Da Lio, Francesca; Rivière, Tristan; Wettstein, Jerome

Integrability by compensation for Dirac equation. (English) Zbl 1496.35195

Trans. Am. Math. Soc. 375, No. 6, 4477-4511 (2022).

The authors consider the Dirac operator acting on the Clifford algebra. Under critical assumptions on the potential and the spinor field, they prove that the equation is subject to an integrability by compensation phenomenon and has a sub-critical behaviour below some positive energy threshold.

Reviewer: Zhipeng Yang (Göttingen)

MSC:

35J46 First-order elliptic systems
30G35 Functions of hypercomplex variables and generalized variables

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