Impacts of anomaly on nuclear and neutron star equation of state based on a parity doublet model

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We examine the role of the \( U(1)_A \) anomaly in a parity doublet model of nucleons which include the chiral variant and invariant masses. Our model expresses the \( U(1)_A \) anomaly by the Kobayashi-Maskawa-'t Hooft (KMT) interaction in the mesonic sector. After examining the roles of the KMT term in vacuum, we discuss its impacts on nuclear equations of state (EOS). The \( U(1)_A \) anomaly increases the masses of the \( \eta' \) and \( \sigma \) mesons and enhances the chiral symmetry breaking. The \( U(1)_A \) anomaly enlarges the energy difference between chiral symmetric and symmetry broken vacuum; in turn, the chiral restoration at high density adds a larger energy density (often referred as a bag constant) to EOSs than in the case without the anomaly, leading to softer EOSs. Including these \( U(1)_A \) effects, we update the previously constructed unified equations of state that interpolate the nucleonic EOS at \( n_B = 2n_0 \) (\( n_0 = 0.16 \text{ fm}^{-3} \); nuclear saturation density) and quark EOS at \( n_B \geq 5n_0 \). The unified EOS is confronted with the observational constraints on the masses and radii of neutron stars. The softening of EOSs associated with the \( U(1)_A \) anomaly reduces the overall radii, relaxing the previous constraint on the chiral invariant mass \( m_0 \). Including the attractive nonlinear \( \rho-\omega \) coupling to improved estimates for the slope parameter in the symmetry energy, our new estimate is \( 400 \text{ MeV} \leq m_0 \leq 700 \text{ MeV} \), with \( m_0 \) smaller than our previous estimate by \( \sim 200 \text{ MeV} \).

I. INTRODUCTION

The chiral \( SU(N_f)_L \otimes SU(N_f)_R \) symmetry in quantum chromodynamics (QCD) and its spontaneous symmetry breaking (SSB) play the key role in describing the low-energy hadron physics, e.g., the soft pion dynamics and the dynamically generated quark masses [1]. The chiral condensates, being the order parameters of the chiral SSB, quantify the degree of the chiral SSB, and also are useful in characterizing states of matter in QCD at finite temperature and/or density [2, 3].

In addition to the dynamical SSB, the current quark mass and the quantum anomaly explicitly break the \( U(1)_A \) symmetry and assist the formation of the chiral condensates [4, 5]. In this paper we study the impact of the \( U(1)_A \) anomaly on the chiral symmetry breaking and examine how it influences nuclear matter equations of state (EOS). While there are many works on nucleonic EOS emphasizing the importance of in-medium interactions among nucleons, in-medium changes of the Dirac sea structure and their impacts on EOS acquire much less attentions. We argue that the \( U(1)_A \) anomaly increases the discrepancy between the chiral symmetry broken and restored phases. In other words, the anomaly increases the bag constant associated with the chiral restoration as shown in Fig. 1. In the context of EOS, a larger bag constant adds the energy density but reduces the pressure, leading to softer EOS.

FIG. 1. The Dirac sea in chiral symmetric (left) and symmetry broken (right) phases. The particle-antiparticle pairs condense to break the chiral symmetry and produce the mass gap \( M \). The mass gap is larger in the presence of the \( U(1)_A \) anomaly. The energy difference in the Dirac sea between the chiral symmetry restored and broken phases defines (a part of) the bag constant.

In the nuclear matter domain, we include the anomaly effects in terms of the Kobayashi-Maskawa-’t Hooft (KMT) interactions[6] for a three-flavor mesonic Lagrangian made of scalar and vector mesons. The KMT interactions relate up-, down-, and strange-quark Dirac sea even before the strangeness appears in a matter. In fact, the chiral restoration for the up- and down-quark sectors assists the chiral restoration for the strange quark sector, possibly changing the masses of hyperons in nuclear matter. Such structural changes in hyperons are
potentially important for matter composition in neutron stars (NSs).

The baryonic part in this work is treated in a parity doublet model (PDM) \[^7,8\] for nucleons in which the ordinary nucleon $N(940)$ and its parity partner $N(1535)$ form a doublet. The novel feature of the PDM is that the nucleon masses include not only the conventional chiral variant mass but also invariant mass ($m_0$) whose existence is supported by the previous lattice QCD simulations\[^9\]. Accordingly, nucleons in the PDM is less sensitive to the chiral condensate or $\sigma$ fields than in conventional linear $\sigma$ models. The PDM for vacuum physics has been studied in Refs.\[^2,11,36,38-41\]. In the latter, the PDM EOS is used from a quark model assuming the quark-hadron-crossover.

As a result, we obtain more relaxed constraints on $\omega$ fields and softens EOS at supra-nuclear densities larger than $\sim 600$ MeV\[^0\], reducing the previous range $\sim 700$ MeV, reducing the previous range by $\sim 200$-300 MeV. We also add the radius constraint from the PSR J0740+6620 for $2\odot$ by $\sim 200-300$ MeV. We also add the radius constraint for applications to NS phenomenology, nuclear EOS in the PDM is extrapolated to densities beyond $n_0$. It has been simply extrapolated \[^35\] or combined with a quark model assuming the quark-hadron-crossover \[^2,11,36,38-41\]. In the latter, the PDM EOS is used up to $2n_0$, and interpolate with the quark EOS at $\geq 5n_0$ via polynomial interpolants. Including the charge neutrality and $\beta$-equilibrium conditions, the unified EOS was confronted with NS constraints from the existence of two-solar mass ($2M_\odot$) NSs \[^12\] and the gravitational waves from the NS merger event GW170817 \[^13,14\]. Based on the upperbound for the NS radii constraint, we previously constrained $m_0$ to rather large values $\sim 600$ MeV, $600$ MeV $\lesssim m_0 \lesssim 900$ MeV.

In this work, we update the constraints by including the $U(1)_A$ anomaly and also include the previously neglected $\rho^2\omega^2$ terms which are usually assumed to be attractive to make EOS softer. Both effects soften EOS at low densities $\sim 1-2n_0$, leading to smaller NS radii. As a result, we obtain more relaxed constraints on $m_0$, $400$ MeV $\lesssim m_0 \lesssim 700$ MeV, reducing the previous range by $\sim 200$-300 MeV. We also add the radius constraint from the PSR J0740+6620 for 2.08$+0.07 M_\odot$ NS, $R_{2.08} = 12.35 \pm 0.75$ km \[^46\] and 12.39$^{+1.30-0.98}$ km \[^17\].

This paper is organized as follows. In Sec. II, we explain the formulation of our model which based on parity doublet structure. In Sec. III, we construct EOS in hadronic matter and quark matter separately and the parameters are determined in Sec. IV. Main results of the analysis are shown in Sec. VI and Sec. VII. In Sec. VIII we show a summary and discussions.

II. FORMULATION

In this section, we construct a model of symmetric nuclear matter.

A. Scalar and pseudoscalar mesons

We first construct an effective Lagrangian for scalar and pseudoscalar mesons based on the $SU(3)_L \times SU(3)_R$ chiral symmetry \[^14,17\] including the effect of $U(1)_A$ anomaly. Quarks transform under $SU(3)_L \times SU(3)_R \times U(1)_A$ symmetry as

\[
q_L \to e^{-iA} g_L q_L, \quad q_R \to e^{+iA} g_R q_R,
\]

with $g_L, g_R \in SU(3)_L, R$ and $\theta_A$ being the transformation parameters. Accordingly, we assign the $U(1)_A$ charge of the left and right handed quarks as $-1$ and $+1$, respectively. The chiral representation of the left handed quark is then given by

\[
q_L : (3,1), \quad q_R : (1,3)+1.
\]

where these $3$ and $1$ in the bracket express the triplet and singlet for $SU(3)_L$ symmetry and $SU(3)_R$ symmetry, respectively. The index indicates the axial charge of the fields. On the other hand, the chiral representation of the right-handed quark is given by

\[
q_R : (1,3)-1.
\]

We introduce a $3 \times 3$ matrix field $\Phi$ for scalar and pseudoscalar mesons as

\[
\Phi_{ij} : (3,\bar{3})-2.
\]

We adopt the meson part of the Lagrangian as

\[
\mathcal{L}_M^{\text{scalar}} = \mathcal{L}_M^{\text{kin}} - V_M - V_{SB},
\]

where

\[
\mathcal{L}_M^{\text{kin}} = \frac{1}{4} \text{tr} \left[ \partial^\mu \Phi \partial^\nu \Phi^\dagger \right],
\]

\[
V_M = -\frac{1}{4} \bar{\mu}^2 \text{tr} \left[ \Phi \Phi^\dagger \right] + \frac{1}{8} \lambda_4 \text{tr} \left[ (\Phi \Phi^\dagger)^2 \right] - \frac{1}{12} \lambda_6 \text{tr} \left[ (\Phi \Phi^\dagger)^3 \right] + \lambda_8 \text{tr} \left[ (\Phi \Phi^\dagger)^4 \right] + \lambda_{10} \text{tr} \left[ (\Phi \Phi^\dagger)^5 \right],
\]

\[
V_{SB} = -\frac{1}{2} c \text{tr} \left[ M^i \Phi^i + M \Phi^i \right],
\]
\[ V_{\text{Anom}} = - B \left[ \det(\Phi) + \det(\Phi^\dagger) \right]. \] (9)

Here \( B \) is the coefficient for the axial anomaly term and \( c \) is the coefficient for the explicit chiral symmetry breaking term with \( \mathcal{M} \) defined as \( \mathcal{M} = \text{diag}\{m_u, m_d, m_s\} \). The above Lagrangian for the meson part is \( U(1)_A \) invariant except the anomaly term. We note that we include only terms with one trace in \( V_M \), which are expected to be of leading order in \( 1/N_c \) expansion. Compared with previous model in Ref. [36], we not only include the anomaly term but also introduce the \( \lambda_8 \) and \( \lambda_{10} \) terms to stabilize the potential in the vacuum.\[1\]

In this work we use a hadronic model only up to \( 2n_0 \) neglecting hyperons. Within mean field treatments adopted in this paper, only the diagonal components are kept. So we reduce \( \Phi \) to

\[ \Phi = \begin{pmatrix} M & 0 \\ 0 & \phi_s \end{pmatrix}_{3 \times 3}, \] (10)

where we keep the abstract notation \( M \) as a \( 2 \times 2 \) matrix field to keep track of the \( SU(2)_L \times SU(2)_R \times U(1)_A \) structure of our model. The meson field under chiral transformation in \( SU(2) \) case is

\[ M \rightarrow g_L M g_R^\dagger, \] (11)

where \( g_L \in SU(2)_L \) and \( g_R \in SU(2)_R \). Then the reduced Lagrangian is

\[ \mathcal{L}_M^{\text{scalar}} = \frac{1}{4} \left( \text{tr} \left[ \partial_\mu M \partial^\mu M^\dagger \right] + \partial_\mu \phi_s \partial^\mu \phi_s^\dagger \right), \] (12)

\[ V_M = - \frac{1}{4} \bar{\mu}^2 \left( \text{tr} \left[ M M^\dagger \right] + \phi_s \phi_s^\dagger \right) + \frac{1}{8} \lambda_4 \left( \text{tr} \left[ (M M^\dagger)^2 \right] + (\phi_s \phi_s^\dagger)^2 \right) + \frac{1}{12} \lambda_6 \left( \text{tr} \left[ (M M^\dagger)^3 \right] + (\phi_s \phi_s^\dagger)^3 \right) + \lambda_8 \left( \text{tr} \left[ (M M^\dagger)^4 \right] + (\phi_s \phi_s^\dagger)^4 \right) + \lambda_{10} \left( \text{tr} \left[ (M M^\dagger)^5 \right] + (\phi_s \phi_s^\dagger)^5 \right), \] (13)

\[ V_{SB} = - \frac{c}{2} \left[ \text{tr} \left[ \mathcal{M}_{2 \times 2} (M + M^\dagger) \right] + m_s (\phi_s + \phi_s^\dagger) \right], \] (14)

\[ V_{\text{Anom}} = - B \left[ \det(M) \phi_s + \det(M^\dagger) \phi_s^\dagger \right]. \] (15)

where \( \mathcal{M}_{2 \times 2} \) = \text{diag}\{\( m_u, m_d \)}\}.

**B. Nucleon parity doublet and vector mesons**

While we treat the mesonic sector including three-flavors, we discuss nucleons only up to \( 2n_0 \) where we assume that hyperons do not enter the system. In the PDM, we assume that nucleons and the chiral partners belong to the representations of \( (2, 1)_{+} \) and \( (1, 2)_{-} \) as

\[ \psi_L^R : (2, 1)_{+}, \quad \psi_R^R : (1, 2)_{+}, \] (16)

\[ \psi_L^S : (1, 2)_{+}, \quad \psi_R^S : (2, 1)_{-}. \] (17)

under \( SU(2)_L \times SU(2)_R \times U(1)_A \) symmetry. In mean field treatments, these fields couple to the two-flavor part in the three-flavor mesonic Lagrangian. Then the nucleon part constructed based on the \( SU(2)_R \times SU(2)_L \times U(1)_A \) symmetry is given by

\[ \mathcal{L}_N = \sum_{i=1,2} \bar{\psi}_i i\gamma^\mu D_\mu \psi_i \]

\[ - g_1 \left( \bar{\psi}_1 L \tau^2 (M^\dagger)^T \tau^2 \psi_1^R + \bar{\psi}_1 R \tau^2 M^T \tau^2 \psi_1^L \right) \]

\[ - g_2 \left( \bar{\psi}_2 L \tau^2 M^T \tau^2 \psi_2^R + \bar{\psi}_2 R \tau^2 M^\dagger \tau^2 \psi_2^L \right) \]

\[ - m_0 \left( \bar{\psi}_1 L \psi_2^R - \bar{\psi}_2 L \psi_1^R - \bar{\psi}_2 R \psi_1^R + \bar{\psi}_2 R \psi_2^R \right), \] (18)

where \( \tau_i (i = 1, 2, 3) \) are the Pauli matrices. The couplings \( g_{1,2} \) are the Yukawa couplings to the scalar fields for \( \psi_{1,2} \) and the origin of the chiral variant masses. Meanwhile \( m_0 \) is the chiral invariant mass which originate from the coupling between \( \psi_1 \) and \( \psi_2 \). In the mean field treatment of \( \sigma \), the mass spectra are given by

\[ m_\pm = \sqrt{m_0^2 + \left( \frac{g_1 + g_2}{2} \right)^2 \sigma^2 + \frac{g_1 - g_2}{2} \sigma}, \] (19)

where \( + \) is for \( N(940) \) and \( - \) for \( N(1535) \) as the mixture of \( \psi_1 \) and \( \psi_2 \) fields. For vanishing \( \sigma \), the masses get degenerated, \( m_\pm \rightarrow m_0 \).

The coupling of vector mesons to nucleons is introduced in the form of the covariant derivatives

\[ D_\mu \psi_{L,R}^{1,2} = (\partial_\mu - iV_\mu) \psi_{L,R}^{1,2}. \] (20)

with \( V_\mu \) general external fields including \( \omega \) and \( \rho \) mesons coupled to baryon number and isospin densities, respectively.

The Lagrangian for vector mesons is based on the hidden local symmetry (HLS) [38, 49]. This part is not affected by the \( U(1)_A \) anomaly. We use the same form as the previous works except addition of the following term

\[ \mathcal{L}_{\omega_\rho} = \lambda_{\omega_\rho} (g_{\omega_\rho})^2 \left( g_{\omega_\rho} \right)^2, \] (21)

where \( \lambda_{\omega_\rho} \) is assumed to be positive, meaning the attractive correlation between the \( \omega \) and \( \rho \) fields. This term assists the appearance of \( \rho \) fields as \( \omega \) fields develop. The \( \omega-\rho \) correlations play important roles in the symmetry energy, as will be discussed in the following section.

**III. NUCLEAR AND QUARK EQUATIONS OF STATE**

In this section, we construct neutron star matter EOS in both hadronic matter part and quark matter part.
A. Nuclear matter EOS

Following Ref. [38], we apply the mean field approximation to the Lagrangian in the last section, and then calculate the thermodynamic potential in the hadronic matter as

\[
\Omega_{\text{PDM}} = V(\sigma, \sigma_s) - V(\sigma_0, \sigma_{s0}) - \frac{1}{2} m^2 \omega^2 - \frac{1}{2} m^2 \rho^2 - \lambda_{\omega\rho} (g_\omega \omega)^2 (g_\rho \rho)^2
\]

\[\quad - 2 \sum_{i=+,-} \sum_{\alpha=\mathbf{p},n} \int \frac{d^3 p}{(2\pi)^3} \left( \mu^*_i - E^i_p \right) .\]

(22)

Here \(i = +, -\) denote for the parity of nucleons and \(E^i_p = \sqrt{p^2 + m_{i}^2}\) is the energy of nucleons with mass \(m_{i}\) and momentum \(p\). The crossing term \(\omega \rho\) interaction is tuned to adjust the slope parameter, see Sec [IV]. The potential \(V(\sigma, \sigma_s)\) of \(\sigma\) and \(\sigma_s\) mean fields is given by

\[
V(\sigma, \sigma_s) = -\frac{1}{2} \mu^2 \left( \sigma^2 + \frac{1}{2} \sigma_s^2 \right) + \frac{1}{4} \lambda_4 \left( \sigma^4 + \frac{1}{2} \sigma_s^4 \right) - \frac{1}{6} \lambda_6 \left( \sigma^6 + \frac{1}{2} \sigma_s^6 \right) + \lambda_8 \left( 2\sigma^8 + \sigma_s^8 \right) + \lambda_{10} \left( 2\sigma^{10} + \sigma_s^{10} \right) - 2B \sigma^2 \sigma_s
\]

\[\quad - 2cm_{n}\sigma + cm_{s}\sigma_s \right). \quad (23)\]

The total thermodynamic potential for the NS is obtained by including the effects of leptons as

\[
\Omega_{\text{H}} = \Omega_{\text{PDM}} + \sum_{l=e,\mu} \Omega_{l} , \quad (24)
\]

where \(\Omega_{l}(l = e, \mu)\) are the thermodynamic potentials for leptons,

\[
\Omega_{l} = -2 \int_{k_F}^{k_F} \frac{d^3 p}{(2\pi)^3} \left( \mu_{l} - E^l_p \right) . \quad (25)
\]

The mean fields here are determined by following stationary conditions:

\[
0 = \frac{\partial \Omega_{\text{H}}}{\partial \sigma} , \quad 0 = \frac{\partial \Omega_{\text{H}}}{\partial \omega} , \quad 0 = \frac{\partial \Omega_{\text{H}}}{\partial \rho} . \quad (26)
\]

We also need to impose the \(\beta\) equilibrium and the charge neutrality conditions,

\[
\mu_e = \mu_\mu = -\mu_Q , \quad (27)
\]

\[
\frac{\partial \Omega_{\text{H}}}{\partial \mu_Q} = n_p - n_l = 0 , \quad (28)
\]

where \(\mu_Q\) is the charge chemical potential. We then have the pressure in hadronic matter as

\[
P_{\text{H}} = -\Omega_{\text{H}} . \quad (29)
\]

B. Quark matter EOS

Following Refs. [2, 50], we use the NJL quark model to describe the quark matter. The model includes three-flavors and \(U(1)_A\) anomaly effects through the quark version of the KMT interaction. The coupling constants are chosen to be the Hatsuda-Kunihiro parameters which successfully reproduce the hadron phenomenology at low energy [2, 51]: \(G^2_\Lambda = 1.835, K^2_\Lambda = 9.29\) with \(\Lambda = 631.4\text{MeV}\), see the definition below. The couplings \(g_\nu\) and \(H\) characterize the strength of the vector repulsion and attractive diquark correlations whose range will be examined later when we discuss the NS constraints.

We can then write down the thermodynamic potential as

\[
\Omega_{\text{CSC}} = \Omega_{e} - \Omega_{s} \left[ \sigma_f = \sigma_f^0, d_j = 0, \mu_q = 0 \right] + \Omega_{e} - \Omega_{e} \left[ \sigma_f = \sigma_f^0, d_j = 0 \right] , \quad (30)
\]

where the subscript 0 is attached for the vacuum values, and

\[
\Omega_{e} = -2 \sum_{i=1}^{18} \int_{p_i}^{\Lambda} \frac{d^3 p}{(2\pi)^3} \epsilon_i , \quad (31)
\]

\[
\Omega_{e} = \sum_{i} \left( 2G\sigma^2_i + Hd^2_i \right) - 4K\sigma_{d}\sigma_{s} - g_\nu n^2_q , \quad (32)
\]

with \(\sigma_f\) are the chiral condensates, \(d_j\) are diquark condensates, and \(n_q\) is the quark density. In Eq. (31), \(\epsilon_i\) are energy eigenvalues obtained from inverse propagator in Nambu-Gorkov bases

\[
S^{-1}(k) = \begin{pmatrix}
\gamma_\mu k^\mu - M + \gamma^0 \hat{\mu} & \gamma_5 \sum_i \Delta_i R_i \\
-\gamma_5 \sum_i \Delta^*_i R_i & \gamma_\mu k^\mu - M - \gamma^0 \hat{\mu}
\end{pmatrix} , \quad (33)
\]

where

\[
M_i = m_i - 4G\sigma_i + K |\epsilon_{ijk}| \sigma_j \sigma_k , \quad (34)
\]

\[
\Delta_i = -2Hd_i , \quad (34)
\]

\[
\hat{\mu} = \mu_q - 2g_\nu n_q + \mu_3 \lambda_3 + \mu_8 \lambda_8 + \mu_Q Q .
\]

\(S^{-1}(k)\) is the \(72 \times 72\) matrix in terms of the color, flavor, spin, and Nambu-Gorkov basis, which has 72 eigenvalues. \(M_u,d,s\) are the constituent masses of \(u,d,s\) quarks and \(\Delta_{1,2,3}\) are the gap energies. The \(\mu_3,8\) are the color chemical potentials which will be tuned to achieve the color neutrality. The total thermodynamic potential including the effect of leptons is

\[
\Omega_{Q} = \Omega_{\text{CSC}} + \sum_{l=e,\mu} \Omega_{l} . \quad (35)
\]

The mean fields are determined from the gap equations,

\[
0 = \frac{\partial \Omega_{Q}}{\partial \sigma_i} = \frac{\partial \Omega_{Q}}{\partial d_i} , \quad (36)
\]

From the conditions for electromagnetic charge neutrality and color charge neutrality, we have

\[
n_j = -\frac{\partial \Omega_{Q}}{\partial \mu_j} = 0 , \quad (37)
\]
After determined all the values, we obtain the pressure
$$P_Q = -\Omega_Q,$$
where \(Q = 3, 8, Q\). The baryon number density \(n_B\) is determined as
$$n_q = -\frac{\partial Q}{\partial \mu_q},$$
(38)
where \(\mu_q\) is 1/3 of the baryon number chemical potential. After determined all the values, we obtain the pressure
$$P_Q = -\Omega_Q.$$
(39)

TABLE I. Physical inputs in vacuum in unit of MeV.

| \(m_s\) | \(m_K\) | \(f_8\) | \(f_K\) | \(m_\omega\) | \(m_\rho\) | \(m_+\) | \(m_-\) |
|-----------|----------|--------|--------|----------|----------|--------|--------|
| 140       | 494      | 92.4   | 109    | 783      | 776      | 939    | 1535   |

TABLE II. Saturation properties used to determine the model parameters: the saturation density \(n_0\), the binding energy \(B_0\), the incompressibility \(K_0\), symmetry energy \(S_0\) and the slope parameter \(L_0\).

| \(n_0\) [fm\(^{-3}\)] | \(B_0\) [MeV] | \(K_0\) [MeV] | \(S_0\) [MeV] | \(L_0\) [MeV] |
|------------------------|----------------|----------------|----------------|----------------|
| 0.16                   | 16             | 240            | 31             | 57.7           |

TABLE III. Values of model parameters determined for several choices of \(\lambda_8 = \lambda_8 f_8^2\). When \(B = 600\) MeV, we only find solutions which satisfy the saturation properties in the range: \(0 \leq \lambda_8' \leq 7.05\), here we list the boundary values as a typical example. \(\lambda_8' = 0\) is the minimum boundary and \(\lambda_8' = 7.05\) is the maximum boundary.

| \(m_0 = 800\) [MeV] | \(\lambda_8 = 0\) (\(\lambda_8' = 7.05\)) |
|---------------------|----------------------------------------|
| \(g_1\)            | 6.99                                   |
| \(g_2\)            | 13.4                                   |
| \(\rho^2 / f_\rho^2\) | 24.83                                 |
| \(\lambda_4\)      | 63.5                                   |
| \(\lambda_6 f_\rho^2\) | 45.27                                 |
| \(\lambda_\omega f_\sigma^2\) | 0.52                                 |
| \(\lambda_{10} f_\sigma^2\) | 0.44                                 |
| \(g_{\omega NN}\)  | 5.12                                   |
| \(g_{\rho NN}\)    | 10.25                                  |

| \(m_0 = 600\) [MeV] | \(\lambda_8 = 0\) (\(\lambda_8' = 7.05\)) |
|---------------------|----------------------------------------|
| \(g_1\)            | 6.99                                   |
| \(g_2\)            | 13.4                                   |
| \(\rho^2 / f_\rho^2\) | 7.22                                   |
| \(\lambda_4\)      | 102.8                                  |
| \(\lambda_6 f_\rho^2\) | 66.23                                 |
| \(\lambda_\omega f_\sigma^2\) | 0.66                                   |
| \(\lambda_{10} f_\sigma^2\) | 0.44                                   |
| \(g_{\omega NN}\)  | 4.17                                   |
| \(g_{\rho NN}\)    | 9.31                                   |

FIG. 2. Restricted combination of \(\lambda_8\) and \(\lambda_{10}\) after fixing the value of \(\sigma_8\) with \(m_0 = 800\) MeV. \(\lambda_8' = \lambda_8 f_8^2\), \(\lambda_{10} = \lambda_{10} f_\sigma^2\).

FIG. 3. The \(B\) dependence of masses of \(\eta\) and \(\eta'\) mesons in the vacuum with unit MeV.

IV. PARAMETER DETERMINATION

In this section, we determine the parameters in the PDM by fitting with the normal nuclear matter properties and the decay constants for different \(m_0\) (summarized in Table I and Table II). It is notified that, for \(B = \lambda_8 = \lambda_{10} = 0\), the present model is exactly the same as Refs. [34, 36] and the model parameters can be determined in the same way. As in the previous works, we use the vector masses \(m_\rho = 776\) MeV and \(m_\omega = 783\) MeV. The parameters \(c_{m_u} = c_{m_d}\) and \(c_{m_s}\) are fixed by the following relations with \(f_\pi\) and \(f_K\) given in Table II

$$2c_{m_u} = m_\rho^2 f_\rho^2, \quad c(m_u + m_s) = m_K^2 f_K^2.$$
(40)

We are left with 11 parameters which will be tuned in the presence of the \(U(1)_A\) anomaly. The mesonic part contains

$$\tilde{\mu}^2, \lambda_4, \lambda_6, \lambda_8, \lambda_{10}, B, \lambda_\omega\rho,$$
(41)

and the nucleonic Lagrangian contains

$$m_0, g_1, g_2, g_{\omega NN}, g_{\rho NN}.$$
(42)
In this paper, we treat \( m_0 \) as a given input and then fix the other parameters. When we present results for \( m_0 \) different from the values in this section, those results are obtained after retuning the above parameters to achieve the same quality of fitting as in the present section, unless otherwise stated.

The mesonic part is constrained by the vacuum physics and nuclear saturation properties. In vacuum, the couplings \((g_1, g_2)\), for a given \( m_0 \), are fixed by demanding \( m_{\text{vac}} = 939 \text{ MeV} \) and \( m_{\text{vac}} = 1535 \text{ MeV} \) through the relation,

\[
m_{\text{vac}} = \sqrt{m_0^2 + \left( \frac{g_1 + g_2}{2} \right)^2 \sigma_0^2} \mp \left| \frac{g_1 - g_2}{2} \right| \sigma_0. \tag{43}
\]

where the \( \sigma \) fields in vacuum are given by

\[
\sigma_0 = f_\pi, \quad \sigma_{s0} = f_K - \frac{f_\pi}{2}. \tag{44}
\]

In order to satisfy these relations on \( \sigma_0 \) and \( \sigma_{s0} \), a proper range of the mesonic parameters in Eq.\( \text{11} \) must be chosen.

There is still large degeneracy among the mesonic parameters. We can break the degeneracy by demanding the mesonic parameters and \((g_{\omega NN}, g_{\rho NN}, \lambda_{\omega} \) to reproduce the saturation properties listed in Table\( \text{1II} \). Then, we are left with the degeneracy related to the choice of parameters \( \lambda_8, \lambda_{10}, \) and \( B \). We show the degeneracy related to \( \lambda_8 \) and \( \lambda_{10} \) in Fig\( \text{6} \) by showing the range to reproduce the above-mentioned saturation properties.

Finally, the parameter \( B \) is strongly correlated with the \( \eta \) and \( \eta' \) masses whose experimental values in the vacuum are

\[
m_{\eta}^{\text{exp}} \simeq 547.9 \text{ MeV}, \quad m_{\eta'}^{\text{exp}} \simeq 957.8 \text{ MeV}. \tag{45}
\]

We fix the parameters to reproduce the above-mentioned vacuum and saturation properties for a given \( B \). We repeat this procedure while increasing \( B \) until the parameters reproduce \( \eta \) and \( \eta' \) masses correctly. The behaviors of \( \eta \) and \( \eta' \) masses as functions of \( B \) are displayed in Fig.\( \text{3} \). The width attached to the curves reflects the different combinations of \( \lambda_8 \) and \( \lambda_{10} \). For \( B = 600 \text{ MeV} \), the masses of \( \eta' \) and \( \eta \) are calculated as

\[
m_{\eta}^{\text{PDM}} = 542 \pm 15 \text{ MeV}, \quad m_{\eta'}^{\text{PDM}} = 962 \pm 20 \text{ MeV}. \tag{46}
\]

In this paper, we take \( B = 600 \text{ MeV} \) as the physical value.

\section{V. EFFECT OF ANOMALY IN MESON SECTOR FOR HADRONIC MATTER}

To study the effect of the anomaly, we perform linear analysis with respect to the variation of \( B \); we weakly vary the value of \( B \) around our physical choice \( B = 600 \text{ MeV} \), leaving the other parameters unchanged. (Within this linear analysis, the results other than \( B = 600 \text{ MeV} \) do not satisfy the saturation properties.)

The vacuum value of \( \sigma \) and \( \sigma_s \) change as shown in Fig\( \text{4} \). The vacuum values of \( \sigma \) and \( \sigma_s \) increase as \( B \) does. This indicates that the anomaly enhances the chiral symmetry breaking, as advertised in the previous sections. The energy density in vacuum is reduced more by the stronger chiral symmetry breaking. When the chiral symmetry is restored, this energy reduction in vacuum is lost, and we have to add more energy density or a bag constant to the EOS in the chiral restored phase.

Another important effect of the anomaly is the increase of \( \sigma \) meson mass, as shown in Fig\( \text{5} \). In the context of nuclear forces, the heavier \( \sigma \) meson mass reduces the range of attractive force and weakens the overall strength; this in turn requires weaker repulsive \( \omega \) interactions to balance with the \( \sigma \) attraction to satisfy the saturation properties. The resultant reduced repulsion leads to a softer nuclear EOS at supra-saturation densities where \( \omega \) dominates over \( \sigma \). In summary, the \( U(1)_A \) anomaly effects softens nuclear EOS at supra-saturation densities.

In Fig\( \text{6} \), we show the density dependence of the energy density for \( B = 580, 600 \) and 620 MeV with \( m_0 = 800 \text{ MeV} \). The energy density overall increases as \( B \) does in whole density region, and the saturation points shift to higher densities. This can be understood by the competition between the \( \sigma \) attraction and \( \omega \) repulsion. In the present linear analyses, increasing \( B \) does not change the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The \( B \) dependence of \( \sigma \) and \( \sigma_s \) for \( m_0 = 500 \text{ MeV} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{The \( B \) dependence of \( m_\sigma \) for \( m_0 = 500 \text{ MeV} \).}
\end{figure}
vector meson mass but increases the mass of \( \sigma \). As a result, the range of \( \sigma \) attraction, \( \sim 1/m_\sigma \), decreases as \( B \) increases, reducing the attractive contributions to the energy density. We also show the energy dependence of the pressure in the Fig. 7 is obtained through

\[
P = \mu_B n_B - \varepsilon, \tag{47}
\]

which indicates that the effect of anomaly softens the equation of state.

VI. EFFECT OF ANOMALY IN NJL-TYPE MODEL FOR QUARK MATTER

In the NJL-type model introduced in Sec. IIIB, the coefficient \( K \) represents the strength of anomaly. Here we gradually decrease the value of \( K \) from \( K \Lambda^3 = 9.29 \) toward 0 with fixing other parameters to study the effect of anomaly. For simplicity, we first set \( H = 0 \) to avoid di-quark condensate. The chiral condensates in the vacuum have the anomaly dependence as in Fig. 8.

Chiral condensates in vacuum increase with increasing \( K \), which is similar to the PDM. This result indicates that the anomaly enhances the chiral symmetry breaking and reduces the ground state energy in vacuum.

In Fig. 9 we show dispersion relations of quarks in the chiral symmetry broken vacuum (left panel) and in the chiral symmetric vacuum (right panel). The approximate chiral symmetry is spontaneously broken by the chiral condensate, and quarks of different chiralities are connected with each other. Then the condensation opens a gap \( M \) in the quark dispersion relation. As a result, the structure of the Dirac sea is changed to generate a non-perturbative QCD vacuum. The difference in energy density between the chiral symmetric Dirac sea and symmetry-broken Dirac sea defines the bag constant \[52\],

\[
\varepsilon_{\text{bag}} = \varepsilon(M_{\text{eff}} = m_q) - \varepsilon(M_{\text{eff}} = M), \tag{48}
\]

where \( M_{\text{eff}} \) is the effective mass of quarks, \( m_q \) is the bare quark mass and \( M \) is the constituent quark mass.

The density dependence of \( \varepsilon_{\text{total}} \) and \( \varepsilon_{\text{bag}} \) are calculated separately as shown in the Fig. 9 for two cases, \( K = 0 \) and \( K = 9.29/\Lambda^3 \). This indicates that \( \varepsilon_{\text{bag}}(K = 0) > \varepsilon_{\text{bag}}(K = 9.29/\Lambda^3) \) at the same density, which implies that the effect of anomaly enhances the bag constant and finally increase the total energy.

From the analysis of the chiral condensates in the vacuum in Fig. 8 anomaly effect lowers the ground state energy of the vacuum. In Fig. 9 we show a schematic view of vacuum structure. The released energy after chiral symmetry restoration is larger with anomaly than without it, then at the same density \( \varepsilon_{\text{bag}}(K = 0) > \varepsilon_{\text{bag}}(K = 9.29/\Lambda^3) \).
We also calculate the density dependence of the relevant pressures in Fig. 10, where $P_{\text{bag}}$ is calculated from $\varepsilon_{\text{bag}}$ using the thermodynamic relation.

$$P_{\text{bag}} = -\varepsilon_{\text{bag}} + \mu_q n_q.$$  \hspace{1cm} (49)

This shows that for same density, $P_{\text{total}} < P_{K=0}^{P_{\text{total}}}$ which is mainly caused by the difference of $P_{\text{bag}}$. In summary, at a given density

$$\varepsilon_{\text{total}} > \varepsilon_{K=0}^{\text{total}}, \quad P_{\text{total}} < P_{K=0}^{\text{total}},$$  \hspace{1cm} (50)

so EOS with a positive $K$ is softer, i.e., $P$ is smaller at a given $\varepsilon$, as shown in Fig. 11.

FIG. 10. Density dependence of $P_{\text{total}}$ and $P_{\text{bag}}$ with $(H, g_V)/G = (0, 0.1)$.

FIG. 11. The energy dependence of pressure for $H/G = 0, g_V/G = 0.1$.

VII. STUDY OF PROPERTIES OF NS

In this section, following Ref. we construct a unified EOS by connecting the EOS obtained in the PDM introduced in Sec. [11A] and the EOS of NJL-type quark model given in Sec. [11B] and solve the TOV equation [53] to obtain the NS mass-radius ($M-R$) relation. As for the interplay between nuclear and quark matter EOS, see, e.g., Ref. [55] for a quick review that classifies types of the interplay.

A. Construction of unified EOS

In our unified equations of state, we use the BPS (Baym-Pethick-Sutherland) EOS [56] as a crust EOS for $n_B \lesssim 0.5 n_0$. From $n_B \simeq 0.5 n_0$ to $2 n_0$ we use our PDM model to describe a nuclear liquid. Beyond the nuclear regime, we assume a crossover from the nuclear matter to quark matter, and use a smooth interpolation to construct the unified EOS. We expand the pressure as a fifth order polynomial of $\mu_B$ as

$$P_1(\mu_B) = \sum_{i=0}^{5} C_i \mu_B^i,$$  \hspace{1cm} (51)

where $C_i (i = 0, \ldots, 5)$ are parameters to be determined from boundary conditions given by

$$\left. \frac{d^n P_i}{(d \mu_B)^n} \right|_{\mu_{BL}} = \left. \frac{d^n P_H}{(d \mu_B)^n} \right|_{\mu_{BL}},$$  \hspace{1cm} (52)

$$\left. \frac{d^n P_i}{(d \mu_B)^n} \right|_{\mu_{BU}} = \left. \frac{d^n P_Q}{(d \mu_B)^n} \right|_{\mu_{BU}},$$  \hspace{1cm} (53)

with $\mu_{BL}$ being the chemical potential corresponding to $n_B = 2 n_0$ and $\mu_{BU}$ to $n_B = 5 n_0$. That is, we demand the matching up to the second order derivatives of pressure at each boundary. The resultant interpolated EOS must satisfy the thermodynamic stability condition,

$$\chi_B = \frac{\partial^2 P}{(d \mu_B)^2} \geq 0,$$  \hspace{1cm} (54)

and the causality condition,

$$\varepsilon_s = \frac{dP}{d\varepsilon} = \frac{n_B}{\mu_B \chi_B} \leq 1,$$  \hspace{1cm} (55)

which means that the sound velocity is less than the light velocity. These conditions restrict the range of quark model parameters $(g_V, H)$ for a given nuclear EOS and a choice of $(n_L, n_U)$.

We exclude interpolated EOSs which do not satisfy the above-mentioned constraints. Similar surveys for the range of $(g_V, H)$ and $(n_L, n_U)$ have been carried out first for APR EOS [57] and for ChEFT EOS [50] in Ref. [58], and more systematically for Togashi EOS [59] in Ref. [59] and for ChEFT EOS [50] in Ref. [51]. The range explored in the present work is largely consistent with the previous works using different nuclear EOSs. Finally we note that the estimate based on non-perturbative massive gluon exchanges favor the estimate of $g_V \sim G$ and $H \sim 1.5 G$ [62].

It is important to note that the constraints become severer for the combination of softer nucleonic EOS and...
stiffer quark EOS. The rapid growth of the stiffness, together with the requirement of $c_s^2 \rightarrow 1/3$ in the high density limit, generally leads to a peak in the sound velocity, as first found phenomenologically in Refs. [40] [41], and later explained microscopically in Refs. [43] [44] with the emphasis on the quark degrees of freedom. The growth of the stiffness in the crossover model is in general quicker than in purely hadronic models, and such features may be studied in gravitational waves from neutron star merger events [45], or in QCD-like theories, e.g., two-color QCD, for which analytical [60] and lattice calculations [62] suggest the rapid stiffening in the crossover domain.

B. Mass-Radius relation

In this section, we study the $M$-$R$ relations of NSs from the unified EOS constructed above. In Ref. [30], where the anomaly in the nuclear EOS is neglected, the chiral invariant mass is constrained to be $600\,\text{MeV} \lesssim m_0 \lesssim 900\,\text{MeV}$. In the present analysis, we improve the analyses in three aspects: (i) we include the anomaly in the nuclear EOS; (ii) we newly include the $\omega^2 \rho^2$-term for flexible tuning of the slope parameter $L$ in the symmetry energy (here we adopt the value $L = 57.7\,\text{MeV}$ as a baseline suggested by Ref. [68]); and (iii) we include a new constraint from the NICER on the radius of $2.1M_\odot$ neutron stars.

We first examine the effects of NJL parameters $(g_V, H)$. For simplicity, we fix parameters in the PDM to $B = 600\,\text{MeV}$, $\lambda_8 = 0, \lambda_{10}' = 0.44$, and $\lambda_{\omega\rho}$ tuned to reproduce $L = 57.7\,\text{MeV}$. We then vary the value of $m_0$ and examine the range of $(g_V, H)$ which is allowed by the causality and thermodynamic stability conditions. The band shown in the Fig. [12] specifies such domains, while the blank part is not allowed. A larger $g_V$ requires a larger $H$. For $m_0 = 800\,\text{MeV}$, the maximum masses for all the combinations are below $2M_\odot$, leading to the conclusion that $m_0 = 800\,\text{MeV}$ should be excluded within the current setup of the PDM parameters.

Next we fix $m_0 = 500\,\text{MeV}$ and vary the value of $\lambda_{\omega\rho}$ or $L$ while the rest of hadronic parameters kept unchanged. The resultant $M$-$R$ relation is shown in Fig. [13] thick curves in the low (high)-mass region indicate the central density of the NS is smaller than $2n_0$ (larger than $5n_0$), and the NS is made from hadronic matter (quark matter). The thin curves on the other hand show that the core is in crossover region. From the figure one sees that the EOSs are softened by the effect of the $\omega^2 \rho^2$-term and the radius for $L = 57.7\,\text{MeV}, M \simeq 1.4M_\odot$ is about $11.2\,\text{km}$ in comparison with the result of $L = 80\,\text{MeV}$ about $12.1\,\text{km}$. There is still a large ambiguity about the values of slope parameter and small slope parameters usually soften the NS EOS, shifting the radius towards smaller values. Precise determination of slope parameter in the future will help us further constrain the NS properties.

In following analysis, we fix the value $L = 57.7\,\text{MeV}$ and examine the effects of anomaly on the $M$-$R$ relation. In Fig. [14(a)], we show how the $M$-$R$ curves change under the $B$ effect. The $(\lambda_8, \lambda_{10}')$ parameters from $m_0 = 400$ to $800\,\text{MeV}$ are fixed to the boundary values in the following analysis, $\lambda_8 = 0, \lambda_{10}' = 0.44$. The NJL parameter $(H, g_V)$ are chosen to have the stiffest two $M$-$R$ curves. In the Fig. [14(a)], because of the softening effect of anomaly, after we set $B = 600\,\text{MeV}$, the stiffest connection for $m_0 = 800\,\text{MeV}$ is unable to satisfy the maximum constraints. In Fig. [14(b)], we show the final results in this work after setting $B = 600\,\text{MeV}$ for different $m_0$ values. We find the final constraints to the chiral invariant mass is changed to be smaller by $\sim 100\,\text{MeV}$ in comparison with the previous constraints in Ref. [30].

VIII. A SUMMARY AND DISCUSSION

In this work, we construct an effective hadronic model in which the effect of strange quark condensate is included in the mesonic sector through the Kobayashi-Maskawa-'t Hooft (KMT)-type interaction reflecting the $U(1)$ axial anomaly. We then study the impact of $U(1)_A$ anomaly on the chiral symmetry breaking in both hadronic and an NJL-type quark modes. In both models the $U(1)_A$ anomaly enhances the chiral symmetry breaking. In the PDM, the anomaly effects increases the effective mass of $\sigma$, and the heavier $\sigma$ meson mass reduces the range of attractive force, weakening the overall strength; this in turn requires weaker repulsive $\omega$ interactions to balance with the $\sigma$ attraction to satisfy the saturation properties. The resultant reduced repulsion leads to a softer nuclear EOS at supra-saturation densities. In the NJL-type model, the anomaly effects lead to large bag constant. Since a larger bag constant adds the energy density but reduces the pressure, the corresponding EOS is softened. We expect that it is a general feature that $U(1)_A$ anomaly softens the NS EOS.

The EOS plays an essential role when determining the NS properties. The NICER analyses of the most massive NS known, PSR J0740+6620, with $M/M_\odot = 2.08 \pm 0.07$ and the radii $R_{2.08} = 12.35 \pm 0.75\,\text{km}$ [40], together with the updated estimate for $R_{1.4} = 12.35 \pm 0.75\,\text{km}$ [40], disfavors strong first order phase transitions in the region between $1.4M_\odot$ and $2.1M_\odot$.

In this case, we assume the hadronic and quark matter are not distinctly different and construct unified EOS for neutron star matter. At present work, we interpolate the EOS obtained in the hadronic model based on the parity doublet structure $(n_B \leq 2n_0)$ and the one in the NJL-type quark model $(n_B \geq 5n_0)$ with crossover in the intermediate region. We found that the unified EOS is also softened by the effect of anomaly due to the softening of the EOS in both hadronic and quark matters. The resultant $M$-$R$ curves are compared with the constraints from GW170817 (LIGO & VIRGO) and PRS J0030+0451 (NICER) as well as the constraint from PRS J0740+6620. From the constraints we restrict the chiral
invariant mass as

\[ 400 \text{ MeV} \lesssim m_0 \lesssim 700 \text{ MeV}. \quad (55) \]

Compared with results without anomaly, \( 500 \text{ MeV} \lesssim m_0 \lesssim 800 \text{ MeV} \), we find that the anomaly softens the EOS, shifting the range of chiral invariant mass towards lower values by 100 MeV. The effects of the \( \omega^2 \rho^2 \) term or \( L \) are also very important when we constrain the chi-
interactions which also break
the

determination of
maximum mass. The typical estimates value is
not only decrease the total radius but also lead to smaller
parameters (different slope parameter. Red curves are connected to the NJL
FIG. 13. Mass-radius relations for
small values of
(1.55, 0.9), (1.5, 0.7).
In this paper, we included the anomaly
B term only
SU

thermore, we can add hyperons to a hadronic model with
chiral symmetry combined with the
U
Uchiral symmetry. We
leave these analyses as future works.

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Appendix A: CHIRAL CONDENSATES

We calculate the chiral condensates in the PDM by differentiating the thermodynamic potential with respect to the current quark masses[11]:

\[ \langle (\bar{u}u + \bar{d}d) \rangle = \frac{\partial \Omega_{\text{PDM}}}{\partial m_q} \]  

(A1)

Then, using the Gell-Mann-Oakes-Renner relation, we obtain

\[ \frac{\langle (\bar{u}u + \bar{d}d) \rangle}{\langle (\bar{u}u + \bar{d}d) \rangle_0} = \frac{\sigma}{f_\pi} \]  

(A2)

Similarly, we obtain

\[ \langle \bar{s}s \rangle = \frac{\partial \Omega_{\text{PDM}}}{\partial m_s} \]  

(A3)

and

\[ \frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_0} = \frac{\sigma_s}{\sigma_{s0}} \]  

(A4)

where \( \sigma_{s0} \) is the mean field \( \sigma_s \) at the vacuum.

In the linear density approximation, the density dependence of the condensates are given by

\[ \langle \bar{q}q \rangle \simeq \langle 0|\bar{q}q|0 \rangle + \rho(N|\bar{q}q|N) = \langle 0|\bar{q}q|0 \rangle + \rho \frac{\Sigma_N}{2m_q}, \]  

(A5)

\[ \langle \bar{s}s \rangle \simeq \langle 0|\bar{s}s|0 \rangle + \rho(N|\bar{s}s|N) = \langle 0|\bar{s}s|0 \rangle + \rho \frac{\Sigma_{sN}}{m_s}, \]  

(A6)

where \( \Sigma_N \) is the \( \pi N \) sigma term and \( \Sigma_{sN} \) is the strange quark sigma term.

In Fig. 15(a), we show the density dependence of \( \langle (\bar{u}u + \bar{d}d) \rangle/(\langle (\bar{u}u + \bar{d}d) \rangle_0 \) determined from the PDM in the neutron star matter. We also plot typical examples of the density dependence of the condensate determined in the linear density approximation where the \( \pi N \) sigma term is taken as \( \Sigma_N = 45, 90 \text{ MeV} \) Ref.[11]. This shows that

In Fig.15(b), we show the density dependence of strange quark chiral condensate compared with the linear density approximation shown by colored bands. In the linear density approximation, we use the value of \( \Sigma_{sN} \) determined by the lattice QCD simulations shown in Table[7] as typical examples. The colored bands in Fig.15(b) are written by taking account of all the errors, e.g. \( \Sigma_{sN} = 40.2 \pm 15.2 \text{ MeV} \) for \( \chi QCD \) [22]. This Fig.15(b) shows that the ambiguity of \( \Sigma_{sN} \) is too large to give a constraint to our model. However, we expect that the precise determination of \( \Sigma_{sN} \) in future will constrain the chiral invariant mass.

| Collaboration | \( \Sigma_{sN}[\text{MeV}] \) |
|--------------|-----------------|
| \chi QCD     | 40.2(11.7)(3.5) |
| ETM          | 41.1(8.2)(7.8)  |
| RQCD         | 35(12)          |
| JLQCD        | 17(18)(9)       |

TABLE V. Values of \( \Sigma_{sN} \) obtained by recent from Lattice QCD simulations.
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(a) $B = 0, 600 \text{ MeV for } m_0 = 500, 800 \text{ MeV}.$

(b) $B = 600 \text{ MeV for different } m_0.$ NJL parameters $(H, gV)/G$ are chosen to be $(1.45, 1.3)_{m_0=400 \text{ MeV}}, (1.6, 1.3)_{m_0=500 \text{ MeV}}, (1.6, 1.3)_{m_0=600 \text{ MeV}}, (1.6, 1.2)_{m_0=700 \text{ MeV}}.$

FIG. 14. Mass-radius relations for different $m_0$ in different parameter setting.

(a) $\langle \bar{u}u + \bar{d}d \rangle/\langle \bar{u}u + \bar{d}d \rangle_0$ (left) and $\langle \bar{s}s \rangle/\langle \bar{s}s \rangle_0$ (right) in the PDM. We use the same parameter choices in Fig. 14(b). In (a), two dotted lines show the density dependence in the linear density approximation with $\Sigma_N = 45, 90 \text{ MeV}$ as typical examples. In (b), the colored bands are drawn in the linear density approximation where the value of $\Sigma_N$ is taken from the lattice QCD results shown in Table V with error bars included: JLQCD(grey band), RQCD(green band), ETM(red band) and $\chi$QCD(orange band).

[1] S. P. Klevansky. The Nambu-Jona-Lasinio model of quantum chromodynamics. Rev. Mod. Phys., 64:649–708, 1992.

[2] Gordon Baym, Tetsuo Hatsuda, Toru Kojo, Philip D. Powell, Yifan Song, and Tatsuyuki Takatsuka. From hadrons to quarks in neutron stars: a review. Rept. Prog. Phys., 81(5):056902, 2018.

[3] Ryugo S. Hayano and Tetsuo Hatsuda. Hadron properties in the nuclear medium. Rev. Mod. Phys., 82:2949, 2010.

[4] G. 't Hooft. Symmetry breaking through bell-jackiw anomalies. Phys. Rev. Lett., 37:8–11, Jul 1976.

[5] G. 't Hooft. How instantons solve the u(1) problem. Physics Reports, 142(6):357–387, 1986.

[6] M. Kobayashi and T. Maskawa. Chiral symmetry and eta-x mixing. Prog. Theor. Phys., 44:1422–1424, 1970.

[7] Carleton DeTar and Teiji Kunihiro. Linear sigma model with parity doubling. Phys. Rev. D, 39:2805–2808, May 1989.

[8] Daisuke Jido, Makoto Oka, and Atsushi Hosaka. Chiral Symmetry of Baryons. Progress of Theoretical Physics, 106(5):873–908, 11 2001.

[9] Gert Aarts, Chris Allton, Simon Hands, Benjamin Jäger, Chrisanthi Praki, and Jon-Ivar Skullerud. Nucleons and parity doubling across the deconfinement transition. Phys. Rev. D, 92(1):014503, 2015.

[10] Takahiro Yamazaki and Masayasu Harada. Chiral partner structure of light nucleons in an extended parity doublet model. Phys. Rev. D, 99:034012, Feb 2019.

[11] Takuya Minamikawa, Toru Kojo, and Masayasu Harada. Chiral condensates for neutron stars in hadron-quark
crossover: From a parity doublet nucleon model to a Nambu–Jona-Lasinio quark model. *Phys. Rev. C*, 104(6):065201, 2021.

[12] D. Jido, T. Hatsuda, and T. Kunihiro. Chiral symmetry realization for even parity and odd parity baryon resonances. *Phys. Rev. Lett.*, 84:3252, 2000.

[13] Y. Nemoto, D. Jido, M. Oka, and A. Hosaka. Decays of 1/2− baryons in chiral effective theory. *Phys. Rev. D*, 57:4124–4135, 1998.

[14] Hiroki Nishihara and Masayasu Harada. Extended Goldberger-Treiman relation in a three-flavor parity doublet model. *Phys. Rev. D*, 92(5):054022, 2015.

[15] Hua-Xing Chen, V. Dmitrasinovic, and Atsushi Hosaka. Baryon fields with \( U(3)_L \times U(3)_R \) chiral symmetry II: Axial currents of nucleons and hyperons. *Phys. Rev. D*, 81:054002, 2010.

[16] Hua-Xing Chen, V. Dmitrasinovic, and Atsushi Hosaka. *mathermBaryons with \( U(3)_L \times U(3)_R \) Chiral Symmetry IV: Interactions with Chiral (3, 3) + (3, 3) Spinless Mesons*. *Phys. Rev. D*, 83:014015, 2011.

[17] Hua-Xing Chen, V. Dmitrasinovic, and Atsushi Hosaka. *mathermBaryons with \( U(3)_L \times U(3)_R \) Chiral Symmetry I: Interaction with Chiral (8, 8) + (8, 8) Vector and Axial-vector Mesons and Anomalous Magnetic Moments*. *Phys. Rev. C*, 85:055205, 2012.

[18] T. Hatsuda and M. Prakash. Parity doubling of the nucleon and first-order chiral transition in dense matter. *Physics Letters B*, 224(1):11–15, 1989.

[19] D. Zschiesche, L. Tolos, Jürgen Schaffner-Bielich, and Robert D. Pisarski. Cold, dense nuclear matter in a su(2) parity doublet model. *Phys. Rev. C*, 75:055202, May 2007.

[20] V. Dexheimer, S. Schramm, and D. Zschiesche. Nuclear matter and neutron stars in a parity doublet model. *Phys. Rev. C*, 72:055803, Feb 2008.

[21] Chihiro Sasaki and Igor Mishustin. Thermodynamics of dense hadronic matter in a parity doublet model. *Phys. Rev. C*, 82:035204, Sep 2010.

[22] Chihiro Sasaki, Hyun Kyu Lee, Won-Gi Paeng, and Manque Rho. Conformal anomaly and the vector coupling in dense matter. *Phys. Rev. D*, 84:034011, Aug 2011.

[23] Susanna Gallicchio, Francesco Giacosa, and Giuseppe Pagliara. Nuclear matter within a dilatation-invariant parity doublet model: The role of the tetraquark at nonzero density. *Nuclear Physics A*, 872(1):13–24, 2011.

[24] J. Steinheimer, S. Schramm, and H. Stöcker. Hadronic \( su(3) \) parity doublet model for dense matter and its extension to quarks and the strange equation of state. *Phys. Rev. C*, 84:045208, Oct 2011.

[25] Won-Gi Paeng, Hyun Kyu Lee, Manque Rho, and Chihiro Sasaki. Dilaton-limit fixed point in hidden local symmetric parity doublet model. *Phys. Rev. D*, 85:054022, Mar 2012.

[26] V. Dexheimer, J. Steinheimer, R. Negreiros, and S. Schramm. Hybrid stars in an \( su(3) \) parity doublet model. *Phys. Rev. C*, 87:015804, Jan 2013.

[27] Won-Gi Paeng, Hyun Kyu Lee, Manque Rho, and Chihiro Sasaki. Interplay between \( \omega \)-nucleon interaction and nucleon mass in dense baryonic matter. *Phys. Rev. D*, 88:105019, Nov 2013.

[28] A. Mukherjee, J. Steinheimer, and S. Schramm. Higher-order baryon number susceptibilities: Interplay between the chiral and the nuclear liquid-gas transitions. *Phys. Rev. C*, 96:025205, Aug 2017.

[29] Daiki Suenaga. Examination of \( N^*(1535) \) as a probe to observe the partial restoration of chiral symmetry in nuclear matter. *Phys. Rev. C*, 97:045203, Apr 2018.

[30] Yusuke Takeda, Youngman Kim, and Masayasu Harada. Catalysis of partial chiral symmetry restoration by \( \Delta \) matter. *Phys. Rev. C*, 97:065202, Jun 2018.

[31] A. Mukherjee, S. Schramm, J. Steinheimer, and V. Dexheimer. The application of the quark-hadron chiral parity-doublet model to neutron star matter. *A&A*, 608:A110, 2017.

[32] Hiroaki Abuki, Yusuke Takeda, and Masayasu Harada. Dual chiral density waves in nuclear matter. *EPJ Web Conf.*, 192:00020, 2018.

[33] Micha Marcenko, David Blaschke, Krzysztof Redlich, and Chihiro Sasaki. Parity doubling and the dense-matter phase diagram under constraints from multi-messenger astronomy. *Universe*, 5(8), 2019.

[34] Yuichi Motohiro, Youngman Kim, and Masayasu Harada. Asymmetric nuclear matter in a parity doublet model with hidden local symmetry. *Phys. Rev. C*, 92:025201, Aug 2015.

[35] Takahiro Yamazaki and Masayasu Harada. Constraint to chiral invariant masses of nucleons from gw170817 in an extended parity doublet model. *Phys. Rev. C*, 100:025205, Aug 2019.

[36] Takuya Minamikawa, Toru Kojo, and Masayasu Harada. Quark-hadron crossover equations of state for neutron stars: Constraining the chiral invariant mass in a parity doublet model. *Phys. Rev. C*, 103:045205, Apr 2021.

[37] Masayasu Harada and Takahiro Yamazaki. Charmed Mesons in Nuclear Matter Based on Chiral Effective Models. *Proceedings of the 8th International Conference on Quarks and Nuclear Physics (QNP2018)*.

[38] Michał Marzenko, David Blaschke, Krzysztof Redlich, and Chihiro Sasaki. Parity Doubling and the Dense Matter Phase Diagram under Constraints from Multi-Messenger Astronomy. *Universe*, 5(8):180, 2019.

[39] Michał Marzenko, David Blaschke, Krzysztof Redlich, and Chihiro Sasaki. Toward a unified equation of state for multi-messenger astronomy. *Astron. Astrophys.*, 643:A82, 2020.

[40] Kota Masuda, Tetsuo Hatsuda, and Tatsuyuki Takatsuka. Hadron-Quark Crossover and Massive Hybrid Stars with Strangeness. *Astrophys. J.*, 764:12, 2013.

[41] Kota Masuda, Tetsuo Hatsuda, and Tatsuyuki Takatsuka. Hadron–quark crossover and massive hybrid stars. *PTEP*, 2013(7):073D01, 2013.

[42] H. T. Cromartie et al. Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar. *Nature Astron.*, 4(1):72–76, 2019.

[43] B. P. Abbott. GW170817: Observation of gravitational waves from a binary neutron star inspiral. *Phys. Rev. Lett.*, 119:161101, Oct 2017.

[44] B. P. Abbott et al. Multi-messenger Observations of a Binary Neutron Star Merger. *Astrophys. J. Lett.*, 848(2):L12, 2017.

[45] B. P. Abbott et al. GW170817: Measurements of neutron star radii and equation of state. *Phys. Rev. Lett.*, 121(16):161101, 2018.

[46] M. C. Miller et al. The Radius of PSR J0740+6620 from Multi-messenger Astronomy. *Universe*, 5(8):180, 2019.

[47] B. P. Abbott et al. GW170817: Measurements of neutron star radii and equation of state. *Phys. Rev. Lett.*, 121(16):161101, 2018.

[48] M. C. Miller et al. The Radius of PSR J0740+6620 from NICER and XMM-Newton Data. *Astrophys. J. Lett.*, 918(2):L28, 2021.

[49] Thomas E. Riley et al. A NICER View of the Mass. Pulsar PSR J0740+6620 Informed by Radio Timing
and XMM-Newton Spectroscopy. Astrophys. J. Lett., 918(2):L27, 2021.

[48] Masako Bando, Taichiro Kugo, and Koichi Yamawaki. Nonlinear realization and hidden local symmetries. Physics Reports, 164(4):217–314, 1988.

[49] Masayasu Harada and Koichi Yamawaki. Hidden local symmetry at loop: A new perspective of composite gauge boson and chiral phase transition. Physics Reports, 381(1):1–233, 2003.

[50] Gordon Baym, Shun Furusawa, Tetsuo Hatsuda, Toru Kojo, and Hajime Togashi. New Neutron Star Equation of State with Quark-Hadron Crossover. Astrophys. J., 885:42, 2019.

[51] Tetsuo Hatsuda and Teiji Kunihiro. QCD phenomenology based on a chiral effective Lagrangian. Phys. Rept., 247:221–367, 1994.

[52] Toru Kojo, Philip D. Powell, Yifan Song, and Gordon Baym. Phenomenological QCD equation of state for massive neutron stars. Phys. Rev. D, 91(4):045003, 2015.

[53] Richard C. Tolman. Static solutions of Einstein’s field equations for spheres of fluid. Phys. Rev., 55:364–373, 1939.

[54] J. R. Oppenheimer and G. M. Volkoff. On massive neutron cores. Phys. Rev., 55:374–381, 1939.

[55] Toru Kojo. QCD equations of state and speed of sound in neutron stars. AAPPs Bull., 31(1):11, 2021.

[56] Gordon Baym, Christopher Pethick, and Peter Sutherland. The Ground state of matter at high densities: Equation of state and stellar models. Astrophys. J., 170:299–317, 1971.

[57] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall. The Equation of state of nucleon matter and neutron star structure. Phys. Rev. C, 58:1804–1828, 1998.

[58] Toru Kojo. Phenomenological neutron star equations of state: 3-window modeling of QCD matter. Eur. Phys. J. A, 52(3):51, 2016.

[59] H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, and M. Takano. Nuclear equation of state for core-collapse supernova simulations with realistic nuclear forces. Nucl. Phys. A, 961:78–105, 2017.

[60] Christian Drischler, Sophia Han, James M. Lattimer, Madappa Prakash, Sanjay Reddy, and Tianqi Zhao. Limiting masses and radii of neutron stars and their implications. Phys. Rev. C, 103(4):045808, 2021.

[61] Toru Kojo, Gordon Baym, and Tetsuo Hatsuda. Implications of NICER for neutron star matter: the QHC21 equation of state. 11 2021.

[62] Yifan Song, Gordon Baym, Tetsuo Hatsuda, and Toru Kojo. Effective repulsion in dense quark matter from nonperturbative gluon exchange. Phys. Rev. D, 100(3):034018, 2019.

[63] Larry McLerran and Sanjay Reddy. Quarkyonic Matter and Neutron Stars. Phys. Rev. Lett., 122(12):122701, 2019.

[64] Toru Kojo. Stiffening of matter in quark-hadron continuity. Phys. Rev. D, 104(7):074005, 2021.

[65] Yong-Jia Huang, Luca Baiotti, Toru Kojo, Kentaro Takami, Hajime Satomi, Hajime Togashi, Tetsuo Hatsuda, Shigehiro Nagataki, and Yi-Zhong Fan. Merger and post-merger of binary neutron stars with a quark-hadron crossover equation of state. 3 2022.

[66] Toru Kojo and Daiki Suenaga. Peaks of sound velocity in two color dense QCD: Quark saturation effects and semishort range correlations. Phys. Rev. D, 105(7):076001, 2022.

[67] Kei Iida and Etsuko Itou. Velocity of Sound beyond the High-Density Relativistic Limit from Lattice Simulation of Dense Two-Color QCD. 7 2022.

[68] Bao-An Li, Bao-Jun Cai, Wen-Jie Xie, and Nai-Bo Zhang. Progress in constraining nuclear symmetry energy using neutron star observables since gw170817. Universe, 7(6), 2021.

[69] Ingo Tews, James M. Lattimer, Akira Ohnishi, and Evgeni E. Kolomeitsev. Symmetry Parameter Constraints from a Lower Bound on Neutron-matter Energy. Astrophys. J., 848(2):105, 2017.

[70] C. Drischler, R. J. Furnstahl, J. A. Melendez, and D. R. Phillips. How Well Do We Know the Neutron-Matter Equation of State at the Densities Inside Neutron Stars? A Bayesian Approach with Correlated Uncertainties. Phys. Rev. Lett., 125(20):202702, 2020.

[71] Brendan T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz. Implications of PREX-2 on the Equation of State of Neutron-Rich Matter. Phys. Rev. Lett., 126(17):172503, 2021.

[72] Yi-Bo Yang, Andrei Alexandru, Terrence Draper, Jian Liang, and Keh-Fei Liu. π N and strangeness sigma terms at the physical point with chiral fermions. Phys. Rev. D, 94(5):054503, 2016.

[73] A. Abdel-Rehim, C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, Ch. Kallidonis, G. Koutsou, and A. Vaquero Aviles-Casco. Direct Evaluation of the Quark Content of Nucleons from Lattice QCD at the Physical Point. Phys. Rev. Lett., 116(25):252001, 2016.

[74] Gunnar S. Bali, Sara Collins, Daniel Richtmann, Andreas Schäfer, Wolfgang Söldner, and André Sternebeck. Direct determinations of the nucleon and pion σ terms at nearly physical quark masses. Phys. Rev. D, 93(9):094504, 2016.

[75] Nodoka Yamanaka, Shoji Hashimoto, Takashi Kaneko, and Hiroshi Ohki. Nucleon charges with dynamical overlap fermions. Phys. Rev. D, 98(5):054516, 2018.