The hierarchical build-up of the Tully–Fisher relation

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ABSTRACT
We use the semi-analytic model GalICS to predict the Tully–Fisher relation in the B, I and K bands, and its evolution with redshift, up to \( z \sim 1 \). We refined the determination of the disc galaxies rotation velocity, with a dynamical recipe for the rotation curve, rather than a simple conversion from the total mass to maximum velocity. The new recipe takes into account the disc shape factor, and the angular momentum transfer occurring during secular evolution leading to the formation of bulges. This produces model rotation velocities that are lower by \( \sim 30 \) km s\(^{-1}\) in case of Milky Way like objects, and \( \leq 20–30 \) km s\(^{-1}\) for the majority of the spirals, amounting to an average effect of \( \sim 20–25 \) per cent. We implemented stellar population models with a complete treatment of the thermally pulsing asymptotic giant branch, which leads to a revision of the mass-to-light ratio in the near-IR. Due to this effect, K-band luminosities increase by \( \sim 0.5 \) at redshift \( z = 0 \) and by \( > 1 \) at \( z = 3 \), while in the I band at the same redshifts the increase amounts to \( \sim 0.3 \) and \( \sim 0.5 \) mag. With these two new recipes in place, the comparison between the predicted Tully–Fisher relation with a series of data sets in the optical and near-infrared, at redshifts between 0 and 1, is used as a diagnostics of the assembly and evolution of spiral galaxies in the model. At redshifts 0.4 \( < z < 1.2 \) the match between the new model and data is remarkably good, especially for later-type spirals (Sb/Sc). At \( z = 0 \) the new model shows a net improvement in comparison with its original version of 2003, and in accordance with recent observations in the K band, the model Tully–Fisher also shows a morphological differentiation. However, in all bands the \( z = 0 \) model Tully–Fisher is too bright. We argue that this behaviour is caused by inadequate star formation histories in the model galaxies at low redshifts. The star formation rate declines too slowly, due to continuous gas infall that is not efficiently suppressed. An analysis of the model disc scalelengths, at odds with observations, hints to some missing physics in the modelling of disc formation inside dark matter haloes.

Key words: galaxies: evolution – galaxies: formation – galaxies: fundamental parameters – galaxies: kinematics and dynamics – galaxies: photometry – galaxies: structure.

1 INTRODUCTION

The theory of galaxy formation has advanced remarkably in the past decade, thanks to the employment of sophisticated methods of investigation such as numerical simulations and semi-analytic models (see e.g. Kauffmann, White & Guiderdoni 1993; Firmani & Avila-Reese 1999; Cole et al. 2000; van den Bosch 2000, 2002; Balland, Devriendt & Silk 2003; Hatton et al. 2003; De Lucia, Kauffmann & White 2004; Baugh et al. 2005; Bower et al. 2006; Menci et al. 2006; Monaco, Fontanot & Taffoni 2007; Cattaneo et al. 2008; Somerville et al. 2008; Trujillo-Gomez et al. 2010). With these tools, the problem of the assembly of structures in the Universe can be addressed, and the vast number of non-linear physical processes that accompany the formation and evolution of galaxies from primordial perturbations can be modelled.

Interesting for galaxy formation are the observations of scaling relations that trace a pattern in the assembly history of the vast
variety of objects that we observe in the Universe. The origin and nature of such relations is at the core of the mechanism of galaxy assembly (Courteau et al. 2007). One of the most firmly established scaling relations is the Tully–Fisher (TF; Tully & Fisher 1977), which correlates the luminosity and the rotation velocity of disc galaxies. The tightness and universality of the relation make it a valid tool to measure extragalactic luminosity distances, since the rotational velocity does not depend on distance or cosmology. But a most interesting aspect of the relation is also the insight it provides on the mechanism of disc assembly (Tonini et al. 2006a).

In the cold dark matter (CDM) scenario for disc galaxy formation (Fall & Efstathiou 1980), the disc forms out of gas that cools inside a dark matter halo. Initially, in equilibrium with the dark matter, the gas collapses into a rotationally supported disc under angular momentum conservation, while the halo evolves adiabatically (Blumenthal et al. 1986). As a consequence, the final equilibrium structure and dynamics of the disc are strongly correlated with those of the dark matter halo, thus providing an insight into the physical halo parameters. In particular, the disc rotation velocity $V_D$, the disc size parametrized through the scalelength $R_D$ and the disc mass $M_D$ are determined by the halo virial properties, $V_{vir} \propto R_{vir} \propto M_{vir}^{1/3}$ (Dutton et al. 2007). The slope of the relation is a function of the radius at which the velocity is measured, mirroring the dark matter mass distribution as a function of galactocentric distance (Yegorova & Salucci 2007).

Interestingly, although both dynamics and baryonic physics enter the TF, the relation is remarkably tight. In particular, galaxies follow the baryonic TF, which links the dynamical properties of disc (stars + gas) and dark matter halo, with a very small scatter (see e.g. McGaugh et al. 2010). In the luminosity versus velocity TF relation, the measured luminosity is given by the stellar emission, and is therefore a proxy for the stellar component. The star formation rate (SFR) and the feedback processes, that regulate the balance between stellar and gas mass, follow a scaling with the dynamical quantities, and provide at the same time a source of scatter.

In galaxy formation models, the supernova feedback efficiency is one of the main parameters driving the slope and zero-point of the TF relation. In fact, being the circular velocity of the disc linked to the halo virial velocity, and therefore to its escape velocity, it is also closely related to the ability of the galaxy to blow a wind. The balance between feedback and star formation sets the amplitude of the TF relation, and its variation across the mass range affects the TF slope (de Lucia et al. 2004; Croton et al. 2006).

Semi-analytic models so far do not have a good grasp of the TF relation. In fact, although both Croton et al. (2006) and de Lucia et al. (2004) reproduce the local J-band TF relation (Giovanelli et al. 1997a,b), this match should not be considered a success because they do not model rotation curves (RCs) in a physically justified way. De Lucia et al. assume the total mass profiles are isothermal (i.e. constant circular velocity), while Croton et al. assume the observed rotation velocity is equal to the maximum circular velocity of the dark matter halo (in the absence of galaxy formation). Both of these assumptions are expected to underpredict the true rotation velocities.

On the other hand, semi-analytic models which actually model the galaxy RCs do not match the TF relation. Among these, GaLICS (Hatten et al. 2003) includes baryons, and Cole et al. (2000), Benson et al. (2003) and Benson & Bower (2010) include baryons, NFW (Navarro, Frenk & White 1997) haloes and halo contraction. In addition, it has been pointed out that semi-analytic models cannot simultaneously account for the mass and angular momentum distribution of disc galaxies in a gas-dynamical context (van den Bosch et al. 2002b; Courteau et al. 2007; Governato et al. 2007). In particular for the modelling of the TF relation, sources of systematic errors other than the feedback implementation include the structure of dark matter haloes inferred from simulations (the halo concentration and density profile for instance), and the modelling of the dynamics of cooling and baryonic collapse inside haloes (see e.g. Tonini et al. 2006a,b, Piontek & Steinmetz 2011).

Of interest in the present paper, the predicted TF relation is also affected by the modelling of the galaxy rotational velocity, and the stellar population models implemented. In this work we improve both these recipes in the GalICS model, and re-address the comparison with the observed TF relation. We consider a broad spectral range from the optical to the near-IR, and in particular we perform the comparison in the K band. We also compare the model and observed evolution of the TF up to redshifts $z \sim 1$, both in the optical and in the near-IR.

In current semi-analytic models, the rotation velocity of disc galaxies is a crude estimate based on the virial velocity of the host dark matter haloes (e.g. de Lucia et al. 2004; Croton et al. 2006), or the sum of a halo + stellar component in spherical symmetry (Hatten et al. 2003). These approximations do not take into account the actual mass profile of the disc, the effects of angular momentum exchange in the galaxy and the dark matter halo expansion due to dynamical friction, and introduce systematic errors both in the zero-point and in the slope of the predicted TF (see Dutton et al. 2007). In support to this point, Monaco et al. (2007) also showed that the observed baryonic TF relation can be matched by hierarchical models if the dark matter halo concentration is lower than predicted by $\Lambda$CDM simulations (for $c = 4$ for halo masses around $10^{12} M_\odot$).

In the present work, we implement for the first time in the semi-analytic model GalICS (Hatten et al. 2003) a recipe for the galaxy RC that takes into account the shape of the disc and the angular momentum exchange between disc and bulge and between the galaxy and the dark matter halo. We do so by modifying the disc and halo parameters following the recipe proposed by Dutton et al. (2007).

The TF relation predicted by galaxy formation models is also determined by the conversion between stellar mass and light, which is performed through the use of stellar population models. In the original version of GaLICS (Hatten et al. 2003) the observed TF relation was not reproduced. In two recent papers (Tonini et al. 2009, 2010) we showed how the predictions of semi-analytic models are greatly affected by the choice of the input stellar populations. In particular, we showed how the use of the Maraston (2005, hereafter M05) models allows GaLICS to reproduce the colours and near-IR luminosities of $z \sim 2$ galaxies, in the mass and age ranges accessible to hierarchical clustering. In this paper, we address the impact of the M05 models on the predicted TF relation in the optical and near-IR, and its evolution with redshift.

This paper is organized as follows. In Section 2, we introduce the galaxy formation model, and we describe the theoretical recipe to derive the galaxy RCs from the dynamical properties of the galaxies and their host dark matter haloes, the stellar population models employed and the method for the selection of model spirals, including the definition of a proxy for the morphological differentiation of these objects. In Section 3, we describe the data samples used to make the comparison between model and observations. In Section 4, we describe the results for the $z = 0$ TF, and its dependence on morphology. In Section 5, we address the evolution of the TF with redshift. Finally, in Section 6 we discuss our results and conclude.
2 THE MODEL

We produce the model galaxies through the hybrid semi-analytic model GalICS (Hatton et al. 2003), and we refer the reader to its original paper for details on the dark matter N-body simulation and the implementation of the baryonic physics. In brief, the model builds up the galaxies hierarchically, and evolves the metallicity consistently with the cooling and star formation history (with the new implementation of the chemical evolution by Pipino et al. 2009). Feedback recipes for supernovae-driven winds and AGN activity are implemented in the code (the latest with the improved version of Cattaneo et al. 2008). Merger-driven morphology evolution and satellite stripping and disruption are taken into account.

In order to compare the model output with the data, we produce mock galaxy catalogues both in rest frame and observer frame, adopting each time the broad-band filter sets of the data available for comparison. The broad-band magnitudes thus obtained are further scattered with Gaussian errors comparable to the observational errors of our data samples (on average $\sigma = 0.1$ mag). Dust extinction is treated in the model, however the observed TF relations that we use for the comparison with data are corrected for internal extinction by the various authors. For this reason, we produce unreddened galaxies for the comparison with rest-frame TF relations. When comparing our model galaxies with observed-frame data (not corrected for reddening), we implement dust extinction with a Calzetti extinction curve and a colour-excess $E(B - V)$ proportional to the SFR for each single galaxy, parametrized as $E(B - V) = 1/3 \left( \log \text{SFR} - 2 \right) + 1/3$ (see Tonini et al. 2010 for a discussion about this choice). Magnitudes are presented in the AB system. We calculate the galaxy rotational velocity at the standard value of $r = 2.2R_D$ (where $R_D$ is the exponential disc scale-length), consistently with the observations used in our comparisons.

The semi-analytic model is rendered for the parameter set $H_0 = 71.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.742$ and $\Omega_{\Lambda} = 0.258$; these parameters are used to compute observed-frame magnitudes. The model absolute magnitudes are not directly dependent on the cosmology (which obviously enters in the physics of the semi-analytic model, but not in the computation of the rest-frame light emission from galaxies); when comparing with multiple sets of data, analysed by different authors with various cosmologies, we re-scale all units to the model $h = H_0/100$.

2.1 Galaxy rotational velocity

We calculate the rotational velocity of each galaxy by building its RC and evaluating it at $r = 2.2R_D$, where $R_D$ is the disc scalelength. Galaxies in the models are systems composed of a dark matter halo and stars distributed in a disc and a bulge; neglecting the gas component (which is not dominant in mass, at least at $z < 1$) the RC is

\[ V_{\text{tot}}^2(r) = V_{\text{halo}}^2(r) + V_{\text{disc}}^2(r) + V_{\text{bulge}}^2(r), \]

with $r$ being the galactocentric distance. For the spheroidal components, i.e. the halo and the bulge, the contribution to the velocity is simply $V^2(r) = \sqrt{GM(r)/r}$. Following the original GalICS recipe, the halo density profile is defined as a truncated isothermal sphere. In the simulation haloes are tidally truncated by the surrounding material (e.g. Maller & Dekel 2002; Dutton & van den Bosch 2009). On the other hand, the ratio can also become greater than unity in case of removal of low angular momentum material (as is illustrated later on in this section). In general, the choice of the halo functional form is a source of uncertainty, and it should be noted that there is an ongoing debate in the literature regarding the shape of dark matter haloes in the presence of baryons, especially in the centre of galaxies where the dark matter component reacts to baryonic mass assembly with either expansion or contraction (see e.g. Tonini, Lapi & Salucci 2006b, Duffy et al. 2010).

The halo is almost entirely pressure-supported, with a mild component, parametrized by $\lambda = |E|^{1/2}/GM^2_H$ in terms of the halo total mass $M$, angular momentum $J$ and binding energy $E$. In simulations the distribution of the halo spin parameter appears to be lognormal, with both median and scatter independent of mass and redshift to a good approximation (see e.g. Bullock et al. 2001; Vitvitska et al. 2002; Hetznecker & Burkert 2006; Bell et al. 2007; Macciò et al. 2007). The median spin parameter for GalICS-relaxed haloes is $\lambda \sim 0.04$. For each galaxy in GalICS, all halo parameters (virial mass and radius, and velocity dispersions) are directly taken from the N-body simulation, and naturally cause some scatter in the halo profile.

The bulge stellar mass distribution follows a Hernquist (1990) profile, characterized by parameters $(1, 4, 1)$, and is assumed to be entirely pressure-supported (see Hatton et al. 2003, for details on bulge formation and geometry).

The disc structure and rotation is determined by the halo spin parameter $\lambda$ following Mo, Mao & White (1998). The disc forms from a baryonic gaseous component that initially has the same specific angular momentum as the halo, a mass $m_D$ in units of the halo mass, and a total angular momentum $J_D$ in units of the halo angular momentum. The disc is assumed to have an exponential surface density profile, with a scalelength defined as

\[ R_D = \frac{1}{\sqrt{2}} \left( \frac{J_D}{m_D} \right) \lambda R_H, \]

where $R_H$ is the halo virial radius (see Mo et al. 1998, for details). In galaxy formation models, the assumption of specific angular momentum conservation during disc formation is widely used, which leads to $J_D/m_D = 1$. This assumption, while useful to simplify the problem and avoid additional degrees of freedom, is often unrealistic. In fact, angular momentum transfer from the infalling baryons to the halo pushes the ratio lower than unity, and both the halo and the disc structure are affected (see Tonini et al. 2006b; also, Piontek & Steinmetz 2011). On the other hand, the ratio can also become greater than unity in case of removal of low angular momentum material (e.g. Maller & Dekel 2002; Dutton & van den Bosch 2009). A discussion on the disc sizes of the model galaxies is to be found in the last section.

In the Hatton et al. (2003) version of GalICS, the disc component of the velocity was treated as $V^2(r) = \sqrt{GM(r)/r}$ (for future reference, this will be called the ‘spheroidal’ velocity model).
However, neglecting the shape factor in the disc gravitational potential leads to an underestimation of the maximum rotational velocity by about 15 per cent, under angular momentum conservation (Binney & Tremaine 2008). For the disc component, in order to take into account the shape factor we assume that

\[ V_{\text{disc}}^2(r) = \frac{GM_D}{2R_D} \left( \frac{r}{R_D} \right)^2 [I_0(K_0 - I_1K_1)]^{1/2}, \]

where \( M_D \) is the total disc mass, and \( I, K \) are modified Bessel functions of first and second kind (Tonini et al. 2006a; Salucci et al. 2007). This recipe effectively increases the rotational velocity of disc-dominated systems, and has therefore an effect on the TF relation especially at the high-mass end. For future reference, we call \( V_{\text{disc}} \) the total rotational velocity when \( V_{\text{disc}} \) is modelled as in equation (4).

Angular momentum conservation is not a realistic scenario also during bulge formation, as pointed out by Dutton et al. (2007). When bulges form through disc instabilities, part of the material in the disc loses its angular momentum, leading to the formation of a pressure-supported component at the centre. The lost angular momentum is transferred to the rest of the disc and the dark matter halo, as seen in numerical simulations (Hohl 1971; Debattista et al. 2006). This affects the value of \( R_D \) through \( j_0/m_0 \) after bulge formation. Following Dutton et al. (2007), after defining \( \Theta = M_{\text{bulge}}/M_{\text{disc}} \), the new disc scalelength is

\[ R_D = \frac{1}{\sqrt{2}} f_k f_s [1 + (1 - f_s)\Theta] \left( \frac{j_0}{m_0} \right) \lambda R_0; \]  

the quantity \( f_s \) expresses the ratio between the specific angular momentum lost by the disc to the halo during bulge formation, to the total specific angular momentum of the baryonic material that went to form disc and bulge. We re-calculate the disc scalelengths of the galaxies in the simulation with this new value, obtaining

\[ R_D^{\text{new}} = R_D^{\text{new}} \left( 1 + (1 - f_s)\frac{M_{\text{bulge}}}{M_{\text{disc}}} \right), \]

and we use the fiducial value \( f_s = 0.25 \) indicated by Dutton et al. (2007). By inserting this new value of \( R_D \) into equation (4), we obtain what we will refer to as the ‘new’ model for the total rotational velocity, \( V_{\text{new}} \), from equation (1).

Fig. 1 shows the comparison between the three models for the RC, ‘new’, ‘discy’ and ‘spheroidal’ (panels from upper to lower, respectively), for a simulated galaxy at \( z = 0 \), which was chosen for the conspicuous bulge (\( M_{\text{bulge}} = 3.2 \times 10^{10} M_\odot \) and \( M_{\text{disc}} = 3.9 \times 10^{10} M_\odot \)). In all panels, the vertical dotted green line shows the radius \( r = 2.2R_0 \) before including the angular momentum transfer from bulge to disc. In the upper panel, the vertical solid green line shows the new location of \( r = 2.2R_0 \) after angular momentum transfer. In all panels, solid thick black lines show the total RC, solid blue lines show the disc component of the velocity and solid red lines show the bulge component. The solid thick black lines show the halo component as a truncated isothermal sphere, while the dashed thin black lines show the halo contribution as an NFW profile. The dashed thick black lines show the total RC if the halo is an NFW.

But the concentration of the bulge mass at the expense of the disc must be accompanied by angular momentum transfer from the bulge to the disc itself, and according to equation (6), the higher the ratio \( M_{\text{bulge}}/M_{\text{disc}} \), the more the disc scalelength consequently increases, leading to the ‘new’ model and a redistribution of the disc mass. The disc expands radially and, since it conserves its mass, it becomes less dense. Its velocity peak broadens and it is pushed outwards, thus stretching the total RC. Due to the decreased density, the disc potential well is lower, and this, together with the rapidly declining bulge contribution, lowers the total velocity measured at the new \( r = 2.2R_0 \). Since this effect if brought about by the presence of the bulge, it is going to be especially important at the high-mass end of the model TF relation, where bulge formation is more frequent (this is a feature of the hierarchical model which will be discussed later on).

In Fig. 1, we also show the halo component of the total RC. The thin solid black line is the velocity profile of the truncated isothermal sphere, while the thin dashed black line is the velocity profile of the NFW halo of same virial parameters. The total RC obtained with the NFW (thick solid black lines) is virtually undistinguishable from the one obtained with the truncated isothermal sphere (solid thick black lines), especially around the region \( r = 2.2R_0 \) and beyond.

The new recipe for the rotation velocity allows for a sharper differentiation of the model RCs depending on the galaxy morphology, a fact that has been long since observed in nature (Salucci et al. 2007). In particular, in the upper panel of Fig. 1 the bump in the RC due to the presence of the bulge component is clearly visible.
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2.2 Stellar population models

The photometry of the model galaxies is produced with the use of the M05 stellar population models, which include a detailed treatment of the Thermally Pulsing Asymptotic Giant Branch phase of stellar evolution. The implementation of the M05 stellar populations in the semi-analytic model significantly improves the predictions for the colours and near-IR luminosities of galaxies, especially at high redshift. The comparison of the model performance when equipped with the M05 and with its original input stellar populations [Project D’Etude Des Galaxies Par Synthese Evolutive (PEGASE); Fioc & Rocca-Volmerange 1997] shows that the inclusion of the Thermally-Pulsating Asymptotic Giant Branch (TP-AGB) produces near-IR luminosities more than 1 mag brighter for a given stellar mass, and $V-K$ colours more than 1 mag redder, matching the observations of high-redshift galaxies (Tonini et al. 2009, 2010; Henriques et al. 2010).

The TP-AGB light emission starts at wavelengths $\lambda > 5000\,\text{Å}$, peaks in the rest-frame $K$ band and affects the nearby bands from the red part of the optical spectrum down to the infrared. This affects both the zero-point and the slope of the predicted TF relation, as a function of redshift. Fig. 3 shows the difference in magnitude between the TF relation predicted with the M05 and the PEGASE runs of the semi-analytic model, in the rest-frame $I$ band (green dots) and $K$ band (red dots), in four redshift bins from 0 to 3. On the $x$-axis we plot $W = 2V_{\text{rot}}$. The difference between the M05 and PEGASE TF in the $I$ band tends to mildly increase with increasing redshift. The median magnitude difference in the redshift bins centred in $z = 0, 1, 2, 3$ is, respectively, $\Delta(TF) \sim 0.15, 0.22, 0.25, 0.2$ mag. At $z = 2, 3$ there is a mild dependence with galaxy mass, so that the difference can go up to $\Delta(TF) \sim 0.35–0.4$ for massive galaxies. In the $K$ band, we find a significant increase of the difference $\Delta(TF)$ with redshift. In the redshift bins centred in $z = 0, 1, 2, 3$ we find the median $\Delta(TF) \sim 0.2, 0.65, 0.8, 0.7$ mag with maximum $\Delta(TF) > 1$ mag for $z \geq 1$. There is no significant trend with galaxy mass.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The difference in the predicted $z = 0$ galaxy rotational velocity at $r = 2.2R_D$ with different assumptions for the disc velocity, as a function of the total ‘new’ rotation velocity. Magenta points: difference between the original Hatton et al. (2003) model and the ‘new’ model. Cyan points: difference between the ‘discy’ model and the ‘new’ model presented here. For each of the models, we plot the total velocity difference $\Delta V$ against the quantity $\log(W = 2V_{\text{rot}})$, which is used for the comparison with observations later on.

The velocity recipe can be further optimized if one takes into account the dark matter halo evolution triggered by the formation of the galaxy. As proposed in Tonini et al. (2006b) and discussed in Dutton et al. (2007), angular momentum exchange between the collapsing baryonic component and the dark matter causes halo expansion, which affects the halo structural shape and ultimately the dark matter contribution to the RC. However, such a recipe would require more fundamental modifications to the prescriptions for the dark matter haloes in the semi-analytic code, and is beyond the scope of this paper.

Fig. 2 illustrates the difference in the predicted total rotational velocity (at $z = 0$) at the radius $r = 2.2R_D$ between the ‘new’ velocity model and the ‘spheroidal’ model (magenta dots), and between the ‘new’ model and the ‘discy’ model (cyan dots), as a function of $\log(W = 2V_{\text{rot}})$, which is used for the comparison with observations later on.

The combined effect of increasing the disc scalelength and decreasing the bulge contribution to the RC is that the ‘new’ model velocity is always lower than the ‘spheroidal’ model velocity, with a difference up to $\sim 30\,\text{km}\,\text{s}^{-1}$ at the high-mass end. On the other hand, the ‘new’ model velocity is also always lower than the ‘discy’ model velocity, by up to $\sim 50\,\text{km}\,\text{s}^{-1}$ at the high-mass end, where bulges contribute significantly to the stellar mass. The difference $\Delta V$ correlates in fact strongly with the total mass, and tends to zero for galaxies with small bulges.

With the new recipe in place, model rotation velocities are lower by $\sim 20–30\,\text{km}\,\text{s}^{-1}$ or less, compared to the ‘spheroidal’ model, for spirals less massive than the Milky Way, and by $\sim 40–50\,\text{km}\,\text{s}^{-1}$ for Milky Way like objects.

In what follows, we adopt the ‘new’ model to compare the model predictions with the data.

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From now on in this work, the M05 models will be used for all the comparison with data.

### 2.3 Selection of model galaxies

The determination of the morphology of the model galaxies is based on the relative contribution of bulge and disc to some galactic property, and can be carried about in various ways. For the purpose of a more comprehensive study, we adopted two different criteria for the selection of the model spiral galaxies.

Hatton et al. (2003) defined as spiral galaxies the objects for which the ratio between the bulge and disc luminosity in the $B$ band did not exceed a given threshold. Simien & de Vaucouleurs (1986) found that this ratio correlates well with the Hubble type (see e.g. Graham & Worley 2008 for an updated study). Following this criterion, if we define the parameter $\gamma = \exp(-L_{\text{bulge}}/L_{\text{disc}})$, the objects with $\gamma > 0.507$ are defined as spirals (see also Baugh, Cole & Frenk 1996). This classification has the advantage to be observationally oriented, but it depends on the spectrophotometric model in use (including dust reddening), and yields no dynamical information.

From the semi-analytic model point of view, we have the information about the mass and size of disc and bulge before we can also classify our galaxies directly in terms of the ratio between the bulge and disc stellar mass to mimic the morphological differentiation. We choose to split our galaxies in five subsamples, defined by $M_{\text{bulge}}/M_{\text{disc}}$ in five intervals: galaxies with $M_{\text{bulge}}/M_{\text{disc}} < 0.25$ loosely correspond to Sc/irregulars, galaxies with $0.25 < M_{\text{bulge}}/M_{\text{disc}} < 0.5$ loosely correspond to Sb spirals, galaxies with $0.5 < M_{\text{bulge}}/M_{\text{disc}} < 0.75$ loosely correspond to Sa spirals and galaxies with $0.75 < M_{\text{bulge}}/M_{\text{disc}} < 1$ are similar to ‘S0’ types. Strictly speaking, observed S0 galaxies are not identified depending on the bulge/disc mass ratio, but are rather selected to be passively evolving discs, based on colours. Our morphology selection does not depend on photometry, but none the less it naturally preserves the correspondence between bulges and old stellar populations, since bulges in the model do not form new stars (except in merger events).

We find that our somewhat arbitrary selection criterion qualitatively matches the observed properties of spirals (K. Masters, private communication). It is also consistent with other prescriptions used in the literature: we re-selected our Sb/Sc spirals following both Croton et al. (2006) and de Lucia et al. (2004), and found that the resulting subsamples coincide with the one selected in terms of $M_{\text{bulge}}/M_{\text{disc}}$. The totality of the spirals in the range $0 < M_{\text{bulge}}/M_{\text{disc}} < 1$ is almost coincident with the sample selected with the Hatton et al. (2003) criterion (with the exception of some massive bulge-dominated objects that Hatton selected as spirals). As $M_{\text{bulge}}/M_{\text{disc}}$ increases, the average age of the stellar populations tends naturally to increase.

The hierarchical nature of the semi-analytic model causes a large number of spiral galaxies to be subject to some kind of perturbations (like gas stripping and strangulation) when they live in dense regions, with the consequent quenching of star formation. In any galaxy identified as a satellite, i.e. infalling into the halo of a central galaxy (which on the contrary sits in the centre of its halo), cooling of fresh gas is shut down, unless received via a merger. A comparison of the model satellites with observed spirals is therefore improper, and as will be shown in Section 4, largely unsuccessful. For this reason, we use only model central galaxies to build the TF.

### 3 DATA SELECTION

The data at our disposal are either in rest frame or observed frame. When data are in rest frame, the absolute magnitudes, evolutionary and k-corrections, and corrections for internal extinction were performed by the various authors. We compare these sets of data with unreddened model galaxies. When data are in observed frame and uncorrected for dust extinction, we compare them with model galaxies to which we apply our reddening recipe.

#### 3.1 Local data

For the $z = 0$ TF relation in the $I$ and $K$ bands, we use the data by Masters et al. (2006) and Masters, Springbob & Huchra (2008). Their $I$-band sample, called Spiral Field $I++$, is a compilation of objects taken from the Spiral Field $I$ and the Spiral Cluster $I$-band catalogues analysed by Giovanelli et al. (1994, 1995, 1997a,b) and Haynes et al. (1999a,b), the SC2 sample (Dale 1998; Dale et al. 1999), data from Vogt (1995) and Catiniella, Haynes & Giovanelli (2005) and the hydrogen line H$_1$ archive from Springbob et al. (2005). The SFI++ contains 5000 galaxies, and Masters et al. (2006) analyse a subsample of data consisting of spirals in the vicinity of nearby clusters. The rotational widths are derived from H$_1$ global profiles when possible (about 60 per cent of the sample), or optical RCs. See Masters et al. (2006) for details about the velocity derivation. These objects are specifically targeted for $I$-band observations, and although the sample is not complete (in magnitude or volume), it includes all types of spirals. However, as a result of various bias corrections, the sample is designed to be dominated by late-type spirals (see Masters et al. 2006). The derived TF relation is consistent with the one published by Giovanelli et al. (1997).

For the $K$ band, the data come from the 2MASS (Two-Micron All-Sky Survey) Tully–Fisher Survey (2MTF), cross-matched with the 2MASS Extended Source Catalog (XSC) including all the galaxies with rotational widths coming either from the Cornell H$_1$ digital archive (Springbob et al. 2005) or from the Cornell data base of Optical Rotation Curves (ORCs) described by Catiniella et al. (2005). The cross-match was performed for objects in the vicinity of the clusters in the SFI++ sample. About 65 per cent of the galaxies in the 2MTF sample belong also to the SFI++ sample. Details about the corrections applied to the rotational widths, to account for inclination, turbulent motions, cosmological broadening and instrumental effects can be found in Masters et al. (2008), along with a detailed list of all the bias corrections.

For both the $I$- and $K$-band samples described above, Masters et al. (2006, 2008) find a morphological dependence of the slope and zero-point of the TF relation.

Further data for the $I$ band, together with the $B$ band at $z = 0$, come from Verheijen (2001). In this case the sample belongs to the Ursa Major cluster (Tully et al. 1996). Only galaxies with a lower inclination limit of 45° are considered, which leaves a sample of 49 galaxies described as the complete sample. Corrections for internal extinction are treated following Tully et al. (1998). In the complete sample, the rotation velocity is estimated from the width of H$_1$ lines, corrected for instrumental resolution, internal turbulent motions and inclination. Further subsamples are defined according to the observed properties of the galaxies; one of these subsamples is called the Rotation Curve sample, containing galaxies for which synthesis imaging data are available, and the shape of the H$_1$ RC is measured. In this case, velocity is also estimated at the maximum of the RC ($V_{\text{max}}$) or in the flat part of the curve ($V_{\text{flat}}$) depending on the curve shape; these estimates are in good agreement with...
the H I-width method (see Verheijen 2001 for details). We also use the relation derived in the B band by Pierce & Tully (1992) for comparison.

3.2 High-redshift data

The evolution of the B-band TF relation up to $z \sim 1.3$ is compared with data from Fernandez-Lorenzo et al. (2009), in a sample belonging to the Deep Extragalactic Evolutionary Probe (DEEP2) survey (Davis et al. 2003). The photometric catalogue (Data Release 1; Coil et al. 2004) features magnitudes corrected for dust extinction (after Tully et al. 1998), and is $k$-corrected following Blanton & Roweis (2007). The rotation velocity has been estimated from optical linewidths of integrated spectra, obtained through line-fitting techniques. The selection of spiral galaxies was performed through visual inspection of Hubble Space Telescope (HST) images of the All-wavelength Extended Groth Strip International Survey (AEGIS), where it overlaps with the DEEP2. Inclination angles of the selected sample are all between 25° and 80°. The sample was purged of E/S0 galaxies and interactive pairs, and includes spirals and irregulars. For comparison, we also take the published TF of Bamford, Aragon-Salamanca & Milvang-Jensen (2006) for field galaxies at $z = 0.33$.

The $K$-band TF up to $z \sim 1.2$ is compared with data from Fernandez-Lorenzo et al. (2010). The sample belongs to the Groth Strip Survey, with spectroscopy provided by the DEEP2 survey (Data Release 3). The photometric data are part the AEGIS survey (Davis et al. 2003, 2007); the $K$-band data were collected with Palomar WIRC (Bundy et al. 2006), and with the Infrared Array Camera on the Spitzer Space Telescope (Barmby et al. 2008). Data are corrected for internal extinction, and galaxy morphology is selected by eye on HST images. As in Fernandez-Lorenzo et al. (2009), the rotational velocity is determined through optical linewidths.

Another sample of optical and near-IR data comes from Böhm & Ziegler (2007). It contains 73 galaxies with photometry in seven bands from the UV to the near-IR. This observational sample was constructed utilizing the multi-band imaging survey of the FORS Deep Field (FDF; Heidt et al. 2003), which comprises deep U, B, g, R, I, J and K photometry performed with the Very Large Telescope and the New Technology Telescope. To extract RCs from the 2D spectra, the $\lambda$-positions (linecentres) of a given emission line were measured by fitting Gaussians stepwise along its spatial profile. Due to the small apparent sizes of the galaxies, the observed RCs are heavily blurred. At redshifts of $z \approx 0.5$, the apparent scale-lengths are of the same order as the slit width and the seeing disc. To correct for these blurring effects, synthetic rotation velocity fields were generated assuming an intrinsic shape with a linear rise of the rotation speed at small radii and a turnover into a regime of constant rotation velocity $V_{\text{rot}}$ at large radii (for details on this prescription as well as tests of various RC shapes, see Böhm et al. 2004). Intrinsic dust absorption was corrected following Tully & Fouqué (1985).

4 $z = 0$ TULLY–FISHER RELATION

In this section, we present the $z = 0$ model TF compared to the observational relations in the B, I and K photometric bands. This choice of bands is functional to test the overall performance of the model in relation to the SFRs and the balance between stellar populations of different ages in our galaxies, as well as the contribution of the TP-AGB light in the near-IR. We show the morphological differentiation of the $K$-band TF relation, which is predicted in the model and present in the data. Moreover, we analyse the relative contributions of our dynamical recipe and the stellar population models in determining the $K$-band TF relation. We compare the performance of the model for central galaxies and satellites. We also discuss a possible origin for S0 galaxies.

4.1 Tully–Fisher relation in $B, I$ and $K$ bands

Fig. 4 shows the predicted $z = 0$ TF relation in the $B$, $I$ and $K$ bands (from left to right); scatter plots are shown in the upper panels, while the median relations are shown in the lower panels. Disc galaxies in the model are selected according to the $M_{\text{bulge}}/M_{\text{disc}}$ criterion: yellow for $M_{\text{bulge}}/M_{\text{disc}} < 0.25$, magenta for $0.25 < M_{\text{bulge}}/M_{\text{disc}} < 0.5$, blue for $0.5 < M_{\text{bulge}}/M_{\text{disc}} < 0.75$ and red for $0.75 < M_{\text{bulge}}/M_{\text{disc}} < 1$. The whole sample of spiral galaxies selected following Hatton et al. (2003) in terms of the relative luminosity of bulge and disc is represented by the cyan dots. The model velocities are taken at the fiducial value of 2.2 $R_0$. In all bands, we compare the model predictions with the observational relation by Verheijen (2001), which is represented by two lines, indicating a different selection criterion for the observed galaxies. The green solid line represents the complete sample with a measured H I global profile, while the green dashed line represents the RC subsample. In the $B$ band, we also plot the observational relation from Pierce & Tully (1992; thick solid black line). In the $I$ and $K$ bands, we compare the model TF with the observational relation by Masters et al. (2006, 2008; black solid line for the relation, and dot–dashed lines for its scatter). The $I$-band TF relation by Masters et al. (2006) is coincident with the relation found by Giovanelli et al. (1997).

For the $B$ band, the plot shows that the model is successful in reproducing the observed TF slope, especially for the later-type disc galaxies, and lies instead on the bright side in terms of the zero-point. The offset with the Pierce & Tully relation is substantial, while it amounts to $0.2–0.4$ mag with respect to the Verheijen relation. Notice the discrepancy between different observational relations as well; in particular, the relation by Pierce & Tully (1992) is on the faint side of the others, and in fact led to a measured value of the Hubble parameter $H_0 \sim 85$ km s$^{-1}$ Mpc$^{-1}$, much larger than more recent results. For this reason, we consider the discrepancy between the model TF and this particular observational relation less significant.

In the $I$ band, the model is able to reproduce the slope of the $I$-band TF relation very well, and is offset in the zero-point by $0.2–0.4$ mag. The model scatter is also comparable to the observed one. The performance of the model TF is worse here than the results of Croton et al. (2006) and de Lucia et al. (2004), who compare their models with the observed relation of Giovanelli et al. (1997). Both the Croton et al. (2006) and de Lucia et al. (2004) models differ from ours in the stellar population models adopted; by using BC03 instead of M05, the TF is shifted towards fainter magnitudes. Moreover, their oversimplification of the rotation velocity recipe shifts the TF towards slower values.

In the $K$ band, the agreement of the model predictions with the data is better than in the other bands. Consistently, the $K$ band is less affected by uncertainties such as dust reddening. Although the model relation is accommodated inside the data scatter, we note the less notice a discrepancy in the slope, which is more evident than in other bands. We can also identify a cut in velocity, around $\log(W) \sim 2.3$, below which the model galaxies are too bright.

Table 1 lists the slopes and zero-points at $\log(W) = 2.4$ of the model TF relations shown in Fig. 4, obtained with power-law fits of the kind $M = a + b \log(W)$, separated by morphological types. Interestingly, in all the bands considered the model shows...
Figure 4. The $z = 0$ TF relation in the $B$, $I$ and $K$ bands (from left to right). The upper panels show the scatter plots, while the lower panels show the median relations. In all panels, model spiral galaxies are selected according to $M_{\text{bulge}}/M_{\text{disc}}$ (see text). Left-hand panel: the $B$-band model TF (left panel) is compared with the observed relation by Verheijen (2001, green solid/dashed lines, see text) and Pierce & Tully (1992; solid black line). Central panel: the $I$-band model TF is compared with the observed one from Masters et al. (2006; black solid line for the relation, and dot-dashed lines for its scatter) and Verheijen (2001; green solid/dashed lines). Right-hand panel: the $K$-band model TF relation is compared with the observed relation by Masters et al. (2008; solid black line for the relation, and dot-dashed lines for its scatter) and Verheijen (2001; green solid/dashed lines).

Table 1. $z = 0$ model TF relations, power-law fits.

| $M_{\text{bulge}}/M_{\text{disc}}$ | [0,0.25] | [0.25,0.5] | [0.5,0.75] | [0.75,1.] | All spirals |
|----------------------------------|---------|---------|---------|---------|-----------|
| $B$ band                         |         |         |         |         |           |
| Slope $\Delta M/\Delta \log W$  | -7.48   | -7.12   | -6.52   | -5.20   | -4.95     |
| Zero-point at $\log W = 2.4$     | -20.76  | -20.48  | -20.15  | -19.91  | -20.18    |
| $I$ band                         |         |         |         |         |           |
| Slope $\Delta M/\Delta \log W$  | -7.51   | -6.66   | -6.43   | -5.33   | -5.37     |
| Zero-point at $\log W = 2.4$     | -21.50  | -21.33  | -21.12  | -20.94  | -21.11    |
| $K$ band                         |         |         |         |         |           |
| Slope $\Delta M/\Delta \log W$  | -7.97   | -7.02   | -6.74   | -5.72   | -5.71     |
| Zero-point at $\log W = 2.4$     | -21.68  | -21.47  | -21.26  | -21.07  | -21.26    |

a net differentiation between morphological types, with the more disc-dominated galaxies following a steeper relation with brighter zero-points in all bands, while bulge-dominated objects show a flatter slope. The morphology dependence of the $K$-band TF will be investigated in detail later on.

Notice the turn in the model TF slope at the high-mass end, for earlier types; this is the combined effect of AGN feedback and bulge formation. On the one hand, in the more massive objects the onset of AGN activity quenches star formation and thus reduces the galaxy luminosity per unit mass, moving the objects below the relation. On the other hand, while at the low-mass end the mechanism of bulge formation is preferentially secular evolution, which does not introduce a large scatter in the galaxy stellar populations, at the high-mass end bulges form primarily through mergers, which contribute to increase in the scatter and raise the mass-to-light ratio. In addition, the larger bulge masses boost the bulge contribution to the RC and increases the rotation velocity. These factors combined push the model galaxies below the relation at the high-mass end, and increase the scatter in the relation. This may also be the cause why the velocity range covered by GalICS spirals is not as wide as the data. The model in fact does not seem to produce late-type galaxies that are massive enough (contrary to both the De Lucia et al. 2004 and Croton et al. 2006 models). If GalICS overpredicts the abundance of large bulges at the high-mass end, then the number of massive spirals evolving into early-types increases, and these objects are lost from the spiral selection. This is mirrored in the progressively larger scatter and flatterening of the slope for earlier-type model spirals.

Overall, although the model produces a reasonable TF slope, its performance is unsatisfactory in the $B$ band, it is marginally acceptable in the $I$ band and it is better, but not perfect, in the
The hierarchical build-up of the TF relation

K band. The interplay between dynamics and baryonic physics that shapes the TF relation is affected by many factors. We argue that we have good control over the dynamical model for the galaxy RCs and the stellar population models, both of which are physically sensible. Other degrees of freedom are introduced by the supernovae feedback recipe, the chemical evolution model, the star formation history of the model galaxies, dust extinction and the modelling of the data to obtain rest-frame quantities (although such adjustments as the k-correction are small for local samples). The increasingly good match between model and data from the optical to the near-infrared suggests two possible causes for the offset.

The first possible explanation is the treatment of dust reddening. The observed relations used for the comparison have been corrected for internal extinction. Verheijen (2001) argues that this correction amounts to up to 1.4 mag in the $B$ band. Fluctuations in the dust corrections can be of the same order of the offset we see between model and data. The better agreement between model and data in the $K$ band is therefore particularly significant in this scenario, since this band is basically unaffected by dust reddening (and in addition the $k$-correction performed on the data to obtained rest-frame magnitudes is relatively small). However, if internal extinction corrections were the main cause of the disagreement in the optical TF, this would imply that in all the observed relations the dust reddening has been systematically underestimated, which seems implausible.

The disagreement between model and data is present in all bands, but it is much worse in the optical/blue bands, and presents a morphology dependence. This suggests that the cause may be found in the balance between the emissions of stellar populations of different ages, which means that the star formation histories in the model galaxies are not realistic. An excess of luminosity like the one we obtain, with a variance from $B$ to $K$, may hint to an excessively slow decline of the SFR towards low redshifts. As will be shown later on, another hint in favour of this argument is the redshift evolution of the TF, the fact that the match between model and data gets better at higher redshifts.

4.2 Morphology dependence of the Tully–Fisher at $z = 0$

From the previous section it is evident that the ratio $M_{\text{bulge}}/M_{\text{disc}}$ plays a role in determining the slope of the model TF. In particular, as the bulge mass increases with respect to the disc, the slope of the model TF decreases and the scatter increases. In general, model late-types agree better with the observed slope. This is encouraging, as the samples of galaxies selected to observe the TF relation are usually biased towards late-type spirals, which tend to feature cleaner RCs, due to stronger emission lines (Masters et al. 2008).

The $M_{\text{bulge}}/M_{\text{disc}}$ ratio is a reasonable proxy for Hubble type, in that it gives a fair representation of the percentage of galaxy mass that is involved in star formation activity (model bulges do not receive cooling of fresh gas). Fig. 5 shows the morphology dependence of the $z = 0$ model TF relation in the $K$ band, according to our bulge/disc stellar mass classification. The results are shown in comparison with data from Masters et al. (2008; dashed black lines and yellow shaded area representing the relation and its scatter), and are divided according to the Hubble type. In the upper panel, we compare observed Sa galaxies with the model galaxies with $0.5 < M_{\text{bulge}}/M_{\text{disc}} < 0.75$ (red dots) and galaxies with $0.75 < M_{\text{bulge}}/M_{\text{disc}} < 1$ (red circles); observed Sb galaxies compared with models galaxies with $0.25 < M_{\text{bulge}}/M_{\text{disc}} < 0.5$ (red dots); observed Sc galaxies compared with models galaxies with $0.25 < M_{\text{bulge}}/M_{\text{disc}}$ (red dots).

![Figure 5](https://example.com/figure5.png)

**Figure 5.** The morphology dependence of the $z = 0$ K-band TF relation. The model predictions are compared with data from Masters et al. (2008) (dashed solid lines, yellow shaded area representing the data scatter). From upper to lower: observed Sa galaxies compared with model galaxies with $0.5 < M_{\text{bulge}}/M_{\text{disc}} < 0.75$ (red dots) and galaxies with $0.75 < M_{\text{bulge}}/M_{\text{disc}} < 1$ (red circles); observed Sb galaxies compared with model galaxies with $0.25 < M_{\text{bulge}}/M_{\text{disc}} < 0.5$ (red dots); observed Sc galaxies compared with models galaxies with $0.25 < M_{\text{bulge}}/M_{\text{disc}}$ (red dots).
0.5 < \frac{M_{\text{bulge}}}{M_{\text{disc}}} < 0.75 \) (red dots), and we also show model galaxies with \( 0.75 < \frac{M_{\text{bulge}}}{M_{\text{disc}}} < 1 \) (red circles); in the middle panel, we compare Sc types with model galaxies with \( 0.25 < \frac{M_{\text{bulge}}}{M_{\text{disc}}} < 0.5 \) (red dots); in the lower panel we compare Sc types with model galaxies with \( 0.25 < \frac{M_{\text{bulge}}}{M_{\text{disc}}} \) (red dots).

The model TF shows a differentiation with galaxy morphology, contributed by the more sophisticated recipe for the model RCs. The same differentiation is shown by the data, with later-type spirals exhibiting a shallower slope. The model TF is consistent with the data, with a good match with Masters et al. (2008) for Sb/Sc spirals (see Table 1), for velocities \( \log(W) > 2.2 \), although the slope is slightly different and the scatter is smaller. The model relation for Sa spirals is on the faint side of the data, with a clear bending of the slope at the high-mass end, due to the increasing bulge/disc mass ratio in the model galaxies. The success of the model in producing a morphology dependence of the TF shows that the balance between bulge and disc emission in the \( K \) band is correctly accounted for thanks to the use of the M05 stellar populations. Moreover, it also shows that the balance between the bulge and disc dynamics is effectively represented by the new RC recipe.

As evident from Fig. 5 and the figures in the previous section, the model TF relation shows a small scatter for late-type galaxies, and an increasingly large scatter as the \( \frac{M_{\text{bulge}}}{M_{\text{disc}}} \) ratio increases. This mirrors the different mechanism for bulge formation in late- and early-type spirals. Small bulges are likely to be formed entirely through secular evolution; they do not alter the RC in a significant way, and their stellar populations are coeval with the ones in the disc. Massive bulges are more likely to be formed via mergers, which introduces a large random factor both in the ratio \( \frac{M_{\text{bulge}}}{M_{\text{disc}}} \) and in the relative ages and chemical composition between disc and bulge stellar populations. The presence of a large bulge manifests itself with a steepening or a bump in the central part of the RC, which increases the rotational velocity at \( 2.2R_e \) at a given magnitude. Moreover, the presence of a massive bulge tends to dim the galaxy emission per unit mass (with respect to a disc-dominated object of the same mass). In addition to increasingly massive bulges, the downward trend at the very high-mass end of the theoretical relation, for the earlier types, is due to the onset of AGN feedback, that quenches star formation, and makes galaxies less luminous per unit mass.

Although we plotted the Masters et al. (2008) relation as a straight line, from their paper it is evident that this trend is visible in the data as well, and is apparent for Sa and Sb types. The model seems to mirror this behaviour, although in the data, it is observed at about \( \log(W) \approx 2.6 \), while GalICS produces it around \( \log(W) \approx 2.4–2.5 \). It is evident that GalICS cannot cover the mass range of the data due in part to the simulation limits; very large spirals are extremely rare in nature, and they occur with 0 probability in the simulation. Moreover, for increasing masses the model galaxies are more likely to experience mergers, which produce larger bulges and increase the \( \frac{M_{\text{bulge}}}{M_{\text{disc}}} \) ratio; galaxies therefore are transformed into earlier types and progressively disappear from the TF relation.

4.3 Relative contribution of dynamics and stellar population models

Fig. 6 summarizes the relative effects of a more precise and realistic model RC, and of more complete stellar populations model. In the four panels, the \( z = 0 \) model TF relation in the \( K \) band is shown for the Sa spirals (with error bars representing its scatter), compared to the corresponding observed TF by Masters et al. (2008) (black solid line, see lower panel of Fig. 5). In each panel, a different combina-

tion dynamical model/stellar population model is shown, between the ‘new’ and ‘spherical’ model for the RC (see Section 2.1) and between M05 and PEGASE stellar population models (see Section 2.2). The effect of different RC models and stellar population models on the TF is of comparable magnitude. The M05 run of the semi-analytic model produces a \( K \)-band TF brighter by \(~\text{0.4 mag}\) than the PEGASE run. The ‘new’ recipe for the rotation velocity shifts the TF by \(~\text{20–30 per cent in velocity},\) depending on the galaxy mass and morphology. There is a certain degeneracy between the dynamics and stellar population models. The ‘new’ dynamical model works better than the ‘spherical’ for both stellar population models.

The analysis carried out so far implies that the model TF is flawed at \( z = 0 \), in the \( B, I \) and \( K \) bands, even after we adopted refined and updated prescriptions for the galaxy RCs and the stellar populations. We use this conclusion to gain a better grasp of more profound problems of the model in reproducing the spiral population, in particular regarding the cooling and star formation histories at low redshifts, as will be discussed further in this work.

4.4 Satellites and the TF relation

Satellite galaxies in the simulation are defined as objects that do not reside in the centre of the parent dark matter halo, but are orbiting around or infalling into a central galaxy. The gas in satellites is stripped because of ram pressure and tidal effects, while the cooling of new gas is prevented. This effectively shuts down star formation in satellite galaxies, which evolve passively unless a merger event takes place. For this reason, model satellite galaxies are ill-suited to be compared with real spirals. For the purpose of clarification and completeness, we show the predicted \( K \)-band TF relation for satellite spirals and compare them to centrals.

Fig. 7 shows the \( z = 0 \) \( K \)-band TF relation for a sample of model spiral galaxies in the range \( 0 < \frac{M_{\text{bulge}}}{M_{\text{disc}}} < 1 \), and additionally split into central (magenta dots) and satellite (blue dots) galaxies. It is clear that satellites do not follow a TF relation, but scatter below the central galaxy relation. The main reason is that satellites are less bright, due to the quenching of the star formation which, even if not affecting directly the \( K \) band, raises the overall mass-to-light ratio. This affects especially the low-mass end of the satellite distribution.

Note that a significant fraction of the galaxies in the Master et al. samples are cluster galaxies, and they follow the same TF relation as the field samples. We can therefore use the TF relation as a tool to evaluate the ability of the model to reproduce the structure of satellite galaxies. We conclude without doubt that the model cannot produce realistic satellites. Again, the main problem resides in the star formation history. Real satellite spiral galaxies, although perturbed when in dense environments, are still star forming, in particular those selected for TF studies. Hierarchical models, in order to reproduce the observed colour–magnitude relation, are forced to shut down star formation in satellites. This highlights a fundamental problem in hierarchical models, which certainly deserves further investigation.

5 STELLAR MASS VERSUS K-BAND TF RELATION: S0 GALAXIES?

There is an ongoing debate about the formation of S0 galaxies. The origin of these objects is still not understood, but current scenarios include massive bulge formation, tidal stripping and star formation quenching as possible causes of a transition from spirals to S0, via a progressive fading of the disc (see Poggianti et al. 2001 and
The hierarchical build-up of the TF relation

Figure 6. Relative contributions of the RC recipe described in Section 2.1 and the stellar population models described in Section 2.2. In the four panels, the $z = 0$ model TF relation in the $K$ band is shown for the model Sa spirals, compared to the corresponding observed TF by Masters et al. (2008) (black solid line, see lower panel of Fig. 5). In each panel, a different combination of dynamical model – stellar population model – is shown, between the ‘new’ and ‘spherical’ model for the RC (see Section 2.1) and between M05 and PEGASE stellar population models (see Section 2.2), as indicated by the labels in each panel.

references therein). In particular, the possibility that S0 galaxies may be spiral galaxies where star formation has been shut down, and the spiral structure has thus become invisible, can be tested with the present study. An interesting consequence of this is the fact that the TF relation should present an offset between S0 and spiral galaxies (Williams, Bureau & Cappellari 2010; Bedregal, Aragon-Salamanca & Merrifield 2006, and references therein), which should be visible in the photometric TF but not in the stellar mass–velocity TF relation. However, Williams et al. (2010) find an offset between observed spirals and S0 both in the $K$-band TF and in the stellar TF, and therefore argue that S0 galaxies are not fading spirals. We use our model to verify whether star formation quenching introduces an offset between the stellar and $K$-band TF relation.

In this section, we show the model TF relation in the $K$ band, compared to the stellar TF relation $M_{\text{tot}}$ versus $V_{\text{rot}}$ for star-forming and nearly passive disc galaxies at $z = 0$. We again differentiate the galaxy morphology through the $M_{\text{bulge}}/M_{\text{disc}}$ ratio. Model disc galaxies with large $M_{\text{bulge}}/M_{\text{disc}}$ and low SFR should in principle present a spectral energy distribution similar to real S0 galaxies. First, the large bulges (built-up preferentially via minor mergers) and the small gaseous mass make these objects the transition point between spirals and ellipticals in the model. Secondly, the low SFRs mimic the quenching due to gas stripping in dense environments as well as the natural fading due to the exhaustion of the gas reservoir.

Fig. 8 shows the comparison between the $z = 0$ stellar TF (left-hand panels) and $K$-band TF (right-hand panels) relations for two subsamples of model galaxies divided according to the instantaneous SFR. In the upper panels, galaxies are selected depending on their total SFR, and triangles show star-forming galaxies with SFR > 3 $M_\odot$ yr$^{-1}$, while circles represent nearly passive galaxies with SFR < 1 $M_\odot$ yr$^{-1}$. In the lower panels instead, galaxies are selected according to their specific SFRs, and triangles represent star-forming galaxies with sSFR > 10$^{-10}$ yr$^{-1}$, while circles represent nearly passive galaxies with sSFR < 3 × 10$^{-11}$ yr$^{-1}$ (the separation between star-forming and nearly passive was taken from Milky Way values as a reference; see Munoz-Mateos et al. 2007). In all panels, galaxies are also colour-coded according to morphology, with cyan circles/filled triangles representing ‘late-type’ spirals with $M_{\text{bulge}}/M_{\text{disc}} < 0.5$, and red circles/filled triangles representing ‘early-type’ spirals with 0.5 < $M_{\text{bulge}}/M_{\text{disc}}$ < 1. Red circles therefore represent the model rendition of S0-like objects (in terms of stellar populations).

When the galaxies are selected according to the total SFR, we find a continuous stellar TF relation, as expected, between star-forming and nearly passive galaxies, while we find a double sequence in the $K$ band, with the star-forming galaxies significantly brighter than their passive counterparts, for a given mass (the difference in mag is ∼0.5, consistent with the findings of Williams et al.). We also
find a morphology segregation in the $K$-band TF, with later-type spirals at the brighter end of the population (except the very top of the mass distribution, where later-types are not found in the model). This segregation tends to disappear in the stellar TF. This result seems to confirm that, indeed, S0 can be considered quiescent discs that are in the process of shutting down the star formation, and in this case the model S0 are represented by the red circles.

If we select the galaxies according to the specific SFR, we find again a different trend between the stellar and $K$-band TF relations. The stellar TF relation shows a net segregation between star-forming and nearly passive galaxies, which form a separate sequence. In addition, there is a net morphology segregation, which puts the earlier-types, nearly passive galaxies at the massive end of the relation for any given velocity. In the $K$-band relation the star-forming, later-type galaxies are clearly on the brighter, lower-mass end of the relation for any given velocity. If we consider that the red circles as the model S0 galaxies, the picture is consistent with S0 galaxies being fading spirals. Earlier types, characterized by massive bulge formation, occupy the high-mass end of the population. The presence of a massive bulge reduces the fraction of galaxy mass that is actively star forming, so that the specific SFR and the luminosity are low compared with later-types of similar mass.

Although more investigation is needed on this topic, we are encouraged to conclude that the scenario that describes S0 galaxies are fading spirals is promising. We note that a direct comparison between our results and the findings of Williams et al. (2009) is problematic, due to the differences between the modelling of the dynamical mass by Williams et al. and the mass–velocity relation in GalICS. Further investigations on these results are under development.

### 6 The Redshift Evolution of the TF Up to $z = 1$

The evolution of the TF is a good test of the galaxy assembly mechanism in the semi-analytic model, and here we present it in the optical and near-IR.

#### 6.1 Evolution of the Optical TF

Fig. 9 shows the redshift evolution of the model rest-frame $B$-band TF relation from $z \sim 0.4$ to $z \sim 0.8$, compared with data from Fernandez-Lorenzo et al. (2009; black solid/dashed lines representing the observed relation and its scatter, in all panels), who derive the galaxy rotational velocity in a somewhat different manner than the rest of the published observational relations, based on optical linewidths (see also Verheijen 2001 for a comparison between TF relations obtained with different velocity estimators). In the $z = 0.4$ panel, data from Böhm & Ziegler (2007) are shown as black triangles, and the relation by Bamford et al. (2006) at $(z) = 0.33$ is also shown for comparison. The colour-coding of the model dots is as in the previous figures.

This figure shows a very good agreement between the model late-type spirals ($M_{\text{bulge}}/M_{\text{disc}} < 0.5$, yellow and magenta points) and the data at $z \sim 0.6$ and $z \sim 0.8$, in slope, zero-point and scatter. At $z \sim 0.4$ the model reproduces the observed slope, but tends to be slightly fainter in zero-point (although still inside the observed $2\sigma$ uncertainty). Again, the model galaxies show a very clear morphological differentiation, and the earlier-types fall off the relation at the high-mass end at all redshifts, which as already discussed is an effect mainly due to bulge formation, and AGN feedback.

The decline of the global SFR with time, together with the migration of the star formation to lower and lower galaxy masses (the downsizing effect), causes the galaxies to become fainter and the slope of the TF to get shallower at lower redshifts (also confirmed by the results of Weiner et al. 2006).

The model optical TF agrees very well with the data at high redshift, and by $z \sim 0.4$ the match starts to fail, where the model galaxies are too faint. By $z \sim 0$, the model is instead too bright (Fig. 4). This seems to confirm our suspicion that the model galaxy star formation histories are the main drivers of the discrepancy (or match) between model and data. At $z > 0.5$, the model spiral galaxies seem to be quite realistic, while for lower redshifts the gas cooling, the accretion of substructures and the supernovae feedback conspire to produce an excessive star formation at odds with observations.

#### 6.2 Evolution of the Near-IR TF Relation

Fig. 10 shows the evolution of the observed-frame $K$-band TF relation, in the redshift range from $z \sim 0.2$ to $z \sim 1.2$. The colour-coding of the model galaxies is the same as in the previous figures. We compare the model predictions with data from Fernandez-Lorenzo et al. (2010; solid black triangles) and Böhm & Ziegler (2007; solid black squares). In this case the observed magnitudes are not corrected for dust extinction, so the model galaxy spectra are reddened according to their SFR, as discussed in Section 2.

The scatter in the data here is large, and we do not attempt to determine the observed slopes and zero-points. None the less, the accord between the model TF and the observations is good in the range $0.4 \leq z \leq 1.2$. Both the observed velocity range and luminosity range are well reproduced. The model galaxies also occupy the same velocity–luminosity space as the data. Moreover, the redshift evolution is reasonably well reproduced.
The hierarchical build-up of the TF relation

Figure 8. Comparison between the stellar and $K$-band TF relations for star-forming and non-star-forming model galaxies. Left-hand panels: model stellar TF relation at $z = 0$. Right-hand panels: model $K$-band TF relation at $z = 0$. Upper panels: galaxies selected depending on their total SFR. Triangles represent star-forming galaxies with SFR $> 3 \, \text{M}_\odot \, \text{yr}^{-1}$ and circles represent nearly passive galaxies with SFR $< 1 \, \text{M}_\odot \, \text{yr}^{-1}$. Lower panels: galaxies selected depending on their specific SFR. Triangles represent star-forming galaxies with sSFR $> 10^{-10} \, \text{yr}^{-1}$ and circles represent nearly-passive galaxies with sSFR $< 3 \times 10^{-11} \, \text{yr}^{-1}$. In all panels, galaxies are colour-coded according to morphology, with cyan circles/filled triangles representing ‘later-types’ with $M_{\text{bulge}}/M_{\text{disc}} < 0.5$, and red circles/filled triangles representing ‘earlier-types’ with $0.5 < M_{\text{bulge}}/M_{\text{disc}} < 1$.

Figure 9. The redshift evolution of the rest-frame $B$-band TF relation from $z \sim 0.4$ to $z \sim 0.8$, compared with data from Fernandez-Lorenzo et al. (2009). Solid/dashed black lines represent the observed relation and its $2\sigma$ uncertainty, in all panels. In the $z = 0.4$ panel, the data from Böhm & Ziegler (2007) are shown as black triangles. The relation by Bamford et al. (2006) at $\langle z \rangle = 0.33$ is also shown (red line). The model galaxies are represented by dots, colour-coded as in the previous figures.
Figure 10. The redshift evolution of the observed-frame $K$-band TF relation from $z \sim 0.2$ to $z \sim 1.2$, compared with data from Fernandez-Lorenzo et al. (2010; solid black triangles) and Böhm & Ziegler (2007; solid black squares). The model galaxies are represented by dots, colour-coded as in the previous figures.

At $z \sim 0.2$, the model galaxies are too bright at the low-mass end, consistently with the results at $z = 0$ (Fig. 4). Following the combined information of the evolution of the TF in the optical and near-IR, we argue that the model star formation histories are realistic down to $z > 0.4$, and odd for lower redshifts.

7 DISCUSSION

It is not a trivial task to disentangle the different physical processes that conspire to produce the TF relation. The slope and zero-point of the relation are determined by the interplay of the dynamics of galaxy assembly inside the dark matter halo, the cooling that regulates the gaseous and stellar content in each halo, the mass accretion and star formation histories and the supernovae feedback, all of which affect the structure and stability of discs as well as the evolution of the luminosity across all bands. Thus, matching the observed TF scaling relation is a fundamental check that the model is correctly predicting the formation and evolution of spirals.

However, a meaningful comparison with the observed TF necessitates some fundamental steps that ensure that there are no systematics that can offset the results. The first step is the correct modelling of the galaxy RCs, which are not directly predicted by the semi-analytic model itself, but need to be determined based on the galaxy fundamental dynamical quantities. The RCs extrapolate spatial information in model galaxies, following a dynamical recipe that is physically motivated and needs to be as sophisticated as possible. In our ‘new’ model, we made use of models for the mass distribution of the galactic components and angular momentum transfer to the disc accompanying bulge formation. It is worth noting that a complementary tool in this sense is represented by smoothed particle hydrodynamics (SPH) simulations, which favour a detailed study of the galaxy structure, and are able to reproduce the TF relation for single galaxies (e.g. Piontek & Steinmetz 2011), even if the shape of the RC often remains unrealistic (see e.g. Governato et al. 2007), and massive spiral galaxies still represent a problem. The sophistication of current SPH simulations also allows for detailed studies of the effects of perturbations such as ram-pressure on the RCs and the star formation (Kronenberg et al. 2008; Kapferer et al. 2009). However, the current resolution limits for this kind of simulations do not allow for cosmological runs and a statistical study.

The second step is the implementation of comprehensive stellar population models, which assures that the photometric side of the scaling relation is not marred by systematic offsets. We showed that this element can introduce a bias as large as 1 mag in the $K$ band, and 0.3-0.4 mag in the $I$ band.

The third step is a consistency check in the comparison with observations that presents challenges not related to the semi-analytic model itself. For instance, in a given band, observed TF relations sometimes are not consistent with one another, and the reason for these discrepancies is probably to be found in the data analysis. In fact, a significant degree of modelling of the raw data is required to produce an observed TF. To name a few, corrections are required for completeness, internal extinction, inclination, average distance and size in case of clusters, morphology, peculiar velocities (see e.g. Masters et al. 2006). In addition, observed-frame magnitudes have to be converted into rest frame through k-correction, and this introduces a further bias in the comparison. As we pointed out in Tonini et al. (2010), it is preferable to translate the semi-analytic model into observed frame and compare the predictions with the apparent magnitudes. This is crucial especially for star-forming objects, because their evolution is fast and the corrections are significant (especially for internal extinction and k-correction).

After the systematics in the modelling and in the comparison with data are taken care of in ‘post-production’, the hierarchical model
can be truly tested, and the problems of the model in reproducing the TF relation can be investigated.

7.1 Star formation histories

The offset between the model and the observed TF relation has the following features: (i) it depends on redshift, with a better agreement between model and data at higher redshift; (ii) at redshift \( z = 0 \) it depends on the photometric band, with a better agreement in the \( K \) band and the worst performance of the model in the \( B \) band; (iii) it presents a morphology dependence across all redshifts and photometric bands. These facts suggest that the star formation history of the model galaxies is unrealistic, after \( z \sim 0.4 \). As a consequence, the predicted ageing of the stellar populations is off, and the emission of stellar populations of different ages is unbalanced. In particular, the decline of star formation with time seems not to be fast enough, with the consequence that the fraction of the total stellar mass constituted of young and intermediate-age stellar populations is too high, a fact that predominantly affects the UV and optical bands, and the near-IR via the TP-AGB light. Note that this does not imply a wrong instantaneous SFR at \( z = 0 \), which in fact is predicted to be consistent with observations (Hatton et al. 2003).

Central galaxies in a hierarchical model never stop accreting material, including fresh gas and gas previously expelled from the galaxy itself. Although this type of star formation history can be chaotic, in most cases at low redshifts GalICS star formation histories show a sufficiently steady decline to be approximated in the form of \( \tau \)-models: \( SFR(t) = SFR_0 \exp(-t/\tau) \). We calculated the predicted luminosity of model galaxies with different star formation time-scales, and found the latter to be a fundamental parameter regulating the relative luminosities in \( B \) and \( K \). For instance, a galaxy of age of 10 Gyr with a star formation history like \( SFR(t) = SFR_0 \exp(-t/10) \) (with a e-folding time \( \tau = 10 \) Gyr) produces a \( B \)-band magnitude brighter by \( \sim 0.83 \) mag compared to a model like \( SFR(t) = SFR_0 \exp(-t/3) \) (with an e-folding time \( \tau = 3 \) Gyr), while it is brighter in the \( K \) band by \( \sim 0.46 \) mag. From Fig. 4 it is evident that a shift of \( \sim 0.83 \) mag in the \( B \) band would produce a much better agreement between model and data, while at the same time, a shift of \( \sim 0.46 \) mag in the \( K \) band would still preserve the accord with the observed relations.

A faster decline of the star formation in time allows the galaxies to fade more rapidly at low redshifts, and brings the predicted TF relation down in luminosity. To size this effect, consider what happens to satellites after the star formation is shut down (Fig. 7). Also, as shown in all previous figures, as more and more of the fraction of galaxy mass is in the inactive bulge component, or AGN feedback kicks in, the galaxy tends to fade and fall below the TF relation.

We argue that, in order to fix this problem, a more radical revision of gas infall and cooling is probably needed. A more detailed investigation of this point is of great interest, but it is beyond the scope of the present study, in that it requires an extended comparison with SFR derived from observations at different redshifts. Note that the derivation of SFR from observations is affected by the adopted stellar population models, dust extinction corrections, star formation history laws, as discussed in Maraston et al. (2010), and the use of inhomogeneous sets of such priors in comparison with our model will lead to significant biases. This line of investigation is currently under development.

In addition to the star formation history, a second-order effect on the optical TF is caused by the chemical evolution model in use. The current version of GalICS implements the Pipino et al. (2009) recipe which, compared to Hatton et al. (2003), produces slightly lower total metallicities, causing the galaxies to be brighter. Depending on age and star formation history, the difference in magnitude amounts to anything between 0 and \( \sim 0.5 \) in the \( B \) band, and less for the \( J \) and \( K \) bands. Accounting for uncertainties in the chemical evolution recipe would only lead to a modest increase of the TF scatter, which would be consistent with the scatter in the data, but it would not shift the overall relation in a significant way.

7.2 Disc scalelength and the disc–halo connection

There is also the possibility that part of the discrepancies that we encountered in our comparison with data are due to a flaw in the modelling of the galactic dynamics. In particular, the galaxy rotation may be offset, regardless of the galaxy type, due to a wrong determination of the galaxy angular momentum. This can be investigated by analysing the disc scalelengths \( R_0 \) produced by the model.

Fig. 11 shows the model disc scalelengths as a function of \( I \)-band luminosity, compared with the relation obtained by Courteau et al. (2007) from observations (solid/dashed lines for the relation and its scatter). The empty dots represent model galaxies in the ‘spheroidal’ model, colour-coded according to \( M_{\text{bulge}}/M_{\text{disc}} \) as in the previous figures. Filled triangles represent the same galaxies after angular momentum transfer from the bulge is evaluated (the ‘new’ model).

It is evident that the model cannot reproduce the Courteau et al. relation. The disc scalelength appears to be too large, and the scatter too big, the latter a problem that was mentioned already in Hatton et al. (2003). More in detail, the model does not reproduce the Courteau et al. relation for the late-type spirals (yellow points), which are mostly untouched by our ‘new’ velocity model. In the old ‘spheroidal’ model, the earlier-type spirals present a marginal agreement with the data, which we argue is accidental. In fact, the values of the disc scalelengths inferred from observations are commonly obtained by fitting the surface brightness profile with an exponential law, and that the result is more accurate the closer the galaxy is to a pure disc. Galaxies with large bulges are not ideal candidates, and they tend to be excluded from the observational samples. The ‘new’ model effectively corrects for the presence of the bulge, moving the earlier-type spirals in the same \( R_0 - L_I \) space as the later-types, in a sense making all galaxies consistently offset with the Courteau et al. relation.

![Figure 11. Model disc scalelengths as a function of \( I \)-band luminosity, compared with the relation obtained by Courteau et al. (2007) from observations (solid/dashed lines for the relation and its scatter). The empty dots represent model galaxies in the ‘spheroidal’ model, colour-coded according to \( M_{\text{bulge}}/M_{\text{disc}} \) as in the previous figures. Filled triangles represent the same galaxies after angular momentum transfer from the bulge is evaluated (the ‘new’ model).](https://academic.oup.com/mnras/article-abstract/415/1/811/989947)
Before analysing the possible causes of the discrepancy, it should be noted that a source of error stems from the use of equation (3) for the determination of \( R_D \). This formula, adopted by Hatton et al. 2003, produces the highest \( R_D \) under angular momentum conservation, in the Mo, Mao and White (MMW) formalism. Additional shape factors that can reduce the value of \( R_D \) are set to unity, but as discussed in Tonini et al. (2006a), their values as inferred from observed RCs are actually close to unity, and vary slowly with disc mass.

As for the cause of the discrepancy with the Courteau data, we offer two explanations.

(i) As discussed in Tonini et al. (2006a) and D’Onghia & Burkert (2004), the halo spin parameter, which is found to vary from \( \frac{\lambda}{m} = 0.04 \) in GalICS to \( \frac{\lambda}{m} = 0.05 \) in other simulations, is too high compared with the one that can be inferred from observations of spirals. If \( \frac{\lambda}{m} = 0.03 \) for instance (the value inferred by Tonini et al. 2006a), then \( R_D \) is smaller by \( \sim 25 \) per cent, which would push our model \( R_D \) towards the Courteau et al. relation. Note that, although \( \lambda \) originates naturally from tidal forces during structure formation, the discrepancy between the simulated and observed \( \lambda \) is not necessarily due to a flaw in the simulations, but rather it hints at a subsequent redistribution of angular momentum between baryons and dark matter.

(ii) As mentioned in Section (2.1), the ratio \( j_D/m_D \) is equal to unity only under angular momentum conservation, which is not a realistic scenario during galaxy formation. In particular, if the baryonic component that collapses to form the protogalaxy is clumpy, it likely dissipates angular momentum in its path to the centre of the dark matter halo, as proposed by Tonini et al. (2006b), and therefore \( j_D/m_D < 1 \). Consequently, the disc scalelength is smaller, and the tension between model \( R_D \) and observations is partially alleviated (\( j_D/m_D \sim 0.75 \) leads to a decrease of \( R_D \) by \( \sim 25 \) per cent). The MMW recipe can therefore be considered a ‘maximal scalelength scenario’, with the real scalelength distribution being centred on smaller values of \( R_D \) because of angular momentum transfer.

Both the effects described above contribute in reducing the size of \( R_D \), and are currently not investigated in semi-analytic models. Note also that such effects do not depend either on redshift or on photometric band, therefore they do not affect our conclusions on the model star formation histories discussed in the previous section.

An additional, largely unknown source of scatter, on both the disc scalelengths and the RC, is represented by the dark matter halo structural evolution. On the one side, readjustments in the halo profile induced by the baryon infall can lead to a dramatic change in the inner density profile (Tonini et al. 2006b), thus altering the contribution of the halo to the total velocity. On the other side, tidal interactions between haloes and mass accretion in the hierarchical assembly repeatedly strip and add material in each halo. There is no standard recipe in semi-analytic models to recalculate the halo equilibrium structure that takes into account all these effects, and since there is no spatial information inside single objects, the mass is the primary driver of halo evolution. The virial radius in particular is not recalculated after every interaction, with the consequence that the halo density profile can oscillate quite a lot. Note that also in \( N \)-body simulations, although all haloes are fitted to a common functional profile (like the NFW or the isothermal sphere), the scatter on the profile is very large, as is the scatter on the mass–virial radius relation (Bullock et al. 2001). The scatter in the virial radius directly translates into a scatter in \( R_D \), while the scatter in the density profile affects the halo velocity. In particular, an overestimation of the disc scalelengths will occur in cases where the mass loss via tidal stripping dominates the halo evolution.

The offset between model and observed disc scalelengths represents an interesting failure of the model, that arguably sheds light on some missing physics. These considerations suggest that baryon cooling and halo structural evolution in the model need some more fundamental revision, which is beyond the scope of the present work.

8 SUMMARY AND CONCLUSIONS

We analysed the predictions of the hierarchical galaxy formation model GalICS on the TF relation and its evolution with redshift. We introduced two new elements in the model: (1) a new recipe for the determination of the galaxy RCs, based on the correct dynamical modelling of the different galactic components, and taking into account the angular momentum exchange between bulge and disc due to secular evolution. (2) The M05 stellar population models that include an exhaustive treatment of the TP-AGB emission, which affects galaxies with ongoing star formation.

Our main conclusions are as follows.

(i) The improvement on the dynamical description of disc galaxies and on the stellar population models impacts the model TF by effects of comparable magnitude; in particular, velocities are shifted to values lower by \( \sim 0–50 \) per cent depending on the galaxy mass and morphology, and the \( K \)-band magnitudes are brighter by \( \sim 0.2 \) mag at redshift zero and up to \( > 1 \) mag at redshift \( z \sim 1 \) and above.

(ii) At redshift \( z = 0 \) the model reproduces the slope of the observed TF for Sb/Sc spirals, in the \( B \) and \( K \) band, but not the zero-point, which is too bright in all bands. In particular, in the \( K \) band the zero-point is too bright for Sb/Sc galaxies at the low-mass end (\( \log(W) < 2.3 \)), although the model galaxies lie within the scatter of the observed TF relation. In the \( B \) and \( I \) bands, the predicted zero-point for Sb/Sc galaxies is too bright across the mass range. We argue that the most probable cause of the discrepancy lies in unrealistic star formation histories at \( z < 0.4 \).

(iii) The model predicts a morphology dependence of the TF relation at all redshifts. At \( z = 0 \) observations in the \( K \) band confirm this trend, and the comparison between model and data is encouragingly good, especially for Sb/Sc spirals. This is an important confirmation that our more sophisticated treatment of the galaxy dynamics is realistic. The model RC now mirrors the morphological differentiation observed in nature, well representing the balance between bulge and disc, and the consequent variations in the predicted TF along the Hubble sequence put in sharper evidence. The model predicts a steeper slope and a smaller scatter for later-type galaxies, and a progressive flattening of the slope and larger scatter for earlier types. This is coherent with the picture of bulge formation in a hierarchical scenario, where the incidence of mergers is significant in the case of massive bulges, while small bulges are more likely to develop via secular evolution. Moreover, with the M05 stellar population models, GalICS is able to correctly reproduce the balance between the bulge and disc emission in the \( K \) band.

(iv) The scatter in the model galaxies increases from late types (Sb/Sc) to early types (Sa/S0), because the main mechanism for bulge formation switches from secular evolution (Sb/Sc) to minor mergers (Sa/S0). At the high-mass end, earlier-type galaxies show a flattening of the slope and tend to fall below the relation due to the presence of massive bulges and AGN feedback.
The study of the TF relation for model satellites galaxies confirms previous hints that the model cannot produce realistic non-central spirals.

The TF relation can be used as a discriminating tool to investigate the origin of S0 galaxies, in the scenario where these objects form from the fading of spirals after star formation shutdown.

The model reproduces the redshift evolution of the TF for later-type galaxies from $z = 0.8$ to $z = 0.4$ in the rest-frame $B$ band, where slope, zero-point and scatter are reproduced for Sb/Sc spirals. The model is consistent with the data in the observed-frame $K$ band at redshifts between $z = 1.2$ and $z = 0.4$. The simultaneous match in the optical and near-IR evolution, although needing more tests with larger data samples, indicates that the mass assembly and star formation histories, as well as the supernovae feedback implementation, are satisfactory at these redshifts.

The model cannot reproduce the whole optical TF evolution from $z = 0$ to $z = 1.2$. We argue that the decline of the star formation is too slow, thus preventing the model from matching simultaneously the local and distant relations.

The model produces disc scalelengths too large compared with observations. We argue that this mirrors a too simplistic recipe of disc formation inside dark matter haloes that does not take into account angular momentum redistribution during the baryonic collapse.

ACKNOWLEDGMENTS

The authors wish to thank the Referee for her/his very useful comments and suggestions, which contributed to improve this work. CT and CM acknowledge the Marie Curie Excellence Team Grant ‘Unimass’ (MEXT-CT-2006-042754) of the Training and Mobility of Researchers programme financed by the European Community. The authors wish to thank Bruno Henriques, Karen Masters, Michele Cappellari, Michael Bureau and Susan Kassin for their interesting suggestions and comments.

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