Isotropic Viscous cosmologies compatible with the standard second law of thermodynamics

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Abstract. A model proposed by M. Giovannini (Phys. Rev. D 61, 108302 (2000)) is considered here. The author has shown that there exists a class of viscous cosmological models which violate the dominant energy condition for a limited amount of time after which they are smoothly connected to the ordinary radiation era (which preserves the dominant energy conditions). It was shown that this violation of the dominant energy condition at an early cosmological epoch may influence the slopes of energy spectra of relic gravitons. However, the bulk viscosity coefficient of these cosmologies became negative during the ordinary radiation era, and then the entropy of the sources driving the geometry decreases with time. We show that for viscous sources with a linear barotropic equation of state we get viscous cosmological models with positive bulk viscous stress during all their evolution, and hence the matter entropy increases with the expansion time.

1. Introduction

The possibility of constructing flat Friedmann–Robertson–Walker (FRW) cosmologies provided with a bulk viscous stress, which induces a violation of the dominant energy condition (DEC) for a limited amount of time at an early cosmological epoch, was realized by Giovannini [2]. This type of models may be connected to some of the recent remarks of Grishchuk [3] concerning the detectability of stochastic gravitational wave background by forthcoming interferometric detectors, such as LIGO, VIRGO, GEO600, LISA [4]. In fact, bulk viscous dissipative processes may influence the slopes of the energy spectra of relic gravitons (generated at the time of violation of the DEC) producing an increasing with frequency in a calculable way. These slopes are crucially related to the sign of the $\rho + p$, where $\rho$ and $p$ are, respectively, the energy density and the pressure density of the cosmic fluid. The requirement that one wants expanding and inflationary universes implies that the energy density of the created gravitons cannot increase with frequency if $\rho + p \geq 0$, i.e. if the DEC is not violated. Previous models which exploit this idea have a phase in their evolution where the matter entropy decreases. Specifically was considered a class of solutions which correspond to a viscous fluid with an equation of state given by $p = -\rho$. In this model the early phase (where the DEC is violated) is smoothly connected to a radiation dominated era. Depending upon the sign of the bulk viscosity coefficient, the entropy of the sources driving the geometry can very well decrease [5, 6].
2. The Giovannini’s model

Consider a flat FRW background

\[ ds^2 = dt^2 - a(t)^2(dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)), \]

(1)

where the Einstein field equations in the presence of the bulk viscosity coefficient \( \xi \) can be written as

\[ H^2 = \frac{\kappa}{3} \rho, \]

(2)

\[ H^2 + \dot{H} = -\frac{\kappa}{6} (\rho + 3P_{\text{eff}}), \]

(3)

the effective pressure is given by

\[ P_{\text{eff}} = p - 3\xi H. \]

(4)

where \( \kappa = 8\pi G \) and \( H = \dot{a}/a \), \( a(t) \) is the scale factor of the flat FRW metric, the overdot represents derivation with respect to the cosmic time coordinate. To have consistent with the notation of the paper [2] we must identify \( M_p^2 = 3/\kappa \) and \( p' = P_{\text{eff}} \).

The equations (2) – (4) imply the energy balance

\[ \dot{\rho} + 3H (\rho + P_{\text{eff}}) = 0. \]

(5)

For have a cosmological model whose evolution violates the DEC only for a finite amount of time, in Ref. [2] it is assumed that

\[ \kappa\xi = \frac{2}{3} \frac{\dot{H}}{H}. \]

(6)

This parametrization reasonable because the amount of violation of DEC is proportional to \( \dot{H} \). Effectively, from Eqs. (2) and (5) we have that for any solution

\[ \rho + P_{\text{eff}} = -\frac{2}{\kappa} \dot{H}, \]

(7)

and then a violation of the DEC implies that \( \dot{H} > 0 \).

To have a cosmology whose early phase (where the DEC is violated) is smoothly connected to a radiation dominated era, Giovannini considers the scale factor given by

\[ a(t) = \left( t + \sqrt{t^2 + t_1^2} \right)^{1/2}, \]

(8)

and then the self-consistent solution takes the form

\[ H = \frac{1}{2\sqrt{t^2 + t_1^2}}, \]

(9)

\[ \kappa\xi(t) = -\frac{2t}{3 (t^2 + t_1^2)}, \quad \kappa\rho(t) = \frac{3}{4 (t^2 + t_1^2)}. \]

(10)
We can see that taking the limit \( t \to \pm \infty \) that \( \xi_{-\infty}(t) > 0, \xi_{+\infty}(t) < 0 \) and \( a_{\pm \infty}(t) \to (\pm t)^{\pm 1/2} \). So \( a(t) \) at the final phase of the whole evolution has the behavior of the radiation dominated era.

Substituting the expression (6) into Eq. (7) we conclude that the Giovannini parametrization implies that the local equilibrium pressure is given by \( p = -\rho \). So for \( t \to +\infty \) the asymptotic solution to Eqs.(8) and (10) is the following exact solution of the field equations (2) – (5):

\[
a(t) = t^{1/2}, \quad \kappa \xi = -\frac{2}{3t}, \quad \kappa p = -\kappa \rho = -\frac{3}{4t^2}. \quad (11)
\]

In fact, the Eqs. (8) and (10) imply that

\[
\kappa P_{\text{eff}} = \frac{(4t^2 - 3\sqrt{t^2 + t_1^2})}{4(t^2 + t_1^2)^{3/2}}, \quad (12)
\]

and then \( P_{\text{eff}} \to \rho/3 \) for \( t \to +\infty \), while the local equilibrium pressure behaves always as \( p = -\rho \).

3. General class of solutions type Giovannini
We shall generalize the Giovannini class of solutions discussed above for include cosmological scenarios which always have a positive viscous coefficient and thus satisfy the standard second law of thermodynamics.

We will search for wider classes of solutions in the presence of viscous sources with a linear barotropic equation of state \( p = \gamma \rho \) for the local pressure. We only study solutions which preserve the form of the scale factor (8) but having a positive bulk viscous stress to take into account an increasing matter entropy during all evolution of the cosmic time.

From Eqs. (2) and (3) we obtain that the bulk viscous coefficient may be written as

\[
\kappa \xi = \frac{2H}{3H} + (\gamma + 1)H. \quad (13)
\]

In fact, the Eq. (13) implies that the parametrization (6) has a state parameter \( \gamma = -1 \). So we can consider viscous cosmological models for which the state parameter \( \gamma \neq -1 \), i.e. we shall find a self–consistent solution for the full set of Einstein field equations. The second term of (13) may be positive and then we can have a non–negative bulk viscosity for all cosmic evolution. We can remark that this is possible for expanding universes (for which \( H > 0 \)) with state parameter \( \gamma > -1 \).

Now from the field equations (2) and (3) we have that

\[
\kappa P_{\text{eff}} = -3H^2 - 2H, \quad (14)
\]

and then this new class of viscous models will have the same effective pressure (12), which behaves as \( P_{\text{eff}} \to \rho/3 \) for \( t \to +\infty \).

The new class of solutions can be written as

\[
a(t) = \left( t + \sqrt{t^2 + t_1^2} \right)^{1/2}, \quad \kappa \rho(t) = \frac{3}{4(t^2 + t_1^2)}, \quad (15)
\]

\[
p = \gamma \rho, \quad \kappa \xi(t) = \frac{3(\gamma + 1)\sqrt{t^2 + t_1^2} - 4t}{6(t^2 + t_1^2)}. \quad (16)
\]
From here we get that the viscous pressure is given by

$$\kappa \Pi = -3\kappa H \xi = \frac{4t - 3(\gamma + 1)\sqrt{t^2 + t_1^2}}{4(t^2 + t_1^2)^{3/2}},$$

and then \( P_{\text{eff}} \) takes the form of Eq. (12).

Now from Eq. (16) we can derive the general behavior of \( \xi \). Observe that its numerator in general can be positive, negative, or zero. It can be shown that, if

$$t_{\text{root}} = \frac{(\gamma + 1)t_1}{\sqrt{(\frac{2}{3} - \gamma)(\gamma + \frac{2}{3})}},$$

the bulk viscous stress is zero. This root is a real one if \(-7/3 \leq \gamma \leq 1/3\), i.e. in this range of the state parameter, the bulk viscous stress has a phase where \( \xi \) is positive (the matter entropy increases with time) and another phase where it is negative (the matter entropy decreases with time). In this case, if \(-7/3 < \gamma < -1\), the value of \( t_{\text{root}} \) is negative and, if \(-1 < \gamma < 1/3\), the value of \( t_{\text{root}} \) is positive. I fact, the solution studied in Ref. [2, 5] lies in this range so, for \(-7/3 \leq \gamma \leq 1/3\), we have a class of Giovannini–like solutions with \( \gamma \neq -1 \).

The solutions for which the matter entropy always increases with cosmic time lie out of the range \(-7/3 \leq \gamma \leq 1/3\). In this case the root (18) does not exist and then for \( \gamma \leq -7/3\), the bulk viscous stress is always negative, and for \( \gamma \geq 1/3\) is always positive.

4. Conclusion

Any viscous cosmology with a constant barotropic state parameter \( \gamma \geq 1/3 \) will have a positive bulk viscous stress, and thus an increasing matter entropy during all cosmic evolution [7].

Finally, notice that we have considered the same form for the background metric (8); thus all calculations reported in Ref. [2] are preserved and we come to the same conclusions concerning the amplification induced by the background (8) in the proper amplitude of the gravitational waves reported in Ref. [2]. It is important to stress here that these results imply that the existence of tilted spectra of relic gravitons can be connected, in the framework of general relativity, with the violation of the DEC induced by a bulk viscosity compatible with the standard second law of thermodynamics.

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