The quark and charged lepton masses and the CKM matrix exhibit an intriguing and hierarchical structure that strongly motivates the search for an underlying mechanism. Remarkably, exponential hierarchies in masses and mixing angles can naturally arise when Standard Model (SM) fields propagate in extra spatial dimensions \([1, 2, 3]\) (see \([4]\) and \([5]\) for related ideas). Most simply, consider a field \(\phi\) in a flat extra dimension \((0 \leq y \leq a)\) with 5D mass parameter \(M\). The observed particles correspond to 4D massless (or “zero-”) modes, \(\phi = \phi^{(0)}(y) e^{ip \cdot x}\), with the 4D momentum satisfying \(p^2 = 0\). \(\phi^{(0)}\) then satisfies

\[
(\Box + M^2)\phi^{(0)}e^{ip \cdot x} = \phi^{(0)}(-\partial_y^2 + M^2)\phi^{(0)} = 0, \tag{1}
\]

subject to boundary effects at \(y = 0, a\). Such effects generically cause one of the two exponential solutions to dominate:

\[
\phi^{(0)}(y) \sim e^{-My} \quad \text{or} \quad e^{+M(y-a)}. \tag{2}
\]

These translate to exponential hierarchies among the 4D effective Yukawa couplings via wavefunction overlaps in matching to the 5D theory.

Naively, this also suggests hierarchical masses and mixings among neutrinos. Of course, recent data, with \(O(1)\) mixing angles and ratios of mass-splittings, is in conflict with this expectation. The coexistence of hierarchical and non-hierarchical mass matrices for closely related particle species is quite non-trivial to explain theoretically. Extra structures and symmetries have been invoked to accommodate this surprising feature \([6]\).

However, in this paper, we will show that closer inspection of the relevant wavefunction overlaps reveals a more interesting range of possibilities which can elegantly capture the apparent “switching” behavior in going from hierarchical charged fermions to non-hierarchical but light neutrinos. Such mechanisms exist for both Dirac and Majorana neutrinos, although they take quite different forms. Our unified treatment of charged fermions and neutrinos has striking implications for well-known approaches to the hierarchy problem such as supersymmetry, warped extra dimensions and strong dynamics.

Let us see how flavor hierarchies emerge from the exponentials \([7]\). We assume no large hierarchies among fundamental 5D parameters, and will try to generate large hierarchies in the 4D effective theory purely via exponentiation \(\sim e^{-Ma}\). We first study Dirac neutrinos so that all fermion masses can be treated uniformly. 5D boundary and bulk Yukawa couplings induce effective 4D Yukawa matrices,

\[
Y_{4D,ij} = \int_0^a dy Y_{5D,ij}(y) H^{(0)}(y) \psi^{(0)}_{L_i}(y) \psi^{(0)}_{R_j}(y), \tag{3}
\]

where \(Y_{5D,ij}(y) \equiv Y_{0,ij} \delta(y) + Y_{\text{bulk},ij} + Y_{a,ij} \delta(y-a)\). Each
of the zero-modes $H^{(0)}, \psi_{L,R}^{(0)}$ satisfies [2] (up to non-exponential normalization factors), where $M$ and the sign of exponent can depend on both SM representation $(q_L, u_R, d_R, \ell_L, e_R, \nu_R, H)$ and generation $i = 1, 2, 3$.

Now, note that the integral [3] is generically exponentially dominated at $y \sim 0$ or $\sim a$. For example, consider a case where both $\psi_{L}^{(0)}$ and $\psi_{R}^{(0)}$ lean away from $H^{(0)}$ as in Fig. 1 and imagine an assortment of $M_{L_i}$ and $M_{R_j}$. There are two cases, the integral [3] being dominated at $y \sim 0$ or $\sim a$, depending on whether $M_{L_i} + M_{R_j} > M_H$ or $< M_H$, respectively:

$$Y_{4D,ij} \sim \int_0^a dy Y_{5D,ij}(y) e^{-(M_{L_i} + M_{R_j})y + M_H(y-a)}$$

\hspace{1cm} (4)

\[ \approx \tilde{Y}_{0,ij} \sim e^{-M_Ha} \]

where $\tilde{Y}_{0,ij}$ is an $O(1)$ linear combination of $Y_{bulk,ij}$ and $Y_{0,ij}$ ($Y_{a,ij}$). The strong inequality in the last line of [4] follows simply from the condition on the exponents in the $y \sim a$ case, $M_{L_i} + M_{R_j} < M_H$. The $y \sim a$ case is the classic model of charged fermion mass matrices, yielding exponential hierarchies in masses and mixings [2, 3]. Note that this would be the only case if $H$ were boundary-localized ($M_H \rightarrow \infty$) as is often assumed in extra-dimensional flavor models. But if that were true, it would strongly suggest that $\nu$ masses and mixings should exhibit hierarchies comparable to charged fermions in stark contrast to data, unless there is some extra rationale for degeneracies among the $M_{L_i}$ and the $M_{R_j}$.

On the other hand, for a generic bulk $H$ ($M_H < \infty$), we can elegantly accommodate $\nu$ data while simultaneously capturing charged fermion hierarchies. For sufficiently large $M_{\nu_{R_j}}$ such that $M_{L_i} + M_{\nu_{R_j}} > M_H$, the $\nu$'s switch to the $y \sim a$ case by [1], which naturally has no large flavor-dependent hierarchies. Note that this does not affect our discussion above regarding charged fermion hierarchies generated at $y \sim a$. Moreover, the last line of [4] means that the $\nu$'s are exponentially lighter than the charged fermions. The structures and relations between the $\nu$ and charged mass matrices are quite robust because they derive from the branching in [1], based on simple inequalities among the $M$'s.

Returning to [3], we could also consider $\psi_{L,R}^{(0)}$ leaning toward $H^{(0)}$ in contrast to Fig. 1. Indeed, an $O(1)$ top Yukawa coupling does not match either exponentially suppressed case in [1], but robustly follows once $t_{L,R}^{(0)}$ both lean towards $H^{(0)}$. Then different leanings for $\psi_{L,R}^{(0)}$ contribute to a smaller bottom quark mass and mixing angles. In this way the simple extra-dimensional framework captures the presence and absence of hierarchies across the range of flavor physics.

The case of Majorana neutrinos works differently. In this case, the smallness of $m_{\nu}$ comes just from its non-renormalizable origins, $\ell_L t_L HH/\Lambda$. As for the non-hierarchical nature of neutrinos, it is a generic consequence precisely when all $t_{L,R}^{(0)}$ lean toward $H^{(0)}$, regardless of the precise $M_{L,R}$:

$$m_{\nu,ij} \sim O(1)_{ij} \frac{v^2}{\Lambda},$$

where $v$ is the weak scale. The hierarchical structure among charged leptons can be generated if $e_{R}^{(0)}$ lean away from $H^{(0)}$.

The minimal experimental implications reduce in the $\nu$ sector to those of the “neutrino mass anarchy” scenario [8], namely, $\theta_{13}$ should be close to the current upper bound $\sim 0.2$, and CP-violating phase(s) should be $O(1)$. More speculatively, at the other end of the spectrum, a 4th SM generation is a natural possibility in the Dirac $\nu$ case, and in order for it to be heavy, their $\psi_{L,R}^{(0)}$ must be leaning toward $H^{(0)}$, just like the top. Thus, we expect large mixing with the 3rd generation, which would dominate the phenomenology.

We must also consider the impact of Kaluza-Klein (KK) excitations. While 4D gauge fields couple flavor-blindly by 4D gauge invariance, gauge KK modes are sensitive to the flavor-dependent profiles of the fermions. Exchanging them will generate flavor-violating 4-fermion operators with strength $\sim \frac{g_{SM}^2}{M_{KK}^2}$ with $M_{KK} \sim a^{-1}$. To avoid excessive flavor-changing neutral currents (FCNCs) from such interactions we need $a^{-1} \approx 1000$ TeV [9]. Therefore, we should address the hierarchy problem between the electroweak scale and at least this high scale. We will consider two solutions, warping the above extra dimension [10] and supersymmetry (SUSY) [11].

The simplest 5D warped spacetime is given by:

$$ds^2 = e^{-2ky} dx_{4D}^2 - dy^2 \quad (0 \leq y \leq a).$$

The curvature scale $k$ is comparable to the typical 5D mass scale, such as $M$ and $a^{-1}$, and is taken to be very high. An exponentially small $v \approx ke^{-ka}$ emerges naturally for $a \gtrsim k^{-1}$, provided that $H^{(0)}$ is sufficiently localized near $y = a$ [12]. The zero-mode profiles continue to be exponentials in $y$, but with the curvature-modified exponents [2]:

$$e^{M_H(y-a)} \rightarrow e^{(k+\sqrt{4k^2+M_H^2})y-a} \quad \text{for } H^{(0)},$$

$$e^{-M_y} \rightarrow \left\{ \begin{align*}
e^{e(k/2-M)y} & \quad \text{for each } \psi_{L,R}^{(0)}.
\end{align*} \right.$$
The curvature-modified stability bound, $M^2_R \geq -4k^2$ [13], has the following important consequence for our Dirac $\nu$ scenario. Plugging (7) into (1) we see that the light non-hierarchical fermions have Yukawa couplings $\lesssim e^{-ka}$, the size of the warped hierarchy itself:

$$Y_{\text{non-hierarchical}} \lesssim \frac{v}{k}. \quad (8)$$

It is intriguing that at the bound, the observed neutrino data with $m_\nu \sim 0.1$ eV then imply a rough hierarchy from $k \sim 10^{15}$ GeV down to the weak scale.

FCNCs mediated by KK excitations are more subtle due to warping [2]. This is because even when all 5D fundamental scales are very large, low-lying KK masses are exponentially light, $M_{KK} \sim ke^{-ka} \sim$ TeV. However, these excitations have extra-dimensional profiles concentrated near $y = a$. Therefore, if the fermions are configured as in Fig. 1, the wavefunction overlaps suppress the flavor-dependent couplings of these KK excitations to SM fermions, similarly to the 4D Yukawa couplings. Such factors can sufficiently suppress hadronic flavor violation for $M_{KK} \sim$ several TeV [14].

Let us now turn to leptons, where $\nu$ data force us to consider FCNCs among charged leptons such as $\mu \to e\gamma$ and $\mu \to e$ conversions in nuclei. Our scenario for Dirac neutrinos is quite analogous to the quark sector. FCNCs are suppressed by wavefunction overlaps dominated at $y \sim a$, comparable to charged lepton Yukawa couplings. Again, $M_{KK} \sim$ several TeV is sufficient to satisfy the current bounds [13]. Because of the switching behavior [4], this physics is divorced from the generation of large $\nu$ mixing angles at $y \sim 0$.

Contrast this to the case where $H$ is exactly boundary-localized ($M_H \to \infty$), so that there is no switching behavior. As in flat spacetime, this case would then lose our explanation for non-hierarchical $\nu$ structure. Furthermore, the $\nu$ mass matrix is then necessarily generated at $y = a$, which exacerbates charged lepton FCNCs due to the large $\nu$ mixings [10].

Our scenario for Majorana $\nu$‘s is completely excluded when warping solves the hierarchy problem. Recall that in this scenario, $\ell_{L_i}^{(0)}$ lean in the same direction as $H^{(0)}$ and therefore the same direction as the KK modes. Consequently, KK-mediated FCNC operators such as $\mu \bar{e}e/M^2_{KK}$ are not suppressed by small wavefunction overlaps. Bounds on $\mu \to 3e$ would then require KK modes heavier than $\sim O(100)$ TeV, grossly at odds with their solving the hierarchy problem.

The successful implementation of our flavor mechanism in warped compactifications extrapolates to a larger class of purely 4D but strongly coupled theories via the AdS/CFT correspondence [17]. The large number of states implied by the extra dimension are reproduced by the many composites of a strong sector which is conformal over a large hierarchy. The 4D theory also contains elementary fermions with SM quantum numbers, $\psi_{L,R}$, outside of the strong sector. The mechanism of partial compositeness (PC) [18] introduces couplings, $\psi_{L,R}O_{R,L}$, to fermionic operators of the strong sector. Electroweak symmetry breaking (EWSB), assumed to occur in the strong sector at scale $v$, is communicated to the $\psi_{L,R}$ via these couplings. This naturally generates hierarchical Dirac fermion mass matrices via running down from a high fundamental scale $\Lambda$:

$$m_{ij} \sim O(1)i_jv \left(\frac{v}{\Lambda}\right)^{\Delta_{L_i}+\Delta_{R_j}-5}, \quad (9)$$

where $\Delta_{L_i}, \Delta_{R_j}$ are scaling dimensions of $O_{L_i,R_j}$. This is the dual of the right-hand branch of [4] after the curvature modification of [7], where $\Delta - 2 \overset{\text{AdS/CFT}}{\rightarrow} M$.

Let us turn to the dual of the left-hand branch of [4]. There must also be composite operators with Higgs quantum numbers (for example, $O_{L,R}$). Let $O_H$ be the operator of this type with the lowest scaling dimension, $\Delta_H$. Then elementary fermions can couple to it bilinearly, $\bar{\psi}_L\psi_R, O_H/\Lambda^{\Delta_H-1}$, leading after EWSB to

$$m_{ij} \sim O(1)i_j\frac{\langle O_H \rangle}{\Lambda^{\Delta_H-1}} \sim O(1)i_jv \left(\frac{v}{\Lambda}\right)^{\Delta_H-1}. \quad (10)$$

In this way, such anarchic contributions to fermion masses follow inevitably from the fermion hierarchies generated by PC. In addition, in strong sectors of large-$N$ type, the gauge singlet operator $O_H^2O_H$ has dimension $\sim 2\Delta_H$. The IR-stability of the strong conformal dynamics against perturbations of this type then requires $2\Delta_H \gtrsim 4$. This implies that fermions masses from (10) are very small, $m_{ij} \lesssim v(v/\Lambda)$, which is the dual of (8). Clearly, for $\Delta_{L_i} + \Delta_{R_j} < \Delta_H + 4$, PC [9] dominates, whereas for $\Delta_{L_i} + \Delta_{R_j} > \Delta_H + 4$, the $O_H$-induced fermion masses [10] dominate. This bifurcation is the dual of [4] (with the modification [7]).

Effects like [10] appear in walking technicolor theories from integrating out extended technicolor (ETC) physics at $\Lambda$, where Yukawa hierarchies follow from a hierarchy of different $\Delta$’s [19]. While the PC and ETC mechanisms have been separately discussed in the literature, the novelty of our framework is that one implies the other, the switching behavior arising out of their competition.

We now turn to the supersymmetric solution for stabilizing the large electroweak hierarchy $a^{-3} \gg v$, but in flat spacetime again. In 5D supersymmetry it is difficult to have non-zero $Y_{\text{bulk}}$, but boundary-localized superpotentials are straightforward. Our flavor mechanisms proceed as before but with $Y_{0,a}$ in [4] now given by just the boundary couplings $Y_{0,a}$.

The nature of the SUSY flavor problem is somewhat transformed by the extra-dimensional flavor structure. The simplest ansatz for weak-scale soft masses that satisfies FCNC constraints is squark and slepton universality.
In purely 4D, there are two options: either (A) universal soft masses emerge from some special mechanism of Planck scale physics, unspoiled by the modest flavor violation of MSSM loop effects in the IR, or (B) universality of soft masses is due to an effective field theory (EFT) mechanism in the IR. But in our 5D picture, option (A) is implausible. While there is a 4D MSSM regime below the compactification scale $a^{-1}$, with flavor violation suppressed by wavefunction overlaps, the 5D regime between the 5D Planck scale and $a^{-1}$ has maximal $O(1)$ flavor-violating interactions and hence loops. Therefore, we are left with option (B).

4D gauge and gravity effects, that are non-local from the 5D perspective, can naturally provide such IR-dominated mechanisms. Gauge mediation (GM) [20], gaugino mediation ($\tilde{g}$M) [21], anomaly mediation (AM) [22], and (D-term dominated) gravity-loop mediation [23] are prime examples. These mechanisms certainly do solve the SUSY flavor problem, but unfortunately then the physics of the superpartners will not contain any experimental traces of the extra-dimensional origins of flavor.

But flavor universality of soft terms need not be exact. One can also have contributions to soft terms which are sensitive to 5D flavor structure, as long as they are subdominant enough to satisfy low-energy flavor tests [24]. An interesting example is the scenario of “Flavorful Supersymmetry” [25]. This can arise in $\tilde{g}$M and AM scenarios in which flavor-blindness depends on sequestering, the separation in extra dimensions of SM matter from the SUSY-breaking sector. Approximate sequestering occurs in the setup of Fig. 1 if SUSY-breaking is localized at $y = a$, where it has small overlap with the zero-modes of the matter superfields. Then, the exponential suppression factors appearing in the non-universal corrections to soft terms are the same as those in Yukawa couplings.

An important observation is that our Majorana $\nu$ scenario is incompatible with this picture of Flavorful SUSY, because the $\ell_L$, then lean towards $y = a$. This implies a maximal breaking of sequestering and $O(1)$ lepton-flavor violation in soft terms, which is disfavored at the TeV scale. As in the warped case above, only our Dirac $\nu$ scenario survives. In contrast, mediation mechanisms which do not require sequestering (e.g. GM) or in which it is achieved independently of zero-mode leannings, can be fully compatible with our Majorana $\nu$ scenario. But, of course, in these cases soft terms will not provide us an imprint of flavor origins. R-symmetric supersymmetry breaking is somewhat exceptional in that the $O(1)$ lepton flavor violation of our Majorana $\nu$ scenario can be consistent with flavor tests, giving a modest hierarchy between the slepton and gaugino masses [26].

The Dirac $\nu$ case introduces new superfields $N_i \equiv (\nu_R, \tilde{\nu}_R)$ to weak-scale physics, whose consequences we discuss for the different mediation mechanisms. These mechanisms all generate soft masses based on the 4D interactions of the sparticle in question. The danger for the sterile $\tilde{\nu}_R$ is that it acquires a very small mass $\ll m_Z$, potentially spoiling the LSP dark matter scenario. In GM, the $\tilde{\nu}_R$ is indeed very light, but the very light gravitino is already problematic for LSP dark matter. In $\tilde{g}$M (or its “flavorful” variant), the very light $\tilde{\nu}_R$ is a new problem. But adding $U(1)_{B-L}$ interactions, higgsed below the compactification scale, generates $m_{\tilde{\nu}_R} \sim O(m_Z)$, making a $\tilde{\nu}_R$ a viable WIMP or super-WIMP [27] dark matter candidate. In AM, $m_{\tilde{\nu}_R} \sim O(m_Z)$ can be generated by $O(1)$ renormalizable superpotential couplings, such as $N^3$, again leading to a $\tilde{\nu}_R$ WIMP. Finally, gravity-loops at the compactification scale [28] can combine with AM, leading to a different LSP from $\tilde{\nu}_R$ that can be a viable and detectable dark matter candidate.

We have studied the impact of our extra-dimensional flavor mechanisms on quite different scenarios for TeV physics. In those in which the TeV physics contains clues to the origins of flavor structure, the Majorana $\nu$ option was disfavored by lepton-flavor violation constraints. In fact, this is more general. The flavor-violating exponential suppression factors, which are the building blocks of Yukawa hierarchies in our 5D picture, can reappear in TeV physics whenever this new physics is localized near $H^{(0)}$. While TeV physics with maximal flavor violation would typically induce too large FCNCs at low energies, these exponential overlaps can suppress them to acceptable levels. However, for our Majorana $\nu$ scenario, these suppression factors are absent since the $\ell_L$, and the new physics (and the Higgs) are localized in the same region of the extra dimension. Therefore, in this type of flavor-sensitive TeV scenario, Dirac neutrinos are preferred.

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