Inverse volume corrections to emergent tachyonic inflation in loop quantum cosmology

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Abstract. The emergent model in the context of loop quantum cosmology with a tachyon scalar field is studied. We find that there is a center equilibrium point in the semiclassical region and a saddle point in the classical region. If the potential of the tachyon field satisfies some conditions, the universe can stay at the center equilibrium point past-eternally and then oscillate infinitely around this point with the tachyon climbing up its potential. Once the potential reaches a critical value, these two equilibrium points coincide with each other and the oscillation phase is broken by an emergent inflation. In order to obtain a successful emergent tachyon inflation, a constraint on $\dot{\phi}^2$ of tachyon is required.

Keywords: physics of the early universe, quantum gravity phenomenology, cosmic singularity, inflation
1 Introduction

The inflationary model is very successful to solving some problems in the standard cosmological model and is consistent with observations of the Cosmic Microwave Background radiation and high redshift surveys [1]. However, the existence of a big bang singularity in the early universe is still an unresolved problem. Some authors have tried to address this problem within the framework of quantum gravity and have suggested some models to avoid this singularity, such as the pre-big bang [2, 3] and ekpyrotic/cyclic scenarios [4] in string/M-theory, but it still remains unclear as to what process could lead to a nonsingular transition from the pre- to post big bang phase.

Recently, in order to establish a singularity-free inflationary model in the context of classical general relativity, a new scenario, an emergent universe [5, 6], has been proposed. In this model, the space curvature is positive and the universe stays past-eternally in an Einstein static state and then evolves into a subsequent inflationary phase. Thus the big bang singularity can be avoided. In addition, researches show that the entropy considerations favor the Einstein static state as the initial state of our universe [7]. However the Einstein static universe in the classical general relativity is unstable. Therefore the universe is extremely difficult to maintain such an initial static state in a long time due to the existence of perturbations, such as the quantum fluctuations.

More recently, the emergent model within the framework of quantum gravity has been studied extensively. In this regard, Mulryne et al. [8] studied the existence and stability of Einstein static state in the context of Loop Quantum Cosmology (LQC) [9, 10] with the inflaton scalar field modified by the inverse volume corrections in Loop Quantum Gravity (LQG) (see [11] for recent reviews), where LQC is an application to cosmology of LQG. The inverse volume is the cube of inverse scale factor and in LQC the correction to the inverse volume arises by exploiting the ambiguity in defining the inverse volume operator which is required to quantize the Hamiltonian constraint [12]. It was found that there is a center equilibrium point in the semiclassical region besides a saddle point in the classical region. If the potential energy of scalar field is less than a critical value, the universe can stay past-eternally at a static state and then enter into an oscillating phase with the scalar field being driven up its potential [13]. As a result, once the potential reaches this critical value, the center equilibrium point coincides with the saddle one, and the oscillating phase is broken and then the universe enters an inflation era. Thus a successful singularity-free inflation model is obtained. However in ref. [8], the author did not consider the inverse volume correction to the gravity, which seems to be ad-hoc. The reason is that the same ambiguity also arises in
the gravitational part of the Hamiltonian constraint. Therefore the inverse volume correction to the gravity should be considered, which may change the dynamics significantly. For the case of the LQC without the inverse volume modification, although there is also a new stable center if the cosmological constant is larger than a critical scale, the universe cannot break the infinite cycles around this center point to naturally enter a subsequent inflationary phase [14]. Thus it will be interesting to study the emergent inflation in LQC with the inverse volume corrections both to matter and gravity. In addition Lidsey and Mulryne [15] studied the dynamics of a scalar field within the context of braneworld scenario proposed by Shtanov and Sahni [16, 17] and found that, with the scalar field climbing up its potential, the cosmic evolution from the past eternal cycle to inflation can occur naturally as in the case of LQC with the scalar field modification. Later, modified stabilities of the Einstein static model in some non-Einstein gravities [18] have also been discussed.

In this paper, we will discuss an emergent inflation in LQC with the inverse volume corrections to both the matter and gravity. A tachyon scalar field is considered since it has a non-canonical kinetic term and has attracted considerable attention for the cosmic inflation at early times and for dark energy and dark matter at late times [19]. In the framework of LQC the tachyon inflation has been studied in the cases of the matter with or without the inverse volume correction and results showed that the super inflation can appear easily in both cases [20, 21].

2 Tachyon in LQC

In a positively curved Friedmann-Robertson-Walker (FRW) background, following the calculations in refs. [22, 23], the effective Hamiltonian for the LQC system with tachyon field can be expressed

\[
H_{\text{eff}} = -\frac{3}{\kappa \gamma^2} S_J(p) \frac{\sin^2(\mu c)}{\mu^2} - \frac{3S_J(p) V^{2/3}}{\kappa} + p^{3/2} \rho_\phi ,
\]

with

\[
S_J(p) = \sqrt{p} S(q) ,
\]

where \( \kappa = 8\pi/M_{\text{pl}}^2 \), \( p = a^2 \), \( J \) is a half-integer, \( \rho_\phi \) is the energy density of tachyon field, \( q = (\frac{\alpha}{\alpha_e})^2 \) with \( \alpha_e = (8\pi \alpha^{1/2} \gamma J)^{1/3} l_{\text{pl}} \) and \( \gamma = 0.2375 \). In eq. (2.2), \( S(q) \) arises from the inverse volume correction to gravity and can be expressed as

\[
S(q) = \frac{4}{\sqrt{q}} \left\{ \frac{1}{10} [(q + 1)^{5/2} + \text{sign}(q - 1)] q - 1^{5/2} - \frac{1}{35} [(q + 1)^{7/2} - |q - 1|^{7/2}] \right\} .
\]

Using the Hamilton’s equation we can obtain the modified Friedmann equation

\[
H^2 = \left( \frac{\kappa}{3} \rho_\phi - \frac{S}{a^2} \right) \left( S + \frac{3S}{\kappa \rho_e a^2} - \frac{\rho_\phi}{\rho_e} \right) ,
\]

where \( \rho_e \simeq 0.82 M_{\text{pl}}^4 \) is the critical LQC energy density. When \( S = 1 \), the above Friedmann equation corresponds to the case without the inverse volume correction in the positively curved universe given in ref. [14]. Since \( H^2 \) must be non-negative, we have the following limits

\[
\frac{3}{\kappa a^2} S \leq \rho_\phi \leq \rho_e \left( S + \frac{3S}{\kappa \rho_e a^2} \right) .
\]
In LQC with the inverse volume correction the energy density and pressure for tachyon matter can be written as [20]

\[ \rho_\phi = \frac{V p^{3/2} |F(q)|^{3/2}}{\sqrt{p^3 |F(q)|^3 - \dot{\phi}^2}} , \]  
(2.6)

\[ p_\phi = -\frac{V p^{3/2} |F(q)|^{3/2}}{\sqrt{p^3 |F(q)|^3 - \dot{\phi}^2}} \left[ 1 + \frac{\dot{\phi}^2 |F(q)|^2}{p^2 |F(q)|^4} \right] . \]  
(2.7)

Here \( V \) is the potential of tachyon field, which must be larger than zero since the energy density must be positive, and

\[ F(q) = a_*^{-2} \left( \frac{8}{77} \left[ 7 \{ (q + 1)^{11/4} - |q - 1|^{11/4} \} - 11q \{(q + 1)^{7/4} - \text{sign}(q - 1) |q - 1|^{7/4} \} \right] \right) ^4 . \]  
(2.8)

By differentiating eq. (2.4) with time and using eqs. (2.6), (2.7) we have

\[ \dot{H} = \frac{1}{2} \left( 3\kappa \rho_\phi \left( 1 - \frac{V^2}{\rho_\phi^2} \right) \frac{d\ln F}{d\ln q} + \frac{2S}{a^2} - \frac{2q}{a^2} \frac{dS}{dq} \right) \left( S + \frac{3S}{\kappa \rho_c a^2} - \frac{\rho_\phi}{\rho_c} \right) - \frac{\kappa}{3} \rho_\phi - \frac{S}{a^2} \left( \frac{2\dot{S}}{\kappa \rho_c a^2} + \frac{6q}{\kappa \rho_c a^2} \right) - \frac{6S}{\kappa \rho_c a^2} - \frac{3\rho_\phi}{\rho_c} \left( 1 - \frac{V^2}{\rho_\phi^2} \right) \frac{d\ln F}{d\ln q} \]  
(2.9)

In the following a constant potential is considered which is a good approximation if the variation of potential is very slow with time.

The equilibrium points of this system are given by the conditions \( \ddot{a} = \dot{a} = 0 \), which imply

\[ a = a_{eq} , \quad H(a_{eq}) = 0. \]  
(2.10)

From eq. (2.4) it is easy to obtain that \( H^2 = 0 \) corresponds to two critical energy densities. We find that the critical density \( \rho = \rho_c \left( S + \frac{3S}{\kappa \rho_c a^2} \right) \) cannot lead to a successful emergent inflation. Therefore in the following we will only consider the critical point \( \rho_\phi = \frac{3S}{\kappa a^2} \). From eq. (2.9), one can obtain the following constraint equations for \( a_{eq} \)

\[ A(a_{eq}) = B(a_{eq}) , \quad B(a) = \frac{2q \frac{dS}{dq} - 2S}{3\kappa \rho_\phi \left( 1 - \frac{V^2}{\rho_\phi^2} \right)} . \]  
(2.11)

Here \( A = \frac{d\ln F}{d\ln q} \). Clearly the condition for the existence of the equilibrium points is that the functions \( A(a) \) and \( B(a) \) intersect. Using eq. (2.6) at the equilibrium point we find

\[ \frac{\dot{\phi}^2}{9} = \frac{4}{3} p^{3/2} |F(q)|^{3/2} \frac{d\ln S}{d\ln q} - \frac{1}{B} . \]  
(2.12)

Since \( \frac{d\ln S}{d\ln q} \) is smaller than 1 as shown in figure 1, the reality condition, \( \dot{\phi}^2 > 0 \), requires that the intersections of functions \( A \) and \( B \) must appear in the range of \( B < 0 \).

In figure 2 we give the curves of functions \( A \) and \( B \). The function \( A \) reaches its maximum \( A_{max} = 4 \) at \( a = 0 \), decreases to its minimum \( A_{min} = -\frac{11}{9} \) at \( a = a_* \), and then asymptotes
Figure 1. The evolutionary curves of function $\frac{d \ln S}{d \ln q}$ against $q$.

Figure 2. The evolutionary curves of functions $A$ (solid line) and $B$ (dotted, dashed and dot-dashed lines) against $a$ with Plank unit. The dotted, dashed, and dot-dashed curves correspond to the function $B$ with $V > V_{\text{crit}}$, $V = V_{\text{crit}}$ and $V < V_{\text{crit}}$, respectively. The vertical long-dashed line denotes the position of $a_{\ast}$.

to $-1$ at $a \to \infty$. The function $B$ is a hyperbola with a single vertical asymptote given by solving $V = \frac{3S}{\kappa a^2}$. However, from the requirement of $\dot{\phi}^2 > 0$, only the down-left branch of this hyperbola plays a role in determining the existence of the equilibrium points.

If the potential satisfies the conditions $0 < V < V_{\text{crit}}$ where $V_{\text{crit}} = \frac{3S}{\kappa a^2} \sqrt{\frac{29}{33} + \frac{4\alpha^2}{33} \frac{d \ln S}{d \ln q}|_{a=a_{\ast}}}$ obtained from the equation $A_{\text{min}} = B$, there are two intersects between the curve of $A$ and the down left branch of $B$. As shown in figure 2, one equilibrium point occurs at the semiclassical region; the other at the classical region. With the increasing of the potential these two intersects become closer and closer. Once $V = V_{\text{crit}}$ they coincide with each other and there is only an equilibrium point. In addition, we find that, in order to obtain two intersect points of functions $A$ and $B$, the kinetic term $\dot{\phi}^2$ of tachyon field must satisfy the following constraint

$$\frac{4}{33} |F|^3 a^6 \left(1 - a^2 \frac{d \ln S}{d \ln q}\right) < \dot{\phi}^2 < a^6 |F|^3,$$

$$\tag{2.13}$$

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Figure 3. The constraint on $\dot{\phi}^2$ given in eq. (2.13) with Plank unit. The vertical long-dashed line denotes the position of $a_\ast$.

which comes from the requirement $0 < V < V_{\text{crit}}$. In figure 3 we give the allowed region of $\dot{\phi}^2$. From this figure, we can see that $a_{\text{eq}}$ cannot be much smaller than $a_\ast$ and the region of $a_{\text{eq}} \lesssim 0.7a_\ast$ is forbidden.

If one obtains the equilibrium points, their stabilities are determined by the eigenvalues coming from linearizing the system near these points. By calculation, we get the eigenvalue $\lambda^2$, but the results are very tedious. We will not give them here. If $\lambda^2 < 0$ the corresponding equilibrium point is a center otherwise it is a saddle. Since $\lambda^2$ is positive for $a > a_\ast$ and negative for $a < a_\ast$, as shown in figure 4, equilibrium points occurring in the classical region are saddles and those in the semiclassical region are centers. Therefore from figure 2 we obtain that in LQC there is a center equilibrium point in the semiclassical region, which originates from the loop quantum effects, and a saddle point in the classical region, which is similar to the classical general relativity.

In figure 5 we give the phase portraits for demonstrating the center equilibrium point and the saddle point. The separatrix is represented by the dashed line. Apparently if the value of the potential is less than $V_{\text{crit}}$ the universe can stay at a stable state past-eternally.
and then undergo an infinite oscillation with the tachyon field climbing up its potential. Once the potential reaches the critical value $V_{\text{crit}}$, the oscillating phase is broken and then the universe enters into an inflationary era.

3 Conclusion

In summary, in LQC with the inverse volume corrections to the tachyon field and the gravity, there are two equilibrium points. The one appearing in the classical region is a saddle point as that in general relativity; the other in semiclassical region is a center which comes from the loop quantum effects. If the potential of tachyon field asymptotes to a positive constant and is less than $V_{\text{crit}}$ as $t \to -\infty$, the universe can stay past-eternally at a stable state. With the tachyon field climbing up its potential slowly, the universe will undergo non-singular oscillations around the center equilibrium point. Once the potential reaches the critical value $V_{\text{crit}}$, the saddle point coincides with the center point and then the cycles are broken by the emergence of an inflation. Thus the universe enters into a de Sitter expansion phase. In addition we find that, in order to obtain a successful emergent tachyon inflation, the allowed region of $\dot{\phi}^2$ at the stable state is constrained. For the normal scalar field the emergent inflation will be studied in the future.

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Figure 5. This figure represents the phase diagram corresponding to figure 2 with the Plank unit, $J = 100$ and $\alpha = 1$. The axes have been compactified using the relations $x(t) = \arctan(H)$ and $y(t) = \arctan(\ln a)$.
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