Abstract. For a given statistic, $A$, the cosmic distribution function, $\Upsilon(\tilde{A})$, is the probability of measuring a value $\tilde{A}$ in a finite galaxy catalog. For statistics related to count-in-cells, such as factorial moments, $F_k$, the average correlation function, $\xi$, and cumulants, $S_N$, the functions $\Upsilon(F_k)$, $\Upsilon(\xi)$, and $\Upsilon(S_N)$ were measured in a large $\tau$CDM simulation. This $N$-body experiment simulates almost the full “Hubble Volume” of the universe, thus, for the first time, it allowed for an accurate analysis of the cosmic distribution function, and, in particular, of its variance $(\Delta A)^2$, the cosmic error. The resulting detailed knowledge about the shape of $\Upsilon$ is crucial for likelihood analyses. The measured cosmic error agrees remarkably well with the theoretical predictions of Szapudi & Colombi (1996) and Szapudi, Bernardeau & Colombi (1998) in the weakly non-linear regime, while the predictions are slightly above the measurements in the highly nonlinear regime. When the relative cosmic error is small, $(\Delta A/A)^2 \ll 1$, function $\Upsilon$ is nearly Gaussian. When $(\Delta A/A)^2$ approaches unity or is larger, function $\Upsilon(\tilde{A})$ is increasingly skewed and well approximated by a lognormal distribution for $A = F_k$ or $A = \xi$. The measured cumulants follow accurately the perturbation theory predictions in the weakly nonlinear regime. Extended perturbation theory is an excellent approximation for all the available dynamic range.

1 Introduction

To confront theory with observations, an accurate understanding of errors is essential. The state of the art maximum likelihood approach requires full information on the distribution function of measurements; the scatter alone is insufficient if the underlying error distribution is non-Gaussian. As shown later, this is often the case in large scale structure studies. What follows, focuses on statistics related to count-in-cells (CIC), in particular, factorial moments, $A = F_k$ (e.g., Szapudi & Szalay, 1993), the averaged two point correlation function, $A = \xi$, and higher order cumulants, $A = S_N$ (e.g., Balian & Schaeffer, 1989). We are concerned with the question: what is the probability of measuring a value $A = \tilde{A}$ in a finite galaxy catalog, $V_i$, with volume $V$? The CIC indicators from a particular realization are influenced by various statistical effects (e.g., Szapudi & Colombi 1996, hereafter SC), which can be approximately separated:

1. Finite volume effects are due to fluctuations of the density field on scales larger than the catalog.
2. Edge effects are caused by the uneven statistical weight given to objects near the survey boundary.
3. Discreteness effects are related to the finite number of galaxies in the catalog, which sample the underlying continuous field.

In a large number of realizations, $1 \leq i \leq N$, the distribution of measurements, the cosmic distribution function $\Upsilon(\tilde{A})$ could be estimated. In most cases, this is unachievable in practice, but $\Upsilon(\tilde{A})$ could still be evaluated theoretically, or via simulations. An important special case is the Gaussian distribution, which is fully characterized by its average, $\langle \tilde{A} \rangle = A$, and its variance, $(\Delta A/A)^2$, the cosmic error.

This contribution concentrates on the properties $\Upsilon$, representing a fraction of the results from a more extended article by Colombi, Szapudi, et al. 1998 based on the latest $N$-body experiment of the VIRGO consor-
sortium. The next section compares the measured cumulants, $S_N$, $1 \leq N \leq 10$, with perturbation theory (PT, e.g., Juszkiewicz et al. 1993; Bernardeau, 1994) and extended perturbation theory predictions (EPT, e.g., Colombi et al. 1996). Section §3 collates the numerical cosmic error with the theoretical predictions of SC, and Szapudi, Bernardeau & Colombi (1998, hereafter SBC). Finally, Section §4 exposes the shape of cosmic distribution function $\Upsilon$.

2 The Underlying Statistics

The algorithm of Szapudi et al. (1998) was employed to extract CICs from a $\tau$CDM simulation with one billion particles in a cubic box of $2000h^{-1}$ Mpc (see the contribution of A. Evrard in the same volume). Note that most of the considerations of this work are quite insensitive to the particular cosmological model, thus similar results are expected for all currently fashionable CDM variants. The virtual (periodic) universe was divided into $C_2 = 16^3$ adjacent cubic subsamples, each of them representing a possible realization of our visible, local universe. 512 cubical sampling cells of size $\ell$ were placed in the full simulation, and in each subsample $\xi_c$, covering the scale range of $0.24h^{-1} \leq \ell \leq 2560h^{-1}$ Mpc. The combined subsamples probed the tail of the CIC probability distribution function, $P_N$, extremely well, down to $P_N \simeq 1.8 \times 10^{-12}$. The measurement in the whole sample is somewhat less accurate, but still state of the art, $P_N \gtrsim 7.5 \times 10^{-9}$.

Figure 1 shows the cumulants as functions of the variance, representing the most accurate measurement in the widest dynamic range to date. In agreement with previous studies (see, e.g., Gaztaña & Baugh, 1995, Szapudi et al. 1998), the results match PT extremely well in the weakly nonlinear regime where $\xi \lesssim 1$, and EPT is an excellent approximation in all the available dynamic range. The 1-loop calculations (not displayed on figure 1) based on spherical model by Fosalba & Gaztaña (1998) show good agreement with the numerical results up to $\xi = 1$.

3 The Cosmic Error

Figure 2 shows the measured cosmic error together with the predictions of SC and SBC. The various theoretical models agree extremely well with the measurements in the weakly nonlinear regime (at least for the $F_h$'s) and tend to be slightly higher than the numerical estimates in the nonlinear regime. EPT yields the closest match to the data. Taken at face value, these results indicate that the hierarchical assumption for the joint moments used by SC and SBC is inaccurate on the smallest scales, and should be corrected in the error calculations, where bivariate distributions play a crucial role.

It is interesting although not surprising to note that factorial moments and cumulants of the same order behave differently in terms of errors. For example, on large scales, where the cosmic error is dominated by edge effects (SC), the cumulants, $\xi^3$, and $S_3$ have larger scatter than the full moments, $F_2$, and $F_3$: the reverse is true on small scales. The error is exactly analogous to the "integral constraint" problem.

4 The Cosmic Distribution Function

On various scales as indicated on each panel, Figure 3 shows $\Upsilon(\bar{A})$ for $A = F_2$, $\xi$, $F_3$, and $S_3$ as a function of the relative difference normalized by the variance, $\delta A/\Delta A$. The measurement is compared to the Gaussian limit, the lognormal distribution (see, e.g., Coles & Jones 1991), and a generalized version of it:

$$\Upsilon(\bar{A}) = \frac{s}{\Delta A x \sqrt{2\pi \eta}} \times \exp \left[ -\frac{(\ln x - \eta/2)^2}{2\eta} \right], \quad (1)$$

with $x = s(\bar{A} - A)/\Delta A + 1$, and $\eta = \ln(1 + s^2)$. The adjustable parameter $s$ is chosen such that the probability distribution function $\Upsilon(\bar{A})$ based

\footnote{The box size was conveniently chosen for the compressed output format provided by Adrian Jenkins to speed up processing.}
Figure 1. The cumulants, $S_N$, measured in the whole $\tau$CDM simulation are shown as functions of the variance $\xi$. The dots, long dashes and short dashes correspond to PT with only terms corresponding to the first order logarithmic derivative of $\xi$, PT with only terms up to the second order logarithmic derivative of $\xi$, and EPT (see, e.g., Bernardeau 1994 and Colombi et al. 1996), respectively. The lower, and upper panels correspond to $3 \leq N \leq 5$, and to $6 \leq N \leq 10$, respectively. The $S_N$’s are increasing with $N$. Finally, the filled, and open symbols correspond to the results from the full simulation, and from the combination of all the subsamples $E_i$, respectively.

has same average, variance and skewness $S = s^3 + 3s$ than the measured $\Upsilon(\tilde{A})$. Of the three choices, function (1) was designed to yield the best fit to the measurements (and it does!) with its three adjustable parameters. Moreover, this approximation also appears to be sufficiently accurate to describe the shape of $\Upsilon(\tilde{A})$, especially for the large $\tilde{A}$ tail. Therefore a third order theory is necessary to determine the shape of $\Upsilon(\tilde{A})$ in the studied dynamic range.

The amount of skewness of the dotted curves in Figure 2 is an indicator of the magnitude of the cosmic error, since the skewness of the lognormal distribution is $S = (\Delta A/A)^2 + 3\Delta A/A$. As we see, function $\Upsilon(\tilde{A})$ is nearly Gaussian when the cosmic error is small, and in general becomes increasingly skewed with $\Delta A/A$. For the factorial moments and $\xi$, the cosmic distribution function is nearly lognormal, thus for these estimators the theory of SC, and SBC is sufficient to characterize the shape of the distribution function.

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Figure 2. The relative cosmic error as a function of scale for the factorial moments (left panels), $\xi$, and the cumulants (right panels) are shown. The symbols represent the cosmic scatter obtained from the measurements in $E_i$, $i = 1, \ldots, 16^3$. The dots, dashes and long dashes display the theoretical predictions, based on the hierarchical models of Szapudi & Szalay (1993), Bernardeau & Schaeffer (1992), and EPT (SBC), respectively. Note that when the cosmic error approaches unity or, is larger, the theoretical calculation is not expected to be valid in the right panels, as it relies on a Taylor expansion in terms of the relative error.
Figure 3. The cosmic distribution function for $F_2$, $\xi$ (6 upper panels), $F_3$ and $S_3$ (6 lower panels) are shown. Three scales are considered in each case, indicated in the upper right corner of each panel, understood in $h^{-1}$ Mpc. The symbols represent the measurements. The errorbars correspond to the measurement error due to the fact we use $C_E = 16^3$ samples as realizations of our local universe. On each panel, the solid curve, the dots, and the dashes corresponds to Gaussian, lognormal and extended lognormal as discussed in the text [equation (1)].