Measurement of $x F_3$, $F_2$ Structure Functions and Gross–Llewellyn Smith Sum Rule with IHEP–JINR Neutrino Detector

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Abstract

The isoscalar structure functions $xF_3$ and $F_2$ are measured as functions of $x$ averaged over all $Q^2$ permissible for the range 6 to 28 GeV of incident (anti)neutrino energy. With the measured values of $xF_3$, the value of the Gross–Llewellyn Smith sum rule is found to be $\int_0^1 F_3\, dx = 2.13 \pm 0.38 \text{ (stat)} \pm 0.26 \text{ (syst)}$. The QCD analysis of $xF_3$ provides $\Lambda_{\overline{MS}}=358\pm59$ MeV. The obtained value of the strong interaction constant $\alpha_S(M_Z) = 0.120_{-4}^{+3}$ is larger than most of the deep inelastic scattering results.

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The data on deep-inelastic $\nu(\bar{\nu})$–scattering in a wide region of momentum transfer provide a reliable basis for precise verification of QCD predictions [1]. In this paper the data on the $xF_3$ and $F_2$ structure functions (SF) are presented for the kinematic region of relatively small momentum transfer $0.55 < Q^2 < 4.0 GeV^2$. The value of the Gross–Llewellyn Smith (GLS) sum rule [2] and $\alpha_S(M_Z)$ are evaluated.

The data samples were obtained from three independent exposures of the IHEP–JINR Neutrino Detector [3] to the wide band neutrino and antineutrino beams [4] of the Serpukhov U70 accelerator. The exposure to the antineutrino beam ($\bar{\nu}_\mu$-exposure) was performed at the proton beam energy $E_p = 70 GeV$, whereas the two $\nu_\mu$-exposures were carried out one at $E_p = 70 GeV$ and the other at $E_p = 67 GeV$. The experimental set–up and selection criteria for CC events are discussed in [5]. We restricted the range of the measurements in $W^2$ to $1.7 GeV^2$ and in $E_\nu(\bar{\nu})$ to $6 < E_\nu(\bar{\nu}) < 28 GeV$. The final number of events and the mean values of $Q^2$, $\langle Q^2 \rangle$ for the three samples are given in Table 1.

The SF were measured as functions of $x$ averaged over all $Q^2$ permissible for the energy range $6 < E_\nu(\nu) < 28 GeV$. Events were binned in intervals of $x$, and values of $xF_3$ and $F_2$ were calculated in these intervals.

The number of $\nu_\mu$ interactions, $n_\nu$, and $\bar{\nu}_\mu$ interactions, $n_{\bar{\nu}}$, in a given bin of $x$ is a linear combination of the 'average' values $\{F_2\}$ and $\{xF_3\}$ of the respective SF in this bin (we assume invariance under the charge conjugation):

$$n_{\bar{\nu}} = a_{\bar{\nu}} \cdot \{F_2\} - b_{\bar{\nu}} \cdot \{xF_3\}$$

$$n_{1,2} = a_{1,2} \cdot \{F_2\} + b_{1,2} \cdot \{xF_3\}. $$

The subscripts 1 and 2 correspond to the $\nu_\mu$-exposures at $E_p = 70 GeV$ and $E_p = 67 GeV$ respectively. The quantities $a_{\nu,\bar{\nu}}$ and $b_{\nu,\bar{\nu}}$ are integrals ('flux integrals') of products of the differential neutrino (antineutrino) flux $\phi^{\nu,\bar{\nu}}(E)$ and known factors depending on the scaling variables $x$, $y$ as foreseen by the standard form of the differential cross-section for deep-inelastic $\nu_\mu(\bar{\nu}_\mu)$-scattering off an isoscalar target:

$$a_{\nu} = N \frac{G^2 M}{\pi} \int \left( 1 - y - \frac{Mxy}{2E} + \frac{1}{2(R+1)} y^2 \right) E \phi^{\nu}(E) dx dy dE,$$

$$b_{\nu} = N \frac{G^2 M}{\pi} \int y \left( 1 - \frac{y}{2} \right) E \phi^{\bar{\nu}}(E) dx dy dE,$$

e etc.

Here $N$ is the number of nucleons in the fiducial volume of the detector and the parameter $R = (F_2 - 2xF_1)/2xF_1$ measures the violation of Callan-Gross relation [6].

The number $n_{\nu,\bar{\nu}}$ of neutrino (antineutrino) interactions in a given $x$-bin was obtained from the measured number of neutrino (antineutrino) events in this bin corrected for acceptance, for smearing effects arising from Fermi motion and measurement uncertainties, for radiative effects (following the prescription given by De
Rújula et al. [7]) and for target non-isoscalarity (assuming \( d_v/u_v = 0.5 \)) [8]). To determine the appropriate correcting factors the Monte–Carlo simulation of the experimental set-up has been carried out using the program CATAS [9]. We used the Buras and Gaemers (BEBC) parametrization [10] for quark distributions. The charmed quark content of the nucleon was assumed to be zero. The kinematic suppression of \( d \to c \) and \( s \to c \) transitions was taken into account assuming slow rescaling [11] and the following charmed and strange quark masses: \( m_c = 1.25 \text{ GeV}, m_s = 0.25 \text{ GeV} \). Fermi motion of nucleons was simulated according to [12]. The details of the Monte–Carlo simulation of the known features of the experimental set-up are discussed in [5] and [13].

The number of interactions in a given bin of \( x \) is subject to kinematic constraints imposed by the cuts in the muon momentum (\( P_\mu > 1 \text{ GeV}/c \)) [5]), in the neutrino energy (\( 6 < E_\nu < 28 \text{ GeV} \)) and in the invariant mass square of the hadronic system (\( W^2 > 1.7 \text{ GeV}^2 \)). These were taken into account in the calculation of the flux integrals by appropriate modification of the volume of integration.

The measured values of \( xF_3 \) and \( F_2 \) are presented in Table 2 and in Figure 1. The systematic errors presented come from the uncertainties of the correcting factors due to the choice of some input quark distributions in the event simulation program CATAS. These systematic uncertainties were estimated by repeating the calculation of the SF using by turns the Field-Feynman [14] and GRV [15] quark distributions. Note that the systematic errors in Table 2 do not include the normalization error of 4\% for \( F_2 \) and 11\% for \( xF_3 \). These normalization errors originate from the uncertainties in the \( \nu_\mu \) and \( \overline{\nu}_\mu \) flux determination [16].

With the values of \( xF_3 \), the GLS sum rule (the integral of \( F_3 \)) has been estimated. Over the interval \( 0.02 < x < 0.65 \) it was calculated by numerical integration of the measured values of \( xF_3 \) weighted by \( 1/x \). The contribution from the regions \( 0 < x < 0.02 \) and \( 0.65 < x < 1 \) was evaluated by integrating over these regions the parametrization of \( xF_3 \) with the values of free parameters obtained from the fit to the data at \( 0.02 < x < 0.65 \). Finally we obtained

\[
\int_0^1 \frac{xF_3(x)}{x} \, dx = 2.13 \pm 0.38 \text{ (stat)} \pm 0.26 \text{ (syst)}. \tag{1}
\]

The systematic error quoted is the quadrature sum of \( \pm 0.24 \) due to \( \nu_\mu \) and \( \overline{\nu}_\mu \) flux uncertainties, and \( \pm 0.09 \) due to the choice of some input quark distributions. In accordance with Table 1 we suppose that the measured value (1) of the GLS sum rule corresponds to the averaged value \( \overline{Q^2} \sim 1.7 \text{ GeV}^2 \).

The experimental data on the \( xF_3 \) were compared with the QCD prediction for \( Q^2\)-evolution by the Jacobi polynomials method in the next-to-leading order QCD approximation [17, 18, 19, 20]. Making QCD analysis of the \( xF_3 \) SF, for the first step we do not discuss the problem of validity of application of perturbative QCD predictions for kinematical region of small \( Q^2 \) as well as the nuclear effects, heavy quarks threshold effects and higher order QCD corrections.

In order to take into account the target mass corrections the Nachtmann moments [21] of \( F_3 \) and \( F_2 \) could be expanded in powers of \( M^2_{\text{nucl.}}/Q^2 \), and retaining
only terms of the order $M_{\text{nucl.}}^2/Q^2$ one could obtain:

$$M_{3(2)}(N, Q^2) = M_{3(2)}^{QCD}(N, Q^2) + \frac{N(N + 1)}{N + 2} \frac{M_{\text{nucl.}}^2}{Q^2} M_{3(2)}^{QCD}(N + 2, Q^2).$$  \hspace{1cm} (2)$$

Here $M_{3(2)}^{QCD}(N, Q^2)$ and $M_{2}^{QCD}(N, Q^2)$ are the Mellin moments of $xF_3$ and $F_2$:

$$M_3(N, Q^2) = \int_0^1 dx x^{N-2} x F_3(x, Q^2),$$

$$M_2(N, Q^2) = \int_0^1 dx x^{N-2} F_2(x, Q^2), \hspace{1cm} N = 2, 3, ...$$  \hspace{1cm} (3)$$

The $Q^2$-evolution of $M_3^{QCD}(N, Q^2)$ and $M_2^{QCD}(N, Q^2)$ is defined \cite{22, 23} by QCD and is presented here for the nonsinglet case for simplicity:

$$M_3^{QCD}(N, Q^2) = \left[ \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{d_N} H_N(Q_0^2, Q^2) M_3^{QCD}(N, Q_0^2), \hspace{1cm} N = 2, 3, ... \hspace{1cm} (4)$$

$$d_N = \frac{\gamma^{(0)}, N}{2 \beta_0},$$

Here $\alpha_s(Q^2)$ is the strong interaction constant, $\gamma^{(0)}_{NS}$ are the nonsinglet leading order anomalous dimensions, and the factor $H_N(Q_0^2, Q^2)$ contains all next-to-leading order QCD corrections \cite{24, 23, 24}.

The unknown coefficients $M_3(N, Q_0^2)$ in (4) could be parametrized as the Mellin moments of some function:

$$M_3^{QCD}(N, Q_0^2) = \int_0^1 dx x^{N-2} A x^b (1 - x)^c (1 + \gamma x), \hspace{1cm} N = 2, 3, ...$$  \hspace{1cm} (5)$$

where the constants $A$, $b$, $c$ and $\gamma$ should be determined from the fit to the data. Having at hand the moments (2) – (5) and following the method discussed in \cite{17, 18, 19} we can write the $xF_3$ SF in the form:

$$xF_3^{N_{\text{max}}}(x, Q^2) = x^\alpha (1 - x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_3^{QCD}(j + 2, Q^2),$$

where $\Theta_n^{\alpha, \beta}(x)$ are the Jacobi polynomials and $c_j^{(n)}(\alpha, \beta)$ are the coefficients of the expansion of $\Theta_n^{\alpha, \beta}(x)$ in powers of $x$:

$$\Theta_n^{\alpha, \beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) x^j.$$

The accuracy of the SF approximation better than $10^{-3}$ is achieved for $N_{\text{max}} = 12$ in a wide region of the parameters $\alpha$ and $\beta$ \cite{18, 19}. 

4
Using nine Mellin moments for SF reconstruction and taking into account target mass corrections we have determined five free parameters $A$, $b$, $c$, $\gamma$ and the QCD parameter $\Lambda_{\overline{MS}}$ (Table 3).

For the $Q^2$ – dependence of the GLS sum rule we can write the following theoretical expression:

$$GLS(Q^2) = 3 \left[ 1 - \alpha_s(Q^2)/\pi + O(\alpha_s^2) - \frac{8}{27} \langle\langle O\rangle\rangle/Q^2 \right]$$

where $\alpha_s$ is the coupling constant in the $\overline{MS}$ scheme. The general structure of the high-twist (HT) term is known from [26]. The evaluation of this term was carried out in [27], $\langle\langle O\rangle\rangle = 0.33 \pm 0.16$ GeV$^2$, and more recently in [28], $\langle\langle O\rangle\rangle = 0.53 \pm 0.04$ GeV$^2$, using the same three-point function QCD sum rules technique.

In order to estimate the uncertainties due to the HT contribution we included a phenomenological term $h(x) = -\frac{8}{27} \langle\langle O\rangle\rangle x$ in the fitting procedure. The first moment of the function $h(x)$ gives some contribution to the GLS sum rule (6) in accordance with [28]. The results of the fit with $\langle\langle O\rangle\rangle = 0.53 \pm 0.04$ GeV$^2$ are presented in Table 3. The value of $\alpha_s(M_Z)$ was calculated for both variants of the fit due to the so-called ‘matching relation’ [29]. We present the GLS sum rule values calculated through (5) with $N = 1$ and with the parameters from Table 3.

We repeated our fit taking into account both the statistical and systematic errors (from Table 2 added in quadrature. With HT from [28], the following estimations have been obtained: $\Lambda_{\overline{MS}} = 359 \pm 71$ MeV, $GLS = 2.66$.

In the singlet case the moments of valence quarks, sea quarks and gluons were parametrized at $Q_0^2$ in the form:

$$M_{q}^{QCD}(N, Q_0^2) = \int_0^1 \frac{dxx}{Q_0^2} \left[ A_v x^b_v (1 - x)^c_v + A_{sea} (1 - x)^{c_{sea}} \right],$$

$$M_{g}^{QCD}(N, Q_0^2) = \int_0^1 \frac{dxx}{Q_0^2} A_g (1 - x)^{c_g}, \quad N = 2, 3, ...$$

Keeping in mind the small number of experimental points we fix $A_g$ from the momentum sum rule $M_{q}^{QCD}(2, Q^2) + M_{g}^{QCD}(2, Q^2) = 1$. Following the results [13] of the QCD analysis of $F_2$ at the momentum transfer $Q^2 = 5$ GeV$^2$ we put $A_{sea} = 0.17$, $c_{sea} = 15$ and $c_g = 9$. The other parameters in (7) as well as $\Lambda$ were determined from the fit of the data in the leading logarithm QCD approximation and were found to be $A_v = 2.49 \pm 0.311$, $b_v = 0.19 \pm 0.02$, $c_v = 2.80 \pm 0.05$, $\Lambda = (517 \pm 17)$ MeV with $\chi^2 = 6.7$ for 6 experimental points and $Q_0^2 = 3$ GeV$^2$. Only statistical errors were taken into account.

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3See [25] for higher order QCD corrections to the GLS sum rule.

4 This shape of $h(x)$ is in qualitative agreement with the theoretical prediction in [30] and experimental estimations in [31] for the $x$ values from Table 2.
Several comments:

- The values (1) and the results on the GLS sum rule in Table 3 are considerably smaller in comparison to the results of previous measurements. (See the summary on the GLS sum rule data in [32] and the latest 3-loop result [33].)

- The parameter $\Lambda_{\overline{MS}}$ is found to be about twice as large as the estimations in [20] and [34]. It is in qualitative agreement with the results of the NLO analysis [35] of the GLS sum rule in the $\overline{MS}$ scheme: $\Lambda_{\overline{MS}}^{(4)} = 317 \pm 23(stat) \pm 99(syst) \pm 62(twist) \text{MeV}$ with HT and $\Lambda_{\overline{MS}}^{(4)} = 435 \pm 20(stat) \pm 87(syst) \text{MeV}$ without HT.

- The illustrative nature of the QCD fit to the data on $F_2$ should be pointed out. The matter is the absence of reliable theoretical predictions for HT contribution to singlet SF. In spite of this, we obtained the momentum fraction carried by quarks in the nucleon, $M^{QCD}_q(2, Q^2) = 0.46$, to be in agreement with the previous measurements.

- The strong interaction constant at the point of Z boson mass is found to be higher than most of the deep inelastic scattering results [36, 37].

- The consideration of the HT contribution decreases $\chi^2$ and appreciably changes the parameters of the fit as well as the GLS sum rule value and $\alpha_s(M_Z)$. For a reliable QCD analysis one must calculate not only the GLS sum rule ($N = 1$) but also the higher SF moments ($N = 2, 3,...$). Using in addition a 3-loop QCD analysis one could expect to improve the estimation of $\alpha_s(M_Z)$.

In conclusion, let us stress once more that the QCD analysis of SF is sensitive to the HT contribution and in the future it should take into account the nuclear effects, heavy quark threshold effects and higher order QCD corrections. We hope to improve the accuracy of our estimations by processing the additional data on deep inelastic scattering obtained with the IHEP–JINR Neutrino Detector in the wide band beams of $\nu_\mu$ and $\overline{\nu}_\mu$. 
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Table 1. Summary of the exposures.

| Beam             | $\bar{\nu}_\mu$ | $\nu_\mu$ | $\nu_\mu$ |
|------------------|------------------|-----------|-----------|
| $E_p$ (GeV)      | 70               | 70        | 67        |
| Final statistics | 741              | 2139      | 3848      |
| $\langle Q^2 \rangle$ (GeV$^2$) | 1.2             | 2.3       |           |
| $x$  | $(Q^2)$ (GeV$^2$) | $F_2$  | stat | syst | $\Delta F_2$ | $xF_3$ | stat | syst |
|------|------------------|--------|------|------|-------------|--------|------|------|
| .052 | .55              | 1.169  | .026 | .047 | .023        | .445   | .044 | .062 |
| .148 | 1.4              | 1.097  | .026 | .022 | .022        | .583   | .044 | .017 |
| .248 | 2.2              | .894   | .023 | .018 | .019        | .622   | .038 | .019 |
| .346 | 2.9              | .576   | .016 | .017 | .013        | .556   | .027 | .011 |
| .447 | 3.4              | .390   | .014 | .012 | .009        | .336   | .023 | .007 |
| .563 | 4.0              | .182   | .008 | .004 | .004        | .177   | .012 | .005 |

Table 2. The isoscalar structure functions $F_2$ and $xF_3$ obtained on the assumption of $R = 0$. The difference $\Delta F_2$ between the values of $F_2$ obtained with $R = .1$ and those obtained with $R = 0$ is also presented. The bin edges are at $x = .0, .1, .2, .3, .4, .5, .65$. 


\[
\langle \langle O \rangle \rangle = 0 \quad \langle \langle O \rangle \rangle = 0.53
\]

|        | \langle \langle O \rangle \rangle = 0 | \langle \langle O \rangle \rangle = 0.53 |
|--------|-------------------------------------|-------------------------------------|
| \chi^2 | 2.8                                 | 2.05                                |
| A      | \(9.28 \pm 1.73\)                   | \(0.90 \pm 0.67\)                  |
| b      | \(1.06 \pm 0.11\)                   | \(0.31 \pm 0.18\)                  |
| c      | \(3.22 \pm 0.31\)                   | \(3.64 \pm 0.21\)                  |
| \gamma | \(-0.90 \pm 0.21\)                  | \(9.53 \pm 5.73\)                  |
| \(\Lambda_{MS}\) [MeV] | \(417 \pm 51\)                      | \(358 \pm 59\)                     |
| GLS sum rule | 1.59                               | 2.63                                |
| \(\alpha_S(M_Z)\) | \(0.123^+_{-4}\)              | \(0.120^+_{-4}\)                  |

Table 3. The results of the NLO QCD fit to the \(xF_3\) SF data for \(f = 4\), \(Q_0^2 = 3\,GeV^2\), \(N_{MAX} = 12\), \(\alpha = 0.7\), \(\beta = 3.0\) with the corresponding statistical errors.
Figure captions.

Fig.1. The $x$-dependence of the isoscalar structure functions $F_2(x)$ and $xF_3(x)$. The statistical and systematic errors are added in quadrature, excluding the normalization error of 4% for $F_2$ and 11% for $xF_3$. The curve is fit of the form $xF_3(x) = A x^b(1-x)^c$. The best fit values of free parameters $A = 5.36\pm1.25\,(stat)$, $b = 0.81\pm0.10\,(stat)$, $c = 3.52\pm0.26\,(stat)$ were obtained using for each $x$-bin the mean $x$ of the bin as the actual $x$-point corresponding to the value of the structure function obtained.
$6 < E_{\nu,\bar{\nu}} < 28$ GeV

$F_2, xF_3$