Mathematical modeling of autonomous single-stage electro-hydraulic servo drive

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Annotation. The study is devoted to mathematical modeling of an autonomous single-stage electro-hydraulic servo drive. Two structural schemes of the drive are considered — simplified and more complex. The drive transient conditions of these two circuits are given.

Introduction
Currently, hydraulic steering machines are widely used in various technical sectors, including those used in aviation and space-rocket hardware. This circumstance is due to several advantages of hydraulic drives in comparison with electric and gas:

- high level of working fluid pressure (up to 56 MPa and more),
- low compressibility of the working fluid,
- significant energy intensity (specific power),
- high speed (dynamic properties),
- relatively small mass and size of hydraulic units.

The disadvantages of hydraulic drives include the complexity of installation, operation and the high cost of a hydropower source, the influence of temperature on the characteristics, technological limitations.

Steering machines according to the type of energy used are divided into manual, gas, electric and hydraulic. Various combinations of these basic types are often used, for example electro-hydraulic. [1]

A steering engine is an electro-hydraulic unit consisting of a hydraulic motor, a hydrodistributor and an electro-hydraulic power steering and designed to transmit, amplify and convert an electrical signal containing control information into the mechanical movement of the rudders or controls. [2]

According to the type of control signal, the hydraulic power actuators are divided into amplifiers with a mechanical and electrical control signal. There are various types of feedbacks in electro-hydraulic servo mechanisms. Rigid feedback on the output coordinate is widely used, which has high tracking accuracy and stability against oscillations.

According to the design of the control element, the hydraulic power actuators is divided into amplifiers with throttling valves of the spool type; type nozzle-flapper; with jet tube; rotary (crane) type; with needle throttle. The most widespread are spool-type hydraulic power steering with a throttle nozzle, and to a lesser extent, with a jet tube.
According to the number of cascades of amplification, the hydraulic power amplifiers is divided into single, double and multi-stage. Single-stage power steering is used when the output requires a relatively small power. In them, the control signal acts on the movable element of the control device. If you want to create more power at the output, then the power steering is included in the hydraulic system with high pressure and flow rate of the working fluid. This leads to an increase in the design dimensions of the power actuator and the power of the control signal, as well as to a decrease in the sensitivity of the power steering. In this case, it is more advantageous to use two- or multi-stage hydraulic power amplifiers in which the power steering of the first amplification stage with a low flow rate and liquid pressure controls the operation of the power steering of the second stage with a higher flow rate and liquid pressure. Thanks to the use of two- and multi-stage amplifiers, it becomes possible to increase the output power with a low-power control signal. [3]

At present, analog electrohydraulic steering machines are widely used as executive bodies of aircraft motion control systems.

A single-stage steering machine is one of the simplest and most common steering machines and is often used to control the direction of the engine relative to two axles [4].

The steering machine consists of a casing, with an electric motor mounted on it and a pump located inside the casing, a control unit, a power mechanism with a feedback sensor. The output rod of the power mechanism moves the controls in accordance with the commands of the control system.

The operation principle of the steering machine (SM) will be considered according to the electrohydraulic circuit diagram (Fig. 1).

![Fig. 1. Schematic diagram of the drive](image)

The electric motor 1 drives the three-gear pump 2, which creates flows of the working fluid directed to the spool plungers 7. In the absence of a signal supplied from the electronic power steering, the liquid flows through the windows opened by the spool plungers. Due to the equality of the area of the windows, the pressure difference in the cavities of the hydraulic cylinder 10 is zero and the piston 11
together with the rod 12 are stationary. In the presence of a signal in the form of a voltage at the ends of the winding of an electromechanical converter (EMC), the rocker 5 rotates clockwise or counterclockwise, depending on the polarity of the signal. The rotation of the rocking chair causes the movement of the spool plungers, which increase the opening of one window and reduce the opening of another. Accordingly, the pressure in one cavity of the hydraulic cylinder decreases, and in the other it increases. Under the action of the force created by the pressure difference in the hydraulic cylinder, the piston 11 moves until the feedback signal from the sensor 13 does not compensate for the input signal. Safety valves 9 installed on pressure lines of the pump limit the excessive pressure increase in the hydraulic cylinder. Elements 3 to 8 form a single-stage hydraulic power steering (HPS) [5].

1. The mathematical model of the hydraulic drive

In the modeling process, the following assumptions were made [5 - 7].

- Since the range of working fluid pressure changes in the cavities of the SM is small ( < 50 kgF/cm²), the dependences of the density, viscosity and bulk modulus of the working fluid on pressure are not taken into account, and their values are assumed to be constant.
- Due to the short duration of the studied processes, the temperature of the working fluid is assumed to be constant.
- Since there is a temperature compensator in the drain cavity of the SM case, the pressure in the drain cavity of the SM is assumed to be constant and equal to atmospheric
- Since the Reynolds number of working fluid flows in the extension orifices of the SM spool hydraulic power steering exceeds 2000 (turbulent flow regime), the flow coefficient of the extension orifices is assumed to be constant.
- Since the length of the connecting pipelines of the SM is not large, their hydraulic resistance is assumed to be small and not taken into account.
- The working edges of the spool plungers and throttle orifices of the sleeves spool hydraulic power steering are ideally sharp; the design of the hydraulic power steering is assumed to be symmetrical, and the dimensions of the extension orifices are exactly the same.
- The resulting non-stationary component of the hydrodynamic force due to the symmetrical design of the hydraulic power steering is assumed to be negligible and not taken into account.
- We neglect the hydrodynamic and hydrostatic forces acting on the spool plungers due to smallness
- Leaks of the working fluid through the safety valves of the SM are assumed to be small and are not taken into account in the mathematical model of the SM.
- The mass of the moving parts of the hydraulic cylinder is neglected in comparison with the reduced mass of the load.
- The force of internal friction in the hydraulic cylinder is taken into account in the sum of the resistance forces of the reduced load.
- The direction of movement of the body of the power hydraulic cylinder SM is back to the direction of movement of its piston.

1.1 The mathematical model of an electro-hydraulic power steering (EHPS)

The equation of the voltage balance in the electric circuit of an electromechanical converter (EMC) SM (in a polarized relay):

\[
L \cdot \frac{dI_k}{dt} + K_e \cdot \frac{d\alpha}{dt} + R \cdot I_k = U_k, \quad (1)
\]

where:
- \(U_k\) — command voltage supplied to the EMC winding;
- \(I_k\) — current flowing through the EMC winding; \(R\) — active resistance of the EMC winding;
- \(L\) — inductance of the EMC winding; \(\alpha\) — angle of rotation of the EMC roller;
- \(K_e\) — coefficient of electromagnetic high-speed communication [V/s/rad].

Transform the equation according to Laplace:
\[
\frac{I_s(T_i \cdot s + 1)}{1/R} = U - K_e \cdot s \cdot \alpha.
\]

where \( T_i = \frac{L}{R} \).

The equation of the balance of moments reduced to the EMC shaft:

\[
J \cdot \frac{d^2 \alpha}{dt^2} + M_{\text{dry}}^d \cdot \text{sign} \left( \frac{d\alpha}{dt} \right) + K_{\text{nv}} \cdot \frac{d\alpha}{dt} + K_{\text{mp}} \cdot \alpha + R_\alpha \cdot F_g = \left( \frac{K_{\text{mi}}}{K_p} \right) \cdot I_s;
\]

where: \( J \) — moment of inertia of the moving parts of the EHPS reduced to the EMC shaft;
\( R_\alpha \) — the arm of the power steering spool;
\( F_g \) — the resulting hydrodynamic force and the stationary component of the hydrodynamic force;
\( M_{\text{dry}}^d \) — the moment of dry friction of motion, reduced to the EMC;
\( K_{\text{nv}} \) — coefficient of viscous friction moment reduced to the EMC shaft;
\( K_{\text{mp}} \) — stiffness coefficient of the spring connected to the EMC shaft;
\( K_{\text{mi}} \) — coefficient of steepness of the moment characteristic of the electromagnetic field;
\( K_p \) — coefficient of alignment of dimensions (\( K_p = 0.0981 \)).

Applying the assumptions, we obtain

\[
J \cdot \frac{d^2 \alpha}{dt^2} + K_{\text{mv-e}} \cdot \frac{d\alpha}{dt} + K_{\text{sm}} \cdot \alpha = \left( \frac{K_{\text{mi}}}{K_p} \right) \cdot I_s;
\]

where: \( K_{\text{mv-e}} \) — equivalent coefficient of the moment of viscous friction.

Transform the equation according to Laplace:

\[
\alpha \cdot \left( \frac{J}{K_{\text{mp}}} \cdot s^2 + \frac{K_{\text{mv-e}}}{K_{\text{mp}}} \cdot s + 1 \right) = \left( \frac{K_{\text{mi}}}{K_p K_{\text{mp}}} \right) \cdot I_s; \tag{2}
\]

The equation of the relationship of the angular displacement of the EMC roller with the linear displacement of the spool plunger:

\[ X_3 = R_\alpha \cdot \alpha. \tag{3} \]

Based on equations (1–3), a structural diagram of the EHPS was constructed: [4, 5].

1.2 Linearized hydraulic drive model (simplified)

The equation of the family of flow-differential characteristics of the EHPS: [7]

\[
Q = K_{QH} X_3 - K_{Qp} \Delta p
\]

where: \( \Delta p = p_1 - p_2 \) — pressure difference between the cavities of the SM;
\( Q \) — low rate of the working fluid between the cavities of the SM.
The coefficients $K_{QX}$ and $K_{Qp}$ are determined by approximating the flow-differential characteristics. The equation of the balance of costs when moving the piston of the power cylinder

$$Q = S_n \frac{dY_p}{dt} + V_o \frac{d(\Delta p)}{2E_c \; dt},$$

where: $Y_p$ — displacement of the piston rod of the power hydraulic cylinder SM;

$$E_c = \frac{B_g}{1 + \frac{V_l}{Vol} + 2 \cdot \frac{F_c^2 \cdot B_g}{V_o \cdot c_{op}}},$$

$V_o$ — the effective volume of the cavity of the power hydraulic cylinder with an average piston position.

Equation of load motion

$$m_l \frac{d^2 Y_l}{dt^2} + K_{fr} \frac{dY_l}{dt} + c_l Y_l = c_{cv} (Y_h - Y_m),$$

where: $c_l$ — positional load coefficient; $K_{fr}$ — coefficient of viscous friction;

$c_{cv}$ — coefficient of the place compliance where the rod of the SM power hydraulic cylinder is connected to the control object (load); $m_l$ — equivalent load mass.

The equation of the balance of forces acting on the piston of the power cylinder SM

$$c_{cv} (Y_h - Y_m) = \Delta p S_n,$$

where

$$\frac{1}{c_e} = \frac{1}{c_o} + \frac{1}{c_{cv}}$$

— coefficient of the place compliance where the rod of the SM power hydraulic cylinder is connected to the control object (load);

$Y_m$ — moving the control object (load).

From the previous equations we get:

$$T_{hd} s(T_c^2 s^2 + 2 \xi c_s + 1) y_m(s) = X_s - K_i y_m(s),$$

$$T_{c} = \sqrt{\frac{m}{c_c}},$$

$$c_e = \frac{2 \cdot F_c^2 \cdot E_c}{V_o (1 + \frac{2F_c^2 E_c}{V_o c_{cv}})},$$

$$\xi = \frac{T_{dc}}{2T_c},$$

$$T_{dc} = \frac{K_{Qp} m}{F_i^2} + \frac{k_{mp} V_o}{2F_c^2 E_c} + \frac{k_{mp}}{c_{cv}}$$

— time constant of relative damping,

$$K_n = \frac{K_{Qp} c_o}{K_{Qp} F_i}$$

— coefficient of internal feedback.

For small liquid leaks through the distributor and small displacements of the spool from the neutral position $K_u$, it is usually small and can be neglected

$$T_{hd} s(T_c^2 s^2 + 2 \xi c_s + 1) y_m(s) = X_s (4)$$
Feedback signal equation:

\[ U_{ac} = K_{ac} (Y_h + Y_c). \]

We have \( c_u \neq c_{cv}, c_u \neq c_o \).

From the equations we get:

\[ U_{ac} (s) = K_{ac} \left( \frac{m}{c_e} + \frac{k_{mp}}{c_e} + 1 \right) y_m (s), \]

\[ \frac{1}{c_e} = \frac{1}{c_o} + \frac{1}{c_{cv}} \] — flexibility of mounting the hydraulic cylinder

The value \( c_e \) is so great that it is possible to exclude the forcing link of the second order and accept \( y_m = y_h \). Then we get:

\[ U_{ac} (s) = K_{poc} y_m (s) (5) \]

The voltage balance equation of the integrator:

\[ U_e = U_{xx} - U_{ac}, (6) \]

where \( U_e \) — error signal.

Based on equations (1–6), a block diagram of the drive is constructed, shown in Fig. 3. [7, 8]

Fig.3. Block diagram of the drive

3. Sophisticated hydraulic drive model (taking into account the effect of external load and displacement of the support of the object)

Accepted assumption: the flow rate \( Q \) depends only on the displacement of the spool \( X_s \), then

\[ Q = K_{QX} X_s. \]

Equation of load motion

\[ m_1 \frac{d^2 Y_l}{dt^2} = c_{cv} (Y_h - Y_m) - K_{mp} \frac{dY_l}{dt} - c_l Y_l - F_l, (7) \]

where \( F_l \) — constantly acting force.

From the equation of the balance of forces acting on the piston of the power hydraulic cylinder, we obtain:

\[ Y_h = \frac{\Delta p S_n}{c_{cv}} + Y_m (8) \]

Substituting (8) in the equation of the balance of the flow rate we get:

\[ Q = S_n \frac{dY_m}{dt} + \frac{V_0}{2E^u} \frac{d(\Delta p)}{dt} m, (9) \]
where $E'_c = \frac{B_g}{1 + \frac{V_i}{V_o} + 2 \cdot \frac{F^2}{V_o \cdot C_k}}$ — reduced modulus of elasticity of the hydraulic cylinder.

Based on equations (1–9), the structural diagram shown in Fig. 4 [5].

2. The results of mathematical modeling.
Using the BMSTU software complex, modeling was carried out, transient drive processes were obtained at various levels of the input signal.

![Fig.4. Block diagram of the drive](image-url)

![Fig.5. Transient drive. 1 - simplified model, 2 complex model (when $U_{in} = U_{max} = 12$ V).](image-url)
Conclusion

In the work, the parameters of an autonomous single-stage electro-hydraulic servo drive are calculated and a mathematical model of such a drive is created. Using the BMSTU software package, modeling was performed, and the dynamic characteristics of the drive were obtained. The results show that the mathematical model of the drive meets the requirements of the system and can be used for further research.

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