INTERNAL CONSTITUTION OF NEUTRON AND STRANGE STARS

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Abstract

In the first of these two lectures I will discuss the rich constitution of neutron stars as a consequence of the Pauli principle which is engaged by the dominance of gravity over the nuclear force. Three especially interesting phenomena are discussed in this context—(1) a mechanism for the formation of low-mass black holes distinct in their mass-range from the black holes formed in the prompt collapse of an entire star, (2) a multilayered crystalline structure consisting of confined hadronic matter embedded in a background of deconfined quark matter (or vice versa) which occupies a many kilometer thick inner region, and (3) a clean and pronounced signal of the formation of quark matter in the interior of neutron stars. In the second lecture I will discuss the strange matter hypothesis, its viability as well as its consequences for compact stars and a new family of white dwarfs with dense nuclear matter central regions some orders of magnitude greater than in ordinary white dwarfs.

1. Neutron Stars

It is interesting to reflect on the consequences of the strong binding of the typical neutron stars that we know as pulsars. The weakest force – gravity – binds a nucleon in a neutron star 10 times more strongly than the strong force binds a nucleon in a nucleus. In doing so it works against the strong short-range repulsion of the nuclear force and against the Fermi pressure. Gravity therefore brings the Pauli principle into play in distributing the conserved baryon number of the star over many baryon species so as to reach the ground state of charge neutral matter.\(^1\) The name “neutron star” therefore has to be understood as a generic name for a star populated by many baryon species, also by quarks and also a mixed phase of confined and deconfined matter that, as I will discuss, arranges itself in a very intricate pattern in the deep interior of the star.

I will discuss several consequences of the rich constellation of compact stars: (1) a mechanism for the formation of low-mass black holes \((M \sim 1.5 - 2 \ M_\odot)\), (2) a multilayered crystalline structure of confined

\(^1\)For degenerate Fermi systems, such as neutron stars and white dwarfs, the Fermi momentum of a given species is related to the density of that species by \(k \propto \rho^{1/3}\). If the corresponding Fermi energy lies higher than the mass of some other Fermion species (modified by interaction energies), it will be favorable for some Fermions of the first species to transform to the second (say by weak interaction). The same number of Fermions distributed now over several species each have a lower Fermi momentum (and Fermi energy) than that of a single species of the same Fermion number. The total energy will be lowered as a result.
and deconfined\(^2\) quark phases and (3) the effect on pulsar braking indices of the deconfinement phase transition as well as the general effect of rotational distortion on inferred magnetic fields and spin-down times of millisecond pulsars.

In this paper we explore possibilities—not certainties. The properties of matter at densities higher than nuclear are essentially unknown, although they are the subjects of investigation at several ultra high energy accelerators. Essentially all we can be fairly confident of are: (1) the equation of state of dense matter obeys the condition of causality, (2) the equation of state also obeys the condition of microscopic stability \(dp/d\epsilon \geq 0\) known as Le Chatelier’s principle and (3) at sufficiently high density, asymptotic freedom of quarks is achieved. Beyond this, a theory of dense matter ought to be firmly anchored to what is known at nuclear density. Within these constraints we explore what is allowed by the laws of physics, in the belief that the laws of nature are likely to be realized in many if not all possible ways in the Universe. The vehicle for the exploration is a covariant nuclear field theory that embraces the above constraints \([1, 2]\).

Gravity compresses matter severely in a star of mass typical of those that are known, say nominally 1.4 M\(_\odot\). Although the strong force resists the compression, matter at the densities typical of the center of a canonical neutron star has a compression energy of several hundred MeV per baryon. Nevertheless, the binding energy per baryon in a canonical star is 100 MeV (the gravitational less the compression energy) and it falls to smaller and even negative values with decreasing mass. These numbers can be read from Figs. 1 and 2. I used to think that the relatively small range in which neutron star masses fell had to do with the creation mechanism, the evolution of massive stars leading to core collapse. And perhaps it does. But inasmuch as a supernova is powered by the transfer of a small fraction of the energy carried by neutrinos which themselves derive their energy from the gravitational binding of the neutron star, it is clear that the binding energy is sufficient to power a supernova only for a narrow range of masses. Theoretically a neutron star of mass as low as 1/10 M\(_\odot\) could exist, but evidently a neutron star of less than about 1/2 M\(_\odot\) could not be made in the typical way.

The rich hyperonized composition of charge neutral equilibrated neutron star matter is illustrated in Fig. 3. For charge neutral matter for which a phase transition to the coexistence phase and pure phase of quark matter occurs, the composition is shown in Fig. 4. Notice the mixed phase region between 4.5 and 7.8 km in which both hadronic matter and quark matter are

\(^2\)By deconfined phase of quarks we mean that quarks are asymptotically free over extended regions. This phase is also called the quark matter phase. By confined phase we mean the phase in which quarks are confined in hadrons.
in equilibrium. I will discuss later the geometrical structure that develops in this region. In each figure the particle populations are shown as a function of radius for the limiting mass star. In the first example, the phase transition to quark matter does not occur because of the choice of parameters, which is after all uncertain. The two cases therefore embrace two possibilities that may be realized in nature. The theory underlying the calculations can be found in Refs. [1, 3, 4, 2].

1.1. THE FIRST TEN SECONDS

The protoneutron star formed in core collapse is hot and very lepton rich and the matter of which it is made is far from its final composition—that of the ground state of cold dense matter. As the star cools and shrinks further during deleptonization, the Fermi energies of neutrons and protons rise with increasing density \( k \propto \rho^{1/3} \). It becomes increasingly favorable with increasing density for neutrons and protons to transform to other baryon number carrying species, hyperons or quarks, through the weak interaction
Figure 3. Neutron star at the mass limit that is composed of hyperonized matter (charge neutral and in general beta equilibrium).

Figure 4. Neutron star at the mass limit for which at low density near the edge a pure hadronic phase exists, interior to which a mixed phase of baryons and quarks exists at intermediate density, and pure quark matter at high density in the inner region of 4.5 km.

\((\tau_{\text{weak}} \sim 10^{-10} \text{ s}) [1, 2]\). The Fermi pressure is relieved as a result, the equation of state is softened, and the matter of the star is less able to resist the pull of gravity. If the collapsing core lies in mass above the limiting mass of the final ground state of cold neutron star matter but below the limiting mass of the hot lepton rich protoneutron star, it will continue to collapse to form a low-mass black hole [5]. But its continued collapse progresses on the time scale of neutrino diffusion (10 s) and is conditioned by neutrino loss. Therefore, unlike the prompt collapse of the entire star to form a black hole of several tens of solar mass, these low-mass black holes are formed after the supernova explosion and neutrino pulse. Depending on the mass function of massive stars and the dependence of core mass on stellar mass, a large fraction of massive stars may end their lives in a supernova explosion and a residual low-mass black hole instead of a neutron star.

To estimate the mass or baryon number window for such events, we may compare neutron star sequences for equilibrated n,p,e matter and fully equilibrated matter corresponding either to hyperonized matter, or to partially or wholly deconfined quark matter. I refer to stars of the latter type as
hybrid stars because they have an ordinary neutron star exterior, an intermediate region of mixed quark and hadronic matter, and possibly a pure quark matter core [6, 4, 7]. Stellar mass as a function of density is shown in Fig. 5. The three stellar sequences illustrated correspond to different degrees of completion of equilibrium. The softening of the equation of state due to hyperonization is quite apparent. (It is also apparent that models of neutron stars which neglect complete equilibrium are unrealistic.)

Figure 6 illustrates stellar mass as a function of baryon number for a protoneutron star (here modeled as a n,p,e star) and a fully equilibrated configuration corresponding to a hyperonized star. The window between H and P corresponds to the mass or baryon number of protostars that will collapse following a supernova and neutrino display to a low-mass black hole.

Figure 5. Three stellar sequences as a function of central density. Three stages of completeness with respect to beta equilibrium are illustrated [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.

Figure 6. Two stellar sequences as a function of baryon number. The one extending to P represents a protostar, and the one to H a fully equilibrated neutron star in its ground state. A core collapse with A falling between H and P will form a low-mass black hole [5].

If the star deleptonizes to a stable mass it will have cooled to the MeV level or less and will essentially be frozen for eternity as far as nuclear transformations are concerned. Those stars that lie close to the mass limit will have a rich baryon population, either hyperons or quarks or both as
illustrated in Figs 3 and 4. The neutrino display accompanying neutron star formation and low-mass black-hole formation will differ in the tale of the signal, the neutrinos suffering an extreme redshift in the latter case.

For reasons not well understood, neutron stars have a high average velocity of about 500 km/s [8]. So would the low-mass black hole formed in the prompt collapse of the protoneutron star. It is interesting to contemplate possible differences in the interaction with the interstellar medium. A neutron stars produces a bow shock fanning out to hyperbolic wings whose visible presence is revealed by the H alpha line. What shock pattern would a high-velocity black hole produce?

1.2. CRYSTALLINE STRUCTURE IN STARS

I turn now to other possible consequences of the high compression of matter in neutron stars—the formation of an unusual crystalline region consisting of confined nuclear matter and deconfined quark matter. For neighboring mass stars the regions of varying crystalline structure extending over a radial distance of many kilometers is illustrated in Figs. 7 and 8. I describe the situation below.

*Figure 7.* Pie section of a hybrid star showing regions of quantum liquid (white areas) and solid regions of various geometric phases [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.

*Figure 8.* Similar to 7 but for a slightly less massive star [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.
A possible phase transition from quarks confined in hadrons to deconfined quark matter in which the quarks are essentially free to move in an extended colorless region was discussed by many authors beginning in the mid seventies [9, 10, 11] and right up to the present. However a profound change has taken place in the understanding of the nature of the phase transition as a result of work that I published in 1991-1992 [4]. Originally, it was imagined that the phase transition was a constant pressure one like the conversion of water into steam. In such phase transitions, the nature of the two phases remains unchanged until the transition is complete from one pure phase to the other. But such phase transitions are a very special case of one-component substances.

Neutron stars are not made from a single-component substance. There are, in fact, two independent components or conserved quantities that characterize the matter of a star—the original baryon number and its net charge. A star has zero net charge because above an infinitesimal ratio of net charge to baryon number the Coulomb force would repel additional charged particles (the Coulomb force being so much stronger than the gravitational). Note that neutrality is a global condition, not a local one. The mistake of enforcing charge neutrality in stellar models as a local constraint has been made time and again. It is not the charge density \( q(r) \) that must vanish but only \( \int q(r)r^2dr \) that must vanish. The latter is a less restrictive condition, and if the internal forces can take advantage of the freedom admitted by global neutrality to achieve a lower energy state, the internal forces will do so. A well known example is an atom which is neutral but has finite charge density.

The appropriate way to express global neutrality of two uniform substances in contact and in equilibrium with each other, such as quark matter and confined nuclear matter, is

\[
4\pi \int_V q(r)r^2dr = (V - V_Q)q_H(\mu_b, \mu_q) + V_Qq_Q(\mu_b, \mu_q) = 0. \tag{1}
\]

Because the substances are uniform in any small locally inertial region \( V \) of the star, the integral over densities that expresses global neutrality takes this simple form. The baryon and electric charge chemical potentials which characterize the state of the phases are the arguments and the \( V \)'s denote volumes. Quark chemical potentials are related to the baryon and charge chemical potentials in the usual way

\[
\mu_u = \mu_c = (\mu_b - 2\mu_q)/3, \quad \mu_d = \mu_s = (\mu_b + \mu_q)/3. \tag{2}
\]

Similarly to (1) the expression for overall baryon conservation within (an unknown) volume \( V \) containing \( B \) baryons is

\[
(V - V_Q)\rho_H(\mu_b, \mu_q) + V_Q\rho_Q(\mu_b, \mu_q) = B \tag{3}
\]
where $\rho$ denotes baryon number density. The Gibbs condition for equilibrium is

$$p_H(\mu_b, \mu_q, T) = p_Q(\mu_b, \mu_q, T)$$  \hspace{1cm} (4)

We have here three equations (1, 3, 4) in the unknowns $\mu_b$, $\mu_q$ and $V$ for any chosen proportion

$$\chi \equiv V_Q/V$$  \hspace{1cm} (5)

of quark phase. Since $\chi$ appears explicitly in the equations that define the solution ($V_Q = V\chi$), the solution changes as the proportion and since the chemical potentials determine the state of quark and hadronic matter, the properties of the phases change with proportion. In particular, the concentrations of electric charge to baryon number changes in each phase as the proportion of quark matter changes. That is

$$c_H \equiv \frac{q_H(\mu_b, \mu_q)}{\rho_H(\mu_b, \mu_q)}, \hspace{1cm} c_Q \equiv \frac{q_Q(\mu_b, \mu_q)}{\rho_Q(\mu_b, \mu_q)}$$  \hspace{1cm} (6)

depend on $\chi$ through the dependence of the chemical potentials on $\chi$. Although the concentrations of charge to baryon number vary in each phase with proportion of phases, overall conservation of charge and baryon number is guaranteed by (1) and (3).

Interesting and possibly far reaching consequences for pulsars follow from the above analysis. Let us enquire as to the nature of the force that drives the system to optimize the charge concentration in the two equilibrium phases. Neutron matter is highly isospin asymmetric. (The proton to neutron ratio is far from unity.) There are two agents that will tend to drive the system to greater symmetry. One is the Fermi energy—any inequality of the Fermi surfaces of neutrons and protons corresponds to a higher energy state than one for which the Fermi surfaces are equal (symmetry). The other agent is the specific preference of the strong interaction for symmetry, namely the coupling of the rho meson to the isospin current of the nucleons. The valley of beta stability, well known in nuclear physics, attests to the preference for symmetry.

On the other hand, charge neutrality is an overriding condition because gravity overwhelms all other forces in a star. Consequently as long as neutron star matter is in the confined phase, it must be highly asymmetric. However, in those inner regions of the star where the pressure or density is high enough, some of the nuclear matter will condense into quark matter in equilibrium with it. The strong isospin asymmetry of neutron star matter can then be relieved by transferring charge and strangeness (mediated by the weak interactions) between the two phases in such amount as minimizes
the energy. Of course, in chemical thermodynamics it is not necessary to enumerate individual reactions nor calculate rates.

For the reason discussed above, regions of nuclear matter will have positive charge and regions of quark matter, negative. The Coulomb force will tend to break up regions of like charge into smaller ones intermixed with regions of opposite charge. The surface energy will resist the breakup. The competition will be resolved when the rarer phase takes on a crystalline order within the dominant phase. The nature of the crystalline form, its size and spacing, will vary as the proportion of the phases because the properties of each phase varies. This was proven above. Therefore the crystal characteristics will vary in the changing pressure environment of a star and therefore as a function of position in the star.

I have verbally described a situation which can and has been defined quite precisely in terms of specific models of nuclear and quark matter. Let us see in a schematic fashion how this can be done (see [7, 2]) for details). The surface energy per unit volume of a quark drop of radius \( r \) in a nuclear background of radius \( R \), chosen so that there is zero net charge in \( R \) (Wigner-Seitz cell) is

\[
E_S/V = \frac{4\pi r^2\sigma}{(4\pi/3)R^3} = \frac{3\sigma\chi}{r} \equiv S(\chi)/r, \tag{7}
\]

where for droplets, \( \chi = (r/R)^3 \) is the volume proportion of the quark phase.

Figure 9. Charge carried on regions of quark and hadronic matter in equilibrium as a function of the proportion of quark matter [7].
Likewise, while more involved to prove [12], the Coulomb energy per unit volume has the form,

$$E_C/V = C(\chi)r^2.$$  \hspace{1cm} (8)

Their sum is a minimum when the size $r$ of the droplets is such that $E_S = 2E_C$. The above equations lead at once to

$$r = \left( \frac{S(\chi)}{2C(\chi)} \right)^{1/3}, \quad R = \frac{r}{\chi^{1/3}}.$$  \hspace{1cm} (9)

Thus at each proportion $\chi$ a definite size of quark drops immersed in the nuclear matter and their spacing is specified. We note that the long-range of the Coulomb force is screened by the formation of the lattice.

The functions $C$ and $S$ and the proportion $\chi$, expressed in terms of the geometry of the one phase immersed in the other, have quite definite forms for each geometry, droplets, rods, and slabs. Also the relationship between $\chi$ and the dimensions characterizing the geometry are quite definite. This is all quite analogous to the sub-nuclear crystal structure of nuclei immersed in an electron gas, which is believed to form the crust of a neutron star [12] and it is somewhat surprising that it took so many years before it was realized that the mixed phase of neutron star matter in equilibrium with quark matter would also form a crystalline lattice.

For a particular choice of nuclear properties within the range defined by experiment, and a particular model of nuclear and quark matter we can see in Fig. 10 how the diameters, spacings and geometry change. Of course the discrete geometries are only idealizations which are interpolated by nature. The thickness and locations of the geometrical phases depend very sensitively on the mass of the star (Figs. 11, 12). This is so because the density distribution in a star of canonical mass is very flat in the central region. Therefore a small change in mass corresponding to a small change in central density, implies a large radial displacement at which a given density is to be found, say the density corresponding to the boundary between pure quark and mixed phase.

The sensitivity of the thickness and location of the crystalline structure is perhaps interesting in connection with pulsar glitches. Glitches imply the existence of solid regions in a star because a purely liquid or gaseous star has no means by which its moment of inertia can do anything but follow smoothly the change in rotational frequency as the star spins down due to energy losses that are generally presumed to be of magnetic-dipolar form. We believe that there is a thin ionic crust on neutron stars which either cracks from time to time or from which pinned superfluid vortex lines slip catastrophically to new sites.
Figure 10. Diameter (D) and spacing (S) of the six idealized geometrical crystalline phases in a hybrid star. Note the suppressed origin. The central core is pure quark matter. (Dashed line is a continuous dimensionality interpolation.)

Now we have reason to believe that in addition to the surface crust there is also a many kilometer thick crystalline region in the interior to which vortex lines could also be pinned. So a vortex line instead of being pinned on the crust at opposite sides of the star could be pinned, one end at the crust, the other in the crystalline core. The great sensitivity of the geometrical nature and thickness of the interior solid would give great individuality to the behavior of different pulsars even of very close mass. One can imagine phenomena involving the sympathetic response of one region to the other. However, I think that it will be very difficult to arrive at semi-quantitative predictions, but perhaps not hopeless.

1.2.1. Evolution of Internal Structure with Pulsar Spin-Down

Another aspect of the deconfinement phase transition is interesting beside the statics discussed above. I refer to the response of the internal structure of the star to changing rotational frequency (see footnote 4). A signal of the changing structure may show up in the braking index of pulsars. We take up that subject in the next section.

At the higher frequencies characteristic of a particular pulsar in its early life, the central density is suppressed compared to what it will become at lower frequency in later life. Imagine a combination of mass and frequency
such that in early life the central density is below the phase transition, but rises above it in later life at a lower frequency. The radial location of boundaries between pure and mixed phases and between geometrical phases will accordingly change with angular velocity $\Omega$. This is shown in Fig. 13. For the particular stellar model and stellar mass, the central density rises to the transition density to pure quark matter at the center of the star at angular velocity of about $\Omega = 1250 \text{ rad s}^{-1}$. Because of the flat profile of neutron stars near their center, the phase boundary moves outward to larger radius with small change in angular velocity. The radius of the star meanwhile shrinks.

1.3. BRAKING INDEX AND INTERNAL STRUCTURE

$^3$From unpublished work of N. K. Glendenning, S. Pei and F. Weber [13, 14].
Pulsar slow down is usually represented by an energy loss equation of the form

\[ \frac{dE}{dt} = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = -C \Omega^{n+1} \]  \hspace{1cm} (10)

where, for magnetic dipole radiation, \( C = \frac{2}{3} m^2 \sin^2 \alpha \), \( n = 3 \), \( m \) is the magnetic dipole moment and \( \alpha \) is the angle of inclination between magnetic moment and rotation axis. We shall refer to \( n \) appearing in the energy-loss equation as the intrinsic index. Other multipoles may participate but it can be expected that one will dominate (magnitude of \( C \)). Other variables may play a role in the radiation from a pulsar over its lifetime but the response of the moment of inertia to the changing rate of rotation will produce its own effect, upon which other variables will superpose theirs.

Usually the above equation is represented by

\[ \dot{\Omega} = -K \Omega^n \]  \hspace{1cm} (11)

which follows from (11) if \( I \) is a constant independent of frequency and therefore of time and where \( K = C/I \) and \( n \) is the braking index. From
these equations follow the well known pulsar spin-down time (or age) and estimate of the magnetic field strength. However, the moment of inertia of a rotating star depends on its frequency and therefore on time. The dependence is very complicated. In General Relativity the very metric of spacetime is affected by the rotation of a star: local inertial frames are set into rotation. The centrifugal effects on the star are measured with respect to the angular frequency of the local frames. These depend on distance from the center of the star. Consequently, not only is a star’s shape flattened as it would be in classical physics, but its internal structure is altered – the distribution of energy density and hence of all other constituents of the star – the location of the thresholds for various baryon species and the boundaries of different phases.\(^4\)

If the frequency dependence, and hence time dependence, of the moment of inertia is taken into account, as it should, especially for rapidly rotating pulsars, the rate of change of angular velocity (11) is replaced by

\[
\dot{\Omega} = -K(\Omega)\Omega^n\left(1 + \frac{I'(\Omega)\Omega}{2I(\Omega)}\right)^{-1}
\]

where \(K\) is no longer a constant because of the angular velocity dependence of \(I\) and \(I' \equiv dI/d\Omega\).

Equation (12) explicitly shows that the angular velocity dependence of \(\dot{\Omega}\) corresponding to any mechanism that absorbs (or deposits) rotational energy such as (10) cannot be a power law, as in (11) with \(K\) a constant. It must depend on the mass and internal constitution of the star through the response of the moment of inertia to rotation. Intuitively it is clear that over the era of observation, and even much longer, the moment of inertia is essentially a constant. This does not alter the fact that the law governing the decay of the angular velocity is (12) and not (11) because \(I'\), which is also constant over any observational era, is nonetheless finite. We shall see that the magnitude of \(I'\Omega/(2I)\) will affect the rate of pulsar spin down differently in different eras of its life because the internal constitution of the star changes with changes in the density or pressure profile caused by the centrifugal force.

The dimensionless measurable quantity \(\Omega\ddot{\Omega}/\dot{\Omega}^2\) equals the intrinsic index of the energy loss mechanism only for \(I = \text{constant}\), or for \(\Omega \to 0\) as can be found from (11). Otherwise we find from (12) that

\[
n(\Omega) = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}.
\]

\(^4\)The usual expression for the moment of inertia in General Relativity is not adequate for our purpose. It ignores the dragging of local inertial frames, the alteration of the metric by rotation, and even the centrifugal flattening. Instead we must use an expression that incorporates these effects as derived by Glendenning and Weber [15, 16].
The measurable braking index $n(\Omega)$ can be very different from $n$; it can even be zero or have negative values depending on the derivatives of the moment of inertia and on $\Omega$. If $I'\Omega$ dominates the other terms in (13), then $n(\Omega)$ will vanish: If $I''\Omega^2 > 6I$ then the observable braking index will be negative for magnetic dipole radiation.

Because the braking index (13) depends explicitly and implicitly on $\Omega$, even if the energy loss mechanism remained unchanged during the entire life of a pulsar, its measured index (13) will change with time, in general continuously, but under circumstances that we discuss later, it can change radically over an era in the life of the star. The right side of (13) reduces to a constant $n$ only if $\Omega = 0$ or $I$ is independent of angular velocity. But this cannot be, except for slow pulsars. The centrifugal force insures the response of $I$ to $\Omega$. Since $I'$ and $I''$ are positive (the moment of inertia increases with $\Omega$ and the centrifugal force grows as the equatorial radius) the braking index is always less than the index $n$ of the energy loss mechanism (10). And the deficit is independent of $n$.

The important question before us now is whether the departure of the braking index $n(\Omega)$ from the intrinsic index $n$ is substantial and for what angular velocity range. How sensitive is $n(\Omega)$ to the internal constitution of stars? In all cases that we have looked at, the braking index for pure magnetic dipole radiation has a value near three for low frequencies like the Crab, but it falls to values less than unity near the Kepler frequency (as a consequence of the universal fact that for a rotating star the moment of inertia is an increasing function of frequency). At about half Kepler (the approximate frequency of the two 1.6 ms pulsars) the braking index is about two. However this does not tell the whole story.

At the frequency of millisecond pulsars, $I'$ has a value of about 0.03 km$^3$s. Therefore the value of the dimensionless parameter $\xi = I'\omega/(2I) \approx 0.4$ for millisecond pulsars as can be read from Fig. 14 (at $\Omega \sim 4000$ rad s$^{-1}$). The rate of change of a millisecond pulsar’s angular velocity is therefore about 70% less for the same $n$ and $K$ as given by (12) compared to (11). And the magnetic field $B$ as given by $\sim (P\dot{P})^{1/2}$ is about 20% larger (larger by $(1 + \xi)^{1/2}$) than the value estimated from (11). The usual dipole age formula is not at all valid for a millisecond pulsar, since the slope of $I$ differs so strongly from zero for such pulsars. One cannot analytically integrate (12) to get a value of the age. The integral of $\dot{\Omega}$ depends implicitly on the structure of the star which is changing over time.

1.3.1. Effect of Continuous Structural Changes

Disregarding the feature at $\Omega \sim 1250$ rad/s until the next section, the general trend in the moment of inertia is shown in Fig. 14. As a pulsar spins down the density at every radial distance increases and the star's
equatorial radius shrinks. Baryon and quark thresholds (see Fig. 3 and Fig. 4) as well as boundaries between confined, mixed and deconfined phases and geometrical phases (see Fig. 10) occur at unique densities (for given model of the equation of state). The boundaries will shift in their radial location as a function of frequency. All such features will have their effect on the moment of inertia of a given baryon mass star, and hence on the braking index as shown in Fig. 15. (We refer here only to the smooth behavior interpolated through the sharp discontinuous behavior at $\Omega \sim 1250$ rad/s.) However these relatively continuous changes do not account for the departure from three of the apparent index of the Crab pulsar and the other three pulsars for which the index has been measured. I presume that these departures reflect on the energy loss mechanism itself. I do not discuss the energy loss mechanism here but only the effects of the changing moment of inertia on the corresponding braking index.

![Figure 14](image1.png)  
**Figure 14.** Moment of inertia as a function of rotational angular velocity. At angular velocities below $\sim 1250$ rads$^{-1}$ a pure quark phase of increasing radius with decreasing frequency (central density) occupies the central region of the star.

![Figure 15](image2.png)  
**Figure 15.** The braking index as a function of rotational angular velocity. The sharp change at $\sim 1250$ rads$^{-1}$ occurs, as with decreasing angular velocity, the core begins to dissolve into quark matter (see Fig. 13.

We summarize the long-term evolution of the pulsar braking index and its dependence on the internal constitution of stars. In all cases, the braking index will be much less than the intrinsic index of the energy-loss mecha-
nism (three in the case of magnetic dipole radiation) for pulsars with short periods, especially millisecond pulsars. For periods of twice Kepler or more, the braking index is close to one unit less than the intrinsic index and increases toward the intrinsic index for very long periods (i.e., most pulsars). However, over a short era (∼ 100,000 y) the braking index may be extremely anomalous and exhibit values that lie anywhere from negative to positive values as a result of the onset of a phase transition and its growth in radial extent. We discuss this next.

1.3.2. Braking Index and the Deconfinement Phase Transition

How can abrupt large-scale changes in the moment of inertia occur? With decreasing rotational frequency the central density of the star increases, and since the density and pressure profiles of neutron stars are very flat in the central region, the radial point at which a given density occurs changes by a considerable fraction of the radius of the star for a very small fractional change in its rotational frequency. If, during the pulsar spin-down, the central density passes from below to above the density for the phase transition, the central region occupied initially by relatively stiff nuclear matter will be replaced by more compressible, and therefore denser, quark matter. The region of quark matter will expand greatly with small decreases in angular velocity because of the flat density profile. The anomalous concentration of mass in the stellar interior occasioned by the phase transition will be mirrored in structural changes in the star such as its size and moment of inertia: the phase transition has ushered in an era in which the star shrinks anomalously as it spins down over time and its mass becomes ever more concentrated near its center—more so than would be the case for a star composed of a simple fluid on which a weakening centrifugal force was acting. The concentration arising from the greater compressibility of quark matter is amplified by its greater gravitational attraction on the outer parts of the star.

At the stage described, the tendency of the star to shrink as the region occupied by quark matter grows in radius, counteracts by angular momentum conservation, the deceleration $\dot{\Omega}$ caused by radiation. The growth of a central region of deconfined quark matter acts, so to speak, as a governor in the mechanical sense, in resisting pulsar spin-down. The structural changes accompanying the phase change thus prolong the epoch over which quark matter engulfs the central region—a situation that is highly favorable for observation of a signal of the transition epoch.

The behavior of the moment of inertia in the frequency range in which the phase transition boundary moves outward in the star is shown in Fig. 16. The temporal development is from large to small moment of inertia. We see that the star actually enters an era in which it spin up for a time!
This is analogous to the situation observed in the rotational spectra of some nuclei. In nuclei it is a phase transition from a normal Fermi gas at high spin to a pair-correlated phase at lower spin that causes a change in the moment of inertia due to a weakening Coreolis interaction.

The particular way in which the deceleration (12) and braking index (13) are effected by the backbending of the moment of inertia (Fig. 16) can be understood with reference to the formulae. In particular, when $I'$ is large and negative (the backend in $I$) the deceleration changes sign—the pulsar spins up. When $I''$, which is related to $I'$ by

$$-I'' = I' \frac{3}{dI^2} \frac{d^2 \Omega}{dI^2} \quad (14)$$

changes from positive to negative infinity at both turning points seen in Fig. 16, the braking index swings from nearly 3 to infinity, to negative infinity and back to nearly 3 (Fig. 17). This is a remarkable signal considering that the braking index is usually thought of as a constant (3 for magnetic dipole radiation).

We emphasize that our calculation does not imply a prediction of the frequency or stellar mass at which the phase transition will occur. Nor
does it even predict that a phase transition will occur at all. The results discussed above pertain to a particular model star. Very little is known about the high-density equation of state. Rather our results show what the signal might be if the transition does occur in neutron stars because of the asymptotic freedom of quarks.

We estimate the plausibility of observing phase transitions in the pulsar population. The duration over which the observable index is anomalous is \( \Delta T \approx -\Delta \Omega / \dot{\Omega} \) where \( \Delta \Omega \) is the angular velocity interval of the anomaly (\( \approx 100 \) rad/s). For a typical period derivative, \( \dot{P} \sim 10^{-16} \), we find \( \Delta T \sim 10^5 \) years. During a typical pulsar’s active lifetime, about \( 10^7 \) yr, the signal (small or negative index) would endure for 1/100 of the lifetime. Given that \( \sim 10^3 \) pulsars are known about 10 of them may be signaling the phase transition.

2. Strange Stars

2.1. THE STRANGE-MATTER HYPOTHESIS

We are so accustomed to the confined phase of hadronic matter (quarks confined in hadrons) that we usually do not question whether it is in fact the absolute ground state of the strong interaction. When we look to the furthest reaches of the Universe we see spectral lines that can be identified with molecules, atoms and nuclei with which we are familiar. Does this not directly inform us that the confined phase is indeed the ground state? The answer in a word is “no”. After all, the absolute ground state of confined hadronic matter is \(^{56}\text{Fe}\). Its binding energy per nucleon is 931 MeV, lower by some eight MeV than the nucleon mass. Yet there is very little iron in the Universe and we know perfectly why this is so. It takes a stellar lifetime to convert a very small fraction of the primordial hydrogen to iron. So the contents of the Universe have little bearing on the question.

For different reasons than this Bodmer (1971) and Witten (1984) hypothesized independently that the absolute ground state is strange quark matter, an approximately equal mixture of the three light flavor quarks, u, d and s [17, 18]. One can grasp the distinct possibility that this state of matter lies quite close in energy per baryon to iron—the ground state of the confined phase. From a variety of nuclear data we are convinced that nuclei are composed of neutrons and protons. Since nuclei are two-flavor objects, this informs us that two-flavor quark matter lies higher in energy per baryon number than the nucleon mass. The available energy scale by which they differ is \( \Lambda_{\text{QCD}} \sim 100 \) to 200 MeV. On the other hand, for the same reason that dense nuclear matter will distribute baryon number over as many baryon species as are energetically available, three-flavor quark matter will lie lower in energy per baryon than two-flavor quark matter at
high density. A Fermi gas estimate of the energy difference is again of the order of \( \Lambda_{\text{QCD}} \) [19]. This places the energy per nucleon of iron and that of dense strange quark matter at about the same value. Lattice QCD is quite unable to predict energies that could be as close as a few percent different. The question can only be answered by observation, and I will argue that pulsars are the most likely sources of the answer. I will discuss limits on rotation of strange stars and neutron stars in this connection.

Needless to say, the question concerning the nature of the true ground state is a fundamental one. As we shall see, there is no sound basis on which either to reject or confirm the hypothesis at the present. The Universe may indeed occupy a metastable though long-lived state.

2.2. VIABILITY OF THE STRANGE-MATTER HYPOTHESIS

2.2.1. Stability of Nuclei to Decay to Strange Nuggets

One might object that the energy argument given above is not satisfactory, since excited states decay to the ground state and we know that the confined state exists. Should a nucleus not then decay forthwith to a strange nugget\(^5\) of the same number of quarks but distributed over the three light flavors in an approximately equal proportion if strange matter were the true ground state? It could not do so except on a time scale of the order larger than the age of the Universe. A nucleus of \( A \) nucleons would have to undergo \( A \) simultaneous weak interactions. One interaction at a time would not do since we know that the first would produce a hypernucleus and the \( \Lambda \) is more massive than the nucleon.

Of course, nuclei of small \( A \) would appear to be candidates for decay by the weak interaction to strange nuggets, yet they do not. This is easily understood. Finite size effects, especially the surface energy, places small-\( A \) nuggets at a higher energy per baryon than large-\( A \) nuggets. Calculations based on the bag model of confinement, though not reliable quantitatively, show that the energy per baryon of strange matter is a decreasing function of baryon number. It is higher than the energy of low-\( A \) nuclei, decreasing from the Lambda mass at \( A=1 \), and approaching an asymptote for large \( A \) that lies below the energy per baryon of iron [20, 21]. So whether or not strange nuggets of large \( A \) or strange matter in bulk is absolutely stable, nuggets of small \( A \) are not.

The density of the strange-matter objects is higher than nuclear density so that the Fermi energy of quarks is larger than the strange-quark mass (\( \sim 150 \text{ MeV} \)). Therefore, all three flavors are about equally populated. Whether the critical \( A \) for which strange nuggets have lower energy than

\(^5\)Strange objects with a number \((3A, A < 10^{50})\) of quarks small enough that gravity is irrelevant are called nuggets – otherwise stars.
the corresponding nucleus is 100 or a 1000, we cannot be sure from bag model calculations. We conclude that low-mass nuclei are protected from decay to strange matter because they have a lower energy than nuggets of the same A. High-mass nuclei are protected because a large number of simultaneous weak interactions (an A’th order weak interaction) would be required to convert them to strange nuggets.

2.2.2. \textit{The Universe and its Evolution}
Since the matter of the very early Universe occupied the deconfined phase, why did matter not remain in this phase? The answer in a word is that the Universe was very “hot”. As the Universe expanded, opening voids in the hot dense quark matter, whose cold dense ground state is, by hypothesis the absolute ground state, the hot quark matter evaporated into nucleons. There remains some debate as to whether large enough objects of strange matter could have cooled before evaporation. But there is general consensus that little if any primordial strange matter survives [22, 23].

For the above reason, the Universe would have evolved along the path of confined quark matter which we see today, and of which we are made. Strange matter, if the hypothesis is true, can be recreated only on the order of stellar lifetimes. Neutron stars, if dense enough in their cores, will dissolve into two-flavor quark matter. Because of the high density, the Fermi energy of two-flavor quark matter will exceed the mass of the strange quark, so that two-flavor quark matter will rapidly weak decay into strange quark matter, one strangeness-changing interaction at a time until the matter is approximately an equal mixture of the three light quark flavors. Being the ground state, and there being no barrier to conversion of dense nuclear matter to strange matter, the whole star will be consumed [24]. It will become a strange star. This is in contrast to the situation discussed above for light nuclei, where single strangeness-changing reactions are endothermic, the Fermi energy in a nucleus being small compared to the strange quark mass.

2.2.3. \textit{Stability of Nuclei to Conversion by Cosmic Strange Nuggets}
Binary compact stars exist and their orbits decay by gravitational radiation [25]. If at least one of the pair is a strange star, the final collision will likely spew forth some strange matter as fragments into the Universe since collisions of neutron stars are expected to do so [26]. Fragmentation usually will produce a preponderance of small fragments. These could have anywhere from the minimum baryon number for which strange nuggets are stable to more massive fragments. Therefore, there would exist a strange nugget component in cosmic rays. Some nuggets would impinge on other stars and in particular on the Earth. (An estimate of the flux can be found
in [19].) What prevents their consuming the hadronic matter with which they come into contact?

Because the strange quark has greater mass than the other two light flavors, quark matter has a slight deficit of strange quarks compared to the others. Consequently, strange nuggets carry positive charge. Strange nuggets and nuclei therefore repel each other. I have estimated the concentration of low-mass strange nuggets in the Earth’s surface layer, taking account of geological mixing to ten kilometer depths, and find that strange nuggets would be very rare objects in earthly samples, much less than \( 10^{-15} \) nuggets per nucleon. The moon has been exposed as long as the earth to these cosmic nuggets, and its surface has been tranquil for most of its life. Moon rock is a more promising source.

2.3. LIMITS TO NEUTRON STAR ROTATION

2.3.1. Absolute Limits

A neutron star at the mass limit can rotate most rapidly of all stars in its sequence since it has the smallest radius. (See Fig. 18). This can be seen from the condition that gravity balances the centrifugal force at the equator. The Kepler angular velocity can be approximated by [27, 28, 29],

\[
\Omega_K \approx \zeta (M/R^3)^{1/2}.
\]

(15)

where \( \zeta \approx 0.625 \). Although this result is Newtonian reduced by the prefactor, it agrees with numerical calculations in General Relativity to better than 10% [28]. We are here interested in establishing the minimum rotational period for a star that is gravitationally bound, by a variational calculation made under conservative physical assumptions [30]. We adopt the following minimal constraints:

1. Einstein’s general-relativistic equations for stellar structure hold.

2. The matter of the star satisfies \( dp/d\epsilon \geq 0 \) which is a necessary condition that a body is stable both as a whole, and also with respect to the spontaneous expansion or contraction of elementary regions away from equilibrium (Le Chatelier’s principle).

3. The equation of state satisfies the causal constraint for a perfect fluid; a sound signal cannot propagate faster than the speed of light, \( v(\epsilon) \equiv \sqrt{dp/d\epsilon} \leq 1 \), which is also the appropriate expression for sound signals in General Relativity [31].

4. The high-density equation of state, whatever it is, matches continuously in energy and pressure to the low-density equation of state of Baym, Pethick and Sutherland [32].

The results of the variational search for the minimum period of neutron stars as a function of their mass is shown in Fig. 19. A canonical neutron
star can have a period not less than 0.3 ms. The actual physical limit is likely to be higher than this because of gravitational wave instabilities. So our result is a most conservative one. We note from Fig. 18 the fine tuning problem required to achieve the limit. The mass window for rapid rotation is extremely narrow.

Figure 18. Generic mass-radius relation for neutron stars. Those that lie below the curves marked 1.6 ms and 0.5 ms can rotate as fast or faster than these periods. [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.

Figure 19. Neutron stars can fall only in the indicated region. All stars are forbidden in the region so marked. Hypothetical stars that are not bound by gravity can fall within the blank region [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.

Even if a pulsar of canonical mass were observed with a period near the limit, it would be implausible to interpret it as a neutron star. The central density would be about 20 times nuclear density. (See Fig. 20.) Considering the charge radius of the proton (0.8 fm), nucleons would be squeezed out of existence into their quark constituents.

2.3.2. Practical Limits
The limit above is obtained under extremely conservative conditions. There is no minimum principle for rotation as there is for energy (mass). A star does not need to be so configured as to rotate fast. So any knowledge additional to that of the four conditions above will raise the lower bound
on period, and realistic equations of state do so considerably [29]. Realistic
equations of state yield stellar models with Kepler frequencies in the 1
ms range. (Those models that have shorter Kepler periods have central
densities so high as to exceed the applicability of models of matter based
on nucleons as constituents.)

There is another instability beside the mass-shedding instability (Ke-
pler) that places a more severe limit on rotation. It is a gravitational wave
instability. It raises the lower limit on rotational periods of neutron stars
by an additional 30% or so [33, 34]. However it is difficult to be precise
as to how much this instability raises the limiting period because of the
uncertainty of the viscosity of dense stellar material.

We conclude that if a period as small as one millisecond were discovered,
it would be quite difficult to reconcile with realistic models of neutron stars.

2.4. LIMITS TO ROTATION OF STRANGE STARS

Strange stars are not bound by gravity but by the strong interaction. Gravity
simply squeezes them, hinders their fission and imposes a limit on their
mass, above which they would collapse. Let us discover the constraint that
rapid rotation places on strange matter [19].

Denote the normal energy density of self-bound matter (the density at
which the internal pressure vanishes) by $\epsilon_b$. A small nugget therefore has
mass

$$M = \frac{4}{3}\pi R^3 \epsilon_b \quad \text{(no gravity)}$$

(16)

so that, unlike neutron stars (or more generally stars bound only by grav-
ity), the mass-radius relation for small mass is

$$R \propto M^{1/3}.$$ (17)

This dependence of the radius on mass has a generically different form from
that of gravitationally bound stars as shown in Fig. 18. We can rearrange
(15) to read [35, 19]:

$$\epsilon_b \geq \frac{3}{4\pi G} \left( \frac{\Omega}{\zeta} \right)^2 = 1.4\epsilon_0 \left( \frac{\text{ms}}{P} \right)^2.$$ (18)

where $\epsilon_0$ denotes the normal density of nuclear matter ($2.5 \times 10^{14}$ g/cm$^3$).
This provides the condition that must be satisfied by the “normal” density
of strange matter, the density at which it is in the equilibrium ground state,
so that it can have a designated period $P$.

The mass-radius relationship for strange stars (self-bound) is entirely
different from that of neutron stars as is shown in Fig. 21. For neutron stars,
the smaller the mass the less gravity compresses the star and the larger it will be. However strange stars (by hypothesis) are bound by the strong interaction and therefore at some particular value of the energy density. Consequently, for small $A$, their radii scale as in (17). Because of the mass-radius relationship of strange objects, which is modified only slightly by gravity for the more massive objects, the entire sequence of strange stars can rotate about as fast as that at the limiting mass. In contrast, the most massive neutron stars can rotate much more rapidly than other members of the sequence as seen from (15). Hence, an observation of a neutron star near the limiting value of the frequency would be a rare occurrence.

2.5. STRANGE-STAR CONFIGURATIONS

Because strange stars are (by hypothesis) bound by the strong interaction, and only additionally by gravity, they have very sharp surfaces. The surface of a star corresponds to vanishing pressure because vanishing pressure can support no overlaying layer of matter against the gravitational attraction.
from within. Therefore the density at the inner edge of the surface is equal to the equilibrium density of strange matter\(^6\)—several times that of nuclear matter. The transition from a high density to zero at the edge of the star occurs in a strong interaction distance (\(\sim 10^{-13}\) cm) since the strong interaction binds the star. At first sight this may seem unusual. But it is quite analogous to nuclei. Both are bound by the strong short-range force and it matters not at all to the skin thickness how much matter lies behind the surface. Bare strange stars, if they exist, have the sharpest surfaces of any conceivable object. Ordinary material objects have surface thicknesses corresponding to the range of molecular forces. Figure 22 shows the density profiles of several strange stars and Fig. 23 compares a neutron star and strange star. Quark populations in a strange star are shown in Fig. 24.

Charm quarks are too massive compared to the chemical potential to be present. It is of some interest to enquire whether any star can contain charm quarks. We compare the baryon mass and gravitational mass of strange stars past the first limiting mass in Fig. 25. We have made a stability analysis and find, as expected, that no configuration past the first mass

\(^6\)Equilibrium implies vanishing pressure.
limit is stable against acoustics modes that would cause its collapse to a black hole [36]. So charm quarks have no present astrophysical interest.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig24}
\caption{Quark populations in a strange star of mass $1.6 \, M_\odot$ [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig25}
\caption{Strange-star sequence showing the baryon and gravitational mass. All stars beyond the first maximum are unstable [2]. Reprinted with permission of Springer–Verlag New York; copyright 1997.}
\end{figure}

2.6. STRANGE STARS WITH NUCLEAR CRUSTS

A strange star has a sharp edge of thickness defined by the range of the strong interaction (cf. Fig. 22). Such ‘bare’ strange stars are unlikely to exist as such. Strange stars can carry a crust of nuclear material which is suspended from contact with the strange star by a strong electric dipole field. This was noted by Alcock, Farhi and Olinto [37] who pointed out that the electrons (which neutralize the positive charge of strange quark matter and are bound to it by the Coulomb attraction) extend several hundred fermis (the de Broglie wavelength) beyond the edge. In consequence, just inside the surface there is a positively charged layer (because strange matter by itself is slightly positively charged). A dipole layer of high voltage is thus created. The surface dipole can support a layer of ordinary matter (which it polarizes) out of contact with the core. The separation gap prevents the conversion of the nuclear surface layer to quark matter.
The gap between the core and its crust of nuclear material is estimated to be of the order of several hundred fermis [37]. Effects of finite temperature have been investigated in ref. [36]. The gap prevents the conversion of the crust to strange matter unless the crust density is too high. The maximum density of the nuclear crust is strictly limited by the neutron drip density $\epsilon_{\text{drip}} \approx 4 \times 10^{11}$ gm/cm$^3$ above which free neutrons would gravitate to the strange core and be converted to quark matter. It is likely that strange stars do have such crusts of various inner crust densities depending on their histories and ages. Interstellar space is not empty.

Thus, strange stars with nuclear crusts form a two-parameter sequence corresponding to the central density and the inner crust density. In practice, we fix the inner density of the crust and vary the central density to generate a corresponding sequence. For a particular equation of state of core and crust material, each pair of such parameters defines a unique stellar structure with a particular mass and radius.

A selection of strange stars with crusts at the limiting density $\epsilon_{\text{drip}}$ are shown in Fig. 26. These are the counterparts to neutron stars. They consist mostly of a massive strange quark core. Because of the gravitational attraction of the core on the crust, the crust is generally thin but grows

Figure 26. Strange stars with nuclear crusts at the drip density: energy density as a function of radial distance from the star’s center for gravitational masses $M/M_\odot = 0.020$ (solid line), 0.20 (dashed), 1.00 (dash-dotted), and 1.50 (dotted). The bag constant is $B^{1/4} = 145$ MeV. (From [15].)
in thickness for smaller cores. Along a sequence of ever smaller cores the mass and thickness of the crust can grow to white-dwarf dimensions. What is unusual about such stars is that the density of nuclear matter in the crust can be as high as the drip density \((4 \times 10^{11} \text{ gm/cm}^3)\) which is very much larger than the central density of white dwarfs \((\epsilon_{\text{wd}} \leq 10^9 \text{ gm/cm}^3)\). Such stars with strange matter cores and layer of dense nuclear material of white dwarf dimensions form a different class of dwarfs, called strange dwarfs \([39]\). Were it not for the gravitational attraction of the dense strange quark core, they would be unstable. A comparison between a white dwarf and strange dwarf is shown in Fig. 27.

A sequence of neutron stars to white dwarfs (and eventually planets) is shown in Fig. 28 together with two members of a continuum of strange objects. The one has inner crust density fixed at the drip density – higher than any white-dwarf density; the other has inner crust density of \(10^8 \text{ g cm}^{-3}\), which is the density of a normal white dwarf. The latter would be stable without the strange core; the former not. What is most remarkable is that the entire sequence of strange objects is stable, from the maximum mass of the compact strange star to the termination of the sequence on the white-dwarf family or at the maximum-mass strange dwarf (whichever occurs first in moving from the maximum-mass strange star toward the dwarfs). This

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Figure 27. Comparison of mass profiles of a strange dwarf (with crust at the neutron drip density) and a white dwarf of mass typical of white dwarfs \((M = 0.6 \, M_\odot)\).

(From [38].)
Figure 28. Neutron star (NS) – white dwarf (wd) sequence, (solid line). Two strange star (SS) – strange dwarf (sd) sequences, for which the inner crust density of nuclear material has the indicated values (in g/cm$^3$). The higher value is the drip density. Vertical bars mark minimum mass stars. Crosses mark termination of the strange star sequences where the strange core shrinks to zero. At those points strange dwarfs become identical to ordinary white dwarfs. (From [39])

was demonstrated by a stability analysis [39]. In contrast, the white dwarf – neutron star sequence has a region of stability ranging over many orders of magnitude in central stellar density from the minimum mass neutron star to the maximum mass white dwarf.

2.6.1. Stability of Strange Dwarfs
The configurations of the neutron star-white dwarf sequence that lie between the points a to b marked on Fig. 28 are unstable. We have carried out a stability analysis for the strange sequence [39]. The result for the fundamental (nodeless) vibrational mode is shown in Fig. 29. When the function $\Phi$ is positive, so is the angular velocity $\omega^2$ of the radial vibrational modes. This corresponds to stability. We see that the entire range of strange objects from the maximum-mass strange star to the maximum-mass strange dwarf are stable.

2.6.2. Strange Dwarfs as Microlensing Candidates
Strange dwarfs could be made in several ways [39]. The capture by main-sequence stars of strange nuggets as a component of cosmic rays, discussed
Figure 29. Pulsation frequencies for the two lowest modes $n = 0, 1$ measured by $\Phi(x) \equiv \text{sgn}(x) \log(1 + |x|)$ where $x \equiv (\omega_n / s)^2$ as a function of central star density in the vicinity of strange dwarfs having inner crust density equal to neutron drip. For $\Phi < 0$, the squared frequency is negative and the mode unstable. (From [39].)

earlier, is one possibility. Main-sequence stars are long-lived, large-area collectors of a cosmic flux of nuggets. Once captured a nugget would gravitate to the center of the star and rest dormant for almost or all of the stellar life time. If the star has mass greater than $\sim 8 \, M_\odot$, the core will contain free neutrons in the last few hours of the existence of the star. Upon core collapse a strange star will be born. If the progenitor mass is smaller than $\sim 8M_\odot$, nuclear burning is incomplete and free neutrons will not be present. A white dwarf is born as a result of vibrational instabilities which expel most of the star in a planetary nebula. The white dwarf is a strange dwarf, having a strange core of baryon number or mass corresponding to the lifetime acquisition of strange nuggets by the main-sequence star. The number of such nuggets will tend to fill the entire space between the dashed and dotted curves of Fig. 28. They represent an enormous ‘phase space’ of low-mass objects—Jupiter mass to several hundredths of a solar mass—that are candidates for microlensing detection.

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