DETERMINING THE RELATIVE SIZE OF THE CP-EVEN AND CP-ODD HIGGS BOSON COUPLINGS TO A FERMION AT THE LHC

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Abstract

We demonstrate that the relative size of the CP-even and CP-odd couplings of a Higgs boson to $t\bar{t}$ can be determined at the LHC at a level that is very useful in discriminating between models, provided that a Higgs signal can be observed at a reasonable $S/\sqrt{B}$ level in the $t\bar{t}h$ (with $h \rightarrow \gamma\gamma$ or $b\bar{b}$) channels. In particular, the CP-even nature of a SM-like Higgs boson’s coupling to $t\bar{t}$ can be confirmed with very substantial statistical significance using the $t\bar{t}h$ ($h \rightarrow \gamma\gamma$) production/decay mode. The key to achieving good discrimination between models is to make full use of the difference in the final state distributions for different ratios of the CP-even to CP-odd couplings. For our analysis, we employ the very convenient and simple optimal analysis procedures derived previously.

1 Introduction

It will be very important to directly determine the CP nature of any Higgs boson ($h$) that is discovered. Since it is almost certain that a substantial number of Higgs boson events will first be available at the LHC, it will be very desirable to use LHC data to verify the CP nature of any observed Higgs without waiting for an $e^+e^-$ collider. The only procedure with a reasonable level of viability proposed to date is to employ weighted integrals of the $pp \rightarrow t\bar{t}hX$ final state \cite{1}. These were shown to provide significant ability to establish that a SM-like Higgs boson has a purely CP-even $t\bar{t}$ coupling. Subsequently, procedures for optimizing the extraction of the relative magnitude of different components of a cross section were developed and their power demonstrated when applied to determining the relative magnitude of the CP-even and CP-odd $t\bar{t}h$ couplings in the process $e^+e^- \rightarrow t\bar{t}h$ \cite{2}. Here, we apply these same techniques to the $t\bar{t}hX$ final state at the LHC and demonstrate substantial improvement as compared to the weighted integral technique of Ref. \cite{1}.
In general, a very meaningful determination of the relative magnitude of the CP-even and CP-odd $t\bar{t}$ couplings of the $h$ will be possible provided only that a signal with reasonable statistical significance is present. The key to the optimal analysis techniques is to take full advantage of all the information available in the cross section as a function of the kinematical variables.

For our analysis, it is important that a relatively pure sample of $t\bar{t}FX$ events can be isolated for one or more final states $F$ into which the Higgs decays. Here, we will consider the $F = \gamma\gamma$ and $F = b\bar{b}$ final states originally discussed in Refs. [3] and [4], respectively. The events will consist of $t\bar{t}hX$ ($h \to F$) signal events and background events. (For a light $h$, our focus here, interference between signal and background amplitudes can be neglected.) The $pp \to t\bar{t}FX$ cross section takes the form

$$\Sigma(\phi) \equiv \frac{d\sigma}{d\phi} = C \left[ a^2 f_a(\phi) + b^2 f_b(\phi) \right] + f_B(\phi), \quad (1)$$

where $\phi$ denotes the final state phase space configuration, $C$ is a coefficient determined by experimental efficiencies, $BF(h \to F)$ etc., and $a$ and $b$ are the CP-even and CP-odd Higgs couplings appearing in the Feynman rule,

$$t\bar{t}h: \quad -\bar{t}(a + ib\gamma_5)\frac{g_{\gamma t}}{2m_W}. \quad (2)$$

(The SM Higgs boson has $a = 1$ and $b = 0$. A purely CP-odd Higgs boson has $a = 0$ and $b \neq 0$.) We note that there is no $ab$ cross term in Eq. (1) since we do not consider any observable sensitive to the spin directions of the top quarks. Further, since both the $a^2$ and $b^2$ terms are of a CP-conserving nature, the functions $f_a$ and $f_b$ do not change when the $t$ and $\bar{t}$ momenta are interchanged.

In Eq. (1), $f_{a,b}(\phi)$ are, in principle, precisely known functions of $\phi$ and have well-determined relative normalization. The only possible uncertainties in the $f_{a,b}$ arise through their dependence on quark/gluon distribution functions; we will assume that the relevant distribution functions will have been determined with sufficient precision, using other LHC data and data from other accelerators, that these uncertainties can be neglected. We also assume that the background contribution $f_B(\phi)$ will be precisely determined using experimental data outside but near the Higgs mass peak in the final state $F$. This precise measurement will include an accurate determination of both the functional form and the normalization, including all experimental efficiencies. Statistical fluctuations in the background within the accepted Higgs mass bin in the state $F$ are automatically incorporated in the procedure we will employ.

# 2 The Optimal Analysis Procedure and Results

As in Ref. [1], we wish to discriminate between models purely on the basis of the difference between the phase space distribution dependence of $f_a$ and $f_b$, not
relying on knowledge of $C$. To this end, it will be convenient to define the rescaled functions $\hat{f}_{a,b,B}(\phi) \equiv f_{a,b,B}(\phi)/I_{a,b,B}$ where $I_{a,b,B} \equiv \int f_{a,b,B}(\phi)d\phi$ so that $\int \hat{f}_{a,b,B} = 1$. (The $\phi$ integral will be restricted by appropriate cuts.) Defining $r \equiv I_b/I_a$ and $\alpha \equiv a^2/(a^2 + rb^2)$, we may rewrite Eq. (1) as

$$\Sigma(\phi) = \sigma_S \left[ \alpha f_\alpha(\phi) + \hat{f}_b(\phi) \right] + \sigma_B \hat{f}_B(\phi) ,$$

where $f_\alpha(\phi) \equiv [\hat{f}_a(\phi) - \hat{f}_b(\phi)]$ ($\int f_\alpha(\phi)d\phi = 0$) and $\sigma_S = C(a^2I_a + b^2I_b)$ is the effective integrated signal cross section. The background cross section is $\sigma_B = I_B$.

Clearly, the ratio $\alpha$ is independent of the overall signal normalization (although errors in $\alpha$ will certainly depend on the signal and background rates). We now use the optimal analysis technique of Ref. [2] to provide a determination of $\alpha$ with the smallest possible error (in the Gaussian statistics approximation).

In general, for $\Sigma(\phi) = \sum_i c_i f_i(\phi) + g(\phi)$ (where $g(\phi)$ and the $f_i(\phi)$ are known functions, including normalization) the determination of the unknown signal coefficients $c_i$ with smallest possible statistical error is given by [2]

$$c_i = \sum_k M^{-1}_{ik} I_k , \quad \text{where} \quad M_{ik} \equiv \int \frac{f_i(\phi)f_k(\phi)}{\Sigma(\phi)}d\phi , \quad \text{and} \quad I_k \equiv \int f_k(\phi)d\phi .$$

The covariance matrix for the $c_i$ is

$$V_{ij} \equiv \langle \Delta c_i \Delta c_j \rangle = M^{-1}_{ij}\sigma_T/N = M^{-1}_{ij}/L_{eff} ,$$

where $\sigma_T = \int \frac{ds}{d\phi}d\phi$ is the integrated cross section and $N = L_{eff}\sigma_T$ is the total number of events, with $L_{eff}$ being the luminosity times efficiency. This result is the optimal one regardless of the relative magnitudes of the different contributions to $\Sigma(\phi)$. In the Gaussian statistics limit, it is equivalent to determining the $c_i$ by maximizing the likelihood of the fit to the full $\phi$ distribution of all the events.

The increase in errors due to statistical fluctuations in the presence of background are implicit in the background contribution to $\Sigma(\phi)$ appearing in Eq. (1), which implies larger $M^{-1}_{ij}$ entries in Eq. (5). From the result of Eq. (5), the $\chi^2$ in the $c_i$ parameter space is then computed as

$$\chi^2 = \sum_{i,j} (c'_i - c_i)V^{-1}_{ij}(c'_j - c_j) = \sum_{i,j} (c'_i - c_i)L_{eff}M_{ij}(c'_j - c_j) ,$$

where, for our theoretical analyses, the $c_i$ are the input model values for which $V_{ij}$ is computed.4

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accepted region. If a subset, $\hat{\phi}$, of the kinematical variables $\phi$ cannot be determined, then the optimal technique can be applied using the variables, $\tilde{\phi}$, that can be observed and the functions $\bar{f}_i(\tilde{\phi}) \equiv \int f_i(\phi)d\tilde{\phi}$.

We apply the above general technique to the specific case in hand by identifying the signal function $f_1 = \sigma_S f_a$ and the known function $g(\phi) = \sigma_S \hat{f}_b(\phi) + \sigma_B \tilde{f}_B(\phi)$. (The normalizations are fixed by the experimental measurements of the signal and background rates, which include all efficiencies etc.) The $\chi^2$ associated with a model yielding a value $\alpha'$ that differs from the value $\alpha$ of the input model is then given by the following simple result:

$$\chi^2 = (\alpha' - \alpha)^2 \frac{S^2}{B} \int d\phi \frac{f_a^2(\phi)}{\sigma_S[\alpha f_a(\phi) + \hat{f}_b(\phi)] + \tilde{f}_B(\phi)} \equiv (\alpha' - \alpha)^2 \frac{S^2}{B} H(\alpha, S/B)^2,$$  \hspace{1cm} (7)

where the expected signal rate $S$ is that estimated (and eventually measured) for the input (true) model. It is important to note that $H(\alpha, S/B)$ depends implicitly on the channel $F$ being considered. Defining the discrimination power, $D \equiv \sqrt{\chi^2}$, we have

$$D = |\alpha' - \alpha| \frac{S}{\sqrt{B}} H(\alpha, S/B)$$  \hspace{1cm} (8)

Note that in the limit of small $S/B$,

$$H(\alpha, S/B) \to H \equiv \left[ \int d\phi \frac{f_a^2(\phi)}{\tilde{f}_B(\phi)} \right]^{1/2}. \hspace{1cm} (9)$$

In applying the above to the $pp \rightarrow t\bar{t}FX$ process, we only consider the kinematical variables associated with the $t\bar{t}F$ portion of the final state, including its $\sqrt{\hat{s}}$ (where $\hat{s}$ is the invariant mass-squared for the $t\bar{t}F$ combination) and overall rapidity. Any variables associated with the internal phase space of $X$ are inclusively summed over as usual (and fall into the $\tilde{\phi}$ category noted above). Also, distributions for the decay products of the $t$-quark and the Higgs boson are not considered.

In order to establish that we are, in fact, dealing with the $t\bar{t}F$ final state and completely determine the kinematical point $\phi$ in phase space, we must be able to reconstruct and identify the $t$ and $\bar{t}$. However, identification of $t$ vs. $\bar{t}$ is not actually necessary since, as noted earlier, our functions $f_a$ and $f_b$ do not change if the $t$ and $\bar{t}$ momenta are interchanged. Thus, in principle we can use either the final state mode in which both the $t$ and $\bar{t}$ decay completely hadronically or that in which one has a leptonic $W$ decay and the other a hadronic $W$ decay.\footnote{\hspace{1cm}2The final state mode in which the $W$'s from both the $t$ and the $\bar{t}$ decay leptonically does not allow full determination of the kinematics of the final state.} In this paper, we choose to focus on the latter (mixed hadronic-leptonic) mode as being possibly somewhat cleaner. However, we anticipate that the purely hadronic final state could be used to further increase our statistics.
In the final state mode in which one $t$ decays leptonically and the other hadronically, there is a potential two-fold ambiguity when determining the longitudinal momentum of the missing neutrino by requiring that the leptonically-decaying $W$ be on-shell. However, the wrong solution gives a reconstructed top-quark mass that differs from the known value, $m_t$. If we assume that energy resolutions and such will allow reconstruction of $m_t$ to within $\pm 10$ GeV for the correct solution, then we can identify the correct solution in those events for which the incorrect solution gives a reconstructed top-quark mass that differs by more than 10 GeV from $m_t$. About 70% of the events that obey the global cuts specified below satisfy this criterion. We accept only such events. The global cuts that we impose on the $t\bar{t}h$ final state are such that there should be reasonable efficiency for the required reconstructions. We require $|y_{t,\bar{t},h}| < 4$. A cut of $p_T > m_h/4$ is applied to the $\gamma$'s or $b$'s in the $F$ final state in order to reduce the size of the background. These are the same cuts as employed in Ref. [1].

We first give results for a Higgs mass of $m_h = 100$ GeV, so as to allow direct comparison with Ref. [1]. (We will discuss later the sensitivity of our results to the $|y|$ cut and to $m_h$.) For the above cuts and this mass, we find $r = 0.288$. (Note that $r$ is independent of the final state channel $F$.)

As already noted, we will consider the $F = \gamma\gamma$ and $F = b\bar{b}$ decay modes for the $h$. The $F = \gamma\gamma$ mode will only be considered in the case where the input model corresponds to a SM-like $h$, since it is only in this case that $BF(h \to \gamma\gamma)$ is likely to be substantial. Following Ref. [3], the LHC ATLAS [5] and CMS [6] collaborations performed detailed simulations of this channel as part of their technical design reports. Based on their results, and assuming an eventual integrated luminosity of $L = 600$ fb$^{-1}$ (summing over the two detectors), Ref. [4] concluded that one could expect $S \sim 130$ and $B \sim 21$ in the $F = \gamma\gamma$ mode for a SM-like $h$ with mass of order 100 GeV. For $F = b\bar{b}$, the best global cuts and associated levels of signal and background have not yet been firmly established. We will adopt the procedure of Ref. [1] of examining results for several different extremes of $S/B$, assuming a certain level of statistical significance $S/\sqrt{B}$. In this way, we avoid making any specific assumption about the branching ratio for $h \to b\bar{b}$ decay, which is, in any case, absorbed into the overall normalization factor $C$ of Eq. (1). For both $F = \gamma\gamma$ and $F = b\bar{b}$, we will follow Ref. [1] and model the background cross section shape $f_B(\phi)$ [see Eq. (1)] using the irreducible $pp \to t\bar{t}\gamma\gamma X$ and $t\bar{t}b\bar{b}X$ background processes, respectively. In the case of $F = \gamma\gamma$, the ATLAS and CMS studies confirm the theoretical claims [3] that the irreducible background should, indeed, be dominant. In the case of $F = b\bar{b}$, minimum-bias and other backgrounds may enter [4]. To the extent that the functional form (in the kinematical variables $\phi$) of the sum of such backgrounds is substantially different from the $f_B(\phi)$ calculated for the irreducible backgrounds, the sensitivities that we compute in this paper as a function of $S/B$ and $S/\sqrt{B}$ will have to be re-evaluated by the experimental collaborations. We re-emphasize that in the actual experiment $f_B(\phi)$ will be directly measured using data for which the $b\bar{b}$ mass is outside the Higgs mass.
In order to assess our ability to determine that a CP-even SM-like Higgs boson is indeed CP-even, we will follow the procedure of Ref. [1] and consider three distinct Higgs coupling cases. We compare [recalling that $\alpha = a^2/(a^2 + r b^2)$]

- I) A Standard-Model-like Higgs boson, with $a \neq 0$ and $b = 0$, i.e. $\alpha_I = 1$, to:
- II) a CP-mixed Higgs boson, with $a = b$, i.e. $\alpha_{II} = 1/(1 + r)$; and,
- III) a pure CP-odd Higgs boson, with $a = 0$, $b \neq 0$, i.e. $\alpha_{III} = 0$.

In order to compare with the results of Ref. [1], we will compute the discrimination powers $D_{1,2} = |\alpha_{II,III} - \alpha_I| (S/\sqrt{B}) H(\alpha_I, S/B)$ defined in Eq. (8). These are a measure of our ability to ascertain that a CP-even SM-like Higgs boson is not CP-mixed or CP-odd, respectively. In comparing, we use only the best-case weighting functions $O_{CP}$ considered in Ref. [1]. For $F = \gamma\gamma$ the best choice for $O_{CP}$ was denoted by $O_{CP} = b_1$; for $F = b\bar{b}$ the best choice was denoted by $O_{CP} = b_4$.

Table 1: $t\bar{t}\gamma\gamma$ channel, $m_h = 100$ GeV: Discrimination powers $D_1$ and $D_2$ for distinguishing a SM-like Higgs from a CP-mixed Higgs and from a CP-odd Higgs, respectively. Results from Ref. [1] are compared to those obtained using the optimal analysis.

| Technique | $b_1$ [1] | optimal |
|-----------|-----------|----------|
| $D_1(B = 0)/\sqrt{S}$ | 0.137 | 0.376 |
| $D_2(B = 0)/\sqrt{S}$ | 0.663 | 1.68 |

| Technique | $b_1$ [1] | optimal |
|-----------|-----------|----------|
| $D_1(S = 130, B = 21)$ | 1.47 | 3.68 |
| $D_2(S = 130, B = 21)$ | 7.10 | 16.5 |

Consider, first, $F = \gamma\gamma$ in the limit of $B = 0$. Table 1 gives $D_{1,2}(B = 0)/\sqrt{S}$, a first measure of the potential power for discrimination between the models; we see a large improvement relative to the $O_{CP} = b_1$ weighting procedure by using the optimal techniques. Table 1 also shows the actual levels of discrimination that can be achieved if we adopt the LHC SM Higgs boson estimates of $S = 130$ and $B = 21$. Of course, since the total number of events available is not large, non-Gaussian effects in the statistics that are not incorporated in our approach will prevent achieving results quite this good.

We turn now to the $F = b\bar{b}$ channel. The discrimination power $D$ for separating models is determined as a function of $S/B$ using Eq. (8) and $H(\alpha_I, S/B)$ (computed for the $b\bar{b}$ channel) as plotted in Fig. 1. In Table 2, we give the number of signal events required to achieve $D_{1,2} = 2$ for $B/S = 50$, and the corresponding $S/\sqrt{B}$ values. Also given are the results obtained using the optimal $O_{CP} = b_4$ observable of Ref. [1]. The improvement using the optimal techniques is very remarkable; $D_{1,2} = 2$ discrimination is possible with far fewer events and much lower $S/\sqrt{B}$ as
Table 2: $t\bar{t}b\bar{b}$ channel, $m_h = 100\text{ GeV}, B/S = 50$: signal event rates, and corresponding $S/\sqrt{B}$ values, required to achieve $D_{1,2} = 2$ using the best $\mathcal{O}_{CP} = b_4$ observable of Ref. [1] as compared to the optimal approach.

|        | $D_1(B/S = 50) = 2$ | $D_2(B/S = 50) = 2$ |
|--------|---------------------|---------------------|
|        | $S$                | $S/\sqrt{B}$       | $S$                | $S/\sqrt{B}$       |
| $b_4$  | 15000              | 17                  | 970                | 4.5                |
| optimal| 1592               | 5.64                | 80                 | 1.26               |

Figure 1: We plot $H(\alpha, S/B)$ for the $t\bar{t}b\bar{b}$ final state as a function of $S/B$ for the three cases: $\alpha = \alpha_I$ (solid); $\alpha = \alpha_{II}$ (dashes); $\alpha = \alpha_{III}$ (dots).

compared to Ref. [1]. For large $B/S$, the results in the $b\bar{b}$ final state for $D_{1,2}$ are easily summarized:

$$D_1 = C_1 \frac{S}{\sqrt{B}}, \quad D_2 = C_2 \frac{S}{\sqrt{B}},$$

with $C_1 = |\alpha_{II} - \alpha_I|H = 0.36$ and $C_2 = |\alpha_{III} - \alpha_I|H = 1.61$, where $H = 1.61$ is the result from Eq. (3). These values of $C_1$ and $C_2$ can be compared to the values 0.091 and 0.455 achieved in Ref. [1] using $\mathcal{O}_{CP} = b_4$; the latter obviously imply much weaker discrimination power.

A special remark is in order regarding the small $S/\sqrt{B} = 1.26$ value required (see Table 2) for $D_2 = 2$ in the $t\bar{t}b\bar{b}$ channel using the optimal approach. If the Higgs boson mass is already known from observation in another channel (e.g. $t\bar{t}\gamma\gamma$), then one can perform the CP analysis for $t\bar{t}b\bar{b}$ events falling into the appropriate $b\bar{b}$ mass bin (assuming mass calibrations are well understood) even though a distinct bump is not observed. In this way, one could confirm and possibly improve CP
results obtained in the channel for which a mass bump is manifest.

Although the $\gamma\gamma$ channel is not likely to be useful for Higgs bosons that do not have a large CP-even component, the $b\bar{b}$ channel is likely to be of very general utility. Light Higgs bosons will almost always have a large branching ratio for decays to $b\bar{b}$. Given the large $t\bar{t}b\bar{b}$ background rate, $B/S$ will generally be large. Thus, it is useful to give the $H(\alpha, S/B)$ values, from which the discrimination power $D$ of Eq. (8) can be computed, for other models. The results for $H(\alpha_{II}, S/B)$ and $H(\alpha_{III}, S/B)$ appear along with those for $H(\alpha_{I}, S/B)$ in Fig. 1. At $S/B = 1/50$, all are of order $\sim 1.55$. They approach the common limit of $H = 1.61$. Using a lower bound of $H(\alpha, S/B) \geq 1.55$, we conclude that a marginal signal at the level of $S/\sqrt{B} = 3$ will result in discrimination power $D \geq 4.5|\alpha' - \alpha|$, regardless of the value $\alpha$ for the input model. This is quite an encouraging result in that for any Higgs boson that is observable in the $t\bar{t}b\bar{b}$ channel one can hope to determine the mix of CP-even and CP-odd Higgs couplings to $t\bar{t}$ at a very useful level.

Table 3: Ratio $r = I_b/I_a$ of cross section contributions arising from the CP-odd $b^2$ term and CP-even $a^2$ term for various $|y|$ cut and $m_h$ (in GeV) choices.

| Case | $|y| < 4$ | $|y| < 2$ | $|y| < 4$ | $|y| < 2$ |
|------|-----------|-----------|-----------|-----------|
|      | $m_h = 100$ | $m_h = 100$ | $m_h = 130$ | $m_h = 130$ |
| $r$  | 0.288     | 0.243     | 0.447     | 0.362     |

Table 4: $t\bar{t}\gamma\gamma$ channel: $D_1$ and $D_2$ discrimination powers obtained using the optimal technique, for the $|y|$ cut and $m_h$ values considered in Table 3.

| Case                             | $|y| < 4$ | $|y| < 2$ | $|y| < 4$ | $|y| < 2$ |
|----------------------------------|-----------|-----------|-----------|-----------|
|                                  | $m_h = 100$ | $m_h = 100$ | $m_h = 130$ | $m_h = 130$ |
| $D_1(B = 0)/\sqrt{S}$            | 0.376     | 0.292     | 0.449     | 0.336     |
| $D_1(S = 130, B = 21)$           | 3.68      | 3.07      | 4.31      | 3.50      |
| $D_2(B = 0)/\sqrt{S}$            | 1.68      | 1.49      | 1.45      | 1.26      |
| $D_2(S = 130, B = 21)$           | 16.5      | 15.7      | 14.0      | 13.2      |

We now explore sensitivity of these results to the $|y|$ cut and to $m_h$. Since the $|y| < 4$ rapidity cut for the $t$, $\bar{t}$ and $h$ may be too generous to allow for full reconstruction in the relatively complicated t$\bar{t}$h final state, we also consider $|y| < 2$. We will explore sensitivity to $m_h$ by giving results for $m_h = 130$ GeV. First, we give the $r = I_b/I_a$ values, appropriate in all the cases considered, in Table 3. Note that as $m_h$ increases the overall signal cross section contribution ($I_a$) of the CP-odd
The $b^2$ term increases relative to the contribution ($I_a$) of the CP-even $a^2$ term, implying larger $r$ values at larger $m_h$. Next, in Table 4 we give the discrimination powers $D_{1,2}$ in the $\gamma\gamma$ mode. For fixed values of $S = 130$ and $B = 21$, the stronger $|y| < 2$ cut gives $D_{1,2}$ values that are not so much smaller than for $|y| < 4$. However, this masks the fact that only 1/3 as many events are accepted for $|y| < 2$ as compared to $|y| < 4$. Reducing $S$ and $B$ by a factor of 3 (for $|y| < 2$) from $S = 130$ and $B = 21$ would yield $D_{1,2}$ values that are a factor of $1/\sqrt{3}$ smaller than given in Table 4. Regarding $m_h$ dependence, note that, keeping $S$ and $B$ constant, $D_1$ ($D_2$) is larger (smaller) for $m_h = 130$ GeV as compared to $m_h = 100$ GeV. The larger $D_1$ is because $|\alpha_{II} - \alpha_I|$ is larger (larger $r$) at $m_h = 130$ GeV. This contrasts with the case of $D_2$ for which $|\alpha_{III} - \alpha_I| = 1$ independent of $m_h$.

Table 5: $t\bar{t}b\bar{b}$ channel: signal event rates, and corresponding $S/\sqrt{B}$ values, required to achieve $D_{1,2} = 2$ using the optimal approach.

| $(|y|, \text{Cut}, m_h)$ | $D_1(B/S = 50) = 2$ | $D_2(B/S = 50) = 2$ |
|-----------------|-----------------|-----------------|
| $(4,100)$       | 1592 $S$        | 80 $S$          |
| $(2,100)$       | 3215 $S$        | 123 $S$         |
| $(4,130)$       | 932 $S$         | 89 $S$          |
| $(2,130)$       | 2018 $S$        | 143 $S$         |

Table 6: $t\bar{t}b\bar{b}$ channel: asymptotic $H$ values and corresponding values of $C_1$ and $C_2$, as defined in Eq. (10).

| Case | $|y| < 4$ $m_h = 100$ | $|y| < 2$ $m_h = 100$ | $|y| < 4$ $m_h = 130$ | $|y| < 2$ $m_h = 130$ |
|------|-----------------|-----------------|-----------------|-----------------|
| $H, C_2$ | 1.61            | 1.29            | 1.53            | 1.20            |
| $C_1$    | 0.360           | 0.252           | 0.473           | 0.319           |

Some comparative results in the $t\bar{t}b\bar{b}$ channel are given in Tables 5 and 6. The $H(\alpha, S/B)$ values for $m_h = 100$ GeV and $|y| < 2$ were given in Fig. 4. For $S/B = 1/50$, one finds $H(\alpha_I, 1/50) = 1.27$, $H(\alpha_{II}, 1/50) = 1.27$, and $H(\alpha_{III}, 1/50) = 1.24$, a significant deterioration from the > 1.55 values found for $|y| < 4$. The $S/B \to 0$ limit is $H = 1.29$, again significantly smaller than the 1.61 value obtained with $|y| < 4$ cuts. More limited acceptance implies that more signal rate (which is harder to achieve with more limited acceptance, e.g. $|y| < 2$ accepts only 1/3 as many events as $|y| < 4$) and larger $S/\sqrt{B}$ is required for good discrimination between the various models. Clearly it is important to accept as much of the phase
space as possible, both to maximize the signal rate and to maximize the region over which shape differences between models can be explored. The trends with $m_h$ are similar to the $\gamma\gamma$ channel, with $D_1$ ($D_2$) being larger (smaller) for $m_h = 130$ GeV as compared to $m_h = 100$ GeV, for the same $S/\sqrt{B}$.

3 The MSSM Higgs Bosons

A Higgs boson sector of particular interest is that of the minimal supersymmetric model (MSSM) \[8\]. If the mass of the CP-odd $A^0$ of the model is $> 150$ GeV, then the light CP-even $h^0$ will be SM-like, and our $t\bar{t}h$ analysis will apply in both the $\gamma\gamma$ and $b\bar{b}$ channels. However, the heavier CP-even $H^0$ and the $A^0$ will probably present a greater challenge. If the tan $\beta$ parameter of the MSSM is near 1, then the $t\bar{t}H^0, t\bar{t}A^0$ couplings, and corresponding LHC production rates, will be substantial for modest $m_{H^0}, m_{A^0}$. Depending upon how easily they can be discovered in the relevant decay channels (such as $H^0, A^0 \rightarrow b\bar{b}, \tau^+\tau^-$, $H^0 \rightarrow h^0 h^0, A^0 \rightarrow h^0 Z$, and, for $m_{H^0}, m_{A^0} \geq 2m_t$, $H^0, A^0 \rightarrow t\bar{t}$ \[4\] and upon how small $S/B$ in each channel is, our $t\bar{t}h$ procedures could be useful. We do not attempt a detailed study here.

If the tan $\beta$ parameter of the model is large and $m_{A^0} \sim m_{H^0} > 150$ GeV, then the $t\bar{t}H^0$ and $t\bar{t}A^0$ couplings are suppressed by a factor of $1/\tan \beta$ and those to $b\bar{b}$ (and $\tau^+\tau^-$) are enhanced by a factor of $\tan \beta$. As a result, the primary LHC production modes are $pp \rightarrow b\bar{b}H^0X, b\bar{b}A^0X$, where both $H^0$ and $A^0$ decay to $b\bar{b}$ ($\tau^+\tau^-$) with a branching fraction of order 0.9 (0.1). In this case, only the $b\bar{b}b\bar{b}$ and $b\bar{b}\tau^+\tau^-$ final states are relevant for detecting the $H^0$ and $A^0$. Both ATLAS \[3\] and CMS \[3\] claim that $S/\sqrt{B} = 5$ discovery of the $H^0, A^0$ is possible (for full ‘three-year’ $L = 300$ fb$^{-1}$ integrated luminosity per detector) in the $b\bar{b}\tau^+\tau^-$ mode provided tan $\beta$ is above a $m_{A^0}$-dependent lower limit ranging from tan $\beta \sim 2$ at $m_{A^0} \sim m_{H^0} \sim 150$ GeV up to tan $\beta \sim 20$ at $m_{A^0} \sim m_{H^0} \sim 500$ GeV. Prospects in the $b\bar{b}b\bar{b}$ final state are more controversial. The $b\bar{b}\tau^+\tau^-$ mode is preferable for our purposes in any case because the $\tau$’s allow us to identify the Higgs boson without any combinatoric uncertainty. The question, then, is whether or not our techniques will allow verification of the CP$=+$ and CP$=-$ nature of the $H^0$ and $A^0$, respectively.

First, and very crucially, we do not need to identify $b$ vs. $\bar{b}$ due to the fact (see earlier) that $f_a(\phi)$ and $f_b(\phi)$ are independent under interchange of the $b$ and $\bar{b}$ momenta. However, there are two significant difficulties. First, in Eq. \[1\] $f_a(\phi) \simeq f_b(\phi)$ in the limit where the quark mass approaches zero. (That is, $\Sigma(\phi) \propto a^2 + b^2$ in this limit, the $a^2 - b^2$ term being suppressed by $\sim m_{\text{quark}}/\sqrt{s}$ in comparison.) Consequently, a very large number of events will be required for a statistically useful determination of the $a^2 - b^2$ coefficient. Second, mass resolution in the $\tau^+\tau^-$ (or $b\bar{b}$) channel will typically not be adequate for separating the $H^0$ and $A^0$ mass peaks, implying that one could at best verify that $a^2 - b^2 = 0$ after averaging over

\[^3\]Their $\gamma\gamma$ branching ratios are typically too small to be useful.
the $H^0$ and $A^0$ (given that the $H^0$ and $A^0$ have the same production rate at large $\tan \beta$). Our first estimates suggest that, for $\tan \beta$ values such that discovery is possible, a statistically meaningful verification of the $a^2 = b^2$ expectation relative to pure $a^2 = 0$ or pure $b^2 = 0$ might be possible, despite the small $m_b/\sqrt{s}$ coefficient multiplying $a^2 - b^2$. A careful study of this issue by the detector collaborations — *i.e.* one that includes all resolutions, minimum-bias backgrounds, and so forth — is needed to properly assess the prospects.

4 Final Remarks and Conclusion

In this paper, we have employed the optimal analysis procedures of Ref. [2] to determine the accuracy with which the relative magnitude of the CP-even and CP-odd $t\bar{t}$ couplings of a light Higgs boson can be determined using the final state kinematical distributions of the $t$, $\bar{t}$ and $h$ relative to one another in LHC $pp \to t\bar{t}h$ events. We have found that the optimal analysis procedure yields a large improvement in the statistical discrimination power ($D \equiv \sqrt{\chi^2}$) as compared to the simpler weighting function techniques previously considered in Ref. [1].

We emphasize that we have employed a discrimination measure that does not make use of the (substantial) sensitivity of the absolute production rate to the CP-even versus CP-odd couplings; this is desirable since absolute rates have additional systematic uncertainties associated with model dependence for branching ratios, experimental efficiencies and the like. However, an estimate or, preferably, experimental measurement of the signal to background ratio is needed in order to compute quantitative results for the $\chi^2$ associated with a model that is different from the input or true model.

It is useful to note that in the actual experimental analysis an equivalent of the optimal analysis procedure employed here would be to determine the likelihood of the fit, as a function of the $a$ and $b$ parameters, to the detailed phase space distribution (allowing the overall normalization to be a free parameter). In the limit of Gaussian statistics, this and the theoretically much more convenient optimal procedure employed here are equivalent: the ratio of the likelihood of a fit in an incorrect model to that in the correct model will be equal to $\exp[-\chi^2]$, where the $\chi^2$ is the $\chi^2$ of an incorrect model choice as computed in the optimal procedure using inputs based on the correct model.

Our conclusion is that if a Higgs signal is observable in the $t\bar{t}h$ ($h \to F$, $F = b\bar{b}$ or $\gamma\gamma$) final states at the LHC, then the relative size of the CP-even and CP-odd couplings of the $h$ to $t\bar{t}$ can be determined at a statistically useful level. In particular, in the case of a SM-like Higgs boson, we find that Higgs CP coupling mixtures that are significantly different from pure CP-even can generally be excluded at a high level of statistical significance.

Finally, we noted that there is reason to hope that the techniques studied here could be employed for studying the CP nature of the heavier Higgs bosons of
the minimal supersymmetric model. In particular, for tan $\beta$ values large enough that a signal is observable in the $b\bar{b}\tau^+\tau^-$ final state coming from $b\bar{b}H^0$ and $b\bar{b}A^0$ production (with $H^0, A^0 \rightarrow \tau^+\tau^-$), one can hope to verify that the overlapping $H^0$ and $A^0$ resonances yield the expected equal mixture of CP-even and CP-odd signals.

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