e-open Sets in $N_{nc}$-Topological Spaces

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Abstract. As a generalization of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been created by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth value, an indeterminacy value and a falsity value. In this paper, we introduce and study a new class of $N_{ne}$ neutrosophic closed set, namely $N_{ne}$ neutrosophic e-closed and $N_{ne}$ neutrosophic e-open sets in neutrosophic topological spaces. Also we study $N_{ne}$ neutrosophic e-interior, $N_{ne}$ neutrosophic e-closure and their properties are discussed.

Keywords and phrases: $N_{ne}$-open sets, $N_{ne}$-closed sets, $N_{ne}$-interior of $H$ and $N_{ne}$-closure of $H$.

1. Introduction

Smarandache’s neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, dynamic, Medicine, Electrical & Electronic, and Management Science and so forth [1, 2, 3, 4, 17, 18]. Topology is an classical subject, as a generalization topological spaces numerous kinds of topological spaces presented throughout the year. Smarandache [13] characterized the Neutrosophic set on three segment Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (nts’s) presented by Salama and Alblowi [10]. Lellies Thivagar et.al. [8] was given the geometric existence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Lellis Thivagar et al. [9] introduced the notion of $N_{n}$-open (closed) sets in $N$ neutrosophic crisp topological spaces. Al-Hamido et al. [5] investigate the chance of extending the idea of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2008, Ekici [6] introduced the notion of e-open sets in topology. In 2020, Vadiel and John Sundar [16] introduced $N$-neutrosophic $\delta$-open, $N$-neutrosophic $\delta$-semiopen and $N$-neutrosophic $\delta$-preopen sets are introduced. In this paper, the notion of $N$-neutrosophic e-open set which is generalization of $N$-neutrosophic $\delta$-semiopen sets and $N$-neutrosophic $\delta$-preopen sets is introduced. Properties and the relationships of $N$-neutrosophic e-open sets and investigated.

2. Preliminaries

Salama and Smarandache [12] presented the idea of a neutrosophic crisp set in a set $X$ and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two
neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty
(resp., whole) set as more than two types. And they studied some properties related to
mutrosophic crisp set operations. However, by selecting only one type, we define the inclusion,
the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a
few properties.

**Definition 2.1** Let $X$ be a non-empty set. Then $H$ is called a neutrosophic crisp set (in short,
ncs) in $X$ if $H$ has the form $H = (H_1, H_2, H_3)$, where $H_1, H_2$, and $H_3$ are subsets of $X$.

The neutrosophic crisp empty (resp., whole) set, denoted by $\phi_n$ (resp., $X_n$) is an ncs in $X$
defined by $\phi_n = (\phi, \phi, X)$ (resp. $X_n = (X, X, \phi)$). We will denote the set of all ncs’s in $X$ as
$nS(X)$.

In particular, Salama and Smarandache [11] classified a neutrosophic crisp set as the
followings.

A neutrosophic crisp set $H = (H_1, H_2, H_3)$ in $X$ is called a neutrosophic crisp set of Type 1
(resp. 2 & 3) (in short, ncs-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$
(resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = X$). $ncS_1(X)$ ($ncS_2(X)$ and
$ncS_3(X)$) means set of all ncs Type 1 (resp. 2 and 3).

**Definition 2.2** Let $H = (H_1, H_2, H_3)$, $M = (M_1, M_2, M_3) \in nS(X)$. Then $H$ is said to be
contained in (resp. equal to) $M$, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1, H_2 \subseteq M_2$ and
$H_3 \subseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$); $H^c = (H_3, H_2, H_1)$; $H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3)$;
$H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cup M_3)$. Let $(A_j)_{j \in J} \subseteq nS(X)$, where $H_j = (H_{j_1}, H_{j_2}, H_{j_3})$. Then
$\bigcap_{j \in J} H_j$ (simply $\bigcap H_j$) $= (\bigcap H_{j_1}, \bigcap H_{j_2}, \bigcup H_{j_3})$; $\bigcup_{j \in J} H_j$ (simply $\bigcup H_j$) $= (\bigcup H_{j_1}, \bigcup H_{j_2}, \bigcap H_{j_3})$.

The following are the quick consequence of Definition 2.2.

**Proposition 2.1** [7] Let $L, M, O \in nS(X)$. Then
(i) $\phi_n \subseteq L \subseteq X_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq M$ if $L \cap M = L$,
(vi) $L \subseteq M$ if $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.

**Proposition 2.2** [7] Let $L, M, O \in nS(X)$. Then
(i) $L \cup L = L$, $L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L$, $L \cap M = M \cap L$ (Commutative laws),
(iii) (Associative laws) : $L \cup (M \cup O) = (L \cup M) \cup O$, $L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws:) $L \cup (M \cap O) = (L \cup M) \cap (L \cup O)$, $L \cap (M \cup O) = (L \cap M) \cup (L \cup O)$,
(v) (Absorption laws) : $L \cup (L \cap M) = L$, $L \cap (L \cup M) = L$,
(vi) (DeMorgan’s laws) : $(L \cup M)^c = L^c \cap M^c$, $(L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L$, $L \cap \phi_n = \phi_n$,
(b) $L \cup X_n = X_n$, $L \cap X_n = L$,
(c) $X_n^c = \phi$, $\phi_n^c = X_n$,
(d) in general, $L \cup L^c \neq X_n$, $L \cap L^c \neq \phi_n$. 
**Proposition 2.3** [7] Let \( L \in ncs(X) \) and let \((L_j)_{j \in J} \subseteq ncs(X)\). Then

(i) \((\bigcap L_j)^c = \bigcup L_j^c, (\bigcup L_j)^c = \bigcap L_j^c\),
(ii) \( L \cap (\bigcup L_j) = \bigcup (L \cap L_j), L \cup (\bigcap L_j) = \bigcap (L \cup L_j)\).

**Definition 2.3** [11] A neutrosophic crisp topology (briefly, *ncots*) on a non-empty set \( X \) is a family \( \tau \) of \( ncs \) subsets of \( X \) satisfying the following axioms

(i) \( \phi, X_n \in \tau \).
(ii) \( H_1 \cap H_2 \in \tau \forall H_1, H_2 \in \tau \).
(iii) \( \bigcup_a H_a \in \tau \), for any \( \{H_a : a \in J\} \subseteq \tau \).

Then \((X, \tau)\) is a neutrosophic crisp topological space (briefly, \( *ncts* \)) in \( X \). The \( \tau \) elements are called neutrosophic crisp open sets (briefly, \( ncos \)) in \( X \). A \( ncs \) \( C \) is closed set (briefly, \( nccs \)) iff its complement \( C^c \) is \( ncos \).

**Definition 2.4** [5] Let \( X \) be a non-empty set. Then \( nc\tau_1, nc\tau_2, \ldots, nc\tau_N \) are \( N\)-arbitrary crisp topologies defined on \( X \) and the collection \( N_{nc}\tau = \{A \subseteq X : A = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j), H_j, L_j \in ncs \}\) is called \( ncs\) topology on \( X \) if the axioms are satisfied:

(i) \( \phi, X_n \in N_{nc}\tau \).
(ii) \( \bigcup_{j=1}^{N} A_j \in N_{nc}\tau \forall \{A_j\}_{j=1}^{N} \subseteq N_{nc}\tau \).
(iii) \( \bigcap_{j=1}^{N} A_j \in N_{nc}\tau \forall \{A_j\}_{j=1}^{N} \subseteq N_{nc}\tau \).

Then \((X, N_{nc}\tau)\) is called a \( ncs\)-topological space (briefly, \( ncots \)) on \( X \). The \( ncs\tau \) elements are called \( ncs\)-open sets (\( ncos \)) on \( X \) and its complement is called \( ncs\)-closed sets (\( nccs \)) on \( X \). The elements of \( X \) are known as \( ncs\)-sets (\( ncs \)) on \( X \).

**Definition 2.5** [5] Let \( (X, N_{nc}\tau) \) be \( ncots \) on \( X \) and \( H \) be an \( ncs \) on \( X \), then the \( ncs \) interior of \( H \) (briefly, \( ncsi(H) \)) and \( ncs \) closure of \( H \) (briefly, \( nccl(H) \)) are defined as

(i) \( ncsi(H) = \bigcup\{A : A \subseteq H \& A is a ncs \text{ in } X\} \) \& \( nccl(H) = \bigcap\{C : H \subseteq C \& C is a ncs \text{ in } X\} \).
(ii) \( ncs\)-regular open \([14]\) set (briefly, \( ncsos \)) if \( H = ncsi(nccl(H)) \).
(iii) \( ncs\)-pre open set (briefly, \( ncsos \)) if \( H \subseteq ncsi(nccl(H)) \).
(iv) \( ncs\)-semi open set (briefly, \( ncsos \)) if \( H \subseteq nccl(ncsi(H)) \).
(v) \( ncs\)-\( \alpha \)-open set (briefly, \( ncsos \)) if \( H \subseteq nccl(ncsi(nccl(H))) \).
(vi) \( ncs\)-\( \gamma \)-open set \([14]\) (briefly, \( ncsos \)) if \( H \subseteq nccl(ncsi(H)) \cup ncci(nccl(H)) \).
(vii) \( ncs\)-\( \beta \)-open set \([15]\) (briefly, \( ncsos \)) if \( H \subseteq nccl(ncci(nccl(H))) \).

The complement of an \( ncsos \) (resp. \( ncsos, ncspos, ncsos, ncs\os, \& ncs\os) \) is called an \( ncs\)-regular (resp. \( ncs\)-semi, \( ncs\)-pre, \( ncs\)-\( \alpha \), \( ncs\)-\( \beta \) \& \( ncs\)-\( \gamma \)) closed set (briefly, \( ncsos \) (resp. \( ncsos, ncspos, ncsos, ncsos, ncsos \& ncsos) \)) in \( X \).

The family of all \( ncsos \) (resp. \( ncsos, ncsos, ncsos, ncsos, ncsos \& ncsos) \) of \( X \) is denoted by \( ncsos(X) \) (resp. \( ncsos(X), ncsos(X), ncsos(X), ncsos(X), ncsos(X), ncsos(X) \& ncsos(X) \)).

**Definition 2.6** [16] A set \( H \) is said to be a
(i) $N_{nc}\delta$ interior of $H$ (briefly, $N_{nc}\delta int(H)$) is defined by $N_{nc}\delta int(H) = \cup \{A : A \subseteq H \& A$ is a $N_{nc}os\}$. 

(ii) $N_{nc}\delta$ closure of $H$ (briefly, $N_{nc}\delta cl(H)$) is defined by $N_{nc}\delta cl(H) = \cup \{x \in X : N_{nc}int(N_{nc}cl(L)) \cap H \neq \emptyset, x \in L \& L$ is a $N_{nc}os\}$. 

**Definition 2.7** [16] A set $H$ is said to be a 

(i) $N_{nc}\delta$- open set (briefly, $N_{nc}\delta os$) if $H = N_{nc}\delta int(H)$. 

(ii) $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta Pos$) if $H \subseteq N_{nc}int(N_{nc}\delta cl(H))$. 

(iii) $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta Sos$) if $H \subseteq N_{nc}cl(N_{nc}\delta int(H))$. 

The complement of an $N_{nc}\delta os$ (resp. $N_{nc}\delta Pos$ & $N_{nc}\delta Sos$) is called an $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre & $N_{nc}\delta$-semi) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta Pcs$ & $N_{nc}\delta Scs$)) in $Y$.

The family of all $N_{nc}\delta os$ (resp. $N_{nc}\delta cs$, $N_{nc}\delta Pos$, $N_{nc}\delta Pcs$, $N_{nc}\delta Sos$ & $N_{nc}\delta Scs$) of $X$ is denoted by $N_{nc}OS(X)$ (resp. $N_{nc}CS(X)$, $N_{nc}POS(X)$, $N_{nc}PCS(X)$, $N_{nc}SOS(X)$ & $N_{nc}SCS(X)$).

3. $e$-open sets in $N_{nc}$-topological spaces

In this segment, some fundamental definitions & properties of $N$-neutrosophic crisp topological spaces are talked about.

**Definition 3.1** Let $H$ be an $N_{nc}s$ on a $N_{nc}ts$ $X$. Then $H$ is said to be a 

(i) $N_{nc}\alpha$-open (briefly, $N_{nc}\alpha os$) set if $H \subseteq N_{nc}cl(N_{nc}\alpha int(H)) \cup N_{nc}int(N_{nc}\alpha cl(H))$. 

(ii) $N_{nc}\alpha$-closed (briefly, $N_{nc}\alpha cs$) set if $N_{nc}cl(N_{nc}\alpha int(H)) \cap N_{nc}int(N_{nc}\alpha cl(H)) \subseteq H$. 

The complement of an $N_{nc}\alpha os$ set is called an $N_{nc}\alpha$ closed (briefly, $N_{nc}\alpha cs$) set in $X$. The family of all $N_{nc}\alpha os$ (resp. $N_{nc}\alpha cs$) set of $X$ is denoted by $N_{nc}\alpha OS(X)$ (resp. $N_{nc}\alpha CS(X)$). The $N_{nc} e$-interior of $H$ (briefly, $N_{nc}e int(H)$) and $N_{nc} e$-closure of $H$ (briefly, $N_{nc}e cl(H)$) are defined as $N_{nc}e int(H) = \cup \{G : G \subseteq H$ and $G$ is a $N_{nc}\alpha$ set in $X\} \& N_{nc}e cl(H) = \cap \{F : H \subseteq F$ and $F$ is a $N_{nc}\alpha$ set in $X\}$. 

**Proposition 3.1** Let $(X, N_{nc}\tau)$ be a $N_{nc}ts$ on $X$. Then the statement hold but the converse need not be true.

(i) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}\alpha os$ (resp. $N_{nc}\alpha cs$).

(ii) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}Pos$ (resp. $N_{nc}Pcs$).

(iii) Every $N_{nc}Pos$ (resp. $N_{nc}Pcs$) is a $N_{nc}\gamma os$ (resp. $N_{nc}\gamma cs$).

(iv) Every $N_{nc}\gamma os$ (resp. $N_{nc}\gamma cs$) is a $N_{nc}\beta os$ (resp. $N_{nc}\beta cs$).

(v) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

(vi) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

(vii) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

(viii) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

(ix) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

(x) Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).

**Proof.** Proof of with examples (i) to (iii), (iv) and (v) to (vi) are proved in [14], [15] and [16]. We prove only (vii) to (x).

(vii) Suppose that $H$ is a $N_{nc}Sos$, then $H \subseteq N_{nc}cl(N_{nc}\delta int(H)) \subseteq N_{nc}cl(N_{nc}\delta int(H)) \cup N_{nc}int(N_{nc}\delta cl(H)).$ Hence $H$ is a $N_{nc}os$. 

4
Lemma 3.1

(iii) Let $H$ be an $N_{nc} Po$. Then $H \subseteq N_{nc}int(N_{nc}cl(H))$ and so $H \subseteq N_{nc}int(N_{nc}cl(H)) \subseteq N_{nc}int(N_{nc}dcl(H))$. Hence $H$ is a $N_{nc}d Po$.

(iv) Let $H$ be an $N_{nc}d Pos$. Then $H \subseteq N_{nc}int(N_{nc}dcl(H)) \subseteq N_{nc}cl(N_{nc}d int(H)) \cup N_{nc}int(N_{nc}dcl(H))$. Hence $H$ is a $N_{nc}e cos$.

(x) It is similar to (iv).

It is also true for their respective closed sets.

Remark 3.1

The following diagram holds for a $N_{nc}H$ of a $N_{nc}ts X$.

\[
\begin{array}{ccc}
N_{nc}os & \rightarrow & N_{nc}e os \\
\downarrow & & \downarrow \\
N_{nc}d pos & \rightarrow & N_{nc}\gamma os \\
\end{array}
\]

\[
\begin{array}{ccc}
N_{nc}os & \rightarrow & N_{nc}dS os \\
\downarrow & & \downarrow \\
N_{nc}e os & \rightarrow & N_{nc}dS os \\
\end{array}
\]

None of these implication is reversible as shown in the following examples.

Example 3.1

Let $X = \{e, d, c, b, a\}$, $\{\phi, X_n, A, B, C\}$, $\{a, b, d, e\}$, $\{a, b, c, b, d, e\}$, $\{a, b, c, d, e\}$, $\{a, b, c, d\}$, $\{a, b, d\}$, $\{a, b, e\}$, $\{a, b, c\}$, $\{a, b, c, d\}$, $\{a, b, c, d, e\}$, $\{a, b, c, d, e\}$. Then,

(i) $\{\{a\}, \{\phi\}, \{b, c, d, e\}\}$ is a $2_{nc}e os$ but not $2_{nc}dS os$.

(ii) $\{\{a\}, \{\phi\}, \{a, b, c, d, e\}\}$ is a $2_{nc}e os$ but not $2_{nc}d Pos$.

(iii) $\{\{a\}, \{\phi\}, \{b, c, d, e\}\}$ is a $2_{nc}dS os$ but not $2_{nc}e os$.

Lemma 3.1

Let $H$ be an $N_{nc}os$ on a $N_{nc}ts X$. Then the following are hold.

(i) $N_{nc}dP cl(H) = H \cup N_{nc}cl(N_{nc}d int(H))$ and $N_{nc}dP int(H) = H \cap N_{nc}int(N_{nc}d cl(H))$.

(ii) $N_{nc}d P cl(N_{nc}d P int(H)) = N_{nc}d P int(H) \cup N_{nc}cl(N_{nc}d int(H))$ and $N_{nc}d P int(N_{nc}d P cl(H)) = N_{nc}d P cl(H) \cap N_{nc}int(N_{nc}d cl(H))$.

(iii) $N_{nc}d S int(H) = H \cap N_{nc}cl(N_{nc}d int(H))$ and $N_{nc}d S cl(H) = H \cup N_{nc}int(N_{nc}d cl(H))$.

(iv) $N_{nc}d int(N_{nc}d S cl(H)) = N_{nc}int(N_{nc}d cl(H))$ and $N_{nc}d int(N_{nc}d P cl(H)) = N_{nc}int(N_{nc}d cl(N_{nc}d int(H)))$.

(v) $N_{nc}d P cl(N_{nc}d S int(H)) = N_{nc}cl(N_{nc}d int(H))$ and $N_{nc}d S cl(N_{nc}d P int(H)) = N_{nc}int(N_{nc}d cl(H))$.

(vi) $N_{nc}d S cl(N_{nc}d S int(H)) = N_{nc}d S int(H) \cup N_{nc}int(N_{nc}d cl(N_{nc}d int(H)))$.

Theorem 3.1

Let $H$ be an $N_{nc}os$ on a $N_{nc}ts X$. Then $H$ is $N_{nc}e$ iff $H = N_{nc}d P int(H) \cup N_{nc}d S int(H)$.

Proof. Let $H$ be $N_{nc}e$. Then $H \subseteq N_{nc}cl(N_{nc}d int(H)) \cup N_{nc}int(N_{nc}d cl(H))$. By Lemma 3.1, we have

\[
N_{nc}d P int(H) \cup N_{nc}d S int(H) = (H \cap N_{nc}int(N_{nc}d cl(H))) \cup (H \cap N_{nc}cl(N_{nc}d int(H)))
\]

\[
= H \cap (N_{nc}int(N_{nc}d cl(H)) \cup N_{nc}cl(N_{nc}d int(H)))
\]

\[
= H.
\]

Conversely, suppose $H = N_{nc}d P int(H) \cup N_{nc}d S int(H)$. By Lemma 3.1, we have

\[
H = N_{nc}d P int(H) \cup N_{nc}d S int(H)
\]

\[
= (H \cap N_{nc}int(N_{nc}d cl(H))) \cup (H \cap N_{nc}cl(N_{nc}d int(H)))
\]

\[
\subseteq N_{nc}int(N_{nc}d cl(H)) \cup N_{nc}cl(N_{nc}d int(H)).
\]

Thus $H$ is $N_{nc}e$ open.
Theorem 3.2 Let $H$ be an $N_{nc}s$ on a $N_{nc}ts$ $X$. Then $N_{nc}\delta cl(H) = N_{nc}\delta Pcl(H) \cap N_{nc}\delta Sc(H)$.

Proof. It is obvious that we have always $N_{nc}\delta cl(H) \subseteq N_{nc}\delta Pcl(H) \cap N_{nc}\delta Sc(H)$.

Conversely, we have

$$N_{nc}\delta cl(H) \supseteq N_{nc}\delta cl(N_{nc}\delta int(N_{nc}\delta cl(H))) \cap N_{nc}int(N_{nc}\delta cl(N_{nc}\delta cl(H)))$$

$$\supseteq N_{nc}\delta cl(N_{nc}\delta int(H)) \cap N_{nc}\delta int(\delta cl(H))$$.

Since $N_{nc}\delta cl(H)$ is $N_{nc}ec$. Hence by Lemma 3.1,

$$N_{nc}\delta Pcl(H) \cap N_{nc}\delta Sc(H) = (H \cup N_{nc}\delta cl(N_{nc}\delta int(H))) \cap (H \cup N_{nc}int(N_{nc}\delta cl(H)))$$

$$\subseteq N_{nc}\delta cl(H)$$.

Theorem 3.3 Let $H$ be an $N_{nc}s$ on a $N_{nc}ts$ $X$. Then $N_{nc}int(H) = N_{nc}\delta Pint(H) \cup N_{nc}\delta Sint(H)$.

Proof. This follows from Theorem 3.2.

Theorem 3.4 Let $H$ be an $N_{nc}s$ on a $N_{nc}ts$ $X$. Then hold.

(i) $H$ is $N_{nc}\delta Po$ iff $H \subseteq N_{nc}\delta Pint(N_{nc}\delta Pcl(H))$

(ii) $H$ is $N_{nc}eo$ iff $H \subseteq N_{nc}\delta Pcl(N_{nc}\delta Pint(H))$.

Proof. (i) Let $H$ be $N_{nc}\delta Po$. Then $N_{nc}\delta Pint(H) = H$ and also $H \subseteq N_{nc}\delta Pint(N_{nc}\delta Pcl(H))$. Conversely, let $H \subseteq N_{nc}\delta Pint(N_{nc}\delta Pcl(H))$. By Lemma 3.1, we have

$$H \subseteq N_{nc}\delta Pint(N_{nc}\delta Pcl(H))$$

$$\subseteq N_{nc}\delta Pint(N_{nc}\delta cl(H))$$

$$= N_{nc}\delta cl(H) \cap N_{nc}int(N_{nc}\delta cl(H))$$

$$= N_{nc}int(N_{nc}\delta cl(H))$$.

Hence, $H$ is $N_{nc}\delta Po$.

(ii) Let $H$ be $N_{nc}eo$. Then $H \subseteq N_{nc}cl(N_{nc}\delta int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))$. By Lemma 3.1, we have

$$H \subseteq (N_{nc}cl(N_{nc}\delta int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))) \cap H$$

$$= (N_{nc}cl(N_{nc}\delta int(H)) \cap H) \cup (N_{nc}int(N_{nc}\delta cl(H)) \cap H)$$

$$\subseteq N_{nc}\delta Pcl(H) \cup N_{nc}cl(N_{nc}\delta int(H))$$

$$= N_{nc}\delta Pcl(N_{nc}\delta Pint(H))$$.

Conversely, suppose $H \subseteq N_{nc}\delta Pcl(N_{nc}\delta Pint(H))$. By Lemma 3.1, we have

$$H \subseteq N_{nc}\delta Pcl(N_{nc}\delta Pint(H))$$

$$= N_{nc}\delta Pint(H) \cup N_{nc}cl(N_{nc}\delta int(H))$$

$$= (H \cap N_{nc}int(N_{nc}\delta cl(H))) \cup N_{nc}cl(N_{nc}\delta int(H))$$

$$\subseteq N_{nc}int(N_{nc}\delta cl(H)) \cup N_{nc}cl(N_{nc}\delta int(H))$$.

Hence, $H$ is $N_{nc}eo$.

Lemma 3.2 Let $H$ be an $N_{nc}s$ on a $N_{nc}ts$ $X$. Then the following statements are hold

(i) $N_{nc}cl(N_{nc}\delta int(H)) = N_{nc}\delta cl(N_{nc}\delta int(H))$, 


(ii) \(N_{nc\,\text{int}}(N_{nc\,\delta}(H)) = N_{nc\,\delta}(N_{nc\,\text{int}}(H))\).

**Theorem 3.5** Let \(H\) be an \(N_{ncs}\) on a \(N_{ncs}\)ts \(X\). Then the following statements are hold

(i) \(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)))\),

(ii) \(N_{nc\,\delta\text{int}}(N_{nc\,\text{cl}}(H)) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)))\),

(iii) \(N_{nc\,\text{int}}(N_{nc\,\delta\text{cl}}(H)) = N_{nc\,\delta}(N_{nc\,\text{int}}(H)) = N_{nc\,\delta}(N_{nc\,\text{int}}(N_{nc\,\delta}(H)))\),

(iv) \(N_{nc\,\text{cl}}(N_{nc\,\delta\text{Sint}}(H)) = N_{nc\,\delta\text{Scl}}(N_{nc\,\delta\text{Sint}}(H))\),

(v) \(N_{nc\,\delta\text{Pint}}(N_{nc\,\text{cl}}(H)) = N_{nc\,\delta\text{Pcl}}(N_{nc\,\delta}(H))\),

(vi) \(N_{nc\,\delta\text{Sint}}(N_{nc\,\text{cl}}(H)) = N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)) \cap N_{nc\,\delta\text{Scl}}(H)\),

(vii) \(N_{nc\,\text{int}}(N_{nc\,\delta\text{Scl}}(H)) = N_{nc\,\delta\text{Sint}}(N_{nc\,\delta\text{Scl}}(H))\),

(viii) \(N_{nc\,\delta\text{Pcl}}(N_{nc\,\text{int}}(H)) = N_{nc\,\text{Pcl}}(N_{nc\,\delta})\),

(ix) \(N_{nc\,\delta\text{Scl}}(N_{nc\,\text{int}}(H)) = N_{nc\,\text{int}}(N_{nc\,\delta\text{cl}}(H)) \cup N_{nc\,\delta\text{Sint}}(H)\).

**Proof.** (i) By Lemma 3.1 and Theorem 3.2
\[
N_{nc\,\text{cl}}(N_{nc\,\text{int}}(H)) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))).
\]

(ii) By Lemma 3.1 and Theorem 3.2, we obtain
\[
N_{nc\,\delta\text{int}}(N_{nc\,\text{cl}}(H)) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\text{int}}(N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H))).
\]

(iii) Follows from (i) and (ii).

(iv) By Lemma 3.1 and Theorem 3.2,
\[
N_{nc\,\text{cl}}(N_{nc\,\delta\text{Sint}}(H)) = N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)) \cap N_{nc\,\delta\text{Scl}}(N_{nc\,\delta\text{Sint}}(H)) = N_{nc\,\text{cl}}(N_{nc\,\delta\text{int}}(H)) \cap N_{nc\,\delta\text{Sint}}(H) \cup N_{nc\,\text{int}}(N_{nc\,\delta\text{Scl}}(N_{nc\,\delta\text{int}}(H))) = N_{nc\,\delta\text{Scl}}(N_{nc\,\delta\text{Sint}}(H)).
\]

(v) By Theorem 3.2, we always have, \(N_{nc\,\delta\text{Pint}}(N_{nc\,\text{cl}}(H)) \subseteq N_{nc\,\delta\text{Pcl}}(N_{nc\,\delta}(H))\). Conversely, by Lemma 3.1 and Theorem 3.2, we obtain
\[
N_{nc\,\delta\text{Pint}}(N_{nc\,\text{cl}}(H)) = N_{nc\,\delta\text{Pcl}}(N_{nc\,\delta\text{Scl}}(H)) \cap N_{nc\,\delta\text{Scl}}(H) = N_{nc\,\text{cl}}(N_{nc\,\delta\text{Pcl}}(H)) \cap N_{nc\,\delta\text{Scl}}(H) = N_{nc\,\delta\text{Pcl}}(H) \cap N_{nc\,\text{int}}(N_{nc\,\delta\text{cl}}(H)) \cap N_{nc\,\delta\text{Scl}}(H).
\]
Thus $N_{nc}\delta P\text{int}(N_{nc}\delta cl(H)) = N_{nc}\delta P\text{int}(N_{nc}\delta cl(H))$.

(vi) Let $H$ be a $N_{nc}$s on $X$. By Theorem 3.2 and Lemma 3.1,

$$N_{nc}\delta S\text{int}(N_{nc}\delta cl(H)) \subseteq N_{nc}\delta S\text{int}(N_{nc}\delta cl(H)) = N_{nc}cl(N_{nc}\delta int(H)).$$

and

$$N_{nc}\delta S\text{int}(N_{nc}\delta cl(H)) \subseteq N_{nc}\delta S\text{int}(N_{nc}\delta S\text{cl}(H)) \subseteq N_{nc}\delta S\text{cl}(H).$$

Thus, $N_{nc}\delta S\text{int}(N_{nc}\delta cl(H)) \subseteq N_{nc}\delta S\text{cl}(H) \cap N_{nc}cl(\delta int(H))$. Conversely by Lemma 3.1 and Theorem 3.2,

$$N_{nc}\delta S\text{int}(N_{nc}\delta cl(H)) = N_{nc}\delta S\text{cl}(H) \cap N_{nc}cl(\delta int(H)).$$

(vii), (viii) and (ix) follow from (iv), (v) and (vi) respectively.

**Proposition 3.2** Let $(X, N_{nc})$ be a $N_{nc}$s. $H$ and $M$ are any two $N_{nc}$s’s in $(X, N_{nc})$. Then the $N_{nc}$-closure and $N_{nc}$-interior operator satisfies the following properties:

(i) $H \subseteq N_{nc}cl(H)$.

(ii) $N_{nc}int(H) \subseteq H$.

(iii) $H \subseteq M \Rightarrow N_{nc}cl(H) \subseteq N_{nc}cl(M)$.

(iv) $H \subseteq M \Rightarrow N_{nc}int(H) \subseteq N_{nc}int(M)$.

(v) $N_{nc}cl(H \cup M) = N_{nc}cl(H) \cup N_{nc}cl(M)$.

(vi) $N_{nc}int(H \cap M) = N_{nc}int(H) \cap N_{nc}int(M)$.

(vii) $(N_{nc}cl(H))^c = N_{nc}int(H^c)$.

(viii) $(N_{nc}int(H))^c = N_{nc}cl(H^c)$.

**Proof.** (i) $N_{nc}cl(H) = \cap \{F : H \subseteq F \& F$ is a $N_{nc}$ set in $X\}$. Thus, $H \subseteq N_{nc}cl(H)$.

(ii) $N_{nc}int(H) = \cup \{G : G \subseteq H \& G$ is a $N_{nc}$ set in $X\}$. Thus, $N_{nc}int(H) \subseteq H$.

(iii)

$$N_{nc}cl(M) = \cap \{F : M \subseteq F \& F$ is a $N_{nc}$ set in $X\} \supseteq \cap \{F : H \subseteq F \& F$ is a $N_{nc}$ set in $X\} \supseteq N_{nc}cl(H).$$

Thus, $N_{nc}cl(H) \subseteq N_{nc}cl(M)$.

(iv)

$$N_{nc}int(M) = \cup \{G : G \subseteq M \& G$ is a $N_{nc}$ set in $X\} \supseteq \cup \{G : G \subseteq H \& G$ is a $N_{nc}$ set in $X\} \supseteq N_{nc}int(H).$$
Thus, $N_{nc}int(H) \subseteq N_{nc}int(H)$.

(v) $N_{nc}ecl(H \cup M)$

$$= \cap \{F : H \cup M \subseteq F \cap F \text{ is a } N_{nc}ec \text{ set in } X\}$$
$$= (\cap \{F : H \subseteq F \cap F \text{ is a } N_{nc}ec \text{ set in } X\}) \cup (\cap \{F : M \subseteq F \cap F \text{ is a } N_{nc}ec \text{ set in } X\})$$
$$= N_{nc}ecl(H) \cup N_{nc}ecl(M).$$

Thus, $N_{nc}ecl(H \cup M) = N_{nc}ecl(H) \cup N_{nc}ecl(M)$.

(vi) $N_{nc}ecl(H \cap M)$

$$= \cup \{G : G \subseteq H \cap M \cap G \text{ is a } N_{nc}eo \text{ set in } X\}$$
$$= (\cup \{G : G \subseteq H \cap M \cap G \text{ is a } N_{nc}eo \text{ set in } X\}) \cap (\cup \{G : G \subseteq M \cap G \text{ is a } N_{nc}eo \text{ set in } X\})$$
$$= N_{nc}ecint(H) \cap N_{nc}ecint(M).$$

Thus, $N_{nc}ecint(H \cap M) = N_{nc}ecint(H) \cap N_{nc}ecint(M)$.

(vii)

$$N_{nc}ecl(H) = \cap \{F : H \subseteq F \cap F \text{ is a } N_{nc}ec \text{ set in } X\}$$
$$\big(N_{nc}ecl(H)\big)^c = \cup \{F^c : H^c \supseteq F^c \cap F \text{ is a } N_{nc}e \text{ set in } X\}$$
$$= N_{nc}ecint(H^c).$$

Thus, $\big(N_{nc}ecl(H)\big)^c = N_{nc}ecl(H^c)$.

(viii)

$$N_{nc}ecint(H) = \cup \{G : G \subseteq H \cap G \text{ is a } N_{nc}eo \text{ set in } X\}$$
$$\big(N_{nc}ecint(H)\big)^c = \cap \{G^c : G^c \subseteq H^c \cap G^c \text{ is a } N_{nc}ec \text{ set in } X\}$$
$$= N_{nc}eccl(H^c).$$

Thus, $\big(N_{nc}ecint(H)\big)^c = N_{nc}eccl(H^c)$.

**Proposition 3.3** Let $(X, N_{nc})$ be any $N_{nc}$ts. $H$ is an $N_{nc}$s’s in $(X, N_{nc})$. Then the properties are true:

(i) $N_{nc}ecl(H) = H$ iff $H$ is an $N_{nc}ec$ set.

(ii) $N_{nc}ecint(H) = H$ iff $H$ is an $N_{nc}eo$ set.

(iii) $N_{nc}eccl(H)$ is the smallest $N_{nc}ec$ set containing $H$.

(iv) $N_{nc}ecint(H)$ is the largest $N_{nc}eo$ set containing $H$.

**Proof.** (i), (iv) are obvious.

**Proposition 3.4** The union (resp. intersection) of any family of $N_{nc}eOS(X)$ (resp. $N_{nc}eCS(X)$) is a $N_{nc}eOS(X)$ (resp. $N_{nc}eCS(X)$).

**Remark 3.2** The intersection of two $N_{nc}eO$’s need not be $N_{nc}eO$.

**Example 3.2** In Example 3.1, the sets $\{e, b, a\}, \{\phi\}, \{d, c\}$ & $\{\{b, c, e\}, \{\phi\}, \{d, a\}$ are $2_{nc}eO$ but the intersection $\{\{b, e\}, \{\phi\}, \{a, c, d\}$ is not $2_{nc}eO$.

**Proposition 3.5** Let $(X, N_{nc})$ be a $N_{nc}$s on $X$. $H$ and $M$ be a $N_{nc}$s on $X$. If $H$ is a $N_{nc}O$ and $M$ is a $N_{nc}eO$, then $H \cap M$ is a $N_{nc}eO$.
Proof.

\[ H \cap M \subseteq H \cap N_{nc}(N_{nc}(H)) \]
\[ \subseteq N_{nc}(H \cap N_{nc}(H)) \]
\[ \subseteq N_{nc}(N_{nc}(H \cap M)). \]

Therefore, \( H \cap M \) is a \( N_{nc}\text{-cos} \).

**Proposition 3.6** Let \( (X, N_{nc}) \) be a \( N_{nc}\text{-sets} \) on \( X \). \( M \) is a \( N_{nc}\text{-subset} \) of \( X \) and \( H \) is a \( N_{nc}\text{-pos} \) on \( X \) such that \( H \subseteq M \subseteq N_{nc}(N_{nc}(H)) \). Then \( M \) is a \( N_{nc}\text{-cos} \).

**Proof.** Since \( H \) is a \( N_{nc}\text{-pos} \), \( H \subseteq N_{nc}(N_{nc}(H)) \). Now

\[ M \subseteq N_{nc}(N_{nc}(H)) \]
\[ \subseteq N_{nc}(N_{nc}(N_{nc}(H))) \]
\[ = N_{nc}(N_{nc}(N_{nc}(H))). \]

Hence \( M \subseteq N_{nc}(N_{nc}(N_{nc}(H))) \). Therefore, \( M \) is a \( N_{nc}\text{-cos} \).

4. Conclusion

In this work, we have introduced some new notions of \( N_{nc} \) open (closed) sets called \( N_{nc}\text{-open} \), \( N_{nc}\text{-closed} \) sets and their respective interior and closure operators are introduced and studied some of their basic properties in the context of \( N_{nc}\text{-sets} \). This can be extended to \( N_{nc}\text{-continuous} \), \( N_{nc}\text{-irresolute} \) function, \( N_{nc}\text{-homeomorphism} \) functions and also a contra field in \( N_{nc}\text{-sets} \).

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