Background check for anomalous like-sign dimuon charge asymmetry

Michael Gronau

Physics Department, Technion - Israel Institute of Technology
Haifa 32000, Israel

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago
Chicago, IL 60637, U.S.A.

The D0 Collaboration has reported an excess of roughly one percent of $\mu^-\mu^-$ pairs over $\mu^+\mu^+$ pairs in $\bar{p}p$ collisions at a center-of-mass energy $\sqrt{s} = 1.96$ GeV at the Fermilab Tevatron, when known backgrounds are subtracted. This excess, if ascribed to CP violation in meson-antimeson mixing of non-strange or strange neutral $B$ mesons, is about 40 times that expected in the Standard Model (SM). We propose a null test, based on a tight restriction on the muon impact parameter $b$, to confirm that this excess is indeed due to $B$ mesons. If the asymmetry is due to anomalous CP violation in $B_s^0-\bar{B}_s^0$ mixing then a tight restriction on $b$ would increase by a factor two the net asymmetry from neutral $B$ mixing, while the sample of dimuons from neutral $B$ decays will be reduced significantly relative to background events.

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I Introduction

The present understanding of CP violation, based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, predicts a very small asymmetry in the yield of same-sign muon pairs due to $b\bar{b}$ production followed by oscillation of a neutral non-strange or strange $B$ meson into its antiparticle. Defining

$$A_{sl}^b \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}}, \quad (1)$$

the D0 Collaboration [1,2] has reported an asymmetry of

$$A_{sl}^b = -0.00957 \pm 0.00251 \text{ (stat)} \pm 0.00146 \text{ (syst)}, \quad (2)$$

about a factor of 40 larger than the SM prediction [3] of

$$A_{sl}^b = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}. \quad (3)$$

This result is interpreted as evidence at the 3.2$\sigma$ level for anomalous CP violation in the mixing of neutral $B$ mesons.
The D0 analysis employs a large number of systematic checks to guarantee the stability of their result. These 16 checks lead to values of \( A_{b_{sl}} \) consistent with the nominal result \( (2) \). All these checks used subsets of data involving about equal fractions of background events (\( F_{b_{sl}} \)) from non-genuine \( b\bar{b} \) pair production. To extend such tests, we suggest a measurement, based in part on a check already performed by D0, to determine if a substantially smaller asymmetry is obtained in a sample depleted in \( b\bar{b} \) pairs. The method employs a reduction of the maximum allowed impact parameter of muon tracks which should reduce the signal more than it reduces sources of background. Basically, we are asking if the D0 experiment can infer zero signal from \( b \) decays when no signal is expected.

In Sec. II we review kinematics of muons in \( B \) decays and show the effect of changing impact parameter selection criteria. The relation between impact parameter and \( D_0 \)’s selection criteria is studied in Sec. III. Implications of strict impact parameter cuts are discussed in Sec. IV while Sec. V concludes.

II Production and decay kinematics

In \( pp \) collisions at the Fermilab Tevatron, \( B \) mesons are produced with a distribution in energy \( E \) such that \( \beta \gamma = \mathcal{O}(1) \), where \( \gamma = 1/\sqrt{1 - \beta^2} = E/m_B \) (see, e.g., Figs. 2.2 and 2.4 of Ref. [4]). The pseudorapidity \( \eta = -\log \tan(\theta_P/2) \), where \( \theta_P \) is the angle with respect to the proton beam, is typically less than \( \mathcal{O}(3) \) in magnitude.

The average path length traversed by a decaying \( B \) meson in its rest frame is about \( c\tau_B = 0.45 \text{ mm} \), where we take \( B_0 \) and \( B_s \) lifetimes to be \( \tau_B = 1.5 \text{ ps} \) and neglect differences between the two. Let \( t \) denote the \( B \) proper decay time in any event. Then the normalized distribution in \( t \) is \( w(t) \equiv (1/\tau) \exp(-t/\tau) \), satisfying \( \int_0^\infty w(t)dt = 1 \).

The proper path length \( ct \) becomes \( \ell = \beta \gamma ct \) in the laboratory frame.

Define an axis \( z^* \) in the \( B \) rest frame pointing in the direction of the boost (with velocity \( \beta \)) from the laboratory frame to the \( B \) rest frame. Let a muon in \( B \) semileptonic decay be emitted with an angle \( \theta^* \) with respect to this axis. Since the \( B \) is spinless, the distribution of the muon will be isotropic in \( \cos \theta^* \). We seek the transformation from this angle to the angle \( \theta \) in the laboratory frame between the \( B \) and muon directions. The impact parameter of the muon track with respect to the primary vertex is then \( \ell \sin \theta \), and its distribution is easily calculated for any value of \( \beta \).

Define the \( x^* \) axis in the \( B \) rest frame to be coplanar with the \( B \) boost direction (the \( z^* \) axis) and the muon direction. Then the Lorentz transformation between momenta in the laboratory frame (unstarred) and the \( B \) rest frame (starred) may be written

\[
\begin{align*}
    p_x &= p_x^* ; \\
    p_z &= \gamma(p_z^* + \beta E^*); \\
    E &= \gamma(E^* + \beta p_z^*) .
\end{align*}
\]

Here \( E^* = (p_x^* + p_z^*)/2, E = (p_x^2 + p_z^2)^{1/2} \); we have neglected the muon mass. Then since \( p_x^* = E^* \sin \theta^*, p_z^* = E^* \cos \theta^*, p_x = E \sin \theta, p_z = E \cos \theta \), we have

\[
\sin \theta = \frac{\sin \theta^*}{\gamma(1 + \beta \cos \theta^*)} . \tag{5}
\]

This reduces, as it should to \( \sin \theta = \sin \theta^* \) for \( \beta = 0 \). Note that this relation does not
Figure 1: Transformation between $\cos \theta^*$ and $\sin \theta$ for $\beta \gamma = 0.7 \ [\beta = 0.573]$ (solid), $\beta \gamma = 1 \ [\beta = 0.707]$ (dashed), and $\beta \gamma = 10 \ [\beta = 0.995]$ (dot-dashed). Note that $\theta \to \pi/2$ when $\cos \theta^* = -\beta$.

depend on the muon energy (in the limit that $m_\mu$ may be neglected). We illustrate the transformation for several illustrative values of $\gamma \beta$ in Fig. 1.

Using this transformation and the isotropy of muon emission in the variable $\cos \theta^*$, one may calculate the average value of $\sin \theta$ and hence the average impact parameter as functions of $\beta \gamma$:

$$\langle b \rangle = \gamma \beta \langle \sin \theta \rangle c \tau,$$

where

$$\langle \sin \theta \rangle = \frac{1}{2} \int_{0}^{\pi} \frac{\sin^2 \theta^* \ d\theta^*}{\gamma(1 + \beta \cos \theta^*)} = \frac{\pi}{2} \frac{1}{1 + \gamma}.$$  

The result is shown in Fig. 2.

Specifically, for $\gamma \beta = (0.5, 1, 2)$ one has $\langle b \rangle = (167, 293, 437) \ \mu$m. Fitting by eye the exponential tail of an impact parameter distribution studied by the CDF Collaboration (see Fig. 6 of Ref. [5]) we estimate $\langle b \rangle = 350 \ \mu$m, which lies in this range. The impact parameter for a given $\gamma \beta$ will be distributed as $W(b) = (1/\langle b \rangle) \exp(-b/\langle b \rangle)$, where $\int_{0}^{\infty} W(b) \, db = 1$. If one excludes values $b > b_0$, the fraction excluded for muons from $B$ decays will be $\int_{b_0}^{\infty} W(b) \, db = \exp(-b_0/\langle b \rangle)$. Thus, the remaining fraction of dimuons from the two $B$ meson decays will be $[1 - \exp(-b_0/\langle b \rangle)]^2$. We show in Table II the remaining fraction of dimuon events for various values of $\langle b \rangle$ and $b_0$. Thus, demanding
Figure 2: Average impact parameter $\langle b \rangle$ as a function of $\gamma \beta$ for a $B$ decay with assumed $c\tau = 450 \mu$m.

Table I: Remaining fraction of dimuon events for various values of average impact parameter $\langle b \rangle$ when events with impact parameter exceeding a value of $b_0$ are discarded. Units are $\mu$m.

| $b_0$ (µm) | 100  | 200  | 300  | 400  | 500  |
|-----------|------|------|------|------|------|
| $\langle b \rangle$ (µm) |      |      |      |      |      |
| 150       | 0.237| 0.542| 0.748| 0.866| 0.930|
| 300       | 0.080| 0.237| 0.400| 0.542| 0.658|
| 450       | 0.040| 0.129| 0.237| 0.347| 0.450|

an impact parameter of less than 100 µm should be enough to significantly reduce the dimuon signal of two $B$ decays in Refs. [1,2]. In the next section we discuss the relation of this criterion to those employed in the D0 analysis.

III Relation to criteria employed by D0

The D0 Collaboration quotes a precision for primary vertex reconstruction of 20 µm in the transverse plane and 40 µm along the beam direction. Requiring an impact parameter of less than 100 µm, as proposed above, thus should not reduce the relative fraction of signals from tracks originating from the primary vertex, including kaons, pions, and (anti)protons misidentified as muons [1]. However, the employed criteria distinguish between the transverse impact parameter relative to the closest primary
vertex (which we shall call \( b_\perp \)) and the longitudinal distance from the point of closest approach to this vertex (which we shall call \( b_\parallel \)). The D0 selection criteria for these variables are \[ b_\perp < 3000 \text{ \( \mu \)m} , \quad b_\parallel < 5000 \text{ \( \mu \)m} \] (8)

We seek the relation between these quantities and the impact parameter \( b \).

Laboratory coordinates \((x, y, z)\) are defined such that the detector axis (very close to the beam axis) is \( z \), the plane of the accelerator is \( x \), and the orthogonal (vertical) direction is \( y \) [6]. The vertex position \((x_0, y_0, z_0)\) is close, but not necessarily equal, to \( x = y = z = 0 \). Define another set of coordinates \((x', y', z')\) for each track such that the transverse point of closest approach (PCA) has coordinates \( x' = b_\perp, y' = z' = 0 \), while the vertex is defined to have \( x' = y' = 0, z' = b_\parallel \).

By definition of the PCA, the linearized approximation to the muon trajectory near the PCA lies in the \( y' - z' \) plane, making an angle \( \psi \) with the \( z' \) axis. The transverse and longitudinal components of muon momentum in the laboratory frame, \( p_\perp \) and \( p_z \), may be used to determine \( \psi \):

\[ p_\perp' = p_\mu \sin \psi , \quad p_\parallel' = p_\mu \cos \psi . \] (9)

Let the distance along the muon track from the PCA be denoted by \( s \). The muon track’s coordinates in the primed system are

\[ y' = s \sin \psi , \quad z' = s \cos \psi . \] (10)

The distance \( d \) of a point along the muon trajectory from the vertex satisfies the relation

\[ d^2 = b_\perp^2 + (s \sin \psi)^2 + (s \cos \psi - b_\parallel)^2 . \] (11)

We seek the minimum value of this quantity with respect to \( s \):

\[ 0 = d(d^2)/ds = 2s - 2b_\parallel \cos \psi , \] (12)

implying \( s = b_\parallel \cos \psi \) and hence

\[ d_{\text{min}} = \left[ b_\perp^2 + (b_\parallel \sin \psi)^2 \right]^{1/2} = b . \] (13)

When \( \psi = 0 \), \( b = b_\perp \), while when \( \psi = \pi/2 \), \( b = \sqrt{b_\perp^2 + b_\parallel^2} \), as expected. It is then a simple matter to construct the impact parameter \( b \) using the D0 track-selection criteria.

### IV Implication of impact parameter cuts

Indeed one of the 16 analysis variations mentioned above (“Test D” [1]) changes the values of the maximum transverse and longitudinal impact parameters to values tighter than (8):

\[ b_\perp < 500 \text{ \( \mu \)m} , \quad b_\parallel < 500 \text{ \( \mu \)m} \] (14)

However, since one always has \( b \geq b_\perp \), one can see from Fig. [2] and Table [1] that this is not expected to reduce the signal very much, and it doesn’t [1]. While the tighter
cuts reduce the number of like-sign dimuon events by a factor of two, the fraction of estimated background events is unchanged within errors. (See Table XIV and Eq. (37) in Ref. [1].)

The much tighter restriction \( b \leq 100 \mu m \) should have a noticeable effect, reducing significantly the dimuon signal from two muonic \( B \) decays relative to background events. The remaining fraction of dimuon signal was calculated in Table I to be 8 and 4 percent for \( \langle b \rangle = 300 \) and 450 microns, respectively. As a second-order effect, we note that a tight restriction on \( b \) implies that the charge asymmetry from the remaining neutral \( B \) decays will be dominated by \( B_s \) mesons which involve a much higher oscillation frequency than \( B^0 \). If the anomalous charge asymmetry observed in Ref. [1, 2] originates in \( B_s - \bar{B}_s \) mixing rather than in \( B^0 - \bar{B}^0 \) mixing then the negative asymmetry from the net signal events would increase by about a factor of two.

In order to demonstrate this effect, let us take for instance \( r_b \equiv b_0/\langle b \rangle = 1/3 \). The net asymmetry from neutral \( B \) meson mixing is given by

\[
A^{b \rightarrow sl}_{sl} = \frac{f_d Z_d}{f_d Z_d + f_s Z_s} A^{d \rightarrow sl} + \frac{f_s Z_s}{f_d Z_d + f_s Z_s} A^{s \rightarrow sl},
\]

where \( A^{d \rightarrow sl}_{sl} \) and \( A^{s \rightarrow sl}_{sl} \) are asymmetries in \( B^0 - \bar{B}^0 \) and \( B_s - \bar{B}_s \) mixing, respectively. We will take \( B^0 \) and \( B_s \) production fractions \( f_d = 0.323, f_s = 0.118 \) [7], noting that our result will not depend on precise values of these parameters. Neglecting small width differences \( (\Delta \Gamma_{d,s}/2\Gamma)^2 \ll 1 \), the parameters \( Z_q \) \( (q = d,s) \) are given by

\[
Z_q = \frac{x_q^2}{1 + x_q^2} - e^{-r_b} \left[ 1 + \frac{x_q \sin(x_q r_b) - \cos(x_q r_b)}{1 + x_q^2} \right],
\]

where \( x_q \equiv \Delta m_q/\Gamma \). Ignoring small errors in neutral \( B \) mass differences, \( x_d = 0.774, x_s = 26.2 \) [7], one finds \( Z_d = 0.0029, Z_s = 0.264 \) implying \( A^{b \rightarrow sl}_{sl} = 0.03 A^{d \rightarrow sl} + 0.97 A^{s \rightarrow sl} \). This result should be compared with \( A^{b \rightarrow sl}_{sl} = 0.51 A^{d \rightarrow sl} + 0.49 A^{s \rightarrow sl} \) [1] (also ignoring errors) obtained with loose restrictions on the muon impact parameter (i.e., \( r_b \gg 1 \)). Thus, a tight restriction on the impact parameter would increase by a factor two the net asymmetry from neutral \( B \) mixing if the asymmetry originates in the \( B_s \) system.

A tight restriction on impact parameter eliminates also a large fraction of non-genuine semileptonic \( B \) decays. This includes hadrons in bottom and charm decays, where the hadrons (kaons, pions and protons) are misidentified as muons, and muons from sequential decays of kaons and pions. In order to provide a null test, or alternatively to demonstrate a nonzero asymmetry from sources other than \( B^0 - \bar{B}^0 \) and \( B_s - \bar{B}_s \) mixing, the number of remaining events, originating mainly in promptly produced hadrons faking muons, must be sufficiently large. We note that a contribution to the same-sign dimuon asymmetry from kaons, dominating over those from pions and protons, must be subtracted from the raw asymmetry in order to obtain the combined CP asymmetry in \( B^0 - \bar{B}^0 \) and \( B_s - \bar{B}_s \) mixing. A positive asymmetry of five percent from kaons was measured in Ref. [1], involving kaons produced at the primary vertex and kaons from bottom and charm decays.
V Conclusion

We have proposed a kinematic test to determine if the anomalous like-sign dimuon charge asymmetry recently reported by the D0 Collaboration [1,2] is due to the decays of short-lived particles such as $B$ mesons. Although charmed particles are also short-lived, they are not expected to contribute much to the like-sign signal. The test involves a reduction of the maximum impact parameter $b_0$ to a value well above the vertex resolution but small compared with that characteristic of $B$ decays. Although we have proposed $b_0 = 100 \mu m$, a dedicated simulation would probably be preferable in order to optimize this parameter.

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References

[1] V. M. Abazov et al. (D0 Collaboration), arXiv:1005.2757 [hep-ex], to be published in Phys. Rev. D.

[2] V. M. Abazov et al. (D0 Collaboration), arXiv:1007.0395 [hep-ex], to be published in Phys. Rev. Letters.

[3] A. Lenz and U. Nierste, JHEP 0706, 072 (2007) [arXiv:hep-ph/0612167].

[4] K. Anikeev et al., “$B$ Physics at the Tevatron: Run II and Beyond,” Fermilab-Pub-01/197, [hep-ph/0201071].

[5] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 77, 072004 (2008) [arXiv:0710.1895 [hep-ex]].

[6] We thank G. Borissov for a helpful communication on this point.

[7] Updated results and references are tabulated periodically by the Heavy Flavor Averaging Group: http://www.slac.stanford.edu/xorg/hfag/rare. See, e.g., E. Barberio et al. arXiv: 0704.3575v1, http://www.slac.stanford.edu/xorg/hfag/results/