Five-Dimensional Aspects of $M$-Theory Dynamics and Supersymmetry Breaking

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ABSTRACT

We discuss the reduction of the eleven-dimensional $M$-theory effective Lagrangian, considering first compactification from eleven to five dimensions on a Calabi-Yau manifold, followed by reduction to four dimensions on an $S_1/Z_2$ line segment at a larger distance scale. The Calabi-Yau geometry leads to a structure of the five-dimensional Lagrangian that has more freedom than the eleven-dimensional theory. In five dimensions one obtains a non-linear $\sigma$ model coupled to gravity, which implies non-trivial dynamics for the scalar moduli fields in the bulk of the $Z_2$ orbifold. We discuss solutions to the five-dimensional equations of motion in the presence of sources localized on the boundaries of the $Z_2$ orbifold that may trigger supersymmetry breaking, e.g., gaugino condensates. The transmission of supersymmetry breaking from the hidden wall to the visible wall is demonstrated in specific models. The rôle of the messenger of supersymmetry breaking may be played by the gravity supermultiplet and/or by scalar hypermultiplets. The latter include the universal hypermultiplet associated with the Calabi-Yau volume, and also the hypermultiplets associated with deformations of its complex structure, which mix in general.

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CERN-TH/98-118
OUTP-98-37-P
May 1998
1 Introduction

The most plausible framework for a Theory of Everything (TOE) is generally agreed to be the theory formerly known as strings, presumably formulated in a suitable non-perturbative manner, termed $M$ theory. In this framework, the string coupling becomes a dynamical field that may be interpreted as an extra spatial dimension. When considered in the strong-coupling limit of the traditional ten-dimensional $E_8 \times E_8$ heterotic string \[1, 2, 3\], $M$ theory appears able to reconcile the bottom-up estimate of the grand unification scale based on low-energy data from LEP and elsewhere with the top-down calculation of the string unification scale based on the Planck mass of the effective four-dimensional gravity \[2, 4, 5, 6, 7\]. In this strong-coupling limit, the additional eleventh dimension becomes large compared with the four-dimensional Planck length. More detailed estimates suggest that it may even be considerably larger than the length scale of grand unification and the distance scale on which six internal dimensions are compactified \[4, 4\].

The prototype formulation of eleven-dimensional $M$ theory that has been studied most extensively in the literature has been that in which the large eleventh dimension has the topology of an $S_1/Z_2$ line segment of length $\pi \rho$, with two ten-dimensional walls located at its ends. This yields a strong-coupling limit of the traditional $E_8 \times E_8$ heterotic string in which one (hidden) $E_8$ factor lives on the ten-dimensional wall at one end of the segment, with the other (observable) $E_8$ factor living on the opposite wall. Supersymmetry then requires that the bulk and the boundary hyperplane theories are not independent. An effective low-energy theory in five dimensions may be obtained by compactification of the six internal dimensions on a small Calabi-Yau space of size $R_{CY} \ll \rho$, which is capable of reducing the observable-sector gauge group to some subgroup of $E_6$, with $E_8$ or a proper subgroup on the hidden wall. The important aspect of this procedure is that the generalized Bianchi identity is satisfied only in the global sense, through the interplay of the sources on both walls. The presence of sources localized on the fixed planes means that in the subsequent dimensional reduction to four dimensions one has to take consistently into account the variation of the bulk fields across the 11th dimension \[2, 4, 8, 9, 10, 11, 12\].

The systematic reduction of the eleven-dimensional $M$-theory Lagrangian to four dimensions has not yet been fully exploited. Since, as described above, $R_5 \gg R_{CY}$, this reduction should proceed in two steps. First the reduction from eleven to five dimensions should be performed. The structure of the Lagrangian in five dimensions is richer than in eleven dimensions, due to effects related to the geometry of the compact Calabi-Yau manifold. In five dimensions, one obtains a non-linear $\sigma$ model coupled to gravity \[13, 14\], which implies non-trivial dynamics of scalar fields in the bulk of the $Z_2$ orbifold. The Calabi-Yau compactification also yields conventional four-dimensional gauge sectors on both walls, with specific couplings to the five-dimensional bulk theory.

The complete derivation of the effective five-dimensional supergravity theory \[15, 16, 17, 12\] is not a straightforward task, as it has to take into account the above-mentioned solution of the eleven-dimensional Bianchi identities and the couplings to the gauge fields on the walls. As indicated by earlier investigations in strings and recently pointed out in the interesting
paper [12], gauged extended supergravities are relevant for the low-energy description of $M$ theory. In contrast to ungauged supergravity theory, gauged supergravities in either five or four dimensions contain a generalized $D$-term potential for the scalar fields, which may contribute by itself to supersymmetry breaking [14, 18]. However, in the present paper we use the simple ungauged $N = 2$ five-dimensional supergravity theory, with couplings derived from $M$ theory via Calabi-Yau compactification [15, 16, 17], as the (toy) model for our discussion. The Bianchi identities are replaced in this model by additional sources, whose origin may be condensation of the gauge field strength and curvature tensors along the compact dimensions. These provide wall sources in the equation of motion for the universal $Z_2$-even field $S$, whose real part represents the volume of the Calabi-Yau space, which varies along the fifth dimension.

The questions we want to ask in this paper are well defined already in this simplified setup, and we believe that the key answers will also be valid in the framework of gauged supergravity [12]. We focus our attention on one important possible origin of sources localized on a boundary of the $Z_2$ orbifold, namely supersymmetry breaking in the hidden sector. If this occurs dynamically via gaugino condensation [19], [20], we expect a coupling of the wall condensate to the bulk fields. These sources would then lead to non-zero modes in the solutions of the five-dimensional equations of motion, which have to be taken carefully into account in the construction of the effective four-dimensional Lagrangian.

At this point arises the important question of the scale of the formation of the condensate, which we shall discuss in more detail later. The original formulation by Horava [21] is based on the assumption that the condensate forms already in eleven dimensions, perhaps due to a hidden-sector gauge group that is strongly coupled already very close to the eleven-dimensional Planck scale $m_{11}$. Noting a difference in the way a boundary gaugino condensate would enter the supersymmetric variations of fermions in the eleven-dimensional $M$-theory Lagrangian and in the ten-dimensional Einstein-Yang-Mills Lagrangian, he concludes that in the eleven-dimensional picture the condensate does allow locally for the existence of a spinor, giving vanishing supersymmetric variations of all fermions. However, this spinor is illegal from the point of view of the full model living on the $Z_2$ orbifold, since it is discontinuous on the visible wall. This is a global obstruction precluding unbroken supersymmetry in the presence of such a ‘hard’ eleven-dimensional condensate.

The phenomenological hurdle to be crossed if supersymmetry breaking is due to a condensate forming close to the Planck scale is that of understanding the dynamical generation of any lower mass scale hierarchically smaller than the Planck scale. From this point of view, a more appealing situation would be if the condensate forms, as often postulated in the earlier days of string phenomenology, at a lower scale $\sim 10^{13}$ GeV. However, this would be below the apparent grand unification scale, and also below the scale of Calabi-Yau compactification of the internal six dimensions. Hence, in this more palatable case the condensation would actually occur in the context of an effective five- or even four-dimensional theory.

One should stress that the two-step dimensional reduction, $11 \rightarrow 5 \rightarrow 4$ dimensions, taking proper account of the dynamics in five dimensions, is a necessary framework for treating any of these three scenarios of the condensate formation. However, they would differ in the exact form
of the sources on the hidden wall. In the first case, the source is determined by the reduction $11 \rightarrow 5$ of a stiff condensate that already exists in eleven dimensions. In the second one, it is legitimate to perform first the reduction of the supersymmetric model from eleven to five dimensions and then to supplement it with a stiff condensate source in five dimensions. Finally, in the third case a stiff condensate should be replaced by an effective boundary superpotential in five dimensions.

The vacuum selection of the five-dimensional $\sigma$ model and the transmission of supersymmetry breaking to the observable wall are determined by the coupling of the $\sigma$ model describing the interactions of the bulk moduli to the boundaries, in the presence of sources on the hidden wall. The issue has already been studied in a toy model with rigid supersymmetry, consisting of a vector hypermultiplet in the five-dimensional bulk coupled to conventional four-dimensional chiral gauge theories on the walls [22, 17]. However, it is known that making five-dimensional supersymmetry local imposes a rather specific pattern of hypermultiplet couplings. It therefore seems opportune to revisit the mechanism for transmitting supersymmetry breaking, incorporating the general features of five-dimensional supergravity as well as the specific constraints that are imposed by Calabi-Yau compactification.

As we recall in more detail in Section 2 below, five-dimensional supergravity theory contains in general a graviton supermultiplet, within which there is a single graviphoton vector state, a number of vector supermultiplets whose couplings are determined by a holomorphic trinomial, and a number of scalar hypermultiplets which parametrize a quaternionic manifold. A characteristic feature of the ungauged five-dimensional supergravity is the complete factorization of the manifolds parametrized by the scalars in the vector and scalar supermultiplets. This means, in particular, that the scalar hypermultiplet fields do not have these vector interactions.

In the case of compactification on a Calabi-Yau manifold, as also set out in Section 2, its topological numbers determine the numbers of vector and scalar supermultiplets: $n_V = h_{1,1} - 1$, $n_S = h_{2,1} + 1$. Moreover, the trinomial characterizing the vector couplings is related to the intersection form of the Calabi-Yau manifold, there is a universal scalar hypermultiplet related to the volume of the Calabi-Yau manifold, and the geometry of the remaining scalar hypermultiplets is related to complex deformations of the Calabi-Yau manifold. The invariance of the eleven-dimensional supergravity theory compactified on an $S_1/Z_2$ line segment under the $Z_2$ symmetry of the orbifold, interpreted as a constraint on the fields present in the model formulated in the ‘upstairs’ picture which we use also in five dimensions, determines through the Calabi-Yau compactification certain parity properties on the five-dimensional fields which we discuss at the end of Section 2.

All Calabi-Yau compactifications yield a universal scalar hypermultiplet, but the properties of the other hypermultiplets associated with the complex structure moduli are quite model-dependent. We develop in Section 3 a simple model with a single non-universal hypermultiplet, that serves to illustrate our subsequent discussion.

In Section 4 we discuss the various possible scenarios for supersymmetry breaking mentioned...
above, that differ in the scale at which it is supposed to originate. We explore in most detail the case where this occurs between the Calabi-Yau and $S_1/Z_2$ compactification scales, which we denote by $R_{\text{CY}}$ and $R_5$ respectively, paying close attention to the possible mechanisms for transmission of the supersymmetry breaking across the five-dimensional bulk. The $Z_2$ parity and Lorentz properties of the different fields tell us which must vanish and which may have non-zero expectation values on the walls. As discussed by [22], the latter are essential for the possible transmission of supersymmetry breaking across the five-dimensional bulk. As we discuss in Section 4, the combination of these parity properties and four-dimensional Lorentz invariance implies that the vector supermultiplets may not transmit supersymmetry breaking directly, at least in the absence of Calabi-Yau deformation. However, this is possible via the gravity and matter hypermultiplets. In particular, the transmission via the hypermultiplets is modulated by the volume of the Calabi-Yau space. Another interesting feature is that, in general, the universal hypermultiplet mixes with the other hypermultiplets associated with complex deformations of the Calabi-Yau manifold, providing these with non-trivial dynamics.

The five-dimensional equations of motion are used in Section 5 to find classical vacuum configurations for sources representing either a stiff condensate or a dynamical condensate, i.e., an effective boundary superpotential for $Z_2$-even moduli, in the simple Calabi-Yau model of Section 3. We explore the effects of the couplings between the different scalar fields in the non-linear $\sigma$ model in the five-dimensional bulk, and related non-linearities in the solutions of the equations of motion with boundary sources. The particular issues we study include the transmission of supersymmetry breaking from the hidden wall to the observable wall, and the choice of vacuum configuration.

Finally, some perspectives for future work are outlined in Section 6.

2 Five-Dimensional Supergravity from Calabi-Yau Compactification

We first recall the general structure of the Lagrangian for ungauged $N = 2$ Maxwell-matter supergravity in five dimensions \cite{13, 14}, especially its hyperplet structure and the geometry of the non-linear $\sigma$ model parametrized by the bosonic fields \cite{13, 14}. We treat the fermions as symplectic-Majorana fields, i.e., there is an even number of them, forming conjugated pairs: $\lambda^a = \Gamma^5(\lambda_a)^*$, where $\lambda^a = \epsilon^{ab}\lambda_b$ and $\lambda_a = \epsilon_{ba}\lambda^b$, with the totally antisymmetric two-index tensor $\epsilon$ defined so that $\epsilon^{12} = \epsilon_{12} = 1$. As is well known the five-dimensional Lagrangian contains a gravity supermultiplet that includes the graviton, two gravitini $\psi^\mu_i$ and a graviphoton, vector supermultiplets that include gauge fields $A^A_\mu$, spin-1/2 fermions $\lambda^a_i$ and spin-0 fields $t^A$, and scalar hypermultiplets that include fermions $\lambda_b$ and spin-0 fields $\sigma^x$. It is frequently convenient to combine the graviphoton and the other vector fields using the indices $I, J, \ldots$. The index $i = 1, 2$ labels supersymmetries and is raised and lowered by the two-dimensional epsilon

\footnote{We use the conventional term $N = 2$, since the symplectic-Majorana gravitini transform as a doublet of the automorphism group of the supersymmetry algebra in five dimensions.}
symbol. The Lagrangian may be written in terms of these physical fields as

\[
S = \int \sqrt{g} \left( -\frac{R}{2} - \frac{1}{2} \bar{\psi}_i^\mu \Gamma^{\mu\nu\lambda} D^\nu \psi_{\lambda i} + \frac{1}{2} \partial^A \partial^B G_{AB} - \frac{1}{4} G_{IJ} F^I F^J + \sqrt{g}^{-1} \frac{1}{48} g_{IJK} \epsilon^A F^I F^K - \frac{1}{2} \bar{\psi}_{i}^\mu \Gamma^{\mu\nu\lambda} D^\nu \psi_{\lambda i} \right) \\
+ \frac{1}{2} g_{xy} \partial \sigma^x \partial \sigma^y - \frac{1}{4} \bar{\lambda}^b \Gamma^\mu D_\mu \lambda_b \\
- \frac{3i}{8 \sqrt{6}} h_I \bar{\psi}_i^\mu (\Gamma^{\mu\nu\alpha\beta} + 2 \delta^{\mu\alpha} \delta^{\nu\beta}) \psi_{\nu i}^a F^I \\
- \frac{i}{2} \bar{\lambda}^a \Gamma^\mu \psi_{\mu i}^a f_a \partial_\alpha \sigma^x \\
+ \frac{i}{2} \bar{\lambda}^b \Gamma^\mu \psi_{\mu i}^b f_b \partial_\sigma^x \\
+ \frac{i\sqrt{6}}{32} h_I \bar{\lambda}^i \Gamma^{\alpha\beta} \lambda_b F_{I}^{\alpha\beta} + \text{(four - fermion terms)}
\]  

(1)

where the first three lines correspond to the gravity, vector and scalar supermultiplets, respectively. By \( F^I \) we denote abelian field strengths of abelian vector bosons which can be present in the theory (at least one - the graviphoton - is always present). We recall that the couplings of the vector supermultiplets are characterized by a trilinear function \( V(X) \) where by \( X \) we denote the scalar components of vector multiplets including the graviphoton, with Chern-Simons terms defined by

\[
V(X) = \frac{1}{6} d_{IJK} X^I X^J X^K
\]

(2)

The coefficients forming a totally symmetric object \( d_{IJK} \) must be constant, i.e. independent of the fields \( t^A \), and all the other couplings in the vector Lagrangian are expressible in terms of them. The real scalars \( t^A \) live on the hypersurface \( V(t) = 1 \), and the kinetic-term metrics for the spin-1 and spin-0 fields are related as follows:

\[
G_{IJ} = -\frac{1}{2} \partial_I \partial_J \ln V|_{V=1} \\
G_{AB} = G_{IJ} \partial_A X^I \partial_B X^J |_{V=1}
\]

(3)

The metric \( g_{xy} \) of the scalar hypermultiplets \( \sigma^x \) is that of a quaternionic manifold, and the symbols \( f_a^b \) are the vielbeins of the metric \( g_{xy} \) on the quaternionic manifold. It is noteworthy that the geometries of the spin-0 fields in the vector and scalar hypermultiplets are completely independent. This has the important phenomenological consequence that the scalar hypermultiplets can have no gauge interactions in ungauged \( N = 2 \) supergravity\(^3\).

We shall need for our subsequent analysis the supersymmetry transformation laws for the various hypermultiplets. These are

\[
\delta e^m = \frac{1}{2} \bar{\epsilon}^i \Gamma^m \psi_{\mu i} \\
\delta \psi_{\mu i} = (D_{\mu} \epsilon)^i - \omega_{xj}^i (\delta \sigma^x) \psi_{\mu i} + \frac{i}{4 \sqrt{6}} h_I (\Gamma^\alpha_{\mu} - 4 \delta^\alpha_{\mu} \Gamma^\beta) \epsilon^i F_{\alpha\beta}^I
\]

(4)

\(^3\)Some are possible in the gauged versions, like those with the graviphoton in the gauged supergravity of [12].
in the case of the gravity supermultiplet,

\[
\delta A_I^\mu = -\frac{1}{2} h^I_\alpha \epsilon^\alpha \Gamma_\mu \lambda^a_i + \frac{i}{4} \sqrt{\delta} h^I_\alpha \bar{\psi}_\mu^i \epsilon_i
\]

\[
\delta \lambda^a_i = -i f^a_A (\partial \lambda^A) \epsilon_i - \omega^b_A (\delta t^A) \lambda^b_i + \frac{1}{4} h^I_\alpha \Gamma_{\mu \nu} F^I_{\mu \nu} \epsilon_i
\]

\[
\delta t^A = \frac{i}{2} f^A_{a \bar{c}} \epsilon^i \lambda_{a}^i
\]

for the vector supermultiplets and the graviphoton, and

\[
\delta \lambda^b = -i f^b_x (\partial \sigma^x) \epsilon_i - \omega^b_x (\delta \sigma^x) \lambda^c
\]

\[
\delta \sigma^x = \frac{i}{2} f^x_{b \bar{c}} \epsilon^i \lambda^b
\]

for the scalar hypermultiplets.

After summarizing the basic facts about general ungauged \( N = 2 \) d=5 matter-Maxwell supergravity, we would like to restrict ourselves to specific models generated through compactifications on Calabi-Yau spaces. More precisely, following \[2, 1, 3\], we take as a starting point eleven-dimensional supergravity coupled to ten-dimensional supersymmetric Yang-Mills theories living on two separated boundary walls. If one makes a field-theoretical compactification on a Calabi-Yau threefold, one obtains a bulk five-dimensional supergravity theory. We expect that this construction should lead to a model that can be described within the general framework \( \{1\} \), but with specific constraints due to its origin in a Calabi-Yau compactification of eleven-dimensional supergravity. These should include constraints on the number of vector hypermultiplets \( A^A_\mu \) and the trilinear geometrical function \( V(t) \) that characterizes their self-couplings, as well as constraints on the number of scalar hypermultiplets and constraints on the quaternionic manifold that describes their geometry \[15, 16\].

Before entering into more detail on these subjects, we recall that the Calabi-Yau manifold is expected to be deformed \[2\]. This is because the generalized Bianchi identity in eleven dimensions is fulfilled only globally, through the interplay between non-zero sources located on both walls, which leads to a non-zero antisymmetric-tensor field background interpolating across the eleventh (to be renamed fifth) dimension between these sources. This non-zero background, combined with the requirement of the vanishing supersymmetry variation of one gravitino on the walls, leads to a non-vanishing correction to the metric. The simplest consequence of this is a linear variation of the Calabi-Yau volume along the \( S^1 / Z_2 \) line segment.

Among the metric deformations, one may choose to restrict oneself to such deformations which are independent of the Calabi-Yau coordinates and depend only on \( x^5 \). This leaves the compact six dimensions a Calabi-Yau space for each \( x^5 \). Then, the natural first step towards the realistic compactification of \( M \) theory down to five dimensions is to proceed with the standard Calabi-Yau compactification \[16, 15\]. The assumption made above has the useful consequence that the corrections to the metric are taken into account as \( x^5 \)-dependent configurations of the bulk moduli fields. However, it should be clear that, as the bulk moduli are part of the non-linear \( \sigma \) model in five dimensions, their vacuum contains more structure than a simple
reconstruction of the original deformation computed in eleven dimensions. This is consistent with the observation made in the forthcoming sections of this paper that the actual vacuum configurations of fields do not have simple linear dependences on $x^5$.

In five dimensions, where we have at our disposal only fields which are true zero modes from the point of view of the Calabi-Yau space, the obvious question is how the non-zero background of the even modulus $S$, representing the Calabi-Yau volume, can get excited. As we have said before, in eleven dimensions the source of the deformation of the volume is the non-zero antisymmetric-tensor field background. In five dimensions, the way to produce the non-zero slope in a field is to couple it to sources located on the boundaries. These sources induce the expected dependence of the volume on the fifth coordinate $S(x^5)$, and so represent the non-zero antisymmetric-tensor background in the effective five-dimensional Lagrangian. As explained in the forthcoming sections of this paper, we take such a background simply into account by assuming additional $\delta$-function sources, located on the walls, in the equation of motion for the field $S$, whose consequences will be discussed later. These additional sources can in principle be incorporated in the effective five-dimensional Lagrangian. For the purpose of this paper we leave them at the level of equations of motion.

Calabi-Yau compactification of $M$ theory yields a five-dimensional supergravity lagrangian of the general form (1), with specific geometrical constraints related to the topological structure of the Calabi-Yau manifold. The indices $A, a$ are $O(h(1,1)−1)$ vector indices, and $I, J$ run over the range $1 \ldots h_{1,1}$. The index $b$ on the fermions belonging to the scalar hypermultiplets runs over the range $1 \ldots 2(h_{2,1}+1)$ and transforms as the fundamental representation of $USp(h_{2,1}+1)$. The index $x$, that counts the real bosonic degrees of freedom in hypermultiplets, runs over the range $1 \ldots 4(h_{2,1}+1)$. These matter fields, when coupled to supergravity, form a quaternionic manifold of real dimension $4(h_{2,1}+1)$, as discussed in the general case of matter coupled to $N=2$ supergravity [23, 13, 18]. The tangent-space metric of this quaternionic manifold is the invariant antisymmetric matrix of the group $USp(1) \times USp(h_{2,1}+1)$. The vielbein $f^a_A$ of the metric of the manifold spanned by the scalar components of the vector superplets may be found using the $SO(h_{1,1} − 1)$ tangent group. The corresponding connections may be formed out of the vielbeins in the standard way, and we denote them by $\omega^j_i$, $\omega^a_b$, ..., respectively.

The reduction of the supersymmetry on the boundary walls to $N=1$ is accomplished, exactly as in the original eleven-dimensional theory, by representing the $Z_2$ geometrical symmetry of the $S^1/Z_2$ orbifold on the fields. Then the $Z_2$-odd components are projected out at the boundaries, leaving a number of degrees of freedom corresponding to simple chiral $N = 1$ supersymmetry on these four-dimensional hyperplanes. In the upstairs picture which we adopt in this paper, this means that we define the action of the $Z_2$ on the bosonic and fermionic fields in the model so as to leave the five-dimensional action invariant. We discuss the $Z_2$ transformation properties of the bulk five-dimensional fields in more detail at the end of this section.

\footnote{In principle, this $Z_2$ symmetry could be embedded non-trivially in the symmetry group of the underlying Calabi-Yau space, which would lead to interesting consequences. However, for the purpose of the present paper we restrict ourselves to the case where the $Z_2$ acts trivially on Calabi-Yau geometry.}

\footnote{However, we mention that these corrections underlie the gauged five-dimensional supergravity in [12].}
We define the Calabi-Yau metric moduli through
\[ i\delta g_{ij} = \sum_{A=1}^{h_{1,1}} \delta M^A V^A_{ij}, \quad \delta g_{ij} = \sum_{\alpha=1}^{h_{2,1}} \delta \bar{Z}^\alpha \bar{b}_{\alpha ij} \] (7)

where the $M^A$ are real moduli corresponding to the deformations of the Kähler class, and the $Z^\alpha$ are complex moduli corresponding to the deformations of the complex structure of the Calabi-Yau manifold, the $V^A_{ij}$ are the harmonic $(1,1)$ forms, and the $\bar{b}_{\alpha ij}$ are harmonic $(2,1)$ forms. In addition, we denote the holomorphic $(3,0)$ form by $\Omega_{ijk}$.

The kinetic-energy terms for the metric moduli can be obtained from the reduction of the eleven-dimensional Einstein-Hilbert action \[15, 16\]. The resulting parts of the five-dimensional action are
\[ S_g = \int \sqrt{g^{(5)}} \left( \frac{1}{2} \partial M^A \partial M^B V \hat{G}_{AB} - VR^{(5)} \right) + VG_{\alpha\bar{\beta}} \partial Z^\alpha \partial \bar{Z}^\beta - \frac{1}{2} (\partial \log V)^2 \] (8)

where $V$ is the volume of the Calabi-Yau three-fold and $g^{(5)}$, $R^{(5)}$, etc., denote gravitational quantities in five dimensions. In order to identify correctly the degrees of freedom and the hypermultiplets to which they belong, one must subtract the volume modulus from the real moduli of the $(1,1)$ type. To this end, we define real fields $t^A$ such that $M^A = t^AV^{1/3}$ and $V(t^A) = 1$. In terms of $t^A$, $V$, $Z^\alpha$, and after a suitable Weyl rescaling of the metric, we obtain
\[ S_g = \int \sqrt{g}(\frac{1}{2} \partial t^A \partial t^B G_{AB} - \frac{R}{2} + G_{\alpha\beta} \partial Z^\alpha \partial \bar{Z}^\beta - \frac{1}{2} (\partial \log V)^2), \] (9)

which has a canonical Einstein-Hilbert term. Denoting the Calabi-Yau manifold by $K$, one may express the metric $G_{\alpha\beta}$ in the form
\[ G_{\alpha\beta} = -i \frac{1}{4} \frac{\int_K b_\alpha \wedge \bar{b}_\beta}{V} \] (10)

and we discuss shortly the form of $G_{AB}$.

The eleven-dimensional antisymmetric tensor $C_{MNP}$ also yields massless fields in five dimensions, which are related to harmonic forms on the Calabi-Yau space. The components of $C_{MNP}$ which have all five-dimensional indices give rise, upon using Hodge duality and the tree-level equations of motion, to a single real scalar which we denote by $S_2$, the components with one five-dimensional index, one holomorphic and one antiholomorphic index give $h_{1,1}$ vector bosons in five dimensions, and the components with two holomorphic and one antiholomorphic index give $h_{2,1}$ complex scalars. Together with the fields coming from the reduction of the metric, including the five-dimensional graviton, these zero modes provide the bosonic components of the gravity hypermultiplet, $h_{1,1} - 1$ vector multiplets, and $h_{2,1} + 1$ hypermultiplets. One of these scalar hypermultiplets contains in its bosonic sector the field $S = S_1 + iS_2$, where $S_1$ corresponds to the Calabi-Yau volume, and the complex scalar $C$ coming from the reduction of the antisymmetric tensor components with three antiholomorphic indices, namely $C_{ijk}$. This tensor is called the universal hypermultiplet, as it is always present in Calabi-Yau compactifications, even when the number $h_{2,1}$ of independent $(2,1)$ forms vanishes.
The remaining hypermultiplets, containing complex scalars coming from the reduction of the metric and antisymmetric tensor field, are non-universal. Since the physics of hypermultiplets will be of importance later in this paper, we write down here for completeness the part of the five-dimensional Lagrangian which contains bosonic fields from the hypermultiplets, including the graviphoton, which is included with the other vector fields:

\[ S_V = \int (\sqrt{-g} \frac{1}{2} \partial^A \partial^B G_{AB} + \sqrt{-g} \frac{1}{4} G_{IJ} F^I F^J + \frac{1}{48} d_{IJK} \epsilon^I F^J F^K ) \]  

(11)

where \( \epsilon \) is the completely antisymmetric tensor in five dimensions, and the real scalars \( t^A \) live on the hypersurface \( \mathcal{V}(t) = 1 \). One has the representation

\[ d_{IJK} = \int_K V^I \wedge V^J \wedge V^K, \]

(12)

in terms of which the metrics \( G_{AB}, G_{IJ} \) are then given by (3). The expressions (12) allow one to identify properly the constraints imposed by Calabi-Yau compactification of eleven-dimensional supergravity on the coefficients in the general five-dimensional supergravity Lagrangian (1) [13].

Before leaving this section, we discuss the \( Z_2 \) properties [8, 17, 12] of various fields in the five-dimensional Lagrangian. These can be read off the eleven-dimensional supergravity Lagrangian, in particular by demanding \( Z_2 \) invariance of the topological \( CGG \) term. Denoting the Minkowski-space coordinates by \( \hat{\mu}, \hat{\nu}, \) etc., the only odd components of the metric are \( g_{\mu 5}, g_{i5}, g_{\bar{i}5} \), e.g. \( g_{\mu 5}(-x^5) = -g_{\mu 5}(x^5) \). The remaining metric components are \( Z_2 \) even. In the case of the antisymmetric tensor field \( C_{MNP} \), the parity assignments are just the opposite: the components without the index 5 are odd, and those with index 5 are even, e.g., \( C_{\mu ij}(-x^5) = -C_{\mu ij}(x^5) \) and \( C_{5ij}(-x^5) = C_{5ij}(x^5) \). Consequently, the moduli scalars \( t, V, Z \) are all even. As for the vectors, their \( A^I_\mu \) components are odd, but their components \( A^I_5 \) are even. The complex scalars coming from components of \( C_{MNP} \) with all indices holomorphic or antiholomorphic are odd, and the remaining real scalar, coming from components with the index structure \( C_{5\mu \nu} \), is even.

We define the \( Z_2 \) action on fermions as follows:

\[
\begin{align*}
\lambda^a(-x^5) &= i \Gamma^5 \lambda_a(x^5) \\
\psi^i_\mu(-x^5) &= i \Gamma^5 \psi_{i\mu}(x^5) \\
\psi^i_5(-x^5) &= -i \Gamma^5 \psi_{i5}(x^5) \\
\epsilon^i(-x^5) &= i \Gamma^5 \epsilon_i(x^5)
\end{align*}
\]

(13)

The advantage of this definition of the action of \( Z_2 \) is that it exchanges left (from the four-dimensional point of view) components of a pair of symplectic spinors between the partners, and similarly the right components. If the spinors \( \lambda^a, a = 1, 2 \), form a symplectic pair, then

\[
\lambda^1_L \rightarrow i \lambda^2_L, \quad \lambda^1_R \rightarrow -i \lambda^2_R
\]

(14)

One should note that, in the representation of the charge conjugation matrix we have chosen, the left and right components of the symplectic spinors are related as follows

\[
\lambda^1_L = (\lambda^2_R)^*, \quad \lambda^1_R = -(\lambda^2_L)^*
\]

(15)
Hence, for instance, one can choose to work with the left components of the symplectic spinors only, and these will contain the full information carried by a pair of symplectic spinors, and provide a nonsinglet representation for the $Z_2$. Now one can easily find the combinations of spinors which are eigenstates of the $Z_2$ parity: the combination $\lambda_L^+ = \lambda_L^1 + i\lambda_L^2$ is an even singlet, whilst $\lambda_L^- = i\lambda_L^1 + \lambda_L^2$ is odd. Similarly, for the fermionic parameters of supersymmetry transformations, we see that $\epsilon_L^+ = \epsilon_L^1 + i\epsilon_L^2$ generates $Z_2$-even transformations, whilst $\epsilon_L^- = i\epsilon_L^1 + \epsilon_L^2$ generates $Z_2$-odd ones. These properties will be important when we discuss supersymmetry on the three-branes. There, the $Z_2$-odd states and supersymmetries are projected out of the spectrum, whilst their derivatives with respect to the odd coordinate $x^5$ are $Z_2$ even, and may sneak in to play a rôle in the four-dimensional physics on the three-branes.

3 The Scalar Hypermultiplet Sector in a Prototype for Calabi-Yau Compactification of $M$ Theory

As we have already remarked, the general form of the moduli space in five-dimensional supergravity is a direct product $M = M_K \times Q$, where $M_K$ is a Kähler manifold and $Q$ is a quaternionic manifold. There is in general a mapping between these two manifolds, called the $s$ map, which means that we only need to know the metric for the Kähler manifold characterizing the dynamics of the complex metric moduli of $(2, 1)$ type to describe the geometry of the full moduli space. Unfortunately, despite the existence of this $s$ map, the general case of a phenomenologically-relevant Calabi-Yau space is very complicated to treat in detail. We therefore discuss a toy model which is not phenomenologically appealing, as its Yukawa couplings vanish, but does allow us to write down a non-trivial, explicit Lagrangian for the hypermultiplets which may be used to discuss issues of phenomenological relevance, such as supersymmetry breaking.

The moduli space for this Calabi-Yau space is given by the product of the Kähler manifold

$$M_K = \frac{SU(1, n)}{U(1) \times SU(n)}$$

with the quaternionic manifold

$$Q = \frac{SU(2, n + 1)}{SU(2) \times SU(n + 1) \times U(1)},$$

where $n$ is the number of hypermultiplets in the theory: in our case, $n = h_{2,1} + 1$, where $h_{2,1}$ is the Hodge number of the Calabi-Yau manifold. In the present case, the quaternionic manifold is in fact also Kähler, which enables us to write down an explicit Kähler potential for the moduli space:

$$K_m = K + \tilde{K},$$

where

$$K(Z, \bar{Z}) = -\log \left(2(1 - Z\bar{Z})\right)$$

10
and
\[ K(Z, \bar{Z}, S, \bar{S}, C_0, \bar{C}_0, C_1, \bar{C}_1) = -\log(S + \bar{S} - \frac{(1 + Z \bar{Z})(C_0 + \bar{C}_0)^2}{1 - ZZ} + \frac{2(Z + \bar{Z})(C_1 + \bar{C}_1)(C_0 + \bar{C}_0) + (1 + Z \bar{Z})(C_1 + \bar{C}_1)^2}{1 - ZZ}). \] (20)

In what follows, we limit ourselves to the case where there is only one non-universal hypermultiplet, i.e., the Hodge number of the corresponding Calabi-Yau space is \( h_{2,1} = 1 \). In our notation, the pair of complex scalars \( (S, C_0) \) belongs to the universal hypermultiplet, and the pair \( (Z, C_1) \) to the single remaining non-universal hypermultiplet.

Using the above Kähler potentials, we can construct the Lagrangian describing the dynamics of the scalar fields:
\[ e^{-1} \mathcal{L} = -K_{Z\bar{Z}} \partial_\mu Z \partial_\mu \bar{Z} - K_{S\bar{S}} \partial_\mu S \partial_\mu \bar{S} - K_{Z\bar{S}} \partial_\mu Z \partial_\mu \bar{S} - K_{Z\bar{C}_i} \partial_\mu Z \partial_\mu \bar{C}_i \] (21)

For comparison with the general formalism outlined above, and for our own further purposes, it is convenient to write the above Lagrangian in the form of the quaternionic metric \( \sigma_{xy} \):
\[ e^{-1} \mathcal{L} = ds^2 = -g_{xy} \partial \sigma^x \partial \sigma^y. \] (22)

In this notation, the indices \( x \) and \( y \) run over all the fields in (21), in the order \( S, C_0, Z, C_1 \). Since the exact metric \( g_{xy} \) has a complicated expression, we limit ourselves to the limit where the fields \( C_i \) and \( Z \) are small, corresponding to a small deformation of the Calabi-Yau manifold.

In this limit, we obtain the simplified expression
\[ g = \begin{bmatrix}
\frac{1}{(S + \bar{S})^2} & -\frac{2(C_0 + \bar{C}_0)}{(S + \bar{S})^2} & 0 & -\frac{2(C_1 + \bar{C}_1)}{(S + \bar{S})^2} \\
-\frac{2(C_0 + \bar{C}_0)}{(S + \bar{S})^2} & \frac{2}{S + \bar{S}} & \frac{2(C_1 + \bar{C}_1)}{S + \bar{S}} & \frac{2(Z + \bar{Z})}{S + \bar{S}} \\
0 & \frac{2(C_1 + \bar{C}_1)}{S + \bar{S}} & \frac{2}{S + \bar{S}} & \frac{2(C_0 + \bar{C}_0)}{S + \bar{S}} \\
-\frac{2(C_1 + \bar{C}_1)}{(S + \bar{S})^2} & \frac{2(Z + \bar{Z})}{S + \bar{S}} & \frac{2(C_0 + \bar{C}_0)}{S + \bar{S}} & \frac{2}{S + \bar{S}}
\end{bmatrix}. \] (23)

when we expand the quaternionic metric up to linear order in all the fields except \( S \).

For our subsequent analysis of supersymmetry breaking, we need vielbeins corresponding to the metric \( g_{xy} \), and also the part of the connection which corresponds to the \( Sp(1) \) subgroup of the tangent group. We introduce the vielbeins in the following way
\[ ds^2 = V^T \bar{\Omega} V, \] (24)
where the \( V \) are vielbeins with the components
\[ V = \begin{bmatrix}
u \\
w \\
r \\
r \\
\bar{w} \\
\bar{v} \\
\bar{u}
\end{bmatrix}, \] (25)
and with the metric

\[ \tilde{\Omega} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \] (26)

Using the metric (23) and the definition (24), we find the following components of the vielbeins:

\[ u = \sqrt{x} dS, \] (27)

\[ v = \frac{n}{\sqrt{x}} dS + \sqrt{y - \frac{n^2}{x}} dC_0, \] (28)

\[ w = \frac{b}{\sqrt{y - \frac{n^2}{x}}} dC_0 + \sqrt{1 - \frac{b^2}{|y - \frac{n^2}{x}|}} dZ, \] (29)

\[ r = \frac{p}{\sqrt{x}} dS + \frac{q - np}{\sqrt{y - \frac{n^2}{x}}} dC_0 + \frac{a - \frac{b(q^2 - np)}{|y - \frac{n^2}{x}|}}{1 - \frac{b^2}{|y - \frac{n^2}{x}|}} dZ + \sqrt{y - \frac{p^2}{x} - \frac{(q - np)^2}{|y - \frac{n^2}{x}|} - \frac{a - \frac{b(q^2 - np)}{|y - \frac{n^2}{x}|}^2}{|1 - \frac{b^2}{|y - \frac{n^2}{x}|}|}} dC_1, \] (30)

where

\[ x = \frac{1}{(S + \tilde{S})^2}, \] (31)

\[ y = \frac{2}{S + \tilde{S}}, \] (32)

\[ n = -\frac{2(C_0 + \tilde{C}_0)}{(S + \tilde{S})^2}, \] (33)

\[ p = -\frac{2(C_1 + \tilde{C}_1)}{(S + \tilde{S})^2}, \] (34)

\[ q = \frac{2(Z + \tilde{Z})}{S + \tilde{S}}, \] (35)

\[ a = \frac{2(C_0 + \tilde{C}_0)}{S + \tilde{S}}, \] (36)

\[ b = \frac{2(S_1 + \tilde{S}_1)}{S + \tilde{S}}. \] (37)

Suppressing \( \sigma \)-model indices \( x \), the connections for this quaternionic manifold are defined as follows

\[ df^{ia} + \omega^i_j f^{ja} + \omega^a_i f^{ib} = 0 \] (38)
The parts of the $Sp(1)$ connection $\omega^i_j$ which are relevant for the projection on the boundary walls are

\[
\begin{align*}
\omega^1_1 &= \frac{1}{4} e^\tilde{K} (dS - d\tilde{S}) + \frac{1}{4} \frac{\tilde{Z} dZ + Z d\tilde{Z}}{1 - |Z|^2} + \ldots \\
\omega^2_1 &= -\frac{1}{4} e^\tilde{K} (dS - d\tilde{S}) - \frac{1}{4} \frac{\tilde{Z} dZ + Z d\tilde{Z}}{1 - |Z|^2} + \ldots \\
\omega^1_2 &= -2 e^{\frac{K + \tilde{K}}{2}} Z dC + \ldots \\
\omega^2_2 &= 2 e^{\frac{K + \tilde{K}}{2}} Z d\tilde{C} + \ldots
\end{align*}
\]

These explicit formulae will be useful in the subsequent discussion of supersymmetry breaking.

4 Scenarios for Supersymmetry Breaking in $M$ Theory

We now address the problem of supersymmetry breaking in $M$ theory using the general five-dimensional framework set out above. The appearance of two compactification scales, namely those of the Calabi-Yau manifold ($R_{CY}$) and of the $S_1/Z_2$ line segment ($R_5$), with $R_{CY} < R_5$, enables us to distinguish three generic possibilities for the origin of supersymmetry breaking. It may originate either (a) in dynamics at a length scale less than $R_{CY}$, or (b) at a distance scale intermediate between $R_{CY}$ and $R_5$, or (c) at a distance scale larger than $R_5$. Examples of scenario (a) include the eleven-dimensional formulation of gaugino condensation and the Scherk-Schwarz mechanism [8, 26]. Scenario (b) may arise if there is supersymmetry breaking in the effective four-dimensional field theory on the hidden wall, e.g., as a result of strong gauge interactions that cause gaugino condensation [20]. Scenario (c) would arise, e.g., if the hidden-sector gauge dynamics becomes strong only at a distance scale larger than $R_5$. In the following we discuss each of these scenarios from the point of view of the five-dimensional effective supergravity theory.

We emphasize in advance that the intermediate five-dimensional supergravity stage is relevant in all these cases, because compactification on a Calabi-Yau space introduces new physics, to some extent compactification-dependent. This has an unavoidable rôle to play, as it couples directly to the trigger for supersymmetry breaking, which is a wall effect, and transmits dynamics between the walls. Hence, even in case (a), the steps to be taken include the compactification of the eleven-dimensional condensate on the Calabi-Yau manifold [11], and the consideration of its physics in connection with that of the $\sigma$ model in the bulk. For the same reason, in case (c), before one integrates out the fifth dimension, one should solve the five-dimensional equations of motion with the fermionic bilinear replaced by the effective superpotential for the even moduli [11]. The fifth dimension does not decouple in a trivial way for length scales larger than its radius.

Let us first consider supersymmetry breaking at short distances. In this case, it is not evident that a field-theoretical description is adequate: one may need to use the whole paraphernalia of extended objects, including strings, membranes, etc.. Nevertheless, one may choose to
investigate the consequences of postulating gaugino condensation within the field-theoretical supergravity framework. In such a scenario of supersymmetry breaking at short distances, the vacuum expectation value of a fermion bilinear would be ‘hard’, in the sense of being independent of the moduli of compactification. If the field-theoretical description is adequate, then the analysis of Horava [21] applies, and should be followed by compactification of the internal six dimensions, and subsequent analysis repeating the procedure described below for the case (b).

Next we explore in detail the possibility (b) that supersymmetry breaking originates at a distance scale intermediate between $R_{CY}$ and $R_5$ (intermediate-scale supersymmetry breaking). In this case, we expect that the origin of supersymmetry breaking may be described by some effective four-dimensional field theory on the hidden wall. We do not discuss here the details how this may arise: strongly-coupled gauge dynamics leading to gaugino condensation may be one option, but there may well be others. In any such scenario, one must discuss how such supersymmetry breaking may be transmitted through the five-dimensional bulk to the observable wall. Here several issues arise, including the couplings of the bulk theory to the walls, whether the bulk theory admits supersymmetry breaking of the type advocated, and the relative magnitudes of the supersymmetry breaking at each end of the $S_1/Z_2$ line segment. We now address each of these issues in turn.

We have defined the bulk theory in the previous sections. However, this theory as it stands contains no potential for scalar fields, but consists of kinetic terms and derivative interactions only. However, we know that compactification on a Calabi-Yau threefold generates non-derivative couplings on both walls, and in particular that non-perturbative effects may create a non-trivial potential for the even bulk moduli, which are legal chiral superfields on the walls. Let us assume that such sources are indeed present in the theory, and assume for the time being that they behave like a covariantly constant condensate, i.e., are proportional to the unique Calabi-Yau three-form $\Omega_{ijk}$, coupling directly to the fifth derivative of the $Z_2$-odd complex scalar in the universal hypermultiplet. A unique form for the five-dimensional coupling of the source to the universal hyperplet is suggested by the reduction of the eleven-dimensional ‘perfect-square’ structure [27] found by Horava [21] to five dimensions. Hence, from the point of view of the dynamical fields in the bulk, the condensate on the wall looks like a $\delta$-function source in the equations of motion for the $Z_2$-odd hyperplet scalars. From the point of view of the Lagrangian, the effective coupling which corresponds to this interpretation is obviously

$$L_{coupling} = -\frac{1}{2} g_{xy} g^{55} (\partial_5 \sigma^x - L \delta(x^5 - \pi \rho) \delta^{x0})(\partial_5 \sigma^y - L \delta(x^5 - \pi \rho) \delta^{y0})$$

(40)

where we assume, as mentioned above, the conventional wisdom that the four-dimensional gaugino condensate must be proportional to the Calabi-Yau $(3,0)$ form $\Omega_{ijk}$. and hence should couple to the $\sigma^{x0}$ belonging to the universal hypermultiplet.

Two types of singular sources will be present in the resulting equations of motion, namely $\delta$ functions and derivatives of these with respect to $x^5$. As the equations of motion are second-order differential equations, $\delta$-function singularities in them tell us that the derivatives of certain

\[6\text{ Although we note that potential terms are present in gauged N=2 supergravity [18], see also [2].} \]
functions are discontinuous across the walls, and the presence of $\partial_5 \delta$ is the sign that a certain function is itself discontinuous across the wall. Quick inspection of the equations reveals that $\partial_5 \delta$-type source appear in the equations of motion for the odd scalar in the universal hypermultiplet. We infer that this scalar suffers a discontinuity across the wall of the form $L \theta(x^5 - \pi \rho)/2$, where $\theta(x)$ is the Heaviside step function (since it is not $Z_2$-even, the limiting values on both sides must have the same magnitude, but opposite signs. On the other hand, $\partial_5 C$ is even and continuous, so either the coefficient of the $\delta$-function singularity in the equation of motion for $C$ must vanish, or the explicit $\delta$ function must be cancelled by the implicit $\delta$ function developed by the solution of the equations along the direction of some other field or its derivative. Similarly, for the even moduli we conclude that either the coefficients of $\partial_5 \delta$-type operators have to disappear, or they are dynamically cancelled, as these fields can only have their derivatives with respect to $x^5$ discontinuous on the walls. The singularities are, for a generic scalar metric, distributed among all equations of motion, and mutual cancellation of all explicit and implicit singular operators is a good consistency check for the solution. It also helps to convert the system of equations with singular sources into the equivalent set of equations defined over the half-circle only, with suitable boundary conditions at the end of that half-circle. Details of this procedure shall be discussed using specific examples in Section 5.

We now explore the forms of supersymmetry breaking admitted by the bulk theory at the ends of the $S_1/Z_2$ line segment. There are important restrictions imposed by the $Z_2$ parity properties of the fields in the effective Lagrangian, since only $Z_2$-even objects may have non-zero vacuum expectation values on the walls. The fact that the $Z_2$ parity of any field $X$ is reversed by taking its derivative in the fifth direction $\partial_5 X$ increases the range of possibilities. It is clear that any candidate vacuum expectation values must also exhibit four-dimensional Lorentz invariance, which remains true for the $\partial_5$ derivatives.

The five-dimensional Lagrangian has the interesting property that all the couplings, except those arising from the Riemannian connection, contain derivatives with respect to five space-time coordinates. Also, in the supersymmetry transformations, the terms on the right-hand sides are proportional to space-time derivatives or are multilinear in fermionic fields. We discard from our discussion the fermionic terms in supersymmetry transformations, on the grounds that their vacuum expectation values would be interpreted as the formation of bound states, which is not plausible, because the only force acting in the bulk is gravity. Terms interesting for supersymmetry breaking must be able to survive the projection from the bulk onto the boundary walls. This means that we must consider the supersymmetry transformations of even combinations of fermions generated by an even combination of susy generators. Equipped with these observations, we now examine the supersymmetry transformation laws for fermions in the different bulk supermultiplets, to see which of them may develop vacuum expectation values that might signal spontaneous supersymmetry breaking on the wall.

In the case of the gravity supermultiplet, one must consider separately the $\psi_{5}^{+}$ components of the gravitino. This is because, from the four-dimensional point of view which we have to assume when discussing eventually the supersymmetry breaking on the walls, they form a symplectic matter fermion, whose even component can in principle participate in supersymmetry breaking. The supersymmetry transformation law for the fifth component of the gravitino
doublet is
\[ \delta \psi^i_5 = (D_5\epsilon)^i - \omega^i_{xj}(\delta \sigma^x)\psi^j_5 + \frac{i}{4\sqrt{6}} h_1(\Gamma^\alpha_5 - 4\delta^\alpha_5\Gamma^\beta_5)\epsilon^i F^J_{\alpha\beta} \] (41)

As we argued earlier, we disregard fermionic terms in this transformation law, and concentrate on the covariant derivative. It contains a part originating from the Riemannian connection, and one related to the $Sp(1)$ connection:
\[ \delta \psi^i_5 = (D_5\omega)^i - \omega^i_{xj}(\partial_5\sigma^x)\epsilon^j + \ldots \] (42)
where the dots stand for the fermionic terms, and for terms containing Maxwell field strengths.

We first consider the first term in (42), and decompose it as
\[ D_5\epsilon^i = \partial_5\epsilon^i + \frac{1}{2}\omega^{ab}_5\Sigma_{ab}\epsilon^i : \Sigma_{ab} = \frac{1}{4}[\Gamma_a, \Gamma_b] \] (43)
For the combination of $\epsilon^i$ which is even under $Z_2$, the first term in (43) is odd, and hence cannot contribute. Expanding
\[ \omega^{ab}_5 = \frac{1}{2}e^{\mu_5}(\partial_5 e^b_\nu - \partial_\nu e^b_5) - \frac{1}{2}e^{ab}(\partial_5 e^a_\nu - \partial_\nu e^a_5) - e^{a\sigma}(\partial_\rho e_{a\sigma} - \partial_\sigma e_{a\rho})e^5_5 \] (44)
we see that the only possible non-vanishing vacuum expectation values would be those appearing in $\omega^{55}_5$, but this vanishes overall by antisymmetry. Turning now to the second term in (42), we note that the important connection here is that associated with the vielbeins $f^a_i$, and that the exact form of $\omega^i_{xj}$ depends on the geometry of the scalar quaternionic manifold in question.

For the simple model analyzed above, the explicit forms of the coefficients can be read off from the expressions given in Section 3, and for the left-handed and $Z_2$-even combination of $\psi^i_5$ one obtains
\[ \delta^+_L \psi^5_5 = \frac{\sqrt{\gamma}}{\sqrt{S + S\sqrt{1 - |Z|^2}}}(\partial_5 C_0 + Z\partial_5 C_1)\epsilon^i_L \] (45)
which need not vanish on the wall. We conclude that the graviton hyperplet may in principle communicate supersymmetry breaking between the walls.

Now, anticipating the need for the four-dimensional interpretation of five-dimensional results, we would like to identify the fields which survive the $Z_2$ projection in terms of 4d multiplets they are going to belong to. From the supersymmetry transformation $\delta e^m_5 = \frac{i}{2}\bar{e}^m\Gamma^m\psi_5$, we see that the four-dimensional superpartner of the even part of $\psi^i_5$ is $e^5_i$ the physical radius of the 5th dimension. The second scalar partner which completes the four-dimensional chiral $R_5$ supermultiplet is the $A_5$ component of the graviphoton. This can be seen, for instance, from the supersymmetry transformation of the 5th component of the graviphoton: $\delta A_5 = \frac{i}{4}\sqrt{6}\bar{\psi}^5_5\epsilon_i$. This means, that the scalar superfield descending on the wall from the 5d gravity supermultiplet corresponds to the overall $T$ modulus identified within the simple truncation of the 5d supergravity to four dimensions in e.g. \cite{8}. The projections of the remaining $h_{(1,1)} - 1$ multiplets containing vector fields shall correspond to the shape moduli supermultiplets called $T^i$, $i = 1, \ldots, h_{(1,1)} - 1$. 16
To determine the possible rôle of these vector supermultiplets, let us inspect the real metric moduli and vector fields. As one can easily check, all of the metric moduli, both real and complex, are even under $Z_2$. Hence, their derivatives with respect to $x^5$ do not exist on the walls, and they cannot be messengers of supersymmetry breaking between the walls. As for gauge bosons, the $A_{5I}^a$ are also even, but the components $A_{5I}^{\hat{a} \mu}$ are odd. Hence, the derivatives $\partial_5 A_{5I}^{\hat{a} \mu}$ could in principle be useful. However, the gauge fields enter the Lagrangian through their field strengths only, and the corresponding components $F_{5I}^{\hat{a} \mu}$ of all gauge field strengths have to vanish in the vacuum in order not to break four-dimensional Lorentz invariance. Specifically, the supersymmetric variations (5) of the fermionic components of the vector supermultiplet are

$$\delta \lambda^a_i = -i f_A^a (\partial_5 t^A) \epsilon_i - \omega_A^{ab} (\partial_5 t^A) \lambda^b_i + \frac{1}{4} h_i^{a \mu \nu} F^{I}_{\mu \nu} \epsilon_i$$

which after suppressing terms which must vanish because of Lorentz invariance gives

$$\delta \lambda^a_i = -i f_A^a (\Gamma^5 \partial_5 t^A) \epsilon_i$$

Since $t^A$ is even, $\partial_5 t^A$ is odd and hence must vanish on the walls. Hence, at leading order in the Calabi-Yau compactification of the $M$-theory Lagrangian, the vector supermultiplets cannot participate in the transmission of supersymmetry breaking between the walls.

Finally, we turn to the scalar matter hypermultiplets. We have argued that the metric moduli, both real and complex, are $Z_2$-even. However, there exist complex scalars arising from the reduction of the internal part of the antisymmetric tensor, $C_{ijk}$, $C_{i\bar{j}k}$, which are $Z_2$-odd, and hence have $\partial_5$ derivatives that are $Z_2$ even. These derivatives are the fields that participate in the transmission of the supersymmetry breaking at the lowest order, and they form the $Z_2$-even bosonic components of the $h_{(2,1)} + 1 N = 2$ hypermultiplets. Consequently, it is a combination of the even fermions from these hypermultiplets which becomes the goldstino eaten up by the four-dimensional gravitino living on the three-brane to form a massive spin-$\frac{3}{2}$ particle. Among the hypermultiplets there is one which is singled out, namely the so-called universal hypermultiplet. It is always present in Calabi-Yau-type compactifications, and it is associated with the unique $(3,0)$ form $\Omega_{ijk}$. The remaining hypermultiplets may be present or not, depending on the particular Calabi-Yau manifold chosen for compactification. However, they cannot be ignored if they are present, since, as discussed in detail in the next section, the $\sigma$-model metric in the hypermultiplet sector is usually highly non-trivial, and the equations of motion for the hypermultiplet scalars in the bulk are consequently non-linearly coupled, mixing together all of them, both even and odd, universal and non-universal.

We now consider the supersymmetry transformation law (5) for the fermions in the scalar hypermultiplets (‘hyperinos’):

$$\delta \lambda^b_i = -i f^{ib}_x (\Gamma^5 \partial_5 \sigma^x) \epsilon_i$$

where we have again suppressed terms that cannot contribute because of Lorentz invariance. Since $\partial_5 \sigma^x$ is even, the first term in (58) may in principle contribute, if the form of the $f^{ib}_x$ is suitable. To see this more explicitly, we consider a single hypermultiplet, and write down the

\footnote{However, it is possible that they may do so indirectly through mixing with the hypermultiplets in the equations of motion.}
exact expression for the even supersymmetry transformation of an even fermion from such a multiplet:

\[ \delta_+ (\lambda_L^1 + i \lambda_L^2) = -\frac{i}{2} (f_{x_1}^1 + f_{x_2}^2 + if_{x_1}^1 - if_{x_2}^2) \partial_5 \sigma^x \epsilon_L^+ \]  

(49)

where the summation over \( x \) is over odd scalar components of the hyperplet. Now, let us call the complex scalars belonging to the universal hypermultiplet \((S, C)\) (the first is even, the second is odd), and the complex scalars belonging to the non-universal hyperlets \((Z^k, C^k)\), \(k = 1, \ldots, h_{(2,1)}\). The even scalars \(Z^k\) are exactly the type \((2,1)\) complex metric moduli. As discussed in [25], once the metric for these metric moduli is known, and using the observation that the full scalar-field space is a quaternionic manifold, one can reconstruct the full metric including the odd-field sector. The task is usually very difficult, but there exist simple examples, one of which we shall discuss in detail in the next section. From the point of view of four-dimensional boundaries, after performing the \(Z_2\)-projection, the universal hypermultiplet shall correspond to the chiral superfield denoted by \(S\) in \([8]\). This supermultiplet couples to the gauge kinetic terms on the walls, thus its scalar component which is the volume of the Calabi-Yau space, sets the tree-level gauge coupling if one ignores deformations. At the level of \(\kappa^{4/3}\) deformations the coefficient of the gauge kinetic terms contains also the scalar part of the 4d field \(T\) which comes down from the 5d gravity supermultiplet, as we have noticed earlier in this section.

To convince oneself that interesting supersymmetry breaking can indeed take place in the hyperplet sector, it is enough to consider the metric near one of the three-branes, i.e., up against the wall. Further, one can expand the metric around vanishing values of the \(Z\)-moduli, and explore order by order whether supersymmetry breaking happens. In fact, it is sufficient to inspect the zeroth-order case, in which the metric simplifies to

\[ ds^2 = -\kappa^2 dZ d\bar{Z} - \frac{1}{(S + \bar{S})^2} dS d\bar{S} - \frac{2}{S + \bar{S}} d\bar{U}^T d\bar{U} \]  

(50)

where \(\bar{U} = (C, C^{1, \ldots, C^{h_{(2,1)}}})\). In this case, one can immediately solve for vielbeins with respect to the tangent-space metric \(\Omega_{ib,jc} = -\frac{1}{2} \epsilon_{ij} \Omega_{bc}\), where \(\Omega_{bc}\) is the constant antisymmetric matrix defining the \(Sp(h_{(2,1)})\) group. Then, using the prescription (49), after some manipulations one finds that the variations of even fermions are of the form

\[ \delta_+ \lambda^+ = \frac{1}{\sqrt{S + \bar{S}}} \tilde{f}_k (\partial_5 C)^k \]  

(51)

where \(k = 0, \ldots, h_{2,1}\), the ‘effective’ vielbeins (numbered here by the ‘effective’ index \(k\)) \(\tilde{f}_k\) are numbers of order one plus corrections \(o(Z)\) and the boundary values of \( (\partial_5 C)^k \) are at this point arbitrary constants, which have no reason to be set to zero. This conclusion holds also in the limit where \(h_{(2,1)} = 0\), i.e., for the universal hypermultiplet alone. Let us note that \(\frac{1}{\sqrt{S + \bar{S}}} \sim \frac{1}{\sqrt{V_{CY}}}\) where \(V_{CY}\) is the Calabi-Yau volume, implying that the magnitude of supersymmetry breaking is directly sensitive to the scale of Calabi-Yau compactification.

---

8The generalization of this formula to the case of more hypermultiplets is obvious.
We therefore see that - in contrast to the vector superplets - both the universal and non-universal hypermultiplets may contribute to supersymmetry breaking. This possibility is very interesting, as it indicates the possibility of non-trivial dynamics for the complex structure moduli, which have been largely ignored in studies of Calabi-Yau compactification of the heterotic string in the weak-coupling limit. We have found that they may play a non-trivial rôle in the strong-coupling limit, represented by a non-trivial dependence on the additional coordinate.

Finally, let us comment on the scenario where supersymmetry breaking occurs at some large distance scale \( R_5 \). This possibility is similar to the ‘historical’ scenario of supersymmetry breaking in the hidden sector discussed extensively in the weak-coupling limit of the heterotic string, where the length of the \( S_1/Z_2 \) line segment is negligibly small. The main point we want to make here is that the procedure developed in this paper should also be used in this case, with the hard fermionic bilinear replaced by the effective superpotential for even moduli \( [11] \). This would lead eventually to the possibility of a dynamical determination of the magnitude of the fifth dimension, but such an application goes beyond the scope of the present paper.

5 Solving the Equations of Motion

We now solve the five-dimensional equations of motion, with the goal of determining how information about physics from the hidden sector located on the hidden wall is fed into the observable sector of the theory. To obtain an answer to this question, one has to define the problem in a more precise way. One possible formulation would be to define five-dimensional observables accessible to the observer imprisoned on the visible wall. The other possibility is to explore the Kaluza-Klein paradigm, and seek an effective four-dimensional model, with the degrees of freedom corresponding to motions along the fifth dimension integrated out, which would summarize from the point of view of the truly four-dimensional observer the contribution of the physics on the hidden wall modulated by the dynamics in the bulk. For the time being, we explore the latter point of view.

In view of the complicated nature of the generic field theory in the bulk, and in the absence of a well-defined non-perturbative model on the hidden wall, we study initially a toy scalar field model without excited gravitaional degrees of freedom, with minimal kinetic terms in the bulk and the simplest possible couplings to four-dimensional operators living on the walls. This helps us to clarify which is the proper way to compute the terms that may break supersymmetry: whether they are truly local operators on the visible wall, or whether one has to compute some non-local averages over the bulk. We conclude that the first of these two proposals is appropriate.

We start with a scalar field that is even under \( Z_2 \), which has been discussed in ten dimensions in [25, 29]. Let the action of the model be (from here on we shall use \( \rho \) to denote the radius of
the fifth dimension)

\[ S(\Phi) = \int d^5x \left( \frac{1}{2} \partial \Phi \partial \Phi - \Phi S(x^5 - \pi \rho) - O \Phi \delta(x^5) \right) \]  

(52)

Here \( O \) is an operator composed of observable fields living on the visible wall at \( x^5 = 0 \), where we assume \( < O >_{\text{vac}} = 0 \) and \( S(x) \) is the function of the four-dimensional coordinates which represents hidden-wall sources, such as a hidden gaugino condensate, which affect the vacuum configuration of the field. As pointed out in [28], the standard Kaluza-Klein procedure is inadequate here, as one encounters non-zero modes in the bulk which contribute to the four-dimensional equations of motion. In our case, the bulk equation of motion is

\[- \square_4 \Phi + \partial^2_5 \Phi = S(x) \delta(x^5 - \pi \rho) + O(x) \delta(x^5) \]  

(53)

The first step is to separate the zero mode, i.e., the part of the field which does not depend on the fifth coordinate. The decomposition should be of the form

\[ \Phi = \phi(x) + \psi(x; x^5) \]  

(54)

but one must formulate a specific prescription for extracting the zero-mode field from the full \( \Phi \) field. The natural and consistent prescription is the requirement that the average over the fifth dimension of the non-zero mode \( \psi \) be zero, namely

\[ \frac{1}{\pi \rho} \int_{0}^{\pi \rho} dx^5 \psi(x; x^5) = 0. \]  

(55)

Next we point out that, in the present case of a \( Z_2 \)-even field, all the information is contained on the half-circle between 0 and \( \pi \rho \), once we apply the correct boundary conditions which, upon using the definite \( Z_2 \) parity, reproduce the singularity structure found in the equation of motion defined on the full circle [9]. In the case at hand, the boundary conditions are

\[ \lim_{x^5 \to \pi \rho} \partial_5 \Phi = -\frac{S}{2} \]

\[ \lim_{x^5 \to 0} \partial_5 \Phi = \frac{O}{2} \]  

(56)

After the above-mentioned splitting, the equation of motion is

\[- \square_4 \phi - \square_4 \psi + \partial^2_5 \psi = 0 \]  

(57)

with the boundary conditions (56).

To proceed, we assume ‘low-energy’ sources, whose derivatives along the space directions are not larger than the typical derivatives along the fifth dimension. Thus one can make for \( \psi \) the series Ansatz of the form

\[ \psi = \sum_{n=0}^{\infty} \psi_n \]  

(58)

\footnote{Analogous reasoning also applies to \( Z_2 \)-odd fields.}
with $\square_4 \psi_n / \partial_5^2 \psi_n < 1$, and write the equation of motion in the form

$$ - \square_4 \psi - \sum_{n=0}^{\infty} \square_4 \psi_n + \sum_{n=0}^{\infty} \partial_5^2 \psi_n = 0 \quad (59) $$

Boundary conditions on $\Phi$ now become boundary conditions on $\psi$. Solving in an obvious way the resulting series of equations, we obtain

$$ \psi_0 = \square_4 \phi \left( \frac{(x^5)^2}{2} - \frac{\pi \rho^2}{6} \right) + \frac{\mathcal{O}}{2} (x^5 - \frac{\pi \rho}{2}) \quad (60) $$

plus, from the boundary conditions, the relation $\square_4 \phi = -\frac{S + \mathcal{O}}{2\pi \rho}$. The remaining terms in the series can easily be computed from the recurrence equations $-\square_4 \psi_n + \partial_5^2 \psi_n = 0$ equipped with trivial boundary conditions. When we substitute the solution $\Phi = \phi + \psi_0$ into the action, and perform the integration over the circle in the fifth dimension, we obtain the effective action

$$ S_{\text{eff}}(\phi) = \int d^4x \left( \pi \rho \partial_\mu \phi \partial^\mu \phi + \frac{\pi \rho^3}{180} (\partial_\mu S \partial^\mu S + \partial_\mu \mathcal{O} \partial^\mu \mathcal{O} - \frac{7}{4} \partial_\mu \mathcal{O} \partial^\mu S) \right) $$

$$ - \int d^4x (\phi + \psi_0)_{x^5=0} \mathcal{O}(x) - \int d^4x (\phi + \psi_0)_{x^5=-\pi \rho} S(x) \quad (61) $$

which, after substituting the values of the known solution for $\psi_0$ at the boundaries, becomes

$$ S_{\text{eff}}(\phi) = \int d^4x \left( \pi \rho \partial_\mu \phi \partial^\mu \phi - \phi (S + \mathcal{O}) + \frac{\pi \rho^3}{180} (\partial_\mu S \partial^\mu S + \partial_\mu \mathcal{O} \partial^\mu \mathcal{O} - \frac{7}{4} \partial_\mu \mathcal{O} \partial^\mu S) \right) $$

$$ + \int d^4x (\mathcal{O}^2 + S^2 - \mathcal{S} \mathcal{O}) \frac{\pi \rho}{6} \quad (62) $$

where the last line has arisen from the boundary terms in (61).

The point to be noticed is that the effective operator that mixes sources on the hidden wall and observable fields on the visible wall is proportional to $\rho$, the distance between the walls. The message of this example applies also to other even fields living in the bulk, such as components of the gravity multiplet. The factor of $\rho$, which is present there also in the presence of boundaries, is in agreement with the prescription for obtaining the four-dimensional gravity action given in [2].

We now consider a second case, which is of particular interest in the context of $M$ theory, namely a field which is odd under the action of $Z_2$. In this case, the Lagrangian which includes a coupling to sources on the hidden wall and to operators consisting of observable fields on the visible wall is

$$ S(\Phi) = \int d^5x \left( \frac{1}{2} \partial_\Phi \partial \Phi + \partial_5 \Phi S \delta(x^5 - \pi \rho) + \partial_5 \Phi \mathcal{O} \delta(x^5) \right) \quad (63) $$

The bulk equation of motion is

$$ - \square_4 \Phi + \partial_5^2 \Phi = S(x) \partial_5 \delta(x^5 - \pi \rho) + \mathcal{O}(x) \partial_5 \delta(x^5) \quad (64) $$

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It is clear that the zero mode, i.e., the part of the field which does not depend on the fifth coordinate, must vanish for the odd fields. The equation of motion for the non-zero mode $\psi(x; x^5)$ coincides then with the full equation of motion, the proper boundary conditions on the half-circle being

$$
\lim_{x^5 \to \pm \pi \rho} \psi = \frac{1}{2} S
$$

Again, using the same method as for the even field, one finds the solution

$$
\psi = \frac{S + O}{2\pi \rho} x^5 - \frac{O}{2} + \psi_1 + \ldots
$$

where the higher terms in the series can easily be computed from the recursive relation $-\Box_4 \psi_{n-1} + \partial_5^2 \psi_n = 0$ with trivial boundary conditions. One should note that if the sources can be regarded as constants, independent of $x$, then the lowest-order solution indicated above is exact.

After substituting the lowest-order solution into the action, one obtains the contribution from the odd field to the effective four-dimensional Lagrangian, including boundary terms:

$$
\delta S_{\text{eff}} = \int d^4x \left( \frac{\partial_\mu S \partial^\mu S}{12} \pi \rho + \frac{\partial_\mu O \partial^\mu O}{12} \pi \rho - \frac{\partial_\mu S \partial^\mu O}{12} \pi \rho \right)
$$

With the help of (66) one computes the $\partial_5$-derivative of $\psi$:

$$
\partial_5 \psi = \frac{S + O}{2\pi \rho} + \ldots
$$

The observation to be made here, which easily generalizes to more realistic models, is that the $\partial_5 \psi$, hence its values computed on the walls as well, is inversely proportional to the distance $\pi \rho$ between the walls. After substituting (68), applied on the walls, into (67) one finds the effective local Lagrangian in the form:

$$
\delta S_{\text{eff}} = \int d^4x \left( \frac{\partial_\mu S \partial^\mu S}{12} \pi \rho + \frac{\partial_\mu O \partial^\mu O}{12} \pi \rho - \frac{\partial_\mu S \partial^\mu O}{12} \pi \rho - \frac{3}{4\pi \rho} (SS + OO + 2SO) \right)
$$

As there is no zero mode, one obtains only an effective coupling of the source to the observable operators, and a contribution to the vacuum energy.

There are two points to be made on the basis of the above arguments. First, the effective coupling of the source to the observable operators is born \textit{locally}, on the visible wall, through the \textit{local} value of the bulk variable, either the field itself or its derivative, as both come from

\footnote{This is immediate in the present case, but would demand solving the actual equations of motion in realistic models}
terms which contain $\delta(x^5)$ coefficients. This is in perfect agreement with the fact that we are using only local solutions of the classical equations of motion to derive the effective action, and no quantum-mechanical effects are involved \[^{11}\]. Hence the whole influence of the bulk physics is encoded in the $x^5$ dependence of the classical trajectory, and the sources on the hidden wall simply play the role of boundary conditions for that classical trajectory. This feature becomes more significant in the case of more complicated non-linear $\sigma$ models in the bulk, when the leading dependence on $x^5$ of the even and odd fields is less simple, with fields varying significantly over the distance between the walls. Secondly, one should note that in the case of the even field the strength of the effective $SO$ interaction grows with the distance $\pi \rho$, whereas in the case of the odd field the strength decreases like $\frac{1}{\pi \rho}$. Finally, one should notice that, in more realistic models, the source $S(x)$ will be in fact a function of ‘boundary’ configurations of the even bulk fields: $S(x) = S(E^i(x; x^5 = \pi \rho))$, which will introduce additional aspects into the analysis. For the moment, we limit ourselves to remarking that, despite these complications, if the local dynamics on the hidden wall produces a stable configuration of the local Lagrangian \[^{\ref{12}}\] coefficients. This is in perfect agreement with the fact that we are

\[^{11}\]In other words, we use the saddle-point trajectory to compute the respective path integral, without including fluctuations even at the Gaussian level.

We now tackle the non-linear $\sigma$ model equations of motion in the bulk. We first warm up with the simple example where the scalars of non-universal hypermultiplets are turned off. As one can quickly compute using the quaternionic metric given in Section 3, the equations of motion for the coupled system of the even scalar $S$ and the odd universal scalar $C$ are (along the real directions)

\[
S''(x^5) + \delta(x^5 - \pi \rho)^2 \left( \frac{4 \vartheta^2 C(x^5)^2}{2C(x^5)^2 - S(x^5)} - \frac{2 \vartheta^2 S(x^5)}{2C(x^5)^2 - S(x^5)} \right) - \frac{4C(x^5)^2 C''(x^5)^2}{2C(x^5)^2 - S(x^5)} = 0
\]

\[
C''(x^5) + \frac{2S(x^5) C''(x^5)^2}{2C(x^5)^2 - S(x^5)} - \frac{4C(x^5)^2 C''(x^5) S'(x^5)}{2C(x^5)^2 - S(x^5)} + \frac{S'(x^5)^2}{2C(x^5)^2 - S(x^5)} = 0
\]

\[
= \frac{4 \vartheta^2 C(x^5)^2 \delta(x^5 - \pi \rho)^2}{2C(x^5)^2 - S(x^5)} - \frac{4C(x^5)^2 C'(x^5)^2}{2C(x^5)^2 - S(x^5)} + \left( \frac{-2 \vartheta C(x^5)^2}{2C(x^5)^2 - S(x^5)} \right) + \frac{\vartheta S(x^5)}{2C(x^5)^2 - S(x^5)} \delta'(x^5 - \pi \rho) + \frac{C'(x^5)^2 S'(x^5)}{2C(x^5)^2 - S(x^5)} = 0
\]

(70)

where we use for convenience a stiff source $\vartheta$, possibly representing a gaugino condensate at short distances, which replaces the general source $S$ from previous paragraphs. As they stand, the equations are defined on the full circle, corresponding to the fifth coordinate $x^5$ varying between $0$ and $2\pi \rho$. However, we know that, because of the definite parities of the fields, all the information is contained on the half-circle $[0, \pi \rho]$. To write the equivalent problem on the half-circle, we need to define the proper boundary conditions which, together with the known parities, reproduce the singular sources located at the ends of the half-circle. As it is easy to see, the proper boundary conditions are

\[
\lim_{x^5 \to 0} C = 0, \quad \lim_{x^5 \to \pi \rho} C = \frac{\vartheta}{2}
\]
\[
\lim_{x^5 \to 0} S' = 0, \quad \lim_{x^5 \to \pi \rho} S' = 0 \quad (71)
\]

One can easily see that the resulting singularity structure on the full circle is the proper one: with the above assignment, \( C' \) develops a singularity at \( x^5 = \pi \rho \), \( \lim_{x^5 \to 0} C = \frac{4}{2} \theta(x^5 - \pi \rho) \), and when one substitutes this into the equations (70) all the singularities indeed cancel among themselves at \( x^5 = \pi \rho \). We note that the solutions to these equations develop singularities inside the perfect square describing the coupling of bulk to the boundary, which cancel exactly the singularity associated with the source. To see this cancellation in the equations on the full circle, it is essential to keep the \( \delta^2 \)-type terms in the equation, despite the appearance that they are of higher order in the bulk-wall coupling.

The structure of the equations (70) is such that one can easily order the terms according to the powers of the moduli \( C \), which can be taken as small as one wishes. The zeroth-order terms in such an expansion are second derivatives \( S'' \), \( C'' \) together with singular sources proportional to \( S \), and to that order the second equation coincides with the simple equation studied in [22]. The system therefore approaches the toy models studied at the beginning of this Section. This explains why the qualitative behaviour of our solutions can be understood in terms of the toy models, and parallels the results obtained in [22].

At this point we would like to turn our attention to the possibility of including a non-trivial background of the even scalar \( S \), whose real part corresponds to the volume of the internal Calabi-Yau space, which varies linearly with \( x^5 \) according to the vacuum solution obtained by Witten [2] in eleven dimensions. This can be achieved by adding sources on the right-hand side of the first of the equations (70). The modified equation with the sources is

\[
S''(x^5) + \delta(x^5 - \pi \rho)^2 \left( \frac{4 \theta^2 C(x^5)^2}{2 C(x^5)^2 - S(x^5)} - \frac{2 \theta^2 S(x^5)}{2 C(x^5)^2 - S(x^5)} \right) - \frac{4 C(x^5)^2 C'(x^5)^2}{2 C(x^5)^2 - S(x^5)} + \frac{2 S(x^5) C'(x^5)^2}{2 C(x^5)^2 - S(x^5)} - \frac{4 C(x^5) C''(x^5) S'(x^5)}{2 C(x^5)^2 - S(x^5)} + \frac{S'(x^5)^2}{2 C(x^5)^2 - S(x^5)} = -\varrho_v \delta(x^5) + \varrho_h \delta(x^5 - \pi \rho) \quad (72)
\]

and the corresponding boundary conditions on the half-circle, replacing (71), are

\[
\lim_{x^5 \to 0} S' = -\frac{\varrho_v}{2}, \quad \lim_{x^5 \to \pi \rho} S' = -\frac{\varrho_h}{2} \quad (73)
\]

where \( \varrho_v, h \) are fixed parameters determined by solving the Bianchi identities. Again, one can check that the singularities cancel between themselves. One notices that the only consistent way to assign a value to the \( \mathbb{Z}_2 \)-odd function on the wall, even if the function has a finite discontinuity, is to give it the value zero at \( x^5 = 0 \) and \( x^5 = \pi \rho \). The results of solving the modified equations (70) in various cases are given in Figs. 1 to 9. In practice, the way solutions are obtained is to assume, among other boundary conditions, a value of \( S(x^5 = 0) \) and a value of \( S'(x^5 = 0) \). The former is unconstrained, the latter is interpreted in the spirit of (73) as the presence of a source \( \varrho_v \). Such boundary conditions generate typically a nonzero \( S'(x^5 = \pi \rho) \). Again, this we simply interpret as a source \( \varrho_h \) located at \( x^5 = \pi \rho \). This approach is sufficient
for our purposes, as we are interested in correlations between qualitative characteristics of the system displayed in the Figures, and not in obtaining any particular values of the sources.

We use our numerical solutions of the equations of motion, with the above boundary conditions, to find the variations of the fields through the bulk, and to determine the boundary values of the interesting quantities on the visible and hidden walls. From the point of view of supersymmetry breaking, the interesting quantities are, as argued above and in Section 4, the local values of derivatives of the $Z_2$-odd fields on the walls, such as $\partial_5 C$ in the present case, modulated by the vielbeins. The vielbeins, as can be checked using expressions given in Section 4, are factors $O(1/\sqrt{2V_{CY}(x^5)})$ times factors of order one. Here we display the leading dependence on $S$ (the $V_{CY}$) of the supersymmetric variations of hyperinos. The solutions presented below generically have supersymmetry broken both on the hidden and on the visible walls.

![Figure 1](image)

Figure 1: The relation between the value of $C'(x^5 = 0)$, measuring the supersymmetry breaking on the visible wall, and the limiting value of $C(x^5 = \pi\rho)$, which equals half of the effective condensate on the hidden wall. Curves (a) and (b) correspond to $S' = 0$ and $S = 1$, 0.05 respectively. Curve (c) is for $S = 0.05$ and the non-zero slope $S'(0) = 0.1$. These results are obtained for the model with only a universal hypermultiplet.

We see in Fig. 1 the relation between the value of $C'(x^5 = 0)$, which measures the supersymmetry breaking on the visible wall, and the limiting value of $C(x^5 = \pi\rho)$, which equals half of the effective condensate on the hidden wall. Curves (a) and (b) correspond to $S' = 0$ and $S = 1$, 0.05 respectively, $\pi\rho = 10$ is assumed in all examples and, as everywhere in this paper, we work in units where the eleven-dimensional Planck scale is equal to 1. The results shown in the figures are generic. One finds that a non-zero hidden condensate generically corresponds to non-zero supersymmetry breaking on the visible wall. Also, in the range where the variables are sufficiently small, corresponding to realistic supersymmetry breaking that is hierarchically smaller than the Planck scale, the scale of the visible supersymmetry breaking grows with the
scale of the condensate. It is also clear, comparing curves (a) and (b) in the Fig. 1, that the change of $S(0)$ does not change the qualitative character of the relation between the quantities, just the numerical magnitude of the effect. The curves (c) tell us what happens when a non-trivial background along the direction of the field $S$ is switched on, see (72) and (73). This corresponds directly to the linear dependence of the volume of the Calabi-Yau space on the fifth (eleventh) coordinate found in [2], since $\Re(S) = V(x^5)$. In our model, this effect is taken into account by switching on the derivatives of $S$ with respect to $x^5$ on the boundary of the semi-circle [12]. We see that assuming $S'(0)$ non-zero makes the dependence of the visible supersymmetry breaking on the condensate become increasingly steeper. Also, this enhances the inhomogeneity of the field configurations in the bulk. Curve (c) is given for $S = 0.05$ and the non-zero slope (quasi-linear $S$ background) $S'(0) = 0.1$. To see how inhomogenous the bulk becomes, we plot the $S$ and $C'$ fields as functions of $x^5$ in Figs. 2 and 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure2.png}
\caption{The $x^5$ dependence of the moduli $S(x^5)$ - curve (a) - and $\frac{C'(x^5)}{\sqrt{23(x^5)}}$ - curve (b). The boundary values of the fields on the visible wall $x^5 = 0$ are $S(0) = C'(0) = 0.05$, $S'(0) = 0$. These results are also for the model with just the universal hypermultiplet.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure3.png}
\caption{The same as in Figure 2, but with $S'(0) = 0.1$.}
\end{figure}

One can easily see that the field configurations are not homogeneous, and not even linear in $x^5$. The non-zero initial slope $S'(0)$ makes the variation of both plotted fields much more pronounced. Simple averaging over the fifth dimension does not commute with the non-linearity of the equations of motion in the bulk, and does not produce particularly natural characteristics of the behaviour of individual fields in the bulk.

\footnote{One should remember the parity properties when going over to the full circle and incorporating singular sources.}
It is straightforward to extend the procedure described above to the case of the system containing one non-universal hyperplet, which includes one odd scalar $C_1$ and one even modulus $Z$, again making use of the quaternionic metric given in Section 4. The boundary conditions which faithfully reproduce the singularity structure in this case are

$$\begin{align*}
\lim_{x^5 \to 0} C &= 0, \quad \lim_{x^5 \to \pi \rho} C = \frac{\vartheta}{2}, \\
\lim_{x^5 \to 0} C_1 &= 0, \quad \lim_{x^5 \to \pi \rho} C_1 = 0, \\
\lim_{x^5 \to 0} S' &= -\frac{\vartheta v}{2}, \quad \lim_{x^5 \to \pi \rho} S' = -\frac{\vartheta h}{2}, \\
\lim_{x^5 \to 0} Z' &= 0, \quad \lim_{x^5 \to \pi \rho} Z' = 0
\end{align*}$$

(74)

where $\vartheta$ is the vacuum expectation value of the condensate. Again, one can solve the full set of equations using these boundary conditions. The results are presented below, and are qualitatively similar to the simpler case. The interesting new phenomenon is to watch is the rôle of the mixing between the two odd fields. It turns out that, even though the non-universal field $C_1$ is assumed not to couple directly to any source of supersymmetry breaking on the wall $^{13}$, the mixing with the remaining fields excites its derivative $\partial C_1$ on the visible wall.

![Figure 4](image1.png)

**Figure 4:** The $x^5$ dependence of $\frac{C_1'(x^5)}{\sqrt{2S(x^5)}}$. Here $C_1'$ is the derivative with respect to $x^5$ of the $Z_2$-odd non-universal hyperplet modulus. Curves correspond to different boundary conditions on the visible wall. Curve (a) corresponds to $S(0) = 1$, $Z(0) = 0.1$, $C_1'(0) = 0.5$, $C'(0) = 0$, curve (b) to $S(0) = 0.5$, $Z(0) = 0.1$, $C_1'(0) = 0.1$, $C'(0) = 0.1$, and (c) to $S(0) = 1$, $Z(0) = 0.1$, $C_1'(0) = 0$, $C'(0) = 0.5$.

*Here and in the subsequent two Figures the model used contains a single non-universal hypermultiplet.*

![Figure 5](image2.png)

**Figure 5:** The evolution of the volume modulus $S$ across the bulk. The boundary conditions for the curves are the same as in Figure 4.

$^{13}$This assumption is not crucial, and there are models where it is relaxed.
It is interesting to note that, even if one starts with vanishing derivative $C'_1$ at one boundary, as in curve (c) in Fig. 4, this derivative is excited in the bulk through the mixing with other moduli in the bulk Lagrangian. The field $S$ (volume modulus) changes visibly across the bulk, as seen in Fig. 5, signalling the necessity of introducing sources on the second wall to maintain consistency between the boundary behaviour on the semicircle and the $Z_2$ parity properties.

In the Figure 6 we have sketched the correlation between boundary value of the derivative $C'_1$ at the visible wall and the vacuum expectation value of the stiff condensate at the hidden wall, if the remaining boundary parameters are fixed. The visible correlation proves the influence of the $\sigma$-model dynamics on the evolution of the fields, as the modulus $C_1$ does not couple directly to the condensate.

![Figure 6](image)

Figure 6: The correlation between boundary value of $C'_1$ at the visible wall and the vacuum expectation value of the stiff condensate at the hidden wall. Curve (a) corresponds to $S(0) = 0.5$, $Z(0) = 0.1$, $C'(0) = 0.1$, and curve (b) differs in the boundary value $C'(0) = 0.01$.

![Figure 7](image)

Figure 7: The vacuum expectation value of the Hamiltonian in the case of the two-piece boundary potential for the volume modulus $S$, as a function of the boundary value of $S$ at the visible wall. The local minimum of the part of the potential on the visible wall is $S_{0,vis} = 4.0$ and on the hidden wall $S_{0,hide} = 1.0$. Here and in the following two Figures the modified model $(75, 76)$ is used.
are with just one potential has two disconnected pieces living on both walls. For simplicity, we take the model perturbative dynamics creates a potential for even moduli living on the walls, and that this $M$ of moduli in the low-energy effective Lagrangian for $S(0) = 3.9$, and $(c)$ - $S(0) = 4.1$, cf. Fig. 7.

Finally, we consider the possibility, very relevant to the possible dynamical stabilization of moduli in the low-energy effective Lagrangian for $M$ theory, that gauge or other non-perturbative dynamics creates a potential for even moduli living on the walls, and that this potential has two disconnected pieces living on both walls. For simplicity, we take the model with just one $Z_2$-even modulus $S$ and one $Z_2$-odd field $C$. The modified equations of motion are

$$S''(x^5) + \frac{1}{(2C(x^5)^2 - S(x^5))}(-2\gamma_{hid}^2 \delta(x^5 - \pi \rho)^2 (S_{0,hid} - S(x^5))^3 (16 C(x^5)^4$$
$$- 2 C(x^5)^2 (S_{0,hid} - 9 S(x^5)) S(x^5) (S_{0,hid} + 3 S(x^5))$$
$$+ 2 (-2 C(x^5)^2 + S(x^5)) C'(x^5)^2 - 4 C(x^5) C'(x^5) S'(x^5) + S'(x^5)^2$$
$$+ \delta(x^5 - \pi \rho) (8 \gamma_{hid} (S_{0,hid} - S(x^5)) (2 C(x^5)^2 + S(x^5))^2 C'(x^5)$$
$$- 8 \gamma_{hid} C(x^5) (S_{0,hid} - S(x^5)) (2 C(x^5)^2 + S(x^5)) S'(x^5))$$
$$+ \frac{1}{(2C(x^5)^2 - S(x^5))}(-2\gamma_{vis}^2 \delta(x^5)^2 (S_{0,vis} - S(x^5))^3 (16 C(x^5)^4$$
$$- 2 C(x^5)^2 (S_{0,vis} - 9 S(x^5)) S(x^5) (S_{0,vis} + 3 S(x^5))$$
$$+ \delta(x^5) (8 \gamma_{vis} (S_{0,vis} - S(x^5)) (2 C(x^5)^2 + S(x^5))^2 C'(x^5)$$
$$- 8 \gamma_{vis} C(x^5) (S_{0,vis} - S(x^5)) (2 C(x^5)^2 + S(x^5)) S'(x^5)) = 0$$

$$C''(x^5) - \gamma_{hid} (S_{0,hid} - S(x^5))^2 \delta'(x^5 - \pi \rho) + \frac{1}{(2C(x^5)^2 - S(x^5))} (4 \gamma_{hid}^2 C(x^5)$$
The two-piece potential for \( S \) that we assume, expanding each part of the effective potential to quadratic order around its respective ‘naive’ minimum, is

\[
\begin{align*}
\delta (x^5 - \pi \rho)^2 & (S_{0, \text{hid}} - 4 C(x^5)^2 - 3 S(x^5)) \\
(S_{0, \text{hid}} - S(x^5))^3 & - 4 C(x^5) C'(x^5)^2 + 2 \gamma_{\text{hid}} \delta (x^5 - \pi \rho) \\
(S_{0, \text{hid}} - S(x^5)) (2 C(x^5)^2 + S(x^5)) (4 C(x^5) C'(x^5) - S'(x^5)) & + C'(x^5) S'(x^5) - \gamma_{\text{vis}} (S_{0, \text{vis}} - S(x^5))^2 \delta (x^5 - \pi \rho) \\
1 & + \frac{1}{(2 C(x^5)^2 - S(x^5))} (4 \gamma_{\text{vis}}^2 C(x^5) \delta (x^5 - \pi \rho)^2) \\
(S_{0, \text{vis}} - 4 C(x^5)^2 - 3 S(x^5)) (S_{0, \text{vis}} - S(x^5))^3 & + 2 \gamma_{\text{vis}} \delta (x^5 - \pi \rho) (S_{0, \text{vis}} - S(x^5)) \\
(2 C(x^5)^2 + S(x^5)) (4 C(x^5) C'(x^5) - S'(x^5))) & = 0 \\
\end{align*}
\]

(76)

We assume that each part of the potential has a local minimum, taken to be \( S_{0, \text{vis}} = 4.0 \) on the visible wall and \( S_{0, \text{hid}} = 1.0 \) on the hidden wall. The coefficients \( \gamma_{\text{vis, hid}} \) are taken to be equal to \(-0.2\) on both walls. In such a situation, when the coefficients of \( \delta \)-like sources depend on boundary configurations of fields which are also subject to bulk dynamics, one expects that both boundary values and bulk configuration are to be chosen in a self-consistent way.

To understand the situation better, we have computed numerically the vacuum expectation value of the total Hamiltonian of the system (excluding gravity) as a function of boundary conditions on the visible wall. We see in Fig. 7 that the local minimum of the Hamiltonian corresponds to the local minimum of the part of the potential living on the visible wall. However, this minimum value is non-zero, signalling an inhomogeneous configuration in the bulk, and, as seen from Fig. 8, it gives boundary values on the second wall that are far away from the local minimum of the second part of the potential for \( S \), which is naively \( S_{0, \text{hid}} = 1.0 \). Of course, the reasoning can be reversed with respect to the walls, and there exists another local minimum of the Hamiltonian, where the boundary value of \( S \) on the hidden wall is close to \( S_{0, \text{hid}} = 1.0 \) but differs from \( S_{0, \text{vis}} = 4.0 \) on the observable wall\(^{14}\). Hence, the existence of the two disconnected pieces of the moduli potential (which may or may not be related to gaugino condensation on the walls) plus the bulk dynamics lead to the interesting phenomenon of an apparently ‘displaced’ moduli vacuum on one of the walls, which is a sign of the existence of the second wall.

We have discussed in this Section supersymmetry breaking at the level of the five-dimensional supergravity. The expectation values of variations of even components of bulk fermions which do not depend on fermionic fields and do not contain four-dimensional space derivatives are identified as possible signatures and measures of supersymmetry breaking. These are bulk variations, but they are continuous across the walls, as are the even fermionic fields, so they constitute legal supermultiplets living on the four-dimensional walls. Similarly to what has been observed in the example of a globally supersymmetric toy model with vector multiplets \(^{22}\), the

\(^{14}\)The local minimum corresponding to the configuration where \( S_{0, \text{vis}} = 4.0 \) gets cancelled is the deeper of the two.
terms we have identified in the supersymmetric variations of fermions are to be interpreted from the point of view of four-dimensional boundary chiral multiplets as gravitational contributions to the $F$ terms of chiral superfields. These contributions are gravitational in the sense that they are suppressed by additional powers of the $m_{11}$ scale, namely $\kappa^{2/3}$, as these terms contain in our example a factor of $\kappa^{2/3}$ due to the gauge coupling to the source on the hidden wall (it is implicit in $S$). In the five-dimensional canonical frame they are also inversely proportional to the distance $\pi \rho$ between the walls, as in (67), (68) and the discussion in Section 3:

$$\delta F_{S,T} = \alpha_{S,T} \frac{S}{\pi \rho \sqrt{V_{CY}}}$$

where $\alpha_{S,T}$ is a coefficient of order one depending on whether we look at supersymmetry variations of hyperinos ($S$) or $\psi_5 (T)$, as seen in (45), (51).

Before attempting to identify the soft terms breaking global supersymmetry on the walls, one should take care of some subtleties. First, although the superfields we discuss here look like true chiral multiplets on the wall, one has to remember that they come from the gravitational bulk sector. The basis we work with in the bulk gives the canonical Einstein-Hilbert and gravitino action in five dimensions, not in four. When going over to four dimensions, one has to perform field and metric redefinitions on the walls, to obtain the field frame which is canonical in four dimensions. The necessary ingredient is then the specific definition of the effective four-dimensional gravitational sector. As we have learned from the toy models discussed in Section 3, the proper way to define effective moduli-charged matter couplings is to compute them on the wall as limiting values of solutions to the equations of motion along the fifth dimension. In principle, gravitational degrees of freedom are no exception, and the same procedure as the one applied in Section 3 to a general $Z_2$-even field should be carried out. Hence, one can carry out the programme of Weyl rescaling the metric, separating out the component $\psi_5^+$ and redefining the four-dimensional gravitino in order to produce a canonical kinetic term for $\psi_5^+$, and correcting suitably the supersymmetry variations to account for redefinitions [30]. The details of this construction of the four-dimensional effective action demand a separate discussion in themselves, and we do not attempt pursue them in the present work. In the present paper we have worked in the canonical five-dimensional basis, and in this basis the goldstino on the wall is a mixture of the $Z_2$-even fermion from the $T$-plet and hyperinos from the $S$ multiplet and from non-universal hypermultiplets.

Finally, we comment further on some of the questions discussed earlier, in particular on the transmission of supersymmetry breaking between the walls, in the light of the corrections to the effective action that arise in the gauged five-dimensional supergravity construction of [12]. We recall briefly that the gauging arises because the vacuum solution has non-vanishing components of the antisymmetric tensor field and its strength, linear in $x^{11}$ to lowest non-trivial order in $\kappa^{2/3}$. Hence, in the construction of the effective five-dimensional theory, one expands the Lagrangian around this non-trivial eleven-dimensional background, and treats five-dimensional zero modes as fluctuations in that non-vanishing background [14]. Upon substituting such an expansion into the topological $C \wedge G \wedge G$ term in the eleven-dimensional supergravity Lagrangian, one finds, among other terms, a new coupling between zero modes of the form $\partial_\mu D A^\mu$, where $D$ is in our language the imaginary part of the complex even scalar $S$ from the universal hypermultiplet,
and in the language of the effective 4d theory on a wall is simply the universal axion. The vector boson $A$ is a particular combination of the $h_{1,1}$ Abelian vector bosons existing in the five-dimensional bulk, whose composition depends on the orientation of the gauge and gravitational instantons with respect to the cohomology basis used to define the zero modes. Note that this term is of order $\mathcal{O}(\kappa^{2/3})$, which is of higher order than the kinetic couplings in the bulk which we have considered up till now in the present paper. This new term breaks the zeroth-order supersymmetry in the bulk, and should be supersymmetrized when one works consistently with the higher-order bulk couplings. The unique way to supersymmetrize such a derivative scalar-vector coupling is to regard it as a part of a covariant derivative in a gauged supergravity model [12]. The particular form of that coupling enables one to identify the Killing vector of the quaternionic manifold isometry which is gauged, and formulae given in [15] can be used to read off the remaining terms in the Lagrangian that are needed to maintain supersymmetry, and the corresponding modifications to the supersymmetry transformation laws. The required terms in the Lagrangian were obtained in [12] from the reduction of the eleven-dimensional Lagrangian with a non-vanishing background. One particularly interesting new term related by supersymmetry to the $\mathcal{O}(\kappa^{2/3})$ term is a potential term, analogous to $D$ terms in four-dimensional supersymmetry, which is of order $\mathcal{O}(\kappa^{4/3})$. Thus, the gauged theory contains new couplings in the bulk, between fields belonging to different multiplets, but they are of higher order in an expansion in powers of $\kappa^{2/3}$ than the couplings we have considered up till now.

We note that the coupling to bulk fields of a source on the wall, such as a gaugino condensate, is already suppressed by a power of $\kappa^{2/3}$. The effects on the transmission of supersymmetry breaking of new terms in the bulk are of higher order, and hence unlikely to change qualitatively the conclusions we have reached working with the leading-order Lagrangian: they would simply contribute additional mixing of the scalars and vectors living in the bulk. One must further check the supersymmetry transformations laws, which are also modified. However, as as has been already noticed in [12], the corrections to the transformations, which are linear in the non-trivial background, are not only of higher order, as is that background, but also discontinuous across the walls, since the background to which they are proportional is itself discontinuous. This means that these corrections do not appear on the walls, and so do not open up any new channels of communication of supersymmetry breaking to the fields living on the visible wall, beyond those already identified in the present paper.

Finally, we observe that the origin of the non-trivial backgrounds of certain five-dimensional zero modes, such as the real part of $S$ which represents the Calabi-Yau volume, is traceable to non-trivial sources living on the walls. Both in the case discussed here and in the gauged supergravity model, these are coupled to zero modes that change quasi-linearly across the bulk. The role of such sources, which we have studied in this paper in the leading-order Lagrangian, continues to hold to leading order also in the presence of the terms associated with the gauging, as do our conclusions.

To summarize this discussion, the gauging of the five-dimensional supergravity induced by a non-trivial background for the antisymmetric tensor field in $\mathcal{O}(\kappa^{2/3})$ induces higher-order corrections to our results, due to additional higher-order mixing between bosonic fields in the bulk. However, our conclusions on the possible patterns of transmission of supersymmetry
breaking remain unaffected by the gauging.

6 Conclusions

We have argued in this paper that a systematic reduction of the eleven-dimensional \(M\)-theory effective Lagrangian should always proceed in two steps. First, the reduction from eleven to five dimensions should be performed. This yields a five-dimensional Lagrangian that is richer in parameters than in eleven dimensions, due to effects related to the geometry of the compact Calabi-Yau manifold. In five dimensions, one always obtains a non-linear \(\sigma\) model for bulk moduli fields, which implies non-trivial dynamics across the \(Z_2\) orbifold. We have presented simple examples of such models, and have analyzed with their help the evolution of the moduli between the walls. In the examples discussed, which belong to the class of ungauged five-dimensional supergravities, the non-trivial configurations in the bulk have to be excited by sources living on the four-dimensional walls.

There exist natural sources which may come from the gauge models living on the boundaries. A favoured example is a condensate of gauge fermions in a strongly-coupled gauge group living on the opposite end of the fifth dimension from the visible four-dimensional sector. The configurations induced by a non-vanishing condensate lead, upon solving the equations of motion in the fifth dimension, to non-vanishing supersymmetry variations of modulini at the second, visible wall. The relevant variations are computed locally on the visible wall, and are inversely proportional to the square root of the Calabi-Yau volume, and directly proportional to vacuum expectation values of derivatives with respect to the fifth coordinate of the \(Z_2\)-odd moduli living in the bulk. We have also given a general argument that the effective strength of the supersymmetry breaking operators induced this way on the visible wall decreases as the inverse of the distance between the walls. The correlations between the magnitude of the visible supersymmetry breaking and the scale of the hidden condensate, and between some other characteristics of the dynamics across the fifth dimension, have been presented in a model containing just a single universal supermultiplet, and in the second case, one which also has one non-universal supermultiplet. One interesting observation concerns the case where a stiff condensate is replaced by effective boundary superpotentials. Then the local vacuum on the observable wall can be visibly different from the naive one computed from the effective moduli potential born directly on the visible wall. The shift of the minima for the moduli is due to the component of the potential on the second, hidden, wall, transmitted by the equations of motion through the bulk.

Our findings are not in disagreement with the eleven-dimensional discussion of [21]. We find that when the condensate is switched on, it induces supersymmetry breaking locally, which cannot be removed by a legal redefinition of the parameter of the supersymmetry transformation. This acts as a source for the \(\sigma\) model in the bulk, and the solution of the bulk equations of motion in the presence of this source is such that the resulting boundary values of the moduli and their derivatives on the observable wall also correspond to a breaking of global supersymmetry restricted to this wall.
In this paper, we have barely scratched the surface of the rich physics opened up by the five-dimensional framework [13, 16, 12] explored here. There are many fascinating open issues such as the exact origin of the supersymmetry-breaking sources on the hidden wall, details of the coupling of the walls to the bulk theory, the effects of the deformation of the Calabi-Yau manifold in the model discussed here, the appearance of non-trivial dynamics for the complex structure moduli, the dynamical choice of the vacuum and the magnitude of the fifth dimension, applications to specific ‘realistic’ Calabi-Yau compactifications, the possible extension to non-Calabi-Yau reductions from eleven to five dimensions, and many more. We can hope that some of the unresolved issues plaguing weak-coupling string models may be illuminated by five-dimensional light.

Acknowledgments:

Z.L. and S.P. are supported in part by the Polish Committee for Scientific Research grant 2 P03B 040 12, and by the M. Curie-Sklodowska Foundation. W.P. would like to thank Professor Graham G. Ross for stimulating discussions.

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