Effect of thresholds on the width of three-body resonances

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Abstract

It has been recently reported an intriguing theoretical result of a narrow three-body resonance with a large available phase space \cite{1}. The resonance was reported in the $N\Lambda\Lambda - \Xi NN$ system near the $\Xi d$ threshold, having a very small width in spite of the open $N\Lambda\Lambda$ channel lying around 23 MeV below the $\Xi NN$ channel. We use first-order perturbation theory as a plausible argument to explain this behavior. We apply our result to realistic local interactions. Other systems involving several thresholds are likely to follow the same behavior.

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I. INTRODUCTION

The coupled $N\Lambda\Lambda - \Xi NN$ system in the dominant S-wave configuration has the quantum numbers $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$, since the coupling between the lower ($N\Lambda\Lambda$) and upper ($\Xi NN$) components of the system is via the $\Lambda\Lambda - \Xi N$ two-body channel with quantum numbers $(i, j^p) = (0, 0^+)$. Therefore, if one adds an additional nucleon also in S-wave, the three-body system will have the quantum numbers $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$. 

The possible existence of a stable bound state of the coupled three-body system $N\Lambda\Lambda - \Xi NN$ was first studied in Refs. [2–4] using the two-body interactions derived in a constituent quark model framework [5]. In that model, the coupled two-body subsystem $\Lambda\Lambda - \Xi N$ in the $(i, j^p) = (0, 0^+)$ channel (the $H$ dibaryon channel) is bound with a binding energy of 6.4 MeV. Thus, in order to search for bound-state solutions of the three-body equations one just needs to calculate the (real) Fredholm determinant for energies below the $HN$ threshold which indeed leads to a stable bound-state solution at about 0.5 MeV below that threshold [2].

However, the most recent analysis of the quark mass dependence of the $H$ dibaryon in $\Lambda\Lambda$ scattering [6, 7] point to the $H$ dibaryon being a resonance above the $\Lambda\Lambda$ threshold. Thus, in order to study possible bound or resonant states of the $N\Lambda\Lambda - \Xi NN$ system one must solve the three-body equations in the complex plane, which makes the numerical problem much harder to deal with. This was done in Ref. [1] using simple separable potentials fitted to the low-energy data of the most recent update of the ESC08 Nijmegen potential [8, 9], that give account of the pivotal results of strangeness $-2$ physics, the NAGARA [10] and the KISO [11] events.

II. FORMALISM

The results of Ref. [1] were obtained taking the nucleon mass as the average of the proton and neutron mass, and the $\Xi$ mass as the average of $\Xi^0$ and $\Xi^-$ mass. Thus, the $\Xi NN$ and $\Xi d$ thresholds are 25.604 MeV and 23.420 MeV above the $N\Lambda\Lambda$ threshold, respectively.
A. The problem

It was found in Ref. [1] that the three-body resonance lies at $E_0 = 23.408 - i 0.045$ MeV measured with respect to the $N\Lambda\Lambda$ threshold, which is 0.012 MeV below the $\Xi d$ threshold. Thus, it is a $N\Lambda\Lambda$ resonance as seen from the lower component or a $\Xi NN$ quasibound state as seen from the upper component. The most interesting feature of this result is the very small width. If one neglects the $\Lambda\Lambda - \Xi N (0, 0^+)$ channel, which is responsible for the coupling between the $N\Lambda\Lambda$ and $\Xi NN$ components, the resonance becomes a bound state of the $\Xi NN$ system at $E_0 = 23.386$ MeV with $E_0$ measured with respect to the $N\Lambda\Lambda$ threshold. Thus, the effect of the $\Lambda\Lambda - \Xi N (0, 0^+)$ channel is to change the three-body eigenvalue by

$$\delta E = 0.022 - i 0.045 \text{ MeV},$$

indicating that the lower three-body channel effectively acts as a perturbation. This is somewhat intriguing since the $\Lambda\Lambda - \Xi N (0, 0^+)$ interaction is not small (see Tables II and III of Ref. [1]).

B. The proposed explanation

In order to provide a plausible argument to explain the above result, we will consider first-order perturbation theory taking the $\Xi NN$ channel as the main interaction and the contribution of the lower channels $N\Lambda\Lambda$ as the perturbation. The argument is very simple. The small binding energy of the $\Xi NN$ system causes the unperturbed wave function to have a very long range in coordinate space while the perturbation is a short-range operator so that the overlap between them is quite small, which results in $\delta E$ being small. Thus, we will calculate

$$\delta E = \frac{\langle \Psi_0 | \delta V | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle},$$

where $| \Psi_0 \rangle$ is the (real) $\Xi NN$ wave function

$$| \Psi_0 \rangle = G_{0}^{\Xi NN} \left( | U_{1;\Xi}^{NN} \rangle + | U_{2;N}^{\Xi N} \rangle + | U_{3;N}^{\Xi N} \rangle \right),$$

with $G_{0}^{\Xi NN}$ the Green’s function for three free particles, $| U_{1;\Xi}^{NN} \rangle$ is the Faddeev component where the two nucleons interact last with the $\Xi$ as spectator, and similarly the other two Faddeev components. They are determined by the last two Eqs. (14) of Ref. [1] neglecting altogether the $(i, j^p) = (0, 0^+)$ channel.
δV is the (complex) perturbation given in lowest order by,

$$
\delta V = \sum_{j=2}^{3} t_{j1;N}^{\Xi-\Lambda\Lambda} G_0^{\Xi\Lambda\Lambda} t_{j1;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Lambda\Lambda-\Xi\Lambda} \cdot i = 2, 3,
$$

(4)

which is shown graphically in Fig. 1. We have used the convention that in both three-body sectors the two identical particles are labeled 2 and 3 with particle 1 being the different one. Using Eqs. (2-4) and taking into account the identity of particles 2 and 3 one obtains,

$$
\langle \Psi_0 | \delta V | \Psi_0 \rangle = 4 \left( \langle U_{1;\Xi}^{NN} | G_0^{\Xi\Lambda\Lambda} t_{31;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Lambda\Lambda-\Xi\Lambda} + \langle U_{3;N}^{\Xi\Lambda} | G_0^{\Xi\Lambda\Lambda} t_{12;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Lambda\Lambda-\Xi\Lambda} \rangle \times t_{3;\Lambda}^{\Lambda\Lambda} (G_0^{\Lambda\Lambda\Lambda} t_{13;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Xi\Lambda\Lambda} | U_{1;\Xi}^{NN} ) + G_0^{\Lambda\Lambda\Lambda} t_{12;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Xi\Lambda\Lambda} | U_{3;N}^{\Xi\Lambda} \rangle \right).
$$

(5)

In Eq. (5), terms of the form \( \langle U_{3;N}^{\Xi\Lambda} | G_0^{\Xi\Lambda\Lambda} t_{31;N}^{\Lambda\Lambda-\Xi\Lambda} \) and \( t_{13;N}^{\Lambda\Lambda-\Xi\Lambda} G_0^{\Xi\Lambda\Lambda} | U_{3;N}^{\Xi\Lambda} \rangle \) do not contribute due to the orthogonality of the spin-isospin states \( \alpha\langle(12)3 | (12)3\rangle_\beta = \delta_{\alpha\beta} \) since the amplitude \( t_{31;N}^{\Lambda\Lambda} \) which belongs to the perturbation corresponds to the two-body channel \( \beta = (0, 0^+) \) while the component \( \langle U_{3;N}^{\Xi\Lambda} | \) of the unperturbed wave function involves only the two-body channels \( \alpha \neq (0, 0^+) \).

The Green’s function that appears in the perturbation term (4) is given explicitly by,

$$
G_0^{\Lambda\Lambda\Lambda}(p_i q_i) = \frac{1}{E - \frac{p_i^2}{2\eta_i} - \frac{q_i^2}{2\nu_i} + i\epsilon},
$$

(6)

where \( p_i \) and \( q_i \) are the Jacobi momenta and \( \eta_i \) and \( \nu_i \) the corresponding reduced masses of the various configurations. Since \( E \) is a positive number this function is singular and moreover it has an imaginary part. The Green’s function attached to the Faddeev components of the unperturbed wave function (3), on the other hand, is given by

$$
G_0^{\Xi\Lambda\Lambda}(p_i q_i) = \frac{1}{E + \Delta E - \frac{p_i^2}{2\eta_i} - \frac{q_i^2}{2\nu_i}},
$$

(7)
with
\[ \Delta E = m_{\Lambda} - m_{\Xi}, \] (8)
so that \( E + \Delta E \) is a small negative number and the function in Eq. (7) is real and it has no singularity although it is sharply peaked at low momenta. In addition, the Faddeev amplitudes \( U_{1;\Xi}^{NN} \) and \( U_{3;N}^{\Xi N} \) are also peaked at low momenta in the \( NN \ (0,1^+) \) and \( \Xi N \ (1,1^+) \) channels, corresponding to the deuteron and \( D^* \) bound states, respectively, which lie very close to threshold. Thus, as mentioned above, the unperturbed wave function has a long range in coordinate space while the perturbation term has the short-range characteristic of hadronic systems. Consequently, the overlap between both terms in Eq. (5) is very small, rendering \( \delta E \) small.

In order to show explicitly this behavior let us consider one of the terms of Eq. (5),
\[ \delta v = \langle U_{1;\Xi}^{NN} | G_0^{\Xi N} t_{31;N}^{N\Xi - \Lambda\Lambda} G_0^{\Lambda\Lambda} t_{12;N}^{\Lambda\Lambda - N\Xi} G_0^{\Xi N} | U_{3;N}^{\Xi N} \rangle. \] (9)

Since in the separable model of Ref. [1] one has that,
\[ t_{31;N}^{N\Xi - \Lambda\Lambda} = g_3^{N\Xi} t_{31;N}^{N\Xi - \Lambda\Lambda} g_1^{\Lambda\Lambda}, \]
\[ t_{12;N}^{\Lambda\Lambda - N\Xi} = g_1^{\Lambda\Lambda} t_{12;N}^{\Lambda\Lambda - N\Xi} g_2^{N\Xi}, \] (10)

FIG. 2: (a) Wave functions \( W_{1;\Xi}^{NN}(q) \) and \( W_{2;N}^{\Xi N}(q) \), in arbitrary units, for the two dominant channels: \( NN \ (0,1^+) \) (solid line) and \( \Xi N \ (1,1^+) \) (dashed line). (b) Real (solid line) and imaginary (dashed line) parts of the diagonal perturbation term \( \delta v'(q,q) \), in arbitrary units.
one can rewrite Eq. (9) as,

\[ \delta v = \langle W_{1;\Xi}^{NN} | \delta v' | W_{2;N}^{\Xi N} \rangle , \]

where

\[ \langle W_{1;\Xi}^{NN} | = \langle U_{1;\Xi}^{NN} G_0^{\Xi NN} g_3^{NN} | , \]

\[ | W_{2;N}^{\Xi N} \rangle = | g_2^{N\Xi} G_0^{\Xi NN} U_{3;N}^{\Xi N} \rangle , \]

\[ \delta v' = \tau_3^{N\Xi - N\Lambda} g_1^{N\Lambda} g_0^{N\Lambda N\Lambda} t_3^{3;N} G_0^{N\Lambda} g_1^{N\Lambda} \tau_{12;N}^{N\Xi - N\Xi} . \]

The expressions (12) depend only on the variables \( q_i \), i.e.,

\[ W_{1;\Xi}^{NN}(q_3) \equiv \langle W_{1;\Xi}^{NN} | q_3 \rangle , \]

\[ W_{2;N}^{\Xi N}(q_2) \equiv \langle q_2 | W_{2;N}^{\Xi N} \rangle , \]

\[ \delta v'(q_3, q_2) \equiv \langle q_3 | \delta v' | q_2 \rangle . \]

We show in Fig. 2(a) the wave functions \( W_{1;\Xi}^{NN}(q) \) and \( W_{2;N}^{\Xi N}(q) \) for the two dominant channels \( NN (0, 1^+) \) and \( N\Xi (1, 1^+) \), respectively, and in Fig. 2(b) the diagonal perturbation term \( \delta v'(q, q) \), where one can see clearly this behavior (note the logarithmic scale of the wave function).

### III. RESULTS

#### A. The separable model

If we now apply the formalism of the previous section to the separable potential model of Ref. [1], it gives

\[ \delta E = 0.014 - i 0.015 \text{ MeV}, \]

which is of the same order of magnitude as the result of the exact calculation given by Eq. (1). This shows that the small value of \( \delta E \), and consequently the very small width, can be understood as resulting from the fact that the \( N\Lambda \Lambda \) channel acts effectively as a perturbation to the \( \Xi NN \) channel when the resonance lies very near the \( \Xi NN \) threshold.

#### B. Application to the local model

The behavior of three-body resonances lying very near the upper channel threshold described in the previous section does not depend on the interactions being separable, but it
is completely general. Thus, we have also used the Malfliet-Tjon type local potentials \[ 12 \] of the \( NN \) subsystem constructed in Ref. \[ 13 \] and those of the \( \Xi N \) subsystem constructed in Ref. \[ 14 \], based in the most recent update of the Nijmegen ESCO8 potentials \[ 9 \]. We show the Fredholm determinant of the \( \Xi NN \) system in Fig. \[ 3 \] for energies very near the \( \Xi d \) threshold, where as one can see the \( \Xi NN \) state lies less than 0.01 MeV above the \( \Xi d \) threshold, so that both the separable and local models predict the resonance very near the \( \Xi d \) threshold and consequently, as we have just shown, will have a very small width.

IV. OUTLOOK

In this letter we have presented a plausible argument to explain the small width of a three-body resonance in a coupled two-channel system lying close to the upper channel in spite of being open the lower one. This is an intriguing result, since the available phase space of the decay channel is quite large, around 23 MeV. We use first-order perturbation theory to explain this behavior. We have applied our result to realistic local interactions.

Let us finally comment that the mechanism we have discussed in this work could also help in understanding the narrow width of some experimental resonances found in the heavy hadron spectra, whose assumed internal structure allow them to split into several different channels \[ 15, 16 \]. It has been explained in Ref. \[ 17 \] how systems with an internal structure
\(QQn\bar{n}\), where \(n\) stands for a light quark, could either split into \((Q\bar{n}) - (nQ)\) or \((QQ) - (n\bar{n})\). For \(Q = c\) or \(Q = b\) the \((QQ) - (n\bar{n})\) threshold is lower than the \((Q\bar{n}) - (nQ)\), the mass difference augmenting when increasing the mass of the heavy quark. Such experimental behavior can be simply understood within the constituent quark model with a Cornell-like potential \[17,18\]. Thus, the possibility of finding meson-antimeson molecules, \((Q\bar{n}) - (n\bar{Q})\), contributing to the heavy meson spectra becomes more and more difficult when increasing the mass of the heavy flavor, due to the lowering of the mass of the \((QQ) - (n\bar{n})\) threshold. This would make the system dissociate immediately. In such cases, the presence of attractive meson-antimeson threshold together with the arguments we have drawn in this work, hint to a possible explanation of a narrow width of some of the \(XYZ\) states lying close to the \((Q\bar{n}) - (n\bar{Q})\) threshold as a meson-antimeson molecule. Similar arguments could be handled for the LHCb pentaquarks, what requires a careful analysis in the models used for the study of these states.

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