Holographic Ricci dark energy as running vacuum

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Abstract
Holographic Ricci dark energy has been proposed ago has faced with problems of future singularity. In the present work we consider the Ricci dark energy with an additive constant in it’s density as running vacuum energy. We have analytically solved the Friedmann equations and also the role played by the general conservation law followed by the cosmic components together. We have shown that the running vacuum energy status of the Ricci dark energy helps to remove the possible future singularity in the model. The additive constant in the density of the running vacuum played an important role, such that, without that, the model predicts either eternal deceleration or eternal acceleration. But along with the additive constant, equivalent to a cosmological constant, the model predicts a late time acceleration in the expansion of the universe, and in the far future of the evolution it tends to de Sitter universe.

1 Introduction

The accelerated expansion of the current universe is a confirmed phenomenon as per the present observations. The observational data from the supernova type Ia (SNeIa) [1, 2], cosmic microwave background [3], large scale structure (LSS) [4], baryon acoustic oscillations [5] and weak lensing [6] are the strong supports for this. The reason for this accelerated expansion is argued to be due to the presence of gravitationally repulsive energy component known as dark energy (DE). The nature and evolution of dark energy is still not clear inspite of the numerous speculation existing in the current literature.

The simplest model for dark energy is by taking it as the vacuum energy or cosmological constant, lead to the standard ΛCDM model of the universe. This approach faces coincidence problem and fine tuning problem [7]. The coincidence problems means that, even though the cosmic evolution of both dark matter and dark energy are different their densities of the same order in the present universe, a thing which is not accounted by the standard models of the universe. The fine tuning problems is that, the theoretically predicted value of the cosmological constant is several orders higher than the observed value. These problems motivated the consideration of various dynamical dark energy models like quintessence [8, 9], kessence [10], and the Chaplygin gas model [11], in which the dark energy density is evolving with time. Another approach in explaining the recent acceleration of the universe, is by modifying the left hand side of the Einstein equation, i.e., the geometry of the space time, which are generally called as modified gravity theories. The dark energy models based on
modified gravity are the so-called f(R) gravity [12], f(T ) gravity [13], Gauss Bonnet theory [14], Lovelock gravity [15], scalar-tensor theories [16] etc.

A class of models which were proposed to alleviate the problems of the ΛCDM model are the dynamical vacuum energy models. Recently much attention has been paid in these models in which dark energy treated as a time varying vacuum in which the density is varying as the universe expands, but equation of state stays constant around -1, and is often termed as running vacuum energy [17, 18, 19]. The main motivation for this approach is arising from vacuum energy predicted by quantum field theory in curved space-time derives from the renormalization group. The vacuum energy predicted from such theories, posses constant equation of state but their density is varying during the evolution. The evolution of such a running vacuum energy is considered in reference [20], where the authors have shown that, the recent acceleration of the universe can be explained with varying density contain an extra constant term, and corresponding equation of state is stands constant around -1. The model is free from future singularity under certain conditions on the model parameters. The recent Plank observations, however point towards the possibility of a phantom equation of state for the dark energy that, \( \omega = -1.49 \pm 0.57 \) \[32\], but the wide error bars makes the phantom nature still doubtful. On the other hand the WMAP observations point towards a value around, \( \omega \sim -0.93 \) \[33\]. But the latest result combining WMAP, BAO, CMB, \( H_0 \) is found to be, \( \omega = -1.073 \pm 0.099 \) \[34\]. Both these results a value around -1 for the equation of state, which can be taken as a concordance one. Moreover the success of ΛCDM model which treat dark energy as the vacuum constant also pointing towards a constant equation of state around \(-1\). In addition to the constant equation of state, the evolutionary nature of the density in running vacuum energy models may safeguard the coevolution of the dark sectors.

In the present work we considered Ricci dark energy as the running vacuum energy. This form of dark energy is derived from holographic principle [26, 27]. The terms in Ricci dark energy is almost similar to that considered in paper [21]. We append the usual form of dark energy with an additional constant term in the density. The paper is organized as follows. After a brief introduction in section 1 we discuss running vacuum energy, and Ricci dark energy as running vacuum energy in section 2. This section also consists of our analysis of the evolution of Hubble parameter, density function and also other relevant cosmological parameter. We conclude in section 3.

## 2 Holographic Ricci dark energy as running vacuum energy

According to holographic principle [23] black holes are the maximally entropic objects of a given region. Hence the total degrees of freedom is bounded by the surface area in Planck units. Cohen et al. [24] have suggested that the total vacuum energy in a region of size \( L \) should not exceed the mass of a black hole...
of the same size, $L^3 \rho_{\text{vac}} \leq L M_p^2$ where $\rho_{\text{vac}}$ is the quantum zero point energy density and $M_p^2 = 1/\sqrt{8\pi G}$ is the reduced Plank mass. Based on this result, Li\cite{25} proposed the holographic dark energy as $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$ where $c^2$ is a dimensionless constant, and $L$ is often called as IR cutoff. Hsu\cite{26} showed that the holographic dark energy with Hubble horizon as IR cutoff does not lead to an accelerating universe. Hence in order to explain an accelerating universe Li suggested the future event horizon as IR cutoff instead of Hubble horizon and particle horizon. Later studies found that in taking future horizon as IR cutoff will lead to causality violation. Hence Gao et al\cite{27} proposed the holographic Ricci dark energy model in which holographic dark energy is proportional to the Ricci scalar.

$$
\rho_\Lambda = -\frac{\alpha}{16\pi} R
$$

(1)

where $R$ is the Ricci scalar curvature $R = -6(\dot{H} + 2H^2 + \frac{k}{a^2})$\cite{28} and $\alpha$ is a constant to be determined, $H = \dot{a}/a$, the Hubble parameter, $k$ is the curvature parameter and $a$ is the scale factor. Later modified form of holographic Ricci dark energy were also been studied\cite{29, 30, 31}. The general behavior of equation of state in these models is that it can assume values greater than or less than $-1$ corresponds to quintessence or phantom nature depending on the model parameter.

Running vacuum energy has got it’s original form as\cite{17},

$$
\rho_\Lambda = n_0 + n_1 H^2 + O(H^4).
$$

(2)

This is the vacuum energy in quantum field theory in curved space-time originating from the renormalization group equation in reference\cite{17}. Only even powers of $H$ are allowed to maintain the general covariance. Since higher order powers are much smaller, they are often neglected. The constant $n_0$ is often replaces the role of the cosmological constant and $n_1$ is a dimensionless parameter given by\cite{17},

$$
n_1 = \frac{3\nu}{8\pi} M_p^2
$$

(3)

where $\nu$ is,

$$
\nu = \frac{1}{6\pi} \sum_i B_i \frac{M_i^2}{M_p^2}
$$

(4)

with $B_i$ as the coefficients computed from the quantum loop contributions of the fields with masses $M_i$. The value of $\nu$ is such that $|\nu| \ll 1$ hence the running vacuum is very close to the true cosmological constant. However small may be the value of $\nu$ it gives a running status for the dark energy.

In this work we considered holographic Ricci dark energy as running vacuum energy which takes the form

$$
\rho_\Lambda(H, \dot{H}) = 3\beta M_p^2 (\dot{H} + 2H^2) + M_p^2 \Lambda_0
$$

(5)

In this equation only the first part on the right hand side corresponds to the conventional Ricci dark energy. The addition of the second term proportional
to $\Lambda_0$ which is a constant, is aiming at to guarantee a transition from an early deceleration to an acceleration phase in the expansion history of the universe. In dealing with entropic dark energy as dynamical vacuum energy in reference [21], the authors considered a similar addition of a constant term to energy density, while in the absence of such a term the universe may undergo either eternal deceleration or acceleration. In the above equation for $\beta=0$, the dark energy density reduces to the cosmological constant as in the case of the original running vacuum equation (2). Unlike the original running vacuum form, there appears a term $\dot{H}$ in the present density equation. But this term will not make generally different from the original form, because, first of all, both $H^2$ and $\dot{H}$ are of same dimension even though they represent different degrees of freedom. Secondly these terms are related through $\dot{H} = -(1 + q)H^2$, where $q$ is the deceleration parameter. During different stages of the cosmic evolution, the $q$ parameter appears roughly a constant. For instance, during radiation dominated phase, the parameter $q = 1$, while for matter dominated phase, $q = 0.5$. Hence $H \sim H^2$ in these cosmic phases. So during these phases there exist a correspondence between the parameter $n_1$ or $\nu$ in the original running vacuum and the $\beta$ parameter in the holographic Ricci running vacuum. However in the later phase of cosmic acceleration the $q$ parameter is undergoing a variation, but due to the smallness of the $\beta$ parameter the $\dot{H}$ term behave almost like the $H^2$ term.

The Friedman equations with radiation, non-relativistic matter and dark energy as cosmic components is given by Eqn (5) are,

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_p^2} \left( \rho_r + \rho_m + \rho_\Lambda(H, \dot{H}) \right)$$

(6)

where $\rho_r$ is the radiation density and $\rho_m$ is density of the non-relativistic matter. The equation representing the cosmic acceleration is given as,

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2} \left( (\rho_i + 3p_i) + 2\rho_\Lambda(H, \dot{H}) \right).$$

(7)

where $\rho_i$ and $p_i$ together representing the density and pressure of the radiation and matter components. Since the dark energy is of the nature of dynamical vacuum energy, it would satisfy the condition,

$$\rho_\Lambda(H, \dot{H}) = -p_\Lambda(H, \dot{H}) = 3\beta M_p^2 (\dot{H}^2 + 2H^2) + M_p^2 \Lambda_0$$

(8)

An important feature of running vacuum energy is it’s decay. For the covariance nature of the theory, Bianchi identity must be satisfied[36], which insure the covariance nature of the equations, and further implies that, it is the total energy density of the entire system of universe is conserved [22], that is a separate conservation law for each individual component is not followed. This effectively take account of the transfer of energy between running vacuum energy density and other components presents in the universe. The conservation equation then takes the form

$$\dot{\rho}_m + \dot{\rho}_r + \dot{\rho}_\Lambda + 3H(\rho_m + \frac{4}{3}\rho_r) = 0.$$

(9)
After substituting the time derivative of the dark energy density, the above equation become,

\[ \dot{\rho}_m + \dot{\rho}_r - \frac{3}{2} \beta \left( \dot{\rho}_m + \frac{4}{3} \dot{\rho}_r \right) = -3H \left( 1 - 2\beta \right) \left( \rho_m + \frac{4}{3} \rho_r \right) \]  

(10)

This means that even in the case allowing an arbitrary interaction \( Q(t) \) between matter and radiation, then any solution of the equations of the form

\[ \dot{\rho}_m = -3H \frac{(1 - 2\beta)}{(1 - \frac{3}{2}\beta)} \rho_m + Q(t) \]  

(11)

\[ \dot{\rho}_r = -4H \rho_r - Q(t) \]

(12)

are simultaneously be the solutions of the equation (10). In stages where either matter or radiation dominated the densities, the equation (10) will be satisfied, that is \( Q(t) \rightarrow 0 \). Hence a simplest version of this is to assume that the total conservation equation reduces to a set of decoupled equations with \( Q = 0 \) at all times, which may be the only way to introduce arbitrary number of cosmic components. Then the evolution of matter during matter dominated era and radiation during radiation dominated era are become

\[ \rho_m = \rho_{m0} a^{-3\xi_m} \]  

(13)

\[ \rho_r = \rho_{r0} a^{-4} \]  

(14)

where \( \xi_m = \frac{(1-2\beta)}{(1-\frac{3}{2}\beta)} \), \( \rho_{m0} \) and \( \rho_{r0} \) are energy density of matter and radiation at the present time respectively. Here it should be noted that, due to the decay of running vacuum, the evolution of the matter density is modified but the radiation follows the conventional behavior that, \( \rho_r \propto a^{-4} \). This may indicate that the running vacuum is substantially coupled with the matter than radiation in the present model. By substituting Eqns (13), (14) and (8) in Eqn (6) we can finally reached to

\[ \frac{\dot{h}}{H_0} = \frac{-1}{\beta} \left( \Omega_{m0} e^{-3\xi_m x} + \Omega_{r0} e^{-4x} + (2\beta - 1) h^2 + \frac{\Lambda_0}{3H_0^2} \right) \]

(15)

where \( \Omega_{m0} = \frac{\rho_{m0}}{3M_p^2 H_0^2} \), \( \Omega_{r0} = \frac{\rho_{r0}}{3M_p^2 H_0^2} \).

By changing the variable from time \( t \) to \( x = \ln a \) the differential equation can be expressed as

\[ \frac{d\dot{h}^2}{dx} = \frac{-2}{\beta} \left( \Omega_{m0} e^{-3\xi_m x} + \Omega_{r0} e^{-4x} + (2\beta - 1) h^2 + \frac{\Lambda_0}{3H_0^2} \right) \]  

(16)

The solution of the above equation is

\[ \dot{h}^2 = \frac{\Omega_{m0}}{\xi_m} e^{-3\xi_m x} + \Omega_{r0} e^{-4x} + \frac{\Lambda_0}{3(1-2\beta)H_0^2} + \Omega_\beta e^{-(4-\frac{2}{\beta})x} \]  

(17)
where \( \Omega_\beta \) is the integration constant. In Eqn (17) the first term is proportional to matter density, second term represent the energy density of radiation and the last two terms together represents running vacuum energy density. This shows that in the far future running vacuum energy density will be the dominating component of the universe, especially at \( z \to -1 \) (see that \( e^{-x} = (1 + z) \) the hubble parameter become a constant. i.e \( h^2 \to \frac{\Lambda_0}{3(1-2\beta)H_0^2} \). It is trivial that for \( h^2 \) is to positive definite in the future the parameter \( \beta < 1/2 \). The first time derivative for the Hubble parameter is obtained by substituting Eq.(17) in eqn.(15)

\[
\frac{\dot{h}}{H_0} = -\frac{3}{2} \Omega_{m_0} e^{-3\xi_m x} - 2\Omega_{r_0} e^{-4x} - \frac{(2\beta - 1)}{\beta} \Omega_\beta e^{-(4-\frac{4}{\beta})x},
\]

(18)

which implies that as \( z \to -1 \) the time derivative, \( \dot{h} \to 0 \), otherwise implies \( h \to constant \). The running vacuum energy density is then obtained by substituting Eq(15) and Eq(17) in Eq(5) as

\[
\rho_\Lambda = \frac{3\beta M_p^2 H_0^2}{2 - 4\beta} \Omega_{m_0} e^{-3\xi_m x} + 3M_p^2 H_0^2 \Omega_\beta e^{-(4-\frac{4}{\beta})x} + \frac{M_p^2 \Lambda_0}{1 - 2\beta}.
\]

(19)

The obtained energy density is depending on energy density of matter but not on the radiation density and the last two terms plays significant role in the future evolution of the universe. It will continue as an exponentially decreasing term if \( \beta > 1/2 \). However for \( \beta > 1/2 \) the last term become negative and as \( z \to -1 \) the entire density \( \rho_\Lambda \) tends to a negative cosmological constant. For \( \beta < 1/2 \) the last term will become positive definite, but the second term will eventually lead to a big rip singularity as \( z \to -1 \). The above equation corresponds to an effective cosmological constant,

\[
\Lambda = \frac{3\beta H_0^2}{2 - 4\beta} \Omega_{m_0} e^{-3\xi_m x} + 3H_0^2 \Omega_\beta e^{-(4-\frac{4}{\beta})x} + \frac{\Lambda_0}{1 - 2\beta}.
\]

(20)

The evolution of this cosmological term depends crucially on the first two terms. The total energy density filling the universe would be, \( \rho_{tot} = \rho_m + \rho_r + \rho_\Lambda \) which results in

\[
\rho_{tot} = 3M_p^2 H_0^2 \Omega_{m_0} e^{-3\xi_m x} + 3M_p^2 H_0^2 \Omega_{r_0} e^{-4x} + \frac{M_p^2 \Lambda_0}{1 - 2\beta} + 3M_p^2 H_0^2 \Omega_\beta e^{-(4-\frac{4}{\beta})x}
\]

(21)

This total energy density satisfies the conservation law

\[
\rho_{tot} + 3H(\rho_{tot} + p_{tot}) = 0
\]

(22)

where the total pressure, \( p_{tot} = p_m + p_r + p_\Lambda \). Here \( p_m = 0, p_r = \frac{4}{3} \rho_r, p_\Lambda = -\rho_\Lambda \). Then the total conservation law takes the form

\[
\Omega_\beta e^{-(4-\frac{4}{\beta})x} = 0
\]

(23)
The above equation is satisfied if the integration constant identically vanishes, i.e. $\Omega_\beta = 0$. This will reduces the running vacuum energy equation (19) to the simple form,

$$\rho_\Lambda = \frac{3\beta M^2_p H_0^2}{2(1 - 2\beta)} \Omega_{m_0} e^{-3\xi_m x} + \frac{M^2_p \Lambda_0}{1 - 2\beta}$$  \hspace{1cm} (24)

For positive definite value for the running vacuum density, the parameter $\beta < 1/2$. For $\beta = 0$ there arise possibility where the density can have negative values in the future and also effectively kills the original expression of the dark energy density. Hence the $\beta$ parameter can be fixed in the range $0 < \beta < 1/2$. The total energy density become,

$$\rho_{\text{tot}} = 3M^2_p H_0^2 \Omega_{m_0} e^{-3\xi_m x} + 3M^2_p H_0^2 \Omega_{r_0} e^{-4x} + \frac{M^2_p \Lambda_0}{1 - 2\beta}$$  \hspace{1cm} (25)

In the far future of the universe as $z \to -1$, the total energy density will be dominated by the constant term, i.e $\rho_{\text{tot}} \to \frac{M^2_p \Lambda}{1 - 2\beta}$. Law itself demands the

It is the constant addition in the dark energy density (i.e, $M^2_p \Lambda$) guarantees the transition of the universe to the late acceleration phase, the integration constant is no longer be essential.

The deceleration parameter, $q = -1 - \frac{\dot{H}}{H^2}$, is evaluated as

$$q = -1 + \frac{3\Omega_{m_0} e^{-3\xi_m x}}{2\Omega_{m_0} e^{-3\xi_m x} + \Omega_{r_0} e^{-4x}}$$  \hspace{1cm} (26)

For a matter dominated phase deceleration parameter $q$ can be approximated as

$$q \sim -1 + \frac{3}{2} \xi_m.$$  \hspace{1cm} (27)

For $\xi_m = 1$, which otherwise implies $\beta = 0$, means contribution of Ricci dark energy is only a constant term, the deceleration $q \sim 0.5$ which corresponds to the behavior of dust like matter dominated universe which is eternally decelerating. For the case, $\xi_m$ is different from 1, at which dark energy consists of both the additive constant and the varying part, see equation (26), the universe may be either eternally decelerating or accelerating. For instance, when $\xi_m > 0.7$ the universe will still be matter dominated and would be eternally decelerating. On the other hand for $\xi_m < 0.7$ the universe will be dominated by dark energy and would be eternally accelerating. These facts indicating that without the constant term $\Omega_{\Lambda_0}$ in the Ricci dark energy density a transition from a decelerating phase to an accelerating phase is impossible. On the other hand if the term $\Omega_{\Lambda_0}$ dominates then $q$ takes the form

$$q = -1 + \frac{3 \Omega_{m_0}}{2 \Omega_{\Lambda_0}} (1 - 2\beta) e^{-3\xi_m x}$$  \hspace{1cm} (28)

For the case, $\xi_m = 1$, which implies $\beta = 0$, the deceleration parameter reduces to

$$q = -1 + \frac{3 \Omega_{m_0}}{2 \Omega_{\Lambda_0}} e^{-3x}$$  \hspace{1cm} (29)
This shows that there can occur a transition from deceleration to acceleration for any positive values of $\xi_m$ as far the dark energy is dominated by the constant term. The equation (28) also shows that in the far future of the evolution of the universe, $z \to -1$, at which the density is dominated by the constant term $\Omega_\Lambda$ and also $e^{-3\xi_m x} = (1 + z)^{3\xi_m x} \to 0$, the deceleration parameter tends to the value -1. That is the universe approach the de Sitter phase as $z \to -1$.

3 Conclusions

In this letter we consider holographic Ricci dark energy plus an additional constant as running vacuum energy. Due to the decay of vacuum in to other possible , satisfying the equation of state $\omega = -1$. Because of the decay of running vacuum, we have consider a general conservation law, satisfied by the entire cosmic components together, which reveals that, the behavior of matter density is modified as, $\rho_m = \rho_{m0} a^{-3\xi_m}$ but the radiation follows the conventional behavior, $\rho_r = \rho_{r0} a^{-4}$. The Hubble parameter were evaluated and also the total energy density. The condition of the total conservation law on the total density, constrain the integration constant appeared while obtaining the hubble parameter to be equal to zero, otherwise the model will eventually leads to a future singularity, where both density and hubble parameter are eventually become infinity, otherwise known as big-rip. The evolution equation of the dark energy density thus appearing will always be positive definite only if the model parameter is in the range, $0 < \beta < 1/2$. On evaluating the deceleration parameter, it is seen that, the model leads to late acceleration, at which the running vacuum dominates over the other cosmic components, only in the presence of the additive constant in the equation of the running vacuum density, but without that the model will leads to either eternal deceleration or acceleration. In the above specified range of the model parameter $\beta$ the model will asymptotically become de Sitter type. As a comparison to the present analysis, it should be noted that, along with considering the entropic dark energy as running vacuum, the authors [21], have analyzed the general evolution of the Ricci dark energy, in which the model will reduce to the de Sitter phase, only if the additive constant to the density is negative. When one consider entropic dark energy as running vacuum[21], the evolution of the running is depends on the radiation component too and the behavior of the radiation component itself is modified due to the running status of the entropic dark energy. While in the present model, first of all, the running vacuum energy is not depending the radiation component, and secondly, the behavior of the radiation component is not modified but follows the conventional behavior.

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