Interference Effects in B-Decays to Flavor-Mixed Neutral Mesons
Clues to Small Amplitudes and CP-Violation

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ABSTRACT

CP violation can be observed in B decays when a given process depends upon interference between two weak amplitudes which have different CP-violating phases. Since most weak decay diagrams have quark lines where each has a definite flavor label, neutral mesons which are flavor mixtures are particularly interesting. Different diagrams can contribute to the different flavor components of the meson, and the wave function itself provides interference.

1. A Flavor Topology Classification

The total amplitude for the decay of a B meson consisting of a b quark and an antiquark denoted by \( \bar{q} \) can be expressed as as the sum of three independent amplitudes with different flavor topologies[1]:

\[
\begin{align*}
B(b\bar{q}) & \rightarrow U_bW^-\bar{q} \rightarrow U_b + \bar{U} + D + \bar{q} \quad (QQ1) \\
B(b\bar{q}) & \rightarrow U_bW^-\bar{q} \rightarrow U_b + \bar{U} + f + \bar{f} \quad (QQ2a) \\
B(b\bar{q}) & \rightarrow U_bW^-\bar{q} \rightarrow D + \bar{q} + f + \bar{f} \quad (QQ2b) \\
B(b\bar{q}) & \rightarrow U_bW^-\bar{q} \rightarrow D + \bar{q} + f + \bar{f} \quad (QQ3)
\end{align*}
\]

where \( U \) denotes a quark of charge (+2/3), \( D \) a quark of charge (-1/3) and \( U_b \) denotes the \( U \) quark produced in the initial \( b \rightarrow U \) transition with the emission of the \( W^- \). The spectator tree diagram (QQ1) has all three flavors produced by the \( b \) decay present in the final state together with the spectator antiquark \( \bar{q} \). The spectator annihilation diagrams (QQ2a) and (QQ2b) have the spectator antiquark \( \bar{q} \) annihilated either by the \( U_b \) or the \( D \) quark produced in the tree diagram. These can arise from a weak annihilation diagram, a weak \( W \)-exchange diagram or a tree diagram followed by final state interactions. Note that (QQ2a) is allowed only if the spectator \( \bar{q} \) is a \( \bar{c} \) or \( \bar{u} \) and that (QQ2b) is allowed only if the spectator \( \bar{q} \) is a \( \bar{c} \) or \( \bar{u} \).
is a $s$ or $\bar{d}$. The penguin diagram (QQ3) contains a loop in which the $U_b$ and $U$ annihilate and appear as a single line in the weak penguin diagram. The same topology can arise in a tree diagram followed by final state interactions.

The particular products of $CKM$ matrix elements in the standard model that contribute to a given decay are completely determined by this topological classification. The $CKM$ product arising in any decay described by diagrams (QQ2) and (QQ3) is the same whether the topology results directly from the weak diagram or from a tree diagram followed by a final state interaction, with the exception of the flavor label of the quark that is created and annihilated in the penguin and is not directly observed. The topology alone cannot determine the detailed dynamics, but can determine the particular products of $CKM$ matrix elements contributing to a particular decay. The following flavor properties result from this classification:

1. In the spectator tree diagram (QQ1) the flavors of the four outgoing quark lines are determined uniquely and completely by the weak vertex.

2. In both the spectator annihilation (QQ2) and penguin (QQ3) diagrams a flavor-symmetric $f\bar{f}$ pair created by gluons appears in the final state. These diagrams come in triplets in which the additional pair is $u\bar{u}, d\bar{d}$ and $s\bar{s}$ with amplitudes having equal magnitudes and a positive relative phase in the SU(3) flavor-symmetry limit. The $s\bar{s}$ amplitude is reduced by SU(3) symmetry breaking due to the quark mass differences. The flavor quantum numbers of the final state are completely determined by the remaining single $q\bar{q}$ pair with well defined non-exotic flavor quantum numbers conserved in all strong final state interactions.

3. Final states having exotic flavor quantum numbers; i.e. containing no $q\bar{q}$ pair of the same flavor, have only spectator tree contributions.

4. For final states containing no quark with the flavor quantum numbers of the incoming spectator quark, only the spectator annihilation diagram can contribute.

5. For final states where both the spectator tree and spectator annihilation diagrams contribute, $f, \bar{q}$ and either $U_b$ or $D$ must have the same flavor. The tree diagram can therefore be turned into a spectator annihilation diagram without changing the weak vertex by closing the lines of the outgoing pair to make a loop annihilating the spectator quark and creating an additional pair with gluons. Thus both diagrams depend upon the same product of weak $CKM$ matrix elements in the standard model.

6. A flavor-mixed neutral meson like $\eta, \eta', \pi^0, \rho^0$ or $\omega$ is produced only in final states containing a $q\bar{q}$ pair of the same flavor. The spectator tree diagram generally contains only one such $q\bar{q}$ pair and in that case produces the three neutral pseudoscalars or vectors with a ratio determined completely by the amplitude of the pair of that flavor in the wave function and corrections for phase space. Thus any deviation from such a ratio; e.g. a larger $\eta'$ production than $\eta$ production or unequal $\rho^0$ and $\omega$ production indicates interference between the spectator tree diagram and one of the other two.

7. The OZI rule forbids the creation of a flavor-mixed neutral meson like $\eta, \eta', \pi^0, \rho^0$ or $\omega$ from the $q\bar{q}$ pair of the same flavor produced by gluons in a quark loop in the spectator.
annihilation or penguin diagram.

8. If the final state contains an $f \bar{f}$ pair as in spectator annihilation or penguin transitions, together with a single light-quark $\bar{q}q$ pair which is not flavor neutral and any number of additional gluons the final state is a flavor-SU(3) octet in the SU(3) symmetry limit. When charge conjugation quantum numbers forbid the SU(3) octet-octet-singlet coupling; e.g. in the spin-zero pseudoscalar-vector final state, the OZI rule is already predicted by SU(3) with no further assumptions.

9. The only way to observe a CP-violating charge-asymmetric quasi-two-body decay in the standard model is by interference between two amplitudes depending upon different CKM matrix elements. This requires interference involving the penguin diagram[2].

There is great interest in finding penguin contributions and CP-violating interference effects, but as yet no firm experimental evidence for penguins. The use of final states containing $\eta$ and $\eta'$ has been discussed[3]. We concentrate here on newer predictions involving $\rho$, $D^o$ and $\bar{D}^o$.

The $\rho^o$ and $\omega$ are equal mixtures with opposite relative phase of the vector quarkonium states $u\bar{u}$ and $d\bar{d}$, denoted respectively by $V_u$ and $V_d$. Equal $\rho$ and $\omega$ production is predicted[4] for any quark diagram leading to a single flavor state, either $V_u$ or $V_d$, together with the interference effect[5] confirmed experimentally[6].

2. $B^o \rightarrow K^o \rho^o$ AND $B^o \rightarrow K^o \omega$ DECAYS

In these decays the Cabibbo-suppressed color-suppressed spectator tree diagram produces $V_u$, while the penguin and all other diagrams via an intermediate $\bar{q}q$ pair have the flavor quantum numbers $\bar{s}d$ and produce $V_d$ in the transitions allowed by flavor SU(3) or OZI,

\[
B^o(\bar{b}d) \rightarrow (\bar{u}u\bar{s})d \rightarrow K^oV_u \tag{ZZ4a}
\]

\[
B^o(\bar{b}d) \rightarrow \bar{s}d \rightarrow K^oV_d \tag{ZZ4b}
\]

\[
\frac{\bar{\Gamma}(B^o \rightarrow K^o \omega)}{\bar{\Gamma}(B^o \rightarrow K^o \rho^o)} = \left| \frac{T + P}{T - P} \right|^2 \tag{ZZ5a}
\]

\[
\frac{\bar{\Gamma}(B^o \rightarrow K^o \omega)}{\bar{\Gamma}(B^o \rightarrow K^o \rho^o)} = \left| \frac{T + \bar{P}}{T - \bar{P}} \right|^2 \tag{ZZ5b}
\]

where $\bar{\Gamma}$ denotes the reduced partial width for the particular decay mode with the dependence of the phase space factor on individual final states removed, $T$, and $P$ denote the contributions to the decay amplitudes (ZZ5a) from the tree and penguin diagrams respectively, $\bar{P}$ denotes the penguin contribution to the charge conjugate amplitudes (ZZ5b) and we use the Nir - Quinn phase convention[7] in which the CP-violating phase appears only in the penguin diagram.
This offers the possibility of detecting the penguin contribution and also measuring the relative phase of penguin and tree contributions, as well as detecting CP violation in a difference between the charge-conjugate ρ/ω ratios (ZZ5a) and (ZZ5b).

$$\frac{\tilde{\Gamma}(B^o \rightarrow K^o\omega)}{\tilde{\Gamma}(B^o \rightarrow K^o\rho^o)} - \frac{\tilde{\Gamma}(\bar{B}^o \rightarrow K^o\omega)}{\tilde{\Gamma}(\bar{B}^o \rightarrow K^o\rho^o)} \approx 4Re \left[ \frac{P - \bar{P}}{T} \right]$$

(ZZ6)

Note, however, that tree diagrams followed by final state interactions exist with the penguin topology and produce the ρ and ω via an intermediate $\bar{s}d$ pair which decays into $K^oV_d$; e.g.

$$B^o \rightarrow D^{*+} + \bar{D}_s \rightarrow K^oV_d$$  (ZZ7a)
$$B^o \rightarrow K^+ + \rho^- \rightarrow K^oV_d$$  (ZZ7b)

The diagram (ZZ7a) depends upon the same CKM matrix elements $V_{bs}$ and $V_{cs}$ as the penguin and will have the same weak phase in the standard model. Therefore the contribution of this diagram to the amplitude $P$ in eqs. (ZZ5) will not interfere with using these relations to obtain CKM matrix elements. However the diagram (ZZ7b) depends upon the same CKM matrix elements $V_{bu}$ and $V_{us}$ as the tree diagram and will have the same weak phase as the amplitude $T$ in eqs. (ZZ5), even though it contributes to the $P$ amplitude in eqs. (ZZ5). This contribution thus does interfere with using the relations (ZZ5) directly to obtain CKM matrix elements, but does not affect the CP violation relation (ZZ6).

Further information can be obtained by looking for the $\rho - \omega$ interference observed in strong reactions[6] in detailed analysis of the $\pi^+\pi^-$ spectrum over the mass range of the $\rho$ resonance. The isospin violating $\omega \rightarrow \pi^+\pi^-$ has a branching ratio of only 2.2%. But the width of the $\omega$ is 8.4 MeV while that of the $\rho$ is 149 MeV[8]. Thus

$$\frac{\tilde{\Gamma}\{\omega \rightarrow (\pi^+\pi^-)_\omega\}}{\tilde{\Gamma}\{\rho^o \rightarrow (\pi^+\pi^-)_\omega\}} = 0.022 \cdot \frac{149}{8.4} \approx 0.39$$

(ZZ8)

where $(\pi^+\pi^-)_\omega$ denotes the $\pi^+\pi^-$ decay mode at the $\omega$ peak. Thus

$$\frac{\tilde{\Gamma}\{B^o \rightarrow K^o\omega \rightarrow K^o(\pi^+\pi^-)_\omega\}}{\tilde{\Gamma}\{B^o \rightarrow K^o\rho^o \rightarrow K^o(\pi^+\pi^-)_\omega\}} = 0.39 \cdot \left| \frac{T + P}{T - P} \right|^2$$  (ZZ9a)

$$\frac{\tilde{\Gamma}\{\bar{B}^o \rightarrow \bar{K}^o\omega \rightarrow \bar{K}^o(\pi^+\pi^-)_\omega\}}{\tilde{\Gamma}\{\bar{B}^o \rightarrow \bar{K}^o\rho^o \rightarrow \bar{K}^o(\pi^+\pi^-)_\omega\}} = 0.39 \cdot \left| \frac{T + \bar{P}}{T - \bar{P}} \right|^2$$  (ZZ9b)
If the two contributions are coherent, the total contribution is given by

\[
\frac{\tilde{\Gamma}\{B^0 \rightarrow \bar{K}^0(\pi^+\pi^-)\omega\}}{\tilde{\Gamma}\{B^0 \rightarrow K^0\rho^0 \rightarrow \bar{K}^0(\pi^+\pi^-)\omega\}} = 1 + 1.25 \cos(\alpha + \phi_{PT}) \cdot \left| \frac{T + P}{T - P} \right| + 0.39 \cdot \left| \frac{T + P}{T - P} \right|^2 \tag{ZZ10a}
\]

where \(\alpha\) is the relative phase of the \(\rho\) and \(\omega\) contributions and \(\phi_{PT}\) is a relative phase defined by

\[
\frac{T + P}{T - P} \equiv \left| \frac{T + P}{T - P} \right| e^{i\phi_{PT}} \tag{ZZ10b}
\]

and similarly for the charge conjugate case (ZZ9b).

Additional information will be obtained if enough statistics are available for observing the detailed behavior of the decay as a function of energy through the resonance. The phase \(\alpha\) will change rapidly in passing through the \(\omega\) resonance and the interference pattern can give information on the phases \(\phi_{PT}\) and \(\phi_{\bar{P}T}\). Any difference between the two indicates \(CP\) violation.

Additional interference effects arise when the final kaon is detected in the \(K_S\) decay mode and the initial \(B\) meson undergoes \(B^0 - \bar{B}^0\) oscillations\[9,10\] There are two independent \(CP\)-violating relative phases: (1) the relative phase of the \(P\) and \(\bar{P}\) amplitudes which expresses the relative weak phase of the penguin and tree contributions; (2) a parameter \(\theta\) \[10\] which expresses the weak phase contribution to the \(B^0 - \bar{B}^0\) mixing relative to the phase of the tree contribution to the decays which has been used to define the relative phase of the \(B^0\) and \(\bar{B}^0\) states. In addition there is the rapidly varying strong phase \(\alpha\) (ZZ10a). Thus measuring these decays both as a function of time and of the invariant mass of the \(\pi^+\pi^-\) system can give interesting information on decay amplitudes and \(CP\) violation.

3. Cascade Decays via \(D^0\) and \(\bar{D}^0\)

CP-violating charge asymmetries can arise in \(B\) decays via different diagrams involving different weak CKM matrix elements into charmed states containing \(D^0\) and \(\bar{D}^0\)[11,12] which then decay into the same final state,

\[
B^\pm \rightarrow K^\pm + D^0 \rightarrow K^\pm + K^+ + K^- \tag{ZZ11a}
\]

\[
B^\pm \rightarrow K^\pm + \bar{D}^0 \rightarrow K^\pm + K^+ + K^- \tag{ZZ11b}
\]

Eq (ZZ11a) involves \(V_{bc}\) and \(V_{su}\); eq. (ZZ11b) involves \(V_{bu}\) and \(V_{sc}\). There are also decays via
\[ \pm D^*; \]
\[ B^\pm \rightarrow K^\pm + D^{*0}(\bar{D}^{*0}) \rightarrow K^\pm + \pi^0 + K^+ + K^- \quad (ZZ12a) \]
\[ B^\pm \rightarrow K^\pm + D^{*0}(\bar{D}^{*0}) \rightarrow K^\pm + \gamma + K^+ + K^- \quad (ZZ12b) \]

Since the \( \pi^0 \) and \( \gamma \) have opposite \( C \) the relative phase of the \( D^0 \) and \( \bar{D}^0 \) produced in the two decay modes are opposite. The opposite charge asymmetry predicted for the two cases can provide a useful experimental check for systematic errors.

4. Charm Decays to \( \rho, \omega, \phi \)

Analogous flavor topologies can be defined in charm decays, with flavor labels having the reverse weak isospin. By analogy with (ZZ5)

\[ \tilde{\Gamma}(D^0 \rightarrow \bar{K}^0 \omega) = (1/2)|T + A|^2 \quad (ZZ13a) \]
\[ \tilde{\Gamma}(D^0 \rightarrow \bar{K}^0 \rho) = (1/2)|T - A|^2 \quad (ZZ13b) \]
\[ \tilde{\Gamma}(D^0 \rightarrow \bar{K}^0 \phi) = \xi^2|A|^2 \quad (ZZ13c) \]

where \( T \) and \( A \) denote the contributions from the tree and annihilation diagrams respectively, and \( \xi \leq 1 \) is an SU(3)-breaking suppression factor. The penguin does not contribute and there is no \( CP \) violation asymmetry. Experimental data\[8\] show a large difference between \( \rho \) and \( \omega \) branching ratios and an appreciable \( \phi \), implying that the two amplitudes \( T \) and \( A \) are comparable. Combining eqs. (ZZ13) and introducing experimental values\[8\] give a triangular inequality

\[ (1.61 \pm 1.21)\% \leq \frac{\tilde{BR}(D^0 \rightarrow \bar{K}^0 \phi)}{2\xi^2} \leq (6.65 \pm 2.29)\% \quad (ZZ14) \]

where \( \tilde{BR}_\omega \) denotes that the branching ratio is normalized to the \( K - \omega \) phase space and needs a phase space correction. The experimental value \((1/2)BR(D^0 \rightarrow \bar{K}^0 \phi) = (0.4 \pm 0.08)\%\) might indicate a violation of the lower bound or large SU(3) breaking. A better measurement of the \( K\omega \) branching ratio would be useful and a measurement of the relative phase of the \( \omega \) and \( \rho \) amplitudes via \( \rho\omega \) interference would enable a direct test of the relations (ZZ13).

An analogous treatment and flavor-topology classification has been given for quasi-twobody \( D^+ \) and \( D_s \) decays\[3\] where \( T \) denotes the sum of contributions from all diagrams in which the spectator \( \bar{d} \) quark remains in the final state and \( A \) denotes the contributions from diagrams in which the spectator quark is annihilated and an additional \( q\bar{q} \) pair with all possible flavors is created by gluons.
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