The modification and Application of Existing Empirical Formula for Predicting Water Inflow Considering the Space of Caverns

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Abstract. The prediction of water inflow in cavern is significant for underground engineering, as the inflow has important influence on construction and drainage system. At present, the empirical formulas for predicting inflow are usually based on a single cavern, but the influence of nearby caverns is not taken into account. In order to consider the influence of spacing and radius on water inflow, the inflow in an underground crude oil storage cavern is analyzed by numerical analysis in this paper. The results show that when the s/r < 20, the spacing has great influence on the inflow; when 20<s/r<60, it has some effect on the inflow; and when s/r>60, it almost has no effect on the inflow (s and r represent spacing between caverns and radius). Based on numerical analysis, empirical formulas for predicting water inflow are modified, and the new formula is proposed considering the influence of the spacing. Finally, field data show that the new formula is more suitable for predicting inflow.

Keywords: Water Inflow, Numerical Analysis, Spacing Ratio, Modified Formula, Oil Depot Cavern

1. Introduction
The prediction of water inflow is unavoidable and difficult in underground engineering. Due to the complexity and uncertainty of rock characteristics, it is difficult to predict the inflow of caverns accurately. The excavation changes the seepage field of the groundwater in the surrounding rock, which will cause many issues in the process of excavation. The inflow in caverns will be the determining factor for supporting system, construction progress and cost of the caverns. Therefore, the accurate prediction of inflow is of great significance for underground engineering. In the research process of this problem, domestic and foreign scholars proposed different methods for predicting water seepage according to their respective research.

Until now, many scholars have done a lot of research on inflow in underground engineering. The most commonly used empirical formula using image well method⁴ was proposed by Harr (1962). Tani
(2003) obtained the seepage field of the zero pressure and constant head of the cavern by mobius transform and Fourier series\cite{2}. Mas (2006) studied the influence of the coupling of fluid and force and the excavation diameter on inflow through numerical analysis\cite{3}. Kolymbas (2007) put forward the analytical solution for the seepage field of deep and shallow cavern under the change of water head around the cavern\cite{4}. Analytical solutions for the seepage field under condition that the boundary water head is zero or constant were put forward by Park (2008) using conformal transformation method\cite{5}. Farhadian (2012) studied the influence of $r/h$, $r$ and $h$ represents cavern radius and water head, on the accuracy of some analytical solutions for determining the water inflow and optimized the inflow formulas\cite{6}. Moon (2010) put forward the analytical solution of seepage field when the water head reduced caused by excavation\cite{7}. Fernandez (2010) proposed an analytical solution for inflow considering the reduction of permeability coefficient as a result of excavation. He proposed a region called lining-like zone shown in figure 1 whose permeability coefficient was smaller than that in undisturbed area. And it was verified by numerical analysis\cite{8}.

![Lining-like zone diagram.](image)

In China, Wang et al (2008) have studied the seepage of the tunnel in the South-to-North Water Transfer Project Area. The research results have realized the seepage prediction of the unsteady seepage tunnel and have been applied to the actual project and achieved good results\cite{9,10}. Li et al (2012) used the finite difference method to predict the seepage of Jiaozhou Bay tunnel in Qingdao during the construction period, providing theoretical support for engineering construction\cite{11}. Zhang et al (2010) used numerical simulation to analyze the distribution law of groundwater seepage field during construction and operation period\cite{12}. Liu et al (2015) used numerical simulation to analyze the distribution law of groundwater seepage field during construction and operation period\cite{13}. In field measurement, Wang et al (2004) combined with a large number of engineering examples to calculate the inflow by forward and inversion methods. The timing of forward and inversion that should be determined according to the construction before and after construction has been proposed\cite{14}.

Although scholars in the global have studied the water inflow considering many different factors, but it has not been comprehensively understand due to the complexity of underground engineering. Field data show that most results obtained by existing empirical formulas are not always accurate. One of the reasons is that empirical formulas have not mentioned the influence of multiple caverns.

In this paper, numerical simulation and theoretical analyses are used to study the inflow under the influence of double-caverns with the background of an underground crude oil storage cavern, and the concept of spacing ratio is introduced to modify the empirical formulas for the first time. Finally, the modified formula is analyzed and verified in the oil depot.

2. Water inflow prediction of underground engineering

Empirical formulas are effective way to predict the inflow in underground engineering. Most of formulas are based on the assumption that the stratum is homogeneous and isotropic porous medium. Caverns have regular shape and water level is steady, so when the model boundary water pressure is 0, the water level is below the ground. Table 1 lists two formulas for estimating the inflow.
Table 1. Empirical formulas of inflow.

| Constant head          | Varying head     |
|------------------------|------------------|
| Harr\(^{[1]}\)         | Moon\(^{[6]}\)   |
| \( q = \frac{2\pi kH}{\ln(2H/r)} \) | \( q = \frac{2\pi k(H-h)}{\ln(2H/r)} \) |

Empirical formula in the first column is applicable for predicting inflow when the water head remains stable during excavation, and the schematic map is shown in Figure 2. The other empirical formula is used for predicting inflow when the water head changes, and the schematic map is shown in Figure 3. In Table 1, \( k \) is the equivalent permeability coefficient of jointed rock mass around cavern, \( q \) is inflow, \( r \) is radius, \( H \) is depth from the groundwater level to the cavern center and \( h \) is water head.

3. Research method

In this paper, the continuous medium theory is used for numerical analysis. The concept of spacing ratio \( s/r \) is introduced for the first time: the ratio is the spacing of two caverns \( s \) to the cavern radius \( r \). Numerical models are established considering the spacing ratio \( s/r \) to analyze the inflow. The modified formulas considering the spacing ratio was proposed using numerical analyses based on empirical formulas.

3.1. Numerical model
The finite element software Abaqus 6.11 is used to simulate the inflow of two caverns during excavation. A two-dimensional axisymmetric plane model with a model size of 300m×300m is adopted. To simplify the calculation, the central axis between the two caverns is the symmetrical axis, where the inflow rate is 0 with the symmetry of model. It is considered that the left boundary is an impermeable boundary. The right boundary of the model is a triangular pressure boundary, and the pressure is determined by the formula $p = \rho gh$, where $p$ is pressure and $g$ is gravitational acceleration. The upper boundary of the model coincides with the groundwater level, and the pressure is 0. The lower boundary pressure of the model is 3 MPa, which equals to 300m water head. As the water head coincides with the ground surface, the model is in saturated and steady state. The assumption are as follows: the model is homogeneous and isotropic; cavern is circular; the support is not constructed; and water pressure in cavern is 0. Figure 4 shows the numerical model.

3.2. Study cases
In cases 1~10, the cavern radius $r$ is 2.5m, the depth $H$ is 50m and the permeability coefficient $k$ is $1\times10^{-4}$ m/d. All parameters of the cases are same but the spacing ratio $s/r$. Table 2 shows the study cases and parameter values.

Table 2. Study cases.
| Number | Spacing ratio $s/r$ | Depth $H/m$ | radius $r/m$ | Permeability coefficient $k/(m/d)$ |
|--------|-------------------|------------|-------------|----------------------------------|
| 1      | 2                 | 50         | 2.5         | $1 \times 10^{-4}$               |
| 2      | 4                 | 50         | 2.5         | $1 \times 10^{-4}$               |
| 3      | 6                 | 50         | 2.5         | $1 \times 10^{-4}$               |
| 4      | 8                 | 50         | 2.5         | $1 \times 10^{-4}$               |
| 5      | 10                | 50         | 2.5         | $1 \times 10^{-4}$               |
| 6      | 20                | 50         | 2.5         | $1 \times 10^{-4}$               |
| 7      | 40                | 50         | 2.5         | $1 \times 10^{-4}$               |
| 8      | 60                | 50         | 2.5         | $1 \times 10^{-4}$               |
| 9      | 80                | 50         | 2.5         | $1 \times 10^{-4}$               |
| 10     | 100               | 50         | 2.5         | $1 \times 10^{-4}$               |

**Figure 5.** Water inflow in study case 1.

**Figure 6.** Water inflow in study case 7.
4. Research results

4.1. Data analysis

The flow vector and contour of the study cases 1, 7 and 10 are shown in Figures 5 to 7. Left side of Figures 5 to 7 are flow vector, in which the arrow direction indicates the direction of the flow and size of the arrow indicates the inflow. It shows that when the spacing ratio s/r is small, inflow of right boundary of carven is larger than left, the excavation of cavern will affect seepage field in the middle area of two caverns. Under the effect of double carvens diversion, the inflow into the caverns from the middle area is less, so that the total inflow in each carven is small.

With the increase of spacing ratio, the influence on the inflow in the middle of the two caverns decreases gradually, and the effect of water diversion in the two caverns also decreases gradually. The inflow from the middle into the caverns gradually increases. When the distance reaches a certain state, double-carvens excavation has no effect on each other and can be considered as a single hole. It can be seen from the vector diagram in Figure 6 that in this condition, the spacing of double-caverns is too large for the cavern, so that it has little effect on the inflow. As shown in the right side of Figures 5 to 7, the inflow in the carven increases with the increase of the spacing ratio, which leads to the increase of inflow into the caverns from the middle of the two caverns.

The inflow obtained from empirical formula and numerical analysis according to study cases 1 to 10 are shown in Figure 8.

As shown in Figure 8, when the spacing ratio is changed, the empirical formula obtains only one inflow, while the inflow of numerical analysis varies: When s/r<20, the spacing ratio has a very significant influence on the inflow, when 20<s/r<60, the spacing ratio has some effect on the inflow and when s/r >60, the spacing ratio almost has no influence on the inflow. It can be seen that when the distance between two carvens is less than 20 times of the radius, the inflow change is obvious, and the influence of spacing ratio on water inflow should be fully considered.

4.2. Formula modification

When the groundwater level remains stable, the empirical formula proposed by Harr is modified by introducing the influence coefficient of multiple caverns, and then the formula is as follows:

\[ q = \frac{2\pi kH}{\ln(2H/r)} \beta \]

In Eq. (1), \( \beta \) is influence coefficient of double-caverns and a function of s/r. Define \( \bar{s} = s / r \), the following two conditions should be met by:

1. When \( \bar{s} \to \infty \), \( \beta = 1 \)
When $s \rightarrow 0$, $\beta = 1/2$

Hyperbolic tangent function can be used to fit the numerical analysis data. That is:

$$\beta = \frac{1}{2} \left[ 1 + \tanh \left( a \left( \frac{s}{r} \right)^b \right) \right]$$

(2)

Where $a, b$ are undetermined constant related to double-caverns.

In the same way, when the underground water level declines, we adjust the empirical formula proposed by Moon, then the flow formula is as follows:

$$q = \frac{2\pi k(H-h)}{\ln(2H/r)} \beta$$

(3)

The Eq. (1) is used to fit the data of numerical analysis and the undetermined parameters $a=0.167$ and $b=0.569$. When the water level remains stable, the flow formula is:

$$q = \frac{\pi kH}{\ln(2H/r)} \left[ 1 + \tan h \left( 0.167 \left( \frac{s}{r} \right)^{0.569} \right) \right]$$

(4)

$$q = \frac{\pi k(H-h)}{\ln(2H/r)} \left[ 1 + \tan h \left( 0.167 \left( \frac{s}{r} \right)^{0.569} \right) \right]$$

(5)

The inflow obtained from modified formula Eq. (4) is shown in Figure 9.

Figure 8 shows that the results obtained from numerical analyses are very close to the results calculated by the modified formula.

5. Engineering application

5.1. Engineering background

One underground water sealed caverns lies in the direction of 5 degrees north by west. Its East-west width is 600 m, and North-south length is 838 m. The design storage capacity is $300 \times 10^4$ m$^3$. It has nine caverns, and is divided into three groups. The length of the cavern is 500–600 m, the width is 20 meters, the height 30 m and the section shape is straight wall and round arch. The distance between the wall of the cavern and the adjacent construction laneway is 25 m, and the distance between two caverns is divided into three cases: 30 m, 50 m and 70 m. The permeability coefficient is about $1 \times 10^{-4}$ m/d determined by pressure water test.

5.2. Flow calculation
This paper uses empirical formula and the modified formula to predict the flow of one underground crude oil storage cavern whose sketch map is shown in Figure 10. The distance between the caverns is 50 m which is selected as the value of spacing and the cross section shape is straight wall and round arch. Assuming that the cross section shape is circle, and the equivalent radius \( r \) of the cavern is 13.3 m in this case, and the initial water table \( H \) is 214 m. After cavern excavation, water level decline \( h \) is 60 m. Figure 11 shows the flow determined by different measures.

![Figure 10. Sketch map of underground water sealed cavern.](image)

Figure 10 shows the inflow obtained by Harr formula, Moon formula, modified Eq. (4) and Eq. (5) with field data respectively. We can see that the prediction results of empirical formula is quite different from the field data. The modified formulas considering the spacing are reasonable than the empirical ones, and the results are most close to the field data when the water level declines. The modified formula is more suitable for prediction inflow than empirical formula under the condition of multiple caverns.

6. Conclusion

In this paper, the inflow after excavation of a double-caverns is studied, Using numerical simulation and empirical formula to analyze the influence of cavern spacing and radius on water inflow, this paper draws a conclusion:

For the first time, consider the effect of the spacing ratio of the cavern on the flow rate. The results of numerical solution show that the inflow of double-caverns determined from numerical analysis is generally less than that from empirical formula: 1) When the \( s/r \) is less than or equal to 20, the spacing ratio has significantly on the inflow; 2) when \( 20 < s/r < 60 \), it has some effect on inflow; 3) and when \( s/r > 60 \), it almost has no effect on the inflow.

When the groundwater level is constant or descends, based on empirical formulas, Through numerical simulation, the formula of water inflow considering the spacing ratio of carven \( s/r \) is proposed first time, which enriched the method of predicting water inflow.
Taking an underground water sealed caverns as the background, This paper uses empirical formula and the modified formula to predict the flow. It can be noted that the modified formula prediction results is more close to field data than existing empirical formulas. The formula proposed in this paper has a better engineering value.

7. Data Availability
The numerical simulation and field engineering data used to support the findings of this study are available from the corresponding author upon request.

8. Conflict of Interests
No conflict of interest exits in the submission of this manuscript, and manuscript was approved by all authors for publication. I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed.

Acknowledgments
The corresponding author is supported by State Grid Corporation headquarters science and technology project (Study on the basic technology of underground pumped storage using abandoned mine caverns) The support of the institution is deeply acknowledged.

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