Strength analysis of composite plates with auxetic honeycomb at static bending by the finite element method

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Abstract. In this paper, three-layer composite plates with continuous outer layers and an auxetic honeycomb interlayer of the chiral type based on rotating circles with tangentially attached rods are modeled. The physical and mechanical properties of the D16 aluminum alloy were chosen as the material properties of the layers of the composite plates. In the course of modeling, the discretization and volume of honeycomb structures in composites were varied at a constant thickness of the layers. Under the conditions of static bending of composite plates, load values were determined, at which maximum stresses in composites were equated to the conditional yield strength. The problem was solved using the finite element method in the framework of the theory of elasticity. The aim of this work is to study the influence of the discretization parameter and volume of the auxetic honeycomb interlayer on the magnitudes of load at yield strength stresses for the proposed composite plates.

1. Introduction
Sandwich composites based on honeycomb structures have a number of advantages over panels of continuous cross-section. In terms of structural applications, sandwich composites demonstrate primarily a reduced mass and sufficient strength, the value of which can be pre-determined by a rational choice of the cell geometry and the relative density of the honeycomb structure [1]. The geometric shape of the cells can form the auxetic behavior of the honeycomb structure, i.e. form a negative Poisson's ratio in the plane, the presence of which entails a number of useful advantages [2, 3]. It is known that auxetic materials have improved indentation resistance, high shear resistance and fracture toughness [3-5], synclastic curvature in bending [6], increased energy absorption [7], and others [3]. When developing sandwich composites with auxetic honeycomb, two variants of orientation of the plane with auxetic behavior are most often used: parallel to the plane [8, 9] or perpendicular to the plane [10, 11] of the composite panel, which directly affects the mechanical properties. The most popular method of manufacturing auxetic honeycomb structures is 3D printing. In this paper, we consider models of three-layer composite plates (figure 1), which include two continuous layers and a honeycomb auxetic interlayer of the chiral type based on rotating circles with tangentially attached rods. The plane, wherein the auxetic behavior of the honeycomb structure is manifested, is parallel to the plane of the composite plate.
To carry out numerical experiments, five chiral structures with different discretization were constructed, where the diameter of the circles \(d\) varied from 0.7 to 1.9 mm in increments of 0.3 mm (figure 2), and the side of the square grid cells \(a\), in the nodes of which the centers of the circles are located, was determined by the coefficient \(a = 1.6d\). The volume of honeycomb structures varied by changing the thickness of their walls \(t\). As general values, the magnitudes of the chiral structure volume with \(d = 0.7\) mm were established with changing the wall thickness \(t\) from 0.05 to 0.4 mm by the increments of 0.0875 mm. The obtained volume values do not lead to the loss of the geometric shape of chiral structures. The total thickness of the composite plate is 2 mm, the thickness of the outer layers and the honeycomb structure is 0.5 mm and 1 mm, respectively. Thus, 25 models of composite plates with five variants of the scale of cells of the honeycomb structure and five variants of the volume for each structure with a certain discretization were constructed. In so doing, 11 models of continuous plates were constructed for numerical experiments by varying the plate thickness from 1 to 2 mm by 0.1 mm increments.

Figure 2. Chiral honeycomb structure based on rotating circles.

2. Calculation model
The analysis of plate strength was carried out in the framework of the elasticity theory by the finite element method in the «COMSOL Multiphysics 5.5» numerical modeling system using the «Structural mechanics» module [12]. A model of a linear elastic body was used to describe the material properties. The plates were fixed by prohibiting the movement of nodes \(u_x, u_y, u_z = 0\) of finite element models which are located on shaded areas (figure 3). \(u_{x,y,z} (0 \leq x \leq x_1, y = 0, 0 \leq z \leq b) = 0\), \(u_{x,y,z} (x_2 \leq x \leq a, y = 0, 0 \leq z \leq b) = 0\), \(u_{x,y,z} (0 \leq x \leq x_t, y = t, 0 \leq z \leq b) = 0\), \(u_{x,y,z} (x_2 \leq x \leq a, y = t, 0 \leq z \leq b) = 0\), \(a = 54\) mm, \(b = 13\) mm, \(t = a/2\), \(x_1 = 12\), \(x_2 = 42\), \(z_1 = 1\), \(z_2 = b - 1\). In case of calculation of composite plates, the external load was uniformly distributed over the nodes on a rectangular area \((l - 0.5 \leq x \leq l + 0.5, y = t, z_1 \leq z \leq z_2)\) with the width of 1 mm and the length of \((b - 2)\) mm. This condition allows one to exclude the false stress concentration at the lateral boundaries of the composite with a smaller width of the interlayer.
Figure 3. The boundary conditions of the plate for three-dimensional problem of theory of elasticity.

In the case of calculating continuous plates, the external load \( F \) was uniformly distributed on a straight-line segment \( \{ x = l, y = t, 0 \leq z \leq b \} \). When constructing a finite element mesh for a continuous plate, quadrilateral prisms were used. Discretization into quadrilateral prisms was performed by creating a two-dimensional mesh of quadrilaterals on the edge of the plate with subsequent broaching (figure 4 (a)). Discretization of composite plates into finite elements was carried out separately for each layer in a similar way when using triangular prisms (figure 4 (b)). The condition of continuity of field variables is established at the layer interfaces of composite plates.

Figure 4. Finite element mesh of three-dimensional models: (a) elements of a continuous plate, (b) elements of a composite plate.

The physical and mechanical properties of aluminum alloy D16 [13] were used as the material properties of composite and continuous plates: elastic modulus \( E = 72 \) GPA, Poisson's ratio \( \mu = 0.33 \), density \( \rho = 2780 \) kg m\(^{-3}\), and conditional yield strength \( \sigma_{0.2} = 290 \) MPa. Under the static bending of composite and continuous plates, the load values \( F \) (N) were determined, at which the maximum stresses according to the Mises criterion were equated to the conditional yield strength \( \sigma_{\text{max}} = \sigma_{0.2} \).

In order to verify the results of calculations of the «COMSOL Multiphysics 5.5» system, additional calculations of continuous plates were performed by the finite element method in displacements using the algorithm for solving a plane problem based on the known equations of elasticity theory [14, 15].
The algorithm of the method was adapted to the calculation of plates, where the stiffness matrix of the finite element $k^e$ is determined by the expression

$$k^e_{i,s} = k^E_{i,s} + k^G_{i,s}$$

(1)

where $k^E$ is the submatrix of normal deformations (2), $k^G$ is the submatrix of shear deformations (3), $r$ and $s$ are numbers of the blocks of the matrices ($r = 1, 2, ..., 4$, $s = 1, 2, ..., 4$),

$$k^E_{i,s} = \frac{Eh}{4(1-\mu^2)} \begin{bmatrix} \gamma \xi_s \xi_s \left(1 + \eta_i \eta_s \right) \frac{3}{3} & \mu \eta_i \xi_s \\ \mu \eta_i \xi_s & \eta_i \eta_s \left(1 + \frac{\xi_s \xi_s}{3} \right) \end{bmatrix}$$

(2)

$$k^G_{i,s} = \frac{Gh}{4} \begin{bmatrix} \eta_i \frac{3}{3} & \eta_i \xi_s \\ \eta_i \xi_s & \gamma \xi_s \xi_s \end{bmatrix}$$

(3)

$E$ is the longitudinal elastic modulus, $G = E/2(1+\mu)$ is the shear modulus, $\mu$ is the Poisson's ratio, $\gamma = b/a$, $a$ and $b$ are dimensions of the sides of a rectangular (finite) element along the $x$- and $y$-axes, respectively, $h$ is the size of the finite element on the $z$-axis, $\xi$ and $\eta$ are dimensionless coordinates of a rectangular element, $\xi_1 = -1$, $\xi_2 = 1$, $\xi_3 = 1$, $\xi_4 = -1$, $\eta_1 = -1$, $\eta_2 = -1$, $\eta_3 = 1$, $\eta_4 = 1$.

The matrix of matching the global numbers of nodes to the local numbers $N$ is constructed according to the principle $N_{m,i} = r$, $r \in 1, 2, ..., 4$, where $m$ is the global node number (figure 5 (a)), $i$ is the number of the finite element (figure 5 (b)), and $r$ is the local node number of the $i$-th finite element (figure 6), if $N_{m,i} \not\in r$, then $N_{m,i} = 0$.

**Figure 5.** The finite element scheme of the plate: (a) global node numbering, (b) finite element numbering.

**Figure 6.** The scheme of the local node numbering of the finite elements.
The extended stiffness matrix $k^{\text{ex}}$ is constructed using the $k_{m,n}^{\text{ex}}(i) = k_{i,s}$ principle, where $r = N_{m,j}$, $s = N_{n,j}$, $m,n \in 1,2,\ldots,c$, $i$ is the number of the finite element, and $c$ is the number of matrix rows $N$, if $r \vee s = 0$ then $k_{i,s} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

The stiffness matrix of the finite element model $K$ is determined by summing the extended stiffness matrices $K = \sum_{i} k^{\text{ex}}(i)$. To account for the external fixation of a node in a finite element model, it is necessary to delete the rows $i_1 = 2m-1$, $i_2 = 2m$ and columns $j_1 = 2m-1$, $j_2 = 2m$ of the stiffness matrix $K$, where $m$ is the global node number.

Node displacements are defined by the expression
\begin{equation}
  u_a = K_a^{-1}P_a
\end{equation}

where $K_a^{-1}$ is the inverse matrix with respect to the stiffness matrix of the model considering the fixed nodes, $P_a$ is the vector of nodal forces, $P_a = \{P_{a_1}, P_{a_2}, \ldots, P_{a_n}\}$ (here and further matrix-row in curly braces means the matrix-column), $n = 2(c-p)$, $P_{a_1}$ and $P_{a_2}$ are the nodal forces along the $x$- and $y$-axes, respectively, $c$ is the number of matrix rows $N$, and $p$ is the number of fixed nodes.

The full displacement vector $u = \{u^{i_1}_a, u^{i_2}_a, \ldots, u^{n_n}_a\}$ is the submatrix of $u$, where $u_{a_b} \neq 0$, $b = 2(c-p)$, $c$ is the number of matrix rows $N$, and $p$ is the number of fixed nodes. The displacement vectors along the $x$- and $y$-axes are defined by the expressions $x = u_a$ and $y = u_a$, respectively, where $m = 2i-1$, $n = 2i$, $i = 1,2,\ldots,s/2$, $u$ is the full vector of displacements, and $s$ is the number of matrix rows in $u$.

The displacement vector of $v$ nodes of the $i$-th finite element is constructed according to the $v_i = \{x_{i_1}, y_{i_1}, x_{i_2}, y_{i_2}, x_{i_3}, y_{i_3}, x_{i_4}, y_{i_4}\}$ principle, where $N_{m,j} = 1$, $N_{m^j,j} = 2$, $N_{m^j,j} = 3$, $N_{m^j,j} = 4$, $x_{m^j}$ and $y_{m^j}$ are the nodal displacements along the $x$- and $y$-axes, respectively.

The strain vector of $\varepsilon$ of the $i$-th finite element is defined by the expression
\begin{equation}
  \varepsilon(i,\xi,\eta) = \beta(\xi,\eta) \cdot v_i
\end{equation}

where $\beta(\xi,\eta)$ is the matrix of the relationship between nodal displacements and strains,
\begin{equation}
  \beta(\xi,\eta) = \frac{1}{2}
  \begin{pmatrix}
  b(1,\eta) & 0 & b(2,\eta) & 0 & b(3,\eta) & 0 & b(4,\eta) & 0 \\
  0 & a(1,\xi) & 0 & a(2,\xi) & 0 & a(3,\xi) & 0 & a(4,\xi) \\
  a(1,\xi) & b(1,\eta) & a(2,\xi) & b(2,\eta) & a(3,\xi) & b(3,\eta) & a(4,\xi) & b(4,\eta)
  \end{pmatrix}
\end{equation}

with $a(r,\xi) = \eta, \eta(1+\xi,\eta)/b$, $b(r,\eta) = \xi, (1+\eta,\eta)/a$, $r = 1,2,\ldots,4$, $a$ and $b$ are dimensions of the sides of a rectangular element.

The nodal stress vector $\sigma$ of the $i$-th finite element is defined by the expression
\begin{equation}
  \sigma(i,\xi,\eta) = \chi \varepsilon(i,\xi,\eta)
\end{equation}

wherein $\chi$ is the matrix of elastic constants, $\xi = \xi, \eta = \eta, r = 1,2,\ldots,4$, and
For the plane problem, the plates were fixed by prohibiting the displacements of nodes $u_x, u_y, u_z = 0$ of finite element models. Fixed nodes are located on shaded areas (figure 7).

\[
\chi = \frac{E}{1-\mu^2} \begin{pmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]  

(8)

For the plane problem, the plates were fixed by prohibiting the displacements of nodes $u_x, u_y, u_z = 0$ of finite element models. Fixed nodes are located on shaded areas (figure 7).

**Figure 7.** Boundary conditions of continuous plates for the plane problem of elasticity theory.

For continuous plates with the thickness of $t = 1.5$ and $t = 2$ mm, the results of calculations via the «COMSOL Multiphysics 5.5» system were verified using the algorithm for solving a plane problem based on the finite element method. In the case of the plane problem for the plate at $t = 1.5$ mm, the finite element scheme shown in figure 5 was used, for the plate with $t = 2$ mm, a similar scheme was used for the number of rectangular elements equal to $i = 54$. As a result of calculations using the algorithm for solving the plane problem, matrices with scalar values of stresses in local nodes of finite elements are obtained. Using the contour graph for the stress matrix of the plate with the thickness $t = 1.5$ mm at $\sigma_{max} = \sigma_{0.2}$, the stress distribution diagram was constructed (figure 8 (a)). In the system «COMSOL Multiphysics 5.5» a similar stress distribution diagram for a plate at $t = 1.5$ mm was obtained (figure 8 (b)). Load value $F_y$ at $\sigma_{max} = \sigma_{0.2}$ as a result of calculation using the algorithm for solving a plane problem for a plate with $t = 1.5$ mm and $t = 2$ mm is equal to 355.7 N and 627 N, respectively, while those obtained via «COMSOL Multiphysics 5.5» are 366.4 N and 660.5 N, respectively. Thus, the difference between the calculation results obtained using the «COMSOL Multiphysics 5.5» system and the algorithm for solving the plane problem for a plate with $t = 1.5$ mm and $t = 2$ mm is 3 % and 5.3 %, respectively. Based on the results of the analysis of the strength of composite and continuous plates, the plate’s solid body volume $V$ (mm³) dependence of the load magnitude $F_y$ at $\sigma_{max} = \sigma_{0.2}$ has been obtained and graphically presented in figure 9.
Figure 8. Stress distribution diagrams for a continuous plate with the thickness of $t = 1.5$ mm at $\sigma_{\text{max}} = \sigma_{0.2}$ obtained by the finite element method using two approaches: (a) the algorithm for solving a plane problem, (b) via «COMSOL Multiphysics 5.5».

![Stress Distribution Diagrams](image)

Figure 9. Diagram of variation in magnitude of load $F_y$ at $\sigma_{\text{max}} = \sigma_{0.2}$ for continuous and composite plates as function of the solid body volume of the plate.

Vertical line-markers correspond to the volume values for composite plates, with the exception of the straight line $V = 1404$ mm$^3$, which is in line with the volume value for the continuous plate with $t = 2$ mm. The graphs show that there is a region with a significant difference in load values $F_y$ at...
The magnitudes of $\sigma_{\text{max}}$ for composite plates relative to a continuous plate with the same volume of solid body $V$. The magnitudes of $F_y$ for composite plates during the discretization of chiral honeycomb with $d = 1.3, 1.6$ and $1.9$ at the same volume $V$ have an insignificant difference within the error range of the finite element method. The use of an auxetic honeycomb interlayer of the chiral type (figure 2) in the manufacture of three-layer composite plates (figure 1) could significantly reduce the consumption of the material used, and therefore decrease the weight of composite plates with an insignificant reduce in strength (figure 9) under static bending conditions (figure 3). With a decrease in the volume of the solid body of honeycomb interlayers, an advantage in strength is observed for composite plates with a lower discretization of chiral structures (figure 9). The considered composite plates can be manufactured using additive technologies.

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