Aquifer water abundance evaluation using a fuzzy-comprehensive weighting method

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Abstract. Aquifer water abundance evaluation is a highly relevant issue that has been researched for many years. Despite prior research, problems with the conventional evaluation method remain. This paper establishes an aquifer water abundance evaluation method that combines fuzzy evaluation with a comprehensive weighting method to overcome both the subjectivity and lack of conformity in determining weight by pure data analysis alone. First, this paper introduces the principle of a fuzzy-comprehensive weighting method. Second, the example of well field no. 3 (of a coalfield) is used to illustrate the method’s process. The evaluation results show that this method is can more suitably meet the real requirements of aquifer water abundance assessment, leading to more precise and accurate evaluations. Ultimately, this paper provides a new method for aquifer water abundance evaluation.

1. Introduction

Coal mining and tunnel construction inevitably expose, damage, and/or disturb roof aquifers. Although it will not induce water inrush when the process exposes the low water abundance zone, it is capable of causing severe water bursts in the high water abundance zone during mining [1]. Therefore, scientific and accurate evaluation of aquifer water abundance distribution features is essential for safe coal mine production and management [2-3]. There are many methods to evaluate questions that contain uncertain factors such as analytic hierarchy process (AHP), fuzzy mathematics, gray theory, entropic weight method, and a neural network method [4-9]. Many scholars have applied these methods in water abundance evaluation, but the assessment methods are still flawed [10-12]. This paper establishes an aquifer water abundance evaluation model that combines fuzzy evaluation with a comprehensive weighting method based on collecting the main controlling factors of aquifer water abundance. The study’s results serve as a theoretical guide for coal mine water inrush prevention and treatment.

2. Fuzzy evaluation method

2.1. Principle of fuzzy evaluation method

The fuzzy evaluation method, based on fuzzy mathematics, is one of the most widely used decision methodologies. It analyzes the relevant factors and their detailed attributions and then grades or classifies them by the maximum membership degree principle. The fuzzy evaluation method can consider many factors and the relationship between them. It is, in fact, a fuzzy mathematics of multi-
target decision-making method. The principle of fuzzy evaluation method is $A \cdot R = B$ (1), where $A$ is the fuzzy vector of factors, representing the weight of each factor set $X = \{x_1, x_2, \ldots, x_n\}$; and $R$ is the fuzzy matrix of factor sets and target set. The target set is defined as $U = \{u_1, u_2, \ldots, u_n\}$, $R = (r_{ij})_{n \times m}$, where $r_{ij}$ is the membership degree of $x_i$ to $u_i$, $i=1,2, \ldots, n$, $j=1,2, \ldots, m$. $B$ is the judging standard that represents the membership degree of the factor sets to target sets when taking a comprehensive consideration of all the factors. $\cdot$ represents matrix multiplication.

2.2. Membership function defining

For a specific fuzzy set, membership functions can operate and process the factor sets quantitatively. Based on the specific conditions of the study area, this paper opted for the Ridge membership function [13].

$$
\mu_{\text{IV}}(u) = \begin{cases} 
1 & u_1 < u \\
0.5 + 0.5 \sin \frac{2\pi}{u_1 - u_2} (u - \frac{3u_1 + u_2}{4}) & \frac{u_1 + u_2}{2} < u \leq u_1 \\
0 & u \leq \frac{u_1 + u_2}{2} 
\end{cases}
$$

(2)

$$
\mu_{\text{III}}(u) = \begin{cases} 
0 & u_1 < u \\
0.5 - 0.5 \sin \frac{2\pi}{u_1 - u_2} (u - \frac{3u_1 + u_2}{4}) & \frac{u_1 + u_2}{2} < u \leq u_1 \\
0.5 + 0.5 \sin \frac{2\pi}{u_1 - u_3} (u - \frac{u_1 + 2u_2 + u_3}{4}) & \frac{u_2 + u_3}{2} < u \leq \frac{u_1 + u_2}{2} \\
0 & u \leq \frac{u_2 + u_3}{2} 
\end{cases}
$$

(3)

$$
\mu_{\text{II}}(u) = \begin{cases} 
0 & u_1 < u \\
0.5 - 0.5 \sin \frac{2\pi}{u_1 - u_3} (u - \frac{u_1 + 2u_2 + u_3}{4}) & \frac{u_2 + u_3}{2} < u \leq \frac{u_1 + u_2}{2} \\
0.5 + 0.5 \sin \frac{2\pi}{u_2 - u_3} (u - \frac{u_2 + 3u_3}{4}) & u_3 < u \leq \frac{u_2 + u_3}{2} \\
0 & u \leq u_3 
\end{cases}
$$

(4)

$$
\mu_{\text{I}}(u) = \begin{cases} 
1 & 0 < u \leq u_3 \\
0.5 - 0.5 \sin \frac{2\pi}{u_2 - u_3} (u - \frac{u_2 + 3u_3}{4}) & u_3 < u \leq \frac{u_2 + u_3}{2} \\
0 & u \leq \frac{u_2 + u_3}{2} 
\end{cases}
$$

(5)
Where $\mu_i(u)$ is the membership function of each factor; $u$ is the actual value of each factor; $u_1, u_2, u_3$ are the standard threshold value of each grade (respectively). The value is determined according to related specification and criterion.

### 2.3. Factor weight definition

There are two methods to get the weight of each factor: the subjective weighting method and the objective weighting method. The subjective weighting method is easily affected by user experience (or lack thereof). When there are too many factors, it is too difficult for users to give an accurate judgment. With the objective weighting method, the weight of each index is obtained mainly by searching for potential links between initial data sets and then analysing any variation trends among them. The objective weighting method can avoid the influence of subjective factors, but it still has a few shortcomings. For instance, the weight of each index may be inconsistent with the actual conditions – in other words, the primary index may not get the largest weight by simple data analysis. A new comprehensive weighting method is proposed in this study; a method that remains as objective as possible and therefore overcomes the subjectivity of previous weighting methods.

#### 2.3.1. Subjective weight obtaining

Subjective weight is obtained by AHP. First, collecting the main controlling factors of the problem and classifying them into several hierarchies, where the primary factors belong to the lowest hierarchy, and the target object belongs to the highest hierarchy. An evaluation system and mathematical model are constructed, based on which a quantitative indicator or a scale indicator is to be established indicating the influence that the various factors have on their superior hierarchy. The user is then asked to grade for each factor; the weight of each factor can be obtained from the judgment matrix of each hierarchy. Finally, the highest hierarchy evaluation target is assessed and we can get the weight of each factor to meet the overall goal [14].

#### 2.3.2. Objective weight obtaining

Taking the most important factor as the reference factor, denoted as $X_0=(x_{10}, x_{20}, \ldots, x_{n0})^T$, the comparison factor denoted as $X_j=(x_{1j}, x_{2j}, \ldots, x_{nj})^T$ ($j=1,2, \ldots, m$), initializing $X_0$ and $X_j$ to obtain the factor matrix $X_0'=(x_{10}', x_{20}', \ldots, x_{n0}')^T$, $X_j'=(x_{1j}', x_{2j}', \ldots, x_{nj}')^T$ ($j=1,2, \ldots, m$) [15]. The relational coefficient of $X_0$ and $X_j$ is represented by

$$\rho_{ij} = \min \left( \min_{i=1}^{m} \frac{x_{i0}' - x_{ij}'}{x_{i0}' - x_{ij}'}, \max_{i=1}^{m} \frac{x_{i0}' - x_{ij}'}{x_{i0}' - x_{ij}'}, \frac{\rho}{\max_{i=1}^{m} \frac{x_{i0}' - x_{ij}'}{x_{i0}' - x_{ij}'}} \right)$$

The relational degree of reference factor $X_0$ and comparison factor $X_j$ are represented [16] by

$$x_j = \frac{1}{n} \sum_{i=1}^{n} x_j, \quad j = 1, 2, \ldots, m$$

The larger the value of $x_j$, the closer the relationship between the comparison factor and the reference factor, and the greater the weight of this factor compared to others [17].

The weight of each factor $a_j$ will be obtained by

$$a_j = x_j \sqrt{\sum_{j=1}^{n} x_j}, \quad j = 1, 2, \ldots, m$$

#### 2.3.3. Synthetic weight obtaining

Using multiple synthetic normalization methods to calculate the synthetic weight of each factor, the synthetic weight can be represented by:

$$\beta_j = (a_j + \omega_j)/2$$
Where $\alpha_j$ is objective weight, $\omega_j$ is objective weight, and $\beta_j$ is synthetic weight, ($j=1, 2, \cdots, m$).

### 2.3.4. Fuzzy-comprehensive weighting method model

The roof aquifer water abundance evaluation model combines the fuzzy comprehensive evaluation method with the synthetic weighting method [18]. This can be represented by:

$$B = \left[ \frac{(\alpha_j \cdot \omega_j)}{2} \left( \alpha_j \cdot \omega_j \right) / 2 \left( \alpha_j \cdot \omega_j \right) / \left( \alpha_j \cdot \omega_j \right) \right] [r_1, r_2, \cdots, r_m]$$

Where $\alpha_j$ is objective weight, $\omega_j$ is objective weight, and $r_{ij}$ is membership degree, $i=1,2,\cdots,n; j=1,2,\cdots,m$. According to the maximum membership degree principle, $B_j= \text{max} (B_1, B_2, \cdots, B_m)$ and the evaluation result is the grade corresponding to $j$.

### 3. Applications

To illustrate the process of the fuzzy-comprehensive weighting method, the no. 3 well field of a particular coal field was used as a case study. The study area is located in the Dongshe District of Erdos City. It shows an irregularity rectangle, covers an area of 125.65 km$^2$, and spreads 12.2 km from north to south and 10.3 km from east to west (on average). According to the hydrogeological prospecting data, the water-bearing medium can be divided into five groups: (I) the loose unconfined Quaternary aquifer, (II) the lower Cretaceous Zhidan group porous confined aquifer, (III) the middle Jurassic Zhiluo Formation fragmental rock porous-fissure confined aquifer, (IV) the mid-lower Jurassic Yan'an Formation fragmental rock porous-fissure confined aquifer, and (V) the upper Triassic Yanchang Formation fragmental rock porous-fissure confined aquifer. Mudstone and sandy mudstone in Anding Formation function as a stable water-resistant layer. Folds and faults structures are poorly developed, and the stratum trend to the west in general with an inclination of 1~3°. The no. 3 well field mainly mines 2-2# coal seam. The direct water filled aquifer, which threatens the mine’s safety, is the aquifer of Zhiluo formation above 2-2# coal seam. The aquifer is thick, rich in water abundance, and it covers the entire well field. The aquifer mainly consists of medium-grained sandstone; some sections are coarse-grained sandstone. The aquifer water abundance of the Zhiluo formation is to be evaluated using fuzzy-comprehensive weighting method.

#### 3.1. Classification of roof aquifer water abundance

After analyzing the geological and hydrogeological conditions of the study area, the main controlling factors of aquifer water abundance were obtained. These include specific capacity, hydraulic conductivity, aquifer thickness, flushing fluid consumption, the thickness ratio of friable rocks to plastic rocks, and the rock quality designation (RQD). Then the roof aquifer water abundance was divided into four grades: (I) low water abundance, (II) medium water abundance, (III) high water abundance, and (IV) higher water abundance. The National Industry Standard, Industry Specifications, and previous studies were also considered to establish the corresponding relationship between different factors and water abundance grade (shown in table 1).

| Factors                                  | Water Abundance Grade |
|------------------------------------------|-----------------------|
| Aquifer                                  | I         II    III   IV |
| Geographical                             | Specific capacity    | 0.001    0.01  0.1  1  |
| Hydrogeological                          | Hydraulic conductivity | 0.01    0.1  1  5  |
| Condition                                | Aquifer thickness    | 1       50  100  150 |
| Aquifer                                  | Flushing fluid consumption | 0.1    0.3  0.7  1  |
| Geological                               | Thickness ratio of friable rocks to plastic rocks | 0       1  10  20  |
| Condition                                | RQD                   | 0.6     0.7  0.8  0.9 |
3.2. Membership degree of each factor
The author selected 15 typical borehole datum in the study area (seen in table 2), each of them including six main controlling factors of aquifer water abundance. The membership degree of each factor was calculated using the fuzzy evaluation method of equation (2) to equation (5). For example, the membership degree of specific capacity in the area located by the borehole XJ-10 is (0, 0.013521953, 0.986478047, 0). Similarly, the membership degree of other factors can be obtained.

Then the fuzzy matrix R can be established.

Table 2. Main controlling factors and parameter values of the Zhiluo group aquifer.

| Drill name | specific capacity | Hydraulic conductivity | Aquifer thickness | flushing fluid consumption | thickness ratio of friable rocks to plastic rocks | Rock quality designation |
|------------|-------------------|------------------------|-------------------|---------------------------|-----------------------------------------------|------------------------|
| XJ-10      | 0.05045           | 0.07451                | 142.66            | 0.15                      | 9.634                                         | 0.7                    |
| XJ-11      | 0.07145           | 0.10445                | 113.37            | 0.12                      | 10.476                                        | 0.829                  |
| XJ-22      | 0.08771           | 0.13375                | 175.71            | 0.4                       | 7.02                                          | 0.518                  |
| XJ-23      | 0.0495            | 0.06033                | 119.67            | 0.38                      | 2.268                                         | 0.739                  |
| XJ-24      | 0.05995           | 0.09238                | 167.75            | 0.14                      | 0.841                                         | 0.711                  |
| XJ-25      | 0.06158           | 0.15521                | 99.65             | 0.66                      | 6.794                                         | 0.612                  |
| XJ-26      | 0.08158           | 0.12384                | 98.13             | 0.26                      | 20.861                                        | 0.743                  |
| XJ-27      | 0.07416           | 0.11203                | 54.19             | 0.67                      | 2.7                                           | 0.688                  |
| XJ-28      | 0.06516           | 0.07497                | 167.75            | 0.35                      | 6.826                                         | 0.778                  |
| XJ-29      | 0.06011           | 0.08313                | 195.08            | 0.23                      | 2.829                                         | 0.829                  |
| XJ-30      | 0.05622           | 0.06697                | 90.56             | 0.47                      | 1.619                                         | 0.836                  |
| XJ-31      | 0.0836            | 0.09183                | 135.52            | 0.31                      | 5.172                                         | 0.827                  |
| XJ-32      | 0.06933           | 0.13187                | 79.21             | 0.26                      | 1.904                                         | 0.865                  |
| XJ-40      | 0.0273            | 0.03054                | 86.6              | 0.11                      | 2.259                                         | 0.808                  |
| XJG-8      | 0.12402           | 0.25916                | 201.89            | 0.66                      | 4.078                                         | 0.603                  |

3.3. Weight of each factor
The subjective weight of each factor can be obtained via the AHP method. The objective weight can be obtained by equation (6) to equation (9). The synthetic weight uses the arithmetic mean of subjective weight and objective weight according to equation (10). Table 3 shows the results.

Table 3. Weights of aquifer water abundance controlling factors.

| Aquifer water abundance | Factors                     | Subjective Weight | Objective Weight | Synthetic Weight |
|-------------------------|-----------------------------|-------------------|------------------|------------------|
|                         | Specific capacity           | 0.3919            | 0.2057           | 0.2988           |
|                         | Hydraulic conductivity      | 0.1801            | 0.1815           | 0.1808           |
|                         | Sandstone thickness         | 0.1949            | 0.1629           | 0.1789           |
|                         | Flushing fluid consumption  | 0.0738            | 0.1392           | 0.1065           |
|                         | Thickness ratio of friable rocks to plastic rocks | 0.0872 | 0.1704 | 0.1288 |
|                         | Rock quality designation    | 0.0721            | 0.1401           | 0.1061           |

3.4. Water abundance evaluation result
Adopting a fuzzy-comprehensive weighting method model and equation (11), the standard that represents the membership degree of the factor sets to target sets is obtained. Then, according to the maximum membership degree principle, fourteen of the fifteen boreholes’ water abundance belong to high water abundance grade III, while XJG-8 borehole belongs to the middle water abundance grade II.
4. Discussion
This paper proposes a new revision of the fuzzy-comprehensive weighting method to evaluate aquifer water abundance. This method is an improvement over previous methods in at least two ways. First, it employs a fuzzy method that can evaluate the aquifer water abundance in a quantity that can also include some qualitative factors. Second, the weight of each main controlling factor considered both the subjective and the objective weights. It is a more scientific and accurate method than prior methods for aquifer water abundance evaluation. However, there are some deficiencies in the evaluation method; to circumvent these, the evaluation standard must be created in conditions where there is no standard threshold value of each grade. This point will be further examined in future research.

5. Conclusion
This paper establishes an improved aquifer water abundance evaluation model by combining fuzzy evaluation with a comprehensive weighting method. The method’s main objective is to collect the main controlling factors of aquifer water abundance. It provides a more effective model of aquifer water abundance evaluation for other areas.

This paper also proposes the application of a synthetic weighting method to factor weight calculation in aquifer water abundance evaluation. This synthetic weighting method overcomes not only the subjectivity but also the lack of conformity of subjective weighting methods, and therefore becomes more suitable to meet the actual requirements of aquifer water abundance evaluation.

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