Hydrodynamic instability of compressible fluid in porous medium

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Abstract. The hydrodynamic Rayleigh –Taylor instability of two superposed compressible fluids in porous medium has been studied. The dispersion relation is derived for such a medium by using normal mode analysis. The RT instability is discussed for various simplified configuration. The effect of porosity and dynamic viscosity has been analyzed and it is observed that porosity and dynamic viscosity have stabilizing effect on the Rayleigh- Taylor instability of compressible fluids.

1. Introduction

The Rayleigh Taylor instability problems attained considerable interest in the last few years, mainly because of its applications in ICF (inertial confinement fusion), space and astrophysical plasma physics. Rayleigh Taylor instability in the compressible medium has been discussed by many authors. Chandrasekhar [1] has given comprehensive treatise which contains solutions of the classical problem of Rayleigh Taylor instability under varying assumptions of hydromagnetics in the incompressible medium. Vandervroot [2] has discussed Rayleigh- Taylor instability for the compressible medium of the fluid. Verma and Verma [3] have investigated the effect of horizontal magnetic field on Rayleigh Taylor instability of compressible fluids. Shivamoggi [4] used barotropy assumption to study the Rayleigh Taylor instability of compressible fluids. Prajapati and Chhajlani [5] have discussed Rayleigh Taylor instability and Kelvin Helmholtz instability with various parameters in porous medium for incompressible fluids. Kango [6] studied Hydrodynamic and hydromagnetic stability of two superposed incompressible Walters B’ viscoelastic fluids in porous medium. In view of the importance of porous and compressible fluids in petroleum engineering, geophysical research and space and astrophysical problems we have discussed the problem of Rayleigh Taylor instability of two superposed compressible fluids in porous medium in the present paper.

2. Linearized equations

We consider that two compressible fluids are separated by a horizontal boundary z = 0 in the porous medium with porosity (ε) and permeability (k1) in presence of gravitational force g (0, 0, -g). The relevant equations of the considered problem are

\[ \left[ \frac{\mu}{\epsilon} \frac{\partial}{\partial t} + \frac{\mu}{k_1} \right] \mathbf{u} = -\nabla \delta \rho + g \delta \rho \quad (1) \]
Where notations have their usual meaning and \( \frac{\partial}{\partial t} \) is representing the speed of sound.

To obtain the solution of linearized equations we assume space and time dependent perturbation

\[
\exp(ik_x x + ik_y y + nt)
\]

Here, \( n \) is the growth rate and \( k^2 = k_x^2 + k_y^2 \), \( k_x \) and \( k_y \) are the wave numbers along x and y axis.

Eq. (1) to (3) becomes in component form as

\[
\begin{cases}
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \rho = 0 \\
\frac{\partial \mathbf{u}}{\partial t} = \nabla \rho + \mathbf{u} \cdot \nabla \rho = \nabla \left[ \frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho \right] = V^2 \left[ \frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho \right]
\end{cases}
\]

Here, \( n \) is the growth rate and \( k^2 = k_x^2 + k_y^2 \), \( k_x \) and \( k_y \) are the wave numbers along x and y axis.

The general solution of above equation is of the form

\[
w = Ae^{q_z z} + Ae^{-q_z z}
\]

Here \( A \) is an arbitrary constant. Applying following boundary conditions on the system of two superposed compressible fluids

(i) The velocity \( w \) must vanish when \( z \) tends to infinity and continuous across the interface.
\( w_1 = w_2 = w_0 \) at \( z = 0 \)

(ii) The total pressure is continuous at the interface.

Now we integrate equation (9) with respect to \( z \) across the interface to satisfy second boundary condition of the system and obtain as follows

\[
\frac{n^2}{\varepsilon} A_1 \left[ \rho_s Q^{-1} Dw \right] + \frac{g w}{\varepsilon} A_1 \rho - g n^2 A_1 \left( \frac{sp w Q^{-1}}{V^2 \varepsilon} \right) = 0 \tag{12}
\]

We obtained a characteristic equation given below.

\[
n^2 \rho_2 s_2 \left( \frac{k^2}{\varepsilon} + \frac{s_2 n^2}{V_2^2} \right)^{-1} \left( \frac{k^2 + \varepsilon s_2 n^2}{V_2^2} - \frac{g^2 k^2}{V_2^2 \varepsilon s_2 n^2} \right)^{1/2} \\
+ n^2 \rho_1 s_1 \left( \frac{k^2}{\varepsilon} + \frac{s_1 n^2}{V_1^2} \right)^{-1} \left( \frac{k^2 + \varepsilon s_1 n^2}{V_1^2} - \frac{g^2 k^2}{V_1^2 \varepsilon s_1 n^2} \right)^{1/2} \\
= \frac{g k^2 \rho_2}{\varepsilon} \left( \frac{k^2}{\varepsilon} + \frac{s_2 n^2}{V_2^2} \right)^{-1} - \frac{g k^2 \rho_1}{\varepsilon} \left( \frac{k^2}{\varepsilon} + \frac{s_1 n^2}{V_1^2} \right)^{-1} 
\]

Here \( s_1 = \frac{I}{\varepsilon} + \frac{\mu}{k \rho_1} \), \( s_2 = \frac{I}{\varepsilon} + \frac{\mu}{k \rho_2} \)

Subscript 1 and 2 represent lower fluid and upper fluid respectively. The dispersion relation (13) is matched with Vandervoot [2] and Verma and Verma [3] and it is found that the dispersion relation is modified due to porosity and permeability of porous medium.

4. Discussion

The dispersion relation (13) is cumbersome to discuss, therefore, the equation is solved by making the assumption that the superposed fluids are highly compressible. This type of assumption has been considered by El- Sayed [7]. Using binomial expansion on dispersion relation (13) we obtain the dispersion relation as

\[
n^6 + n^5 \left[ \frac{4 \varepsilon v'}{k_1} \left( \beta' + \frac{I}{\beta'} \right) \right] + n^4 \left[ 4 V'^2 k^2 + V'^2 k^2 \left( \beta' + \frac{I}{\beta'} \right) + \frac{3 \varepsilon v'^2}{k_1^2} \left( \beta' + \frac{I}{\beta'} + 2 \right) \right] \\
+ n^3 \left[ 6 \varepsilon v'^2 V'^2 k^2 \left( \beta' + \frac{I}{\beta'} + 2 \right) + 2 \varepsilon v'^3 \left( \beta' + \frac{I}{\beta'} + 2 \right) \right] \\
+ n^2 \left[ 6 \varepsilon v'^2 V'^2 V'^2 k^2 \left( \beta' + \frac{I}{\beta'} + 2 \right) + 2 V'^4 k^4 \left( \beta' + \frac{I}{\beta'} + 2 \right) - g^2 k^2 \right] \\
+ n \left[ \frac{g^2 k^2 \varepsilon v'}{k_1} \left( \beta' + \frac{I}{\beta'} \right) + 2 \varepsilon v'^2 V'^2 k^3 \left( \beta' - \frac{I}{\beta'} \right) + 4 \varepsilon v'^3 V'^4 k^4 \left( \beta' + \frac{I}{\beta'} + 2 \right) \right] \\
+ \left[ 2 g V'^4 k^5 \left( \beta' - \frac{I}{\beta'} \right) - g^2 V'^2 V'^2 k^4 \left( \beta' + \frac{I}{\beta'} \right) \right] = 0
\]

Here \( \nu' = \frac{\mu}{(\rho_1 + \rho_2)} \), \( V'^2 = \frac{p}{(\rho_1 + \rho_2)} \), \( \beta' = \frac{\rho_1}{\rho_2} \)

To carry out the effect of porosity and dynamic viscosity on the growth rate of RT instability of compressible fluids in porous medium the dispersion relation is found in dimensionless form by using following substitution in equation (14)

\[
n^* = \frac{n}{\sqrt{gk}}, \ V'^* = \frac{V'}{gk}, \ k^* = k \sqrt{gk}, \ \nu'^* = \frac{\nu'}{\sqrt{gk}}
\]
We have solved dimensionless dispersion assuming arbitrary values of parameters and plotted graphs between growth rate ($n^*$) and wave number ($k^*$). Rayleigh Taylor instability in variation of porosity and dynamic viscosity.

The behavior of dynamic viscosity is observed through the figure (1) which has been plotted by taking $k_1 = 0.1$, $\varepsilon = 0.1$, $V^* = 1.4$, $\beta_1 = 0.5$ and $\nu^* = 0.1$, 0.2, and 0.3 and it has been found that the growth rate decreases on increasing the value of dynamic viscosity that indicate that the dynamic viscosity has stabilizing influence on the growth rate of Rayleigh Taylor instability.

From Figure 2 we have seen the effect of porosity on the unstable mode of Rayleigh Taylor instability of compressible fluids. The figure is plotted by putting $k_1 = 0.1$, $\nu^* = 1.2$, $k_1 = 0.1$, $V^* = 1.4$, $\beta' = 0.5$ and $\varepsilon = 0.1$, 0.2, and 0.3 in dimensionless equation. It has been noticed that porosity has stabilizing effect.

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References

[1] Chandrasekhar S 1961 Hydrodynamic and Hydromagnetic stability (Oxford: Clarendon Press)
[2] Vandervoot P O 1961 Astrophys. J. 134 699
[3] Verma Y K and Verma Pratibha 1963 Proc. Nat. Inst. Sci. India A 29 309
[4] Shivamoggi B K 2012 Z. Angew. Math. and Phys. 63 521
[5] Prajapati R P and Chhajlani R K 2010 J. Porous Media 13(9) 765
[6] Kango S K 2011 Adv. Theor. Appl. Mech. 4 113
[7] El-Sayed M F 2003 Eur. Phys. J. D 23 391