On the Contribution of low-mach, high-beta Shocks to the Cosmic Ray Spectrum

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Abstract. Astrophysical shocks accelerate particles through the Fermi acceleration process, which involves a charged particle repeatedly crossing the shock after being reflected by the local magnetic field and gaining momentum. Eventually, the particles reach relativistic speeds and can be observed as cosmic rays. This is a self-sustaining interaction because the presence of non-thermal particles in the shock-region causes instabilities in the magnetic field, which in turn allow the magnetic field to reflect the particles. This process has been studied extensively in the case of high-Mach, low-$\beta$ shocks, such as those that are found in stellar wind collisions and supernovae. However, there are astrophysical shocks, such as those that occur in colliding galaxy clusters, that are characterized by a low sonic Mach number, combined with a high plasma-beta. So far, these shocks have been largely neglected, and little is known about their ability to accelerate particles. Using a combined PIC-MHD code, we have performed a series of numerical simulations of low-Mach, high-beta shocks, to investigate the interaction between the particles and the magnetic field under such conditions. We find that even low-Mach shocks are capable of accelerating charged particles. However, due to the behaviour of the magnetic field, the process tends to be relatively inefficient, reducing the effective contribution to the cosmic ray spectrum. Furthermore, the interaction tends to radically change the nature of the shock itself, which indicates that further study is required to quantify the shocks’ long-term behaviour.

1. Introduction

When two galaxy clusters come into close proximity with each other, the intra-cluster medium interacts, forming shocks that are characterized by a low sonic Mach number ($M_S$) and a high plasma-$\beta$ (with $\beta$ the magnetic energy density divided by the thermal energy density) [1, 2]. At this time it is unknown whether such shocks contribute to the cosmic ray (CR) spectrum (e.g. see [3, 4, 5]), either by generating non-thermal particles in their own shock front or by re-accelerating relativistic particles that received their initial acceleration from shocks somewhere inside the galaxy cluster (stellar wind collisions, supernovae, gamma-ray bursts, active galactic nuclei etc.).

The general nature of the process by which shocks accelerate particles is well known. Numerical models have shown that a small percentage of the ions that cross the shock become non-thermal (e.g.[6, 7, 8]). The interaction between these particles and the local magnetic field causes instabilities, which, in turn, initiate the diffusive shock acceleration (DSA) process that continues to accelerate these particles through repeated shock crossings [9, 10, 11]. However,
these simulations have focused primarily on of high-Mach, low-β shocks, such as the ones found in stellar win collisions and expanding supernovae, rather than the low-Mach, high plasma-β shocks that are typical for galaxy cluster interactions.

Recent particle-in-cell (PIC) simulations [12] show that at least some low-Mach, high-β shocks are capable of injecting supra-thermal ions as well, depending on the exact shock parameters. However, because of the numerical cost of the PIC method, they could not follow the process long enough to determine whether the DSA process can function under these circumstances and accelerate these particles to relativistic speeds, thereby contributing to the CR spectrum.

A second question that needs answering is whether these shocks can re-accelerate particles that are already moving at relativistic speeds. These particles would be generated inside the galaxies that make up a galaxy cluster, and escape once they reach relativistic velocities. As they depart from the galaxy cluster, these particles would eventually reach the cluster collision shock and could, theoretically, receive a second acceleration. Such a multi-stage acceleration was reported for electrons [13] and may work for ions as well. However, the density of such particles is likely to be very small, and we need to investigate whether they are capable of triggering the instabilities that are required to maintain the DSA process.

2. Method

2.1. Numerical approach

Although the PIC method has been highly successful at simulating the behaviour of non-thermal plasmas, it comes at a high computational cost because the number of particles necessary fully simulate the plasma quickly becomes prohibitive for all but the largest computational facilities. In order to be able to follow the movement of particles as they interact with a shock, while at the same time maintaining high computational efficiency, we use a combined PIC-MHD method. This method is a variation of the two-fluid approach, assuming that the plasma is mostly thermal, but containing a small non-thermal component. As long as this condition is satisfied, the thermal plasma can be simulated with the traditional fluid-based magnetohydrodynamics (MHD) method. The non-thermal component of the plasma has to be treated as a collection of individual particles but, because they make up a (relatively) small part of the total mass, the absolute number of particles can be kept (relatively) low, thereby reducing the computational burden. These two components exist in the same volume and interact through the electro-magnetic field. (We assume that the gas is non-collisional, rendering direct interactions between particles irrelevant.)

We base our code on the MPI-AMRVAC code [14], which is a finite-volume, fully conservative MHD code that solves the conservation equations of (relativistic-)MHD on a grid with adaptive mesh refinement. Into this code we have introduced a module that traces the motion of charged particles using the using a relativistic form of the Boris-method [15] based on the MHD quantities from the beginning of the MHD time-step.

The back-reaction of the non-thermal particles on the thermal plasma is treated in a self-consistent way: Because the non-thermal particles carry an electric charge, the electromagnetic field exerts a force on them and vice-versa. Furthermore, the motion of supra-thermal particles generates a current, which in turn changes the local electromagnetic field, leading to a change in Ohm’s law [16, 17]. The relevant equations, as well as their derivation and the method used to solve them, can be found in [16, 17].

2.2. Simulation setup

We start our simulation by defining a physical space of $180 \times 30 R_l$, with $R_l$ the Gyro-radius defined by the upstream magnetic field and the initial velocity of the non-thermal particles. This box is filled with a thermal gas that flows in from the right-hand boundary, passes through shock, halfway in the grid and leaves the box at the left-hand boundary.
For the initial conditions of the gas, we use the Rankine-Hugoniot conditions of a standing shock [18]. We run two simulations. The first with an upstream velocity (in the downstream medium’s rest frame) of $0.027c$ and a sonic Mach number of $M_S = 2.0$, the second with an upstream velocity (in the downstream medium’s rest frame) of $0.052c$ and a sonic Mach number of $M_S = 3.2$. In both simulations, we place the magnetic field at a $13^\circ$ angle with the flow with the field-strength set to create an upstream plasma-\(\beta = 100\). These parameters are copied from the input used by [12].

It should be noted that the combined PIC-MHD method considers the shock itself to be an MHD phenomenon (i.e. a discontinuity in the thermal plasma). It follows that we cannot simulate the mechanism that accelerates particles out of the thermal distribution as they cross the shock because we lack the micro-physics required to model this process. Instead, we assume a fixed injection rate of 0.3 percent for the $M_S = 2.0$ model and 0.4 percent for the $M_S = 3.2$ model, so that 0.3 and 0.4 percent, respectively, of the particles traversing the shock become supra-thermal. The latter number is based directly on the results obtained with a PIC simulation by [12]. The former is an order of magnitude larger than the results obtained by [12]. However, this is necessary to test the model. [12] found no significant particle injection rate for the $M_S = 2.0$ model. Repeating that with the combined PIC-MHD method would be pointless because the simulation would simply maintain the unperturbed standing shock. Therefore we have chosen to artificially increase the injection rate to determine whether it is possible to start the DSA given a sufficient non-thermal particle density. This simulation setup was demonstrated previously in [19].

The supra-thermal injection is represented by introducing the particles isotropically (in the post-shock rest-frame) at the location of the shock with 3 times the pre-shock velocity [17, 19]. We limit the supra-thermal component to protons and assume that the electrons remain fully thermalized.

### 3. Result

Figure 1 presents the results of the $M_S = 2.0$ simulation, showing (from top to bottom) the magnetic field strength relative to the unperturbed upstream magnetic field strength ($B_0$), the non-thermal plasma density relative to the thermal plasma density after a simulation time of $t = 20,000 R_l/c$. Despite the artificially enhanced injection rate, there is no sign of a disturbance of the upstream magnetic field. Although the field strength varies, the field-lines remain straight. As a result, the particles will no be reflected back toward the shock, and therefore no DSA can take place. Figure 2 presents the results of the $M_S = 3.2$ simulation, showing the same variables after the same simulation time. Contrary to the $M_S = 2.0$ case, the $M_S = 3.2$ simulation shows signs of variation in the magnetic field direction, both upstream (to the right of the shock) and downstream (to the left of the shock), as well as the strength. The magnetic field lines are clearly deformed, indicating the presence of instabilities that can, at least in theory, reflect particles back toward the shock, thereby starting the DSA process.

The spectral energy distributions (SEDs) of the particles in the grid reflect the characteristics of the plasma and the magnetic field. As shown in Fig. 3, which shows the energy distribution as a function of the particle energy, the $M_S = 2.0$ simulation shows no evidence of acceleration. The particles, which were injected at a fixed velocity have spread out, with some particles losing energy while others gained energy. However, the average particle energy has remained similar to the original injection energy and both the gain and loss are small. By contrast, the SED for the $M_S = 3.2$ model shows clear evidence of particle acceleration, with a high energy slope, indicating that the particles are being subjected to the DSA process.
Figure 1. From top to bottom: absolute magnetic field strength relative to the unperturbed upstream field ($B_0$), supra-thermal gas density relative to thermal gas density, and thermal gas density relative to the unperturbed upstream thermal gas density ($\rho_0$) as well as the magnetic field lines at $t = 20000 R_l/c$ for the model with $M_S = 2.0$. Although the field strength shows some light variation, the direction of the field lines remains unperturbed.

Figure 2. Similar to Fig. 1 but for the model with $M_S = 3.2$. The perturbation in the field strength is much larger and the field lines show local variations in direction. The thermal gas density also shows local variations. This indicates the possibility for DSA.

4. Increasing the box-size

Although the simulations in the previous section showed that DSA of protons can take place in the $M_S = 3.2$ model, the quantitative value of these models is limited by the small size of the box. This limits the number of times that a particle can cross the shock before escaping through either the upstream or the downstream boundary. Therefore, we repeat the $M_S = 3.2$ simulation with an elongated box ($2400 \times 15 R_l$).

Figure 4, which shows the same characteristics as Figs. 1-2 for this new simulation after a simulation time of $t = 20000 R_l/c$ shows once again the presence of instabilities in the magnetic field and thermal plasma. These are so-called 'streaming instabilities, which were analyzed by [12] who determined that they show mixed characteristics of both the resonant and non-resonant streaming instability. The instabilities are stronger than for the original simulation (Fig. 2). Because the box is longer, the instabilities can be triggered far from the shock and have more time to grow before they reach the shock surface.

The presence of these instabilities allows the DSA process to accelerate particles effectively to higher velocities. Figure 5 shows the scaled particle SED as a function of $(\gamma − 1)$, with $\gamma$ the Lorentz factor, for easier comparison to [12]. At higher energies, the SED forms a straight line, extending to higher energies in the log-log plot. This line, which is indicative of a powerlaw distribution, continues to $\gamma = 2$, or, in the case of protons, an energy of $\gamma mc^2 \simeq 1.8$ GeV. If a particle, travelling at such a speed collides with a thermal ion, the end result would be the production gamma-radiation through pion decay, which would be observable as diffuse gamma radiation to instruments such as Fermi-LAT. However, observations, such as [20], have found no sign of such radiation.

Beyond $\gamma = 2$, the negative slope of the SED increases. This has likely three reasons: 1)
Figure 3. The particle SEDs for the $M_s = 2.0$ (purple) and $M_s = 3.2$ (green) shocks, showing the particle energy distribution as a function of the energy relative to the injection energy ($E_0$). Whereas the $M_s = 2.0$ shock shows no significant acceleration, at the $M_s = 3.2$ shock, the particles are being accelerated and reach energies of up to ten times the injection energy. Image Credit: [19]

Even in the larger box, the particles will ultimately escape. Because the fastest particles travel the longest distances, they reach the outer boundaries first and are preferentially removed from the simulation. 2) The fixed powerlaw distribution for DSA mechanism applies to either fully non-relativistic and fully relativistic particles [11], but the slope changes. The protons in this particular model are in the process of crossing from non-relativistic to relativistic speeds and therefore the slope of the SED will change at high energies. 3) The powerlaw index of the SED depends strongly on the shock conditions, in particular, the compression rate. As we shall see in the next section, the characteristics of the shock in this simulation are not invariable.

5. Variations in shock conditions

Figure. 4 shows that the upstream instabilities can reach the point where they fundamentally change the nature of the shock. The first parameter to change is the angle between the magnetic field and the flow. The simulation was started with this angle fixed at 13° in the upstream medium, making this a quasi-parallel flow. However, Fig. 6, which presents the angle between the magnetic field and the flow immediately upstream of the shock as a function of the position along the shock surface at the same moment in time as Figs. 4-5 shows that this is no longer representative. Instead, the angle between field- and flow-direction varies considerably in space, and therefore, inevitably, in time. The variations become so large (with an absolute angle of 45°+) that the shock can locally change its nature from quasi-parallel to quasi-perpendicular.

The changes in the magnetic field, in turn, influence the dynamic behaviour of the upstream plasma. Although the initial magnetic field was weak, and therefore unable to influence the motion of the plasma, the variation in field strength and direction caused by the instabilities are sufficient to change this. As the loops in the magnetic field enter the shock, they are compressed to the point where the magnetic tension in the field lines becomes sufficiently strong to counteract the compressing force, thereby slowing down the flow. This alters the sonic Mach number of the shock as shown in Fig. 7.
The shock conditions no longer being static, it is inevitable that their varying nature starts to influence the particle SED. Particles that repeatedly cross the shock will encounter different local conditions depending on time and place. Although at first glance the presence of large-scale instabilities seems beneficial to the DSA process, there is a secondary effect that has not been taken into account in these simulations: the influence of the shock conditions on the non-thermal particle injection rate.

[12] demonstrated that the injection rate depends on the shock conditions with injection rates decreasing rapidly for shocks with Mach numbers below $M_S = 2.25$. The influence of the angle between the magnetic field and the flow is as yet not fully determined but is known to play a part as well. E.g. PIC simulations by [12] showed that even for quasi-perpendicular shocks supra-thermal particles were injected in significant numbers but, unlike in quasi-parallel shocks, they failed to trigger the upstream instabilities that are required for DSA. [6, 7] found similar results for high-Mach, low-$\beta$ shocks using the PIC-Hybrid method. [17], using the combined PIC-MHD method to repeat the work by [6, 7] did find DSA. In each case, the models assumed a constant angle, rather than a magnetic field that varies in space and time. Therefore, they are not representative of the situation encountered.

Figure 4. Similar to Figs. 1-2 but for the $M_S = 3.2$ simulation with an elongated box. The instabilities are more extreme than for an identical simulation in a shorter box (Fig. 2) because they can start further from the shock and therefore have a longer time to grow.

Figure 5. The particle SED for all particles in the simulation box at the same time as Fig. 4. This shows the particle distribution as a function of the Lorentz factor $\gamma$ minus one. The powerlaw slope, indicating DSA, extends to approximately $\gamma = 2$.

6. Conclusions
Our models confirm the results obtained by [12], which showed that low-Mach, high-$\beta$ shocks, such as those observed for galaxy cluster collisions, can accelerate ions, causing them to dissociate from the thermal plasma. We have also shown that, under the right circumstances, the DSA process can continue this acceleration to propel the ions to relativistic speeds, and contribute to the CR spectrum. However, the interaction between the non-thermal particles and the magnetic field, which is required to generate the instabilities that allow the DSA process to occur, causes large-scale disturbances in the upstream medium, thereby changing the shock conditions to the point that the nature of the shock itself changed. Further investigation is required to determine whether this will eventually counteract the DSA process by reducing the particle injection rate at the shock. A paper showing these results in more detail is currently under review by MNRAS. Further research will include a time-dependent injection rate. A secondary line of investigation,
which will look at the ability of these shocks to re-accelerate relativistic particles, is planned for the near future.

Figure 6. The angle between the magnetic field and the upstream flow as a function of its position along the shock surface at the same time as Figs. 4-5. Because of the instabilities, the angle varies to such an extent in both space and time that it can become quasi-perpendicular.

Figure 7. The sonic Mach number ($M_S$) at shock as a function of its position along the shock surface at the same moment in time as Figs. 4-6. The compression of the magnetic field loops has lead to a change in local magnetic field tension, which starts to counteract the compressing force. This changes the velocity of the flow and the local gas temperature, leading to variations in the sonic Mach number.

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