The influence of the edge density fluctuations on electron cyclotron wave beam propagation in tokamaks

N Bertelli¹, A A Balakin², E Westerhof¹, O E Garcia³, A H Nielsen⁴ and V Naulin⁴

¹FOM-Institute for Plasma Physics Rijnhuizen, Association EURATOM-FOM, Trilateral Euregio Cluster, Nieuwegein, The Netherlands, www.rijnhuizen.nl
²Institute of Applied Physics RAS, Nizhny Novgorod, Russia
³Department of Physics and Technology, University of Tromsø, N-9037 Tromsø, Norway
⁴Association EURATOM-Risø National Laboratory, Technical University of Denmark, OPL-128 Risø, PO Box 49, DK-4000 Roskilde, Denmark

E-mail: N.Bertelli@rijnhuizen.nl

Abstract. A numerical analysis of the electron cyclotron (EC) wave beam propagation in the presence of edge density fluctuations by means of a quasi-optical code [Balakin A. A. et al, Nucl. Fusion 48 (2008) 065003] is presented. The effects of the density fluctuations on the wave beam propagation are estimated in a vacuum beam propagation between the edge density layer and the EC resonance absorption layer. Consequences on the EC beam propagation are investigated by using a simplified model in which the density fluctuations are described by a single harmonic oscillation. In addition, quasi-optical calculations are shown by using edge density fluctuations as calculated by two-dimensional interchange turbulence simulations and validated with the experimental data [O. E. Garcia et al, Nucl. Fusion 47 (2007) 667].

1. Introduction

One of the goals of electron cyclotron current drive (ECCD) in tokamak plasmas is the control of magnetohydrodynamic (MHD) instabilities. The effectiveness of this control depends strongly on the achievable localization of the EC driven current density and therefore it is essential to study the various physics effects determining this localization. In this work, we investigate the effects of density fluctuations associated, in particular, with edge turbulence on the EC wave beam propagation as it may affect the beam width and consequent power deposition and ECCD profile. Such effects are expected to be particularly relevant when the beam propagates over a large distance between the fluctuations and the EC resonance layer. This is the case in ITER, in which the distance between scrape-off layer (SOL) density fluctuations and the EC resonance layer is larger than one meter. An immediate consequence of crossing the turbulent edge region will be a perturbation of the phase front of the beam resulting, in turn, in a perturbation of the wave vector spectrum. As the beam propagates, this can result in a broadening of the beam itself. In the literature, this effect has generally been studied by making use of ray tracing codes based on the geometrical optics approximation [1, 2]. The conventional ray- and beam-tracing codes typically are based on the expansion assuming the different length scales as \( \lambda \ll w \ll L \), where \( \lambda \)
is the wavelength, $w$ the wave beam width and $L$ the typical inhomogeneity scale of the dielectric response of the plasma. The presence of the SOL density fluctuations, also known as blobs which can reach the same size of the width of wave beam [3], limits strongly the applicability of geometrical optics. In order to properly account for the effect of the density perturbations, a quasi-optical beam tracing code, as presented in Ref. [4] assuming only $L, w \gg \lambda$ ($\lambda$ is the wavelength), is required. Here, a systematic numerical analysis of the effects of the edge density fluctuations on the EC beam propagation is presented by using this quasi-optical beam tracing code.

This paper is structured as follows. The main features of the quasi-optical beam tracing code are presented in section 2. Section 3 addresses the relation between the density fluctuations and the perturbations in the phase front of the beam. Quasi-optical calculations are presented in section 4 for an ITER-like beam propagating in vacuum taking into account a single harmonic phase fluctuation, as would result also from a single harmonic density fluctuation in a narrow layer. An analysis of the effective beam intensity is performed in the case of (i) this single harmonic oscillation and (ii) the phase perturbation resulting from a narrow layer of edge density fluctuations as calculated by two-dimensional interchange turbulence simulations [3].

2. Quasi-optical beam tracing code
The quasi-optical beam tracing code solves a parabolic wave equation, which can be seen as a generalization to anisotropic media with spatial dispersion of the parabolic equation for wave propagation in isotropic media of Fock and Leontovich [5]. The equation takes the following form [4]

\[ U = u(\tau, \xi) \exp(ik_0\Psi), \quad \frac{\partial u(\tau, \xi)}{\partial \tau} = ik_0\hat{H}_0[u(\tau, \xi)]. \]  

(1)

In equation (1), $U$ is the electric field defined in analogy to the geometrical optics assumption, namely, where $u$ is a slowly varying amplitude on the scale of wavelength and $\Psi$ is the fast oscillating phase. The second equation in (1) describes the evolution of the scalar beam amplitude $u$. In addition, $\tau$ is a parameter along the reference ray acting in the role of arc length and $\xi = (\xi_1, \xi_2)$ are coordinates across the beam propagation ($\xi = 0$ on the reference ray). In other words, $\{\tau, \xi\}$ represents an accompanying curvilinear coordinate system of the beam. For more details about this equation, the reader is referred to [4].

This quasi-optical code is capable of treating correctly wave beams of arbitrary shape in the presence of strongly inhomogeneous dispersion and absorption as well as in media with strong spatial dispersion [4]. In this work we make use of this quasi-optical code in order to study the effects of the density fluctuations on the EC wave propagation.

3. Density and phase fluctuations
Crossing the narrow turbulent edge region, also known as scrape-off layer (SOL) [3], results in a perturbation of the phase front of the beam modifying the wave vector spectrum. It is assumed that the density fluctuations are weak, i.e., not strong enough to deflect significantly the geometrical optics ray on the scale of the fluctuations themselves. The slowly varying beam amplitude is now written as $u(\tau, \xi) = \bar{u}(\tau, \xi)e^{i\delta\varphi}$ where $\bar{u}$ is the amplitude in the case without fluctuations. The geometrical optics approximation can then be used to find, through a simple integration along the beam path across the turbulence layer (SOL), the perturbation of the beam phase, $\delta\varphi$, as

\[ \delta\varphi(\xi) = k_0 \int_{\text{SOL}} \frac{\delta H(\xi, \tau)}{\partial p_\tau} d\tau. \]  

(2)

Here, $\delta H$ is the variation of the Hamiltonian due to the presence of the density fluctuations and $p_\tau$ is the normalized longitudinal component of the wave vector. Equation (2) has been
Figure 1. QO beam profiles affected by a harmonic phase perturbation of the type (3) for \( \varphi_0 = \pi/2, \theta = 0 \) and \( w\kappa = 3 \) (a), \( w\kappa = 1 \) (b), \( w\kappa = 0.3 \) (c). Figures (d), (e) and (f) show the 3D beam profiles.

implemented in the quasi-optical code with a Hamiltonian which can take into account the SOL density fluctuations.

4. Numerical results

4.1. Harmonic phase fluctuations

In order to understand the influence of the presence of the phase fluctuations (related to the density fluctuations as described above) on the beam propagation, we start by analyzing a simplified model with phase perturbations given by a single harmonic oscillation,

\[
\delta \varphi = \varphi_0 \cos(\kappa \xi + \theta),
\]

where \( \varphi_0 \) is the amplitude of the fluctuation, \( \kappa^{-1} \) its scale length, and \( \theta \) its phase. Such phase perturbations can be seen as resulting from a single harmonic density fluctuation in a narrow layer. Note that \( \kappa \) is perpendicular to the magnetic equilibrium field since along the magnetic field lines the density fluctuations are mostly homogeneous. First we study the case with an amplitude of \( \varphi_0 = \pi/2 \). Three different scale lengths \( \kappa^{-1} \) are considered: (i) \( w\kappa \ll 1 \), i.e., large scale as compare to the width of beam waist, \( w \); (ii) \( w\kappa = 1 \), i.e., the intermediate case; (iii) \( w\kappa \gg 1 \), i.e., small-scale fluctuations. Four different phases \( \theta \) are considered: (i) \( \theta = 0 \), representing a defocusing fluctuation at the beam axis; (ii) \( \theta = \pi/2 \), representing deflecting fluctuation; (iii) \( \theta = \pi \), representing a focusing fluctuation; (iv) random phase \( \theta \). For all calculations, ITER beam parameters from the Upper Port Launcher lower steering mirror are
used. In particular, the width of the beam waist is \( w = 2.1 \) cm and the distance to the beam focus inside the plasma \( D_{\text{focus}} = 1.62 \) m, which corresponds to the location of the EC resonance layer [6].

4.2. Perturbed beam profiles

The propagation of the beam after being affected by a harmonic phase perturbation of the type (3) and amplitude \( \varphi_0 = \pi/2 \) is illustrated in Figures 1-4. Figure 1 shows the case of a defocusing phase \( \theta = 0 \) for small-scale fluctuations (a), an intermediate case (b) and large-scale fluctuations (c). In Figure 1, it appears that (i) for small-scale fluctuations, the beam is fragmented; (ii) in the intermediate case, there is an additional defocusing of the beam together with a forward shift in the location of the waist beam; (iii) for \( w\kappa = 0 \), the influence of fluctuations is practically negligible. In Figure 2, which shows the case for a deflecting phase \( \theta = \pi/2 \), it appears that, for small-scale fluctuations there is again a fragmented beam but now without symmetry with respect to the beam axis, while intermediate scales show a clear deflection of the beam. The large-scale fluctuation is seen to cause only a very small deflection. Figure 3 shows for the case of a focusing phase \( \theta = \pi \), a broadening of the beam for small-scale fluctuations while for \( w\kappa = 0.3 \) again no effect is seen. On the contrary, for intermediate scales, the beam is clearly focused and

\[\begin{align*}
\text{Figure 2. The same as Figure 1 for } \varphi_0 = \pi/2, \theta = \pi/2 \text{ and } w\kappa = 3 \text{ (a), } w\kappa = 1 \text{ (b), } w\kappa = 0.3 \text{ (c).}
\end{align*}\]
the location of the waist is shifted backward. Note that this shift is exactly an opposite effect compared to the case of Figure 1(b). Finally, Figure 4 shows an average of the beam profiles over random phase $\theta$. It mainly appears that there is on average (i) a broadening for small-scale fluctuations; (ii) a weak defocusing of the beam for the intermediate case and (iii) practically no effect for large-scale fluctuations.

4.3. Effective beam intensity

Another important aspect to analyze is the beam intensity ($I \propto |\bar{u}|^2$) which is the more relevant number for the power absorption and, in turn, the current drive. Figure 5 shows the effective beam intensity averaged over the phase $\theta$, $I_{\text{eff}}$, normalized to the unperturbed intensity, $I_0$, as a function of the amplitude, $\varphi_0$, of the phase fluctuations. In particular, the normalized effective beam intensity is shown as obtained in the near, mid and far field, namely, for $\tau^* = 0.2, 0.4 \text{ m}$, $\tau^* = 2 \text{ m}$ and $\tau^* = 6 \text{ m}$, respectively ($\tau^*$ is the distance between the edge of the plasma and the position where the intensity is calculated). Four fluctuation scales are considered: $w_\kappa = 10, 3, 1, \text{ and } 0.3$. One can see that (i) in the near field ($\tau^* = 0.2, 0.4 \text{ m}$) the decrease of the normalized effective beam intensity is less than in the far field ($\tau^* = 2, 6 \text{ m}$); (ii) in the far field, a strong decrease of normalized effective beam intensity is found for small-scale fluctuations while small effects appear when the fluctuation scale is larger; (iii) for large-scale fluctuations ($w_\kappa = 0.3$) no significant effects can be seen for all cases.

4.4. Scrape-off layer turbulence

Density fluctuations in the TCV scrape-off layer have been investigated by probe measurements and compared with a two-dimensional interchange turbulence simulation. In particular, these numerical simulations are in excellent agreement with the TCV experimental measurements [3, 7]. Here, we make use of these SOL density fluctuation data in order to illustrate the possible consequences on the effective beam intensity. The phase perturbation generated by the passing of the beam through the turbulence SOL is obtained by integration of equation (2) substituting the 2D density perturbation predicted by the turbulence code. The resulting phase perturbation is rescaled in order to study the dependence of the amplitude of the SOL turbulence. The averaged effect of the turbulence is obtained by averaging the perturbed wave beam over its realizations as obtained for each of the predicted 2D turbulence density field at different times as well as at all possible spatial phases. Figure 6 shows the normalized effective beam intensity as a function of the the root mean square amplitude $||\delta\phi||$ of the fluctuations. The

![Figure 4](image_url)

**Figure 4.** The same as Figure 1 for $\varphi_0 = \pi/2$ and $w_\kappa = 3$ (a), $w_\kappa = 1$ (b), $w_\kappa = 0.3$ (c). Here, QO beam profiles are averaged over random phase $\theta$. 

Figures 5. Effective beam intensity, $I_{\text{eff}}$, normalized to the unperturbed intensity, $I_0$, as a function of the amplitude of the harmonic fluctuations for beams averaged over random phase $\theta$ and for $\kappa \kappa = 10$ (a), $\kappa \kappa = 3$ (b), $\kappa \kappa = 1$ (c), $\kappa \kappa = 0.3$ (d). The effective intensity is calculated in the near field ($\tau^* = 0.2, 0.4$ m, brown and red curves, respectively), in the mid field ($\tau^* = 2$ m, blue curve) and in the far field ($\tau^* = 6$ m, green curve).

5. Summary and Conclusions
The power deposition and driven current density profile can be affected by perturbations of the wave beam due to the edge density fluctuations. Calculations with a quasi-optical code [4] show that, for a simplified model corresponding to a single harmonic fluctuation, this effect can lead to a considerable broadening of the beam and a reduction of the beam intensity, in particular, for fluctuations at scales smaller than or comparable to the beam waist. Furthermore, a TCV-like turbulence case has been investigated confirming that this effect can be relevant particularly when the beam is expected to propagate over a large distance before it reaches the EC resonance. In a future work, ITER simulations with self consistent predictions of ITER edge turbulence will

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Figure 6. Normalized effective beam intensity as a function of the root mean square of the amplitude $||\delta\varphi||$ of the SOL density fluctuations as obtained from turbulence code [3]. The effective intensity is shown in the near/mid/far field as in Figure 5.

be performed taking into account also the ballooning nature of the SOL density fluctuations, which leads to a reduction of the edge plasma turbulence in front of the location of the ITER ERCH Upper Port Launcher with respect to turbulence in front of the Equatorial Launcher.

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