Allocation of seats in the European Parliament and a degressive proportionality.

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Abstract

Distribution of seats in The European Parliament postulated by Treaty of Lisbon should be degressively proportional. The meaning of degressively proportional concept can be found in two principles annexed to the draft of European Parliament resolution. The first, referred as the principle of fair division, states that „the larger the population of a Member State, the greater is entitlement to a large number of seats”. The other condition, referred to as the principle of relative proportionality, holds that „the larger the population of a country, the more inhabitants are represented by each of its Members of the EU”. We postulate a clear and fair method which determines uniquely a distribution of seats in the European Parliament which fulfil the requirements of degressive proportionality.

More generally, let \( l_i \) be any non-increasing sequence of real positive numbers. We say that a sequence of natural numbers \( m_i \) is degressively proportional with respect to the sequence \( l_i \), if \( m_i \) and \( l_i/m_i \) are non-increasing sequences. Our method can be instrumental in uniquely determining a degressively proportional sequence \( m_i \) with respect to \( l_i \) which fulfils given conditions.

Keywords: fair division, relative proportionality, distribution function of discrete measure.

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1. Introduction

The European Parliament is one of the most important institutions of the European Union based on representations of members states. Principles of seats distribution in The EP have changed with subsequent EU enlargement stages. Due to large distribution of population between individual member states, no proportional method can be employed in seat distribution. Therefore another approach to apportionment was postulated. The postulate was expressed in Article 9a paragraph 2 of the Treaty of Lisbon. The article states that:

„The European Parliament shall be composed of representatives of the Union’s citizens. They shall not exceed seven hundred and fifty in number, plus the
Representations of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats". (Treaty of Lisbon [1]).

The meaning of the concept of degressive proportionality can be found in two principles annexed to the draft of European Parliament resolution. The first, referred as the principle of fair division, states that „the larger the population of a Member State, the greater is entitlement to a large number of seats”. The other condition, referred to as the principle of relative proportionality, holds that „the larger the population of a country, the more inhabitants are represented by each of its Members of the EU”. (Lamassoure and Severin [2]).

A formal approach to the definition of degressive proportionality was studied by Ramirez-Palmarez-Marquez [4] and Lyko-Cegielka-Dniestrański-Misztal [3]. Let $n$ represent the number of Members States, $l_i$ - the population of the $i$-th member, and $m_i$ - the number of mandates offered to the Member State. Suppose that $l_1 > l_2 > \ldots > l_n$. Then the sequence $m_i$ is degressively proportional with respect to the sequence $l_i$ if it is non-increasing and satisfies the following condition:

$$\frac{l_1}{m_1} > \frac{l_2}{m_2} > \ldots > \frac{l_n}{m_n}. \tag{1}$$

The present composition of the European Parliament does not satisfy the principles of the degressive proportionality. Distribution of seats in the European Parliament postulated by Committee on Constitutional Affairs members Lamassoure and Severin does indeed fulfill the requirements of degressive proportionality. The main problem is to find a clear and fair method (acceptable for all Member States) which determines uniquely a sequence $m_i$ degressively proportional with respect to $l_i$.

For a real number $x$ we denote by $\lceil x \rceil$ the least integer $\geq x$. All the following sequences are sequences of length $n$ or $n-1$, where $n \geq 2$ is a fixed number. The definition of degressive proportionality can be slightly extended as follows. Let $l_i$ be any fixed non-increasing sequence of real positive numbers. We say that a sequence of natural numbers $m_i$ is **degressively proportional with respect to the sequence $l_i$**, if $m_i$ is non-increasing and

$$\frac{m_1}{l_1} \leq \frac{m_2}{l_2} \leq \ldots \leq \frac{m_n}{l_n}. \tag{1}$$

It Theorem 1(a) we prove that a sequence of natural numbers $m_i$ is degressively proportional with respect to $l_i$ if and only if it is defined inductively

$$m_1 = M,$$

$$m_i = \min \left( \left\lceil \frac{m_{i-1}}{l_{i-1}} \right\rceil + a_i, m_{i-1} \right), \quad \text{for } 2 \leq i \leq n, \tag{2}$$

for some sequence $a_i \geq 0$ and a constant $M \in \mathbb{N}$. Note that a sequence $M_i$ defined inductively in Theorem 1(b) is degressively proportional with respect to $l_i$ and is smaller, then any other such sequence with the first element $\geq M$. The pair $(M, \{a_i\})$ will be called the **initial condition for the sequence $m_i$**.
Fix constants $M, Y \in \mathbb{N}$ such that $Y \geq \sum M_i$. We suggest the following method of the choice of a sequence $m_i$ degressively proportional with respect to $l_i$ and satisfying the inequalities

$$
\sum m_i \leq Y \quad \text{and} \quad m_1 = M.
$$

Fix a sequence $a_i \geq 0$. For every $c \geq 0$, let $m_i(c)$ denote the degressively proportional sequence with respect to $l_i$, with the initial condition $(M, \{ca_i\})$.

In Theorem 2(a) we prove that all the functions $m_i: c \mapsto m_i(c), 1 \leq i \leq n$, and the function $\Phi = \sum m_i$, are left-continuous and non-decreasing on the positive half line $[0, \infty)$. Hence, there exists a positive number $c_Y$ such that

$$
\Phi(c_Y) = \max \{\Phi(c) : \Phi(c) \leq Y\}.
$$

We postulate the choice of the sequence $m_i(c_Y)$ dominating all of other sequences degressively proportional with respect to $l_i$, with initial condition $(M, \{ca_i\})$, for every $c > 0$, and satisfying inequalities (3).

In Theorem 3(b) we determine the constant $\delta$ such that $\Phi(\delta)$ is the largest value of the function $\Phi$. If $Y \in [\Phi(0), \Phi(\delta)]$, then Theorem 4(b) provides the lower bound of the value $\Phi(c_Y)$. This implies

$$
Y - (n - 2) \leq \sum m_i(c_Y) \leq Y.
$$

The problem of computing the number $c_Y$ is equivalent to finding some points (not necessarily the discontinuity points) at which the function $\Phi$ takes its successive values. We present two methods which lead to finding such points:

(a) If $\Phi(c_0)$ is not the largest value of $\Phi$, then $c_0 + \beta(c_0)$ is the first discontinuity point belonging to $[c_0, \infty)$, where $\beta(c_0)$ is the constant described in Theorem 5.

(b) For every $c \geq 0$, the function $\Phi$ has in $[c, c + \gamma(c)]$ at most one point of discontinuity, where $\gamma(c)$ is the constant of Theorem 6. If $\Phi(c) < \Phi(c + \gamma(c))$, then $\Phi(c)$ and $\Phi(c + \gamma(c))$ are consecutive values of $\Phi$. We are able, therefore, to find consecutive values of $\Phi$ without knowing of the discontinuity points of $\Phi$.

Hence, if $\Phi(c)$ is not the largest value of the function $\Phi$, then

$$
\Phi(c) \quad \text{and} \quad \Phi(c + \beta(c) + \gamma(c + \beta(c)))
$$

are consecutive values of $\Phi$ (see Remark 4). For a real number $x$ we denote by $\lfloor x \rfloor_k$ the decimal representation of $x$ up to $k$ digits after the decimal point. It is convenient to choose $k$ such that

$$
\lfloor(1 - l_i/l_{i-1})/a_i\rfloor_k > 0, \text{ for } a_i \neq 0 \text{ and } l_i \neq l_{i-1},
$$

(cf. the constants $\gamma_3$ and $\gamma$ of Theorem 6). Using Excel we can easily compute

$$
\Phi(c) \quad \text{and} \quad \Phi(c + [\beta(c)]_k + [\gamma(c + [\beta(c)]_k)]_k).
$$
which are practically different (accordingly, consecutive values of \( \Phi \)).

Let us come back to the initial problem of distribution of sets in the European Parliament. In this case \( n = 27 \), \( l_1 > l_2 > \ldots > l_{27} \) is a sequence of populations of Member States, and \( M = 96 \) is the number of mandates offered to Germany. The number \( a_i, 2 \leq i \leq n \), can be regarded as the degree of preference of the \( i \)-th Member State. Since a natural intention of the European community is to offer a fair representation to all members, we consider the case when \( a_i \) is a constant sequence, say \( a_i \equiv 1 \), as reflecting this intention. For each \( c \geq 0 \), let \( m_i(c) \) be a degressively proportional sequence with respect to \( l_i \), with the initial condition \((96, \{c\})\).

In the columns of Table 1 we present the values of the functions \( m_i : c \mapsto m_i(c) \), \( 1 \leq i \leq 27 \), and the function \( \Phi = \sum m_i \) at the points

\[
\begin{align*}
c_1 &= 1.11, c_2 = 1.140625, c_3 = 1.25731913, \\
c_4 &= 1.5, c_5 = 1.555, c_6 = 1.6, c_7 = 1.7,
\end{align*}
\]

respectively. Notice that

\[
(\sum m_i(c_k), m_{27}(c_k)) = (736, 5), (751, 5), (757, 6), \quad \text{for } k = 1, 2, 3.
\]

Using Excel and methods (a) and (b), we can prove that the numbers

\[
736, 751, 757, 758, 773, 779, 784
\]

are all values of the function \( \Phi = \sum m_i \) which are in the interval \([736, 784]\) (see Examples).

2. Main result

Let \( l_i \) be a non-increasing sequence of real positive numbers. In Theorem 1 we characterize all degressively proportional sequences with respect to \( l_i \).

\textbf{Theorem 1.} Let \( l_i > 0 \) be any non-increasing sequence.

(a) A sequence of natural numbers \( m_i \) is degressively proportional with respect to \( l_i \) if and only if it is defined inductively by \((3)\) for some sequence \( a_i \geq 0 \) and a constant \( M \in \mathbb{N} \).

(b) The following sequence

\[
\begin{align*}
M_1 &= M, \\
M_i &= \left\lfloor \frac{M_{i-1}l_i}{l_{i-1}} \right\rfloor, \quad \text{for } 2 \leq i \leq n.
\end{align*}
\]

is a non-increasing minorant, that is, it is smaller than any other sequence with the first element \( \geq M \) which satisfies condition \((7)\).
Proof. If \( m_i \) is a sequence of natural numbers defined inductively by (2), then
\[ m_i \geq m_{i-1} \frac{l_i}{l_i-1}. \]
Hence, \( m_i \) satisfies (1). If \( m_i \) is a non-increasing sequence of natural numbers which satisfies (1), then
\[ a_i = m_i - m_{i-1} \frac{l_i}{l_i-1} \geq 0. \]
Hence, \( m_i \) is defined inductively by (2).

From \( M_i \frac{l_i}{l_i-1} \leq M_{i-1} \), we see that \( M_i \) is non-increasing. Let \( v_i \) be a sequence of natural numbers which satisfies (1), and \( v_1 \geq M \). We prove (b) by induction. Assume that \( M_i \leq v_i \). Then,
\[ \frac{M_i}{l_i} \leq \frac{v_i}{l_i} \leq \frac{v_{i+1}}{l_{i+1}}. \]
Hence, \( M_{i+1} = [M_i \frac{l_{i+1}}{l_i}] \leq v_{i+1} \), which concludes (b).

Remark 1. Without loss of generality we can assume (see (2)) that the se-
Hence, $\Phi$ are non-decreasing. We only need to show the following implications:

For all $2 \leq i \leq n$, if $l_i = l_{i-1}$ then $a_i = 0$. \hspace{1cm} (4)

**Theorem 2.** Let $l_i > 0$ be any non-increasing sequence, $M \in \mathbb{N}$, and $a_i \geq 0$. For every $c \geq 0$, let $m_i(c)$ denote a degressively proportional sequence with respect to $l_i$, with the initial condition $(M, \{ca_i\})$. Set

$$A_i(c) = \frac{m_{i-1}(c)l_i}{l_{i-1}} + ca_i,$$

(5)

for $2 \leq i \leq n$, and $c \geq 0$.

Then all the functions $m_i: c \mapsto m_i(c)$, and the function $\Phi = \sum m_i$, are left-continuous and non-decreasing on the positive half line $[0, \infty)$.

**Proof.** It is easily seen that all the functions $m_i$, $2 \leq i \leq n$, and the function $\Phi$ are non-decreasing. We only need to show the following implications:

(i) for every $1 \leq i < n$, if $m_i(c - \varepsilon) = m_i(c)$ for sufficiently small $\varepsilon > 0$, then $[A_{i+1}(c - \varepsilon)] = [A_{i+1}(c)]$ for another sufficiently small $\varepsilon > 0$.

(ii) for every $1 \leq i < n$ and $0 \leq c \leq d$, if $[A_{i+1}(c)] = [A_{i+1}(d)]$ and $m_i(c) = m_i(d)$, then $m_{i+1}(c) = m_{i+1}(d)$.

Proof (i). We proceed by induction on $i$. Assume that $m_i(c - \varepsilon) = m_i(c)$ for sufficiently small $\varepsilon > 0$. Then for another sufficiently small $\varepsilon > 0$ we obtain

$$[A_{i+1}(c)] - 1 < A_{i+1}(c - \varepsilon) = A_{i+1}(c) - \varepsilon a_{i+1} \leq [A_{i+1}(c)].$$

Hence, $[A_{i+1}(c - \varepsilon)] = [A_{i+1}(c)]$.

Proof (ii). We proceed by induction on $i$. Assume that $[A_{i+1}(c)] = [A_{i+1}(d)]$ and $m_i(c) = m_i(d)$. Then,

$$m_{i+1}(c) = \min ([A_{i+1}(c)], m_i(c)) = \min ([A_{i+1}(d)], m_i(d)) = m_{i+1}(d),$$

which completes the proof. \hfill \Box

**Theorem 3.** Let $l_i > 0$ be any non-increasing sequence, $M \in \mathbb{N}$, and $a_i \geq 0$. For every $c \geq 0$, let $m_i(c)$ denote a degressively proportional sequence with respect to $l_i$, with the initial condition $(M, \{ca_i\})$.

(a) Suppose that $c_0 \geq 0$ satisfies the following condition:

For every $2 \leq i \leq n$, $m_i(c_0) = m_{i-1}(c_0)$ or $a_i = 0$.

Then all functions $m_i: c \mapsto m_i(c)$, $1 \leq i \leq n$, and the function $\Phi = \sum m_i$ are constant on the half line $[c_0, \infty)$.

(b) Set

$$\delta = M \max \left\{ \frac{1}{a_i} \left( 1 - \frac{l_i}{l_{i-1}} \right) : a_i \neq 0 \right\}.$$

The function $\Phi$ takes all its values in the interval $[0, \delta]$. 

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Proof. Fix \( c \geq c_0 \). Let \( A_i(c) \), \( 2 \leq i \leq n \), be the sequence defined by (2). We proceed by induction on \( i \). Assuming \( m_i(c) = m_i(c_0) \), we will prove it for \( i+1 \). If \( m_{i+1}(c) = m_i(c_0) \), then

\[
m_{i+1}(c) = \min ([A_{i+1}(c)], m_i(c)) = \min ([A_{i+1}(c)], m_{i+1}(c_0)) = m_{i+1}(c_0).
\]

The last equality follows from \([A_{i+1}(c)] \geq [A_{i+1}(c_0)] \geq m_{i+1}(c_0)\). If \( a_{i+1} = 0 \), then \([A_{i+1}(c)] = [A_{i+1}(c_0)]\). Hence, \( m_{i+1}(c) = m_{i+1}(c_0)\).

We turn to condition (b). If \( a_i \neq 0 \), then, by the definition of the number \( \delta \), we have

\[
\delta \geq \frac{M}{a_i} \left( 1 - \frac{l_i}{l_{i-1}} \right) \geq \frac{m_i-1(\delta)}{a_i} \left( 1 - \frac{l_i}{l_{i-1}} \right).
\]

Hence,

\[
A_i(\delta) = \frac{m_i-1(\delta)l_i}{l_{i-1}} + \delta a_i \geq m_i-1(\delta).
\]

Accordingly,

\[
m_i(\delta) = \min ([A_i(\delta)], m_{i-1}(\delta)) = m_{i-1}(\delta).
\]

Hence, by (a), the function \( \Phi \) is constant on the half line \([\delta, \infty)\). \( \square \)

Theorem 4. Let \( l_i > 0 \) be any non-increasing sequence, \( M \in \mathbb{N} \), and suppose that \( a_i \geq 0 \) is different from the zero sequence \((0, \ldots, 0)\) satisfying the condition (3). For every \( c \geq 0 \), let \( m_i(c) \) denote a degressively proportional sequence with respect to \( l_i \), with the initial condition \((M, \{ca_i\})\).

(a) Then,

\[
\alpha = \min \left\{ \frac{1}{a_i} \left( 1 - \frac{l_i}{l_{i-1}} \right) : a_i \neq 0 \right\} > 0.
\]

For every \( 2 \leq i \leq n \), and \( c \geq 0 \),

\[
m_i(c + \alpha) \leq m_i(c) + 1.
\]

(b) For every natural number \( Y \in [\Phi(0), \Phi(\delta)] \), there is a point \( x \in [0, \delta] \) such that

\[
Y - (n - 2) \leq \Phi(x) \leq Y.
\]

Proof. Since \( a_i \) is different from the zero sequence and satisfies implication (4), then \( \alpha > 0 \). The second part of condition (a) is proved by induction on \( i \). Fix \( c \geq 0 \). Let \( A_i(c) \), \( 2 \leq i \leq n \), be a sequence defined by (3). Suppose that \( m_i(c + \alpha) \leq m_i(c) + 1 \). If \( a_{i+1} \neq 0 \), then, by the definition of the number \( \alpha \),

\[
A_{i+1}(c + \alpha) \leq A_{i+1}(c) + \frac{l_{i+1}}{l_{i}} + \alpha a_{i+1} \leq [A_{i+1}(c)] + 1.
\]

If \( a_{i+1} = 0 \), then the above equalities are also satisfied. Hence,

\[
m_{i+1}(c + \alpha) = \min ([A_{i+1}(c + \alpha)], m_{i}(c + \alpha))
\]

\[
\leq \min ([A_{i+1}(c)] + 1, m_{i}(c) + 1) = m_{i+1}(c) + 1.
\]
Let $Y \in [\Phi(0), \Phi(\delta)]$. If the function $\Phi$ is not constant on the interval $[0, \delta]$, then there is a point of discontinuity $c_k \in [0, \delta]$ such that $\Phi(c_k) \leq Y < \Phi(c_k + \alpha)$. By (a), for every $2 \leq i \leq n$,

$$m_i(c_k + \alpha) \leq m_i(c_k) + 1.$$  

Hence,

$$\Phi(c_k) \leq Y < \Phi(c_k + \alpha) \leq \Phi(c_k) + n - 1,$$

which concludes (b).

\[ \square \]

**Remark 2.** In order to determine a point $x$ satisfying assertions of Theorem 4(b) we do not need to find the discontinuity points of $\Phi$. The point $x$ can be found by consecutive dividing intervals into intervals of equal length after \( \log_2 \lceil \delta / \alpha \rceil \) steps (starting from the interval $[0, \delta]$ and ending with the interval of length less than or equal to $\alpha$).

**Theorem 5.** Let $l_i > 0$ be any non-increasing sequence, $M \in \mathbb{N}$, and $a_i \geq 0$. For every $c \geq 0$, let $m_i(c)$ denote the degressively proportional sequence with respect to $l_i$, with the initial condition $(M, \{a_i\})$. Suppose that $\Phi(c_0)$ is not the largest value of the function $\Phi$: $c \mapsto \Phi(c) = \sum m_i(c)$. Set

$$J = J(c_0) = \{2 \leq i \leq n : m_i(c_0) < m_{i-1}(c_0) \text{ and } a_i \neq 0\},$$

$$\beta = \beta(c_0) = \min \left\{ \frac{1}{a_i} ([A_i(c_0)] - A_i(c_0)) : i \in J \right\},$$

where $A_i(c_0)$, $2 \leq i \leq n$, is a sequence defined by $[0, \infty)$.

Then $J \neq \emptyset$ and $c_0 + \beta$ is the first discontinuity point belonging to $J$ such that $\beta a_j = [A_j(c_0)] - A_j(c_0)$. We first prove

(i) for every $1 \leq i \leq n$, $m_i(c_0 + \beta) = m_i(c_0),$

(ii) if $i < j$, then $m_i(c_0 + \beta + \varepsilon) = m_i(c_0)$, for sufficiently small $\varepsilon > 0$,

(iii) $m_j(c_0 + \beta + \varepsilon) = m_j(c_0) + 1$ for sufficiently small $\varepsilon > 0$.

Proof (i). We proceed by induction on $i$. Suppose that $m_i(c_0 + \beta) = m_i(c_0)$. If $i + 1 \in J$, then, by the definition of the constant $\beta$, we obtain

$$A_{i+1}(c_0 + \beta) = A_{i+1}(c_0) + \beta a_{i+1} \leq [A_{i+1}(c_0)].$$

Since $[A_{i+1}(c_0 + \beta)] = [A_{i+1}(c_0)]$, we have $m_{i+1}(c_0 + \beta) = m_{i+1}(c_0)$. In the case $i + 1 \notin J$, the proof is analogous to the proof of Theorem 3(a).

Proof (ii). We proceed by induction on $i < j$. Assume that $i + 1 < j$ and $m_i(c_0 + \beta + \varepsilon) = m_i(c_0)$, for sufficiently small $\varepsilon > 0$. If $i + 1 \in J$, then, by the definition of the constant $\beta$ and by that of the number $j$, $\beta a_{i+1} < [A_{i+1}(c_0)] - A_{i+1}(c_0)$. Hence, for another sufficiently small $\varepsilon > 0$, we obtain

$$A_{i+1}(c_0 + \beta + \varepsilon) = A_{i+1}(c_0) + (\beta + \varepsilon) a_{i+1} < [A_{i+1}(c_0)].$$

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Since \([A_{i+1}(c_0 + \beta + \varepsilon)] = [A_{i+1}(c_0)]\), we have \(m_{i+1}(c_0 + \beta + \varepsilon) = m_{i+1}(c_0)\). In the case \(i + 1 \not\in J\), the proof is analogous to the proof of Theorem 3(a).

Proof (iii). By (ii), \(m_{j-1}(c_0 + \beta + \varepsilon) = m_{j-1}(c_0)\), for sufficiently small \(\varepsilon > 0\). Hence, by the definition of the number \(j\), we have

\[
A_j(c_0 + \beta + \varepsilon) = A_j(c_0) + (\beta + \varepsilon)a_j = [A_j(c_0)] + \varepsilon a_j < [A_j(c_0)] + 1.
\]

Therefore, for sufficiently small \(\varepsilon > 0\), we obtain

\[
m_j(c_0 + \beta + \varepsilon) = \min([A_j(c_0 + \beta + \varepsilon)], m_{j-1}(c_0)) = \min([A_j(c_0)] + 1, m_{j-1}(c_0)) = m_j(c_0) + 1.
\]

The least equality follows from \(m_j(c_0) < m_{j-1}(c_0)\).

By (i), \(\Phi(c_0 + \beta) = \Phi(c_0)\), while by (iii), \(\Phi(c_0 + \beta + \varepsilon) > \Phi(c_0)\), for \(\varepsilon > 0\), and the proof is complete. \(\Box\)

**Remark 3.** If \(l_i\) and \(a_i\) are sequences of rational numbers, then all the points of discontinuity of \(\Phi = \sum m_i\) are rational too.

**Theorem 6.** Let \(l_i > 0\) be any non-increasing sequence, \(M \in \mathbb{N}\), and suppose that \(a_i \geq 0\) is different from the zero sequence \((0, \ldots , 0)\) satisfying the condition \((\ref{condition})\). For every \(c \geq 0\), let \(m_i(c)\) denote the degressively proportional sequence with respect to \(l_i\), with the initial condition \((M, \{c a_i\})\). For \(c \geq 0\), set

\[
\omega = \omega(c) = \min \left\{ \frac{1}{a_i} ([A_i(c)] - A_i(c)) : a_i \not= 0 \right\},
\]

\[
J_1 = \left\{ 2 \leq i \leq n : A_i(c) + \frac{l_i}{l_{i-1}} < [A_i(c)] \text{ and } a_i \not= 0 \right\},
\]

\[
J_2 = \left\{ 2 \leq i \leq n : [A_i(c)] \leq A_i(c) + \frac{l_i}{l_{i-1}} \text{ and } a_i \not= 0 \right\},
\]

\[
J_3 = \left\{ 2 \leq i \leq n : \frac{1}{a_i} ([A_i(c)] - A_i(c)) = \omega \text{ and } i \in J_2 \right\},
\]

\[
\gamma_1 = \min \left\{ \frac{1}{a_i} \left( [A_i(c)] - A_i(c) - \frac{l_i}{l_{i-1}} \right) : i \in J_1 \right\},
\]

\[
\gamma_2 = \min \left\{ \frac{1}{a_i} ([A_i(c)] - A_i(c)) : i \in J_2 \setminus J_3 \right\},
\]

\[
\gamma_3 = \min \left\{ \frac{1}{a_i} \left( 1 - \frac{l_i}{l_{i-1}} \right) : i \in J_3 \right\},
\]

where \(A_i(c)\), \(2 \leq i \leq n\), is a sequence defined by \((\ref{sequence})\).

Then,

\[
\gamma = \gamma(c) = \min \{ \gamma_k : J_k \neq \emptyset \} > 0.
\]

The function \(\Phi : c \mapsto \Phi(c) = \sum m_i(c)\) has in \([c, c + \gamma(c)]\) at most one point of discontinuity.

**Proof.** Since \(a_i\) is different from the zero sequence, the set \(J_1 \cup J_2\) is not empty. Hence, by implication \((\ref{discontinuity})\), \(\gamma > 0\). We now turn to the next part of the proof.
By Theorem 5, we may assume that $\omega(c) \leq \beta(c) < \gamma(c)$. It suffices to show that $\Phi$ is constant at the interval $(c + \omega, c + \gamma]$. To this purpose, we prove that one of the following conditions is satisfied for every $2 \leq i \leq n$:

(j) $m_i$ equals $m_i(c)$ on the interval $(c + \omega, c + \gamma]$,

(jj) $m_i$ equals $m_i(c) + 1$ on the interval $(c + \omega, c + \gamma]$.

We proceed by induction. Suppose that condition (j) or (jj) holds for $i$. We first prove that one of the following conditions holds for $i + 1$.

(k) $[A_{i+1}]$ equals $[A_{i+1}(c)]$ on the interval $(c + \omega, c + \gamma]$,

(kk) $[A_{i+1}]$ equals $[A_{i+1}(c)] + 1$ on the interval $(c + \omega, c + \gamma]$.

If $i + 1 \in J_1$, then, for every $x \in (c + \omega, c + \gamma]$,

$$A_{i+1}(c) \leq A_{i+1}(x) \leq A_{i+1}(c) + \frac{l_{i+1}}{l_i} + (x - c)a_{i+1} \leq [A_{i+1}(c)].$$

If $i + 1 \in J_2$ and condition (j) (respectively (jj)) holds for $i$, then, for every $x \in (c + \omega, c + \gamma]$,

$$A_{i+1}(c) \leq A_{i+1}(x) = A_{i+1}(c) + (x - c)a_{i+1} \leq [A_{i+1}(c)]$$

(or

$$[A_{i+1}(c)] < A_{i+1}(x) = A_{i+1}(c) + \frac{l_{i+1}}{l_i} + (x - c)a_{i+1} \leq [A_{i+1}(c)] + 1,$$

respectively). If $i + 1 \in J_3$, then, for every $x \in (c + \omega, c + \gamma]$,

$$[A_{i+1}(c)] < [A_{i+1}(x)] + (x - c - \omega)a_{i+1} = A_{i+1}(c) + (x - c)a_{i+1}
\leq A_{i+1}(x) \leq A_{i+1}(c) + \frac{l_{i+1}}{l_i} + (x - c)a_{i+1} \leq [A_{i+1}(c)] + 1.$$

If $a_{i+1} = 0$, then

$$A_{i+1}(x) = \begin{cases} A_{i+1}(c) & \text{for } m_i(x) = m_i(c), \\ A_{i+1}(c) + \frac{l_{i+1}}{l_i} & \text{for } m_i(x) = m_i(c) + 1. \end{cases}$$

We proceed to show that condition (j) or (jj) holds for $i + 1$. If condition (j) holds for $i$, and condition (k) holds for $i + 1$, then, for every $x \in (c + \omega, c + \gamma]$, we have

$$m_{i+1}(x) = \min ([A_{i+1}(x)], m_i(x)) = \min ([A_{i+1}(c)], m_i(c)) = m_{i+1}(c).$$

If condition (jj) holds for $i$, and condition (kk) holds for $i + 1$, then, for every $x \in (c + \omega, c + \gamma]$, we have

$$m_{i+1}(x) = \min ([A_{i+1}(x)], m_i(x))
= \min ([A_{i+1}(c)] + 1, m_i(c) + 1) = m_{i+1}(c) + 1.$$
If condition (j) holds for \( i \), and condition (kk) holds for \( i + 1 \), then, for every \( x \in (c + \omega, c + \gamma] \), we have

\[
m_{i+1}(x) = \min \left( \lceil A_{i+1}(c) \rceil + 1, m_i(c) \right) = \begin{cases} 
m_{i+1}(c) & \text{for } \lceil A_{i+1}(c) \rceil \geq m_i(c), 
m_i(c) + 1 & \text{for } \lceil A_{i+1}(c) \rceil < m_i(c). \end{cases}
\]

If condition (jj) holds for \( i \), and condition (k) holds for \( i + 1 \), then, for every \( x \in (c + \omega, c + \gamma] \), we have

\[
m_{i+1}(x) = \min \left( \lceil A_{i+1}(c) \rceil, m_i(c) + 1 \right) = \begin{cases} 
m_{i+1}(c) & \text{for } \lceil A_{i+1}(c) \rceil \leq m_i(c), 
m_i(c) + 1 & \text{for } \lceil A_{i+1}(c) \rceil > m_i(c), \end{cases}
\]

which completes the proof.

\[ \square \]

**Remark 4.** We conclude from Theorems 5 and 6 that if \( \Phi(c) \) is not the largest value of the function \( \Phi \), then

\[ \Phi(c) \text{ and } \Phi(c + \beta(c) + \gamma(c + \beta(c))) \]

are consecutive values of \( \Phi \).

### 3. Examples

Recall that for a real number \( x \) we denote by \( [x]_8 \) the decimal representation of \( x \) up to eight digits after the decimal point. Suppose \( l_1 > l_2 > \ldots > l_{27} \) is the sequence of populations of Member States of the European Parliament. Let \( m_i(c), c \geq 0 \), be a degressively proportional sequence with respect to \( l_i \), with the initial condition \((96, \{c\})\), and \( \Phi(c) = \sum m_i(c) \).

Suppose we want to find \( 1.11 < c_2 < c_3 \), such that \( \Phi(1.11), \Phi(c_2), \Phi(c_3) \) are consecutive values of \( \Phi \). This can be accomplished in the following steps:

**S1.** Set \( c_1 = 1.11 \). Find \( \lfloor \beta(c_1) \rfloor_8 \) (cf. Table 3).

**S2.** Set \( d_1 = c_1 + \lfloor \beta(c_1) \rfloor_8 \). Find \( \lfloor \gamma(d_1) \rfloor_8 \) (cf. Table 4).

**S3.** Set \( c_2 = d_1 + \lfloor \gamma(d_1) \rfloor_8 \). Find \( \lfloor \beta(c_2) \rfloor_8 \) (cf. Table 5).

**S4.** Set \( d_2 = c_2 + \lfloor \beta(c_2) \rfloor_8 \). Find \( \lfloor \gamma(d_2) \rfloor_8 \) (cf. Table 6).

**S5.** Set \( c_3 = d_2 + \lfloor \gamma(d_2) \rfloor_8 \). Find \( \lfloor \beta(c_3) \rfloor_8 \) (cf. Table 7).

**S6.** Set \( d_3 = c_3 + \lfloor \beta(c_3) \rfloor_8 \).

By Theorem 5, \( \Phi \) is constant on the interval \([c_1, d_1], [c_2, d_2]\) and \([c_3, d_3]\). Since \( \Phi(d_1) < \Phi(c_2) \) and \( \Phi(d_2) < \Phi(c_3) \), Theorem 6 shows that \( \Phi \) has exactly one discontinuity point in \([d_1, c_2]\), and in \([d_2, c_3]\) too.
|       | $l_i$ | $A_i$ | $[A_i]$ | $M_i$ | $1 - \frac{l_i}{l_{i-1}}$ |
|-------|-------|-------|---------|-------|-----------------------------|
| 1. Germany | 82.438 |       | 96      |       |                             |
| 2. France  | 62.999 | 73.36306072 | 74 | 74 | 0.235801451 |
| 3. Gr. Britain | 60.393 | 70.93893554 | 71 | 71 | 0.041365736 |
| 4. Italy   | 58.752 | 69.07078635 | 70 | 70 | 0.027172023 |
| 5. Spain   | 43.758 | 52.13541667 | 53 | 53 | 0.255280333 |
| 6. Poland  | 38.157 | 46.21602907 | 47 | 47 | 0.127999452 |
| 7. Romania | 21.610 | 26.61818277 | 27 | 27 | 0.433655686 |
| 8. Netherlands | 16.334 | 20.46801834 | 21 | 21 | 0.244146229 |
| 9. Greece  | 11.125 | 14.30298763 | 15 | 15 | 0.318905351 |
| 10. Portugal | 10.570 | 14.25168539 | 15 | 15 | 0.049887640 |
| 11. Belgium | 10.511 | 14.91627247 | 15 | 15 | 0.005581835 |
| 12. Czech Rep. | 10.251 | 14.62896014 | 15 | 15 | 0.024735991 |
| 13. Hungary | 10.077 | 14.74539069 | 15 | 15 | 0.016973954 |
| 14. Sweden | 9.048  | 13.46829414 | 14 | 14 | 0.102113724 |
| 15. Austria | 8.266  | 12.79000884 | 13 | 13 | 0.086427940 |
| 16. Bulgaria | 7.719  | 12.13972901 | 13 | 13 | 0.066174662 |
| 17. Denmark | 5.427  | 9.139914497 | 10 | 10 | 0.296929654 |
| 18. Slovak Rep. | 5.389 | 9.929979731 | 10 | 10 | 0.097902027 |
| 19. Finland | 5.256  | 9.753290965 | 10 | 10 | 0.024679904 |
| 20. Ireland | 4.209  | 8.007990868 | 9  | 9  | 0.199200913 |
| 21. Lithuania | 3.403 | 7.276550249 | 8  | 8  | 0.191494417 |
| 22. Latvia  | 2.295  | 5.395239495 | 6  | 6  | 0.325550636 |
| 23. Slovenia | 2.003  | 5.236001397 | 6  | 6  | 0.127233115 |
| 24. Estonia | 1.345  | 4.028956565 | 5  | 5  | 0.328502339 |
| 25. Cyprus  | 0.766  | 2.847583643 | 3  | 3  | 0.430483271 |
| 26. Luxemburg | 0.469 | 1.836814621 | 2  | 2  | 0.387728466 |
| 27. Malta   | 0.405  | 1.727078891 | 2  | 2  | 0.136460554 |
| $\Phi(0)$  |        |        |         |      | 645                          |

Table 2: In column 5 of Table 2 the values of minorant $M_i$ are put, cf. Theorem 1(c). Note that $|1 - l_i/l_{i-1}|_8 > 0$, for $2 \leq i \leq 27$. 

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|       | \( l_i \) | \( A_i \)  | \( [A_i] \) | \( m_i(c_1) \) | \( [A_i] - A_i \) |
|-------|----------|-----------|-----------|--------------|-----------------|
| 1. Germany | 82.438   | 74.47306072 | 75 | 75 | 0.526939276 |
| 2. France   | 62.999   | 74.47306072 | 75 | 75 | 0.526939276 |
| 3. Gr. Britain | 60.393 | 73.0075698 | 74 | 74 | 0.992430197 |
| 4. Italy     | 58.752   | 73.0075698 | 74 | 74 | 0.900729720 |
| 5. Spain     | 43.758   | 56.22458333 | 57 | 57 | 0.775416667 |
| 6. Poland    | 38.157   | 50.81403126 | 51 | 51 | 0.185968737 |
| 7. Romania   | 21.610   | 29.99356003 | 30 | 30 | 0.006439972 |
| 8. Netherlands | 16.334 | 23.78561314 | 24 | 24 | 0.214386858 |
| 9. Greece    | 11.125   | 17.45627158 | 18 | 18 | 0.543728419 |
| 10. Portugal  | 10.570   | 18.21202247 | 19 | 19 | 0.787977528 |
| 11. Belgium   | 10.511   | 19.00952696 | 20 | 20 | 0.990473037 |
| 12. Czech Rep. | 10.251 | 18.66475216 | 19 | 19 | 0.335247836 |
| 13. Hungary   | 10.077   | 18.80446883 | 19 | 19 | 0.195531168 |
| 14. Sweden    | 9.048    | 17.27195296 | 18 | 18 | 0.728047038 |
| 15. Austria   | 8.266    | 17.55429708 | 18 | 18 | 0.445702918 |
| 16. Bulgaria  | 7.719    | 17.91885555 | 18 | 18 | 0.081144447 |
| 17. Denmark   | 5.427    | 13.76526023 | 14 | 14 | 0.234733774 |
| 18. Slovak Rep. | 5.389 | 15.01197162 | 16 | 16 | 0.988028377 |
| 19. Finland   | 5.256    | 14.76448135 | 15 | 15 | 0.235518649 |
| 20. Ireland   | 4.209    | 12.32118721 | 13 | 13 | 0.678812785 |
| 21. Lithuania | 3.403    | 11.62057258 | 12 | 12 | 0.379427417 |
| 22. Latvia    | 2.295    | 9.202859242 | 10 | 10 | 0.797140758 |
| 23. Slovenia  | 2.003    | 9.837668845 | 10 | 10 | 0.162331155 |
| 24. Estonia   | 1.345    | 7.82497609  | 8  | 8  | 0.175072391 |
| 25. Cyprus    | 0.766    | 5.666133829 | 6  | 6  | 0.333866171 |
| 26. Luxemburg | 0.469    | 4.783629243 | 5  | 5  | 0.216370757 |
| 27. Malta     | 0.405    | 5.427697228 | 6  | 6  | 0.572302772 |

\( \Phi(c_1) \) 736

Table 3: \( c_1 = 1.11 \) and \( |\beta(c_1)|_8 = 0.00643997 \). In column 6 of Table 3 the values of the sequence \( m_i(c_1) \) are put.
|    | $l_1$ | $A_i$ | $[A_i]$ | $m_i(d_1)$ | $[A_i] - A_i$ | $[A_i] - A_i - \frac{l_1}{l_{i-1}}$ |
|----|------|------|--------|------------|---------------|----------------------------------|
| 1  | Germany | 82.438 | 96     | 96         | 520499306    | -0.243699244                     |
| 2  | France  | 62.999 | 74.47950609 | 75       | 0.592759 | -0.243699244                     |
| 3  | Gr. Britain | 66.393 | 73.01400977 | 74       | 0.980590227 | 0.027355963                     |
| 4  | Italy   | 58.752 | 73.10571025 | 74       | 0.894289750 | -0.078538226                     |
| 5  | Spain   | 43.758 | 56.23102330 | 57       | 0.768976697 | 0.024185030                     |
| 6  | Poland  | 38.157 | 50.82047123 | 51       | 0.179528767 | -0.692471781                     |
| 7  | Romania | 21.610 | 30.00000000 | 30       | 1.69589E-09 | -0.566344313                     |
| 8  | Netherlands | 16.334 | 23.79205311 | 24       | 0.207946888 | -0.54706883                     |
| 9  | Greece  | 11.125 | 17.46271155 | 18       | 0.537288449 | -0.143806200                     |
| 10 | Portugal| 10.570 | 18.21846244 | 19       | 0.781535758 | -0.168574801                     |
| 11 | Belgium | 10.511 | 19.01596693 | 20       | 0.984033067 | -0.010350898                     |
| 12 | Czech Rep. | 10.251 | 18.67119213 | 19       | 0.328807866 | -0.646456144                     |
| 13 | Hungary | 10.077 | 18.81090880 | 19       | 0.189091198 | -0.793934849                     |
| 14 | Sweden  | 9.048  | 17.27839293 | 18       | 0.721607068 | -0.176279208                     |
| 15 | Austria | 8.266  | 17.56073705 | 18       | 0.439262948 | -0.474309112                     |
| 16 | Bulgaria| 7.719  | 17.92529552 | 18       | 0.074704477 | -0.859120831                     |
| 17 | Denmark | 5.427  | 13.77106200 | 14       | 0.228293804 | -0.47767542                     |
| 18 | Slovak Rep. | 5.389 | 15.018441159 | 16       | 0.981588407 | -0.011490566                     |
| 19 | Finland | 5.256  | 14.77092132 | 15       | 0.229078679 | -0.746241417                     |
| 20 | Ireland | 4.209  | 12.32762718 | 13       | 0.672372815 | -0.128426271                     |
| 21 | Lithuania | 3.403 | 11.62701255 | 12       | 0.372087447 | -0.435518136                     |
| 22 | Latvia  | 2.295  | 9.209929212 | 10       | 0.790700788 | 0.116295851                     |
| 23 | Slovenia| 2.003  | 9.401408815 | 10       | 0.155891185 | -0.716875700                     |
| 24 | Estonia | 1.345  | 7.831367579 | 8        | 0.168632421 | -0.502860339                     |
| 25 | Cyprus  | 0.766  | 5.672573799 | 6        | 0.327426201 | -0.242090528                     |
| 26 | Luxembourg | 0.469 | 4.790699213 | 5        | 0.20930787 | -0.402340753                     |
| 27 | Malta   | 0.405  | 5.434137198 | 6        | 0.565862802 | -0.297676644                     |

Table 4: $d_1 = c_1 + |\beta(c_1)| = 1.11643997$. Since $J_3 = \{7\}$ and $|1-l_7/l_6| = 0.43365568$ (see Table 2), we have $|\gamma(d_1)| = 0.02418503$. 
|        | \( l_i \) | \( A_i \) | \( \lfloor A_i \rfloor \) | \( m_i(c_2) \) | \( \lfloor A_i \rfloor - A_i \) |
|--------|------------|-----------|----------------|----------------|----------------|
| 1. Germany | 82.438 | 96 | 75 | 75 | 0.496314276 |
| 2. France  | 62.999 | 74.50368572 | 74 | 74 | 0.961805197 |
| 3. Gr. Britain | 60.393 | 73.03819480 | 74 | 74 | 0.870104720 |
| 4. Italy | 58.752 | 73.12989528 | 74 | 74 | 0.744791667 |
| 5. Spain | 43.758 | 56.25520833 | 57 | 57 | 0.496314276 |
| 6. Poland | 38.157 | 50.84465626 | 51 | 51 | 0.155343737 |
| 7. Romania | 21.610 | 30.02415803 | 31 | 31 | 0.975814972 |
| 8. Netherlands | 16.334 | 24.57209191 | 25 | 25 | 0.427908087 |
| 9. Greece | 11.125 | 18.16799123 | 19 | 19 | 0.832008770 |
| 10. Portugal | 10.570 | 19.19275983 | 20 | 20 | 0.807240169 |
| 11. Belgium | 10.511 | 20.03457013 | 21 | 21 | 0.965429872 |
| 12. Czech Rep. | 10.251 | 19.67064117 | 20 | 20 | 0.329358826 |
| 13. Hungary | 10.077 | 19.81811988 | 20 | 20 | 0.181880121 |
| 14. Sweden | 9.048 | 18.20046424 | 19 | 19 | 0.799535762 |
| 15. Austria | 8.266 | 18.49849414 | 19 | 19 | 0.501505858 |
| 16. Bulgaria | 7.719 | 18.88330586 | 19 | 19 | 0.116694139 |
| 17. Denmark | 5.427 | 14.49896157 | 15 | 15 | 0.501038428 |
| 18. Slovak Rep. | 5.389 | 16.03559460 | 17 | 17 | 0.964405404 |
| 19. Finland | 5.256 | 15.77042645 | 16 | 16 | 0.229573553 |
| 20. Ireland | 4.209 | 13.15261130 | 14 | 14 | 0.847388699 |
| 21. Lithuania | 3.403 | 12.45970317 | 13 | 13 | 0.540296834 |
| 22. Latvia | 2.295 | 9.907889179 | 10 | 10 | 0.092110821 |
| 23. Slovenia | 2.003 | 9.868293845 | 10 | 10 | 0.15706155 |
| 24. Estonia | 1.345 | 7.855552609 | 8 | 8 | 0.144447391 |
| 25. Cyprus | 0.766 | 5.696758829 | 6 | 6 | 0.303241171 |
| 26. Luxemburg | 0.469 | 4.814254243 | 5 | 5 | 0.185745757 |
| 27. Malta | 0.405 | 5.458322228 | 6 | 6 | 0.541677772 |

Table 5: \( c_2 = d_1 + \lfloor \gamma(d_1) \rfloor_s = 1.140625 \) and \( \lfloor \beta(c_2) \rfloor_s = 0.09211082 \). In column 6 of Table 5 the values of the sequence \( m_i(c_2) \) are put.
\[ l_i, A_i, [A_i], m_i(d_2), [A_i] - A_i, [A_i] - A_i - \frac{l_i}{l_{i-1}} \]

|       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|
| 1. Germany | 82.438 |       |       |       | 96    |
| 2. France  | 62.999 | 74.59579654 | 75 | 75 | 0.404203456 | -0.359995094 |
| 3. Gr. Britain | 60.393 | 73.13030562 | 74 | 74 | 0.869694377 | -0.088939887 |
| 4. Italy   | 58.752 | 73.22206010 | 74 | 74 | 0.779939900 | -0.194834076 |
| 5. Spain   | 43.758 | 56.34731915 | 57 | 57 | 0.652680847 | -0.092110820 |
| 6. Poland  | 38.157 | 50.93676708 | 51 | 51 | 0.063232917 | -0.808767631 |
| 7. Romania | 21.610 | 30.11629585 | 31 | 31 | 0.883704152 | -0.317359837 |
| 8. Netherlands | 16.334 | 24.66429273 | 25 | 25 | 0.335797267 | -0.420056505 |
| 9. Greece  | 11.125 | 18.26010205 | 19 | 19 | 0.739879750 | 0.05803301 |
| 10. Portugal | 10.570 | 19.28487065 | 20 | 19 | 0.715129349 | -0.23493011 |
| 11. Belgium | 10.511 | 20.12680955 | 21 | 21 | 0.873190521 | -0.12099112 |
| 12. Czech Rep. | 10.251 | 19.76275199 | 20 | 20 | 0.237249492 | -0.190461334 |
| 13. Hungary | 10.077 | 19.91023070 | 20 | 20 | 0.089769301 | -0.893256745 |
| 14. Sweden  | 9.048  | 18.29257506 | 19 | 19 | 0.707424942 | -0.504177022 |
| 15. Austria | 8.266  | 18.59060496 | 19 | 19 | 0.400395038 | -0.504177022 |
| 16. Bulgaria | 7.719  | 18.97541668 | 19 | 19 | 0.024583319 | -0.909249990 |
| 17. Denmark | 5.427  | 14.5907239 | 15 | 15 | 0.408927608 | -0.29424738 |
| 18. Slovak Rep. | 5.389  | 16.12770542 | 17 | 17 | 0.872294584 | -0.120703390 |
| 19. Finland | 5.256  | 15.86253727 | 16 | 16 | 0.137462733 | -0.287857364 |
| 20. Ireland | 4.209  | 13.24472212 | 14 | 14 | 0.755277879 | -0.045521208 |
| 21. Lithuania | 3.403  | 12.55181399 | 13 | 13 | 0.448186014 | -0.360319569 |
| 22. Latvia  | 2.295  | 9.999999999 | 10 | 10 | 1.33412E-09 | -0.674404935 |
| 23. Slovenia | 2.003  | 9.969404665 | 10 | 10 | 0.039595335 | -0.831171550 |
| 24. Estonia | 1.345  | 7.947663429 | 8 | 8 | 0.052365671 | -0.619156189 |
| 25. Cyprus  | 0.766  | 5.788869649 | 6 | 6 | 0.211130351 | -0.358386378 |
| 26. Luxemburg | 0.469  | 4.906365063 | 5 | 5 | 0.093634937 | -0.518636603 |
| 27. Malta   | 0.405  | 5.550433048 | 6 | 6 | 0.449566952 | -0.413972494 |

Φ(d_2) 751

Table 6: \( d_2 = c_2 + [\beta(c_2)]_9 = 1.23273582 \). Since \( J_3 = \{22\} \) and \([1 - l_{22}/l_{21}]_9 = 0.32559506 \) (see Table 2), we have \([\gamma(d_2)]_9 = 0.02458331\).
| Rank | Country        | $l_i$  | $A_i$       | $[A_i]$ | $m_i(c_3)$ | $[A_i] - A_i$ |
|------|---------------|--------|------------|---------|------------|--------------|
| 1    | Germany       | 82.438 | 74.62037985| 75      | 75         | 0.379620146  |
| 2    | France        | 62.999 | 73.15488893| 74      | 74         | 0.845111067  |
| 3    | Gr. Britain   | 60.393 | 73.15488893| 74      | 74         | 0.753410590  |
| 4    | Italy         | 58.752 | 73.24658941| 74      | 74         | 0.628097537  |
| 5    | Spain         | 43.758 | 56.37190246| 57      | 57         | 0.038649607  |
| 6    | Poland        | 38.157 | 50.96135039| 51      | 51         | 0.859120842  |
| 7    | Romania       | 21.610 | 30.14087916| 31      | 31         | 0.311213957  |
| 8    | Netherlands   | 16.334 | 24.68878604| 25      | 25         | 0.690546039  |
| 9    | Greece        | 11.125 | 18.28468536| 19      | 19         | 0.715314640  |
| 10   | Portugal      | 10.570 | 19.30945396| 20      | 20         | 0.848735742  |
| 11   | Belgium       | 10.511 | 20.15126426| 21      | 21         | 0.212664696  |
| 12   | Czech Rep.    | 10.251 | 19.78733530| 20      | 20         | 0.859120842  |
| 13   | Hungary       | 10.077 | 19.93481401| 20      | 20         | 0.682841632  |
| 14   | Sweden        | 9.048  | 18.31715837| 19      | 19         | 0.575141607  |
| 15   | Austria       | 8.266  | 18.61518827| 19      | 19         | 0.384811728  |
| 16   | Bulgaria      | 7.719  | 18.99999999| 19      | 19         | 0.64021E-09  |
| 17   | Denmark       | 5.427  | 14.61565570| 15      | 15         | 0.84344298   |
| 18   | Slovak Rep.   | 5.389  | 16.15228873| 17      | 17         | 0.847711274  |
| 19   | Finland       | 5.256  | 15.88712058| 16      | 16         | 0.112879423  |
| 20   | Ireland       | 4.209  | 13.26930543| 14      | 14         | 0.730694569  |
| 21   | Lithuania     | 3.403  | 12.57639730| 13      | 13         | 0.423602704  |
| 22   | Latvia        | 2.295  | 10.02458331| 11      | 11         | 0.975416691  |
| 23   | Slovenia      | 2.003  | 10.85775486| 11      | 11         | 0.142254140  |
| 24   | Estonia       | 1.345  | 8.643739499| 9       | 9          | 0.356260501  |
| 25   | Cyprus        | 0.766  | 6.382969688| 7       | 7          | 0.617030312  |
| 26   | Luxemburg     | 0.469  | 5.543219913| 6       | 6          | 0.456780087  |
| 27   | Malta         | 0.405  | 6.438555804| 7       | 7          | 0.561444196  |

| $\Phi(c_3)$ | 757 |

Table 7: $c_3 = d_2 + \lfloor \gamma(d_2) \rfloor = 1.25731943$. In column 6 of Table 7 the values of the sequence $m_i(c_3)$ are put. Since $m_{16}(c_3) = m_{15}(c_3)$, we have $\lfloor \beta(c_3) \rfloor = 0.0386496$. 

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References

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