Superfluidity in neutron-star matter

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Received: date / Revised version: date

Abstract. Correlated basis function perturbation theory and the formalism of cluster expansions have been recently employed to obtain an effective interaction from a state-of-the-art nuclear Hamiltonian. We report the results of a study of the superfluid gap in pure neutron matter, associated with formation of Cooper pairs in the $^1S_0$ sector. The calculations have been carried out using an improved version of the CBF effective interaction, in which three-nucleon forces are taken into account using a realistic microscopic model. Our results show that the superfluid transition occurs at densities corresponding to the neutron star inner crust, and that inclusion of three-nucleon interactions leads to a sizable reduction of the energy gap at the Fermi surface.

PACS. 26.60.-c Nuclear matter aspects of neutron stars – 21.30.-x Nuclear forces

1 Introduction

In neutron stars, both conditions for the occurrence of superfluidity in fermion systems, i.e. strong degeneracy and the existence of an attractive interaction between the constituents of matter, are believed to be fulfilled.

The onset of a superfluid (and/or superconducting) phase does not have a significant impact on the equation of state, determining the equilibrium properties of the star, except in the very low density region of the crust. The condensation energy – i.e. the difference between the energies of the normal and superfluid states associated with the formation of Cooper pairs – is in fact negligible with respect to the typical binding energies of the normal phase \cite{1}.

The main effect of the superfluid transition is the opening of an energy gap at the Fermi surface. This leads to a reduction of the phase space available to particles undergoing scattering processes, which in turn results in a strong modification of the neutrino emission, scattering and absorption rates, as well as of the transport coefficients, including the shear viscosity and thermal conductivity. As a consequence, a quantitative understanding of the superfluid phase transition is required to study both neutron star cooling and the stability of rotating stars, which is largely determined by dissipative processes \cite{2,3}.

The effective interaction approach has long been recognized as well suited for the development of a unified description of equilibrium and non-equilibrium properties of nuclear matter, based on realistic models of nuclear dynamics at microscopic level \cite{4,5}.

In recent implementations of this approach, the effective interaction has been derived from highly realistic phenomenological Hamiltonian – strongly constrained by the available data – within the formalism of Correlated Basis Function (CBF) \cite{6,7,8}. Unlike the bare nucleon-nucleon force, the effective interaction is well behaved at short distances, and can be used to carry out perturbative calculations in the basis of eigenstates of the non-interacting system.

Existing applications of the CBF effective interaction include calculations of the shear viscosity and thermal conductivity coefficients of neutron matter \cite{7,9}, as well as the nuclear matter response to neutrino interactions \cite{6,8,10,11}.

In this Letter, we report the results of a calculation of the superfluid gap associated with the formation of Cooper pairs in the $^1S_0$ channel of pure neutron matter (PNM), performed using the CBF effective interaction of Ref.\cite{8}.

2 The CBF effective interaction

The formalism of nuclear many-body theory provides a consistent framework, suitable for treating the non-perturbative nature of nucleon-nucleon (NN) interactions. Within this approach, nuclear matter is modeled as a collection of point-like particles, the dynamics of which are dictated by the Hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \ldots ,$$

(1)
where \( p_i \) and \( m \) denote the momentum of the \( i \)-th nucleon and its mass, respectively, \( v_{ij} \) is the NN interaction potential and the ellipses refer to the presence of interactions involving three or more nucleons.

The NN potential \( v_{ij} \) reduces to the Yukawa one-pion exchange potential at large distances, while its behavior at short and intermediate range is determined by a fit of deuteron properties and NN scattering phase shifts.

Performing perturbative calculations in the basis of eigenstates of the non interacting system requires the replacement of the bare NN potential with a well behaved effective interaction, that can be obtained from either \( G \)-matrix perturbation theory [5] or the correlated basis function (CBF) formalism [6,7].

Within the CBF approach, non perturbative effects are taken into account replacing the states of the non-interacting system, i.e. Fermi gas states \( |n_{FG}\rangle \) in the case of uniform nuclear matter, with a set of correlated states, defined as (see, e.g., Ref. [12])

\[
|n\rangle = \frac{F|n_{FG}\rangle}{\langle n_{FG}|F|n_{FG}\rangle^{1/2}}.
\]

(2)

The operator \( F \), embodying the correlation structure induced by the NN interaction, is written in the form

\[
F = S \prod_{ij} f_{ij},
\]

(3)

where \( S \) is the symmetrization operator accounting for the fact that, as the operator structure of the two-body correlation functions \( f_{ij} \) reflects the complexity of the NN potential, in general \([f_{ij}, f_{kl}] \neq 0\).

The new basis defined by Eq. (2) can be employed to perform perturbative calculations with the bare NN potential. However, the same formalism can be also exploited to obtain an effective interaction, suitable for use with the Fermi gas basis [6,7].

The CBF effective interaction \( v_{\text{eff}} \) is defined by the relation

\[
\langle H \rangle = \frac{\langle 0|H|0\rangle}{\langle 0|0 \rangle} = \langle 0_{FG}| \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij}^{\text{eff}} |0_{FG}\rangle,
\]

(4)

where \( |0_{FG}\rangle \) and \( |0\rangle \) denote the Fermi gas and correlated ground state, respectively, and \( H \) is the nuclear hamiltonian of Eq. (1).

In Refs. [6,7] the left hand side of Eq. (4) has been evaluated using a truncated version of the of the state-of-the-art Argonne \( v_{18} \) potential [13,14]. The calculation of the ground state expectation value has been carried out performing a cluster expansion and including the contributions of two-nucleon clusters. In Ref. [7] the effects of three- and many-nucleon interactions have been also taken into account using the density dependent potential originally proposed in Ref. [15].

An improved CBF effective attraction has been recently derived by the authors of Ref. [8], who explicitly included three-body cluster contributions to \( \langle H \rangle \). This scheme allows for a more realistic treatment of three-body forces, which are known to play a critical role in determining both the spectra of few-nucleon systems [16] and the saturation properties of isospin symmetric nuclear matter [17], based on a realistic description at microscopic level.

The CBF effective interaction of Ref. [8] has been obtained from a nuclear Hamiltonian consisting of the \( v'_{6} \) NN potential [14], supplemented with the UIX three-nucleon potential [18]. The \( v'_{6} \) potential accounts for deuteron properties and S-wave NN scattering phase shifts, while the UIX potential, including a Fujita-Miyazawa two-pion exchange attractive term and a purely phenomenological repulsive term, is designed to reproduce the properties of the three-nucleon bound states.

The energy per particle of both symmetric nuclear matter and pure neutron matter obtained from \( v_{\text{eff}} \) in the Hartree-Fock approximation turns out to be in good agreement with the results of highly advanced many-body approaches [8].

The solid line of Fig. 1 shows the radial dependence of the CBF effective interaction in the \( S = 0, T = 1 \) sector. The solid and dot-dash lines correspond to the CBF effective interaction of Ref. [8] and to the bare Argonne \( v'_{6} \) potential. The inset shows a blow up of the attractive part of the effective interaction, obtained with (solid line) and without (dashed line) inclusion of the three-nucleon potential.

3 Results

For any baryon density \( \rho = k_F^3/3\pi^2 \), where \( k_F \) denotes the Fermi momentum, the gap equation corresponding to \( S \)-wave coupling in cold PNM

\[
\Delta(k) = -\frac{1}{\pi} \int k'^2 d\epsilon - \frac{v(k, k') \Delta(k')}{(\xi^2(k') + \Delta^2(k'))^{1/2}},
\]

(5)
In Eq. (5) has been solved using the algorithm discussed in Ref. [19].

\[ v(k, k') = \int r^2 dr j_0(kr)e^{\xi(r)}j_0(k'r) , \]

where \( j_0(x) = \sin(x)/x \) and \( e^{\xi(r)} \) is the NN potential (see Fig. 1), while

\[ \xi(k) = e(k) - \mu , \]

\( e(k) \) and \( \mu = e(k_F) \) being the energy of a particle carrying momentum \( k \) and the chemical potential, respectively.

The calculation has been carried out using the CBF effective interaction of Ref. [5], with and without inclusion of the UIX three-nucleon potential in the nuclear Hamiltonian. The resulting effective interactions are compared in the inset of Fig. 1. The single particle spectra \( e(k) \) have been consistently computed from the CBF effective interactions within the Hartee-Fock approximation. For comparison, the calculation has been also performed using the bare \( v_0' \) potential and the kinetic energy spectrum.

The superfluid gap at the Fermi surface, \( \Delta(k_F) \), is displayed in Fig. 2 as a function of the Fermi momentum. The dot-dash line shows the results obtained using the bare \( v_0' \) potential, while the solid and dashed lines correspond to calculations carried out using the CBF effective interaction, with and without inclusion of three-nucleon forces, respectively.

It appears that, while the range of Fermi momentum in which \( \Delta(k_F) \neq 0 \) is about the same for all potentials under consideration, the inclusion of three-nucleon forces leads to a sizable reduction of the gap. Their effect, while being hardly noticeable in the \( r \)-dependence of the effective interaction (see inset of Fig. 1), turns out to be sizable in the matrix element \( \langle k | v_{\text{eff}} | k' \rangle \) at \( |k| = |k'| = k_F, \) \( v_F \), driving the solution of the gap equation. For example, at \( k_F = 0.9 \text{ fm}^{-1} \), roughly corresponding to the maxima of the curves of Fig. 2, the inclusion of the UIX potential reduces \( v_F \) by more than 25%.

4 Conclusions

We have carried out a calculation of the superfluid gap in PNM, associated with the formation of Cooper pairs of neutrons in states of total spin \( S = 0 \) and relative angular momentum \( \ell = 0 \). The interaction in this channel, which dominates the attractive component of the neutron-neutron force, has been described using an effective interaction derived from the state-of-the-art models of the two- and three-nucleon potentials referred to as Argonne \( v_0' \) and Urbana IX, within the CBF formalism.

The main advantage of our approach lies in the fact that, unlike the effective interactions based on the mean field approximation, the CBF effective interaction reduces to the bare interaction in the zero-density limit. As a consequence, it can be used to carry out consistent calculations of a number of different properties of neutron star matter, thus allowing for a comprehensive description of equilibrium and non-equilibrium properties of neutron stars.

We find that a superfluid phase develops in the region of low density, \( \rho < \rho_0 \), where \( \rho_0 \sim 2.7 \times 10^{14} \text{g/cm}^3 \) is the equilibrium density of isospin symmetric nuclear matter. As the maximum of the superfluid gap occurs at density \( \rho \sim 0.15\rho_0 \), our work is relevant to the description of the neutron gas in the region of the inner crust, extending from \( \rho \sim 4 \times 10^{13} \text{g/cm}^3 \) to \( \rho \sim 2 \times 10^{14} \text{g/cm}^3 \).

In the case of \( ^1S_0 \) pairing, the critical temperature of the superfluid transition can be estimated from the value of the gap at \( T = 0 \). The resulting maximum values are in the range \( T_c \sim 1 \text{–} 2 \text{ MeV} \), corresponding to \( \sim 1 \text{–} 2 \times 10^{36} \text{K} \).

Our results, while being interesting in their own right, should be regarded as a first step towards a comprehensive description of the superfluid and superconducting phases of neutron stars. The interaction between neutrons coupled to total spin \( S = 1 \) and angular momentum \( \ell = 1 \) is also attractive. The formation of Cooper pairs of neutrons with these quantum numbers is expected to occur at densities \( \rho > \rho_0 \) typical of the neutron star core. The appearance of a superfluid phase in this region would strongly affect the dissipative processes determining the stability of rotating stars. In addition, the small fraction – typically less that \( \sim 10% \) – of protons are also expected to become superconducting, thus affecting dissipative processes driven by electromagnetic interactions with electrons and muons. The extension of the formalism employed in our work to study neutron superfluidity in the \(^3P_2 \) channel and proton superconductivity does not involve any conceptual difficulties.
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