Complexity analysis of interconnected pore using hydraulic tortuosity

Hessel Juliust, M. Nashih Amien, Genki Tanod Pantouw, Fourier Dzar Eljabbar Latief*

Kelompok Keilmuan Fisika Bumi dan Sistem Kompleks, Institut Teknologi Bandung, Jalan Ganesha no. 10 Bandung 40132, Indonesia

E-mail: fourier@fi.itb.ac.id

Abstract. The complexity of the pore structure has been analysed using hydraulic tortuosity. The hydraulic tortuosity is defined as the ratio between the total curvature angles of the fluid flowpath to the distance between two facing sides of the sample. The fluid flowpaths in the connected pore are represented by streamlines which are calculated from the velocity map obtained from fluid flow simulation by means of Lattice Boltzmann Method. The streamline that is calculated with Euler Methods produce coordinates of the flowpath in the connected pore which are then used to calculate the tortuosity. The streamline are verified in a simple porous medium. The complexity analysis is verified on three simple models showing significant differences in complexity levels. The streamline generated using euler methods is suitable for the porous medium. The relation of the tortuosity and the permeability is tested in two porous medium that the permeability is known. It is found that the more complex the pore structure, the greater the tortuosity value. The hydraulic tortuosity is also inversely proportional to the permeability.

1. Introduction
Determining the physical properties of rocks is very important to understand the characteristics of a reservoir. Rocks have several physical parameters, such as porosity, tortuosity and permeability [1]. Physical parameters to be discussed in this study is tortuosity. A common definition of tortuosity in porous media is the ratio of the length of the pore path to the length of the porous medium [2]. Tortuosity can be distinguished as geometric, hydraulic, electric and diffusive models tortuosity [3], depend on the need of their basic concepts of geometry, fluid mechanics, electrodynamics and diffusion equations. From these concepts, there is a similarity where tortuosity is highly related to the complexity of the interconnected pore inside the medium.

The more winding a path, the more complex the porous media structure will be. Tortuosity is used to described complexity level which can also be attributed to the resistance of a porous medium to drain a fluid [4]. In some cases, however, the tortuosity calculated by Carman's tortuosity calculation method showed the same value for different levels of complexity were made [5]. Therefore, tortuosity calculation is required with other methods for porous media. Tortuosity can represent not only the length of the path, but also of the complexity of the path.

* To whom any correspondence should be addressed.
1.1 Tortuosity

Tortuosity is commonly used to analyze flow properties in porous medium [2]. Tortuosity by Carman definition is the ratio of the length of the path formed by the fluid flow in the pore to the length of the porous medium side. Hydraulic tortuosity (\(\tau\)) is the dimensionless parameter in the Kozeny-Carman semi-empirical calculations to determine the permeability and geometric relations of the medium.

\[
\kappa = \frac{\phi^3}{c s^2} = \frac{\phi^3}{\beta \tau s^2}
\]  

(1)

Hydraulic tortuosity is described as the ratio of the length of the path formed by the fluid flow in the pore (\(\lambda\)) to the length of the porous medium side, as follows:

\[
\tau = \frac{\lambda}{L}
\]  

(2)

Figure 1 shows the difference between the hydraulic tortuosity (Figure 1 (a)) and geometric tortuosity (Figure 1 (b)). The hydraulic tortuosity uses a path where a fluid is more likely to flow. Meanwhile the geometric tortuosity takes the shortest distance [6].

![Figure 1](image_url)

**Figure 1.** (a) Hydraulic tortuosity path; (b) Geometric tortuosity path

This Kozeny-Carman relation in equation (1) is semi-empirical, and it is very suitable to a porous medium which was generated from a randomly packed spheres [7]. In this study, the method of tortuosity calculation used is not contained in equation \(\tau = \frac{\lambda}{L}\) (2), but the method of tortuosity calculation by Dougherty [8], as follows:

\[
\tau = \frac{\sum_{i=0}^{n} \theta_i}{L} \text{[m}^{-1}\text{]}
\]  

(3)

The path deflection angle (\(\theta_i\)) to determine the value of tortuosity is added. This tortuosity value has dimensions (m\(^{-1}\)), but according to Hillel [9] tortuosity is a dimensionless parameter and the equation becomes [10]:

\[
\tau = \frac{\sum_{i=0}^{n} \theta_idl_i}{L}
\]  

(4)

![Figure 2](image_url)

**Figure 2.** The flow illustration with path length is the same as different complexity, but with the same tortuosity value

This is done because with equation (2) we get the same tortuosity value even though it is made different level of complexity, as shown in Figure 2 which is a string with the same length but different complexity. A simple analysis can be done qualitatively using simple paths as shown in Figure 2.
Samples (a), (b), (c), (d), and (e) have the same tortuosity if equation (2) is used while they will have different values when equation (4) is used. The tortuosity of a straight line is equal to 1 if the tortuosity calculation is used by equation (2) while it is equal to 0 in equation (4). This explains that equation 4 is more suitable to be used in calculating tortuosity because the smallest value 0 is in flow with absolutely no complexity.

1.2 Trajectory of fluid flow
To calculate hydraulic tortuosity in porous medium, trajectory tracking is required. In this study streamline is used as a means of tracking the trajectory, where the streamlines represent the most likely to flow. Streamline is a line that always aligns with vector speed [11]. Seen in Figure 3. there is a vector field indicated by a red line in the example and the resulting streamline is blue.

![Figure 3. Streamline example on velocity vector field](image)

1.3 Streamline Equation
To determine the streamline equation, the vector velocity is defined as \( \vec{V} \).

\[
\vec{V}(x, y, z) = u\hat{i} + v\hat{j} + w\hat{k}
\]  

(5)

Next defined the streamlined countercurrent vector.

\[
d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}
\]  

(6)

This vector is always parallel to the velocity vector, then:

\[
d\vec{s} \times \vec{V} = 0.
\]  

(7)

\[
(w\,du - v\,dz)\hat{i} + (u\,dz - w\,dx)\hat{j} + (v\,dx - y\,dy)\hat{k} = 0
\]  

(8)

Next, separation of each component into 0 gives 3 equations that define the streamline, into equation (9).

\[
\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.
\]  

(9)

Therefore to make it easier to solve, streamline equation is solved with Numerical calculation.

It is necessary to obtain the coordinate of streamline at each point, with a predefined starting point \( x_0, y_0 \) and \( z_0 \). It can be seen in Figure 3 that the starting point can determine the next point which will then be streamlined.

In numerical integration, the starting point is used as the initial value and in streamline tracing, called seed. To solve the streamlined equations, numerical integration is required and here euler integration is used.
2. Methods
To obtain velocity vector, fluid flow is simulated by Lattice Boltzmann method using PALABOS library. This simulation is used to obtain vector velocity values at each pixel point.

2.1 Obtaining Streamline from velocity vectors
After obtaining velocity vectors at each coordinate point, a streamline line will be determined on the porous medium. To solve the streamline equation contained in equation (9) the euler method is used. The first to obtain a streamline a seed value is required. The seed value is the starting point of the line to determine where the streamline line will be created. The value of this seed consists of the values at x coordinates, y coordinates and z coordinates. The seed is then used to find the coordinates next iteration. To obtain the coordinates Euler's integration method is applied by using the previous coordinates and multiplying the speed in that direction with the time step. At this stage, the coordinates, and the velocity vectors are function of position in 3D space ($\vec{V}(x, y, z)$). The euler integration is performed iteratively inside the boundary plane until a maxium iteration is reached. This is done because the streamline is limited to the space observed. If the condition is met, the coordinates are rounded. For more details the algorithm is described in Figure 4.

\[
\theta = \cos^{-1}\left(\frac{\sum_{i=0}^{n} v_{ix}^i x + v_{iy}^i y + v_{iz}^i z}{\sqrt{\sum_{i=0}^{n} v_{ix}^2 + v_{iy}^2 + v_{iz}^2}}\right)
\]

(10)

From the angle and line coordinates obtained then the hydraulic tortuosity value is calculated based on equation 4. Hydraulic tortuosity is calculated for all streamlines which will then be averaged. The overall stages of hydraulic tortuosity calculation are shown in Figure 5.
There will be a comparison of the tortuosity values at three levels of medium that the higher the level the more qualitatively complex the fluid flow path will be. There will be also a comparison of the tortuosity value in a digital sample of porous medium with the same porosity which both of its permeability already known.

3. Samples
There will three types of sample that will be tested in this study. Figure 6 (a) shows a simple three dimensional medium model in order to verify the streamline tracing. Figure 6 (b), (c), and (d) is a comparison of the complexity of a medium. Figure 6 (c), is more complex than Figure 6 (b) while Figure 6 (d) is more complex than Figure 6 (c). Figure 6 (e) and (f), show the digital sample of porous medium with the same porosity which both of its permeability already known, the first one is sample a and the second one is sample b.

![Figure 6](image.png)

**Figure 6.** (a) Example of simple medium model; (b) sample level 1 to compare of the complexity of a medium; (c) sample level 2 to compare of the complexity of a medium; (d) sample level 3 to compare of the complexity of a medium; (e) Digital sample which permeability already known (sample a); (f) Digital sample which permeability already known (sample b)

4. Results and Discussion

4.1 Streamline Tracing
It could be seen in Figure 7 that this method of streamline tracing give a suitable result that we used in Figure 6. The porous medium is wide at the inlet and the outlet but narrow in the middle of the medium and so is that the Streamline.

![Figure 7](image.png)

**Figure 7.** (a) Result of streamline tracing in simple medium model; (b) Result of streamline tracing in simple medium model (Streamline only)

**Figure 8.** (a) Streamline of sample a; (b) Streamline of sample b

4.2 Comparison of three different complexity level medium
Here, we will compare the tortuosity between three levels of medium that qualitatively the complexity is higher when the level is raised. We could see the model in Figure 6 and the results in Table 1.

| Level | 1    | 2    | 3    |
|-------|------|------|------|
| Mean Tortuosity | 0.0012 | 0.003 | 0.004 |

**Table 1.** Tortuosity of different complexity level of a medium
The results show that the more complex the flowpath of the fluid (higher the level), the higher the tortuosity value will be so it show that tortuosity that calculated this way could measure the complexity of the porous medium.

4.3 Tortuosity value in a digital sample of porous medium

There are two digital sample of porous medium in Figure 6 which its permeability is already known (sample (a) and sample (b)). The streamline of these porous medium is shown in Figure 8 and the comparison of permeability is shown in Table 2.

From the Kozeny-Carman semi-empirical equation it is well known that higher tortuosity is related to a lower the permeability. It is also proven in this computational experiment from the two samples with same porosity but with different permeabilities. It is proven that the sample a path of fluid flow looks more complex and results in a high tortuosity obtained a low permeability. In contrast to sample b that has a low tortuosity and has a high permeability.

### Table 2. Comparison with the mean tortuosity with the known permeability

| Sample | Mean Tortuosity | Permeability |
|--------|----------------|--------------|
| a      | 0.581          | 5.272        |
| b      | 0.001          | 81.66        |

5. Conclusion

From the simple porous medium model it is found that streamline could represent the trajectory of the fluid flow. Based on the results from the simple paths and complex porous media, the hydraulic tortuosity is able to describe the complexity that is the more complex the connected pore path, the greater the tortuosity value, and vice versa. It also proven that this measurement of tortuosity can be used as a parameter to determine the complexity of the connected pore structure of the porous medium. The hydraulic tortuosity is inversely proportional to the permeability.

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