In-medium modification of $P$-wave charmonia from QCD sum rules

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We investigate the changes of the masses and widths of $\chi_{c0}$ and $\chi_{c1}$ in hot gluonic matter near $T_c$ and in nuclear medium using QCD sum rules. As in the previous works for the $J/\psi$ and $\eta_c$, in-medium effects are incorporated through the changes of gluon condensates. Twist-2 terms for the $^3P_0(\chi_{c0})$ and $^3P_1(\chi_{c1})$ are also included for the first time. The results show that larger mass shifts and width broadenings take place as compared to the $S$-wave states. As the critical change take place near $T_c$, related measurements can reveal critical phenomenon in QCD.

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I. INTRODUCTION

Spectral property of charmonia in medium can provide experimental information about the confinement nature of QCD. At high temperature, hadronic matter undergoes a phase transition into the deconfined phase. Recent experimental data measured at Relativistic Heavy Ion Collider (RHIC) reveals that the deconfined matter is strongly interacting. It was previously expected that the production of $J/\psi$ will be suppressed by the color screening in the deconfined matter formed in the initial stages of heavy ion collisions. On the other hand, recent experimental observation shows non-trivial suppression pattern that are not easily understood by a simple suppression mechanism. An important fact in $J/\psi$ production is that they are not only produced directly but also through decay from higher mass states such as $\psi'$ and $\chi_c$. Since such excited states are likely to dissolve at just above the critical temperature $T_c$ whereas the ground state does at $1.6T_c$, as have been shown by lattice calculations, sequential suppression scenario has been proposed, in which the initial suppression comes from the disappearance of the feed down from $\psi'$ and $\chi_c$ followed by the suppression of directly produced $J/\psi$. In previous works, two of us (K.M and S.H.L) have studied the properties of $J/\psi$ and $\eta_c$ using QCD sum rules, and have shown that their masses and widths change following the critical behavior of gluon condensates, which are extracted from lattice QCD. In this paper, we apply the same formalism to the $\chi_{c0}$ and $\chi_{c1}$ states to investigate how the properties of these states are affected by the QCD phase transition, and explore the consequences in the experimental measurements. We also study the nuclear matter case which may serve as a testing ground for the phase transition through precursor phenomena.

The paper organized as follows; in the next section, we briefly summarize our formalism and present new Wilson coefficients. In section III we present quantitative results of in-medium change of masses and widths of $\chi_{c0}$ and $\chi_{c1}$. Section IV is devoted to discussion and summary.

II. FORMALISM

QCD sum rules are based on the current correlation function and the dispersion relation. In this paper, we consider the scalar current $j^S = \bar{c}c$ and axial vector current $j^A_\mu = (\gamma_\mu q_g/q^2 - g_{\mu\nu}\bar{c}c)\gamma^\nu\gamma_5c$ for $\chi_{c0}$ and $\chi_{c1}$, respectively. For finite temperature we start with the correlation function

$$\Pi^j(q) = i \int d^4xe^{iqx}\langle Tj(x)j(0)\rangle_T,$$

with $\langle \cdots \rangle_T$ being the Gibbs average $\langle O(x)\rangle_T = \text{Tr}(e^{-\beta H}O)/\text{Tr}(e^{-\beta H})$. For the scalar, there is only one independent polarization function, but for the axial vector correlation, there are two independent polarization with respect to the medium. This is so because when the axial vector meson is moving with a finite three momentum $q$ with respect to the medium at rest, the response will be different depending on whether the polarization is parallel or perpendicular to the three momentum. Hereafter, we assume that both the medium and the $\bar{c}c$ pair are at rest so that $q = (\omega, 0)$. Then, for both the scalar and axial vector meson, there will be only one independent mode. For the axial vector correlation function, one can extract the scalar correlation function through $\Pi^S(q) = \Pi^S(q)/q^2$ and $\Pi^A(q) = -\Pi^A(q)/(3q^2)$. Taking $q^2 < 0$, the dispersion relation relates the correlation function with the spectral density $\rho(s)$:

$$\Pi^j(\omega^2) = \int_0^\infty du \frac{\rho(u)}{u^2 - \omega^2}.$$

In the QCD sum rules, we calculate the correlation function by means of the operator production expansion (OPE) with condensates while we model the hadronic spectral density with a pole and continuum. In the present calculation, we neglect the continuum part in the hadronic spectral density since we can suppress the contribution by means of the moment sum rules. Recently

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the role of the zero mode in the spectral density has been emphasized in several literature \cite{15, 16} since it gives a constant contribution to the imaginary time correlator from which the spectral density is reconstructed via maximum entropy method in lattice QCD. This comes from Landau damping effect, i.e., scattering of the current with on-shell quarks in medium and has been known as the scattering term \cite{17} in the framework of QCD sum rules. This term is also related to the transport coefficients as they are the terms responsible for the long range order surviving at zero momentum and energy of the current. The scattering term exists whenever the thermal excitation, thermal charm quark in our present case, couples directly to the current, and contributes to the spectral density at zero energy in the kinematical limit of $q \to 0$ with a strength proportional to the thermal charm quark density at finite temperature \cite{18}. It can be shown that the same scattering term can be obtained in the OPE side by resuming the thermal charm quark contribution to the zeroth order charm quark operators, that did not have direct charm quark contribution at zero temperature as they were converted to gluonic operators in the heavy quark expansion \cite{19}. Therefore, while the scattering term contribution is non-negligible in the spectral density, similar term in the OPE will cancel its contribution and thus will not be considered further in this work. In a sense, it is the advantage of the QCD sum rule method to artificially turn off the effects of thermal charm quarks altogether and concentrate on the thermal gluonic effects on the pole. For confined phase, there is no on-shell quarks and we do not have to consider the scattering term from the beginning since we have only glueballs other than the heavy quarkonia in this phase. Therefore, we will neglect the scattering terms in both the confined and deconfined phase.

For heavy quarkonia, the moment sum rule gives a systematic procedure. The moment is defined as

$$M_n(Q^2) = \frac{1}{n!} \left( \frac{d}{dq^2} \right)^n \tilde{\Pi}^I(q^2) \bigg|_{q^2=-Q^2}. \quad (3)$$

Noting that the expansion parameter in the OPE for heavy quarkonia is $\Lambda_{QCD}^2/(4m_c^2 + Q^2)$ and focusing on temperature as large as $\Lambda_{QCD}$ so that the expansion parameter is not much modified from the vacuum value, we can truncate the OPE at the leading order perturbative correction and the dimension four gluon condensates, with the dimension four twist-2 gluon condensates being a new addition as compared to the vacuum case. In this approach, all the temperature effects in the OPE are encoded in the temperature dependence of operators. Therefore the moment \cite{3} for the OPE side becomes, for $\xi = Q^2/4m_c^2$,

$$M_n(\xi) = A_n(\xi)[1 + a_n(\xi)\alpha_n(\xi) + b_n(\xi)\phi_b + c_n(\xi)\phi_c], \quad (4)$$

where $A_n$, $\alpha_n$, $b_n$, and $c_n$ are the Wilson coefficients for the bare loop, perturbative radiative correction, scalar gluon condensate and twist-2 gluon condensate, respectively. In Eq. (4), $\phi_b$ and $\phi_c$ have temperature dependencies through the gluon condensates. They have been extracted from lattice calculation of pure SU(3) theory and shown in Ref. \cite{12}. Here we would like to stress that they are not order parameters in a strict sense but their behavior more or less reflect the phase transition, i.e., sudden change across the critical temperature $T_c$.

As for the Wilson coefficients, $A_n$, $\alpha_n$ and $b_n$ are listed in Ref. \cite{20}. $c_n$ for the scalar and axial currents are calculated for the first time in this work by making use of the background field technique \cite{21}. Defining the scalar functions $\tilde{\Pi}^S(q) = \Pi^S(q)/q^2$ and $\tilde{\Pi}^A(q) = -\frac{1}{m^2}\Pi^A_{\mu\nu}$ for the scalar channel and the axial vector channel correlation function, respectively, we find that the correction term for the twist-2 gluon operators in OPE becomes

$$\Delta \tilde{\Pi}^S(q) = \left\{ \frac{\alpha_s}{\pi} G_{\alpha\beta\mu} G^{\alpha\beta\mu} \right\} \left( \frac{q^2 g_3}{Q^2} \frac{1}{Q^4} \right) \left[ \frac{3}{2} J_2(y) - \left( 1 - \frac{1}{3} y \right) J_1(y) - \frac{1}{2} \right], \quad (5)$$

$$\Delta \tilde{\Pi}^A(q) = \left\{ \frac{\alpha_s}{\pi} G_{\alpha\beta\mu} G^{\alpha\beta\mu} \right\} \left( \frac{q^2 g_3}{Q^2} \frac{1}{Q^4} \right) \left[ \frac{3}{2} J_2(y) + \frac{4}{3} y J_1(y) - 6 \right], \quad (6)$$

where $y = Q^2/m^2$ and $J_n(y) = \int_0^1 dx [1 + x(1-x)y]^{-n}$. Therefore $c_n$ is given by

$$c_n^S(\xi) = b_n^S(\xi) + \frac{4n(n+1)}{3(1+\xi)} \left[ \frac{1 + x(1-x)y}{F(n+ \frac{1}{2}, n + \frac{3}{2}; \rho)} \right], \quad (7)$$

$$c_n^A(\xi) = b_n^A(\xi) + \frac{4n(n+1)}{3(1+\xi)} \left[ \frac{1 + x(1-x)y}{F(n+ \frac{1}{2}, n + \frac{3}{2}; \rho)} \right], \quad (8)$$

where $\rho = \frac{Q^2}{m^2}$ and $F(a,b;c;z)$ denotes the hypergeometric function $\text{}_2F_1(a,b;c;z)$. One can see that $c_n$ can be expressed as a sum of $b_n$ and an additional term, as in the case of the vector and pseudoscalar case \cite{22}, and that the additional terms of the scalar and axial vector channel are identical.

Masses and widths are extracted in the same manner.
as in the previous works for $J/\psi$ and $\eta_c$.\cite{11,12}

Taking ratio of the moments and equating the OPE side with the phenomenological side, we find

$$\frac{M_{n-1}}{M_n}\big|_{\text{OPE}} = \frac{M_{n-1}}{M_n}\big|_{\text{phen.}}. \quad (9)$$

In the phenomenological side, we simply express the pole term with the relativistic Breit-Wigner form and neglect the continuum contribution. Namely, we put

$$\rho(s) = \frac{1}{\pi} \frac{f_0 \Gamma \sqrt{s}}{(s - m^2)^2 + s\Gamma^2}. \quad (10)$$

This simplification is justified by taking the moment since it suppresses the high energy contribution in the phenomenological side. While the continuum slightly modifies the mass value, its contribution does not affect the in-medium change from the vacuum value, which is the interest of this paper. Also, the thermal factor $\tanh(\sqrt{s}/2T)$ is put to unity since we are considering temperatures much lower than the mass of charmonium.

### III. RESULTS

![Figure 1](image1.png)

**FIG. 1**: (Color online). Masses in the $\Gamma \to 0$ limit as a function of $n$ and $T/T_c$. The upper panel is for $\chi_{c0}$ and the lower for $\chi_{c1}$.

Following Ref. \cite{20}, we work with $\xi = 2.5$ to validate the OPE since the Wilson coefficients are larger than those of the $S$-wave. The parameters are the same as in Ref. \cite{11}, with $\alpha_s(\xi = 1) = 0.21$ and $m_c(\xi = 1) = 1.24$ GeV. Since we have neglected the continuum, vacuum masses are a little larger than the experimental data. However, it does not affect the results of medium-induced changes discussed in what follows, as we will discuss later.

Figure 1 shows the masses in the $\Gamma \to 0$ limit calculated from $m^2 = M_{n-1}/M_n|_{\text{OPE}} - 4m^2\xi$ at various temperatures. At large $n$, the OPE breaks down, while at small $n$, the continuum slightly effects the in-medium change from the vacuum value, which is the interest of this paper. Also, the thermal factor $\tanh(\sqrt{s}/2T)$ is put to unity since we are considering temperatures much lower than the mass of charmonium.

Solving Eq. (9) with respect to $m$ and $\Gamma$ for the minimum of OPE side moment ratio at each temperature, we obtain the relation between mass shift and width broadening. The results are displayed in Fig. 2. As in the $S$-wave cases, a linear relation is also seen in the $P$-wave quarkonia. In the small mass shift and large broadening case at $T = 1.03T_c$, the relation shows some deviations from the linear relation. This comes from the fact that the stability is achieved at large $n$ which gives strong dependence of the moment ratio on $n$. As mentioned before, convergence of OPE is not so good. Strictly speaking, physical parameters extracted from the sum rules should not strongly depend on the external parameters, $\xi$ and $n$. In this case, however, $\Gamma$ at $\delta m = 0$ depends on $n$ while $\delta m$ does not. Therefore the results of the width might be less reliable. In reality there should be
mass shifts in this temperature region caused by change of the chromo-electric field, \textit{i.e.}, second-order Stark effect \cite{24,25}. Hence, a small mass shift with large width broadening would not be a realistic combination.

To see the temperature dependence, we depict the temperature dependence of the maximum mass shift and width broadening in Fig. 5. One can see that both \(\chi_{c0}\) and \(\chi_{c1}\) exhibit a clear critical behavior with respect to the abrupt change of the gluon condensates. This qualitative feature was also seen in the S-wave \cite{12} cases, but in the present case, the spectral changes are roughly a factor of two larger. Especially the maximum mass shifts reach \(\sim 150\) MeV at \(T = T_c\) and more than 400 MeV at \(T = 1.03T_c\). Lattice calculation suggests \cite{8} that the \(\chi_{c0}\) and \(\chi_{c1}\) will dissolve before \(T = 1.1T_c\). However, as is evident also in the lattice data, our result suggest that just before it dissolves, the spectral density will be greatly modified and the mass greatly reduced slightly above \(T_c\), which are more promising signals for direct observation in future experiments than the correspondingly smaller mass shift expected for \(J/\psi\).

We also calculate the mass shift and width broadening in nuclear matter. This can be done by replacing the change of the gluon condensates with those for the nuclear matter \cite{12,22}. The result of mass shift and width broadening is shown in Fig. 6. The maximum mass shift is around 17 MeV, which is more than two times larger than that obtained previously for the S-wave charmonia. The difference between \(\chi_{c0}\) and \(\chi_{c1}\) is around 1 MeV.

\section{Summary and Discussions}

In this paper, we extend the previous QCD sum rules analyses for the S-wave charmonia to \(P\)-waves. The results are qualitatively similar to those of \(S\)-waves but
show larger spectral changes. Due to the absence of convergence of the OPE, our approach is limited to temperatures above but close to $T_c$. Note that our results do not necessarily imply the absence of the bound states; spectral function obtained in lattice calculations does not have a good resolution of the width. Although potential models are extensively studied with potentials extracted from lattice calculations \cite{26,27,28,29}, it is not straightforward to implement the critical change into potentials at fixed temperatures, and also ambiguities exist in the choice of the potential.

One may wonder if the continuum part in the hadronic spectral function that we have neglect in this paper can affect the result. As described before, contribution from this part can be suppressed by making use of the moment sum rule prescription. However, this suppression lead to too large $n$ of the moment at $T \geq 1.03T_c$, which results in the non-monotonic behavior of the width in Fig. 5. We will be able to improve it by taking the continuum into account. In this case, however, the continuum threshold $s_0$ in the phenomenological side appears as a free parameter. This can be fixed so that mass agrees with the experimental value in the vacuum case, while we have no criteria in the case of medium. This means the change of the OPE in medium might be attributed to the change of the threshold. Nevertheless, present calculation has general consequences because it corresponds to results at $s_0 \to \infty$ limit. Therefore, Fig. 11 shows the maximum mass at each temperatures, as it is well known in the charmonium sum rule \cite{20} that the extracted mass increases with increasing $s_0$. Consequently, if one adjusts $s_0$ so as to reproduce the experimental value at zero temperature and then keep it fixed as temperature changes, the analysis will result in similar amount of spectral changes. If one allows $s_0$ to change with temperature, the mass shifts will be larger than those obtained at $s_0 \to \infty$. Even in this case, there are clear evidence of the change of the masses at $T > 1.02T_c$ at which masses are smaller than experimental values. Further discussion on the effect of the continuum will be given in a future publication \cite{30}. Future works also include directly comparing the QCD sum rule result with the $T$-dependence of the Euclidean correlation functions observed in lattice QCD calculations.

In this paper, we utilized the gluon condensates extracted from lattice gauge theory of the pure gluonic system. Since there are many differences between the quenched and full QCD, one may wonder if the present results are relevant for realistic situations. First, the effect of dynamical quarks alters thermodynamic properties. Namely, transition temperature reduces to 170-200 MeV \cite{31} while it is around 260 MeV for pure SU(3) \cite{32}. On the other hand, what is important for the present analysis is the the quantitative change in magnitude of the condensate near $T_c$. Figure 21 of Ref. \cite{12} shows the behavior of the scalar gluon condensates of full QCD case after the fermionic contribution is subtracted out. One sees that the amount of decrease from the vacuum value is almost the same as in the pure gauge case while the change around $T_c$ becomes rather moderate. Considering the fact that the present lattice results have been performed on quite coarse lattice for which the hadron spectrum is still distorted and thus the thermodynamic quantities in the low temperature region are not properly calculated the changes of gluon condensate could

Figure 5: (Color online). Upper: mass shift as a function of $T/T_c$. Lower: Width broadening as a function of $T/T_c$. Circles and triangles denote the $\chi_{c0}$ and $\chi_{c1}$, respectively.

Figure 6: Relation between mass shift and width broadening in the nuclear matter. Solid and dashed lines stand for $\chi_{c0}$ and $\chi_{c1}$, respectively.
be larger below \( T_c \). This means that the spectral changes set in at much lower temperature below \( T_c \) due to the large number of degree of freedom in the hadronic phase and the amount of change at \( T_c \) is almost the same. Although not available at present, the twist-2 gluon part will also change a little as it is proportional to the entropy density of the gluons, which will not be effected greatly by the dynamical quarks. Hence, our present result could change a little but the main features will remain if dynamical quarks are introduced.

The results contain rich physical consequences relevant to recent and future experiments. First of all, a large fraction of \( J/\psi \) comes from \( \chi_c \). The in-medium modification of \( \chi_c \) will affect the final yields of \( J/\psi \). If both \( J/\psi \) and \( \chi_c \) can exist slightly above \( T_c \), \( \chi_c \) with modified mass and width will decay into \( J/\psi \). This will give further enhancement of \( J/\psi \) production. Moreover, if the charmonia will be produced statistically \( \chi_c \) at the same hadronization temperature, the larger mass shift of \( \chi_c \) will lead to a larger enhancement of the \( \chi_c \) compared to \( J/\psi \) and then will result in the change of the production ratio \( N_{J/\psi}/N_{\chi_c} \) in nucleus-nucleus collisions. Alternatively, since the width of \( \chi_c \) can be large enough to decay inside medium, it might provide a chance to directly measure the large mass shift. This is experimentally a challenging subject, which requires the reconstruction of the \( J/\psi + \gamma \) invariant mass distribution with extremely fine resolution, but if successful will provide a direct evidence of the critical phenomenon in QCD. Our result for the nuclear matter can be a testing ground of this idea. The maximally expected mass shift around 15 MeV may not be large, but enough to be observable in the anti-proton project at FAIR \[33\]. This will be another challenge for future experiments.

To conclude, we have studied the in-medium spectral change of \( \chi_0 \) and \( \chi_1 \) with the QCD sum rule approach developed in Ref. \[11\]. Our results show that the change in these states are more prominent than previously studied S-wave states, and this result will provide another means to probe critical phenomena in QCD.

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