Robust PI protective tracking control of decentralized-power trains with model uncertainties against over-speed and signal passed at danger

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Abstract
Controlling the movements of trains to desired target speed and distance without breaking through safety regions is of primary importance in practical applications for safety reasons. In the classical train control and protection framework, automatic train operation regulates the speed and distance with respect to tracking desired ones under the supervision of automatic train protection, an independently operating subsystem, to prevent the phenomenon of over-speed and signal passed at danger. This motivates to develop an integrated control scheme combining functions of control and protection, achieving protective tracking control against over-speed and signal passed at danger doubtlessly. Meanwhile, computationally inexpensive control structure is desired for practical applications due to limited computing resource provided by on-board computer on trains. In this paper, a robust control with PI structure and protective tracking is proposed for decentralized-power trains regardless of model uncertainties. Specifically, the circumvent problem of over-speed and signal passed at danger is formulated as prescribed performance control. It is proved rigorously that the proposed approach results in stable closed-loop system, and finally, comparative simulation results are given to demonstrate the effectiveness and advantages.

1 INTRODUCTION

In the last decades, control design for various kinds of trains’ automatic train operation (ATO) has been greatly advanced and developed to improve the performance due to high requirements of multiple objectives, such as energy-saving, riding comfort, and so on [1]. The major function of ATO system is to drive the train automatically with respect to tracking target speed curve versus distance, based on preallocated timetable information obtained from the trackside. While, automatic train protection (ATP) system takes charge of continually checking that the speed of a train is compatible with the permitted speed allowed by trackside signals and the position of a train is compatible with the position authorized by moving authority unit, including automatic stop at certain signal aspects, if not, ATP activates a timely emergency brake to slow down or stop the train.

In the fields of advanced control design, there are numerous reported method done by worldwide scholars. To name a few, intelligent methods including fuzzy systems and neural networks are designed for ATO system in [2–8], adaptive methods for ATO system are developed in [9–14], optimal train control algorithms are reported in [15–20], cooperative control and scheduling methods are given in [21–28]. Despite these well established theoretical results, there are still two important issues from both aspects of practical applications and theoretical analysis can be considered for further improvements: (i) due to the limited computing sources that can be provided by on-board computers equipped on trains, how to design a computationally inexpensive yet efficient control algorithm for practical applications in the presence of model uncertainties? (ii) the violations of permitted speed and position allowed by trackside signals and moving authority unit bring in unsafe factors, and a naturally raised question is, how to design a proper control
algorithm combining the functions of “tracking” and “protection” in an integrated one, and thus avoid unexpectedly triggered emergency braking implemented by ATP doubtlessly. The aim of this work is to give a systematic solution to dispose these issues.

Some previous efforts have been made to keep the states or tracking errors in some predefined allowed regions. An adaptive control that provides an arbitrarily good transient and steady-state response is designed in [29] for a class of minimum phase linear system, which is capable of forcing tracking error to be confined by pre-specified constant after a pre-specified period of time and ensuring overshoot to be bounded by pre-specified upper bound, containing a LTI compensator and switching mechanism. Inspired by this success, a tracking control with prescribed transient behaviour, later named as funnel control [30, 31], is proposed [32], which releases the conditions that controlled plant is of relative degree one and extends to a class of nonlinear systems. The idea of funnel control is to construct an adaptive gain which exhibits large values in the condition that error variable is close-enough to the funnel boundary, and theoretical results have been reported for nonlinear systems with relative degree one [33, 34], relative degree two [30], and high-order nonlinear systems by backstepping methodology [35]. To further present a simplified scheme circumventing the complexity problem of backstepping, a prescribed performance control (PPC) methodology is proposed to also achieve the funnel control result by incorporating the allowed performance function into the original error variable, which has been employed to neural approximation-based linearisable MIMO nonlinear systems [36], neural approximation-based strict feedback systems [37], MIMO affine nonlinear systems [38]. In order to further simplify the control structure, an approximation-free low-complexity PPC is proposed in [39] because of that approximating the “ideal controller” by existing neural networks or fuzzy systems is hard task in practical applications.

In this paper, we propose a robust PI tracking control for a class of decentralized-power trains incorporating the target speed versus position tracking and over-speed and signal passed at danger protection simultaneously. To better mimic the dynamic property of decentralized-power trains, a multiple-mass model is adopted, with \( n \) cars connected by \( n-1 \) couplers. With comparison to existing literatures, the developed controller exhibits the following twofold newly significant features.

1. Since ATP subsystem in practical applications is generally assumed to be capable of guaranteeing the safe concerns of trains independently, methods reported in existing literatures do not consider the “protection” boundaries consequently, thus, unexpected emergency brakes are ineluctable with 100% certainty when implementing control schemes in [2–28]. Our previous work [40] presents the first attempt to address control design with protection constraints, which is developed based on a simple single-mass model. Alternatively, the proposed robust PI protective control in this paper, partially inspired by low-complexity PPC [39] and ATO control with protection constraints [40], is on the basis of multiple-mass model reflecting the decentralized-power property of currently operated trains in most railway lines, and capable of circumventing the problems of over-speed and signal passed at danger with absolute certainty.

2. Approximation and identification based methods are actually computationally expensive for most industrial applications and trains are no exception, because of the reliability-oriented aim, not computing performance-oriented, is desired for on-board computers. In this sense, although well-established and proved theoretical results are obtained, the methods in [29–38] are not applicable to the control of trains due to limited computing resource. Meanwhile, the approximation-free low-complexity PPC [39] ensures satisfactory tracking performance if and only if monotonically decreasing boundary functions are adopted in transforming the tracking error variables, while the safety boundaries against over-speed and signal passed at danger for trains are generally sectionalized constants. In order to improve the closed-loop convergence rate and ultimately steady tracking performance without monotonically decreasing boundary functions, this paper addresses the tracking control problem with PI structure without requiring the prior accurate information of dynamic model, yielding a robust scheme against model uncertainties.

The rest of this paper is organized as follows. Some necessary and formulated problem are given in Section 2. Section 3 presents the main results of this paper, including equivalence analysis of safe protection and prescribed performance, detailed control design steps and rigorous closed-loop stability analysis. Comparative simulation results are shown in Section 4 to show the effectiveness and advantages of the proposed control by implementing to Beijing subway Yizhuang line, containing fourteen stations and thus thirteen operating intervals. Section 5 ends this paper by conclusions.

2 PROBLEM FORMULATION AND PRELIMINARIES

2.1 Model description

A decentralized-power train with \( n \) cars connected by \( n-1 \) couplers can be modelled as the following differential equation [28, 41, 42]

\[
\begin{align*}
\mathcal{M} \ddot{\mathcal{P}} + \mathbf{A} + (B + \mathbf{K}^e) \dot{\mathcal{P}} + C \dot{\mathcal{P}}^2 + M I (q) \sin(\Theta(P)) \\
+ \mathbf{K}^c \mathcal{P} + \mathbf{K} \mathcal{L} &= \mathcal{P}
\end{align*}
\]

(1)

where \( \mathcal{M} = \text{diag}(m_1, m_2, ..., m_n) \in \mathbb{R}^{n \times n} \) is the mass matrix of sequential cars (including powered-cars and trailer-cars) on trains, \( \dot{\mathcal{P}} = [\dot{p}_1, \dot{p}_2, ..., \dot{p}_n]^T \in \mathbb{R}^n \) is the acceleration vector of all cars, \( \mathbf{A} + B \mathcal{P} + C \dot{\mathcal{P}}^2 \) is the basic operation and aerodynamic resistances with \( \mathbf{A} = [a_1, a_2, ..., a_n]^T \in \mathbb{R}^n \), \( B = \text{diag}(b_1, b_2, ..., b_n) \),
is the gradient forces vector, and \( \mathbf{v} \) is the speed vector,
\[
\mathbf{v} = \mathbf{p} = [p_1, p_2, \ldots, p_n]^T = [v_1, \ldots, v_n]^T \in \mathbb{R}^n
\]
is the speed vector,
\[
\mathbf{I} (\mathbf{g}) = \text{diag}(g_1, g_2, \ldots, g_n) \in \mathbb{R}^{n \times n}, g \cong 9.8 \text{ m/s}^2
\]
is the acceleration of gravity,
\[
\sin(\Theta(\mathbf{p})) = [\sin(\Theta(p_1)), \sin(\Theta(p_2)), \ldots, \sin(\Theta(p_n))]^T \in \mathbb{R}^n
\]
is the gradient forces vector, and
\[
\mathbf{P} = [p_1, p_2, \ldots, p_n]^T \in \mathbb{R}^n
\]
is the position vector, \( \mathbf{K}^d \) is the coupled-damping coefficients matrix with
\[
\mathbf{K}^d = \begin{bmatrix}
-k_1^d & k_1^d & k_2^d & \cdots & k_{n-1}^d & -k_n^d \\
 k_1^d & -k_1^d & -k_2^d & \cdots & -k_{n-1}^d & k_n^d \\
 \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 k_{n-1}^d & -k_{n-1}^d & k_n^d & \cdots & \cdots & \cdots
\end{bmatrix}_{\alpha \times n},
\]
is the length vector of jarless couplers with \( \mathbf{L} = [l_1, l_2, \ldots, l_{n-1}, 0]^T \in \mathbb{R}^n \), \( \mathbf{F} = [F_1, F_2, \ldots, F_n]^T \in \mathbb{R}^n \) is the input forces, including traction and braking ones, vector implemented on cars. Choose the \( \mathbf{U} = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^n := \mathcal{M}^{-1} \mathbf{F} \) as the control input vector to be designed, then, the formulated problem can be given as follows.

### 2.1.1 Problem Formulation

The objective of this work is to design a state feedback robust PI control \( \mathbf{U} = \mathbf{F}(\mathbf{P}, \mathbf{p}) \in \mathbb{R}^n \) to ensure the following targets: (i) for the given coupled reference position and speed vectors \( \mathbf{p}_r(t) \) and \( \mathbf{v}_r(t) = \mathbf{p}_r(t)' \), the controlled trains can track the reference trajectories vectors with the tracking errors \( \mathbf{P} - \mathbf{p}_r \) and \( \mathbf{V} - \mathbf{v}_r \) tunable characterized by control parameters explicitly and can be adjusted to as small as possible, therein, \( \mathbf{p}_r = [p_{r1}, p_{r2}, \ldots, p_{rn}]^T \in \mathbb{R}^n, p_{r2} = p_{r1} + l_1, \ldots, p_{r2} = p_{r1} + l_{n-1}, \mathbf{v}_r = [v_1, v_2, \ldots, v_n]^T \in \mathbb{R}^n, p_{1r} \) and \( v_r \) are obtained from off-line optimization [43, 44]. In this study, the length of cars is omitted since no dynamics is introduced by constant lengths. (ii) considering the safe manipulation and over-speed protection of trains, the error variables vectors \( \mathbf{P} - \mathbf{p}_r \) and \( \mathbf{V} - \mathbf{v}_r \) are kept between specified regions characterized by upper and lower boundaries invariably. And (iii) all the resulted closed-loop signals are guaranteed to be bounded.

### 2.2 Assumptions and lemmas

In this paper, the following assumptions and lemmas are used throughout the whole work.

**Assumption 1.** The reference position and speed trajectories vectors \( \mathbf{P}_r \) and \( \mathbf{V}_r \) are bounded versus time and \( C^1 \) functions with \( \mathbf{P}_r = \mathbf{V}_r \).

**Assumption 2.** Depending on diversified speed sensors and location techniques, the state variables of a controlled train, including position \( \mathbf{P} \) and speed \( \mathbf{V} \), are available for measurements and feedback to design feasible control.

**Lemma 1**[45]. In allusion to the initial value issue \( \eta(t) = \varphi(\mathbf{t}, \eta(t)) \), \( \eta(0) = \eta^0 \in \Lambda_\eta \), where \( \eta : \mathbb{R}_+ \times \Lambda_\eta \rightarrow \mathbb{R}^n \) and \( \Lambda_\eta \subset \mathbb{R}^n \) is an open yet non-empty set. In condition that (i) \( \eta \) is locally Lipschitz function over \( \eta \), (ii) \( \varphi \) is continuous on \( t \) for all \( \eta \in \Lambda_\eta \), and (iii) \( \varphi \) is locally integrable function over \( t \) for all \( \eta \in \Lambda_\eta \), one has that a unique maximal solution \( \eta : [0, T) \rightarrow \Lambda_\eta \) of \( \eta(t) = \varphi(\mathbf{t}, \eta(t)) \) over the interval \( [0, T) \), where \( T \in \{R_+, \infty\} \) holds. As a result, \( \eta(t) \in \Lambda_\eta, \forall \ t \in [0, T) \). Moreover, provided that \( T < \infty \), one can find a time moment \( t^* \in [0, T) \) satisfying \( \eta(t^*) \notin \Lambda_\eta^* \) exists for any \( \Lambda_\eta^* \subset \Lambda_\eta \).

### 3 MAIN RESULTS

#### 3.1 Equivalence of protective safe operation and prescribed performance

A schematic diagram showing the relationship of tracking operation and safe protection is given in Figure 1, from where it is observed the permitted speed and authorized position boundaries are characterized by the supervised speed curve versus position (the red solid line, denoted as \( T' \)), including ceiling speed, braking curve, and stopping point. In the meantime, a delay-free operation lower boundary (the blue dot-dash line, denoted as \( T' \)) is used to avoid large excursion from operating plan and disruption. While, the green solid line (denoted as \( T' \)) outputs the target position and speed curves, defining
the reference trajectories to drive a train to track. During the whole running process, it is desired that the actual running curve $T$ of trains track $T'$ with zero or small constant error and $T' < T < T''$, which is equivalent to $T'' := T' - T' < T - T' := T''$ with $T'$ and $T''$ defined as the allowed error boundaries. In consequence, the safe protection control of trains against over-speed and signal passed at danger phenomenon.

**Step I-a:** Supposing that the initial position of all cars of a train $P^0 = P(0)$ are located in the MA-permissible regions, that is, $E_p \vdash > 0$ with $E_{p,i}^r - E_{p,i}^l$ with $E_{p,i}^r$, $E_{p,i}^l$, and $E_{p,i}$ being the $i$th elements of

$$I(P^0) = [p_{p0}(0), \ldots, p_{p0}(0)]^T \in \mathbb{R}^n, \quad P^0 \in \mathbb{R}^n,$$

and $P^0 \in \mathbb{R}^n$, respectively. Design the intermediate virtual controller as:

$$\alpha(P, P_t) := [\alpha_1(p_1, p_{1,t}), \ldots, \alpha_n(p_n, p_{n,t})]^T$$

$$= -\mathcal{K}_{\text{pro,1}} S_p(t) - \mathcal{K}_{\text{int,1}} \int_0^t S_p(s) ds \quad (2)$$

with

$$\mathcal{K}_{\text{pro,1}} = \begin{bmatrix} k_{\text{pro,1,1}} & \cdots & k_{\text{pro,1,n}} \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad k_{\text{pro,1,i}} \in \mathbb{R}^+, \quad \forall i = 1, \ldots, n$$

and

$$\mathcal{K}_{\text{int,1}} = \begin{bmatrix} k_{\text{int,1,1}} & \cdots & k_{\text{int,1,n}} \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad k_{\text{int,1,i}} \in \mathbb{R}^+, \quad \forall i = 1, \ldots, n,$$

with $k_{\text{int,1,i}} \in \mathbb{R}^+$, $\forall i = 1, \ldots, n$, respectively. It is noticed that an initial presupposed condition is required to design the feasible intermediate virtual control. Such an initial condition is easy to satisfy in practical applications since a train is not allowed to

3.2 Robust PI prescribed performance control design

**Step I-a:** Define the position tracking error vector as

$$E_p = P - P_t = [e_{p1,0}, \ldots, e_{pn,0}]^T \in \mathbb{R}^n.$$  

We make use of a nonlinear mapping function

$$\mathcal{N}_1(\#) := \ln \left( \frac{1 + \#}{1 - \#} \right)$$

with

$$\mathcal{N}_1(\#) = \ln \left( \frac{1 + e_{p1,0}/\rho_p}{1 - e_{p1,0}/\rho_p} \right), \ldots, \ln \left( \frac{1 + e_{pn,0}/\rho_p}{1 - e_{pn,0}/\rho_p} \right)^T \in \mathbb{R}^n,$$

with which the original position error vector $E_p$ is converted into the new coordinate $S_p$. Therein, the $\rho_p$ takes charges of receiving the moving authority (MA) related information via ground-side equipments, that is, $P$ is always kept under upper bound $P_t + I\rho_p$, $I = [1, \ldots, 1]^T$ to circumvent the signal passed at danger phenomenon.

**Step I-b:**
break through the boundary of MA grimly and the initial states are also confined in such boundary.

**Step II-a:** Define the speed tracking error vector as

\[
E_v = V - V_i - \alpha(P, P_i) \\
= [\epsilon_v,1, \ldots, \epsilon_v,n]^T \\
= [v_1 - v_i - \alpha(p_1, p_{1,i}), \ldots, \\
v_n - v_i - \alpha(p_n, p_{n,i} + \sum_{i=1}^{n-1} l_i)]^T 
\in \mathbb{R}^n \tag{3}
\]

In a same manner as Step I-a, we use \(N'_i : (-1, 1) \to \mathbb{R} \) with \(N'_i(\bullet) = \ln(\frac{1 + \bullet}{1 - \bullet}) \) to perform the error transformation,

\[
S_v = [\epsilon_v,1, \ldots, \epsilon_v,n]^T \\
= [\ln \left(\frac{1 + \epsilon_v,1}{\rho_v}\right), \ldots, \ln \left(\frac{1 + \epsilon_v,n}{\rho_v}\right)]^T 
\in \mathbb{R}^n,
\]

with which the original position error vector \(E_v\) is converted into the new coordinate \(S_v\). Therein, the \(\rho_v\) takes charges of receiving the speed restriction related information from automatic train protection (ATP) via ground-side equipment, that is, \(V\) is always kept under

\[
V_i + \alpha(P, P_i) + \rho_v
\]

to circumvent the over-speed phenomenon. It is noticed that the precondition of over-speed free is that no signal passed at danger happens which has actually been guaranteed in previous steps by constraining \(P\) as

\[-I\rho_p < P - P_i < I\rho_p\]

**Step II-b:** Supposing that the initial speed of all cars of a train \(V^0 = V(0)\) are located in the speed restriction-permissible regions, that is, \(E_{\rho_p,i} > \left|E_{\rho,i} - E_{\rho,i}^0 - E_{\alpha(p,p_i + \sum_{i=1}^{n-1} l_i)}\right|\) with \(E_{\rho_p,i}, E_{\rho,i}, E_{\alpha(p,p_i + \sum_{i=1}^{n-1} l_i)}\) being the \(i\)th elements of \(I(\varphi^0) = [\varphi, (0), \ldots, \varphi, (0)]^T \in \mathbb{R}^n, V^0 \in \mathbb{R}^n, V^0_i \in \mathbb{R}^n, \) and \(\alpha(P, P_i) \in \mathbb{R}^n\), respectively. Design the control input as:

\[
U(P, P^T, V, V_i) = -K_{\text{pro},2} S_v(t) - K_{\text{int},2} \int_0^t S_v(s) ds \tag{4}
\]

with

\[
K_{\text{pro},2} = \begin{bmatrix} k_{\text{pro},2,1} & \cdots & k_{\text{pro},2,n} \end{bmatrix} \in \mathbb{R}^{n \times n}, k_{\text{pro},2,i} \in \mathbb{R}^+, \forall i = 1, \ldots, n
\]

and

\[
K_{\text{int},2} = \begin{bmatrix} k_{\text{int},2,1} & \cdots & k_{\text{int},2,n} \end{bmatrix} \in \mathbb{R}^{n \times n}, k_{\text{int},2,i} \in \mathbb{R}^+, \forall i = 1, \ldots, n
\]

with \(k_{\text{int},2,i} \in \mathbb{R}^+, \forall i = 1, \ldots, n\), respectively. It is also noticed that an initial presupposed condition is required to design the feasible control input. Such an initial condition is also easy to satisfy in practical applications since a train is not allowed to break through the boundary of speed restriction grimly and the initial states are also confined in such boundary.

**Remark 1.** By utilizing the PPC method, to be specific, the non-linear mapping function \(N'_i : (-1, 1) \to \mathbb{R} \) with

\[
N'_i(\bullet) = \ln \left(\frac{1 + \bullet}{1 - \bullet}\right)
\]

to perform the error transformation, the constrained control problem to circumvent the over-speed and signal passed at danger becomes an unconstrained one by incorporating these constraint boundaries and original error variables into new coordinates \(S_p\) and \(S_v\). In this sense, the following control design problem can be solved without considering the non-differentiatiable points caused by hard constraints raised by over-speed and signal passed at danger, no extra techniques, such as hyperbolic switching control [46], pure hyperbolic function-based control [47], are required and thus the control design procedure is simplified immensely. Meanwhile, though model uncertainties exist directly in the considered model (1), it is noticed that the control design procedures are independent of dealing with uncertainties using adaptive method [10], neural approximation-based control [11], disturbance observer-based method [48], and so on. In this sense, under proper and easily-satisfied practical assumptions, the proposed control method is model-free against model uncertainties even with over-speed and signal passed at danger constraints. Moreover, the proposed control is also applicable in the presence of external disturbances by modifying theoretical analysis parts and keeping the control structure unchanged, which is discussed in the following Section 3.4 in details.

**Theorem 1.** Consider a class of decentralized-power trains with model uncertainties (1) obeying Assumptions 1 and 2. Supposing that a decentralized-power train is initially located satisfying the conditions

\[
E_{\rho_p,i} > \left|E_{\rho,i} - E_{\rho,i}^0\right|
\]

and

\[
E_{\rho,i} > \left|E_{\rho,i} - E_{\rho,i}^0\right| - E_{\alpha(p,p_i + \sum_{i=1}^{n-1} l_i)} \tag{5}
\]

with \(E_{\rho_p,i}, E_{\rho,i}, E_{\rho,i}^0, E_{\alpha}, E_{\alpha}\)
and

$$E_{\alpha(P,\rho)+\sum_{i=1}^{n} b^0}$$

being the $i$th elements of

$$I(\rho_p^0) = [\rho_p(0), ..., \rho_p(0)]^T \in \mathbb{R}^n, P_t^0 \in \mathbb{R}^n, P^0 \in \mathbb{R}^n,$$

$$I(\rho_v^0) = [\rho_v(0), ..., \rho_v(0)]^T \in \mathbb{R}^n, V_t^0 \in \mathbb{R}^n, V^0 \in \mathbb{R}^n,$$

and $\alpha(P, P_t) \in \mathbb{R}^n$, respectively. The proposed robust PI control (2) and (4), shown in Figure 2, is capable of guaranteeing the solve of formulated problems in Section 2 regardless of the model uncertainties.

3.3 Stability analysis and proof of Theorem 1

To proceed the proof of Theorem 1, we first introduce the following two denotations:

$$\Theta_p := [\theta_{p,1}, ..., \theta_{p,n}]^T$$

$$\Theta_v := [\theta_{v,1}, ..., \theta_{v,n}]^T$$

and

$$\Theta_p := [\theta_{p,1}, ..., \theta_{p,n}]^T$$

$$(P - P_t) \otimes I(\rho_p)^{-1} = (P - P_t) \otimes I(\rho_p)^{-1}$$

with $\otimes$ denoting a multiplier exporting $\mathbb{R}^{\otimes 1} \otimes \mathbb{R}^{\otimes 1} \rightarrow \mathbb{R}^{\otimes 1}$ by internal multiply operation using

$$1 \cdot I, 1 = [1, \ldots, 1], I = \text{diag}(1, \ldots, 1_{\otimes}), I(\rho_p)^{-1}$$

and

$$I(\rho_v)^{-1} = \begin{bmatrix} 1 & \ldots & 1 \\ \rho_v & \ldots & \rho_v \\ & \otimes & \\ & & \otimes \\ & & \otimes \\ & & \otimes \\ & & \otimes \end{bmatrix}^T$$

which means

$$P = \Theta_p \otimes I(\rho_p) + P_t$$

$$V = \Theta_v \otimes I(\rho_v) + V_t + \alpha(P, P_t)$$

and

$$I(\rho_p) = \begin{bmatrix} \rho_p, \ldots, \rho_p \\ & \otimes \\ & & \otimes \\ & & \otimes \\ & & \otimes \\ & & \otimes \end{bmatrix}^T$$
and

\[ I(\rho_v) = \left[ \rho_v, \ldots, \rho_v \right]^T. \]

Therein, the \( \rho_p \) and \( \rho_v \) are also known as the position prescribed performance function (PPF) and speed PPF, respectively. With respect to the new coordinates \( \Theta_p \) and \( \Theta_v \), the new closed-loop dynamics of a decentralized-power train can be rewritten as

\[
\dot{\Theta}_p := \Phi_p(\Theta_p, \Theta_v, t)
\]

\[
= \left[ \Theta, \Theta I(\rho_v) + \alpha(P, P_t) - \Theta_p \Theta I(\rho_p) \right] \Theta I(\rho_p)^{-1}
\]

\[
= \left[ \Theta, \Theta I(\rho_v) - K_{pro1} S_p(t) - K_{int1} \int_0^t S_p(\tau) d\tau - \Theta_p \Theta I(\rho_p) \right] \Theta I(\rho_p)^{-1}
\]

\[
\dot{\Theta}_v := \Phi_v(\Theta_p, \Theta_v, t)
\]

\[
= \left[ U - M^{-1} A - M^{-1}(B + K^d) P - M^{-1} CP^2 \right] \Theta I(\rho_p)^{-1}
\]

\[
- \Theta \left[ K_{pro2} S_v(t) - K_{int2} \int_0^t S_v(\tau) d\tau - M^{-1} A \right] \Theta I(\rho_p)^{-1}
\]

\[
+ \left[ -M^{-1} (B + K^d) (\Theta, \Theta I(\rho_v) + \dot{V}_v + \alpha) \right] \Theta I(\rho_p)^{-1}
\]

\[
+ \left[ -M^{-1} C (\Theta, \Theta I(\rho_v) + \dot{V}_v + \alpha)^2 \right] \Theta I(\rho_p)^{-1}
\]

\[
+ \left( -I(\rho) \sin \Theta(P) \right) \Theta I(\rho_p)^{-1}
\]

\[
+ \left[ -M^{-1} K^v (\Theta, \Theta I(\rho_v) + \dot{P}_v) - M^{-1} K^L \right] \Theta I(\rho_p)^{-1}
\]

\[
+ \left[ -\dot{V}_v - \dot{\alpha} - \Theta, \Theta I(\rho_v) \right] \Theta I(\rho_p)^{-1}.
\]

(7a)

Define

\[
\Theta := \begin{bmatrix} \Theta_p \\ \Theta_v \end{bmatrix} \quad \text{and} \quad \Phi(\Theta_p, \Theta_v, t) := \begin{bmatrix} \Phi_p(\Theta_p, \Theta_v, t) \\ \Phi_v(\Theta_p, \Theta_v, t) \end{bmatrix},
\]

(7)

can be rewritten in a following compact form:

\[
\dot{\Theta} = \Phi(\Theta_p, \Theta_v, t)
\]

(8)

What follows, the proof of Theorem 1 would be completed by three steps.

\textbf{Step A:} Let us denote \( \Lambda_\Theta = \Lambda_{\Theta_p} \times \Lambda_{\Theta_v} \) with

\[
\Lambda_{\Theta_p} = \Lambda_{\Theta_v} = \left[ (-1, 1), \ldots, (-1, 1) \right]^T,
\]

which is nonempty and open vector set. According to the principles mentioned in the above design steps, the initial conditions are

\[
E_{\rho_p,i} > |E_{\rho_p,i} - E_{\rho_p,i}|
\]

and

\[
E_{\rho_v,i} > |E_{\rho_v,i} - E_{\rho_v,i}|
\]

with

\[
E_{\rho_p,i}, E_{\rho_v,i} \in \mathbb{R}^n
\]

and \( E_{\rho_p,i}, E_{\rho_v,i} \in \mathbb{R}^n \) being the \( i \)-th elements of \( I(\rho_p, 0) = [\rho_p(0), \ldots, \rho_p(0)]^T \in \mathbb{R}^n, P^0 \in \mathbb{R}^n, P_t^0 \in \mathbb{R}^n, I(\rho_v, 0) \)

\[
\in \mathbb{R}^n, \quad \Phi(\Theta_p, \Theta_v, t) = 0 \quad \text{for all} \quad i = 1, \ldots, n
\]

meaning that \( \Theta_p^0 \in \Lambda_{\Theta_p} \) and \( \Theta_v^0 \in \Lambda_{\Theta_v} \).

It is noticed that \( M^{-1} A + M^{-1}(B + K^d) P + M^{-1} CP^2 + I(\rho) \sin \Theta(P) + M^{-1} K^P + M^{-1} K^L \) containing terms of constants and functions vectors with respect to \( P \) and \( P_t \) is a smooth functions vector versus \( P \) and \( P_t \) and time. As the receiving on-board terminals of information from MA and ATP, \( I(\rho_p) \in \mathbb{R}^n \) and \( I(\rho_v) \in \mathbb{R}^n \) are also ensured to be smooth functions vectors. According to Assumption 1, \( P_n, P_t = V_t \) and \( V_v \) are smooth functions vectors. Further, the components of proposed intermediate virtual control \( \alpha(P, P_t) \) and \( U(P, P_t, V, V_v) \) are also smooth functions vectors over \( \Theta \).

Based on these observations and Lemma 2, we can reach the conclusion that an one and only maximal solution \( \Theta : [0, T) \rightarrow \mathbb{R}^n \).
\[ \Lambda_{\Theta} \] of \( \hat{\Theta} = \Phi(\Theta_p, \Theta_v, t) \) over \( [0, T) \) guaranteeing \( \Theta(t) \in \Lambda_{\Theta} \), \( \forall t \in [0, T) \), to be specific, \( \Theta_p \in \Lambda_{\Theta_p} \) and \( \Theta_v \in \Lambda_{\Theta_v} \) are all true for all \( t \in [0, T) \).

**Step B-I:** Choose the positive-definite Lyapunov function

\[ V_p = \frac{1}{4} \tilde{S}_p(t)^T \tilde{S}_p, \]

and deduce the time derivative of \( V_p \) along with the closed-loop dynamic equations (7a), we can obtain

\[
V'_p = ( \tilde{S}_p(t) \tilde{S}_p(t)^T ) \cdot (1 - \Theta_p \tilde{S}_p(t))^{-1} \\
= -k_{\text{pro},1} \cdot ( S_p(t) \otimes S_p(t) ) + \Theta_p \tilde{S}_p(t) - \Theta_p \otimes I(\rho_p) \tilde{S}_p(t) \\
\cdot \left[ (1 - \Theta_p \tilde{S}_p(t))^{-1} \otimes I(\rho_p)^{-1} \right] \\
\cdot \left[ \min \left\{ k_{\text{pro},1}, \ldots, k_{\text{pro},1,n} \right\} \right] \cdot \left\| S_p(t) \right\| \\
\cdot \left\| (1 - \Theta_p \tilde{S}_p(t))^{-1} \otimes I(\rho_p)^{-1} \right\| \tag{9}
\]

It has been proved in the above Step A that \( \Theta_p \in \Lambda_{\Theta_p} \) is true for all \( t \in [0, T) \), meanwhile, \( \rho_p > 0 \) is the indispensable condition to avoid the singularity problem of nonlinear transformation function

\[ \mathcal{N}_p(\bullet) = \ln \left( \frac{1 + \bullet}{1 - \bullet} \right). \]

Based on these facts, each element of vector

\[ (1 - \Theta_p \tilde{S}_p(t))^{-1} \otimes I(\rho_p)^{-1} \]

is guaranteed to be strictly positive with

\[ \begin{align*}
1 : = & \left[ 1, \ldots, 1 \right]^T \\
& \text{n times}
\end{align*} \]

Let us denote

\[ H_1 : = \Theta_p \otimes I(\rho_p) - k_{\text{int}} \int_0^t S_p(s)ds - \Theta_p \otimes I(\rho_p). \]

Based on Assumption 1 and extreme value theorem, one can find an unknown yet bounded vector \( H_1^T \) satisfying \( \| H_1 \| \leq \delta_1 \)

with \( \delta_1 \) and \( \delta_1 \)

being the \( i \)th elements of \( H_1^T \) and \( H_1^T = [\delta_1, \ldots, \delta_1]^T \) and \( H_1^T = [\delta_1, \ldots, \delta_1]^T \), respectively. It yields

\[
V'_p \leq \left[ -k_{\text{pro},1} \cdot ( S_p(t) \otimes S_p(t) ) + S_p(t) \otimes H_1^T \right]^T \\
\cdot \left[ (1 - \Theta_p \tilde{S}_p(t))^{-1} \otimes I(\rho_p)^{-1} \right] \\
\leq -\left\| S_p(t) \right\| \cdot \left\| (1 - \Theta_p \tilde{S}_p(t))^{-1} \otimes I(\rho_p)^{-1} \right\| \tag{10}
\]

It is not hard to obtain one sufficient condition to guarantee the negative property of \( V_p \) is \( \| \rho_p \| > \frac{\| \rho \|}{\min \{ k_{\text{pro},1}, \ldots, k_{\text{pro},1,n} \}} \) for all \( i = 1, \ldots, n \), this means

\[ \| \rho_p \| > \min \left\{ k_{\text{pro},1}, \ldots, k_{\text{pro},1,n} \right\} \]

\[ \therefore \rho_p > 0 \]

with \( \rho_p = \rho_p(0) \) denoting the initial value of \( \rho_p(t) \). Noting the proposition of intermediate virtual controller \( \alpha(P, P_v) = -k_{\text{pro},1} S_p(t) - k_{\text{int}} \int_0^t S_p(s)ds \) is a smoothly defined function vector with respect to \( S_p \) and is ensured to be bounded over \( t \in [0, T) \) based on extreme value theorem. Recalling \( \mathcal{V} = \Theta_v \otimes I(\rho_v) + \mathcal{V}_r + \alpha(P, P_v) \) and \( \Theta_v \in \Lambda_{\Theta_v} \), one ensures \( \mathcal{V} \) is bounded over \( t \in [0, T) \) using Assumption 1. The inverse function vector of

\[ S_p = \left[ \ln \left( \frac{1 + e_{p,1}/\rho_p}{1 - e_{p,1}/\rho_p} \right), \ldots, \ln \left( \frac{1 + e_{p,n}/\rho_p}{1 - e_{p,n}/\rho_p} \right) \right]^T \]

versus \( \Theta_p \) can be obtained as

\[ \Theta_p = \left[ e_{p,1} - 1, \ldots, e_{p,n} - 1 \right] \cdot \frac{1}{e_{p,1} + 1, \ldots, e_{p,n} + 1} \]

herein, the \( i \)th element can be written as

\[ e_{p,i} - 1 \]

for all \( i = 1, \ldots, n \). We discuss this equation in two perspectives. (i) it is easy to know that \( e_{p,i} > 0 \) and thus

\[ \frac{2}{e_{p,i} + 1} > 0 \]

are always true, consequently,

\[ 1 - \frac{2}{e_{p,i} + 1} < 1, \]

and (ii) along with

\[ \epsilon_{p,i} \to -\infty, \epsilon_{p,i} \to 0 \]

but unequal to 0, and further

\[ 1 - \frac{2}{e_{p,i} + 1} > -1. \]

\textsuperscript{1} By \( \Theta_v \in \Lambda_{\Theta_v} \) it means that the \( i \)th element of \( \Theta_v \) pertains to \( i \)th element of \( \Lambda_{\Theta_i} \), i.e. \( (-1, 1). \) Similar arguments are applicable to the \( \Theta_v \in \Lambda_{\Theta_i} \).
It follows
\[
-1 < \begin{bmatrix}
\varepsilon_{\text{p},1}^{t-1} \\
\varepsilon_{\text{p},1}^{t} \\
\vdots \\
\varepsilon_{\text{p},n}^{t-1} \\
\varepsilon_{\text{p},n}^{t}
\end{bmatrix}
\begin{bmatrix}
0_{p,1} \\
0_{p,1} \\
\vdots \\
0_{p,n} \\
0_{p,n}
\end{bmatrix}
= \Theta_p^t := \begin{bmatrix}
0_{p,1} \\
0_{p,1} \\
\vdots \\
0_{p,n} \\
0_{p,n}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{\text{p},1}^{t} \\
\varepsilon_{\text{p},1}^{t} \\
\vdots \\
\varepsilon_{\text{p},n}^{t} \\
\varepsilon_{\text{p},n}^{t}
\end{bmatrix}
\begin{bmatrix}
0_{p,1} \\
0_{p,1} \\
\vdots \\
0_{p,n} \\
0_{p,n}
\end{bmatrix}
:= \Theta_p^U
\]
which is equivalent to
\[
-1 \cdot \rho_p < \varepsilon_p^t < 1 \cdot \rho_p, \forall t \in [0, T).
\]
Based on these arguments, the time derivative
\[
\dot{\alpha}(P, P_\text{t}) = \frac{\partial \alpha}{\partial \rho_p} \begin{bmatrix}
\alpha_1(P, P_\text{t}) \\
\vdots \\
\alpha_n(P, P_\text{t})
\end{bmatrix} + \alpha(P, P_\text{t}) - \Theta_p \otimes \Theta_p \otimes I(\rho_\text{p}) \otimes I(\rho_\text{p})^{-1},
\]
is bounded vector over \( t \in [0, T) \) in line with above-mentioned analysis.

**Step B-II:** Choose the positive-definite Lyapunov function
\[
\dot{V}_\nu = \frac{1}{4} S_\nu^T S_\nu,
\]
and deduce the time derivative of \( V_\nu \) along with the closed-loop dynamic equations (7b), we can obtain
\[
\dot{V}_\nu = \left( S_\nu \otimes \Theta_\nu \right)^T (1 - \Theta_\nu \otimes \Theta_\nu)^{-1}
\]
\[
= \left[ -\mathcal{K}_{\text{pro},2} \cdot \left( S_\nu(t) \otimes S_\nu(t) \right) + \mathcal{M}^{-1} \cdot (\mathcal{M} \otimes S_\nu(t)) \right] 
\]
\[
- \mathcal{K}_{\text{int},2} \cdot \left( \int_0^t S_\nu(s) \mathcal{I} S_\nu(s) \text{d}s \right) - \mathcal{M}^{-1} \cdot (\mathcal{M} \otimes S_\nu(t)) 
\]
\[
+ \left( -\mathcal{M}^{-1} (B + \mathcal{K}) (\Theta_\nu \otimes I(\rho_\nu) + \nu_\nu + \alpha) \right) \mathcal{I} S_\nu(t) 
\]
\[
+ \left( -\mathcal{M}^{-1} C (\Theta_\nu \otimes I(\rho_\nu) + \nu_\nu + \alpha) \right)^2 \mathcal{I} S_\nu(t) 
\]
\[
- I(\rho_\nu) \sin(\Theta(P)) \mathcal{I} S_\nu(t) 
\]
\[
+ \left( -\mathcal{M}^{-1} \mathcal{K}' (\Theta_\nu \otimes I(\rho_\nu) + P_\text{t}) - \mathcal{M}^{-1} \mathcal{K} \mathcal{L} \right) \mathcal{I} S_\nu(t) 
\]
\[
+ \left( -\dot{\nu}_\nu - \alpha - \Theta_\nu \otimes I(\rho_\nu) \right) \mathcal{I} S_\nu(t) 
\]
\[
\cdot \left( 1 - \Theta_\nu \otimes \Theta_\nu \right)^{-1} \mathcal{I} I(\rho_\nu)^{-1}
\]
It has been proved in the above Step A that \( \Theta_\nu \in A_{\theta,\nu} \) is true for all \( t \in [0, T) \), at the same time, \( \rho_\nu > 0 \) is the indispensable condition to avoid the singularity problem of nonlinear transformation function
\[
\mathcal{N}_1(\star) = \ln \left( \frac{1 + \star}{1 - \star} \right).
\]
By this facts, it is known that each element of vector
\[
(1 - \Theta_\nu \otimes \Theta_\nu)^{-1} \mathcal{I} I(\rho_\nu)^{-1}
\]
is ensured to be strictly positive with
\[
1 := [1, ..., 1]^T,
\]
Let \( \mathcal{H}_2 \) being a sunamation of following terms
\[
-\mathcal{K}_{\text{int},2} \cdot \int_0^t S_\nu(s) \mathcal{I} S_\nu(s) \text{d}s, -\mathcal{M}^{-1} \cdot A_\nu, -\mathcal{M}^{-1} \cdot (B 
\]
\[
+ \mathcal{K}' (\Theta_\nu \otimes I(\rho_\nu) + \nu_\nu + \alpha (P, P_\text{t})), -\mathcal{M}^{-1} C (\Theta_\nu \otimes I(\rho_\nu) 
\]
\[
+ \nu_\nu + \alpha)^2, -\mathcal{M}^{-1} \mathcal{K}' (\Theta_\nu \otimes I(\rho_\nu) + P_\text{t}), 
\]
\[
- I(\rho_\nu) \sin(\Theta(P)) \mathcal{I} S_\nu(t), 
\]
and \( -\Theta_\nu \otimes I(\rho_\nu) \). Invoking Assumption 1 and extreme value theorem again, one can find an unknown yet bounded vector \( \mathcal{H}_2 \) satisfying \( |b_2| \leq b_2^T \) with \( b_2 \) and \( b_2^T \) being the \( i \)th elements of \( \mathcal{H}_2 \) and \( \mathcal{H}_2^T \), \( \mathcal{H}_2 = [b_2, ..., b_2^T]^T \) and \( \mathcal{H}_2^T = [b_2, ..., b_2^T]^T \), respectively. It follows
\[
\dot{V}_\nu \leq \left[ -\mathcal{K}_{\text{pro},2} \cdot \left( S_\nu(t) \otimes S_\nu(t) \right) + S_\nu(t) \otimes \mathcal{H}_2 \right]^T 
\]
\[
\leq -\| S_\nu(t) \| \cdot \left( \min \left\{ \mathcal{K}_{\text{pro},2,1}, ..., \mathcal{K}_{\text{pro},2,n} \right\} \| S_\nu(t) \| - \| \mathcal{H}_2 \| \right)
\]
\[
\cdot \left( 1 - \Theta_\nu \otimes \Theta_\nu \right)^{-1} \mathcal{I} I(\rho_\nu)^{-1} \right] \cdot \left( 1 - \Theta_\nu \otimes \Theta_\nu \right)^{-1} \mathcal{I} I(\rho_\nu)^{-1} \right]
\]
\[
\cdot \left( 1 - \Theta_\nu \otimes \Theta_\nu \right)^{-1} \mathcal{I} I(\rho_\nu)^{-1}
\]
It is not hard to obtain one sufficient condition to guarantee the negative property of \( \dot{V}_\nu \) is \( |\dot{\nu}_\nu| > \frac{b_2^T}{\min \left\{ \mathcal{K}_{\text{pro},2,1}, ..., \mathcal{K}_{\text{pro},2,n} \right\}} \) for all \( i = 1, ..., n \), this means
\[
|\dot{\nu}_\nu(t)| \leq \dot{\nu}_\nu^T := \max \left( \frac{\dot{\nu}_\nu^T}{\min \left\{ \mathcal{K}_{\text{pro},2,1}, ..., \mathcal{K}_{\text{pro},2,n} \right\}} \right)
\]
for all $i \in \{0, T\}, i = 1, \ldots, n$, with $e_{v,i}(0)$ denoting the initial value of $e_{v,i}(t)$. The inverse function vector of $S_e$ can be obtained as

$$
\Theta_e = \begin{bmatrix}
\text{int}(\frac{1 + e_{v,i}/P_e}{1 - e_{v,i}/P_e}), \ldots, \text{int}(\frac{1 + e_{v,s}/P_e}{1 - e_{v,s}/P_e})
\end{bmatrix}^T
$$

versus $\Theta_e$ can be obtained as

$$
\Theta_e = \begin{bmatrix}
\frac{\dot{e}_{v,1}}{v_{v,1}} - 1, \ldots, \frac{\dot{e}_{v,s}}{v_{v,s}} - 1
\end{bmatrix}^T,
$$

herein, the $i$th element can be written as

$$
\frac{\dot{e}_{v,i}}{v_{v,i}} - 1 = 1 - \frac{2}{\frac{\dot{e}_{v,i}}{v_{v,i}} + 1}
$$

for all $i = 1, \ldots, n$, which can be discussed as two cases. (i) it is easy to know that $\frac{\dot{e}_{v,i}}{v_{v,i}} > 0$ and thus

$$
\frac{2}{\frac{\dot{e}_{v,i}}{v_{v,i}} + 1} > 0
$$

are always true, consequently,

$$
1 - \frac{2}{\frac{\dot{e}_{v,i}}{v_{v,i}} + 1} < 1,
$$

and (ii) along with $e_{v,i} \rightarrow -\infty$, $\dot{e}_{v,i} \rightarrow 0$ but unequal to 0, and further

$$
1 - \frac{2}{\frac{\dot{e}_{v,i}}{v_{v,i}} + 1} > -1.
$$

In conclusion, one obtains

$$
-1 < \begin{bmatrix}
\frac{\dot{e}_{v,1}}{v_{v,1}} - 1 \\
\vdots \\
\frac{\dot{e}_{v,s}}{v_{v,s}} - 1
\end{bmatrix} = \Theta_v \leq \begin{bmatrix}
\frac{\dot{\theta}_L_{v,1}}{\theta_L_{v,1}} \\
\vdots \\
\frac{\dot{\theta}_L_{v,s}}{\theta_L_{v,s}}
\end{bmatrix} \leq \begin{bmatrix}
\frac{\dot{\theta}_U_{v,1}}{\theta_U_{v,1}} \\
\vdots \\
\frac{\dot{\theta}_U_{v,s}}{\theta_U_{v,s}}
\end{bmatrix} = \Theta_v^U
$$

which is equivalent to

$$
-1 \cdot \rho_v < \mathcal{E}_v < 1 \cdot \rho_v, \forall i \in [0, T).
$$

Finally, it is not hard to conclude that

$$
\mathcal{U}(\mathcal{P}, \mathcal{P}_r, \mathcal{V}, \mathcal{V}_e)
$$

is also bounded for all $i \in [0, T)$ steadily.

Step C: From above proof steps, one knows $\Theta_v^L \leq \Theta_v \leq \Theta_v^U$ and $\Theta_v^L \leq \Theta_v \leq \Theta_v^U$ for all $i \in [0, T)$, that is,

$$
\Theta = \begin{bmatrix}
\Theta_v^L \\
\Theta_v \\
\Theta_v^U
\end{bmatrix} \in \Lambda_v^* = [\Theta_v^L, \Theta_v^U],
$$

which is nonempty and compact sets vector with $\Theta_v^L = [\theta_v^L, \theta_v^L]^T$ and $\Theta_v^U = [\theta_v^U, \theta_v^U]^T$. In this sense, one knows $\Lambda_v^* \subset \Lambda_v$. Suppose that $\mathcal{T} < +\infty$, one can find a time moment $\mathcal{T}^* \in [0, \mathcal{T})$ satisfying $\Theta(\mathcal{T}^*) \notin \Lambda_v^*$, it is an obvious contradiction with foregoing conclusions. Therefore, $\mathcal{T}$ is unbounded, that is, $\mathcal{T} = +\infty$. Finally, it is concluded that $-1 \cdot \rho_p < \mathcal{E}_v < 1 \cdot \rho_p$ and $-1 \cdot \rho_v < \mathcal{E}_v < 1 \cdot \rho_v$ hold for all $i \geq 0$.

The proof completes.

Remark 2. From the above proof, it is known that a train is well controlled without violating potential signal passed at danger via guaranteeing $-1 \cdot \rho_p < \mathcal{E}_p < 1 \cdot \rho_p$, therein, $\rho_p$ is the receiving terminal of tolerable bound which is bigger than signal passed at danger bound certainly. In this sense, the circumvention of signal passed at danger is well guaranteed. Based on the definition $\mathcal{E}_v = \mathcal{V} - \mathcal{V}_e - \alpha$, it is known $-1 \cdot \rho_v < \mathcal{V} - \mathcal{V}_e - \alpha < 1 \cdot \rho_v$, that is to say, $-1 \cdot \rho_v + \alpha^L < \mathcal{V} - \mathcal{V}_e < 1 \cdot \rho_v + \alpha^U$ with $\alpha^L$ and $\alpha^U$ being the minimum and maximum numerical values of $\alpha$ respectively. It is known circumvention of over-speed, equivalent to the constrained problem of $\mathcal{V} - \mathcal{V}_e$ can be guaranteed in the situation that a train is not very far from the target position at the begin running time, or a train will running aggressively to follow target position. In practice, the initial position and speed of trains are generally relative zero, and target distance-to-go curves, containing position versus time and speed versus time, are obtained using offline relative zero, and target distance-to-go curves, containing position versus time and speed versus time, are obtained using offline optimization with zero initial and terminal values. In this sense, the proposed scheme is applicable to whole process operation optimization of trains among stations with relatively small calculative values of $\alpha^L$ and $\alpha^U$.

Remark 3. For practical decentralized-power trains, not all cars are powered-cars and trailer-cars only provide braking control, i.e. negative input of $u_i$ if $i$th car is trailer one. To cope with this issue without destroying the continuous control signals, we design a nonlinear gain

$$
\frac{-\tanh(u_{i,e} \cdot u_{i}(t)) + 1}{2}
$$

for $i$th trailer-car with $u_{i,e}$ being positive constant. Speaking mathematically,

$$
\frac{-\tanh(u_{i,e} \cdot u_{i}(t)) + 1}{2} u_{i} = u_{i} + \frac{-\tanh(u_{i,e} \cdot u_{i}(t)) - 1}{2} u_{i}
$$

with

$$
\frac{-\tanh(u_{i,e} \cdot u_{i}(t)) - 1}{2} u_{i}
$$
being continuous function and thus bounded. As a result, the introductions of above-mentioned nonlinear gains to trailer-cars do not damage the theoretical analysis, and this improves the robustness of proposed controller.

### 3.4 Further discussions on external disturbances

In practical situations, external disturbances are inevitable due to the changes of running environments and various kinds of
abrupt factors. In the presence of external disturbances, the original dynamic equation (1) can be rewritten as follows:

$$
\mathbf{M} \ddot{\mathbf{P}} + \mathbf{A} + (\mathbf{B} + \mathbf{K}^d) \dot{\mathbf{P}} + \mathbf{C} \dot{\mathbf{P}}^2 + \mathbf{M} \dot{\mathbf{q}} \sin(\Theta(\mathbf{P})) + \mathbf{K}^\nu \mathbf{P} + \mathbf{K} \mathbf{L} + \mathbf{I}(1)\mathbf{D} = \mathbf{F}
$$

where the denotations are exactly the same as (1), while the extra term $\mathbf{I}(1)\mathbf{D}$ represents the external disturbance vector with $\mathbf{I}(1) = \text{diag}(1, \ldots, 1) \in \mathbb{R}^{n \times n}$ and $\mathbf{D} = [d_1, \ldots, d_n]^T \in \mathbb{R}^{n \times n}$, therein, $d_i, i = 1, \ldots, n$ denotes the external disturbance imposed on the $i$th car. Now, we will discuss the feasibility of main
FIGURE 6 Partial comparative simulation results of position tracking performance from CQ to WYS. Red dash lines are obtained via proposed PI protective control, blue dot lines are obtained via pre-per control, azure lines are obtained via prop control. (a) Powered-cars: CQ to CQS. (b) Trailer-cars: CQ to CQS. (c) Powered-cars: CQS to JHR. (d) Trailer-cars: CQS to JHR. (e) Powered-cars: JHR to TSR. (f) Trailer-cars: JHR to TSR. (g) Powered-cars: TSR to RCES. (h) Trailer-cars: TSR to RCES. (i) Powered-cars: RCES to RJES. (j) Trailer-cars: RCES to RJES. (k) Powered-cars: RJES to WYS. (l) Trailer-cars: RJES to WYS.

results in Theorem 1 subject to external disturbances as twofold cases.

Case 1: $I(1)D$ is continuous-time defined bounded constants or functions vector. In this case, we can treat the composite term $A + I(1)D$ as a new vector as $A' = A + I(1)D$, which satisfies the assumptions, i.e. locally Lipschitz, continuous, locally integrable, mentioned in Lemma 2. Meanwhile, the extreme value theorem is applicable to $A'$ directly in above theoretical analysis. In this sense, the main results in Theorem 1 and corresponding closed-loop analysis are applicable to this case.
FIGURE 7  Partial comparative simulation results of position tracking performance from WYS to SJZ. Red dash lines are obtained via proposed PI protective control, blue dot lines are obtained via pre-per control, azure lines are obtained via prop control. (a) Powered-cars: WYS to YWG. (b) Trailer-cars: WYS to YWG. (c) Powered-cars: YWG to YZB. (d) Trailer-cars: YWG to YZB. (e) Powered-cars: YZB to FP. (f) Trailer-cars: YZB to FP. (g) Powered-cars: FP to SRG. (h) Trailer-cars: FP to SRG. (i) Powered-cars: SRG to XC. (j) Trailer-cars: SRG to XC. (k) Powered-cars: XC to SJZ. (l) Trailer-cars: XC to SJZ.
FIGURE 8  Partial comparative simulation results of speed tracking performance from CQ to WYS. Red dash lines are obtained via proposed PI protective control, blue dot lines are obtained via pre-per control, azure lines are obtained via prop control. (a) Powered-cars: CQ to CQS. (b) Trailer-cars: CQ to CQS. (c) Powered-cars: CQS to JHR. (d) Trailer-cars: CQS to JHR. (e) Powered-cars: JHR to TSR. (f) Trailer-cars: JHR to TSR. (g) Powered-cars: TSR to RCES. (h) Trailer-cars: TSR to RCES. (i) Powered-cars: RCES to RJES. (j) Trailer-cars: RCES to RJES. (k) Powered-cars: RJES to WYS. (l) Trailer-cars: RJES to WYS
FIGURE 9  Partial comparative simulation results of speed tracking performance from WYS to SJZ. Red dash lines are obtained via proposed PI protective control, blue dot lines are obtained via pre-per control, azure lines are obtained via prop control. (a) Powered-cars: WYS to YWG. (b) Trailer-cars: WYS to YWG. (c) Powered-cars: YWG to YZB. (d) Trailer-cars: YWG to YZB. (e) Powered-cars: YZB to FP. (f) Trailer-cars: YZB to FP. (g) Powered-cars: FP to SRG. (h) Trailer-cars: FP to SRG. (i) Powered-cars: SRG to XC. (j) Trailer-cars: SRG to XC. (k) Powered-cars: XC to SJZ. (l) Trailer-cars: XC to SJZ.
FIGURE 10 Comparative simulation results of position tracking performance from WYS to SJZ. Red dash lines are obtained via proposed PI protective control, blue dot lines are obtained via pre-per control, azure lines are obtained via prop control. (a) CQ to CQS. (b) CQS to JHR. (c) JHR to TSR. (d) TSR to RCES. (e) RCES to RJES. (f) RJES to WYS. (g) WYS to YWG. (h) YWG to YZB. (i) YZB to FP. (j) FP to SRG. (k) SRG to XC. (l) XC to SJZ.
directly without modifying any control structure and theoretical analysis.

**Case 2:** \( I(1)D \) is piecewise-time defined bounded constants or functions vector. In this case, a proper assumption is needed to circumvent the self-Zeno behaviour of piecewise \( I(1)D \), that is, there are finite number, denoted as \( N_e \), of switching constants \( t_{i,j} \) with \( i = 1, ..., n \) being the sequential number of car and \( j = 1, 2, ..., \) being the times of switching in the \( j \)th external disturbance channel. By this proper assumption, it is easy to know that \( I(1)D \) can be reconstructed as right-continuous function vector as

\[
I(1)D = \begin{cases} 
I(1)D_1, & t \in [0, \min \{t_{i,j}\}], \\
I(1)D_2, & t \in [\min \{t_{i,j}\}, \min \{t_{i,j}\}], \\
\vdots & \\
I(1)D_{N_e}, & t \in [\max \{t_{i,j}\}, \max \{t_{i,j}\}] 
\end{cases}
\]

where \( \min \{t_{i,j}\} \), \( \min_{2nd}\{t_{i,j}\} \), \( \min_{2nd}\{t_{i,j}\} \) and \( \max\{t_{i,j}\} \) are all continuous-time defined bounded constants or functions vector, which is the same as previous **Case 1**. The following theoretical analysis can be done by these minor modifications based on the main results of Theorem 1. Using previous arguments, it is easy to know \( I(1)D_1 \) satisfies locally Lipschitz, continuous and locally integrable properties claimed in Lemma 2, the extreme value theorem is also applicable to \( I(1)D_1 \) without doubt. Therefore, the main results in Theorem 1 and corresponding closed-loop analysis are applicable to this case for \( t \in [0, \min\{t_{i,j}\}] \). After the time period \( t \in [0, \min\{t_{i,j}\}] \) ends, \( \min\{t_{i,j}\} \) is treated as, actually is, the new initial time moment for period \( t \in [\min\{t_{i,j}\}, \min_{2nd}\{t_{i,j}\}] \). Due to the right-continuous property of \( I(1)D \), it is not hard to know that the main results in Theorem 1 and corresponding closed-loop analysis are applicable to this case for \( t \in [0, \min\{t_{i,j}\}] \cup [\min\{t_{i,j}\}, \min_{2nd}\{t_{i,j}\}] \), equivalently, \( t \in [0, \min\{t_{i,j}\}] \). By repeating this procedure and invoking the finite property of switching time moments, one concludes that the main results in Theorem 1 are applicable to this case for

\[
t \in [0, \min\{t_{i,j}\}] \cup [\min\{t_{i,j}\}, \min_{2nd}\{t_{i,j}\}] \cup \cdots \cup [\max\{t_{i,j}\}, \max\{t_{i,j}\}]
\]

which is a subset of \([0, T]\). Until now, the remaining analysis procedures of Theorem 1 can be applied to this case without any modifications.
to the ones in the proposed robust PI protection control, as is shown in the Algorithm 1. The control parameters are chosen as $\mathcal{K}_{pp,1} = \text{diag}(5, \ldots, 5)$, $\mathcal{K}_{mc,1} = \text{diag}(5, \ldots, 5)$, $\mathcal{K}_{pp,2} = \text{diag}(25, \ldots, 25)$, and $\mathcal{K}_{mc,2} = \text{diag}(5, \ldots, 5)$. We first set the position PPF as $\rho_p = 5$ and speed PPF $\rho_v = 0.5$, the obtained comparative results are shown in Figures 4(b–e), 6, 7, 8, and 9. From where it is observed that the proposed PI protective tracking control in Theorem 1 performs better than the proportional control and pioneering prescribed performance control in the aspect of small scale both position and speed tracking errors. Secondly, we chose more “tighter” PPFs as $\rho_p(t) = (5-0.3)e^{-t^2} + 0.3$ and $\rho_v(t) = (0.5-0.1)e^{-0.5t^2} + 0.1$, similar results proving the advantages of proposed PI protective tracking control are obtained, for simplicity, we only present the zoom-in views of comparative tracking errors in powered cars in Figures 4(f) and 10.

Based on the above results and chosen PPFs, one knows that both these algorithms guarantee the circumvention of violating the PPFs by choosing proper control parameters. In order to illustrate the protective tracking control under such situation, we
defined a protective performance index as \( T_{i,j}^p = \frac{\max|e_{i,j}(t)|}{\rho_p(\infty)} \) and \( T_{i,j}^\rho = \frac{\max|\rho_{i,j}(t)|}{\rho_r(\infty)} \). Therein, \( i = 1, \ldots, 8 \) is the index of cars, and \( j = 1, \ldots, 13 \) is the index of station sections, respectively. \( \rho_p(\infty) \) and \( \rho_r(\infty) \) are ultimate settings of PPFs. It is easy to understand that the smaller values these indexes lead to higher safety level, i.e. more protective tracking control performance. The plots of \( T_{i,j}^p \) are given in Figure 11 under the same axis ranges (to save pages limit, the plots of \( T_{i,j}^\rho \) are omitted), therein, Figure 11(a)–(c) and (d)–(f) are the results under settings of PPFs \( \{\rho_p = 5, \rho_r = 0.5\} \) and \( \{\rho_p(t) = (5 - 0.3)e^{-t} + 0.3, \rho_r(t) = (0.5 - 0.1)e^{-2t} + 0.1\} \) respectively. From Figure 11(a)–(c), one observes that the prescribed performance control and controller proposed in this paper guarantee almost the same protective tracking control performance, which is better than the single proportional control; from Figure 11(d)–(f), one clearly know that the proposed controller in this paper performs best, which is in line with the above-mentioned theoretical analysis that not monotonically decreasing boundary PPFs are required in this work on comparison with pioneering prescribed performance control methodology [30]. By above-mentioned discussions and comparative simulation results, one concludes that the proposed PI control scheme in Theorem 1 performs better errors convergence, which demonstrates the effectiveness and advantages of proposed method.

5 | CONCLUSIONS

The robust PI protective tracking control has been investigated for a class of decentralized-power trains in the presence of model uncertainties. By “protective” tracking, it means over-speed and signal passed at danger are totally avoided, meanwhile, satisfactory tracking performance with respect to target speed curve versus position is achieved. The proposed control is also computationally inexpensive since PI structure is adopted using transformed errors, which incorporate the boundaries allowed by trackside equipments and tracking errors. The closed-loop stability of resulted systems is guaranteed with rigorously mathematical analysis using Lyapunov theory. Applications of proposed control to Beijing subway Yizhuang line are presented, and comparative results are given to demonstrate the effectiveness and advantages.

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