The study on lashing ware location of vibration reduction model of detuned blade disc structure

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Abstract. The studies on vibration localization of periodic structure are mainly based on displacement modes at present, while the strain modes have not been applied to the research of vibration localization. Consequently, it was imagined to apply the strain modes to vibration localization based on a finite element model of bladed disk with shrouds, and the vibration localization factors based on strain modes and displacement modes were compared with the condition that the bladed disk is mistuning. The result shows that the vibration localization based on strain modes is much more sensitive to stiffness mistuning than the vibration localization based on displacement modes; what’s more, the strain mode shapes are different with the displacement modes at the same mode order. The result is important to the design of periodic structure with snubbing mechanism.

1. Introduction
The periodic structure is the axial symmetry system formed by several identical substructure arranging along circumference, such as satellite antenna or bladed disk structure [1, 2]. Theoretically all the substructure of periodic structure is same, but mistuning exists inevitably in reality [3]. The mistuning makes vibration energy of periodic structure concentrate in a few blades, reducing fatigue life heavily. [4, 5]

Vibration localization is the important part of vibration mechanics of periodic structure, generally reflecting characteristics of global dynamics of periodic structure, measuring concentration degree of vibration energy of mistuning periodic structure. The research mainly includes this two parts, modal localization and vibration response localization. The modal localization includes modal shape of structure, characteristics of frequency veering, description of modal localization degree, etc. The vibration response localization includes maximum amplitude of forced response of structure, degree description of vibration response localization, etc [6].

For a long time, in the process of studying vibration localization, about two hundred references retrieved are based on displacement modes. Studies on applying strain modal to vibration localization are insufficient. There are many reasons. One important reason is that the studies on periodic structure such as bladed disk focus on free blades-turbine disc structure at present. Its vibration amplitude of blades is big. The vibration localization based on strain modal can reflect concentration degree of mistuning bladed disk structure. However, as the further studies develops, many periodic structures with snubbing mechanism such as bladed disk with shrouds, grouped bladed disk with shrouds and bladed disk with lashing wire are the objects of research. Due to its low sensitivity, displacement modes sometimes cannot reflect vibration localization of mistuning bladed disk structure precisely. In
contrast, as the second mode \cite{7}, strain modal is more sensitive \cite{8} than displacement modes during structural damage detection. Furthermore, it has many methods of application \cite{9, 10}. the structural damage is a typical stiffness mistuning in the periodic structure. Therefore, it’s important to study on application of strain modal and vibration localization of bladed disk with snubbing mechanism.

This paper takes bladed disk with shrouds as example. Based on a finite element mode, it studies vibration localization of stiffness mistuning bladed disk based on strain modal by simulating analysis and compares the former with traditional vibration localization of periodic structure based on displacement modes. This paper develops research methods for vibration localization of periodic structure and also provides theoretical guidance for the stiffness mistuning of periodic system with snubbing mechanism.

2. Finite element mode

2.1. Specification of mode

The finite element mode used for calculation in this paper is shown in the Figure 1. Modeling finite element of bladed disk system is based on simulating the maximum actual condition. This paper refers medium-length bladed disk of a certain type of gas turbine and establishes a bladed disk with shrouds mode.

The top of medium blades mode fixes its position by the shroud contact. The key parameters of bladed disk system are listed as follows: the external radius of disk is 600mm. The radius of axle hole is 50mm. The length of blade is 500mm. It’s variable aspect ratio. Through adopting laying off many times. Quadrilateral blades with shroud is used. The elasticity modulus of blades disk $E = 2 \times 10^5$ MPa. Intensity: $\rho = 7850$ kg/m$^3$.

2.2. Mesh generation

In view of different structural characteristics of all the components of bladed disk and accuracy and efficiency of calculation, this paper generates mesh for turbine disc by using tetrahedral solid 187 element with intermediate node, for blades and blades with shrouds by using hexahedral solid 45 element; the contact coupling of blades with shrouds working face is simulated by face-to-face contact. After mesh generation, the bladed disk has 3360 elements.

![Figure 1. Finite element mode.](image)

2.3. The processing of boundary condition

The axial constraint is applied to the axle hole of turbine disc.

3. Mode localization of bladed disk with shrouds

The frequency veering curve of periodic structure is drawn based on corresponding relationship between natural frequency and nodal diameter. According to the study \cite{11}, the mode in the frequency veering region is very sensitive to blades mistuning and results in dramatic change in mode localization. It’s based on the complex plane mapping theory \cite{12}. For drawing the frequency veering curve of bladed disk and studying the mode localization of bladed disk, this paper studies the frequency of top 100 mode order of bladed disk and corresponding mode vibration by simulation calculation. The statistical top 20 mode order are shown in Table 1.
Table 1. Statistical chart of mode vibration.

| Order | Frequency | Displacement modes shape | Strain modal shape |
|-------|-----------|---------------------------|-------------------|
| 1     | 36.063    | 0 nodal -circle           | 1 nodal -circle   |
| 2     | 39.085    | 1 nodal -diameter         | 1 nodal -diameter |
| 3     | 39.096    | 1 nodal -diameter         | 1 nodal -diameter |
| 4     | 57.897    | 2 nodal -diameter         | 2 nodal -diameter |
| 5     | 57.917    | 2 nodal -diameter         | 2 nodal -diameter |
| 6     | 60.479    | 0 nodal -circle           | 2 nodal -circle   |
| 7     | 127.69    | 3 nodal -diameter         | 3 nodal -diameter |
| 8     | 127.69    | 3 nodal -diameter         | 3 nodal -diameter |
| 9     | 197.98    | 1 nodal -circle 1 nodal -diameter | 1 nodal -circle 1 nodal -diameter |
| 10    | 198.06    | 1 nodal -circle 1 nodal -diameter | 1 nodal -circle 1 nodal -diameter |
| 11    | 215.43    | 1 nodal -circle           | 1 nodal -circle   |
| 12    | 235.04    | 4 nodal -diameter         | 4 nodal -diameter |
| 13    | 235.06    | 4 nodal -diameter         | 4 nodal -diameter |
| 14    | 282.88    | 1 nodal-circle 2 nodal -diameter | 1 nodal-circle 2 nodal -diameter |
| 15    | 282.9     | 1 nodal -circle 2 nodal -diameter | 1 nodal -circle 2 nodal -diameter |
| 16    | 326.16    | 1 nodal -circle           | 2 nodal -circle   |
| 17    | 360.72    | 5 nodal -diameter         | 5 nodal -diameter |
| 18    | 360.75    | 5 nodal -diameter         | 5 nodal -diameter |
| 19    | 373.71    | 1 nodal -diameter         | 1 nodal -diameter |
| 20    | 373.79    | 1 nodal -diameter         | 1 nodal -diameter |

As shown in Table 1, classified by nodal -diameter and nodal -circle, there exists inconsistency in distribution of strain mode vibration and displacement mode vibration, specifically, consistent in node-diameter mode and inconsistent in nodal -circle mode.

![Mode vibration](image1.png)

(a) Mode vibration at the 1st mode order

(b) Mode vibration at the 16th mode order

(c) Mode vibration at the 2nd mode order

(d) Mode vibration at the 5th mode order

Figure 2. Mode vibration of nodal -diameter and nodal -circle.

The nodal diameter is the diameter of the amplitude with value 0 in the mode vibration of periodic structure. Similarly, the nodal -circle is the circle of the amplitude with value 0 in the mode vibration.
On condition that there are N nodal -diameter in the mode vibration, it is called mode vibration with N nodal -diameter. There exists N nodal circle, the mode vibration is called mode vibration with N nodal -circle. The nodal -circle is also called 0 nodal -diameter. The nodal -diameter and nodal -circle have been applied to many mode studies on periodic structures. The figures above show some typical nodal -diameter and nodal -circle. The figure on the left side in each one is displacement modes. The figure on the right side is strain modal at the same mode order.

Figure 2(a) shows displacement mode vibration with 0 nodal-circle and strain mode vibration with 1 nodal -circle. Figure 2(b) shows displacement mode vibration with 1 nodal -circle and strain mode vibration with 2 nodal -circle. Figure 2(c) shows displacement mode vibration with 1 nodal -diameter and strain mode vibration with 1 nodal -diameter. Figure 2(d) shows displacement mode vibration with 2 nodal -diameter and strain mode vibration with 2 nodal -diameter. This shows that there exists different mode vibration of strain modal and displacement modes.

The frequency veering curve of bladed disk drawn is shown in the figure below.

**Figure 3.** Frequency veering curve.

It can be seen from the frequency veering region indicated in the Figure 3 that the bladed disk structure with mode order 17 and 28 are sensitive modal mode order.

**Figure 4.** The maximum amplitude of strain modal of each blade.
The detuning strength (standard deviation of detuning) with 3% stiffness mistuning is exerted on the blades of tuned mode. The bladed disk with mode order 17 is extracted in the context of stiffness mistuning.

Figure 5. The maximum amplitude of displacement modes of each blade.

The Figures 4 and 5 show that there exists mode localization in the bladed disk with shrouds in the context of stiffness mistuning. For quantifying sensitive degree influenced by two modes localization, parameters of modal localization defined by Reference [6] are quoted as follow:

\[
L = \sqrt{\frac{|X|_{\text{max}}^2 - \frac{1}{N - 1} \sum_{i=1,i\neq j}^{N} X_i^2}}{rac{1}{N - 1} \sum_{i=1,i\neq j}^{N} X_i^2}}
\]  

(1)

According to the above parameters, N is the number of blades, J is the serial number of blades of the maximum vibration displacement, its maximum amplitude displacement is |X|_{\text{max}}.

Table 2 lists parameter values of modal localization identified by the mode order which is the most sensitive to mistuning influenced by pre-torsion mistuning of bladed disk.

Table 2. The parameter values of modal localization of bladed disk.

| 17\textsuperscript{th} mode order | Parameter values of modal localization |
|----------------------------------|--------------------------------------|
| Based on displacement modes      | 1.576                                 |
| Based on strain modal            | 12.526                                |

Table 2 shows that strain modal localization is more sensitive to mistuning than displacement modes localization.

4. Vibration response localization of bladed disk with shrouds

In the gas turbine, blades’ exciting force features in period. Therefore, harmonic signal can be used to simulate approximately exciting force of blades. For sake of simple analysis, concentrated force with 18 degrees of equiamplitude but phase difference is exerted on the upper end of each blade, shown in the Figure 6 below. Harmonic response method is employed to conduct response analysis.
Figure 6. Harmonic response analysis mode.

The tuned bladed disk mode and that with stiffness mistuning are respectively conducted by response analysis. The maximum displacement amplitude is extracted. Vibration localization factors are used to calculate vibration response localization of bladed disk based on two modes. The vibration localization factors are shown in the formula below.

\[
L = \frac{|A_{\text{mis max}}| - |A_{\text{tun max}}|}{|A_{\text{tun max}}|} \times 100\%
\]  

(2)

In the above formula, \(A_{\text{mis max}}\) is the maximum response amplitude of mistuning periodic structure. \(A_{\text{tun max}}\) is the maximum response amplitude of the tuned periodic structure.

The results are shown in the Table 3.

Table 3. The maximum amplitude of vibration response and localization.

| 17th mode order | Based on displacement modes | Based on strain modal |
|-----------------|-----------------------------|-----------------------|
| With the condition of tuning | 2.1231 | 2.6477 |
| Stiffness mistuning | 2.4113 | 9.815 |
| Localization factors | 13.572% | 270.7% |

Table 3 explains that the vibration response localization based on strain modal is more sensitive to stiffness mistuning than the vibration response localization based on displacement modes. It can be seen from the comparison between the ratio of parameter values of modal localization in Chart 2 and the ratio of vibration response localization factors, strain modal is more applicable to vibration response localization.

5. Conclusions

This paper attempts to apply strain modal to measure the vibration localization of periodic structure, based on a finite element mode of bladed disk with stubbing mechanism, analyzes the sensitive degree of stiffness mistuning with the condition of vibration localization factors based on strain modes and displacement modes were compared. The results show that vibration localization based on strain modal, especially vibration response localization is more sensitive to mistuning that displacement modes. Consequently, vibration localization based on strain modal is more applicable to study mistuning bladed disk with stubbing mechanism. Moreover, the strain mode shapes are different with the displacement modes at the same mode order.

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