Requirements for Investigating the Connection Between Lyman Alpha Absorption Clouds and the Large-Scale Distribution of Galaxies

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ABSTRACT

We model the requirements on observational data that would allow an accurate determination of the degree of association between Lyman $\alpha$ absorbers and peaks in the redshift distribution of galaxies (large-scale structures like clusters of galaxies). We compare simulated distributions of low-redshift Lyman $\alpha$ absorption systems, constrained to be consistent with the distribution observed with HST, with the large-scale distribution of galaxies determined from pencil-beam redshift surveys. We estimate the amount of observational data required from catalogues of Lyman $\alpha$ absorbers and galaxies to allow a statistically significant test of the association of absorbers with large-scale structures of galaxies.

We find that for each line-of-sight observed for Ly$\alpha$ absorption lines (assuming that the entire redshift range out to $z \simeq 0.4$ is observable), redshifts must be obtained for at least $\sim 18$ galaxies brighter than $M_B = -18$ and having redshifts between 0.2 and 0.4. Based on the redshift surveys used in this study, a search radius of $\sim 10'$ from the quasar line-of-sight is required. This will ensure that all peaks in the galaxy redshift distribution are represented by at least one galaxy in the observed sample. If Lyman $\alpha$ absorbers are intrinsically uncorrelated with galaxies, we find that $\sim 8$ lines-of-sight must be observed to show that the distributions are different at the 95% confidence level. However, if a fraction of the Lyman $\alpha$ absorbers are distributed with the peaks in the galaxy distribution, $\sim 38$ lines-of-sight must be mapped for the distribution of both Lyman $\alpha$ absorbers and galaxies in order to determine the fraction of absorbers distributed with the peaks of the galaxy distribution to an accuracy of 10%.

Subject headings: quasars:absorption lines, galaxies:large-scale structure
1. Introduction

Ly\(\alpha\) absorption clouds, observed in the spectra of quasars, are numerous and detectable to high redshift. As intervening absorption systems, they are tracers of the evolution of the gaseous content of the Universe. By understanding the extent to which these absorbers are associated with clusters and large-scale structures of galaxies, we hope to trace the evolution of these structures as well.

The connection between low redshift Ly\(\alpha\) absorbers at \(z \leq 0.5\) and their high redshift counterparts is presently unclear. The high redshift Ly\(\alpha\) absorbers display very little velocity correlation. They have been proposed to originate from intergalactic clouds and, consistent with the absence of clustering, would not be associated with galaxies (Sargent et al. 1980). Ground-based studies of these absorbers have shown strong evolution in their number density (e.g., Bechtold 1994). The situation at low redshifts may be different. The study of Ly\(\alpha\) absorbers at low redshifts has been made possible through the use of the Hubble Space Telescope (HST) to obtain ultraviolet spectra of moderate redshift quasars. These observations have revealed a larger number of absorbers than was initially expected based on the extrapolation of the evolution observed at higher redshifts (Bahcall et al. 1991; Morris et al. 1991; Bahcall et al. 1993a). They appear to be more clustered than high redshift absorbers, although to a lesser degree than galaxies (Bahcall et al. 1995). The relationship between Ly\(\alpha\) clouds observed at low redshifts and large-scale galaxy structures can in principle be determined directly by comparing the observed distribution of the Ly\(\alpha\) absorbers detected in QSO spectra with the distribution of galaxies in the fields of the same quasars.

There have been several examples of individual Ly\(\alpha\) absorption lines which appear to be associated with galaxies lying at the same redshift. Such matches have been found in the directions of the quasars H1821+643 (Bahcall et al. 1992) and PKS 0405−123 (Spinrad et al. 1993) with absorbers having EW > 0.32 \(\text{\AA}\). The matches typically are found to have impact parameters ranging from \(70\, h^{-1}\text{kpc}\) to \(160\, h^{-1}\text{kpc}\) (\(h=H_0/100\, \text{km/s/Mpc}\)) and lie within 500 km/s of the Ly\(\alpha\) absorption line redshift. More recently, Lanzetta et al. (1995) have surveyed galaxy redshifts in the fields of several quasars observed by HST and found 11 galaxy/absorption system matches with impact parameters \(<160\, h^{-1}\text{kpc}\). They conclude that at least \(0.32 \pm 0.10\) Ly\(\alpha\) absorption lines are due directly to intervening galaxies having large halo regions of hydrogen gas. While such studies will help determine if some individual galaxies are associated with Ly\(\alpha\) absorbers, they do not allow us to investigate the relationship between the absorbers and the large-scale structures of galaxies. These studies lack complete redshift sampling of the galaxies in the individual quasar fields and cover limited regions (in angular extent) around each quasar. As a result, the complete line-of-sight distribution of galaxies is not well determined on size scales comparable to that of clusters of galaxies and it is not generally possible to determine if the observed galaxies are part of larger structures (groups or clusters of galaxies).

There is evidence that Ly\(\alpha\) clouds could be distributed randomly with respect to regions of
high galaxy density. In the direction of the quasar 3C273, one of the strongest Lyα lines is found in a region where the nearest luminous galaxy is 10 Mpc away (Morris et al. 1993). The Morris et al. study also found that in a region of high galaxy concentration in the same field, no Lyα lines are found within 36 Mpc. Stocke et al. (1995) have made similar findings using the CfA galaxy redshift survey in the direction of Mrk 501. They find no galaxies brighter than $M_B=-16$ within $100h_{75}^{-1}$ kpc of a $>4\sigma$ Lyα absorption line detection. Although the galaxy coverage is very good for these studies, they probe a relatively small path length in redshift space and therefore intercept fewer strong Lyα lines (observed equivalent widths greater than 0.32 Å) that can then be compared with the galaxy distribution. In general, all the Lyα lines studied in the Stocke et al. work are weaker than the typical Lyα lines observed and studied at higher redshift.

In this paper we investigate what data are necessary to adequately determine the extent and nature of the association between the large-scale galaxy structures and Lyα absorbers observed along the sightlines to distant quasars. Obviously, catalogues of both absorbers and the large-scale structures are needed. Not as clear is the exact nature and size that these databases must take. In the case of the absorption line catalogue we will assume that for the near future the largest and most complete catalogue available will be that being compiled by the HST Quasar Absorption Line Key Project (Bahcall et al. 1993a; Bahcall et al. 1995). Their observations will provide a large and homogeneous catalogue of strong Lyα absorption lines toward over 80 quasars with redshifts between 0.15 and 1.9. However, the number of Lyα absorbers along each individual line-of-sight is greatly reduced from what we see at high redshift and the entire redshift path length for each line-of-sight is not observed for every quasar (in fact most of the quasars were not observed below 1600 Å). Given this catalogue of Lyα absorbers, we have conducted various simulations designed to determine what is required of a redshift survey in the fields of these quasars in order to test the following hypotheses: 1) low redshift Lyα absorbers are uncorrelated with the large-scale structures of galaxies traced by the peaks in the galaxy distribution; 2) some fraction of the Lyα absorbers is associated with the peaks in the galaxy distribution with the remaining absorbers being unassociated. Specifically, we determine the characteristics and number of the galaxies that need to have measured redshifts along each sightline and the total number of QSO sightlines that must be studied in order either to falsify the first hypothesis or to determine the fraction of associated absorbers if the second hypothesis is valid.

2. Analysis

Several factors make the investigation of the relationship between strong absorbers and large-scale structures difficult at redshifts less than 0.2. First, there are extremely few strong Lyα absorption systems at very low redshift (intrinsically, the volume density is low as a practical consequence of the limited amount of path length observed at redshifts less than 0.2). Second, the volume of space surveyed at low redshift is relatively small, yielding few large-scale structures
for comparison. Third, those structures which are present are difficult to identify without a very wide field (on the order of a one degree diameter) redshift survey. As a result, most of the information in studying the relationship between absorbers and large scale structures (at least for the strong lines contained in the Key Project database) will come by comparing the cluster and group distribution to the absorbers at redshifts between 0.2 and 0.5.

In this section we try to determine the rough characteristics (e.g. number of galaxies in each field, how bright, over what redshift range, for how many quasar fields) an incomplete redshift survey would need in order to test either of the hypotheses stated at the end of the introduction.

2.1. Sampling the Galaxy Distribution

If only a small number of galaxy redshifts are obtained near the sightline of a quasar, the redshift distribution of these galaxies does not show obvious peaks and voids and therefore does not yield much information about the location in redshift space of large-scale structures of galaxies. We are interested in determining the minimum number of galaxies which would allow us to best estimate the location of large-scale structures in redshift space. In this section, we determine the minimum number of galaxies which must be observed near the quasar line-of-sight satisfying the following criteria: 1) the subset has the largest possible fraction of its galaxies in the “peak” regions in redshift space and 2) every peak in the true galaxy distribution is represented by at least one galaxy in the subset.

To model the effects of incomplete sampling of the true distribution of galaxies we simulated limited observations of “true” distributions as defined from two very extensive galaxy redshift surveys along different sightlines. The components of these surveys are described in Tables 1 and 2 (Peterson et al. 1986; Broadhurst et al. 1993; Colless et al. 1990; Broadhurst 1994). One survey is in the direction 1043+00 and contains a total of 213 galaxy redshifts. The pencil beam diameter of the survey is about $\sim32'$ corresponding to $6.8h^{-1}\text{Mpc}$ at $z=0.25$. The survey samples galaxies down to an absolute magnitude of $M_B \sim -18$ out to $z \sim 0.54$ where we have assumed $H_o = 75 \text{ km/s/Mpc}$. The second survey is in the direction of the South Galactic Pole at 0055-28 and contains 161 galaxy redshifts. The diameter of the survey beam is about $\sim21'$ corresponding to $4.6h^{-1}\text{Mpc}$ at $z=0.25$. This survey samples galaxies down to an absolute magnitude of $M_B \sim -18$ out to $z \sim 0.46$. Both surveys include a much broader cone covering about 3°, but extending to redshifts of only $\sim 0.1$. The nominal completeness of the various components of each survey is listed in the tables. These surveys provide us with an empirical test bed for modeling the limitations of incomplete redshift surveys in representing the large-scale structures contained in the survey volume.

Figures 1a and b show the redshift distributions of these two surveys collapsed along a single line-of-sight in redshift space. There are obvious peaks in the distributions noted by Broadhurst et
al. (1990) as the pencil-beam intersects large-scale structures. The survey pencil-beam diameters are close to optimal for detecting wall-like topologies on scales comparable to those revealed in the CfA surveys (Szalay et al. 1991). Since we are interested in the large-scale distribution of galaxies, we have conducted our tests using the galaxy distribution in redshift space alone without considering the effects of different impact parameters of the individual galaxies to the QSO sightline. Over the angular fields we are considering, the distribution of the galaxies in redshift space provides ample information about the locations of groups and clusters of galaxies along a sightline. Our tests were designed to simulate actual “observations” of galaxies along these lines-of-sight through randomly selected subsamples of the total data set representing the objects whose redshifts are obtained in a given limited redshift survey. Using these surveys as a representation of the “true” universe, we will investigate how well selected subsets of the sample represent the large-scale distribution of the galaxies including the incidence of clusters or peaks in the redshift distribution.

Our first step is to define a weighting function in redshift space that indicates association with peaks in the redshift surveys. For the redshift distributions of Figures 1a and b, we have chosen a histogram representation with bin size of $\Delta z = 0.01$ so that a typical galaxy cluster or group in the data would be sampled by at least 2 bins. The location and width of the peaks were determined mainly through visual inspection based on apparent over-densities in the galaxy distribution and the width of those over-densities at half of their maximum height in the histogram. The peak locations and widths can also be identified by determining the local noise level (standard deviation) within a 3 bin radius of the assumed peak. Gaussian statistics were used to determine the standard deviation from 6 bins consisting of 3 bins on either side of the peak, excluding those associated with nearby peaks. Our “peaks” are those bins which are $\geq 3\sigma$ above this noise level. Once the location and width of peaks in each galaxy distribution has been determined, we can define our weighting function. The weighting function is designed so that regions in redshift space that are “associated” with a peak have a weight of 1.0 and regions that are well outside the peaks have weights of 0. We can consider each peak as a Gaussian shape having a full-width at half-maximum equal to the peak width. The “peak” regions in our weighting function are then defined as the peak center $\pm 1\sigma$ and the function value within these regions is 1.0. Beyond $\pm 1\sigma$ the function value behaves as a Gaussian, trailing off towards 0 in the “void” regions of redshift space (see Figures 2a and b). In this way, each peak has a finite width in the weighting function.

The weighting function can be used as a measure of the degree of concentration of any subsample of galaxies to the peaks. To do so, the galaxies in a subsample are assigned an initial delta function of unit amplitude, then weighted by multiplication with the galaxy distribution functions in Figures 2a and b for each of the two redshift surveys respectively. The mean function value for the weighted sample measures the concentration in peaks of the redshift distribution, or the averaged probability that an individual galaxy in the sample has a redshift associated with a peak. Obviously, not all galaxies in the redshift surveys fall within a peak according to the weighting function of Figure 2. To satisfy the first criterion mentioned at the beginning of this
section, we investigated what constraints could be placed on subsample selection to allow a higher fraction of the galaxies to fall within the “peaks”. These constraints will point to a sampling strategy for determining the redshift peaks in newly observed samples with the best attainable reliability.

Figures 1a and b show the galaxy distributions with all of the galaxies from each of the smaller surveys providing us with our “true” map of the universe. The low z peaks are more easily identified due to the inclusion of the 3° diameter field which extends to only z<0.1. However, the deep portion of these surveys, necessary in order to consider comparison between the absorbers and large-scale structures, only covers the inner 20′ to 30′ of each survey. The hatched region in Figures 1a and b represents the galaxy distribution within this smaller cone. When only the smaller angular field is considered, it becomes difficult to identify the low redshift peak in the distribution. Since the data set we are using to represent the “true” universe does not allow us to define a peak at low redshift, and for the reasons described at the beginning of this section, we place a lower limit of z=0.2 for identifying peaks in cones of 30′ or less. An upper limit at z=0.4 is imposed by the limitations for obtaining redshifts for a reasonable sampling of the galaxy luminosity function with 4-m class telescopes. Therefore, we have little information from the two surveys used here about the true galaxy distribution above z=0.4 and cannot determine how well galaxies beyond this redshift represent the peaks and voids of the distribution.

Intrinsically bright galaxies have a somewhat higher probability of being in the peaks of the distribution. Supporting evidence is found in the fact that eliminating the data from the redshift surveys having z < 0.2 increases the mean function value determined from the remaining galaxies for both redshift surveys. The less luminous objects observable at low redshifts, seem to be more uniformly distributed. Limiting the observed galaxy absolute magnitude range for a random sampling of the galaxies in the field improves the probability that a given galaxy is a member of a peak in the true galaxy distribution. The redshift surveys we are using have galaxy absolute magnitudes ranging from $M_B \simeq -18$ to $M_B \simeq -21$ based on their apparent magnitudes with $H_0=75$ km/s/Mpc. If we exclude the few galaxies which are fainter than $M_B = -18$, the concentration in peaks is increased. While raising this lower limit may increase the concentration further it also greatly decreases the number of observable galaxies, i.e. there aren’t enough luminous galaxies available to locate and define the peaks in the galaxy distribution. We therefore have chosen to constrain the range of galaxy absolute magnitude to $M_B \leq -18$ which is $\sim 0.44L^*$ (Marzke et al. 1994). This corresponds to an apparent magnitude of $B \leq 23$ at z=0.4. Eliminating galaxies with $M_B > -18$ increases the mean function value from the remaining galaxies in each of the two redshift surveys.

Once we have maximized the fraction of galaxies falling within the peaks in an “observed” subset, we can determine the minimum size subsample that retains this same fraction of galaxies associated with peaks in the galaxy distribution. In addition, these subsets must also have at least one galaxy in the redshift range of each peak in the true galaxy distribution. In this way, we can make certain that all peaks in the true galaxy distribution are represented.
To do this, we simulated “observations” of the true universe by randomly selecting galaxies in the test surveys through sub-cones 5’ in diameter using numerous cones to span each survey out to the largest effective angular size of the survey. For our purposes, simulating observations of galaxies at significant redshift (out to z=0.4), the “effective angular size” is constrained to be the largest angle in each test survey for which galaxy redshifts out to at least z=0.4 are available. For this reason we limit our simulations to the the inner 35’ x 32’ of the 1043+00 survey and the inner 23’ x 10’ of the South Galactic Pole survey. This same simulated observational procedure was repeated with increasing subcone sizes until the maximum size of the survey was reached.

For each “observation”, the galaxies found within the cone were assigned an initial delta function of unit amplitude, then weighted by multiplication with the galaxy distribution function in Figures 2a and b. By averaging the weighted amplitudes of all of the galaxies observed within a cone, a mean function value is determined. For example, if half of the galaxies in a cone fall within 1σ of the central redshift of a peak and the other half fall completely outside, the mean function value for that cone is 0.5, indicating a 50% chance that a given galaxy in that subset was selected from a peak in the true galaxy distribution.

Figures 3a and b show the number of galaxies observed in a cone vs. the mean function value for each of the two redshift surveys with the data sampling limits as described above. Each dot represents a different sub-cone of the total survey within which n galaxies have been observed. These plots contain cone sizes from 5’ in diameter to the largest angular size possible within the survey limits; 35’ x 32’ for the 1043+00 survey and 23’ by 10’ for the SGP survey. It is clear that if all of the galaxies in these surveys which are between the stated redshift and magnitude limits are observed, then the mean function value is 0.68 for the 1043+00 survey and 0.74 for the South Galactic Pole survey. As we examine galaxy sets containing fewer and fewer observed galaxies, the typical deviation from the mean function value for a given subset becomes greater than ∼5% for subsets containing fewer than ∼18 galaxies. For samples with sizes below this limit, the mean function value, expressing the probability that a given galaxy in that subset lies within 1σ of a peak in the true distribution, becomes very uncertain.

These simulations of galaxy observations along a specific line-of-sight suggest that the probability of observed galaxies lying in redshift peaks reaches a maximum between 0.68 and 0.74 where we have limited our redshift range to 0.2≤z≤0.4 and placed a lower limit on the absolute magnitude of $M_B \leq -18$ corresponding to an apparent magnitude of $B \leq 23$ at z=0.4. A subsample of at least ∼18 galaxy redshifts are needed for a representative sample where ∼70% of the galaxies fall within 1σ of a peak in the true galaxy distribution. The angular field of view necessary to obtain this minimum number of galaxy redshifts is r ≃ 11’ in the 1043+00 survey and r ≃ 7’ in the South Galactic Pole survey. These spatial ranges correspond to sampling regions of space ∼2.2 Mpc in size at z=0.3.

The angular diameters quoted here, however, are dependent on the completeness and efficiency for the redshift surveys we used. The selection efficiency for obtaining galaxy redshifts
is critical in determining the angular diameter of the cone required to measure a sufficient number of objects. A simple integration of the galaxy luminosity function (Marzke et al. 1994), in a truncated cone of $r = 10' \prime$ and redshift limits of 0.2 to 0.4 suggests that we would find $\sim$120 galaxies with $M_B \leq -18$, about six times the number of galaxies within the same constraints for the surveys used here. It is therefore possible to limit the angular area required for search around each quasar by increasing the efficiency with which redshifts are obtained. There is a lower limit set by the physical area subtended by the large-scale structures themselves. One might consider a Mpc or so as the minimum diameter below which the search becomes more relevant to individual objects rather than clusters or associations. A field with a radius of 4' at $z=0.2$ would encompass over 1 Mpc and provide enough galaxies for redshift measurement to meet our criterion given a $\sim$100% efficiency for obtaining redshifts. With a $\sim$60% efficiency, the minimum number of galaxy redshifts could be obtained within a 5' radius field.

There are $\sim$10-12 sets containing 18 or more galaxies in each of the two redshift surveys. These subsamples can then be binned in redshift space in the same manner as the entire redshift survey. If we consider each galaxy in these subsets as a peak, we find that each peak, defined from the total galaxy distribution within the $0.2 \leq z \leq 0.4$ redshift range, is represented. By assuming that each galaxy represents a peak in the galaxy distribution, all true peaks are located. However, only $\sim$70% of the galaxies are associated with true peaks; approximately 30% of the galaxies will actually lie in the “void” regions of the true galaxy distribution. We find that overestimating the number of peaks by 30% is the minimum error which can be attained in defining peaks in redshift space while also minimizing the total number of galaxy redshifts obtained.

### 2.2. Comparing the Galaxy and Lyman Alpha Cloud Distributions

Comparison of the distributions of galaxies and absorbers requires not only the sample of galaxies, for which the determination of large-scale structure was discussed in the previous section, but also a sample of absorbers. For this paper we will assume that low redshift Ly$\alpha$ absorption systems detected on lines-of-sight to quasars at redshifts greater than 0.4 are consistent in distribution and number with what has been observed by the HST Quasar Absorption Line Survey (Bahcall et al. 1993a). Therefore, determining the number of Ly$\alpha$ clouds necessary to test the two hypotheses stated in the introduction can be restated as determining the minimum number of lines-of-sight that need to be observed for absorbers and galaxies.

We are assuming that the absorption lines found along a line-of-sight will be drawn from the simplified line distribution function

$$\frac{dN}{dz} = \left( \frac{dN}{dz} \right)_o (1 + z)^\gamma$$

with $(dN/dz)_o = 18$ and $\gamma = 0.3$ as determined from HST observations by Bahcall et al. (1993a) based on the detection of Ly$\alpha$ lines having rest equivalent widths of 0.32$\,\AA$ or greater. To model
the case of no association between large-scale structure and absorbers, simulated samples of Ly$\alpha$ absorbers were generated with no velocity correlations on small scales and with a line-of-sight density evolution with redshift as defined above. Eq. 1 was used to determine the probability that a line would exist ($\Delta N$) within a certain redshift bin ($\Delta z$). The redshift bin size was chosen to be the resolution element size for the HST Quasar Absorption Line Survey which is $\Delta v=270$ km/s. We then divided the redshift space between 0.2 and 0.4 into bins of this size. A random number generator was used to produce a number between 0 and 1 for each bin. If that number was less than the probability $\Delta N$ determined for that bin, a Ly$\alpha$ line would be generated at that redshift. This same procedure was repeated to simulate line lists from many lines-of-sight with the total distribution with redshift consistent with the global distribution found by Bahcall et al.

To obtain a quantitative measure of association between the peaks in the galaxy distribution and absorbers, the redshift distribution of each random absorption line list was then weighted by the galaxy redshift distribution function (see Figures 2a and b). We compare the distribution of Ly$\alpha$ line function values to similarly computed distributions of function values for subsets of galaxy redshifts drawn from the 1043+00 and SGP surveys. The number of galaxies along each sightline was chosen in a manner consistent with the discussion in section 2.1, providing the minimum number of galaxy redshifts to represent the peaks in the galaxy distribution. The dotted line in Figure 1 shows the arbitrarily normalized galaxy selection function for the $0.2 \leq z \leq 0.4$ range for each survey based on a Schechter luminosity function (Marzke et al 1994). The effects of this function are ignored in our simulations when choosing galaxy subsets for comparison with the Ly$\alpha$ line lists since variations of this function are small over this redshift range.

As the number of sightlines increases, the number of galaxies in the comparison sample must also increase. Since the previous experiment indicates that $\sim 18$ galaxies are necessary to characterize adequately the galaxy distribution along a single sight line, we conducted this test assuming that the minimum number of galaxy redshifts will be measured for each sightline. A set of 18 galaxies is compared with the Ly$\alpha$ line list for a single sight line, a set containing 36 galaxies is compared with 2 Ly$\alpha$ line lists, etc. Since the galaxies in these distributions are drawn from the redshift range of $z=0.2$ to 0.4, the Ly$\alpha$ line lists generated contain lines within this range which results in an average of 3.9 absorbers per line list.

The Kolmogorov-Smirnov test (KS test) was used to find the level of confidence at which the distribution of weighting function values for observed galaxies is different from that of the locally Poissonian distribution of Ly$\alpha$ lines. Figure 4 shows this KS probability as the number of observed sightlines increases. To show that the two distributions are different at the 90% confidence level, at least 6 lines-of-sight must be observed or $\sim 24$ Ly$\alpha$ lines in total. At least 8 lines-of-sight are necessary for 95% confidence and 12 are required to show that they are different at the

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3 All tests comparing the Ly$\alpha$ line distributions to that of galaxies along a sightline were performed separately using both redshift surveys. Because the results from each survey were consistent with one another, we will only present results for the 1043+00 survey in the text and figures.
99% confidence level. This exercise demonstrates that the distribution of Ly\(\alpha\) lines in redshift space, constructed to show locally Poissonian statistics, obviously differs from a representative distribution of galaxies which shows clustering. If the Ly\(\alpha\) lines are distributed in this way, we would need to study only \(\sim 8\) to 12 sightlines for absorbers (observed at the level of the HST Key Project observations) and galaxies (with a limited redshift survey as discussed above) in order to recognize that the absorbers and large-scale structures of galaxies are not distributed in the same manner.

Next we explored the possibility that only a fraction of Ly\(\alpha\) lines are randomly distributed while the remainder are associated with the peaks in the galaxy distribution. We compared the degree of concentration in the peaks for Ly\(\alpha\) line lists generated from the power-law equation to that for Ly\(\alpha\) lines distributed like the peaks in the galaxy distribution. Remember, however, that unless we have a very large number of galaxy redshifts along the quasar sightline, we cannot know the location of actual peaks in the galaxy distribution with absolute certainty. Our earlier simulations indicate that with subsets of at least \(\sim 18\) galaxies (meeting the selection criteria detailed above and assuming each of these galaxies is located in a “peak”) we overestimate the number of peaks by 30%. The weighting function values (which measure the degree of association with peaks) for these generated Ly\(\alpha\) lines must therefore be corrected for a 30% excess in apparent associations. We chose to do that in the simulations by generating an “associated” line list from the “observed” galaxy sample, then applying the “true” weighting function to assign zero weight to the 30% which are spurious associations.

The peak associated Ly\(\alpha\) lines are generated in much the same way as those with global properties characterized by Eq. 1 as described earlier in this section. The only difference is that we have substituted the probability function of Eq. 1 with an empirically determined probability function representing the peaks. In other words, rather than using the pure power-law equation for \(dN/dz\) in Eq. 1 to determine the probability that a line will be placed in a particular \(\Delta z\) redshift bin, we are using the global trend for \(dN/dz\) modified by the location and width of peaks as determined by subsets of galaxies containing at least \(\sim 18\) galaxies. This distribution assumes the same number density of low-redshift Ly\(\alpha\) lines as described by Eq. 1 but redistributed on small scales to correlate with the peaks.

The Monte-Carlo generated Ly\(\alpha\) lines were then weighted by the true galaxy distribution functions in Figures 2a and b so that each line received a value based on its redshift location with respect to true peaks in the galaxy distribution. Figure 5 shows the normalized cumulative distributions of these function values for 10000 Ly\(\alpha\) lines distributed with no association to the peaks in the galaxy distribution (solid line) and 10000 Ly\(\alpha\) lines distributed like the peaks in the galaxy distribution (dashed line). Within the distribution of random Ly\(\alpha\) lines there are many that fall at or near a function value of 0 since many random lines will fall between peaks based on our weighting function in Figures 2a and b. For the peak-distributed lines (dashed line), note that fewer fall at 0 and many more have function values of 1.0 since they have been generated to be associated with peaks in the galaxy distribution. Still, some have function values at or near 0 due
to the fact that the peak-associated lines have been generated based on peak locations as defined from subsets of ~18 galaxies. Since only 70% of these galaxies actually fall in a true peak, we have generated some peak-associated lines at redshifts where an actual peak doesn’t exist. These lines will have values at or near 0 since they don’t fall in the “true” peak regions in redshift space. Any mixture of the two distributions in Figure 5 will produce a cumulative distribution located between them determined by the percentages of each parent population contained within it.

The aim of our test is to characterize to what accuracy we can determine the composition of an observed distribution of Lyα lines taken from several lines-of-sight based on the proximity of each Lyα line to peaks in the galaxy distribution. In other words, if we obtain a sample of Lyα line redshifts, how accurately can we determine the fractional contribution of each of the two parent distributions, purely unclustered lines and galaxy peak associated lines, and what is the minimum number of Lyα lines which must be observed to make this determination? Our approach differs from that taken by Bahcall et al (1993a) who calculated the number of Lyα lines necessary to detect clustering within the Lyα sample alone. We are interested in determining the number of Lyα lines needed to identify a population clustered with galaxies when we are able to make use of additional information about the distribution of the galaxies along the same sight lines as the absorbers. For our simulation the distribution of galaxies is determined from the simulated limited redshift surveys described in section 2.1.

Our approach is to generate samples of lines for which the fractional contribution of each parent distribution is known. The sample is then compared through a KS test to a set of distributions with a range in fractional composition of the two parents (Figure 5). This computation produces a distribution of KS probability values peaking at the fractional mixture of the two parents which best fits the sample. Since the KS test determines the probability that two distributions are different, a large KS probability indicates a lack of difference or a “best fit”.

Many random draws of samples containing the same number of lines, for example, in a 50:50 ratio will produce a range of KS probability distributions where the best fit will vary around ~50%. To take into account the variation in peak location and width of the KS probability distribution, we generated distributions for 100 random draws for each sample of Lyα lines and determined the mean fractional composition value and its variance at the KS probability peak. The range in fractional composition of parent populations which includes 95% of the random draws is represented by the mean peak value ± 2 times the standard deviation of the distribution. We computed this range for samples containing various total numbers of Lyα lines and fractional compositions. We find that the standard deviation for Lyα line sets containing the same total number of lines remained roughly the same regardless of composition value.

Figure 6 reveals that as more Lyα lines are considered, the range of fractional composition values decreases allowing for the true composition to be more accurately determined. At 150 Lyα lines, approximately 38 lines-of-sight, enough Lyα absorption lines are observed within the $0.2 \leq z \leq 0.4$ redshift range to determine the fractional composition to within 10% of the true value
for \(\sim 95\%\) of the generated samples. The accuracy improves slowly as the number of lines-of-sight increases. If fewer than 5 sightlines are observed, these tests suggest that it is impossible to determine the composition of the sample, i.e. what fraction are associated with large-scale structures if some fraction are random with respect to these structures. 5 sightlines are equivalent to only \(\sim 20\) Ly\(\alpha\) lines in the redshift range being considered. We find that the measurement of Ly\(\alpha\) absorbers from at least 38 sightlines is necessary (equivalent to 150 Ly\(\alpha\) lines), in addition to the minimum number of galaxy redshifts needed, to show that the population of Ly\(\alpha\) absorbers is composed of a mixture of those which are distributed like the peaks in the galaxy distribution and those which are uncorrelated with the peaks in the galaxy distribution.

3. Discussion

Recent surveys to identify and obtain redshifts for galaxies projected near quasar sightlines have been designed to study the relationship between absorbers and individual galaxies. Other surveys have concentrated on the association of quasars with host galaxy clusters. Often the field of view is limited by the cassegrain spectrograph with multi-object capability. Such samples can serve as the starting point for defining the larger-scale structures. They may not be adequate to do so in their current form; they were not designed to be.

Two examples from current surveys illustrate the point. The quasar PKS 0405-123 has been observed with HST revealing 14 Ly\(\alpha\) absorbers within the redshift range of \(z=0.081\) to 0.540 (Bahcall et al. 1993b). Recently, Ellingson et al. (1994) published a list of 29 galaxy redshifts in this field ranging from \(z=0.16\) to 0.66. This redshift survey covers a field 5.9' by 3.8' around the quasar and is 78% complete to \(r=21.5\). The redshift range of this survey is comparable to the surveys used in our significance tests. In the optimal redshift range determined for the surveys used in our simulations \((z=0.2\) to 0.4), 10 galaxy redshifts are measured which meet our absolute magnitude criterion of \(M_B \leq -18\). We assume B-R colors for these galaxies of up to 1.75 (Colless et al. 1990). Figure 3 indicates that a sample of 10 galaxy redshifts does not meet the minimum sample size requirements to ensure that 70% of the galaxies will fall within \(1\sigma\) of a peak in the galaxy redshift distribution.

As another example, consider the available observations of the field of the quasar 3C 351. This quasar has also been observed with HST revealing \(\sim 16\) Ly\(\alpha\) lines within the redshift range of \(z=0.092\) to 0.370 (Bahcall et al. 1993a). Included in the survey of Lanzetta et al. (1995) are redshifts for 10 galaxies in this field, ranging from \(z=0.07\) to 0.370. Their survey of this field covers \(\sim 5'\) in diameter and is 57% complete down to \(r=21.5\). Within their sample, 4 galaxies meet our redshift range and absolute magnitude criteria. Again, the small sample size does not ensure that 70% of these galaxies lie within \(1\sigma\) of a peak in the galaxy redshift distribution.

More extensive redshift coverage of galaxies in wider fields around these quasars would allow
for a better determination of the large-scale distribution of galaxies along the sightline. Although some observations of galaxies in the fields of many more of the HST observed quasars exist, not enough redshifts have been measured to yield statistically significant results. Ideally, it would be necessary to obtain enough galaxy redshifts along the line-of-sight to a quasar having a redshift at or beyond \( z=0.4 \) to satisfy the requirements determined in section 2.1. In this way, all the peaks in the galaxy distribution out to the QSO redshift would be represented with an overestimation of 30%. We would then need to obtain these surveys in the fields of at least \( \sim 8 \) QSO’s for which HST spectra are available in order to test the hypothesis that the \( \text{Ly} \alpha \) absorption clouds are uncorrelated with the peaks in the galaxy distribution. It would be necessary to obtain these surveys in the fields of \( \sim 38 \) QSO’s with HST spectra to determine to 10% accuracy what fraction of the \( \text{Ly} \alpha \) absorbers are associated with the peaks in the galaxy distribution.

4. Conclusions

We have conducted various numerical experiments designed to investigate the significance with which the association between \( \text{Ly} \alpha \) absorption clouds and the large-scale distribution of galaxies can be determined. We have found that in pencil-beam redshift surveys extending to redshifts of \( z\sim0.5 \), the maximum probability of selecting a subsample of galaxies such that every peak in the distribution is represented by at least one galaxy occurs when redshifts for at least 18 galaxies are obtained between \( z=0.2 \) and 0.4 with \( M_B \leq -18 \) and drawn from an angular radius of \( \sim 10' \) around the quasar line-of-sight. The limited cone size of the surveys used in these simulations is the main factor pushing us to the \( z \geq 0.2 \) region to sample volumes of space large enough to detect large-scale structures. Based upon these redshift surveys, it would be necessary to have at least \( \sim 18 \) galaxy redshifts in each quasar field to populate all the peaks in the galaxy distribution along the line-of-sight. Without knowing the true galaxy distribution along a sightline, we conclude that \( \sim 70\% \) of the \( \sim 18 \) galaxy redshifts measured will fall within \( \sim 1\sigma \) of a true peak in the galaxy distribution. A typical sightline must be surveyed in a cone of radius \( \sim 10' \) to get this number of redshifts down to \( B=23 \) at \( z=0.4 \) for the surveys used in this study, typical of a 4-meter class telescope redshift survey.

If the \( \text{Ly} \alpha \) absorption clouds are uncorrelated with the peaks in the galaxy distribution, we find that at least 8 lines-of-sight must be observed to show that the distribution of galaxies and that of the absorbers is different at the 95% significance level. However, if some fraction of the \( \text{Ly} \alpha \) absorbers is distributed like the peaks in the galaxy distribution and some fraction is uncorrelated, we find that \( \sim 38 \) lines-of-sight must be observed to determine the fraction (to 10% accuracy) of absorbers which are distributed like the galaxies.

Our test results clearly indicate that more data are needed in order to draw reliable conclusions about the extent and nature of the association between \( \text{Ly} \alpha \) absorption clouds and the
peaks in the galaxy distribution along the line-of-sight. Fortunately, several research groups are actively obtaining redshifts for galaxies in the fields of quasars observed with HST.

We would like to thank Tom Broadhurst for providing the redshift surveys used in this paper in electronic form. Thanks also to Gary Schmidt, Joe Shields, Jill Bechtold and Ata Sarajedini for helpful conversations. We thank Simon Morris for his valuable comments and suggestions for improving this paper. B.T.J. acknowledges support for this work by NASA through grant number HF-1045.02-93A from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Incorporated, under NASA contract NAS5-2655.
Table 1. Redshift Survey Data for 1043+00 Field

| # of Galaxies | $z$ range   | $B$ range   | field size   | completeness | ref. |
|---------------|-------------|-------------|--------------|--------------|------|
| 70            | 0.0 to 0.097| 13.95 to 17.19 | 209.3’ x 213.2’ | 74%          | 1    |
| 53            | 0.0 to 0.213| 17.32 to 19.69 | 34.3’ x 33.5’  | ~80%         | 4    |
| 108           | 0.0 to 0.438| 19.70 to 20.77 | 36.8’ x 31.8’  | ~80%         | 4    |
| 32            | 0.0 to 0.543| 21.05 to 22.50 | 5.3’ x 12.0’   | 81%          | 3    |

Table 2. Redshift Survey Data for South Galactic Pole Field

| # of Galaxies | $z$ range   | $B$ range   | field size   | completeness | ref. |
|---------------|-------------|-------------|--------------|--------------|------|
| 75            | 0.0 to 0.133| 13.82 to 17.50 | 208.8’ x 209.0’ | 52%          | 1    |
| 59            | 0.0 to 0.444| 20.50 to 21.50 | 22.8’ x 9.5’   | 84%          | 2    |
| 27            | 0.0 to 0.564| 20.71 to 22.42 | 5.3’ x 12.0’   | 80%          | 3    |

References. — (1) Peterson et al. 1986; (2) Broadhurst, Ellis & Shanks 1993; (3) Colless et al. 1990; (4) Broadhurst 1994.
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FIGURE CAPTIONS

Figure 1 - The redshift distribution of galaxies in the 1043+00 survey (a) and the South Galactic Pole survey (b). The hatched region represents the galaxies within the inner $35' \times 32'$ for the 1043+00 survey and the inner $23' \times 10'$ for the SGP survey. These are the regions used in the simulations. The dotted line is the arbitrarily normalized selection function for galaxies based on a Schechter luminosity function (see text in Section 2.2).

Figure 2 - Peak-defining function based on the distributions for each of the two surveys in Figure 1. The location and width of each peak was determined by smoothing the galaxy distribution to find the average background galaxy level and defining the peaks as any points which fell above $1\sigma$ of this background. Regions of redshift space within $1\sigma$ of a peak in the galaxy distributions of Figure 1 have function values of 1.0. Beyond $1\sigma$, the function value behaves as a Gaussian.

Figure 3 - Number of observed galaxies vs. the mean function value for each of the two surveys in Figures 1 and 2. The mean function value is an average of the $n$ observed galaxies’ function values as determined from Figure 2.

Figure 4 - KS probability that the distribution of function values for simulated Lyα absorption lines is different from the distribution of function values for a subset of galaxies as a function of the number of sightlines observed. For a single sightline, $\sim 3.9$ Lyα lines are observed in the redshift range 0.2 to 0.4. A small KS probability indicates that the two distributions are different.

Figure 5 - Cumulative distributions of the function values for Monte-Carlo generated Lyα lines distributed as a power-law with no small scale velocity correlation (solid line) and Lyα lines distributed like the peaks in the galaxy distribution as defined from the minimum number of galaxies necessary in an incomplete redshift survey ($\sim 18$) (dashed line). The function value is determined for each simulated Lyα line from the function shown in Figure 2.

Figure 6 - Twice the standard deviation in the fractional composition of a sample of Lyα lines as a function of the number of Lyα lines in the sample (see text). At $\sim 150$ Lyα lines (corresponding to $\sim 38$ lines-of-sight) enough absorption lines are observed within the $z=0.2$ to 0.4 redshift range to determine the fractional composition to within $\pm 10\%$ of the true value for $\sim 95\%$ of the generated samples.
2 x Std. Dev. in Fractional Composition