What happens to the quantum Hall effect when magnetic-field-induced spin-density wave moves

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The influence of the motion of a magnetic-field-induced spin-density wave (FISDW) on the quantum Hall effect in a quasi-one-dimensional conductor is studied theoretically. In the ideal case of a free FISDW, it is found that the counterflow of the FISDW precisely cancels the quantum Hall current, so the resultant Hall conductivity is zero. In real systems, the Hall conductivity should vanish at the high frequencies, where the pinning and the damping can be neglected, and the dynamics of the FISDW is dominated by inertia.

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It is known experimentally and understood theoretically that the magnetic-field-induced spin-density-wave (FISDW) state, observed in the (TMTSF)$_2$X organic quasi-one-dimensional conductors, exhibits the quantum Hall effect. In the theoretical explanation of this effect, it is assumed that the FISDW is pinned and acts on electrons as a static potential. The Hall conductivity, calculated in the presence of this potential at zero temperature, is quantized.

On the other hand, under certain conditions, the density wave in a quasi-one-dimensional conductor can move. It is interesting to study how this motion would influence the quantum Hall effect. Since the density-wave condensate can move only along the chains, at first sight, this purely one-dimensional motion cannot contribute to the Hall effect which is essentially a two-dimensional effect. Nevertheless, it is shown below that in the case of the FISDW, unlike in the case of a regular charge- or spin-density wave (CDW/SDW), a nonstationary motion of the condensate does produce a non-trivial contribution to the Hall conductivity. This effect is found at zero temperature in the absence of normal carriers and has the same origin as the quantum Hall effect. In the ideal system where the FISDW is not pinned or damped, the contribution due to the FISDW motion precisely cancels the bare quantum Hall term, so that the resultant Hall conductivity is zero. In real systems, this effect should manifest itself at high enough frequencies where the dynamics of the FISDW is dominated by inertia, and the pinning and the damping can be neglected. On the other hand, the effect cannot be observed in the DC measurements where the FISDW can be depinned by strong electric field. In this paper, we present only a heuristic, semiphenomenological outline, whereas a systematic derivation will be given elsewhere. Some of these results were briefly reported also in other conference proceedings.

Let us consider a two-dimensional system where electrons are confined to the chains parallel to the $x$-axis, and the spacing between the chains along the $y$-axis is equal to $b$. Magnetic field $H$ is applied along the $z$-axis perpendicular to the $(x,y)$-plane. The system is in the FISDW state at zero temperature. Let us consider first the case where the electric field $E_y$ is applied perpendicular to the chains. The electron Hamiltonian can be written as

$$
\mathcal{H} = \frac{\hbar^2 k_x^2}{2m} + \Delta_0 \cos(Q_xx + \Theta) + 2t_b \cos(k_yb - qx + \omega_yt).
$$

(1)

Here $\hbar = h/2\pi$ is the Planck constant, $m$ is the electron mass, $k_x$ and $k_y$ are the electron wave vectors along the chains, and $t_b$ is the amplitude of tunneling between the chains. In the gauge $A_y = Hx - cE_yt$ and $A_x = A_z = 0$, the magnetic and the transverse electric fields appear in the Hamiltonian through the Peierls–Onsager substitution $k_y \rightarrow k_y - eA_y/\hbar c$, so $q_x = ebH/\hbar c$ and $\omega_y = ebE_y/\hbar$ ($e$ is the electron charge and $c$ is the velocity of light). The FISDW potential is represented by the second term in Eq. (1) with the amplitude $\Delta_0$ and the phase $\Theta$. It is well known that the longitudinal wave vector of the FISDW is equal to $Q_x = 2k_F - Nq_x$, where $k_F$ is the Fermi wave vector and $N$ is an integer. For simplicity, we set the transverse wave vector of the FISDW to zero.

We see that, in the presence of the magnetic field $H$, the hopping term in Eq. (1) acts as a potential, periodic along the chains with the wave vector $q_x$ proportional to $H$. In the presence of the transverse electric field $E_y$, this potential moves along the chains with the velocity $\omega_y/q_x = cE_y/H$ proportional to $E_y$. This velocity is nothing but the drift velocity in crossed electric and magnetic fields. The FISDW potential may also move along the chains, in which case its phase $\Theta$ depends on time $t$, and the velocity of the motion is proportional to the time derivative $\dot{\Theta}$. Since we are interested only in a spatially homogeneous motion of the FISDW, we assume that $\Theta$ depends only on $t$ and not on the coordinates $x$ and $y$. We also assume that both potentials move very slowly, adiabatically, which is the case when the electric...
field is sufficiently weak.

Now, we are going to calculate the current along the chains produced by the motion of the potentials. Since there is an energy gap at the Fermi level, following the arguments of Laughlin [3] we can say that an integer number of electrons $N_1$ is transferred from one end of a chain to another when the FISDW potential shifts by its period $L_1 = 2\pi/Q_x$. The same is true for the motion of the hopping potential with an integer $N_2$ and the period $L_2 = 2\pi/q_x$. Suppose that the first potential shifts by an infinitesimal displacement $dx_1$ and the second by $dx_2$. The total transferred charge $dq$ would be the sum of the prorated amounts of $N_1$ and $N_2$:

$$dq = eN_1 \frac{dx_1}{L_1} + eN_2 \frac{dx_2}{L_2}.$$  \hspace{0.5cm} (2)

Now, suppose that both potentials are shifted by the same displacement $dx = dx_1 = dx_2$. In this case, we can also write that

$$dq = e\rho dx,$$ \hspace{0.5cm} (3)

where $\rho = 4k_F/2\pi$ is the concentration of electrons. Equating (2) and (3) and substituting the expressions for $\rho$, $L_1$, and $L_2$, we find the following Diophantine-type equation:

$$4k_F = N_1(2k_F - Nq_x) + N_2q_x.$$ \hspace{0.5cm} (4)

Since $k_F/q_x$ is, in general, an irrational number, the only solution of Eq. (4) for the integer $N_1$ and $N_2$ is $N_1 = 2$ and $N_2 = N_1N = 2N$.

Dividing Eq. (4) by the time increment $dt$ and the distance between the chains $b$, we find the density of current along the chains, $j_x$. Taking into account that according to Eq. (1) the displacements of the potentials are related to their phases: $dx_1 = -\partial\Theta/Q_x$ and $dx_2 = \omega_y dt/q_x$, we find the final expression for $j_x$:

$$j_x = -\frac{e}{\pi b} \dot{\Theta} + \frac{2Ne^2}{\hbar} E_y.$$ \hspace{0.5cm} (5)

The first term in Eq. (5) represents the contribution of the FISDW motion, the so-called Fröhlich conductivity $\dot{\Theta}$. The second term describes the quantum Hall effect. The integer number $N$ in the quantized Hall conductivity $\sigma_{xy} = 2Ne^2/\hbar$ is the same as that in the FISDW wave vector $Q_x = 2k_F - Nq_x$.

To complete solution of the problem, it is necessary to find how $\dot{\Theta}$ depends on $E_y$. For this purpose, we need the equation of motion of $\Theta$, which can be derived once we know the Lagrangian of the system, $L$. Two terms in $L$ can be readily recovered taking into account that the current density $j_x$ given by Eq. (5) is the variational derivative of the Lagrangian with respect to the electromagnetic vector-potential $A_x$: $j_x = e\delta L/\delta A_x$. Written in the gauge-invariant form, the recovered part of the Lagrangian is equal to

$$L_1 = -\sum_{i,j,k} Ne^2 \frac{e}{2\pi \hbar c} \varepsilon_{ijk} A_i F_{jk} - e \frac{\hbar}{2N} \Theta E_x,$$ \hspace{0.5cm} (6)

where $\varepsilon_{ijk}$ is the antisymmetric tensor with the indices $i,j,k = t,x,y$; $A_i$ and $F_{jk}$ are the vector-potential and the tensor of the electromagnetic field, and $E_x \equiv F_{tx}$ is the electric field along the chains. The first term in Eq. (6) is the so-called Chern–Simons term responsible for the quantum Hall effect [4]. The second term describes the interaction of the density-wave condensate with the electric field along the chains [5].

Lagrangian (6) should be supplemented with the kinetic energy of the FISDW condensate, $K$. The FISDW potential itself has no inertia because it is produced by the instantaneous Coulomb interaction between electrons, so $K$ originates completely from the kinetic energy of the electrons which are confined under the FISDW gap. The latter energy is proportional to the square of their average velocity, which, in turn, is proportional to the electric current along the chains:

$$K = \frac{\pi \hbar b}{4v_F e^2} j_x^2,$$ \hspace{0.5cm} (7)

where $v_F$ is the Fermi velocity. Substituting Eq. (6) into Eq. (7), expanding, and omitting an unimportant term proportional to $E_y^2$, we obtain the second part of the Lagrangian of the system:

$$L_2 = \frac{\hbar}{4\pi bv_F} \dot{\Theta}^2 - \frac{eN}{2\pi v_F} \Theta E_y.$$ \hspace{0.5cm} (8)

The first term in Eq. (8) is the same as the kinetic energy of a purely one-dimensional density wave [6] and is not specific to the FISDW. The most important is the second term which describes the interaction of the FISDW motion and the electric field perpendicular to the chains. This term is allowed by symmetry in the considered system and has the structure of a mixed vector–scalar product:

$$v_\Theta [E \times H].$$ \hspace{0.5cm} (9)

Here, $v$ is the velocity of the FISDW which is proportional to $\dot{\Theta}$ and is directed along the chains, that is, along the $x$-axis. The magnetic field $H$ is directed along the $z$-axis, thus, allowing the electric field $E$ to enter only through the component $E_y$. Comparing formula (8) with the second term in Eq. (7), one should take into account that the magnetic field enters the second term implicitly, through the integer $N$, which depends on $H$ and changes sign when $H$ changes sign.

Varying the total Lagrangian $L = L_1 + L_2$, given by Eqs. (6) and (8), with respect to $\Theta$, we find the equation of motion of $\Theta$:

$$\ddot{\Theta} = -\frac{2ev_F}{\hbar} E_x + \frac{eNb}{\hbar} E_y.$$ \hspace{0.5cm} (10)
In Eq. (10), the first two terms constitute the standard one-dimensional equation of motion of the density wave \[ \dot{\Theta} + \frac{1}{\tau} \dot{\Theta} + \omega_0^2 \Theta = -\frac{2eN}{h} E_x + \frac{eN}{h} \dot{E}_y, \] where \( \tau \) is the relaxation time and \( \omega_0 \) is the pinning frequency. Solving Eq. (12) via the Fourier transformation produces the Hall conductivity \( \sigma_{xy} \) as a function of frequency:

\[ \sigma_{xy}(\omega) = \frac{2N e^2}{h} \frac{\omega_0^2 - i\omega/\tau}{\omega_0^2 - \omega^2 - i\omega/\tau}. \]

The absolute value of the Hall conductivity, \( |\sigma_{xy}| \), computed from Eq. (13) is plotted in the Fig. 1 as a function of \( \omega/\omega_0 \) for \( \omega_0\tau = 2 \). As we can see in the Figure, the Hall conductivity is quantized at zero frequency and has a resonance at the pinning frequency. At the higher frequencies, where the pinning and the damping can be neglected and the system effectively behaves as an ideal, purely inertial system considered above, the Hall conductivity does decrease toward zero.

The frequency dependence of the Hall conductivity in regular semiconductor quantum Hall systems was measured using the technique of crossed wave guides [8,9]. Unfortunately, no such measurements were performed in the FISDW systems. These measurements would be extremely interesting. To give a crude estimate of the required frequency range, we quote the value of the pinning frequency \( \omega_0 \sim 3 \text{ GHz} \sim 0.1 \text{ K} \sim 10 \text{ cm} \) for a regular (not magnetic-field-induced) SDW in (TMTSF)\(_2\)PF\(_6\) [10].

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