Interactions of Unparticles with Standard Model Particles

Shao-Long Chen∗ and Xiao-Gang He†

Department of Physics and Center for Theoretical Sciences,
National Taiwan University, Taipei, Taiwan

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Abstract

We study interactions of unparticles $U$ of dimension $d_U$ due to Georgi with Standard Model (SM) fields through effective operators. The unparticles describe the low energy physics of a non-trivial scale invariant sector. Since unparticles come from beyond the SM physics, it is plausible that they transform as a singlet under the SM gauge group. This helps tremendously in limiting possible interactions. We analyze interactions of scalar $U$, vector $U^\mu$ and spinor $U^s$ unparticles with SM fields and derivatives up to dimension four. Using these operators, we discuss different features of producing unparticles at $e^+e^-$ collider and other phenomenologies. It is possible to distinguish different unparticles produced at $e^+e^-$ collider by looking at various distributions of production cross sections.

∗Electronic address: shaolong@phys.ntu.edu.tw
†Electronic address: hexg@phys.ntu.edu.tw
In a scale invariant theory in four space-time dimensions, there are no particles with a non-zero mass. In our real world, there are plenty of particles with non-zero masses. If scale invariance plays a role in nature, it must have been broken at some high energy scale beyond the Standard Model (SM) scale. At low energy, our world is described by the SM. At high energy there may be both scale invariant sector and also other sectors which do not have scale invariant such as the SM fields. Recently Georgi proposed an interesting idea to describe possible scale invariant effect at low energies, termed unparticle [1]. Based on a specific model scale invariant theory by Banks and Zaks [2], Georgi argued that operators \( O_{BZ} \) made of BZ fields may interact with operators \( O_{SM} \) made of SM fields at some high energy scale by exchange particles with large masses, \( M_U \), with the generic form \( O_{SM} O_{BZ}/M_U^k \). At another scale \( \Lambda_U \) the BZ sector induce dimensional transmutation, below that scale the BZ operator \( O_{BZ} \) matches on to unparticle operator \( O_U \) with dimension \( d_U \) and the unparticle interaction with SM particles at low energy has the form \( \lambda \Lambda_U^{4-d_SM-d_U} O_{SM} O_U \). Here \( d_SM \) is the dimension of the operator \( O_{SM} \).

An unparticle looks like a non-integral \( d_U \) dimension invisible particle. Depending on the nature of the original operator \( O_{BZ} \) and the transmutation, the resulting unparticles may have different Lorentz structure. We will indicate an unparticle acts like a Lorentz scalar as \( O_U \), a vector as \( O_U^{\mu} \) and a spinor as \( O_U^{s} \). If all interactions are perturbative, one maybe able to calculate the dimension \( d_U \) and also the coupling \( \lambda \). But the matching from the BZ physics to the unparticle physics will be a complicated strong interaction problem to deal with. One can work with the effective coupling \( \lambda \) for phenomenology which has been practiced by many.

For detailed studies, one also needs to know how an unparticle interacts with SM particles. Recent studies have focused on several low dimension operators [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The unparticle interactions with the SM particles are through exchange of some other heavy particles of mass \( M_U \). Therefore the form of the interaction is basically determined by the nature of the heavy particle. If it is a SM singlet, the unparticle \( O_U \) resulting from the transmutation should also transform under the SM gauge group as a singlet. One cannot rule out other possibilities. There are many ways that the SM fields can couple to an unparticle. If the unparticle is a SM singlet, the possibilities are limited since the SM fields have to form SM singlet first. In this work we concentrate on possible interactions of unparticles with the SM particles assuming that unparticles transform as SM
singlets and study some implications.

In the following we list operators composed of SM fields and derivatives with dimensions less than or equal to 4 invariant under the SM gauge group.

**Scalar** $O_{\mu}^\mu$ couplings:

- **a)** Couplings with gauge bosons
  \[
  \lambda_{g9} \Lambda_{\mu}^{-d_{ut}} G_{\mu\nu} G_{\nu\mu} O_{\mu}, \quad \lambda_{w_9} \Lambda_{\mu}^{-d_{ut}} W_{\mu\nu} W_{\nu\mu} O_{\mu}, \quad \lambda_{b_{9}} \Lambda_{\mu}^{-d_{ut}} B_{\mu\nu} B_{\mu\nu} O_{\mu},
  \]
  \[
  \tilde{\lambda}_{g9} \Lambda_{\mu}^{-d_{ut}} \tilde{G}_{\mu\nu} \tilde{G}_{\nu\mu} O_{\mu}, \quad \tilde{\lambda}_{w_9} \Lambda_{\mu}^{-d_{ut}} \tilde{W}_{\mu\nu} \tilde{W}_{\nu\mu} O_{\mu}, \quad \tilde{\lambda}_{b_{9}} \Lambda_{\mu}^{-d_{ut}} \tilde{B}_{\mu\nu} \tilde{B}_{\mu\nu} O_{\mu},
  \]  

- **b)** Coupling with Higgs and Gauge bosons
  \[
  \lambda_{h_9} \Lambda_{\mu}^{-d_{ut}} (H^+ D_{\mu} H) \bar{\mu}^\mu O_{\mu},
  \]
  \[
  \lambda_{d_{h_9}} \Lambda_{\mu}^{-d_{ut}} (D_{\mu} H) (D^\mu H) O_{\mu},
  \]
  \[
  \lambda_{d_{h_9}} \Lambda_{\mu}^{-d_{ut}} (D_{\mu} H) (D^\mu H) O_{\mu},
  \]

- **c)** Couplings with fermions and gauge bosons
  \[
  \lambda_{Q_9} \Lambda_{\mu}^{-d_{ut}} Q_L \gamma_\mu \gamma_\mu D^\mu Q_L O_{\mu}, \quad \lambda_{U_9} \Lambda_{\mu}^{-d_{ut}} U_R \gamma_\mu D^\mu U_R O_{\mu}, \quad \lambda_{D_9} \Lambda_{\mu}^{-d_{ut}} D_R \gamma_\mu D^\mu D_R O_{\mu},
  \]
  \[
  \lambda_{L_9} \Lambda_{\mu}^{-d_{ut}} \bar{L}_L \gamma_\mu \gamma_\mu L_L O_{\mu}, \quad \lambda_{E_9} \Lambda_{\mu}^{-d_{ut}} \bar{E}_R \gamma_\mu \gamma_\mu E_R O_{\mu}, \quad \lambda_{D_9} \Lambda_{\mu}^{-d_{ut}} \bar{D}_R \gamma_\mu \gamma_\mu D_R O_{\mu},
  \]
  \[
  \tilde{\lambda}_{Q_9} \Lambda_{\mu}^{-d_{ut}} Q_L \gamma_\mu \gamma_\mu Q_L O_{\mu}, \quad \tilde{\lambda}_{U_9} \Lambda_{\mu}^{-d_{ut}} U_R \gamma_\mu U_R O_{\mu}, \quad \tilde{\lambda}_{D_9} \Lambda_{\mu}^{-d_{ut}} D_R \gamma_\mu D_R O_{\mu},
  \]
  \[
  \tilde{\lambda}_{L_9} \Lambda_{\mu}^{-d_{ut}} \bar{L}_L \gamma_\mu \gamma_\mu L_L O_{\mu}, \quad \tilde{\lambda}_{E_9} \Lambda_{\mu}^{-d_{ut}} \bar{E}_R \gamma_\mu \gamma_\mu E_R O_{\mu}, \quad \tilde{\lambda}_{D_9} \Lambda_{\mu}^{-d_{ut}} \bar{D}_R \gamma_\mu \gamma_\mu D_R O_{\mu},
  \]
  \[
  \lambda_{Y_9} \Lambda_{\mu}^{-d_{ut}} \bar{Y}_R \gamma_\mu \gamma_\mu O_{\mu},
  \]

- **d)** Couplings with fermions and Higgs boson
  \[
  \lambda_{Y_9} \Lambda_{\mu}^{-d_{ut}} Q_L H U_R O_{\mu}, \quad \lambda_{Y_9} \Lambda_{\mu}^{-d_{ut}} Q_L \tilde{H} D_R O_{\mu},
  \]
  \[
  \lambda_{Y_9} \Lambda_{\mu}^{-d_{ut}} L_L H \gamma_\mu R_O O_{\mu}, \quad \lambda_{Y_9} \Lambda_{\mu}^{-d_{ut}} L_L \tilde{H} E_R O_{\mu},
  \]

**Vector** $O_{\mu}^\mu$ couplings:

- **a)** Couplings with fermions
  \[
  \lambda'_{Q_9} \Lambda_{\mu}^{-d_{ut}} \bar{Q}_L \gamma_\mu Q_L O_{\mu}, \quad \lambda'_{U_9} \Lambda_{\mu}^{-d_{ut}} \bar{U}_R \gamma_\mu U_R O_{\mu}, \quad \lambda'_{D_9} \Lambda_{\mu}^{-d_{ut}} \bar{D}_R \gamma_\mu D_R O_{\mu},
  \]
  \[
  \lambda'_{L_9} \Lambda_{\mu}^{-d_{ut}} \bar{L}_L \gamma_\mu \gamma_\mu L_L O_{\mu}, \quad \lambda'_{E_9} \Lambda_{\mu}^{-d_{ut}} \bar{E}_R \gamma_\mu \gamma_\mu E_R O_{\mu}, \quad \lambda'_{D_9} \Lambda_{\mu}^{-d_{ut}} \bar{D}_R \gamma_\mu \gamma_\mu D_R O_{\mu},
  \]

- **b)** Couplings with Higgs boson and Gauge bosons
  \[
  \lambda'_{h_9} \Lambda_{\mu}^{-d_{ut}} (H^+ D_{\mu} H) O_{\mu}^\mu, \quad \lambda'_{p_9} \Lambda_{\mu}^{-d_{ut}} B_{\mu\nu} \bar{\gamma}^\nu O_{\mu},
  \]

**Spinor** $O_{\mu}^\mu$ couplings:

- **a)**
  \[
  \lambda_{\nu} \Lambda_{\mu}^{-d_{ut}} \bar{\nu}_R O_{\mu}^\mu, \quad \lambda_{\nu} \Lambda_{\mu}^{-d_{ut}} L_L H O_{\mu}^\mu.
  \]  

Here $G, W$ and $B$ are the $SU(3)_C, SU(2)_L$ and $U(1)_Y$ gauge fields, respectively. $Q_L, U_R, D_R,
\( L_L, E_R \) are the SM left-handed quark doublet, right-handed up-quark, right-handed down-quark, left-handed lepton doublet and right-handed charged lepton, respectively. In the above we also included the right handed neutrino \( \nu_R \) which might be needed from neutrino oscillation data.

The scalar \( U \) unparticle has the largest number of operators. In this class of interactions, the lowest SM dimension operators is the coupling of \( U \) to two Higgs fields, \( H^\dagger H O_U \). The second lowest operator involves two right handed neutrinos, \( \bar{\nu}_R^C \nu_R O_U \). The rest have the same dimensions with the SM fields and derivatives forming dimension four operators. In the following we point out some interesting features.

The operator \( H^\dagger H O_U \) with a low dimension \( \Lambda_{U}^{2-d_U} \) may have the best chance to show up at low energies. An effect is that when the Higgs field develops a non-zero vacuum expectation value (vev) \( \langle H \rangle = v/\sqrt{2} \) as required by gauge symmetry breaking and generation of SM particle masses, there is a tadpole coupling \( \lambda_{hh} \Lambda_{U}^{2-d_U} v^2 / 2 \) of unparticle to vev which introduces a scale to the unparticle sector. This interaction will cause the unparticle sector to be pushed away from its scale invariant fixed point and the theory become non-scale invariant at some low scale. Below that the unparticle sector presumably becomes a traditional particle sector [13]. We note that this may not be necessarily true if one also include the other operator \( (H^\dagger H)^2 O_U \). This term also has a tadpole coupling of unparticle to vev. It is given by \( \lambda_4 h \Lambda_{U}^{2-d_U} \). If \( \lambda_{hh} \Lambda_{U}^{2-d_U} + \lambda_4 h v^2 / 2 = 0 \), the tadpole will be removed. One then has

\[
\lambda_{hh} \Lambda_{U}^{2-d_U} H^\dagger H O_U + \lambda_4 h \Lambda_{U}^{2-d_U} (H^\dagger H)^2 O_U = \frac{1}{4} \lambda_4 h \Lambda_{U}^{2-d_U} (h^4 + 4vh^3 + 5v^2 h^2 + 2v^3 h) O_U. \tag{7}
\]

Here, we have removed the would-be Goldstone boson in the Higgs field and \( h \) is the physical Higgs field. The operators above will induce mixing between \( h \) and the scalar unparticle. A physical Higgs may oscillate into \( O_U \) and disappear. We should note that the cancellation mechanism discussed above is by assumption. We are not able to find a symmetry to guarantee it and it may not be stable. Another possibility is that these couplings cannot be generated such that unparticle physics effect can still show up at low energies. More studies are needed.

The operator \( \bar{\nu}_R^C \nu_R O_U \) involves right-handed neutrino interaction \( \nu_R \) with an unparticle. If \( \nu_R \) is heavy, there is no observable effect. If \( \nu_R \) turns out to be a light sterile neutrino, one may see some effects in neutrino decays, a heavier \( \nu_R \) may decay into a lighter \( \nu_R \) and \( O_U \). Such effects may be difficult to observe.
There are six operators involve $O_U$ and gauge particles. The interactions with gluon fields can produce $O_U$ at hadron colliders through $gg \rightarrow gU$, $q\bar{q} \rightarrow gU$ and $gq \rightarrow qU$. The operators with $W$ and $B$ can produce $U$ at a photon collider through $\gamma\gamma \rightarrow U$, $\gamma e \rightarrow \gamma U$, and also interesting signature in $WW$ scattering.

The operators in class c) have rich phenomenology. Several of the the operators have been studied in flavor changing decay of a heavy fermion to a light fermion plus an unparticle such as $t \rightarrow u(c) + U$, meson and anti-meson mixing, and other flavor changing decays. These operators can also produce $U$ at hadron and $e^+e^-$ colliders. We will come back to this later in discussing how to distinguish different types of unparticles.

The operators in class d) involve an unparticle with SM Yukawa terms. These will open new decay channels for the Higgs and the top quark with an unparticle in the final state. They can also induce $e^+e^- (q\bar{q}) \rightarrow hU$.

There are less operators involve the vector $U^\mu$ and SM particle. The class a) operators are the most studied ones. Similar to class c) operators for scalar unparticle couplings, they can induce $t \rightarrow u(c) + U$, meson and anti-meson mixing and can also produce $U$ at hadron and $e^+e^-$ colliders. The first operator in class b) can induce $h$ and unparticle mixing, and the second operator can induce $B$ and unparticle mixing.

The unparticle may have spinor structure under the Lorentz group. There are only two operators for spinor unparticle and SM particle interactions. The operator $\bar{\nu}_R O_U^e$ has the lowest dimension in the whole list. This operator will mixing the unparticle with right-handed neutrino. Deviations of neutrino oscillation pattern may be the best place for looking for unparticle effects. The operator $\bar{L}_L H O_U^e$ can induce mixing between left-handed neutrino and an unparticle. Again this will affect neutrino mixing and also cause the PMNS mixing matrix to be not the usual $3 \times 3$ unitary matrix for three left handed neutrinos. This operator will cause Higgs to decay into a neutrino and an unparticle.

There are a lot of interesting phenomenology which can be carried out using the above listed interactions. Besides the production of unparticles, there are also virtual effects of unparticles. In the rest of the paper we concentrate on the possibility of distinguishing whether an $O_U$ or an $O_U^e$ is produced through $e^+e^-$ collider through $e^+e^- \rightarrow \gamma(Z) + U$, $\gamma(Z) + U^\mu$.

The Feynman diagrams for the above processes are shown in Fig. 1. For $U$ production, the operators $\lambda_{bb} B^{\mu\nu} B_{\mu\nu} O_U$, $\lambda_{ww} W^{\mu\nu} W_{\mu\nu} O_U$, $\tilde{\lambda}_{ww} \tilde{W}^{\mu\nu} W_{\mu\nu} O_U$, $\tilde{\lambda}_{bb} \tilde{B}^{\mu\nu} B_{\mu\nu} O_U$ contribute
through s-channel, and the operators $\lambda_{LL} L L_\gamma \mu D^\mu L L O_\mu$ and $\lambda_{E E} E R_\gamma \mu D^\mu E R O_\mu$ contribute through by $u, t$-channels. The operators $\bar{L} L_\gamma \mu L L \partial^\mu O_\mu$ and $\bar{E} R_\gamma \mu E R \partial^\mu O_\mu$ do not contribute. For $O_\mu$ production, the operators $L L_\gamma \mu L L O_\mu$ and $L L_\gamma \mu \bar{L} \mu$ contribute through $u, t$-channels.

Carrying out the phase integral for the unparticle, we find that for $e^+ e^- \rightarrow \gamma(Z) + O_\mu(P_\mu)$, the cross section is given by

$$\frac{d\sigma}{dE_\gamma} = \frac{1}{2s} \frac{1}{|M|^2} \frac{E_\gamma A_{d\mu}(P_\mu^2)^{d\mu-2}}{16\pi^3} d\Omega,$$

where $|M|^2$ is the initial spin averaged matrix element squared. $\Omega$ is the photon solid angle.

In the above, we have followed Ref.[1] using $A_{d\mu} = \theta(p_\mu^0)\theta(p_\mu^2)(P_\mu^2)^{d\mu-2}$ for bosonic unparticle phase space factor, with $A_{d\mu} = (16\pi^5/2(2\pi)^{2d\mu})\Gamma(d\mu + 1/2)/\Gamma(d\mu - 1)\Gamma(2d\mu))$.

While for the processes $e^+ e^- \rightarrow Z + O_\mu$, the cross section is given by

$$\frac{d\sigma}{dE_Z} = \frac{1}{2s} \frac{1}{|M|^2} \frac{\sqrt{E_Z^2 - m_Z^2} A_{d\mu}(P_\mu^2)^{d\mu-2}}{16\pi^3} d\Omega.$$

Here $\Omega$ is the $Z$ solid angle.

The matrix elements squared for different operators are given by:

$$|M|^2(e^+ e^- \rightarrow \gamma U) = \frac{e^2 s(u^2 + t^2)}{N_{d\mu}^2} f(\lambda_{ww}, \lambda_{bb}) ,$$

$$|M|^2(e^+ e^- \rightarrow Z U) = \frac{e^2 s[(t - m_Z^2)^2 + (u - m_Z^2)^2 + 2s m_Z^2]}{N_{d\mu}^2} g(\lambda_{ww}, \lambda_{bb}) ,$$

$$|M|^2(e^+ e^- \rightarrow \gamma U) = \frac{e^2 s(u^2 + t^2)}{N_{d\mu}^2} f(\bar{\lambda}_{ww}, \bar{\lambda}_{bb}) ,$$

$$|M|^2(e^+ e^- \rightarrow Z U) = \frac{e^2 s[(t - m_Z^2)^2 + (u - m_Z^2)^2 - 2s m_Z^2]}{N_{d\mu}^2} g(\bar{\lambda}_{ww}, \bar{\lambda}_{bb}) ,$$

FIG. 1: The Feynman diagrams for $U$ and $U^\mu$ productions through $e^+ e^- \rightarrow \gamma(Z) + U^{(\mu)}$. For $U^\mu$ production, only diagrams (c) and (d) contribute.

[Diagram of Feynman diagrams for $e^+ e^- \rightarrow \gamma(Z) + U^{(\mu)}$.]
FIG. 2: Normalized photon (Z boson) energy spectrum and angular distribution of $e^+e^- \rightarrow \gamma + U$ for $d_U = 1.5$ at $\sqrt{s} = 200$ GeV. Dashed, solid, dotted and dot-dashed curves represent the contributions from the operators with couplings $\lambda_{LL,EE}$ ($\lambda_{LL}$ and $\lambda_{EE}$ give same distributions), $\lambda_{LL,EE}'$ ($\lambda_{LL}'$ and $\lambda_{EE}'$ give same distributions), $\lambda_{ww,bb}$ and $\tilde{\lambda}_{ww,bb}$, respectively. Note that for the left panel, the curves are identical for the contributions from $\lambda_{ww,bb}$, $\tilde{\lambda}_{ww,bb}$. We have imposed $|\cos \theta_{\gamma,Z}| < 0.97$ and $E_\gamma > 5$ GeV.

\[ |\mathcal{M}|^2(e^+e^- \rightarrow \gamma U) = \frac{2e^2(\lambda_{LL}^2 + \lambda_{EE}^2)}{(\Lambda_U^2)^{d_U-1}} \frac{s}{\Lambda_U^2}, \]

\[ |\mathcal{M}|^2(e^+e^- \rightarrow Z U) = \frac{2g^2}{\cos^2 \theta_w} \frac{(\lambda_{LL}^2(\frac{1}{2} - \sin^2 \theta_w)^2 + \lambda_{EE}^2 \sin^4 \theta_w)}{(\Lambda_U^2)^{d_U-1}} \frac{s + ut}{\Lambda_U^2}, \]

\[ |\mathcal{M}|^2(e^+e^- \rightarrow \gamma U^\mu) = \frac{e^2(\lambda_{LL}^2 + \lambda_{EE}^2)}{(\Lambda_U^2)^{d_U-1}} \frac{u^2 + t^2 + 2sP_U^2}{ut}, \]

\[ |\mathcal{M}|^2(e^+e^- \rightarrow Z U^\mu) = \frac{g^2}{\cos^2 \theta_w} \frac{(\lambda_{LL}^2(\frac{1}{2} - \sin^2 \theta_w)^2 + \lambda_{EE}^2 \sin^4 \theta_w)}{(\Lambda_U^2)^{d_U-1}} \frac{(u^2 + t^2 + 2s(P_U^2 + m_Z^2) - P_U^2m_Z^2(\frac{u}{t} + \frac{t}{u}))}{ut}, \]

where

\[ f(\lambda_{ww}, \lambda_{bb}) = \frac{(\lambda_{ww} \sin^2 \theta_w + \lambda_{bb} \cos^2 \theta_w)^2}{s^2} \]
\[ g(\lambda_{ww}, \lambda_{bb}) = \frac{(\lambda_{ww} - \lambda_{bb})^2 \sin^2 2\theta_w}{s(s - m_Z^2)} \]

The above matrix elements squared would give very different energy and angular distributions for \( U \) and \( U^\mu \) productions since the \( s, u \) and \( t \) parameters appear in different combinations. The photon and Z boson angular and energy distributions are plotted taking \( d_U = 1.5 \) and \( \sqrt{s} = 200 \) GeV, in Fig. 2, relevant for LEP II data, and also 500 GeV, in Fig. 3, relevant for ILC for illustration. In both Figs. 2 and 3, we plot the distributions with different couplings setting others to zero. Since the electron mass is small, the cross section diverges at \( \cos \theta = \pm 1 \) for \( U^\mu \) production due to \( u \) and \( t \) appear in the denominators if electron mass is neglected, but finite for \( U \) production. This provides a clear way to distinguish \( U \) and \( U^\mu \) production as can be seen from Figs. 2 and 3. The energy distributions can also provide useful information which can also be seen from Figs. 2 and 3. For this case we made a cut with \( |\cos \theta_{\gamma,Z}| < 0.97 \) to avoid the divergence for \( U^\mu \) production at \( |\cos \theta_{\gamma,Z}| = 1 \).

The above discussion show that the study of various distributions may be able to provide information about the type of unparticles at ILC if a large number of events can be obtained. One needs, however, to see if existing constraints have already ruled out such possibilities. We now comment on constraints on the relevant coupling of the operators.

Presently there is a direct constraint from LEP II data [17] at \( \sqrt{s} = 207 \) GeV on the cross section for \( e^+e^- \rightarrow \gamma X^0 \) where \( X^0 \) is invisible. With the cuts \( E_\gamma > 5 \text{GeV} \) and \( |\cos \theta| < 0.97 \), the cross section \( \sigma \) is constrained to be \( \lesssim 0.2 \text{ pb} \) at 95% C.L. Interpreting \( X^0 \) as an unparticle, bounds can be obtained for combinations of parameters \( \lambda_i \) and \( \Lambda_U \). The corresponding bounds on various cross sections we are interested can then be obtained. We list them in Table I for \( d_U = 1.5 \).

No observation of \( e^+e^- \rightarrow \gamma + U \) at LEP II may be due to too low event number. With larger integrated luminosity at ILC, unparticle effects may be observed. If the scale of unparticle physics is close to the upper bound, with an integrated luminosity of 100 \( fb^{-1} \),
FIG. 3: Normalized photon (Z boson) energy spectrum and angular distribution of $e^+e^- \rightarrow \gamma + U$ for $d_U = 1.5$ at $\sqrt{s} = 500$ GeV.

The event numbers can reach more than $1.93 \times 10^4$ ($1.3 \times 10^3$) and $1.07 \times 10^4$ ($6.19 \times 10^2$) for $e^+e^- \rightarrow \gamma(Z)+U$ with $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV, respectively. With an integrated luminosity of 500 $fb^{-1}$, the event numbers would be five times larger. The ILC would be able to study the detailed distributions discussed earlier, and provide crucial information on the properties of the unparticles.

There are also several non-collider laboratory constraints directly related to the operators have been studied including g-2 of electron[4, 5, 8] and invisible positronium decays[8]. We find that positronium decays into an $U$ may provide a stronger bound than that at LEP II.

The operators with couplings $\lambda_{LL,EE}$ and $\lambda'_{LL,EE}$ contribute to para-positronium to unparticle (p-Ps → $U$)) and ortho-Positronium to unparticle (o-Ps → $U$) decays directly at the tree level, respectively. The 90% C.L. experimental bounds on these decay branching ratios[18], $4.3 \times 10^{-7}$ and $4.2 \times 10^{-7}$, then lead to [19]

\[
A_{dU} |\lambda_{LL} - \lambda_{EE}|^2 (2m_e/\Lambda_U)^{2dU} < 9.2 \times 10^{-9},
\]
\[
A_{dU} |\lambda'_{LL} + \lambda'_{EE}|^2 (2m_e/\Lambda_U)^{2(dU-1)} < 2.0 \times 10^{-12}. \tag{12}
\]

In the above, our bound on scalar unparticle scale is different than that obtained in Ref. [8]
is due to the fact that we have required the unparticles to couple to SM invariant operators such that the couplings scale as $\Lambda^{-2d_U}$ while in Ref. [8] they scale as $\Lambda^{-2(d_U-1)}$.

The constraints on $\lambda_{LL,EE}$ are much weaker than that from LEP II. The constraints on $\lambda_{LL,EE}'$ are, however, much stronger. Assuming no cancellation between $\lambda_{LL}'$ and $\lambda_{EE}'$, the bounds would imply that the cross sections for $e^+e^- \rightarrow \gamma(Z)+U$ to be less than $5.5 \times 10^{-5}(3.4 \times 10^{-6})$ pb at $\sqrt{s} = 200$ GeV and $2.9 \times 10^{-5}(2.3 \times 10^{-6})$ pb at $\sqrt{s} = 500$ GeV for $d_U = 1.5$. If true, the unparticle effects due these operators will not be able to be studied at ILC. However, with larger $d_U$, it is still possible. For example, for $d_U = 1.88$, one can get more than $4.6 \times 10^4(1.2 \times 10^3)$ and $5.1 \times 10^4(2.6 \times 10^3)$ $e^+e^- \rightarrow \gamma(Z)+U$ events at $\sqrt{s} = 200$ GeV and 500 GeV, respectively with the integrated luminosity of 100 $fb^{-1}$.

We should also note that since the constraint is proportional to $\lambda_{LL}'+\lambda_{EE}'$, if there is a cancellation such that this quantity is small, but individual $\lambda_{LL,EE}'$ is not small, the cross sections for $e^+e^- \rightarrow \gamma(Z)+U$ can still be large and unparticle physics effects can still be studied at ILC.

No constraints on $\lambda_{ww,bb}(\tilde{\lambda}_{ww,bb})$ can be obtained from a positronium decays into an unparticle. However, one can obtain constraints from o-Ps$\rightarrow\gamma+U$ decay through the diagrams shown in Fig. 1. We have carried out such study using formulae in Ref. [20] for $\Upsilon \rightarrow \gamma+U$ with appropriate replacement for parameters. We find that the constraints are
much weaker than that from LEP II data.

In summary, unparticle physics due to scale invariance leads to very rich collider and flavor phenomenology. Under the scenario that the unparticle stuff transforms as a singlet under the SM gauge group, we listed possible operators involving interactions of scalar $U$, vector $U^\mu$ and spinor $U^s$ unparticles with the SM fields and derivatives up to dimension four and discussed some phenomenology related to these operators. We find that the interactions of unparticle with Higgs sector and lepton sector are quite interesting. We also find that $e^+e^-$ collider can provide useful information for scalar and vector unparticles.

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