Nonperturbative renormalization group approach to Lifshitz critical behaviour

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Abstract – The behaviour of a $d$-dimensional vectorial $N=3$ model at a $m$-axial Lifshitz critical point is investigated by means of a nonperturbative renormalization group approach that is free of the huge technical difficulties that plague the perturbative approaches and limit their computations to the lowest orders. In particular being systematically improvable, our approach allows us to control the convergence of successive approximations and thus to get reliable physical quantities in $d=3$.

Introduction. – Lifshitz critical behaviour (LCB) [1] (see also [2–5]) occurs when a disordered phase encounters both a homogeneous ordered phase and a spatially modulated ordered phase with a modulation wave vector $q_{\text{mod}} \neq 0$. In the general case the vector $q_{\text{mod}}$ spans a $m$-dimensional subspace of the $d$-dimensional space with $0 \leq m \leq d$. For a $N$-component order parameter the universal behaviour at criticality is completely determined by the set $(m, d, N)$. LCB has been proposed to occur in many systems including magnetic models (notably the ANNNI model [6]), liquid crystals, microemulsions, polymer mixtures, ferroelectrics, high-$T_c$ superconductors, see [4,5] for reviews. In the domain of magnetic materials there has been a growing activity in the search for LCB behaviour. A clear-cut LCB has been found in ternary uranium silicide (UPD$_\tau$) [7] and, possibly, in the manganese phosphide (MnP) [8]. One can thus expect accurate determinations of the critical quantities from experiments in a near future.

From the theoretical point of view the simplest model displaying LCB can be obtained by generalizing the Hamiltonian, or action, relevant to study the usual vectorial ferromagnetic-paramagnetic phase transition. Let us consider a $N$-component vector field $\phi(x)$ in a $d$-dimensional space. The coordinates $x$ are decomposed into a parallel component $x_\parallel \in \mathbb{R}^m$ and an orthogonal component $x_\perp \in \mathbb{R}^{d-m}$, i.e., $x = (x_\parallel, x_\perp)$. The action allowing a LCB reads

$$
\Gamma[\phi] = \int d^{d-m}x_\parallel d^m x_\parallel \left\{ \frac{Z_\parallel}{2} (\partial_\parallel^2 \phi)^2 + \frac{Z_\perp}{2} (\partial_\perp \phi)^2 + \rho_0 \left( \partial_\parallel \phi \right)^2 + u \left( \phi^2 - \kappa \right)^2 \right\},
$$

(1)

where $\partial_\parallel$ and $\partial_\perp$ stand for the derivatives in the corresponding directions. The coupling constants $Z_\parallel$, $Z_\perp$ and $u$ are supposed to be always positive while $\rho_0$ and $\kappa$ are allowed to change sign. The coupling $\kappa$ stands for a magnetization occurring in the —homogeneous— ordered phase. From a mean-field analysis one observes that, for $\rho_0 > 0$, when the coefficient $\tau = -\omega \kappa$ in front of $\phi^2$ varies from a positive to a negative value the system undergoes a phase transition from a disordered to a homogeneous ordered phase while for $\rho_0 < 0$ a transition occurs, for some $\tau = \tau_\ast$, from a disordered to a modulated ordered phase. The two transition lines join at the Lifshitz point which, within a mean-field analysis, is located at $\tau = \rho_0 = 0$.

The salient property characterizing LCB is that of anisotropic scale invariance (ASI). Indeed, at the Lifshitz point, because of the absence of $(\partial_\parallel^2 \phi)^2$ term, the scaling dimensions in the $\perp$ and $\parallel$ directions differ. In particular

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the two-point correlation functions scale as [4,5]

\[ \Gamma^{(2)}(q_\perp \to 0, q_\parallel = 0) \sim q_\perp^{-2 - \eta_{c2}}, \]

\[ \Gamma^{(2)}(q_\perp = 0, q_\parallel \to 0) \sim q_\parallel^{-2 - \eta_{c4}}, \]  

which define the two anomalous scaling dimensions \( \eta_{c2} \) and \( \eta_{c4} \). On the other hand, for a generic scaling operator, one expects the following asymptotic behaviour under a scale transformation:

\[ \mathcal{O}(s q_\perp, s^\theta q_\parallel) \sim s^{-\Delta} \mathcal{O}(q_\perp, q_\parallel) \] when \( s \to 0 \), where \( \Delta \) is the scaling dimension associated to the operator \( \mathcal{O} \), \( \theta \) being the anisotropy critical exponent.

In particular, for the two-point function one has:

\[ \Gamma^{(2)}(s q_\perp, s^\theta q_\parallel) \sim s^{2 - \eta_{c2}} \Gamma^{(2)}(q_\perp, q_\parallel) \] when \( s \to 0 \). This behaviour, together with eq. (2), provides the relation

\[ \theta = 2 - \frac{\eta_{c2}}{4 - \eta_{c4}}. \]  

Finally two critical exponents, \( \nu_{c4} \) and \( \nu_{c2} \), characterize the behaviour of the correlation lengths near criticality:

\[ \xi_\parallel \propto \tau^{-\nu_{c4}} \] and \[ \xi_\perp \propto \tau^{-\nu_{c2}} \]

with

\[ \nu_{c4} = \theta \nu_{c2}. \]  

ASI occurs in many contexts: equilibrium critical phenomena of anisotropic systems, like those described by action (1) at a Lifshitz point or, e.g., in the crumple-to-tubule transition in anisotropic membranes [9,10], as well as in dynamical critical phenomena at and away from equilibrium (see [4]). In quantum field theory an intensive activity has been developed towards theories for which Lorentz invariance is broken at high energy by high-order derivative terms in the spatial directions (see [11] for a review). In these “Lifshitz-type” theories the presence of anisotropy between temporal and spatial directions drastically improve the UV behaviour and renormalizability properties. These ideas have been further extended towards anisotropic scale invariant gravity [12] and cosmology [13]. Finally a theory of local scale invariance (LSI) has been introduced [14] both for equilibrium and out-of-equilibrium phenomena leading to conjecture exact expressions for the two-point correlators of anisotropic systems. While Monte Carlo results [15] have been claimed to agree with these predictions, in a very recent work [16] the predictions of the LSI theory of [14] were challenged. Specifically, ref. [16] found that the epsilon expansions of some scaling functions obtained from a two-loop expansion about the upper critical dimension are inconsistent with the predictions of [14] and [15].

Nonperturbative renormalization group approach. – In this context it is clear that an efficient and systematically improvable approach of anisotropic systems, and in particular of LCB, is needed. From this point of view one has to emphasize that the available, perturbative, techniques are especially in trouble. Let us start with the weak-coupling \( \epsilon \)-expansion. A first problem is that going from isotropic to anisotropic systems shifts the upper critical dimension from \( d_{uc} = 4 \) to \( d_{uc} = 4 + m/2 \). This means that even for the minimal nontrivial value of \( m \), equal to one, the \( \epsilon \)-expansion implies to deal with the large value \( \epsilon = 3/2 \) when computing the critical properties in \( d = 3 \). Assuming that the series obtained are Borel-summable, which is not guaranteed, getting reliable physical quantities thus implies computing, at least up to four or five-loop order. But then one faces a second and important problem. As emphasized in [4,17,18] the real-space free propagator used to perform the \( \epsilon \)-expansion takes a very complicated form, known as Fox-Wright generalized hypergeometric functions, that leads to enormous [19] technical difficulties. This explains why the weak-coupling \( \epsilon \)-expansion results have been very controversial during a long time [18,20] and that it took almost twenty years to fill the gap between early one-loop order results [1] and the complete two-loop order computation [4,17,18,21]. For this reason it is extremely unlikely that higher-order contributions will be obtained in a near future. Similar difficulties occur within a large-\( N \) approach (see [22] and [23]) and it is only very recently [19] that consistency between this large-\( N \) approach at order \( O(1/N) \) and the weak-coupling expansion at two-loop order has been firmly established. Finally note that it is also possible, in principle, to investigate LCB by means of a low-temperature approach in the vicinity of the lower critical dimension which, for \( N > 1 \) components system, is given by \( d_{lc} = 2 + m/2 \). This has been done at one-loop order by Sak and Grest [24]. However, as in the \( O(N) \) model, the series obtained within a low-temperature approach are generally suspected to be non–Borel-summable and thus of no practical use.

We investigate here the LCB by means of a nonperturbative renormalization group (NPRG) approach. Our computation is based on the concept of running effective action [25] (see [26–30] for reviews), \( \Gamma_k[\phi] \), a functional of the \( N \)-component vector field \( \phi(x) \) that describes the effective physics at a coarse-grained scale \( k \). Technically the index \( k \) stands for a running scale that separates the high-momentum modes, with \( q > k \), from the low-momentum ones, with \( q < k \) and \( \Gamma_k[\phi] \) represents a coarse-grained free energy where only fluctuations with momenta \( q \geq k \) have been integrated out. The running of \( k \) towards \( k = 0 \) thus corresponds to gradually integrate over all fluctuations. The \( k \)-dependence, RG flow, of \( \Gamma_k \) is provided by an exact—albeit one-loop—evolution equation [25]:

\[ \frac{\partial \Gamma_k}{\partial t} = \frac{1}{2} \text{Tr} \left\{ \left( \Gamma^{(2)}_k + R_k \right)^{-1} \frac{\partial R_k}{\partial \tau} \right\}, \]  

where \( t = \ln k/\Lambda \), \( \Lambda \) being some microscopic, lattice, scale. The trace in (5) involves \( d \)-dimensional momentum integral over a momentum \( q \) as well as a summation over vectorial indices. The function \( R_k(q) \) realizes the split between low- and high-momentum degrees of freedom.

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while $\Gamma^{(2)}_k$ represents the second functional derivative of $\Gamma_k$ with respect to $\phi$, i.e. the inverse field-dependent propagator. Considering $\Gamma_k$ in its full generality eq. (5) provides an exact RG flow for the coupling constants associated to any power of $\phi$ and of its derivatives.

There are several major advantages in using eq. (5). First, one deals with an one-loop equation while the computations are naturally performed in momentum space. In this way we avoid all the technical difficulties encountered within the perturbative approaches that are associated to the multi-loop structure and the complexity of the propagator in real space. Second, the equation being nonperturbative in the coupling constants entering in the effective action (the $\phi^4$-like coupling constant $u$, the temperature $T \sim 1/\kappa$ as the parameter $1/\kappa$) the approach overcomes a major problem of the perturbative theory: the need to resum, if possible, the perturbative renormalized series. Third this technique is systematically improvable without conceptual or technical difficulty. Let us develop these last two points. Equation (5), although exact is not exactly solvable. One thus has to consider truncations of $\Gamma_k[\phi]$ and, thus, approximations of eq (5). Different kinds of approximations are allowed which keep the nonperturbative and one-loop character of the equation untouched. A very useful and efficient approximation is based on an expansion of $\Gamma_k[\phi]$ in powers of both fields and field derivatives. A derivative expansion is particularly justified to investigate critical phenomena whose physics is dominated by low momenta and thus by low powers of the field derivatives. A field expansion is justified by both its general character and its rapid convergence [31] apparently without need of resummation procedure, as attested by several studies involving Ising model [32], frustrated magnets [28,33], randomly dilute Ising model [34], membranes [10,35] and other [27]. Moreover, in the context of anisotropic systems a field expansion is particularly suitable since the physical dimension of $d = 3$ is close to the lower critical dimension of (vectorial) anisotropic systems $d_{lc} = 2 + m/2$ in the vicinity of which, the RG flow (5), together with the ansatz (1), is one-loop exact. One can thus expect the ansatz (1), or a bit more sophisticated ansatz, to provide very sensible results in $d = 3$. In this article we provide the RG equations for the coupling constants entering in the action (1) while we have computed with powers of the field up to order $\phi^2$. The validity of this approach is then checked by studying both the cut-off independence of the physical quantities and their behaviour when the field content is enriched. We show in particular that converged critical quantities are obtained using a limited number of powers of the field. Note that a NPRG approach of LCB in the Ising case has already been performed in [36] using a full but local potential approach of the Polchinski equation, thus neglecting the anomalous dimension. We investigate here the behaviour of the vectorial $N = 3$ case providing both the critical exponents $\nu_4$ and $\nu_2$ together with the anomalous scaling dimensions $\eta_4$ and $\eta_2$.

**Renormalization group equations.** – The flow equations for the coupling constants $\kappa$, $u$, $\rho_0$ entering in (1) are obtained, as usual [27,28] by appropriate functional derivatives of the RG equation (5). One defines the dimensionless quantities using the scale $\kappa_l$ (see footnote 2): $\tau = Z^{(m+4)/4}Z_m^{1/4}\kappa_l^{(m+4-2d)/2}$, $\tau = Z^{(m+4)/4}Z_m^{1/4}(2d-m-8)/2$ and $\rho_0 = Z^{1/2}Z_m^{1/2}k_{\perp}^{-1}\rho_0$ and their flow reads, with $t = \ln k_{\perp}/\Lambda$,

$$
\partial_t \tau = -(d - m + \theta(m + \eta_4 - 4)) \tau + (N - 1) \tau^{(0)} + 3 \tau^{(2)},
$$

$$
\partial_t \tau = -(d - m + \theta(m + 2 + \eta_4 - 8)) \tau + 2 \tau^{(0)} (N - 1) \tau^{(0)} + 9 \tau^{(4)},
$$

$$
\partial_t \rho_0 = \theta(\eta_4 - 2) \rho_0 + \frac{1}{m} \left( \frac{1}{\nu^2} \left( \tau^{(0)} - \tau^{(2)} \right) \right),
$$

while the running anomalous dimensions $\eta_2 = -\partial_t \ln Z_\tau$ and $\eta_4 = -(1/\theta) \partial_t \ln \tau$ are given by

$$
\eta_2 = \frac{1}{\kappa} \left( \tau^{(0)} + \tau^{(2)} \right) - \frac{1}{2 \nu^2} \left( \tau^{(0),0} - \tau^{(0),0} \right),
$$

$$
\eta_4 = \frac{1}{6 \theta (m + 2)} \left[ \left( m + 2 \right) \nu^2 \tau^{(2)} \left( \tau^{(0),0} - \tau^{(0),0} \right) - 9 \frac{1}{2} \left( \tau^{(2),0} - \tau^{(2),0} \right) + 9 \nu \tau \left( \tau^{(2),0} + \tau^{(2),0} \right) - 8 \nu^2 \tau^{(2)} \left( \tau^{(2),0} - \tau^{(2),0} \right) + 4 \nu^3 \tau^{(2)} \left( \tau^{(2),0} + \tau^{(2),0} \right) + 6 \nu^2 \tau^{(2)} \left( \tau^{(2),0} - \tau^{(2),0} \right) - 12 \nu^3 \tau^{(2)} \left( \tau^{(2),0} - \tau^{(2),0} \right) \right),
$$

where, in eqs. (6) and (7), $\tau^{(i),0}, \tau^{(i),0}, \tau^{(i),0}, \tau^{(i),0}$ are dimensionless “threshold functions” (see [27]) $t_\alpha^{(a,b)}$, $M_\alpha^{(a,b)}$, $S_\alpha^{(a,b)}$, $T_\alpha^{(a,b)}$, $U_\alpha^{(a,b)}$ that are given by

$$
A_\alpha^{(a,b)} = \frac{\partial}{\partial t} \int d^m q f \left( t, \left( q^{(2)} + m^2 q^2 \right)^{1/2} \right),
$$

where $K_b = -\Gamma \gamma_{(b)} (4 \pi q^{(m-d)/2})$. \( \gamma_{(b)} = (m + b - d)/2 \), $P(q_{(i)}) = Z_i q_{(i)}^0 + \rho_0 q_{(i)}^0 + K_{\alpha,0}(q_{(i)})$, $m_\perp = 4 \kappa u$, $m_\perp = 0$ and where $\partial_t \alpha$ only acts on $R_{\alpha,b}$. In eq. (8) the function $F(q_{(i)})$ is given by $d, d^2 q_{(i)}^2, d^2 P/d^2 q_{(i)}^2, d^2 P/d^2 q_{(i)}^2$, and $d^2 P/d^2 q_{(i)}^2$ for $I$, $M$, $S$, $T$, and $U$, respectively. These threshold functions encode the nonperturbative content of the approach since, as is clear from (8), they are nonpolynomial functions of the coupling constants $u$ and $\kappa$ entering in the squared “mass” $m_{\perp}^2$. Note that the threshold functions $A_\alpha^{(a,b)}$ of eq. (8) are integrals over $q_{(i)}$ only. Indeed since the momenta $q_{\perp}$, or the derivative $\partial_{\perp}$, enters only quadratically in action (1) one can exactly perform the integration over the $d - m$ orthogonal degrees of freedom in the RG equations (6) and (7).

\[1\]The scale $k_l = k^2$ could be chosen as well.
Physical results. – Let us discuss the RG equations (6) and (7). The one-loop structure of eq. (5) together with action (1) allow, when the eqs. (6) and (7) are expanded in powers of the suitable coupling constant, to recover all the one-loop results obtained perturbatively. The weak-coupling results obtained in the vicinity of the upper critical dimension [1] are easily recovered by performing an expansion in powers of the coupling constant \( u \) in the vicinity of \( d_{uc} = 4 + m/2 \). In the same way one obtains the large-\( N \) results at dominant order [37]. More importantly for our purpose we recover the low-temperature \( T \) results obtained in the vicinity of the lower critical dimension \( d_{lc}(m) = 2 + m/2 \) by Sak and Grest [24] using a large-\( R \) expansion since \( \kappa \sim 1/T \). We get the flow of \( \pi : \partial_\eta \pi = -2\epsilon \pi + \tilde{C}(N - 2) \) with \( \tilde{C} = \Gamma[m/4]/2^d\pi^{d/2}\Gamma[m/2] \) and \( \epsilon = d - d_{lc}(m) \) with the running anomalous dimensions: \( \eta_4 = \tilde{C}/\pi \) and \( \eta_2 = \tilde{C}\theta/\pi \). At the fixed point one has \( \pi^* = \tilde{C}(N - 2)/2\epsilon \) and \( \eta_4^* = 2\epsilon/(N - 2) \) and \( \eta_2^* = \eta_4^*/2 \) which coincide exactly with the expressions of Sak and Grest. While fully expected, this result is particularly valuable for anisotropic systems for which the lower critical dimension \( d_{lc}(m = 1) = 2.5 \) is especially close to the physical dimension \( d = 3 \).

We now specialize to the \( N = 3 \) case, and thus, \( m = 1 \). One finds a nontrivial fixed point with two directions of instability — corresponding to LCB — in any dimension between \( d_{lc} = 2.5 \) and \( d_{uc} = 4.5 \). Figure 1 displays the curves \( \eta_4 \) and \( \eta_2 \) as functions of \( d \), that call for several remarks. First they show the ability of the NPRG approach to interpolate smoothly between \( d_{lc} \) and \( d_{uc} \). Second they confirm, by a direct investigation in \( d = 3 \) and for \( N = 3 \), the salient fact that LCB in \( d = 3 \) is characterized by a negative value of \( \eta_4 \). This result also points out the limits of the perturbative, large-\( N \) or low-temperature, approaches that lead to a positive value of \( \eta_4 \) in \( d = 3 \). We now focus on the \( d = 3 \) case. For each fixed point the critical exponents \( \nu_4, \nu_2, \eta_4 \) and \( \eta_2 \) are computed and optimized [31,38].

To do this one considers one (or more) family of cut-off functions indexed by a real parameter \( \lambda : R^\lambda_4(q_0) \). Typically one has considered a cut-off function of the form \( R^\lambda_4(q_0) = \lambda Z_{eff}/(\exp(q_0^4/k_{eff}^4) - 1) \). For each family one varies \( \lambda \) in order to find stationary values of the critical quantities. Stationarity is a condition that must necessarily be fulfilled by any putative physical quantity to ensure its quasi-independence with respect to both the cut-off function and the truncation used [31]. However an explicit study of the convergence is necessary to get trustable results. This has been realized by adding successively powers of the field up to order \( \phi^{12} \). Doing this we have been able, for all critical exponents, at almost any order of the field expansion \(^2\) to find stationary values. This is illustrated, for instance, in figs. 2 and 3 which represent the critical exponents \( \nu_4, \nu_2, \eta_4 \) and \( \eta_2 \) in the vicinity of their stationary values for different truncations of the action.

Note that the critical exponents vary very smoothly with the parameter \( \lambda \) around the stationary points which indicates a very weak dependence of the results with respect to the cut-off function used. This fact has been confirmed by using other families of cut-off functions that lead to the same results. More importantly figs. 2 and 3 also indicate a rapid convergence of the physical quantities when powers of the field \( \phi \) are added. Between order

\(^2\) The \( \phi^6 \) case seems to be special in the sense that it does not exhibit clear stationary values.

Fig. 1: (Colour on-line) The anomalous dimensions \( \eta_4 \) and \( \eta_2 \) as functions of the dimension \( d \) using a field truncation up to \( \phi^6 \).

Fig. 2: (Colour on-line) The exponent \( \nu_2 \) as function of \( \lambda \) for truncations from \( \phi^4 \) (lower curve) to \( \phi^{12} \) (upper curve). Stationary points are indicated by black diamonds.

Fig. 3: (Colour on-line) The anomalous dimension \( \eta_2 \) as function of \( \lambda \) for truncations from \( \phi^4 \) (upper curve) to \( \phi^{12} \) (lower curve). Stationary points are indicated by black diamonds.
results at order \( O(e^2) [18] \) and the large-\( N \) results at order \( 1/N [22,23] \). Note that the error bars on the values of the critical exponents are evaluated i) from the direct analysis of the convergence of the field expansion for a given critical exponent when more and more powers of the fields are added ii) from the discrepancy that is observed between the values of a critical exponent according to the fact that it is obtained through a direct optimization or if it is obtained through optimization of the combination of the critical exponents leading to it through the scaling relation (3) and (4). An important outcome of our approach is that the critical exponents found here strongly differ from those obtained within the weak-coupling perturbative approach [18]. This discrepancy is not surprising as the perturbative results have been obtained only at low orders and yet our computation, although based on a different kind of approximation, shows the importance of taking account of several orders to reach converged results. Finally, we note that, amazingly, our critical exponents \( \nu_4 \) and \( \nu_2 \) are close to the values computed within a very recent large-\( N \) expansion [23], contrary to \( \eta_4 \) and \( \eta_2 \).

**Conclusion.** – We have shown that the NPRG approach provides the critical exponents of the Lifshitz point while avoiding all the drastic difficulties encountered using perturbative approaches. Moreover our approach is systematically improvable, as explicitly shown through the present study of the convergence of the field expansion. Although they should play a less important role, higher derivative terms can be treated along the same lines. Finally our work could stimulate, in particular, extensive numerical works as well as investigation of high-quality magnetic compound to confirm the adequacy of our quantitative predictions.

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