On chiral medium with zero permittivity and permeability values

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Abstract. The possibility of propagation of electromagnetic waves of circular polarization in a Tellegen isotropic chiral medium with zero values of dielectric permittivity and magnetic permeability (chiral nihility) is considered. The refractive indices of normal waves are analyzed using the forward and backward wave identifier. In the presence of dissipative losses, the existence of two different normal waves in such a medium is impossible.

1. Introduction
The beginning of the third Millennium coincided with the emergence of interest in metamaterials – artificial composite media, the properties of which are determined mainly not by the composition, but by the structure of the incoming ingredients. As a rule, they have a pronounced periodic structure. If the period of the structure is small in comparison with the characteristic length of the electromagnetic or acoustic wave propagating in the material, such a metamaterial is considered as a macroscopically continuous medium with effective values of material parameters, such as dielectric permittivity ($\varepsilon$) and magnetic permeability ($\mu$). In the most general form, metamaterials are bianisotropic media, which are described by means of tensor material parameters. In comparison with natural media, these substances can be very different from them in terms of effective values of material parameters, which is a prerequisite for the emergence of new, previously unseen wave manifestations. In the same connection, some restrictions on permissible values of material parameters or their combinations, which previously seemed impossible or of no practical interest, were revised [1]. For exotic media was proposed special names, such as “antivacuum” ($\varepsilon = -1$, $\mu = -1$), “nihility” ($\varepsilon = 0$, $\mu = 0$) [2].

The efforts of researchers were focused primarily on the study of the so-called double-negative media. Actually, these isotropic media were considered theoretically for the first time in the USSR and subsequently became known in the West mainly due to the work of V.G. Veselago [3, 4], who used the term “left-handed media”. The unusual phenomenon of negative refraction, which is observed on the interface with the left-handed media, led to the discovery and discussion of many new and tempting phenomena (a flat perfect lens that is not sensitive to the diffraction limit, object masking, optical illusions, etc.). These effects are due to the fact that in transparent media with real but negative values, not forward but backward normal waves are excited, as indicated by L.I. Mandel’shtam in the 40s of the XX century [5]. Veselago differently approached the description of negative refraction, suggesting that the refractive index $n$ of the medium with $\varepsilon < 0$ and $\mu < 0$ be considered negative. This view has led to another naming of such medium as “medium with negative refractive index” or “negative-index metamaterial” (NIM).

Real metamaterials, in which the real parts of both permittivity and permeability are negative, have significant heat losses, which reduces their efficiency. J. Pendry [6] and S.A. Tretyakov with co-authors
[7, 8] proposed methods for obtaining the backward wave in an isotropic chiral medium with positive permittivity and permeability. Moreover, the possibility of an exotic medium called “chiral nihility” has been claimed [7]. In contrast to the non-chiral “nihility,” in such a medium with a single non-zero material chiral parameter (κ), electromagnetic waves can still propagate, and such a medium retains the properties of a biisotropic chiral medium as a birefringent medium with two normal waves. The simplicity of material relations combined with the variety of wave manifestations attracted the attention of theorists to this model medium (for example, [9–11]).

However, there is a disturbing circumstance that calls into question the principle feasibility of such a medium. Among the three equal systems of constitutive relations that are used to describe electromagnetic waves in an isotropic chiral medium (Tellegen relations, Post relations, Drude–Born–Fedorov relations), the state of “chiral nihility” is possible only in one Tellegen system. The purpose of this report is to clarify the conditions for normal waves in the medium “chiral nihility,” using an approach unrelated to the concept of “negative refractive index”.

2. A nonchiral isotropic medium

We first consider a nonchiral isotropic lossless medium. The dispersion equation of plane monochromatic normal waves with circular frequency \( \omega \), for the mathematical description of which the complex exponential function \( e^{i(kr-\omega t)} \) is applied, has the form

\[
\mathbf{n}^2 = \varepsilon \mu ; \quad n = \frac{k}{k_0}, \quad k_0 = \frac{\omega}{c},
\]

where \( c \) is the velocity of light in vacuum. The refraction vector \( \mathbf{n} \) unlike the wave vector \( \mathbf{k} \) is directly related to the refractive index by the formula

\[
n = \sqrt{\mathbf{n} \cdot \mathbf{n}} = |\mathbf{n}| > 0.
\]

Quadratic equation (1) has two solutions, differing in sign, but by the condition (2) of them should choose the arithmetic square root

\[
n = \sqrt{\varepsilon \mu}.
\]

Thus, it is always necessary to exclude from consideration negative values for the refractive index as not satisfying the condition (2). In order to distinguish between two complementary media that have the same refractive index, as well as common \( |\varepsilon| \) and \( |\mu| \), it is necessary to use the wave type identifier [12]:

\[
a = \mathbf{n} \cdot \hat{s} = \begin{cases} +1, & \text{если } \mathbf{n} = \mathbf{s} \\ -1, & \text{если } \mathbf{n} = -\mathbf{s} \end{cases}
\]

where the unit vectors \( \mathbf{n} \) and \( \hat{s} \) characterize the direction of motion of the wave phase front and the wave energy flow, respectively. Thus, the forward waves correspond to the value \( a = +1 \), and the backward waves – the value \( a = -1 \). It is also obvious that for a transparent medium

\[
a = \text{sgn } \varepsilon = \text{sgn } \mu.
\]

If the isotropic medium has losses, the refractive index becomes a complex value. The sign of the imaginary part of the index is chosen in accordance with the principle of ultimate absorption (“the principle of redemption” by G.D. Malyuzhinets), according to which the energy of the wave field decreases with increasing distance from the source. With the selected time factor, the imaginary additive is always assumed to be positive for forward waves, but for backward waves its sign changes. Generically, we can write

\[
n = n_r + i n_l, \quad n_r > 0, \quad n_l > 0.
\]
The identifier $a$ in the absorbing medium is calculated by the formula $a = \text{sgn} \left( \varepsilon_r / |\varepsilon| + \mu_r / |\mu| \right)$ [13].

3. The Tellegen chiral medium

Let us now turn to the Tellegen chiral medium. The reduced (dimensionless) wave numbers are calculated by the formula

$$n_{1,2} = \pm \kappa.$$  \hspace{1cm} (7)

The chirality parameter in a transparent medium is a real value, positive or negative depending on the type of chirality of the medium. So far $|\kappa| < n$, which is typical for natural media and conventional chiral mixtures, both numbers $n_{1,2}$ are positive. In metamaterials, strong chirality ($|\kappa| > n$) can be realized, and in this case one of the numbers in (7) turns out to be a negative value, and the corresponding normal wave becomes a backward wave. The strong chirality condition is also achieved by lowering the refractive index $n$ of the equivalent nonchiral medium. In the extreme case, assuming $n$ equal to zero, get “chiral nihility”. This medium is thus characterized by two normal circularly polarized waves with a common refractive index $n = |\kappa|$, and always one of them is a forward wave and the other is a backward wave. It should, however, be borne in mind that the wave impedance of such a medium is calculated by the formula

$$Z = \lim_{\varepsilon, \mu \rightarrow 0} \sqrt{\frac{\mu}{\varepsilon}},$$  \hspace{1cm} (8)

that is, in the general case is an uncertain quantity.

In the presence of dissipative losses in the medium, the chirality parameter becomes a complex value: $\kappa = \kappa_r + i \kappa_i$. Let us assume for certainty that $\kappa_r > 0$. According to (6), the refractive index of the first mode of the medium “chiral nihility” $\bar{\nu}_1 = \nu_1 = +\kappa = \kappa_r + i \kappa_i$ corresponds to the forward wave provided that $\kappa_i > 0$, and corresponds to the backward wave under the opposite condition $\kappa_i < 0$. The medium in question is a passive medium, and the chirality parameter is its only material parameter. Therefore, the negative loss option $\kappa_i < 0$ should be excluded from the analysis. The refractive index of the second mode $\bar{\nu}_2$ should be calculated by the formula

$$\bar{\nu}_2 = \nu_2 \text{sgn} (\text{Re} \nu_2) = (-\kappa_r - i \kappa_i)(-1) = \kappa_r + i \kappa_i.$$  \hspace{1cm} (9)

Since the wave with index $\bar{\nu}_1$ is a forward wave, the wave with index $\bar{\nu}_2$ must be the backward wave, which by definition has $\text{Im} (\bar{\nu}_2) < 0$. From formula (9) it follows that in fact $\text{Im} (\bar{\nu}_2) > 0$. A contradiction is achieved, which indicates the impossibility of the presence of two different normal waves in a dissipative medium called “chiral nihility”. Since the loss parameter $\kappa_i$ can be taken as small as necessary, then making a limit transition ($\kappa_i \rightarrow 0$) on it, we come to the conclusion about the complete physical incorrectness of the “chiral nihility” model for isotropic chiral medium.

4. Summary

The mathematical model of the isotropic chiral medium described by the Tellegen constitutive relations with degenerate (zero) values of dielectric permittivity and magnetic permeability (“chiral nihility”) is internally contradictory, since at vanishingly small heat losses it does not provide the existence of two normal waves (forward wave and backward wave). Therefore, it is physically incorrect, and it should not be used in problems of electrodynamics of complex media and metamaterials.
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