Effect of dissipation on the decay-rate phase transition

Soo-Young Lee\textsuperscript{a}, Hungsoo Kim\textsuperscript{b}, D.K.Park\textsuperscript{a}, Chang Soo Park\textsuperscript{c}, Jae Kwan Kim\textsuperscript{b}

\textsuperscript{a} Department of Physics, Kyungnam University, Masan, 631-701, Korea.

\textsuperscript{b} Department of Physics, Korea Advanced Institute of Science and Technology, Taejon, 305-701, Korea.

\textsuperscript{c} Department of Physics, Dankook University, Cheonan, 330-714, Korea.

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Abstract

A general condition for sharp transition of decay rate from quantum to thermal regimes is derived in dissipative tunneling models when position-dependent mass is involved. It is shown that the effect of dissipation in general changes the order of the phase transition. Especially, for the models with constant mass the Ohmic dissipation enlarges the range of parameters for first-order phase transitions. In the case of second-order phase transition the Ohmic dissipation suppresses the decay rate near the transition temperature($T_c$). For the super-Ohmic case the dissipation yields an opposite effects to the Ohmic dissipation within exponential approximation.
I. INTRODUCTION

It is well known that the decay of the metastable states at zero temperature is determined by pure quantum tunneling process whose dynamics is described by classical configurations called bounce in Euclidean space [1]. Since, however, the decay rate is determined at high temperature by thermal activation which corresponds to classical configuration called sphaleron [2], there exists some temperature $T_c$ at which the transition from classical- to quantum-dominated decay occurs. This phase transition problem was firstly discussed by Affleck [3] within quantum mechanics. He demonstrated that under certain assumptions for the shape of the barrier the transition between the thermal and quantum regimes is dominated by solutions with a finite period in the Euclidean time that smoothly interpolate between the zero-temperature bounce and the static high-temperature sphaleron. The transition is thus a second-order one. Chudnovsky [4], however, has shown that the type of the phase transition in the crossover from thermal activation to thermally assisted quantum tunneling is completely dependent on the shape of the potential barrier. He also has shown that the order of the phase transition is easily conjectured by $P$-vs-$E$ graph, where $P$ and $E$ are Euclidean period and energy, respectively. The sharp crossover between the thermal and thermally assisted tunneling occurs when $P = P(E)$-curve possesses a minimum at $E = E_c$, which is different from the energy of sphaleron solution. Based on Chudnovsky’s observation the sharp first-order transitions are found at spin tunneling systems with [5,6] and without [7] external magnetic field.

Recently, a sufficient criterion for the first-order phase transition in decay problem of the metastable state is obtained by carrying out the nonlinear perturbation near the sphaleron solution in the two-dimensional string model [8]. Inspired by spin-tunneling problem, the result of Ref. [8] is subsequently extended to the quantum mechanical model when mass is position-dependent [9].

The purpose of this paper is to derive a general criterion for the sharp first-order phase transition in the quantum mechanical tunneling models when the position-dependent mass and dissipation are involved. We first consider a system with Ohmic dissipation in which the spectral density $J(\omega)$ is linearly proportional to frequency, i.e., $J(\omega) = \alpha \omega$ where $\alpha$ is the dissipation coefficient. In this case Caldeira and Leggett already derived the effective action in their seminal paper [10]. We will use their result for the derivation of the criterion. The extension to the case of super-Ohmic dissipation where $J(\omega) = \alpha \omega^3$ is also examined. In the super-Ohmic case a counter term corresponding to the deformation of potential due to environment is introduced. The absence of the counter term in the Ohmic case is due to our use of the result of Ref. [10], in which a proper counter term is already used for the Ohmic dissipation. Since we work at Euclidean space, we use a prescription for the dissipation term which does not break the time-reversal symmetry in the both Ohmic and super-Ohmic cases. Carrying out the nonlinear perturbation we will derive the general condition for the first-order phase transition in the dissipative quantum mechanical system when mass is position-dependent in Sec.II. It is found that the effect of dissipation is only deformation of the eigenvalues for the temporal fluctuation operator $\hat{l}$ defined in this section. To get some physical intuition we will apply this general criterion to the simple quantum mechanical models in Sec.III. Brief conclusion and the direction of the future research in this field will be given at the final section.
II. CRITERION OF FIRST-ORDER PHASE TRANSITION FOR DISSIPATIVE QUANTUM SYSTEMS

The effect of dissipation on quantum tunneling is investigated in Ref. [10] by introducing the infinite number of oscillators as an environment and assuming the linear coupling between the environment and system. We follow their formalism to obtain the effective action for the dissipative system.

The effective action in Euclidean space is given as

\[
S[q(\tau)] = \int_0^T \left[ \frac{1}{2} M(q) \dot{q}^2 + V(q) + \delta V(q) \right] d\tau + \frac{1}{2} \int_{-\infty}^{\infty} d\tau' \int_0^T d\tau \gamma(\tau - \tau') \{ q(\tau) - q(\tau') \}^2, \tag{1}
\]

where

\[
\gamma(\tau - \tau') \equiv \frac{1}{2\pi} \int_0^\infty J(\omega) e^{-\omega|\tau - \tau'|} d\omega. \tag{2}
\]

Here, the mass of quantum system is generally taken as position-dependent which is motivated from spin tunneling models [5–7]. The final non-local term in Eq.(1) represents the effect of dissipation. It is emphasizing to note that the counter term \( \delta V(q) \) is introduced in Eq.(1) [11]. In fact, the above effective action without \( \delta V(q) \) has been derived in consideration of a counter term which cancels divergence in the case of Ohmic dissipation. However, for the cases of non-Ohmic dissipation new divergence is expected to appear, so that appropriate counter term may have to be needed. Shortly, it will be shown that this is a generic case.

Now, we take the spectral density generally as

\[
J(\omega) = \alpha \omega^n \tag{3}
\]

where \( n \) is an positive integer. Then, the effective action can be written as

\[
S[q(\tau)] = \int_0^T \left[ \frac{1}{2} M(q) \dot{q}^2 + V(q) + \delta V(q) \right] d\tau + \frac{\alpha \Gamma(n + 1)}{4\pi} \int_{-\infty}^{\infty} d\tau' \int_0^T d\tau \frac{\{ q(\tau) - q(\tau') \}^2}{|\tau - \tau'|^{n+1}}, \tag{4}
\]

where \( \Gamma(n) \) is the Gamma function. From this effective action, the equation of motion is given by

\[
M(q) \ddot{q} + \frac{1}{2} M'(q) \dot{q}^2 - \frac{\alpha \Gamma(n + 1)}{\pi} \int_{-\infty}^{\infty} d\tau' \frac{q(\tau) - q(\tau')}{|\tau - \tau'|^{n+1}} = V'(q) + \delta V'(q), \tag{5}
\]

where the prime denotes the derivative with respect to \( q \). In the following subsections we will derive the criterion of the first-order phase transition in the cases of Ohmic dissipation and super-Ohmic dissipation \( (J(\omega) = \alpha \omega^3) \).

A. The Ohmic dissipation

In the case of Ohmic dissipation, the spectral density is linearly proportional to frequency. The equation of motion is, thus, Eq.(5) with \( n = 1 \). The dissipation term can be rewritten...
in terms of the Fourier transform partner of \( q(\tau) \), i.e., \( \tilde{q}(\omega) \), which makes the equation of motion to be
\[
M(q)\ddot{q} + \frac{1}{2} \frac{\partial M(q)}{\partial q} \dot{q}^2 - \alpha \int_{-\infty}^{\infty} d\omega \tilde{q}(\omega) |\omega| e^{i\omega \tau} = V'(q).
\]
(6)

In the derivation of this equation we take a prescription for the dissipation term
\[
- \frac{\alpha}{\pi} \int_{-\infty}^{\infty} d\tau' \frac{q(\tau) - q(\tau')}{(\tau - \tau')^2} \rightarrow - \frac{\alpha}{\pi} \int_{-\infty}^{\infty} d\tau' \frac{q(\tau) - q(\tau')}{(\tau - \tau' + i\epsilon)(\tau - \tau' - i\epsilon)},
\]
which preserves the time-reversal symmetry, and \( \delta V' = 0 \) as mentioned before. Since, in fact, the dissipation breaks the time-reversal symmetry in real space-time, another prescription was used at Ref. [11]. However, in Euclidean space the dissipation does not yield damping motion, which justifies our choice of the prescription.

Now, following Ref. [9], let us determine the type of transition by expanding the equation of motion at the sphaleron solution \( q_s \) as
\[
q(\tau) = q_s + a\eta_1(\tau)
\]
or equivalently
\[
\tilde{q}(\omega) = \tilde{q}_s + a\tilde{\eta}_1(\omega),
\]
where \( a \) represents an oscillation amplitude near sphaleron solution. Since it is sufficient in determining the order of phase transition to consider only the solutions near sphaleron, we can assume \( a \) is very small constant. Substituting these expressions into Eq.(6) the equation of motion within the first order of \( a \) becomes
\[
(\hat{l} - \hat{h})\eta_1(\tau) = 0,
\]
(10)
where operators \( \hat{l} \) and \( \hat{h} \) are defined as
\[
\hat{l} = M(q_s) \frac{d^2}{d\tau^2} - \frac{\alpha}{2\pi} \int d\omega |\omega| e^{i\omega \tau} \int d\tau e^{-i\omega \tau},
\]
\[
\hat{h} = V''(q_s),
\]
(11)
respectively. In order to solve this equation, we take a trial solution
\[
\eta_1(\tau) = \cos \Omega \tau
\]
(12)
and equivalently
\[
\tilde{\eta}_1(\omega) = \frac{1}{2}(\delta(\Omega - \omega) + \delta(\Omega + \omega)).
\]
(13)
Using this trial solution the frequency \( \Omega \) near the sphaleron solution is obtained within the first order of the amplitude \( a \) as
\[
\Omega_0^{(1)} = \pm \frac{1}{2} \left[ -\frac{\alpha}{M(q_s)} + \sqrt{\left(\frac{\alpha}{M(q_s)}\right)^2 + 4\omega_s^2} \right]
\]
(14)
\[ \omega_s = -\frac{V''(q_s)}{M(q_s)}. \]  

(15)

Comparing \(|\Omega^{(1)}|\) with \(\omega_s\) which is \(\alpha \to 0\) limit of \(|\Omega^{(1)}|\), it is easy to understand that the Ohmic dissipation reduces the frequency. Since in the case of second-order phase transition the transition temperature \(T_c\) is determined by this \(|\Omega^{(1)}|\), i.e., \(T_c = |\Omega^{(1)}| / 2\pi\), this result implies that the Ohmic dissipation decreases the transition temperature. As shown in Fig.1, the decrease of \(T_c\) means increase of action value and, then, suppression of decay rate near \(T_c\). We will, however, show that the super-Ohmic dissipation gives the opposite effect on decay rate within exponential approximation.

Next order calculation can be conducted by taking

\[ q(\tau) = q_s + a\eta_1(\tau) + a^2\eta_2(\tau). \]

(16)

Then, we find equation of motion for \(\eta_2(\tau)\) as

\[ a(\hat{l} - \hat{h})\eta_2(\tau) = - (\hat{l} - \hat{h})\eta_1(\tau) + aW_1(\tau), \]

(17)

where

\[ W_1(\tau) = (\Omega^2 M'(q_s) + \frac{1}{2} V'''(q_s)) \cos^2 \Omega \tau - \frac{1}{2} \Omega^2 M'(q_s) \sin^2 \Omega \tau. \]

(18)

Since \((\hat{l} - \hat{h})\) is an hermitian operator, taking a scalar product with \(|\eta_1>\) on both sides of the equation of motion yields

\[ a(\hat{l}(\Omega) - \hat{h}(\Omega)) < \eta_1 | \eta_2 >> = - < \eta_1 | \hat{l} - \hat{h} | \eta_1 > + a < \eta_1 | W_1 >, \]

(19)

where

\[ l(\Omega) = -\Omega^2 M(q_s) - \alpha | \Omega |, \]

\[ h(\Omega) = V''(q_s). \]

(20)

As usual perturbation theory, \(< \eta_1 | \eta_2 >\) is zero. Furthermore, the second term of the right-hand side in Eq.(19) is zero because of \(\tau\)-integration. Then, the above equation(Eq.(19)) is identical with Eq.(14). Therefore, within the second order of \(a\) we can not find the variation of the frequency from \(\Omega^{(1)}\), i.e.,

\[ \Omega^{(2)}_O = \Omega^{(1)}_O. \]

(21)

Before calculating next order frequency \(\Omega^{(3)}_O\), we would like to evaluate \(\eta_2(\tau)\) explicitly for later use. This is achieved directly from Eq.(17), which yields

\[ \eta_2(\tau) = (\hat{l} - \hat{h})^{-1}W_1(\tau) = g_1 + g_2 \cos 2\Omega \tau, \]

(22)

where
\[ g_1 = -\frac{\Omega^2 M'(q_s) + V'''(q_s)}{4V''(q_s)}, \]
\[ g_2 = -\frac{3\Omega^2 M'(q_s) + V'''(q_s)}{4[4M(q_s)\Omega^2 + \alpha | \Omega | V''(q_s)]}. \]  

(23)

Now, taking a third order correction into \( q(\tau) \) as
\[ q(\tau) = q_s + a\eta_1(\tau) + a^2\eta_2(\tau) + a^3\eta_3(\tau), \]
and inserting the above expression into Eq.(6), the equation of motion for \( \eta_3(\tau) \) is straightforwardly obtained as
\[ a^2(\hat{l} - \hat{h})\eta_3 = -(\hat{l} - \hat{h})\eta_1 - a(\hat{l} \cdot \hat{h})\eta_2 + aW_1(\tau) + a^2W_2(\tau), \]

(25)

where
\[
W_2(\tau) = (\Omega^2 g_1 M'(q_s) + g_1 V'''(q_s)) \cos \Omega \tau \\
+ (5\Omega^2 g_2 M'(q_s) + g_2 V'''(q_s)) \cos 2\Omega \tau \cos \Omega \tau \\
+ \frac{1}{2}(\Omega^2 M''(q_s) + \frac{1}{3}V''') \cos^3 \Omega \tau \\
- 2\Omega^2 g_2 M'(q_s) \sin \Omega \tau \sin 2\Omega \tau \\
- \frac{1}{2}\Omega^2 M''(q_s) \cos \Omega \tau \sin^2 \Omega \tau.
\]

Scalar product with \( \eta_1 \) in Eq.(25) yields an equation
\[-(l(\Omega) - h(\Omega)) < \eta_1 | \eta_1 > + a^2 < \eta_1 | W_2 > = 0 \]

(27)
in which we can determine \( \Omega_0^{(3)} \). Since the first-order phase transition between thermal and thermally assisted quantum tunneling regimes occurs when the period of solution near the sphaleron increases with approaching to the sphaleron solution, we get a condition for sharp transition by \( | \Omega_0^{(3)} | > | \Omega_0^{(1)} | \). This is identical to the condition that the value of the left-hand side of Eq.(27) at \( \Omega = | \Omega_0^{(1)} | \) is negative. Therefore, we finally obtain the criterion of first-order phase transition as
\[
\Omega_0^{(1)} = [(g_1 + \frac{3}{2}g_2)M'(q_s) + \frac{1}{4}M''(q_s)] + (g_1 + \frac{1}{2}g_2) V'''(q_s) + \frac{1}{8} V''''(q_s) < 0, \]

(28)

where \( g_1 \) and \( g_2 \) are evaluated at \( \Omega = \Omega_0^{(1)} \). Since the dissipative coefficient is involved at \( \Omega^{(1)} \), \( g_1 \), and \( g_2 \), the criterion of first-order phase transition is generally different from non-dissipative case. Particularly, when particle mass is constant, the dissipation coefficient \( \alpha \) appears only in \( g_2 \), and from the fact \( V''(q_s) < 0 \) we can conclude that the Ohmic dissipation enhances the possibility for the occurrence of the sharp first-order transition.

B. The super-Ohmic dissipation

In this subsection we consider the criterion for the first-order phase transition when the dissipation is super-Ohmic, i.e., \( J(\omega) = \alpha \omega^3 \), which appears in the analysis of macroscopic magnetization tunneling [12].
The dissipation term in Eq. (5) with \( n = 3 \) can be written again in terms of \( \tilde{q}(\omega) \). After integrating it at complex \( \tau' \)-plane by using same prescription, one can find the equation of motion (Eq.(5)) to be in the form

\[
M(q) \ddot{q} + \frac{1}{2} \frac{\partial M(q)}{\partial \dot{q}} \dot{q}^2 + \alpha \int_{-\infty}^{\infty} d\omega \tilde{q}(\omega) | \omega |^3 e^{i\omega \tau} = V'(q). \tag{29}
\]

In deriving Eq.(29) we used \( \delta V' = \frac{3\alpha}{2} \ddot{q} \) to cancel a divergence which appears in the course of integration. The presence and absence of divergence in super-Ohmic and Ohmic cases, respectively, are because that we start with the effective action (Eq.(4)) as mentioned in the previous section.

The remaining calculation for super-Ohmic case is equivalent to that for the Ohmic case. The differences are followings. The \( \hat{l} \) operator is changed in this case into

\[
\hat{l} = M(q_s) \frac{d^2}{d\tau'^2} + \frac{\alpha}{2\pi} \int d\omega | \omega |^3 e^{i \omega \tau} \int d\tau e^{-i \omega \tau}, \tag{30}
\]

and the frequency near the sphaleron solution within the first order of the amplitude \( a, \Omega_S^{(1)} \), becomes the root of the equation

\[
M(q) \Omega^2 - \alpha | \Omega |^3 + V'' = 0. \tag{31}
\]

This equation tells that for the second-order phase transition the transition temperature \( T_c = | \Omega_S^{(1)} | /2\pi \) becomes higher than that of the non-dissipative case, \( \omega_s /2\pi \). This means that contrary to the Ohmic dissipation case, the tunneling rate near \( T_c \) is enhanced by the super-Ohmic dissipation.

Next order calculation shows that \( \eta_2 \) has a same form with the previous case (Eq.(28)), where \( g_1 \) is given at Eq.(23) and \( g_2 \) is changed in the form

\[
g_2 = -\frac{3\Omega^2 M'(q_s) + V'''(q_s)}{4[4M(q_s)\Omega^2 - \alpha | 2\Omega |^3 + V''(q_s)]}. \tag{32}
\]

Finally, the criterion of first-order phase transition is equivalent to Eq.(28) except that \( g_2 \) is replaced by Eq.(32) and Eq.(28) should be evaluated at \( \Omega = \Omega_S^{(1)} \) instead of \( \Omega_O^{(1)} \). It is, then, evident that when particle mass is constant the super-Ohmic dissipation reduces the possibility for the occurrence of the sharp first-order phase transition within exponential approximation.

### III. APPLICATION TO QUANTUM MECHANICAL TUNNELING MODELS

#### A. Asymmetric double well case

Consider a usual double well potential with a symmetry-breaking term proportional to \( q^3 \), i.e.,

\[
V(q) = \frac{1}{2}(q^2 - 1)^2 - f q^3. \tag{33}
\]
This type of potential is frequently used in field theories for the application to cosmology \cite{13,14}. In this case the sphaleron solution is

\[ q_s = 0. \quad (34) \]

We assume that particle mass \( M \) is constant, so that the derivative of mass are zero. Substituting the derivatives of the potential at \( q_s \)

\[ V''(q_s) = -2, \quad V'''(q_s) = -6f, \quad V''''(q_s) = 12 \quad (35) \]

and Eq.(33) into Eq.(28), it is easy to obtain following criterion

\[ \frac{\alpha}{2} \left[ \sqrt{\left( \frac{\alpha}{M} \right)^2 + \frac{8}{M}} - \frac{\alpha}{M} \right] > \frac{5}{2} \frac{1}{2(3f^2 + 1)} \quad (36) \]

for the first-order phase transition when the dissipation is Ohmic. Unfortunately, for any values of positive \( \alpha \) and \( M \) the left-hand side cannot exceed 2, i.e., always second order. Hence, dissipation does not change the order of the phase transition. This result is maintained even in the super-Ohmic case.

We can also obtain the criterion for a double well potential with \( q \) asymmetric term as

\[ V(q) = \frac{1}{2}(q^2 - 1)^2 - Fq. \quad (37) \]

In this case, the result is very similar to \( q^3 \) case. There is no first-order phase transition. This result can be expected since the both asymmetric potentials Eqs.(33) and (37), when the dissipation term is ignored, give actions proportional to each other, which can be easily shown by a appropriate translation of \( q \) and scalings of \( q \) and \( \tau \).

\section*{B. Case for spin tunneling-inspired model}

Consider an hamiltonian

\[ H = \frac{p^2}{2M(\phi)} + V(\phi), \quad (38) \]

where

\[ M(\phi) = \frac{1}{2K_1(1 - \lambda \sin^2 \phi)}, \quad (39) \]

and

\[ V(\phi) = K_2S(S + 1)\sin^2 \phi. \quad (40) \]

Although this hamiltonian can be derived by the coherent representation from the hamiltonian of spin tunneling model \cite{8}

\[ H = K_1S_z^2 + K_2S_y^2 \quad (41) \]
where $K_1$ and $K_2$ represent an anisotropic constants and $\lambda = K_2/K_1$, we will not go through the content of physics on spin tunneling in this paper. Instead, we will restrict ourselves into the discussion on the effect of dissipation in the hamiltonian, Eq.(38).

Since the sphaleron solution is simply

$$\phi_s = \frac{\pi}{2},$$

it is easy to obtain

$$V''(\phi_s) = -2K_2S(S+1), \quad V'''(\phi_s) = 0, \quad V''''(\phi_s) = 8K_2S(S+1),$$

$$M(\phi_s) = \frac{1}{2K_1(1-\lambda)}, \quad M'(\phi_s) = 0, \quad M''(\phi_s) = -\lambda \frac{1}{K_1(1-\lambda)^2}. \quad (43)$$

Hence the use of Eq.(28) for the Ohmic dissipation yields the condition for first-order phase transition as

$$\alpha \sqrt{S(S+1)} < \frac{2\lambda - 1}{1 - \lambda}. \quad (44)$$

This result is shown in Fig.2. It is shown that, unlike the case of constant mass, the effect of the Ohmic dissipation reduces the range of parameters for the first-order phase transition.

Since it is impossible to get an analytic expression of criterion for the super-Ohmic case, one has to resort to the numerical calculation. The type of transition in the parameter space is given at Fig.3. The effect of the super-Ohmic dissipation enlarges the parameter range for the first-order phase transition, which is opposite behavior to the Ohmic case.

It is worthwhile noting that the above criterion may not be realized in real spin system. In order to study the effect of dissipation in the real spin system, one has to derive the effective hamiltonian from the original hamiltonian(Eq.(41)) with the dissipation term fully through coherent representation or particle mapping [15]. Since this procedure in general produces a complicated dissipation term, our cirterions does not hold in this case. Therefore, what we have done in this subsection is to obtain the general criterion for the sharp transition for the model hamiltonian (Eq.(38)) with Ohmic and super-Ohmic dissipation and to show the possibility of the change in type of phase transition due to dissipation.

### IV. CONCLUSION

Performing the nonlinear perturbation, the general condition for the sharp first-order phase transition of decay rate between thermal regime and thermally assisted quantum tunneling regime has been derived when position-dependent mass and dissipation are involved. It has been shown that in the models with constant mass Ohmic dissipation enhances the possibility for the occurrence of the sharp transition, while super-Ohmic dissipation reduces it. Application to two simple quantum mechanical models is given at the previous section. In the asymmetric double well case the phase transition is always second order regardless with or without dissipation. In this case, by comparing $\Omega_O^{(1)}$ and $\Omega_S^{(1)}$ with $\omega_s$ one can perceive that the Ohmic and super-Ohmic dissipation suppresses and enhances the decay rate.
respectively, near the transition temperature $T_c$ within exponential approximation. Similar feature at zero temperature has been reported in Ref. [1].

For the case of $J(\omega) = \alpha \omega^2$ it is not clear how to give a proper prescription which does not break the time-reversal symmetry. This difficulty does not seem to originate from the property of Euclidean space. In real time space, a retarded Green function is generally chosen in order to break the time-reversal symmetry. However, when $J(\omega) = \alpha \omega^2$, in spite of using the retarded Green function, the time-reversal symmetry is not broken.

The generalization of our result, Eq. (28), to nonlinear coupling between an environment and system and general type of dissipation might be highly non-trivial. In this case the dissipation term in the equation of motion, Eq. (3), is proportional to

$$\frac{\alpha}{\pi} \int_{-\infty}^{\infty} d\tau' [q(\tau) - q(\tau')] z_2 \frac{z_1}{|\tau - \tau'|},$$

where $z_1$ and $z_2$ are some constants. Hence, the integration with respect to $\tau'$ through Fourier transform as was done in Eq. (3) is impossible. In our opinion the only way to break through this difficulty is to rely on the numerical analysis. This will be discussed elsewhere. It is also interesting to apply the present method to the spin tunneling problems. Since dissipation arises from the coupling with an environment which contains quasi-particles like photons or phonons, it is possible to construct a Hamiltonian which contains the spin-environment coupling. Then, by integrating over the environmental degrees of freedom and with the help of coherent representation or particle mapping, one can derive an effective Hamiltonian in which the effect of dissipation is fully involved as a non-local term. From this procedure, one can investigate the effect of the dissipation on the phase transition in spin tunneling problem. This will also be discussed elsewhere.

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FIGURES

FIG. 1. The second-order phase transition. The decrease of transition temperature means the increase of action value near $T_c$, indicating suppression of the decay rate.

FIG. 2. The parameter domains for first and second-order transitions in the hamiltonian (Eq.(40)) with Ohmic dissipation.

FIG. 3. The parameter domains for first and second-order transitions in the hamiltonian (Eq.(40)) with super-Ohmic dissipation.
Fig. 1
Fig. 2

The diagram shows the variation of $(S(S+1))^{1/2}$ against $\lambda$ for different values of $\alpha$. The curves indicate the transition between first-order and second-order phases.
Fig. 3

\[ S(S+1) \]

\[ \lambda \]

\( \alpha = 0 \)
\( \alpha = 0.1 \)
\( \alpha = 0.2 \)

Second-order
First-order