Effective criteria for determining the number of measurements selection

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Abstract. This paper describes a method for determining the number of measurements, which allows you to increase the accuracy of measurements, as well as reduce the number of measurements in cases where the variance is small. In the same cases, when the share of random error is significant, additional measurements allow you to avoid an erroneous result, which can lead to a defective product.

1. Introduction

When measuring the parameters of high-precision products, there is a need for multiple measurements of the same value. This happens often when assessing the possible random inaccuracy of measurements show the possibility of such value random error, in which the solution of the measurement problem is incorrect, and that the probability of its occurrence cannot be neglected. For example, control results based on a measurement result with a single observation may be erroneous since it may contain too large a random component of the inaccuracy of measurements [1], [2], [3].

For multiple observations, the average value of the n observed values is taken as the measurement result. The basis for this solution of the measurement problem is the hypothesis that random errors of the same magnitude, but different in sign, occur equally often. Then, after eliminating the misses, you can calculate the average, in which there will be no random errors. And if the systematic component of the measurement error is excluded from this average, then this average can be taken as a real value [4], [5], [6]. However, equal probabilities of the appearance of equal-sized and different-sign values of the measured quantity do not mean that they will appear in a specific measurement experiment with a limited number of observations. This determines the main task of planning measurements with multiple observations: how many observations are necessary so that only the permissible part of their scattering range remains in their average value? To solve this problem, it is necessary to know in advance the law of distribution of random errors. In some cases, for example, in the educational process, it is assumed that the spread of errors occurs according to the normal distribution law (Gauss). The same is often done with relatively low requirements for the measurement process when the features of the laws of distribution of random variables can be ignored. Then the average of n observations, taken as random (due to the presence of some part of the random error of observations in it) will have a Student distribution [7],[8]. If we assume about the value of the ratio of the error in estimating the mathematical expectation to the mean square deviation $\frac{e}{s}$ and set the confidence probability P, then using the ratio $t/\sqrt{n} = e/s$, we can determine the number of observations from the table 1.
Table 1. The ratio of the error in estimating the mathematical expectation to the mean square deviation for different confidence probabilities.

| n  | ϵ/s when P is equal to |
|----|------------------------|
|    | 0.90       | 0.95       | 0.98       | 0.99       |
| 5  | 0.899      | 1.15       | 1.50       | 1.80       |
| 6  | 0.793      | 1.00       | 1.28       | 1.51       |
| 7  | 0.715      | 0.890      | 1.13       | 1.32       |
| 8  | 0.657      | 0.816      | 1.02       | 1.19       |
| 9  | 0.611      | 0.754      | 0.940      | 1.08       |
| 10 | 0.574      | 0.706      | 0.873      | 1.00       |
| 11 | 0.541      | 0.633      | 0.820      | 0.936      |
| 12 | 0.515      | 0.630      | 0.774      | 0.881      |
| 13 | 0.491      | 0.598      | 0.734      | 0.833      |
| 14 | 0.471      | 0.572      | 0.700      | 0.794      |
| 15 | 0.453      | 0.550      | 0.672      | 0.762      |

For example, it is necessary to determine the required number of observations $n$, if the errors of observations have a normal distribution law, and the reliability of the measurement result should be at least 0.95. Then, to determine $n$, we assume that the error in estimating the mathematical expectation $\epsilon$ is equal to the mean square deviation $s$ of the results of $n$ observations. That is $\frac{t}{\sqrt{n}} = \frac{\epsilon}{s} = 1$, we can also determine from the table that $n = 6$.

However, when measuring high-precision products, the random component does not obey the normal distribution law. In such cases, it is impossible to limit the known method for determining the number of measurements, since the random component of the measurement error may be large. Then, if there are not enough measurements, the measurement result may lead to erroneous conclusions.

In this case, it seems appropriate to switch to nonparametric methods using the idea of sequential analysis. As the evaluation criteria of the selection using restrictions on the variance in average; increment the fluctuations in the average; the mean square fluctuation; the increment of the mean square fluctuations. These criteria can be used separately or in combination. This approach is methodically more convenient when there are sufficient grounds for choosing a model of distribution laws. Restrictions on the confidence interval can be used as a criterion. To use these criteria, Bauman Moscow State Technical University developed software. Research on models of the stability of these criteria and their comparative effectiveness has shown that their use leads to a reduction in the probability of errors.

2. The main part

The development of information processing technology also affects the change in measurement methods. The content of the concept of "measurement" includes, in general, procedures for planning, extracting information and processing it. The physical direction in measurement theory considers measurement as a process based on a comparison with a unit of physical quantity. In this case, there is inevitably a certain error. This comparison depends on the sensitivity of the process to disturbing factors. At the same time, it turns out that the better we want to make a comparison, the more sensitive the measurement process is to perturbations.

The use of mathematical methods for processing the results of observations allows in some cases to identify the necessary measurement information. However, this is almost always impossible with scheduling errors.

Measurement planning is the stage of managing it with incomplete knowledge of the mechanism of the phenomenon or otherwise-in conditions of uncertainty. Various recommendations and training manuals suggest determining the number of measurements before they are taken. Such planning requires
a fairly accurate assumption about the distribution law and, at best, an assumption about the ratio of the error of the mean and the root-mean-square. In addition, the planning methodology does not involve an assessment of the correctness of the choice and does not provide for the use of current information. The latter, of course, is associated with the automation of data processing, and, consequently, with the development of algorithms.

In the proposed algorithm, one of the following criteria is used for the final selection of \( n \): a limit on the confidence interval; a limit on the difference of two adjacent standard deviations in order (on the fluctuation of the standard deviation), that is \( s_n - s_{n-1} \). When the confidence interval limit is selected as the criterion, after performing a pre-selected number of observations, the confidence interval is calculated and compared with the selected acceptable value. If the calculated value of the confidence interval does not exceed the allowed value, no further observations are made. If it exceeds, then continue observations until the confidence interval decreases to an acceptable value. As an acceptable value, you can take the permissible measurement error or part of it, if you are not sure that a systematic error is excluded. Similarly, the number of observations is determined when using the restriction on the variation of the standard deviation as a criterion. The limits can be set to a relative fluctuation in percentages: \( \frac{s_n - s_{n-1}}{s_n} \cdot 100\% \).

In this case, when measuring a certain value, the required number of observations will primarily depend on the results obtained. If, because of processing the obtained data, it is revealed that the proportion of the random component of the error is small, then many measurements will not be required. If there is a significant variance in the data obtained, the algorithm will request that the measurements continue until enough data is obtained to decide.

Let us consider the operation of the algorithm on the example of multiple measurements of the length of the general normal of the gear wheel. Since for multiple measurements, the number of observations made must be at least 3, the calculation is carried out after the third value. In this example, the obtained values were entered into the program manually, but when working in an enterprise, such an algorithm can be easily implemented in the software.

![Figure 1](image-url)

The graph shows that after 6 measurements, the fluctuations in the standard deviation become insignificant, so when working according to a given algorithm, after the three subsequent values do not exceed the confidence interval, the command to stop the measurements will be given.

For comparison, we will perform multiple measurements of the length of the general normal of the used wheel. Due to the wear of the working surface of the tooth profile, the variance of the obtained values will be higher, so in accordance with the calculation algorithm, more measurements will be required to determine the length of the general normal with the required accuracy. As can be seen from figure 2, fluctuations in the standard deviation become insignificant only after 22 measurements.
If you need to perform more accurate calculations, you can use several criteria at the same time. Currently, the algorithm uses the following criteria:

\( T_1 \)- fluctuations in the average: 
\[
T_1(n) = \left| \frac{x(1)_{av} - x(n+1)_{av}}{x(n)_{av}} \right|
\]

\( T_2 \)- change of the average fluctuations: 
\[
T_2(n) = T_1(n) - T_1(n + 1)
\]

\( T_3 \)- fluctuations in the mean square deviation: 
\[
T_3(n) = \frac{S(n) - S(n+1)}{S(n)}
\]

\( T_4 \)- change of the mean square deviation: 
\[
T_4(n) = \frac{D(n) - D(n+1)}{D(n)}
\]

where 
\[
S(n) = \sqrt{\frac{1}{1-n} \sum_{i=1}^{n} (x(i) - x(n))_{cp}^2}
\]

\[
D(n) = x(n)_{av} = \bar{x}_{av} \pm t_{av} \frac{S(n)}{\sqrt{n}}
\]

That is, depending on the task, you can apply one criterion, two or all at the same time. An example of determining the number of measurements based on two criteria is shown in figure 3.

Also, when working with the algorithm, it is possible to set different restrictions for different criteria, for example, by setting higher requirements for fluctuations in the mean square than for fluctuations in the average value.

Using this method of determining the number of measurements allows you to increase the accuracy of measurements, as well as reduce the number of measurements in cases where the variance is small. In the same cases, when the proportion of random error is significant, additional measurements allow you to avoid an erroneous result, which can lead to a defective product.

**3. Conclusion**

After accumulating statistical data on the operation of this algorithm on specific equipment, it is possible to expand this study. In addition to reducing the impact of the random fraction of the measurement error, it is possible to use these criteria to identify an unaccounted systematic error. When checking parts in
production, the operation of machine tools and other equipment creates unwanted vibrations, which have a significant impact on the results of high-precision measurements.

This algorithm can be efficiently used in the automation of the measurement process in CNC machine tools and coordinate measuring machines.

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