A Tachyon Field around the Black Hole

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Abstract

We study the effects of the presence of the tachyon field around the black hole. We show that in presence of the tachyon field, unlike the ordinary canonical scalar field, the time evolution of the black hole mass depends on the potential of this field. By considering several types of potential, we study the behavior of the black hole mass and its time evolution and find some interesting results. We find that the presence of the tachyon field causes the accretion of the mass into the black hole. We also show that with linear and hilltop potentials, in some ranges of the parameters space, the mass of the black hole can decrease even without any Hawking radiation.

Key Words: Tachyon Field, Black Hole, Mass Accretion
1 Introduction

Given that in the current universe there are many black holes that have different masses, the study of black holes, as the thermodynamical systems, has attracted a lot of attention. As regards some massive black holes have originated from primordial black holes, one of the interesting aspects in the study of the black holes is the accretion of the matter into the black hole. Although the standard argument assumes that the mass of the primordial black holes remains constant once they are formed [1, 2, 3], the authors of Ref. [4] have shown that this is not necessarily true. They have studied scenarios in which the accretion of the black holes’ mass occurs. Satisfying the null energy condition causes that the mass of the black hole never decreases in a classical way, but even increases [5]. However, by considering the quantum processes, there would be the Hawking radiation leading to the mass decrement of the black hole [6, 7]. By emitting the Hawking radiation, the black holes can be evaporated completely. So, any black hole in its evolution has been affected by both decrement (arising from the Hawking radiation) and increment (due to the accretion of the matter-energy) of the mass. If the Hawking radiation dominates, the black hole evaporates. If the accretion dominates, the black hole grows.

In Ref. [4], it is discussed that the primordial black holes might have grown by absorbing the existing scalar field in the universe, leading to the mass increment of the black holes. Also, in Refs. [8, 9] it has been shown that by considering the phantom scalar field, the mass of the black hole effectively decreases. In Ref. [10], the authors have considered the nonminimally coupled canonical scalar field and studied the evolution of the black hole mass. They have shown that the accretion of the nonminimally coupled scalar field causes the mass to decrease when the black hole mass is smaller than a certain critical value. Note that, this result came up without exist the phantom energy in the model.

Another interesting scalar field that can be responsible for both the initial and late time acceleration phase of the universe is the tachyon field [11, 12, 13, 14, 15, 16, 17, 18]. In this paper, instead of considering the phantom field or non-minimal coupling, we adopt the tachyon field to study the accretion of the black hole. We show that when we consider the tachyon field in the theory, the time derivative of the black hole mass would be corresponding to the potential of this field. Note that, in the minimal “simple canonical scalar field”, there is no dependence on the potential in the time derivative of the black hole mass [4]. In this respect, in our model, it is possible to study the accretion of the mass in the minimally coupled scalar field with different potentials. On the other hand, the authors of Refs [19, 20, 21] have shown that a negative potential can lead to interesting cosmological results. In this regard, we show that by adopting the suitable negative potential, there would be a decrease in the black hole mass, at least in some ranges of the model’s parameter space. This result is obtained without presenting a nonminimal coupling. Note that, the negative potential for the tachyon field leads to the negative energy density. Although currently there is no evidence for the role of the negative energy density in our universe, some realizations of the relativistic quantum field theories predict this possibility [22, 23]. Also, some authors have studied the models with negative energy density and have shown that, at least at a theoretical level, it leads to interesting cosmological implications [24, 25]. Therefore, it seems inspiring to consider both negative and positive potentials for the tachyon field.

With these preliminaries, the paper is organized as follows. In section 2, we consider the tachyon field around the black hole. By using the energy-momentum tensor of the tachyon field and also its equation of motion, we seek the effects of this field on the mass evolution of the black hole. In
this section, we obtain an expression for the time evolution of the black hole mass in terms of the tachyon field’s potential. In section 3, we study the evolution of the black hole mass by considering several types of potential. We show that by considering the tachyon field around the black hole, it is possible to have accretion of the mass into the black hole. We also show that, with linear and hilltop potentials, the mass of the black hole can decrease without considering the Hawking radiation. In section 4, we summarize our work.

2 Tachyon Field around the Black Hole

With a tachyon field, we deal with the following action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - V(\phi) \sqrt{1 - 2X} \right], \tag{1} \]

where \( R \) is the Ricci scalar, \( \kappa \) is the gravitational constant, and \( V(\phi) \) is the potential of the tachyon field. Also, \( X \) is defined as \( X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \). It should be noticed that, as it has been demonstrated in Ref. [26], if we consider \( X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \) the tachyon field becomes a phantom. However, in this work we don’t consider this case. Note that, in Ref. [4], where a canonical scalar field has been considered, the authors have studied both zero and non-zero potentials. However, in the case with a tachyon field, considering the zero potential removes the effect of the tachyon field entirely.

The interesting point is that, although our considered tachyon field is not a usual phantom, depending on the values of the potential, it is possible for the tachyon field to behave like a phantom. In fact, from equation (3) we have \( \rho + p = -\frac{VX}{\sqrt{1 - 2X}} \), where \( \rho = T_{00} \) and \( p = T_{ii} \) with \( i = 1, 2, 3 \) (see also Ref. [27], where it has been shown that it is possible to express density and pressure in terms of Lagrangian and its derivatives). Considering that the parameter \( X \) is given by \( X = \frac{1}{2} \dot{\phi}^2 \), the tachyon field violates the null energy condition for positive values of the potential. In this way, although the tachyon field is different from the usual phantom field, for the positive values of the potential, the tachyon field has the “phantom-like” behavior. On the other hand, for the negative values of the potential, we have \( \rho + p > 0 \), meaning the tachyon field doesn’t violate the null energy condition. While, in this case, the weak energy condition is violated (since we have \( \rho < 0 \)), and in some realizations of the relativistic quantum field theories this violation is predicted and possible.

Now, we consider the Schwarzschild coordinates and obtain the spherically symmetrical version
of (2) as
\[\ddot{\phi} - \left(1 - \frac{2M}{r}\right) \frac{1}{r^2} \partial_r \left[\left(1 - \frac{2M}{r}\right) r^2 \partial_r \phi\right] = -\frac{V'}{V} \left(1 - \frac{2M}{r}\right) \left[1 - \left(1 - \frac{2M}{r}\right)^{-1} \dot{\phi}^2 + \left(1 - \frac{2M}{r}\right) \left(\partial_r \phi\right)^2\right],\] (4)

where a dot shows the derivative of the parameter with respect to the time. We should find spherically symmetric solutions to the equation (4) with the following boundary condition according to the Bondi accretion process,
\[\phi(t, r \to \infty) = \phi_c(t),\] (5)

where \(\phi_c(t)\) shows the cosmological evolution of the scalar field. In fact, we assume the black hole lies on a local asymptotically flat space with boundary condition (5). This is because, in comparison with the cosmological length or time scales, the black hole is very small [28]. After we find a solution \(\phi(r, t)\) for equation (4), satisfying the boundary condition, we can obtain its energy flux through the horizon that is completely absorbed by the black hole. In this regard, in Schwarzschild coordinates, the rate of accretion of the tachyon field onto the black hole is given by
\[\dot{M} = \oint_{r = 2M} r^2 T_t^r d\Omega.\] (6)

If we adopt Eddington-Finkelstein coordinates \((v, r)\), which is regular on the horizon, the solution of equation (4) in stationary configuration is given by
\[\phi(v, r) = C_1 + C_2 \left[v - r + 2M \log \left(\frac{2M}{r}\right)\right],\] (7)

where \(C_1\) and \(C_2\) are constant. In [29], the authors have found a specific form of lagrangian leading to the stationary solutions. This means that they have obtained a form of lagrangian for which the action is invariant under shift symmetry \(\phi \to \phi + \lambda\). However, with the lagrangian form given in (1), we follow a quasi-stationary approach [31]. In the quasi-stationary approach, we consider a slowly rolling scalar field with \(V'(\phi) \approx 0\) and \(\dot{\phi} \approx 0\) which resemble the slow-roll conditions in inflation. Although the equation (4) is apparently different from the corresponding ones in Refs. [10, 31], we can consider the right-hand side of that equation as an effective term and write
\[\ddot{\phi} - \left(1 - \frac{2M}{r}\right) \frac{1}{r^2} \partial_r \left[\left(1 - \frac{2M}{r}\right) r^2 \partial_r \phi\right] = -V'_{\text{eff}},\] (8)

where
\[V'_{\text{eff}} = \frac{V'}{V} \left(1 - \frac{2M}{r}\right) \left[1 - \left(1 - \frac{2M}{r}\right)^{-1} \dot{\phi}^2 + \left(1 - \frac{2M}{r}\right) \left(\partial_r \phi\right)^2\right].\] (9)

Note that, in equation (5) and forthcoming equations, a dot on the parameter means the derivative over the Eddington-Finkelstein coordinate \(v\) which is equal to derivative over time.
As emphasized in Ref. [31], the solution of equation (8) is rather independent of the form of the potential. In this regard, the field configuration at the black hole horizon can be approximated as [31, 10, 30]

\[ \phi(v, r) = \phi_c \left( v - r + 2M \log \left( \frac{2M}{r} \right) \right), \] (10)

where \( \phi_c \) has been introduced in boundary condition (5). Now, by substituting the solution (10) in the equation (8), we get

\[ \left[ 1 + \frac{2M}{r} + \left( \frac{2M}{r} \right)^2 + \left( \frac{2M}{r} \right)^3 \right] \ddot{\phi} = -V'_{\text{eff}}, \] (11)

where we now have the following expression for \( V'_{\text{eff}} \)

\[ V'_{\text{eff}} = V' \left[ 1 - \left( 1 - \frac{16M^4}{r^4} \right) \dot{\phi}^2 \right]. \] (12)

Note that, since we have considered the slowly varying tachyon field, \( \dot{\phi} \) is almost a constant, and the approximated solution is valid as long as \( \ddot{\phi} \) and \( V'_{\text{eff}} \) are very close to zero.

To study the time evolution of the black hole mass, we should find \( T_{tr} \). From equations (3) and (10) we obtain

\[ T_{tr} = \frac{(2M)^2 V \dot{\phi}_c^2}{\sqrt{1 - \left[ 1 + \frac{2M}{r} + \left( \frac{2M}{r} \right)^2 + \left( \frac{2M}{r} \right)^3 \right] \dot{\phi}^2}}. \] (13)

The important point about equation (13) is the presence of the potential. When there is a canonical scalar field, in both minimally or nonminimally coupled cases, the potential is absent in \( T_{tr} \). However, if we consider the tachyon field, the potential is presented in \( T_{tr} \). This is an interesting issue. The reason is that since \( T_{tr} \) is potential dependent, by considering just the several types of potential we can study the time evolution of the black hole mass.

In a quasi-stationary approach, it is possible to parameterize the field absorbed at the black hole horizon, by the field approximated from infinity as [28, 10, 30]

\[ \phi_c(t) \approx \phi_\infty + \dot{\phi}_\infty (t - t_0), \] (14)

with \( \phi_\infty \) and \( \dot{\phi}_\infty \) to be constant parameters. In fact, in this regard, we consider \( r \gg 2M \) limit and approximate the field from the infinity to the black hole horizon. Now, from equations (6), (13) and (14) we get

\[ \dot{M} = \frac{16\pi M^2 V \dot{\phi}_\infty^2}{\sqrt{1 - \left[ 1 + \frac{2M}{r} + \left( \frac{2M}{r} \right)^2 + \left( \frac{2M}{r} \right)^3 \right] \dot{\phi}_\infty^2}}. \] (15)

Since we consider \( r \gg 2M \) limit, for small values of \( \dot{\phi}_\infty \), we can rewrite equation (15) as

\[ \dot{M} = \left[ 16\pi M^2 V \dot{\phi}_\infty^2 \right] \left[ 1 + \frac{1}{2} \left[ 1 + \frac{2M}{r} + \left( \frac{2M}{r} \right)^2 + \left( \frac{2M}{r} \right)^3 \right] \dot{\phi}_\infty^2 \right]. \] (16)
Also, in the following, we just keep the terms in $\dot{M}$ which are of the order of $M^2$. This means that we keep the two first terms of the second bracket of the equation (16). In the following, using equation (16), we study the evolution of the black hole mass.

3 Evolution of the Black Hole Mass

To find $M$ from equation (16), we follow Ref. [10] and consider $\dot{M}$ as

$$
\dot{M} = f(t) M^2, 
$$

where, in our case,

$$
f(t) = 16\pi V \phi_\infty^2 \left(1 + \frac{1}{2} \phi_\infty^2\right). 
$$

Note that, $f(t)$ in equation (18) depends on the potential. However, by considering the canonical scalar field, even the non-minimally coupled one, there is no dependence on the potential [4, 10]. From equations (17) and (18) we see that a negative potential can cause decreasing in the black hole mass even in the absence of Hawking radiation. This is an interesting issue and we discuss it in the next subsections.

Now, the solution of equation (17) is given by

$$
M(t) = \frac{M_0}{1 - M_0 \mathcal{F}(t)}, 
$$

where $M_0$ is the mass at time $t_0$ and $\mathcal{F}$ is defined as

$$
\mathcal{F}(t) = \int_{t_0}^{t} f(t') dt'. 
$$

Since the mass is positive, so there is always a constraint on equation (19) as $M_0 \mathcal{F}(t) < 1$. In this step, we should specify the form of the potential. We consider four types of potential as linear, quadratic, natural, and hilltop potentials. In the following, we study each case separately.

3.1 Linear Potential

The first potential which we consider is the linear potential as $V \sim \phi$ [32]. By this potential, we find the function $f(t)$ as

$$
f(t) = 8 \phi_\infty^2 \left(\phi_\infty^2 + 2\right) \pi \left(\phi_\infty (t - t_0) + \phi_\infty\right), 
$$

and $\mathcal{F}(t)$ as

$$
\mathcal{F}(t) = 16 \pi \phi_\infty^2 \left(1 + \frac{1}{2} \phi_\infty^2\right) \left(\phi_\infty \left(\frac{1}{2} t^2 - t_0 t\right) + \phi_\infty t\right). 
$$
By substituting equations (21) and (22) in equation (17) and (19), we can obtain the black hole mass and its time evolution as follows

\[
M = \frac{M_0}{-16\pi\dot{\phi}_\infty^2\left(1 + \frac{1}{2}\dot{\phi}_\infty^2\right)\left(\dot{\phi}_\infty\left(\frac{1}{2}t^2 - t_0t\right) + \phi_\infty t\right)M_0 + 1},
\]

(23)

\[
\dot{M} = \frac{16\pi\left(\phi_\infty(t-t_0) + \phi_\infty\right)\dot{\phi}_\infty^2\left(1 + \frac{1}{2}\dot{\phi}_\infty^2\right)M_0^2}{\left(-16\pi\dot{\phi}_\infty^2\left(1 + \frac{1}{2}\dot{\phi}_\infty^2\right)\left(\dot{\phi}_\infty\left(\frac{1}{2}t^2 - t_0t\right) + \phi_\infty t\right)M_0 + 1\right)^2}.
\]

(24)

Equation (23) sets the following constraint on the model’s parameter space in the linear potential case

\[
16\pi\dot{\phi}_\infty^2\left(1 + \frac{1}{2}\dot{\phi}_\infty^2\right)\left(\dot{\phi}_\infty\left(\frac{1}{2}t^2 - t_0t\right) + \phi_\infty t\right)M_0 < 1.
\]

(25)

Therefore, for the numerical study of the model, we should consider the above constraint. In this regard, we perform numerical analysis on this constraint to find the suitable ranges of the parameters. To perform this analysis, we should specify the parameters \(\phi_\infty\) and \(\dot{\phi}_\infty\). Given that these parameters are arbitrary constant, we consider two cases as \(\phi_\infty = 0.1\dot{\phi}_\infty\) and \(\phi_\infty = -0.1\dot{\phi}_\infty\). We also adopt \(M_0 = 1\) and \(t_0 = 0\). By these adoptions of the parameters, we perform numerical analysis on the parameter space of \(\phi\) and \(t\), satisfying the constraint (25). The results are shown in figure 1, where the parameter \(\chi\) represents the left-hand side of the constraint (25). The region of \(\phi_\infty\) and \(t\) leading to \(\chi < 1\) are viable. This figure helps us to adopt appropriate values of parameter \(\phi_\infty\) and therefore \(\dot{\phi}_\infty\), to study the black hole mass and its time evolution numerically. In figure 2, we plot the evolution of the black hole masse for \(\phi_\infty = 0.1\dot{\phi}_\infty\) and \(\phi_\infty = -0.1\dot{\phi}_\infty\), for some sample values of \(\phi_\infty\) as 0.3, 0.1, -0.1 and -0.3. These adopted values of \(\phi_\infty\) satisfy the constraint (25). From figure 2, we see that by adopting \(\phi_\infty = 0.1\dot{\phi}_\infty\), \(\phi_\infty = 0.1\) and \(\phi_\infty = 0.3\), we get accretion of the tachyon field into the black hole and the black hole mass increases. By adopting \(\phi_\infty = 0.1\dot{\phi}_\infty\), \(\phi_\infty = -0.1\) and \(\phi_\infty = -0.3\), the black hole mass decreases even without considering the Hawking radiation. In the case with \(\phi_\infty = -0.1\dot{\phi}_\infty\), for all adopted values of \(\phi_\infty\), the tachyon field at first causes increasing of the black hole mass, then it decreases the mass. This interesting result is obtained because of the fact that with \(\phi_\infty = 0.1\dot{\phi}_\infty\), for \(\phi_\infty = -0.1\) and \(\phi_\infty = -0.3\), and with \(\phi_\infty = -0.1\dot{\phi}_\infty\), for all adopted values of \(\phi\), the potential can be negative leading to negative values of \(\dot{M}\). The evolution of \(\dot{M}\) versus time is shown in figure 3, for both cases with \(\phi_\infty = 0.1\dot{\phi}_\infty\) and \(\phi_\infty = -0.1\dot{\phi}_\infty\). We conclude that, by considering the tachyon field with the linear potential, it is possible to have a decrease in the black hole mass without considering the Hawking radiation. Note that, in the ranges of the parameters space leading to black hole mass decrease, we have \(\rho + p > 0\). This means that, in these ranges, the null energy condition is not violated. Therefore, in these ranges, the tachyon field is not a phantom. In this case, we face the mass decrease of the black hole because of the negative energy density of the tachyon field.
Figure 1: Presentation of the range of parameters $\phi$ and $t$, for the linear potential, satisfying the constraint \( (25) \). The upper panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the lower panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. Also, the parameter $\chi$ represents the left-hand side of equation \( (25) \).
Figure 2: Time evolution of the black hole mass for different values of $\phi_\infty$, with linear potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$.

Figure 3: Time evolution of $\dot{M}$ for different values of $\phi_\infty$, with linear potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. 
3.2 Quadratic Potential

The next potential which we study is the quadratic potential as \( V \sim \phi^2 \). With the quadratic potential, we find the following expressions

\[
f(t) = 8\pi \left( \phi_\infty^2 + 2 \right) \left( \phi_\infty (t - t_0) + \phi_\infty \right)^2 \phi_\infty^2 ,
\]

and

\[
F = \frac{16}{3} \pi \phi_\infty \left( 1 + \frac{1}{2} \phi_\infty^2 \right) \left( \phi_\infty t - \phi_\infty t_0 + \phi_\infty \right)^3.
\]

With these expressions and by using equations (17) and (19), we get

\[
M = \frac{M_0}{3 - 8\pi \left( (t - t_0) \phi_\infty + \phi_\infty \right)^3 \phi_\infty \left( \phi_\infty^2 + 2 \right) M_0},
\]

\[
\dot{M} = \frac{72 \left( \phi_\infty^2 + 2 \right) \phi_\infty^2 \pi \left( (t - t_0) \phi_\infty + \phi_\infty \right)^2 M_0^2}{\left( -3 + 8\pi \left( (t - t_0) \phi_\infty + \phi_\infty \right)^3 \phi_\infty \left( \phi_\infty^2 + 2 \right) M_0 \right)^2}.
\]

To have the positive black hole mass, from equation (28), the following constraint should be satisfied

\[
8\pi \left( (t - t_0) \phi_\infty + \phi_\infty \right)^3 \phi_\infty \left( \phi_\infty^2 + 2 \right) M_0 < 3.
\]

This means that, to the numerical study of the model, we should consider the parameters’ values satisfying the constraint (30). Here also, we perform a numerical analysis which is shown in figure 4. As the linear case, we have considered \( \phi_\infty = 0.1 \phi_\infty \) and \( \phi_\infty = -0.1 \phi_\infty \) and also adopted \( M_0 = 1 \) and \( t_0 = 0 \). As figure shows, with \( \phi = 0.1 \phi \), the constraint (30) implies some constraint on the parameter \( \phi \). However, with \( \phi_\infty = -0.1 \phi_\infty \), there is no constraint on \( \phi_\infty \) and all considered values of \( \phi_\infty \) are viable. Now, we can study the evolution of the black hole mass in the quadratic potential case. In this regard, we analyze the time evolution of the black hole mass by using equation (28). The results are shown in figure 5. As figure shows, for both cases with \( \phi_\infty = 0.1 \phi_\infty \) and \( \phi_\infty = -0.1 \phi_\infty \) and with quadratic potential, the black hole mass increases continually. Correspondingly, the value of \( \dot{M} \) in every time is positive. This issue has been shown in figure 6. As a result, the accretion of the tachyon field with quadratic potential into the black hole leads to an increase in the black hole mass. Note that, with a quadratic potential, we have \( \rho + p < 0 \), leading to violation of both null and weak energy conditions. This means that a tachyon field with quadratic potential behaves like a phantom (despite it is not a usual phantom field).

3.3 Natural Potential

With the natural potential \( V \sim \Lambda^4 \left[ 1 + \cos \left( \frac{\phi}{l} \right) \right] \), we find the function \( f(t) \) as

\[
f(t) = 8\pi \Lambda^4 \left[ 1 + \cos \left( \frac{\phi_\infty (t - t_0) + \phi_\infty}{l} \right) \right] \phi_\infty^2 \left( \phi_\infty^2 + 2 \right),
\]
Figure 4: Presentation of the range of parameter $\phi_{\infty}$ and $t$, for the quadratic potential, satisfying the constraint (30). The left panel is corresponding to the case with $\phi_{\infty} = 0.1 \phi_{\infty}$ and the right panel is corresponding to the case with $\phi_{\infty} = -0.1 \phi_{\infty}$. Also, the parameter $\chi$ represents the left hand side of equation (30).
Figure 5: Time evolution of the black hole mass for different values of $\phi_\infty$, with quadratic potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$.

Figure 6: Time evolution of $\dot{M}$ for different values of $\phi_\infty$, with quadratic potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. 
and \( F \) as
\[
F = 8 \pi \Lambda^4 \dot{\phi}_\infty^2 \left( \dot{\phi}_\infty^2 + 2 \right) \left[ t + \frac{l}{\phi_\infty} \sin \left( \frac{\dot{\phi}_\infty}{l} t - \frac{\dot{\phi}_\infty t_0 - \phi_\infty}{l} \right) \right].
\] (32)

Now, from equations (17), (19), (31) and (32), we find the black hole mass and its time evolution in the natural potential case as follows
\[
M = M_0 \left[ -8 \pi \Lambda^4 \phi_\infty^2 \left( \phi_\infty^2 + 2 \right) \left[ t + \frac{l}{\phi_\infty} \sin \left( \frac{t \dot{\phi}_\infty}{l} - \frac{\dot{\phi}_\infty t_0 - \phi_\infty}{l} \right) \right] M_0 + 1 \right],
\] (33)
\[
\dot{M} = \left[ 8 \pi \Lambda^4 \phi_\infty^2 \left( \phi_\infty^2 + 2 \right) M_0^2 \left( 1 + \cos \left( \frac{\phi_\infty}{l} (t - t_0) + \phi_\infty \right) \right) \right] \left[ -8 \pi \Lambda^4 \phi_\infty^2 \left( \phi_\infty^2 + 2 \right) \left[ t + \frac{l}{\phi_\infty} \sin \left( \frac{t \dot{\phi}_\infty}{l} - \frac{\dot{\phi}_\infty t_0 - \phi_\infty}{l} \right) \right] M_0 + 1 \right]^2.
\] (34)

For the natural potential case, equation (33) gives the following constraint on the model’s parameters
\[
8 \pi \Lambda^4 \phi_\infty^2 \left( \phi_\infty^2 + 2 \right) \left[ t + \frac{l}{\phi_\infty} \sin \left( \frac{t \dot{\phi}_\infty}{l} - \frac{\dot{\phi}_\infty t_0 - \phi_\infty}{l} \right) \right] M_0 < 1.
\] (35)

The parameter space satisfying the constraint (35) is shown in figure 7, for two cases with \( \phi_\infty = 0.1 \dot{\phi}_\infty \) and \( \phi_\infty = -0.1 \dot{\phi}_\infty \). To plot this figure, we have considered \( \Lambda = 1, l = 1, M_0 = 1 \) and \( t_0 = 0 \). As the figure shows, for both cases, there are some values of the parameters which satisfy the constraint (35) and can be used in numerical analysis of \( M \) and \( \dot{M} \). Here, we consider \( \phi_\infty = -0.3, -0.1, 0.1 \) and 0.3. With these adopted values of \( \phi_\infty \), we study the evolution of the black hole mass versus time for both cases with \( \phi_\infty = 0.1 \dot{\phi}_\infty \) and \( \phi_\infty = -0.1 \dot{\phi}_\infty \). The results are shown in figure 8. This figure shows that, as time goes, the mass of the black hole increases. This means that with natural potential we always have the accretion of the mass into the black hole and there is no evaporation or decreasing of the black hole mass. This result is verified by figure 9, where we have plotted the time evolution of \( \dot{M} \). When we adopt a natural potential for the tachyon field, we find \( \rho + p < 0 \). Therefore, in this case also, both null and weak energy conditions are violated. In this regard, it seems that a tachyon field with a natural potential behaves like a phantom.

### 3.4 Hilltop Potential

With the hilltop potential which is defined as \( V \sim V_0 \left[ 1 - \left( \frac{r}{R} \right)^q \right] \), we find the following expression for \( f(t) \)
\[
f(t) = 16 \pi V_0 \left[ 1 - \left( \frac{\phi_\infty (t - t_0) + \phi_\infty}{\mu} \right)^q \right] \phi_\infty^2 \left( 1 + \frac{1}{2} \phi_\infty^2 \right).
\] (36)
Figure 7: Presentation of the range of parameter $\phi_\infty$ and $t$, for the natural potential, satisfying the constraint (35). The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. Also, the parameter $\chi$ represents the left hand side of equation (35).
Figure 8: Time evolution of the black hole mass for different values of $\phi_\infty$, for the natural potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$.

Figure 9: Time evolution of $\dot{M}$ for different values of $\phi_\infty$, for the natural potential. The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. 
Also, the hilltop potential gives $F$ as

$$F = 8\pi V_0 \dot{\phi}^2 \left( \dot{\phi}^2 + 2 \right) \left( t - \frac{l}{\phi_\infty} \left( \frac{\dot{\phi}_\infty t}{\mu} + \frac{\dot{\phi}_\infty t_0 + \phi_\infty}{\mu} \right)^{1+q} \right).$$

By having the above equations and using equations (17) and (19), we find the black hole mass as follows

$$M = M_0 \left[ 1 - 8\pi V_0 \dot{\phi}^2 \left( \dot{\phi}^2 + 2 \right) \left( t - \frac{\mu}{\phi_\infty} \left( \frac{\dot{\phi}_\infty t - \dot{\phi}_\infty t_0 + \phi_\infty}{\mu} \right)^{1+q} \right) M_0 \right],$$

and its time evolution as

$$\dot{M} = \left[ 16\pi V_0 \dot{\phi}^2 \left( 1 + \frac{1}{2} \dot{\phi}^2 \right) M_0^2 \left( 1 - \left( \frac{\dot{\phi}_\infty (t - t_0) + \phi_\infty}{\mu} \right)^q \right) \right]$$

$$\left[ 1 - 8\pi V_0 \dot{\phi}^2 \left( \dot{\phi}^2 + 2 \right) \left( t - \frac{\mu}{\phi_\infty} \left( \frac{\dot{\phi}_\infty t - \dot{\phi}_\infty t_0 + \phi_\infty}{\mu} \right)^{1+q} \right) M_0 \right]^2.$$

For the hilltop potential case, equation (38) gives the following constraint on the model’s parameters

$$8\pi V_0 \dot{\phi}^2 \left( \dot{\phi}^2 + 2 \right) \left( t - \frac{\mu}{\phi_\infty} \left( \frac{\dot{\phi}_\infty t - \dot{\phi}_\infty t_0 + \phi_\infty}{\mu} \right)^{1+q} \right) M_0 < 1.$$

The values of the model’s parameter which is used to perform numerical analysis on the model with hilltop potential should satisfy the constraint (40). The acceptable ranges of the parameters are shown in figure 10, for two cases with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and $\phi_\infty = -0.1 \dot{\phi}_\infty$. In this figure, the parameter $\chi$ is the left-hand side of the constraint (40). The viable regions of $\phi_\infty$ and $t$ are those that lead to $\chi < 1$. By using these viable values, we study the evolution of the black hole mass versus time numerically. The results are shown in figure 11. Our numerical analysis shows that, in the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and for $\phi_\infty = 0.1$ and $\phi_\infty = 0.3$, the presence of the tachyon field causes the black hole mass first increases and then decreases by time. Therefore, with hilltop potential, it is possible to have a decreasing black hole mass without considering the Hawking radiation. In the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$, if we consider $\phi_\infty = -0.1$ and $\phi_\infty = -0.3$, it is possible to have decrease in the mass of the black hole. However, for $\phi_\infty = -0.3$, the evolution of the black hole mass follows a strange pattern and is not favor. The time evolution of $\dot{M}$ for both cases with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and $\phi_\infty = -0.1 \dot{\phi}_\infty$ is shown in figure 12. This figure also confirms that, for $\phi_\infty = 0.1 \dot{\phi}_\infty$, $\phi_\infty = 0.1$ and $\phi_\infty = 0.3$ and also for $\phi_\infty = -0.1 \dot{\phi}_\infty$, $\phi_\infty = -0.1$ and $\phi_\infty = -0.3$, the mass of the black hole decreases even there is no Hawking radiation.

Here also, in the ranges of the parameters space leading to black hole mass decrease, we have $\rho + p > 0$. Therefore, in these ranges, the null energy condition is not violated. This means that, in these ranges, the tachyon field is not a phantom. As the linear potential case, the mass decrease of the black hole occurs due to the negative energy density of the tachyon field.
Figure 10: Presentation of the range of parameter $\phi_\infty$ and $t$, with hilltop potential, satisfying the constraint (40). The left panel is corresponding to the case with $\phi_\infty = 0.1 \dot{\phi}_\infty$ and the right panel is corresponding to the case with $\phi_\infty = -0.1 \dot{\phi}_\infty$. Also, the parameter $\chi$ represents the left hand side of equation (40).
Figure 11: Time evolution of the black hole mass for different values of $\phi_{\infty}$, with hilltop potential. The left panel is corresponding to the case with $\phi_{\infty} = 0.1\dot{\phi}_{\infty}$ and the right panel is corresponding to the case with $\phi_{\infty} = -0.1\dot{\phi}_{\infty}$.

Figure 12: Time evolution of $\dot{M}$ for different values of $\phi_{\infty}$, with hilltop potential. The left panel is corresponding to the case with $\phi_{\infty} = 0.1\dot{\phi}_{\infty}$ and the right panel is corresponding to the case with $\phi_{\infty} = -0.1\dot{\phi}_{\infty}$. 
4 Summary

In this paper, we have studied the accretion of the black hole mass in the presence of the tachyon field. Given that the previous works on the black hole mass accretion were with canonical scalar fields, in both the minimally and non-minimally coupled cases, we have considered a non-canonical scalar field on this issue. As regards the energy-momentum tensor of the tachyon field has non-diagonal components, the potential of the field is involved in the equations corresponding to the mass and its time evolution. In this regard, there is no need for the non-minimal coupling between the tachyon field and gravity. We have used four types of potential as linear, quadratic, natural, and hilltop potentials. To study the black hole mass, we have used the fact that some interesting cosmological solutions are corresponding to the negative potentials. In this regard, we have used the parameter space leading to both the negative and positive potentials. We have restricted the values of the model’s parameters by implying the constraint \( M \geq 0 \), where \( M \) is the black hole mass. To show those values of the model’s parameters satisfying the constraint \( M \geq 0 \), we have plotted some figures which gave us the viable ranges of the parameters. We have used these viable ranges to study the time evolution of the parameters \( M \) and \( \dot{M} \). By numerical analysis of \( M \) and \( \dot{M} \), we were wonder whether black hole mass decreases or not. Our numerical study has shown that if we consider the linear potential, for the cases with \( \phi_\infty = \pm 0.1 \dot{\phi}_\infty \), \( \phi_\infty = 0.1 \) and \( \phi_\infty = 0.3 \) we have accretion of the mass into the black hole. However, with \( \phi_\infty = \pm 0.1 \dot{\phi}_\infty \), \( \phi_\infty = -0.1 \) and \( \phi_\infty = -0.3 \), the black hole decreases without considering the Hawking radiation.

Another interesting point is that the tachyon field with linear and hilltop potentials, in some ranges of the parameter space leading to the black hole mass decrease, satisfies the null energy condition. In the cases with quadratic and natural potentials, the null energy condition is violated and the tachyon field has “phantom-like” behavior.

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