Mass spectra of bottom-charm baryons

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In this paper, we investigate the mass spectra of bottom-charm baryons systematically, where the relativistic quark model and the infinitesimally shifted Gaussian basis function method are employed. Our calculation shows that the \(\rho\)-mode appears lower in energy than the other excited modes. According to this feature, the allowed quantum states are selected and a systematic study of the mass spectra for \(\Xi_{bc}\) (\(\Xi_{bc}\)) and \(\Omega_{bc}\) (\(\Omega_{bc}\)) families is performed. The root mean square radii and quark radial probability density distributions of these baryons are analyzed as well. Next, the Regge trajectories in the \((J, M^2)\) plane are successfully constructed based on the mass spectra. At last, we present the structures of the mass spectra, and analyze the difficulty and opportunity in searching for the ground states of bottom-charm baryons in experiment.

Key words: Bottom-charm baryons, Mass spectra, Relativistic quark model.

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I. Introduction

Quantum chromodynamics (QCD) predicts the existence of baryons with two heavy quarks \((b, c)\) and one light quark, known as the doubly heavy baryons (DHBs). DHBs can be divided into three groups: double-charm baryons \((\Xi_{cc} \text{ and } \Omega_{cc})\), double-bottom baryons \((\Xi_{bb} \text{ and } \Omega_{bb})\), and bottom-charm baryons \((\Xi_{bc} \text{ and } \Omega_{bc})\). The study of DHBS contributes to an in-depth understanding of the heavy quark symmetry, chiral dynamics, fundamental theory of the strong interaction, and models inspired by QCD.
The experimental research of DHBs was full of twists and turns. The $\Xi_{cc}^{++}$ baryon was first observed by the SELEX collaboration in 2002 [1]. But, other experiments have failed to confirm this so far. The experimental turning point for DHBS research occurred in 2017. Observations of $\Xi_{cc}^{++}$ baryons were reported by the LHCb collaboration that year and have been confirmed several times since [2–4]. Now, the $\Xi_{cc}^{++}$ baryon has become the first DHB collected in the new PDG data [5]. It opened the door to the experimental detection of DHBs, and people expected to find more DHBs experimentally. However, in the next experiments of searching for the $\Xi_{bc}^{0}$, $\Omega_{bc}^{0}$ and $\Xi_{bb}^{+}$ baryons, the LHCb collaboration observed no significant signals in the invariant mass range of 6.7 $\sim$ 7.3 GeV. So far, the DHB containing bottom quark has not yet been discovered experimentally.

The zero result of searching for the bottom-charm baryons has aroused great concern. Many theoretical efforts have been carried out to predict the production in the collider [9–16], the decay properties [17–29] and the accurate mass spectrum [30–63] of DHBs, so as to provide more powerful theoretical supports to the related experiments in the near future.

Recently, we have developed an approximate method [64, 65] to analyze the singly heavy baryon spectra systematically in which the relativistic quark model [66, 67] and the infinitesimally shifted Gaussian (ISG) basis function method [68] are employed. The result indicates that this method is reasonable and effective in the singly heavy baryon spectroscopy. Additionally, it is worth pointing out that these calculations are based on a uniform set of parameters [64, 65]. Then, we extended this method to study the mass spectra of the double charm baryons ($\Xi_{cc}$ and $\Omega_{cc}$) [69] and the double bottom baryons ($\Xi_{bb}$ and $\Omega_{bb}$) [70]. In the following sections, the systematic calculation of the bottom-charm baryon spectra by this method will be performed.

This paper is organized as follows. In Sect.II, the calculation method used in this work is briefly introduced. In Sect.III, we present the calculation results of the bottom-charm baryons, including the root mean square (r.m.s.) radii, mass spectra, quark radial probability density distributions, Regge trajectories and spectral structure, and give discussions about the results. Sect.IV is reserved for our conclusions.

II. Phenomenological methods adopted in this work

In our calculations, the relativistic quark model is employed to investigate the full mass spectra. In order to improve the computational accuracy and efficiency, the ISG method is also adopted in our studies. The relevant technical details can be found in references [64–68]. Next, we mainly introduce the selection of Jacobi coordinates and the structure of functions for bottom-charm baryons. The DHB is regarded as a three-quark system and the related calculation is performed in the Jacobi coordinates.
as shown in Fig.1. There are three channels of Jacobi coordinates, which are defined as

\[ \rho_i = r_j - r_k, \]
\[ \lambda_i = r_i - \frac{m_j r_j + m_k r_k}{m_j + m_k}, \]

where \( i, j, k = 1, 2, 3 \) (or replace their positions in turn). \( r_i \) and \( m_i \) stand for the position vector and the mass of the \( i \)th quark, respectively.

\[\Xi_{bc} = \frac{1}{\sqrt{2}} (bc - cb) q,\]
\[\Xi'_{bc} = \frac{1}{\sqrt{2}} (bc + cb) q.\]

Here \( q \) denotes up quark (u) or down quark (d). For \( \Omega_{bc} \) and \( \Omega'_{bc}, q \) is replaced by strange quark.

In channel 3, \( l_{\rho 3} \) (denoted in short as \( l_{\rho} \)) stands for the orbital angular momentum between the two heavy quarks, and \( l_{\lambda 3} \) (denoted in short as \( l_{\lambda} \)) represents the one between the bottom-charm quark pair and the light quark. For a quantum state in this work, the spatial wave function is combined with the spin function as follow,

\[ |l_{\rho} l_{\lambda} L s j J M_J \rangle = \left\{ (|l_{\rho} m_{\rho}|l_{\lambda} m_{\lambda})L \times (|s_1 m_{s_1}|s_2 m_{s_2})s_J \right\} \times |s_3 m_{s_3}\rangle \}_{JM_J}. \]
III. Numerical results and discussion

3.1 \( \rho \)-mode

As usual, \( nL(J^P) \) is used to describe a baryon state. For the excited states \( (L \neq 0) \), there exist several \( |l_\rho l_\lambda LsJMJJ \rangle \) states under the condition of \( L = l_\rho + l_\lambda \). They may be divided into the following three modes: (1) The \( \rho \)-mode with \( l_\rho \neq 0 \) and \( l_\lambda = 0 \); (2) The \( \lambda \)-mode with \( l_\rho = 0 \) and \( l_\lambda \neq 0 \); (3) The \( \lambda-\rho \) mixing mode with \( l_\rho \neq 0 \) and \( l_\lambda \neq 0 \). As an approximation, we take no account of the mixing of these modes, and note that the most likely mode to be observed experimentally should be that with lower energy. With this in mind, we need to analyze which mode has the lowest energy in our calculations.

Considering the excitation energies of the \( 1D(\frac{3}{2}^+, \frac{5}{2}^+) \) states for \( \Xi_{QQ}^\prime \) and \( \Xi_{QQ} \) as functions of \( m_2 \), we investigate the trend of these three modes in the heavy quark limit. Meanwhile, the dependence of excitation energies on \( m_2 \) of the \( \rho \)-mode is compared with that of the other two modes. Being \( m_c = 1.628 \) GeV and \( m_b = 4.977 \) GeV in the actual calculations below, we set \( m_1 = (m_c/m_b)m_2 \) here to keep them in proportion. From Fig.2, one can see that the excitation energies of the \( \rho \)-mode are significantly lower than those of the other two modes when \( m_2 \) increases from 2.0 GeV to 5.0 GeV. This suggests that the \( \rho \)-mode appears lower in energy than the other two modes in the heavy quark limit for bottom-charm baryons. Therefore, we only study the \( \rho \)-mode in this work. Additionally, the excitation energy differences between the \( 1D(\frac{3}{2}^+, \frac{5}{2}^+) \) states in the \( \rho \)-mode get closer to each other with increasing \( m_2 \) as shown in Fig.2, which is consistent with the heavy quark spin symmetry [71].

3.2 Mass spectra, r.m.s. radii and quark radial probability density distributions

In the \( \rho \)-mode, the complete mass spectra of the \( \Xi_{bc}^\prime, \Xi_{bc}, \Omega_{bc}^\prime \) and \( \Omega_{bc} \) baryons with quantum numbers up to \( n = 4 \) and \( L = 4 \) are calculated. The corresponding r.m.s. radii and part of the quark radial probability density distributions are computed as well. The detailed results are presented in Tables I-IV and Figs.3-6.

From Tables I-IV, one can find some general features as follows: (1) For the spin-doublet states with the same \( j \), the \( J = j + \frac{1}{2} \) state is higher in energy than the \( J = j - \frac{1}{2} \) state. (2) For the same \( L \), the mass splitting among different states becomes larger and larger with the increase of \( j \). For example, Table I shows the mass differences (splittings) of the \( 1D \) doublets with \( j = 1, 2, 3 \) are 21 MeV, 35 MeV and 50 MeV, respectively. So, the mass splitting of the bottom-charm baryons shows the same feature as those of the double-charm and -bottom baryons [69, 70]. Moreover, by comparing the mass splittings of these three groups of the DHBs, one can find that this mass splitting is inversely
FIG. 2: (Color online) The dependence of excitation energies on \( m_2 \) for different modes of \( \Xi'_{QQ} \) and \( \Xi_{QQ}' \), where \( m_1 = (m_c/m_b)m_2 \) and \( m_3 = m_u(d) \). The excitation energies are measured from the ground states \( 1S(\frac{1}{2})^+_{j=1} \) of \( \Xi'_{QQ} \) and \( 1S(\frac{1}{2})^+_{j=0} \) of \( \Xi_{QQ}' \), respectively.

proportional to the total mass of the heavy quark pair. (3) The mass difference between two adjacent radial excited states gradually decreases with increasing \( n \). This is a general property for singly and doubly heavy baryons in our calculations.

The radial probability densities \( \omega(r_\rho) \) and \( \omega(r_\lambda) \) in a three-quark system can be defined below,

\[
\omega(r_\rho) = \int |\Psi(r_\rho, r_\lambda)|^2 dr_\lambda d\Omega_\rho, \\
\omega(r_\lambda) = \int |\Psi(r_\rho, r_\lambda)|^2 dr_\rho d\Omega_\lambda,
\]

where \( \Omega_\rho \) and \( \Omega_\lambda \) stand for the solid angles spanned by vectors \( r_\rho \) and \( r_\lambda \), respectively. Through the analyses of the quark radial probability density distributions in Figs.3-6 and the r.m.s. radii in Tables I-IV, one can find some interesting properties.

(1) The \( \langle r_\rho^2 \rangle^{1/2} \) value of a ground state is smaller than the corresponding \( \langle r_\lambda^2 \rangle^{1/2} \) value. This means that the two heavy quarks are bonded tightly. By using the results of our previous papers \cite{69, 70} and examining the r.m.s. radii of these three groups of DHBs, one can find the \( \langle r_\rho^2 \rangle^{1/2} \) value is also inversely proportional to the total mass of the heavy quark pair.

(2) Tables I-IV show that when \( n \) is fixed, \( \langle r_\rho^2 \rangle^{1/2} \) values become larger with increasing \( L \). Figs.3-6 also show a consistent change trend for the distribution of the \( r_\rho^2 \omega(r_\rho) \) (solid lines), especially in the case of \( n=1 \) or 3, where the peak of \( r_\rho^2 \omega(r_\rho) \) is significantly shifted outward with increasing \( L \).

(3) The curves in Fig.3 are very similar to those in Fig.4 and the values of the r.m.s. radii in Tables I and II are also very close to each other for the same \( nL(J^P)_j \) state. This suggests that the flavor
symmetry has only a little effect on the shapes of the bottom-charm baryons and their mass values.

(4) Although the difference of the curves in Fig.3 (or 4) and Fig.5 (or 6) is very small, for the same quantum state, the apparent differences can be seen in the $\langle r^2 \rangle^{1/2}$ values and the mass values for $\Xi_{bc}$ and $\Omega_{bc}$ as shown in Table I and III. This reflects the different contributions to these physical quantities from up (down) quark and strange quark in bottom-charm baryons.

![Graph showing quark radial probability density distributions]

FIG. 3: (Color online) Quark radial probability density distributions for some $nL$ states in the $\Xi_{bc}'$ family. The solid line denotes the probability density with $r_\rho$, and the dash line the one with $r_\lambda$.

3.3 Regge trajectories

As an effective phenomenological approach, the Regge trajectory \[ M^2 = \alpha J + \beta, \] can help ones to predict the evolution trend of hadron mass spectra. In turn, it could deepen our understanding of the hadron structure by testing the universality of the Regge theory.

In this paper, the following definition for the $(J, M^2)$ Regge trajectories is used.

As shown in Fig.7, the group of $\Xi_{bc}'(NP)$ with the natural parity of $(-1)^{J-1/2}$ is composed of $S(1/2^+)_j=1$, $P(3/2^-)_j=1$, $D(5/2^+)_j=2$, $F(7/2^-)_j=3$ and $G(9/2^+)_j=4$ states. The group of $\Xi_{bc}'(UP)$ with the
| $l_p$ $l_\lambda L s j$ | $nL(J^P)$ | $\langle r_x^2 \rangle^{1/2}$ | $\langle r_y^2 \rangle^{1/2}$ | mass | $l_p$ $l_\lambda L s j$ | $nL(J^P)$ | $\langle r_x^2 \rangle^{1/2}$ | $\langle r_y^2 \rangle^{1/2}$ | mass |
|----------------|----------------|----------------|----------------|-------|----------------|----------------|----------------|----------------|-------|
| 0 0 0 1 1 | 1$S(\frac{1}{2}^+)$ | 0.379 | 0.469 | 6952 | 1$D(\frac{7}{2}^+)$ | 0.749 | 0.566 | 7470 |
| | 2$S(\frac{1}{2}^+)$ | 0.691 | 0.555 | 7346 | | 2$D(\frac{7}{2}^+)$ | 0.952 | 0.602 | 7747 |
| | 3$S(\frac{1}{2}^+)$ | 0.429 | 0.831 | 7455 | 2$D(\frac{7}{2}^+)$ | 0.792 | 0.925 | 7922 |
| | 4$S(\frac{1}{2}^+)$ | 1.072 | 0.610 | 7673 | | 4$D(\frac{7}{2}^+)$ | 1.521 | 0.685 | 8011 |
| 0 0 0 1 1 | 1$S(\frac{3}{2}^+)$ | 0.382 | 0.488 | 6980 | | 1$F(\frac{3}{2}^-)$ | 0.888 | 0.549 | 7598 |
| | 2$S(\frac{3}{2}^+)$ | 0.694 | 0.574 | 7368 | | 2$F(\frac{5}{2}^-)$ | 1.022 | 0.570 | 7862 |
| | 3$S(\frac{3}{2}^+)$ | 0.432 | 0.841 | 7470 | | 3$F(\frac{5}{2}^-)$ | 0.941 | 0.922 | 8051 |
| | 4$S(\frac{3}{2}^+)$ | 1.080 | 0.628 | 7692 | | 4$F(\frac{7}{2}^-)$ | 1.678 | 0.663 | 8199 |
| 1 0 1 0 1 | 1$P(\frac{1}{2}^-)$ | 0.566 | 0.504 | 7223 | | 1$F(\frac{3}{2}^-)$ | 0.899 | 0.592 | 7642 |
| | 2$P(\frac{1}{2}^-)$ | 0.838 | 0.561 | 7538 | | 2$F(\frac{3}{2}^-)$ | 1.035 | 0.611 | 7907 |
| | 3$P(\frac{1}{2}^-)$ | 0.609 | 0.872 | 7705 | | 3$F(\frac{3}{2}^-)$ | 0.946 | 0.952 | 8082 |
| | 4$P(\frac{1}{2}^-)$ | 1.321 | 0.637 | 7840 | | 4$F(\frac{3}{2}^-)$ | 1.668 | 0.700 | 8142 |
| 1 0 1 0 1 | 1$P(\frac{3}{2}^-)$ | 0.571 | 0.523 | 7247 | | 1$G(\frac{3}{2}^+)$ | 1.029 | 0.572 | 7759 |
| | 2$P(\frac{3}{2}^-)$ | 0.841 | 0.579 | 7559 | | 2$G(\frac{3}{2}^+)$ | 1.097 | 0.579 | 8024 |
| | 3$P(\frac{3}{2}^-)$ | 0.610 | 0.884 | 7719 | | 3$G(\frac{3}{2}^+)$ | 1.078 | 0.946 | 8199 |
| | 4$P(\frac{3}{2}^-)$ | 1.326 | 0.654 | 7856 | | 4$G(\frac{3}{2}^+)$ | 1.806 | 0.675 | 8236 |
| 2 0 2 1 1 | 1$D(\frac{1}{2}^-)$ | 0.728 | 0.534 | 7431 | | 1$G(\frac{5}{2}^+)$ | 1.039 | 0.614 | 7800 |
| | 2$D(\frac{1}{2}^-)$ | 0.940 | 0.572 | 7708 | | 2$G(\frac{5}{2}^+)$ | 1.121 | 0.621 | 8067 |
| | 3$D(\frac{1}{2}^-)$ | 0.777 | 0.902 | 7896 | | 3$G(\frac{5}{2}^+)$ | 1.085 | 0.975 | 8228 |
| | 4$D(\frac{1}{2}^-)$ | 1.522 | 0.660 | 7986 | | 4$G(\frac{5}{2}^+)$ | 1.786 | 0.710 | 8267 |
| 2 0 2 1 1 | 1$D(\frac{3}{2}^-)$ | 0.733 | 0.552 | 7452 | | 1$G(\frac{7}{2}^+)$ | 1.030 | 0.566 | 7751 |
| | 2$D(\frac{3}{2}^-)$ | 0.943 | 0.590 | 7728 | | 2$G(\frac{7}{2}^+)$ | 1.097 | 0.572 | 8017 |
| | 3$D(\frac{3}{2}^-)$ | 0.778 | 0.915 | 7909 | | 3$G(\frac{7}{2}^+)$ | 1.080 | 0.942 | 8193 |
| | 4$D(\frac{3}{2}^-)$ | 1.521 | 0.676 | 8001 | | 4$G(\frac{7}{2}^+)$ | 1.806 | 0.668 | 8229 |
| 2 0 2 1 2 | 1$D(\frac{1}{2}^-)$ | 0.731 | 0.528 | 7425 | | 1$G(\frac{9}{2}^+)$ | 1.043 | 0.620 | 7803 |
| | 2$D(\frac{1}{2}^-)$ | 0.941 | 0.566 | 7703 | | 2$G(\frac{9}{2}^+)$ | 1.129 | 0.628 | 8073 |
| | 3$D(\frac{1}{2}^-)$ | 0.781 | 0.899 | 7892 | | 3$G(\frac{9}{2}^+)$ | 1.088 | 0.980 | 8230 |
| | 4$D(\frac{1}{2}^-)$ | 1.522 | 0.654 | 7981 | | 4$G(\frac{9}{2}^+)$ | 1.780 | 0.714 | 8270 |
| 2 0 2 1 2 | 1$D(\frac{3}{2}^-)$ | 0.740 | 0.559 | 7460 | | 1$G(\frac{11}{2}^+)$ | 1.032 | 0.560 | 7743 |
| | 2$D(\frac{3}{2}^-)$ | 0.946 | 0.596 | 7736 | | 2$G(\frac{11}{2}^+)$ | 1.100 | 0.566 | 8011 |
| | 3$D(\frac{3}{2}^-)$ | 0.784 | 0.920 | 7915 | | 3$G(\frac{11}{2}^+)$ | 1.083 | 0.939 | 8186 |
| | 4$D(\frac{3}{2}^-)$ | 1.521 | 0.681 | 8006 | | 4$G(\frac{11}{2}^+)$ | 1.804 | 0.662 | 8223 |
| 2 0 2 1 3 | 1$D(\frac{5}{2}^-)$ | 0.737 | 0.523 | 7420 | | 1$G(\frac{13}{2}^+)$ | 1.047 | 0.626 | 7807 |
| | 2$D(\frac{5}{2}^-)$ | 0.943 | 0.561 | 7700 | | 2$G(\frac{13}{2}^+)$ | 1.141 | 0.634 | 8079 |
| | 3$D(\frac{5}{2}^-)$ | 0.788 | 0.896 | 7890 | | 3$G(\frac{13}{2}^+)$ | 1.093 | 0.984 | 8232 |
| | 4$D(\frac{5}{2}^-)$ | 1.524 | 0.648 | 7976 | | 4$G(\frac{13}{2}^+)$ | 1.769 | 0.717 | 8273 |
unnatural parity of $(-1)^{J+1/2}$ is composed of $P(\frac{1}{2}^-)_{J=1}$, $D(\frac{3}{2}^+)_{J=2}$, $F(\frac{5}{2}^-)_{J=3}$ and $G(\frac{5}{2}^+)_{J=4}$ states. For $\Xi_{bc}$ family, the remaining states in Table I can also be put into these lines, because their mass values are very near those states with the same $L(J^P)$. The situation is similar for $\Xi_{bc}$, $\Omega'_{bc}$ and $\Omega_{bc}$ families. From Fig.7, one can see that most of the data points fall on the trajectory lines.
FIG. 6: (Color online) Same as Fig. 3, but for the $\Omega_{bc}$ family.

By comparing the slope values in Table V with those of the double-charm and -bottom baryons [69, 70], we find a phenomenon: The slopes of the two lines with $n = 1$ and $n = 3$ are roughly the same. However, the slope of the line with $n = 2$ is different from those of the other two lines, and this difference becomes bigger with increasing the total mass of the heavy quark pair.

FIG. 7: (Color online) $(J, M^2)$ Regge trajectories for the $\Xi_{bc}^\prime$ ($\Xi_{bc}$) and $\Omega_{bc}^\prime$ ($\Omega_{bc}$) families and $M^2$ is in GeV$^2$. $NP$ denotes the natural parity, and $UP$ the unnatural parity.
3.4 Shell structure of the mass spectra

The spectral structures of the $\Xi_{bc}'$ ($\Xi_{bc}$) and $\Omega_{bc}'$ ($\Omega_{bc}$) baryons with $L \leq 2$ are presented in Figs. 8 and 9, respectively. As shown in Fig. 8, there are 3 members of the $1S$-wave states for the $\Xi_{bc}$ and $\Xi_{bc}'$ families. The calculated masses are $6952$ MeV for $1S(\frac{1}{2}^+)_{j=1}$ state, $6955$ MeV for $1S(\frac{1}{2}^+)_{j=0}$ state, and $6980$ MeV for $1S(\frac{3}{2}^+)_{j=1}$ state, respectively. One can see that the masses of these three states are relatively close, and the $1S(\frac{1}{2}^+)_{j=0}$ state of $\Xi_{bc}$ lies between the $1S(\frac{3}{2}^+)_{j=1}$ doublet states of $\Xi_{bc}'$. This suggests that there is a certain difficulty in the identification of the three $1S$ states in experiment. On the other hand, good news is that there lies a big gap (about $240$ MeV) between the $1S$ and $1P$ sub-shells as shown in Fig. 8. This implies the experimental measurement of the $1S$ states for the $\Xi_{bc}'$ and $\Xi_{bc}$ baryons could be done cleanly. The same is true for $\Omega_{bc}'$ and $\Omega_{bc}$ baryons as shown in Fig. 9, where the calculated masses are $7053$ MeV for $1S(\frac{1}{2}^+)_{j=1}$ state, $7055$ MeV for $1S(\frac{1}{2}^+)_{j=0}$ state and $7079$ MeV for $1S(\frac{3}{2}^+)_{j=1}$ state, respectively.

At last, the calculated masses of the $1S$ states in this work are compared with those given by some other theoretical methods as shown in Table VI. From Table VI, one can see that the masses by the other methods are mainly distributed in the range of $6800\sim7100$ MeV for $\Xi_{bc}'$ and $\Xi_{bc}$ families, and $6900\sim7200$ MeV for $\Omega_{bc}'$ and $\Omega_{bc}$ families. With the above masses, the average value for each $1S$ state is calculated. Table VI shows that our calculated masses are close to the average values in general.

![FIG. 8: (Color online)Shell structure of the $\Xi_{bc}'$ ($\Xi_{bc}$) family.](image-url)
IV. Conclusions

In this work, by using the relativistic quark model and the ISG method, we investigate the bottom-charm baryon spectra systematically. In the ρ-mode, we obtain the mass spectra of the Ξ_{bc}' (Ξ_{bc}) and Ω_{bc}' (Ω_{bc}) families. The related r.m.s. radii and quark radial probability density distributions are investigated as well, from which we learn more about the structure of bottom-charm baryons.

Based on the obtained mass spectra, we construct successfully the Regge trajectories in the (J, M²) plane. We find the slopes of the lines with n = 2 differ from those of the other lines with n = 1 and n = 3, and the difference changes regularly with increasing total mass of the two heavy quarks.

At last, the mass spectral structures of the Ξ_{bc}' (Ξ_{bc}) and Ω_{bc}' (Ω_{bc}) families are presented. We analyze the features of the mass spectral structures, and discuss the difficulty and opportunity of the experimental measurements for the 1S states in Ξ_{bc}' (Ξ_{bc}) and Ω_{bc}' (Ω_{bc}) families. We also compare our calculated masses of the 1S states with those given by some other theoretical methods. It turns out that our results are close to their average values in general.

FIG. 9: (Color online) Same as Fig.8, but for the Ω_{bc}' (Ω_{bc}) family.
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| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 0 0 0 0 |       |     |     | 1S(\(\frac{1}{2}^+\)) | 0.370 | 0.479 | 6955 |
|       |       |     |     | 2S(\(\frac{3}{2}^+\)) | 0.683 | 0.569 | 7351 |
|       |       |     |     | 3S(\(\frac{1}{2}^+\)) | 0.422 | 0.834 | 7451 |
|       |       |     |     | 4S(\(\frac{1}{2}^+\)) | 1.060 | 0.621 | 7677 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 1 0 1 0 |       |     |     | 1P(\(\frac{1}{2}^+\)) | 0.559 | 0.514 | 7251 |
|       |       |     |     | 2P(\(\frac{1}{2}^+\)) | 0.835 | 0.572 | 7545 |
|       |       |     |     | 3P(\(\frac{1}{2}^+\)) | 0.599 | 0.878 | 7707 |
|       |       |     |     | 4P(\(\frac{1}{2}^+\)) | 1.315 | 0.648 | 7847 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 1 0 1 1 |       |     |     | 1P(\(\frac{3}{2}^+\)) | 0.562 | 0.503 | 7220 |
|       |       |     |     | 2P(\(\frac{3}{2}^+\)) | 0.836 | 0.561 | 7535 |
|       |       |     |     | 3P(\(\frac{3}{2}^+\)) | 0.604 | 0.871 | 7702 |
|       |       |     |     | 4P(\(\frac{3}{2}^+\)) | 1.317 | 0.637 | 7838 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 1 0 1 1 |       |     |     | 1P(\(\frac{5}{2}^+\)) | 0.566 | 0.522 | 7243 |
|       |       |     |     | 2P(\(\frac{3}{2}^+\)) | 0.839 | 0.578 | 7556 |
|       |       |     |     | 3P(\(\frac{3}{2}^+\)) | 0.605 | 0.883 | 7716 |
|       |       |     |     | 4P(\(\frac{3}{2}^+\)) | 1.322 | 0.654 | 7855 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 1 0 1 2 |       |     |     | 1P(\(\frac{7}{2}^+\)) | 0.572 | 0.499 | 7221 |
|       |       |     |     | 2P(\(\frac{3}{2}^+\)) | 0.841 | 0.556 | 7536 |
|       |       |     |     | 3P(\(\frac{5}{2}^+\)) | 0.616 | 0.869 | 7706 |
|       |       |     |     | 4P(\(\frac{5}{2}^+\)) | 1.327 | 0.632 | 7837 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 1 0 1 2 |       |     |     | 1P(\(\frac{7}{2}^+\)) | 0.580 | 0.530 | 7261 |
|       |       |     |     | 2P(\(\frac{7}{2}^+\)) | 0.846 | 0.585 | 7571 |
|       |       |     |     | 3P(\(\frac{7}{2}^+\)) | 0.618 | 0.890 | 7730 |
|       |       |     |     | 4P(\(\frac{7}{2}^+\)) | 1.334 | 0.660 | 7865 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 2 0 2 0 |       |     |     | 1D(\(\frac{1}{2}^+\)) | 0.733 | 0.529 | 7426 |
|       |       |     |     | 2D(\(\frac{1}{2}^+\)) | 0.942 | 0.567 | 7704 |
|       |       |     |     | 3D(\(\frac{1}{2}^+\)) | 0.783 | 0.899 | 7893 |
|       |       |     |     | 4D(\(\frac{1}{2}^+\)) | 1.523 | 0.654 | 7981 |

| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass (MeV) |
|-------|-------|-----|-----|-----|--------|-------------|-------------|-------------|
| 2 0 2 0 |       |     |     | 1P(\(\frac{5}{2}^+\)) | 0.742 | 0.559 | 7461 |
|       |       |     |     | 2P(\(\frac{5}{2}^+\)) | 0.948 | 0.596 | 7737 |
|       |       |     |     | 3P(\(\frac{5}{2}^+\)) | 0.786 | 0.920 | 7916 |
|       |       |     |     | 4P(\(\frac{5}{2}^+\)) | 1.521 | 0.681 | 8006 |

TABLE II: The root mean square radii (fm) and mass spectra (MeV) of the \(\Xi_{bc}\) family.
### TABLE III: The root mean square radii (fm) and mass spectra (MeV) of the $\Omega^+_c$ family.

| $l_\nu$ $l_\lambda$ $L$ $s$ $j$ | $nL(J^P)$ | $\langle r^2_{\text{rms}} \rangle^{1/2}$ | $\langle r^2_{\text{bc}} \rangle^{1/2}$ | Mass (MeV) | $l_\nu$ $l_\lambda$ $L$ $s$ $j$ | $nL(J^P)$ | $\langle r^2_{\text{rms}} \rangle^{1/2}$ | $\langle r^2_{\text{bc}} \rangle^{1/2}$ | Mass (MeV) |
|---|---|---|---|---|---|---|---|---|---|
| 0 0 0 1 1 | $1S(\frac{1}{2}^+)$ | 0.371 | 0.429 | 7053 | $\ldots$ | $1S(\frac{1}{2}^+)$ | 0.738 | 0.522 | 7578 |
| | $2S(\frac{1}{2}^+)$ | 0.675 | 0.526 | 7453 | $\ldots$ | $2D(\frac{3}{2}^+)$ | 0.942 | 0.558 | 7856 |
| | $3S(\frac{1}{2}^+)$ | 0.438 | 0.771 | 7554 | $\ldots$ | $3D(\frac{3}{2}^+)$ | 0.787 | 0.872 | 8022 |
| | $4S(\frac{3}{2}^+)$ | 1.039 | 0.581 | 7786 | $\ldots$ | $4D(\frac{3}{2}^+)$ | 1.521 | 0.639 | 8131 |
| 0 0 0 1 1 | $1S(\frac{1}{2}^+)$ | 0.375 | 0.446 | 7079 | $\ldots$ | $1F(\frac{1}{2}^−)$ | 0.877 | 0.510 | 7716 |
| | $2S(\frac{1}{2}^+)$ | 0.677 | 0.542 | 7474 | $\ldots$ | $2F(\frac{1}{2}^−)$ | 1.011 | 0.532 | 7979 |
| | $3S(\frac{1}{2}^+)$ | 0.441 | 0.782 | 7568 | $\ldots$ | $3F(\frac{1}{2}^−)$ | 0.933 | 0.871 | 8155 |
| | $4S(\frac{3}{2}^+)$ | 1.048 | 0.596 | 7803 | $\ldots$ | $4F(\frac{1}{2}^−)$ | 1.685 | 0.621 | 8286 |
| 1 0 1 0 1 | $1P(\frac{1}{2}^−)$ | 0.557 | 0.465 | 7331 | $\ldots$ | $1F(\frac{1}{2}^−)$ | 0.889 | 0.546 | 7754 |
| | $2P(\frac{1}{2}^−)$ | 0.829 | 0.524 | 7649 | $\ldots$ | $2F(\frac{1}{2}^−)$ | 1.021 | 0.567 | 8018 |
| | $3P(\frac{1}{2}^−)$ | 0.607 | 0.818 | 7806 | $\ldots$ | $3F(\frac{1}{2}^−)$ | 0.938 | 0.900 | 8183 |
| | $4P(\frac{1}{2}^−)$ | 1.301 | 0.599 | 7958 | $\ldots$ | $4F(\frac{1}{2}^−)$ | 1.678 | 0.652 | 8263 |
| 1 0 1 0 1 | $1P(\frac{3}{2}^−)$ | 0.561 | 0.481 | 7353 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.019 | 0.532 | 7879 |
| | $2P(\frac{3}{2}^−)$ | 0.832 | 0.539 | 7666 | $\ldots$ | $2G(\frac{1}{2}^−)$ | 1.078 | 0.539 | 8141 |
| | $3P(\frac{3}{2}^−)$ | 0.608 | 0.830 | 7819 | $\ldots$ | $3G(\frac{1}{2}^−)$ | 1.069 | 0.896 | 8303 |
| | $4P(\frac{3}{2}^−)$ | 1.308 | 0.613 | 7973 | $\ldots$ | $4G(\frac{1}{2}^−)$ | 1.822 | 0.631 | 8364 |
| 2 0 2 1 1 | $1D(\frac{1}{2}^−)$ | 0.718 | 0.494 | 7543 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.029 | 0.568 | 7914 |
| | $2D(\frac{1}{2}^−)$ | 0.932 | 0.534 | 7822 | $\ldots$ | $2G(\frac{1}{2}^−)$ | 1.095 | 0.575 | 8178 |
| | $3D(\frac{1}{2}^−)$ | 0.771 | 0.849 | 7997 | $\ldots$ | $3G(\frac{1}{2}^−)$ | 1.074 | 0.925 | 8330 |
| | $4D(\frac{1}{2}^−)$ | 1.516 | 0.618 | 8109 | $\ldots$ | $4G(\frac{1}{2}^−)$ | 1.808 | 0.661 | 8390 |
| 2 0 2 1 1 | $1D(\frac{3}{2}^−)$ | 0.723 | 0.510 | 7562 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.020 | 0.526 | 7872 |
| | $2D(\frac{3}{2}^−)$ | 0.935 | 0.548 | 7839 | $\ldots$ | $2G(\frac{1}{2}^−)$ | 1.079 | 0.534 | 8135 |
| | $3D(\frac{3}{2}^−)$ | 0.773 | 0.862 | 8010 | $\ldots$ | $3G(\frac{1}{2}^−)$ | 1.070 | 0.892 | 8298 |
| | $4D(\frac{3}{2}^−)$ | 1.518 | 0.632 | 8122 | $\ldots$ | $4G(\frac{1}{2}^−)$ | 1.822 | 0.626 | 8359 |
| 2 0 2 1 2 | $1D(\frac{4}{2}^−)$ | 0.720 | 0.489 | 7538 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.033 | 0.572 | 7917 |
| | $2D(\frac{4}{2}^−)$ | 0.933 | 0.529 | 7818 | $\ldots$ | $2G(\frac{1}{2}^−)$ | 1.102 | 0.580 | 8183 |
| | $3D(\frac{4}{2}^−)$ | 0.775 | 0.846 | 7994 | $\ldots$ | $3G(\frac{1}{2}^−)$ | 1.078 | 0.929 | 8332 |
| | $4D(\frac{4}{2}^−)$ | 1.518 | 0.613 | 8105 | $\ldots$ | $4G(\frac{1}{2}^−)$ | 1.803 | 0.665 | 8392 |
| 2 0 2 1 2 | $1D(\frac{5}{2}^−)$ | 0.729 | 0.516 | 7569 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.038 | 0.577 | 7920 |
| | $2D(\frac{5}{2}^−)$ | 0.937 | 0.553 | 7847 | $\ldots$ | $2G(\frac{1}{2}^−)$ | 1.111 | 0.586 | 8189 |
| | $3D(\frac{5}{2}^−)$ | 0.778 | 0.866 | 8015 | $\ldots$ | $3G(\frac{1}{2}^−)$ | 1.082 | 0.933 | 8333 |
| | $4D(\frac{5}{2}^−)$ | 1.519 | 0.636 | 8126 | $\ldots$ | $4G(\frac{1}{2}^−)$ | 1.795 | 0.667 | 8394 |
| 2 0 2 1 3 | $1D(\frac{5}{2}^−)$ | 0.726 | 0.485 | 7535 | $\ldots$ | $1G(\frac{1}{2}^−)$ | 1.038 | 0.577 | 7920 |
| \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass | \( l_\rho \) | \( l_\lambda \) | \( L \) | \( s \) | \( j \) | \( nL(J^P) \) | \( \langle r_{\rho}^2 \rangle^{1/2} \) | \( \langle r_{\lambda}^2 \rangle^{1/2} \) | Mass |
|-----|-----|-----|-----|-----|------|-------|-------|------|-----|-----|-----|-----|-----|------|-------|-------|------|
| 0 0 0 0 | 1S(\( \frac{1}{2}^+ \)) | 0.363 | 0.438 | 7055 | 1F(\( \frac{3}{2}^- \)) | 0.874 | 0.515 | 7722 |
| 0 0 0 0 | 2S(\( \frac{1}{2}^- \)) | 0.666 | 0.538 | 7457 | 2F(\( \frac{5}{2}^- \)) | 1.010 | 0.537 | 7984 |
| 0 0 0 0 | 3S(\( \frac{1}{2}^+ \)) | 0.433 | 0.774 | 7550 | 3F(\( \frac{1}{2}^- \)) | 0.929 | 0.875 | 8160 |
| 0 0 0 0 | 4S(\( \frac{1}{2}^- \)) | 1.026 | 0.591 | 7788 | 4F(\( \frac{3}{2}^- \)) | 1.686 | 0.627 | 8241 |
| 1 0 1 0 | 1P(\( \frac{1}{2}^- \)) | 0.549 | 0.474 | 7337 | 1F(\( \frac{3}{2}^- \)) | 0.882 | 0.541 | 7749 |
| 1 0 1 0 | 2P(\( \frac{1}{2}^- \)) | 0.826 | 0.534 | 7655 | 2F(\( \frac{5}{2}^- \)) | 1.016 | 0.562 | 8011 |
| 1 0 1 0 | 3P(\( \frac{1}{2}^- \)) | 0.597 | 0.824 | 7807 | 3F(\( \frac{1}{2}^- \)) | 0.933 | 0.895 | 8180 |
| 1 0 1 0 | 4P(\( \frac{1}{2}^- \)) | 1.293 | 0.608 | 7964 | 4F(\( \frac{3}{2}^- \)) | 1.681 | 0.648 | 8261 |
| 1 0 1 1 | 1P(\( \frac{1}{2}^- \)) | 0.552 | 0.464 | 7327 | 1F(\( \frac{3}{2}^- \)) | 0.876 | 0.510 | 7715 |
| 1 0 1 1 | 2P(\( \frac{1}{2}^- \)) | 0.827 | 0.524 | 7647 | 2F(\( \frac{5}{2}^- \)) | 1.010 | 0.532 | 7978 |
| 1 0 1 1 | 3P(\( \frac{1}{2}^- \)) | 0.602 | 0.817 | 7803 | 3F(\( \frac{1}{2}^- \)) | 0.932 | 0.871 | 8155 |
| 1 0 1 1 | 4P(\( \frac{1}{2}^- \)) | 1.296 | 0.599 | 7957 | 4F(\( \frac{3}{2}^- \)) | 1.686 | 0.621 | 8236 |
| 1 0 1 1 | 1P(\( \frac{3}{2}^- \)) | 0.557 | 0.480 | 7349 | 1F(\( \frac{3}{2}^- \)) | 0.887 | 0.546 | 7754 |
| 1 0 1 1 | 2P(\( \frac{3}{2}^- \)) | 0.830 | 0.539 | 7665 | 2F(\( \frac{5}{2}^- \)) | 1.020 | 0.567 | 8017 |
| 1 0 1 1 | 3P(\( \frac{3}{2}^- \)) | 0.603 | 0.829 | 7816 | 3F(\( \frac{3}{2}^- \)) | 0.937 | 0.900 | 8183 |
| 1 0 1 1 | 4P(\( \frac{3}{2}^- \)) | 1.303 | 0.613 | 7971 | 4F(\( \frac{3}{2}^- \)) | 1.678 | 0.652 | 8263 |
| 1 0 1 2 | 1P(\( \frac{5}{2}^- \)) | 0.563 | 0.460 | 7330 | 1F(\( \frac{3}{2}^- \)) | 0.880 | 0.505 | 7710 |
| 1 0 1 2 | 2P(\( \frac{5}{2}^- \)) | 0.832 | 0.519 | 7648 | 2F(\( \frac{5}{2}^- \)) | 1.012 | 0.526 | 7974 |
| 1 0 1 2 | 3P(\( \frac{5}{2}^- \)) | 0.614 | 0.816 | 7807 | 3F(\( \frac{5}{2}^- \)) | 0.936 | 0.868 | 8151 |
| 1 0 1 2 | 4P(\( \frac{5}{2}^- \)) | 1.309 | 0.595 | 7957 | 4F(\( \frac{5}{2}^- \)) | 1.685 | 0.616 | 8231 |
| 1 0 1 2 | 1P(\( \frac{7}{2}^- \)) | 0.571 | 0.488 | 7366 | 1F(\( \frac{3}{2}^- \)) | 0.894 | 0.552 | 7759 |
| 1 0 1 2 | 2P(\( \frac{7}{2}^- \)) | 0.837 | 0.544 | 7679 | 2F(\( \frac{5}{2}^- \)) | 1.025 | 0.572 | 8024 |
| 1 0 1 2 | 3P(\( \frac{7}{2}^- \)) | 0.616 | 0.835 | 7829 | 3F(\( \frac{7}{2}^- \)) | 0.943 | 0.904 | 8186 |
| 1 0 1 2 | 4P(\( \frac{7}{2}^- \)) | 1.318 | 0.618 | 7981 | 4F(\( \frac{5}{2}^- \)) | 1.675 | 0.655 | 8266 |
| 2 0 2 0 | 1D(\( \frac{1}{2}^+ \)) | 0.723 | 0.489 | 7539 | 1G(\( \frac{3}{2}^+ \)) | 1.021 | 0.526 | 7872 |
| 2 0 2 0 | 2D(\( \frac{3}{2}^+ \)) | 0.934 | 0.529 | 7819 | 2G(\( \frac{5}{2}^+ \)) | 1.079 | 0.534 | 8135 |
| 2 0 2 0 | 3D(\( \frac{5}{2}^+ \)) | 0.777 | 0.847 | 7995 | 3G(\( \frac{7}{2}^+ \)) | 1.071 | 0.892 | 8297 |
| 2 0 2 0 | 4D(\( \frac{7}{2}^+ \)) | 1.519 | 0.613 | 8105 | 4G(\( \frac{9}{2}^+ \)) | 1.821 | 0.626 | 8358 |
| 2 0 2 0 | 1D(\( \frac{3}{2}^+ \)) | 0.731 | 0.516 | 7570 | 1G(\( \frac{5}{2}^+ \)) | 1.034 | 0.573 | 7917 |
| 2 0 2 0 | 2D(\( \frac{5}{2}^+ \)) | 0.938 | 0.554 | 7848 | 2G(\( \frac{7}{2}^+ \)) | 1.103 | 0.581 | 8183 |
| 2 0 2 0 | 3D(\( \frac{7}{2}^+ \)) | 0.781 | 0.867 | 8016 | 3G(\( \frac{9}{2}^+ \)) | 1.078 | 0.929 | 8331 |
| 2 0 2 0 | 4D(\( \frac{9}{2}^+ \)) | 1.519 | 0.636 | 8126 | 4G(\( \frac{11}{2}^+ \)) | 1.802 | 0.664 | 8392 |
TABLE V: Fitted values of the slope ($\alpha$) and intercept ($\beta$) of the Regge trajectories for the $\Xi'_{bc}$ ($\Xi_{bc}$) and $\Omega'_{bc}$ ($\Omega_{bc}$) families.

| Trajectory | $\alpha$(GeV$^2$) | $\beta$(GeV$^2$) | $\alpha$(GeV$^2$) | $\beta$(GeV$^2$) |
|------------|-------------------|-------------------|-------------------|-------------------|
|            | $\Xi'_{bc}$        | $\Xi_{bc}$        | $\Omega'_{bc}$    | $\Omega_{bc}$    |
| $n = 1(NP)$| $3.099 \pm 0.191$ | $47.409 \pm 0.548$ | $3.193 \pm 0.198$ | $48.800 \pm 0.568$ |
| $n = 2(NP)$| $2.780 \pm 0.059$ | $52.778 \pm 0.170$ | $2.832 \pm 0.065$ | $54.354 \pm 0.187$ |
| $n = 3(NP)$| $3.005 \pm 0.179$ | $54.660 \pm 0.515$ | $3.054 \pm 0.181$ | $56.129 \pm 0.520$ |
| $n = 1(UP)$| $2.632 \pm 0.097$ | $51.014 \pm 0.222$ | $2.739 \pm 0.102$ | $52.540 \pm 0.234$ |
| $n = 2(UP)$| $2.483 \pm 0.009$ | $55.595 \pm 0.020$ | $2.556 \pm 0.016$ | $57.256 \pm 0.037$ |
| $n = 3(UP)$| $2.581 \pm 0.097$ | $58.237 \pm 0.222$ | $2.637 \pm 0.098$ | $59.776 \pm 0.225$ |
|            | $\Xi_{bc}$        | $\Xi_{bc}$        | $\Omega_{bc}$    | $\Omega_{bc}$    |
| $n = 1(NP)$| $3.097 \pm 0.183$ | $47.415 \pm 0.526$ | $3.193 \pm 0.191$ | $48.796 \pm 0.550$ |
| $n = 2(NP)$| $2.770 \pm 0.048$ | $52.812 \pm 0.137$ | $2.823 \pm 0.055$ | $54.379 \pm 0.159$ |
| $n = 3(NP)$| $3.021 \pm 0.184$ | $54.600 \pm 0.527$ | $3.068 \pm 0.187$ | $56.074 \pm 0.537$ |
| $n = 1(UP)$| $2.643 \pm 0.107$ | $50.984 \pm 0.244$ | $2.754 \pm 0.113$ | $52.496 \pm 0.258$ |
| $n = 2(UP)$| $2.495 \pm 0.020$ | $55.563 \pm 0.046$ | $2.562 \pm 0.024$ | $57.237 \pm 0.054$ |
| $n = 3(UP)$| $2.588 \pm 0.110$ | $58.210 \pm 0.252$ | $2.644 \pm 0.111$ | $59.749 \pm 0.255$ |
TABLE VI: The predicted masses (in MeV) of 1S states in this work and some other theoretical methods.
The average values (\(\bar{m}\)) are calculated based on the mass values referenced here. The difference values in the
brackets are calculated from \(\bar{m}\).

| State | 1S(\(\frac{1}{2}^+\)) (Ξ_{bc}) | 1S(\(\frac{1}{2}^+\)) (Ξ'_{bc}) | 1S(\(\frac{1}{2}^+\)) (Ω_{bc}) | 1S(\(\frac{1}{2}^+\)) (Ω'_{bc}) | 1S(\(\frac{1}{2}^+\)) (Ω'_{bc'}) |
|-------|-------------------------------|-------------------------------|---------------------------|-------------------------------|---------------------------|
| 30    | 6948(35)                      | 6973(-1)                      | 6922(-9)                  | 7047(27)                      | 7066(-8)                  | 7011(-15)                |
| 33    | 6880(-33)                     | 6980(6)                       | 6970(39)                  | 6960(-60)                     | 7060(-14)                 | 7050(24)                 |
| 35    | 6934(21)                      | -                             | -                         | 7033(13)                      | -                         | -                        |
| 37    | 6805(-108)                    | 6835(-139)                    | 6787(-144)                | 6906(-114)                    | 6930(-144)                | 6893(-133)               |
| 38    | 6953(40)                      | 7044(70)                      | 7015(84)                  | 7064(44)                      | 7142(68)                  | 7116(90)                 |
| 39    | -                             | -                             | 6930(-1)                  | -                             | -                         | 7017(-9)                 |
| 41    | 6890(-23)                     | 6930(-44)                     | 6930(-1)                  | 7010(-10)                     | 7040(-34)                 | 7040(14)                 |
| 43    | 6958(45)                      | 6991(17)                      | -                         | 7137(117)                     | 7170(96)                  | -                        |
| 46    | 6915(2)                       | 7003(29)                      | -                         | -                             | -                         | -                        |
| 47    | 6820(-93)                     | 6900(-74)                     | 6850(-81)                 | 6930(-90)                     | 7000(-74)                 | 6970(-56)                |
| 48    | 6800(-113)                    | 6850(-124)                    | 6870(-61)                 | 6980(-40)                     | 7020(-54)                 | 7050(24)                 |
| 50    | 6950(37)                      | 7020(46)                      | 7000(69)                  | 7050(30)                      | 7110(36)                  | 7090(64)                 |
| 55    | 7014(101)                     | 7064(90)                      | 7037(106)                 | -                             | -                         | -                        |
| 59    | 6988(75)                      | 7083(109)                     | -                         | 7103(83)                      | 7200(126)                 | -                        |
| 60    | 6914(1)                       | -                             | 6933(2)                   | -                             | -                         | -                        |
| 63    | 6920(7)                       | 6986(12)                      | -                         | -                             | -                         | -                        |

\(\bar{m}\) 6913 6974 6931 7020 7074 7026

Our 6952(39) 6980(6) 6955(24) 7053(33) 7079(5) 7055(29)