Volume, Coulomb, and volume-symmetry coefficients of nucleus incompressibility in the relativistic mean field theory with the excluded volume effects

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ABSTRACT
The relation among the volume coefficient $K$ (=incompressibility of the nuclear matter), the Coulomb coefficient $K_c$, and the volume-symmetry coefficient $K_{vs}$ of the nucleus incompressibility are studied in the framework of the relativistic mean field theory with the excluded volume effects of the nucleons, under the assumption of the scaling model. It is found that $K = 300 \pm 50\text{MeV}$ is necessary to account for the empirical values of $K$, $K_c$, and $K_{vs}$, simultaneously, as is in the case of the point-like nucleons. The result is independent on the detail descriptions of the potential of the $\sigma$-meson self-interaction and is almost independent on the excluded volume of the nucleons.
To determine the incompressibility $K$ of nuclear matter from the giant monopole resonance (GMR) data, one may use the leptodermous expansion [1] of nucleus incompressibility $K(A,Z)$ as follows.

$$K(A,Z) = K + K_{sf} A^{-1/3} + K_{vs} I^2 + K_c Z^2 A^{-4/3} + \cdots ; \quad I = 1 - 2Z/A, \quad (1)$$

where the coefficients, $K$, $K_{sf}$, $K_{vs}$ and $K_c$ are volume coefficient (incompressibility of nuclear matter), surface term coefficient, volume-symmetry coefficient and Coulomb coefficient, respectively. The higher terms is omitted in eq. (1).

Although there is uncertainty in the determination of these coefficients by using the present data, Pearson [2] pointed out that there is a strong correlation between $K$ and $K_c$. (See table 1.) Similar observations are done by Shlomo and Youngblood [3].

Table 1

According to this context, Rudaz et al. [4] studied the relation between incompressibility and the skewness coefficient by using the generalized version of the relativistic Hartree approximation [5]. The compressional and the surface properties are studied by Von-Eiff et al. [6-8] in the framework of the relativistic mean field theory (RMF) of the $\sigma$-ω-ρ model with the nonlinear $\sigma$ terms. They found that low incompressibility ($K \approx 200$MeV) and a large effective nucleon mass $M^*$ at the normal density ($0.70 \leq M^*/M \leq 0.75$) are favorable for the nuclear surface properties [8], where $M$ is a free nucleon mass. On the other hand, using the same model, Bodmer and Price [9] found that the experimental spin-orbit splitting in light nuclei supports $M^* \approx 0.60 M$. Furthermore, the result of the generator coordinate calculations for breathing-mode GMR by Stoitsov, Ring and Sharma [10] suggests $K \approx 300$MeV. It seems that there are two suggestions, large ($> 0.7M$) or small ($< 0.7M$) $M^*$, and there are also two suggestions, large ($\sim 300$MeV) or small ($\sim 200$MeV) $K$.

In refs. [11,12,13], we studied the relation between $K$ and $K_c$ in detail, by using RMF with the nonlinear $\sigma$ terms [14], by using RMF with the nonlinear $\sigma$ and $\omega$ terms [15], and by using RMF with the nonlinear $\sigma$ term and the excluded volume effects (EVE) of nucleons[16]. One of our finding in those studies is that, under the assumption of the scaling model [1], $K = 300 \pm 50$MeV is favorable to account for $K$, $K_c$ and $K_{vs}$, simultaneously, in any case of three models. The conclusion seemed to be common in the RMF, although, in the three analyses, we assume the quartic-cubic potential as the $\sigma$ meson self-interaction. In recent paper[17], we have reexamined the conclusion by using RMF with the nonlinear $\sigma$ terms, and by using the one with the nonlinear $\sigma$ and $\omega$ terms, in more general way in which the result does not depend on the detail description of the $\sigma$ meson self-interaction $U(\phi)$. It is found that the conclusion $K = 300 \pm 50$MeV is necessary for any type of $U(\phi)$ in both cases. It is also found that the result hardly depends on the strength of the $\omega$ meson self-interaction. It seems that
this conclusion is not drastically changed in the use of any type of RMF and the
scaling model. In this paper, we reexamine the conclusion by using the RMF
with the nonlinear \( \sigma \) terms and EVE of the nucleons \([16, 13]\), in the general way
in which the result does not depend on \( U(\phi) \) as in ref. \([17]\). The reason why we
restrict our discussions to \( K, K_c, \) and \( K_{vs} \) is that the general discussions, which
are independent of the detail of the model ( e.g., types of the interactions, values
of the parameters in the Lagrangian, etc.) are possible to a considerable extent,
since, as is shown below, these quantities are almost analytically estimated by
using the result for the nuclear matter, if we assume the scaling model \([1]\). We
also remark that our discussions do not depend on the \( \sigma \) meson mass, since
\( K, K_c, \) and \( K_{vs} \) are able to be calculated by the result for the nuclear matter.

We use RMF with EVE of nucleons \([16, 13]\). For the Lagrangian density,
we use the \( \sigma \)-\( \omega \)-\( \rho \) model with the nonlinear \( \sigma \) terms. The Lagrangian density
consists of four fields, the nucleon \( \psi \), the scalar \( \sigma \)-meson \( \phi \), the vector \( \omega \)-meson \( V_\mu \), and the vector-isovector \( \rho \) meson \( b_\mu \), i.e.,
\[
L_{N\sigma\omega\rho} = \bar{\psi}(i\gamma_\mu \partial^\mu - M)\psi
+ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\sigma^2 \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 V_\mu V^\mu - \frac{1}{4} B_{\mu\nu} \cdot B^{\mu\nu} + \frac{1}{2} m_\rho^2 b_\mu \cdot b^\mu
+ g_s \bar{\psi}\gamma_\mu \psi V^\mu - g_\omega \bar{\psi}\gamma_\mu \gamma_5 V^\mu \frac{1}{2} \cdot b^\mu \psi - U(\phi) ;
\]
where \( m_\sigma, m_\omega, m_\rho, g_s, g_\omega, \) and \( g_\rho \) are \( \sigma \)-meson mass, \( \omega \)-meson mass, \( \rho \)-meson mass, \( \sigma \)-nucleon coupling, \( \omega \)-nucleon coupling, and \( \rho \)-nucleon coupling, respectively. The \( U(\phi) \) is a nonlinear self-interaction potential of \( \sigma \) meson field \( \phi \). For example, in ref. \([13]\), we have used the quartic-cubic terms of \( \phi \) as in ref. \([14]\), i.e.,
\[
U(\phi) = \frac{1}{3} b \phi^3 + \frac{1}{4} c \phi^4,
\]
where \( b \) and \( c \) are the constant parameters which are determined phenomeno-
logically. However, in this paper, we do not give an explicit expression of \( U(\phi) \)
and discuss the problem in more general way, without any assumption of \( U(\phi) \),
as in ref. \([17]\).

In RMF of the point-like nucleons, the baryon density is given by
\[
\rho^B = \frac{\lambda}{3\pi^2} k_F^3,
\]
where \( k_F \) is Fermi momentum and \( \lambda = 2 \) in the nuclear matter. In the model
with the EVE \([16, 13]\), the volume \( V \) for the \( N \) body system of nucleons in
configurational space is reduced to the effective one, \( V - NV_n \), where \( V_n \) is the
excluded volume of a nucleon. According to this modification for the volume, the baryon density $\rho$ is given by

$$\rho = \frac{\tilde{\rho}}{1 + V_n \tilde{\rho}},$$

(5)

where $\tilde{\rho}$ has the same expression as $\rho_{pt}$ for the given $k_F$. In a similar way, the scalar density is given by

$$\rho_s = \frac{\tilde{\rho}_s}{1 + V_n \tilde{\rho}},$$

(6)

where $\tilde{\rho}_s$ has the same expression as the scalar density of the system of the point-like nucleons, and is given by

$$\tilde{\rho}_s = \lambda \left[ \frac{2\pi^2}{\lambda^2} \frac{M^*}{k_F} \sqrt{k_F^2 + M^{*2}} - M^{*2} \ln \left( \frac{k_F + \sqrt{k_F^2 + M^{*2}}}{M^*} \right) \right].$$

(7)

For the detail description of this model, see refs. [16, 13].

In the scaling model [1], $K, K_c$ and $K_{vs}$ in eq. (1), are given by

$$K = 9\rho_0^2 \frac{\partial^2 E_b}{\partial \rho^2} |_{\rho = \rho_0},$$

(8)

$$K_c = \frac{3}{5R_0} \left( \frac{9K'}{K} + 8 \right),$$

(9)

$$K_{vs} = K_{sym} - L \left( \frac{9K'}{K} + 6 \right),$$

(10)

where $\rho, \rho_0, E_b$ and $q_{el}$ are the baryon density, the normal baryon density, the binding energy per nucleon, and the electric charge of proton, respectively, and $R_0 = [3/(4\pi \rho_0)]^{1/3}$,

$$K' = 3\rho_0^3 \frac{d^3 E_b}{d \rho^3} |_{\rho = \rho_0},$$

(11)

$$L = 3\rho_0 \frac{dJ}{d \rho} |_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 \frac{d^2 J}{d \rho^2} |_{\rho = \rho_0} : \quad J = \frac{1}{2} \rho^2 \frac{\partial^2 E_b}{\partial \rho^2} |_{\rho=0}. \quad (12)$$

The quantity such as $K'$ is sometimes called "skewness".

In RMF with EVE, $L$ and $K_{sym}$ are given by [7]

$$L = \frac{3\rho}{8} \alpha \rho^2 + \frac{\rho}{2} \left( \frac{2k_F k'_F}{E_F^2} - \frac{k^2_F E''_F}{E_F^2} \right) |_{\rho = \rho_0},$$

(13)

and

$$K_{sym} = \frac{3}{2} \rho^2 \left( \frac{2k_F^2}{E_F} + \frac{2k_F k'_F E''_F}{E_F^2} - \frac{4k_F k'_F E''_F}{E_F^3} + \frac{2k^2_F E''_F}{E_F^3} - \frac{k^2_F E'''_F}{E_F^3} \right) |_{\rho = \rho_0},$$

(14)
where \( \alpha_{\rho} = g_{\rho}/m_{\rho} \),

\[
\begin{align*}
  k_{F}' &= \frac{dk_F}{d\rho} = h \frac{k_F}{3\rho}, \\
  k_{F}'' &= \frac{d^2k_F}{d\rho^2} = \frac{2h^2}{3\rho}, \\
  k_{F}''' &= \frac{d^3k_F}{d\rho^3} = 2h^2(\frac{1}{3}\rho - \frac{1}{3}k_F), \\
  h &= 1 + V_n\bar{\rho}, \quad (15)
\end{align*}
\]

\[
E_F' = \sqrt{k_F^2 + M'^2}, \quad E_F'' = \frac{dE_F'}{d\rho}, \quad E_F''' = \frac{d^2E_F'}{d\rho^2}. \quad (16)
\]

We remark that equations for the point-like nucleon [17] are given, if we put \( V_n = 0 \) in eq. (15).

At \( \rho = \rho_0 \), \( E_F' \) and \( E_F'' \) are related to \( K \) and \( K' \) in the following relations, respectively.

\[
E_F' = \frac{k_F}{E_F}, \quad (17)
\]

\[
M'' = \frac{dM^*}{d\rho} = \left[ \frac{K}{3\rho^2} + \frac{\alpha^2}{m_v} - h \frac{k_F}{E_F} k_F' \right] / (h M^* - V_n\bar{\rho}), \quad \alpha_v = \frac{g_v}{m_v}, \quad (18)
\]

\[
E_F''' = \left[ \frac{K + K'}{3\rho^2} - V_n (h^2 E_F' - \rho_s' M'^* - t\bar{\rho}) \right] / (h - \frac{E_F}{M^*} V_n\bar{\rho}), \quad (19)
\]

\[
t = \frac{(E_F'^2 - k_F'^2 - k_F'' M'^* - k_F^2) / M^*}{M^*}, \quad (20)
\]

\[
\rho_s' = \frac{d\rho_s}{d\rho} = h^2 \frac{M^*}{E_F} + \frac{\lambda M^*}{2\pi^2} \left[ 2k_F M'^* + \frac{2k_F M'^*}{E_F} - 3M'^2 \ln \left( \frac{k_F + E_F}{M^*} \right) \right]. \quad (21)
\]

Furthermore, at \( \rho = \rho_0 \), \( C_v \) and \( C_p \) are also related to \( M^* \) as follows.

\[
\alpha_v^2 = \frac{1}{\rho} \left( M - a_1 - E_F^* - V_n P(E_F^*, M^*) \right), \quad (22)
\]

[13] and

\[
\alpha_p^2 = \frac{8}{\rho} \left( a_4 - \frac{k_F^2}{6E_F^*} \right), \quad (23)
\]

[18] where \( P(E_F^*, M^*) \) is given by replacing \( M \) by \( M^* \) in the pressure \( P(\sqrt{k_F^2 + M^2}, M) \) of the free nucleon system, and \( a_1 \) and \( a_4 \) are the binding energy and the symmetry energy at \( \rho = \rho_0 \), respectively. We remark that eqs. (9), (10), and (13)~(23) have no explicit dependence on \( U(\phi) \).

From eq. (9), \( K' \) are determined, if \( \rho_0 \), \( K \) and \( K_c \) are given. Therefore, from the eqs. (9), (10), and (13)~(23), it is seen that \( K_{us} \) is determined, if \( \rho_0 \), \( a_1 \), \( K \), \( K_c \), \( V_n \) and \( M^* \) are given, without giving the detail descriptions for \( U(\phi) \). Using these equations, we calculate \( K_{us} \). In the calculations, we put \( \rho_0 = 0.15\text{fm}^{-3}, \ a_1 = E_0(\rho_0) = 15.75\text{MeV} \) and \( a_4 = J(\rho_0) = 30.0\text{MeV} \).

We restrict \( R_n(= [3V_n/4\pi]^{1/3}) \) in the region of \( 0.8\text{fm} \), because larger value of \( R_n(\gtrsim 0.9\text{fm}) \), which is close to the average nucleon spacing \( R_0(\sim 1.1\text{fm}) \) in normal nuclear matter, may cause divergences in calculations. For \( K \) and \( K_c \), we use the values in table 1. We assume that \( M^* = 0.5M \sim 0.93M \), the
phenomenologically acceptable values. (The upper bound for $M^*$ is gotten, if we put $C_v = 0$ and $k_F \sim 1.4 \text{fm}^{-1}$ in eq. (21). We remark that $k_F$ is larger in RMF with EVE than in the case of the point-like nucleons.) In fig. 1, we show $K_{vs}$ as a function of $M^*$ for two sets of $K$ and $K_c$ in table 1, at $R_n = 0.8 \text{fm}$ and at $R_n = 0.6 \text{fm}$, comparing them with the result in the case of the point-like nucleons. In the fig. 1(a), $K_{vs}$ decreases as $M^*$ increases in any case of $R_n$. In that case (in the case of $K = 300 \text{MeV}$ and $K_c = -3.990 \text{MeV}$), $K_{vs} = -247(-292, -298) \sim 176(-53, -115) \text{MeV}$ for $R_n = 0.8(0.6, 0) \text{fm}$. These values are in good agreement with the corresponding empirical values in table 1. We also remark that the uncertainty of $K_{vs}$ in changing $M^*$ is not so larger than the empirical error bar of $K_{vs}$ in table 1. In the case of $K = 350 \text{MeV}$ and $K_c = -7.274 \text{MeV}$ (fig. 1(b)), $K_{vs}$ shows more complicated behaviors in changing $M^*$. However, the uncertainty of $K_{vs}$ is comparable to the magnitude of the empirical error bar of $K_{vs}$ in table 1. In this case, $K_{vs} = -652(-673, -671) \sim -484(-610, -632) \text{MeV}$ for $R_n = 0.8(0.6, 0) \text{fm}$. These values are somewhat smaller than the empirical ones. In table 2, we summarize the range of the calculated $K_{vs}$ for each set of $K$ and $K_c$ in table 2. Comparing the table 1 and table 2, we see that $K = 300 \pm 50 \text{MeV}$ is necessary to account for $K$, $K_c$ and $K_{vs}$, simultaneously in any case of $R_n$, as in the case of the point-like nucleon ($R_n = 0$) [17]. To say more exactly, in many cases of $K$ and $M^*$, the repulsive effect of EVE tends to make $K_{vs}$ larger, on the contrary to the attractive effects of the vector meson self-interaction (VSI) [12, 17], in which $K_{vs}$ tends to becomes smaller by introducing VSI. However, these effects are not so large enough to change the conclusion.

Fig. 1(a),(b), Table 2(a),(b),(c)

We remark the following three points.

1. The results are independent of the form of $U(\phi)$, since eqs. (9), (10), and (13)∼(23) are required for any type of $U(\phi)$.

2. The question, whether there are coupling parameters, which reproduce the set of $K$ and $K_c$ in table 1, or not, is still open, and the answer for the question depends on the detail descriptions of $U(\phi)$. For example, if we use the quartic-cubic potential (3), we could not find the coupling parameters, which reproduce $K = 200 \text{MeV}$ and $K_c = 2.577 \text{MeV}$ at any $R_n$ [13]. Also, using eq. (3), we get the parameter set for $K = 300 \text{MeV}$ and $K_c = -3.990 \text{MeV}$ only at $M^* = 0.83M$, in the case of the point-like nucleons ($R_n = 0$)[11]. (We remark that the value of $\rho_0$ is slightly different from the one used in this paper. However, the results hardly depend on $\rho_0$. ) In those cases, the number of the parameters may not be large enough to reproduce the empirical value well, and, if the higher terms of $\phi$ are added to (3), the wider range of $K$ and $M^*$ may be available. However, $K = 300 \pm 50 \text{MeV}$ is necessary to reproduce the empirical values of $K$, $K_c$, and $K_{vs}$ simultaneously, for any type of $U(\phi)$ and for any $R_n(= 0 \sim 0.8 \text{fm})$. 

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(3) The results do not have a strong-dependence on $\rho_0$, $a_1$ and $a_4$, since the calculated $K_{vs}$ is much more sensitive to the ratio $K'/K$ than to those quantities.

In summary, we have studied $K$, $K_c$, and $K_{vs}$ by using the relativistic mean field theories based on the $\sigma$-$\omega$-$\rho$ model with the nonlinear $\sigma$ term and the excluded volume effects. It is found that, at any $R_n(\leq 0.8\text{fm})$, $K = 300\pm50\text{MeV}$ is necessary to account for the empirical values of $K$, $K_c$, and $K_{vs}$ at the same time, as is in the cases of the point-like nucleons [17]. The result is independent on the detail descriptions of $U(\phi)$. To see this result with the one in ref. [17], it seems that the conclusion that $K = 300\pm50\text{MeV}$ is necessary to account for $K_{vs}$ is not drastically changed, if we use any type of the relativistic mean-field theory and the scaling model. The reason is probably that the calculated $K_{vs}$ is most sensitive to the ratio $K'/K$, which is adjusted to the empirical values. This general conclusion $K = 300\pm50\text{MeV}$ has also good agreement with the result of the Dirac-Brueckner-Hartree-Fock calculation [19], with the result in ref. [10], and with the earlier work by Sharma [20].

Acknowledgment: One(H. K.) of the authors would like to thank Prof. M.M. Sharma for useful discussions and suggestions. The authors gratefully acknowledge the computing time granted by the Research Center for Nuclear Physics (RCNP).

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Table and Figure Captions

Table 1
The sets of the empirical values of $K$, $K_c$, and $K_{vs}$ in the table 3 in ref. [2]. (According to the conclusion in ref. [2], we only show the data in the cases of $K = 150 \sim 350$MeV.) All quantities in the table are shown in MeV.

Table 2
Range of the calculated $K_{vs}$ using the sets of $K$ and $K_c$ in table 1 as inputs, if we put $M^* = 0.5M \sim 0.93M$. (a) The result in the case with excluded volume effects with $R_n = 0.8$fm. (b) The result in the case with excluded volume effects with $R_n = 0.6$fm. (c) The result in the case of the pointlike nucleons. ($R_n = 0$): In each table, "upper bound", "mean value", and "lower bound" mean that the results are obtained by using the upper bound, the mean value, and the lower bound of $K_c$ in table 1, respectively. All quantities in the table are shown in MeV.

Fig. 1 $K_{vs}$ as a function of $M^*$. (a) The cases of $K = 300$MeV and $K_c = -3.990$MeV. (b) The cases of $K = 350$MeV and $K_c = -7.724$MeV: In each figure, the solid line is the result in the case of the point-like nucleons, and the dotted and the dashed lines are the results in the cases with excluded volume effects with $R_n = 0.6$fm and with $R_n = 0.8$fm, respectively.
| Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
|-------|-------|-------|-------|-------|
| $K$   | 150.0 | 200.0 | 250.0 | 300.0 | 350.0 |
| $K_c$ | $5.861 \pm 2.06$ | $2.577 \pm 2.06$ | $-0.7065 \pm 2.06$ | $-3.990 \pm 2.06$ | $-7.274 \pm 2.06$ |
| $K_{ls}$ | $66.83 \pm 101$ | $-46.94 \pm 101$ | $-160.7 \pm 101$ | $-274.5 \pm 101$ | $-388.3 \pm 101$ |

Table 1

| upper bound | mean value | lower bound |
|-------------|------------|-------------|
| $K = 150$  | 1211~2562  | 709~1752    |
| $K = 200$  | 808~1902   | 307~1092    |
| $K = 250$  | 406~1242   | -96~431     |
| $K = 300$  | 4~582      | -498~229    |
| $K = 350$  | -399~-79   | -944~889    |

Table 2(a)

| upper bound | mean value | lower bound |
|-------------|------------|-------------|
| $K = 150$  | 1021~1962  | 569~1276    |
| $K = 200$  | 659~1404   | 206~719     |
| $K = 250$  | 296~847    | -156~161    |
| $K = 300$  | -66~290    | -521~396    |
| $K = 350$  | -428~268   | -959~880    |

Table 2(b)

| upper bound | mean value | lower bound |
|-------------|------------|-------------|
| $K = 150$  | 966~1755   | 531~1118    |
| $K = 200$  | 618~1237   | 182~601     |
| $K = 250$  | 269~721    | -167~84     |
| $K = 300$  | -80~204    | -522~433    |
| $K = 350$  | -431~313   | -951~865    |

Table 2(c)
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