Systematic investigation on the stability of doubly heavy tetraquark states

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We systematically investigate the stability of the doubly heavy tetraquark states $[QQ][qq]$ ($Q = c$ and $b$, $q = u$, $d$, and $s$) within the framework of the color flux-tube model involving a multibody confinement potential, $\sigma$-exchange, one-gluon-exchange and one-Goldstone-boson-exchange interactions. Our numerical analysis indicates that the states $[bb][ud]$ with $0^+$ and $[bb][us]$ with $1^+$ are the most promising stable states against strong interactions. The states $[cc][ud]$ with $0^+$, $[bc][ud]$ with $0^0$, $0^+$, and $1^+$, and $[bb][ud]$ with $0^+$ and $1^+$ as stable states are also predicted in the color flux-tube model. The dynamical mechanism producing those stable doubly heavy tetraquark states are discussed in the color flux-tube model.

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I. INTRODUCTION

Searching for exotic hadrons beyond conventional $q\bar{q}$-meson and $qqq$-baryon pictures is an extremely meaningful topic in hadronic physics because they may contain more abundant low-energy strong interaction information than ordinary hadrons. A large amount of new hadron states have been observed since the BELLE collaboration’s discovery of the charmonium-associated state $X(3872)$ in 2003 \cite{bib1}. Many of new hadron states can not be accommodated in the conventional $Q\bar{Q}$-meson framework, such as the charged $Z_c^\pm$ states, which must have a smallest quark component $ccud$ due to carrying one unit charge. Tetraquark states $Q\bar{Q}qq$ have therefore attracted much attention from theoretical physicists to describe the internal structure of new hadron states \cite{bib1}. Most of new hadron states can be accommodated in the picture of tetraquark states just matching their experimental data with model values of $Q\bar{Q}qq$ \cite{bib2}. Even so, none of new hadron states is lower than its threshold up to now and can therefore decay into $QQ$ and $q\bar{q}$ via strong interactions \cite{bib2}.

The theoretical explorations on the question whether or not the doubly heavy tetraquark states $[QQ][q\bar{q}]$ or $[QQ][qq]$ can exist as stable states against breakup into two $Q\bar{q}$ mesons was pioneered in the early 1980s \cite{bib3}. From then on, a lot of attention has been payed to the states in various phenomenological methods, such as the MIT bag model \cite{bib4}, constituent quark model \cite{bib5}, chiral perturbation theory \cite{bib6}, string model \cite{bib7}, lattice QCD \cite{bib8}, and QCD sum rule approach \cite{bib9}. A large amount of researches indicated that the state $[bb][ud]$ with $0^+$ is stable against strong interactions in various theoretical framework. However, the state has not been determined because of a lack of experimental information about the strength of the interaction between two heavy quarks.

The discovery of the doubly charmed baryon $\Xi_{cc}$ by the LHCb Collaboration at CERN has provided the crucial experimental input which allows this issue to be finally resolved \cite{bib10}. Subsequently, the enthusiasms of the theoretical investigation on the doubly heavy tetraquark states $[QQ][q\bar{q}]$ are stimulated again to search for possible stable tetraquark states \cite{bib11,bib12,bib13}. Undoubtedly, the stability of the doubly heavy tetraquark states are model dependent, see Table II. More investigations from the different theoretical point of view should therefore be very necessary to present a comprehensive understanding on the properties of the states, which must be benefit to the future experimental searches for stable doubly heavy tetraquark states.

Recently, we have developed a color flux-tube model (CFTM) based on the lattice QCD picture and the traditional quark models \cite{bib22,bib23}. The most salient feature of this model is a multibody confinement potential instead of a two-body one proportional to the color charge in the traditional quark models \cite{bib24}. We systematically investigate the hidden charmed states observed in recent years within the framework of the CFTM. It can be found that many of hidden charmed states as compact tetraquark states can be accommodated in the CFTM, especially the charged tetraquarks $Z_c^\pm$. The multibody color flux-tube dynamical mechanism is seem to be propitious to describe multiquark states from the phenomenological point of view \cite{bib2}. We have therefore a great ambition to research the properties of the doubly heavy tetraquark states under the hypothesis of diquark-antidiquark picture within the framework of CFTM. We concentrate more on the mass spectrum of doubly heavy tetraquark states than on their decay properties and we investigate the dynamical mechanism bunching quarks together and affecting their binding energy. This work is attempted to broaden the theoretical horizon in the properties of the doubly heavy tetraquark states and may provide some valuable clues to the experimental establishment of the tetraquark states in the future.

This paper is organized as follows. After the introduction section, the introduction of the CFTM is briefly given in Sec. II. The construction of the wavefunctions
of the doubly heavy tetraquark states are shown in Sec. III. The numerical results and discussions of the stable doubly heavy tetraquark states are presented in Sec. IV. The last section is devoted to list a brief summary.

II. THE COLOR FLUX-TUBE MODEL

This model assumption is inspired by the lattice QCD picture and the traditional constituent quark models, which includes one-gluon-exchange potential $V_{ij}^B$, one-Goldstone-boson-exchange potential $V_{ij}^B$, σ-meson exchange potential $V_{ij}^C$ and a multibody confinement potential $V_{min}^C$. More details of the CFTM can be found in our previous paper [2]. The model Hamiltonian for the doubly heavy tetraquark states used here is presented as follows,

$$
H_4 = \sum_{i=1}^{4} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_C + \sum_{i>j}^{4} V_{ij} + V_{min}^C,
$$

$$
V_{ij} = V_{ij}^G + V_{ij}^B + V_{ij}^\sigma. \tag{1}
$$

The codes of the quarks (antiquarks) in the doubly heavy tetraquark states $[Q\bar{Q}][q\bar{q}]$ are orderly assumed to be 1, 2, 3 and 4, respectively. Their positions are denoted as $r_1, r_2, r_3$ and $r_4$. $T_C$ is the center-of-mass kinetic energy of the states and should be deducted; $p_i$ and $m_i$ are the momentum and mass of the $i$-th quark (antiquark), respectively.

According to double Y-shaped color flux-tube structures of the tetraquark state with diquark-antiquark configuration, a four-body quadratic confinement potential instead of linear one used in the lattice QCD can be written as,

$$
V^C = K \left[ (r_1 - y_{12})^2 + (r_2 - y_{12})^2 + (r_3 - y_{34})^2 \right. \\
+ \left. (r_4 - y_{34})^2 + \kappa_d (y_{12} - y_{34})^2 \right], \tag{2}
$$

Analytical formulas illustrating the differences between linear and quadratic confinement potentials can be found in Ref. [23]. However, the numerical comparison studies between two confinement potentials showed that the inaccuracy of this replacement is quite small [24, 27].

Two Y-shaped junctions $y_{12}$ and $y_{34}$ are variational parameters, which can be determined by taking he minimum of the confinement potential. The parameters $K$ and $\kappa_dK$ are the stiffnesses of a three-dimension and $d$-dimension color flux-tube, respectively. The relative stiffness parameter $\kappa_d$ is equal to $\frac{C_d}{C_3}$, where $C_d$ is the eigenvalue of the Casimir operator associated with the SU(3) color representation $d$ at either end of the color flux-tube, such as $C_3 = \frac{2}{3}$, $C_6 = \frac{10}{3}$, and $C_8 = 3$.

The minimum of the confinement potential $V_{min}^C$ can be obtained by taking the variation of $V^C$ with respect to $y_{12}$ and $y_{34}$, and it can be expressed as

$$
V_{min}^C = K \left[ R_1^2 + R_2^2 + \frac{\kappa_d}{1 + \kappa_d} R_3^2 \right], \tag{3}
$$

The canonical coordinates $R_i$ have the following forms,

$$
R_1 = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad R_2 = \frac{1}{\sqrt{2}}(r_3 - r_4), \quad R_3 = \frac{1}{\sqrt{4}}(r_1 + r_2 - r_3 - r_4), \quad R_4 = \frac{1}{\sqrt{4}}(r_1 + r_2 + r_3 + r_4). \tag{4}
$$

The use of $V_{min}^C$ can be understood here as that the gluon field readjusts immediately to its minimal configuration. The interactions $V_{ij}^B$ and $V_{ij}^\sigma$ only occur between light quarks, while the interaction $V_{ij}^G$ is universal. They take their standard forms and are listed in the following,

$$
V_{ij}^B = \frac{g_{ch}^2 m_3^3}{4\pi 12m_i m_j} \frac{\Lambda_3^2 - m_3^2}{\Lambda_3^2} \sigma_{ij}, \tag{5}
$$

$$
V_{ij}^\sigma = \frac{\alpha_s}{4} \frac{\lambda^i_j \cdot \lambda^{ij}}{m_j^2} \left( 1 - \frac{2\pi \delta(r_{ij}) \sigma_{ij} \cdot \sigma_j}{3m_i m_j} \right), \tag{6}
$$

$$
V_{ij} = -\frac{g_{ch}^2}{4\pi} \frac{\alpha_s}{\Lambda_2^2 - m_\sigma^2} \left( Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right). \tag{7}
$$

where $\chi$ stands for $\pi$, $K$ and $\eta$. $Y(x) = e^{-x}/x$. $\alpha_s$ is the running strong coupling constant and takes the following form [29],

$$
\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln \left( (\mu_{ij}^2 + \mu_0^2)/\Lambda_0^2 \right)}, \tag{8}
$$

where $\mu_{ij}$ is the reduced mass of two interacting particles. The function $\delta(r_{ij})$ in $V_{ij}^G$ should be regularized [30],

$$
\delta(r_{ij}) = \frac{1}{4\pi r_{ij}^2 (\mu_{ij})} e^{-r_{ij}/\tilde{r}_0(\mu_{ij})}, \tag{9}
$$

where $\tilde{r}_0(\mu_{ij}) = \tilde{r}_0/\mu_{ij}$. $\Lambda_0$, $\alpha_0$, $\mu_0$ and $\tilde{r}_0$ are adjustable model parameters.

The starting point of the model study on the multi-quark states is to accommodate ordinary hadrons in the model in order to fix model parameters. The mass parameters $m_\sigma$, $m_\chi$ and $m_\eta$ in the interaction $V_{ij}^\sigma$ take their experimental values. The cutoff parameters and the mixing angle $\theta_P$ take the values in the work [27]. The mass parameter $m_\sigma$ in the interaction $V_{ij}^G$ can be determined through the PCAC relation $m_\sigma^2 \approx m_3^2 + 4 m_{u,d}^2$ [31]. The chiral coupling constant $g_{ch}$ can be obtained from the $\pi NN$ coupling constant through

$$
g_{ch}^2 = \left( \frac{3}{5} \right) \frac{2 g_{NNN}^2 m_{u,d}^2}{4 \pi m_N}. \tag{10}
$$
All model parameters in the CFTM and the mass spectra of the ground states of mesons obtained by fitting the experimental data were presented in Ref. [2]. Once the meson masses are obtained, one can calculate the threshold of the doubly heavy tetraquark states \([QQ][q\bar{q}]\) simply by adding the masses of two \(Q\bar{q}\) mesons to identify the stability of the tetraquark states against strong interaction.

### III. WAVEFUNCTIONS OF THE DOUBLY HEAVY HEAVY STATES

The properties of the doubly heavy tetraquark states can be obtained using a complete wavefunction which includes all possible flavor-spin-color-spatial channels that contribute to a given well defined parity, isospin, and total angular momentum. Within the framework of the diquark-antidiquark configuration, the trial wavefunction of the doubly heavy tetraquark state \([QQ][q\bar{q}]\) can be constructed as a sum of the following direct products of color, isospin \(\eta_i\), spin \(\chi_s\) and spatial \(\phi\) terms

\[
\Phi_{I M_1 J M_J}^{[QQ][q\bar{q}]} = \sum_\alpha \xi_\alpha \left[ \left( \left[ \phi_{I M_1}^{[QQ]}(R) \chi_{s_{M_1}} \right]_{J_{M_1} a b} \right) \chi_{s_{M_J}} \chi_{s_{M_J}} \right]_{J M_J}^{[QQ][q\bar{q}]} \times \\
\times \left[ \eta_{a_{M_1}} \eta_{b_{M_J}} \right]_{I M_1 J M_J} \chi_{s_{M_1}} \chi_{s_{M_J}} \chi_{s_{M_J}} \chi_{s_{M_J}}
\]

The subscripts \(a\) and \(b\) in the intermediate quantum numbers represent the diquark \([QQ]\) and antidiquark \([q\bar{q}]\), respectively. The summing index \(\alpha\) stands for all possible flavor-spin-color-spatial intermediate quantum numbers.

The relative spatial coordinates \(r\), \(R\) and \(X\) and center of mass \(R_c\) in the tetraquark state \([QQ][q\bar{q}]\) can be defined as,

\[
\begin{align*}
    r &= r_1 - r_2, \\
    R &= r_3 - r_4, \\
    X &= \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4}, \\
    R_c &= \frac{m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4}{m_1 + m_2 + m_3 + m_4}.
\end{align*}
\]

The corresponding angular excitations of three relative motions are, respectively, \(l_a\), \(l_b\) and \(l_c\). The parity of the doubly heavy tetraquark states \([QQ][q\bar{q}]\) can therefore be expressed in terms of the relative orbital angular momentum associated with the Jacobi coordinates as \(P = (-1)^{l_a + l_b + l_c}\). It is worth mentioning that this set of coordinate is only a possible choice of many coordinates and however most propitious to describe the correlation of two quarks (antiquark) in the diquark (antidiquark) and construct the symmetry of identical particles. In order to obtain a reliable solution of few-body problem, a high precision numerical method is indispensable. The Gaussian Expansion Method (GEM) [32], which has been proven to be rather powerful to solve few-body problem, is therefore used to study four-quark systems in present work. According to the GEM, any relative motion wave function can be written as,

\[
\phi_{I M_1 J M_J}^G(z) = \sum_{n=1}^{n_{max}} c_n N_m z^l e^{-\nu_n z^2} Y_{lm}(\mathbf{z})
\]

More details of the relative motion wave functions can be found in the paper [32].

The color representation of the diquark maybe antisymmetrical \([QQ]_{\bar{3}}\), or symmetrical \([QQ]_6\), whereas that of the antidiquark maybe antisymmetrical \([\bar{q}\bar{q}]_{\bar{3}}\), or symmetrical \([\bar{q}\bar{q}]_6\). Coupling the diquark and the antidiquark into an overall color singlet according to color coupling rule have two ways: \([([QQ]_{\bar{3}})[\bar{q}\bar{q}]_6]_1\) (good diquark) and \([[QQ]_6[\bar{q}\bar{q}]_{\bar{3}}]_1\) (bad diquark). In general, the interaction in the good diquark is attractive, whereas the interaction in the bad diquark is repulsive. Anyway, a real physical state should be their mixture because of the coupling between two color configurations.

The spin of the diquark \([QQ]\) is coupled to \(s_a\) and that of the antiquarks \([q\bar{q}]\) to \(s_b\). The total spin wave function of the doubly heavy tetraquark state \([QQ][q\bar{q}]\) can be written as \(S = s_a \oplus s_b\). Then we have the following basis vectors as a function of the total spin \(S\).

\[
S = \begin{cases} 
    0 = 1 \oplus 1 & 
    1 \oplus 0 \\
    1 = 1 \oplus 1, 1 \oplus 0, & 
    0 \oplus 1 \\
    2 = 1 \oplus 1
\end{cases}
\]

With respect to the flavor wavefunction, we only consider \(SU_f(2)\) symmetry in the present work. The quarks, \(s, c\) and \(b\), have isospin zero so they do not contribute to the total isospin. The flavor wave functions of the antidiquark consisting of \(\bar{u}\) and \(d\) quarks are similar to those of spin.

Taking all degrees of freedom of identical particles in the diquark (antidiquark) into account, the Pauli principle must be satisfied by imposing restrictions on the quantum numbers of the basis states. Such as the color-antisymmetrical tetraquark state \([cc\bar{Q}\bar{q}]_{\bar{3}}\), the quantum numbers must satisfy the relations \((-1)^{l_a + l_b + s_a} = 1\) and \((-1)^{l_b + l_c + s_b} = 1\). But for the color-symmetrical tetraquark state \([cc]_6[\bar{u}\bar{d}]_{\bar{6}}\), the quantum numbers must satisfy the relations \((-1)^{l_a + l_b + s_a} = 1\) and \((-1)^{l_b + l_c + s_b} = -1\). On the contrary, the situation of non-identical particles is extremely simple because of no any restrictions.

### IV. NUMERICAL RESULTS AND ANALYSIS

The converged numerical results of the doubly heavy tetraquark states \([QQ][q\bar{q}]\) within the framework of the CFTM can be obtained through solving the four-body Schrödinger equation with the Rayleigh-Ritz variational principle.

\[
(\mathbf{H}_4 - E_4)\psi_{I M_1 J M_J}^{[QQ][q\bar{q}]} = 0.
\]
A tetraquark state should be stable against strong interaction if its energy lies below all possible two-meson thresholds. We express the lowest threshold of the doubly heavy tetraquark \([Q\bar{Q}][q\bar{q}]\) as \(T_{\text{min}}^{M_1 M_2}\), where \(M_1\) and \(M_2\) stand for two \(Q\bar{Q}\) mesons. The binding energy of the doubly heavy tetraquark states can be therefore defined as

\[
E_b = E_A - T_{\text{min}}^{M_1 M_2} \quad (14)
\]

to identify whether or not a tetraquark state is stable against strong interactions. If \(E_b \geq 0\), the tetraquark state can fall apart into two mesons. If \(E_b < 0\), the strong decay into two mesons is forbidden and therefore the decay must be weak or electromagnetic interaction.

| Flavor | \(I^J\) | \(n^{\pi^+}\) | \(L_J\) | Masses | \(T_{\text{min}}^{M_1 M_2}\) | \(E_b\) |
|--------|--------|-----------|--------|--------|------------------|--------|
| \([cc][\bar{u}\bar{d}]\) | 01\(^*\) | 0\(^*\)\(S_1\) | 3719 ± 12 | \(DD^*\) | 197 | 150 |
| 01\(^*\) | 0\(^*\)\(P_1\) | 3931 ± 12 | \(DD\) | 228 | |
| \([bc][\bar{u}\bar{d}]\) | 01\(^*\) | 0\(^*\)\(S_0\) | 10 \pm 1 | \(DD\) | 187 | |
| 01\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(DD^*\) | 148 | |
| \([bb][\bar{u}\bar{d}]\) | 00\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(D^* B^*\) | 9 | |
| 00\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(D^* B^*\) | 9 | |
| \([cc][\bar{u}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(DD_s\) | 282 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(D^* D_s\) | 94 | |
| \([bc][\bar{u}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(D^* B_s\) | 35 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(D^* B_s\) | 35 | |
| \([bb][\bar{u}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(BB_s\) | 108 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(BB_s\) | 108 | |
| \([cc][\bar{s}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(D\bar{D}_s\) | 335 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(D\bar{D}_s\) | 335 | |
| \([bc][\bar{s}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(B\bar{B}_s\) | 232 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(B\bar{B}_s\) | 232 | |
| \([bb][\bar{s}\bar{s}]\) | 0\(^*\) | 0\(^*\)\(S_0\) | 11 \pm 7 | \(B\bar{B}_s\) | 68 | |
| 0\(^*\) | 0\(^*\)\(S_1\) | 4017 ± 7 | \(B\bar{B}_s\) | 68 | |

In the following, we discuss the properties of the doubly heavy tetraquark states \([Q\bar{Q}][q\bar{q}]\) to search for all possible stable states against strong interactions in the CFTM. In order to obtain the lowest states with positive parity, we assume that three relative motions are in a relative S-wave in the doubly heavy states. In the case of the lowest states with negative parity, we assume that the angular excitation of the relative motion occur in not \(l_b\) and \(l_c\), but \(l_a\), namely \(l_a = 1\), \(l_b = l_c = 0\).

The binding energy of the doubly heavy tetraquark states can be therefore defined as

\[
E_b = E_A - T_{\text{min}}^{M_1 M_2} \quad (14)
\]

to identify whether or not a tetraquark state is stable against strong interactions. If \(E_b \geq 0\), the tetraquark state can fall apart into two mesons. If \(E_b < 0\), the strong decay into two mesons is forbidden and therefore the decay must be weak or electromagnetic interaction.

The binding energies of the doubly heavy tetraquark states within various theoretical methods are presented in Table II, in which "..." represents that the corresponding state was not researched by authors. It is extremely obvious that the state \([bb][\bar{u}\bar{d}]\) with \(01^+\) has a distinguished strong binding, above 100 MeV, in the absolutely majority of work and must therefore be the most promising stable doubly heavy tetraquark state against dissociation into two heavy-light mesons via strong interaction. Its strange partner, \([bb][\bar{u}\bar{s}]\) with \(\frac{1}{2}1^+\), also has a binding energy from a few to dozens of MeV in all of the investigations with exception of Ebert's work \([33]\), which lies slightly, about 13 MeV, above the \(B^* B_s\) threshold. In this way, the state \([bb][\bar{u}\bar{s}]\) with \(\frac{1}{2}1^+\) stands a good chance of existence as a bound state. It is strongly suggested that the two extremely possible stable states against strong interactions should be explored in experiments in the near future.

In addition to the two doubly heavy tetraquark states \([bb][\bar{u}\bar{d}]\) with \(01^+\) and \([bb][\bar{u}\bar{s}]\) with \(\frac{1}{2}1^+\), the existence of other states in Table II as stable states against strong interactions are obviously model dependent. The state \([cc][\bar{u}\bar{d}]\) with \(01^+\) lies below, greater than 100 MeV, the threshold \(DD^*\) only in the CFTM and the chiral quark models \([34,35]\). Other results on the state are higher than the threshold \(DD^*\). In the case of the states \([bc][\bar{u}\bar{d}]\) with \(00^+\), \(01^+\) and \(12^+\), Sakai et al described them as \(D^{(*)}B^{(*)}\) molecule states with binding energies about 20–60 MeV \([36]\). QCD sum rule research indicated that the extracted masses for both the scalar and axial vector \([bc][\bar{q}\bar{q}]\) tetraquark states are also below the open-flavor thresholds \(DB\) and \(DB^*\). Lattice QCD study shown the existence of a strong-interaction-stable tetraquark \([bb][\bar{u}\bar{d}]\) with \(01^+\) below \(DB^*\) threshold in the range of 15 to 61 MeV \([20]\). In the CFTM, the states \([bc][\bar{u}\bar{d}]\) with \(00^+\) and \(01^+\) can be depicted as deeply bound states with binding energies 136 MeV and 171 MeV, respectively. The state \([bc][\bar{u}\bar{d}]\) with \(12^+\) as a bound state have...
TABLE II: The stable doubly heavy tetraquark states \([QQ][q\bar{q}]\) in various methods, two results in Ref. \([33]\) for two different sets of parameters \(C_1\) and \(C_2\), unit in MeV.

| States Flavor | \(I_J^P\) | \(T^\text{inv}_{13}\) | Ours \(\text{CTFM}\) | Others \(\text{CTFM}\) |
|--------------|-----------|----------------|----------------------|----------------------|
| \([cc][ud]\) | 01\(^+\)  | \(-150\)       | 7                    | 64                   |
| \([bc][ud]\) | 00\(^+\)  | \(-136\)       | -11                  | 95                   |
| \([bc][ud]\) | 01\(^+\)  | \(-171\)       |                      | 56                   |
| \([bc][ud]\) | 12\(^+\)  | \(-4\)         |                      | 90                   |
| \([bb][ud]\) | 01\(^+\)  | \(-278\)       | -215                 | -212                 |
| \([bb][ud]\) | 12\(^+\)  | \(-30\)        |                      | 23                   |
| \([bb][us]\) | \(\frac{1}{2}\) \(^+\) | -49        | -48                 | -7                   |
| \([bb][ud]\) | 01\(^-\)  | -114           |                      | 13                   |

TABLE III: The energies of all stable states with the color configurations \([QQ][q\bar{q}][s\bar{s}]_1\) and \([QQ][q\bar{q}][a\bar{a}]_1\) in the CFTM, unit in MeV.

| Flavor | \(I_J^P\) | \(3. \otimes 3\) | \(6. \otimes 6\) | Coupling |
|--------|-----------|-----------------|-----------------|----------|
| \([cc][ud]\) | 01\(^+\) | 3731 ± 12       | 4007 ± 8        | 3719 ± 12|
| \([bc][ud]\) | 00\(^+\) | 6996 ± 12       | 7262 ± 8        | 6990 ± 12|
| \([bc][ud]\) | 01\(^+\) | 7003 ± 12       | 7304 ± 7        | 6997 ± 12|
| \([bc][ud]\) | 12\(^+\) | 7299 ± 7        |                  | 7299 ± 7 |
| \([bb][ud]\) | 01\(^+\) | 10283 ± 12      | 10583 ± 8       | 10282 ± 12|
| \([bb][ud]\) | 12\(^+\) | 10572 ± 7       |                  | 10572 ± 7 |
| \([bb][us]\) | \(\frac{1}{2}\) \(^+\) | 10629 ± 9    |                  | 10629 ± 9 |
| \([bb][ud]\) | 01\(^-\) | 10404 ± 12      | 10847 ± 7       | 10404 ± 12|

TABLE IV: The average distance \(r_{ij}^2\) between the \(i\)-th and \(j\)-th particle in the stable states, unit in fm.

| Flavor | \(I_J^P\) | \(E_b\) | \(r_{12}^2\) | \(r_{34}^2\) | \(r_{56}^2\) | \(r_{13}^2\) | \(r_{14}^2\) | \(r_{15}^2\) | \(r_{16}^2\) |
|--------|-----------|---------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \([cc][ud]\) | 01\(^+\) | -150 | 0.65 | 0.78 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 |
| \([bc][ud]\) | 00\(^+\) | -136 | 0.53 | 0.78 | 0.91 | 0.83 | 0.83 | 0.83 | 0.83 |
| \([bc][ud]\) | 01\(^+\) | -171 | 0.55 | 0.78 | 0.92 | 0.84 | 0.84 | 0.84 | 0.84 |
| \([bc][ud]\) | 12\(^+\) | -4 | 0.56 | 1.13 | 1.06 | 0.98 | 0.98 | 0.98 | 0.98 |
| \([bb][ud]\) | 01\(^+\) | -278 | 0.42 | 0.77 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 |
| \([bb][ud]\) | 12\(^+\) | -30 | 0.42 | 1.13 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| \([bb][us]\) | \(\frac{1}{2}\) \(^+\) | -49 | 0.42 | 0.89 | 0.90 | 0.76 | 0.76 | 0.90 |
| \([bb][ud]\) | 01\(^-\) | -114 | 0.65 | 0.77 | 0.89 | 0.89 | 0.89 | 0.89 | 0.89 |

a slight binding, about 4 MeV. In this way, our conclusion on three heavy states \([bc][ud]\) is qualitatively consistent with that of Sakai. Karliner also predicted that the state \([bc][ud]\) with 00\(^+\) lies below the threshold \(DB\) about 11 MeV \([11]\). The heavy state \([bb][ud]\), the partner of \([bc][ud]\) with 12\(^+\), can exist as a stable state with binding energy about 30 MeV, which is not supported by existing results on the doubly heavy tetraquark states so far. With respect to the state \([bb][ud]\) with 1\(^-\), the CFTM predicts that it lies below the \(BB\) threshold about 114 MeV. The energy of this state is higher, just 1 MeV, than the threshold in Ref. \([38]\). Very recently, Pfleumner et al predicted the doubly heavy tetraquark state \([bb][ud]\) with 1\(^-\) as a resonance higher 17 MeV than the \(BB\) threshold applying lattice QCD potentials \([32]\). In general, the heavy states \([QQ][ud]\) with \(I = 0\) are easier than the states with \(I = 1\) to form bound states in the CFTM.

All possible stable doubly heavy tetraquark states should be, in general, the admixture of the two color configurations \([QQ][q\bar{q}][s\bar{s}]_1\) and \([QQ][q\bar{q}][a\bar{a}]_1\) under the diquark-antidiquark picture as a working hypothesis. Theoretically, the magnitude of their mixing through color-magnetic interaction is governed by the order of \(1/m^2_{q\bar{q}}\). Special attention is therefore payed to the role quantitatively played by the two color configurations in the CFTM. The energies of all possible stable doubly heavy tetraquark states with the two color configurations and their coupling results are given in Table III. It can be found that the configuration \([QQ][q\bar{q}][s\bar{s}]_1\) dominates the energy of the doubly heavy tetraquark states. The mixing process pushes the energy of the states down a little comparing with that of the configuration \([QQ][q\bar{q}][a\bar{a}]_1\). The heavier the heavy quark pair \([QQ]\), the stronger the effect. For the \([cc]\) and \([bc]\) sections, it is just more than ten MeV and several MeV, respectively. In the \([bb]\) section, the color configuration \([bb][a\bar{a}][q\bar{q}]_1\) has almost no any effect on the masses of the states and can be ignored in the CFTM. Therefore, the color configuration \([QQ][q\bar{q}][a\bar{a}]_1\) absolutely dominates the behavior of the doubly heavy tetraquark states in the course of investigation on their properties \([34, 40]\). However, the color configuration \([q\bar{q}][a\bar{a}][q\bar{q}]_1\) must be taken into account in the researches on the light tetraquark states \([34]\).

Regarding to the stable doubly heavy tetraquark states \([bb][a\bar{a}][s\bar{s}]_1\) and \([cc][a\bar{a}][a\bar{a}]_1\) with 01\(^+\), the two heavy diquarks \([bb][a\bar{a}]\) and \([cc][a\bar{a}]\) must have spin one because the flavor and orbit are symmetrical, the color-spin-orbit-isospin combination is \((c_a, s_a, l_a, i_a) = (3, 1, 0, 0)\). The antidiquark \([a\bar{a}]\) couples into spin and isospin zero, the color-spin-orbit-isospin combination is \((c_a, s_a, l_a, i_a) = (3, 0, 0, 0)\). For the heavy diquarks \([bb][a\bar{a}]\) and \([cc][a\bar{a}]\), the color-magnetic interaction is therefore weak repulsive. However, the large masses admit two heavy quarks to approach each other as short as possible because kinetic energy is inversely proportional to the quark mass. Meanwhile, the Coulomb interaction is attractive in the diquark \([QQ]\). The heavier the heavy quark, the stronger the Coulomb interaction, the shorter the distance, see Table IV. In the limit of heavy quark, the diquark \([QQ]\) gradually shrink into a pointlike particle, which is qualitatively consistently with the conclusion in Quigg's
work \textsuperscript{17} with the exception of attractive Coulomb interaction, there exists strong attractive interactions in the antidiquark \([\bar{u}d]_{3}\) with spin and isospin zero generated through one-Goldstone-boson-exchange (mainly \(\pi\)) and color-magnetic interaction. This conclusion is also hold for the state \([bc]_{3}[\bar{u}d]_{3}\) with \(I^J=1^+\). In this way, the interaction in the doubly heavy states \([QQ][\bar{u}d]\) with \(I^J=1^+\) become strong gradually with the increase of the mass ratio \(\frac{m_{Q}}{m_{\pi}}\), which was pointed out by many investigations on natures of the doubly heavy tetraquark states and is strengthen again by the present work \textsuperscript{3,29,38,41}.

The states \([bc]_{3}[\bar{u}d]_{3}\) with \(I^J=0^+\) is allowed because of no symmetry restriction on the diquark \([bc]\). In the contrary to the diquarks \([bb]\) and \([cc]\), the color-magnetic interaction in the diquark \([bc]\) is weak attractive due to its spin \(s_b=0\). Therefore, the energy of the state \([bc]_{3}[\bar{u}d]_{3}\) with \(I^J=0^+\) is 7 MeV lower than that of the state \([bc]_{3}[\bar{u}d]_{3}\) with \(I^J=1^+\). With respect to the state \([bb]_{3}[\bar{u}d]_{3}\) with \(I^J=1^+\), which involves one angular excitation allowed to occur between two \(b\)-quarks because of their large masses. Meanwhile, the diquark \([bb]\) has spin zero so that the color-magnetic interaction is weak attractive. Therefore, the state with \(I^J=0^+\) has a lower mass than that of other states with negative parity.

In one word, there exists strong attractive interactions coming from the Coulomb interaction, the color-magnetic interaction and one Goldstone boson exchange (mainly \(\pi\)), which are more than 200 MeV, in the stable doubly heavy tetraquark states \([QQ]_{3}[\bar{u}d]_{3}\) with \(I^J=0^+\), see Table V. Lattice QCD simulations on the doubly heavy tetraquark states \([QQ][\bar{u}d]\) also indicated that the phase shifts in the isospin singlet channels suggest attractive interactions growing as \(m_{Q}\) decreases \textsuperscript{12}. The state \([bb]_{3}[\bar{u}\bar{s}3]_{c}\) with \(I^J=1^+\) is analogous to the state \([bb]_{3}[\bar{u}d]_{3}\) with \(I^J=1^+\) with the exception of the one Goldstone boson exchange (\(K\) and \(\eta\)). The magnitude of the attractive interaction is weaker than the state \(I^J=1^+\) because of no \(\pi\)-meson exchange interaction in the state with \(I^J=1^+\).

In order to quantitatively understand the dynamical mechanism forming the stable doubly heavy tetraquark states, we calculate the contributions coming from the different piece of the Hamiltonian in the CFTM to the binding energies of the stable states, which are presented in Table V. One can find that the most of the binding energies come from meson exchange interactions, which are equal to the values in the tetraquark states. Once meson exchanges are switched off, some of stable states will vanish with the exception of the states \([bb]_{3}[\bar{u}d]\) with \(I^J=1^+\) and \(1^+\) and the state \([bb][\bar{u}s]\) with \(I^J=1^+,\) which become into weak bound states with binding energy of several and a dozen MeV. The reason is that the meson exchange between two light quarks in the states \([QQ][\bar{q}q]\) does not occur in the threshold consisting of two \(Q\bar{q}\) mesons. Therefore, the doubly heavy tetraquark states \([QQ][\bar{q}q]\) provide an ideal field to research the interaction between the different quark interactions because the chiral symmetry is explicitly broken in the heavy sector but it is spontaneously broken in the light one.

The color-magnetic interaction also plays an important role in the formation of the stable doubly heavy tetraquark states with isospin zero, which makes contributions to the binding energies ranging from 54 MeV to 208 MeV, see Table V. The Coulomb interaction, independence of spin and isospin, universally produces extremely strong attractive interactions ranging from 550 MeV to 719 MeV in the stable doubly heavy tetraquark states, which can be understood by the small separations between any two particles \(r_{ij}\), specifically for the distance of two heavy quarks \(r_{12}^{-2}\), see Table IV. However, the Coulomb interaction has no direct contribution to the binding energies of the heavy tetraquark states \([QQ][\bar{q}q]\), see Table V. It can be found that the contri-
contributions to the binding energies from the color-magnetic and Coulomb interactions amplify with the increase of the mass ratio $\frac{m_q}{m_i}$ in the group of states $[QQ][qq]$ with the same $[qq]$ and $IJP$, such as the group $[cc][\bar{u}\bar{d}]$, $[bc][\bar{u}\bar{d}]$ and $[bb][u\bar{d}]$ with $01^+$. The states $[bc][\bar{u}\bar{d}]$ and $[bb][u\bar{d}]$ with $12^+$ as stable states against strong interactions should be emphasized because of weak attractive meson exchange and repulsive color-magnetic interactions, which are different from the binding mechanism of the states with $I = 0$. The kinetic energies make great contributions to the binding energies in the states with $12^+$, see Table V. The reason is that the repulsive color-magnetic interaction and the motions of quarks prevent any two quarks from approaching each other. Meanwhile, the magnitude of the $\pi$-meson exchange in the states $[QQ][u\bar{d}]$ with $12^+$ $(\langle \sigma_1 \cdot \sigma_2 \rangle \langle F_i \cdot F_j \rangle = 1)$ weaken to the $\frac{1}{2}$ of that in the states $[QQ][\bar{u}\bar{d}]$ with $01^+$ $(\langle \sigma_1 \cdot \sigma_2 \rangle \langle F_i \cdot F_j \rangle = 9)$. In this way, any two quarks should sit at far from each other, see Table IV, so that the kinetic energies greatly reduce, about 400 MeV, comparing with those of the states with $01^+$. For the same reasons, the energies of other states $[QQ][\bar{q}\bar{q}]$ with $2^+$ in Table I is just a little higher than their corresponding threshold. With respect to the state $[[bb][u\bar{s}]]$ with $\frac{1}{2}1^+$, one-Goldstone-boson-exchange interaction cannot provide large attraction because of lacking of $\pi$-meson exchange interaction. The system also reduces its kinetic energy to strengthen the stability of the system.

The quark model with twobody quadratic confinement potential by means of the Casimir scaling and other interactions in the CFTM is directly extended to the heavy tetraquark states $[QQ][qq]$, see Table VI, the energies are in general higher about 100 MeV than those given by the CFTM because the Casimir scaling will lead to anticonfinement for some color structure in the multiquark system 43. Meanwhile, the model with twobody confinement potential is also known to be flawed phenomenologically because it leads to power law van der Waals forces between color-singlet hadrons, which will disappear automatically by taking account into the flip-flop potential in the LQCD simulation on the tetraquark states 44. Comparing with the twobody confinement potential, the fourbody confinement potential based on the lattice picture push down the energy of the tetraquark states, about 100 MeV, and can provide the binding energies ranging from over 30 MeV to about 100 MeV, which is therefore universal dynamical mechanism forming stable doubly tetraquark states in the CFTM. In other words, some states with binding energies below 100 MeV are not stable states anymore in the model with twobody confinement potential.

V. SUMMARY

We systematically study the doubly heavy tetraquark states $[QQ][qq]$ with diquark-antidiquark picture in order to search for all possible stable states against strong interactions in the CFTM with a multibody confinement potential, $\sigma$-exchange, one-gluon-exchange and one-Goldstone-boson-exchange interactions. The CFTM model predicts that the tetraquark states $[cc][\bar{u}\bar{d}]$ with $01^+$, $[bc][\bar{u}\bar{d}]$ with $00^+$, $01^+$, and $12^+$, $[bb][u\bar{d}]$ with $01^+$, $01^+$ and $12^+$, $[bb][q\bar{s}]$ with $\frac{1}{2}1^+$ are stable states against strong interactions. The tetraquark states $[bb][u\bar{d}]$ with $01^+$ and $[bb][q\bar{s}]$ with $\frac{1}{2}1^+$ are the most promising stable doubly heavy tetraquark states, which should be explored in experiments in the near future. The strong decays of these stable doubly heavy tetraquark states are kinematically forbidden if they really exist. However, they can decay only weakly or electromagnetically and therefore they must have a small decay width.

The color configuration $[[QQ][\bar{q}\bar{q}][\bar{s}][\bar{s}]]_1$ dominates the properties of the doubly heavy tetraquark states, in which the diquark $[QQ]$ can be regarded as a basic building block because of their small sizes. The Coulomb interaction is very strong attractive and greatly reduce the energies of the doubly heavy tetraquark states $[QQ][\bar{q}\bar{q}]$. However, it has no direct contribution to the binding energies of the bound states. The multibody confinement potential based on the color flux-tube picture employs a collective degree of freedom whose dynamics play an important role in the formation of the bound states, which push down the energies of the doubly heavy tetraquark states about 100 MeV comparing the twobody one.

The doubly heavy tetraquark states $[QQ][u\bar{d}]$ with $I = 0$ are strong bound states and have the binding energies of the order of 100 MeV mainly coming from color-magnetic interaction and one-Goldstone-boson-exchange interaction. The doubly heavy tetraquark states $[QQ][\bar{u}\bar{d}]$ with $12^+$ are weak bound states because of weak meson exchange and repulsive color-magnetic interactions, which is formed mainly by reducing their kinetic energies. The strange state $[[bb][\bar{u}\bar{s}]]$ with $\frac{1}{2}1^+$ is similar to the state $[[bb][u\bar{d}]]$ with $01^+$ and however a relative weak bound state because one-Goldstone-boson-exchange interaction cannot provide large attraction because of lacking of $\pi$-meson exchange interaction. The state also reduces its kinetic energy to strengthen the stability of the system.

Until now, none of stable doubly heavy states $[QQ][\bar{q}\bar{q}]$ has been observed in experiments and therefore more comprehensive investigations on their properties are still needed. The experimental detection and analysis of the doubly heavy tetraquark states will undoubtedly provide an invaluable opportunity to severely check the availability of the different theoretical models and, therefore, will allow one to makes more reliable theoretical predictions on the exotic hadronic spectra.

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