Regarding the evaluation of car stability during lateral slips

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**Abstract.** The purpose of this paper is to highlight the analysis of the way in which the automobile can enter the state of instability when curving and the influence of some construction parameters (position of the weight centre, axle track, elasticity of the tires) and functional parameters (speed, acceleration, braking) which can influence the dynamic behavior. In this paper, there are described some dynamic simplified models of the automobile when curving, the way in which the experimental determinations have been made when a real automobile is involved in many circular routes with speeds and rays that we know. This paper also describes the methodology of study and the devices used as equipment for the automobile during the study on routes in order to register some specific parameters important for limiting the possibility of the appearance of lateral slips.

1. Introduction
If at the current estimation of motor vehicles performance and in particular cars performance, among the terms most frequently used, in relation to engine performance (peak power, hourly consumption or specific consumption, etc.) or to the entire motor vehicle performance (acceleration, maximum speed, start-up time and space, braking time and space, transmission organization and composition, type of brakes, steering and suspension, fuel consumption at 100 km, facilities and equipment used, etc.) information or opinions regarding the limits of stability provision, that the motor vehicle is able to achieve in certain driving regimes, are found only seldom.

We consider that it is difficult to specify such performance and limit criteria, considering the diversity of travel regimes that an automobile may have throughout its life. However, given the importance of maintaining the stability of the car under any travel conditions, such criteria become increasing necessary to be determined and specified since the projecting phase of the vehicle.

It is harmful and uneconomical for a motor vehicle not to be technically compliant all the time, but it is extremely hazardous for it to lose stability, because then the control of the car in question is lost.

2. Motor vehicle driving in turn
The quantitative estimation of turning characteristics is quite difficult because of the undetermined static pattern of the motor vehicle. The lateral deviation angles of the four wheels $\delta_1e$, $\delta_1i$, $\delta_2e$ and $\delta_2i$, each depending on several factors, render the location of the steering area, in which the dynamic steering center
is found, extremely difficult. However, based on experimental research, the specialized literature provides a series of information with regard to the steering characteristics.

In addition to the classical cases of insufficient or excessive turning, we use the so-called own control (subcommand, override), which characterizes the change of the motor vehicle rotational speed around the vertical axis, for the travel on a circular trajectory.

The main parameter for assessing the motor vehicle capacity to undergo trajectory modifications becomes the ratio between the angular speed of the motor vehicle longitudinal axis and the steering angle of the wheels, thus emphasizing the delay or the advance of the reaction of the car in relation to the steering angle of the wheels.

For driving on a circle with \( R_c \) radius (figure 1), the machine with a normal steering capacity corresponds to the case of rigid wheels (figure 1, position a). With the increase of the normal acceleration, due to the centrifugal force, the angles of lateral deviation \( \delta_1 \) and \( \delta_2 \) increase to the same extent.

\[
\delta_1 = \delta_2 = \delta
\]

\[
v_1^2 = v_c^2 + g \cdot \delta
\]

\[
\theta_1 > \theta > \theta_2
\]

\[
R_c \leq R
\]

For the motor vehicles with an insufficient steering characteristic, the front axle will involve a deviation movement toward the outside of the curve; the car does not enter the turn and tends to maintain a straight trajectory (figure 1, position b). The rotation speed around the vertical axis decreases with the increase of the normal acceleration and in order to maintain the turning radius \( R_c \), the steering wheels must be turned to the angle \( \theta_1 > \theta \).

For the motor vehicles with an excessive steering characteristic, the rear axle slips toward the outside of the curve, the car tends to leave the imposed trajectory toward the outside of the curve (figure 1, position c). The rotation speed increases with the normal acceleration of the car and in order to maintain the turning radius \( R_c \), the steering angle of the wheels must be \( \theta_2 < \theta \).

The sensitivity of the car when turning the wheels decreases for understeering cars, the critical understeering characterizing the final refusal to enter the turn and increases in oversteering motor vehicles. The critical oversteering appears at the continuous decrease of the turning radius after a spiral.

We can affirm that cars with an excessive steering have a critical speed and overspeeding leads to a steering instability, while understeering cars have a characteristic speed but it is not critical.
If a transversal force (figure 2) is experienced by a car in straight travel, transversal force which can be the wind force $F_v$, in case of neutral steering, the car moves still in a straight line, but inclined in relation to the initial direction to the lateral deviation angle $\delta$; in the event of understeering, the movement becomes curvilinear, the curve being in the same direction as the force (the angle between the speed and the transversal force will be under 90°), the curvature center becoming $O_\delta$; in the case of oversteering, the curvilinear travel is performed on a trajectory with a reverse curvature in relation to the force (the angle between the speed and the transversal force will be higher than 90°), the curvature center becoming $O_\delta$.

The critical speed is determined starting from the dynamic turning radius

$$R_\delta = \frac{L}{\theta + (\delta_2 - \delta_1)}$$

From the relationship of normal acceleration: $a_n = R \cdot \omega_a^2 = \frac{V_a^2}{R_c}$, where $V_a$ is the speed of the motor vehicle; $\omega_a$ – the angular speed for trajectory crossing by the motor vehicle.

The inertial forces $F_{cy}$ and $F_i$ are equal to the product between the mass of the car $m$ and the acceleration of the center of gravity - normal $a_{cn}$ and tangential $a_{ct}$. For the determination of these accelerations we consider the movement of the car in relation to a rectangular system of axles x and y (figure 3)

According to figure 3 we can write: $\omega = \frac{d \sigma}{dt}$; $\frac{d \omega}{dt} = a_\omega = \frac{d^2 \sigma}{dt^2}$

where:
- $\omega$ is the angular speed of the car while turning;
- $\sigma$ is the angle between the axis AB and the axis x;
- $\frac{d\omega}{dt}$ is the angular acceleration of the car.

**Figure 3.** The movement of the car in relation to a system of axes.

We understand that axis AB is not parallel to the axis of the coordinate OX as presented in figure 3.

We determine the projections $V_x$ and $V_y$ of the speed $V_c$ of the car’s center of gravity on the axes $x$ and $y$. We consider the positive sense of these speeds in the direction of the axes and after the decomposition of speed $V_c$ into speeds $V_{cn}$ and $V$ we obtain:

$$v_x = v \cdot \cos \sigma + v_{cn} \cdot \sin \sigma$$
$$v_y = v_{cn} \cdot \cos \sigma - v \cdot \sin \sigma$$

(2)

After differentiating these equations in relation to time, we obtain the expressions of the center of gravity accelerations on the axes $x$ and $y$:

$$a_{cx} = \frac{d^2v_x}{dt^2} = \frac{d}{dt} \cdot \cos \sigma - v \cdot \sin \sigma + \frac{dv_{cn}}{dt} \cdot \sin \sigma + v_{cn} \cdot \omega \cdot \cos \sigma$$
$$a_{cy} = \frac{d^2v_y}{dt^2} = \frac{d}{dt} \cdot \cos \sigma - v_{cn} \cdot \omega \cdot \sin \sigma - \frac{dv_{cn}}{dt} \cdot \sin \sigma - v \cdot \omega \cdot \cos \sigma$$

(3)

At the same time, the accelerations $a_{cx}$ and $a_{cy}$ of the car’s center of gravity, having the same direction as the system of axes, can be determined directly as projections of the accelerations $a_c$ and $a_{cn}$.

If the speed $V_c$ decreases, then:

$$a_{cx} = a_c \cdot \cos \sigma - a_{cn} \cdot \sin \sigma$$
$$a_{cy} = -a_{cn} \cdot \cos \sigma - a_c \cdot \sin \sigma$$

(4)

After solving the equations (3) and (4) we obtain the final expressions for the accelerations of the center of gravity of the car:
Using the graphical recording equipment and some transducers mounted on the car, we recorded the variation in time of the steering wheels turning angles from the initial straight direction.

Based on these graphics we determined for each trajectory the bypass time $t_0$ from the moment of wheels steering and until the end of the maneuver, after which the oscillation amplitude of the longitudinal axis of the car becomes equal to or less than 1.5°.

It is also possible to perform a statistical processing of the graph, by calculating the total steering angle of the wheels and the average value of the angular speed of rotation of the longitudinal axis of the car, which confronts with the subjective estimation of the experimenting driver.

For this purpose the car moves with speeds of 10, 20, 30 km/h in such a way that the outer front wheel stamp on curvilinear trajectories with radii of 10, 15 and 20 m respectively, which are marked on the testing platform (figure 4.).

\[
\begin{align*}
    a_{en} &= \omega \cdot v - \frac{dv_{en}}{dt} \\
    a_{ef} &= \frac{dv}{dt} + \omega \cdot v_{en}
\end{align*}
\]

$\Delta_1$, $\Delta_2$, $\Delta_3$, $\Delta_4$, $\Delta_5$, $\Delta_6$.

\[ R_1 = 10 \text{ m}, \quad R_2 = 15 \text{ m}, \quad R_3 = 20 \text{ m} \]

\[
\begin{align*}
    \Delta_1 &= 45 \text{ m} \\
    \Delta_2 &= 35 \text{ m} \\
    \Delta_3 &= 150 \text{ m} \\
    \Delta_4 &\quad \text{Main direction deviation}
\end{align*}
\]

**Figure 4.** Moving on a circular trajectory.

The movement on the circular trajectory with a radius of 15 m, was carried out with steady speeds of 20, 25 and 30 km/h, each distance being crossed for 5 times. Figure 5 represents the movement of the car on the circular trajectory obtained with the values generated by the GPS component of the measuring system. The trajectory is represented in geographical coordinates, latitude and longitude, expressed in minutes.

The data obtained has been stored on the SD card, in a specific format of the measurement and acquisition system, this data constituting the database required for processing with RACELOGIC - VBOX TOOLS software, in order to convert it into a format supported by the Microsoft Excel application (Annexes). During the trials 26 rows of values were recorded on the SD card, following that for the subsequent processing such values to be totally or partly activated depending on the objective pursued.
Figure 5 Circular trajectory in geographical coordinates.

After carrying out the trials, the trajectory shown in figure 5 was represented based on the initial values having as input values the geographical coordinates expressed in minutes. With the calculated data we were able to draw the car’s movement trajectory by using the values of the wheels steering angles.

Figure 6 presents the car’s movement trajectory, obtained after the experimental data processing, having as calculation parameters the values of the steering angles of the wheels.

Figure 6. Trajectory representation in Cartesian coordinates.

3. Data measuring and acquisition equipment
The data measuring and acquisition equipment is composed of: GPS VBOX module (1), data digital conversion module RLVBMIM01 (2), accelerometer and the yaw sensor (3), memory card type CD (4) -
which stores the measured sizes in real time based on the GPS component, GPS antenna (5), switch for marking the positions of the car ordered by the operator (6).

![Figure 7. Measuring and acquisition equipment.](image)

Table 1 shows a part of the actual values recorded by the equipment used during car’s driving on a circular trajectory with a speed of 20km/h.

| X  | Speed [km/h] | Lat. acc. [m/s²] | Long. acc. [m/s²] | Latit. [min] | Long. [min] | Yaw angle speed [rad/s] | Dist. [m] | Tens. [V] |
|----|--------------|------------------|------------------|--------------|-------------|----------------------|----------|----------|
| 0  | 21.31        | -0.327645851     | -0.50986591      | 2699.30531   | -1482.20514 | 0                    | 138.17   | 1.808020592 |
| 0.1| 21.39        | -0.475865404     | 0.022660707      | 2699.30506   | -1482.20542 | 0.593055556         | 142.67   | 1.796801567 |
| 0.2| 21.17        | -0.369450618     | -0.062316944     | 2699.30479   | -1482.20568 | 1.184166667         | 146.2    | 1.781267643 |
| 0.3| 20.67        | -0.171676398     | -0.141629418     | 2699.30453   | -1482.20592 | 1.765277777         | 147.88   | 1.770911694 |
| 0.4| 20.5         | -0.406405024     | -0.048154002     | 2699.30426   | -1482.20613 | 2.337083333         | 151.89   | 1.767459631 |
| 0.5| 20.2         | -0.47635484      | -0.084977651     | 2699.30398   | -1482.20632 | 2.902361111         | 156.66   | 1.758829713 |
| 0.6| 20.1         | -0.355745912     | -0.028325884     | 2699.30377   | -1482.20647 | 3.462083333         | 160.24   | 1.756240726 |
| 0.7| 19.33        | -0.286690923     | -0.218109304     | 2699.30342   | -1482.20666 | 4.009722222         | 163.24   | 1.755377769 |
| 0.8| 19.21        | -0.297257305     | -0.033991106     | 2699.30313   | -1482.20676 | 4.545               | 166.37   | 1.755377769 |
| 0.9| 19.27        | -0.342008577     | 0.01699553       | 2699.30285   | -1482.20678 | 5.079444444         | 169.96   | 1.755377769 |
| 1  | 19.31        | -0.338899916     | 0.011330353      | 2699.30256   | -1482.20683 | 5.615277777         | 173.51   | 1.754514694 |
| 1.1| 19.09        | -0.26331219      | -0.062316944     | 2699.30227   | -1482.20687 | 6.148611111         | 176.3    | 1.752788782 |
| 1.2| 19.32        | -0.351492267     | 0.065149532      | 2699.30198   | -1482.20688 | 6.682083333         | 179.98   | 1.751925707 |
| 1.3| 19.69        | -0.351409706     | 0.104805769      | 2699.30168   | -1482.20686 | 7.223888889         | 183.59   | 1.751925707 |
| 1.4| 20.19        | -0.292458854     | 0.141629418      | 2699.30138   | -1482.20682 | 7.777777778         | 186.52   | 1.752788782 |
| 1.5| 20.16        | -0.400661318     | -0.008497765     | 2699.30108   | -1482.20675 | 8.338194445         | 190.54   | 1.751925707 |
| 1.6| 20.44        | -0.346605816     | 0.079312474      | 2699.30078   | -1482.20665 | 8.902083333         | 193.97   | 1.752788782 |
| 1.7| 20.69        | -0.362096717     | 0.070814709      | 2699.30049   | -1482.20653 | 9.473333334         | 197.51   | 1.754514694 |
| 1.8| 20.64        | -0.415302874     | -0.014162942     | 2699.30019   | -1482.20638 | 10.047361111        | 201.58   | 1.757966757 |
| 1.9| 20.76        | -0.252477854     | 0.03399106       | 2699.29991   | -1482.20626 | 10.623611111        | 204.04   | 1.764007688 |
| 2  | 20.97        | -0.362850152     | 0.059484356      | 2699.29962   | -1482.20582 | 11.209444444        | 207.54   | 1.7778157 |
| 2.1| 20.98        | -0.437702224     | 0.002832588      | 2699.29935   | -1482.20578 | 11.784583333        | 211.76   | 1.789034605 |
Figure 8 includes the representation of graphical records of values measured by means of the apparatus VBox during the experimental testing: the motor vehicle’s travelling speed, longitudinal and lateral acceleration, yaw rate of the car to travel on the circle, the coordinates (longitude and latitude) of the testing place, the distance travelled during a test.

Figure. 8. Graphical records of the values measured during the experimental testing

4. Conclusions
This work had as objective to determine the limit traveling speeds of a real motor vehicle on known circular trajectories, in order to be able to establish the limit conditions for the occurrence of car’s side slippage. This allowed, for actual testing conditions (motor vehicle, turning radius known), the possibility to determine the limit speeds for entry in turning at which the car’s side slipping may occur. After the processing and interpretation of the results, it comes out that the analytical functions (using Table Curve software) determined for all three travel trajectories of travel have the same equation, with different coefficients, with an accuracy of 0.9998; from these relations it results that, for any type of car, the lateral stability limit is determined according to the traveling speed.

For all tests performance we used vehicles in the same state of equipment, corresponding to the "standard" vehicle, situation in which, based on specific values, mathematical functions were generated with the help of which, by extrapolation, we can predict car’s behavior as regards stability up to reaching the limits of safety.

Starting from the data obtained it can be assessed that, if the vehicle is fitted with equipment for measuring in real time the values of the sizes that contribute to the maintaining of stability, regardless of
the loading and travel regime, it will be possible to warn the driver but also to put into service certain systems able to prevent the loss of stability.

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