Network strategies in election campaigns

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Abstract. This study considers a simple variation of the voter model with two competing parties. In particular, we represent the case of political elections, where people can choose to support one of the two candidates or to remain neutral. People operate within a social network and their opinions depend on those of the people with whom they interact. Therefore, they may change their opinions over time, which may mean supporting one particular candidate or none. Candidates attempt to gain people’s support by interacting with them, whether they are in the same social circle (i.e. neighbors) or not. In particular, candidates follow a strategy of interacting for a time with people they do not know (that is, people who are not their neighbors). Our analysis of the proposed model sought to establish which network strategies are the most effective for candidates to gain popular support. We found that the most suitable strategy depends on the topology of the social network. Finally, we investigated the role of charisma in these dynamics. Charisma is relevant in several social contexts, since charismatic people usually exercise a strong influence over others. Our results showed that candidates’ charisma is an important contributory factor to a successful network strategy in election campaigns.

Keywords: network dynamics, random graphs, networks, interacting agent models
1. Introduction

In recent years, opinion dynamics [1] has attracted the attention of many scientists and several models have been developed to study the formation and spreading of opinions (e.g. [2–7]). Interactions among individuals and the topology of their networks play a fundamental role in these dynamics [1, 8, 9]. One of the simplest models of opinion dynamics is the voter model [3, 10–12], which describes a set of agents who change their opinions over time by interacting among themselves. The voter model allows us to represent the development of consensus where different opinions are present. From a physical perspective this model can generally identify phase transitions in the system, such as from a disordered state to an ordered one [13, 14], although as shown by [15], non-linear dynamics, in which the final phase of the system is characterized by the coexistence of different opinions, can also be introduced. Moreover, the voter model can be applied in several ways, to identify a particular characteristic or to analyze the behavior that occurs in real systems, such as political elections [16–18] and competitions more generally [19–21]. In this work, we focus on political elections and introduce a variant of the classic voter model to study the best strategies for gaining popular support. In the proposed model, there are two competitors (or candidates), each trying to convince a community of agents. Agents in their turn are either neutral or have a preference for one candidate. Therefore, we consider a system in which there are three possible opinions [22]. Agents operate within a network and they change their opinions over time under the influence of their neighbors. During the evolution of the system, candidates try to influence the opinion of agents by connecting with them for a time. In particular, agents temporarily connected with candidates consider them as normal members of their social circle (i.e. their neighbors) during the time that they are forming their next opinion. It is therefore very important for each candidate to identify the agents most able to generate these connections. In this context, the most able are those that enable support for the candidate to increase as fast as possible across the whole population. It is worth noting that the dynamics described are
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based on the structure of underlying adaptive networks [23–25]. We compared different network strategies that are used to select agents. Numerical simulations showed a correlation between the best strategy and the topology of the agent network. We also investigated whether the definition of network strategies should take candidates’ charisma into consideration, as this is a fundamental quality in social contexts. The remainder of the paper is organized as follows. Section 2 introduces the proposed model. Section 3 shows the results of numerical simulations. The final section 4 presents conclusions.

2. The model

A simple variant of a voter model involves two competitors, e.g. two candidates during an election campaign. Candidates aim to gain popular support in a population of agents. Agents in their turn operate in a network within which they can interact with others. Moreover, each agent has an opinion, which may either be neutral or favouring one candidate. Opinions are mapped to states: agents in state 0 are neutral, whereas agents in states 1 and 2 have a preference for candidates 1 and 2, respectively. Although three-state voter models have been analyzed by other authors (see [26]), the classic version contains only two opinions, so the two-state variant has been much more widely studied. Agents change their opinion over time under the influence of their neighbors (obviously, candidates’ views never change). At time \( t = 0 \), all agents are in a neutral state (i.e. 0), with the exception of the two candidates, who are in states 1 and 2, respectively. At each time step, agents change their state (i.e. opinion) according to the following transition probabilities:

\[
p_{x \rightarrow y} = \sigma_y - \sum_{i=0}^{2} \sigma_i
\]

where \( p_{x \rightarrow y} \) is the transition probability to change from the \( x \)th state to the \( y \)th state and \( p_x \) the probability to remain in the same state. The value of \( \sigma_y \) is computed as \( \sigma_y = n_y/n_t \), where \( n_y \) is the number of neighbors in the \( y \)th state and \( n_t \) is the total number of neighbors (i.e. the degree of the agent). \( p_x \) is computed based on the densities \( \sigma_i \) of neighbors having all the feasible states different from the \( x \)th state. At each time step, the agents’ states are defined using a weighted random selection with transition probabilities (equation (1)) used as weights. Therefore, the development of the system is described by the following equations:

\[
\begin{align*}
N_0(t+1) &= N \cdot \left( \sum_{i=1}^{N} \sigma_i^0(t) - \sum_{i=1}^{N} \sigma_i^1(t) + \sigma_i^2(t) \right) + N_0(t) \\
N_1(t+1) &= N \cdot \left( \sum_{i=1}^{N} \sigma_i^1(t) - \sum_{i=1}^{N} \sigma_i^0(t) + \sigma_i^2(t) \right) + N_1(t) \\
N_2(t+1) &= N \cdot \left( \sum_{i=1}^{N} \sigma_i^2(t) - \sum_{i=1}^{N} \sigma_i^1(t) + \sigma_i^0(t) \right) + N_1(t)
\end{align*}
\]

where \( \sigma^0_i, \sigma^1_i, \sigma^2_i \) are the densities of neighbors with states 0, 1, and 2, respectively.
where $N_x$ is the number of agents in the $x$th state and $\sum_{i=1}^{\omega} \omega_i = x$ indicates the $i$th agent having a state $\omega_i$ equal (or different) to $x$. Candidates do not change their state, their aim being to gain popular support. In particular, they network with agents outside their social circle in order to affect their transition probabilities. These network connections only last for one time step and each candidate generates, every time, a number of connections equal to their degree (i.e. the number of their neighbors). Therefore, agents who are temporarily connected with a candidate compute their transition probabilities as follows:

$$p_{x\rightarrow y} = \sigma_y^t$$

$$p_x = 1 - p_{x\rightarrow y}$$

where $\sigma_y^t$ is the temporal density of those in the $y$th state (i.e. the state of the candidate who contacted the agent), computed as:

$$\sigma_y^t = \frac{n_y + 1}{n_t + 1}$$

The development of the system can therefore be described by the following equations:

$$N_1(t + 1)^T = N_1(t + 1) + k_1 \left[ \sum_{j=1}^{k_1} \sigma_1^{A_1,j}(t) \right] - k_2 \left[ \sum_{j=1}^{k_2} \sigma_2^{A_2,j}(t) \right]$$

$$N_2(t + 1)^T = N_2(t + 1) + k_2 \left[ \sum_{j=1}^{k_2} \sigma_2^{A_2,j}(t) \right] - k_1 \left[ \sum_{j=1}^{k_1} \sigma_1^{A_1,j}(t) \right]$$

$$N_0(t + 1)^T = N - N_1(t + 1)^T - N_2(t + 1)^T$$

where $N_x(t + 1)^T$ is the number of agents in the $x$th state, considering the temporal connections, and $k_1$, $k_2$ the degree of candidates 1 and 2, respectively. The exponent of $\sigma_x^t$ in equation (6), i.e. $A_x[j]$, represents the $j$th agent among those selected by the $x$th candidate to generate connections at time $t$. During the election campaign, at each time step, candidates have to select the most useful agents to generate connections, using one of the following network strategies:

- **S0.** Random selection;
- **S1.** Random weighted selections, using the degree of agents as weights;
- **S2.** 2nd-degree connections: agents at distance 2 (i.e. neighbors of their social circle);
- **S3.** 3rd-degree connections: agents at distance 3.

Figure 1 shows an example of two candidates generating a temporal connection by using strategies S2 and S3 respectively. Strategies S0 and S1 can be
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defined as ‘global strategies’, as candidates consider the whole network when selecting agents. Moreover, by using strategy $S_1$ agents with high degree are more likely to be selected. On the other hand, strategies $S_2$ and $S_3$ can be defined as ‘local strategies’, as candidates select agents by considering only the small part of the network immediately around them (i.e. friends of friends, and so on). In order to establish whether it is possible to identify the best network strategy, we analyzed the proposed model using scale-free networks and small-world networks to connect the agents.

3. Results

We performed many numerical simulations of the proposed model in our attempt to identify the best network strategy for gaining popular support. Agents were arranged in scale-free networks generated by the Barabasi–Albert model (BA model hereinafter) [27], and in small-world networks generated by the Watts–Strogatz model (WS hereinafter) [28]. In order to obtain small-world networks, we started from a 2D regular lattice with 6 neighbors per node, then we rewired with probability $\beta = 0.1$ each edge at random. Finally, both kinds of network (i.e. scale-free and small-world) have a number of agents $N = 10^4$, provided with an average degree $\langle k \rangle = 6$. Further simulations were performed in scale-free networks with $N > 10^4$, to observe the effect of a greater number of hubs (see Appendix). Recall that scale-free networks generated by the BA model have a degree of distribution $P(k)$ characterized by a scaling parameter $\gamma \approx 3$. In order to compare network strategies, we looked at the number of agents with a preference for each candidate and the number of neutral agents. In particular, we analyzed the variation of the density $\rho$ of agents in these three states.

Figure 1. Two candidates (i.e. red and green nodes) generate a temporal edge that is indicated by a dotted line, following a strategy: the red node uses strategy $S_2$ (i.e. it selects 2nd-degree connections), whereas the green node uses strategy $S_3$ (i.e. it selects 3rd-degree connections).
Analysis of the curves \((\rho, t)\), representing agents in different states shows that the number of neutral agents first falls to zero after about \(5.5 \times 10^3\) time steps in scale-free networks and after about \(1.2 \times 10^4\) time steps in small-world networks. After that, in both kinds of network, the system seems to reach a steady state characterized by small fluctuations of densities between states 1 and 2. Moreover, two important points, called \(T_1\) and \(T_2\), can be identified in the curves \((\rho, t)\). These points occur at the intersections between the density of neutral agents and those of agents in the other states. Point \(T_1\) is the intersection between neutral agents and agents with a preference for candidate 1, whereas \(T_2\) is the intersection between neutral agents and agents with a preference.
for the other candidate (i.e. 2). As discussed below, points $T_1$ and $T_2$ can be used to compare network strategies.

### 3.1. Comparison among network strategies

A useful parameter when comparing network strategies is the difference of densities over time $\Delta \rho$ between agents in state 1 and agents in state 2—see figure 4. The topology of the agents’ network seems to play a crucial role, as shown in panels (c) and (d) of figure 4 relating to scale-free and small-world networks, respectively. In particular, strategy $S_2$ is better than strategy $S_3$ in scale-free networks, but the opposite is true of small-world networks (i.e. strategy $S_3$ is better).
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Values of $\text{avg} (\Delta \rho)$ highlight that, in scale-free networks, local strategies are better than global ones, the best being $S_2$. As regards global strategies, $S_1$ is much better than $S_0$. On the other hand, in small-world networks, we found that the best strategy is $S_3$ followed by strategy $S_1$. In this case also, therefore, a local strategy is more effective than global ones. It is interesting to note that strategy $S_2$ yields optimal results in scale-free networks, but it is the worst strategy (among those analyzed) in small-world networks. When we compared global strategies, we found that $S_1$ is

Figure 4. Difference between densities of agents $\Delta \rho$ in states 1 and 2, when different strategies are adopted by the two candidates. Results are averaged over 20 different realizations. (a) Results achieved in scale-free networks, when the candidates use strategies $S_3$ and $S_1$ respectively. (b) Results achieved in small-world networks, when the candidates use strategies $S_3$ and $S_1$ respectively. (c) Results achieved in scale-free networks, when the candidates use strategies $S_3$ and $S_2$ respectively. (d) Results achieved in small-world networks, when the candidates use strategies $S_3$ and $S_2$ respectively. We therefore calculated the average value of $\delta \rho$, comparing all strategies in both kinds of networks - see panel (a) of figure 5. As discussed above, points $T_1$ and $T_2$ of diagrams ($\rho$, $t$) provide information about the rate at which candidates achieve global support. In particular, as shown in panel (b) of figure 5, we calculated the difference $T_2 - T_1$ for each curve ($\rho$, $t$).
always better than $S_0$ in particular in scale-free networks, due to the presence of hubs (i.e. nodes with a high degree). Further information is provided by the histogram $(T_2 - T_1)$ in panel (b) of figure 5, which enables us to determine which strategies gain popular consensus most quickly. As discussed before, after the number of neutral agents falls to zero, the system reaches a near-steady state, with small differences between the density of agents in states 1 and 2. Therefore, as the time is an important variable in competitions such as political elections [29], a good strategy enables consensus to be achieved in a small number of time steps. As a result of this analysis, we found that the best strategies, identified in the histogram $\text{avg}(\Delta \rho)$, are also the fastest. Furthermore, it is worth highlighting the fact that, although $S_2$ is weaker than global strategies in small-world networks (according to values of $\text{avg}(\Delta \rho)$), it produces a rapid increase when it comes to global consensus. Hence, in the event that the time variable is critical (i.e. candidates have little time to achieve consensus), local strategies are better than global ones.

3.2. Charismatic competitors

According to recent studies [30, 31], charisma plays an important role for politicians aiming to achieve popular consensus. More generally, charisma is relevant in several social contexts as charismatic people are able to exercise a strong influence over others. We therefore investigated the proposed model taking the charisma of candidates into account. In particular, we modified the transition probabilities of temporarily connected agents as follows:

$$p_{x \rightarrow y} = \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{2} & \text{if } x \neq 0 \end{cases}$$

Figure 5. Comparison between results achieved in scale-free networks (blue bars) and those achieved in small-world networks (red bars), when the network strategies vary. Results are averaged over 20 different realizations. (a) $\text{avg}(\Delta \rho)$, i.e. average difference between density of agents in states 1 and 2. (b) Difference between points $T_2$ and $T_1$, indicated in the inset (enlargement of a diagram $(\rho, t)$).
whereas, $p_x$, i.e. the probability that temporarily connected agents remain in the same state is always computed by equation (4). A charismatic candidate always gains the support of neutral agents, but has a 50% probability of gaining the support of agents who prefer his/her opponent. Figure 6 shows the results achieved in both kinds of network (i.e. scale-free and small-world), varying the network strategies adopted by candidates.

It is interesting to note that the presence of charismatic competitors strongly affects the results. In particular, the histogram $(\Delta \rho)$ (panel (a) of figure 6) shows that global strategies are better than local ones in both kinds of network. In scale-free networks the best strategy is $S_1$ whereas in small-world networks the best one seems to be $S_0$. Notwithstanding, we observed from the histogram $(T_2 - T_1)$ that in scale-free networks there are small temporal differences between strategies. Therefore, from this point of view, all strategies are similar. In small-world networks, however, we found that the best strategies are also the fastest ones. Finally, even when the presence of charismatic competitors is taken into account, the topology of networks still affects results.

4. Discussion and conclusions

This study has analyzed network strategies for gaining popular support where there are two competing candidates. In our simple variation of the voter model, agents change opinion according to transition probabilities computed considering...
the opinions of their social circle (i.e. their neighbors). Moreover, candidates are also allowed to interact temporarily with agents who do not belong to their social circle, with the aim of affecting their opinion. The proposed model is therefore based on an adaptive network. In particular, at each time step, candidates use a network strategy to select a number of agents, equal to their degree, to generate temporal connections. Candidates can choose between global strategies of random selection and weighted random selection (to select agents with a high degree), and local strategies of 2nd-degree connections and 3rd-degree connections. Simulations were carried out with agents arranged in scale-free networks and small-world networks. The results show that the topology of networks strongly affects the outcomes of the model. In particular, in scale-free networks the best strategy for selection of agents is $S_2$ (i.e. 2nd-degree connections). On the other hand, in small-world networks strategy $S_3$ (i.e. 3rd-degree connections) is more effective. In general, we found that local strategies are more advantageous than global ones in both kinds of network. In particular, although strategy $S_2$ does not appear suited to small-world networks, it is faster than either of the global strategies. Recall that the term ‘fast’, in this context, is used to identify strategies that enable an increase in global support in a small number of time steps. Furthermore, we performed simulations taking account of ‘charismatic’ candidates. We used the probability of their convincing temporarily connected agents as a measure of charisma. In particular, this probability is 1 in the event they interact with neutral agents, whereas it is equal to 0.5 in the event they interact with agents that have a preference for their opponent. We found that global strategies are better than local ones. In small-world networks both histograms, i.e. $\Delta \rho$ and $T_2 - T_1$, show that $S_0$ and $S_1$ are better than local strategies, and moreover, they yield similar results. On the other hand, in scale-free networks, global strategies are still better than local ones but, from a temporal perspective, the differences are small. All strategies permit many agents to be convinced in a similar number of time steps. In conclusion, our results highlight that both the topology of the agent network and the charisma of competitors should be considered when planning for a successful strategy during election campaigns.

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Appendix A

In this section, we report the results achieved in the proposed model using scale-free networks with $N = 5 \times 10^4$ agents. The effects of hubs (i.e. nodes with a high degree) in these dynamics can also be evaluated. As indicated in figure A1, on a qualitative level, the results are similar to those achieved in smaller scale-free networks—see figures 2 and 4.
We observed that when \( N \) (i.e. the number of agents) is increased, the number of time steps to reduce neutral agents to zero also increases. In particular, with \( N = 5 \times 10^4 \), the density of neutral agents falls to zero after about \( 1.8 \times 10^4 \) time steps, while with \( N = 10^4 \) it takes about \( 5.5 \times 10^3 \) time steps. Figure A2 shows results related to parameters \( \text{avg}(\Delta \rho) \) and \( T_2 - T_1 \), which are similar to those achieved in scale-free networks with \( N = 10^4 \).

Therefore, we can state that when there are more hubs in the agent network (considering the scale-free configuration), on a qualitative level, the outcomes of the proposed model are similar to those achieved in the main analysis.
Figure A2. Results achieved in scale-free networks with $N = 5 \times 10^4$ agents, varying the network strategy. Left, $\text{avg}(\Delta \rho)$, i.e. average difference between densities of agents in states 1 and 2. Right, difference between points $T_2$ and $T_1$. Results are averaged over 20 different realizations.

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