The search algorithm for discrete mode of electron plasma

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Abstract. The question about the shielding of external electric field electron plasma is examined. The search algorithm for discrete modes of electron plasma have been described. The case of arbitrary degree of degeneracy of the electron gas have been considered. Special attention is paid to the case when the degree of degeneracy of the electron gas is large. Have been demonstrated an effective way of finding the discrete mode in this case.

1. Introduction

The question about the response of the plasma on the electric field has a long history [1-7]. But a number of fundamental issues still remain unresolved. It refers to the nature of the shielding of electric field by plasma, with varying degrees of degeneration of the electron gas.

In earlier works [8, 9], the authors concerned the question of the existence of a plasma mode. This mode is also known as a Debye mode. This mode is a discrete root of the dispersion function. In the present work are described in detail the search algorithm of this discrete root. Our results are correct for the general case of plasma with an arbitrary degree of degeneracy of the electron gas.

We proceed from the analytical solution of the problem of the behavior of plasma in an alternating electric field. From this solution it is possible to find the dispersion function. Properties of dispersion functions determine the behavior of discrete mode. Our goal is to find the algorithm that gives the possibility to define a discrete mode according to the properties of the dispersion functions.

2. Statement of the problem and the main equations

In electronic plasma with arbitrary degree of degeneracy the dispersion function is as follows:

$$\Lambda(z) = \Lambda(z, \Omega, \varepsilon) = 1 + \frac{1}{w_0 \eta_1^2} \int_{-\infty}^{+\infty} \frac{\eta_1^2 - tz}{t - z} k(t, \alpha) \, dt.$$  \hspace{1cm} (1)

Here $\Omega$ is the frequency of the external field divided by the plasma frequency, $\varepsilon$ is the collision frequency divided by the plasma frequency, $w_0$, $\eta_1^2$ – some complex constants that depend on plasma parameters, $k(t, \alpha)$ is a real core, depending on the plasma parameters, $z$ is a complex variable, integration is carried out on the real axis.

To establish the existence of finite complex zeros of the dispersion function, we use the argument principle. As on the real $y$ axis the dispersion functions have a peculiarity, then to eliminate this peculiarity it is necessary to conduct a cut along the specified axis. Because of the evenness of the dispersion function we can examine only the upper half-plane as the zeros, if they exist, will differ only by a sign. Take in the upper half plane contour $\gamma_{E}$. It goes in a positive direction and consists of a semicircle of radius $R=1/\varepsilon$ and of the line $\gamma_{E}$, separated from the real axis ($-\infty; +\infty$) at a distance $\varepsilon$.
Contour arbitrarily closely approaches the real axis when \( \epsilon \to 0 \). Because of the argument principle the number of zeros in the region bounded by the contour is equal to

\[
N = \frac{1}{2\pi i} \oint \frac{d\ln \Lambda(z)}{z}.
\]

Let us turn to the limit in this equality when \( \epsilon \to 0 \). Take into account that the dispersion function is analytical in a neighbourhood of an infinitely distant point, and also the property \( \Lambda^\ast(\cdot z) = \Lambda(\cdot z) \). Then we get

\[
N = \frac{1}{2\pi i} \oint \frac{d\Lambda^\ast(z)}{\Lambda(z)}.
\]

Consider depending on a parameter \( \mu \), the curve \( G(\mu) = \Lambda^\ast(\mu)/\Lambda(\mu) \). The number of zeros \( N \) is equal to the number of revolutions of the curve around the origin. Separate out of function \( G(\mu) \) the real and imaginary parts. Represent it in the form:

\[
G(\mu) = \frac{(w_0 - 1)\eta_1^2 + (\eta_1^2 - \mu^2)\lambda(\mu) + is(\mu)(\eta_2^2 - \mu^2)}{(w_0 - 1)\eta_1^2 + (\eta_2^2 - \mu^2)\lambda(\mu) - is(\mu)(\eta_2^2 - \mu^2)}
\]

Here \( s(\mu) = \pi \mu/2 \).

Given that

\[
w_0 = 1 - i\frac{\alpha}{\nu}, \quad \eta_1^2 = \epsilon(\nu - i\Omega)r(\alpha),
\]

we obtain

\[
P^\pm(\mu) = \Omega^2 r(\alpha) + (\mu^2 - \epsilon^2 r(\alpha))\lambda(\mu) \pm s(\mu)\epsilon\Omega r(\alpha),
\]

\[
Q^\pm(\mu) = \epsilon\Omega r(\alpha)(\lambda(\mu) + 1) \pm s(\mu)(\epsilon^2 r(\alpha) - \mu^2).
\]

Select the real and imaginary parts of the function \( G(\mu) \):

\[
G(\mu) = \frac{G_1(\mu)}{G_0(\mu)} + i \frac{G_2(\mu)}{G_0(\mu)}
\]

Here

\[
G_0(\mu) = (P^+(\mu))^2 + (Q^+(\mu))^2,
\]

\[
G_1(\mu) = P^-(\mu)P^+(\mu) + Q^-(\mu)Q^+(\mu),
\]

\[
G_2(\mu) = P^+(\mu)Q^-(\mu) - P^-(\mu)Q^+(\mu).
\]

Consider the curve \( L \), defined implicitly by parametric equations:

\[
L: G_1(\mu, \Omega, \epsilon) = 0, \quad G_2(\mu, \Omega, \epsilon) = 0.
\]

Expressing these equations in the explicit form of \( \epsilon \) and \( \Omega \), we obtain parametric equations for the curve \( L(\alpha, \epsilon, \Omega) \).

On the borders of change of parameter \( \mu \) must mention, because there is a critical point \( \mu^* \) from which the parametric equations for the curve have solutions. In other words, to implement the curve \( L \), the parameter \( \mu \) should vary from this value, \( \mu^* \) to infinity.

After some transformations, we obtain the resulting expression for \( \Omega^2 = L_1, \epsilon^2 = L_2 \):

\[
L(\alpha): \quad \Omega = \sqrt{L_1(\mu, \alpha)}, \quad \epsilon = \sqrt{L_2(\mu, \alpha)}, \quad \mu^* \leq \mu \leq +\infty,
\]

where

\[
L_1(\mu, \alpha) = \frac{s_0(\alpha)}{s_2(\alpha)} \cdot \frac{\mu^2[\lambda(\mu, \alpha)(1 + \lambda(\mu, \alpha)) + s^2(\mu, \alpha)]}{(\lambda(\mu, \alpha)[1 + \lambda(\mu, \alpha)]^2 + s^2(\mu, \alpha)}.
\]
 Functions \( L_1(\mu,\alpha) \) and \( L_2(\mu,\alpha) \) determine in an explicit parametric form a curve \( L \) that is the boundary of the region \( D^+ \) (the area where a discrete root of the dispersion function exists) (fig.1). With the increase of the parameter \( \mu \) movement along the curve happens from right to left. If \( \mu \to \mu^* \), this corresponds to infinitely large values of \( \varepsilon \) and \( \Omega \). Further, the curve approaches the origin, at some point (around \( \varepsilon=1.8 \)). The numerator of \( L_1(\mu,\alpha) \) becomes zero, the curve touches the axis \( \varepsilon \). Then the curve behaves as a smooth line, and for \( \varepsilon \to 0 \) the corresponding value for \( \Omega \to 1, \mu \to +\infty \). It is possible to plot the curve \( G(\mu) \), which illustrate conclusions about the presence of the root. If \( (\Omega, \varepsilon) \in D^+ \), the curve \( G(\mu) \) rotates around the origin, and if \( (\Omega, \varepsilon) \in D^- \) the closed curve does not passes the origin. In [3] and [8], a study was conducted at the border, when \( (\Omega, \varepsilon) \in L \). Below we will analyze analogous cases to this problem.

Since the physical meaning have non-negative frequency of the electric field and the frequency of collisions in the plasma, then the image of the curve \( L \) can be limited to the first quarter of the plane \( (\Omega, \varepsilon) \). In Fig. 1 one can see how the curve \( L \) divides it into two regions, \( D^+ \) and \( D^- \). Figure 2 shows the behavior of the curve \( G(\mu) \) for the two cases of the parameters \( (\Omega, \varepsilon) \). In the first case the curve is constructed for the parameters from the region \( D^- \). It is seen that it does not capture the origin of coordinates. In the second case, the parameters derived from the field \( D^+ \). Then the curve passes once the origin of coordinates. This means that in the upper half-plane we have one root \( z_0 \). The reverse root \(-z_0 \) locates in the lower half-plane.
Fig. 2 Curve G(μ), built for parameters from different areas. Curve 1 is the case of the (Ω, ε) ∈ D, ε=0.002, Ω=1.4, the curve 2 – the case (Ω, ε) ∈ D+, ε=0.002, Ω=1.1.

Because of the evenness of the dispersion functions of its zeros differ only in sign. For z₀ accept the one whose real part is positive. This zero meets the following solution:

\[ \varphi_{z_0}(x, \mu) = \exp \left( -\frac{w_0 x}{z_0} \right) \frac{E_0 z_0 \mu - \eta_1^2}{w_0 z_0 - \mu}, \quad e_{z_0}(x) = \exp \left( -\frac{w_0 x}{z_0} \right) E_0. \]

3. The dependence of the Debye mode from the problem parameters

The root of the dispersion equation corresponding to the Debye mode z₀ can be computed as follows:

\[ z_0^2 = z_1^2 + \frac{\Lambda(z_1)}{\Lambda_{\infty} X(z_1) X(-z_1)}, \tag{2} \]

where z₁ is any point selected from the region D⁺ \(\setminus\) γ, \(\Lambda_{\infty} = 1 - 1/w₀ + w₀/\nu w₀^2\).

The function X(z) has the following form:

\[ X(z) = \frac{1}{z} \exp \left( \frac{1}{2\pi i} \int_0^{+\infty} \ln G(t) - 2\pi i t - z \frac{dt}{t - z} \right) \]

It is interesting to see what happens for high chemical potentials when increasing the frequency of the external fields interspersed region D⁺ and D⁻.

Fig. 3 Curve L with α=50.
Figure 3 represents a curve L when α=50. It is possible see how narrow the interval of values of the frequency of the external field, where the discrete root will disappear, and then appear with increasing Ω. According to the formula (2) it is possible to find the value of roots for each part of the region D^{-}. The results of the calculations are depicted in figure 4.

![Figure 4](image)

**Fig. 4.** The locus of point, which is the root of the dispersion function with α=50 and the collision frequency of electrons in the plasma, equal to ω_p.

The graph for this case is depicted in figure 4. The left end of the loop corresponds to movement from zero in the direction of increasing the frequency of the external field. When approaching the boundary of a domain D^{+} graph of root approaches the real axis, the root becomes part of a continuous spectrum. Moving further in the field of D the root is not observed. If it enters again in the area D^{+} loop continues to move in the first quarter of the complex plane. When approaching the border of the D^{+} graph of root again "sits" on the axis. Moreover, the curve looks smooth, as if it was not part of the D region. This is due to the narrowness of the transition region D^{-} between two areas of existence of Debye mode.

### 4. Conclusion

The algorithm of finding of discrete mode (Debye mode) at screening the external alternative electric field presents. The efficiency of this algorithm is demonstrated by finding Debye mode for the case of the electron gas with a high degree of degeneration. It is shown that the Debye mode shows in this case the unusual behavior. This fact may be important for describing the interaction of electromagnetic radiation with plasma.

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