Abstract  We discuss applications of gauge/gravity duality to describe the spectrum of light hadrons. We compare two particular 5-dimensional approaches: a model with an infrared deformed Anti-de Sitter metric and another one based on a dynamical AdS/QCD framework with back-reacted geometry in a dilaton/gravity background. The models break softly the scale invariance in the infrared region and allow mass gap for the field excitations in the gravity description, while keeping the conformal property of the metric close to the four-dimensional boundary. The models provide linear Regge trajectories for light mesons, associated with specially designed infrared gravity properties. We also review the results for the decay widths of the $f_0$ s into two pions, as overlap integrals between mesonic string amplitudes, which are in qualitative agreement with data.

Keywords  Hadrons · AdS/QCD · back-reacted geometries

1 Introduction

The AdS/CFT correspondence\cite{1} map $N = 4$ Super Yang-Mills (SYM) theory in 4D flat space into Type IIB string field theory in 10D space-time $AdS_5 \times S_5$. This mapping is such that the strong-coupling regime from one theory is equivalent to the perturbative regime of the other. The hope is to develop analytical tools, suitable for the perturbative regime, to treat the nonperturbative and strongly interacting region of quantum chromodynamics (QCD).

The holographic principle applied to QCD would allow to describe the strong interaction and nonperturbative dynamics using gravity weak-coupling perturbative methods. However, the basic difficulty to use this method to analyze strong-force physics lies on the fact that the gauge theory within the AdS/CFT duality is very different from QCD. In short, $N=4$ SYM theory has a conformal symmetry, whereas QCD breaks this symmetry at low energies and also the $N=4$ SYM theory is supersymmetric, whereas QCD does not have this symmetry. Consequently, one should modify the AdS geometry to build a realistic gravity dual of QCD. Applications of gauge/gravity dualities to QCD-like gauge theories either start from specific D-brane setups in ten- (or five-) dimensional supergravity and derive the corresponding gauge theory properties, or try to guess a suitable background and to improve it in bottom-up framework by comparing the predictions to QCD data.

A top-down approach to seek for a QCD dual starts by considering D-branes in the theory, which break supersymmetry in part and introduces flavor. One example is $N_f$ D7 probe branes ($D3 − D7$ model\cite{2}), which brings flavor physics to AdS/CFT. The supergravity side is dual to a four-dimensional $N = 2$ supersymmetric large-$N$ gauge theory. Even that, a running coupling constant is found in a type II B $N = 2$ theory, it still does not has confinement. There is a vast literature addressing these
issues, see e.g. the review [3]. The failure in getting hadronic spectra suggests that other strategies in the search for the QCD dual should be envisaged with the guidance of experimental data to shape, even in a phenomenological way, the holographic representation of QCD.

Polchinski and Strassler [4] gave a first step along these lines. They introduced an infrared (IR) hard-wall in the fifth dimension, which makes a slice of AdS where a boundary condition at the QCD scale requires discrete string modes. Following this work, many applications to study the colorless spectrum from the strong interaction properties of QCD have been pursued (see e.g. glueballs in [3, 4] and light hadrons in [5]). Although, the hard-wall model accounts qualitatively for many aspects of hadron physics [3, 4, 5], it does not lead to linear Regge trajectories. This difficulty was solved by the introduction of soft-wall model [8] with AdS$_5$ geometry and an additional inert dilaton background field. It leads to linear Regge trajectories, $m^2_{n,S} \sim n + S$, for light-flavor mesons of spin $S$ and radial excitation $n$. Regge behavior can also be achieved by IR deformations of the AdS$_5$ metric [9, 10].

Despite the success of the soft-wall models with AdS$_5$ geometry, the resulting vacuum expectation value (vev) of the Wilson loop does not exhibit the characteristic area-law behavior of a confining theory. The AdS$_5$ metric with the associate conformal invariance property does not confine by the Wilson loop analysis [11, 12]. Moreover one can raise the issue that the soft-wall model background with a nontrivial dilaton and AdS$_5$ metric are not a solution of Einstein equations. The analysis of the 5d coupled dilaton-gravity Einstein equations was performed by Csaki and Reece [13] (see also [14]) using the superpotential formalism. They concluded that to solve those equations with a linear confining background of the dilaton soft wall model it would require the addition of new degrees of freedom, like in particular a tachyon. The tachyon-dilaton-graviton model was pursued in ref. [15].

In order to overcome these difficulties without the introduction of a tachyon field, in [16] was proposed a Dynamical AdS/QCD model, solution of the dilaton-gravity equations, that leads to a linear confining background. For the lower excitations, where experimental data exists an approximate linear trend is found.

The basic amplitudes in the AdS$_5$ deformed inspired models, for which dynamics provide a whole excitation spectrum, have been recently promoted to the status of a wave function. This novel conceptual development concerns the connection between the valence light-front wave function and the mesonic string amplitude associated with classical fields propagating in Anti-de Sitter space [17]. The key point is to realize that the string mode eigenvalue equation, which carries the breaking of the conformal symmetry induced either by an infrared boundary condition or by an effective potential through a coupling of the meson field to a dilaton and gravity, is analogous to a valence eigenmode equation. In this way, the holographic realization of the mass squared operator equation for the valence wave function is achieved. The free mass squared operator is identified and also an effective potential which carries the complexity of the coupling of the valence sector with the full light-front Fock space determined by the QCD hamiltonian embedding at the same time confinement. The identification suggested access the content of the so called iterated resolvent method [18] proposed as a way to bridge the valence physics with the full QCD dynamics in the light-front Fock space.

We add that the duality between gauge theories and classical gravity [1] have been successfully used in many different contexts. We focus in hadronic physics, and mention one example that nonconformal holography has also been applied to describe the elliptic flow in relativistic heavy ion collisions [19].

In this contribution, we discuss two particular applications of gauge/gravity duality to describe the spectrum of light hadrons [11, 16]. One model has an infrared deformed Anti-de Sitter metric [10] and the another one is based on a dynamical AdS/QCD framework [16]. Both models break softly the scale invariance in the IR region bringing a gap for the field excitations in the gravity description, while keeping the conformal property of the metric at the four-dimensional boundary. As we have discussed these models provide linear Regge trajectories for light mesons, associated with specially designed infrared gravity properties. Furthermore, within the dynamical AdS model the decay widths of the $f_0'S$ into two pions computed as overlap integrals between mesonic string amplitudes [20] is found in qualitative agreement with data [21].

2 Infrared deformed AdS/QCD model

In the following, we discuss the holographic calculation of hadron spectrum from two different perspectives [10] and [16, 20]. The basic idea is that a hadron can be described by classical fields propagating...
The effective potential of the Sturm-Liouville equation for a deformed AdS reads:

\[ V(z) = m_n^2 \psi_n, \] (2)

where \( m_n \) is the hadron mass. For the particular case of the AdS metric \( A(z) = \log(z) \) the eigenvalue problem has a continuum spectra. For mesons dual to modes of a scalar (\( S \)) and vector (\( V \)) fields, the effective potentials are respectively written as:

\[ V_S(z) = \frac{1}{2\tau} \left( \frac{15}{4} + m_n^2 \right), \quad V_V(z) = \frac{1}{2\tau} \left( \frac{3}{4} + m_n^2 \right), \quad V_B(z) = \frac{m_5}{2\tau} (m_5 \mp 1), \] (3)

where the sign \( \pm \) depends on the fermion/baryon chirality \( i\gamma^5 \psi \). Within the AdS/CFT correspondence the full mode amplitude should behave as \( z^\tau \), where \( \tau = \Delta - \sigma \) (conformal dimension minus spin) is the twist dimension for the corresponding interpolating operator that creates the given state configuration [4]. The five-dimensional mass is chosen to fit the twist dimension as i) \( m_5 = 1 \) for scalars, ii) \( m_5 = \sqrt{\tau/2} \) for vector and iii) \( m_5 = \tau - 2 \) for mesons. The outcome of the potentials (3) is a continuum spectrum.

The discrete spectrum solution of (2) demands a change of the AdS metric background independent of the hadron. In this direction fluctuations of the background in a 10-dimensional supergravity could be much more appealing, however it is still missing a mass gap [27].

The asymptotically AdS_{5} metrics which give linear Regge trajectories depends on the quantum number \( L \), suggesting that the 5D description could be not enough to permit modes in an universal metric background independent of the hadron. In this direction fluctuations of the background in a 10-dimensional supergravity could be much more appealing, however it is still missing a mass gap [27].
with \( m_5 = \tau - 2 \) and the confining effective potential \( \Phi \) is delivered through \( A \) by an analytical form of the metric given by:

\[
A = -\log \left( \frac{1}{\lambda z} + \frac{\lambda z}{L+1} \right).
\]

(7)

Note that the metric has the quantum number \( L \) dependence. For mesons the correspondent form of the metric is so far found by numerical means and the details were discussed in \([10]\).

### 3 Dynamical AdS/QCD model

The bottom-up proposal of a dynamical AdS/QCD model with a dilaton-gravity back-reacted geometry \([16]\), came to address the issue raised by the vev of the Wilson loop in non-deformed AdS, metric, which does not exhibit the characteristic area-law behavior that a linearly confining static quark-antiquark potential would generate. A second, common shortcoming of a soft-wall background is that it is not a solution of the 5D Einstein equations (see e.g. \([14]\)). Within our proposal of a back-reacted geometry of the dilaton-gravity model the Regge trajectory of the mesonic mass spectrum presents an approximate linear behavior. The action for five-dimensional gravity coupled to a dilaton field is:

\[
S = \frac{1}{2k^2} \int d^5x \sqrt{g} \left( -R - V(\Phi) + \frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi \right),
\]

(8)

where the metric is given by \( A \), \( k \) is the Newton constant and \( V(\Phi) \) is the scalar field potential. The equations of motion are a coupled set of Einstein equations with solutions satisfying the following relations

\[
\Phi' = \sqrt{3A'^2 + 3A''}, \quad V(\Phi) = \frac{3e^{2A}}{2} \left( A' - 3A'' \right),
\]

(9)

which completely determines the dilation properties. This is the strategy of \([16]\) to built the dilaton-gravity background with the metric satisfying the scale invariance in the UV region and the criteria of Wilson loop area-law in the IR region, which says that \( A z^t \ (t \geq 1) \) \([14]\). These constraints are the guiding physics that the phenomenologically parameterized metric should satisfy.

**Mesons with spin.** The 5D action for a gauge field \( \phi_{M_1...M_S} \) of spin \( S \) in the dilaton-gravity background is given by \([8]\)

\[
I = \frac{1}{2} \int d^5x \sqrt{g} e^{-\Phi} \left( \nabla_M \phi_{M_1...M_S} \nabla^N \phi_{M_1...M_S} \right).
\]

(10)

As in \([8]\) and \([28]\) the axial gauge is used and new spin fields \( \tilde{\phi}_{M_1...M_S} = e^{2(S-1)\Phi} \phi_{M_1...M_S} \) introduced. The substitution \( \phi_n = e^{B/2} \psi_n \) in the mode equation gives it in the Sturm-Liouville form of eq. \( 2 \), with \( V_M(z) = \frac{B^2(z)}{4} - \frac{B''(z)}{2} \), where \( B = A(2S-1) + \Phi \). The choice of the warp factor

\[
A(z) = \log(z\Lambda_{QCD}) + \frac{1 + \sqrt{3}}{2S + \sqrt{3}} - \frac{(z\Lambda_{QCD})^2}{1 + e^{1+e(z\Lambda_{QCD})}},
\]

(11)

reproduces the linear Regge trajectories \([16]\), with the associated dilaton field and potential obtained from eqs \([9]\). An analytical approximation to the spectrum for \( A_{QCD} = 0.3 \) GeV is (in units of GeV)

\[
m_{n,S}^2 \simeq \frac{1}{10} (11n + 9S + 2), \quad (n \geq 1)
\]

(12)

which implements the approximate universality of the linear trajectory slopes for light flavors explicit.

**Scalar and pseudoscalar mesons.** We write the metric as

\[
A(z) = \log(z\Lambda_{QCD}) + \frac{(\xi z\Lambda_{QCD})^2}{1 + e^{1+e(1+\xi z\Lambda_{QCD})}},
\]

(13)

to describe the \( f_0 \) and pion families with a single parameter \( \xi \). For the \( f_0 \) family we found \( \xi = 0.58 \) \([20]\). Comparing to the vector sector, the slope of the Regge trajectory for the scalar excitations is reduced. This means that the size of the scalar ground state \( f_0(600) \) should be larger than the size of
other light mesons ground states, see the left-frame of fig. 1, where we compared the $f_0(600)$ and $\pi$ reduced amplitudes. We mention that, scalar mesons were also analyzed in \cite{29} and \cite{30}). The pion Regge trajectory has an experimental slope of about 1 GeV$^2$ and near twice the value found for the $f_0$ family. Therefore, the scaling factor of the holographic coordinate for pseudoscalars is larger than the one corresponding to the $f_0's$. Indeed by equating IR effective potentials of the pion and higher spin meson $\xi = 0.76$ is found, and a value of $\xi = 0.88$ allows a fine-tuning of the pion mass trajectory. The small pion mass is implemented by changing the 5D mass as $m_5^2 \rightarrow m_5^2 - \lambda z^2$ with $\lambda = 2.19$ GeV$^2$\cite{31}. The $f_0's$ partial decay width into $\pi\pi$ can be obtained from the overlap integral of the normalized string amplitudes (Sturm-Liouville form) in the holographic coordinate dual to the scalars ($\psi_n$) and pion ($\psi_{\pi}$) states,

\begin{equation}
    h_n = \sqrt{\frac{\lambda}{\Lambda_{QCD}}} \int_0^\infty dz \psi^2(z)\psi_n(z),
\end{equation}

where the parameter $\lambda$ gives the natural strength of the transition amplitude as it is the scale for the coupling between the pion and a scalar field, as has been obtained through the pion mass shift. The factor of $\Lambda_{QCD}$ is introduced to provide the correct dimension of the decay width. We find that $\lambda \Lambda_{QCD} = 13$ GeV$^2$, for $\Lambda_{QCD} = 0.3$ GeV, giving the results shown in the left-frame of fig. 1 compared to the experimental data. First, $f_0(980)$ has a mixing angle of $20^\circ$\cite{32} to fit the partial width. Furthermore, we observe that $f_0 \rightarrow \pi\pi$ decay width decrease fast as radial excitation of the scalar increase. The overlap between the the pion wave functions and the excited $f_0$ modes with nodes (see left-frame of fig.1) explains such a decrease\cite{33}.

4 Summary

We discussed some aspects of the bottom-up strategy to reveal aspects of the duality between gravity and QCD, by phenomenological studies of the light-hadron spectrum and decay of scalar mesons. Two particular 5D approaches were addressed. One model is built based on an infrared deformed Anti-de Sitter metric\cite{10}. The other model has a dynamical AdS/QCD framework with back-reacted geometry in a dilaton/gravity background\cite{16}. We stress that the methods used for the dynamical AdS/QCD model, solution of the five-dimensional Einstein-dilaton equations applies to essentially all asymptotically AdS$_5$ with a Poincaré-invariant boundary. Both holographic models break the scale invariance in the infrared region and allow to describe the light-meson spectrum with a gravity description. They keep the conformal property of the metric close to the four-dimensional boundary. Linear Regge trajectories for light mesons are associated with specially designed infrared metric properties. The results obtained for the decay widths of the $f_0 \rightarrow \pi\pi$s, as overlap integrals between mesonic string amplitudes, compared to the data seems encouraging. A challenge remains for the description of baryons within the dynamical AdS/QCD framework, which decouples from the dilaton field and for them it is enough a polynomial metric, which breaks the area-law criteria.
References

1. Maldacena, J.: The Large N Limit of Superconformal Field Theories and Supergravity. Adv. Theor. Math. Phys. 2, 231-252 (1998)
2. Karch, A., Katz, E.: Adding flavor to AdS/CFT. JHEP 0206, 043 (2002)
3. Erkelenz, J., et al.: Mesons in Gauge/Gravity Duals - A Review. Eur. Phys. J. A 35, 81-133 (2008)
4. Polchinski, J., Strassler, M. J.: Hard Scattering and Gauge/String Duality. Phys. Rev. Lett. 88, 031601 (2002)
5. Boschi-Filho, H., Braga, N.: Gauge / string duality and scalar glueball mass ratios. JHEP 0305, 009 (2003)
6. Boschi-Filho, H., Braga, N.: QCD/string holographic mapping and glueball mass spectrum. Eur. Phys. J. C 32, 529 (2004)
7. de Téramond, G. F., Brodsky, S. J.: Hadronic spectrum of a holographic dual of QCD. Phys. Rev. Lett. 94, 0201601 (2005)
8. Karch, A., Katz, E., Son, D.T., Stephanov, M.A.: Linear Confinement and AdS/QCD. Phys. Rev. D 74, 015005 (2006)
9. Kuperstein, S., Sonnenschein, J.: Non-critical, near extremal AdS(6) background as a holographic laboratory of four dimensional YM theory. JHEP 0411, 026 (2004)
10. Forkel, H., Beyer, M., Frederico, T.: Linear square-mass trajectories of radially and orbitally excited hadrons in holographic QCD. JHEP 0707, 77 (2007)
11. Maldacena, J.: Wilson loops in large N field theories. Phys. Rev. Lett. 80, 4859 (1998)
12. Rey, S.J., Yee, J.T.: Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity. Eur. Phys. J. C 22, 379 (2001)
13. Csaki, C., Reece, M.: Toward a systematic holographic QCD: A Braneless approach. JHEP 0506, 062 (2007)
14. Gursoy, U., Kiritsis, E., Nitti, F.: Exploring improved holographic theories for QCD: Part II. JHEP 0802, 019 (2008)
15. Batell, B., Gherghetta, T.: Dynamical Soft-Wall AdS/QCD. Phys. Rev. D 78, 026002 (2008)
16. de Paula, W., Frederico, T., Forkel, H., Beyer, M.: Dynamical holographic QCD with area-law confinement and linear Regge trajectories. Phys. Rev. D 79, 075019 (2009)
17. Brodsky, S.J., de Téramond, G.F.: Hadronic spectra and light-front wavefunctions in holographic QCD. Phys. Rev. Lett. 94, 0201601 (2006)
18. Pauli, H.C., Brodsky, S.J.: Solving Field Theory in One Space One Time Dimension. Phys. Rev. D 32, 2001 (1985)
19. Noronha, J., Gyulassy, M., Torrieri, G.: Conformal Holography of Bulk Elliptic Flow and Heavy Quark Quenching in Relativistic Heavy Ion Collisions. Phys. Rev. C 82, 054903 (2010)
20. de Paula, W., Frederico, T.: Scalar mesons within a dynamical holographic QCD model. Phys. Lett. B 693, 287-291 (2010)
21. Nakamura, K., et al.: (Particle Data Group) J. Phys. G 37, 075021 (2010)
22. Anisovich, A.V., Anisovich, V.V., Sarantsev, A.V.: Systematics of q anti-q states in the (n, M^2) and (J, M^2) planes. Phys. Rev. D 62, 051502 (2000)
23. Klempt, E., Richard, J.M.: Baryon spectroscopy. Rev. Mod. Phys. 82, 1095-1153 (2010)
24. Vega, A., Schmidt, I.: Modes with variable mass as an alternative in AdS/QCD models with chiral symmetry breaking. Phys. Rev. D 82, 115023 (2010)
25. Forkel, H., Klempt, E.: Diqark correlations in baryon spectroscopy and holographic QCD. Phys. Lett. B 679, 77-80 (2009)
26. de Téramond, G.F., Brodsky, S.J.: Light-Front Holography and Gauge/Gravity Duality: The Light Meson and Baryon Spectra. Nucl.Phys.Proc.Suppl. 199, 89-96 (2010)
27. Bianchi, M., de Paula, W.: On exact symmetries and massless vectors in holographic flows and other flux vacua. JHEP 1004, 113 (2010)
28. Katz, E., et al.: Tensor mesons in AdS/QCD. Phys. Rev. D 74, 086004 (2006)
29. Vega, A., Schmidt, I.: Scalar hadrons in AdS_5 × S^5. Phys. Rev. D 78, 017703 (2008)
30. Colangelo, P., De Fazio, F., Gianuzzi, F., Jugeau, F., Nicotri, S.: Light scalar mesons in the soft-wall model of AdS/QCD. Phys. Rev. D 78, 055009 (2008)
31. de Paula, W., Frederico, T.: Scalar Spectrum from a Dynamical gravity/gauge model. Int. J. Mod. Phys. D 19, 1351-1356 (2010)
32. Bediaga, I., Navarra, F., Nielsen, M.: The Structure of f_0(980) from charmed mesons decays. Phys. Lett. B 579, 59-66 (2004)
33. de Paula, W., Frederico, T.: Mesonic Spectrum from a Dynamical Gravity/Gauge model. Nucl.Phys.Proc.Suppl. 199, 113-118 (2010)