Probing New Physics through $\mu - e$ Universality in $K \to \ell \nu$

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The recent NA48/2 improvement on $R_K = \Gamma(K \to e\nu)/\Gamma(K \to \mu\nu)$ emphasizes the role of $K_{12}$ decays in probing the $\mu - e$ universality. Supersymmetric (SUSY) extensions of the Standard Model can exhibit $\mu - e$ non-universal contributions. Their origin is twofold: those deriving from lepton flavor conserving couplings are subdominant with respect to those arising from lepton flavor violating (LFV) sources. We show that $\mu - e$ non-universality in $K_{12}$ is quite effective in constraining relevant regions of SUSY models with LFV (for instance, supergravities with a see-saw mechanism for neutrino masses). A comparison with analogous bounds coming from $\tau$ LFV decays proves the relevance of the measurement of $R_K$ to probe LFV in SUSY.

INTRODUCTION

High precision electroweak tests represent a powerful tool to probe the Standard Model (SM) and, hence, to constrain or obtain indirect hints of new physics beyond it. Kaon and pion physics are obvious grounds where to perform such tests, for instance in the well studied $\pi_{2\ell}$ ($\pi \to \ell \nu_l$) and $K_{2\ell}$ ($K \to \ell \nu_l$) decays, where $l = e$ or $\mu$. Unfortunately, the relevance of these single decay channels in probing the SM is severely hindered by our theoretical uncertainties, which still remain at the percent level (in particular due to the uncertainties on non perturbative quantities like $f_\pi$ and $f_K$). This is what prevents us from fully exploiting such decay modes in constraining new physics, in spite of the fact that it is possible to obtain non-SM contributions which exceed the high experimental precision which has been achieved on those modes.

On the other hand, in the ratios $R_\pi$ and $R_K$ of the electronic and muonic decay modes $R_\pi = \Gamma(\pi \to e\nu_l)/\Gamma(\pi \to \mu\nu_l)$ and $R_K = \Gamma(K \to e\nu_l)/\Gamma(K \to \mu\nu_l)$, the hadronic uncertainties cancel to a very large extent. As a result, the SM predictions of $R_\pi$ and $R_K$ are known with excellent accuracy $^1$ and this makes it possible to fully exploit the great experimental resolutions on $R_\pi$ $^2$ and $R_K$ $^2$ $^3$ to constrain new physics effects. Given our limited predictive power on $f_\pi$ and $f_K$, deviations from the $\mu - e$ universality represent the best hope we have at the moment to detect new physics effects in $\pi_{2\ell}$ and $K_{2\ell}$.

The most recent NA48/2 result on $R_K$:

$$R_K^{\text{exp}} = (2.416 \pm 0.043_{\text{stat.}} \pm 0.024_{\text{syst.}}) \times 10^{-5} \quad \text{NA48/2}$$

which will further improve with current analysis, significantly improves on the previous PDG value:

$$R_K^{\text{SM}} = (2.44 \pm 0.11) \times 10^{-5}.$$ 

This is to be compared with the SM prediction which reads:

$$R_K^{\text{SM}} = (2.472 \pm 0.001) \times 10^{-5}.$$ 

Denoting by $\Delta r^{\mu - e}_{NP}$ the deviation from $\mu - e$ universality in $R_K$ due to new physics, i.e.:

$$R_K = \frac{R_K^{\text{SM}}}{1 + \Delta r^{\mu - e}_{NP}},$$

the NA48/2 result requires (at the $2\sigma$ level):

$$-0.063 \leq \Delta r^{\mu - e}_{NP} \leq 0.017 \quad \text{NA48/2}.$$ 

In this Letter we consider low-energy minimal SUSY extensions of the SM (MSSM) with $R$ parity as the source of new physics to be tested by $R_K$ $^4$. The question we intend to address is whether SUSY can cause deviations from $\mu - e$ universality in $K_{12}$ at a level which can be probed with the present attained experimental sensitivity, namely at the percent level. We will show that i) it is indeed possible for regions of the MSSM to obtain $\Delta r^{\mu - e}_{NP}$ of $O(10^{-2})$ and ii) such large contributions to $K_{12}$ do not arise from SUSY lepton flavor conserving (LFC) effects, but, rather, from LFV ones.

At first sight, this latter statement may seem rather puzzling. The $K \to e\nu_\ell$ and $K \to \mu\nu_\ell$ decays are LFC and one could expect that it is through LFC SUSY contributions affecting differently the two decays that one obtains the dominant source of lepton flavor non-universality in SUSY. On the other hand, one can easily guess that, whenever new physics intervenes in $K \to e\nu_\ell$ and $K \to \mu\nu_\ell$ to create a departure from the strict SM $\mu - e$ universality, these new contributions will be proportional to the lepton masses; hence, it may happen (and, indeed, this is what occurs in the SUSY case) that LFC contributions are suppressed with respect to the LFV ones by higher powers of the first two generations lepton masses (it turns out that the first contributions to $\Delta r^{\mu - e}_{NP}$ from LFC terms arise at the cubic order in $m_\ell$, with $\ell = e, \mu$). A second, important reason for such result is that among the LFV contributions to $R_K$ one can select those which involve flavor changes from the first two lepton generations to the third one with the possibility of picking up terms proportional to the tau-Yukawa coupling which can be large.
in the large $\tan\beta$ regime (the parameter $\tan\beta$ denotes the ratio of Higgs vacuum expectation values responsible for the up- and down- quark masses, respectively). Moreover, the relevant one-loop induced LFV Yukawa interactions are known \( \Box \) to acquire an additional tan $\beta$ factor with respect to the tree level LFC Yukawa terms. Thus, the loop suppression factor can be (partially) compensated in the large $\tan\beta$ regime.

Finally, given the NA48/2 $R_K$ central value below the SM prediction, one may wonder whether SUSY contributions could have the correct sign to account for such an effect. Although the above mentioned LFV terms can only add positive contributions to $R_K$ (since their amplitudes cannot interfere with the SM one), it turns out that there exist LFC contributions arising from double LFV mass insertions (MI) in the scalar lepton propagators which can destructively interfere with the SM contribution. We will show that there exist regions of the SUSY parameter space where the total $R_K$ arising from all such SM and SUSY terms is indeed lower than $R_K^{SM}$.

\( \mu - e \) UNIVERSALITY IN $\pi \to \ell\nu$ AND $K \to \ell\nu$ DECAYS

Due to the V-A structure of the weak interactions, the SM contributions to $\pi\ell_2$ and $K\ell_2$ are helicity suppressed; hence, these processes are very sensitive to non-SM effects (such as multi-Higgs effects) which might induce an effective pseudoscalar hadronic weak current.

In particular, charged Higgs bosons ($H^\pm$) appearing in any model with two Higgs doublets (including the SUSY case) can contribute at tree level to the above processes inducing the following effects \( \Box \):

\[
\frac{\Gamma(M \to \ell\nu)}{\Gamma_{SM}(M \to \ell\nu)} = \left[ 1 - \tan^2\beta \left( \frac{m_{s,d}}{m_u + m_{s,d}} \right) \frac{m^2_M}{m^2_H} \right]^2
\]

where $m_u$ is the mass of the up quark while $m_{s,d}$ stands for the down-type quark mass of the $M$ meson ($M = K, \pi$). From Eq. (2) it is evident that such tree level contributions do not introduce any lepton flavour dependent correction. The first SUSY contributions violating the $\mu - e$ universality in $\pi \to \ell\nu$ and $K \to \ell\nu$ decays arise at the one-loop level with various diagrams involving exchanges of (charged and neutral) Higgs scalars, charginos, neutralinos and sleptons. For our purpose, it is relevant to divide all such contributions into two classes: i) LFC contributions where the charged meson $M$ decays without FCNC in the leptonic sector, i.e. $M \to \ell\nu\tau$; ii) LFV contributions $M \to \ell\nu\nu$, with $i$ and $k$ referring to different generations (in particular, the interesting case will be for $i = e, \mu$, and $k = \tau$).

THE LEPTON FLAVOUR CONSERVING CASE

One-loop corrections to $R_\pi$ and $R_K$ include box, wave function renormalization and vertex contributions from SUSY particle exchange. The complete calculation of the $\mu$ decay in the MSSM \( \Box \) can be easily applied to the meson decays. It turns out that all these LFC contributions yield values of $\Delta R^{e-\mu}_{SM}$, which are much smaller than the percent level required by the achieved experimental sensitivity. Indeed, a typical $\Delta R^{e-\mu}_{SUSY}$ induced by (charged and neutral) Higgs exchanges is of order

\[
\Delta R^{e-\mu}_{SUSY} \sim \alpha_2 \left( \frac{m^2_\mu - m^2_e}{m^2_H} \right) \tan^2\beta ,\]

where $H$ denotes a heavy Higgs circulating in the loop. Then, even if we assume particularly favorable circumstances like $\tan\beta = 50$ and arbitrary relations among the Higgs boson masses, we end up with $\Delta R^{e-\mu}_{SUSY} \lesssim 10^{-6}$ much below the percent level of experimental sensitivity.

The charginos/neutralinos sleptons ($\ell_{e,\mu}$) contributions to $\Delta R^{e-\mu}_{SUSY}$ are of the form

\[
\Delta R^{e-\mu}_{SUSY} \sim \frac{\alpha_2}{4\pi} \left( \frac{m^2_\mu - m^2_e}{m^2_\mu + m^2_e} \right) \frac{m^2_\nu}{M^2_{SUSY}} ,
\]

where we considered all SUSY masses involved in the loops to be of $\mathcal{O}(M_{SUSY})$. The degeneracy of slepton masses (in particular those of the first two generations) severely suppresses these contributions. Even if we assume a quite large mass splitting among slepton masses (at the 10% level for instance) we end up with $\Delta R^{e-\mu}_{SUSY} \lesssim 10^{-4}$. For the box-type non-universal contributions we find similar or even more suppressed effects compared to those we have studied.

On the other hand, one could wonder whether the quantity $\Delta R^{e-\mu}_{SUSY}$ can be constrained by the pion physics. In principle, the sensitivity could be even higher: from

\[
R^{e-\mu}_{SM} = (1.2354 \pm 0.0002) \cdot 10^{-4}
\]

and by making a comparison with the SM prediction

\[
R^{e-\mu}_{SM} = (1.2354 \pm 0.0002) \cdot 10^{-4}
\]

one obtains (at the $2\sigma$ level)

\[-0.0107 \leq \Delta R^{e-\mu}_{NP} \leq 0.0022 .
\]

Unfortunately, even in the most favorable cases, $\Delta R^{e-\mu}_{SUSY}$ remains much below its actual exp. upper bound.

In conclusion, SUSY effects with flavor conservation in the leptonic sector can differently contribute to the $K \to \ell\nu\ell_\nu$ and $K \to \mu\nu\ell_\nu$ decays, hence inducing a $\mu - e$ non-universality in $R_K$, however such effects are still orders of magnitude below the level of the present exp. sensitivity on $R_K$. The same conclusions hold for $R_\pi$. 

\[
-0.0107 \leq \Delta R^{e-\mu}_{NP} \leq 0.0022 .
\]
THE LEPTON FLAVOUR VIOLATING CASE

It is well known that models containing at least two Higgs doublets generally allow flavour violating couplings of the Higgs bosons with the fermions $\tilde{R}$. In the MSSM such LFV couplings are absent at tree level. However, once non holomorphic terms are generated by loop effects (so called HRS corrections) and given a source of LFV among the sleptons, Higgs-mediated (radiatively induced) $V_{\ell R \ell L}$ LFV couplings are unavoidable. These effects have been widely discussed in the recent literature through the study of several processes, namely $\tau \to \ell_j \ell_k \ell_l$, $\mu - e$ conversion in nuclei, $B \to \ell_j \ell \tau$, $H \to \ell_j \ell_k$ and $\ell_i \to \ell_j \gamma$.

In this Letter we analyze the LFV decay channels of the purely leptonic $\pi^\pm$ and $K^\pm$ decays and discuss a possible way to detect LFV SUSY effects through a deviation from the $\mu - e$ universality. One could naively think that SUSY effects in the LFV channels $M \to \ell_i \nu_k$ are further suppressed with respect to the limits on the LFC ones. On the contrary, we show that charged Higgs mediated SUSY LFV contributions, in particular in the kaon decays into an electron or a muon and a tau neutrino, can be strongly enhanced.

The quantity which now accounts for the deviation from the $\mu - e$ universality reads:

$$R_{\pi,K}^{LFV} = \frac{\sum_i \Gamma(\pi^0(K) \to e\nu_i)}{\sum_i \Gamma(\pi^0(K) \to \mu\nu_i)} \quad i = e, \mu, \tau,$$

with the sum extended over all (anti)neutrino flavors (experimentally one determines only the charged lepton flavor in the decay products).

The dominant SUSY contributions to $R_{\pi,K}^{LFV}$ arise from the charged Higgs exchange. The effective LFV Yukawa couplings we consider are (see Fig. 1):

$$\ell H_{\pm} \nu_\tau \to \frac{g_2}{\sqrt{2}} m_\tau \Delta_R^M \tan^2 \beta \quad \ell = e, \mu. \quad (5)$$

Crucial to our result is the quadratic dependence on $\tan \beta$ in the above coupling: one power of $\tan \beta$ comes from the trilinear scalar coupling in Fig.1, while the second one is a specific feature of the above HRS mechanism.

The $\Delta_R^M$ terms are induced at one loop level by the exchange of Bino (see Fig.1) or Bino-Higgsino and sleptons. Since the Yukawa operator is of dimension four, the quantities $\Delta_R^M$ depend only on ratios of SUSY masses, hence avoiding SUSY decoupling. In the so called MI approximation the expression of $\Delta_R^M$ is given by:

$$\Delta_R^M \simeq \frac{\alpha_1}{4\pi} \mu M_1 m_\tau^2 \delta_R^{M} \left[ I(M_1^2, \mu^2, m_\tau^2) - (\mu \leftrightarrow m_\ell) \right] \quad (6)$$

where $\mu$ is the the Higgs mixing parameter, $M_1$ is the Bino ($B$) mass and $m_\tau$ stands for the left-handed (right-handed) slepton mass matrix entry. The LFV MIs, i.e. $\delta_R^{M}(X) = (\bar{m}_2)^2_{X}/m_X^2$ ($X = L,R$), are the off-diagonal flavor changing entries of the slepton mass matrix. The loop function $I(x,y,z) = dI(x,y,z)/dz$, where $I(x,y,z)$ refers to the standard three point one-loop integral which has mass dimension -2. Following the thorough analysis in 13, it turns out that $\Delta_R^M \lesssim 10^{-3}$.

Making use of the LFV Yukawa coupling in Eq. (5), it turns out that the dominant contribution to $\Delta_{K,\mu}^{F S U S Y}$ reads:

$$R_{K}^{LFV} \simeq R_{K}^{SM} \left( 1 + \frac{m_4^4}{M_H^2} \frac{m_2^2}{m_2^2} |\Delta_R^{M}|^2 \tan^6 \beta \right). \quad (7)$$

In Eq. (7) terms proportional to $\Delta_R^M$ are neglected given that they are suppressed by a factor $m_2^2/m_4^2$ with respect to the term proportional to $\Delta_R^{M}$. Taking $\Delta_R^{M} \simeq 5 \cdot 10^{-4}$ accordingly to what said above, $\tan \beta = 40$ and $M_H = 500$GeV we end up with $R_{K}^{LFV} \simeq R_{K}^{SM} (1 + 0.013)$. We see that in the large (but not extreme) tan $\beta$ regime and with a relatively heavy $H^\pm$, it is possible to reach contributions to $\Delta_{F S U S Y}^{F S U S Y}$ at the percent level thanks to the possible LFV enhancements arising in SUSY models.

Turning to pion physics, one could wonder whether the analogous quantity $\Delta_{\pi}^{F S U S Y}$ is able to constrain SUSY LFV. However, the correlation between $\Delta_{\pi}^{F S U S Y}$ and $\Delta_{K,\mu}^{F S U S Y}$:

$$\Delta_{\pi}^{F S U S Y} \simeq \left( \frac{m_d}{m_u + m_d} \right)^2 \frac{m_4^4}{m_k^2} \Delta_{F S U S Y}^{K,\mu} \quad (8)$$

clearly shows that the constraints on $\Delta_{K,\mu}^{F S U S Y}$ force $\Delta_{\pi}^{F S U S Y}$ to be much below its actual exp. upper bound.

Obviously, a legitimate worry when witnessing such a huge SUSY contribution through LFV terms is whether the bounds on LFV tau decays, like $\tau \rightarrow e X$ (with $X = \gamma, \eta, \mu \mu$), are respected. Higgs mediated $Br(\tau \rightarrow \ell X)$ and $\Delta_{F S U S Y}^{F S U S Y}$ have exactly the same SUSY dependence; hence, we can compute the upper bounds of the relevant LFV tau decays which are obtained for those values of the SUSY parameters yielding $\Delta_{F S U S Y}^{F S U S Y}$ at the percent level. We obtain $Br(\tau \rightarrow e X) \lesssim 10^{-10}$. Given the exp. upper bounds on the LFV $\tau$ lepton decays, we conclude that it is possible to saturate the upper bound on $\Delta_{F S U S Y}^{F S U S Y}$ while remaining much below the present and expected future upper bounds on such LFV decays. There exist other SUSY contributions
to LFV $\tau$ decays, like the one-loop neutralino-charged slepton exchanges, for instance, where there is a direct
dependence on the quantities $\delta_{RR}^{\ell}$. Given that the
existing bounds on the leptonic $\delta_{RR}^{\ell}$ involving transitions
to the third generation are rather loose [16], it turns out
that also these contributions fail to reach the level of exp.
sensitivity for LFV $\tau$ decays.

**ON THE SIGN OF $\Delta r_{\mu}^{\tau-\mu}$**

The above SUSY dominant contribution to $\Delta r_{\mu}^{\tau-\mu}$
arises from LFV channels in the $K \to e\nu$ mode, hence
without any interference effect with the SM contribution.
Thus, it can only increase the value of $R_K$ with respect
the SM expectation. On the other hand, the recent
NA48/2 result exhibits a central value lower than $R_K^{SM}$
(and, indeed, also lower than the previous PDG central
value). One may wonder whether SUSY could account
for such a lower $R_K$. Obviously, the only way it can is
through terms which, contributing to the LFC $K \to \nu\nu$
channels, can interfere (destructively) with the SM con-
tribution. We already commented that SUSY LFC contribu-
tions are subdominant. However, one can envisage
the possibility of making use of the large LFV contribu-
tions to give rise to LFC ones through double LFV MI
that, as a final effect, preserves the flavour.

To see this point explicitly, we derive the corrections
to the LFC $H^{\pm}\nu_\ell$ vertices induced by LFV effects

$$\ell H^{\pm} \nu_\ell \to g_2 \frac{m_\ell}{\sqrt{2} M_W} \tan\beta \left( 1 + \frac{m_\tau}{m_\ell} \Delta^{\ell}_{RL} \tan\beta \right), \quad (9)$$

where $\Delta^{\ell}_{RL}$ is generated by the same diagram as in Fig. 1
but with an additional $\delta_{LL}^{\ell \ell}$ MI in the sneutrino propa-
gator. In the MI approximation, $\Delta^{\ell}_{RL}$ is given by

$$\Delta^{\ell}_{RL} \simeq -\frac{\alpha_3}{4 \pi} \mu M_1 \mu^2 R \delta^{\ell \ell} \delta^{3 \ell \ell} \tilde{I}''(M_2^2, m_1^2, m_2^2), \quad (10)$$

where $\tilde{I}''(x, y, z) = \tilde{I}''(x, y, z)/dy dz$. In the large slep-
ton mixing case, $\Delta^{\ell}_{RL}$ terms are of the same order of
$\Delta_{R}^{\ell}$ [18]. These new effects modify the previous $R_K^{LFV}$
expression in the following way:

$$R_K^{LFV} \simeq R_K^{SM} \left[ 1 - \frac{m_{K}^{2} m_{\ell}}{m_{H}^{2} m_{e}} \Delta_{R}^{\ell} \tan^{2}\beta \right]^2 + \frac{m_{K}^{4}}{m_{H}^{2}} \left( \frac{m_{\ell}^{2}}{m_{e}^{2}} \right) \Delta_{R}^{\ell} \tan^{2}\beta. \quad (11)$$

In the above expression, besides the contributions reported
in Eq. 7, we included also the interference between
SM and SUSY LFC terms (arising from a double
LFV source). Setting the parameters as in the ex-
ample of the above section and if $\Delta_{R}^{\ell} = 10^{-4}$ we get
$R_K^{LFV} \simeq R_K^{SM} (1 - 0.032)$, that is just within the expected exp.
resolution reachable by NA48/2 once all the avail-
ble data will be analyzed. Finally, we remark that the
above effects do not spoil the pion physics constraints.

**CONCLUSIONS**

Our Letter shows that, rather surprisingly, a precise
measurement of the flavor conserving $K_{e2}$ decays may
shed light on the size of LFV in new physics. Since neu-
trino masses and oscillations clearly point out that lepton
flavor number is violated and since new physics (for in-
stance supersymmetric versions of models with see-saw
mechanism for neutrino masses [18]) is known to have
the potentiality for (large) enhancements of such LFV
with respect to the SM, we emphasize the importance of
further improving the exp. sensitivity on $R_K$ as a partic-
cularly interesting probe of such new physics effects.

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