The research on current rush to air mass in delivery pipeline system with YOUNGS-VOF method

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Abstract: Water hammer with gas often occurs in long delivery pipeline system larger than ordinary water hammer, surface tracking between air and water is important and difficult. In this paper, flow rush to interception air-mass in long water conveyance pipe system is researched on base on surface in between water and gas tracking method of staggered grid dispersed with YOUNGS-VOF, the limited compressibility method of viscous incompressible fluid control equations, and numerical method of ideal gas state equation. The numerical results show that YOUNGS-VOF method can be used to deal with the interface and water hammer is consistent with the theory analysis.

1 Introduction

Restricted by topographic conditions, in the process of long-distance water conveyance pipeline arrangement, due to abnormal conditions such as long-term shutdown, gas will inevitably be contained in local water conveyance pipeline. When the gas cannot be discharged smoothly, the water hammer pressure produced in the pipeline will be much larger than the conventional water hammer pressure. Gas-liquid two-phase transient flow in water transmission pipeline is very complex. In the transient process, gas-liquid two-phase push each other, and the interphase interface is constantly moving and changing, including material transformation and heat exchange between gas and liquid. If some necessary assumptions are made, such as ignoring the heat exchange between gas and liquid, simplifying the equation of state of gas, or only taking the liquid flow field and pressure change as the research object, the analysis and research can be carried out in two-dimensional or even three-dimensional numerical model, and the local flow pattern of water conveyance system can be described in detail, and the numerical results obtained can be satisfactory. Compared with the one-dimensional mathematical model[1] used before, there will be no small progress, which will provide theoretical reference for the optimal design and operation safety of water conveyance system. The cross-section of common water pipelines is mostly round, and the expression of two-dimensional mathematical model of water flow in Cartesian coordinates is complicated in the solving process. Considering that the velocity of fluid in the pipeline is mainly in the direction of pipeline axis, it can be assumed that the cross-section of the pipeline is rectangular. Using N-S equation, mass conservation equation and fluid transfer equation of two-dimensional viscous incompressible fluid in Cartesian coordinates to simulate the two-phase transient flow in the water pipeline is beneficial to the further study.

VOF method has solved the problem of free interface tracking simulation since it was proposed, which has aroused great interest of scholars at home and abroad. While improving it one after another, it has also been applied to practical projects and achieved many achievements. Its application is
embodied in many disciplines and fields such as chemistry, power, water conservancy, machinery, aerospace, etc., and it solves the problem of two-phase flow with free surface of motion well. In addition to solving the problems in the above fields, there are still many multiphase flow problems involving the free surface in water conservancy and hydropower projects, which can be solved by VOF method. HIRT first proposed to approach the free surface with a horizontal line, and to determine the horizontal and vertical directions of the free surface inclination in the cell grid according to the volume function value of adjacent fluids. Although it is quite different from the actual free surface, it has a pioneering significance. FLAIR and YOUNGS et al. used oblique segments to simulate the free surface in the cell, and determined the slope and coefficient of the free surface line according to the fluid volume number in the adjacent grid. However, the free surface in the adjacent two cells was reconstructed by Flair, while Youngs used oblique segments to approximate each free surface element. Obviously, the free surface obtained in this way is close to the actual one, while KIM adopts the conic curve to simulate the free surface, and the free surface obtained is more refined. Fig. 1 is a reconstruction of the actual fluid configuration using several commonly used VOF methods. Fig.1-a is the actual interface. FLAIR, YOUNGS, HIRT and KIM free surface reconstruction methods were used to describe Fig. 1-a, Fig. 1-b ~ e. Fig. 1-b and c use oblique segments to simulate the free surface, and the results obtained are more consistent with the actual results.
Fig.1 Several commonly used VOF methods for interface reconstruction

2. Two-dimensional physical model and governing equations

2.1. Physical model
As shown in the figure, the water pipeline is horizontally placed, with a control valve connected to the reservoir at the left end in the middle of the pipeline. The water pipeline above the valve is full of water, with a distance of L1 from the valve. The water pipeline connected behind the valve is full of sealed atmospheric gas, and its initial characteristics are consistent with those of the outside atmosphere. The length of the pipeline behind the valve is L2, the end of the pipeline is rigidly closed, and the height of the pipeline is H. There is a pool or reservoir at the upstream of the pipeline.
2.2. Fluid control equation

In gas-liquid two-phase flow, the density of atmosphere is usually much smaller than that of liquid. Therefore, in some cases, the influence of gas is usually not considered in gas-liquid two-phase flow, so as long as the governing equation of liquid is listed, when Reynolds number Re in the flow field is not large, the original variable model of N-S equation is adopted, and when Reynolds number Re is large, the turbulent model should be adopted. According to the above physical model, the original variable model of N-S equation is adopted here\[2\][3].

2.2.1. Momentum equation

Momentum equation of two dimensional incompressible fluid:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= g_x - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial x^2} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= g_y - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial y^2}
\end{align*}
\]  

(1)

In the formula, \(u, v\) represents the velocity in the \(x, y\) direction, \(p\) represents the pressure at any point of the fluid, and \(\nu\) represents the dynamic viscosity coefficient of the fluid.

2.2.2. Momentum equation

The continuity equation of water flow contains no pressure term, which is written by using the concept of artificial compressibility as follows:

\[
\frac{1}{pc\rho} \frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(2)

The purpose of adding the \(\frac{1}{pc\rho} \frac{\partial p}{\partial t}\) term is to make the calculated \(v\) and \(u\) satisfy the continuous equation and \(c\) express the isentropic sound velocity in liquid.

2.2.3. Fluid volume transfer equation

The transfer equation of viscous incompressible fluid in conservation form can be written as:

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = 0
\]  

(3)

According to the principle of VOF method, at a certain time, if \(f=1\), it means that the unit is occupied by the specified phase fluid and is a fluid unit. If \(f=0\), the unit has no specified phase fluid, and is called an empty unit relative to the specified phase fluid; if \(0<f<1\), the unit is an interface unit containing two-phase substances. In this paper, the improved YOUNGS-VOF method[4][5][6] and discrete formula (3) are adopted.

2.2.4. Equation of state of ideal gas

In the calculation process, it is assumed that the changes of pressure, volume and temperature of gas phase conform to the ideal gas state equation.

\[
P_v = \text{Const}
\]  

(4)
3. Solving the governing equation

3.1. Momentum equation discretization

The staggered grid is shown in Figure 3, the pressure and volume functions are placed at the center of the grid, the velocity defines the midpoint of four sides, and the discrete equation\(^7\)[\(^8\)] is as follows.

\[
U_{i+1/2,j}^{n+1} = U_{i+1/2,j}^n + \delta t \left[ g_n - \frac{\rho u_{i+1/2}^{n+1} - \rho u_{i+1/2}^n}{\rho g_{i+1/2}^n} - FUX - FUY + VISX \right] 
\]

\[
V_{j+1/2}^{n+1} = V_{j+1/2}^n + \delta t \left[ g_j - \frac{\rho v_{j+1/2}^{n+1} - \rho v_{j+1/2}^n}{\rho g_{j+1/2}^n} - FVX - FVY + VISY \right] 
\]

\[
\delta u_{i+1/2} = \frac{1}{2}(\delta_{u,i} + \delta_{u,i+1}) \quad \delta v_{j+1/2} = \frac{1}{2}(\delta_{v,j} + \delta_{v,j+1}) 
\]

\(FUX, FUY, FVX, FVY\) are difference schemes of the convection terms \(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\). \(VISX, VISY\) are difference schemes of viscous terms \(\nu \nabla^2 u\) , \(\nu \nabla^2 v\). A mixed eccentric scheme of upwind difference scheme and central difference scheme is adopted for convection term, and a standard central difference scheme is adopted for viscous term.

![Fig.3 Quantities definition on staggered grid](image)

3.2. Continuity equation discretization

Discrete continuity equations are written in iterative form.

\[
\left( \delta p_{ij} \right)^{n+1} = -c^2 \delta t D_{ij}^{n+1} 
\]

\[
D_{ij}^{n+1} = \left( u_{i+1/2}^{n+1} - u_{i+1/2}^n \right) / \delta x + \left( v_{j+1/2}^{n+1} - v_{j+1/2}^n \right) / \delta y
\]

In the formula, superscript \(m+1\) represents the number of iterations, and \(n+1\) represents the variable at this moment.

3.3. Processing of boundary conditions

According to the principle of source and sink, the boundary condition on the solid wall adopts the non-slip boundary condition. The free surface element pressure interpolation is as follows.

\[
p_{s,ij} = (1 - \beta) p_s + \beta p_c
\]

In the formula, \(\beta = \frac{d_c}{d}\) is called interpolation coefficient, as shown in Fig. 4, \(p_s\) is called the center pressure of interpolation unit, \(p_c\) is called a given pressure on free surface, no consideration is given to the surface tension on the free surface. \(d\) is called the average distance from interpolation unit center to free surface, and \(d_c\) is called the distance from interpolation unit center to free surface unit center. The velocity on the boundary of free surface element satisfies the continuity equation.
3.4. Processing of boundary conditions

Assuming the pressure field, the flow field is solved according to equations (5) and (6), and the increment of the pressure field is obtained according to equations (7) and (8). When the difference between the increment of the pressure and the numerical flow field before and after is small, the pressure and velocity in the solution area can be obtained at the next moment.

4. Dam-break flood simulation

4.1. Basic parameters of physical model

| Table1 Basic parameters in the water conservancy system |
|---------------------------------------------------------|
| Length of pipeline from reservoir to valve L1 (m)      | 1.2 |
| Length of pipeline from valve to closed end L2(m)      | 0.3 |
| Pipe height h(m)                                      | 0.15 |
| coefficient of kinematic viscosity $\nu$ (m²/s)       | 0.0000012 |
| Density of water $\rho$(kg/m³)                        | 1000 |
| acceleration of gravity $g$(m/s²)                     | 9.806 |
| Gas polytropic index $n$                              | 1.0 |
| Reservoir water level H(m)                            | 10 |
| time step $\Delta T$(s)                               | 0.001 |
| Pipeline segmentation(M×N)                            | 20×50 |

4.2. Numerical results

Assuming that the valve in the pipeline is fully opened instantly, the pressure difference exists between the fluid and gas in contact with the gas phase in the pipeline, and the water flow moves forward. The water flow field and gas-liquid interface at specified time are shown in Figure 5-9. In the figure, the abscissa indicates the length and height of the pipeline, the white area in the pipeline is occupied by gas, and the rest is occupied by fluid. The arrow in the figure indicates the velocity of fluid particles at this point, and the gas-liquid interface is represented by black line. Due to the gravity pressure difference between the upper and lower boundaries of the fluid in the pipeline, the moving direction of the fluid particles is below the horizontal line, and there is little difference between the upper and lower particle velocities at the initial moment, so the interface is almost vertical as shown in Fig. 5. With the increase of velocity difference, the interface expands almost into an inclined straight line. When the water flow contacts the closed end, the lower water flow velocity decreases sharply, while the upper water flow velocity changes little. With the constant increase of air mass pressure, the water flow velocity decreases continuously. When the interface velocity is almost 0, the interface shape is shown in Fig. 7. Because the air mass pressure is greater than the water flow pressure, the air
mass pushes the upper water flow back, but the lower water flow speed increases and rises along the closed end. The interface shape is shown in Figures 8 and 9. In this process, the pressure of gas is constantly increasing, and when it reaches the maximum value, it is constantly decreasing.

4.3. Study on the Change of Water Filling Length and Air Mass Pressure
The calculation results show that the water hammer with gas in the water pipeline can produce much higher pressure than the ordinary pure water hammer. When the head of the upper reservoir is 10m, the peak pressure produced by the impact of pipeline water flow on air mass can reach more than 35m. When the air mass length in the pipeline is constant, the water filling length in front of the valve is changed. It is assumed that the air mass length in the pipeline behind the valve is 0.3m, and when the water filling length in front of the valve is 1.5m, 2.0m and 2.5m respectively, the peak pressure of water impacting the closed air mass can reach 36.3 m (curve B), 35.6 m (curve C) and 35.2 m (curve D) respectively. The calculation results show that when the air mass length in the pipeline is kept constant and the water filling length of the pipeline before the valve is changed, the peak pressure of the air mass does not change very much, and remains at about 35m, which is consistent with the peak pressure calculated by the one-dimensional characteristic line model.
Fig. 10 Air mass pressure vs time with water body length in pipeline

5. Conclusion

(1) The VOF method can track the change of two intersecting interfaces in the process of gas-liquid transient, and the shape of free surface obtained by numerical simulation is more in line with the actual situation than the conventional one-dimensional characteristic line algorithm.

(2) Two-dimensional model can also verify that the pressure peak of high-pressure water flow impacting air mass in pipeline is much larger than that of water hammer under conventional conditions, and the pressure peak often occurs in the first cycle.

(3) When the amount of gas in the pipeline is constant, the length of the pipeline has little to do with the pressure peak produced by the water hammer with gas in the pipeline, but the pressure peak of the water hammer with gas in the pipeline has a great relationship with the gas volume in the pipeline.

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