Compensation of Subdivision Error of Capacitance Encoder Based on Lissajous Graphic Analysis

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Abstract: With the development of measurement technology in the direction of precision, high speed, automation, integration, intelligence, economy, non-contact and multifunction, it has simple structure, small size, low energy consumption and strong adaptability. And other advantages have been widely used. However, the sensor of the capacitive encoder works in the state of low voltage and low current, and is extremely susceptible to the influence of the external electric field and magnetic field, causing problems such as input signal distortion and system failure. With the increasing requirements of the manufacturing industry for stability and accuracy, it is of great significance to analyze the capacitive coding error and improve the accuracy. This article analyzes the error generated by the capacitive encoder, enumerates the influence of DC deviation, amplitude deviation, phase deviation, and high-order harmonics on the interpolation error, and compensates the error signal for the aspects that cause larger errors. The sine and cosine signals are close to the ideal state.

1. Introduction
Capacitive sensor is a new type of displacement sensor developed in the early 1980s. Photoelectric and grating encoders occupy the main share with their excellent accuracy and resolution, but their structure is complex and expensive, and they are mainly used for precision measurement. Compared with it, the capacitance grid has the advantages of low cost, high speed and strong environmental adaptability under the condition of equal accuracy, so it develops rapidly and is highly valued and widely used by people.

2. Error analysis
The capacitive grid sensor outputs fine and coarse sine and cosine signals without any error. The coarse channel only cares about the running direction of the capacitive encoder, and has almost no effect on the accuracy, so the error is allowed to be larger, and the fine channel determines the conversion accuracy of the capacitive encoder, so here we only discuss the sine and cosine signals of the fine channel. Fine channel output sine and cosine signal:

\[
\begin{align*}
U_{FS} &= U_{REF} + K_1 \sin \alpha \\
U_{FC} &= U_{REF} + K_1 \cos \alpha
\end{align*}
\]

(1)
Among them, \( a \) is the displacement of the fine channel rotation of the capacitive grid encoder, UREF is the DC component, and its value is 1.5V, and the maximum signal voltage range is 0~3V.

The error of the capacitive grid encoder mainly includes: DC component error, sine-cosine signal amplitude error, sine-cosine signal phase quadrature error and harmonic error, the following is a specific analysis.

2.1 DC component error
Under different conditions of input power and changes in external temperature, the sine and cosine DC component signals will be unequal, which will cause interpolation errors. Assuming that the DC components of both sine and cosine signals have errors, the sine DC component is used as the standard, and the cosine DC component error is \( \delta \), then the cosine voltage signal expression:

\[
U_{CS} = (1 + \delta)U_{REF} + k_1 \cos \alpha
\]  
(2)

The calculation angle formula is:

\[
\theta = \tan^{-1} \left( \frac{\sin \alpha}{\frac{\delta U_{REF}}{k_1} + \cos \alpha} \right)
\]  
(3)

The system error is:

\[
\delta = \alpha - \theta = \alpha - \tan^{-1} \left( \frac{\sin \alpha}{\frac{\delta U_{REF}}{k_1} + \cos \alpha} \right)
\]  
(4)

2.2 Amplitude error
Changes in the power supply voltage, resistance, or outside temperature will cause the gain of the amplifier to change, which will cause interpolation errors. Assuming that there is an error in the amplitude of the sine signal and the cosine signal, the amplitude of the sine signal is taken as the standard, and the amplitude error of the cosine signal is set to \( r \), then the cosine voltage signal expression:

\[
U_{CS} = U_{REF} + k_1(1 + \gamma) \cos \alpha
\]  
(5)

The calculation angle formula is:

\[
\theta = \tan^{-1} \frac{k_1 \sin \alpha}{(1 + \gamma)k_1 \cos \alpha} = \tan^{-1} \frac{\tan \alpha}{1 + \gamma}
\]  
(6)

The system error is:
2.3 Error of phase quadrature
In the case of external temperature changes, the phase of the induced sine and cosine signals will not be orthogonal, resulting in interpolation errors. Assuming that the phase of the sine signal and the cosine signal are not orthogonal, the phase of the sine signal is taken as the standard, and the phase error of the cosine signal is set to $\beta$, then the cosine voltage signal expression:

$$U_{CS} = U_{REF} + k_1 \cos(\alpha + \beta)$$  \hspace{1cm} (8)

The calculation angle formula is:

$$\theta = \tan^{-1} \frac{k_1 \sin \alpha}{k_1 \cos(\alpha + \beta)} = \tan^{-1} \frac{\sin \alpha}{\cos(\alpha + \beta)}$$  \hspace{1cm} (9)

The system error is:

$$\varepsilon = \alpha - \theta = \alpha - \tan^{-1} \frac{\sin \alpha}{\cos(\alpha + \beta)}$$  \hspace{1cm} (10)

2.4 Error of signal harmonics
The sine and cosine signals have harmonic signals under signal filtering and external interference, which can bring subdivision errors. Assuming that the sine and cosine signals have high-order harmonic signals, then:

$$U_{FS} = U_{REF} + \sum_{i=1}^{\infty} k_i \sin(i\alpha)$$  \hspace{1cm} (11)

$$U_{CS} = U_{REF} + \sum_{i=1}^{\infty} k_i \cos(i\alpha)$$  \hspace{1cm} (12)

The calculation angle formula is:

$$\theta = \tan^{-1} \frac{\sum_{i=1}^{\infty} k_i \sin(i\alpha)}{\sum_{i=1}^{\infty} k_i \cos(i\alpha)}$$  \hspace{1cm} (13)

The system error is:

$$\varepsilon = \alpha - \theta = \alpha - \tan^{-1} \frac{\sum_{i=1}^{\infty} k_i \sin(i\alpha)}{\sum_{i=1}^{\infty} k_i \cos(i\alpha)}$$  \hspace{1cm} (14)

3. Error compensation
By deriving the formulas of the DC component error, amplitude error, phase quadrature error and harmonic error of the sine and cosine signal, the mathematical expression of each error and the maximum angle error are obtained. The signal quality of the capacitive grid sensor is not good. The output sine and cosine signal contains DC component error, amplitude error, phase quadrature error and harmonic component error. The total error is expressed by the root mean square method:
When only considering the error of the DC component, when the error $\delta$ is 2%. $\varepsilon_{\text{max}1} = 0.02^\circ$

When only considering the error of the amplitude, when the error $r$ is 2%. $\varepsilon_{\text{max}2} = 0.01^\circ$

When only considering the phase quadrature error, when the error $\beta$ is 0.5°. $\varepsilon_{\text{max}3} = 0.05^\circ$

When only considering the error of signal harmonics within the third order, when the values of the second and third harmonic coefficients $K_2$ and $K_3$ are less than 2%. $\varepsilon_{\text{max}4} = 0.035^\circ$

Then the total error is $\Delta \varepsilon = 0.065^\circ$.

After calculation, the degree of non-quadrature of the sine-cosine voltage signal and the positive harmonic error will cause a larger error to the capacitive encoder. The error is very large, and signal compensation must be used to reduce the system error, otherwise the capacitive encoder cannot meet the requirements of conversion accuracy.

3.1 Amplitude compensation

\[
\begin{align*}
U_{FS} &= U_{REF} \cdot K_1 \sin(\alpha) \\
U_{CS} &= U_{REF} \cdot K_1 \cos(\alpha)
\end{align*}
\]

Figure 2. Schematic diagram of amplitude error

Suppose the amplitude of sine is $r$, the amplitude of cosine is $a$, and the deviation of cosine amplitude in Y axis direction is $a$. Due to the existence of error, when the position is at point $N$, the true position angle, and the error angle is $\Delta \theta$. To derive the relationship between $\Delta \theta$ and $a$, use the law of sine in the error triangle $PON$ to know:

\[
\frac{r}{\sin(\angle PNO)} = \frac{a}{\sin(\Delta \theta)}
\]

Because:

\[
\Delta \theta = \arctan \left[ \frac{\sin(2\theta_r)}{\cos(2\theta_r) + \frac{dx}{dr} - 1} \right]
\]

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\]

$\theta_r = \arctan \left[ \frac{y}{\sqrt{x^2+y^2}} \right]$
Figure 3 sinusoidal distribution

$\Delta \theta$ has four extreme values in one cycle. Suppose the function of the first quadrant is $f_1(\theta_r)$, and the other quadrants are in sequence are $f_2(\theta_r), f_3(\theta_r), f_4(\theta_r)$. The four functions have similar laws and have:

$$f_2(\theta_r) = f_1(\pi - \theta_r), f_3(\theta_r) = f_1(\theta_r - \pi), f_4(\theta_r) = f_1(2\pi - \theta_r)$$

Take the same in the amplitude error $K_4 = \frac{2r}{dr} - 1, \theta_k = 2\theta_r$, and then perform the derivative to get the extreme value

$$(\Delta \theta)' = \begin{bmatrix} 1 \\ \sin \theta_k \cos \theta_k \end{bmatrix} \begin{bmatrix} \sin \theta_k \\ \frac{1}{K_1 + \cos \theta_k} \end{bmatrix}$$

$$(\Delta \theta)' = \frac{1 + K_1 \cos \theta_k}{1 + 2K_1 \cos \theta_k + K_1^2} \quad (20)$$

Therefore, when $2\theta_r = \theta_k = \pm \pi/2, \Delta \theta$ takes the extreme value.

3.2 Compensation of harmonic components

The angle error is periodic, and the error data can be processed by the harmonic analysis method. The harmonic expansion of the error function.

$$E(t) = a_0 + \sum_{k=1}^{\infty} \left[ a_k \cos \left( \frac{2\pi k}{N} t \right) + b_k \sin \left( \frac{2\pi k}{N} t \right) \right] = a_0 + \sum_{k=1}^{\infty} \left[ c_k \sin \left( \frac{2\pi k}{N} t + \varphi_k \right) \right]$$

$$c_k = \sqrt{a_k^2 + b_k^2}$$

$$\varphi_k = \arctan \left( \frac{b_k}{a_k} \right) \quad (21)$$

Is the coefficient of each harmonic, and is the amplitude and phase of each harmonic. From the discrete sampling sequence, the coefficients of the expansion of the function can be obtained by the above formula.

$$E(\Delta t \cdot n) = a_0 + \sum_{k=1}^{\infty} a_k \cos \left( \frac{2\pi k}{N} n \right) + b_k \sin \left( \frac{2\pi k}{N} n \right)$$

$$a_0 = \frac{1}{N} \sum_{n=0}^{N-1} E(\Delta t \cdot n) \quad a_k = \frac{2}{N} \sum_{n=0}^{N-1} E(\Delta t \cdot n) \cos \left( \frac{2\pi k}{N} n \right) \quad (22)$$

$$b_k = \frac{2}{N} \sum_{n=0}^{N-1} E(\Delta t \cdot n) \sin \left( \frac{2\pi k}{N} n \right)$$

$$E(\theta_1) - a_0 = a_1 \cos \left( \frac{2\pi}{n} \cdot 1 \right) + b_1 \sin \left( \frac{2\pi}{n} \cdot 1 \right) + a_2 \cos \left( \frac{2\pi}{n} \cdot 1 \right) + b_2 \sin \left( \frac{2\pi}{n} \cdot 1 \right) + \ldots$$

$$+ a_i \cos \left( \frac{2\pi}{n} \cdot i \right) + b_i \sin \left( \frac{2\pi}{n} \cdot i \right)$$

$$E(\theta_n) = a_0 + a_1 \cos \left( \frac{2\pi n}{N} \cdot 1 \right) + b_1 \sin \left( \frac{2\pi n}{N} \cdot 1 \right) + a_2 \cos \left( \frac{2\pi n}{N} \cdot 1 \right) + b_2 \sin \left( \frac{2\pi n}{N} \cdot 1 \right) + \ldots$$

$$+ a_i \cos \left( \frac{2\pi n}{N} \cdot i \right) + b_i \sin \left( \frac{2\pi n}{N} \cdot i \right) \quad (23)$$
n is the circle equal fraction when correcting the error, and i is the corresponding harmonic calculation order.

\[ E' = [E(\theta_1) - a_0, E(\theta_2) - a_0, ... , E(\theta_n) - a_0]^T \]

\[ A = \begin{bmatrix}
\cos \left(1 \cdot \frac{2\pi}{n} \cdot 1\right) & \cos \left(1 \cdot \frac{2\pi}{n} \cdot i\right) & \cdots & \cos \left(1 \cdot \frac{2\pi}{n} \cdot i\right) \\
\sin \left(1 \cdot \frac{2\pi}{n} \cdot 1\right) & \sin \left(1 \cdot \frac{2\pi}{n} \cdot i\right) & \cdots & \sin \left(1 \cdot \frac{2\pi}{n} \cdot i\right)
\end{bmatrix} \]

\[ X = [a_1, a_2, ..., a_i, b_1, b_2, ..., b_i]^T \]

\[ E' = AX \] (24)

Solving this matrix equation can get the value of the harmonic coefficients of each order, and the equation group belongs to the overdetermined equation group, and the least square solution is obtained, in the form:

\[ X = (A^T A)^{-1} A^T E' \] (25)

The order that has a greater influence on the error is selected by the phase size, and the numerical expression of the harmonic compensation is obtained.

4. Conclusions
With the advancement of technology, we pay more attention to the accuracy of sensors. This paper studies the possibility of error generated by capacitive encoders and analyzes the effects of DC deviation, amplitude deviation, phase deviation, and high-order harmonics on the interpolation error, and analyzes the error signal in terms of causing larger errors. Compensates the error signal for the aspects that cause larger errors. The sine and cosine signals are close to the ideal state. Thereby improving the accuracy of capacitive encoder.

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