Generation and evolution of stable stellar magnetic fields in young A-type stars

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Abstract While the presence of magnetic fields on low-mass stars is attributed to a dynamo process essentially driven by convective motions, the existence of magnetic fields on intermediate-mass stars has very likely other reasons. Presuming that the fields we see are nearly constant in time, the paper focuses on the generation of stable magnetic configurations at the early stages of stellar evolution. The convective processing of an initial magnetic field during the pre-main-sequence phase is studied in a very simple model star. Azimuthal magnetic fields are found to be typical remnants in the upcoming radiative envelope after the convection has receded.

1. Introduction

The observational evidence of magnetic fields on intermediate-mass stars is reviewed elsewhere in this issue. The vast majority of them appear to be invariable. Nevertheless, they must change at least during relatively short intervals on the evolutionary time-scale as the star does change its state at some points during lifetime.

The following paper will deal with the evolution of magnetic fields in radiative envelopes into stable configurations resembling the observed geometries. The second part will illustrate first simulations of the processing of an initial magnetic field in the convective phase of an intermediate-mass star and the implications for the remnants of that phase.

2. Stable magnetic configurations

Intuitively, a good option for very slowly evolving magnetic fields are force-free fields for which currents are parallel to the magnetic field lines everywhere in the star (and possibly also outside the star). In such a situation zero Lorentz forces do not cause any flows driven by magnetic fields. Since finite conductivity of the stellar plasma leads to some dissipation of the currents, the fields will change on the Ohmic time-scale which is extremely
long given the microscopic plasma magnetic diffusivity of the order of $10^2$–$10^4$ cm$^2$/s (Spitzer 1962). A structure of a typical spot size of $10^{10}$ cm faces a microscopic diffusion time of about 1 Gyr.

One solution for stable, force-free fields was found by Chandrasekhar & Kendall (1957) which are stable even at the presence of diffusion. The constraint though is that the stability only holds true if the sphere is contained in a perfectly conducting medium with zero exterior fields. A vacuum condition on the sphere’s surface destroys the stability of that specific solution and the Chandrasekhar–Kendall functions are subject to a diffusive instability.

3. **Evolution into quasi-stable configurations**

If force-free fields are the topologies we observe, magnetic fields must have evolved into those stable configurations at some stage of stellar evolution, most likely very early on. It is very difficult to draw a complete theoretical picture starting from the first stages of star formation, which is connected inevitably with magnetic fields, including accretion and early convection, to the final settling of the intermediate-mass star.

A few authors have addressed the evolution of initial conditions into quasi-stable magnetic configurations in numerical simulations. Braithwaite & Nordlund (2006) started with the generation of a helical, quasi-stable field with roughly equal poloidal and toroidal field strengths. This is compatible with the findings of enhanced stability of such combinations by Wright (1973) which were later expanded by Braithwaite (2009). Later in the simulations by Braithwaite & Nordlund, diffusion gradually opens poloidal field lines to the exterior, thereby reducing the space for toroidal field inside the star and giving rise to instability. The study was extended by Braithwaite (2008) as to generate predominantly non-axisymmetric magnetic fields, starting from a random field distribution mimicking the remnants from an early convective evolutionary stage. The evolution into the stable topology takes place on a few Alfvén time-scales, while the final configuration changes very slowly on an Ohmic time-scale.

An alternative route has been drawn by Arlt & Rüdiger (2011a,b) who use an initially differentially rotating star to generate strong toroidal magnetic fields in the radiative zone. Such fields are prone to the Tayler instability predominantly delivering non-axisymmetric configurations. The initial differential rotation is presumed to be due to rotational braking.
of the star (theoretically by Stępień 2000; observationally by Alecian et al. 2013c). The braking may open a route to the discrimination between normal A stars and Ap stars since only the ones with enormous braking (a) are slow like Ap stars and (b) have strong enough toroidal fields to go through the Taylor instability and show non-axisymmetric fields. (Since strong braking probably requires strong fields, the concept may be circular reasoning though! – See Sect. 5 for an alternative.) The magnetic-field and velocity fluctuations generated by the instability reduce the differential rotation in the star and the toroidal fields are no longer sustained. The non-axisymmetric perturbations which have grown up to this point are then subject only to Ohmic diffusion. The scenario has been confirmed by fully compressible simulations by Szklarski & Arlt (2013) showing that a given braking of the star may lead to a non-axisymmetric instability of internal toroidal fields.

The observations by Alecian et al. (2013a) of a rapid field change in a Herbig Ae star may be interpreted such that the aligned dipole observed until 2009 is a remnant from the convective dynamo of the pre-main-sequence life (a shell is enough to provide this), while in 2010/11, the instability set in and created non-axisymmetric fields which have been observable since. They have become invariable, since no dynamo any longer sustains a field that can become unstable. The instability remnants are non-axisymmetric and decay on the very long Ohmic time-scale.

Both types of simulations – the ones by Braithwaite et al. and the ones by Arlt et al. – show a relatively quick change of magnetic-field topology on the Alfvén time-scale and end with very, very slowly changing non-axisymmetric fields as to represent Ap star magnetic fields. The actual scenarios are not too different; it is mostly the creations of initial conditions that make the simulations differ.

4. **Dynamos in Ap stars**

Theoretically, stationary magnetic fields can also be maintained by a dynamo process, i.e. by the sustained induction through flows. If certain differential rotation and meridional circulation are continuously generated by radiation in the radiative envelope, we may assume the flow as given and study possible dynamo action. A simple axisymmetric flow is in principle able to drive a dynamo (Dudley & James 1989).

The flows, however, need to be specially designed: the circulation used
by Dudley & James (1989), for example, was not consistent with the
differential rotation they imposed. A consistent circulation would have
the opposite orientation and does not drive a dynamo. Also, as soon as a
convective (diffusive) core not penetrated by the flow is introduced, dynamo
action becomes much more difficult to be excited (Arlt 2008). While these
simple flows deliver simple field topologies they would probably not explain
the diversity of surface field patterns found on Ap stars, even if dynamo
action is feasible. The k.o. criterion is the growth time which, for such
‘laminar dynamos’, is on the time-scale of Ohmic dissipation as compared
to turbulent dynamos growing on much shorter time-scales.

A turbulent dynamo may indeed be at play in the convective core
of the star and has been studied in a number of simulations. While a
large-scale poloidal field can be generated to thread the radiative zone,
the field strength are below kGauss-level already at 0.3 stellar radii (the
top of what was possible to be computed in a global domain, cf. Brun
et al. 2005). An additional imposed field mimicking the effect of a fossil
field led to a more efficient dynamo action and super-equipartition fields
(as compared to the convective motions) in the convective core, reaching
300 kG strength (Featherstone et al. 2009). Further studies will be needed
to show whether these fields can deliver the spots on the surfaces of Ap
stars. Given the stability of the observed spots, the time-scales on which
the simulated flux-rise will change spots can be a decisive criterion for the
concept of a core dynamo providing most of the magnetic flux.

5. Early convection

The question of the origin of the magnetic energy contained in an Ap star
and the selection problem discriminating normal A stars from magnetic
ones certainly requires models and simulations of the pre-main-sequence
evolution of intermediate-mass stars. We expect fully convective phases for
stars up to about 2.5 M⊙ (Alecian, this issue).

A mean-field dynamo model in a convection zone whose depth dimin-
ishes with time was set up by Kitchatinov et al. (2001). Their last section
addresses the remaining rms field in the radiative zone below the convec-
tion zone. Because of the non-axisymmetry of the dynamo solutions of
deep convection zones, the fields remaining in the radiative zone are of the
order of a gauss only.

Here we tried to follow the processing of an initial magnetic field
Figure 1. Velocity field in a vertical cross-section through the pre-main-sequence model of convective processing of an initial large-scale field. Fewer grid points were plotted near the inner boundary for clarity. The time of this snap-shot is $t = 0.001$ diffusion times; the bottom of the convection zone is at $r = 0.56$ at that time.
in a direct numerical simulation including convection. Two basic initial conditions are compared: a homogeneous magnetic field \( B_0 \) parallel to the rotation axis, and a homogeneous magnetic field perpendicular to the rotation axis, which we will call “equatorial” in the following. In other words, if we imagine a Cartesian coordinate system \((x, y, z)\) in which the \( z \)-axis is aligned with the rotation axis of the star, one of the initial fields is \((0, 0, B_0)\), and the other is \((B_0, 0, 0)\).

An almost full spherical computational domain is used (inner radius 0.1 stellar radii) while the MHD equations are reduced to the Boussinesq approximation to be able to at least tentatively cover evolutionary timescales with the simulations. The dimensionless equations solved are

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} &= -(\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla p + (\nabla \times \mathbf{B}) \times \mathbf{B} + \text{Ra}(r, t)\Theta r + \text{Pm} \Delta \mathbf{u} \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{u} \times \mathbf{B}) + \Delta \mathbf{B} \\
\frac{\partial \Theta}{\partial t} &= -\mathbf{u} \cdot \nabla \Theta - \mathbf{u} \cdot \nabla T + \frac{\text{Pm}}{\text{Pr}} \Delta \Theta,
\end{align*}
\]

where \( \mathbf{u}, \mathbf{B}, \Theta, \) and \( p \) are the velocity, magnetic field, temperature fluctuations, and pressure, respectively. The equations are solved with the MHD code by Hollerbach (2000) and are normalized by the radius of the star, \( R_* \), and the magnetic diffusivity, \( \eta \), leading to times measured in diffusion times, \( \tau_{\text{diff}} = R_*/\eta \) and the dimensionless numbers \( \text{Pm} = \nu/\eta \), \( \text{Pr} = \nu/\chi \), and \( \text{Rm} = R_*^2 \Omega_*/\eta \), where \( \nu \) is the viscosity, \( \chi \) the thermal diffusivity and \( \Omega_* \) the angular velocity of the star (at the pole). \( \text{Ra} = g\alpha \Delta T d^3/\chi \eta \) is a modified Rayleigh number with \( g \) being the gravitational acceleration, \( \alpha \) the thermal expansion coefficient, \( \Delta T \) the temperature difference between the inner radius \( r_i \) and the outer radius \( r_o \), and \( d \) is the gap width, \( r_o - r_i \). The temperature fluctuations measure the deviations from the purely conductive profile \( T = (1 - r_o/r)/(1 - r_o/r_i) \). The linear terms are solved implicitly in a spectral space, while nonlinear terms computed in real space on a grid with 240, 240, and 102 points in the radial, latitudinal, and azimuthal directions, respectively.

The buoyancy is a function of radius and time being \( \zeta(r, t) = 1/2 + \{1 + \text{erf}[(r - r_c)/0.02]\} \) where \( r_c(t) \) is the radius of the bottom of the convection zone. It depends on time by \( r_c(t) = (t/t_0)^{1/4} \) where \( t_0 = 0.01 \tau_{\text{diff}} \), which is the time by which the outer convection zone disappeared. The Rayleigh number in the convection zone was \( \text{Ra} = 2 \cdot 10^8 \). We do not simulate the
onset of the core convection.

The magnetic Reynolds number $R_m$ enters the equations by setting an initial rotation to the velocity field for which $R_m = 5 \cdot 10^4$, while the exact dimensionless formulation is a differential rotation of the form $u_\phi(t = 0) = R_m r \sin \theta / \sqrt{1 + (r \sin \theta)^4}$, i.e. constant rotation on cylinders. Note that realistic stellar $R_m$ and $Ra$ are orders of magnitude higher, meaning that global numerical simulations always assume much larger diffusivities than the microscopic plasma diffusivities in stars. This is typically interpreted as an unresolved turbulence below the resolution scale of the simulations. However, rotating, stratified turbulence has not only diffusive effects on the large-scale quantities. This is why such an argumentation is very weak, and we may miss substantial physics when not resolving the smallest scales. Computing facilities are very far away from being able to resolve these scales, unfortunately. With this warning being kept in mind, let us see what is coming out of the simulations anyway.

Figure 2 shows the temporal evolution of the magnetic field components in the computational domain averaged over the volume at $0.1 < r < 0.5$ for a homogeneous vertical initial field and a homogeneous equatorial initial field. The bottom of the convection zone crosses the upper radial limit at $t = 0.000625$. Note the different time-scale of the two graphs since it was much more difficult to run the simulation for the vertical initial field which in fact has not yet reached the time when the convection zone vanishes entirely ($t = 0.01$). The average kinetic energy density is $5 \cdot 10^8$, for comparison, due to the initial rotation with the total angular momentum being conserved by the numerical scheme.

The lower graph of Fig. 2 also shows the squares of the signed averages of $B_\phi$ in the two hemispheeres indicating a substantial large-scale field. The pure winding-up of the equatorial field would lead to zero averages of the signed $B_\phi$ because of the non-axisymmetry (and it is in fact zero in the beginning while the rms is not).

The Alfvén speed of the internal magnetic field reaches roughly 7% of the rotational velocity which is, for a star rotating with 20 km/s at 0.5 stellar radii (as a rough guess from the five magnetic HAeBe stars from Alecian et al. 2013b), about 200 kG. Note, however, that the initial field was already 50 kG in this scaling. Still, the storage of a factor four stronger field in the radiative zone is a promising result given the very crude character of the simulations presented here. The 200 kG fields may already be strong enough to explain the kGauss fields emerging on the
Figure 2. Average unsigned magnetic field in the interior of the computational domain versus time for an initially vertical homogeneous magnetic field (top) and an initially horizontal equatorial magnetic field (bottom). The lower graph also shows the average signed field for the northern and southern hemispheres as dotted lines.
surfaces of Ap stars as observed.

The interesting outcome here is that the vertical initial field leads to equal energies in the three magnetic field components, while the equatorial initial field leads to a dominance of the azimuthal field. Combinations of equally strong poloidal and toroidal fields are known to have the highest stability, whereas purely toroidal fields will become unstable more easily. There is no unique link between our energy plot and poloidal and toroidal fields, but at least it is an indication that the initial magnetic field orientation gives a discriminating criterion for the later evolution into stars with magnetic surfaces and stars with very weak or zero fields.

6. Outlook

We would like to stress the importance of studying the pre-main-sequence evolution of intermediate-mass stars in terms of convective processing of magnetic fields and possible remnants in the emerging radiative zones. I am sure we will find dynamo fields in the youngest Herbig stars which need to be linked to the presence and absence of magnetic fields on main-sequence stars.

It will be interesting to improve the observations of the differential rotation of A stars as compared to Ap stars at different evolutionary stages. Ap stars appear to have no surface differential rotation (e.g. Lüftinger et al. 2010). Normal A stars showed no surface differential rotation in the Fourier analysis of spectral line profiles (Ammler-von Eiff & Reiners 2012) beyond 7400 K, while Kepler light curves indicate differential rotation of up to 15 per cent (Balona 2013). Since the presence of internal large-scale magnetic fields has a great impact on the differential rotation of a radiative star, the precise knowledge of the rotation profiles, especially for young A stars, would be highly beneficial.

Acknowledgements. The author would like to thank the organizers of the conference for the kind invitation.

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