Role of the $f_1(1285)$ state in the $J/\psi \to \phi KK^*$ and $J/\psi \to \phi f_1(1285)$ decays

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We study the role of the $f_1(1285)$ resonance in the decays of $J/\psi \to \phi KK^*$ and $J/\psi \to \phi f_1(1285)$. The theoretical approach is based on the results of chiral unitary theory where the $f_1(1285)$ resonance is dynamically generated from the $K^+\bar{K}^–$ c.c. interaction. In order to further test the dynamical nature of the $f_1(1285)$ state, we investigate the $J/\psi \to \phi KK^*$ decay close to the $KK^*$ threshold and make predictions for the ratio of the invariant mass distributions of the $J/\psi \to \phi KK^*$ decay and the $J/\psi \to \phi f_1(1285)$ partial decay width with all the parameters of the mechanism fixed in previous studies. The results can be tested in future experiments and therefore offer new clues on the nature of the $f_1(1285)$ state.

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I. INTRODUCTION

The $f_1(1285)$ resonance [$I^G(J^{PC}) = 0^+(1^{++})$] is an axial-vector state with mass $M_{f_1} = 1281.9 \pm 0.5 \text{ MeV}$ and total decay width $\Gamma_{f_1} = 24.2 \pm 1.1 \text{ MeV}$ [1]. This state is described as a $q\bar{q}$ state within the quark model [2,3]. On the other hand, the $f_1(1285)$ is also suggested to be a dynamically generated state made from the single channel $KK^*$ interaction in the chiral unitary approach [4]. As shown in Ref. [5], because the $f_1(1285)$ resonance has positive $G$ parity, it cannot couple to other pseudoscalar–vector channels. For reasons of parity it can also not decay into two pseudoscalar mesons. Thus, since the resonance is located below the $KK^*$ mass threshold, its observation is difficult in two body decays. Indeed, the main decay channels of the $f_1(1285)$ are $4\pi$ (branching ratio $= 33\%$), $\eta\pi\pi$ ($52\%$), and $\pi KK^*$ ($9\%$).

While Nature is probably more complicated and the $f_1(1285)$ state might have components of either type (see discussions in Ref. [6]), two comments are in order. First, the fact that states of different nature is possible does not mean that there should be a duplication of states with the same quantum numbers corresponding to each type of structure. The different structures mix and at the end it is a particular mixture what gives rise to the observed states. These features were well described in Refs. [7,8] for the $\sigma$ ($f_0(500)$) meson. One starts with a seed of $q\bar{q}$ and lets it couple to $\pi\pi$ components respecting unitarity of the $\pi\pi$ interaction. At the end, a physical state develops in which the original seed has been eaten up by the meson cloud, which becomes the dominant component of the wave function. The other comment is that, depending on the reaction, one or the other component will evidence itself more clearly, and in the present case, where we have a $KK^*$ produced at the end, it is quite clear that it is this component the one which will show up.

In Refs. [9,10], the decays of $f_1(1285) \to \eta\pi^0\pi^0$ and $f_1(1285) \to \pi KK$ were studied using the picture in which the $f_1(1285)$ is dynamically generated from the single channel $KK^*$ interaction. The theoretical predictions are compatible with the experimental measurements. Very recently, the production of the $f_1(1285)$ resonance in the reaction $K^-p \to f_1(1285)\Lambda$ within an effective Lagrangian approach was studied in Ref. [11] based on the results obtained in chiral unitary theory. The theoretical calculations are in agreement with the experimental data which provides further support for the molecular structure of the $f_1(1285)$ state.

On the experimental side, in Refs. [12,13], the decay of $J/\psi \to \phi f_1(1285)$ was studied from the $J/\psi \to \phi(\pi^+\pi^-)$ and $J/\psi \to \phi\eta\pi^+\pi^-$ decays by the DM2 Collaboration, while in Ref. [14], the branching fraction of $J/\psi \to \phi KK^*$ was measured from the decay of $J/\psi \to \phi KK\pi$ by the BES Collaboration. Because the $J/\psi$ and the $\phi$ mesons have quantum numbers $0^+(1^{--})$ and $0^-(1^{--})$, respectively, the decay $J/\psi \to \phi KK^*$ constitutes the ideal reaction to look for the $f_1(1285)$ state, with quantum numbers $0^+(1^{++})$, coupling to an $s$ wave $KK^*$ pair. However, since the $f_1(1285)$ is located below the $KK^*$ threshold, it will contribute to the region close to the threshold of $KK^*$.

In the present work, following the formalism of Ref. [6], we study the decays of $J/\psi \to \phi KK^*$ and $J/\psi \to \phi f_1(1285)$ with the picture that the $f_1(1285)$ resonance is dynamically generated from the single channel $KK^*$ interaction.
This paper is organized as follows. In Sec. II we discuss the formalism and the main ingredients of the model. In Sec. III we present our main results and, finally, a short summary and conclusions are given in Sec. IV.

II. FORMALISM

We want to study the role of the $f_1(1285)$ state, which is dynamically generated by the $K$ and $K^*$ interaction, in the $J/\psi \rightarrow \phi K^*$ decay. In the chiral unitary approach of Ref. [5], the $f_1(1285)$ resonance was obtained by solving the Bethe-Salpeter equation in the $K K^*$ channel to obtain the scattering amplitude

$$ t = \frac{v}{1 - vG}, \tag{1} $$

where $v$ is the $KK^* \rightarrow KK^*$ transition potential and $G$ is the loop function for the propagators of the $K$ and $K^*$ mesons given in Ref. [6]. The $v$ and $G$ depend on the invariant mass $M_{inv}$ of the $KK^*$ system, and hence the scattering amplitude $t$ is also dependent on $M_{inv}$. The loop function $G$ is divergent, and it can be regularized both with a cutoff prescription or with dimensional regularization in terms of a subtraction constant [19]. In this work we will make use of the cutoff regularization scheme, which introduces a cutoff parameter $q_{max}$. The cutoff is tuned to get a pole of the $t$ matrix at the mass (1281.3 MeV) of the $f_1(1285)$. This provides the coupling $g_{f_1} = 7555$ MeV of the resonance to the $KK^*$ channel (see more details in Ref. [6]). With the explicit expressions for $v$ and $G$ taken from Ref. [6], we obtain a good description of the $f_1(1285)$ resonance using a cutoff $q_{max} = 990$ MeV, as in Ref. [6].

For $J/\psi \rightarrow \phi KK^*$, the decay mechanism is shown in Fig. 1. To take into account the final state interaction of the $KK^*$ pair, we have to consider the resumation of the diagrams shown in the figure.

According to the diagrams in Fig. 1, the transition matrix for the process $J/\psi \rightarrow \phi KK^*$ can be given by

$$ T_{J/\psi \rightarrow \phi KK^*} = V_P C_s \left[ 1 + G(M_{inv}^2 t(M_{inv}) \right] = V_P C_s \frac{t(M_{inv})}{v(M_{inv})}, \tag{2} $$

where the last equality follows from Eq. (1). The $V_P$ and $C_s$ are the bare production vertex and the spin structure (the spin of $K^*$ together with the one of the $\phi$ must give the spin of $J/\psi$: $1 \otimes 1 \rightarrow 1$) factor for $J/\psi \rightarrow \phi KK^*$. We assume that this bare vertex is of a short range nature, i.e., just a coupling constant in the field theory language.

The spin structure of the $J/\psi$, $K^*$, and $\phi$ coupling can be written as

$$ C_s = \epsilon_{ijk} \epsilon_i(J/\psi) \epsilon_j(\phi) \epsilon_k(K^*). \tag{3} $$

Summing and averaging $C_s^2$ over final and initial polarizations of the vector mesons we find

$$ \sum \sum C_s^2 = \frac{2}{3} \left( 3 + \frac{p_{\phi}^2}{m_{\phi}^2} + \frac{p_{K^*}^2}{m_{K^*}^2} \right), \tag{4} $$

where $p_{\phi}$ and and $p_{K^*}$ are the $\phi$ and $K^*$ momenta in the $J/\psi$ rest frame, respectively,

$$ p_{\phi} = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_{\phi}^2, M_{inv}^2)}{2M_{J/\psi}}, \tag{5} $$

$$ p_{K^*} = \frac{\lambda^{1/2}(M_{J/\psi}^2, m_{K^*}^2, M_{inv}^2)}{2M_{J/\psi}}, \tag{6} $$

where $M_{\phi K}$ is the invariant mass of $\phi K$ system, and $\lambda(x,y,z)$ is the Kähler or triangle function.

We can easily get the $KK^*$ invariant mass spectrum for the $J/\psi \rightarrow \phi KK^*$ as [20–22]:

$$ \frac{d\Gamma_{J/\psi \rightarrow \phi KK^*}}{dM_{inv}} = \frac{V_P^2}{(2\pi)^3} \frac{M_{inv}}{v(M_{inv})} \left| t(M_{inv}) \right|^2 \times \int_{M_{inv}}^{M_{max}} \sum_{M_{\phi K}} C_s^2 M_{\phi K} dM_{\phi K}. \tag{7} $$

For a given value of $M_{inv}$, the range of $M_{\phi K}$ is defined as,

$$ M_{\phi K}^{max} = \sqrt{(E_K + E_\phi)^2 - \left( \sqrt{E_K^2 - m_K^2} - \sqrt{E_\phi^2 - m_\phi^2} \right)^2}, $$

$$ M_{\phi K}^{min} = \sqrt{(E_K + E_\phi)^2 - \left( \sqrt{E_K^2 - m_K^2} + \sqrt{E_\phi^2 - m_\phi^2} \right)^2}, $$

where $E_K = (M_{inv}^2 - m_{K^*}^2 + m_K^2)/2M_{inv}$ and $E_\phi = (M_{J/\psi}^2 - M_{inv}^2 - m_{\phi}^2)/2M_{inv}$. We are interested in the production of the $f_1(1285)$ resonance, the relevant mechanism is depicted diagrammatically in Fig. 2 and we have

$$ T_{J/\psi \rightarrow \phi f_1(1285)} = V_P C_s' G(M_{f_1}, g_{f_1}), \tag{8} $$

where the spin factor $C_s'$ is easily obtained. We must recall that the coupling of $f_1(1285)$ to $KK^*$ c.c. is given by $g_{f_1} \epsilon_i(f_1) \epsilon_i(K^*)$. Contracting the two $\epsilon(K^*)$ in the $K^*$ propagator in Fig. 2 we have

$$ C_s' = \epsilon_{ijk} \epsilon_i(J/\psi) \epsilon_j(\phi) \epsilon_k(f_1). \tag{9} $$

Then, the partial decay width of $J/\psi \rightarrow \phi f_1(1285)$ is given by

$$ \Gamma_{J/\psi \rightarrow \phi f_1(1285)} = \frac{V_P^2 G^2(M_{f_1}, g_{f_1})^2 p_\phi^2}{M_{J/\psi}^2} \sum \sum C_s'^2, \tag{10} $$

with

$$ \sum \sum C_s'^2 = \frac{2}{3} \left( 3 + \frac{p_{\phi}^2}{M_{f_1}^2} + \frac{p_{\phi}^2}{m_{\phi}^2} \right). \tag{11} $$
and $p'_\phi$ is the $\phi$ meson momentum obtained in the $J/\psi$ rest frame which is

$$
p'_\phi = \frac{\lambda^{1/2}(M^2_{J/\psi}, m^2_\phi, M^2_{f_1})}{2M_{J/\psi}}.
$$

The chiral theory cannot provide the value of the constant $V_P$ in Eqs. (7) and (10), however, if we divide $d\Gamma/dM_{inv}$ by $\Gamma_{J/\psi \rightarrow \phi f_1(1285)}$, the constant $V_P$ is cancelled, and we can make precise predictions for the ratio $R_\Gamma$ as,

$$
R_\Gamma = \frac{d\Gamma_{J/\psi \rightarrow \phi K K^*}/dM_{inv}}{\Gamma_{J/\psi \rightarrow \phi f_1(1285)}}.
$$

This ratio is relevant because it has no free parameters (all the parameters are fixed by previous works) and, thus, it is a prediction of the theory. The shape, as well as the absolute values of the ratio $R_\Gamma$ for the $K K^*$ mass distribution, can be compared with the experimental measurements.

III. NUMERICAL RESULTS AND DISCUSSION

In Fig. 3 the numerical results of $R_\Gamma$ as a function of the invariant mass $M_{inv}$ of the $K K^*$ system are shown. The solid curve stands for the theory prediction and the dotted curve stands for the phase space. For evaluating the contributions of the phase space, we replace $t(M_{inv})/v(M_{inv})$ of Eq. (2) by a constant, thus removing any effect of the $M_{inv}$ dependence of the $f_1(1285)$ resonance. Then we tune this constant such that the $M_{inv}$ integrated $R_\Gamma$ in the range of energies from the $K K^*$ threshold to 1.7 GeV is the same as the one evaluated with the explicit resonance formalism.

In addition, in Fig. 3 we also show the results which are obtained without considering the spin structure factor by the dashed curve in Fig. 3. We see that the structure factor gives a small effect to our predictions and could be neglected.

We see a clear threshold enhancement in Fig. 3 which is caused by the contributions of the $f_1(1285)$ state below threshold, which is dynamically generated by the $K K^*$ interaction. The theoretical predictions can be tested by future experiments.

Actually, the range of the invariant mass of $K K^*$ in the decay of $J/\psi \rightarrow \phi K K^*$ is from the threshold of $K K^*$ up to 2.077 GeV ($M_{J/\psi} - m_\phi = 2077$ MeV), however, we cannot go so far because the chiral theory works well about 200 – 300 MeV from the threshold, hence we con-
sider only the range of 300 MeV above the $\bar{K}K^*$ threshold as shown in Fig. 3.

On the other hand, experimentally we have, from Ref. [1],

\[ Br(J/\psi \to \phi\bar{K}K^*) = (2.18 \pm 0.23) \times 10^{-3}, \]  
\[ Br(J/\psi \to \phi f_1(1285)) = (6.00 \pm 3.16) \times 10^{-4}. \]  

Note that we have corrected the branching ratio $Br(J/\psi \to \phi f_1(1285)) = (2.6 \pm 0.5) \times 10^{-4}$ quoted in the PDG which we found is misquoted. In Ref. [1], the PDG information is obtained, the peak around 1297 MeV of the $\eta\pi^+\pi^-$ mass distribution was attributed to the $f_1(1285)$ with a width of 10 \pm 8 MeV. They obtain $Br(J/\psi \to \phi X(1297)) \times Br(X(1297) \to \eta\pi^+\pi^-) = (2.1 \pm 0.5 \pm 0.4) \times 10^{-4}$. Taking now into account that $Br(f_1(1285) \to \eta\pi^+\pi^-) = (35 \pm 15)\%$ from the PDG, we obtain $Br(J/\psi \to \phi f_1(1285)) = (6.00 \pm 3.16) \times 10^{-4}$ as shown in Eq. (15).

Note that we do not use the value of $Br(J/\psi \to \phi f_1(1285)) = (3.2 \pm 0.6 \pm 0.4) \times 10^{-4}$ presented in Ref. [17], which was obtained from the decay of $J/\psi \to \phi 2(\pi^+\pi^-)$. Very recently, in Ref. [22], the branching fraction $Br(J/\psi \to \phi f_1(1285), f_1(1285) \to \eta\pi^+\pi^-)$ was measured, with the result $(1.2 \pm 0.06 \pm 0.14) \times 10^{-4}$. Taking this value into account, and the $Br(f_1(1285) \to \eta\pi^+\pi^-)$ used before, we get $Br(J/\psi \to \phi f_1(1285)) = (3.43 \pm 1.53) \times 10^{-4}$. This value is consistent within errors with what we have obtained before.

From Eqs. (14) and (15) we obtain

\[ R = \frac{Br(J/\psi \to \phi\bar{K}K^*)}{Br(J/\psi \to \phi f_1(1285))} = 3.6 \pm 2.0. \]  

One might think we should compare our theoretical result, $R \approx (M_{\bar{K}K^*} - M_{KK^*}) R_t \Gamma_{\text{inv}}$, to the experimental result in Eq. (16), but, as discussed before, we take the $\bar{K}K^*$ to $\bar{K}K^*$ scattering amplitude $t(M_{\text{inv}})$ from the chiral unitary approach, and we can not go too far from the $\bar{K}K^*$ threshold. Furthermore, there could be also other contributions from higher mass states with spin-parity $J^P = 1^+$ and $2^+$ at higher invariant mass region of $\bar{K}K^*$. These higher states will not contribute too much to the lower energy region and hence will not affect our predictions here. On the other hand, note that the experimental results of Ref. [17] were obtained in the 1980s and only few signal events were observed. Further improvement can be done in the future at BESIII or BelleII. The future experimental observation of the mass distribution $R_t$ would provide very valuable information on the mechanism of the $J/\psi \to \phi\bar{K}K^*$ decay.

IV. SUMMARY

In summary, we have studied the decays of $J/\psi \to \phi\bar{K}K^*$ and $J/\psi \to \phi f_1(1285)$ with the theoretical approach which is based on results of chiral unitary theory where the $f_1(1285)$ resonance is dynamically generated from the $K^*\bar{K}$ - $c.c.$ interaction. The ratio $R_t = \frac{\Gamma_{J/\psi \to \phi\bar{K}K^*}/\Gamma_{\text{inv}}}{\Gamma_{J/\psi \to \phi f_1(1285)}}$ as a function of invariant mass $M_{\text{inv}}$ of $\bar{K}K^*$ is predicted. A clear threshold enhancement in Fig. 8 compared with the phase space appears, which is caused by the presence of the $f_1(1285)$ state below threshold. The experimental observation of this mass distribution would then provide very valuable information to check our predictions and the basic nature of the $f_1(1285)$ resonance.

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