Measurements of Intensity Distributions in the Approach to Localization

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We report measurements of intensity distributions of transmitted microwave radiation in quasi-1D samples with lengths $L$ as large as the localization length $\xi$. In contrast to negative exponential statistics found in the diffusive limit, the distribution falls as a stretched exponential of power $1/2$ and becomes nearly log-normal as $L$ approaches $\xi$. We confirm the relationship between the moments and full distributions of the intensity and total transmission obtained by Kogan and Kaveh using random matrix theory. Good agreement is found when this relationship is used to compare measurements of intensity distribution in strongly absorbing samples with lengths as large as $\xi$ with calculations by Brouwer of the transmission distribution. The variances of the measured distributions increase superlinearly near the localization threshold confirming the coexistence of localization and absorption and the suitability of these variances as measures of the closeness to the localization threshold.

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As the localization threshold is approached, fluctuations in key transmittance quantities become as large as the ensemble average values of these quantities. Thus a comprehensive description of mesoscopic transport should provide the full distribution of transmittance quantities and the relationships between them. In order of increasing spatial averaging, key transmittance quantities are the intensity, corresponding to the transmission coefficient $T_{ab}$ for incident mode $a$ and outgoing mode $b$, the total transmission, $T_a = \sum_b T_{ab}$, and the transmittance, $T = \sum_{ab} T_{ab}$. In the absence of absorption, the transmittance equals the dimensionless conductance $g = N\ell/L$, where $N$ is the number of transverse modes and $\ell$ is the transport mean free path. Localization in quasi-1D samples is achieved when $g = 1$ at $L = \xi = N\ell$. In this Letter, we focus on the intensity distribution, which is the key distribution in statistical optics. We demonstrate its relationship to the distribution of total transmission, find the scaling of the variance of the intensity and total transmission up to $L = \xi$, and determine the extent to which absorption influences localization.

In the diffusive limit, the degree of long-range intensity correlation is small and the intensity distribution is well approximated by the Rayleigh distribution. For polarized detection, this corresponds to negative exponential statistics, $P(s_{ab}) = \exp(-s_{ab})$, where $s_{ab} = T_{ab}/<T_{ab}>$ is the intensity normalized to its ensemble average value. In previous work, deviations from negative exponential behavior have been observed and ascribed to long-range intensity correlation. In these studies, fluctuations as large as $s_{ab} \sim 10 \sim g$ were observed. In the present work, fluctuations as large as fifty times $g$ are observed in samples with lengths $L \sim \xi$.

The intensity distribution is studied in a quasi-1D geometry, which is equivalent to the electronic case of a thin wire. Thouless argued that, the level width $\Delta \nu$ in a wire at $T = 0$ should become smaller than the level spacing $\Delta \nu$ since $\Delta \nu$ is proportional to the inverse of the travel time and so falls as $1/L^2$ in the diffusive limit, whereas $\Delta \nu$ is the inverse of the density of states and so falls as $1/L$. As a result, the modes in adjacent sections of the wire should not overlap, and electrons should become localized.

The question arises as to whether radiation can be localized in the presence of absorption. In this case, the level width falls as $1/L$, just as the level spacing does. In previous measurements in absorbing samples, the variance of the normalized total transmission, $<s_{ab}>=T_a/<T_a>$, var($s_a$), was found to scale sublinearly with $L$. If the attenuation length due to absorption, $L_a$, serves as a cutoff length for localization, then var($s_a$) would approach an asymptotic limit as $L$ increases. On the other hand, if localization can be achieved in absorbing samples, then var($s_a$), which is essentially the degree of correlation in the intensity of different outgoing modes, should increase superlinearly as $L$ approaches $\xi$. This might occur, since the wave remains temporally coherent in the presence of absorption. Weaver has shown in a 2-D simulation that the introduction of absorption does not disrupt the spatial localization of acoustic waves in closed systems, though the overall energy decreases exponentially with time. In recent calculations, Brouwer found that for diffusive waves the prefactor multiplying $L/\xi$ in the expression for var($s_a$) drops from $\frac{2}{3}$ to $\frac{1}{2}$ as the ratio $L/L_a$ increases. The behavior of this quantity, however, was only considered for lengths considerably less than the localization length.
Here we report measurements of intensity transmitted through random waveguides with $L \leq \xi$, but $> L_a$. We expect that modes in this sample are completely mixed and the degree of intensity correlation between different modes is constant. Wave propagation in this sample should therefore be described by random matrix theory (RMT) [13]. Recently, Kogan and Kaveh used RMT to obtain a relationship between the moments of intensity and total transmission in nonabsorbing quasi-1D samples. [14] They find,

$$\langle s_{ab}^n \rangle = n! \langle s_a^n \rangle,$$

(1)

This leads to a relationship between the distributions of intensity and total transmission, [14,17]

$$P(s_{ab}) = \int_0^{\infty} \frac{ds_a}{s_a} P(s_a) \exp \left( -\frac{s_{ab}}{s_a} \right).$$

(2)

Since the distribution of total transmission can be calculated from the distribution of the eigenvalues of the transmission matrix, [16,18,19] these relations provide a basis for calculating the intensity moments and distributions from RMT.

The intensity distribution is obtained from measurements of the field transmitted through an ensemble of random configurations of 1.27 cm-diam. polystyrene spheres inside a copper tube. Tubes with diameters $d = 5.0$ and 7.5 cm and lengths up to 520 cm are used. The samples have filling fractions of 0.52 and 0.55 for $d = 5.0$ and 7.5 cm, respectively. The sample tube is rotated between successive measurements to produce new configurations of scatterers. At least two thousand sample configurations were used for each distribution. Field spectra were taken from 16.8 to 17.8 GHz in steps of 1 MHz using a Hewlett-Packard 8722C network analyzer. The radiation is coupled into and out of the sample by 0.4 cm wire antennas placed 0.5 cm from the ends of the sample. In order to ensure that the distributions were not distorted by noise, it was necessary to use an amplifier with an output power of 40 W for samples with lengths greater than 200 cm so that the average intensity was at least three hundred times the noise. In the frequency range of the measurements, $\ell \sim 5$ cm [20]. A fit of measurements of the field autocorrelation function with frequency shift to theory [21] gives $L_a = 34 \pm 2$ cm and $D = (3.03 \pm 0.21) \times 10^{10}$ cm$^2$/s, where $D$ is the diffusion coefficient. The localization length for the samples with $d = 5$ cm is found below to be $\xi = 551 \pm 18$ cm.

In Fig. 1, we present the intensity distributions for two samples with $L/\xi \sim 0.1$ and 0.4. Calculations for diffusive waves in the absence of absorption have predicted that for $s_{ab} \gg g$, the intensity distribution falls as $\exp(-2\sqrt{gs_{ab}})$ [13]. For the samples measured, we find $P(s_{ab}) \sim \exp(-2\sqrt{gs_{ab}})$ in the tail of the intensity distribution. The values of $\gamma$ obtained from a fit to the tail of the measured distributions is within 20% of the parameter $g' = \frac{2}{\text{var}(s_a)}$ [15] which equals $g$ in the absence of absorption. The fit of a stretched exponential to the distribution for $L/\xi = 0.4$ in the range of $s_{ab}$ from 10 to 18 is shown in Fig. 2 and gives $\gamma = 2.9$, which is close to value of $g'$ of 3.06 for this sample.

The measured intensity distributions are compared to the transform of the measured transmission distributions [15] for the corresponding samples using Eq. (3). The transforms shown as solid lines in Fig. 1, are in good agreement with the measured intensity distributions. A comparison of the moments of intensity and transmission is shown in Fig. 3. We find an increasing deviation of the ratios $\langle s_{ab}^n \rangle / \langle s_a^n \rangle$ from the value of unity expected from Eq. (1) as $n$ increases. We find, however, that agreement with Eq. (1) is dramatically improved when the moments are calculated using the asymptotic expressions in the diffusive limit for the intensity and transmission distributions beginning from the point at which the measured distribution have their first zero. The asymptotic expressions for the intensity distribution $\exp(-2\sqrt{g s_{ab}})$ is substituted for the measured distribution for values of $s_{ab}$ between 20 and 150, whereas the asymptotic exponential expression $\exp(-g s_a)$ [16,18,19] is substituted for the measured transmission distribution for values of $s_a$ between 5 and 25. The improved agreement indicates that the extent to which the measured ratio of moments is in accord with Eq. (1) is largely limited by the range of intensity and transmission values measured, which depends on the number of configurations on which measurements were made.

Applying Eq. (3) to the second moments gives

$$\text{var}(s_{ab}) = 2\text{var}(s_a) + 1.$$ 

(3)

The same expression can be obtained from a perturbation calculation up to order $1/g$ [18,22]. Our measurements confirm the prediction of RMT that Eq. (3) is independent of the value of $g$ and is correct up to order $1/N$.
Intensity statistics at the localization threshold are studied in measurements at $L = 520$ cm and $d = 5$ cm and shown in Fig. 4a. Values of $s_{ab}$ as large as 50 are observed. The distribution is seen in Fig. 4b to be nearly log-normal, in agreement with predictions for localized radiation [14].

Using Eq. (2), we compare these measurements of the intensity distribution to random matrix calculations of the transmission distribution in the presence of absorption [14]. In order to compare the intensity distribution to theory, however, $\xi$ must be determined. Far from the localization threshold, in the absence of absorption and internal reflection, $\text{var}(s_a)$ and $\xi$ are related by [14,15,32]

$$\text{var}(s_a) = \frac{2L}{3\xi}.$$  \hspace{1cm} (4)

To find $\xi$ in samples in which corrections due to absorption, localization and internal reflection cannot be ignored, we first obtain the intensity distribution for the equivalent samples without absorption using the measured spectra in our absorbing samples. The procedure used is based on the work by Yossef [3] who proposed that for $\omega \tau_a \gg 1$ and when the dispersion is neglected the field in absorbing media differs from that in the absence of absorption only by a factor of $\exp(t/2\tau_a)$, where $\tau_a = L_a^2/Ds$ is the exponential attenuation time due to absorption. We first obtain the time response $E(t)$ to a narrow gaussian pulse in time by Fourier transforming the product of the measured spectrum and a broad gaussian in the frequency domain. The time dependent field is then multiplied by $\exp(t/2\tau_a)$ and the so modified time spectra are transformed back into the frequency domain. The intensity distribution is then calculated using these field spectra. This procedure could be applied, however, only for $L \leq 150$. For longer samples the signal in time is buried in the noise level for large $t$ and thus a significant part of the field in the absence of absorption cannot be recovered. The intensity distributions corrected for absorption are in good agreement with transforms of the diffusive result for the distributions of total transmission calculated in Refs. [16,18] and give values for the parameter $g'$ [1] equal to the value of $g$, as expected in the absence of absorption. We also find that the average transmission obtained from the spectra corrected for absorption is consistent with the expected scaling as $(L + 2z_b)^{-1}$, where $z_b$ is the diffusion extrapolation length due to internal reflection [3]. These results confirm the ability of this approach to statistically eliminate absorption in the diffusive limit. The influence of internal reflection upon $\text{var}(s_a)$ in the absence of absorption can be accounted for by substituting $\bar{L} = L + 2z_b$ for $L$ in Eq. (6). We next account for the leading order correction to $\text{var}(s_a)$ due to nonlocal correlation. In the absence of absorption, the variance is increased by an additional factor of $(1 + 2L/\xi)$ [1] to yield,

$$\text{var}(s_a) = \frac{2\bar{L}}{3\xi} + \frac{4\bar{L}^2}{15\xi^2}.$$ \hspace{1cm} (5)

A fit of Eq. (3) to the data corrected for absorption using $\xi$ and $z_b$ as fitting parameters gives $\xi = 551 \pm 18$ cm and $z_b = 5.25 \pm 0.31$ cm. The value of $z_b$ obtained is consistent with the value of this parameter for the same samples in the frequency range between 18 and 19 GHz. [20]

The dependence of $\text{var}(s_a)$ upon $L$ with and without absorption is shown in Fig. 5. The solid curve represents the result of the calculations in the diffusive regime ($L/\xi < 1$) by Brouwer [14] which account for absorption and the thin dashed curve shows the fit of Eq. (5) to the data corrected for absorption. The values of $\text{var}(s_a)$ are calculated from $\text{var}(s_{ab})$ using Eq. (4). For lengths up to 200 cm, the result from the measurements, show sublinear behavior which is consistent with the results from total transmission measurements [14] and the calculations in Ref. [14]. The deviation from the solid line increases for larger lengths and may reflect localization corrections that were not included in the theory. For strongly absorbing samples ($L \gg L_a$), Brouwer finds a log-normal distribution for the total transmission with $<\text{ln}T_a> = -L/L_a - 3L/4\xi - \ln N$ and $\text{var}(\text{ln}T_a) = L/2\xi$. [14] Using these relations, we calculate the values of $\text{var}(s_a)$ in the regime of strong absorption. The result is shown by a thick dashed line. We note that this curve has a physical meaning for $L \gg L_a$ only. In this regime, it shows significant corrections due to localization in qualitative agreement with the results from the measurements. Thus, we can associate the increase of $\xi\text{var}(s_a)/\bar{L}$ for these samples with the transition to localization. We now compute $P(s_{ab})$ for the sample with $L = 520$ cm using the transmission distribution calculated in Ref. [14] and the values of $\xi$ and $L_a$ found here. The calculated intensity distribution is presented as the solid line in Fig. 4a and is in good agreement with the measurements.

In conclusion, we find that the intensity distribution for $s_{ab} \gg g$ is described by a stretched exponential to power 1/2 and that the distribution for $L \sim \xi$ is close to log-normal. We confirm experimentally
the relationships obtained by Kogan and Kaveh between the moments and full distributions of intensity and total transmission. These relations unify the statistical description of local and spatially averaged transmittance quantities. Our measurements demonstrate that the statistics of wave transport is only marginally affected by absorption and that absorption does not substantially inhibit localization. The ability to reach the localization threshold using a quasi-one-dimensional sample is an extension to classical waves of the suggestion by Thouless that electrons will always be localized in sufficiently long wires at low temperatures. These results show that the variances of the intensity or transmission are reliable measures of the impact of localization upon transport in random media.

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FIGURES:

Fig. 1. Probability distribution of intensity for samples with $L/\xi \sim 0.1$ and 0.4; the samples dimensions are (a) $d = 7.5$ cm, $L = 100$ cm, and (b) $d = 5.0$ cm, $L = 200$ cm, respectively. The solid lines represent distributions obtained from measured transmission distributions [9] using Eq. (2).

Fig. 2. Fit of a stretched exponential of power 1/2 to the tail of the intensity distribution ($L/\xi \sim 0.4$).

Fig. 3. Comparison between the moments of intensity and total transmission ($L/\xi \sim 0.4$): • moments obtained from the measurements, ○ moments calculated from the extended distributions.

Fig. 4. Intensity distribution for the sample with $L = 520$ cm and $d = 5.0$ cm. The solid line in part (a) shows the distribution obtained from a transform of the total transmission distribution for this sample calculated using the expressions from Ref. [14]. The dashed line in part (b) represents a normal distribution.

Fig. 5. Dependence of $\text{var}(s_a)$ upon $L$. The different symbols represent: • results obtained from the measurements, ○ results from the data corrected for absorption. Note that the curves obtained using the theoretical expressions in Ref. [14] represent the ratio $\xi \text{var}(s_a)/L$, not $\xi \text{var}(s_a)/\tilde{L}$.
