Constraining Cosmological Parameters with Observational Data
Including Weak Lensing Effects

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In this paper, we study the cosmological implications of the 100 square degree Weak Lensing survey (the CFHTLS-Wide, RCS, VIRMOS-DESCART and GaBoDS surveys). We combine these weak lensing data with the cosmic microwave background (CMB) measurements from the WMAP5, BOOMERanG, CBI, VSA, ACBAR, the SDSS LRG matter power spectrum and the Type Ia Supernova (SNIa) data with the “Union” compilation (307 sample), using the Markov Chain Monte Carlo method to determine the cosmological parameters, such as the equation-of-state (EoS) of dark energy $w$, the density fluctuation amplitude $\sigma_8$, the total neutrino mass $\sum m_\nu$ and the parameters associated with the power spectrum of the primordial fluctuations. Our results show that the $\Lambda$CDM model remains a good fit to all of these data. In a flat universe, we obtain a tight limit on the constant EoS of dark energy, $w = -0.97 \pm 0.041 (1\sigma)$. For the dynamical dark energy model with time evolving EoS parameterized as $w_0(a) = w_0 + w_a (1 - a)$, we find that the best-fit values are $w_0 = -1.064$ and $w_a = 0.375$, implying the mildly preference of Quintom model whose EoS gets across the cosmological constant boundary during evolution. Regarding the total neutrino mass limit, we obtain the upper limit, $\sum m_\nu < 0.471$ eV (95% C.L.) within the framework of the flat $\Lambda$CDM model. Due to the obvious degeneracies between the neutrino mass and the EoS of dark energy model, this upper limit will be relaxed by a factor of 2 in the framework of dynamical dark energy models. Assuming that the primordial fluctuations are adiabatic with a power law spectrum, within the $\Lambda$CDM model, we find that the upper limit on the ratio of the tensor to scalar is $r < 0.35$ (95% C.L.) and the inflationary models with the slope $n_s \geq 1$ are excluded at more than 2 $\sigma$ confidence level. In this paper we pay particular attention to the contribution from the weak lensing data and find that the current weak lensing data do improve the constraints on matter density $\Omega_m$, $\sigma_8$, $\sum m_\nu$, and the EoS of dark energy.

I. INTRODUCTION

Cosmological observations play a crucial role in understanding our universe. With the accumulation of observational data from CMB measurements, large scale structure (LSS) surveys and SNIa observations as well as the improvements of the data quality, the accurate constraints on cosmological parameters can be expected from the data analysis. Recently WMAP group has released the five-year data of temperature and polarization power spectra \textsuperscript{1,2,3}. And the Arcminute Cosmology Bolometer Array (ACBAR) experiment has also published its new CMB temperature power spectrum \textsuperscript{4}. These new CMB data can strengthen the constraints on the cosmological parameters, especially for those associated with the inflationary models \textsuperscript{1,2,5}. Furthermore, the Supernova Cosmology Project has made an unified analysis of the world’s supernovae datasets and presented a new compilation “Union” (307 sample) \textsuperscript{6} which includes the recent samples of SNIa from SNLS and ESSENCE Survey, as well as some older datasets. In the literature \textsuperscript{6,7,8}, these data has been widely used to constrain various cosmological models. However, one should keep in mind that the degeneracies of cosmological parameters exist in almost all cosmological observations, \textit{i.e.}, they are not sensitive to single parameters but to some specific combinations of them. It is therefore highly necessary to combine different probes to break parameter degeneracies so as to achieve tight constraints. Furthermore, different observations are affected by different systematic errors, and it is thus helpful to reduce potential biases by combining different probes.

Weak gravitational lensing which is directly related to the dark matter distribution, the geometry and the dynamics of the universe provides an useful way to break the degeneracies mentioned above. Cosmic shear, \textit{i.e.} the distortion of images of the high-redshift source galaxies through the tidal gravitational field of the large-scale matter distribution in the universe, provides a powerful probe of the mass fluctuations in the universe and gives a direct measurement of the matter power spectrum down to the non-linear regime. Since the mass distribution and its late time evolution are sensitive to the dark energy and the neutrino mass, we expect to get interesting information about both through the study of cosmic shear.

Recently, the cosmic shear analysis of the 100 square degree weak lensing survey which includes the Canada-France-Hawaii Telescope Legacy Survey (CFHTLSWide)\textsuperscript{6}, the Garching-Bonn Deep Survey (GaBoDS)\textsuperscript{10}, the Red-Sequence Cluster Survey (RCS)\textsuperscript{11}, and the VIRMOS-DESCART survey.
(VIRMOS)\textsuperscript{12} has been presented by Ref.\textsuperscript{13}. These combined surveys cover 113 degree\textsuperscript{2} which is the largest sky coverage so far. With these data, there are some studies on the cosmological implications recently in the literature \textsuperscript{15, 16, 17, 18, 20, 21}. In this paper we study the constraints on cosmological parameters from the weak lensing cosmic shear as well as the current cosmological observational data, such as CMB, LSS and SN Ia. Specifically, we focus on the effects from the weak lensing.

Our paper is organized as follows: In Section II we describe the method and the latest observational datasets we use in this paper; Section III contains our main global fitting results on the cosmological parameters and the last section is the summary.

\section{II. METHOD AND DATA}

In our study, we perform a global analysis using the publicly available MCMC package CosmoMC\textsuperscript{22}. We assume the purely adiabatic initial conditions. Our most general parameter space is:

\[ P = (\omega_b, \omega_c, \Theta_s, \tau, w_0, w_a, f_\nu, n_s, A_s, \alpha_s, r) , \]  

where \( \omega_b \equiv \Omega_b h^2 \) and \( \omega_c \equiv \Omega_c h^2 \), in which \( \Omega_b \) and \( \Omega_c \) are the physical baryon and cold dark matter densities relative to the critical density, \( \Theta_s \) is the ratio (multiplied by 100) of the sound horizon to the angular diameter distance at decoupling, \( \tau \) is the optical depth to re-ionization, \( f_\nu \) is the dark matter neutrino fraction at present, namely,

\[ f_\nu = \frac{\rho_\nu}{\rho_{dm}} = \frac{\Sigma m_\nu}{93.105 \text{ eV} \; \Omega_\nu h^2} \]  

where \( \rho_{dm} \) is the matter density of the universe.

The primordial scalar power spectrum \( P_\chi(k) \) is parameterized as \textsuperscript{23}:

\[ \ln P_\chi(k) = \ln A_\chi(k_{0}) + (n_s(k_{0}) - 1) \ln \left( \frac{k}{k_{0}} \right) + \frac{\alpha_s}{2} \left( \ln \left( \frac{k}{k_{0}} \right) \right)^2 , \]  

where \( A_\chi \) is defined as the amplitude of initial power spectrum, \( n_s \) measures the spectral index, \( \alpha_s \) is the running of the scalar spectral index and \( r \) is the tensor to scalar ratio of the primordial spectrum. For the pivot scale we set \( k_{0} = 0.05 \; \text{Mpc}^{-1} \). Moreover, \( w_0 \) and \( w_a \) are the parameters of dark energy EoS, which is given by \textsuperscript{24}:

\[ w_{de}(a) = w_0 + w_a(1 - a) , \]  

where \( a = 1/(1 + z) \) is the scale factor and \( w_{de} = -dw/da \) characterizes the “running” of EoS (RunW henceforth). The ΛCDM model has \( w_0 = -1 \) and \( w_a = 0 \). For the dark energy model with a constant EoS, \( w_a = 0 \) (WCDM henceforth). When using the global fitting strategy to constrain the cosmological parameters, it is crucial to include dark energy perturbations \textsuperscript{25, 26, 27}. In this paper we use the method provided in Refs.\textsuperscript{27, 28} to treat the dark energy perturbations consistently in the whole parameter space in the numerical calculations.

For the weak lensing likelihood statistic, we mainly consider the shear correlation function \( \xi \) following Ref.\textsuperscript{13}. The shear correlation functions are defined as

\[ \xi_+(\theta) = \langle \gamma_\ell^x(\theta + \theta) \rangle + \langle \gamma_\ell^y(\theta + \theta) \rangle, \]

\[ \xi_-(\theta) = \langle \gamma_\ell^x(\theta + \theta) \rangle - \langle \gamma_\ell^y(\theta + \theta) \rangle, \]  

where the shear \( \gamma = (\gamma_\ell^x, \gamma_\ell^y) \) is rotated into the local frame of the line joining the centres of each galaxy pair separated by \( \theta \). The shear correlation function \( \xi_+ \) is related to the convergence power spectrum through

\[ \xi_+^2(\theta) = \frac{1}{2\pi} \int_{0}^{\infty} \left[ \int_{0}^{r_H} \frac{dr}{r^2} \right] \frac{\theta_0^2}{2} , \]

where \( r_H \) is the Hubble constant, \( f_K(r) \) is the comoving angular diameter distance out to a distance \( r \) (\( r_H \) is the comoving horizon distance), \( a(r) \) is the scale factor, and \( n(r,z) \) is the redshift distribution of the sources\textsuperscript{2}. \( P_\Delta \) is the 3-dimensional mass power spectrum computed from a non-linear estimation of the dark matter clustering, and \( k \) is the 2-dimensional wave vector perpendicular to the line-of-sight. \( P_A \) evolves with time, hence depends on the co-moving radial coordinate \( r \).

The lensing signal can be split into E mode and B mode, and the E and B shear correlation functions are given by

\[ \xi_E(\theta) = \frac{\xi_+(\theta) + \xi_-(\theta)}{2} , \quad \xi_B(\theta) = \frac{\xi_+(\theta) - \xi_-(\theta)}{2} . \]  

\footnote{Available at: \url{http://cosmologist.info/cosmomc/}}

\footnote{In our calculations, we use the redshift distribution given in the EQ (11) of Ref.\textsuperscript{13} and the best fit values of the nuisance parameters \( a, b \) and \( c \) presented in their Table 2. We have numerically checked that the correlations coefficients between \( a, b, c \) parameters and cosmological parameters are small enough to be neglected. So the main global analysis results presented in this paper are obtained with the fixed nuisance parameters. Similar conclusions about varying \( a, b \) and \( c \) are also given in Ref.\textsuperscript{19}.}
where

\[ \xi^0(\theta) = \xi_-(\theta) + 4 \int_0^\infty \frac{d\theta}{\theta} \xi_-(\theta) - 12 \theta^2 \int_0^\infty \frac{d\theta}{\theta} \xi_-(\theta). \]  

(9)

In this paper we use the E mode correlation as the data vector. The corresponding likelihood function is given by

\[ \mathcal{L} = \frac{1}{\sqrt{2\pi}^n |C|} \exp \left[ -\frac{1}{2} (\xi_E - m) C^{-1} (\xi_E - m)^T \right], \]

(10)

where \( n \) is the number of angular scale bins and \( C \) is the \( n \times n \) shear covariance matrix. We take the covariance matrix given by Ref. \[13\] which includes the statistical noise, the residual B-mode and the sample variance.

In the computation of the CMB, we include the WMAP5 temperature and polarization power spectra with the routine for computing the likelihood supplied by the WMAP team\(^3\). We also include some small-scale CMB measurements, such as BOOMERanG \[29\], CBI \[30\], VSA \[31\] and the newly released ACBAR data \[3\]. For the Large Scale Structure information, we use the Sloan Digital Sky Survey (SDSS) luminous red galaxy (LRG) sample \[32\]. The supernova data we use are the recently released “Union” compilation of 307 sample \[\tilde{0}\] In the calculation of the likelihood from SNe Ia, we marginalize over the relevant nuisance parameter \[33\].

Furthermore, we make use of the Hubble Space Telescope (HST) measurement of the Hubble parameter \( H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) by a Gaussian likelihood function centered around \( h = 0.72 \) and with a standard deviation \( \sigma = 0.08 \) \[34\].

### III. NUMERICAL RESULTS

In this section we present our global fitting results. The most relevant cosmological parameters for the weak gravitational lensing signal are the matter density \( \Omega_m \) and the matter power spectrum normalization parameter \( \sigma_8 \). On the other hand, other cosmological parameters related to the primordial power spectrum from inflation and the expansion history of the universe can also affect the weak lensing effects. Thus we expect that weak lensing data can provide important constraints on these parameters as well.

First of all, we consider the flat \( \Lambda \)CDM model. To see the effect of the current weak lensing data, we calculate and compare the results for the two cases with and without the lensing data included. In Figure 1 we present the constraints on \( \Omega_m \) and \( \sigma_8 \). A significant improvement from the cosmic shear data can be seen clearly. We obtain \( \Omega_m = 0.25 \pm 0.011 \) and \( \sigma_8 = 0.80 \pm 0.016 \) at 1\( \sigma \) C.L. from the combined CMB, LSS, SN and weak lensing cosmic shear data in \( \Lambda \)CDM model. Compared with the results with no weak lensing data included, the constraints are improved by 45% and 40% for \( \Omega_m \) and \( \sigma_8 \), respectively.

In the following, we will present the constraints on the neutrino mass, the equation of state of dark energy and the parameters associated with inflation models respectively.

#### A. Neutrino Mass

From the neutrino oscillation experiments, such as the atmospheric neutrino experiments \[35\] and the solar neutrino experiments \[36\], we know that neutrinos are massive. Therefore they can leave some unique imprints on cosmological observables, such as the CMB anisotropies

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\(^3\) Available at the LAMBDA website: [http://lambda.gsfc.nasa.gov/](http://lambda.gsfc.nasa.gov/)
and LSS [37]. Thus cosmological observations can provide crucial complementary information on the absolute neutrino masses.

Besides the influence on the expansion history of the universe, the existence of massive neutrinos affect the structure formation process. Massive neutrinos contribute to the dark matter composition of the universe. Because of their relatively large thermal velocities, they damp the perturbations within their free streaming scale, and suppress the matter power spectrum at small scale by roughly $\Delta P / P \sim -8\Omega_\nu / \Omega_m$ [38]. Such a suppression of the power spectrum depends on the Jeans length of neutrinos, which decreases with time as the neutrino thermal velocity decreases. Since the cosmic shear can measure the matter power spectrum down to small-scale non-linear regime, it can be used to significantly improve the constraint on the neutrino mass.

Within the $\Lambda$CDM model, from Table I one can read out 95% upper limit of the total neutrino mass derived from the current observations, CMB+LSS+SN, $\sum m_\nu < 0.663$ eV (95% C.L.), which is consistent with the recent results from WMAP5 group [1, 2]. By taking into account the weak lensing cosmic shear data, the constraints can be tightened to $\sum m_\nu < 0.471$ eV (95% C.L.).

The degeneracies between the neutrino mass and other cosmological parameters, such as the EoS of dark energy $w$ and the running of spectral index $w_a$, are known to exist. We thus also consider the neutrino mass limit in the framework of the dynamical dark energy model. One can see from the blue curve in Figure 2 that the degeneracy [25, 39, 41] causes a much looser limit on the neutrino mass with $\sum m_\nu < 1.11$ eV (95% C.L.).

B. Equation of State of Dark Energy

For the dark energy component, in Table II we list the constraints on the EoS, dark energy density and the Hubble constant in different dark energy models. In order to emphasize the contribution from the cosmic shear measurements, we compare the following two cases: combining CMB, LSS and SN Ia with or without weak lensing data. We find that in both cases the $\Lambda$CDM model remains a good fit to the data.
TABLE II. Constraints on the dark energy EoS and some background parameters from the observations. Here we have presented the mean and the best fit values, which are obtained from the cases with and without the weak lensing data, respectively.

| Parameter | $w_0$ | $w_a$ | $\Omega_{\text{de}}$ | $H_0$ |
|-----------|-------|-------|----------------------|-------|
| ACII flat | Mean w/ wl | Mean w/o wl | BestFit w/ wl | BestFit w/o wl |
| WCDM flat | BestFit | $-0.967$ | $-0.992$ | 0 | 0 | 0.741 | 0.742 | 71.15 | 71.53 |
| RunW flat | BestFit | $-1.064$ | $-1.066$ | 0.375 | 0.514 | 0.737 | 0.738 | 70.74 | 70.01 |

Considering dynamical dark energy models in flat universe, we first explore the constraints on the constant EoS of dark energy, $w$. In Fig 5 we show the one dimensional probability distributions and the two dimensional cross correlation of $w$ and the present dark matter density, $\Omega_m$. One can find that the weak lensing measurements provide a significant improvement, notably seen in the two-dimensional constraints on $\Omega_m$ and $w$. The current observations including the weak lensing data yield a strong constraint on the constant EoS of dark energy, $w = -0.973 \pm 0.0413$ (1 $\sigma$). This result is similar to and somewhat tighter than the limit from WMAP5 [1], $w = -0.972^{+0.061}_{-0.060}$ (1 $\sigma$). It excludes some of the quintessence models, for example, the tracker model which predicts $w \sim -0.7$ [42] at 5 $\sigma$.

![Figure 5](image-url)

**FIG. 5:** One dimensional constraints on the inflationary parameters $n_s$, $A_s$, $\alpha_s$ and $r$ from the current observations in flat ACII model. The black solid lines are from data sets CMB + LSS + SN Ia and the red dashed lines are obtained by taking into account the weak lensing data.

C. Parameters of the primordial fluctuations

In this section we present the constraints on the parameters related to the primordial fluctuations, which are closely associated with inflationary models.

The results are shown in Fig 6. Within the ACII model, we have $n_s = 0.953 \pm 0.0202$ (1 $\sigma$), which excludes the scale-invariant spectrum, $n_s = 1$, and the spectra with a blue tilt, $n_s > 1$, at more than 2 $\sigma$ confidence level. When considering the gravitational waves, the latest observational data yield the 95% upper limit of tensor-to-scalar ratio $r < 0.346$.

We also explore the constraint on the running of the spectral index. From all the combined data, we obtain a limit on the running of the spectral index with $\alpha_s = -0.0231 \pm 0.0185$ (1 $\sigma$) for the ACII model. The error is dramatically reduced comparing with the previous results [47, 48], beneficial from the more accurate observational data. No significant evidence for large runnings of the
spectral index is found. Our results also show that the current lensing data do not improve the constraints on these parameters significantly.

IV. SUMMARY

In this paper we have studied the constraints on the cosmological parameters from the latest observational data including CMB, LSS, SN Ia and weak lensing effects. We have paid particular attention to the additional contributions from the 100 square degree cosmic shear data. Our results show that the current weak lensing data do help to improve the constraints on $\Omega_m$, $\sigma_8$, the total neutrino mass $\Sigma m_\nu$, and the constant EoS of dark energy. For other parameters, they do not add too much value due to their large statistical errors.

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