HI gas content of SDSS galaxies revealed by ALFALFA: implications for the mass-metallicity relation and the environmental dependence of HI in the local Universe

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Abstract

The neutral hydrogen (HI) gas is an important barometer of recent star formation and metal enrichment activities in galaxies. We develop a novel statistical method for predicting the HI-to-stellar mass ratio $f_{\text{HI}}$ of galaxies from their stellar mass and optical colour, and apply it to a volume-limited galaxy sample jointly observed by the Sloan Digital Sky Survey and the Arecibo Legacy Fast ALFA survey. We eliminate the impact of the Malmquist bias against HI-deficient systems on the $f_{\text{HI}}$ predictor by properly accounting for the HI detection probability of each galaxy in the analysis. The best-fitting $f_{\text{HI}}$ predictor, with an estimated scatter of 0.272 dex, provides excellent description to the observed HI mass function. After defining an HI excess parameter as the deviation of the observed $f_{\text{HI}}$ from the expected value, we confirm that there exists a strong secondary dependence of the mass-metallicity relation on HI excess. By further examining the 2D metallicity distribution on the specific star formation rate vs. HI excess plane, we demonstrate that the metallicity dependence on HI is more fundamental than that on specific star formation rate. In addition, we show that the environmental dependence of HI in the local Universe can be effectively described by the cross-correlation coefficient between HI excess and the red galaxy overdensity $\rho_{\text{cc}} = -0.18$. This weak anti-correlation also successfully explains the observed dependence of HI clustering on $f_{\text{HI}}$. Our method provides a useful framework for learning HI gas evolution from the synergy between future HI and optical galaxy surveys.

Key words: galaxies: evolution — galaxies: formation — galaxies: abundances — galaxies: ISM — galaxies: statistics — cosmology: large-scale structure of Universe

1 INTRODUCTION

The neutral hydrogen (HI) gas represents a key intermediate stage in baryon cycling, between the initial accretion from the diffuse circumgalactic or intergalactic medium (Sancisi et al. 2008; Tumlinson et al. 2017) and the formation of dense molecular clouds that directly fuel star formation (Kennicutt & Evans 2012; Lada et al. 2012; Leroy et al. 2013). The variation of the HI gas reservoir usually precedes the colour transformation of galaxies induced by star formation and quenching (Baldry et al. 2004; Faber et al. 2007), while regulating the metallicity of the interstellar medium (ISM) together with galactic outflows (Dalcanton 2007; Matteucci 2012). In this paper, we develop a statistical framework for connecting the HI gas mass detected by ALFALFA (Haynes et al. 2011) to the stellar mass and optical colours of galaxies observed in SDSS (York et al. 2000), and explore the physical drivers of gas-phase metallicity and the environmental dependence of HI within this framework.

As the most important measure of the HI content of a galaxy, the HI-to-stellar mass ratio $f_{\text{HI}}$ (hereafter referred to as HI fraction) has been found to correlate with the optical colour with a scatter of $0.4$ dex (Kannappan 2004). Subsequently, Zhang et al. (2009) built a photometric estimator of $f_{\text{HI}}$ by introducing an additional scaling of $f_{\text{HI}}$ with the $i$-band surface brightness, reducing the scatter to $0.31$ dex. Li et al. (2012) later extended the $f_{\text{HI}}$ estimator by using a linear combination of four parameters (including stellar mass, stellar surface mass density $\mu_*$, NUV-$r$ colour, and the $g$–$i$ colour gradient), resulting a slightly improved scatter of $0.3$ dex and a more accurate match to the high-$f_{\text{HI}}$ systems observed by ALFALFA. Alternatively, non-linear predictors have been recently developed using machine learning algorithms, which usually require training over a large number of
HI-detected systems (Teimoorinia et al. 2017; Rafieferantsoa et al. 2018).

However, current HI surveys like ALFALFA are relatively shallow in depth, and are thus systematically biased against low-\(\text{HI}\) systems at any given redshift. Consequently, any \(\text{HI}\) predictor inferred or trained exclusively from systems above the HI detection threshold would be plagued by the Malmquist bias, overestimating the \(\text{HI}\) systems for systems that are missed by the HI survey. Such Malmquist bias can be partially alleviated by observing a smaller volume to a higher depth in HI. For example, using a roughly \(\text{HI}\)-limited but significantly smaller sample (GALEX Arecibo SDSS Survey), Catinella et al. (2010) constructed a \(\text{HI}\) predictor using the linear combination of NUV-r colour and \(\mu\), resulting in a scatter of \(-0.3\) dex (see also Catinella et al. 2013). Without having to trade volume for depth, we develop a new method to eliminate the Malmquist bias when predicting \(\text{HI}\) from the stellar mass and colour of SDSS galaxies, by properly accounting for the ALFALFA detection probability of each SDSS galaxy in the analysis.

Beyond \(\text{HI}\), the metal abundance within the gas serves as the fossil record of the chemical enrichment history, reflecting the complex interplay between star formation and gas accretion during the baryon cycling (Peebles et al. 2014). For star-forming galaxies, gas-phase metallicity is tightly correlated with stellar mass with a scatter of 0.1 dex in the oxygen-to-hydrogen abundance ratio, forming the well-known mass-metallicity relation (MZR; Tremonti et al. 2004). It has been suggested that the star formation rate (SFR) could drive the scatter in MZR — there exists a so-called fundamental metallicity relation (FMR), which manifests as a strong secondary dependence of the MZR on SFR (Mannucci et al. 2010; Lara-López et al. 2010; Andrews & Martini 2013). Various theoretical models have subsequently been proposed to explain the MZR, assuming SFR is the main process that shaped the MZR (Peeples & Shankar 2011; Davé et al. 2012; Dayal et al. 2013; Lilly et al. 2013; Zahid et al. 2014). However, the physical driver of the scatter in MZR is still under debate, and the existence of FMR depends on the systematic uncertainties in the metallicity estimator and potential biases in the sample selection (Yates et al. 2012; Salim et al. 2014; Telford et al. 2016).

Besides star formation, it is reasonable to expect that gas accretion plays a role in regulating the metallicity of the ISM. Indeed, Bothwell et al. (2013) showed that the MZR of \(-4000\) ALFALFA galaxies exhibits a strong secondary dependence on HI mass, with HI-rich galaxies being more metal poor at fixed stellar mass. Applying a principal component analysis over \(-200\) galaxies compiled from several molecular gas surveys, Bothwell et al. (2016b) further argued that the underlying driver of MZR is the molecular gas mass, and the FMR is merely a by-product of molecular FMR via the Kennicutt-Schmidt law (Bothwell et al. 2016a). More recently, by stacking the HI spectra of star-forming galaxies along the MZR, Brown et al. (2018) confirmed the strong anti-correlation between HI mass and gas-phase metallicity at fixed stellar mass, providing further evidence that the scatter in the MZR is primarily driven by fluctuations in gas accretion. To ascertain whether SFR or HI mass is the more fundamental driver, we will present a comprehensive analysis of metallicity, SFR, and HI mass for a large sample of galaxies jointly observed by SDSS and ALFALFA.

In addition to the optical properties of each galaxy, the HI gas reservoir also depends on the large-scale density environment. For example, it is long known that satellite galaxies in massive halos are deficient in HI (Haynes et al. 1984; Boselli & Gavazzi 2006; Yoon & Rosenberg 2015; Jaffé et al. 2015), due to processes like the ram-pressure and tidal stripping (Gunn & Gott 1972; Merritt 1983; Moore et al. 1996; Abadi et al. 1999; McCarthy et al. 2008; Kronberger et al. 2008; Bekki 2009). Gas accretion history may be tied to the halo growth history, which is known to be correlated with the large-scale environment (Fakhouri & Ma 2010). The environmental dependence of cosmic HI distribution can be predicted using semi-analytic models (Fu et al. 2010; Xie et al. 2018) and hydro-dynamic simulations (Davé et al. 2017), or statistically accounted for within the halo model (Guo et al. 2017; Obuljen et al. 2018). However, a quantitative description of the environmental dependence of HI is still lacking. In our analysis, we quantify this dependence using the cross-correlation coefficient \(\rho_{cc}\) between HI excess and galaxy overdensity, and develop three independent approaches to measuring \(\rho_{cc}\) directly from data.

This paper is organized as follows. We briefly describe the data and the joint SDSS-ALFALFA sample in § 2, and introduce our likelihood model in § 3. We present our main findings on the mass-metallicity relation in § 4 and the environmental dependence of HI in § 5. We conclude by summarizing our results and looking to the future in § 6.

Throughout this paper, we assume the WMAP9 cosmology (Hinshaw et al. 2013) for distance calculations. All the length and mass units in this paper are scaled as if the Hubble constant were 100\(\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}\). In particular, all the separations are co-moving distances in units of \(h^{-1}\text{Mpc}\), and the stellar and HI mass are both in units of \(h^{-2}\text{M}_\odot\). We use \(\lg x = \log_{10} x\) for the base-10 logarithm.

## 2 DATA

### 2.1 SDSS Volume-Limited Stellar Mass Sample

We make use of the final data release of the Sloan Digital Sky Survey (SDSS DR7; York et al. 2000; Abazajian et al. 2009), which contains the completed data set of the SDSS-I and the SDSS-II. In particular, we obtain the Main Galaxy Sample (MGS) data from the dr72 large-scale structure sample bright0 of the “New York University Value Added Catalogue” (NYU–VAGC), constructed as described in Blanton et al. (2005). The bright0 sample includes galaxies with \(10 < m_r < 17.6\), where \(m_r\) is the \(r\)-band Petrosian apparent magnitude, corrected for Galactic extinction. We apply the “nearest-neighbour” scheme to correct for the 7% galaxies that are without redshift due to fibre collision, and use data exclusively within the contiguous area in the North Galactic Cap and regions with angular completeness greater than 0.8.

We employ the stellar mass and gas-phase metallicity estimates from the latest MPA/JHU value-added galaxy catalogue\(^1\). The stellar masses were estimated based on fits to

\(^1\) [http://home.strw.leidenuniv.nl/~jarle/SDSS/](http://home.strw.leidenuniv.nl/~jarle/SDSS/)
the SDSS photometry following the philosophy of Kauff- 
mann et al. (2003a) and Salim et al. (2007), and as-
suming the Chabrier (Chabrier 2003) initial mass func-
tion (IMF) and the Bruzual & Charlot (2003) SPS model.
The MPA/JHU stellar mass catalogue is then matched to
the NYU-VAGC sample (Tremonti et al. 2004). Blue galaxies that are detected in ALFALFA are ad-
ditionally marked by blue crosses. The SDSS sample is volume-
limited with $\log(M_*/h^{-2}M_\odot) \geq 9.4$ and $z \in [0.016, 0.04]$. The region enclosed by the rectangular box is further highlighted in Figure 2.

From the bright0 catalogue, we select a volume-
limited sample of 14,140 galaxies with log-stellar mass $\log(M_*/h^{-2}M_\odot) \geq 9.4$ and redshift range $z \in [0.016, 0.04]$, which forms the basis sample for our joint analysis with ALFALFA (as will be described below). Although the ALFALFA survey robustly detected HI sources up to $z \sim 0.05$ before encountering the radio frequency interference (RFI), we choose a slightly lower redshift limit of $z_{\text{max}}=0.04$ so that the volume-limited sample can reach a lower threshold in stellar mass, hence a higher fraction of HI detections within the SDSS sample.

2.2 The ALFALFA $\alpha\,$100 HI Sample

The Arecibo Fast Legacy ALFA (ALFALFA; Haynes et al. 2011) survey is a blind extragalactic HI survey conducted using the seven-horn Arecibo L-band Feed Array (ALFA) onboard the 305-m Arecibo telescope. In order to reveal the faint-end population of the HI mass function in the local Universe ($z\leq0.05$), ALFALFA mapped $\sim 7000$ deg$^2$ of two contiguous high Galactic latitude regions between 2005 and 2011, searching for HI line emission across the entire frequency range between 1335 and 1435 $MHz$. We make use of the final data release ($\alpha\,$100; Haynes et al. 2018), which contains $\sim 31500$ sources up to $z = 0.06$. Due to the minimal overlap between the ALFALFA and SDSS footprints in the southern Galactic cap, we will focus exclusively on the northern Galactic region in our joint analysis of the two surveys.

Each HI detection is characterised by its angular po-
sition on the sky, radial velocity, velocity width $W_0$, and integrated HI line flux density $S_21$. The HI mass of each system can be estimated as

$$M_{HI} = 2.356 \times 10^5 D^2 S_{21} \left[h^{-2}M_\odot\right],$$

where $D$ is the distance to the source measured in units of $h^{-1}$Mpc, and $S_{21}$ in units of Jy km s$^{-1}$. Each detection is then assigned a detection category code depending on several reliability indicators, including the signal-to-noise ratio (S/N) of the detection and whether there exists an optical counterpart identified in other surveys (mainly SDSS). In particular, the “Code 1” sources (25434) are reliable detections with S/N above 6.5, while a subset of those below 6.5 are assigned “Code 2” (6068) due to having identified optical counterparts. In general, the Code 2 detections are also highly reliable despite having a relatively low S/N. Therefore, we will include both categories of detections in our analysis, and develop a likelihood model to self-consistently account for the non-detections.

2.3 Cross-matched SDSS and ALFALFA Sample

To study the HI content of optically selected galaxies, we cross-match the SDSS bright0 and the ALFALFA $\alpha\,$100 catalogues across their shared footprint in the northern Galactic cap. For each SDSS galaxy we first find all its potential HI counterparts by adopting a search radius of 36 arcsec, 80% larger than the typical centroiding uncertainty of ALFALFA ($\sim 20$ arcsec; Haynes et al. 2011). We flag it as an HI non-detection if no ALFALFA source is found within 36 arcsec of that galaxy; If the search returns one or multiple HI candidates, we then examine if the radial velocity of each candidate falls within $\pm 600$ km s$^{-1}$ of the SDSS redshift. To ensure that the match is unique, we always choose the closest HI source (in 2D) when there are multiple candidates left after the two passes.

After the cross-match, each SDSS galaxy in the volume-
limited stellar mass sample described in §2.1 is either de-
tected in ALFALFA with a reliable HI mass, or an HI non-
detection due to the lack of an HI emission with S/N above 6.5. For each SDSS galaxy detected in HI, we characterise the HI richness of the system by defining an HI-to-stellar mass ratio $f_{HI} = M_{HI}/M_*$, which we refer to as HI fraction through-
out the rest of the paper. For the galaxies that are not de-
tected in HI, we emphasize that the resulting Malmquist bias not only has to be properly accounted for in the analysis, but they should also provide important clues as to which kind of SDSS galaxies (in terms of $M_*$ and $g-r$) are intrinsically more likely to be HI-deficient than those that are observed.

We plan to focus on the star-forming population in

\[ \text{http://egg.astro.cornell.edu/alfalfa/} \]
our analysis, as the majority of quenched galaxies are not detected in ALFALFA and those detected in HI do not follow the same gas scaling relations as the star-forming ones (Boselli et al. 2014). Therefore, we divide galaxies into quenched (red) and star-forming (blue) based on their $g-r$ colours ($K$-corrected to $z=0.1$). We use broad-band colours rather than the star formation rate (SFR), because we are interested in building a photometric estimator of HI fraction that do not rely on high S/N spectroscopic observations. For the same reason, we did not remove the 1,650 type 2 Active Galactic Nucleus (AGN) candidates from the blue sample using the BPT diagnostics, which rely on emission line indices that usually require high S/N spectra. Following Zu & Mandelbaum (2016), we adopt a stellar mass-dependent colour cut to divide galaxies into red and blue,

$$(g-r)_{cut}(M_\star) = 0.8 \left( \frac{\lg M_\star / h^{-2} M_\odot}{10.5} \right)^{0.6},$$

indicated by the gray dashed lines in Fig. 3 (described further below).

To summarize, our volume-limited SDSS-ALFALFA joint sample includes 8,721 red and 5,419 blue galaxies with stellar mass above $\lg (M_\star / h^{-2} M_\odot) = 9.4$ and redshifts between 0.016 and 0.04. We will focus exclusively on this joint sample throughout the rest of the paper. Figure 1 show the distribution of red (red dots) and blue (blue circles) galaxies of our joint sample across the shared footprint between SDSS and ALFALFA. Among the 5,419 blue galaxies, 3,258 (60%) of them were detected in HI by ALFALFA (blue crosses; including both Code 1 and 2 detections), and 2,161 are non-detection in ALFALFA, respectively. The region enclosed by the gray rectangular box is further highlighted in Figure 2, which indicates the redshift and RA distribution of red galaxies (red dots) and blue galaxies with five different levels of $f_{\text{HI}}$ (colour-filled circles): HI non-detection (orange), $\lg f_{\text{HI}} < -0.5$ (green), $-0.5 \leq \lg f_{\text{HI}} < -0.25$ (cyan), $-0.25 \leq \lg f_{\text{HI}} < 0$ (blue), and $0 \leq \lg f_{\text{HI}} < 0.5$ (purple). The Coma cluster can be clearly seen as the dominant structure at $z\sim0.023$ and RA-13$^h$.

Figure 3 shows the colour-mass diagrams of eight different redshift bins ($\Delta z = 0.003$) between $z=0.016$ and 0.04. In each panel, the red dots above the gray dashed line (Equation. 2) represent the quenched/red galaxies, while the colour-filled and blue open circles below are blue galaxies with and without detection in HI, respectively. The colour-coding of the filled circles indicate the value of $\lg f_{\text{HI}}$, as described by the colourbar shown in the top left panel. The inset panels show the observed stellar mass functions of total (gray histograms), blue (blue), and HI-detected blue (yellow) galaxies at respective redshifts. The total and blue histograms stay roughly unchanged across the eight redshift bins, a manifest of the high stellar mass-completeness of the SDSS volume-limited sample. However, the yellow histograms decrease substantially at the low mass end towards higher redshifts, signaling the strong Malmquist bias of the HI detection rate in ALFALFA.

Within the blue population shown in Figure 3, the HI non-detections mainly occupy the high-$g-r$ and low-$M_\star$ corner of the so-called “blue cloud” at low redshifts (top panels), but spread out to the entire cloud in the highest redshift bin (bottom right panel). Among the HI-detected galaxies, the HI fraction exhibits strong decreasing trends with both $g-r$ and $M_\star$ in all panels. The trend with $g-r$ is likely real, because at fixed $M_\star$ the non-detections are preferentially redder and have a smaller average $M_\text{HI}$ (hence $f_{\text{HI}}$) than those detected in HI. However, it is unclear whether the trend with

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Figure 2. Redshift distribution of galaxies in the SDSS-ALFALFA joint sample ($120^h<\text{RA}<245$ and $24^h<\text{DEC}<82$; highlighted by the rectangular box in Figure 1). Red dots and coloured circles indicate the positions of the red and blue SDSS galaxies, respectively. Different sizes and colours of the circles correspond to the five levels of HI gas fraction $f_{\text{HI}}$ observed by ALFALFA (from “non-detection” to $f_{\text{HI}}>1$), indicated by the legend on the top right. The dominant structure at $z\sim0.023$ and RA-13$^h$ is the Coma cluster.
\[ M_\ast \text{ seen in each panel is physical — at fixed colour the non-detections have on average lower } M_\ast \text{ and lower } M_\text{HI} \text{ than those detected in HI, but the two populations may have similar } f_\text{HI}. \text{ In the next Section, we will build a rigorous likelihood model to quantify the level of intrinsic correlation between } f_\text{HI} \text{ and } M_\ast, \text{ despite the obscuration caused by the Malmquist bias.}

3 METHODOLOGY

3.1 HI Fraction Predictor and Detection Probability Model

Inspired by the two roughly independent trends of \( \lg f_\text{HI} \) with \( \lg M_\ast, \) and \( g-r \) seen in Figure 3, we construct a linear mixture model for the HI fraction predictor (HI-FP):

\[ \lg f_\text{HI} = a \times \lg M_\ast + b \times (g-r) + c + \sigma_{\lg f_\text{HI}} \times \epsilon, \tag{3} \]

where \( a, b, \) and \( c \) are the three parameters that determine \( \langle \lg f_\text{HI} | M_\ast, g-r \rangle \) (i.e., the expected value of \( \lg f_\text{HI} \) for any galaxy with given \( M_\ast \) and \( g-r \)), while \( \epsilon \) is a Gaussian random variable with a zero mean and a unit variance.

Note that \( \sigma_{\lg f_\text{HI}}^2 \) is the quadratic sum of the intrinsic scatter and the 1-\( \sigma \) measurement uncertainty. However, since the measurement error on \( f_\text{HI} \) reported by ALFALFA is rather uniform across the sample (\( \pm 0.05 \) dex), we do not treat the two scatter components separately. In addition, we assume a constant log-normal scatter about the mean HI fraction at fixed \( M_\ast \) and \( g-r \). We have tried incorporating

\[ f(\lg M_{\ast \text{HI}} | z) = \begin{cases} f(\lg M_{\ast \text{HI}} | z) / f(12 | z) & \text{if } \lg M_{\ast \text{HI}} \leq 12, \\ 1 & \text{if } \lg M_{\ast \text{HI}} > 12, \end{cases} \tag{4} \]

and

\[ f(\lg M_{\ast \text{HI}} | z) = \frac{(\lg M_{\ast \text{HI}} - 7)^{\mu_z}}{(\lg M_{\ast \text{HI}} - 7)^{\mu_z} + (\lg M_{\ast \text{HI},z} - 7)^{\mu_z}}, \tag{5} \]

where \( \lg M_{\ast \text{HI},z} \) is the characteristic logarithmic HI mass at which the detection rate equals to 50% at redshift \( z \), while \( \mu_z \) controls the slope of the decline from 100% at high \( M_{\ast \text{HI}} \) to 0% at low \( M_{\ast \text{HI}} \). To compute \( \lg M_{\ast \text{HI},z} \) and \( \mu_z \) at arbitrary redshift \( z \), we choose three pairs of \( (\lg M_{\ast \text{HI},z}, \mu_z) \) at \( z = 0.016, 0.028, 0.04 \) as free parameters, and use the cubic spline (Press et al. 1992) method to smoothly interpolate the two parameters of \( f(\lg M_{\ast \text{HI}} | z) \).

Combining the HI fraction predictor and the HI detection probability model, we now have ten parameters \( \theta = \{a, b, c, \sigma_{\lg f_\text{HI}}, \lg M_{\ast \text{HI},0}, \mu_0, \lg M_{\ast \text{HI},1}, \mu_1, \lg M_{\ast \text{HI},2}, \mu_2\} \),

\[ \text{Figure 3. Colour-mass diagrams of the SDSS-ALFALFA joint sample at eight different redshift slices (between } 0.016 \text{ and } 0.04 \text{ with } \Delta z = 0.003). \text{ In each panel, red dots indicate the red galaxies, defined as those above the colour cut (gray dashed line) on the diagram. Below the colour cut, blue open circles indicate the blue galaxies that are not detected in ALFALFA, while the colour-filled circles are the blue galaxies detected with different HI gas fractions } f_\text{HI}, \text{ colour-coded by the colour bar in the top left panel. The inset panel inside each panel shows the stellar mass functions of the total (gray), blue (blue), and ALFALFA-detected blue (gold) galaxies. While the stellar mass functions of the total and blue galaxies remain unchanged with redshift, the HI-detection completeness of ALFALFA decreases rapidly with increasing redshift.} \]
4.0 (Equation 3) and the HI incompleteness (Equation 4), our analysis.

\[ P_i = \text{product of the likelihood function} \]

ability distribution function (PDF) of \( \theta \) where the subscripts

3.2 Likelihood Model

Our input data \( D \) consist of \( N=5,419 \) blue galaxies in the SDSS-ALFALFA joint sample, each observed with three features \( \{M_*, g-r, z\} \). Among the \( N \) galaxies, \( n=3,258 \) of them are observed with reliable HI fraction \( f_{\text{HI}} \), while the rest \( N-n \) are non-detections. Our goal is to derive the posterior probability distribution function (PDF) of \( \theta \) given \( D \), \( P(\theta | D) \), i.e., the product of the likelihood function \( P(D | \theta) \) and the prior \( P(\theta) \). We adopt flat priors on all the ten parameters in our analysis.

Armed with the models for the HI fraction predictor (Equation 3) and the HI incompleteness (Equation 4), we can derive the likelihood function \( P(D | \theta) \) analytically by decomposing it into two components,

\[ P(D | \theta) = P(D_{\text{det}} | \theta) \times P(D_{\text{non-det}} | \theta), \tag{6} \]

where

\[ P(D_{\text{det}} | \theta) = \prod_{i=1}^{n} f_{\text{det}}(\log M_{\text{HI}} | z^i, \theta) \times P(\log M_{\text{HI}} | M_*^i, g^i-r^i, \theta) \]

\[ P(D_{\text{non-det}} | \theta) = \prod_{j=1}^{n-n} \int_{0}^{\log M_{\text{HI}}^\text{max}} [(1 - f_{\text{det}}(\log M_{\text{HI}} | z^j, \theta)) \times P(\log M_{\text{HI}} | M_*^j, g^j-r^j, \theta) \times \log M_{\text{HI}}^j] \tag{8} \]

describe the \( n \) HI-detected galaxies and the \( N-n \) non-detections, respectively. In the above equations, \( P(\log M_{\text{HI}} | M_*^i, g^i-r^i, \theta) \) is the probability distribution of a galaxy having a log-HI mass \( \log M_{\text{HI}} \) given its stellar mass and colour, and can be calculated as

\[ P(\log M_{\text{HI}} | M_*^i, g^i-r^i, \theta) = \frac{1}{2\pi \sigma_{\log M_{\text{HI}}}^i} \times \exp \left\{ \frac{[\log M_{\text{HI}} - \log M_* - a \log M_* - b(g-r) - c]^2}{2\sigma_{\log M_{\text{HI}}}^2} \right\} \tag{9} \]

For computing the likelihood for the non-detections in Equation 8, we adopt an integration limit of \( \log M_{\text{HI}}^\text{max}=12 \), consistent with the parameterisation of detection rate in Equation 4.

3.3 Posterior Results

We apply the likelihood model to the SDSS-ALFALFA joint blue galaxy sample, and infer the posterior distributions of the ten parameters using the Markov Chain Monte-Carlo (MCMC) method \textit{emcee} (Foreman-Mackey et al. 2013). After marginalizing over the six nuisance parameters that describe the HI detection rate, we obtain the posterior PDFs of the four key parameters that determine the HI fraction predictor \( \{a, b, c\} \) and the log-normal scatter \( \sigma_{\log f_{\text{HI}}} \), as shown in Figure 4.

The diagonal panels of Figure 4 show the marginalized 1D posterior PDFs for each of the four key parameters, and the 1-\( \sigma \) constraints are \( a=-0.328 \pm 0.015 \), \( b=-1.492 \pm 0.046 \), \( c=3.662 \pm 0.139 \), and \( \sigma_{\log f_{\text{HI}}} = 0.272 \pm 0.004 \), respectively. In the off-diagonal panels, the red, magenta, and blue contour lines enclose the 68%, 95%, and 98% confidence regions, respectively. The strong correlation between \( a \) and \( c \) indicates that the model could potentially explain away some of the apparent trend of \( f_{\text{HI}} \) with \( M_* \), by invoking a strong Malmquist bias, but the fact that \( P(a | D) \) diminishes rapidly to zero around \( a=-0.28 \) demonstrates that a negative intrinsic correlation between \( f_{\text{HI}} \) and \( M_* \) at fixed \( g-r \) is still necessary for interpreting the data.

We simultaneously derive 1-\( \sigma \) constraints on the six nuisance parameters that describe the HI detection probability (not shown on Figure 4), including \( \log M_{\text{HI}}=8.564 \pm 0.054 \), \( \mu_0=7.463 \pm 1.810 \), \( \log M_{\text{HI}}=9.170 \pm 0.010 \), \( \mu_1=14.386 \pm 0.920 \), \( \log M_{\text{HI}}=9.427 \pm 0.013 \), and \( \mu_2=30.822 \pm 2.270 \).

From the best-fitting key parameters, we can predict the underlying HI mass function of the volume-limited sample by summing the probability distribution of HI mass of all galaxies in the sample. The results for eight different redshift bins are shown as the black solid curves in Figure 5. In comparison, blue histograms with errorbars are the observed HI mass functions from ALFALFA. The ratios between the histograms and the solid curves, i.e., the inferred detection rate, are shown as the gray dashed lines (with shaded uncertainty bands) in the bottom sub-panels, while the blue
solid curves are the predictions from the best-fitting detection rate parameters. At the lowest redshift (top left of Figure 5; \(z = 0.018\)), the observed HI source number counts are in good agreement with the model prediction above \(\lg M_{\text{HI}} > 8.5\), indicating that ALFALFA is capable of detecting most of the galaxies with \(\lg M_{\text{HI}} > 9.4\) as HI sources. As the redshift increases (from left to right, top to bottom), the ALFALFA survey missed progressively more and more HI-rich systems — at \(z = 0.04\) (bottom right), ALFALFA only detected the most HI-rich systems and is highly incomplete at \(\lg M_{\text{HI}} > 9.5\).

Figure 6 provides a more visually-appealing way of showing the impact of Malmquist bias on ALFALFA detections in eight different redshift bins. In each panel, the gray circles show the underlying distribution of star-forming galaxies at that redshift on the \(f_{\text{HI}}\) vs. \(\langle f_{\text{HI}}\rangle\) plane, predicted from the best-fitting model,

\[
\langle f_{\text{HI}}\rangle = -0.328 \lg M_* - 1.492 (g-r) + 3.662,
\]

with a scatter of \(\sigma_{\lg f_{\text{HI}}} = 0.272\) dex. The ALFALFA-detected galaxies (blue dots) are similarly distributed at low redshifts (\(z < 0.02\)) compared to the gray circles, but land primarily above the one-to-one line (dashed line) at high redshifts (\(z > 0.03\)) — ALFALFA preferentially detected galaxies that have excess HI mass than expected. Without correctly accounting for the Malmquist bias in the ALFALFA data, one would derive an HI fraction predictor that systematically over-predicts the average HI mass in galaxies (by more than 0.15 dex at \(z < 0.04\)).

Finally, for any given galaxy with observed stellar mass and colour, we can now predict the expected value of its HI mass fraction (Equation 10) that is statistically consistent with the overall abundance of HI galaxies detected and missed by ALFALFA. Beyond the mean HI fraction, the inferred amount of scatter (0.272 dex) is dominated by the intrinsic scatter, as the contribution from measurement uncertainties is very small (~0.05 dex). This intrinsic scatter (\(0.272^2 - 0.05^2 = 0.267\) dex) was driven by a myriad of physical processes involved in the build-up and depletion of individual HI reservoir, which were inevitably linked to the galactic chemical evolution and the large-scale density environment. Therefore, we will explore the connection between HI and the gas-phase metallicity and red galaxy overdensity in the next two sections.

4 WHAT DRIVES THE MASS-METALLICITY RELATION?

4.1 Relative Metallicity, Relative sSFR, and HI Excess

The mass-metallicity relation (MZR) is a tight scaling relationship between the stellar mass \(M_*\) and the gas-phase metallicity \(Z_{\text{gas}}\) of star-forming galaxies, with a scatter of
-0.1 dex in the distribution of $Z_{\text{gas}}$ at fixed $M_\ast$. Defined as $Z_{\text{gas}}=12+\log(O/H)$, $Z_{\text{gas}}$ is essentially a measure of the oxygen to hydrogen number density ratio in the ISM. Therefore, at fixed $M_\ast$, a galaxy can be perturbed away from the median MZR either by varying the oxygen abundance in the ISM via star formation and galactic outflow, or by modifying the gas reservoir via gas inflow and stripping. To gain more insight on the physical driver of the MZR, we will adopt this perturbative viewpoint and measure the variation of metallicity with small changes in SFR and HI gas fraction.

The star-forming galaxies also form a relatively narrow sequence on the $\log s\text{SFR}-M_\ast$ plane (a.k.a., star-formation main sequence; SFMS). Taking advantage of the MZR and SFMS, we can define a relative metallicity $z_g$ and a relative sSFR $\phi$ as the deviations of the observed metallicity and logarithmic sSFR from the two mean scaling relations (each normalized by the scatter at fixed $M_\ast$), respectively. Specifically, we define $z_g$ and $\phi$ as follows,

$$z_g = \frac{Z_{\text{gas}} - \langle Z_{\text{gas}} \mid M_\ast \rangle}{\sigma_{Z_{\text{gas}}|M_\ast}},$$

(11)

and

$$\phi = \frac{\log s\text{SFR} - \langle \log s\text{SFR} \mid M_\ast \rangle}{\sigma_{\log s\text{SFR}|M_\ast}},$$

(12)

where $\langle Z_{\text{gas}} \mid M_\ast \rangle$ and $\langle \log s\text{SFR} \mid M_\ast \rangle$ are the mean MZR and SFMS, respectively. $\sigma_{Z_{\text{gas}}|M_\ast}$ and $\sigma_{\log s\text{SFR}|M_\ast}$ are the two corresponding scatterers at fixed $M_\ast$. After switching to $z_g$ and $\phi$ defined above, we can now directly compare the relative amount of metals and star formation of two arbitrary galaxies, regardless of how different their stellar masses are.

Similarly, the HI fraction of star-forming galaxies follows a tight scaling relationship about the best-fitting $\langle f_{\text{HI}} \rangle$ model (Equation 10) with a scatter of $\sigma_{f_{\text{HI}}} = 0.272$ dex. To quantify the tendency of a galaxy to have excess or deficit amount of HI relative to its expected value, we define an “HI excess” parameter $\gamma$,

$$\gamma = \frac{\log f_{\text{HI}} - \langle f_{\text{HI}} \rangle}{\sigma_{f_{\text{HI}}}},$$

(13)

where $f_{\text{HI}}$ and $\langle f_{\text{HI}} \rangle$ are the observed and expected HI-tostellar mass ratios, respectively.

### 4.2 2D Relative Metallicity Map

To study the 2D dependence of $z_g$ on $\phi$ and $\gamma$, we selected from the SDSS-ALFALFA joint sample a subset of HI-detected galaxies that have also high S/N spectra observed by SDSS, so that each of those galaxy has all three properties (i.e., metallicity, sSFR, and HI mass) robustly measured. We eliminated the spurious metallicity measurements due to AGNs by imposing the BPT selection criteria for star-forming galaxies defined by Kauffmann et al. (2003b). After the selections, we have 1,913 galaxies in the subsample.

The left two panels of Figure 7 summarizes the individual dependences of MZR on $\phi$ (top left) and $\gamma$ (bottom left), respectively. In each panel, the gray dotted curve and shaded band indicate the mean MZR and its scatter; The five coloured solid lines are the mean MZR of galaxies in five quintiles of $\phi$ (top) or $\gamma$ (bottom), colour-coded by the colour bar on the top left of each panel. The two inset panels show the segregated distributions of galaxy quintiles on the SFMS diagram and the $f_{\text{HI}}$ vs. $\langle f_{\text{HI}} \rangle$ plane, respectively. For the MZR dependence on $\phi$ shown in the top left panel,"
The lowest-\(\phi\) quintile exhibits the lowest average metallicity, while the metallicity trend with the four higher-\(\phi\) quintiles are less clear. The lowest-\(\phi\) galaxies may have insufficient star formation to chemically enrich the entire gas reservoir, or preferentially live in systems with an excess of HI gas that dilutes the metallicity. Conversely, metallicity and HI excess exhibit a strong anti-correlation in the bottom left panel, where the five MZRs of the subsamples form a well-defined decreasing sequence with increasing \(\gamma\). This anti-correlation can be naturally explained if HI excess is the underlying driver of MZR.

The observed metallicity trend with \(\phi\) indicates that there exists some weak positive correlation between \(\phi\) and \(z_g\), in apparent contradiction with the so-called “fundamental metallicity relation” (FMR), which states that the gas-phase metallicity is anti-correlated with star formation rate. This apparent discrepancy can be explained by the combination of three factors: 1) we adopted the Bayesian metallicity estimates from Tremonti et al. (2004), which is known to show a non-monotonic trend of metallicity with SFR in this mass range (Yates et al. 2012); 2) we binned galaxies by \(\phi\), i.e., the ranking order of sSFR at fixed \(M_*\), rather than their absolute values of SFR; 3) we employed a volume-limited sample of galaxies, therefore eliminating the systematic bias in the MZR measurement due to high-SFR outliers (Telford et al. 2016). A detailed demonstration of the explanation is beyond the scope of this paper, and we will expand our investigation of FMR in a future work (Zu et al. in prep).

To understand the strong metallicity trend with \(\gamma\) and the lack of such a trend with \(\phi\), we measure the 2D relative metallicity distribution of star-forming HI galaxies on the \(\gamma\) vs. \(\phi\) plane, as shown on the right panel of Figure 7. The colour of each pixel represents the average relative metallicity \(Z_{\text{gas}}\) at given \(\gamma\) and \(\phi\), colour-coded by the vertical colour bar on the right. The dashed contour highlights the high number density region that encloses 90% of the galaxies, revealing a weak anti-correlation between \(\gamma\) and \(\phi\). Clearly, the relative metallicity exhibits a strong dependence on HI excess, so that galaxies with high \(\gamma\) have significantly higher \(z_g\) than those with low \(\gamma\), regardless of their relative sSFR. Conversely, the relative metallicity shows little dependence on \(\phi\) at fixed \(\gamma\), suggesting that SFR is a secondary driver of the gas-phase metallicity of a galaxy compared to the amount of excess HI gas in that system. Therefore, the lack of a clear metallicity trend with \(\phi\) in the top left panel of Figure 7 is expected, and the lowest metallicity exhibited by the lowest-\(\phi\) quintile can be entirely attributed to the anti-correlation between \(\gamma\) and \(\phi\), which maps the lowest-\(\phi\) quintile galaxies to the highest-\(\gamma\) (hence the lowest \(z_g\)) galaxies.

To summarize, our result in Figure 7 provides strong evidence that the scatter in the MZR is primarily tied to the amount of excess HI gas in galaxies \(\gamma\), rather than the relative star formation rate \(\phi\). It is commonly believed that

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**Figure 7.** Top left: Dependence of the mass-metallicity relation (MZR) on specific star formation rate. The inset panel shows the distribution of galaxies on the sSFR-\(M_*\) diagram, with each quintile colour-coded by the colour bar on the top left corner. The MZR of each quintile is indicated by the curve of the same colour in the main panel. The gray dotted curve and the shaded band indicate the median MZR and the scatter, respectively. Bottom left: Similar to the top left panel, but for the dependence of MZR on HI excess. Right: Distribution of the average relative metallicity \(Z_{\text{gas}}\) (normalized by the scatter in the MZR; indicated by the vertical colour bar on the right) on the relative sSFR (\(\phi\)) vs. HI excess (\(\gamma\)) plane. The dashed contour highlights the high number density region that encloses 90% of the sample. Clearly, the variation of relative metallicity \(Z_{\text{gas}}\) across the 2D plane is primarily driven by changes in the HI excess \(\gamma\).
the MZR is mainly shaped by the balance between metal loss due to outflows and metal production by stellar nucleosynthesis yield, both of which are tied to star formation. However, our result suggests that the dilution effect due to inflows may have played a more important role in shaping the gas-phase metallicity than the direct modification of metal abundance. Therefore, we emphasize that it is necessary to explicitly track the evolution of gas reservoir in the analytic or semi-analytic exploration of the MZR.

5 ENVIRONMENTAL DEPENDENCE OF HI

5.1 Red Galaxy Overdensity

Our HI fraction predictor is a function of stellar mass and optical colour, without any direct dependence on the galaxy environment. Although stellar mass and colour both depend strongly on the environment, the time scale over which the environment modifies these two quantities is much longer than that of gas stripping processes. In particular, the HI gas discs of satellite galaxies can be rapidly stripped off by the ram pressure of the hot intra-cluster gas upon infall, while their star formation activities can still last for a couple Gyrs after infall (Wetzel et al. 2013; Simha et al. 2014). For those satellite galaxies, there would be no detectable change in the mass or colour of their stellar component despite a sudden drop in the HI fraction (see the paucity of HI-rich galaxies in the Coma cluster in Figure 2). Therefore, we expect that the scatter in the HI fraction predictor is at least partly driven by the galaxy environment.

Ideally, we would prefer using halo mass as the main environment indicator, as ram pressure scales with the product of the hot halo gas density and the infall velocity squared, both of which depend on halo mass. Furthermore, the observed small scatter in the so-called “Baryonic Tully-Fisher Relation” (McGaugh et al. 2000; Lelli et al. 2016) implies that there exists a strong correlation between HI gas mass and halo mass at fixed stellar mass. However, observationally we can only measure the average halo mass for an ensemble of galaxies (Mandelbaum et al. 2016), but not yet for individual systems. Alternatively, one can employ the empirical group catalogue and estimate halo masses from the abundance matching method (AM), albeit with significant scatter between the AM and true halo masses. Using the Yang et al. SDSS group catalog, Yoon & Rosenberg (2015) found strong radial variation of the HI detection fraction inside the most massive groups. However, they failed to detect any dependence of HI fraction on the AM halo mass, likely due to the combination of scatter and poor statistics (i.e., small number of rich groups).

Inspired by the observed strong correlation between the red galaxy overdensity and halo mass (Rozo & Rykoff 2014; Zu & Mandelbaum 2016, 2018), we adopt the red galaxy overdensity δ_red as a proxy for galaxy environment, defined as

\[ \delta_{\text{red}} = \frac{N_{\text{obs, red}}}{N_{\text{ran, red}}} \]  

(14)

where \( N_{\text{obs, red}} \) and \( N_{\text{ran, red}} \) are the number counts of observed and random red galaxies within a cylindrical volume centered on that blue galaxy. The cylinder has an aperture radius

\[ R = 2h^{-1}\text{Mpc} \]  

and line-of-sight height \( \Delta z = \pm 600\text{km s}^{-1} \). To correct for the survey masks and boundary effect when computing \( \delta_{\text{red}} \), we calculate \( N_{\text{ran}} \) from a random galaxy catalogue that has the same redshift and angular selection functions as the observed red galaxy sample. The cylinder dimension is chosen roughly to match the projected radius and the Fingers-of-God effect of massive cluster. We have verified that our conclusions are insensitive to the choice of cylinder dimension.

If part of the scatter in \( \log f_{\text{HI}} \) is driven by \( \delta_{\text{red}} \), both the HI detection rate \( f_{\text{det}} \) and the average HI excess \( \gamma \) of blue galaxies should depend on \( \delta_{\text{red}} \), so that galaxies that live in high-\( \delta_{\text{red}} \) environments are more likely to be missed by ALFALFA and have a lower \( \gamma \) than those in low-\( \delta_{\text{red}} \) regions. As expected, the open circles in top and bottom panels of Figure 8 show the observed declining trend of \( f_{\text{det}} \) and \( \gamma \) with increasing \( \delta_{\text{red}} \), respectively. The dashed lines are the predictions from the null assumption that there is no dependence of \( \gamma \) on \( \delta_{\text{red}} \), so that the Spearman’s rank correlation coefficient \( \rho_{cc} \) between \( \gamma \) and \( \delta_{\text{red}} \) is zero. In the \( \rho_{cc}=0 \) model, we randomly draw \( \gamma \) values from a standard normal distribution and assign mock \( M_{\text{HI}} \) values to the blue galaxies using \( \log M_{\text{HI}} = \log M_* + (\log f_{\text{HI}}) + \gamma \sigma_{\log f_{\text{HI}}} \). We then pass the mock \( M_{\text{HI}} \) through the best-fitting detection probability model at each redshift, thereby generating a mock SDSS-ALFALFA joint sample free of any environmental dependences. Unsurprisingly, the dashed lines show no trend with \( \delta_{\text{red}} \) in either

![Figure 8](image-url)

Figure 8. Top: HI detection rate as a function of \( \delta_{\text{red}} \). Open circles with errorbars show the HI detection rate in ALFALFA, while the dashed and solid lines are the predictions from two HI mocks that assume different cross-correlation coefficients between HI excess and \( \delta_{\text{red}} \). Bottom: Similar as above, but for the average HI excess as a function of \( \delta_{\text{red}} \). Again, the \( \rho_{cc} = -0.18 \) mock successfully reproduces the ALFALFA observation.

\[ R = 2h^{-1}\text{Mpc} \]  

and line-of-sight height \( \Delta z = \pm 600\text{km s}^{-1} \). To correct for the survey masks and boundary effect when computing \( \delta_{\text{red}} \), we calculate \( N_{\text{ran}} \) from a random galaxy catalogue that has the same redshift and angular selection functions as the observed red galaxy sample. The cylinder dimension is chosen roughly to match the projected radius and the Fingers-of-God effect of massive cluster. We have verified that our conclusions are insensitive to the choice of cylinder dimension.

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Figure 9. Projected cross-correlation functions $w_p$ between HI-detected galaxies and the red (top row) and blue (bottom row) galaxies in SDSS. In each row, the HI-detections are divided into four bins based on their HI gas fraction $f_{HI}$, from HI-poor (left two panels; $[0.01, 0.32]$, $[0.32, 0.56]$) to HI-rich (right two panels; $[0.56, 1.00]$, $[1.00, 3.16]$). In each panel, symbols with errorbars show the $w_p$ between SDSS red/blue galaxies and the ALFALFA detections, while the dashed and solid circles indicate the expected $w_p$ from the two HI mocks with different cross-correlation coefficients between HI excess and $\delta_{red}$, $\rho_{cc} = 0$ and $-0.18$, respectively. The HI mock with a weak anti-correlation ($\rho_{cc} = -0.18$) successfully reproduces the observed projected correlation functions between red/blue and HI galaxies.

5.2 Clustering Dependence on $f_{HI}$

For a more comprehensive study of the environmental dependence of HI, we measure the projected cross-correlation functions $w_p$ between the HI-detected galaxies and the red vs. blue galaxies in SDSS. The projected correlation function is computed as

$$w_p(r_p) = \frac{\int_{r_p}^{r_{\max}} \rho^2(r_p, r_{\pi}) \, dr_{\pi}}{\int_{r_p}^{r_{\max}} \rho^2(r_p, r_{\pi}) \, dr_{\pi}},$$

(15)

where $r_p$ and $r_{\pi}$ are the projected and perpendicular distances between a pair of galaxies, and $\rho^2$ is the redshift-space cross-correlation function between HI and SDSS galaxies. We adopt an integration limit of $r_{\max} = 40 h^{-1}\text{Mpc}$ to reduce the impact due to peculiar motions. The 2D correlation function $\xi^2$ is computed using the Davis & Peebles estimator (Davis & Peebles 1983),

$$\xi(r_p, r_{\pi}) = \frac{N_R / N_D - 1}{H D / H R},$$

(16)

where HD is the number count of HI-SDSS galaxy pairs separated by $(r_p, r_{\pi})$, and HR is the number count of pairs between an HI-detected galaxy and a point in the SDSS random catalogue. $N_P$ and $N_R$ are the number of objects in the observed and random SDSS catalogues. We adopt the Davis & Peebles estimator because it only requires random catalogues for the SDSS samples, for which we have well-defined window functions and masks (but not for ALFALFA). For the SDSS galaxies, we use two separate sets of randoms for the blue and red galaxies, each with ten times the size of the observed sample. We compute the measurement uncertain-

panel, and the $\rho_{cc}=0$ model is thus strongly disfavored by the ALFALFA observations.

To infer the cross-correlation coefficient $\rho_{cc}$ between $\gamma$ and $\delta_{red}$, one method is to find the value of $\rho_{cc}$ that reproduces the observed $f_{det}(\delta_{red})$ in the top panel of Figure 8. Similarly, we generate the $\rho_{cc}<0$ mocks by imposing a negative Spearman’s rank correlation between $\gamma$ and $\delta_{red}$, while keeping the standard normal distribution of $\gamma$ intact. Using a simple minimum $\chi^2$ estimation that takes into account the Jackknife uncertainties of $f_{det}$, we find that the $\rho_{cc} = -0.18$ model (solid line in the top panel) provides the best-fit to the observed $f_{det}(\delta_{red})$.

A second method for inferring $\rho_{cc}$ is to find the best-fitting model that reproduces the observed $f_{det}(\delta_{red})$ in the bottom panel of Figure 8. We repeat a similar $\chi^2$ fitting procedure using the $f_{det}(\delta_{red})$ data, which also yield the best-fitting value of $\rho_{cc} = -0.18$ (solid line in the bottom panel). The excellent consistency between the two independent methods of inferring $\rho_{cc}$ is highly non-trivial, as the first method relies more on the robustness of our best-fitting detection probability model, whereas the second method depends more on the correctness of the best-fitting HI fraction predictor. Therefore, this consistency not only confirms the existence of an environmental dependence of HI in the local Universe, but also demonstrates the efficacy of our comprehensive model for interpreting the SDSS-ALFALFA joint dataset.

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ties using 100 Jackknife subsamples defined over spatially contiguous patches on the sky. We refer readers to Zu & Mandelbaum (2015) for technical details in the construction of random catalogues and \( w_p \) computation.

Similarly, we also compute \( w_p \) between SDSS galaxies and the two mock HI samples constructed in § 5.1, i.e., mock ALFALFA observations assuming different levels of correlation between \( \delta_{\text{red}} \) and \( \gamma \) (\( \rho_{cc} = 0 \) and \(-0.18\)). The \( \rho_{cc} = -0.18 \) mock has shown excellent agreement with the observed \( f_{\text{red}}(\delta_{\text{red}}) \) and \( \langle \delta_{\text{red}} \rangle \) in Figure 8, but comparison between the mock and SDSS-ALFALFA \( w_p \) measurements would serve as a third and more stringent test of the \( \rho_{cc} \) parameter. (Isochrone et al. 2011), the HI sky will be observed to a much higher sensitivity than ever before. \(\chi^2_{\text{red}} \) was derived from three independent tests illustrated in Figure 8 and Figure 9. We note that this inferred correlation between \( \gamma \) and \( \delta_{\text{red}} \) can also be used as an interesting constraint of other models of galaxy formation.

6 CONCLUSION

In this paper, we develop a statistical method to infer the HI-to-stellar mass ratio \( f_{\text{HI}} \) of galaxies from their stellar mass and optical colour, using a volume-limited galaxy sample jointly observed by SDSS and ALFALFA. Compared to the traditional methods, the key feature of our method is its capability of removing the Malmquist bias against low-\( f_{\text{HI}} \) systems in ALFALFA, via a self-consistent modelling of the HI detection rate of each galaxy observed in SDSS. The best-fitting HI fraction predictor has an estimated scatter of 0.272 dex, slightly smaller than the \( \sim 0.30 \) dex reported by traditional methods.

To explore the impact of gas accretion on gas-phase metallicity, we define an HI excess parameter \( \gamma \) as the deviation of the observed \( \log f_{\text{HI}} \) from the expected value (normalized by scatter). We discover a strong secondary dependence of the mass-metallicity relation on \( \gamma \), echoing the findings of Bothwell et al. (2013) and Brown et al. (2018). This secondary dependence defines a fundamental metallicity relation of HI, similar to the fundamental metallicity relation of the star formation rate (Mannucci et al. 2010; Lara-López et al. 2010; Andrews & Martini 2013).

By taking advantage of two tight scaling relations, i.e., the mass-metallicity relation and the star formation main sequence, we define the relative metallicity and relative \( s\text{SFR} \) of each galaxy as the (normalized) deviations from the two respective mean relations. To elucidate the underlying driver of the scatter in the MZR, we examine the 2D relative metallicity distribution on the relative \( s\text{SFR} \) vs HI excess plane. We find that the variation of relative metallicity is primarily driven by the change in HI excess, so that galaxies with higher HI excesses always have lower relative metallicities, regardless of the difference in relative \( s\text{SFR} \). This 2D metallicity map provides strong evidence that the metallicity dependence on HI is more fundamental than on SFR.

Furthermore, the HI excess also depends on the large-scale overdensity environment. Using the red galaxy overdensity \( \delta_{\text{red}} \) as a measure of the large-scale environment, we demonstrate that there exists a weak anti-correlation between HI excess and \( \delta_{\text{red}} \) in the SDSS-ALFALFA joint sample. From the dependence of detection rate and HI excess on \( \delta_{\text{red}} \), we infer the cross-correlation coefficient \( \rho_{cc} \) between the two quantities to be \(-0.18\). The \( \rho_{cc} = -0.18 \) model also successfully reproduces the dependence of HI clustering on \( f_{\text{HI}} \). We believe this anti-correlation can be largely explained by the ram pressure and tidal stripping of HI gas discs in cluster environments (but see Wang et al. 2018).

With the advent of exciting HI surveys like the Square Kilometer Array (SKA; Maartens et al. 2015) and the Five-hundred-meter Aperture Spherical Telescope (FAST; Nan et al. 2011), the HI sky will be observed to a much higher...
depth within a significantly larger volume than ALFALFA. Our method will provide a viable path to the synergy between the next-generation HI surveys and upcoming optical surveys, e.g., the Bright Galaxy Survey program within the Dark Energy Spectroscopic Instrument (DESI; DESI Collaboration et al. 2016). In particular, we expect the method to provide valuable insight into the evolution of HI gas and metallicity in cluster environments (Peng & Maiolino 2014; Li et al. 2018) and the dependence of HI on large-scale tidal environments (Liao & Gao 2018; Alam et al. 2018).

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