Suppression of heating by long-range interactions in periodically driven spin chains

Devendra Singh Bhakuni,1 Lea F. Santos,2 and Yevgeny Bar Lev1

1Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
2Department of Physics, Yeshiva University, New York, New York 10016, USA

We propose a mechanism to suppress heating in periodically driven many-body quantum systems by employing sufficiently long-range interactions and experimentally relevant initial conditions. The mechanism is robust to local perturbations and does not rely on disorder or high driving frequencies. Instead, it makes use of an approximate fragmentation of the many-body spectrum of the non-driven system into bands, with band gaps that grow with the system size. We show that when these systems are driven, there is a regime where decreasing the driving frequency decreases heating and entanglement build-up. This is demonstrated numerically for a prototypical system of spins in one dimension, but the results can be readily generalized to higher dimensions.

Periodically driven quantum systems continue to produce fascinating physics and phenomena inaccessible to their static counterparts. Some notable examples include the Kapitza pendulum [1], dynamical localization [2–4], Floquet topological insulators [5–7], dynamical phase transitions [8], induced many-body localization (MBL) [9–13], and Floquet time-crystals [14–18]. However, a key obstacle to realizing new phases of matter in driven systems is that typically the drive heats up the system to a featureless infinite-temperature state where all correlations and observables become trivial [19–22].

In one-dimensional systems, heating can be suppressed with the inclusion of sufficiently strong disorder, which leads to the formation of the Floquet-MBL phase [13, 20, 23–25]. Alternatively, heating can be suppressed at any dimension, whether the system is clean or disordered, by considering driving frequencies greater than the single-particle excitation energy, such that the absorption of a photon from the drive will always result in a multiparticle process [26–32]. Under these conditions the system will spend a significant amount of time in a nontrivial metastable state – a phenomenon called Floquet-prethermalization [26–32]. It has been recently demonstrated with nuclear spins using nuclear magnetic resonance techniques [33] and with ultracold atoms in a driven optical lattice [34].

If the driving frequencies are smaller than the single-particle excitation energy, the system can efficiently absorb energy from the drive, which results in fast heating to infinite temperature [35]. But is this the fate of all driven quantum systems? In this Letter, we show that the answer is negative. Heating can actually be suppressed in any dimension and for frequencies smaller than the single-particle excitation if the system has sufficiently long-range interactions.

The physics of non-driven systems with power-law decaying interactions, \( r^{-\alpha} \) (where \( r \) is the distance between two bodies), has gained considerable attention due to experimental realizations in trapped ions [36–40], where the range of the interactions can be tuned. A particularly intriguing regime is \( \alpha < d \) (\( d \) being the dimension of the system), where conventional thermodynamics does not apply [41]. Power-law decaying interactions occur in various systems, from spin glasses and magnetically frustrated systems to atomic, molecular, and optical systems [42–46]. They are associated with phenomena that are absent for neighboring interactions [47–54]. They are known to affect transport [55–61], destroy many-body localization [62–68], and facilitate the propagation of correlations [53, 54, 69–71].

While for \( \alpha > d \), the physics is many times only quantitatively different from the physics of systems with local interactions (\( \alpha \to \infty \)), novel physics often emerges for slowly decaying interactions, \( \alpha < d \). An example is the emergence of a Hilbert space fragmentation into weakly connected subspaces. If the dynamics starts in one of these subspaces, it can be effectively described by a local Hamiltonian for a long time [72, 73], so despite the presence of long-range interactions, features that are usually associated with short-range interactions may be observed, such as the logarithmic growth of entanglement [52, 74], light-cone evolution [72, 75], and self-trapping [76]. On the other hand, if the initial state spans multiple subspaces, the dynamics violates the generalized Lieb-Robinson bound and leads to the instantaneous spread of correlations [36–38, 54].

The behavior of periodically driven systems with power-law decaying interactions was studied in [77–80]. For \( \alpha > d \) and large driving frequencies, exponentially slow heating and the emergence of Floquet prethermalization were obtained [26–29, 78]. In this prethermal regime, a novel non-equilibrium phase of matter dubbed the prethermal time-crystal [79], which is similar to the MBL-time crystal [14–16], has been argued to exist. For \( \alpha < d \), the general expectation is that to achieve a prethermal plateau, the system needs both to be in the high-frequency regime and to be finite. The second condition arises since the single-particle excitation energy increases with system size and therefore for fixed frequency, the prethermal plateau shrinks as the system size increases [78].

In this Letter, we show that it is in fact possible to suppress heating in systems with long-range interactions in the low-frequency regime, where the driving frequen-
cies are smaller than the single-particle excitation energy. This can be done by taking advantage of the effective fragmentation of the Hilbert space, which is induced by interactions with $\alpha < d$, and by selecting initial states within one of those approximate subspaces, such that the energy absorption from the drive becomes ineffective. In this way, we can achieve prethermal phases whose lifetimes grow as the system size increases and which are viable at any dimension. We demonstrate this behavior by numerically examining the dynamics of the half-chain entanglement entropy and the energy absorption in a spin chain with $\alpha < 1$.

Model.—We consider a long-range interacting spin chain of length $L$ described by the Hamiltonian,

$$
\hat{H}_0 = J_z \hat{V} + J_x \sum_{\langle i, j \rangle} \hat{\sigma}_i^x \hat{\sigma}_j^x + h_x \sum_{i=1}^{L} \hat{\sigma}_i^z;
$$

(1)

where $\hat{\sigma}_i^x,y,z$ are Pauli operators, $J_z$ is the strength of the long-range term $\hat{V}$ and we set $J_x = 1$, $J_y$ corresponds to the strength of nearest-neighbor interactions in the $x$-direction, and $h_x$ is the amplitude of a transverse magnetic field. The operator norm of the long-range term is $\|\hat{V}\| \sim L^{2-\alpha}$ for $\alpha < 1$, such that it becomes dominant in the thermodynamic limit. Nevertheless, even in this limit, the dynamics is not given by $\hat{V}$ for almost all initial states, and is highly nontrivial, since the model stays non-integrable for any value of $\alpha$. While one can make $\hat{V}$ extensive by proper rescaling [81–83], this rescaling does not naturally occur in the experiments, where finite systems are studied [17, 36, 37, 84]. We therefore do not consider this rescaling in our work.

The static Hamiltonian $\hat{H}_0$ is periodically driven by the following time-dependent perturbation,

$$
\hat{H}_1 (t) = \text{sgn} (\sin (\omega t)) \left( h_y \sum_{i=1}^{L} \hat{\sigma}_i^y + h_z \sum_{i=1}^{L} \hat{\sigma}_i^z \right),
$$

(2)

such that the total Hamiltonian is $\hat{H} (t) = \hat{H}_0 + \hat{H}_1 (t)$. Here, $\omega = 2\pi / T$ is the driving frequency, $\text{sgn} (\cdot)$ is the sign function, $T$ is the driving period, and $h_y$ and $h_z$ are the magnitudes of the magnetic fields along the $y$- and $z$-directions, respectively. We use a square-wave driving to closely follow the experiment with trapped ions in Ref. [40], and also since it is computationally more efficient than a continuous time-varying drive. However, the results presented here should be insensitive to the choice of the driving protocol. We explore the dynamics of the driven system, $\hat{H} (t)$, with $\alpha < 1$.

To study the heating dynamics, we use the numerically exact Krylov subspace techniques to evolve the system in time [85]. Due to the lack of symmetries, we have to consider the entire Hilbert space of dimension $2^L$, so we analyze system sizes up to $L = 22$. We investigate the energy density of the static system measured with respect to the initial state,

$$
\epsilon (t) \equiv \frac{1}{L} \text{Tr} \left[ (\hat{\rho} (0) - \hat{\rho} (t)) \hat{H}_0 \right],
$$

(3)

where $\hat{\rho} (t)$ is the density matrix as a function of time, and the half-chain entanglement entropy,

$$
S (t) = - \text{Tr} [\hat{\rho}_A (t) \ln \hat{\rho}_A (t)],
$$

(4)

where $\hat{\rho}_A (t) = \text{Tr}_B \hat{\rho} (t)$ is the reduced density matrix of the subsystem $A$ consisting of $L/2$ spins.

Heating suppression.— Figure 1 shows the evolution with time of the entanglement entropy and the energy density for $L = 20$, different frequencies, and the initial state $|\psi (0)\rangle = |11 \cdots 11011 \cdots 11\rangle$, where all the spins, except the one in the middle, point up. For most frequencies in Fig. 1, the entanglement entropy exhibits three distinct regimes: an initial growth for a short time, which is followed by the emergence of a long-lived prethermal state (Floquet-prethermalization), where $S (t)$ saturates to a plateau value $S_p$ (horizontal black solid line), after which the entropy finally reaches an infinite-temperature value (black dashed line) corresponding to the result by Page, $S_{\text{Page}} = (L \ln 2 - 1) / 2$ [86]. The dependence of the behavior of the energy density on the frequency is comparable to that for the entropy, it remains constant during

![Figure 1](image-url)
the prethermal phase and eventually goes to its infinite-temperature value at long-times. Those distinct dynamical stages in Fig. 1 were observed before in Ref. [78], where high driving frequencies were considered and the dynamics started with initial product states in the z-direction with an equivalent number of spins pointing up and down. But in stark contrast with previous studies, we find that below a certain frequency value, we can extend the prethermal phase and postpone heating by decreasing the driving frequency, as shown in Figs. 1 (b,d). Contrary to past studies for which the heating time increases monotonically with the frequency, we have now a non-monotonic dependence. For frequencies $\omega \gtrsim 11$, the heating time grows as $\omega$ increases [Fig. 1 (a,c)], but for a range of frequencies with $\omega < 11$, the heating time shrinks as $\omega$ increases [Fig. 1 (b,d)]. For frequencies close to $\omega \sim 11$, the system heats up very quickly, hinting on a resonant behavior.

To show the frequency dependence more explicitly, we define the heating time $\tau^*$ as the time when the entanglement entropy reaches a half-way mark between its prethermal plateau and its asymptotic value, $S(\tau^*) \equiv S_p + [S_{\mathrm{Page}} - S_p]/2$, which is indicated with dot-dashed red lines in Figs. 1 (a,b). We see in Fig. 2 (a) that, as expected, for $\omega > 11$ the heating time increases as we increase the driving frequency, however, within a range of values for $\omega < 11$, the heating time increases as we decrease $\omega$ and these results further improve as the system size grows. A similar qualitative picture is obtained also for the heating time calculated from the energy density $\epsilon (t)$. As we explain next, this unusual dependence on the frequency is a consequence of the effective fragmentation of the Hilbert space verified for the non-driven system when $\alpha < 1$ [72].

Energy bands.— To better understand the Hilbert space fragmentation of the static system, let us first examine the long-range term $\hat{V}$ of $\hat{H}_0$ [Eq. (1)], which for $\alpha = 0$ can be written in terms of the collective spin operator $\hat{M}_z = \sum_1^L \hat{\sigma}_i^z/2$ as $\hat{V} = 2M_z^2 - L/2$. The energy spectrum of $\hat{V}$ consists of degenerate bands with the energies

$$E_b = 2 \left( \frac{L}{2} - b \right)^2 - \frac{L}{2}, \quad b = 0, 1, ..., \frac{L}{2},$$

where $b$ indicates the number of spins pointing down in the z-direction, and we designate the corresponding energy band as the band $b$. Since the energy of a product state with $b$ down-spins is equal to the energy of a state with $L - b$ down-spins, each band $E_b^{(L)}$ degenerate for $b < L/2$. For $0 < \alpha < 1$, the degeneracy within each band of the spectrum of $\hat{V}$ is partially lifted, but the different subspaces are still separated in energy. We define the energy gap between two nearby bands as $\Delta_b \equiv E_b - E_{b-1}$, which can be obtained analytically. The gap between the bands $b = 0$ and $b = 1$ can be calculated as

$$\Delta_1 = \sum_{r=1}^{L-1} \frac{2}{r^\alpha} \sim \frac{2}{1 - \alpha} L^{1 - \alpha}. \quad (6)$$

One sees that the gap increases monotonically with system size for $\alpha < 1$. Similarly, we can obtain $\Delta_2$,

$$\Delta_2 = \left( \sum_{r=2}^{L-1} \frac{2}{r^\alpha} \right) - 2 \left( \sum_{r=1}^{L/2-1} \frac{2}{r^\alpha} + \left( \frac{2}{L} \right)^\alpha \right) \sim \frac{2(2 - 2\alpha)}{1 - \alpha} L^{1 - \alpha}, \quad (7)$$

which also increases with the system size, although $\Delta_2 < \Delta_1$.

The other terms of the static Hamiltonian $\hat{H}_0$ couple the states of $\hat{V}$. The $J_z$ term connects states within the same band and states from band $b$ to bands $b \pm 1$, while the $h_x$ term connects states of band $b$ to bands $b \pm 1$. However, if the values of $J_z$ and $h_x$ are smaller than the...
gap between the bands, they cannot effectively couple them. Furthermore, the numerical calculations for the values of $\Delta_1$ and $\Delta_2$ for $\hat{H}_0$ with $J_x, h_x \neq 0$ approach the gaps between the bands of $\hat{V}$ in the limit $L \to \infty$, as shown in Fig. 3. This implies that the dynamics starting from an initial state within one band gets confined to that approximate subspace for a time that grows with the system size [72].

**Resonant transition.**— The periodic driving of $\hat{H}_0$ tries to establish transitions between the different bands, but for this to happen efficiently it must deposit an amount of energy on par with the gap between the bands, $\omega \approx \Delta_1$. For the initial state considered in Fig. 1, the most relevant bands are $b = 0, 1$ and 2, with the corresponding gaps $\Delta_1$ and $\Delta_2$. To see the dependence of the heating time on the dominant gap more clearly, we rescale the driving frequency by the largest gap, $\Delta_1$, as shown in Fig. 2 (b). We see that the heating time $\tau^*$ reaches its smallest value when $\omega \approx \Delta_1$, because at this point we hit a resonant transition that leads to fast heating. This can be directly observed also in Fig. 1, where the fastest heating is indeed verified for $\omega \approx \Delta_1 \approx 11$. Another drop in the value of $\tau^*$ occurs when $\omega \approx \Delta_2$, which may be due to a multiple photon process. Since a square-wave drive contains multiple harmonics, higher-order transitions might occur. While such processes, are suppressed at high enough frequencies, at very low frequencies, they could give rise to a non-monotonic dependence of the thermalization time on the frequency as is indeed observed in Fig. 2. Next, we explain what causes the suppression of heating as the frequency increases above $\Delta_1$ and, especially, when it decreases within the range $\Delta_2 < \omega < \Delta_1$.

**Non-monotonic frequency dependence.**— The maximum energy required to flip one spin for any initial state, scales like $J_{\text{eff}} = \sum_k r^{-\alpha} \sim L^{1-\alpha}$, with the system size. For the initial state considered above, $J_{\text{eff}}$ coincides with the largest gap between the energy bands, $J_{\text{eff}} = \Delta_1$. In the high frequency regime, $\omega \gg J_{\text{eff}}$, we expect slow heating, as indeed observed in Fig. 1 (a,c). For $\omega < J_{\text{eff}}$ one might have expected fast heating to occur, however, because $\Delta_1 = J_{\text{eff}}$, one photon from the drive is not sufficient to induce a transition from the band $b = 1$ of the initial state to a neighboring band and the dynamics gets confined to the initial band for a long time, leading to heating suppression and the emergence of the prethermal phase in Fig. 1 (a,c)]. In this case, increasing the frequency, $\omega \to \Delta_1$, induces heating due to the approach to the resonant condition. Therefore, heating suppression can be achieved by going away from the resonant frequency either by increasing [Fig. 1 (a,c)] or decreasing [Fig. 1 (b,d)] the driving frequency.

To demonstrate that for frequencies off-resonance to the gap the dynamics is indeed confined for long-times to the band of the initial state, we calculate the fidelity corresponding to the probability to find the evolved state within the initial band,

$$F_b(t) = \text{Tr} \left[ \hat{\rho}(t) \hat{P}_b \right], \quad \hat{P}_b = \sum_k |V^b_k\rangle \langle V^b_k|, \quad (8)$$

where $\hat{P}_b$ is the projector to the initial band spanned by the states $|V^b_k\rangle$. In Fig. 4, we plot the time $\tau_F$ that it takes for the fidelity to decay to half of its initial value for various frequencies and starting from an initial state in the band $b = 1$. We obtain a behavior very similar to that for the heating time $\tau^*$: the fidelity decays fast for frequencies close to the gap value, $\omega \approx \Delta_1$, and as we move away from it, $\tau_F$ increases significantly. This corroborates our claim that the suppression of heating and the emergence of Floquet-prethermalization, that we observe, are indeed a result of the confinement of the dynamics to the initial band.

**Discussion.**— We demonstrate that in periodically driven spin systems with long-range interactions, heating can be strongly suppressed not only with driving frequencies larger than the energy it costs to flip a single spin, but also with frequencies smaller than that energy. This is due to the formation of energy bands in the many-body spectrum of the static system, which get further apart as the system size increases. If the system is initialized within one band and the drive is off-resonance with the gap between the bands, then heating is significantly suppressed. This results in a non-monotonic dependence of the heating time on the frequency. For frequencies larger than the gap, increasing the frequency suppresses heating, while for frequencies below the gap, increasing the frequencies *enhances* heating.

Our results therefore provide a robust way to suppress heating even for small driving frequencies, which can be tested in experiments with ion traps [36, 40]. While in this Letter, due to numerical limitations, we have explored a one-dimensional system, our results should hold for any dimension, provided $\alpha < d$. 

![Figure 4](image_url)

**Figure 4.** Half-time decay of the fidelity, $\tau_F$, as a function of the rescaled frequency $\omega/\Delta_1$. The two vertical lines indicate the gaps $\Delta_2$ (dashed-dot) and $\Delta_1$ (dashed) for $L = 22$. As in Fig. 1: initial state $|\psi(0)\rangle = |11 \cdots 11011 \cdots 11\rangle$, $\alpha = 0.67$, $J_x = 0.69$, $h_x = 0.23$, $h_y = 0.21$, and $h_z = 0.19$. 


In the future, it would be interesting to see if constraining a long-range interacting system to a certain energy band allows to obtain, at least a transient, time-crystalline behavior, which has been ruled out for $\alpha < d$ [79]. It would be also interesting to study the effect of aperiodic drives on heating in such systems [87, 88].

This research was supported by a grant from the United States-Israel Binational Foundation (BSF, Grant No. 2019644), Jerusalem, Israel, and the United States National Science Foundation (NSF, Grant No. DMR-1936006), and by the Israel Science Foundation (grants No. 527/19 and 218/19). D.S.B acknowledges funding from the Kreitman fellowship.

[1] Pyotr Leonidovich Kapitza, “Dynamical stability of a pendulum when its point of suspension vibrates, and pendulum with a vibrating suspension,” Collected papers of PL Kapitza, 2, 714–737 (1965).
[2] D. H. Dunlap and V. M. Kenkre, “Dynamic localization of a charged particle moving under the influence of an electric field,” Phys. Rev. B 34, 3625–3633 (1986).
[3] D. H. Dunlap and V. M. Kenkre, “Dynamic localization of a particle in an electric field viewed in momentum space: Connection with Bloch oscillations,” Phys. Lett. A 127, 438–440 (1988).
[4] André Eckardt, Martin Holthaus, Hans Luger, Alessandro Zenesini, Donatella Ciampini, Oliver Morsch, and Ennio Arimondo, “Exploring dynamic localization with a Bose-Einstein condensate,” Phys. Rev. A 79, 013611 (2009).
[5] Jérôme Cayssol, Balázs Dóra, Ferenc Simon, and Roderich Moessner, “Floquet topological insulators,” Phys. stat. sol. (b) 201, 100–162 (2013).
[6] A. Gómez-León and G. Platero, “Floquet-Bloch theory and topology in periodically driven lattices,” Phys. Rev. B 89, 195116 (2014).
[7] Mark S. Rudner, Netanel H. Lindner, Erez Berg, and Michael Levin, “Anomalous edge states and the bulk-edge correspondence for periodically driven two-dimensional systems,” Phys. Rev. X 3, 031005 (2013).
[8] V. M. Bastidas, C. Emary, B. Regler, and T. Brandes, “Nonequilibrium quantum phase transitions in the Dicke model,” Phys. Rev. Lett. 108, 043603 (2012).
[9] Soonwon Choi, Dmitry A. Abanin, and Mikhail D. Lukin, “Dynamically induced many-body localization,” Phys. Rev. B 97, 100301 (2018).
[10] Luca D’Alessio and Anatoli Polkovnikov, “Many-body energy localization transition in periodically driven systems,” Ann. Phys. (N. Y). 333, 19–33 (2013).
[11] Eyal Bairey, Gil Refael, and Netanel H. Lindner, “Driving induced many-body localization,” Phys. Rev. B 96, 020201 (2017).
[12] Devendra Singh Bhakuni, Ritu Nehra, and Auditya Sharma, “Drive-induced many-body localization and coherent destruction of Stark many-body localization,” Phys. Rev. B 102, 024201 (2020).
[13] Achilleas Lazarides, Arnab Das, and Roderich Moessner, “Fate of Many-Body Localization under Periodic Driving,” Phys. Rev. Lett. 115, 030402 (2015).
[14] Dominic V. Else, Bela Bauer, and Chetan Nayak, “Floquet Time Crystals,” Phys. Rev. Lett. 117, 090402 (2016).
[15] Vedika Khemani, Achilleas Lazarides, Roderich Moessner, and S. L. Sondhi, “Phase Structure of Driven Quantum Systems,” Phys. Rev. Lett. 116, 250401 (2016).
[16] N. Y. Yao, A. C. Potter, I. D. Potirniche, and A. Vishwanath, “Discrete Time Crystals: Rigidity, Criticality, and Realizations,” Phys. Rev. Lett. 118, 030401 (2017).
[17] J. Zhang, P. W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G.Pagano, I. D. Potirniche, A. C. Potter, A. Vishwanath, N. Y. Yao, and C. Monroe, “Observation of a discrete time crystal,” Nature 543, 217–220 (2017).
[18] A. Kshetrimayum, J. Eisert, and D. M. Kennes, “Stark time crystals: Symmetry breaking in space and time,” Phys. Rev. B 102, 195116 (2020).
[19] Luca D’Alessio and Marcos Rigol, “Long-time behavior of isolated periodically driven interacting lattice systems,” Phys. Rev. X 4, 041048 (2014).
[20] Pedro Ponte, Anushya Chandran, Z. Papić, and Dmitry A. Abanin, “Periodically driven ergodic and many-body localized quantum systems,” Ann. Phys. (N. Y). 353, 196–204 (2015).
[21] David J. Luitz, Yevgeny Bar Lev, and Achilleas Lazarides, “Absence of dynamical localization in interacting driven systems,” SciPost Phys. 3, 029 (2017).
[22] Achilleas Lazarides, Arnab Das, and Roderich Moessner, “Equilibrium states of generic quantum systems subject to periodic driving,” Phys. Rev. E 90, 012110 (2014).
[23] Pedro Ponte, Z. Papić, François Huveneers, and Dmitry A. Abanin, “Many-body localization in periodically driven systems,” Phys. Rev. Lett. 114, 140401 (2015).
[24] Dmitry A. Abanin, Wojciech De Roeck, and François Huveneers, “Theory of many-body localization in periodically driven systems,” Ann. Phys. (N. Y). 372, 1–11 (2016).
[25] Pranjal Bordia, Henrik Lütschen, Ulrich Schneider, Michael Knap, and Inmanuel Bloch, “Periodically driving a many-body localized quantum system,” Nat. Phys. 13, 460–464 (2017).
[26] Dmitry A. Abanin, Wojciech De Roeck, and François Huveneers, “Exponentially Slow Heating in Periodically Driven Many-Body Systems,” Phys. Rev. Lett. 115, 256803 (2015).
[27] Dmitry A. Abanin, Wojciech De Roeck, Wen Wei Ho, and François Huveneers, “Effective Hamiltonians, prethermalization, and slow energy absorption in periodically driven many-body systems,” Phys. Rev. B 95, 014412 (2017).
[28] Takashi Mori, Tomotaka Kuwahara, and Keiji Saito, “Rigorous Bound on Energy Absorption and Generic Relaxation in Periodically Driven Quantum Systems,” Phys. Rev. Lett. 116, 120401 (2016).
[29] Dmitry Abanin, Wojciech De Roeck, Wen Wei Ho, and François Huveneers, “A Rigorous Theory of Many-Body Prethermalization for Periodically Driven and Closed Quantum Systems,” Commun. Math. Phys. 354, 809–827 (2017).
[30] Simon A. Weidinger and Michael Knap, “Floquet prethermalization and regimes of heating in a periodically driven, interacting quantum system,” Sci. Rep. 7, 1–10 (2017).
Recently, Floquet-prethermalization was achieved away from the high frequency limit in a model with short-range interactions by imposing special constraints on the driving protocol [89].

Philip Richerme, Zhe-Xuan Gong, Aaron Lee, Crystal Senko, Jacob Smith, Michael Foss-Feig, Spyridon Michalakis, Alexey V. Gorshkov, and Christopher Monroe, “Non-local propagation of correlations in quantum systems with long-range interactions,” Nature 511, 198–201 (2014).

Jens Eisert, Mauritz Van Den Worm, Salvatore R. Manmana, and Michael Kastner, “Breakdown of quasilocalization in long-range quantum lattice models,” Phys. Rev. X 3, 1–16 (2013).

J. Schachenmayer, B. P. Lanyon, C. F. Roos, and A. J. Daley, “Entanglement growth in quench dynamics with variable range interactions,” Phys. Rev. X 3, 076401 (2013).

Benedikt Kloss and Yevgeny Bar Lev, “Spin transport in systems with Long-range Interactions,” Phys. Rev. Lett. 104, 240403 (2010).

R. Bachelard, C. Chandre, D. Fanelli, X. Leoncini, and S. Ruffo, “Abundance of regular orbits and nonequilibrium phase transitions in the thermodynamic limit for long-range systems,” Phys. Rev. Lett. 101, 260603 (2008).

R. Bachelard and D. Fanelli, “Spreading of correlations in long-range interacting quantum systems,” Phys. Rev. Lett. 111, 207202 (2013).

P. Hauke and L. Tagliacozzo, “Spread of correlations in long-range interacting quantum systems,” Phys. Rev. Lett. 111, 207202 (2013).

L. S. Levitov, “Decoherence of vibrational modes caused by electric dipole interaction,” Phys. Rev. Lett. 64, 547–550 (1990).

M. A. Nielsen and I. L. Chuang, “Quantum information and computation,” Cambridge University Press, 2010.

Bo Yan, Steven A. Moses, Bryce Gadway, Jacob P. Covey, Kaden R. A. Hazzard, Ana Maria Rey, Deborah S. Jin, and Jun Ye, “Observation of dipolar spin-exchange interactions with lattice-confined polar molecules,” Nature 501, 521–525 (2013).

R. Islam, C. Senko, W. C. Campbell, S. Korenblit, J. Smith, A. Lee, E. E. Edwards, C. C. J. Wang, J. K. Freericks, and C. Monroe, “Emergence and frustration of magnetism with variable-range interactions in a quantum simulator,” Science 340, 583–587 (2013).

Joseph W. Britton, Brian C. Sawyer, Adam C. Keith, C. C. J. Wang, James K. Freericks, Hermann Uys, Michael J. Biercuk, and John J. Bollinger, “Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins,” Nature 484, 489–492 (2012).

N. D. Mermin and H. Wagner, “Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models,” Phys. Rev. Lett. 17, 1133–1136 (1966).

Michael Kastner, “Nonequivalence of ensembles for long-range quantum spin systems in optical lattices,” Phys. Rev. Lett. 104, 240403 (2010).

D. Mukamel, S. Ruffo, and N. Schreiber, “Breaking of ergodicity and long relaxation times in systems with long-range interactions,” Phys. Rev. Lett. 95, 240604 (2005).

N. D. Mermin and H. Wagner, “Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models,” Phys. Rev. Lett. 17, 1133–1136 (1966).

David Wei, Jun Rui, S. L. Sondhi, Vedika Khemani, R. Islam, C. Senko, Jacob Smith, A. Lee, E. E. Edwards, C. C. J. Wang, J. K. Freericks, and C. Monroe, “Emergence and frustration of magnetism with variable-range interactions in a quantum simulator,” Science 340, 583–587 (2013).

Joseph W. Britton, Brian C. Sawyer, Adam C. Keith, C. C. J. Wang, James K. Freericks, Hermann Uys, Michael J. Biercuk, and John J. Bollinger, “Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins,” Nature 484, 489–492 (2012).

N. D. Mermin and H. Wagner, “Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models,” Phys. Rev. Lett. 17, 1133–1136 (1966).

D. Mukamel, S. Ruffo, and N. Schreiber, “Breaking of ergodicity and long relaxation times in systems with long-range interactions,” Phys. Rev. Lett. 95, 240604 (2005).

G. L. Celardo, J. Barré, F. Borgonovi, and S. Ruffo, “Time scale for magnetic reversal and the topological nonconnectivity threshold,” Phys. Rev. E 73, 011108 (2006).

R. Bachelard, C. Chandre, D. Fanelli, X. Leoncini, and S. Ruffo, “Abundance of regular orbits and nonequilibrium phase transitions in the thermodynamic limit for long-range systems,” Phys. Rev. Lett. 101, 260603 (2008).

Michael Kastner, “Nonequivalence of ensembles for long-range quantum spin systems in optical lattices,” Phys. Rev. Lett. 104, 240403 (2010).

J. Schachenmayer, B. P. Lanyon, C. F. Roos, and A. J. Daley, “Entanglement growth in quench dynamics with variable range interactions,” Phys. Rev. X 3, 1–16 (2013).

Jens Eisert, Mauritz Van Den Worm, Salvatore R. Manmana, and Michael Kastner, “Breakdown of quasilocalization in long-range quantum lattice models,” Phys. Rev. Lett. 111, 260401 (2013).

P. Hauke and L. Tagliacozzo, “Spread of correlations in long-range interacting quantum systems,” Phys. Rev. Lett. 111, 207202 (2013).

L. S. Levitov, “Decoherence of vibrational modes caused by electric dipole interaction,” Phys. Rev. Lett. 64, 547–550 (1990).

M. A. Nielsen and I. L. Chuang, “Quantum information and computation,” Cambridge University Press, 2010.

N. D. Mermin and H. Wagner, “Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models,” Phys. Rev. Lett. 17, 1133–1136 (1966).

David Wei, Jun Rui, S. L. Sondhi, Vedika Khemani, R. Islam, C. Senko, Jacob Smith, A. Lee, E. E. Edwards, C. C. J. Wang, J. K. Freericks, and C. Monroe, “Emergence and frustration of magnetism with variable-range interactions in a quantum simulator,” Science 340, 583–587 (2013).

Joseph W. Britton, Brian C. Sawyer, Adam C. Keith, C. C. J. Wang, James K. Freericks, Hermann Uys, Michael J. Biercuk, and John J. Bollinger, “Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins,” Nature 484, 489–492 (2012).

N. D. Mermin and H. Wagner, “Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models,” Phys. Rev. Lett. 17, 1133–1136 (1966).

D. Mukamel, S. Ruffo, and N. Schreiber, “Breaking of ergodicity and long relaxation times in systems with long-range interactions,” Phys. Rev. Lett. 95, 240604 (2005).
disordered systems induced by the long-range many-body interaction,” arXiv preprint cond-mat/0611387 (2006).

[63] Alexander L. Burin, “Localization in a random XY model with long-range interactions: Intermediate case between single-particle and many-body problems,” Phys. Rev. B 92, 104428 (2015).

[64] Wojciech De Roeck and François Huveneers, “Stability and instability towards delocalization in many-body localization systems,” Phys. Rev. B 95, 155129 (2017).

[65] K. S. Tikhonov and A. D. Mirlin, “Many-body localization transition with power-law interactions: Statistics of eigenstates,” Phys. Rev. B 97, 214205 (2018).

[66] Sarang Gopalakrishnan and David A. Huse, “Instability of many-body localized systems as a phase transition in a nonstandard thermodynamic limit,” Phys. Rev. B 99, 134305 (2019).

[67] Sabyasachi Nag and Arti Garg, “Many-body localization in the presence of long-range interactions and long-range hopping,” Phys. Rev. B 99, 224203 (2019).

[68] Shihadhi Roy and David E. Logan, “Self-consistent theory of many-body localisation in a quantum spin chain with long-range interactions,” SciPost Phys. 7, 42 (2019).

[69] Zhe Xuan Gong, Michael Foss-Feig, Spyridon Michalakis, and Alexey V. Gorshkov, “Persistence of locality in systems with power-law interactions,” Phys. Rev. Lett. 113, 030602 (2014).

[70] Leonardo Mazza, Davide Rossini, Manuel Endres, and Rosario Fazio, “Out-of-equilibrium dynamics and thermalization of string order,” Phys. Rev. B 90, 020301 (2014).

[71] Michael Foss-Feig, Zhe Xuan Gong, Charles W. Clark, and Alexey V. Gorshkov, “Nearly linear light cones in long-range interacting quantum systems,” Phys. Rev. Lett. 114, 157201 (2015).

[72] Lea F. Santos, Fausto Borgonovi, and Giuseppe Luca Celardo, “Cooperative Shielding in Many-Body Systems with Long-Range Interaction,” Phys. Rev. Lett. 116, 250402 (2016).

[73] G. L. Celardo, R. Kaiser, and F. Borgonovi, “Shielding and localization in the presence of long-range hopping,” Phys. Rev. B 94, 144206 (2016).

[74] Alessio Lerose and Silvia Pappalardi, “Origin of the slow growth of entanglement entropy in long-range interacting spin systems,” Phys. Rev. Res. 2, 012041 (2020).

[75] David Maximilian Storch, Mauritz Van Den Worm, and Michael Kastner, “Interplay of soundcone and supersonic propagation in lattice models with power law interactions,” New J. Phys. 17, 063021 (2015).

[76] H. N. Nazareno and P. E. de Brito, “Long-range interactions and nonextensivity in one-dimensional systems,” Phys. Rev. B 60, 4629–4634 (1999).

[77] Wen Wei Ho, Ivan Protopopov, and Dmitry A. Abanin, “Bounds on Energy Absorption and Prethermalization in Quantum Systems with Long-Range Interactions,” Phys. Rev. Lett. 120, 200601 (2018).

[78] Francisco Machado, Gregory D. Meyer, Dominic V. Else, Chetan Nayak, and Norman Y. Yao, “Exponentially Slow Heating in Short and Long-range Interacting Floquet Systems,” Phys. Rev. Res. 1, 033202 (2019).

[79] Francisco Machado, Dominic V. Else, Gregory D. Kahanamoku-Meyer, Chetan Nayak, and Norman Y. Yao, “Long-Range Prethermal Phases of Nonequilibrium Matter,” Phys. Rev. X 10, 011043 (2020).

[80] Tomotaka Kuwahara, Takashi Mori, and Keiji Saito, “Floquet-Magnus theory and generic transient dynamics in periodically driven many-body quantum systems,” Ann. Phys. (N. Y). 367, 96–124 (2016).

[81] Michael Kastner, “Diverging equilibration times in long-range quantum spin models,” Phys. Rev. Lett. 106, 130601 (2011).

[82] Romain Bach Bard and Michael Kastner, “Universal threshold for the dynamical behavior of lattice systems with long-range interactions,” Phys. Rev. Lett. 110, 170603 (2013).

[83] Michael Kastner, “N -scaling of timescales in long-range N -body quantum systems,” J. Stat. Mech. Theory Exp. 2017, 014003 (2017).

[84] Brian Neyenhuis, Jiehang Zhang, Paul W. Hess, Jacob Smith, Aaron C. Lee, Phil Richerme, Zhe Xuan Gong, Alexey V. Gorshkov, and Christopher Monroe, “Observation of prethermalization in long-range interacting spin chains,” Sci. Adv. 3, e1700672 (2017).

[85] David J. Luitz and Yevgeny Bar Lev, “The ergodic side of the many-body localization transition,” Ann. Phys. 529, 1600350 (2017).

[86] Don N. Page, “Average entropy of a subsystem,” Phys. Rev. Lett. 71, 1291–1294 (1993).

[87] Hongzheng Zhao, Florian Mintert, Roderich Moessner, and Johannes Knolle, “Random Multipolar Driving: Tunably Slow Heating through Spectral Engineering,” Phys. Rev. Lett. 126, 040601 (2021).

[88] Takashi Mori, Hongzheng Zhao, Florian Mintert, Johannes Knolle, and Roderich Moessner, “Rigorous bounds on the heating rate in thue-morse quasiperiodically and randomly driven quantum many-body systems,” Phys. Rev. Lett. 127, 050602 (2021).

[89] Christoph Fleckenstein and Marin Bukov, “Prethermalization and thermalization in periodically driven many-body systems away from the high-frequency limit,” Phys. Rev. B 103, L140302 (2021).