V^e$ and $g_A^e$ are as in Eq. (51), but with $F_2(s^2)$ replaced by $F_1(s^2, \kappa)$ of Eq. (49). The experimental values of $g_V^e$ and $g_A^e$ are shown in Table 4. Note that the CHARM II collaboration has recently measured the ratio $g_V^e/g_A^e$, in which the dependence on $F_1(s^2, \kappa)$ cancels.

### 3.3.3 Atomic Parity Violation

Atomic parity violation arises through the interference of the electromagnetic and weak interactions. The parity violating couplings $C_{1i}$ and $C_{2i}$ are defined by

$$
-\mathcal{L}^{eq} = \frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i} \bar{e}_L \gamma_\mu \gamma_5 q^i_{\gamma} \gamma^\mu q^i_{\gamma} + C_{2i} \bar{e}_L \gamma_\mu q^i_{\gamma} \gamma^\mu q^i_{\gamma}\right].
$$

(53)

Including mixing, these couplings are given by

$$
C_{1i} = 2 \left(\frac{\hat{G}_\mu}{G_\mu}\right) a_e v_i, \quad C_{2i} = 2 \left(\frac{\hat{G}_\mu}{G_\mu}\right) v_e a_i,
$$

(54)

where the vector and axial couplings have been defined in Eq. [13] and $\hat{G}_\mu/G_\mu$ in Eq. [12]. $C_{1u}$ and $C_{1d}$ are measured in parity violating transitions in cesium; the combination $C_{2u} - \frac{1}{2}C_{2d}$ has been determined in polarized $e-D$ scattering at SLAC. All the experimental values are given in Table 4.

### 3.4 Neutral Currents ($Z$ Peak)

The very accurate measurements at LEP put strong constraints on the mixing of ordinary and exotic fermions, particularly as regards the $\tau$-lepton and heavy quarks. Here I will present the experimental data on the decay widths of the $Z^0$ as well as the forward-backward asymmetries for leptons and heavy flavours. The material in this subsection comes entirely from Ref. [4].
3.4.1 \( Z^0 \) Decay Widths

Taking into account all radiative corrections, at one loop the partial width for the decay \( Z^0 \to f \bar{f} \) is \([12]\)

\[
\Gamma_{Z \to f \bar{f}} = N_c^f \frac{M_Z}{12\pi} \sqrt{2} G_\mu M_Z^2 \rho_f \left( v_f^2 + a_f^2 \right) \left( 1 + \delta_{QED}^f \right) \left( 1 + \delta_{QCD}^f \right),
\]

where \( N_c^f = 3(1) \) for quarks (leptons), \( \delta_{QCD}^f \) is the QCD correction for hadronic final states, and \( \delta_{QED}^f \) is an additional photonic correction. Fermion mixing effects appear in two places – first, in the vector and axial couplings \( v_f \) and \( a_f \) (see Eq. \([15]\)), and also in the effective weak mixing angle which appears in the vector coupling. This weak mixing angle is renormalized by electroweak effects \([11]\):

\[
s_{eff}^2(f) = \frac{1}{2} \left[ 1 - \left( 1 - \frac{G_\mu}{G_\mu} \frac{4A}{\rho M_Z^2} \left( \frac{1}{1 - \Delta \alpha} + \Delta \rho_{rem} \right) \right) \right].
\]

As in the renormalized expression for \( M_W \) (Eq. \([33]\)), mixing effects appear indirectly in \( G_\mu/\hat{G}_\mu \) (Eq. \([32]\)). There are also electroweak corrections in the \( \rho_f \) term: \( \rho_f = \rho + \Delta \rho_{rem}^f \), where \( \rho \) contains all large \( t \)-quark effects and is universal, and \( \Delta \rho_{rem}^f \) (and \( \Delta \rho_{rem}^\tau \) above) include all the nonuniversal flavour-dependent corrections. In doing the fits, all corrections have been taken into account, including the finite mass effects for heavy fermions.

The experimental values of the five partial widths \( \Gamma_{Z}, \Gamma_{h}, \Gamma_{e}, \Gamma_{\mu} \) and \( \Gamma_{\tau} \) \([10]\) have large correlations among themselves. The widths are all shown in Table 5.

3.4.2 Leptonic Asymmetries

On resonance, the forward-backward asymmetry in the process \( e^+e^- \to Z^0 \to f \bar{f} \) takes the form

\[
A^F_{FB} = 3 \frac{v_e a_e}{v_e^2 + a_e^2} \frac{v_f a_f}{v_f^2 + a_f^2}.
\]

As in the partial widths, mixing effects enter both in the vector and axial couplings (Eq. \([15]\)), and in the renormalized effective weak mixing angle (Eq. \([56]\)). In the fits, all QED and QCD (for hadronic final states) corrections have been included \([11]\). The \( \tau \) polarization asymmetry has also been measured at LEP. This asymmetry is written

\[
A^\text{pol}_{\tau} = -2 \frac{v_\tau a_\tau}{v_\tau^2 + a_\tau^2}.
\]

The experimental values \([24]\) for all leptonic asymmetries are given in Table 5.

3.4.3 Heavy Flavours

The partial widths for \( Z^0 \to b \bar{b} \) \([25]\) and \( c \bar{c} \) \([26]\) have also been measured. These are listed in Table 5. The forward-backward asymmetries for these final states have also been measured.
Table 5: Partial widths (given in MeV) and asymmetries measured at the Z peak. The correlations among the measurements marked with a † have been taken into account in the fits [4]. Also displayed are the axial couplings $a_{b,c}^\gamma Z$ and the charm asymmetries with $D^*$ tagging $A_{c,D^*}^{\gamma Z}$, all measured off resonance.

For $b\bar{b}$, there is a peculiarity which must be taken into account. Due to the fact that neutral $B$-mesons can oscillate into $\bar{B}$-mesons, the observed asymmetry is not the true asymmetry but must be corrected:

$$A_{b}^{FB} = \frac{A_{b}^{FB \text{obs}}}{1 - 2\chi_B},$$

(59)

where $\chi_B$ is a measure of the probability for $B-\bar{B}$ oscillations. Experimentally, this parameter has been found to be $\chi_B = 0.146 \pm 0.016$ [28]. The forward-backward asymmetries for both $b\bar{b}$ (corrected) and $c\bar{c}$ final states is given in Table 5.

Finally, the forward-backward asymmetries for $b\bar{b}$ and $c\bar{c}$ final states have also been measured at lower energies at PEP and PETRA. In this region, the asymmetries $A_{b,c}^{\gamma Z}$ include interference between the $\gamma$ and the $Z^0$, and essentially measure the product of axial couplings $a_e a_{b,c}$. Both final states are tagged using high $p$ and $p_T$ leptons, leading to large correlations between the two measurements [29]. Due to the correlations, these data are only used for those fits where only one mixing angle at a time is allowed to vary. For $c\bar{c}$, there is an additional tagging method not applicable for $b\bar{b}$, namely using $D^*$'s [30]. The results using
this method are used in all the fits. The experimental data are shown in Table 5. Note that the axial couplings \( a_b \) and \( a_c \) \([31]\) are given in the case of lepton tagging, while for \( D^* \) tagging the forward-backward asymmetry is shown \([30]\).

4 Constraints

In the section, I present the constraints which the experimental data shown in the previous section place on the mixing between ordinary and exotic fermions. I will show the results of two fits. In the first (the ‘individual fit’), only one mixing angle at a time is allowed to be nonzero, and in the second (the ‘joint fit’) all mixing angles vary simultaneously.

In both fits, the constraints are obtained by using a least-squares method. One complication is that the mixing angles are bounded, that is, \( 0 \leq s_{L,R}^2 \leq 1 \). In order to deal with this, the following procedure is used. For each parameter \( s_i^2 \), the \( \chi^2 \) distribution is calculated. Then, assuming a probability distribution

\[
P(s_i^2)ds_i^2 = N_i e^{-\chi^2(s_i^2)/2}ds_i^2,
\]

in which \( N_i^{-1} = \int_0^1 e^{\chi^2(s_i^2)/2}ds_i^2 \) (i.e. \( N_i \) is chosen such that \( P(s_i^2) \) is properly normalized in the domain \([0,1]\)), the 90% C.L. upper bounds on the \( s_i^2 \) are calculated from \( P(s_i^2) \).

Despite the large number of parameters, the experimental data is comprehensive enough to constrain all mixing angles. The results of the individual and joint fits are shown in Table 6, which is taken from Ref. [4]. In the ‘Source’ column of this Table are listed those observables which are most important for constraining the mixing angles in the individual fits. However, in the joint fit it is possible to evade the bounds from these observables through fine-tuned cancellations between different mixings. In this case, other observables, which depend on different combinations of the mixings, become important. These new observables, which are denoted by a * in Table 6, are typically less precise, so that the constraints in the joint fit are somewhat weaker than those in the individual fit.

From this Table it is evident that the neutral current data at the \( Z \) peak is especially important for bounding all mixings. For the first generation fermions and the \( \mu \) and \( \nu_\mu \), the low-energy charged and neutral current results (particularly \( \nu q \) and \( eq \) scattering) are also useful. In addition, the asymmetries off the \( Z \) peak are helpful in constraining the mixing angles of the \( c \)- and \( b \)-quarks.

I must again stress that the data used to obtain these constraints are already a bit out of date. For example, only the 1990 LEP data was used; the inclusion of the 1991 LEP data would surely strengthen most of the bounds somewhat. The most important new development is in \( \tau \)-decays. The value of \( (g_\tau/g_e)^2 \) shown in Table 3 differs from its standard model value of 1 by about 1.5 standard deviations. However, the latest measurements of the \( \tau \) mass and lifetime have removed this discrepancy \([32]\). Thus, the limits on \( (s_{L,R}^\tau)^2 \) shown in Table 6, which depend on the old value of \( (g_\tau/g_e)^2 \), should be taken with a grain of salt – the new bounds are probably quite a bit better.
Table 6: 90% C.L. upper limits on mixing angles for individual fits (one angle at a time is allowed to vary) and joint fits (all angles allowed to vary simultaneously) [3]. Observables which are most important for the constraints are shown in the ‘Source’ column (those quantities which contribute only in the joint fits are tagged with an asterisk). $s_{\mu R}^{LEP}$ and $s_{\mu R}^{NC}$ refer to the weak mixing angle as extracted in neutral current measurements at the Z peak and at low energy, respectively. See the text for a discussion of the bounds on $(s_R^s)^2$, $(s_R^b)^2$ and $(s_L^\mu)^2$ (marked with a †).
In all fits, $\Lambda_e = \Lambda_\mu = \Lambda_\tau$ has been assumed. Furthermore, in the individual fit, $\Lambda = 2$ was taken. Note that, in this fit, only the neutrino mixings can depend on $\Lambda$. Since $(s_L^{\nu_e})^2$ and $(s_L^{\nu_\mu})^2$, are bounded mainly by charged current data, the dependence on $\Lambda$ is minimal. On the other hand, the constraint on $(s_L^{\nu_\tau})^2$ does depend on $\Lambda$: $(s_L^{\nu_\tau})^2 < 0.098, 0.032, 0.015$ for $\Lambda = 0, 2, 4$. (As I said in the previous paragraph, these numbers should not be taken too seriously. However, even with the new data, the strong dependence of $(s_L^{\nu_\tau})^2$ on $\Lambda$ will persist.)

The constraints on $(s_R^s)^2$ and $(s_R^b)^2$ are considerably weaker than those of other angles due to a peculiarity of the observables which bound them. These mixing angles are constrained mainly by the LEP observables, which depend on the couplings $v_qa_q$ and $v_q^2 + a_q^2$ ($q = s, b$). However, for $(s_R^b)^2 \simeq 0.3$, the $s^4$ terms cancel against the $s^2$ terms. Thus there are two minima in the $\chi^2$ distribution, centred around 0 and 0.3. The 90% C.L. bounds of Table 6 are obtained by integrating over both regions. The restriction to the region centred at no mixing gives stronger bounds, $(s_R^s)^2 \lesssim 0.09$ and $(s_R^b)^2 \lesssim 0.10$.

The bounds on most mixing angles in Table 6 are quite stringent. However, one might argue that the exotic fermions which give rise to these mixings necessarily appear in models with other forms of new physics, extra $Z$’s for instance, and that these new effects might weaken significantly the mixing limits. This seems quite unlikely, given the number and variety of constraints. In fact, such a study has been done [33], in the context of $E_6$ and $SO(10)$ models. In this paper, the effects of $Z$-$Z'$ mixing and fermion mixing were analyzed simultaneously. In general, the presence of an extra $Z$ did not much alter the mixing limits. Although not a proof, this analysis lends support to the idea that, regardless of the model, it is rather difficult to evade the constraints on the mixing of ordinary and exotic fermions found in Table 6.

5 Conclusions

In this chapter, I have discussed the constraints which precision measurements put on the mixing of ordinary and exotic fermions. Exotic fermions are defined as new fermions whose left- or right-handed components transform in a non-standard way under $SU(2)_W$, that is, $L$ singlets and/or $R$ doublets. Excluding noncanonical colour and electric charge assignments, there are 4 types of exotic fermions – mirror fermions, vector singlets and doublets, and new Weyl neutrinos.

In general, mixing between ordinary and exotic fermions will lead to flavour-changing neutral currents among the light particles, which are extremely well constrained experimentally. However, if one chooses fine-tuned directions in mixing parameter space such that each ordinary fermion mixes with its own exotic fermion, then the bounds from FCNC can be evaded. I have developed the formalism which describes this mixing – there is one mixing angle per $L$ and $R$ charged fermion. For neutrinos, the situation is more complicated due to the possibility of Majorana masses and the fact that there is no experimental evidence
against FCNC involving neutrinos. Nevertheless, because the final neutrinos in any process are unobserved, it is possible to describe mixing in the neutrino sector by one angle, plus one auxiliary parameter, per ordinary neutrino species.

There are enough constraints from low-energy charged and neutral current data, as well as the experimental results from LEP, to constrain all mixing angles. I have described two types of fits. In the first, all mixing angles but one are set to zero, and the non-zero angle is constrained. In the second all angles are allowed to be non-zero simultaneously. The results are shown in Table 6. In the individual fit, most of the mixing parameters \((s^f)^2\) are constrained to be of order 1%, with some of the angles (such as those for \(e_{L,R}, u_L, d_L, \mu_L\) and \(\nu_{\mu L}\)) quite a bit smaller. The two exceptions are \(s_R\) and \(b_R\), whose mixings are bounded to be only about 0.3. In the joint fit, due to the possibility of accidental cancellations among the mixings, the limits are weakened. Typically, the constraints are relaxed by a factor of 2-3, but this factor can be as much as 6-8 in a few cases.

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