Lepton Flavour Violation in charged leptons within SUSY-seesaw

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From works
E. Arganda, M.H. and J. Portolés
JHEP06(2008)079 LFV semilep. $\tau$ decays

E. Arganda, M.H. and A. Teixeira
JHEP10(2007)104 $\mu - e$ conv. in nuclei

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Motivation

★ Lepton Flavour Violation (LFV) occurs in Nature: \( \nu_i - \nu_j \) oscill.

★ LFV is very sensitive to SUSY: if \( m_\nu \) from seesaw with Majorana \( \nu_R \) \( \Rightarrow Y_\nu \) can be \( \mathcal{O}(1) \). Large \( Y_\nu \) induce, via SUSY loops, large LFV rates

★ We focus here on LFV semilep \( \tau \) decays, \( \tau \rightarrow \mu PP \) and \( \tau \rightarrow \mu P \), and \( \mu - e \) conv. in nuclei which 1) complement previous studies of LFV leptonic \( \tau \) and \( \mu \) decays and 2) have more sensitivity to Higgs sector

★ Exp. bounds for the processes of interest here (Belle-BABAR,SINDRUM):

\[
\begin{align*}
\text{BR}(\tau \rightarrow \mu \eta) &< 5.1 \times 10^{-8} \\
\text{BR}(\tau \rightarrow \mu \eta') &< 5.3 \times 10^{-8} \\
\text{BR}(\tau \rightarrow \mu \pi) &< 5.8 \times 10^{-8} \\
\text{BR}(\tau \rightarrow \mu \rho) &< 2 \times 10^{-7} \\
\text{BR}(\tau \rightarrow \mu \phi) &< 1.3 \times 10^{-7} \\
\text{CR}(\mu - e, \text{Au}) &< 7 \times 10^{-13} \\
\text{BR}(\tau \rightarrow \mu \pi^+ \pi^-) &< 4.8 \times 10^{-7} \\
\text{BR}(\tau \rightarrow \mu \pi^0 \pi^0) &< \text{no bound} \\
\text{BR}(\tau \rightarrow \mu K^+ K^-) &< 8 \times 10^{-7} \\
\text{BR}(\tau \rightarrow \mu K^0 \bar{K}^0) &< \text{no bound} \\
\text{CR}(\mu - e, \text{Ti}) &< 4.3 \times 10^{-12}
\end{align*}
\]

Present: Some LFV semilep. decays already competitive with lep. ones
Future: \( \mu - e \) conv. in Ti, the most challenging \( 10^{-18} \) (PRISM/PRIME)

★ Few predictions of these processes in the previous literature:
Some not in the SUSY-seesaw context (i.e no connection with \( \nu \) physics);
Others not complete (i.e not all loops); Hadronisation treated differently
Our work presented here:

- Predictions of LFV rates within SUSY-seesaw for:
  - $\tau \to \mu P$, $P = \eta, \eta', \pi$
  - $\tau \to \mu PP$, $PP = \pi^+\pi^-, \pi^0\pi^0, K^+K^-, K_0\bar{K}_0$
  - $\tau \to \mu V$, $V = \rho, \phi$ (related to $\tau \to \mu PP$)
  - $\mu - e$ conversion in different nuclei: Ti, Au, ...

- Full one-loop computation of LFV rates
- Require compatibility with $\nu$ data
- Compare with present LFV bounds
- Explore sensitivity to SUSY, and seesaw parameters
- Found higher sensitivity to Higgs sector in these processes than in LFV lep. decays, $\tau \to 3\mu, ..$
- Found a set of simple formulas that approximate well the full result and are useful for comparison with data
1-loop diagrams in $\tau \rightarrow 3\mu, \tau \rightarrow \mu P, \tau \rightarrow \mu PP, \tau \rightarrow \mu V, \mu \rightarrow e$
Framework for LFV

• Use seesaw (Type I) for $\nu$ mass generation

• Within Constrained MSSM $+$ $3\nu_R$ (Majorana) $+$ $3\tilde{\nu}_R$
  Two scenarios for soft parameters at $M_X = 2 \times 10^{16}$ GeV:
  * Universal soft parameters: CMSSM-seesaw
    $(M_0, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu)); \tan \beta = v_2/v_1$
  * Non-universal soft Higgs masses: NUHM-seesaw
    $(M_0, M_{1/2}, M_{H_1}, M_{H_2}, A_0, \tan \beta, \text{sign}(\mu))$

• LFV generated by 1-loop running from $M_X$ to $M_Z$
  Full RGEs including $\nu$ and $\tilde{\nu}$ sectors (No Llog approx)

• Mass eigenstates for all SUSY and Higgs particles (No MI approx)

• Numerical estimates:
  * SPheno 2.2.2 (W.Porod) for int. of RGEs and SUSY spectrum
  * Additional subroutines for all LFV processes (by us)
    Also subroutines for checks of BAU, EDM and $(g-2)_\mu$
Framework for Hadronisation

- **We use Chiral Perturbation Theory (χPT)**
  It realizes nicely the large $N_C$ expansion of $SU(N_C)$ QCD and is the appropriate scheme to describe strong ints of PG Bosons $P = \pi, K, \eta$.

- $\text{BR}(\tau \to \mu P), P = \pi, \eta, \eta'$, from leading $O(p^2)$ χPT. Results in terms of $F_\pi$ and $m_P$ (assume $\eta_8 - \eta_0$ mix. ang. $\theta \simeq -18^o$ for $P = \eta, \eta'$).
- $\text{BR}(\tau \to \mu PP), PP = \pi^+\pi^-, K^+K^-, K_0\bar{K}_0$ from χPT plus contributions from resonances ($R_{\chi T}$). Results in terms of $F_\pi, m_P$ and well established form factors $F^{PP}_V(s), (G.\text{Ecker et al. PLB223}(1989)425)$.

\[
F^{\pi\pi}_V(s) = F(s) \exp \left[ 2 \text{Re}(\tilde{H}_{\pi\pi}(s)) + \text{Re}(\tilde{H}_{KK}(s)) \right] \\
F(s) = \frac{M^2_\rho}{M^2_\rho - s - iM_\rho \Gamma_\rho(s)} \left[ 1 + \left( \frac{M^2_\omega}{M^2_\rho} - \gamma \frac{s}{M^2_\rho} \right) \frac{s}{M^2_\omega - s - iM_\omega \Gamma_\omega} \right] \frac{\gamma s}{M^2_\rho - s - iM_\rho \Gamma_\rho(s)} , \\
\tilde{H}_{PP}(s) = \frac{s}{F^2_\pi} \left[ \frac{1}{12} \left( 1 - 4 \frac{m^2_P}{s} \right) J_P(s) - \frac{k_P(M_\rho)}{6} + \frac{1}{288\pi^2} \right] , \sigma_P(s) = \sqrt{1 - 4 \frac{m^2_P}{s}} \\
J_P(s) = \frac{1}{16\pi^2} \left[ \sigma_P(s) \ln \frac{\sigma_P(s) - 1}{\sigma_P(s) + 1} + 2 \right] , \frac{k_P(\mu)}{2\pi^2} = \frac{1}{32\pi^2} \left( \ln \frac{m^2_P}{\mu^2} + 1 \right)
\]
Framework for $\mu - e$ conversion in nuclei

- We follow the general parameterisation and approxs of Kuno & Okada Rev.Mod.Phys.73(01)151

- Equal proton and neutron densities in the nucleus; non-relativistic $\mu$ wave function for the 1s state; neglect momentum dependence of nucleon form factors

- $\mu - e$ conv. rate compared to muon capture rate, as a function of: $Z, N$ number of p and n in nucleus; $Z_{\text{eff}}$ effective atomic charge, $F_p$ nuclear matrix element. We compute isoscalar and isovector couplings $g^{(0)}, g^{(1)}$ from the full set of 1-loop diagrams. $Z_{\text{eff}}, F_p, \Gamma_{\text{capt}}$ for various nuclei from Kitano, Koike, Okada, PRD66(02)096002.

$$\text{CR}(\mu - e, \text{Nucleus}) = \frac{m_{\mu} G_F^2 \alpha^3 Z_{\text{eff}}^4 F_p^2}{8 \pi^2 Z} \times \left\{ \left| (Z + N) \left( g^{(0)}_{LV} + g^{(0)}_{LS} \right) + (Z - N) \left( g^{(1)}_{LV} + g^{(1)}_{LS} \right) \right|^2 + \left| (Z + N) \left( g^{(0)}_{RV} + g^{(0)}_{RS} \right) + (Z - N) \left( g^{(1)}_{RV} + g^{(1)}_{RS} \right) \right|^2 \right\} \frac{1}{\Gamma_{\text{capt}}}$$. 
Seesaw parameters versus neutrino data

SeeSaw eq.: \( m_\nu = -m_D^T m_N^{-1} m_D \); 3 light \( \nu \) (\( \sim \nu_L \)), 3 heavy \( N \) (\( \sim \nu_R \))

Solution:

\[
    m_D = Y_\nu v_2 = i \sqrt{m_N^{\text{diag}}} R \sqrt{m_\nu^{\text{diag}}} U_{MNS}^\dagger
\]

[Casas, Ibarra ('01)]

\( R \) is a 3 \( \times \) 3 complex matrix and orthogonal

\[
    R = \begin{pmatrix}
    c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\
    c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\
    s_2 & s_1 c_2 & c_1 c_2
    \end{pmatrix},
    c_i = \cos \theta_i, s_i = \sin \theta_i, \theta_{1,2,3} \text{ complex}
\]

Parameters: \( \theta_{ij}, \delta, \alpha, \beta, m_\nu, m_{N_i}, \theta_i \) (18); \( m_{N_i}, \theta_i \) drive the size of \( Y_\nu \)

Hierarchical \( \nu \)'s: \( m_{\nu_1}^2 << m_{\nu_2}^2 = \Delta m_{\text{sol}}^2 + m_{\nu_1}^2 << m_{\nu_3}^2 = \Delta m_{\text{atm}}^2 + m_{\nu_1}^2 \)

2 Scenarios

- Degenerate \( N \)'s
  \( m_{N_1} = m_{N_2} = m_{N_3} = m_N \)
- Hierarchical \( N \)'s
  \( m_{N_1} << m_{N_2} << m_{N_3} \)
Our choice of input parameters

Constrained MSSM +3ν_R + 3\bar{\nu}_R + seesaw

• CMSSM:

\begin{align*}
M_0, M_{1/2}, A_0 \; (\text{at } M_X \sim 2 \times 10^{16} \text{ GeV}) \\
\tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle} \; (\text{at EW scale}) \\
\text{sign}(\mu) \; (\mu \text{ derived from EW breaking})
\end{align*}

• NUHM: (\(M_0, M_{1/2}, M_{H_1}, M_{H_2}, A_0, \tan \beta, \text{sign}(\mu)\))

Choose \(M_0 = M_{1/2}, M_{H_1}^2 = M_0^2(1 + \delta_1), M_{H_2}^2 = M_0^2(1 + \delta_2)\)

• Seesaw parameters

\begin{align*}
m_{\nu_{1,2,3}} \; (\text{set by data}) \\
m_{N_{1,2,3}} \; (\text{input}) \\
U_{MNS} \; (\text{set by data}) \\
R(\theta_1, \theta_2, \theta_3) \; (\text{input})
\end{align*}

• For numerical estimates:

\begin{align*}
(\Delta m^2_{12})_{\text{sol}} = 8 \times 10^{-5} \text{ eV}^2 \\
(\Delta m^2_{23})_{\text{atm}} = 2.5 \times 10^{-3} \text{ eV}^2 \\
\theta_{12} = 30^o; \; \theta_{23} = 45^o; \; \delta = \alpha = \beta = 0; \; 0 \leq \theta_{13} \leq 10^o
\end{align*}

250 \text{ GeV} < M_0, M_{1/2} < 1000 \text{ GeV}, \; -500 \text{ GeV} < A_0 < 500 \text{ GeV}

5 < \tan \beta < 50, \; -2 < \delta_{1,2} < 2
Competing LFV $\tau \to \mu + \ldots$ decays:

$\tau \to \mu \gamma$, $\tau \to 3\mu$, $\tau \to \mu PP$, $\tau \to \mu P$, $\tau \to \mu V$, ..

I) Common/Non-common aspects

II) Comments on $\tau \to \mu \gamma$ and $\tau \to 3\mu$
Phenomenological parameter for LFV in the tau-muon sector: $\delta_{32}$

Within the Mass Insertion (MI) approx.,

$$\text{BR}(\tau \to \mu + ...) \propto |\delta_{32}|^2$$

In SUSY-Seesaw scenarios, the contributions from neutrinos and sneutrinos to the slepton mass matrix entry 32, in the Leading Logarithmic Approximation (Llog) are parameterized by:

$$\delta_{32} = \frac{\Delta \tilde{M}_{32}^2}{M_{\text{SUSY}}^2} = -\frac{1}{8\pi^2} \frac{(3 M_0^2 + A_0^2)}{M_{\text{SUSY}}^2} \left( Y_\nu^\dagger L Y_\nu \right)_{32}$$

$$L_{ii} = \log(M_X/m_{N_i}); \ M_{\text{SUSY}} \text{ is an average SUSY mass}$$

The relation with neutrino physics comes in,

$$v_2^2 (Y_\nu^\dagger L Y_\nu)_{32} = L_{33} m_{N_3} \left[ (\sqrt{m_{\nu_3} c_1 c_2 c_{13} c_{23}} - \sqrt{m_{\nu_3} s_1 c_2 c_{12} s_{23}}) \\
(\sqrt{m_{\nu_3} c_1^* c_2^* s_{23}} + \sqrt{m_{\nu_3} s_1^* c_2^* c_{12} c_{23}}) \right]$$

$$+ L_{22} m_{N_2} \left[ (\sqrt{m_{\nu_2} (s_1 c_3 - c_1 s_3) c_{23}} + \sqrt{m_{\nu_2} (s_1 s_2 s_3 - c_1 c_3) c_{12} s_{23}}) \\
(\sqrt{m_{\nu_2} (s_1^* c_3 - c_1^* s_3 s_{23}}) + \sqrt{m_{\nu_2} (c_1^* c_{3}^* - s_1^* s_2^* s_{23}^*) c_{12} c_{23}) \right]$$

$$+ L_{11} m_{N_1} \left[ (\sqrt{m_{\nu_1} (s_1 s_3 - c_1 s_3) c_{12} c_{23}} + \sqrt{m_{\nu_2} (s_1 s_2 c_3 + c_1 s_3) c_{12} s_{23}}) \\
(\sqrt{m_{\nu_1} (s_1^* s_{1}^* - s_1^* s_2^* s_{23}^*) c_{12} c_{23}} - \sqrt{m_{\nu_2} (s_1^* s_{2}^* c_{3}^* + c_1^* s_{3}^*) c_{12} c_{23})}$$
Most relevant seesaw param.: $m_{N_3}$ if $\nu_R$ hierarchical ($m_N$ if degenerate)

$\text{BR} \sim \left|\delta_{32}\right|^2 \sim \left|m_{N_3}\log m_{N_3}\right|^2$ (larger BRs than for deg.) (same all decays)

Next relevant seesaw parameter: $\theta_i$ (same all decays)

Ex.: $\text{BR} \times 10 - 100$ if $\theta_2$: $0 \rightarrow 2.9e^{i\pi/4}; \left|\delta_{32}\right| \sim O(1); Y_{\nu} \sim O(1)$

Most relevant SUSY parameter: $\tan \beta$ (same all decays)

$\text{BR} \sim (\tan \beta)^2$ if $\gamma$-dominated; $\sim (\tan \beta)^6$ if H-dominated (not same all dec.)

Some BRs reach exp. lim. at large $m_{N_3}$, large $\tan \beta$ and large $\theta_i$
The most competitive LFV tau decay: $\tau \to \mu \gamma$

From our previous study, JHEP11(2006)090, presented at Tau06, Pisa

$$(-\pi/4 \lesssim \text{arg}\theta_1 \lesssim \pi/4, \ 0 \lesssim \text{arg}\theta_2 \lesssim \pi/4),$$

(SP1a: $M_0 = 100$ GeV, $M_{1/2} = 250$ GeV, $A_0 = -100$ GeV, $\tan \beta = 10$, $\mu > 0$)

Present: $\mu \to e\gamma$ more competitive than $\tau \to \mu\gamma$, except if very small $\theta_{13}$

MEGA bound, $\text{BR}(\mu \to e\gamma) < 10^{-11}$, already disfavours $m_{N_3} \gtrsim 10^{14}$ GeV

Conclusion: For a given SPS, $\tau \to \mu\gamma$ sets upper bounds on $m_{N_3}$ that, if small $\theta_{13}$, are competitive with those from $\mu \to e\gamma$.

BUT: both are insensitive to Higgs!!. Next: Some LFV semileptonic tau decays do!!
Sensitivity to Higgs if and only if:
light Higgs and heavy SUSY

| $\delta_2$ | $M_0 = M_{1/2}$ | $A_0$, tan $\beta$ | $\theta_i$, $\theta_{13}$ |
|------------|----------------|------------------|-------------------|
| 0          | 250 GeV        | 0, 50            | 0, 5°             |
| 0.5        | 500 GeV        | 0, 50            | 0, 5°             |
| 1          | 850 GeV        | 0, 50            | 0, 5°             |
| 1.5        | 850 GeV        | 0, 50            | 0, 5°             |

Universality

NUHM-seesaw predicts **light Higgs particles** even for large $M_0 = M_{1/2} = M_{\text{SUSY}}$

Ex.: for $M_{\text{SUSY}} = 850$ GeV, tan $\beta = 50$, $A_0 = 0$, $\delta_1 = -1.8$, $\delta_2 = 0$, we find:

- light Higgs: $m_{H^0} = 127$ GeV, $m_{h^0} = 123$ GeV, $m_{A^0} = 127$ GeV, $m_{H^+} = 155$ GeV
- heavy SUSY: $m_{\tilde{t}_1} = 734$ GeV, $m_{\tilde{\nu}_1} = 971$ GeV, $m_{\tilde{\chi}^-_1} = 687$ GeV, $m_{\tilde{\chi}^0_1} = 362$ GeV
We find no sensitivity to Higgs in $\tau \rightarrow 3\mu$ either

$\tau \rightarrow 3\mu$ is highly dominated by $\gamma$ diags., even at low $m_{A^0}$ and large $M_{SUSY}$

Present bound on $BR(\tau \rightarrow 3\mu)$ disfavours $m_{N_3} \gtrsim 10^{14}$ GeV, $\tan \beta \gtrsim 50$, $M_{SUSY} \lesssim 300$ GeV

$\tau \rightarrow \mu \gamma$ still more competitive than $\tau \rightarrow 3\mu$; $BR(\tau \rightarrow 3\mu) \simeq 2 \times 10^{-3} BR(\tau \rightarrow \mu \gamma)$
Results for LFV semilep. tau decays

\[ \tau \rightarrow \mu, \gamma, Z^0 \rightarrow \mu, h^0, H^0 \]

\[ \tau \rightarrow \mu, Z^0 \rightarrow \mu, A^0 \]
Relevant contributions to $\tau \rightarrow \mu PP$ in NUHM

- $\tau \rightarrow \mu K^+ K^-, \tau \rightarrow \mu K^0 \bar{K}^0$: HKK coup. large $\sim m_K^2$ ($\chi PT$: $B_0 m_s = m_K^2 - \frac{1}{2} m_\pi^2$)
  Both KK channels $\gamma$-dominated at low $M_{SUSY}$, $H$-dominated at large $M_{SUSY}$

- $\tau \rightarrow \mu \pi^+ \pi^-$: $\gamma$-dominated; $\tau \rightarrow \mu \pi^0 \pi^0$: only $H$ contributes.
  $H\pi\pi$ coup. small $\sim m_\pi^2$ ($\chi PT$: $B_0 m_{u,d} = \frac{1}{2} m_\pi^2$)
\( \tau \rightarrow \mu \text{PP} \): CMSSM versus NUHM

**CMSSM:** \( \gamma \) dominance at all \( M_{\text{SUSY}} \)

**NUHM:** \( H^0 \) enters at large \( M_{\text{SUSY}} \)

\* In scenarios with light \( H \) and heavy SUSY (NUHM) we find SUSY non-decoupling

\* \( \text{BR}(\tau \rightarrow \mu \pi^+\pi^-) \sim \text{exp.bound} > \text{BR}(\tau \rightarrow \mu K^+K^-) \gtrsim \text{BR}(\tau \rightarrow \mu K^0\bar{K}^0) \gg \text{BR}(\tau \rightarrow \mu \pi^0\pi^0) \)

hierarchy from dominant electromag. contrib. and relative phase space suppression
\[ \tau \to \mu P \ (P = \eta, \eta', \pi) \ \text{and} \ \tau \to \mu V \ (V = \rho, \phi) \]

CMSSM: SUSY decoupling

NUHM: SUSY non-decoupling in \( \tau \to \mu P \)

\* Large \( \text{BR}(\tau \to \mu \rho) \) (\( \sim \exp \) bound). Next \( \tau \to \mu \phi \). Also \( \tau \to \mu \eta(\eta') \) in NUHM

\* \( \tau \to \mu \eta \) and \( \tau \to \mu \eta' \) largely dominated by \( A^0 \) Higgs contribution
Approx. formulae for LFV semilep. $\tau$ decays

Valid at large $\tan \beta$ and MI: agreement with full results within a factor of 2

\[
\text{BR}(\tau \to \mu \eta)_{H_{\text{approx}}} = 1.2 \times 10^{-7} |\delta_{32}|^2 \left( \frac{100}{m_{A^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6 \sim \frac{1}{7} \times \text{BR}_{\text{Sher PRD66}(2002)57301}
\]

\[
\text{BR}(\tau \to \mu \eta^\prime)_{H_{\text{approx}}} = 1.5 \times 10^{-7} |\delta_{32}|^2 \left( \frac{100}{m_{A^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6 \sim 100 \times \text{BR}_{\text{Brignole–Rossi NPB701(04)}}
\]

\[
\text{BR}(\tau \to \mu \pi)_{H_{\text{approx}}} = 3.6 \times 10^{-10} |\delta_{32}|^2 \left( \frac{100}{m_{H^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6 \sim \text{BR}_{\text{Brignole–Rossi}}
\]

\[
\text{BR}(\tau \to \mu \pi^0 \pi^0)_{H_{\text{approx}}} = 1.3 \times 10^{-10} |\delta_{32}|^2 \left( \frac{100}{m_{H^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6
\]

\[
\text{BR}(\tau \to \mu \pi^+ \pi^-)_{H_{\text{approx}}} = 2.6 \times 10^{-10} |\delta_{32}|^2 \left( \frac{100}{m_{H^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6
\]

\[
\text{BR}(\tau \to \mu K^+ K^-)_{H_{\text{approx}}} = 2.8 \times 10^{-8} |\delta_{32}|^2 \left( \frac{100}{m_{H^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6 \sim \frac{1}{50} \times \text{BR}_{\text{Chen–Geng PRD74(2006)}}
\]

\[
\text{BR}(\tau \to \mu K^0 \bar{K}^0)_{H_{\text{approx}}} = 3.0 \times 10^{-8} |\delta_{32}|^2 \left( \frac{100}{m_{H^0}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^6
\]

\[
\text{BR}(\tau \to \mu \pi^+ \pi^-)_{\gamma_{\text{approx}}} = 3.7 \times 10^{-5} |\delta_{32}|^2 \left( \frac{100}{M_{\text{SUSY}}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^2 \text{ dominant for all } M_{\text{SUSY}}
\]

\[
\text{BR}(\tau \to \mu K^+ K^-)_{\gamma_{\text{approx}}} = 3.0 \times 10^{-6} |\delta_{32}|^2 \left( \frac{100}{M_{\text{SUSY}}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^2 \text{ dominant if } M_{\text{SUSY}} \leq 300 \text{ GeV}
\]

\[
\text{BR}(\tau \to \mu K^0 \bar{K}^0)_{\gamma_{\text{approx}}} = 1.8 \times 10^{-6} |\delta_{32}|^2 \left( \frac{100}{M_{\text{SUSY}}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^2 \text{ dominant if } M_{\text{SUSY}} \leq 250 \text{ GeV}
\]

\[
\text{Compare to } \text{BR}(\tau \to \mu \gamma)_{\text{approx}} = 1.5 \times 10^{-2} |\delta_{32}|^2 \left( \frac{100}{M_{\text{SUSY}}(\text{GeV})} \right)^4 \left( \frac{\tan \beta}{60} \right)^2 > \text{semil if } M_{\text{SUSY}} < 1500 \text{GeV}
\]
Comparison approximate/full results (I)

- For $\text{BR}(\tau \rightarrow \mu \eta)$:
  - $m_N = (10^{10}, 10^{11}, 10^{14})$ GeV
  - $M_0 = M_{1/2} = 250$ GeV, $A_0 = 0$
  - $\theta_2 = 2.9e^{i\pi/4}$, $\theta_1 = \theta_3 = 0$
  - $\delta_1 = -2.4$, $\delta_2 = 0.2$

- For $\text{BR}(\tau \rightarrow \mu \pi^+ \pi^-)$:
  - $m_N = (10^{10}, 10^{11}, 10^{14})$ GeV
  - $M_0 = M_{1/2} = 250$ GeV, $A_0 = 0$, $\tan \beta = 50$
  - $\theta_2 = 2.9e^{i\pi/4}$
  - $\delta_1 = -2.4$, $\delta_2 = 0$

- For $\text{BR}(\tau \rightarrow \mu K^+ K^-)$:
  - $m_N = (10^{10}, 10^{11}, 10^{14})$ GeV
  - $M_0 = M_{1/2} = 250$ GeV, $A_0 = 0$, $\tan \beta = 50$
  - $\theta_2 = 2.9e^{i\pi/4}$
  - $\delta_1 = -2.4$, $\delta_2 = 0$
Comparison approximate/full results (II)

Full

Approx.

\( \text{BR (}\tau \rightarrow \mu \gamma) \)

\( m_N = (10^{10}, 10^{11}, 10^{14}) \text{ GeV} \)

\( A_0 = 0, \tan \beta = 50, \delta_2 = 2.9e^{-i/4} \)

\( \delta_1 = -2.4, \delta_2 = 0 \)

\( m_{N1} = 10^{10} \text{ GeV}, m_{N2} = 10^{11} \text{ GeV} \)

\( M_{\text{SUSY}} = 750 \text{ GeV} \)

\( \theta_2 = 2.9e^{-i/4}, \delta_1 = -2.4, \delta_2 = 0-0.2 \)

Approx. (60, 10^{15})

Approx. (50, 3 \times 10^{14})

Approx. (60, 10^{17})

Full (50, 10^{14})

Approx. (50, 10^{14})

Higgs (\tan \beta, m_{N3}(\text{GeV})) = (50, 10^{14})
Maximum sensitivity to Higgs sector found in $\tau \rightarrow \mu \eta$ and $\tau \rightarrow \mu \eta'$

$\text{BR}(\tau \rightarrow \mu \eta)$ at exp. bound for $m_{N_3} = 10^{15}$ GeV, $\tan \beta = 60$, $\theta_2 = 2.9e^{i\pi/4}$

Next relevant channel in sensitivity to Higgs sector is $\tau \rightarrow \mu K^+K^-$ (but still below exp. bound)
Results for $\mu - e$ conversion in nuclei
$\mu - e$ conversion in nuclei: CMSSM versus NUHM

First estimates of $\text{CR}(\mu - e, \text{Nuclei})$ did not include H-contrib. (Hisano et al PRD53(1996)2442)

CMSSM: $\gamma$ dominance for all $M_{\text{SUSY}}$

NUHM: $H^0$ dominance if $H^0$ light

$\star$ NUHM $\Rightarrow$ heavy SUSY spectra do not decouple in $\mu - e$ conversion due to H

$\text{CR}(\mu - e, \text{Au})$ above present experimental bound even for heavy SUSY
**Sensitivity to Higgs sector in $\mu-e$ conv. in nuclei**

- NUHM: Noticeable sensitivity to the Higgs sector if $H_0$ is light, due to large couplings of Higgs to strange quarks in nucleon/nuclei ($\propto m_s$)

- Ratio of $\mu-e$ to $\mu\rightarrow e\gamma$ can be a factor 10 larger in NUHM than in CMSSM

- Found useful approximate formula, if H-dominated, valid at large $\tan\beta$ and MI approx.

\[
\text{CR}(\mu-e, \text{Nucleus})_{H_{\text{approx}}} \simeq \frac{m_\mu^5 G_F^2 \alpha^3 Z_{\text{eff}}^4 F_p^2}{8\pi^2 Z} (Z+N)^2 \left| g_{LS}^{(0)} \right|^2 \frac{1}{\Gamma_{\text{capt}}},
\]

\[
g_{LS}^{(0)} = \frac{g^2}{48\pi^2} G_s^{(s,p)} \frac{m_\mu m_s}{m_{H_0}^2} \delta_{21}(\tan\beta^3)
\]

Numerical estimates of $\text{CR}(\mu-e, \text{Ti})_{H_{\text{approx}}}$ OK with Kitano et.al. PLB575(2003)300
**Future prospects for \( \mu - e \) conversion in nuclei**

**Degenerate** \( \nu_R \)

- \( \nu_R \) in degenerate mass region:
  - \( m_N = 10^{10} \text{ GeV} \), \( m_{N2} = 10^{11} \text{ GeV} \)
  - \( \theta_{13} = 5° \), \( \theta_i = 0 \)

- **Hierarchical** \( \nu_R \)
  - \( m_{N1} = 10^{10} \text{ GeV} \)
  - \( m_{N2} = 10^{11} \text{ GeV} \)

**Challenging:** if sensitivity \( \sim 10^{-18} \) reached: \( m_N \) down to \( 10^{12} \text{ GeV} \) will be tested

- CR(\( \mu - e \)) very sensitive to \( \theta_{13} \), mainly for hierarchical \( \nu_R \) (as \( \mu \rightarrow e\gamma \) and \( \mu \rightarrow 3e \))

A future measurement of \( \theta_{13} \) can help in searches of LFV in \( \mu - e \) sector
Semileptonic tau decays complement nicely the searches for LFV in $\tau - \mu$ sector, in addition to $\tau \rightarrow \mu \gamma$. The future prospects for $\mu - e$ conversion in Ti are the most challenging for LFV. Both processes allow to test the Higgs sector (better than $\tau \rightarrow 3\mu$), besides the SUSY and seesaw sectors.
Additional transparencies
Predictions for other SPS points

Similar for SPS1a,1b. Slightly worse prospects for SPS2,3. SPS5 the worst.

SPS4 the most restrictive one (due to $\tan \beta = 50$):

Present bounds from $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ already exclude $m_{N_3} \gtrsim 10^{14}$ GeV!!
Comparing predictions for various LFV decays

Hierarchical $m_{N_i}$ and complex $\theta_i$

$(m_{N_1}, m_{N_2}, m_{N_3}) = (10^8, 2 \times 10^8, 10^{14})$ GeV, $\arg(\theta_1) = 0, \pi/10, \pi/8, \pi/6, \pi/4$ ($\theta_2 = \theta_3 = 0$)

SPS 4

\begin{itemize}
  \item BRs for $0 < |\theta_i| < \pi$, $0 < \arg\theta_i < \pi/2$ can increase up to $10^2 - 10^4$ respect to $\theta_i = 0$
  \item BRs above present experimental bounds: mainly $\mu \to e\gamma$, $\mu \to 3e$ and $\tau \to \mu\gamma$
  \item Similar results for $\theta_2$. BRs nearly constant with $\theta_3$ in the case of hier. N's
\end{itemize}
Constraints from 'viable' BAU

BAU requires complex $R \neq 1$ $\Rightarrow$ complex $\theta_i \neq 0$. Most relevantly $\theta_2$

$n_B/n_\gamma \in \text{interval} \Rightarrow (\text{Re}(\theta_2), \text{Im}(\theta_2)) \in \text{area ('ring')}$ (WMAP in darkest ring)

Implications for LFV

☆ 'viable' BAU $\leftrightarrow n_b/n_\gamma \in [10^{-10}, 10^{-9}]$ (WMAP $\sim 6.1 \times 10^{-10}$,'06)

BAU [disfav]-[fav]-[disfav]-[fav]-[disfav] pattern in $0 < |\theta_2| < 3$

The BAU [fav] windows occur at small ($\neq 0$) $|\theta_2| \lesssim 1.5$

☆ smaller $|\theta_2|$ $\Rightarrow$ smaller LFV rates

☆ The existence, location and size of the windows depend on $m_{N_1}$

$m_{N_1} \sim O(10^{10})$ GeV BAU [fav] windows at $|\theta_2| \sim O(1) \text{ and } |\theta_2| \sim O(10^{-2})$

$m_{N_1} \sim O(10^{9})$ GeV only one window at $|\theta_2| \sim O(5 \times 10^{-1})$
\( \Delta a^{\text{SUSY}}_{\mu} \in [10^{-8}, 10^{-9}] \): compatible with \( a^\text{EXP}_\mu - a^\text{SM}_\mu = 3.32 \times 10^{-9} \) (3.8\sigma)
SUSY SPS points (I)

**SPS1a**

\[
\begin{align*}
M_0 &= 100 \text{ GeV} \\
M_{1/2} &= 250 \text{ GeV} \\
A_0 &= -100 \text{ GeV} \\
\tan \beta &= 10 \\
\mu &> 0
\end{align*}
\]

**SPS1b**

\[
\begin{align*}
M_0 &= 200 \text{ GeV} \\
M_{1/2} &= 400 \text{ GeV} \\
A_0 &= 0 \text{ GeV} \\
\tan \beta &= 30 \\
\mu &> 0
\end{align*}
\]

**SPS2**

\[
\begin{align*}
M_0 &= 1450 \text{ GeV} \\
M_{1/2} &= 300 \text{ GeV} \\
A_0 &= 0 \text{ GeV} \\
\tan \beta &= 10 \\
\mu &> 0
\end{align*}
\]
SUSY SPS points (II)

**SPS3**
- $M_0 = 90$ GeV
- $M_{1/2} = 300$ GeV
- $A_0 = 0$ GeV
- $\tan \beta = 10$
- $\mu > 0$

**SPS4**
- $M_0 = 400$ GeV
- $M_{1/2} = 300$ GeV
- $A_0 = 0$ GeV
- $\tan \beta = 50$
- $\mu > 0$

**SPS5**
- $M_0 = 150$ GeV
- $M_{1/2} = 300$ GeV
- $A_0 = -1000$ GeV
- $\tan \beta = 5$
- $\mu > 0$