I. INTRODUCTION

Ultra-relativistic collisions at the Large Hadron Collider (LHC) at CERN, Switzerland, have been instrumental in understanding the sublime nature of the microscopic world at very high energies. One of the most astounding facets of the microscopic realm is a relatively new state of matter called Quark Gluon Plasma (QGP), which is expected to be formed in such collisions. QGP has partons (quarks and gluons) as the degrees of freedom, and exists at very high temperature and/or baryon density. Earlier, it was believed that there wouldn’t be any QGP formation in $pp$ collisions. However, recent studies indicate possible formation of QGP droplets in high multiplicity $pp$ collisions [1-3]. Such reasons compel us to study high energy $pp$ collisions with ever-increasing intrigue. Understanding the behavior of matter in the hadronic phase of these type of collisions, and having the knowledge about various thermodynamical quantities involved is very useful. Isothermal compressibility ($\kappa_T$), multiplicity fluctuations and speed of sound ($c_s$) are such thermodynamic quantities that tell us about the fascinating behaviours of the system. $\kappa_T$ gives us information about how the volume of a system changes with the change in pressure at constant temperature $T$. It tells us about the deviation of a real fluid from a perfect fluid. For a perfect fluid, $\kappa_T = 0$; which means the fluid is incompressible. However, no such fluid exists in nature. But, recent findings have shown that QGP behaves as a nearly perfect fluid, having the lowest $\kappa_T$ estimated till now [5]. This also complements previous findings of the ratio of shear viscosity and entropy ($\eta/s$) from ADS/CFT calculations which gives a lower bound (KSS bound) to the ratio [6]. The elliptic flow measurements from heavy-ion collisions at RHIC have found that the medium formed in such collisions gives $\eta/s$ value closer to the KSS bound, which might suggest that QGP is almost a perfect fluid [7-8].

Isothermal compressibility is linked to the multiplicity fluctuations and it can be expressed in terms of free energy with respect to pressure [9,10]. This multiplicity fluctuation is a property of Grand Canonical ensemble (GCE). In thermodynamical limit however, the microcanonical, canonical and grand canonical ensembles become equivalent [11,12]. So in principle, the study of multiplicity fluctuation should give us an idea about the statistical behaviour of $pp$ collisions at different charged particle multiplicity. The study of speed of sound will help us to have a proper idea about the equation of state (EOS) of the system. It plays a crucial role in the hydrodynamical evolution of the matter created in the collisions. It also affects the momentum distributions of the particles created in the collision systems. Observations from heavy-ion collisions have proved that the speed of sound is different in three different phases in the evolution of the collision systems, namely the QGP phase, the mixed phase and the hadronic phase. For an ideal gas, the value of squared speed of sound, $c_s^2$ is 1/3, whereas for a hadron gas the value is around 1/5 [13]. The expansion time scale of the system is a measure of the speed of

Multiplicity Dependence of Isothermal Compressibility, Multiplicity Fluctuation and Speed of Sound in $pp$ collisions at $\sqrt{s} = 7$ TeV

Dushmanta Sahu, Sushanta Tripathy*, and Raghunath Sahoo†

Discipline of Physics, School of Basic Sciences, Indian Institute of Technology Indore, Simrol, Indore 453552, India

Archita Rani Dash

Department of Physics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632014, India

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In order to understand the detailed dynamics of the systems produced in $pp$ collisions, it is essential to know about the Equation of State (EoS) and various thermodynamic properties using identified particles. In this work, we study the isothermal compressibility, multiplicity fluctuations and speed of sound of the system by considering differential freeze-out scenario. We have used a thermodynamically consistent Tsallis non-extensive statistics to have a better explanation for the dynamics of $pp$ collision systems. Isothermal compressibility gives us a clear idea about the deviation of the system from a perfect fluid. By studying the multiplicity fluctuation as a function of $dN_{ch}/d\eta$, we show how the high multiplicity collisions differ from the low multiplicity collisions from the view point of statistical ensembles. The speed of sound in the system as a function of $dN_{ch}/d\eta$ gives us a vivid picture of the dynamics of the system. The results show quite an intuitive perspective on high multiplicity $pp$ collisions and give us a limit of $dN_{ch}/d\eta \gtrsim (10 - 20)$ after which a change in the dynamics of the system may be observed.

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sound which is given by \( \tau_{\text{exp}}^{-1} \sim \frac{1}{\tau_{\text{rel}}} \). Here, \( \epsilon \) is the energy density of the system and \( \tau \) is the relaxation time – the time a system takes to come to equilibrium. While the collision time scale is given by \( \tau_{\text{coll}}^{-1} \sim n \sigma v \), where \( n \) is density of particles, \( \sigma \) is collision cross section and \( v \) is the particle velocity. For a system to be in thermal equilibrium, the expansion time scale must be greater than the collision time scale. However, we are taking differential freeze-out scenario, so it will be interesting to see how the speed of sound varies for different hadronic species as a function of charged particle multiplicity.

A large number of particles are produced in high energy collisions, which demands us to take a statistical approach to study the QCD matter, the particle production and the thermodynamics of the systems. The transverse momentum \( (p_T) \) of the final state particles produced in high energy collisions are expected to follow a thermalized Boltzmann-Gibbs (BG) distribution. However, it is experimentally observed that the \( p_T \)-spectra at the RHIC [14, 15] and LHC [16–19] energies show deviation from thermalized Boltzmann distribution. Higher contribution of pQCD effects are responsible for this deviation and the spectra are better described by a combination of Boltzmann-type exponential and pQCD inspired power-law distribution. Although a first principle derivation of Tsallis non-extensive distribution [20] is still a question, empirically it has been very successful in describing the \( p_T \)-spectra in hadronic collisions. This is used for getting the multiplicity and thus particle ratios in experimental papers. There are vast theoretical developments in this front to bring up the physics messages, which include considering systems formed in hadronic collisions as away from equilibrium, system thermodynamics etc. Although there are various forms of Tsallis distribution function used in literature, a thermodynamically consistent distribution function is used in this paper to describe the \( p_T \)-spectra in LHC \( pp \) collisions [21]. The deviation from equilibrium is denoted by a parameter \( q \), and \( q = 1 \) denotes the equilibrium condition (BG scenario). At high charged particle multiplicity in high energy collisions, \( q \) tends to 1, which is an indication that the system has attained global equilibrium. The extracted thermodynamic parameters [22] such as temperature, \( T \) and non-extensivity parameter, \( q \) are obtained for different multiplicity classes, which are then used to have the estimation of isothermal compressibility, multiplicity fluctuation (related to criticality in the system) and the speed of sound in the medium (related to the equation of state).

In this paper, we have studied the isothermal compressibility, the multiplicity fluctuations and speed of sound in high energy \( pp \) collisions. In section II, we have given a brief formulation for isothermal compressibility, multiplicity fluctuations and \( c_s^2 \) using non-extensive statistics. In section III, the results and discussions are given. Finally we have summarized our findings in section IV.

II. FORMULATION

A thermodynamically consistent Tsallis distribution function is given as,

\[
f = \frac{1}{1 + (q - 1) \left( \frac{E - p \cdot u - \mu}{T} \right) \beta^{-q}}, \tag{1}
\]

where \( E \) is the particle energy, \( p \) is the four-momentum, \( u \) is the fluid velocity, and \( T \) and \( \mu \) are the temperature and the chemical potential, respectively. The thermodynamical quantities in non-extensive statistics are calculated as [23],

\[
n = g \int \frac{d^3p}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right] \beta^{-q}, \tag{2}
\]

\[
\epsilon = g \int \frac{d^3p}{(2\pi)^3} E \left[ 1 + (q - 1) \frac{E - \mu}{T} \right] \beta^{-q}, \tag{3}
\]

\[
P = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right] \beta^{-q}. \tag{4}
\]

\( n, \epsilon \) and \( P \) are the number density, energy density and pressure of hadrons, respectively. \( q \) is the particle degeneracy. Also, the non-extensive entropy density is given by,

\[
s = \frac{\epsilon + P - \mu n}{T}. \tag{5}
\]

From thermodynamics, the isothermal compressibility is defined as [24],

\[
\kappa_T = -\frac{1}{V} \frac{\partial P}{\partial V} \bigg|_{T}, \tag{6}
\]

where \( V, P \) and \( T \) are the volume, pressure and temperature of the system. In terms of number fluctuations and average number, isothermal compressibility can be defined as,

\[
\left\langle (N - \langle N \rangle)^2 \right\rangle = \text{var}(N) = \frac{T\langle N \rangle^2}{V} \kappa_T, \tag{7}
\]

where \( N \) is the particle multiplicity. From basic thermodynamic relation, we have

\[
\left\langle (N - \langle N \rangle)^2 \right\rangle = VT \frac{\partial n}{\partial \mu}. \tag{8}
\]

Thus, from Eq.7 and Eq.8 we derive the expression,

\[
\kappa_T = \frac{\partial n/\partial \mu}{n^2}. \tag{9}
\]
where,
\[
\frac{\partial n}{\partial \mu} = \frac{gq}{T} \int \frac{d^3p}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{\frac{1-2q}{2}} \tag{10}
\]

At the LHC energies, the baryochemical potential, \( \mu \) is almost zero and for our studies, we can set \( \mu = 0 \) in the calculations. To have a better understanding about a system, it is important to know the Equation of State (EoS) which is given by the speed of sound in that system. Speed of sound square is given by [25],
\[
c_s^2 = \left( \frac{\partial P}{\partial \epsilon} \right)_{s/n}. \tag{11}
\]

It can be further written as,
\[
c_s^2 = \frac{\partial P}{\partial T} \frac{1}{\partial \epsilon}. \tag{12}
\]

III. RESULTS AND DISCUSSION

In this work, we have estimated the isothermal compressibility of a hot and dense hadron gas formed in \( pp \) collisions by taking Eq.9 into account. We have taken \( T \) and \( q \) values from Tsallis distribution function by fitting the \( p_T \)-spectra of produced identified particles in a differential freeze-out scenario [25]. Fig.1 shows the variation of \( \kappa_T \) of the identified particles as a function of charged particle multiplicity. We observe that \( \kappa_T \) decreases with the increase in \( dN_{ch}/d\eta \). This result is in agreement with our previous findings [3]. For lighter particles, the values of \( \kappa_T \) are lower and the values increase as the mass of the particles increase till a certain \( dN_{ch}/d\eta \). Pion being the lightest meson, has the lowest \( \kappa_T \) as compared to the others. This is because, \( \kappa_T \) is inversely proportional to the number density of the system. As pion number density is highest in a collision system, its isothermal compressibility is the lowest. However, as it is clearly seen, for higher charged particle multiplicity (\( dN_{ch}/d\eta \) \( \gtrsim \) 10 - 20), the \( \kappa_T \) of all the hadrons converge together and show only a slight variation from each other. Beyond this threshold limit, a QGP-like medium formation is expected regardless of the collision systems [27]. It is worth mentioning here that a differential kinetic freeze-out scenario becomes a single freeze-out as has been discussed in heavy-ion collisions for a multiplicity threshold of \( dN_{ch}/d\eta \gtrsim \) 10 - 20, which is one of the important findings of the present study.

The isothermal compressibility of water at room temperature is reported to be \( 6.62 \times 10^{12} \) fm\(^3\)/GeV [28]. We have estimated the \( \kappa_T \) of hadron gases to be around 10 fm\(^3\)/GeV at high charged particle multiplicity. This value is still larger than the isothermal compressibility found for QGP-like medium in our previous work, which was found to be \( \sim 0.3 \) fm\(^3\)/GeV [5].

Figure 2 shows the variation of multiplicity fluctuations as a function of charged particle multiplicity. We clearly see that with \( dN_{ch}/d\eta \), the multiplicity fluctuations increases for all identified particles. The pion being the lightest and the most abundant species in the system, its multiplicity fluctuation is also the highest. The rest of the particles follow this trend according to their abundances in the system. For heavy-ion collisions, Grand Canonical Ensembles (GCE) are broadly used. Previously, it was thought that \( pp \) collisions are explained by Canonical Ensembles (CE). But recent studies have shown that there can be possible formation of QGP droplets in high multiplicity \( pp \) collisions. So for such systems, the low \( dN_{ch}/d\eta \) region suggests a canonical ensemble and for high \( dN_{ch}/d\eta \) the system might behave as a Grand Canonical Ensemble, where there is a possibility of QGP-like medium formation [2]. We know that for CE, there is no number fluctuation, however for GCE, we should observe considerable number fluctuation. This can be seen from Fig.2. At low charged particle multiplicity, the number fluctuation is less and it increases with increase in \( dN_{ch}/d\eta \).

The speed of sound squared \( (c_s^2) \) being related to the equation of state of the system (EoS), is one of the important thermodynamic observables. In Fig. 3 we have plotted the \( c_s^2 \) as a function of charged particle multiplicity. We observe that \( c_s^2 \) increases with the increase in \( dN_{ch}/d\eta \). Here we see a mass ordering between the identified particles. As pion is the most abundant particle in the system, the \( c_s^2 \) for pion is the highest. These observations complement the previous estimations [29, 30]. The other particles follow the same trend accordingly. At low \( dN_{ch}/d\eta \), the speed of sound is lower. This is because the \( c_s^2 \) depends on the density of the system. For low charged particle multiplicity, the density is lower, and it increases slowly with increase in \( dN_{ch}/d\eta \). However, after a certain charged particle multiplicity, the density of the system doesn’t change [31]. We observe that there

![FIG. 1: (Color online) \( \kappa_T \) as a function of charged particle multiplicity for \( pp \) collisions at \( \sqrt{s} = 7 \) for different final state particles.](image-url)
is a transition in the behavior of the $c_s^2$ plot at around $dN_{ch}/d\eta \gtrsim 10 - 20$, and almost becomes constant after this limit. This gives us a hint about a change in the dynamics of the systems after certain charged particle multiplicity. The squared value of the speed of sound in air at room temperature is $1.3 \times 10^{-12}$ and in distilled water at room temperature it is around $2.5 \times 10^{-11}$ [32]. With comparison to these, the $c_s^2$ of the hadron gases we found here is very high. This indicates the fact that, the hadron gas systems formed in high energy collisions are very dense mediums and at higher charged particle multiplicities, the value of $c_s^2$ tend towards 1/3, which means the medium behaves like almost an ideal gas.

In summary,

1. We have estimated the isothermal compressibility of the hadron gases in $pp$ collision system by considering differential freeze-out scenario. We observed that $\kappa_T$ of the systems decreases with the increase in the charged particle multiplicity. This suggests that at higher charged particle multiplicity, the system is less compressible.

2. The isothermal compressibility becoming independent of particle species around $dN_{ch}/d\eta \gtrsim 10 - 20$ is an indication of a transition from differential to single kinetic freeze-out, as is observed in heavy-ion collisions. This observed threshold in the final state charged particle density is an important finding in view of the scaling observed in the LHC energies across different collision species.

3. We have also estimated the number fluctuations in the hadron gases in $pp$ collision systems. We have found that for low charged particle multiplicity, the number fluctuation is less and it increases with the increase in $dN_{ch}/d\eta$. This indicates that with increase in $dN_{ch}/d\eta$, after a threshold of 10-20, there is a transition from canonical ensemble to a grand canonical ensemble description of the system.

4. The speed of sound squared for different particles are estimated. We see a range of values from 0.15 to 0.32 for different particles, with pion having the highest $c_s^2$.

5. In all our findings, we have observed a limit of $dN_{ch}/d\eta \gtrsim 10 - 20$ after which the system appears to be going through some change in its dynamics. This limit of charged particle multiplicity may suggest a requirement for the possible formation of QGP droplets in high multiplicity $pp$ collisions [33, 34].

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