$p - p'$ System with $B$ Field,  
Branes at Angles and Noncommutative Geometry

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Abstract

We study the generic $p - p'$ system in the presence of constant NS 2-form $B_{ij}$ field. We derive properties concerning with the noncommutativity of D-brane worldvolume, the Green functions and the spectrum of this system. In the zero slope limit, a large number of light states appear as the lowest excitations in appropriate cases. We are able to relate the energies of the lowest states after the GSO projection with the configurations of branes at angles. Through analytic continuation, the system is compared with the branes with relative motion.

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1This work is supported in part by the Grant-in-Aid for Scientific Research (10640268) and Grant-in-Aid for Scientific Research fund (97319) from the Ministry of Education, Science and Culture, Japan, and in part by the Japan Society for the Promotion of Science for Young Scientists.

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I. Introduction

There are several reasons to expect that noncommutative geometry may play a pivotal role in the developments of string theory. The developments which have already taken place include an effective worldvolume description of D-branes in terms of noncommuting coordinates [1]. One may mention the recent proposal by Connes, Douglas and Schwarz that, in the small volume limit, the compactification of Matrix theory on a torus with background $C_{-ij}$ should be described by noncommutative Yang-Mills theory on the dual torus [2, 3]. The T-duality in string theory is found to have a counterpart in noncommutative geometry called Morita equivalence [4]. Much work has been done, following these developments. (See [5] for references.) Let us mention here the work on instantons on noncommutative space. In order to remove the singularities of the instanton moduli space, one needs to introduce a nonvanishing parameter in the ADHM construction of instantons [6]. Nekrasov and Schwarz noticed that the ADHM construction of instantons on noncommutative space recovers this modification so that the smooth moduli of the instantons are the moduli space of instanton on noncommutative space [7].

These ideas have recently been extended by Seiberg and Witten [5]. Noncommutativity of D-brane worldvolumes has been shown to emerge from the open string Green function in the presence of constant rank-two antisymmetric background field $B_{ij}$. In fact, at the boundaries of an open string, the two-point Green function is

$$\langle x^i(\tau)x^j(\tau') \rangle = -\alpha' G^{ij} \log(\tau - \tau')^2 + \frac{i}{2} \theta^{ij} \epsilon(\tau - \tau') , \quad (1.1)$$

where $\tau \equiv |z|$, $G^{ij}$ is an open string metric and $\theta^{ij}$ is a noncommutative parameter measured by

$$[x^i(\tau), x^j(\tau)] = i\theta^{ij} . \quad (1.2)$$

As mentioned above, the D-brane worldvolume is a noncommutative space. In the zero slope limit $\alpha' \rightarrow 0$ while keeping the open string metric $G^{ij}$ and the noncommutative parameter $\theta^{ij}$ finite, the operator product becomes a $*$ product of noncommutative geometry, which is associative and translationally invariant. Properties of the D-branes can be described by an effective action on the noncommutative space. Several other topics have been investigated in [5].

In the D0-D4 system with $B_{ij}$ background, in particular, it has been found that a large number of light states emerge when $P f(B) < 0$. In the zero-slope limit, the system can be effectively considered as the configuration in which there is a continuous distribution of D0’s in the D4 worldvolume. The properties of the system can then be treated as those of D0-D0 pairs. The existence of a large number of light states could be interpreted as fluctuations of
the one end point of the open string stretching between two D0s, keeping the other end fixed. As the D0-D4 system without $B_{ij}$ field can be understood in terms of small instantons [8, 9], the low lying D0-D4 excitation spectrum with $B_{ij}$ should correspond to small fluctuations around point-like instantons. But now the instantons live in noncommutative $R^4$.

In this paper, we consider the generic $p-p'$ system in the presence of $B$ field. In the next section, we will derive the value of the parameter $\theta$ which measures noncommutativity, the Green function and the excitation spectrum. In [8], it has been shown that in the presence of constant $B_{ij}$ the D-brane worldvolume should be noncommutative. The analysis is based on the open string Green function in the background [14]. The end point of the open string probes the worldvolume structure of D-brane. This suggests that we may investigate the D$p$ worldvolume via D$p$-D$p'$ open string. We find that although the open string Green function is quite different from the one in eq.(1.1) where only D$p$-branes are considered, the noncommutative structure is the same. Our calculation gives a consistency check of the formalism: in D$p$-D$p'$ system, we can study the space structure either with D$p$-D$p$ open string or with D$p$-D$p'$ open string. The two approach should give the same answer. This might indicate the existence of a universal description of the noncommutativity of D$p$-brane worldvolumes rather than the one by the open string. We find that a number of light states emerge in the zero slope limit in some appropriate cases. Since the D0-D2 system in the presence of the $B$ field can be used as a building block of the Green functions and the description of the states in terms of the modes, we will discuss this case in some detail.

Another theme on the $p-p'$ system with constant $B$ field which we observe is its equivalence to the system of two D-branes at angles after T-duality. Configurations of D-branes at angles have been studied in [10, 11]. It has been noticed that in general such configurations break all of the supersymmetries and only for some special configurations a fraction of them survives. One such example is two D2 branes intersecting at right angles. Via T-duality, it is related to the D0-D4 system. We provide appropriate identifications between $\nu$ s in the $p-p'$ system originated from the canonical form of $B$ and angles in the T-dualized system, paying attention to the GSO projection and BPS configuration. It is found that the energies of the ground state after the GSO projection and the lowest excitations in the $p-p'$ system are closely related to the condition for supersymmetry in the system of branes at angles. The above identifications measure how far the system is from a BPS configuration. In the dual picture, the light states correspond to the small fluctuations around the BPS configuration. For example, in the case of D0-D4, under our identification, the energies of the lowest GSO states are proportional to $\phi_1 + \phi_2$. When $Pf(B) < 0$, the T-dualized picture is a D2 almost parallel to another D2, in the $\alpha' \to 0$ limit, and a large number of light states appear. And when $Pf(B) > 0$, the T-dualized picture is two almost anti-parallel D2 branes. The tachyon appears as the lowest state and has energy proportional to $\pi$. There are no light
states in the limit of $\alpha' \to 0$. The value of $\phi_1 + \phi_2$ tells us whether we are near or far from a BPS configuration. In fact, even without taking the zero slope limit, we can know from the dual picture in which case our \( p-p' \) system keeps a fraction of supersymmetries. In the case of D0-D4, as long as $B_{12} = -B_{34}$, the identification leads to $\phi_1 + \phi_2 = 0$, which is the condition for supersymmetry in the case of two D2 branes at angles, so that our system is BPS although the worldvolume of D4 now is noncommutative. In the case D0-D4 with $B_{ij}$, the BPS condition requires that $B_{12} = B_{34}$. The careful treatment of the generic D0-D$p$ ($p = 2, 4, 6, 8$) system will be discussed in section III.

In section IV, we compare our \( p-p' \) system with $B_{ij}$ background with two D-branes with relative motion. The dynamics of D-branes with relative motion has also been noted for some time \[15, 16\]. It has been related to an open string pair production in a constant electric background \[17\]. Therefore, for generic $B_{\mu\nu}$ background, the \( p-p' \) system can be treated either as the two D-branes with relative motion or as those with relative orientation depending upon whether the time direction is included or not.

The one-loop vacuum amplitude has been evaluated. The dependence on $\nu$ tells us in which case we have a nearly supersymmetric configuration. The effect of $B_{\mu\nu}$ can be seen from the investigation of either motion D-branes or D-branes at angles. This means the dynamics of D-branes come from the introduction of $B_{\mu\nu}$ field.

II. \( p-p' \) System with $B_{ij}$ Field

In \[5\], it has been shown that two D$p$-branes in the presence of constant $B_{ij}$ field can still preserve one-half of the supersymmetries and the excitations from an open string between the two D$p$-branes are unchanged. The only effect of $B$ field is to make its worldvolume noncommutative. In order to describe the D-brane worldvolume geometry, it is expedient to introduce the parameter $\theta_{ij}$ which measures noncommutativity and the open string metric $G_{ij}$. In the zero-slope limit, the tower of string excitations are projected out and we obtain an effective description in terms of finite $\theta$ and $G$. The low energy effective action of the D$p$-brane is super-Yang-Mills theory defined on a noncommutative space \[5\].

In the case of \( p-p' \) system, we find that supersymmetry can not be kept generically and
tachyons are ubiquitous. As $B_{ij}$ can be set to its canonical form

$$B_{ij} = \frac{\epsilon}{2\pi \alpha'} \begin{pmatrix} 0 & b_1 & 0 & 0 & 0 \\ -b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 & \vdots \\ 0 & 0 & -b_2 & 0 & \vdots \\ 0 & 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad (2.1)$$

and the space-time is flat with metric

$$g_{ij} = \epsilon \delta_{ij}, \quad (2.2)$$

we can reduce the problems of the mode expansions and the Green functions in the $p-p'$ system to those in the D0-D2 system. Let us focus on this case for a while.

We take the D2 worldvolume to extend in the $x^0, x^1, x^2$ directions. We define $Z \equiv x^1 + ix^2$. The boundary conditions on the D0-D2 bosonic open strings $Z$ and $\overline{Z}$ are

$$\partial_\sigma Z|_{\sigma=0} = \partial_\sigma Z + b \partial_\tau Z|_{\sigma=\pi} = 0, \quad (2.3)$$

The mode expansion of $Z$ and that of $\overline{Z}$ satisfying the above boundary conditions are

$$Z = i \sqrt{\frac{\alpha'}{2}} \sum_n (z^{n+\nu} - \overline{z}^{n+\nu}) \frac{\alpha_{n+\nu}}{n+\nu}, \quad (2.4)$$

$$\overline{Z} = i \sqrt{\frac{\alpha'}{2}} \sum_n (z^{-n-\nu} - \overline{z}^{-n-\nu}) \frac{\overline{\alpha}_{-n-\nu}}{n-\nu}, \quad (2.5)$$

where

$$e^{2\pi i \nu} = \frac{1 + ib}{1 - ib}, \quad 0 \leq \nu < 1, \quad (2.6)$$

and $z = e^{\tau + i \sigma}$ (Im$z \geq 0$). When $b$ is small or goes to $\pm \infty$, $\nu$ can be approximated by

$$\nu \approx \begin{cases} -\frac{1}{\pi b}, & b \to -\infty \\ \frac{1}{2} + \frac{b}{\pi}, & b \approx 0 \\ 1 - \frac{1}{\pi b}, & b \to \infty \end{cases} \quad (2.7)$$

From quantization, we know the commutation relation of $\alpha$ and $\overline{\alpha}$:

$$[\alpha_{n+\nu}, \overline{\alpha}_{-m-\nu}] = -\frac{2}{\epsilon} (n + \nu) \delta_{m+n}. \quad (2.8)$$

The oscillator ground state $|0\rangle$ implements the conditions

$$\alpha_{n+\nu}|0\rangle = 0, \quad n < 0, \quad (2.8)$$
\[ \bar{\alpha}_{m+\nu}|0\rangle = 0 \, , \quad m \geq 0 \, . \]  
(2.9)
in the current notation. (Notice the minus sign of the right hand of eq. (2.7)). We have two groups of creation operators, the one consisting of \( \alpha_{n+\nu} \) with \( n \geq 0 \) and the other of \( \bar{\alpha}_{m+\nu} \) with \( m < 0 \). We will find that the excitation energies are different between these two groups.

The Virasoro generators of the \( Z^{-Z} \) system are given by

\[ L_m = \frac{\epsilon}{2} \sum_{n} \alpha_{n+\nu} \bar{\alpha}_{m+n+\nu} \, . \]  
(2.10)
The ground state energy is

\[ E(\nu) = \sum_{n=1}^{\infty} (n - \nu) = \frac{1}{24} - \frac{1}{2} \left( \nu - \frac{1}{2} \right)^2 \, . \]  
(2.11)
From the eigenvalues of \( L_0 \) action, we can read off the excitation energies. The two lowest excitations are \( \alpha_{\nu} \) with its energy \( \nu \) and \( \bar{\alpha}_{-1+\nu} \) with its energy \( 1 - \nu \). Which one is the lowest depends on whether \( \nu \) is bigger or smaller than one-half, or equivalently on whether \( b \) is positive or negative.

We have the D0-D2 open string with one end on the D0 brane and the other on the D2 brane. To evaluate the parameter \( \theta \) which measures the noncommutativity on the D2 worldvolume, let us find the commutator between \( Z \) and \( \bar{Z} \) at the D2 endpoint. This leads us to calculate the two-point function between \( Z \) and \( \bar{Z} \). From the mode expansion eq. (2.4), we find

\[ \langle 0|Z(z)\bar{Z}(z')|0\rangle = -\frac{\alpha'}{2} \sum_{n=0}^{\infty} \frac{\alpha_{n+\nu}}{n+\nu} \left( z^{n+\nu} - z^{n+\nu} \right) \sum_{m>0} \frac{\bar{\alpha}_{m+\nu}}{m+\nu} \left( z^{-m-\nu} - z^{-m-\nu} \right) |0\rangle \]
\[ = -\frac{\alpha'}{\epsilon} \sum_{n=0}^{\infty} \frac{1}{n+\nu} \left[ \left( \frac{z}{z'} \right)^{n+\nu} - \left( \frac{z}{z'} \right)^{n+\nu} - \left( \frac{\bar{z}}{\bar{z}'} \right)^{n+\nu} + \left( \frac{\bar{z}}{\bar{z}'} \right)^{n+\nu} \right] \]  
(2.12)
where the infinite series can be written in terms of the hypergeometric function

\[ \Phi(z, 1, \nu) = \sum_{n=0}^{\infty} \frac{z^n}{n+\nu} = \nu^{-2} F_1(1, \nu; 1 + \nu; z) \, , \quad |z| < 1 \, . \]  
(2.13)
Similarly, we can obtain the two-point function \( \langle 0|\bar{Z}(z)Z(z')|0\rangle \). The commutator at the end point \( \sigma = \pi \) is

\[ [Z, \bar{Z}] = -\frac{\alpha'}{\epsilon} \sum_{n\in\mathbb{Z}} \frac{1}{n+\nu} \left( 2 - e^{2i\nu \pi} - e^{-2i\nu \pi} \right) = \frac{2\pi \alpha'}{\epsilon} \cdot \frac{2b}{1+b^2} \, , \]
\[ [x^1, x^2] = i \frac{2\pi \alpha'}{\epsilon} \cdot \frac{b}{1+b^2} \, . \]  
(2.14)
Eq. (2.14) can be compared with eq. (2.5) of [3]:
\[
\theta^{ij} = 2\pi\alpha' \left( \frac{1}{g + 2\pi\alpha'B} \right)^{ij}_A ,
\]
where \(( \ )_A\) denotes the antisymmetric part of the matrix. Substituting eqs. (2.1) and (2.2), one has
\[
\theta^{12} = \frac{2\pi\alpha' - \epsilon}{1 + b^2} .
\]
Eq. (2.14) and eq. (2.16) agree. The Green function of the Dp-Dp\'-system and that of the D0-D2 system have the same noncommutativity.

The open string metric \(G^{ij}\) on the D2 worldvolume can be extracted from the Green function evaluated at the end point \(\sigma = \pi\). We find
\[
\langle 0 | Z(z) Z(z') | 0 \rangle_{\sigma = \pi} = \frac{\alpha'}{\epsilon} \sum_{n=1}^{\infty} \frac{1}{n - \nu} \left( e^{\tau'} \right)^{n-\nu} \left( 2 - e^{2i\nu\pi} - e^{-2i\nu\pi} \right) .
\]
From the property
\[
\lim_{z \to 1} \Phi(z, 1, \nu) / [ -\log(1 - z) ] = 1 ,
\]
we find that the singular behaviour of eq. (2.17) as \(\tau \to \tau'\) is logarithmic:
\[
\langle 0 | Z(z) Z(z') | 0 \rangle_{\sigma = \pi} \sim -\frac{\alpha'}{2\epsilon} \left( 2 - e^{2i\nu\pi} - e^{-2i\nu\pi} \right) \log \left( e^{\tau} - e^{\tau'} \right)^2 ,
\]
\[
\langle 0 | x^1(z) x^1(z') | 0 \rangle_{\sigma = \pi} \sim -\frac{\alpha'}{4\epsilon} \left( 2 - e^{2i\nu\pi} - e^{-2i\nu\pi} \right) \log \left( e^{\tau} - e^{\tau'} \right)^2 .
\]
Eq. (2.19) can be compared with eq. (2.5) in [3]:
\[
G^{ij} = \left( \frac{1}{g + 2\pi\alpha'B} \right)^{ij}_S ,
\]
where \(( \ )_S\) denotes the symmetric part of the matrix. From eqs. (2.1) and (2.2) we obtain
\[
G^{11} = G^{22} = \frac{1}{\epsilon(1 + b^2)} .
\]
Comparing this with the prefactor of the logarithm in eq. (2.19), we find that the open string metric obtained in our system is exactly the same as that in [3].

We conclude that the noncommutativity of the worldvolumes of the D-branes and the open string metric can both be probed, using the generic \(p-p'\) system.

\(^{3}\)Please note that the time variables \(\tau\) and \(\tau'\) in [3] mean respectively \(e^{\tau}\) and \(e^{\tau'}\) in this section.
Let us now turn to the worldsheet fermions in the NSR formalism. First, define
\[ \Psi = \psi^1 + i\psi^2, \quad \bar{\Psi} = \psi^1 - i\psi^2, \]
(2.22)
\[ \Phi = \bar{\psi}^1 + i\bar{\psi}^2, \quad \bar{\Phi} = \bar{\psi}^1 - i\bar{\psi}^2. \]
(2.23)

The boundary conditions for the worldsheet fermions can be determined by demanding the worldsheet supersymmetry. We find that, in the Ramond sector, the worldsheet fermions obey
\[ \Psi + \Phi|_{\sigma=0} = (\Psi - \Phi) - ib(\Psi + \Phi)|_{\sigma=\pi} = 0, \]
(2.24)
\[ \bar{\Psi} + \bar{\Phi}|_{\sigma=0} = (\bar{\Psi} - \bar{\Phi}) + ib(\bar{\Psi} + \bar{\Phi})|_{\sigma=\pi} = 0. \]
(2.25)

The mode expansion on the upper half plane is
\[ \Psi = \sum_{n \in \mathbb{Z}} d_{n+\nu} z^{n+\nu - \frac{1}{2}}, \quad \Phi = - \sum_{n \in \mathbb{Z}} d_{n+\nu} z^{n+\nu - \frac{1}{2}}, \]
(2.26)
\[ \bar{\Psi} = \sum_{n \in \mathbb{Z}} \bar{d}_{-n+\nu} z^{-n+\nu - \frac{1}{2}}, \quad \bar{\Phi} = - \sum_{n \in \mathbb{Z}} \bar{d}_{-n+\nu} z^{-n+\nu - \frac{1}{2}}. \]
(2.27)

The total ground state energy vanishes in the Ramond sector due to the cancellation between bosons and fermions. As in the bosonic case, the excitations come from two types of oscillators $d$ and $\bar{d}$, and the two lowest ones have energy $E = \nu$ and $E = 1 - \nu$ respectively.

In the NS sector, the boundary conditions change to
\[ \Psi + \Phi|_{\sigma=0} = (\Psi + \Phi) - ib(\Psi - \Phi)|_{\sigma=\pi} = 0, \]
(2.28)
\[ \bar{\Psi} + \bar{\Phi}|_{\sigma=0} = (\bar{\Psi} + \bar{\Phi}) + ib(\bar{\Psi} - \bar{\Phi})|_{\sigma=\pi} = 0. \]
(2.29)

The mode expansion on the upper half plane reads
\[ \Psi = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_{r+\nu} z^{r+\nu - \frac{1}{2}}, \quad \Phi = - \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_{r+\nu} z^{r+\nu - \frac{1}{2}}, \]
(2.30)
\[ \bar{\Psi} = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \bar{b}_{-r+\nu} z^{-r+\nu - \frac{1}{2}}, \quad \bar{\Phi} = - \sum_{r \in \mathbb{Z} + \frac{1}{2}} \bar{b}_{-r+\nu} z^{-r+\nu - \frac{1}{2}}. \]
(2.31)

The contribution to the ground state energy from the NS fermions is $-E(\nu - \frac{1}{2})$ and in general cannot cancel the bosonic contribution. Two kinds of excitations can be ordered according to the energies they carry:
\[ \nu > \frac{1}{2} : \nu - \frac{1}{2}, \frac{3}{2} - \nu, \frac{1}{2} + \nu, \frac{5}{2} - \nu, \frac{3}{2} + \nu, \cdots. \]
(2.32)
\[ \nu < \frac{1}{2} : \frac{1}{2} - \nu, \frac{1}{2} + \nu, \frac{3}{2} - \nu, \frac{3}{2} + \nu, \frac{5}{2} - \nu, \cdots. \]
(2.33)
The total ground state energy in the NS sector is
\[ E_0 = 3E(0) - 3E\left(\frac{1}{2}\right) + E(\nu) - E\left(\left|\nu - \frac{1}{2}\right|\right) \]
\[ = -\frac{1}{4} - \frac{1}{2}\left|\nu - \frac{1}{2}\right| \]
\[ = \begin{cases} 
-\frac{1}{2}\nu & \nu > \frac{1}{2} \\
\frac{1}{2}\nu - \frac{1}{2} & \nu < \frac{1}{2} \end{cases} \]  
(2.34)

The first excited state has an energy
\[ E_1 = E_0 + \left|\nu - \frac{1}{2}\right| = -\frac{1}{4} + \frac{1}{2}\left|\nu - \frac{1}{2}\right| . \]  
(2.35)

We rewrite the energies of these two states collectively as
\[ E^\pm = -\frac{1}{4} \pm \frac{1}{2}\left(\nu - \frac{1}{2}\right) , \]  
(2.36)

i.e.
\[ E^+ = \frac{1}{2}(\nu - 1) , \quad E^- = -\frac{1}{2}\nu \]  
(2.37)

Due to the GSO projection, only one of these two states survive. We determine which state we should project out in the following way. As we will discuss later, D0-branes are induced on the D2-brane worldvolume in the zero-slope limit [3]. When \( Pf(B) < 0 \), we identify them with D0’s not \( \overline{D0} \)’s, following the argument in [3]. Using this definition of the D0-brane, we take the GSO projection so that the system becomes D0-D2 not \( \overline{D0} \)-D2. In the other cases which we will investigate in the following, we will take the GSO projection in the same manner.

By definition, \( E^- \) is chosen as the ground state after the GSO projection while \( E^+ \) is left out in the entire region of \( \nu : 0 \leq \nu < 1 \). We can restate this by saying that, when \( b \) is negative, we keep \( E_1 \) and leave \( E_0 \) out and that, when \( b \) is positive, we keep \( E_0 \) and leave \( E_1 \) out. From this latter point of view, it can be said that, in the two regimes of \( \nu \), the GSO projections are opposite. We will see that the pattern of classification of states seen in eq. (2.37) will hold in the generic \( p-p' \) system.

For small positive \( b \),
\[ E_0 \approx -\frac{1}{2}\left(\frac{1}{2} + \frac{b}{\pi}\right) < 0 , \]  
(2.38)

and we have a tachyonic state after the GSO projection. For small negative \( b \), we have
\[ E_0 = \frac{1}{2}\nu - \frac{1}{2} \approx -\frac{1}{4} + \frac{b}{2\pi}; \quad E_1 \approx -\frac{1}{2}\left(\frac{1}{2} + \frac{b}{\pi}\right) , \quad E_2 \approx 3\left(\frac{1}{2} + \frac{b}{\pi}\right) . \]  
(2.39)
The ground state is projected out and the part of the remaining first excitations is still tachyonic.

We are, however, more interested in the situation in the zero slope limit, \( i.e. |b| \to \infty \) limit. We would like to know if some light states survive. This is in parallel to the discussion of D0-D4 system in \[5\]. We will see that a large number of light states appear for \( b \) negative.

As \( b \to +\infty \), the ground state energy becomes

\[
E_0 \approx -\frac{1}{2} \left( 1 - \frac{1}{\pi b} \right),
\]

being of order 1. Clearly this is tachyonic and is the same order as the energy of the tachyon in the D0-D\( \overline{D}0 \) system. In the language of \[5\], the large positive \( b \) induces a large number of D0s in the D2, and the D0 can annihilate one of these D0s. All of the one-particle excitations are projected out and the lowest two-particle excitation has energy

\[
E = E_0 + 1 = \frac{1}{2} \left( 1 - \frac{1}{\pi b} \right),
\]

which is positive, and also of order 1. This state cannot become light in the zero-slope limit. We conclude that our system has no light state for \( b \) positive.

For \( b \to -\infty \), we have a different story. The first two one-particle excitations have energies

\[
E_1 \approx \frac{1}{2\pi b}, \quad E_2 \approx -\frac{3}{2\pi b},
\]

and their mass squared are finite as \( \alpha' \to 0 \). Apart from these two light states, we obtain eight more states by acting on \(|0\rangle\) with the creation operators which carry the lowest energy \( \frac{1}{2} \) and which are obtained from the NS fermion partners of \( x^{0,3,\ldots,9} \). These states have energies

\[
E_i \approx -\frac{1}{2\pi b},
\]

surviving the \( \alpha' \to 0 \) limit. We can further act on these states with an arbitrary polynomial consisting of \( \alpha_{\nu} \) with its energy \( \nu = -\frac{1}{\pi b} \). This also leads to a finite energy state in the zero-slope limit. We conclude that, for \( b \) negative, we have a large number of light states in the \( \alpha' \to 0 \) limit. The one end of the open string is fixed and the other end can be located anywhere in the D2 worldvolume. We have a continuous distribution of D0s in D2.

The appearance of a large number of light states in the zero-slope limit for \( b \) negative can also be understood in the framework of two D-branes intersecting at angles. As we will show in the next section, when \( \nu \) is about zero, the corresponding angle is about zero. We have a configuration of two D1 branes with very small relative angle, which is nearly supersymmetric. The excitations are almost massless.
This completes our discussion of the D0-D2 system with $B_{ij}$ background. The D0-D4 system with $B_{ij}$ background has been discussed in [5] and we will recapitulate this case very briefly for the sake of completeness. The four lowest energy states including the ground state have energies

$$E^+ = \pm \frac{1}{2} (\nu_1 + \nu_2 - 1), \quad E^- = \pm \frac{1}{2} (\nu_1 - \nu_2).$$

By definition, the GSO projection keeps the states with energies $E^+$ and leaves out the states with energies $E^-$ in the entire region of $\nu_1$ and $\nu_2$. When $P_f(B) > 0$, the D0-D4 system is tachyonic. The tachyon mass squared is of order $1/\alpha'$ in the $\alpha' \to 0$ limit. It implies that this is a standard D0-D0 tachyon. When $P_f(B) < 0$, and after the GSO projection, the lowest state and the first three excitations survive the $\alpha' \to 0$ limit. As in the D0-D2 case, the one-particle excitations from the lowest fermionic partners of the other directions and the multiparticle excitations of arbitrary varieties consisting of the lowest bosonic mode give light states in the zero-slope limit. We have a large number of light states here.

Let us now turn to the D0-D6 case. In the D0-D6 system with $B_{ij}$ field, the ground state in the NS sector has energy

$$E_0 = \frac{1}{4} - \frac{1}{2} \left( |\nu_1 - \frac{1}{2}| + |\nu_2 - \frac{1}{2}| + |\nu_3 - \frac{1}{2}| \right).$$

Each excitation contributes to the energy by $|\nu_i - \frac{1}{2}|$. The energies of the lowest excitations are

$$E = \frac{1}{4} \pm \frac{1}{2} |\nu_1 - \frac{1}{2}| \pm \frac{1}{2} |\nu_2 - \frac{1}{2}| \pm \frac{1}{2} |\nu_3 - \frac{1}{2}|.$$  \hspace{1cm} (2.46)

In fact, these eight states may be classified into two groups each consisting of four states. The one group has energies

$$E^+ = \left\{ \frac{1}{2}(-\nu_1 - \nu_2 - \nu_3 + 2), \frac{1}{2}(\nu_1 + \nu_2 - \nu_3), \frac{1}{2}(\nu_1 - \nu_2 + \nu_3), \frac{1}{2}(-\nu_1 + \nu_2 + \nu_3) \right\},$$

and the other group has energies

$$E^- = \left\{ \frac{1}{2}(\nu_1 - \nu_2 - \nu_3 + 1), \frac{1}{2}(-\nu_1 + \nu_2 - \nu_3 + 1), \frac{1}{2}(-\nu_1 - \nu_2 + \nu_3 + 1), \frac{1}{2}(\nu_1 + \nu_2 + \nu_3 - 1) \right\}. \hspace{1cm} (2.48)$$

By definition, the GSO projection keeps the states with energies $E^+$ and projects out the states with energies $E^-$. With this choice, we see if some light states exist.
When $Pf(B)$ is positive, namely, either all of $b_i$ s are positive, or two are negative and the one is positive, $E^+$ is about $\pm \frac{1}{2}$. We have a tachyonic state and three excited states, all with mass squared being of order 1 in the unit of $1/\alpha'$. These states do not survive the $\alpha' \to 0$ limit. When $Pf(B)$ is negative, we have two possibilities: either all of $b_i$ s are negative, or two of them are positive and the one is negative.

In the former case,

$$E^+ \approx \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} + \frac{1}{\pi b_3} + 2 \right),$$

in the zero-slope limit. The first state in eq. (2.49) coming from the three-particle excitation is very heavy and the remaining three states coming from the one-particle excitations are light. In fact, we have three more light states coming from the one-particle excitations which carry energies

$$\frac{1}{2} \left( -\frac{3}{\pi b_1} - \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( -\frac{1}{\pi b_1} - \frac{3}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{3}{\pi b_3} \right).$$

Similarly, the lowest modes of the NS fermions from the directions without $B_{ij}$ carry energy $\frac{1}{2}$. These give rise to light states with energy

$$E_i = \frac{1}{2} (\nu_1 + \nu_2 + \nu_3) \approx \frac{1}{2} \left( -\frac{1}{\pi b_1} - \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right).$$

One can also act on these states with an arbitrary polynomial consisting of the lowest bosonic creation operators $\alpha_{\nu_i} (i = 1, 2, 3)$. This gives us a large number of states with finite energy in the $\alpha' \to 0$ limit as each $\alpha_{\nu_i}$ has energy $\nu_i \approx -\frac{1}{\pi b_i}$.

In the latter case, we may take $b_1, b_2$ positive and $b_3$ negative without losing generality. We obtain

$$E^+ \approx \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + 3 \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - 3 \frac{1}{\pi b_3} \right).$$

Obviously the second state comes from the three-particle excitations and is very heavy. The other three states are light. Three more light states come from the one-particle excitations with energies

$$\frac{1}{2} \left( \frac{3}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{3}{\pi b_2} - \frac{1}{\pi b_3} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{3}{\pi b_3} \right).$$

As in eq. (2.51), we have

$$E_i \approx \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} \right).$$
The lowest bosonic modes are $\alpha_{-1+\nu_1}, \alpha_{-1+\nu_2}$ and $\alpha_{\nu_3}$, each of which carries energy
\[ \frac{1}{\pi b_1}, \quad \frac{1}{\pi b_2}, \quad \text{and} \quad -\frac{1}{\pi b_3} \] respectively. These again give rise to a large number of light states. We conclude that, in the $Pf(B) < 0$ case, there always exist a large number of light states in the $\alpha' \to 0$ limit.

The D0-D8 system with $B_{ij}$ field can be studied in the same way. In the NS sector, the ground state energy is
\[ E_0 = \frac{1}{2} - \frac{1}{2} \sum_{i=1}^{4} |\nu_i - \frac{1}{2}|. \] and the lowest excitations have energies
\[ E = \frac{1}{2} + \frac{1}{2} \sum_{i=1}^{4} (\pm) |\nu_i - \frac{1}{2}|. \] These sixteen states including the ground state can be classified into two groups each consisting of eight states. The energies are
\[
E^+ = \begin{cases}
\frac{1}{2}(-\nu_1 - \nu_2 - \nu_3 - \nu_4 + 3), \\
\frac{1}{2}(\nu_1 + \nu_2 - \nu_3 - \nu_4 + 1), \\
\frac{1}{2}(\nu_1 - \nu_2 + \nu_3 - \nu_4 + 1), \\
\frac{1}{2}(\nu_1 - \nu_2 - \nu_3 + \nu_4 + 1), \\
\frac{1}{2}(-\nu_1 + \nu_2 + \nu_3 - \nu_4 + 1), \\
\frac{1}{2}(-\nu_1 + \nu_2 - \nu_3 + \nu_4 + 1), \\
\frac{1}{2}(\nu_1 + \nu_2 + \nu_3 + \nu_4 + 1), \\
\frac{1}{2}(\nu_1 + \nu_2 + \nu_3 + \nu_4 - 1), \\
\end{cases} 
E^- = \begin{cases}
\frac{1}{2}(\nu_1 - \nu_2 - \nu_3 - \nu_4 + 2), \\
\frac{1}{2}(-\nu_1 + \nu_2 - \nu_3 - \nu_4 + 2), \\
\frac{1}{2}(-\nu_1 - \nu_2 + \nu_3 - \nu_4 + 2), \\
\frac{1}{2}(\nu_1 - \nu_2 - \nu_3 + \nu_4 + 2), \\
\frac{1}{2}(\nu_1 + \nu_2 + \nu_3 - \nu_4), \\
\frac{1}{2}(\nu_1 + \nu_2 - \nu_3 + \nu_4), \\
\frac{1}{2}(\nu_1 - \nu_2 + \nu_3 + \nu_4), \\
\frac{1}{2}(-\nu_1 + \nu_2 + \nu_3 + \nu_4), \\
\end{cases}. \] The GSO projection selects the states with energies $E^+$. It is not difficult to find that, for $Pf(B) > 0$, we have no light state and the mass squared of the tachyonic state is of order 1 in the unit of $1/\alpha'$. When $Pf(B) < 0$, we always have a large number of light states in the $\alpha' \to 0$ limit. Let us first consider the case with one positive and three negative $b_i$'s. Taking $b_1 > 0$ and $b_2, b_3, b_4 < 0$, one can find eight light states in the lowest excitations with energies:
\[
\frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} + \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{1}{\pi b_1} - \frac{1}{\pi b_2} + \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} - \frac{1}{\pi b_2} + \frac{1}{\pi b_3} + \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{3}{\pi b_1} - \frac{1}{\pi b_2} - \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \quad \frac{1}{2} \left( \frac{3}{\pi b_1} - \frac{1}{\pi b_2} - \frac{1}{\pi b_3} + \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{1}{\pi b_1} - \frac{1}{\pi b_2} - \frac{3}{\pi b_3} - \frac{1}{\pi b_4} \right), \quad \frac{1}{2} \left( \frac{1}{\pi b_1} - \frac{1}{\pi b_2} - \frac{3}{\pi b_3} + \frac{3}{\pi b_4} \right). \]
From the bosonic sector, we find the single particle excitations with energies
\[ 1 - \nu_1 \approx \frac{1}{\pi b_1}, \quad -\frac{1}{\pi b_2}, \quad -\frac{1}{\pi b_3}, \quad -\frac{1}{\pi b_4}. \] (2.60)

For the case with three positive and one negative \( b \)'s, for example, \( b_1, b_2, b_3 > 0, b_4 < 0 \), we have
\[
\begin{align*}
\frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} + \frac{1}{\pi b_3} + \frac{1}{\pi b_4} \right), & \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} - \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{1}{\pi b_1} - \frac{1}{\pi b_2} + \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), & \quad \frac{1}{2} \left( -\frac{1}{\pi b_1} + \frac{1}{\pi b_2} + \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{3}{\pi b_1} + \frac{1}{\pi b_2} + \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), & \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{3}{\pi b_2} + \frac{1}{\pi b_3} - \frac{1}{\pi b_4} \right), \\
\frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} + \frac{3}{\pi b_3} - \frac{1}{\pi b_4} \right), & \quad \frac{1}{2} \left( \frac{1}{\pi b_1} + \frac{1}{\pi b_2} + \frac{1}{\pi b_3} - \frac{3}{\pi b_4} \right). 
\end{align*}
\] (2.61)

From the bosonic sector, we obtain the one-particle excitations with energies
\[ \frac{1}{\pi b_1}, \quad \frac{1}{\pi b_2}, \quad \frac{1}{\pi b_3}, \quad -\frac{1}{\pi b_4}, \] (2.62)
which give rise to a large number of light states in the \( \alpha' \to 0 \) limit.

**III. The Relation with the Branes at Angles**

It has been realized for some time that the system of the two D-branes at angles is T-dual to the \( p-p' \) system with background \( \mathcal{F} = F + B \) \[10, 11\]. For example, in the case of D2-D2 at angles, the BPS configuration is T-dual either to the D0 bound to D4 as a self-dual instanton or to the D0 bound to D4 as an anti-self-dual instanton. Now working on the \( p-p' \) system in the presence of \( B_{ij} \) field, we reconsider its relation with the configuration of the branes at angles, focusing on the BPS configuration, the existence of light states and the moduli of noncommutative instantons. The properties of the system of branes at angles have also been discussed in \[12\].

In the generic \( p-p' \) system with \( B_{ij} \) field, the boundary condition of the \( p-p' \) open string is
\[ \partial_{\tau} x^i|_{\sigma=0} = g_{ij} \partial_{\sigma} x^j + 2\pi i \alpha' B_{ij} \partial_{\tau} x^j|_{\sigma=\pi} = 0. \] (3.1)

Let us first consider the D0-D2 case, \( i.e. i, j = 1, 2 \), as the boundary conditions of the generic \( p-p' \) system reduce to those of this case. Via T-duality, we will relate this case to the system of two D1-branes intersecting at an angle.
We study an open string ending on two D1-branes at an angle $\phi$ in the $(x^1, x^2)$-plane. We take one of them to be aligned along the $x^1$ direction. In terms of the coordinates $Z = x^1 + ix^2$, the boundary conditions are [13]

$$\sigma = 0, \quad \partial_\sigma \text{Re}(Z) = \partial_\tau \text{Im}(Z) = 0, \quad \sigma = \pi, \quad \partial_\sigma \text{Re} [\exp(i\phi)Z] = \partial_\tau \text{Im} [\exp(i\phi)Z] = 0. \quad (3.2)$$

Writing in components, eq. (3.2) becomes

$$\sigma = 0, \quad \partial_\sigma x^1 = 0, \quad \partial_\tau x^2 = 0, \quad \sigma = \pi, \quad \partial_\sigma (x^2 + bx^1) = 0, \quad \partial_\tau (bx^2 - x^1) = 0, \quad (3.3)$$

where

$$\tan \phi = -\frac{1}{b}. \quad (3.4)$$

After taking T-duality in the $x^1$ direction

$$\partial_\sigma x^1 \leftrightarrow -i \partial_\tau x^1, \quad (3.5)$$

the boundary condition becomes identical to that of the D0-D2 system with $B_{ij}$ [13], which we wanted to show.

From eq. (3.4), we obtain

$$\phi = \begin{cases} \nu \pi, \\ \nu \pi - \pi \end{cases}. \quad (3.6)$$

This means that we have two choices of angles corresponding to the same $b$ or $\nu$. Which identification we should adopt depends on the ways the GSO projection is made. For the systems of D$p$-D$p$ at angles, we take the GSO projection so that when all the angles $\phi_i$ vanish we have supersymmetric systems, i.e. the systems of D$p$-D$p$ not D$p$-D$p$. We have to fix the identification between the angles $\phi_i$ and $\nu_i$ in such a way as the situation is consistent with the GSO projections taken in the last section.

In what follows we will fix the relations between the angles and $\nu$’s for all cases discussed in the last section. We will show that the energies of the ground states surviving the GSO projection that we obtained in the last section are described in the forms related to the BPS conditions for the systems of the D-branes at angles.

Let us look at the D0-D2 system more closely. Taking T-duality, we obtain D1-D1 at an angle $\phi$. Since the system must be supersymmetric when $\phi = 0$, the GSO projection (2.37) tells us to identify $\phi$ with $\nu$ in the following way,

$$\phi = \nu \pi. \quad (3.7)$$
It follows that the energy $E^-$ in (2.37) of the state surviving the GSO projection is expressed as

$$E^- = -\frac{1}{2\pi} \phi .$$

(3.8)

For $b < 0$ we have $\phi \approx 0$ in the zero-slope limit. The system becomes almost supersymmetric, *i.e.* the D1-D1 system, and the ground state is nearly massless. The bosonic fluctuations generated by $\alpha_\nu$'s with energy $\nu = \phi/\pi \approx 0$ give us a large number of light states in this limit. On the other hand, for $b > 0$ we have $\phi \approx \pi$. The system becomes non-supersymmetric, *i.e.* almost D1-\(\overline{\text{D}1}\) system, and the ground state becomes tachyonic.

As we pointed out in the last section, when $\nu$ varies from 0 to 1 the level crossing happens between the ground state and the first excited state. This yields the apparent flip of the GSO projection. Combining this fact and the identification (3.7), we are able to explain how we can convert the D1-D1 system ($\phi = 0$) continuously to the D1-\(\overline{\text{D}1}\) system ($\phi = \pi$) while they have the GSO projections opposite to each other.

As the next example, let us consider the D0-D4 case with $B$ field. When we take T-duality in the $x^1$- and the $x^3$-directions, the system becomes D2-D2 at angles $\phi_1$ and $\phi_2$ in $(x^1, x^2)$- and $(x^3, x^4)$-planes respectively. For the consistency with the GSO projection (2.44) we should identify the angles with $\nu$'s as follows:

$$\phi_1 = \nu_1 \pi, \quad \phi_2 = \nu_2 \pi - \pi .$$

(3.9)

Among the supersymmetry conditions for the system of D2-D2 at angles

$$\phi_1 + \phi_2 \equiv 0, \quad \phi_1 - \phi_2 \equiv 0 \pmod{2\pi} ,$$

(3.10)

the former equation is that for the present D0-D4 system. The energies $E^\pm_\pm$ in (2.44) of the states surviving the GSO projection are expressed as

$$E^\pm_\pm = \pm \frac{1}{2\pi} (\phi_1 + \phi_2) .$$

(3.11)

For $P f(B) > 0$ we have $\phi_1 + \phi_2 \approx \pm \pi$ in the zero-slope limit. We have an almost D2-D2 system and the ground state is tachyonic. For $P f(B) < 0$ we obtain $\phi_1 + \phi_2 \approx 0$ in the zero-slope limit. We obtain an almost D1-D1 system. The lowest energy states are almost massless and the bosonic fluctuations with energies proportional to the very small angles give us many light states in this limit.

On the other hand, if we consider the D0-D4 system, we should choose the GSO projected states with energies $E^\pm_\pm = \pm \frac{1}{2} (\nu_1 - \nu_2)$, which is proportional to $\phi_1 - \phi_2$ under identification $\phi_i = \nu_i \pi$. Note that the condition for supersymmetry now turns out to be the latter equation in eq. (3.10). Therefore, only when $P f(B) > 0$, we have a large number of light states coming
from the $D0-\bar{D}0$ pairs. It can be interpreted as the fluctuations around the supersymmetric configuration in the picture of branes at angles.

Another point which is worth mentioning is that the supersymmetric condition, either of the form of $\phi_1 + \phi_2 = 0$ for D0-D4 or of the form of $\phi_1 - \phi_2 = 0$ for $D0-\bar{D}0$, is closely related to the moduli of instantons or anti-instantons on noncommutative $R^4$. As argued in [3], in the case of D0-D4 with $ Pf(B) < 0$, the tachyon mass squared vanishes when $b_1 + b_2 = 0$ and the system is supersymmetric. From the dual picture of branes at angles, this correspond to $\phi_1 + \phi_2 = 0$ under our identification (3.9). As for $D0-\bar{D}0$, when $b_1 = b_2$, the system becomes supersymmetric and BPS, representing a point on noncommutative anti-instanton moduli space.

Next we consider the D0-D6 system with $B$ field. When we take T-duality in three of the spatial directions along the D6-brane worldvolume, we obtain the system of D3-D3 at angles $\phi_1$, $\phi_2$ and $\phi_3$. By repeating the considerations on supersymmetry and the GSO projection similar to those of the D0-D2 and the D0-D4 cases, we find that we should choose one of the following identifications:

$$\phi_i = \nu_i \pi \quad (i = 1, 2, 3),$$

and its permutations. In both identifications, the energies $E^+$ of the states surviving the GSO projection, listed in (2.47), are expressed as

$$2\pi E^+ \equiv \begin{cases} (-\phi_1 - \phi_2 - \phi_3) \\ (\phi_1 + \phi_2 - \phi_3) \\ (\phi_1 - \phi_2 + \phi_3) \\ (-\phi_1 + \phi_2 + \phi_3) \end{cases} \pmod{2\pi}.$$

From this we find that when the states become massless, the condition that supersymmetry for the system of D3-D3 at angles be unbroken is satisfied. This means that at least one of the following equations holds:

$$\phi_1 \pm \phi_2 \pm \phi_3 \equiv 0 \pmod{2\pi}.$$

Finally let us consider the case of D0-D8. Taking T-duality in four of the spatial directions along the D8-brane worldvolume, we obtain the system of the D4-D4 at angles $\phi_i$ ($i = 1, \ldots, 4$). In this case we should choose one of the following identifications:

$$\phi_1 = \nu_1 \pi - \pi, \quad \phi_i = \nu_i \pi \quad (i = 2, 3, 4),$$

$$\phi_1 = \nu_1 \pi, \quad \phi_i = \nu_i \pi - \pi \quad (i = 2, 3, 4).$$
and the permutations of them. As in the three previous cases, supersymmetry is partially recovered when some states of energy $E^+$ listed in eq. (2.58) become massless. Therefore at least one of the following conditions for supersymmetry is satisfied:

$$
\phi_1 + \phi_2 + \phi_3 + \phi_4 \equiv 0, \quad (\text{mod } 2\pi),
\phi_1 + \phi_2 - \phi_3 - \phi_4 \equiv 0, \quad (\text{mod } 2\pi),
\phi_1 - \phi_2 - \phi_3 + \phi_4 \equiv 0, \quad (\text{mod } 2\pi),
\phi_1 - \phi_2 + \phi_3 - \phi_4 \equiv 0, \quad (\text{mod } 2\pi).
$$

(3.16)

Obviously, for $P f(B) < 0$ a large number of light states appear in the zero-slope limit and for $P f(B) > 0$ the system is not supersymmetric and the T-dualized picture is almost D4-D4 pair.

### IV. One-Loop Amplitudes and the Branes with Relative Motion

In this section we will relate the $p$-$p'$ system in the presence of $B_{ij}$ field with moving D-branes. As is well known, the system of the D-branes with relative motion has a connection with D-branes in a constant electric background [15, 17, 12]. In the last section we considered the D-branes in a constant magnetic field, which can be related to D-branes at angles by T-duality. Therefore in a generic $B_{\mu\nu}$ background, the $p$-$p'$ system can be either thought of as D-branes with relative motion or the ones with relative orientation. This fact reminds us of the motion of a charged particle in the presence of constant electric field or magnetic field.

Suppose that the two D-branes move with a relative velocity $v$. The mode expansion of an open string satisfying the appropriate boundary condition has almost the same form as eq. (2.4) but with a pure imaginary parameter:

$$
i\epsilon \equiv \frac{\arctan(v)}{\pi} = \nu,
$$

(4.1)

where $\nu$ is the parameter in the mode expansion (2.4). This identification shows that the two problem can be treated in one way.

As an illustration, let us consider the open string one-loop vacuum amplitude for the D0-D2 case. It takes the form

$$
A \sim \int_0^\infty \frac{d(t_{NN+1})}{(2\pi)^{t_{NN+1}}} \sum_i \int_0^\infty \frac{dt}{t} \exp \left[ -2\pi\alpha' t \left( k^2 + M_i^2 \right) \right]
$$

$$
= \sum_i \int_0^\infty \frac{dt}{t} \left( 32\pi^2 \alpha' t \right)^{-\frac{1}{2}} \exp \left( -2\pi\alpha' t M_i^2 \right),
$$

(4.2)

where

$$
M_i^2 = \frac{y^2}{4\pi^2 \alpha'^2} + \frac{1}{\alpha'} \sum \text{(oscillators)},
$$

(4.3)
with \( y \) being the distance between two D-branes. We denote by \( \sharp NN \) the number of directions in which both ends of the open string obey the Neumann boundary condition. Here \( \sharp NN = 0 \). Then the amplitude is

\[
A \sim \int_0^\infty \frac{dt}{t} \left( 32\pi^2 \alpha' t \right)^{-\frac{1}{2}} \exp \left( -\frac{ty^2}{2\pi\alpha'} \right) \cdot B \times F ,
\]

with

\[
B = (\eta(it))^{-6} \left\{ -i e^{\nu^2\pi t} \frac{\eta(it)}{\vartheta_{11}(i\nu t; it)} \right\} ,
\]

\[
F = \frac{1}{2} \left\{ \prod_{a=1}^4 Z_0^0(\nu_a; it) - \prod_{a=1}^4 Z_1^0(\nu_a; it) - \prod_{a=1}^4 Z_1^1(\nu_a; it) \right\} ,
\]

\[
= \left[ Z_1^1 \left( \nu; \frac{i}{2} it \right) \right]^4 ,
\]

where \( \nu = \nu_1 \neq 0 \), and \( \nu_{2,3,4} = 0 \). \( B \) is the contribution from the bosonic oscillators and \( F \) is the one from fermionic parts. We have introduced

\[
Z^\alpha_{\beta}(\nu; it) = \frac{\vartheta_{\alpha\beta}(i\nu t; it)}{e^{\nu^2\pi t} \eta(it)} .
\]

Let us focus on the long range potential between the branes. We approximate eq. (4.4) in the limit of \( \alpha' \to 0 \) and small \( t \). Using the modular property of Jacobi function, we find that

\[
\vartheta_{11}(i\nu t; it) = -it^{-\frac{1}{2}} \exp(\nu^2\pi t) \vartheta_{11} \left( \nu; \frac{i}{2} t \right) ,
\]

\[
\vartheta_{11} \left( \nu; \frac{i}{2} t \right) = -2q^{\frac{1}{4}} \sin(\nu\pi) \prod_m (1 - q^m) (1 - zq^m) (1 - z^{-1} q^m) ,
\]

where

\[
q = e^{-\frac{t^2}{\pi^2}} , \quad z = e^{2i\pi \nu} .
\]

In the small \( t \) limit, this leads to

\[
\vartheta_{11}(i\nu t; it) \approx 2it^{-\frac{1}{2}} \exp(\nu^2\pi t) \sin(\nu\pi) \exp \left( -\frac{\pi}{4t} \right) .
\]

Similarly, by using the modular transformation of the Dedekind eta function, we find that in this limit

\[
\eta(it) \approx t^{-\frac{1}{2}} \exp \left( \frac{\pi}{12t} \right) .
\]

Gathering all of the contributions from the bosonic and the fermionic sectors, we obtain

\[
A \propto \frac{\sin^4 \left( \frac{\nu\pi}{2} \right)}{\sin \nu\pi} \int_0^\infty dt \frac{2\pi^3}{\pi\alpha' t^2} \exp \left( \frac{ty^2}{2\pi\alpha'} \right) .
\]
We can read off the potential in the zero-slope limit from the amplitude (4.14). For $b < 0$ i.e. $\nu \approx 0$, we have a quite small potential showing that the system is in an almost supersymmetric configuration. For $b > 0$, $\nu$ tends to 1. The potential becomes very large. This indicates that the system is far from being supersymmetric.

Acknowledgements:

We are grateful to T. Yokono for useful discussions.

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