Violation of no-signaling in higher-order quantum measure theories

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More general probability sum-rules for describing interference than found in quantum mechanics (QM) were formulated by Sorkin in a hierarchy of such rules. The additivity of classical measure theory corresponds to the second sum-rule. QM violates this rule, but satisfies the third and higher sum-rules. This evokes the question of whether there are physical principles that forbid their violation. We show that under certain assumptions, violation of higher sum-rules allows for superluminal signaling.

I. INTRODUCTION

The structure of QM is famously rigid: it is exactly unitary and linear. Measurement outcomes are described by the quadratic Born probability rule. Multi-particle systems are characterized by a tensor product structure. Modifying QM even slightly, for example by introducing nonlinear observables [1] or nonunitary evolution [2] leads to implausible consequences, such as the violation of the no-signaling principle [3,9] or the efficient solution of computationally hard problems [2,10]. Here it may be noted that the nonlinearity or nonunitarity mentioned above pertains to the basic dynamical equations, and do not refer to the nonlinearity occurring in effective theories such as the Gross-Pitaevski equation [11] or the Euler-Heisenberg Lagrangian [12], or to the nonunitarity occurring in open quantum systems [13]. From a foundational perspective, understanding what modifications to QM are tenable is relevant to one of the open problems in modern theoretical physics: namely, the unification of QM with general relativity for creating a unified theory of quantum gravity. This presumably involves generalizing either theory in hope of a better match with the current version of the other.

One approach to studying variants of QM is via the post-quantum framework of generalized probability theories [14] or considering the properties of correlations shared between two or more observers [15]. In another approach due to Sorkin [16], and actively under study [17,18], QM is placed within a hierarchy of probability measure theories, whose members can be distinguished operationally using a generalization of Young’s double slit experiment [16]. In such an experiment, one assigns a probability measure to a set of pathways belonging to a particle being detected at a given position, with slits A and B, in which one or both slits may be left open. For any point on the screen, we can write down the three quantities $P_{\psi_A}$, $P_{\psi_B}$, and $P_{\psi_C}$, respectively. Here ‘∧’ is the Boolean AND operator. For quantum probability, the interference term:

$$I_2(A,B) \equiv P(A \land B) - P(A) - P(B)$$

is non-vanishing, i.e., the 2-sum rule $I_2(A,B) = 0$, fails [16], meaning that probabilities with individual slits being open are not additive.

On the other hand, the quantum mechanical Born rule satisfies the 3-sum rule, in that the three-term interference

$$I_3(A,B,C) \equiv P(A \land B \land C) - P(A \land B) - P(A \land C) - P(B \land C) + P(A) + P(B) + P(C)$$

(2)

vanishes. Here $P(A \land B \land C)$ is the probability to detect a particle at a given position, with slits A, B and C open. In QM, suppose $\psi_j$ ($j = A, B, C$) is the amplitude that a particle propagates from slit j to point x on the screen, then we have $P(A \land B \land C) \equiv |\psi_A + \psi_B + \psi_C|^2$, $P(A \land B) = |\psi_A + \psi_B|^2$, etc., and $P_A = |\psi_A|^2$, etc. Substituting these into Eq. (2), we find that $I_3(ABC) = 0$, implying that third-order (and higher-order interference) are absent in the hierarchy of sum-rules defined as follows. Informally, this is because interference occurs through mixing of pairs of paths and not triplets or quadruplets of paths.

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The validation of the $N$-sum rule requires the vanishing of the $N$-th order interference term

$$I_N(A_1, A_2, \cdots, A_N) \equiv P \left( \bigwedge_j A_j \right) - \sum P([N-1]-sets)$$

$$+ \sum P([N-2]-sets) - \cdots (-1)^{N-1} \sum_{j=1}^{N} P(A_j),$$

where $\sum P([N-1]-sets)$ is the sum of probabilities over all choices of $(N-1)$ open slits, etc [10]. Experiments to date place a stringent upper bound on such a term [19–21].

It is of interest to know whether such modifications to QM can accommodate other properties of QM, considered to be fundamental, since if this were not so, then this incompatibility could be used as an axiomatic basis [22] to rule out higher order interference. Here we prove that, under certain assumptions, such higher order super-quantum interferences indeed lead to superluminal signaling.

In developing this hierarchical framework, Sorkin had ignored contributions from non-classical i.e., looped paths [10]. In a recent work, one of us has investigated the effect of including such paths in the calculation [23]. We find that taking into account non-classical paths in a triple slit problem, does indeed generate a non-zero third order interference term. However, in our calculations, we still hold the Born rule to be true and essentially complete the textbook picture which tends to ignore non-classical paths. Thus, our results [23] do not contradict the claims of the current work, where Sorkin’s no-looped path assumption is adopted.

Another assumption is that the state space structure of quantum mechanics holds good, with only the dynamical part altered to accommodate a new interference recipe. In this context, it is of interest to note a recent work [24], where the authors present a formalism to realize the higher-order interferences, with states being represented by tensors of correspondingly larger number of indices. In contrast to our approach, their method allows for the possibility of non-quantum states. It will be interesting to investigate the consequence from our signaling point of view for such generalized theories.

The remaining article is arranged as follows. In Section II, we briefly recall the no-signaling principle. Whereas certain postquantum theories, such as those discussed in Refs. [13, 15] are explicitly designed to be non-signaling, the status of signaling in Sorkin’s formalism has not been studied, as far as we know. An argument is presented in Section III for why a violation of no-signaling may result from deviations from the Born rule under the assumption of standard properties of the state space of QM and its tensor product structure. It is not straightforward to apply this result to the Sorkin formalism because the latter only specifies how quantum interference can be generalized, while remaining mute on whether or what other alterations to the theory accompany this. Under the stated assumptions, a specific realization of nonlocal signaling in the Sorkin formalism is considered and demonstrated in Section IV using a non-maximally entangled state in a sufficiently high-dimensional space. Our result does not rule out modifications to QM along the lines envisaged by Sorkin, but suggests regimes where such effects may be relevant. We briefly adumbrate this point and conclude in Section V.

II. NO-SIGNALING

No-signaling is a fundamental feature of QM as we understand it, and implies that a signal does not travel from one point to another except through the physical communication of a particle. It is in fact a feature of non-relativistic QM, but compatible with relativity. Einstein-Podolsky-Rosen (EPR) correlations interpreted “realistically” imply a nonlocal influence, demonstrated by the violation of Bell-type inequalities [25], but this nonlocality cannot be used for signaling.

Formally, no-signaling is the statement that the reduced density operator of a system is unaffected by local operations on another system with which it may be entangled. Given the joint density operator $\rho_{AB}$ of two particles $A$ and $B$, and any local operation $E_j$ on particle $B$ (a local unitary, a measurement or a general positive operator valued measure), let $\rho'_{AB}$ be the joint state after the operation. We have

$$\rho'_A = \text{Tr}_B \left( \sum_j E_j \rho_{AB} E_j^\dagger \right) = \text{Tr}_B \left( \sum_j E_j^\dagger E_j \rho_{AB} \right) = \rho_A,$$

since we necessarily have that $\sum_j E_j^\dagger E_j = I$, by completeness. The above proof of no-signaling presumes the validity of taking partial trace, which is related to the Born rule, in the sense that it is the unique function of the density operator of a composite system that is compatible with the Born rule and the tensor product structure [20]. The implications for signaling of relaxing this rule are now considered.
III. NO-SIGNALING AND NON-CONTEXTUALITY

By Gleason’s theorem \cite{27}, non-Bornian probabilities are contextual. Such probabilities can be the basis for a nonlocal signaling mechanism \cite{28}. We study this point in some detail. Consider a composite system \( S \) described by the Hilbert space \( \mathcal{H}_S \equiv \mathcal{H}_A \otimes \mathcal{H}_B \). A local unitary operation on side \( A \) has the form \( \epsilon \equiv v_A \otimes I_B \), where \( I_B \) is the identity operation in \( \mathcal{H}_B \). And similarly \( \epsilon' \equiv v_A' \otimes I_B \). We form the partition:

\[
\mathcal{H}_S = \bigoplus_k \mathcal{H}_A \otimes \mathcal{B}_k \equiv \bigoplus_j \mathcal{J}_k,
\]

where \( \mathcal{B}_k = \bigoplus_j \mathcal{J}_k \). It follows that each partition \( \mathcal{J}_k \) is an invariant subspace under \( \epsilon \) and under \( \epsilon' \), in the sense that if a composite system exists in \( \mathcal{J}_k \), then it is not shifted out of \( \mathcal{J}_k \) if subjected to \( \epsilon \) or \( \epsilon' \).

Let \( \Phi_k \equiv (\Phi(A) \otimes \Phi(B))_j \) be a fiducial, separable basis for the complement \( \mathcal{J}_k \). According to the assumption of non-contextuality in the sense of Gleason \cite{27}, the probability measure \( \mu[\mathcal{J}_k] \) associated with \( \mathcal{J}_k \) should be independent of whether the basis of measurement in \( \mathcal{J}_k \) is chosen to be \( \epsilon (\Phi_k) \) or \( \epsilon' (\Phi_k) \). We have therefore

\[
\mu[\mathcal{J}_k|\epsilon(\Phi_k)] = \mu[\mathcal{J}_k|\epsilon'(\Phi_k)] \equiv \mu[\mathcal{J}_k].
\]

In other words, the probability associated with \( \mathcal{J}_k \) is independent of whether the basis of measurement is completed in the complementary state space by \( \epsilon (\Phi_k) \) or \( \epsilon' (\Phi_k) \). Now,

\[
\mu[\mathcal{J}_k|\epsilon(\Phi_k)] = \sum_{a=1}^{\dim_A} \sum_{b \in \beta_k} \text{Prob}(A = a, B = b) \equiv \text{Prob}_B(k|\epsilon) \quad (7a)
\]

\[
\mu[\mathcal{J}_k|\epsilon'(\Phi_k)] = \sum_{a' = 1}^{\dim_A} \sum_{b \in \beta_k} \text{Prob}(A' = a', B = b) \equiv \text{Prob}_B(k|\epsilon'), \quad (7b)
\]

where \( A \) and \( A' \) are random variables representing basis elements in \( v_A (\Phi(A)) \) and \( v_A' (\Phi(A)) \) and \( \beta_k \) is the set of dimensions in \( \Phi(B) \) that span \( \mathcal{B}_k \). Here \( \text{Prob}_B(jk\xi) \) is the probability for Bob to obtain outcome \( k \) in the \( \xi \)-context (\( \xi = \epsilon, \epsilon' \)). Suppose that probabilities were contextual, and that we are able to find \( k \) such that

\[
\mu[\mathcal{J}_k|\epsilon(\Phi_k)] \neq \mu[\mathcal{J}_k|\epsilon'(\Phi_k)]. \quad (8)
\]

Then it follows from Eq. \( 7 \) that

\[
\text{Prob}_B(k|\epsilon) \neq \text{Prob}_B(k|\epsilon'), \quad (9)
\]

which represents a violation of no-signaling. The existence of contextualty does not necessarily entail that we can find such \( k \) in a composite system.

However, if \( A \) and \( B \) are entangled, \( A \) can remotely steer \( B \)’s ensemble. These ensembles which are unitarily equivalent in standard QM may become inequivalent when non-Bornian probabilities are allowed in an ensemble-dependent way, resulting in a signal. This turns out to be the origin of the signaling we obtain in Section IV A. On the other hand, if \( B \) produces such different ensembles locally, he may not produce a signal, because the contextuality is local. This is discussed immediately below.

Following an idea presented in Ref. \cite{28}, we may first try to use a single-particle system of sufficiently high dimension to demonstrate signaling. To test the violation of the 3-sum rule, we pass the particle through a beam splitter, of which one of the packets is either subjected to a 3-slit diaphragm or it is not. One may ask if the particle statistics in the other arm is affected by the choice, given the violation of the 3-sum rule. If yes, it would form the basis of a nonlocal signaling. However, the violation may simply cause a local redistribution of probabilities observed in the screen behind the slits, and not result in signaling. In the entanglement-based scheme considered in Section IV A, a similar conspiracy by local redistribution can thwart a signal when \( B \), rather than \( A \), determines the realized ensemble.

IV. THE TRIPLE- AND HIGHER-SLIT EXPERIMENTS AND SIGNALING

Entanglement is shared between Alice and Bob. The latter may employ a multiple-slit interference experiment, and correspondingly, his particle is of sufficiently high dimension. Alice’s is a spin-1/2 particle, which ensures that its
FIG. 1: Charlie distributes entanglement (10) to Alice and Bob, who performs a multi-slit interference experiment. In the scheme of Section IV A, the number of slits $N \geq 4$ and Alice observes her particle, while Bob measures his in the slit or screen basis. In the scheme of Section IV B, the number of slits $N \geq 3$ and Bob observes his particle always in the screen basis, while Alice measures hers in the computational or Hadamard basis.

measurement statistics are unaffected even if there is a deviation from Born rule due to the presence of higher order interference terms in the Sorkin framework.

We may begin by considering (maximal) entanglement between the modes of Bob’s particle, that are assumed to be localized at each slit, and corresponding modes of Alice’s particle. The impasse we are met with here is that the tight correlation will render Bob’s modes incoherent, precluding a test of higher-order interference. On the other hand, making Bob’s modes fully coherent renders them disentangled from Alice’s ones, and hence Alice powerless to remotely prepare Bob’s ensemble. What is required thus is non-maximal entanglement between Alice’s and Bob’s particles, which provides a trade-off between required coherence and remote control.

The scheme below requires a system of dimensionality $2 \times 4$. We present a more general version for a $2 \times N (N > 3)$ system. Charlie creates a non-maximally entangled state between a qubit and an $N+1$-dimensional particle, of the form:

$$|\Psi\rangle_{AB} = \sum_{j=0}^{N-1} \alpha_j |0\rangle_A |j\rangle_B + \alpha_N |1\rangle_A |N\rangle_B.$$  

(10)

where $\{|j\rangle\}$ constitutes modes that are sufficiently localized in the transverse direction. In this state, the entanglement is such that the first $N$ modes are coherent with each other (the off-diagonal terms are non-vanishing in the density operator when represented in this basis), while the last mode is incoherent from them.

Charlie now distributes the entanglement to Alice and Bob, such that Alice receives the first particle in $|\Psi\rangle_{AB}$ and Bob, who is spatially distant from her, receives the second particle. The state in Alice’s station, which is the reduced density operator of the first particle, is

$$\rho_A = \left(\sum_{j=0}^{N-1} |\alpha_j|^2\right) |0\rangle\langle 0| + |\alpha_N|^2 |1\rangle\langle 1|$$

$$\rho_B = \sum_{j,k=0}^{N-1} \alpha_j \alpha_k^* |j\rangle\langle k| + |\alpha_N|^2 |N\rangle\langle N|.$$  

(11)

Bob’s particle is passed through a set-up consisting of a diaphragm with $N+1$ slits, aligned to receive the transversely localized modes $|j\rangle$. We may thus regard $\{|j\rangle\}$ as the ‘slit basis’ of Bob (Figure 1). Measuring his particle in this basis, Bob leaves Alice’s particle in the state $\rho_A^{\text{slit}} = \rho_A$. 
For simplicity, the diffraction resulting from slit passage may be modelled by a discrete Fourier transform [26]:

$$U_f^j |j\rangle = \frac{1}{\sqrt{N+1}} \sum_{k=0}^{N} e^{2\pi ij k/(N+1)} |k\rangle,$$

where the output basis is assumed to refer to the screen. We find:

$$\left( I_A \otimes U_f^j \right) |\psi\rangle_{AB} = \sum_{j=0}^{N-1} \alpha_j |0\rangle_A \frac{1}{\sqrt{N+1}} \sum_{k=0}^{N} e^{2\pi ij k/(N+1)} |k\rangle_B + \alpha_N |1\rangle_A \frac{1}{\sqrt{N+1}} \sum_{k=0}^{N} e^{2\pi iN k/(N+1)} |k\rangle_B$$

$$= \frac{1}{\sqrt{N+1}} \sum_{k=0}^{N} \left[ \left( \sum_{j=0}^{N-1} e^{2\pi ij k/(N+1)} \alpha_j \right) |0\rangle_A + e^{2\pi iN k/(N+1)} \alpha_N |1\rangle_A \right] |k\rangle_B$$

$$= \sum_{k=0}^{N} [Y_k |0\rangle + Z_k |1\rangle] |k\rangle = \sum_{k=0}^{N} |\psi_k\rangle_A |k\rangle_B,$$

(13)

Bob measures his particle in the state (13) in the ‘screen basis’ \{|k\rangle_B\}. The probability with which he detects each \(|k\rangle_B\) determines his fringe pattern and is given by the norm of each mode in Eq. (13):

$$B_k = \| |\psi_k\rangle |^2 = \frac{1}{N+1} \left( \sum_{j,j'} e^{2\pi i(j-j') k/(N+1)} \alpha_j^* \alpha_{j'} + |\alpha_N|^2 \right),$$

(14)

from which it follows that \(\sum_{j=0}^{N} B_j = 1\), using the identity \(\frac{1}{N+1} \sum_{k=0}^{N} e^{2\pi i k/(N+1)} = \delta_{00}\). We note that the structure of the fringe pattern, as an incoherent sum of the contribution from the last slit and a coherent contribution from the slits 0 through \(N-1\), is due to the entanglement (Figure 1).

A. On signaling from Bob to Alice

Let the state of Alice’s particle conditioned on Bob’s measuring in the slit basis be denoted \(\rho_A^{\text{slit}}\). Denoting the normalized version of \(|\psi_j\rangle\) by \(|\phi_j\rangle \equiv \frac{1}{\sqrt{B_j}} |\psi_j\rangle\), we find that the state of the first particle, conditioned on the measurement of the second in the screen basis, is:

$$\rho_A^{\text{scr}} = \sum_j B_j |\phi_j\rangle_A \langle \phi_j | = \sum_j |\psi_j\rangle_A \langle \psi_j | = \rho_A.$$

(15)

We thus have \(\rho_A^{\text{scr}} = \rho_A^{\text{slit}}\), which is the statement of no-signaling in standard QM.

On the other hand, if \(n\)th-order interference \((3 \leq m \leq N)\) can occur, then the \(B_j\)’s in Eq. (14), which are obtained by the usual Born quadratic formula, must be replaced by \(B_j'\) such that \(\sum_j B_j' = \sum_j B_j = 1\). The exact form of \(B_j'\) doesn’t matter. It suffices to note that under a different sum rule, in general \(B_j' \neq B_j\).

We thus have in place of Eq. (15),

$$\rho_A^{\text{scr}} = \sum_j B_j' |\phi_j\rangle_A \langle \phi_j | = \sum_j B_j' \frac{|\psi_j\rangle_A \langle \psi_j |}{B_j} \neq \rho_A$$

(16a)

in general. By construction, Bob’s density operator \(\rho_A^{\text{scr}}\) is normalized, though not necessarily linearly related to \(\rho_A\). We note that \(\rho_A^{\text{slit}}\) is unaffected even if we assume a non-standard Sorkin prescription, since during measurement in the slit basis, only a single path (and not three or more paths) contributes to each possible detection. Thus, the usual Born recipe for calculating probabilities will hold good.

Suppose we allow that in addition to \(B_j\), the projected state \(|\phi_j\rangle_A\) also must be replaced by the equivalent \(|\phi_j'\rangle_A\) such that \(\sum_j B_j' \langle \phi_j | A | \phi_j' \rangle A = \sum_j |\psi_j\rangle \langle \psi_j |\). By the Hughston-Jozsa-Wootters theorem [29], Bob’s measurement is equivalent to measuring in the rotated basis \(U_{jk} |k\rangle\), where \(\sqrt{B_j'} |\phi_j'\rangle = \sum_k U_{jk} \sqrt{B_k} |\phi_k\rangle\), followed by standard quantum measurement, as seen by direct substitution. That \(U\) defined above is indeed unitary may be verified by taking the norm of the r.h.s, and verifying that it yields 1 when summed over \(j\), only when \(U\) has this property. In other words, we still really remain within the scope of the Born rule.
Bob can thus potentially transmit a superluminal signal to Alice by remotely preparing the state \( \rho_A \) or \( \rho_A^{\text{scr}} \), by measuring in the slit basis or the screen basis. However, it turns out that the conspiracy of local redistribution of probability may nullify this signal. We have that \( B_k = |Y_k|^2 + |Z_k|^2 \). Suppose we have that:

\[
B'_k = |Y'_k|^2 + |Z'_k|^2 \text{ such that } \sum_{k=0}^{N-1} |Y_k|^2 = \sum_{k=0}^{N-1} |Y'_k|^2.
\]  

(17)

In other words, the probabilities of \( |k\rangle \)’s correlated with \( |0\rangle_A \) are redistributed amongst themselves, so that the probability for Alice to observe \( |0\rangle_A \) or \( |1\rangle_A \) is unaltered.

### B. On signaling from Alice to Bob

As noted earlier, the nullification of the above signal through local redistribution can be attributed to the interferometric observer (Bob) choosing the ensemble. To amend this, we now propose that Alice makes the choice, while Bob observes his screen. In particular, Alice measures her particle either in the computational basis or Hadamard basis. In the latter case, Eq. (13) becomes:

\[
|\Psi'\rangle_{AB} = \frac{1}{\sqrt{2}} \sum_{k=0}^{N} [(Y_k + Z_k) |0\rangle_A + (Y_k - Z_k) |1\rangle_A] |k\rangle_B
\]

\[
= \frac{1}{\sqrt{N+1}} \sum_{k=0}^{N} \left( \sum_{j=0}^{N} e^{2\pi i j k/(N+1)} \alpha_j \right) \frac{1}{\sqrt{2}} [(|0\rangle_A + |1\rangle_A) - 2 e^{2\pi i N k/(N+1)} \alpha_N |1\rangle_A] |k\rangle_B.
\]

(18)

This entails an incoherent sum of two \((N+1)\)-path interference terms as observed by Bob. By contrast, if Alice measures in the computational basis, then from Eq. (14), we find that the screen probability is an incoherent sum of an \(N\)-path interference pattern and a singleton contribution. The Born rule naturally ensures that the resulting two ensembles are equivalent in that they give rise to the same density operator. It is not obvious that this equivalence should hold when the ensembles deviate from the standard rule according to two distinct non-Bornian recipes.

We illustrate this with a simple example for the case of a composite system with dimensionality \(2 \times 3\), which is smaller than that required in Section [IV.A]. Let the state shared between Alice and Bob be

\[
|\chi\rangle = |0\rangle_A (\alpha|0\rangle_B + \beta|1\rangle_B) + |\gamma|_A |2\rangle_B. 
\]

(19)

As before, the action of Bob’s triple-slit is modelled by the discrete Fourier transform

\[
F = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & e^{i\omega} & e^{-i\omega} \\ 1 & e^{-i\omega} & e^{i\omega} \end{pmatrix},
\]

(20)

where \( \omega = 2\pi/3 \). His particle’s reduced density operator at the screen is given by

\[
\sigma = \frac{1}{3} \left( (|\alpha + \beta|^2 + |\gamma|^2) |0\rangle_B\langle 0| + (|\alpha + e^{i\omega}\beta|^2 + |\gamma|^2) |1\rangle_B\langle 1| + (|\alpha + e^{-i\omega}\beta|^2 + |\gamma|^2) |2\rangle_B\langle 2| \right)
\]

(21)

which is an incoherent sum of amplitude contribution from the first two slits and the third. State \( \sigma \) will remain unaltered even if the 3-sum rule is violated, because in the two incoherent sectors of \( |\chi\rangle \) (that correlated with \( |0\rangle_A \) and that with \( |1\rangle_A \), at most only 2 paths are available for interference.

On the other hand, under Alice’s Hadamard transformation, state \( |\chi\rangle \) transforms to:

\[
|\chi'\rangle = \frac{|0\rangle_A}{\sqrt{2}} (\alpha|0\rangle_B + \beta|1\rangle_B + \gamma|0\rangle) + \frac{|1\rangle_A}{\sqrt{2}} (\alpha|0\rangle_B + \beta|1\rangle_B - \gamma|0\rangle).
\]

(22)

Assuming violation of the 3-sum rule, here we have the possibility for 3-path interference in each incoherent sector. When particle \( B \) is subjected to the triple-slit, the joint state is:

\[
|\chi''\rangle = \frac{1}{\sqrt{6}} |0\rangle_A (|0\rangle_B [\alpha + \beta + \gamma] + |1\rangle_B [\alpha + e^{i\omega}\beta + e^{-i\omega}\gamma] + |2\rangle_A [\alpha + e^{-i\omega}\beta + e^{i\omega}\gamma])
\]

\[
+ \frac{1}{\sqrt{6}} |1\rangle_A (|0\rangle_B [\alpha + \beta - \gamma] + |1\rangle_B [\alpha + e^{i\omega}\beta - e^{-i\omega}\gamma] + |2\rangle_A [\alpha + e^{-i\omega}\beta - e^{i\omega}\gamma])
\]

(23)
Because of violation of the 3-sum rule, the probability to detect at each point on the screen is not necessarily given as the incoherent sum of the squared law term but of some other function $F$, $G$, etc., of the amplitude contributions received from the three slits. For example, in Eq. (23) the probability for outcome $|\psi\rangle_A|\psi\rangle_B$ will not be $\frac{1}{6}|\alpha + \beta + \gamma|^2$ but some other function $F$ of this amplitude sum. Likewise, with the probability to obtain $|\psi\rangle_A|\psi\rangle_B$.

As an instance, for outcome $|\psi\rangle_B|\psi\rangle_B$ on the screen, no-signaling requires $F$ and $G$ such that

$$\frac{1}{2}|\alpha + \beta + \gamma|^2 + \frac{1}{2}|\alpha + \beta - \gamma|^2 = \frac{1}{2}F(\alpha + \beta + \gamma) + \frac{1}{2}G(\alpha + \beta - \gamma),$$

where the l.h.s is just the $\frac{1}{3}(|\alpha + \beta|^2 + |\gamma|^2)$ coefficient of $|\psi\rangle_B|\psi\rangle_B$ in Eq. (21). For general $\alpha, \beta, \gamma$ it is clear that the only prescription that achieves this equality is the usual quadratic Born recipe.

V. DISCUSSIONS AND CONCLUSIONS

We have shown that under certain assumptions, modifications to local quantum measurement in the form of violations of the 3-sum or higher-sum rules, is incompatible with no-signaling. Local redistribution of probabilities can potentially thwart the signal when a single particle is used or, in the case of nonlocal correlations, when the ensemble choice is made by the interferometric observer. However, when nonlocal quantum correlations between two particles are used by the remote observer to steer the ensemble realized by the interferometric observer, then local redistribution is no longer a barrier to signaling under violation of higher-sum rules. In such a case, the two Bob ensembles, which will be unitarily equivalent under the assumption of the Born rule, may no longer be equivalent. While we may thus presumably exclude violation of these higher sum-rules in the normal macroscopic regime probed by experiments so far, where relativistic causality holds, these deviations from the quantum postulates could be relevant in the sufficiently microscopic regime where quantum gravity effects arise.

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