Computational study of the three dimensional autoresonance acceleration of electrons cloud in magnetic fields varying on time

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Abstract. There are many particle acceleration mechanisms using electromagnetic waves and external magnetic fields where the electron cyclotron resonance condition is preserved. Among these mechanisms, the gyroresonant acceleration proposed by K. Golovanivsky consists of the 2D acceleration of electrons by a circular polarized standing transversal electric wave and a homogeneous magnetic field, which changes on time to compensate the increase of relativistic factor as the electrons are accelerated. By using an analytical model, he obtained a set of differential equations describing both the evolution of the electron energy and the phase shift between the particle velocity and the electric field component of the microwave field. The 3D simulation of the electrons bunch acceleration by the TE$_{111}$ cylindrical mode of 2.45 GHz in frequency and the strength of 100 kV/m, said acceleration mechanism is numerically studied. In our numerical scheme, the inhomogeneous magnetic field is produced by a set of 4 current coils, wherein the currents in the central coils change linearly on time. In the transversal mid plane, the initial magnetic field is fitted to the classical resonance value. The concentration of the electron bunch is $n_e = 2.6 \times 10^8$ cm$^{-3}$, so the simple particle approximation is used. The electron trajectories, its velocities, and energies are obtained from the numerical solution of the relativistic Newton-Lorentz equation. Finally, the number of accelerated electrons was determined, identifying the favorable regions for the acceleration regime. These results can be useful for the design of gyroresonant accelerator devices. It is worth mentioning that said accelerators can be used as X-ray sources when accelerated electrons hit a metallic target.

1. Introduction

In 1980, K. Golovanivsky proposed a new circular accelerator scheme based on the self-sustaining of electron cyclotron resonance (ECR), known as gyroresonant accelerator (GYRAC). This acronym is also frequently used to designate said acceleration mechanism [1]. Golovanivsky presented an analytical model to obtain a set of differential equations describing both the evolution of the electron energy and the phase shift between the particle velocity and the electric field. Because the electrons can be accelerated up to high energies in small spaces, this acceleration scheme has been extensively studied [2-7]. In fact, the GYRAC mechanism has served as support for the study of other acceleration schemes as spatial autoresonance acceleration (SARA) [8].

We show the results of the 3D simulation of the electrons cloud acceleration in a TE$_{111}$ cylindrical cavity affected by an axis-symmetric inhomogeneous magnetic field which grows
slowly in time. The magnetic field is obtained by a set of 4 current coils, wherein the currents
in the central coils change linearly on time. In our numerical calculations, we consider a cavity
of 2.45 GHz in frequency and the electric field strength of 100 kV/cm [8]. In order to obtain
the classical resonance conditions at the transversal mid plane, the initial magnetic field is fitted to
the classical resonance value $B_0(= m_e \omega/e)$, equal to 0.1 T.

In this work we describes the physical scheme used for the GYRAC realization and some
theoretical basis and used simulation model for the 3D electron acceleration, which is based on
the numerical solution of the relativistic Newton-Lorentz equation for the electron acceleration
based on GYRAC mechanism. In order to validate the results of our numerical scheme,
we compare our results with those obtained by solving the set of differential equations of
Golovanivsky model, which describe the evolution of both the electron energy and the phase-
shift between the electron velocity and the electric field. We present the results of the electrons
cloud acceleration in GYRAC regime, whose concentration is $n_e = 2.6 \times 10^8$ cm$^{-3}$, so the simple
particle approximation is used. The adequate temporal and spatial configuration of the magnetic
field to accelerate the electrons cloud is calculated.

These results can be useful for the design of gyroresonant accelerator devices. It is worth
mentioning that said accelerators can be used as X-ray sources when accelerated electrons hit
a metallic target. The X-ray sources are widely used in medical applications, the study of
crystalline substances, among others.

2. Theoretical formalism

2.1. Physical scheme

Figure 1(a) shows the physical system used for the realization of the GYRAC mechanism. A
resonant cavity (1) is placed inside of a mirror magnetic trap formed by a set of four current coils:
the external DC current coils (2) whose currents circulate counterclockwise respect to the cavity
axis, which is taken as the z-axis, and the central coils (3) whose currents circulate clockwise
and increase in time. In this way the external magnetic field magnitude grows slowly in time.
A magnetically confined electrons bunch is accelerated under self-sustained ECR conditions by
the right-hand circularly polarized electric field component of the TE$_{111}$ microwave field. This
field is shown in red arrows in Figure 1(b) in both the transverse plane and longitudinal plane
of the cavity. The electric field rotates at a frequency $\omega$, which is indicate by the green arrow in
Figure 1(b).

2.2. Gyroresonant accelerat mechanism

The GYRAC mechanism uses a uniform external magnetic field increasing slowly in time, $\vec{B}_0(t)$,
to maintain the resonance condition $\omega = \omega_c$ as the electrons gain energy; being $\omega$ the microwave
field frequency and $\omega_c = eB_0(t)/\gamma m_e$ the electron cyclotron frequency. Where $e$ and $m_e$ are the
charge and mass of an electron, respectively; and $\gamma$ is the Lorentz factor.

In order to describe the electron dynamics, said mechanism was initially studied taken into
account only the electric field component of the TE$_{111}$ microwave field, $\vec{E}_{hf}$ [9]. It is worth
mentioning that for the physical scheme showed in Figure 1, such approximation is valid only at
the transverse midplane of the cavity; where both the magnetic field produced by the set of coils,
$\vec{B}^c(\vec{r}, t)$, can be approximated to the uniform field $\vec{B}_0(t)$; and the magnetic field component of the
TE$_{111}$ microwave field is nil. In the transverse midplane of the cavity, the cylindrical components
of $\vec{E}_{hf}$ can be approximate by the Equation (1) and Equation (2).

$$E_{r hf} = E_0 \sin \varphi$$  \hspace{1cm} (1)
$$E_{\theta hf} = E_0 \cos \varphi,$$  \hspace{1cm} (2)
where $E_0$ is the electric field strength at the center of the cavity and $\varphi$ is the phase-shift between the electron velocity and the electric field. The Equation (3) describes the uniform external magnetic field,

$$\vec{B}_0(t) = B_0[1 + b(t)]\hat{z}, \quad (3)$$

where $B_0 = m_e\omega/e$, corresponds to the magnetic field value to obtain classical resonance and $b(t)$ is a dimensionless function used to describe the growth of the magnetic field, being $b(0) = 0$.

The set of differential Equation (4) and Equation (5) describes the evolution of both the electron energy and the phase shift between the electron velocity and the electric field component of the microwave field, respectively, in the 2D electron acceleration through the GYRAC mechanism.

$$\frac{d\gamma}{d\tau} = -g_0\left(1 - \frac{1}{\gamma^2}\right)^{\frac{1}{2}} \cos\varphi \quad (4)$$

$$\frac{d\varphi}{d\tau} = g_0 \left(\gamma^2 - 1\right)^{-\frac{1}{2}} \sin\varphi + [b(\tau) - \gamma + 1] \gamma^{-1}, \quad (5)$$

where $g_0 = E_0/B_0c$ represents the dimensionless electric field strength, $c$ is the speed light, and $\tau = \omega t$ the dimensionless time.

In real systems, as plasmas in mirror magnetic traps, electrons move in the 3D space, where the space inhomogeneities effects of the external magnetic field become significant. This systems only can be theoretically studied through numerical simulation techniques.

**Figure 1.** Physical scheme model for the realization of the GYRAC mechanism: (a) Mutual disposition of the resonant cavity (1) and the set of current coils (2 and 3) used for the GYRAC mechanism. (b) TE$_{111}$ electric field component of the microwave field (see the red arrows). $\omega$ correspond to the angular frequency of the field. The curved green arrow indicates the direction of rotation respect to the cavity axis, which is taken as z axis.

3. Simulation model

The energetic evolution of the non-interacting electrons bunch and their dynamics can be obtained from the solution of the relativistic Newton-Lorentz equation, which in a dimensionless form can be written as the Equation (6).
\[
\frac{d\vec{u}}{d\tau} = \vec{g} + \frac{\vec{u}}{\gamma} \times \vec{b},
\]

(6)

where, \(\vec{u} = \vec{p}/m_e c\) is the electron momentum, \(\vec{g} = E^{hf}(\vec{r}, t)/(-B_0 c)\) is the exact electric field component of the \(TE_{111}\) cylindrical microwave field. \(\vec{b} = B(\vec{r}, t)/(-B_0)\) is the magnetic field at the electron position in the time \(t\), being \(B(\vec{r}, t) = B_c(\vec{r}, t) + B^{hf}(\vec{r}, t)\). Here \(B_c\) is the magnetostatic field produced by the set of the current coils (See Figure (1)) and \(B^{hf}\) correspond to the magnetic field component of the \(TE_{111}\) cylindrical mode. \(\tau = \omega t\) is the dimensionless time, and finally \(\gamma = (1 + u^2)^{1/2}\) is the Lorentz factor.

In our simulations we use the analytic expressions describing \(E^{hf}\) and \(B^{hf}\) [9]. It is worth mentioning that for the electrons cloud simulations we considered a low electrons concentration, \(n_e = 2 \times 10^8 \text{ cm}^{-3}\), so the self-generated electromagnetic field contribution is not significant.

Under a finite difference scheme, the Equation (6) can be written as Equation (7).

\[
\frac{\vec{u}^{n+1/2} - \vec{u}^{n-1/2}}{\Delta \tau} = \vec{g}^n + \frac{\vec{u}^{n+1/2} - \vec{u}^{n-1/2}}{2\gamma^n} \times \vec{b}^n,
\]

(7)

Which is solved by using the Boris-Buneman algorithm [10]. The positions in each simulation time step are calculated using Equation (8).

\[
x^{n+1} = x^n + \frac{\vec{u}^{n+1/2} \Delta \tau}{\gamma^{n+1/2}}
\]

Here \(\gamma^{n+1/2} = [1 + (u^{n+1/2})^2]^{1/2}\). Such positions are expressed in units of the relativistic Larmor radius \(r_l = c/\omega\).

For the electron bunch simulation, the axis-symmetric magnetic field produced by the set of current coils is numerically calculated from the Biot-Savart law at both the initial and final times. The external magnetic field at any other time is calculated via linear interpolation. In this way, a linear growth of the magnetic field is obtained.

4. Results and discussion

4.1. Results validation. 2D acceleration

In order to check our numerical model, we compare the obtained results from the numerical solution of Newton-Lorentz Equation for a single electron accelerated through the GYRAC mechanism with those obtained by solving the set of differential Equation (4) and Equation (5). For this case, we consider an electric field strength equal to 0.3 kV/cm. The parameters for the external magnetic field described by the Equation (3) are chosen as: \(B_0 = 0.1\ \text{T}\) and \(b(\tau) = \alpha \tau^2\), with \(\alpha = 10^{-5}\). The initial conditions for the electron are chosen as: \(x = 0.1\ \text{mm}\) and \(y = 1\ \text{mm}\), \(v_x = 6.15\ \text{mm/s}\) and \(v_y = 1.165\ \mu\text{m/s}\).

For the numerical solution of the set of differential Equation (4) and Equation (5), we use the initial conditions \(\gamma(0) = 1.000019\) and \(\phi(0) = 3.166\ \text{rad}\). Said conditions are the same used in [1].

The Figure 2 shows the electron energy evolution in units of \(m_e c^2\) as a function of the normalized time calculated from the two methods previously described. The red line corresponds to the values calculated from the numerical solution of the relativistic Newton-Lorentz equation and the black line to those obtained from the numerical solution of the set of differential Equation (4) and Equation (5).

The good agreement between these results guarantee the validity of our numeral model. We can see a small difference between the results obtained from the two methods, which is
attributed to use the exact expressions of the electric field for the numerical solution of the relativistic Newton-Lorentz equation instead of a simple standing plane-wave approximation.

Figure 2. The evolution of the energy in units of $m_e c^2$ as a function of the normalized time. The red line corresponds to the values calculated from the numerical solution of the relativistic Newton-Lorentz equation and the black line to those obtained from the numerical solution of the set of differential Equation (4) and Equation (5). One can see that the electron energy grows almost monotonically from the initial time up to $\tau \sim 550$, when the electron energy is about about 1.38 MeV ($\gamma \sim 3.7$). This corresponds to a state where the cyclotron resonance conditions are preserved.

4.2. Acceleration of an electron bunch

In order to simulate the electron bunch acceleration by using a linear growing of the external magnetic field, $B = \alpha \tau$, with $\alpha \simeq 7 \cdot 10^{-4}$ in the center of the cavity, we calculate it at the times $t = 0$ and $t = 4.65 \mu s$ by using the parameters presented in Table 1.

Table 1. Parameters of coils to achieve electron cyclotron resonance condition at the initial time.

| Coil | Inner radius (cm) | Outer radius (cm) | Length (cm) | Current density (A/mm²) | $z$ (cm) |
|------|-------------------|-------------------|-------------|-------------------------|---------|
| 1    | 4.6               | 11                | 4.4         | 5                       | -3      |
| 2    | 4.6               | 11                | 4.4         | 5                       | 13      |
| 3    | 4.6               | 11                | 1           | -8.7                    | 1.5     |
| 4    | 4.6               | 11                | 1           | -8.7                    | 8.5     |
At the instant \( t = 0 \), an spherical electron cloud of radius \( R_{\text{bunch}} = 2.28 \text{ cm} \) and the concentration \( n_e = 2.6 \times 10^8 \text{ cm}^{-3} \) is located in the center of the cavity, being the electrons placed in random positions. We simulated this system through 10,000 superparticles (1 SP = 3.91 × 10⁵ electrons) in a similar way as it is used in particle-in-cell (PIC) simulations [11-13]. Of course, in the present case the effect of the self consistent fields are not significant. The computational simulation was carried out for two different distributions of Maxwell speeds (case 1: 2 eV and case 2: 5 keV).

Figure 3(a) and Figure 3(b) show r-z diagrams to describe the temporal evolution of an electron bunch accelerated through the GYRAC mechanism, by using either an initial temperatures equal to 2 eV or 5 keV, respectively. It is shown that the number of electrons accelerated at high energies through the GYRAC mechanism is greater in case 1 than in case 2. The red dots in Figure 3(a) and Figure 3(b) correspond to electrons in the energy range 14 keV - 16 keV. It should be noted that, for both the case 1 and case 2, the accelerated electrons up to high energies are located at the midplane of the cavity, \( z = 5 \text{ cm} \). This happens because the appropriate physical conditions to accelerate the electrons through the GYRAC mechanism were fitted for said transverse plane.

**Figure 3.** Temporal evolution of the electron cloud in the GYRAC mechanism, with maximum thermal speed of injection of (a)2 ev and (b)5 keV. In figure b it is observed that the amount of points is smaller, this indicates that many of the particles left the cavity, but regardless of the injection rate in the two experiments a similar maximum energy was achieved.

### 5. Conclusions

The realized numerical experiment shows that the obtained mirror-like magnetic trap can be used to accelerate an electron bunch up to energies of 16 MeV by by using the cylindrical \( TE_{111} \) mode through the GYRAC mechanism. The results show that a electron bunch can be captured in the resonance regime. The obtained results can be useful for the design of gyroresonant accelerator devices. It is worth mentioning that said accelerators can be used as X-ray sources when accelerated electrons hit a metallic target.

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