Bounded particle interactions driven by a nonlocal dual Chern-Simons model

Van Sérgio Alves,1 E. C. Marino,2 Leandro O. Nascimento,3 J. F. Medeiros Neto,1 Rodrigo F. Ozela,1 and Rudnei O. Ramos4

1Faculdade de Física, Universidade Federal do Pará, 66075-110 Belém, PA, Brazil
2Instituto de Física, Universidade Federal do Rio de Janeiro, 21941-972 Rio de Janeiro, RJ, Brazil
3Faculdade de Ciências Naturais, Universidade Federal do Pará, 68800-000 Breves, PA, Brazil
4Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, 20550-013 Rio de Janeiro, RJ, Brazil

Quantum electrodynamics (QED) of electrons confined in a plane and that yet can undergo interactions mediated by an unconstrained photon has been described by the so-called pseudo-QED (PQED). PQED is the (2+1)-dimensional version of the equivalent dimensionally reduced original QED. In this work, we show that PQED with a nonlocal Chern-Simons term is dual to the Chern-Simons Higgs model at the quantum level. We apply the path-integral formalism in the dualization of the Chern-Simons Higgs model to first describe the interaction between quantum vortex particle excitations in the dual model. This interaction is explicitly shown to be in the form of a Bessel-like type of potential in the static limit. This result per se opens exciting possibilities for investigating topological states of matter generated by interactions, since the main difference between our new model and the PQED is the presence of a nonlocal Chern-Simons action. Indeed, the dual transformation yields an unexpected square root of the d’Alembertian operator, namely, $(\sqrt{-\Box})^{-1}$ multiplied by the well-known Chern-Simons action. Despite the nonlocality, the resulting model is still gauge invariant and preserves the unitarity, as we explicitly prove. Finally, when coupling the resulting model to Dirac fermions, we then show that pairs of bounded electrons are expected to appear, with a typical distance between the particles being inversely proportional to the topologically generated mass for the gauge field in the dual model.

I. INTRODUCTION

Quantum vortices are excitations that play an important role in condensed matter systems, such as superfluids, superconductors, and in many other systems with a $U(1)$ symmetry [1]. In (2+1)-dimensions, vortices represent stable and mobile excitations that can be characterized, at a classical level, by a discontinuity in the field. This discontinuity, generally associated with a vortex, can be characterized by a quantized topological charge $q = \pm 1, \pm 2, \cdots$, called vorticity. This implies the increase of $2\pi q$ in the phase of the field, after any closed circuit around the vortex. Because of its topological stability, vortices are not destroyed by thermal fluctuations around the critical temperature. Usually, the thermal energy necessary to destroy a vortex is much above the ground state energy. Since vortices are stable and defined in terms of topological charges, it is usual to consider (2+1)-dimensional vortices as charged quasiparticles. The position of a quasiparticle is taken as the central position of the vortex, and its charge is considered to be the topological charge of the vortex itself. This analogy is at the core of the so-called dual models and it has been extensively studied in literature (for a review, see e.g. Ref. [1] and also Refs. [2][3] for some recent work).

In general, well-known (2+1)-dimensional models, such as the Maxwell-Higgs model [5] and similar models [6] have vortex-like solutions that, under appropriate dual transformations, lead to a representation of quantized point vortices, interacting through a gauge field. This rises the question about other types of interactions that may emerge among the vortices driven by different gauge fields in (2+1)-dimensional systems. Although constrained in a spatial plane, it is reasonable to expect that quasiparticles, here representing charged vortices, could interact among themselves through a gauge field that is not constrained to the spatial plane. This situation is similar to the case of PQED [7][8], where the electronic interactions are mediated by an unconstrained photon. This model has been derived from a dimensional reduction of the well-known QED. Because of this, a nonlocal term (in both space and time) emerges in the gauge-field term, namely, $\left(F_{\mu\nu}\right)^2 \rightarrow \left(F_{\mu\nu}\right)^2/\sqrt{\Box}$. This nonlocal term is, essentially, an effect of the dimensional reduction and it yields the Coulomb interaction among static particles in the plane. Furthermore, PQED has been shown to be an useful tool for describing electronic interactions in graphene, where electrons are described by the Dirac equation [9][10]. To the best of our knowledge, the possibility of describing interacting quantum vortices, through a PQED-like model, has not been investigated until now.

In this work, we start from an Abelian Chern-Simons-Higgs (CSH) model. The reason is threefold. First, it is one of the simplest planar models that exhibits a phase transition between a vortex condensed and noncondensed phases [11], thus, it has been a prototype for studies in terms of duality transformations [12]. Second, as a Chern-Simons (CS) model, it has some strong motivations for applications in the context of planar condensed matter systems [13][14]. Finally, it is well-known that the CS term provides a mass for the Maxwell field in the plane without breaking gauge symmetry; this mass is usually called a topological mass. Thereafter, we apply a set of dual transformations in the CSH theory, yielding a nonlocal model that, as we are going to show, combines

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the usual PQED with a nonlocal Chern-Simons action, both of them coupled to the matter current. Since this current describes the vortex current in the dual model, we conclude that our model describes vortex interactions at both the classical and the quantum levels. It is also worth to mention that recently a mass parameter has been introduced in PQED through dimensional reduction of the so-called Proca QED [15]. Here, nevertheless, the mass is generated into a system that is planar from the very beginning, hence, interactions are different even at the static limit. Having such effective construction applied, in particular to dual models, would certainly extend the applicability of these models and their use for describing real physical systems, where the dual descriptions in terms of point-like vortical quasiparticles are considered.

The remainder of this work is organized as follows. In Sec. II, we show how to describe interacting vortices by means of a nonlocal theory similar to PQED, but with the addition of an extra nonlocal and very similar to the Chern-Simons term. This term is shown to be spontaneously generated within our approach and, thenceforth, we calculate the static potential between the quantum vortices and also in the case of matter fermion fields, demonstrating that the latter can form bounded states.

In Sec. III, we prove that the resulting nonlocal model derived in the previous section is unitary. Finally, in Sec. IV we give our final remarks and discuss possible generalizations and applications of our model.

II. THE NONLOCAL ACTION FOR VORTEXES

We start by considering an Abelian CSH model. This model is given in terms of a complex scalar field $\phi$ and a gauge field $A_\mu$, whose Euclidean lagrangian density in (2+1)-dimensions is

$$L_{\text{Eucl}}[A_\mu, \phi, \phi^*] = -i \frac{\theta}{2} \epsilon_{\mu\nu\gamma} A_\mu F_{\nu\gamma} + |D_\mu \phi|^2 + V(|\phi|),$$

(2.1)

where $D_\mu \equiv \partial_\mu + ieA_\mu$ is the covariant derivative, $\theta$ is the Chern-Simons parameter, and $V(|\phi|)$ is a spontaneous symmetry breaking potential. The explicit form for the potential is not necessary to be specified here, only that it has at symmetry breaking a vacuum expectation value for $\phi$ given by $|\langle \phi \rangle| \equiv \rho_0$. We would like to show how in the process of dualization, where the vortex degrees of freedom become explicit, a nonlocal theory can be naturally found, with a gauge-field sector similar to that of the PQED [12, 13], yet with some fundamental differences as far as the resulting Chern-Simons term is concerned.

The field equations, derived from the action in Eq. (2.1), allow for nontrivial solutions with a vortex form [19]. These vortices can be associated with a singularity in the phase of $\phi$, and this can be seen as follows. First, we write the complex scalar field $\phi$ in a polar form, namely, $\phi = \rho \exp(i\chi)/\sqrt{2}$, where both $\rho$ and $\chi$ are real fields. On the other hand, we may decompose the phase into two parts, i.e., $\chi(x) = \chi_{\text{reg}}(x) + \chi_{\text{sing}}(x)$, where $\chi_{\text{reg}}$ and $\chi_{\text{sing}}$ are its regular and singular parts, respectively. In this case, the vortex current $J_\mu$ will be associated with $\chi_{\text{sing}}(x)$ and it can be written as [13, 17]

$$J^\mu = \frac{1}{2\pi} e^{\mu\nu\gamma} \partial_\nu \partial_\gamma \chi_{\text{sing}}.$$

(2.2)

This is a general feature. Next, let us look for an effective action for the gauge-field that mediates the interactions between the current $J^\mu$. We will apply the path-integral formalism for calculating this action.

The partition function of the model in Eq. (2.1) reads

$$Z = \int DA_\mu D\phi D\phi^* \exp \left\{ - \int d^3x L_{\text{Eucl}}[A_\mu, \phi, \phi^*] \right\}$$

$$= \int DA_\mu D\rho \left( \prod_x \rho \right) Z[\rho, A_\mu] \times \exp \left\{ - \int d^3x \left[ -\theta \frac{1}{2} \epsilon_{\mu\nu\gamma} A_\mu \partial_\nu A_\gamma + \frac{1}{2} (\partial_\mu \rho)^2 + V(\rho) \right] \right\},$$

(2.3)

where we have defined

$$Z[\rho, A_\mu] = \int D\chi \exp \left\{ - \int d^3x \left[ \frac{1}{2} \rho^2 (\partial_\mu \chi + eA_\mu)^2 \right] \right\}.$$

(2.4)

The functional integral over $\chi$ in Eq. (2.4) can be rewritten in terms of functional integrals over $\chi_{\text{reg}}$ and $\chi_{\text{sing}}$, respectively. Thereafter, we introduce an auxiliary-vector field $C_\mu$ that satisfies

$$Z[\rho, A_\mu] = \int D\chi_{\text{sing}} D\chi_{\text{reg}} DC_\mu \left( \prod_x \rho^{-3} \right)$$

$$\times \exp \left\{ - \int d^3x \left[ \frac{1}{2\rho^2} C_\mu^2 - iC_\mu (\partial_\mu \chi_{\text{reg}}) - iC_\mu (\partial_\mu \chi_{\text{sing}} + eA_\mu) \right] \right\}.$$

(2.5)

We can now perform the functional integration over $A_\mu$ in Eq. (2.5). Note that the integral over $\chi_{\text{reg}}$ gives the constraint $\partial_\mu C_\mu = 0$. This constraint can be respected by writing $C_\mu$ as a new gauge field $h_\mu$, dual to $C_\mu$, through the relation [13, 17]

$$C_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\gamma} \partial_\nu (K^{1/2} h_\gamma),$$

(2.6)

where $K$ is an arbitrary constant with mass dimension. This situation is analogous to the case of Electrodynamics, where a null divergence of the magnetic field $\partial_\mu B_\mu = 0$ implies that we can write $\vec{B}$ as the rotational of a vector $\vec{A}$, such that $B_i = \epsilon_{ijk} \partial_j A_k$. As in the case of Electrodynamics, the final form of $h_\mu$ will depend on the Euler-Lagrange equation for this field and its corresponding boundary conditions. The solution in Eq. (2.6) has been discussed in details in Ref. [17]. Here, we will show that
a new class of solution is also allowed, which will lead to our main conclusions in this paper.

We note that the constraint \( \partial_\mu C_\mu = 0 \) is still obeyed if we consider that \( C_\mu \) has an explicit dependence on a generalization of the \( \text{d'Alembertian} \) differential operator \( \Box \), i.e., \( K \) does not need to be a constant. This dependence may even involve negative powers of \( \Box \), which resembles the case of PQED \([7,18]\). This observation is key for our main result that we will deduce in the following. Therefore, Eq. (2.6) can then be written in a more general form as

\[
C_\mu = \frac{C}{2\pi} \epsilon_{\mu \nu \gamma} \partial_\nu \left[ \frac{1}{(-\Box)^{n/2}} a_\gamma \right], \tag{2.7}
\]

where \( C \) and \( n \) are, in principle, arbitrary constants. Fortunately, the apparent arbitrariness in the power of \( \Box \) can be removed by considering unitarity arguments, which will restrict \( n \) to only two well-defined values, namely, \( n = 0, 1/2 \), similar to the case studied in Ref. [8], and that we will show explicitly in Sec. III to also be satisfied in the present case as well. Proceeding with our derivation, one notices that the dual gauge field \( a_\gamma \) is of the form \( \partial_\mu a_\gamma = \epsilon_{\mu \nu \gamma} a_\nu \) in a convolution sense \([8]\), i.e.,

\[
C_\mu(x) = \frac{C}{2\pi} \epsilon_{\mu \nu \gamma} \partial_\nu \int d^3 x' \frac{d^3 k}{(2\pi)^3} \frac{e^{-ik \cdot (x-x')}}{(k^2)^{n/2}} a_\gamma(x'). \tag{2.8}
\]

Then, by using Eq. (2.7) in Eq. (2.5), we obtain that the path integral over \( C_\mu \) is replaced by an integration over \( a_\mu \). Hence, we can rewrite Eq. (2.5), after performing the integration over \( \chi_{\text{reg}} \), as

\[
Z[\rho, A_\mu] = \int \mathcal{D}\chi_{\text{sing}} \mathcal{D} a_\mu \left( \prod_x \rho^{-3} \right) \times \exp \left\{ - \int d^3 x \left\{ \frac{C^2}{16\pi^2 \rho^2} |f_{\mu \nu}|^2 + \frac{1}{(-\Box)^{n/2}} f_{\mu \nu} - i e C \epsilon_{\nu \gamma} F_{\nu \gamma} \left[ \frac{1}{(-\Box)^{n/2}} a_\mu + i C J_\mu a_\mu \right] \right\}, \tag{2.9}
\]

where we have defined \( f_{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \) and

\[
J_\mu(x) = \int d^3 x' \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-ik \cdot (x-x')}}{(k^2)^{n/2}} \mathcal{J}_\mu(x'), \tag{2.10}
\]

with \( \mathcal{J}_\mu \) given by Eq. (2.2). Note that only for \( n = 0 \) can the current \( J_\mu \) be understood as the vortex current. By using Eq. (2.9) in Eq. (2.3), the functional integration over \( A_\mu \) can now be performed, which leads to the result

\[
Z = \mathcal{N} \int \mathcal{D}\chi_{\text{sing}} \mathcal{D} a_\mu \mathcal{D} \rho \left( \prod_x \rho^{-2} \right) \exp(-S), \tag{2.11}
\]

where \( \mathcal{N} \) is a normalization constant and

\[
S = \int d^3 x \left\{ \frac{C^2}{16\pi^2 \rho^2} |f_{\mu \nu}|^2 + \frac{1}{(-\Box)^{n/2}} f_{\mu \nu} - i e C \epsilon_{\nu \gamma} \epsilon_{\mu \rho} \partial_\nu \left[ \frac{1}{(-\Box)^{n/2}} a_\rho + i C J_\mu a_\mu \right] + \frac{1}{2} (\partial_\mu \rho)^2 + V(\rho) \right\}, \tag{2.12}
\]

is our effective action \( S = S[\rho, \chi_{\text{sing}}, a_\mu] \). Note that, although \( \rho \) has nontrivial dynamics, only the \( a_\mu \)-field couples to the current \( J_\mu \), which is related to vortices excitations through Eq. (2.10). Our main result, therefore, is exact up to this point.

Next, we will consider the approximation \( \rho \approx \rho_0 \neq 0 \), where \( \rho_0 \) is given by the minimum value of the spontaneous symmetry breaking potential \( V(\rho) \). This is similar to the approach adopted in Refs. [19, 20] and it is analogous to the London approximation, usually considered in condensed matter physics. In this case, we apply the arbitrariness of the constant \( C \) in Eq. (2.7) in order to conveniently fix it as \( C = \sqrt{8\pi^2} \rho_0 \). Using this in Eq. (2.12) then yields

\[
S = \int d^3 x \left\{ \frac{1}{4} f_{\mu \nu} \left[ \frac{2}{(-\Box)^{n/2}} f_{\mu \nu} - i m e \epsilon_{\mu \rho} \epsilon_{\nu \gamma} \partial_\gamma \left[ \frac{2}{(-\Box)^{n/2}} a_\rho + i C J_\mu a_\mu \right] \right. \right\}, \tag{2.13}
\]

where we have also defined \( m = 2e^2 \rho_0^2 / \theta \), that plays the role of the Chern-Simons parameter in our resulting dual theory.

One observes that the first term in the right-hand side of Eq. (2.13) for \( n = 1/2 \), is exactly the gauge field sector of the PQED \([7]\). Surprisingly, however, our dual transformation provided also with a nonlocal Chern-Simons action, which has no analogy in PQED. When \( n = 0 \), we have an analogous of planar Maxwell-Chern-Simons theory. In this case, the main effect of the mass \( m \) would be to generate a mass for the dual gauge field \( a_\mu \) (see, e.g., Ref. [21] for the case of the usual Chern-Simons action). In our case, the mass term will also be responsible for the massive degree of freedom displayed by the dual gauge field \( a_\mu \), for which the mass is defined by the parameter \( m \). This can be easily seen from the classical field equation for \( a_\mu \) derived from the dual action Eq. (2.13),

\[
\left( \partial_\nu \partial_\nu + m^2 \right) (-\Box)^{-n/2} f_\mu = 0, \tag{2.14}
\]

where \( f_\mu = \epsilon_{\mu \nu \gamma} \partial_\nu a_\gamma \). We can say that Eq. (2.13) is one way for realizing a massive PQED model.

The massive behavior of the dual gauge field can also be seen from its free propagator. By adding a gauge-fixing term proportional to a constant \( \alpha \) and, after a straightforward calculation, we can find the free Feynman propagator \( \Delta_\mu(k) \) for the dual gauge field \( a_\mu \),

\[
\Delta_\mu(k) = \left[ P_{\mu \nu}(k) + \Delta_\mu^{GF}(\alpha, k) \right] D_F(k)/4, \tag{2.15}
\]
form of the gauge-field propagator, namely, \( V \) Yuka potential in the plane). In all of these cases, the in QED and PQED (see, e.g., Ref. [15] for the case of the ond kind. In the short-range limit \( mr \to 0 \), \( J \) -proportional terms. In-

\[ P_{\mu\nu}(k) = \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu + m\epsilon_{\mu\nu\alpha}k^\alpha}{k^2 + m^2}, \quad \text{(2.16)} \]

\[ D_F(k) = \frac{1}{(k^2)^{1-n}}, \quad \text{(2.17)} \]

and

\[ \Delta_{\mu\nu}^G(\alpha, k) = \frac{1}{\alpha} \frac{k_\mu k_\nu}{k^2}. \quad \text{(2.18)} \]

Note that we can identify the physical mass pole at \( -k^2 = m^2 \), where the negative sign here is due to the Euclidean metric. This massive aspect of our model also heals the infrared divergence, usually associated with massless photons as in QED.

### A. On the interacting potential for vortices and for matter fields

Next, we would like to clarify the physical meaning of our model given by Eq. (2.13). The simplest scenario is the one with static vortices, where \( J_\mu(x) \to \delta_0 J_\mu(x) \delta^2(\tau) \). This exactly the case of static charges in QED and PQED (see, e.g., Ref. [15] for the case of the Yukawa potential in the plane). In all of these cases, the potential interaction \( V(r) \) is given by the Fourier transform of the gauge-field propagator, namely,

\[ V(r) = \frac{1}{2\pi^2} \int \frac{d^2k}{e^{i\mathbf{k}\mathbf{r}}} G_{00}(k_0 = 0, k), \quad \text{(2.19)} \]

where \( G_{00}(k) \) is the time-component of the gauge-field propagator that mediates interactions among vortices. This, nevertheless, is not equal to \( \Delta_{00}(k) \), because of Eq. (2.10), which shows that the matter current is, actually, \( (\Box)^{-n/2} \) multiplied by the vortex current.

Let us consider here \( n = 1/2 \) for the sake of comparison with PQED and, as we are going to see explicitly in Sec. III, this is a value fully consistent with the unitarity condition when applied to the present model. After integrating out \( a_\mu \) in Eq. (2.13) and replacing \( J_\mu \to J_\mu / (\Box)^{1/4} \), we find that \( G_{\mu\nu}(k) = \Delta_{\mu\nu}(k)/(k^2)^{1/2} \). Note that because of charge conservation, i.e., \( \partial_\mu J^\mu = 0 \), hence, all of the gauge-dependent terms vanish and we only need to consider the \( \delta_{00} \)-proportional terms. Indeed, using \( J_0 \to J_0 / (\Box)^{1/4} \), we find that \( G_{00}(k) = 1/(k^2 + m^2) \). A result that applied to Eq. (2.19) yields

\[ V(r) = \frac{1}{2\pi} K_0(mr), \quad \text{(2.20)} \]

where \( K_0(mr) \) is the modified Bessel function of the second kind. In the short-range limit \( mr \ll 1 \), we find a logarithmic potential \( V(r) \approx \ln(mr) \), while in the long-range limit \( mr \gg 1 \), we obtain an exponential decay, given by \( V(r) \approx e^{-mr}/\sqrt{mr} \). These results resemble the case of planar QED with the Chern-Simons term. This is because the Chern-Simons parameter behaves like a mass term in this case. On the other hand, the case with \( n = 0 \) has been studied in Ref. [19]. There, again, one would obtain that the Bessel potential is generated at the tree level. This only upholds that our dual transformations are indeed well performed, which is quite satisfying.

Although we are concerned with interacting vortices, the theory in Eq. (2.13) could be applied for other types of matter current. In fact, the coupling between the current \( J_\mu \) and the dual gauge field \( a_\mu \), seen in the last term in Eq. (2.13), is exactly of the expected form, between matter fields and a gauge field in general. For instance, let us consider \( J_\mu = \psi \gamma_\mu \psi \), which applies for the Dirac field. In this case, using Eq. (2.19) with \( G_{00}(k) = \Delta_{00}(k) \), we obtain that the interaction now reads

\[ m^{-1}V(mr) = \frac{1}{2\pi mr} - \frac{1}{4}[I_0(mr) - L_0(mr)], \quad \text{(2.21)} \]

where \( I_0(mr) \) is the modified Bessel function of the first kind, \( L_0(mr) \) is the modified Struve function and \( m = 2e^2\rho_0^2/\theta \) is assumed to be positive. Surprisingly, we find that bound-states may be formed close to a critical distance, given by \( mr_c \approx 2.229 \), which is the minimum of Eq. (2.21), i.e., \( V(mr) = 0 \) at \( mr = mr_c \). Around this position, the potential is similar to a quantum harmonic oscillator and quantized energy levels are expected to appear. We believe that these pairs of bounded electrons may have a more deep application in superconductivity, as an analogy to the well-known Cooper pairs, generated by interactions of the electrons with mechanical vibrations of the lattice. Here, our mechanism relies only on the effects of Coulomb potential plus a nonlocal Chern-Simons action, whose nonlocality is the same as in PQED. This another source of binding is likely to be relevant for calculating non-BCS-superconductors phases. We believe that this is a quite relevant and important result of this work.

In the following section, we analyze the relation between the possible values for the exponent \( n \) of the box operator appearing in the equations derived above when constructing the dual model. We show how the value for \( n \) is fixed when requiring the resulting dual model to preserve unitarity.

### III. Unitarity of the Dual Action

In this section, we demonstrate that the only choices of \( n \) in Eq. (2.13) that will lead to a unitary theory are \( n = 0 \) and \( n = 1/2 \). The strategy we will adopt, in order to verify the unitarity of our model is to prove that the optical theorem is obeyed. For this, we will follow closely the procedure employed in Ref. [8]. We start by writing the scattering operator as \( S = 1 + iT \) and consider its matrix elements between initial and final states, \( |i\rangle \) and \( |f\rangle \), as

\[ S_{if} = \langle i|S|f\rangle = \delta_{if} + iT_{if}(2m)^2\delta^4(k_i - k_f), \quad \text{(3.1)} \]
where $T_{ji}$ is defined by the relation $T_{ji}(2\pi)^3\delta^3(k_i - k_f) = \langle i | T | f \rangle$. The unitarity of the $S$-matrix then implies in

$$T_{ji} - T_{ji}^\dagger = i \sum_n (2\pi)^3\delta^3(k_i - k_f)T_{in}T_{j\dagger n}, \quad (3.2)$$

and where in the above equation we have inserted the complete set of states $|n\rangle$. Putting $|i\rangle = |f\rangle$, we can replace $T_{ii}$ by the Feynman propagator $\Delta_F (T_{ii} = \Delta_F)$. In this case, Eq. (3.2) reads

$$\Delta_F (x) - \Delta_F (x) = i \int d\Phi (2\pi)^3\delta^3(0) \times \int \frac{d^3x}{(2\pi)^3} \Delta_F^\ast(x)\Delta_F (x - x_n), \quad (3.3)$$

where $d\Phi$ is the phase-space factor, related to the characteristic time scale of the system $T$ as: $\int d\Phi (2\pi)^3\delta^3(0) = T\gamma$, where $\gamma$ is determined through dimensional considerations [8]. The propagator of our model is given by $\Delta_{0,\mu\nu} = F_{\mu\nu}D_F/4$, where

$$D_F(t, \vec{r}) = \int \frac{d\omega}{2\pi} \int \frac{d^2\vec{k}}{(2\pi)^2} \exp(i\vec{k} \cdot \vec{r} - i\omega t) \frac{\delta^2(\omega - |\vec{k}|^2 + i\varepsilon)}{\omega - |\vec{k}|^2 + i\varepsilon}^{-n}. \quad (3.4)$$

Taking the Fourier transform of the above expression and using the fact that the transform of a convolution is a product, we can write the unitarity condition given in Eq. (3.3) in momentum space as

$$\Delta_F^\ast(\omega, \vec{k}) - \Delta_F (\omega, \vec{k}) = iT^{2(n-1)}D_F^\ast(\omega, \vec{k})D_F(\omega, \vec{k}). \quad (3.5)$$

Using now Eq. (3.4), we can write Eq. (3.5) as

$$\frac{1}{(\omega^2 - |\vec{k}|^2 - i\varepsilon)^{1-n}} - \frac{1}{(\omega^2 - |\vec{k}|^2 + i\varepsilon)^{1-n}} = 2i\text{Im} \left[ (\omega^2 - |\vec{k}|^2 + i\varepsilon)^{1-n} \right] \left[ (\omega^2 - |\vec{k}|^2)^2 + \varepsilon^2 \right]^{1-n} \left[ (\omega^2 - |\vec{k}|^2)^2 + \varepsilon^2 \right]^{1-n} \quad (3.6)$$

Equation (3.6) is the same as derived in Ref. [8] and it has been shown in that reference that it admits a constant $T$ solution only for $n = 0$ and $n = 1/2$, which are the only cases where the theory is unitary. These results obtained for $D_F$ can also be straightforwardly extended to $P_{\mu\nu}D_F/4$, as long as $P^2 = P$. This is precisely the case in our model. Therefore, it follows that the only choices of $n$ that lead to an unitary theory are $n = 0$ and $n = 1/2$. This concludes our proof. Finally, we note that the choice $n = 0$ leads to a Maxwell-Chern-Simons-Higgs theory, which was discussed in Ref. [17], while the case $n = 1/2$ leads to the following model,

$$S = \int d^4x \left[ \frac{1}{4} f_{\mu\nu} \left( \frac{2}{\sqrt{-\Box}} \right) f_{\mu\nu} + \frac{i}{2} \epsilon_{\mu\nu\gamma} a_{\gamma} \partial_\nu \left( \frac{2}{\sqrt{-\Box}} \right) a_\mu + iCa_{\mu}J_\mu \right]. \quad (3.7)$$

IV. CONCLUSIONS

Planar theories are relevant either because of comparison with quantum chromodynamics (QCD) at low energies or applications in condensed matter physics. Planar QED has been shown useful for comparison with QCD because it has a confining logarithmic potential. On the other hand, PQED has been shown an ideal tool for describing electronic interactions in two-dimensional materials [9] [10]. In this theory, electrons are constrained to a plane and interact through electromagnetic fields, which can also propagate out of the plane. Nevertheless, the model itself is entirely defined in (2+1)-dimensions, which gives its nonlocal feature. We note, however, that when in a superconductor phase, photons inside for example a graphene layer, are expected to become massive, due to the Anderson-Higgs mechanism; in this situation, a necessary mass term for the gauge field in PQED is absent a priori. Within the realm of gauge fields, a mass term is usually written as either a Proca or a Chern-Simons action. While the former is not gauge invariant, the latter preserves this invariance and has a topological nature, which is known for describing topological defects such as vortices.

In this work, we derive an extension of the PQED, where it includes a nonlocal Chern-Simons-like massive term for the gauge field. Our model emerges from a dual transformation of the planar CS model, where the vortex degrees of freedom are made explicit. Because we describe vortex excitations as point-like quasiparticles, we have that their interactions are mediated by a (dual) gauge field. The resulting theory is then shown to admit a PQED-like term $f_{\mu\nu} [2/\sqrt{-\Box}] f_{\mu\nu}/4$, but it is also conveyed by a “pseudo-Chern-Simons” term, given by $(m/2)\epsilon_{\mu\nu\gamma} a_{\gamma} [2/\sqrt{-\Box}] a_\mu$. This type of structure is quite novel and, despite their nonlocal nature, we have proved that it is unitary. The emerging of a pseudo-Chern-Simons like term leads to a very rich structure of the gauge-field propagator. Indeed, it has a branch cut, as a result of the presence of the d’Alambertian differential operator, and it has a massive pole away from the branch cut. When coupled to Dirac fermions, our model yields a confining potential, which can generate pairs of bounded electrons in the static limit at positions $mr_c \simeq 2.229$. On the other hand, the interactions among vortices are quickly screened due to an exponential decay
with an effective interaction length, given by $\xi = m^{-1}$. We shall discuss the quantum corrections as well as the possible applications in condensed matter physics of this model elsewhere.

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