Conductance and localization in disordered wires:
role of evanescent states

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This paper extends an earlier analytical scattering matrix treatment of conductance and localization in coupled two- and three Anderson chain systems for weak disorder when evanescent states are present at the Fermi level. Such states exist typically when the interchain coupling exceeds the width of propagating energy bands associated with the various transverse eigenvalues of the coupled tight-binding systems. We calculate reflection- and transmission coefficients in cases where, besides propagating states, one or two evanescent states are available at the Fermi level for elastic scattering of electrons by the disordered systems. We observe important qualitative changes in these coefficients and in the related localization lengths due to ineffectiveness of the evanescent modes for transmission and reflection in the various scattering channels. In particular, the localization lengths are generally significantly larger than the values obtained when evanescent modes are absent. Effects associated with disorder mediated coupling between propagating and evanescent modes are shown to be suppressed by quantum interference effects, in lowest order for weak disorder.

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I. INTRODUCTION

In a recent paper (hereafter referred to as I) we developed a scattering matrix analysis of Landauer conductance and of localization and reflection to coupled two- and three chain systems for weak disorder, using the Anderson tight-binding model. The full quasi-1D scattering set up consists, as usual, of the system of coupled disordered chains of length $L$ connected to electron reservoirs by semi-infinite leads composed of non-disordered coupled chains attached at both ends. The leads carry the current which is incident on the disordered sample, together with the currents which are reflected or transmitted by the sample in modes of properly defined quantum channels at the Fermi level.

The analytical treatment of I was carried out for the case where the states at the Fermi energy in the various channels available for scattering of the injected electrons all belong to subbands of propagating states (i.e. the case where all channels are propagating). Thus, in this case all reflected- or transmitted electrons carry current. On the other hand, there may exist Fermi energy domains where the states at the Fermi level corresponding to incident electrons in a particular propagating subband include evanescent state solutions of the Schrödinger equations for some channels (evanescent channels) besides propagating solutions for others.

In practice, Fermi energy domains such that propagating states in some channels coexist at the Fermi level with evanescent states in other channels are found when the interchain coupling exceeds the width of the subbands of propagating states in the leads. Evanescent states do not carry current, as is well-known. They may however influence the current transport indirectly via their coupling, induced by the disorder, to propagating states at the Fermi level in other channels.

The disorder-induced indirect effect of an evanescent channel in the Landauer conductance and in the related localization length is quite distinct from the direct effect due to the disappearance of a conducting channel when the Fermi level is moved across the edge of its propagating subband into an evanescent domain. Indeed, this process completely suppresses the primary (direct) effect which existed at Fermi energies within the propagating subband of the considered channel, where the latter was on the same footing as the other propagating channels coupled to it by the disorder.

The object of this paper is to complement the discussion in I by a detailed analytical study of Landauer conductance and of localization lengths, as well as of reflection coefficients in Fermi energy domains where the relevant scattering modes in some channels are evanescent or non-propagating. As discussed above evanescent modes generally affect the conductance both directly, in that their existence implies the absence of corresponding (current carrying) propagating modes, and indirectly via their coupling to the other propagating modes.

To our knowledge very few studies of the effect of evanescent modes in the conductance or the resistance of disordered wires have been published, for models which are different from the Anderson tight-binding model. Bagwell studied in detail current transmission amplitudes and electrical conductance as a function of Fermi energy for electrons scattered from a single $\delta$-function defect (and also numerically, for scattering from a finite range scatterer) in an otherwise non-disordered multichannel wire. On the other hand, Cahay et al. analysed the effect of evanescent states on the resistance of the two-dimensional random array of elastic scatterers numerically, using the scattering-matrix formalism of Datta, Cahay and Mc Lennan.
The scarcity of studies of the effects of evanescent modes (and the fact that the subject is not mentioned in any of the recent reviews or monographs discussing multichannel systems) does not distract from their intrinsic interest. It suffices to recall that the leads in the quantum conductance problem act as electron waveguides which define a basis for the scattering matrix of the multichannel disordered region. This indicates a similarity between the conductance problem and the study of optical waveguides where e.g. it is known that both propagating and evanescent wave solutions of the electromagnetic wave equation must be superposed in order to correctly describe the fields near sources or obstacles in an otherwise perfect waveguide. On the other hand, in view of the generality of the Anderson model in the context of localization and transport in disordered systems, the present study of the role of evanescent modes should be of particular interest. Also, besides their relevance in various experimental situations, few channel tight-binding systems are interesting because they can be discussed analytically for weak disorder, with the same degree of accuracy as corresponding 1D-systems.

In Sec. II.A we recall the Schrödinger tight-binding equations for the two- and three coupled chain systems studied in I. We briefly discuss the diagonalization of the interchain coupling terms carried out in I, which leads to the description of the leads in terms of independent channels for scattering of plane wave- and evanescent wave modes. In the three chain case we distinguish between equidistant chains on a planar strip with free boundary conditions and a system of equidistant coupled linear chains on a cylindrical surface. In Sec. II.B we recall the basic formulæ for the Landauer two-point conductance and for the Lyapounov exponent of the conductance (inverse localization length). These expressions are similar to those used in I except that the summations over channels in which an electron may be transmitted (reflected) are restricted to the only propagating channels. In Sec. III we summarize the main points of the determination of the transfer- and scattering matrices in these models referring to I for the detailed forms of these matrices (with proper adaptations for the case where propagating as well as evanescent channels are present at the Fermi level). The final analytic expressions for the averaged coefficients of transmission- and reflection in propagating channels as well as expressions for localization lengths are included in Sec. IV. The discussion of these results together with further remarks is given in Sec. V.

II. COUPLED TWO- AND THREE CHAIN DISORDERED WIRES

A. Tight-binding models in channel bases

Our \(N\)-chain model of a wire consists of parallel linear chains of \(N_L\) disordered sites each (of spacing \(a = 1\) and length \(L = N_L a\)) connected at both sides to semi-infinite non-disordered \(N\)-chain leads.

The coupled two-chain Anderson model \((N = 2)\) is defined by the Schrödinger equation in matrix form

\[
\begin{pmatrix}
\phi_{n+1}^1 + \phi_{n-1}^1 \\
\phi_{n+1}^2 + \phi_{n-1}^2 \\
\phi_{n+1}^3 + \phi_{n-1}^3
\end{pmatrix} =
\begin{pmatrix}
E - \epsilon_{1n} & -h & 0 \\
-h & E - \epsilon_{2n} & -h \\
0 & -h & E - \epsilon_{3n}
\end{pmatrix}
\begin{pmatrix}
\phi_n^1 \\
\phi_n^2 \\
\phi_n^3
\end{pmatrix},
\]

where the \(\phi_m^i\) denote the wave-function amplitudes at sites \(m\) on the \(i\)th chain, \(h\) is a constant matrix element for an electron to hop transversally between a site \(n\) on chain 1 and the nearest-neighbour site \(n\) on chain 2. The site energies \(\epsilon_{im}\) are random variables associated with the sites \(1 \leq m \leq N_L\) of the disordered chain \(i\), and \(\epsilon_{im} = 0\) on the semi-infinite ideal chains defined by the sites \(m > N_L\) and \(m < 1\), respectively. The above energies, including \(E\), are measured in units of the constant hopping rate along the individual chains.

The coupled three chain model \((N = 3)\) is defined in a similar way by a set of tight-binding equations, whose actual form depends, however, on interchain boundary conditions. For free boundary conditions which correspond to arranging the parallel equidistant chains with nearest-neighbour interchain coupling on a planar strip, the Schrödinger equation is

\[
\begin{pmatrix}
\phi_{n+1}^1 + \phi_{n-1}^1 \\
\phi_{n+1}^2 + \phi_{n-1}^2 \\
\phi_{n+1}^3 + \phi_{n-1}^3
\end{pmatrix} =
\begin{pmatrix}
E - \epsilon_{1n} & -h & 0 \\
-h & E - \epsilon_{2n} & -h \\
0 & -h & E - \epsilon_{3n}
\end{pmatrix}
\begin{pmatrix}
\phi_n^1 \\
\phi_n^2 \\
\phi_n^3
\end{pmatrix},
\]

with the sites in the disordered sections of length \(L = N_L a\) and in the semi-infinite ideal chain sections labelled in the same way as in the two-chain case. On the other hand, in the case of periodic boundary condition which correspond to equidistant linear chains on a cylindrical surface, the tight-binding equations are

\[
\begin{pmatrix}
\phi_{n+1}^1 + \phi_{n-1}^1 \\
\phi_{n+1}^2 + \phi_{n-1}^2 \\
\phi_{n+1}^3 + \phi_{n-1}^3
\end{pmatrix} =
\begin{pmatrix}
E - \epsilon_{1n} & -h & -h \\
-h & E - \epsilon_{2n} & -h \\
-h & -h & E - \epsilon_{3n}
\end{pmatrix}
\begin{pmatrix}
\phi_n^1 \\
\phi_n^2 \\
\phi_n^3
\end{pmatrix}.
\]
An important aspect of the scattering matrix analysis of conductance in the above systems is the definition of bases for the scattering matrix which correspond to independent channels at the Fermi energy $E$. For this purpose one first transforms the Schrödinger equations (1-3) to bases in which the interchain hopping terms are diagonalized. This transformation, discussed in I, replaces the coupled tight-binding equations describing the leads by decoupled equations corresponding to independent chain systems or channels. The transformed Schrödinger equations for the leads in the various systems under consideration are

$$
\begin{pmatrix}
\psi_{n+1}^1 + \psi_{n-1}^1 \\
\psi_{n+1}^2 + \psi_{n-1}^2
\end{pmatrix} =
\begin{pmatrix}
E - h & 0 \\
0 & E + h
\end{pmatrix}
\begin{pmatrix}
\psi_n^1 \\
\psi_n^2
\end{pmatrix},
$$
(4)

for the two-chain system

$$
\begin{pmatrix}
\psi_{n+1}^1 + \psi_{n-1}^1 \\
\psi_{n+1}^2 + \psi_{n-1}^2 \\
\psi_{n+1}^3 + \psi_{n-1}^3
\end{pmatrix} =
\begin{pmatrix}
E - \sqrt{2}h & 0 & 0 \\
0 & E & 0 \\
0 & 0 & E + \sqrt{2}h
\end{pmatrix}
\begin{pmatrix}
\psi_n^1 \\
\psi_n^2 \\
\psi_n^3
\end{pmatrix},
$$
(5)

for the three-chain system with the free boundary conditions,

$$
\begin{pmatrix}
\psi_{n+1}^1 + \psi_{n-1}^1 \\
\psi_{n+1}^2 + \psi_{n-1}^2 \\
\psi_{n+1}^3 + \psi_{n-1}^3
\end{pmatrix} =
\begin{pmatrix}
E - 2h & 0 & 0 \\
0 & E + h & 0 \\
0 & 0 & E + h
\end{pmatrix}
\begin{pmatrix}
\psi_n^1 \\
\psi_n^2 \\
\psi_n^3
\end{pmatrix}, \quad n < 1 \quad \text{or} \quad n > N_L,
$$
(6)

for the three-chain system with periodic boundary conditions. The transformations $\hat{U}^{-1}$ to the new amplitude vectors

$$
\begin{pmatrix}
\psi_n^1 \\
\psi_n^2 \\
\vdots \\
\psi_n^j \\
\vdots
\end{pmatrix} = \hat{U}^{-1}
\begin{pmatrix}
\varphi_n^1 \\
\varphi_n^2 \\
\vdots \\
\varphi_n^j \\
\vdots
\end{pmatrix},
$$
(7)

are given explicitly in I, as are the transformed Schrödinger equations for the disordered regions (equations (8.b-10.b) of I) in which the disordered channels are now coupled by the disorder.

The bases for the definition of transfer matrices and the corresponding scattering matrices in the following section are provided by the Bloch wave (propagating)- and evanescent mode solutions of the Schrödinger equations (4-6) of the form

$$
\psi_n^j \equiv \psi_{n,\pm}^j \sim e^{\pm ikjn}.
$$
(8)

The propagating solutions for the various channels $j$ correspond to real wavenumbers $k_j$ and exist in energy subbands defined by

$$
2 \cos k_1 = E - h,
2 \cos k_2 = E + h,
$$
(9)

for the two-channel systems,

$$
2 \cos k_1 = E - \sqrt{2}h,
2 \cos k_2 = E,
2 \cos k_3 = E + \sqrt{2}h,
$$
(10)

for the three-channel system with free boundary conditions, and, finally,

$$
2 \cos k_1 = E - 2h,
2 \cos k_2 = 2 \cos k_3 = E + h,
$$
(11)
for the periodic three-channel system. On the other hand, the evanescent mode solutions for the various channels correspond to imaginary wavenumbers

\[ k_j = i\kappa_j, \quad \cos k_j = \cosh \kappa_j, \]

in [8] and in [9,11] i.e. to exponentially growing or decaying solutions,

\[ \psi^j_n \equiv \psi^j_{n,\pm} \sim e^{\pm i\kappa_j n}. \]  

The evanescent modes in a given channel correspond to energies lying outside the energy band of propagating states in this channel.

### B. Conductance and localization

We describe the conductance of a multichannel disordered wire by the Landauer two-probe formula\(^2,9\)

\[ g = \frac{2e^2}{h} \text{Tr}(\hat{t}^\dagger \hat{t}) . \]  

Here \( \hat{t} \) denotes the transmission matrix associated with the \( M \leq N \) propagating channels of the \( N \)-channel system, at the considered Fermi energy,

\[ \hat{t} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1M} \\ t_{21} & \cdots & \cdots & \cdots \\ \vdots & \cdots & \cdots & \cdots \\ t_{M1} & t_{M2} & \cdots & t_{MM} \end{pmatrix} , \]  

where \( t_{ij} \) is the amplitude transmitted in a propagating channel \( i \) at the Fermi level at one end of the wire when there is an incident amplitude in the (propagating) channel \( j \) at the other end. As in I the localization length is determined from the rate of exponential decay of the conductance\(^4,9\), namely

\[ \frac{1}{L_c} = - \lim_{N_L \to \infty} \frac{1}{N_L} \langle \ln g \rangle , \]  

where averaging over disorder may be used, as usual, because of the self-averaging property of \( \ln g \).

The reason for restricting to the propagating channels in the double sum \( \text{Tr}(\hat{t}^\dagger \hat{t}) \) in [14] is that evanescent modes at the Fermi level do not contribute to the current. However, since the conductance of the wire is defined as the current which flows through it divided by the electro-chemical potential difference between the reservoirs connected to the disordered region by ideal leads, the evanescent modes may play an indirect role through their effect on the electrochemical potentials\(^9,10\). Such effects have been discussed by Bagwell\(^5\) for electrons scattered from a single defect in an ideal quasi-1D wire. Here we consider similar effects for scattering from coupled chains of random atomic sites.

### III. SCATTERING MATRIX ANALYSIS OF TRANSPORT

The analysis of transmission and reflection by the disordered region in I focused on the special case where all the scattering channels at the Fermi energy are assumed to be propagating. This situation is encountered typically when the interchain hopping parameter is less than half the width of the band of propagating states in the various channels i.e. \( |h| < 2 \). In the opposite case, \( |h| > 2 \), both evanescent and propagating channels will generally exist at the Fermi level.

At the level of the general formalism only minor modifications of the analysis of I are required in order to incorporate the case where evanescent scattering channels may exist. For discussing this case we shall thus closely follow the treatment of I and refer to the latter for most of the details. The first step of the analysis involves defining transfer matrices for the disordered region of the quasi-1D systems described by (8.b-10.b) of I. These transfer matrices are used in a second step for obtaining scattering matrices giving transmission- and reflection amplitudes which allow us to study the conductance and related properties.
A. Transfer matrices

Transfer matrices $\tilde{Y}_n$ for thin slices enclosing only a single site $n$ per channel of the systems described by (8.b-10.b) of I are defined by rewriting these equations in the form

$$
\begin{pmatrix}
\psi^1_{n+1} \\
\psi^1_n \\
\psi^2_{n+1} \\
\psi^2_n \\
\vdots \\
\vdots 
\end{pmatrix} = \tilde{Y}_n
\begin{pmatrix}
\psi^1_{n} \\
\psi^1_{n-1} \\
\psi^2_{n} \\
\psi^2_{n-1} \\
\vdots \\
\vdots 
\end{pmatrix} .
$$  

(17)

The matrix $\tilde{Y}_n \equiv \tilde{X}_{0n}$ of dimension 4 for the case $N = 2$ and the matrices $\tilde{Y}_n \equiv \tilde{X}_n'$, $\tilde{X}_n''$ of dimension 6 for the cases $N = 3$ with free boundary conditions, respectively, are given explicitly in equations (14-16) of I.

The next step consists in transforming the $\tilde{Y}_n$ matrices in bases constituted in general by Bloch wave solutions (equation (8)) of (4-6) for some channels and evanescent wave solutions (13) for the remaining ones at the Fermi level.

Finally, the transfer matrices for the disordered wires of lengths $L = N_L a$ are products of transfer matrices in the mixed Bloch wave-evanescent wave bases associated with the $N_L$ individual thin slices,

$$
\tilde{Y}_L = \prod_{n=1}^{N_L} \tilde{Y}_n .
$$  

(19)

As in I, the atomic site energies in (18) are assumed to be independent gaussian random variables with zero mean and correlation

$$
\langle \varepsilon_{in} \varepsilon_{jm} \rangle = \varepsilon_0^2 \delta_{ij} \delta_{mn} .
$$  

(20)

For weak disorder it is thus sufficient to explicitate (14) to linear order in the site-energies in order to study averages to lowest order in the correlation (20). The latter implies indeed that different slices in (14) are uncorrelated. Under some notational provision the final transfer matrices are given by (30) ($N = 2$) and (32) ($N = 3$) of I for the real or imaginary wavenumbers $k_1, k_2, k_3$ in (9-11) at an arbitrarily chosen Fermi energy.

B. Scattering matrices

The scattering of plane waves (reflection and transmission) at and between the two ends of the random quasi-1D systems is governed by the $S$-matrix,

$$
\tilde{S} = \begin{pmatrix}
\hat{r}^- & \hat{t}^-
\hat{t}^+ & \hat{r}^+
\end{pmatrix} .
$$  

(21)
where

\[ \hat{f}^{±±} = \begin{pmatrix} t_{11}^{±±} & t_{12}^{±±} & \cdots \\ t_{21}^{±±} & t_{22}^{±±} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} , \]  

(22)

and

\[ \hat{r}^{±±} = \begin{pmatrix} r_{11}^{±±} & r_{12}^{±±} & \cdots \\ r_{21}^{±±} & r_{22}^{±±} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} . \]  

(23)

Here \( t_{ij}^{±±}(t_{ij}^{−−}) \) and \( r_{ij}^{−−}(r_{ij}^{±±}) \) denote the transmitted and reflected amplitudes in channel \( i \) (which may be either propagating or evanescent) when there is a unit flux incident from the left (right) in a current carrying channel \( j \). Left to right- and right to left directions are labelled + and -, respectively. The \( S \)-matrix expresses outgoing wave amplitudes in terms of ingoing ones on either side of the quasi-1D disordered wire via the scattering relations

\[ \begin{pmatrix} 0 \\ 0' \end{pmatrix} = S \begin{pmatrix} I \\ I' \end{pmatrix} . \]  

(24)

Here \( I \) and \( I' \) (0 and 0') denote ingoing (outgoing) amplitudes at the left and right sides of the disordered region, respectively. It follows from current conservation that e.g. for a unit flux which is incident from the right in channel \( i \) one has

\[ \sum_{j=1}^{M} (|t_{ji}^{−−}|^2 + |r_{ji}^{−−}|^2) = 1 , \]  

(25)

where the summation is restricted to the \( M \leq N \) current carrying channels at the Fermi level. Likewise, one also has for propagating channels \( j \),

\[ \sum_{j=1}^{M} (|t_{ji}^{±±}|^2 + |r_{ji}^{±±}|^2) = 1 \]  

(26)

Note that in the case where channels are propagating the current conservation implies that the scattering matrix \([21]\) is unitary (\( SS^+ = I \)). This is no longer true, of course, in the presence of evanescent channels. As was shown in I the components of the out- and ingoing waves column vectors in \([21]\) are given by the quantities \( a_{1,0}, a_{2,0}, \ldots, a_{N,0}, a_{1,N_L}, a_{2,N_L}, \ldots, a_{N,N_L} \) and \( a_{1,0}^+, a_{2,0}^+, \ldots, a_{N,0}^+, a_{1,N_L}, a_{2,N_L}, \ldots, a_{N,N_L} \), respectively, defined by components of wave transfer column vectors

\[ \{a_{1,n-1}^+, a_{1,n-1}^+, a_{2,n-1}^+, \ldots\} \equiv \hat{W}^{-1}\{\psi_{n-1}^1, \psi_{n-1}^1, \psi_{n-1}^1, \ldots\} \]  

(27)

and obey the transfer matrix equation for the wires,

\[ \{a_{1,N_L}^+, a_{1,N_L}^+, a_{2,N_L}^+, \ldots\} = \hat{Y}_L\{a_{1,0}, a_{1,0}, a_{2,0}, \ldots\} \]  

(28)

Here the in- and outgoing waves include, in particular, evanescent waves evolving in directions towards- or away from the disordered wires. The components of the out- and ingoing waves column vectors in \([21]\) are thus \( a_{1,0}, a_{2,0}, \ldots, a_{N,0}, a_{1,L}, a_{2,L}, \ldots, a_{N,L} \) and \( a_{1,0}^+, a_{2,0}^+, \ldots, a_{N,0}^+, a_{1,L}, a_{2,L}, \ldots, a_{N,L} \), respectively. With the so defined vectors of outgoing and incoming amplitudes, the \( S \)-matrix is obtained by rearranging the equation \([25]\) so as to bring them in the form \([24]\). The details of this somewhat lengthy calculation are explicated in I. The explicit forms of the scattering matrices, Eqs (46-47) and (48, 48.a-48.f) of I, for \( N = 2 \) and \( N = 3 \), respectively, are expressed in terms of transfer matrix elements which are themselves defined in terms of general parameters given by (22.a), (24) and (25) of I with wavenumbers, real or pure imaginary, defined by \([9, 11]\) above. These \( S \)-matrices readily yield the transmission and reflection submatrices in \([21]\).
IV. RESULTS

In the appendix of I we obtained the explicit expressions of transmission- and reflection coefficients, \(|t_{ij}^-|^2\) and \(|r_{ij}^+|^2\), in terms of tight-binding parameters defining the transfer matrices of the disordered wires for cases where all channels are propagating at the Fermi level. In the appendix of the present paper we discuss the analogous expressions in more general cases where, some of the channels are conducting while the others are evanescent at the Fermi level. This requires explicating the transmission- and reflection amplitude coefficients \(|t_{ij}^-|^2\) and \(|r_{ij}^+|^2\) for arbitrary wavenumbers, real or imaginary, for which the tight-binding parameters in (22.a) and (24-25) of I may thus be complex. In particular, the coefficients \(|t_{ij}^-|^2\) and \(|r_{ij}^+|^2\) for conducting channels, which are relevant for studying the conductance and related transport properties, are influenced by the evanescent channels via the disorder mediated coupling between the two types of channels.

A. Two-channel wires

The two bands of propagating states in (9) for real \(k_1, k_2\) are non-overlapping for \(|h| > 2\). In this case, to a propagating state in channel 1 corresponds an evanescent state at the same energy in channel 2 and vice versa. For the two-channel system the transmission- and reflection coefficients \(|t_{ij}^-|^2\) and \(|r_{ij}^+|^2\) are given explicitly in (A.1-A.3) in the appendix, for the case where channel 1 is propagating and channel 2 is evanescent. By expanding these expressions to second order in the tight-binding parameters (A.3) and averaging over the disorder, using (20), we obtain

\[
\langle |t_{11}^-|^2 \rangle = 1 - \frac{N_L \varepsilon_0^2}{8 \sin^2 k_1} ,
\]

(29)

\[
\langle |r_{12}^+|^2 \rangle = \frac{N_L \varepsilon_0^2}{8 \sin^2 k_1} ,
\]

(30)

which obey the current conservation relation (25) for the conducting channel. From (29), (14) and (16) it then follows that the weak disorder localization length is

\[
\frac{1}{L_c} = \frac{\varepsilon_0^2}{16 \sin^2 k_1} .
\]

(31)

The above results will be discussed further in Sec. V.

B. Three-channel wires

Before discussing coefficients of reflection and transmission of plane waves in various propagating channels when evanescent modes are present, we first identify the various cases where both types of modes exist in the leads at the Fermi level, for free-boundary- and periodic systems, respectively.

Free boundary conditions: if the subbands of propagating states in (10) for real \(k_1, k_2, k_3\) do not overlap, that is if

\[
|h| > 2\sqrt{2} ,
\]

(32)

to an incident electron in a propagating state of energy \(E\) in a particular channel correspond evanescent states at the same energy in the other two channels. On the other hand, if the two outer bands 1 and 3 in (10) do not overlap while the inner band overlaps with the outer bands i.e. for

\[
\sqrt{2} < |h| < 2\sqrt{2} ,
\]

(33)

there will be an evanescent mode present at the Fermi level for energies \(E\) lying within the overlap regions of the inner and outer bands.
\textit{Periodic boundary conditions:} If the propagating band of channel 1 and the two degenerate bands of channels 2 and 3 in \cite{11} do not overlap i.e. for
\begin{equation}
    h > \frac{4}{3} ,
\end{equation}
then the waves at the Fermi level in channels 2 and 3 which correspond to an incident wave in the propagating band 1 are evanescent. Conversely, for incident wave energies lying within the degenerate bands 2 and 3 the modes of the same energies in channel 1 will be evanescent. The same is true for $|h| < \frac{4}{3}$ for incident waves of Fermi energy within the non-overlapping parts of the band of channel 1 and the bands of channels 2, 3, respectively.

In the following we will discuss successively the cases where one or two evanescent modes are present at the Fermi level for the two types of three-channel systems. For convenience the expression of transmission and reflection coefficients in terms of transfer matrix elements of the disordered regions given in (32) of I (under the proviso of \cite{11}) is discussed in the appendix and displayed in detail for the intrachannel transmission coefficients ($|t_{jj}^-|^2$, $j = 1, 2, 3$ in \[A.10\]-\[A.12\]).

1. Free boundary condition

a. Two evanescent modes
Because of the symmetric arrangement of the propagating bands about $E = 0$ we distinguish the cases where the Fermi level lies in the central $k_2$-band or within one of the outer bands.

For energies within the $k_2$-band, we obtain from (A.11),
\begin{equation}
    \langle |t_{22}^-|^2 \rangle = 1 - \frac{N_L \varepsilon_0^2}{8 \sin^2 k_2} ,
\end{equation}
and from (48.a), (48.e) and (32) of I,
\begin{equation}
    \langle |r_{22}^+|^2 \rangle = \frac{N_L \varepsilon_0^2}{8 \sin^2 k_2} ,
\end{equation}
using (20) and (A.8). It then follows from (16) that
\begin{equation}
    \frac{1}{L_c} = \frac{\varepsilon_0^2}{16 \sin^2 k_2} .
\end{equation}
The current conservation property \[25\] for propagating channels is clearly obeyed.

On the other hand, for Fermi energies in the $k_1$-band we get
\begin{equation}
    \langle |t_{11}^-|^2 \rangle = 1 - \frac{3 N_L \varepsilon_0^2}{32 \sin^2 k_1} ,
\end{equation}
using (20) and (A.8), and from (48.a), (48.e) and (23) of I,
\begin{equation}
    \langle |r_{11}^+|^2 \rangle = \frac{3 N_L \varepsilon_0^2}{32 \sin^2 k_1} ,
\end{equation}
which shows that \[25\] is obeyed. In this case we obtain for the localization length
\begin{equation}
    \frac{1}{L_c} = \frac{3 \varepsilon_0^2}{64 \sin^2 k_1} .
\end{equation}

b. One evanescent mode
As a typical case we choose the Fermi level within the overlap region of the \( k_1 \) and \( k_2 \)-bands, so that the matching mode in channel 3 is evanescent i.e. \( k_3 = i\kappa_3 \). In this case we obtain successively from (A.10) and (A.11), using (2.0) and (A.8),

\[
\langle |t_{11}^-|^2 \rangle = 1 - \frac{N_L\varepsilon_0^2}{32\sin k_1} \left( \frac{3}{\sin k_1} + \frac{4}{\sin k_2} \right), \quad (41)
\]

\[
\langle |t_{22}^-|^2 \rangle = 1 - \frac{N_L\varepsilon_0^2}{8\sin k_2} \left( \frac{1}{\sin k_2} + \frac{1}{\sin k_1} \right), \quad (42)
\]

and from (48.a-48.b) and (48.e-48.f) and (32) of I,

\[
\langle |t_{12}^-|^2 \rangle = \langle |t_{21}^-|^2 \rangle = \frac{N_L\varepsilon_0^2}{16\sin k_1\sin k_2}, \quad (43)
\]

and

\[
\langle |r_{11}^-|^2 \rangle = \langle |r_{22}^-|^2 \rangle = \frac{3N_L\varepsilon_0^2}{32\sin^2 k_1}, \quad (44)
\]

\[
\langle |r_{22}^-|^2 \rangle = \frac{N_L\varepsilon_0^2}{8\sin^2 k_2}, \quad (45)
\]

\[
\langle |r_{12}^-|^2 \rangle = \langle |r_{21}^-|^2 \rangle = \frac{N_L\varepsilon_0^2}{16\sin k_1\sin k_2}. \quad (46)
\]

An important check on the correctness of equations (41-46) is to note that, again, they verify the fundamental sum rules for the currents. The localization length for weak disorder obtained from the above results is

\[
\frac{1}{L_c} = \varepsilon_0^2 \left( \frac{3}{4\sin^2 k_1} + \frac{1}{\sin^2 k_2} + \frac{1}{\sin k_1\sin k_2} \right). \quad (47)
\]

2. Periodic boundary conditions

For periodic boundary conditions, the current conservation properties (25.a) can be obeyed only if the disorder is restricted to identical realizations in the chains 1 and 2 i.e.

\[
\varepsilon_{1n} = \varepsilon_{2n}, \quad n = 1, 2 \ldots N_L, \quad (48)
\]

while the random site energies on chain 3 remain independent. This was shown in detail in I, in the case where all three channels are conducting but remains true as well when evanescent channels are present, as expected.

For energies in the propagating band of channel 1 in cases where there is no overlap with the degenerate propagating bands 2 and 3 (or for energies in the non-overlapping part of band 1 when this band partly overlaps the bands 2 and 3) one has evanescent states at the Fermi level in channels 2 and 3. In this case we obtain, using (20) and (A.9), in (A.10),

\[
\langle |t_{11}^-|^2 \rangle = 1 - \frac{5N_L\varepsilon_0^2}{36\sin^2 k_1}, \quad (49)
\]

and in (48.a), (48.e) and (32) of I,

\[
\langle |r_{11}^-|^2 \rangle = \frac{5N_L\varepsilon_0^2}{36\sin^2 k_1}. \quad (50)
\]
By inserting (49) in (16) and expanding to linear order in the correlation $\varepsilon_0^2$ we find for the localization length

$$\frac{1}{L_c} = \frac{5\varepsilon_0^2}{72 \sin^2 k_1} .$$  \hspace{1cm} (51)

On the other hand, for Fermi energies such that the degenerate channels 2 and 3 are conducting and channel 1 is evanescent we find, from (A.11-A.12),

$$\langle |t_{22}^-|^2 \rangle = \langle |t_{33}^-|^2 \rangle = 1 - \frac{N_L\varepsilon_0^2}{4 \sin^2 k_2} ,$$ \hspace{1cm} (52)

and from (48.c), (48.e) and (32) of I,

$$\langle |t_{23}^-|^2 \rangle = \langle |t_{32}^-|^2 \rangle = \frac{N_L\varepsilon_0^2}{18 \sin^2 k_2} ,$$ \hspace{1cm} (53)

using (20) and (A.9). Furthermore, from (48.a), (48.e) and (32) of I together with (20) and (A.9) we get,

$$\langle |r_{22}^-|^2 \rangle = \langle |r_{33}^-|^2 \rangle = \frac{5N_L\varepsilon_0^2}{36 \sin^2 k_2} ,$$ \hspace{1cm} (54)

and

$$\langle |r_{23}^-|^2 \rangle = \langle |r_{32}^-|^2 \rangle = \frac{N_L\varepsilon_0^2}{18 \sin^2 k_2} .$$ \hspace{1cm} (55)

The current conservation relation (25-25.a) are obeyed again and by using the above results in (16) we find

$$\frac{1}{L_c} = \frac{7\varepsilon_0^2}{72 \sin^2 k_2} .$$ \hspace{1cm} (56)

V. DISCUSSION AND CONCLUDING REMARKS

A simple interpretation of the results of Sect. IV for reflection- and transmission coefficients in two- and three channel systems in the presence of evanescent channels results from the comparison with the results of I for the case where all channels are propagating at the Fermi level. Thus we find that equations (29) and (30) for the two-channel systems ($N=2$) where the channel 2 is evanescent, follow exactly from equation (52) and (55) of I by suppressing the effect of the channel 2 (or, equivalently, assuming this channel to be absent). Similarly, we find that equations (35) and (36) for the $N=3$ case with free boundary conditions follow from (62) and (68) of I by ignoring the effects of channels 1 and 3, which are now assumed to be evanescent. In the same way (38) and (39) follow from (61) and (67) of I by suppressing the effects of channels 2 and 3. Finally, in the case of a single evanescent channel 3 the equations (41-46) for the free boundary model follow from (61-63), (67-68) and (70) of I, respectively, by ignoring the effect of the channel 3. Exactly similar conclusions are obtained by comparing the results of Sec. IV for the $N=3$ case with periodic boundary conditions with the corresponding results of Sec. IV of I.

As we now discuss these simple properties of the reflection and transmission coefficients in Sec. IV are partly the consequence of destructive quantum interference, for sufficiently weak disorder of the terms describing the disorder-mediated coupling between propagating- and evanescent modes at the Fermi level in the corresponding amplitudes of transmission and reflection. This is illustrated e.g. in the equations (A.10-A.12) for the random intrachannel transmission coefficients $|t_{jj}^-|^2$: the final double sum in each one of these expressions involves couplings between the propagating channel $j$ and at least one evanescent channel assumed to be present at the Fermi Level. Now, since the terms $m = n$ (the only once which survive in the averaging with (20)) in the sums over coupling terms of propagating and evanescent modes are pure imaginary (see parameters in (A.8) and (A.9) for $k_j$ real and $k_i$ imaginary), they add up to zero in the intensity coefficients (A.10-A.12). In other words, these coupling terms, while existing in the transmission amplitudes (as they arise from corresponding terms in (A.7)) interfere destructively as a result of their special phases.
On the other hand, it is clear that the absence of coupling effects between propagating- and evanescent modes in the results of Sec. IV is specific to our weak disorder approximation. Indeed corresponding perturbative coupling terms at higher orders in the parameters (A.8) and (A.9) in the transmission- and reflection amplitudes would have different phases, yielding non-vanishing effects in the intensity coefficients. Similar higher order coupling effects (of fourth order in the random potential) between a single propagating mode and evanescent modes for a narrow strip-shaped pure wire with a random boundary ("surface") potential have recently been discussed by Makarov and Tarasov.

The localization lengths when evanescent modes are included, namely equation (31) in the two-chain wire, equations (57), (61) and (67) for the three chain wire with free boundary conditions and, finally, equations (51) and (56) for the three chain case with periodic boundary conditions differ qualitatively from the results of I (equations (58), (73) and (86) in I, respectively) for the usually considered case where evanescent states are absent at the Fermi level. The numerical coefficients in the above expressions indicate that localization lengths in the presence of evanescent states are generally enhanced by factors which vary typically between 1 and 2, with respect to corresponding values in I when evanescent states are absent. Strongly enhanced localization lengths in the presence of evanescent modes have also been observed in the numerical calculations of Cahay et al. for a different model.

The results in Sec. IV for averaged reflection coefficients in the presence of evanescent modes may be used for deriving mean free paths for elastic scattering of an electron, using the formula

\[
\ell_e = \frac{L_c}{2} = \frac{1}{MN_c} \sum_{i,j=1}^{M} |t_{ij}|^2 ,
\]

where \( M \) is the number of propagating channels in the quasi-1D conductor. By inserting the averaged reflection coefficients for the various quasi-1D systems discussed in Sec. IV, we find in all cases that

\[
\langle |t_{ij}|^2 \rangle = \langle |t_{ij}^-|^2 \rangle = \frac{1}{2},
\]

where \( L_c \) stands for the corresponding localization lengths for weak disorder given by (51), (57), (61), (67), (58) and (52), respectively. These \( L_c \)-values determine \( \ell_e \) in the Born approximation for scattering by the disorder. We note that for systems with a single propagating channel (58) coincides formally with the relation between the localization length and the mean free path in a one-dimensional chain obtained by Thouless. On the other hand, for systems with two propagating channels (54) is analogous to the general formula \( L_c = M\ell_e \), of Thouless for a quasi-1D system with \( M \) propagating channels. The equation (52) shows that, for weak disorder, the enhancement of the elastic mean free path due to evanescent states at the Fermi level is proportional to the corresponding enhancement of the localization length.

The results of Sec. IV also allow us to express the conditions for the validity of the weak disorder scattering analysis of transport in quasi-1D systems in a precise physical form. Clearly our treatment is valid for intrachannel transmission coefficients close to unity (near-transparency) and sufficiently low reflection- and interchannel transmission coefficients. Roughly speaking this requires \( L = N_L a \leq L_c \) (a condition which can be made more precise for the various systems and Fermi energy domains by combining the results for the \( \langle |t_{ij}^-|^2 \rangle \) in Sec. IV and in I with the corresponding expressions for \( L_c \)), which corresponds to the weak localization or (quasi) metallic regime. As is well-known, this regime allows one to find the correct expression for the localization length for weak disorder (\( \varepsilon_0^2 << 1 \)). This has been demonstrated, in particular, for the Anderson model for a one-dimensional chain where the localization length has been calculated analytically both in the weak- and in the strong localization regime (where all states are localized on the scale of \( L \) i.e. \( L > L_c \)) for weak disorder: the expressions obtained for both cases are identical and coincide with the familiar Thouless formula, Eq. (51) of I.

In the present paper and in I, we have determined averaged conductances which allowed us to find localization lengths. It would be interesting to generalize these analyses to study conductance fluctuations and the ubiquitous universal conductance fluctuations in the considered coupled Anderson chain systems. In particular, since conductance fluctuations involve quartic terms in the tight-binding parameters (A.8, A.9) they would clearly be influenced by the disorder mediated coupling between evanescent- and propagating modes mentioned earlier in this section. On the other hand, it would be interesting to study localization in many-chain tight-binding models of quasi-1D wires. The exact analytical results obtained in this paper and in I for two- and three chain systems would be useful limiting cases for future treatments of localization in many-channel tight-binding wires.

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APPENDIX: CALCULATION OF TRANSMISSION AND REFLECTION COEFFICIENTS

As in I we confine ourselves to discussing transmission- and reflection amplitude coefficients \( t_{ij}^- \) and \( r_{ij}^+ \) associated with outgoing waves at the left of the disordered region.

1. Two-channel wires

In the case where \( k_1 \) is real and \( k_2 = i\kappa_2 \) is pure imaginary we are interested in finding

\[
| t_{11}^- |^2 = \frac{|X_{44}|^2}{|\delta|^2} , \quad | r_{11}^+ |^2 = \frac{|X_{21}|^2}{|\delta|^2} , \tag{A.1}
\]

where \( X_{ij} \) denotes the matrix elements of the transfer matrix of the disorder region defined by (30) in I and \( \delta = X_{22}X_{44} - X_{24}X_{42} \). The generalization of (A.3) of I yields

\[
|\delta|^2 e^{-2\kappa_2 N_L} = 1 + 2\text{Im} \sum_{m=1}^{N_L} a_{2m} + \sum_{m,n=1}^{N_L} \left[ a_{1m}a_{1n} + a_{2m}a_{2n}^* + (b_m\hat{b}_n e^{-(ik_1 + i\kappa_2)(m-n)} + \text{c.c.}) \right]
\]

\[
|X_{44}|^2 e^{-2\kappa_2 N_L} = 1 + \sum_{m,n} a_{2m}a_{2n}^* \tag{A.2}
\]

\[
|X_{21}|^2 = \sum_{m,n} a_{1m}a_{1n} \cos 2(m-n)k_1
\]

where

\[
a_{1n} = \frac{\varepsilon_{1n} + \varepsilon_{2n}}{4\sin k_1} , \quad a_{2n} = \frac{\varepsilon_{1n} + \varepsilon_{2n}}{4\sinh \kappa_2} , \tag{A.3}
\]

\[
b_n = \frac{\varepsilon_{2n} - \varepsilon_{1n}}{4\sqrt{i}\sin k_1 \sinh \kappa_2} .
\]

2. Three-channel wires

The transmission- and reflection amplitudes of interest defined by the matrices \( \hat{S}_1 \) and \( \hat{S}_3 \) in (48.a) and (48.e) of I yield

\[
| t_{ij}^- |^2 = \frac{|\alpha_{ij}|^2}{\Delta^2} , \tag{A.4}
\]

where the \( \alpha_{ij} \) are given by quantities \( \beta_k \) defined in I: \( \alpha_{11} = \beta_1, \alpha_{12} = \beta_4, \alpha_{13} = \beta_7, \alpha_{21} = \beta_2, \alpha_{22} = \beta_5, \alpha_{23} = \beta_6, \alpha_{31} = \beta_3, \alpha_{32} = \beta_6, \alpha_{33} = \beta_9 \).

To second order in the site energies the expression (A.4) reduce to

\[
| r_{11}^+ |^2 = e^{ik_1 N_L} | Y_{21} |^2 , \quad | r_{12}^- |^2 = e^{ik_1 N_L} | Y_{23} |^2 , \quad | r_{13}^- |^2 = e^{ik_1 N_L} | Y_{25} |^2 ,
\]

\[
| r_{21}^+ |^2 = e^{ik_2 N_L} | Y_{41} |^2 , \quad | r_{22}^- |^2 = e^{ik_2 N_L} | Y_{43} |^2 , \quad | r_{23}^- |^2 = e^{ik_2 N_L} | Y_{45} |^2 ,
\]

\[
| r_{31}^- |^2 = e^{ik_3 N_L} | Y_{61} |^2 , \quad | r_{32}^- |^2 = e^{ik_3 N_L} | Y_{63} |^2 , \quad | r_{33}^- |^2 = e^{ik_3 N_L} | Y_{65} |^2 . \tag{A.5}
\]

Here the \( Y_{ij} \) are matrix elements of the transfer matrix (32) in I of the disordered region and the \( \beta_i \)'s and \( \Delta \) are quadratic and cubic forms in these elements defined in (48.e) and (48.f) of I. The exponential coefficients in (A.5) differ from unity for imaginary wavenumbers.

In the three-channel case we have a variety of different domains of Fermi energies where besides one propagating channel at least, there exist different evanescent channels, both for free boundary conditions and for periodic boundary conditions. Therefore we wish to explicitate the expressions (A.3) and (A.4) to second order in forms valid for arbitrary
wavenumbers $k_1, k_2, k_3$, real or pure imaginary, which implies that the tight-binding parameters involved in the transfer matrix (32) of I will be generally complex too.

On the other hand, for weak disorder, we only require the explicit forms of the quantities in (A.4-A.5) to second order in the site energies or, equivalently, to second order in the tight-binding parameters (24-25) of I. These explicit forms follow trivially for the reflection coefficients in (A.5) and for the interchannel transmission coefficients in (A.4) since the elements of the reflection matrix $i^{--}$ given by (48.a) of I as well as the off-diagonal elements of the transmission matrix $i^{--}$ given by (48.c) are proportional, to lowest order, to off-diagonal elements of the transfer matrix (32) in I, which are linear in the site energies. Also, as indicated earlier the form (32) of I of this matrix remains valid if, even in the considered case of complex wavenumbers and corresponding complex tight-binding parameters in (24), the symbols $O^*$, with $O \equiv s_j, u_j, v_{ij}, \omega_{ij}$, are taken to mean replacement of the exponent coefficients $i = \sqrt{-1}$ in (31) of I by $-i$. For these reasons we refrain from further explicitating the reflection coefficients and the interchannel transmission coefficients in (A.4-A.5).

We now turn to the discussion of the explicit forms of the intrachannel transmission coefficients, $|\tilde{t}_{jj}^{--}|^2, j = 1, 2, 3$, which play an important role and whose evaluation to second order requires more effort. Using the definition (48.e) and the explicit expressions of transfer matrix elements in (32) of I we obtain successively

\[
\begin{align*}
\beta_1 &= -e^{-i(k_1+k_3)N_L} \left[ 1 - i \sum_m b_{2m}(1 - i \sum_n a_{3n}) + \sum_{m,n} d_m q_n e^{i(m-n)(k_3-k_2)} \right], \\
\beta_2 &= e^{-i(k_1+k_3)N_L} \left[ 1 - i \sum_m a_{1m}(1 - i \sum_n a_{3n}) + \sum_{m,n} p_m q_n e^{i(m-n)(k_1-k_3)} \right], \\
\beta_3 &= -e^{-i(k_1+k_3)N_L} \left[ 1 - i \sum_m a_{1m}(1 - i \sum_m b_{2m}) + \sum_{m,n} c_m f_n e^{i(m-n)(k_2-k_1)} \right].
\end{align*}
\]  

Similarly we evaluate the cubic form $\Delta$ defined in (48.f) and (45) of I to second order in the tight-binding parameters in (32) of I. This yields

\[
\Delta = -e^{-i(k_1+k_2+k_3)N_L} \left[ 1 - i \sum_m (a_{1m} + b_{2m} + a_{3m}) - \sum_{m,n} \left( a_{1m} b_{2n} + a_{3m} b_{2n} + a_{1m} a_{3n} + g_m p_n e^{i(m-n)(k_3-k_1)} + d_m q_n e^{i(m-n)(k_3-k_2)} + c_m f_n e^{i(m-n)(k_2-k_1)} \right) \right].
\]  

The atomic tight-binding parameters given in (24-25) of I are:

\[
\begin{align*}
a_{1n} &= \frac{\varepsilon_{1n} + 2\varepsilon_{2n} + \varepsilon_{3n}}{8 \sin k_1}, \quad a_{3n} = \frac{\varepsilon_{1n} + 2\varepsilon_{2n} + \varepsilon_{3n}}{8 \sin k_3}, \\
b_{2n} &= \frac{\varepsilon_{1n} + \varepsilon_{3n}}{4 \sin k_2}, \quad c_n = f_n = \frac{\sqrt{2}(\varepsilon_{1n} - \varepsilon_{3n})}{8 \sqrt{\sin k_1 \sin k_2}}, \\
d_n = q_n &= \frac{\sqrt{2}(\varepsilon_{1n} - \varepsilon_{3n})}{8 \sqrt{\sin k_1 \sin k_3}}, \quad g_n = p_n = \frac{\varepsilon_{1n} - 2\varepsilon_{2n} + \varepsilon_{3n}}{8 \sqrt{\sin k_1 \sin k_3}},
\end{align*}
\]  

for free boundary conditions, with the wavenumbers, real or imaginary, defined by (10) and

\[
\begin{align*}
a_{1n} &= \frac{\varepsilon_{1n} + 2\varepsilon_{2n} + \varepsilon_{3n}}{6 \sin k_1}, \quad a_{3n} = \frac{\varepsilon_{2n} + \varepsilon_{3n}}{6 \sin k_2}, \\
b_{2n} &= \frac{2\varepsilon_{1n} + \varepsilon_{3n}}{6 \sin k_2}, \quad c_n = \frac{\varepsilon_{1n} - \varepsilon_{3n}}{6 \sqrt{\sin k_1 \sin k_2}}, \quad g_n = \frac{\varepsilon_{2n} - \varepsilon_{3n}}{6 \sqrt{\sin k_1 \sin k_2}}, \\
d_n = q_n &= \frac{\varepsilon_{3n} - \varepsilon_{2n}}{6 \sin k_2}, \quad f_n = \frac{2\varepsilon_{1n} - \varepsilon_{2n} - \varepsilon_{3n}}{6 \sqrt{\sin k_1 \sin k_2}}, \\
p_n &= -\frac{-\varepsilon_{1n} + 2\varepsilon_{2n} - \varepsilon_{3n}}{6 \sqrt{\sin k_1 \sin k_2}}, \quad q_n = \frac{\varepsilon_{3n} - \varepsilon_{1n}}{6 \sin k_2}.
\end{align*}
\]
for periodic boundary conditions where the wavenumbers are given by (11). Finally we insert (A.6) and (A.7) in (A.4) for the intrachannel transmission coefficients $|t_{jj}|^2$ and evaluate the resulting expressions to second order in the site energies. This yields the relatively simple final expressions

$$
|t_{11}^-|^2|e^{-ik_1N_L}|^2 = 1 - 2 \text{ Im} \sum_m a_{1m} + \sum_{m,n} [a_{1m}a_{1n}^* - 2 \text{ Re} (a_{1m}a_{1n})]
$$

$$
- \sum_{m,n} \left[ (g_m p_n e^{i(m-n)(k_3-k_1)} + c.m f_n e^{i(m-n)(k_2-k_1)}) + c.c. \right], \quad (A.10)
$$

$$
|t_{22}^-|^2|e^{-ik_2N_L}|^2 = 1 - 2 \text{ Im} \sum_m b_{2m} + \sum_{m,n} [b_{2m}b_{2n}^* - 2 \text{ Re} (b_{2m}b_{2n})]
$$

$$
- \sum_{m,n} \left[ (d_m q_n e^{i(m-n)(k_3-k_2)} + c.m f_n e^{i(m-n)(k_2-k_1)}) + c.c. \right], \quad (A.11)
$$

$$
|t_{33}^-|^2|e^{-ik_3N_L}|^2 = 1 - 2 \text{ Im} \sum_m a_{3m} + \sum_{m,n} [a_{3m}a_{3n}^* - 2 \text{ Re} (a_{3m}a_{3n})]
$$

$$
- \sum_{m,n} \left[ (g_m p_n e^{i(m-n)(k_3-k_1)} + d_m q_n e^{i(m-n)(k_3-k_2)}) + c.c. \right], \quad (A.12)
$$

where summations over $m$ and $n$ run independently from $m = 1$ to $m = N_L$ and from $n = 1$ to $n = N_L$.

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