THE CORRELATION FUNCTION OF FLUX-LIMITED X-RAY CLUSTERS

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ABSTRACT

We show that the spatial correlation function of a flux-limited sample of X-ray selected clusters of galaxies will exhibit a correlation scale that is smaller than the correlation scale of a volume-limited, richness-limited sample of comparable apparent spatial density. The flux-limited sample contains clusters of different richesses at different distances: poor groups are found nearby and rich clusters at greater distances. Since the cluster correlation strength is known to increase with richness, the flux-limited sample averages over the correlations of poor and rich clusters. On the other hand, a volume-limited, richness-limited sample has a minimum richness threshold, and a constant mixture of richesses with redshift. Using the observed correlation scale for rich \((R \geq 1)\) clusters, \(r_o(R \geq 1) = 21 \pm 2h^{-1}\text{Mpc}\) that was determined from previous volume-limited studies, we derive for the ROSAT flux-limited X-ray cluster sample \(r_o(\text{flux-limited}) \approx 14h^{-1}\text{Mpc}\), in agreement with the recently observed value of \(13.7 \pm 2.3h^{-1}\text{Mpc}\).

Cosmology: large-scale structure of Universe – galaxies: clusters of – X-rays: galaxies
1. INTRODUCTION

The correlation function of clusters of galaxies constrains cosmological models for the formation and evolution of structure in the universe (White et al. 1987; Bahcall 1988; Suginoara & Suto 1991; Bahcall & Cen 1992; Croft & Efstathiou 1993). The cluster correlation function is known to be stronger than the correlation function of galaxies; the correlation scale of the rich and rare clusters of richness class \( R \geq 1 \) is \( r_o(R \geq 1) = 21 \pm 2h^{-1}\text{Mpc} \) (Bahcall & Soneira 1983; Postman et al. 1992; Peacock & West 1992), while the galaxy correlation scale is \( r_o(g) = 5h^{-1}\text{Mpc} \) (Groth & Peebles 1977). The cluster correlation strength is observed to increase with cluster richness: rich, rare clusters exhibit stronger correlations than the more numerous poor clusters and groups (Bahcall & Soneira 1983; Bahcall & West 1992). All samples of clusters studied so far, from poor groups of galaxies to the richest \( R \geq 1 \) and \( R \geq 2 \) Abell clusters, including the intermediate richness clusters observed by the new automated cluster surveys of the EDCC (Nichol et al. 1992) and APM (Dalton et al. 1992) are consistent with a universal richness-dependent correlation (Bahcall & West 1992). Large-scale cosmological N-body simulations of galaxy clusters also show the same dependence of the cluster correlation function on richness (Bahcall & Cen 1992; Croft & Efstathiou 1993). This richness-dependent correlation function applies to complete richness-limited samples of clusters (i.e., clusters above a given richness threshold); it represents the underlying spatial distribution of a system of clusters of a given richness class.

Recently, the spatial correlation function of a sample of X-ray clusters of galaxies detected in a flux-limited survey of the ROSAT X-ray satellite was reported (Romer et al. 1993). The sample contains all X-ray sources above a given
X-ray flux threshold that are associated with a local galaxy density enhancement. A correlation length of \( r_o = 13.7 \pm 2.3h^{-1}\text{Mpc} \) was determined for the sample. Romer et al. (1993) conclude, from a direct comparison of this correlation length and the larger length of the rich \( R \geq 1 \) clusters, that inconsistencies exist between the two results, and that the \( R \geq 1 \) cluster correlation scale has been overestimated.

In the present letter we show that a flux-limited sample, such as the X-ray sample described above, differs significantly from a richness-limited sample (such as the \( R \geq 1 \) clusters from which the \( 21h^{-1}\text{Mpc} \) scale length was obtained). The flux-limited sample is a "richness-mixed" sample; it contains, by definition, poor clusters nearby and rich clusters at greater distances. Since cluster correlations depend on richness, a simple comparison between the correlation properties of a flux-limited and a richness-limited sample is inappropriate. Here we show that, based on the richness-dependent correlation function and a correlation scale of \( r_o(R \geq 1) = 21 \pm 2h^{-1}\text{Mpc} \) for \( R \geq 1 \) clusters, the expected correlation scale for the above flux-limited X-ray sample is \( r_o(\text{flux-limited}) \approx 2/3r_o(R \geq 1) = 14h^{-1}\text{Mpc} \), as is indeed observed. This first X-ray selected ROSAT cluster sample thus confirms the richness-dependent cluster correlation function.

2. THE CORRELATION FUNCTION OF A FLUX-LIMITED X-RAY SAMPLE

The richness-dependent cluster correlation function is represented by (Bahcall & Soneira 1983; Bahcall & West 1992)

\[
\xi_{cc}(r) \approx 4Nr^{-1.8},
\]

where \( \xi_{cc} = Ar^{-1.8} = (r/r_o)^{-1.8} \) is the standard form of the correlation function.
(with an amplitude $A$ and a correlation scale $r_o$), and $N$ is the median richness of the cluster sample. Relation (1) applies to complete volume-limited, richness-limited samples (all clusters above a threshold richness limit); it represents the spatial distribution of clusters of a given richness class. Similarly, the cluster correlation amplitude also depends on the mean separation of clusters, $d$ (where $d = n^{-1/3}$, and $n$ is the space-density of the cluster sample). This led to the universal dimensionless cluster correlation function (Szalay & Schramm 1985; Bahcall & West 1992)

$$\xi_{cc}(d) = 0.2(r/d)^{-1.8} = (r/0.4d)^{-1.8}, \quad \text{i.e.,} \quad r_o \approx 0.4d . \quad (2)$$

The above-described universal correlation function is seen both in observations and in model simulations (Bahcall & Cen 1992). It applies to complete volume-limited, richness-limited samples, where $n$ and $d$ represent the underlying density and mean separation of a complete system of clusters above a given richness threshold. All the principal cluster samples analyzed to-date for which relations (1) and (2) apply are volume and richness limited (e.g., Abell, Zwicky, APM, EDCC clusters, and groups; see summary in Bahcall & West 1992).

Recently, a flux-limited correlation function of X-ray clusters was determined by Romer et al. (1993). The sample includes all X-ray sources above a flux threshold of $F_x \geq 10^{-12}$ergs cm$^{-2}$ s$^{-1}$. A total of 161 sources associated with some enhancement in the galaxy density distribution are detected in a 3100 deg$^2$ region; 128 of these systems have measured redshifts. Romer et al. find $r_o = 13.7 \pm 2.3h^{-1}$Mpc for the correlation function of this flux-limited redshift sample (for a correlation slope of $-1.9 \pm 0.4$). They contrast this correlation scale with the value of $r_o = 21 \pm 2h^{-1}$Mpc observed for the rich $R \geq 1$ clusters.
However, the volume-limited and the flux-limited samples are not expected to yield the same correlation scales, since the criteria for inclusion in the two samples are different. The volume-limited sample includes all clusters above a given richness threshold; the flux-limited sample includes a mixture of richesses that is a function of redshift. The *average* observed cluster density, $n$, and mean separation, $d$, of the flux-limited sample are *not* representative of a given richness system, and thus can not be applied in relations (1) and (2).

What would the correlation function of such a flux-limited sample be if the underlying cluster correlation is represented by the richness-dependent universal correlation function (relations 1 and 2)? We address this question below.

An observational relation exists, as expected theoretically, between the X-ray luminosity of clusters and cluster temperature, or mass. Henry & Arnaud (1991) find that, for a Hubble constant of $H_o = 100h\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ (where $h = 1$ will be used throughout), $L_x(\text{Bol}) = 2.5 \times 10^{42}T^{2.7 \pm 0.4}$, where $L_x(\text{Bol})$ is the bolometric X-ray luminosity of the cluster in erg s$^{-1}$, and $T$ is the intracluster gas temperature in keV. Edge & Stewart (1991) find similar results, with $L_x(\text{Bol}) = 2.5 \times 10^{42}T^{2.5}$ (see also Henry et al. 1992 and David *et al.* 1993 for similar relations). Converting the bolometric luminosity to the ROSAT observed energy band of 0.1-2.4 keV (for the typical range of cluster temperatures $T \sim 2 - 10$ keV), we find $L_x(0.1 - 2.4\text{keV}) \approx 2.8 \times 10^{42}T^{2 \pm 0.4}\text{ergs s}^{-1}$. The virial mass of a cluster is proportional to the temperature, $T$, or to the square of peculiar velocity in the cluster, $\sigma$: $M \propto T \propto \sigma^2$ (Sarazin 1988; Bahcall & Cen 1993; Lubin & Bahcall 1993). Lubin and Bahcall find, on average, $\sigma = 400T^{0.5}$ km s$^{-1}$ for the best $\sigma \propto T^{0.5}$ observed relation. The virial cluster mass within $1.5h^{-1}\text{Mpc}$ of the cluster center, assuming an isothermal density profile, is then
$M(< 1.5 h^{-1}\text{Mpc}) = 2\sigma^2 R(1.5)/G \approx 1.1 \times 10^{14} T(\text{keV})$. Combining the above relations we find

$$L_x(0.1 - 2.4 \text{keV}) \approx 2.2 \times 10^{42} (M/10^{14} M_\odot)^{2\pm0.4} \text{ergs s}^{-1}. \quad (3)$$

The above relation is consistent with the theoretically expected dependence resulting from the thermal bremsstrahlung origin of the X-ray emission: $L_x(\text{Bol}) \propto M_{\text{gas}}^2 T^{0.5} \propto M^{2.5}$ (for an approximately constant size of the X-ray emitting region, and $M_{\text{gas}} \propto M$). This yields $L_x(0.1 - 2.4) \propto M^2$, consistent with eq. (3).

Using eq. (3) above, the observed X-ray flux of a cluster is

$$F_x(0.1 - 2.4 \text{keV}) \approx 2.2 \times 10^{42} (M/10^{14} M_\odot)^{2\pm0.4}/4\pi d^2 \text{ergs cm}^{-2} \text{ s}^{-1}, \quad (4)$$

where $d$ is the luminosity-distance of the cluster. The flux is therefore proportional, approximately, to $F \propto (M/d)^2$. For a flux-limited sample, the nearby clusters have a low mass (and thus a low richness) threshold, and the distant clusters have a richer threshold. In a flux-limited sample the number density of clusters decreases faster for poor clusters than for rich clusters. A volume-limited sample, on the other hand, has a cluster density that remains constant (in comoving coordinates) for each richness class.

To calculate the expected correlation function of a flux-limited X-ray cluster sample, we use N-body simulations that match the correlation function of the observed richness threshold cluster samples ($R \geq 1$ clusters, EDCC clusters, and APM clusters). We use a large-scale Particle-Mesh code with a box size of $400 h^{-1}\text{Mpc}$ to simulate the evolution of the dark matter. The box contains $500^3$ cells and $250^3 = 10^{7.2}$ dark matter particles. The spatial resolution is
0.8h^{-1}\text{Mpc}. Details of the simulations are discussed in Cen (1992) and Bahcall & Cen (1992). A model that reproduces the observed mass-function of clusters, as well as the observed correlation function of the richness-threshold \( R \geq 1 \), APM, and EDCC clusters is a low-density, unbiased CDM model (Bahcall & Cen 1992). This model, with \( \Omega = 0.2, h = 0.5 \), and no bias \( (b = 1) \) produces the observed richness-dependent universal cluster correlation function (eq. 1-2), and is consistent with other cluster properties such as their mass-function.

Clusters are selected in the simulation box using an adaptive linkage algorithm following the procedure described in Suto, Cen & Ostriker (1992) and Bahcall & Cen (1992); the cluster mass within \( 1.5h^{-1}\text{Mpc} \) is determined. In order to extend the volume-limited sample of the underlying cluster distribution to distances of the most distant X-ray clusters observed \( (z \leq 0.25) \), a mosaic of eight \( 400h^{-1}\text{Mpc} \) simulation boxes are used, corresponding to \( 800h^{-1}\text{Mpc} \) on a side. Within this larger mosaic box, \( \sim 3000 R \geq 1 \) clusters are identified in the simulation (with \( n = 6 \times 10^{-6} h^3\text{Mpc}^{-3} \)).

Each cluster of mass \( M \) (within \( 1.5h^{-1}\text{Mpc} \)) is assigned an X-ray luminosity as given by relation (3), and an X-ray flux—to an observer at the corner of the box—as given by (4). The flux threshold of the Romer et al. (1993) sample is then applied. All clusters in the \( 800h^{-1}\text{Mpc} \) simulation box with X-ray flux above this threshold are identified; they correspond to a flux-limited X-ray sample similar to the one observed.

The redshift distribution of the X-ray clusters in both the observed and the simulated flux-limited samples are presented in Figure 1. Two simulated cases are shown: one corresponds to relation (4) (with a 15\% lower amplitude in the \( F_x - M^2 \) relation in order to match the observed average co-moving density of
X-ray clusters in the range $cz = 5000$ to $50000$ km s$^{-1}$, $n \sim 5 \times 10^{-6} h^3 \text{Mpc}^{-3}$); the other corresponds to a somewhat shallower slope than given in relation (4), $F_x \propto M^{1.7}$, with an amplitude that yields an average comoving density of $\sim 8 \times 10^{-6} h^3 \text{Mpc}^{-3}$. Both simulations yield results that are consistent with the redshift distribution of the observed flux-limited sample of clusters. Furthermore, we find that plausible variations in relation (4) do not produce significant changes in the results.

The mean richness of the X-ray clusters as a function of redshift is presented in Figure (2). In the flux-limited simulations, there is a strong increase of richness with redshift. By contrast, the mean richness in volume-limited samples is constant. Approximately 30% of the clusters are poor ($R \leq 0$), consistent with the observed sample of Romer et al. (1993).

The correlation function of the simulated flux-limited sample is presented in Figure 3. It is compared with the X-ray cluster observations of Romer et al. (1993). The agreement between the observations and simulations is excellent. The simulations yield $r_o \approx 14 h^{-1} \text{Mpc}$ for the flux-limited sample, consistent with the observed $r_o = 13.7 \pm 2.3 h^{-1} \text{Mpc}$. The correlation function of the simulated volume-limited rich $R \geq 1$ clusters (with $n = 6 \times 10^{-6} h^3 \text{Mpc}^{-3}$, $d = 55 h^{-1} \text{Mpc}$) is also shown, for comparison; this function matches well the observed $R \geq 1$ correlations, with $r_o \approx 21 h^{-1} \text{Mpc}$ (Bahcall & Soneira 1983; Postman et al. 1992; Peacock & West 1992).

Figure 3 shows that the flux-limited sample exhibits a lower correlation amplitude than the volume-limited, richness-limited sample of comparable average number density. This is expected due to the “richness-mixed” nature of the flux-limited sample. In the present case, the correlation scales of the flux-limited and
the richness-limited samples satisfy

\[ r_o(\text{X-ray flux-limited}) \approx \frac{2}{3} r_o(R \geq 1), \]

with \( r_o(R \geq 1) \approx 21 h^{-1} \text{Mpc} \) and \( r_o(\text{X-ray flux-limited}) \approx 14 h^{-1} \text{Mpc} \), as observed. The results are insensitive to reasonable variations in the \( L_x(M_{\text{cluster}}) \) relation. As an example, we present in Fig. 3 the correlation function for the case \( L_x(0.1 - 2.4) = 3.5 \times 10^{42} (M/10^{14} M_\odot)^{1.7} \text{ergs s}^{-1} \), which also matches the observed cluster density distribution (Fig. 1).

3. CONCLUSIONS

We show that the spatial correlation function of a flux-limited sample of X-ray selected clusters of galaxies exhibits a correlation scale that is smaller than the correlation scale of a volume-limited, richness-limited sample of comparable average number density (Fig. 3). This is expected due to the “richness-mix” nature of the flux-limited sample, which contains poor clusters nearby and rich clusters farther away.

We show that the correlation scale of the new flux-limited X-ray cluster sample from ROSAT (Romer et al. 1993) is expected to be \( r_o(\text{X-ray flux-limited}) \approx \frac{2}{3} r_o(R \geq 1) \approx 14 h^{-1} \text{Mpc} \), as observed. We conclude that the new observations of X-ray clusters from ROSAT exhibit a correlation function that is consistent with, and is actually predicted from, the richness-dependent cluster correlation and \( r_o(R \geq 1) \approx 21 h^{-1} \text{Mpc} \).

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FIGURE CAPTIONS

Fig. 1– Redshift distribution of the flux-limited X-ray clusters. The X-ray cluster observations of Romer et al. (1993) are shown by the faint histogram. Two simulated samples (§2) are represented by the dark and dashed histograms. $dN(z)$ represents the relative number of clusters (i.e., fraction of total) in each redshift bin. (A volume-limited sample yields $dN(z) \propto z^2$).

Fig. 2– The mean richness class of the flux-limited X-ray cluster samples as a function of redshift. The two simulated samples (Fig. 1, §2) are shown. Also presented, for comparison, are the horizontal lines for the volume-limited, richness-limited $R \geq 1$ and $R \geq 0$ clusters.

Fig. 3– Cluster correlation function: the expected difference between the cluster correlation function of a volume-limited $R \geq 1$ cluster sample (faint line, with $r_o \approx 21h^{-1}\text{Mpc}$), and the relevant flux-limited X-ray cluster correlations ($F_x \geq 10^{-12}\text{ergs cm}^{-2}\text{ sec}^{-1}$; dark and dashed lines, $r_o \approx 14h^{-1}\text{Mpc}$). The flux-limited sample is expected to exhibit weaker correlations than the $R \geq 1$ complete sample. The observed flux-limited X-ray cluster correlations (Romer et al. 1993), shown by the solid dots, are consistent with the expected flux-limited correlation function and $r_o(R \geq 1) \approx 21h^{-1}\text{Mpc}$.
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