DERIVATION OF STOCHASTIC ACCELERATION MODEL CHARACTERISTICS FOR SOLAR FLARES FROM
RHESSI HARD X-RAY OBSERVATIONS

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ABSTRACT

The model of stochastic acceleration of particles by turbulence has been successful in explaining many observed features of solar flares. Here, we demonstrate a new method to obtain the accelerated electron spectrum and important acceleration model parameters from the high-resolution hard X-ray (HXR) observations provided by RHESSI. In our model, electrons accelerated at or very near the loop top (LT) produce thin target bremsstrahlung emission there and then escape downward producing thick target emission at the loop footpoints (FPs). Based on the electron flux spectral images obtained by the regularized spectral inversion of the RHESSI count visibilities, we derive several important parameters for the acceleration model. We apply this procedure to the 2003 November 3 solar flare, which shows an LT source up to 100–150 keV in HXR with a relatively flat spectrum in addition to two FP sources. The results imply the presence of strong scattering and a high density of turbulence energy with a steep spectrum in the acceleration region.

Key words: acceleration of particles – Sun: flares – Sun: X-rays, gamma rays

Online-only material: color figures

1. INTRODUCTION

It is well established that the impulsive phase hard X-ray (HXR) emission of solar flares is produced by bremsstrahlung of nonthermal electrons spiraling down the flare loop while losing energy primarily via Coulomb collisions (Brown 1971; Hudson 1972; Petrosian 1973). Thus, HXR observations provide the most direct information on the spectrum of the radiating electrons and perhaps on the mechanism responsible for their acceleration. The common practice to extract this information has been to use the parametric forward fitting of HXR spectra to emission by an assumed radiating or injected electron spectrum (e.g., Holman et al. 2003), usually a power law with breaks and cutoffs (or plus a thermal component). A more direct connection was established between the observations and the acceleration process first by Hamilton & Petrosian (1992), fitting to high spectral resolution but narrowband observations (Lin & Schwartz 1987), and later by Park et al. (1997), fitting to broadband observations (e.g., Marschhäuser et al. 1994; Dingus et al. 1994). This was done in the framework of stochastic acceleration (SA) by plasma waves or turbulence.

However, it is preferable to obtain this information non-parametrically by some inversion techniques first attempted by Johns & Lin (1992). Recently, Piana et al. (2003) and Kontar et al. (2005) applied regularized inversion techniques to obtain the radiating electron flux spectra from the spatially integrated photon spectra observed by RHESSI (Lin et al. 2002). This is an important advance but it gives the spectrum of the effective radiating electrons summed over the whole flare loop, but not the spectrum of the accelerated electrons. This difference arises because high spatial resolution observations, first from Yohkoh (Masuda et al. 1994; Petrosian et al. 2002) and now from RHESSI (e.g., Liu et al. 2003), have shown that, essentially for all flares, in addition to the emission from the loop footpoints (FPs) (e.g., Hoyng et al. 1981), there is substantial HXR emission from a region near the loop top (LT). Thus, the total radiating electron spectrum is a complex combination of the accelerated electrons at the LT and those present in the FPs after having been modified by transport effects.

It is therefore clear that separate inversion of the LT and FP photon spectra to electron spectra would provide more direct information on the acceleration mechanism. More recently, Piana et al. (2007) have applied the regularized inversion technique to the RHESSI data in the Fourier domain (Hurford et al. 2002) to obtain electron flux spectral images. The goal of this Letter is to demonstrate that with the resulting spatially resolved electron flux spectra at the LT and FPs one can begin to constrain the acceleration model parameters directly.

In the following section, we present a brief review of the relation between the derived electron flux images and the characteristics of the SA model, and in Section 3 we apply this relation to a flare observed by RHESSI. A brief summary and our conclusions are presented in Section 4.

2. ACCELERATION AND RADIATION

The observations of distinct LT and FP HXR emissions, with little or no emission from the legs of the loop, point to the LT as the acceleration site and require enhanced scattering of electrons in the LT. Petrosian & Donaghy (1999) showed that the most likely scattering agent is turbulence which can also accelerate particles stochastically. In fact, SA of the background thermal plasma has been the leading mechanism for acceleration of electrons (e.g., Hamilton & Petrosian 1992; Miller et al. 1996; Park et al. 1997; Petrosian & Liu 2004; Grigis & Benz 2006; Bykov & Fleishman 2009) and ions (e.g., Ramaty 1979; Mason et al. 1986; Mazur et al. 1995; Liu et al. 2004, 2006; Petrosian et al. 2009), and is the most developed model in terms of comparing with observations.

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2.1. Particle Kinetic Equation

In this model, one assumes that turbulence is produced at or near the LT region (with background electron density \( n_{LT} \), volume \( V \), and size \( L \)). In the presence of a sufficiently high density of turbulence, the scattering can result in a mean scattering length or time (\( \tau_{\text{scat}} \)) smaller than \( L \) or the crossing time (\( \tau_{\text{cross}} = L/v \)), leading to a nearly isotropic pitch angle distribution (Petrosian & Liu 2004). The general Fokker–Planck equation for the density spectrum \( N(E) \) of the accelerated electrons, averaged over the turbulent acceleration region, simplifies to

\[
\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2}[D_{\text{EE}}N] - \frac{\partial}{\partial E}[\langle A(E) - \dot{E}_L(E) \rangle N] - \frac{N}{T_{\text{esc}}(E)} + \dot{Q}(E),
\]

(1)

where \( D_{\text{EE}}(E) \) and \( \langle A(E) \rangle \) are the energy loss rate and the direct acceleration rate by turbulence, respectively. \( \dot{E}_L \) is the particle energy loss rate, and \( \dot{Q}(E) \) and \( N(E)/T_{\text{esc}}(E) \) describe the rate of injection of (thermal) particles and escape of the accelerated particles from the acceleration region. For electrons of energies below \( \sim 1 \text{ MeV} \), which are of interest here, Coulomb collisions\(^3\) dominate the energy loss rate:

\[
\dot{E}_L = \dot{E}_{\text{Coul}} = 4\pi r_0^2 m_e c^3 n_{LT} \ln \Lambda/\beta,
\]

(2)

where \( \ln \Lambda \) is the Coulomb logarithm taken to be 20 for solar flare conditions. Following Petrosian & Liu (2004), we approximate the escape time as

\[
\tau_{\text{esc}}(E) \simeq \tau_{\text{cross}}(1 + \tau_{\text{cross}}/\tau_{\text{scat}}),
\]

which smoothly connects the two limiting cases of \( \tau_{\text{cross}}/\tau_{\text{scat}} \gg 1 \) and \( \ll 1 \). The mean scattering time is related to the pitch angle diffusion rates (Dung & Petrosian 1994; Pryadko & Petrosian 1997) due to both Coulomb collisions \( (D_{\text{mb}}^{\text{Coul}} \text{ and } \tau_{\text{scat}}^{\text{Coul}}) \) and turbulence \( (D_{\text{mb}}^{\text{turb}} \text{ and } \tau_{\text{scat}}^{\text{turb}}) \) as

\[
\tau_{\text{scat}}(E) = \frac{1}{8} \int_{-1}^{1} \frac{(1 - \mu^2)^2}{D_{\text{mb}}^{\text{Coul}}(\mu, E) + D_{\text{mb}}^{\text{turb}}(\mu, E)} d\mu.
\]

(3)

Similarly, we can define the scattering times \( \tau_{\text{Coul}}^{\text{Coul}} \) and \( \tau_{\text{scat}}^{\text{turb}} \) for each process alone. For Coulomb collisions, \( D_{\text{mb}}^{\text{Coul}} = \frac{2(1-\mu^2)}{\gamma+1} \frac{\dot{E}_{\text{Coul}}}{E} \). For turbulence, \( D_{\text{mb}}^{\text{turb}} \), like \( D_{\text{EE}} \), depends on the spectrum of turbulence and on the background plasma density, composition, temperature, and magnetic field (see Schlickeiser 1989; Dung & Petrosian 1994; Pryadko & Petrosian 1997, 1998; Petrosian & Liu 2004). Since these coefficients determine the spectrum of the accelerated electrons, one can then constrain some aspects of the acceleration mechanism if an accurate spectrum of the electrons can be derived from observations.

2.2. LT and FP Spectra

The accelerated electrons in the (LT) acceleration region with a flux spectrum \( F_{LT}(E) = v N(E) \) produce thin target bremsstrahlung emission (photons s\(^{-1}\) keV\(^{-1}\)):

\[
J_{LT}(\epsilon) = n_{LT} V \int_{\epsilon}^{\infty} F_{LT}(E) \sigma(\epsilon, E) dE,
\]

(4)

\[\text{For SA, } A(\epsilon) = D_{\text{EE}}(\epsilon)/E + dD_{\text{EE}}/dE, \text{ where } \frac{\dot{\epsilon}}{\epsilon} = (2 - \gamma^{-2})/(1 + \gamma^{-2}), \gamma = 1 + E/m_e c^2 = 1/\sqrt{1 - \beta^2} \text{ is the Lorentz factor, and } v = \beta \text{ is the electron velocity.}\]

\[\text{At higher energies, synchrotron loss must be included in } \dot{E}_L.\]

\[\text{where } \sigma(\epsilon, E) \text{ is the angle-averaged bremsstrahlung cross section (Koch & Motz 1959). The escaping electrons with flux } F_{FP}(E) = N(E)L/T_{\text{esc}} \text{ produce thick target bremsstrahlung emissivity (coming mostly from the FPs; see Petrosian 1973; Park et al. 1997):}\]

\[
J_{FP}(\epsilon) = nV \int_{\epsilon}^{\infty} F_{FP}(E) \sigma(\epsilon, E) dE,
\]

(5)

where \( n \) is the density and \( F_{FP} \) is the effective radiating electron flux spectrum at the FPs,

\[
F_{FP}(E) = vN_{FP} = \frac{v(E)}{E_L(n)} \int_{E}^{\infty} \frac{N(E')}{T_{\text{esc}}(E')} dE'.
\]

(6)

Since \( \dot{E}_L \propto n \), the FP photon spectrum is independent of density. In what follows, we evaluate Equations (5) and (6) using the LT density \( n_{LT} \).

2.3. Acceleration Model Parameters

Regularized spectral inversion of RHESSI count visibilities gives the electron visibilities (Piana et al. 2007), which can then be used to construct images of electron flux (multiplied by line-of-sight column density). From these images, we extract the spatially resolved spectra, \( F_{LT}(E) \) at the LT and \( F_{FP}(E) \) at the FPs. Thus, we can obtain the accelerated electron spectrum \( N(E) \) at the thin target LT. Also from differentiation of Equation (6) we derive the escape time as

\[
\tau_{\text{esc}}(E) = -\frac{\tau_L(E) F_{LT}/F_{FP}}{d\ln F_{FP}/d\ln \epsilon + 2/(\gamma + \gamma^*)} \equiv \tau_L(E)\xi(E),
\]

(7)

where the FP index \( \delta_{FP} = 1/(\gamma + \gamma^*) = -d\ln \sigma(E)/d\ln \epsilon \), and \( \tau_L(E) = E/\dot{E}_L \) is the Coulomb loss time at the LT. The function \( \xi(E) \) is an observable quantity representing the ratio \( T_{\text{esc}}/\tau_L \). In the above derivation, we have used the relativistic form of electron velocity \( v(\epsilon) \).

Given \( T_{\text{esc}}(E) \), from its relation to \( \tau_{\text{cross}} \) and \( \tau_{\text{scat}} \), we obtain the mean scattering time as

\[
\tau_{\text{scat}}(E) \simeq \tau_{\text{cross}}(1 + \tau_{\text{cross}}/\tau_{\text{scat}}),
\]

which is valid for \( T_{\text{esc}} > \tau_{\text{cross}} \). Disentanglement of \( \tau_{\text{scat}} \) from \( \tau_{\text{cross}} \) is complicated (Equation (3)) at energies when turbulence and Coulomb collisions contribute equally to \( \tau_{\text{scat}} \). However, if turbulence dominates the pitch angle diffusion, then to the first order we can write \( \tau_{\text{scat}}^{\text{turb}} \simeq \tau_{\text{scat}}^{\text{Coul}}(1 + \tau_{\text{cross}}/\tau_{\text{scat}}^{\text{Coul}}) \) and obtain some average value of \( D_{\text{mb}}^{\text{turb}} \). Furthermore, given \( N(E) \) we can in principle derive the other Fokker–Planck coefficients, namely \( A(\epsilon) \) and \( D_{\text{EE}} \) (see Equation (1)). Therefore, we can reach a consistent picture of the acceleration process due to turbulence and begin to make inroads into the spectrum and the nature of turbulence itself.

3. APPLICATION: THE 2003 NOVEMBER 3 FLARE

As a first demonstration, we apply our new procedure to the 2003 November 3 solar flare (X3.9 class) during the nonthermal peak, in which we find a hard LT source (extending above 100 keV in HXR) distinct from the thermal loop in addition to two FP sources.\(^4\) In Figure 1, we show the electron flux images

\[\text{\(^4\) Q. Chen & V. Petrosian (2010a, in preparation) present HXR observations of this flare and argue that the high-energy LT source should not be an artifact of the pulse pileup effect.}\]
up to 250 keV, which also show a loop at low energies, and one LT and two FPs at higher energies. In Figure 2 (top panel), we show the electron spectra \( N_{LT} E^2 F(E) \), where \( N_{LT} = n_{LT} L \) is the LT column density. The LT flux spectrum can be fitted by a power law with an index \( \delta_{LT} = 2.1 \) and \( \delta_{FP} = 2.8 \) below and above the break energy \( E_b = 91 \pm 3 \) keV. It is clear that the total radiating electron spectrum differs significantly from the (LT) accelerated electron spectrum.

Given the above LT and FP electron flux spectra, we derive the energy dependence of the escape time (Equation (7)). The LT density can be estimated as \( n_{LT} \simeq \sqrt{EM/L^3} \simeq 0.5 \times 10^{11} \) cm\(^{-3} \), where the LT size \( L \simeq 10^9 \) cm is obtained from the LT angular size, and the emission measure \( EM \simeq 0.2 \times 10^{59} \) cm\(^{-3} \) is obtained from spectral fitting of the LT thermal emission. As in Figure 2 (bottom panel), the escape times increase slowly with energy and can be fitted by either a power law,

\[
T_{esc}(E) = 0.3 \left( \frac{E}{100 \text{ keV}} \right)^\kappa, \quad \kappa = 0.83 \pm 0.10, \tag{8}
\]

or a broken power law with a break at \( E_b = 118 \pm 37 \) keV, and the indices \( \kappa_1 = 0.62 \pm 0.23 \) and \( \kappa_2 = 1.09 \pm 0.25 \). The fact that the escape time should be longer than the crossing time yields an upper limit on \( N_{LT} \), which is satisfied by the above LT density and size.

We then calculate the mean scattering time in the LT region. Except at the lowest energy, the Coulomb contribution is small so that the scattering time thus calculated can be attributed to turbulence. The scattering time due to turbulence (see Section 2.3) can be fitted by a power law above \( \sim 40 \) keV:

\[
\tau_{\text{scat}}^\text{turb} = 0.016 \left( \frac{E}{100 \text{ keV}} \right)^{-\lambda}, \quad \lambda = 1.90 \pm 0.14. \tag{9}
\]

4. SUMMARY AND DISCUSSION

In this Letter, we describe a new method to directly obtain the model parameters for SA of particles by turbulence in solar flares from regularized spectral inversion of the high-resolution \textit{RHESSI} HXR data (Piana et al. 2007). We have argued that particle acceleration takes place at or near the LT region. The accelerated electrons produce thin target emission at the LT region undergoing Coulomb collisions and producing thick target emission. In this model, the LT and FP electron spectra are connected by

(A color version of this figure is available in the online journal.)
the escape process from the LT region (Equation (6)), thus allowing us to determine the energy dependence of the escape time. Our method has the advantage that one can now constrain the model parameters uniquely rather than just satisfying the consistency between the model and the data as commonly done by forward fitting routines. This method can be applied to flares with simultaneous HXR emission from the LT and FP sources.

We have applied our method to the 2003 November 3 flare, in which we can obtain the electron flux images for both the LT and FPs up to 250 keV. The LT accelerated electron flux spectrum can be fitted by a power law, and the effective radiating flux spectrum at the FPs is better fitted by a broken power law. From these spectra we derive the energy variation of the escape time and the scattering time. As seen in Figure 2, the turbulence scattering time is relatively short and decreases with energy. A short scattering time may arise from a high density of turbulence energy \( E_\text{turb} \), with the exact relationship depending also on the magnetic field \( B \), and the spectral index \( q \) and minimum wave number \( k_{\text{min}} \) of turbulence. A high level of turbulence also implies efficient acceleration which generally means a flat spectrum for the accelerated electrons, which is the case for the current flare. The energy dependences of \( r_\text{esc} \) and \( D_{\text{HE}} \) are also a function of these characteristics of turbulence; at high energies, they are determined primarily by the spectral index of turbulence (see Dung & Petrosian 1994; Pryadko & Petrosian 1997, 1998, 1999; Liu et al. 2006).

For the usually assumed Kolmogorov \( q = 5/3 \) or Iroshnikov–Kraichnan \( q = 3/2 \) turbulence spectra, one expects the scattering time to increase with energy as \( E^{2-q} \), which translates into an escape time varying roughly as \( T_{\text{esc}} \propto 1/\sqrt{E} \) at high (but nonrelativistic) energies. The energy dependences of \( T_{\text{esc}} \) and \( r_{\text{esc}} \) obtained here require a steeper turbulence spectrum \( (q > 3) \) at high wavenumbers. Such a steep spectrum can be present beyond the inertial range where damping is important (e.g., Jiang et al. 2009). The electron energies and the wave-particle resonance condition determine the wavevector of the accelerating plasma waves. This relation depends primarily on the plasma parameter \( \alpha \propto \sqrt{\pi}/B \) (e.g., Petrosian & Liu 2004). Thus, given the magnetic field and plasma density we can determine the wavevectors for transition from the inertial to the damping ranges of turbulence.

It should, however, be emphasized that the results obtained here may not be representative of typical flares. More commonly flares have much softer LT emission, which would give an escape time decreasing (and scattering time increasing) with energy, consistent with a low level and a flat spectrum of turbulence.

The exact relation between the derived quantities \( (N(E), T_{\text{esc}}, r_{\text{esc}}, \text{and } r_{\text{turb}}) \) and the turbulence characteristics \( (E_{\text{turb}}, B, q, \text{etc.}) \) is complicated and depends on the angle of propagation of the plasma waves with respect to the magnetic field and other plasma conditions. In future, we will apply these procedures to more flares (Q. Chen & V. Petrosian, 2010b, in preparation) and deal with these relations explicitly.

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