Quantum Image Representation Methods Using Qutrits

Ankit Khandelwal
TCS Research
Tata Consultancy Services
Bengaluru, India
ankit27.kh@gmail.com

M Girish Chandra
TCS Research
Tata Consultancy Services
Bengaluru, India
m.gchandra@tcs.com

Abstract—Quantum Image Processing is a recent highlight in the quantum computing field. All previous methods for representing the images as quantum states were defined using qubits. One Quantum Image Representation (QIR) method using qutrits is present in the literature. Inspired by the qubit methods and the higher state-space available for qutrits, multiple QIR methods using qutrits are worked out in this paper. The ternary quantum gates required for the representations are described, and then the implementation details for five qutrit-based QIR methods are given. All the methods have been simulated in software, and example circuits are provided.

Index Terms—Qutrits, Quantum image representation, Quantum computation

I. INTRODUCTION

Quantum Image Processing research is in a nascent stage. Several qubit QIR methods are available in the literature. A review summarising many of them is also available [1]. In particular, FRQI [2], FRQCI [3], MCQI [4] and QRCI [5] methods are considered to provide their qutrit extensions in this paper. To the best of our knowledge, these extensions are novel. An approach to extend the NEQR [6] method was previously followed to define the QTRQ [7] method.

It has been well known for a long time that it is, in principle, possible to use qudits with more than two states to carry out Quantum Information Processing [8]. Compared to qubit, qudit systems provide a more extensive state-space to store and process information and can provide certain advantages such as reduction of the circuit depth/complexity [9]–[11]. In the present work, we are restricting our focus to qutrit-based computing. It is useful to note that there have been physical implementations of qutrit systems [12], [13]; Rigetti has also announced experimental access to qutrits. While Cirq already provides qudit simulation functionality, it is also under development at PennyLane [15].

The paper is organised as follows. The qutrit quantum gates used in implementing the representations are defined in Section II. The steps to implement the new qutrit-based QIR methods are defined and described in Section III. Conclusion and pointers for future work are provided in Section IV.

II. QUTRIT GATES

The qutrit gates used in the implementations are described in this section.

A. Single Qutrit Ternary Gates

A corresponding qutrit gate \( A^{(ij)} \) can be defined for any qubit gate \( A \) by defining \( A^{(ij)} \) such that the action of \( A \) is on \((i,j)\) basis states only \((i,j \in \{0,1,2\}, i \neq j)\) [16]. This allows us to define three \( X \) gates for qutrits:

\[
X^{(01)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

\[
X^{(02)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}
\]

\[
X^{(12)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
\]

By combining the action of these gates, two more single qutrit gates can be constructed:

1) Single-Shift Gate: The action of this gate on a basis state is given by:

\[
[+1] |x\rangle = |(x + 1) \mod 3\rangle
\]

The gate is defined as:

\[
[+1] = X^{(01)} X^{(12)}
\]

2) Double-Shift Gate: The action of this gate on a basis state is given by:

\[
[+2] |x\rangle = |(x + 2) \mod 3\rangle
\]

The gate is defined as:

\[
[+2] = X^{(12)} X^{(01)}
\]

This construction can be used to define Hadamard \( H^{(ij)} \) gates too. But, these versions do not create an equal superposition state.
3) **Ternary Hadamard Gate:** The qudit extension of the qubit Hadamard gate as defined in [7] is considered in this paper.

\[ H = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi i}{3}} & e^{\frac{4\pi i}{3}} \\ 1 & e^{\frac{4\pi i}{3}} & e^{\frac{2\pi i}{3}} \end{pmatrix} \] (8)

The action of the Hadamard gate on the basis states is given by:

\[ H |0\rangle = \frac{1}{\sqrt{3}} (|0\rangle + |1\rangle + |2\rangle), \] (9)

\[ H |1\rangle = \frac{1}{\sqrt{3}} (|0\rangle + e^{\frac{2\pi i}{3}} |1\rangle + e^{\frac{4\pi i}{3}} |2\rangle), \] (10)

\[ H |2\rangle = \frac{1}{\sqrt{3}} (|0\rangle + e^{\frac{4\pi i}{3}} |1\rangle + e^{\frac{2\pi i}{3}} |2\rangle), \] (11)

This version of the qutrit Hadamard gate can thus be used on the |0\rangle state to get to an equal superposition of the three basis states without any phase.

4) **Ternary Rotation Gates:** The definition of elementary one-qutrit gates as given in [16] is considered.

\[ R^{(ij)}_\alpha (\theta) = \exp \left( -\frac{i}{2} \theta \sigma^{(j)}_\alpha \right), j < k, \alpha \in \{x, y, z\} \] (12)

\[ \sigma^{(j)}_x = |j\rangle \langle j| - |k\rangle \langle k| \] (13)

\[ \sigma^{(j)}_y = |j\rangle \langle k| + |k\rangle \langle j| \] (14)

\[ \sigma^{(j)}_z = -i |j\rangle \langle j| + i |k\rangle \langle k| \] (15)

Note that these rotation gates can also be obtained by using the previous construction.

The construction can also be used to get extensions of general single-qubit rotation gate \( U(\theta, \phi, \delta) \). For example:

\[ U^{(12)}(\theta, \phi, \delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \frac{\delta}{2} & -e^{i\phi} \sin \frac{\delta}{2} \\ 0 & e^{i\phi} \sin \frac{\delta}{2} & e^{i(\delta+\phi)} \cos \frac{\delta}{2} \end{pmatrix} \] (16)

5) **Ternary Identity Gate:** The identity gate for qutrits is the 3D identity matrix.

\[ I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \] (17)

Its action on a state |\psi\rangle is given as:

\[ I |\psi\rangle = |\psi\rangle \] (18)

### B. Controlled Qutrit Gates

Control qutrits for ternary gates can be in any of the three basis states. Thus, for each single-qutrit gate, three different controlled variations of two-qutrit gates (the control qutrit can be in any of |0\rangle, |1\rangle or |2\rangle state), nine different three qutrit gates (the control qutrits can be in any of |00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle or |22\rangle state) and so on are possible. The decomposition of any such gate can be assured using the universality of qudit logic [10].

![Fig. 1: Grayscale and RGB images considered for example in the paper.](image)

### III. QUTRIT QUANTUM IMAGE REPRESENTATION METHODS

Some QIR methods using qudits are described in this section. Cirq has been used to simulate these methods. Cirq provides basic functionality to use qudits by allowing the use of custom quantum gates that can be defined for any dimension qudit. The gates used in the paper are not natively available and were defined with the definitions given in the previous section before use, another small contribution to the area through this work. The images considered for encoding are of dimension \( 3^n \times 3^n \).

#### A. **FQRI**

Inspired by the qubit FRQI method, the flexible qutrit representation of quantum images (FQRI) is proposed. The image information is encoded in a quantum state given by the formula:

\[ |I(\theta)\rangle = \frac{1}{3^n} \sum_{i=0}^{3^{2n}-1} (\cos \theta_i |0\rangle + \sin \theta_i |1\rangle + 0 |2\rangle) \otimes |i\rangle \] (19)

2n + 1 qutrits are required to encode a \( 3^n \times 3^n \) image with 2n qutrits used to encode the location information, and the remaining qutrit is used to encode the pixel values. The pixel values are encoded in the \( \theta_i \) values where \( \theta_i \in [0, \pi/2] \). For an 8-bit grayscale image, the pixel values are in the range \([0, 255]\), which are then scaled to be in \( \theta \)’s range.

The state can be prepared in two steps. First, the qutrit Hadamard gate \((H)\) is applied on the 2n qutrits to create a superposition state. Then \( R^{(01)}_y (2\theta_i) \) gates controlled on the 2n location qutrits are applied. Formally:

1) Begin with \( |0\rangle \otimes (2n +1) \) state.
2) Apply transform \( \mathcal{H} = I \otimes H^{\otimes 2n} \):

\[ \mathcal{H}|0\rangle \otimes (2n +1) = \frac{1}{3^n} [0] \otimes \sum_{i=0}^{3^{2n}-1} |i\rangle = |H\rangle \] (20)

### TABLE I: The pixel values for the RGB image in Fig. 1b. The R values taken alone make the pixel values for the grayscale image in Fig. 1a.

| R   | G   | B   |
|-----|-----|-----|
| 0   | 72  | 192 |
| 1   | 255 | 204 |
| 2   | 133 | 79  |
| 0   | 204 | 252 |
| 2   | 252 | 20  |

| Y   | X   |
|-----|-----|
| 0   | 1   |
| 2   | 0   |
| 1   | 2   |
The flexible qutrit representation of RGB quantum images (FQRRI) is proposed here. In this encoding, 2n+1 qutrits are used to encode RGB images. Again assuming 8-bit images, all R, G and B pixel values will be in the range [0, 255]. The image is encoded in the quantum state given by:

\[
|I(\theta)\rangle = \frac{1}{3^n} \sum_{i=0}^{3^{2n}-1} \left( \cos \theta^{gb}_i \cos \theta^{gr}_i |0\rangle + \sin \theta^{gb}_i |1\rangle + \cos \theta^{gr}_i \sin \theta^{gr}_i |2\rangle \right) \otimes |i\rangle
\]  

(23)

Here, \(\theta^{gb}_i\) and \(\theta^{gr}_i\) encode the pixel values of G, B and G, R channels, respectively, of the \(i^{th}\) pixel. The three values are encoded in two angles using the equations:

\[
\theta^{gb}_i = \frac{(G_i \mod 16) \times 256 + B_i}{4095} \times \frac{\pi}{2}
\]  

(24)

\[
\theta^{gr}_i = \frac{(G_i \mod 16) \times 256 + R_i}{4095} \times \frac{\pi}{2}
\]  

(25)

The circuit diagram pertinent to FQRRI representation of the image in Fig. 1a is provided in Fig. 8.

The decoding of the encoded image is probabilistic and depends on the number of shots used. The final state can be measured in the computational basis to get counts of all basis states. The \(\theta_i\) values can be calculated from the probabilities.

This encoding works well for grayscale images. As only two of the three possible states of the pixel value qutrit are being used, this encoding can be hypothesised to be implemented with a qubit-qutrit hybrid model where 2n qutrits are used for encoding the pixel location and a qubit is used for pixel values.

As mentioned earlier, all three states of the pixel value qutrit can be utilised to encode RGB information of colour images. This method is described in the following encoding.

**B. FQRRI**

The flexible qutrit representation of RGB quantum images (FQRRI) is proposed here. In this encoding, 2n+1 qutrits are used to encode RGB images. Again assuming 8-bit images, all R, G and B pixel values will be in the range [0, 255]. The image is encoded in the quantum state given by:

\[
|I(\theta)\rangle = \frac{1}{3^n} \sum_{i=0}^{3^{2n}-1} \left( \cos \theta^{gb}_i \cos \theta^{gr}_i |0\rangle + \sin \theta^{gb}_i |1\rangle + \cos \theta^{gr}_i \sin \theta^{gr}_i |2\rangle \right) \otimes |i\rangle
\]  

(23)

The circuit diagram pertinent to FQRRI representation of the image in Fig. 1a is provided in Fig. 8.

The decoding of the encoded image is probabilistic and depends on the number of shots used. The final state can be measured in the computational basis to get counts of all basis states. Each measurement will provide us with a ternary string. From this string, the last 2n values give us the pixel location, and from the first value, the probability of getting \(|0\rangle, |1\rangle\) and \(|2\rangle\) states can be calculated. Let’s call these probabilities \(p^b_i, p^r_i\) and \(p^r_i\) for the \(i^{th}\) pixel.

\[
\theta^{gb}_i = \sin^{-1} \left( 3^n \times \frac{p_i^b}{2} \right)
\]  

(26)

\[
\theta^{gr}_i = \tan^{-1} \left( \frac{p_i^r}{p_i^0} \right)
\]  

(27)

This gives,

\[
B_i = \left( \theta^{gb}_i \times 4095 \times \frac{2}{\pi} \right) \mod 256
\]  

(28)

\[
R_i = \left( \theta^{gr}_i \times 4095 \times \frac{2}{\pi} \right) \mod 256
\]  

(29)

\[
G_i = \left[ \left( \theta^{gb}_i \times 4095 \times \frac{2}{\pi} \right) \mod 256 \right] \times 16
\]  

(30)

To create the state in (23), the steps to be followed are:

1) Begin with \(|0\rangle^{\otimes(2n+1)}\) state.

2) Apply transform \(\mathcal{H} = I \otimes H^{\otimes2n}\):

\[
\mathcal{H}|0\rangle^{\otimes(2n+1)} = \frac{1}{3^n} |0\rangle \otimes \sum_{i=0}^{3^{2n}-1} |i\rangle = |H\rangle
\]  

(31)

3) Apply 2n controlled \(R^{(01)}_y(2\theta^{gb}_i)\) gates (\(\mathcal{R}^{gb}_i\)) and 2n controlled \(R^{(02)}_y(2\theta^{gr}_i)\) gates (\(\mathcal{R}^{gr}_i\)) for each pixel state \(|i\rangle\):

\[
\mathcal{R}^{gb}_i = \left( I \otimes \sum_{j=0,j\neq i}^{3^{2n}-1} |j\rangle \langle j| + R^{(01)}_y(2\theta^{gb}_i) \otimes |i\rangle \langle i| \right)
\]  

(32)

\[
\mathcal{R}^{gr}_i = \left( I \otimes \sum_{j=0,j\neq i}^{3^{2n}-1} |j\rangle \langle j| + R^{(02)}_y(2\theta^{gr}_i) \otimes |i\rangle \langle i| \right)
\]  

(33)

\[
\left( \prod_{i=0}^{3^{2n}-1} \mathcal{R}^{gr}_i \mathcal{R}^{gb}_i \right) |H\rangle = |I(\theta)\rangle
\]  

(34)

The circuit diagram for FQRRI of the image in Fig. 1b is shown in Fig. 9.
C. FQRQCI

The qubit FRQCI method uses the phase to encode an extra value in the quantum state. This still leaves two angles to encode three values. This, like FQRRI, needs more shots to decode the image accurately. The extra state available in qutrits can be used to encode the three pixel values in three different angles. This inspires the flexible qutrit representation of quantum color images (FQRQCI).

As before, $2n + 1$ qutrits are required to encode an RGB image. The quantum state of the image using this encoding is given by the formula:

$$|I(\theta)\rangle = \frac{1}{3^n} \sum_{j=0}^{3^{2n}-1} \left( \cos \theta_j^r |0\rangle + \sin \theta_j^r \cos \theta_j^g |1\rangle + e^{i \theta_j^b} \sin \theta_j^g |2\rangle \right) \otimes |j\rangle$$

where,

$$\theta_j^r = \frac{R_j}{255} \times \frac{\pi}{2}$$
$$\theta_j^g = \frac{G_j}{255} \times \frac{\pi}{2}$$
$$\theta_j^b = \frac{B_j}{255} \times \frac{\pi}{2}$$

To create the state in (35), the steps to be followed are:

1) Begin with $|0\rangle^{\otimes (2n+1)}$ state.
2) Apply transform $\hat{H} = I \otimes H^{\otimes 2n}$:

$$\hat{H}|0\rangle^{\otimes (2n+1)} = \frac{1}{3^n} |0\rangle \otimes \sum_{j=0}^{3^{2n}-1} |j\rangle = |H\rangle$$

3) Apply 2n controlled $R^0_{y}\left(2\theta_j^r\right)$ gates ($R^r_j$) and 2n controlled $U^{(12)}\left(2\theta_j^g, \theta_j^b, 0\right)$ gates ($U^{gb}_j$) for each pixel state $|j\rangle$:

$$R^r_j = \left(I \otimes \sum_{k=0, k \neq j}^{3^{2n}-1} |k\rangle \langle k| \right) + R^0_{y}\left(2\theta_j^r\right) \otimes |j\rangle \langle j|$$

$$U^{gb}_j = \left(I \otimes \sum_{k=0, k \neq j}^{3^{2n}-1} |k\rangle \langle k| \right) + U^{(12)}\left(2\theta_j^g, \theta_j^b, 0\right) \otimes |j\rangle \langle j|$$

$$\prod_{j=0}^{3^{2n}-1} U^{gb}_j R^r_j |H\rangle = |I(\theta)\rangle$$

D. MCQRI

The qubit MCQI method uses $2n + 3$ qubits to encode colour images with $2n$ qubits used to encode pixel location, two qubits used to specify which colour to encode and the last qubit to encode the pixel value. The extra third state in qutrits can be used to specify which colour to use in a single qutrit. Thus, only $2n + 2$ qutrits can be used to encode RGB images. Inspired by this, the multi channel qutrit representation for quantum images (MCQRI) is proposed.

The quantum state of the image is given by the formula:

$$|I(\theta)\rangle = \frac{1}{3^n} \sqrt{3} \sum_{i=0}^{3^{n}-1} \left( \cos \theta_i^{r} |00\rangle + \cos \theta_i^{g} |01\rangle + \cos \theta_i^{b} |02\rangle \right) \sin \theta_i^{r} |10\rangle + \sin \theta_i^{g} |11\rangle + \sin \theta_i^{b} |12\rangle \otimes |i\rangle$$

The pixel values are encoded in the $\theta_j^r$ values where $\theta_j^i \in [0, \pi/2]$ by scaling. The following steps are followed to get to this state:

1) Begin with $|0\rangle^{\otimes (2n+2)}$ state.
be denoted by $p_1$ and $p_2$ respectively. The last $3^{2n+1}$ values will be zero. Now,
\[ e^i = \frac{1}{2} \cos^{-1} (3^{2n+1} \times (p_1^i - p_2^i)) \] (54)

These $e^i$ values are the $\theta^i_j$ values from (46). Again, this array can be divided into three equal parts. The first part corresponds to R values, next to G values and the last to B values which can then be scaled back to the range $[0,255]$ to obtain the original image.

One can also think of implementing this as a qubit-qutrit hybrid model with $2n+1$ qutrits and one qubit.

### E. QRCIQ

All the previous encodings had probabilistic decoding. The encoding considered here provides a deterministic decoding process. The QTRQ method is the qutrit extension of the qubit NEQR method. The quantum representation model of color digital images using qutrits (QRCIQ), the qutrit extension of the qubit QCRI method, is proposed.

The encoding requires $2n+2+3$ qutrits to encode RGB images. Here, $2n$ qutrits are required to encode pixel location, the next two qutrits are used to encode the bit plane number, and the remaining three qutrits encode the bit plane value. Assuming 8-bit images, the quantum state is given by the formula:
\[ |I\rangle = \frac{1}{3^{n+1}} \sum_{b=0}^{5} \sum_{i=0}^{3^{n-1}-1} |R^i_y,C^i,G^i,B^i_r\rangle \otimes |i\rangle \] (55)

Here, $b$ represents the bit plane number and $R^i_y,C^i,G^i,B^i_r$ is the RGB bit value of the $i^{th}$ pixel of $b^{th}$ plane. The bit plane values are in $\{0,1,2\}$. The state is prepared using the following steps:

1) Begin with $|0\rangle^{\otimes(2n+2+3)}$ state.
2) Apply transform $\mathcal{H} = I \otimes I \otimes I \otimes H^{\otimes2n+2}$:
\[ \mathcal{H}|0\rangle^{\otimes(2n+2)} = \frac{1}{3^{n+1}} |000\rangle \otimes \sum_{j=0}^{3^{n-1}-1} |j\rangle = |H\rangle \] (56)

3) If $C^i_b = 0$, where $C \in \{R,G,B\}$, do nothing.
4) If $C^i_b = 1$, apply $[+1]$ gate on the $C$ qutrit controlled by $2n+2$ qutrits with control state $|b\rangle \otimes |i\rangle$.
5) If $C^i_b = 2$, apply $[+2]$ gate on the $C$ qutrit controlled by $2n+2$ qutrits with control state $|b\rangle \otimes |i\rangle$.

The circuit diagram for QRCIQ of the image in Fig. 1b is shown in Fig. 12.

The decoding is deterministic as after measuring the state in the computational basis, the mere presence of a basis state provides all information it encodes. The bit string of the basis state can be decoded to give the pixel location, bit plane number and the bit plane pixel value. It is important to note here that extra states will be present in the final state corresponding to unused bit planes. As two qutrits can encode $3^2 = 9$ bit planes and only six are required for an 8-bit image using ternary strings, three bit planes are empty, corresponding
to \( b = 6, 7 \) or 8. Thus, while decoding, these states need to be discarded. It’s possible to envision that these available “slots” can be populated to provide extra information about the images or facilitate processing or error correction.

**Remark**

The encodings that have a probabilistic decoding, use inverse trigonometric functions to calculate the encoded pixel values. These functions have a finite domain, for example, \( \sin^{-1} : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}] \). The values calculated using a finite number of shots may fall outside the domain of these functions. Thus, the values need to be clipped to stay in the domain to get a valid result. The accuracy of the calculated values can be improved by increasing the number of shots. While QRCIQ does not have a probabilistic decoding and does not depend on the number of shots used, it still requires a minimum number of shots to observe all basis states at least once. Once all states have been observed, increasing the number of shots will have no effect on the decoding.

**IV. CONCLUSION AND FUTURE WORK**

A few implementations to represent images as quantum states using qutrits have been described. All of these were validated using simulations with Cirq. While the decoding is probabilistic for most of them and depends on the number of shots used, it is also important to note that decoding the image is not the only use case for these representations. Different image processing tasks can be performed on the final quantum state [17]–[20]. Machine learning tasks like classification can also be performed on the qutrit quantum representation of the images as performed with qubit QIRs [21, 22]. A summary of the provided methods is shown in Table II. The resource requirement increases slower than the qubit case and is always lower except for trivial cases. This can be seen from Fig. 7.

**TABLE II: Comparison of the different quantum image representation methods presented. The number of required qutrits are for an image of size \( 3^n \times 3^n \).**

| QIR   | Color   | Required Qutrits | Decoding Method | Equation |
|-------|---------|------------------|-----------------|----------|
| FQRRI | RGB     | \( 2n + 1 \)     | Probabilistic   | Eq. 19   |
| FORRI | Grayscale | \( 2n + 1 \)   | Probabilistic   | Eq. 23   |
| FQRQCI| RGB     | \( 2n + 1 \)     | Probabilistic   | Eq. 35   |
| MCQRI | RGB     | \( 2n + 2 \)     | Probabilistic   | Eq. 46   |
| QRCIQ | RGB     | \( 2n + 5 \)     | Deterministic   | Eq. 55   |

Fig. 6: Decoded images using QRCIQ using different number of shots.

**Fig. 7: Number of qubits or qutrits required to represent an image.**

While from the universality of qudit logic, the decomposition of the multi-qutrit gates used can be assured, the decompositions have not been looked at in this paper. Research can be done to find optimal decompositions of the gates. More representations can be studied to utilise the extra state available in qutrits. Appropriate use of the empty slots can also be studied. Comparison of different representations on their effectiveness in various image processing tasks can also be explored.

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Fig. 8: The circuit diagram for the FQRI representation for the image in Fig. 1a.

Fig. 9: The circuit diagram for the FQRRI representation for the image in Fig. 1b.

Fig. 10: The circuit diagram for the FQRQCI representation for the image in Fig. 1b.

Fig. 11: The circuit diagram for the MCQRI representation for the image in Fig. 1b.

Fig. 12: The circuit diagram for the QRCIQ representation for the image in Fig. 1b.