Power-Law Entropy Corrected Holographic Dark Energy Model

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Among various scenarios to explain the acceleration of the universe expansion, the holographic dark energy (HDE) model has got a lot of enthusiasm recently. In the derivation of holographic energy density, the area relation of the black hole entropy plays a crucial role. Indeed, the power-law corrections to entropy appear in dealing with the entanglement of quantum fields in and out the horizon. Inspired by the power-law corrected entropy, we propose the so-called “power-law entropy-corrected holographic dark energy” (PLECHDE) in this Letter. We investigate the cosmological implications of this model and calculate some relevant cosmological parameters and their evolution. We also briefly study the so-called “power-law entropy-corrected agegraphic dark energy” (PLECADE).

I. THE MODEL

One of the dramatic candidate for dark energy, that arose a lot of enthusiasm recently, is the so-called “holographic dark energy” (HDE) proposal. This model is based on the holographic principle which states that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume [1] and it should be constrained by an infrared cutoff [2].

On this basis, Li [3] suggested the following constraint on its energy density $\rho_D \leq 3c^2 M_p^2 / L^2$, the equality sign holding only when the holographic bound is saturated. In this expression $c^2$ is a dimensionless constant, $L$ denotes the IR cutoff radius and $M_p = (8\pi G)^{-1/2}$ stands for the reduced Plank mass. Based on cosmological state of holographic principle, proposed by Fischler and Susskind [4], the HDE models have been proposed and studied widely in the literature (see e.g. [5–9] and references therein). The HDE model has also been tested and constrained by various astronomical observations [10, 11] as well as by the anthropic principle [12]. It is fair to claim that simplicity and reasonability of HDE model provides more reliable frame to investigate the problem of dark energy rather than other models proposed in the literature. For example, the coincidence problem can be easily solved in some models of HDE based on the fundamental assumption that matter and HDE do not conserve separately [13].

It is worthy to note that the definition and derivation of holographic energy density ($\rho_D = 3c^2 M_p^2 / L^2$) depends on the entropy-area relationship $S \sim A \sim L^2$ of black holes, where $A$ represents the area of the horizon [2]. However, this definition of HDE can be modified due to the power-law corrections to entropy which appear in dealing with the entanglement of quantum fields in and out the horizon [14]. The power-law corrected entropy takes the form [15]

$$S = \frac{A}{4G} \left[ 1 - K_\alpha A^{1-\alpha/2} \right],$$

where $\alpha$ is a dimensionless constant whose value is currently under debate, and

$$K_\alpha = \frac{\alpha (4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}},$$

where $r_c$ is the crossover scale. The second term in Eq. (1) can be regarded as a power-law correction to the area law, resulting from entanglement, when the wavefunction of the field is chosen to be a superposition of ground state and exited state [14]. The entanglement entropy of the ground state obeys the Hawking area law. Only the excited state contributes to the correction, and more excitations produce more deviation from the area law [16] (also see [17] for a review on the origin of black hole entropy through entanglement). This lends further credence to entanglement as a possible source of black hole entropy. The correction term is also more significant for higher excitations [14]. It is important to note that the correction term falls off rapidly with $A$ (see the discussion in favor of $\alpha > 2$ in the below) and hence in the semi-classical limit (large $A$) the area law is recovered. So for large black holes the correction term falls off rapidly and the area law is recovered, whereas for the small black holes the correction is significant. This can be interpreted as follows: for large area, i.e., at low energies, it is difficult to excite the modes and hence, the ground state modes contribute to most of the entanglement entropy. However, for small horizon area, a large number of field modes can be excited and contribute significantly to the correction causing large deviation from the area law.

Inspired by the power-law corrected entropy relation [14], and following the derivation of HDE [18] and entropy-corrected holographic dark energy (ECHDE) [19], we can easily obtain the energy density of the so-called “power-law entropy-corrected holographic dark energy”...
(PLECHDE), namely
\[ \rho_D = 3c^2 M_p^2 L^{-2} - \beta M_p^2 L^{-\alpha}. \] (3)

In the special case \( \beta = 0 \), the above equation yields the well-known holographic energy density. The significant of the corrected term in various regions depends on the value of \( \alpha \). When \( \alpha = 2 \) the two terms can be combined and one recovers the ordinary HDE density. Let us consider the case with \( \alpha > 2 \) and \( \alpha < 2 \) separately. In the first case where \( \alpha > 2 \) the corrected term can be comparable to the first term only when \( L \) is very small. Indeed, it was argued that \( \alpha \) should be ranges as \( 2 < \alpha < 4 \). However, the satisfaction of the generalized second law of thermodynamics for the universe with the power-law corrected entropy (1) implies that the case \( \alpha < 2 \) should be rejected [13].

II. NON-INTERACTING CASE

We consider the non-flat Friedmann-Robertson-Walker (FRW) universe which is described by the line element
\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \] (4)
where \( a(t) \) is the scale factor, and \( k \) is the curvature parameter with \( k = -1, 0, 1 \) corresponding to open, flat, and closed universes, respectively. The corresponding Friedmann equation takes the form
\[ H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_m + \rho_D), \] (5)
where \( \rho_m \) and \( \rho_D \) are the energy density of dark matter and dark energy, respectively.

It is important to note that in the literature, various scenarios of HDE have been studied via considering different system’s IR cutoff. In the absence of interaction between dark matter and dark energy in flat universe, Li [20] discussed three choices for the length scale \( L \) which is supposed to provide an IR cutoff. The first choice is the Hubble radius, \( L = H^{-1} \), which leads to a wrong equation of state, namely that for dust. The second option is the particle horizon radius. In this case it is impossible to obtain an accelerated expansion. Only the third choice, the identification of \( L \) with the radius of the future event horizon gives the desired result, namely a sufficiently negative equation of state to obtain an accelerated universe. However, as soon as an interaction between dark energy and dark matter is taken into account, the first choice, \( L = H^{-1} \), in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem [13]. It was also argued [9] that in a non-flat universe the natural choice for IR cutoff could be the apparent horizon radius \( r_A = 1/\sqrt{H^2 + k/a^2} \) provided the interaction is taken into account. In recent years, some new infrared cut-offs have also been proposed in the literature. In [20], the authors have added the square of the Hubble parameter and its time derivative within the definition of holographic dark energy. While in [21], the authors proposed a linear combination of particle horizon and the future event horizon. In this section, following [22], as system’s IR cutoff we choose the radius of the event horizon measured on the sphere of the horizon, defined as
\[ L = ar(t), \] (6)
where the function \( r(t) \) can be obtained from the following relation
\[ \int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^\infty \frac{dt}{a} = R_h. \] (7)
Solving the above equation for the general case of the non-flat FRW universe, we have
\[ r(t) = \frac{1}{\sqrt{k}} \sin y, \] (8)
where \( y = \sqrt{k} R_h/a \). We also define as usual, the fractional energy densities such as
\[ \Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{3M_p^2 H^2}{\rho_m}, \] (9)
\[ \Omega_k = \frac{\rho_k}{\rho_{cr}} = \frac{k}{H^2 a^2}, \] (10)
\[ \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{3M_p^2 H^2}{\rho_D}. \] (11)

Now we can rewrite the Friedmann equation in the following form
\[ 1 + \Omega_k = \Omega_m + \Omega_D. \] (12)
Using the definitions of \( \Omega_D \) and \( \rho_D \), we obtain a useful relation
\[ HL = \sqrt{\frac{3c^2 - \beta L^{2 - \alpha}}{3\Omega_D}}. \] (13)
Taking derivative with respect to the cosmic time \( t \) from Eq. (6) and using Eqs. (5) and (13) we obtain
\[ \dot{L} = HL + a \dot{r}(t) = \sqrt{\frac{3c^2 - \beta L^{2 - \alpha}}{3\Omega_D}} - \cos y. \] (14)
Consider the FRW universe filled with dark energy and pressureless matter which evolves according to their conservation laws
\[ \dot{\rho}_D + 3H \rho_D (1 + w_D) = 0, \] (15)
\[ \dot{\rho}_m + 3H \rho_m = 0, \] (16)
where \( w_D \) is the equation of state parameter of dark energy. Differentiating (3) with respect to time and using
Eq. (14) we find
\[ \dot{\rho}_D = (-6c^2M_p^2L^{-3} + \alpha\beta M_p^2L^{-\alpha - 1}) \times \left[ \frac{3c^2 - \beta L^2 - \alpha}{3\Omega_D - \cos y} \right]. \] (17)

Inserting this equation in conservation law (15), we obtain the equation of state parameter
\[ w_D = -1 + \frac{1}{3} \left[ \frac{6c^2 - \alpha\beta L^2 - \alpha}{3c^2 - \beta L^2 - \alpha} \right] \times \left[ 1 - \sqrt{\frac{3\Omega_D}{3c^2 - \beta L^2 - \alpha} \cos y} \right]. \] (18)

It is important to note that in the limiting case \( \beta = 0 \) Eq. (18) reduces to its respective expression in ordinary HDE
\[ w_D = -1 + \frac{2\sqrt{\Omega_D}}{3c} \cos y. \] (19)

For completeness, we give the deceleration parameter
\[ q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}, \] (20)

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. Taking the time derivative of the Friedmann equation (5) and using Eqs. (12), (15) and (16) we obtain
\[ q = \frac{1}{2} \left[ 1 + \Omega_k + 3\Omega_D w_D \right]. \] (21)

Substituting \( w_D \) from Eq. (18), we get
\[ q = \frac{1}{2} \left[ 1 + \Omega_k - 3\Omega_D + \Omega_D \left( \frac{6c^2 - \alpha\beta L^2 - \alpha}{3c^2 - \beta L^2 - \alpha} \right) \times \left( 1 - \sqrt{\frac{3\Omega_D}{3c^2 - \beta L^2 - \alpha} \cos y} \right) \right]. \] (22)

When \( \beta = 0 \), Eq. (22) restores the deceleration parameter for standard HDE model \( \beta = 0 \)
\[ q = \frac{1}{2} \left[ 1 + \Omega_k - \frac{\Omega_D}{2} - \frac{\Omega_D^{3/2}}{c} \cos y. \right. \] (23)

### III. INTERACTING CASE

The above study can also be performed for the interacting case. In the absence of a symmetry that forbids the interaction there is nothing, in principle, against it. Indeed, microphysics seems to allow enough room for the interacting \( \beta = 0 \). Taking the interaction into account, the continuity equations read
\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \] (24)
\[ \dot{\rho}_m + 3H\rho_m = Q, \] (25)

where \( Q \) represents the interaction term which can be, in general, an arbitrary function of cosmological parameters like the Hubble parameter and energy densities \( Q(H\rho_m, H\rho_D) \). The simplest choice is \( Q = 3b^2H(\rho_m + \rho_D) \) with \( b^2 \) is a coupling constant \( [23, 20] \), although more general phenomenological interaction terms can be used \( [27] \). The positive \( b^2 \) is responsible for the transition from dark energy to matter and vice versa for negative \( b^2 \). Sometimes this constant is taken in the range \([0, 1]\) \[28\]. Note that if \( b^2 = 0 \) then it represents the non-interacting FRW model while \( b^2 = 1 \) yields complete transfer of energy from dark energy to matter. Recently, it is reported that this interaction is observed in the Abell cluster A586 showing a transition of dark energy into dark matter and vice versa \[29\]. Observations of cosmic microwave background and galactic clusters show that the coupling parameter \( b^2 < 0.025 \), i.e. a small but positive constant of order unity \[30\], a negative coupling parameter is avoided due to violation of thermodynamical laws. Therefore the theoretical interacting models are phenomenologically consistent with the observations. It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction \( Q \).

Inserting Eq. (17) in (24) and using relation (13) we obtain the equation of state parameter
\[ w_D = -1 + \frac{1}{3} \left( \frac{6c^2 - \alpha\beta L^2 - \alpha}{3c^2 - \beta L^2 - \alpha} \right) \left[ 1 - \sqrt{\frac{3\Omega_D}{3c^2 - \beta L^2 - \alpha} \cos y} \right] - \frac{b^2(1 + \Omega_k)}{\Omega_D}. \] (26)

If we define, following \[31\], the effective equation of state parameter as
\[ w_D^{\text{eff}} = w_D + \frac{\Gamma}{3H}, \] (27)

Here \( \Gamma = 3b^2(1 + u)H \), where \( u = \rho_m/\rho_D \) is the energy density ratio of two dark components. Then, the continuity equation (24) for the dark energy can be written in the standard form
\[ \dot{\rho}_D + 3H\rho_D(1 + w_D^{\text{eff}}) = 0. \] (28)

Substituting Eq. (26) in Eq. (27), we find
\[ w_D^{\text{eff}} = -1 + \frac{1}{3} \left( \frac{6c^2 - \alpha\beta L^2 - \alpha}{3c^2 - \beta L^2 - \alpha} \right) \times \left[ 1 - \sqrt{\frac{3\Omega_D}{3c^2 - \beta L^2 - \alpha} \cos y} \right] \] (29)
Finally, we examine the deceleration parameter. Substituting $w_D$ from Eq. (36) in Eq. (21), we get

$$q = \frac{1}{2} \left[ 1 + \Omega_k - 3\Omega_D + \Omega_D \left( \frac{6c^2 - \alpha \beta L^{2-\alpha}}{3c^2 - \beta L^{2-\alpha}} \right) \right] \times \left( 1 - \sqrt{\frac{3\Omega_D}{3c^2 - \beta L^{2-\alpha} \cos y}} \right) - 3\beta^2 (1 + \Omega_k).$$

**IV. PLECHDE WITH HUBBLE HORIZON AS IR CUT-OFF**

In this section we consider PLECHDE with $L = H^{-1}$ as an IR-cutoff in a flat FRW universe. This cutoff is particularly relevant for the very early universe undergoing a hypothetical phase of inflation - a very brief period of exponential expansion. After the end of inflationary phase, the universe evolved subsequently through the radiation and matter phases. In the last two stages, the Hubble horizon is replaced with the future event horizon $R_h$ as a dynamical cutoff. Consequently the power-law-correction to HDE becomes negligible until the beginning of late time cosmic acceleration. Note that the HDE with the Hubble Horizon as cutoff cannot generate late time accelerated expansion and only dynamical future event horizon can serve this purpose. Therefore the choice of Hubble horizon to generate cosmic acceleration is restricted to the early universe. In this case, the energy density of PLECHDE can be rewritten as

$$\rho_D = 3c^2 M_p^2 H^2 - \beta M_p^2 H^\alpha.$$  (31)

Differentiating with respect to time we obtain

$$\dot{\rho}_D = \dot{H} H M_p^2 \left( 6c^2 - \alpha \beta H^{\alpha-2} \right).$$  (32)

Taking the time derivative of Friedmann equation (19) for the flat universe ($k = 0$) and using the continuity equations (15) and (16), we get

$$\dot{H} = -\frac{\rho_D}{2M_p^2} (1 + u + w_D).$$  (33)

Inserting Eqs. (32) and (33) in Eq. (16) we can easily obtain the equation of state parameter of PLECHDE

$$w_D = -1 - \frac{(6c^2 - \alpha \beta H^{\alpha-2}) u}{6(c^2 - 1) - \alpha \beta H^{\alpha-2}}.$$  (34)

It is worth noting that in the absence of correction term ($\beta = 0$) the above equation reduces to

$$w_D = -1 - \frac{c^2}{c^2 - 1} u,$$  (35)

while from the Friedmann equation we find $u = 1/c^2 - 1$. Substituting this relation in Eq. (35) we obtain $w_D = 0$, which is a wrong equation of state for dark energy and cannot derive the acceleration of the universe expansion [6]. However, as one can see from Eq. (34) in the presence of the power-law correction term, the identification of IR-cutoff with Hubble radius, $L = H^{-1}$, can lead to accelerated expansion.

**V. POWER-LAW ENTROPY-CORRECTED NEW AGEGRAPHIC DARK ENERGY**

A so-called agegraphic dark energy (ADE) is originated from uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. The ADE model assumes that the observed DE comes from the spacetime and matter field fluctuations in the universe. However, the original ADE [32] model had some difficulties. For example it suffers from the difficulty to describe the matter-dominated epoch, there is no inflation attractor using ADE even if the entropy corrections are applied to it and its density falls as the universe expands unlike typical dark energy candidates including cosmological constant and quintessence [19]. Therefore, a new model of ADE was proposed by Wei and Cai [33], while the time scale was chosen to be the conformal time instead of the age of the universe. The new ADE (NADE) contains some new features different from the original ADE and overcome some unsatisfactory points. The ADE models have been examined and studied in ample detail (see e.g. [34–37] and Refs. therein). The energy density of the NADE is given by [33]

$$\rho_D = \frac{3n^2 M_p^2}{\eta^2},$$  (36)

where the conformal time $\eta$ is given by

$$\eta = \int_0^\alpha \frac{da}{Ha^2}.$$  (37)

Here, we would like to propose the so-called “power-law entropy-corrected agegraphic dark energy” (PLECADE) whose $L$ in Eq. (19) is chosen to be the conformal time $\eta$. Therefore, we write down the energy density of PLECADE as

$$\rho_D = \frac{3n^2 M_p^2 \eta^{-2} - \beta M_p^2 \eta^{-\alpha}}{\beta}.$$  (38)

To be more general we consider the interacting case. Taking the time derivative of Eq. (38) and using the relation $\dot{\eta} = 1/\alpha$ we find

$$\dot{\rho}_D = -\frac{6n^2 M_p^2}{a\eta^3} + \frac{\alpha \beta M_p^2}{a\eta^{\alpha+1}}.$$  (39)

Using definition (11) as well as (38) we obtain

$$H \eta = \sqrt{\frac{3n^2 - \beta \eta^{2-\alpha}}{3\Omega_D}}.$$  (40)

Substituting Eq. (35) in (21) and using relation (12) and (40) we find the equation of state parameter of PLECADE as

$$w_D = -1 + \frac{\sqrt{3\Omega_D}}{3a\sqrt{n^2 - \beta \eta^{2-\alpha}}} \left( \frac{6n^2 - \alpha \beta \eta^{2-\alpha}}{3n^2 - \beta \eta^{2-\alpha}} \right) - \frac{b^2 (1 + \Omega_k)}{\Omega_D}.$$  (41)
In the limiting case $\beta = 0$, one recovers the equation of state parameter of usual NADE, namely
\[
w_D = -1 + \frac{2\sqrt{\Omega_D}}{3na} - \frac{b^2(1 + \Omega_k)}{\Omega_D}.
\] (42)

VI. CONCLUSION

It has been shown that the origin of black hole entropy may lie in the entanglement of quantum fields between inside and outside of the horizon [14]. Since the modes of gravitational fluctuations in a black hole background behave as scalar fields, one is able to compute the entanglement entropy of such a field, by tracing over its degrees of freedom inside a sphere. In this way the authors of [14] showed that the black hole entropy is proportional to the area of the sphere when the field is in its ground state, but a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. For large horizon areas, these corrections are relatively small and the area law is recovered.

Motivated by the power-law corrected entropy, we proposed the so-called “power-law entropy-corrected holographic dark energy” (PLECHDE) in this Letter. We calculated some relevant cosmological parameters such as the equation of state and deceleration parameter of the PLECHDE and various other dark energy candidates modeled by using scalar fields. Besides the Einstein’s gravity, it can be extended to Brans-Dicke chameleon cosmology and $f(R)$ gravity. Moreover it would be interesting to differentiate PLECHDE from other dark energy candidates by checking the corresponding statefinder parameters. These issues are now under consideration and will be addressed elsewhere.

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