Neutral currents in a $SU(4)_L \otimes U(1)_Y$ gauge model with exotic electric charges

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Abstract

The weak currents with respect to the diagonal neutral bosons $Z, Z', Z''$ of a specific $SU(4)_L \otimes U(1)_Y$ gauge model are computed in detail for all the fermion families involved therein. Our algebraical approach, which is based on the general method of solving gauge models with high symmetries proposed several years ago by Cotăescu, recovers in a non-trivial way all the Standard Model values for current couplings of the traditional leptons and quarks, and predicts plausible values for those of the exotic fermions in the model.

PACS numbers: 12.15.Mm; 12.60.Cn; 12.60.Fr.

Key words: 3-4-1 gauge models, neutral currents

1 Introduction

In this letter we focus on the neutral currents of a specific gauge model based on the enlarged local group $SU(3)_C \otimes SU(4)_L \otimes U(1)_Y$ (in short, 3-4-1). In the last decade, this has been a very visited extension of the Standard Model \[1\] - \[3\] for it can accommodate in a natural manner some new phenomena such as neutrino oscillation (with resulting tiny masses) or extra neutral bosons to be detected at LHC. As usual, the electro-weak sector undergoes a spontaneous symmetry breakdown (SSB) up to the $U(1)_{em}$ residual one of electromagnetic interaction. Although, some older versions of the left-right symmetric models based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ \[4\] - \[7\] seem to presume the $SU(4)_L \otimes U(1)_Y$ electro-weak approach, the latter has been in its own right largely studied \[8\] - \[22\] by taking into account both its “exotic electric charge” version and “no exotic electric charge” version. This split can be made on different ways \[18\] - \[20\], but here we resort to the algebraical method proposed by Cotăescu \[23\] and worked out by the author in some recent papers \[17\] \[20\] \[24\] on 3-4-1 models.

General method The general method mainly consists of a particular geometrization of the scalar sector of the gauge model of interest. Since, the local gauge group $SU(N)_L \otimes U(1)_Y$ is taken into consideration, the fermion families will be nothing
but the irreducible unitary representations (irrep) of this group (with a proper hypercharge assignment so that all the anomalies cancel) and the gauge bosons (associated to the generators of the corresponding Lie algebra of the symmetry group) couple the fermions through covariant derivatives in the usual manner. Also, a minimal Higgs mechanism (mHm) is designed to spontaneously break this symmetry up the the universal electromagnetic one ($U(1)_{em}$). The above mentioned geometrization of the scalar sector is intended to supply - once the SSB took place - the non-degenerate boson mass spectrum. A step further, one can obtain the general expressions for the electric and neutral currents - i.e. the electric and neutral charges of the fermions with respect to all physical bosons in the model - by just setting a versor choice in a general Weinberg transformation (gWt) that makes the job of reaching the physical basis from the gauge one through a special rotation $\omega \in SO(N-1)$, as the electromagnetic field $A^{\mu}_{em}$ has been separated. That is, $\omega$ diagonalizes the mass matrix of the neutral bosons and it takes particular shapes from one particular model to another.

All the details of the general approach can be found in Ref. [23] and they are taken for granted without insisting here on deducing them one more time. The resulting formulæ are finally adapted to the particular model we work on.

**Electric and Neutral Charges**  

The charges of the particles are identified with the resulting coupling coefficients of the currents with respect to the well-determined physical bosons. We are interested in diagonal bosons (those associated with diagonal generators of the gauge group - $D^\rho$ - in the irrep $\rho$, in our notation). Hence, according to Eqs. (58) - (59) in Ref. [23], the spinor multiplet $L^\rho$ (of the irrep $\rho$) presents the following electric charge matrix

\[ Q^\rho = g \left[ (D^\rho \cdot \nu) \sin \theta + y^\rho \cos \theta \right], \]  

(1)

and the $n-1$ neutral charge matrices

\[ Q^\rho(Z^i) = g \left[ D^\rho_k - \nu_k(D^\rho \cdot \nu)(1 - \cos \theta) - y^\rho \nu_k \sin \theta \right] \omega^k_i, \]  

(2)

corresponding to the $n-1$ neutral physical fields, $Z^i_{\mu}$. We note that all the other gauge fields, namely the charged bosons $A^{\mu}_{\mu}$, have the same coupling, $g/\sqrt{2}$, to the fermion multiplets.

2 **SU(4)$_L$ $\otimes$ U(1)$_Y$ model with exotic electric charges**

We work here within the framework of the particular 3-4-1 model with exotic electric charges (see Ref. [20] [24] for the electric charge assignment), based on the versor setting $\nu_1 = 1$, $\nu_2 = 0$, $\nu_3 = 0$ - Case 1 in the Ref. [20]. In what follows we denote the irreps by $\rho = (n_{color}, n_{\rho}, y^\rho_{ch})$ indicating the genuine chiral hypercharge $y_{ch}$ instead of $y$ of the general method. This replacement $y \rightarrow \frac{g}{2}y_{ch}$ exploits a specific relation among the two gauge couplings $g$ (of the $SU(N)_L$) and $g'$ (of the $U(1)_Y$). Evidently, it was initially designed to simplify the work by dealing with only one gauge coupling (instead of two) in computations of the general method.
Fermion sector  With these assumptions, the multiplets of the 3-4-1 model of interest here will be denoted by \((n_{\text{color}}, n_{\mu}, g_{\nu}^{\alpha})\), and one gets the following particle content.

Lepton families

\[
f_{\alpha L} = \left( \begin{array}{c}
 e_{\alpha}^c \\
 \nu_{\alpha}^c \\
 N_{\alpha}
\end{array} \right) \sim (1, 4, 0)
\]  

Quark families

\[
Q_{iL} = \left( \begin{array}{c}
 J_i \\
 u_i \\
 d_i
\end{array} \right) \sim (3, 4^*, -1/3) \quad Q_{3L} = \left( \begin{array}{c}
 J_3 \\
 -b \\
 T
\end{array} \right) \sim (3, 4, +2/3)
\]  

(3)  

As in all cases when the general method was applied to particular 3-4-1 models \([17, 20, 24]\) the first gauge coupling \(g\) is identical to the SM first coupling.

Boson sector  The electro-weak boson sector is essentially determined by the generators of the Lie algebra associated to the gauge group \(SU(4)_L \otimes U(1)_Y\). We use here the standard generators \(T_a\) of the \(su(4)\) algebra, so that the Hermitian diagonal generators of the Cartan subalgebra are, in order, \(D_1 = T_3 = \frac{1}{2} \text{Diag}(1, -1, 0, 0)\), \(\tilde{D}_2 = T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(1, 1, -2, 0)\), and \(D_3 = T_{15} = \frac{1}{\sqrt{6}} \text{Diag}(1, 1, 1, -3)\) respectively. In this basis, the gauge fields are \(A_\mu^a\) and \(A_\mu \in su(4)\), that is

\[
A_\mu = \frac{1}{2} \left( \begin{array}{cccc}
 D_\mu^1 & \sqrt{2}X_\mu & \sqrt{2}X_\mu' & \sqrt{2}K_\mu \\
 \sqrt{2}X_\mu^* & D_\mu^2 & \sqrt{2}W_\mu & \sqrt{2}K_\mu' \\
 \sqrt{2}X_\mu'^* & \sqrt{2}W_\mu^* & D_\mu^3 & \sqrt{2}Y_\mu \\
 \sqrt{2}K_\mu'^* & \sqrt{2}K_\mu & \sqrt{2}Y_\mu^* & D_\mu^4
\end{array} \right),
\]  

(7)  

with neutral diagonal bosons: \(D_\mu^1 = A_{\mu}^3 + A_{\mu}^8/\sqrt{3} + A_{\mu}^{15}/\sqrt{6}\), \(D_\mu^2 = -A_{\mu}^3 + A_{\mu}^8/\sqrt{3} + A_{\mu}^{15}/\sqrt{6}\), \(D_\mu^3 = -2A_{\mu}^8/\sqrt{3} + A_{\mu}^{15}/\sqrt{6}\), and \(D_\mu^4 = -3A_{\mu}^{15}/\sqrt{6}\) respectively. Apart from the charged Weinberg bosons, \(W\), there are new charged bosons, \(K, K', X, X'\).

with \(\gamma = 1, 2, 3\) and \(i = 1, 2\). Capital letters denote exotic fermions, while small ones the fermions predicted by the SM. One can easily check that all the anomalies cancel by an interplay among families.

In addition, the connection between the \(\theta\) angle of our method \([23]\) and \(\theta_W\) (the Weinberg angle from SM) was inferred \([20]\): \(\sin \theta = 2 \sin \theta_W\) that points to the coupling relation: \(g'/g = \sin \theta_W / \sqrt{1 - 4 \sin^2 \theta_W}\).
Note that $X$ is doubly charged coupling different chiral states of the same charged lepton (the so called “bilepton”), while $Y$ is neutral.

The next step is the diagonalization of the mass matrix Eq. (33) in Ref. [24]. By that parameter choice - namely, matrix $\eta$ in the scalar sector - and by imposing that $m(Z) = m(W)/\cos \theta_W$ (from the SM) is eigenvalue of that matrix, one gets the matrix $\omega$. It is:

$$\omega = \begin{pmatrix}
\frac{\sqrt{1 - 4\sin^2 \theta_W}}{2\cos \theta_W} & -\frac{\sqrt{3}}{2\cos \theta_W} & 0 \\
\frac{\sqrt{3}}{2\cos \theta_W} & \frac{\sqrt{1 - 4\sin^2 \theta_W}}{2\cos \theta_W} & 0 \\
0 & 0 & 1
\end{pmatrix}.$$ (8)

**Neutral Charges** Evidently, the two subject to diagonalization are - $Z^1 = Z$ of the SM, and $Z^2 = Z'$ respectively. They exhibit the following neutral charges (after working out in Eq.(2) the versor choice $\nu_1 = 1$, $\nu_2 = 0$, $\nu_3 = 0$ and the coupling matching $g'/g = \sin \theta_W/\sqrt{1 - 4\sin^2 \theta_W}$ and $\sin \theta = 2\sin \theta_W$):

$$Q^\rho(Z^1) = g \left[ \left(D^\rho_1 \sqrt{1 - 4\sin^2 \theta_W} - y_{\rho,1}^\rho \frac{2\sin^2 \theta_W}{\sqrt{1 - 4\sin^2 \theta_W}} \right) \omega^1_1 + D^\rho_2 \omega^2_1 \right],$$ (9)

$$Q^\rho(Z^2) = g \left[ \left(D^\rho_1 \sqrt{1 - 4\sin^2 \theta_W} - y_{\rho,2}^\rho \frac{2\sin^2 \theta_W}{\sqrt{1 - 4\sin^2 \theta_W}} \right) \omega^1_2 + D^\rho_2 \omega^2_2 \right],$$ (10)

while the heaviest neutral boson - $Z^3 = Z''$, in our notation - will couple the fermion representations through:

$$Q^\rho(Z^3) = g D^\rho_3.$$ (11)

Now, it is a matter of some algebra - assuming that $\omega^1_1 = \omega^2_2 = \frac{\sqrt{1 - 4\sin^2 \theta_W}}{2\cos \theta_W}$ and $\omega^2_1 = -\omega^1_2 = \frac{\sqrt{3}}{2\cos \theta_W}$ along with the particular assignments of the fermion representations ($\rho$) - Eqs. (8) - (10) - and the neutral charges of all fermions in the 3-4-1 model of interest here are computed. They are summarized in the Table. We took into account that at the SM level $e = g\sin \theta_W$ holds.

### 3 Conclusions

In this letter, we have presented in detail the neutral currents of a particular 3-4-1 gauge model based on the general approach proposed by Cotăescu [23]. As one can observe by inspecting the Table, all the SM values are naturally recovered, while the two new neutral bosons of this model exhibit particular features. Namely, $Z'$ is leptophobic, since $\sin \theta_W \simeq 0.223$, but supplies very large couplings for quarks. The third neutral boson
Table 1: Coupling coefficients of the neutral currents in 3-4-1 model with exotic electric charges

| Particle | Coupling ($\epsilon / \sin 2\theta_W$) | $Z \rightarrow ff$ | $Z' \rightarrow ff$ | $Z'' \rightarrow ff$ |
|----------|----------------------------------------|-----------------|-----------------|-----------------|
| $\nu_eL, \nu_\mu L, \nu_\tau L$ | 1 | $-\sqrt{1 - 4 \sin^2 \theta_W / \sqrt{6}}$ | $\cos \theta_W / \sqrt{6}$ |
| $e_L, \mu_L, \tau_L$ | $2 \sin^2 \theta_W - 1$ | $-\sqrt{1 - 4 \sin^2 \theta_W / \sqrt{6}}$ | $\cos \theta_W / \sqrt{6}$ |
| $e_R, \mu_R, \tau_R$ | $-2 \sin^2 \theta_W$ | $2\sqrt{1 - 4 \sin^2 \theta_W / \sqrt{6}}$ | $\cos \theta_W / \sqrt{6}$ |
| $N_eL, N_\mu L, N_\tau L$ | 0 | 0 | $-\sqrt{1 - 4 \sin^2 \theta_W / \sqrt{6}}$ |
| $u_L, c_L$ | $1 - \frac{2}{3} \sin^2 \theta_W$ | $\frac{1}{\sqrt{3}} \left(\frac{1 - 2 \sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}\right)$ | $-\frac{1}{\sqrt{6}} \cos \theta_W$ |
| $d_L, s_L$ | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $\frac{1}{\sqrt{3}} \left(\frac{1 - 2 \sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}\right)$ | $-\frac{1}{\sqrt{6}} \cos \theta_W$ |
| $t_L$ | $1 - \frac{2}{3} \sin^2 \theta_W$ | $-\frac{1}{\sqrt{3}} \sqrt{1 - 4 \sin^2 \theta_W}$ | $\frac{1}{\sqrt{6}} \cos \theta_W$ |
| $b_L$ | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $-\frac{1}{\sqrt{3}} \sqrt{1 - 4 \sin^2 \theta_W}$ | $\frac{1}{\sqrt{6}} \cos \theta_W$ |
| $T_L$ | $-\frac{4}{3} \sin^2 \theta_W$ | $-\frac{1}{\sqrt{3}} \sqrt{1 - 4 \sin^2 \theta_W}$ | $-\frac{1}{\sqrt{6}} \cos \theta_W$ |
| $D_{1L}, D_{2L}$ | $\frac{2}{3} \sin^2 \theta_W$ | $\frac{2}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | $\sqrt{1 - 4 \sin^2 \theta_W / \sqrt{6}}$ |
| $u_R, c_R, t_R, T_R$ | $-\frac{4}{3} \sin^2 \theta_W$ | $\frac{4}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | 0 |
| $d_R, s_R, b_R, D_{iR}$ | $+\frac{2}{3} \sin^2 \theta_W$ | $\frac{2}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | 0 |
| $J_{1L}, J_{2L}$ | $\frac{8}{3} \sin^2 \theta_W$ | $-\frac{8}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | $-\cos \theta_W / \sqrt{6}$ |
| $J_{1R}, J_{2R}$ | $\frac{8}{3} \sin^2 \theta_W$ | $-\frac{8}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | 0 |
| $J_{3L}$ | $-\frac{10}{3} \sin^2 \theta_W$ | $\frac{10}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | $\cos \theta_W / \sqrt{6}$ |
| $J_{3R}$ | $-\frac{10}{3} \sin^2 \theta_W$ | $\frac{10}{\sqrt{3}} \frac{\sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}}$ | 0 |
"Z" seems to make no distinction among the SM fermions, being at the same time quite a vector like interaction with respect to the SM fermions, while for the exotic particles it is strongly parity-violating.

These results are the exact couplings of the fermion and they do not need any supplemental mixing angle since this job have been done as an inner step by the method itself through the general Weinberg transformation. The resulting values are important for some specific decays and weak processes, but these computations are beyond the scope of this letter.

References

[1] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
[2] S. L. Glashow, Nucl. Phys. 22, 579 (1961).
[3] A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No.8), ed. N. Svartholm (Almqvist and Wiksell, 1968) p.367.
[4] C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).
[5] R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975).
[6] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D 11, 1502 (1975).
[7] R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 2558 (1975).
[8] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50, R34 (1994).
[9] F. Pisano and V. Pleitez, Phys. Rev. D 51, 3865 (1995).
[10] A. Doff and F. Pisano, Mod. Phys. Lett. A 14, 1133 (1999).
[11] A. Doff and F. Pisano, Phys. Rev. D 63, 097903 (2001).
[12] Fayyazuddin and Riazuddin, JHEP 0412, 013 (2004).
[13] W. A. Ponce, D. A. Gutierrez and L. A. Sanchez, Phys. Rev. D 69, 055007 (2004).
[14] L. A. Sanchez, F. A Perez and W. A. Ponce, Eur. Phys. J C 35, 259 (2004).
[15] L. A. Sanchez, L. A. Wills-Toro and J. I. Zuluaga, Phys. Rev. D 77, 035008 (2008).
[16] Riazuddin and Fayyazuddin, Eur. Phys. J C 56, (2008).
[17] A. Palcu, arXiv: 0902.3756 [hep/ph].
[18] A. Doff and F. Pisano, Mod. Phys. Lett. A 15, 1471 (2000).
[19] W. A. Ponce and L. A. Sanchez, Mod. Phys. Lett. A 22, 435 (2007).
[20] A. Palcu, arXiv: 0902.1301 [hep/ph] - to be published in Mod. Phys. Lett. A.
[21] S. Sen, arXiv: 0901.2240 [hep/ph].

[22] T. Li, F. Wang and J. M. Yang, arXiv: 0901.2161 [hep-ph].

[23] I. I. Cotăescu, *Int. J. Mod. Phys. A* **12**, 1483 (1997).

[24] A. Palcu, arXiv: 0902.1828 [hep-ph v2] - to be published in Mod. Phys. Lett. A.