Electroweak Radiative Corrections, Born Approximation, and Precision Tests of the Standard Model at LEP

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ABSTRACT

We have examined the evidence for the electroweak radiative corrections in the LEP precision data along with the intriguing possibility that the QED corrections only may be sufficient to fit the data. We find that the situation is very sensitive to the precise value of $M_W$. While the world average value of $M_W$ strongly favors nonvanishing electroweak radiative corrections, the QED corrections alone can account for the data within 2σ in the context of the standard model. We discuss how the precision measurements of $M_W$ can provide a decisive test for the standard model with radiative corrections and give a profound implication for the existence of t-quark and Higgs scalar.

1Permanent address and supported in part by the USDOE contract DE-FG02-91ER40688-Task A
Recently much interests have been paid to the electroweak radiative corrections (EWRC) and precision tests of the standard model thanks to the accurate data obtained at LEP [1-7]. There have been numerous articles published on the subject as has been documented in [7,8]. The LEP data are generally regarded as the success of the standard model and as the evidence for the nonvanishing EWRC [9]. However, Novikov, Okun, and Vysotsky [10] have argued recently that the experimental data from LEP on the electroweak parameters as defined in the standard model could be explained by the Born approximation with $\alpha(M_Z^2)$ instead of $\alpha(0)$ and the corresponding redefinition of the weak mixing angle $\sin^2 \theta$ instead of $\sin^2 \theta_W$ and that the genuine EWRC are yet to be observed. In particular the electroweak Born predictions are claimed to be within $1\sigma$ accuracy of all electroweak precision measurements made at LEP. This is very interesting because no Born approximation in any precision test has ever produced such an impressive description of all available data in our memory.

In this paper, we reexamine this claim and test if the present LEP data [1-5] can indeed be accounted for by the QED corrections only of the full one-loop EWRC, i.e., by the electroweak Born approximation (EWBA). In order to do so, we have firstly considered the case of the pure QED corrections by consistently turning off the non-photonic one-loop contributions coming from the weak interaction origin in the full one-loop EWRC and then compared the results with the full one-loop EWRC with the aid of the ZFITTER program [11] but with a few modifications such as using an improved QCD correction factor and making the best $\chi^2$ fit to the data.

Since the basic lagrangian contains the bare electric charge $e_0$, the renormalized physical charge $e$ is fixed by a counter term $\delta e$; $e_0 = e + \delta e$. The counter term $\delta e$ is determined by the condition of the on-shell charge renormalization in the $\overline{MS}$ or on-shell scheme. It is well known that the charge renormalization in the conventional QED fixes the counter term by the renormalized vacuum polarization $\hat{\Pi}^\gamma(0)$ and one can evaluate $\hat{\Pi}^\gamma(q^2) = \hat{\Sigma}^{\gamma\gamma}(q^2)/q^2$ from the photon self energy $\hat{\Sigma}^{\gamma\gamma}(q^2)$, for example, by the dimensional regularization method. This gives at $q^2 = M_Z^2$ the total fermionic contribution of $m_f \leq M_Z$ to the real part $\text{Re}\hat{\Pi}^\gamma(M_Z^2) = -0.0602(9)$, so that the running charge defined as

$$
e^2(q^2) = \frac{e^2}{1 + \text{Re}\hat{\Pi}^\gamma(q^2)}$$

(1)

gives $\alpha(M_Z^2) = 1/128.786$ in the on-shell scheme if the hyperfine structure constant $\alpha = e^2/4\pi = 1/137.0359895(61)$ is used. The concept of the running charge, however, is scheme dependent [12]: the $\overline{MS}$ fine structure constant at the $Z$-mass scale is given by

$$\hat{\alpha}(M_Z) = \alpha/[1 - \Pi^\gamma(0)|_{\overline{MS}} + 2 \tan \theta_W (\Sigma^{\gamma Z}(0)/M_Z^2)_{\overline{MS}}]$$

(2)
In this case, one can show \( \hat{\alpha}(M_Z) = (127.9 \pm 0.1)^{-1} \), which is different by some 0.8 % from the on-shell \( \alpha(M_Z^2) \), as \( \hat{\alpha}(M_Z) \) gets the weak gauge boson contributions also.

Taking just the QED one-loop contributions of the photon self-energy in the full EWRC is equivalent to the EWBA with the effective \( \alpha(M_Z^2) \) instead of \( \alpha \) in the sense of Novikov et al. But unlike our QED case, they substituted \( \alpha(M_Z^2) \) also for \( \alpha \) in the Coulomb correction factor \( R_{QED} \) originating from the real photon emissions from the external states.

The electroweak parameters are evaluated numerically with the hyperfine structure constant \( \alpha \), the four-fermion coupling constant of \( \mu \)-decay, \( G_\mu = 1.16639(2) \times 10^{-5}\text{GeV}^{-2} \), and \( Z \)-mass, \( M_Z = 91.187(7)\text{GeV} \). Numerical estimate of the full EWRC requires the mass values of the leptons, quarks, Higgs scalar and \( W \)-boson besides these quantities. While \( Z \)-mass is known to an incredible accuracy from the LEP experiments largely due to the resonant depolarization method, the situation with the \( W \)-mass is desired to be improved, i.e., \( M_W = 80.22(26) \) GeV [13] vs. the CDF measurement \( M_W = 79.91(39) \) GeV [14] and \( M_W/M_Z = 0.8813(41) \) as determined by UA2 [15]. One has, in the standard model, the on-shell relation

\[
\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2},
\]

while the four-fermion coupling constant \( G_\mu \) can be written as

\[
G_\mu = \frac{\pi \alpha}{\sqrt{2} M_W^2} \left( 1 - \frac{M_W^2}{M_Z^2} \right)^{-1} \left( 1 - \Delta r \right)^{-1}
\]

so that \( \Delta r \), representing the radiative corrections, is given by

\[
\Delta r = 1 - \left( \frac{37.28}{M_W} \right)^2 \frac{1}{1 - M_W^2/M_Z^2}.
\]

We note from Table 1(Cases 1,2) that the radiative correction \( \Delta r \) is very sensitive to the value of \( M_W \). Mere change in \( M_W \) by 0.39% results as much as a 28% change in \( \Delta r \).

On the other hand, the QED contribution to \( \Delta r \) is \( (\Delta r)_{QED} = -\text{Re} \hat{\Pi}^\gamma(M_Z^2) = 0.0602 \). Thus we see from Table 1(Cases 1,2) that the QED portion of \( \Delta r \) is a major component of the radiative corrections, particularly in the case of CDF \( M_W \), for which the QED contribution is already within 3.34% of the needed \( \Delta r \) and is close enough to be within the experimental uncertainty. However, with the current world average value \( M_W = 80.22 \) GeV, the QED corrections leave 34.4% that is to be accounted for by the weak interaction corrections.

Using \( M_Z \) and \( \sin^2 \theta_W \) instead of \( M_W \), \( \Delta r \) can also be expressed as

\[
\Delta r = 1 - \frac{\left( \frac{\alpha}{\sqrt{2} G_\mu} \right)}{M_Z^2 \cos^2 \theta_W \sin^2 \theta_W} = 1 - \frac{0.16714}{\cos^2 \theta_W \sin^2 \theta_W}.
\]
Table 1: Dependence of the radiative correction $\Delta r$ on the values of $M_W$ (Cases 1,2) and of $\sin^2 \theta_W$ (Cases 3,4).

Table 1(Cases 3,4) shows the estimates of $\Delta r$ as well as $M_W$ for two values of the on-shell weak-mixing angle, i.e., $\sin^2 \theta_W = 0.2319$ [16] based on quark charge asymmetry or forward-backward asymmetry measurements at LEP and $\sin^2 \theta_W = 0.2257$ [5] as summarized by LEP collaborations. We see that precise determination of the on-shell value of $\sin^2 \theta_W$ can also constrain the needed radiative correction and the value of $M_W$, thus providing another crucial test for the evidence of the EWRC in the standard model.

We have examined the full EWRC to the nine observables of the Z-decay and $M_W$ as shown in Table 2 and 3. Full details with complete results of theoretical formula of renormalization and full EWRC will be presented elsewhere [17]. These parameters are calculated with a modified ZFITTER program in which the best $\chi^2$ fit to the data is searched and the gluonic coupling constant $\tilde{\alpha}_s(M_Z^2) = 0.123 \pm 0.006$ is used in the improved QCD correction factor [18] $R_{\text{QCD}} = 1 + 1.05 \tilde{\alpha}_s + 0.9(\pm 0.1) \left( \frac{\alpha_s}{\pi} \right)^2 - 13.0 \left( \frac{\alpha_s}{\pi} \right)^3$ for all quarks that can be produced in the $Z \rightarrow f \bar{f}$ decay. The partial width for $Z \rightarrow f \bar{f}$ is given by

$$\Gamma_f = \frac{G_F M_Z^3}{\sqrt{2} 24\pi} \beta R_{\text{QED}} c_f R_{\text{QCD}}(M_Z^2) \left\{ [(\hat{v}_f^2)^2 + (\hat{a}_f^2)^2] \times \left( 1 + 2 \frac{m_f^2}{M_Z^2} \right) - 6 (\hat{a}_f^2)^2 \frac{m_f^2}{M_Z^2} \right\} \quad (7)$$

where $\beta = \beta(s) = \sqrt{1 - 4m_f^2/s}$ at $s = M_Z^2$, $R_{\text{QED}} = 1 + \frac{3}{4} Q_f^2$ and the color factor $c_f = 3$ for quarks and 1 for leptons. Here the renormalized vector and axial-vector couplings are defined by $\hat{a}_f^2 = \sqrt{\rho_f^2 2a_f^2} = \sqrt{\rho_f^2 2I_f^Z}$ and $\hat{v}_f^2 = \tilde{a}_f^2 [1 - 4 |Q_f| \sin^2 \theta_W \kappa_f^Z]$ in terms of the familiar notations [11, 12]. Note that the QED correction $(\Delta r)_{\text{QED}}$ is included in the couplings through $\sin^2 \theta_W$ via (3) and (5) and all other non-photonic loop corrections are grouped in $\rho_f^Z$ and $\kappa_f^Z$ as in [11,17,19]. Thus the case of the QED corrections only, i.e., the EWBA can be achieved simply by setting $\rho_f^Z$ and $\kappa_f^Z$ to 1 in the vector and axial-vector couplings. Numerical results for the best $\chi^2$ fit to the experimental parameters of Z-decay are shown in Tables 2 and 3 for $M_W = 79.91 \pm 0.39$ GeV and $M_W = 80.22 \pm 0.26$ GeV respectively as experimental inputs. They correspond to the values that give the best $\chi^2$
| Parameter | Experiment | Pure QED correc. | Full EW | Full EW | Full EW |
|-----------|------------|-----------------|---------|---------|---------|
| $m_t$ (GeV) | 150 | 120 | 138 | 158 |
| $m_H$ (GeV) | 60 ≤ $m_H$ ≤ 1000 | 60 | 300 | 1000 |
| $M_W$ (GeV) | 79.91 ± 0.39 | 79.94 | 80.10 | 80.10 | 80.13 |
| $\Gamma_Z$ (MeV) | 2488.0 ± 7.0 | 2488.4 | 2489.0 | 2488.9 | 2488.8 |
| $\Gamma_{bb}$ (MeV) | 383.0 ± 6.0 | 379.4 | 377.4 | 376.5 | 375.4 |
| $\Gamma_{ll}$ (MeV) | 83.52 ± 0.28 | 83.47 | 83.53 | 83.53 | 83.63 |
| $\Gamma_{had}$ (MeV) | 1739.9 ± 6.3 | 1740.3 | 1738.8 | 1738.2 | 1737.7 |
| $R(\Gamma_{bb}/\Gamma_{had})$ | 0.220 ± 0.003 | 0.218 | 0.217 | 0.217 | 0.216 |
| $R(\Gamma_{had}/\Gamma_{ll})$ | 20.83 ± 0.06 | 20.85 | 20.82 | 20.81 | 20.78 |
| $\sigma_P^h$ (nb) | 41.45 ± 0.17 | 41.41 | 41.37 | 41.38 | 41.40 |
| $g_V$ | -0.0372 ± 0.0024 | -0.0372 | -0.0341 | -0.0334 | -0.0334 |
| $g_A$ | -0.4999 ± 0.0009 | -0.5000 | -0.5003 | -0.5005 | -0.5006 |
| $\sin^2 \theta_W$ | 0.2321 | 0.2314 | 0.2284 | 0.2283 | 0.2278 |
| $\Delta r$ | 0.0623 | 0.06022 | 0.05162 | 0.05131 | 0.04967 |

Table 2: Numerical results including full EWRC for nine experimental parameters of the $Z$-decay and $M_W$. The case of pure QED corrections only, i.e., EWBA is shown also for comparison. Each pair of $m_t$ and $m_H$ represents the case of the best $\chi^2$ for the given input $m_H$ and experimental $M_W = 79.91 ± 0.39$ GeV.

| Parameter | Experiment | Pure QED correc. | Full EW | Full EW | Full EW |
|-----------|------------|-----------------|---------|---------|---------|
| $m_t$ (GeV) | 150 | 126 | 142 | 160 |
| $m_H$ (GeV) | 60 ≤ $m_H$ ≤ 1000 | 60 | 300 | 1000 |
| $M_W$ (GeV) | 80.22 ± 0.26 | 79.94 | 80.13 | 80.13 | 80.15 |
| $\Gamma_Z$ (MeV) | 2488.0 ± 7.0 | 2488.4 | 2490.2 | 2489.7 | 2489.3 |
| $\Gamma_{bb}$ (MeV) | 383.0 ± 6.0 | 379.4 | 377.3 | 376.4 | 375.3 |
| $\Gamma_{ll}$ (MeV) | 83.52 ± 0.28 | 83.47 | 83.53 | 83.53 | 83.63 |
| $\Gamma_{had}$ (MeV) | 1739.9 ± 6.3 | 1740.3 | 1739.6 | 1738.2 | 1738.1 |
| $R(\Gamma_{bb}/\Gamma_{had})$ | 0.220 ± 0.003 | 0.218 | 0.217 | 0.217 | 0.216 |
| $R(\Gamma_{had}/\Gamma_{ll})$ | 20.83 ± 0.06 | 20.85 | 20.82 | 20.81 | 20.78 |
| $\sigma_P^h$ (nb) | 41.45 ± 0.17 | 41.41 | 41.38 | 41.39 | 41.40 |
| $g_V$ | -0.0372 ± 0.0024 | -0.0372 | -0.0341 | -0.0334 | -0.0334 |
| $g_A$ | -0.4999 ± 0.0009 | -0.5000 | -0.5003 | -0.5005 | -0.5006 |
| $\sin^2 \theta_W$ | 0.2261 | 0.2314 | 0.2284 | 0.2279 | 0.2278 |
| $\Delta r$ | 0.0448 | 0.06022 | 0.04975 | 0.04991 | 0.04985 |

Table 3: The same as Table 2 but for the experimental $M_W = 80.22 ± 0.26$ GeV.
fit in each case. Also included in the Tables 2 and 3 is the case of pure QED corrections, i.e., the EWBA, as well as the output $\sin^2 \theta_W$ and $\Delta r$, for comparison. We see that the contributions of the weak corrections are generally small and the QED portion of the radiative corrections seems to be close to the experimental values within the uncertainty of the current measurements. The smallness of the weak corrections is achieved by a precarious compensation of two large contributions i.e., between those of t-quark and Higgs scalar. In general, the radiative correction parameter $\Delta r$ can be written as

$$\Delta r = (\Delta r)_{\text{QED}} - \frac{c_W^2}{s_W^2} \Delta \rho + \Delta r_{\text{rem}}$$

where $c_W^2$ and $s_W^2$ denote $\cos^2 \theta_W$ and $\sin^2 \theta_W$. Main contribution to $\Delta \rho = \rho - 1$ is from the heavy t-quark through the mass renormalizations of weak gauge bosons $W$ and $Z$, while there is a part in $(\Delta r)_{\text{rem}}$ containing also the t-quark and Higgs scalar contributions. The near absence of the weak interaction contributions to the radiative corrections is more impressive for $M_W = 79.91$ GeV than for $M_W = 80.22$ GeV. At closer examination, however, the EWBA in the latter case over-estimates the radiative corrections and the full one-loop EWRC fair better.

In order to complete the analysis of the global fit to the data, $m_H$ is allowed to vary in the range $60-1000$ GeV. We find the best fit to the data is obtained by $m_t = 142^{+16}_{-14}$ GeV, the central value being the best $\chi^2$ case of $m_H = 300$ GeV in Table 3. The upper and lower bounds on $m_t$ as a function of $m_H$ are shown in Fig. 1 and Fig. 2 for the two experimental $M_W$ at several confidence levels (CL). For example, in the case of experimental $M_W = 80.22$ GeV, the upper bound on $m_t$ is 198 (181) GeV at 95% (68%) CL, while the lower bound on $m_t$ is 112 (135) GeV [20] at 95(68)% CL that come from $m_H = 1000$ GeV.

Note from Tables 2 and 3 that the best global fits to the data give a rather stable output $M_W = 80.13 \pm 0.03$ GeV if the full EWRC are taken into account, which is to be contrasted to the output $M_W = 79.94$ GeV for the EWBA, for either experimental $M_W$ value. Also $\sin^2 \theta_W = 0.2279 \pm 0.0005$ in the case of the full EWRC is to be compared to $\sin^2 \theta_W = 0.2314$ in the case of QED corrections only. Clearly one needs better precisions on the measurements of $M_W$. While the current world average value of $M_W$ supports strongly for the evidence of the full EWRC in the LEP data, the Born approximation appears to be in fair agreement, i.e., within $2\sigma$, with the data at the present precisions. In order to dismiss the Born approximation, an improvement of better than 100 MeV in the error of $M_W$ over the current data is desired. If $M_W$ turns out to be definitely at around 79.94 GeV with such precision, then the QED correction is all that has been observed at LEP and one is cultivating the null result of the EWRC to produce the range of t-quark mass as pointed out in [10]. However, if $M_W$ is definitely around 80.13 GeV.
with the desired precision, the existing LEP data are the evidence for the nonvanishing electroweak radiative corrections.

We have examined the results of the best $\chi^2$ fit to the precision measurements of the Z-decay parameters at LEP and $M_W$ in the standard model with the full EWRC as well as those of the EWBA, i.e., the QED corrections only with the aid of a modified ZFITTER program. We find that the Born approximation is in agreement with the data within $2\sigma$ level of accuracy at the present state of precision while the world average value of $M_W$ clearly supports for the evidence of the nonvanishing EWRC in the LEP data. Further precision measurement of $M_W$ can provide a real test of the standard model as it will give a tight constraint for the needed amount of the EWRC providing a profound implication to the mass of t-quark and Higgs scalar in the context of the standard model. As long as t-quark remains unobserved, either amount of the radiative corrections, i.e., $\Delta r \simeq 0.05$ with the full EWRC and $\Delta r = \Delta \alpha \simeq 0.06$ with the QED corrections only, can fit the data more or less equally well. But if $M_W$ is determined within 100 MeV uncertainty, $\Delta r$ within the context of the standard model will be tightly constrained to distinguish the evidence for the radiative corrections that can discriminate the mass range of the t-quark and Higgs scalar, thus providing a crucial test for and even the need of new physics beyond the standard model.

Acknowledgements

One of us (KK) would like to thank the Center for Theoretical Physics, Seoul National University (CTPSNU) and the Korea Advanced Institute of Science and Technology (KAIST) where parts of the work were done and LPTPE, Université P.&M. Curie where the work was completed for the kind hospitality during his sabbatical stay. Also the authors would like to thank Professors Hi-sung Song, Jae Kwan Kim, R. Vinh Mau and other colleagues at CTPSNU, KAIST, and LPTPE for the stimulating environment and supports and in particular Professor M. Lacombe for checking the numerical computations.

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**Figure Captions**

**Fig. 1**: The mass ranges of $m_t$ and $m_H$ at several Confidence Levels for $M_W = 79.91$ GeV.

**Fig. 2**: The same as that of Fig. 1 but for $M_W = 80.22$ GeV.
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