Possible Scheme of Turbulent Potential Flow Free Expansion Below the Non-Pressure Pipe

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Abstract. The paper substantiates a general scheme for solving two-dimensional free spreading problem in terms of open water potential flow below the non-pressure pipe. The first link. Since a uniform flow at its outlet from a rectangular pipe with its free spreading is coupled with a non-uniform flow of a general form, then, using a theorem from the general theory of two-dimensional in terms of turbulent water potential flows, the conclusion is given: since a straight-line characteristic is always the boundary between a uniform and non-uniform flow, then only a simple wave can directly adjoin the uniform flow area. Simple waves serve as a transitional form from the uniform flow to the general non-uniform flow. Therefore, it is not entirely correct to pose the free flow spreading boundary problem, trying to satisfy only the boundary conditions and obtain an analytical solution without taking into account the intermediate flow “simple wave”. The second important link in solving the problem is a general flow choice. And it can be selected from the intermediate flow condition found by the authors in the velocity hodograph plane. Let us call this flow the “type A flow”. "Type A flow" satisfies all the requirements of the flow spreading process function. When increasing, i.e. $\tau$ tends to 1, flow depth $h$ tends to 0; velocity $V$ tends to the maximum. The flow depth is greater on the symmetry axis than when moving along the equipotential to the flow free boundary. Using these two links, it is possible to apply a flow coupling scheme and solve the boundary problem of determining the entire spectrum of parameters for the potential flow spreading. The results obtained in this work can be used by the designers of hydraulic structures, which length is relatively short and where the flow resistance forces can be neglected.

1. Introduction
The previously obtained results of the research on turbulent potential flow free expansion were reflected in the works [1-5]. This work is relevant in connection with the possibility of increasing the model adequacy to the real process and an increase in the number of options for conjugating various two-dimensional in terms of water flows. A model scheme of free flow spreading, based on the uniform flow conjugation with a radial flow by means of a transition section (a simple wave) is proposed in the work [6]. However, as experiments and observations of the real flow confirm, when it is freely spread into a wide diversion channel along the flow symmetry axis, the depths and velocities’
distribution does not quite satisfy radial spreading, at which the depths and velocities do not change in the direction perpendicular to the flow symmetry axis.

2. Main part

Therefore, in this paper, instead of the radial flow as a general-arrangement flow, we propose to take the flow of the following form \( A \):

\[
\begin{align*}
\psi (\tau, \theta) &= A \frac{\sin \theta}{\tau^{1/2}}; \\
\phi (\tau, \theta) &= A \frac{h_0}{H_0} \frac{\cos \theta}{\tau^{1/2} (1 - \tau)},
\end{align*}
\]  

(1)

where \( A \) – is a constant;

\( \psi (\tau, \theta) \) – is the current function;

\( \phi (\tau, \theta) \) – is the potential function;

\( H_0 = h_0 + \frac{V_0^2}{2g} \) – defines the constant in D. Bernoulli’s integral;

\( h_0, V_0 \) – denote the velocity depth and magnitude at the flow point characteristic;

\( \theta \) – is the angle characterizing the local velocity vector’s direction;

\( \tau \) – is the speed-dependent parameter:

\[
\tau = \frac{V^2}{2gH_0}. 
\]  

(1a)

The design of the analytical solution in the form (1) satisfies the main system of equations of a plan turbulent flow potential two-dimensional in the plane of the velocity hodograph [7, 8]:

\[
\begin{align*}
\frac{\partial \phi}{\partial \tau} &= \frac{h_0}{2H_0} \cdot \frac{3\tau - 1}{\tau (1 - \tau)} \cdot \frac{\partial \psi}{\partial \theta}; \\
\frac{\partial \phi}{\partial \theta} &= 2 \frac{h_0}{H_0} \cdot \frac{\tau}{1 - \tau} \cdot \frac{\partial \psi}{\partial \tau},
\end{align*}
\]  

(2)

By the direct differentiation (1), it is possible to verify that this is the solution of the system (2).

Taking the solution \( A \) as a general-arrangement flow, the free flow spreading scheme can be proposed in the scheme form (Fig.1).
The purpose of this work is to substantiate the flow interface scheme. Scheme I shows a section of uniform flow $h = h_0$; $V = V_0$; $\theta = 0$. This area is bounded with the lines $KK_1$, $K_A'$, $A_0'M_0$, $M_0A_0$, $A_0K$ and has already been verified both theoretically and experimentally.

The lines $KA_0$, $K_A'\ A_0$ – are the segments of straight lines, the length of which $X_0$ is defined in the work [9].

$A_0M_0$, $A_0'M_0$ are the segments of straight-line characteristics of the second class separating a uniform flow I from a simple wave II and the general-arrangement flow $A$.

Plot II is the site where $M_0M_0'$ is a characteristic of the first class; $A_0M_0$, $A_1M_1$ are a characteristic of the second class in a simple wave - straight line segments.

Plot III is a general-arrangement flow $A$.

Transition section II is a simple expansion wave, substantiated theoretically in [6, 10-17] as a necessary flow for conjugation of a uniform flow with a general-arrangement flow $A$.

The flow $A$ has been selected from the following considerations. It is transformed into a general-arrangement radial flow at infinity at $\tau \to 1$.

In the vicinity of the point $M_0$ and further in the plot III among the characteristics of the first class, it most qualitatively fits the conjugation with a uniform flow and reflects the fact that the depth is maximum on the flow symmetry axis, and along the equipotential, i.e. the depths are smoothly decreased in the flow transverse direction (which is confirmed by the experimental studies of the flow free spreading process).

The characteristic of the first class in the velocity hodograph plane has the form [7, 8]:

$$\theta = \arctg \sqrt{\frac{3\tau - 1}{1 - \tau}} - \sqrt{3} \cdot \arctg \left( \frac{1}{1 - \tau} \cdot \sqrt{3} \cdot \arctg \left( \frac{3\tau - 1}{1 - \tau} \right) \right) + C_1,$$

where
\[ C_1 = \arctg \sqrt{\frac{3\tau_0 - 1}{1 - \tau_0}} - \sqrt{3} \cdot \arctg \left( \sqrt{\frac{3\tau_0 - 1}{1 - \tau_0}} \right). \] (4)

The constant \( C_1 \) is determined from the condition that the characteristic passes through the point \( M_0 \) with the following parameters:

\[ \tau = \tau_0; \quad \theta = 0. \] (5)

In this case, the maximum flow spreading angle is determined from (3) at \( \tau = 1 \):

\[ \theta_{\text{max}} = C_1 + \sqrt{3} - 1 \cdot \frac{\pi}{2}. \] (6)

The characteristics of the second class are determined by the condition of their intersection with the characteristics of the first class at the points \( M_i(\tau_i, \theta_i) \) and the direction of which is determined by the wave angle in the physical region:

\[ \alpha_i = \arcsin \left( \sqrt{\frac{1 - \tau_i}{2\tau_i}} \right). \] (7)

To determine the flow parameters in the general-arrangement flow area \( A \) in the flow plan, it is necessary to use a differential complex relationship between the flow plan and the velocity hodograph plane [7, 8, 17):

\[ dz = \left( d\varphi + i \frac{h_0}{h} d\psi \right) \cdot \frac{1}{V} \cdot e^{i\theta}, \] (8)

where \( z = x + iy \) is the complex coordinate in the plane of the flow physical plane;

\[ i = \sqrt{-1} \] defines the imaginary unit [18].

So, in principle, the problem of determining the flow free spreading parameters spectrum is solved analytically and can be an element of a uniform flow conjugation with a radial flow and an element in conjunction of two uniform flows.

3. Conclusions

The proposed model is more promising compared to the previously proposed model of a turbulent flow free spreading, in which the flow radial spreading was assumed as a general-arrangement flow, since it is qualitatively closer to the real process of flow spreading.

The derivation of specific algorithms for determining the flow parameters’ spectrum is a consequence of the proposed scheme and will be published in the authors’ following editions.

4. References

[1] Kokhanenko V N, Kelekhsaev D B, Kondratenko A I and Evtushenko S I 2019 Two-dimensional motion equations in water flow zone IOP Conf. Series: Materials Science and Engineering 698(6) 066026 DOI:10.1088/1757-899X/698/6/066026

[2] Kokhanenko V N, Kelekhsaev D B, Kondratenko A I and Evtushenko S I 2019 A System of Equations for Potential Two-Dimensional In-Plane Water Courses and Widening the Spectrum of Its Analytical Solutions AIP Conference Proceedings 2188 050017 DOI: 10.1063/1.5138444

[3] Kokhanenko V N, Kelekhsaev D B, Kondratenko A I and Evtushenko S I 2020 Solution of equation of extreme streamline with free flowing of a torrential stream behind rectangular pipe IOP Conf. Series: Materials Science and Engineering 775(1) 012134 DOI:10.1088/1757-899X/775/1/012134
[4] Kokhanenko V N, Kelekhsaev D B, Kondratenko A I and Evtushenko S I 2019 Solution of Equations of Motion of Two-Dimensional Water Flow J. Construction and Architecture vol 7. Issue 3 (24) p 5-12 DOI: 10.29039/2308-0191-2019-7-3-5-12

[5] Kokhanenko V N, Burtsveva O A, Evtushenko S I, Kondratenko A I and Kelekhsaev D B 2019 Two-Dimensional in Plan Radial Flow (NonPressure Potential Source) J. Construction and Architecture vol 7. Issue 4 (25) p 74-78 DOI 10.29039/2308-0191-2019-7-4-74-78

[6] Yemtsev B T 1967 Two-dimensional turbulent flows (Moscow.: Energoizdat Publ.) p 212

[7] Kokhanenko V N, Volosukhin Ya V, Shiryaev V V and Kokhanenko N V 2007 Modeling of one-dimensional and two-dimensional open water flows (Rostov-on-Don: South Federal University Publishing House) p 168

[8] Kokhanenko V N, Volosukhin Ya V, Lemeshko N G and Papchenko N G 2013 Modeling of turbulent two-dimensional in terms of water flows (Rostov-on-Don: South Federal University Publishing House) p 180

[9] Kokhanenko V N, Kelekhsaev D B 2019 The problem solution of determining the equation of the extreme streamline and parameters along it, taking into account Xd section Research results - 2019: materials of the IV National conference of the teaching staff and scientific workers Plavtov South-Russian State Polytechnic University (NPI) Novocherkassk Russia p 113-117

[10] Vysotsky L I 1960 Hydraulic calculation of scattering trampolines by the method of longitudinal approximations (MCEU named after V.V. Kuibyshev Publ.)

[11] Sherenkov I A 1958 Calculation of the spreading turbulent flow behind the output heads of culverts Joint Workshop on Hydropower and Water Management Issue 1 (Kharkiv)

[12] Sherenkov I A 1957 Spreading of a turbulent flow behind the outlet heads of culverts under railway embankments Works of the Kharkov Institute of Railway Engineers transport named after CM. Kirov Issue 30

[13] Meleschenko N T 1948 Planned problem of hydraulics of open flows Bulletin of Vedeneev VNIIG, JSC Vol 36

[14] Suntsov N N 1958 Analogy Methods in Aerodynamics (Fizmatgiz Publ.)

[15] Ippen A T 1951 Mechanics of supercritical flow Transactions ASCE vol 116

[16] Rose H 1958 Fluid mechanics for hydraulic engineers (Moscow: Gosenergoizdat Publ.) p 368

[17] Alexandrova M S 2020 Method of Analogies between hydraulics of two-dimensional water flows and gas dynamics J. Construction and Architecture vol 8 Issue 2(27) pp 11-18 DOI 10.29039/2308-0191-2020-8-2-11-18

[18] Korn G and Korn T 1970 Handbook of mathematics for scientists and engineers (Moscow: Nauka Publ.) p 720