Quark Substructure and Isobar Effects on Deuteron Form-Factors

E. Lomon

aCenter for Theoretical Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, MA 02139

Elastic ed scattering, with deuteron polarization, up to high momentum transfer provides detailed information on the deuteron wave function. This determines the range dependence of the orbital and spin components of the one- and two-body currents, restricting contributions of isobar and meson-exchange currents and of quark/gluon degrees of freedom, as well as the nucleon component. The R-matrix boundary condition model combines all these effects, predicting nucleon-nucleon reactions and the deuteron form-factors simultaneously. A brief description of the model is followed by a comparison of its results with data, emphasizing the restrictions placed on the model by ed elastic form-factors.

1. INTRODUCTION

There is now elastic electron-deuteron scattering data determining the electric form-factor \( A(q^2) \) up to 6 (GeV/c)^2\(^2\)[1], the magnetic form-factor \( B(q^2) \) up to 2.8 (GeV/c)^2[2], and the tensor-polarization form-factor \( t_{20}(q^2) \) up to 1.8 (GeV/c)^2[3]. These data restrict orbital and spin components of the deuteron wave function at scales as small as 0.2 fm, at which distance quark degrees-of-freedom (d.o.f.), isobar components and meson-exchange currents all have a significant role.

The R-matrix boundary condition method[4,5] provides a hybrid quark/gluon and hadron model, incorporating all the above contributions. Only a few of the parameters are not predetermined by data independent of the nucleon-nucleon (NN) interaction and symmetry requirements. The remaining few are almost all determined by NN scattering data. Essentially one parameter is free to determine the behavior of the three independent elastic electron-deuteron form-factors (edff) over the large range of momentum-transfers, \( q \). This parameter determines the relative amount of \( \Delta \Delta(7D_1) \) and \( \Delta \Delta(3D_1) \) in the deuteron, which profoundly affects the \( q \) dependence of the spin and convective currents[6]. The NN scattering is not sensitive to the ratio, but only to the sum.

Following a review of the R-matrix method and its application to the NN system, three specific models for the \( I = 0, J^p = I^+ \) sector, of different levels of completeness, will be
compared with the NN scattering and edff data. From these one can extrapolate to a model that represents all the data.

2. THE $R$-MATRIX BOUNDARY CONDITION MODEL

At high momentum-transfer (short range) the running coupling constant of QCD is small, permitting a perturbative description in terms of current quarks and gluons (asymptotic freedom). At low momentum-transfer (long range) nonperturbative effects produce clustering into color singlet hadrons (confinement). The transition between these extremes has been shown to occur over a small range of the running coupling constant\(^7\), and therefore over a short distance. The $R$-matrix method\(^8\) is well suited to this situation in which two regions, each well represented by a different approximate Hamiltonian, have their wave functions connected by a boundary condition at the separating surface.

For the QCD application, in which confinement requires the quark wave function to be small at the transition boundary, the suitable form of the $R$-matrix equation is\(^4,5\)

$$r_0 \left( \frac{\partial \psi_\alpha(r,W)}{\partial r} \right)_{r_0} = \sum_{\beta} f_{\alpha\beta}(W) \psi_\beta(r_0,W) \quad (1)$$

with

$$f_{\alpha\beta}(W) = f_{\alpha\beta}^0 + \sum_i \frac{\rho_i^{\alpha\beta}}{W - W_i} \quad (2)$$

in which $\psi_\alpha$ is the exterior wave function for hadron-pair channel $\alpha$, $W$ is the total energy, the poles $W_i$ are the energies of a complete set of internal states vanishing at $r_0$, and the residues $\rho_i^{\alpha\beta}$ are given by absolute squares of the internal wave function derivatives at the boundaries. Thus the $f_{\alpha\beta}(W)$ are meromorphic functions with real poles of positive residue. The residues can be expressed as

$$\rho_i^{\alpha\beta} = -r_0 \frac{\partial W_i}{\partial r_0} \xi_i^\alpha \xi_i^\beta \quad (3)$$

where the fractional parentage coefficients $\xi_i^\alpha$ are geometric coefficients expressed in terms of Clebsch-Gordon coefficients of the spin/flavor/color space of the given quark configuration. Therefore only the $f_{\alpha\beta}^0$, representing the effective constant of distant poles and the pole at infinity, are free parameters.

The hadrons at $r > r_0$ interact via hadron exchange potentials, as given by known hadron masses and coupling constants fixed by independent experiments or symmetry conditions.

The separation radius, $r_0$, must be within the range of asymptotic freedom ($\leq 0.85$ of the equilibrium radius of the interior bag model) because of the sensitivity of $\rho_i^{\alpha\beta}$ to the derivatives at $r_0$ of the internal wave function. Eq. (1) also requires that $r_0$ satisfy $\psi_2(r_0,W_i(r_0)) = 0$. Using the low energy data, the latter condition fixes the value of $r_0$ with some precision, giving a value consistent with the first condition for the Cloudy bag model, but not for the MIT bag model\(^4,5\), ruling out the latter for the multi-hadron domain. The Cloudy bag model determines $r_0 = 1.05$ fm. The model then gives a good detailed fit to NN data for $T_{\text{Lab}} \leq 0.8$ GeV\(^9\) and also is consistent with some evidence for
the lowest $I = 1$ exotic resonance, $J^P = 0^+$ at 2.70 GeV. These resonances, produced
near the $f$-poles at $W_i$ correspond to multi-quark configurations other than the minimal
$q\bar{q}$ and $q^3$ configurations. The lowest in the NN system is the $I = 0, J^P = 1^+$ at 2.63 GeV.

3. DEUTERON PREDICTIONS

3.1. Interior Wave Function

The first $f$-pole in the NN system, corresponding to the $[q(1s_\frac{1}{2})]^6$ configuration, is in
the $I = 0, J^P = 1^+$ state, 0.76 GeV above the deuteron mass. As the width of the exotic
resonance is only 0.03 GeV, the $f_{\alpha\beta}$ are nearly constant. It has been shown\[11\] that the
interior wave function vanishes for constant $f$, and the actual probability of being in the
interior has been estimated to be $\leq 0.004$\[6\]. This implies a large “hole” in the deuteron
wave function, which has long been noted as a property of the edff and is also embodied
in the effective “hard core” of NN scattering. In our model this apparent repulsion arises
from the rapid change in the effective d.o.f. at $r_0$. However at the energy $W_i$ there is
“matching” of the interior and exterior wave functions, resulting in a substantial interior
probability and a resonance.

The fact that $r_0$ is 2-3 times larger than the cores of repulsive core models is com-
penated by the discontinuous increase of wave functions at $r_0$, so that the experimental
effective range parameters can be produced by both types of core effect. However the
models differ at higher NN scattering energies for large $q$ edff.

3.2. Exterior Wave Function

In the $I = 0, J^P = 1^+$ system, the NN ($^3S_1$) and NN ($^3D_1$) states are coupled to
each other and to isobar channels by meson exchange potentials as well as the $f$-matrix.
Because of their low threshold mass and strong tensor coupling, the $\Delta\Delta(^3S_1)$, $\Delta\Delta(^3D_1)$
and $\Delta\Delta(^3D_1)$ channels are most important and are included in all our models. NN
($^3S_1$) and $\Delta\Delta(^3S_1)$ states have nonvanishing $\xi^\prime_\alpha$ with the $[q(1s_\frac{1}{2})]^6$ quark configuration,
contributing to the $f$-pole residue. The $NN^\prime(1440)(^3S_1)$ channel also has a low threshold,
and with its large width is of next importance to the deuteron. But over the energy
range which includes the first exotics, other channels are also of some importance and the
$NS_{11}(1535)(^1P_1)$, $NS_{11}(1650)(^1P_1)$ and $\Delta S_{31}(1620)(^3P_1)$ are included in our recent work.
These channels modify the best choice of $f^0_{\alpha\beta}$ for the lower threshold channels, but have
negligible components in the deuteron.

3.3. Determining the $f^0_{\alpha\beta}$ for Three Models

The $f^0_{\alpha\beta}$ are sharply restricted by fitting the NN scattering data, which in the deuteron
sector requires a fit to the $np(^3S_1 - ^3D_1)$ phase parameters $\delta$ and $\eta(^3S_1)$, $\delta$ and $\eta(^3D_1)$
and $\epsilon_1$ for $T_{Lab} \leq 0.8$ GeV. For the models $C'$ and $D'$\[12\], which have only NN and $\Delta\Delta$
channels, the lower energy behavior of the $\delta$’s and $\epsilon_1$ determine the NN sector $f_{\alpha\beta}$, while
the energy dependence for 0.4 GeV $< T_{Lab} < 0.8$ GeV fixes those coupling the NN and $\Delta\Delta(^3S_1)$ sectors, and the NN to the sum of $\Delta\Delta(^3D_1)$ and $\Delta\Delta(^7D_1)$ sectors. The last
two have the same threshold behavior, affecting the elastic scattering in the same way.
Only detailed $\Delta$-production data could separate them, so the ratio is free to adjust to the
edff. The magnetic form factor $q$-dependence is very sensitive to the ratio because of the
opposite spin and convection currents of these states\[8\].
The model $C'$ did not consider quark configurations. Without $f$-poles the best choice of separation radius was $r_0 = 0.74$ fm. Model $D'$ included the lowest $f$-pole with $r_0 = 1.05$ fm as discussed above. As shown previously, $C'$ and $D'$ give equivalent fits to the $\delta$'s while case $D'$ is a better fit to $\epsilon_1$.

When the other isobar channels are included, the $NN^*(1440)$, because of its larger width, modifies the energy dependence and increases the inelasticity. This reduces the required coupling to the $\Delta\Delta$ channels. This model $E$ results in a better fit to $\bar{\epsilon}_1, \delta^{(3D_1)}$ and to the $\eta$'s (Fig. 1).

Figure 1. The phase parameters for $I = 0$, $J^P = 1^+$, $np$ scattering. Model $E$ (solid curves); SAID SP00 phase parameters (dashed curves); Bugg 1990–1991 phase parameters (squares).

3.4. The EDFF Predictions

In previous work, the edff for models $C'$ and $D'$ were calculated with the nonrelativistic, coupled channel impulse approximation (IA) and the meson-exchange current (MEC) terms of $\pi$, $\rho$, and $\omega$, “pair” corrections and the $\rho\pi\gamma$ term to first relativistic order. For the IA the isobar form factors are assumed proportional to the nucleon electromagnetic form-factors. In all cases the MEC corrections to the isobar channels are neglected. Both Höhler et al. (HO) and Gari-Krümpelmann (GK) nucleon form factors were used. The results (Figs. 3-6 of [12]) are seen to be a good to $A(q^2)$ for the HO choice and to $B(q^2)$ for the GK choice. This is not inconsistent as $A(q^2)$ is dominated by the nucleon electric form-factor and $B(q^2)$ by the nucleon magnetic form-factor. The $t_{20}(q^2)$ predictions were consistent with the very low $q$ experimental results available at the time.

Here we present, versus the extended data range of the edff, the results of models $C'$, $D'$ and $E$ where the first order relativistic correction has been added to the impulse approximation and the second order relativistic corrections have been included in the MEC[16,17]. For model $E$ the ratio of $\Delta\Delta(7^2D_1)$ to $\Delta\Delta(3^2D_1)$ coupling to the NN sector was guided by the $C'$ and $D'$ model ratios. It has not yet been optimized to the data.
Figure 2. $A(q^2)$: Data points are described in Ref. [1]. Model $C'$ (HO) (solid line); model $C'$ (GK) (dash-dash); model $D'$ (HO) (dash-dot); model $D'$ (GK) (dot-dot); model $E$ (HO) (long dashes); model $E$ (GK) (dash-dot-dot).

Figure 3. $t_{20}(q^2)$: Data points described in Ref. [3]. Model $C'$ (solid line); model $D'$ (dash-dot); model $E$ (long dashes). The dependence on nucleon emff (HO or GK) is negligible.

Also the balance of $\Delta\Delta$ and $NN^*(1440)$ coupling to $NN$ and the value of the $N^*(1440)$ magnetic moment, unknown from independent data, have not been varied.

Table 1 shows the results of model $E$ for the static properties of the deuteron. For models $C'$ and $D'$ the results are as stated in [12] except for a small relativistic change in $Q_{\text{deut}}$.

| Model | $BE$ (MeV) | $P_D$ (%) | $P_{\Delta 5}$ (%) | $P_{\Delta 3}$ (%) | $P_{\Delta 7}$ (%) | $P_{N^*}$ (%) | $Q (fm^2)$ | $\mu_D (\mu)$ |
|-------|------------|-----------|--------------------|--------------------|--------------------|-------------|------------|--------------|
| Model $E$ | 2.2247 | 5.21 | .006 | 3.24 | 1.79 | 0.71 | .273 | .860 |
| Exp $\ell$ | 2.2246 | | | | | | .286 | .857 |

*The higher mass channels have negligible probability.*

$A(q^2)$ is shown in Fig. 2. It is seen that model $E$ with either choice of nucleon form-factors fits the data reasonably well for $q^2 < 2.5$ (GeV/c)$^2$, but is only large enough for larger $q^2$ when the GK choice is made. Models $C'$ and $D'$ on the other hand are better with the HO choice for $q^2 < 2.5$ (GeV/c)$^2$, but at 6 (GeV/c)$^2$ only model $C$ with GK is large enough.

$t_{20}(q^2)$ is shown in Fig. 3. For the momentum transfer range there is negligible sensitivity to the choice of nucleon form-factors, as these cancel in the ratio of quadrupole to...
monopole electric amplitudes which dominates $t_{20}$. The result is however very sensitive to the model used. The simple constant $f$-matrix model, $C'$, gives a good fit over the whole range of $q$. Model $D'$ puts the maximum of $t_{20}$ at much too small a momentum transfer. This is related to the large amplitude of the $L = 2$, $\Delta \Delta$ states in this model. Model $E$, with intermediate $\Delta \Delta$ components, has the maximum of $t_{20}$ between that of models $C'$ and $D'$.

The $B(q^2)$ for models $C'$ and $D'$ is similar to that of [12] with minima at slightly smaller $q$. However the minimum of $B(q^2)$ for model $E$ is at much too small a value ($q^2 = 1.3 (\text{GeV}/c)^2$).

4. CONCLUSIONS

The $R$-matrix boundary condition model $E$, with all the relevant isobar channels, reproduces the $np(^3S_1 - ^3D_1)$ scattering phases well up to $T_{\text{Lab}} \leq 0.8 \text{ GeV}$. It also reproduces very well the static properties of the deuteron and $A(q^2)$ for $q^2 \leq 6 (\text{GeV}/c)^2$. It does not fit $t_{20}(q^2)$ as well as the simpler model $C'$ (although it is better than model $D'$), and has the first minimum of $B(q^2)$ at much too small a value. To correct the full model ($E$) for $B(q^2)$, the ratio of the $\Delta \Delta(^7D_1)$ to $\Delta \Delta(^3D_1)$ couplings to the NN sector needs to be varied. That, and perhaps a further substitution of $N^*(1440)$ coupling for $\Delta \Delta$ coupling to the NN sector may also correct the fit to the maximum of $t_{20}(q^2)$.

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