GAUGE APPROACH TO GRAVITATION AND REGULAR BIG BANG THEORY

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Field theoretical scheme of regular Big Bang in 4-dimensional physical space-time, built in the framework of gauge approach to gravitation, is discussed. Regular bouncing character of homogeneous isotropic cosmological models is ensured by gravitational repulsion effect at extreme conditions without quantum gravitational corrections. The most general properties of regular inflationary cosmological models are examined. Developing theory is valid, if energy density of gravitating matter is positive and energy dominance condition is fulfilled.

I. INTRODUCTION

The Hot Bing Bang scenario built on a base of Friedmannian homogeneous isotropic cosmological models (HICM) of general relativity theory (GR) is the foundation of relativistic cosmology. Well known physical laws allow to describe physical processes in evolving Universe beginning from the time $t_1 \sim 10^{-4} s$ from the Bing Bang corresponding to the start of cosmological expansion (in the case of Friedmannian HICM the moment $t = 0$ corresponds to cosmological singularity), when the energy density of gravitating matter was comparable with nuclear density and the temperature $T \sim 10^{11} K$. Observable confirmation of theoretical predictions of primordial nucleosynthesis of light elements, explanation of large scale structure formation of the Universe are important achievements of the Hot Bing Bang theory. To describe gravitating matter and physical processes in evolving Universe at $t < t_1$ one has to use modern theory of fundamental physical interactions, unified models of particle physics, that is the object of cosmology of early Universe. The creation of inflationary paradigm, which permits to solve a number of problems of standard cosmological scenario, in particular, to explain the homogeneity and isotropy of the Universe at initial stages of cosmological expansion is important achievement of early Universe cosmology $^1$. The further progress in cosmology of early Universe is connected with the fact that gravitational interaction in the case of usual gravitating systems with positive values of energy density $\rho$ and pressure $p$ in the frame of GR as well as Newtons gravitation theory has the character of attraction, but not repulsion. Although in the case of gravitating systems with negative pressure (for example, massive and nonlinear scalar fields) the gravitational interaction in GR can have the repulsion character, however, the PCS can not be solved by taking such systems into account in the frame of GR $^2$ $^3$. There were many attempts to resolve PCS in the frame of GR as well as other classical theories of gravitation (see $^4$ $^5$ and Refs given herein). A number of regular cosmological solutions was obtained in the frame of metric theories of gravitation and also other theories, in the frame of which gravitation is described by using more general geometry than the Riemannian one. In connection with this, note that the resolution of PCS means not only obtaining regular cosmological solutions, but also excluding singular solutions of cosmological equations, as a result generic feature of cosmological solutions has to be its regular character. Moreover, gravitation theory and cosmological equations have to satisfy the correspondence principle with GR in the case of usual gravitating systems with sufficiently small energy densities and weak gravitational fields excluding nonphysical solutions. The greatest part of existent attempts to resolve the PCS does not satisfy indicated conditions $^5$.

Usually the Universe evolution in the frame of standard cosmological scenario is considered beginning from the time $t$ greater than the Planckian time $t_p \sim 10^{-43} s$, when the energy density $\rho$ and temperature $T$ were smaller than the Planckian ones: $\rho_p \sim 9 \cdot 10^{112} \frac{1}{m^3}$ and $T_p \sim 10^{32} K$. Then according to general opinion quantum gravitational effects are negligible and gravitational field can be described classically. On the contrary, at Planckian conditions ($\rho \geq \rho_p$, $T \geq T_p$) one means that quantum gravitational effects must be essential, and possibly the PCS can be solved by taking these effects into account. So, in order to solve the PCS the idea of quantum birth of the Universe was introduced $^6$ $^7$. There are several realizations of this idea in the frame of quantum cosmology by using Wheeler-DeWitt equation (see re-
The appearing closed micro-universe is transformed then into macro-universe by virtue of inflation. Although singular state with divergent energy density is absent in such model, the problem of the beginning in the time of the Universe remains. The theory of quantum micro-universe was developed further in the frame of loop quantum gravity. The space quantization permits to continue solution for micro-universe of closed and flat type to the past and obtain bouncing solution containing the compression stage before the expansion. Note, this scenario has some vagueness, in particular: if at compression stage the Universe was a macro-object filled by gravitating matter what physical factors lead to transformation of macro-universe into micro-universe before a bounce, what is a flat model with finite volume?

The PCS was discussed also in the frame of other candidate for quantum gravity theory - string theory/M-theory. Some cosmological scenarios with evolution stage before the Big Bang - Pre-Big Bang scenario and ekpyrotic scenario were proposed. Note that HICM in a four-dimensional Einsteinian frame, generally speaking, are singular. The obtaining of regular bouncing solutions is connected with violation of condition of energy density positivity for gravitating matter. So, in Ref. 11 non-local negative scalar field potential is introduced, in Ref. 15 a component with negative energy density is used. To build oscillating cosmological model, the specific negative scalar field potential was introduced in Refs. 13, 14. Although the energy density does not diverge by transition from compression to expansion, the singularity connected with vanishing of the scale factor remains.

As it was shown in a number of our papers (see and Refs given herein), the applying of gauge approach to gravitation permits to build regular Big Bang theory by satisfying positivity property of energy density for gravitating matter. The present paper is devoted to consideration of principal features of developing approach and to further analysis of regular inflationary cosmology. In Section 2 generalized cosmological Friedmann equations for HICM in the frame of gauge approach to gravitation are introduced. In Section 3 these equations are applied to analyze inflationary cosmological models filled by scalar fields and usual gravitating matter. As illustration of regular Big Bang theory in Section 4 particular inflationary cosmological model is discussed.

II. GAUGE APPROACH TO GRAVITATION AND GENERALIZED COSMOLOGICAL FRIEDMANN EQUATIONS

As it is known, the local gauge invariance principle is the basis of modern theory of fundamental physical interactions. The theory of electro-week interaction, quantum chromodynamics, Grand Unified models of particle physics were built by using this principle. From physical point of view, the local gauge invariance principle establishes the correspondence between certain important conserving physical quantities, connected according to the Noether’s theorem with some symmetries groups, and fundamental physical fields, which have as a source corresponding physical quantities and play the role of carriers of fundamental physical interactions. The applying of this principle to gravitational interaction leads, generally speaking, to generalization of Einsteinian theory of gravitation. Metric theories of gravitation including GR, in the frame of which the energy-momentum tensor is a source of gravitational field, can be introduced in the frame of gauge approach by using the localization of 4-parametric translation group. Because the localized translation group is, in fact, the group of general coordinate transformations, from this point of view the general covariance of GR plays dynamical role. At the same time the Lorentz group (group of tetrad Lorentz transformations) in GR and other metric theories of gravitation does not play any dynamical role from the point of view of gauge approach, because corresponding Noether’s invariant in these theories is identically equal to zero.

If one means that the Lorentz group plays the dynamical role in gauge field theory and the Lorentz gauge field exists in the nature, in this case we obtain with necessity the gravitational theory in the Riemann-Cartan space-time (see, for example, 16, 17). Corresponding theory is known as Poincare gauge theory of gravitation (PGTG). Gravitational field variables in PGTG are the tetrad (translational gauge field) and Lorentz connection (Lorentz gauge field); corresponding field strengths are torsion and curvature tensors. As sources of gravitational field in PGTG are covariant generalizations of energy-momentum and spin tensors. Unlike gauge Yang-Mills fields, for which the Lagrangian is quadratic in the gauge fields, for which the Lagrangian is quadratic in the gauge field strengths, gravitational Lagrangian of PGTG can include also linear in curvature term (scalar curvature), which is necessary to satisfy the correspondence principle with GR.

The first attempt to apply the simplest PGTG - Einstein-Cartan theory - in order to solve the PCS was made in Refs. 21, 22. By using some classical model for spinning matter, non-singular cosmological solutions were obtained. However, it was shown later, these solutions have model character and critically depend on spinning matter description; by another spin description (for example, by means of Dirac field) the cosmological singularity does not disappear. Moreover, because in the case of spinless matter the Einstein-Cartan theory is identical to GR, all singular solutions for Friedmannian HICM are exact solutions of Einstein-Cartan theory of gravitation. The next step to apply the PGTG in order to solve the PCS was made in Ref. 22. In the frame of PGTG based on general expression of gravitational Lagrangian \( L_G \) including both a scalar curvature and different in-
variants quadratic in the curvature and torsion tensors gravitational equations for HICM were deduced, these equations lead to the following generalized cosmological Friedmann equations (GCFE) 34:

\[
\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[ R \sqrt{1 + \alpha (\rho - 3p)} \right] \right\}^2 = \frac{8 \pi}{3M_p^2} \frac{\rho + \frac{4}{3} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)},
\]

\[
\frac{\dot{R} + R \left( \ln \sqrt{1 + \alpha (\rho - 3p)} \right)}{R} = -\frac{4 \pi}{3M_p^2} \frac{\rho + 3p - \frac{9}{2} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)},
\]

where \( k = +1, 0, -1 \) for closed, flat, open models respectively, \( \alpha \) is indefinite parameter with inverse dimension of energy density, \( M_p \) is Planckian mass, a dot denotes differentiation with respect to time. (The system of units with \( h = c = 1 \) is used). Note that Eqs. (1)-(2) are valid also in the frame of the most general gauge theory of gravitation – metric-affine theory 22 24. From Eqs. (1)-2 follows the conservation law in usual form

\[
\dot{\rho} + 3H (\rho + p) = 0,
\]

where \( H = \frac{k}{R} \) is the Hubble parameter. Besides cosmological equations (1)-(2) gravitational equations of PGTG lead to the following relation for the torsion function \( S \)

\[
S = -\frac{1}{4} \frac{d}{dt} \ln |1 + \alpha (\rho - 3p)|.
\]

Before we start discussing inflationary cosmology on a base of GCFE (1)-(2), let us make several remarks about these equations. At \( \alpha \to 0 \) the GCFE (1)-(2) are transformed into Friedmann cosmological equations of GR. The value of \( \alpha^{-1} \) determines the scale of extremely high energy densities. Solutions of GCFE (1)-(2) coincide practically with corresponding solutions of GR if the energy density is small \(|\alpha (\rho - 3p)| \ll 1 \) \((p \neq \frac{1}{3} \rho)\). The difference between GR and PGTG can be essential at extremely high energy densities \(|\alpha (\rho - 3p)| \gtrsim 1 \). Ultrarelativistic matter \((p = \frac{1}{3} \rho)\) and gravitating vacuum \((p = -\rho)\) with constant energy density are two exceptional systems because Eqs. (1)-(2) are identical to Friedmann cosmological equations of GR in these cases independently on values of energy density and \( S = 0 \). The behaviour of solutions of Eqs. (1)-(2) depends essentially on the following restriction on equation of state of gravitating matter at extreme conditions: \( p > \frac{1}{3} \rho \) or \( p < \frac{1}{3} \rho \). Because of strong nucleon interaction we have for nuclear matter \( p > \frac{1}{3} \rho \) 24; we will put that this restriction is valid also for gravitating matter at extreme conditions, and the scale of extremely high energy densities determined by \( \alpha^{-1} \) surpasses nuclear density. From physical point of view, we can suppose that the value of \( \alpha^{-1} \) is smaller than the Planckian energy density, but it will be shown later, the behaviour of solutions of GCFE does not depend on this assumption.

The GCFE lead to restrictions on admissible values of energy density. In fact, if energy density \( \rho \) is positive and \( \alpha > 0 \), from Eq. (1) in the case \( k = +1 \), 0 follows the relation:

\[
Z \equiv 1 + \alpha (\rho - 3p) \geq 0.
\]

The condition (5) is valid not only for closed and flat models, but also for cosmological models of open type \((k = -1) \) (see below). In the case of models filled by gravitating matter with equation of state \( p = p(\rho) \), it is easy to obtain the solution of the system of Eqs. (1) and (3) in quadratures 26; these solutions are regular in metrics, Hubble parameter, its time derivative and have bouncing character. The transition from compression to expansion takes place by reaching limiting energy density defined by the following condition \( Z = 0 \). At first the conclusion on possible existence of limiting energy density for gravitating systems, close by which the gravitational interaction has the character of repulsion, was obtained in Ref. 22. Later the idea on limiting energy density as “universal law of the nature” was discussed by M.A. Markov 27, and the value of limiting energy density was postulated to be equal to the Planckian one. According to (5) the value of limiting energy density in our case depends on parameter \( \alpha \) and equation of state for gravitating matter at extreme conditions and can be essentially smaller than the Planckian one 35.

III. INFLATIONARY COSMOLOGICAL MODELS AND ITS PROPERTIES

Now by using GCFE (1)-(2) we will study homogeneous isotropic models filled by interacting scalar field \( \phi \) minimally coupled with gravitation and gravitating matter with equation of state in general form \( p_m = p_m(\rho_m) \). (The generalization for the case with several scalar fields can be made directly). Then the energy density \( \rho \) and pressure \( p \) take the form

\[
\rho = \frac{1}{2} \dot{\phi}^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2} \dot{\phi}^2 - V + p_m,
\]

where scalar field potential \( V = V(\phi, \rho_m) \) includes the interaction between scalar field and gravitating matter. In the most important particular case of radiation (ultrarelativistic matter) the expressions of \( V(\phi, \rho_m) \) can be obtained by taking into account temperature corrections for given scalar field potentials 1 and the following relation for energy density \( \rho_m \sim T^4 \). Because the form of
The relation (5) determines the following restriction on Universe. By using scalar field equation in homogeneous properties of inflationary cosmological solutions for early scalar field potential, our analysis will be made without its concretization. Our main aim will be to investigate properties of inflationary cosmological solutions for early Universe. By using scalar field equation in homogeneous isotropic space

$$\dot{\phi} + 3H\phi = -\frac{\partial V}{\partial \phi} \tag{7}$$

we obtain from Eqs. (3), (6), (7) the conservation law for gravitating matter

$$\dot{\rho}_m \left(1 + \frac{\partial V}{\partial \rho_m}\right) + 3H (\rho_m + p_m) = 0. \tag{8}$$

By using Eqs. (6)–(3) the GCFE (11)–(12) can be transformed to the following form

$$\left\{ H \left[ Z + 3\alpha\phi^2 + \frac{3\alpha}{2} Y \right] + 3\alpha \frac{\partial V}{\partial \phi} \right\}^2 + \frac{k}{R^2} Z^2 = \frac{8\pi}{3M_p^2} \left[ \rho_m + \frac{1}{2} \phi^2 + V + \frac{1}{4} \alpha \left(4V - \phi^2 + \rho_m - 3p_m\right)^2 \right] Z, \tag{9}$$

$$\dot{H} \left[ Z + 3\alpha \left(\phi^2 + \frac{1}{2} Y\right) \right] Z + H^2 \left\{ \left[ Z - 15\alpha\phi^2 + \frac{3\alpha}{2} Y \right] - 18\alpha^2 \left(\phi^2 + \frac{1}{2} Y\right)^2 \right\} - 12\alpha H \dot{\phi} \left\{ 3\alpha \frac{\partial V}{\partial \phi} \left(\phi^2 + \frac{1}{2} Y\right) \right\} + \left[ \frac{\partial V}{\partial \phi} + \frac{9}{8} \frac{\rho_m + p_m}{1 + \frac{\partial V}{\partial \rho_m}} \frac{\partial^2 V}{\partial \phi \partial \rho_m} \left(1 + \frac{1}{3} \frac{d}{d \rho_m} (p_m + 2V)\right) \right] Z \right\} + 3\alpha \left[ \frac{\partial^2 V}{\partial \phi^2} \phi^2 - \left(\frac{\partial V}{\partial \phi}\right)^2 \right] Z - 18\alpha^2 \left(\frac{\partial V}{\partial \phi}\right)^2 \phi^2 \right.$$}

$$= \frac{8\pi}{3M_p^2} \left[ V - \phi^2 - \frac{1}{2} (\rho_m + 3p_m) + \frac{1}{4} \alpha \left(4V - \phi^2 + \rho_m - 3p_m\right)^2 \right] Z, \tag{10}$$

where $Z = 1 + \alpha \left(4V - \phi^2 + \rho_m - 3p_m\right)$ and $Y = \alpha \frac{\rho_m + p_m}{1 + \frac{\partial V}{\partial \rho_m}} (3p_m - 4V)$. The formula (4) takes the form

$$S = \frac{3\alpha}{2Z} \left[ H \left(\phi^2 + \frac{1}{2} Y\right) + \frac{\partial V}{\partial \phi}. \right]. \tag{11}$$

The relation (5) determines the following restriction on admissible values of variables for scalar field and gravitating matter:

$$\dot{\phi}^2 \leq 4V + \alpha^{-1} + \rho_m - 3p_m. \tag{12}$$

Now let us introduce the 3-dimensional space $P$ with axes $(\phi, \dot{\phi}, \rho_m)$. The domain of admissible values of scalar field $\phi$, time derivative $\dot{\phi}$ and energy density $\rho_m$ in space $P$ determined by (12) is limited by bound $L$ defined as

$$Z = 0 \text{ or } \dot{\phi} = \pm \left(4V + \alpha^{-1} + \rho_m - 3p_m\right)^{\frac{1}{2}}. \tag{13}$$

From Eq. (11) the Hubble parameter on the bound $L$ is equal to

$$H_L = \frac{-\frac{\partial V}{\partial \phi}}{\dot{\phi}^2 + \frac{1}{2} Y}. \tag{14}$$

Let us consider the most important general properties of cosmological solutions of GCFE (11)–(12). At first note, by given initial conditions for variables $(\phi, \dot{\phi}, \rho_m)$
and also in the case \( k = \pm 1 \) for \( R \) there are two different solutions corresponding to two values of the Hubble parameter following from Eq. (9):

\[
H_{\pm} = \frac{-3\alpha \frac{\partial V}{\partial \phi} \phi \pm \sqrt{D}}{Z + 3\alpha \left( \frac{\phi^2}{2} + \frac{1}{2} Y \right),}
\]

where

\[
D = \frac{8\pi}{3M_p^2} \left[ \rho_m + \frac{1}{2} \phi^2 + V \right] + \frac{1}{4} \alpha \left[ 4V - \phi^2 + \rho_m - 3\rho_m \right] \left( Z - \frac{k}{R^2} Z^2 \right).
\]

(15)

Obviously, the expression (16) for \( H_{\pm} \) will be regular, if \( Y \geq 0 \). This relation is valid, in particular, for all models filled by gravitating matter with \( \rho_m \geq \frac{2}{3} \) and scalar fields with potentials applying in chaotic inflation theory. Unlike GR, the values of \( H_+ \) and \( H_- \) in GTG are sign-variable and, hence, both solutions corresponding to \( H_+ \) and \( H_- \) can describe the expansion as well as the compression in dependence on its sign. Below we will call solutions of GCFFE corresponding to \( H_+ \) and \( H_- \) as \( H_+ \)-solutions and \( H_- \)-solutions respectively. Note that at asymptotics like GR the sign of \( H \) is negative and the sign of \( H_+ \) is positive. In points of bound \( L \) we have \( D = 0 \), \( H_+ = H_- \) and the Hubble parameter is determined by (17). If initial conditions correspond to asymptotics of \( H_- \)-solution, then unlike GR by compression stage the derivative \( \phi \) does not diverge and by reaching the bound \( L \) the transition from \( H_- \)-solution to \( H_+ \)-solution takes place. In fact, by using the following formula for \( H_\pm \)-solutions

\[
\dot{Z} = 6\alpha \frac{\frac{\partial V}{\partial \phi} Z \pm \sqrt{D}}{Z + 3\alpha \left( \frac{\phi^2}{2} + \frac{1}{2} Y \right)},
\]

(17)

it is easy to show that

\[
\lim_{Z \to 0} H_{\pm} = \dot{H}_L + \frac{2\pi}{3\alpha M_p^2} \rho_m + \rho_m + \frac{\phi^2}{2}.
\]

(18)

From (15) follows that in points of bound \( L \) the derivatives \( \dot{H}_+ \) and \( \dot{H}_- \) are equal and its values do not depend on the model type, as a result we have the smooth transition from \( H_- \)-solution to \( H_+ \)-solution on bound \( L \), and corresponding cosmological solutions for all types models are regular in metrics, Hubble parameter and its time derivative. Note, that in points of bound \( L \) conditions of uniqueness of solutions of Eqs. (14)–(15) are not fulfilled, as a result there are specific solutions (7-28), trajectories of which are situated on the bound \( L \) and have with \( H_+ \)-solutions common points, \( H_- \)-solutions reach the bound \( L \) and \( H_+ \)-solutions originate from them, the surface \( Z = 0 \) is envelope in space \( P \) for cosmological solutions (30). According to Eq. (17) and (18) the function \( S \) has the following asymptotics for \( H_+ \) and \( H_- \)-solutions at \( Z \to 0 \):

\[
\lim_{Z \to 0} S = -\frac{1}{4} \lim_{Z \to 0} \frac{\dot{Z}}{Z} \sim \pm Z^{-\frac{1}{2}}.
\]

(19)

Unlike flat and open models, for which \( H_+ = H_- \) only in points of bound \( L \) and regular inflationary models include \( H_+ \)– and \( H_- \)-solutions reaching bound \( L \), in the case of closed models the regular transition from \( H_- \)-solution to \( H_+ \)-solution is possible without reaching the bound \( L \). It is because by certain value of \( R \) according to (15)–(16) we have \( H_+ = H_- \) in the case \( Z \neq 0 \). Such models are regular also in torsion . Regular inflationary solution of such type was considered in Ref. [29].

All discussed cosmological solutions have bouncing character, but in the presence of scalar fields a bounce takes place not in points of bound \( L (Z = 0) \). In order to study the behaviour of cosmological models at a bounce, let us analyze extreme points for the scale factor \( R(t) \): \( R_0 = R(0), H_0 = H(0) = 0 \). (This means that in the case of \( H_+ \)-solutions \( H_{+0} = 0 \) and in the case of \( H_- \)-solutions \( H_{-0} = 0 \)). Denoting values of quantities at \( t = 0 \) by means of index "0", we obtain from (9)–(10):

\[
\frac{k}{R_0^2} Z_0^3 + 9\alpha^2 \left( \frac{\partial V}{\partial \phi} \right)_0^2 \phi_0^2 = \frac{8\pi}{3M_p^2} \left[ \rho_{m0} + \frac{1}{2} \phi_0^2 + V_0 + \frac{1}{4} \alpha \left( 4V_0 - \phi_0^2 + \rho_{m0} - 3\rho_{m0} \right)^2 \right] Z_0, \qquad (20)
\]

\[
\dot{H}_0 = \left\{ \frac{8\pi}{3M_p^2} \left[ V_0 - \phi_0^2 - \frac{1}{2} \left( \rho_{m0} + 3\rho_{m0} \right) + \frac{1}{4} \alpha \left( 4V_0 - \phi_0^2 + \rho_{m0} - 3\rho_{m0} \right)^2 \right] -3\alpha \left[ \left( \frac{\partial^2 V}{\partial \phi^2} \right)_0^2 \phi_0^2 - \left( \frac{\partial V}{\partial \phi} \right)_0^2 \right] + 18\alpha^2 \left( \frac{\partial V}{\partial \phi} \right)_0^2 \phi_0^2 Z_0^{-1} \right\} \left\{ Z_0 + 3\alpha \left[ \phi_0^2 + \frac{1}{2} \phi_0^2 \right] \right\}^{-1}, \qquad (21)
\]
where \( Z_0 = 1 + \alpha \left( 4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} \right) \). A bounce point is described by Eq. \( 20 \), if the value of \( \dot{H}_0 \) is positive.

By using Eq. \( 20 \) we can rewrite the expression of \( \dot{H}_0 \) in the form

\[
\dot{H}_0 = \left\{ \frac{8\pi}{M_p^2} \left[ V_0 + \frac{1}{2} (\rho_{m0} - p_{m0}) + \frac{1}{4} \alpha \left( 4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} \right)^2 \right] - 3\alpha \left[ \left( \frac{\partial^2 V}{\partial \phi^2} \right)_0 \dot{\phi}_0^2 - \left( \frac{\partial V}{\partial \phi} \right)_0^2 \right] - \frac{2k}{R^2} Z_0 \right\} \left( Z_0 + 3\alpha \left[ \dot{\phi}_0^2 + \frac{1}{2} V_0 \right] \right)^{-1}. \tag{22}
\]

We see from \( 22 \) unlike GR the presence of gravitating matter satisfying the energy dominance condition \( (\rho_m \ll \rho_m) \) does not prevent from the bounce realization \([37]\). Eq. \( 20 \) determines in space \( P \) extremum surfaces depending on the value of \( \alpha \) and in the case of closed and open models also parametrically on the scale factor \( R_0 \).

In the case of various scalar field potentials applying in inflationary cosmology the value of \( \dot{H}_{+0} \) or \( \dot{H}_{-0} \) is positive on the greatest part of extremum surfaces, which can be called "bounce surfaces" \([35]\). By giving concrete form of potential \( V \) and choosing values of \( R_0, \dot{\phi}_0, \phi_0 \) and \( \rho_{m0} \) at a bounce, we can obtain numerically particular bouncing solutions of GCFE for various values of parameter \( \alpha \). Like GR, if initial value of scalar field at the beginning of cosmological expansion is not small \((\phi \geq 1M_p) \), corresponding solution is inflationary cosmological solution containing in addition to inflationary stage also compression stage, transition stage from compression to expansion and a stage after inflation with oscillating regime for scalar field. For given scalar field potential properties of regular inflationary cosmological solutions depend on initial conditions at a bounce and parameter \( \alpha \). Numerical analysis of such solutions in the case of simplest scalar field potentials applying in chaotic inflation was carried out in Ref. \( 32 \). Note that by equal initial conditions characteristics of inflationary stage in developing theory coincide with that of GR \( 32 \).

The analysis of GCFE shows, that some properties of cosmological solutions depend essentially on parameter \( \alpha \), i.e. on the scale of extremely high energy densities. From physical point of view interesting results can be obtained, if the value of \( \alpha^{-1} \) is much less than the Planckian energy density \( 20 \), i.e. in the case of large values of parameter \( \alpha \) (by imposing \( M_p = 1 \)). In order to investigate cosmological solutions at the beginning of cosmological expansion in this case, let us consider the GCFE by supposing that

\[
\left| \alpha \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) \right| \gg 1, \tag{23}
\]

\[
\rho_m + \frac{1}{2} \dot{\phi}^2 + V \ll \alpha \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right)^2.
\]

Note that the second condition \( 24 \) does not exclude that ultrarelativistic matter energy density can dominate at a bounce. We obtain:

\[
\frac{k}{R^2} \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right)^2 + \left\{ H \left[ 4V + 2\dot{\phi}^2 + \rho_m - 3p_m + \frac{3}{2} Y \right] + 3 \frac{\partial V}{\partial \phi} \right\}^2 = \frac{2\pi}{3M_p^2} \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right)^3, \tag{24}
\]

\[
\dot{H} \left( 4V + 2\dot{\phi}^2 + \rho_m - 3p_m + \frac{3}{2} Y \right) \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) + H^2 \left\{ \left[ 4V - 16\dot{\phi}^2 + \rho_m - 3p_m \right. \right.
\]

\[
\left. + \frac{3}{2} Y - \frac{9}{2} \frac{Y}{1 + \frac{dV}{dp_m}} \right\} \left( 1 + \frac{dp_m}{dp_m} + \frac{\rho_m + p_m}{1 + \frac{dV}{dp_m}} \right) \frac{\partial^2 V}{\partial p_m^2} \frac{d^2}{dp_m^2} (3p_m - 4V)
\]

\[
\times \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) - 18 \left( \dot{\phi}^2 + \frac{1}{2} Y \right)^2 \right\}
\]
\[-12H_0 = \left\{ \frac{\partial V}{\partial \phi} + \frac{9}{8} \frac{\rho_m + p_m}{(1 + \frac{\partial V}{\partial p_m})^2} \frac{\partial^2 V}{\partial \phi \partial p_m} \left( 1 + \frac{1}{3} \frac{d}{d p_m} (p_m + 2V) \right) \right\} \]

\times \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) + 3 \frac{\partial V}{\partial \phi} \left( \dot{\phi}^2 + \frac{1}{2} \right) \}

\left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right)

\times \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right)^3. \quad (25)\]

According to Eq. 24, the Hubble parameter in considered approximation is equal to

\[H_\perp = \left[ 4V + 2\dot{\phi}^2 + \rho_m - 3p_m + \frac{3}{2} Y \right]^{-1} \]

\[\times \left[ -3 \frac{\partial V}{\partial \phi} \dot{\phi} \pm \left| 4V - \dot{\phi}^2 + \rho_m - 3p_m \right| \sqrt{\frac{2\pi}{3M_p^2} \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) - \frac{k}{R^2}} \right] \quad (26)\]

and extreme points of the scale factor are determined by the following condition

\[k \frac{R_0^2}{\dot{\phi}_0^2} + 9 \left[ \frac{ \left( \frac{\partial V}{\partial \phi} \right)_{\dot{\phi}_0} }{4V - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0}} \right]^2 \]

\[= \frac{2\pi}{3M_p^2} \left( 4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} \right). \quad (27)\]

From Eq. 24, the time derivative of the Hubble parameter at extreme points is

\[\dot{H}_0 = \left\{ \frac{2\pi}{3M_p^2} \left( 4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} \right) \right\}^2 \]

\[\times \left[ -3 \left( \frac{\partial^2 V}{\partial \phi^2} \right)_{\dot{\phi}_0} - \left( \frac{\partial V}{\partial \phi} \right)_{\dot{\phi}_0}^2 \right] + \frac{18 \left( \frac{\partial V}{\partial p_m} \right)_{\dot{\phi}_0} \dot{\phi}_0^2}{4V_0 - \dot{\phi}_0^2 + \rho_{m0} - 3p_{m0}} \]

\times \left[ 4V_0 + 2\dot{\phi}_0^2 + \rho_{m0} - 3p_{m0} + \frac{3}{2} Y \right]^{-1}. \quad (28)\]

From Eqs. 24–27 follows that in approximation the dynamics of inflationary cosmological models at a bounce does not depend on parameter \(\alpha\). In the case of models containing at a bounce ultrarelativistic matter and scalar fields, the dynamics in considered approximation does not depend on ultrarelativistic matter, if interaction terms in scalar field potential can be neglected.

The analysis given in this Section shows that existence of limiting bound \(L\) and bounce surface in space \(P\) ensures regular character of inflationary cosmological solutions in metrics, Hubble parameter and its time derivative. Characteristic feature of inflationary cosmological models is the presence of the stage of regular transition from compression to expansion. The duration of this stage is several order smaller than duration of inflationary stage. If we take into account that duration of inflationary stage in chaotic inflation is extremely small \([1]\), we can tell that our inflationary models correspond to regular Big Bang or Big Bounce.

IV. PARTICULAR REGULAR INFLATIONARY COSMOLOGICAL MODEL

As illustration of discussed theory regular cosmological models in the simplest particular case will be considered in this Section. We will consider models including noninteracting scalar field with potential \(V(\phi)\) and ultrarelativistic matter \((p_m = \frac{1}{3} \rho_m)\) \([11]\). In this case the bound \(L\) in space \(P\) is reduced to two cylindrical surfaces \(\dot{\phi} = \pm \left( 4V + \alpha^{-1} \right)^{\frac{1}{2}}\). Bounce surface is reduced also to cylindrical surfaces in the case under consideration, when the scale of extremely high energy densities is much smaller than the Planckian energy density (see 27). In connection with this we will consider instead of space \(P\) the plane of variables \((\phi, \dot{\phi})\) and intersections of the bound \(L\) and bounce surface with this plane. We have in this plane two bound \(L_{\perp}\)–curves and in the case of flat models two bounce curves \(B_1\) and \(B_2\) determined by equation 29

\[4V_0 - \dot{\phi}_0^2 = 3 \left( \frac{M_p^2}{2\pi} V_0^2 \dot{\phi}_0^2 \right)^{\frac{1}{2}}. \]
Each of two curves $B_{1,2}$ contains two parts corresponding to vanishing of $H_+$ or $H_-$ and denoting by $(B_{1+}, B_{2+})$ and $(B_{1-}, B_{2-})$ respectively. If $V'$ is positive (negative) in quadrants 1 and 4 (2 and 3) on the plane $(\phi, \dot{\phi})$, the bounce will take place in points of bounce curves $B_{1+}$ and $B_{2+}$ ($B_{1-}$ and $B_{2-}$) in quadrants 1 and 3 (2 and 4) for $H_+$-solutions ($H_-$-solutions) (see Fig. 1). To analyze flat bouncing models we have to take into account that besides regions lying between curves $L_{\pm}$ and corresponding bounce curves the sign of values $H_+$ and $H_-$ for applying potentials is normal: $H_+ > 0$, $H_- < 0$.

The Hubble parameter $H_+$ is negative in regions between curves $(L_+ + B_{1+})$, $(L_- + B_{2+})$, and the value of $H_-$ is positive in regions between curves $(L_+ + B_{1-})$, $(L_- + B_{2-})$. As it was noted above any cosmological solution has to contain both $H_-$- and $H_+$-solution and regular transition from $H_-$-solution to $H_+$-solution takes place in points of $L_+$ where $H_+ = H_-$. In the case of open and closed models Eq. (27) determines 1-parametric family of bounce curves with parameter $R_0$.

Bounce curves of closed models are situated in region between two bounce curves $B_1$ and $B_2$ of flat models, and in the case of open models bounce curves are situated in two regions between the curves: $L_+$ and $B_1$, $L_-$ and $B_2$. In general case, when approximation (28) is not valid, bounce surface in space $P$ of cosmological models including scalar field and ultrarelativistic matter determined by Eq. (28) in space $P$ depends on parameter $\alpha$ and it is not more cylindric surface. The situation concerning cosmological solutions of Eqs. (26–11) does not change.

Below particular bouncing cosmological inflationary solution for flat model by using scalar field potential in the form $V = \frac{1}{2}m^2\phi^2$ ($m = 10^{-6}M_p$) is given. The solution was obtained by numerical integration of Eqs. (1), (11) and by choosing in accordance with Eq. (28) the following initial conditions at a bounce: $\phi_0 = \sqrt{2}10^3 M_p$, $\dot{\phi}_0 = \sqrt{3.96757141} \alpha (\alpha = 10^{14} M_p^{-4})$; radiation energy density is negligibly small, initial value of $R_0$ can be arbitrary. The dynamics of the Hubble parameter and scalar field is presented for different stages of bounces by solving in Figs. 24 (by choosing $M_p = 1$). The transition stage from compression to expansion (Fig. 2) is essentially asymmetric with respect to the point $t = 0$ because of $\phi_0 \neq 0$. In course of transition stage the Hubble parameter changes from maximum in module negative value at the end of compression stage to maximum positive value at the beginning of expansion stage. The scalar field changes linearly, the derivative $\dot{\phi}$ grows at first being positive to maximum value $\dot{\phi} \sim \phi_0$ and then the value of $\dot{\phi}$ decreases and becomes negative. In the case of presence of radiation, which can give the main contribution to energy density at a bounce, the duration of transition stage changes. Quasi-de-Sitter inflationary stage and quasi-de-Sitter compression stage are presented in Fig. 3. As was noted above, characteristics of inflationary stage do not depend practically on parameter $\alpha$ and coincide with that of GR. The amplitude and frequency of oscillating scalar field after inflation (Fig. 3) are close to that of GR, however, the behaviour of the Hubble parameter after inflation in considering case with large value of parameter $\alpha$ is essentially noneinsteinian, at first the Hubble parameter oscillates near the value $H = 0$, and later the Hubble parameter becomes positive and decreases with the time like in GR. Before quasi-de-Sitter compression stage there are also oscillations of the Hubble parameter and scalar field not presented in Figs. 24. Ultrarelativistic matter, which could dominate at a bounce has negligibly small energy densities at quasi-de-Sitter stages. At the same time the gravitating matter could be at compression stage in more realistic bouncing models. As it follows from our consideration regular character of such inflationary cosmological models has to be ensured by cosmological equations of PGTG.

The interaction between scalar fields and radiation leads to quantitative corrections of considered cosmological models. In accordance with Eq. (8) temperature corrections for scalar field potentials change the connection between scale factor $R$ and radiation energy density and can be also essential for more late stages of cosmological evolution, when energy densities are sufficiently small and consequences of PGTG and GR coincide. In particular, these corrections can give additional contribution to the effect of acceleration of cosmological expansion (if relics scalar fields exist).

Note that bouncing character have solutions not only in classical region, where scalar field potential, kinetic energy density of scalar field and energy density of gravitating matter do not exceed the Planckian energy density, but also in regions, where classical restrictions are not fulfilled and according to accepted opinion quantum gravitational effects can be essential.

V. CONCLUSION

As we see, the applying of gauge approach to gravitational interaction permits to build consequent field theoretical scheme in the frame of 4-dimensional physical space-time, which is free of principal difficulties of GR by description of early Universe. Satisfying the correspondence principle with GR in the case of gravitating systems with rather small energy densities, generalized cosmological Friedmann equations lead to conclusion, that at extreme conditions gravitational interaction has the repulsion character in the case of usual gravitating systems with positive values of energy density satisfying the energy dominance condition. This means, there is not necessity to refuse fundamental physical requirement of energy density positivity for physical matter. The solution of PCS is obtained by classical description of gravitational field without quantum gravitational corrections. Moreover, from the point of view of developing approach,
the Planckian era could be absent by evolution of our Universe. Unlike loop quantum cosmology, the Universe is macro-object at all stages of its evolution including the transition stage from compression to expansion. To build a realistic cosmological model we have to know the content and properties of gravitating matter at different...
stages of its evolution. It is of principal interest to investigate physical processes at the beginning of cosmological expansion depending on limiting energy density and limiting temperature, and to obtain observable physical consequences depending on parameter $\alpha$. From physical point of view, it is interesting also the building of cosmological models by breakdown of its homogeneity and isotropy.

FIG. 4: The stage after inflation.

\[ 10^2 \phi \]

\[ 8.74 \]

\[ 8.74 \]

\[ 8.76 \]

\[ 10^{-9} t \]

\[ 10^{-9} t \]

\[ 8.73 \]

\[ 8.74 \]

\[ 8.75 \]

\[ 8.76 \]

\[ 10^2 \phi \]

\[ 8.74 \]

\[ 8.74 \]

\[ 8.76 \]

\[ 10^{-9} t \]

\[ 10^{-9} t \]

\[ 8.73 \]

\[ 8.74 \]

\[ 8.75 \]

\[ 8.76 \]

\[ 10^2 \phi \]

\[ 8.74 \]

\[ 8.74 \]

\[ 8.76 \]

\[ 10^{-9} t \]

\[ 10^{-9} t \]

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\[ 8.75 \]

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\[ 10^2 \phi \]

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\[ 10^{-9} t \]

\[ 10^{-9} t \]

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\[ 10^2 \phi \]

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\[ 10^{-9} t \]

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\[ 10^2 \phi \]

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\[ 10^2 \phi \]

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\[ 10^2 \phi \]

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\[ 8.76 \]

\[ 10^2 \phi \]

\[ 8.74 \]

\[ 8.74 \]

\[ 8.76 \]

\[ 10^{-9} t \]

\[ 10^{-9} t \]
The GCFE lead to limiting energy density also in the case $\alpha < 0$ if at extreme conditions $p < \frac{1}{4}\rho$. \[32\]

From mathematical point of view the appearance of specific solutions is connected with the fact, that the Eqs. (1)-(2) were multiplied by $Z^2$ when its transformation to the (9)-(10) was performed. Specific solutions can be excluded by corresponding transformation of Eq. (10).

In GR a bounce is possible only in closed models if the following condition $V_0 - \phi_0^2 - \frac{1}{4}(\rho_{\text{mo}} + 3p_{\text{mo}}) > 0$ takes place.

If $\alpha^{-1} \ll M_p^4$ the derivative $\dot{H}_0$ is negative in the neighbourhood of origin of coordinates in space $P$ that leads to appearance of oscillating solutions of GCFE.\[31\]

The neighbourhood of origin of coordinates is not considered in this approximation, the behavior of extremum curves near origin of coordinates was examined in Ref. 31, 32.