A simple calculation method for determination of equivalent square field

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ABSTRACT

Determination of the equivalent square fields for rectangular and shielded fields is of great importance in radiotherapy centers and treatment planning software. This is accomplished using standard tables and empirical formulas. The goal of this paper is to present a formula based on analysis of scatter reduction due to inverse square law to obtain equivalent field. Tables are published by different agencies such as ICRU (International Commission on Radiation Units and measurements), which are based on experimental data; but there exist mathematical formulas that yield the equivalent square field of an irregular rectangular field which are used extensively in computation techniques for dose determination. These processes lead to some complicated and time-consuming formulas for which the current study was designed. In this work, considering the portion of scattered radiation in absorbed dose at a point of measurement, a numerical formula was obtained based on which a simple formula was developed to calculate equivalent field. Using polar coordinate and inverse square law will lead to a simple formula for calculation of equivalent field. The presented method is an analytical approach based on which one can estimate the equivalent square field of a rectangular field and may be used for a shielded field or an off-axis point. Besides, one can calculate equivalent field of rectangular field with the concept of decreased scatter radiation with inverse square law with a good approximation. This method may be useful in computing Percentage Depth Dose and Tissue-Phantom Ratio which are extensively used in treatment planning.

Key words: Equivalent square field, irregular field, Percentage Depth Dose, scattered beam, Tissue-Phantom Ratio

Introduction

Determination of equivalent square field of a rectangular or shielded field (in order to calculate PDD, TAR) is one of the prevalent methods of dose determination on the central axis of radiation beam. There exist different approaches to determine equivalent field, such as empirical table published by ICRU,[1] in which non-presented values are obtained by interpolation or computational methods which perform via a simple empirical relation \( \frac{a}{b} \) as and some other methods which are not clearly based on the concept of equivalent field.[2] The concept of equivalent square field is usually utilized to calculate percentage depth dose and similar quantities in radiotherapy, which in most dose calculation softwares and algorithms is used for irregular fields too. The above-mentioned formula is used extensively in optimizing these tables by considering scattered radiation and beam quality.[3-5] Many attempts have been made to calculate depth dose independent of equivalent field, and some algorithms (e.g. ALFARD) have been developed[4] in which equivalent field is one of its Kernel’s major factors[6-8] and is used to calculate absorbed dose at an off-axis point.[9] Various methods of dose calculating have been investigated by Tudor et al. in ellipsoidal fields in which it was concluded that dose calculation based on the concept of equivalent field results in a lower percentage error.[10]

The aim of the present paper is to prove a formula based on the reduction in scattered beam due to inverse square law at a point on the central axis of beam, based on which one can extract equivalent square field of a rectangular field.
Materials and Methods

Considering a rectangular field with dimensions of \(a \times b\), absorbed dose at point A is due to primary and scattered beams. So, one can consider that absorbed dose at point A is caused by square field with dimensions of \(a \times a\) and two strips at the corners with dimensions of \(a \times \frac{(b-a)}{2}\) exist at both side of center [Figure 1].

On the other hand, the \(a \times b\) rectangular field is equivalent to \(x \times x\) square field. So, if \(a \times a\) field is omitted from \(x \times x\) field (necessarily \(x \times x > a \times a\)) [Figure 2], scattered portion from the rest of the field is equivalent to scattered radiation resulted from the two above-mentioned \(a \times \frac{(b-a)}{2}\) fields. In other words,

\[
S(x^2-a^2) = S(a(b-a)) \quad \text{.....(1)}
\]

where \(S\) is scattered beam, \(x^2-a^2\) is stripped peripheral surface in Figure 2, and \(a(b-a)\) is total stripped surface in Figure 1.

Using Cartesian system to compute scattered radiation at point A from stripped surface [Figure 3], one can obtain an expression as follows:

\[
S_{a(b-a)} = 4 \int_{0}^{\frac{\sqrt{2}}{2}} \int_{0}^{\frac{\sqrt{2}}{2}} dx dy, \quad r^2 = x^2 + y^2 \quad \text{.....(2)}
\]

Solving the above integral using Mathematica math software (Wolfram Research Inc., IL, USA) redounds to the following solution:

\[
S_{a(b-a)} = \sum_{k=0}^{\infty} \frac{\Gamma(k)}{\Gamma(k+a+2)} \left( 4 \pi \Phi(a, a+2) \right) - 4 a^2 \text{Catalan} \quad \text{.....(3)}
\]

Catalan = \(\sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)^2} \approx 0.915966\)

\[
\Phi(z, s, a) = \sum_{k=0}^{\infty} \frac{z^{k}}{(a+k)^s}
\]

Although the above answer seems to be complicated, since all variables in Equation (3) are numerical, the answer is easily obtained by computer for each desired value of \(a\) and \(b\).

Omitting midmost area in Figure 1 prevents the singularity might happen due to above integration, when \(r\) reaches to zero. The main problem arises when the effect of scattered beams in Figure 2 is to be calculated; if the above procedure follows in Cartesian coordinate system, a non-linear equation similar to Equation (3) obtains with respect to \(x\) in which it is not possible to eliminate \(x\) from it. This problem is resolved using polar coordinates system instead of Cartesian coordinate system. To do so, the central hollow square is divided into four trapeziums, as shown in Figure 3, and the effect of scattered beams at point A is calculated as the following equation:

\[
S(x^2-a^2) = 8 \int_{0}^{\frac{\pi}{4}} \int_{0}^{\frac{x}{2 \cos \theta}} \frac{1}{r^2} r dr d\theta \quad \text{.....(4)}
\]

\[
S(x^2-a^2) = 2 \pi \ln\left(\frac{x}{a}\right) \quad \text{.....(5)}
\]

Comparing the above simple logarithmic equation with Equation (3), an expression for the scattered radiation obtains as follows:

\[
2 \pi \ln\left(\frac{x}{a}\right) = S(a(b-a)) \quad \text{.....(6)}
\]
In the above equation, $x$ is the dimension of the equivalent square field and can be easily calculated as:

$$x = a e^{-\frac{2\pi}{b}}$$

.....(7)

Results

A comparison of published tables by ICRU [Table 1] with tables obtained from Equation (7) [Table 2] and from [Table 3][11] shows that in large fields, which are usually used in radiotherapy, calculated values in Table 1 are very close to the values in Table 2. For instance, if we consider a $20 \times 10$ field, a value of 13 is obtained from Table 1 and values of 13.1 and 13.3 are obtained from Tables 2 and 3, respectively.

Discussion

In large fields, values obtained from suggested method [Table 2] are much closer to the values suggested by ICRU [Table 1] than the values computed by conventional

| Table 1: Equivalent values for fields with dimensions from 2 cm to 30 cm published by ICRU |
|---|
| **Long Axis** | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 2.7 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 3.1 | 4.8 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 3.4 | 5.4 | 6.3 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 3.6 | 5.8 | 7.5 | 8.9 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 3.7 | 6.1 | 7.9 | 8.6 | 9.9 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 3.8 | 6.3 | 8.4 | 10.1 | 11.6 | 12.9 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 3.9 | 6.5 | 8.6 | 10.5 | 12.2 | 13.7 | 14.9 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 4 | 6.6 | 8.9 | 10.8 | 12.7 | 14.3 | 15.7 | 16.9 | 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 4 | 6.7 | 9 | 11.1 | 13 | 14.7 | 16.3 | 17.7 | 18.9 | 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 4 | 6.8 | 9.1 | 11.3 | 13.3 | 15.1 | 16.8 | 18.3 | 19.7 | 20.9 | 22 | 0 | 0 | 0 | 0 | 0 |
| 24 | 4.1 | 6.8 | 9.2 | 11.5 | 13.5 | 15.2 | 17.2 | 18.8 | 20.3 | 21.9 | 22.9 | 24 | 0 | 0 | 0 | 0 |
| 26 | 4.1 | 6.9 | 9.3 | 11.6 | 13.7 | 15.7 | 17.5 | 19.2 | 20.9 | 22.6 | 23.4 | 24.9 | 26 | 0 | 0 | 0 |
| 28 | 4.1 | 6.9 | 9.4 | 11.7 | 13.8 | 15.9 | 17.8 | 19.6 | 21.3 | 22.9 | 24.4 | 25.7 | 27 | 28 | 0 | 0 |
| 30 | 4.1 | 6.9 | 9.4 | 11.7 | 13.9 | 16 | 18 | 19.9 | 21.7 | 23.3 | 24.9 | 26.4 | 27.7 | 29 | 30 |

| Table 2: Values obtained for equivalent field using equation (7) |
|---|
| **Long Axis** | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 2.63 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 2.91 | 4.77 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 3.06 | 5.26 | 6.84 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 3.16 | 5.58 | 7.44 | 8.87 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 3.22 | 5.81 | 7.88 | 9.55 | 10.9 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 3.27 | 5.98 | 8.23 | 10.08 | 11.62 | 12.91 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 3.31 | 6.12 | 8.5 | 10.51 | 12.22 | 13.68 | 14.92 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 3.34 | 6.23 | 8.72 | 10.86 | 12.72 | 14.32 | 15.72 | 16.93 | 18 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 3.36 | 6.31 | 8.9 | 11.16 | 13.14 | 14.87 | 16.4 | 17.75 | 18.94 | 20 | 0 | 0 | 0 | 0 | 0 |
| 22 | 3.38 | 6.39 | 9.05 | 11.41 | 13.5 | 15.35 | 17 | 18.46 | 19.77 | 20.95 | 22 | 0 | 0 | 0 | 0 |
| 24 | 3.4 | 6.45 | 9.18 | 11.62 | 13.81 | 15.77 | 17.52 | 19.1 | 20.51 | 21.79 | 22.95 | 24 | 0 | 0 | 0 |
| 26 | 3.41 | 6.5 | 9.29 | 11.81 | 14.08 | 16.13 | 17.98 | 19.66 | 21.88 | 22.56 | 23.81 | 24.95 | 26 | 0 | 0 |
| 28 | 3.42 | 6.54 | 9.39 | 11.97 | 14.32 | 16.45 | 18.39 | 20.16 | 21.77 | 23.25 | 24.59 | 25.83 | 26.96 | 28 | 0 |
| 30 | 3.43 | 6.58 | 9.47 | 12.11 | 14.53 | 16.74 | 18.76 | 20.61 | 22.31 | 23.87 | 25.3 | 26.62 | 27.84 | 28.96 | 30 |
methods [Table 3], the accuracy of which has been accepted by some treatment planning software and texts.[11] Besides, many dose correction methods in their analytical part use conventional method mentioned above to calculate equivalent square field through the correction of scattered beams. However, it is clear that in small enough fields, the error will be considerably high in suggested and conventional methods [Tables 2 and 3]; this error has been removed by applying correction coefficients to $2ab$.[12]

Recommendations for researchers based on the capabilities of the above formula

The power of the proposed method with respect to the previous methods is its possibility to calculate the equivalent field for shielded fields. Most computational methods and formulas used in treatment planning for determination of equivalent square field and dose calculation provide a fixed equivalent field for a shielded field without considering the shield position in the field; this is while the scattered beams reaching point A depend on shield position and consequently the equivalent fields must be different [Figure 4]. As it is obvious from this figure, although the shield position and so the scatter is different, the equivalent square field for both conditions is the same.

This will be important when working with fields and shields of large dimensions. However, this will be easily achieved by the proposed method for obtaining Equation (7) by separating a central square field to calculate equivalent field and scattered beams considering inverse square law. Besides, in multi-leaf collimators [Figure 5], if the dimensions of each leaf are known, this technique may be easily used to calculate the dose and equivalent square field (Equation (8)).

$$S_{(x^2-a^2)} = \sum_i S_{a,b_i}$$

\[\text{.....(8)}\]
Where $a_i$ and $b_j$ are dimensions of area outside the $a \times a$ square, which totally results in the desired field shape, this is a potential capability which needs more detailed work to be practically useful. The importance of the proposed method is that it enables one to calculate dose at an arbitrary off-axis point provided this point does not place on the sides of the rectangular field.

It is possible to consider the effects of beam quality by calculation of attenuation in phantom and inserting the $e^{-\mu r}$ factor in Equations (2 and 4); considering the dependence of attenuation coefficient on energy and atomic number, the proposed formula might become more appropriate with results much closer to real conditions, which needs a brief simulation and phantom measurement.

## Conclusion

The presented method is an analytical approach based on which one can estimate the equivalent square field of a rectangular field and may be used for a shielded field or an off-axis point. Besides, one can calculate equivalent field of rectangular field with the concept of decreased scatter radiation with inverse square law with a good approximation. This method may be useful in computing PDD and TPR which are extensively used in treatment planning.

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