Quantum transfer through a non-Markovian environment under frequent measurements and Zeno effect

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We study transitions of a particle between two wells, separated by a reservoir, under the condition that the particle is not detected in the reservoir. Conventional quantum trajectory theory predicts that such no-result continuous measurement would not affect these transitions. We demonstrate that it holds only for Markovian reservoirs (infinite band-width \( \Lambda \)). In the case of finite \( \Lambda \), the probability of the particle’s inter-well transition is a function of the ratio \( \nu/\Lambda \), where \( \nu \) is the frequency of measurements. This “scaling” tells us that in the limit of \( \nu \to \infty \), the measurement freezes the initial state (Quantum Zeno effect), whereas for \( \Lambda \to \infty \) it does not affect the the particle’s transition across the reservoir. It also supports a simple explanation of the Zeno effect entirely in terms of the energy-time uncertainty relation, with no explicit use of the projection postulate. Experimental tests of our predictions are discussed.

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It is well-known that the unitary evolution of quantum system is interrupted by measurement, so that the subsequent evolution of a system depends on the measurement record. Frequent measurements with intervals \( \Delta t \) are of special interest. Then in the limit of \( \Delta t \to 0 \), they freeze the particle’s motion (Quantum Zeno effect). This result is a consequence of the projection postulate applied to sequential measurements.

The Zeno effect looks very surprising, since it reveals the dynamical impact of the projection postulate on quantum motion. Instead one can try to attribute the Zeno effect to the influence of the measurement devices. On first sight this cannot be the case. Indeed, due to the interaction with detectors the system acquires the energy \( \hbar /\Delta t \), according to the energy-time uncertainty relation. As such, it is natural to accelerate an equalization of the particle, instead of its freezing – the anti-Zeno effect. Nevertheless, as demonstrated in this paper the Zeno effect can be entirely attributed to the energy-time uncertainty relation, without the explicit use of the projection postulate. It would make the Zeno effect much less surprising and moreover, quite expectable.

The concept of continuous measurement is inherent in the Quantum Trajectory approach (“informational” evolution), which treats quantum motion, based on the results of intermediate measurements. It is therefore natural to investigate the Zeno-effect dynamics in this framework. A most pronounced example of the informational evolution, was proposed for a two-state system (qubit), coupled to a continuously monitored reservoir under condition that no-signal is registered there. It was predicted that the qubit can change its state despite the null-result measurements. This was confirmed in experiments with a superconducting “phase” qubit measured via tunneling.

In this paper we study a different arrangement, with two distant localized states, connected by a common reservoir under continuous null-result monitoring. Predictions based on the Quantum Trajectory approach for this case are even more dramatic: the system can display transition between these two localized states via the reservoir, although the latter is under continuous null-result monitoring. This result is highly counterintuitive and is a clear contradiction with the Zeno effect. We therefore perform here a detailed quantum mechanical analysis of these undetectable transitions. This will allow us to understand the nature of this phenomenon and its relation to the Quantum Trajectory approach and to the Zeno effect.

Consider two quantum dots coupled to the reservoir, monitored by an external detector, as shown in Fig. 1(a). Alternatively, one can monitor the charge of two quantum dots by a nearby Point-Contact (PC) detector, Fig. 1(b). Then the current \( I \) flowing through the point-contact, increases when electron tunnels to the reservoir. In order to make this setup fully equivalent to that on Fig. 1(a), the point-contact detector must be placed symmetrically with respect to the dots. The system is described by the following Hamiltonian:

\[
H = E_1 |1\rangle \langle 1| + E_2 |2\rangle \langle 2| + \sum_r E_r |r\rangle \langle r|
+ \sum_r \left[ (\Omega_{1r}|r\rangle \langle 1| + \Omega_{2r}|r\rangle \langle 2|) + H.c. \right]
\]  

(1)

where \( \Omega_{1(2)r} \) are tunneling couplings of the upper and lower dot to the reservoir. The states in the dots \(|1(2)\rangle\)
the electron is initially in the linear superposition of the states $|1'(2')\rangle$. After the time interval $\Delta t$ the wave function becomes

$$|\Psi(\Delta t)\rangle = \alpha_1 |1'\rangle + \alpha_2 [1 - iH\Delta t - \frac{1}{2}H^2(\Delta t)^2 + \cdots ] |2'\rangle$$

(3)

where we expanded the evolution operator $\exp(-iH\Delta t)$ up to the second order in $\Delta t$. (From now we adopt the units where $\hbar = 1$). The null-result measurement in the reservoir implies that the electron's wave function is projected on the two-dot subspace, $|\Psi(\Delta t)\rangle \to \bar{Q}|\Psi(\Delta t)\rangle$, where $Q = (|1'\rangle\langle 1'| + |2'\rangle\langle 2'|)/N$ and $N$ is a normalization factor. Therefore

$$|\Psi_1\rangle = \bar{Q}|\Psi(\Delta t)\rangle = \frac{1}{N_1} \left[ \alpha_1 |1'\rangle + \alpha_2 (1 - C(\Delta t)^2) |2'\rangle \right]$$

(4)

where $C = \frac{1}{2} \sum_r \left( \Omega_{1r}^2 + \Omega_{2r}^2 \right)$ and $N_1^2 = 1 - 2\alpha_2^2 C(\Delta t)^2$.

After $n$ subsequent null-result measurements during time $t$, with $n = t/\Delta t$, we find

$$|\Psi_n\rangle = \frac{1}{N_n} \left[ \alpha_1 |1'\rangle + \alpha_2 (1 - C(\Delta t)^2)^n |2'\rangle \right]$$

(5)

where $N_n = \sqrt{1 - 2n\alpha_2^2 C(\Delta t)^2}$. Thus in the limit of $\Delta t \to 0$ and $t=\text{const}$, one obtains

$$|\Psi_n\rangle \to \alpha_1 |1'\rangle + \alpha_2 |2'\rangle \equiv |\Psi(0)\rangle.$$  

(6)

This means that the continuous null-result monitoring of the reservoir, Fig. 1(a), or an indirect monitoring as in Fig. 1(b), reveals the Zeno effect by preventing the electron's inter-dot transitions.

On the other hand, the conditional electron's dynamics can be studied by the Quantum Trajectory method, designed for such type of problems. Surprisingly, by applying this method one arrives to the opposite conclusion: the continuous null-result monitoring of the reservoir would not prevent the electron's inter-dot transitions. Let us explain this point with more details.

According to the theory of continuous quantum measurement, the evolution of the density matrix, $\rho_c(t)$, conditioned on the measurement-result, can be written as

$$d\rho_c = dN(t) \mathcal{G}[a] \rho_c - dt \mathcal{H} \left[ iH_S + \Gamma a / 2 \right] \rho_c,$$

(7)

where $H_S = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2|$ and $dN(t) = 0$ or 1, being the measurement record of electron number detected in the reservoir during the time interval $(t, t+dt)$. The super-operators $\mathcal{G}$ and $\mathcal{H}$ are defined in [3] as, $\mathcal{G}[a] \rho_c = a\rho_c a^\dagger - \rho_c$, and $\mathcal{H}[x] \rho_c = x\rho_c + \rho_c x^\dagger - (x + x^\dagger)\rho_c$, where $\langle \cdots \rangle \equiv \text{Tr}[\cdots] \rho_c$. In our case $a = |0\rangle\langle 1| + |0\rangle\langle 2|$, where the state $|0\rangle$ corresponds to empty quantum wells with the electron is inside the reservoir. Here $\Gamma$ is the

![FIG. 1: (Color online) Two quantum dots are coupled to a common reservoir: (a) The electron tunnels to the reservoir, monitored by an external detector; (b) The electron's tunneling is monitored by a Point-Contact detector. The detector current $I$ increases whenever the electron leaves the dots.](image)
tunneling rate from each of the dots to the reservoir. (For simplicity, we took \( \Gamma_1 = \Gamma_2 = \Gamma \).)

If we register only the events with no electron in the reservoir, then \( dN(t) = 0 \) in Eq. (10). As a result the corresponding conditional density matrix \( \rho_c(t) \) is given by the equation

\[
\dot{\rho}_c = -i[H_S, \rho_c] + \Gamma \left( \langle a^\dagger a \rangle \rho_c - \frac{1}{2} \{ a^\dagger a, \rho_c \} \right) \tag{8}
\]

Despite the non-linearity of this equation it can be solved analytically (1). One finds from this solution that the occupation probabilities for the dots in the asymptotic limit, \( \bar{\rho} = \rho_c(t \to \infty) \) are equal, \( \bar{\rho}_{11} = \bar{\rho}_{22} \) and coinciding with \( \bar{\rho}_{1,2} \), Eq. (2), obtained without any intermediate monitoring of the reservoir. This implies that the inter-dot transitions through the reservoir are not interrupted by the continuous measurement. In fact, this result might be foreseen, since the measurement time \( \Delta t \) does not appear in the Quantum Trajectory equation (1), thus indicating that the Zeno effect is not reflecting in this equation. In order to understand the disagreement between predictions of the Quantum Trajectory approach and the Zeno effect, we present below detailed quantum mechanical analysis of the continuous null-result measurements for the setup in Fig. 1.

Consider the electron wave function, written as

\[
|\Psi(t)\rangle = b_1(t)|1\rangle + b_2(t)|2\rangle + \sum_r b_r(t)|r\rangle, \tag{9}
\]

where \( b_{1,2,r}(t) \) are the probability amplitudes of finding the electron in the dots and reservoir. Substituting Eq. (9) into the Schrödinger equation, \( i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H|\Psi(t)\rangle \) and performing the Laplace transform, \( \tilde{b}(\omega) = \int_0^\infty b(t) \exp(\omega t)dt \), we obtain the following system of algebraic equations for \( \tilde{b}(\omega) \)

\[
(\omega - E_j)\tilde{b}_j(\omega) - \sum_r \Omega_{jr}\tilde{b}_r(\omega) = ib_j(0) \tag{10a}
\]

\[
(\omega - E_r)\tilde{b}_r(\omega) - \Omega_{1r}\tilde{b}_1(\omega) - \Omega_{2r}\tilde{b}_2(\omega) = 0 \tag{10b}
\]

where \( j = 1, 2 \). The r.h.s. of these equations reflects the initial conditions, corresponding to the electron localized in the dots. Substituting \( \tilde{b}_r(\omega) \) from Eq. (10a) into Eq. (10b) and replacing \( \sum_r \to \int g(E_r)E_r dE_r \), where \( g(E_r) \) is the density of states, we obtain

\[
(\omega - E_j)\tilde{b}_j(\omega) - \sum_{j'} \mathcal{F}_{jj'}(\omega) \tilde{b}_{j'}(\omega) = i b_j(0), \tag{11}
\]

where

\[
\mathcal{F}_{jj'}(\omega) = \int \frac{\Omega_{jj'} \Omega_{jj'} g(E_r)}{\omega - E_r} dE_r \tag{12}
\]

The time-dependent amplitudes \( b_{1,2}(t) \) are obtained by the inverse Laplace transform.

In many calculations the density of reservoir states, \( \varrho(E_r) \), is taken energy-independent, so called wide-band limit (“Markovian” reservoir). Here we consider a finite-band spectrum by taking the density of states in the Lorentzian form,

\[
\varrho(E_r) = \frac{\varrho_0 \Lambda^2}{E_r^2 + \Lambda^2}, \tag{13}
\]

while the coupling amplitudes are energy independent, \( \Omega_{jj'} = \Omega_j \). Then we obtain \( \mathcal{F}_{jj'}(\omega) = \frac{\Lambda \Gamma_{jj'}}{2(\omega + i\Lambda)} \), where \( \Gamma_j = 2\pi \varrho_0 \Omega_j^2 \). Using this result we find for the vector \( \{ \tilde{b}_1(\omega), \tilde{b}_2(\omega) \}^T = \tilde{U}(\omega)\{ b_1(0), b_2(0) \}^T \), where \( \tilde{U}(\omega) = i/|\omega - \tilde{H}(\omega)| \) and

\[
\tilde{H}(\omega) = \begin{pmatrix} E_1 + \frac{\Lambda \Gamma_1}{2(\omega+i\Lambda)} & \frac{\Lambda \Lambda_{12}}{2(\omega+i\Lambda)} \\ \frac{\Lambda \Lambda_{12}}{2(\omega+i\Lambda)} & E_2 + \frac{\Lambda \Gamma_2}{2(\omega+i\Lambda)} \end{pmatrix} \tag{14}
\]

Finally, by applying the inverse Laplace transform, we obtain \( \{ b_1(t), b_2(t) \}^T = U(t)\{ b_1(0), b_2(0) \}^T \), where the evolution operator \( U(t) = \int_{-\infty}^{\infty} \tilde{U}(\omega)e^{-i\omega t}d\omega/(2\pi) \).

A null-result measurement in the reservoir, quantum mechanically, collapses the entire wave function onto subset of the dot’s states,

\[
|\Psi(t)\rangle \to \frac{1}{N}\{ b_1(t)|1\rangle + b_2(t)|2\rangle \} = \frac{1}{N}U(t)|\Psi(0)\rangle \tag{15}
\]

where \( N \) is a normalization factor. Then after \( n \) such null-result measurements in the reservoir with subsequent time interval \( \Delta t = t/n \), the final state of the system is given by

\[
|\Psi_n(t)\rangle = \frac{1}{\sqrt{N_n}}U^n(\Delta t)|\Psi(0)\rangle \tag{16}
\]

where \( N_n = N_n N_{n-1} \cdots N_2 N_1 \). Here \( N_j \) denotes the normalization factor associated with the \( j \)th measurement. Obviously, \( N_n \) is a normalization of the state \( U^n(\Delta t)|\Psi(0)\rangle \) at the final stage.

Consider the electron initially occupying the upper dot in Fig. 1. Let us evaluate the electron’s survival probability at time \( t \), subjected to the null-result monitoring of the reservoir with frequency \( \nu = 1/\Delta t \). For simplicity we consider the case of \( \Gamma_1 = \Gamma_2 = \Gamma \). The results are displayed in Fig. 2 for two different values of the band-width, \( \Lambda = 3\Gamma \) (lines) and \( \Lambda = 10\Gamma \) (symbols) and different observation frequencies: \( \nu = 0.5\Lambda \) (dash lines and symbols ○), \( \nu = 5\Lambda \) (dot lines and symbols △), \( \nu = 50\Lambda \) (dash-dotted lines and symbols •). The solid lines (and symbols •) show the result of unitary evolution (with no intermediate null-result measurements, except for only the last one at \( t \)). In this case the conditional survival probability is given by \( P_1(t) = \langle b_1(t)^2 \rangle/\left[ \langle b_1(t)^2 \rangle^2 + \langle b_2(t)^2 \rangle^2 \right] \), where \( b_{1,2}(t) \) are obtained from the inverse Laplace transform of Eq. (11). The two panels correspond to: (a) dots’ levels are aligned with the Lorentzian center of the density.
of states, $E_1 = E_2 = 0$; (b) dots’ levels are shifted from the Lorentzian center, $E_1 = E_2 = 3\Lambda$. One finds that in all figures the symbols lie on the lines, corresponding to the same $\nu/\Lambda$, thus displaying a perfect scaling with this ratio.

It is clear from Fig. 2 that the conditional survival probability $P_1(t)$ constantly increases with $\nu/\Lambda$, except for non-aligned levels, Fig. 2(b). There in some cases one can observe an acceleration of the inter-dot transition (the anti-Zeno effect). For instance, Fig. 2(b), shows that for $\nu = 0.5\Lambda$ and $t \sim 20(1/\Gamma)$, probability of the inter-dot transition increases with respect to no intermediate-measurement case (solid line). However, in the limit $\nu \to \infty$, one always recovers the Zeno effect.

On the other hand by increasing $\Lambda$ we diminish the effect of frequent measurements of the reservoir. Finally, in the limit of $\Lambda \to \infty$, $P_1(t)$ displays the unitary evolution, without any effect of frequent intermediate measurements. It means the Zeno effect disappears for reservoirs with infinite band-width (Markovian reservoirs).

A general behavior of the conditional survival probability and in particular the scaling in $\nu/\Lambda$, displayed in Fig. 2, is in full agreement with a simple explanation of the Zeno effect, based on the energy-time uncertainty relation. It can be illustrated by a following example. Consider continuous monitoring of electron oscillations between two dots with aligned or misaligned levels, Fig. 3. If the levels are aligned, $E_1 = E_2$, Fig. 3(a), the amplitude of the inter-well transitions is one. Each measurement, however, transfers the energy $\Delta E \sim 1/\Delta t$, that makes the levels misaligned. Then probability of the inter-dot transmission drops as $\sim \Omega_0^2/(\Delta E)^2$, where $\Omega_0$ is the Rabi frequency for the aligned levels. Thus, in the limit of continuous measurement, $\Delta t \to 0$, the electron will be permanently localized in the left dot.

In the case of misaligned levels, Fig. 3(b), the interaction with detector makes the levels partially in resonance for $\Delta t \sim 1/[E_2 - E_1]$. As a result, probability of the inter-dot transitions increases by the measurement, so that we find the anti-Zeno effect [1]. However, with the decrease of $\Delta t$, the amplitude of energy fluctuations increases, so that the energy levels are off the resonance again. As a result the Zeno effect will be always revealed if the measurement time is small enough.

A similar effect of the energy-time uncertainty relation due to continuous monitoring of the reservoir takes place in the setup of Fig. 3. Indeed, the probability of transmission to the reservoir is maximal when the dot’s levels are in resonance with the peak of the density of states ($E_r = 0$). If the reservoir is monitored with frequency $\nu$, Fig. 3(a), its energy spectrum is shifted up or down by $\Delta E = \nu$. Equivalently, for the setup, shown in Fig. 3(b), the energy levels of the both dots are simultaneously shifted by the same energy ($\nu$), whereas the reservoir spectrum remains unshifted by the measurements. As a result, the energy levels of the quantum dots are off-resonance with the density of state peak, so probability of the inter-dot transmission through the reservoir falls down. In the limit $\nu \to \infty$, the energy-shift becomes so large, that the corresponding density of states is zero. As a result, the electron remains locked in its initial state.

For Markovian reservoirs, however, the density of states is constant ($\Lambda \to \infty$). Therefore the shift of the reservoir spectrum (or the dots levels in Fig. 3(b)) by the measurement is irrelevant for the transition rates. As a result, we expect no Zeno effect at all [2]. This does not stay in a contradiction with its derivation, Eqs. (5)-(6), based on the assumption that the evolution operator can be expanded in powers of $\Delta t$. Indeed, this assumption is not valid for Markovian reservoirs, since the evolution operator is singular at $\Delta t = 0$. It appears, for instance in divergency of the coefficient $C$ in Eq. (5) for Markovian case.

One can easily recognize that the above arguments are totally consistent with the scaling in the ratio $\nu/\Lambda$, dis-
played in Fig. 4. Indeed, the density of reservoir states, Eq. (13), is a function of \((E_r/\Lambda)^2\), where the main contribution to the transition rates from the dots to reservoir is coming from \(E_r = E_{1,2}\). In the case of measurement, the energy levels in the reservoir (or in the dots) are shifted by \(|\Delta E| = \nu\). Then the transition rates depends on \((E_{1,2} \pm \nu)^2/\Lambda^2\). If the energy levels \(E_{1,2}\) are scaled in \(\Lambda\), one finds that probability of the inter-dot transition is a function of \(\nu/\Lambda\), in the agreement with Fig. 4.

It follows from our results that the informational evolution (quantum trajectory), which disregards the Zeno effect, is consistent with our analysis only for Markovian reservoirs. In the case of non-Markovian reservoirs, the measurement frequency together with the band-width should enter any equations for conditional transmission probabilities. We believe that the scaling in \(\nu/\Lambda\), which has a clear physical meaning, could be very useful for a possible extension of the quantum trajectory approach to the non-Markovian case [14].

We have demonstrated that the Zeno effect has nothing paradoxical in its nature by explaining it through the energy-time uncertainty relation. Indeed, the large energy transfer does not necessarily destabilize the system due to the short-time measurements. This could happen only if there exist available reservoir states with such large energies. Otherwise the system cannot move since any quantum transitions between states with largely different energies are strongly suppressed. For instance, it can take place for reservoirs with a finite band-with, Eq. (13). For Markovian reservoirs, however, the Zeno effect is not expected.

It is quite remarkable that precisely the absence of the Zeno effect can result in paradoxical behavior of a quantum system under continuous measurement. Indeed, consider again the setup in Fig. 3. Note that the two dots are connected only through the reservoir, Eq. (1). Nevertheless, for the Markovian case \((\Lambda \to \infty)\), the electron can make transitions between the two dots without any record in the reservoir, even though the latter is continuously monitored. This appears to be a “teleportation” phenomenon in its literal meaning, namely undetectable matter transfer between two distant places [8]. A similar phenomenon has been discussed earlier for different systems, but without continuous monitoring [2].

An experimental realization using the PC detector, shown in Fig. 3(b), looks very promising. This detector is proven very efficient for single-electron monitoring [3]. The measurement time can be varied by increasing the signal \((\Delta I)\) through increase of the voltage. Alternatively, one can use another measurement device, for instance a Single-Electron transistor [4]. In any case, a simultaneous monitoring of two dots in this type of experiments looks easier than continuous monitoring of the reservoir.

In conclusion, we have presented a quantum-mechanical analysis of electron transfer through a non-Markovian reservoir under continuous null-result monitoring. We found that the results differ from predictions of the Quantum Trajectory method, except in the Markovian case. This suggests that the Quantum Trajectory method should be modified for non-Markovian environments by including explicitly the measurement time. The latter should appear in combination with the reservoir band-width. We also proposed a simple explanation of Zeno effect without explicit use of the projection postulate. Finally we discussed the undetectable quantum transfer through a continuum and its relation to the Zeno effect.

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