Precise Positions of Five Major Uranian Satellites During 2008–2014 Based on Gaia EDR3*

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Received 2022 April 14; revised 2022 May 21; accepted 2022 May 27; published 2022 July 6

Abstract

The five major Uranian satellites (Ariel, Umbriel, Titania, Oberon, and Miranda) were observed by a 1.56 m telescope at Sheshan Station of Shanghai Astronomical Observatory during 2008–2014 and a total of 1915 positions of these five satellites were presented in this paper. Since all five satellites are close to Uranus, their positions are affected by the uneven background, which is caused by the halo of Uranus. The median filtering method is used to remove the influence of the halo of the bright Uranus, which also made the rate of target detection increase by 30%–100%, especially for Miranda. Gaia EDR3 is used as the reference catalog when calculating positions of the five satellites. A comparison between our positions with the theoretical positions of the satellites from IMCCE is given. Such precise positions over a long time will be very helpful to improve the orbit parameters of the five major Uranian satellites.

Supporting material: machine-readable table

1. Introduction

Although Uranus is far from the Earth and its exploration history is only decades since Voyager 2 passed Uranus in 1986, its strange behavior has deeply attracted the attention of astronomers. First, the angle between Uranus’ rotation axis and revolution axis is as high as 97°77, which means Uranus rotates almost horizontally, and then revolves around the Sun. Second, when Voyager 2 passed Uranus, it measured the magnetic field direction of Uranus. The angle between the direction of its magnetic field and its rotation axis is as high as 59°, and the overall distribution is very uneven, and it has multiple poles (Smith et al. 1986; Hussmann et al. 2006).

Major planets in the solar system are thought to be formed in the solar nebula, which is a disk-shaped cloud of gas and dust left in the formation of the Sun. Therefore, the rotation and rotation directions of major planets in the early solar system should be roughly the same, while Uranus has a large deflection of its rotation axis and its major satellites deviate at the same angle. It is easy to think that this is caused by one collision of another celestial body. Uranus’ satellites are also produced in the collision event, and the mass of the collision body should be relatively large. Recent research Ida et al. (2020) has given strong evidence to this theory. Based on simulations, in the early days of the solar system, a giant ice block hit Uranus, overturning Uranus and causing it to tilt. The mass of the ice block is equivalent to 1–3 times that of the earth. And Uranus’ weird magnetic field may be the aftermath of the impact event.

The period of Uranus rotating the Sun is 84 yr. When Voyager 2 visited Uranus in 1986, the Sun was illuminating the southern hemisphere of Uranus. About 40 yr later, now the north parts of Uranus are visible. This raises interest for studies on the newly illuminated surfaces of Uranus. Are they different from the south? Are there any new features that can tell us more about the forces and physics that modeled the satellites surfaces? These topics are in the scope of future space missions to be launched from 2030 on, to arrive at the Uranus system from 2040 on.

The orbit parameters of five major Uranian satellites have played an important role in the work of Ida et al. (2020), while the precision of orbit parameters relies on both precision and time coverage of the positions of satellites (Desmars & Arlot 2009; Jacobson 2014; Emelyanov 2018). Considering ephemeral work is important for the planning and successful execution of missions in the space exploration of Uranus and its satellite systems, it is necessary to observe these five satellites with ground-based telescopes. Their space observations are mainly from Voyager 2 and Hubble telescope. Because these satellites are bright, ground-based optical telescopes can make long-term observations. Many observatories around the world have devoted some time to observations (Stone Ronald 2000; Camargo et al. 2015; Yizhakevych et al. 2016; Xie et al. 2019; Camargo et al. 2022; Veiga et al. 1987, 2003).

In this paper, we focus on the five major satellites of Uranus, Oberon, Titania, Ariel, Umbriel, and Miranda. The parameters of these five satellites are shown in Table 1. Our research group have focused on the Uranian satellites since 1990 and has

* Released on 2021 March 1st.

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acquired charge coupled device (CCD) images data using the ground-based telescope (Shen et al. 2002; Qiao et al. 2013; Yan et al. 2016). For the CCD image processing of these satellites near the bright planets, many scholars have used several methods to fit the planets’ halo separately, generally using high-order polynomials such as symmetric circle and ellipse to fit, and then calculated the positions of the satellites with high precision (Pascu et al. 1985, 1986; Veiga & Martins 1995; Stone Ronald 2000; Peng et al. 2002). Assafin et al. (2008) have developed a prototype of a coronagraph of simple design making use of good quality commercial optical systems. Pilot tests made with this coronagraph with a telescope of 1.6 m show that it is possible to obtain signal-to-noise ratios of 50 or higher for 10 s exposure time, for satellites as close as 2 Uranus radii from the planet center. The method used in this paper is specially developed by our group for detecting the faint satellites near bright planets. It has been applied to the work about the image processing of Neptunian satellites (Zhang et al. 2021, 2022), and will be used to process Saturnian satellites in the future.

The processing of the observed CCD images during 2008–2014 were carried out by the latest CCD image processing software developed by our group with the help of Gaia EDR3 catalog (Gaia Collaboration et al. 2020). This batch of observed positions were compared with the positions from ephemeris (DE431+Lainey et al. 2015; Lainey 2008; Arlot et al. 2017). These long time-span and high-precision observation data are of important significance for orbit improvement of each satellite as well as subsequent deep space exploration.

The content of this paper is arranged as follows: in Section 2, the basic information of the observation and the preprocessing method are introduced. In Section 3, the process of astrometric reduction and results are given. In Section 4, comparison of our positions with the ephemeris is given. Section 5 is conclusion.

2. Observations and Measurement

2.1. Observations

When the opposition of Uranus happened in 2008–2014, the 1.56 m astrometric reflector at the Shanghai Astronomical Observatory (N31°096, E121°184, H97m, IAU code 337) was used to observe Uranus and its five major satellites, Ariel, Umbriel, Titania, Oberon, and Miranda. Table 2 shows the main parameters of the telescope and CCD camera.

Due to the large brightness difference between Uranus and its surrounding satellites, as well as the fact that the CCD we used did not have the function of antiblooming, if the exposure time is too long, the image of bright Uranus will become saturated and have a strong halo, possibly causing the images of the satellites nearby to become degraded. On the other hand, if the exposure time is too short, it is hard to get high signal–noise rate (SNR) images of faint satellites and get enough reference stars. Therefore, suitable exposure times were considered before observation; the exposure time ranged from 20 to 120 s depending on the different weather condition and the relative positions of the satellites and Uranus. At the same time, for all observations, binning 2 x 2 was used to increase the SNR of star images and decrease the readout time, so the spatial resolution of the single pixel of the image is 0′′64. The Figure 1(a) shows the sample image obtained on 2011 September 24th, and each satellite can be seen vaguely.

2.2. Measurement

As can be seen in Figure 1(a), the halo of Uranus is so large that it covers the area of nearby satellites. If using normal methods deals with such images directly, not only the success rate of the detecting faint satellite, but also the accuracy of positions of the satellites will be influenced. For this batch of images, the methods mentioned in our previous work (Zhang et al. 2021) were used again. First, we generated a background image by median filtering using the original image (see Figure 1(a)), and then deducted the entire background image from the original image to obtain new images (see Figure 1(b)). Red circles in the Figure 1(a) and (b) are the targets detected in the images by the same method of the star detection, but the images are preprocessed by different methods. Figure 1(a) is preprocessed with the traditional process (correction of bias, dark, and flat), while another method described in our previous paper (Zhang et al. 2021) is used in Figure 1(b). It can be

| Major Satellites (1) | Ariel (UI) (2) | Umbriel (UII) (3) | Titania (UIII) (4) | Oberon (UIV) (5) | Miranda (UV) (6) |
|----------------------|----------------|-------------------|-------------------|-----------------|-----------------|
| Mass (10^{18}kg)    | 12.9           | 12.2              | 34.2              | 28.8            | 0.66            |
| Radius (km)         | 581.1 × 577.9 × 577.7 | 584.7             | 788.9             | 761.4           | 240 × 234.2 × 232.9 |
| Mean density (kg m^{-3}) | 1,500         | 1,460             | 1,660             | 1,560           | 1,200           |
| Visual geometric albedo | 0.39          | 0.21              | 0.27              | 0.23            | 0.32            |
| Semimajor axis (10^{7} km) | 190.90        | 266.00            | 436.30            | 583.50          | 129.90          |
| Semimajor axis (Uranian Radii) | 7.469    | 10.41             | 17.07             | 22.83           | 5.082           |
| Orbital period (days) | 2.520379      | 4.144176          | 8.705867          | 13.463234       | 1.413479        |
| Inclination (degrees) | 0.04          | 0.13              | 0.08              | 0.07            | 4.34            |
| Eccentricity         | 0.0012         | 0.0039            | 0.0011            | 0.0014          | 0.0013          |

Note. 
*: https://nssdc.gsfc.nasa.gov/planetary/factsheet/uraniansaftfact.html

| Tele. and CCD (1) | Parameters (2) |
|-------------------|----------------|
| Diameter of primary mirror | 156 cm |
| Focal length       | 15600 mm       |
| Size of CCD array (pixels) | 2112 × 2048 |
| Size of pixel      | 24 μm          |
| Angular resolution of pixel | 0′′32 |
| Field of view      | 11′′2 × 10′′8  |
| Bandpass of CCD (nm) | 300–1100 |

Table 1
The Parameters of the Five Major Satellites

| Major Satellites | Ariel (UI) | Umbriel (UII) | Titania (UIII) | Oberon (UIV) | Miranda (UV) |
|------------------|------------|---------------|----------------|--------------|--------------|
| Mass (10^{18}kg) | 12.9       | 12.2          | 34.2           | 28.8         | 0.66         |
| Radius (km)      | 581.1 × 577.9 × 577.7 | 584.7       | 788.9          | 761.4        | 240 × 234.2 × 232.9 |
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| Orbital period (days) | 2.520379 | 4.144176      | 8.705867       | 13.463234    | 1.413479     |
| Inclination (degrees) | 0.04 | 0.13          | 0.08           | 0.07         | 4.34         |
| Eccentricity       | 0.0012    | 0.0039        | 0.0011         | 0.0014       | 0.0013       |

Table 2
Parameters of The Telescope Used for Observation of Five Major Uranian Satellites
seen that the number of targets detected in Figure 1(b) is significantly more than that detected in Figure 1(a). In particular, faint satellites (as Umbriel and Miranda) near the bright Uranus in Figure 1(b) are detected. They are also exactly the targets that we focus on. As shown in Figure 1(c), this figure is an amplification of the region near Uranus in Figure 1(b).

Figure 2 shows a three-dimensional diagram of the location and flow of each star images in the region of Figure 1(c). To better display their locations, we selected this image, which had an exposure time of 120 s. In Figure 2, it is evident that both Umbriel and Miranda are in the halo of Uranus in the original images. Using the method mentioned above, they can be well split out, and precise positions of them could be obtained.

In order to verify that the preprocessing method used in this article is effective, we take the images on 2011 September 22 and compare the preprocessing of the images. Table 3 gives the results of using the image processing method of this paper, and the results from the traditional image processing method are also given. From Table 3, it can be seen that only Ariel, Titania, and Oberon can be detected with traditional method, and the number of successful detection is relatively small, but Umbriel and Miranda were not able to be detected. In the observed images of this night, Titania and Oberon are far from Uranus and are almost unaffected by its halo, so the positioning accuracy of the two methods is similar. Using the method described in this paper, 22 positions of Ariel can be detected and the positioning accuracy of its R.A. and decl. is, respectively, 0.037″ and 0.044″, which is equivalent to 1/15 pixels. However, only two positions can be successfully extracted using traditional processing methods. Umbriel and Miranda are very close to Uranus, so it was heavily affected by the halo and quite few data is available for the two satellite by now. Therefore, it is very important to obtain more observations for them. Table 3 shows that more positions of these two satellites can be obtained by the image processing method from our group. It can be concluded that the preprocessing method used in this paper is helpful to detecting and positioning the faint satellites near bright planets. It is worth noting that the image preprocessing method used here is not effective for precise photometry of satellites. Because this method of image preprocessing is related to region brightness, the brightness of the star image will be affected after image preprocessing.
3. Astrometric Reduction and Results

For the processing of the CCD images, the same automatic program developed by our group were carried out (Zhang et al. 2021). All reference stars and the five major satellites of Uranus were detected automatically by the program, and x–y coordinates were calculated. Then reference stars were confirmed by positions from the Gaia catalog (Gaia Collaboration et al. 2020) and images of the satellites were identified with the predicted positions from their ephemeris of DE431 + Lainey et al. (2015). The positions of five major satellites of Uranus were calculated by standard astrometric process.

Table 4 presents an extract of the observed positions of the five major satellites. The full table will be available at the Institut de mécanique céleste et de calcul des éphémérides (IMCCE). Here the R.A. and decl. are in topocentric coordinates, and the time of observation is given in UTC.

4. Comparison with Ephemeris

In order to check the accuracy of our results and to analyze the difference between the existing ephemeris and observed positions, we compared the position data after image processing with the existing satellite ephemeris. The O–C data are given for each satellite in Tables 5–9. Here O means the observed position, and C is the theoretical position that were predicted from the ephemeris (DE431 + Lainey et al. 2015).

Figure 3 shows the offset distribution for different satellites corresponding to Tables 5–9. Different colors were used to represent the data obtained from the different observation missions.

Here is the summary about the above comparison:

1. A total of 386 Ariel positions, 336 Umbriel positions, 541 Titania positions, 594 Oberon positions, and 58 Miranda positions were obtained.
Table 4
An Extract from The List of The Observed Positions of The Five Major Uranian Satellites

| Sat. | Year | Month | Day(UTC) | α(hms) | δ(dms) | Site |
|------|------|-------|----------|--------|--------|------|
| U1   | 2011 | 09    | 24.656840| 0 10 14.6541| +00 16 23.700| 337  |
| U1   | 2011 | 09    | 24.659109| 0 10 14.6352| +00 16 23.595| 337  |
| U2   | 2011 | 09    | 24.659109| 0 10 14.5489| +00 16 17.620| 337  |
| U2   | 2011 | 09    | 24.659109| 0 10 14.5363| +00 16 17.620| 337  |
| U3   | 2011 | 09    | 24.656840| 0 10 15.7311| +00 15 49.901| 337  |
| U3   | 2011 | 09    | 24.659109| 0 10 15.7168| +00 15 49.905| 337  |
| U4   | 2011 | 09    | 24.656840| 0 10 14.6338| +00 16 49.070| 337  |
| U4   | 2011 | 09    | 24.661134| 0 10 14.5943| +00 16 48.882| 337  |
| U5   | 2011 | 09    | 24.659109| 0 10 15.1741| +00 16 2.934 | 337  |
| U5   | 2011 | 09    | 24.659109| 0 10 15.1688| +00 16 2.825 | 337  |

Note. U1, U2, U3, U4, and U5, respectively, represent Ariel, Umbriel, Titania, Oberon, and Miranda. The decimal day corresponds to the middle time of each exposure. Right ascension and decl. are topocentric and are referred to the mean equator and equinox of J2000.0. Sat. is the number of satellites. Y, M, and D, respectively, represent the year, month and day. Site is the code of the observing site.

(This table is available in its entirety in machine-readable form.)

Table 5
Results for Ariel

| Year | Date | N | Δα(°) | Δδ(°) | σα(°) | σδ(°) |
|------|------|---|-------|-------|--------|--------|
| 2008 | 0808 | 25 | 0.089 | 0.043 | 0.070 | 0.091 |
| 2009 | 0817 | 16 | 0.028 | 0.018 | 0.127 | 0.089 |
| 2010 | 0820 | 2 | 0.046 | 0.079 | 0.060 | 0.052 |
| 2011 | 0814 | 15 | 0.133 | 0.092 | 0.078 | 0.062 |
| 2012 | 0816 | 64 | 0.018 | 0.042 | 0.038 | 0.034 |
| 2013 | 0817 | 54 | 0.043 | 0.004 | 0.047 | 0.043 |
| 2014 | 0824 | 42 | 0.011 | 0.071 | 0.049 | 0.040 |
| 2015 | 0825 | 18 | 0.031 | 0.022 | 0.061 | 0.069 |
| Total | 16 | 386 |        |        |        |        |

Note. O is the observed position of Ariel from this paper, C is the ephemerides based on DE431+ Lainey et al. (2015) ephemeris. Columns: year of observation; number of positions; mean offset in R.A.; mean offset in decl.; standard deviation of the offset in R.A.; standard deviation of the offset in decl. The O−C in R.A. are already multiplied by the cosine of the decl.

Table 6
Results for Umbriel

| Year | Date | N | Δα(°) | Δδ(°) | σα(°) | σδ(°) |
|------|------|---|-------|-------|--------|--------|
| 2008 | 0808 | 22 | 0.082 | 0.063 | 0.089 | 0.083 |
| 2009 | 0817 | 16 | 0.085 | 0.045 | 0.118 | 0.146 |
| 2010 | 0820 | 2 | 1.142 | 0.026 | 0.193 | 0.058 |
| 2011 | 0824 | 14 | 0.073 | 0.222 | 0.046 | 0.046 |
| 2012 | 0825 | 22 | 0.029 | 0.025 | 0.044 | 0.044 |
| 2013 | 0825 | 18 | 0.031 | 0.022 | 0.061 | 0.069 |
| Total | 19 | 336 |        |        |        |        |

Note. O is the observed position of Umbriel from this paper, and C is the ephemerides based on DE431+ Lainey et al. (2015) ephemeris. Columns: same as Table 5.

2. Due to the different weather conditions, both 2008 Aug and 2014 Aug observation missions obtained only one day observation data. These data were of good quality. Although the number of observations in 2008 September and 2009 August are large, their accuracy is slightly bad. There are many CCD frames captured in the two observational missions in 2010 and the positions are of high accuracy, especially for the mission in 2010 September. According to the observational notebook, the weather was good during this observational mission. The exposure time did not need to be very long, and the halo of Uranus had a relatively small effect to the surrounding satellites.

3. The mean values in Tables 5–9 presented a large difference between the observed positions and the existing ephemeris, which also indicated that the orbits of these five satellites still need to be improved. The more precise ephemeris will provide better data support for the study of solar system evolution and future deep space exploration.
5. Conclusion

A total of 1915 precise positions of five major Uranian satellites during 2008–2014 were obtained based on Gaia EDR3 in this paper. These 7 yr time-span observations will be very helpful for improvement of orbit parameters of both Uranus and its five major satellites. All these data will be available in electronic form at the IMCCE. To remove the influence of Uranus’ halo on the positions of surrounding faint satellites, a CCD image preprocessing algorithm based on the median filtering method was applied, and good results were achieved, as well as more satellites submerged by halos were detected successfully, especially Miranda. The method developed by our group will be applied to the processing of CCD images of Saturnian satellites in the future.
The authors are grateful to the staff at the Sheshan station of the Shanghai Astronomical Observatory for their assistance, especially to S. H. Wang and H. J. Pan for providing us so much convenience throughout our observations. This work has made use of data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC; https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions; in particular, the institutions participating in the Gaia Multilateral Agreement. This work was carried out with the financial support of the National Science Foundation of China (grant Nos. 11803019, 12073062).

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Table 7

| Year | Date | N | $\Delta - C_1$ ($^\circ$) | $\Delta - C_2$ ($^\circ$) | $\sigma_1$ ($^\circ$) | $\sigma_2$ ($^\circ$) |
|------|------|---|----------------|----------------|----------------|----------------|
| 2008 | 0808 | 24 | 0.163 | 0.022 | 0.073 | 0.084 |
| 2009 | 0917 | 9 | $-0.067$ | $-0.073$ | 0.108 | 0.089 |
| 2010 | 0922 | 16 | 0.003 | $-0.061$ | 0.092 | 0.064 |
| Total | 4 | 58 |

Table 8

| Year | Date | N | $\Delta - C_1$ ($^\circ$) | $\Delta - C_2$ ($^\circ$) | $\sigma_1$ ($^\circ$) | $\sigma_2$ ($^\circ$) |
|------|------|---|----------------|----------------|----------------|----------------|
| 2008 | 0808 | 22 | 0.133 | 0.034 | 0.058 | 0.063 |
| 2009 | 0901 | 29 | $-0.036$ | $-0.026$ | 0.133 | 0.113 |
| Total | 19 | 541 |

Table 9

| Year | Date | N | $\Delta - C_1$ ($^\circ$) | $\Delta - C_2$ ($^\circ$) | $\sigma_1$ ($^\circ$) | $\sigma_2$ ($^\circ$) |
|------|------|---|----------------|----------------|----------------|----------------|
| 2010 | 0917 | 9 | $-0.067$ | $-0.073$ | 0.108 | 0.089 |
| 2011 | 0922 | 16 | 0.003 | $-0.061$ | 0.092 | 0.064 |
| Total | 4 | 58 |

Note. $O$ is the observed position of Titania from this paper, and $C$ is the ephemerides based on DE431+ Lainey et al. (2015) ephemeris. Columns: same as Table 5.

Note. $O$ is the observed position of Oberon from this paper, and $C$ is the ephemerides based on DE431+ Lainey et al. (2015) ephemeris. Columns: same as Table 5.