Calculation Method of Productivity for Shale Oil in Volumetric Fractured Horizontal Wells

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ABSTRACT: In order to evaluate quantitatively the productivity of shale oil in fractured horizontal wells and in view of the influence of non-Darcy seepage and pressure sensitivity on shale oil productivity, the methods of conformal transformation and superposition theory of complex potential are used to deduce the productivity formula in volumetric fractured horizontal wells with start-up pressure gradient and pressure sensitivity and to calculate quantitatively the productivity of each fracture. In addition, the influence factors on productivity, such as the fracture length, number of fractures, pressure sensitivity coefficient, starting pressure gradient, and other factors, have been analyzed as well. The study results indicate that the productivity of horizontal wells decreases with the increase of the start-up pressure gradient and pressure sensitivity coefficient. With the increase of the half-length and number of fractures, the productivity of fractured horizontal wells will increase, but the amplitude of increase decreases gradually. This method can be used for productivity calculation and optimization design of the fracturing parameter in volumetric fractured horizontal wells.

1. INTRODUCTION

The shale oil resources are rich in continental basins of China, regard as an important strategic replacement area for increasing oil and gas reserves and production.5 By referring to the development concept of shale oil and gas in North America, great progress has been made in exploration and development of shale oil and gas, and successive discovery has been made in the Ordos Basin, the Junggar Basin, and the Songliao Basin, which indicates the good prospects for exploration and development of shale oil and gas in China.6−8 At present, the development of shale oil is mainly by horizontal wells and multi-stage fracturing technologies to increase the volume of production in single well.

Given that most foreign shale oil reservoirs are interlayer-type or mixed-type shale oil reservoirs of marine or lacustrine facies, the shale oil of China’s continental basins is fundamentally different. Taking the Gulong shale oil as an example, it is a combination of source and reservoir, in situ accumulation of oil and gas, with no migration or only micro-nano-scale migration during the hydrocarbon generation process, and with narrow pore throat and large specific surface area. These characteristics lead to greater influence of the molecular force on the solid–liquid surface. Once the displacement pressure gradient reaches a certain critical value, fluid can flow, which is a non-Darcy seepage. Meanwhile, the rock skeleton of the reservoir will undergo plastic deformation, with the change in pressure, resulting in smaller pore throat, and lower permeability; thus, the pressure sensitivity of the reservoir needs to be considered.9−13 The existence of start-up pressure gradient and pressure sensitivity will lead to the reduction of single-well production, and the impact of both on productivity must be taken into account.

Scholars at home and abroad have carried out multi-faceted research studies on the productivity of horizontal wells.14−27 Among them, Lang et al.14 proposed in 1994 the potential theory and superposition principle to conduct production prediction in fractured horizontal wells; Li et al.15 proposed in 1996 a performance prediction method in the fractured horizontal well and analyzed the factors that affect the performance of hydraulic fractured horizontal wells; Zifei et al.16 proposed in 1996 a steady-state solution formula for vertical fractures in rectangular oil reservoirs using conformal transformation and deduced the productivity formula of horizontal wells in fractured reservoirs; Zerzar et al.17 deduced the productivity equation for fractured wells with limited conductivity in 2003; and Zheng18 proposed in 2013 that a
local coordinate system was introduced to establish a horizontal well productivity calculation model that considers the uneven distribution of fractures based on conformal transformation. Liu19 and others put forward in 2013 the concept of the productivity-stress sensitivity index and established a productivity-stress sensitivity equation that considers permeability and porosity. Zhang et al.20 proposed in 2015 the productivity formula of oil wells in low permeability reservoirs based on the consideration of influence of the starting pressure gradient and media deformation coefficient. Zeng et al.21 proposed in 2018 a mathematical model of multi-stage fractured horizontal wells that comprehensively considers start-up pressure gradient, partial penetration fractures, and reservoir heterogeneity. These methods are mainly aimed at low-permeability reservoirs, and both volume fracturing have not been considered, and the combined influence of non-Darcy flow and pressure sensitivity on the productivity of oil wells have not been considered yet. In shale oil reservoirs, the permeability is extremely low, while the interaction between fluid and rock is strong, showing non-Darcy seepage characteristics. Fluid can flow only when the driving pressure gradient is greater than the starting pressure gradient;22,23 at the same time, change in reservoir pressure will cause change in permeability of shale oil reservoirs,24 which should not be overlooked. Based on the reality that shale oil reservoirs are mainly reformed by volume fracturing, with consideration of the non-Darcy flow characteristics and pressure sensitivity, we derived the calculation method of volume fracturing productivity for horizontal wells. The productivity change in horizontal wells has been studied by considering start-up pressure gradient, pressure sensitivity, and the combined effects of both. In addition, the influence of fracture half-length and number of fractures on productivity has been analyzed. This method provides a basis for the prediction of production in horizontal wells for shale oil reservoir of this type.

2. CALCULATION FORMULA OF PRODUCTION BY HORIZONTAL WELL CONSIDERING THE INFLUENCE OF NON-DARCY AND PRESSURE SENSITIVITY

2.1. Establishment of a Single Fracture Production Model. According to the characteristics of the shale oil in the study area, it is assumed that fractures are infinite conductivity ones, and the oil layer boundary is a constant pressure one. In addition, it is assumed that it is a steady-state seepage, and the physical properties of crude oil are the same, with staged fracturing in horizontal wells. The schematic diagram of the fracturing model is shown in Figure 1.

Supposing that there is an oil layer with a thickness of \( h \) and a permeability of \( K \), among which a horizontal well with a length of \( L \) penetrates \( N \) fractures with equidistance distribution. The potential distribution of a single fracture is derived from potential theory, and the productivity formula for a horizontal well with multiple fractures is derived from the superposition principle.25–27 As shown in Figure 2, the \( y \)-axis is the direction of the horizontal segment. \( x \) is the direction of single fracture, which is perpendicular to the horizontal section.

![Figure 1. Schematic diagram of the fracturing model for horizontal wells.](https://doi.org/10.1021/acsomega.1c07057)

![Figure 2. Distribution of a single fracture.](https://doi.org/10.1021/acsomega.1c07057)

![Figure 3. Schematic diagram of the conformal transformation.](https://doi.org/10.1021/acsomega.1c07057)

Take the transformation function, that is

\[
    z = x_f \cosh \omega
\]  
(1)

Among them, there is \( z = x + iy \) in the \( z \)-plane, and there is \( \omega = \xi + i\eta \) in the \( \omega \)-plane.

Therefore

\[
    x + iy = x_f \cosh(\xi + i\eta)
\]  
(2)

Therefore, the corresponding relationship of coordinates is as follows

\[
    \begin{align*}
        x &= x_f \cosh \xi \cos \eta \\
        y &= x_f \sinh \xi \sin \eta
    \end{align*}
\]  
(3)

According to the trigonometric function relationship: \( \cos^2 \eta + \sin^2 \eta = 1 \), we can get

\[
    \left( \frac{x}{x_f \cosh \xi} \right)^2 + \left( \frac{y}{x_f \sinh \xi} \right)^2 = 1
\]  
(4)

By use of \( x, y, \) and \( x_f \) to represent \( \xi, \) we can simplify formula 4.
\[ \xi = \text{arccosh} \left( \sqrt{\frac{1}{2} x^2 + \frac{1}{2} y^2 + \frac{1}{4} \left( \frac{x^2}{x_i^2} + \frac{y^2}{x_i^2} \right) - \frac{1}{4} \left( \frac{x^2}{x_i^2} + \frac{y^2}{x_i^2} \right)} \right) \]

(5)

The expression of the potential function is as follows

\[ \Phi = \frac{qB}{2\pi h} \xi + C \]

(6)

where \( \Phi \) is the potential function, \( q \) is the fracture production, \( h \) is the thickness of the oil layer, \( B \) is the volume coefficient of crude oil, and \( C \) is a constant.

By substituting formula 5 into formula 6, we can obtain the potential of a single fracture in the z-plane, namely

\[ \Phi = \frac{qB}{2\pi h} \text{arccosh} \left( 1 \frac{x - x_0}{x_i} + \frac{y - y_0}{x_i} + \frac{1}{2} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) - \frac{1}{4} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) \right) + C \]

(7)

Assuming that the coordinates of the midpoint of any fracture on the z-plane are \((x_0, y_0)\), then, formula 7 can be rewritten as

\[ \Phi = \frac{qB}{2\pi h} \text{arccosh} \left( 1 \frac{x - x_0}{x_i} + \frac{y - y_0}{x_i} + \frac{1}{2} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) - \frac{1}{4} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) \right) + C \]

(8)

To reorganize formula 9, that is

\[ \Phi = \frac{qB}{2\pi h} \text{arccosh} \left( 1 \frac{x - x_0}{x_i} + \frac{y - y_0}{x_i} + \frac{1}{2} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) - \frac{1}{4} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) \right) + C \]

(9)

To order

\[ Y(x, y) = \text{arccosh} \left( 1 \frac{x - x_0}{x_i} + \frac{y - y_0}{x_i} + \frac{1}{2} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) - \frac{1}{4} \left( \frac{x - x_0}{x_i^2} + \frac{y - y_0}{x_i^2} \right) \right)^{1/2} \]

(10)

Formula 10 is the expression of the potential of a single vertical fracture in a horizontal well.

2.1.1. Calculation Formula of Horizontal Well Production

By Considering the Influence of Non-Darcy Seepage When It Comes to a Single Fracture. When only the influence of non-Darcy seepage is considered, that is, the starting pressure gradient is considered,\(^{28-30}\) then, the specific expression is as follows

\[ G = \alpha \left( \frac{K}{\mu} \right)^{-n} \]

(11)

where \( G \) is the starting pressure gradient, MPa/m; \( K \) is the average permeability of formation, mD; \( \mu \) is the fluid viscosity, mPa s; \( \alpha_i \), and \( n \) is the regression coefficient.

The equation of motion of the oil phase is

\[ v_o = -K_n\left( \frac{dp}{dr} - G \right) \]

(12)

The continuity equation of the oil phase with stable seepage is

\[ \left( \frac{\partial v_o}{\partial x} + \frac{\partial v_o}{\partial y} + \frac{\partial v_o}{\partial z} \right) = 0 \]

(13)

To substitute formula 12 into formula 13, the basic differential equation for the oil phase with stable seepage is

\[ \nabla \left( \frac{K_n}{\mu} \nabla (p - Gr) \right) = 0 \]

(14)

To order \( \mu_h = \left( \frac{K_n}{\mu} \right)^{-1} \)

\[ \Phi = \frac{K_n}{\mu_h} (p - Gr) \]

(15)

Formula 15 can be substituted into formula 14

\[ \nabla^2 \Phi = 0 \]

(16)

The new variable \( \Phi \) satisfies the Laplace equation, the solution of which is the potential function, where \( \mu_h \) is the visual fluid viscosity, mPa s.

At a point \((x_0, y_0)\) at the boundary, the boundary pressure is \( p_e \) at this time, and the potential function is \( \Phi_e \). The coordinates of the midpoint of any existing fracture is \((x_0, y_0)\), the production is \( q \), the pressure at this point is \( p_i \), with the consideration of the starting pressure gradient, simultaneous formula 15 can be obtained

\[ \Phi_e = \frac{K_n}{\mu_h} (p_e - GY_e) \]

(17)

\[ \Phi_i = \frac{K_n}{\mu_h} (p_i - GY_i) \]

(18)

Formula 17 minus 18 can be sorted out
\[ \Phi_e - \Phi_i = \frac{K}{\mu} (p_e - p_i - G(\chi_e - \chi_i)) \]
\[ = \frac{K}{\mu} \left( p_e - p_i - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \right) \]  
(19)

To substitute formula 10 into formula 19, the following can be obtained
\[ \frac{qB}{2\pi h} [Y(x_e, \chi_e) - Y(x_i, \chi_i)] \]
\[ = \frac{K}{\mu} \left( p_e - p_i - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \right) \]  
(20)

To organize formula 20 to get
\[ q = \frac{2nk\bar{K}e^{-\alpha_i(p_e - p_i)}}{\mu B} \left( p_e - p_i - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \right) \]
\[ Y(x_e, \chi_e) - Y(x_i, \chi_i) \]  
(21)

Formula 21 is the calculation formula of horizontal well production by considering the influence of non-Darcy flow.

2.1.2. Calculation Formula of Horizontal Well Production by Considering Pressure Sensitivity When It Comes to a Single Fracture. When considering the only pressure-sensitive effect, the specific expression is as follows:
\[ K = K_e e^{-\alpha(p_e - p_i)} \]  
(22)

where \( K_e \) is the original permeability, mD; \( \alpha \) is the pressure sensitivity coefficient, MPa\(^{-1}\); \( p_e \) is the original formation pressure, MPa; and \( p_{i\text{ord}} \) is the bottom hole flowing pressure, MPa.

Same as the previous derivation, at a point \((x_e, \chi_e)\) at the boundary, by considering the pressure-sensitive effect, the following formula is
\[ \Phi_e - \Phi_i = \frac{K}{\mu} (p_e - p_i) - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \]  
(23)

In the same way, to substitute formula 10 into formula 23, we can get
\[ \frac{qB}{2\pi h} [Y(x_e, \chi_e) - Y(x_i, \chi_i)] = \frac{K_e e^{-\alpha_i(p_e - p_i)}}{\mu} (p_e - p_i) \]  
(24)

To organize formula 24, we can get
\[ q = \frac{2nk\bar{K}e^{-\alpha_i(p_e - p_i)}}{\mu B} \left( p_e - p_i \right) \]
\[ Y(x_e, \chi_e) - Y(x_i, \chi_i) \]  
(25)

Formula 25 is the calculation formula of horizontal well production by considering pressure sensitivity.

2.1.3. Calculation Formula of Horizontal Well Production by Considering the Combined Influence of Both Non-Darcy Seepage and Pressure Sensitivity When It Comes to a Single Fracture. When the above two conditions exist at the same time, the non-Darcy seepage and pressure sensitivity are considered to affect horizontal well production. The derivation idea is the same as the foregoing. At this time, at a point \((x_e, \chi_e)\) at the boundary, by considering the influence of non-Darcy seepage and pressure sensitivity, the following formula is concluded
\[ \Phi_e - \Phi_i = \frac{K}{\mu} (p_e - p_i - G(\chi_e - \chi_i)) \]
\[ = \frac{K_e e^{-\alpha_i(p_e - p_i)}}{\mu} \left( p_e - p_i - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \right) \]  
(26)

To substitute formula 10 into formula 26 and sort it out, we can get
\[ q = \frac{2nk\bar{K}e^{-\alpha_i(p_e - p_i)}}{\mu B} \left( p_e - p_i - \alpha \left( \frac{\bar{K}}{\mu} \right)^n (\chi_e - \chi_i) \right) \]
\[ Y(x_e, \chi_e) - Y(x_i, \chi_i) \]  
(27)

Formula 27 is the calculation formula of horizontal well production by considering the combined influence of both non-Darcy and pressure sensitivity.

2.2. Establishment of the Multi-Fracture Production Model. Now supposing that a horizontal well with a length of \( L \) has \( N \) multiple vertical fractures without consideration of the productivity of the horizontal well in the unfractured area, the productivity of these \( N \) fractures can be solved according to the principle of potential superposition.

There are three situations as follows, namely, by considering non-Darcy seepage effects, only considering the influence of pressure sensitivity and by considering the combined influence of both non-Darcy seepage and pressure sensitivity effects. In each case, different parity of the number \( N \) will be discussed, respectively.

2.2.1. Calculation Formula of Horizontal Well Production by Considering the Influence of Non-Darcy Seepage When It Comes to Multiple Fractures.

(1) When the number of fractures \( N \) is an odd number, the coordinates of the fracture midpoint are \((0, 0), (0, \pm d), (0, \pm 2d), \ldots, (0, \pm Nd_d)\), among which \( N_d = (N - 1)/2 \), and the distance between fractures is \( d = L/N \), the productivity of the \( i \)th fracture is \( q_{i d} \) and \( x_{i d} \) is the half-length of the \( i \)th fracture; the distribution of the fractures is shown in Figure 4.

Therefore, formula 9 can be written as the potential \( \Phi(x, y) \) of a certain point in the space as

![Figure 4. Distribution map in the coordinate system when the number of fractures is odd.](image-url)
\[
\Phi(x, y) = \sum_{i=-N_0}^{N_0} q_i B \frac{x}{x_i} \text{arcosh} \left( \frac{\sqrt{2}}{2} \left[ 1 + \left( \frac{x}{x_i} \right)^2 + \left( \frac{y - id}{x_i} \right)^2 \right] \right) + C
\]

To order

\[
\sqrt{2} \left( 1 + \left( \frac{x}{x_i} \right)^2 + \left( \frac{y - id}{x_i} \right)^2 \right)^{1/2}
\]

Therefore, formula 28 can be simplified to

\[
\Phi(x, y) = \frac{B}{2\pi h} \sum_{i=-N_0}^{N_0} q_i Y_i(x, y) + C
\]

To suppose that the vertical intersection coordinate of the \(j\)th fracture and horizontal well is \((0, y_j)\), where \(y_j = jd\), the pressure at this point is the bottom hole flowing pressure \(p_{wf}\), the pressure at the boundary \((0, y_e)\) is \(p_e\) where \(y_e = re\) by considering the influence of the starting pressure gradient, there is

\[
\Phi(0, y_e) - \Phi(0, y_j) = \frac{K_{f}}{\mu} (p_e - p_{wf} - G(y_e - y_j))
\]

To substituting formula 29 and 11 into formula 30 for further collation, we can get

\[
\frac{B}{2\pi h} \sum_{i=-N_0}^{N_0} q_i Y_i(0, y_e) - \frac{B}{2\pi h} \sum_{i=-N_0}^{N_0} q_i Y_i(0, y_j) = \frac{K_{f}}{\mu} (p_e - p_{wf} - G(y_e - y_j))
\]

That is

\[
\sum_{i=-N_0}^{N_0} q_i [Y_i(0, y_e) - Y_i(0, jd)] = \frac{2\pi K_f h}{\mu B} (p_e - p_{wf} - \alpha (\frac{K}{\mu})^{-n} (r_e - jd))
\]

By solving the above equations, the horizontal well productivity can be obtained as

\[
Q = \sum_{i=-N_0}^{N_0} q_i
\]

(2) When the number of fractures \(N\) is an even number, to make the calculation easier, \(d = L/2N\) is taken. At this time, \(d\) is the half the distance between fractures, where \(N_0 = N - 1\), the coordinates of the midpoint of each fracture are \((0, \pm d), (0, \pm 3d), \ldots, (0, \pm N_0 d)\), and the fracture distribution is shown in Figure 5. Same as the above calculation method, the equation can be obtained at this time as follows

\[
\sum_{i=-N_0}^{N_0} q_i [Y_i(0, r_e) - Y_i(0, jd)] = \frac{2\pi K_f e^{-\alpha (\frac{K}{\mu})^{-n} h} (p_e - p_{wf} - \alpha (\frac{K}{\mu})^{-n} (r_e - jd))}{\mu B}
\]

Among which \(i\) increases from \(-N_0\) to \(N_0\) at a rate of 2 each time during the summation process.

Similarly, the horizontal well productivity is

\[
Q = \sum_{i=-N_0}^{N_0} q_i
\]
where \(i\) increases from \(-N_0\) to \(N_0\) at a rate of 2 each time during the summation process.

2.2.3. Calculation Formula of Horizontal Well Production by Considering the Combined Influence of Non-Darcy Seepage and Pressure Sensitivity When It Comes to Multiple Fractures. When considering the starting pressure gradient and pressure sensitivity of the reservoir, formula 30 can be rewritten as follows

\[
\Phi(0, y) - \Phi(0, y_i) = \frac{K}{\mu}(p_e - p_j) = \frac{K}{\mu}e^{-\alpha_i(y - y_i)}(p_e - p_j - G(y - y_i))
\]

When the number of fractures \(N\) is an odd number, to substitute formula 29 and formula 11 into formula 39, we can get

\[
\sum_{i=-N_0}^{N_0} q[Y(0, r_i) - Y(0, r_{i+1})] = \frac{2\pi K e^{-\alpha_i(y - y_i)}}{B \mu}(p_e - p_j - \alpha_i(\frac{K}{\mu})^{-\alpha}(r_e - r_{i+1}))
\]

At this time, if the summation calculation is conducted, we assume the increase speed of \(i\) is 1 unit. Similarly, when the number of fractures \(N\) is an even number, the expression form of the productivity equation is the same as eq. 40. At this time, if summation calculation is conducted, we assume the increase rate of \(i\) is 2 units, from \(-N_0\) increasing to \(N_0\).

3. CASE ANALYSIS

3.1. Case Analysis of a Typical Well. According to the actual situation of a horizontal well in Xinjiang Oilfield, relevant parameters are selected, as shown in Table 1.

| Name                  | Symbol | Value  |
|-----------------------|--------|--------|
| oil layer thickness (m) | \(h\)  | 13.0   |
| length of horizontal section (m) | \(L\)  | 1200   |
| formation pressure (MPa) | \(p_e\) | 36.7   |
| bottom hole pressure (MPa) | \(p_{wb}\) | 15.0   |
| crude oil density (g/cm³) | \(\rho\) | 0.888  |
| average formation permeability (×10⁻³ μm²) | \(\tilde{K}\) | 0.13   |
| oil layer permeability (×10⁻³ μm²) | \(K_e\) | 0.01   |
| seam spacing (m) | \(d\)  | 15     |
| half-length of the fracture (m) | \(x_1\) | 180    |
| supply radius (m) | \(r_e\) | 600    |
| crude oil viscosity (mPa·s) | \(\mu\) | 50.27  |
| crude oil volume coefficient | \(B\)  | 1.124  |

Assuming that the fracturing of the horizontal well is divided into 28 stages, with three clusters in one section and an average section spacing of 42.8 m and the fracture cluster spacing of 15 m, the horizontal section is not reperforated, and thus, the fluid will first flow into fractures from formation and then flow into the wellbore along fractures. Therefore, the production of fractured horizontal wells is the sum of production of each fracture. According to formula 40, the production of fractures is calculated. The coordinates and production of each fracture in the coordinate system are shown in Table 2. From Table 2, it can be seen that the production from each fracture is not the same, with the highest production at the end fractures and the lowest production at the middle fractures. The relationship between bottom hole flowing pressure and production is shown in Figure 6, where the regression coefficient of starting pressure gradient \(\alpha_1\) is 0.0023, and the stress sensitivity coefficient \(\alpha_2\) is 0.03 MPa⁻¹.

If the influence of starting pressure gradient and pressure sensitivity is not considered, the formula of horizontal well production can be obtained by rewriting formula 40

\[
\sum_{i=-N_0}^{N_0} q[Y(0, r_i) - Y(0, r_{i+1})] = \frac{2\pi K h}{\mu B}(p_e - p_j)
\]

If only the influence of the starting pressure gradient is considered, the horizontal well production can be calculated according to formula 34. In the same way, if only the influence of reservoir pressure sensitivity is considered, formula 38 is the calculation formula of horizontal well production.

Through numerical calculation, the relationship between bottom hole flow pressure and production under the above conditions can be obtained, as shown in Figure 6.

It can be seen from Figure 6 that if the influence of the starting pressure gradient and stress sensitivity are not considered, when the bottom hole flow pressure is the same, the productivity of horizontal wells will be greater than that when the influence of both are considered. The calculated productivity of horizontal wells will be smaller than that when only one situation is considered. In the initial stage, the influence of non-Darcy seepage on the productivity of horizontal wells is greater than that of pressure sensitivity if only one situation is considered. In the later stage, it is quite the contrary.

3.2. Multi-Well Verification. When the bottom hole flow pressure is 15 MPa, the production calculated by formula is 31.5949 t/d, which is close to the actual production volume of 33.04 t/d. With the same method, calculations were also carried out on several other horizontal wells within the studied area. The comparison of calculation results is shown in Table 3. There is little difference between the values calculated by the mathematical model and the actual results.

4. ANALYSIS OF THE MAIN FACTORS AFFECTING PRODUCTIVITY

4.1. Influence of Starting Pressure Gradient and Pressure Sensitivity on Productivity. When we keep other data unchanged but change the regression coefficient \(\alpha_1\) of the start-up pressure gradient, that is, taking 0.002, 0.0046, and 0.0069, we can calculate the relationship between bottom hole flow pressure and production by changing the regression coefficient of the starting pressure gradient, as shown in Figure 7. It can be seen that the production of horizontal wells will decrease to a certain extent with the increase in start-up pressure. When only the start-up pressure gradient is changed, the production of horizontal wells decreases with the increase in the regression coefficient of the start-up pressure gradient under the same bottom hole flow pressure. As shown in Figure 7, the difference between line a and line c can be used to obtain a scatter plot of the relative reduction of daily oil production.
production when only the starting pressure gradient is changed.

By the same abovementioned method, we have studied the influence of pressure sensitivity on productivity under different conditions by changing only the pressure sensitivity coefficient. Here, $\alpha$ is 0.03, 0.06, and 0.09 MPa$^{-1}$, respectively. The relationship between bottom hole flow pressure and production with different pressure sensitivity coefficients is shown in Figure 7. It can be seen that when only the pressure sensitivity effect is considered, the production of the oil well decreases with the increase in the pressure sensitivity coefficient. Under the condition of a certain pressure sensitivity coefficient, the production of horizontal wells increases with the decrease in bottom hole flowing pressure, and the increasing rate decreases with the decrease in bottom hole flowing pressure. As shown in Figure 7, the difference between line a and line e can be used to obtain a scatter plot of the relative reduction of daily oil production when only the pressure sensitivity is changed. As the bottom hole pressure decreases, the change trend of the triangle point is larger than that of the circle point, which further indicates that the influence of pressure sensitivity on production of horizontal wells is more significant than that of the start-up pressure gradient.

### 4.2. Influence of Fracture Half-Length on Productivity
To study the influence of different fracture half-lengths on the productivity of horizontal wells, by selecting the fracture half-lengths of 40, 45, 50, 60, 70, 80, 120, 160, 200, 240, and 280, 360 m, respectively, we have calculated the daily production of horizontal wells and drawn the relationship curve between fracture half-length and daily oil production and growth rate of production, as shown in Figure 8. It can be seen from Figure 8 that the daily production of horizontal wells increases with the increase in fracture half-length, but the increased value becomes smaller and smaller. When the
fracture half-length reaches 160 m, the growth rate of daily production reaches about 10%; when the fracture half-length reaches more than 160 m, the increase in daily production slows down. Therefore, it is concluded that the fracture length can be appropriately increased to improve the productivity of horizontal wells. As shown in Figure 8, the upper limit of the fluctuation of the growth rate of production is not more than 2% when the growth rate of production slows down.

4.3. Influence of the Number of Fractures on Productivity. To study the influence of the number of fractures on the productivity of horizontal wells, by taking the number of fractures as 16, 20, 24, 28,..., 96, 100, 104 respectively, we have calculated the daily production of horizontal wells and drawn the relationship between the number of fractures and daily oil production and growth rate of production. The curve is shown in Figure 9. It can be seen from Figure 9 that the production of horizontal wells increases gradually with the increase in the number of fractures. The increase in the number of fractures also indicates the decrease in fracture spacing. The calculation shows that when the number of fractures reaches more than 60 strips, the increase change in horizontal well productivity decreases. When the growth of production slows down, the upper limit of the fluctuation of the growth rate of production is between 2 and 3%.

5. RESULTS AND DISCUSSION

In summary, the starting pressure gradient and pressure sensitivity have exerted a certain degree of influence on the productivity of shale oil in horizontal wells. When start-up pressure gradient and pressure sensitivity are considered, the calculated productivity of the horizontal well is lower than that without consideration. When considering only the starting pressure gradient, the production of horizontal wells decreases accordingly as starting pressure gradient increases. When considering only the pressure sensitivity, the higher the pressure sensitivity coefficient is, the lower the production of horizontal wells is. The study finds that the influence of pressure sensitivity on productivity of horizontal wells is more significant than that of the start-up pressure gradient with the decrease in bottom hole pressure. The fracture half-lengths affects the production of horizontal wells as well. By increasing appropriately, the length of the fractures and the productivity of horizontal wells can be increased. From the perspective of the growth rate of production, when the number of fractures increases to a certain level, the upper limit of the fluctuation of the growth rate is greater than the half-length of the fractures. The more the number of fractures is, the greater the production of horizontal wells is. This indicates that dense fractures should be distributed as much as possible to increase the production of a single well.

6. CONCLUSIONS

By considering comprehensively the start-up pressure gradient and pressure sensitivity, based on the principles of conformal transformation and complex potential superposition, a mathematical model for prediction of shale oil productivity in fractured horizontal well has been established. In addition,
the influence factors for productivity, such as fracture length, number of fractures, pressure sensitivity coefficient, starting pressure gradient and others factors, have been analyzed. Based on the results presented, the following conclusions have been obtained:

1) Both start-up pressure gradient and pressure sensitivity affect the production of shale oil in horizontal wells. As the bottom hole pressure decreases, the influence of pressure sensitivity on productivity of horizontal wells is more significant than that of start-up pressure gradient.

2) The influence of fractures on the production of shale oil in horizontal well has been analyzed. The production of horizontal wells increases with the increase in fracture half-lengths, but the increased value of production decreases gradually. From the perspective of the growth rate of production, when the number of fractures increases to a certain level, the upper limit of the fluctuation of the growth rate is greater than the half-length of the fractures. Increasing the number of fractures is conducive to improving productivity, which indicates that dense fractures as much as possible could increase the production of a single well.

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Notes
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■ ABBREVIATIONS

\( x \) real part of complex number \( z \), dimensionless
\( y \) imaginary part of complex number \( z \), dimensionless
\( \omega \) \( \omega \) plane’s complex number, dimensionless
\( \xi \) real part of complex number \( \omega \), dimensionless
\( \eta \) imaginary part of complex number \( \omega \), dimensionless

\( x_f \) half-length of the fracture, m
\( \Phi \) potential function, dimensionless
\( q \) fracture production, t/d
\( h \) the thickness of oil layer, m
\( B \) the volume coefficient of crude oil, dimensionless
\( C \) the constant, dimensionless
\( G \) starting pressure gradient, MPa/m
\( \mu \) the fluid viscosity, mPa s
\( r \) the well radius, m
\( \alpha_i \) \( i \) is the regression coefficient, dimensionless
\( v_o \) oil phase seepage velocity, m/s
\( v_{oso} \), \( v_{osy} \), \( v_{osz} \) seepage velocity of the oil phase in \( x, y \), and \( z \) directions, m/s
\( K_{so} \) the relative permeability of the oil phase, mD
\( \mu_o \) the visual fluid viscosity, mPa s
\( K_i \) the original permeability, mD
\( \alpha_s \) the pressure sensitivity coefficient, MPa\(^{-1}\)
\( P_o \) the original formation pressure, MPa
\( P_{hf} \) the bottom hole flowing pressure, MPa
\( P_{fj} \) the flowing pressure of \( j \)th fracture, MPa
\( L \) the horizontal section length of the horizontal well, m
\( N \) number of fractures, integer
\( d \) the distance between fractures, m
\( x_0 \) the half-length of the \( i \)th fracture and \( x_0 = x_0 \) m
\( q_i \) the productivity of the \( i \)th fracture, t/d
\( r_e \) supply radius, m

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