Retailer’s EOQ model considering demand and holding cost of the defective items under carbon emission tax

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Abstract. This paper presents a retailer’s inventory model considering imperfect production process and material handling that cause some defective items. The defective items possess a fraction of its original utility. Therefore, after a quality inspection, the retailer holds them in a different area until they are sold entirely. The proposed model considers the demand and holding cost of the defective items. It also incorporates carbon emission costs from transportation and storage activities. The objective function is to maximize the expected total profit, which simultaneously minimizes the total carbon emissions. A numerical example illustrates the model implementation. From this data set, the optimum order quantity is 1770.1 units, and the backorder quantity is 580.3 units, which give an expected total profit of $1,227,945 per year and expected total emissions of 0.765 tonCO₂/year. Further analysis shows a trade-off between economic and environmental performance. Incorporating carbon emission costs into an EOQ model will reduce the expected total carbon emissions. However, it also causes a reduction in the expected total profit.

Keywords: EOQ, defective items, demand, carbon emission

1. Introduction
In many manufacturing environments, machine reliability and human error are part of existing production and material handling problems. They may cause some defective items with considerable economic consequences. This situation affects inventory decisions, such as in economic order quantity (EOQ) models. Salameh and Jaber [1] and Cardénas-Bárron [2] constructed a fundamental approach to address this situation by setting a quality inspection. Then, Maddah and Jaber [3] made some improvements concerning the probability function of the defective items. Many researchers assume that the defected items can be sold as a single lot [1], [2], [3], [4], [5], while Teng and Hsu [6] identified a situation in which the defective items are gradually depleted following customer demand. In real business, the demand rate of defective items is lower than the demand rate of a good product. Other researchers were allowing shortages and assuming either full or partial backorder [6], [7], [8], [9], [10].

The green logistics system is an emerging research topic [11]. Recently, studies on inventory models with concern on environmental impact attracted many researchers. Battini et al. [12] identified some sources of emissions in inventory management activity including material handling, transportation, storage, sales, and waste disposal. Therefore, inventory decision models may incorporate carbon tax and carbon cap policies [13], [14]. Previous studies show that carrying more inventory may not be the wrong decision as it can lower the total emissions by reducing energy usage. Chen et al. [15] developed EOQ...
models to study the effect of various carbon emission policies. They considered emissions from ordering and inventory holding activities and found a possibility of higher emission reduction percentage compared to the increase in cost. Hence, there are possibilities of a trade-off between economic and environmental performance [16]. Wang and Ye [17] proved the benefit of incorporating carbon emissions into decision making in terms of lower carbon emissions and cost. They compared the total carbon emissions resulted from the EOQ and just-in-time (JIT) approach. Recently, Taleizadeh et al. [18] considered carbon emissions, shortages, and sales discounts. The objective is to optimize the order cycle time and selling price.

Recent research simultaneously considers the effect of defective items and carbon emissions in an EOQ model. Kazemi et al. [19] identified a lower total expected profit when an EOQ model incorporates carbon emission costs. Besides, the model will also result in a smaller order size. Wee and Daryanto [20] examined EOQ models when there is a percentage of defective items in a received lot considering carbon emissions costs and a full backorder. Sarkar et al. [21] allowed the repairment of the defective items. In their proposed model, the holding cost and carbon emission from holding the perfect and repaired items are different.

### 2. Methods

Besides maximizing its profit, a retailer is willing to reduce the environmental impact of its logistics activities by optimizing the lot size and shortage backorder. An extended EOQ model is studied, taking into consideration the probability, demand rate, and holding cost of the defective items. Further, carbon emission cost from delivering and holding the items, and backorder cost are counted. Figure 1 illustrates the inventory model for one replenishment cycle. The proposed model is constructed based on the model from Maddah and Jaber [3], Teng and Hsu [6], and Wee and Daryanto [20].

![Inventory level with defective items sale](image)

The notations for all parameters and variables are listed below:

- \( T \) order cycle time (year); (decision variable)
- \( y^{*} \) optimum order quantity (units); (decision variable)
- \( b^{*} \) optimum backorder quantity (units); (decision variable)
- \( d_{1} \) rate of customer demand for good quality items (units/year)
- \( d_{2} \) rate of customer demand for defective items (units/year); \( d_{1} > d_{2} \)
- \( \alpha \) rate of defective items per lot \( y \),
- \( f(\alpha) \) probability density function (pdf.) of \( \alpha \)
rate of quality inspection (units/year); \(i >> d_t\)

t_i \quad \text{quality inspection period (year)}

\(P_p\) \quad \text{selling price of good quality items ($/unit)}

\(P_d\) \quad \text{selling price of the defective items ($/unit)}; \(P_p > P_d\)

\(C_c\) \quad \text{cost of ordering per cycle ($/cycle)}

\(C_p\) \quad \text{per unit purchasing cost ($/unit)}

\(C_i\) \quad \text{per unit quality inspection cost ($/unit)}

\(C_{h1}\) \quad \text{unit cost for holding good quality items ($/unit/year)}

\(Ch_2\) \quad \text{unit cost for holding defective items ($/unit/year)}

\(C_b\) \quad \text{penalty cost due to backorder ($/unit)}

\(C_d\) \quad \text{fixed cost per order delivery ($/cycle)}

\(C_f\) \quad \text{variable cost from fuel consumption ($/liter)}

\(C_t\) \quad \text{carbon tax ($/tonCO_2)}

l \quad \text{delivery distance (km)}

F \quad \text{direct emission from fuel usage (tonCO_2/liter)}

E \quad \text{indirect emission from electricity usage (tonCO_2/kWh)}

w \quad \text{product weight (ton/unit)}

a \quad \text{fuel efficiency for empty truck (liter/km)}

b \quad \text{fuel efficiency from truckload (liter/km/ton)}

\(c\) \quad \text{average fuel consumption per unit inventory (kWh/unit)}

\(ETR\) \quad \text{expected total revenue ($)}

\(ETC\) \quad \text{expected total cost ($)}

\(ETPU\) \quad \text{expected total profit per unit time ($)}

\(E[.]\) \quad \text{expected value operator of \(\alpha\)}

Further, the proposed model works under some assumptions:

1. The retailer plans an item with constant demand.
2. A particular probability density function of the defective items is known.
3. The inspection rate is much higher than the demand rate so that no shortage during the inspection.
4. After inspection, the retailer keeps the defective items in a different area. They will be sold at a lower price and have a lower demand rate.
5. Shortages are allowed and completely backordered.

Figure 1 shows that the retailer receives \(y\) units at the beginning of the cycle. During the period \([0, t_i]\), the inventory level depleted due to customer demand \(d_t\). At the same time, the retailer performs a quality inspection.

\[t_i = y/i\]

At time \(t_i\), the inspection ends, and the defective items \(\alpha y\) being separated into a different area. Inventory level of the perfect items is continuously decreasing due to customer demand \(d_t\) and reaches zero at time \(t_i\). Meanwhile, the inventory level of the defective items also decreases in a lower rate \(d_2\) and reaches zero at time \(t_2\). Further, shortages are accumulated during \(t_2\) with \(d_1\) rate and will be completely backordered at the beginning of the next cycle.

From Fig. 1, \(T = t_1 + t_2\) and \(T = (1 - \alpha)y/d_t\). Due to the probability of defective items, \(E[T] = (1 - E[\alpha])y/d_t\). The \(ETR\) comes from the total sales of the perfect as well as the defective items. Hence,

\[ETR = (1 - E[\alpha])yP_p + E[\alpha]yP_d\quad(1)\]

\(ETC\) per cycle = ordering cost \((C_i)\) + purchasing cost \((C_2)\) + inspection cost \((C_i)\) + holding cost of perfect items \((C_i)\) + holding cost of defective items \((C_i)\) + backorder cost \((C_b)\) + transportation cost \((C_t)\)
\[ ETC = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 \]  

In which \( C_1 = C_o, \) \( C_2 = yC_p, \) and \( C_3 = yC_t. \)

The amount of inventory per cycle for perfect items is similar to Wee et al. [8]. The inventory holding cost considers the traditional holding cost for perfect items \( (C_h) \) and carbon emission cost \( (cEC_t) \) due to energy consumption in holding the inventory. Therefore, for the perfect items

\[ C_4 = (C_{h1} + cEC_t) \left( \frac{1}{2} \frac{(y - E[\alpha]y - b)^2}{d_1} + \frac{E[\alpha]y^2}{i} \right) \]  

The amount of inventory per cycle for defective items is similar to Teng and Hsu [6]. Because \( t_3 = E[\alpha]y/d_1, \) one has

\[ C_5 = (C_{h2} + cEC_t) \left( \frac{(E[\alpha]y)^2}{2d_2} \right) \]  

The backorder cost is

\[ C_6 = \frac{1}{2} \frac{C_b b^2}{d_1} \]  

The delivery activity implies transportation costs (e.g., setup, fuel, etc.) and carbon emission costs from fuel combustion. The transportation cost consists of a fixed cost \( (C_d + 2aFC_t) \) and a variable cost \( (bFC_fy), \) which depends on the delivery quantity. Hence, the carbon emission cost also has a fixed cost \( (2aFC_t) \) and a variable cost \( (bFC_fy). \) Therefore,

\[ C_7 = C_d + 2aFC_f + bFC_fy + 2aFC_t + bFC_fy \]  

Finally, the ETC per cycle becomes

\[ ETC = C_o + yC_p + yC_t + (C_{h1} + cEC_t) \left( \frac{1}{2} \frac{(y - E[\alpha]y - b)^2}{d_1} + \frac{E[\alpha]y^2}{i} \right) + (C_{h2} + cEC_t) \left( \frac{(E[\alpha]y)^2}{2d_2} \right) \]

\[ + \frac{1}{2} \frac{C_b b^2}{d_1} + C_d + 2aFC_f + bFC_fy + 2aFC_t + bFC_fy \]

From Maddah and Jaber [3],

\[ ETPU = \frac{ETR - ETC}{E[T]} \]

Therefore,

\[ (1 - E[\alpha])yP_p + E[\alpha]yP_d - C_o - yC_p - yC_t - (C_{h1} + cEC_t) \left( \frac{1}{2} \frac{(y - E[\alpha]y - b)^2}{d_1} + \frac{E[\alpha]y^2}{i} \right) \]

\[ - (C_{h2} + cEC_t) \left( \frac{(E[\alpha]y)^2}{2d_2} \right) - \frac{1}{2} \frac{C_b b^2}{d_1} - C_d - 2aFC_f - bFC_fy - 2aFC_t - bFC_fy \]

\[ \frac{1}{1 - E[\alpha]} \]

Taking the second derivative of \( ETPU \) with respect to \( b \) and \( y, \) one has

\[ \frac{\partial^2 ETPU}{\partial b^2} = - \frac{(C_{h1} + cEC_t + C_b) \frac{1}{y}}{1 - E[\alpha]} \]  

\[ \frac{\partial^2 ETPU}{\partial y^2} = \left( \frac{b^2}{y^3} (C_{h1} + cEC_t + C_b) + \frac{2d_1}{y^3} (C_o + C_d + 2aFC_t + 2aFC_f) \right) \frac{1}{E[\alpha] - 1} \]
\[
\frac{\partial^2 ETPU}{\partial b \partial y} = \frac{b}{y^2} \frac{C_{h1} + cEC_t + C_b}{1 - E[\alpha]} \quad (11)
\]

\[
\left(\frac{\partial^2 ETPU}{\partial b \partial y}\right)^2 - \left(\frac{\partial^2 ETPU}{\partial b^2}\right) \left(\frac{\partial^2 ETPU}{\partial y^2}\right) = - \frac{2d_1(C_{h1} + cEC_t + C_b)(C_o + C_d + 2d_1EC_t + 2d_1C_t)}{y^4} \frac{1}{(E[\alpha] - 1)^2} \leq 0 \quad (12)
\]

From equations (9), (10), and (12), the profit function \(ETPU\) is strictly concave. Further, setting the first derivative of \(ETPU\) with respect to \(b\) and \(y\) equal to zero, \(b^*\) and \(y^*\) can be obtained as follow:

\[
b^* = \frac{(C_{h1} + cEC_t)y'(1 - E[\alpha])}{C_{h1} + cEC_t + C_b} \quad (13)
\]

\[
y^* = \sqrt{\frac{2d_1C_o + 2d_1C_d + 4d_1alC_t + 4d_1alFC_t + C_b^2 + C_{h1}b^2 + cEC_t^2)}{d_2}}
\]

Further, the ETE per unit time from inventory holding and delivery activities can be subtracted from Eq. (7) as

\[
ETE = \left(\frac{2}{c} \left(\frac{1}{d_1} (y - E[\alpha]y - b)^2 + \frac{E[\alpha]y^2}{b} + cE \frac{E[\alpha]y^2}{2d_2} + 2alF + blwPy \right) \left(\frac{d_1}{1 - E[\alpha]}\right)\right) \quad (15)
\]

### 3. Result and discussion

A set of data is provided to illustrate the research problem and its solution. The data is adapted from Teng and Hsu [6], with some additional transportation and carbon emissions parameters from Wee and Daryanto [20], as follows:

- \(d_1 = 50,000\); \(d_2 = 30,000\); \(P_b = 50\); \(P_d = 40\); \(i = 175,200\); \(w = 0.01\) ton/unit;
- \(C_o = 100\); \(C_b = 20\); \(C_p = 25\); \(C_{h1} = 10\); \(C_{h2} = 8\); \(C_t = 0.5\); \(C_t = $75/tonCO_2\); \(C_d = $100/delivery\)
- \(l = 10\) km; \(F = $0.75/L\); \(a = 27\) L/100km; \(b = 0.57\) L/100km/ton;
- \(c = 1.44\) kWh/unit; \(F = 2.6 \times 10^3\) tonCO\(_2\)/L; \(E = 0.5 \times 10^3\) tonCO\(_2\)/kWh;

The percentage of defective items \(\alpha\) is assumed to have a uniform distribution with

\[
f(\alpha) = \begin{cases} 
25, & 0 \leq \alpha \leq 0.04 \\
0, & \text{otherwise} \end{cases}, \quad E[\alpha] = 0.02
\]

Solving the maximizing function of \(ETPU\) using equations (13) and (14) simultaneously, one has \(b^* = 580.3\) units and \(y^* = 1770.1\) units with the expected total profit $1,227,945 per year. Further, the expected total emissions are 0.765 tonCO\(_2\)/year.

Suppose that there is no carbon tax regulation (i.e., \(C_t = 0\)). Solving the problem simultaneously, one has \(b^* = 577.8\) units and \(y^* = 1768.8\) units with the \(ETPU = $1,228,002\) per year and \(ETE = 0.766\) tonCO\(_2\)/year.

The above results show that the proposed inventory model has successfully reduced the expected total carbon emissions by incorporating carbon emission into an EOQ model. However, it results in a
reduction in the expected total profit. A similar lower expected total profit situation was gained by Kazemi et al. [19] in their study.

4. Conclusion
This study proposes an EOQ model to solve a profit maximization problem of a retailer that willing to reduce the environmental impact of its inventory decisions in terms of total carbon emissions. The model incorporated a percentage of defective items in a received lot, followed by inspection and separation of the defective items. It considers the sales rate and holding cost for both the perfect and defective items and a complete backorder. Optimal order quantity and maximum backorder level are stated. A numerical example is presented, and the result shows that incorporating carbon emissions into an EOQ model may reduce total carbon emissions although there is a reduction in the expected total profit.

The proposed model can be extended for cases with a partial backorder and items with deterioration [22], [23]. Besides, this study considered a deterministic customer demand for defective items with a known selling price. Therefore, future research could determine the optimal price for the defective items to increase the demand rate.

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