On the radii of hot strange stars

A G Alaverdyan, G S Hajyan
Yerevan State University, Alex Manoogian str. 1, Yerevan, 0025, Armenia
E-mail: gevorg.hajyan@gmail.com

Abstract. The equilibrium states of hot strange stars are studied. The equation of state of hot strange quark matter is determined on the basis of the MIT bag model. It is shown that for hot strange quark stars the mass-central density and mass-radius relations do not depend on the central temperature if quarks are ultrarelativistic. For these relations the weight of the thermal energy is of fundamental significance. That explains the nature of change in the radius of a strange star when it cools down.

1. Introduction

The dependence of the radii of neutron stars on their temperature is generally small and is frequently neglected. The studies of the evolution of the hot white dwarfs (WD) and neutron stars (NS) assume that their radii are constant in time [1, 2, 3, 4, 5]. For central temperatures $T_c < 10^{10}$K the constant radius approximation is valid. The cooling time of a NS from an initial central temperature $T_c = 10^{10}$K to a value $T_c = 10^9$K is about a year [2, 3, 5]. The cooling time from $T_c = 10^{11}$K to $T_c = 10^{10}$K is even less – of the order of from several minutes to several hours. Initial temperatures of neutron stars and hot strange stars (hereafter HSS) can be of the order of $T_c \sim 10^{12}$K if the major part of the gravitational energy of the progenitor star is converted into heat.

In Ref. [6] sequences of isothermal strange stars (SS) were computed. The equation of state (EoS) of hot strange quark matter (HSQM) was taken to be the one of the MIT bag model [7]. For simplicity, the presence of electrons in Ref. [6] was ignored, and the quark-gluon interaction constant was assumed to vanish. The calculations were performed for the cases where the bag parameter $B$ depends on the quark density and where it does not depend on this density. From these calculations it follows that the higher the temperature the smaller is the maximum mass for any given isothermal sequence (see Figs. 7 and 8 in Ref. [6]). However, because the pressure of matter at constant density of particles is always increasing with the increase of temperature, the temperature dependence of the mass maximum found in Ref. [6] appears to be strange. Similar strange behavior is observed for the radii of some configuration as a function of the temperature.

In Ref. [6] the EoS of HSQM is determined in a complicated way. We believe that for this equation of state it was sufficient to use the known results for the thermal corrections to the energy and pressure of a degenerate Fermi gas [8]. The absence in Ref. [6] of the explanation of the strange temperature dependences of the maximum mass and radius of HSS, as well as calculations for stars with a fixed baryonic number, motivated us to reconsider the problem of determining of the integral parameters of strange quark star at non-zero temperature.
2. Problem statement, the equation of state

The purpose of this study is to determine

(i) the equilibrium configurations of hot strange stars,
(ii) the dependence of the radius and mass of an SS, with fixed baryonic number on temperature
(iii) the amount of energy lost during the cooling of HSS.

To solve these problem we use the MIT bag model of quark matter [7] in its simplest form: quarks are ultrarelativistic, the constant of the quark-gluon interaction \( \alpha_c = 0 \), and the bag parameter \( B \) is constant. The energy density and pressure within this model, including the leading order thermal corrections to the zero-temperature ideal gas result [8] are given by

\[
\begin{align*}
\epsilon &= \sum \epsilon_i(\mu_0, T) + B, \\
P &= \sum P_i(\mu_0, T) - B, \\
\epsilon_i &= 3\mu_i^4 [ 1 + (m_i/\mu_i)^2 + (2\pi^2/3) \cdot (T/\mu_i)^2 ] / 4\pi^2, \\
P_i &= \mu_i^4 [ 1 - (m_i/\mu_i)^2 + (2\pi^2/3) \cdot (T/\mu_i)^2 ] / 4\pi^2, \\
n_i &= (\mu_i^2 - m_i^2)^{3/2} / \pi^2.
\end{align*}
\]

Here \( m_i \) is the mass, \( \mu_i \) is the chemical potential, \( \epsilon_i \) and \( P_i \) are the energy density and pressure and \( n_i \) is the density of quarks at \( T = 0 \). The index \( i \) labels the quarks with fixed discrete quantum numbers (i.e. color, flavor). Here and below we work with natural units where \( \hbar = k = c = 1 \).

Because the quarks are assumed to be ultrarelativistic, we can set in Eqs. (1)-(3) \( m_i = 0 \). This approximation is well satisfied for the light \( u \) and \( d \) flavors, but holds only approximately for the \( s \) quark \( (m_s \approx 150) \) MeV. The non-zero mass the \( s \) quark leads to a violation of symmetry in the chemical composition. It is entirely due to this that in the local electrical neutrality implies the presence of electrons in SQM. In Ref. [6] the mass of the \( s \)-quark is taken into account but at the same time it is assumed that electrons are absent. The electrical neutrality is ensured by the condition

\[ n_u = (n_d + n_s)/2. \]

We consider such an approach to be inconsistent and erroneous. In SS the Fermi energy of quarks reaches values of the order of \( 500 - 600 \) MeV, which is 3-4 times larger than \( m_s \). Therefore, in a first approximation \( m_s \) may be neglected. In this approximation electrons are absent, so the condition of beta-equilibrium between quarks follows from the equality of their chemical potentials \( \mu_i(n_i, T) \):

\[ \mu_u(n_u, T) = \mu_d(n_d, T) = \mu_s(n_s, T). \]

It is easy to show that these conditions are equivalent to the conditions:

\[ \mu_{0u}(n_u) = \mu_{0d}(n_d) = \mu_{0s}(n_s). \]

In the state of thermodynamic equilibrium, the chemical potentials depend on the temperature and on baryonic number density \( n = (n_u + n_d + n_s)/3 \). To summarize, our models assumes that \( \alpha_e = 0, B = 0, \) and \( m_s = 0, \) therefore the concentrations and the chemical potentials of all the quarks are equal. Introducing \( \mu \equiv \mu_0 i \) we obtain from (1)-(4)

\[ P = (\epsilon - 4B), \]

i. e., within our approximations the pressure depends only on the energy density of HSQM.
3. The complete system of equations of the equilibrium state of HSS

The equilibrium structure of HSS is determined by the equations of hydrostatic equilibrium [the Tolman-Oppenheimer-Volkoff (TOV) equations]

\[
\frac{dP}{dr} = -\frac{G(P + \varepsilon)(m + 4\pi r^3 P)}{r^2 (1 - 2G m/r)}, \quad (10)
\]

\[
\frac{dm}{dr} = 4\pi r^2 \varepsilon, \quad (11)
\]

where \( G \) is the gravitational constant, \( r \) is the radial coordinate, \( m \) is the mass within the sphere of the radius \( r \). These equations should be supplemented by the equation for the baryonic charge \( N(r) \) in the sphere of the radius \( r \)

\[
\frac{dN}{dr} = 4\pi r^2 n \cdot (1 - 2Gm/r)^{-1/2}, \quad (12)
\]

as well as by the condition of isothermality

\[
T \cdot \exp(\nu/2) = \text{const.} \quad (13)
\]

where \( g_{00} \) is the temporal metric coefficient in the Schwarzschild metric. In Ref. [6] it was assumed that \( T = \text{const.} \) in the entire star. However, in a strong gravitational field the quantity that remains constant within the star (either NS or SS) is \( T \sqrt{g_{00}} \). Equations (12)-(13) should be supplemented by the equation for the field \[9\]

\[
\frac{d\nu}{dP} = -\frac{2/(\varepsilon + P)}{1-2Gm/r}. \quad (14)
\]

In our case from (9), (13) and (14) we obtain for \( T(r) \)

\[
T(r) = T_c \left[ \frac{P(r) + B}{P_c + B} \right]^{1/4}, \quad (15)
\]

where \( T_c \) is the central temperature.

4. Numerical results and discussion

For given \( T_c \) and \( P_c \) we solved Eqs. (10)-(12) to obtain sequences of isothermal HSS. In our numerical calculations we assume the value of the bag constant \( B = 80 \text{ MeV} \). Since within the approximations outlined above the HSQM pressure depends only on the energy density the dependence of the mass of stellar configurations \( M(\rho_c) \) on the central density \( \rho_c = \rho_c / c^2 \) of matter is independent of the central temperature of the star. This statement applies also to the radius-mass relationship \( R(M) \). In contrast, the full baryonic number \( N_0 \) depends on the central energy density as well as the central temperature \( T_c \). Figure 1 shows the dependence of the mass on the central density \( \rho_c \) in g/cm³ and Fig. 2 shows the dependence of the radius \( R \) on the mass. According to the static stability criterion, the configurations on iso-entropic sequences are stable up to the first maximum. At the maximum of these curves the stability is lost [10, 11]. Since all the HSS including the iso-entropic ones are on the same \( M(\rho_c) \) curve (Fig.1), all the configurations up to the maximum are stable. For the isothermal series from Ref. [6] this stability criterion is not applicable. To determine the point of loss of stability on these curves more research is needed. Below, only stable HSS will be considered. Let us find out how the radius of HSS changes when it cools. The dependence of the radius of SS and NS on their mass are very different. Self-confining SQM at zero pressure has a density \( \rho_0 = 4B / c^2 = 5.7 \cdot 10^{14} \text{g/cm}^3 \) (see
For small masses $R \sim M^{1/3}$, as the gravitational forces are weak for a significant compression of SQM. In Fig. 2 this dependence is shown by the dashed curve. With the increase of mass, gravitational forces are growing and the radius SS grows slower than $R \sim M^{1/3}$. For the masses $M > 1.56M_\odot$ gravity is already so strong that the familiar from neutron stars form of the $R(M)$ curve is restored. In Fig. 2 the point $b$ ($M = 1.56M_\odot$) separates these two areas. For different models of SQM and variants of the EoS the $R(M)$ dependence for cold SS was obtained in many studies, in particular in Refs. [12, 13, 14, 15]. As it has already been stated for the hot and cold SS $R(M)$ curves coincide. When a HSS cools its total baryonic number $N_0$ remains constant, and the mass decreases as the star loses energy. If at the initial moment HSS is in the region $ab$ of the $R(M)$ curve, the cooling star will move down along this curve towards the point $a$. The final position of the SS on the $R(M)$ curve is determined only by the numerical value $N_0$. For small masses (the area close to the point $a$ in Fig. 2) $R \sim (N_0/n_0)^{1/3}$, where $n_0$ is the baryonic number of SQM at $P = 0$. From (5) and (11) it follows that as the temperature decreases $n_0$ increases. Consequently, the radius of the HSS decreases. If the HSS is located close to the point $b$ in Fig. 2 and its mass $M < 1.56M_\odot$ it mass will decrease as it cools down.

Although for these masses the gravitational forces are decreasing, the decrease of pressure and the increase of $n_0$ are dominant which again leads to compression of the star. Thus, the HSS located in the region $ab$ in Fig. 2 will get compressed as they cool.

Consider a HSS which is born in a supernova explosion and has a high temperature. If the HSS at the formation moment is in the region $bc$ of the $R(M)$ curve (see Fig. 2) it will expand as it cools, although its temperature and pressure will fall. At first glance, it is strange that when the pressure is reduced the star expands. In such cases, as a rule, stars compress, as in the case for example for hot white dwarfs. However, in the case of HSS the role of mass reduction due to the loss of energy during the cooling is much more important than for ordinary stars. The weakening of gravity is so significant that even with a decrease of pressure the star expands. If these HSS evolve along the $R(M)$ curve up to the point $b$, then they only expand. Otherwise, after passing the point $b$, the expansion will turn into compression.

Figures 3, 4 and 5 we show the dependence of the change of the radius on the central temperature $T_c$ for HSS with baryonic numbers $N_0 = (0.65; 2; 2.4) \cdot 10^{57}$. In these figures we also show the initial and final values of the masses (in units of the solar mass) and the radii (in kilometers). For example a HSS with baryonic number $N_0 = 6.5 \cdot 10^{56}$ and initial parameters

\[\alpha_c = 0, B = 80 \text{ MeV}/f^2, \; m_1 = 0\]
\[ M = 0.77 M_\odot, \ T_c = 80 \text{ MeV} \simeq 9.28 \cdot 10^{11} \text{ K} \] and \[ R = 8.27 \text{ km} \] loses 38 percent of its mass and monotonically compresses to \[ R_0 = 7.23 \text{ km} \] in the course of cooling down to \[ T_c = 0 \] (see Fig. 3).

The two other configurations \( (N_0 = 2 \cdot 10^{57} \) and \( N_0 = 2.4 \cdot 10^{57} \) with initial temperatures \( 47.5 \text{ MeV} \) and \( 22.5 \text{ MeV} \) respectively have the maximum masses \( M = 1.7 M_\odot \). The configuration with \( N_0 = 2 \cdot 10^{57} \) during the cooling at first expands by \( \sim 100 \) m, and then compresses by \( \sim 450 \) m. The third configuration only expands. We conclude that the strange behavior of the radius during the cooling of the HSS with \( M > 1.56 M_\odot \) is due to the weakening of the gravitational field caused by the reduction of the mass of the star.

5. **Comparison with the results of Ref. [6]**
There is a fundamental difference between our results and those of Ref. [6]. While in this study all isothermal sequences have the same \( M = M(\rho_c) \), in Ref. [6] not only they don’t coincide but also have significant differences in the numerical values (Figs. 7-12 and Table 1 and 2 of Ref. [6]).

The reason for these differences may be only the non-zero value of the mass of the strange quark \( m_s \). In Ref. [6] the value of \( m_s \) used in numerical calculations is not given. However, when \( m_s = 0 \) the conditions of chemical equilibrium (8) and electro-neutrality (6) [Eqs. (8), (9), (12)]

![Figure 3](image1.png)

**Figure 3.** The change of the radius \( R - R_0 \) of the hot strange star with the barionic number \( N_0 = 6.5 \cdot 10^{56} \) at cooling (explanations in text).

![Figure 4](image2.png)

**Figure 4.** The change of the radius \( R - R_0 \) of a hot strange star with the baryonic number \( N_0 = 2 \cdot 10^{57} \) during the cooling (see for explanation in text).

![Figure 5](image3.png)

**Figure 5.** The change of the radius \( R - R_0 \) of a hot strange star with the baryonic number \( N_0 = 2.4 \cdot 10^{57} \) during cooling.
in Ref. [6] are incompatible. Indeed, according to (5) and (8) \( n_u = n_d \), because \( m_u = m_d = 0 \), and from (5) and (6) \( n_u = n_s \), which at the same chemical potentials and \( m_s \neq 0 \) is impossible.

The obtained results qualitatively remain valid also in the case when the bag constant \( B \) depends on the concentration of barionic number. Surely the duration of the high temperature stage of HSS is very short. Neutrino energy losses, bremsstrahlung of electrons on the surface of the SS, electron-positron pair emission, etc. quickly cool the star [16]. However, we should keep in mind that if all the cold SS were formed by the above mentioned scenario, the maximum mass of the existing cold SS will be below the maximum value \( M(\rho_c) \). The HSS with a high temperature and maximum mass loses mass during cooling. From Fig. 4 and 5 it follows that the difference between the highest possible value of cold SS and maximum \( M(\rho_c) \) can be significant.

We plan to study this problem in a separate study.

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