Addendum to: Dynamical torsion suppression in Brans–Dicke inflation and Lorentz violation (Eur Phys J C (2022)): Einstein–Cartan–Brans–Dicke–Maxwell universe with a chiral dynamo?

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Received: 27 April 2022 / Accepted: 3 July 2022
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Abstract We have recently shown that in Einstein–Cartan–Brans–Dicke (ECBD) gravity, torsion effects are not present in today’s universe because they are suppressed in a four-dimensional system, as in the bulk of five-dimensional braneworld endowed with torsion. In this addendum, we naturally introduce new features on the ECBD Maxwell Lagrangian dynamics deriving the chiral dynamo equation, and studying its physical properties. Magnetogenesis results are derived in this ECBDM universe. In particular we show, in the present universe, torsional magnetic fields of the second-order in the ohmic resistivity behave as $B_{\text{ECBD}} = 10^{-13}$ Gauss, which is exactly the estimate made by Miniati et al. (Phys Rev Lett, 2018) for the axion QCD magnetic field in the present universe. We found this at 1 pc coherent length, whereas they found it at 20 pc. To obtain this result, we consider a strong magnetic field of the order of the Biermann battery $10^3 G$. This seems to show that our model of ECBDM gravity can be used with success in inflationary magnetogenesis. To this end we might consider the same Lagrangian

$$L_{\text{ECBDM}} = \int d^4x \sqrt{-g} \left( -\phi R + \omega \partial_\mu \phi \partial^\mu \phi - \frac{\phi^3}{4} + J^\mu A_\mu \right)$$

where $\mu = 0, 1, 2, 3$ and $F^2 = F_{\mu\nu}F^{\mu\nu}$ is the electromagnetic field tensor squared, and ECBDM is the Einstein–Cartan–Brans–Dicke–Maxwell, where we add the electromagnetic Lagrangian without the axial anomaly term. Note that, torsion is not coupled with the EM fields. In this section we couple scalar fields to torsion via non-minimal coupling [10]. R in this Lagrangian is the Ricci–Cartan scalar. The new EM terms introduced here now allow us to introduce the chiral dynamo problem. The chiral total current is

$$J = \sigma [E + v \times B] + \mu_3 B$$

The chiral current is superposed on the electric current, where $\sigma = \eta^{-1}$, where $\sigma$ is the electrical conductivity and $\eta$ is the electrical resistivity. Now,

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{4\pi}{\omega} \left( \rho - 3p - \frac{8\pi \sigma^2}{\phi} \right)$$

where $\phi$ is the inflaton field and $H = \frac{\dot{a}}{a}$ is the Hubble expansion, while $a$ is the cosmic scale factor. The Friedmann–

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Robertson–Walker (FRW) metric is given by
\[ ds^2 = dt^2 - a^2(t) \left[ dx^2 + dy^2 + dz^2 \right] \] (4)

Units are used where, the gravitational constant \( G = 1 \), and the Ricci–Cartan scalar \( R \) is given by
\[ R = [\dot{S}_0 - 6S_0^2 + 12H^2] \] (5)

Now let us obtain the first field equations from the Euler–Lagrange (EL) equation for the cosmic scale \( a \),
Making use of expression (9) and only considering Hubble parameter \( H \) up to \( O(H^2) \), we obtain
\[ \phi^{-1} \mathcal{L}_{\text{ECBDM}} + 8H^2(1 + 2S_0) - [2H^2 + \dot{H}] = 0 \] (13)

Taking the ECBD Lagrangian in the form
\[ \phi^{-1} \mathcal{L}_{\text{ECBD}} \approx 12H^2 - \omega \left( \frac{\dot{\phi}}{\phi} \right)^2 \] (14)

Note that from this equation that if we constrain ECBDM cosmology to the case where expansion rebounds, universe expansion stops, \( (H = 0) \) and torsion is constant, this assumption can be used as a boundary condition to determine the BD parameter \( \omega \). The last expression reduces to
\[ \omega \left( \frac{\dot{\phi}}{\phi} \right)^2 + S_0^2 = 0 \] (15)

Assuming this specific boundary condition where inflation turns into deflation, one obtains from the last expression that
\[ S_0 = \sqrt{\omega t}^{-1} \] (16)

from previous equations. Then, torsion is non-constant, and substitution of the last expression squared into (14) yields the result
\[ \left( \frac{\dot{\phi}}{\phi} \right) = S_0 = \sqrt{\omega t}^{-1} \] (17)

This expression shall be very useful to solve chiral dynamo equation in the next section. Note that expression (16) above is quite important, due to the fact that torsion is determined in terms of the BD parameter \( \omega \). This agrees with the status quo that general relativity is a torsionless equation, since GR is characterized by \( (\omega = 0) \).

2 Helical chiral dynamos in Einstein–Cartan–Brans–Dicke–Maxwell gravity

In this section we shall finally derive the chiral dynamo equation in the case of the ECBDM model of the previous section. We recall that though we do not start from the coupling of torsion to EM fields; the magnetic field may be computed in terms of torsion, as we shall see in this section. Here in differential forms notation \( F = dA \) where \( A = (A_i dx^i) \) is the magnetic potential four-dimensional one form, where no torsion is present. Variation of the four-dimensional potential in the above Lagrangian is
\[ \partial_i (a^3 \phi^3 F^{ij}) = a^3 \phi^3 J_{5+c}^{ij} \] (18)
where electric current $J_c$ is given by

$$J_{(s+c)} = \mu_3 B + \sigma (E + v \times B)$$  \hspace{1cm} (19)$$

First current is the chiral, and the second is the normal electric current. Substitution of this current into the expression (19) yields

$$\left( \lambda^2 - \lambda \mu_5 \right) \left[ \nabla \times \left( \frac{\phi}{\phi} + H \right) - \sigma \right] = \sigma \nabla \times [v \times B]$$

which from the Bianchi identity

$$\partial_i F_{jk} = 0$$

we obtain the Faraday effect

$$\nabla \times E = - \partial_t B$$

After some algebra we obtain the chiral dynamo equation

$$\partial_t B = - \eta (\lambda^2 - \lambda \mu_5) [1 + 2 \eta (H + \omega t^{-1}) - i v \cdot k] B$$

where we have used Eq. (17) of the previous section. Since in the early universe the conductivity is very high and the ohmic resistivity is quite low, we may keep only terms up to first order in resistivity and drop terms like $O(\eta^2)$ and compute second-order contributions of the ohmic resistivity or diffusivity by the end of this section, where we compare both results. Under this assumption chiral dynamo equation reduces to

$$\partial_t B = - \eta (\lambda^2 - \lambda \mu_5) B$$

We now assume that chirality dominates over the relation between the magnetic helicity parameter and the chiral chemical potential, such as $\mu_5 \geq \lambda$. This allows us to express the last chiral dynamo equation in the form

$$\partial_t B = \eta \lambda \mu_5 B$$

and then now it is much easier to find the solution of this simple dynamo equation. This solution can be approximated by

$$B \approx B_{\text{seed}}(\eta^2 \lambda \mu_5 t)$$

where $B_{\text{seed}}$ is the seed magnetic field from the early universe, such as the adiabatic equilibrium field of the ECBDM model for cosmology. One obtains in the present universe a magnetic field of $10^{-13} \mu G$ from a seed field of the order of $10^{41} G$ obtained by Berera in the early universe, one would be able to obtain, instead an axionic or magnetic field in the present universe of $10^{-13}$. Using our solution, a magnetic field of $B_{\text{axion}} = 10^{-3} \mu G$, which is a field found at the core of some galaxies is found. We note that the ECBDM universe seems to be a promising model to improved results for magnetic fields from torsion, which are able to seed galactic dynamos at a reasonable coherence length. It is also important to note that, here we have not address back reaction into the system, which would require a metric of non-Friedmannian nature, because we are not using the axial anomaly term $E\cdot B$, and most of the magnetic fields obtained are very weak to back-react on the isotropic homogeneous universe considered.

3 Conclusions

In this addendum we have investigated the magnetogenesis in the case of the ECBDM model for cosmology. One obtains in the present universe a magnetic field of $10^{-13}$ Gauss from a second order in the ohmic diffusivity in chiral dynamo equation. With GUT seed fields of the order of $10^{41} G$ obtained by Berera in the early universe, one would be able to obtain, instead an axionic magnetic field in the present universe of $10^{-13}$. Using our solution, a magnetic field of $B_{\text{ECBD}}(\eta^2) = 10^{-3} \mu G$, which is a field found at the core of some galaxies is found. We note that the ECBDM universe seems to be a promising model to improved results for magnetic fields from torsion, which are able to seed galactic dynamos at a reasonable coherence length. It is also important to note that, here we have not address back reaction into the system, which would require a metric of non-Friedmannian nature, because we are not using the axial anomaly term $E\cdot B$, and most of the magnetic fields obtained are very weak to back-react on the isotropic homogeneous universe considered. Of course, solutions of several Lagrangians can be used in the near future to test the model dependence of the electrodynamics which we were using to further investigate magnetogenesis in Riemann–Cartan spacetime [24]. We were told very recently that Bamba et al. [25] have investigated helical magnetogenesis with a reheating phase and high-order curvature baryogenesis without torsion. It would be interesting if we tried to add torsion couplings even non-minimally with magnetogenesis along their lines. This work may appear in the near future. More on GR magnetogenesis may be found in the last reference [26]. Actually, Bamba et al. [28] have
investigated the non-helical magnetic fields in the reheating phase in higher-order curvature coupling. This is an interesting subject to work and to review in order to perform extensions to modified gravity magnetogenesis.

**Acknowledgements** We would like to express our gratitude to Universidade do Estado do Rio de Janeiro for partial financial support. Thanks are also due to Carl Brans for helpful discussions on his Brans–Dicke cosmology and Yu N Obukhov for discussions on the subject of this paper. Thanks are due to my wife Ana Paula Teixeira Araujo for her constant support.

**Data Availability Statement** This manuscript has associated data in a data repository. [Authors’ comment: No new data have been produced during this study. All used data are available upon reasonable request from L C Garcia de Andrade.]

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