A new compound control method for sine-on-random mixed vibration test

Buyun Zhang, Ruochen Wang and Falin Zeng
Automotive Engineering Research Institute, Jiangsu University, Zhenjiang, China

Abstract. Vibration environmental test (VET) is one of the important and effective methods to provide supports for the strength design, reliability and durability test of mechanical products. A new separation control strategy was proposed to apply in multiple-input multiple-output (MIMO) sine on random (SOR) mixed mode vibration test, which is the advanced and intensive test type of VET. As the key problem of the strategy, correlation integral method was applied to separate the mixed signals which included random and sinusoidal components. The feedback control formula of MIMO linear random vibration system was systematically deduced in frequency domain, and Jacobi control algorithm was proposed in view of the elements, such as self-spectrum, coherence, and phase of power spectral density (PSD) matrix. Based on the excessive correction of excitation in sine vibration test, compression factor was introduced to reduce the excitation correction, avoiding the destruction to vibration table or other devices. The two methods were synthesized to be applied in MIMO SOR vibration test system. In the final, verification test system with the vibration of a cantilever beam as the control object was established to verify the reliability and effectiveness of the methods proposed in the paper. The test results show that the exceeding values can be controlled in the tolerance range of references accurately, and the method can supply theory and application supports for mechanical engineering.

1 Introduction
Mixed mode vibration environmental test (MMVET) reproduces the real complex vibration environments of mechanical equipment or products in laboratory, and the purpose of MMVET is to supply strong guarantee for performance design of products, structural optimization, reliability and durability. Sine on random (SOR) test is a typical kind of MMVET, which is based on low level of broadband random signal superposition of one or several sinusoidal signals, such as the body vibration of helicopters and the engine vibration of vehicles [1-3]. The research on SOR at present focus on control strategy and its application of multiple input multiple output (MIMO) vibration system, including the separation of mixed signals, control algorithm of random and sine vibration, etc. Yuan [4] separated the mixed signals by digital tracking filter software method, and applied it to vibration control system. The principle of the method was simple and easy to operation, but the separation precision is low. Engelhardt [5] used Vold-Kalman filtering method to separate random and sine mixed signals, and the precision of the method is high, but the separation process is complex and can’t meet the real-time control requirement of MMVET. Zhang [6] used correlation integral method to separate mixed signals in time domain, which had higher precision and speed, and it could effectively avoid the frequency domain identification leak problem. On the aspect of control algorithm of random and sine vibration test, Underwood and Smallwood [7-8] has obtained some achievements on the signal generating
methods, test conditions tailoring, and control algorithm of MIMO vibration test. Cui [9] studied the setting of reference power spectral density (PSD) and pointed out that the coupling of control points was one key factor of control effect; and then proposed cross proportional control algorithm for MIMO random vibration test, which improved self PSD control effect. Jiang [10] proposed PID controller design method based on internal modes according to modern control theory and applied it to MIMO vibration random test. From the present study, most control strategy focused on the whole frequency domain. But at the actual vibration test the PSD value at most frequencies could meet the requirements of reference, only at some frequencies the value exceeded the reference. Therefore, the traditional control methods were low efficiency and had complex control process.

This paper analyzed the relationship of PSD elements between inputs and outputs according to MIMO linear vibration theory, and then described the reference spectrum matrix by PSD elements, based on which the Jacobi control algorithm was proposed. On the other hand, to avoid the impact influence of excessive correction, a new method called Impendence control was put forward in MIMO sine vibration test. At last, the two-input two-output SOR vibration test system was established on a cantilever beam, the test results showed that the method proposed in this paper was effective and reliable.

2 Control strategy

2.1 The separation control theory

Random signal is a kind of uncertain signal. Its vibration level is low and the frequency band is broad. Usually the PSD is regarded as the control object at random vibration test, and in MIMO systems the PSD matrix contain both self-PSD and cross-PSD elements, which would be set by different products and vibration environmental conditions. The response signal is controlled into ±3 dB of tolerance range by updating the excitation signals according to some control algorithm. The sinusoidal signal of mixed signals is a few points of host signal, and the control objects are amplitude and phase of the signal at someone frequency. Due to the big error between the response and reference value, the excessive correction of excitation signals usually cause impact influence to vibration devices. So the sine vibration control method should consider both the control precision and speed. The strategy of SOR vibration environmental test is to divide the mixed signals into two parts, random and sine, and control respectively as shown in Figure 1. When the signals meet the reference values, the final control results could be obtained by combining the adjusted signals.

![Figure 1. Separation control method of SOR vibration test](image)

\[ x = x_c + \sum_{k=1}^{N} a_k \sin(\omega_k t + \phi_k) \] (1)

where \( N \) is the number of sine signals, \( a_k \) is the amplitude of the \( k^{th} \) signal, \( \phi_k \) is the phase, and \( \omega_k = 2\pi f_k \).

The correlation integral method is adopted for the mixed signal separation. The random part and sine part of mixed signals are independent, so the mean value of product of any sinusoidal signal and
random signal is zero. Suppose the frequency of identified signal is \( \omega_m \), so the real part of integral function can be calculated by

\[
\begin{align*}
    f_{m,R} &= \frac{2}{T_m} \int_0^{T_m} x \sin(\omega_m t) dt = \frac{2}{T_m} \left[ x_r + \sum_{k=1}^{\infty} a_k \sin(\omega_m t + \phi_k) \right]. \\
    \sin(\omega_m t) dt &= \frac{2}{T_m} \int_0^{T_m} a_m \sin(\omega_m t + \phi_m) \sin(\omega_m t) dt + \frac{2}{T_m} \sum_{k=1}^{\infty} \frac{N}{k} \int_0^{T_m} \sin(k \omega_m t) dt \\
    a_k \sin(\omega_m t + \phi_k) &= a_m \cos(\phi_m)
\end{align*}
\]

where \( T_m \) is the period of identified signal. In the similar way, the imaginary part can be calculated by

\[
\begin{align*}
    f_{m,I} &= \frac{2}{T_m} \int_0^{T_m} x \cos(\omega_m t) dt = a_m \sin(\phi_m) \\
    \text{from equation (2) and (3) we can get}
\end{align*}
\]

\[
\begin{align*}
    a_m &= \sqrt{(f_{m,R})^2 + (f_{m,I})^2} \\
    \phi_m &= \arctan \left( \frac{f_{m,I}}{f_{m,R}} \right)
\end{align*}
\]

After identifying the amplitude and phase of sine signal, the random signal can be separated from the mixed signal, and the two group signals would be controlled respectively.

3 Control algorithm

3.1 Jacobi method of random vibration test

PSD matrix is regarded as the reference spectrum of MIMO random vibration test. Jacobi method corrects the self-PSD, phase and coherence function of the reference PSD matrix. The paper derives the method formula systematically by taking two-input two-output vibration system for example. Suppose there are two stationary broadband random signals denote as \( d_1 \) and \( d_2 \), the coherence function is \( \gamma_{d1d2} \) and the phase is \( \theta_d \), then the driving PSD is

\[
S_y = \begin{bmatrix} S_{d,11} & S_{d,12} \\ S_{d,21} & S_{d,22} \end{bmatrix} = \begin{bmatrix} S_{d,11} & S_{d,21}^* \\ S_{d,12} & S_{d,22} \end{bmatrix} \begin{bmatrix} \gamma_d \sqrt{S_{d,11}S_{d,22}e^{-j\theta_d}} \\ e^{j\theta_d} \end{bmatrix}
\]

where \( S_{d,11} \) is self-PSD of \( d_1 \), and \( S_{d,22} \) is self-PSD of \( d_2 \). \( S_{d,21} = \gamma_d \sqrt{S_{d,11}S_{d,22}e^{-j\theta_d}} \) is cross-PSD expressed by coherence function and phase of two group signals, \( S_{d,21}^* \) is the conjugate of \( S_{d,21} \).

According to linear vibration theory we can get

\[
S_y = HS \quad H^* = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} S_{d,11} & S_{d,21}^* \\ S_{d,12} & S_{d,22} \end{bmatrix} \begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21}^* & h_{22}^* \end{bmatrix}
\]

where \( H \) is frequency response function (FRF), and superscript \( H \) denotes conjugate and transpose. The expansion equation (6) can be written as
Suppose the test is on test, the response obtained from the equation usually can’t meet the reference requirements. Denote the amplitude and phase are the mainly control elements which is relationship of excitation and response in frequency domain. Combining equation (9), (10) and (11), then

\[ S_{y,k} = J_k S_{d,k} \]  

(9)

3.2 Impendence method of sine vibration test

The amplitude and phase are the mainly control objects in MIMO sine vibration test. According to relevant standards, we set the reference value denoted as \( \mathbf{R} \), and the FRF of structure is \( \mathbf{H} \). Based on the relationship of excitation and response in frequency domain, we can get the initial theoretical excitation which is \( \mathbf{D}_0 = \mathbf{H}^{-1} \mathbf{R} \). As similar to the error generating mechanism in random vibration test, the response obtained form the equation usually can’t meet the reference requirement. Denote \( \mathbf{G} = \mathbf{H}^{-1} \) as the impedance matrix of vibration system, then \( \mathbf{D}_0 = \mathbf{G} \mathbf{R} \). Suppose the \( k^{th} \) error of response and reference as

\[ \Delta \mathbf{E}_{r,k} = \mathbf{S}_{d,k} - \mathbf{S}_{y,k} \]  

(10)

Set \( k+1^{th} \) driving PSD is

\[ \mathbf{S}_{d,k+1} = \mathbf{S}_{d,k} + \Delta \mathbf{S}_{d,k} \]  

(11)

Combining equation (9), (10) and (11), then

\[ \mathbf{S}_{d,k+1} = \mathbf{S}_{d,k} + \mathbf{J}_{k} (\mathbf{S}_{d,k}) \Delta \mathbf{E}_{r,k} \]  

(12)

4 Test verification

A two-input two-output SOR mixed mode vibration test system is established with an aluminum cantilever beam as the research object to verify the Jacobi Impedence control method. The devices need in the test are 2 desks of Labworks PA-138 power amplifier, 2 Labworks ET-139 vibrators, 3 PCB 333B32 acceleration sensors, 1 set of data acquisition and transmission system of VXI Plus & Play, and 1 computer. Test site is shown in Figure 2.
Figure 2. Two-input two-output verification test system

The reference values would be divided into two parts as random PSD and amplitudes of sine signals. The specific values set as Table 1.

| Test type   | Freq./Hz | P.1       | P.2       |
|-------------|----------|-----------|-----------|
| Random test | 20~100   | 3dB/oct   | 3dB/oct   |
|             | 100~1000 | 10^{-6}g^2/Hz | 10^{-6}g^2/Hz |
|             | 1000~2000| 10^{-6}g^2/Hz | -4dB/oct      |
| Sine test   | 50       | 0.19g     | 0.19g     |
|             | 150      | 0.15g     | 0.15g     |

Before random vibration test the FRF of the beam should be estimated by H1 method. Load the initial excitation to the vibrator, we can get the random response PSD as shown in Figure 3. and Figure 4.

![Figure 3. The initial response PSD of P.1](image)

![Figure 4. The initial response PSD of P.2](image)

The two figures show that even though the PSD of the two points can meet the reference tolerance of ±3dB, at some frequencies the values are bigger than the references before controlling. For example, in 50Hz, 150Hz, 282Hz and 1553Hz, the values exceed the reference over 6dB, which can’t be used to reliability test later. The results controlled by Jacobi algorithm show in Figure 5 and Figure 6.
It can be seen from the figures that values at most frequencies are controlled in the ±3dB tolerance range of reference PSD, only in 50Hz and 150Hz of point 1 the values are bigger than the 6dB. This is because the higher level sine vibration influence at these two frequencies. At point 2, the value at 1553Hz is bigger than the 6dB, but all the other values can meet the requirements of test. Again at the same time, sine vibration test should be controlled, and the control results are shown in the table below.

| Loc. | Frq. Hz | Ref. | Uncontrolled g | Controlled g | % error |
|------|---------|------|----------------|--------------|--------|
| P.1  | 50      | 0.19 | 0.2426         | 0.1987       | 4.5    |
|      | 150     | 0.15 | 0.1782         | 0.1475       | 1.7    |
| P.2  | 50      | 0.19 | 0.2135         | 0.1845       | 2.9    |
|      | 150     | 0.15 | 0.1764         | 0.1479       | 1.4    |

From the control results, we can conclude that the two response amplitudes of different frequency are controlled within 5% of references, and the precision is high enough to satisfy the engineering demand.

5 Conclusion
Correlation integral method was used to separate the mixed signals. It is significant strategy that carrying out random and sine vibration test respectively, and the separation accuracy of mixed signals has a great influence on compound control precision. The correlation integral method to separate the mixed signals has higher precision and it is used in the test system.

The paper proposed a new method called Jacobi Impedence algorithm to be applied in MIMO SOR vibration test. Jacobi matrix was put forward to random test, and the method focused on the elements of PSD to improve the control efficiency greatly. Sine vibration test control formula was deduced and
the impedance algorithm was proposed based on compression factor. The method improves the control precision and guarantees convergence rate.

The two-input two-output SOR vibration test system with a cantilever beam was established to verify the proposed control method. The results show that random response spectrum by Jacobi control algorithm could be modified to ±3dB of reference tolerance range, and sine amplitude responses could be controlled within 5% of the reference. The control algorithm is effective and reliable, and can provide theoretical basis for vibration control engineering.

Acknowledgement
This work was supported by the Natural Science Foundation of the Higher Education Institutions of Jiangsu Province (16KJD460001), Jiangsu University Senior talent start-up funding scientific research projects (15JDG166).

References
[1] Trimboli,D.F, Oak,R. & Mich. (1997). Sine on random data analysis method for simulating engine vibration. United States Patent: 5675505.
[2] Sun, C.K. (2010). Research on MIMO sine on random vibration control system. Nanjing: Nanjing University of Aeronautics & Astronautics.
[3] Kihm, F. Halfpenny, A. & Ferguson, N.S. (2015). Fatigue life from sine-on-random excitations. *Procedia Engineering*, 101, 235-242.
[4] Yuan,H.J. & Li, C.R. (2000). Study of the sine-on-random vibration control. *Acta Aeronautica et Astronautica Sinica*, 21(4), 383-384.
[5] Engelhardt,C. Baker,M. & Mouron, A. et al. (2012). Separation of sine and random components from vibration measuremenets. *Sound and Vibration*, 7, 6-11.
[6] Zhang, B.Y . Chen, H.H. & He, X.D. (2014). Separation method of sinusoidal and random components in MIMO mixed vibration test. *Journal of Vibration Engineering*, 27(6), 920-925.
[7] Underwood,M.A. (2008). Testing civil structures using multiple shaker excitation techniques. *Sound and Vibration*, 42(4),10-15.
[8] Smallwood,D.O. (2013). Minimum input trace for multiple input multiple output linear systems. *Journal of IEST*, 56(2),57-67.
[9] Cui, X.L. Chen, H.H. & He, X.D. (2010). Cross proportional control algorithm for MIMO random vibration test. *Journal of Nanjing University of Aeronautics & Astronautics*, 42(4),429-434.
[10] Jiang, S.Y. (2011). Study and application of Multi-Exciter Multi-Axis vibration control. Nanjing: Nanjing University of Aeronautics & Astronautics.