Fitting and circuit modeling to the attenuation characteristics of logging cable

Jingfu Yan, Qingyan Zou, Dejun Liu
China University of Petroleum, Beijing 102249, China

Abstract. In order to get a previous understanding about data transmission effect through a long well-logging cable (about 7000 m), a lumped parameter circuit simulating the attenuation characteristics of well-logging cable was designed in this paper. Based on the actual attenuation measurement at every discrete frequency point, the analytic expression of transfer function of the cable was firstly obtained by nonlinear fitting method, then a physically realizable circuit model corresponding to the transfer function was thus established. According to the simulation results, the circuit model shows good fitting effect to the cable attenuation characteristics.

1 Introduction
In a whole set of well-logging equipment, the cable with the length of several kilometers is a necessary component. It not only plays the role of bearing the weight of the underground instrument, but also acts as the data transmission channel from the downhole to the ground. On the other hand, the signal attenuation characteristic of well-logging cables varies depending on types, lengths, as well as manufacturers, and the difference is even significant in some cases. Hence, the cable's influence on data transmission must be carefully considered when designing the data transceiver circuit. Unfortunately, the common organization is not likely to implement the indoor experiment to test the transceiver performance using a long well-logging cable due to its bulk and high cost. The usual way is designing a cable simulator whose data transmission performance is similar to that of actual cable and taking it as the simulated channel, and lastly doing the field experiment if the indoor result is good enough. Therefore, the coincidence level of transmission characteristic between the cable simulator and the actual cable will directly affect the overall debugging process of the transceiver circuit.

Before designing the cable simulator, the parameters or the attenuation characteristic of the actual cable must be understood clearly. There are usually two methods to obtain them[1], theoretical calculation and test. In most cases, the parameter concluded from theoretical calculation varies significantly because of the impact of environment, materials, production processes and other factors. This paper gets the attenuation characteristic of well-logging cable using the test method and the specific steps are as follows: the sweep signal with constant amplitude was applied to one end of the well-logging cable, and the voltage signal was measured at the other end, thus the attenuation value at each discrete frequency point could be easily calculated. This paper illustrates how to find a circuit model to the well-logging cable according to the above attenuation values.

2 Relationship between amplitude - frequency characteristic and zero - pole point location
System transfer function can be defined as
\[ H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \]  

(1)

In order to analyze the Amplitude - Frequency characteristic, the formula above can be converted into the zero - pole point form, and let \( s = j\omega \), the function can be expressed as

\[ H(j\omega) = \frac{K \prod_{j=1}^{m} (j\omega - z_j)}{\prod_{i=1}^{n} (j\omega - p_i)} \]

Any factor \((j\omega - p_i)\) in the denominator of the above equation corresponds to a vector that is directed to the point \( j\omega \) on imaginary axis by a pole point \( p_i \); while any factor \((j\omega - z_j)\) in the numerator corresponds to a vector that is directed to the point \( j\omega \) on imaginary axis by a zero point \( z_j \). For any zero \( z_j \) and pole \( p_i \), the corresponding complex vector can be expressed as

\[ j\omega - z_j = N_j e^{j\varphi_j} \]
\[ j\omega - p_i = M_i e^{i\varphi_i} \]

So the amplitude of transfer function is:

\[ |H(j\omega)| = K \frac{N_1 N_2 \cdots N_m}{M_1 M_2 \cdots M_n} \]

When \( \omega \) moves along the imaginary axis, the modulus of each vector will change accordingly, the amplitude-frequency characteristic curve can therefore be generated.

3 Solution to the transfer function of well-logging cable

For a long well-logging cable, transmission line theory should be used for analysis and its exact model should be a distributed parameter circuit. However, for a specific well-logging cable, the signal attenuation characteristic is definite at each frequency, which provides the possibility to acquire the cable characteristic by means of actual measurements, and then to find the possible transfer function.

3.1 Preprocessing of raw data

The normalized data derived from the measured amplitude attenuation values at each discrete frequency was shown in Figure 1. The attenuation characteristic generally shows the pattern that the attenuation becomes serious when the signal frequency increases and vice versa. Specifically, when the frequency is less than 30 KHz, the amplitude attenuation characteristic changes significantly and fluctuates distinctly. When the frequency is higher than 30 KHz, the curve shows the placid trend of monotonically decreasing. Meanwhile, the signal whose frequency exceeds 300KHz is unlikely to be transmitted effectively through the well-logging cable for its serious attenuation of about 60dB.

![Figure 1: the amplitude attenuation values vs. frequency](image-url)
In order to reflect the variation of attenuation characteristic more accurately, more values were measured in the lower frequency band contrast to the higher frequency. But for the iterative algorithm in fitting method, the data should be equal interval sampled. In order to meet the demands of iterative operation, and also take the accuracy of fitting algorithm into account, the original non-uniform measurement data need to be equally spaced or interpolated. This design chooses 1 KHz as the starting point and 4 KHz as the frequency interval. Part of the data is shown in table 1 for illustration.

Table 1: Part of pre-processed data

| Frequency (KHz) | Gain  |
|----------------|-------|
| 1              | 0.96875 |
| 13             | 0.256873 |
| 25             | 0.183575 |
| ...            | ... |
| 265            | 0.0123153 |
| 277            | 0.0123457 |
| 289            | 0.0123457 |

3.2 Introduction to fitting method

The least squares method is a mathematical optimization technique for finding the optimum solution to unknown parameters. It can minimize the sum of error squares between the obtained data and the ideal data. So the least squares method can be used here for curve fitting. In particular, for experimental data \((x_i, y_i)\), \((i = 1, 2, ..., n)\), find the function \(f(x, \theta)\), which makes the sum of squares of the deviation between the values of this function at the point \(x_i\) \((i = 1, 2, ..., n)\) and the observed data \(y_i\) \((i = 1, 2, ..., n)\) is minimized. i.e. the function \(f(x, \hat{\theta})\) satisfies the following conditions:

\[
\min \sum_{i=1}^{n} (f(x_i, \hat{\theta}) - y_i)^2 = \sum_{i=1}^{n} (f(x_i, \hat{\theta}) - y_i)^2
\]

Where \(\theta\) is the unknown parameter and \(\hat{\theta}\) is the best parameter determined by the least squares method. For this design, the experimental data is shown in table 1. The target function is just the transfer function of the well-logging, whose form is shown in the formula (1), and the coefficient of the polynomial fraction in the transfer function is the parameter to be determined. The fitting process of this design uses the function NLINFIT included in MATLAB. In order to compromise between the calculation complexity and the fitting effect to the transfer function, the order of the numerator polynomial in the transfer function has been determined as 4 and the order of denominator polynomial is 5. The NLINFIT algorithm belongs to nonlinear least squares fitting method, it is easy to get the closer results for unknown parameter. In the process of using the function, choice of the initial value of each coefficient is very important, here all initial values are set 1. The specific implementation process is as follows:

1) Creation of a new function in MATLAB, the specific program are as follows:
   \[
   f = \text{function } \text{myfun(a,x)} \]
   \[
   f = (a(1) \cdot x^4 + a(2) \cdot x^3 + a(3) \cdot x^2 + a(4) \cdot x + a(5)) / (a(6) \cdot x^5 + a(7) \cdot x^4 + a(8) \cdot x^3 + a(9) \cdot x^2 + a(10) \cdot x + a(11));
   \]

2) Fitting by using NLINFIT function, and the specific program are as follows:
   \[
   a0 = \text{[1 1 1 1 1 1 1 1 1 1];}
   \]
   \[
   \text{for } i=1:200
   \]
   \[
   a = \text{nlinit(x,y,@myfun,a0);}
   \]
   \[
   a0 = a;
   \]

end
Figure 2 shows the contrast between the preprocessed data and the amplitude-frequency characteristic curve of the transfer function. As can be seen from the figure, the coincidence is quite good. In order to quantitatively describe the difference between the fitting result and the original data, the fitting effect is evaluated by the index of SSE (Sum of Squares of Errors), SSE is defined as

$$SSE = \sum_{i=1}^{N} [y(i) - \hat{y}(i)]^2$$

By calculation, the SSE value of this design is 0.002. It is shown that the NLINFIT function turns out to be satisfying in this design. According to further observation, it also shows good relevance with the original non-uniform measured data. At this point, the corresponding transfer function obtained through fitting operation can be expressed as

$$H(s) = \frac{-2.1s^4 + 1.1 \times 10^5 s^3 + 4.6 \times 10^6 s^2 + 4.1 \times 10^7 s + 1}{8 \times 10^{-4} s^4 - 12.4s^3 + 6.5 \times 10^3 s^2 + 2 \times 10^6 s^2 - 3.9 \times 10^4 s + 1}$$

Figure 2: the preprocessed data and the amplitude-frequency characteristic curve of the transfer function

Since the coefficients of the denominator polynomial in the transfer function contain negative numbers, which means there are poles located in the right half of s domain, indicating that the system is unstable and cannot be realized physically. Therefore, it is necessary to adjust the pole in the right half plane to the left of the imaginary axis symmetrically. At the same time the amplitude-frequency characteristic of the system will not be affected. So the final transfer function is expressed as

$$H(s) = \frac{-2.1s^4 + 1.1 \times 10^5 s^3 + 4.6 \times 10^6 s^2 + 4.1 \times 10^7 s + 1}{8 \times 10^{-4} s^4 + 12.8s^3 + 6.48 \times 10^3 s^2 + 3.2 \times 10^6 s^2 + 3.9 \times 10^4 s + 0.96}$$

(2)

4 Circuit model design

In order to obtain the corresponding circuit model of the transfer function, the formula (2) is decomposed into the form of partial fraction expansion:

$$H(s) = \frac{-3188s + 1.4 \times 10^7}{s^2 + 15506s + 8.1 \times 10^7} + \frac{2090}{s + 276} + \frac{1468}{s + 219} + \frac{5.4 \times 10^{-6}}{s + 2.5 \times 10^{-7}}$$

(3)

The last item in formula (3) is much smaller than others, so it can be ignored when designing the simulated circuit and only the first three items of $H(s)$ should be considered. Where, the first item is second-order system, and the second and third items are first-order system.

According to circuit theory, the first and second order transfer functions can be realized by the first and second order circuits respectively. Figure 3 shows the commonly used circuit model.

![Figure 3: First-order and Second-order Circuit Model](image)

The transfer function of the first-order circuit model in figure 3 (a) can be expressed as
The transfer function of the second-order circuit model in figure 3 (b) can be expressed as

$$H(s) = \frac{U_{in}(s)}{U_{out}(s)} = \frac{1}{RCs + 1}$$  \hspace{1cm} (4)

The transfer function of the second-order circuit model in figure 3 (b) can be expressed as

$$H(s) = \frac{U_{in}(s)}{U_{out}(s)} = \frac{RCs}{LCs^2 + RCs + 1}$$  \hspace{1cm} (5)

$$H(s) = \frac{U_{in}(s)}{U_{out}(s)} = \frac{1}{LCs^2 + RCs + 1}$$  \hspace{1cm} (6)

After comparing the form of each item in formula (3) with formula (4), (5) and (6), the corresponding circuit model of every item can be easily found as long as the appropriate values of R, L and C are selected.

Except passive components, operational amplifiers were used in this design, which not only can isolate each level circuit to avoid the interaction between them, but also make it easy to perform signal additive and subtraction operations. From the establishment of the circuit model, it can be seen that the values of R, L and C in each part of the model is not unique. After calculation and simulation for many times, the circuit model corresponding to the formula (2) was finally obtained and shown in Figure 4.

In Figure 5, the polyline represents the amplitude-frequency characteristic based on the original measured data, and * indicates the simulation result of the model circuit. Because the attenuation of the actual cable fluctuates seriously in the lower frequency band, there are some deviation between the attenuation characteristic of the model circuit and the original data. However, in the higher frequency range, the fitting effect is very good and can meet the demand well for the attenuation characteristic of the logging cable itself is relatively smooth.

**Figure 4**: The Circuit Model Corresponding to Formula (2)
Figure 5: the Amplitude-Frequency Characteristics of the Circuit in Figure 4

5 Conclusion
Based on the measurement and analysis to the signal amplitude attenuation at discrete frequency, the transfer function of the well-logging is found by using the nonlinear fitting function in MATLAB, and then the circuit model corresponding to the transfer function is established. Compared with the method of measuring cable parameters directly by measurement device, this testing - fitting - design method can obtain more accurate circuit model. Of course, this design is only the modeling to the amplitude-frequency characteristic, and does not take the phase-frequency characteristic into account. To establish a circuit model that conforms to the complete frequency characteristics of the well-logging cable, it is necessary to further consider the fitting algorithm, the order of transfer function, the selection of parameters’ initial value in the iterative algorithm, etc.

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