Frequency Optimization of a Splash Shield Based on Topology Optimization and Reverse Engineering

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Abstract. Natural frequency optimization and structure redesign of a bicycle splash shield are completed with the combination of reverse engineering and topology optimization. Based on reverse engineering, the three-dimensional model of the splash shield is established. The maximum first natural frequency is taken as the design objective, the maximum acceleration response and minimum volume are taken as constraints, and the material density is taken as design variable. The variable density method is used to realize the optimization of the splash shield. The results show that: the first natural frequency increased by about 5 times under all constraints after optimization; the combination of optimization design and reverse engineering technology is an effective method of structural design.

1. Introduction
Since the concept of frequency optimization appeared in the 1980s, researchers have put forward various algorithms, which are widely used in various fields. Moreover, with the combination of reverse engineering and topology optimization technology, engineers can redesign the existing structure and often get satisfactory results. In order to make the several natural frequencies of the structural system reach the predetermined requirements, the sequential unconstrained minimization techniques (SUMT) is used to optimize the approximation, and the satisfactory results are obtained[1]. For the dynamic optimization process of the complex coupled system, the fixed-interface mode synthesis method is introduced to improve the efficiency of the optimization[2]. Then, the progressive structure optimized method is introduced for structural dynamical optimization with frequency constraint[3]. Because of the good efficiency and accuracy, the generalized second order sensitivity and the re-deleting procedure is used for complex frequency optimization problems[4]. Also, the evolutionary structural optimization(ESO) method is used to research the optimization of the three-dimensional structural multiple frequency[5]. Furthermore, sequential response surface model and genetic algorithm are introduced to the problem of the free vibration frequency[6]. Then, A dynamic multi-objective optimal scheme for continuum topology is proposed based on bi-direction evolutionary structural optimization (BESO) [7]. As the strong grid dependence problem for the evolution criterion of ESO or BESO method, the smooth bi-directional evolutionary structural optimization (SBESO), as a variant of the BESO procedure, is proposed to overcome this disadvantage[8]. With the development of optimization theory and algorithm, many practical engineering problems have been solved effectively. A frequency optimization design and disturbance response analyses are conducted for large flexible deployed antenna to be insensitive to the
disturbance[9]. And, frequency optimization of the rigid-flexible coupling dynamic model is executed to increase the natural frequency[10]. Also, frequency optimization is executed in the design of crankshaft of automobile engine[11] and railway passenger car[12] to improve the natural frequency. In this paper, the natural frequency optimization and structure redesign of a bicycle splash shield are completed with the combination of reverse engineering and topology optimization. And the first natural frequency is improved obviously under all constraints after optimization.

2. Reverse design
Firstly, the point cloud data of the bicycle splash shield is obtained by three-dimensional scanning, and the finite element model is established. Two preparations are needed: surface treatment and reference point pasting. Because the surface of the splash shield is black, it needs increase brightness to improve the scanning effect. Then, several reference points are pasted for combination of multi-view scanning data, as shown in figure 1. The point cloud data of the splash shield is obtained as shown in figure 2.

![Figure 1. Surface treatment of the splash shield](image1)

![Figure 2. Point cloud data of the splash shield](image2)

After removal of error points, noise reduction, smooth, the point cloud data is converted into the high quality model which consists of many triangular patches (figure 3). Subsequently, the surface model of the splash shield is built, as shown in figure 4.

![Figure 3. The polygon mesh of the splash shield](image3)

![Figure 4. The structure model of the splash shield](image4)

3. Determination of dynamic environment
When the bicycle crosses the speed hump, the barrier will give the bicycle an excitation, which is transmitted to the splash shield through the tire and structure, as shown in figure 5 and figure 6. The excitation is attenuated when transmitted to the splash shield due to the existence of tire stiffness, structure stiffness and damping devices. But these factors aren’t to be considered because the structural stiffness optimization of the splash shield is mainly discussed in this paper. Furthermore, it supposed that the excitation directly acted on the connection between the splash shield and the bicycle structure. Then the simplified dynamic model of the splash shield is got in figure 7. Also, the geometric parameters of the typical speed hump are obtained in figure 8.
After the geometric parameters of the speed hump are determined, the movement of the bicycle over the speed hump mainly depends on the speed of the vehicle. The pure rolling and sliding of the tire also exist at the same time. And when the speed increases, the tire could jump up. In order to simplify the analysis process, the following assumptions are made:

1. The tire always keeps contact with the speed hump when crossing it, and the vertical velocity and acceleration components of tire at point A and B are 0;
2. The ascending and descending sections of the speed hump follow the sine acceleration motion law, as shown in figure 9;
3. Because the motion law of the ascending section is consistent with that of the descending section, only the excitation effect of the ascending section on the splash shield is analysed.

According to the assumed sinusoidal acceleration law of motion, the acceleration, velocity and displacement function can be expressed as follows:

\[ a_y = \frac{h \omega^2}{2\pi} \sin \omega t = a_\omega \sin \omega t \]  

(1)
(2)

\[
v_y = \frac{h \omega}{2\pi} (1 - \cos \omega t) = v_a (1 - \cos \omega t)
\]

(3)

\[
s_y = \frac{h \omega}{2\pi} (t - \frac{\sin \omega t}{\omega}) = s_a (t - \frac{\sin \omega t}{\omega})
\]

Where \( a_y, v_y, s_y \) are acceleration component, velocity component and displacement component along Y direction, \( a_a, v_a, s_a \) are the amplitudes of \( a_y, v_y, s_y \), \( h \) is the height of the speed hump, \( \omega \) is the circular frequency of the excitation signal, \( \omega = 2\pi f = 2\pi/T \). \( T \) is the excitation signal period, which is the same as the time that the tire passes through the ascending section of the speed hump. It can be approximately calculated as follows:

\[
T \approx \frac{AB}{v_x}
\]

Where \( v_x \) is bicycle speed on the level road.

The bicycle speed for ordinary people is in the range of 10km/h-30km/h, that is, 2.78m/s-8.33m/s. On this basis, five cases are taken for analysis, as shown in table 1. It can be seen that the frequency of the acceleration excitation mainly concentrated below 100Hz.

| Case | \( L_{AB} \) (mm) | \( V \) (m/s) | \( t \) (s) | \( f \) (Hz) |
|------|-------------------|--------------|------------|-------------|
| 1    | 0.5               | 0.224        | 4.47       |
| 2    | 1                 | 0.112        | 8.94       |
| 3    | 111.8             | 3            | 0.037      | 26.83       |
| 4    | 5                 | 0.022        | 44.72      |
| 5    | 8                 | 0.014        | 71.55      |

4. Dynamic analysis

Modal analysis and frequency response analysis of the splash shield are executed subsequently. Modal analysis is a method to study the dynamic characteristics of structures, which is widely used in the field of engineering vibration. With the modal analysis, the vibration characteristics of the structure in a certain frequency range could be got. The basic principle is to transform the physical coordinates in the vibration differential equation of linear time invariant system into modal coordinates, and to decouple the equations into a set of independent equations described by modal coordinates and modal parameters, so as to obtain the modal parameters of the system.

Frequency response analysis is a method to calculate the structural response under steady-state vibration excitation. The dynamic response of splash shield under excitation could be obtained by frequency response analysis. There are two different numerical methods to evaluate the frequency response: direct frequency response analysis and modal frequency response analysis. In the modal frequency response analysis, structural vibration modes are used to reduce the problem dimensions.

\[
\{x\} = \{\phi\} \{\xi(\omega)\} e^{i\omega t}
\]

(5)

The variables are transformed from physical coordinates \( \{u(\omega)\} \) to modal coordinates \( \{\xi(\omega)\} \) with formula (5). If damping is not considered, the undamped simple harmonic equation of motion at the external frequency could be obtained.

\[
-\omega^2[M]\{x\} + [K]\{x\} = \{P(\omega)\} e^{i\omega t}
\]

(6)

Then, the vibration equation with modal coordinates is got by combining the equation (5) and equation (6).

\[
-\omega^2[M][\phi]\{\xi(\omega)\} + [K][\phi]\{\xi(\omega)\} = \{P(\omega)\}
\]

(7)

In order to decouple the vibration equation, the equation (7) multiples it by \([\phi]^T\):
Where $\mathbf{M}$ is modal (generalized) mass matrix, $\mathbf{K}$ is modal (generalized) stiffness matrix, $\mathbf{P}$ is modal force vector.

Furthermore, the vibration equations with generalized mass matrix and generalized stiffness matrix could be expressed by using the orthogonality of modal shape. Then, the equations are written as a series of uncoupled single degree of freedom systems:

$$ -\omega^2 \mathbf{m}_i \ddot{\xi}_i(\omega) + \mathbf{k}_i \dot{\xi}_i(\omega) = \mathbf{P}_i(\omega) $$

Where $\mathbf{m}_i$ is the $i$-th model mass, $\mathbf{k}_i$ is the $i$-th model stiffness, $\mathbf{P}_i$ is the $i$-th model force.

Because of the existence of damping matrix $\mathbf{B}$, the direct frequency method is used to solve the coupling problem of modal coordinate representation:

$$ \left[ -\omega^2 \mathbf{M} + \mathbf{B} \right] \ddot{\xi}(\omega) + \left[ \mathbf{K} \right] \dot{\xi}(\omega) = \left[ \mathbf{P}(\omega) \right] $$

(10)

Model damping is used to decouple the vibration equation, $b_i = 2m_i\omega\xi_i$. Equation (10) could be expressed as follows:

$$ -\omega^2 \mathbf{m}_i \ddot{\xi}_i(\omega) + i\omega b_i \dot{\xi}_i(\omega) + \mathbf{k}_i \dot{\xi}_i(\omega) = \mathbf{P}_i(\omega) $$

(11)

Then, the response of each mode is calculated as follows:

$$ \ddot{\xi}_i(\omega) = \frac{\mathbf{P}_i(\omega)}{-\omega^2 \mathbf{m}_i + i\omega b_i + \mathbf{k}_i} $$

(12)

The splash shield material is polypropylene (PP), which is a thermoplastic synthetic resin with excellent performance. It is a colourless and transparent thermoplastic light-weight general-purpose plastic with an elastic modulus of 896 MPa, a Poisson's ratio of 0.41 and a density of 1.04 g/cm³. First, the finite element model of the splash shield (figure 10) is established to constrain all the degrees of nodes at the connection of the splash shield and bicycle structure. Based on this, the modal analysis is carried out and the natural frequency of the splash shield less than 100Hz is given, as shown in figure 11.

Figure 10. The FEM model of splash shield
(a). 1th mode, f=2.23Hz  
(b). 2th mode, f=3.37Hz  
(c). 3th mode, f=16.26Hz  
(d). 4th mode, f=34.88Hz  
(e). 5th mode, f=35.96Hz  
(f). 6th mode, f=52.39Hz  
(g). 7th mode, f=67.46Hz  
(h). 8th mode, f=87.58Hz

Figure 11. The mode analysis of splash shield

From figure 11, it can be seen that there are more natural frequencies of splash shield within 100Hz, which means that splash shield will produce more resonance under the excitation of speed hump. According to table 1, with the change of bicycle speed, the excitation frequency of speed hump changes in the range of 0 to 100Hz. In this paper, the model response analysis method is used to analyse the frequency response of speed hump. The finite element model for frequency response analysis is shown in figure 12, and the analysis result is shown in figure 13.
According to figure 13, under the splash shield excitation, node 1721 has the largest response at the frequencies of 2.23Hz and 35.96Hz, corresponding to the first and second vertical bending modes respectively.

5. Topology optimization

The excitation of the speed hump will cause the vibration of the splash shield in the vertical direction, which will make the splash shield close to the tire or even touch the tire. In order to avoid this situation, two methods can be adopted: (1) when installing the splash shield, the height of the installation position can be increased to keep it away from the tire. But at different speeds, the vibration amplitude is different, and the installation distance required is different. Moreover, when the installation distance is too large, the effect of the splash shield preventing throwing water and mud will be weakened; (2) optimize the rigidity of the splash shield to improve the natural frequency, and reduce the vibration amplitude. This method has good applicability for different speeds. Therefore, in this paper, the second method is used to optimize the stiffness of the splash shield with the element density as the design variable and the first vertical bending vibration frequency as the objective. The objective function and constraints can be mathematically expressed as follows:

\[
\begin{align*}
\text{Max } & \ f \\
\text{Find } & \ \rho = \left( \rho_1, \rho_2, \ldots, \rho_n \right)^T \\
\text{S.t. } & \ \alpha \lambda_i \geq f \quad i = 1, 2, \ldots, N_\lambda \\
& \sum_{j=1}^n V_j \rho_j = V \leq 0 \quad j = 1, 2, \ldots, n \\
& 0 \leq \rho_{\min} \leq \rho_j \leq \rho_{\max} \quad j = 1, 2, \ldots, n \\
& (K - \lambda \rho \rho \rho \rho M) \cdot [\phi_i] = 0
\end{align*}
\]

where \( \rho \) is the design variable, \( n \) is the total number of finite element modal of the splash shield, \( N_\lambda \) is the total number of modes selected for analysis, \( \alpha = 0.95 \), \( V_j \) is the volume of the element \( j \), \( \rho_j \) is the density of the element \( j \), \( V \) is the total volume of the design domain, \( K \) is the global stiffness matrix, \( M \) is the global mass matrix, \( \phi_i \) is the \( i \)-th orthogonal eigenvector, \( f \) is the first order natural frequency. Employing the solid isotropic material with penalization (SIMP [13-15]) method, the problem is relaxed for density to have any value between 0 and 1 with small lower bound of \( \rho_{\min} = 0.001 \) to avoid singularities when calculating for equilibrium.
The optimized element density is shown in figure 14, the change curve of the first vertical bending vibration frequency is shown in figure 15, and the comparison of the splash shield vibration frequency before and after optimization is shown in table 2.

![Figure 14. The topology optimization results of the splash shield](image1)

![Figure 15. The change curve of the first vertical bending vibration frequency](image2)

![Figure 16. Finite element model for static stress analysis](image3)

![Figure 17. The results of the static stress analysis](image4)

| Mode | origin natural frequency (Hz) | optimized natural frequency (Hz) |
|------|-------------------------------|---------------------------------|
| 1    | 2.23                          | 6.72                            |
| 2    | 3.37                          | 8.92                            |
| 3    | 16.26                         | 19.69                           |
| 4    | 34.88                         | 33.21                           |
| 5    | 35.96                         | 42.50                           |
| 6    | 52.39                         | 68.01                           |
| 7    | 67.46                         | 68.91                           |
| 8    | 87.58                         | 77.77                           |

It can be seen from figure 16 that the element density of splash shield is the largest in the root area, which is also the connection area with the structure. It can be seen from Table 2 that the frequency increase after topology optimization is not obvious. In order to analyse this phenomenon, a concentrated force of 1N is applied to the outer end of the splash shield, and all freedom of nodes in the connecting area are constrained, as shown in figure 16, and the static analysis is carried out, as shown in figure 17.
The stress in the root area is the largest as shown in the figure 17, and there is a stress concentration phenomenon in this area, which is consistent with the previous topology optimization results. Further analysis shows that when the load is transferred from the splash shield body through the transition area to the connection area. Also it can be seen from the figure 18 that the stiffness of the connection is very weak. Then, an effective transition area between the body and the connection area is redesigned to ensure the continuity of the force transmission path, as shown in figure 19. The modal analysis of the improved splash shield is shown in table 3. It can be found that the frequency within 100Hz has been significantly reduced and the frequency has been significantly increased. According to this, frequency response analysis is carried out, as shown in figure 20. And the response is significantly reduced.

Table 3. Natural frequency of redesigned splashed shield

| Mode | origin natural frequency (Hz) | redesigned natural frequency (Hz) |
|------|-------------------------------|----------------------------------|
| 1    | 2.23                          | 11.92                            |
| 2    | 3.37                          | 16.02                            |
| 3    | 16.26                         | 33.14                            |
| 4    | 34.88                         | 52.61                            |
| 5    | 35.96                         | 71.22                            |
| 6    | 52.39                         | 106.55                           |
| 7    | 67.46                         | 112.24                           |
| 8    | 87.58                         | 122.92                           |

Figure 18. Load transmission path analysis of splash shield
Figure 19. The redesigned splash shield
Figure 20. The frequency response of Node 1721 at the redesigned splashed shield
6. Conclusions

The stiffness optimization of the structure could be realized based on topology optimization and reverse engineering technologies. This method is applied to the design of a splash shield, and the results show that:

1) the first natural frequency increased by about 5 times under all constraints after optimization;
2) the combination of optimization design and reverse engineering technology is an effective method of structural design.

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