A semi-synchronous label propagation algorithm with constraints for community detection in complex networks

Jia Hou Chin & Kuru Ratnavelu

Community structure is an important feature of a complex network, where detection of the community structure can shed some light on the properties of such a complex network. Amongst the proposed community detection methods, the label propagation algorithm (LPA) emerges as an effective detection method due to its time efficiency. Despite this advantage in computational time, the performance of LPA is affected by randomness in the algorithm. A modified LPA, called CLPA-GNR, was proposed recently and it succeeded in handling the randomness issues in the LPA. However, it did not remove the tendency for trivial detection in networks with a weak community structure. In this paper, an improved CLPA-GNR is therefore proposed. In the new algorithm, the unassigned and assigned nodes are updated synchronously while the assigned nodes are updated asynchronously. A similarity score, based on the Sørensen-Dice index, is implemented to detect the initial communities and for breaking ties during the propagation process. Constraints are utilised during the label propagation and community merging processes. The performance of the proposed algorithm is evaluated on various benchmark and real-world networks. We find that it is able to avoid trivial detection while showing substantial improvement in the quality of detection.

Over the decade, network analysis has been widely applied in various research fields such as biology, transportation, sociology and bibliometric studies1–4. Complex networks possess features that provide insight into these properties, with a majority of the real-world complex networks consisting of a network feature called the community structure. A community in a complex network is defined as a set of nodes that are densely connected to each other in a group, while they are loosely connected with the rest of the network5. Naturally, nodes with similar attributes will be more likely to form a community. Thus, in principal, one can acquire the functions, traits or properties of a group of individuals by investigating a community. Given the practicality of studying the community structure in complex networks, community detection emerges as a popular research topic. Consequently, a large number of community detection algorithms have been developed for the purpose of uncovering the community structure in complex networks6.

The label propagation algorithm (LPA)7 was first introduced in 2007, as a community detection algorithm that requires less computational time. The objective of the LPA is to allocate each node into a community with the most number of its neighbouring nodes. The simplicity and near linear complexity of the LPA makes it feasible to detect communities in huge networks with millions of nodes. However, there are some pronounced issues in the LPA that affect its performance. The randomness that is induced in its update sequences and tie breaking processes cause the LPA to return multiple detections, thus making it a non-deterministic detection algorithm. Furthermore, in networks with a weak community structure, the LPA is unable to detect any meaningful community. As a consequence, the LPA detects only one community (trivial detection) in such networks.

The relative simplicity of LPA, coupled with these issues, led scientists to seek improvements and enhancements in this algorithm. Leung et al.8 introduced link preferential and hop attenuation to handle the tie breaking cases. Modularity was implemented into the LPA by Barber and Clark9, while Liu and Murata10 further improved it by merging the detected communities to further increase the modularity. Xie et al.11 proposed a modified LPA, called the speaker-listener based LPA (SLPA), that can detect overlapping communities. Aside from their SLPA,
Xie and Szymanski12 also developed LabelRank, a LPA variant that implemented a Markov Cluster Algorithm in order to stabilise LPA. They further improved the LabelRank with an update rule, hence speeding up the LPA13. A modified LPA that utilises a prediction of the percolation transition was proposed by Zhang et al.14. That algorithm is able to delay the formation of monster size communities, which reduces the chance of trivial detection. The NIBLPA15 is a node-influence based LPA that tackles the randomness issue in the LPA. The influence scores, of the nodes in a network, are used to determine the update sequences as well as breaking ties between multiple communities during the propagation processes. The most recent LPA variants include the SpeakEasy16, LINSIA17 and CLPA-GNR18 algorithms. SpeakEasy is a LPA variant that specialises in the detection of overlapping communities in biological networks. It employs both the neighbouring and global information of a network so that the combination of that information can yield accurate detection. The ability of LINSIA to control the propagation processes allows it to reveal hub and outlier nodes, apart from detecting overlapping communities in a network. CLPA-GNR is a modified LPA that implements constraints at different stages of the algorithm, while updating the solo and grouped nodes separately. Furthermore, it can obtain deterministic detections in undirected and unweighted networks by removing the randomness in the LPA. Even though much effort has already been expended in getting rid of the disadvantages and in enhancing the advantages of the LPA, the improvement of the LPA in terms of robustness and stability remains an open question.

Semi Synchronous Constrained Label Propagation Algorithm (SSCLPA)

The implementation of constraints and fixed update sequences in the CLPA-GNR allow it to produce accurate and deterministic detection. However, the drawback of the CLPA-GNR is its tendency of obtaining trivial detection in networks with weak community structure18. Hence, in this work, we address this issue and propose a new LPA variant called SSCLPA, which is an improved CLPA-GNR. The proposed algorithm can detect disjoint communities in undirected and unweighted networks. It draws on the essence of the CLPA-GNR such as the constraints and fixed update sequence, and further enhances them. As a result, the SSCLPA is able to avoid trivial detection and still be able to obtain deterministic and accurate detection.

Similar to its predecessor, the proposed algorithm consists of constraints that are applied at various stages of the algorithm. The restrictions are gradually relaxed towards the end of the algorithm. A new constraint is implemented in the SSCLPA, where communities that reach certain threshold of strength values are exempted from the propagation or merging processes. This new form of constraint is crucial in delaying the formation of monster size communities, hence allowing the growth of other communities. By limiting the growth of specific communities, the chances of getting trivial detection can be eliminated. Instead of the mutual neighbour score (MNS) that is used in the CLPA-GNR, the Sorensen-Dice index (SDI) is implemented as the similarity score in SSCLPA. Similar to the function of the MNS in the CLPA-GNR, SDI is used in the early stage of the algorithm to detect initial communities. Aside from that, it can substitute the capacity score in the CLPA-GNR to break ties between multiple communities during the propagation processes.

In both the CLPA-GNR and SSCLPA, nodes are categorised into two types, namely the solo and grouped nodes. But, unlike in the CLPA-GNR where all the propagation processes are asynchronous, the solo nodes undergo synchronous updates while grouped nodes are subjected to asynchronous updates in the SSCLPA. The synchronous updates of solo nodes can speed up the propagation process without sacrificing the accuracy of the detection. There are also difference in the rules for the update sequence in CLPA-GNR and SSCLPA. In the CLPA-GNR, the degree of the nodes is the only criterion in deciding the update sequences. However, in the SSCLPA, the number of neighbouring nodes that are also solo nodes is also taken into account.

In general, the SSCLPA will do an initial detection by using similarity score, which creates large amount of small communities. The propagation process involves the propagation of the communities, while the merging process attempts to reduce the number of communities by merging them. These processes are repeated throughout the algorithm until convergence in the labels is achieved. The details of the SSCLPA are explained in the Method section.

Results

The SSCLPA is tested on various benchmark networks before it is implemented on any real-world network. Three types of benchmark networks are employed in this study, namely the Lancichinetti-Fortunato-Radicchi (LFR)19,20, Girvan-Newman (GN)2, and Relaxed Caveman (RC)21,22 benchmark networks. It must be noted that at this time both the benchmark and real-world networks are undirected and unweighted with disjoint communities. The evaluation criterion for networks with ground truth communities is the normalised mutual information (NMI)23. The value of the NMI ranges from 0 to 1, where NMI = 1 when two partitions are identical. If a partition is totally independent of another partition, then NMI = 0. On the other hand, the modularity (Q)24 and modularity density (Qsd)25 are used to evaluate the quality of detection in networks without the ground truth communities. A good detection yields high values of Q and Qsd.

The performance of the proposed algorithm is also compared to the other community detection methods: LPA7, CLPA-GNR18, GANXis (or SLPA)11,26,27, the Ronhovde and Nussinov algorithm (RN)28, Blondel29, and Infomap30. LPA is the original label propagation algorithm, while the CLPA-GNR and GANXis are the aforementioned LPA variants. In addition, the RN is a spin–glass type Potts model community detection algorithm. This algorithm is not only good in detecting heterogeneous sized communities in a network, but it is also free from resolution-limit. The Blondel algorithm is an effective modularity maximisation detection method, that can detect communities heuristically in a relatively short computational time. Lastly, Infomap detects communities by optimising the map equation while minimising the description length of a random walker. As GANXis, LPA and Blondel do not produce a deterministic detection, they are executed 100 times for each network and the detection that yields the highest Q value is chosen for the purpose of comparison.
The Lancichinetti-Fortunato-Radicchi benchmark (LFR).

The LFR benchmark networks are the most popular benchmark networks for community detection algorithms, as they contain features that are common in the real-world networks. Furthermore, the degree of nodes and the sizes of the communities in the generated LFR synthetic networks are always heterogeneous. One of the most important parameters in the LFR networks is the mixing parameter, $\mu$, which represents the average percentage of edges that connect a pair of nodes from different communities. Note that the strength of the community structure decreases as the value of $\mu$ increases.

We generated 4 groups of LFR networks in this investigation. The average and maximum degrees are fixed at $k_{avg} = 20$ and $k_{max} = 50$. On the other hand, the exponent for the degree sequence and the exponent for the community size distribution are fixed at $\gamma = 2$ and $\beta = 1$. The rest of the parameters are depicted in Table 1.

Table 1. Summary of the generated LFR networks.

| Networks | $N$  | $C_{min}$ | $C_{max}$ | $\mu$          |
|----------|------|-----------|-----------|----------------|
| SNSC     | 1000 | 10        | 50        | 0.1 to 0.8     |
| SNLC     | 1000 | 20        | 100       | 0.1 to 0.8     |
| LNSC     | 5000 | 10        | 50        | 0.1 to 0.8     |
| LNLC     | 5000 | 20        | 100       | 0.1 to 0.8     |

Figure 1. The NMI comparison on the undirected and unweighted LFR benchmark networks, with various network sizes and community sizes. The notations $N$, $k$ and $C$ refer to the size of networks, average degree and the size of communities, respectively. See also the caption on the plots.

Lancichinetti-Fortunato-Radicchi benchmark (LFR). The LFR benchmark networks are the most popular benchmark networks for community detection algorithms, as they contain features that are common in the real-world networks. Furthermore, the degree of nodes and the sizes of the communities in the generated LFR synthetic networks are always heterogeneous. One of the most important parameters in the LFR networks is the mixing parameter, $\mu$, which represents the average percentage of edges that connect a pair of nodes from different communities. Note that the strength of the community structure decreases as the value of $\mu$ increases.

We generated 4 groups of LFR networks in this investigation. The average and maximum degrees are fixed at $k_{avg} = 20$ and $k_{max} = 50$. On the other hand, the exponent for the degree sequence and the exponent for the community size distribution are fixed at $\gamma = 2$ and $\beta = 1$. The rest of the parameters are depicted in Table 1.

Figure 1 shows the results for the various detection algorithms on the 4 groups of LFR networks. Algorithms that yield NMI = 0 indicate that those algorithms can only detect a single community in the networks. Thus, the
detection is trivial and the algorithms cannot uncover any community structure in those networks. The present algorithm manages to detect community structure in all the groups of the LFR networks, even in networks with high $\mu$ values. The other LPA based algorithms, including the original LPA, get trivial detection when $\mu \geq 0.6$. Infomap also faces a similar problem at high $\mu$ values. We highlight that not only does the SSCLPA detect communities at high $\mu$ values ($\mu \geq 0.7$), the quality of the detection is also good. For instance, in the SNSC case (Fig. 1(A)), the $\text{NMI}$ of the present algorithm is significantly higher than the other algorithms at $\mu = 0.7$. The $\text{NMI}$ values of the SSCLPA in the SNLC, LNSC and LNLC cases (Fig. 1(B–D)) are close to 1, which indicates near perfect detection. Although the performance of the present algorithm is not as good as RN at $\mu = 0.8$ in most of the networks, the quality of its detections is still highly acceptable. One of the reasons for the discrepancies between the SSCLPA and RN algorithms is the number of detected communities in networks with $\mu = 0.8$. The number of detected communities in those networks by using the present algorithm is always smaller than the number of ground truth communities. On the contrary, the RN yields a large number of small size communities in those networks. Nonetheless, the overall performance of SSCLPA is good and it outperforms the other LPA based algorithms when applied to the LFR benchmark networks.

**Girvan-Newman benchmark (GN).** There are 128 nodes, which are divided equally into 4 communities with 32 nodes each, in the GN networks. The parameters, $P_{\text{in}}$ and $P_{\text{out}}$, refer to the probabilities of defining an edge as an intra-community or inter-community edge. It is important for $k_{\text{avg}} \geq 16$ when deciding the values of $P_{\text{in}}$ and $P_{\text{out}}$. GN networks can be generated by using the LFR networks generator, where $\mu$ is used to represent $P_{\text{in}}$ and $P_{\text{out}}$.

The following parameters are employed to generate the GN networks with the LFR networks generator: $N = 128$, $k_{\text{avg}} = k_{\text{max}} = 16$, $C = 32$, $\gamma = \beta = 0$, and $\mu = 0–0.8$.

The comparison between the results of the SSCLPA and the other algorithms in a GN benchmark is depicted in Fig. 2(A). Once again, the SSCLPA yields the best detection across the $\mu$ values among the LPA based algorithms in this benchmark test. Furthermore, it outperforms Infomap and its performance is on par with Blondel. Although the $\text{NMI}$ values of the proposed algorithm are again not as high as the RN at $\mu = 0.8$, the SSCLPA still manages to detect 4 communities in those GN networks. On the other hand, similar to the LFR cases, RN detects far more communities than the number of ground truth communities in those networks.

**Relaxed Caveman benchmark (RC).** The RC benchmark networks have 512 nodes with 16 communities of highly heterogeneous size. Initially, a RC network consists of 16 isolated $k$-cliques. Similar to the role of $\mu$ in the LFR benchmark networks, here a parameter known as the Degradation ($D$)$^{31}$ is implemented to progressively weaken the community structure of the network. As the value of $D$ increases, the number of intra-community edges that are converted to inter-community edges is increased. The value of $D$ is varied from 10% to 80% in this work. As shown in Fig. 2(B), the performance of the SSCLPA is exceptional in the RC benchmark networks. Generally, it has higher a $\text{NMI}$ than all the LPA based algorithms. Moreover, instead of getting trivial detection like the other LPA based algorithms at the higher $D$ values, the results of the proposed algorithm are comparable to those from Blondel which has the best performance in the RC networks. We note that the RN shows a sudden spike in the $\text{NMI}$ value at $D = 70\%$. Apparently, this phenomenon is caused by the tendency of RN to detect large number of communities in networks with weak community structure. This tendency of RN renders its detection unreliable in finding meaningful communities in those kind of networks.

**Real-World Networks.** The real-world networks that are used in this study are summarised in Table 2 and the detection results are depicted in Tables 3, 4 and 5. These networks are often employed in the testing of...
community detection algorithms. As shown in Table 3, the detection performance of the SSCLPA agrees fairly well with the ground truth communities in the Zachary, Dolphins and Football networks. In fact, it achieves the highest NMI value in the Football network and it is also the second best algorithm in the Zachary network. Its performance in the Pol-books network is less than desirable, but this is compensated for by having the second highest Qds value (see Table 5).

Undoubtedly, the Blondel algorithm, which focuses on the optimisation of the Q value, can obtain the highest Q values in almost all of the real-world networks (see Table 4). Nonetheless, the performance of the SSCLPA in terms of the Q values is acceptable, considering that its Q values are within 6% of the highest Q values in networks with sizes lesser than 10000 nodes, except for the Email network, which is ~8% lower than then best Q value. It can be observed that the SSCLPA underperforms in term of the Q in large networks such as the Pretty Good Privacy, Astro-ph and Brightkite networks. However, the SSCLPA has the second highest Qds after the Qds of the GANXiS in those networks (see Table 5). Furthermore, it is worth noting that its Qds is higher than those of the Infomap and Blondel algorithms. In general, the proposed algorithm outperforms most of the other algorithms except for the GANXiS in term of Qds. Instead, it can be observed that the SSCLPA has the best detection performance in terms of Qds in the Football and Jazz networks.

The computational time of various community detection algorithm is compared in Table 6. Since in general the CLPA-GNR runs slower than the SSCLPA, we do not report the computational time of the CLPA-GNR. As a

| Networks     | Nodes | Edges | Ground Truth |
|--------------|-------|-------|--------------|
| Zachary52    | 34    | 78    | Yes          |
| Dolphins13   | 62    | 159   | Yes          |
| Pol-books14  | 105   | 441   | Yes          |
| Football14   | 115   | 613   | Yes          |
| Jazz16       | 198   | 2742  | No           |
| E. coli17    | 418   | 519   | No           |
| Email17      | 1133  | 5451  | No           |
| Power11      | 4941  | 6494  | No           |
| Pretty Good Privacy18 | 10680 | 24316 | No           |
| arXiv Astro-ph19 | 18771 | 198050 | No           |
| Brightkite40 | 58228 | 214078 | No           |

**Table 2. Summary of the real-world networks considered in this study.**

| Networks     | SSCLPA | CLPA-GNR | LPA | GANXiS | Infomap | RN | Blondel |
|--------------|--------|----------|-----|--------|---------|----|---------|
| Zachary      | 0.707  | 0.837    | 0.602 | 0.707 | 0.699 | 0.631 | 0.687 |
| Dolphins     | 0.616  | 0.488    | 0.645 | 0.458 | 0.537 | 1.000 | 0.645 |
| Pol-books    | 0.493  | 0.552    | 0.554 | 0.462 | 0.537 | 0.574 | 0.569 |
| Football     | 0.969  | 0.955    | 0.933 | 0.951 | 0.952 | 0.969 | 0.903 |

**Table 3. The NMI of the real-world networks with ground truth communities.** The best detection algorithm for each network is shown in bold.

| Networks     | SSCLPA | CLPA-GNR | LPA | GANXiS | Infomap | RN | Blondel |
|--------------|--------|----------|-----|--------|---------|----|---------|
| Zachary      | 0.415  | 0.303    | 0.416 | 0.415 | 0.402 | 0.406 | 0.420 |
| Dolphins     | 0.525  | 0.513    | 0.527 | 0.513 | 0.525 | 0.379 | 0.527 |
| Pol-books    | 0.518  | 0.514    | 0.526 | 0.519 | 0.527 | 0.527 | 0.527 |
| Football     | 0.601  | 0.579    | 0.605 | 0.603 | 0.603 | 0.601 | 0.603 |
| Jazz         | 0.420  | 0.282    | 0.443 | 0.442 | 0.443 | 0.288 | 0.445 |
| E. coli      | 0.735  | 0.749    | 0.772 | 0.757 | 0.707 | 0.771 | 0.777 |
| Email        | 0.525  | 0.520    | 0.558 | 0.540 | 0.536 | 0.008 | 0.570 |
| Power        | 0.884  | 0.888    | 0.818 | 0.797 | 0.811 | 0.343 | 0.934 |
| Pretty Good Privacy | 0.798 | 0.852 | 0.821 | 0.804 | 0.857 | — | 0.880 |
| arXiv Astro-ph | 0.334 | 0.294   | 0.537 | 0.570 | 0.581 | — | 0.614 |
| Brightkite   | 0.538  | -        | 0.640 | 0.631 | 0.441 | — | 0.664 |

**Table 4. The Q values of the real-world networks.** The best detection algorithm for each network is shown in bold.
are sets of nodes and edges. In the and , respectively. The labels of , denoted by , are not yet assigned into any community yet, while grouped nodes mean otherwise. Solo and grouped nodes are SSCLPA, nodes are divided into two types, known as the solo and grouped nodes. Solo nodes refer to nodes that Brightkite networks. The reported time for the LPA is the sum of 100 runs on both networks. On the other hand, the reported time for the LPA variant that yields deterministic detection, the SSCLPA runs in reasonable time in large networks. In general, the LPA can run faster than the proposed algorithm, but the time that are depicted in the table is the total time for 100 runs. Thus, it is not guaranteed that the LPA can produce its best detection within those runs. In fact, larger number of runs is needed by LPA as the size of the networks increases. GANXiS, which is also a LPA variant, requires longer computational time despite its good performance in terms of and in those networks. For instance, GANXiS consumes 1741s to produce 10 detection results. Although the Infomap and Blondel algorithms can complete the detection within 10 s, but their low values in those networks cannot be overlooked.

Discussion
The SSCLPA addresses both the randomness and trivial detection issues in the LPA. It inherits the prominent features of our earlier CLPA-GNR and further enhances them. In particular its update sequences are fixed based on the degree and the number of solo neighbouring nodes. Furthermore, the SDI features of our earlier CLPA-GNR and further enhances them. In particular its update sequences are fixed based on the degree and the number of solo neighbouring nodes. Furthermore, the SDI values of the real-world networks cannot be overlooked.

Table 5. The values of the real-world networks. The best detection algorithm for each network is shown in bold.

| Networks        | SSCLPA | CLPA-GNR | LPA   | GANXiS | Infomap | RN   | Blondel |
|-----------------|--------|----------|-------|--------|---------|------|---------|
| Zachary         | 0.235  | 0.182    | 0.234 | 0.235  | 0.217   | 0.240| 0.230   |
| Dolphins        | 0.207  | 0.187    | 0.187 | 0.216  | 0.213   | 0.136| 0.187   |
| Pol-books       | 0.216  | 0.183    | 0.192 | 0.225  | 0.199   | 0.190| 0.191   |
| Football        | 0.491  | 0.482    | 0.449 | 0.473  | 0.474   | 0.491| 0.417   |
| Jazz            | 0.232  | 0.187    | 0.215 | 0.221  | 0.220   | 0.205| 0.213   |
| E. coli         | 0.132  | 0.116    | 0.142 | 0.154  | 0.087   | 0.154| 0.116   |
| Email           | 0.076  | 0.057    | 0.050 | 0.059  | 0.088   | 0.015| 0.041   |
| Power           | 0.070  | 0.055    | 0.161 | 0.149  | 0.003   | 0.124| 0.019   |
| Pretty Good     | 0.160  | 0.064    | 0.153 | 0.160  | 0.018   | —   | 0.031   |
| Privacy         | 0.130  | 0.097    | 0.115 | 0.145  | 0.099   | —   | 0.027   |
| arXiv Astro-ph  | 0.037  | -        | 0.027 | 0.044  | 0.006   | —   | 0.011   |

Table 6. The computational time of various detection algorithms on the Astro-ph and Brightkite networks. The reported time for the LPA is the sum of 100 runs on both networks. On the other hand, the reported time for the GANXiS is the sum of 100 on the Pretty Good Policy and arXiv Astro-ph networks, while 10 runs on the Brightkite networks.

| Networks        | SSCLPA | LPA   | GANXiS | Infomap | Blondel |
|-----------------|--------|-------|--------|---------|---------|
| Pretty Good     | 24 s   | 7 s   | 557 s  | 1 s     | 0.25 s  |
| Privacy         | 169 s  | 83 s  | 2444 s | 5 s     | 0.22 s  |
| Brightkite      | 290 s  | 76 s  | 1741 s | 8 s     | 0.37 s  |

LPA variant would also be interesting to implement different criteria on the exemption of communities, in order to observe the effects of exempting different communities on the final outcomes of the detections. Since the CLP processes are synchronous, they are readily parallelisable. The extension of the SSCLPA into the directed or weighted networks is highly possible, and it is a top priority in our future work.

Methods
Let be a network where and are sets of nodes and edges. In the SSCLPA, nodes are divided into two types, known as the solo and grouped nodes. Solo nodes refer to nodes that are not yet assigned into any community yet, while grouped nodes mean otherwise. Solo and grouped nodes are denoted by and , respectively. The labels of and are updated sepa-
Main Processes. Exempted Community (EC). It is common to find that after a few iterations of the LPA, some of the detected communities are far stronger than most of the others. These communities will grow exponentially in the later stages of the LPA. Eventually, this phenomenon may lead to trivial detection, where the LPA only detects a single community that consists of all the nodes in a network. In order to prevent this kind of detection, communities that exceed a strength threshold are exempted from the propagation and merging processes. By doing so, the other communities have the chance to grow without competing with those communities.

Given that \( C = \{ c_i : i \in \mathbb{I} \} \) and \( V_C = \{ v_d : d \in C, i \in \mathbb{I} \} \) as the set of communities and their corresponding members in the communities, the term \( k_{IN}(V_C) \) is defined as the intra-community degree. For instance, \( k_{IN}(v_{ci}) = 5 \) shows that \( v_{ci} \) is connected to 5 other member nodes in community \( c_i \). Then, the strength value of the member nodes in the communities is defined as:

\[
s_C(V_C) = \frac{k_{IN}(V_C)}{k(V_C)} \quad (1)
\]

where \( k(V_C) \) is the degree value of the corresponding member nodes. As consequence, the strength value of the communities can be obtained:

\[
S_C = s_C(V_C) = \frac{\sum s_C(V_C)}{|V_C|} \quad (2)
\]

where \(|V_C|\) is the number of member nodes in a community.

Finally, \( C_{EC} \) is defined as a set of communities with \( S_{C_{EC}} > \alpha \). The parameter, \( \alpha \), is the strength value that determines the number of communities to be exempted and \( \alpha \propto S_C \). In general, a lower \( \alpha \) value brings about more exempted communities and we note that this process is executed prior to other processes as \( C_{EC} \) plays a crucial part in those processes.

Constrained Label Propagation (CLP). The CLP is a label propagation process that assigns \( V_S \) into detected communities. Nodes are updated synchronously in CLP unless stated otherwise. Let node \( v_{sj} \) be the targeted node. A community \( c_i \) is eligible to claim \( v_{sj} \) if it fulfils the following conditions:

1. Let \(|E_{v_{sj}c_i}|\) be the total number of edges between \( v_{sj} \) and \( c_i \). Then \(|E_{v_{sj}c_i}| = \max |E_{v_{sj}c_i}|\)
2. \( c_i \notin C_{EC} \)
3. If \( Q0 \) is the minimum \( k_{IN}(V_c) \) and \( QI \) is the first quartile of \( k_{IN}(V_c) \), then \(|E_{v_{sj}c_i}|\) must be larger or equal to \( Q0 \) or \( QI \). This condition is defined differently at various stages of the algorithm.

CLP Condition (1) is simply the label propagation condition of the original LPA. However, this condition does not always increase or retain the strength of the communities after the solo nodes enter the communities.
Therefore, CLP Condition (3) is implemented for this purpose. Finally, CLP Condition (2) ensures that the target solo nodes do not enter the exempted communities.

If there is a tie between multiple communities, the mean of the SDI of the competing communities is compared. For example, let there be a tie between communities $c_1$ and $c_2$. The targeted node $v_{S1}$ is connected to $c_1$ and $c_2$ via $v_{1, p}$, $v_{1, q}$, $v_{2, p}$, and $v_{2, q}$, respectively. If the SDI of $v_{S1}$ with $v_{1, p}$ and $v_{2, q}$ is higher than that for $v_{S1}$ with $v_{1, q}$ and $v_{2, p}$, it will enter $c_1$, provided that the CLP Condition (3) is satisfied. The labels of the nodes remain the same if there is a tie in the SDI.

**Grouped Nodes Reallocation (GNR).** The function of the GNR is to check the validity of $V_{GR}$ and reallocate them if necessary. The GNR is identical to the CLP process, but it is implemented on the group nodes $V_{GR}$ instead of the solo nodes $V_{S}$. So, similar to the CLP, the GNR needs to fulfill the conditions that are applied on the CLP before $V_{GR}$ can be reallocated from one community to another. If there is a tie between multiple communities, the mean of the SDI of the competing communities is compared. In contrast to the CLP, the GNR is an asynchronous label propagation process. As a consequence, $V_{GR}$ are updated asynchronously. Since the purpose of the GNR is to strengthen the detected communities, it is usually implemented after the CLP or GM processes.

**Groups Merging (GM).** Start with the largest community in descending order, a couple of communities, $c_i$ and $c_j$, can be merged if the following conditions are met:

![Figure 3. The flowchart of the SSCLPA.](image-url)
1. \( c, c' \notin C_{EC} \).
2. Let \(|E_{dp}|\) be the number of edges between a couple of communities, where \( d, p \in C \) and \( d \neq p \). Then, this condition is defined as \( |E_{dp}| = \max |E_{c,p}| \).
3. Let \(|E_{c|^IN}|\) be the total number of intra-community edges in the communities. Two ratios are obtained, where \( \text{RatioA} \) is the average number of edges in \( c \) and \( \text{RatioB} \) is the average number of edges between \( c \) and \( c' \). Then this condition is defined as:

\[
\text{RatioA} = \frac{|E_{c|^IN}|}{|V_c|}  \tag{3}
\]

\[
\text{RatioB} = \frac{|E_{c'^|IN}|}{|V_{c'}|}  \tag{4}
\]

\[
\text{RatioB} - \text{RatioA} \leq 1  \tag{5}
\]

GM Condition (3) is a relaxed version of the merging condition in the CLPA-GNR\(^{18}\) where it allows more communities to be merged. Nonetheless, this merging strategy can still maintain the strength of the merged communities to some extent.

**Main Stages.** The flowchart of the SSCLPA is depicted in Fig. 3. The details of the stages and their corresponding processes are explained in the following subsections. In general, each stage is a combination of variations in the CLP, GNR and GM processes.

**Stage 1: Initialisation.** In this stage, the nodes are assigned unique initial labels and the SDI is calculated.

- Initial labeling: Every single node in a network is given a unique label.
- SDI calculation: The SDI of all the pairs of nodes, \( x \) and \( y \), in an undirected and unweighted network is calculated:

\[
SDI_{xy} = SDI_{yx} = \frac{2|b_x \cap b_y|}{|b_x| + |b_y|}  \tag{6}
\]

where \( |b_x| \) and \( |b_y| \) are the number of neighbouring nodes of \( x \) and \( y \) respectively. It must be noted that \( |b_x| \) does not include node \( y \) and vice versa. The term \( |b_x \cap b_y| \) represents the number of mutual neighbouring nodes of \( x \) and \( y \).

**Stage 2: Initial Detection.** This stage aims to detect as many communities as possible in a network. Nodes with one degree are grouped with their sole neighbouring nodes to form communities. Then, more communities are found by using the highest to highest (HTH) SDI Grouping. The sizes of the detected communities are increased by using CLP 1 and the communities are strengthened by GNR 1.

- k1 grouping: Nodes with one degree are assigned the label of its sole neighbouring node.
- HTH SDI Grouping: Solo nodes \( V_s \), in ascending order of degree, are assigned into communities where the member nodes of the communities have the highest SDI score with each other. Let \( B_x = \{b_{i}: i \in \mathbb{R}\} \) be a set of neighbouring nodes of node \( x \), and the highest SDI score of node \( x \) is defined as \( SDI_{x|^max} = \max_{E_{c|^IN}} |SDI_{sc}| \). Node \( x \) will be assigned into a community with a set of nodes, \( V_M \subset V \) if the following conditions are satisfied:

\[
SDI_{xV_M} = SDI_{x|^max}  \tag{7}
\]

\[
SDI_{V_Mx} = SDI_{V_M|^max}  \tag{8}
\]

- CLP 1: This is an asynchronous CLP so \( V_s \) is updated according to the predefined update sequence. In this CLP, a community \( c \) must have at least one member node which has the highest SDI score with the target solo node \( v_S \), \( SDI_{v_S|x} = SDI_{x|^max} \). CLP Condition (2) is not required here. CLP Condition (3) is defined as \( |E_{x,y}c| \geq Q1 \). The labels of the nodes remain the same if there is a tie between multiple communities.
- GNR 1: CLP Condition (2) is not required in this GNR. CLP Condition (3) is defined as \( |E_{x,y}c'| \geq Q1 \) here. The labels of the nodes again remain the same if there is a tie between multiple communities.
Stage 3: Label Propagation and Communities Merging 1. In this stage, both the CLP 2 and GNR 2 processes are executed iteratively in order to further increase the sizes of the communities. Every time the CLP 2 is executed, it is followed by the GNR 2 for enhancement purpose. In order to reduce the large number of communities that are detected in Stage 2, GM1 and GM2 are introduced in this stage. Similarly, GNR 2 is executed after the CLP and GM processes in order to strengthen the detected communities.

- CLP 2 & GNR 2: Here we execute the CLP and GNR processes with CLP Condition (3) of $|v_{Sj}| \geq Q1$.
- GM 1: Execute the GM process on communities with more than 3 member nodes, $C_2 = \{v_S: i \in \mathbb{R}, |V_{Sj}| > 3\}$. This step prevents the communities with lesser than 4 member nodes from disrupting the merging process, which have the potential of forming monster size communities. The labels of the nodes remain the same if there is a tie between multiple communities.
- GM 2: Unlike GM 1, all the detected communities, regardless of their sizes, can be merged. However, a new condition where the modularity score does not decrease after the merging is added in GM 2. This additional condition controls the merging of the communities with lesser than 4 member nodes. The labels of the nodes again remain the same if there is a tie between multiple communities.

Stage 4: Iterative Merging. In Stage 4, GM 1 and GM 2 are executed iteratively to further reduce the number of communities, until the networks reach a stability where the number of communities cannot be further reduced.

Stage 5: Label Propagation and Communities Merging 2. In the last stage, STL-CLP is used to boost the size of the communities that do not grow in size during the previous stages. Then, most of the the $V_{Sj}$, if not all, will be assigned into communities by using CLPA 3/4/5/6. In order to do so, the constraints on the CLP 3/4/5/6 are gradually relaxed from CLP 3 to 6. Furthermore, the remaining communities will be merged for the last time by using GM 3. As usual, GNR 3/4/5 are used to strengthen the communities after the CLP and GM processes. Similar to the CLP 3/4/5/6, the constraints in the GNR 3/4/5 are gradually relaxed. In networks with a weak community structure, some of the processes are omitted in order to avoid trivial detection. This procedure is explained in the legend of Fig. 3.

- Smallest to largest CLP (STL-CLP): Start from the smallest and proceed to the largest communities, a community $c_i$ will absorb the solo node $v_{Sj}$ into the community as long as $v_{Sj}$ is connected with $V_{ci}$, and CLP Conditions (2) and (3) $|E_{v_{Sj}c_i}| \geq Q1$ are satisfied. CLP 1 and CLP 2 do not always allow the growth of small communities, as those communities often fail to compete for solo nodes in CLP 1 & 2. This is a CLP which prioritises the growth of the small communities.
- CLP 3/4/5/6 and GNR 3/4/5: In these label propagation processes, a solo node $v_{Sj}$ has the chance to enter an exempted community provided that the number of edges from the solo node to the exempted communities, $|E_{v_{Sj}c_i}|$, is the highest amongst all the communities that are connected to the solo node, $|E_{v_{Sj}c_i}| = \max |E_{v_{Sj}c_j}|$. In addition, $|E_{v_{Sj}c_i}|$ must be 2 times higher than the second highest number of connection from the solo node to the other communities. As mentioned earlier, the constraints on the CLP 3/4/5/6 and GNR 3/4/5 are gradually relaxed in Stage 5. This can be done by modifying CLP Condition (3) for each CLP and GNR. Aside from that, CLP Condition (3) is defined differently depending on whether the $c_i$ is an exempted community or not. Thus, CLP Condition (3) in the CLP 3/4/5/6 and GNR 3/4/5 is defined as follow:For CLP 3 and GNR 3:

$$|E_{v_{Sj}c_i}| \geq Q2, \quad \text{if} \quad c_i \in C_{EC}$$
$$|E_{v_{Sj}c_i}| \geq Q1, \quad \text{if} \quad c_i \notin C_{EC}$$

(9)

For CLP 4 and GNR 4:

$$|E_{v_{Sj}c_i}| > Q2, \quad \text{if} \quad c_i \in C_{EC}$$
$$|E_{v_{Sj}c_i}| > Q1, \quad \text{if} \quad c_i \notin C_{EC}$$

(10)

For CLP 5 and GNR 5:

$$|E_{v_{Sj}c_i}| \geq Q1, \quad \text{if} \quad c_i \in C_{EC}$$
$$|E_{v_{Sj}c_i}| \geq Q0, \quad \text{if} \quad c_i \notin C_{EC}$$

(11)

where $Q0$, $Q1$ and $Q2$ are the minimum, first quartile and median of the $k_{IN}(V_{Sj})$. Finally, the CLP Condition (3) is omitted in CLP 6.
- GM 3: Generally, this process is very similar to GM 2, except that it can handle ties between multiple communities. In case of a tie, the values of RatioB – RatioA for each pair of communities are compared. The pair of communities with the min $|\text{RatioB – RatioA}|$ is merged, provided that the modularity does not decrease after
the merging. Furthermore, it is a free-for-all GM where all the communities, including the exempted communities, have the chance to merge. Hence, GM Condition (1) is omitted in this process.

**Time Complexity.** The time complexity of the initial labelling is represented by \(O(|V|)\). The calculation of the SDL allocation of one degree nodes, and the HTH SDL grouping run on \(O(|E|)\). As the solo and grouped nodes are updated separately in the CLP and GNR processes, the time complexity of the propagation process is also split. In general, the time complexity of the CLP and GNR are \(O(|E|)\) and \(O(|E_{GR}|)\), where \(O(|E|) \leq O(|E_{GR}|)\). \(O(|E_{GR}|) \leq O(|E|)\) and \(O(|E|) + O(|E_{GR}|) = O(|E|)\). Most of the time, the CLP or GNR process is coupled with the EC and GNR processes. Thus, the time complexity of the EC-GM-GNR process is \(O(|E| + 2|E|)\). Stage 4 of the proposed algorithm is iterative. Let \(t_{m} \) be the number of iterations before Stage 4 reaches a convergence in the labels. The time complexity of Stage 4 is \(t_{m} \times \left(2 \times O(t(|E_\text{S}| + 2|E|)) + O(t(|E_\text{S}| + |E|)) + O(t(|E_{GR}|))\right)\). By referring to the algorithm flowchart (see Fig. 3), the time complexity are \(O(|V|) + O(|E|)\), \(2 \times O(|E|) + O(t(|E_\text{S}|)) + O(t(|E_{GR}|))\), \(2 \times O(t(|E_\text{S}|)) + 2 \times O(t(|E_{GR}|)) + 2 \times O(t(|E_\text{S}| + |E|))\) and \(5 \times O(t(|E_\text{S}|)) + 5 \times O(t(|E_{GR}|)) + O(|E|)\) for Stage 1, 2, 3 and 5, respectively.

**References**

1. Piñero, J., Berenstein, A., Gonzalez-Perez, A., Chernomorets, A. & Furlong, L. I. Uncovering disease mechanisms through network biology in the era of next generation sequencing. Scientific Reports 6, 24570 EP – http://dx.doi.org/10.1038/srep24570 (2016).
2. Ding, R., Ujum, N., Hamid, H. b. & Wu, J. Complex network theory applied to the growth of kuala lumpur’s public urban rail transit network. PLoS ONE 10, 1–22 http://dx.doi.org/10.1371/journal.pone.0119210 (2015).
3. Weng, L., Menczer, F. & Ahn, Y.-Y. Virality prediction and community structure in social networks. Scientific Reports 3, 2522 EP – http://dx.doi.org/10.1038/srep02522 (2013).
4. Patt, C. K., Ujum, E. A. & Ratnavelu, K. The structure of collaboration in the journal of finance. Scientometrics 85, 849–860 http://dx.doi.org/10.1007/s11192-010-0254-0 (2010).
5. Girvan, M. & Newman, M. E. J. Community structure in social and biological networks. Proceedings of the National Academy of Sciences 99, 7821–7826 http://www.pnas.org/content/99/12/7821.abstract (2002).
6. Fortunato, S. Community detection in graphs. Physics Reports 486, 75–174 http://www.sciencedirect.com/science/article/pii/S0370157309002841 (2010).
7. Raghavan, U. N., Albert, R. & Kumara, S. Near linear time algorithm to detect community structures in large-scale networks. Phys. Rev. E 76, 036106 http://link.aps.org/doi/10.1103/PhysRevE.76.036106 (2007).
8. Leung, I. X. Y., Hui, P., Liu, P. & Crowcroft, J. Towards real-time community detection in large networks. Phys. Rev. E 79, 066107 http://link.aps.org/doi/10.1103/PhysRevE.79.066107 (2008).
9. Barber, M. J. & Clark, J. W. Detecting network communities by propagating labels under constraints. Phys. Rev. E 80, 026129 http://link.aps.org/doi/10.1103/PhysRevE.80.026129 (2009).
10. Liu, X. & Murata, T. Advanced modularity-specialized label propagation algorithm for detecting communities in networks. Physica A: Statistical Mechanics and its Applications 389, 1493–1500 http://www.sciencedirect.com/science/article/pii/S0378437109010152 (2010).
11. Xie, J., Szymanski, B. K. & Liu, X. Slpa: Uncovering overlapping communities in social networks via a speaker-listener interaction dynamic process. In 2011 IEEE 11th International Conference on Data Mining Workshops (ICDMW), 344–349 (2011).
12. Xie, J. & Szymanski, B. Labelrank: A stabilized label propagation algorithm for community detection in networks. In Network Science Workshop (NSW), 2013 IEEE 2nd, 138–143 (2013).
13. Xie, J. & Szymanski, B. Community detection using a neighborhood strength driven label propagation algorithm. In Network Science Workshop (NSW), 2011 IEEE, 188–195 (2011).
14. Zhang, A. et al. Detecting community structures in networks by label propagation with prediction of percolation transition. The Scientific World Journal 2014 (2014).
15. Xing, Y. et al. A node influence based label propagation algorithm for community detection in networks. The Scientific World Journal 2014 (2014).
16. Gaiteri, C. et al. Identifying robust communities and multi-community nodes by combining top-down and bottom-up approaches to clustering. Scientific Reports 5, 16361 http://www.ncbi.nlm.nih.gov/pmc/articles/PMC4637843/ (2015).
17. Wu, T., Guo, Y., Chen, L. & Liu, Y. Integrated structure investigation in complex networks by label propagation. Physica A: Statistical Mechanics and its Applications 448, 68–80 http://www.sciencedirect.com/science/article/pii/S0378437115010122 (2016).
18. Chin, J. H. & Ratnavelu, K. Detecting community structure by using a constrained label propagation algorithm. PLoS ONE 11, 1–21 http://dx.doi.org/10.1371/journal.pone.0145154 (2016).
19. Lancichinetti, A., Fortunato, S. & Radicchi, F. Benchmark graphs for testing community detection algorithms. Phys. Rev. E 78, 046110 http://link.aps.org/doi/10.1103/PhysRevE.78.046110 (2008).
20. Lancichinetti, A. & Fortunato, S. Benchmarks for testing community detection algorithms on directed and weighted graphs with overlapping communities. Phys. Rev. E 80, 066118 http://link.aps.org/doi/10.1103/PhysRevE.80.066118 (2009).
21. Watts, D. J. & Strogatz, S. H. Collective dynamics of small-world networks. Nature 393, 440–442 http://dx.doi.org/10.1038/30918 (1998).
22. Schaeffer, S. E. Graph clustering. Computer Science Review 1, 27–64 http://www.sciencedirect.com/science/article/pii/S1574013707000020 (2007).
23. Danon, L., Daz-Guilera, A., Duch, J. & Arenas, A. Comparing community structure identification. Journal of Statistical Mechanics: Theory and Experiment 2005, P09008 http://stacks.iop.org/1742-5468/2005/i=09/a=P09008 (2005).
24. Newman, M. E. J. & Girvan, M. Finding and evaluating community structure in networks. Phys. Rev. E 69, 026113 http://link.aps.org/doi/10.1103/PhysRevE.69.026113 (2004).
25. Chen, M., Nguyen, T. & Szymanski, B. K. On measuring the quality of a network community structure. In Social Computing (SocialCom), 2013 International Conference on, 122–127 (2013).
26. Xie, J. & Szymanski, B. K. Towards Linear Time Overlapping Community Detection in Social Networks, chap. Advances in Knowledge Discovery and Data Mining: 16th Pacific-Asia Conference, PAKDD 2012, Kuala Lumpur, Malaysia, May 29 – June 1, 2012, Proceedings, Part II, 25–36 (Springer Berlin Heidelberg, Berlin, Heidelberg, 2012). http://dx.doi.org/10.1007/978-3-642-30220-6_3.
27. Xie, J., Kelley, S. & Szymanski, B. K. Overlapping community detection in networks: The state-of-the-art and comparative study. ACM Comput. Surv. 45, 43:1–43:35 http://doi.acm.org/10.1145/2301654.2301657 (2013).

28. Ronhovde, P. & Nussinov, Z. Local resolution-limit-free potts model for community detection. Phys. Rev. E 81, 046114 http://link.aps.org/doi/10.1103/PhysRevE.81.046114 (2010).

29. Blondel, V. D., Guillaume, J.-L., Lambiotte, R. & Lefebvre, E. Fast unfolding of communities in large networks. Journal of Statistical Mechanics: Theory and Experiment 2008, P10008 http://stacks.iop.org/1742-5468/2008/i=10/a=P10008 (2008).

30. Rosvall, M. & Bergstrom, C. T. Maps of random walks on complex networks reveal community structure. Proceedings of the National Academy of Sciences 105, 1118–1123 http://www.pnas.org/content/105/4/1118.abstract (2008).

31. Aldecoa, R. & Marn, I. Deciphering network community structure by surprise. PLoS ONE 6, e24195 http://dx.doi.org/10.1371 (2011).

32. Zachary, W. W. An information flow model for conflict and fission in small groups. Journal of Anthropological Research 33, 452–473 http://www.jstor.org/stable/3629752 (1977).

33. Lusseau, D. The emergent properties of a dolphin social network. Proceedings of the Royal Society of London B: Biological Sciences 270, S186–S188 (2003).

34. Evans, T. S. Clique graphs and overlapping communities. Journal of Statistical Mechanics: Theory and Experiment 2010, P12037 (2010). http://stacks.iop.org/1742-5468/2010/i=12/a=P12037.

35. Gleiser, P. M. & Danon, L. Community structure in jazz. Advances in Complex Systems 6, 565–573 http://www.worldscientific.com/doi/abs/10.1142/S0219525903001067 (2003).

36. Shen-Orr, S. S., Milo, R., Mangan, S. & Alon, U. Network motifs in the transcriptional regulation network of escherichia coli. Nat Genet 31, 64–68 http://dx.doi.org/10.1038/ng881 (2002).

37. Guimerà, R., Danon, L., Diaz-Guilera, A., Giralt, F. & Arenas, A. Self-similar community structure in a network of human interactions. Phys. Rev. E 68, 065103 http://link.aps.org/doi/10.1103/PhysRevE.68.065103 (2003).

38. Boguñá, M., Pastor-Satorras, R., Diaz-Guilera, A. & Arenas, A. Models of social networks based on social distance attachment. Phys. Rev. E 70, 056122 (2004).

39. Leskovec, J., Kleinberg, J. & Faloutsos, C. Graph evolution: Densification and shrinking diameters. ACM Trans. Knowledge Discovery from Data 1, 1–40 (2007).

40. Cho, E., Myers, S. A. & Leskovec, J. Friendship and mobility: User movement in location-based social networks. In Proc. Int. Conf. on Knowledge Discovery and Data Mining. 1082–1090 (2011).

Acknowledgements
The authors would like to acknowledge Professor Michael Brunger of Flinder University for his careful reading of the paper and for some useful suggestions. This project is supported by University of Malaya HIR Grant UM.C/625/1/HIR/MOHE/SC/13. J.H.C. also wants to acknowledge the support of University of Malaya HIR GRAS. The funding bodies had no role in the study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Author Contributions
Both authors wrote the main manuscript text. J.H.C. prepared all the figures and tables. Both authors conceived and designed the algorithm. J.H.C. collected, processed and analysed the data. Both authors reviewed the manuscript.

Additional Information
Competing Interests: The authors declare no competing financial interests.

How to cite this article: Chin, J. H. and Ratnavelu, K. A semi-synchronous label propagation algorithm with constraints for community detection in complex networks. Sci. Rep. 7, 45836; doi: 10.1038/srep45836 (2017).

Publisher’s note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

© The Author(s) 2017