Corner states of light in photonic waveguides

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The recently established paradigm of higher-order topological states of matter has shown that not only edge and surface states but also states localized to corners, can have robust and exotic properties. Here we report on the experimental realization of novel corner states made out of visible light in three-dimensional photonic structures inscribed in glass samples using femtosecond laser technology. By creating and analysing waveguide arrays, which form two-dimensional breathing kagome lattices in various sample geometries, we establish this as a platform for corner states exhibiting a remarkable degree of flexibility and control. In each sample geometry we measure eigenmodes that are localized at the corners in a finite frequency range, in complete analogy with a theoretical model of the breathing kagome. Here, measurements reveal that light can be ‘fractionalized’, corresponding to simultaneous localization to each corner of a triangular sample, even in the presence of defects.

Very recently, a paradigm shift in the theory of topological phases occurred when it was realized that the presence of crystalline symmetries can lead to robust lower-dimensional topological boundary states localized to the corners of two- (2D) or three-dimensional (3D) lattices or the hinges of 3D lattices. These higher-order topological phases are protected by crystal symmetry and, as such, their protection does not carry the same robustness as ‘ordinary’ topological states. Instead they depend on crystalline symmetries and can be viewed as a variant of crystalline topological insulators. The first experimental realizations of such higher-order boundary modes have been achieved very recently: the quadrupole topological insulator in topologic circuits, mechanical metamaterials and microwave circuits, and the corner modes (predicted to appear in breathing kagome lattices) in acoustic metamaterials.

Here, we demonstrate a radically different realization of breathing kagome corner modes using optical waveguides inscribed in glass samples using femtosecond laser technology (Fig. 1a). This platform has previously been used to probe other topological band structures. In particular, chiral edge states akin to those of Chern insulators have been observed by making use of helical waveguides and protected photonic mid-gap defect modes have been probed in distorted honeycomb arrays, while localized bulk modes have been reported in Lieb lattice arrays.

Our kagome structures are arranged in two different geometries, the rhombus and equilateral triangle shown in Fig. 1, and our measurements demonstrate that light can be confined to corners/edges and guided with exquisite control by varying the relative waveguide separation and by tuning the wavelength of the light. These results are remarkably robust and persist in the presence of certain defects, thus paving the way for novel applications of light based on the corner states. Here, we first describe the breathing kagome lattice model and its corner states, then detail how this is realized within our waveguide set-up, and present and elaborate on the experimental results. Technical details on the experiments are provided in the Methods.

In the waveguide set-up, the propagation of light is governed by the paraxial equation (Supplementary Section 1), which in the limit of spatially sharp carving and weak evanescent coupling between the waveguides is accurately modelled by a tight-binding Hamiltonian whose hopping parameters depend on the set-up and the wavelength, ω, of the light. We use this to experimentally simulate the dynamics of the breathing kagome tight-binding model, which is composed of a network of corner-sharing triangles as displayed in Fig. 1b,c with rhombic and triangular sample geometries, respectively. The lattice has two inequivalent lattice spacings, d₁ (dashed lines) and d₂ (solid lines), leading to two effective tunnelling strengths, t₁ ≈ e⁻ᵈ₁/λ, where d = d₁(λ) is set by the experiment and has the dimensions of length. We proceed, assuming that these two nearest-neighbour hopping parameters, t₁ and t₂, are sufficient to understand the breathing kagome system for a range of short enough wavelengths, which is fully consistent with the outcome of our experiments, as discussed in the following. In this limit, the effective tight-binding Hamiltonian is

\[ H = t₁ \sum_{\langle i,j \rangle \in \Lambda} aᵢ^† a_j + t₂ \sum_{\langle i,j \rangle \in V} a_i^† a_j \]  

with \( a_i^† \) creating a state on site \( i \), where the two sums are over the neighbouring waveguides in up- and down-triangles (solid and dashed lines in Fig. 1b,c). This set-up naturally supports zero-energy states (red bands, Fig. 1c,f) exponentially localized at corners, which for the rhombus are captured by

\[ |ψ_{\text{corner}}⟩ = N \sum_{m,m'} (−t₁/t₂)^{m+m'} a^†_{\Lambda,m,m'} |0⟩ \]

where \( N \) is a normalization factor and \( a^†_{A,m,m'} \) creates a state on the A sublattice (red, Fig. 1b) in the unit cell numbered \((m,m')\) (starting with \((0,0)\) at the lower left corner). Although no longer exactly solvable in the finite triangular geometry, the corner states are, to an exponentially good approximation, obtained in the \(|t₁/t₂| < 1\) region by the three \( C_3 \) invariant combinations of the rhombus states (see Supplementary Section 3 for more details).

We set \( d₁ = 12 \mu m \) and \( d₂ = 7 \mu m \), such that \(|t₁/t₂| < 1\), and we expect a localized corner state in the lower left corner of the rhombus geometry and similarly in each corner of the triangle in a finite-wavelength window (at small \( λ \)). At wavelengths away from this window the effective tight-binding model is no longer accurately described by equation (1), so there is no reason to expect localized corner states and we consequently expect to observe an essentially random light pattern. The measured light intensity distributions, fully confirming this picture, are displayed in Fig. 2 for the rhombic (top panels) and triangular (bottom panels) lattices, where the corner...
The wavefunctions for the representative parameter choice $t_d$, total weight of the zero-energy, correspond to forming an equal-amplitude superposition of three finite-sized eigenstates, each being close to the $C_s$-invariant combinations of the rhombus corner states, but whose energies are split at finite size. Thus, as we probe the time evolution, the dynamically acquired phases imply relative phase shifts $\Delta \phi = \Delta E \times t$, which should generically lead to light at all corners given that either the time, that is, the length of the waveguides, or the energy splitting, $\Delta E$, is large enough to observe appreciable relative phase shifts $\Delta \phi$. By studying waveguide arrays with smaller lattice spacing and fewer waveguides, which naturally have larger $\Delta E$, we are clearly able to observe such ‘fractionalization’ of light to all three corners of the triangle sample geometry (Fig. 4), in accordance with the theoretical prediction in Fig. 1g. As illustrated in Fig. 5, this holds even in the presence of defects, where this phenomenology is shown to be robust with respect to the corner at which the light is injected.

We corroborated our findings by studying several differently prepared samples as detailed in Supplementary Sections 5 and 6. Several of these samples include significant imperfections, yet exhibit corner states in a finite frequency window as long as we are in the topological regime $|t_1/t_2| < 1$ ($|d_1/d_2| > 1$). This is related to the fact that, although there are local operators that immediately remove the corner states from zero energy, there are (near) zero-energy corner states similar to equation (2) under more general conditions than one may naively expect. This, in turn, relates to the corner states’ provenance being from local destructive interference rather than from fine-tuning and the fact that the interference is a salient feature of the kagome lattice when
Fig. 3 | Observation of corner states in the rhombic lattice of a waveguide array (d₁ = 12 μm and d₂ = 7 μm) with edges consisting of 11 waveguides. 

a. Microscope image of the rhombus lattice. b–d. CCD camera images of light emerging at the output facet of the rhombus lattice waveguide array. 

Coherent light at a wavelength of 720 nm is injected into the waveguide at the corner (indicated by a red circle). The key observation here is that the light stays confined to the corner hosting a corresponding localized corner state of the breathing kagome model, as in b, while it spreads out through the lattice when there is no such corresponding state, as in c and d. The light propagation length is 5 cm. The image in a is on a 1:1 scale with the experimental images (b–d).

Fig. 4 | Observation of the ‘fractionalized’ corner states in a triangular lattice of a waveguide array with d₁ = 11 μm and d₂ = 6 μm with edges consisting of six waveguides. 

a. Microscope image of the triangle lattice. b–d. CCD camera images of light emerging at the output facet of the triangular lattice of the waveguide array. 

Coherent light at a wavelength of 720 nm is injected into the waveguide at the corner indicated by a red circle. The light propagation length is 5 cm. The image in a is on a 1:1 scale with the experimental images in b–d.

Fig. 5 | Observation of the ‘fractionalized’ corner states in a triangular lattice with a defect in a waveguide array with d₁ = 11 μm and d₂ = 6 μm with edges consisting of six waveguides. 

a. Microscope image of the triangular lattice. b–d. CCD camera images of light emerging at the output facet of the triangular lattice of the waveguide array. 

Coherent light at a wavelength of 720 nm is injected into the waveguide at the corner indicated by a red circle, and the defect, the missing waveguide, is marked by a green circle. The light propagation length is 5 cm.

In this Letter, we provide a realization of kagome-based corner states in an optical waveguide set-up. In fact, the breathing kagome model that we realize is remarkably simple, requiring only straight waveguides and hosting corner states of light in the visible part of the spectrum. This is in glaring contrast to the much more intricate set-ups with spiralling optical waveguides needed to realize ordinary first-order topological states. It also contrasts with the recent beautiful, albeit more complicated, photonic realization of protected mid-gap states located near the corners of a distorted honeycomb structure, which requires auxiliary waveguides to excite the zero-energy modes, whereas, in our set-up, directly injecting light at the corner waveguide suffices. Moreover, the ‘fractionalization’ of light observed in the triangular geometries is a striking new feature of our work. Our set-up thus not only provides an optical realization of novel physical phenomena, but its implementation is remarkably simple and stable, hence drastically enhancing the prospects of incorporating these ideas in photonic technological devices, where the confinement of light plays a central role.

Note added in proof: Independent of this work, other photonic realizations of corner states were simultaneously reported, albeit in 2D set-ups, which are entirely different from our 3D coupled waveguides.

Online content

Any methods, additional references, Nature Research reporting summaries, source data, statements of code and data availability and associated accession codes are available at https://doi.org/10.1038/s41566-019-0519-y.

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Author contributions
E.J.B. initiated the research. E.K.K. and E.J.B. derived the theoretical results. A.E.H., A.M., G.A. and M.B. designed and carried out the experiment and performed the data analysis. M.B. supervised the experimental part. E.J.B. and E.K.K. wrote the main text. M.B. and A.E.H. wrote the experimental part. All authors discussed the results and contributed to the final version of the manuscript.

Competing interests
The authors declare no competing interests.

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Methods

Fabrication of waveguide structures. The topological photonic waveguide lattice structures were fabricated using a pulsed femtosecond laser (BlueCut femtosecond laser from Menlo Systems). The femtosecond laser produces light pulses centred at a wavelength of 1,030 nm, having a duration of 350 fs. In this experiment, we used a repetition rate of 1 MHz. The waveguides were written in a Corning EAGLE2000 alumino-borosilicate glass sample with dimensions of $L = 50 \text{ mm}$, $W = 25 \text{ mm}$, $h = 1.1 \text{ mm}$. To inscribe the waveguide structures, pulses of 210 nJ were focused using a ×50 objective with a numerical aperture (NA) of 0.55. The waveguides were written at a depth between 70 and 175 $\mu$m under the surface according to the designed structure, while the sample was translated at a constant speed of 30 mm s$^{-1}$ by a high-precision three-axis translation stage (A3200, Aerotech). The fabricated waveguides support a Gaussian single mode in a region that well includes our entire scanning range from ~700 to 800 nm with a mode field diameter ($1/e^2$) of ~6–8 $\mu$m (see Supplementary Section 2 for details). The propagation losses are estimated to be ~0.3 dB cm$^{-1}$ and the birefringence is on the order of $7 \times 10^{-5}$. After the structures were written in the glass sample, the lateral facets were carefully polished to optical quality with a roughness of 0.1 $\mu$m. The samples were examined using a microscope, and each individual waveguide input, output and position was verified in accordance with the designed structure.

Measurements. For observation of the topological isolation of the waveguide structures, the coherent light beam from a tunable laser (Cameleon Ultra II, Coherent) was launched into the glass sample using a ×100 objective of 0.9 NA, which is sufficient for individual excitation of each waveguide composing the structure, while the output light of the glass sample was collected with a ×100, high-NA objective. A CCD camera was used to collect the image profile of each individual waveguide forming the topological structure.

Data availability

The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.