Folding Automaton for Trees

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Abstract. In this paper we observed the definition of folding technique in graph theory and we derived the corresponding automaton for trees. Also derived some propositions on symmetrical structure tree, non-symmetrical structure tree, point symmetrical structure tree, edge symmetrical structure tree along with finite number of points. This approach provides to derive one edge after n’ number of foldings.

1. Introduction

The class of Automaton contains many enthusiastic and convoluted gatherings. Its exhaustive investigation was begun with Automaton with modest number of states. Automaton assumes an important role in principle of calculation, compiler development, manmade brainpower, travelling and formal confirmation. Additionally finite Automaton are utilized as a part of content preparing, compilers, and equipment [1]. Tree Automaton perceive sets of trees, Many structures in software engineering are trees, which incorporates XML archives, Computations of parallel projects with fork/join Values of mathematical information composes in practical dialects. Tree Automaton can be utilized to characterize XML diagram dialects, Model-check parallel projects, Analyze useful projects [1]. The investigation of finite Automaton (FA) adequate, or programmed, structures started in progress by Hodgson (Th’eories d’ecidables standard mechanize fini, Annales de Sciences Math’ematiques, 1983), and after that carried on in Khoussainov and Nerode [2].

A tree automaton is a sort of state machine. Tree Automaton manage tree structures instead of the strings of more regular state machines. The accompanying article manages branching tree Automaton, which compare to regular languages of trees. As with traditional Automaton, finite tree Automaton (FTA) can be either a deterministic automaton or not. As indicated by how the automaton forms the input tree, finite tree Automaton can be of two sorts: (a) base up, (b) top down. This is a critical issue, as in spite of the fact that non-deterministic (ND) top-down and ND base up tree Automaton are proportionate in expressive power, deterministic top down Automaton are entirely less effective than their deterministic base up counterparts, since tree properties indicated by deterministic top down tree Automaton can just depend upon path properties. Deterministic base up tree Automaton are as capable as ND tree Automaton. Another augmentation is to consider unordered trees and they are identified with
tree Automaton modulo equational hypotheses. We design the redaction of new chapter covering Tree Automaton for Unranked Trees, Unordered Trees and in general Tree Automaton for Trees modulo equational hypotheses [3].

2. Preliminaries

Definition 2.1. Finite Automaton [4]
An finite automaton is represented formally by the 5-tuple \((Q, \Sigma, \delta, q_0, F)\) where \(Q\) is a finite set of states.
\(\Sigma\) is a finite set of Input symbols.
\(\delta\) is the transition function, that is, \(\delta : Q \times \Sigma \rightarrow Q\).
\(q_0\) is the initial state.
\(F\) is a set of final states of \(Q\).

Definition 2.2. Tree Automaton [4]
A (non-deterministic) finite tree automaton (NFTA) over alphabet \(F\) is a 4-tuple \(A = (Q, F, Q_f, \Delta)\) where \(Q\) is a finite set of states, \(Q \cap F_0 = \emptyset, Q_f \subseteq Q\) is a set of final states, \(\Delta\) is a set of transition rules of the form \(f(q_1, \cdots, q_n) = q\) where \(f \in F_n\) and \(q, q_1, \cdots, q_n \in Q\) and there is no explicit initial state.

Definition 2.3. Trees [5]
A tree is an undirected graph in which any two vertices are connected by exactly one path. In other words, any acyclic connected graph is a tree.

Folding Automaton for Trees
Tree with non-symmetry structure [6]
Tree with 4’ vertices

![Figure 1: Graph and Automaton](image)

Transition Table

|     | \(q_0\) | \(q_1\) | \(q_2\) | \(q_3\) |
|-----|--------|--------|--------|--------|
| \(q_0\) | -      | 0      |        | -      |
| \(q_1\) | 1      | -      | 1      | 1      |
| \(q_2\) | -      | 0      |        | -      |
| \(q_3\) | -      | 0      |        | -      |

Folding Graph
Type I: Top to Bottom (TTB)
Transition Table

|     | \(q_0\) | \(q_1\) |
|-----|--------|--------|
| \(q_0\) | -      |
| \(q_1\) | 1      |
Here, edge folding is possible. Since edge symmetry is available.

**Type II: Bottom to Top (BTT)**

a)

Here, edge folding is possible. Since edge symmetry is available.

b)

Here, edge folding is possible. Since edge symmetry is available.

**Transition Table**

|       | $q_0$ | $q_1$ |
|-------|-------|-------|
| $q_0$ | -     | 0     |
| $q_1$ | 1     | -     |

**Transition Table**

|       | $q_0$ | $q_1$ |
|-------|-------|-------|
| $q_0$ | -     | 0     |
| $q_1$ | 1     | -     |
Here, edge folding is possible. Since edge symmetry is available.

**Proposition 1:** For Any tree in graph theory corresponding folding Automaton technique need not be the same. Since from the above two types the construction of folding Automaton have been taken in 2 different ways namely, top to bottom (TTB) and bottom to top (BTT).

**Tree with symmetry structure [6]**

Tree with 6’ vertices

![Figure 5: Graph and Automaton](image)

**Transition Table**

|        | $q_0$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ |
|--------|-------|-------|-------|-------|-------|
| $q_0$  | -     | 0     | -     | -     | 1     |
| $q_1$  | 1     | -     | 1     | -     | -     |
| $q_2$  | -     | 0     | -     | -     | -     |
| $q_3$  | -     | 0     | -     | -     | -     |
| $q_4$  | 0     | -     | -     | -     | -     |
| $q_5$  | 0     | -     | -     | -     | -     |

**Folding Graph**

Both BTT (Bottom to Top) and TTP (Top to Bottom)

![Figure 6: Folding Graph and Folding Automaton](image)

**Transition Table**

|        | $q_0$ | $q_1$ |
|--------|-------|-------|
| $q_0$  | -     | 0     |
| $q_1$  | 1     | -     |

Here, edge folding is possible. Since edge symmetry is available.

**Proposition 2:** When construction of TTP (or) BTT Automaton is always coincide for the above type of tree structure having folding concept.(i.e.) when degree of 2’ vertices equal with one another. Then, BTT and TTB having same folding technique.

**Tree generated from one point [6]**

Tree with 5’ vertices Transition Table

|        | $q_0$ | $q_1$ |
|--------|-------|-------|
| $q_0$  | -     | 0     |
| $q_1$  | 1     | -     |
Figure 7: Graph and Automaton

|   | $q_0$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ |
|---|---|---|---|---|---|
| $q_0$ | - | 0 | - | - | - |
| $q_1$ | 1 | - | 0 | 1 | - |
| $q_2$ | - | 1 | - | - | - |
| $q_3$ | - | 0 | - | - | - |
| $q_4$ | - | 1 | - | - | - |

FOLDING GRAPH
Both BTT (Bottom to Top) and TTP (Top to Bottom)

Figure 8: Folding Graph and Folding Automaton

Transition Table

|   | $q_0$ | $q_1$ |
|---|---|---|
| $q_0$ | - | 0 |
| $q_1$ | 1 | - |

Here, edge folding is possible. Since edge symmetry is available.

**Proposition 3:** When construction of TTB (or) BTT Automaton is always coincide for the above type tree structure having folding concept (i.e) when degree of one vertex having more power than all other vertices (deg(2) = 4 is vertex 2 in the dominating vertex), then, BTT (Bottom to Top) and TTP (Top to Bottom) having same folding technique.
Tree generated from one point and one or more edges connected
a) Tree generated from one point [6]

Tree with 9’ vertices graph Transition Table

|   | q0 | q1 | q2 | q3 | q4 | q5 | q6 | q7 | q8 |
|---|----|----|----|----|----|----|----|----|----|
| q0 | -  | 0  | -  | -  | 0  | 1  | 1  | 1  | 1  |
| q1 | 1  | -  | 0  | 0  | -  | -  | -  | -  | -  |
| q2 | -  | 1  | -  | -  | -  | -  | -  | -  | -  |
| q3 | -  | 1  | -  | -  | -  | -  | -  | -  | -  |
| q4 | 1  | -  | -  | -  | -  | -  | -  | -  | -  |
| q5 | 0  | -  | -  | -  | -  | -  | -  | -  | -  |
| q6 | 0  | -  | -  | -  | -  | -  | -  | -  | -  |
| q7 | 0  | -  | -  | -  | -  | -  | -  | -  | -  |
| q8 | 0  | -  | -  | -  | -  | -  | -  | -  | -  |

Folding Graph: Type II (BTT):

|   | q0 | q1 |
|---|----|----|
| q0 | -  | 0  |
| q1 | 1  | -  |

Here, edge folding is possible. Since edge symmetry is available.

**Proposition 4:** When construction of TTB (or) BTT Automaton is always coincide for the above type tree structure having folding concept.(i.e) when degree of one vertex having more power than all other vertices (deg(1) = 6 i.e, vertex 1 in the dominating vertex).then, BTT (Bottom to Top) and TTP (Top to Bottom) having same folding technique.
b) Tree generated from one (or) more edges connected [6]:

Tree with 10' vertices graph:

Folding Graph: Type II (BTT):

![Folding Graph: Type II (BTT)](image)

Transition Table

|   | $q_0$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | $q_9$ |
|---|------|------|------|------|------|------|------|------|------|------|
| $q_0$ | -    | 0    | -    | -    | 0    | 1    | 1    | 1    | 1    |       |
| $q_1$ | 1    | -    | 0    | -    | 0    | -    | -    | -    | -    |       |
| $q_2$ | -    | 1    | -    | 0    | -    | -    | -    | -    | -    |       |
| $q_3$ | -    | -    | 1    | -    | -    | -    | -    | -    | -    |       |
| $q_4$ | 1    | -    | -    | -    | -    | -    | -    | -    | -    |       |
| $q_5$ | 0    | -    | -    | -    | -    | -    | -    | -    | -    |       |
| $q_6$ | 0    | -    | -    | -    | -    | -    | -    | -    | -    |       |
| $q_7$ | 0    | -    | -    | -    | -    | -    | -    | -    | -    |       |
| $q_8$ | 0    | -    | -    | -    | -    | -    | -    | -    | -    |       |
| $q_9$ | 0    | -    | -    | -    | -    | -    | -    | -    | -    |       |

Folding Graph: Type II (BTT):

![Folding Graph: Type II (BTT)](image)

Figure 12: Folding Graph and Folding Automaton

Transition Table

|   | $q_0$ | $q_1$ |
|---|------|------|
| $q_0$ | -    | 0    |
| $q_1$ | 1    | -    |

Here, edge folding is possible. Since edge symmetry is available.

**Proposition 5:** From (a) and (b), we observed that addition of edge / edges in tree provides link between 2 different Automaton to get the same folding technique. And also from the above graphs, when construction of TTB (or) BTT Automaton is always coincide for the structure having folding concept.

**3. Conclusion**

We intend different applications which is available in Automaton theory for generalization of graphs which is connected. The relation of connected graph and automaton is observed for
trees. Also observed the folding technique is applied to obtain folding Automaton for trees. And we observed that the folding Automaton for symmetrical tree structure is coincide for both Automaton namely TTB(top to bottom) and BTT(bottom to top).while for non-symmetrical tree structure ,corresponding folding Automaton need not be the same but edge symmetry is available for both cases(TTB,BTT).then, for point symmetrical tree structure and edge symmetrical tree structure provides the link between two different Automaton to get the same folding technique and finally for both tree structures after folding edge symmetry is derived.

4. Future Work
In future, this approach may be extended for some special named connected graphs also.

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