Optimal PMU Placement for Outage Detection and Identification using Genetic Algorithm

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Abstract—Phasor Measurement Unit (PMU) technology is increasingly used for real-time monitoring applications, especially line outage detection and identification (D&I) in the power system. Current outage D&I schemes either assume a full PMU deployment or a partial PMU deployment with a fixed placement. However, the placement of the PMUs has a fundamental impact on the effectiveness of the D&I scheme. Building on a dynamic relationship between the bus voltage phase angle and active power, we formulated the optimal PMU placement problem for outage D&I as an optimization problem readily solvable by a heuristic algorithm. We tested the formulation using a genetic algorithm and simulated outages of IEEE 39 bus system. The optimal placement found produces a better D&I result of single-line outages than a randomly scattered, tree-like, and degree-based placements.

Index Terms—Genetic algorithm, optimal placement, outage detection, outage identification, phasor measurement unit (PMU).

I. INTRODUCTION

Power systems are critical infrastructures essential to modern livelihood. It is incredibly complex because of the extensive geographical scale, fast dynamics, and high operational standards. There is also increasing volatility in power systems due to the integration of distributed energy resources. Independent system operators (ISOs) demand more intelligent real-time monitoring tools to detect and locate abnormal events and minimize the economic impact of such events. One common but extensively researched abnormal event in power systems is transmission line outage. Line outages can happen due to various reasons, such as severe weather conditions, equipment failures, component wear and tear.

Outage dynamics propagate through the system in a time scale of milliseconds, and traditional supervisory control and data acquisition is not able to capture these dynamics [1]. Recognizing its real-time monitoring capability, ISOS are progressively installing more phasor measurement units (PMUs) on their power grids. PMUs are devices installed at substations, capable of recording high-precision, high-fidelity, and GPS time-synchronized measurements. An industry-grade PMU could measure substation current and voltage phasors with a total vector error of less than 1% according to the IEEE C37.118.1-2011 standard. The sampling frequency could also reach between 30 to 60 samples per second. Since its introduction, Researchers have been studying PMU technology for tasks such as dynamic state estimation, stability control, and fault detection. See [2] for a comprehensive review of PMU applications in power system.

Recent literatures focus on using PMU data for real-time line outage detection and identification (D&I) in power systems [3]–[8]. However, they either assume PMUs are installed on all the buses, or a limited number of PMUs are installed on pre-determined locations. Due to the economic and data-handling constraint, ISOs need to work with a limited number of PMUs, i.e., some parts of the system are unobservable. The placement of these PMUs can influence the effectiveness of outage D&I scheme. Therefore, it is necessary to investigate the optimal PMU placement (OPP) problem for the specific applications.

OPP problems are combinatorial optimization problems since there are $2^N$ possible placements for a system with N buses. Traditionally, many researchers focus on finding placements optimal for network observability, using variants of heuristic algorithms, e.g., genetic algorithm [9], simulated annealing [10], and Tabu search [11]. Recently, OPP problems are studied for dynamic state estimation [12], bad data detection [13], as well as anomaly detection and localization [14], [15]. In this work, we formulated the OPP problem as an optimization problem where the placement found ensures a minimal approximation error between our dynamic system model and the actual system behavior. This formulation finds explicitly a placement such that the accuracy of line outage D&I can be improved given a limited number of PMUs.

II. METHODOLOGY

A. System Model with Limited PMUs

Given a power system with $N$ buses connected by $L$ transmission lines, the power grid network can be modeled as a graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, $\mathcal{N} = \{1, 2, \ldots, N\}$, and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ where $\mathcal{N}$ is the set of N buses and $\mathcal{E}$ is the set of L transmission lines. For every bus, let $P$ be the net active power, $Q$ be the net...
reactive power, \( V \) be the nodal voltage magnitude, and \( \theta \) be the phase angle. Power flows in the network can be described by the AC power flow model:

\[
P_m = V_m \sum_{n=1}^{N} V_n Y_{mn} \cos(\theta_m - \theta_n - \alpha_{mn}), \quad (1a)
\]

\[
Q_m = V_m \sum_{n=1}^{N} V_n Y_{mn} \sin(\theta_m - \theta_n - \alpha_{mn}), \quad (1b)
\]

for bus \( m = 1, 2, \ldots, N \). \( Y_{mn} \) is the magnitude of the \((m,n)\)th complex admittance of the bus admittance matrix when it is written in the exponential form. Linearizing and retaining the real part of Eqn. \[1\] in the same way as \[8\], we obtain a discrete-time dynamic relationship between \( P \) and \( \theta \), two \((N-1)\)-vectors by removing the reference bus, as:

\[
\Delta P_k = J(\theta_{k-1}) \Delta \theta_k, \quad (2)
\]

where \( \Delta P_k = P_k - P_{k-1} \) and \( \Delta \theta_k = \theta_k - \theta_{k-1} \), the difference between two consecutive timestamps. The elements of the \( J \) matrix by partial differentiation are:

\[
\frac{\partial P_m}{\partial \theta_n} = V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \alpha_{mn}), \quad m \neq n, \quad (3a)
\]

\[
\frac{\partial P_m}{\partial \theta_m} = -\sum_{n=1}^{N} \frac{\partial P_m}{\partial \theta_n}. \quad (3b)
\]

Assuming that, under a normal operating condition, active power fluctuations are due to random changes in electricity demand. We can model the active power mismatch by a Gaussian distribution, \( \Delta P_k \sim N(0, \sigma^2 \Delta t I) \), where \( \sigma^2 \) is pre-determined and \( \Delta t \) is the sampling interval. Therefore, we have

\[
\Delta \theta_k \sim N(0, \sigma^2 (J(\theta_{k-1})^T J(\theta_{k-1}))^{-1}). \quad (4)
\]

Suppose \( K < N \) PMUs are installed on selected buses. Therefore, certain parts of the system are not directly observable, resulting in a degree of information loss as compared to the full PMU deployment case. In particular, for the \( N - K \) buses, we do not observe their bus voltage phase angles and magnitudes. For a full PMU case, every element of the \( J \) matrix can be computed and updated with new PMU measurements. However, this would not be the case for a limited PMU deployment. For example, if there is no PMU installed on bus \( m \), the off-diagonal element \( \frac{\partial P_m}{\partial \theta_j} \) would not be computable, and the summation in the diagonal element \( \frac{\partial P_m}{\partial \theta_m} \) is also affected. This inaccuracy in the \( J \) matrix has an impact on the effectiveness of the relationship \[4\] describing the system’s dynamic behavior.

**B. Genetic Algorithm for Optimal Placement**

Let \( S(n_p) = [x_1, \ldots, x_N] \) denote a fixed PMU placement of \( n_p \) PMUs on a power network of \( N \) buses. In particular, \( x_i = 0 \) if the \( i_{th} \) bus does not have a PMU, and \( x_i = 1 \) if it has, for \( i = 1, \ldots, N \). Given a fixed placement \( S(n_p) \), we define the optimal placement is the one that minimizes the discrepancy between the Jacobian matrix under a full PMU deployment, \( J(\theta) \), and the one under a limited PMU deployment, \( J_S(n_p)(\theta) \). Since the \( J \) matrix is time-variant and dependent on \( \theta \), we assume \( \theta \) follows a probability distribution \( H \) on a close interval of \([-\pi, \pi]\). To quantify the discrepancy over the distribution of \( \theta \), we let

\[
\delta_S(n_p) = \int \left( \| J(\theta) \| \| V - |J_S(n_p)(\theta)| \| V \right) \ dH(\theta), \quad (5)
\]

be the integral of the absolute difference between the Frobenius norms of the two matrices where the norm is defined as

\[
|A|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2}. \quad (6)
\]

Therefore, given a fixed number of PMUs, \( n_p \), the OPP problem is now an optimization problem that can be written as

\[
\min_{S(n_p)} \delta_S(n_p)
\]

\text{s.t.} \quad S(n_p) = [x_1, \ldots, x_N]

\[
\sum_{i=1}^{N} x_i = n_p \quad \forall i \in \{0,1\}
\]

The actual distribution of the phase angles is unknown. We use \( \theta \) under the steady-state condition to approximate the integration. We use a meta-heuristic method, genetic algorithm (GA), to solve this optimization problem. GA is a type of optimization algorithms inspired by the natural reproduction and evolutionary process. These algorithms are more effective than random searches and more efficient than exhaustive searches \[17\]. The major components of a GA consist of a fitness function, an initial population, a mutation and crossover mechanism, and a next-generation selection mechanism. Using \( \theta \) and \( V \) under a normal condition, we evaluate the fitness of a given placement by \[5\]. A mutation of one such individual is defined to be a position shuffle between a random pair of digits with a low probability. Finally, by repeatedly selecting the best one out of three random individuals from the current population, a fixed number of them forms the next generation. This process repeats for a fixed number of generations. In the last generation, the placement with the optimal fitness is chosen as the optimal solution. See Algorithm \[11\] for the details of the GA procedure.

**C. PMU Placement Evaluation**

Here we briefly describe the outage D&I scheme developed in our previous work to evaluate the different PMU placement \[4\]. Since the \( J(\theta) \) matrix is determined by the network topology, a line outage would change the structure of the matrix. Let \( J_{0}(\theta) \) and \( J_{1}(\theta) \) represent the Jacobian with no outage and with outage at line \( \ell \). We can formulate the

\[1\] For more details about the D&I scheme, please refer to \[8\].
Algorithm 1 Genetic algorithm for optimal placement

1: procedure GA(θ)
2: Randomly generate N placements
3: g ← 0
4: while g ≠ 50 do \( \triangleright \) Run for 50 generations
5: Evaluate \( \delta_S(n) \) of every individual
6: Select 100 individuals based on tournament of size 3
7: for all individuals do \( \triangleright \) Create next generation
8: Mutate the individual with probability of 0.2
9: end for
10: g ← g + 1
11: end while
12: return individual with the lowest \( \delta \) \( \triangleright \) The optimal placement
13: end procedure

**TABLE I**

| Placement Strategy         | Placement (Bus) |
|----------------------------|-----------------|
| Scattered                  | 1, 2, 5, 7, 9, 11, 13, 14, 16, 17, 19, 21, 23, 24, 26, 28, 30, 32, 34, 37 |
| Tree                       | 2-5, 7-9, 11-19, 21, 26-28 |
| Degree-based               | 1-8, 10, 11, 13, 14, 16, 17, 19, 22, 23, 25, 26, 29 |
| GA-generated               | 2-5, 8, 10-12, 14-18, 21-25, 27, 35 |

The detection problem as a sequential hypothesis testing problem has the following null and alternative hypothesis:

\[
H_0: \Delta \theta[k] \sim N(0, \sigma^2(J_0(\theta)^T J_0(\theta))^{-1}), \quad (8a)
\]

\[
H_1: \Delta \theta[k] \sim N(0, \sigma^2(J_\ell(\theta)^T J_\ell(\theta))^{-1}), \ell \in \mathcal{L}, \quad (8b)
\]

where \( \mathcal{L} \) is the set of all possible single-line outages. We adopt a generalized likelihood ratio (GLR) control chart approach, which repeatedly evaluates the likelihood of an outage against the likelihood of no outage. The GLR approach detects an outage at time \( D \) where

\[
D = \inf \left\{ k \geq 1: \max_{\ell \in \mathcal{L}} W_{\ell,k} \geq c \right\}. \quad (9)
\]

\( W_{\ell,k} \) is the monitoring statistic of outage scenario \( \ell \), and \( c \) is a pre-determined threshold corresponding to a certain false alarm rate constraint. Following the detection, we identify the tripped line by considering the top three candidates, \( \ell(1), \ell(2), \) and \( \ell(3) \), such that

\[
W_{\ell(1),D} \geq W_{\ell(2),D} \geq W_{\ell(3),D} \geq W_{y,D}, \quad (10)
\]

for all \( y \in \mathcal{L} \). For our placement evaluation, we are concerned with whether the outage can be detected, i.e., \( D \) is less than the simulation duration, and whether the true tripped line is one of the top three lines identified, i.e., \( \ell \in \{\ell(1), \ell(2), \ell(3)\} \).

**III. SIMULATION STUDIES**

**A. Simulation Setting**

Assuming 20 PMUs are available, we test the GA-generated PMU placement as well as two other placements on single-line outages of the IEEE 39 bus New England system \[18\]. Outage dynamics are simulated using the open-source simulation platform COSMIC \[19\]. The sampling frequency of the installed PMUs is assumed to be 30 samples per second. For other outage-related simulation details, please refer to \[6\]. For the parameters in the GA, we set the number of generations to 50, the mutation probability of the individual to 0.2, and the bus index shuffling probability to 0.05. A tournament of size three is used to select the next generation population, set at a size of 100. Three other placement strategies are compared to the optimal placement found by the proposed approach:

1) Scattered placement: The PMUs are randomly scattered across the whole network.
2) Tree placement: The PMUs form a tree with no cycles.
3) Degree-based placement: The bus nodes are ranked based on their degree of connection, e.g., a bus connected to six other buses has a degree of 6. Top 20 buses are equipped with a PMU.

**B. Simulation Results**

See Table I for details of the PMU placements under different strategies and Fig. 1 for an illustration on the test power system\[2\]. Fig. 2 shows the GA-generated placement and it achieved an objective value where \( \delta^*_S(n) = 166 \). The GA-generated placement resembles a spanning tree, albeit with cycles. Bus 16 and bus 18 have central positions in the graph, and they are connected through three and four edges, respectively. While the majority of the PMU buses are connected in a single graph, there is also a separate tree, i.e., bus 10, 11, and 12.

1) **Impact of the placement strategies**: We compare the outage D&I performance of different PMU placements using a heat map with empirical likelihoods as entries. The horizontal axis represents the line identified by the identification scheme, and the vertical axis represents the actual tripped line. The cells of the heat map are color-coded based on the empirical likelihood of identifying the respective line outage based on 1000 line outage simulations. 0 on the horizontal axis indicates a missed detection. A perfect identification would have value 1 on all diagonal cells and 0 everywhere else. The results for a full PMU deployment, a scattered placement, a tree placement, and a degree-based placement are shown in Fig. 3. Fig. 4 shows the performance of the placement found by the GA.

a) **Likelihood of missed detection**: The likelihood of missed detection means how likely the detection scheme would miss the outage under a given PMU placement. The likelihoods are displayed as the first column of all the heat maps where actual outages are identified as 0 by the detection scheme. The GA-generated placement shows a similar missed detection performance as compared to that of a full PMU deployment. There is a different degree of likelihood for missed
Fig. 1. Three different PMU placements on the 39 bus system where the black dots represent the PMU’s.

Fig. 2. GA-generated placement.

Fig. 3. Heat map showing the D&I performance under different placement strategies.

Fig. 4. Identification results for GA-generated placement.

detection across many lines under the scattered placement. The degree-based placement could not detect outages at line 18 to 36 with high probabilities, suggesting that a placement solely based on the topology graph connections might not be adequate for outage detection.

b) Likelihood of correct identification: The likelihood of correct identification means how likely the tripped line could be accurately identified. This aspect could be analyzed by looking at the diagonal entries of the heat maps. For the GA-generated placement, its performance matches that of the full deployment for most of the outages. Line 12 to 15 are not accurately identified, likely due to the lack of PMUs nearby. The scattered placement does not perform well, as seen from Fig. 3b. Many of the outages were not located. The reason is likely that many PMUs are isolated in this placement scenario, as seen from Fig. 1a. The isolation likely results in more inaccuracies between the Jacobian under a full and a partial PMU deployment. The tree placement, on the other hand, produced a decent identification performance with some
inaccuracies towards the last part, as seen from Fig. 3c.

2) Impact of the number of PMUs installed: A fewer number of PMUs available corresponds to a higher degree of information loss. To quantify the impact, we implement the proposed GA for five different number of PMUs. Fig. 5 shows the objective values of the best 30 placements found in GA respective to the given number of PMUs. The individual placements in the last GA generation may not be unique. Therefore, many top placements are the same. We can observe a considerable gap in objective values between the optimal placement and the non-optimal placements. The gap is especially significant when there are only 10 PMUs available. On the other hand, the objective values for optimal placements under the case of 20, 25, and 30 PMUs are close to each other, indicating a diminishing return to the number of PMUs. This phenomenon suggests that it would be worthwhile to investigate a minimum number of PMUs required to achieve a particular D&I performance.

IV. CONCLUSION

In this work, we formulated the optimal PMU placement problem as an optimization problem that seeks to minimize the difference between the Jacobian matrix under a full PMU deployment case and a limited PMU deployment case. We adapted the GA to solve the optimization problem and illustrated the approach with the IEEE 39 bus system. The results shows that the GA-generated placement has a better D&I performance than the scattered, tree, and degree-based placement with 20 available PMUs. We also observed a diminishing return when more PMUs are available. A potential further research direction is to incorporate the power network information to direct the placement search.

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