INFRARED REGION OF QCD AND CONFINING STRINGS

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Abstract

Gauge field configurations appropriate for the infrared region of QCD are proposed in a submanifold of $su(3)$. Some properties of the submanifold are presented. Using the usual action for QCD in the absence of quarks, confinement of these configurations is realized as in the London theory of Meissner effect. Choosing a representation for the monopole field strength, a string action corresponding to the effective gauge theory action in the infrared region, is obtained. This confining string action contains the Nambu-Goto term, extrinsic curvature action and the Euler characteristic of the string world sheet.

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I. Introduction

QCD, the gauge theory of strong interaction based on the gauge group $SU_c(3)$, exhibits asymptotic freedom [1] in the high energy region and in this region, the success of the perturbation theory (due to the small coupling strength of the QCD running coupling constant) has been supported by experimental results [2]. In the infrared region where the coupling becomes strong, the perturbation theory cannot be used and the mechanism of confinement of the gluons and the quarks is not completely understood. 't Hooft [3] and Mandelstam [4] suggested that magnetic monopoles must play a crucial role in the mechanism of confinement. Color confinement is possibly due to a dual Meissner effect [4,5,6]. This suggests that in the infrared region, some other variables other than $A^a_\mu$ may be more relevant, and Mandelstam [7] suggested that the monopole plasma is probably a way of parameterizing the ground state of the confining phase. The idea of 't Hooft [3] is to make an Abelian projection and this has been discussed by Mandelstam [7]. In particular, Kondo [8] has examined this for $SU(2)$ gauge theory to obtain an Abelian projected effective field theory. Here the non-Abelian degrees of freedom are integrated and the relation between the non-Abelian gauge fields and the monopole configuration is not clear. Recently, Faddeev and Niemi [9] have proposed a set of variables for describing the infrared region of 4-dimensional $SU(2)$ and $SU(N)$ gauge theory, continuing the earlier works of Corrigan et.al., [10] and Cho [11]. They [9] propose a non-linear sigma model action as relevant in the infrared region of Yang-Mills theory. The author has recently proposed [12] gauge field configurations for the infrared region of QCD with $SU(3)$ as the gauge group, in which the magnetic confinement of gluons and quarks have been realized as in the London theory of Meissner effect. A dual version produces the electric confinement as well, thereby explicitly demonstrating the scenario proposed by Nambu [6].

While this field theoretic approach to confinement is based upon the attempts to identify relevant gauge field configurations as in [9, 12], there is another approach based upon the string representation as pointed out by Polyakov [13]. Starting from the action for compact $U(1)$ gauge theory, he [13] obtains the action for the confining strings as the rigid string action (which contains the extrinsic curvature action besides the Nambu-Goto action) in the large Wilson loop limit. It is to be noted here that compact
U(1) gauge theory must include the monopole configurations \cite{14}. This important connection \cite{13} between the gauge fields (including monopole configurations) and strings has been extended to 4-dimensional case by Diamantini, Quevedo and Trugenberger \cite{15} by introducing the antisymmetric Kalb-Ramond fields \cite{16} interacting with the monopoles. These studies \cite{13,15} reveal two important features. First, the string action so obtained contains besides the Nambu-Goto action, the extrinsic curvature action, earlier proposed by Polyakov \cite{17} and independently by Kleinert \cite{18} as relevant to describe QCD strings. Second, the coefficient of the extrinsic curvature action is found to be negative, as desired by Kleinert and Chervyakov \cite{19} to improve the stability of the theory.

It is the purpose of this paper to first identify the relevant gauge field configurations in the infrared region of QCD and to obtain an action \cite{12} which naturally contains the monopole configurations, exhibiting confinement and then to obtain a string representation of this low energy effective action by going to the Euclidean version. In Section.II, the proposed \cite{12} SU(3) gauge field configuration appropriate in the infrared region of QCD are briefly reviewed and some properties of the submanifold are obtained in Section.III. An effective action exhibiting confinement is obtained in Section.IV. The string representation of this action (in Euclidean space) is obtained in Section.V. The results are summarized in Section.VI.

II. Gauge Field Configurations for the Infrared Region of QCD.

The suggestions of 'tHooft \cite{3}, Mandelstam \cite{4,7} and Polyakov \cite{13} that the low energy region of QCD must contain monopole configurations to provide a mechanism of confinement, can be implemented by proposing the SU(3) gauge field configurations $A^{a}_{\mu}$ satisfying

$$D^{ab}_{\mu} \omega^{b} = \partial_{\mu} \omega^{a} + g f^{acb} A^{c}_{\mu} \omega^{b} = 0,$$

(1)

where $\omega^{a}$ is an SU(3) octet vector $\in$ SU(3)/Z3 and are chosen such that

$$\omega^{a} \omega^{a} = 1,$$

$$d^{abc} \omega^{b} \omega^{c} = \frac{1}{\sqrt{3}} \omega^{a},$$

(2)
with $d^{abc}$ as the symmetric Gell-Mann $SU(3)$ tensors. Now, in the infrared region, the gauge group manifold is not the complete $su(3)$ group manifold spanned by arbitrary $\omega^a$'s but a submanifold spanned by those $\omega^a$'s satisfying (2) and the relevant gauge field configurations $A_\mu^a$ are determined by (1). The second relation in (2) is very special to $SU(3)$ and will be crucial in what follows.

A solution to (1) is

$$A_\mu^a = C_\mu \omega^a - \frac{4}{3g} f^{abc} \omega^b \partial_\mu \omega^c,$$

(3)

where $C_\mu$ is arbitrary. To show that (3) solves (1), we made use of the following relations

$$\omega^a \partial_\mu \omega^a = 0,$$
$$d^{abc} (\partial_\mu \omega^b) \omega^c = \frac{1}{2\sqrt{3}} \partial_\mu \omega^a,$$

(4)

which follow from (2), and the relation [21]

$$f^{abc} f^{edc} = \frac{2}{3} (\delta_{ae} \delta_{bd} - \delta_{ad} \delta_{be}) + d^{ace} d^{bdc} - d^{ade} d^{bcd},$$

(5)

for the $f$ tensors. We find a useful relation for the $\omega^a$'s in (2) as

$$\frac{4}{3} f^{abc} f^{edc} \omega^b \omega^e \partial_\mu \omega^d = -\partial_\mu \omega^a,$$

(6)

which will be used subsequently. The gauge field configuration (3) and the submanifold determined by (2) are proposed as relevant to describe the infrared region of QCD.

It is therefore very essential to show, in view of [3,4,7,13], that the proposed configurations (3) admits monopoles necessary for confinement. This will be shown in the Section IV. Before this, we note two important consequences that follow from (1). A mass term $m^2 A_\mu^a A_\mu^a$ for the gauge fields usually cannot be added to the lagrangian due to its gauge non-invariance. Since under a gauge transformation the field changes as $\delta A_\mu^a = D_\mu^b \omega^b$, if
we consider gauge transformations within the submanifold (2), then such a
term is allowed in view of (1). Second, if we choose a Lorentz covariant gauge
\[ \partial_\mu A_\mu^a = 0, \]
for the \( A_\mu^a \)'s in (3), then there will be no Gribov ambiguity [22]
in fixing the gauge, in view of (1), as long as we are within the submanifold,
since the gauge variation of the gauge fixing condition \( \partial_\mu \delta A_\mu^a = \partial_\mu (D_\mu^a \omega^b) \)
identically vanishes. In this way, the non-propagating ghost requirement of
t'Hooft [3] is satisfied by the choice of (1).

III. SOME PROPERTIES OF THE SUBMANIFOLD.

In this section, we consider the finite gauge transformations in the sub-
manifold defined by (2), [23]. The finite gauge transformations here, will be
shown become linear in \( \omega^a \)'s. The second relation in (2) upon using the first
relation becomes \( d_{abc} \omega^a \omega^b \omega^c = \frac{1}{\sqrt{3}} \). It is known [21,24] that, given one octet
vector \( \omega^a \), there exists at most two linearly independent octets. They are \( \omega^a \)
itself and \( d_{abc} \omega^b \omega^c \). Also, it is equally known that at most two independent
\( SU(3) \) invariants can be formed. They are \( \omega^a \omega^a \) and \( d_{abc} \omega^a \omega^b \omega^c \). In view of
(2), we now have only one octet since \( d_{abc} \omega^b \omega^c \) is taken to be \( \frac{1}{\sqrt{3}} \omega^a \) and the
two said \( SU(3) \) invariants become constants 1 and \( \frac{1}{\sqrt{3}} \) respectively. These
characterize the submanifold.

Consider a gauge transformation generated by
\[ U = \exp \left( i \omega^a \frac{\lambda_a}{2} \right), \]
for \( \omega \)'s satisfying (2) and \( \lambda_a \)'s are the Gell-Mann \( SU(3) \) matrices. We shall
make use of the basic relation for the \( \lambda \)- matrices, viz.,
\[ \lambda_a \lambda_b = i f^{abc} \lambda_c + \frac{2}{3} \delta^{ab} + d^{abc} \lambda_c, \]
where \( f^{abc} \) and \( d^{abc} \) are the usual anti-symmetric and symmetric \( SU(3) \) ten-
sors respectively. Expanding the exponential in (7), we have
\[ U = 1 + i \omega^a \frac{\lambda_a}{2} - \frac{1}{2!} \frac{1}{4} (\omega^a \lambda_a)^2 \]
\[ - \frac{i}{3!} \frac{1}{8} (\omega^a \lambda_a)^3 + \frac{1}{4!} \frac{1}{16} (\omega^a \lambda_a)^4 + \cdots. \]
Using (8), it follows

\[(\omega^a \lambda_a)^2 = \frac{2}{3} + \frac{1}{\sqrt{3}} \omega^a \lambda_a,\]
\[(\omega^a \lambda_a)^3 = \frac{2}{3\sqrt{3}} + \omega^a \lambda_a,\]
\[(\omega^a \lambda_a)^4 = \frac{2}{3} + \frac{5}{3\sqrt{3}} \omega^a \lambda_a,\]
\[(\omega^a \lambda_a)^5 = \frac{10}{9\sqrt{3}} + \frac{11}{9} \omega^a \lambda_a,\]

and so on. Substituting these in \(U\), we have,

\[U = (1 - \frac{1}{2!} \frac{1}{6} - i \frac{1}{3!} \frac{1}{12\sqrt{3}} + \frac{1}{4!} \frac{5}{144\sqrt{3}} + \cdots)\]
\[+ (\frac{i}{2} - \frac{1}{2!} \frac{1}{4\sqrt{3}} - i \frac{1}{3!} \frac{1}{8} + \frac{1}{4!} \frac{5}{48\sqrt{3}} + \cdots) \omega^a \lambda_a\]
\[= \alpha + \beta \omega^a \lambda_a,\]  

(9)

where \(\alpha\) and \(\beta\) are (finite) complex constants. \(U^{-1}\) can be easily found to be \(\alpha^* + \beta^* \omega^a \lambda_a\) (where \(*\) stands for complex conjugation) and so \(UU^{-1} = I\), gives the conditions,

\[\alpha \alpha^* + \frac{2}{3} \beta \beta^* = 1,\]
\[\alpha \beta^* + \beta \alpha^* + \frac{1}{\sqrt{3}} \beta \beta^* = 0.\]  

(10)

Thus, the finite gauge transformation \(U\) becomes linear in \(\omega\). This is a special feature of the submanifold defined by (2). Thus a parameterization of \(SU(3)\) in terms of a single real octet vector \(\omega^a\) is achieved by (9). It has been shown by Macfarlane, Sudbery and Weisz [21] that any special unitary matrix \(U\) written as \(\exp(iA)\) with \(A\) hermitian and \(A = \omega^a \lambda_a\), can be written as \(U = u_0 + i u_a \lambda_a\), where \(u_a = a \omega_a + b d_{abc} \omega^b \omega^c\), with \(u_0, a, b\) as functions of the two \(SU(3)\) invariants, \(\omega^a \omega^a\) and \(d_{abc} \omega^a \omega^b \omega^c\). In our choice, these invariants are taken as constants 1 and \(\frac{1}{\sqrt{3}}\) and this gives (9) for \(U\). We will relate this \(U\) to the rotation matrix of a subgroup of rotations in eight dimensional Euclidean space \(E_8\).
Now, consider a finite gauge transformation due to $U$ in (9). Under a finite gauge transformation, we know that the gauge field $A_\mu = A_\mu^a \frac{\lambda_a}{2}$ transforms as

$$A_\mu \rightarrow A_U^\mu = \frac{1}{i} (\partial_\mu U) U^{-1} + U A_\mu U^{-1}. \quad (11)$$

To check the notation and the factors, we find the infinitesimal version of (11) using (7), is

$$\delta A^a_\mu = D^{ac}_\mu \omega_c, \quad (12)$$

as it should be. In view (1), $\delta A^a_\mu = 0$. Here we want to show that for finite gauge transformations (9) with (10), also $\delta A_\mu = 0$, i.e.,

$$A_U^\mu = A_\mu, \quad (13)$$

for those $A^a_\mu$’s satisfying (1).

We shall make use of the standard relations among $f$ and $d$ tensors [21], viz.,

$$f^{i\ell m} d^{mjk} + f^{jm} d^{imk} + f^{klm} d^{ijm} = 0, \quad (14)$$

and

$$f^{ijm} f^{klm} = \frac{2}{3} (\delta^{ik} \delta^{j\ell} - \delta^{i\ell} \delta^{jk}) + d^{ikm} d^{jlm} - d^{ikm} d^{jkm}. \quad (15)$$

Starting from (9), we have

$$(\partial_\mu U)^{-1} = \beta (\partial_\mu \omega^a) \lambda_a (\alpha^* + \beta^* \omega^b \lambda_b)$$

$$= \beta \alpha^* (\partial_\mu \omega^a) \lambda_a + i \beta \beta^* (\partial_\mu \omega^a) \omega^b f^{abc} \lambda_c$$

$$+ \beta \beta^* (\partial_\mu \omega^a) \omega^b d^{abc} \lambda_c, \quad (16)$$

where $(\partial_\mu \omega^a) \omega^a = 0$, following from the first relation in (2) is used. Also, the second relation in (2) gives

$$d^{abc} (\partial_\mu \omega^b) \omega^c = \frac{1}{2\sqrt{3}} (\partial_\mu \omega^a). \quad (17)$$
and so
\[
(\partial_{\mu}U)^{-1} = \left(\beta^* + \frac{1}{2\sqrt{3}}\beta^*\right)(\partial_{\mu}\omega^{a})\lambda_{a} + i\beta^*(\partial_{\mu}\omega^{a})\omega^{b}f^{abc}\lambda_{c}. \tag{18}
\]

We are interested in the transformation of the $A_{\mu}^{a}$'s satisfying (1) and so using (1) in (18), we have
\[
(\partial_{\mu}U)^{-1} = \left(\beta^* + \frac{1}{2\sqrt{3}}\beta^*\right)f^{abc}\omega^{b}A_{\mu}^{c}\lambda_{a}
+ i\beta^*\left(f^{apq}\omega_{p}^{\mu}A_{\mu}^{q}\right)\omega^{b}f^{abc}\lambda_{c}. \tag{19}
\]

Using (15) for simplifying $f^{apq}f^{abc}$, (19) becomes
\[
(\partial_{\mu}U)^{-1} = \left(\beta^* + \frac{1}{2\sqrt{3}}\beta^*\right)f^{abc}\omega^{b}A_{\mu}^{c}\lambda_{a}
+ i\beta^\ast\left\{\frac{2}{3}A_{\mu}^{a}\lambda_{a} - \frac{2}{3}\omega^{b}\omega^{c}A_{\mu}^{b}\lambda_{c} + \frac{1}{\sqrt{3}}\omega^{a}d^{pca}A_{\mu}^{q}\lambda_{c}
- dqbadqca\omega_{p}\omega^{b}A_{\mu}^{q}\lambda_{c}\right\}, \tag{20}
\]
where the relations in (2) have been used.

Now, we consider the evaluation of $UA_{\mu}U^{-1}$ using (9).
\[
UA_{\mu}U^{-1} = (\alpha + \beta\omega^{a}\lambda_{a})A_{\mu}^{b}\frac{\lambda_{b}}{2}\left(\alpha^* + \beta^\ast\omega^{c}\lambda_{c}\right),
= \frac{1}{2}(\alpha + \beta\omega^{a}\lambda_{a})\left\{\alpha^\ast\lambda_{b} + i\beta^\ast f^{bcd}\omega^{c}\lambda_{d} + \frac{2}{3}\beta^\ast\omega^{b}
+ \beta^\ast d^{pca}\omega^{p}\omega^{b}\lambda_{c}\right\}, \tag{21}
\]
using (8). Expanding further and using (8), we have
\[
UA_{\mu}U^{-1} = \frac{1}{2}\left\{\alpha\alpha^\ast\lambda_{b} + i\alpha\beta^\ast f^{bcd}\omega^{c}\lambda_{d} + \frac{2}{3}\alpha\beta^\ast\omega^{b}
+ \alpha\beta^\ast d^{pca}\omega^{p}\omega^{b}\lambda_{d} + \beta\alpha^\ast\omega^{a}(if^{abc}\lambda_{c} + \frac{2}{3}\delta_{ab} + d^{abc}\lambda_{c})
+ i\beta^\ast f^{bcd}\omega^{c}\omega^{a}(if^{ade}\lambda_{e} + \frac{2}{3}\delta_{ad} + d^{ade}\lambda_{e})
+ \frac{2}{3}\beta^\ast\omega^{a}\omega^{b}\lambda_{c}
+ i\beta^\ast d^{pca}\omega^{p}\omega^{b}(if^{ade}\lambda_{e} + \frac{2}{3}\delta_{ad} + d^{ade}\lambda_{e})\right\}, \tag{22}
\]
In here there are fourteen terms. The sixth term and the third term are like-terms; the seventh and the fourth are like-terms, and the fifth and the second are like-terms. The ninth term vanishes identically. In the thirteenth term, the second relation in (2) is used. Then we find,

\[ UA_\mu U^{-1} = \frac{1}{2} \left\{ \alpha \alpha^* \lambda_b + i(\alpha \beta^* - \beta \alpha^*) f^{bcd} \omega^c \lambda_d + \frac{2}{3}(\alpha \beta^* + \beta \alpha^*) \omega^b \right. \\
+ (\alpha \beta^* + \beta \alpha^*) d^{bcd} \omega^c \lambda_d \\
+ i\beta^* \left( i f^{ade} f^{bcd} \omega^a \omega^c \lambda_e + f^{bcd} d^{ade} \omega^c \omega^d \lambda_e \right) \\
+ \left. \frac{2}{3} \beta \beta^* \omega^a \omega^b \lambda_a \\
+ \beta \beta^* \left( i f^{ade} d^{bcd} \omega^a \omega^c \lambda_e + \frac{2}{3\sqrt{3}} \omega^b + d^{bcd} d^{ade} \omega^a \omega^c \lambda_e \right) \right\} A^b_\mu. \] (23)

Now, the coefficient of \( \omega^b \) terms in (23) is \( \frac{2}{3} (\alpha \beta^* + \beta \alpha^* + \frac{1}{\sqrt{3}} \beta \beta^*) \), which is zero due to the second relation in (10). Using the relation (14) to rewrite the eighth term \( f^{ade} f^{bcd} \) as \( -f^{bad} d^{edc} - f^{cad} d^{ebd} \), the term involving \( f^{cad} \) vanishes due to the symmetry of \( \omega^a \omega^c \) in \( a \) and \( c \). Then the remaining of the eighth term cancels with the sixth term in (23). In the fifth term involving \( f^{ade} f^{bcd} \), we make use of (15). Then (23) becomes,

\[ UA_\mu U^{-1} = \frac{1}{2} \left\{ \alpha \alpha^* \lambda_b + i(\alpha \beta^* - \beta \alpha^*) f^{bcd} \omega^c \lambda_d + (\alpha \beta^* + \beta \alpha^*) d^{bcd} \omega^c \lambda_d \\
- \beta^* \left( \frac{2}{3} \lambda_b - \frac{2}{3} \omega^b \omega^c \lambda_e + \frac{d^{bcd} \omega^c \lambda_e}{\sqrt{3}} - d^{abc} d^{bcd} \omega^c \omega^c \lambda_e \right) \\
+ \frac{2}{3} \beta \beta^* \omega^a \omega^b \lambda_a + \beta \beta^* d^{bcd} d^{ade} \omega^a \omega^c \lambda_e \right\} A^b_\mu. \] (24)

Here, the two terms involving the two \( d \)-tensors are the same and so,

\[ UA_\mu U^{-1} = \frac{1}{2} \left\{ \alpha \alpha^* \lambda_b + i(\alpha \beta^* - \beta \alpha^*) f^{bcd} \omega^c \lambda_d + (\alpha \beta^* + \beta \alpha^*) d^{bcd} \omega^c \lambda_d \\
- \frac{2}{3} \beta \beta^* \lambda_b + \frac{4}{3} \beta \beta^* \omega^a \omega^b \lambda_a - \beta^* \frac{1}{\sqrt{3}} d^{bcd} \omega^d \lambda_e \\
+ 2 \beta \beta^* d^{abc} d^{bcd} \omega^a \omega^c \lambda_e \right\} A^b_\mu. \] (25)

From (20) and (25), we have,

\[ \frac{1}{i}(\partial_\mu U) U^{-1} + UA_\mu U^{-1} = -i(\beta \alpha^* + \frac{1}{2\sqrt{3}} \beta \beta^*) f^{abc} \omega^b \lambda_a A^c_\mu \]
\[
+ \beta\beta^* \left\{ \frac{2}{3} A^a_\mu \lambda_a - \frac{2}{3} \omega^b \omega^c A^b_\mu \lambda_c \right. \\
+ \frac{1}{\sqrt{3}} d^{pca} \omega^a A^a_\mu \lambda_c - d^{pba} d^{pca} \omega^b \omega^a \lambda_c \right\} \\
+ \frac{1}{2} (\alpha \alpha^* \lambda_b + i(\alpha \beta^* - \beta \alpha^*) f^{bcd} \omega^c \lambda_d \\
+ (\alpha \beta^* + \beta \alpha^*) d^{bca} \omega^c \lambda_d - \frac{2}{3} \beta \beta^* \lambda_b + \frac{4}{3} \beta \beta^* \omega^b \lambda_a \\
- \frac{\sqrt{3}}{2} (d^{a} f^{bcd} \omega^d \lambda_c + 2 \beta \beta^* d^{abd} d^{bcd} \omega^c \lambda_e) A^b_\mu. \tag{26}
\]

Now, consider the terms involving two \(d\)-tensors. They are,

\[
(-d^{pba} d^{pca} \omega^p \omega^b A^q_\mu \lambda_c + d^{pbd} d^{bcd} \omega^a \omega^c A^b_\mu \lambda_e).
\]

By changing the summation indices in the second term above, viz., \(b \rightarrow q\), \(a \rightarrow b\), \(c \rightarrow p\), \(e \rightarrow c\) in that order, the two terms cancel each other.

Then, the terms involving one \(d\)-tensor are,

\[
\beta\beta^* d^{pca} \omega^a A^a_\mu \lambda_c + \frac{1}{2} (\alpha \beta^* + \beta \alpha^*) d^{bca} \omega^c \lambda_d A^b_\mu - \frac{\beta \beta^*}{2 \sqrt{3}} d^{bca} \omega^d \lambda_e A^b_\mu.
\]

By rearranging the indices, this becomes,

\[
\frac{1}{2} (\alpha \beta^* + \beta \alpha^* + \frac{1}{\sqrt{3}} \beta \beta^*) d^{pca} \omega^a A^a_\mu \lambda_c,
\]

which vanishes due to the second relation in (10). Now, consider the terms involving one \(f\)-tensor. They are,

\[
- i(\beta \alpha^* + \frac{\beta \beta^*}{2 \sqrt{3}}) f^{abc} \omega^b A^c_\mu \lambda_a + \frac{i}{2} (\alpha \beta^* - \beta \alpha^*) f^{bcd} \omega^c \lambda_d A^b_\mu.
\]

In the second term, make \(b \leftrightarrow c\), \(d \rightarrow a\) so that the above expression becomes,

\[
- \frac{i}{2} (\alpha \beta^* + \beta \alpha^* + \frac{1}{\sqrt{3}} \beta \beta^*) f^{abc} \omega^b A^c_\mu \lambda_a,
\]

which vanishes due to the second relation in (10). Finally, the terms involving two \(\omega\)'s in (26) cancel each other, leaving

\[
\frac{1}{\ell} (\tilde{\partial}_\mu U)^{-1} + U A_\mu U^{-1} = \frac{1}{\ell} (\alpha \alpha^* + \frac{2}{3} \beta \beta^*) A^a_\mu \lambda_a = \frac{1}{2} A^a_\mu \lambda_a,
\]

\[
= A_\mu, \tag{27}
\]

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using the first relation in (10).

Thus when \( U = \exp\{i\omega^a \lambda_a/2\} \) finite transformation is considered, \( U \) becomes \( \alpha + \beta \omega^a \lambda_a \), linear in \( \omega^a \) in view of (2), with \( \alpha \) and \( \beta \) satisfying (10). This finite transformation leaves \( A_\mu \) invariant, for those \( A^a_\mu \)'s satisfying (1). Therefore, within the submanifold defined by (2), the gauge fields satisfying (1) can have a mass term, even for finite transformations.

It will be useful to consider the converse of this result. In order to this, we ask the question: What will be the gauge field configurations that remain unchanged under finite gauge transformations? Since under a finite gauge transformation \( U \), \( A_\mu \) transforms as,

\[
A_\mu \to A^U_\mu = \frac{1}{i}(\partial_\mu U)U^{-1} + U A_\mu U^{-1},
\]

the answer is given by

\[
A_\mu = \frac{1}{i}(\partial_\mu U)U^{-1} + U A_\mu U^{-1}.
\]

By right multiplying the above expression by \( U \), we have

\[
\frac{1}{i}\partial_\mu U + [U, A_\mu] = 0,
\]

whose infinitesimal version is (1). This means that for those \( A_\mu \)'s and \( U \) satisfying (28), the mass term remain invariant. In order to give explicit expressions for \( A^a_\mu \) and \( U \), the transformation \( U \) is chosen by those \( \omega \)'s satisfying (2). Then (9) gives the required \( U \). The choice of \( \omega \)'s satisfying (2) and \( A^a_\mu \) in (3) determined by (1), produce magnetic monopole configurations in the QCD action [12].

It is known that the tensor indices taking eight values in \( d \) and \( f \) are tensor indices associated with the adjoint group \( SU(3)/Z(3) \) of \( SU(3) \). The tensors \( d \) and \( f \) associated with \( SU(3)/Z(3) \), are cartesian tensors in eight real dimensions [24]. Given a single octet vector \( \{\omega^a\} \) and the tensors \( d \) and \( f \), it is known that at most two linearly independent octets can be formed. They are \( \omega^a \) itself and \( d_{abc}\omega^b\omega^c \). Then at most two independent \( SU(3) \) invariants
can be formed, which are taken as,
\[ \omega^a \omega^a, \]
\[ d_{abc} \omega^a \omega^b \omega^c. \]  
(29)

A geometric meaning can be given to the invariants. If we associate a \(3 \times 3\) matrix \(A\) with \(\omega^a\) as \(A = \omega^a \lambda_a\), where \(\lambda^a\)'s are the Gell-Mann matrices, then \(Tr(A^2)\) and \(det(A)\) give the above two invariants. In our choice made in (2), we have taken the two invariants as constants.

A study of the relationship of the adjoint group SU(3)/Z(3) to the subgroup of rotation group \(R8\) which leaves invariant the length \(\omega^a \omega^a\) of the real eight component vector \(\omega^a\) and the cubic invariant \(d_{abc} \omega^a \omega^b \omega^c\) has been made by Macfarlane [24]. Here we will give the main results for our choice (2). The eight components \(\omega^a\) can be taken to describe a point of \(E8\). Rotations in \(E8\) are real linear transformations
\[ \omega^a \rightarrow \omega'^a = R_{ab} \omega^b. \]  
(30)

Invariance of the length leads to \(RR^T = I\). We take \(det(R) = 1\). To relate \(SU(3)\) to a subgroup of \(R8\), the group of rotations in \(E8\), associate with each point in \(E8\), a \(3 \times 3\) traceless hermitian matrix \(A\)
\[ A = \omega^a \lambda_a, \]  
(31)
an element of the algebra of \(SU(3)\). Transformations of \(E8\) induced by \(U \in SU(3)\) transformation
\[ A \rightarrow A' = UAU^{-1}, \]  
(32)
can be shown to give [24]
\[ R_{ab} = \frac{1}{2} Tr\{\lambda_a U \lambda_b U^{-1}\}. \]  
(33)

For \(U\) in (9) and using the relations in (10), we find explicitly,
\[ R_{ab} = (1 - \beta \beta^*) \delta_{ab} + 2 \beta \beta^* \omega_a \omega_b - \sqrt{3} \beta \beta^* d_{abc} \omega_c \\
+ if_{abc} \omega_c (\alpha \beta^* - \beta \alpha^*). \]  
(34)
It is verified that $R_{ab}R_{ac} = \delta_{bc}$, i.e., the transformation is orthogonal. It is seen that $R_{ab}\omega_b = \omega_a$. Thus the rotation leaves $\omega^a\omega^b$ invariant. It is seen that the invariance of $d_{abc}\omega^a\omega^b\omega^c$ is verified by showing $d_{abc}R_{ap}R_{bq}R_{cr} = d_{pqr}$.

Now, we examine the property of an octet $B = \omega^a\lambda_a$ in the submanifold. This will shed some light on the type of monopole configurations discussed in the next section. In order for this, we consider the "characteristic equation" for the $3 \times 3$ matrix $B$. It is [21] (for the choices in (2), namely the two $SU(3)$ invariants taken as 1 and $\frac{1}{\sqrt{3}}$, constants.) given by $x^3 - x - \frac{2}{3\sqrt{3}} = 0$ with $x$ as the eigenvalues of $B$. The eigenvalues are found to be $-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$, showing that in its diagonalized form, $B$ is $-\lambda_8$-like. In this way, the monopole configurations in section.IV, will be $\lambda_8$-like and the submanifold defined by (2), picks up this configuration. It is possible to realize the $\lambda_3$-like configuration, by going out of the submanifold, which we will not do in this study.

IV. LOW ENERGY EFFECTIVE ACTION.

The field strength $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + g f^{abc}A^{b}_{\mu}A^{c}_{\nu}$ for the $A^{a}_{\mu}$ in (3) is calculated as

$$F^{a}_{\mu\nu} = (\partial_{\mu}C_{\nu} - \partial_{\nu}C_{\mu}) \omega^a - \frac{8}{3g} f^{abc} (\partial_{\mu}\omega^b)(\partial_{\nu}\omega^c) + \frac{16}{9g} f^{abc} f^{fde} \omega^e \omega^c \partial_{\mu}\omega^d \partial_{\nu}\omega^m.$$  \hspace{1cm} (35)

Use of the Jacobi identity for $f$’s [21] and the relation (6) for the last term in (35) gives

$$F^{a}_{\mu\nu} = (\partial_{\mu}C_{\nu} - \partial_{\nu}C_{\mu}) \omega^a - \frac{4}{3g} f^{abc} (\partial_{\mu}\omega^b)(\partial_{\nu}\omega^c) - \frac{16}{9g} f^{abc} f^{def} \omega^e \omega^f (\partial_{\mu}\omega^d)(\partial_{\nu}\omega^m).$$ \hspace{1cm} (36)

Now using $f^{edf} f^{cmb} = \frac{2}{3}(\delta_{ec}\delta_{dm} - \delta_{em}\delta_{cd}) + \delta^{ed} \delta^{fmb} - \delta^{deb} \delta^{fmd}$ and the relations (2), the last term in (36) can be shown to vanish, leaving

$$F^{a}_{\mu\nu} = (\partial_{\mu}C_{\nu} - \partial_{\nu}C_{\mu}) \omega^a - \frac{4}{3g} f^{abc} (\partial_{\mu}\omega^b)(\partial_{\nu}\omega^c).$$ \hspace{1cm} (37)
Consistent with (1), the above field strength $F_{\mu \nu}^a$ is "$SU(3)$ parallel" to $\omega^a$, i.e.,

$$f^{abc} \omega^b F_{\mu \nu}^c = 0,$$

which can be verified by using (4) and the relation among $f$'s. Unlike the case of $SU(2)$ [10,11], this does not imply that $F_{\mu \nu}^a$ is along $\omega^a$.

The QCD action in the absence of quarks,

$$S = -\frac{1}{4} \int d^4x (F_{\mu \nu}^a)^2,$$  

becomes

$$S = -\frac{1}{4} \int d^4x \left\{ f_{\mu \nu}^2 - \frac{8}{3g} f_{\mu \nu} \left( f^{abc} \omega^a \partial_\mu \omega^b \partial_\nu \omega^c \right) \right\} + \frac{16}{9g^2} f^{abc} f^{a'bc'} \partial_\mu \omega^b \partial_\nu \omega^c \partial_\rho \omega^d \partial_\sigma \omega^d,$$  

when (37) is used and where

$$f_{\mu \nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.$$  

It will be convenient to rescale the $\omega^a$'s as $g^{\frac{1}{3}} \omega^a$ and then in the strong coupling limit, the above action becomes

$$S \simeq -\frac{1}{4} \int d^4x \left\{ f_{\mu \nu}^2 - \frac{8}{3} f_{\mu \nu} \left( f^{abc} \omega^a \partial_\mu \omega^b \partial_\nu \omega^c \right) \right\},$$  

showing the Abelian dominance in the infrared region of QCD. Denoting $f^{abc} \omega^a \partial_\mu \omega^b \partial_\nu \omega^c = X_{\mu \nu}$, we see that $f_{\mu \nu}$ is coupled to $X_{\mu \nu}$. It follows that $\partial_\mu X_{\mu \nu} = f^{abc} \omega^a \partial_\mu (\partial_\nu \omega^b \partial_\nu \omega^c) \neq 0$, and the dual $\overline{X}_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} X_{\alpha \beta}$ violates the Bianchi identity,

$$\partial_\mu \overline{X}_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} f^{abc} \partial_\mu \omega^a \partial_\alpha \omega^b \partial_\beta \omega^c \neq 0.$$  

This along with

$$-\frac{2}{3} \int S \epsilon_{\mu \nu \alpha \beta} f^{abc} \omega^a \partial_\alpha \omega^b \partial_\beta \omega^c \, dx^\mu \wedge dx^\nu = -\frac{4}{3} \int S \overline{X}_{\mu \nu} \, dx^\mu \wedge dx^\nu,$$  

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a topological invariant, imply that magnetic monopoles are present in (42). In this way, the proposed gauge field configurations (3) give an action (42) which contains the monopole configurations. In the attempts [9] to relate the infrared region of QCD to a non-linear sigma model by expressing $A^a_\mu$ in terms $n^a$ (the sigma model fields), it is to be observed that the QCD action (39) will produce quartic terms involving the derivatives of $n^a$. The quadratic term in the sigma model action, namely, $\partial_\mu n^a \partial_\mu n^a$ can come only from the mass term $A^a_\mu A^a_\mu$. So, such models [9] implicitly assume the presence of a mass term for $A^a_\mu$ fields in the QCD lagrangian. Such a mass term, if included, will induce a mass term for the $C_\mu$ fields in our approach [12]. In the partition function $Z$ for the action (42), after introducing a mass term for the $C_\mu$-field with mass $m$ (for instance by Coleman-Weinberg mechanism by introducing complex scalars minimally coupled to $C_\mu$ as in [5,7,11]), a functional integration over $C_\mu$ produces an effective lagrangian [12]

$$\mathcal{L}_{\text{eff}} = -\frac{8}{9} \partial_\mu X_{\mu\nu} \left(\Box - \frac{m^2}{2}\right)^{-1} \partial_\rho X_{\rho\nu},$$

(45)
a form identical to the London case of magnetic confinement, as in an ordinary superconductor due to Meissner effect.

V. A STRING REPRESENTATION OF THE EFFECTIVE ACTION.

In order to find a string representation of the field theoretic action (42), we first consider its dual form. Introducing the dual field strength $G_{\mu\nu}$ dual to $f_{\mu\nu}$, we have

$$Z = \int [dC_\mu][G_{\mu\nu}] \exp\left\{ \int \left( -\frac{1}{4} G^2_{\mu\nu} + \frac{1}{2} G_{\mu\nu} f_{\mu\nu} - \frac{2}{3} f_{\mu\nu} X_{\mu\nu} \right) d^4 x \right\}. \quad (46)$$

The functional integral over $G_{\mu\nu}$ produces the partition function for (42). From (46), variation with respect to the $C_\mu$-field ($f_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$) gives

$$\partial_\nu \{ G_{\mu\nu} - \frac{4}{3} X_{\mu\nu} \} = 0,$$

which is solved for $G_{\mu\nu}$ as

$$G_{\mu\nu} = \epsilon_{\mu\nu\lambda\sigma} \partial_\lambda \tilde{A}_\sigma + \frac{4}{3} X_{\mu\nu}, \quad (47)$$

where the field $\tilde{A}_\mu$ serves as dual to $C_\mu$. Using (47) in (46), we eliminate the
\[ C_\mu \text{-field to obtain} \]
\[
Z = \int [d\tilde{A}_\mu] \exp \left[ \frac{1}{4} \left( (\partial_\lambda \tilde{A}_\sigma - \partial_\sigma \tilde{A}_\lambda)^2 + \frac{8}{3} X_{\mu\nu} \epsilon_{\mu\nu\lambda\sigma} \partial_\lambda \tilde{A}_\sigma \right) \right] + \frac{16}{9} X_{\mu\nu} X_{\mu\nu} \, d^4x. \tag{48}
\]

This action possesses dual \( U(1) \) invariance and coincides with the Abelian projected effective theory of QCD based on \( SU(3) \) in its dual form. We introduce a mass term for the \( \tilde{A}_\sigma \)-field which can arise from Coleman-Weinberg mechanism by invoking complex scalars coupled to \( \tilde{A}_\sigma \) as in the works of [3,4,11]. By functionally integrating the \( \tilde{A}_\sigma \)-field, we obtain an effective action
\[
S_{\text{eff}} = \int \left\{ -\frac{8}{9} \partial_\lambda \tilde{X}_{\lambda\sigma} (\Box - \frac{m^2}{2})^{-1} \partial_\rho \tilde{X}_{\rho\sigma} - \frac{4}{9} \tilde{X}_{\mu\nu} \tilde{X}_{\mu\nu} \right\} d^4x, \tag{49}
\]
apart from a constant (divergent) factor not involving the fields. This action suggests a dual confinement of gluons in the submanifold as in the London theory of Meissner effect. Second, we go over now to the Euclidean space.

From (43) and (44) it follows that \( \tilde{X}_{\mu\nu} \) represents the field strength of magnetic monopole. We make the following choice for \( X_{\mu\nu} \),
\[
X_{\mu\nu}(x) = \int d^2 \xi \delta^4(x - y) [y_\mu, y_\nu], \tag{50}
\]
where \([y_\mu, y_\nu] = \epsilon^{ab} \frac{\partial y_\mu}{\partial \xi^a} \frac{\partial y_\nu}{\partial \xi^b} ; a, b = 1, 2, y_\mu(\xi_1, \xi_2)\) represents the position of a point on the worldsheet swept by the string of the monopole and \( \xi_1, \xi_2 \) are the local coordinates of the worldsheet. In here, we have the Nambu’s picture [6] in mind that the magnetic flux lines terminate at the end points (quarks). Such a form for \( X_{\mu\nu} \) has been earlier suggested by Wentzel [25]. Quarks exchange the gluons and in the infrared region, the proposed gauge field configuration (3) produces an action (42) which contains \( X_{\mu\nu} \). Thus it is consistent with the picture that the magnetic flux lines connect two quarks or quark-antiquark pair.

Now using \( \tilde{X}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} X_{\alpha\beta} \), it is found that
\[
\int \partial_\lambda \tilde{X}_{\lambda\sigma} (\Box - \frac{m^2}{2})^{-1} \partial_\rho \tilde{X}_{\rho\sigma} d^4x = -\frac{1}{2} \int X_{\alpha\beta} X_{\alpha\beta} d^4x - \frac{m^2}{4} \int X_{\alpha\beta} (\Box - \frac{m^2}{2})^{-1} X_{\alpha\beta} d^4x, \tag{51}
\]
and $X_{\mu\nu} X_{\mu\nu} = X_{\mu\nu} X_{\mu\nu}$. Then Eqn.49 becomes, (Euclidean version),

$$S_{\text{eff}} = -\frac{2m^2}{9} \int X_{\mu\nu}(\Box - \frac{m^2}{2})^{-1} X_{\mu\nu} d^4x.$$  \hspace{1cm} (52)

It is to be noted that the second term on the right hand side of Eqn.49 is exactly cancelled by the first term in (51). We now use the representation for $X_{\mu\nu}$ (50) and denote $[y_\mu, y_\nu] = \sigma_{\mu\nu}(y)$. It is easy to verify that

$$\sigma_{\mu\nu}(x) \sigma_{\mu\nu}(x) = 2g,$$

where $g$ is the determinant of the induced metric $g_{ab} = \partial_a X^\mu \partial_b X^\mu$ on the string worldsheet. The operator $(\Box - \frac{m^2}{2})^{-1}$ acting on $\delta^4(x-y)$ can be evaluated using the procedure outlined by Diamantini, Quevedo and Trugenberger [15], by derivative expansion of the Green function of the operator above. Introducing an ultra-violet cut-off $\Lambda$ (corresponding to finite thickness of the worldsheet), the above action (52) becomes

$$S_s = \frac{2}{9} \frac{m^2}{4\pi} K_0\left(\frac{m}{2\Lambda}\right) \int \sqrt{g} d^2\xi - \frac{2}{9} \frac{\Lambda^2}{8\pi m^2} \int \sqrt{g} g^{ab} \partial_a t_{\mu\nu} \partial_b t_{\mu\nu} d^2\xi + \frac{2}{9} \frac{\Lambda^2}{m^2} \int \sqrt{g} R d^2\xi,$$

(54)

where $t_{\mu\nu} = \frac{1}{\sqrt{g}} \sigma_{\mu\nu}$ and $R$ is the scalar curvature of the worldsheet. Here the first term is identified with the area term or the Nambu-Goto action. The second term is the familiar extrinsic curvature action (see the book by A.M.Polyakov in Ref.17, page.284) and the last term is the topological Euler characteristic of the worldsheet. Since the action (52) has no coupling parameters in the strong coupling limit, the coefficients in front of the second third terms are numbers while the coefficient in front of the first term represents the string tension. The divergent integrals in the ultra-violet cut-off momentum $\Lambda$ are approximated by $\frac{\Lambda^2}{m^2}$. Eqn.54 represents the string action for the infrared region of QCD and as we have shown a realization of confinement as in the London theory of Meissner effect, the above action is the confining string action. This agrees with Ref.15, although they start from Kalb-Ramond action with monopoles while we obtain (54) from pure QCD action in the infrared region characterized by the submanifold (2).

VI. Conclusions
SU(3) gauge field configurations appropriate for the infrared region of QCD are proposed in submanifold of su(3). The usual action of QCD then contains monopole configurations interacting with $f_{\mu \nu}$. A dual version is constructed and is shown to give the confinement of these field configurations as in Meissner effect. A string representation is obtained by choosing a representation for $X_{\mu \nu}$ in terms of the geometry of the string worldsheet and by going over to the Euclidean version.

The string action which corresponds to confinement consists of the Nambu-Goto term, extrinsic curvature action and the Euler characteristic of the worldsheet, showing the inevitable occurrence of the extrinsic curvature action. In this connection between the gauge field theory and the string theory, the origin of the extrinsic curvature action is in the derivative expansion of the Green function for the four dimensional operator $(\Box - \frac{m^2}{2})^{-1}$ acting on $X_{\mu \nu}$ which contains the 4-dimensional Dirac delta function. The leading term however gives the Nambu-Goto action. Since the gauge field action has been shown to have confinement of these gauge fields, one can expect that the resulting string action also to exhibit confinement in the infrared region. This has been shown by an explicit calculation of the quantum one-loop partition function of both the Nambu-Goto action and extrinsic curvature action by Viswanathan and Parthasarathy [26], in which the partition function has been found to be that of a modified Coulomb gas and in the infrared limit, there are long range interaction responsible for confinement while in the ultra-violet limit, the flux lines curl up to vortices leading to the non-confining phase of the system.

Having realized a mechanism of confinement in the infrared region of QCD, it is pertinent to address the issue of chiral symmetry breaking as these two issues are related. It has already been shown by Nair [27] and Nair and Rosenzweig [28] that quarks in the monopole background break chiral symmetry, i.e., a non-vanishing value for $\langle \bar{\psi} \psi \rangle$ has been obtained in the monopole background, the mechanism being similar to that of Callan [29] and Rubakov [30], except that in QCD, this does not violate flavour number. As we have a monopole configuration realized in (42), by choosing a specific form for $X_{\mu \nu}$, chiral symmetry breaking can be realized along the lines of [27] and [28].
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References.

1. H.D.Politzur, Phys.Rev.Lett. 30 (1973) 1346.
   D.J.Gross and F.Wilczek, Phys.Rev.Lett. 30 (1973) 1343.
   G.t’Hooft, (unpublished) 1972; See hep-th/9812203.

2. T.Greenshaw, M1 Collaboration and A.Doyle. ZEUS collaboration in
   the Proceedings of the Sixth International Workshop in DIS and QCD,
   April 1998; Brussels, Belgium.

3. G.t’Hooft, Nucl.Phys. B190 (1981) 455.

4. S.Mandelstam, Phys.Rev. D19 (1978) 2391.

5. G.t’Hooft, in High Energy Physics Proceedings, 1975, Edited by A.Zichichi;
   Nucl.Phys. B138 (1978)1; B153 (1979) 141.

6. Y.Nambu, Phys.Rev. D10 (1974) 4262; Phys.Rep. C23 (1975) 250.

7. S.Mandelstam, in the Proceedings of the Monopole Meeting, Trieste,
   Italy, 1981. Edited by N.S.Craige, P.Goddard and W.Nahm; World
   SScientific. 1982.

8. K.-I.Kondo, Phys.Rev. D58 (1998) 105016, 105019.

9. L.Faddeev and A.J.Niemi, hep-th/9807069 9812090.

10. E.Corrigan and D.Olive, Nucl.Phys. B110 (1976) 237.
    E.Corrigan, D.Olive,D.B.Fairlie and J.Nuyts, Nucl.Phys. B106 (1976)
    475.

11. Y.M.Cho, Phys.Rev. D21 (1980) 1080; D23 (1981) 2415; Phys.Rev.Lett.
    44 (1980) 1115.

12. R.Parthasarathy, hep-th/9902027.
13. A.M. Polyakov, Nucl. Phys. B486 (1997) 23.

14. A.M. Polyakov, Phys. Lett. B59 (1975) 82.

15. M.C. Diamantini, F. Quevedo and C.A. Trugenberger, Phys. Lett. B396 (1997) 115.

16. M. Kalb and P. Ramond, Phys. Rev. D9 (1974) 2273.

17. A.M. Polyakov, Nucl. Phys. B268 (1986) 406.
   A.M. Polyakov, *Gauge Fields and Strings*, Harwood Academic Publishers, Chur, 1987.

18. H. Kleinert, Phys. Lett. B174 (1986) 335; Phys. Rev. Lett. 58 (1987) 1915.

19. H. Kleinert, hep-th/9601030.

20. H. Kleinert and A.M. Chervyakov, Phys. Lett. B381 (1996) 286.

21. A.J. Macfarlane, A. Sudbury and P.H. Weise, Comm. Math. Phys. 11 (1968) 77.

22. V.N. Gribov, Nucl. Phys. B139 (1978) 1.

23. R. Parthasarathy, hep-th/9903060.

24. A.J. Macfarlane, Comm. Math. Phys. 11, (1968) 91.

25. G. Wentzel, Supplement of Prog. Theor. Phys. Nos. 37, 38 (1966) 163.

26. K.S. Viswanathan and R. Parthasarathy, Phys. Rev. D51 (1995) 5830.

27. V.P. Nair, Phys. Rev. D28, (1983) 2673.

28. V.P. Nair and C. Rosenzweig, Phys. Lett. 131B, (1983) 434; Phys. Lett. 135B, (1984) 450; Phys. Rev. D31, (1985) 401.

29. C. Callan, Phys. Rev. D25, (1982) 2141; Phys. Rev. D26, (1982) 2058.

30. V. Rubakov, JETP Lett. 33, (1981) 644; Nucl. Phys. B203, (1982) 2058.

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