Integration of Reliability Concept into Soil Tillage Machine Design*

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The main interest of designers is not only to determine the amplitudes of tillage forces but also the type of their distributions under different soil mechanical properties with object of achieving reliable designs. Whereas, the deterministic design approach consists in achieving designs without considering the randomness of the design parameters that may lead to non reliable designs. In this work, we establish a statistical study on the randomness of the soil properties in collaboration with Cranfield University. This new study can be considered as a useful database for agricultural equipment design fields. Here, we take into account the uncertainties of soil mechanical properties that have big effects on tillage forces. The tillage forces are calculated in accordance with analytical model of McKyes and Ali with some modifications to include the effect of both soil-metal adhesion and tool speed. The distributions of soil-tool forces are next established to design soil tillage equipments such as shank chisel plow. The reliability index is then calculated using two deferent methods (Monte Carlo method and Hasofer and Lind approach). The Hasofer and Lind approach provides the structural reliability level with a low computing time relative to the Monte Carlo approach.

Keywords: probabilistic design, reliable design, reliability index, soil-tillage forces.

1- Introduction
Accurate prediction of soil-tool forces is of great value to the designers of soil tillage equipments [1]. However, there are many soil cutting models available that could be used to predict the forces acting on the tillage tool [2]. Analytical and...
Numerical methods are proposed to achieve this aim. The soil-tool forces in the analytical methods are function of three categories of variables, 1) soil mechanical properties, 2) tool parameters, and 3) parameters of tillage operating. In these models, the soil mechanical parameters are taken as constants, and the researchers attempted to accomplish the relationships between the soil-tool forces on the one hand, and the tool and tillage operating parameters on the other hand [3]. While the researchers worked to simulate soil material behavior under loading of tillage tools, in the finite elements methods, two various theoretical bases have been introduced, namely the curve-fitting technique and the elastic-perfectly plastic assumption. The two FEM modeling methods have considered Young's modulus of elasticity and Poisson's ratio as constants. They also assumed a homogenous soil body during the FEM analysis [4]. However, Fielke in [5] had studied the effect of Poisson's ratio on tillage forces. In fact, soil is not a continuous and homogeneous mass but a three-phase medium consisting of solid, liquid, and gaseous particles. Therefore soil mechanical properties are both vertically and horizontally variable [6]. In this work, we establish a statistical study of some soil mechanical parameters that can be considered as random parameters in order to integrate the uncertainty into the soil tillage equipment design. Here, we present in Table 1, an efficient database for agricultural equipment designers.

### Table 1

| No. | \( \gamma \) (kN/m³) | \( C \) (kPa) | \( \phi \) | \( \sigma \) (kPa) | \( C_s \) (kPa) |
|-----|-----------------|--------------|--------|-----------------|---------------|
| 1   | 14.70           | 04.60        | 37.5   | 11.9            | 0.01          |
| 2   | 10.80           | 00.10        | 34.0   | 14.4            | 3.29          |
| 3   | 14.61           | 02.26        | 35.0   | 15.2            | 2.20          |
| 4   | 15.70           | 07.19        | 35.0   | 15.9            | 2.70          |
| 5   | 14.34           | 06.30        | 37.3   | 22.0            | 2.50          |
| 6   | 11.00           | 11.90        | 29.8   | 17.2            | 0.00          |
| 7   | 14.50           | 06.00        | 28.8   | 18.3            | 0.00          |
| 8   | 13.20           | 23.00        | 33.1   | 18.8            | 0.00          |
| 9   | 14.12           | 08.90        | 35.0   | 18.8            | 2.31          |
| 10  | 16.19           | 12.80        | 32.0   | 19.8            | 0.18          |
| 11  | 13.05           | 16.70        | 35.0   | 19.9            | 0.21          |
| 12  | 16.38           | 15.50        | 22.0   | 20.0            | 0.29          |
| 13  | 13.73           | 06.00        | 23.0   | 21.6            | 0.35          |
| 14  | 14.02           | 23.00        | 27.1   | 22.0            | 0.31          |
| 15  | 16.98           | 10.50        | 30.8   | 22.0            | 3.25          |
| 16  | 16.38           | 31.75        | 42.0   | 22.0            | 5.27          |
| 17  | 14.02           | 12.10        | 30.2   | 22.3            | 3.22          |
| 18  | 15.30           | 13.30        | 36.5   | 22.4            | 3.21          |
| 19  | 15.79           | 20.50        | 35.0   | 23.0            | 0.00          |
| 20  | 17.66           | 05.00        | 38.0   | 23.0            | 0.00          |
| 21  | 19.62           | 10.20        | 32.5   | 23.1            | 0.00          |
| 22  | 19.00           | 20.40        | 31.8   | 23.3            | 0.00          |
| 23  | 14.71           | 13.90        | 30.3   | 23.5            | 0.60          |
| 24  | 14.91           | 15.50        | 39.3   | 23.8            | 0.00          |
| 25  | 15.30           | 15.30        | 32.6   | 24.0            | 0.00          |
| 26  | 15.01           | 06.70        | 31.4   | 24.1            | 0.00          |
| 27  | 14.62           | 11.70        | 29.2   | 24.5            | 0.00          |
| 28  | 12.80           | 07.00        | 31.4   | 24.7            | 0.00          |
| 29  | 13.50           | 17.00        | 37.6   | 25.0            | 0.00          |
| 30  | 13.23           | 16.00        | 26.6   | 23.2            | 0.00          |
| 31  | 16.98           | 11.70        | 27.4   | 27.3            | 6.66          |
| 32  | 18.05           | 19.50        | 28.4   | 29.0            | 8.00          |
2- Mechanical Properties of Soil

The working part of tillage equipment (ex: plow bottoms in moldboard plows, disk blades in disk plows) receiving energy form the tractor or other source works the soil and changes its state and properties. To determine the tillage tool effect on the soil, we should determine the distribution type for various soil mechanical properties. The mechanical properties of soil are important for soil-working, that is, properties which affect the nature of the process, hence the properties which have effects on the forces acting on the tillage tool are:

1. **Soil bulk density** is defined as the weight divided by the volume. Mechanical behavior of soil is influenced by any changes that occur in soil bulk density, that means any changes of soil bulk density will directly affect the amplitudes of soil tillage forces.

2. **Angle of internal friction**: It exhibits the existence of friction force between soil particles. Its values are affected by soil porosity, moisture content, normal stress, and grain size distribution.

3. **Angle of external friction**: External friction force is the resistance or reaction to a force imposed externally to cause one surface sliding over the other under the standard pressure conditions. The angle of external friction can be determined using an apparatus and the Coulomb's concept of friction coefficient.

4. **Cohesion** is defined as the force that holds two particles of the same kind together. Kepner in [7] found that the cohesion and the internal friction are parameters of shear, as indicated in the following equation, \(\tau = c + \sigma \tan(\phi)\).

5. **Adhesion**: Adhesion is defined as the force of attraction between two unlike bodies. In the case of soils, adhesion is due to the film of moisture between the soil particles and the contacting surface of the soil. The force of adhesion is due to the surface tension of water, and consequently it depends upon the value of surface tension and moisture content.

In the Appendix below, a table of 32 samples is presented as experimental studies of the above five soil mechanical properties. This data can be helpful to establish a soil tillage force model.

3- Soil tillage forces

There are lots of methods and models used to predict the forces acting on the tillage tool. However, the majority of researchers used to apply the general earth pressure model proposed by Reece [8]. The total force acting on the tillage tool can be written as follows:

\[
P = P_\gamma + P_c + P_{ca} + P_q + P_a,
\]

Here, \(P\) is the total soil cutting force acting on the tillage tool (\(kN\)), \(P_\gamma\) is the force acting on the tillage tool caused by soil gravity (\(kN\)), \(P_c\) is the force acting on the tillage tool caused by cohesion (\(kN\)), \(P_{ca}\) is the force acting on the tillage tool caused by adhesion (\(kN\)), \(P_q\) is the force acting on the tillage tool caused by surcharge pressure (\(kN\)), and \(P_a\) is the force acting on the tillage tool caused by tool speed (\(kN\)). The different force components are given by the following equations:

\[
P_\gamma = \gamma d^2 w N_1,
\]

\[
P_c = c d w N_{ca},
\]

\[
P_{ca} = c_a d w N_{ca},
\]

\[
P_q = q d w N_q,
\]

\[
P_a = \gamma v^2 d w N_a,
\]

with:

\(\gamma\): Soil bulk density (\(kN/m^3\)), \(d\): Tool working depth (\(m\)), \(c\): Soil internal cohesion (\(kPa\)), \(c_a\): Soil-metal adhesion (\(kPa\)), \(q\): Surcharge pressure at the soil surface (\(kPa\)), \(v\): Forward tool speed (\(m/s^2\)), \(w\): Tool width (\(m\)), \(N_1\): Inertial coefficient (dimensionless), \(N_{ca}\): Adhesion coefficient (dimensionless), \(N_c\): Cohesion coefficient (dimensionless), \(N_q\): Surcharge pressure coefficient (dimensionless), \(N_a\): Gravity coefficient (dimensionless)

The coefficients (\(N_1, N_{ca}, N_c, N_q, N_a\)) are defined according to the soil failure model. In this work, we use McKyes and Ali's model [8] to determine these \(N\)-factors. We select this model according to its simplicity and accuracy [9]. McKyes and Ali assumed that the soil failure surface from the tool tip to the soil surface was linear, and made an unknown angle \(\beta\), with the soil surface, Fig. 1a. The forward distance of the failure crescent from the blade on the surface was assumed to be equal to the radius \(r\) of the crescent.
The forces acting on the soil segment are illustrated in Fig. 1b, including the effects of the density of soil $\gamma$, the internal friction angle $\phi$, the soil-metal friction angle on the blade surface $\delta$, the soil cohesion $c$, the soil-metal adhesion $c_a$ and the surface surcharge pressure $q$.

According to McKeys and Ali's model, Eq. (1) can be written:

$$ P = \frac{1}{2} r \left[ 1 + \frac{2s}{3w} \right] \sin (\beta_r + \phi) + c \frac{\cos (\phi)}{\sin (\delta)} \left[ 1 + \frac{s}{w} \right] + ... $$

$$ + q \frac{r}{d} \sin (\beta_r + \phi) \left[ 1 + \frac{s}{w} \right] + c_a \left[ \sin (\beta_r + \phi) - \cot (\alpha) \cos (\beta_r + \phi) \right] - $$

$$ - \gamma^2 \left[ \frac{\sin (\beta_r + \phi)}{\cot (\alpha)} \right] \frac{\cos (\beta_r + \phi)}{\tan (\beta_r) \cot (\alpha)} \left[ 1 + \frac{s}{w} \right] \sin (\alpha + \beta_r + \delta + \phi) $$

Here, $r$ is the distance from the blade to the forward failure plan (m), given by:

$$ r = d \left[ \cot (\alpha) + \cot (\beta_r) \right] $$

$s$ is the width of the side crescent (m), given by:

$$ s = d \sqrt{\cot^2 (\beta_r) + 2 \cot (\alpha) \cot (\beta_r)} $$

$\alpha$ is the Rack angle of the tool from the horizontal (deg), $\delta$ is the angle of the soil failure zone (deg), $\delta$ is the angle of soil-metal friction (deg) and $\phi$ is the angle of internal soil friction (deg).

The calculated force in Eq. (7) is a function of the unknown angle $\beta_r$. McKyes and Ali obtained this angle $\beta_r$ by minimizing the dimensionless term of gravity $N$. The drought (H) and vertical force (V) are obtained by combining $P$ with force of adhesion [9] as follows:

$$ H = P \sin (\alpha + \delta) + c_a d v \cot (\alpha) $$

$$ V = P \cos (\alpha + \delta) - c_a d v $$

4- Reliability Analysis

The safety is the state in which the structure is able to fulfill all the functioning requirements (e.g. strength and serviceability) for which it is designed. To evaluate the failure probability with respect to the chosen failure scenario, a limit state function $G(x,y)$ is defined by the condition of good functioning of the structure. The limit between the state of failure $G(x,y) < 0$ and the state of safety $G(x,y) > 0$ is known as the limit state function $G(x,y) = 0$ (Fig.2a). Here, $x$: the vector of deterministic variables, and $Y$: the vector of random variables. The limit state function plays an important role in the
development of the structural reliability analysis methods, and it can be in simple or complicated form. The reliability analysis methods are developed corresponding to the limit states of various types and complexity. Here, two methods can be used to evaluate failure probabilities. The first method is based on the Monte Carlo simulation technique. The second is called Hasofer and Lind approach [11].

4-1 Monte Carlo simulation technique
Perhaps the most famous early use of this technique was by Enrico Fermi in 1930, it took this name during World War II by von Neumann. It is possible to calculate the probability of failure for both the explicit and implicit limit state function with only a little background in probability and statistics. Haldar and Mahadevan [12] summarized this technique in six essential steps: 1) defining the problem in terms of all the random variables; 2) quantifying the probabilistic characteristic of all the random variables in terms of their probability density function and the corresponding parameters; 3) generating values of these variables; 4) evaluating the problem deterministically for each set of realizations of all the random variables, or simply numerically experimentation; 5) extracting probabilistic information from N such realizations; and 6) determining the accuracy and efficiency of the simulation.

It is known that a value of $G(x,y)$ less than zero indicates failure. Let $N_f$ be the number of simulation cycles when $G(x,y)$ is less than zero and let $N$ be the total number of simulation cycles. Therefore, an estimate of the failure probability can be expressed as:

$$P_f = \frac{N_f}{N}. \quad (12)$$

4-2 Hasofer and Lind Method
They proposed to work in the space of standard independent Gaussian variables instead of the space of physical variables. The transformation from the physical variables $y$ to the normalized variables $u$ (depending on the distribution law of the random variable) is given by: $u = T(y)$ or $y = T^{-1}(u)$ (Probabilistic Transformation). The reliability index $\beta$ proposed by Hasofer and Lind is defined as the minimum distance from the origin of the axes in the reduced coordinate system to the limit state surface (Fig. 3b). It can be found by the following optimization problem:

$$\beta = \min_u \left( u^T u = \min \sum_i u_i^2 \right) \quad \text{subject to} \quad H(u) \leq 0, \quad (13)$$

where the limit state function takes the form $H(u) = G(x,y)$ in the normalized space. According to FORM technique [13], the relationship between the probability of failure $P_f$ and the reliability index is approximate as follows:

$$P_f = \Phi(-\beta), \quad \text{(14)}$$

where $\Phi(.)$ is the standard Gaussian cumulated function given as follows:

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} e^{-\frac{z^2}{2}} dz. \quad (15)$$

![Fig. 2: The transformation between the physical space and the normalized one.](image-url)
5- Numerical Application on A shank chisel plow

5-1 Modeling of soil mechanical property distributions

To determine the distribution types of soil mechanical properties, we consider (32) samples of soil mechanical properties for different soil types presented in Appendix. First, we model the histograms and the probability density functions of various studied properties as illustrated in Figs.3a to 3e.

Next, we determine the corresponding distribution types according to the shape of the histogram and parameters as illustrated in Table 2. Here, $\mu$ and $\xi$ are the shape and scale parameters of a lognormal distribution, $k$ and $\lambda$ are, respectively, the scale and shape parameters of the Weibull distribution, and $\eta$ is the scale parameter of an exponential distribution.

| Variable       | Type       | Distribution Parameters |
|----------------|------------|-------------------------|
| $\gamma$ (kN/m$^3$) | Lognormal  | $\mu = 2,703, \xi = 0,135$ |
| $c$ (kPa)       | Weibull    | $k = 13,924, \lambda = 1,777$ |
| $\varphi$ ($^\circ$) | Lognormal  | $\mu = 3,467, \xi = 0,146$ |
| $\delta$ ($^\circ$) | Weibull    | $k = 22,909, \lambda = 7,047$ |
| $c_a$ (kPa)     | Exponential| $\eta = 0,716$          |
5-2 Modeling of soil tillage force distributions

Chisel plows illustrated in Fig. 4a can be used primarily to realize the weed control, the seedbed preparation, and other secondary tillage operations.

Their functionalities are to shatter, mix, and aerate the soil with a little soil inversion and a little coverage of plant residue. The soil engaging tools are shanks equipped with shovels.

Fig. 4. Five Shank Chisel plow (a), and a typical shank chisel plow (b)

The distribution of soil forces are established only for a shank chisel plow, Fig. 4b. In this study, we select a simple chisel plow containing shovels as narrow tines in order to calculate the soil-tool forces in applying McKyes and Ali’s model.

Tool and operating parameters of a shank chisel plow are: Rake angle - \( \alpha = 45^\circ \), Tool width - \( w = 0.05(m) \), Tillage Depth - \( d = 0.25(m) \), Tool Speed - \( v = 1.67(m/s) \).

We elaborate the histograms and probability density functions of the horizontal and vertical forces as shown in Fig. 5., and their probabilistic characteristics are presented in Table 3.

Fig. 5: Histogram and probability density function of horizontal force (a), and of vertical force (b)
Table 3

Probabilistic characteristics of tillage forces

| Force Type | Distribution Type | Distribution Parameters | Mean Value | Standard Deviation |
|------------|-------------------|-------------------------|------------|-------------------|
| $P_H$ (kN) | Lognormal         | $\mu = 0.815$ , $\xi = 0.421$ | 2.463      | 1.044             |
| $P_V$ (kN) | Lognormal         | $\mu = 0.052$ , $\xi = 0.415$ | 1.032      | 0.427             |

5-3 Reliability analysis of a shank chisel plow

The performance criterion, related to the mechanical resistance of tillage machines is determined by the difference between the allowable stress and the maximum stress. Therefore, the limit state function that defined the safe region can be written using the following equation:

$$G(x, y) = \sigma_{\text{max}} - \sigma_w \leq 0,$$

(16)

Here, $x$ is the vector of deterministic variables and $y$ is the vector of random variables, $\sigma_w$ is the allowable stress and $\sigma_{\text{max}}$ is the maximum stress is given by:

$$\sigma_{\text{max}} = \frac{6}{bh^2} \left[ \left( L_2 + L_4 \right) P_H + \frac{L_4}{\tan(\alpha)} P_V \right] + \frac{1}{bh} P_H.$$

(17)

The limit state function of the simplified shank model, illustrated in Fig. 6, is a function of the following variables as:

$$G(x, y) = f(P_H, P_V, \alpha, b, h, L_1, L_2, L_4).$$

(18)

Using the horizontal and vertical force equation, we get and and the input of geometrical parameters of the studied shank: $L_1=600$ mm, $L_2=350$ mm, $L_3=15$ mm, $L_4=75$ mm, $\alpha = 45^\circ$, $b=32$ mm, $h=58$ mm. Here, the corresponding maximum stress value equals to: $\sigma_{\text{max}} = 63.99$ (MPa).

Fig. 6: A schematic drawing of the chisel plow shank with acting forces

The variability in the horizontal and vertical forces was determined in the earlier Section. The variability in the rake angle $\alpha$ was considered during the determination of the variability in tillage forces, so it is considered in this study as constant. The probability distributions of $b$ and $h$ were defined as uniform distributions with lower and upper bounds based on the manufacturing accuracy of $\pm 0.1$ mm. Furthermore, we assumed that $L_4$ and $\sigma_w$ have normal distributions with coefficient of variations equal to 0.05. The dimensions $L_1$ and $L_2$ were considered as deterministic variables in the reliability analysis. The statistical parameters for the random variables are presented in Table 4.

The Monte Carlo Simulation Technique necessitates a high computing time to evaluate the probability of failure but it is considered as a reference method. We use the Hasofer and Lind method as an approximate method to find the probability of failure with a low computing time. Using this method, the optimization problem (13) is carried out in the normalized space and the relationship between the probabilities of failure can be approximate using equations (14) and (15). Table 5 presents the reliability results of the studied problem.
Table 4

| Variable | Distribution Type | Distribution Parameter |
|----------|-------------------|------------------------|
| b (mm)   | Uniform           | \( L_a = 31.9 \), \( U_b = 32.1 \) |
| h (mm)   | Uniform           | \( L_a = 57.9 \), \( U_b = 58.1 \) |
| \( L_1 \) (mm) | Deterministic     | \( L_1 = 600 \) |
| \( L_2 \) (mm) | Deterministic     | \( L_2 = 350 \) |
| \( L_4 \) (mm) | Normal            | \( m_{L_4} = 75 \), \( \sigma_{L_4} = 3.75 \) |
| \( P_h \) (kN) | Lognormal        | \( \mu = 0.815 \), \( \zeta = 0.421 \) |
| \( P_v \) (kN) | Lognormal        | \( \mu = 0.052 \), \( \zeta = 0.0415 \) |
| \( \sigma_w \) (MPa) | Normal           | \( m_{\sigma_w} = 235 \), \( \sigma_{w} = 11.75 \) |

Table 5

| Method                    | \( P_f \) | \( \beta \) |
|---------------------------|----------|-----------|
| Monte Carlo Method        | 0.0520%  | 3.279     |
| Hasofer-Lind Method       | 0.0501%  | 3.289     |

Table 6 shows the design point at the limit state and mean value of different variables. We note that the horizontal and vertical forces have the biggest effect on the failure mode.

| Variable | Mean Value | Design Point |
|----------|------------|--------------|
| b (mm)   | 32         | 31,998       |
| h (mm)   | 58         | 57,998       |
| \( L_4 \) (mm) | 75       | 74,674       |
| \( P_h \) (kN) | 2,643    | 8,925        |
| \( P_v \) (kN) | 1,032    | 3,308        |
| \( \sigma_w \) (MPa) | 235       | 230,325      |

When using Monte Carlo simulation technique, it is possible to estimate the probability of failure without taking into account the shape of the limit state function. Here, we can guarantee the accuracy level for a big number of iterations that leads to a high computational time to evaluate the failure probability. However, Hasofer and Lind method approximates the limit state in a normalized space to evaluate the failure probability. We have found that the results of both methods are almost similar.

6- Conclusions
The originality of this work is to integrate the uncertainties on the soil mechanical properties in order to carry out reliable designs. This study provides the structural designers in agricultural machine area. The statistical studies can efficiently improve the design quality, especially the safety criteria. It also consists in evaluating the failure probability formulations with object of computing the reliability index using various existing methods such as Monte-Carlo simulation, Hasofer-Lind's one. As a perspective work, we seek to integrate the reliability-based design optimization that defines the best compromise between cost and safety in considering our resulting reliability indices as target (allowable) reliability indices.
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