Predicting charged lepton flavor violation from gauge symmetry

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The simplest realization of the inverse seesaw mechanism in a $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge theory offers striking flavor correlations between rare charged lepton flavor violating decays and the measured neutrino oscillations parameters. The predictions follow from the gauge structure itself without the need for any flavor symmetry. Such tight complementarity between charged lepton flavor violation and oscillations renders the scenario strictly testable.

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Preliminaries

Beyond the discovery of the Higgs boson [1, 2] no signs of genuine new physics have shown up so far at high energies. However, the existence of new physics has been established with the discovery of neutrino oscillations [3, 4], implying the existence of lepton flavor violation and nonzero neutrino masses. Unraveling the origin of the latter constitutes one of the main challenges of particle physics. While the prevailing view is that neutrino masses arise from physics associated with unification, the strong CP problem by including in an elegant way a global $CP$ symmetry, extended with a global $U(1)$ and accompanied by three generations of neutral fermions, is without the need for any flavor symmetry. Such tight complementarity between charged lepton flavor violation and oscillations renders the scenario strictly testable with the coming generation of LFV searches.

The Model

We consider a variant of the model introduced in [9] in which neutrinos get masses via the inverse seesaw mechanism instead of quantum corrections. The model is based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry, extended with a global $U(1)_C$ to consistently define lepton number. We also invoke an auxiliary parity symmetry in order to ensure a realistic quark mass spectrum. The model contains three generations of lepton triplets ($\psi_L$), two generations of quark triplets ($Q_{1,2}^L$), one generation of quark anti-triplet ($Q_{1}^R$), along, of course, with their iso-singlet right-handed partners, and accompanied by three generations of neutral fermion singlets ($S$). The gauge symmetry breaking is implemented through three scalar anti-triplets ($\phi_{1,2,3}$), the particle content of the model is summarized in table (I). The fundamental fermions interact through the exchange of 17 gauge bosons: the 8 gluons of $SU(3)_C$, the 8 “weak” $W_i$ bosons associated to $SU(3)_L$ (4 of which form 2 electrically charged bosons, and the rest are neutral), and the $B$ boson associated to $U(1)_X$.

The lepton representations in table (I) can be decomposed as:

$$\psi_L = \left( \begin{array}{c} \ell^- \\ \nu \\ N^c_L \end{array} \right)_{L}^{e,\mu,\tau},$$

where we identify $N^c_L \equiv (\nu_R)^c$ [8].

¹ For other inverse seesaw constructions in 3-3-1 scenarios see [13].
TABLE I: Particle content of the model. Here \( U_R \equiv \{ u_R, c_R, t_R \} \) and \( D_R \equiv \{ d_R, s_R, b_R \} \).

| \( SU(3)_C \) | \( SU(3)_L \) | \( U(1)_X \) | \( U(1)_L \) |
|---|---|---|---|
| \( \phi \) | \( 3^* \) | \( 3 \) | \( - \frac{1}{3} - 1 \) |
| \( t_R \) | \( 3^* \) | \( 1 \) | \( 0 + \frac{1}{3} + \frac{2}{3} + \frac{1}{3} \) |
| \( Q_L \) | \( 3 \) | \( 3 \) | \( 0 + \frac{1}{3} + \frac{1}{3} \) |
| \( L \) | \( 3 \) | \( 1 \) | \( 0 + \frac{1}{3} + \frac{1}{3} \) |
| \( 3^* \) | \( 3^* \) | \( 3^* \) | \( 3^* \) |

In the scalar sector, on the other hand, we have:

\[
\begin{align*}
\phi_1 &= \left( \begin{array}{c}
\phi_1^0 \\
\phi_1^- \\
\phi_1^+
\end{array} \right), & \phi_2 &= \left( \begin{array}{c}
\phi_2^0 \\
\phi_2^- \\
\phi_2^+
\end{array} \right), & \phi_3 &= \left( \begin{array}{c}
\phi_3^0 \\
\phi_3^- \\
\phi_3^+
\end{array} \right).
\end{align*}
\]

After electroweak symmetry breaking, the electric charge and lepton number assignments of the particles of the model follow from the action of the operators:

\[
Q = T_3 + \frac{1}{\sqrt{3}} T_8 + X; \quad (3)
\]

\[
L = \frac{4}{\sqrt{3}} T_8 + L. \quad (4)
\]

The relevant terms in the Lagrangian for leptons are:

\[
-L_{\text{lep}} = y^\nu \bar{\psi}_L \psi_R \phi_1 + y^a \bar{\psi}_L \psi_R \phi_3 + m_S \phi_3 + \frac{m_S}{2} S + \text{h.c.},
\]

where \( y^\nu \) and \( y^a \) are generic \( 3 \times 3 \) matrices, while \( y^a \) is anti-symmetric and \( m_S \) is the \( 3 \times 3 \) Majorana mass term for the singlets \( S \) (symmetric, due to the Pauli principle). In full generality, the scalars of the model are allowed to take vacuum expectation values (VEVs) in the following directions \( \phi_1^T = (k_1, 0, 0) / \sqrt{2} \), \( \phi_2^T = (0, k_3, n) / \sqrt{2} \), and \( \phi_3^T = (0, k_3, n') / \sqrt{2} \). However, in order to recover the SM as a low energy limit, we assume the hierarchy \( k_{1,2,3} \ll n, n' \). Moreover, we assume: \( k_3 = n' = 0 \), which together with the \( Z_2 \) symmetry guarantees the existence of a simple pattern of realistic quark masses.

Notice that since \( \phi_3^0 \) is singlet under the \( SU(2)_L \) subgroup contained in \( SU(3)_L \), the VEV \( n \) will control the four new gauge bosons masses and break \( SU(3)_L \) to \( SU(2)_L \). On the other hand, \( SU(2)_L \otimes U(1)_Y \) is broken at the electroweak scale by the \( k_1 \) and \( k_2 \) VEVs down to the electromagnetic \( U(1)_Q \) symmetry.

**Neutrino masses and inverse seesaw mechanism**

The presence of the small term \( \overline{S}S \), in eq. (5), explicitly breaks \( U(1)_L \) and provides the seed for lepton number violation leading to neutrino masses via the inverse seesaw mechanism.

\[
\begin{align*}
\psi_L &\equiv (u_L, c_L, t_L) \\
\bar{t}_R &\equiv (d_R, s_R, b_R) \\
\bar{Q}_L &\equiv (\pm \frac{1}{\sqrt{3}} Q_L, 0) \\
\bar{L} &\equiv (\pm \frac{1}{\sqrt{3}} L, 0)
\end{align*}
\]

**FIG. 1:** The branching ratio of the decay \( \mu \to e\gamma \) versus \( \sin^2 \theta_{13} \) for \( M = 100 \text{ GeV} \), \( 500 \text{ GeV} \), and \( 1 \text{ TeV} \). We take \( \tilde{m} = 10^{-21} \text{ GeV} \). The vertical band is the 3σ range reported in [18]. The other mixing angles are randomly taken within their 3σ range [18].

Indeed, after spontaneous symmetry breaking of the electroweak gauge group, we get the following \( 9 \times 9 \) neutrino mass matrix, in the basis \( (\nu, N, S) \) [14]:

\[
\mathcal{M} = \begin{pmatrix}
0 & m_D & 0 \\
0 & 0 & M \\
m_S & M & m_D
\end{pmatrix},
\]

where \( m_D \equiv \sqrt{2} k_1 y^\nu \) and \( M \equiv \frac{1}{\sqrt{2}} n y^a \). The inverse seesaw-induced light neutrino masses can be written as [14]:

\[
m_\nu = m_D \left(M^T\right)^{-1} m_S M^{-1} m_D. \quad (7)
\]

Here, the matrix \( M \) can be taken diagonal without loss of generality. Using this freedom and taking into account that \( m_D \) is anti-symmetric, eq. (7) can be expressed in terms of an effective symmetric \( 3 \times 3 \) matrix, \( \tilde{M}^{-1} \equiv M^{-1} m_S M^{-1} \), as:

\[
m_\nu = - m_D \tilde{M}^{-1} m_S M^{-1} m_D \equiv - m_D \tilde{M}^{-1} m_D. \quad (8)
\]

A simple implication of the antisymmetry of the “Dirac” entry \( m_D \) is that \( \text{Det}(m_\nu) = 0 \), so that the lightest neutrino is massless.

**Lepton flavor violation predictions**

Let us now proceed to a simple parameter counting. On the left-hand side of eq. (7) one has 5 independent

\[\text{Note that the matrix in eq. (6) does not depend on the conditions imposed on the VEVs. Indeed, even if } k_3 \neq 0 \text{ the resulting linear seesaw term } [15–17] \text{ would give only a subleading contribution } \sim m_\nu (M_W / n)^2.\]
complex parameters, since $\text{Det}(m_\nu) = 0$. In contrast, on the right-hand side of eq. (7) one has 9 independent complex parameters: 3 in $m_D$, and 6 elements in $M$. Therefore, we have 4 (complex) relations among the parameters ($y^a$, $y^\nu$, and $m_S$). One can choose as free parameters the 3 off-diagonal entries of $M^{-1}$, together with a global scaling factor $\tilde{m}$ defined through:
\[
m_D^{ij} = \tilde{m}^{-1} \left( m_\nu^{ij} - m_\nu^{i1} m_\nu^{1j} \right). \tag{9}
\]
From eq. (7), we can see that $\tilde{m}$ scales as $\sqrt{m_\nu^3 m_S}/M^2$, so that for $m_S \approx 10\,\text{eV}$, $M \approx 1\,\text{TeV}$, and neutrino masses of $\mathcal{O}(0.1)\,\text{eV}$, we obtain $\tilde{m} \approx 10^{-22}\,\text{GeV}$. In contrast, the diagonal entries of $M^{-1}$ are functions of its off-diagonal elements and $m_D$. We emphasize that eq. (9) is not an ansatz, but the most general solution of eq. (8).

Such a parameterization makes explicit the direct relation between charged LFV observables and neutrino oscillations parameters, which is a characteristic feature of our model. Indeed, LFV in this model arises from the term:
\[
-\mathcal{L}_{\text{LFV}} = y^a \psi_L^T C^{-1} \psi_L \phi + \text{h.c.}, \tag{10}
\]
which depends solely upon the coupling $y^a$, hence $m_D$. Using eq. (9) together with $m_\nu = U_\nu^T m_\nu^{\text{diag}} U_\nu$, where $U_\nu$ is the leptonic mixing matrix in its standard parameterization in terms of three mixing angles and the Dirac phase ($\delta$), we can express the relevant coupling for LFV as:
\[
y^a = \begin{pmatrix}
0 & -e^{-i\delta} \tan \theta_{13} & \sin \theta_{12} \\
-e^{i\delta} \tan \theta_{13} & 0 & -\cos \theta_{12} \\
-\sin \theta_{12} & \cos \theta_{12} & 0
\end{pmatrix}
\times \frac{\sqrt{\Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2}}{2k_1 \tilde{m}} e^{-i\delta} \sin \theta_{13} \cos \theta_{13}
\times \begin{cases}
\sqrt{\Delta m_{\text{sol}}^2} & \text{for NH} \\
\sqrt{\Delta m_{\text{atm}}^2} & \text{for IH}
\end{cases}.
\tag{11}
\]
It is remarkable that the Yukawas $y^a$ relevant for determination of LFV rates are, up to a global scaling factor, fully determined by the parameters measured in neutrino oscillation experiments. This allows us to make definite predictions for LFV observables that can be used to provide an unambiguous test of the model.

**Radiative $\ell_i \to \ell_j \gamma$ decays**

For definiteness we focus here on flavour-changing leptonic (radiative) decays to show the predictive power of the model. This probe constitutes one of the most important tests of new physics and has been actively sought after in many experiments. The branching ratio (BR) of the decay of the charged lepton $\ell_i \to \ell_j \gamma$ is:
\[
\text{BR}(\ell_i \to \ell_j \gamma) = \frac{m_{\ell_i}^5 |(y^a F y^a)_{ij}|^2}{\Gamma_{\ell_i}}, \tag{12}
\]
where $\Gamma_{\ell_i}$ is the total decay width of $\ell_i$, and $F$ is a function that depends on the masses and mixings of all the particles running inside the loop (summation over the different contributions is implicit here). We have three different classes of contributions: i) loops mediated by the new heavy gauge bosons. These are suppressed due to the large scale of the breaking of $SU(3)_L$ compared to $M_W$; ii) contributions from the exchange of a charged scalar whose mass $\sim \sqrt{f}$ depends on an unknown trilinear coupling, $f \phi \phi \phi_3$, appearing in the scalar potential; and finally iii) the “standard” loop, mediated by the SM $W$ boson and neutrinos. Assuming $f \sim \nu$, the only sizeable contribution is the “standard” loop, and therefore $F$ is function of the neutrinos and their mixings only.

This branching ratio depends on the neutrino mixing parameters, the global scaling factor $\tilde{m}$, and on the neutrino mass hierarchy. As can be seen in eq. (11), the off-diagonal entries in $y^a$ are larger in the case of inverted mass hierarchy (IH) by a factor $\sqrt{\Delta m_{\text{atm}}^2/\Delta m_{\text{sol}}^2}$ with respect to normal hierarchy (NH) so that, for the same input parameters, one has larger LFV effects in IH.

From figure (1) one sees that $\mu \to e\gamma$ may take place with sizeable rates, larger than current limits. Given the expected sensitivities of upcoming experiments one expects that the detection of this and other muon number violating processes might become feasible.

Since the BR depends on a global multiplicative factor, it is interesting to consider the ratio of branching ratios of LFV lepton decays. It follows from eq. (12) that:
\[
\frac{\text{BR}(\ell_i \to \ell_j \gamma)}{\text{BR}(\ell_k \to \ell_n \gamma)} = \frac{m_{\ell_i}^5 |(y^a F y^a)_{ij}|^2/\Gamma_{\ell_i}}{m_{\ell_k}^5 |(y^a F y^a)_{kn}|^2/\Gamma_{\ell_k}}. \tag{13}
\]
For the simplest case of nearly degenerate right-handed neutrinos, the $F$ functions are all equal and cancel out in the fraction. In this case, eq. (13) depends exclusively
on the ratios of $|y^a y^b|$, i.e., only on the neutrino mixing angles. The main advantage of considering the ratio of branching ratios and a quasi degenerate spectrum is that this leads to clean predictions which do not depend on the neutrino mass hierarchy nor the loop functions.

Indeed, in this simplified scenario, by combining eq. (11) with eq. (13) we obtain the following predictions:

$$BR(\mu \rightarrow e\gamma) = \frac{m_\tau^2 \Gamma_\tau}{m_\mu^2 \Gamma_\mu} \sin^2 \theta_{12} \cot^2 \theta_{13} \approx 76,$$

$$BR(\tau \rightarrow e\gamma) = \cot^2 \theta_{12} \approx 2,$$

(14)

(15)

where we have used the best-fit values for the neutrino parameters as derived in the global fit of neutrino oscillations given in [18]. So, when the right-handed neutrino spectrum is degenerate, the model predicts $BR(\mu \rightarrow e\gamma) \gg BR(\tau \rightarrow \ell_i \gamma)$. Therefore, given the expected sensitivities for $\tau$ LFV decays \(^3\), the simple observation of $\tau \rightarrow \ell_i \gamma$ in one (or several) of the near future experiments would rule out our simplest degenerate right-handed neutrino hypothesis. The viable alternative scenario in such cases would be a hierarchical right-handed neutrinos spectrum, implying a non-vanishing contribution of the $F$ loop functions in the ratio of BRs.

We compute the various relevant LFV observables using FlavorKit [20] \(^4\). As an illustration in figure (2) we show the ratio of the BR of $\tau$ and $\mu$ decays to $e\gamma$, namely $BR(\mu \rightarrow e\gamma)/BR(\tau \rightarrow e\gamma)$ as a function of the solar mixing parameter $\sin^2 \theta_{12}$. The other oscillation parameters are varied randomly within their 3$\sigma$ ranges [18]. Similarly, the ratio of the BR of leptonic $\tau$ decays, i.e., $BR(\tau \rightarrow \mu\gamma)/BR(\tau \rightarrow e\gamma)$ depends mainly on the solar mixing parameter.

For other LFV processes such as $\ell_i \rightarrow 3 \ell_j$ and $\mu - e$ conversion in nuclei, our results are qualitatively similar to the ones found in standard low-scale seesaw models [23]. Loops including neutrinos give the most important contributions, leading to LFV rates comparable to the ones for the radiative decay $\ell_i \rightarrow \ell_j \gamma$. This will be of special relevance due to the expected sensitivities in the coming experiments [24]. The complete study of all LFV processes is, however, beyond the scope of this letter.

**Conclusions and discussion**

In summary, we have shown how a simple variant of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ electroweak gauge symmetry implementing the inverse seesaw mechanism implies striking flavor correlations between rare charged lepton flavor violating decays and the measured neutrino oscillations parameters. The predictions follow simply from the enlarged gauge structure without any imposed flavor symmetry. Such tight complementarity between charged LFV and neutrino oscillations renders the scenario strictly testable. A more detailed study of other LFV processes will be taken up elsewhere. The scheme also has a non-trivial structure in the quarks sector since, thanks to the anomaly cancelation requirements, the Glashow-Iliopoulos-Maiani mechanism breaks down, leading to a plethora of flavor-changing neutral currents in the quark sector [25, 26]. Last but not least, the model presents a rich structure of new physics at the TeV scale that could be potentially studied in the coming run of the LHC.

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\(^3\) The expected Belle II sensitivities for $\tau$ radiative decays are around $10^{-9}$ [19], whereas the current MEG bound on $BR(\mu \rightarrow e\gamma)$ is many orders of magnitude stronger, $BR < 5.7 \cdot 10^{-13}$.

\(^4\) This is a computer tool based on SARAH [21] and SPheno [22], that increases their capability to handle flavor observables.

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