Time-reversal symmetry in optics

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Abstract

The utilization of time-reversal symmetry in designing and implementing (quantum) optical experiments has become more and more frequent over the last few years. We review the basic idea underlying time-reversal methods, illustrate it with several examples and discuss a number of implications.

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1. Introduction

Time-reversal symmetry is a fundamental concept in physics. Based on everyday life observations, time-reversal symmetry is far from being obvious. It often seems to be broken in that many evolutions apparently occur only in one direction in time, i.e. they cannot run backwards. In such cases, irreversibility comes about because the reverse direction occurs only with a forbiddingly small probability. One might call such processes thermodynamically irreversible but they are certainly not irreversible in principle. If we could control all degrees of freedom we would retrieve time-reversal symmetry. Only in very special cases in particle physics does there seem to be a real violation of time-reversal symmetry. Only in very special cases in particle physics does there seem to be a real violation of time-reversal symmetry [1]. In all other cases, time-reversal symmetry is preserved and one may take advantage of it.

Here, we concentrate on the field of optics and quantum optics. Optics is governed by Maxwell’s equations, which obey time-reversal symmetry [2]. As outlined in a problem example below, already simple laboratory tasks may be optimized based on this property. But also in cutting edge problems of modern optics, one can benefit from taking guidance from time-reversal symmetry. One example is the efficient absorption of a single photon by a single atom. For perfect excitation of the atom, the photon should have the shape of the time-reversed version of a spontaneously emitted photon. This holds true in free space [3–5], in a waveguide [6] and for atoms in a resonator [7]. We will discuss the free space absorption problem example in more depth below. Time-reversal symmetry arguments also play a key role in the storage and retrieval of photons in atomic ensembles [8, 9].

Methods based on time-reversal symmetry have been applied successfully in focusing electromagnetic radiation onto non-quantum sources, e.g. for microwave emitters embedded in a cavity [10, 11]. Recently, sub-diffraction limited focusing was demonstrated in the optical domain in the absence of a source by focusing through a scattering medium [12]: the wave front impinging onto a lens has been shaped such that it resembles the phase conjugate, i.e. time-reversed version of the wave front that is generated by a hypothetical source emitting a wave through the scattering medium. Another recent example of the application of time-reversal techniques is the determination of the optimal wave front for a new variant of 4π microscopy [13].

The paper is outlined as follows: in the next section, we highlight the relation between phase conjugation and time reversal. Then, the application of time-reversal techniques in optics is discussed for several different cases. Finally, some issues related to phase conjugation of quantum states of light are reviewed.

2. Time reversal and phase conjugation

We highlight the relation between time reversal and phase conjugation (see e.g. [14]). For this purpose, we express an electrical field $\hat{E}(\vec{r}, t)$ by its spectral components $\hat{A}(\vec{r}, \omega)$:

$$\hat{E}(\vec{r}, t) = \int_0^{\infty} d\omega \left[ \hat{A}(\vec{r}, \omega) e^{i\omega t} + \hat{A}^*(\vec{r}, \omega) e^{-i\omega t} \right].$$ (1)
Time reversal is equivalent to replacing $t$ by $-t$. This results in

$$\tilde{E}(\vec{r}, -t) = \int_0^\infty d\omega \left[ \tilde{A}(\vec{r}, \omega) e^{-i\omega t} + \tilde{A}^*(\vec{r}, \omega) e^{i\omega t} \right].$$

Thus, time reversal is equivalent to complex conjugating all spectral amplitudes:

$$t \rightarrow -t \iff \tilde{A}(\vec{r}, \omega) \rightarrow \tilde{A}^*(\vec{r}, \omega).$$

A graphical representation is given in figure 1.

We illustrate the equivalence stated above in a few examples. Consider an electromagnetic wave with an exponentially decreasing amplitude, $E(t) \sim \exp(-\beta t + ivt) \cdot \theta(t) \cdot \theta(-t)$ being the Heaviside function. The spectral amplitudes follow a Lorentzian distribution with $A(\omega) \sim (\beta + i(\omega - v))^1$. Complex conjugation gives $A^*(\omega) \sim (\beta - i(\omega - v))^1$, which is the spectral distribution of a wave with an exponentially increasing amplitude, $E(t) \sim \exp(+\beta t + ivt) \cdot \theta(-t)$. The latter is clearly the time-reversed version of the former. This example is relevant in the problem of exciting a single atom with a single photon in different geometries [4–7] as well as in the absorption of light pulses by an empty Fabry–Perot resonator [15] (see figure 2 for illustration).

As an even simpler example, consider a monochromatic (outward moving) spherical wave $E(\vec{r}) \sim \exp(-i k r + i v t) / r$. Phase conjugating $A(\vec{r}, \omega) \sim \exp(-i k r) / r \cdot \delta(\omega - v)$ gives the amplitude distribution of a wave of the same frequency but moving inwards: $\tilde{E}(\vec{r}) \sim \exp(i k r + i v t) / r$. This illustrates that time reversal is equivalent to inversion of motion in all degrees of freedom [16].

As a practical application, we mention the temporal inversion of picosecond pulses with distorted temporal profile by phase conjugation in a nonlinear medium [17].

### 3. Examples in classical optics

A simple example of the application of time-reversal methods is the coupling of light into a standard (single-mode) optical fiber. One usually says that one has to mode match the beam which is to be coupled to the optical mode of the fiber in order to achieve 100% coupling efficiency. However, this perfect mode matched light field is nothing else than the time-reversed version of the light field that is emitted from the fiber end and collimated by the coupling lens. Of course, upon time reversal the spatial profile remains the same and for a monochromatic beam also the temporal profile is not altered. The only change is in the direction of propagation. Time reversal is equivalent to inversion of motion in all degrees of freedom, i.e. the only difference compared with the wave originating from the fiber is the opposite sign of the wave vector. If the lens used for coupling exhibits aberrations the spatial profile of the time-reversed version must be shaped such that the aberrations introduced by the lens are precorrected: the spatial phase distribution is the complex conjugate of the wave collimated by the lens. This case is included in the spatial dependence of the amplitudes in equations (1) and (2).

Another example is the coupling of light to a Fabry–Perot resonator. What was stated in [7] for resonators with atoms inside is, of course, also true for an empty resonator. So consider a resonator consisting of two perfectly reflecting mirrors. Let a wave be traveling back and forth between the two mirrors. If at $t = 0$ the reflectivity of one of the mirrors is suddenly decreased, light leaks out of the cavity. The field amplitude measured outside the cavity drops exponentially with the cavity decay constant. Upon time reversal, the light would propagate back into the resonator with an exponentially increasing amplitude. Therefore, the optimum light pulse that is coupled into an empty cavity is an exponentially increasing one (see [15]).

An example of an apparent lack of time-reversal symmetry is an optical isolator (see [18, 19]). Such a device is based on the rotation of the polarization vector of the electric field by the Faraday effect. Typically an isolator consists of an entrance polarizer, the Faraday rotator and an exit polarizer with its transmission axis rotated 45° away from that of the entrance polarizer. The strength of the magnetic field of the Faraday rotator is designed such that it perfectly rotates the input polarization onto the axis of the exit polarizer. If now a
beams is back reflected onto the exit polarizer with a proper state of polarization (i.e. the time-reversed version of a wave leaving the isolator), it passes the exit polarizer and is rotated 45° by the Faraday rotator. However, it is rotated such that it is blocked by the entrance polarizer, because the polarization direction rotation due to the Faraday effect is independent of the direction of light propagation. In the time-reversal picture this is to be expected, since it is only the evolution in the degrees of freedom of the wave that has been reversed. Hence, also the current flow generating the magnetic field of the Faraday rotator (which is, of course, also a degree of freedom) has to be reversed in order to recover time-reversal symmetry.

4. A quantum optics example

Next, we discuss a quantum optical example: the absorption of a single photon by a single atom in free space. The process of perfect absorption can be understood as the time-reversed version of spontaneous emission. On spontaneous emission from the excited atomic state, the electromagnetic field (assumed to be in a vacuum state before the emission of the photon) goes into the state of an outward-moving single-photon dipole wave packet with an exponentially decaying temporal mode profile. The time-reversal argument now suggests that perfect absorption will be achieved by an inward-moving dipole wave [3] with an exponentially increasing temporal profile [4]. The latter requirement has been checked in a theoretical simulation [5], where the atom has been excited with a one-photon Fock state, which is a superposition of single-frequency-mode Fock states with a Lorentzian weight. The first requirement demands illumination from full solid angle, which would require, e.g., an infinitely deep parabolic mirror as the focusing device. Therefore, any finite focusing optics may enable absorption close to but never exactly at 100%. The same holds true for the temporal domain, where in principle infinitely long pulses are required. However, a pulse length of about 4 atomic lifetimes already boosts absorption probabilities to 99.9% [5].

In practice, any focusing device may exhibit aberrations. Since upon spontaneous emission the wave front emitted by the atom is disturbed by the aberrations of the redirecting optics, the time-reversed version must exhibit wave front aberrations of opposite sign. In other words, correcting for the aberrations of the focusing optics is nothing else than making use of the equivalence between time reversal and phase conjugation.

The arguments put forward so far are related to the optical degrees of freedom. For perfect absorption, also the atomic degrees of freedom have to be reversed. In principle, upon emission of a photon, momentum conservation predicts a recoil of the atom. Before measuring or absorbing the photon, the momenta of the photon and the ground state atom are entangled (see figure 3). Using the time-reversal argument one would have to generate an incoming single-photon dipole wave entangled with an incoming atomic center of mass motion wave. This is clearly a task beyond current technology. Therefore, the idea is to take guidance from the Mössbauer effect: tightly trapping the atom at one position in space in a volume well within the Lamb–Dicke regime will effectively enlarge the mass of the atom. In the regime of quantized harmonic oscillation of the atom, the photon momentum will be transferred to the whole macroscopic trap, the resulting motion of which is then negligible. For suitable trap parameters, see, e.g., [20]. A detailed theoretical discussion of the recoil problem in spontaneous emission can be found in [21].

5. Phase conjugation of quantum states of light

In the example of the time-reversed version of spontaneous emission, one might have the idea of creating the time-reversed photon by direct phase conjugation of the spontaneously emitted one. While optical phase conjugation is technically demanding in the classical domain, one has to cope with its fundamental noise properties in the quantum domain.

There are several implications when phase conjugating classical light fields. One is related to the spectrum of the light pulse to be phase conjugated. As has been shown by Ou et al [22], the pulse shape reflected by a phase conjugating mirror may become independent of the incident pulse if the incident pulse length is much shorter than the response time of the phase conjugating mirror. This should not pose a problem. The pulse length of a spontaneously emitted photon is practically of the order of a few excited-state lifetimes, which is at least of the order of nanoseconds for most of the atomic dipole transitions. The response time of phase conjugating mirrors is considerably shorter, as evidenced by the successful reversal of picosecond pulses [17].

In contrast to that, the long pulse length does pose a technical problem. Pulse durations of several nanoseconds correspond to spatial pulse lengths of the order of 1 m. For a successful phase conjugation, the pulse should fit completely into the conjugating material, which is possible in principle but not very practical given the above parameters.

A more serious obstacle has been put forward by Yamamoto and Haus [23] and by Gaeta and Boyd [24] who showed that the phase conjugation process induces excess quantum noise. This excess noise is related to the quantum noise limit of phase-insensitive optical amplifiers as explained in the next paragraph. Both conjugation and amplification are non-unitary operations and can only be
implemented imperfectly, i.e. by introducing noise. Unitary operation can be retrieved by embedding the process in a higher dimensional Hilbert space, i.e. by using additional, auxiliary field modes [25].

One might be tempted to reduce the task of phase conjugating a light field to implementing the transformation \( \hat{a} \to \hat{c} = a^+ \) much like an amplifier would require \( \hat{a} \to \hat{c} = \sqrt{G} a^+ \), \( G \) being the power gain. However, neither of these operators fulfills the commutator relation \( \{ \hat{c}, \hat{c}^\dagger \} = 1 \). The problem is cured by allowing for an auxiliary field mode \( \hat{b} \) with \( \{ \hat{a}, \hat{b} \} = 0 \). The relations we are looking for are described by the following Bogoliubov transformation:

\[
\hat{a} \to \hat{c} = \gamma_1 \hat{a} + \gamma_2 \hat{a}^\dagger + \gamma_3 \hat{b} + \gamma_4 \hat{b}^\dagger. \tag{4}
\]

For phase conjugation \( \gamma_2 \) and \( \gamma_4 \) have to be different from zero, \( \hat{a} \to \hat{c} = \gamma_2 \hat{a}^\dagger + \gamma_4 \hat{b} \). Using the field commutator for \( \hat{c} \), one finds the relation \( |\gamma_2|^2 = |\gamma_4|^2 - 1 \). Without loss of generality we assume that \( \gamma_2 \) and \( \gamma_4 \) are real, yielding \( \gamma_4 = \sqrt{|\gamma_2|^2 + 1} \).

Some terms in the Hamilton operator for the total field, \( \hat{c}^\dagger \hat{c} + 
\frac{1}{2} \), do not preserve energy: one term, e.g., leads to the creation of two photons, one each in modes \( \hat{a} \) and \( \hat{b} \). By adding a (quantized) pump field all terms in the Hamiltonian can be modified to be energy conserving. But this step is not needed when the only purpose is to estimate the noise added by phase conjugation. Here, we recall the relation between the variances for the amplitude quadratures \( X_i = \frac{1}{2} (\hat{c}^\dagger \hat{c} + i \hat{a}^\dagger \hat{a}) \), \( i = a, b, c \), of the input, auxiliary and output fields, respectively, the variance being defined as \( \langle \Delta X_i^2 \rangle = \langle X_i^2 \rangle - \langle X_i \rangle^2 \). If the auxiliary field is in the ground state, the calculation yields

\[
\langle \Delta X_i^2 \rangle = \gamma_2^2 \langle \Delta X_a^2 \rangle + \gamma_4^2 (\gamma_2^2 + 1) \langle \Delta X_b^2 \rangle. \tag{5}
\]

This quantifies the added noise. If the gain in the phase conjugating process is to be \( \gamma_2^2 = 1 \), phase conjugation adds two units of vacuum noise:

\[
\langle \Delta X_c^2 \rangle = 2 \langle \Delta X_b^2 \rangle. \tag{6}
\]

This result has been reported earlier by Caves [26] and Cerf and Iblisdir [27]. Thus one can say that there is no universal noiseless phase conjugation much like there is no universal ‘NOT” operation among the single-qubit gates [28, 29].

However, given enough \textit{a priori} information about a state, the phase conjugated field may nevertheless be generated artificially with 100% fidelity. In conclusion, we underline the usefulness of the time-reversal symmetry concept as a powerful tool in designing optical experiments.

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