Probabilistic Evaluation of the Extent of the Aquifer – Case Study

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1. Introduction

Environmental researchers often analyze phenomena and objects which can be determined as “poorly-defined” (Fisher 1999). Due to mathematical rules of sets these are the objects which are difficult to be assigned to a specific class of objects in compliance with dichotomic rules of binary (Aristotelian) logic. The extents of lithofacial, stratigraphical and tectonic units are represented in the cartographic studies based on the point or local reconnaissance performed in the field. With respect to distances, those limits are of probable course, more or less similar to the natural boundary. Error assessment of graphic presentation of the geological units has not been expressed in values yet. There is even a lack of approximate estimation of probability to define the unit borders. A similar problem can be identified in hydrogeology. There we have to evaluate homogenic areas and units with similar properties of groundwaters or aquifers. Hydrogeologic cartography offers diversified studies, due to the credibility of used data. It directly finds reflection in the accuracy and likelihood of estimation of the extent of groundwater bodies, their amounts and quality.

In environmental researches, the proper use of the information (or the lack of the information) leads to searching for way to represent this kind of data. It is argued (Leung & Leung, 1993) that the application of Boolean logic (the all-or-nothing system) in the GIS design causes the following problems: a) it imposes artificial precision on intrinsically imprecise information, graded spatial phenomena and processes, b) it fails to determine and communicate to users the extent of imprecision and error, c) it is inappropriate to human cognition, perception and thinking processes, which are generally embedded with imprecision (Leung & Leung 1993).

The aquifer is a good illustration of a “poorly-defined” object (fig. 1). This result from the lack of information on its extent (especially for the confined aquifer), facies changes within the aquifer and various definitions of the aquifer.
In order to describe correctly “poorly-defined” objects in modeling proper methods should be found. They should allow intermediate values to be defined between conventional evaluations like 1 and 0, true or false. For describing “poorly-defined” objects we can use one of the multi-valued logic such as “fuzzy logic” (Zadeh, 1965), kernel-based probability density function estimation (Brundson, 1995) or other probability methods such as Bayesian theory or Dempster – Shafer theory (Shafer, 1976; Klr and Yuan 1997; Eastman 1999b). This paper attempts to evaluate the extent of the unconfined aquifer in a nonparametric – probabilistic scale with help of last one.

The main study objective was to evaluate the probability that an unconfined aquifer may be found in each pixel location in a surface represented in the studied area. Due to a large amount of data IDRISI software was used to achieve the aim.

2. The area of research

The research area of 1300 km² in the east part of the Pomeranian Lakeland in Poland was chosen for a testing procedure. This area lies completely within the limits of the last (Veichselian) glaciation. Along with the relatively slight hypsometric differentiation, the relief of the studied area is characterized by a few forms of fluvioglacial and glacial source. The main form is outwash sediments (the Wda sandur) and a moraine plateau (Fig. 2).
There are only Cenozoic water bearing strata recognized within the log wells. The Pleistocene water bearing layers are the major aquifer for the studied area. It consists from of the unconfined aquifer and a few confined aquifers (Fig. 3).

The (hydro)geologic recognition of research area is rather shallow and diversified due to uninhabited area. These is shown on the Fig. 4
Figure 3. Geologic cross-section along line A-A’ in figure 4.

Figure 4. Map of depth of geologic recognition of the research area.

3. Methodology

The Dempster-Shafer theory (Shafer, 1976) is an extension of Bayesian probability theory. This theory makes a distinction between probability and ignorance and allows for the
expression of ignorance in uncertainty management (Lee et al., 1987; Klir & Yuan, 1995). The basic assumption of Dempster-Shafer theory are that ignorance exists in the body of knowledge, and that the belief for hypothesis is not necessary to the complement of the belief for its negation. By using the “belief functions” to represent the uncertainty of hypothesis, the theory releases some of the axioms of probability theory. The resulting system becomes a superclass of probability theory. However, it suffers from the need for large numbers of probability assignments and from the need for independence assumptions (Malczewski, 1999). Unlike Bayesian probability analysis, D-S theory explicitly recognizes the possibility of ignorance in the evaluation, i.e. the incompleteness of knowledge or evidence in the hypothesis (Eastman, 1999).

The research objective was performed on IDRISI (ver. Andes) raster based software program. In IDRISI, the BELIEF module (Fig. 8) can be used to implement the Dempster-Shafer theory. BELIEF constructs and stores the current state of knowledge for the full hierarchy of hypotheses formed from a frame of discernment (also called state space). BELIEF first requires that the basic elements in the frame of discernment be defined. As soon as the basic elements are entered, all hypotheses in the hierarchical structure will be created in the hypothesis list. For each line of evidence entered, basic probability assignment images (in the form of real number images with a 0 – 1 range) are required with an indication of their supported hypothesis.

4. The development of knowledge base

The research question guides us to define the frame of discernment – it includes two elements [present] and [absent]. The hierarchical combination of all possible hypotheses, therefore, includes [present], [absent] and [present, absent]. We are most interested in the result generated for the hypothesis [present]. The final results produced for the hypothesis [present] are dependent on how all evidences are related together in the process of aggregation.

Given knowledge about existing wells and given expert knowledge about the occurrences of aquifers, each evidence is transformed into a layer representing likelihood that a aquifer exists. The aggregated evidence produces results that are used to predict the presence of an aquifer and evaluate the impact of each line of evidence to the total body of knowledge.

For study several bitmaps and pixel maps were prepared. At the beginning each map included separately: point, line or area data which all confirm or deny the occurrence of the aquifer in a dichotomy scale 0 and 1. In the next stage, the information on each map was changed due to the prepared membership function. As a result, the pixel map with values from 0 to 1 was obtained. Finally, all the maps (information layers) were put to the BELIEF module and probability map was compute.

4.1. Data input for the unconfined aquifer

There is significant difference between analyzing the extent of the unconfined or confined aquifer with the use of the GIS methods. With the exception of the wells as the best indicator of existing aquifers, there is far more indirect evidence of occurrences for the unconfined
aquifer than the first one. For example, they are: springs, rivers, lakes, the area of extent of alluvial or outwash deposits. There is high probability that the unconfined aquifer will be close to these forms (Kachnic, 2010).

The author focused here only on the unconfined aquifer. Still, GIS methods are a tool for, by and large, two dimensional data. And there is no advanced GIS raster program for analyzing three dimensional data yet, required for analyzing the confined aquifer.

For estimating the extent of the unconfined aquifer in a probabilistic scale the following data was selected:

a. locations of wells and boreholes,
b. area of the extent of the outwash and moraine plateau,
c. course of main rivers and lakes,
d. map of depth to the water table in the area where there are no impermeable sediments on the terrain surface.

4.2. Creating probability maps (fuzzyfication)

The stage of fuzzyfication is a procedure, which allows for converting a discrete image (bitmap) into images with a probabilistic (nonparametric) scale. The reliability of the obtained maps depends on the applied parameters of fuzzyfication controlled by a membership function. For researches the following assumptions were taken:

4.2.1. Probability for background

Initially, for the whole research area the background value was assumed as constant 0.5. That means there is no proof for the existing of unconfined aquifer and there isn’t evidence for the lack of the aquifer in research area.

4.2.2. Membership function for wells

Wells are the best point markers of the aquifer. For these features, the area in the close vicinity of the wells obtained high likehood. The map with locations of the wells with unconfined conditions were rasterised and all the pixels where there were wells, obtained the value “one”. The pixel values are high in the area calculated by means of empirical formula, and finally, the pixel value decreased down to the level of the background (Fig. 5).

The empirical formula was applied as one of the assumptions for the extent of the unconfined aquifer. That was a formula:

\[ R = 2s\sqrt{k \ast H} \]  \hspace{1cm} (1)

known as the Kusakin formula (Bear, 1979; Hölting, 1996).

Where R – is a radius in [m] of depression cone; s – the maximum depression observed in a well [m], k – the coefficient of permeability, in [m/24h] and H – thickness of the aquifer in the well log, [m].
The Kusakin formula isn’t a good assumption for the extent of the aquifer and should be established as the better one.

### 4.2.3. Membership function for wells and boreholes with lack of unconfined aquifer

The value “0” was assigned for the pixels where boreholes exist and there is not an unconfined aquifer noticed. In the vicinity of those pixels, probability increases from “0” to the value of background for the range 300 [m] (Fig. 6). The above distance was established subjectively as the optimal one after analyzing the geological and hydrogeological cross-section from the research area.

### 4.2.4. Membership function for boreholes with observed an unconfined aquifer

The value “1” was assigned for the pixels where boreholes exist and there is an unconfined aquifer noticed. In the vicinity of those pixels, probability increases from “1” to the value of background for the range 300 [m].
4.2.5. Membership function for the area of outwash sediments and moraine plateau

In the research area the main body of the unconfined aquifer is associated with fluvioglacial outwash (a sandur). The area of the outwash extent was digitized from the Geological Map of Poland in a scale 1:200 000 (Butrymowicz et al., 1978). The rest of the area was classified as a logic negation, which means the area without sand sediments on the terrain surface (i.e. moraine plateau). The value “0.8” was assigned to all the pixels which represent the area of outwash sediments and the river valley (Fig. 2). For the remaining area a constant value “0.3” was established a priori.

![Graph of the membership function for area of outwash and moraine plateau.]

4.2.6. Membership function for area in the vicinity rivers and lakes

Rivers and lakes are hydrologic objects with frequent connection to the aquifer, especially the unconfined aquifer. Close to a river or a lake there are often sand sediments with the aquifer, therefore, this closeness to water indicates the plausibility for the aquifer. Only rivers that are longer than 5 km and lakes with the area bigger than 1 ha were analyzed. Simple statistical methods were used in order to develop the relationship between the distance to water and the locations of the wells. On the basis of that procedure the author found that there should be higher likelihood (the value of 0.8) in the zone 200 [m] from the river banks or lake shores (Fig. 8).

The information about the depth to the water table and the extent of the unit “a” where there are no impermeable sediments on the terrain surface, was taken from the computer Hydrogeological Map of Poland in a scale 1:50 000 (HMofP). HMofP is a new kind of a map, prepared and stored in GIS (Geomedia) system as a multisheet map (Paczyński et al., 1999). From 1994 to 2004, 1069 sheets covering the whole of Poland were made. The map is based on the concept of the main usable aquifer which is a productive aquifer meeting the following criteria: thickness at least 5 m, transmissivity at least 50 m²/24h, and potential discharge of a well at least 10 m³/h. All data is kept in 19 information vector layers, which contain among others: topographic situation, well and spring locations, type of the aquifer, water quality classes, aquifers pollution risk classes, land use and hydrodynamic
information, e.g. hydraulic head, groundwater flow directions and transmissivity distribution (Herbich, 2005; Fert et al., 2005).

Figure 8. Graph of the membership function for hydrological objects.

5. Stage of calculating

All prepared pixel maps were put to the BELIEF module (Fig. 9).

Figure 9. The window of BELIEF module of IDRISI program.
After processing in the BELIEF module a set of maps was generated. These were maps of the degree to which evidence provided concrete support for the hypothesis (belief) (Fig. 9) and the degree to which that evidence did not refute the hypothesis (plausibility).

Figure 10. The probabilistic map of the extent of the unconfined aquifer in a research area.
6. Conclusion

The limits of geological and hydrogeological units (structure) presented on the cartographic studies include often significant errors due to poor of recognition. Those errors are highest in the area recognized by drilling holes, i.e. point recognition.

The purpose of the methodology presented herein is to produce a probabilistic information layer of the extent of the unconfined aquifer in the studied area (Fig. 10). It is an attempt to use Dempster-Shafer theory in hydrogeology. Taking into account the fuzzy set theory, the author proposed the assessment of hydrogeological structure extent based upon hydrogeological boreholes where pumping test had been carried out.

Additional information for the probabilistic map come from hydrological, geomorphological investigations and data from other geological reconnaissance point. The accuracy of such map is largely determined by the reliability of database information and established membership functions.

The author treat the generated maps as a supplement to a classic set of information concerning hydrogeology and which provides a new form of a map layer which can be useful for decision-makers. The statistical description of the pixel value on the result map may be used for the assessment of reliability of groundwater model and as decision support for sustainable groundwater management.

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