The Nature of Subproton Scale Turbulence in the Solar Wind

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The nature of subproton scale fluctuations in the solar wind is an open question, partly because two similar types of electromagnetic turbulence can occur: kinetic Alfvén turbulence and whistler turbulence. These two possibilities, however, have one key qualitative difference: whistler turbulence, unlike kinetic Alfvén turbulence, has negligible power in density fluctuations. In this Letter, we present new observational data, as well as analytical and numerical results, to investigate this difference. The results show, for the first time, that the fluctuations well below the proton scale are predominantly kinetic Alfvén turbulence, and, if present at all, the whistler fluctuations make up only a small fraction of the total energy.

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Introduction.—Despite many years of observations, the nature of small scale fluctuations in the solar wind remains under debate. In particular, there are several conflicting theories for the turbulence at scales smaller than the proton gyroradius. In this Letter, we present new observations, theory and numerical simulations to determine the types of fluctuations present between the ion and electron scales.

Theoretical descriptions of plasma turbulence can be categorized as weak or strong. Weak turbulence theory involves fluctuations that do not change significantly during each interaction, so that they retain their linear wave mode properties, allowing their energy spectrum to be derived analytically [e.g., 1–7]. If the turbulence is strong, the non-linear terms in the dynamical equations are comparable to the linear terms and the fluctuations fully decay in one interaction (a situation also known as critical balance [8]). Therefore, qualitative properties of the linear modes may still be present, even in strong turbulence. Indeed, observations in the solar wind, in which the turbulence is thought to be strong, display many properties that are similar to those of the linear waves [e.g., 9–12].

Knowing the types of fluctuations present is central to understanding the nature of the turbulence. For example, in the solar wind at scales larger than the proton gyroradius, the fluctuations display properties similar to Alfvén [13] waves [e.g., 9]: perpendicular magnetic fluctuations much larger than parallel magnetic fluctuations ($\delta B_\perp \gg \delta B_\parallel$), velocity and normalized magnetic fluctuations of similar amplitude $\delta v \sim \delta b$, and frequent times of strong correlation between $v$ and $b$. The phase speed of the fluctuations was also suggested to be similar to the Alfvén speed [10]. These properties are used to justify the application of Alfvénic turbulence theory to the solar wind. Interestingly, however, the magnetic fluctuations are slightly larger than the velocity fluctuations $\delta b \gtrsim \delta v$, in both the solar wind and MHD turbulence simulations [14, 15, and references therein], showing that in strong turbulence there can be quantitative differences to the linear wave relationships.

At smaller scales, around the proton gyroradius and below, the situation is less clear and different wave modes have been suggested to be relevant. Two possibilities are kinetic Alfvén waves (KAWs) [10, 16–36], and whistler waves [6, 24, 34, 37–54]. Since the non-linear equations that they derive from have a similar form, the turbulence energy spectra, obtained from dimensional arguments, are the same.

Previous attempts to distinguish these possibilities in observations considered fluctuations around the proton scale, rather than well below, and led to contradictory or uncertain conclusions. Measurements of the normalized reduced magnetic helicity [11, 16, 39, 55–58] indicate that the proton scale fluctuations are generally right handed in the plasma frame, which was initially interpreted as due to the presence of whistler waves [39]. The KAW, however, is also right handed [18, 22, 59], so this is not a useful distinguishing measure [26]. Bale et al. [10] suggested the ratio of electric to magnetic fluctuations at the proton scale to be consistent with the KAW, rather than whistler, dispersion. Although Salem et al. [60] concluded that this ratio alone was not enough to make the distinction. Various authors have used the amplitude of the parallel, compared to the perpendicular, magnetic fluctuations [33, 47, 58, 60–64], although they reached different conclusions, partly because there is not a large difference between the modes and different definitions of the parallel direction were used [62]. Finally, $k$-filtering, a multi-spacecraft optimization technique, has led to contradictory findings [65–67] and cannot currently be used far below the proton scale, due to the available spacecraft separations.

In this Letter, we present a new measure to clearly distinguish the nature of the fluctuations well below the proton scale. Applying this to solar wind observations, and comparing the result to theory and numerical simulations, shows that the fluctuations between ion and electron scales are predominantly kinetic Alfvén turbulence, rather than whistler turbulence.

Theory.—In a collisionless plasma of beta $\beta \sim 1$, both the KAW and whistler wave can be excited for perpendicular scales between the ion and electron gyroradii, $1/\rho_i \ll k_\perp \ll 1/\rho_e$, [e.g., 35]. While some properties of these electromagnetic modes are qualitatively similar, such as the dispersion relation, magnetic compressibility and helicity, there is one key difference. The KAW is low frequency compared to the
ion thermal speed, $\omega \ll k_{\parallel}v_{th,i}$, so the ions can fluctuate and are involved in the dynamics, along with the electrons. The whistler wave, however, is high frequency, $\omega \gg k_{\parallel}v_{th,i}$, so the ions are not able to move fast enough and, due to quasi-neutrality, the electron density fluctuations are also negligibly small. This can be seen, for example, in the numerical solutions of Gary and Smith [47] and the analytical solutions of Boldyrev et al. [35].

It is possible, however, for whistler waves to generate density fluctuations if their frequency is not asymptotically large, but close to $k_{\parallel}v_{th,i}$ so that the ions can still fluctuate. To produce density fluctuations, $\delta n/n_0$, comparable to the magnetic fluctuations, $\delta B/B_0$, would require in this case all of the energy to be at $k_{\parallel}d_i \approx 1$ (where $d_i$ is the ion inertial length) for any $k_{\parallel}$. A slight spread of energy over $k_{\parallel}$ would significantly decrease the density amplitude according to $(\delta n/n_0)^2/(\delta B/B_0)^2 \sim \frac{1}{\lambda^2}(k_{\parallel}d_i)^{-4}$ [35]. However, for $k_{\parallel}d_i \approx 1$ the whistler modes would be strongly damped by the ions: the damping rate is $\gamma/\omega_0 = -2\sqrt{\pi}\beta_i^{-1} x \exp(-x^2)$, where $x = k_{\parallel}d_i/\sqrt{\pi}$ [35], so for them to remain undamped at $\beta_i \sim 1$ would require $x \gtrsim 2$. For such values of $x$ these modes would have very small density fluctuations: $(\delta n/n_0)^2/(\delta B/B_0)^2 \lesssim 0.03$. This difference between the density fluctuations in whistlers and KAWs is the basis of our technique to distinguish the nature of the fluctuations.

A natural normalization for the density and magnetic fluctuations in kinetic Alfvén turbulence is [22, 36]

$$\hat{\delta n} = \left(1 + \frac{T_i}{T_e}\right)^{\frac{1}{2}} \frac{v_s}{v_A} \left[1 + \left(\frac{v_s}{v_A}\right)^2 \left(1 + \frac{T_i}{T_e}\right)\right]^{\frac{1}{2}} \frac{\delta n_0}{n_0}, \tag{1}$$

$$\hat{\delta B} = \frac{\delta B}{B_0}, \tag{2}$$

where $T_i$ and $T_e$ are the ion and electron temperatures, $v_s = \sqrt{T_i/m_i}$ is the ion acoustic speed, $m_i$ is the ion mass, $v_A$ is the Alfvén speed, $n_0$ is the mean density and $B_0$ is the mean magnetic field strength. With this normalization, the KAW has equal density and perpendicular magnetic fluctuation amplitudes, $\delta n_{\perp} = \delta n$, independent of the wavevector. For the whistler wave, $\delta B_{\perp} \gg \delta n$ for the reasons discussed above. We note that these equations do not include temperature anisotropies and the small population of alpha particles which drifts with respect to the protons in the solar wind. While these features can affect such normalizations [15], they are not significant, compared to the error estimates, for the data intervals considered here.

**Results.**—The ARTEMIS spacecraft [68] have measured the density and magnetic fluctuations in the solar wind at 1 AU with sufficient resolution to test these predictions. The spacecraft potential fluctuations measured by EFI [69] were used to infer the electron density fluctuations [70, 71] and the magnetic fluctuations were measured by SCM [72]. For the normalization factors and kinetic scales, FGM [73] was used for the DC magnetic field and the ESA ground moments [74] were used for the particle densities, bulk velocities and temperatures.

Figure 1 shows the spectra of electron density and magnetic fluctuations, normalized according to Equations 1 and 2, measured by ARTEMIS-P2 on 11th October 2010 from 00:21 to 01:14 UT. They have a similar shape to such kinetic scale spectra measured by other spacecraft [e.g., 61, 75]. The magnetic field spectrum uses FGM data below 2 Hz and SCM data above 2 Hz. Noise due to harmonics of the spacecraft spin frequency (0.30 Hz), clock frequencies (8/32 Hz) and sidebands due to the spin modulation of the clock frequencies were removed by deleting the affected portions of the spectrum. There is additional noise at higher spacecraft-frame frequencies ($f_{sc} > 10$ Hz), but this is outside our range of current interest. The density fluctuation noise floor is marked with a horizontal dotted line and the SCM noise floor with a dash-dotted line. Proton and electron gyroradii, $\rho_{i,e}$, and inertial lengths, $d_{i,e}$, are marked assuming the Taylor [76] hypothesis. It can be seen that between the ion and electron scales, the normalized density and magnetic fluctuations are of similar amplitude, $\delta \tilde{n} \sim \delta \tilde{B}$. As discussed above, this suggests that the turbulence is kinetic Alfvén in nature, rather than whistler turbulence.

The same analysis was performed on all of the the 17 intervals used by Chen et al. [70, 71]. This number is limited since the spacecraft need to be in the free solar wind [77] and in burst mode. The proton beta for these intervals covers the range $0.29 \leq \beta_i \leq 3.7$, and the electron beta $0.40 \leq \beta_e \leq 5.5$. For each interval, the average “kinetic Alfvén ratio”, defined as $\delta \tilde{n}^2/\delta B_{\perp}^2$, was calculated over the range $2.5 < f_{sc} < 7.5$ Hz. This range was chosen because it is high enough to avoid spacecraft spin effects in the magnetic field spectrum, low enough to avoid the instrument noise floors and is between the proton and electron scales $5 \lesssim k\rho_i \lesssim 14$. Since the total magnetic energy, $\delta B^2 = \delta B_{\perp}^2 + \delta B_{\parallel}^2$, was measured with the SCM data, the per-
out, although the energy in such fluctuations cannot gener-
ally be more than a few per cent. Finally, there may be other
physics occurring in the solar wind at kinetic scales, such as
instabilities, that is not captured in the simulation. The mean
values of the kinetic Alfvén ratio, along with their standard
error of the mean are given in Table I.

The final comparison performed between the solar wind
and the simulation is the correlation between the spectral in-
dices of the different fields. Scatterplots of the spectral in-
dices measured in the solar wind intervals over the range
2.5 < \( f_{sc} < 7.5 \) Hz and in the simulation snapshots taken
from [32] with fitting range 5 < \( k < 15 \), are shown in Figure
2c.d. Again, the spread in values is smaller in the simulation,
as expected, but the linear correlation coefficients \( r \) (given in
Table I, along with the 95% confidence intervals) are similar:
there is a mild positive correlation. This similarity is further
evidence that the kinetic scale fluctuations in the solar wind
can be described by kinetic Alfvén turbulence.

**Discussion.**—We have shown that between ion and electron
scales in the solar wind, the kinetic Alfvén ratio is \( \delta n^2/\delta b_{\perp}^2 = 0.75 \), as expected for kinetic Alfvén turbulence, rather than
whistler turbulence, in which the ratio should be smaller by
more than an order of magnitude. Although the density and
perpendicular magnetic fluctuations are of similar amplitude,
there is a slight excess of magnetic energy in both the solar
wind observations and the kinetic Alfvén turbulence simu-
lation. This suggests that strong kinetic Alfvén turbulence,
while having some properties of KAWs, can produce quan-
titative differences, similarly to the excess magnetic energy
seen at MHD scales [e.g., 14, 15].

In the estimates of whistler compressibility discussed ear-
lier, the results of linear theory were used. Due to the critical
balance condition, one may expect this estimate to approx-
imately hold for strong turbulence as well. However, such
an assumption may not be necessary as there is a reason to
believe that if whistler turbulence is present in this case, it
would be weak. Indeed, at \( k_{\perp} \rho_i = 7.1 \) for the interval in Fig-
ure 1, the fluctuation amplitude is \( \delta B/B_0 = 0.026 \). Strong
turbulence requires \( \delta B/B_0 = k_{\parallel}/k_{\perp} \), giving \( k_{\parallel}/\rho_i = 0.18 \)
and a propagation angle of \( \theta = 88.5^\circ \). For whistlers to exist
at such \( k_{\perp} \) requires \( k_{\|}/k_{\perp} > \sqrt{2} \beta_i/(k_{\perp} \rho_i) \), that is, \( \theta < 75^\circ \),
since \( \beta_i = 1.96 \) here. Since the required angles are larger, this
means that if the turbulence is strong, it cannot be whistler
turbulence. Having \( \theta = 75^\circ \) would mean \( \delta B/B_0 \ll k_{\parallel}/k_{\perp} \) so
whistler turbulence would be in the weak regime. The fact that
the fluctuations in the solar wind are strongly non-Gaussian in
this range [64, 78, 79] lends further support to the strong ki-

### Table I. Comparison of solar wind data and kinetic Alfvén turbulence simulation

|                                | \( \delta n^2/\delta b_{\perp}^2 \) | \( r \) |
|--------------------------------|------------------------------------|--------|
| solar wind                     | 0.75 \( _{−0.17}^{+0.22} \)        | 0.46 \( _{−0.49}^{+0.17} \) |
| simulation                     | 0.786 \( _{−0.004}^{+0.004} \)      | 0.52 \( _{−0.22}^{+0.17} \) |

FIG. 2. Histograms of the kinetic Alfvén ratio in (a) the solar wind
and (b) kinetic Alfvén turbulence simulation. Scatterplots of spec-
tral indices in (c) the solar wind and (d) kinetic Alfvén turbulence
simulation.
netic Alfvén turbulence interpretation.

Since solar wind turbulence at MHD scales is predominantly Alfvénic, with around 10% of the energy in the slow mode fluctuations and very little in the fast mode, it makes sense that the transition is to kinetic Alfvén, rather than whistler, turbulence [12]. This is also consistent with other observations, such as the fluctuations being anisotropic with \( k_\perp > k_\parallel \) [61], having a significant, rather than negligible, parallel electric field spectrum [80], and the flattening of the density spectra at ion scales [71], which is thought to be due to the enhanced compressibility of kinetic Alfvén turbulence [23]. Since such fluctuations are relatively low frequency, the Taylor hypothesis can be used to relate the spacecraft-frame frequency spectra to the wavenumber spectra of theory and simulations. Indeed, the measured spectral indices of density and magnetic fluctuations \( \sim -2.7 \) are similar to those in kinetic Alfvén turbulence simulations [28, 32].

If the frequency of the supraion scale fluctuations becomes close to the ion cyclotron frequency \( \omega - \Omega_i \sim \Omega_i / \sqrt{k_\perp \rho_i} \), it has been proposed that ion-Bernstein modes may couple to the turbulent cascade [20, 81, 82]. For a kinetic Alfvén cascade, this would happen when \( k_\parallel v_{th,i} \sim \Omega_i / (k_\perp \rho_i) \). For collisionless damping of these modes to be negligible, the ratio \( k_\parallel v_{th,i} / (\omega - \Omega_i) \sim 1/\sqrt{k_\perp \rho_i} \) should be much smaller than one. It then follows that in the asymptotic limit \( k_\perp \rho_i \gg 1 \), the ion-Bernstein modes occupy a narrow band in the frequency space, which may reduce their coupling to the kinetic Alfvén cascade [e.g., 20, 81]. For moderate values of \( k_\perp \rho_i \), on the other hand, these modes are relatively strongly damped compared to the kinetic Alfvén modes [82], which also reduces their energetic relevance. A quantitative estimate of the level of ion-Bernstein fluctuations driven by kinetic Alfvén turbulence must await further observations and fully kinetic numerical treatment.

The nature of the supraion scale fluctuations has some important implications, such as understanding plasma heating. For example, since the fluctuations are kinetic Alfvén turbulence, the cyclotron resonance may not be as important relative to other damping mechanisms [22]. Kinetic Alfvén turbulence may also generate particular types of structures, such as 2D sheets [32], which are important for understanding heating if it occurs preferentially at such structures [83, 84]. Determining the nature of supraion scale turbulence is also relevant to other astrophysical plasmas, such as the ionized interstellar medium [85] and hot accretion flows [86], which may have similar turbulence properties but are not as well measured. Understanding supraion scale turbulence in the solar wind, such as the fact that it is predominantly kinetic Alfvén in nature, can provide insight into the dynamics and heating of such plasmas.

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