Gravitational wave speed: Implications for models without a mass scale

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The recent report that the gravitational wave speed equals the light speed puts strong constraints on the anisotropic stress parameter of many modified gravity models, a quantity that is directly observable through large-scale structure. We show here that models without a mass scale completely escape these constraints. We discuss a few relevant cases in detail: Brans-Dicke theory, nonlocal models, and Galileon Lagrangian.

I. INTRODUCTION

The recent detection of gravitational waves (GW) from a neutron star merger [1] had a big impact in the field of modified gravity theories [2–7]. For the first time we observed a multi-messenger event, with a simultaneous detection of gravitational waves and an associated optical counterpart. This observation placed a constraint on the speed of gravitational waves $c_T$ with an unprecedented accuracy of $|c_T/c - 1| \leq 1 \times 10^{-15}$ [8], where $c$ is the speed of light. Therefore, at least at the present time, gravitational waves undoubtedly travel at the speed of light.

The GW speed constitutes a smoking gun evidence for several modified gravity theories where modifications in the gravitational sector affect the propagation of the gravitational degrees of freedom. Many of these models fall within the so-called Horndeski gravity, which is the most general four-dimensional scalar-tensor theory with second order equations of motion [9] (although recently Horndeski gravity has been generalized to include higher-order derivative terms while keeping only one additional scalar degree of freedom [10–13]). Some of the terms present in the Horndeski action lead to a modification of gravitational wave speed which puts them, and the theories they define, in great tension with the aforementioned constraint on $c_T$ [5]. In passing, let us notice that the constraints on modified gravity models disappear if in the frame in which baryons are uncoupled the gravity sector is standard; this leaves completely free the coupling between dark matter and dark energy. Here, however, we restrict our attention to universally-coupled models.

Modifications of gravitational wave propagation are also linked to the large scale structure formation in modified gravity theories, as first shown in Refs. [14, 15]. Hence, by constraining the speed of gravitational waves we can also constrain $\eta$. The latter, for scalar perturbations in the Newtonian gauge with line element

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2(1 + 2\Phi) \delta_{ij} dx^i dx^j,$$

(1)

where $a(t)$ is the cosmological scale factor, is defined as the ratio of Bardeen potentials $\Phi$ and $\Psi$, i.e.,

$$\eta \equiv -\frac{\Phi}{\Psi}. \tag{2}$$

Furthermore, one can also define the effective Gravitational coupling $Y$, in momentum space, as

$$Y \equiv -\frac{2k^2\Psi}{a^2\rho_m\delta_m}, \tag{3}$$

where $\rho_m$ and $\delta_m$ are the matter energy density and the matter density contrast, respectively [1].

In Refs. [14, 15] it was shown that in the quasi-static approximation $\left(\hat{k} \equiv k/Ha \gg 1\right)$, for scales much smaller than the mass $M$ of scalar field fluctuations $k \ll M$, the gravitational slip parameter takes the value $\eta = c_T^{-2}$. Hence, by using the fact that gravitational waves, at least presently, travel with the speed of light, one obtains that $\eta$ should also be equal to unity at those scales [3], i.e. it should recover its General Relativity (GR) value. However, in this work we want to develop the suggestion advanced in [3] that this constraint on $\eta$ is applicable only for those modifications of gravity which introduce a new mass scale into the theory: when a theory of gravity is not supplemented with a new mass scale, $\eta$ is a) scale-independent, b) in general different from unity, and c) left unconstrained by the GW constraint $c_T = c$.

1 All the perturbation quantities in this paper are meant to denote the positive-definite root-mean-squares of the corresponding random variables.
In the case of Horndeski theory, in the quasi-static approximation and inside the Jeans length of the scalar field \((k_\text{J} \gg 1)\), we have for the gravitational slip parameter \(\eta\) and the effective gravitational constant \(Y\) the following expressions [16–18]

\[
\eta = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right), \quad Y = h_1 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right).
\] (4)

Here, the functions \(h_{1–5}\) express the modification introduced by Horndeski gravity compared to GR, and are scale independent functions constructed from the Horndeski functional parameters and their derivatives with respect to the scalar field \(\phi\) and its canonical kinetic term \(X = -g_{\mu\nu}\phi^\mu \phi^\nu / 2\), i.e. \(h_1 \equiv h_1(z) \equiv h_1(\phi, X)\). One can find explicit expressions for the Horndeski parameters \(h_i\) in the Appendix of Ref. [16]. It turns out that the Horndeski function \(h_2\) in this approximation has a simple relation with the speed of gravitational waves, namely \(h_2 = c_T^{-2}\). For the case of \(\Lambda\)CDM cosmology we have that \(h_{1,2} = 1\) and \(h_{3,4,5} = 0\), hence \(\eta\) is identically equal to unity.

However, from Eq. (4) it should be clear that in the limit \(k^2 h_4, k^2 h_5 \gg 1\), it is possible to have \(\eta = h_2 h_4/h_5\). This implies a value of the slip that is generally different from unity and currently unconstrained by the speed of gravitational waves. As we will argue forward for theories which do not introduce any additional mass scale besides the Planck mass, a similar conclusion holds at all the scales within the quasi-static approximation.

II. A PROTOTYPICAL EXAMPLE: BRANS-DICKE MODEL

First, we will focus on one of the most well-known examples of modified theories of gravity, Brans-Dicke [19]. In the Jordan frame, its action can be written as

\[
S_{\text{BD}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ \phi \left( -\omega_{\text{BD}} (\partial \phi)^2 - V(\phi) \right) + S_m \right],
\] (5)

where \(S_m\) corresponds to the matter action, \(\omega_{\text{BD}}\) is the Brans-Dicke parameter and \(V(\phi)\) is the self-interacting potential that can be taken, for instance, as a cosmological constant. From Eq. (5) it should be clear that the dimensionless scalar field \(\phi\) performs an effective rescaling of the gravitational constant. The General Relativity limit of the theory is thus recovered when \(\omega_{\text{BD}} \to \infty\) and \(\phi \to 1\). Currently, the tightest constraints available on the Brans-Dicke parameter come from Solar-System tests, placing a lower bound of 40000 on \(\omega_{\text{BD}}\) [20, 21]. On cosmological scales, this bound is relaxed by approximately one order of magnitude [22, 23] but, on the other hand, it applies also to the case in which baryons are uncoupled.

Despite its apparent simplicity, Brans-Dicke introduces an interesting phenomenology across all periods of cosmic history. At early times, it is well established that the BD scalar field follows distinct attractor solutions: it remains frozen during radiation domination and follows a power-law of the scale factor \(a\) during matter domination. Given the role of \(\phi\), the rescaling of the cosmological gravitational constant \(G_{\text{cosm}} = G_N / \phi\) produces a shift in the background expansion history compared to \(\Lambda\)CDM, that is suppressed as we approach \(z = 0\) if one fixes the present-day value of the scalar field to \(\phi_0 \approx 1\).

The modifications introduced by the Brans-Dicke theory also extend to the linear evolution of the gravitational potentials, \(\Phi\) and \(\Psi\). In the regime of validity of the quasi-static approximation, the theory predicts a non-trivial value for the slip between the gravitational potentials given by [18]

\[
\eta = \frac{1 + \omega_{\text{BD}} + \phi(M_a/k)^2}{2 + \omega_{\text{BD}} + \phi(M_a/k)^2},
\] (6)

where \(M^2 \approx V_\phi\) is the mass scale introduced by the Brans-Dicke theory. It is evident that if \(M = 0\) as, for instance, for the case of a constant potential \(V(\phi)\), the mass scale is removed and \(\eta\) becomes scale independent. If we take into account the cosmological constraints on the Brans-Dicke parameter, roughly \(\omega_{\text{BD}} < 1000\) [22, 23], \(\eta\) could still deviate from unity at the \(10^{-3}\) level.

III. NONLOCAL MODELS

As another less trivial example, we choose the Deser-Woodard (DW) nonlocal gravity model where the new terms added to the Einstein-Hilbert action do not introduce any additional dimension scale into the theory [24]. Indeed the action of the DW model is given by

\[
S_{\text{DW}} = \frac{1}{16\pi G} \int \sqrt{-g} d^4x \left[ R + R f \left( \frac{R}{M} \right) \right] + S_m.
\] (7)
In this action the modification of gravity is achieved by the inclusion of the term \( f(R/\Box) \). The function \( f \) is a general analytic function and should be chosen to reproduce a cosmologically valid evolution of our Universe, both at the background and perturbation levels [25, 26]. The absence of a new scale within the DW model is explained by the fact that the Ricci scalar \( R \) and d’Alambert operator \( \Box \) have the same dimension so the argument of the function \( f \) is a dimensionless quantity. Phenomenological studies of the DW model are performed usually by localizing it [27, 28]. This is done by introducing two scalar fields \( X \) and \( U \) defined through the following differential equations

\[
\Box X \equiv R, \quad \Box U \equiv f' R, \quad \tag{8}
\]

where \( f' \) stands for the derivative of the function \( f \) with respect to its argument. Using auxiliary fields \( X \) and \( U \) we can translate the initial nonlocal theory \( \Box \) into a local multi-scalar one. Investigation of the DW model at the background level have been performed in a way to reproduce exact \( \Lambda \)CDM behavior. This has been achieved by choosing a particular structure for the function \( f \) [26]. After fixing the structure of the function \( f \) there are no more free parameter in the model, and we can proceed to investigate the behavior of the DW model at the perturbation level. The perturbative studies of the DW model show that it has a distinct behavior from \( \Lambda \)CDM at this level. Moreover, from the observational point of view it can also be favored over \( \Lambda \)CDM model [27, 28]. The gravitational slip parameter for the DW model has the structure

\[
\eta = -\frac{1 + U + f - 4f'}{1 + U + f - 8f'^2}. \quad \tag{9}
\]

This expression shows that, in the quasi-static approximation, the gravitational slip parameter does not have a scale dependence and can have a value different from unity. In Ref. [27] it was found numerically \( \eta \approx -0.65 + 1.79a^2 - 0.54a^3 \). So for the DW model in the quasi-static approximation there is no limit where we recover the GR value of \( \eta \).

The scale independence of \( \eta \) in the DW model can be directly understood from the structure of the perturbation equations. Here, for simplicity, we show just the perturbation of the equation corresponding to the auxiliary field \( X \). However, the arguments presented below also hold for the other fields introduced in the theory. For the linear perturbation \( \delta X \) of the auxiliary field \( X \) one has the following equation

\[
\delta \ddot{X} + 3H \delta \dot{X} + \frac{k^2}{a^2} \delta X = \tag{10}
-6H \dot{\Phi} - \dot{\Psi} (\dot{\Psi} + 3 \dot{\Phi}) \left( \dot{X} + 6H \right) - 2 \frac{k^2}{a^2} (\Psi + 2 \Phi),
\]

where an over-dot denotes the derivative with respect to the cosmic time \( t \) and the Hubble function is defined as \( H \equiv a/a \). As can be easily recognized from Eq. (10), due to the fact that in the quasi-static approximation \( k \gg 1 \), we do not have any additional momentum scale in the theory, we cannot compensate the contribution of the term \( \dot{k}^2 (\delta X + 2 \Psi + 4 \Phi) \); hence, as a result from Eq. (10), we get

\[
\delta X = -2 \left( \Psi + 2 \Phi \right). \quad \tag{11}
\]

This outcome [29, 30] demonstrates that, for the DW model in the quasi-static approximation, the perturbation of the auxiliary field \( X \) does not have a scale dependence. One also obtains a similar result for the perturbed auxiliary field \( U \), which is given by \( \delta \dot{U} = f' \delta X \). On the other hand, from the perturbed spatial components of the Einstein equations for the DW model, in the absence of the anisotropic stress, one obtains the following relation between the Bardeen potentials and the perturbations of the auxiliary fields

\[
\Psi + \Phi + f' \delta X + \delta U + (\Psi + \Phi) (f + U) = 0. \quad \tag{12}
\]

By using Eq. (12) and the expressions for \( \delta X \) and \( \delta U \) we obtain the gravitational slip parameter (9) for the DW model. So we once more confirm that \( \eta \) for the DW model in the quasi-static approximation is a scale-independent variable, which in general has a value different from unity, and cannot be constrained by the measurements of \( \eta \).

The speed of gravitational waves within the DW model has been first obtained in Ref. [31], where the author shows that the gravitational waves in the DW model propagate with the speed of light and have two orthogonal polarizations as in the case of GR. Then, the DW model modifies only the gravitational waves amplitude compared to GR, which is due to a different rate of the background expansion in this model. This might lead to other observational effects (see [29, 30]).
IV. SHIFT SYMMETRY: COVARIANT GALILEON THEORY

We close our discussion with an example of the Galileon theory which is given by the following action [32]

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R + \frac{1}{2} \sum_{i=1}^{5} c_i L_i \right] + S_m, \tag{13} \]

where \( (c_i, \{i, 1, 5\}) \) are the free parameters of the model and the corresponding five covariant Lagrangians \( L_i \) are defined as

\[
\begin{align*}
L_1 &= \tilde{M}^3 \phi, \\
L_2 &= (\nabla \phi)^2, \\
L_3 &= (\Box \phi)(\nabla \phi)^2 / \tilde{M}^3, \\
L_4 &= (\nabla \phi)^2 \left[ 2 (\Box \phi)^2 - 2 \phi_{\mu\nu}\phi^{\mu\nu} - R (\nabla \phi)^2 / 2 \right] / \tilde{M}^2, \\
L_5 &= (\nabla \phi)^2 \left[ (\Box \Phi)^3 - 3 (\Box \phi) \phi_{\mu\nu}\phi^{\mu\nu} + 2 \phi_{\mu\nu}\phi_{\rho\sigma}\phi^{\mu\nu}\phi^{\rho\sigma} - 6 \phi_{\mu\nu}\phi^{\mu\nu}\phi^{\rho} G_{\nu\rho} \right].
\end{align*}
\]

Here, the parameter \( \tilde{M} \) should not be confused with a mass term since it is the coefficient of a term which is linear in the Galileon field \( \phi \).

The Galileon theory defined by the above mentioned action is invariant under the field transformation

\[ \phi \to \phi + b_{i\mu} x^\mu + C, \tag{17} \]

where \( b_{i\mu} \) and \( C \) are arbitrary constants. This symmetry is also known as a Galileon symmetry. Being invariant under shift symmetry, the Galileon theory does not introduce any additional mass scale into the theory. Hence, in agreement with the above discussed examples, the Galileon theory in the quasi-static approximation will have a scale-independent gravitational slip parameter generally different from unity. Indeed, in this case the linear perturbation equations have the structure [33]

\[
\begin{align*}
B_6 \Phi + B_7 \delta \phi + B_8 \Psi &= 0, \\
D_7 \Phi + D_9 \delta \phi + D_{10} \Psi &= 0,
\end{align*}
\]

where \( B_i \) and \( D_i \) are scale-independent functions that depend on \( H, c_i, \tilde{M} \) and on the Galileon field \( \phi \) and its time derivatives. Now, by using Eqs. (18) and (19) to subtract the perturbation of the Galileon field \( \delta \phi \), we obtain for the gravitational slip parameter the following expression

\[ \eta = \frac{D_{10} B_7 - D_9 B_8}{D_7 B_7 - D_9 B_6}. \tag{20} \]

So in agreement with the examples presented in the previous sections we observe that for the Galileon theory in the quasi-static approximation the gravitational slip parameter again does not have a scale dependence and in general will not approach the GR value \( \eta = 1 \). However, as it has been shown in Ref. [5], the gravitational wave speed put stringent constraints on the structure of the functions \( B_i \) and \( D_i \). Namely, it has been shown that in order to pass the gravitational wave constraints the constants \( c_4 \) and \( c_5 \) in the action (13) must be vanishing. Under these conditions the functions \( B_i \) and \( D_i \) in Eqs. (18) and (19) reduce to

\[
\begin{align*}
B_7 &= 0, & B_6 &= B_8 = 2M_{pl}^2, \\
D_9 &= c_2 - 4c_3 H \tilde{\phi} / \tilde{M}^3 - 2c_3 \tilde{\phi} / \tilde{M}^3.
\end{align*}
\]

Inserting these expressions into Eq. (20) we get \( \eta \) to be exactly unity. This case is therefore trivial, since for \( c_4, c_5 = 0 \) there is no modification of gravity at all.

V. CONCLUSIONS

The recent precise measurement of the GW speed/light speed ratio killed several extended forms of modified gravity models, e.g. those in which the coupling of a scalar field to gravity is not conformal. Even duly remarking that the constraint applies only at the present time and only to models in which both dark matter and baryons are coupled, this historical observation has several important consequences. One of these is that the observable anisotropy parameter \( \eta \) has to be unity at scales larger than the field mass scale. As noted in Ref. [3], however, this applies only insofar the modification of gravity introduces a new mass scale in the theory. In this paper we point out that when no new
mass scale is introduced (besides the Planck mass) then \( a \) \( \eta \) becomes scale-independent, \( b \) \( \eta \) is in general different from unity, and \( c \) it is not necessarily constrained by the GW speed.

To explore this scenario, we discussed three models in which no mass scale arises: standard Brans-Dicke, nonlocal DW model, and Galileon Lagrangians. In the first two cases \( c_T \) is predicted to be exactly unity, but \( \eta \) is left unconstrained, scale-independent, and in general different from unity. For the Galileon Lagrangian, \( \eta \) is indeed scale-independent, but the requirement \( c_T = 1 \) imposes that the gravity sector is a standard one and therefore \( \eta = 1 \). Except for this case, our study confirms that \( \eta \) remains a very informative modified gravity parameter even after the GW events and should be a prime target for observational studies moving forward.

ACKNOWLEDGMENTS

We acknowledge support from DFG through the project TRR33 “The Dark Universe.”

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