Chirality in Coulomb-blockaded quantum dots

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We investigate the two-terminal nonlinear conductance of a Coulomb-blockaded quantum dot attached to chiral edge states. Reversal of the applied magnetic field inverts the system chirality and leads to a different polarization charge. As a result, the current–voltage characteristic is not an even function of the magnetic field. We show that the corresponding magnetic-field asymmetry arises from single-charge effects and vanishes in the limit of high temperature.

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Introduction.—The Onsager-Casimir symmetry relations establish that the linear-response transport is even under reversal of an external magnetic field. It is then of fundamental interest to investigate the conditions under which one can see deviations from the Onsager symmetries as one enters the nonlinear regime. Recently, it has been shown that in nonlinear mesoscopic transport there arise magnetic-field asymmetries entirely due to the effect of electron-electron interactions in the nonlinear regime. These works have been focused on quantum dots with large density of states and connected to leads via highly conducting openings (typically, quantum point contacts supporting more than one propagating mode). Recent experiments by Zumbühl et al.\textsuperscript{3,4} on large chaotic cavities are in good agreement with theory. In non-linear transport magnetic-field-asymmetries can occur under a wide variety of conditions.\textsuperscript{2} In particular, in our work, we considered a quasi-localized level separated from the leads with tunnel barriers but neglected single-charge effects. Therefore, it is natural to ask whether magnetic field asymmetries are visible in the Coulomb-blockade regime.\textsuperscript{9,10,11} Since Coulomb energies can be much larger than the energy scales for quantum interference, magnetic-field asymmetries induced by single electron effects should be visible at much higher temperatures.

The electrostatic approach used in the classical model of Coulomb blockade\textsuperscript{12} predicts a potential difference $U_d$ between the quantum dot (QD) and the reservoirs which depends on the QD charge $Q_d$, \begin{equation}
\phi_d = \frac{Q_d}{C_{\Sigma}} + \phi_{\text{ext}},
\end{equation}
and on an external potential $\phi_{\text{ext}}$ related to the polarization charge $Q_{\text{ext}}$ externally induced by nearby reservoirs and gates: \begin{equation}
\phi_{\text{ext}} = \frac{Q_{\text{ext}}}{C_{\Sigma}} = \frac{\sum_{\alpha} C_{\alpha} V_{\alpha}}{C_{\Sigma}},
\end{equation}
where the sum extends over all leads. This model assumes a uniform screening potential described by the QD (geometric) capacitance couplings $C_{\alpha}$ with the contacts. The total capacitance of the equivalent circuit is thus $C_{\Sigma} = \sum_{\alpha} C_{\alpha}$.

Consider now a two-terminal sample in the quantum Hall regime (see Fig.\textsuperscript{1} with one edge state running along each side (top and bottom). With the help of gates it is possible to create in the center a potential hill which behaves as a tunable quasi-localized state coupled to edge states acting as source and drain reservoirs. The resulting antidot connects the edge states in two ways (i) scattering coupling, in which electrons tunnel from the edge states to the antidot, and (ii) electrostatic coupling, in which the antidot screening potential feels the repulsion through capacitive couplings: $C_1$ ($C_2$) between the dot and the upper (lower) edge state. The system has a definite chirality determined from the magnetic field direction (upward or downward) since, e.g., the upper edge state originates from the left terminal for a given field $+B$ but carries current from the right terminal for the opposite field direction $-B$. Thus, the non-equilibrium polarization charge becomes $Q_{\text{ext}}(+B) = C_1 V_1 + C_2 V_2$ and $Q_{\text{ext}}(-B) = C_2 V_1 + C_1 V_2$, which is clearly magnetic-field asymmetric whenever the capacitance coupling is asymmetric. Thus, we expect that the current traversing the dot is not an even function of $B$.

The qualitative argument above can be traced back to the oddness of the Hall potential. We investigate now the effect in detail to give precise predictions for the de-

![FIG. 1: Sketch of the system under consideration. The antidot is coupled to chiral edge states via tunnel barriers acting as leaky capacitors. A back-gate contact controls the dot occupation with a capacitive coupling $C_g$. When the magnetic field is reversed, both edge states invert their propagating direction.](image)
dependence of the magnetic-field asymmetry on temperature, bias and gate voltages.

Model.—Electrons from lead $\alpha$ tunnel onto the dot via the edge states with a transmission probability characterized by a Breit-Wigner resonance with a width $\Gamma_\alpha$. We assume that transport is governed by transitions between QD ground states, which is a good approximation when both temperature and bias voltages are much smaller than any excitation energy. Then, the scattering matrix $S^\alpha\beta$ for the transition from $Q_d = N - 1$ electrons to $Q_d = N$ electrons when an electron is transmitted from lead $\alpha$ to lead $\beta$, 

$$S^\alpha\beta(E) = \delta_{\alpha\beta} - i \frac{\Gamma^N_{\alpha} \Gamma^N_{\beta}}{E - \mu_d(N)} + i \Gamma^N/2, \quad (3)$$

has a complex pole with a real part associated to the QD electrochemical potential $\mu_d(N)$. The total resonance width is proportional to $\Gamma^N = \sum_\alpha \Gamma^N_\alpha$. The widths fluctuate according to the Porter-Thomas distribution but in what follows we neglect intradot correlation effects in $\Gamma$ and take it as energy independent, which works well provided bias variations are much smaller than the barrier height.

We emphasize that the scattering matrix is not only a function of the carrier’s energy $E$ but also depends on the full electrostatic configuration via $\mu(N) = E(N) - E(N - 1)$, where $E(N)$ is the ground-state energy of a $N$-electron QD,

$$E(N) = \sum_{i=1}^{N} \varepsilon_i + \frac{(N e)^2}{2 C_\Sigma} - e N \sum_\alpha \frac{C_\alpha}{C_\Sigma} \varepsilon_\alpha. \quad (4)$$

In Eq. (4), $E(N)$ consists of two terms. First, the kinetic energy is a sum over QD single-particle levels $E_k = \sum_{i=1}^{N} \varepsilon_i$ arising from confinement. These levels may be, in general, renormalized due to coupling to the leads: $\varepsilon_N \to \varepsilon_N + (\Gamma^N/\pi) \ln [(D - E)/(D + E)]$ with $D$ the bandwidth assuming flat density of states in the leads. The renormalization term is a slowly increasing function of $E$ and can be safely neglected. Therefore, the kinetic energy is invariant under reversal of $B$. The second contribution to $E(N)$ is the potential energy $U(N)$ which depends on the charge state of the dot and the set of applied voltages including nearby gates. We assume that the dot is in the presence of a back-gate potential $V_g$ which controls the number of electrons at equilibrium via the capacitance coupling $C_g$ (see Fig. 1). Then, the QD charge, which is quantized to a value $Q_d = -Ne$ in the Coulomb valleys, determines the QD potential from the discretized Poisson equation,

$$C_1(\phi_d - V_1) + C_2(\phi_d - V_2) + C_g(\phi_d - V_g) = -Ne, \quad (5)$$

which amounts to the Hartree approximation, disregarding exchange and pairing effects. These effects might be important in certain situations but we shall see below that this level of approximation already suffices to obtain a sizable magnetic-field asymmetry.

FIG. 2: (Color online). Magnetic-field asymmetry of the differential conductance versus gate voltage for different temperatures. We set $C_1 + C_2 = C_g = 0.5$ ($C_\Sigma = 1$), asymmetry factor $\eta = 0.5$, $\Delta = 0.1 U$, $T = 0.002 U$ and $V = 0.005 U$ ($U = e^2/C_\Sigma$). Inset: Coulomb-blockade oscillations of the linear conductance $(V = 0)$ for the same parameters and $k_B T = 0.01 U$.

Equations (11) and (12) are readily derived from Eq. (5). Then, we find that the QD potential energy reads

$$U(N, +B) = \frac{N^2 U}{2} - e N \left( \frac{C_1}{C_\Sigma} V_1 + \frac{C_2}{C_\Sigma} V_2 + \frac{C_g}{C_\Sigma} V_g \right), \quad (6)$$

where $C_\Sigma = C_1 + C_2 + C_g$ and $U = e^2/C_\Sigma$. We now reverse the magnetic field:

$$U(N, -B) = \frac{N^2 U}{2} - e N \left( \frac{C_2}{C_\Sigma} V_1 + \frac{C_1}{C_\Sigma} V_2 + \frac{C_g}{C_\Sigma} V_g \right). \quad (7)$$

From Eqs. (6) and (7) it is clear that the QD electrochemical potential shows a magnetic-field asymmetry, $\Phi_\mu = (\mu(N, +B) - \mu(N, -B))/2$, given by

$$\Phi_\mu = \frac{C_2 - C_1}{2 C_\Sigma}(V_1 - V_2). \quad (8)$$

Since $\mu(N)$ determines the position of the differential conductance resonance, it follows that the $I-V$ characteristics of the antidot is asymmetric under $B$ reversal. We remark that this model assumes full screening of the charges injected in the dot, i.e. the local potential neutralizes the excess charge: $C_\alpha \ll e^2/\varepsilon_\alpha$ with $\varepsilon_\alpha$ the density of states of edge state $\alpha$. Deviations from this limit would probably decrease the size of the asymmetry. Finally, we emphasize that magnetic field asymmetries develop only to the extent that capacitive interactions with surrounding contacts are considered.

Results.—The current around the $N - 1 \to N$ resonance for spinless electrons reads

$$I_N(B) = -\frac{e}{h} \int dE \left( S_{12}^N \right)^\dagger S_{12}^N \left[ f_1(E) - f_2(E) \right], \quad (9)$$

where the scattering matrix $S$ from Eq. (3) depends on $B$ because the QD potential response is asymmetric under
$B$ reversal, as shown above. $f(E)$ is the Fermi function and we take $V_1 = -V_2 = V/2$. Our goal is to calculate the asymmetry,
\[ \Phi_G = \frac{G_N(+B) - G_N(-B)}{2}, \]  
(10)
of the differential conductance $G_N = dI_N/dV$.

In the classical Coulomb-blockade regime, one neglects quantum fluctuations in $Q_d$. Since the coupling to the leads causes a finite lifetime of the QD charges, $Q_d$ is quantized only when $k_BT \gg \Gamma^N$. Furthermore, one assumes that there is no overlap between the distinct resonances, thereby the mean level spacing in the dot $\Delta \gg \Gamma$. Hence, we expand Eq. (1) to leading order in $B$ and obtain $G_N(V)$ for $B > 0$:
\[ G_N(V, +B) = -\frac{e^2}{h} \frac{\Gamma_L^N \Gamma_R^N}{4C\Sigma k_BT \Gamma_N} \left[ y_2(V) + y_1(-V) \right], \]  
(11)
with
\[ y_\alpha(V) = (C_\alpha + C_g/2) \cosh^{-2} \left( \frac{\tilde{\xi}_N + eV C_\alpha + C_g/2}{2k_BT} \right), \]  
(12)
for $\alpha = 1, 2$ where $\tilde{\xi}_N = \xi_N - E_F + \mathcal{U}(N-1/2) - eC_gV_g/C\Sigma$ with $E_F$ the Fermi energy in the leads. For $B < 0$ one must make in Eq. (11) the replacement $1 \rightarrow 2$ and $V \rightarrow -V$. Then, our expression predicts a magnetic-field asymmetry which arises only in the nonlinear conductance (for voltages $V \neq 0$) and only due to electrostatic interactions with the leads. For $V = 0$ we reproduce the expression of the linear conductance $G_0 = G(V = 0)$ as a function of $V_g$ while $G_0$ is independent on the sign of $B$, thus fulfilling the Onsager relation. Sharp Coulomb-blockade peaks are observed in the oscillating $G_0$ as a function of $V_g$ when $k_BT \ll e^2/C\Sigma$; (see inset of Fig. 2).

We illustrate the behavior of $\Phi_G$ in Figs. 2 and 3. We define a capacitance asymmetry factor,
\[ \eta = \frac{C_1 - C_2}{C_1 + C_2}. \]  
(13)
Clearly, $\Phi_G$ is nonzero only for asymmetric couplings. In Fig. 2 we show $\Phi_G$ as a function of the back-gate voltage $V_g$ for a finite bias and different temperatures. For simplicity, we set $E_F = 0$ and take uniformly spaced levels: $\Delta = \tilde{\xi}_N = \xi_{N-1}$ independent of $N$ (in reality, levels are Wigner-Dyson distributed). The curve is periodic since $\Phi_G$ reflects the periodicity of the conductance. The asymmetry vanishes exactly at the degeneracy points, i.e., at gate voltages $V_g = e(N - 1/2)/C_g + \xi_{N} C_S/eC_g$ or simply $V_g = e(N - 1/2)/C_g$ for $\Delta \ll T_d$, where the conductance is maximum as $\tilde{\xi}_N = 0$. Importantly, $|\Phi_G|$ reaches the maximum value on both sides of the degeneracy point and then decreases in the Coulomb-blockade valley, where the charge is fixed, because no transport is permitted. For very low voltages ($eV \ll k_BT$) a compact analytic expression can be found:
\[ \Phi_G = -\frac{e}{h} \frac{\Gamma_L^N \Gamma_R^N}{4N k_BT} \frac{\eta eV}{k_BT} \times \frac{C_S - C_\alpha}{C_S} \cosh^{-2} \left( \frac{\tilde{\xi}_N}{2k_BT} \right) \tanh \left( \frac{\tilde{\xi}_N}{2k_BT} \right). \]  
(14)

We find that the maxima of $|\Phi_G|$ take place approximately at $\tilde{\xi}_N = k_BT$, i.e., for gate voltages of the order of $k_BT$ away from the degeneracy point. This explains as well why the maxima (minima) shift to lower (higher) values of $V_g$ with increasing $T$. Moreover, it is worthwhile to note that the asymmetry effect vanishes overall in the high-$T$ regime. This implies that when temperature is higher than the interaction $e^2/C\Sigma$, transport is mediated by thermal fluctuations only, which are $B$-symmetric. We note in passing that our results are formally related to the voltage asymmetry that arises in a quantum dot which is more coupled to, say, the left lead than to the right lead. As a consequence, the conductance measured at forward bias differs from the backward bias case.

Figure 3 presents the nonequilibrium conductance as a function of the bias voltage at a fixed $V_g$ corresponding to one maximum in Fig. 2. The asymmetry increases rapidly with voltage and this increase is sharper for increasing capacitance asymmetry.

In Ref. 3 we distinguished between capacitive asymmetry and scattering asymmetry, the latter arising from asymmetric tunnel couplings $\Gamma_L^N \neq \Gamma_R^N$. Both asymmetries can be varied independently by changing the height and width of the tunnel barrier separating the dot and the edge states. This distinction was possible because the problem could be solved exactly at all orders in the coupling $\Gamma^N$ (coherent tunneling). When the dot is Coulomb-blockaded, tunneling is sequential and tunnel couplings are treated to first order ($\Gamma^N$ is the lowest energy scale). Thus, the effect of a tunnel asymmetry is trivially incorporated in our equations since $\Gamma_L^N \Gamma_R^N / \Gamma^N = (1 - \xi^2) / 4\Gamma^N$ with the scattering asymmetry factor $\xi = (\Gamma_L^N - \Gamma_R^N) / \Gamma^N$. However, in the classical treatment of Coulomb blockade given here, the asymmetry $\Phi_G$ vanishes when $\eta = 0$ independently of $\xi$. To include quantum fluctuations is a difficult task since the charge $Q_d$ is not simply $Ne$ and the self-consistent procedure to find the dot potential becomes involved. In the absence of Coulomb blockade effects, but in the presence of a Hartree potential, the task can be solved to all orders in $\Gamma$.

Cotunneling processes contribute to the conductance to order $\Gamma^2$. Thereby a residual asymmetry is expected around the conductance minimum. We consider elastic cotunneling, which is the dominant off-resonance mechanism at low bias when $k_BT \ll \Gamma$, as experimentally demonstrated. Elastic cotunneling consists of the virtual tunneling of an electron in a coherent fashion without leaving the dot in an excited state. Hence, our theory for transport between ground states is applicable. For definiteness, we investigate the minimum between
the $N = 1$ and $N = 2$ resonances. Due to large denominators in Eq. (3), we can use $T = 0$ Fermi functions in Eq. (3) and expand in powers of $\Gamma$. The resulting conductance goes as $(\Gamma/U)^2$. In the inset of Fig. 3 we plot the numerical result of the asymmetry of the cotunneling conductance, $\Phi_c$, as a function of $V$ for gates voltages around the conductance minimum, which represents the electron-hole (e-h) symmetry point. Interestingly enough, $\Phi_c$ changes sign about the minimum and exactly vanishes (not shown) at the e-h symmetry point since charge fluctuations are quenched there (the mean charge is 1/2 per channel). For $E_F = \varepsilon_1 + \Delta/2$ the $G_0$ minimum takes place at $Q_g = C_g V_g = +e$. Then, to leading order in $(Q_g/e - 1)$ we find

$$
\Phi_c = -\frac{e^2}{h} \frac{\Gamma_L \Gamma_R}{\Gamma} \frac{192\eta(C_\Sigma - C_g)\mu eV}{C_\Sigma(\Delta + U)^4} \left(\frac{Q_g}{e} - 1\right),
$$

valid in the limit $eV \ll U$ and $k_B T \ll \Delta < U$. This expression reproduces the effects discussed above and is in remarkable agreement with the numerical results (see inset of Fig. 3).

Thus far we have neglected the spin degeneracy. When $T$ is further lowered, spin-flip cotunneling processes lead to Kondo effects and the corrections of the conductance become of the order of $e^2/h$. Notably, a dependence on the bias polarity due to asymmetric couplings has been observed. Therefore, one might expect a large magnetic-field asymmetry. However, recent works have emphasized the robustness of the e-h symmetry point in the Kondo regime against external disturbances which would suggest that also the magnetic field asymmetry vanishes at this point.

**Conclusions.**—We have demonstrated that careful consideration of the interaction between a quantum dot and the edge states to which it is coupled leads to an out-of-equilibrium charging which is asymmetric under magnetic-field reversal. Crucial to this result is the chirality of the polarization charge. Obviously, any model generating an uneven polarization charge would similarly and quite generally predict an asymmetry. Importantly, the temperature scale of the magnetic field asymmetry we find is determined by the Coulomb charging energy. Consequently, the effect reported here should be readily observable in a wide range of systems.

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