Optical polarization of the Crab pulsar: precision measurements and comparison to the radio emission

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ABSTRACT
The linear polarization of the Crab pulsar and its close environment was derived from observations with the high-speed photopolarimeter Optical Pulsar TIMing Analyser at the 2.56-m Nordic Optical Telescope in the optical spectral range (400–750 nm). Time resolution as short as 11 μs, which corresponds to a phase interval of 1/3000 of the pulsar rotation, and high statistics allow the derivation of polarization details never achieved before. The degree of optical polarization and the position angle correlate in surprising details with the light curves at optical wavelengths and at radio frequencies of 610 and 1400 MHz. Our observations show that there exists a subtle connection between presumed non-coherent (optical) and coherent (radio) emissions. This finding supports previously detected correlations between the optical intensity of the Crab and the occurrence of giant radio pulses. Interpretation of our observations requires more elaborate theoretical models than those currently available in the literature.

Key words: radiation mechanisms: non-thermal – instrumentation: polarimeters – techniques: polarimetric – pulsars: general – pulsars: individual: the Crab pulsar.

1 INTRODUCTION
The Crab pulsar and its pulsar wind nebula (PWN) are two of the most intensively studied objects in the sky. The compact remnant of SN1054, a cornerstone of high-energy astrophysics, is one of the youngest and the most energetic pulsar, and its pulsed emission has now been detected throughout the electromagnetic spectrum from about 10 MHz (Bridle 1970) up to >25 GeV with some evidence (3.4σ) of pulsed emission above 60 GeV (Aliu et al. 2008). The PWN was detected even up to energies of ∼100 TeV (Aharonian et al. 2004; Allen 2007; Allen et al. 2007). The pulsar and its nebula are predominantly sources of non-thermal radiation (synchrotron, curvature and inverse Compton processes), which is indicated not only by the broad-band spectral continua but also by the strong polarization of these emissions. The outstanding brightness of the Crab across the electromagnetic spectrum makes it the ideal target to investigate polarization at all wavelengths where suitable instrumentation is available.

Before we focus on the Crab, we will introduce the subject of polarization in pulsars by a short review of the total population: most pulsars were discovered in the radio regime (presently, about 2000 objects are listed in the Australia Telescope National Facility Pulsar Catalogue; Manchester et al. 2005) and for a large fraction of them polarization studies were performed (e.g. Gould & Lyne 1998; Karastergiou & Johnston 2006). Most radio pulsars were found to show strong linear polarization, often accompanied by a characteristic swing of the position angle (PA) in an S-like shape near the pulse centre. This swing is interpreted in the ‘rotating vector model’ (RVM; Radhakrishnan & Cooke 1969) as a projection of the magnetic field line at the point of emission on to a plane perpendicular to the observer’s sight line. The point of emission is usually assumed to be in the polar cap region of the pulsar where a regular dipolar field line points with a small angle (beamwidth) towards the observer. The free parameters of this simple geometrical model are the inclination angle between the axes of rotation and magnetic dipole and the viewing angle between the line of sight and the rotation axis. Analyses of radio polarization from many pulsars (e.g. references in Lyne & Graham-Smith 2006) showed that most sources can be described in a polar cap model with a RVM and the intrinsic polarization could be as high as 100 per cent. Several pulsars, however, especially at radio frequencies above several GHz, show reduced or complex
Table 1. High-energy polarization measurements of pulsars and their nebulae. PD is given as a percentage and PA in degrees (north, 0° to east, 90°).

| Band                      | Pulsar/polarization | Nebula (near PSR) | Ref. |
|---------------------------|---------------------|-------------------|------|
| Optical (V ~ 16.6)        | Phase-resolved (see Fig. 5) | 9.7 ± 0.1 per cent; 139±8 ± 0.2 | [1], [2], [3] |
|                           | Phase-averaged 9.8 ± 0.1 per cent and 109.5 ± 0.1 | (<5 arcsec from PSR) | [3] |
|                           | Phase-averaged after DC subtraction 5.5 ± 0.1 per cent and 96.4 ± 0.1 | [3] |
| UV                        | Phased-resolved, similar to optical – | | |
|                           | PD and PA (MP and IP) | | |
| X-ray                     | Only upper limits | 19.2 ± 0.9 per cent; 155±8 ± 1.4 | [5], [6] |
| Hard X-ray/ soft γ-ray    | Off-pulse (phase: 0.52–0.88) >72 per cent; 120.6 ± 8.5 | | [7] |
|                           | Phase-averaged 47 per cent +19 per cent; 100° ± 11° | | [7] |
| γ-ray                     | Off-pulse (phase: 0.5–0.8) 46 ± 10 per cent; 123° ± 11° | | [8] |
| Optical (V~22.5)          | Phase-averaged: ~5 per cent, no error quoted; phase-resolved: <15 per cent | 5.6 ± 1.0 per cent; 70° ± 5° | [9], [10], [11] |
|                           |                       |                       |      |
| Optical (V ~ 23.6)        | Phase-averaged: 9.4 ± 4 per cent, 146° ± 11° | | [9], [12] |
|                           |                       |                       |      |
| Optical (V ~ 25)          | Double peak light curve; PD bridge ~100 per cent, peaks ~0 per cent | | [13] |
|                           | PA sweeps in agreement with RVM | | |
|                           |                       |                       |      |
| Optical (V ~ 25.7)        | Phase-averaged: ~10.4 per cent (very uncertain, no error quoted) | | [9] |

[1] S88, [2] Słowikowska, Rudak & Kanbach (2008), [3] this paper, [4] Graham-Smith et al. (1996), [5] Silver et al. (1978), [6] Weisskopf et al. (1978), [7] Forot et al. (2008), [8] Dean et al. (2008), [9] Wagner & Seifert (2000), [10] Middleditch, Pennyacker & Burns (1987), [11] Chanan & Helfand (1990), [12] Mignani et al. (2007), [13] Kern et al. (2003),

polarization signatures with indications of depolarization or switching between modes of polarization. These complications could arise from the subtle and often highly variable and unstable nature of the coherent processes underlying the radio emission.

The situation changes when going to higher energies, i.e. optical, X- and γ-ray regimes. Incoherent single-particle radiation processes in the magnetosphere provide the high-energy pulsar emission (thermal emission from the neutron star surface is not considered here). Precise measurements of polarization degree (PD) and PA as a function of pulsar phase in a wide range of energies should provide deep insight into the pulsars emission mechanisms, particle spectra and emission site topologies. Fast X- and γ-ray polarimetry from space-borne instruments are presently of very limited sensitivity. Results have therefore been reported only for the brightest pulsar and its PWN, i.e. the Crab. In the optical domain, the situation is somewhat reversed: here, the polarimeters are quite sensitive but the optical magnitudes of most pulsars are exceedingly faint. Therefore, polarization has only been measured for five of the 14 known optical pulsars (Table 1). For three of them, phase-resolved measurements were performed. The Crab, being the brightest object, has a multitude of optical polarization measurements, whereas B0540—69 and B0656+14 have only marginal results. For the remaining two, only phase-averaged results are available.

The scarcity of optical polarization measurements, either in a phase-averaged mode or with time resolution of the pulsar rotation, is clearly due to the large difference in magnitude going from the brightest pulsar, i.e. the Crab, to the next, B0540—69, which is ~6 mag fainter. Therefore, only for the Crab pulsar have fully phase-resolved polarization measurements been possible. The first optical phase-resolved linear polarization observations of the Crab pulsar (Wampler, Scargle & Miller 1969; Cocke et al. 1970; Kristian et al. 1970) showed that the polarization angle sweeps through each peak and the PD first decreases and then increases within each pulse, reaching the minimum shortly after the pulse peak. These early observations were limited to the main pulse (MP) and inter pulse (IP) phase ranges only. Interpretation of the polarization pattern behaviour has been made in terms of geometrical models (Radhakrishnan & Cooke 1969; Wampler et al. 1969; Cocke, Ferguson & Muncaster 1973; Ferguson 1973; Ferguson, Cocke & Gehrels 1974), some of which offer the possibility of determining the magnetoospheric location of the radiation source (e.g. Cocke et al. 1973; Ferguson et al. 1974).

For a long time, it was thought that optical radiation of the Crab pulsar persisted only through both peaks, the MP and IP, and in the bridge region between them, but not in the phase range following the IP and preceding the MP. Several phase-resolved imaging observations (Peterson et al. 1978; Percival et al. 1993; Golden et al. 2000b) showed, however, that radiation persists throughout the whole pulsar rotation. They found that the minimum intensity, consistent with that coming from an unresolved source at the pulsar position, occurs immediately before the start of the MP within a phase range of approximately 0.779 and 0.845. This level, often called the direct current (DC)

2 Direct current or continuous current, in this case, refers to a continuous emission component that could be present throughout the whole rotational phase.

3 The terms: PD and p as well as PA and θ are used interchangeably.
during both off-pulse phase ranges (bridge and DC). However, for the DC phase range the results were not conclusive. The PD was at a level of 70 and 47 ± 10 per cent and the PA was ~118° and ~130° for Jones et al. (1981) and S88, respectively.

Weisskopf et al. (1978) measured the linear polarization of the X-ray flux from the Crab nebula at energies of 2.6 and 5.2 keV, independent of any contribution from the pulsar. Within a field of view of 3°, the X-rays are polarized at a level of 19.2–19.5 per cent and a PA of 156°–152°. For the pulsar itself, only upper limits could be derived (Silver et al. 1978). No evidence for an energy dependence of the X-ray polarization was found. Close to the pulsar the nebular optical polarization is quite uniform (at ~9–11 per cent) but the PA changes steadily with radial distance: 2–3 arcsec from the pulsar the mean value is around 140°, exceeding 155° and becoming very location dependent beyond 5 arcsec (McLean, Aspin & Reitsema 1983). The X-ray results for the nebula are in good agreement with the optical polarization measurements which yield a 19 per cent polarization at 162° for the central region of the nebula within 0.486 arcmin radius (Oort & Walraven 1956). The similarity of the optical and X-ray results indicates that the polarization is independent of energy over a broad spectral range. Recently, γ-ray phase-resolved (off-pulse phase region, i.e. 0.5–0.8) polarization measurements of the Crab nebula and pulsar were reported by Dean et al. (2008) for the energy range 100 keV –1 MeV. The PD was found at 46 ± 10 per cent and the PA is 123° ± 11°, which indicates that this energetic radiation is dominated by emission from the inner nebula around the pulsar. This result has been confirmed by Forot et al. (2008) by using the IBIS (Imager on Board the Integral Satellite) instrument on board of the INTEGRAL telescope to obtain the linearly polarized emission for energies between 200 and 800 keV and to extend polarization characteristics for pulses and bridge phases (Table 1).

We structure the paper as follows: firstly, we describe our instrumentation (Section 2). In Sections 3 and 4 the observations, data reduction methodology and measurements of polarimetric and photometric standards are described. In Section 5, we present the results of the Crab nebula and its pulsar. Also in Section 5 we show the pulsar polarization characteristics after the subtraction of an unpulsed component and discuss time alignment between optical and radio wavelengths. In Section 6, we present a discussion of theoretical models and compare them to our measurements and end with a summary and conclusions.

2 INSTRUMENTATION

Our goal was to obtain polarization characteristics of the Crab pulsar with very high time resolution as a function of the pulsar rotational phase. We used the 2.56-m Nordic Optical Telescope (NOT)4 for these measurements. This facility does not only provide a mirror with a reasonably large collecting area, but also has an excellent performance in offset guidance and telescope control, which is important for instruments with very small entrance apertures. Moreover, contrary to many larger optical telescopes, guest instruments can be used at NOT. For the Crab pulsar observations carried out during 2003 November 23–27, we used the high-speed photopolarimeter Optical Pulsar TIMing Analyser (OPTIMA)5. We briefly describe the system below. An extended description of the OPTIMA instrument is presented by Kanbach et al. (2003, 2008).

2.1 OPTIMA

The basic requirement for a high-speed photometer is to select and time tag individual photons from a celestial source with high efficiency and precision. In the case of OPTIMA, which was primarily designed for studying the optical light curves of faint pulsars and other highly variable targets, the source flux is isolated in the focal plane of the telescope by the use of an optical fibre pickup which acts as a diaphragm. The target fibre is central to a hexagonal bundle of identical fibres which measure the sky background and, in the case of the Crab nebula, the close environment of the pulsar (Fig. 1). OPTIMA uses commercial single-photon counting modules with avalanche photodiodes (APDs).6 These APD counters have quantum efficiencies (QEs) peaking at 70 per cent for λ ~ 700 nm and a wide response with QE > 20 per cent from 440 to 980 nm.

The light from the telescope at the Cassegrain focus is incident on a slanted mirror with an embedded bundle of optical fibres. Optionally, filters or a rotating polarizer can be inserted into the incoming beam. The field around the fibres, visible in the mirror (2 × 3 arcmin²), is imaged with a target acquisition camera (Apogee Instruments AP6e). We use the following nomenclature for the fibre channels: channel 0 for the central fibre; channels 1 to 6 (counterclockwise) for the ring fibres and channel 7 for a sky background fibre, located about 1 arcmin offset from the target fibre. For the observations, we used tapered fibres with 320 μm entrance aperture and an exit pupil of 100 μm. The single fibre size in the focal plane is equivalent to a 2.344 arcsec resolution at the 2.56-m NOT telescope. Although the pure silica core fibres transmit light from 400 to ~950 nm with better than 99 per cent, the net transmission of the tapered fibres is lower and appears to range between 60 and 90 per cent. Since the photon counters also have slightly different responses, the fibre–counter combinations are calibrated on a ‘flat-field’, normally the sky background (see Table 2).

The timing of individual photons is controlled by signals from the global positioning system (GPS) to an absolute accuracy of ~2 μs, although the readout system limits the resolution to ~4 μs. The OPTIMA detector is operated with two computers and is autonomous except for the need to have a good telescope guiding system.

4 http://www.not.iac.es/telescope/technical-details.html
5 http://www.mpe.mpg.de/gamma/instruments/optima/
6 Perkin–Elmer SPCM-AQR-15FC
Figure 1. An enlargement of the HST image of the inner Crab nebula source: R. Romani, private communication. North is up, east is left. The pulsar is identified with the lower/right of the two stars near the geometric centre of the nebula. A small arc-like feature (the inner knot) which is clearly resolved in the HST image is located 0.65 arcsec to the SE of the pulsar. The OPTIMA fibre bundle, centred on the pulsar and scaled with the NOT focal plane scale, is overplotted.

Table 2. Fibre flat-field correction factors.

| Channel | Factor          |
|---------|-----------------|
| 0       | 1.000 ± 0.012   |
| 1       | 1.009 ± 0.013   |
| 2       | 1.074 ± 0.013   |
| 3       | 0.899 ± 0.012   |
| 4       | 0.850 ± 0.011   |
| 5       | 0.956 ± 0.012   |
| 6       | 1.140 ± 0.014   |

2.2 Rotating polarization filter

For the present observations, OPTIMA was equipped with a rotating polarization filter (RPF) in the incoming beam, fully covering all fibre channels and the CCD imager. The polarizing filter (type 10K by Spindler & Hoyer) was mounted on a precision roller bearing and was rotated with typical frequencies of a few Hz (the averaged frequency over the whole observations was 3.4 Hz). Thus, incoming linearly polarized light was modulated at twice the rotation frequency of the filter. The reference position of the filter was given by a signal from a magnetic switch (Hall sensor), and was registered, timed and stored in a separate DAQ (data acquisition) channel in the same way as photon events. The position of the polarizing filter for any photon event was then derived by interpolating the photon arrival time between the preceding and the following Hall sensor signal, i.e.

\[ \alpha_{\text{RPF}}(\text{TOA}) = \frac{t_{\text{TOA}} - t_b}{t_a - t_b} \times 360^\circ, \]

where \( \alpha_{\text{RPF}}(\text{TOA}) \) is the RPF angle at which the event was observed, \( t_{\text{TOA}} \) is the event time of arrival (TOA) and \( t_b \) and \( t_a \) are the recorded times of the Hall signal sensor before and after \( t_{\text{TOA}} \). Since, during one turn of the RPF, all possible polarization angles are measured two times, for all values bigger than 180° exactly 180° was subtracted.

Slight irregularities in the rotational frequency of the RPF that occur on time-scales longer than fractions of a second, e.g. due to supply voltage drifts or mechanical resistance changes in the bearing and motor, can thus be corrected with sufficient accuracy. The RPF was tested in the lab with unpolarized and linearly polarized light to ensure and prove that the OPTIMA fibres and detectors have no intrinsic systematic response to polarized light (Kellner 2002).

The polarizing filter modulates the incoming light effectively only over a wavelength range of about 470–750 nm. Since the APD response (QE > 20 per cent) extends from about 440 nm to 980 nm and no wavelength information of the individual recorded events is available, it is necessary to block radiation outside the filter modulation range. Such photons, especially towards the near IR, are not modulated and would decrease the estimate for the degree of polarization. Therefore, an IR blocking filter that cuts the wavelength range above 750 nm was inserted. The method to derive the polarization characteristics follows the approach described by Sparks & Axon (1999). Details are given in Appendix A of this paper.
3 OBSERVATIONS

The Crab observations were performed on 2003 November 23–27 (52966–52971 MJD). They consist of 160 pointings of 10 min each. About one third of the total number of counts falls in the central fibres, whereas around 5 to 10 per cent are registered in each of the background fibre (channels marked from 1 to 6 in Fig. 1). The count rate for the central fibre decreases when the seeing deteriorates and photons spill out into the ring channels (1–6). In order to screen for observation intervals of acceptable seeing, we apply a cut on the fraction of total counts in channel 0 at the level of 30 per cent. After screening for good seeing and proper pointing, our data base resulted in 83 files, equivalent to 13 h and 43 min of total exposure.

4 DATA REDUCTION

4.1 Flat-field correction

For flat-field correction factors of all channels, we used two sets of dark-sky observations, pointing at $\alpha = 00^h07^m02^s00$, $\delta = +73^\circ03'28"00$. Both observations were taken on November 25. The data acquisition started at 21:39:10 and 21:49:13 UTC. The exposure time amounted to 600 and 300 s for the first and second observation, respectively. The detected count rates behind the filters ranged from 180 cps for channel 4 up to 240 cps for channel 6. To obtain the flat-field correction factors, we assumed that this value was 1.0 for the central fibre, and we scaled the count rates of the other fibres to the central one. We binned the data in 10 s intervals, and then calculated the average and standard deviation for each single channel of the ring fibres. The resulting numbers are shown in Table 2.

4.2 The HST polarization standards

In order to understand the intrinsic polarization and response of the instrument, we performed observations of stars with well-known polarization. Since OPTIMA is very sensitive and limited in its capacity for data acquisition at high rates (count rates above 37 kcps lead to notable but correctable pileup effects; Mühlegger 2006), we selected three of the faintest stars from the Turnshek et al. (1990) list of calibration objects, a highly polarised star, a weakly, nearly unpolarised, star, and one photometric standards (Table 3). The first, BD+64 106, is a highly polarized (PD $\sim$ 6 per cent) star, whereas the second, G191B2B, has a very low degree of polarization on the level of 0.1 per cent. As still both are quite bright by OPTIMA standards, we therefore performed measurements of a fainter photometric standard, expecting its light not to be polarized.

We observed BD+64 106 on November 25 and 27 (Table 4). The observing conditions during the two exposures were very different, requiring a detailed discussion of the results. On November 25, the average seeing was about 1.3 arcsec (RosoDIMM measurements). Weather conditions were worse during the second night and seeing was most likely more than 2.0 arcsec (no RosoDIMM information available). This difference in seeing causes different count rates in the central (190 and 150 kcps, respectively, for the two nights) and ring fibres ($\lesssim$20 and $\lesssim$50 kcps). Values of PD and PA for all channels and for all sets of observations are gathered in Table 4. It is important to note that, even if the central fibre was affected by pileup, the calculated PA for each single observation of BD+64 106 is quite constant. This is caused by the fact that the PA, being the phase of the maximum of the modulated incoming light, does not strongly depend on the pileup effect. Saturation is, however, important when the amplitude of the RPF modulation, corresponding to the degree of polarization, is to be measured. For the central fibre, there is a significant difference in PD between the first and second data set (Table 4). When the dispersion of the stellar image due to seeing was larger, the star light was more smoothly distributed among all channels, therefore an increase of PD in the central fibre is observed. Seeing conditions, being equivalent to the count rates in the ring, did not affect significantly the measured values of PD in the ring fibres. Only in channel 1 the polarization is smaller during the second night of observations, because it was the most saturated ring channel.

We conclude that for the purpose of calibrating the north direction of the instrument it is accurate and valid to use the values obtained for BD+64 106 from the central fibre (Table 4, bold-faced text). The averaged PA amounts to 96.74 $\pm$ 0.37. It was obtained by shifting the

| Table 3. HST polarization and photometric standards. |
|---------------------------------------------------|
| Name/
| spectral type |
| Coordinates FK5 | $HST$ Polarization standards | Comments |
| BD+64 106 | 00$^h$07$^m$36$^s$70 | $V = 10.34$ |
| B1V | +64$^\circ$51'34"9 | $p = 5.65 \pm 0.053$ per cent |
| | | $\theta = 96:8$ |
| G191B2B | 05$^h$05'30"61 | $V = 11.79$ |
| WD | +52$^\circ$49'51"9 | $p = 0.09 \pm 0.048$ per cent |
| WD | 03$^h$48'50"20 | $HST$ Photometric standard |
| WD | $-00^h58'31"2$ | $V = 14.06$ |
intrinsic RPF angles (the position of the Hall sensor which indicates the origin of the intrinsic angles) by 92°. The angles θ given here and hereafter in the text and tables are the E-vector PAs relative to celestial north (N to E). Due to the pileup in the central fibre, the best estimate of the PD of BD+64 106 comes from the sum of the light detected in the ring fibres. The obtained value, 4.72 ± 0.30 per cent, is somewhat smaller than the value given by Turnshek et al. (1990), 5.65 ± 0.053 per cent, but one should take into account the systematic problems encountered with such a bright standard star. In this case, the bad seeing is an advantage, because it naturally defocuses the target.

We then performed observations of the unpolarized Hubble Space Telescope (HST) standard G191B2B. Data for this target were taken on November 26 with seeing around 1.0 arcsec. As expected, we found a very small PD for this star, and thus the PA has little meaning. During these observations, the count rate was about 150 and ≲40 kcps in the central and ring fibres, respectively. This means that the central fibre was severely affected by pileup and the result 0.05 ± 0.02 per cent (Table 5) might be biased, even if it is in good agreement with the value 0.09 ± 0.048 per cent given by Turnshek et al. (1990). The averaged value of PD from the background fibres is higher and amounts to 0.33 ± 0.02 per cent. Most probably, it is partly contaminated by the polarized sky background.

Table 5. Measurements of the HST polarization standard G191B2B and the HST photometric standard GD50.

| Obs. date | G191B2B | GD50 |
|-----------|---------|------|
| Time (UTC) | 2003 November 26 | 2003 November 27 |
| Expo. (s) | 21:33:08 | 21:52:17 |
| 0 | 0.04 | 0.05 ± 0.02 | 0.08 |
| 1 | 0.37 | 0.38 ± 0.13 | 0.30 |
| 2 | 0.75 | 0.67 ± 0.19 | 0.44 |
| 3 | 0.84 | 0.71 ± 0.17 | 0.61 |
| 4 | 0.30 | 0.53 ± 0.18 | 0.62 |
| 5 | 0.16 | 0.31 ± 0.19 | 0.23 |
| 6 | 0.14 | 0.20 ± 0.16 | 0.53 |
| 1–6 | 0.35 | 0.33 ± 0.02 | 0.22 |

Note. Date and time of observations, as well as exposure length in seconds are given in the first, second and third rows, respectively. Obtained values of PD, p, are given in per cent for each OPTIMA detector channel from 0 to 6. Additionally, last row (assigned as ‘1–6’) gives p values obtained after averaging the Stokes parameters from all ring channels. Mean gives p and standard deviations after averaging all existing data of G191B2B from November 26. In the last column, the results of GD50 measurements are given.
Table 6. The Crab pulsar radio ephemeris.

| Parameter               | Value          |
|-------------------------|----------------|
| RA (J2000), $\alpha_{2000}$ | $05^h34^m31^s972$ |
| Dec. (J2000), $\delta_{2000}$ | $+22^\circ00'52^\prime07''$ |
| Valid range (MJD)       | 52944–52975    |
| Epoch, $t_0$ (TDB MJD)  | 52960.000000296 |
| $v_0$ (Hz)              | 29.8003951530036 |
| $v_0(10^{-10}$ Hz s$^{-1}$) | $-3.73414$ |
| $v_0(10^{-20}$ Hz s$^{-2}$) | 1.18 |

The fainter reference star (GD50) ensured count rates lower than the pileup threshold (Table 3). It is a photometric standard, therefore we expect its light not to be polarized. It was observed on November 27 with rates in the central channel of 10–15 kcps and in the ring channels $\lesssim$1 kcps. Its PD is below 1 (measured in the central fibre), whereas the light in the ring fibres, due to the sky background polarization, is polarized about three times higher (Table 5). However, the sky background could be even more highly polarized, because the ring count rates were made up of about 75 per cent spill out from the unpolarized star and of 25 per cent polarized sky background.

4.3 Raw data binning for the Crab analysis

For each incoming and detected photon, the OPTIMA data acquisition system stores its TOA in a compact proprietary binary format with a resolution of 4 $\mu$s. By using the OPTIMA system software (Straubmeier 2001), one can obtain TOAs in units of Julian Date (JD). In order to apply the rotational model of the pulsar derived from radio observations to our optical photons, it is required to measure the TOAs in an ‘inertial observer frame’. Using the NOT position coordinates, the Crab pulsar coordinates (Table 6) and as planetary ephemeris the Jet Propulsion Laboratory Solar System Ephemeris DE200 tabulations (Standish 1982), we transformed the recorded TOAs to TOAs at the Solar system barycentre, the inertial frame commonly used for this purpose.

Parameters of the rotational model of the Crab pulsar (Table 6) obtained from the radio observations are published each month by the Jodrell Bank Observatory pulsar group in the Crab Pulsar Monthly Ephemeris$^7$ (Lyne, Pritchard & Graham-Smith 1993). For each recorded event (TOA), the corresponding pulsar phase and the phase of the RPF are calculated. We sort the individual events into a three-dimensional data array. The first dimension is given by the rotational phase of the pulsar (binned with various resolutions), the second one by the phase of the polarization filter at each photon TOA (binned in 1$^\circ$ intervals) and the third dimension is the fibre number or number of DAQ channel (seven channels).

To obtain an ‘unpolarized’ light curve, the events spread out over the RPF phase are all added up. A time resolution of 67 $\mu$s, i.e. 500 bins per rotational period of the Crab pulsar, ensures a good signal-to-noise (S/N) ratio even for the phase ranges with very low intensity. The Crab light curve in its raw form, i.e. the measured count rate in the central fibre without background subtraction, is shown in the top panel of Fig. 2. The same light curve after background subtraction (method described in the next section) is shown in the lower panel of Fig. 2 with a logarithmic scaling to enhance the visibility of the DC level. We indicate the components MP, IP, non-zero intensity level between two peaks – bridge and the DC region, previously known as the so called ‘off-pulse’ component. For our further analysis, we define the DC component as the counts between phases 0.7729 and 0.8446, which is in accordance with the findings of Percival et al. (1993). The higher S/N ratio in the peaks allows us to use a better time resolution of up to 3000 bins per period, i.e. 11 $\mu$s bin interval, for these phase ranges (Figs 6 and 7).

5 RESULTS FOR THE CRAB NEBULAE AND PULSAR

5.1 Nebular contribution

The Crab synchrotron nebula is a relativistic magnetized plasma that is powered by the spin-down energy of the pulsar. It was the first recognized astronomical source of synchrotron radiation. The synchrotron nature of the radiation was confirmed by optical polarization observations (Woltjer 1957). The conversion efficiency of the nebula is quite high, with 10–20 per cent of the spin-down energy released by the pulsar appearing as synchrotron radiation. The inner synchrotron nebula is a region consisting of jets, a torus of X-ray emission, small-scale variations in polarization and spectral index and complexes of sharp wisps. Most theoretical models associate the sharp wisps seen at visible and radio wavelengths with the location of the shock wave between the pulsar and the synchrotron nebula. Closer to the outer boundary of the nebula are the filaments – the chemically enriched material ejected during the supernova explosion observed by Chinese astronomers in 1054.

Observations of the Crab nebula with ground-based telescopes are fundamentally limited by atmospheric seeing. Observations from space with the Wide Field and Planetary Camera 2 (WFPC2) on board of the HST (Hester et al. 1995) with 0.1 arcsec angular resolution,
Figure 2. The Crab pulsar light curve obtained from photons recorded by the OPTIMA central fibre APD (channel 0). The components of the light curve are indicated as follows: MP; IP; non-zero intensity level between MP and IP, i.e. bridge as well as the DC region, previously known as the ‘off-pulse’ component in the phase range 0.7729–0.8446. The upper panel shows the raw light curve including counts from the nebula and the sky background. In the lower panel, the light curve is shown in a logarithmic scale after subtraction of the nebular and sky backgrounds. This display shows clearly the DC intensity level. Two rotation periods are shown for clarity.

However, revealed details of the nebular environment of the Crab pulsar that are important in the context of this paper and which we summarize in the following section.

They discovered a bright knot of visible emission located 0.65 arcsec south–east of the pulsar, along the axis of the system (Fig. 1). This inner knot, along with a second similarly sharp but fainter knot located at a distance of 3.8 arcsec from the pulsar (hereafter outer knot), lies at an approximate PA of ∼115° north to east. Both knots are aligned with the X-ray and optical jet to the south–east of the pulsar and are elongated in the dimension roughly perpendicular to the jet direction, with lengths of about half an arcsecond. Both the inner and outer knot appear to be present but not well resolved in the images of the Crab nebula previously taken by ground based telescopes. Fig. 1 is an enlargement of the co-added HST WFPC2 images of 12 observations of the Crab nebula taken in between 2000 and 2001 with the F547M filter (observations group index 6 of Ng & Romani 2006, kindly supplied by Roger Romani, private communication). The pulsar is identified with the lower/right of the two stars near the geometric centre of the nebula. The OPTIMA fibre bundle centred on the pulsar and scaled with the NOT focal plane scale is overplotted. It is clear that we are not able to resolve the Crab pulsar from the inner knot within the central fibre. Moreover, with seeing being larger than 1.0 arcsec the pulsar light is somewhat spread into the ring channels. Evidence of this effect is seen in the light curves when the intensities measured by individual ring fibres are folded with the pulsar phase and pulsed emission is detected. Additionally, there may be contributions of photons coming from the outer knot (not very clearly seen in the Fig. 1), but an indication of it can be seen on the south–east side in between the fibres 3 and 4. It is also noteworthy to mention that the ring fibres see different patches of nebulosity. One third of all counts are recorded in the central fibre. The background fibres contribute to the total number of counts with a percentage of 9, 10, 14, 11, 12 and 11 per cent for channels from 1 to 6, respectively.

To obtain the polarization characteristics of the pulsar neighbourhood, we assumed that within the DC phase range the contribution of the pulsar emission to the ring fibres is minimal. The integrated pulsed contribution in each fibre (spill out from the pulsar) with respect to the total counts in the fibre is on the level of 3.1, 4.5, 4.4, 2.5, 2 and 1.9 per cent for the channels from 1 to 6, respectively. Therefore, during our background (nebula) calculations we consider only light coming within the ‘off-pulse’ phase range, i.e. 7 per cent of the whole rotational cycle of the Crab pulsar. The resulting PD and PAs for each of the single OPTIMA apertures are given in Table 7, as well as illustrated in the
Table 7. The Crab nebula’s PD and PA in the DC phase range of the pulsar rotation.

| Channel | \( p \) (per cent) | \( \theta \) (°) |
|---------|----------------------|-----------------|
| 0       | 15.15(6)             | 127.8(1)        |
| 1       | 9.05(10)             | 141.3(3)        |
| 2       | 10.30(9)             | 142.5(3)        |
| 3       | 8.77(7)              | 148.4(2)        |
| 4       | 9.35(9)              | 136.5(3)        |
| 5       | 11.50(8)             | 133.1(2)        |
| 6       | 10.26(8)             | 137.9(2)        |
| \(<1\>–6\) | 9.71(8)             | 139.8(2)        |
| \(0–<1\>–6\) | 33.08(22)          | 118.8(2)        |

Table 8. PD and PA at three selected Crab pulsar phases, i.e. the phases of both optical maxima, IP (0.398) and MP (0.993), as well as the radio phase (0.000). Values before and after DC component subtraction are shown (binning 1000 per cycle).

| Phase  | PD (per cent) | PA (°) | After DC subtraction |
|--------|---------------|--------|----------------------|
| 0.398  | 7.6 ± 0.3     | 109.9 ± 1.1 | 5.8 ± 0.3  | 106.0 ± 1.6 |
| 0.993  | 5.9 ± 0.2     | 119.8 ± 0.8 | 5.3 ± 0.2  | 119.9 ± 0.9 |
| 0.000  | 3.3 ± 0.2     | 149.6 ± 1.6 | 3.0 ± 0.2  | 157.5 ± 1.7 |

Figure 3. Polarization of the Crab nebula as measured by each single fibre at the minimum phase of the pulsar light curve. It is a graphic representation of the results given in Table 7. The axes are calibrated in arcseconds centred on the pulsar. The scaled OPTIMA fibre bundle is overplotted on the figure given in S88. The aperture of a single fibre is 2.35 arcsec. The solid red lines show the polarization detected over the OPTIMA apertures, accepting only photons arriving in the DC phase interval. The dashed red line in ‘Ch 0’ shows the average polarization derived from the ring of apertures (Ch 1 to Ch 6) and is representative of the larger-scale polarization of the nebula. The dashed blue line in the centre accounts for the polarization needed to explain the difference between the polarization measured in the central 2.35 arcsec and the larger-scale average. As such it could represent the polarization of light coming from the inner knot and the DC level of the pulsar. The thin black lines are the results that S88 determined with a polarimeter based on a rotating half-wave plate, a Foster prism and an aperture of 2 arcsec.

Fig. 3. We compared our results with previous ones by overplotting them on the polarization sky map of the very close pulsar neighbourhood presented by S88. By averaging the Stokes parameters over all background channels, we get \( p = 9.7 \pm 0.1 \) per cent and \( \theta = 139.8 \pm 0.2 \) for the region surrounding the pulsar. Close to the pulsar, the nebular polarization is quite uniform (~9–11 per cent) but the PAs change steadily with radial distance, 2–3 arcsec from the pulsar the mean value is around 140°, but beyond 5 arcsec the PA exceeds 155° and becomes very position dependent (McLean et al. 1983).
5.2 Polarization characteristics of the Crab pulsar

The Crab pulsar is detected at all phases of rotation, i.e. also in the so-called ‘off-pulse’ phase with an intensity of about 2.13 per cent compared to the maximum intensity of the MP (Fig. 2). As mentioned before, the measured level of the DC component differs from author to author. For example, very early measurements performed by Peterson et al. (1978) give the ‘unpulsed background’ from the Crab pulsar on the level of 3.6 per cent of the main peak intensity. Much lower values were obtained by Jones et al. (1981) and later by S88: 0.6 and 1.2 per cent, respectively. In addition, Percival et al. (1993) claim that the ‘off-pulsed’ flux has an intensity less than 0.9 per cent of the peak flux [visible and ultraviolet (UV) data from HST, 2σ upper limit], whereas a fractional flux derived by Golden et al. (2000a) from photometric analysis gives ~1 per cent.

For the background (nebula and sky) subtraction, we took the averaged Stokes parameters \(I_{DC}, Q_{DC}, U_{DC}\) recorded in the ring fibres over the ‘off-pulse’ phase. The pulsar Stokes parameters \(I, Q, U\); Fig. A1) as a function of its rotational phase are derived after subtracting the steady nebular component from the central channel measurements. The colour-coded Stokes parameters \(Q, U\) as a vector diagram are shown in Fig. 4. Colours refer to the pulse phase as indicated in the inset and the scale is such that \(I = 100\) at the maximum light, i.e. maximum of the MP. The MP and IP maxima are indicated with black open squares. Points belonging to the MP phases follow an outer ellipse (upper panel), whereas those belonging to the IP an inner one (bottom panel). In both cases, the direction of increasing pulsar phase is counterclockwise. Noteworthy is that already from the Stokes parameters one can see that there is a sudden change in the pattern near the radio phase, i.e. where the red points change to the black ones. As the next step of the data analysis, the polarization characteristics, the PA and the degree of polarization of the \(E\)-vector, are calculated from the Stokes parameters (equation A2). Results plotted with a different time resolution are presented in Figs 5 and 6.

5.3 Polarization characteristics of the Crab pulsar after DC subtraction

The apparent constancy of the PA within the phase range 0.78–0.84 (Fig. 5, upper-left panel) may suggest that the optical emission from the Crab pulsar consists of two components – pulsed and unpulsed. The pulsed component is characterized by a highly variable PA and PD.
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The unpulsed, i.e. DC component, is characterized by constant intensity on the level of 2.13 per cent of the MP intensity, fixed $\theta \sim 119^\circ$ and a degree of polarization on the level of 33 per cent. Its source of emission is unclear but one can consider either an origin in the pulsar magnetosphere, in the pulsar wind zone, or in the nebular features close to the pulsar. It is also possible that the inner knot (located only 0.65 arcsec apart from the pulsar and being a persistent feature throughout the sequences of the HST images; Hester et al. 2002) contributes to the ‘off-pulse’ emission, although Golden et al. (2000a) claim that the Crab image during the off-phase is compatible with an unresolved point source. Assuming that the unpulsed component is present at all phase angles and has constant polarization, we obtained the polarization characteristics of the ‘pulsed component’ separately by subtracting the respective Stokes parameters $I_{\text{DC}}, Q_{\text{DC}}, U_{\text{DC}}$. Results are presented in Fig. 5 (right-hand column) and Fig. 7. After subtracting the ‘unpulsed component’, the PA and PD in the phases where the intensity is very low are not well defined. The values of $\theta$ and $p$ become very noisy because the Stokes parameters go basically to zero for these rotational phases.

The PD of the ‘off-pulse’ component obtained in this work (33.1 $\pm$ 0.2 per cent) differs from the values presented by other authors: e.g. Jones et al. (1981), $\sim$70 per cent; S88, $\sim$47 $\pm$ 10 per cent. The reasons for these discrepancies could be systematic resulting from the instruments or observational procedures, due to the definitions used for the ‘off-pulse’ phase interval, or intrinsic to the Crab pulsar. In terms of systematics, the measurements of Jones et al. (1981) and S88 were performed with single-aperture photometers, where the observations with the pulsar in the aperture and the observations of the nebular background were done sequentially and possibly under changing seeing conditions. In the OPTIMA measurement, seven apertures were recorded simultaneously, and we think this leads to a more secure background subtraction. The ‘off-pulse’ phase interval was defined by S88 as an interval of 20 per cent of the pulsar rotation around the minimum of the light curve, whereas we used only $\sim$7 per cent for this range. The estimate of S88 for the background polarization at the position of the pulsar, $\text{PD} = 8.1 \pm 0.6$ per cent and $\text{PA} = 152^\circ \pm 2^\circ$, differs from our results given in Table 7. In their interpolation, they included nebula measurements out to a distance of about 4 arcsec from the pulsar (Fig. 3 of S88), while we only average out to about 3 arcsec with the fibres. In the N-S direction, S88 have some large PA values far from the pulsar and that might be reflected in the interpolation for the Crab position. As regards to the possible intrinsic changes in the Crab pulsar and its closest environment, we note that the predicted decrease of the optical...
luminosity is \(\sim 1\) per cent each 2 years (Pacini 1971). Therefore, one would expect a reduction of 0.13 mag over the 22 years between Jones et al. (1981) and our observations. The consequences for the polarization of the emission are, however, not known.

5.4 Alignment between optical and radio wavelengths

Precise timing of pulsar light curves throughout the electromagnetic spectrum can be used to constrain theories of the spatial distribution of various emission regions and their specific propagation delays. In the radio regime, this concept has been applied in the so-called frequency mapping analysis. From recent observations at different energies, it became clear that the Crab pulsar emission maxima of MP and IP are not aligned in phase at different wavelengths, from radio to the \(\gamma\) energy range. Comparing the visible and UV light curves obtained from \textit{HST}, Percival et al. (1993) were among the first to show that the phase separation between the two peaks, ipso facto the phases of the pulse

![Figure 6](image1.png)

**Figure 6.** Polarization characteristics of the Crab pulsar – PA (top row) and PD (bottom row) – as a function of rotational phase. Data binning is 1000 and 3000 bins per cycle (corresponding to 33.5 and 11 \(\mu\)s per bin) for odd and even columns, respectively. The first two columns are zoomed around the MP phase (using different scales), whereas the next two are zoomed around the IP phase. Dot–dashed vertical line indicates the optical MP maximum phase (\(= 0.993\)), as well as the IP maximum phase (\(= 1.398\)). Solid vertical line indicates the radio peak phase (\(= 1.0\)). For clarity, the optical light curve of the Crab pulsar is overplotted (solid line).

![Figure 7](image2.png)

**Figure 7.** Same as in Fig. 6 but after DC subtraction.
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Figure 8. Optical polarization characteristics of the Crab pulsar (PA – top row, PD – bottom row) compared with the pulsar radio profiles. Profiles obtained at radio frequencies of 1400 and 610 MHz are shown as a solid line and dashed line, respectively. Left-hand column shows zoom around the MP phase, whereas right-hand column around the IP phase. Points indicate the optical polarization measurements, while the dotted line shows the optical intensity profile.

maxima, changes with energy. Since then, many authors have measured this effect in an attempt to understand its relation to and impact on the emission mechanism. However, the techniques used by different authors to measure the phase separation (i.e. the phases of the peak maxima) have varied. Therefore, this might cause method-dependent biases. Eikenberry & Fazio (1997) showed that the peak-to-peak separation appears to be a more or less smooth function of energy from infrared to γ-ray energies. The separation decreases from $0.4087 \pm 0.0003$ to $0.398 \pm 0.003$ with energy over the range from $0.5\,\text{eV}$ to $1\,\text{MeV}$, respectively. There is some evidence of a turnover or a break in this trend at energies of $0.7\,\text{eV}$ ($H$ bandpass filter). No default method exists for determining the position of the peak of the profile. For our purpose and using our high-statistics light curve, it was enough to determine the peak phases just by looking for the maximum intensity. This gives us the phase values for the MP: $0.993 \pm 0.001$ and the IP: $0.398 \pm 0.001$. Our peak-to-peak separation is therefore $0.4050 \pm 0.0014$. It is in very good agreement with the values obtained by Eikenberry & Fazio (1997) for the visual passband, i.e. $0.4057 \pm 0.0003$, and from a previous OPTIMA observation of $0.4060 \pm 0.0003$ (Straubmeier 2001).

Simultaneously with the optical observations, we performed radio observations at Jodrell Bank observatory, which allows us to compare these wavelength ranges. Pulsed radio emission at 610 and 1400 MHz is detectable only around the peaks of emission due to the strong background from the nebula. Fig. 8 shows the radio light curves. At 610 MHz, the MP is preceded by the so-called precursor peak, which is mostly visible at lower frequencies and is not detected at 1400 MHz. The nature of this radio precursor is unclear – some researchers proposed that it is the proper emission from the polar cap of the pulsar while the main peaks are generated higher up in the magnetosphere (e.g. Rankin 1990). Recently, Petrova (2008) showed that different components in the Crab pulse profile may be induced by the scattering from different harmonics of the particle gyrofrequency that takes place at different magnetospheric altitudes. This and the rotational effect give rise to the components. In this model, the low-frequency component is formed by the scattering from the first harmonic of the gyrofrequency into the state below the resonance, whereas the precursor is formed by the scattering between the states below the resonance. This model is well supported by the radio polarization data.

As can be seen in Fig. 8, the optical maxima of the MP are leading the radio pulse. A similar lead has also been found at X- and γ-ray energies. The reported values of the time lag are as follow: $344 \pm 40\,\mu\text{s}$ (Rots, Jahoda & Lyne 2004, RXTE data); $280 \pm 40\,\mu\text{s}$ (Kuiper et al. 2003, INTEGRAL data) and $241 \pm 29\,\mu\text{s}$ (Kuiper et al. 2003, EGRET data). The uncertainty of the latter value does not include the EGRET absolute timing uncertainty of better than $100\,\mu\text{s}$. At optical wavelengths, the situation is quite different and does not provide such a coherent picture. The quantitative amount of the lag between optical and radio peaks, and whether it exists at all and at all times, is controversial when we compare results obtained in different investigations. Several authors reported the optical peak leading the radio peak by a time shift of $140 \pm 78\,\mu\text{s}$ (Sanwal 1999), $100 \pm 20\,\mu\text{s}$ (Shearer et al. 2003) and $255 \pm 21\,\mu\text{s}$ recently found by Oosterbroek (2008). On the other hand, Golden et al. (2000b) reported that the optical pulse trails the radio pulse by about $80 \pm 60\,\mu\text{s}$. Additionally, Romani et al. (2001) concluded
that both, radio and optical, peaks are coincident to better than 30 μs, but this analysis did not take into account the uncertainty of the radio ephemeris being on the level of 150 μs in the error calculations.

From our measurements, we conclude that the optical main peak is leading the radio peak by a time shift of 235 ± 68 μs. The uncertainty in this value is composed from an uncertainty of 33 μs in the determination of the optical peak of the MP and from 60 μs in the radio ephemeris. Our value of the optical phase difference between the MP and IP of 0.4050 ± 0.0014 is consistent with the latest optical measurements carried out by Oosterbroek et al. (2006), who obtained 0.4054 ± 0.0004. Both values, as it has already been shown by Eikenberry & Fazio (1997), are not consistent with the X-ray results, e.g. obtained from RXTE data by Rots et al. (2004) of 0.4001 ± 0.0002. This implies that the details of the pulse profile in X-rays and in the optical domain are different. In a simple geometrical model (ignoring relativistic effects), a time shift of ~235 μs indicates that possibly the optical radiation is formed ~70 km higher in the magnetosphere than the radio emission. The difference in phase of 0.007 could also be interpreted as an angle between the radio and optical beam of ~2.5 (neglecting aberration and magnetic sweepback).

Another striking correlation between the radio intensity profile and the optical polarization can be seen in Figs 8 and 9: the radio precursor seems to be perfectly aligned with the bump in the degree of optical polarization. During this phase of the leading wing of the optical MP, the PA change is also characterized by a nearly linear swing. At the present stage of modelling the coherent and incoherent emissions from a pulsar magnetosphere, where both processes are generally treated independent of each other and mutual interactions have not been investigated in depth, it is premature to speculate on the origin of these observational results.

6 SUMMARY AND DISCUSSION

The Crab pulsar emits highly anisotropic radiation which spans a wide range of wavelengths, from radio to extreme γ-rays. Knowledge of polarization characteristics of this radiation is of fundamental importance in our attempts to find the mechanisms responsible for Crab’s magnetospheric activity. Good-quality X-ray and γ-ray polarimetry with satellite observatories is expected to be available for pulsar studies (among other types of objects) in the near future. Present-day state of instrumentation allows to carry out optical polarimetry of this object with unprecedented quality. Our project to study the Crab pulsar with OPTIMA at NOT is, to the best of our knowledge, the most recent and most complete one. The observations with 11 μs time resolution are an order of magnitude better than the previous best observations. We have completely resolved the polarization characteristics of both peaks of the Crab pulsar, MP and IP, in the optical passbands (see Fig. 6). Moreover, we were able to better characterize the polarized emission between the peaks, i.e. the bridge as well as the DC (‘off-pulse’) region. We find that the MP of the Crab pulsar arrives 235 ± 68 μs before the peak of the radio pulse.
Optical polarization of the Crab pulsar

(i) The polarization characteristics of both, the MP and the IP components, are quite similar.
(ii) The PD reaches a minimum at phase close to the radio main peak; the minimum is not aligned with the optical peak.
(iii) There is a well-defined bump in the PD on the rising flank of the MP.
(iv) There is an indication of such a bump also for the IP (especially after DC subtraction).
(v) The PA swings through a large angle in both peaks: after subtraction of the DC component, the angle swing is 130° and 100° for MP and IP, respectively.
(vi) The PA at the bridge and ‘off-pulse’ phases is constant.
(vii) The PA slope changes dramatically at phases 0.993 (MP maximum) and 1.0 (radio peak).
(viii) The trailing wing of the MP (phase range 1.0–1.03) shows a linearly increasing degree of polarization. This feature turns into a bump shape after DC subtraction. There is a slight indication of the same behaviour for the trailing wing of the IP.

The phase-averaged PD of the Crab pulsar amounts to 9.8 ± 0.1 per cent with a PA of 109.5 ± 0.1. After the DC subtraction, it is 5.5 ± 0.1 per cent and 96.4 ± 0.1, respectively. We note that the DC-subtracted average PA is closely aligned with the pulsar proper motion direction (PA = 278° ± 3°, an equivalent to 98° ± 3°; Ng & Romani 2008), and off by 27.6 ± 0.2 to the pulsar’s spin axis (Ng & Romani 2008).

Minimum PD occurs at the phase of 0.999, very close to the radio pulsar phase, where F = 3.3 ± 0.2 per cent, 149°6 ± 1°6 before the DC subtraction and p = 3.0 ± 0.2 per cent, 157°5 ± 1°7 after DC subtraction.

Because the peaks separation changes with energy, there is no well-defined ‘zero’ radio phase for the IP, i.e. the IP radio phase depends on the observational radio frequency. Moreover, it is not possible to state the exact phase at which the PD reaches minimum in the phase range 1.42–1.44 – it is more like a plateau than a well-defined and sharp minimum. On the other hand, there might be an indication of a small bump-shaped structure in this phase range, similar to the one observed for the MP in the phase range 1.0–1.05 (see Fig. 7, bottom row). Thus, we can only remark on some general trends.

During the IP, the minimum value of p is on the level of 5 per cent, before DC subtraction. Similar to the MP case, it is also shifted (with respect to the phase of the optical maximum) to the trailing wing of the pulse. The minimum PD changes after DC subtraction to ~2.13 per cent. This means that the situation is inverted. Before the DC subtraction, the minimum of PD is reached during the MP, whereas after this subtraction it is observed during the IP.

The Crab is the pre-eminent example of a high-energy source with polarized optical emission – if the pulsar is not phase resolved, the polarization averages out to about 10 per cent, similar to other optical pulsars. Polarization can be generally expected if the radiating particles are confined, e.g. by magnetic forces, to anisotropic distributions. The detection of polarized objects in the field of unidentified γ-ray sources could therefore be a valuable tracer to aid identification.

Our results agree generally well with previous measurements (e.g. S88; Kanbach et al. 2005), but they show details with much better definition and statistics. The behaviour of PA as a function of phase observed for the Crab pulsar at optical wavelengths (Fig. 5) differ from those observed at radio wavelengths (e.g. Moffett & Hankins 1999; Karastergiou, Jessner & Wielebinski 2004; Słowikowska et al. 2005). Two factors may be responsible for this difference: different propagation effects and different intrinsic emission mechanisms. In particular, the former factor plays an essential role in such high-energy emission models like the outer gap model or the two-pole caustic model: in both models, high-energy emission comes from a very wide range of altitudes, contrary to radio emission which originates within a narrow range of altitudes. For comparison, the light curves and polarization characteristics obtained within the framework of three high-energy magnetospheric emission models of pulsars, i.e. the polar cap model, the two-pole caustic model and the outer gap model are shown in Fig. 10 (Dyks, Harding & Rudak 2004a,b). The two-pole caustic model (Dyks & Rudak 2003) predicts fast swings of the PA and minima in the PD, similar to what is observed. The polar cap and outer gap models do not reproduce the observational polarization characteristics of the Crab pulsar adequately. Another model, placing the origin of the pulsed optical emission from the Crab in a striped pulsar wind zone, has been proposed by Petri & Kirk (2005). This model features also polarization characteristics that bear a certain resemblance to the observations. The agreement is, however, much better after subtracting the DC component (see Fig. 6 of Petri & Kirk 2005).

Recently, Takata, Chang & Cheng (2007) attempted to simultaneously reproduce all known high-energy emission properties of the Crab, including its optical polarization characteristics within the framework of a modified outer gap model. The model is restricted to synchrotron emission due to secondary and tertiary electron–positron pairs which are expected in different spatial locations of the three-dimensional gap; internal polarization characteristics are calculated with particular care. Yet, the calculated polarization properties for optical light hardly reproduce the observed properties. However, similarly in the case of the two-pole caustic model (Dyks et al. 2004a,b), the polarization characteristics obtained in Takata et al. (2007) become more consistent with the Crab optical data after the DC component is subtracted (as mentioned in the previous section). An important outcome of Takata et al. (2007) is that it offers the energy dependence of the polarization features, covering the energy range between 1 eV and 10 keV.

The models of pulsar magnetospheric activity are based on various (sometimes ad hoc) assumptions and different boundary conditions. Those lead to the model differences in the macro scale (spatial extent of accelerators and emitting regions) as well as in the micro scale (specific radiative processes). The former include polar gaps, slot gaps, caustic gaps, outer gaps and striped winds. The latter include e.g. curvature radiation, synchrotron radiation and inverse Compton scattering. In consequence, the models differ significantly in the resulting
Figure 10. Comparison between observations and models. The left-hand column shows results of our measurements of the Crab pulsar (DC subtracted) optical polarization characteristics, i.e. light curve (Stokes I), PA and PD as a function of pulsar phase (from top to bottom, respectively). In the next four columns, the optical light curve, PA and PD calculated for the following high-energy radiation models are shown: the two-pole caustic model; the polar cap model and the outer gap model (Dyks et al. 2004b), as well as the striped wind model (Pétri & Kirk 2005). The latter model is calculated for two Lorentz factors, 20 and 50. They are shown as solid and dashed lines, respectively. (We thank to Jarek Dyks and John Kirk for supplying the numerical values.)

‘observed’ radiation properties: light curves, energy spectra and – last but not least – polarization (Romani & Yadigaroglu 1995; Dyks et al. 2004a,b; Pétri & Kirk 2005; Takata et al. 2007).

Linear polarization characteristics in the high-energy domain (optical, X-rays and γ-rays) is considered as a powerful tool which may lead to a breakthrough in our understanding of pulsar emission mechanism. For this reason, the optical high time-resolved polarization properties obtained for the Crab pulsar have attracted particular attention due to their uniqueness. Some models, like the two-pole caustic model (Dyks et al. 2004a,b), the outer gap model (Romani & Yadigaroglu 1995; Takata et al. 2007) or the striped pulsar wind model (Pétri & Kirk 2005) are able to reproduce (very roughly) some of these properties, e.g. (i) and (v). However, a fully convincing explanation of the properties listed from (i) to (viii) is beyond the reach of all above-mentioned models.

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APPENDIX A

Here, we present step by step our polarization data analysis based on the method described by Sparks & Axon (1999). The RPF inside the OPTIMA instrument provides polarimetric data that represent a series of ‘images’ of an object taken through 180 sets of linear polarizers, when we bin the continuous rotation of the RPF into discrete 1° intervals. A single polarizer is not a 100 per cent perfect polarizer, but its characteristics are well established, and this is essential for the chosen data analysis method. From an input data set of 180 independent intensities \( I \) (measured in counts) and their errors \( \sigma_i = \sqrt{I} \), corresponding to a set of observations through 180 identical but not perfect polarizers, we derive the Stokes parameters, following the case of \( n \) polarizers after Sparks & Axon (1999).

Linearly polarized light requires measurement of three quantities to be fully characterized. There are various ways of expressing this. The most common one involves the total intensity of the light \( I \), the degree of polarization \( p \) and the PA \( \phi \). An intermediate stage between the input data and the solution of polarization quantities are the Stokes parameters \( (I, Q, U) \) that are related through

\[
Q = Ip \cos 2 \theta, \quad U = Ip \sin 2 \theta, \tag{A1}
\]
or equivalently,

\[
p = \frac{(Q^2 + U^2)^{1/2}}{I}, \quad \theta = \frac{1}{2} \arctan \left( \frac{U}{Q} \right). \tag{A2}
\]

These quantities describe all intrinsic properties of linearly polarized radiation. We would like to underline that they should not be confused with the properties of the polarizing elements of the polarimeter, which in our case are the properties of the measured intensities in each 1° intervals of the RPF, i.e. in each of 180 polarizers. These three quantities that characterize fully the behaviour or response of the linearly polarizing element are:

(i) its overall throughput (hereafter \( t \), in particular to unpolarized light;
(ii) its efficiency as a polarizer (hereafter \( \epsilon \)), i.e. the ability to reject and accept polarized light of perpendicular and parallel orientations and
(iii) the PA of the polarizer (hereafter \( \phi \)).

There are a variety of conventions commonly used to present these quantities (Mazzuca, Sparks & Axon 1998). Here, we adopt the convention after Sparks & Axon (1999), i.e. the output intensity of a beam with input Stokes parameters \( (I, Q, U) \) passing through a polarizing element is given by

\[
l_k = \frac{1}{2} t_k [I + \epsilon_k (\cos 2 \phi_k Q + \sin 2 \phi_k U)], \tag{A3}
\]

where \( t_k \) is related to the throughput of unpolarized light, \( \epsilon_k \) is the efficiency of the polarizer and \( \phi_k \) is the PA of the polarizer \( k \). During our measurements, we always use the same polaroid, just the PA of the filter changes, therefore \( \epsilon_k = \epsilon = 0.998, t_k = t = 0.32 \) (reference: Polarization Filter Type VIS 4 K, Linos Photonics), and \( \phi_k \) takes values from 0 to 179 with 1° steps.

Following the equations given by Sparks & Axon (1999), we define the new three-component vector for the effective measurements:

\[
I''_1 = \sum \frac{l_k t_k}{\sigma_k^2}, \quad I''_2 = \sum \frac{l_k t_k \epsilon_k \cos 2 \phi_k}{\sigma_k^2}, \quad I''_3 = \sum \frac{l_k t_k \epsilon_k \sin 2 \phi_k}{\sigma_k^2}. \tag{A4}
\]

The error estimate for each measured intensity (=counts) is based on Gaussian statistics; therefore, \( I_k = \sigma_k^2 \) and the corresponding factors reduce to unity. The results of calculating the effective intensity components \( I''_1, I''_2 \) and \( I''_3 \) are shown in Fig. A2. In our case, these values are constant. \( I''_1 \) is just a sum over the same 180 polarizers with the same throughput to unpolarized light, i.e. \( t = 0.32 \). Therefore, we get: \( I''_1 = 180 \times 0.32 = 57.6 \). Whereas, \( I''_2 = 180 \epsilon \sum \cos 2 \phi_k \) and \( I''_3 = 180 \epsilon \sum \sin 2 \phi_k \), where \( \sum \) denotes a sum over index \( k \) of the 180 polarizers. Knowing that \( \epsilon = 0.998 \) and the integrals of \( \cos 2 \phi_k \) and \( \sin 2 \phi_k \) over the range from 0° to 180°, we expect \( I''_2 \sim I''_3 \sum \cos 2 \phi_k \) and \( I''_2 \sim I''_3 \sum \sin 2 \phi_k \), within available numerical precision, to be very close to zero.

Similarly, we can define a three-component vector of effective transmittances

\[
t''_1 = \sum \frac{t_k^2}{\sigma_k^2}, \quad t''_2 = \sum \frac{t_k^2 \epsilon_k \cos 2 \phi_k}{\sigma_k^2}, \quad t''_3 = \sum \frac{t_k^2 \epsilon_k \sin 2 \phi_k}{\sigma_k^2}, \tag{A5}
\]

and the vector of effective efficiencies

\[
\epsilon''_1 = \frac{1}{\sum t_k^2/\sigma_k^2} \sqrt{\left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \cos 2 \phi_k \right)^2 + \left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \sin 2 \phi_k \right)^2},
\]

\[
\epsilon''_2 = \frac{1}{\sum t_k^2 \epsilon_k \cos 2 \phi_k/\sigma_k^2} \sqrt{\left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k^2 \cos 2 \phi_k \right)^2 + \left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k^2 \sin 2 \phi_k \cos 2 \phi_k \right)^2},
\]

\[
\epsilon''_3 = \frac{1}{\sum t_k^2 \epsilon_k \sin 2 \phi_k /\sigma_k^2} \sqrt{\left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k^2 \sin 2 \phi_k \cos 2 \phi_k \right)^2 + \left( \sum \frac{t_k^2}{\sigma_k^2} \epsilon_k^2 \sin^2 2 \phi_k \right)^2}.
\]
as well as a vector of effective PAs

\[
\phi_i' = \frac{1}{2} \arctan \left( \frac{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \sin 2\phi_k}{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \cos 2\phi_k} \right),
\]

\[
\phi_p' = \frac{1}{2} \arctan \left( \frac{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \sin 2\phi_k \cos 2\phi_k}{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \cos 2\phi_k \cos 2\phi_k} \right),
\]

\[
\phi_q' = \frac{1}{2} \arctan \left( \frac{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \sin 2\phi_k}{\sum \frac{t_k^2}{\sigma_k^2} \epsilon_k \sin 2\phi_k \cos 2\phi_k} \right).
\]

The effective intensity, transmittances, efficiencies and PAs are shown in Fig. A2. By making these substitutions, the solution for the Stokes vector is given by

\[
(I, Q, U) = B \begin{pmatrix}
I_p'/(0.5I') \\
Q'_p/(0.5I') \\
U'_p/(0.5I')
\end{pmatrix}
\]

where

\[
B = \begin{pmatrix}
\epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1') & \epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1') & \epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1') \\
\epsilon_1'' \sin 2\phi_1'' - \epsilon_1'' \sin 2\phi_1' & \epsilon_1'' \sin 2\phi_1'' - \epsilon_1'' \sin 2\phi_1' & \epsilon_1'' \sin 2\phi_1'' - \epsilon_1'' \sin 2\phi_1' \\
\epsilon_1'' \cos 2\phi_1'' - \epsilon_1'' \cos 2\phi_1' & \epsilon_1'' \cos 2\phi_1'' - \epsilon_1'' \cos 2\phi_1' & \epsilon_1'' \cos 2\phi_1'' - \epsilon_1'' \cos 2\phi_1'
\end{pmatrix} / \Omega
\]

and

\[
\Omega = \epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1') + \epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1') + \epsilon_1'' \epsilon_1'' \sin(2\phi_1'' - 2\phi_1').
\]

We calculated the covariance matrix (Fig. A3) defined as the inverse of matrix C given by equation 8 in Sparks & Axon (1999). Following the error propagation equation, we calculate the uncertainties of the PD and the PA according to

\[
\sigma_p^2 \simeq \left( \frac{\partial p}{\partial Q} \right)^2 \sigma_Q^2 + \left( \frac{\partial p}{\partial U} \right)^2 \sigma_U^2 + \left( \frac{\partial p}{\partial I} \right)^2 \sigma_I^2 + 2\sigma_{QI} \left( \frac{\partial p}{\partial Q} \right) \left( \frac{\partial p}{\partial I} \right) + 2\sigma_{UI} \left( \frac{\partial p}{\partial U} \right) \left( \frac{\partial p}{\partial I} \right) + 2\sigma_{QU} \left( \frac{\partial p}{\partial Q} \right) \left( \frac{\partial p}{\partial U} \right)
\]

\[
\sigma_q^2 \simeq \sigma_\phi^2 \left( \frac{\partial \phi}{\partial Q} \right)^2 + \sigma_\phi^2 \left( \frac{\partial \phi}{\partial U} \right)^2 + \sigma_\phi^2 \left( \frac{\partial \phi}{\partial I} \right)^2 + 2\sigma_{Q\phi} \left( \frac{\partial \phi}{\partial Q} \right) \left( \frac{\partial \phi}{\partial I} \right) + 2\sigma_{U\phi} \left( \frac{\partial \phi}{\partial U} \right) \left( \frac{\partial \phi}{\partial I} \right) + 2\sigma_{QU} \left( \frac{\partial \phi}{\partial Q} \right) \left( \frac{\partial \phi}{\partial U} \right).
\]

where appropriate standard deviations are the components of the covariance matrix. In general, the diagonal terms dominate the uncertainties. Covariant terms make the maximum contribution on the level of 5 and 1 per cent to the \( \sigma_p \) and \( \sigma_\phi \), respectively.

Figure A1. Stokes parameters I, Q, U derived from 180 polarizers as a function of the Crab pulsar rotation phase (1 cycle = 500 bins). Normalization was such that I = 100 at the maximum and Q and U are normalized to I. These values corresponds to the values shown on the QU plane in the Fig. 5.
Figure A2. From left to right: effective intensity $I_1''$, $I_2''$, $I_3''$, transmittance $t_1''$, $t_2''$, $t_3''$, efficiency $\epsilon_1''$, $\epsilon_2''$, $\epsilon_3''$ and PA $\phi_1''$, $\phi_2''$, $\phi_3''$ derived from 180 polarizers as a function of the Crab pulsar rotational phase (1 cycle = 500 bins).
Figure A3. Covariance matrix as a function of the Crab pulsar rotational phase (1 cycle = 500 bins).

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