The thermal conductivity $\kappa$ of ultraclean YBa$_2$Cu$_3$O$_7$ was measured at very low temperatures in magnetic fields up to 13 Tesla.\footnote{1} In zero field, $\kappa/T$ rises from its universal limit $\kappa_{\text{00}}/T$ very fast as a function of temperature which, according to the quasiclassical calculation for a d-wave superconductor,\footnote{2} indicates either an extremely small normal state impurity scattering rate $\Gamma = 1/2\tau$ in the unitary limit (phase shift $\rho = \pi/2$), or an unrealistically large $\Gamma$ for the Born approximation (phase shift $\rho = 0$). As a function of field, $\kappa$ initially increases very rapidly reaching nearly twice its universal limit $\kappa_{\text{00}}$ and remains almost unchanged up to 13 Tesla. The theory of Vekhter and Houghton,\footnote{3} which includes scattering of quasiparticles at the vortices via Andreev scattering, yields a rise proportional to $\sqrt{H}$ in agreement with the measurements for a less pure sample of YBCO.\footnote{4} Recently the data of Ref. 1 have been accounted for by assuming a phase shift $\rho$ for impurity scattering slightly less than the unitary limit, $\pi/2$, and adding to the extremely small impurity scattering rate $\Gamma$ a phenomenological quasiparticle scattering due to Andreev scattering at the superfluid flow of the vortices.\footnote{5}

In the present paper we use the theory of Ref. 3 and the equivalent theory of Ref. 6 to solve the problem of the occurrence of a plateau-like feature in the field dependence of $\kappa$ in YBCO. These theories are based on the theory of Brandt, Pesch, and Tewordt (BPT-theory)\footnote{7} where the spatial average of the Gorkov Green’s function $G$ for the Abrikosov vortex lattice was calculated. The expression for the thermal conductivity which has been derived from this Green’s function\footnote{8,5} can explain the measured field dependence of $\kappa$ for different field directions for superconducting spin-triplet pairing states with vertical or horizontal nodes in Sr$_2$RuO$_4$. A simplified version of the BPT-theory which is based on the quasiclassical Green’s function $g$,\footnote{8,5} referred to here as the P-approximation, yields the expression of Ref. 3 for the thermal conductivity $\kappa \equiv \kappa_{xx}$ in the vortex state with the field perpendicular to the basal plane for a d-wave superconductor. We find that the expressions for the thermal conductivity derived from the BPT-Gorkov or from the quasiclassical Green’s functions (Refs. 6 and 3) yield to a very good approximation the same results for all reduced fields $h = H/H_{c2}$, impurity scattering rates $\delta = \Gamma/\Delta_0$, and phase shifts $\rho$. Here $\Delta_0$ is the amplitude of the d-wave order parameter.

To save space we omit here the full expression for $\kappa$ and concentrate on the discussion of the Andreev scattering which is derived from first principles. This contribution to the quasiparticle scattering is contained in the expression for $\text{Im}\xi_0$ in the denominator of the $\omega$-integral for $\kappa$ which also includes the well-known factor $\omega^2 \operatorname{sech}^2(\omega/2T)$.\footnote{6} Here $\xi_0$ is the position of the pole of the BPT-Green’s function $G$ as a function of the normal state energy $\xi$ measured from the Fermi energy. The equation for the zero $\xi_0$ of the denominator of $G$ yields $\text{Im}\xi_0 = \gamma_i + \gamma_A$, where $\gamma_i$ is the total impurity scattering rate and $\gamma_A$ is the imaginary part of the quasiparticle energy $-\Sigma_A$ at $\xi_0$. The kernel of Gorkov’s integral equation for $G$ in the spatial representation yields

$$\Sigma_A(r_1, r_2; \omega) = -\Delta(r_1)\Delta^*(r_2)G^0(r_1 - r_2; -\omega),$$  \hspace{1cm} (1)$$

where $\Delta(r)$ is Abrikosov’s vortex lattice order parameter. From this expression it is clear that $\gamma_A = -\text{Im}\Sigma_A$ is the scattering rate for converting a quasiparticle at $\Delta^*(r_2)$ by Andreev reflection into a quasihole and then back into a quasiparticle at $\Delta(r_1)$. The Fourier transform of $\Sigma_A(r_1 - r_2; \omega)$ yields\footnote{7}

$$\Sigma_A(p, \omega) = -i\sqrt{\pi}\Delta^2(\Lambda/v_\perp)w[(\omega + i\gamma_i + \xi_p)\Lambda/v_\parallel].$$  \hspace{1cm} (2)$$

Here $\Delta^2$ is the spatial average of $|\Delta(r)|^2$, $\Lambda = (2eH)^{-1/2}$ is of the order of the vortex lattice constant, $v_\perp(p)$ is the Fermi velocity component perpendicular to the applied field $H$, and $w$ is Dawson’s integral. In the limit $\omega \to 0$...
corresponding to the $T \to 0$ limit of $\kappa/T$, one obtains explicit expressions for $\text{Im}\xi_0 = \gamma_i + \gamma_A(\xi_0)$ and $\kappa/\kappa_n$. In the limit $v_\perp \to 0$, i.e., $\mathbf{v}(\mathbf{p}) \parallel \mathbf{H}$, Eq.(2) tends to the self energy of the BCS Green’s function. In the limit $\Lambda \to \infty$, i.e., $H \to 0$, the expressions for $\kappa$ in the BPT-approximation\(^8\) and in the equivalent P-approximation\(^4\) tend correctly to the expression for $\kappa$ which was first derived in Ref. 10.

In the following we present results for a d-wave pairing state in a field perpendicular to the base plane where the Abrikosov vortex lattice state is multiplied by $\cos(2\phi)$. The impurity scattering rate $\gamma_i$ is calculated self-consistently in the t-matrix approximation for the self energy $\Sigma_i$:

$$\gamma_i = \text{Re}(\Sigma_i); \quad \Sigma_i = \Gamma \tilde{g}(\omega) \left[ \cos^2 \rho + \tilde{g}^2 \sin^2 \rho \right]^{-1} ;$$

$$g(\omega, \phi) = \left[ 1 - i \sqrt{\pi} (2\Delta \lambda / \nu)^2 \cos^2(2\phi) w' (z) \right]^{-1/2} ;$$

$$z = 2(\omega + i \Sigma_i) \Lambda / \nu; \quad \Lambda = (2eH)^{-1/2}.$$  

Here $g$ is the quasi-classical Green’s function in the P-approximation\(^8\) and $\tilde{g}$ is the angular average of $g$. The field dependence of $\Delta$ is approximately given by $\Delta = \Delta_0 \sqrt{T - h}$, where $h = H/H_{c2}$.

In Fig.1(a) we show the ratio of $\kappa_{xx} \equiv \kappa$ to the normal state conductivity $\kappa_n$ vs $h = H/H_{c2}$ for $\omega = 0$ corresponding to the limit $T \to 0$, and the density of states $N/N_0 = \text{Re} \tilde{g}(0)$, for reduced normal state scattering rates $\delta = \Gamma / \Delta_0 = 0.1, 0.01, 0.001$, and 0.0001 in the Born approximation (phase shift $\rho = 0$). The occurrence of a plateau in the low field range for all scattering rates is demonstrated more clearly in Fig.1(b). In Fig.2(a) we show our corresponding results for $\kappa/\kappa_n$ and $N/N_0$ for a phase shift $\rho = 0.495\pi$ which is very close to the unitary limit $\rho = \pi/2$ and has been used in Ref. 5 to fit the data of Ref. 1. Fig.2(b) shows more clearly that a plateau in the low field range is reached only approximately in the limit of very small scattering rates, here $\delta = 0.001$ and 0.0001. The latter value is of the order of magnitude of the value that has been used in Ref. 5 to fit the data of Ref. 1. We find that the field dependence of $\kappa/\kappa_n$ is very nearly the same as the field dependence of the ratio of the scattering rates in the normal and vortex states, i.e., the angular average of $\Gamma / (\gamma_i + \gamma_A)$.

This means, according to Figs.1(a) and 2(a), that the Andreev scattering rate $\gamma_A$ is much larger than the impurity scattering $\Gamma$ up to fields just below $H_{c2}$, and that the ratio $\gamma_A / \Gamma$ at fixed $h$ increases with decreasing scattering rate $\delta$.

We now attempt to explain the second feature of the experiments of Ref. 1, i.e., that $\kappa/T$ becomes almost temperature independent up to about 0.6 K for constant fields above 1 Tesla up to 13 Tesla. In Fig.1(c) we show our results for $\kappa(\omega)/\kappa_n$ vs $\tilde{\Omega} = \omega / \Delta_0$ for $\delta = 0.1$ and 0.05 and several fields $h = 0.1, 0.06,$ and 0.02 in the Born approximation ($\rho = 0$). Here, $\kappa(\omega)/\kappa_n$ is the factor multiplying $(\omega / T)^2 \text{sech}^2(\omega / 2T)$ in the normalized integral over $d(\omega / T)$ for $\kappa / \kappa_n$. Since $(\omega / T)^2 \text{sech}^2(\omega / 2T)$ has a maximum at $\omega / T = 2.4$, one has $\tilde{\Omega} = \omega / \Delta_0 \approx 2.4(T / \Delta_0)$, and thus $\kappa(\omega) / \kappa_n$ yields approximately the dependence of $\kappa / \kappa_n \propto \kappa / T$ as a function of $\tilde{\Omega} \approx 2.4(T / \Delta_0)$. We have divided $\kappa(\omega) / \kappa_n$ by $\delta$ because $\kappa_n / \delta \sim (\pi / 2) \kappa_00$, where $\kappa_00$ is the universal conductivity limit. One recognizes from Fig.1(c) that $\kappa(\omega) / \kappa_n \delta$ is almost constant in the range from $\tilde{\Omega} = 0$ to 0.1 which means that $\kappa / T$ is almost constant up to about $T / T_c \sim 0.1$, and that the curves for constant fields $h = 0.1, 0.06,$ and 0.02 lie close together for each of the two scattering rates $\delta = 0.1$ and 0.05. In Fig.2(c) we show the corresponding results for the phase shift $\rho = 0.495\pi$ and scattering rate $\delta = 0.0001$, and in Fig.3(a) the results for scattering rate $\delta = 0.001$ and the same phase shift and fields. Again these results indicate that $\kappa/T$ is nearly constant as a function of $T$ at least for the higher fields $h = 0.1$ and 0.06. However, these curves for the almost unitary limit are not as close to each other as those for the Born approximation in Fig.1(c). This is in disagreement with the data for the temperature dependence of $\kappa / T$ for fixed fields between 0.8 and 13 Tesla which lie on about the same level.\(^1\)

We have also calculated $\kappa(\omega) / \kappa_n \delta$ in the limit of zero field. The expression for $\kappa$ in Ref. 6 yields in the limit $H \to 0$, or $\Lambda \to \infty$, the following result:

$$\frac{\kappa(\omega)}{\kappa_n \delta} = \int_0^{2\pi} \frac{d\phi}{\pi} \cos^2(\phi) \frac{1}{\text{Im} \left[ \Omega^2 - \cos^2(2\phi) \right]} \frac{1}{2} \left( 1 + \frac{|\tilde{\Omega}|^2 - \cos^2(2\phi)}{|\Omega^2 - \cos^2(2\phi)|} \right) ;$$

$$\tilde{\Omega} = \Omega + i \Sigma_i / \Delta_0; \quad g(\tilde{\Omega}, \phi) = \tilde{\Omega} \left[ \Omega^2 - \cos^2(2\phi) \right]^{-1/2} ; \quad \Omega = \omega / \Delta_0 .$$  

The impurity self energy $\Sigma_i$ is calculated self-consistently with the help of Eq.(3). Eq.(6) agrees with the general strong-coupling result of Ref. 10. In Fig.2(c) we have plotted our results for $\kappa(\omega) / \kappa_n \delta$ for phase shift $\rho = 0.495\pi$ and scattering rate $\delta = 0.0001$, and in Fig.3(a) we show the results for the same phase shift and scattering rates $\delta = 0.001$,
0.01, and 0.1. One sees from these figures that for $H = 0$ the ratio $\kappa(\omega)/\kappa_0 \delta \sim (2/\pi)\kappa(\omega)/\kappa_{00}$ tends for $\Omega \to 0$, or $T \to 0$, correctly to the value $(2/\pi)$, and that this function rises approximately proportional to $\Omega^2$ with a slope that decreases for increasing $\delta$. The zero field and constant field curves in Figs.2(c) and 3(a) for $\delta = 0.0001$ and 0.001 are qualitatively similar to the data for the $T$ dependence of $\kappa/T$ in zero and constant fields. However, the values of $\kappa/T$ in units of $\kappa_{00}/T$ shown in Fig.2(c) for the constant fields $h =$0.1, 0.06, and 0.02 are an order of magnitude too large in comparison to the experimental values. The corresponding values for $\delta = 0.001$ shown in Fig.3(a) are much smaller. These values can be reduced further to reach the experimental value of about 2 with a reasonable increase in the factor $\alpha$ multiplying the quantity $\Delta\Lambda/v$ [see Eqs. (2) and (4)] for a d-wave pairing state. Here $\alpha \propto v_2/v$ where $v_2$ is the slope of the d-wave gap at the node. The dashed curves in Fig.3(a) correspond to the value $\alpha = 5$. These curves are reduced to those in Fig.3(b) by using the value $\alpha = 31$. The upper dashed curves in Fig.2(c) for $\delta = 0.0001$ also correspond to $\alpha = 5$ and they are reduced to the lower dashes curves by taking the same value of $\alpha = 31$ as for $\delta = 0.001$ in Fig.3(b). A still much larger value of $\alpha$ is needed to reach the experimental value $\kappa/\kappa_{00} \sim 2$. That corresponds to an unrealistically large value of $v_2/v$. We thus conclude that the measured $T$ dependence of $\kappa/T$ for ultraclean YBCO in zero field, and finite fields up to 13 Tesla, can be qualitatively explained by our theory with the parameter values $\rho = 0.495\pi$ and $\delta = 0.001$ and 0.0001. However, the corresponding curves for the field dependence of $\kappa/\kappa_n$ in the $T \to 0$ limit [see Fig.2(b)] do not rise as abruptly to the plateau-like value for increasing field as the experimental curve. Contrary to the nearly unitary limit, in the Born approximation the field dependence of $\kappa/\kappa_n$ shown in Fig.1(b), and the temperature dependence of $\kappa/T$ [see Fig.1(c)], are, for example for $\delta = 0.1$, in much better agreement with the experiments of Ref. 1. However, the $T$ dependence of $\kappa/T$ for zero field is in total disagreement with the data: it rises steeply for very small $\Omega$ and then tends to a constant value of about $4\kappa_{00}/T$ for $T$ up to about 0.1 [see Fig.1(c)].

We have also investigated intermediate values of the impurity scattering phase shift, for example, $\rho = 0.4\pi$ which has been used to fit the data for the microwave conductivity in YBCO films. We find that the measured field dependence of the thermal conductivity is better described with $\rho = 0.4\pi$ and $\delta = 0.001$ or 0.0001 than with $\rho = 0.495\pi$ and the same $\delta$’s. However, the $\Omega$ dependence of $\kappa/\kappa_0$ for zero field is quite far from a $\Omega^2$ dependence: it rises steeply to a maximum and then decreases slowly for increasing $\Omega$. Thus phase shifts intermediate between the Born approximation and the unitary limit do not seem useful for explaining the experiments of Ref. 1.

The aim of the theory for $\kappa$ in Ref. 6 was to explain the measurements of $\kappa$ in Sr$_2$RuO$_4$ for fields perpendicular to the ab plane and for rotating in-plane fields. Recently, $\kappa$ was measured in Sr$_2$RuO$_4$ in zero field down to very low temperatures. This low-temperature behavior of $\kappa$ demonstrates the universal character of the heat transport due to a gap with nodes and suggests strong impurity scattering with a phase shift close to $\pi/2$. Since our present theory for a d-wave superconductor also applies to a spin triplet t-wave pairing state with vertical or horizontal line nodes, we conclude that our results for phase shifts $\rho = 0.495\pi \simeq \pi/2$ also apply to Sr$_2$RuO$_4$. The $T$ dependence of $\kappa/T$ for $\delta = 0.01$ and 0.1 shown in Fig.3(a) for zero field is similar to the data in Ref. 13. The curves for $\kappa/\kappa_n$ vs $H/H_{c2}$ for $\delta = 0.01$ and 0.1 in Fig.2(a) lie far below the curves for $\delta = 0.5$ and 0.2 obtained in the Born approximation in Ref. 6.

In summary, we have calculated the thermal conductivity for the Abrikosov vortex lattice state with d-wave pairing which automatically includes Andreev scattering due to the self energy in Gorkov’s integral equation for the Green’s function. The most important result of the present paper is that the BPT-approximation based on the Gorkov equations yields very nearly the same results for $\kappa$ as the P-approximation based on the equations for the quasiclassical Green’s functions. These results for $\kappa$ correspond to previous results for the density of states $N/N_0$ for d-wave pairing in the vortex state. In Ref. 14 it was shown that the BPT- and P-approximations yield nearly the same results as the solutions of the quasiclassical equations for applied fields between $H_{c2}$ and $H_{c1}$ while the Doppler shift approximation gives rise to substantial errors. In agreement with Ref. 3 we find that, for impurity scattering in the Born limit, $\kappa$ exhibits a steep initial increase as a function of field and then becomes almost constant (see Fig.1) while, in the almost unitary limit, $\kappa$ rises like $\sqrt{H}$ (see Fig.2). The latter result disagrees with the results of Ref. 5 where, in the almost unitary limit, $\kappa$ rises initially very rapidly and then becomes almost constant with a value in units of the universal conductivity $\kappa_{00}$ close to the experimental value. The reasons for the discrepancy are apparently the following. First, the renormalization of the impurity scattering self energy in the t-matrix approximation is carried out in Ref. 5 by employing the zero-field density of states while, in Ref. 3 and here [see Eqs.(3) and (4)], the density of states is calculated self-consistently for finite field. Second, the Andreev scattering rate $\gamma_A$ is approximated in Ref. 5 by $bE_H$ where $E_H \sim v/L$ [see Eq.(5)] is the magnetic field energy and $b$ is a parameter which is fitted to the data of Ref. 1. This expression for the Andreev scattering rate is taken in analogy to the expression derived by the method of the Doppler shift followed by an averaging over a single vortex. However, the expression for $\gamma_A$ derived in Ref. 15 contains an additional exponential factor which changes the field dependence considerably. In our approach $\gamma_A = -\Im \Sigma_A$ where $\Sigma_A$ is the self energy in the Gorkov equation which has the form for Andreev scattering by the spatial variation of the complex order parameter $\Delta(r)$ of the total Abrikosov vortex lattice [see Eq.(1)]. It is
interesting that the exponential function arising from the imaginary part of Eq.(2) has, in the absence of impurity scattering \( \gamma_i \), a similar form as that of Ref. 15 considered as a function of frequency, field, and angle \( \theta = \angle(p, H) \). Our \( \gamma_A \) in the absence of impurity scattering is shown as a function of \( \theta \) for several different fields and frequencies in Fig.5 of Ref. 16. It should be noted that here \( \gamma_A \) depends also on \( \gamma_i \) [see Eq.(2)] where both quantities are calculated self-consistently together with Eqs.(3) - (5). Although the theory of Ref. 7 was originally derived for applied fields \( H \) near \( H_{c2} \), it has been shown to work well over the entire range of linear magnetization,\(^{17}\) and it has been tested down to \( H_{c1} \) for a d-wave superconductor by comparison with the solutions of the quasiclassical equations.\(^{14}\)

Let us now briefly discuss why our theory in its present form fails to explain the experiments of Ref. 1 consistently for zero and finite fields. The measured rapid growth of the zero field \( \kappa/T \) with \( T \) in ultraclean YBCO can, in agreement with Ref. 5, indeed be explained only by assuming a very small impurity scattering rate \( \delta = \Gamma/\Delta_0 \sim 10^{-3} \) to \( 10^{-4} \) and a phase shift close to the unitary limit \( \pi/2 \) for isotropic scatterers. The observed sudden onset of a "plateau" in \( \kappa/T \) as a function of field \( H \) requires, for this phase shift limit, the assumption of a very small \( \delta \leq 10^{-3} \) because this makes the region of \( \sqrt{H} \)-behavior small and \( \kappa \) beyond this region nearly constant as a function of \( H \). The third feature of the experiments, i.e., that \( \kappa/T \) is independent of \( T \), is also satisfied. However, the saturation values of \( \kappa/T \) in units of \( \kappa_{00}/T \) are too large and lie too far apart for different fields in comparison to the experimental values. The Born approximation (phase shift zero) with \( \delta \sim 0.1 \) to 0.01 yields a much better description of the measured field dependence of \( \kappa \), i.e., a steep initial increase followed by a plateau, and the saturation values of \( \kappa/T \) in units of \( \kappa_{00}/T \) for different fields are in fair agreement with the data. However, the calculated \( T \) dependence of \( \kappa/T \) in zero field is quite different from the observed temperature dependence if one assumes isotropic impurity scattering in the Born limit. This is also the case for intermediate phase shifts between \( \rho = 0 \) and \( \rho = \pi/2 \), for example for \( \rho = 0.4\pi \) which has been used to fit microwave conductivity data.\(^{11}\)

In conclusion, we have shown that the measured field dependence of \( \kappa/T \) in ultraclean YBCO,\(^{1} \) in particular the plateau, can be better described by the Born approximation than by the almost unitary phase shift limit for impurity scattering. The use of the almost unitary limit is however necessary \(^{5} \) in order to explain the observed rapid increase of \( \kappa/T \) with \( T \) in zero field with the model of point-like impurity scattering with a single phase shift. It is possible that the deficiencies of the present theory may be due to this over-simplified model. Similar difficulties in explaining microwave conductivity measurements in very clean YBCO have led to consideration of impurity potentials with finite range. This gives rise to considerable changes in the density of states at low frequencies due to renormalization of the d-wave gap.\(^{18}\) It seems possible that such effects could lead to a better description of the \( T \) dependence of \( \kappa/T \) in zero field in the Born approximation. On the other hand we believe that the theory of Andreev scattering contained in the theory of \( \kappa \) in Ref. 3 need not be altered because it agrees with our theory where the Andreev scattering is evident from the form of the self energy in the Gorkov equations.

Our results for the unitary limit and scattering rates \( \delta \sim 0.01 \) and 0.1 also apply to spin triplet states with vertical or horizontal line nodes in Sr\(_2\)RuO\(_4\). Indeed, the zero field measurements at very low \( T \) strongly suggest a phase shift close to \( \pi/2 \).\(^{13}\)

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FIG. 1. 1a) Thermal conductivity ratio, $\kappa/\kappa_n$, vs $h = H/H_{c2}$ at $T = 0$ in the Born approximation for the reduced scattering rates $\delta = \Gamma/\Delta_0 = 0.1, 0.01, 0.001,$ and $0.0001$ (solid curves, from top to bottom) and the density of states at the Fermi energy, $N/N_0$ vs $h$ (dashed curves from top to bottom).
Fig. 1b

FIG. 2. 1b) The same as 1(a) but on a reduced scale.
Fig. 1c

FIG. 3. 1c) The factor in the integrand of the ω-integral for $\kappa$, $\kappa(\omega)/\kappa_\eta\delta$, vs $\Omega = \omega/\Delta_0$. Upper curves for $\delta = 0.05$ in the Born approximation for $h = 0.1, 0.06, \text{and} 0.02$ and lower curves for $\delta = 0.1$ and the same $h$ (from top to bottom). Solid curve for $h = 0$ and $\delta = 0.1$. 
FIG. 2a  \( \kappa/\kappa_n \) vs \( h \) at \( T = 0 \) for impurity scattering phase shift \( \rho = 0.495\pi \) and \( \delta = 0.1, 0.01, 0.001, \) and 0.0001 (solid curves, from top to bottom), and \( N/N_0 \) vs \( h \) (dashed curves from top to bottom).
Fig. 2b

FIG. 5. 2b) The same as 2(a) but on a reduced scale for $\delta = 0.01, 0.001, \text{and } 0.0001$. 
Fig. 2c

FIG. 6. 2c) $\kappa(\omega)/\kappa_0\delta$, vs $\Omega$ [see notation of Fig.1(c)] for phase shift $\rho = 0.495\pi$ and $\delta = 0.0001$. Dashed curves for $h = 0.1$, 0.06, and 0.02 (from top to bottom). Upper curves for $\alpha = 5$ and lower curves for $\alpha = 31$. Solid curve for $h = 0$. 
Fig. 7. 3a) Factor in the $\omega$-integral for $\kappa / \kappa_n \delta$, vs $\Omega = \omega / \Delta_0$, for phase shift $\rho = 0.495\pi$. Solid curves for $h = 0$ and $\delta = 0.001$, 0.01, and 0.1 (from left to right). Dashed curves for $\delta = 0.001$, $\alpha = 5$, and $h = 0.1$, 0.06, and 0.02 (from top to bottom).
FIG. 8. 3b) Dashed curves for $\delta = 0.001$, $\alpha = 31$, and $h = 0.1, 0.06, \text{ and } 0.02$ (from top to bottom). Solid curve for $h = 0$. 

Fig. 3b