The t-J model in the large N limit (N denotes the number of spin components) yields a pseudogap phase in the underdoped region which is related to a d-wave charge density wave (d-CDW). We present results for the doping dependence of the superconducting and d-CDW order parameters as well as for collective excitations in the presence of these two order parameters. We argue that the electronic Raman spectrum with B1g symmetry probes the amplitude fluctuations of the d-CDW at zero temperature.

The superconducting transition temperature $T_c$ in the cuprates shows a maximum at optimal doping $\delta = \delta_c$ and a monotonic decrease towards lower or higher dopings. On the other hand, the excitation gap in the one-particle spectrum, often called pseudogap, increases monotonically with decreasing doping, remains finite above $a$ monotonic decrease towards lower or higher dopings. On the other hand, the excitation gap in the one-particle spectrum, often called pseudogap, remains finite above $T_c$ in the underdoped region, and has d-wave symmetry. Generalizing the t-J model from 2 to N spin components it has been shown [1] that such a pseudogap arises in the large N limit of the t-J model due to an instability of the normal or the superconducting state with respect to a d-CDW (sometimes also called flux or bond-order wave). In the following we present results for the symmetry broken states, namely the doping dependence of the superconducting and d-CDW order parameters and the dynamics of d-wave charge density fluctuations in the presence of these order parameters. The relevance of a d-CDW for high-$T_c$ superconductors has recently also been discussed in Ref. [2].

In the case of the t-J model the d-CDW order parameter $\Phi$ is defined as

$$\Phi = -2tJ/N_c \sum_{k\sigma} \gamma(k) \langle \tilde{c}^\dagger_{kσ} \tilde{c}_{k+Qσ} \rangle. \quad (1)$$

$J$ is the Heisenberg coupling, $\tilde{c}^\dagger$, $\tilde{c}$ are creation and annihilation operators for electrons under the constraint that double occupancies of lattice sites are excluded, $N_c$ is the number of primitive cells, $\langle ... \rangle$ denotes an expectation value, and $Q$ is the wave vector of the d-CDW. $\gamma(k)$ is equal to $(\cos(k_x) - \cos(k_y))/2$. Keeping only the instantaneous contribution in the effective interaction $\tilde{V}$ (3) the order parameter $\Delta$ for d-wave superconductivity is

$$\Delta = 2(J - V_c)/N_c \sum_k \gamma(k) \langle \tilde{c}^\dagger_{k+Q} \tilde{c}_{k} \rangle. \quad (2)$$

$V_c$ is a repulsive nearest-neighbor Coulomb potential which is needed to stabilize the d-CDW with respect to phase separation $\tilde{V}$. In the presence of the above two order parameters the operators $\langle \tilde{c}^\dagger_{k,\uparrow}, \tilde{c}_{k,\downarrow}, \tilde{c}^\dagger_{k+Q,\uparrow}, \tilde{c}_{k-Q,\downarrow} \rangle$ are coupled leading to the following Green’s function matrix $\tilde{G}$

$$\tilde{G}^{-1}(\omega, k) = \begin{pmatrix} \omega - \epsilon(k) & -\Delta(k) & -i\Phi(k) & 0 \\ -\Delta(k) & \omega + \epsilon(k) & 0 & i\Phi(k) \\ -i\Phi(k) & 0 & \omega - \epsilon(k) & -\Delta(k) \\ 0 & -i\Phi(k) & -\Delta(k) & \omega + \epsilon(k) \end{pmatrix} \quad (3)$$

$\epsilon(k)$ is the one-particle energy, $\epsilon(k) = -(\delta t + \alpha J)(\cos(k_x) + \cos(k_y)) - 2t\delta\cos(k_x)\cos(k_y) - \mu$, with $\alpha = 1/N_c \sum_q \cos(q_x) f(\epsilon(q))$. $f$ is the Fermi function, $\delta$ the doping away from half-filling, $\mu$ a renormalized chemical potential, $t$ and $t'$ are nearest and second-nearest neighbor hopping amplitudes, $\omega$ a (complex) frequency, and $\tilde{k} = k - Q$. $\Phi(k)$ and $\Delta(k)$ are equal to $\Phi\gamma(k)$ and $\Delta\gamma(k)$, respectively.

### References

1. Competition between superconductivity and the pseudogap phase in the t-J model

R. Zeyher and A. Greco

*MPI-FKF, Stuttgart, Germany,*

b *FCEIA and IFIR(UNR-CONICET), 2000-Rosario, Argentina*

(March 22, 2022)

The superconducting transition temperature $T_c$ in the cuprates shows a maximum at optimal doping $\delta = \delta_c$ and a monotonic decrease towards lower or higher dopings. On the other hand, the excitation gap in the one-particle spectrum, often called pseudogap, increases monotonically with decreasing doping, remains finite above $T_c$ in the underdoped region, and has d-wave symmetry. Generalizing the t-J model from 2 to N spin components it has been shown [1] that such a pseudogap arises in the large N limit of the t-J model due to an instability of the normal or the superconducting state with respect to a d-CDW (sometimes also called flux or bond-order wave). In the following we present results for the symmetry broken states, namely the doping dependence of the superconducting and d-CDW order parameters and the dynamics of d-wave charge density fluctuations in the presence of these order parameters. The relevance of a d-CDW for high-$T_c$ superconductors has recently also been discussed in Ref. [2].

In the case of the t-J model the d-CDW order parameter $\Phi$ is defined as

$$\Phi = -2tJ/N_c \sum_{k\sigma} \gamma(k) \langle \tilde{c}^\dagger_{kσ} \tilde{c}_{k+Qσ} \rangle. \quad (1)$$

$J$ is the Heisenberg coupling, $\tilde{c}^\dagger$, $\tilde{c}$ are creation and annihilation operators for electrons under the constraint that double occupancies of lattice sites are excluded, $N_c$ is the number of primitive cells, $\langle ... \rangle$ denotes an expectation value, and $Q$ is the wave vector of the d-CDW. $\gamma(k)$ is equal to $(\cos(k_x) - \cos(k_y))/2$. Keeping only the instantaneous contribution in the effective interaction $\tilde{V}$ (3) the order parameter $\Delta$ for d-wave superconductivity is

$$\Delta = 2(J - V_c)/N_c \sum_k \gamma(k) \langle \tilde{c}^\dagger_{k+Q} \tilde{c}_{k} \rangle. \quad (2)$$

$V_c$ is a repulsive nearest-neighbor Coulomb potential which is needed to stabilize the d-CDW with respect to phase separation $\tilde{V}$. In the presence of the above two order parameters the operators $\langle \tilde{c}^\dagger_{k,\uparrow}, \tilde{c}_{k,\downarrow}, \tilde{c}^\dagger_{k+Q,\uparrow}, \tilde{c}_{k-Q,\downarrow} \rangle$ are coupled leading to the following Green’s function matrix $\tilde{G}$

$$\tilde{G}^{-1}(\omega, k) = \begin{pmatrix} \omega - \epsilon(k) & -\Delta(k) & -i\Phi(k) & 0 \\ -\Delta(k) & \omega + \epsilon(k) & 0 & i\Phi(k) \\ -i\Phi(k) & 0 & \omega - \epsilon(k) & -\Delta(k) \\ 0 & -i\Phi(k) & -\Delta(k) & \omega + \epsilon(k) \end{pmatrix} \quad (3)$$

$\epsilon(k)$ is the one-particle energy, $\epsilon(k) = -(\delta t + \alpha J)(\cos(k_x) + \cos(k_y)) - 2t\delta\cos(k_x)\cos(k_y) - \mu$, with $\alpha = 1/N_c \sum_q \cos(q_x) f(\epsilon(q))$. $f$ is the Fermi function, $\delta$ the doping away from half-filling, $\mu$ a renormalized chemical potential, $t$ and $t'$ are nearest and second-nearest neighbor hopping amplitudes, $\omega$ a (complex) frequency, and $\tilde{k} = k - Q$. $\Phi(k)$ and $\Delta(k)$ are equal to $\Phi\gamma(k)$ and $\Delta\gamma(k)$, respectively.

Expressing the expectation values in the order parameters by $G$ and using Eq.(3) one obtains coupled equations for the two order parameters. The thick solid and dashed lines in Fig. 1 show the numerically determined doping dependence of $\Phi$ and $\Delta$ at zero temperature, calculated for $t'/t = -0.35$, $J/t = 0.3$, $V_c/t = 0.06$, and $Q = (\pi,\pi)$. The energy unit is $t$. The thin solid and dashed lines show $\Phi$ and $\Delta$ for non-interacting order parameters putting the second order parameter to zero. In this case the onset of $\Phi$ is much higher than in the interacting case, especially
if $t'$ is nonzero. This reduction in the onset of $\Phi$ reflects the competition of the two order parameters. The superconducting order parameter $\Delta$ increases monotonically with decreasing doping in the non-interacting case but is heavily suppressed by $\Phi$ below the onset of $\Phi$ in the interacting case. For $0.05 < \delta < 0.15$ both order parameters are non-zero so that superconductivity and d-CDW coexist throughout this region.

The eigenvalues of the matrix $G^{-1}$ give the one-particle energies in the presence of the two order parameters. The corresponding density of states $\rho(\omega)$ is shown in Fig. 2 for three different dopings. In the upper diagram, corresponding to the overdoped case with $\Phi = 0$, the usual density for a d-wave superconductor is seen. The lowest diagram in Fig. 2 corresponds to the underdoped region dominated by the d-CDW. The zero in $\rho(\omega)$ occurs somewhat above the chemical potential $\omega = 0$ and $\rho(\omega)$ is rather asymmetric around this point. The diagram in the middle of Fig. 2 describes a slightly underdoped case where both order parameters are of similar magnitude. $\rho(\omega)$ reflects here both gaps and possesses additional structures due to the geometry of the two-dimensional Fermi surface and the Umklapp processes induced by $Q$.

FIG. 1. Order parameters $\Phi$ and $\Delta$ as a function of doping in units of $t$ at $T = 0$ in the interacting (thick lines) and non-interacting (thin lines) case.

FIG. 2. One-particle density of states for three different dopings.

The above calculations were performed assuming a commensurate $Q = (\pi, \pi)$. In reality the d-CDW is at low temperatures and a finite doping incommensurate $Q = (\pi, q)$ with $q$ different from $\pi$. Writing $q = \pi - x$ the incommensuration $x$ was approximately calculated from the leading instability in $k$-space using a linear approximation, $x$ is shown in Fig. 3 as a function of $\delta$. The left diagram in this figure corresponds to $\Delta = 0$, i.e., the thin line in Fig. 1. In this case $x$ assumes the rather large value of about 0.6 at the onset of the d-CDW, but then decays rapidly to about 0.1 with decreasing doping. The right diagram in Fig. 3 illustrates that superconductivity not only depresses the onset of the d-CDW but also the incommensuration $x$ resulting in a practically commensurate d-CDW in the underdoped regime. In our calculations $x$ has been determined from that momentum where the d-wave charge susceptibility in the superconducting or normal state shows the
strongest divergence.

**FIG. 3.** Deviation $x$ of one wave vector component of the d-CDW from $\pi$ as a function of doping in the absence (left diagram) and presence (right diagram) of superconductivity.

Let us denote by $\chi(q, \omega)$ the response function associated with the d-wave density operator $\rho(q) = 1/N_c \sum_{k\sigma} \gamma(k)e_{k+q,\sigma}^\dagger e_{k\sigma}$. At large $N$ $\chi$ is given by $\chi(q, \omega) = \chi^{(0)}(q, \omega)/(1 + J\chi^{(0)}(q, \omega))$ which holds exactly at $q = (0,0)$ and $(\pi, \pi)$ and in a good approximation for a general $q$. For a general $q$ the explicit expression for $\chi^{(0)}$ is rather involved, so we give it here only for the special case $q = (0,0)$,

$$\chi^{(0)}(\omega) = P_{11,11}(\omega) - P_{12,21}(\omega) - P_{13,31}(\omega) + P_{14,41}(\omega),$$

(4)

$$P_{ij,kl}(\omega) = \frac{2T}{N_c} \sum_{k,n,\gamma} \gamma^2(k)G_{ij}(i\omega_n + \omega, k)G_{kl}(i\omega_n, k).$$

(5)

The left diagram in Fig. 4 shows the negative imaginary part of $\chi^{(0)}$ (dashed curves) and of $\chi$ (solid curves) at $q = (0,0)$ and three different dopings. The curves illustrate that the Heisenberg term as the residual interaction strongly scatters the quasi-particle excitations across the pseudogap shifting most of the spectral weight down into bound states inside the gap. In the overdoped case (upper diagram in Fig. 4) this bound state describes an exciton state inside the superconducting gap. In the strongly underdoped case (lowest diagram) the bound state corresponds to the amplitude mode of the d-CDW probed at a

**FIG. 4.** Left diagram: Correlation function for d-wave density fluctuations at $q = (0,0)$ in the free (dashed lines, called $B_{1g}^{(0)}$) and interacting (solid lines, called $B_{1g}$) case; Right diagram: dispersion of the main peak in the d-wave density fluctuation spectrum along $(q,q)$ for three different dopings.
wave vector $(\pi, \pi)$ away from the wave vector of the d-CDW, i.e., at $q = (0, 0)$. With decreasing doping the frequency of the peak increases monotonically and thus is tight to the total pseudogap. The solid curves in Fig.3 are proportional to the $B_{1g}$ spectra of electronic Raman scattering in the cuprates and are in good agreement with the experimental data [4]. In particular, one can understand why the $B_{1g}$ peak does not probe the superconducting but the pseudogap.

In the right-hand diagram of Fig.4 we plotted the position of the main peak in the d-wave fluctuation spectrum along $q = (q, q)$ for three different dopings. In the overdoped ($\delta = 0.25$) and the slightly overdoped ($\delta = 0.15$) cases only one well-pronounced peak in the spectrum was obtained describing collective d-wave density fluctuations. Its dispersion is rather weak away from $(\pi, \pi)$. Near $(\pi, \pi)$ a well-pronounced soft mode develops if the doping approaches the onset of the d-CDW from above. Below the onset the soft mode hardens with decreasing doping and its peak position is near or somewhat below the value $2\Phi$ in analogy with BCS-theory. In the strongly underdoped case with $\delta = 0.077$ the momentum region around $(\pi, \pi)$, where the peak position is roughly given by $2\Phi$, is rather small. In the neighborhood of $q \sim 2$ the fluctuation spectrum develops two peaks and the spectral weight shifts from the upper to the lower peak with decreasing momentum. The lower peak shows only little dispersion towards smaller momenta and lies somewhat above the corresponding peaks at larger dopings.

In conclusion, we found that the underdoped regime of the t-J model at large $N$ is characterized by the competition between d-wave superconductivity and a d-wave CDW. This competition is especially strong for a finite $t'$: For instance, for $t'/t = -0.35$ the d-CDW onset shifts from about $\delta = 0.29$ to $\delta = 0.14$ due to superconductivity. The wave vector of the d-CDW is very close to the commensurate value $(\pi, \pi)$, except near its onset. The d-wave density fluctuation spectrum is determined by collective effects. It exhibits a pronounced soft mode behavior near the onset of the CDW but shows only little dispersion far away from $(\pi, \pi)$. The fluctuation spectrum at zero momentum agrees well with data from electronic Raman scattering, in particular, with respect to its dependence on doping.

The authors thank Secyt and the BMBF (Project ARG 99/007) for financial support and P. Horsch for a critical reading of the manuscript.

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