Stable optical lateral forces from inhomogeneities of the spin angular momentum

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Transverse spin momentum related to the spin angular momentum (SAM) of light has been theoretically studied recently and predicted to generate an intriguing optical lateral force (OLF). Despite extensive studies, there is no direct experimental evidence of a stable OLF resulting from the dominant SAM rather than the ubiquitous spin-orbit interaction in a single light beam. Here, we theoretically unveil the nontrivial physics of SAM-correlated OLF, showing that the SAM is a dominant factor for the OLF on a nonabsorbing particle, while an additional force from the canonical (orbital) momentum is exhibited on an absorbing particle due to the spin-orbit interaction. Experimental results demonstrate the bidirectional movement of 5-μm-diameter particles on both sides of the beam with opposite spin momenta. The amplitude and sign of this force strongly depend on the polarization. Our optofluidic platform advances the exploitation of exotic forces in systems with a dominant SAM, facilitating the exploration of fascinating light-matter interactions.

INTRODUCTION

Light carries momenta that can be transferred to objects and exert optical forces. Since the seminal work by Ashkin et al. (1), the optical radiation pressure force (2) and gradient force (1) have enabled numerous physical and biological applications (3–8). Optical "tractor beams" provide an additional degree of freedom to optical micromanipulation by exploiting the dominant forward scattering (9–13). The optical lateral force (OLF), which represents a particular category of force perpendicular to the wave vector (14–16), has attracted a great deal of interest due to its intriguing physics and potential applications in enantiomer sorting (17–19). By virtue of the spin-orbit coupling from the circularly polarized beam (20–22), the OLF can be generated on an achiral particle placed above or at the surface, which breaks the mirror symmetry of the light scattering (14, 15, 23, 24). Linearly polarized beams have also been able to exert reversible OLFs on chiral microparticles by lateral momenta transfer (17) or plasmonic directional side scattering (25). Aside from using interfaces, OLFs can emerge on chiral particles in complex optical fields, for instance, wave interference (26–28) and evanescence waves (18). Despite these extensive explorations, few attempts have been devoted to achieving an OLF on an achiral particle using a simple light beam in a homogeneous environment.

An elliptically polarized beam, when tightly focused, transfers the spin momentum into the canonical (or orbital) momentum by the spin-orbit interaction to move the metallic particle in a circular orbit (29–32). Recently, the extraordinary transverse spin (33–41), which is orthogonal to the wave vector, has been theoretically investigated in such highly focused optical field. Although the nonequilibrium and unstable transverse force from the transverse spin might have been observable in a focused circularly polarized beam, the dominant optical gradient force could attract the particle to a stable trapping position where no transverse force exists. Meanwhile, this transient transverse force can also be overwhelmed by thermal fluctuations or the spin-orbit interaction (36). Thus, it cannot be truly considered a stable OLF. Besides, the recently reported orbital of a particle in a focused ring-shaped field with a radial intensity gradient was unambiguously attributed to the spin-to-orbit conversion (31), whereas the comprehensive analysis of the transverse force from a system with dominant spin angular momenta (SAMs) still remains elusive as highly focused beams induce predominant spin-orbit interactions (29). In addition to highly focused beams, the transverse spin and force have been theoretically discovered in evanescent waves (15, 32) and structured beams (35). The measurements of the transverse spin and force were reported with scanning a particle through the focal plane of the beam (42, 43), using a nanocantilever (44), or a three-dimensional force spectroscopy (45). Transverse spin was experimentally demonstrated in structured electromagnetic guided waves, such as surface Bessel beams and Airy beams (46). The spin was also found in the unpolarized paraxial light (47), in the evanescent waves (48), in surface acoustic waves (49), and using an atomic force microscope tip for the quantum sensing of photonic spin density (50). A summary of the transverse spin and potential force effects can be found in table S1. Despite ubiquitous efforts, to the best of our knowledge, there is no direct demonstration of the spin momentum–induced stable OLF in a single optical beam without the contribution of the prevalent spin-orbit interaction.

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Here, we demonstrate that a stable spin-correlated OLF on a nonabsorbing sphere is possible to achieve even when using a single beam in a homogeneous environment. The beam is loosely focused into a line-shaped spot with widths much larger than the wavelength, thus significantly diminishing the force from the spin-orbit interaction. The OLF from the inhomogeneity of the SAM can appear in the loosely focused beam. The particle is immersed in a liquid medium with the same refractive indices as top and bottom quartz slides, mimicking a perfect homogeneous environment. Theoretical investigations reveal that the spin momentum (proportional to the curl of the SAM, $S$) dominantly contributes to the OLF on a nonabsorbing nanoparticle. Experimental results demonstrate that 5-μm-diameter particles move bidirectionally on both sides of the beam with opposite spin momenta. The amplitude and direction of the OLF can be robustly controlled by different helicities of light. Our optofluidic strategy provides a novel opportunity to harness exotic optical forces on various kinds of particles (e.g., high index, metal, and chiral) from the optical spin and other intrinsic properties of light.

**RESULTS**

**Principle and theoretical modeling of the spin-angular-momentum-dependent optical lateral force**

To investigate the intriguing OLF in a single beam in a homogeneous environment, a laser beam (wavelength $\lambda = 532$ nm) is focused into a line-shaped optical field as illustrated in Fig. 1A, whose widths in two orthogonal directions are controlled by two cylindrical lenses (fig. S1). The particle is immersed in a liquid medium, which is sandwiched by two glass slides. To ensure a homogeneous environment and eliminate the interference from the reflected light, the liquid polyethylene

![Fig. 1. Origin of the spin-correlated optical lateral force in a single line-shaped beam.](https://www.science.org)
glycol is delicately chosen to have the same refractive index as the glass (quartz, refractive index equals 1.46) (Fig. 1A). Because of the line-shaped light field, the SAM, S, visualized by the loops in Fig. 1A, is nonlinearly distributed in the x direction and decays from the center of the beam (x = 0). All loops rotate anticlockwise (top view), indicating that the sign of S only depends on the helicity of the beam and does not change through the whole field. Meanwhile, the different decaying directions of S in the x direction result in opposite spin momentum densities \( p_S \), which can be expressed as \(32, 51\)

\[
\begin{align*}
    p_S = p_S^e + p_S^m = \frac{1}{2} \nabla \times S
\end{align*}
\]

\[\begin{align*}
    = \frac{g}{4\omega_0} \text{Im} \left[ \nabla \times \left( \frac{1}{\mu} F^* \times F \right) + \nabla \times \left( \frac{1}{\varepsilon} H^* \times H \right) \right]
\end{align*}\]

where \( p_S^e \) and \( p_S^m \) are spin momenta from electric and magnetic field contributions, \( g = (8\pi^{-1}) \) in Gaussian units, and \( \varepsilon \) and \( \mu \) are the permittivity and permeability of the medium, respectively. To investigate the \( p_S \)-dependent OLF, the electric field of the focused in-paraxial wave beam can be approximately written as

\[
\begin{align*}
    E &= A(\cos \theta \hat{x} + i \sin \theta \hat{y}) \exp \left[ -\left( \frac{x^2}{w_x^2} + \frac{y^2}{w_y^2} \right) \right] \exp(-ikz)
\end{align*}
\]

where \( A \) is the wave amplitude, \( \hat{x} \) and \( \hat{y} \) are unit vectors of the corresponding axes, and \( k \) is the wave number in the medium; \( \theta \) is the angle between the electric field of the light beam and the fast axis of the quarter–wave plate (also the \( x \) axis); \( w_x \) and \( w_y \) are the half focal widths in the \( x \) and \( y \) directions, respectively (Fig. S1). In the experiment, \( w_x \) and \( w_y \) are 15 and 500 \( \mu \)m, respectively, which are much larger than \( \lambda \) (532 nm), so we can ignore the beam divergence and largely reduce the effect of the spin–orbit interaction. The explicit expression (52) of the optical force can be written as \( F = F_e + F_m + F_{em} \) where \( F_e, F_m \) and \( F_{em} \) express the electric dipole force, magnetic dipole force, and force from the interference between the electric and magnetic dipoles, respectively. \( F_e \) or \( F_m \) emerges from the contributions of the electric or magnetic part of the inhomogeneous energy distribution (gradient force) and the orbital momentum density (see eqs. S12 and S16). After an explicit derivation (notes S1 and S2), \( F_e = 0 \) because of the vanished gradient force and electric orbital momentum when the particle is placed at \( y = 0 \) (see eq. S15). Thus, the OLF becomes

\[
\begin{align*}
    F_{OLF}(x, 0) &= F_m + F_{em}
\end{align*}
\]

\[\begin{align*}
    = \frac{g \delta}{\omega} \left[ \text{Im}(\alpha_m) \frac{c^3}{\omega} \frac{k^2}{3} \text{Re}(\alpha_e \alpha_m) \left(p_S^m + p_O^m\right) \right]
\end{align*}\]

\[\begin{align*}
    + \frac{2k^2c^2 \sin \theta \cos \theta}{\omega \mu_0 \varepsilon w_x w_y} \left[ \text{Im}(\alpha_m) \frac{k^2 w_y^2}{3e} \text{Re}(\alpha_e \alpha_m) \left(k^2 + \frac{1}{w_x^2} \right) \right] \exp \left( -\frac{2x^2}{w_x^2} \right) \hat{y}
\end{align*}\]

where \( \alpha_e \) and \( \alpha_m \) are the electric and magnetic polarizabilities, \( p_O^m \) is the magnetic orbital momentum density, \( c \) is the speed of light in vacuum, and \( \omega \) is the angular frequency of light. For a nonabsorbing particle, \( \text{Im}(\alpha_m) = 0 \), resulting in \( F_{OLF} = F_{em} \), the force is only related to \( \text{Re}(\alpha_e \alpha_m) \) \( (p_S^m + p_O^m) \) \( (p_S^m = 0 \) according to eq. S33). Intriguingly, for different orientation angles \( \theta \) of the quarter–wave plate, the sign of the OLF oscillates with a period of \( \pi \) (Eq. 3; see also fig. S2). Thus, from Eq. 1, by plotting the distributions of magnetic SAM \( S^m \) and the magnetic spin momentum \( p_O^m \) is negligible when \( w_y \gg \lambda \); see discussions below), we can infer opposite directions of the OLF on the two sides of the beam center (\( x = 0 \)) in Fig. 1 (A and B). According to Eqs. 1 and 2

\[
\begin{align*}
    S^m = \frac{gA^2 c^2 k}{\mu_0 \omega^2} \exp \left[ -2 \left( \frac{x^2}{w_x^2} + \frac{y^2}{w_y^2} \right) \right]
\end{align*}\]

\[\begin{align*}
    \left( 2 \cos^2 \theta - \frac{2 \sin^2 \theta \hat{y}}{w_x^2} + k \sin \theta \cos \theta \hat{z} \right)
\end{align*}\]

where \( z \) is the unit vector of the \( z \) axis. Note that the spin–to–orbital angular momentum conversion has been widely found in highly focused beams and interpreted using the helicity-dependent geometric phase (29, 31, 53–55). This conversion induces an orbital momentum \( p_0 \) and consequently the rotation optical force to move particles in a circular orbit (eqs. S12, S16, and S18). However, in the loosely focused beam described by Eq. 2, the contribution of the orbital momentum can be hugely diminished according to Eq. 3, which shows that the magnetic dipole force term related to \( p_O^m \) is zero because \( \text{Im}(\alpha_m) = 0 \) for a nonabsorbing particle. Only the force from the interference between the electric and magnetic dipoles contributes to the OLF. In this case, the OLF has contributions from both \( p_S^m \) and \( p_O^m \), with a ratio of \( k^4/(w_x^4) \) (see Eq. 3), which can be \( \gg 1 \) when \( w_y \gg \lambda \), meaning that the force from the spin momentum is much larger than that from the orbital momentum. Therefore, we can conclude that the OLF comes predominantly from the inhomogeneity of the SAM or the spin momentum rather than spin–orbit interactions in this loosely focused beam with \( w_y \gg \lambda \). The force from the orbital momentum can be prominent when \( w_y \sim \lambda \) in a highly focused beam or when the particle has absorption [i.e., \( \text{Im}(\alpha_m) \neq 0 \) for a metallic particle; see eq. S30].

The intensity gradient of the beam in the \( x \) direction gives rise to an inhomogeneous distribution of the SAM, as shown in Fig. 1B. The SAM \( S \) decays from the center of the beam along the \( ±x \) and \( ±y \) directions, which also generates opposite distributions of the spin momentum \( p_S \) on both sides of the beam axis, as shown in Fig. 1C. Consequently, the OLF emerges from the spin momentum according to Eq. 3. Thus, the general origin of this SAM-dependent optical force, not just limited to the OLF here, is as follows: Any helicity beam with the SAM gives rise to a SAM inhomogeneity–dependent optical force when the beam has an electric or magnetic gradient that induces a spin momentum \( p_S \) (Eq. 1). This particular force is parallel to the spin momentum vector, which consists of both the electric \( (p_S^e) \) and magnetic \( (p_S^m) \) contributions. The OLF can be in the same or opposite direction of \( p_S \), depending on the sign of \( \text{Re}(\alpha_e \alpha_m) \) (see Eq. 5). In most typical scenarios, the SAM inhomogeneity–dependent force only considers the electric spin momentum \( p_S^e \) on an electric dipole (36, 57), which is usually very weak and can be easily overwhelmed by the optical force from the rotating Poynting vectors or the orbital momentum in vortex beams (58, 59), the spin–orbit interaction–induced orbital momentum in highly focused beams (29, 33), and the most common optical gradient forces in various interference fields (35, 60). The magnetic spin momentum \( p_S^m \) plays a crucial role in the spin-dependent magnetic dipole force and the force from the interference between the electric and magnetic dipoles, which is commonly neglected. Despite the rarely unveiled and complex nature of this SAM inhomogeneity–dependent force, it
has already ubiquitously existed in various configurations, which we summarize in table S1.

Before being focused by two cylindrical lenses, a plane wave after the quarter–wave plate always has a homogeneous $S^\text{hom}$, regardless of the different orientation angles $\theta$ (Fig. 1B). The OLF does not occur even for different amplitudes of $S^\text{hom}$, under, for instance, $\theta = \pi/8$ (fig. S5) and $\theta = \pi/4$ [left-handed circular polarization (LCP)], as well as different signs of $S^\text{hom}$ under $\theta = \pi/4$ and $\theta = 3\pi/4$ [right-handed circular polarization (RCP)]. In this case of a plane wave, only the radial pressure force works on the particle because $p_\text{S} = \frac{1}{2} \mathbf{V} \times \mathbf{S} = 0$ ($32, 33$), which induces a zero OLF, as indicated in Eq. 3. After focusing (bottom row of Fig. 1D), the inhomogeneity of $S^\text{hom}$ (Eq. 4) induces a negative OLF for $\theta = \pi/4$, and a positive OLF for $\theta = 3\pi/4$, on the particle placed at $x > 0$. Meanwhile, opposite OLFs occur for the same $\theta$ when the particle is placed at $x < 0$. As shown in Eq. 4 and Fig. 1D, $S^\text{hom}$ has components in the $x, y,$ and $z$ directions after focusing, indicating that, in general, the spin-correlated force has three components. This force in the $x$ direction depends on the inhomogeneity of the SAM in the $y$ direction and can be neglected for left and right circularly polarized beams when $w_y \gg w_x$. This is because the OLF is in the same or the opposite direction of the spin momentum $p_\text{S}$ (Eq. 3), which is proportional to the curl of the SAM, i.e., $p_\text{S} = \frac{1}{2} \mathbf{V} \times \mathbf{S}$. Therefore, the larger curl of the field gives rise to a larger spin-dependent force. The spin-dependent force in the $x$ direction when the particle is placed at $x = 0$ can be easily obtained from Eq. 3, as shown in eq. S40, showing that the spin-dependent force in the $x$ direction is much smaller than that in the $y$ direction because $p_\text{S} \gg p_\text{S}^x$, caused by $w_y \gg w_x$. The optical radiation pressure force toward the $-z$ direction is balanced by the supporting force from the substrate because the particle resides on the surface of the substrate. Consequently, we only need to consider the $y$-component force (OLF) induced by the inhomogeneity of the SAM, manifesting this configuration as a paradigm for probing the spin-dependent optical force.

**Simulation of the optical lateral force**

The mathematical derivation of the OLF (Eq. 3) is only applicable to the dipole particle with a radius $a \ll \lambda$. To assess the magnitude of the OLF for larger particles beyond the dipole limit, for instance, $a \sim \lambda$, we performed systematic numerical simulations using the Minkowski stress tensor (see Materials and Methods for simulation details) in the commercial finite element software COMSOL. To eliminate the influence of the optical gradient force in the $y$ direction, we placed the particles at the symmetric axis $y = 0$ and moved them from the region “$x < 0$” to “$x > 0$.” We simulate the OLF, $F_y$, and the force in the $x$ direction, $F_x$ (composed of the optical gradient force and the spin-correlated force in the $x$ direction), with different $w_y$ to investigate the effect of the beam expansion in the $y$ direction (Fig. 2A). The amplitude of $F_y$ rises when $w_y$ is increased from 1 to 5 $\mu$m and stops rising when $w_y$ is further increased to infinity. $w_y \rightarrow \infty$ results in $F_y \rightarrow 0$ in Eq. 2, meaning that the beam does not diverge in the $y$ direction. The beam becomes a pure plane wave when $w_y = w_x \rightarrow \infty$. The sign of the OLF turns from negative to positive when the particle moves from the region $x < 0$ to $x > 0$, coinciding with the right-handed beam, while $F_x$ attracts the particle to the central point $(x = 0)$. The maximum $F_y$ for $w_x = 500$ nm is almost 50 times larger than the maximum OLF, but this ratio can be significantly reduced when the beam is expanded in the $x$ direction (e.g., $w_y = 15$ $\mu$m).

We then consider a practical case with the parameters used in the experiment, e.g., $w_x = 15$ $\mu$m and $w_y = 500$ $\mu$m. As shown in Fig. 2B, the OLF is much smaller than that in Fig. 2A because of the decreased field gradient, even if the maximum amplitudes of intensity (at $x = 0$ and $y = 0$; see fig. S1) are the same. The left-handed ($\theta = \pi/4$) and right-handed ($\theta = 3\pi/4$) circular polarizations induce the opposite forces with the same amplitude, indicating the spin-correlated characteristics (force from the curl of $S^\text{hom}$). The sign of the OLF on a nonabsorbing particle (refractive index equals 1.56) reverses from positive to negative for $\theta = \pi/4$ when crossing from the region $x < 0$ to $x > 0$, in agreement with Eq. 3 and the plot of $p_\text{S}^m$ in Fig. 1B. Because $w_x$ and $w_y$ are 15 and 500 nm in the experiment, respectively, the gradient force in the $y$ direction is expected to be tens of times weaker than that in the $x$ direction. The effect from the gradient force in the $y$ direction can be further mitigated when the particle size is at the microscale, which can be approximately predicted from the dipole model: The optical gradient force and spin-correlated OLF are proportional to $a^2$ and $a^6$ ($a$ is the particle radius), respectively. Thus, increasing the particle radius will markedly increase the ratio of the OLF and the gradient force. Meanwhile, the increased size significantly increases the magnitude of OLF on the particle (Fig. 2C), thus reducing the influence of the Brownian motion, facilitating the observation of the OLF (61). The maximum radius used for the simulation of the OLF is 500 nm because the OLF is very sensitive to the mesh used in the simulation (see Materials and Methods for simulation details). The OLF can be continuously tuned with the orientation angles of the quarter–wave plate when the particles are placed at $x = 400$ nm, as shown in Fig. 2D. When $\theta = 0$ and $\pi/2$, the beams are linearly polarized and the corresponding OLFs are both zero. The OLFs have maximum amplitudes but opposite signs for two circularly polarized beams ($\theta = \pi/4$ and $3\pi/4$). Recently, Ginis et al. (62) found that the maxima and minima of the OLF may not necessarily be accompanied by circularly polarized beams. We should also expect this effect to potentially occur in our configuration. We fix the ellipticity angle on the Poincaré sphere to $\pi/2$ (quarter–wave plate) and only consider the orientation angle $\theta$ in this work; thus, the maximum OLF here corresponds to a circularly polarized beam. In sharp contrast to the OLF, $F_y$ remains negative with all orientation angles when the particle is at $x > 0$ (see fig. S6). Notably, the OLFs on particles larger than the dipole ($a \sim \lambda$) are calculated rigorously using the Minkowski stress tensor in COMSOL, which increase steadily when increasing the particle size rather than exponentially as indicated by $\text{Re}(e_\alpha e_\mu^\alpha)$ in the dipole model (Eq. 3), which can be expressed as (32, 63)

$$\text{Re}(e_\alpha e_\mu^\alpha) = \frac{k^2 a^8}{30} \left| \frac{e_\rho - e_\epsilon}{e_\rho + 2e_\epsilon} \right|^2 \left[ \text{Re}(e_\rho) + 2 \right]$$

(5)

where $e_p$ and $\mu_\rho$ are the permittivity and permeability of the particle, respectively. As seen from Eq. 5, $\text{Re}(e_\alpha e_\mu^\alpha) > 0$ for a nonabsorbing particle. $\text{Re}(e_\alpha e_\mu^\alpha)$ can be $< 0$ for a metallic particle, such as the gold particle, whose $\text{Re}(e_\rho) < -2$. Consequently, the OLFs on a nonabsorbing particle and a gold nanoparticle are in the opposite and the same direction of $p_\text{S}^m$, respectively, as indicated by Eqs. 3 and 5.

**Experimental demonstration of optical lateral forces**

To facilitate the observation of OLF, we illuminate the particle in a homogeneous environment (see Figs. 1A and 3) with a line-shaped ($w_x = 15$ $\mu$m and $w_y = 500$ $\mu$m) obliquely incident beam at an incident angle of 20°. The beam coming from the laser (532 nm,
power of 2 W, Laser Quantum, mpc 6000) initially passes through a 4f system (composed of two lenses with focusing lengths of 15 and 200 mm), so the beam gets expanded for a better focusing quality. Two cylindrical lenses are then used to focus the beam into a line shape. After crossing the 

\[ w_y = 5 \mu m \]

the beam remains perfect inside water, which could be observed using a charge-coupled device (CCD) camera (DS-Ri1, Nikon) to capture the image of the particle. More information about the setup can be found in Materials and Methods.

Two intriguing scenarios can be observed in the experiment (Fig. 4, A and B). Scenario 1 occurs when the particle is initially located on the left of the beam and near the beam center (\( x = 0 \)), and it experiences an optical radiation pressure force (due to the oblique incidence) and a gradient force both pointing to the +x direction (Fig. 4A). Meanwhile, the OLF deflects the particle in the +y direction. The large \( w_y (500 \mu m) \) and 5-\( \mu m \)-diameter particle size used in the experiment ensure that the OLF can dominate the movement of the particle in the \( x \) direction by weakening the effect from the optical gradient force. Although the maximum diameter of the particle used in the simulation is 1 \( \mu m \), which is limited by the large consumed memory of our computer (over 300 Gbytes), the trajectories of particles with different sizes by the OLF follow a similar trend (see Materials and Methods for simulation details).
than that in water under the same OLF because the drag force is greater in a moving velocity 10 times slower in the polyethylene glycol beams, as shown in Fig. 4 (D and E, respectively). The left-handed (spin $\sigma^-$) beam forces the particle to move in the $+y$ and $-y$ directions on the left and right sides of $x = 0$, respectively. The movement trajectory was exactly the opposite for the right-handed (spin $\sigma^+$) beam. In contrast, the trajectories in the $x$ direction for both spins followed the same trend of continuously moving particles toward the $+x$ direction until they reach the balance points of optical radiation pressure force (rightward), and gradient force (leftward).

The experimentally recorded trajectories in the $x$ and $y$ directions in scenario 1, under the illuminations of left-handed and right-handed beams, are shown in Fig. 4 (D and E, respectively). The left-handed (spin $\sigma^-$) beam forces the particle to move in the $+y$ and $-y$ directions on the left and right sides of $x = 0$, respectively. The movement trajectory was exactly the opposite for the right-handed (spin $\sigma^+$) beam. In contrast, the trajectories in the $x$ direction for both spins followed the same trend of continuously moving particles toward the $+x$ direction until they reach the balance points of optical radiation pressure force and gradient force. The $y$-direction trajectories of the 5-$\mu$m-diameter particles in scenario 2 showed a 0.5- to 1-$\mu$m displacement after 150 s (Fig. 4F), corresponding to an OLF of 2.5 to 5 fN, predicted using the equation $F_{OLF} = F_{drag} = 6\pi\eta v a$, where $F_{drag}$ is the fluidic drag force, $\eta$ is the viscosity of the polyethylene glycol medium ($\eta = 0.0161$ Pa-s), and $v$ is the velocity of the particle. The running average of 20 points is used to smooth the raw data to show the moving trend of particles more clearly (66). The viscosity of polyethylene glycol was over 10 times larger than the commonly used water ($10^{-3}$ Pa-s) in optical tweezing experiments, resulting in a moving velocity 10 times slower in the polyethylene glycol than that in water under the same OLF because the drag force increases linearly with the viscosity. The high viscosity also significantly suppresses the Brownian motion of the particle (5), preventing it from completely overwhelming the OLF and facilitating the observation of the OLF. The Brownian motion is still a non-negligible factor, which induces stochastic signals that greatly influence trajectories of particles as discussed in note S3.

The measured maximum OLF on 5-$\mu$m-diameter particles located on the right side of the beam axis ($x = 0$) under different laser powers is shown in Fig. 5A. When illuminated with left-handed and right-handed beams, particles experienced optical forces in the $-y$ and $+y$ directions, respectively. The OLF increased linearly with laser power, confirming the proposed experimental strategy. The OLF dropped to near zero when the laser power was decreased to 400 mW, due to the friction between the surface and particle, as well as the inertia of particle in the high-viscosity polyethylene glycol medium. We also varied the quarter-wave plate angle $\theta$ to investigate the OLF for different illumination polarizations. The corresponding measured OLF is shown in Fig. 5B, fitted using a sine curve (Eq. 3 and Fig. 2D), showing a relatively good agreement between the theory and experiment.

**Fig. 3. Experimental configuration.** (A) Experimental setup. M, mirror; L1, lens 1 (200 mm); L2, lens 2 (15 mm); HWP, half-wave plate (532 nm); QWP, quarter-wave plate (532 nm); CL1, cylindrical lens 1 (300 mm); CL2, cylindrical lens 2 (75 mm); AL, air lens; DM, dichroic mirror; BS, beam splitter; NF, notch filter (532 nm). (B) Illustration of a particle in a homogeneous environment. Medium: Polyethylene glycol (PEG), refractive index equals 1.46. Up and down quartz glass slides: refractive indices equal 1.46.
and experiment. The measured OLF has a period of π and maximum negative and positive values for the left-handed (θ = 45° or π/4) and right-handed (θ = 135° or 3π/4) beams, respectively, showing the exquisite capability to control the movement direction and velocity of the particle transversely, using the spin-correlated OLF in a single laser beam.

DISCUSSION

In addition to intriguing cases demonstrated here, one could also expect other fascinating OLF phenomena in the proposed experimental platform. For instance, instead of negative OLF values for particles residing on the right side of the beam with the LCP, the OLF will be positive when Re(εp) < −2 and Im(εp) → 0 (fig. S13).
case of an enhanced positive OLF, the contribution from $F_m$ is negligible compared with $F_{em}$, showing the important role of the interference between the electric and magnetic dipoles, which is often neglected in the calculation of optical forces. Meanwhile, by exciting multipoles in a high-index nanoparticle (e.g., silicon), one could also expect enhanced spin-correlated OLF (67). Note that the OLF remains negative in the entire parameter space of permittivities of the particle and the surrounding media (fig. S14), even when $\varepsilon_p < \varepsilon$, which usually changes optical gradient force from attractive to repulsive. This behavior can be seen from Eqs. 3 and 5, which show that the sign of the OLF remains constant when $\text{Im}(\varepsilon_p) = 0$ (nonabsorbing particle). Moreover, the developed approach can be applied to investigate the spin-correlated force on special particles (e.g., chiral and composite particles) in a homogeneous environment.

Our approach circumvents the existing restrictions for the generation of stable OLFs, which usually require an interface to break the symmetric light scattering, structured light fields such as interference waves and evanescent waves, or special particles such as chiral particles or helicity-sensitive structures (20). By theoretically investigating the stable OLF in a loosely focused line-shaped beam, the spin momentum was found to be the predominant factor contributing to the force on a symmetric nonabsorbing nanoparticle, while the canonical momentum can also induce a much weaker OLF and exert an extra magnetic force on a metallic nanoparticle. The sign of OLF on nonabsorbing and metallic particles could be opposite, considering their positive and negative real parts of permittivities, respectively (see Eqs. 3 and 5). The 5-μm-diameter microparticle was used in the experiment to enlarge the OLF and mitigate the influence of the Brownian motion. The studied particles were experimentally demonstrated to move bidirectionally on both sides of the beam with velocities and directions strongly dependent on the helicity of light.

Because SAM inhomogeneity–dependent systems exist ubiquitously, such as evanescent waves and standing waves (see table S1 for more configurations), one could expect additional experimental demonstrations of this particular type of force. In general, more attention should be paid to this force when investigating the optical force in a light field with SAM, especially when the field is not tightly focused. In this case, the SAM inhomogeneity–dependent force may compete with the force from the spin-orbit interaction and even the optical gradient force. The remarkable transverse spin shown here provides a degree of freedom to achieve the force from fundamental properties of light and can also find promising applications in controllable directional scattering and efficient spin-direction couplers (29, 38, 41, 48). This study opens up an avenue for using the dominant spin momentum of a simple, loosely focused single light beam for manipulating particles and optical forces, rather than systems with strong spin-orbit interactions. It also sheds light onto the development of nontrivial optofluidic approaches for optical sorting and probing.

**MATERIALS AND METHODS**

**Simulation of the light wave and optical lateral forces**

The light wave and rigorous simulation of OLF were performed using a finite element method in COMSOL v.5.4. The large particle size (5 μm) used in the experiment generates a large OLF and reduces the effect of the optical gradient force because the optical gradient force and spin-correlated OLF are, in the dipole approximation, proportional to $a^2$ and $a^3$, respectively. The large size can also mitigate the effect from the Brownian motion. The 5-μm-diameter particle needs to be placed away from the beam center to investigate OLF along the x direction; thus, the simulation requires a model dimension >5 μm, which is far beyond the computational capability of our available workstation with 384 Gbytes of RAM; therefore, the numerical simulations were performed for smaller particles. The SAM or spin-correlated OLF is very sensitive to the mesh and integral boundary surrounding the particle, especially when the particle is close to the beam center. The surrounding integral boundary should be less than 10 nm away from the particle and the maximum mesh should be less than 4 nm to obtain a correct OLF. The mesh of the particle should also be much smaller than the usual simulations. Therefore, we set the gap between the integral boundary and the particle surface to 5 nm. The maximum mesh of the space between them is set to 2 nm, and the maximum meshes of particles are set to 5, 10, 10, 20, and 20 nm for particle radii of 25, 50, 100, 250, and 500 nm, respectively. In addition, the maximum mesh of the medium is set to 0.1 μm. All these configurations consume over 300 Gbytes of computer memory. The electric field is defined using Eq. 2. The optical forces are calculated by integrating the Maxwell stress tensor in the Minkowski form over the integral surface as

$$\langle F_{\text{OLF}} \rangle = \frac{1}{8} \langle \hat{T} \cdot \hat{n} \rangle \, dA \quad (6)$$

$$\langle T_{ij} \rangle = \frac{1}{2} \left[ D_i E_j + B_j H_i - \frac{1}{2} (D \cdot E^* + B \cdot H^*) \delta_{ij} \right] \quad (7)$$

Here, the symbol $\langle \cdot \rangle$ means the time average over an oscillation period, $\hat{n}$ is the unit outward normal to the integral surface, and $\delta_{ij}$ is the Kronecker delta. The integral of the Minkowski stress tensor is directly coded in COMSOL and evaluated from the field information (Eq. 7) by solving Maxwell equations in COMSOL.

**Sample preparation and experimental setup**

A suspension of 5-μm-diameter polystyrene particles (Polysciences) was dispersed in polyethylene glycol (Sigma-Aldrich). A single small drop of the solution (~10 μl) was gently dripped from a 20-μl syringe (World Precision Instruments) onto the surface of a quartz cover slide. Subsequently, another quartz cover slide was placed on top of the medium to prevent disturbance from the external air currents and ensure a flat surface for the incident light. The refractive index of polyethylene glycol was measured to be 1.46, which was the same as the quartz cover slide. The particle inside polyethylene glycol was then considered to be in a homogeneous environment. The experimental setup (Fig. 3) consisted of external optical components connected to a microscope, which was customized to have three independently mobilized axes. Light from the argon ion laser (532 nm, power of 2 W, Laser Quantum, mpc 6000) was obliquely incident onto the sample at an angle of 20°. The beam was focused into a line-shaped light beam using two cylindrical lenses with focal lengths of 300 and 75 mm. A rotating half-wave plate and a quarter-wave plate were allowed for tuning the polarization of light. Particles were illuminated by a quasi-homogeneous white light source (Nikon) for visualization through a 10× microscope objective (NA of 0.3, Nikon). A 532-nm notch filter (Thorlabs) was used to remove most of the scattered laser light. The movements of particles were recorded by a CCD camera (DS-Ri1, Nikon) with a frame rate of ~15 frames per second.
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Stable optical lateral forces from inhomogeneities of the spin angular momentum

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