Ionosphere influence on success rate of GPS ambiguity resolution in a satellite formation flying

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Abstract. Satellite formation flying is one of the most promising technologies for future space missions. The distribution of sensors and payloads among different satellites provides more redundancy, flexibility, improved communication coverage, among other advantages. One of the fundamental issues in spacecraft formation flying is precise position and velocity determination between satellites. For missions in low Earth orbits, GPS system can meet the precision requirement in relative positioning, since the satellite dynamics is modeled properly. The key for high accuracy GPS relative positioning is to resolve the ambiguities to their integer values. Ambiguities resolved successfully can improve the positioning accuracy to decimetre or even millimetre-level. So, integer carrier phase ambiguity resolution is often a prerequisite for high precision GPS positioning. The determination of relative position was made using an extended Kalman filter. The filter must take into account imperfections in dynamic modeling of perturbations affecting the orbital flight, and changes in solar activity that affects the GPS signal propagation, for mitigating these effects on relative positioning accuracy. Thus, this work aims to evaluate the impact of ionosphere variation, caused by changes in solar activity, in success rate of ambiguity resolution. Using the Ambiguity Dilution of Precision (ADOP) concept, the ambiguity success rate is analyzed and the expected precision of the ambiguity-fixed solution is calculated. Evaluations were performed using actual data from GRACE mission and analyzed for their performance in real scenarios. Analyses were conducted in different configurations of relative position and during different levels of solar activity. Results bring the impact of various disturbances and modeling of solar activity level on the success rate of ambiguity resolution.

1. Introduction
Formation flying is one of the most promising technologies for future space missions. The sensor and payload distribution among different satellites provides more redundancy, exibility, improve communications and surveillance coverage and new applications which are not possible with a single satellite [1]. This distributed approach provides enhanced performance and operational advantages.

One of the fundamental issues of formation flying spacecraft is determination of the state (position and velocity) among satellites belonging to formation. For missions in low earth orbit (LEO), the Global Positioning System (GPS) can meet the accuracy requirement in relative positioning, since satellites in LEO are covered by the GPS constellation. GPS receivers have a relatively low cost compared with other navigation sensors and their performance is generally reliable and is always available. Therefore, GPS is often regarded as the main instrument for navigation in satellite formation missions. The key for high accuracy GPS relative positioning is to resolve the ambiguities to their integer values. Ambiguities resolved successfully can
improve the positioning accuracy to decimetre or even centimetre-level. So, integer carrier phase ambiguity resolution is often a prerequisite for high precision GPS positioning [2].

Solar activity level varies with a period of 11 years on average, known as the solar cycle. A peak in solar activity affects the behavior of the ionosphere, causing disturbances in the propagation of signals from GPS (ionospheric scintillation) and reducing positioning accuracy. The next peak of solar activity (cycle 24) is foreseen for May, 2013 [4].

Thus, this work aims to evaluate the impact of ionosphere variation, caused by changes in solar activity, in success rate of ambiguity resolution. Using the Ambiguity Dilution of Precision (ADOP) factor, the ambiguity success rate is analyzed and the expected precision of the ambiguity-fixed solution is calculated.

This work uses real data delivered from GRACE mission. The GRACE mission (Gravity Recovery and Climate Experiment) is made up two identical satellites (GRACE A and GRACE B) in a leader-follower formation, orbiting Earth at the same orbital plane, with inclination of 89°. Initial altitude was about 500 km above Earth’s surface, after launch. Due to atmospheric drag, the altitude decreases to about 300 km. The nominal separation between the satellites is 220 km.

Results show the influence of ionosphere activity and satellite separation on success rate of GPS ambiguity resolution, using single and double frequency GPS measurements.

2. Measurement models

Ambiguity resolution will be tested in two situations: i) solving L1 ambiguities only; ii) solving L1 and L2 ambiguities. The following models, which are given referred to the state vector (geometric distance, in double difference form, and ambiguities), are linear. The measurement of carrier phase in single difference form is shown in Fig. 1. The double difference measurements are obtained by subtracting two single difference measurement referred to two different satellites.

\[
\begin{bmatrix}
\rho \\
\phi_1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
1 & \lambda_1
\end{bmatrix}
\begin{bmatrix}
D \\
a_1
\end{bmatrix} + \epsilon
\]

where \( \rho \) is double differenced pseudorange, \( \phi_1 \) is double differenced carrier phase (in distance units) in L1, \( \lambda_1 \) is L1 wavelength, \( D \) is geometric distance in double difference form, \( a_1 \) is double difference integer ambiguity and \( \epsilon \) represents unmodeled errors.

Using L1 and L2 carrier measurements, the observation model for \( a_1 \) and \( a_2 \) ambiguities is given by:

\[
\begin{bmatrix}
\rho \\
\phi_1 \\
\phi_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
1 & \lambda_1 & 0 \\
1 & 0 & \lambda_2
\end{bmatrix}
\begin{bmatrix}
D \\
a_1 \\
a_2
\end{bmatrix} + \epsilon
\]

Figure 1. Carrier phase measurement in single difference form.
where \( \rho \) is double differenced pseudorange, \( \phi_1 \) and \( \phi_2 \) are double differenced carrier phase measurements in L1 and L2, whose wavelengths are \( \lambda_1 \) and \( \lambda_2 \), respectively, \( a_1 \) and \( a_2 \) are double differenced integer ambiguity, and \( \epsilon \) represents unmodeled errors.

If baseline is greater than about 5 km, ionospheric effect becomes more significant. An ionospheric parameter is inserted together with geometric distance and ambiguities in state vector. In this case, it is introduced a pseudo-observation of ionospheric error \( I_0 \), zero-valued and variance \( \sigma^2_0 \) in each epoch [5]. This pseudo-observation helps narrow the ionospheric error to reasonable values during the initial phase of the filter, leading to distinguish between the ambiguities and ionospheric error. Therefore, it is expected the position estimate will not be influenced by the ionospheric error. The measurement model, with the introduction of this pseudo-observation, is described by:

\[
\begin{bmatrix}
\rho \\
I_0 \\
\phi_1 \\
\phi_2
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & -1 & \lambda_1 & 0 \\
1 & -\beta & 0 & \lambda_2
\end{bmatrix}
\begin{bmatrix}
D \\
I \\
0 \\
a_1 & a_2
\end{bmatrix} + \epsilon
\]

(3)

where \( \beta = (\lambda_2/\lambda_1)^2 = (77/60)^2 \).

The estimation is carried out in three steps: \( i \) float solution; \( ii \) integer solution; \( iii \) correction of parameters from resolved integer ambiguities.

In the first step, the integer constraint \( a \in \mathbb{Z}^n \) is disregarded on the ambiguities and a standard estimation procedure is performed. As a result, the float solution is obtained, which is a real-valued estimate of \( a \) and \( b \), together with their covariance matrix:

\[
\begin{bmatrix}
\hat{a} \\
\hat{b}
\end{bmatrix},
\begin{bmatrix}
P_a & P_{ab} \\
P_{ba} & P_b
\end{bmatrix}
\]

(4)

where \( P_a \) is ambiguities covariance matrix regarding to \( \hat{a} \) estimate, \( P_b \) is the partition regarding to \( \hat{b} \) estimate, and \( P_{ab} \) and \( P_{ba} \) are matrix of cross correlated terms.

In the second step, the float ambiguity estimate \( \hat{a} \) is used to compute the corresponding integer ambiguity estimate, denoted as \( \hat{a} \). Once the integer ambiguities are computed, the third step consists in correcting the float estimate of real-valued parameters \( b \). As a result, the fixed solution is obtained:

\[
\hat{b} = \hat{b} - P_{\hat{b}a}P_{\hat{a}}^{-1}(\hat{a} - \hat{a})
\]

(5)

If the success rate, \( i.e. \), probability that estimated integer ambiguities are equal the true ambiguities, is sufficiently close to 1, the precision of the fixed solution can be described by the following covariance matrix:

\[
P_b = P_b - P_{\hat{b}a}P_{\hat{a}}^{-1}P_{\hat{ab}}
\]

(6)

2.1. Ambiguity Dilution of Precision

The success of ambiguity resolution process depends on quality of float ambiguity estimation. This quality can be measured by covariance matrix of float ambiguities. However, instead analyzing each entry of covariance matrix (Eq. 6), [9] introduced the Ambiguity Dilution of Precision (ADOP) concept as a way of quality measure. It is a scalar value, easier to evaluate than taking each entry of covariance matrix. The ADOP concept is defined as:

\[
ADOP = \left(\sqrt{|P_{\hat{a}}|}\right)^{1/n}
\]

(7)

where \( P_{\hat{a}} \) is ambiguities covariance matrix, \( n \) is its order, and \( | \cdot | \) means the determinant of a matrix. ADOP is a scalar quantity given in cycles unit. This ADOP value not only depends on the variances of the ambiguities, but also on their covariances.
The ADOP is equal to the geometric average of the sequential conditional standard deviations of the ambiguities. This follows from the fact that the determinant of the covariance matrix of the ambiguities is equal to the product of the n sequential conditional variances. The ADOP is thus a simple measure for the average precision of the ambiguities [7].

The ADOP is invariant against the choice of ambiguity parametrization. ADOP does not change when the definition of the ambiguities is changed, due to all admissible ambiguity transformations can be shown to have a determinant equal to one. If the ambiguities are transformed by:

$$\tilde{\mathbf{z}} = \mathbf{Z}^T \tilde{\mathbf{a}}; \quad P_{\tilde{\mathbf{z}}} = \mathbf{Z}^T P_{\mathbf{a}} \mathbf{Z}$$

(8)

The determinant of the transformed ambiguity covariance matrix remains invariant, since:

$$|P_{\tilde{\mathbf{z}}}| = |\mathbf{Z}^T P_{\mathbf{a}} \mathbf{Z}| = |\mathbf{Z}^T||P_{\mathbf{a}}||\mathbf{Z}| = |P_{\mathbf{a}}|$$

(9)

once $|\mathbf{Z}| = 1$ for all admissible transformations. Because of this property, using the decorrelating $Z$-transform of the LAMBDA method [8, 3], the ADOP remains the same.

Since the ADOP gives a good approximation to the average precision of the ambiguities, it also provides a good approximation to the integer ambiguity success rate:

$$P(\tilde{\mathbf{a}} = \mathbf{a}) \leq \left[ 2\Phi \left( \frac{1}{2ADOP} \right) - 1 \right]^n$$

(10)

where $\Phi(x)$ is the standard normal cumulative distribution function:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2}v^2 \right) dv$$

(11)

In general, if ADOP is smaller than about 0.12 cycles, the success rate becomes larger than 0.999, while for ADOP smaller than 0.14 cycles, the success rate is always better than 0.99 [7].

3. Results

All tests were carried out using real data from GRACE mission. GRACE satellites performed a maneuver of switching positions on December 10th, 2005. At this event, the relative distance between satellites was less than 10 km for several hours. So, the first data set was chosen at this period of closest approximation. A second approaching maneuver was carried out on July 14th, 2014, and the second data set was chosen at this date. The period of first data set was close to a solar minimum, while second data set was close to a solar maximum [6]. It was assumed that ambiguities can be considered resolved if success rate is greater than 0.97.

Observation models in Section 2 were used to compute, using a Kalman filter, float ambiguities and their covariance matrix for the two data sets. For all tests, pseudorange standard deviation was set to 1.0 m and standard deviation of L1 and L2 carrier phase measurement was set to 0.005 m.

For the first test, graphs in Figs. 2(a) and 2(b) show ADOP value along time of observation for L1 only and L1 and L2 frequencies, respectively.

As seen on Fig. 2(a), ADOP reaches values as low as 1 cycle, giving a success rate of only 0.003. This means ambiguity cannot be resolved successfully in this time span. Otherwise, using double frequency measurements, ADOP reaches 0.1 cycle, so ambiguities can be resolved after 180 seconds, with success rate of 0.97. Figures 3(a) and 3(b) show success rates for single and double frequency measurements. In this test, ionosphere pseudo-observation standard deviation was set to 0.04 m.

The second test shows that using only L1 frequency, it was not possible to resolve ambiguities within observation time, as in the first test. However, using L1 and L2 frequencies, ambiguities
could be resolved after 280 seconds (success rate of 0.97). *ADOP* values are shown on graphs of Figs. 4(a) and 4(b). Ionosphere pseudo-observation standard deviation was set to 0.25 m, due to the higher solar activity. One factor which can explain the slower ambiguity resolution is the ionosphere was more disturbed, once solar activity was increasing. So ionosphere pseudo-observation standard deviation was set to 0.25 m in this case. Thus, the success rate took a longer time to reach 0.97. Figures 5(a) and 5(b) show the success rate for second data set.

4. Conclusion

Two tests were conducted using real GPS data from GRACE mission. First data set was chosen in a period of minimum solar activity. Second data set was chosen in a period of higher solar activity, and when the satellites were close each other.

Both tests showed that L1 only measurements are not sufficient to resolve ambiguities in neither case. However, using L1 and L2 measurements, ambiguities could be resolved in both cases. The tests showed that in first test, ambiguities could be resolved faster than in second test. Lower solar activity causing less disturbances in ionosphere can explain this fact.

These results are important in developing filters for future navigation applications that use GPS receivers to determine relative position of satellites in formation flying configurations, supporting filter parameters tuning, in particular on ionospheric error components.
Figure 4. Second data set: ADOP values for (a) L1 only; (b) L1 and L2 frequencies.

Figure 5. Second data set: Success rate values for (a) L1 only; (b) L1 and L2 frequencies.

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