Anisotropic flow in ultra-relativistic heavy ion collisions

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Abstract

A sign reversal of the directed flow parameter $v_1$ in the central rapidity region in Au+Au collisions at $\sqrt{s} = 200$ AGeV is predicted. This anti-flow is shown to be linked to the expansion of the hot matter created. In line with this observation the predicted elliptic flow parameter $v_2$ of various particle species is linked to the mean free path of these particles.

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The exploration of the transverse collective flow is the earliest predicted observable to probe heated and compressed nuclear matter \[1\]. Its sensitivity to the equation of state (EoS) might be used to search for abnormal matter states and phase transitions \[2, 3\].

Until now, the study of directed and anisotropic flow in high energy nuclear collisions is attracting large attention from both experimentalists and theorists \[4–9\]. Flow in general is sensitive to the equation of state \[5, 6, 10–12\] which governs the evolution of the system created in the nuclear collision. Elliptical flow \[10, 13–20\] (i.e. squeeze-out except for a reversed sign in the observable) is especially sensitive to the early time scales of the reaction. It might serve as a keyhole to the (non-)equilibrium dynamics of the strongly interacting matter even before hadronization.

In fluid dynamics, the transverse collective flow is intimately connected to the pressure \(P(\rho, S)\) (which in turn depends on the density \(\rho\) and the entropy \(S\)) of the matter in the reaction zone \[21\]:

\[
P_{x} = \int_{t} \int_{A} P(\rho, S) \, dA \, dt.
\]

Here \(dA\) represents the surface element between the participant and spectator matters and the total pressure is the sum of the potential pressure and the kinetic pressure. Thus, the transverse collective flow depends directly on the equation of state, \(P(\rho, S)\).

In the particle cascade picture employed in transport theoretical approaches, the collective expansion of the system created during a heavy-ion collision also implies space-momentum correlations in particle distributions at freeze-out. However, it is unclear how this collective motion is connected to the hydrodynamical flow behaviour of the matter, since one deals with a system out of equilibrium. Nevertheless, a strong space-momentum correlation in the particle emission pattern persists: This means that - on average - particles created on the left side of the system move in the left direction and particles created on the right side move in the right direction. We will show that the rapidity dependence of directed flow of nucleons and pions can be used to address this space-momentum correlation experimentally. Therefore, it can yield an undisturbed view into the properties of the expanding...
A sketch of a semi-central heavy-ion collision is shown in Fig. 1, from before the collision (a) to the resulting distributions of $\langle x \rangle$ and $\langle p_x \rangle$ shown in (c). In Fig. 1a the projectile and target are shown before the collision in rapidity (horizontal axis) and coordinate space (vertical axis, labeled x) - the off-set between the nuclei is given by the impact parameter. In Fig. 1b the overlap region of the collision is shifted due to interactions towards the midrapidity region. The spectators are only slightly affected by the collisions and remain essentially at target and projectile rapidity. The undressing of the participating nucleons results in a rapidity shift of the nucleons and leads to the creation of hot matter along the beam axis (shown in Fig. 1c: as grey area around the horizontal axis). The subsequent expansion of this highly excited vacuum leads to a positive space-momentum correlation and the nucleons are pushed away from the beam axis. Thus, nucleons around central rapidities sitting at negative $x$ in coordinate space will receive negative $\langle p_x \rangle$-push, while those at positive $x$ acquire a positive $\langle p_x \rangle$. This results in a wiggle structure in the rapidity dependences of $\langle p_x \rangle$ (or $v_1$, respectively) which is depicted in Fig. 4 (and will be discussed in detail below).

The shape of the wiggle, the magnitude of $v_1$ and the rapidity range, depend on the pressure exerted by the partons and color fields creating the space-momentum correlation and therefore on the equation of state of the hadronizing matter. Indeed, comparing the UrQMD model predictions to RQMD calculations which include a rope mechanism to mimic parton interactions in the early stage, the non-interacting strings employed in the UrQMD approach result in smaller space-momentum correlations and thus in a smaller ‘anti-flow’ around midrapidity.

These arguments firmly establish the prediction of a change of sign of the directed flow at mid-rapidity. The scenario is tested quantitatively in Fig. 2 and 3 for Au+Au collisions at $\sqrt{s} = 200 \text{ AGeV}$ at minimum trigger bias, i.e. integrated over all impact parameters, using the Ultra-Relativistic Quantum Molecular Dynamics (UrQMD 1.2) model in cascade mode. To quantify directed flow, the first Fourier coefficient $v_1$, of the particle
azimuthal distribution is used. At a given rapidity and transverse momentum the coefficient is determined by \[25\]:

\[
v_1 = \left\langle \frac{p_x}{p_t} \right\rangle.
\] (2)

Similarly the second Fourier coefficient is determined from \[25\]:

\[
v_2 = \left\langle \frac{p_x^2}{p_t^2} - \frac{p_y^2}{p_t^2} \right\rangle,
\] (3)

which will be investigated later in this paper.

Figure 2 shows the UrQMD calculations of \(v_1\) for nucleons, lambda's and anti-protons in Au+Au collisions at the full RHIC energy \(\sqrt{s} = 200\) AGeV. Indeed, the shape of \(v_1(y)\) is for nucleons and lambda's at mid-rapidity consistent with the picture described above. Anti-protons show a strong anti-correlation with the protons at forward/backward rapidities. This indicates the presence of anti-baryon absorption in nuclear matter even at ultra-relativistic energies. Zooming into the midrapidity region (Fig. 4): \(v_1\) shows a weak negative slope at mid-rapidity for protons. The distribution of \(\Lambda\)'s is in shape and magnitude similar to those of the protons. For larger rapidities, the \(v_1\) values show the well-known 'bounce-off' of the nucleons around projectile and target rapidity. Note that the recently predicted wiggle in hydrodynamical calculations [19,18] has a very different source: That wiggle occurs only at small impact parameters and if a phase transition to a QGP is included. The QGP equation of state is a prerequisite to reach the stopping needed to create a tilted source and the stall of the \(p_x\) flow. The predicted wiggle in this Letter does not assume a QGP equation of state. Another difference is that our predictions rely on partial transparency.

Pions and kaons are produced particles and their space-rapidity correlation is different from that of nucleons as shown in Fig. 2 and Fig. 4. Shadowing by nucleons at central rapidities might also lead to an observable signature in the pion and kaon directed flow. However, due to the small number of participating nucleons in semi-peripheral collisions, this signature does only show up near target and projectile rapidities. It does not lead to a wiggle in the \(v_1\) of pions at midrapidity. Compared to the baryons and to the pions, the kaons
show only very weak flow over the whole inspected rapidity range. At very forward/backward
rapidities, the pions and kaons seem to follow the nucleons: The light mesons are 'boiled-off'
the excited spectator matter, thus following the nucleons bounce-off flow pattern closely.

The elliptic flow of matter at central rapidities is another interesting observable which
yields crucial information about the interaction strength and the mean free path of the
matter created at midrapidity. This information can be directly observed in the strength
of the $v_2$ parameters at midrapidity: Fig. 3 shows the impact parameter dependence of the
elliptic flow parameter for various particle species at midrapidity ($|y| \leq 1$). The impact
parameter bins are: $b \leq 3$ fm, $3 \leq b \leq 6$ fm, $6 \leq b \leq 9$ fm and $9 \leq b \leq 12$ fm. A clear
maximum of the elliptic flow in all particle species is observed for semi-peripheral collision.
Our speculation about the formation of transverse flow in those ultra-relativistic collision
is also supported by the transverse momentum dependence of $v_2$ (in min. bias Au+Au
reactions) as depicted in Fig. 6: A strong increase of the ellipticity parameter with $p_T$ is
predicted which signals the existence of radial expansion.

Figs. 7 and 8 shows the anisotropic flow of nucleons, lambdas, anti-protons, pions and
kaons as a function of rapidity for minimum biased Au+Au collisions at the full RHIC
energy ($\sqrt{s} = 200$ AGeV). The elliptic flow of all inspected hadrons shows a prominent dip
at central rapidities. This indicates a region of low ‘pressure’ (or small interaction strength,
to be more specific). Comparing the elliptic flow parameters of baryons (Fig. 7) with those
of the mesons (Fig. 8) shows that the mesons acquire $2/3$ of the baryon elliptic flow. This
scaling with the geometrical quark model cross section may indicate a connection between
elliptic flow and in-medium cross section of the hadrons as will be discussed below.

The appearance of this dip in $v_2(y)$ might appear to be counter-intuitive. However,
it points towards a distinct feature of the model dynamics in the early stage, namely the
pre-equilibrium string dynamics and interactions on the parton level. Fig. 9 shows that
the $v_2$ parameter is closely related to the formation time of particles in the string picture.
A standard default setting of formation time results in an average formation time (in the
string rest frame) of 1 fm/c. Consequently, the particles created in the initial stage of the
collision are not allowed to interact during this time. With particle velocities near the speed of light, these particles do have a mean free path on the order of 1 fm at midrapidity. If the formation time - and therefore the mean free path - is decreased, the elliptic flow parameter \( v_2 \) increases. In the limit of a vanishing mean free path (hydro limit) the elliptic flow in the present model becomes maximal and it is quantitatively consistent with hydrodynamical predictions [26]. In contrast, increasing the formation time (mean free path) results in a vanishing \( v_2 \), in line with the limit given by Hijing calculations without quenching [27].

Thus, the strength of the anisotropic flow of pions is directly connected to the mean free path of the particles (partons, hadrons) forming the hot midrapidity region. The measurement of \( v_2 \) might therefore yield valuable information about the transport properties of QCD-matter, like the interaction frequencies and the viscosity of the excited partonic and hadronic matter at RHIC energies. Especially \( \Omega \) baryons, with their small hadronic cross section, are supposed to measure QGP properties without any additional disturbance due to the hadronic phase.

In this Letter we have shown that a combination of space-momentum correlations and radial expansion results in a wiggle in the rapidity dependence of directed flow in high energy nucleus-nucleus collisions. Since the magnitude of the wiggle depends on the radial expansion strength, which in turn is given by the equation of state, its observation provides unique insight into the properties of the excited matter created around midrapidity. A peak in the centrality dependence of \( v_2 \) is observed, while a dip in the rapidity dependence of the elliptic flow parameter is predicted. It has been demonstrated that the strength of the elliptic flow parameter is directly proportional to the characteristic mean free path of the matter formed in the central rapidity region. Both the wiggle and the minimum in the elliptic flow appear at central rapidities, hence they are accessible to the STAR experiment at RHIC in the near future.
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FIG. 1. A schematic sketch of a semi-central heavy-ion collision in progressing time (a) to (c) and how the excited matter (grey area in (c)) leads to the antiflow of nucleons at central rapidities. In these figures the vertical axis is the coordinate along the impact parameter direction and the horizontal axis is the rapidity axis.
FIG. 2. Directed flow parameter $v_1$ of protons, lambdas and anti-protons as a function of rapidity in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min.bias.
FIG. 3. Directed flow parameter $v_1$ of kaons and pions as a function of rapidity in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min.bias.
FIG. 4. Antiflow of protons in the central rapidity region as observed in the directed flow parameter $v_1$ at central rapidities in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min. bias.
FIG. 5. Elliptic flow parameter $v_2$ of pions, kaons, protons, anti-protons and lambda’s at midrapidity ($|y| \leq 1$) as a function of centrality in Au+Au reactions at $\sqrt{s} = 200$ AGeV.
FIG. 6. Elliptic flow parameter \( v_2 \) at midrapidity (\(|y| \leq 1\)) as a function of transverse momentum in Au+Au reactions at \( \sqrt{s} = 200 \text{ AGeV} \), min. bias.
FIG. 7. Elliptic flow parameter $v_2$ of protons, lambdas and anti-protons as a function of rapidity in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min. bias.
FIG. 8. Elliptic flow parameter $v_2$ of kaons and pions as a function of rapidity in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min. bias.
FIG. 9. Relation between the elliptic flow parameter $v_2$ at midrapidity and the mean free path (formation time) of the particles in Au+Au reactions at $\sqrt{s} = 200$ AGeV, $b = 7$ fm.