On Phantom Thermodynamics

S. H. Pereira and J. A. S. Lima

Departamento de Astronomia, Universidade de São Paulo
Rua do Matão, 1226 - 05508-900, São Paulo, SP, Brazil

Abstract

The thermodynamic properties of dark energy fluids described by an equation of state parameter \( \omega = p/\rho \) are rediscussed in the context of FRW type geometries. Contrarily to previous claims, it is argued here that the phantom regime \( \omega < -1 \) is not physically possible since that both the temperature and the entropy of every physical fluids must be always positive definite. This means that one cannot appeal to negative temperature in order to save the phantom dark energy hypothesis as has been recently done in the literature. Such a result remains true as long as the chemical potential is zero. However, if the phantom fluid is endowed with a non-null chemical potential, the phantom field hypothesis becomes thermodynamically consistent, that is, there are macroscopic equilibrium states with \( T > 0 \) and \( S > 0 \) in the course of the Universe expansion. Further, the negative value of the chemical potential resulting from the entropy constraint \( (S > 0) \) suggests a bosonic massless nature to the phantom particles.

PACS numbers: 98.80.-k

*Electronic address: spereira@astro.iag.usp.br
†Electronic address: limajas@astro.iag.usp.br
I. INTRODUCTION

Several kinds of complementary astronomical observations indicate that the Universe is expanding in an accelerated form and that the transition (from a decelerating to an accelerating regime) occurred at a redshift of the order of unity \[1, 2\]. In the context of general relativity, an accelerating stage and the associated dimming of type Ia Supernovae are usually explained by assuming the existence of an exotic substance with negative pressure sometimes called dark energy. Actually, for dark energy dominated models, the scale factor evolution is governed by the equation \(3\ddot{a}/a = -4\pi G(\rho + 3p)\), and means that a hypothetical component with negative pressure satisfying \(p < -\rho/3\) may accelerate the Universe (\(\ddot{a} > 0\)).

There are many candidates to represent this extra non-luminous relativistic component. In the case of XCDM cosmologies, for instance, it can be phenomenologically described by an equation of state (EoS) of the form \[3\]

\[p = \omega \rho,\]

where \(p\) and \(\rho\) denotes the pressure and energy density, respectively, and \(\omega\) is a constant negative parameter. The case \(\omega = -1\) corresponds to a positive cosmological constant, or vacuum energy, while for \(\omega < -1\) we have the so called phantom dark energy regime \[4\], or phantom fluids\(^1\).

The case for a phantom dominated Universe has been first suggested with basis on SN-Ia analysis alone which favor \(\omega < -1\) more than cosmological constant or quintessence \[5\]. A more precise observational data analysis (involving CMB, Hubble Space Telescope, type Ia Supernovae, and 2dF data sets) allows the equation of state \(p = \omega \rho\) with a constant \(\omega\) on the interval \([-1.38, -0.82]\) at the 95% C. L. \[6\].

From a theoretical point of view, the study of phantom regime is also a very interesting subject mainly due to a long list of pathologies. Initially, it was criticized by several authors due to issues of stability \[7\] which must be added to some weird properties, like the possibility of superluminal sound speed, as well as the violation of some classical energy conditions \[8\].

\(^1\) Indeed, in the standard lines of the thermodynamic, we will see that it does not make sense to speak of phantom fluids for systems with null chemical potential.
In particular, since \((p + \rho < 0)\) one may see that it violates the strong and dominant energy conditions. Further, the energy density of a phantom field increases along the cosmic evolution thereby causing a super accelerating universe which will end in a doomsday state dubbed \textit{Big Rip} \cite{9} which is of type I singularity according to the Barrow classification scheme \cite{10}. Such a \textit{Big Rip} singularity corresponds to \(\rho \to \infty\) at a finite time in the future which presumably will be avoided only if one considers possible effects from quantum gravity. A quantum treatment on the phantom regime has been discussed by several authors \cite{11}.

Another interesting point concerns the study of the spectral distribution and some related thermodynamical properties of the phantom fluid, like their temperature and entropy. We have two different approaches to study the thermodynamic of phantom fluids. The first, based on a somewhat ambiguous thermodynamic deduction \cite{12} (see discussion in section II), was given by González-Díaz and C. L. Sigüenza \cite{13}, which claimed that the temperature of phantom fluids in a Friedmann-Robertson-Walker (FRW) geometry should be negative and defined by the scaling law

\[
T \sim (1 + \omega)a^{-3\omega},
\]

where \(a(t)\) is the scale factor (note the negative prefactor, \(1 + \omega\), multiplying the power of the FRW scale function). By adopting such a temperature reinterpretation, it was possible to keep the entropy of the phantom field positive definite as required by its probabilistic definition in the context of statistical mechanics. In a second approach, a group of authors \cite{14, 15} have advocated that the temperature of any dark energy component is always positive definite obeying the evolution law

\[
T \sim a^{-3\omega},
\]

and, more important, that the existence of phantom fluids is not thermodynamically consistent because its co-moving entropy is negative since \(S \sim (1 + \omega)T^{1/\omega}a^3\). In this approach, a possible way to save the phantom regime is to introduce a negative chemical potential to the fluid \cite{16}, so that the phantom hypothesis is recovered. A chemical potential has also been recently introduced in the context of dark energy \(k\)-essence models described in terms of a self interacting complex scalar field \cite{17}.

In this note we have the intention to shed some light on this discussion, by favoring a
phantom component with positive temperature, and, under certain thermodynamic conditions, with positive entropy.

II. THERMODYNAMIC ANALYSIS OF DARK ENERGY FLUIDS

For simplicity, let us now consider that the homogeneous and isotropic FRW universe model is dominated by a separately conserved dark energy fluid described by the EoS (1). Following standard lines (see, for instance, Kolb and Turner [18]), the combination of the first and the second law of thermodynamics applied to a co-moving volume element of unit coordinate volume and physical volume $V = a^3$, implies that

$$TdS = d(\rho V) + pdV \equiv d[(\rho + p)V] - Vdp,$$

where $\rho$ and $p$ are the equilibrium energy density and pressure. The integrability condition,

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T},$$

leads to the following relation between the energy density and pressure ($\rho$ and $p$ depends only on the temperature)

$$dp = \frac{\rho + p}{T}dT,$$

which also follows directly from the equilibrium expression for the pressure and energy density. Substituting (6) into (4), we have the differential entropy definition,

$$dS = \frac{1}{T}d[(\rho + p)V] - (\rho + p)V \frac{dT}{T^2} = d\left[\frac{(\rho + p)V}{T} + C\right],$$

where $C$ is a constant (from now on fixed to be zero). Therefore, up to an additive constant, the entropy per co-moving volume must be defined by

$$S \equiv \frac{(\rho + p)}{T}V,$$

a result that remains valid regardless the number of spatial dimensions. Actually, for a multidimensional Universe model, the unique difference is that instead $V = a^3$, one must write $V = a^n$ for $n$-spatial dimensions in the above entropy formula. On the other hand, if the dark fluid expands adiabatically, $dS = 0$, or equivalently

$$d\left[\frac{(\rho + p)V}{T}\right] = 0,$$
which means that the entropy $S$ per co-moving volume is conserved. The same definition of entropy follows from the energy conservation law, $d(\rho V) + pdV = 0$, which can be rewritten as

$$d((\rho + p)V) = Vdp.$$  \hspace{1cm} (10)

As expected, by inserting (6) into (10) one obtains (9). Now, using the equation of state (1), we may write the entropy density on the form

$$s \equiv \frac{S}{V} = \frac{\rho + p}{T} = (1 + \omega)\frac{\rho}{T},$$  \hspace{1cm} (11)

which defines the entropy density in terms of the temperature for a dark energy fluid. All the above results are very well known, and the unique loss of generality comes from the fact that the chemical potential of the dark energy fluid was assumed to be zero from the very beginning.

At this point we would like to call attention for a paper published by Youm [12] related to the entropy of an Universe with $n$-spatial dimensions. He assumed that Eq. (11) defines the temperature in terms of the constant entropy thereby getting the scaling law (2). Later on, this approach was adopted by González-Díaz and C. L. Sigüenza [13] giving origin to the idea of negative temperature in the phantom regime ($\omega < -1$). This interpretation has been subsequently considered in many different contexts (see, for instance, [19] and Refs. therein).

As it appears, this is a very controversial approach since (11) defines the entropy density and not the temperature as assumed by the quoted authors. In order to discuss this point we recall that (11) can also be obtained in a very clear manner through the Tisza-Callen axiomatic approach. Actually, by postulating that the entropy (or the energy) is a homogeneous first order function of the extensive parameters, $S(\lambda U, \lambda V, \lambda N) = \lambda S(U, V, N)$, one obtains the so-called Euler relation [20]

$$TS = U + pV - \mu N,$$  \hspace{1cm} (12)

where $\mu$ is the chemical potential and $U = \rho V$ is the internal energy. Therefore, if the chemical potential is zero, one obtains (11). Note also that by taking the infinitesimal variation of the Euler relation (12) and combining with the second law one obtains the
Gibbs-Duhem relation

\[ SdT = V dp - N d\mu, \]  

which reduces to (6) when the chemical potential is zero (after inserting the entropy expression given by the above Euler relation).

This is the basics for any homogeneous substance described by the standard thermodynamics. More important still for the discussion here, the temperature as defined by

\[ \frac{1}{T} = \left( \frac{\partial S}{\partial U} \right)_{V,N}, \]  

is always positive definite for the equilibrium states. Therefore, if the energy density in the cosmological FRW context is positive (weak energy condition) one may conclude from (8), or directly from (11), that the entropy for a phantom fluid ($\omega < -1$) is negative definite, and, therefore, such a component is thermodynamically forbidden. Note also that all dark energy fluids with $\omega > -1$ have positive entropies, a result obtained before the Supernovae observations [14]. In addition, once the dependence of the energy density on the scale factor $\rho(a)$ is established for an expanding adiabatic Universe, the expression for the entropy itself determines the temperature evolution law as happens for the cosmic background radiation ($\omega = 1/3$). Naturally, this approach to determine the temperature law is not valid if the system evolves through a sequence of non-equilibrium states as happens when bulk viscosity [21] or irreversible matter creation [22] mechanisms are taken into account. It should also be remarked that the temperature evolution law can also be obtained even when the hypothesis that the energy density and pressure are functions only on the temperature and does not need to be explicitly used as discussed above. This approach will be discussed in the next section by using only local variables in the FRW background.

III. TEMPERATURE EVOLUTION LAW IN THE FRW GEOMETRY

The equilibrium thermodynamic states of a relativistic simple fluid obeying the $\omega$-EoS can be completely characterized by the conservation laws of energy, the number of particles, and entropy. In terms of specific variables, $\rho$, $n$ (particle number density) and $s$ (entropy
density) the above quoted laws for a FRW type background can be expressed as
\[
\dot{\rho} + 3(1 + \omega)\rho \frac{\dot{a}}{a} = 0, \quad \dot{n} + 3n \frac{\dot{a}}{a} = 0, \quad \dot{s} + 3s \frac{\dot{a}}{a} = 0, \tag{15}
\]
with general solutions of the form:
\[
\rho = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+\omega)}, \quad n = n_0 \left(\frac{a_0}{a}\right)^3, \quad s = s_0 \left(\frac{a_0}{a}\right)^3, \tag{16}
\]
where \(\rho_0, n_0, s_0\) and \(a_0\) are present day (positive) values of the corresponding quantities. On the other hand, the quantities \(p, \rho, n\) and \(s\) are related to the temperature \(T\) by the Gibbs law
\[
nT d\left(\frac{s}{n}\right) = d\rho - \rho + p \frac{dn}{n}, \tag{17}
\]
and from the Gibbs-Duhem relation \([13]\) there are only two independent thermodynamic variables, say \(n\) and \(T\). Therefore, by assuming that \(\rho = \rho(T, n)\) and \(p = p(T, n)\), one may show that the following thermodynamic identity must be satisfied
\[
T \left(\frac{\partial p}{\partial T}\right)_{n} = \rho + p - n \left(\frac{\partial \rho}{\partial n}\right)_{T}, \tag{18}
\]
an expression that remains locally valid even for out of equilibrium states \([23]\). Now, inserting the above expression into the energy conservation law as given by \([15]\) one may show that the temperature satisfies
\[
\frac{\dot{T}}{T} = \left(\frac{\partial p}{\partial \rho}\right)_{n} \frac{\dot{n}}{n} = -3\omega \frac{\dot{a}}{a}, \tag{19}
\]
and assuming \(\omega \neq 0\) a straightforward integration yields
\[
n = n_0 \left(\frac{T}{T_0}\right)^\frac{1}{3\omega} \quad \Leftrightarrow \quad T = T_0 \left(\frac{a}{a_0}\right)^{-3\omega}. \tag{20}
\]
In the standard fluid description, the temperature appearing in the above expressions is positive regardless of the value of \(\omega\). Note also that the temperature evolution law is completely independent of the entropy density. The above expressions also imply that for a given co-moving volume \(V\) of the fluid, the product \(T^\frac{1}{3}V\) remains constant and must characterize the equilibrium states (adiabatic expansion). At this point, the above temperature law, \(T \sim a^{-3\omega}\), should be compared with the one proposed in Refs. \([12, 13]\), namely, \(T \sim (1 + \omega)a^{-3\omega}\). It shows that the prefactor \((1 + \omega)\) in the temperature law is completely
artificial, and, therefore, it has no physical meaning. Moreover, the entropy expression as given by the Euler relation with $\mu = 0$, is just telling us that the phantom fluid is thermodynamically forbidden because the entropy of a dark energy fluid becomes negative for $\omega < -1$.

In an attempt to turn acceptable a phantom fluid with negative temperature, the authors of Ref. [13] comment on some quantum mechanical systems with negative temperatures. Actually, the possibility of negative values of temperature has been discussed by several authors [24, 25, 26]. From Eq. (14) one may conclude that the temperature may be negative if the entropy diminishes while the internal energy grows. This may happens, for instance, in some condensed matter system when the energy spectrum is limited from above thereby presenting population inversion phenomenon as required for the operation of semiconductor lasers [27]. Such an effect for paramagnetic systems of nuclear moments in a crystal were studied in detail by Purcell and Pound [28]. However, as remarked by Izquierdo and Pavón [29], all models of phantom energy models proposed so far in literature assume some type of scalar field with no upper bound on their energy spectrum. Moreover, while population inversion is a rather transient phenomenon, the phantom regime is supposed to last for many eons. In a point of fact, bodies of negative temperature would be completely unstable and in principle they cannot exist naturally in the Universe, except in some singular states of a system [30]. Such states are out of equilibrium (different from the analysis assumed in Refs. [12, 13]). They can be produced only in certain very unique systems, specifically in isolated spin systems, and they spontaneously decay away [20].

The considerations presented in the two previous sections may induce someone to think that phantom fluids cannot exist in nature or that the statistical mechanics and thermodynamics need to be somewhat generalized, as for instance, by adopting the non-extensive framework proposed by Tsallis [31]. However, it should be recalled that all the results above discussed are valid only if the chemical potential of the phantom fluid is identically zero.
IV. SAVING THE PHANTOM HYPOTHESIS

As we have argued, the concept of negative temperature is not a reasonable physical or mathematical solution to save the phantom hypothesis. Therefore, the important question now is how a phantom fluid may exist with temperature and entropy positives. In principle, it should be nice if such a problem might be solved in the framework of the standard thermodynamics and statistical mechanics. As far as we know, the unique possibility available to us is to introduce a new thermodynamic degree of freedom, namely, the chemical potential, a quantity appearing naturally in the Euler and Gibbs-Duhem relations.

For one component fluid the Gibbs free energy $G$ is defined by:

$$G(T, p, N) \equiv U + pV - TS,$$

with differential

$$dG = -SdT + Vdp + \mu dN,$$

yielding for the chemical potential

$$\mu = \left(\frac{\partial G}{\partial N}\right)_{p,T}.$$  

Now, by using relations (1), (16) and (20) it is straightforward to show that

$$G = \left[(1 + \omega)\frac{\rho_0 T}{n_0 T_0} - \frac{s_0}{n_0} T\right] N,$$

and

$$\mu = \left[(1 + \omega)\rho_0 - s_0T_0\right] \frac{T}{n_0 T_0} \equiv \mu_0 \frac{T}{T_0},$$

where we have defined $\mu_0 \equiv [(1 + \omega)\rho_0 - s_0T_0]/n_0$ as the present day value of the chemical potential. We see that, for the phantom regime ($\omega < -1$), we have $\mu_0 < 0$ as well as $(\partial \mu/\partial T) < 0$. Moreover, for $T \to 0$, we have that $\mu$ increases toward zero.

Therefore, if $\mu$ is different from zero, one may show that the entropy [8] must be replaced by (see also Eq. (12))

$$S(T, V) = \left[\left(1 + \omega\right)\rho_0 - \mu_0 n_0\right] \left(\frac{T}{T_0}\right)^{1/\omega} V.$$  

(26)
The basic conclusion here is that in order to keep the entropy positive definite, the following constraint must be satisfied \[16\]:

\[ \omega \geq \omega_{\text{min}} = -1 + \frac{\mu_0 n_0}{\rho_0}, \quad (27) \]

which introduces a minimal value to the \(\omega\)-parameter below which the entropy becomes negative. Therefore, only in this case a dark energy component in the phantom regime \((\omega_{\text{min}} < -1)\) is endowed with a positive entropy as required from Boltzmann’s microscopic definition \((S = k_B \ln W)\).

It is also worth noticing that such a condition can also be obtained by a completely different approach, namely, by studying the accretion of a phantom fluid with non-zero chemical potential by a black hole with basis on the generalized second law of thermodynamics \[32\]. Still more important, since the chemical potential of an ideal relativistic bosonic gas satisfies \(\mu \leq mc^2\), we see that Eq. \((25)\) plus the entropy constraint \((S > 0)\) suggest a bosonic massless nature to the phantom particles.

V. CONCLUDING REMARKS

Contrary to the claims of some authors \[13\], we have shown here that the temperature of a dark energy fluid must be always positive definite in the range \(\omega > -1\), and that the phantom regime \((\omega < -1)\) is thermodynamically forbidden, because its entropy is negative. Based on a straightforward thermodynamic analysis of the dark energy regime in the FRW metric we have demonstrated that the true thermodynamic temperature evolution law is given by the form \(T \sim a^{-3\omega}\) as previously derived \[14, 15\]. Finally, we have also advocated that in order to give some physical meaning to the phantom regime one needs to include a negative chemical potential. Only in this case, the entropy of the phenomenological phantom fluid hypothesis is not in contradiction with the probabilistic definition of the thermodynamic entropy.
Acknowledgments

The authors would like to thank V. C. Busti, J. V. Cunha, J. F. Jesus, A. C. Guimarães, R. Holanda and R. C. Santos for helpful discussions. JASL is partially supported by CNPq and FAPESP (Brazilian Research Agencies) under Grants 304792/2003-9 and 04/13668-0, respectively. SHP is supported by CNPq No. 150920/2007-5.

[1] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999); P. Astier et al., Astron. Astrophys. 447, 31 (2006); A. G. Riess et al., Astro. J. 659, 98 (2007).
[2] D. N. Spergel et al. Astrophys. J. Suppl. Ser. 170, 377 (2007); S. W. Allen et. al., [arXiv:0706.0033] (2007).
[3] T. Padmanabhan, Phys. Rept. 380, 235 (2003); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003); J. A. S. Lima, Braz. Jour. Phys. 34, 194 (2004), [astro-ph/0402109]; J. S. Alcaniz, Braz. J. Phys. 36, 1109 (2006); V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D 15, 2105 (2006); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).
[4] R. R. Caldwell, Phys. Lett. B 545, 23 (2002); S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003); B. McInnes, [astro-ph/0210321]; V. Faraoni, Int. J. Mod. Phys. D 11, 471 (2002); P. F. Gonzalez-Diaz, Phys. Rev. D 68, 021303 (2003); Y.S. Piao and E. Zhou, Phys. Rev. D 68, 083515 (2003); M. Sami and A. Toporensky, [gr-qc/0312009]; J. M. Cline, S. Jeon, G. D. Moore, [hep-ph/0311312]; R. Silva, J. S. Alcaniz and J. A. S. Lima, Int. J. Mod. Phys. D 16, 469 (2007).
[5] P. S. Corasaniti, M. Kunz, D. Parkinson, E. J. Copeland, and B. A. Bassett, Phys. Rev. D 70, 083006 (2004); U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. 354, 275 (2004).
[6] A. Melchiorri, L. Mersini-Houghton, C. J. Odman, and M. Trodden, Phys. Rev. D 68, 043509 (2003).
[7] P. H. Frampton, J. M. Cline, S. Y. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004); P. Singh, M. Sami and N. Dadhich, Phys.Rev. D 68, 023522 (2003).

[8] A. E. Schulz, M. J. White, Phys. Rev. D 64, 043514 (2001); J. G. Hao and X. Z. Li, Phys. Rev. D 67, 107303 (2003); S. Nojiri and S. D. Odintson, Phys. Lett. B 562, 147 (2003); Phys. Lett. B 565, 1 (2003); J. Santos, J. S. Alcaniz and M. J. Rebovca, Phys. Rev. D 74, 067301 (2006).

[9] R. R. Caldwell, M. Kamionkowski, N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003); J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, Phys. Rev. D 68, 023510 (2003); J. Santos and J. S. Alcaniz, Phys. Lett. B 619, 11 (2005); M. Szydlowski, O. Hrycyna and A. Krawiec, JCAP 0706, 010 (2007); R. C. Santos and J. A. S. Lima, Phys. Rev. D 77 083505 (2008).

[10] J. D. Barrow, Class. Quant. Grav. 21, L79 (2004). See also S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005).

[11] V. K. Onemli, R. P. Woodard, Phys. Rev. D 70, 107301 (2004); E. O. Kahya, V. K. Onemli, Phys. Rev. D 76 043512 (2007), E. M. Barboza Jr., N. A. Lemos, Gen. Rel. Grav. 38 1609 (2006).

[12] D. Youm, Phys. Lett. B 531, 276 (2002).

[13] P. F. González-Díaz and C. L. Sigüenza, Nucl. Phys. B 697, 363 (2004); Phys. Lett. B 589, 78 (2004).

[14] J. A. S. Lima and A. Maia Jr., Phys. Rev. D 52, 56 (1995); Int. J. Theor. Phys. 34, 9 (1995), [gr-qc/9505052]; J. A. S. Lima and J. Santos, Int. J. Theor. Phys. 34, 143 (1995); J. A. E. Carrillo, J. A. S. Lima, A. Maia Jr., Int. J. Theor. Phys. 35, 2013 (1996), [hep-th/9906016].

[15] J. A. S. Lima and J. S. Alcaniz, Phys. Lett. B 600, 191 (2004), [astro-ph/0402265].

[16] J. A. S. Lima and S. H. Pereira, Phys. Rev. D 78, 083504 (2008), [arXiv:0801.0323].

[17] N. Bilić, Fortschr. Phys. 56, 363 (2008), [arXiv:0806.0642].
[18] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley (1990).

[19] J. A. Freitas Pacheco and J. E. Horvath, Class. Quant. Grav. **24**, 5427 (2007).

[20] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed., John Wiley & Sons, New York, (1985).

[21] G. L. Murphy, Phys. Rev. D **48**, 4231 (1973); J. D. Barrow, Phys. Lett B **180**, 335 (1986); M. Morikawa and M. Sasaki, Phys. Lett. B **165**, 59 (1985); T. Padmanabhan and S. M. Chitre, Phys. Lett. A **120**, 433 (1987); J. A. S. Lima, R. Portugal and I. Waga, Phys. Rev. D **37**, 2755 (1988); J. A. S. Lima and A. S. Germano, Phys. Lett. A **170**, 373 (1992); J. Casas-Vazquez and D. Jou, Rep. Prog. Phys. **43**, 1937 (2003).

[22] I. Prigogine *et al.,* Gen. Rel. Grav., **21**, 767 (1989); M. O. Calvao, Lima, J. A. S. and Waga, Phys. Lett. A **162**, 233 (1992); W. Zimdhal and D. Pavón, Phys. Lett. A **176**, 57 (1993); W. Zimdahl and D. Pavón, Mon. Not. R. Astr. Soc. **266**, 872 (1994); W. Zimdahl and D. Pavón, Gen. Rel. Grav. **26**, 1259 (1994); J. Gariel and G. Le Denmat, Phys. Lett. A **200** 11 (1995); J. A. S. Lima, A. S. M. Germano and L. R. W. Abramo, Phys. Rev. D **53**, 4287 (1996), gr-qc/9511006; J. A. S. Lima and L. R. W. Abramo, Class. Quant. Grav. **13**, 2953 (1996), gr-qc/9606067; J. A. S. Lima, Gen. Rel. Grav. **29**, 805 (1997), gr-qc/9605056; J. S. Alcaniz and J. A. S. Lima, Astron. and Astrophys. **349**, 729 (1999), astro-ph/9906410; W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavon, Phys. Rev. D **64** 063501 (2001), astro-ph/0009353; P. Gopakumar and G. V. Vijayagovindan, Int. J. Mod. Phys. D **15**, 321 (2006); Y. Quinang, T-J. Zhang and Z-L Yi, Astrop. Spac. Sci. **311**, 407 (2007); J. A. S. Lima, E. F. Silva and R. C. Santos, arXiv:0807.3379.

[23] S. Weinberg, Astrop. J. **168**, 175 (1971); R. Silva, J. A. S. Lima and M. O. Calvão, Gen. Rel. Grav. **34**, 865 (2002), gr-qc/0201048.

[24] N. F. Ramsey, Phys. Rev. **103**, 20 (1956).

[25] P. T. Landsberg, Phys. Rev. **115**, 518 (1959).

[26] N. Bloembergen, Am. J. Phys. **41**, 325 (1973).

[27] P. T. Landsberg, *Thermodynamics and Statistical Mechanics*, Dover, New York, (1990).

[28] E. M. Purcell and R. V. Pound, Phys. Rev. **81**, 279 (1951).

[29] G. Izquierdo and D. Pavón, Phys. Lett. B **633**, 420 (2006).
[30] L. D. Landau and E. M. Lifshitz, *Statistical Physics* Part 1, 3rd ed., Pergamon Press, New York, (1985).

[31] C. Tsallis, J. Stat. Phys. 52, 479 (1988); J. A. S. Lima, R. Silva and A. R. Plastino, Phys. Rev. Lett. 86, 2938 (2001), [cond-mat/0101030]; J. A. S. Lima, R. Silva and J. Santos, Astron. Astrophys. 396, 309 (2002), [astro-ph/0109474].

[32] J. A. S. Lima, S.H. Pereira, J. E. Horvath and D. C. Guariento, [arXiv:0808.0860].