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Replies to Comments on If-Thenism

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I am hugely grateful for these provocative and illuminating comments. My thanks to all N commentators (N ≈ 13). I will have something to say about each contribution, but the overall organization will be thematic. A reminder first of the issues we’re wrestling with.

1. Introduction

A relatively undemanding claim \( \varphi \) sometimes implies, to all appearances, a more demanding one \( \psi \). That is a donkey, in Dretske’s famous example, appears to imply, since zebras cannot be mules, that It is not a cleverly disguised mule. There are eight pawns on each side appears to require the existence of (sixteen) abstract objects. It’s been reported that The Russians possess lurid video of the president-elect, according to information gathered by a former MI6 operative. This implies, if ‘information’ is factive, that The Russians do possess lurid video of the president-elect. \( \psi \) seems in each case to stick its neck out further than \( \varphi \) did in some direction. There is the appearance of a further claim that cannot be settled just by settling whether \( \varphi \).

What does it mean for \( \psi \) to make a further claim? To call \( \psi \) more demanding just pushes the question back a step. In what does its greater demandingness consist? Perhaps \( \psi \)’s demands are more onerous. But we have to be careful here. More “onerous” demands seem like they might more easily go unmet; but that runs contrary to our assumption that \( \psi \) is implied by \( \varphi \). Perhaps \( \psi \) just visits its demands on more of the world. Perhaps it invites further questions, even if the answers are predetermined.

I suspect there is truth in all these ideas, but our predicament in any case is the same.

An article of common sense has been taken hostage by some unhinged philosophical drifter. There are three main styles of response. We may

(1) decide that \( \psi \) must be not as demanding a claim as we’d thought
(2) decide that \( \psi \) must be a more demanding claim than we’d thought
(3) insist that \( \psi \) really is more demanding, despite being implied by \( \varphi \).

The first approach is taken by people like Carnap, Alston, and Thomasson. I called them ‘boosters’ in the paper—a better term might be ‘deflationists.’ The second approach, call it ‘sceptical’ or ‘inflationist,’ is taken by error theorists like Field, mad dog realists like Lewis, and various sorts of fictionalist.\(^1\) The third or ‘defiant’ approach

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\(^1\) A fictionalist reconstruction is called for because \( \varphi \)’s literal content is so demanding.
is taken by people like Dretske and Nozick—ψ goes further only in the sense of being harder to know—and sometimes by me—ψ is about more than φ.

All these groups have some explaining to do. Why is ψ experienced as more demanding than φ? Deflationists think our expectations have been ramped up by bad philosophy. Defiantists try to put the demandingness on an axis other than truth. ψ seems less precarious, according to the inflationist, because it aspires only to some form of acceptability. Acceptability might mean that it works well as an inference ticket; or that it is true in a certain story; or that φ-assuming-that-ψ is true.

There is no one best way of resolving a hostage crisis. I like the third approach (defiance) for problems of epistemic closure [Yablo 2017], and have used the first in defence of naïve realism about secondary qualities [Yablo 1995]. But today we are exploring if/thenism, a version of the second strategy. If/thenists are those in the second camp who read φ-assuming-that-ψ as If φ then ψ.

Of course, conditionals can be read in various ways as well. If you are going to be an if/thenist, you would do well, I claim, to read the conditional incrementally, as expressing what φ adds to ψ.

2. Classic If/Thenism

Horgan in an excellent, under-discussed paper proposes a counterfactual version of if/thenism. Say φ is Flowers tend to have a Fibonacci number of petals; its stripped-down content for Horgan is

Flowers would have a Fibonacci number of petals, were there MOs.

(MOs are mathematical objects; I will sometimes just say ‘numbers.’) I had two worries about this proposal. Who is to say that flowers would not have had fewer petals, if there were numbers? I do not believe they would have, but a translation scheme should not be taking bets on such things. The other worry was about explanation. Gomorrah was destroyed because the number of righteous men there was less than 10—not because the number of righteous men would have been less than 10, if numbers had existed.

Horgan is aware of the first worry, and may in Rosen’s view already have answered it:

The if-thenist should identify the conditional in the vicinity of a mathematical claim φ as the more complicated counterfactual: If things had been just as they are in concrete respects and numbers had existed, it would have been that φ [Rosen, section 1].

This seems plausible for the case at hand. But the if/thenist might want to operate in realms where we don’t have a word like ‘concrete’ to indicate the subject matter that we are trying to get right. It may be clear only what we are not trying to get right (pure abstracta, e.g.).

If φ’s usable content had always to be marked out in positive terms, then Horgan’s approach might be fine. But I at least want the option of marking it out negatively as, for instance, φ’s strongest implication that is silent on whether ψ. (This turns out to be another way of formulating the definition of φ→ψ—to write it as a conditional, ψ→φ—presented in the target article [Yablo 2012].)

When should partial content be defined positively, in terms of what it is about, and when negatively, in terms of what it is not about? There are technical issues here about which sorts of contents can be accessed which ways. φ’s strongest implication that is not
about whether $\psi$ is negative on its face, but it is equivalent in certain cases to something positive: what $\varphi$ says about $m$, $m$ a subject matter that is suitably orthogonal to whether $\psi$.

Rosen responds on Horgan’s behalf to the explanatory worry, too. That worry assumed that The number of righteous men would have been $< 10$, if there were numbers and There were fewer than ten righteous men were competing explanations of the destruction of Gomorrah. But they are not competing if they express the same proposition.

Yablo wants the ‘real content’ of a mixed mathematical claim to be a claim that is not about numbers at all: In our simple-minded example, There are no unicorns. It’s entirely possible that his construction yields that result. But then the real content is also what Horgan suggested: If there had been numbers (and things had been concretely as they are), the number of unicorns would have been zero. After all, in an intensionalist framework these are the same proposition.

Horgan’s counterfactual for these purposes is $\omega \& \gamma \supset \varphi$, where $\omega$ says there are numbers (or MOs) and $\gamma$ that things remain concretely the same. Call that $\varphi_H$ for short. The conditional I would use is $\omega \supset \varphi$, or $\varphi_Y$. Are their real contents the same?

They probably are if the real content of $\chi$ is its intensional content $|\chi|$—the set of $\chi$-worlds—and Rosen is right that this is how I usually talk. But $\chi$’s real content might also be (I talk this way too at times) $\chi$’s directed content $\|\chi\|$, which adds in $\chi$’s subject matter (its truthmakers and falsemakers).

What should real content be in Rosen’s argument on behalf of Horgan? The argument assumes that if $\varphi_Y$ is not about numbers, then $\varphi_H$ (if true in the same worlds) is not about numbers either. But, Horgan’s conditional could still be about more than mine even if $|\varphi_H|$ is $|\varphi_Y|$; for subject matter lives one level up from intensional content. The argument’s premise (agreement in real content) takes real content to be intensional. But, we can draw conclusions about $\varphi_H$’s subject matter only if it is not intensional.

This is a short reprieve, if $\|\varphi_H\| = \|\varphi_Y\|$, which turns on whether $\varphi_H$ and $\varphi_Y$ are true/false for the same reasons. Let $\varphi$ again be The # of righteous men $< 10$. $\varphi_Y$ is true because of the fact of three righteous men. It is not clear that that fact even necessitates $\omega \& \gamma >$ The # of righteous men is under 10.

Somewhere in logical space there’s a world of Cartesian beings with detachable minds. I don’t know whether minds count as concrete, but imagine first that they don’t. Then Horgan’s counterfactual may be false. Had there been disembodied minds in a concrete duplicate of $w$, then (assuming the minds are male and righteous) there would have been more righteous men. One might on the other hand try to understand ‘concrete’ more broadly, to encompass everything non-mathematical. But what is it for things to be in $w$ just as they are in “other than mathematical respects”? It’s for $\alpha \sim \omega$ is true in $w$, where $\alpha$ is the full actual truth and $\omega$ is as before. If that is right then Horgan and I may have more in common than our intensions.

3. Shortest-Path Conditionals

How do we get the fact of three Fs (exactly) to necessitate If $\omega$, then the #Fs $< 10$? What that fact does clearly necessitate is the material conditional $\omega \supset \#F$s $< 10$. The material conditional is for familiar reasons too weak, but here is a way to strengthen it: Require $\psi \supset \varphi$ to be true otherwise than by $\psi$ being false. This takes us more or less to the subtractive, or incremental, or ‘shortest-path’ conditional $\psi \supset \varphi$. (The ‘$\supset$’ is meant to evoke the ‘$\sim$’ in $\varphi \sim \psi$.)
This idea of a remainder, when $\psi$ is subtracted from $\varphi$, or the surplus content of $\varphi$ over $\psi$, can seem fanciful. The commentators raise questions both about remainders as such, and the recipe that’s supposed to produce them. To pick five: The recipe is obscure; it outsources too much to truthmakers; remainders don’t always exist; remainders are not unique ($\varphi \sim \psi$ could be a bunch of things); and the definition misfires if $\psi$ is necessary, or impossible.

I take these worries seriously. Obscurantism is not the method of true philosophy, and it is not clear what a non-obscure form of subtraction would be. Colyvan’s photographic model can serve as a touchstone here; for photographic subtraction unquestionably ‘works’ and yields definite results.

Radiologists have to deal with entangled images in which the object of interest is occluded in part by other objects, for example, a bony structure blocks our view of blood vessels. To clean the entangled image up, they make an image (the ‘mask’) of the intervening entities and overlay the original with a negative of that mask. A dumbed-down digital formulation has us subtracting mask-y grayscale values, pixel by pixel, from grayscale values in the original.\(^2\)

One aspect of the model seems exactly right. Images have contents, just like sentences. They represent their objects as being a certain way. But although our interest is in the contents, the operation by which one image is subtracted from another abstracts away from content. We do not ask, ‘what would the entangled image say, if it were silent on issues addressed by the mask?’ One computes the remainder pixel by pixel, replacing $i$’s value at a given $<x,y>$ with what it adds to $k$’s value at the same point.

If logical subtraction is conceived on the first model, it will seem mysterious. One has no idea in most cases how to put into words what $\psi$ says that was not already said by $\varphi$. But that’s the wrong model. Logical remainders are constructed in a pointwise, bottom-up, fashion, like photographic remainders. $\varphi \sim \psi$’s intension is calculated world by world by assigning it a truth-value in each one.

Obviously the calculations are not strictly analogous. $i \sim k$’s value at a point is obtained by subtracting one number from another. The truth-value of $\varphi \sim \psi$ depends not just on whether $\varphi$ and $\psi$ are true in $w$, but how they are true (or false) there. This depends in turn on the kinds of facts that obtain in $w$. The question is whether $\psi \supset \varphi$ has in $w$ a ‘targeted truthmaker,’ and mutatis mutandis for false.

### 4. Existence

This is enough to get us started on the five complaints. I agree that we can’t for the most part directly work out what a stronger statement adds to a weaker one; we wouldn’t know where to begin on such a project. But the idea that we should have to depends on conceiving subtraction as a content-driven, top-down affair. The actual recipe is pointwise and bottom-up.

Doubts about the recipe are one source of concern about the existence of remainders. Also though we may feel that certain would-be remainders shouldn’t exist—what for instance does *This is red* add to *Something is red*?—which then breeds doubts about the recipe. If subtraction were a well-defined operation, it would yield remainders even in the ‘bad’ cases. But then a content would exist the whole idea of which is absurd.

\(^2\) 3 = black, = dark grey, 1 = light grey, 0 = white.
If it is no knock on image subtraction if we can’t always make sense of \( i \sim \kappa \), why should it be a problem for logical subtraction that we can’t always find meaning in \( \varphi \sim \psi \)? It is the opposite of a problem in fact. For some remainders are genuinely funky. This is something that a theory of subtraction ought to explain. Consider for instance

1a: Tom is red and bulgy \( \sim \) Tom is red
1b: Tom is scarlet \( \sim \) Tom is red
2a: Everything is bulgy \( \sim \) Tom is bulgy
2b: Tom is bulgy \( \sim \) Something is bulgy

Two of these, (1a) and (2a), make good sense. (1a) is true just where Tom is bulgy, and (2a) is true where everything is bulgy except possibly for Tom. The two that don’t are not entirely un evaluable. (1b) is true in red-worlds where Tom is scarlet, and false where it’s another shade of red. (2b) is true in worlds where Tom is bulgy, and false where something is bulgy but not Tom. Quite generally \( \varphi \sim \psi \) is true (false) in those \( \psi \)-worlds where \( \varphi \) is true (false).

That (1b) and (2b) are not wholly un evaluable suggests that there is a contentful remainder even in the bad cases. Something goes wrong in these cases. But it’s not the definition we gave of subtraction. The definition confirms our judgments and in some sense explains them.

Why would (1b) be undefined in worlds where Tom isn’t red? It is true in \( w \) only if Tom is red \( \supset \) Tom is scarlet has a Tom is red-compatible truthmaker there. But it is hard to think of a fact short of Tom being scarlet that entails Tom is red \( \supset \) Tom is scarlet without falsifying Tom is red. This is why (1b) fails to be true in worlds where Tom isn’t red; the fact of Tom being scarlet cannot obtain in such worlds. It is false in \( w \) only if Tom is red \( \supset \) Tom is scarlet has a Tom is red-compatible truthmaker in \( w \). But it is hard to think of a fact short of Tom being, say, crimson that entails Tom is red \( \supset \) Tom is not scarlet without falsifying Tom is red. The remainder’s falsity, like its truth, requires Tom to be red.

Not everyone finds this line of thought as compelling as I do. A proper notion of subtraction would be defined only on deserving inputs, Steinberg contends, noting that on Fine’s account,

\[ A \sim B \text{ only exists ... if each (exact) truthmaker for } A \text{ [is] the fusion of some } B \text{-independent part } c \text{ and a } B \text{-ish part } b. \]

I think that two issues are getting confused here: (i) whether \( \sim \) is defined on \( \varphi \) and \( \psi \), and (ii) whether \( \varphi \sim \psi \) is defined on \( w \). Steinberg maintains in effect that the answer to (i) should be NO unless the answer to (ii) is always (for each \( w \)) YES. This sets an absurdly high bar, and prevents \( \varphi \sim \psi \) from serving as a diagnostic of how extricable \( \psi \) is from \( \varphi \).

Steinberg counts it a point in favour of Fine’s requirement that it explains why I raised my arm\( \sim \)It went up

 strike[s] us as barely intelligible ... [this is because] truthmakers for the proposition that I raise my arm aren’t plausibly conceived of as fusions of two independent states of affairs one of which concerns my arm’s going up.

But, when did intuitive first-pass intelligibility become the test of existence for contents? A better test is evaluableability. I raised my arm\( \sim \)It went up passes it; it is false for instance in worlds where I am brain dead or determined to keep my arm by my side. Likewise \( \varphi \sim \psi \) may be true in certain \( \neg \psi \) worlds and undefined in others, as,

Witches didn’t kill Kennedy\( \sim \)Witches exist is true in worlds where Kennedy wasn’t killed. I don’t know how an all-or-nothing approach explains this.
There is always at least a minimal remainder, I claim, evaluable only in $\psi$-worlds. The point of having remainders in the bad cases is that extricability comes in degrees. Minimal remainders stand at one end of the spectrum; they are not evaluable in any $\neg\psi$-worlds. The existence of a maximal remainder, defined in all $\neg\psi$ worlds, means that $\psi$ is fully extricable from $\varphi$. How the cases line up in between—how far into the $\neg\psi$-region $\varphi \sim \psi$ is defined—can be instructive about $\psi$’s relation to $\varphi$, for instance, about the extent to which factorization is possible. $\varphi \sim \psi$ may be of interest even in the bad cases, if it is evaluable around here. And so on. Why would we want to throw these theoretical benefits away by insisting on maximal remainders every time?

5. Exportability

A related issue is raised by Leng. If we are going to be leaning on false (or suspicious) assumptions, then we need to explain at some point how those assumptions are meant to be discharged. Pincock speaks in this connection of

the ’export’ challenge for mathematical fictionalism, the challenge being to provide rules that will indicate, for a given context, which claims can be extracted from the fiction and taken literally as claims about the actual world. [Pincock 2012: 252]

Rosen goes some way toward meeting this challenge when he defines $\psi$’s nominalistic content as the proposition true in $\psi$-worlds and concrete duplicates thereof. Balaguer constructs it out of the worlds that ’hold up their end of the bargain.’ Rayo goes further, showing how to tease conditions on worlds apart from the objects that help us frame those conditions. But the crowd still cries out for more. Leng appreciates that I am trying to lay my hands, not just on a set-of-worlds nominalistic content, but a specification (in terms of targeted truthmakers) of what it is about the worlds that earns them membership in the set. But she doubts that truth-grounds are dividable up in this way. Perhaps nothing less than ’the full concrete core of a world’ can verify the relevant instances of $\omega \supset \varphi$.

The problem here stems in part from the presumed necessity of mathematical objects. The nominalist has got to go along with this presumption, Leng thinks, for dialectical reasons:

When the alternatives that are being considered are worlds where there do or do not exist abstract objects, the worry is that all the relevant questions against the platonist will be begged... If we take the content of ... $\varphi$, but (perhaps) for $\omega$, to be picked out by some range of possible worlds, these will all be $\omega$-worlds, so such a picture will not distinguish between the content of $\varphi \sim \omega$ and the content of $\varphi$...

I have two responses, the first rhetorical. The nominalist wants to show how things could work compatibly with nominalistic assumptions, in the hope that their model proves the most satisfactory. To expect them to work within the opposite assumptions disallows this; it denies them the opportunity to show that nothing is left unexplained.

Compare Hume on miracles. Reports of wondrous events are explicable, he tries to convince us, in terms of ordinary human gullibility. Imagine someone objecting that this begs the question against theists who think that God would not allow that degree of gullibility. Leng’s platonist is perhaps a bit like that theist.

Rayo goes some way toward meeting this challenge when he de...

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rao [2008], Melia’s idea of ’weaselling away the math’ is also important if perhaps less explicit. I take the same rough line as Melia but with more bells and whistles.
The construction of surplus content has two parts that I have been somewhat running together. First we look for the feature of certain $\psi$-worlds whereby they are $\varphi \& \psi$ rather than $\neg \varphi \& \psi$. Then we ask which $\neg \psi$ (or ‘away’) worlds possess this feature. Leng is right that the second step can’t get off the ground if $\psi$ is necessary. But the first step still goes through. Even if all worlds are $\psi$, one can still look for the feature $X$ of certain $\psi$-worlds that makes them also $\varphi$. One can ask, for instance, what distinguishes number-worlds where the number of stars is finite from those where the number is not finite. The answer is not that they contain numbers, since so do the others. It’s that they have finitely many stars.\footnote{An example where the subtracted $\psi$ is clearly necessary may help. That vixens are necessarily female does not prevent us from asking what it is about certain Vixens are female-worlds that makes them moreover Vixens are female & Dogs bark-worlds.}

A notion of set is sometimes suggested that allows more than one empty set, according to the type of object that it would have contained, if objects of that type existed. The SET of dodos agrees in membership with the SET of happy dodos, but they are empty for different reasons. The SET of worlds with finitely many stars is distinguishable from the SET with finitely many stars and numbers, even if they agree in their membership. Once this is taken on board, we may want to recognize numberless worlds after all, so as to make $\varphi \& \omega$ vs $\neg \varphi \& \omega$ into a set-level distinction. We’d then be back where we started, with away-worlds, albeit of a type not considered possible by all platonists.

6. Truthmakers

I doubt that the full concrete core of $w$ is needed to make it true there that $\omega \supset \#(petals)$ is Fibonacci. But perhaps truthmakers are not so easily identified. Rosen makes a useful suggestion about this. If we are amenable to grounding, we can think of a sentence’s truthmaker(s) as the fact(s) in which its truth is grounded. To get remainders into the picture, let $\tau$ be a $\psi$-ground of $\varphi$ iff whenever $\psi$ and $\tau$ both obtain, they together ground $\varphi$. Then

$$(RR) \varphi \sim \psi \text{ is true in } w \text{ iff some } \psi\text{-ground for } \varphi \text{ obtains in } w.$$ 

$$(RR)$$ tells us what it takes for $\psi \sim \psi$ to be true in a world. ‘Does it tell us which proposition $\psi \sim \psi$ is?’ Rosen asks. ‘That depends on whether the identity of a proposition is fixed by its truth-conditions’ (section 3). A proposition’s identity for me is not fixed by its truth-conditions; we need in addition a subject matter. But one is close at hand (the positive component, anyway); Rosen’s $\psi$-grounds can be $\psi \sim \psi$’s truthmakers.

How does (RR) compare to the existing evaluation rule for $\psi \sim \psi$? A truthmaker $\tau$ for $\psi \supset \phi$ in $w$ is targeted iff (1) $\tau$ does not entail $\neg \psi$, and (2) $\tau$ ‘makes the most of $\psi$’.

$$(RY) \varphi \sim \psi \text{ is true in } w \text{ iff } \psi \supset \phi \text{ has, and } \psi \supset \neg \psi \text{ lacks, a targeted truthmaker in } w$$

(Vice versa for falsity.) (RY) agrees with (RR) if

$$(EQ) \tau \text{ is a ttm for } \psi \supset \phi \text{ iff whenever } \psi \text{ and } \tau \text{ both obtain, they together ground } \varphi.$$ 

$$(EQ)$$ is not implausible. I have questions though in both directions. Suppose $\psi$ and $\varphi$ are $P \lor Q$ and $(P \lor Q) \supset R$. Then $\tau$ should intuitively be $R$—or rather $[R]$, the fact that $R$. (RR) yields this result only if $[P \lor Q]$ and $[R]$ ground $(P \lor Q) \supset R$. If $P$ is true and $Q$ false,
though, we may think it is \([P]\) and \([R]\) that do the grounding. Going in the other
direction, do \([P \& Q]\) and \([Q \& R]\) ground \(P \& Q \& R\)? If so, then \([P \& Q]\)
comes out a targeted truthmaker for \(Q \& R \supset P \& Q \& R\). This is the wrong result since \([P]\)
makes better use of \(Q \& R\) than \([P \& Q]\) does. If \([P \& Q]\) and \([Q \& R]\) aren’t joint
grounds—if the shared conjunct prevents this—then we are in effect requiring \(\tau\) to make the most
of \(\psi\). And then it begins to look like a targeted truthmaker.

What is it for \(\tau\) to make the most of \(\psi\)? The idea, as Rosen says, is that no competitor
uses more of \(\psi\). The amount \(\tau\) uses of \(\psi\) is measured by how easily it could have got by
with less—with a consequence (part, really) \(\psi^\circ\) of \(\psi\).

\[
\tau^* \text{ uses more of } \psi \text{ than } \tau \text{ iff } \\
(i) \tau \text{ always entails } \psi^\circ \text{ when } \tau^* \text{ does; but} \\
(ii) \tau^* \text{ does not always entail } \psi^\circ \text{ when } \tau \text{ does.}
\]

305 Now, there is a danger of going overboard with the idea of using as much of \(\varphi\) as possible.
\(X\) is red and round \(\sim X\) is red ought to be \(X\) is round. But does \([X\text{ is round}]\)
really make the most of the antecedent? What about the weaker ‘fact’ \([X\text{ is red }\supset X\text{ is red and round}]\)?

Of course one could say (and many people do say) that there is no such fact; and/or that such a fact
would not be a truthmaker. But neither move is required, for \([R \supset R \& O]\)
does not use more of \(R\) than \([O]\) in the first place. One would need an \(R^-\) such that \([O]\)
did and \([R \supset R \& O]\) did not entail \(R^- \supset (R \& O)\). But \([O]\) doesn’t entail any conditional
of this type; truth is not preserved in an \(O\)-world where \(R^-\) and \(R\) is false.

Someone might propose \([R \supset R \& O]\) as a targeted truthmaker in its own right. This
gets by the letter of the definition, but violates the spirit. The reason truthmakers for
\(\varphi \sim \psi\) are required to be compatible with \(\psi\) (clause (i)) is to prevent \(\psi\)’s falsity from
playing a role in why \(\varphi \sim \psi\) is true.\(^5\) If \([R \supset R \& O]\) is allowed as a truthmaker for
\(R \supset R \& O\), then \([\sim R]\) does play a role (at one remove) in why \(R \supset \sim (R \& O)\) is true, by being
the reason \([R \supset R \& O]\) obtains. If material conditionals are to be allowed as truthmakers,
(i) should be revised to (i'): Truthmakers and their makers can't imply \(\sim \psi\). This
assumes that \(R \supset R \& O\) is grounded in the fact that \(\sim R\), and Rosen might question this.

He says about a related example that

If \([\psi \equiv \varphi]\) is a targeted truthmaker for \(\psi \supset \varphi\), then \(\varphi \sim \psi\) turns out to be \(\psi \equiv \varphi\) ... the identity of
\(\varphi \sim \psi\) then turns on the obscure question of whether \([\psi \equiv \varphi]\) is a truthmaker for \(\psi \supset \varphi\) [section 5],

adding that 'Intuition is muted on whether \([\psi \equiv \varphi]\) is a truthmaker for \(\psi \supset \varphi\). It may
be muted, but it's not silent. \([P \& Q]\) is not in anyone's view a truthmaker for \(Q\); \([P]\) is
just along for the ride. But then \((P \& Q) \lor (R \& S)\) is not a truthmaker for \(Q \lor R\) either.
The first of these is to the second as \([\psi \equiv \varphi]\) is to \(\psi \supset \varphi\).

Rosen is right that \([\psi \equiv \varphi]\) (if allowed as a truthmaker) gets past my existing condition (i)
on targeting. It is blocked, I think, by the extension of (i) to truthmaker-makers;
for \([\psi \equiv \varphi]\) holds on account of \(\sim \psi\), which conflicts with \(\psi \supset \varphi\)'s antecedent. I agree in
a way that 'this arcane problem ... brings out the extent to which the account depends
on substantive judgments about truth-making.' It brings out that the dependence is less
than one might have thought. Substantive judgments don’t come into it, only antiskeptical judgments.

\(^5\) Neither should \(\psi\)'s truth play a role in why \(\varphi \sim \psi\) is true, which is the reason for clause (ii).
7. Uniqueness

Rosen wonders whether remainders are uniquely determined. Ulatowski raises this question as well but in a different setting. The official definition of remainder is point-wise and bottom-up. But I do sometimes help myself to top-down heuristics. There is the enthymeme heuristic:

(EN) The remainder is the $\rho$ that completes the enthymeme $\psi, \rho \therefore \varphi$ in a maximally $\varphi$-beholden way.

And there is a heuristic in terms of difference-making:

(DM) The remainder is the $\delta$ that distinguishes $\varphi&\psi$-worlds from $\neg\varphi&\psi$-worlds.

What sets Pete called-worlds where he wins apart from Pete called-worlds where he loses? That he had the better hand in those worlds. What completes the enthymeme Pete called, $\rho \therefore$ Pete won in a maximally Pete called-beholden way? Again that Pete had the better hand.

Ulatowski thinks there might be other $\rho$'s that 'complete the enthymeme in a maximally $\psi$-beholden way.' And one might wonder in a similar spirit whether there is a single best choice of difference-maker. I believe that it is Someone shot Kennedy that makes the difference, among Oswald-didn't-do-it worlds, between those where someone else shot Kennedy, and no one else did. The following statements would serve equally well, according to Ulatowski:

1. A third gunman on the grassy knoll shot Kennedy
2. Mafioso shot Kennedy
3. Martians shot Kennedy

[any of these could be] the feature [making] Oswald-didn't-do-it worlds [into]
Some-one-other-than-Oswald-shot-Kennedy worlds [section 4]

Actually none of these can be the remainder. Let me explain. 'Maximally $\varphi$-beholden' applies in the first instance to truthmakers $\tau$; $\tau$ should be consistent with $\varphi$ and make the most of it. For the remainder $\rho$ to be maximally $\psi$-beholden is for all its truthmakers to have that property. A third gunman shooting Kennedy is a truthmaker for $\rho$, along with Johnson’s shooting Kennedy, Castro’s doing it, and so on. The proposition with all and only facts of the form $[X \text{ shot Kennedy}]$ as truthmakers is the proposition that someone shot Kennedy. That is the intuitive remainder and the one delivered by my account.

Ulatowski links the question of $\rho$'s identity up with a topic not much discussed in the target article: Partial truth. The connection seems to be this. $\rho$ is intended in many applications to capture the truth in $\varphi$, a statement that is overall false (since its consequence $\psi$ is false). Or perhaps it should capture something in $\varphi$ that is more nearly true than $\psi$ as a whole is. Ulatowski wonders how we are going 'to rule out content that makes the remainder farther-from-truth.' But, remainders are just as likely to be wholly false as partly true. Truthiness is not a factor in their identity. Ulatowski comes around to this at the end, conjecturing that 'an expression being maximally $\psi$-beholden might be completely independent of its being truthlike.'

8. Fictionalism

Incremental if-thenism is, as Kim notes, a descendant of Walton-style fictionalism. How does Walton get Holmes wrote a monograph on cigar ash to express, at the level of
real content, that Holmes wrote such a monograph according to the stories? Readers are caught up in a make-believe game \( G \) in which the stories are true. \( \varphi \)'s real content, relative to the game, is the real-world fact that makes \( \varphi \) pretence-worthy in \( G \). The real-world fact that makes it pretence-worthy that Holmes did so-and-so, in a game where the stories are treated as true, is this: The stories say that he did so-and-so.\(^6\)

Kim makes it into a desideratum for incremental content that \( \varphi \sim \psi \) should express the same proposition as \( \varphi \) expresses according to Walton, in a game where we pretend that \( \psi \). She doubts for two reasons that the desideratum is met. One starts from the fact that incremental conditionals are not ‘conservative’; \( A \sim \rightarrow C \) can differ in truth-value from \( A \rightarrow \rightarrow D \) even if \( C \) and \( D \) are equivalent over \( A \)-worlds. The consequents of

(i) \( Al \) is right about whether \( p \sim \rightarrow p \)

(ii) \( Al \) is right about whether \( p \rightarrow \rightarrow Al \) says that \( p \)

hold in the same antecedent-worlds. But (i) is correct if \( Al \) says that \( p \), and (ii) is correct if \( p \). (\( Al \) can say that \( p \) even if \( p \) is not the case.) This kind of behaviour is NOT seen, Kim thinks, with According to the story that \( A \), \( C \) and According to the story that \( A \), \( D \).

We ordinarily consider [them] equivalent when \( C \) and \( D \) coincide on \( A \). Fictionalism adopts our ordinary understanding of truth in fiction. Yablo's f-thenism does not treat their corresponding conditionals, \( A \sim \rightarrow C \) and \( A \rightarrow \rightarrow D \) as equivalent when \( C \) and \( D \) coincide on \( A \).

It is true that there are theories of fictional truth that have the result Kim is alleging. \( C \), according to \( F \) is true on Lewis's theory iff \( C \) holds in the nearest worlds where \( F \) is told as known truth. If \( C \) and \( D \) hold in the same \( F \)-worlds, they hold in the same worlds where \( F \) is told as known truth. So one is true according to \( F \) iff the other is.

But this is often seen as a problem for theories like Lewis's. Why should it be true in all fictions \( F \) that someone knows \( F \), and tells it? An absurdist fiction like 'Six Characters in Search of an Author' might perhaps feature itself as a character, but it's not an everyday thing.\(^7\) But then one should not expect the following to agree as a rule in truth-value.

\[ H: \text{According to [a certain] book, nothing changes.} \]
\[ H^*: \text{According to the book, the book says that nothing changes.} \]

If the whole text of the book is Nothing changes, then \( H \) is true but not \( H^* \). Even where a book does pronounce on its content, it may get the content wrong. Suppose we add to the book just mentioned a second sentence: Books never take a stand on whether things change. \( H \) would be true in that case too, but \( H^* \) false; the book portrays itself as agnostic on the matter.

### 9. Necessity

A number of commentators agree with Leng that sense can be made of \( \varphi \sim \psi \) only if \( \psi \) is contingent. We know how to subtract \( Al \) is here from Everyone is here. But when the subtrahend is necessary,

\(^6\) The stories say that \( P \) is also the \( \rho \) that completes The stories are true, \( \rho \). \( \vdash \). \( P \).

\(^7\) The Holmes stories may be special in this respect, insofar as Watson is said to be writing the chronicle we are reading.
it is not clear how to understand the approach. For then there are no worlds to consider where ψ is false, and we can ask what would be left of φ ... [Thomasson, section 1].

I don’t see why ψ has to be false in w for there to be a sensible question there of what φ adds to it. (φ→ψ doesn’t imply ¬ψ; it only fails to imply φ.) Also we are not supposed to be 'asking' what would be left of φ in the first place. The recipe is supposed to tell us what is left, on the basis of our answers to why is w a φ¬ψ-world rather than a ¬φ¬ψ-world?8

This is a question entirely about ψ-worlds; it ignores ¬ψ worlds rather than presupposing them. Numberless worlds may indeed come in handy, if we want a number-free witness to the fact that that #(dragons)>0 worlds differ from #(dragons) = 0 worlds in containing dragons. But a witness is not essential. The condition of containing dragons is weaker even by platonists’ lights than the condition of containing dragons and numbers.9

10. Impossibility

Nominalists do not usually take numbers to be impossible. But they are impossible (or worse) if nominalists are correct, according to Felka, and Armour-Garb and Kroon. They take inspiration here from Kripke’s suggestion that without unicorns, there can be no propositions about unicorns, and hence no worlds where such propositions are true, and Sorensen’s analogous suggestion about God.

This is a good challenge! But let me push back a bit. ‘Unicorn’ is a natural-kind term, and ‘God’ is a name. The challenge assumes that ‘number’ needs an externalist semantics along the lines of names and kind-terms. Can’t it just mean: entity suited by nature to serving as a measure of cardinality, and that has no more to its nature than that?

A second premise is this: ‘Insofar as a [sentence] fails to express a proposition, the sentence lacks a truth-value relative to any possible world’. Kripke says this in places. But he disputes it in more places. He disputes it, for instance, in his work on truth. The Liar sentence λ fails to express a proposition, according to Kripke. Neither it seems does its conjunction with 0 = 1. The latter is false, though, on account of 0’s relations to 1.10

Kripke (Kripke, et al. [1974, p. 479]) expresses sympathy elsewhere for Strawson’s (eventual) view that ‘King of France’-sentences can be something like false. We do often boggle at these sentences, but not always:

[i]f you put it to [someone] very categorically, ... first specifying an armament program to make it relevant and then saying ‘The present king of France will invade us’, the guy is going to say ‘No!’, right?

He has doubts even about the application to unicorn-talk (‘Vacuous Names’, Kripke [2011, p. 68]):

How can the statement that unicorns exist not really express a proposition, given that it is false? ... it is not sufficient just to be able to say that it is false, one has to be able to say under what circumstances it would have been true, if any.

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8 Officially: what is a targeted truthmaker in w for ψ⊂φ.
9 Admittedly not all platonists (Rayo [2013]).
10 It is gappy on the weak Kleene scheme, which is why most people prefer strong Kleene.
He thinks we are right to call it false. The question is how it can be false in the absence of a proposition.

One not outrageously implausible line on all this is that to count as false, a sentence \( S \) need only be false about a certain subject matter [Yablo 2014]. Unicorns exist counts as false in the empty world because it is wrong about how many objects there are. Numbers exist counts as false in a nominalistic world because it is wrong about how many abstract objects there are. Sentences like these count as true in \( w \) if their negations count as false; that is, there is no relevant subject matter they get wrong.

11. Availability of the Incremental Reading

Incremental if-thenism is wrongly so-called if conditionals do not admit of an incremental reading. I argue mostly by example that they do, for instance, that there is a reading of

(i) If that guy's the murderer, then he's insane
(ii) If that guy's the murderer, the murderer is insane

where they stand or fall respectively with

(i') The murderer is insane
(ii') That guy is insane.

—(i) is true and (ii) false if the murderer is insane, but that guy is not insane. Dohrn observes that we can maintain these judgments together only by rejecting the antecedent; that guy can't be the murderer. This seems right. He concludes that 'there is no intuition that Yablo's exemplary pairs disagree in truth-value' [section 3]. My conclusion would have been that the intuition doesn't sit well with that guy's being the murderer. This commits me, as Dohrn says, to a reading

where the conditionals constituting any single pair actually disagree in truth-value although they agree (their consequents have the same truth-value) in 'at home' worlds [worlds where the antecedent holds].

He finds this an unlikely scenario. But, it is familiar already from Gibbardian stand-offs. Alice knows that Russell was the author of Principia Mathematica if it was single-authored; for she knows that Russell was an author. Benny knows on a similar basis that Whitehead was the author if it was single-authored. Pooling their information they decide that the book had more than one author. Alice and Benny may indeed lose interest in their conditionals, when they realize the common antecedent is false. But that isn't to say the conditionals are now false. Both of these sound pretty good to me even after rejecting the antecedent: If Hegel was right about the number of planets, it was 7 (for that was his view); If Hegel was right about the number of planets, he thought it was not 7 (for it wasn't).

12. Scope

Bueno observes that his own approach does 'not depend on an admittedly complex procedure of “subtracting Peano Arithmetic’s consistency from its truth”' [Yablo 2016: 18]. Neither does mine! True(PA)~Con(PA) was intended as a doomed attempt at subtraction. It’s evaluable mainly in worlds where PA is consistent. Con(PA) is not very
extricable from $\text{True}(PA)$.\footnote{If there is a crisis here—PA’s consistency is a further question even though PA’s truth implies it—it will have to be handled some other way.} (I don’t say it’s not extricable at all. $\text{True}(PA) \sim \text{Con}(PA)$ is false, I would think, in empty worlds.)

If we can’t subtract PA’s consistency from its truth, what about its ontological commitments? Incremental if-thenism seems on shaky ground here, too. What could possibly be left of *Perfect numbers exist* when we cancel out the implication that numbers exist? Perhaps the cash value of $\varphi$ is a metalogical fact about theoremhood in a certain theory.

If there were structures satisfying the mathematical principles of a theory $M$, $\varphi$ would be true in those structures. [section 3]

This resembles the type of (non-incremental) if-thenism once advocated by Putnam. Field defends a related view but about mathematical knowledge [Field 1984]. I count as knowing that $\varphi$, say that perfect numbers exist, if I know that $\Box(\text{PA} \supset \varphi)$, where $\Box$ expresses logical necessity. Bueno thinks that real content cannot be explained in this way, which seems right. For one thing, the proposal applies equally well to *algebraic* theories where truth talk seems out of place. Why on this view would we be more inclined to judge theorems of arithmetic true than theorems of Group Theory or Boolean Algebra?

One notable difference here is that theorems of arithmetic line up nicely with logical truths (not truths of metalogic) in a way that algebraic theorems do not. That $2 + 1 = 3$ corresponds to the logical fact that three things are $F \lor G$, if two are $F$ and another is $G$.\footnote{I sketch a translation manual in Yablo [2002].} The notorious problem with this approach is that (certain) arithmetical *falsehoods*, too, map into truths, if the number of objects is finite.\footnote{As I learned in 2002 from Rosen. Rayo [2013] greatly extends the result.}

Plebani proposes that the ‘new’ if-thenism may be able to get around this problem. Let $\alpha$ be Peano Arithmetic, the second-order version. $\alpha \sim \varphi$ is true in $w$, according to our definition, just if

1. $\alpha \supset \varphi$ has a targeted truthmaker in $w$
2. $\alpha \sim \varphi$ lacks a targeted truthmaker in $w$

If $\varphi$ is a truth of arithmetic, then $\alpha \supset \varphi$ is true as a matter of logic; its targeted truthmaker is the ‘null fact’ $\top$. Assuming no such truthmaker can be found for $\alpha \supset \neg \varphi$, $\alpha \sim \varphi$ is true in each $\top$-world, that is, every world whatsoever. (The surplus content of an arithmetical falsehood is for similar reasons always false.) Plebani’s translation scheme thus takes each arithmetical truth to a conditional $\alpha \sim \varphi$ that is true for logical reasons.

All arithmetical truths map, on Plebani’s scheme, to statements true for the same reason ($\top$). Plebani sometimes allows himself a different formulation that does better in this respect:

*If-thenism lives off the synergistic relations between $\alpha$ and $\pi$ [$\pi = \text{Infinitely many primes exist}$]. The reason why it is true that if a certain sequence of objects is an omega sequence, then it has infinitely many objects occupying prime number positions is that *omega sequences include infinitely many primes*… The nature of omega sequences (conceived as a fact) explains why $\alpha \supset \pi$ (and also $\alpha \sim \pi$) is true at any world [section 4].*
The generic fact here about \(\omega\)-sequences is distinct from the one expressed by *Almost all primes are odd*. Facts about \(\omega\)-sequences ‘as such’ are insensitive to the actual number of objects in a world. A translation into generics thus avoids the notorious problem too. This may bear on Steinberg’s objection that it is
doubtful that the proposition literally expressed by *The number of even primes is 1* can be conceptualized as a conjunction of the proposition that there are numbers with some other proposition that is independent of there being numbers. [1]

The generic fact about \(\omega\)-sequences that they feature one even prime does seem to be independent, though, of the existence of numbers—sosein is independent of sein—and their conjunction is at least a candidate for the role of ‘literal content of The number of even primes is 1.’

Issues of scope are raised also by Thomasson. She does not want everyday objects like statues to get drawn into arcane debates about mereological composites as a class. The incrementalist asks, as ever, what is the result if we subtract from *There is a statue in that region* the fact, or postulate, of composite material objects? Thomasson remarks that ‘There are reasons for doubting whether this works’ (2014: 493). But then how are we going to free the hostage?

Thomasson is ready with an answer: \(\psi\) is not as demanding as we have come to think. Composite objects cannot (as an analytic matter) fail to exist given that particles are arranged statue-wise in certain regions, and/or it is statuing in those regions.

I agree with this, except for the part about the analytic guarantee. Isn’t it enough that material objects obviously exist? Sceptical doubts are not incoherent, they’re just crazy, like doubts about the external world, or the past. One could try to redefine the past, 1984-style, as whatever is preserved in records and memories. This would answer the ‘5 minute world’ sceptic, but it’s a serious overreaction to that sceptic. (Re)defining composite objects so that they come for free with statuing has, for me, some of the same flavour. One could raise about the past, too, a question Thomasson presses about statues—what more could you want, to put the moon landing beyond doubt? It has been beyond doubt all along. Nothing as dramatic as analytic entailment is called for in either case.

If-thetism does not aspire to free every hostage, and certainly not everything that’s been treated as a hostage. But it aspires to free some hostages, that is, more than one of them. This bears on the suggestion by Bueno and Finn that *The number of dragons = 0* does not imply *Numbers exist* in the first place; it implies only that *There are numbers* in an ontologically neutral sense of ‘there are’.

Finn calls the position defiantism, but we use the term differently. My defiantist accepts both of two incompatible-seeming appearances: That \(\psi\) is more demanding than \(\varphi\), and that it’s implied by \(\varphi\). Finn’s defiantist operates with two \(\psi\)’s, one more demanding (because ontologically committal) than \(\varphi\), and the other (carrying only a ‘quantifier commitment’) implied by \(\varphi\). There is again a question of scope. Observing that

In all of Yablo’s examples below [a list soon follows] we have an unproblematic truth \(\varphi\) apparently entailing an ontological claim \(\psi\) [section 1]

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14 ‘Yablo himself elsewhere expresses doubt that there is “anything useful” left when the metaphysical aspects of ordinary object sentences are stripped away’[section 1.1].
he suggests that this ‘show[s] the hostage crises to be ontological.’ The ‘examples below’ were ontological, true. But other examples are not, like I am thinking about Thales, therefore Thales existed, and I received the information that P, therefore P. Non-ontological crises are sometimes of a piece with their ontological counterparts, aren’t they? If not then I have to agree with Finn that I am barking up the wrong tree. If so, though, then if-thenism may have something to offer. “Neutral quantifiers” are only ontologically neutral. But not all hostage crises are existential crises. A topic-neutral notion like subtraction looks to be pitched at the right level of generality

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