Color-evaporation-model calculation of 
$e^+e^- \rightarrow J/\psi + c\bar{c} + X$ at $\sqrt{s} = 10.6$ GeV

Daekyoung Kang, Jong-Wan Lee, and Jungil Lee

Department of Physics, Korea University, Seoul 136-701, Korea

Taewon Kim

Department of Physics, KAIST, Daejon 305-701, Korea

Pyungwon Ko

School of Physics, KIAS, Seoul 130-722, Korea

(Dated: November 5, 2018)

Abstract

Measurements by the Belle Collaboration of the cross section for inclusive $J/\psi$ production in $e^+e^-$ annihilation have been a serious challenge to current heavy-quarkonium theory. Especially, the measured cross sections for exclusive $J/\psi + \eta_c$ and inclusive $J/\psi + c\bar{c} + X$ differ from nonrelativistic QCD predictions by an order of magnitude. In order to check if other available alternative theory can resolve such a large discrepancy, we calculate the cross section for inclusive $J/\psi + c\bar{c} + X$ based on the color-evaporation model. As a phenomenological model, the color-evaporation model is still employed to predict cross sections for inclusive quarkonium production in various processes. Our results show that the color-evaporation-model prediction is even smaller than the nonrelativistic QCD prediction by an order of magnitude. The resultant color-evaporation-model prediction for $J/\psi + c\bar{c} + X$ fraction in the inclusive $J/\psi$ production cross section is 0.049, while the empirical value measured by the Belle Collaboration is 0.82.

PACS numbers: 13.66.Bc, 12.38.Bx, 14.40.Gx
Spin-triplet $S$-wave charmonium states such as $J/\psi$ have been clean probes of both perturbative and nonperturbative feature of quantum chromodynamics (QCD). Systematic studies based on first principles became possible after the introduction of the nonrelativistic QCD(NRQCD) factorization formalism \[1\], an effective field theory of QCD. However, still there are several challenges to quarkonium physics research. One of the most interesting problems is regarding inclusive $J/\psi$ production in $e^+e^-$ annihilation at $B$ factories. The Belle Collaboration has measured the cross section for exclusive $J/\psi + \eta_c$ production by observing a peak in the momentum spectrum of inclusive $J/\psi$ signals that corresponds to the 2-body final state $J/\psi + \eta_c$ \[2\]. The measured cross section is by an order of magnitude larger than the predictions of NRQCD \[3, 4\]. Interesting proposals \[5, 6, 7\] to resolve the problem were disfavored by a recent analysis by the Belle Collaboration \[8\]. Having failed in explaining the result using lowest-order perturbative calculations, one may guess that higher-order corrections in strong coupling $\alpha_s$ may be very large \[9\]. If it is true, perturbative expansion is not a proper method to predict the cross section. Recently, we proposed that the measurement of the cross section for the $e^+e^-$ annihilation into four charm hadrons may provide us with a strong constraint in determining the origin of the large discrepancy \[10\] and corresponding experimental analysis is being carried out. In the Belle analysis \[2\] they also reported the cross section for the inclusive $J/\psi + c\bar{c}$ production at the center-of-momentum (c.m.) energy $\sqrt{s} = 10.6$ GeV. From the measured inclusive cross sections $\sigma(e^+e^- \rightarrow J/\psi + D^{*+} + X) = 0.53^{+0.19}_{-0.15} \pm 0.14$ pb and $\sigma(e^+e^- \rightarrow J/\psi + D^0 + X) = 0.87^{+0.32}_{-0.28} \pm 0.20$ pb they extracted the cross section for $J/\psi + c\bar{c} + X$ \[2\] based on the Lund model for fragmentation of a charm quark into the $D$ mesons \[11\]. The resulting cross section is $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17$ pb \[2\], which is larger than NRQCD predictions \[12, 13, 14\] by an order of magnitude. For example, an NRQCD prediction given in Ref. \[12\] is about 0.07 pb. The Belle Collaboration also calculated the ratio $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)/\sigma(e^+e^- \rightarrow J/\psi + X) = 0.82 \pm 0.15 \pm 0.14$ \[2, 15\]. This remarkable result reveals the fact that $e^+e^- \rightarrow J/\psi + c\bar{c}$ is the dominant source for inclusive $J/\psi$ production \[16\] in $e^+e^-$ annihilation at $B$ factories. Again, the result is remarkably larger than available NRQCD predictions. Two-photon mediated process $e^+e^- \rightarrow J/\psi + c\bar{c}$ of order $\alpha^4$ \[17\] has been calculated using NRQCD in order to check if the photon-fragmentation effect is significant like the process $e^+e^- \rightarrow J/\psi + J/\psi$ \[3, 6\]. Unlike the exclusive double $J/\psi$ production the photon-fragmentation contribution was found to be negligible compared to the single-photon process of order $\alpha^2\alpha_s^2$ in $e^+e^- \rightarrow J/\psi + c\bar{c}$ \[17\]. Different approaches were also introduced using a large $K$-factor \[18\] and the nonperturbative quark-gluon-string model \[19\].

In this paper, we calculate the cross section for inclusive $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ process at the c.m. energy $\sqrt{s} = 10.6$ GeV using the color-evaporation model (CEM) \[20, 21, 22, 23\]. Predictions of the quark-hadron-duality model, which is similar to the CEM, are available \[24, 25\] and they underestimate the cross section for $e^+e^- \rightarrow J/\psi + c\bar{c}$ like that of NRQCD. The quark-hadron-duality model assumes that only a color-singlet $c\bar{c}$ state evolves into a $J/\psi$, while the CEM sums over all possible spin and color states. Thus the CEM calculation will probe the importance of the missing color-octet component. The hard process for the inclusive production of $J/\psi + c\bar{c}$ is approximated by $e^+e^- \rightarrow c\bar{c}c\bar{c} \bar{c}$ leading order in strong coupling $\alpha_s$. Invariant mass of the $c\bar{c}$ pair evolving into the final $J/\psi$ is restricted from $2m_c$ to open charm threshold $2m_D$. Multiplying the CEM parameter $F_{J/\psi}$ to the hard-scattering cross section, the CEM prediction for the inclusive $J/\psi + c\bar{c}$ production cross section is obtained. The CEM parameter $F_{J/\psi}$ represents the probability of the $c\bar{c}$ pair...
evolving into a $J/\psi$. Our results show that the CEM prediction is significantly smaller than that of NRQCD, which already underestimates the empirical cross section by an order of magnitude. The process is well distinguished from many other hadronic processes, where CEM predictions are not that far away from NRQCD ones. Comprehensive reviews on the CEM can be found in Refs. [26, 27].

In the CEM the cross section for inclusive $J/\psi$ production is obtained by integrating differential cross section for $c \bar{c} + X$ over the invariant mass $m_{c \bar{c}}$ of the $c \bar{c}$ pair from $2m_c$ to $D\bar{D}$ threshold, $2m_D$.

$$\sigma(J/\psi + X) = F_{J/\psi} \int_{2m_c}^{2m_D} \left( \frac{d\sigma_{c\bar{c}}}{dm_{c\bar{c}}} \right) dm_{c\bar{c}},$$

(1)

where $\sigma_{c\bar{c}}$ is the cross section for $e^+e^- \rightarrow c\bar{c} + X$. The CEM parameter $F_{J/\psi}$ in Eq. (1) represents the probability for the $c\bar{c}$ pair to evolve into the final $J/\psi$ inclusively. This phenomenological parameter is fitted to the data from hadronic experiments which have a large number of data samples. Thus the uncertainties of the parameter come from parton distribution function and renormalization/factorization scale as well as the charm-quark mass $m_c$.

Before we proceed to consider the inclusive $J/\psi + c\bar{c} + X$ production, we must address the point that the CEM is a model valid only for inclusive production of a heavy quarkonium. The parton cross section $\sigma_{c\bar{c}}$ in Eq. (1) includes all possible spin and color states for a $c\bar{c}$ pair. The final quarkonium state is always accompanied by light hadrons because the quantum numbers for $c\bar{c}$ and the quarkonium can be different in general. Therefore, it is impossible to apply the CEM to predict the exclusive two-charmonium production in $e^+e^-$ annihilation.

In order to apply the CEM to calculate the cross section for the inclusive $J/\psi + c\bar{c}$ production in $e^+e^-$ annihilation, we have to calculate the invariant-mass distribution of a $c\bar{c}$ pair created in the process $e^+e^- \rightarrow c\bar{c}c\bar{c} + X$ and follow the way shown in Eq. (1). In leading order in strong coupling $\alpha_s$, $c\bar{c}c\bar{c}$ can be produced at order $\alpha_s^2 \alpha_s^2$. There are two topologically distinct Feynman diagrams generating two pairs of $c\bar{c}$, which are shown as $\mathcal{M}_1$ and $\mathcal{M}_2$ in Fig. (a) and (b), respectively. Momenta for the involving particles are assigned as $e^-(k_1)e^+(k_2) \rightarrow c(p_1)\bar{c}(p_2)c(p_3)\bar{c}(p_4)$. The amplitude for the two diagrams shown in Fig. (1) are

$$-i\mathcal{M}_i = \frac{(4\pi)^2 e_c \alpha_s}{s(p_2 + p_3)^2} \bar{u}_c(k_2)\gamma_\alpha u_c(k_1) \bar{u}(p_3)T^\alpha\gamma_\beta v(p_2) \bar{u}(p_1)T^\beta H_{i}^\alpha v(p_4),$$

(2)

where $s = (k_1 + k_2)^2$, $e_c = \frac{2}{3}$ is the fractional electric charge of the charm quark, and $\alpha$ is the SU(3) color index for the virtual gluon. The vector indices $\alpha$ and $\beta$ are for the virtual...
photon and the gluon, respectively. We suppress the spin and color indices of the charm quarks in Eq. (2). For \( i = 1 \) or 2 the tensors \( H_i^{a\beta} \) in Eq. (2), which are matrices in spinor space, are defined by

\[
H_1^{a\beta} = \gamma^\beta \Lambda(p_1 + p_2 + p_3)\gamma^a, \\
H_2^{a\beta} = \gamma^a \Lambda(-p_2 - p_3 - p_4)\gamma^\beta,
\]

(3a)

where \( \Lambda(p) = (\not{p} + m_c)/(p^2 - m_c^2) \). There are 6 more Feynman diagrams that can be obtained from the two amplitudes \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \) by exchanging two charm quarks and two antiquarks, respectively, as

\[
\mathcal{M}_3 = -P_{1+3}\mathcal{M}_1, \quad \mathcal{M}_4 = -P_{1+3}\mathcal{M}_2, \\
\mathcal{M}_5 = -P_{2+4}\mathcal{M}_1, \quad \mathcal{M}_6 = -P_{2+4}\mathcal{M}_2, \\
\mathcal{M}_7 = +P_{1+3}P_{2+4}\mathcal{M}_1, \quad \mathcal{M}_8 = +P_{1+3}P_{2+4}\mathcal{M}_2.
\]

(4)

where \( P_{i+j} \) is the operator exchanging two particles with momentum indices \( p_i \) and \( p_j \) shown in Fig. 1. The signs of \( \mathcal{M}_3 \) through \( \mathcal{M}_8 \) in Eq. (4) are determined by the antisymmetry of Fermi statistics in exchanging identical fermions among the final-state particles.

In the process \( e^+e^- \rightarrow c\bar{c}c\bar{c} \), there are four ways to pick a pair of \( c\bar{c} \), which will evolve into the final \( J/\psi \) inclusively. We choose the \( c\bar{c} \) pair with momenta \( p_1 \) and \( p_2 \). Then we integrate over the invariant mass \( m_{12} = \sqrt{(p_1 + p_2)^2} \) of the pair from \( 2m_c \) to \( 2m_D \). The invariant mass \( m_{34} = \sqrt{(p_3 + p_4)^2} \) of the other pair is integrated over the whole phase space, which runs from \( 2m_c \) to \( \sqrt{5 - 2m_{12}} \). Because we distinguish two charm quarks and two anti-charm quarks simultaneously, we do not multiply the statistical factor \((1/2)^2\) for identical particles in the phase space integration. Corresponding CEM prediction for the inclusive process \( e^+e^- \rightarrow c\bar{c}c\bar{c} + X \) is calculated using the formula

\[
\sigma(J/\psi + c\bar{c} + X) = F_{J/\psi} \int_{2m_c}^{2m_D} \left( \frac{d\sigma}{dm_{12}} \right) dm_{12}, 
\]

(5)

where the differential cross section \( \frac{d\sigma}{dm_{12}} \) in Eq. (5) is defined by

\[
\frac{d\sigma}{dm_{12}} = \frac{1}{32(2\pi)^8 s^{3/2}} \int_{2m_c}^{\sqrt{s} - m_{12}} dm_{34} \int d\Omega_{12}^* d\Omega_{34}^* \sum |\mathcal{M}|^2,
\]

(6)

where \( \mathcal{M} = \sum_{i=1}^{8} \mathcal{M}_i \). The summation notation \( \sum \) in Eq. (6) stands for averaging over initial spin states and summation over final color and spin states. The solid angle \( d\Omega \) is for the three momentum \( P \) of \( p_1 + p_2 \) in the \( e^+e^- \) c.m. frame. Remaining two solid-angle elements of the form \( d\Omega_{12}^* \) are for the three momentum \( p_1^* \) in the rest frame of \( p_1 + p_2 \), where \( \{i,j\} = \{1,2\} \) or \( \{3,4\} \). Integrating over the eight variables in Eqs. (5) and (6), we get the CEM prediction for the cross section \( \sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) \). The cross section for \( e^+e^- \rightarrow c\bar{c}c\bar{c} + X \) can be obtained if we remove the \( F_{J/\psi} \) in Eq. (5), replace the cut \( 2m_D \) in Eq. (5) by \( \sqrt{s} - 2m_c \), and multiply the statistical factor \((1/2)^2\) to consider two pairs of identical particles in the final state. The process \( e^+e^- \rightarrow c\bar{c}c\bar{c} \) has been considered in Ref. [10]. We compute the \( \sum |\mathcal{M}|^2 \) in Eq. (6) using REDUCE [23] and carry out the phase-space integral in Eqs. (5) and (6) making use of the adaptive Monte Carlo routine VEGAS [29]. As a check, we carry out the same calculation using CompHEP [30]. Our analytic result for \( \sum |\mathcal{M}|^2 \) and numerical values for the total cross section agree with those obtained by using CompHEP.

Let us summarize our theoretical inputs. Following recent quarkonium calculations in Refs. [3, 5, 6], we use the electromagnetic coupling \( \alpha = 1/137 \) and the next-to-leading
order pole mass $m_c = 1.4 \pm 0.2$ GeV for the charm-quark mass. The strong coupling constant $\alpha_s$ depends on the typical scale in the hard-scattering cross section. If we use the renormalization scale $\mu = 2m_c$ with $m_c = 1.4$ GeV, the value is $\alpha_s(2m_c) \approx 0.26$. The value is appropriate for the gluon-fragmentation contribution. However, a $c\bar{c}$ pair from different quark lines can evolve into the $J/\psi$. In this case, the momentum transfer carried by the internal gluon can be as large as $\sqrt{s}/2 = 5.3$ GeV to get $\alpha_s(\sqrt{s}/2) \approx 0.21$. The latter value has been used for the exclusive $J/\psi + \eta_c$ production in Ref. [3] because the momentum scale is exactly $\sqrt{s}/2$. In our numerical analysis we take into account the uncertainty in the scale and allow the variation $\alpha_s = 0.23 \pm 0.03$. Because the cross section is proportional to $\alpha_s^2$, the uncertainty from the strong coupling is about $\pm 30\%$. Finally, we have to fix the numerical value for the CEM parameter $F_{J/\psi}$ for directly produced $J/\psi$. The CEM parameter $F_{J/\psi}^{inc}$ for inclusive $J/\psi$ production, which includes feed-down from higher charmonium resonances, has been fitted to the data from $pp$ and $pA$ collisions. They have small statistical errors but depend on parton distribution functions, renormalization/factorization scale, and $m_c$. A recent set of the values can be found in Refs. [26, 27, 31, 32]. In Table 3.7 of Ref. [27] several values are given depending on parton distribution function and $m_c$. The values are $F_{J/\psi}^{inc} = 0.0248$ for $m_c = 1.4$ GeV, 0.0229 for $m_c = 1.3$ GeV, and 0.0144 $\sim$ 0.0155 for $m_c = 1.2$ GeV. The fraction of direct $J/\psi$ is 0.62 as shown in Table 3.6 of Ref. [27].

The values for directly produced $J/\psi$ are then $F_{J/\psi} = 0.015$ for $m_c = 1.4$ GeV, 0.014 for $m_c = 1.3$ GeV, and about 0.01 for $m_c = 1.2$ GeV. In Ref. [33], authors extracted the parameter using the open charm production in the CEM framework to estimate the QCD correction to the hadroproduction of charmonium. The values in Ref. [33] are consistent with the photoproduction data [33, 34]. For $m_c = 1.45$ GeV, the authors of Ref. [33, 34] found $F_{J/\psi}^{inc} = (0.43 \sim 0.5)/9$ which results in $F_{J/\psi} = 0.030 \sim 0.034$. Roughly taking into account the difference between the two fits, we take $F_{J/\psi} = 0.025 \pm 0.010$ for $m_c = 1.4$ GeV. The uncertainty from the CEM parameter is about $\pm 40\%$. The fits in Refs. [33, 34] favor larger values, while the fits in Ref. [27] prefer smaller ones. As shown in Ref. [27], numerical value for $F_{J/\psi}$ depends on $m_c$. We take into account the $m_c$ dependence of the $F_{J/\psi}$ described above by making a linear fit to the values given in the Table 3.7 of Ref. [27]. Resultant fit is $F_{J/\psi} = 0.025 + 0.047 (\frac{m_c}{GeV} - 1.4)$.

In Fig. [2] we show the differential cross section $d\sigma/dm_{12}$ in Eq. (8) multiplied by the CEM parameter $F_{J/\psi} = 0.025$ with $m_c = 1.4$ GeV and $\alpha_s = 0.23$. The area of the shadowed region in Fig. [2] corresponds to the inclusive cross section $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$. If we increase the $m_c$, the cross section decreases because the left end point of the phase space for $m_{12}$ shifts toward the cut $2m_D = 2 \times 1.87$ GeV and the height of the peak decreases. The cross section gets larger if we choose a smaller $m_c$. The $m_c$ dependence of our CEM predictions for the inclusive cross section $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$ at $\sqrt{s} = 10.6$ GeV is shown in Fig. [3]. The band represents the uncertainty $\approx \pm 30\%$ coming from the strong coupling $\alpha_s = 0.23 \pm 0.03$. In Fig. [3] we use the $m_c$-dependent CEM parameter $F_{J/\psi} = 0.025 + 0.047 (\frac{m_c}{GeV} - 1.4)$. Considering roughy $\pm 40\%$ uncertainty in $F_{J/\psi}$ and $\pm 30\%$ uncertainty in strong-coupling dependence, our prediction has roughly $\pm 50\%$ uncertainty neglecting $m_c$ dependence, which is well described in Fig. [3].

In previous NRQCD calculations for $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ authors used larger values for the strong coupling $\alpha_s$ [12, 13, 14]. For example, $\alpha_s(2m_c) = 0.28$ was used in Ref. [12] to get the NRQCD prediction for the cross section $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.07$ pb at $\sqrt{s} = 10.6$ GeV. In order to have a fair comparison with the NRQCD predictions let us replace our $\alpha_s$ with $\alpha_s = 0.28$. Then our predictions given in Figs. [2] and [3] should be
FIG. 2: Differential cross section $F_{J/\psi} d\sigma / dm_{c\bar{c}}$ in fb/GeV with respect to the invariant mass $m_{c\bar{c}} = m_{12}$ of $c\bar{c}$ for $e^+e^-$ annihilation into $c\bar{c}c\bar{c}$, where $m_c = 1.4$ GeV, $\alpha = 1/137$, $\alpha_s = 0.23$, and $F_{J/\psi} = 0.025$. The area of the shadowed region with $2m_c \leq m_{c\bar{c}} \leq 2m_D$ stands for the CEM prediction for $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$.

FIG. 3: The CEM prediction for the inclusive cross section $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$ in fb at $\sqrt{s} = 10.6$ GeV as a function of $m_c$. The $m_c$-dependent CEM parameter $F_{J/\psi} = 0.025 + 0.047 \left( \frac{m_c}{1.4} \right)$ is used and the band represents the uncertainty from the strong coupling $\alpha_s = 0.23 \pm 0.03$ only.

increased by a factor of $(0.28/0.23)^2 \approx 1.5$. The upper bound of our CEM cross section is about 12 fb which is less than the NRQCD prediction by a factor of 6. The underestimation of the cross section in the CEM may be due to the limited phase space for the invariant mass $m_{c\bar{c}}$ because of the small center-of-momentum energy. Further studies of invariant-mass spectra for hadronic processes will reveal if our guess is valid.

Another important feature of our result is that the CEM prediction is less than that of the quark-hadron-duality model. The prediction of the quark-hadron-duality model is about 60 fb [24, 25] which is similar to the NRQCD predictions [12, 13, 14]. The quark-
hadron-duality model collects only color-singlet contributions while the CEM includes all the color degrees of freedom. In the CEM the invariant mass of the $c\bar{c}$ pair is integrated over $2m_c < m_{c\bar{c}} < 2m_D$, which is less than the region $2m_c < m_{c\bar{c}} < 2m_D + (0.5 \sim 1 \text{ GeV})$ for the quark-hadron-duality model. If we increase the upper limit for the $m_{c\bar{c}}$ by 1 GeV, the CEM cross section at $m_c = 1.4 \text{ GeV}$ becomes 12 fb, which is still smaller than the quark-hadron-duality-model prediction by a factor of 5. A reason for this difference may be due to the long-distance factor $F_{J/\psi}$. We use the value for the $F_{J/\psi}$ extracted from hadronic collisions, while the quark-hadron-duality model uses a different method.

Before closing our discussion, let us comment on the ratio $\frac{\sigma(e^+e^- \to J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \to J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14$ measured by the Belle Collaboration\[15\]. The empirical value is remarkably larger than the prediction of NRQCD. Unlike NRQCD, the nonperturbative factor $F_{J/\psi}$ in the CEM prediction for the ratio cancels and the ratio is depending only on $\alpha_s$ and $m_c$. Therefore CEM prediction for the ratio should be more reliable than the absolute value for the cross sections. The inclusive $J/\psi$ production rate can be calculated by considering the parton process $e^+e^- \to c\bar{c}g$ in the leading order in $\alpha_s$. Note that $e^+e^- \to c\bar{c}$ channel is forbidden because $m_{c\bar{c}} = \sqrt{s} \gg 2m_D$. According to our recent analysis \[35\], CEM prediction for the cross section is

$$\sigma(e^+e^- \to J/\psi + X) \approx 95 \text{ fb},$$

where we use $F_{J/\psi} = 0.025$ and $\alpha_s(2m_c) = 0.26$ with $m_c = 1.4 \text{ GeV}$. Resulting ratio becomes

$$\frac{\sigma(e^+e^- \to J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \to J/\psi + X)} \bigg|_{\text{CEM}} \approx 0.062.$$\[8\]

The empirical rate measured by the Belle Collaboration was obtained with the cut $|p_{J/\psi}^*| > 2.0 \text{ GeV}$ of the three-momentum of the $J/\psi$ in the $e^+e^-$ c.m. frame. If we impose the same cut to our prediction, our prediction for the ratio becomes $\approx 0.049$, which is less than the value given in Eq. \[8\] by about 21%. The ratio is again significantly smaller than the experimental value $\approx 0.82$.

In summary, we have calculated the CEM prediction for the inclusive cross section for $e^+e^- \to J/\psi + c\bar{c} + X$ using the CEM parameter $F_{J/\psi}$ fitted to the data from $pp$ and $pA$ collisions\[27, 31, 32\] and from photoproduction\[34\]. Our CEM prediction for the cross section $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$ is about 9.7 fb at $m_c = 1.2 \text{ GeV}$, 5.8 fb at $m_c = 1.4 \text{ GeV}$, and 2.2 fb at $m_c = 1.6 \text{ GeV}$. The cross section is, at least by a factor of 6, smaller than the NRQCD prediction, which is already smaller than the empirical value by an order of magnitude. Thus the color-evaporation model severely underestimates the inclusive $J/\psi + c\bar{c} + X$ cross section in $e^+e^-$ annihilation measured by the Belle Collaboration.

Acknowledgments

We would like to thank Geoff Bodwin for introducing us the problem studied in this work. We also thank Eric Braaten and Geoff Bodwin for valuable suggestions. PK is supported in part by KOSEF through CHEP at Kyungpook National University. JL is supported by
a Korea Research Foundation Grant(KRF-2004-015-C00092).

[1] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995); 55, 5853(E) (1997).
[2] K. Abe et al. [BELLE Collaboration], Phys. Rev. Lett. 89, 142001 (2002).
[3] E. Braaten and J. Lee, Phys. Rev. D 67, 054007 (2003) [arXiv:hep-ph/0211085].
[4] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Lett. B 557, 45 (2003) [arXiv:hep-ph/0211181].
[5] G. T. Bodwin, J. Lee and E. Braaten, Phys. Rev. Lett. 90, 162001 (2003) [arXiv:hep-ph/0212181].
[6] G. T. Bodwin, J. Lee, and E. Braaten, Phys. Rev. D 67, 054023 (2003) [arXiv:hep-ph/0212352].
[7] S. J. Brodsky, A. S. Goldhaber and J. Lee, Phys. Rev. Lett. 91, 112001 (2003) [arXiv:hep-ph/0305269].
[8] K. Abe et al. [Belle Collaboration], Phys. Rev. D 70, 071102 (2004) [arXiv:hep-ex/0407009].
[9] K. Hagiwara, E. Kou and C. F. Qiao, Phys. Lett. B 570, 39 (2003) [arXiv:hep-ph/0305102].
[10] D. Kang, J.-W. Lee, J. Lee, T. Kim, and P. Ko, [arXiv:hep-ph/0412224] Phys. Rev. D, in press.
[11] T. Sjostrand, Comput. Phys. Commun. 82, 74 (1994).
[12] P. L. Cho and A. K. Leibovich, Phys. Rev. D 54, 6690 (1996) [arXiv:hep-ph/9606229].
[13] F. Yuan, C. F. Qiao and K. T. Chao, Phys. Rev. D 56, 321 (1997) [arXiv:hep-ph/9703438].
[14] S. Baek, P. Ko, J. Lee and H. S. Song, J. Korean Phys. Soc. 33, 97 (1998) [arXiv:hep-ph/9804455].
[15] K. Abe et al. [Belle Collaboration], BELLE-CONF-0331, contributed paper, International Europhysics Conference on High Energy Physics (EPS2003), Aachen, Germany (2003).
[16] K. Abe et al. [BELLE Collaboration], Phys. Rev. Lett. 88, 052001 (2002).
[17] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D 68, 031501 (2003) [arXiv:hep-ph/0305084].
[18] K. Hagiwara, E. Kou, Z. H. Lin, C. F. Qiao and G. H. Zhu, Phys. Rev. D 70, 034013 (2004) [arXiv:hep-ph/0401246].
[19] A. B. Kaidalov, JETP Lett. 77, 349 (2003) [Pisma Zh. Eksp. Teor. Fiz. 77, 417 (2003)] [arXiv:hep-ph/0301246].
[20] H. Fritzsch, Phys. Lett. B 67, 217 (1977).
[21] F. Halzen, Phys. Lett. B 69, 105 (1977).
[22] M. Gluck, J. F. Owens and E. Reya, Phys. Rev. D 17, 2324 (1978).
[23] V. D. Barger, W. Y. Keung and R. J. N. Phillips, Phys. Lett. B 91, 253 (1980).
[24] V. V. Kiselev, A. K. Likhoded and M. V. Shevlyagin, Phys. Lett. B 332, 411 (1994) [arXiv:hep-ph/9408407].
[25] A. V. Berezhnoy and A. K. Likhoded, Phys. Atom. Nucl. 67, 757 (2004) [Yad. Fiz. 67, 778 (2004)] [arXiv:hep-ph/0303145].
[26] N. Brambilla et al., [arXiv:hep-ph/0412158.
[27] M. Bedjidian et al., [arXiv:hep-ph/0311048.
[28] A. C. Hearn, REDUCE User’s Manual v. 3.7 (The RAND Corporation, Santa Monica, 1999) (Email:reduce@rand.org).
[29] G. P. Lepage, J. Comput. Phys. 27, 192 (1978).
[30] E. Boos et al. [CompHEP Collaboration], Nucl. Instrum. Meth. A 534, 250 (2004)
[31] R. Vogt, arXiv:hep-ph/0203151.
[32] R. Vogt, Heavy Ion Phys. 18, 11 (2003) arXiv:hep-ph/0205330.
[33] J. F. Amundson, O. J. P. Eboli, E. M. Gregores and F. Halzen, Phys. Lett. B 372, 127 (1996) arXiv:hep-ph/9512248.
[34] J. F. Amundson, O. J. P. Eboli, E. M. Gregores and F. Halzen, Phys. Lett. B 390, 323 (1997) arXiv:hep-ph/9605295.
[35] D. Kang, J.-W. Lee, J. Lee, and T. Kim, in preparation.