Endogenous Capital Utilization in CGE Models: A Mongolian Application with the PEP-1-1 Model

BY RAGCHAASUREN GALINDEV a AND BERNARD DECALUWÉ b

This paper extends the PEP-1-1 model (a static computable general equilibrium, or CGE, model for small open economies) to incorporate variable capital utilization. It argues that CGE models with fixed sectoral capital may underestimate the impact of shocks in the short run by ignoring industries' adjustment of their capital utilization rate (or intensity of use) in response to changes in their economic environment. The model is calibrated to a 2014 Mongolian social accounting matrix. An increase in the export price of coal is considered as a shock for demonstration purposes. Compared to the standard PEP-1-1 model the impact of the shock is larger in the expanded model. In addition, the results of the PEP-1-1 model are derived as a special case of the model involving capital utilization.

JEL codes: D58, Q33.

Keywords: CGE model; Variable (endogenous) capital utilization, Mongolian economy.

1. Introduction

Computable general equilibrium (CGE) models are widely used for policy and external shock analysis (e.g., the standard PEP models,1 the IFPRI model,2 the CoPS models,3 the MAMS model of Lofgren et al., (2013) and the GTAP models4). This paper investigates short-run closure for single-country, open-economy models in which the capital stock is fixed at the sectoral level and the aggregate labor supply is endogenous. In this environment, the abundance of labor is a key factor that determines the impact of shocks on aggregate variables such as gross

---

1 See https://www.pep-net.org/pep-standard-cge-models.
2 See http://www.ifpri.org/publication/standard-computable-general-equilibrium-cge-model-gams-0.
3 See https://www.copsmodels.com/.
4 See https://www.gtap.agecon.purdue.edu/models/default.asp.
domestic product (GDP). The issue here is that the impact of a shock could be underestimated for reasonably elastic aggregate labor supply curve because another important shock-amplifying mechanism – the fact that firms adjust the speed or intensity of use of their capital (capital utilization rate) depending on the economic conditions – is largely overlooked.

An economically meaningful direct measure of capital utilization is the workweek of capital (the number of hours a week) as used in Shapiro (1986).\textsuperscript{5} Empirical evidence suggests that capital utilization estimates vary across time and industries.\textsuperscript{6} Moreover, according to Beaulieu and Mattey (1998), the workweek is pro-cyclical (i.e., positively correlated with the overall business cycle) with a varying degree of correlation from one manufacturing industry to the next.\textsuperscript{7}

The concept of variable (or endogenous) capital utilization is a key feature of modern business cycle economics models, such as the real business cycle (RBC) and dynamic stochastic general equilibrium (DSGE) models (e.g., Greenwood et al., 1988; Burnside and Eichenbaum, 1996; King and Rebelo, 1999; Baxter and Farr, 2005; Christiano et al., 2005; Smets and Wouters, 2007). In these models, capital utilization is considered a part of capital services augmenting the capital stock in the aggregate production function. Its cost is captured by a convex depreciation function. Its optimality condition is determined alongside those of capital and labor. Like the mechanisms for endogenous labor supply (ranging from the labor-leisure tradeoff to various reasonable grounds for sticky nominal prices), variable capital utilization is a shock amplification mechanism. Early RBC models such as Prescott (1986) are criticized for technology shocks having to be unreasonably large to produce observed business cycle fluctuations. More specifically, the Solow residual cannot be entirely considered as a productivity shock because it contains other information such as endogenous capital utilization. Later contributions, however, consider variable capital utilization so that even small and plausible shocks are able to generate realistic business cycle fluctuations.\textsuperscript{8} There seems to be a gap in the CGE model literature regarding this issue being macroeconomic models at the national level.

\textsuperscript{5} See Beaulieu and Mattey (1998) for a literature review of other capital utilization metrics, such as electricity and material use, and their empirical relevance.

\textsuperscript{6} Shapiro (1986) found that the average workweek of capital in the U.S. manufacturing sector is slightly over 50 hours out of 168 hours per week for the period 1952-1982, implying a utilization rate of a little over 30%.

\textsuperscript{7} They also found that the average workweek in manufacturing for the period 1974-1992 was 97 hours per week (which equates to a utilization rate of about 58%) with large variations across manufacturing industries in the U.S.

\textsuperscript{8} Generating reasonable business cycles means that the impulse responses, the standard deviations, and the cross- and auto-correlations of model-generated macroeconomic variables are close to their observed counterparts.
In CGE models with a fixed sectoral capital stock, the only factor that can alter sectoral output is labor. If a sector is largely capital intensive (i.e., the capital income share of the value added is large), the extent to which its output can change is limited, and change can happen only by hiring a large amount of additional workers. On the other hand, the rate of return on capital has to adjust unrealistically for a given stock of capital to respond to relatively large shocks. If the elasticity of substitution between labor and capital is relatively low, the relative price of factors needs to be adjusted accordingly for the firm to still be able to utilize fixed capital stock. For instance, a large negative shock could cause the rate of return on capital to be negative, in which case the model has no solution. Figure 1 shows the quarterly time series of coal production, sales and export volumes in Mongolia. It is apparent there is considerable short-run cyclical volatility. The coal sector in Mongolia is largely capital intensive (the capital share of the value added was 62.5% in 2014), so its production cannot change much in response to a demand shock in the standard model with fixed sectoral capital without imposing technology shocks or artificially adjusting capital.

Figure 1. Coal production, sales and export volumes in Mongolia.

Source: Mongolian Customs Authority; Mineral Resources and Petroleum Authority.

This paper develops the concept of variable capital utilization in a CGE model. We extend the PEP-1-1 model of Decaluwé et al., (2013), which is a static single-country model, by calibrating it to a 2014 Mongolian social accounting matrix (SAM). We model endogenous capital utilization in accordance with the business cycle literature, in which the capital utilization rate augments the stock of capital

---

9 Under such circumstances, either the stock of capital must be reduced arbitrarily or the model parameters must be adjusted.
in the production function so that the benefit of varying utilization is captured by marginal productivity while its cost is embedded in the accelerated depreciation of capital. We augment the stock of capital in each sector with the utilization rate in the PEP-1-1 model’s value added function and use the same convex cost function of utilization as Baxter and Farr (2005). The cost minimization problem for producers incorporates the capital utilization rate alongside capital and labor inputs. The utilization rate’s optimality condition determines its response to a shock, which depends primarily on the elasticity of capital depreciation cost. Since variable capital utilization is a short-run phenomenon, we assume that capital is sector-specific and total labor supply is perfectly elastic at an exogenous real wage (i.e., unemployment is endogenous at a fixed real wage). There is a lack of evidence regarding the elasticity of capital depreciation cost at the sectoral level, so we apply the same benchmark value as Baxter and Farr (2005) to all sectors and conduct a sensitivity analysis with different values. For sufficiently high elasticity values, the PEP-1-1 model is derived as a special case because the capital utilization rate remains invariant to shocks.

One could examine the short-run impact of any individual shock or a group of shocks in the literature, such as government spending, preference, tax, export demand, world prices, productivity, investment or savings and COVID-19. For demonstration purposes, we consider a 25% increase in the world price of export coal. For a given value of the cost of capital utilization parameter, the results of the model with capital utilization are much larger in magnitude than those of the standard model. For example, in the benchmark case, the coal sector’s output was more than doubled in the expanded model, whereas it increased 5.63% in the standard model. As a result, real GDP increased 3.95% in the expanded model rather than 0.43% in the standard model. The reason is that the capital utilization rate acts as a production factor like labor and adjusts to the shock. In the limiting case in which the marginal cost of depreciation with respect to the capital utilization rate is vertical, the results of the standard model are derived.

One could argue that similar results can be generated from the standard model by either fixing the rental price and endogenizing capital as in Lemelin et al., (2010) or assuming a positive relationship between the rental rate and the supply of capital as in Dixon and Rimmer (2010). These approaches pertain to the extensive margin of capital while our approach focuses on the intensive margin. Contrary to our approach, these approaches can be considered ad hoc. In that

---

10 We consider two types of coal – domestic and export.
11 By the “standard” model in this paper, we mean the PEP-1-1 model, and we use them interchangeably.
12 Alternatively, one could consider long-run closure in CoPS-type static models in which the gross rate of return on investment (the ratio of rental price to the price of new capital) is fixed so that capital is endogenous.
respect, variable capital utilization could provide a rationale for them. From a practical point of view, changes in the utilization rate can be abrupt and exceed its physical limits (or boundary conditions). To solve this issue, we propose a method for imposing the boundary conditions in the simulations. We find that the magnitude of change in the variables derived from the interior solution is reduced when the boundary conditions are imposed.

This paper is organized as follows. In the next section, we discuss the standard PEP-1-1 model and its extension with variable capital utilization. In Section 3, we outline the data and calibrations used for the newly introduced variables and parameters. Section 4 simulates the expanded model with a shock in various scenarios and compares the results with those of the standard model. Section 5 concludes the paper.

2. Models

2.1 The standard PEP-1-1 model

The standard PEP-1-1 model is described in detail in Decaluwé et al., (2013). In brief, activities are nested, and each level uses a production function with constant returns to scale. More specifically, at the first level, production is modeled by a Leontief function composed of value added and intermediate consumption. At the next level, value added is specified by a function with constant elasticity of substitution (CES) between labor and capital inputs. Each activity can produce multiple commodities that are aggregated by a constant elasticity of transformation (CET) function. Finally, the amounts to sell domestically and export are governed by a CET function and relative prices. Each activity pays various production-related taxes.

Total demand (household, investment, government spending and intermediate) for each commodity is a CES function of domestically produced and imported quantities. Households receive a fraction of total capital and labor income from the industries, which is allocated to direct taxes, consumption and savings. Household demand for each commodity is governed by a linear expenditure system. Firms receive a fraction of total capital income and allocate it to direct taxes, transfers to other agents and savings.

The government receives direct taxes from households and firms as well as indirect taxes on transactions (sales taxes, import duties, export taxes and production taxes). Its income is allocated to transfers to households, firms and the rest of the world (ROW), spending on goods and services, and savings.

13 For example, suppose the workweek of a plant is 112 hours (two 8-hour shifts a day for 7 days) in the business-as-usual scenario. The interior solution suggests that the utilization rate increases by 100% (i.e., four 8-hours shift a day), which is physically impossible.
The ROW receives import income, transfers and fractions of capital and labor income. It spends on exports, makes transfers to domestic agents and invests in the domestic economy (foreign savings). World export and import prices are given for each commodity. Export demand for each commodity is an iso-elastic function of its relative price. The current account balance is fixed in the domestic currency. This in turn determines foreign savings to maintain equilibrium in the balance of payments. The exchange rate is the numeraire.

The sum of the savings of households, firms, the government and the ROW determines the total investment expenditure. Spending on each commodity for investment purposes is a constant fraction of the total investment expenditure.

In the general equilibrium, total domestic demand for domestic commodities equates to total domestic supply, and total labor supply is equal to total labor demand plus the savings-investment and balance of payments equilibrium conditions.

2.2 The model with variable capital utilization

Let us now discuss variable capital utilization. The value added in each industry is given by the following CES function:

\[
VA_j = B_j^{VA} \left[ \beta_j^{VA} LD_j^{-\rho_j^{VA}} + (1 - \beta_j^{VA})(U_j KD_j)^{-\rho_j^{VA}} \right]^{-\frac{1}{\rho_j^{VA}}} \tag{1}
\]

where \( VA_j \) is the value added, \( B_j^{VA} \) is the productivity level, \( LD_j \) is labor, \( KD_j \) is capital stock, \( U_j \) is the capital utilization rate, \( \beta_j^{VA} \) is the share parameter and \( \rho_j^{VA} \) is the elasticity parameter in industry \( j \). Each industry’s capital “services” are the product of its capital utilization rate and stock of capital. To determine capital utilization endogenously, there needs to be a cost associated with it. Following the tradition of business cycle models, we specify the cost of capital utilization in terms of depreciation. The idea is that the more intensively capital is utilized, the faster it depreciates. More specifically, this cost is a convex function of the utilization rate as in Baxter and Farr (2005):

\[
\delta_j = a_j + \frac{b_j}{1+\varphi_j} U_j^{1+\varphi_j}, \quad \varphi_j > 0 \tag{2}
\]

where \( \delta_j \) is the depreciation rate, \( a_j \) and \( b_j \) are parameters, and \( \varphi_j \) is the elasticity of marginal depreciation with respect to the utilization rate: \( \varphi_j = U_j \delta_j''(U_j)/\delta_j'(U_j) > 0 \).

The utilization rate has a subjective boundary condition – i.e., \( U_j \leq U_j^{max} \) – and \( U_j^{max} \) is set based on the available information. For example, in a particular application, the interior solution of the model suggests that the utilization rate doubles (100% increase) in a sector. The utilization rate under normal conditions corresponds to a workweek of 112 hours (that is, two 8-hour shifts a day, 7 days a week). According to the interior solution, it has to increase to 224 hours (or four 8-
hour shifts per day). In reality, however, only one shift can be added. This corresponds to  \( u_{j}^{\text{max}} = 1.33 \).

The industry’s profit maximization problem can be written as:

\[
\max_{KD_j, LD_j, U_j} = PVA_j V_A - W_j LD_j - (RN_j + PINDEX_j \delta_j) KD_j
\]

subject to (1), (2) and \( U_j \leq U_j^{\text{max}} \) where \( PVA_j \) is the price of the value added, \( W_j \) is the nominal wage rate, \( RN_j \) is the net rental rate and \( PINDEX_j \) is a price index that represents the monetary cost of replacing a unit of capital and also ensures the nominal homogeneity of the model. The gross rental price, \( R_j \), in the standard model is now determined by

\[
R_j = RN_j + PINDEX_j \delta_j
\]

The real wage rate and real rental price are determined by \( W_j/PVA_j \) and \( R_j/PVA_j \) respectively.

The first-order conditions with respect to labor, capital and the capital utilization rate are given by Equations (4), (5) and (6), respectively, as follows:

\[
\frac{\beta_j^VA_j V_A}{(1-\beta_j^VA_j)} \rho_j^{VA} + (1-\beta_j^VA_j)(U_j KD_j)^{-\rho_j^{VA}} LD_j^{\rho_j^{VA}+1} = \frac{w_j}{PVA_j}
\]

\[
\frac{\beta_j^VA_j LD_j^{-\rho_j^{VA}} + (1-\beta_j^VA_j)(U_j KD_j)^{-\rho_j^{VA}}}{U_j^{\rho_j^{VA}+1} KD_j} = \frac{R_j}{PVA_j}
\]

\[
\frac{\beta_j^VA_j LD_j^{-\rho_j^{VA}} + (1-\beta_j^VA_j)(U_j KD_j)^{-\rho_j^{VA}}}{b_j U_j^{\rho_j^{VA}+1} KD_j} = \frac{\delta_j}{PVA_j}
\]

In Equations (4) and (5), the marginal product of labor and marginal product of capital are equal to the real wage rate and real rental price, respectively. The expression on the left side of Equation (6) is the marginal benefit of the capital utilization rate, which is a downward-sloping curve due to the law of diminishing marginal product, while the one on the right side of it is the real marginal cost, which is an upward-sloping curve. The curvature is captured by the elasticity parameter \( \phi_j \). The intersection of these two curves determines the optimal capital utilization rate.\(^{14}\)

Equations (4) and (5) can be rearranged as follows:

\[
\frac{LD_j}{KD_j} = \left( \frac{\beta_j^VA_j R_j U_j^{\rho_j^{VA}}}{(1-\beta_j^VA_j) W_j U_j^{\rho_j^{VA}}} \right)^{\sigma_j^{VA}}
\]

where \( \sigma_j^{VA} = 1/(1 + \rho_j^{VA}) \) is the elasticity of substitution between labor and capital services. Equation (6) can be rewritten as follows:

\(^{14}\)See Figure A1 in the Appendix for a description of the marginal cost (MC) and marginal benefit (MB) curves for different values of elasticity of substitution between labor and capital.
Since the expression on the left side of Equation (8) is the real rental price according to Equation (5), it can be further simplified as:

\[
R_j = b_j U_j^{\rho_j + 1} PINDEX_j.
\]  

(9)

To solve the model, we replace the value added equation with Equation (1) and the optimal ratio of capital and labor with Equation (7), and add Equations (2) and (8) to the standard PEP-1-1 model’s code. The rest of the code remains the same. We follow a simple strategy to impose the boundary condition \( U_j \leq U_j^{max} \). In the case of a given shock, we first obtain the interior solution of the model and check if the utilization rate in each industry violates the boundary condition. If it does not, the interior solution becomes the model solution and the simulation results can be used. If it does, the utilization rate (or rates) that generate such an outcome must be identified. Then, it (they) can be exogenously set equal to its (their) boundary rate and the remaining utilization rates can be left endogenous when solving the model again.

### 3. Data and calibrations

#### 3.1 Data

The models are calibrated to a Mongolian SAM. We constructed the SAM using 2014 data from the supply and use table, the balance of payments and the government budget obtained from the National Statistical Office.

Table A1 in the Appendix shows the macro SAM as a share of the nominal GDP in 2014, which was MNT 22.2 trillion. As can be seen in the table, household consumption represents more than half of the GDP (57%), while the current government’s expenditures equate to 13% of the GDP. Gross fixed capital formation and inventory changes jointly account for 35% of the GDP. The value of both exports and imports are more than half of the GDP (52% and 56%, respectively). The economy is equally intensive in both capital and labor – the value of payments to capital owners and employee compensation are 45.3% and 44.8% of the GDP, respectively. Value added accounts for 90.2% of the GDP, and the remaining 9.8% is from net indirect taxes on commodities (7.7%), import duties (1.6%) and net taxes on production (0.5%).

The accounts in the detailed SAM include 17 sectors and commodities, two production factors (capital and labor), three types of institutions (the private sector...

15 The PEP-1-1 model is designed to accommodate multiple types of capital. We consider only one type of capital when demonstrating our concept.
three types of taxes (income tax, import duties and taxes on commodities), and savings (investment) accounts divided into public investment, mining, private investment and changes in inventory.

Production structure: The livestock and trade sectors contribute most to labor income, while the metal ores sector contributes most to capital income. The manufacturing, metal ores and other mining sectors are highly intensive in capital, while the livestock, public administration, education and health sectors are most intensive in labor (see Table A2).

Trade structure: Table A3 in the Appendix shows that metal ores account for more than half of total exports, while the majority of imports (74%) are manufacturing commodities. Commodities such as export coal, metal ores and other mining products are almost entirely exported. Most manufacturing and accommodation commodities are imported. On the other hand, trade and public administration are not bought and sold internationally.

Demand structure: Table A4 shows the demand structure for each commodity. The majority of accommodation is consumed by households. Public administration, education and health are mostly consumed by the government. Almost all domestic coal and other mining products are used as intermediate inputs for production. Electricity and financial activities are mainly used as intermediate inputs as well. Trade is a 100% margin commodity, while 14% of transport services are used as margin. Construction services are mainly used for investment purposes.

Investment/savings structure: More than half of total investment is funded by household savings. The ROW and the government fund 33% and 12% of total investment, respectively. Forty-four percent and 37% of total investment is allocated to funding private and public investment, respectively, for gross fixed capital formation.

Structure of household (private sector) income and expenditures: The main sources of income for households are capital ownership and labor, which jointly contribute about 87% of their total income. Households spend most of their income (59.2%) on commodities. Fourteen percent of it goes to the government as direct taxes, and another 5% of it is transferred to the government as non-tax payments. Transfers to the ROW are relatively small (1.5%), while savings equates to about 20% of total income.

Structure of government income and expenditures: The government receives the majority of its revenue from households (including firms) as direct taxes (47%) and transfers (16.7%). Just over a quarter (27%) of it comes from commodity taxes. Other sources of income are relatively small. Almost half of the government’s

---

Note that households and firms are aggregated in our SAM. This means that “household” savings covers all the savings of the private sector – households and firms.
budget is spent on purchasing goods and services. Thirty-seven percent of its budget is allocated to households as transfers. Savings account for 14% of its total budget and are used to fund its capital expenditures.

3.2 Calibrations

The calibrations are standard, as in the PEP-1-1 model, except for those related to capital utilization. In the base year, where all variables end with \( O \), we normalize that the utilization rate is equal to unity in all sectors, i.e., \( U_{Oj} = 1 \) as in the modern business cycle models.\(^{17}\) This represents the length of the workweek in either the business-as-usual (BAU) scenario or the base year in question.

Consequently, Equations (1), (2), (7) and (8) can be written as follows in the base year:

\[
V_{AOj} = b^V_{j} \left[ \beta^V_{j} LDO_{j}^{-\rho^V_{j}} + (1 - \beta^V_{j}) KDO_{j}^{-\rho^V_{j}} \right]^{-\frac{1}{\rho^V_{j}}},
\]

\[
\delta_{Oj} = a_{j} + \frac{b_{j}}{1 + \phi_{j}},
\]

\[
\frac{LDO_{j}}{KDO_{j}} = \frac{\left( \beta^V_{j} \sigma^V_{j} \right)}{\left(1 - \beta^V_{j} \right) W_{Oj}^{V}},
\]

\[
\left(1 - \beta^V_{j}\right) V_{AOj}^{V} + (1 - \beta^V_{j}) KDO_{j}^{-\rho^V_{j}} = b_{j} \frac{INDEXO_{j}}{PVAO_{j}}.
\]

We set \( W_{Oj} = 1 \) and \( INDEXO_{j} = PIXCONO = 1 \) where \( PIXCONO \) is the consumption price index. In 2014, the annual average nominal interest rate was 19.6% in local currency according to the central bank of Mongolia. Assuming that the rental rate of capital in each sector is equal to the nominal interest rate \( R_{Oj} = 0.196 \) helps to determine \( KDO_{j} \). Next we calibrate \( a_{j} \), \( b_{j} \), \( \delta_{Oj} \) and \( \phi_{j} \). We set the latter two in line with the relevant CGE literature. The condition \( U_{Oj} = 1 \) determines the depreciation rates in the BAU scenario, \( \delta_{Oj} \). We use the values from Dixon and Rimmer’s (2002) MONASH model,\(^{18}\) which are given in the following table.

\(^{17}\) In business cycle economics, models are calibrated in the steady-state equilibrium condition (or balanced growth path), where shocks take their average values (e.g., King and Rebelo, 1999).

\(^{18}\) See [https://www.copsmodels.com/](https://www.copsmodels.com/). More specifically, we use the depreciation rates from the motor vehicle tariff application in the demonstration version of RunMONASH. The MONASH model is described in detail in Dixon and Rimmer (2002).
Table 1. Depreciation rates by sector in the BAU scenario

| Sector                  | Rate  |
|-------------------------|-------|
| Crops                   | 0.066 |
| Livestock               | 0.081 |
| Export coal             | 0.078 |
| Domestic coal           | 0.078 |
| Metal ores              | 0.083 |
| Other mining            | 0.083 |
| Electricity             | 0.074 |
| Manufacturing           | 0.087 |
| Construction            | 0.086 |
| Trade                   | 0.070 |
| Accommodation           | 0.065 |
| Transportation          | 0.103 |
| Financial activities    | 0.054 |
| Public administration   | 0.052 |
| Education               | 0.052 |
| Health                  | 0.059 |
| Other services          | 0.060 |

Evidence of the parameterization of $\varphi_j$ is rare. Among a panel of US firms from 21 manufacturing industries for the period 1949-1985, Basu and Kimball (1997) found that the 95% confidence interval of the estimate is [-0.2, 2]. For purely economic reasons, it should be positive. From what we have seen above, the smaller the elasticity value, the flatter the marginal depreciation curve of the utilization rate and, hence, the greater the response of utilization to changes in demand. In aggregate business cycle models, values are commonly considered equal to unity or less than unity. For instance, Baxter and Farr (2005) consider $\varphi_j = 1$, $\varphi_j = 0.1$ and $\varphi_j = 0.05$ in the aggregate model. We consider $\varphi_j = 1$ for all sectors, meaning that marginal depreciation is a linear function of the utilization rate. In the sensitivity analysis, we consider $\varphi_j = 0.5$ and $\varphi_j = 2$. At the moment, we cannot find any evidence of sector-specific values being used for $\varphi_j$. It is also evident from Equation (9) that $b_j = R_0j$ as $U_0j = 1$ and $INDEXOj = 1$. We can use $\delta O_j$, $b_j$ and $\varphi_j$ in Equation (2) to obtain $a_j = \delta O_j - b_j/(1 + \varphi_j)$.

---

19 The aggregate business model requires smaller productivity shocks as elasticity decreases, which is a desirable feature according to Baxter and Farr (2005) and others.
4. Scenario and simulation results

As mentioned earlier, we consider short-run closure, which is characterized by the following conditions.

1. The nominal exchange rate is the numeraire and fixed at unity.
2. The stock of capital in each sector is fixed, $KD_j = KDO_j$.
3. Labor is mobile, $W_j = W$ and the total labor supply is endogenous at a fixed real wage, $WO/PXCONO$.
4. Nominal government spending is fixed at the initial value.
5. The nominal current account deficit is fixed at the initial value.
6. The minimum household consumption of each commodity is fixed at the initial value.
7. The world import and export prices of each commodity are fixed.
8. The savings, tax and transfer rates are fixed.
9. Nominal labor and capital income from the ROW are fixed.
10. The stock variation of each commodity is fixed.

Like the standard model, the expanded model with variable capital utilization generates the initial data and the SAM when there is no shock to the above exogenous variables. We refer to this as the BAU scenario. The current model also passes the nominal homogeneity test if we multiply the exogenous nominal and price variables in Equations (4), (5), (7) and (9) above by the same magnitude.

4.1 Scenario

We consider a scenario in which the world price of export coal increases by 25%. All the remaining exogenous variables are fixed at their initial values. For the sensitivity analysis, we consider the following three cases for the elasticity of marginal depreciation with respect to capital utilization: $\varphi_j = 2$, $\varphi_j = 1$ and $\varphi_j = 0.5$. As $\varphi_j$ decreases, the marginal depreciation curve gets flatter and, hence, capital utilization becomes more responsive. We present the simulation results without the boundary conditions on utilization rates in Subsection 4.2 and with them in Subsection 4.3.
4.2 Interior solutions

Table 2 shows the macroeconomic results of the two models for comparison. In the case of short-run closure with sector-specific capital and abundant total labor supply, real GDP at base prices increases 0.43% because of a 1.06% increase in labor supply in the standard model. In the expanded model with variable capital utilization, on the other hand, the changes in variables become significantly larger as the degree of responsiveness of the utilization rate increases (or the elasticity of marginal depreciation cost decreases). One can see that real GDP at base prices increases 2.17%, 3.95% and 7.36% as the elasticity of marginal depreciation cost decreases, which leads to 0.96%, 2.18% and 4.88% increases in the weighted average capital utilization rate, respectively. In general, the changes in variables are larger in the expanded model than in the standard model. In addition, the magnitude of the changes increases as the value of $\varphi_j$ gets smaller. The reason for this is simply that capital utilization acts as a production input and its availability is governed by the value of $\varphi_j$. One can also see that the results of the standard model and the expanded model are the same when $\varphi_j = \infty$. One noticeable difference between the two models is nominal government savings. It is calculated as the difference between government income and nominal current expenditures (spending on goods and services plus transfers to other agents). Government income is endogenous, while spending on goods is fixed and the sum of transfers is indexed so that government savings adjust to increases in government income and the consumption price index.

Table 2. Macroeconomic variables (% change from BAU)

|                              | Standard model | Model with variable capital utilization |
|------------------------------|---------------|-----------------------------------------|
| $\varphi_j = \infty$        |               |                                         |
| Nominal GDP at market prices | 2.31          | 4.83                                    |
| Real GDP at base prices      | 0.43          | 2.17                                    |
| Employment                   | 1.06          | 3.18                                    |
| Average utilization rate     | 0.00          | 0.96                                    |
| $\varphi_j = 2$             |               |                                         |
| Nominal GDP at market prices |               |                                         |
| Real GDP at base prices      |               |                                         |
| Employment                   |               |                                         |
| Average utilization rate     |               |                                         |
| $\varphi_j = 1$             |               |                                         |
| Nominal GDP at market prices |               |                                         |
| Real GDP at base prices      |               |                                         |
| Employment                   |               |                                         |
| Average utilization rate     |               |                                         |
| $\varphi_j = 0.5$           |               |                                         |
| Nominal GDP at market prices |               |                                         |
| Real GDP at base prices      |               |                                         |
| Employment                   |               |                                         |
| Average utilization rate     |               |                                         |

(Continued)

20 One could derive the results of the standard model from the expanded model in two ways. The first way is to assign very high values to $\varphi_j$ so that the marginal cost of capital utilization is large enough and the utilization rate does not respond to shocks. Alternatively, set $U_j = 1$ and inactivate the optimality condition of the capital utilization rate for each industry.
Table 2. Macroeconomic variables (% change from BAU) (Continued)

|                      | Standard model | Model with variable capital utilization |
|----------------------|----------------|----------------------------------------|
|                      | \( \varphi_j = \infty \) | \( \varphi_j = 2 \) | \( \varphi_j = 1 \) | \( \varphi_j = 0.5 \) |
| Nominal investment   | 2.71           | 5.93                                   | 8.90               | 14.10              |
| Nominal government income | 2.18         | 4.95                                   | 7.48               | 11.84              |
| Nominal household income | 2.01         | 4.12                                   | 6.01               | 9.25               |
| Nominal exports      | 2.26           | 5.68                                   | 8.81               | 14.24              |
| Nominal imports      | 1.69           | 4.47                                   | 7.01               | 11.42              |
| Nominal government savings | 13.78      | 31.53                                  | 48.22              | 77.66              |
| Consumption price index | 0.50        | 1.04                                   | 1.37               | 1.69               |
| GDP deflator         | 1.93           | 2.65                                   | 3.09               | 3.49               |
| Nominal wage rate    | 0.50           | 1.04                                   | 1.37               | 1.69               |

As can be seen in Table 3, sectoral capital utilization rates change significantly in response to the shock. Again, the magnitude of utilization rate changes is greater for flatter marginal depreciation curves (i.e., smaller \( \varphi_j \)) and vice versa. Notably, the export coal sector’s capital utilization rate increases 177.06% for \( \varphi_j = 0.5 \). This implies that the workweek of capital becomes 2.77 times longer in this sector. Whether this is actually possible is an empirical question.

Table 3. Capital utilization rates by sector (% change from BAU)

|                      | \( \varphi_j = 2 \) | \( \varphi_j = 1 \) | \( \varphi_j = 0.5 \) |
|----------------------|---------------------|---------------------|---------------------|
| Crops                | 0.48                | 1.24                | 3.11                |
| Livestock            | 0.90                | 2.01                | 4.44                |
| Domestic coal        | 0.95                | 2.24                | 5.24                |
| Export coal          | 47.62               | 94.46               | 177.06              |
| Metal ores           | -1.40               | -3.22               | -6.39               |
| Other mining         | -0.65               | -1.60               | -3.42               |
| Manufacturing        | 0.28                | 1.18                | 3.79                |
| Electricity          | 0.33                | 0.81                | 1.97                |
| Construction         | 1.95                | 4.32                | 9.14                |
| Trade                | 0.91                | 2.32                | 5.54                |

(Continued)
Table 3. Capital utilization rates by sector (% change from BAU) (Continued)

| Sector                | $\varphi_j = 2$ | $\varphi_j = 1$ | $\varphi_j = 0.5$ |
|-----------------------|-----------------|-----------------|-------------------|
| Transportation        | 1.51            | 3.98            | 9.44              |
| Accommodation         | -0.09           | -0.05           | 0.41              |
| Financial activities  | 0.83            | 2.09            | 4.96              |
| Public administration | -0.16           | -0.25           | -0.20             |
| Education             | 0.31            | 0.79            | 2.01              |
| Health                | 0.26            | 0.66            | 1.70              |
| Other services        | 0.88            | 2.29            | 5.60              |

Table 4 shows the changes in total production by sector. In general, the changes observed in the standard model are amplified in the model with capital utilization. Both negative and positive changes are larger in magnitude. In the benchmark case with $\varphi_j = 1$, the export coal sector’s production more than doubles in the model with capital utilization, whereas it increases 5.63% in the standard model. If capital depreciation grows slowly, as is the case when $\varphi_j = 0.5$, its production increases 186.51%. This signifies the importance of capital utilization at the sectoral level. Whether this can happen in a particular application depends on the level of production in the base year and the boundary condition. The changes in the other sectors are much smaller.

Table 4. Changes in total production by sector (% change from BAU)

| Standard model    | Model with variable capital utilization |
|-------------------|----------------------------------------|
|                   | $\varphi_j = \infty$                  | $\varphi_j = 2$ | $\varphi_j = 1$ | $\varphi_j = 0.5$ |
| Crops             | 0.20                                   | 0.80            | 1.66             | 3.63              |
| Livestock         | 0.08                                   | 0.99            | 2.12             | 4.55              |
| Domestic coal     | 0.14                                   | 1.09            | 2.41             | 5.44              |
| Export coal       | 5.63                                   | 54.85           | 102.81           | 186.51            |
| Metal ores        | -0.12                                  | -1.62           | -3.47            | -6.62             |
| Other mining      | -0.02                                  | -0.69           | -1.65            | -3.48             |
| Manufacturing     | -0.11                                  | 0.48            | 1.62             | 4.50              |
| Electricity       | 0.33                                   | 0.93            | 1.53             | 2.87              |
| Construction      | 1.81                                   | 4.47            | 7.16             | 12.21             |
| Trade             | 0.89                                   | 2.80            | 4.77             | 8.51              |
| Transportation    | 0.70                                   | 3.79            | 7.05             | 13.18             |
| Accommodation     | -0.17                                  | -0.26           | -0.09            | 0.61              |
| Financial activities | 0.31                               | 1.55            | 3.01             | 6.07              |
| Public administration | -0.32                              | -0.54           | -0.56            | -0.32             |

(Continued)
Table 4. Changes in total production by sector (% change from BAU) (Continued)

| Standard model | Model with variable capital utilization |
|----------------|----------------------------------------|
| \( \phi_j = \infty \) | \( \phi_j = 0.5 \) | \( \phi_j = 1 \) | \( \phi_j = 2 \) |
| Education | 0.50 | 1.06 | 1.75 | 3.24 |
| Health | 0.43 | 0.91 | 1.50 | 2.79 |
| Other services | 0.32 | 1.54 | 3.16 | 6.69 |

Table 5 also shows that the impact on rental prices, \( R_j \), is much larger in the expanded model. The nominal rate of return in the BAU scenario is 19.6%. For instance, it increases to 44.1% in the export coal sector when \( \phi_j = 2 \) in response to the shock. Again, the change in absolute value gets larger as the response of capital utilization increases. In the standard model, it increases 154.78% and absorbs the increase in the price of export coal as capital in this sector is fixed. In the capital utilization version of the model, however, it increases more than that because the increase in the capital utilization rate shifts up the marginal productivity of capital. It reflects the increase in the depreciation cost.

Table 5. Gross rental prices by sector (% change from BAU)

| Standard model | Model with variable capital utilization |
|----------------|----------------------------------------|
| \( \phi_j = \infty \) | \( \phi_j = 0.5 \) | \( \phi_j = 1 \) | \( \phi_j = 2 \) |
| Crops | 1.11 | 2.50 | 3.90 | 6.47 |
| Livestock | 2.10 | 3.80 | 5.49 | 8.54 |
| Domestic coal | 2.41 | 3.95 | 5.98 | 9.80 |
| Export coal | 154.78 | 225.05 | 283.33 | 368.98 |
| Metal ores | -1.03 | -3.16 | -5.06 | -7.89 |
| Other mining | -0.09 | -0.91 | -1.84 | -3.48 |
| Manufacturing | 0.19 | 1.88 | 3.78 | 7.53 |
| Electricity | 0.87 | 2.05 | 3.01 | 4.72 |
| Construction | 3.39 | 7.06 | 10.32 | 15.95 |
| Trade | 1.37 | 3.82 | 6.14 | 10.27 |
| Transportation | 1.45 | 5.68 | 9.61 | 16.43 |
| Accommodation | 0.32 | 0.77 | 1.27 | 2.33 |
| Financial activities | 1.21 | 3.57 | 5.66 | 9.36 |
| Public administration | 0.24 | 0.56 | 0.86 | 1.40 |
| Education | 0.92 | 1.98 | 2.98 | 4.77 |
| Health | 0.84 | 1.82 | 2.71 | 4.30 |
| Other services | 1.37 | 3.73 | 6.06 | 10.36 |
Bear in mind that part of the nominal rental rate is the depreciation cost associated with utilization – \( R_j = RN_j + PINDEX_j \delta_j \). The net rental price, \( RN_j \), is shown in Table 6. In the standard model, it is calculated as the difference between the nominal rental price and the fixed depreciation rate multiplied by the consumption price index: \( R_j = RN_j + PINDEX_j \delta O_j \). We can see that the net rental rates are in a comparable range. In the export coal sector, the net rental price increases less as \( \phi_j \) increases, resulting in the cost of capital depreciation growing faster than the gross rental rate.

| Table 6. Net rental prices by sector (% change from BAU) |
|--------------------------------------------------------|
| Standard model | Model with variable capital utilization |
| \( \phi_j = \infty \) | \( \phi_j = 2 \) | \( \phi_j = 1 \) | \( \phi_j = 0.5 \) |
| Crops | 1.4 | 2.5 | 3.3 | 4.1 |
| Livestock | 3.2 | 4.2 | 4.9 | 5.6 |
| Domestic coal | 3.7 | 4.3 | 5.2 | 6.2 |
| Export coal | 256.8 | 249.1 | 235.5 | 205.0 |
| Metal ores | -2.2 | -3.8 | -4.2 | -3.8 |
| Other mining | -0.5 | -1.2 | -1.4 | -1.3 |
| Manufacturing | -0.1 | 2.0 | 3.5 | 5.2 |
| Electricity | 1.1 | 2.1 | 2.7 | 3.3 |
| Construction | 5.6 | 8.2 | 9.3 | 10.2 |
| Trade | 1.8 | 3.9 | 5.1 | 6.1 |
| Transportation | 2.5 | 7.6 | 10.0 | 12.0 |
| Accommodation | 0.2 | 0.8 | 1.3 | 2.0 |
| Financial activities | 1.5 | 3.4 | 4.3 | 5.2 |
| Public administration | 0.1 | 0.6 | 1.0 | 1.6 |
| Education | 1.1 | 1.9 | 2.5 | 3.1 |
| Health | 1.0 | 1.8 | 2.3 | 2.9 |
| Other services | 1.8 | 3.6 | 4.7 | 5.9 |

Table 7 shows a similar picture for employment by sector. In response to the shock, total labor supply increases 1.06%, 3.18%, 5.17% and 8.79% in the corresponding versions of the expanded model (see Table 2). Again we see a similar picture as above – the versions with more responsive capital utilization experience larger changes.
Table 7. Employment by sector (% change from BAU)

|                     | Standard model | Model with variable capital utilization |
|---------------------|----------------|-----------------------------------------|
|                     | $\varphi_j = \infty$ | $\varphi_j = 2$ | $\varphi_j = 1$ | $\varphi_j = 0.5$ |
| Crops               | 0.49           | 1.25          | 2.24          | 4.38          |
| Livestock           | 1.27           | 2.36          | 3.65          | 6.27          |
| Domestic coal       | 0.38           | 1.33          | 2.69          | 5.78          |
| Export coal         | 20.45          | 72.51         | 122.13        | 206.78        |
| Metal ores          | -0.46          | -2.24         | -4.47         | -7.31         |
| Other mining        | -0.12          | -0.91         | -1.91         | -3.76         |
| Manufacturing       | -0.46          | 1.11          | 2.98          | 6.72          |
| Electricity         | 0.55           | 1.34          | 2.03          | 3.47          |
| Construction        | 4.34           | 8.02          | 11.16         | 16.54         |
| Trade               | 1.30           | 3.68          | 1.86          | 9.90          |
| Transportation      | 1.41           | 6.16          | 10.26         | 17.11         |
| Accommodation       | -0.27          | -0.35         | -0.12         | 0.72          |
| Financial activities| 1.06           | 3.35          | 5.31          | 8.85          |
| Public administration| -0.39         | -0.63         | -0.63         | -0.34         |
| Education           | 0.62           | 1.24          | 1.99          | 3.54          |
| Health              | 0.51           | 1.02          | 1.65          | 2.99          |
| Other services      | 1.30           | 3.57          | 5.82          | 10.01         |

4.3 Boundary conditions on capital utilization

Let us translate the changes in utilization rates into duration of the workweek of capital. In the export coal sector, the duration may seem excessive even for the reasonable values of $\varphi_j$ that we consider. Suppose that the workweek of capital in this sector is 84 hours (12 hours a day, 7 days a week) in the BAU scenario. Then, a 100% increase in the workweek reaches the physical limit of a 168-hour workweek. Similarly, if the workweek is 112 hours (that is, two 8-hour shifts a day, 7 days a week) in the BAU scenario, the change in utilization rate cannot exceed 33.3% (one extra shift). In this case, for example, the results of the $\varphi_j = 2$ scenario are not possible, as the physical limit of the workweek (or the boundary condition on the utilization rate) is met. In this sense, the standard CGE model without variable capital utilization can be considered the extreme case in which utilization rates cannot change.

The utilization rates in the BAU scenario can be assumed to be either normal (i.e., averages of long periods) or year-specific (i.e., depend on the base year data). Regardless, the concept of capital utilization requires a CGE modeler to obtain more information about $U_{O_j}$ and $U_{j}^{max}$.
We now outline a simple approach to impose boundary conditions on utilization rates.\textsuperscript{21} Let us consider the $\varphi_j = 0.5$ scenario. Suppose that we know $U_{excoal}^{\text{max}}$ and determine that the export coal sector’s utilization rate exceeds its boundary. Let us consider two cases: $U_{excoal}^{\text{max}} = 1$ and $U_{excoal}^{\text{max}} = 1.5$. The utilization rate obviously cannot increase in the former case, but it can increase up to 50% in the latter. We know that the simulation results generated under such circumstances using the model without boundary conditions (i.e., the interior solution results) are unattainable. Hence, we must impose the boundary condition. In doing so, we make $U_{excoal}$ exogenous and fix it at the boundary rate, meaning $U_{excoal} = U_{excoal}^{\text{max}}$. The utilization rates of the other sectors remain endogenous. The results are shown in the following tables. For comparison purposes, we also include the results of the standard model and those of the model without boundary conditions ($\varphi_j = 0.5$).

### Table 8. Macroeconomic variables (% change from BAU)

|                      | Standard model | Model with variable capital utilization |
|----------------------|----------------|----------------------------------------|
|                      | $\varphi_j = \infty$ | $U_{excoal}^{\text{max}} = 1$ | $U_{excoal}^{\text{max}} = 1.5$ | No boundary |
| Nominal GDP at market prices | 2.31 | 2.15 | 4.84 | 11.10 |
| Real GDP at base prices     | 0.43 | 0.69 | 2.70 | 7.36 |
| Employment                 | 1.06 | 1.06 | 3.31 | 8.79 |
| Nominal investment          | 2.71 | 2.57 | 6.02 | 14.10 |
| Nominal government income   | 2.18 | 2.03 | 4.96 | 11.84 |
| Nominal household income    | 2.01 | 1.83 | 4.07 | 9.25 |
| Nominal exports             | 2.26 | 2.06 | 5.64 | 14.24 |
| Nominal imports             | 1.69 | 1.52 | 4.41 | 11.42 |
| Nominal household savings   | 2.01 | 1.83 | 4.07 | 9.25 |
| Nominal government savings  | 13.78 | 13.48 | 32.61 | 77.66 |
| Consumption price index     | 0.50 | 0.23 | 0.67 | 1.69 |

The results of the $U_{excoal}^{\text{max}} = 1$ scenario are comparable to those of the standard model. However, the utilization rates in the sectors other than export coal are endogenous in the former case, so that the results are slightly different (see Table 8). In the $U_{excoal}^{\text{max}} = 1.5$ scenario, the changes are significantly larger as the

\textsuperscript{21} One could solve this by defining it as a mixed complementarity problem in the GAMS (General Algebraic Modeling System).
utilization rates have more room to adjust (see Table 9). See Tables A8, A9 and A10 in Appendix for more results.

Table 9. Capital utilization rates (% change from BAU)

| Sector               | $U_{excoa}^{max} = 1$ | $U_{excoa}^{max} = 1.5$ | No boundary |
|----------------------|----------------------|-------------------------|-------------|
| Crops                | 0.62                 | 1.37                    | 3.11        |
| Livestock            | 0.84                 | 1.93                    | 4.44        |
| Domestic coal        | 0.77                 | 2.09                    | 5.24        |
| Export coal          | 0.00                 | 50.00                   | 177.06      |
| Metal ores           | -0.85                | -2.56                   | -6.39       |
| Other mining         | -0.41                | -1.34                   | -3.42       |
| Manufacturing        | 0.51                 | 1.48                    | 3.79        |
| Electricity          | -0.22                | 0.71                    | 1.97        |
| Construction         | 1.72                 | 3.97                    | 9.14        |
| Trade                | 0.71                 | 2.14                    | 5.54        |
| Transportation       | 0.65                 | 3.22                    | 9.44        |
| Accommodation        | 0.11                 | 0.20                    | 0.41        |
| Financial activities | 0.52                 | 1.82                    | 4.96        |
| Public administration| -0.03                | -0.09                   | -0.20       |
| Education            | 0.47                 | 0.94                    | 2.01        |
| Health               | 0.39                 | 0.79                    | 1.70        |
| Other services       | 0.78                 | 2.21                    | 5.60        |

5. Conclusion

In this paper, we extend the PEP-1-1 model of Decaluwé et al., (2013) with endogenous capital utilization to show that CGE models with predetermined sector-specific capital may underestimate the impact of shocks in the short run. The idea behind variable capital utilization is that capital utilization is a part of capital services that can be endogenously altered by industries depending on their economic environment – i.e., industries can use their machines at greater or lesser intensity. Capital utilization augments the amount of physical capital in the value added function and is a component of the Solow residual in growth accounting. In that sense, it generates decreasing marginal benefits for industries. On the other hand, it incurs a cost in terms of capital depreciation. As in the business cycle literature, cost is a convex function of the utilization rate. The optimal capital utilization rate responds to shifts in marginal cost and benefits.

To demonstrate the applicability of the concept, we calibrate the standard model to the 2014 Mongolian SAM and consider a shock in the form of a 25% increase in the world price of export coal. We show that the standard PEP-1-1 model can be
derived as a special case of our model when the cost of depreciation with respect to capital utilization is too sensitive. For a given elasticity value governing the sensitivity of capital depreciation, we find that endogenous capital utilization amplifies the impact of the shock more than the standard model does. We also introduce a practical method to simulate the expanded model when the interior solution of the model exceeds the physical limit of the utilization rate.

This extension of CGE models for short-run analysis could be considered by other researchers, provide a rationale for considering endogenous capital in short-run closure for applied work and call for more empirical research on parameter estimation.

Acknowledgements

This work was carried out as part of an institutional support project with financial and scientific support from the Partnership for Economic Policy (PEP) and funding from the United Kingdom’s Department for International Development (DFID or UK Aid) and the Government of Canada through the International Development Research Center (IDRC).

The authors thank their PEP network colleagues as well as three anonymous referees and the editors of the JGEA for their comments.
References

Baxter, M., and D. D. Farr (2005). Variable capital utilization and international business cycles. *Journal of International Economics* 65, 335-347.

Beaulieu, J. J., and J. Mattey (1998). The Workweek of Capital and Capital Utilization Manufacturing. *Journal of Productivity Analysis* 10, 199-223.

Burnside, C., and M. Eichenbaum (1996). Factor-Hoarding and the Propagation of Business-Cycle Shocks. *American Economic Review* 86, 1154-1174.

Christiano, L. J., M. Eichenbaum and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1-44.

Decaluwé, B., A. Lemelin, V. Robichaud and H. Maisonnave (2013). “The PEP Standard Single-Country Static CGE model”, Partnership for Economic Policy.

Dixon, P. B., and M. T. Rimmer (2002). Dynamic, General Equilibrium Modelling for Forecasting Policy: A Practical Guide and Documentation of MONASH. North-Holland.

Dixon, P. B., and M. T. Rimmer (2010). Simulating the U.S. Recession With or Without the Obama Package: The Role of Excess Capacity. General Paper No. G-193.

Greenwood, J., Z. Hercowitz and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review* 78, 402-417.

King, R. G., and S. T. Rebelo (1999). Resuscitating real business cycles, in B. Taylor and M. Woodford (eds.). *Handbook of Macroeconomics* 1B, 927-1007.

Lemelin, A., V. Robichaud and B. Decaluwé (2010). “What can a CGE say about the housing bubble burst crisis and global imbalances?” Partnership for Economic Policy.

Lofgren, H., M. Cicowiez and C. Diaz-Bonilla (2013). MAMS – A Computable General Equilibrium Model for Developing Country Strategy Analysis. *Handbook of Computable General Equilibrium Modeling*, Elsevier.

Prescott, E. C. (1986). Theory Ahead of Business Cycle Measurement. *Carnegie-Rochester Conference Series on Public Policy* 15, 11-44.

Shapiro, M. D. (1986). Capital Utilization and Capital Accumulation: Theory and Evidence. *Journal of Applied Econometrics* 1(3), 211-234.

Smets, F., and R. Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97(3), 586-606.
Appendix

Figure A1 describes the marginal cost (MC) and marginal benefit (MB) curves of the capital utilization rate in the metal ores sector for an illustration purpose. In doing so, we depict them at the initial points with $KDO = 20,097,587; LDO = 661.960; VAO = 2,631,327; UO = 1; PVAO = 1$. We set $\varphi = 1$ (MC-1) and $\varphi = 0.5$ (MC-2). We consider a range of values for $\sigma^V_A$ – 0.3 (MB-1), 0.8 (MB-2), 1.1 (MB-3), 3.0 (MB-4), 5.0 (MB-5) and 10.0 (MB-6) which gives different values for $\rho^V_A, \beta^V_A$ and $B^V_A$. Note that all intercepts at $UO = 1$, the MB curve gets flatter as $\sigma^V_A$ gets larger and the MC curve gets steeper as $\varphi$ increases.

![Figure A1. Marginal cost and benefit curves of the capital utilization rate](image-url)
Table A1. Macro SAM (% of GDP)

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | Labor | 0.4 | 44.8 |     |     |     |     |     |     |     |     |     |     |     |
| 2 | Capital | 0.0 | 45.3 |     |     |     |     |     |     |     |     |     |     |     |
| 3 | Households | 43.9 | 39.0 | 10.4 |     |     |     |     |     |     |     |     |     |     |
| 4 | Government |     |     |     | 13.4 | 1.6 | 7.7 | 0.3 | 0.5 | 0.0 | 28.2 |     |     |     |
| 5 | TD  | 13.4 |     |     |     |     |     |     |     |     |     |     |     |     |
| 6 | TM  |     |     |     |     |     |     |     |     | 1.6 |     |     |     |     |
| 7 | TI  |     |     |     |     |     |     |     |     | 7.7 |     |     |     |     |
| 8 | ROW | 1.4 | 6.3 | 1.5 | 0.8 |     |     |     |     |     |     |     |     |     |
| 9 | Sectors |     |     |     |     | 137.7 | 48.1 |     |   | 185.8 |     |     |     |     |
| 10 | Commodities | 56.6 | 13.0 |     |     |     |     |     |     |     |     |     |     |     |
| 11 | Export |     |     |     |     |     |     |     |     |     |     |     |     | 51.6 |
| 12 | Investment | 19.6 | 4.1 |     |     |     |     |     |     |     |     |     |     | 35.2 |
| 13 | VSTK |     |     |     |     |     |     |     |     |     |     |     |     | 6.6 |
| 14 | TOTAL | 45.3 | 45.3 | 95.7 | 28.2 | 13.4 | 1.6 | 7.7 | 66.4 | 185.8 | 220.1 | 51.6 | 35.2 | 6.6 |

Notes: TD denotes direct taxes, TM is import duties, TI is other indirect taxes, ROW stands for the rest of the world and VSTK denotes stock variations.

Table A2. Production structure (%)

| Sector           | Labor | Capital | Value added | Value added/total | Intensity |
|------------------|-------|---------|-------------|-------------------|-----------|
|                  |       |         |             |                   | Labor     | Capital |
| Crops            | 1.3   | 1.7     | 1.5         | 41.6              | 43.1      | 56.9    |
| Livestock        | 23.7  | 3.2     | 13.4        | 77.6              | 87.9      | 12.1    |
| Domestic coal    | 0.3   | 0.5     | 0.4         | 27.9              | 37.5      | 62.5    |
| Export coal      | 0.8   | 1.4     | 1.1         | 27.9              | 37.5      | 62.5    |
| Metal ores       | 6.6   | 19.6    | 13.1        | 40.7              | 25.2      | 74.8    |
| Other mining     | 1.0   | 5.3     | 3.2         | 37.1              | 16.0      | 84.0    |
| Manufacturing    | 4.9   | 14.5    | 9.7         | 36.4              | 25.0      | 75.0    |
| Electricity      | 2.4   | 1.6     | 2.0         | 36.6              | 59.6      | 40.4    |
| Construction     | 4.2   | 5.7     | 4.9         | 22.0              | 42.1      | 57.9    |
| Trade            | 17.5  | 8.0     | 12.7        | 64.7              | 68.5      | 31.5    |
| Transportation   | 7.9   | 8.0     | 8.0         | 43.5              | 49.5      | 50.5    |
| Accommodation    | 1.2   | 0.7     | 1.0         | 38.6              | 63.7      | 36.3    |
| Financial activities | 3.0 | 7.3     | 5.2         | 78.3              | 29.0      | 71.0    |
| Public administration | 7.3 | 1.8     | 4.5         | 59.9              | 80.4      | 19.6    |
| Education        | 8.0   | 1.9     | 5.0         | 76.5              | 80.6      | 19.4    |
| Health           | 3.5   | 0.6     | 2.1         | 60.6              | 85.0      | 15.0    |
| Other services   | 6.2   | 18.4    | 12.3        | 61.1              | 25.2      | 74.8    |
| Total            | 100.0 | 100.0   | 100.0       |                   | 49.7      | 50.3    |
Table A3. Trade structure (%)

| Commodity      | Export share | Import share | Export intensity<sup>22</sup> | Import penetration |
|----------------|--------------|--------------|------------------------------|--------------------|
| Crops          | 0.3          | 0.7          | 4.5                          | 11.5               |
| Livestock      | 3.6          | 0.2          | 11.4                         | 0.8                |
| Domestic coal  | -            | 0.0          | -                            | 0.2                |
| Export coal    | 7.4          | -            | 100.0                        | -                  |
| Metal ores     | 53.2         | 0.0          | 99.4                         | 0.5                |
| Other mining   | 11.5         | 0.2          | 92.7                         | 16.4               |
| Electricity    | 0.0          | 1.9          | 0.1                          | 20.0               |
| Manufacturing  | 13.6         | 73.9         | 24.3                         | 67.2               |
| Construction   | 0.3          | 1.3          | 0.8                          | 3.6                |
| Trade          | -            | -            | -                            | -                  |
| Accommodation  | 2.9          | 5.2          | 62.4                         | 77.7               |
| Transportation | 3.6          | 3.4          | 10.9                         | 11.9               |
| Financial activities | 0.3 | 2.1 | 2.1 | 16.8 | |
| Public administration | - | - | - | - |
| Education      | 0.1          | 1.4          | 0.9                          | 11.7               |
| Health         | 0.0          | 0.8          | 0.5                          | 13.5               |
| Other services | 3.1          | 9.0          | 6.8                          | 19.6               |
| Total          | 100.0        | 100.0        |                              |                    |

Table A4. Domestic demand structure (%)

| Commodity       | Household consumption | Government consumption | Intermediate consumption | Margin | GFCF | Stock variation | Total demand |
|-----------------|-----------------------|------------------------|--------------------------|--------|------|----------------|--------------|
| Crops           | 42.7                  | -                      | 57.2                     | -      | 0.0  | 0.1            | 100.0        |
| Livestock       | 19.0                  | -                      | 34.4                     | -      | 23.6 | 23.0           | 100.0        |
| Domestic coal   | 11.2                  | -                      | 96.8                     | -      | 9.9  | -              | 100.0        |
| Metal ores      | -                     | -                      | 63.9                     | -      | -    | 36.1           | 100.0        |
| Other mining    | 1.0                   | -                      | 95.8                     | -      | -    | 3.2            | 100.0        |
| Electricity     | 8.6                   | -                      | 91.4                     | -      | -    | -              | 100.0        |
| Manufacturing   | 35.6                  | -                      | 50.8                     | -      | 9.9  | 3.7            | 100.0        |
| Construction    | 0.3                   | -                      | 26.0                     | -      | 73.6 | -              | 100.0        |
| Trade           | -                     | -                      | 100.0                    | -      | -    | -              | 100.0        |
| Accommodation   | 66.9                  | 0.2                    | 32.9                     | -      | -    | -              | 100.0        |

<sup>22</sup> Exports excluding taxes and margins, i.e., at base prices.
Table A4. Domestic demand structure (%) (Continued)

| Commodity     | Household consumption | Government consumption | Intermediate consumption | Margin | GFCF | Stock variation | Total demand |
|---------------|-----------------------|------------------------|--------------------------|--------|------|----------------|--------------|
| Transportation| 30.9                  | 0.4                    | 54.6                     | 14.1   | -    | -              | 100.0        |
| Financial     | 14.6                  | -                      | 85.4                     | -      | -    | -              | 100.0        |
| Public        | 3.3                   | 88.9                   | 7.7                      | -      | -    | -              | 100.0        |
| Education     | 43.2                  | 53.8                   | 3.0                      | -      | -    | -              | 100.0        |
| Health        | 34.2                  | 59.2                   | 6.5                      | -      | -    | -              | 100.0        |
| Other services| 32.6                  | 4.8                    | 58.1                     | -      | 4.5  | -0.0           | 100.0        |

Notes: GFCF stands for gross fixed capital formation.

Table A5. Investment/savings structure (%)

| Source          | Allocation          | Allocation          |
|-----------------|---------------------|---------------------|
| Households      | 55.5                | Private investment  |
| Government      | 11.5                | Public investment   |
| Rest of the world| 32.9               | Changes in inventory|
| Total           | 100.0               | Total               |

Table A6. Household income and expenditures (%)

| Household income       | Household expenditures |
|------------------------|------------------------|
| Wages                  | 45.9                   | Consumption         |
| Capital income         | 40.8                   | Direct taxes        |
| Transfers from the     | 10.8                   | Transfers to the government|
| Transfers from the ROW | 2.5                    | Transfers to the ROW|
|                        |                        | Savings             |
| Total                  | 100.0                  | Total               |

Table A7. The government budget (%)

| Government revenue    | Government expenditures |
|-----------------------|-------------------------|
| Transfers from households| 16.7                   | Transfers to households|
| Direct taxes (TD)     | 47.3                    | Transfers to the ROW |
| Import duties (TM)    | 5.7                     | Public consumption   |
| Export taxes          | 0.0                     | Savings              |
| Net taxes on products (TI)| 27.4               |                      |
| Transfers from the ROW| 1.1                     |                      |
| Net taxes on production| 1.8                    |                      |
| Total                 | 100.0                   | Total               |
### Table A8. Boundary condition: Changes in production (% change from BAU)

|                     | Standard model | Model with variable capital utilization |
|---------------------|----------------|----------------------------------------|
|                     |                | $U_{exc}^{max} = 1$ | $U_{exc}^{max} = 1.5$ | No boundary |
| Crops               | 0.20           | 0.72                     | 1.60                     | 3.63         |
| Livestock           | 0.08           | 0.86                     | 1.97                     | 4.55         |
| Domestic coal       | 0.14           | 0.80                     | 2.17                     | 5.44         |
| Export coal         | 5.63           | 5.68                     | 57.41                    | 186.51       |
| Metal ores          | -0.12          | -0.89                    | -2.66                    | -6.62        |
| Other mining        | -0.02          | -0.42                    | -1.36                    | -3.48        |
| Manufacturing       | -0.11          | 0.61                     | 1.76                     | 4.50         |
| Electricity         | 0.33           | 0.32                     | 1.03                     | 2.87         |
| Construction        | 1.81           | 2.27                     | 5.26                     | 12.21        |
| Trade               | 0.89           | 1.08                     | 3.25                     | 8.51         |
| Transportation      | 0.70           | 0.89                     | 4.44                     | 13.18        |
| Accommodation       | -0.17          | 0.17                     | 0.29                     | 0.61         |
| Financial activities| 0.31           | 0.64                     | 2.21                     | 6.07         |
| Public administration| -0.32         | -0.05                    | -0.14                    | -0.32        |
| Education           | 0.50           | 0.75                     | 1.51                     | 3.24         |
| Health              | 0.43           | 0.64                     | 1.29                     | 2.79         |
| Other services      | 0.32           | 0.93                     | 2.63                     | 6.69         |

### Table A9. Boundary condition: Gross rental prices by sector (% change from BAU)

|                     | Standard model | Model with variable capital utilization |
|---------------------|----------------|----------------------------------------|
|                     |                | $U_{exc}^{max} = 1$ | $U_{exc}^{max} = 1.5$ | No boundary |
| Crops               | 1.11           | 1.16                     | 2.75                     | 6.47         |
| Livestock           | 2.10           | 1.49                     | 3.59                     | 8.54         |
| Domestic coal       | 2.41           | 1.39                     | 3.84                     | 9.80         |
| Export coal         | 154.78         | 156.34                   | 231.98                   | 368.98       |
| Metal ores          | -1.03          | -1.05                    | -3.17                    | -7.89        |
| Other mining        | -0.09          | -0.39                    | -1.35                    | -3.48        |
| Manufacturing       | 0.19           | 1.00                     | 2.92                     | 7.53         |
| Electricity         | 0.87           | 0.56                     | 1.74                     | 4.72         |
| Construction        | 3.39           | 2.83                     | 6.73                     | 15.95        |
| Trade               | 1.37           | 1.30                     | 3.92                     | 10.27        |
| Transportation      | 1.45           | 1.21                     | 5.57                     | 16.43        |

(Continued)
Table A9. Boundary condition: Gross rental prices by sector (% change from BAU)  
(Continued)

| Sector              | Standard 1 | Standard 2 | Standard 3 | Standard 4 |
|---------------------|------------|------------|------------|------------|
| Accommodation       | 0.32       | 0.40       | 0.97       | 2.33       |
| Financial activities| 1.21       | 1.02       | 3.43       | 9.36       |
| Public administration| 0.24      | 0.18       | 0.53       | 1.40       |
| Education           | 0.92       | 0.93       | 2.09       | 4.77       |
| Health              | 0.84       | 0.82       | 1.86       | 4.30       |
| Other services      | 1.37       | 1.40       | 4.03       | 10.36      |

Table A10. Boundary condition: Employment by sector (% change from BAU)

| Sector            | Standard model | Model with variable capital utilization | No boundary |
|-------------------|----------------|----------------------------------------|-------------|
|                   |                | $U_{excoa}^{max} = 1$ | $U_{excoa}^{max} = 1.5$ |                 |
| Crops             | 0.49           | 0.87                                   | 1.93        | 4.38        |
| Livestock         | 1.25           | 1.18                                   | 2.71        | 6.27        |
| Domestic coal     | 0.38           | 0.85                                   | 2.30        | 5.78        |
| Export coal       | 20.45          | 20.66                                  | 75.60       | 206.78      |
| Metal ores        | -0.46          | -0.98                                  | -2.94       | -7.31       |
| Other mining      | -0.12          | -0.45                                  | -1.47       | -3.76       |
| Manufacturing     | -0.46          | 0.90                                   | 2.61        | 6.72        |
| Electricity       | 0.55           | 0.39                                   | 1.25        | 3.47        |
| Construction      | 4.34           | 3.04                                   | 7.06        | 16.54       |
| Trade             | 1.30           | 1.25                                   | 3.77        | 9.90        |
| Transportation    | 1.41           | 1.14                                   | 5.70        | 17.11       |
| Accommodation     | -0.27          | 0.20                                   | 0.35        | 0.72        |
| Financial activities| 1.06       | 0.92                                   | 0.30        | 8.85        |
| Public administration| -0.39 | -0.06                                  | -0.16       | -0.34       |
| Education         | 0.62           | 0.82                                   | 1.65        | 3.54        |
| Health            | 0.51           | 0.69                                   | 1.38        | 2.99        |
| Other services    | 1.30           | 1.37                                   | 3.90        | 10.01       |