The Quest for the Golden Activation Function

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Abstract. Deep Neural Networks have been shown to be beneficial for a variety of tasks, in particular allowing for end-to-end learning and reducing the requirement for manual design decisions. However, still many parameters have to be chosen in advance, also raising the need to optimize them. One important, but often ignored system parameter is the selection of a proper activation function. Thus, in this paper we target to demonstrate the importance of activation functions in general and show that for different tasks different activation functions might be meaningful. To avoid the manual design or selection of activation functions, we build on the idea of genetic algorithms to learn the best activation function for a given task. In addition, we introduce two new activation functions, ELiSH and HardELiSH, which can easily be incorporated in our framework. In this way, we demonstrate for three different image classification benchmarks that different activation functions are learned, also showing improved results compared to typically used baselines.

1 Introduction

Deep Neural Networks (DNNs) (see e.g., [1, 2]) have recently become very popular and are now successfully applied for a wide range of applications. However, as more complex and deeper networks are of interest, strategies are required to make neural network training more efficient and more stable. While for instance initialization (e.g., [3, 4]) and normalization techniques (e.g., [5]) are well studied, an also relevant and important factor is often neglected: the role of activation functions (AFs). Even though recent work demonstrated that AFs are of high relevance (see e.g., [6, 7, 8, 9, 10, 11]), due to is simplicity and reliability most deep learning approaches use Rectified Linear Units (ReLU) [12] as nonlinear activation functions.

Initially, due to their universal approximation properties the research in this field was mostly concentrated on squashing functions such as Sigmoid and Tanh [13]. However, training DNNs using such functions suffers from the vanishing gradient problem [14]. To overcome this problem, various non-squashing functions were introduced, where the most notable example is Rectified Linear Unit (ReLU) [12]. In particular, as the derivative of positive inputs in ReLU are one, the gradient cannot vanish. In contrast, as all negative values are mapped to zero, there is not information flow in DNNs for negative values. This problem is known as dying ReLU.
To deal with this problem, various generalizations of ReLU such as Leaky ReLU [13] have been proposed. Similarly, Exponential Linear Units (ELU) [8] do not only eliminate the bias shift in the succeeding layers, but also push the mean activation value towards zero by returning a bounded exponential value for negative inputs. Although, showing competitive results, ELU is not backed by a very strong theory. A theoretically proven extension, Scaled Exponential Linear Units (SeLU) [6], makes DNN learning more robust. It fact, it is shown that the proposed self-normalizing network converges towards a normal distribution with zero mean and unit variance.

A different direction was pursued in [7], finally introducing the Swish activation function. Different search spaces are created by varying the number of core units used to construct the activation function, and an RNN is trained to search the state space for a novel activation function. The proposed approach shows competitive results for both shallow and deep neural networks. Recently, a theoretic proof and justification for the design have been given in [16], showing that Swish propagates information better than ReLU.

In this way, existing approaches to estimate activation functions for DNN learning are lacking theoretical foundation, are based on complex theory, which is hard to understand in the context of practical applications, or are based on inefficient search schemes, which still require to manually set several parameters. To overcome these problems, as first contribution, we propose an approach based on ideas of Genetic Algorithms [17].

In particular, building on neuro-evolutionary algorithms [18], starting from simple initial activation functions more and more complex functions can be obtained over time, which are better suited for a given task. In contrast, to brute-force search strategies, the search space is explored in a more efficient way, drastically reducing the training effort. In addition, we propose to define piece-wise functions, better representing the desired properties. In fact, this idea can easily be included in the proposed learning framework.

As second contribution, based on recent theoretical findings [16], we introduce two new activation functions, namely, ELiSH and HardELiSH, which have shown to be competitive compared to existing approaches as well as very useful within the proposed framework. To demonstrate the benefits of our learned activation functions, we applied our approach for three different object classification benchmark data sets, varying in size and complexity, and run it using two different network architectures. The results clearly demonstrate that using the proposed approach better results can be obtained as well as that for different tasks different activation functions are useful.

The reminder of the paper is structured as follows: First, in Sec. 2, we discuss the related work in the context of Genetic Algorithms for Neural Networks. Next, in Secs. 3 and 4, we introduce the new activation functions and the new neuro-evolutionary algorithm for learning task-specific activation functions. Then, in Sec. 5, we give a detailed experimental evaluation of our approach and discuss the findings. Finally, in Sec. 6, we summarize and conclude our work.
2 Related Work

Neuroevolution, i.e., applying evolutionary algorithms (EAs) in the optimization of DNNs [19], is a vital field of research. In general, there are two main directions. First, optimizing training parameters such as hyper-parameters [20] or weights [21, 22]. In the latter case, in contrast to methods like gradient descent, also global optima can be estimated. Second, evolving an optimal DNN topology, which, however, is not straightforward. Therefore, existing approaches follow two strategies: constructive [23] and destructive [24]. Constructive methods start from a simple topology and gradually increase the complexity until an optimality criterion is satisfied. In contrast, destructive approaches start from an initially complex topology and incrementally reduce the unnecessary structures.

Recently, co-evolution of topology and weights (TWEANNs) has shown to be more effective and efficient. The most successful related approach NEAT [23]. NEAT follows the constructive strategy and gradually evolves a simple DNN topology towards unbounded complexity by adding nodes and connections between them while preserving the optimality of topology. Due to its success, there have been several extensions of NEAT. For instance, in two extensions, DeepNEAT and CoDeepNEAT, have been proposed. In contrast to NEAT, in DeepNEAT a node represents a layer and consists of a table of hyper-parameters (i.e., number of neurons) related to it. In CoDeepNEAT, two populations (modules and blueprints) are initialized separately, where a module is a graph and represents a shallow DNN. A blueprint has also a graph structure and consists of nodes pointing out to specific module species. Both modules and blueprints evolve in parallel, and, finally, the modules and blueprints are combined to build up the topology of the DNN.

Similarly, [27] explored a CNN architecture via Cartesian Genetic programming (CGP) for image classification, where also high level functions such as convolution or pooling operations are implemented. Recently, [28] proposed a constructive hierarchical genetic representation approach for evolving DNN topologies. Initialized with small populations of primitives such as convolutional and pooling operations at the bottom of the hierarchy, the topology gets more and more complex by adding evolved primitives into graph structure.

So far most attentions have been drawn to TWEANNs, however, we are interested in evolving activation functions, which was only of limited interest up to now. The ideas closest to ours are Hyper-NEAT [29] and HA-NEAT [30]. Hyper-NEAT is a NEAT extension to evolve connective compositional pattern-producing networks (CPPNs) to estimate the weights of the ANN. In fact, the geometry of patterns can be represented by a composition of functions. Similarly, HA-NEAT [30] extends NEAT to evolve activation functions of neurons, topology, and weights, resulting in a heterogeneous network. In contrast, we fixed the topology and evolved the piece-wise activation functions on layer level. The proposed candidate solutions are more complicated (advanced) than those of HA-NEAT. More importantly, the complexity of evolved activation functions is, in contrast to HA-NEAT, unbounded. Nevertheless, it is possible to evolve our idea along with topology.
3 ELiSH: Exponential Linear Sigmoid Squashing

Based on recent findings in [7] and [16], in the following, we introduce two new activation functions, ELiSH and HardELiSH, which will then also be applied in our evolutional framework. In particular, the design of these activation functions was motivated by the recently proposed Swish activation function [7]:

\[
y(x) = \frac{x}{1 + e^{-x}}.
\]

(1)

In fact, Swish possess various properties desirable for activation functions. In fact, the function is unbounded above, bounded below, non-monotonic, and smooth [7]. In addition, in [16] it was also shown that it provides a good information flow through a DNN. In general, [7] and [16] identified a family of activation functions in the form of \( y(x) = x \cdot \text{sigmoid}(x) \), which improves the information propagation and does not suffer form the vanishing gradient problem. In this way, we introduce the new Exponential Linear Sigmoid Squashing (ELiSH) activation function:

\[
y(x) = \begin{cases} 
  \frac{x}{1 + e^{-x}} & x \geq 0 \\
  \frac{e^x - 1}{1 + e^{-x}} & x < 0.
\end{cases}
\]

(2)

From Eq. (2) it is clear that ELiSH shares the properties of Swish, as its negative part is a multiplication of ELU and Sigmoid, while sharing the same positive part with Swish. Similarly, we introduce HardELiSH as a multiplication of HardSigmoid and ELU in negative part and HardSigmoid and Linear in positive part:

\[
y(x) = \begin{cases} 
  x \times \max(0, \min(1, (x + 1)/2)) & x \geq 0 \\
  (e^x - 1) \times \max(0, \min(1, (x + 1)/2)) & x < 0.
\end{cases}
\]

(3)

Moreover, we would like to take advantage of compositional functions. For example in Swish, Sigmoid improves the information flow and Linear avoids a vanishing gradient, which is also the main motivation to design ELiSH and HardELiSH. Both activation functions and their derivatives are shown in Figure 1.

4 Evolving Piece-wise Activation Functions

The goal of this work is estimate non-linear activation functions better suited for specific tasks. To this end, we build on two ideas. First, as negative and positive inputs have a different influence on learning, we propose to use split activation functions. Second, as the search space can be very large, we propose to build on the ideas of genetic algorithms to allow for a more efficient search.

\[^1\]See Table 1 for the definition of these functions.
4.1 Genetic Algorithms

Genetic Algorithms (GA) (see e.g., [31]) can be seen a population-based meta-heuristic to solve problems in the field of stochastic optimization. In particular, we are given a large set of candidate solutions, referred to as population, but we do not now how to approach the global optimum. In this way, the main idea is to evolve a population to a better solution.

The evolution typically starts from a population consisting of randomly selected candidate solutions. These are called individuals and are described by a set of properties (gens), which can be altered by three bio-inspired operations: (a) selection, (b) crossover, and (c) mutation. Selection is the simple process of selecting individuals according to their fitness. In contrast, crossover is a stochastic operator, exchanging information between two individuals (often called parents: mom and dad) to form a new offspring. Similarly, mutation is also a stochastic operator that helps to increase the diversity of the population by randomly choosing one or more genes in an offspring and changing them.

Then, an iterative process, where an iteration is referred to as generation, the fitness of each individual is evaluated. Based on their fitness, we select a set of parents solutions for breeding. Subsequently, we apply breeding operators on pairs of individuals to generate new pairs of offsprings. Eventually, we update the population with the set of parents and bred offsprings. This process is repeated until a predefined number of generations or an optimality criterion is met.

4.2 Genetic Operators for Activation Functions

In our case, targeting to evolve piecewise activation functions, our populations consists of individuals representing an activation function, where a gene is either the left or the right part of an activation function. This is illustrated in Figure 2.

To evolve activation functions as described above, we need first to introduce new operators, representing our problem, Inheritance and Hybrid. The latter
operator combines the parents’ activation functions by means of mathematical operators. When applying the crossover operator we stochastically choose between Inheritance and Hybrid as shown in Algorithm 3.

**Inheritance** The Inheritance operator is intended to inherit genes from both parents. The first (second) offspring inherits its left activation function from the mom (dad), and its right activation function from the dad (mom). Thus, the operator is defined in a similar way as a one point crossover operator, however, the cutoff point is predetermined (i.e., we are dealing with functions). This is illustrated in Figure 3.

**Hybrid Crossover** The Hybrid crossover operator is proposed to combine multiple activation functions. As for Inheritance crossover, the cutoff point is fixed. Using a randomly selected mathematical operator, the first (second) offspring combines mom’s and dad’s (dad’s and mom’s) negative part of the activation function to form its own negative part. Subsequently, the first (second) offspring’s positive part of the activation function is formed via a combination of mom’s and dad’s (dad’s and mom’s) positive part. This is illustrated in Figure 4.

**Mutation Operator** The mutation operator randomly chooses a gene and then replaces it with a randomly selected predefined activation function. In fact, this operator helps our GA algorithm to keep exploring the search space for new activation functions. This is illustrated in Figure 5.
4.3 Evaluating an Activation Function

The Hybrid crossover operator results in a hybrid activation function that we evaluate by parsing according to the following grammar:

\[
\text{expression} := f \mid \text{operation}, \text{expression}, \text{expression} \\
\text{operation} := + \mid - \mid \times \mid / \mid ^\wedge \mid \min \mid \max \mid f \circ g \\
f := \text{ELiSH} \mid \text{HardELiSH} \mid \text{Swish} \mid \text{ReLU} \mid \text{ELU} \mid \text{SeLU} \ldots 
\]

where \( f \) represents the set of candidate solutions. The list is not fixed, and we can easily add additional operations and candidate solutions \( f \).

Example: Given an activation function generated by Hybrid crossover:

\[
(\max : (+ : (\min : \text{ELU} : \text{ReLU}) : \text{Swish}) : (\times : \text{ELU} : \text{Linear})).
\]

Using Eq. 4, we parse above activation function as shown in Figure 6 to compute the equivalent infix expression:

Fig. 6: The parse tree of \( \max((\min(\text{ELU,ReLU}) + \text{Swish}), (\text{ELU} \times \text{Linear})) \).

4.4 Learning Activation Functions

Having defined the newly defined genetic operators and having explained the evaluation, we can now introduce the overall evolutionary approach, which is summarized in Algorithm 1. Initially, we generate a population of random activation functions (Line 3). Next, using the evaluate operator (Line 4), the fitness of each individual is determined by train and test performance of a DNN. Indeed, the DNN uses an individual as its activation function. Then, we select a set of parent activation functions based on their fitness for breeding (Line 7). To generate new activation functions (Line 11), we apply a new crossover operator as defined in Section 4.2 and afterwards the mutation operator. Similarly, we update our population with the set of parents and bred offsprings and continue to the next generation. This procedure is iterated until a pre-pre-defined optimality criterion is met.
Algorithm 1 Genetic Algorithm

1: procedure GA(population-size)
2: .population ← ∅
3: .population ← Initialize(population-size)
4: Evaluate(population)
5: repeat
6:  children ← ∅
7:  parents ← SELECT(population, 15%)
8:  for i ≤ (population-size − |parents|)/2 do
9:    increment i by one
10:   (offsprings ← CROSSOVER(mom, dad)
11:   for offspring ∈ offsprings do
12:     offspring ← MUTATE(offspring)
13:   Evaluate(offsprings)
14:  children ← children ∪ offsprings
15:  population ← parents ∪ children
16: until termination condition
18: return population

Algorithm 2 Selection Operator

1: procedure SELECT(population, n)
2:  parents ← top n% of population
3:  for individual ∈ population − parents do
4:    c ← toss a coin
5:    if c is heads then
6:      parents ← parents ∪ MUTATE(individual)

Algorithm 3 Crossover

1: procedure CROSSOVER(mom, dad)
2:  c ← toss a coin
3:  if c is heads then
4:    return Inheritance(mom, dad)
5:    return Hybrid(mom, dad)

5 Experimental Results

The purpose of our experiments is threefold. First, we would like to show that for different tasks different choices of activation functions are meaningful. Second, we demonstrate the generality of the evolved activation functions by applying them using a different architecture. Third, we show that the best performing activation functions are similar, representing a specific characteristics of the data. In particular, we run experiments on three different object classifications benchmarks (i.e., CIFAR-10\(^2\), CIFAR-100\(^2\), and Tiny ImageNet\(^3\)), differing in number of classes, number of samples, and complexity, and by using two different DNN architectures (i.e., preactivation-ResNet \(^32\) and VGG \(^33\)).

5.1 Experimental Setup and Implementation Details

Similar to \(^3\), we run our Genetic Algorithm based learning strategy on more shallow architectures, i.e., ResNet38 for CIFAR-10 and ResNet20 for CIFAR-100 and Tiny ImageNet. The thus explored activation functions are then used for training deeper networks, i.e., Resnet56 \(^32\). In addition, to demonstrate that the obtained activation functions are of more general interest, the selected functions are additionally applied for training classifiers based on VGG-16 \(^33\). To this end, we used the default parameters for both architectures. However, to avoid random effects, all networks have been initialized using the same initialization \(^34\); moreover, to keep the computational effort feasible\(^3\), the batch size was set to 32.

\(^2\)https://www.cs.toronto.edu/~kriz/cifar.html
\(^3\)https://tiny-imagenet.herokuapp.com/
\(^4\)The experiments were carried out on a standard PC (Core-i7, 64GB RAM) with two Titan-X GPUs attached.
Our implementation for evolutionary learning builds on DeepEvolve\textsuperscript{5}, a neuroevolution framework developed to explore the optimal DNN topology for a given task. In our case, we fixed the DNN topology and defined the search space based on the activation functions. Throughout all experiments, we used a population size of 40 and evolved the population over 8 generations. The considered candidates for the initial population are shown in Table 1 and Figure 7.

### Table 1: Candidate piece-wise activation functions. Please note, by $y(x < 0)$ and $y(x \geq 0)$ we indicate the (left and right)-piece, respectively.

| Activation Function | Expression |
|---------------------|------------|
| 1. HardELiSH        | $y(x < 0) = \max(0, \min(1, (x + 1)/2)) \times (e^x - 1)$  
$y(x \geq 0) = x \times \max(0, \min(1, (x + 1)/2))$ |
| 2. ELiSH            | $y(x < 0) = (e^x - 1)/(1 + e^{-x})$ and $y(x \geq 0) = x/(1 + e^{-x})$ |
| 3. Swish            | $y(x) = x/(1 + e^{-x})$ |
| 4. ReLU             | $y(x) = \max(x, 0)$ |
| 5. ELU              | $y(x < 0) = e^x - 1$ and $y(x \geq 0) = x$ |
| 6. SeLU             | $y(x < 0) = \lambda \alpha(e^x - 1)$ and $y(x \geq 0) = \lambda x$ |
| 7. Softplus         | $y(x) = \ln(1 + e^x)$ |
| 8. HardSigmoid      | $y(x) = \max(0, \min(1, (x + 1)/2))$ |
| 9. Sigmoid          | $y(x) = 1/(1 + e^{-x})$ |
| 10. Sin             | $y(x) = \sin(x)$ |
| 11. Linear          | $y(x) = x$ |

![Fig. 7: Piecewise activation functions as defined in Table 1](https://github.com/jliphard/DeepEvolve)
5.2 Quantitative Results

First, we evolved a set of candidate activation functions using our GA-based approach using ResNet-38 and used the evolved activation functions using ResNet-56 on CIFAR-10. The thus obtained results in terms of classification accuracy for the best performing solutions are shown in Table 2. In addition, we give a comparison to three different baselines, namely ReLU, ELU, and SeLU, which have proven to work well for a wide range of applications. It can be seen from Table 2 that the best results can be obtained using HardELiSH (93.13\%) and the activation function consisting of a combination of a multiplication of HardELiSH and Swish in the positive part and Swish in the negative part (93.02\%). In general, it can be recognized that the top 6 evolved activation functions are outperforming the baselines.

| Accuracy | Activation Function |
|----------|---------------------|
| 1. 93.13\% | \( y(x < 0) = \text{HardELiSH} \) |
| 2. 93.02\% | \( y(x < 0) = \text{HardELiSH} \circ \text{Swish} \) and \( y(x \geq 0) = \text{Swish} \) |
| 3. 92.89\% | \( y(x < 0) = \text{Sin} \) and \( y(x \geq 0) = \text{Swish} + \text{Swish} \) |
| 4. 92.83\% | \( y(x) = \text{Swish} \) |
| 5. 92.83\% | \( y(x) = \text{ELiSH} \) |
| 6. 92.26\% | \( y(x < 0) = \text{Swish} \times \text{ELiSH} \times \text{Sin} \) and \( y(x \geq 0) = \text{ReLU} \) |
| 7. 92.43\% | \( y(x) = \text{ReLU} \) |
| 8. 91.45\% | \( y(x) = \text{ELU} \) |
| 9. 91.43\% | \( y(x) = \text{SeLU} \) |

Table 3: Performance of top six explored functions for Cifar10-VGG16.

| Accuracy | Activation Function |
|----------|---------------------|
| 1. 93.23\% | \( y(x) = \text{ELiSH} \) |
| 2. 92.89\% | \( y(x) = \text{HardELiSH} \) |
| 3. 92.76\% | \( y(x < 0) = \text{Swish} \times \text{ELiSH} \times \text{Sin} \) and \( y(x \geq 0) = \text{ReLU} \) |
| 4. 91.35\% | \( y(x < 0) = \text{Sin} \) and \( y(x \geq 0) = \text{Swish} + \text{Swish} \) |
| 5. 91.39\% | \( y(x < 0) = \text{HardELiSH} \circ \text{Swish} \) and \( y(x \geq 0) = \text{Swish} \) |
| 6. 90.86\% | \( y(x) = \text{Swish} \) |
| 7. 93.00\% | \( y(x) = \text{ReLU} \) |
| 8. 92.88\% | \( y(x) = \text{SeLU} \) |
| 9. 92.60\% | \( y(x) = \text{ELU} \) |

Additionally, we run the same experiment using the VGG-16 framework and show the results in Table 3. Even though the AFs have not been trained for this architecture, we get competitive results: Similarly, we get the best results using
ELiSH (71.25%), once again followed by HardELiSH. However, as the AFs have not been evolved for the VGG architecture, the gap compared to the baselines is smaller or even vanishing. For better understanding, we also illustrate the top 6 activation functions for CIFAR-10 in Figure 8.

Next, we run the same experiments on CIFAR-100, however, to reduce the computational effort, building on a ResNet-20 during evolution. The corresponding results for ResNet-56 and VGG-16 are given in Table 4 and Table 5. It can be seen in Table 4 that ELiSH (74.65%) and the compositional function of max(Sin, HardELiSH) and Sin in the negative part and Swish in the positive part (74.31%) show the best performances for ResNet56. For VGG, as demonstrated in Table 5, the activation function consisting of HardELiSH in the negative part and (SeLU + Linear) in the positive part (71.25%) and Swish (71.23%) yield the best results. These results show that for negative inputs HardELiSH, Sin and the combinations of them come up during evolution. In Figure 9, also for CIFAR-100 we show an illustration of the top 6 evolved activation functions.

Table 4: Performance of top six explored functions for CIFAR100-ResNet56.

| Accuracy | Activation Function                                                                 |
|----------|-------------------------------------------------------------------------------------|
| 1. 74.65% | $y(x) = \text{ELiSH}$                                                             |
| 2. 74.31% | $y(x < 0) = \max(\text{Sin, HardELiSH}) \circ \text{Sin}$ and $y(x \geq 0) = \text{Swish}$ |
| 3. 74.09% | $y(x < 0) = \text{Sin}$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear}) \circ \text{ReLU}$ |
| 4. 74.05% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear})$ |
| 5. 73.98% | $y(x) = \text{Swish}$                                                              |
| 6. 73.61% | $y(x < 0) = \max(\text{Sin, HardELiSH})$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear})$ |
| 7. 73.31% | $y(x) = \text{ReLU}$                                                               |
| 8. 72.58% | $y(x) = \text{ELU}$                                                                |
| 9. 71.57% | $y(x) = \text{SeLU}$                                                               |

Table 5: Performance of top six explored functions for CIFAR100-VGG16.

| Accuracy | Activation Function                                                                 |
|----------|-------------------------------------------------------------------------------------|
| 1. 71.25% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear})$ |
| 2. 71.23% | $y(x) = \text{Swish}$                                                              |
| 3. 70.80% | $y(x < 0) = \max(\text{Sin, HardELiSH})$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear})$ |
| 4. 70.77% | $y(x < 0) = \max(\text{Sin, HardELiSH}) \circ \text{Sin}$ and $y(x \geq 0) = \text{Swish}$ |
| 5. 70.74% | $y(x < 0) = \text{Sin}$ and $y(x \geq 0) = \max(\text{SeLU, SeLU + Linear}) \circ \text{ReLU}$ |
| 6. 70.70% | $y(x) = \text{ELiSH}$                                                              |
| 7. 71.12% | $y(x) = \text{ELU}$                                                                |
| 8. 70.74% | $y(x) = \text{ReLU}$                                                               |
| 9. 70.59% | $y(x) = \text{SeLU}$                                                               |
Finally, we run the same experiments on Tiny ImageNet using the same setup as used for CIFAR-100. The thus obtained results for ResNet-56 and VGG-16 are given in Table 6 and Table 7. It can be seen from Table 6 that the activation function with the combination HardELiSH in the negative part and min(ELU, Swish) in the positive part (57.53%) and ELiSH (57.34%) demonstrate the best performances. As can be seen from Table 7, ELiSH also provides good results for VGG16 (52.30%). It seems that that Swish and Linear (and a combination of them) in the positive part are the best fitting activation functions for this dataset. In addition, we show an illustration of the top 6 explored activation functions in Figure 10.

Table 6: Performance of top six explored functions for ImageNet-Resnet56.

| Accuracy | Activation Function |
|----------|---------------------|
| 1. 57.53% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \min(\text{ELU, Swish})$ |
| 2. 57.34% | $y(x) = \text{ELiSH}$ |
| 3. 57.07% | $y(x) = \text{Swish}$ |
| 4. 56.68% | $y(x < 0) = \min(\text{ELiSH, ReLU})$ and $y(x \geq 0) = (\text{ReLU} + \text{ELiSH})$ |
| 5. 56.62% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \text{ELU}$ |
| 6. 56.32% | $y(x < 0) = \text{Swish} + \min(\text{ELiSH, ReLU})$ and $y(x \geq 0) = (\text{ReLU} + \text{ELiSH})$ |
| 7. 57.27% | $y(x) = \text{ReLU}$ |
| 8. 55.32% | $y(x) = \text{ELU}$ |
| 9. 50.09% | $y(x) = \text{SeLU}$ |

Table 7: Performance of top six explored functions for ImageNet-VGG.

| Accuracy | Activation Function |
|----------|---------------------|
| 1. 52.30% | $y(x) = \text{ELiSH}$ |
| 2. 52.19% | $y(x) = \text{Swish}$ |
| 3. 51.77% | $y(x < 0) = \min(\text{ELiSH, ReLU})$ and $y(x \geq 0) = (\text{ReLU} + \text{ELiSH})$ |
| 4. 51.48% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \min(\text{ELU, Swish})$ |
| 5. 51.41% | $y(x < 0) = \text{Swish} + \min(\text{ELiSH, ReLU})$ and $y(x \geq 0) = (\text{ReLU} + \text{ELiSH})$ |
| 6. 51.34% | $y(x < 0) = \text{HardELiSH}$ and $y(x \geq 0) = \text{ELU}$ |
| 7. 51.70% | $y(x) = \text{ReLU}$ |
| 8. 51.10% | $y(x) = \text{ELU}$ |
| 9. 49.54% | $y(x) = \text{SeLU}$ |

5.3 Discussion

The results presented above clearly show that for three different problems, totally different activation functions are estimated. In fact, from the obtained results
we can recognize two different kinds of activation functions considering their positive part only: (a) non expansion (contraction) and (b) expansion mapping, which can be considered as a function approximator [35]. In general, an expansion mapping expands changes in input values, while a contraction mapping is less sensitive to changes in input values.

For CIFAR-10, with exception of activation function #3 (Table 2) all evolved activation functions for positive inputs are contraction mappings. This can be explained by the fact that some classes build multi-modal clusters, resulting in large intra-class distances, which might cause misclassifications. A contraction mapping like Swish, however, helps to reduce this effect, improving the classification accuracy.

For CIFAR-100, in contrast, we observe more expanding mappings. If the instances of a class are close to each other, this allows for better exploiting discriminative power. The expanding character of activation functions can especially be recognized from repetitive piecewise functions in the negative part, as can be seen from Figure 8.

For Tiny ImageNet, a more complex benchmark, expansion and non expansion mappings can be recognized. This can, again, be seen from the negative part of the activation functions. When the rate decay on negative side is exponential like in activation function #4 (Table 6) the tendency is toward expanding, otherwise it is identity.

![Activation Functions](image)

Fig. 8: Top six explored functions for CIFAR-10.
y(x ≥ 0) = max(Linear, Sin) y(x ≥ 0) = max(SeLU, SeLU + Linear) y(x < 0) = ELiSH y(x < 0) = Sin

y(x ≥ 0) = max(SeLU, Swish) y(x ≥ 0) = Swish y(x < 0) = max(Sin, HardELiSH) y(x < 0) = max(Sin, HardELiSH) ◦ Sin

Fig. 9: Top six explored functions for CIFAR-100.

y(x ≥ 0) = min(ELU, Swish) y(x ≥ 0) = ReLU + ELiSH y(x ≥ 0) = ELU y(x < 0) = HardELiSH y(x < 0) = min(ELiSH, ReLU) y(x < 0) = HardELiSH

y(x) = ELiSH y(x ≥ 0) = ReLU + ELiSH y(x < 0) = Swish + min(ELiSH, ReLU) y(x) = Swish

Fig. 10: Top six explored functions for TinyImageNet.

6 Conclusion and Future Work

Even though deep learning approaches allow end-to-end learning for a variety of applications, there are still many parameters which need do be manually set. An important parameter, which is often ignored, is the choice of activation functions. Thus, we tackled this problem and studied the importance of activation functions when learning DNN models for classification. In particular, our contribution is threefold. First, we introduced two new activation functions based on theoretical considerations. Second, we introduced a genetic algorithm based evolving procedure to learn the best activation function for a given task. Third, we gave a detailed evaluation and a discussion on our findings. Future work will include a more thorough experimental evaluation and an analysis of the effect of activation functions on the computational complexity during DNN learning.
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