Efficient Model Assisted Probability of Detection Estimations in Eddy Current NDT

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Abstract—In this article, an efficient 3D eddy current nondestructive evaluation (ECNDE) solver is proposed to make estimations for probability of detection (PoD) study. The singular value decomposition (SVD) works as the recompression technique to improve the overall performance of the adaptive cross approximation (ACA) algorithm-based boundary element method (BEM) ECNDE solver. Through the benchmark tests, the proposed solver is demonstrated to be an efficient simulation tool for model assisted PoD study with significant memory and CPU time savings while maintaining a good accuracy.

Keywords—adaptive cross approximation (ACA) algorithm, boundary element method (BEM), eddy current testing (ECT), model assisted probability of detection (MAPOD), singular value decomposition (SVD).

I. INTRODUCTION

Nondestructive testing/evaluation (NDT/E) refers to test the defects or damages of materials without destroying the service performance of the tested object [1]. This non-invasive method is widely used in aerospace, civil engineering and nuclear industry [2]. Probability of detection (PoD) is an important method to evaluate the reliability of NDT system [3]. At the beginning, PoD was obtained only by experiment, but it takes a lot of time. Therefore, model assisted probability of detection (MAPoD) was proposed, and a large amount of data required for PoD calculation are obtained through simulation model [4].

As a fast algorithm, the adaptive cross approximation (ACA) algorithm can be used to compress low rank matrices, which can greatly reduce the memory and CPU time requirements for BEM matrices used in solving eddy current NDT problems [5]. In ACA algorithm, the low rank matrix $Z$ would be approximated by the multiplication of the $U$ and $V$ matrices, and the redundancies can be removed by the algebraic recompression technique which is called the singular value decomposition (SVD) [6].

In this article, the data required for MAPoD study are obtained from the novel ACA-SVD accelerated BEM solver, which is different from the conventional eddy current MAPoD study. The accuracy and efficiency of the proposed solver is validated, and the PoD curve is plotted for eddy current NDT problem.

II. FORMULATION

A. BEM model

In this work, the large amounts of data required for PoD study are obtained from ACA-SVD based BEM solver, instead of time-consuming experiments. The formulation selected is the Stratton-Chu formulation which contains both the tangential and normal components of the electromagnetic fields explicitly. The Stratton-Chu formation can be used with low frequency and high conductivity approximations, because in the operating frequency range of eddy current NDT, the conduction currents in the metal are greater than the displacement currents. The details of BEM based ECNDE solver can be found in [7].

B. ACA-SVD algorithm

The entire impedance matrix obtained from BEM is not rank deficient, while, there are rank deficient submatrices due to the nature of Green function. To find the low rank matrices, firstly, the object under detect is enclosed in a cube and partitioned into smaller blocks until each one contains certain number of unknowns. Then, the block pairs are classified into near and far block ones based on the distance between blocks. The near block pairs are fully computed by BEM while the far block ones are approximated by the ACA-SVD algorithm. The basic procedures of ACA algorithm can be found in [5]. There may contain redundancies in $U$ and $V$ matrices and the redundancies can be removed by the singular value decomposition. With the QR decompositions $U = Q_u R_u$ and $V^T = Q_v R_v$, the products of $R_u$ and $R_v$ matrices can be decomposed by SVD as $R_u R_v^T = \tilde{U} \tilde{\Sigma} \tilde{V}$. The original matrix $Z$ is approximated as $Z = \tilde{U} \times \tilde{V}$, where $\tilde{U} = Q_u \tilde{U}$ and $\tilde{V} = \tilde{V} Q_v$. The SVD works as the post compression technique to reduce the storage in ACA algorithm.
III. NUMERICAL RESULTS

A. Validation of the ACA-SVD based BEM solver

The proposed ACA-SVD based BEM solver is validated by placing a coil with the finite cross section on the conductor plate to predict the impedance changes. The plate has a conductivity of 21.8 MS/m, a thickness of 65 mm, the inner and outer radius of the coil are 7.04 mm and 12.4 mm respectively, the liftoff is 3.43 mm, the height is 5.04 mm, the number of turns is 556, and the detection frequency is 20 kHz. Good agreements of the impedance variations among the ACA-SVD, ACA, BEM, analytical, semi-analytical methods and experiment are achieved, and the relative differences among them are smaller than 1% in both real and imaginary parts of the impedance variations. As show in Table 1 and Table 2, it can be found that both the memory requirements of far block interactions and the CPU time per iteration decrease as the truncated error of SVD increases with maintaining the tolerance of ACA algorithm.

| TABLE I. MEMORY REQUIREMENTS OF THE FAR BLOCK INTERACTIONS. |
|------------------|------------------|------------------|------------------|------------------|
| τ                | ACA              | ACA-SVD e=10^-4 | ACA-SVD e=10^-3 | ACA-SVD e=10^-2 | ACA-SVD e=10^-1 |
| 10^-3            | 120.3            | 119.0            | 104.9            | 56.6             |
| 10^-2            | 171.7            | 151.1            | 108.2            | 56.8             |
| 10^-1            | 225.8            | 154.6            | 108.5            | 56.9             |

| TABLE II. CPU TIME PER ITERATION. |
|------------------|------------------|------------------|------------------|------------------|
| τ                | ACA              | ACA-SVD e=10^-4 | ACA-SVD e=10^-3 | ACA-SVD e=10^-2 | ACA-SVD e=10^-1 |
| 10^-3            | 0.037            | 0.0368           | 0.0368           | 0.0356           | 0.0287           |
| 10^-2            | 0.044            | 0.0438           | 0.0404           | 0.0353           | 0.0293           |
| 10^-1            | 0.049            | 0.0473           | 0.0420           | 0.0353           | 0.0294           |

B. PoD analysis

The specific details of PoD can be referred to [3]. For the PoD analysis, a finite thickness plate with a rectangular surface flaw is studied, which has a thickness of 12.22 mm with the conductivity 30.6 MS/m. The flaw lengths range from 0.1 mm to 1 mm in the step of 0.1 mm, with the depth 5 mm and width 0.28 mm. The coil with finite cross section has the inner radius 9.34 mm, outer radius 18.4 mm, and 408 turns. The flaw features, which are the maximum impedance variations of the coil, are predicted at 7000 Hz. To account for the uncertainty due to the imperfection in inspections, lift-off variation is taken as the uncertain parameter, which is assumed to be normal distribution with mean 2 mm and variance 0.5 mm.

The PoD curve with a lower 95% confidence bound is shown in Fig.1. The threshold value is 7 mΩ. The three key PoD metrics \( \alpha_{50} \), \( \delta_{90} \) and \( \delta_{90/95} \) are 0.579 mm, 0.669 mm, and 0.678 mm, respectively.

IV. CONCLUSION

In this work, in order to obtain the data for MAPoD study, a novel ACA-SVD based BEM eddy 3D current NDE solver is proposed. The accuracy and efficiency are validated in the benchmark. Then a finite thickness plate with a rectangular surface defects is analyzed for MAPoD study. It shows that the proposed solver is an excellent simulation tool to make estimations for MAPoD study.

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