Vector meson masses in hot nuclear matter: the effect of quantum corrections

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The medium modification of vector meson masses is studied taking into account the quantum correction effects for the hot and dense hadronic matter. In the framework of Quantum Hadrodynamics, the quantum corrections from the baryon and scalar meson sectors were earlier computed using a nonperturbative variational approach through a realignment of the ground state with baryon-antibaryon and sigma meson condensates. The effect of such corrections was seen to lead to a softer equation of state giving rise to a lower value for the compressibility and, an increase in the in-medium baryonic masses than would be reached when such quantum effects are not taken into account. These quantum corrections arising from the scalar meson sector result in an increase in the masses of the vector mesons in the hot and dense matter, as compared to the situation when only the vacuum polarisation effects from the baryonic sector are taken into account.

PACS number: 21.65.+f,21.30.+y

I. INTRODUCTION

The effective masses of the vector mesons (ω, ρ and φ) in the hot and dense matter have attracted a lot of interest in the recent past, both experimentally [1,2] and theoretically [3–7]. Brown and Rho came with the hypothesis that the vector meson masses drop in the medium according to a simple (BR) scaling law [3], given as

\[ m_{V}^*/m_{V} = f_{\pi}^{*}/f_{\pi}, \]

where \( f_{\pi} \) is the pion decay constant and asterix refers to in-medium quantities. There have been approaches based on QCD sum rules [4,5] which confirm such a scaling hypothesis [4] with a saturation scheme that leads to a delta function at the vector meson pole and a continuum for higher energies for the hadronic spectral function. It is however seen that such a simple saturation scheme for finite densities does not work and a more realistic description for the hadronic spectral function is called for [6]. Using an effective Lagrangian model to calculate the hadronic spectral function, it is seen that such a universal scaling law is not suggested for in-medium vector meson masses [7].

In the conventional hadronic models [8,9], on the other hand, the masses of the vector mesons stay constant or increase slightly, in the mean field approximation, i.e., when the polarization from the Dirac sea is neglected [9]. With the inclusion of quantum corrections from the baryonic sector, however, one observes a drop in the vector meson masses in the medium in the Walecka model [10–12]. This medium modification of the vector meson masses plays an important role in the enhanced dilepton yield [13] for masses below the ρ resonance in the heavy ion collision experiments [14,15]. It has been emphasized recently that the Dirac sea contribution (taken into account through summing over baryonic tadpole diagrams in the relativistic Hartree approximation (RHA)) dominates over the fermi sea contribution and \( m_{\omega}^*/m_{\omega} < 1 \) is caused by a large dressing of \( NN \) cloud in the medium [12]. It was earlier demonstrated in a nonperturbative formalism that a realignment of the ground state with baryon-antibaryon condensates was equivalent to the relativistic Hartree approximation [14]. The ground state for the nuclear matter was extended to include sigma condensates to take into account the quantum correction effects from the scalar meson sector. Such a formalism includes multiloop effects and is self consistent. The methodology was then generalized to consider hot nuclear matter [15] as well as to the situation of hyperon-rich dense matter [16]. In the present work, we study the effect of such quantum corrections on the in-medium vector meson masses.

We organize the paper as follows. We first briefly recapitulate the nonperturbative framework used for studying the quantum correction effects in hot nuclear matter in section II. The medium modification of the ω and ρ meson masses are considered in section III. We also examine the effect of quantum corrections on the in-medium vector meson masses in dense hyperon-rich matter in section IV, where it is observed that the medium modification to the strange vector meson (φ) is small compared to that of the vector ω meson. This is because, in the hyperonic matter with quantum effects taken into account [16], the masses of the hyperons remain rather insensitive to density, unlike the in-medium nucleon masses which change appreciably with density, and further, because the φ-mesons do not couple
to the nucleons $|\Psi\rangle$. On the other hand, $\omega$ meson mass gets contribution also from the nucleonic sector. In section V, we summarize the results of the present work and discuss possible outlook.

II. QUANTUM VACUUM IN HOT NUCLEAR MATTER

We briefly recapitulate here the vacuum polarisation effects arising from the nucleon and scalar meson fields in hot nuclear matter in a nonperturbative framework [13]. The method of thermofield dynamics (TFD) is used here to study the “ground state” (the state with minimum thermodynamic potential) at finite temperature and density within the Walecka model. The temperature and density dependent baryon and sigma masses are also calculated in a self-consistent manner in the formalism. The ansatz functions involved in such an approach are determined through minimisation of the thermodynamic potential.

The Lagrangian density in the Walecka model is given as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu)\psi + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \lambda \sigma^4 + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)(\partial^\mu \omega^\nu - \partial^\nu \omega^\mu).$$

(1)

In the above, $\psi$, $\sigma$, and $\omega_\mu$ are the fields for the nucleon, $\sigma$, and $\omega$ mesons with masses $M$, $m_\sigma$, and $m_\omega$ respectively. The quartic coupling term in $\sigma$ is necessary for the sigma condensates to exist, through a vacuum realignment [14]. We retain the quantum nature of both the nucleon and the scalar meson fields, whereas, the vector $\omega$–meson is treated as a classical field, using the mean field approximation for $\omega$–meson, given as $(\omega^\mu) = \delta^\mu_\nu \omega_\nu$. The reason is that without any quartic or any other higher order term for the $\omega$-meson, the quantum effects generated due to $\omega$-meson through the present variational ansatz turns out to be zero.

The Hamiltonian density can then be written as

$$\mathcal{H} = \mathcal{H}_N + \mathcal{H}_\sigma + \mathcal{H}_\omega,$$

(2)

with

$$\mathcal{H}_N = \bar{\psi}(-i\alpha \cdot \nabla + \beta M)\psi + g_\sigma \sigma \bar{\psi}\psi,$$

(3a)

$$\mathcal{H}_\sigma = \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} \sigma (-\nabla^2)\sigma + \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^4,$$

(3b)

$$\mathcal{H}_\omega = g_\omega \omega_0 \bar{\psi}\psi - \frac{1}{2} m_\omega^2 \omega_0^2.$$

(3c)

We may now write down the field expansion for the nucleon field $\psi$ at time $t = 0$ as [14]

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int [U_r(k)c_{Ir}(k) + V_r(-k)\tilde{c}_{Is}(-k)] e^{i \mathbf{k} \cdot \mathbf{x}} d\mathbf{k},$$

(4)

with $c_{Ir}$ and $\tilde{c}_{Is}$ as the baryon annihilation and antibaryon creation operators with spins $r$ and $s$ respectively, and $U$ and $V$ are the spinors associated with the particles and antiparticles respectively [14]. Similarly, we may expand the field operator of the scalar field $\sigma$ in terms of the creation and annihilation operators, at time $t = 0$ as

$$\sigma(x, 0) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{\sqrt{2\omega(k)}} (\mathbf{a}(\mathbf{k}) + \mathbf{a}^\dagger(-\mathbf{k})) e^{i \mathbf{k} \cdot \mathbf{x}}.$$

(5)

In the above, $\omega(k) = \sqrt{k^2 + m_\sigma^2}$. The perturbative vacuum is defined corresponding to this basis through $\mathbf{a} \mid \text{vac}\rangle = 0 = c_{Ir} \mid \text{vac}\rangle = \tilde{c}_{Ir}^\dagger \mid \text{vac}\rangle$.

To include the vacuum polarisation effects for hot nuclear matter, we shall now consider a trial state with baryon–antibaryon and scalar meson condensates and then generalize the same to the finite temperatures and densities [14]. We thus explicitly take the ansatz for the trial state as

$$|F\rangle = U_r U_F \mid \text{vac}\rangle,$$

(6)
with \[
U_F = \exp \left[ \int d\mathbf{k} \ f(\mathbf{k}) \ c_{I\nu}(\mathbf{k}) \ a_{rs} \hat{c}_{Is}(-\mathbf{k}) - h.c. \right] \tag{7}
\]

Here \(a_{rs} = u_{I\nu}^\dagger(\sigma \cdot \hat{\mathbf{k}}) v_{Is}\) and \(f(\mathbf{k})\) is a trial function associated with baryon-antibaryon condensates. For the scalar meson sector, \(U_{\sigma} = U_{II}U_I\) where \(U_i = \exp(B_i^\dagger - B_i)\), \((i = I, II)\). Explicitly the \(B_i\) are given as

\[
B_i^\dagger = \int d\mathbf{k} \sqrt{\frac{\omega(\mathbf{k})}{2}} f_\sigma(\mathbf{k}) a_i^\dagger(\mathbf{k}), \quad B_{II}^\dagger = \frac{1}{2} \int d\mathbf{k} g(\mathbf{k}) a_i^\dagger(\mathbf{k}) a_i^\dagger(-\mathbf{k}). \tag{8}
\]

In the above, \(a_i^\dagger(\mathbf{k}) = U_I a(\mathbf{k}) U_I^{-1} = a(\mathbf{k}) - \sqrt{\frac{\omega(\mathbf{k})}{2}} f_\sigma(\mathbf{k})\) corresponds to a shifted field operator associated with the coherent state \([14, 17]\) and satisfies the usual quantum algebra. Further, to preserve translational invariance \(f_\sigma(\mathbf{k})\) has to be proportional to \(\delta(\mathbf{k})\) and we take \(f_\sigma(\mathbf{k}) = \sigma_0 (2\pi)^{3/2} \delta(\mathbf{k})\). \(\sigma_0\) corresponds to a classical field of the conventional approach \([14]\). Clearly, the ansatz state is not annihilated by the operators, \(c, \tilde{c}^\dagger\) and \(a\). However, one can define operators, \(\tilde{d}, \tilde{d}^\dagger\) and \(b\), related through a Bogoliubov transformation to these operators, which will annihilate the state \(|F\rangle\).

We next use the method of thermofield dynamics \([18]\) to construct the ground state for nuclear matter at finite temperature. Here the statistical average of an operator is written as an expectation value with respect to a ‘thermal state,’ which is a coherent state \([14, 17]\) and satisfies the usual quantum algebra. Further, to preserve translational invariance \(f_\sigma(\mathbf{k})\) has to be proportional to \(\delta(\mathbf{k})\) and we take \(f_\sigma(\mathbf{k}) = \sigma_0 (2\pi)^{3/2} \delta(\mathbf{k})\). \(\sigma_0\) corresponds to a classical field of the conventional approach \([14]\). Clearly, the ansatz state is not annihilated by the operators, \(c, \tilde{c}^\dagger\) and \(a\). However, one can define operators, \(\tilde{d}, \tilde{d}^\dagger\) and \(b\), related through a Bogoliubov transformation to these operators, which will annihilate the state \(|F\rangle\).

The temperature-dependent unitary operators \(U_{\sigma}(\beta)\) and \(U_F(\beta)\) are given as \([18]\)

\[
U_{\sigma}(\beta) = \exp \left( \frac{1}{2} \int d\mathbf{k} \left[ \theta_-(\mathbf{k}, \beta) \ d_{I\nu}^\dagger(\mathbf{k}) \ d_{I\nu}^\dagger(-\mathbf{k}) + \theta_+(\mathbf{k}, \beta) \ \tilde{d}_{I\nu}^\dagger(\mathbf{k}) \ \tilde{d}_{I\nu}^\dagger(-\mathbf{k}) \right] - h.c. \right), \tag{9}
\]

and

\[
U_F(\beta) = \exp \left( \int d\mathbf{k} \left[ \left( \gamma(2\pi)^{-3} \int d\mathbf{k} (k^2 + M^*^2)^{1/2} (\sin^2 \theta_- + \sin^2 \theta_+) \right) \ c_{I\nu}(\mathbf{k}) \ a_{rs} \tilde{c}_{Is}(-\mathbf{k}) - h.c. \right) \right). \tag{10}
\]

The underlined operators are the operators corresponding to the doubling of the Hilbert space that arise in thermofield dynamics method. We shall determine the condensate functions \(f(\mathbf{k})\) and \(g(\mathbf{k})\), and the functions \(\theta_+(\mathbf{k}, \beta), \ \theta_-(\mathbf{k}, \beta)\) and \(\theta_+(\mathbf{k}, \beta)\) of the thermal vacuum through minimisation of the thermodynamic potential. The thermodynamic potential is given as

\[
\Omega \equiv -p = \epsilon - \frac{1}{\beta} \mathcal{S} - \mu \rho_B, \tag{12}
\]

where \(\epsilon\) and \(\mathcal{S}\) are the energy- and entropy- densities of the thermal vacuum, and \(\rho_B\) is the baryon density. The ansatz functions used in the definition of the thermal vacuum are determined through the minimisation of the thermodynamic potential. Then subtracting out the pure vacuum contribution and carrying out the renormalisation procedures for the baryonic and scalar meson sectors \([18]\), we obtain the expression for the energy density as

\[
\epsilon_{\text{ren}} = \epsilon^{(N)}_{\text{finite}} + \Delta \epsilon_{\text{ren}} + \epsilon_\omega + \Delta \epsilon_\sigma, \tag{13}
\]

with

\[
\epsilon^{(N)}_{\text{finite}} = \gamma (2\pi)^{-3} \int d\mathbf{k} (k^2 + M^*^2)^{1/2} \sin^2 \theta_+ - \sin^2 \theta_+ \tag{14a}
\]

\[
\Delta \epsilon_{\text{ren}} = -\frac{\gamma}{16 \pi^2} (M^*^4 \ln \left( \frac{M^*}{M} \right) + M^3 (M - M^*) - \frac{7}{2} M^2 (M - M^*)^2 + \frac{13}{3} M (M - M^*)^3 - \frac{15}{12} (M - M^*)^4 \tag{14b}
\]
\[ \epsilon_\omega = g_\omega \omega_0 \rho_B^{ren} - \frac{1}{2} m_\omega^2 \omega_0^2, \]  

(14c)

\[ \Delta \epsilon_\sigma = \frac{1}{2} m_\sigma^2 \sigma_0^2 + 3 \lambda_R \sigma_0^2 + \frac{M_\sigma^4}{64 \pi^2} \left( \ln \left( \frac{M_\sigma^2}{m_R^2} \right) - \frac{1}{2} \right) - 3 \lambda_R I_f^2 \]

\[- \frac{M_{\sigma,0}^4}{64 \pi^2} \left( \ln \left( \frac{M_{\sigma,0}^2}{m_R^2} \right) - \frac{1}{2} \right) + 3 \lambda_R I_f^2 \]

(14d)

as the mean field result, contribution from the Dirac sea, and contributions from the \( \omega \) and \( \sigma \) mesons respectively. In the above, \( \sin^2 \theta_\pi \) are the distribution functions for the baryons and antibaryons given through

\[ \sin^2 \theta_\pi = \frac{1}{\exp(\beta (\epsilon^*(k) + \mu^*)) + 1}. \]  

(15)

with \( \epsilon^*(k) = (k^2 + M^*)^{1/2} \) and \( \mu^* = \mu - g_\omega \omega_0 \) as the effective energy and effective chemical potential, where the effective nucleon mass \( M^* = M + g_\sigma \sigma_0 \). The baryon number density after subtracting out the pure vacuum contribution is given as

\[ \rho_B^{ren} = \gamma (2\pi)^{-3} \int dk (\sin^2 \theta_- - \sin^2 \theta_+). \]  

(16)

In the expression for energy density arising from the scalar meson sector, the field dependent effective sigma mass, \( M_\sigma(\beta) \), satisfies the gap equation in terms of the renormalised parameters as

\[ M_\sigma(\beta)^2 = m_R^2 + 12 \lambda_R \sigma_0^2 + 12 \lambda_R I_f(M_\sigma(\beta)), \]  

(17)

where,

\[ I_f(M_\sigma(\beta)) = \frac{M_\sigma(\beta)^2}{16 \pi^2} \ln \left( \frac{M_\sigma(\beta)^2}{m_R^2} \right) + \frac{1}{(2\pi)^3} \int dk \frac{\sin^2 \theta_\sigma(k, \beta)}{(k^2 + M_\sigma(\beta)^2)^{1/2}}. \]  

(18)

and,

\[ \sin^2 \theta_\sigma = \frac{1}{e^{\beta \omega_\sigma(k, \beta)} - 1}; \quad \omega_\sigma(k, \beta) = (k^2 + M_\sigma(\beta)^2)^{1/2}. \]  

(19)

In equation (14a), \( M_{\sigma,0} \) and \( I_{f0} \) are the expressions as given by eqs. (17) and (18) with \( \sigma_0 = 0 \). We might note here that the gap equation given by (17) is identical to that obtained through resumming the daisy and superdaisy graphs [19] and hence includes higher order corrections from the scalar meson field.

The thermodynamic potential, \( \Omega \), given by equation (12), after subtracting out the vacuum contributions, is now finite and is given in terms of the meson fields, \( \rho_0 \) and \( \omega_0 \). Extremisation of the thermodynamic potential with respect to the meson fields \( \sigma_0 \) and \( \omega_0 \) give the self–consistency conditions for \( \sigma_0 \) (and hence for the effective nucleon mass, \( M^* = M + g_\sigma \sigma_0 \)), as

\[ \frac{d(\Delta \epsilon_\sigma)}{d\sigma_0} + \frac{\gamma}{(2\pi)^3} g_\sigma \int dk \frac{M^*}{(k^2 + M^*)^{1/2}} (\sin^2 \theta_- + \sin^2 \theta_+) + \frac{d(\Delta \epsilon_{ren})}{d\sigma_0} = 0 \]

(20a)

and, for the vector meson field, \( \omega_0 \), as

\[ \omega_0 = \frac{g_\omega}{m_\omega^2} \frac{\gamma}{(2\pi)^3} \int dk (\sin^2 \theta_- - \sin^2 \theta_+), \]

(20b)

where \( \sin^2 \theta_\pi \) are the thermal distribution functions for the baryons and antibaryons, given through equation (15).

In Eq. (20a), the first term includes contribution from the scalar meson condensates. In the mean field approximation for the scalar mesons in the linear Walecka model, \( \lambda_R=0 \) and the energy density from the sigma meson is \( \frac{1}{2} m_\sigma^2 \sigma_0^2 \). This then corresponds to the relativistic Hartree approximation, with the last term in (20a) being the contribution arising from the vacuum polarisation effects from the baryonic sector.

In the next section, we shall consider the meson properties (\( \omega \) and \( \rho \)), calculated from the meson self–energy, as modified in the medium due to its coupling to nuclear excitations.
III. IN-MEDIUM MASSES FOR $\omega$ AND $\rho$ VECTOR MESONS

We now examine the medium modification to the masses of the $\omega$- and $\rho$-mesons including the quantum correction effects in the hot nuclear matter in the relativistic random phase approximation. The interaction vertices for these mesons with nucleons are given as

$$\mathcal{L}_{\text{int}} = g_V \left( \bar{\psi} \gamma_{\mu} \tau^a \psi V^\mu_a - \frac{\kappa_V}{2 M_N} \bar{\psi} \sigma_{\mu\nu} \tau^a \psi \partial^{\nu} V^\mu_a \right)$$  \hspace{1cm} (21)

where $V^\mu_a = \{ \omega^\mu, \rho^\mu \}$, $M_N$ is the free nucleon mass, $\psi$ is the nucleon field and $\tau_a = \{ 1, \vec{\tau} \}$, $\vec{\tau}$ being the Pauli matrices. $g_V$ and $\kappa_V$ correspond to the couplings to the vector and tensor interactions to the nucleon fields.

The vector meson self energy is expressed in terms of the nucleon propagator modified by the quantum effects. This is given as

$$\Pi^{\mu\nu}(k) = -\gamma_I g_V^2 \frac{i}{(2\pi)^4} \int d^4p Tr \left[ \Gamma^{\mu\nu}_V(k) G(k) \Gamma^{\nu\mu}_V(-k) G(p+k) \right],$$  \hspace{1cm} (22)

where, $\gamma_I=2$, is the isospin degeneracy factor for nuclear matter. In the above, $\Gamma^{\mu\nu}_V$ represents the meson-nucleon vertex function obtained from (21) and is given by

$$\Gamma^{\mu\nu}_V(k) = \gamma^\mu \tau_a - \frac{\kappa_V}{2 M_N} \sigma^{\mu\nu} i k^\nu \tau_a \hspace{1cm} (23)$$

The nucleon propagator in the medium is given as

$$G(k) = \left( \gamma^\mu \tilde{k}_\mu + M^*_N \right) \left[ \frac{1}{k^2 - M^*_N^2 + i\epsilon} + 2\pi i \delta(k^2 - M^*_N^2) \eta(\tilde{k} \cdot u) \right]$$

$$= G_F(k) + G_D(k),$$  \hspace{1cm} (24)

where, $\tilde{k} \equiv (k^0 + \Sigma^0_V, \vec{k})$ and the effective nucleon mass is $M^*_N = M_N + \Sigma_S$, where the vector and scalar self energies are given in terms of the expectation values for $\omega$ and $\sigma$ fields as $\Sigma^0_V = -g_\omega \omega_0$ and $\Sigma_S = g_\sigma \sigma_0$. In the above, $\eta(p \cdot u) = \theta(p \cdot u) f_{FD}(z) + \theta(-p \cdot u) f_{FD}(-z)$, with, $f_{FD}(z) = (1 + e^z)^{-1}$, $z = (p \cdot u - \mu^*)/T$ and $u^\mu$ is the four-velocity of the thermal bath. The expectation values, $\sigma_0$ and $\omega_0$, in the presence of quantum corrections, are determined self-consistently through the equations (20a) and (20b) for the hot nuclear matter [14].

The vector meson self energy can be written as the sum of two parts

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_F + \Pi^{\mu\nu}_D.$$  \hspace{1cm} (25)

where,

$$\Pi^{\mu\nu}_F = -2i g_V^2 \int \frac{d^4p}{(2\pi)^4} Tr \left[ \Gamma^{\mu\nu}_V(k) G_F(p) \Gamma^{\nu\mu}_V(-k) G_F(p+k) \right],$$  \hspace{1cm} (26a)

$$\Pi^{\mu\nu}_D = -2i g_V^2 \int \frac{d^4p}{(2\pi)^4} Tr \left[ \Gamma^{\mu\nu}_V(k) G_D(p) \Gamma^{\nu\mu}_V(-k) G_D(p+k) \right]$$

$$+ \Gamma^{\mu\nu}_V(k) G_D(p) \Gamma^{\nu\mu}_V(-k) G_F(p+k)$$

$$+ \Gamma^{\mu\nu}_V(k) G_F(p) \Gamma^{\nu\mu}_V(-k) G_D(p+k).$$  \hspace{1cm} (26b)

$\Pi^{\mu\nu}_F$ is the contribution arising from the vacuum polarisation effects, described by the coupling to the $N\bar{N}$ excitations. The shift in the vector meson mass occurs through processes like $V \rightarrow N\bar{N} \rightarrow V$, where $N$ represents nucleons in the medium modified due to the quantum corrections. This Feynman part of the self energy, $\Pi^{\mu\nu}_F$ is divergent and needs renormalization. For the $\omega$ meson, the tensor coupling is generally small as compared to the vector coupling to the nucleons [11], and hence is neglected in the present work. We use dimensional regularization to separate the divergent parts. The renormalized vacuum polarization tensor for the $\omega$-meson is then given as [20],

$$\Pi^{\tau\tau}_{\tau\tau}(k^2) = \frac{g_\omega^2}{\pi^2} k^2 \left\{ \Gamma(2 - n/2) \int_0^1 z(1-z) \right.$$  \hspace{1cm} (27)

$$- \int_0^1 dz z(1-z) \ln \left[ M_N^2 - k^2 z(1-z) \right] \right\} - \xi,$$
in which the last term arises from a counter term added in the Lagrangian given as

\[ \mathcal{L}_{ct} = -\frac{1}{4}\xi V^\mu{}^\nu V_{\mu\nu}. \]  

(28)

The renormalization condition to determine \( \xi \) is

\[ \Pi_{\xi}^{ren}(k^2)|_{MN} = 0. \]  

(29)

We finally arrive at

\[ \Pi_{\xi}(k^2) = \frac{1}{3} Re(\Pi_{\xi}^{ren})^\mu = -\frac{g^2}{\pi^2}k^2 \int_0^1 dz z(1-z) \ln \left[ \frac{M_N^2 - k^2z(1-z)}{M_N^2 - k^2z(1-z)} \right]. \]  

(30)

Because of the tensor interaction, the vacuum self energy for the \( \rho \) meson is not renormalizable. We employ a phenomenological subtraction procedure \[10,11\] to extract the finite part using the renormalization condition

\[ \frac{\partial^n\Pi_{\rho}(k^2)}{\partial(k^2)^n}|_{MN} = 0, \]  

(31)

with \( n = 0, 1, 2, \cdots \infty \). Using dimensional regularization and the above subtraction procedure, we arrive at the following expression

\[ \Pi_{\rho}(k^2) = -\frac{g^2}{\pi^2}k^2 \left[ I_1 + M_N^2 - k^2z(1-z) \right] \]  

(32)

where,

\[ I_1 = \int_0^1 dz z(1-z) \ln \left[ \frac{M_N^2 - k^2z(1-z)}{M_N^2 - k^2z(1-z)} \right], \]  

(33)

\[ I_2 = \int_0^1 dz \ln \left[ \frac{M_N^2 - k^2z(1-z)}{M_N^2 - k^2z(1-z)} \right]. \]  

(34)

In a hot and dense medium, because of Lorentz invariance and current conservation, the ground state structure of the polarization tensor takes the form

\[ \Pi^{\mu\nu} = \Pi_T(k_0, \vec{k})A^{\mu\nu} + \Pi_L(k_0, \vec{k})B^{\mu\nu}, \]  

(35)

where the two Lorentz invariant functions \( \Pi_T \) and \( \Pi_L \), characterizing the transverse and longitudinal projection tensors. These are obtained through the contractions

\[ \Pi_L = -\frac{k^2}{|\vec{k}|^2} u^\mu u^\nu \Pi_{\mu\nu}, \]  

(36a)

\[ \Pi_T = \frac{1}{2} \left( \Pi_{\mu\nu} - \Pi_L \right) \]  

(36b)

with \( u_\mu \) being the four velocity of the thermal bath. In the above, \( A^{\mu\nu} \) and \( B^{\mu\nu} \) are the transverse and longitudinal projection operators. The dispersion relation for the longitudinal (transverse) mode is obtained as

\[ k_0^2 - |\vec{k}|^2 - m_V^2 + Re\Pi_L^{(T)}(k_0, |\vec{k}|) + Re\Pi_T^{(L)}(k_0, |\vec{k}|) = 0 \]  

(37)

The in-medium mass for the vector meson (\( m_V^* \)) is defined as the lowest zero of equation \(37\) in the limit \( \vec{k} \to 0 \). In this limit, \( \Pi_T^{(E)} = \Pi_L^{(E)} = \Pi^{(E)} \) and we have \( \Pi^{(E)} \).
\[ \frac{1}{3} \Pi_\mu^\nu = \Pi = \Pi^D + \Pi^F, \]

where the density dependent part for the self energy is given as

\[ \Pi^D(k_0, \vec{k} \to 0) = -\frac{4g_\omega^2}{\pi^2} \int p^2 dp F(|\vec{p}|, M_N^*) \left[ \sin^2 \theta_-(\mu^*, T) + \sin^2 \theta_+(\mu^*, T) \right] \]

with

\[ F(|\vec{p}|, M_N^*) = \frac{1}{\epsilon^*(p)(4c^*(p)^2 - k_0^2)} \left[ \frac{2}{3} (2|\vec{p}|^2 + 3M_N^*)^2 + k_0^2 \right] \left\{ \frac{\kappa_V}{2M_N} \right\} \]

\[ + \frac{2}{3} \left( \frac{\kappa_V}{2M_N} \right)^2 (|\vec{p}|^2 + 3M_N^*)^2 \]

where \( \epsilon^*(p) = (p^2 + M_N^*|^2)^{1/2} \) is the effective energy for the nucleon. The effective mass of the vector meson is then obtained by solving the equation

\[ k_0^2 - m_V^2 + \text{Re} \Pi(k_0, \vec{k} = 0) = 0. \]

As already stated, the \( \omega \)NN tensor coupling is generally small (e.g. \( \kappa_\omega \simeq 0.1 \) in vector dominance model, whereas, \( g_\omega \simeq 10 \)). Hence, we neglect the tensor coupling for the \( \omega \) meson, in the present calculations. The vector coupling \( g_\omega \), along with the scalar coupling \( g_s \) is fixed from the nuclear matter saturation properties.

For the nucleon-rho couplings, we use the vector and tensor couplings as obtained from the N-N forward dispersion relation \([11,20,21]\). These couplings, however, do not take into account the medium effects. We also consider the modification of the \( \rho \)-meson for the case when the \( \rho \) vector coupling to the nucleon is determined from the symmetry energy for the nuclear matter, thus taking into account the medium dependence of the coupling. But since the medium dependence of the tensor coupling is not yet known, we take it as a parameter and examine the effect of it on the in-medium \( \rho \) meson mass. With the couplings as described above, we consider the temperature and density dependence of the \( \omega \) and \( \rho \) meson masses in the hot nuclear matter modified due to quantum corrections.

In the next section, we shall consider effect of quantum corrections on the strange meson (\( \phi \)-meson) as coupled to the baryons through a vector coupling in hyperon-rich matter \([16]\). The medium modification for \( \phi \)-meson mass occurs due to their coupling to the hyperons, as they do not couple to the nucleons by OZI rule \([11]\). It is observed that the medium modification of the \( \phi \)-meson mass is rather small as compared to the mass of the vector \( \omega \). This is because of the fact the latter has a significant contribution from the coupling to the nucleon sector, and the \( \phi \)-meson has only contribution from coupling to hyperons which are rather insensitive to the medium effects.

**IV. \( \phi \)- AND \( \omega \)- MESON MASSES MODIFICATION IN HYPERON-RICH DENSE MATTER**

The interaction for the vector mesons (\( \phi \) and \( \omega \)) with the baryons (B=N,Λ, Σ±0, Ξ0,−) is given through the interaction Lagrangian

\[ \mathcal{L}_{int} = \sum_B g_{\phi B} \bar{\psi}_B \gamma^\mu \psi_B \phi^\mu \]

\[ \text{(42)} \]

The strange vector meson, \( \phi \), couples to the hyperons, but not to the nucleons \([11]\). Hence \( \phi \)-mesons do not have any mass modification in nuclear matter, but get modified in strange hadronic matter. The hyperon-rich dense matter has been studied earlier in a nonperturbative treatment \([16]\) including the vacuum polarizations from the baryons (nucleons and hyperons) and scalar meson sector. This has been done in the context of structure of neutron stars and so the condition of electrical charge neutrality has been imposed for the study of the strange hadronic matter. In the present section, we study how the vector mesons (\( \omega \) and \( \phi \)) are modified in such a medium.

The vector meson mass in the medium obtained as a solution of the dispersion relation \([11]\), now gets modified due to contributions from the hyperons, and is given as,

\[ k_0^2 - m_V^2 + \sum_B \text{Re} \Pi_B(k_0, \vec{k} = 0) = 0. \]

\[ \text{(43)} \]
To solve the above equation for the in-medium meson mass, we take the meson-hyperon couplings as determined through the SU(3) symmetry\textsuperscript{16,23} given as

\[
\frac{1}{2} g_{\omega N} = \frac{1}{2} g_{\omega \Xi} = g_{\omega \Sigma} = \frac{1}{3} g_{\omega N}
\]
\[
2 g_{\phi N} = 2 g_{\phi \Xi} = g_{\phi \Sigma} = -\frac{2\sqrt{2}}{3} g_{\omega N}
\]

In the following section, we discuss the results obtained in the present calculations and discuss the effect of quantum corrections arising from the scalar meson sector over those from the baryonic sector (corresponding to the relativistic Hartree approximation).

V. RESULTS AND DISCUSSIONS

We first discuss the effect of quantum corrections on the in-medium nucleonic properties for the hot nuclear matter\textsuperscript{15}. In fig. 1, we plot the temperature dependence of the nucleon mass for various values of the baryon density. The quantum corrections arising from the sigma field has the effect of the softening of the equation of state, and, a higher value for the effective nucleon mass as compared to the RHA. The effective nucleon mass was seen to increase with the vacuum polarisation effects arising from the baryons as compared to the MFT calculations\textsuperscript{14}. Though for \( \rho = 0 \), the effective nucleon mass decreases monotonically with temperature, for higher values of densities, the nucleon mass first increases and then decreases as a function of temperature\textsuperscript{24}. The variation in the nucleon mass is very slow upto a temperature, \( T \approx 150 \text{ MeV} \), beyond which there is a fast decrease. Though qualitative features are same for the in-medium mass with inclusion of quantum effects from the scalar meson sector, it is observed that such effects lead to a higher value for the nucleon mass.

We next study the temperature and density dependence of the vector mesons in the hot nuclear matter. In figure 2, we plot the \( \omega \) meson mass as a function of the temperature for various densities, with the \( \omega N \) coupling along with the scalar coupling \( g_{\sigma} \), determined from the nuclear matter saturation properties. In particular, for RHA, \( C_{\pi}^2 = g_{\sigma}^2 M_N^2 / m_{\omega}^2 = 114.7 \), and with quantum corrections from scalar mesons for self coupling, \( \lambda_R = 1.8 \), \( C_{\pi}^2 = 96.45 \), respectively\textsuperscript{14}. In Walecka model, the Dirac sea has been shown to have a significant contribution over the Fermi sea, leading to a substantial drop in the omega meson mass in the nuclear matter\textsuperscript{10,11,20}. Specifically, at saturation density at \( T = 0 \), the decrease in the omega meson mass from the vacuum value, is around 150 MeV in RHA, whereas with sigma quantum effects, the drop in the mass reduces to around 117 MeV. A similar reduction in the mass drop, as compared to the RHA was also observed when scalar field quantum corrections were included within an one loop approximation\textsuperscript{8,23}, unlike the approximation adopted here, which is self-consistent and includes multi-loop effects. This means that the quantum corrections do play an important role in the medium modification of the vector meson masses. In the present work, it is seen that the quantum corrections from the scalar mesons, over those arising from the baryonic sector, have the effect of giving rise to a higher value for the \( \omega \)-meson mass. This again is related to the fact that effective nucleon mass is higher when such quantum effects are taken into account.

In figure 3, we illustrate the medium modification for the \( \rho \) meson mass with the vector and tensor couplings to the nucleons being fixed from the NN forward scattering relation\textsuperscript{1,20}. The values for these couplings are given as \( g_{\sigma N}^2 / 4\pi = 0.55 \) and \( \kappa_{\rho} = 6.1 \). We notice that the decrease in the \( \rho \) meson with increase in temperature is much sharper than that of the \( \omega \) meson. Such a behaviour of \( \rho \)-meson undergoing a much larger medium modification was also observed earlier\textsuperscript{20}, with the relativistic Hartree approximation for the nucleons. This indicates that the tensor coupling plays a significant role for the \( \rho \) meson, which is absent for the \( \omega \) meson. With inclusion of quantum corrections from the scalar meson, the qualitative comparison between the \( \omega \) and \( \rho \) vector mesons remains the same, though these effects are seen to lead to larger values for the vector meson masses.

In the vector and tensor couplings of the \( \rho \) meson to the nucleons as determined from the NN forward scattering processes\textsuperscript{21}, as considered above, any medium dependence of these couplings have not been taken into account. We shall next consider the mass modification for the \( \rho \) meson, with the nucleon \( \rho \) coupling, \( g_\rho \) as determined from the symmetry energy coefficient, \( a_{\text{sym}} \), given as\textsuperscript{25}

\[
a_{\text{sym}} = \frac{2}{m_\rho} \cdot \frac{k_F^3}{12\pi^2} + \frac{k_F^2}{6(k_F^2 + M_N^2)^{1/2}}
\]

where \( k_F \) is the Fermi momentum of nuclear matter at saturation density, \( \rho_0 \). In our calculations, we choose \( a_{\text{sym}} = 32.5 \text{ MeV} \), which gives the value for the nucleon-rho meson coupling as \( g_\rho = 6.82, 7.0 \), for RHA and \( \lambda_R = 1.8 \), respectively. However, in considering such a medium dependence of the vector coupling, it is assumed that the
temperature dependence of such coupling is much smaller compared to its density dependence, and hence has not been taken into account in the present work. We take the tensor coupling as a parameter in our calculations. since the medium dependence of the tensor coupling is not known as yet. The in-medium mass of the $\rho$ meson is plotted in figure 4 for baryon density $\rho/\rho_0=0.5$. It is observed that the $\rho$ meson mass has a strong dependence on the tensor coupling. For the same tensor coupling $\simeq 6$, the $\rho$ meson mass is much lower ($M_\rho^*/M_\rho \simeq 0.33$) when the medium dependence of the vector coupling constant is taken into account through the symmetry energy, as compared to the value ($M_\rho^*/M_\rho \approx 0.65$), when $g_\rho$ is fixed from the N-N forward scattering processes.

In figure 5, the effective baryon masses, $M_B = M_B + g_\sigma \sigma$, in the hyperon matter are plotted as a function of the baryon density at zero temperature, for RHA, and, with quantum corrections also from scalar mesons. The variation of the hyperon masses are slower as compared to the nucleon masses. This is a reflection of a smaller coupling of hyperons to the scalar sigma field \cite{10,12}. Further, we note that the decrease in the effective baryon mass with density, is slower when quantum corrections from the sigma mesons through condensates are taken into account as was the case for nuclear matter \cite{14}.

The medium modification for the vector mesons ($\omega$ and $\phi$) in the hyperonic matter are plotted in figure 6. One observes that the strange meson $\phi$ has smaller modification to the mass as compared to that of the $\omega$ meson. This is due to the fact that $\phi$-meson does not couple to the nucleons and also, the hyperon masses are rather insensitive to the changes in the baryon densities as may be seen in figure 5. The strange meson ($\phi$) mass modification observed as small compared to the $\omega$ and $\rho$ meson masses is in line with the earlier observations \cite{1,3,4}.

VI. SUMMARY

To summarize, we have considered in the present paper, the vector meson mass modifications due to quantum correction effects in the hot and dense matter. The baryonic properties as modified due to such effects subsequently determine the vector meson masses in the hot and dense hadronic matter. It has been recently emphasized that the Dirac sea contribution dominates over the fermion sea, thus implying that the vacuum polarisation effects arising from the baryonic sector do play an important role in the vector meson properties in a medium. In the present work, we study the effect of quantum corrections arising from the scalar meson over those resulting from the baryon sector (given through the summing over baryonic tadpole diagrams in the relativistic Hartree approximation). The quantum corrections from $\sigma$-meson give rise to a higher value for the vector meson masses. Also, it is observed that the strange vector meson ($\phi$) has smaller medium modification in the hyperon matter as compared to the $\omega$-meson similar to earlier observations. This is a reflection of the facts that, firstly, the hyperon masses have much smaller modification as compared to that of nucleons and, also, secondly, the $\phi$-meson does not couple to the nucleons, whereas $\omega$-meson does have appreciable contribution from the coupling to the nucleons. The vector meson properties being modified in a medium have an important role to play in the dilepton production in relativistic heavy ion collisions and it will be interesting to investigate how the dilepton yield gets affected due to the quantum corrections in the hot and dense matter. This and related problems are under investigation.

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FIG. 1. Effective nucleon mass as a function of the temperature for various values of the baryon densities for RHA and \( \lambda_R = 1.8 \). The quantum corrections from the scalar meson sector leads to an increase in the in-medium nucleon mass.
FIG. 2. In-medium $\omega$-meson mass as a function of temperature. The sigma meson quantum effects lead to an increase in the vector meson mass.

FIG. 3. In-medium $\rho$-meson mass as a function of temperature and density for $g^2_\rho/4\pi = 0.55$, $\kappa_\rho=6.1$. The decrease is sharper than that of the $\omega$ meson. The quantum corrections from scalar meson lead to a higher meson mass.
FIG. 4. In-medium $\rho$-meson mass as a function of temperature and density, with the vector coupling $g_\rho$ fitted from the symmetry energy for the nuclear matter, and for various values of the tensor coupling. The quantum corrections from scalar meson increase the vector meson mass.

FIG. 5. Effective baryonic masses as a function of density for hyperonic matter. The nucleon masses vary much faster than the hyperon masses. The quantum effects lead to an increase in the effective masses as compared to RHA.
FIG. 6. Effective vector meson masses ($\phi$ and $\omega$) plotted as a function of density for hyperon matter for RHA and $\lambda_R=1.8$. The variation in the strange meson mass is rather slow as compared to the $\omega$ meson.