Double Beta Decay Constraint on Composite Neutrinos

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Abstract

Neutrinoless double beta decay \( (\beta\beta)_0 \nu \) occurs through the magnetic coupling of dimension five, \( \lambda_W/m_{\nu^*} \), among the excited electron neutrino \( \nu^* \), electron and \( W \) boson if \( \nu^* \) is a massive Majorana neutrino.

\[
\left( \frac{\lambda_W m_W}{m_{\nu^*}} \right)^2 \left| \frac{m_{\nu^*}}{m_W} + \frac{2}{m_{\nu^*} + 1} \right| - \frac{0.129}{m_{\nu^*} / m_W} < 2.13 \times 10^{-2},
\]

where \( \lambda_W \) is the relative strength, \( m_{\nu^*} \) is the composite scale and \( m_* \) is the mass of excited neutrino. If the coupling is not small, i.e., \( \lambda_W > 1 \) and \( m_{\nu^*} = m_* \), we find \( m_* > 3.4m_W \) which is the most stringent limit.

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1 Introduction

If neutrinos are composite particles, there exist the excited neutrinos which couple to the ground state leptons by the dimension five magnetic coupling[1],[2]. This interaction is expressed as[3]

\[ L_{\text{int}} = g \frac{\lambda_W}{m_{\nu^*}} \bar{e} \sigma^{\mu\nu} (\eta_L^* R + \eta_R^* L) \nu^* \partial_\mu W^- + h.c., \]  

(1)

where \( \nu^* \) is a heavy excited electron neutrino, \( L = (1 - \gamma_5)/2, \) \( R = (1 + \gamma_5)/2, \) \( m_{\nu^*} \) is the mass dimension which is of order the mass of \( \nu^* \), i.e., \( m_* \). This interaction is derived by the \( SU(2) \times U(1) \) gauge invariance and parameters \( \eta_L \) and \( \eta_R \) are normalized by

\[ |\eta_L|^2 + |\eta_R|^2 = 1. \]  

(2)

The extensive search of \( \nu^* \) have been made by many groups[4] and found that \( m_{\nu^*} > 91 \text{GeV} \) by assuming that \( \lambda_Z > 1 \) which is the coupling for \( \nu^* \to eZ \) decay similarly defined to \( \lambda_W \).

The purpose of this paper is to explore the mass range \( m_{\nu^*} > m_Z \) by using the neutrinoless double beta decay \( (\beta\beta)_0 \nu \) by assuming that \( \nu^* \) is a massive Majorana neutrino. Then the \( (\beta\beta)_0 \nu \) decay occurs through \( \nu^* \) exchange. Since \( \nu^* \) enters as a virtual state, we can investigate heavy \( \nu^* \).

In our previous paper[5], we started from the interaction in Eq.(1) and derived the effective four point interaction between leptons and hadrons as

\[ L_{\text{eff}} = -G_{\text{eff}} \bar{\nu} \sigma^{\mu\nu} (\eta_L^* R + \eta_R^* L) \nu^* \partial_\mu J^\dagger_\nu + h.c., \]  

(3)

where \( J^\dagger_\nu \) is the hadronic current and

\[ G_{\text{eff}} = 2G_F \frac{g\lambda_W}{m_{\nu^*}}. \]  

(4)

From this effective interaction with the chirality selection rule \( \eta_L \eta_R = 0 \), the half-life of the neutrinoless double beta decay was derived[5] and is given by

\[ T^{-1} = 4 \left( \frac{\lambda m_A}{m_{\nu^*}} \right)^4 \left( \frac{m_A}{m_e} \right)^2 G_{01} \left| \frac{m_e}{m_A} M_{GT,N} + \frac{m_A}{m_*} \left( \frac{g_V}{g_A} \right)^2 M'_F - \frac{2}{3} M_{GT} - \frac{1}{3} M_T \right|^2, \]  

(5)
which disagreed with the old result by Panella and Srivastava[6]. By comparing this formula to the Heidelberg-Moscow data[7] for $^{76}$Ge decay, we found[6]

$$\left(\frac{\lambda m_A}{m_{\nu^*}}\right)^2 \left| \frac{m_s}{m_A} + 7.2 \cdot 10^{-2} \frac{m_A}{m_s} \right| < 1.4 \cdot 10^{-8}. \quad (6)$$

The formula in Eq.(5) and also the constraint in Eq.(6) are valid as far as we consider the effective interaction in Eq.(3).

Recently, Panella, Carimalo, Srivastava and Widom pointed out[8] that propagators of W-boson exchange should not be contracted as far as one sticks to the interaction in Eq.(1). This is true since the mass of excited neutrinos $m_*$ is greater than $m_W$. They calculated[8] the half-life formula of the neutrinoless double beta decay and claimed that my calculation is wrong. Here, I say that my calculation is correct as far as we consider the effective interaction in Eq.(3). Anyway, I feel to be obliged to repeat the computation starting from the interaction in Eq.(1).

In Sec. 2, we show the computation of the decay and give the half-life formula. In Sec. 3, the numerical analysis will be given. Summary is presented in Sec.4.

## 2 Decay formula of neutrinoless double beta decay

In the fourth order perturbations of the interaction in Eq.(1), the $\beta\beta_0\nu$ decay takes place and the S-matrix for this decay is given by

$$S = -i \frac{G_{eff}^2 m_W^4}{2(2\pi)^{12}} \int \frac{dx_1dx_2dx_3dx_4dq_1dq_2}{q^2 - m_s^2 - m_W^2} e^{-iq(x_1-x_2)} e^{-iq_1(x_1-x_3)} e^{-iq_2(x_2-x_4)} q_1\mu q_2\nu'$$

$$\times T(\bar{e}(x_1)\sigma^{\mu\nu} \sigma^{'\mu\nu'} [m_*(\eta_L^* R + \eta_R^* L) + \gamma_\mu q^\mu \eta_L^* \eta_R^*] e^C(x_2)) T(J_\nu^\dagger(x_3) J_{\nu'}^\dagger(x_2)),$$

$$\quad (7)$$

where $e^C$ is the charge conjugation of $e$, i.e., $e^C = C\bar{e}^T$.

In the following computation, we take the S-wave function for electrons which is given by

$$< 0 | e(x) | p > = \psi_S(\epsilon)e^{-i\epsilon x^0}; \quad \psi_S(\epsilon) = \sqrt{\frac{\epsilon + m}{2\epsilon}} \left( \frac{\chi_s}{\sqrt{\epsilon + m}} \right)^T F_0(Z, \epsilon), \quad (8)$$
where $\epsilon$ is the energy of electron and $F_0(Z, \epsilon)$ is the relativistic Coulomb factor defined in Eq.(3.1.25) in Ref.9. The S-wave function is independent of the space coordinate. After integrating $x_1, x_2$ and then $q_1, q_2$, we find

\[ S_{fi} = \frac{G^2 m_W^4}{2(2\pi)^3} \int dx_3 dx_4 \int dq \frac{e^{-i(qx_3-x_4)}}{q^2 - m^2_s} < N_f | T(J^+_\mu(x_3)J^+_{\nu}(x_4)) | N_i > \]

\[ \times \left\{ \frac{e^{i(x_3^0 + x_4^0)}}{((q + k_1)^2 - m^2_W)((q + k_2)^2 - m^2_W)} \right\} (-q + k_1)_{\mu}(q + k_2)_{\nu} \]

\[ \times \bar{\psi}_S(\epsilon) \sigma^{\mu\nu} \sigma^{\mu'\nu'} [m_s(\eta_L^2 R + \eta_R^2 L) + \gamma_{\nu} q_{\mu} \eta_L^2 \eta_R^2] \psi_C(\epsilon_2) \]

\[ -(\epsilon_1 \leftrightarrow \epsilon_2) , \]

(9)

where $k_i = (\epsilon_i, 0)$.

Next we perform the $q^0$ integration. There are six poles at $\pm (E_s - i\epsilon), \epsilon_1 \pm (E_W - i\epsilon)$ and $-\epsilon_2 \pm (E_W - i\epsilon)$. The poles in the lower-half plain contribute when $x_3^0 - x_4^0 > 0$ and those in the upper-half plain do when $x_3^0 - x_4^0 < 0$. By performing the $q^0$ integration, the time ordering for hadronic currents is dealt automatically. We find

\[ S_{fi} = -\frac{G^2 m_W^4}{4(2\pi)^3} \int dx_3 dx_4 \int dq e^{i(qx_3-x_4)} e^{i(x_3^0 + x_4^0)} \]

\[ \times \bar{\psi}_S(\epsilon) \sigma^{\mu\nu} \sigma^{\mu'\nu'} [m_s(\eta_L^2 R + \eta_R^2 L) + \gamma_{\nu} q_{\mu} \eta_L^2 \eta_R^2] \psi_C(\epsilon_2) \]

\[ \times [\theta(x_3^0 - x_4^0) < N_f | J^+_{\mu}(x_3) | n > < n | J^+_{\nu}(x_4) | N_i > A_{\mu\nu'} \]

\[ + \theta(x_4^0 - x_3^0) < N_f | J^+_{\mu}(x_3) | n > < n | J^+_{\nu}(x_4) | N_i > B_{\mu\nu'} \]

\[ -(\epsilon_1 \leftrightarrow \epsilon_2) , \]

(10)

where

\[ A_{\mu\nu'} = \frac{(q_s - k_1)_{\mu}(q_s + k_2)_{\nu} e^{-iE_s(x_3^0 - x_4^0)}}{E_s((E_s - \epsilon_1)^2 - E_W^2) [ (E_s + \epsilon_2)^2 - E_W^2 ]} \]

\[ + \frac{q_{\nu} (q_W + k_1 + k_2)_{\mu} e^{-i(E_W + \epsilon_1)(x_3^0 - x_4^0)}}{E_W[(E_W + \epsilon_1)^2 - E_W^2][(E_W + \epsilon_1 + \epsilon_2)^2 - E_W^2]} \]

\[ + \frac{(q_{\nu} - k_1 - k_2)_{\mu} q_{\nu} (q_W - k_1)_{\nu} e^{-i(E_W - \epsilon_2)(x_3^0 - x_4^0)}}{E_W[(E_W - \epsilon_2)^2 - E_W^2][(E_W - \epsilon_1 - \epsilon_2)^2 - E_W^2]} , \]
Then, we perform the integration immediately since we use the non-relativistic approximation of hadronic current, where \( \vec{r} \) with the value \( n \) is the position of the j-th nucleon in the nucleus and \( j \) is the position of the j-th nucleon in the nucleus and \( j \) and similarly for \( E_W, q_W \) and \( \vec{q}_W \).

Here \( E_* = \sqrt{m_*^2 + q^2} \), \( q_* = (E_*, \vec{q}) \) and \( \vec{q}_* = (E_*, -\vec{q}) \) and similarly for \( E_W, q_W \) and \( \vec{q}_W \).

Then, we perform the \( x_3^0 \) and \( x_4^0 \) integration. The \( x_3 \) and \( x_4 \) integration can be made immediately since we use the non-relativistic approximation of hadronic current,

\[
J'_\mu(\vec{x}) = \sum_j J^j_\mu(j) \delta(\vec{x} - \vec{r}_j), \quad J'_\mu(j) = \gamma_j^+ (g_V g_{\rho_0} + g_A \delta_{\mu\nu} g_j^I) F(q^2),
\]

where \( \vec{r}_j \) is the position of the j-th nucleon in the nucleus and \( F(q^2) \) is the form factor defined by

\[
F(q^2) = \left( \frac{1}{1 + (q^2/m_A^2)} \right)^2,
\]

with the value \( m_A = 0.85 \text{GeV} \). By these integration, we obtain the energy conservation \( E_i = E_f + \epsilon_1 + \epsilon_2 \) and the energy denominators. We find

\[
R_{fi} = \frac{G_{eff}^2 m_A^4}{\sqrt{2}} \bar{\psi}_S(\epsilon_1) \sigma^{\mu\nu} \sigma^{\mu'\nu'} [m_*(\eta_L^{*2} R + \eta_R^{*2} L) + \gamma_\mu q^\nu \eta_L \eta_R^*] \psi_S(\epsilon_2) \times \sum_{j \neq l} \int \frac{d\vec{q}}{(2\pi)^3} e^{-\vec{q} \cdot \vec{r}_j} \sum_n <N_f|J^j_\nu(\vec{r}_j)|n> <n|J^l_\nu(\vec{r}_l)|N_i > C_{\mu\mu'}
\]

\[
- (\epsilon_1 \iff \epsilon_2),
\]

where \( \epsilon_n = E_n - (E_i + E_f)/2 \) and

\[
C_{\mu\mu'} = \frac{(q_* - k_1)_\mu (q_* + k_2)_\mu'}{(E_* + \epsilon_n - \epsilon_1 - \epsilon_2) E_* [(E_* + \epsilon_1)^2 - E_W^2][(E_* + \epsilon_2)^2 - E_W^2]} + \frac{q_W (q_W + k_1 + k_2)_{\mu'}}{(E_W + \epsilon_n + \epsilon_1 + \epsilon_2) E_W [(E_W + \epsilon_1)^2 - E_W^2][(E_W + \epsilon_1 + \epsilon_2)^2 - E_W^2]} + \frac{(q_W - k_1 - k_2)_{\mu'} q_W}{(E_W + \epsilon_n - \epsilon_1 - \epsilon_2) E_W [(E_W - \epsilon_1)^2 - E_W^2][(E_W - \epsilon_1 - \epsilon_2)^2 - E_W^2]}
\]

\[
D_{\mu\mu'} = \frac{(q_* + k_1)_\mu (q_* - k_2)_\mu'}{(E_* + \epsilon_n + \epsilon_1 - \epsilon_2) E_* [(E_* + \epsilon_1)^2 - E_W^2][(E_* + \epsilon_2)^2 - E_W^2]} + \frac{q_W (q_W + k_1)_{\mu'}}{(E_W + \epsilon_n - \epsilon_1 - \epsilon_2) E_W [(E_W - \epsilon_1)^2 - E_W^2][(E_W - \epsilon_1 - \epsilon_2)^2 - E_W^2]} + \frac{(q_W - k_1)_{\mu'} q_W}{(E_W + \epsilon_n - \epsilon_1 + \epsilon_2) E_W [(E_W + \epsilon_1)^2 - E_W^2][(E_W + \epsilon_1 + \epsilon_2)^2 - E_W^2]}
\]
\[
\begin{align*}
&+ \frac{(\bar{q}_W + k_1 + k_2)_\mu \bar{q}_W \nu'}{(E_W + \epsilon_n + \frac{\epsilon_1 + \epsilon_2}{2})E_W[(E_W + \epsilon_2)^2 - E_0^2][(E_W + \epsilon_1 + \epsilon_2)^2 - E_0^2]} \\
&+ \frac{\bar{q}_W \nu(\bar{q}_W - k_1 - k_2) \mu'}{(E_W + \epsilon_n - \frac{\epsilon_1 + \epsilon_2}{2})E_W[(E_W - \epsilon_1)^2 - E_0^2][(E_W - \epsilon_1 - \epsilon_2)^2 - E_0^2]}
\end{align*}
\] (15)

Hereafter, we consider the case where the chirality selection rule is satisfied, i.e., \(\eta_L \eta_R = 0\). So far we took (a1) the S-wave function for electron wave function and thus the total angular momentum taken by electrons are 0 or 1 so that the \(0^+ \rightarrow 0^+\) and \(0^+ \rightarrow 1^+\) transitions are allowed in general. Then, we used (a2) the non-relativistic approximation of the hadronic current because we are dealing with the allowed transition. Next, we make (a3) the closure approximation where \(\epsilon_n\) is replaced by the average value \(<\epsilon_n> = \mu_0 m_e\). Then, the sum of the intermediate states can be taken. Then, we find

\[
\sum_n <N_f|J^I_\nu(\bar{r}_j)|n><n|J^I_\nu(\bar{r}_i)|N_i> = \sum_n <N_f|J^I_\nu(\bar{r}_j)|n><n|J^I_\nu(\bar{r}_i)|N_i>
\]

so that \(C_{\mu\nu'}\) and \(D_{\mu\nu'}\) enter in \(S_{fi}\) as their sum.

Now, we concentrate on the \(0^+ \rightarrow 0^+\) transition. Since \(J_\nu(\bar{r}_j)\) is an parity even operator, only the \(mu = \mu' = 0\) and \(\mu = k, \mu' = k'\) \((k, k' \in \{1, 2, 3\})\) parts contribute to the \(0^+ \rightarrow 0^+\) transition because \(\bar{q}\) is an odd parity operator. Now, we find that \(C_{00} + D_{00}\) and \(C_{kk'} + D_{kk'}\) are even functions with respect to the exchange of \(\epsilon_1\) and \(\epsilon_2\). Then, the first part and the \((\epsilon_1 \leftrightarrow \epsilon_2)\) part are combined as

\[
R_{fi} = \frac{G^2_{\text{eff}} m_W^4}{4\sqrt{2}} \bar{\psi}_S(\epsilon_1)\{\sigma^{\mu\nu}, \sigma^{\mu'\nu'}\} m_+(\eta_L^{\mu 2} R + \eta_R^{\mu 2} L)\psi_S^C(\epsilon_2) \int \frac{d\vec{q}}{(2\pi)^3} e^{i\bar{q}\cdot \bar{r}_{ji}}
\]

\[
\times \sum_{j \neq l} <N_f|J^I_\nu(\bar{r}_j)|J^I_\nu(\bar{r}_i)|N_i> (C_{\mu\nu'} + D_{\mu\nu'})
\] (17)

Now we use the identity \(\{\sigma^{\mu\nu}, \sigma^{\mu'\nu'}\} = 2(g^{\mu\nu'}g^{\mu'\nu} - g^{\mu\nu}g^{\mu'\nu'} - i\epsilon^{\mu\nu\mu'\nu'}\gamma_5)\). Then, we find

\[
R_{fi} = -\frac{G^2_{\text{eff}} m_W^4}{2\sqrt{2}} \bar{\psi}_S(\epsilon_1)(\eta_L^{\mu 2} R + \eta_R^{\mu 2} L)\psi_S^C(\epsilon_2) \int \frac{d\vec{q}}{(2\pi)^3} e^{-\bar{q}\cdot \bar{r}_{ji}}
\]

\[
\times \sum_{j \neq l} <N_f|C_{00} + D_{00}|J^I_j(\bar{r}_j) \cdot J^I_l(\bar{r}_i)>
\]
\begin{equation}
\frac{1}{N_i} \rangle \),
\end{equation}

where we used \( C_{kk'} + D_{kk'} = q_k g_k' (C + D) \).

Now, we expand \( C_{\mu'\mu} \) and \( D_{\mu'\mu} \) with respect to the small quantities \( \epsilon_i/E_* \), \( \epsilon_i/E_W \), \( \epsilon_i/(m_*^2 - m_W^2) \) and take the leading order terms. Here we assume \( m_* > m_W \) and used the fact that \( \epsilon_i \simeq \text{a few MeV} \). Then, we obtain

\begin{equation}
C_{00} + D_{00} \simeq - \frac{\mu_0 m_e (E_* + 2E_W)}{E_* E_W (E_* + E_W)^2} \simeq - \frac{\mu_0 m_e (m_* + 2m_W)}{m_* m_W^3 (m_* + m_W)^2},
C + D \simeq \frac{2}{E_*^2 E_W^4} \simeq \frac{2}{m_*^2 m_W^4}.
\end{equation}

Here we used the fact that the momentum \( q \) is effectively cut off by \( m_A \) due to the form factor \( F(q^2) \).

Then, we use

\begin{equation}
\int \frac{d\vec{q}}{(2\pi)^3 (\vec{q}^2 + m_A^2)^4} e^{i\vec{q} \cdot \vec{r}} = \frac{1}{4\pi m_A^6} \frac{1}{r} F_N, \quad \int \frac{d\vec{q}}{q_k q_l (2\pi)^3 (\vec{q}^2 + m_A^2)^4} e^{i\vec{q} \cdot \vec{r}} = \frac{1}{4\pi m_A^6} \frac{11}{r^3} [\delta_{kl} F_4 - (3\hat{x}_k \hat{x}_l - \delta_{kl}) F_5],
\end{equation}

where with \( x_A - m_A r \)

\begin{align*}
F_N &= \frac{x_A}{48} (3 + 3x_A + x_A^2) e^{-x_A}, \\
F_4 &= \frac{x_A}{48} (3 + 3x_A - x_A^2) e^{-x_A}, \\
F_5 &= \frac{x_A}{48} e^{-x_A}
\end{align*}

Then, we find

\begin{equation}
R_{fi} = \frac{(G_{eff} m_A g_A)^2}{4\pi \sqrt{2} R} \psi_S(\epsilon_1)(\eta^2 R + \eta^2 L) \psi_S^{\Gamma}(\epsilon_2) F_0(Z + 2, \epsilon_1) F_0(Z + 2, \epsilon_2) \times \left\{ \frac{\mu_0 m_e m_W (m_* + 2m_W)}{2 (m_* + m_W)^2} M_{GT,N} - \frac{m_A^2}{m_*^2} \left( \frac{g_V}{g_A} \right)^2 M'_F - \frac{2}{3} M'_{GT} - \frac{1}{3} M'_{T} \right\}.
\end{equation}

where

\begin{equation}
M_{GT,N} = \langle N_f | \sum_{n \neq m} \tau^{(+)}_n \tau^{(+)}_m \bar{\sigma}_n \cdot \bar{\sigma}_m (R_{nm}) F_N(x_A) | N_i \rangle,
\end{equation}

6
\[M'_F = \langle N_f | \sum_{n \neq m} \tau_n^{(+)\tau_m^{(+)}} \left( \frac{R}{\tau_{nm}} \right) F_4(x_A) | N_i \rangle, \]

\[M'_{GT} = \langle N_f | \sum_{n \neq m} \tau_n^{(+)\tau_m^{(+)}} \bar{\sigma}_n \cdot \bar{\sigma}_m \left( \frac{R}{\tau_{nm}} \right) F_4(x_A) | N_i \rangle, \]

\[M'_T = \langle N_f | \sum_{n \neq m} \tau_n^{(+)\tau_m^{(+)}} \left\{ 3(\bar{\sigma}_n \cdot \bar{r}_{nm})(\bar{\sigma}_m \cdot \bar{r}_{nm}) - \bar{\sigma}_n \cdot \bar{\sigma}_m \right\} \left( \frac{R}{\tau_{nm}} \right) F_5(x_A) | N_i \rangle, \]

After taking the spin sum and performing the phase-space integration, we find the half-life of the transition of the neutrinoless double beta decay due to the heavy composite neutrino for \(0^+ \rightarrow 0^+\) transition as

\[T^{-1} = 4 \left( \frac{\lambda m_A}{m_{\nu^*}} \right)^4 \left( \frac{m_A}{m_e} \right)^2 G_{01} \left| \left( \frac{1}{2} \mu_0 m_e m_W (m_* + 2m_W) \right) \right| \left( \frac{m_A}{m_* + m_W} \right)^2 M_{GT,N} \]

\[- \frac{m_A}{m_*} (\frac{g_V}{g_A})^2 M'_{F} - \frac{2}{3} M'_{GT} - \frac{1}{3} M'_{T} \right|^2, \]

where \(G_{01}\) is the phase space factor defined in Eq.(3.5.17a) in the paper by Doi, Kotani and Takasugi[9]. The above expression is obtained by taking \(| \eta_L |^4 + | \eta_R |^4 = 1\) which is valid because of the chirality conservation.

### 3 Constraint from neutrinoless double beta decay

In the following, we analyze the constraint on the coupling by using the experimental half-life limit of \(^{76}\text{Ge}\) which is measured by the Heidelberg-Moscow collaboration[4],

\[T(0^+ \rightarrow 0^+ : ^{76}\text{Ge}) > 5.6 \cdot 10^{24} yr \quad 90\% c.l.. \]

We derive the constraint on composite parameters from this data. By using the values of nuclear matrix elements obtained by Hirsch, Klapdor-Kleingrothaus and Kovalenko[10],

\[M_{GT,N} = 0.113, \quad M'_F = 3.06 \cdot 10^{-3}, \quad M'_{GT} = -7.70 \cdot 10^{-3}, \quad M'_T = -3.09 \cdot 10^{-3}, \]

the phase space factor \(G_{01} = 6.4 \cdot 10^{-15}/yr\) given in Ref.6 and \(g_A/g_V = 1.25\), we find the half-life of the \(0^+ \rightarrow 0^+\) transition for \(^{76}\text{Ge}\) is

\[T^{-1} = 3.9 \times 10^{-22} \left( \frac{\lambda W m_W}{m_{\nu^*}} \right)^4 \left( \frac{m_*}{m_W} + 2 \right) \left( \frac{m_*}{m_W} + 1 \right)^2 - \frac{0.129}{m_*/m_W} \right|^2, \]
Then, we find the constraint
\[
\left( \frac{\lambda_W m_W}{m_{\nu^*}} \right)^2 \left| \frac{m_*}{m_W} + \frac{2}{(m_W/m_W + 1)^2} - \frac{0.129}{m_W/m_W} \right| < 2.13 \times 10^{-2}. \quad (31)
\]

4 Discussions

Firstly, we shall discuss about the excited neutrino mass. We consider the case that the composite scale \( m_{\nu^*} \) is the same as the excited neutrino mass \( m_* \). In Fig.1, we show the limit of \( m_* \) for \( \lambda_W > 1 \). From this figure, we find
\[
m_* > 3.4m_W \quad (\lambda_W > 1). \quad (32)
\]

Firstly, we compare our new result with the old result which is estimated by using the effective interaction in Eq.(3). By comparing Eq.(28) and Eq.(5), we find that the second term in the parenthesis is the same as the old one, while the first term is different by the cancellation among the poles due to \( \nu^* \) and W propagator.

Next, we compare our result with the one by Panella et al.[8]. One difference is the coefficients of mass factors of nuclear matrix elements. They used the assumption \( m_* \gg m_W \). Our coefficients agree with them in this limit. We evaluated without this assumption to see the behavior near \( m_W \). Our formula is valid for \( m_* \gg m_A = 0.85\text{GeV} \). Another difference is the overall factor. Our decay rate formula is about sixteen times larger than their formula for \( m_* \gg m_W \) case where we can compare with their formula.

As a result, we find the factor 2 stringent limit on the composite scale \( m_{\nu^*} \) or the relative coupling \( \lambda_W \). These can be seen from Fig.2 and Fig.3. In Fig.2, we showed the lower bound on \( \Lambda_c \equiv m_{\nu^*}/\sqrt{2} \) for \( \lambda_W > 1 \). The lower bound at \( m_* = 6m_W \) is about 0.15 while their bound is about 0.08. In Fig.3, we showed the upper bound on \( \lambda_W \) when \( \Lambda_c \equiv m_{\nu^*}/\sqrt{2} = 1\text{TeV} \). Again our bound is about factor two severer than their bound.

In summary, we computed the half-life formula of the neutrinoless double beta decay for the \( 0^+ \rightarrow 0^+ \) transition. We made the systematic analysis without assuming \( m_* \gg m_W \). Our result differs from the one by Panella et al., some mass factors due to their assumption \( m_* \gg m_W \) and the normalization difference about sixteen times.
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Figure Captions:

Fig.1: The lower bound on the excited neutrino mass $m_\nu$ for $\nu_{\nu} = m_\nu$ and $\lambda_W > 1$. The allowed mass range is the region $m_\nu > 3.4m_W$.

Fig.2: The lower bound on the composite scale $\Lambda_c \equiv m_{\nu_\nu}/\sqrt{2}$ as a function of $m_\nu/m_W$.

Fig.3: The upper bound on the relative coupling $\lambda_W$ for $\Lambda_c \equiv m_{\nu_\nu}/\sqrt{2} > 1$TeV as a function of $m_\nu/m_W$. 