Simulations of vortices in a star-shaped plate with an artificial pin

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Abstract. Although a triangular vortex lattice is stable in a bulk type-II superconductor, exotic vortex configurations are expected to appear in a small superconducting plate. Theoretical calculations on vortex structures in a star-shaped superconducting plate have been given in our preceding work. In this work, we extended our theoretical studies to the case of having an artificial pin. We performed the Ginzburg-Landau (GL) calculations systematically to compare with the pin-free case by using the finite element method. We found that a vortex tends to accommodate preferentially in an artificial pin in the star-shaped plate. We found a systematic evolution of vortex structure with increasing magnetic field. We compare our theoretical calculations with vortices in a star-shaped Mo\textsubscript{80}Ge\textsubscript{20} plate with an artificial pin and without an artificial pin obtained by a scanning SQUID microscope. We reconstructed the vortex image on the sample surface by using the inverse Biot-Savart law and the Fourier transformation.

1. Introduction
Interplay relationship between the magnetic field penetration depth $\lambda$ and the sample size $a$ of a bulk sample ($\lambda \ll a$) yields to form a triangular lattice in type II superconductors as revealed originally by Abrikosov [1]. However, a situation is subjected to change drastically when one considers a small-sized superconductor ($\lambda \approx a$). Fertile features of having exotic vortex patterns would appear due to the vortex-vortex interaction and the vortex-boundary interaction, and the vortex-current interaction. Choice of sample geometry is also crucial to consider the possible fascinating configurations. We consider that some of exotic vortex patterns may serve as the basis of utilizing various superconducting

Figure 1. Vortex images taken by scanning SQUID microscopy by using a star-shaped Mo\textsubscript{80}Ge\textsubscript{20} plate. (a) the star-shaped Mo\textsubscript{80}Ge\textsubscript{20} film with no artificial pin (b) the star-shaped Mo\textsubscript{80}Ge\textsubscript{20} plate with an artificial pin at the
devices. The symmetric changes in vortex structures have been investigated extensively by using the regular polygons with mirror symmetric lines. For instance, with the same vorticity $L$, the vortex configuration is different from each other between square and triangle plates [2]. Both theoretical and experimental approaches have used to explore the distribution of vortices. The nonlinear Ginzburg–Landau theory and the London approximation have been often used for such purposes [3] [4] [5]. In Figure 1, we show the vortex images of a star-shaped Mo$_{80}$Ge$_{20}$ amorphous thin plate with an artificial pinning center obtained by a scanning superconducting quantum interference device (SQUID) microscope in our group [6] [7]. In this work, we extended our theoretical studies to the case of having an artificial pin. We performed the Ginzburg-Landau (GL) calculations systematically to compare with the pin-free case by using the finite element method. We performed experiments on controlling vortices in a star-shaped superconductor containing a pinning center. It is also meaningful to investigate the nature of vortex profile under the presence of the pinning center in view of theoretical approach. In this study, we aim at answering a question of whether or not vortex control can be realized by introducing a pin. In addition, we reconstruct a clear vortex image on the sample surface from the observed image by applying the inverse Fourier transformation method.

2. Theoretical

It is interesting to consider the results obtained by scanning SQUID microscopy on star-shaped Mo$_{80}$Ge$_{20}$ plate films with artificial pinning center [7]. We performed the Ginzburg-Landau calculations of star-shaped superconducting small plates systematically to compare with experimental findings. The nonlinear Ginzburg-Landau free energy used for the calculations is given by

$$F(\Delta, A) = \int d\Omega \left[ \frac{1}{2} (\sqrt{\beta} |\Delta|^2 + \frac{\alpha}{\sqrt{\beta}})^2 + \frac{1}{4m} \left( \frac{1}{2} \nabla^2 + \frac{2e}{c} A \right) \Delta^2 + \frac{|| \nabla \times A - H ||^2}{8\pi} + \frac{1}{8\pi} (\nabla A)^2 \right]$$

(1)

where $\Delta(r)$ is an order parameter of a superconductor, $A$ is a vector potential and $H$ is the external magnetic field. The parameter $\beta$ is a positive constant, and $\alpha$ depends on temperature as $\alpha = \alpha(0)(1 - T/T_c)$. We consider that a temperature of the system is well below the transition temperature $T_c$ to reflect an experimental condition while the Ginzburg-Landau equation is applicable to temperatures near $T_c$. We note that our preceding studies showed that the extensive application of the Ginzburg-Landau equation to temperatures somewhat lower than $T_c$. We used the finite element method [8], where the model consists of a number of triangular elements to specify an order parameter $\Delta$, a vector potential $A$, and a temperature $T$ locally as

$$\Delta(x,y) = N_1(x,y) \Delta_1 + N_2(x,y) \Delta_2 + N_3(x,y) \Delta_3$$

(2)

$$A(x,y) = N_1(x,y) A_1 + N_2(x,y) A_2 + N_3(x,y) A_3$$

(3)

$$N_i(x,y) = (a_i + b_i x + c_i y) / 2S_e \quad (i = 1,2,3)$$

(4)

where $\Delta_i$ and $A_i$ are the values of the order parameter and the magnetic vector potential at the $i$-th node, respectively. The relations $a_i = x_i y_j - x_j y_i$, $b_i = y_j - y_i$, $c_i = x_j - x_i$, are cyclically defined using coordinates of nodes $(x_i, y_i)$ of the triangular element and $S_e$ is an area of the element.
3. Results and discussion

We calculated the distribution of vortex in a star-shaped with inscribed circle radius $r = \lambda_0$ and circumscribed circle radius $R = 2.61 \lambda_0$. The size (radius) of pinning center is chosen as $0.16 \lambda_0$. The value of the Ginzburg-Landau parameter is $\kappa = 10$ (type-II superconductors). The magnetic field is applied in perpendicular to the star-shaped plate at a temperature of $T/T_c = 0.5$. This is because $T_c$ of Mo$_{80}$Ge$_{20}$ is 7.3 K and our experimental measurements are performed at $T = 4$ K ($T/T_c = 0.55$). Applied magnetic field $H$ was represented by a dimensionless magnetic field of $h = H/H_{c2}$ reduced by the upper critical field $H_{c2}$. Figures 3 and 4 show the comparison of theoretical distribution with experimental vortex images. Figure 3 is comparative studies on the case of no artificial pin. Figure 4 is comparative studies in the presence of central pin. The systematic evolution of vortex configuration can be seen as a function of the magnetic field. We also compare the numerical calculations with the experimental images in Figure 3 and 4. One finds that there are in reasonable agreements between theoretical calculations and experimental data. Therefore, we consider that it is very useful to conduct such researches by comparing both theoretical and experimental approaches with each other in the field of the mesoscopic superconductivity. Further increase of the magnetic field shows an interesting feature in the theoretical vortex images. In Figure 3, at $L=1$, vortex appears at the center. At $L=2$, vortex distribution has the two-fold symmetry.

Figure 3. Comparative studies on the case of no artificial pin. (a) $L=0$; (b) $L=1$; (c) $L=2$; (d) $L=3$; (e) $L=4$; (f) $L=5$; (g) $L=6$; (h) $L=7$.

Figure 4. Comparative studies on the case of central pin. (a) $L=0$; (b) $L=1$; (c) $L=2$; (d) $L=3$; (e) $L=4$; (f) $L=5$; (g) $L=6$; (h) $L=7$. 
At $L=3$, vortices form a triangular. At $L=4$, vortices form a cross. This distribution has the five-fold symmetry axis. At $L=5$, the vortex distribution has the five-fold symmetry. At $L=6$, vortices form a shell structure of $L=(1,5)$. At $L=7$, vortices form a shell structure of $L=(2,5)$. In Figure 4, at $L=1$, vortex appears at the center. This result is similar to the case obtained for the mode without pinning center. At $L=2$, one vortex appears at the center. At $L=3$, one vortex appears at the center, but it is not in good agreement with experience result. At $L=4$, one vortex appears at the center, but it is different from that observed in experimental results. At $L=5$, one vortex appears at the center. This is different from the case without pinning center. This result is different from the case without pinning center results. At $L=6$, one vortex appears at the center. This result is similar to the case of the experimental results. At $L=7$, one vortex appears at the center. From these results, we found a reasonable agreement between experiment and theory.

4. Processing images
We try to improve vortex images by considering the following items. Magnetic flux from tiny-sized magnetic moment spreads quickly with increasing distance from the surface. Demagnetization effect of the superconducting pick-up coil also affects the local field distribution around the coil. The current distribution on the surface can be refined by applying the inverse Fourier transformation to raw data of local magnetic field distribution. Figure 5 comparatively shows that original images and processing images. According to our studies, we not three remarks. First, noise was fairly removed in the processed image. Second, the spatial resolution was remarkably improved after conducting data procession. Third, the position of vortices becomes clear due to enhanced shapeless of the image.

Figure 5. Original data versus processed data.

5. Conclusion
We developed theoretical calculations of the star-shaped plate with pinning systematically. We conclude based on theoretical calculation that a pin can control the vortex distribution. Although the agreement between theory and experiment is fair while it seems to be difficult to obtain the stable (reproducible) distribution for vortices $L = 3$ and 4. We demonstrated to improve the clearness of vortex images by using the inverse Biot-Savart-law algorithm.

Acknowledgment
This work is supported by Grant-in-Aid (No.26800192, No. 23226019, No.16H02450) from JSPS.

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