Characterizations of the medium in jet quenching calculations.

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1 Introduction

One of the major discoveries of the heavy-ion program at the Relativistic Heavy-Ion Collider (RHIC) has been the observed suppression of high transverse momentum hadrons when compared to the yield of similar hadrons in p-p collisions scaled up by the expected number of binary collisions [12]. In p-p collisions, such hadrons are produced in the fragmentation of high pt jets produced in hard scatterings. The presence of a dense medium in the space-time development of the partonic shower from such jets and in turn leads to a medium modification of the fragmentation distribution of the hadrons [29].

The presence of a hard jet introduces a large energy scale within the process and allows for a calculation of the medium modification using the methods of perturbative QCD (pQCD). Following the early attempts of Baker-Dokshitzer-Mueller-Peigné-Schi and Zakharov (BDMS) [5, 6, 7, 8, 11], such calculations have grown in both sophistication and in the number of di erent observables that they are applied to. The majority of current approaches to the energy loss of light partons may be divided into four major schemes that are often referred to by the names of the original authors:

- Higher Twisted scheme (HT) [2, 14, 15, 16, 17]
- Path integral approach to the opacity expansion by Aamodt, Salgado and Wiedemann (BDMS) [8, 11]
- Finite temperature ed theory approach by Aamodt, Moore and Yndurain (AMY) [24, 25, 26, 27, 28]
- Reaction Operator approach to the opacity expansion by Gyulassy, Levai and Vitev (GLV) [29, 30, 31, 32, 33]

All these schemes utilize slightly di erent approximations regarding the various scales involved in the calculation and so what di erent quantitative pictures of the medium. It will be demonstrated in the companion publication of Ref. [34], that, using these di erent formalisms to compute the medium modification of high pt hadrons in an identical medium leads to rather similar predictions for experimental observables. In these proceedings, we outline some of the main differences between the theoretical formalisms of the di erent schemes in turn.

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2 Hard scattering and the medium modi ed fragmentation function

In the collision of two heavy-ions, there occasionally occurs a hard scattering between two initial partons which leads to two back-to-back outgoing partons with larger transverse momentum. These encounter multiple scattering in the produced medium leading to a medium modification of the distribution of hadrons emanating from these partons. In the computation of this medium modification, all schemes utilize a factorized approach where the leading order cross section to produce a hadron h with transverse momentum p_T (rapidity between y and y + dy) may be expressed as a convolution of initial nuclear structure functions G_A^A (x_A) G_P^P (x_P), initial state nuclear e ections such as shadowing and Cronin e ections are understood to be included to produce partons with momentum fractions x_h, x_p, a hard partonic cross section to produce a high transverse
m onentum parton c with a transverse m onentum $p$ and a m edium m odi ed fragm entation function for the nal hadron $D^h_c(z)$, 
\[
\frac{d^2 Q}{dydp} = \frac{1}{d} \int dx_a dx_b G^h_a(x_a)G^h_b(x_b) 
\]

In the vicinity of m di-rapidity, $z = p_T = \hat{p}$ and $\hat{t} = (p_x)^2$ ($P$ is the average incom ing m onentum of a nu-
cleon in nucleus A). The entire e ect of energy loss is concentrated in the calculation of the m odi cation to the fragm entation function. The four m odels of energy loss are in a sense four schemes to estimate this quantity from perturbative QCD calculations.

While the terminology (m edium m odi cation) used to describe the change in the fragm entation function seems to indicate that the m edium has in uenced the actual pro cess of the form ation of the nal hadrons from the partonic cloud, this is not the case. All computations simply demonstrate the change in the gluon radiation spectrum from a hard parton due to the presence of the m edium. The hadronization of the hard parton is always assum ed to oc cur in the vacuum after the parton, with degraded energy, has escaped from the m edium. Note that some of the hard gluons radiated from the hard parton will also encounter similar m edium effects in m edium and may endure vacuum hadronization after escaping from the m edium. Differences between form alisms also arise in the inclusion of hadrons from the fragm entation of such sub-leading gluons; whereas in approaches which com pute the change in the distribution of nal partons (such as AMY) or the change in the distribution of nal hadrons (such as HT), hadrons from sub-leading gluons are implicitly included, for form alisms which com pute the energy loss of the leading parton (such as ASW), do not include such sub-leading corrections.

To better appreciate the approxim ation schemes, one may introduce a set of scales (see Fig. 1): $E$ or $p^+$, the forward energy of the jet; $Q^2$, the vir tuality of the initial jet-parton; $m$, the m onentum scale of the m edium and $L$, its spatial extent. Most of the di erences between the various schemes may be reduced to the di erent relations between these various scales assum ed by each scheme as well as by how each scheme treats or approximates the structure of the m edium. In all schemes, the forward energy of the jet far exceeds the m edium scale, $E$. 

3. Single gluon emission and scattering in the m edium

The rst step in energy loss c alculations is to com pute the e ect of a single gluon em ission o a hard jet in the m edium. The m ajor theoretical di erences between the various schemes arise in this calculation. It is in this step that di erent assum ptions regarding the m edium (in di erent form alisms) are introduced. In the next section, the

![Fig. 1. A schematic picture of the various scales involved in the m edium c alculations of jets in dense m atter.](image)

Single emission kernel will be repeated to com pute the e ect of multiple emissions. In most cases, this involves a certain phenomenological picture and also introduces further di erences between the di erent approaches.

3.1 Higher tw ist approach

The origin of the higher tw ist (HT) approxim ation schem e lies in the calculations of m edium enhanced higher tw ist corrections to the total cross-section in D eep-Inelastic Scattering (DIS) o a large nucleus [35]. In those calculations, the authors com puted a certain class of power corrections to the total leading tw ist cross sections, which, though suppressed by powers of the hard scale $Q^2$, are enhanced by the extent of the m edium. In the case of high $p_T$ hadron production one identi es and resums contributions to the single hadron inclusive cross section.

One presumes that the produced jet has a very large forward energy $E$ which is much larger than its vir tuality $Q$ (which limits the transverse m onentum of the radiated gluon, $k_T$), which in turn is much larger than the characteristic m onentum scale in the m edium, i.e., $E >> k_T$. This hierarchy is then applied to the com putation of multiple Feynman diagrams such as the one in Fig. 2. This diagram represents the process of a hard virtual quark produced in a hard collision, which then radiates a gluon and then scatters a soft m edium gluon with transverse m onentum $q_T$ prior to exiting the m edium and fragm ent into hadrons. Even at the order considered, there exist various other contributions which involve scattering of the initial quark, or the soft gluon off, prior to radiation as well as scattering of the radiated gluon itself. All such contributions are com bined coherently to calculate the m edium c alculation to the fragm entation function directly.

The hierarchy of scales allows one to use the collinear approxim ation to factorize the fragm entation function and its m odi cation from the hard sc attering cross section. Thus, even though such a m odi ed fragm entation function is derived in DIS, it may be generalized to the kinematics of a heavy-ion collision. Diagrams where the outgoing parton scatters o the m edium gluons, such as those in Fig. 2, produce a m edium dependent additive contribution to the
vacuum fragmentation function, which may be expressed as,

$$D_j(z_iq^2) = \frac{Z_q^2}{2} \frac{d^3x}{k_t^2} \frac{z_i}{2} \int_0^1 P_{ji}(x;P \cdot k_t^2)D_j(z_i) \frac{z_i}{X} 5; (2)$$

In the above equation, \( P_{ii} \) represents the medium-modified splitting function of parton \( i \) into parton \( j \) where a medium-on-impact fraction \( x \) is left in parton \( j \). The new medium-on-impact fraction \( x_i = k_r^2 = \frac{P \cdot p' \times (1 - x)}{k_t^2} \), where the radiated gluon or quark carries away a transverse medium-on-impact \( k_r^2 \). \( P \) is the incoming medium-on-impact of a nucleon in the nucleus and \( p \) is the medium-on-impact of the virtual photon. The medium-modified splitting functions \( S \) may be expressed as a product of the vacuum splitting function \( P_{ii} \) and a medium-dependent factor,

$$P_{ii} = P_{ii} \frac{Z_r^2}{2} \frac{d^3x}{C_k} \frac{1}{k_t^2} f(x_i); (3)$$

where \( C_k \) is the representation-dependent Casimir and \( N_c \) is the number of colors. The medium-on-impact of the soft gluons is represented by the factor \( k_r^2 \). The distance \( \Delta s \) is the distance between the origin of the jet and the location of its scattering, which is limited by the length of the medium \( L \). The function \( f(x_i) \) depends on the number of scatterings per radiated gluon included and encodes the medium-interference effects such as the Landau-Pomeron exchange. The jet encounters multiple scattering, which is given as \( [1] \)

$$q(x) = \frac{4}{2n_c} C_R \left( \frac{Z}{N_c} \right) \frac{d^2}{d^2k_t} \frac{1}{(2 \pi)^2} f(x_i); (4)$$

The opacity of the medium \( \gamma \) is a function of the jet energy \( \gamma \). The medium-on-impact of a nucleon in the medium under consideration is limited by space-time location. The expectation is to be taken in the medium under consideration. Any space-time dependence is essentially included in the implicit expectation. The gluons which contribute to \( q \) do not have to be the entropy carriers of the system. In applications to cold nuclear matter, these gluons constitute the virtual gluon cloud inside the nucleons. In the case of a deconfined quark-gluon plasma, these may be the entropy carrying gluons or virtual excitations within these degrees of freedom. Where gluons in the jet scatter off depend on the scale of the hard jet. It is in the mediumly obvious from Eq. [6] that \( q \) is a function of the jet energy \( p \). Note that \( p \) is not integrated out. The actual dependence on \( p \) depends on the medium in question. In the case of cold nuclear media, a quark-gluon plasma, the dependence is logarithmic. There is also a logarithmic dependence on the virtuality of the jet, which sets in due to radiation corrections to the definition in Eq. [4]. A box, as demarcated in Ref. [15], \( q \) may even possess a tensorial structure if the medium is not isotropic. In the calculations of the current manuscript, both the dependence on the energy and virtuality of the jet will be ignored. The medium will be assumed to be isotropic. The value of \( q \) quoted should thus be considered as approximations to the full functional form.

### 3.2 Opacity expansion approach

Unlike the higher twist scheme, which is set up to directly calculate the full distribution of hadrons, opacity expansion approaches such as the Glendenning-Levay-Lavignac (GLLV) scheme \([26, 30, 31]\), and the Abreu-Gaitero-Pineda scheme (AGAP) scheme \([19, 28, 29] \) were constructed primarily to deal with the problem of energy loss of the leading parton in dense deconfined matter. Both these schemes assume that the medium is composed of heavy almost static color scattering centers which are well separated, in the sense that the mean free path of a jet is much greater than the color screening length of the medium \([4]\). The opacity of the medium \( n \), which constitutes the expansion parameter of these calculations, quantifies the number of scatterings per collision and is given by \( n = L_c / \gamma \), where \( L_c \) is the thickness of the medium. The difference between the two approaches of the GLLV and the AGAP arises from how these tend to expand in \( n \). In the GLLV formalism, one constructs a recursive operator expansion in opacity, whereas in the AGAP approach such a path integral over opacity is formalized. The solution of the recursive operator approach in the GLLV allows for an order-by-order expansion in opacity. The path-integral in the AGAP approach has been solved analytically in two limits: the one-scattering approximation, equivalent to a zeroth-order in opacity calculation in the GLLV approach and...
in the multiple scattering approximation where all orders in opacity have been resummed. In this article (as well as in the companion [34] where results of calculation will be compared with experimental data), the focus will lie on the path integral approach of ASW.

The path integral approach for the energy loss of a hard jet propagating in a colored medium was first introduced in Ref. [8]. It was later dem onstrated to be equivalent to the well-known BDMPS approach [5, 6, 7] in the multiple scattering limit. ASW represents the current most widespread, variant of this approach. In this scheme, a hard, on-shell parton traversing a dense medium full of heavy scattering centers will undergo multiple transverse scatterings of order \( p^\alpha \). It will in the process split into an outgoing parton and a radiated gluon which will also scatter multiply in the medium. The radiated gluon, induced by the multiple scattering, has a transverse momentum \( k_T \) (determined from the HT approach). The propagation of the incoming (outgoing) partons as well as that of the radiated gluon in this background color field may be expressed in terms of effective Green's functions \( G(q; x, y) \) (for quark or gluon) which obey the ob jective Dyson-Schwinger equation,

\[
G(q; x, y) = G_0(q; x, y) + i \int \frac{d^4q'}{Z(q')} G(q') A_0(x, y) G(q'; y, z)^2 G_0(q', z; z_0)^2 G(z; z_0)
\]

where \( G_0 \) is the free Green's function and \( A_0 \) represents the color potential of a scattering center in the medium. The solution for the above interacting Green's function involves a path ordered Wilson line which follows the potential from the location \( (x, y) \) to \( (z, z_0) \). Expanding the expression for the radiation cross section to order \( A_0^2 \) corresponds to an expansion up to \( n^\alpha \) order in opacity.

Taking the high energy limit and the soft radiation approximation (\( x \ll 1 \)), one focuses on isolating the leading behavior in \( x \) that arises from the large number of interference diagrams at a given order of opacity. As a result of the approximations made, one recovers the BDMPS condition that the leading behavior in \( x \) is contained solely in gluon re-scattering diagrams. This results in the expression for the inclusive energy distribution for gluon radiation of an in-medium produced parton as [36],

\[
\frac{dx}{d\omega y} = \int d^2 q T_g T_g T_g \left( \frac{Z(q)}{2} \right)^2 d\omega y T_g T_g T_g \int_0^1 d\alpha \left( \begin{array}{c} \alpha \\ 1 - \alpha \end{array} \right) G(\omega y; \omega y, \omega y, \omega y) \frac{d\omega y}{d\omega y}
\]

where, as always, \( k_T \) is the transverse momentum of the radiated gluon and \( x \) is its forward momentum. The vectors \( y \) and \( u \) represent the transverse locations of the emission of the gluon in the medium and the momentum conjugate where \( y \) and \( u \) represent the longitudinal positions. The density of scatterers in the medium at location \( \omega y \) and the scattering cross section is \( \sigma(\omega y) \). In this form, the opacity is obtained as \( \omega y \) over the extent of the medium.

Numerical implementations of this scheme have focused on two separate regimes. In one case, \( \omega y \) is replaced with a dipole form \( C r^2 \) and one solves the harmonic oscillator path integral. This corresponds to the case of multiple soft scatterings of the hard probe. In the limit of a static medium with a very large length, one obtains the simple form for the radiation distribution [8],

\[
\frac{d\omega y}{d\omega y} = \left( \frac{p}{\omega y} \right)^2 \left( \frac{1}{\omega y} \right)^2 \text{ for } \omega y > \omega y_c,
\]

where \( \omega y_c = \omega r \). The distribution \( \omega y \) is called the characteristic frequency of the radiation. Up to constant factors, this is equal to the mean energy lost in the medium, \( \omega y \). The only tunable parameter is \( r \), where \( L \) is the length of the medium and \( \omega y \) is the jet transport coefficient, defined as the transverse momentum per unit length. For jet radiation in finite mediums, the mean \( \omega y \) may be performed to obtain \( \omega y = \omega r^2 = 2 \), where \( L \) is the length of the medium. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned. For a dynamical medium of finite extent, the characteristic frequency and the overall \( \omega y \) are tuned.

In the other extreme, one expands the exponent as a series in \( \omega y \); keeping only the leading order term corresponds to the picture of gluon radiation associated with a single scattering. In this second form, the analytical results of the ASW scheme formally approach those of the GLV reaction operator expansion [39]. In either case, the gluon emission intensity distribution has been found to be rather similar; once scaled with the characteristic frequency in each case.

3.3 Finite temperature field theory approach

In this scheme, often referred to as the AMoD-AM (A M Y) approach, the energy loss of hard jets is considered in an extended medium in equilibrium at asymptotically high temperature \( T \). Due to asymptotic freedom, the coupling constant \( g \) ! at such high temperatures and a power counting scheme of mass poles from the ability to identify a hierarchy of parametrized separated scales \( g T, g^2 T, g^3 T, \ldots \). In this limit, it becomes possible to construct an effective field theory of soft modes, i.e., \( g T \), by summing contributions from hard bops with \( p \). This results in the effective propagators and vertices [39].

One assumes a hard on-shell parton, with energy sufficiently large that the term is suppressed, traversing such a medium, 
undergoing soft scatterings with mom ementum transfers $q_T \to$ other hard partons in the medium. Such soft scatterings induce collinear radiation from the parton, with a transverse mom ementum of the order of $q_T$. The form action time for such collinear radiation $t_i(T)\approx 1$ is of the same order of magnitude as the mean free time between soft scatterings [42]. As a result, multiple scatterings of the incoming (outgoing) parton and the radiated gluon need to be considered to get the leading order gluon radiation rate. One essentially calculates the imaginary parts of internal order ladder diagrams such as those shown in Fig. 3, this is done by means of integral equations [26].

The imaginary parts of such ladder diagrams yield the 1/2 decay rates of a hard parton $a$ into a radiated gluon and another parton $b$ [26]. These decay rates are then used to evolve hard quark and gluon distributions from the initial hard collisions, when they are form ed, to the time when they exit the medium, by means of a Fokker-Planck like equation [27], which is written schematically as,

$$\frac{dP_a(p)}{dt} = \sum_b \frac{X}{P_b(p+k)} \frac{d_{ab}(p+k,p)}{d_{ab}(p+k,p)} \int_{-\infty}^{\infty} dP_a(p) \frac{d_{ab}(p+k,p)}{d_{ab}(p+k,p)} \cdots$$

(8)

The use of an effective theory for the description of the medium and the propagation of the jet, makes this approach considerably more systematic than the two previous approaches: both the properties of the jet and the medium are described using the same hierarchy of scales. It remains the only approach to date which naturally includes partonic feedback from the medium, i.e., processes where a thermal quark or gluon may be absorbed by the hard jet. In contrast to ASW and HT, this approach also (naturally) includes any changing interactions in the medium. Elastic energy loss may also be incorporated within the same basic formalism. However, since the AMY scheme assumes a thermalized partonic medium, its applicability is somewhat limited: it cannot compute the quenching of jets in the con ned sector. As a result, energy loss in cold con ned nuclear matter as well as in the hadronic phase of heavy-ion collisions cannot be computed in this model. The o -shellness or virtuality of all jets is considered to be similar to that of hard partons in the medium, as a result, interference between vacuum and medium induced radiation is also not considered.

In realistic calculations, the total perature of the medium is usually set by the underlying hydrodynam ic simulation (see Ref. [34] for details). While in the HT or ASW formalism, an A nsatz is made for the one tunable par ameter $q$ and its relation to $T$, in the AMY formalism, $q$ may be calculated directly from a knowledge of the tem perature and the strong coupling constant $g$ (or $\alpha_s$). This is due to the precise picture of the medium used; that of a hot plasma of quarks and gluons. In realistic simulations, the coupling is, in principle, unknown and becomes the primary parameter. This is then tuned by comparison to one data point.

4.1 Higher twist schem e

In subsection 3.1, the medium modified fragm entation function calculated in Eq. (3) included only one gluon em ission in the medium. Any remaining gluon emissions occurred in the vacuum and were included in the renormalization of the vacuum fragmentation function. Unlike the remaining from as, the results from just the single gluon emission in the medium yield a medium modified fragmentation function and are already computes with experiment.

In reality, one expects multiple emissions to occur in the medium compared to escape into the vacuum and further emissions in the vacuum. One starts with a vacuum fragmentation function at a low scale and insists that the parton exit in the medium with a certain virtuality. Emissions from the scale up to the scale may be included by using the standard vacuum: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [42, 43, 44].
Emissions in the medium account for the remaining evolution from the scale up to the scale $Q$. To compute this in-medium evolution, the medium modified fragmentation function from single gluon emission in Eq. (3) is now generalized to an evolution equation in virtuality of the propagating parton [see Ref. [43] for details], i.e.,

$$
\frac{d^2 \Phi^h_{ij}(z, p_T^2)}{d^2 \log(M^2)} = Z_i Z_j \left( \frac{N_c^2 - 1}{2} \right) C_F \left( k_{ij}^2 + k_{ij}^2 \right) \int_{q^2}^{z} \frac{f(x, y, z, k_{ij}^2)}{p_T^2} \text{d}x \text{d}y.
$$

The initial conditions to this differential equation are provided by the fragmentation functions at scale $Q$. The resulting medium modified fragmentation functions include both vacuum and in-medium induced emissions from the scale $Q$ down to the scale. Further emissions occur solely in the vacuum. This medium modified fragmentation function may now be convoluted with the cross section to produce a hard parton [as in Eq. (1)] to get the naldistribution of hadrons. Computation of the medium modified fragmentation function in an evolving medium such as the deconfinement phase in a quark-gluon plasma involves further calculational details presented in Ref. [34]. As the HT formalism is setup to directly calculate the naldistribution of hadrons, the parton and medium profiles along that path, which loses energy due to its passage through the medium.

For the parton to lose a finite fraction of its initial energy, multiple gluons are emitted. Each such emission at a given energy is assumed to be independent and a probabilistic scheme is set up, within which the jet loses an energy fraction $\varepsilon$ in $n$ steps with a Poisson distribution (22, 24),

$$
P_n(\varepsilon^P) = \frac{e^{-\varepsilon^P} \varepsilon^P^n}{n!} z = \sum_{i=1}^{n} \frac{\text{dN}_{ij}}{\text{d}x_i} (x_0 - x_i); \quad \text{(10)}
$$

where $\text{dN}_{ij}$ is the mean number of gluons radiated per coherent interaction set. Summing over $n$ gives the probability $P(\varepsilon)$ for an incident jet to lose a mean energy fraction due to its passage through the medium.

This probability distribution is then used to model a medium modified fragmentation function, by shifting the energy fraction available to produce a hadron as well as accounting for the phase space available after energy loss (The fragmentation function used is that of a vacuum fragmentation function). The medium modified fragmentation function is thus defined as (22, 24),

$$
\mathcal{D}(z, p_T^2) = Z_1 \int_{0}^{1} d P \text{d}z \left( \frac{\varepsilon^P}{1} \right). \quad \text{(11)}
$$

The above medium modified fragmentation function is then used in a factorized formalism as in Eq. (1) to calculate the nald hadronic spectrum. Additional details related to the computation of the energy loss probability distribution are given in Ref. [34].

### 4.3 Finite temperature field theory scheme

In subsection 4.2, the opacity expansion was used to calculate the differential spectrum for single gluon radiation from a hard parton. The calculations were carried out in the soft gluon limit, i.e., $\varepsilon = 0$. The next step is to calculate the probability for the leading hard parton to radiate a finite energy $E = p_T^0$. After losing this energy, the degraded hard parton escapes the medium and fragments in vacuum into a shower of hadrons.

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5 Discussions and conclusions

In these proceedings, the different underlying theoretical mechanisms used in some of the prevalent jet energy loss calculations have been outlined. Specific attention was paid to how each jet resolves the medium and on the properties of the medium which controls the medium cation of the hard jet. In all cases, the jet medium cation from alias may be reduced to a form which depends on only one parameter: this is the transport coefficient $\eta$ defined as the transverse momentum gained by a hard parton per unit length traversed in a dense medium. While in the ASW form alias, $\eta$ is the sole tunable parameter in the HT form alias, $\eta$ depends on the gluon distribution strength correlation in Eq. (3) and may be calculated from a known density of the medium and the coupling constant $\alpha_s$ in the AMY form alias.

In all cases, the jet is assumed to fragment outside the medium. As a result, all form aliases use a medium modulated fragmentation function, which uses a vacuum fragmentation function as input. While in the ASW and the HT form aliases, the medium cation is con puted in both decon sod and con sod phases, due to the assum ptions made in the AMY form alias, the medium cation in this form alias occurs only in the decon sod phase. The medium cation in the con sod phase is assumed to be small and ignored. While both the HT and the ASW form aliases include contributions from interference with vacuum radiation, these are ignored in the AMY scheme. The AMY approach however includes contributions from them all feedback which has so far not been straightforwardly included in the HT and ASW form aliases. The consistent setup of the AMY form alias also allows for the most natural extension to include elastic energy loss. While in the strict interpretation of heavy scattering centers in the ASW (and GLV) form alias, elastic energy loss is identically zero, the inclusion of elastic loss requires additional assum ptions about the medium in the HT approach. Extensions to the GLV form alias to include mobile scattering centers, and thus, both include elastic energy loss and modify the formation of radiative energy loss are currently underway. Similar extensions in the HT approach are also being carried out. However, given the incompleteness of the di rare form alias, elastic energy loss was not discussed in these proceedings, nor will be included in the realistic com parison presented in Ref. [34].

While the description of the di rare form alias in these proceedings have not included the effects of a dynamical medium, realistic calculations of jet medium cation in heavy-ion collisions do include such effects. The medium cation depends on the path traversed by a given jet. This in turn depends on the origin of the jet and the direction of travel in the medium. The details related to this problem, as it applies to the di rare form alias, will be presented in the companion paper [34]. As a result, complements to experimental data will also be carried out in this reference as well.

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