A step toward CNO solar neutrinos detection in liquid scintillators

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Abstract

The detection of CNO solar neutrinos in ultrapure liquid scintillator detectors is limited by the background produced by Bismuth-210 nuclei that undergo $\beta$-decay to Polonium-210 with a lifetime of $\sim 7$ days. Polonium-210 nuclei are unstable and decay with a lifetime equal to $\sim 200$ days emitting $\alpha$ particles that can be also detected. In this letter, we show that the Bi-210 background can be determined by looking at the time evolution of $\alpha$-decay rate of Po-210, provided that $\alpha$ particle detection efficiency is stable over the data acquisition period and external sources of Po-210 are negligible. A sufficient accuracy can be obtained in a relatively short time. As an example, if the initial Po-210 event rate is $\sim 2000 \text{ cpd}/100\text{ton}$ or lower, a Borexino-like detector could start discerning CNO neutrino signal from Bi-210 background in $\Delta t \sim 1\text{yr}$. 
1 Introduction

One of the main goals of the present and next generation ultrapure liquid scintillator detectors, such as KamLAND [1], Borexino [2], SNO+ [3] and LENA [4], is the determination of the neutrino fluxes produced by the CNO cycle in the Sun.

Despite being sub-dominant in the Sun, the CNO cycle [5] has a key role in astrophysics, being the prominent source of energy in more massive stars and in advanced evolutionary stages of solar like stars, see [6]. The evaluation of CNO efficiency is connected with various interesting problems, like e.g. the determination of globular clusters age [7] from which we extract a lower limit to the age of the Universe. At the moment, we still miss a direct observational evidence for CNO energy generation in the Sun. We only have a loose upper limit on CNO luminosity, $L_{\text{CNO}}$, obtained by combining the results of the various solar neutrino experiments. The most recent Solar Standard Model (SSM) [8] predicts the CNO contribution to the Sun’s luminosity to be equal to about 0.7%. Experimental bounds report $L_{\text{CNO}} < 3\%$ at 99% C.L., see e.g. [9, 10, 11]. The detection of CNO solar neutrinos would clearly provide a direct test of the CNO cycle efficiency.

The measurement of the CNO solar neutrino flux can also provide clues to solve the so called “solar composition problem”. The flux is, in fact, directly related to the abundance of carbon, nitrogen and oxygen in the Sun. The photospheric abundances of these (and other) heavy elements have been recently re-determined [12, 13, 14], indicating that the sun metallicity is lower than previously assumed [15]. Solar models that incorporate these lower abundances are no more able to reproduce the helioseismic results. Detailed studies have been done to solve this discrepancy, see e.g. [16, 17], but a definitive solution of this problem is still missing.

The above points show the importance of CNO solar neutrinos detection which, however, is a very difficult task. Not only the flux is relatively low, but also their energy is not large. The neutrinos produced in the CNO cycle have continuous energy spectra with endpoints at about $\sim 1.5$ MeV [1]. Differently from the monochromatic Be and pep solar neutrinos, they do not produce specific spectral features that permit to extract them unambiguously from the background event spectrum in high purity liquid scintillators. In particular, as it is shown in Fig.1, the electrons produced by the $\beta-$decay of Bismuth-210 to Polonium-210 have a spectrum that is similar to that produced by CNO neutrinos. As a consequence, spectral fits are able to determine only combined “Bismuth+CNO” contribution, as it is done e.g. by Borexino in [2, 11].

In order to remove this degeneracy, we propose a simple method to determine the Bi-210 decay rate which is based on the relationship between the Bi-210 and Po-210 abundances. Polonium-210, which is the Bismuth-210 daughter, is unstable and decays with a lifetime $\tau_{\text{Po}} \sim 200$ days emitting a monochromatic $\alpha$ particle that can be easily detected. In the absence of Bismuth-210, the $\alpha-$decay rate of Po-210 nuclei follows the exponential decay law, $n_{\text{Po}}(t) \propto \exp(-t/\tau_{\text{Po}})$. The deviations from this behaviour can be used to determine the $\beta-$decay rate of Bi-210 nuclei. The only required assumptions are that the $\alpha$ particle detection efficiency is stable and external sources of Po-210 are negligible during the data acquisition period.

The plan of the paper is the following. In the next section, we determine the relationship between the $\alpha-$decay rate of Po-210, $n_{\text{Po}}(t)$, and the $\beta-$decay rate of Bi-210, $n_{\text{Bi}}(t)$. In sect. 3, we estimate the accuracy $\Delta n_{\text{Bi}}$ of the Bi-210 decay rate determination as a function of the detector mass $M$, the data

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1 The dominant components of the CNO neutrino flux are the so called N-neutrinos produced by $^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e$ with an endpoint at $E_{\nu} = 1.20$ MeV and the O-neutrinos produced by $^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_e$ with an endpoint at $E_{\nu} = 1.73$ MeV.
acquisition period $\Delta t$ and the initial polonium decay rate $n_{\text{Po},0}$. In sect. 4, we discuss the implications of the proposed approach for CNO neutrino signal extraction. In sect. 5, we summarize our results.

## 2 The relationship between Polonium and Bismuth

Bismuth-210 and Polonium-210 are both daughters of $^{238}\text{U}$. Bismuth-210 is produced by the slow decay of Lead-210 which has a lifetime equal to $\tau_{\text{Pb}} = 32.3$ y. It then undergoes a $\beta^-$ decay to Polonium-210:

$$^{210}\text{Bi} \rightarrow ^{210}\text{Po} + e^- + \nu_e \quad (1)$$

with a lifetime $\tau_{\text{Bi}} = 7.232$ d. The electrons produced in the decay have a continuous spectrum with an endpoint $E_{\text{max}} = 1.16$ MeV that lays over the event spectrum produced by CNO neutrinos, see Fig.1. In the analysis of the 192 days data [11], Borexino obtained an event rate equal to about 20 cpd/100tons [11] for the combined contribution provided by Bismuth+CNO neutrinos. For reference, we consider that the expected CNO neutrino signal is equal to about 5 cpd/100ton as predicted by the SSM and the LMA-MSW scenario (see sect. 4 for details).

Polonium-210 is also unstable and undergoes $\alpha$ decay to Lead-206:

$$^{210}\text{Po} \rightarrow ^{206}\text{Pb} + \alpha \quad (2)$$

with a lifetime $\tau_{\text{Po}} = 199.634$ d. The $\alpha$ particles produced in the decay have an energy $E_\alpha = 5.3$ MeV, but they are observed at a much lower effective energy, due to the large quenching factor of alpha
particles in liquid scintillators. They produce a sharp peak superimposed to the low energy part of the \(^7\text{Be}\) neutrino event rate at a visible energy \(E_{\text{vis}} \simeq 0.5\) MeV, see Fig.1.

Naively, one could expect that secular equilibrium between Bismuth-210 and Polonium-210 is obtained, which would imply that the rate of \(\beta\) decays of Bi-210 is equal to the rate of \(\alpha\) decays of Po-210. However, this is not the case in practical situations. Borexino, in fact, observes a Po-210 abundance which is about a factor 100 larger than what implied by secular equilibrium and that corresponds to an initial alpha decay rate of the order \(\sim 8 \times 10^3\) cpd/100ton [18]. The large counting rate permits to determine the Polonium activity with a high accuracy.

We remark that the \(\alpha\) particles emitted in the decay, beside producing the very distinctive spectral feature shown in Fig.1, can be directly identified by using a pulse shape discrimination. As it is shown in [2, 11] a high purity liquid scintillator could have very good \(\alpha - \beta\) discrimination properties. A figure-of-merit based on the Gatti discrimination technique [20] has been used in Borexino to produce an \(\alpha\)-subtracted spectrum where the Po-210 background is removed.

The relationship between the Po-210 and the Bi-210 abundances can be quantified in a very simple way. We have that:

\[
\frac{dN_{Po}}{dt} = -\frac{N_{Po}(t)}{\tau_{Po}} + \frac{N_{Bi}(t)}{\tau_{Bi}} + S_{Po}(t)
\]

where \(N_{Po}(t)\) (\(N_{Bi}(t)\)) is the number of Polonium-210 (Bismuth-210) nuclei per unit mass in the detector at a time \(t\), and \(S_{Po}(t)\) indicates any possible external source of polonium-210 (i.e. not related to \(^{238}\text{U}\) decay chain). The solution of Eq. (3) can be written as:

\[
N_{Po}(t) = N_{Po,0} \exp(-t/\tau_{Po}) + \tau_{Po} \left< \frac{N_{Bi}(t)}{\tau_{Bi}} + S_{Po}(t) \right>
\]

where \(N_{Po,0}\) is the number of Po-210 nuclei per unit mass at the time \(t = 0\) and the symbol \(\left< f(t) \right>\) indicates the time “average”:

\[
\left< f(t) \right> = \frac{1}{\tau_{Po}} \int_0^t dt' \ f(t-t') \exp(-t'/\tau_{Po})
\]

for the generic function \(f(t)\).

We assume that, during a data acquisition period, after liquid scintillator purification, external sources of Polonium can be neglected, i.e. \(S_{Po}(t) \simeq 0\). In this assumption, the \(\alpha\)-decay rate of Polonium and the \(\beta\)-decay rate of Bismuth, given by:

\[
n_{Po}(t) \equiv N_{Po}(t)/\tau_{Po} \\
n_{Bi}(t) \equiv N_{Bi}(t)/\tau_{Bi}
\]

respectively, follow the relation:

\[
n_{Po}(t) = n_{Po,0} \exp(-t/\tau_{Po}) + \left< n_{Bi}(t) \right>.
\]

The above equation provides all the ingredients that are necessary for our analysis. It shows that, if we are able to measure \(n_{Po}(t)\) as a function of time with a high accuracy, we can provide an estimate for \(n_{Bi}(t)\), which is the main background for CNO neutrinos detection.
3 Determination of the Bi-210 event rate

The main observation of this letter is that, due to the high statistics of Po-210 events, the beta-activity of Bi-210 nuclei can be extracted from the time evolution of Polonium-210 counting rate in a relatively short time (i.e. comparable with Polonium lifetime). It is not necessary, in particular, to wait that secular equilibrium between Polonium-210 and Bismuth-210 is reached, which would require much longer times.

This point can be quantified with straightforward analytical considerations by making the following exercise. We assume that the Bi-210 event rate is constant\(^2\), obtaining from Eq.\((7)\):

\[
\begin{align*}
n_{\text{Po}}(t) &= \left[ n_{\text{Po},0} - n_{\text{Bi}} \right] \exp\left( -t/\tau_{\text{Po}} \right) + n_{\text{Bi}}.
\end{align*}
\] (8)

We imagine to collect the Po-210 events\(^3\) for a total period equal to \(\Delta t\). We then divide this time interval into two large bins from \([0, \Delta t/2]\) and \([\Delta t/2, \Delta t]\). According to the time evolution implied by Eq.\((8)\), the number of events collected in the two bins are:

\[
\begin{align*}
N_1 &= \varepsilon M \left[ \left( n_{\text{Po},0} - n_{\text{Bi}} \right) \tau_{\text{Po}} \left( 1 - e^{-\Delta t/2\tau_{\text{Po}}} \right) + n_{\text{Bi}} \frac{\Delta t}{2} \right] \\
N_2 &= \varepsilon M \left[ \left( n_{\text{Po},0} - n_{\text{Bi}} \right) \tau_{\text{Po}} e^{-\Delta t/2\tau_{\text{Po}}} \left( 1 - e^{-\Delta t/2\tau_{\text{Po}}} \right) + n_{\text{Bi}} \frac{\Delta t}{2} \right]
\end{align*}
\] (9) (10)

where \(M\) indicates the detector mass and \(\varepsilon\) indicates the detection efficiency (averaged over the integration period) that we assume not to vary significantly.

\(^2\)This hypothesis corresponds to assuming that there are no sources of Lead-210 and Bismuth-210 in the detector and it is done here only to consider a specific situation and to illustrate quantitatively the proposed method. We remark that, in the analysis of the experimental data, it is not necessary to make \textit{a-priori} assumptions on the time evolution of \(n_{\text{Bi}}(t)\) since this can be determined from the experimental data themselves, as it is discussed in sect\(^4\) and appendix.

\(^3\)The Po-210 counting rate can be determined either by fitting the low energy part of the event spectrum or by selecting Po-210 events by pulse shape discrimination technique, see [20]. The optimal approach has to be chosen on experimental basis.
In the absence of the Bismuth-210 contribution, the ratio of events collected in the two bins is equal to $N_2/N_1 = \exp(-\Delta t/2\tau_{Po})$. The deviations of $N_2/N_1$ from this value can be used to measure the beta activity of Bi-210. We have, in fact:

$$\left[N_2 - N_1 e^{-\Delta t/2\tau_{Po}}\right] = \varepsilon M n_{Bi} \frac{\Delta t}{2} \left(1 - e^{-\Delta t/2\tau_{Po}}\right).$$

By propagating the statistical errors $\Delta N_2 = \sqrt{N_2}$ and $\Delta N_1 = \sqrt{N_1}$, one is able to estimate the accuracy $\Delta n_{Bi}$ of the determination of the Bi-210 decay rate. In the assumption that $n_{Po,0} \gg n_{Bi}$, we obtain:

$$\Delta n_{Bi} \simeq \sqrt{n_{Po,0}/\tau_{Po} M f(\Delta t)}$$

where we considered $\varepsilon \sim 1$. The function $f(\Delta t)$ is explicitly given as:

$$f(\Delta t) = \left(2\tau_{Po}/\Delta t\right) e^{-\Delta t/2\tau_{Po}} \left[1 + e^{-\Delta t/2\tau_{Po}}\right]^{-1}.$$

We see that the relevant parameters are the initial Po-210 event rate $n_{Po,0}$, the detector mass $M$ and the total time of the measure $\Delta t$. In Fig. 2, we show the iso-countour lines for $\Delta n_{Bi}$ in the plane ($n_{Po,0}, \Delta t$) for two values of the detector mass $M = 100$ ton and $M = 1000$ ton. For an initial Polonium activity equal to $n_{Po,0} = 2000$ cpd/100 ton, we reach an accuracy comparable to the expected CNO signal, $n_{CNO} \simeq 5$ cpd/100 ton, after a time $\Delta t \sim 300d$ ($\Delta t \sim 150d$) for a 100 ton (1000 ton) detector. We remark that an important assumption in the above analysis is that the detection efficiency $\varepsilon$ is known and stable over the period of the measure. Variations and/or errors in $\varepsilon$ clearly provide additional sources of uncertainties.

These analytic estimates have been checked numerically by simulating random Po-210 events, according to the time evolution expressed by Eq. (8), in detectors with masses $M = 100$ ton and $M = 1000$ ton. We assumed that the Bi-210 event rate is equal to $n_{Bi} = 20$ cpd/100 ton, while the initial Po-210 event rate have been varied in the range $n_{Po,0} = 2000 - 8000$ cpd/100 ton in order to study the dependence of our final results on the initial polonium contamination. We binned the data over five days and then extracted $n_{Bi}$ and $n_{Po,0}$ by fitting the data over an observation period $\Delta t$ with the functional form in Eq. (8). The accuracy $\Delta n_{Bi}$ for the reconstructed Bi-210 event rate is reported in Table 1. We see that the numerical results agree within $\sim 15\%$ with the analytical expression (12).

| $n_{Po,0}$ | $0.5$ year | $1$ year | $1.5$ year | $0.5$ year | $1$ year | $1.5$ year |
|-----------|-----------|----------|-----------|-----------|----------|-----------|
| 2000      | 10        | 2.9      | 1.3       | 3.3       | 0.9      | 0.4       |
| 4000      | 15        | 4.1      | 1.8       | 4.6       | 1.3      | 0.6       |
| 8000      | > 20      | 5.7      | 2.5       | 5.6       | 1.8      | 0.8       |

Table 1: The absolute uncertainty $\Delta n_{Bi}$ in the Bi-210 event rate determination, reconstructed by performing a fit over an observation period $\Delta t$ to simulated data. All rates are expressed in cpd/100ton. We assume that the true Bi-210 event rate is $n_{Bi} = 20$ cpd/100ton.
4 Implications for CNO solar neutrino extraction

The possibility to determine the Bi-210 event rate represents an important step forward in the direction of CNO solar neutrino detection, as can be immediately understood by looking at the expected event spectrum in liquid scintillator detectors in Fig.1. The spectrum was calculated by assuming the solar neutrino fluxes predicted by [8] and the oscillation parameters corresponding to the LMA-MSW solution of the solar neutrino problem given by [19]. The integrated rates are
\[ n_{\text{Be}} = 50 \text{ cpd/100ton} \] from beryllium neutrinos, \[ n_{\text{pep}} = 2.7 \text{ cpd/100ton} \] for pep neutrinos and \[ n_{\text{B}} = 0.48 \text{ cpd/100ton} \] for boron neutrinos. The CNO neutrino signal is taken as \[ n_{\text{CNO}} = 5.1 \text{ cpd/100ton} \] that corresponds to the prediction obtained by using the high photospheric metal abundances of [15]. For comparison, by using the low photospheric values of [13] one obtains the lower rate \[ n_{\text{CNO}} = 3.6 \text{ cpd/100ton} \]. The background levels have been estimated considering the Borexino results as a reference (see e.g. [11]). The Kripton-85 and Carbon-11 event rate are chosen as \[ n_{\text{Kr}} = 25 \text{ cpd/100ton} \] and \[ n_{\text{C11}} = 25 \text{ cpd/100ton} \], respectively. The Bi-210 event rate is taken as \[ n_{\text{Bi}} = 20 \text{ cpd/100ton} \], while the (initial) Po-210 event rate is chosen as \[ n_{\text{Po,0}} = 2000 \text{ cpd/100ton} \]. This value is lower than what initially obtained in Borexino [18] but we believe that it represents a reasonable goal for next runs and next generation experiments. Finally, we modelled the detector energy resolution by a Gaussian function with an energy dependent width \[ \sigma(E) = 0.05 \text{ MeV} \sqrt{E/1 \text{ MeV}} \] [11].

We see that the CNO neutrino fluxes are not expected to produce significant and/or recognizable features anywhere in spectrum (see Fig.1). As a consequence, spectral fits are unable to constrain the CNO signal. This is emphasized in the left panel of Fig.3, where we display the results of a spectral fit to simulated data in the plane \( (n_{\text{Bi}}, n_{\text{CNO}}) \). In the simulation, we considered a detector mass \( M = 100 \text{ ton} \) and a total observation period equal to \( \Delta t = 1 \text{ yr} \), during which all the components have been assumed to be constant in time except the Po-210 contribution that was modulated in time according to Eq.(8).

In the fit, we considered all the signals and backgrounds as free parameters, except the pp, pep and Boron neutrinos contributions which have been fixed to their expected values. We see from Fig.3 that the CNO signal is basically unconstrained. The origin of the strong anti-correlation with the Bi-210 background is understood on a quantitative basis by considering that the Bi-210 and CNO components give the dominant contribution to the spectrum only in a narrow energy range at \( E \sim 0.8 \text{ MeV} \). The signal produced in this window is proportional to the combination:

\[ n_\beta = n_{\text{CNO}} + \xi n_{\text{Bi}} \]  

where the factor \( \xi = 0.67 \) is the ratio between the normalized Bi-210 and CNO spectra evaluated at \( E \sim 0.8 \text{ MeV} \). The data allow, thus, a good determination of \( n_\beta \) with a poor reconstruction of \( n_{\text{CNO}} \) and \( n_{\text{Bi}} \).

The degeneracy between Bi-210 and CNO neutrinos is removed when we use the time evolution of the Po-210 contribution to determine the Bi-210 background. By performing a fit to the simulated data in the domain of energy and time and taking advantage of Eq.(8) between Po-210 and Bi-210 rates, we obtain the considerable improvement in the CNO signal determination that is shown in the right panel of Fig.3. The final uncertainty is \( \Delta n_{\text{CNO}} \approx 2.1 \text{ cpd/100ton} \) and it is essentially determined by the accuracy of the Bi-210 event rate determination according to \( \Delta n_{\text{CNO}} \sim \xi \Delta n_{\text{Bi}} \), as can be estimated by using the \( \Delta n_{\text{Bi}} \) values given in tab.1 and/or in Eq.(12).

\footnote{This plot has been obtained by fitting simultaneously the time and energy distribution of simulated data without considering the possibility to identify Po-210 events by pulse shape discrimination [20], since this additional information do not introduce relevant improvements in our analysis. This possibility, however, should not be overlooked because it may become important in the analysis of real data, depending on the experimental conditions.}
The above results indicate that present generation liquid scintillator experiment, like e.g. Borexino, already have the potential to probe the CNO neutrino flux, provided that they are stable for sufficiently long time (∼1yr) and/or the initial polonium contamination can be made sufficiently low. Considering, moreover, that the uncertainty scales as \( \Delta n_{\text{Bi}} \propto \sqrt{n_{\text{Po},0}/M} \), we note that: i) future Kton-scale detectors, like e.g. SNO+, will be able to start discriminating between high and low metallicity solar models; ii) the initial Po-210 contamination \( n_{\text{Po},0} \sim 8000 \text{cpd}/100\text{ton} \) reported Borexino by [18] corresponds, anyhow, to 1\( \sigma \) extraction of CNO solar neutrinos.

A final remark is important. In our analysis, we assumed that the Bi-210 event rate is constant in time. This assumption was done to consider a specific situation but it is not necessary in the analysis of real data, since the time evolution of \( n_{\text{Bi}}(t) \) can be determined from the experimental data themselves. A simple strategy can be the following. By performing spectral fits to the signal at different times \( t \), we determine the quantity \( n_\beta(t) \) defined in Eq.(14). This allow us to obtain Bi-210 rate as a function of time, according to:

\[
 n_{\text{Bi}}(t) = \frac{1}{\xi} [n_\beta(t) - n_{\text{CNO}}] 
\]

where the unknown factor \( n_{\text{CNO}} \) is constant in time (we neglect the small seasonal modulation of neutrino fluxes due to earth orbit eccentricity). The presence of a time varying \( n_{\text{Bi}}(t) \) does not invalidate the possibility to perform our analysis, since the only requisite for the validity of Eq.(7) is that there are no external Polonium-210 sources. We can use, in fact, Eq.(15) in Eq.(7) to obtain the following expression:

\[
 n_{\text{Po}}(t) - \langle n_\beta(t) \rangle = [n_{\text{Po},0} + n_{\text{CNO}}] \exp(-t/\tau_{\text{Po}}) - n_{\text{CNO}} 
\]

that allows to determine \( n_{\text{CNO}} \) from a two parameters fit to the “observable” quantity \( n_{\text{Po}}(t) - \langle n_\beta(t) \rangle \) with no additional assumptions. Alternatively, we can compare the behaviour of \( n_\beta(t) \) with expectations. A constant value is expected when external sources of Bismuth-210 and Lead-210 are negligible. A linear growth of \( n_{\text{Bi}}(t) \) (and thus of \( n_\beta(t) \)) is, instead, produced by a slow increase of the Radon contamination of the detector (see appendix for details). We can assume an arbitrary functional form for \( n_{\text{Bi}}(t) \) that agrees with the observed \( n_\beta(t) \) and fit the data in the time and energy domain, using Eq.(7) in place of Eq.(8) to discriminate Bi-210 from CNO neutrino contribution.

5 Conclusions

In this letter we have proposed a simple and general method that allows to measure the flux of CNO neutrinos in massive high radio-purity liquid scintillator detectors by means of determining the Bi-210 background. We summarize here the main points of our analysis:

i) We have discussed the relationship between Polonium-210 and Bismuth-210, see Eq.(7), showing that the deviations of the \( \alpha \)-decay rate of Po-210 from the normal exponential decay law, \( n_{\text{Po}}(t) \propto \exp(-t/\tau_{\text{Po}}) \), can be used to determine the beta activity of Bi-210.

ii) We have estimated the expected accuracy of the Bi-210 event rate determination as a function of the relevant parameters that are the initial polonium contamination of the detector, the detector mass and the total observational time. Our results are summarized by Eq.(12) and/or tab.1. A good accuracy can be obtained in relatively short time, comparable to the Po-210 lifetime \( \tau_{\text{Po}} \sim 200 \text{d} \).

iii) We have discussed the implications of our approach for CNO solar neutrino detection. We have shown that, if the initial Po-210 event rate is \( \sim 2000 \text{cpd}/100\text{ton} \) or lower, a Borexino-like detector could start discerning CNO neutrino signal in \( \Delta t \sim 1\text{yr} \). Future Kton-scale detectors, like e.g. SNO+, in the same time interval, could begin to discriminate between high and low metallicity solar models.
Figure 3: The 1-σ and 2-σ allowed regions obtained by a fit to simulated data, assuming a detector mass $M = 100$ ton and an observation period $\Delta t = 1$ yr. In the left panel, we consider only information contained in the energy distribution of the events. In the right panel, we perform a fit in the domain of energy and time, taking into account the relationship between Po-210 and Bi-210 expressed by Eq. (8). The blue dashed line corresponds to the condition $n_\beta = \text{const}$, see Eq. (14).

The determination of the Bi-210 event rate could represent an important step toward CNO solar neutrinos detection in high purity liquid scintillator experiments. The only required assumptions are that $\alpha$-particle detection efficiency is stable over the data acquisitions period and external sources of Po-210 are negligible. A special effort on the experimental side in this direction is worthwhile in the next future.
A Lead-Bismuth-Polonium 210 decay chain

The decay chain is described by the system of differential equations:

\[
\begin{align*}
\frac{dN_1(t)}{dt} &= -\frac{N_1(t)}{\tau_1} + S_1(t) \\
\frac{dN_2(t)}{dt} &= -\frac{N_2(t)}{\tau_2} + \frac{N_1(t)}{\tau_1} + S_2(t) \\
\frac{dN_3(t)}{dt} &= -\frac{N_3(t)}{\tau_3} + \frac{N_2(t)}{\tau_2} + S_3(t)
\end{align*}
\] (17)

where, to keep notation simple, we use the index 1 is for Lead-210, the index 2 for Bismuth-210 and the index 3 for Polonium-210. The functions \(S_i(t)\) describe source terms and the initial conditions are expressed as:

\[N_i(0) = N_{i,0}\] (18)

These equations can be treated as a decoupled system of equations in the form

\[
\frac{dN_i(t)}{dt} = -\frac{N_i(t)}{\tau_i} + \xi_i(t)
\] (19)

that is readily solved as follows:

\[N_i(t) = N_i e^{-\frac{t}{\tau_i}} + \tau_i \langle \xi(t) \rangle_i
\] (20)

where we used the notation

\[\langle \xi(t) \rangle_i = \frac{1}{\tau_i} \int_0^t e^{-\frac{t-t'}{\tau_i}} \xi(t')dt'
\] (21)

that can be thought of as a suitable average over an interval of time of the order of \(\tau_i\) preceding the time \(t\). Some explicit expressions are given in the following.

The solution of the homogeneous system (i.e., with source terms set to zero) with the boundary condition \(N_i(0) = N_{i,0}\) is:

\[
\begin{align*}
\overline{N}_1(t) &= e^{-\frac{t}{\tau_1}} N_{1,0} \\
\overline{N}_2(t) &= e^{-\frac{t}{\tau_1}} N_{1,0} \frac{\epsilon_{21}}{1-\epsilon_{21}} + e^{-\frac{t}{\tau_2}} \left( N_{2,0} - N_{1,0} \frac{\epsilon_{21}}{1-\epsilon_{21}} \right) \\
\overline{N}_3(t) &= e^{-\frac{t}{\tau_1}} N_{1,0} \frac{\epsilon_{31}}{(1-\epsilon_{31})(1-\epsilon_{31})} - e^{-\frac{t}{\tau_2}} \left( N_{2,0} - N_{1,0} \frac{\epsilon_{31}}{1-\epsilon_{31}} \right) \frac{1}{1-\epsilon_{32}} + e^{-\frac{t}{\tau_3}} \left[ N_{3,0} + \left( N_{2,0} - N_{1,0} \frac{\epsilon_{31}}{1-\epsilon_{31}} \right) \frac{1}{1-\epsilon_{32}} \right]
\end{align*}
\] (22)

where we used the notation

\[\epsilon_{ij} = \frac{\tau_i}{\tau_j}\] (23)

and considered the inequality \(\tau_1 \gg \tau_3 \gg \tau_2\). The inhomogeneous (full) system admits the following solution satisfying the boundary condition \(\delta N_i(0) = 0\):

\[
\begin{align*}
\delta N_1(t) &= \tau_1 \langle S_1(t) \rangle_1 \\
\delta N_2(t) &= \tau_2 \langle S_2(t) + \langle S_1(t) \rangle_1 \rangle_2 \\
\delta N_3(t) &= \tau_3 \langle S_3(t) + \langle S_2(t) + \langle S_1(t) \rangle_1 \rangle_2 \rangle_3
\end{align*}
\] (24)

Thus, due to the linearity of the system, the most general solution in this case is just

\[N_i(t) = \overline{N}_i(t) + \delta N_i(t)
\] (25)
As an application, we discuss the case when we do not have sources of Bismuth or Polonium but only of Lead (this corresponds to the practical situation in which the Radon content of the detector increases with time, since Radon-222 converts rapidly in Lead-210). We consider the simplest case, namely when the source is a constant:

\[ S_1(t) = S_1, \quad S_2(t) = S_3(t) = 0 \]  

(26)

In this assumption, eqs. (24) can be explicitly calculated obtaining:

\[
\begin{align*}
\delta N_1(t) &= S_1 \tau_1 (1 - e^{-\frac{t}{\tau_1}}) \\
\delta N_2(t) &= S_1 \tau_2 \left( 1 - e^{-\frac{t}{\tau_1}} \frac{1}{1-\epsilon_{21}} + e^{-\frac{t}{\tau_2}} \frac{\epsilon_{21}}{1-\epsilon_{21}} \right) \\
\delta N_3(t) &= S_1 \tau_3 \left[ 1 - e^{-\frac{t}{\tau_1}} \frac{1}{(1-\epsilon_{21})(1-\epsilon_{31})} - e^{-\frac{t}{\tau_2}} \frac{\epsilon_{21}}{(1-\epsilon_{21})(1-\epsilon_{31})} + e^{-\frac{t}{\tau_3}} \frac{\epsilon_{21}\epsilon_{31}}{(1-\epsilon_{23})(1-\epsilon_{31})} \right]
\end{align*}
\]

(27)

Now consider the fact \( \tau_2 \ll \tau_3 \ll \tau_1 \) and the limit \( \tau_2 \ll t \ll \tau_1 \), which applies to the typical conditions of data taking, \( t \) being on the scale of one year. We get the approximated solutions:

\[
\begin{align*}
N_{\text{Pb}}(t) &\approx N_{\text{Pb},0} + t \left( S_{\text{Pb}} - \frac{N_{\text{Pb},0}}{\tau_{\text{Pb}}} \right) \\
N_{\text{Bi}}(t) &\approx \frac{\tau_{\text{Bi}}}{\tau_{\text{Pb}}} N_{\text{Pb}}(t) \\
N_{\text{Po}}(t) &\approx N_{\text{Po},0} \exp \left( -\frac{t}{\tau_{\text{Po}}} \right) + \tau_{\text{Po}} \left( \frac{N_{\text{Bi}}(t)}{\tau_{\text{Bi}}} \right) \text{Po}
\end{align*}
\]

(28)

where we used the notations adopted in the paper to increase readability. If the Lead-210 source is sufficiently intense, i.e. \( S_{\text{Pb}} > \frac{N_{\text{Pb},0}}{\tau_{\text{Pb}}} \), we expect a linear increase of the Bi-210 beta activity in the detector, \( n_{\text{Bi}}(t) = \frac{N_{\text{Bi}}(t)}{\tau_{\text{Bi}}} \). When the Lead-210 source can be neglected, we obtain instead \( n_{\text{Bi}}(t) \simeq \text{const.} \).
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