BOSE-EINSTEIN CORRELATIONS IN CASCADE PROCESSES AND NON-EXTENSIVE STATISTICS

O.V. UTYUZH AND G. WILK

The Andrzej Soltan Institute for Nuclear Studies; Hoża 69; 00-689 Warsaw, Poland
E-mail: utyuzh@fuw.edu.pl and wilk@fuw.edu.pl

Z. WLODARCZYK

Institute of Physics, Pedagogical University; Konopnickiej 15; 25-405 Kielce, Poland
E-mail: wlod@pu.kielce.pl

We discuss the effect of nonextensivity of the emitting source on the Bose-Einstein correlations (BEC). This is done numerically by comparing cascade hadronization model (CAS), which is known to exhibit fractal structure in both space-time and phase-space, with its equivalent obtained from the information theory approach (MaxEnt), in which hadronization proceeds uniformly in the phase-space. To this end we have developed a new method of accounting for BEC in Monte Carlo event generators, which preserves all kinematics of the hadronization process.

Some time ago problem of sensitivity of Bose-Einstein correlations (BEC) to the possible fractal structure of emitting hadronic source has been formulated. Recently we have addressed this problem numerically by modelling such source in terms of the cascade process taking place both in phase-space and in space-time. In addition to the expected feature of intermittency observed in its multiparticle distributions it also shows a kind of modified Lévy distribution in space-time variables, which can be interpreted as a signal of nonextensivity arising due to the space-time fractal structure of such source. In this context it is worth to note recent investigations of the possible continuous emission from the otherwise hydrodynamically described expanding hadronic source, which leads also to a kind of foam-like (multifractal) space-time structure noticeably influencing the BEC. As we have demonstrated, BEC are, indeed, sensitive to the nonextensivity parameter $q$ which influences spacio-temporal development of the cascade process, i.e., its fractality. They are also sensitive to the particular ("slow" or "fast") way in which cascade develops (quantified by the mean life-time of the cascade link parameter $\tau$). However, in we did not compare explicitly nonextensive (such as cascade) approach to hadronization with an extensive one, our analysis concerned properties of the cascade process itself. Such comparison will be performed here. However, the "afterburner" method of incorporating BEC into event generators used in is not satisfactory for this purpose because
it changes the original event, making comparison of events coming from different hadronization models very difficult. The more satisfactory weighting procedures are, on the other hand, too much complicated for our purposes. The same is true for procedures relying on shifting of momenta, which in addition also change the initial energy-momentum balance.

We shall propose instead a new, simple procedure of modelling BEC, which preserves both the energy-momenta of all produced secondaries and their total multiplicity distributions (as well as their intermittency pattern). It changes, however, the charge allocation (if any) of the produced secondaries, preserving at the same time both the total charge of the initial source and multiplicities of charged and neutral secondaries resulting from a given event generator. Suppose that in the $l$th event ($l = 1, \ldots, N_{\text{event}}$) our generator provides us with $n_l = n_l^+ + n_l^- + n_l^0$ particles. Keeping their energy-momenta and spacio-temporal positions intact we shall now allocate to them anew the charges and this will be done in the following way:

1. One chooses randomly, with weights proportional to $p_l^+ = n_l^+/n_l$, $p_l^- = n_l^-/n_l$ and $p_l^0 = n_l^0/n_l$, the SIGN (from: "+", 
-" or "0") and attaches it to the particle ($i$) chosen randomly from particles produced in this event and not yet reassigned new charges.

2. One calculates distances in momenta, $\delta_{ij}(p) = |p_i - p_j|$, between the chosen particle ($i$) and all other particles still without signs and arranges them in order of the ascending $\delta_{ij}(p)$ with $j = 1$ denoting the nearest neighbour of particle ($i$). To each $\delta_{ij}(p)$ an appropriate weight $P(i, j)$ is then assigned (the form of which will be discussed below).

3. One selects from a uniform distribution a random number $r \in (0, 1)$. If $n_{\text{SIGN}} > 0$, i.e., if there are still particles of given SIGN with not reassigned charges, one checks the previously selected particles in ascending order of $j$ and if $r < P(i, j)$ then charge SIGN is assigned also to the particle ($j$), the multiplicity of particles with this SIGN is reduced by one, $n_{\text{SIGN}} = n_{\text{SIGN}} - 1$, and next particle, $j = j + 1$, is selected from that bunch. If the new $n_{\text{SIGN}} = 0$ one returns to point (1) but with the updated values of probabilities $p_l^+$, $p_l^-$ and $p_l^0$. However, if $r > P(i, j)$ then one returns to (1), again with the updated values of $p_l^+$, $p_l^-$ and $p_l^0$. Procedure finishes when $n_+ = n_- = n_0 = 0$, in which case one proceeds to the next event.

It is important to realize that the above method of choice of particles of the same SIGN leads to a geometrical (Bose-Einstein) distribution of particles in
this group (cell) (for $P(ij) = P = \text{const}$ its mean multiplicity equals $P/(1-P)$, in fact because it contains also $n = 0$, therefore in the algorithm it will be greater by 1). In this way one accounts for the bosonic character (Bose-Einstein statistics) of the produced particles and after application of this procedure they show strong tendency to occupy the same cell in the phase-space (defined, for example as wave packet in the momentum space centered on the mean momentum of selected particles). That this will show up in the 2-particle BEC correlation function $C_2(Q) = \frac{\sigma(p_1,p_2)}{\sigma(p_1)\sigma(p_2)} > 1$ for $Q = |p_1 - p_2| \to 0$ was already demonstrated in statistical model based on information theory (with conservation of charges imposed). One of the main parameters of this model was the size of phase-space cells containing particles of the same charge, which was therefore fixed and the same for all events. In our case both the sizes of phase-space cells and their number depend crucially on the weight parameters $P(ij)$ and can therefore vary both from event to event and also in a given event. Notice that, because we do not limit a priori the number of particles which can be put into given cell, we are, in fact, getting in this way automatically BEC of all orders. It means that $C_2$ presented in Figs. 1 and 2 are calculated in the environment of multiparticle BEC and as such can exceed 2 at some circumstances (as demonstrated in 9).

It is instructive to realize what kind of dynamical picture this method corresponds to in the case of cascade model. Notice that we do not change initial energy-momentum flow here, however, we do profoundly change the initial charge flow resulting from our event generator. Taking any example of cascade described in 3, allocating to final secondaries charges according to the proposed procedure and working then out charge flow backwards, one encounters strong charge fluctuations with multiple charges occurring in branching vertices, which were not present there originally (at the same time total charge of the whole system is at every cascade step always equal to the initial charge of the mass $M$ initiating cascade). This observation is, in fact, a general one: precisely the allowance for such charge fluctuations leads to the occurrence of like-charge bunching, which in turn, are interpreted as effect of BEC. In our case one could argue that such multicharged vertices should be introduced into the scheme of the hadronization cascade process itself (i.e., already into our Monte Carlo event generator). This is, however, an impossible task because it leads to unsurmountable problems with their subsequent proper deexcitation to single charged final particles. The relaxation of control over the initial charge flow (or lack of such altogether) is therefore necessary condition of applicability of the proposed algorithm.

As in all other approaches it will be important what kind of weight factors
$P(ij)$ we shall choose. Two such choices will be demonstrated here: $P(ij) = \text{const} = 0.5$ and $P(ij) = \exp[-\frac{1}{2} \delta_{ij}^2(x) \cdot \delta_{ij}^2(p)]$. This later form uses the available information on the particle production provided by event generator. One can argue that this is, in a sense, a “the most natural form” because of the following: if particles $(ij)$ would be described by the wave packets in the space-time, their widths would follow momentum separation $\delta_{ij}(p)$ and the corresponding probability distribution in $\delta_{ij}(x)$ would be of the above gaussian form.

Figure 1. Comparison of BEC for CAS (left panels) and MaxEnt (right panels) types of the emitting one-dimensional sources of masses $M = 100$, 40 and 10 GeV for constant value of parameter $P = 0.5$ (see text for details).

Figure 2. The same as in Fig. 1 but for the “most natural” choice of $P$ and for two different types of cascade evolution characterized by constant $\tau = 0.2$ fm - full symbols and mass-dependent $\tau = 1/M$ - open symbols (see text for details).

With this algorithm we can now easily compare the truly fractal source of hadronization provided by the cascade (CAS) with the most simple one corresponding to the instantaneous hadronization in the whole available phase space, as - for example - provided by the maximalization of the information entropy approach (MaxEnt). This is done in the following way: to each CAS event characterized by the multiplicity $n_i$ one builds the corresponding MaxEnt event according to the procedure outlined in (i.e., one calculates the corresponding Lagrange multiplier $\beta_i$ or “temperature” $T_i = 1/\beta_i$). Using now the same multiplicities, $n_i$, $n_i^{(+)}$, $n_i^{(-)}$ and $n_i^{(0)}$, as in CAS one calculates...
the corresponding BEC. The results for constant weight $P$ are given in Fig. 1 whereas Fig. 2 contains results using the most natural choice of the weights $P(ij)$. In the case on MaxEnt we argue that it is given by the following form: $P(ij) = \exp \left[ -\frac{(p_i-p_j)^2}{2\mu_T^2} \right]$. Here $p_i,j$ are the momenta of the particles considered, $\mu_T$ denotes their transverse mass, $k$ is Boltzmann constant and $T_l$ is the mentioned above "temperature" of the $l$th event.

Table 1. List of parameters $\gamma$, $\lambda$ and $R$ (in fm) fitting data shown in Figs. 1 and 2 by the following formula: $C_2(Q) = \gamma \cdot [1 + \exp(-R \cdot Q)]$.

| Model used: | $M$ | CAS | MaxEnt | CAS | MaxEnt | CAS | MaxEnt |
|------------|-----|------|--------|------|--------|------|--------|
| Fig. 1     | 10 GeV | 0.84 | 0.72 | 0.92 | 0.81 | 0.97 | 0.88 |
|            | 40 GeV | 1.42 | 1.49 | 1.25 | 1.26 |
|            | 100 GeV | 1.05 | 0.87 | 1.25 | 1.10 |
| Fig. 2     | 10 GeV | 0.76 | 0.80 | 0.95 | 1.00 | 0.97 | 1.02 |
| (\tau = 0.2) | CAS | MaxEnt | CAS | MaxEnt | CAS | MaxEnt |
| Fig. 2     | 40 GeV | 1.53 | 1.58 | 0.41 | 0.59 | 0.21 | 0.33 |
| (\tau = 1/M) | 100 GeV | 0.67 | 0.94 | 0.60 | 1.34 | 0.46 | 1.32 |
|            | 10 GeV | 0.49 | —    | 0.82 | —    | 0.93 | —    |
|            | 40 GeV | —    | 1.27 | —    | 0.55 | —    |
|            | 100 GeV | —    | 0.65 | —    | 0.65 | —    |

Our preliminary results are presented in Figs. 1 and 2 for one-dimensional case only (first 3-dimensional analysis of CAS is provided in [5]). The reference event for CAS was the original cascade itself, which do not shows any BEC effect at all (actually, the mixed-event method applied to MaxEnt gives in this case identical result). To facilitate estimations of differences between pictures presented in Figs. 1 and 2, we have listed in Table 1 parameters of simple exponential like parametrization of these results. There are some features worth of noticing. Fig. 1 demonstrates that constant (i.e., independent on the details of the hadronizing source given by event generators) weights lead to very similar BEC pattern in both types of models. It is given entirely by the number of particles of the same charge in a given cell, which depends on $P$ (the bigger $P$ the more particles and bigger $C_2(Q = 0)$; smaller $P$ leads to increasing number of cells, which results in decreasing $C_2(Q = 0)$, as was already noticed in [4]). Only by making $P$ depending on the details of hadronization process, as in Fig. 2, we start to see differences between models. But even in this case they are rather weak and $C_2$ depends on the parameters of the hadronizing source only as much as they influence the number of elementary cells and multiplicities in them. Therefore the "size" $R$ listed in Table 1 corresponds to the size of elementary cell rather than to the size of the hadronizing source (in fact for $\tau = 0.2$ fm the real size of CAS source grows from 0.29 fm for $M = 10$ GeV to 1.61 fm for $M = 100$ GeV and for $\tau = 1/M$...
from 0.12 fm to 0.62 fm, respectively). There is noticeable difference between CAS and MaxEnt cases for "slow" cascades ($\tau = 0.2$ fm). For "fast" ones (with $\tau = 1/M$, where particles are produced much earlier and more uniformly in space-time) both the extensive MaxEnt and nonextensive CAS schemes lead to similar BEC. More detailed analysis of this approach, with its application also to the 3-dimensional case will be presented elsewhere.

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