HAWKING-LIKE EFFECTS AND UNRUH-LIKE EFFECTS: TOWARD EXPERIMENTS

HARET C. ROSU

Instituto de Física, Universidad de Guanajuato, Apdo Postal E-1143, León, Gto, Mexico

The Hawking effect and the Unruh effect are two of the most important predictions in the theoretical physics of the last quarter of the 20th century. In parallel to the theoretical investigations there is great interest in the possibility of revealing effects of this type in some sort of experiments. I present a general discussion of the proposals to measure the Hawking and Unruh effects and/or their 'analogues' in the laboratory, and I make brief comments on each of them. The reader may also find the various physical pictures corresponding to the two effects which were applied to more common phenomena, and vice versa.

1. Introduction

The most famous scientific formula seems to be \( E = mc^2 \), which was settled by Einstein. However, the following more recent formula is at least of the same rank

\[
T = \frac{\hbar}{2\pi ck} \cdot a
\]

including three fundamental physical constants, and the rather geometrical constant \( \pi \). \( T \) is a quantum field temperature parameter identified with the thermodynamic temperature and \( a \) is the acceleration parameter in the physical system under investigation. It was Hawking\(^1\) in 1974, who first obtained an equivalent of Eq. (1.1) for Schwarzschild black holes\(^\star\), being followed in 1975 by Davies\(^\#\), who provided a discussion of the quite similar scalar particle production phenomena in Rindler metric using a mirror model, and as a matter of fact, one can find Eq.(1.1) clearly derived in 1976 in a seminal paper of Unruh\(^4\), who applied the so-called quantum/particle detector method. The acceleration parameter is the surface gravity in the black hole case (\( \kappa = c^4/4GM \)), and the proper acceleration of a quantum system (most often the electron) in the latter case. The radiated spectrum in these two cases may be considered as exactly of black body type (up to some distorsion due to the transmission through the surrounding potential barrier for black holes), almost forcing

\(^\star\)For a concise look at black hole physics before the Hawking effect, see Ref.\(^\|\).
one to accept a truly thermal/thermodynamic interpretation as very natural. However, other viewpoints may be favoured, e.g., coherent effects, Casimir-like effects, ambiguity in defining positive and negative modes, instanton effects. There are standard methods to get these fundamental thermal-like field effects, e.g., Euclidean Green’s functions, Bogolubov mixing coefficients, construction of individual wave packets, renormalized energy-momentum tensor, instanton techniques, analytic mappings, thermofield dynamics, and even more classical derivations (these latter ones may be considered to be at variance with the relationship between the Hawking effect and the Weyl trace anomaly). These effects are very general, since they are a direct consequence of the Fulling nonuniqueness of canonical quantization in curved spacetimes. They must appear in the framework of any quantum/stochastic field theory, perhaps with some specific non-trivial features. Various aspects of these effects have been revealed in the vast literature that has been accumulated over the years; the interested reader is directed to some well-known review papers. However, we have to point out that we still lack a dedicated book although these effects are discussed with various degree of detail in black hole and/or quantum field theory in curved space-time books, as well as in general relativity books (e.g., Frolov and Novikov, Caltech-membrane book, Birrell and Davies, Wald) Probably we should wait first for a dedicated conference/symposium. Perhaps, it is interesting to recall the concept of thermal radiation from nothing elaborated by Kandrup in the cosmological context, which one may also take into account for the case of Hawking radiation and Unruh radiation, if nothing means zero point energy. In fact, Hawking and Unruh effects may be considered (and have been considered) as intellectual surprises in the sense given by Peierls, i.e., they could have been predicted much earlier (for how much earlier there are different opinions!).

The main goal of this survey paper is to present the experimental settings that were proposed so far to detect such thermal-like effects, attaching short comments to each of them. In cgs system Eq. (1.1) turns into:

$$T = 4 \times 10^{-23} \cdot a$$

(1.2)

and therefore we need accelerations greater than $10^{20} \cdot g_\oplus$ ($g_\oplus$ is the mean Earth surface gravity) in order to have a ‘heat-bath’ quantum vacuum at the level of only one Kelvin. We are facing extremely small thermal-like quantum noises requiring on the scale of terrestrial laboratories extremely strong non-adiabatic perturbations to be applied. That is, such perturbations are not built up at a constant rate from zero to their final value over a time interval which is long as compared to inverse quantum frequencies, and therefore one is changing the occupation probabilities, i.e., directly the populations of the quantum states. Non-adiabatic transitions are a necessary prerequisite of all sorts of heat, including those to be discussed next. Generally, the Hawking effect is considered to show up in the realm of astrophysics, whereas the Unruh effect is more appropriate to an extremely non-perturbative and non-linear
Indeed, the radiated Unruh power is $4.1 \times 10^{-118} a^4$ as compared with the Larmor power which is $5.7 \times 10^{-51} a^2$ in cgs units and one could see that the two radiations are comparable for $a = 3 \times 10^{30} g_{\odot}$. Such accelerations are produced by electric and/or magnetic fields that are one order of magnitude beyond the critical electrodynamical field $F = m^2 c^3 / e \hbar \approx 1.32 \times 10^{16} V/cm \approx 4.41 \times 10^{13} G$ at which the spontaneous electron-positron pair production in vacuum starts on. Thus Unruh effect will be competed by non-linear electrodynamic effects (first of all nonlinear Thomson scattering). On the other hand, Nikishov and Ritus on account of processes induced by charged particles in an electric field, provided arguments against the Unruh heat-bath concept. Their point is that on the length scale of quantum pair-production processes, one cannot encounter a constant acceleration field, so that in general the occurring pair production processes are not thermal/Unruh-like. In their paper, they worked out probability exponents for various cases of pair production in a constant electromagnetic field and showed that as a rule these are not ‘thermodynamic’ ones, i.e., linear in the excitation energy. For instance, they considered the probability of the process $q_1 \rightarrow q_2 + q_3$ in the case of weakly differing accelerations of the products, and obtained a thermodynamic exponential with an effective excitation temperature which is twice the Unruh temperature. This is why they called such a case an ideal detector of uniformly accelerated motion. It is also quite well-known the opinion of these authors that Rindler states cannot be produced in Minkowski space without sources of infinite power on the event horizons. Another argument against the physical realization of coordinate systems such as the Rindler one was provided by Padmanabhan, who, by analogy with QED, concluded that any physically realizable coordinate system can differ from the Minkowski coordinates only in a finite region of spacetime.

One should also recall the debated point of extrapolating Lorentz invariance to extremely high energies that was emphasized by Jacobson in the black hole case. In fact, it is well known that the black body spectrum is the only distribution that is Lorentz and even conformal invariant. Indeed, the usual Bose distribution is just the form in the rest frame of the Lorentz scalar $1 / \{ \exp(\beta pv) - 1 \}$, where $v$ is the four velocity of the thermal bath.

With the purpose of relating Hawking effect and Unruh effect to Everyday Physics/Earth laboratories, we shall use the most simple/intuitive vocabulary at our disposal throughout the paper.

To close the Introduction, a comment about the title. By Hawking-like and Unruh-like effects we mean effects of Hawking and Unruh type, i.e., quite similar effects which need only minor modifications (in my opinion, the latter condition is not well satisfied in some cases I’ll be reviewing next). In order to test scientific ideas that are applied in domains much beyond our present technologies, we have to find equivalents to those ideas that might be tested in the laboratory. One can encounter quite a few papers written with this purpose in the literature. In this regard, the list of ‘experimental’ proposals to follow is a clear illustration of such an idea. At the same time, the inverse action is also at one’s disposal, that is, applying...
frameworks of ordinary effects to Black Hole Physics.

The organization of the paper is as follows: the next section contains a short presentation of the original derivations of the effects; all sections thereafter deal with the analogs and the model experiments suggested for Hawking and Unruh effects. Although not exhaustive, a quite broad material is brought together in order to imprint on the reader what might be a useful global view on many topics.

2. Hawking’s and Unruh’s Paradigms

This section is included in the review for self-consistency reasons, otherwise it follows closely published text.

The remarkable results of Hawking and Unruh belong, together with the Penrose effect, and the superradiance, to the ‘epoch of effects’ that occurred in black hole physics in early 1970s. At the present time, these effects are standard theoretical paradigms in quantum field theory, astrophysics, and cosmology. In this section we remind the basic ideas used by the two authors in their seminal papers on the topic.

During 1974-1976, Hawking dealt with the Klein-Gordon equation for a massless scalar field in a Schwarzschild metric and he used naturally an in-out formalism for radial null rays that define a one-to-one mapping between past null infinity and future null infinity as required by an asymptotic observer. Since the usual coordinates of the background become singular at the horizon, one should consider there the Kruskal-Szekeres regular set, which is related to the Schwarzschild coordinates by transformations reminding the Langer transformation in ordinary semiclassical physics. Hawking propagated the scalar normal modes by the method of geometrical optics and used the method of Bogolubov coefficients (at that time already well established in quantum cosmology/cosmological particle production, for a review see Ref.[177]) to obtain his famous conclusion that black holes are normal objects.

An incoming null ray $v = \text{const}$, originating on $I^-\bar{\text{v}}$ propagates through the gravitational background to become an outgoing null ray $u = \text{const}$, arriving on $I^+\bar{u}$ at a value $u = F(v)$. Similarly, and this was the procedure of Hawking, one can trace a null ray from a constant $u$ on $I^+\bar{u}$ to a $v = G(u)$ on $I^-\bar{v}$, where the function $G$ is the inverse of $F$. Getting one of these functions is the clue towards all the physical results. By this geometrical ray-tracing (or constant phase tracing), Hawking obtained in the Schwarzschild case (Newton, Planck, Maxwell, Boltzmann constants all set equal to unity)

$$G(u) = -C \exp[(4M)^{-1}u] + v_0$$

where $C$ and $v_0$ are constants. The latter constant denotes the ingoing ray that reaches the horizon at the moment of its formation. For quantization, one needs complete sets of mode functions, that is ingoing and outgoing wave packets constructed from massless spherical waves. The wave packets are solutions of the scalar equation in the Schwarzschild background geometry if the gravitational backscattering is neglected. Even so, the exponential increase with advanced time of the
mean frequency of the wave packets at $I^-$ is a well-known disturbing feature in the intermediate stages of the calculation. The quantization, carried on wave packet mode functions on the two asymptotic infinities, brings in wave packet creation and annihilation operators and the Bogolubov mixing coefficients can be calculated using the ray tracing formula to change conveniently the variables. For a recent presentation of the formalism with interesting discussions of the localization issue see. The main result is the following relation between the two Bogolubov coefficients

$$|\alpha_{\omega \omega'}|^2 = e^{2\pi \omega / \kappa} |\beta_{\omega \omega'}|^2$$  \hspace{1cm} (2.2)

which is exactly the relation required at the level of Bogolubov coefficients to obtain the emission of a blackbody spectrum.

By a straightforward application of the Hawking formalism to the Rindler wedge equipped with a reflecting mirror placed to the right of the origin, Davies obtained the result that an observer moving with uniform acceleration $a$ sees the fixed surface of the mirror radiating a thermal spectrum with a temperature $a/2\pi$, that may be considered as a close variant of the Unruh effect.

An essential mathematical result that one needs in order to discuss the Unruh effect refers to writing the Minkowski vacuum state as an entangled Rindler vacuum state

$$|0\rangle_M = C \left[ \prod_i \exp(e^{-2\pi \omega M r^i l^i}) \right] |0\rangle_R$$  \hspace{1cm} (2.3)

where $C$ is a normalization constant, the indicial set $(i)$ is $(i) = (\omega_R, k)$, the creation operators $r^i$ and $l^i$ live only in the right and left Rindler wedge respectively, and the other subscripts are obvious. Gerlach has written interesting papers on this vacuum ‘superfluidity’.

In the third section of his “Notes on black-hole evaporation” Unruh presented a clear analysis of the behavior of particle/quantum detectors under acceleration in flat spacetime with two remarkable findings

(i) A particle detector will react to states which have positive frequency with respect to the detector proper time, not with respect to any universal time.

(ii) The process of detection of a field quanta by a detector, defined as the exciting of the detector by the field, may correspond to either the absorption or the emission of a field quanta when the detector is an accelerated one.

These fundamental conclusions are reached by investigating two types of detectors. The first one, a box containing a Schrödinger particle in its ground state, which is said to have detected a quanta of a massless scalar field if the detector is found in a state other than its ground state at some late time. The second one is a relativistic model describing the interaction of two complex scalar fields, one of which is the detector field $\Psi$ of mass $M_d$ coupled via a real scalar field $\Phi$ to an ‘excited’ state representing the second scalar field $\phi$ of mass $M > M_d$. The detector field $\Psi$ is said to have detected a $\Phi$ quantum if it changes into the second, excited field $\phi$ at some late time.
The box detector is held fixed on a line of constant acceleration $a$ in Rindler space (or else, the box moves with constant linear acceleration in Minkovski space). In this case the Schrödinger equation for the particle in the box has an additional potential $m\zeta a$, where $\zeta = (2a)^{-1}[4a^2\rho - 1]$ is the proper length coordinate in the Rindler polar coordinate $\rho$, assumed small within the box. Unruh performed a calculation of the lowest-order transition rate per unit proper time for an interaction of the form $\epsilon \Phi \psi$, where $\psi$ belongs to the set $\psi_j = \exp(-iE_j\tau/2a)$ of detector states.

In addition, expanding $\Phi$ in Rindler modes yields a result containing two factors. One may be interpreted to represent the destruction of one of the Rindler particles in the product of Rindler states which exist in the Minkowski vacuum (see Eq. (2.3)) by the detection process. The other factor represents the ‘sensitivity’ (or efficiency) of the detector to a Rindler mode. Unruh obtained the final result (the vacuum heat bath) by passing to Minkovski creation and annihilation operators, and by evaluating the ‘sensitivity’ in a WKB approximation. The result essentially comes out from the fact that the detector measures frequencies with respect to its proper time which is not the same for all geodesic detectors in accelerated reference systems.

Before ending this section, perhaps it is worthwhile to mention the association of thermal effects with the ‘above barrier’ reflection coefficient that has been noticed only by few authors. The point is that the semiclassical ‘above barrier’ reflection coefficient is exponentially small and therefore allows the emergence of ‘thermal’ backscatterings. I have commented on the importance of this standpoint elsewhere, it allows the interesting development of considering new thermal regimes for black holes within the WKB approach with two turning points. In solid state physics these WKB thermal effects are, e.g., field emission and thermionic emission of electrons from solid surfaces.

3. Hawking Effect in Astrophysics

Hawking radiation is insignificant for stellar mass black holes and only primordial black holes (PBHs), i.e., those having a mass smaller than $M_c = 10^{15}g$ (this is the mass of a common Earth mountain) could have a detectable Hawking luminosity. The point is that as early as 1976 Hawking and Page concluded that mountain-mass black holes (they would have to be hadron-sized objects) formed in some way in the very early Universe (phase transitions, bubble collisions, string collapses, Zel’dovich- Harrison density perturbations) are at the present epoch in their final evaporation stages (denoted as Hawking explosions). The temperature of the PBHs possessing the critical mountain-mass is around 14 MeV, and they have been emitting on a time scale comparable with the lifetime of the Universe at the peak black body photon radiation located at 14 MeV. Consequently, the most simple experiment is to measure the photon flux in the tens of MeV range by means of satellite-borne detectors. This has been done already in 1977-1978 but the measured $\gamma$ flux was seen to fall with energy as $E^{-2.5}$ without any evidence for a photon excess in the
vicinity of 14 MeV. The negative result was turned into a well-known limit on the number of PBHs per logarithmic mass interval at the critical mass, the so-called Hawking-Page [HP] bound of 1976

\[ N = \frac{dn}{d(ln M)} |_{M=M_c} < 10^5 \ (10^{11}) \ pc^{-3} \]  

(3.1)

and the HP explosion rate

\[ \frac{dn}{dt} = \frac{3\alpha(M_c)}{M_c^3} \ N = 2.2 \cdot 10^{-10} \ N \ pc^{-3} \ yr^{-1}. \]

(3.2)

The latter, however, is strongly dependent on the cosmological parameters (spacial curvature and Hubble \( h_0 \) parameter), the astrophysical premises (e.g., that PBHs have clustered- second figure in Eq. (3.1)- or not into galaxies- first figure in Eq. (3.1)), and the particle physics parameter \( \alpha(M_c) \) which is a measure of the degrees of freedom coming from particle physics.

In 1989, Halzen and Zas\(^48\) reanalyzed the MeV limit on the number of critical PBHs by taking into account the particle degrees of freedom of the standard model of quarks and leptons. They obtained an increase of one order of magnitude in their density Eq. (3.1), and of two orders of magnitude in their explosion rate Eq. (3.2). On an intuitive base, the revised HP limit says that there cannot be more than about 2000 explosions per \( pc^3 \) per year assuming galactic clustering of the critical PBHs in our galactic neighborhood. At higher energies, the observed spectrum from PBHs is a convolution of the fundamental emission spectrum with the quark fragmentation functions, resulting in a power law at energies above a few TeV in the last seconds before explosion. It is worth noting however that multiparticle production at accelerators revealed that gluonic branching processes may well be dominant over quark branching\(^49\). In order to obtain reliable theoretical results on the extremely high-energy spectrum emitted by PBHs, one needs insights into the general mathematical theory of branching processes as applied to multiparticle production\(^50\).

Apparently, there exist real possibilities for detecting Hawking bursts in the TeV and PeV range to sky depths not excluded by the HP bound by means of the new generation of air shower arrays (e.g., CYGNUS, CASA) and Cherenkov telescopes (e.g., the Whipple Telescope on Mount Hopkins, Arizona). For further details we refer the reader to some literature\(^51,52,53,54,55,56\).

The last data taken with the CYGNUS detector between 1989 September and 1993 January in search for one-second bursts of ultrahigh energy gamma rays from arbitrary located point sources in the northeen sky finds no evidence for such bursts. Moreover, it sets the most restrictive upper limit at the moment of \( 8.5 \times 10^5 pc^{-3} \ yr^{-1} \) at the confidence level of 99% for the rate-density of evaporating PBHs, assuming them uniformly distributed in the Sun neighborhood.\(^58\) The CYGNUS detector is located in Los Alamos, New Mexico, and consists of 108 scintillation counters of 1 \( m^2 \) each, deployed over 22000 \( m^2 \). The mean primary energy for gamma-ray-initiated
events is 50 TeV. The information provided by the CYGNUS array is gathered from a larger volume of space, being complementary to that of atmospheric Cherenkov telescopes which are able to probe greater distances.

In the same astrophysical context, suppose that one day we will be almost sure that a certain object or group of objects are black holes, perhaps surrounded by some material shells. Of course, we shall have some sort of power spectra from them and we would like to determine the horizon area temperature distribution. Taking the black body origin as granted (it can be argued that the overhorizon correlations are precisely such that the spectrum comes out as of a black body), we will face the inverse black/grey body problem for a quantum (horizon) lightlike surface. This problem is not at all trivial even for classical surfaces. Some hints may be found in some of my works, where I considered it as an inverse Moebius problem, on the lines of the developments due to N.-x. Chen. Also, the coherence characteristics of black hole sources are basically unknown at the present time, although some conclusions may be drawn from the squeezing picture of the Hawking radiation.

To conclude this section, we remind Chapline’s discussion on the connections of PBHs and hadron physics. A number of authors have studied low mass (mini) black holes in the particle physics perspective. Also, Turner’s old question whether PBHs might be the source of the cosmic ray antiprotons has been reanalysed by MacGibbon. In the same cosmic-ray context, Greenberg and Burns commented on the ionization tracks and ranges of small black holes, however without taking into account the Hawking radiation. This might work in the case of some kind of black hole relics/remnants. As a matter of fact, this last paragraph may be thought of as a connection with Section 13 below.

4. Hydrodynamical Hawking Effect

In a Physical Review Letter of 1981, Unruh showed that a thermal spectrum of sound waves should be given out from the sonic horizon/Mach shock wave in transsonic fluid flow. Starting with the equations of motion of an irrotational fluid (i.e., Navier-Stokes and the continuity equation) and linearizing them, the perturbations of the flow can be described by a massless scalar field in a metric determined by the background fluid velocity field. This metric looks like a Schwarzschild metric when a spherically symmetric, stationary convergent flow exceeding smoothly the speed of sound at some radius (the horizon radius) is considered. That is, in the radial outward direction the velocity of sound is

\[ v_r = -c + \alpha (r - R) + O((r - R)^2) \]  

and the radial part of the background fluid metric is

\[ ds^2 = \frac{\rho_0(R)}{c} \left[ 2\alpha (r - R) d\tau^2 - \frac{dr^2}{2\alpha (r - R)} \right] \]  

where \( \alpha = \frac{\partial v_r}{\partial r} \) is the radial velocity gradient. One may recognize this metric due to the velocity/stream potential as similar to the metric near a Schwarzschild horizon.
After quantising the sonic comoving field, Unruh finds near the sonic horizon the following time dependence of the phonon modes

$$\phi_\omega \approx (t - t_0 + \text{const})^{i\omega/\alpha} \quad (4.3)$$

which is similar to the $$(t - t_0)^{i\kappa\omega}$$ dependence near the Schwarzschild horizon with respect to a freely falling observer. The thermal flux of phonons would be at a temperature

$$T = \frac{\hbar}{2\pi k} \cdot \frac{\partial v}{\partial r} \simeq 10^{-2} K \cdot \frac{\partial v}{\partial r} \quad (4.4)$$

Since the transsonic transition is usually accompanied by turbulent instabilities, one would expect the sonic thermal-like spectrum at the spherical shock to be much under any experimental detection. Indeed, in order to have a Planck spectrum peaked at only 1 K, the gradient of the velocity at the shock has to reach 100 m/s per Å. This estimate is very disappointing. It is almost sure that a simple atomic fluid cannot support such huge gradients. However, the situation may change in the case of superfluids, as first argued by Comer. Already in the summer of 1991, Volovik wrote a paper with Schopohl on the analogy between Schwinger pair production and superfluidity ($^3$He-B) and he is actively pursuing the analogy project between quantized vortices and black holes. For other important discussions of quasiparticle pair creation in unstable superflow, the reader is directed to Elser’s papers.

Jacobson discussed the fluid flow analogy in the context of the question “would a black hole radiate if there were a Planck scale cutoff in the rest frame of the hole?” Trying to give an answer, Jacobson developed an interesting superfluid black hole model which certainly has to be further elaborated. Indeed, one may work out hybridization mechanisms between surface and bulk modes (ripplons and rotons) in what may be an attractive physical picture for subtle problems in black hole physics.

It is worthwhile to recall also the Planck aether substratum/vortex sponge of Winterberg which may be useful despite the ancient (19th century) quaint picture. A great deal of superfluid and vortex turbulence literature may also be looked upon in the above perspective.

The Mach horizons deserve further studies from the point of view of their thermal-like effects, because together with Cherenkov horizons, are the closest material structures to the black hole lightlike horizons one can think about.

Furthermore, since collapse may be reduced to appropriate boundary conditions on the past horizon (see Unruh’s “Notes” of 1976), more should be known on outgoing boundary conditions for dispersive waves in hydrodynamics. A good paper on these lines belongs to Israeli and Orzsag.

In addition, Hayward has recently discussed an outgoing spherically symmetric light-cone evolving according to the Einstein equations. Hamiltonian formulations for relativistic superfluids should be taken into account as powerful formalisms for
investigating phenomena of Hawking and Unruh type.

5. Unruh Effect in Storage Rings

J.S. Bell (‘the quantum engineer’) and his co-workers, J.M. Leinaas and R.J. Hughes, have imagined another experimental scheme connected to the Unruh effect. During 1983-1987 they published a number of papers on the idea that the depolarising effects in electron storage rings could be interpreted in terms of Unruh effect. However, the incomplete radiative polarization of the electrons in storage rings has been first predicted in early sixties in the framework of QED. Besides, it is known that the circular vacuum noise does not have the same universal thermal character as the linear Unruh noise. This appears as a ‘drawback’ of the ‘storage ring electron thermometry’, not to mention the very intricate spin physics. Keeping in mind these facts, we go on with further comments, following the very clear discussion of Leinaas.

The circular acceleration in the LEP machine is \( a_{LEP} = 3 \times 10^{22} \text{g} \cdot \text{s}^{-2} \) corresponding to the Unruh temperature \( T_U = 1200 \text{K} \). It is a simple matter to show that an ensemble of electrons in a uniform magnetic field at a nonzero temperature will have a polarization expressed through the following hyperbolic tangent

\[
P_U = \tanh \left( \frac{\pi G}{2} \beta \right).
\]

For the classical value of the gyromagnetic factor \((G = 2)\) and for highly relativistic electrons \((\beta = 1)\), \( P_U = \tanh \pi = 0.996 \), beyond the limiting polarization of Sokolov and Ternov \( P_{lim} = \frac{8}{\sqrt{3}} \approx 0.924 \).

On the other hand, the function \( P_U(G) = \frac{1 - e^{-\pi G}}{1 + e^{-\pi G}} \) is very similar, when plotted, to the function \( P_{DK}(G) \), which is a combination of exponential and polynomial terms in the anomalous part of the gyromagnetic factor of the electron, and it was obtained through QED calculations by Derbenev and Kondratenko. The difference is merely a shift of the latter along the positive G-axis with about 1.2 units. As shown by Bell and Leinaas, when the Thomas precession of the electron is included in the spin Hamiltonian a shift of 2 units is obtained for \( P_U(G) \). This suggests a more careful treatment of spin effects arising when one is going from the lab system to the circulating coordinate frame. A new spin Hamiltonian was introduced by Bell and Leinaas with a more complicated structure of the field vector in the scalar product with the Pauli matrices. This complicated structure takes into account the classical external fields, the quantum radiation field and the fluctuations around the classical path. Within this more complete treatment, Bell and Leinaas were able to get, to linear order in the quantum fluctuations, a Thomas-like term and a third resonant term directly related to the vertical fluctuations in the electron orbit, which are responsible for the spin transitions. The resonance factor, denoted \( f(G) \), induces an interesting variation of the beam polarization close to the resonance. As \( \gamma \) passes through it from below, the polarization first falls from 92% to −17%, and then it increases again to 99% before stabilizing to 92%. This is the only clear difference from the standard QED. Such resonances induced by the vertical fluctuations of the orbit have been considered before in the Russian literature but
focusing strictly on their depolarizing effect. Their nature is related to the fact that the Fourier spectrum of the energy jumps associated with the quantum emission processes contains harmonics giving the usual resonance condition. As emphasized by Bell and Leinaas, a more direct experimental demonstration of the circular Unruh noise would be the measurement of the vertical fluctuations. However, this will clearly be a very difficult task since such fluctuations are among the smallest orbit perturbations. At the same time, the measurement of the polarization variation close to the narrow resonance, in particular the detection of polarizations exceeding the limiting one, will make us more confident in the claims of Bell and Leinaas. It is worth mentioning that the rapid passage through the resonance does not change the polarization, while a slow passage reverses it but does not change the degree of polarization. Therefore only an intermediate rate with respect to a quantum emission time scale of passing through the resonance will be appropriate.

Barber and Mane,\textsuperscript{84} have shown that the DK and BL formalisms for the equilibrium degree of radiative electron polarization are not so different as they might look, and they also obtained an even more general formula for $P_{eq}$ than DK and BL ones. On the base of their formula they estimated as negligible the BL increase near the resonance.

The basic experimental data on spin depolarizing effects remain as yet those measured at SPEAR at energies around 3.6 GeV in 1983. Away from the resonant $\gamma'$s the maximum polarization of Sokolov and Ternov was confirmed.\textsuperscript{85}

We are going to address now some spin physics effects in external fields of critical strength. As was stated in the Introduction, Unruh effect may show up in such fields as a kind of thermal background for some nonlinear phenomena with thresholds in that energy region. In the spin physics context the detailed structure of the electron mass operator $M$ has to be known for such fields. We refer the reader to the paper of Bayer \textit{et al.}\textsuperscript{86} where one can find expressions for the real part of $M$ (related to the anomalous magnetic moment of the electron) and the imaginary part (related to the probability of emission).

Ternov\textsuperscript{87} provided a quantum generalization of the BMT evolution equation including the effects of Zitterbewegung and of the gradients of the magnetic field, expected to become important in the critical regime.

One should mention the quasiclassical trajectory coherent states introduced by Bagrov and Maslov\textsuperscript{88}.\textsuperscript{89} These states have been used in obtaining another generalization of the BMT equation for the electron spin in the case of an arbitrary external torsion field\textsuperscript{89}.

A recent paper of Cai, Lloyd and Papini\textsuperscript{90} claims that the Mashhoon effect due to the spin-rotation coupling is stronger than the circular Unruh effect (spin-acceleration coupling) at all accelerator energies in the case of a perfect circular storage ring. However, the comparison is not at all a straightforward one.

Bautista\textsuperscript{91} solved the Dirac equation in Rindler coordinates with a constant magnetic field in the direction of acceleration and showed that the Bogolubov coefficients of this problem do not mix up the spin components. Thus there is no spin
polarization due to the acceleration in this case.

In our opinion, the real importance of considering Unruh effect at storage rings is related to clarifying radiometric features of the synchrotron radiation. There is a strong need to establish radiometric standards in spectral ranges much beyond those of the cavity/blackbody standards, and synchrotron radiation has already been considered experimentally from this point of view. Quantum field thermality is intrinsically connected to the KMS condition. This is a well-known skew periodicity in imaginary time of Green’s functions expressing the detailed balance criterion in field theory. However, the task is to work out in more definite terms the radiometric message of the KMS quantum/stochastic processes.

6. Unruh Effect and Geonium Physics

The very successful Geonium physics could help detecting the circular thermal-like vacuum noise. The proposal belongs to J. Rogers and apparently it is one of the most attractive. The idea of Rogers is to place a small superconducting Penning trap in a microwave cavity. A single electron is constrained to move in a cyclotron orbit around the trap axis by a uniform magnetic field (Rogers figure is $B = 150$ kGs). The circular proper acceleration is $a = 6 \times 10^{21} g_\oplus$ corresponding to $T = 2.4$ K. The velocity of the electron is maintained fixed ($\beta = 0.6$) by means of a circularly polarized wave at the electron cyclotron frequency, compensating also for the irradiated power. The static quadrupole electric field of the trap creates a quadratic potential well along the trap axis in which the electron oscillates. The axial frequency is 10.5 GHz (more than 150 times the typical experimental situation) for the device scale chosen by Rogers. This is the measured frequency since it is known that the best way of observing the electron motion from the outside world (Feynman’s “rest of the Universe”) is through the measurement of the current due to the induced charge on the cap electrodes of the trap, as a consequence of the axial motion of the electron along the symmetry axis. At 10.5 GHz the difference in energy densities between the circular noise and the universal linear noise are negligible (see Fig. 2 in Rogers’ work). Actually, Rogers used the parametrization for the spectral energy density of a massless scalar field as given by Kim, Soh and Yee. Their calculation is based on the Wightman two-point functions (recall that in quantum optics this is equivalent to not assuming the rotating wave approximation) and yields the following result:

$$\frac{de}{d\omega} = \frac{\hbar}{\pi^2 c^3} \left[ \frac{\omega^3}{2} + \gamma \omega_c^2 x^2 \sum_{n=0}^{\infty} \frac{\beta^{2n}}{2n+1} \cdot \sum_{k=0}^{n} (-1)^k \frac{(n-k-x)^{2n+1}}{k!(2n-k)} \cdot \Theta[n-k-x] \right] \quad (6.1)$$

where $\gamma$ is the relativistic gamma factor, $x = \omega/\gamma \omega_c$, $\omega_c = eB/\gamma mc$ is the cyclotron frequency, and $\Theta$ is the Heaviside step function. The power spectral density at the axial frequency is only $\partial P/\partial f = 0.47 \cdot 10^{-22}$ W/Hz, and may be assumed to be almost the same as the electromagnetic spectral energy density. This power is resonantly transferred to the $TM_{010}$ mode of the microwave cavity and a most
sensible cryogenic GaAs field-effect transistor amplifier should be used to have an acceptable signal-to-noise ratio of $S/N = 0.3$. According to Rogers, the signal can be distinguished from the amplifier noise in about 12 ms.

In conclusion, very stringent conditions are required in the model experiment of Rogers. Top electronics and cryogenic techniques are involved as well as the most sophisticated geonium methods. Taking into account the high degree of precision attained by geonium techniques, one may think of Rogers’ proposal as one of the most feasible. The critique of this proposal is similar to that in storage rings, namely that the circular Unruh effect is not universal, depending also on the electron velocity. Also, Levin, Peleg, and Peres studied the Unruh effect for a massless scalar field enclosed in a two-dimensional circular cavity concluding that the effects of finite cavity size on the frequencies of normal modes of the cavity (Casimir effect) ignored by Bell et al, and by Rogers are in fact quite important.

A better experimental setting for detecting vacuum noises by means of a trapped quantum detector (electron) may well be the cylindrical Penning trap, for which the trap itself is a microwave cavity. In this case small slits incorporating choke flanges divide high-conductivity copper cavity walls into the required electrode Penning configuration, including two compensation electrodes. The driven axial resonance for this configuration has already been observed with almost the same signal-to-noise ratio as in hyperbolic Penning traps. By means of these cylindrical cavity traps, a more direct coupling to the cavity modes may be achieved, especially in the weak coupling regime, where the cyclotron oscillator and the cavity mode cannot form normal modes, and therefore other nonlinear effects are not coming into play. The cylindrical $TM_{010}$ mode is essentially a zero-order Bessel function in the radial direction and has no modes along the z axis. The price to pay in the case of the cylindrical trap is a loss of control on the quality of the electrostatic quadrupole potential.

7. Hawking Effect and Casimir Effect

There exist strong similarities between Hawking-like effects (black hole physics in general) and the Casimir effect. Indeed, the global structure of the spacetime manifold is what really matters for Hawking-like effects, and makes the general features of Hawking’s result to be met already in the moving mirror models.

The Hawking effect might be looked upon as a Casimir effect if one argues as follows. The causal constraints generate peculiar surfaces (horizons) that may be considered as some kind of boundaries. Price and Fabbri have shown long ago that the gravitational field of a black hole creates an effective potential barrier acting as a good conductor in the low frequency range and blocking the high multipoles of the high frequency electromagnetic waves. The barrier is very well localized near $r = 1.5R_h = 3M$. For low frequency waves ($\omega \leq \omega_c = (2/27)^{1/2} \frac{1}{M}$) there are two real turning points for all partial waves). According to Nugayev, who elaborated in more detail on the analogy between the Hawking effect and the
Casimir effect, all the ‘thermal’ radiation is born in the spatial region between the two turning points. Nugayev’s goal was to investigate the ‘particle production’ by a black hole in terms of temperature corrections to the Casimir effect, which are due to the interaction of the radiation with the surface of the potential barrier. He claimed that the two turning points are at different temperatures \(T_{\text{inner}} > T_{\text{outer}}\) and therefore a (Casimir) energy flow from the inner to the outer turning spherical region should occur. This flow of Casimir energy makes the area of the horizon to shrink at precisely the rate consistent with the energy flux at spatial future infinity. Nugayev also argues that the virtual particles are turned into real ones by the small but infinite tail of the potential barrier beyond the maximum which lies at \(3M\).

Making use of the Casimir picture, Nugayev predicted in another work two regimes of black hole evaporation, an anomalous skin effect regime at low temperatures and a normal skin effect at higher temperatures. However, one should keep in mind that no analogy is complete. As was first shown by Ford, although the vacuum energy density in bounded space may have thermal representations (see also Ref.[106]), the spectrum of the Casimir effect is not at all thermal. This may be seen when one is revealing the contribution of each frequency interval to the Casimir energy by means of weighting functions, as Ford has proven.

Let us also mention the connection found by Burinskii, who developed a model for the material of the Kerr-Newman metric source based on the usually neglected volume Casimir energy.

8. Unruh Effect and Nonadiabatic Casimir Effect

An experimental equivalent of a fast moving mirror might be a plasma front created when a gas is suddenly photoionized. This is the proposal of Yablonovitch. The argument is that the phase shift of the zero-point electromagnetic field transmitted through a plasma window whose index of refraction is falling with time (from 1 to 0) is the same as when reflected from an accelerating mirror. Consider the case of hyperbolic motion. Since the velocity is

\[
v = c \tanh(a \tau/c)
\]

where \(\tau\) is the observer’s proper time, the Doppler shift frequency will be

\[
\omega_D = \omega_0 \sqrt{\frac{1 - v/c}{1 + v/c}} = \omega_0 \exp(-a \tau/c)
\]

and consequently a plane wave of frequency \(\omega_0\) turns into a wave with a time-dependent frequency. Such waves are called chirped waves in nonlinear optics and acoustics. Eq. (8.2) represents an exponential chirping valid also for black holes. For an elementary discussion of Doppler shift for accelerated motion see Ref.[108]. It is worthwhile to mention that in the semiclassical treatment of black hole physics one is usually dealing with chirped signals, since the WKB functions are generally of
variable wavelength, and by meeting supplementary conditions on their derivatives they are made to look as much as possible like fixed linear combinations of plane waves. On the other hand, in the case of wave packets one is always working with the average frequency of the wave packets (see the second paper of Jacobson\cite{1}, or the paper of Frolov and Novikov on the dynamical origin of black hole entropy\cite{12}).

The technique of producing plasma fronts/windows in a gas by laser breakdown, and the associated frequency upshifting phenomena (there are also downshifts) of the electromagnetic waves interacting with such windows are well settled since about twenty years, and blue shifts of about 10% have been observed in the transmitted laser photon energy.

In his paper, Yablonovitch works out a very simple model of a linear chirping due to a refractive index linearly decreasing with time, \( n(t) = n_0 - \dot{n}t \), implying a Doppler shift of the form \( \omega \rightarrow \omega[1 + \frac{\dot{n}}{n}t] \sim \omega[1 + \frac{a}{c}t] \). To have accelerations \( a = 10^{20}g\), the laser pulses should be less than 1 picosecond. Even more promising may be the nonadiabatic photoionization of a semiconductor crystal in which case the refractive index can be reduced from 3.5 to 0 on the timescale of the optical pulse. As discussed by Yablonovitch, the pump laser has to be tuned just below the Urbach tail of a direct-gap semiconductor in order to create weakly bound virtual electron-hole pairs which contribute a large reactive component to the photocurrent since they are readily polarized. The background is due to the bremsstrahlung emission produced by real electron-hole pairs, and to diminish it one needs a crystal with a big Urbach slope (the Urbach tail is an exponential behavior of the absorption coefficient).

On the other hand, Yablonovitch remarked that the experimental interpretation is highly ambiguous, since one may consider the phenomenon to be a single-cycle microwave squeezing and/or an inverse quadratic electro-optic effect with zero-point photons as input waves, and more theoretically as Unruh effect and nonadiabatic Casimir effect. Besides, one should notice the difference between the laboratory and the black hole/hyperbolic chirping. The former is linear, whereas the latter is exponential.

The ‘plasma window’ of Yablonovitch was criticized in the important paper by Dodonov, Klimov, and Nikonov [DKN]\cite{11} on the grounds that we are not in the case of exponentially small reflection coefficient as required to get a Planck spectrum from vacuum fluctuations. At the general level, one may argue that nonstationary Casimir effects may produce some deformed Planck distributions, and only in particular cases purely Planck distributions. As a matter of fact, depending on the nonstationarity, one may obtain very peculiar photon spectra, and this might be of great interest in applied physics. DKN showed explicitly that an exponential ‘plasma window’, for which presumably the modulation ‘depth’ is the effective Unruh temperature, does not produce a Planck spectrum. However, for a parametric function displaying the symmetric Epstein profile one can get in the adiabatic limit a ‘Wien’s spectrum’ with the effective temperature given by the logarithmic derivative of the variable magnetic permeability with respect to time. According to DKN this
corresponds to a ‘dielectric window’ and not to a ‘plasma window’. The experimental realization of nonstationary Casimir effects are either resonators with moving walls, as first discussed by Moore\textsuperscript{112} or resonators with time-dependent refractive media as discussed by DKN. On the lines of Yablonovitch, Hizhnyakov\textsuperscript{113} studied the sudden changes of the refractive index caused by the excitations of a semiconductor near a band-to-band transition in the infrared by a synchronously pumped subpicosecond dye laser, and also referred to the analogy with Hawking and Unruh effects. Very recently, C.K. Law\textsuperscript{114} combined the moving walls of Moore with the dielectric medium with time-varying permittivity in a one-dimensional electromagnetic resonant cavity, obtaining an effective quadratic Hamiltonian, which is always required when we want to discuss nonstationary ‘particle production’ effects.

Another interesting solid-state black hole emitting analog has been put forth by Resnik\textsuperscript{115} and refers to surfaces of singular electric and magnetic permeabilities.

9. Unruh Effect and Channeling

An Erevan group\textsuperscript{116} has proposed to measure the Unruh radiation emitted in the Compton scattering of the channeled particles with the Planck spectrum of the inertial crystal vacuum. The proposal is based on the fact that the crystallographic fields are acting with large transverse accelerations on the channeled particles.

The estimated transverse proper acceleration for positrons channeled in the (110) plane of a diamond crystal is $a = 10^{25}\gamma \text{ cm/s}^2$, and at $\gamma = 10^8$ one could reach $10^{33}\text{ cm/s}^2 = 10^{30}\text{ g}$.\textsuperscript{120}

Working first in the particle instantaneous rest frame, the Erevan group derived the spectral angular distribution of the Unruh photons in that frame. By Lorentz transformation to the lab system they got the number of Unruh photons per unit length of crystal and averaged over the channeling diameter. At about $\gamma = 10^8$ the Unruh intensity, i.e., the intensity per unit pathlength of the Compton scattering on the Planck vacuum spectrum becomes comparable with the Bethe-Heitler bremsstrahlung ($dN_\gamma/dE \propto 1/E$, and mean polar emission angle $\theta = 1/\gamma$).

Incidentally, there is a parallel with some experiments\textsuperscript{117,118,119} performed at LEP, where the scattering of the LEP beam from the thermal photon background in the beam pipe has been measured (the black body photons emitted by the walls of the pipe have a mean energy of 0.07 eV). Fortunately the effect is too small to affect the lifetime of the stored beams.

An eye should be kept open on such phenomena like parametric x-ray production by relativistic particles in crystal\textsuperscript{120,121} as well as on other crystal-assisted phenomena.\textsuperscript{122}

In another work of the armenian group\textsuperscript{123} the same type of calculations was applied to estimating the Unruh radiation generated by TeV electrons in a uniform magnetic field as well as in a laser field. The Unruh radiation becomes predominant over the synchrotron radiation only when $\gamma = 10^9$ for $H = 5 \cdot 10^7\text{ Gs}$ and consequently it is impossible to detect it at the SLC. Supercolliders with bunch structure
capable of producing magnetic fields of the order $10^9 G$ are required. Pulsar magnetospheres are good candidates for considering such a Unruh radiation.

A circularly polarized laser field seems more promising since in this case the Unruh radiation could be detectable at lower magnetic fields and energies ($\gamma = 10^7$). This is due to the fact that the proper centripetal acceleration of the electron is

$$a = 2\omega \eta \sqrt{1 + \eta^2},$$

where $\omega$ is the frequency of the electromagnetic wave, and $\eta = e\epsilon/m\omega$ ($\epsilon$ being the amplitude of the field).

10. Hawking-like Effects and Free Electron Lasers (FELs)

In principle, FELs might be a means to put into evidence Unruh radiation as well as Hawking radiation.\(^{124,125}\)

We first recall that in general relativity, it is well known the so-called complexification trick/procedure,\(^{124}\) which leads to new solutions of Einstein equations from a given solution. In particular, A. Peres\(^{126}\) has shown long ago that by the complexification of the isotropic Schwarzschild metric one could get a gravitational tachyon, i.e., a super-light, extended ($r = 2M$) gravitational source.\(^{127}\) (one may also call it a quasi-Minkowski metric with its deviation from flatness of the form $f(Z - uT)$, with $u > 1$, Maxwell constant is unity). According to Peres such procedures have been discussed by N. Rosen already in 1954.\(^{128}\) By this technique a closed horizon is changed into an open (cone-like) horizon. The Mach cone and the Cherenkov shock wave are common examples of open horizons. As far back as 1910, a paper of H. Bateman has the title “Transformations of coordinates which can be used to transform one physical problem into another.”\(^{129}\) Jacobson and Kang\(^{130}\) have recently investigated the conformal invariance of black hole temperature. They showed that this is fulfilled for stationary black holes under those conformal transformations being the identity at infinity.

On the other hand, the electromagnetic emission always makes an important contribution to the radiation of a black hole horizon.\(^{131}\) With this in mind we have to add to the complexification trick a second trick, in which gravitation is supposed to be equivalent to an optical medium. This is an old but not very used method (Einstein was aware of it, and the initiators were W. Gordon, I. Tamm, and L.I. Mandelstam, who wrote papers in the twenties). The interested reader may consult some more recent literature.\(^{132}\) It is as if in this case gravitation gets rid of its fundamental character turning into the constitutive equations of a dielectric medium with a variable refractive index. The constitutive relations of gravitational media are

$$\begin{align*}
D_i & = \epsilon_{ik}E_k - (\vec{G} \times \vec{H})_i \\
B_i & = \mu_{ik}H_k + (\vec{G} \times \vec{E})_i
\end{align*}$$

where $\epsilon_{ik} = \mu_{ik} = -(g)^{1/2}g^{ik}/g_{00}$ and $G_i = -g_{0i}/g_{00}$. The case $g_{0i} \neq 0$ is related to birefringence. For FRW metrics one should use the following form of the gravitational dielectric parameters

$$\epsilon_{ik} = \mu_{ik} = f(\rho)\delta_{ik}$$  \hspace{1cm} (10.3)
The functional form of $f(\rho)$ depends on the cosmological and/or black hole model one has in mind. For example in the de Sitter case $f(\rho) \propto (1 + \rho^2/4R^2)^{-1}$, we see that the dielectric medium is just the Maxwell fisheye lens. This is a spherical lens with an index of refraction that can be written down in the form

$$n(\rho) = \frac{n_0}{1 + \alpha^2 \rho^2}$$

for $\rho < R$. The constant $\alpha$ gives the constant optical gradient. At the present time the GRIN (graded index) technology is at the level of $\alpha = 0.1-0.2 \text{ mm}^{-1}$. GRIN spheres were obtained for the first time in 1986 by means of a modified suspension polymerization technique.

In our previous works, we gave some hints for studying the electromagnetic radiation of the black hole horizon and the Unruh effect on the equivalent scheme of a Cherenkov-Walsh FEL with a GRIN lining. In such a FEL a relativistic electron beam of very good quality passes over a thin dielectric guide or through a channel in it, interacting with the axial component of the TM modes of the guiding structure. The stimulated emission occurs in the modes with a phase velocity slightly less than the velocity of the beam.

There might be a chance for Hawking-like effects to be seen in this experimental configuration if and only if the lining structure is chosen to be a GRIN material with a very high optical gradient (one may think of quartz and fused silica which are common materials in GRIN optics). Of course, the $\gamma$ of the beam should be very high. In this setting, Hawking-like noise would be related to the waveguide dispersion of the liner. Also gas-loaded FELs considered by Pantell’s group should be taken into account as well as plasma lining. The estimated temperature of the Cherenkov wake in an inhomogeneous lining material is

$$T = \frac{\hbar c}{2\pi k} \frac{dn}{d\rho}$$

The present-day optical gradients ($0.2 \text{ mm}^{-1}$) could generate a thermal effect of only 0.7 K. Besides, there are many sources of noise in FEL devices (the most common is the shot noise) and moreover, a lot of other phenomena are waiting to be better understood before addressing more exotic and minute effects. For example Sessler, in his 1989 CAS lecture “Prospects for the FELs” speaks about an untoward number of new effects and discusses superradiance, plasma self-focusing, chirping, and quantum mechanical behavior for electrons and photons in FEL settings, so clearly it will be very difficult to disentangle either ‘Hawking’ or ‘Unruh’ effect by means of FELs.

In addition, Becker and collaborators have commented on testing the photon-photon sector of quantum electrodynamics (i.e., nonlinear effects in QED) with bright short-wavelength FELs with a high repetition rate.

Finally, different types of strophotron FEL configurations, which are based on
11. Unruh Effect and Anomalous Doppler Effect (ADE)

When studied with the detector method, the Unruh effect for a detector with internal degrees of freedom is very close to the anomalous Doppler effect (ADE), since in both cases the quantum detector is radiating ‘photons’ while passing onto the upper level and not on the lower one. It is worthwhile to note that the ADE-like concept has been used by Unruh and Wald, without referring to it explicitly, when they have considered the Unruh effect for a uniformly accelerated quantum detector looked upon from the inertial reference frame. Their main and well-known conclusion was that emission in an inertial frame corresponds to absorption from the Unruh’s ‘heat bath’ in the accelerated frame. Essentially one may say the following.

(i) For the observer placed in the noninertial frame the ‘photon’ is unobservable (it belongs to the left wedge in the Rindler case).

(ii) When the observer places himself in an inertial reference frame, he is able to observe both the excited quantum detector (furnishing at the same time energy to it) and the ‘photons’. By writing down the energy-momentum conservation law he will be inclined to say that the ‘photons’ are emitted precisely when the detector is excited.

There is not much difference between the discussion of Unruh and Wald and some Russian papers distributed over more than 40 years belonging to Ginzburg and Frank. A quantum derivation of the formula for the Doppler effect in a medium has been given by these authors already in 1947, and more detailed discussion has been provided by Frank in the seventies and eighties. See also the recent review paper of Ginzburg.

Neglecting recoil, absorption, and dispersion (a completely ideal case) the elementary radiation events for a two-level detector with the change of the detector proper energy denoted by $\delta \epsilon$ are classified according to the photon energy formula,

$$\hbar \omega = -\frac{\delta \epsilon}{D \gamma}$$  \hspace{1cm} (11.1)

where $\gamma$ is the relativistic velocity factor ($\gamma > 1$) and $D$ is the Doppler directivity factor

$$D = 1 - \frac{(vn/c) \cos \theta}{D \gamma}$$  \hspace{1cm} (11.2)

The discussion of signs in Eq.(11.1) implies 3 cases as follows:

D $> 0$ for normal Doppler effect (NDE, $\delta \epsilon < 0$)
D $= 0$ for Cherenkov effect (CE, $\delta \epsilon = 0$, undetermined case)
D $< 0$ for anomalous Doppler effect (ADE, $\delta \epsilon > 0$).

Consequently, for a quantum system endowed with internal degrees of freedom the stationary population of levels is determined by the probability of radiation in the ADE and NDE regions. The possibility of doing population inversion by means of ADE has been tackled in the Russian literature since long ago. A quantum system
with many levels propagating superluminally in a medium has been discussed for the first time by Ginzburg and Fain in 1958. The inverse population of levels by means of ADE or a combination of ADE and acceleration may be enhanced whenever the ADE region is made larger than the NDE region. This is possible, e.g., in a medium with a big index of refraction. Naryshkina, found already in 1962 that the radiation of longitudinal waves in the ADE region is always greater than in the NDE region, but apparently her work remained unnoticed until 1984, when Nemtsov wrote a short note on the advantage of using ADE longitudinal waves to invert a quantum system propagating in an isotropic plasma. The same year, Nemtsov and Eidman demonstrated inverse population by ADE for the Landau levels of an electron beam propagating in a medium to which a constant magnetic field is applied. More recently, Kurian, Pirojenko and Frolov [KPF] have shown that in certain conditions (for certain range of the parameters), a detector moving with constant superluminous velocity on a circular trajectory inside a medium may be inverted too. Bolotovsky and Bykov have studied the space-time properties of ADE on the simple case of a superluminous dipole propagating in uniform rectilinear motion in a nondispersive medium. These authors are positive with the separate observation of the ADE phenomenon for this case.

The radiation of a uniformly moving superluminal neutral polarizable particle has been studied by Meyer. Frolov and Ginzburg, remarked that this case is an analog of ADE due to zero-point fluctuations of electric polarizability.

Moreover, we can modify the index of refraction in the Doppler factor in such a manner as to get the ADE conditions already at sublight velocities. In this way a more direct link to the Unruh effect is available, as has been shown also by Brevik and Kolbenstvedt. These authors studied in detail the DeWitt detector moving through a dielectric nondispersive medium with constant velocity as well as with constant acceleration, giving in first order perturbation theory formulas for transition probabilities and rates of emitted energy.

Let us mention here that one way to look at negative energy waves in plasma physics is to consider them as a manifestation of induced ADE elementary events discussed in the book of Nezlin. As a matter of fact, a number of authors have already dealt with the problem of amplification and generation of electromagnetic waves based on ADE in the field of quantum electronics. For details on the nonlinear instabilities in plasmas related to the existence of linear negative energy perturbations expressed in terms of specific creation and annihilation operators, and also for a discussion of the complete solution of the three-oscillator case with Cherry-like nonlinear coupling, one should consult the Trieste series of lectures delivered by Pfirsch. Also, Baryshevskii and Dubovskaya considered ADE processes for channeled positrons and electrons. Moreover, Kandrup and O’Neill investigated the hamiltonian structure of the Vlasov-Maxwell system in curved background spacetime with ADM splitting into space plus time, showing the importance of neg-
ative energy modes for time-independent equilibria.

12. Hawking-like Effects and Squeezing

Why is it that in the inertial vacuum we have only zero point fluctuations but when changing to the coordinates of a noninertial reference frame, the new vacuum states, appropriately defined, are thermal-like states containing real photons? Where do the real photons come from?!

In our opinion, the most natural answer to such a paradox is given in the context of squeezing. Any noninertial vacuum, no matter how it is defined, is a squeezed vacuum with respect to the inertial one. The squeezing parameter is related to the boost transformation from inertial to noninertial coordinates. The point is that squeezed vacuum states have a nonzero mean photon number

\[ <n> = \sinh^2 r \]  \hspace{1cm} (12.1)

where \( r \) is the squeezing parameter characterizing the boost transformation. Consequently, any noninertial/gravitational vacuum is no longer a true vacuum, in the sense of having no real particles, and the paradox is solved in a very convenient way.

The squeeze parametrization of the Bogolubov coefficients allows one to accept the idea that some real photons show up in the long quadrature of the squeezed zero point fluctuations of a noninertial/gravitational vacuum. From this squeezing perspective, I do not favor the opinion of Barut and Dowling \[161\] that noninertial thermal baths do not contain real photons. Their claim is that the photons are still bound to the body of the quantum noninertial detector, though turned into dressed states. Of course, the relationship between squeezing and dressed-state polarization (a variant of vacuum polarization) is an interesting open problem for quantum physics in general, to which the concept of decoherence may have a substantial contribution.

I recall here that already in 1976 Hawking wrote down the Bogolubov transformations for the Schwarzschild black holes as follows \[162\]

\[
\begin{align*}
a^{(1)}_\omega &= j_\omega \\
a^{(3)}_\omega &= (1 - x_b)^{-1/2}(h_\omega - x_b^{1/2} g^\dagger_\omega) \\
a^{(4)}_\omega &= (1 - x_b)^{-1/2}(g_\omega - x_b^{1/2} h^\dagger_\omega)
\end{align*}
\]

where \( x_b = \exp(-8\pi GM_\omega) \) is the single Bogolubov parameter of the problem. The \( a \) operators are annihilation operators for modes having zero Cauchy data on the past null infinity and a suitable time dependence on the past horizon, whereas the right hand side annihilation and creation operators correspond to a different basis of three orthogonal families taking into account the fact that an observer at future null infinity can measure only components of the modes outside the future horizon. Grishchuk and Sidorov \[163\] used the Bogolubov transformations obtained
by Hawking to show that the \textit{in} and \textit{out} states are related by a two-mode squeeze operator with the squeezing parameter in each mode given by

\[
\tanh^2 r = \exp (-8\pi GM\omega)
\]  

(12.2)

Moreover, the two-mode SBH squeeze operator \(S(r, \theta)\) has an EPR form, \textit{i.e.}, \(S(r, \pi)\), with \(r\) given by Eq. (12.2). This squeeze operator is to be applied to the \(y\) and \(w\) quasiparticles in Hawking's notation, \textit{i.e.}, those having zero Cauchy data on the past infinity and complementary zero Cauchy data on the horizon in terms of the affine parameter. Then one might think of the equivalence of the black hole spacetimes with some nonlinear optical media in which parametric down conversion phenomena have been put into evidence and represent an extremely active research field. I am tempted to call just \textit{spacetime squeezing} the black hole squeezing, unless one think of it as the concept used by Bialynicka-Birula team some time ago for the squeezing due to the most general case of nonuniform and time dependent linear electromagnetic media. As discussed in Section 10, the \textit{gravitational media} corresponding to the cosmological models are usually gyrotropic, with equal permittivity and permeability tensors. Other examples are the \textit{factorized media} of [DKN], \textit{i.e.}, media with space-time factorized dielectric permittivity and magnetic permeability, which for the time being have no gravitational or more common analogs. Moreover, Yurke and Potasek have shown in the quantum optical context that parametric interactions resulting in the two-mode squeezing provide a mechanism for thermalization whenever one is observing only one mode of a two-mode squeezed vacuum. The generalization to the black hole case is straightforward and provides a reasonable explanation for the overwhelmingly discussed black-hole information paradox. An equivalent of Eq. (12.1) for parametric processes is

\[
\langle n \rangle = \sinh^2 (\Omega \kappa t/4)
\]  

(12.3)

where \(\Omega\) is the frequency of the resonant field with respect to which the parametric processes are achieved, \(\kappa\) is the 'depth' of the modulation, and \(t\) is time. However, the electrodynamic particle production processes in the laboratory involves weak nonlinear parametric phenomena, and for the time being one can make only a formal comparison with the powerful parametric processes required to really put into evidence Hawking and/or Unruh effect.

A strong claim that laboratory squeezing in fibers is equivalent to Unruh effect has been made by Grishchuk, Haus, and Bergman [GHB]. To accomplish laboratory optical squeezing one needs to suppress classical noise and phase match the vacuum wave with the exciting source. These two conditions are very well satisfied by working with fibers. However, one should be aware of the fact that the optical and the material Schroedinger equations, despite their similarities, have also some essential differences, as they apply to different situations.

Highly interesting open issues are the connections among photodetection theory, squeezed states, and accelerated detectors on the lines of Klyshko. A model electron detector similar to the DeWitt monopole detector has been considered.
some time ago by Cresser, who on its base developed a theory of electron detection and photon-photoelectron correlations in two-photon ionization.  

A paper of J.T. Wheeler, on the so-called gravitationally squeezed light is also to be mentioned before ending this section. Wheeler derived an estimate for the amplitude of the squeezing in the case of a beam of coherent light propagating in a gravitational field. For an earthbound experiment his formula is $A = 8g_\gamma/5\omega c$, and so $\omega = 10^{15}$ Hz implies the minute figure $A = 5 \cdot 10^{-23}$. Applying the same formula to the Unruh effect, it might be possible to observe squeezing of photons emitted from particle colliders with an amplitude less than one percent for the same $\omega$ and accelerations of $10^{21} \text{ cm/s}^2$. In the case of black holes, J.T. Wheeler asserted that squeezing may be used to tell how many times a given photon had orbited the black hole close to the $r = 3M$ limit.

It will also be of interest to look at the antibunching properties of black hole radiation (a property of the fourth order correlation function), whereas squeezing is a property of second order ones. Usually, the statistics of a beam is characterized by the Mandel parameter $Q$. The $-1$ value of this parameter corresponds to pure states.

13. Unruh Effect and Hadron Physics

We would like to mention here one of the first applications of Hawking-like effects, namely to explain the thermal spectrum in the transverse energy of the produced particles observed in high-energy collisions. Salam and Strathdee have considered Hawking effect of Kerr-Newman black solitonic solutions in strong gravity to be responsible for the $E_T$ thermal spectrum. Hosoya applied moving mirror effects to the thermal gluon production, and recently the armenian group estimated the contribution of Unruh effect to the soft photon production by quarks, as entailed into the observed anomalous low $p_T$ photons in $K^+p$ interactions at $P = 70 \text{ GeV/c}$.

The idea of relating the hadronic temperature to the Unruh effect is rather old. One way to introduce a hadronic temperature is in terms of Lorentz-squeezed hadrons. Also, Dey et al. related the Unruh temperature to the observed departure from the Gottfried sum rule for the difference of the proton and neutron structure functions in deep inelastic electron scattering. In fact, some of these considerations are not far from the way Nikishov and Ritus tackled the electromagnetic cases.

Other vivid pictures have to do with the relationship between the limiting Hagedorn temperature/maximal acceleration and the Hawking temperature, the space-time duality symmetry, and the role played by strings in the last stages of black hole evaporation.

14. Conclusions and Perspectives

I provided a heuristic survey of the various proposals made so far to detect
the class of thermal-like vacuum noises, commonly known as Hawking effect and Unruh effect. The proposals enumerated herein suggest a transition from a pure *gedanken phase* to a real *experimental one*, but it is fair to say that we are still far from those precise statements required by the definite experimental action. This research field is extremely rich covering a large range of physical situations, and I tried to touch upon its many facets from a global and rather pedestrian standpoint. Of course, these effects, as measured on some analogs in terrestrial laboratories, are extremely tiny. Nevertheless, the analogies developed over the years showed that other fields of physics may have a contribution to the better understanding of the two effects. Moreover, as a corollary, those fields of physics enriched themselves with some unconventional pictures. However, one should be always aware of the ambiguity of interpreting the produced effects as a more direct consequence of the employed experimental method rather than in terms of sophisticated theoretical effects. In other words, the question of the most natural interpretation is always the most stringent one when considering analogies from the experimental point of view.

Perhaps one of the best applications of these conceptual effects is in the areas of optical and electrodynamical radiometry, since they clearly possess those universal qualities usually asked for in those fields. My feeling at the end of the survey is that actually the goal is not so much to try to measure a ‘Hawking’ or a ‘Unruh’ effect. Being ideal concepts/paradigms, what we have to do in order to put them to real work is to make them interfere with the many more ‘pedestrian’ viewpoints.

Another topic to be considered in more detail in the future is the connection between Berry’s phase and the noninertial/gravitational thermal-like effects. Indeed, Berry phase can be related to the so-called Wigner angle (Lorentz transformations in nonparallel directions do not commute involving a rotation angle) and also to the Thomas precession (measuring the time rate of Wigner rotations, and usually associated to spin-orbit couplings) which in turn could also be considered in the class of squeezing phenomena.

Last but not least, the clear-cut aspects of Unruh effect in the realm of nonlinear (multiphoton) quantum electrodynamical effects (the case of Hawking effect is similar within the squeezing perspective) should be further studied taking into account the ‘quasi-feasibility’ of some proposed experimental schemes. As Prof. Keith McDonald recently communicated to the author, *it is useful to continue looking for new ways to explore such effects*. At the same time, we shouldn’t be overenthusiastic about these highly ideal effects; the nonlinear physics is extremely rich in all sorts of effects coming into play at some curious length and time scales that might be assembled from various combinations of the coefficients in some nonlinear partial differential equations, that usually enter the mathematical description of the complicated physical processes that we were writing here about.

Anyway, the correspondence between semiclassical electrodynamics and semiclassical gravity within the pair creation regime should be further studied in order to clarify their similarities and differences, and to appreciate better to what extent
a substantial amount of particle production might be well described semiclassically. In this spirit, we draw attention to a recent paper of Blencowe, who introduced and studied in some detail an electrodynamical model, that one might call a ‘QED-Centauro’ phenomenon: an electrically neutral spherical object, entailing an equal number densities of positive and negative charges exploding in such a way that the negative charges leave the bubble as an expanding spherical shell. Spontaneous pair creation analogous to the Hawking effect occurs when the potential energy difference between the shell and the core exceeds $2m_e$, where $m_e$ is the electron rest mass. Stephens tried to draw an analogy between the one loop approximation of the pair production in a uniform electric field and Hawking effect. Myhrvold commented on thermal radiation from accelerated electrons.

As for the semiclassical gravity, one should notice the recent line of attack suggested by Kuo and Ford and by Calzetta and Hu in terms of a generalized Langevin equation describing in Brownian manner the statistical behavior of test particles moving in the fluctuating gravitational field. There are several advantages of such an approach, among which a more transparent interpretation of back reaction processes. It would be of interest to see what will be the clarifying points brought in for Hawking effect in such a picture.

There is lately a debate concerning the boundary conditions for the Unruh effect. It points to a serious drawback of all previous studies based only on restricting the domain of definition of the fields by light cone (causal) boundary conditions. It has been shown that the basic property of hermiticity of the Hamiltonian of quantum field theories requires a particular supplementary boundary condition at the origin (i.e., the point in common for the left and right Rindler wedges) which has not been considered in deriving the ‘universal’ Unruh effect giving the relationship between the Rindler and Minkowski quantization schemes. On the other hand, Grib argued that in general the light cone boundary conditions are not of “impenetrable wall” type. The light cones are characteristic surfaces for wave equations and causal conditions on them do not violate the wave equations. Therefore a connection between the fields living in different regions of the Minkowski space is possible. Of course, the matter of interpretation of this connection is another deal. Moreover, for the present author the behaviour of a noninertial particle detector in empty Minkovski space is in many regards more important and related indeed to real physics.

Acknowledgements
This work was supported in part by the CONACyT Project 458100-5-25844E.

References

1. Introduction

[1] S.W. Hawking, Nature 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975)
[2] R. Ruffini and J.A. Wheeler, Physics Today 24, 30 (January 1971)
[3] P.C.W. Davies, J. Phys.A: Math. Gen. 8, 609 (1975)
[4] W.G. Unruh, Phys. Rev. D 14, 870 (1976)
[5] K. Freese, C.T. Hill, M. Mueller, Nucl. Phys. B 255, 693 (1985)
[6] C.T. Hill, Nucl. Phys. B 277, 547 (1986)
[7] T.D. Lee, Prog. Theor. Phys. Suppl. 85, 271 (1985)
[8] T.D. Lee, Nucl. Phys. B 264, 437 (1986)
[9] L.N. Pringle, Phys. Rev. D 39, 2178 (1989)
[10] T. Padmanabhan, Ap. Sp. Science 83, 247 (1982)
[11] S.M. Christensen and M.J. Duff, Nucl. Phys. B 146, 11 (1978)
[12] G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15, 2738 (1977)
[13] G.W. Gibbons and M.J. Perry, Proc. R. Soc. London A 358, 467 (1978)
[14] A.S. Lapedes, J. Math. Phys. 19, 2289 (1978)
[15] R.M. Wald, Commun. Math. Phys. 45, 9 (1975)
[16] P.C.W. Davies, S.A. Fulling, and W.G. Unruh, Phys. Rev. D 13, 2720 (1976)
[17] N. Sanchez, Phys. Rev. D 24, 2100 (1981)
[18] W. Israel, Phys. Lett. A 57, 107 (1976)
[19] T.H. Boyer, Phys. Rev. D 21, 2137 (1980) and D 29, 1089 (1984); D.C. Cole, Phys. Rev. D 31, 1972 (1985) and D 35, 562 (1987); A. Higuchi and G.E.A. Matsas, Phys. Rev. D 48, 689 (1993)
[20] S.M. Christensen and S.A. Fulling, Phys. Rev. D 15, 2088 (1977); for a recent review of the Weyl anomaly see M.J. Duff, Class. Quantum Grav. 11, 1387 (1994)
[21] S.A. Fulling, Phys. Rev. D 7, 2850 (1973)
[22] B.S. DeWitt, Phys. Rep. 19, 295-357 (1975)
[23] C.J. Isham, Annals of N.Y. Ac. Sci. 302, 114-157 (1977)
[24] D.W. Sciama, P. Candelas, D. Deutsch, Adv. Phys. 30, 327-366 (1981)
[25] S. Takagi, Prog. Theor. Phys. Suppl. 88, 1-142 (1986)
[26] S.A. Fulling and S.N.M. Ruijsenaars, Phys. Rep. 152, 135-176 (1987)
[27] V.L. Ginzburg and V.P. Frolov, Usp. Fiz. Nauk 153, 633-674 (1987)
[28] I.D. Novikov and V.P. Frolov, *Physics of Black Holes*, (Kluwer, Dordrecht, 1989)
[29] K.S. Thorne, R.H. Price, and D.A. Macdonald, Eds., *Black Holes: The Membrane Paradigm*, (Yale Univ. Press, New Haven and London, 1986)
[30] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, (Cambridge Univ. Press, 1982, reprinted 1989)
[31] H.E. Kandrup, Phys. Lett. B 215, 473 (1988)
[32] R. Peierls, *Surprises in Theoretical Physics*, (Princeton Univ. Press, Princeton, 1979)
[33] K.T. McDonald in AIP Conf. 130, 23 (1985); “Proposal for experimental studies of nonlinear QED”, DOE/ER/3072-38 (1986)
[34] A.I. Nikishov and V.I. Ritus, JETP 94, 31 (1988) [Sov. Phys. JETP 68, 1313-1321 (1988)]
[35] T. Padmanabhan, Phys. Rev. Lett. 64, 2471 (1990)
[36] M.J. Bowick, “The Cosmological Kibble Mechanism in the Laboratory: String Formation in Liquid Crystals”, preprint Syracuse (1992); Acta Phys. Pol. B 24, 1301 (1993)
[37] Steven B. Giddings, “Analogue of the Aharonov-Bohm Effect for Black Holes and Strings” in *Quantum Coherence*, Proc. Intern. Conf. on Fundamental Aspects of Quantum Theory, pp 82-91, (World Scientific, 1990)

\*2. Hawking's and Unruh's Paradigms*

[38] R. Penrose, Riv. Nuovo Cimento 1, 252 (1969)
[39] Ya.B. Zel'dovich, Sov. Phys. JETP 35, 1085 (1972); A.A. Starobinskii, Sov. Phys. JETP 37, 28 (1973)
[40] L. Parker, Fundam. Cosmic Physics 7, 201 (1982)
3. Hawking Effect in Astrophysics

[46] D.N. Page, S.W. Hawking, Astrophys. J. 206, 1 (1976)
[47] C.E. Fichtel, G.A. Simpson, and D.J. Thomson, Astrophys. J. 222, 833 (1978)
[48] F. Halzen, E. Zas, “Search for TEV Hawking Radiation”, preprint Madison MAD-PH-528 (December 1989)
[49] B. Mueller, Duke University Report No. Duke-TH-92-36 (1992)
[50] R.C. Hwa, in Hadronic Multiparticle Production, Advanced Series on Directions in High Energy Physics, Vol. 2, Ed. P. Carruthers, World Scientific, 1988; S.G. Matinyan and E.B. Prokophrenko, Phys. Rev. D 48, 5127 (1993); for an old paper on black hole emission processes in the high energy limit see: B. Carter et al., Astron. & Astrophys. 52, 427 (1976)
[51] F. Halzen, E. Zas, J.H. MacGibbon, T.C. Weekes, “Search for Gamma-Rays from BHs”, preprint Madison MAD-PH-575 (July 1990).
[52] Jane MacGibbon, B.R. Webber, Phys. Rev. D 41, 3052 (1990)
[53] Jane H. MacGibbon and B.J. Carr, Astrophys. J. 371, 447 (1991)
[54] Jane H. MacGibbon, “Quark and Gluon Jet Emission from PBHs: (2) The Lifetime Emission”, preprint NASA-Goddard Space Flight Center 91-001 (1991)
[55] Jane H. MacGibbon, “Cosmic Rays from PBHs”, preprint NASA-Goddard Space Flight Center 91-016 (1991)
[56] A.F. Grillo, “Point Sources of High Energy Cosmic Rays”, Lectures at the Second School on “Non-Accelerator Particle Astrophysics”, ICTP-Trieste, 3-14 June 1991.
[57] P.V. Ramana Murthy, “VHE and UHE Gamma Ray Astronomy”, Lectures at the Second School on “Non-Accelerator Particle Astrophysics”, ICTP-Trieste, 3-14 June 1991.
[58] D.E. Alexandreas et al., Phys. Rev. Lett. 71, 2524 (1993)
[59] G.'t Hooft, Physica Scripta T 36, 247 (1991)
[60] H. Rosu, Nuovo Cimento B 108, 1333 (1993)
[61] N.-x. Chen Phys. Rev. Lett. 64, 1193 (1990)
[62] G.F. Chapline, Phys. Rev. D 12, 2949 (1975)
[63] M. Visser, Mod. Phys. Lett. A 8, 1661 (1993); C.F.E. Holzhey and F. Wilczek, Nucl. Phys. B 380, 447 (1992)
[64] M.S. Turner, Nature 297, 379 (1982)
[65] G. Greenstein and J.O. Burns, Am. J. Phys. 52, 531 (1984)

4. Hydrodynamical Hawking Effect

[66] W.G. Unruh, Phys. Rev. Lett. 46, 1351 (1981); Phys. Rev. D 51, 2827 (1995); For more recent works, see M. Visser, Class. Quant. Grav. 15, 1767 (1998) (gr-qc/9712010, v2), Phys. Rev. Lett. 80, 3436 (1998) (gr-qc/9712010); D. Hochberg and J. Perez-Mercader, Phys. Rev. D 55, 4880 (1997) (gr-qc/9609043)
[67] G.L. Comer, “Superfluid Analog of the Davies-Unruh Effect”, preprint Racah, November 1991; for a standard textbook see: S.J. Putterman, Superfluid hydrodynamics (North Holland, Amsterdam, 1974)
5. Unruh Effect in Storage Rings

[77] J.S. Bell, J.M. Leinaas, Nucl. Phys. B 212, 131 (1983)
[78] J.S. Bell, R.J. Hughes, J.M. Leinaas, Z. Phys. C 28, 75 (1985)
[79] J.S. Bell, J.M. Leinaas, Nucl. Phys. B 284, 488 (1987)
[80] K.T. McDonald, in Proc. 1987 IEEE Part. Accel. Conf. vol. 2, 1196 (1987)
[81] J. Leinaas, “Hawking Radiation, the Unruh Effect and the Polarization of Electrons”, Europhysics News 22, 78 (1991)

6. Unruh Effect and Geonium Physics

[95] J. Rogers, Phys. Rev. Lett. 61, 2113 (1988)
[96] L.S. Brown and G. Gabrielse, Rev. Mod. Phys. 58, 233 (1986)
[97] S.K. Kim, K.S. Soh, and J.H. Yee, Phys. Rev. D 35, 557 (1987); J.R. Letaw and J.D. Pfautsch, Phys. Rev. D 24, 1491 (1981); S. Hacyan and A. Sarmiento, Phys. Lett. B 179, 287 (1986); G. Denardo and R. Percacci, Nuovo Cimento B 48, 81 (1978)
[98] H. Rosu, Nuovo Cimento B 111, 507 (1996)
[99] O. Levin, Y. Peleg, and A. Peres, J. Phys. A: Math. Gen. 26, 3001 (1993)
[100] J. Tan and G. Gabrielse, Appl. Phys. Lett. 55, 2144 (1989); Phys. Rev. A 48, 3105 (1993)
• 7. Hawking Effect and Casimir Effect
[101] R.H. Price, Phys. Rev. D 5, 2419 (1972)
[102] R. Fabbri, Phys. Rev. D 12, 933 (1975)
[103] R.M. Nugayev, Commun. Math. Phys. 111, 579 (1987)
[104] R.M. Nugayev, Phys. Rev. D 43, 1195 (1991)
[105] L.H. Ford, Phys. Rev. D 38, 528 (1988); Phys. Rev. A 48, 2962 (1993)
[106] G. Cocho, S. Hacyan, A. Sarmiento, F. Soto, Int. J. Theor. Phys. 28, 699 (1989)
[107] A. Ya. Burinskii, Phys. Lett. B 216, 123 (1989)

• 8. Unruh Effect and Nonadiabatic Casimir Effect
[108] E. Yablonovitch, Phys. Rev. Lett. 62, 1742 (1989)
[109] W. Cochran, Am. J. Phys. 57, 1039 (1989)
[110] V. Frolov and I. Novikov, Phys. Rev. D 48, 4545 (1993)
[111] V.V. Dodonov, A.B. Klimov, and D.E. Nikonov, Phys. Rev. A 47, 4422 (1993)
[112] G.T. Moore, J. Math. Phys. 11, 2679 (1970)
[113] V.V. Hizhnyakov, Quantum Opt. 4, 277 (1992)
[114] C.K. Law, Phys. Rev. A 49, 433 (1994)
[115] B. Resnik, Phys. Rev. D 62, 044044 (2000), gr-qc/9703077

• 9. Unruh Effect and Channeling
[116] S.M. Darbinian, K.A. Ispirian, A.T. Margarian, preprint Yerevan Phys.Inst. YERPHY-1188(65)-89 (August 1989)
[117] B. Dehning, A.C. Melissinos, F. Perrone, C. Rizzo, G. von Holtey, Phys. Lett. B 249, 145 (1990)
[118] C. Bini, G. De Zorzi, G. Diambrini-Palazzi, G. Di Cosimo, A. Di Domenico, P. Gauzzi, D. Zanello, “Scattering of Thermal Photons by a 46 GeV Positron Beam at LEP”, preprint CERN-PRE-91-64 (February 1991)
[119] Unauthored, “LEP Takes its Temperature”, CERN Courier 31, 2 (March 1991)
[120] I. Ya. Dubovskaya et al., J. Phys.: Condens. Matter 5, 7771 (1993)
[121] A. Caticha, Phys. Rev. B 45, 9541 (1992)
[122] A. Bellacem et al., Phys. Lett. B 177, 211 (1986); X. Artru, Phys. Lett. A 128, 302 (1988); A. Bellacem, N. Cue, and J.C. Kimball Phys. Lett. A 111, 86 (1985)
[123] S.M. Darbinian, K.A. Ispiryan, M.K. Ispiryan, A.T. Margaryan, JETP Lett. 51, 110 (1990)

• 10. Hawking-like Effects and FELs
[124] H. Rosu, preprint Magurele IFA-FT-355 (April 1989, revised at ICTP in November 1991)
[125] H. Rosu, preprint Magurele IFA-FT-367 (December 1989, revised at ICTP in November 1991)
[126] E.T. Newman and A.I. Janis, J. Math. Phys. 6, 915 (1965)
[127] A. Peres, Phys. Lett. A 31, 361 (1970)
[128] N. Rosen, Bull. Res. Council Israel 3, 328 (1954)
[129] L.S. Schulman, Nuovo Cimento B 2, 38 (1971); J.R. Gott III, Nuovo Cimento B 22, 49 (1974)
[130] H. Bateman, Proc. Lond. Math. Soc. (2) 8, 469 (1910)
[131] T. Jacobson and G. Kang, Class. Quantum Grav. 10, L201 (1993)
[132] D.N. Page, Phys. Rev. D 13, 198 (1976)
[133] J. Plebański, Phys. Rev. 118, 1396 (1960); A.M. Volkov, A.A. Izmost’ev, and G.V. Skrotskii, JETP 59, 1254 (1970) [Sov. Phys. JETP 32, 686 (1971)]; B. Mashhoon,
11. Unruh Effect and Anomalous Doppler Effect

- W.G. Unruh and R.M. Wald, Phys. Rev. D 29, 1047 (1984); A. Higuchi, G.E.A. Matsas, and D. Sudarsky, Phys. Rev. D 46, 3450 (1992) and D 45, R3308 (1992)
- V.L. Ginzburg and I.M. Frank, Dokl. Akad. Nauk 56, 583 (1947)
- I.M. Frank, Usp. Fiz. Nauk 129, 685 (1979); Vavilov-Cherenkov Radiation. Theoretical Aspects (Nauka, Moskow, 1988)
- V.L. Ginzburg, in Progress in Optics XXXII, Ed. E. Wolf (Elsevier, 1993) [Russian version in FIAN 176 (Nauka, Moskow, 1986)]. The 1993 version is updated and has a new section with comments on acceleration radiation.
- V.P. Frolov and V.L. Ginzburg, Phys. Lett. A 116, 423 (1986)
- V.L. Ginzburg and V.M. Fain, JETF 35, 817 (1958)
- L.G. Naryshkina, JETF 43, 953 (1962)
- B.E. Nemtsov, Pis’ma v JTF 10, 588 (1984)
- B.E. Nemtsov and V. Ya. Eidman, JETF 87, 1192 (1984)
- V.E. Kurian, A.V. Pirojenko, V.P. Florov, “On the Possibility of Population Inversion in Quantum Systems Moving Uniformly on a Circle in a Medium”, FIAN preprint 142 (1988); KSF 10, 54 (1988)
- B.M. Bolotovski and V.P. Bykov, Radiofizika 32, 386 (1989)
- P.P. Meyer, J. Phys. A: Math. Gen. 18, 2235 (1985)
- I. Brevik and H. Kalbenstvedt, Nuovo Cimento B 103, 45 (1989)
- M.V. Nedin, Dynamics of Beams in Plasmas, in Russian, (Energoizdat, 1982)
- N.S. Ginzburg, Radiofizika 22, 470 (1979); A.N. Didenko et al., Pis’ma v JTF 9, 1207 (1983); M.V. Kuzelyev and A.A. Rukhadze, Fizika Plasmy 15, 1122 (1989)
- D. Pfirsch, “Negative-energy Modes in Collisionless Kinetik Theories and their Possible Relation to Nonlinear Instabilities”, series of lectures at the Plasma College, ICTP-Trieste (June 1991)
- V.G. Bar'yshchevskii and I. Ya. Dubovskaya, Sov. Phys. Dokl. 21, 741 (1976)
- H.E. Kandrup and E. O’Neill, Phys. Rev. D 48, 4534 (1993)

12. Hawking-like Effects and Squeezing

- L.P. Grishchuk and Y.V. Sidorov, Phys.Rev. D 42, 3413 (1990);
- A.O. Barut and J.P. Dowling, Phys. Rev. A 41, 2277 (1990)
- S. Hawking, Phys. Rev. D 14, 2460 (1976)
- Z. Białynicka-Birula and I. Białynicki-Birula, J. Opt. Soc. Am. B 4, 621 (1987)
- B. Yurke and M. Potasek, Phys. Rev. A 36, 3464 (1987)
[165] L. Grishchuk, H.A. Haus, and K. Bergman, Phys. Rev. D 46, 1440 (1992), especially Section VIII, where a nonlinear Mach-Zehnder configuration is proposed for generating radiation from zero-point fluctuations interacting with a moving index grating, and where it is claimed the similarity with the Unruh effect; H.A. Haus, private communication (1993)

[166] D.N. Klyshko, Phys. Lett. A 154, 433 (1991)

[167] J.D. Cresser, J. Opt. Soc. Am. B 6, 1492 (1989)

[168] J.T. Wheeler, Gen. Rel. Grav. 21, 293 (1989)

[169] C.W. Gardiner, Phys. Rev. Lett. 70, 2269 (1993)

- 13. Unruh Effect and Hadron Physics

[170] A. Salam and J. Strathdee, Phys. Lett. B 66, 143 (1977)

[171] A. Hosoya, Prog. Theor. Phys. 61, 280 (1979)

[172] M. Horibe, Prog. Theor. Phys. 61, 661 (1979)

[173] S.M. Darbinian, K.A. Ispirian, A.T. Margarian, preprint Erevan YERPHI-1314(9)-91 (1991)

[174] S. Barshay and W. Troost, Phys. Lett. B 73, 437 (1978)

[175] D. Han, Y.S. Kim, M.E. Noz, Phys. Lett. A 144, 111 (1990)

[176] J. Dey, M. Dey, L. Tomnio, and M. Schiffer, Phys. Lett. A 172, 203 (1993)

[177] R. Parentani and R. Potting, Phys. Rev. Lett. 63, 945 (1989)

[178] H. Verlinde, “String Theory and BHs”, Lectures at Spring School on String Theory and Quantum Gravity, ICTP-Trieste (April 1991)

- 14. Conclusions and Perspectives

[179] H. Han, Y.S. Kim, M.E. Noz, Phys. Rev. A 37, 807 (1988)

[180] R. Chiao, Nucl. Phys. (Proc. Suppl.) B 6, 327 (1989)

[181] M. Kugler and S. Shtrikman, Phys. Rev. D 37, 934 (1988)

[182] D. Han, E.E. Hardekopf, Y.S. Kim, Phys. Rev. A 39, 1269 (1989)

[183] H. Mathur, Phys. Rev. Lett. 67, 3325 (1991); Geometrical phases for relativistic particles were first considered by I. Bialynicki-Birula and Z. Bialynicka-Birula, Phys. Rev. D 35, 2383 (1987) and T.F. Jordan J. Math. Phys. 28, 1759 (1987)

[184] K. McDonald, private communication (1993)

[185] M. Blencowe, Phys. Rev. D 48, 4800 (1993); Some papers on QED pair production in strong fields are: F. Cooper and E. Mottola, Phys. Rev. D 40, 456 (1989); F. Cooper, “Dynamical Approach to Pair Production from Strong Fields”, LA-UR-92-2753 (1992); E. Mottola, “Pair Production and Back Reaction in Strong Fields: Numerical Results”, LA-UR-92-3073; H. Rumpf, Helvetica Physica Acta 53, 85 (1980); G. Baur and C.A. Bertulani, Phys. Rep. 163, 299 (1986); S. Keller and R.M. Dreizler, Ann. Phys. 229, 252 (1994); D.C. Ionescu, Phys. Rev. A 49, 3188 (1994); J. Rau, “Pair production in the quantum Boltzmann equation”, Phys. Rev. D 50, 6911 (1994) The classic papers in this field are: J. Schwinger, Phys. Rev. 82, 664 (1951); W. Heisenberg and H. Euler, Z. Phys. 98, 714 (1936); For a book see: W. Greiner, B. Mueller, and J. Rafelski, Quantum Electrodynamics of Strong Fields (Springer, Berlin, 1985)

[186] C.R. Stephens, Ann. Phys. 193, 255 (1989)

[187] N.P. Myhrvold, Ann. Phys. 160, 102 (1985)

[188] Chung-I Kuo and L.H. Ford, Phys. Rev. D 47, 4510 (1993)

[189] E. Calzetta and B.I. Hu, Phys. Rev. D 49, 6636 (1994)

[190] A.M. Fedotov, V.D. Mur, N.B. Narozhnyi, V.A. Belinskii, B.M. Karnakov, Phys. Lett. A 254, 126 (1999); N.B. Narozhnyi et al., hep-th/9906181; D. Oriti, Nuovo Cimento B 115, 1005 (2000), [gr-qc/9912082]

[191] A.A. Grib, JETP Lett. 67, 95 (1998)
[192] H.C. Rosu, Nuovo Cimento B 115, 1049 (2000), gr-qc/9912056