Swimmers’ wakes are not reliable indicators of swimming performance

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The structure of swimmers’ wakes is often assumed to be an indicator of swimming performance. Here, we discuss three cases where this assumption fails. In general, great care should be taken in deriving any conclusions about swimming performance from the wake flow pattern.

I. INTRODUCTION

When swimmers propel themselves through a fluid, they leave a distinct pattern of fluid flow in their wakes analogous to the footprints of terrestrial animals [1]. The flow structures in the wake carry much information with them; for example, a proper control volume analysis can translate the velocity and stress fields in the fluid to forces on the swimmer. Through the advent of particle image velocimetry in particular, flow structures are furthermore much more accessible to scientists than the musculature of swimmers, offering a non-intrusive way to estimate the forces produced and energy expended by swimmers. This offers some motivation for why, it seems, studies of swimming animals so often show the flow structures in the swimmers’ wakes [2]. (They are also quite visually-pleasing.)

The present letter, however, is a cautionary one. Here, we re-interpret results in the literature and offer three cases demonstrating that the wakes of swimmers can be entirely misleading when trying to assess the propulsive performance of swimmers. Throughout, we employ a flapping rectangular foil as a model for fins and flukes, as is typically done [3]. In the first case, we show that significant changes in the wake can be associated with no changes in propulsion; in the second case, we show that small changes in the wake can be associated with large changes in propulsion; and in the third case, we show that changes in the pattern and self-interaction of the wake are associated with changes in propulsion that are captured by simple models that are agnostic to the state of the wake.

II. FIRST CASE: VORTEX SPACING

An often-cited mechanism for how flapping foils produce thrust is based on the wake mechanics [4]. As a foil flaps and moves forward, it leaves behind a staggered array of vortices in its wake. A common pattern is shown schematically in figure 1 where two opposite-sign vortices are shed per flapping period, arranged such that those with a counterclockwise orientation are positioned above those with a clockwise orientation (other patterns are possible; we will return to this point later). The vortices induce flow that takes the form of a meandering jet that increases the streamwise momentum of the fluid opposite to the direction of travel. By action-reaction, the fluid imparts a thrust force onto the foil.

Based on this mechanism, it seems intuitive that the spacing of the vortices should dictate how much thrust is produced. Suppose we run three experiments: in each experiment, the flapping frequency and amplitude are the same, but from one experiment to the next we increase the forward speed of the foil. What happens to the thrust? The resulting wakes are sketched in figure 1 with speed increasing from top to bottom. When the vortices are spaced closer horizontally, the induced velocity has a larger component in the streamwise direction, and we should expect that more thrust is produced.

In fact, despite significant changes in the spacing of the vortices in the wake, all three experiments will produce the same mean thrust. It has been shown indirectly [5] and directly [6, 7] that for flapping motions representative of those seen in nature, the mean thrust $T$ is independent of the swimming speed of the animal, instead being proportional to the density of the fluid $\rho$, the area of the foil $A$, and the square of the velocity of the trailing edge $V$, so that

$$T \sim \rho V^2 A,$$

(1)
FIG. 2. Drag-producing wake.

where ~ indicates a proportionality. In the three experiments we have described, the horizontal spacing of the vortices is dictated by the ratio of the swimming speed and the frequency of flapping, \( U_\infty / f \). Since the mean thrust is independent of the swimming speed, it should be clear that the spacing of vortices in the wake cannot reliably give an indication of the mean thrust produced. The vertical spacing of vortices in the wake is also commonly used to make conclusions about thrust production. As described previously, when the counterclockwise-oriented vortices are positioned above the clockwise-oriented vortices, we expect the foil to produce thrust; a wake with this arrangement of vortices is called a reverse von Kármán vortex street and is often termed a “thrust-type” wake \([8]\). Following the same logic, we expect a foil to produce drag when the vortices have the opposite arrangement, as sketched in figure 2; a wake with this arrangement of vortices is called a von Kármán vortex street and is often termed a “drag-producing” wake \([8]\). When the vortices are in line, we expect no net horizontal force; this arrangement of vortices is inaccurate, however. Generally speaking, the drag-thrust transition occurs when the wake is already a “thrust-type” wake \([10]\), since some excess streamwise fluid momentum is needed to overcome profile drag or velocity fluctuations and pressure differences in the control volume \([11, 12]\). Furthermore, even “drag-producing” wakes have been observed to produce thrust \([10]\). In this context, the recent work by Lagopoulos et al. \([13]\) offers an alternative method to distinguish drag- and thrust-producing behavior based on kinematic inputs instead of the vortex arrangement. Despite intuition, vortex spacing, either in the horizontal or vertical direction, is not a reliable indicator of thrust production.

III. SECOND CASE: REYNOLDS NUMBER

The Reynolds number \( Re = U_\infty c/\nu \) measures the strength of inertial forces relative to the strength of frictional forces in a flow. Here, \( c \) is the chord length of the foil, and \( \nu \) is the kinematic viscosity of the fluid. For flapping foils, the effects of the Reynolds number are typically not considered (for example, in \([14]\) the authors use results from flow visualizations captured at \( Re = 1100 \) to make conclusions about swimming performance at \( Re = 40,000 \)). This is likely because Reynolds number effects are presumed to be small compared to kinematic effects, which are, of course, strong. In addition, the structure of the wake typically has a rather weak dependence on the Reynolds number: in studies spanning a large range of Reynolds numbers, geometries, and kinematics, the authors have shown that the basic process of vortex formation and the establishment of the wake are not significantly affected by the Reynolds number \([15–19]\). Increasing the Reynolds number tends to lead to the appearance of some small-scale structures and a sharpening of flow structures, but the basic sketch drawn in figure 1 does not change. Based solely on the wake, we would not expect much of a change in swimming performance with Reynolds number.

Nevertheless, the efficiency of propulsion turns out to be quite sensitive to the Reynolds number, especially with regard to its optimal value. (Here, we use the Froude efficiency \( \eta = C_T/C_P \), where \( C_T = T/\rho U_\infty^2 A \) is the mean thrust coefficient and \( C_P = P/\rho U_\infty^3 A \) is the mean power coefficient.) The efficiency’s sensitivity to the Reynolds number was shown analytically in \([7]\) and confirmed by simulations in \([20]\). To explain why this is so, we first note that the optimal efficiency coincides with low thrust (see \([7]\) for details). Changing the Reynolds number will lead to a small change in thrust, which changes the efficiency by

\[
\Delta \eta = \frac{\partial \eta}{\partial C_T} \Delta C_T = \frac{\eta}{C_T} \Delta C_T. \tag{2}
\]

Even though the change in thrust may be small, the change in optimal efficiency will be large because it coincides with low thrust. Changing the Reynolds number may not change the wake much, which would lead us to believe that swimming performance is hardly affected, but the efficiency may change substantially. The wake is therefore not a reliable indicator of efficiency.

IV. THIRD CASE: VORTEX PATTERN AND INTERACTIONS

Although the reverse von Kármán vortex street is the most commonly encountered wake vortex pattern, many other patterns are possible. In figure 3 we have sketched some of the patterns observed in the experiments and computations of \([10]\); even more exotic patterns have been observed. With wildly varying wake patterns, we may expect to see large differences in swimming performance as the wake transitions from one pattern to another.

In fact, we do not. There is “no evidence of particular vortex patterns having a distinct effect on force measurements” \([21]\). As kinematic parameters are varied, all metrics of swimming performance vary smoothly, even when the wake transitions from one pattern to another. The insensitivity of swimming performance to the type of
The insensitivity of swimming performance to vortex pattern may be due to the fact that by the time the pattern develops, the vortices have advected far enough downstream so as to not have a large effect on the forces on the foil. For instance, Young and Lai \cite{24} compared the swimming performance of flapping foils whose wakes were allowed to deform according to the induced velocity field with those that were not. The authors found no difference in swimming performance, indicating that although the location where vorticity is shed (the trailing edge) is important, the subsequent development of the vortex pattern is not. This suggests that the interactions between vortices have little bearing on swimming performance.

The importance of vortex interactions was also addressed in \cite{25}. There, the authors investigated intermittent swimming motions, where the foil alternates between one period of flapping and rest; the duty cycle gives the proportion of time spent flapping. Each burst of flapping releases a group of vortices into the wake, and the duty cycle determines the spacing between the groups. When the duty cycle is low, groups of vortices are independent of each other. As the duty cycle increases, the groups of vortices move closer and should interact more strongly; in this sense, the duty cycle provides a way to control the strength of vortex interactions. The authors found that the time-averaged thrust and power (averaged over flapping and rest time) simply scaled linearly with the duty cycle, that is, they are independent of duty cycle when averaged only over the time that the foil flaps. The individual bursts of vortices are therefore effectively independent of each other even as the duty cycle tends toward unity (at least as far as swimming performance is concerned). Vortex interactions, and the resulting vortex patterns, apparently have little bearing on swimming performance.

V. CONCLUSION

The wake behind swimmers is often looked to as an indicator of swimming performance. We have offered three cases to the contrary. Significant differences in the wake may cause no changes in swimming performance, insignificant changes to the wake may cause great changes in swimming performance, and the pattern and self-interaction of the wake have little bearing on swimming performance, dispelling the notion that there is a preferred pattern of vortices. This is not to say that the wake is not informative (indeed, swimming performance can be recovered from wake measurements when a control volume analysis is properly performed, but doing so requires information that is difficult to obtain in experiments, e.g. velocity-pressure correlations); we merely point out that conclusions based on the wake can be misleading, and that great care should be taken.

FIG. 3. Vortex patterns from \cite{10}.

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[7] D. Floryan, T. Van Buren, and A. J. Smits, Efficient cruising for swimming and flying animals is dictated by fluid drag, Proceedings of the National Academy of Sciences 115, 8116 (2018).
[8] M. Sfakiotakis, D. M. Lane, and J. B. C. Davies, Review of fish swimming modes for aquatic locomotion, IEEE Journal of Oceanic Engineering 24, 237 (1999).
[9] K. D. Jones, C. M. Dohring, and M. F. Platzer, Experimental and computational investigation of the Knoller-Betz effect, AIAA journal 36, 1240 (1998).
[10] A. Andersen, T. Bohr, T. Schnipper, and J. H. Walther, Wake structure and thrust generation of a flapping foil in two-dimensional flow, Journal of Fluid Mechanics 812 (2017).
[11] R. Ramamurti and W. Sandberg, Simulation of flow about flapping airfoils using finite element incompressible flow solver, AIAA journal 39, 253 (2001).
[12] D. G. Bohl and M. M. Koochesfahani, MTV measurements of the vortical field in the wake of an airfoil oscillating at high reduced frequency, Journal of Fluid Mechanics 620, 63 (2009).
[13] N. S. Lagopoulos, G. D. Weymouth, and B. Ganapathisubramani, Universal scaling law for drag-to-thrust wake transition in flapping foils, Journal of Fluid Mechanics 872 (2019).
[14] J. M. Anderson, K. Streitlien, D. S. Barrett, and M. S. Triantafyllou, Oscillating foils of high propulsive efficiency, Journal of Fluid Mechanics 360, 41 (1998).
[15] K. Ohmi, M. Coutanceau, T. P. Loc, and A. Dulieu, Vortex formation around an oscillating and translating airfoil at large incidences, Journal of Fluid Mechanics 211, 37 (1990).
[16] K. Ohmi, M. Coutanceau, O. Daube, and T. P. Loc, Further experiments on vortex formation around an oscillating and translating airfoil at large incidences, Journal of Fluid Mechanics 225, 607 (1991).
[17] H. Dong, R. Mittal, M. Bozkurttas, and F. Najjar, Wake structure and performance of finite aspect-ratio flapping foils, in 43rd AIAA Aerospace Sciences Meeting and Exhibit (2005) p. 81.
[18] R. T. Jantzen, K. Taira, K. O. Granlund, and M. V. Ol, Vortex dynamics around pitching plates, Physics of Fluids 26, 053606 (2014).
[19] U. Senturk and A. J. Smits, Numerical simulations of the flow around a square pitching panel, Journal of Fluids and Structures 76, 454 (2018).
[20] U. Senturk and A. J. Smits, Reynolds number scaling of the propulsive performance of a pitching airfoil, AIAA Journal , 1 (2019).
[21] A. W. Mackowski and C. H. K. Williamson, Direct measurement of thrust and efficiency of an airfoil undergoing pure pitching, Journal of Fluid Mechanics 765, 524 (2015).
[22] D. Floryan, T. Van Buren, C. W. Rowley, and A. J. Smits, Scaling the propulsive performance of heaving and pitching foils, Journal of Fluid Mechanics 822, 386 (2017).
[23] T. Van Buren, D. Floryan, and A. J. Smits, Scaling and performance of simultaneously heaving and pitching foils, AIAA Journal , 1 (2018).
[24] J. Young and J. C. S. Lai, Mechanisms influencing the efficiency of oscillating airfoil propulsion, AIAA Journal 45, 1695 (2007).
[25] D. Floryan, T. Van Buren, and A. J. Smits, Forces and energetics of intermittent swimming, Acta Mechanica Sinica 33, 725 (2017).