Curvatons in the minimally supersymmetric standard model

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Abstract

Curvaton is an effectively massless field whose energy density during inflation is negligible but which later becomes dominant. This is a novel mechanism to generate the scale invariant perturbations. I discuss the possibility that the curvaton could be found among the fields of the minimally supersymmetric standard model (MSSM), which contains a number of flat directions along which the renormalizable potential vanishes. The requirements of late domination and the absence of damping of the perturbations pick out essentially a unique candidate for the MSSM curvaton. One must also require that inflation takes place in a hidden sector. If the inflaton energy density can be radiated into extra dimensions, many constraints can be relaxed, and the simplest flat direction consisting of the Higgses $H_u$ and $H_d$ would provide a working example of an MSSM curvaton.
I. INTRODUCTION

The recent WMAP observations \cite{1} of the temperature fluctuations of the cosmic microwave background provide convincing evidence for a period of cosmic inflation in the early Universe. The observed perturbation spectrum is nearly scale invariant and the total density $\Omega_{\text{tot}} = 1$, as predicted by many models of inflation (for a review, see \cite{2}). These are usually based on the slow rolling of a scalar field (the inflaton) in a flat potential. The non-zero potential energy acts as dark energy, causing the Universe to expand exponentially, while quantum fluctuations of the inflaton field are magnified and eventually, when the inflaton decays, are imprinted on the decay products.

The large dark energy of the early Universe provides us with three things: superluminal stretching of space needed to solve the horizon problem; quantum fluctuations of scalar fields which may seed density perturbations; and, once the primordial dark energy decays, the origin of all matter. Scalar field inflation can model all these three things but also has some obvious shortcomings; in particular, it is totally unclear how, exactly, the Standard Model (SM) degrees of freedom are produced. One is required to refer to ad hoc couplings of the inflaton to some other fields, which then couple to the SM fermions and bosons. Indeed, despite the apparent success of simple inflationary models based on slowly evolving scalar fields, there is no compelling theory of inflation.

Conventionally one relates the spectrum of density perturbations to the properties of the inflaton potential. However, it has become clear that this is not a necessary condition for a successful inflationary scenario. In curvaton models \cite{3,4,5} the primordial dark energy induces quantum fluctuations in an effectively massless field whose energy density during inflation is negligibly small but which later becomes dominant and then by decaying imprints the primordial perturbation on matter fields. Such a possibility has recently received a great deal of attention \cite{6,7,8,9,10,11,12,13}. Because during inflation the curvaton is massless at the scale of the Hubble rate $H$, it will be subject to fluctuations which have a spectrum that is almost identical to the spectrum of the inflaton field and therefore almost scale invariant. However, as the curvaton is subdominant during inflation, its perturbations will not be adiabatic but perturbations in the entropy density, i.e. isocurvature perturbations. When the curvaton decays, the isocurvature perturbations will be converted to the usual adiabatic perturbations of the decay products, which thus should ultimately contain also
SM degrees of freedom.

Literature abounds with schemes of multifield inflation, but the curvaton models differ from these in one important respect. It provides a logical separation between the field responsible for density perturbations and for the geometrical stretching of space (such a separation was already apparent in pre-Big Bang models, see e.g. [14]). This separation opens the door to the possibility of truly understanding the origin of all matter. Indeed, an exciting possibility could be that the curvaton is just one of the fields of the Minimally Supersymmetric Standard Model (MSSM) [8, 9, 10, 12] and that hence cosmology could be indirectly tested also by particle physics experiments.

II. THE CURVATON PARADIGM

During inflation, any effectively massless scalar field is subject to fluctuations similar to the conventional inflaton with a spectrum

$$P^{1/2}(k) \sim \frac{H_*^2}{2\pi}$$

where \(H_*\) is the value of the Hubble parameter during inflation (at the horizon exit \(k = aH\)). By definition, during inflation the energy density of the inflaton must dominate. Therefore the fluctuations in other scalar fields do not perturb space-time. Such isocurvature perturbations may be converted to adiabatic density perturbations if the subdominant scalar at some later stage becomes dominant and then decays, as first pointed out in [15]. This idea was revived in the context of pre-Big Bang models [3] and was then applied to conventional inflation [4, 5] and dubbed as the curvaton mechanism.

Curvatons have been considered in the context of supersymmetric theories [8, 9, 10, 12, 16], and both axions [17] and pseudo-Nambu-Goldstone bosons [18] have been suggested as curvaton candidates. Curvaton dynamics, reheating and observational constraints have been discussed in [19]. Brane-inspired scenarios have been coupled to the curvaton idea in [20, 21, 22].

In its simplest realisation, a curvaton model can be written as

$$V(\varphi, \phi) = V_{inf}(\varphi) + \frac{1}{2}m^2\phi^2$$

where \(\varphi\) is the inflaton and \(\phi\) the curvaton. \(V_{inf}\) is the inflaton potential with \(V_{inf} \simeq H_*^2 M_P^2 \simeq \rho_{inf}\), where \(M_P\) is the Planck mass. While inflation lasts, the curvaton field remains fixed
at some initial value \( \phi_* \) (in more complicated potentials, it may change albeit typically very slowly). The curvaton’s subdominance requires \( V_{inj} \gg m^2 \phi_*^2 \), and the curvaton is effectively massless during inflation provided \( m^2 \ll H_*^2 \). Then the curvaton perturbation reads \( \delta \phi_* \simeq H_* / 2\pi \). The perturbation will be gaussian if the mean value of the field is larger than the perturbation itself, \( \phi_*^2 \gg H_*^2 / 4\pi^2 \).

Once inflation is over, the inflaton field begins to oscillate about the minimum of the potential and then decays. The curvaton field starts to move away from \( \phi_* \) and when \( m^2 \simeq H^2 \), the curvaton starts to oscillate. Oscillations behave effectively as cold matter (in some background) and eventually overwhelm the energy density of the inflaton decay products. The curvaton must decay before the onset of nucleosynthesis so that the decay rate \( \Gamma_\phi > H_{nuc} \simeq 10^{-40} M_P^2 \). A negligible curvature perturbation due to inflaton means that curvaton models predict no detectable primordial gravitational waves in the CMB.

The details depend on the actual form of the curvaton potential as well as on the inflaton decay mechanism. The motion of the curvaton field after inflation is particularly important as it can affect the amplitude of the initial perturbation \( \delta \phi_* / \phi_* = \delta \phi / \phi \) only for a quadratic potential). Curvaton dynamics has been discussed in \([7]\) where it was found that if non-renormalizable terms dominate, there exists an attractor solution which can erase the curvaton perturbations. However, the conclusion depends somewhat on the initial conditions.

### III. FLAT DIRECTIONS IN THE MSSM

An obvious candidate for a massless curvaton would be a flat direction of the MSSM (for a review, see \([23]\)). These are described in terms of order parameters, which are combinations of squarks, sleptons and Higgses, which in the limit of exact supersymmetry (SUSY) have a vanishing potential. The existence of such flat directions is a consequence of gauge invariance and supersymmetry. An example of an MSSM flat direction is

\[
H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix},
\]

where \( H_u \) is the Higgs, \( L \) a slepton and \( \phi \) is a complex field parameterizing the flat direction. All the other fields are set to zero. The MSSM flat directions have all been classified in \([24]\).
and can be presented in terms of composite gauge invariant monomial operators\(^1\).

Flatness is lifted by SUSY breaking and by non-renormalizable terms \([24, 26]\) of the form \(W = \lambda \Phi^n / nM^{n-3}\), where \(M\) is the large cutoff scale, usually taken to be the Planck scale, and \(n \gtrsim 4\) is the dimensionality of the non-renormalizable operator. The operators of interest lifting the flatness turn out to have dimensionalities \(n = 4, 6, 7,\) and \(9\).

There can also arise terms induced by the expansion of the Universe which depend on the Hubble parameter \(H\). The generic form of the potential for an MSSM flat direction during inflation is \([26, 27]\)

\[
V(\phi) = (m_\phi^2 - CH^2)|\phi|^2 + \left[ (a \lambda_n H + A_\phi) \lambda_n \frac{\dot{\phi}^n}{nM^{n-3}} + \text{h.c.} \right] + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}},
\]

(3.2)

Here \(\Phi = \phi e^{i\theta}/\sqrt{2}\). The first term in Eq. (3.2) includes the Hubble-induced and low energy soft mass terms. The second term includes the Hubble-induced and low energy SUSY breaking \(A\) terms. The Hubble-induced terms are present only for F-term inflation; in the case of D-term inflation, they are absent (for a discussion, see \([23]\)). The last term is the contribution from the non-renormalizable superpotential. The coefficients \(|C|, a, \lambda_n \sim \mathcal{O}(1)\), and the coupling \(\lambda \approx 1/(n - 1)!\).

During inflation the MSSM scalar fields will fluctuate along the flat directions and, as long as these remain effectively massless, form condensates. Which direction among the many possible flat directions is chosen by the fluctuations is a matter of chance. Note however that once one particular flat direction starts to develop a condensate, its existence will generally speaking lift the flatness of the other potential directions (although there are some examples of directions which are simultaneously flat, e.g. directions which involve only squarks and only leptons). Because the observable Universe originates from a single horizon patch, there is typically only one MSSM condensate in our Universe.

IV. FLAT DIRECTIONS AS CURVATONS

The \(A\)-terms in Eq. (3.2) are responsible for generating a global charge for the condensate. This is important in the context of Affleck-Dine baryogenesis \([28]\) and Q-balls \([29]\), but for the present considerations it suffices to focus on the amplitudes only. A generic form of the

\(^1\) Flat directions can also be constructed from more than two MSSM scalars, see \([25]\).
flat direction potential contains a renormalizable mass term and a non-renormalizable part and reads thus

\begin{align}
V(\phi) &= \frac{1}{2} m_\phi^2 \phi^2 + V_{NR}, \quad (4.1) \\
V_{NR} &= \frac{\lambda^2 \phi^{2(n-1)}}{2^{n-1} M^{2(n-3)}}, \quad (4.2)
\end{align}

where $m_\phi$ could also include a Hubble-induced term. However, in that case the amplitude of the fluctuations of the flat direction dies out completely during inflation. For the curvaton mechanism to work one must therefore require that the inflation model is such that $O(H)$ terms are not induced. One example is the SUSY D-term inflation, which naturally leads to a vanishing Hubble-induced mass term \[30\]. Another example invokes the so-called Heisenberg symmetry on the Kähler manifold \[31\]. In the latter case, no mass term appears in the tree-level potential, although a harmlessly small negative mass squared is induced at the one-loop level. No-scale supergravity theories belong to this class.

Let us simply assume that for one reason or another, during inflation the flat direction does not receive any Hubble-induced mass. Hence the mass term in Eq. (4.2) is of the order of the SUSY breaking with $m_\phi \sim \text{TeV} \ll H_*$. However, this is not yet sufficient for the curvaton scenario to work. One should also make sure that the inflaton fluctuations do not contribute significantly to the adiabatic density perturbations. This means that the Hubble parameter during inflation should obey $H_* \sim \rho_{\text{inf}}^{1/2} M_P < 10^{14} \, \text{GeV}$. (Needless to say, the energy density of the flat direction should be negligible compared to that of the inflaton, $\rho_\phi \ll \rho_{\text{inf}}$.) The (isocurvature) fluctuation of the flat direction reads then $\delta \phi \sim H_*/2\pi$, and if $\delta \phi/\phi_* \sim H_*/\phi_* \sim 10^{-5}$, one obtains the right order of magnitude for the density perturbation, provided there is no later damping of the curvaton perturbation. Here $\phi_*$ is the amplitude during inflation, and can be estimated by $V''(\phi_*) \sim H_*^2$.

Simple analysis shows \[8\] that during inflation the curvaton is slow-rolling in the non-renormalizable potential $V_{NR}$. Thus, the Hubble parameter and the amplitude of the field read, respectively,

\begin{align}
H_* &\sim \lambda^{-\frac{1}{n-3}} \delta^{\frac{n-2}{n-3}} M_P, \quad (4.3) \\
\phi_* &\sim \lambda^{-\frac{1}{n-3}} \delta^{\frac{1}{n-3}} M_P, \quad (4.4)
\end{align}

where $\delta \equiv \delta \phi/\phi_* \sim H_*/\phi_*$. 6
After inflation, the inflaton field first oscillates and then decays. The Hubble rate starts to decrease from its value $H_*$ during inflation and the MSSM curvaton moves towards the origin of the field space and begins to oscillate when $H \sim m_\phi$. As a consequence, its energy density relative to the inflaton decay products starts to grow (assuming now for simplicity that the inflaton has decayed before the curvaton oscillations begin). If the decay products consist of (MS)SM radiation, there may arise important thermal corrections in the effective curvaton potential. In fact, it has been argued that the flat direction decays by thermal scatterings before its domination and/or never dominates the energy density of the Universe [9, 11].

These difficulties can be avoided by taking the inflaton sector to be hidden and completely decoupled from the observable one [9, 10]. Since the inflaton must be very weakly coupled to ordinary particles, this seems rather a natural assumption which have in the conventional inflationary picture has been precluded only because of the need to somehow generate the SM degrees of freedom. A hidden inflaton would decay into (light) particles in the hidden sector, and the inflaton decay products would not come into thermal contact with the MSSM particles originating from the eventual curvaton decay.

Such a radical proposal may be quite welcome, because in spite of the success of cosmic inflation in explaining the flatness and the homogeneity of the Universe, there is not a single realistic particle physics model which would give rise to an inflaton field as a natural consequence of first principles. In almost all cases the inflaton is a gauge singlet by construction and the contrived slope of the inflaton potential is simply adjusted to fit the observations. The coupling of such a singlet to the SM degrees of freedom is usually set by hand. In this regard the MSSM flat directions can play a significant role. A natural outcome of the MSSM curvaton dynamics is the direct reheating of the Universe with the MSSM degrees of freedom.

V. CONSTRAINTS ON THE MSSM CURVATON

The MSSM curvaton should satisfy two conditions: 1) its energy density should dominate the Universe at the time of its decay; 2) the perturbations generated during inflation should not be damped by the subsequent dynamics.

Let us begin by addressing the first issue. As already mentioned above, the oscillation of
the flat direction starts when \( H \sim m_\phi \sim \text{TeV} \), and the amplitude at that time is

\[
\phi_{\text{osc}} \sim \left( \frac{m_\phi M^{n-3}}{\lambda} \right)^{\frac{1}{n-2}}. \tag{5.1}
\]

If the decay of the inflaton occurs earlier, at the onset of curvaton oscillations the Universe will be dominated by hidden radiation. The curvaton must nevertheless grow to dominate the energy when it decays. Since the evolution of the energy density of the flat direction is \( \propto a^{-3} \propto H^{-3/2} \), one can find that the curvaton density equals the density of hidden radiation \( \rho_h \sim H_{\text{EQ}}^2 \) when

\[
H_{\text{EQ}} \sim m_\phi \left( \frac{m_\phi M^{n-3}}{\lambda M_p} \right)^{\frac{4}{n-2}}. \tag{5.2}
\]

The curvaton should decay later than this so that the decay rate \( \Gamma_\phi \sim f^2 m_\phi < H_{\text{EQ}} \), where \( f \) is some Yukawa (or gauge) coupling. Hence one obtains a constraint on coupling strength \( f \). It turns out that the constraint is virtually identical also in the case when the inflaton decays after the curvaton oscillation has already started, and that only the \( n \geq 7 \) cases satisfy the decay-after-domination condition, with \( n = 7 \) being marginal.

One may relax this condition if the hidden sector inflaton decays not into radiation but into some fluid with a general equation of state \( p = w \rho \). Assuming the hidden fluid dominates the density of the Universe at the time when the curvaton starts to oscillate, one finds the constraint

\[
f < \left[ \left( \frac{m_\phi}{\lambda M_p} \right)^{\frac{1}{n-2}} \right] \frac{1}{2w}. \tag{5.3}
\]

This is depicted for \( n = 4, 6, 7, \) and \( 9 \) in Fig. 1. As one can see, the \( n = 9 \) direction is essentially the only usable for the hidden radiation case, but even \( n = 6 \) directions can be available if the hidden sector fluid has a stiff equation of state \( (w = 1) \). Notice that \( n = 4 \) directions cannot dominate the Universe and hence are completely ruled out as curvaton candidates.

Let us now turn to discuss the evolution of perturbations. In an \( m_\phi^2 \phi^2 \) potential both the homogeneous and (linear) perturbation parts obey the same type of equation so that the ratio \( \delta \phi/\phi \) remains a constant. However, the MSSM curvaton is rolling in the non-renormalizable part of the potential. The equations of motion for the superhorizon modes of the homogeneous and fluctuation parts are written respectively as

\[
3H \dot{\phi} + V'(\phi) = 0, \tag{5.4}
\]
Using Eq. (4.2) one then finds

$$\frac{\delta \phi}{\phi} \sim \left(\frac{\delta \phi}{\phi}\right)_i \left(\frac{\phi}{\phi_i}\right)^{2(n-2)}.$$  \hspace{1cm} (5.6)

where $i$ denotes the initial values. During inflation, there is essentially no damping, but after inflation the field amplitude decreases as $\phi \sim (HM^3/\lambda)^{1/(n-2)}$ as the Hubble parameter changes from $H_*$ to $m_\phi$. Hence there is a possibility for a large suppression of the curvaton perturbation generated during inflation \cite{9}. It turns out \cite{9} that because of insufficient energy density and/or damping of fluctuations, MSSM flat directions cannot act as curvatons if $n \leq 7$. In contrast, for the $n = 9$ flat direction the condensate will dominate the energy density of the Universe before it decays, and if initially $\phi \leq 0.3M_P$, which does not appear to be an unreasonable assumption, the condensate perturbations will not be considerably damped during inflaton oscillations. This result may be considered as a proof of existence for an MSSM curvaton.

VI. DUMPING INFLATON ENERGY OUT OF THIS WORLD

With the advent of brane world scenarios, where the Universe is regarded as a three dimensional hypersurface embedded in a higher dimensional bulk, it has become possible to couple the inflaton to degrees of freedom that are not visible in our Universe. (For a
review of brane world cosmology, see \[32\]). Therefore in the context of MSSM curvations, which require the inflaton decay products to reheat some hidden sector, it is of interest to ask whether the whole of the primordial dark energy could be dumped out of the visible Universe. The local density of the inflaton decay products should also be diluted; in other worlds, the decay products should not remain in the vicinity of the brane. This could be achieved by redshifting the decay products into infinity of a warped 5th dimension \[22\].

To this end, consider a simple Randall-Sundrum-type \[33\] scenario with a three dimensional hypersurface carrying MSSM degrees of freedom. The MSSM brane is embedded in a 5 dimensional space (the bulk) with a non-factorizable metric

\[
d s^2 = e^{-2k|z|}(z)\eta_{\mu\nu}dx^\mu dx^\nu - dz^2, \tag{6.1}
\]

where \(\eta_{\mu\nu}\) is the four dimensional Minkowski metric. We take the extra dimension to be infinite. The brane is located at \(z = 0\). The action consists of the 4D brane with tension \(\sigma\) and a 5D bulk with a negative cosmological constant which is related to the brane tension by fine tuning. The warp factor present in the metric is \(k = 4\pi G_5 \sigma/3\), where \(G_5\) is the 5D Newton’s constant. The important point is that the geometry (6.1) has a particle horizon at \(z = \infty\): a particle that escapes from the brane and moves along a geodesic travels from \(z = 0\) to \(z = \infty\) in a finite proper time \(\tau = \pi/2k\).

Let us assume that the inflaton \(\phi\) is a true 4D brane field, with a homogeneous distribution that dominates the energy density in the early Universe on the observable brane and gives rise to a period of inflation. Once inflation comes to an end, the inflaton will decay, but instead of reheating the brane degrees of freedom, let us assume that it couples to the bulk scalar fields \(\psi\) alone\(^2\), and hence decays into Kaluza-Klein (KK) modes of the bulk degrees of freedom. This happens through an effective coupling

\[
\sqrt{g(z)} h \phi(x) [\chi(0, m)\chi(0, m')] \psi_m(x) \psi_{m'}(x); \tag{6.2}
\]

where \(h\) is the coupling strength, \(g(z)\) the metric and \(\chi(0, m)\) are the \((z\) dependent\) wave functions of a KK modes of mass \(m\). One may estimate the total decay rate as \[22\]

\[
\Gamma_\phi = \int_0^{m_\phi} \int_0^{m_\phi} \frac{dm}{k} \frac{dm'}{k} \frac{h^2}{k^2} \left[\frac{\chi(0, m)\chi(0, m')}{m_\phi}\right]^2 \approx \frac{h^2}{32} \left(\frac{m_\phi}{k}\right)^2 m_\phi. \tag{6.3}
\]

\(^2\) For instance, by virtue of some global quantum number carried by the inflaton and the bulk fields but not by the MSSM fields.
The KK mass dependence of the effective couplings indicates that the inflaton would preferably decay into the heavy modes \( k \sim M_P \), i.e. those with the largest momentum along the fifth dimension. As the bulk modes carry momentum along the fifth dimension, they would simply fly into the empty bulk and towards infinity, taking the inflaton energy away from the brane. This process can be thought of as a hot radiating plate cooling down by emitting its energy into the cold surrounding space.

As the original energy density of the inflaton field is redshifted away from the brane into the extra dimension, only the tail of the density distribution would be felt by the brane. It has been pointed out, however, that the energy emitted from the brane would eventually collapse to a black hole at the end of the 5th dimension \[34\]. The presence of the black hole changes the brane expansion by introducing a new contribution to the Friedmann equation which behaves as \( 2M_{BH}/a^4 \), where \( a \) is the brane scale factor and \( M_{BH} \) is a parameter interpreted as the 5D "mass" of the black hole \[35\]. This term acts as a dark energy, which, provided that \( M_{BH} \) is small, has a subleading role in the early Universe.

There are some string theoretical motivations for a brane-world scenario with warped infinite dimension involving \( \bar{D}3 \) and \( D3 \) branes that annihilate at an adS throat, as discussed in \[22\]. A similar string inflation model requiring no slow-roll has been presented in \[21\].

VII. MSSM HIGGSES AS CURVATONS

If the inflaton reheats the bulk but not the brane, then MSSM condensates do not decay because of thermal background and will naturally dominate the energy density once the primordial dark energy has been radiated away. In that case several flat directions are good curvaton candidates, including \[10\], the simplest \( n = 4 \), \( H_uH_d \)-direction with

\[
H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}.
\] (7.1)

Only large amplitudes are relevant for cosmological purposes so that one can ignore the \( \mu \)-term and the Higgs mass terms. Then the effective potential for the \( H_uH_d \) flat direction can be written as

\[
V(\phi) = \lambda^2 \frac{|\Phi|^6}{M_p^2},
\] (7.2)
where $\lambda$ is a constant of $O(1)$ which we set to unity for simplicity. Writing $V''(\phi_*) = \beta^2 H_*^2$ and $H_*/\phi_* = \delta$ one requires that $\beta \ll 1$ and the perturbation $\delta \sim 10^{-5}$. Thus one obtains

$$\phi_* \sim \beta \delta M_P, \quad H_* \sim \beta \delta^2 M_P. \quad (7.3)$$

One then finds the scale of inflationary dark energy to be $V^{1/4}_{\text{inf}} \sim (H_* M_P)^{1/2} \sim \beta^{1/2} \delta M_P$.

Once the condensate decays, it will reheat the universe. The maximum temperature is $T_{\text{max}} \sim [V(\phi_*)]^{1/4}$. After inflation the amplitude of the flat direction is very large so one expects both fermionic and bosonic preheating \cite{36}. (Preheating in the context of MSSM flat direction has been discussed in \cite{37}). The $q$-parameter in this case is given by \cite{10}

$$q \sim \frac{f^2 \phi_*^2}{\omega^2} \sim f^2 \left( \frac{M_p}{\phi_*} \right)^2 \gg 1, \quad (7.4)$$

where $\omega = \sqrt{V''(\phi_*)}$. Once MSSM fermions and bosons are produced, the energy density of (non-thermal) radiation evolves as $\rho_{\text{rad}} = \rho_f + \rho_b \propto a^{-4}$, while the residual oscillation of the flat direction decreases as $\rho_{\phi} \propto a^{-9/2}$, where $a(t)$ is the scale factor of the universe.

After the mass term in the effective potential of the flat direction dominates, $\rho_{\phi} \propto a^{-3}$. The equality of the energy densities occurs when $\phi = \phi_{\text{eq}} \sim (\beta \delta)^{-1} m$, where $m$ is $O(100)$ GeV. If the decay of the flat direction takes place before this time, the contribution to the total radiation density is subdominant. This happens if

$$f \gtrsim \sqrt{8\pi} \left( \frac{\phi_{\text{eq}}}{M_p} \right)^{\frac{1}{2}} \sim \sqrt{8\pi} (\beta \delta)^{-\frac{1}{2}} \left( \frac{m}{M_p} \right)^{\frac{1}{2}}. \quad (7.5)$$

For $\beta = 0.1$ and $\delta = 10^{-5}$, RHS reads $\sim 10^{-4}$. Since the Higgs has couplings much larger than $10^{-4}$ (e.g. gauge couplings), the conclusion is that the $H_uH_d$ flat direction decays well before the equality time. Hence the amount of radiation is totally determined by the preheating era. One finds \cite{10} that the reheat temperature $T_{\text{RH}} \lesssim T_{\text{max}} \sim 10^9 \left( \frac{\beta}{0.1} \right)^{\frac{3}{2}} \left( \frac{\delta}{10^{-5}} \right)^{\frac{3}{2}}$ GeV, so that the gravitino problem \cite{38} is automatically avoided (for a generic $n = 6$ ($n = 7$) direction the maximum reheat temperature would be $10^{13}$ ($10^{14}$) GeV).

The spectral index of the CMB temperature perturbations can be evaluated as \cite{4}

$$n_s - 1 = 2 \frac{\dot{H}_*}{H_*^2} + 2 \frac{V''(\phi_*)}{H_*^2}. \quad (7.6)$$

For $H_uH_d$ the change of the Hubble parameter is negligible and one finds \cite{10} $n_s - 1 \approx 0.007$ for $\beta = 0.1$. Note that there is a dependence on the Higgs potential, which in principle could be determined in the laboratory by extracting the relevant N-point scattering amplitudes from the data.
VIII. DISCUSSION

If inflation takes place in a hidden sector, the $n = 9$ flat direction $QuQue$ can act as a curvaton and produce the required spectrum of density perturbation. Since the MSSM curvaton consists of squark and slepton fields, its decay will naturally give rise to both baryons and cold dark matter in the form of neutralinos. A possible complication is that instead of decaying directly the condensate may first fragment into Q-balls \cite{23, 29}. This is also the generic behaviour of the $n = 9$ flat direction in, say, a no-scale model (but does not happen for the 3rd generation $QuQ_3ue$ direction if $\tan\beta \leq 3$) \cite{9}. Another complication could be the fact that since the MSSM curvaton is a complex field, its phase will also be subject to perturbations. These correspond to isocurvature fluctuations of the global charges $B$ and/or $L$ and may give rise to a baryonic isocurvature perturbation which is too high \cite{12}. However, $QuQue$ happens to have a vanishing $A$-term and is therefore safe from this constraint \cite{9}.

The implicit assumption here has been that the inflaton decay products determine the early evolution of the Universe before the curvaton becomes dominant. As discussed here, with the advent of extra dimensional theories such as brane worlds, this is no longer a logical necessity. If our Universe is a brane-like object embedded in a higher dimensional bulk, it is possible that the primordial dark inflaton energy could be radiated out of the brane into the bulk. The analogue would be a a hot plate emitting its energy into the surrounding cold space. It has been argued that such a situation could be found within the brane world scenarios with a warped, infinite extra dimension \cite{22}. In such a case the inflaton decay products would move rapidly away from the vicinity of the brane and redshift into the infinity of the 5th dimension. Then almost any MSSM flat direction could be the curvaton, including the simplest one based on the Higgses $H_u$ and $H_d$ \cite{10}. One would then find a spectral index very close to 1 but with a weak dependence on the Higgs potential.

It is an open problem whether dumping of the inflaton energy into extra dimensions can be realised in a string theoretical context. Meanwhile, the MSSM curvaton remains an interesting possibility for the origin of all matter and density perturbations that could, at least in principle, be testable also in the laboratory.
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[1] D. N. Spergel et al., Astrophys. J. Suppl. 148:175 (2003).
[2] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999).
[3] K. Enqvist and M. S. Sloth, Nucl. Phys. B 626, 395 (2002).
[4] D.H. Lyth and D. Wands, Phys. Lett. B 524, 5 (2002).
[5] T. Moroi and T. Takahashi, Phys. Lett. B 522, 215 (2001) [Erratum, ibid. B 539, 303 (2002)].
[6] D. H. Lyth, C. Ungarelli and D. Wands, Phys. Rev. D 67:023503 (2003); K. Dimopolous and D. H. Lyth, hep-ph/0209180; D. H. Lyth and D. Wands. Phys. Rev. D 68:023503 (2003); K. Dimopoulos, D. H. Lyth, A. Notari and A. Riotto, JHEP 0307:053 (2003).
[7] K. Dimopoulos, G. Lazarides, D. H. Lyth and R. Ruiz de Austri, Phys. Rev. D 68:123515 (2003).
[8] K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. Lett. 90, 091302 (2003).
[9] K. Enqvist, A. Jokinen, S. Kasuya and A. Mazumdar, Rev. D 68:103507 (2003).
[10] K. Enqvist, S. Kasuya and A. Mazumdar, hep-ph/0311224.
[11] M. Postma, Phys. Rev. D67:063518 (2003); astro-ph/0305101
[12] K. Hamaguchi, M. Kawasaki, T. Moroi and F. Takahashi, hep-ph/0308174; S. Kasuya, M. Kawasaki and F. Takahashi, Phys. Lett. B 578:259 (2004).
[13] N. Bartolo and A. R. Liddle, Phys. Rev. D 65:121301 (2003); M. Bastero-Gil, V. Di Clemente and S. F. King, Phys. Rev. D 67:103516 (2003); M. Postma, Phys. Rev. D 67:063518 (2003); C. Gordon and A. Lewis, Phys. Rev. D 67:123513 (2003); hep-ph/0305134; J. McDonald, Phys. Rev. D 68:043505 (2003); JCAP 0312:005,2003; hep-ph/0310126 A. Mazumdar, hep-th/0310162; M. Axenides, K. Dimopoulos, hep-ph/0310194 M. Giovannini, hep-ph/0310024 David H. Lyth, Phys. Lett. B 579, 239 (2004); Eung Jin Chun, Konstantinos Dimopoulos, David Lyth, hep-ph/0402059
[14] J. E. Lidsey, D. Wands and E. J. Copeland, Phys. Rept. 337, 343 (2000); M. Gasperini and
G. Veneziano, Phys. Rept. 373, 1 (2003).

[15] S. Mollerach, Phys. Rev. D 42, 313 (1990); A. D. Linde and V. Mukhanov, Phys. Rev. D 56, 535 (1997).

[16] J. McDonald, hep-ph/0310126

[17] M. S. Sloth, Nucl.Phys. B 656, 239 (2003); K. Dimopoulos, G. Lazarides, D. Lyth, and R. Ruiz de Austri, JHEP 0305:057 (2003); Eung Jin Chun, K. Dimopoulos, and D. Lyth, hep-ph/0402059

[18] K. Dimopoulos, D. H. Lyth, A. Notari, A. Riotto, JHEP 0307:053 (2003); R. Hofmann, hep-ph/0208267

[19] N. Bartolo, and A. R. Liddle, Phys.Rev. D 65:121301 (2002); C. Gordon, and A. Lewis, Phys.Rev. D 67:123513 (2003); N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. D69:043503 (2004) ; Bo Feng, and Mingzhe Li, Phys.Lett. B 564, 169 (2003); M. Bastero-Gil, V. Di Clemente, and S. F. King, hep-ph/0311237; C. Gordon, and Karim A. Malik, astro-ph/0311102; S. Gupta, K. A. Malik, and D. Wands, astro-ph/0311562

[20] K. Dimopoulos, Phys. Rev. D68:123506 (2003).

[21] L. Pilo, A. Riotto and A. Zaffaroni, hep-th/0410004.

[22] K. Enqvist, A. Mazumdar, and A. Pérez-Lorenzana, hep-th/0403044

[23] K. Enqvist and A. Mazumdar, Phys. Rept. 380, 99 (2003).

[24] T. Gherghetta, C. Kolda, and S.P. Martin, Nucl. Phys. B 468, 37 (1996).

[25] Kari Enqvist, Asko Jokinen, and Anupam Mazumdar, JCAP 0401:008 (2004).

[26] M. Dine, L. Randall, and S. Thomas, Nucl. Phys. B 458, 291 (1996).

[27] M. Dine, L. Randall, and S. Thomas, Phys. Rev. Lett. 75, 398 (1995).

[28] I. Affleck and M. Dine, Nucl. Phys. B 249, 361 (1985).

[29] A. Kusenko, and M. E. Shaposhnikov, Phys. Lett. B 418, 46 (1998); G. R. Dvali, A. Kusenko, and M. E. Shaposhnikov, Phys. Lett. B 417, 99 (1998); K. Enqvist and J. McDonald, Phys. Lett. B 425, 309 (1998); K. Enqvist and J. McDonald, Nucl. Phys. B 538, 321 (1999).

[30] C. F. Kolda and J. March-Russell, Phys. Rev. D 60:023504 (1999).

[31] M. K. Gaillard, H. Murayama, and K. A. Olive, Phys. Lett. B 355, 71 (1995).

[32] F. Quevedo, Class. Quant. Grav. 19, 5721 (2002).

[33] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999); L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999)
[34] A. Hebecker and J. March-Russell, Nucl. Phys. B 608, 375 (2001).
[35] C. Barceló, M. Visser, Phys. Lett. B482, 183 (2000).
[36] L. Kofman, A. D. Linde, and A.A. Starobinsky, Phys. Rev. D 56, 3258 (1997); P. B. Greene and L. Kofman, Phys. Rev. D 62, 123516 (2000).
[37] M. Postma and A. Mazumdar, JCAP 0401:005 (2004).
[38] J. Ellis, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. B 145, 181 (1984); A. L. Maroto and A. Mazumdar, Phys. Rev. Lett. 84, 1655 (2000); R. Kallosh et al., Phys. Rev. D 61:103503 (2000).