Optical Lattice Design Assisted by Non-Hermitian Hamiltonians

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Abstract. A brief introduction to non-Hermitian arrays of coupled waveguides is presented. The PT-symmetric dimer is revisited for the sake of clarity. It belongs to the class of photonic lattices with underlying SO(2, 1) symmetry that have been shown to provide all-optical conversion from phase to amplitude.

1. Introduction

In the last decade, arrays of coupled photonic waveguides have provided a robust and flexible platform for the optical simulation of standard quantum phenomena [1] thanks to the development of advanced fabrication techniques [2]. Reciprocally, the ability to mimic Hermitian Hamiltonians has inspired the development of passive photonic integrated circuits serving practical purposes [3, 4].

In recent years, the idea of non-Hermitian quantum mechanics [5–7] has also impacted the design of optical devices [8–16]. Here, a brief summary on the simplest non-Hermitian photonic element, which is the PT-symmetric dimer [17–20], is presented and used as a guiding example to uncover its relation to optical finite realizations of the Lorentz group [21]. Optical finite realizations of SO(2, 1) and SO(3, 1) may prove helpful in the design of all-optical photonic integrated circuits, such as the recently proposed PT-symmetric device providing conversion from amplitude (phase) to phase (amplitude) modulation [22].

2. Revisiting the PT-symmetric dimer

A lossy two-waveguide coupler is described by the following non-Hermitian coupled-mode differential equation system [17],

\[-i\partial_z \begin{pmatrix} E_0(z) \\ E_1(z) \end{pmatrix} = \begin{pmatrix} n_0 & g \\ g & n_1 \end{pmatrix} \begin{pmatrix} E_0(z) \\ E_1(z) \end{pmatrix},\]

where the complex field amplitudes at the j-th waveguide are given by \( E_j \), the effective complex refractive indices by \( n_j \), the effective real waveguide coupling by \( g \), and the notation \( \partial_z \) is used for the derivative with respect to \( z \). For the sake of simplicity, let us assume that the real part of the effective refractive indices are identical, \( n_j = \alpha_R + i\alpha_j \) with \( \alpha_x \in \mathbb{R} \), and write,

\[ E_j(z) = e^{i\alpha_R z} e^{-\alpha_j z} \mathcal{E}_j(z), \]

where \( \mathcal{E}_j(z) \) represents the solutions of complex order for the PT-symmetric dimer.

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Figure 1. Experimental realization of the PT-Symmetric dimer in three different architectures: (a) lossy waveguides, (b) pumped waveguides, and (c) coupled whispering gallery microcavities.

that is, we consider the complex fields up to a common phase induced by the real part of the effective refractive index, $\alpha_R$, and attenuation (amplification) induced by a positive (negative) bias,

$$\alpha_b = \frac{1}{2} (\alpha_0 + \alpha_1).$$

(3)

Under this assumption, the mode-coupling differential equation set becomes PT-symmetric,

$$-i \partial_z \begin{pmatrix} E_0(z) \\ E_1(z) \end{pmatrix} = \begin{pmatrix} i\alpha & g \\ g & -i\alpha \end{pmatrix} \begin{pmatrix} E_0(z) \\ E_1(z) \end{pmatrix},$$

(4)

where the new real constant,

$$\alpha = \frac{1}{2} (\alpha_0 - \alpha_1),$$

(5)

can be interpreted as effective linear, $i\alpha$, or gain, $-i\alpha$, at each waveguide. This device is known in the literature as the PT-symmetric linear dimer and it has been experimentally demonstrated in lossy waveguides [18], pumped waveguides [19], and whispering-gallery mode microcavities [20].

Following standard techniques [23], it is straightforward to calculate the propagation through the PT-symmetric dimer,

$$\begin{pmatrix} E_0(z) \\ E_1(z) \end{pmatrix} = \begin{pmatrix} \cosh \Omega z - \frac{\alpha}{\Omega} \sinh \Omega z & \frac{i g}{\Omega} \sinh \Omega z \\ \frac{i g}{\Omega} \sinh \Omega z & \cosh \Omega z + \frac{\alpha}{\Omega} \sinh \Omega z \end{pmatrix} \begin{pmatrix} E_0(0) \\ E_1(0) \end{pmatrix},$$

(6)

This uncovers three possible regimes: (i) a PT-symmetric phase for values $\alpha < g$ delivering a real propagation parameter $\Omega = \sqrt{g^2 - \alpha^2}$ where the dimer behaves like a periodic oscillator, Fig 2(a), (ii) a completely degenerate phase, $\alpha = g$ and $\Omega = 0$, where the dimer behaves like a directional amplifier with a power law, Fig 2(b), and (iii) a broken phase, $\alpha > g$ yielding an imaginary $\Omega$, where the dimer becomes a directional amplifier with an exponential law [21], Fig 2(c).

3. A finite representation of the Lorentz group

Let us borrow Dirac notation for vectors and matrices [24] and rewrite the differential equation set for the PT-symmetric dimer,

$$i \partial_z |\mathcal{E}\rangle = 2 \left( i\alpha \hat{J}_0 + g \hat{J}_x \right) |\mathcal{E}\rangle,$$

(7)

where the operators $\{ \hat{J}_0, \hat{J}_x, \hat{J}_y \}$ form the $SU(2)$ group, which $j = 1/2$ representation delivers Eq. (4). This equation can also be written in terms of the $SO(2,1)$ group, which is equivalent to the 2+1D Lorentz group [21],

$$i \partial_z |\mathcal{E}\rangle = 2 \left( \alpha \hat{K}_0 + g \hat{K}_x \right) |\mathcal{E}\rangle,$$

(8)
where we have just made imaginary some elements of $SU(2)$, \( \{ \hat{K}_0, \hat{K}_x, \hat{K}_y \} \equiv \{ i\hat{J}_0, \hat{J}_x, i\hat{J}_y \} \). It is straightforward to note that any of the experimental architectures where the $PT$-symmetric dimer has been realized \([17–20,25]\) may be tailored to produce optical finite representation of the Lorentz group with different Bargmann parameter, \( j = 1/2, 1, 3/2, \ldots \). For example, starting from the $PT$-symmetric dimer, which is the \( j = 1/2 \) representation of $SO(2,1)$, we can include an extra lossy waveguide, pumped waveguide, or pumped whispering mode cavity, and obtain a \( j = 1 \) representation of the $SO(2,1)$ group, and so on. Any device belonging to this class will show the directional oscillator and amplifier with power law and exponential laws behaviors described above \([21]\).

4. All-optical $PT$-symmetric phase to amplitude conversion

The directional oscillator and amplifier behavior of $PT$-symmetric devices is well known and studied. Here we will bring forward a different application. The \( j = 1 \) optical representation of $SO(2,1)$, which is a planar three-waveguide coupler described by the mode-coupling set \([21]\),

\[
-i \partial_z \begin{pmatrix} \mathcal{E}_0(z) \\ \mathcal{E}_1(z) \\ \mathcal{E}_2(z) \end{pmatrix} = \begin{pmatrix} i\alpha & g & 0 \\ g & 0 & g \\ 0 & g & -i\alpha \end{pmatrix} \begin{pmatrix} \mathcal{E}_0(z) \\ \mathcal{E}_1(z) \\ \mathcal{E}_2(z) \end{pmatrix},
\]

or in Dirac notation,

\[
-i \partial_z |\mathcal{E}(z)\rangle = \mathbb{H}|\mathcal{E}(z)\rangle.
\]

Introducing a scaled propagation distance, \( \zeta = g\zeta \), and a ratio between effective linear gain or loss with respect to the coupling parameter, \( \xi = g/\alpha \), yields a mode-coupling matrix described by the propagation parameter \( \Omega = \sqrt{2 - \xi^2} \) and propagation ruled by the equation \([21]\),

\[
|\mathcal{E}(z)\rangle = \begin{cases} 
\begin{pmatrix} 1 + \frac{i}{\sin \Omega} \mathbb{H} + \frac{1}{\Omega^2} (\cos \Omega \xi - 1) \mathbb{H}^2 \end{pmatrix} |\mathcal{E}(0)\rangle, & \xi \neq \sqrt{2}, \\
\begin{pmatrix} 1 + i\xi \mathbb{H} - \frac{1}{2} \xi^2 \mathbb{H}^2 \end{pmatrix} |\mathcal{E}(0)\rangle, & \xi = \sqrt{2}.
\end{cases}
\]

Now, let us take a directional oscillator, \( \xi < \sqrt{2} \), stop propagation at the scaled distance,

\[
\zeta_f = \frac{1}{\Omega} \arccos \left( \xi^2 - 1 \right),
\]

and consider a two-waveguide input where the fields have the same power but different phases,

\[
|\mathcal{E}(0)\rangle = \begin{pmatrix} e^{i\phi} \\ 1 \\ 0 \end{pmatrix} \mathcal{E}.
\]
It is straightforward to calculate the output,

$$|\mathcal{E}(\zeta_f)| = \begin{pmatrix} 0 \\ -\mathcal{E} \end{pmatrix},$$

and note that the power at the last waveguide varies with respect to the initial phase difference,

$$|\mathcal{E}_2(\zeta_f)|^2 = [1 + 4\xi (\xi - 1) \sin \phi] \mathcal{E}^2.$$

Thus, it is possible to provide phase to amplitude conversion in a finite optical representation of the 2+1D Lorentz group. The converse has also been demonstrated [22].

5. Conclusion

We have revisited the simplest non-Hermitian photonic device, which is the PT-symmetric dimer, in order to show its relation with the class of photonic lattices with underlying SO(2, 1) symmetry. We brought forward the possibility to use this class beyond the conventional directional oscillators and amplifiers behavior to convert phase (amplitude) modulation into amplitude (phase) modulation.

Acknowledgments

The author acknowledges fruitful discussion with J. Guerrero, R. El-Ganainy and A. J. Stoffel.

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