Fuzzy-set analysis of models of temperature deformation of thin-walled elements with elliptic boundaries in industrial and aerospace structures

S V Storozhev¹, V I Storozhev², V E Bolnokin³, Duong Minh Hai⁴ and D I Mutin³

¹Donbass National Academy of Civil Engineering and Architecture, 2, Derzhavina street, Makeyevka, 286123, DPR, Ukraine
²Donetsk National University, 24, University street, Donetsk, 283001, DPR, Ukraine
³Mechanical Engineering Research Institute of the Russian Academy of Sciences, 4, Malyy Haritonievsky, Moscow, 101000, Russian Federation
⁴Naval Technical Institute, 9, Mac Quyet Str, Hai Phong City, Duong Kinh Dist, Viet Nam

E-mail: stvi@donnu.ru

Abstract. A theoretical numerical-analytical fuzzy-set methodology to take into account the influence of scatter errors in the values of the initial physical, mechanical and geometric parameters on the estimates of the endogenous characteristics of temperature stresses in models of bending and flat temperature deformation of plate-type thin-walled structural elements with elliptical boundaries in industrial and aerospace equipment was developed. The methodology is based on the use of the alpha-level form of the heuristic principle of generalization in the transition to fuzzy-multiple arguments in the analytical representations of deterministic models for the estimated indefinite indicators of temperature bending moments and circumferential thermomechanical stresses at the boundaries of elliptical holes in thin plates. The results of calculations using the developed technique are presented. The proposed method is intended for use in theoretical and applied research on assessing the reliability and strength of thin-walled plate structural elements of aerospace engineering, industrial equipment and building structures under thermomechanical influences for conditions of uncertainty.

1. Introduction
Structural elements in the form of thin circular and elliptical plates, plates with small holes of elliptical and circular shapes, are common structural elements of aerospace vehicles and industrial equipment. As a result, increased requirements on the estimates of the strength and reliability of these structural elements are imposed. The most relevant models of the stress state of structural elements under consideration include models of their temperature deformation [1 - 5]. In particular, these are models of bending deformation of one-connected elliptical-shaped plates caused by temperature differences on opposite planar faces; models of symmetrical in thickness thermal stressed state of plates with holes under the action of heat fluxes in their plane, and a number of other models for structures of this type. Methods of analysis of these models for modern high-tech applications should by provide a correctly accounting of the influence of existing scatter in the values of the physical-mechanical and
geometric parameters of these structures on the characteristics of fields of temperature stresses arising in them.

If the initial information on scattering errors for exogenous parameters in models of bending or flat temperature deformation of thin plates is correct statistical information, then, in principle, probabilistic-stochastic analysis methods to take into account the effects of scatter can be used [6]. However, in most cases, there are no extensive correct statistics on the scatters that should be taken into account. In this regard, the description of uncertain endogenous parameters and the subsequent determination of endogenous parameters for models of thermal deformation of thin-walled structural elements using methods of the theory of fuzzy sets that impose less stringent requirements on the type of source information can be implemented [7 - 9]. In this context, the aim of this study is to develop a theoretical numerical-analytical fuzzy-multiple methodology to take into account the influence of scatter errors in the values of the initial parameters on the estimates of the endogenous characteristics of temperature stresses when analyzing the models of thermal deformation of thin-walled elements with elliptical boundaries for industrial and aerospace structures. Such an alternative methodology uses the calculated ratios of deterministic versions of the considered models, in which, further, on the basis of various schemes for applying the heuristic principle of generalization, a transition to fuzzy-set arguments is realized [10].

2. The basic relations of deterministic versions of the studied models

The first of the models under consideration is the applied model of thermoelastic bending of thin plates from anisotropic composite materials of the orthorhombic class based on Kirchhoff hypotheses, the theoretical numerical-analytical study of which is effectively implemented using the methods of complex potentials of generalized complex variables.

The basic relations of the deterministic version of this model for a structural element in the form of a one-connected anisotropic plate of an orthorhombic class with a thickness $h$ that occupies a region $V = \{(x_1, x_2) \in S, -h/2 \leq x_1 \leq h/2\}$ in the coordinate space $Ox_1x_2x_3$ are formulated. The boundary elliptical contour $L$ of the median plane of the plate $S$ has semiaxes $a, b$.

Plate bending is caused by the action of a stationary temperature field linearly varying in its thickness $T(x_1, x_2, x_3) = T_0(x_1, x_2), \quad (1)$

and external forces on flat faces $x_3 = \pm h/2$ are absent. The case of a rigidly fixed lateral surface of the plate is considered. On the lateral boundary surface $\Gamma = \{(x_1, x_2) \in L, x_3 \in [-h/2, h/2]\}$ the condition

$$(\tau_0(x_1, x_2))_3 = T_0 \quad (2)$$

is satisfied. In the posed problem, the averaged characteristics of the stress state - bending moments $M_1, M_2$, twisting moment $H_{12}$, and transverse forces $N_1, N_2$, obtained using the theory of functions of generalized complex variables [5] – are constant values and have representations

$$M_1 = -\gamma_1 T_0, \quad M_2 = -\gamma_2 T_0, \quad H_{12} = N_1 = N_2 = 0, \quad (3)$$

$$\gamma_1 = D_{11} \alpha_1 + D_{12} \alpha_2, \quad \gamma_2 = D_{12} \alpha_1 + D_{22} \alpha_2, \quad (4)$$

$$D_{11} = E_1 h^3 / (12(1 - \nu_1 \nu_2)), \quad D_{22} = E_1 h^3 / (12(1 - \nu_1 \nu_2)),$$

$$D_{12} = \nu_1 E_1 h^3 / (12(1 - \nu_1 \nu_2)) = \nu_2 E_2 h^3 / (12(1 - \nu_1 \nu_2));$$

$E_1, E_2, \nu_1, \nu_2$ – Young’s modules and Poisson’s ratios of the plate material; $\alpha_1, \alpha_2$ – coefficients of thermal expansion of the plate material along elastic-equivalent directions. Thus, in accordance with relations (3), (4), the analyzed endogenous characteristics of bending moments $M_1, M_2$ are described by functional dependencies.
The second of the considered models, to which a fuzzy-sets technique for analyzing the influence of uncertainty factors of exogenous parameters is developed, is a model of describing the concentration of mechanical stresses on the contour of an elliptical hole with semiaxes $a, b$ in thin isotropic plate. The generalized plane stress state of the plate is due to the action of a directed at an angle $\beta$ to the coordinate axis $Ox_1$ symmetric in thickness $h$ heat flux with an average intensity $q$, which corresponds to the distribution of the average temperature function $T(x_1, x_2)$ in plane of the plate

$$T(x_1, x_2) = q(x_1 \cos \theta + x_2 \sin \theta).$$

According to [4], the stresses at a point with an angular coordinate $\theta$ on the openings contour with a parametric description

$$x_1 = a \cos \theta, \quad x_2 = b \sin \theta,$$

are determined by the representation

$$\sigma_\theta = F_{\sigma_\theta}(\alpha, q, a, b, E, \beta, \theta) = \alpha q ER [(1 - k^2) \cos(\theta + \beta) - (1 + k^2) \cos(\theta - \beta)][1 - \cos 2\theta + k^2 (1 + \cos 2\theta)]^{-1}. \tag{8}$$

In the representation (8) $k = b/a; R = (a + b)/2; \alpha_r$ - temperature coefficient of linear expansion of the plate material; $E$ - Young’s modulus of the plate material. Representations (5) and (8) are the basis for the application of a fuzzy-multiple methodology for the assessing of indicators of the influence of uncertainty in the form of scatter errors in the values of the initial parameters of the considered models.

3. Obtaining of a fuzzy-set representations for endogenous parameters of models

The hypothesis that is introduced in further research is the interpretation of the values of the initial physical-mechanical and geometric parameters with scatter errors in the form of trapezoidal normal fuzzy intervals.

In the applied model of thermoelastic bending of a structural element in the form of a thin elliptical plate from an anisotropic composite material of the orthorhombic class for exogenous parameters $a, b, h, E_1, E_2, v_1, v_2, G, T_0, \alpha_1, \alpha_2$ fuzzy-interval descriptions $\tilde{a}, \tilde{b}, \tilde{h}, \tilde{E}_1, \tilde{E}_2, \tilde{v}_1, \tilde{v}_2, \tilde{G}, \tilde{T}_0, \tilde{\alpha}_1, \tilde{\alpha}_2$ in the form are introduced

$$\tilde{a} = (a_1, a_2, a_3, a_4), \quad \tilde{b} = (b_1, b_2, b_3, b_4), \quad \tilde{h} = (h_1, h_2, h_3, h_4), \tag{9}$$

$$\tilde{E}_1 = (E_{11}, E_{12}, E_{13}, E_{14}), \quad \tilde{E}_2 = (E_{21}, E_{22}, E_{23}, E_{24}), \quad \tilde{v}_1 = (v_{11}, v_{12}, v_{13}, v_{14}),$$

$$\tilde{v}_2 = (v_{21}, v_{22}, v_{23}, v_{24}), \quad \tilde{G} = (G_1, G_2, G_3, G_4), \quad \tilde{T}_0 = (T_{01}, T_{02}, T_{03}, T_{04}), \quad \tilde{\alpha}_1 = (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}),$$

$$\tilde{\alpha}_2 = (\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}).$$

Using representations (9) and the rules of fuzzy-interval arithmetic [10 - 12], the following expressions can be obtained in turn:

$$\tilde{D}_{11} = (\tilde{E}_{11} h_1^4)/(12(1 - v_{11}v_{21})), \quad \tilde{E}_{12} h_2^4)/(12(1 - v_{12}v_{22})), \tag{10}$$

$$\tilde{E}_{13} h_3^4)/(12(1 - v_{13}v_{23})), \quad \tilde{E}_{14} h_4^4)/(12(1 - v_{14}v_{24})), \quad \tilde{D}_{22} = (\tilde{E}_{22} h_1^4)/(12(1 - v_{21}v_{21})), \quad \tilde{E}_{23} h_3^4)/(12(1 - v_{23}v_{23})), \quad \tilde{E}_{24} h_4^4)/(12(1 - v_{24}v_{24})), \quad \tilde{E}_{21} h_1^4)/(12(1 - v_{21}v_{21})).$$
The principle of generalization

\[ D_{12} = \frac{(v_1 E_1 h_1^3) / (12(1 - v_{11} v_{22}))}{v_{12} E_{12} h_{12}^3} / (12(1 - v_{12} v_{22}))}, \]
\[ v_{13} E_{13} h_{13}^3 / (12(1 - v_{13} v_{23}))}, v_{14} E_{14} h_{14}^3 / (12(1 - v_{14} v_{23})). \]
\[ \tilde{\gamma}_1 = ((\alpha_{11} + v_{11} \alpha_{21}) E_{11} h_{11}^3 / (12(1 - v_{11} v_{21}))), (\alpha_{12} + v_{12} \alpha_{22}) E_{12} h_{12}^3 / (12(1 - v_{12} v_{22}))), \]
\[ (\alpha_{13} + v_{13} \alpha_{23}) E_{13} h_{13}^3 / (12(1 - v_{13} v_{23}))), (\alpha_{14} + v_{14} \alpha_{24}) E_{14} h_{14}^3 / (12(1 - v_{14} v_{23}))), \]
\[ \tilde{\gamma}_2 = ((\alpha_{21} + v_{21} \alpha_{11}) E_{21} h_{21}^3 / (12(1 - v_{11} v_{21}))), (\alpha_{22} + v_{22} \alpha_{12}) E_{22} h_{22}^3 / (12(1 - v_{12} v_{22}))), \]
\[ (\alpha_{23} + v_{23} \alpha_{13}) E_{23} h_{23}^3 / (12(1 - v_{13} v_{23}))), (\alpha_{24} + v_{24} \alpha_{14}) E_{24} h_{24}^3 / (12(1 - v_{14} v_{23}))). \]

\[ \tilde{\mathbf{M}}_1 = -T_{01} (\alpha_{11} + v_{11} \alpha_{21}) E_{11} h_{11}^3 / (12(1 - v_{11} v_{21})), -T_{02} (\alpha_{12} + v_{12} \alpha_{22}) E_{12} h_{12}^3 / (12(1 - v_{12} v_{22})), \]
\[ -T_{03} (\alpha_{13} + v_{13} \alpha_{23}) E_{13} h_{13}^3 / (12(1 - v_{13} v_{23})), -T_{04} (\alpha_{14} + v_{14} \alpha_{24}) E_{14} h_{14}^3 / (12(1 - v_{14} v_{23})). \]
\[ \tilde{\mathbf{M}}_2 = -T_{01} (\alpha_{21} + v_{21} \alpha_{11}) E_{21} h_{21}^3 / (12(1 - v_{21} v_{11})), -T_{02} (\alpha_{22} + v_{22} \alpha_{12}) E_{22} h_{22}^3 / (12(1 - v_{22} v_{12})), \]
\[ -T_{03} (\alpha_{23} + v_{23} \alpha_{13}) E_{23} h_{23}^3 / (12(1 - v_{23} v_{13})), -T_{04} (\alpha_{24} + v_{24} \alpha_{14}) E_{24} h_{24}^3 / (12(1 - v_{24} v_{14}))). \]

In particular, for a plate made of a composite fibrous material - fiberglass [5], when setting fuzzy-interval initial parameters in the form

\[ a = (194l, 198l, 201l, 203l), b = (94l, 97l, 101l, 102l), \]
\[ E = (3.8l, 4l, 4.2l, 4.6l), \]
\[ h = (3.8l, 4l, 4.2l, 4.6l), \]
\[ E_1 = (3.78E, 3.81E, 3.83E, 3.91E), \]
\[ E_2 = (0.96E, 1.0E, 1.05E, 1.12E), \]
\[ v_1 = (0.071, 0.072, 0.074, 0.076), \]
\[ v_2 = (0.26, 0.28, 0.29, 0.3), \]
\[ G = (0.378E, 0.383E, 0.385E, 0.392E), \]
\[ \alpha_{11} = (0.66\alpha, 0.69\alpha, 0.71\alpha, 0.74\alpha), \]
\[ \alpha_{12} = (3.75\alpha, 3.80\alpha, 3.82\alpha, 3.94\alpha), \]
\[ T_{01} = (260, 275, 284, 295) \times 10^{-7}[\text{м}], E_n = 10^6[\text{Па}], \alpha_1 = 10^{-5}[\text{град}^{-1}], \]
\[ \mu_E(M_1), \mu_E(M_2). \]

To analyze the uncertainties factors in the model of the thermomechanical stresses concentration on the contour of an elliptical opening under the influence of a heat flux the fuzzy-interval descriptions hypothesis for the values of the initial parameters with scatter errors is also accepted and the modified \( \alpha \)-level form of the heuristic principle of generalization is used. For exogenous parameters \( \alpha, q, a, b, E, \beta \) in relation (8) are introduced fuzzy-interval descriptions \( \tilde{\alpha}, \tilde{q}, \tilde{a}, \tilde{b}, \tilde{E}, \tilde{\beta} \) of the form

\[ \tilde{\alpha} = (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}), \tilde{q} = (q_1, q_2, q_3, q_4), \tilde{a} = (a_1, a_2, a_3, a_4), \tilde{b} = (b_1, b_2, b_3, b_4), \tilde{E} = (E_{11}, E_{12}, E_{13}, E_{14}), \tilde{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4), \]
\[ (12) \]
and corresponding representations in the form of decompositions into subsets of the $\alpha$-level

$$\alpha_i = \bigcup_{\alpha\in[0,1]} [\alpha_{i\alpha}, \bar{\alpha}_{i\alpha}], \bar{\alpha}_{i\alpha} = (1-\alpha)\alpha_{i1} + \alpha\alpha_{i2}, \alpha_{i\alpha} = \alpha\alpha_{i3} + (1-\alpha)\alpha_{i4},$$

$$\bar{q} = \bigcup_{\alpha\in[0,1]} [q_{\alpha\alpha}, \bar{q}_{\alpha\alpha}], \bar{q}_{\alpha\alpha} = (1-\alpha)q_{\alpha1} + \alpha q_{\alpha2}, \bar{q}_{\alpha\alpha} = \alpha q_{\alpha3} + (1-\alpha)q_{\alpha4},$$

$$\bar{a} = \bigcup_{\alpha\in[0,1]} [a_{\alpha\alpha}, \bar{a}_{\alpha\alpha}], \bar{a}_{\alpha\alpha} = (1-\alpha)a_{\alpha1} + \alpha a_{\alpha2}, \bar{a}_{\alpha\alpha} = \alpha a_{\alpha3} + (1-\alpha)a_{\alpha4},$$

$$b = \bigcup_{\alpha\in[0,1]} [b_{\alpha\alpha}, \bar{b}_{\alpha\alpha}], \bar{b}_{\alpha\alpha} = (1-\alpha)b_{\alpha1} + \alpha b_{\alpha2}, \bar{b}_{\alpha\alpha} = \alpha b_{\alpha3} + (1-\alpha)b_{\alpha4},$$

$$\bar{E} = \bigcup_{\alpha\in[0,1]} [E_{\alpha\alpha}, \bar{E}_{\alpha\alpha}], \bar{E}_{\alpha\alpha} = (1-\alpha)E_{\alpha1} + \alpha E_{\alpha2}, \bar{E}_{\alpha\alpha} = \alpha E_{\alpha3} + (1-\alpha)E_{\alpha4},$$

$$\bar{\beta} = \bigcup_{\alpha\in[0,1]} [\beta_{\alpha\alpha}, \bar{\beta}_{\alpha\alpha}], \bar{\beta}_{\alpha\alpha} = (1-\alpha)\beta_{\alpha1} + \alpha \beta_{\alpha2}, \bar{\beta}_{\alpha\alpha} = \alpha \beta_{\alpha3} + (1-\alpha)\beta_{\alpha4}.$$

For an endogenous characteristic in the form of a parametric fuzzy set $\tilde{\sigma}_q(\theta)$ taking into account the properties

$$\partial F_{\varepsilon_0} / \partial \alpha > 0, \partial F_{\varepsilon_0} / \partial q > 0, \partial F_{\varepsilon_0} / \partial E > 0,$$

based on the application of the modified $\alpha$-level form of the heuristic principle of generalization a representation is obtained

$$\tilde{\sigma}_q(\theta) = \bigcup_{\alpha\in[0,1]} [\tilde{\sigma}_{0\alpha}(\theta), \sigma_{0\alpha}(\theta)],$$

$$\sigma_{0\alpha}(\theta) = \inf_{\alpha\in[\alpha_{\alpha\alpha}, \alpha_{\alpha\alpha}]} \{F_{\varepsilon_0}(\alpha_{\alpha\alpha}, q_{\alpha\alpha}, a, b, E_{\alpha\alpha}, \beta, \theta),$$

$$\beta_{\alpha\alpha} = \inf_{\beta_{\alpha\alpha}} [\beta_{\alpha\alpha}, \ldots, \beta_{\alpha\alpha}],$$

$$\bar{\beta}_{\alpha\alpha} = \sup_{\beta_{\alpha\alpha}} [\beta_{\alpha\alpha}, \ldots, \beta_{\alpha\alpha}].$$

An example of the implementation of the developed method is considered for the case of influence of heat flux in a plate containing an elliptical opening and made of a composite polymer material when accounting scatter errors values of physical-mechanical and geometric parameters

$$\alpha = (3.85\alpha_1, 4.0\alpha_2, 4.05\alpha_3, 4.20\alpha_4), \quad \bar{E} = (3.48\varepsilon_1, 3.51\varepsilon_2, 3.53\varepsilon_3, 3.61\varepsilon_4),$$

$$\bar{a} = (9.4l_{\alpha1}, 9.8l_{\alpha2}, 10.1l_{\alpha3}, 10.3l_{\alpha4}), \quad \bar{b} = (19.4l_{\alpha1}, 19.7l_{\alpha2}, 20.1l_{\alpha3}, 20.2l_{\alpha4}),$$

$$\beta = (1.047\varepsilon_{\alpha1}, 1.042\varepsilon_{\alpha2}, 1.047\varepsilon_{\alpha3}, 1.053\varepsilon_{\alpha4}), \quad L = 10^{-3}[m], \quad E_s = 10^9[Pa], \quad \alpha_\tau = 10^{-5}[\text{grad}^{-1}], \quad \beta_\tau = 1[\text{rad}].$$

The fuzzy characteristic $\tilde{q}$ is defined as $\tilde{q} = (234q_1, 240q_2, 245q_3, 252q_4), \quad q_\tau = 1[\text{grad} / m].$

Membership functions $\mu_{\epsilon_0}(\sigma_q)$ for fuzzy-set characteristics $\tilde{\sigma}_q(\theta)$ obtained for values $\theta=0$ and $\theta=\pi$ using representation (15) are presented in figures 3, 4.

The estimates obtained as a result of the analysis of the considered model allow us to draw reasonable conclusions about the ranges of the most reliable deviations in the values of the analyzed indicators of the intensity of bending moments and stress concentration levels at various points of the opening contour for the considered scatter errors of physical-mechanical parameters, as well as about the boundaries of possible values of concentration indicators taken at a minimum level of confidence.
4. Conclusion
The implemented development of a theoretical numerical-analytical fuzzy-set methodology for taking into account the influence of scatter errors in the values of the initial physical, mechanical and geometric parameters on the estimates of the endogenous characteristics of temperature stresses in the analysis of models of bending and flat temperature deformation of thin-walled structural elements of the plate type with elliptical boundaries is intended for using in theoretical and applied research evaluating the reliability and durability of thin wall plate structural elements of aerospace equipment, industrial equipment and building structures at temperature impacts. The technique allows using expert-type information and incomplete statistical data to obtain design estimates, based on which membership functions for fuzzy sets can be obtained in the form of normal trapezoidal fuzzy intervals.

References
[1] Constanda C 2014 Mathematical Methods for Elastic Plate (London: Springer)
[2] Murakami Y 2017 Theory of Elasticity and Stress Concentration (Chichester, West Sussex, United Kingdom: John Wiley & Sons)
[3] Savin G 1961 Stress concentration around the holes (Oxford: Pergamon Press)
[4] Savin G 1968 Stress concentration around the holes (Kiev: Naukova Dumka)
[5] Kosmodamiansky A S and Kaloerov S A 1983 Thermal stresses in multi-connected plates (Kiev: Visha Shkola)
[6] Lomakin V A 1970 Statistical problems of the mechanics of solid deformable bodies (Moscow: Nauka)
[7] Anastassiou G A 2010 Fuzzy Mathematics: Approximation Theory (Berlin, Heidelberg: Springer-Verlag)
[8] Hanss M 2005 Applied Fuzzy Arithmetic: An introduction with Engineering Application (Berlin Heidelberg: Springer-Verlag)
[9] Sonbol A H and Fadali M S 2012 TSK Fuzzy Function Approximators: Design and Accuracy Analysis IEEE Trans. Syst. Man and Cybern 42 702-12
[10] Vyskub V G, Mutina E I, Storozhev S V and Storozhev V I 2019 Model of fuzzy estimation of mechanical stress concentration for aerospace and industrial flat structures with polygonal holes of uncertain curvature at rounded corner points Journal of Physics: Conf. Series: Materials Science and Engineering 537 022013 doi:10.1088/1757-899X/537/2/022013
[11] Grzegorzewski P and Mrówka E 2005 Trapezoidal approximations of fuzzy numbers Fuzzy Sets Syst 153 115-35
[12] Ban A I, Coroianu L C and Grzegorzewski P 2011 Trapezoidal approximation and Aggregation Fuzzy Sets Syst 177 45-59