PARAMETRIC TENSION BETWEEN EVEN AND ODD MULTPOLE DATA OF THE WMAP POWER SPECTRUM: UNACCOUNTED CONTAMINATION OR MISSING PARAMETERS?

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ABSTRACT

There exists power contrast in even and odd multipoles of the WMAP power spectrum at low and intermediate multipole ranges. This anomaly is explicitly associated with the angular power spectrum, which is heavily used for cosmological model fitting. Having noted this, we have investigated whether even (odd) multipole data is individually consistent with the WMAP concordance model. Our investigation shows that the WMAP concordance model does not make a good fit for even (odd) multipole data set, which indicates parametric tension between even and odd multipole data set. Noting that tension is highest in primordial power spectrum parameters, we have additionally considered a running spectral index, but found that tension increases to even a higher level. We believe these parametric tensions may be indications of unaccounted contamination or imperfection of the model. 

Key words: cosmic background radiation – methods: data analysis

Online-only material: color figures

1. INTRODUCTION

For the past years, there have been great successes in the measurement of cosmic microwave background (CMB) anisotropy by ground and satellite observations (Runyan et al. 2003; Ade et al. 2008; Nolta et al. 2009; Hinders et al. 2009; Hinshaw et al. 2009; Dunkley et al. 2009; Pryke et al. 2009; Reichardt et al. 2009; Larson et al. 2010; Jarosik et al. 2010). By comparing the angular power spectrum of the CMB anisotropy with theoretical predictions, we may impose strong constraints on cosmological models (Liddle & Lyth 2000; Dodelson 2003; Mukhanov 2005; Weinberg 2008). In spite of remarkable goodness of fit (Komatsu et al. 2009, 2010), there are some features of the Wilkinson Microwave Anisotropy Probe (WMAP) data, which are not well explained by the WMAP concordance model (Chiang et al. 2003; de Oliveira-Costa et al. 2004; Copi et al. 2004, 2009, 2010; Eriksen et al. 2004a; Cruz et al. 2005; Land & Magueijo 2005; Kim & Naselsky 2009a, 2009b, 2010a, 2010b; Bennett et al. 2010). In particular, the power contrast anomaly between even and odd multipoles is explicitly associated with the angular power spectrum, which is mainly used to fit cosmological models (Land & Magueijo 2005; Kim & Naselsky 2010a, 2010b; Gruppuso et al. 2010; Bennett et al. 2010). Having noted this, we have investigated whether even (odd) multipole data set is consistent with the WMAP concordance model. Our investigation shows that there exists some level of tension, which may be an indication of unaccounted contamination or missing ingredients in the assumed parametric model such as the flat ΛCDM model.

2. EVEN (ODD) MULTPOLE DATA AND COSMOLOGICAL MODEL FITTING

We may consider CMB anisotropy as the sum of even and odd parity functions:

\[ T(\hat{n}) = T^+(\hat{n}) + T^-(\hat{n}), \]

where

\[ T^+(\hat{n}) = \frac{T(\hat{n}) + T(-\hat{n})}{2}, \]

\[ T^-(\hat{n}) = \frac{T(\hat{n}) - T(-\hat{n})}{2}. \]

Using the parity property of spherical harmonics \( Y_{lm}(\hat{n}) = (-1)^l Y_{lm}(-\hat{n}) \) (Arfken & Weber 2000), it is straightforward to show

\[ T^+(\hat{n}) = \sum_{l=even} \sum_{m} a_{lm} Y_{lm}(\hat{n}), \]

\[ T^-(\hat{n}) = \sum_{l=odd} \sum_{m} a_{lm} Y_{lm}(\hat{n}). \]

Obviously, the power spectra of even and odd multipoles are associated with \( T^+(\hat{n}) \) and \( T^-(\hat{n}) \), respectively. Given the ΛCDM model, we do not expect any distinct features between even and odd multipoles. However, power contrast between even and odd multipoles of the WMAP temperature (TT) power spectrum has been reported (Land & Magueijo 2005; Kim & Naselsky 2010a, 2010b; Gruppuso et al. 2010; Bennett et al. 2010). At lowest multipoles (\( 2 \leq l \leq 22 \)), there is odd multipole preference (i.e., power excess in odd multipoles and deficit in even multipoles; Land & Magueijo 2005; Kim & Naselsky 2010a, 2010b; Gruppuso et al. 2010), and even multipole preference at intermediate multipoles (\( 200 \leq l \leq 400 \); Bennett et al. 2010). Additionally, we have investigated temperature and E mode (TE) correlation, and noticed odd multipole preference at 100 \( \lesssim l \lesssim 200 \) and even multipole preference at 200 \( \lesssim l \lesssim 400 \), though its statistical significance is not high enough, due to low the signal-to-noise ratio of polarization data. Not surprisingly, these power contrast anomalies are explicitly associated with the angular power spectrum data, which are mainly used to fit cosmological models. Having noted this, we have investigated whether the even (odd) multipole data set is consistent with the concordance model.

For a cosmological model, we have considered ΛCDM + SZ effect + weak lensing, where cosmological parameters are \( \lambda \in \{ \Omega_0, \Omega_c, \tau, n_s, A_s, A_{ls}, H_0 \} \). For data constraints, we have used the WMAP 7 year TT and TE power spectrum data, which have been estimated from the internal linear combination map, and cut-sky V- and W-band maps (Larson et al. 2010). It should be noted that we have used the WMAP power spectrum via the WMAP team’s likelihood code.
In order to use only even (odd) multipole data, we have made slight modifications to the WMAP team’s likelihood code, where the Blackwell–Rao estimator and the MASTER pseudo-Cl estimator are modified, respectively, for low multipoles (\( l \leq 32 \)) and high multipoles (\( l > 32 \); Eriksen et al. 2004b; Wandelt et al. 2001; Hivon et al. 2002; Larson et al. 2010). Note that we have also re-derived the Fisher matrix in accordance with even (odd) multipole data subset. Additionally, the beam and point source corrections are modified accordingly. Hereafter, we shall denote WMAP CMB data of whole, even, and odd multipoles by \( D_0 \), \( D_2 \), and \( D_3 \), respectively. Note that even/odd multipole splittings are made for TT and TE power spectrum up to the multipoles of WMAP sensitivity (i.e., \( l \leq 1200 \) for TT and \( l \leq 800 \) for TE). Using \texttt{CosmoMC} with the modified WMAP likelihood code, we have explored the parameter space on a message passing interface cluster with six chains (Lewis & Bridle 2002, 2006; Eriksen et al. 2004b; Larson et al. 2010). For the convergence criterion, we have adopted the Gelman and Rubin’s “variance of chain means” and set the R-1 statistic to 0.03 for stopping criterion (Gelman & Rubin 1992; Brooks & Gelman 1998).

In Figure 1, we show the marginalized likelihood of parameters, which are obtained from the run of a \texttt{CosmoMC} with \( D_0 \), \( D_2 \), and \( D_3 \), respectively.

In Table 1, we show the best-fit parameters and 1σ confidence intervals, where \( \lambda_2 \) and \( \lambda_3 \) denote the best-fit values of \( D_2 \) and \( D_3 \), respectively. The parameter set \( \lambda_0 \) is the best-fit values of whole data \( D_0 \) and accordingly corresponds to the WMAP concordance model. As shown in Figure 1 and Table 1, we find non-negligible tension especially in the parameters of the primordial power spectrum. It is worth noting that the best-fit spectral index of even multipole data (i.e., \( D_2 \)) is close to a flat spectrum (i.e., \( n_s = 1 \)), while the result from the whole data rules out the flat spectrum by more than 2σ.

There is a likelihood-ratio test, which allows us to determine the rejection region of an alternative hypothesis, given a null hypothesis and an alternative hypothesis, we may investigate whether two sets of parameters are consistent with each other. Therefore, we have evaluated the following in order to assess parametric tension:

\[
L(\lambda_1 | D_1) / L(\lambda_1 | D_0)
\]

where parameter sets \( \lambda_1 \) and \( \lambda_2 \) correspond to a null hypothesis and an alternative hypothesis, respectively.

In Table 2, we show the likelihood ratio, where the quantities used for the numerator and denominator are indicated in the uppermost row and leftmost column, respectively. As shown by \( L(\lambda_0 | D_2) / L(\lambda_2 | D_2) \) and \( L(\lambda_0 | D_3) / L(\lambda_3 | D_3) \), the WMAP concordance model (i.e., \( \lambda_0 \)) does not make a good fit for even (odd) multipole data set. Besides, there exists a significant tension between two data subsets, as indicated by very small values of \( L(\lambda_2 | D_2) / L(\lambda_2 | D_3) \) and \( L(\lambda_2 | D_2) / L(\lambda_2 | D_3) \). The parameter likelihood, except for \( A_{sz} \), follows the shape of Gaussian functions, as shown in Figure 1. For a likelihood of Gaussian shape, the likelihood ratios 0.1353 and 0.0111 correspond to 2σ and 3σ significance levels, respectively. From Table 2, we may see that most of the ratios indicate \( \sim 2\sigma \) tension or even higher.

As discussed previously, the tension is highest in parameters of primordial power spectrum, which may be an indication of missing parameters in primordial power spectrum (e.g., a running spectral index). Therefore, we have additionally considered a running spectral index \( d\tau_k/d\ln k \) and repeated our investigation. Surprisingly, we find that tension increases to even

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**Table 1**: Cosmological Parameters (\( \Lambda CDM + sz + lens \))

| Parameter | \( \lambda_0 \) | \( \lambda_2 \) | \( \lambda_3 \) |
|-----------|-----------------|-----------------|-----------------|
| \( \Omega_m h^2 \) | 0.0226 ± 0.0006 | 0.0231 ± 0.0008 | 0.0217 ± 0.0008 |
| \( \Omega_b h^2 \) | 0.112 ± 0.006 | 0.109 ± 0.008 | 0.115 ± 0.008 |
| \( n_s \) | 0.9837 ± 0.0147 | 0.9913 ± 0.0157 | 0.9859 ± 0.0151 |
| \( \lambda \) | 3.185 ± 0.047 | 3.132 ± 0.065 | 3.239 ± 0.062 |
| \( H_0 \) | 70.53 ± 2.48 | 71.73 ± 3.59 | 69.68 ± 3.47 |
| \( A_{sz} \) | 1.891019 ± 0.0226 | 0.1690183 ± 0.006 | 0.890411 ± 0.008 |

**Table 2**: The Likelihood Ratio: \( \Lambda CDM + sz + lens \)

| Denominator | \( L(\lambda_0 | D_0) \) | \( L(\lambda_2 | D_2) \) | \( L(\lambda_3 | D_3) \) |
|-------------|-----------------|-----------------|-----------------|
| \( L(\lambda_0 | D_2) \) | 1 | 0.076 | 0.0099 |
| \( L(\lambda_2 | D_2) \) | 1 | 2 × 10^{-4} | 1 |
| \( L(\lambda_3 | D_3) \) | 0.16 | 0.0022 | 1 |

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**Figure 1**: Marginalized likelihood of cosmological parameters (\( \Lambda CDM + SZ + lens \)), given whole or even (odd) multipole data.

(A color version of this figure is available in the online journal.)
a higher level. We show the marginalized parameter likelihoods and the likelihood ratios in Figure 2 and Table 3, where we find that tension is also highest in the primordial power spectrum parameters.

3. DISCUSSION

The WMAP power contrast anomaly between even and odd multipoles is explicitly associated with the angular power spectrum data, which are mainly used to fit a cosmological model. Having noted this, we have investigated whether even (odd) low multipole data set is consistent with the WMAP concor-

dance model. Our investigation shows that there exists some level of tension. Noting tension is highest in primordial power spectrum parameters, we have additionally considered the running of a spectral index $d_n/s/d\ln k$, but found that tension increases to even a higher level. These parametric tensions may be indications of unaccounted contamination or missing ingredients of the assumed parametric model (i.e., the flat ΛCDM without a running spectral index). Therefore, we believe that these parametric tensions deserve further investigation. The Planck surveyor data, which possess a wide frequency coverage and systematic distinct from the WMAP, may allow us to resolve this tension.

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