Determining non-linear characteristics of a concentrating solar collector according to the experiment design

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Abstract. A linear concentrating solar air collector with internal multiple-fin array was built. The collector was designed as an air heater for the system of house heating with an all-year-round heat accumulation. In order to conduct transient type numerical simulations of the year-long work cycle of the system, it was necessary to know the working characteristic of the solar collector. To determine the characteristics, some factors for experimental research were selected. The authors decided to conduct experimental research on the basis of fractional $3^k$ factorial design for three factors. A set-up was built and then, a series of experiments were conducted according to the adopted experiment design. A working characteristic in the form of a quadratic polynomial was determined, and a diagram was prepared. The obtained results were then discussed. The obtained research method and the determined formula met the needs of further numerical modelling of the year-long work of the heating system containing the examined solar collector.

1. Introduction

Solar air heaters are the subject of research, whose main stream relates to flat air collectors, and its complement are concentrating collectors. The research of flat collectors usually relates to the filling, which changes the structure of flow and conditions of heat exchange for the given amount of air flow [1, 2, 3, 4]. The research of flat collectors rarely involves experimental research for a variable amount of flowing air [5, 6]. The research of linear concentrating collectors as air heaters are even scarcer and relate to collectors with a linear conical concentrator for the specified amount of flowing air [7] and parabolic concentrator for the variable amount of flowing air [8].

Today, energy analyses of the year-long effects of solar system performance are mostly done with transient programs, like TRNSYS [9, 10]. These programs use solar collectors’ specifications, determined by producers according to standard procedures included, usually, in the American norm ASHRAE [11] as well as in others [12, 13]. The methods used relate to the determination of efficiency depending on irradiation and the temperature difference between the heated liquid and the surroundings. This method of determining the efficiency assumes a constant value of the liquid's flow through the collector. Constant value of the liquid's flow enables maximisation of the process' efficiency, but it limits the obtained liquid temperature – it would be higher in case of a reduced flow. In order to make allowance for the influence of liquid flow, a full mathematical model of the processes taking place is usually developed.

While making numerical analyses in transient type programs, it is convenient to use the mathematical function allowing for the flow of liquid also in case when the studied solar system uses such a mode of
work of the variable flow collector. It is convenient to use the methodology of experiment design [14] to determine the mathematical function, which enables to reduce the necessary number of experiments.

This paper presents the procedure used to determine non-linear working characteristic of the developed parabolic linear concentrating collector with the use of factorial design for three factors. Fractional 3\(^{k}\) factorial design [15] was used. The purpose of the tested solar concentrating collector is to perform the function of an air heater in the conceptual heating system with a long-term thermal storage. The idea of internally finned absorber was previously investigated by authors in [16], where the analytical model was shown in detail.

2. Solar collector
The authors built the prototype of a solar concentrating collector in accordance with their technological capabilities. Polymethyl (PMMA) plate with high quality aluminium coating was used as the concentrator's material. It was attached to the construction of fins, in which the shape of a parable had been cut with a laser. The parable was fitted to the solar concentrator's optical requirements. An aluminium cylinder with an internal multiple-fin array was used as the air heater. The fins' role is to transfer heat to the air flowing inside of them. The heat is taken from the external surface, which is irradiated by sun. The internal fins were made in two halves of the cylinder (see Fig.1). Both parts of the absorber were sealed and joined with bolts.

![Figure 1. Internally finned absorber made of aluminium as the air heater.](image)

The absorber's external surface was covered with black silicone paint, resistant to high temperature. It was then put into an evacuated co-axial pipe vessel made of borosilicate glass. Both sides of the glass casing are tipped with the absorber's mounting rings made of polytetrafluoroethylene (PTFE). Detailed data of the solar collector are presented in Table 1.

| collector | heated medium | air |
|-----------|---------------|-----|
| concentration ratio | 19.1 |
| tracker rotation ability | horizontal & vertical |
| concentrator | profile line | parabolic |
| concentrated irradiation angle | 180° |
| plate material | PMMA, 2 mm |
| reflector material | aluminium layer |
| width | 1.2 m |
The collector was mounted on the tracker’s head, to enable moving towards the direction of the Sun. The whole piece was then mounted on a mobile metal frame (see Fig. 2).

**Figure 2.** Solar collector mounted on tracker’s head with a mobile frame (initial test photo).
3. Collector efficiency

For the purpose of further deliberations, an idea was adopted to define the solar collector's efficiency as the ratio of solar irradiation energy to the energy received in the absorber and transferred to the air flowing through the absorber. It was agreed that the energy received by the ventilator is not taken into consideration. Thus:

$$\eta = \frac{P_r}{P_a}$$

(1)

where:
- $\eta$ – thermal efficiency (-),
- $P_r$ – thermal power of radiation (W),
- $P_a$ – thermal power absorbed (W).

The energy of solar irradiation falling on the collector:

$$P_r = A_{ap} \cdot I_b$$

(2)

where:
- $A_{ap}$ – aperture surface area (m$^2$),
- $I_b$ – beam irradiation (W/m$^2$).

The thermal energy transferred to the flowing air:

$$P_a = V \cdot c_p \cdot \rho (t_{out} - t_{in})$$

(3)

where:
- $c_p$ – specific heat of air (J/kg K),
- $t_{in}$ – air temperature at absorber inlet (°C),
- $t_{out}$ – air temperature at absorber outlet (°C),
- $V$ – volumetric air flow (m$^3$/s),
- $\rho$ – density of air (kg/m$^3$).

The following factors influence the performance of the concentrating solar collector: the method of heat exchange in the absorber, the level of concentration, optical quality of the mirror, the type of absorbing coatings used and the type of thermal insulation used. These factors were not subject to any change in case of the discussed research, as they resulted from the adopted technology of the collector. The collector's performance depends also on a number of the liquid's and the surrounding's thermal and flow parameters. Generally, the collector's performance enhancements when:
- solar irradiation $I_b$ increases
- air temperature at the collector's inlet $t_{in}$ decreases
- air flow $V$ increases
- ambient temperature $t_{amb}$ increases
- ambient air's velocity $w_{amb}$ decreases

In the adopted concept of a building's thermal system with heat accumulation, ambient temperature $t_{amb}$ is approximately constant (about 20°C), ambient air's velocity $w_{amb}$ practically equals zero. Hence, three variables were considered independent variables: $I_b$, $t_{in}$, $V$. In its general form, the collector's performance is the function of these three independent variables:

$$\eta(I_b, t_{in}, V)$$

(4)
4. Experiment design

It was assumed, that function (4) is not a linear function, but a quadratic polynomial, it is however expected that the independent variables interact. For such interactions, $3^k$ factorial design is used, whose factors adopt three value levels. The variables occur at levels:

- higher level, marked as $+I$;
- medium level, also called basic or zero level, marked as $0$;
- lower level, marked as $-I$.

Hartley's plan [15] was chosen from among many known three-level experiment designs, as one of the most convenient ones. Hartley's plan is a fractional design - 11, instead of $3^3 = 27$ experiments have to be done for the discussed function (4). It significantly reduces the time and cost of experimental research. The values of independent variables for further 11 experiments create a hyper-cubic space, shown in Fig. 3 and in table 1 for any variables $I_b$, $t_{in}$, $V$.

![Hyper-cubic space of the variable’s value of Hartley’s experiment design](image)

**Table 2.** Variable’s value (-1/0/+1) of Hartley’s experiment design.

| reference number of experiment | $I_b$ | $t_{in}$ | $V$       |   |
|-------------------------------|------|---------|----------|---|
| 1                             | 0    | 0       | 0        |   |
| 2                             | -1   | 0       | 0        |   |
| 3                             | +1   | 0       | 0        |   |
| 4                             | 0    | -1      | 0        |   |
| 5                             | 0    | +1      | 0        |   |
| 6                             | 0    | 0       | -1       |   |
| 7                             | 0    | 0       | +1       |   |
| 8                             | +1   | -1      | -1       |   |
| 9                             | -1   | +1      | -1       |   |
| 10                            | +1   | +1      | +1       |   |
| 11                            | -1   | -1      | +1       |   |

Hartley experiment plan is used to generate a quadratic equation describing the searched parameter with the help of three independent variables, selected beforehand. In the examined case, a series of experiments conducted according to the experiment design enables deriving a polynomial:
\[ \eta = a_1 + a_2I_b + a_3I_b^2 + a_4t_{in} + a_5t_{in}^2 + a_6V + a_7V^2 + \\
+ a_8I_b t_{in} + a_9I_b V + a_{10}t_{in} V + a_{11}I_b t_{in} V \]  
(5)

with visible 11 coefficients \( a_1 \) to \( a_{11} \).

**Table 3. Assumed value of factors.**

| variable | -1   | 0    | +1   |
|----------|------|------|------|
| \( I_b \) (W/m\(^2\)) | 300  | 600  | 900  |
| \( t_{in} \) (°C)   | 20   | 80   | 140  |
| \( V \) (m\(^3\)/s) | 0.002| 0.004| 0.006|

A three-level experiment design requires adoption of defined values for each level -1/0/+1 of factors. Values adopted for further research are shown in Table 3. The adopted levels of temperature changeability result from the prognosed temperature ranges of the working medium. The indicated temperatures enable the analysis of the solar collector work both in the open and closed cycle. The levels of radiation changeability are related to its availability. 900 W/m\(^2\) is the average maximum value available with a cloudless sky, whereas below 300 W/m\(^2\) the work of collectors with follow-up system is little cost-effective.

The changes of the flowing air value are closely related to the absorber’s geometry and flow resistance related thereto. It was the subject of the author’s analysis in their previous study [16].

### 5. Experimental set-up

The existing set-up was adjusted according to the requirements of the experiment design. The plan of the adjusted set-up is shown in Fig. 4. Parabolic concentrator PC directs the beam irradiation BR on the absorber’s pipe AB. The irradiation \( I_b \) can be regulated with the blender B containing one or two layers of plastic mesh. It is possible to fix one or two layers of mesh, which enables a three-level regulation of irradiation intensity \( I_b \). Using one layer of mesh reduced BR direct irradiation to approximately 45\%, the use of a double mesh layer reduced irradiation BR to approximately 33\%. The value of direct irradiation \( I_b \) is measured with a pyranometer PR.

![Figure 4. Scheme of experimental set-up.](image-url)
The mesh was fixed on a special frame, shown in Fig. 5. The mesh used was a 1 mm thick plastic, with holes of 18 mm in diameter, distributed hexagonally. Double layers of the mesh were moved (see the draft in the left-bottom corner of Fig. 5).

Two Kipp&Zonen pyranometers (premium quality) were mounted on the tracker near the concentrator frame. During the measuring of radiation under shutter B, the pyranometer PR was being moved by hand, to make the measurement uniform. Direct irradiation BR was measured as the difference between total irradiation and dispersed irradiation. The item that was used to make a shadow over the pyranometer PR was a non-transparent disc PB held at the distance of approximately 0.5 m. Ambient air of temperature \( t_{amb} \) is sucked in by an air fan F. The fan F is powered by a voltage regulator VR\(_1\), which enabled the change of volumetric air flow \( V \). Three values of volumetric air flow \( V \) were used, measured by an anemometer VM at the air's outlet to the surroundings. The device VM was used to measure the air of temperature \( t_{amb} \) at the beginning of the day of research.

The air flows from the fan F through the electric heater EHT of maximum power of approximately 1.7 kW, powered by voltage regulator VR\(_2\). It enables the change of temperature of the air coming out of the heater EHT, independently of ambient temperature \( t_{amb} \). Heated air flows around the thermometer, which enables the measurement of air temperature \( t_{in} \). The heater's EHT power was adjusted in such a way, as to maintain the assumed ambient temperature \( t_{in} \). The air flows into the absorber, where it is heated to temperature \( t_{out} \). This temperature is measured with a thermometer at the outlet from the absorber AB. The level of power on the regulator VR\(_2\) was determined experimentally for each combination of temperature \( t_{in} \) and volumetric air flow \( V \).

The experimental set-up met the requirements of conducting research according to the experiment design requirements. A three-level regulation of three parameters: \( I_b, t_{in}, V \) was possible.

6. Experiment

Experimental research was carried out at Wroclaw University of Technology in Poland, during summer. The sky was completely cloudless during the research. The value of irradiation \( I_b \) ranged from 830 to 955 W/m\(^2\); ambient temperature \( t_{amb} \) ranged from 19 to 22°C, the velocity of ambient air \( W_{amb} \) ranged from 0 to 0.22 m/s. These conditions were considered sufficient for the experiment. Considerable weight of the absorber resulted in the situation that a steady state of heat transfer was achieved only after...
approximately 20 minutes. Due to that reason, all 11 experiments took 4 hours, from 10 AM to 14 PM to conduct. Table 5 shows measured values of the inputs and outputs as well as the results of calculations done according to formulas (2)-(4). As the factual value of direct irradiation changed throughout the day, the assumed values of 300-600-900 W/m² were not exactly achieved - the biggest deviation was 55 W/m². Air temperature \( t_\text{in} \) differed from the assumed values in the range of 20-80-140 °C – the biggest deviation was 6.9 °C. It was caused by air temperature fluctuation during the process of obtaining the steady state. The deviation of air flow did not exceed 5% in relation to the assumed 0.002-0.004-0.006 m³/s. It was assumed, that the obtained deviations are acceptable and allow to conduct further calculations according to the adopted experiment design.

Table 4. Acquired uncertainties for the parameters measured.

| ±Δ\( t \) °C | ±Δ\( I \) W/m² | ±Δ\( V \) m³/s |
|----------------|----------------|----------------|
| 0.1            | 20             | 1%             |

The analysis of uncertainty for the equipment used in calculation of such parameters as temperature, volume flux and density of solar radiation was done according to procedures described in [17]. Uncertainties adopted for the calculated values are shown in Table 4, and extended uncertainty for thermal efficiency in Table 5.

Table 5. Measured and calculated values of the conducted experiment.

| reference number of experiment | \( I_\text{b} \) (W/m²) | \( t_\text{in} \) (°C) | \( V \) (m³/s) | \( t_\text{out} \) (°C) | \( P_\text{a} \) (W) | \( P_\text{r} \) (W) | \( \eta \) (-) | \( U(\eta) \) (-) |
|-------------------------------|--------------------------|------------------------|-----------------|------------------------|------------------|-----------------|---------------|---------------|
| 1                             | 623                      | 83.1                   | 0.0042          | 104.5                  | 392.5            | 97.6            | 0.249         | 0.020         |
| 2                             | 286                      | 79.2                   | 0.0042          | 83.8                   | 180.2            | 21.0            | 0.116         | 0.016         |
| 3                             | 905                      | 78.4                   | 0.0042          | 114.4                  | 570.2            | 164.1           | 0.288         | 0.020         |
| 4                             | 583                      | 24.0                   | 0.0042          | 53.9                   | 367.3            | 136.3           | 0.371         | 0.036         |
| 5                             | 592                      | 143.4                  | 0.0042          | 148.7                  | 373.0            | 24.2            | 0.065         | 0.005         |
| 6                             | 603                      | 77.8                   | 0.0019          | 117.2                  | 379.9            | 81.3            | 0.214         | 0.029         |
| 7                             | 580                      | 82.5                   | 0.0063          | 97.0                   | 365.4            | 99.2            | 0.271         | 0.020         |
| 8                             | 955                      | 22.3                   | 0.0019          | 129.0                  | 601.7            | 220.0           | 0.366         | 0.049         |
| 9                             | 323                      | 136.7                  | 0.0019          | 126.3                  | 203.5            | -21.4           | -0.105        | 0.015         |
| 10                            | 934                      | 139.2                  | 0.0063          | 158.6                  | 588.4            | 132.7           | 0.225         | 0.011         |
| 11                            | 312                      | 21.5                   | 0.0063          | 33.2                   | 196.6            | 80.0            | 0.407         | 0.056         |

7. Determining the characteristic and discussion

The results of experiment measurements and the results of calculations shown in Table 5 were entered into a calculation sheet. Polynomial coefficients were determined by means of matrix calculations on rules described in [15]. These coefficients result from 11 combinations, which have been given proper value in Table 2. Values of polynomial coefficients and values of thermal efficiency calculated from (5) are shown in Table 6. The efficiency values obtained from Hartley’s plan are very similar to the efficiency determined on the basis of experimental measurements. The divergence falls within the standard uncertainty.
Table 6. Values of polynomial coefficients and thermal efficiency from Hartley plan.

| number | value                | $\eta$ (-) |
|--------|----------------------|------------|
| 1      | -6.60296732E-02     | 0.249      |
| 2      | -1.94465583E-03     | 0.116      |
| 3      | -3.88024809E-06     | 0.288      |
| 4      | 9.82072808E-04      | 0.384      |
| 5      | -5.20303890E-07     | 0.065      |
| 6      | 7.83563135E+01      | 0.214      |
| 7      | -7.28312615E+02     | 0.271      |
| 8      | 2.18807413E-07      | 0.356      |
| 9      | -5.07440590E-01     | -0.105     |
| 10     | -8.60792172E-02     | 0.225      |
| 11     | 7.71987334E-04      | 0.407      |

With the full formula (5) it is possible to present the efficiency of the studied solar collector graphically. Fig. 6 shows the collector’s thermal efficiency $\eta$ depending on the air inlet temperature $t_{in}$ and volumetric air flow $V$ for the three values of irradiation. The main part of the diagram is made of the efficiencies obtained for direct irradiation $I_b=600$ W/m$^2$. It is visible, that with air temperature increase $t_{in}$, the collector's thermal efficiency $\eta$ decreases, and that it increases with the increase of volumetric air flow $V$. Dotted line marks the location of efficiency diagrams for 300 and 900 W/m$^2$ of beam irradiation $I_b$. It is apparent that efficiency increases with the increase of $I_b$. Presented results are in line with general rules for heat transfer in solar collector.

It was agreed that the obtained equation (5) with coefficients shown in Table 6 will be used for further modelling of the solar installation’s year-long operation.

Figure 6. Working characteristic – thermal efficiency versus beam radiation, air flow and air temperature.
8. Conclusion
The authors have presented research procedure used to determine the working characteristic of a solar concentrating collector as air heater. They have indicated three significant factors and chosen the experiment design. Three value levels have been determined for each of the factors, to carry out the experiment. An experimental set-up was built, allowing to conduct the planned series of experiments. A series of experiments have been carried out and the measurements enabled calculation of coefficients of polynomial being the equivalent of the studied collector's working characteristics. A few chosen diagrams of the collector's working characteristic have been developed and it has been assumed that their shape is correct and complies with the general rules of heat exchange. It has been assessed that:

- the working characteristic of the studied solar collector was achieved, in the form of a quadratic polynomial of three independent input variables,
- the achieved characteristic is useful for further research of the modelling of the solar installation's year-long operation,
- the procedure used is relatively easy to conduct for other solar collectors.

Appendix
Nomenclature:

- $A_{ap}$ – aperture surface area (m$^2$)
- $a_i$ – polynomial coefficient (-)
- $c_p$ – specific heat of air (J/kg K)
- $I_b$ – beam irradiation (W/m$^2$)
- $P_r$ – thermal power of radiation (W)
- $P_a$ – thermal power absorbed (W)
- $t_{amb}$ – ambient temperature (°C)
- $t_{in}$ – air temperature at absorber inlet (°C)
- $t_{out}$ – air temperature at absorber outlet (°C)
- $U$ – extended uncertainties (-)
- $V$ – volumetric air flow (m$^3$/s)
- $w_{amb}$ – ambient air velocity (m/s)
- $\rho$ – density of air (kg/m$^3$)
- $\eta$ – thermal efficiency (-)

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