The influence of color temperature mismatch in star simulator on positioning accuracy and magnitude measurement by star sensor

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**Abstract.** During the calibrating of star sensor, the calibration accuracy is greatly affected by the mismatch between the color temperature of the light and the to-be-measured star, which further affects the attitude measurement accuracy. This paper studied the near-infrared spectra of stars with different color temperatures, and analyzed the errors on star positioning and magnitude measurement of star sensor due to the color temperature mismatch. The results showed that in the central field of view, the spot centroid deviation caused by spectral mismatch is smaller than that in the edge field of view. And the defocus of the imaging surface also affects the spot centroid deviation. Besides, when calibrating with 6000K color temperature light, the maximum measurement error can reach -1.9126 magnitude.

1. **Introduction**

The star sensor [1] images stars through optical system and image sensor. With the star sensor operating, the stellar centroid positioning accuracy and magnitude measurement accuracy are important performance indicators, which directly affects the star identification rate.

The star simulator [2] is named as the light for calibration [3] of the star sensor. The color temperature mismatch between star simulator and the to-be-measured star will affect the calibration of the star sensor from two aspects, including star positioning error [4] and magnitude measurement error [5].

Wu Ziming’s [6] simulation experiment verified that the centroid deviation of the three spectral stars [7] reached 0.24 pix, and the practical one proved that the stellar centroid deviation was 0.18 pix. Liu Haibo [8] also studied the effect of color temperature difference on the positioning accuracy of stars through simulation, and the positioning error increases with the increase of the field of view. Feng Chen’s[9] simulation verified that the relative error of magnitude measurement can reach 7.9\% due to spectral mismatching. However, the current researches concentrate on the invisible light field less generally, and less on reciprocal action of magnitude measurement and positioning errors.
In recent years, the all-sky time star sensor[10] has mostly adopted near-infrared optical detection methods, which can improve the star sensor’s ability to detect stars during the day. Based on the simulation of the optical system of the star sensor, this paper quantitatively analyses the mismatch of the color temperature of the to-be-measured star and the calibrated light in the near-infrared range, as well as the effect of view field of incident light and defocus of the imaging surface on spot centroid measurement.

2. The effect of color temperature mismatch on the accuracy of star positioning
The star simulator simulates the spectrum of a certain color temperature light to calibrate the star sensor. In the same field of view, when the star sensor measures single stars with different color temperatures, the positions of the spot centroids on the imaging surface of the detector are not completely consistent, and the measured centroid angles are also inconsistent. The error is shown in Figure 1.

![Mismatched color temperature](image1)

Figure 1. Schematic diagram of star positioning error of star sensor for star simulator color temperature mismatch

In the experiment of the chapter, the star color temperature simulated by the star simulator is 2550K, 3490K, 4766K, 6516K, 8910K, and 31054K. The steps include optical system modelling, energy calculation of detector pixel, calculation of centroid and angle deviation.

2.1. Optical system modeling
The optical system used in this experiment has a focal length \( f = 29.887 \text{mm} \), \( 
 \frac{F}{
 \# = 3 \), and the optical system model is shown in Figure 2. The pixel number of the detector is set to 128×128, the pixel size is 1μm×1μm, and the quantum efficiency[11] is \( R(\lambda) \), as shown in Figure 3.

![Optical system model](image2)

Figure 2. Optical system model

![Detector quantum efficiency](image3)

Figure 3. Detector quantum efficiency

The stellar emission spectrum can be regarded as the black-body radiation one, given by Planck’s Law:
\[
P(\lambda, T) = \frac{2\pi e^2}{\lambda^3 \left[ \exp\left(\frac{hc}{\lambda k_B T}\right) - 1 \right]}
\]

where, \( T \) is the absolute temperature of the black body (K), \( \lambda \) is the wavelength (nm), \( h=6.626\times10^{-34}\) J·s is the Planck constant, \( c=2.997\times10^8\) m/s is the speed of light in vacuum, \( k_B=1.38\times10^{-23}\) J/K is Boltzmann constant.

2.2. Detector pixel energy calculation
In ZEMAX, the input spectrum of the optical system is composed of a series of single-wavelength light, and the energy \( E(\lambda_i, x, y) \) received by the single-wavelength light of 900nm~1700nm in each pixel is obtained. The pixel is shown in the figure 4 and the pixel energy distribution is shown in Figure 5. The energy \( M_T(x, y) \) generated by the polychromatic light in the detector pixel is

\[
M_T(x, y) = \sum \rho_i \cdot R(\lambda_i) \cdot E(\lambda_i, x, y)
\]

where, \( p_i \) single-wavelength light matching coefficient, \( R(\lambda) \) is the quantum efficiency of the detector.

2.3. Calculation of centroid and angle deviation
When the color temperature is \( T \), the center of mass coordinates \((I_x, I_y)\) can be obtained according to \( M_T(x, y) \), Which is used for calculating the star position deviation: \( \Delta s \):

\[
\Delta s = \sqrt{(I_{x0} - I_{x1})^2 + (I_{y0} - I_{y1})^2}
\]

where, \((I_{x0}, I_{y0})\) is the centroid coordinates of the calibration light with a color temperature of 6000K, and \((I_{x1}, I_{y1})\) is the centroid coordinates of the to-be-measured stellar light.

The angular deviation of the star position is \( \theta \)

\[
\theta = \frac{\Delta s \cdot m}{f} \cdot \frac{180}{\pi} \cdot 3600
\]

where, \( m \) is the size of the pixel, \( f \) is the focal length of the system, \( \theta \) is the deviation angle.

2.4. The influence of the field of view and defocusing amount on the deviation of the centroid angle
In order to study the role of the field of view of the color temperature affecting the star positioning accuracy of the star sensor, the light incident field of view is set to \((0°,0°)\), \((0°,2°)\), \((0°,4°)\). The optical paths of different fields of view are shown in Figure 6.
3. The influence of color temperature mismatch on magnitude measurement error

The star sensor has a specific response range and curve when measuring the magnitude of stars. A star sensor calibrated with a light with a color temperature of 6000K is used to measure the irradiance of a stellar light with the same color temperature, and the measurement results will not produce errors due to the calibration of the response curve of the detector. However, the spectral distribution of color and radiation of stars in the universe is different. The measurement results must be bound to errors when a star sensor with a spectral response curve is used to measure the magnitude.

3.1. 0-magnitude light spectral distribution

In astrophotometry, the irradiance of stars received outside the earth’s atmosphere is used to express the brightness of different stars. Namely, the magnitude of stars is used to measure the brightness of stars. Every 5 magnitudes, the brightness of the stars differs by 100 times. Visual magnitude \( m_v \) is

\[
m_v = -2.5 \log \left( \frac{E_0}{S_\text{eye}(\lambda)} \right) + 2.5 \log E_0 \tag{5}
\]

where, \( E(\lambda) \) is the irradiance distribution of the measured starlight at the wavelength \( \lambda \), \( S_\text{eye}(\lambda) \) is the view function, and \( E_0=2.65 \times 10^{-8} \)lx is the illuminance value of 0 magnitude star.

\( E(\lambda) \) is proportional to \( P(\lambda, T) \). The spectral distribution of each color temperature at 0 magnitude, that is, \( E(\lambda) \) when \( m_v=0 \):
Figure 8 shows the 0-magnitude spectral curves with color temperatures of 6000K, 3000K, 4000K, 5000K, 7000K, 8000K, 9000K.

3.2. magnitude measurement

If the spectral radiation curve distribution of the calibration light used in the laboratory is $f_0(\lambda)$, the quantum efficiency response curve of the detector is $R(\lambda)$. The magnitude $m$ detected by the star sensor is:

$$m = -2.5 \log \left( \frac{\int_{\lambda'} f_T(\lambda) \cdot R(\lambda) d\lambda \int_{\lambda'} f_0(\lambda) d\lambda}{\int_{\lambda'} f_0(\lambda) \cdot R(\lambda) d\lambda} \right) + C$$

(7)

where, $C$ is a constant, $f_T(\lambda)$ is the spectral radiation curve distribution of the target light.

When $T=6000K$, the value of $C$ can be obtained by substituting $f_T(\lambda)=f_0(\lambda)$ and $m=0$ into Equation (7). With the value of $C$ obtained, $f_T(\lambda)$ ($T=2550K$, 3490K, 4766K, 6516K, 8910K, 31054K) is respectively substituted into Equation (7) for calculating the value of $m$ (measured magnitude) of each color temperature light.

4. Results and analysis

4.1. The influence of the field of view on the positioning accuracy of stars

In chapter 2, the energy received by the center pixel occupies most of the energy. By taking the center pixel with the $45 \times 45$ pixels and the pixel size with $1 \mu m \times 1 \mu m$, the centroid angle deviation is calculated according to Equation (4).

Table 1. The deviation angle of the centroid of the diffuse spot at different fields of view (based on 6000K)

| Color Temperature | Field of View | (0°, 0°) | (0°, 2°) | (0°, 4°) |
|-------------------|---------------|---------|---------|---------|
| 2550K             | 1.47E-13      | 0.0584  | 0.1312  |
| 3490K             | 9.78E-14      | 0.0294  | 0.0646  |
| 4766K             | 0             | 0.0100  | 0.0215  |
| 6516K             | 0             | 0.0029  | 0.0061  |
| 8910K             | 4.89E-14      | 0.0112  | 0.0238  |
| 31054K            | 4.89E-14      | 0.0247  | 0.0517  |

The color temperature mismatch affects the centroid position of dispersed spot on the imaging surface of the star sensor from table 1. When the field of view is (0°, 0°), the centroid angle deviation is only 10-13 orders of magnitude. With the same color temperature, the farther away the star is from the central field of view, the greater the centroid deviation is. When the field of view is (0°, 4°), the maximum centroid angle deviation is 0.1312°. With the same field of view, if the color temperature is on the same side of the calibrated color temperature. The greater the color temperature difference is, the greater the centroid deviation is. For example, when the field of view is (0°, 4°), with the color temperature increasing from 2550K, 3490K to 4766K, the centroid angle deviation gradually decreases; with the color temperature increasing from 6516K, 8910K to 31054K, the centroid angle deviation gradually increases.
4.2. The effect of defocusing of the imaging surface on the positioning accuracy of stars

Table 2 shows the deviation angles of stars of various color temperatures with different defocus.

| Color temperature | Deviation angle (mm) |
|-------------------|----------------------|
| 2550K             | -0.217 -0.058 0 0.042 0.142 0.217 |
| 3490K             | 0.367 0.226 0.131 0.060 0.045 0.363 |
| 4766K             | 0.114 0.108 0.065 0.032 0.026 0.234 |
| 6516K             | 0.002 0.010 0.006 0.003 0.003 0.092 |
| 8910K             | 0.000 0.037 0.024 0.013 0.012 0.122 |
| 31054K            | 0.033 0.079 0.052 0.029 0.028 0.293 |

From Table 2, it can be found that the defocus has a greater or lesser impact on the positioning accuracy of stars. When \( T=2550K \) and the defocus is -0.217mm, the positioning error of the star is increased by 3 times. But when the star color temperature is close to the calibration color temperature, the star positioning error is relatively small.

4.3. The influence of color temperature mismatch on magnitude measurement

The measured magnitude of each color temperature light with an apparent magnitude of 0 is shown in Figure 9(1). It can be seen that when light with different color temperatures are used to simulate a 0-magnitude star with an actual color temperature of 6000K, the maximum magnitude calibration error can reach -1.9126 magnitudes. The closer the actual star is to the calibration light, the smaller the magnitude error is. If the light with 2550K color temperature is used as the calibration light, the magnitude measurement error is up to 2.7717 magnitude, shown in Figure 9(2).

![Figure 9. Magnitude measurement error](image)

- (1) calibrated light color temperature 6000K
- (2) calibrated light color temperature 2550K

5. Conclusion

This paper, using the optical software for simulation analysis, discussed the influence of the mismatch of the color temperature of the to-be-measured star and the calibration light on the star positioning error and magnitude measurement.

To begin with, the influence of color temperature mismatch on star positioning error is related to the field of view and defocus of the optical system. The star positioning error caused by the mismatch between the star's color temperature and the color temperature of the calibrated light is firstly due to the quantum response efficiency of the star sensor, and it is also affected by the field of view and the defocus in the optical system. Therefore, selecting a star sensor with appropriate quantum response efficiency, as well as selecting a star close to the central field of view and a stable optical system, all
help to reduce the error effect caused by the color temperature mismatch. However, if the stellar color temperature matches the calibration color temperature, some factors about the optical system like the field of view have little effect on the positioning accuracy.

In addition, the color temperature mismatch will also cause accuracy errors in the magnitude measurement of the stars. The main reason is that the quantum efficiency of the star sensor responds differently to the spectrum of different wavelengths. When the star's spectrum differs greatly, the magnitude measurement result of the star sensor must be inaccurate.

In summary, through simulation calculations, this paper verifies that the color temperature mismatch between the to-be-measured star and the calibration light is an important factor affecting the star positioning and magnitude measurement of the star sensor. Furtherly, selecting a calibration light that is completely consistent with the spectrum of the to-be-measured light to calibrate the star sensor will greatly improve the testing and calibration capabilities of the star sensor.

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