On apparent faster-than-light behavior of moving electric fields

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Abstract

For every observer, however distant, the electric field of a uniformly moving charge is always directed away from, or points towards, the instantaneous present position of the charge and not away from, or towards, the retarded position at which the observer sees it (due to the finite speed of light). This fact is a well-established consequence of, among others, the application of the Liénard-Wiechert potentials, and its significance for fundamental physics is probably not fully appreciated. Here we show how and why this property has non-negligible consequences for what we take for granted about the relativity of simultaneity and faster-than-light communication. In particular, if we consider two opposite electric charges whose distance shrinks to zero at a constant velocity (shrinking electric dipole), then the cancellation of the total field seems to be instantaneous everywhere in space and in every inertial reference frame. A simple variant of the shrinking electric dipole setup appears to allow a sort of faster-than-light communication of information. Our results provide simple theoretical support to the conclusions of recent experiments on the propagation speed of Coulomb and magnetic fields. It would also be interesting to explore any possible connection between our findings and quantum
non-locality.

**Keywords:** special relativity · relativity of simultaneity · electromagnetism · electric dipole · faster-than-light propagation · Gauss’s law · non-locality

## 1 Introduction

It is a well-known result of special relativity that nothing generated at one point in space (mass, energy, or information) can reach another point at speed exceeding the speed of light in vacuum \( c \). This fact derives from the application to all bodies of the relativistic velocity-addition formula obtained ultimately by imposing the constancy of the speed of light in every reference frame [1]. Einstein directly addressed the issue of an upper limit on speeds in a paper on special relativity published in 1907 [2]. There, he stated that “any assumption of the spreading of an effect with a velocity greater than the speed of light is incompatible with the theory of relativity”. By applying the velocity-addition formula, he showed that if one assumes the contrary, then one “would have to consider as possible a transfer mechanism whose use would produce an effect which *precedes* the cause (accompanied by an act of will, for example)” [emphasis in the original]. Perhaps, this is one of the first instances (if not the first) of causality violation associated with the possibility of superluminal motion. In a brief calculation, Einstein showed that if an effect propagates through a material medium faster than light, then the interval of time \( T \) needed by that effect to cover a distance between, say, point A and point B in the material turns out to be a negative number. This is interpreted as an instance of the “effect which precedes the cause”. We shall return to causality violation in Section 4.

However, it is well known that wave and illumination fronts may exceed the speed of light if they are not tied to mass or to transmitting locally produced information: consider, for instance, an illuminated spot from a lighthouse moving along a distant mountain wall, the propagation of shadows, or the illumination front of any intrinsically variable source of light (see, for instance, [3, 4]). In all these cases, neither mass nor energy (or information)

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\(^1\)Charged particles moving inside a dielectric medium can easily have a velocity higher than that of light in that medium. However, this is not regarded as a real superluminal motion.
originates at one end of the traveled distance and moves to the other end.

In the present paper, however, we shall show how and why the behavior of the electric field generated by charges moving at a constant velocity has non-negligible consequences for what we take for granted about the relativity of simultaneity and faster-than-light communication. In Section 2, we recall the well-know and established physical fact that the electric field of a uniformly moving charge is measured by a distant observer as always directed away from, or pointing towards, the actual, instantaneous position of the charge and not away from, or towards, its retarded position. We then describe the thought experiment of a shrinking electric dipole and show that there are situations in which the cancellation of the dipole field is instantaneous everywhere in the surrounding space. In Section 3, we propose a simple variant of the shrinking electric dipole thought experiment, and, with the application of Gauss’s law, we show how it has non-negligible consequences for the relativity of simultaneity and seems to allow faster-than-light communication of information. In Section 4, we summarize and discuss our findings, also in the context of recent experimental results.

2 The shrinking electric dipole

When a point charge \( q \) moves at a uniform velocity \( v \), the electric field is anywhere in space always directed away from, or points towards, the instantaneous present position of the charge. That means that while a distant observer sees the charge in a position that is retarded with respect to the present position (owing to the finite speed of light), he actually measures the field as directed away from, or pointing towards, the actual, present position. This peculiar feature derives directly from electromagnetism. It can be derived from the Liénard-Wiechert potentials or, equivalently, from the Lorentz transformations of fields and space-time coordinates [5-7], or even by appealing to the principle of relativity [8]. Following Purcell’s or Jackson’s derivations [5,7], the electric field of a point charge \( q \), moving with uniform velocity \( v \), can be expressed in terms of the charge’s instantaneous present position as

\[
E = \frac{q}{4\pi\epsilon_0} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{3/2}} d^3, \tag{1}
\]
Figure 1: The point charge $q$ moves at a constant velocity $\mathbf{v}$. If the magnitude of $\mathbf{v}$ is such that we cannot neglect the term $v^2/c^2$, the electric field becomes more intense at right angles to the motion than in the direction of the motion, see Eq. (1).

$$\text{where } \epsilon_0 \text{ is the vacuum permittivity, } c \text{ is the speed of light, } d \text{ is the radial vector from the charge's present position to the observation point, and } \theta \text{ is the angle between distance } d \text{ and velocity } \mathbf{v} \text{ (Fig. 1).}$$

Quoting Purcell’s words almost verbatim, it means that if $q$ passed the origin of the system $S$ at precisely 12:00 p.m., $S$ time, an observer stationed anywhere in the system $S$ would report that the electric field in his vicinity was pointing, at 12:00 p.m., exactly radially from the origin (see Fig. 1).

In the following experiment, we make use of this property, and for the sake of clarity, we refer to it as the field property.

Consider a small metallic sphere with charge $-Q$ and a larger metallic sphere with charge $+Q$. The larger sphere is hollow and has two small holes along a diameter such that the smaller sphere can slide inside. These two charge holders are separated by the distance $2R$ at initial time $t = 0$ s and are forced to move towards each other at a constant velocity $v$ relative to a distant, stationary observer (see Fig. 2). If two oppositely charged bodies were left to move freely, they would accelerate towards each other, and
Figure 2: Pictorial representation of the shrinking electric dipole thought experiment.
the field property described in the text for uniform motion would not hold in principle. Therefore, they are somehow forced to move at a constant velocity. The measure of time refers to the frame of the stationary observer. The two charge holders generate two distinct electric fields. Furthermore, the spheres are oriented such that when they meet, the smaller sphere can slide completely inside the larger one without friction and thus without suffering any deceleration. Let the distance between the charge holders and the observer be considerably less than $cT$, where $T$ is the interval of time taken by the charge holders to reach the meeting point from their starting positions. This condition guarantees that the system is ‘relaxed’ when the experiment starts. Any physical information has had the time to reach the observer, and the field has had ample time to spread all over the space where the experiment takes place. Notice that the use of charge holders with finite spatial extension is only functional to make the interpenetration process more clearly understandable. According to special relativity, the geometry of moving extended bodies is very complex, and we do not deal with this further complexity here. The reader has to imagine all the bodies involved as being actually point-like.

The system described here represents a sort of shrinking electric dipole, and it has already been sketched in [8] and [9].

For the sake of derivation, let us assume in this Section that during the interpenetration, the larger and the smaller sphere do not enter in electrical contact and retain their charge.

Owing to the field property described above, the observer measures at his location a total field that is the vector sum of the two fields directed away from, or pointing towards, the actual, instantaneous positions of the moving charges, no matter how distant these charges are from the observer (Fig. 2-A).

Now, consider what happens when the smaller sphere goes entirely into the larger one and their centers overlap. Although the total charge does not go to zero (no electrical contact), the total field in the proximity of the sphere becomes equal to zero (superposition of equal and opposite electric fields).

What does happen at the same instant of time close to the distant observer? In every phase of the process leading to the interpenetration, the total field measured by the observer is the vector sum of the two fields always directed away from, or pointing towards, the actual, instantaneous positions of the moving charges (principle of superposition). Exactly at the complete interpenetration (overlapping centers), the sum of the two fields measured
by the observer is then equal to zero. In the proximity of the observer, it is as if we have two equal fields pointing in opposite directions generated by two opposite and independent charges at the same place (see Fig. 2-B). 

With reference to Eq. (1) and Fig. 2, the total electric field $E_{tot}(t)$ in the proximity of the observer is equal to

$$E_{tot}(t) = E_-(t) + E_+(t) =$$

$$= \frac{-Q}{4\pi\varepsilon_0[(-R + vt)^2 + d^2]} \left\{ 1 - \frac{v^2}{c^2} \left( \frac{d^2}{(-R + vt)^2 + d^2} \right) \right\}^{3/2} \hat{r}_-(t) +$$

$$+ \frac{Q}{4\pi\varepsilon_0[(R - vt)^2 + d^2]} \left\{ 1 - \frac{v^2}{c^2} \left( \frac{d^2}{(R - vt)^2 + d^2} \right) \right\}^{3/2} \hat{r}_+(t), \quad t \in \left[ 0; \frac{R}{v} \right]$$

(2)

where $\hat{r}_+(t)$ and $\hat{r}_-(t)$ are the unit vectors of the distances $r_+(t) = \sqrt{(R - vt)^2 + d^2}$ and $r_-(t) = \sqrt{(-R + vt)^2 + d^2}$ that separate the charge holders from the observer, and $\sin^2 \theta(t) = \frac{d^2}{(-R + vt)^2 + d^2} = \frac{d^2}{(R - vt)^2 + d^2}$.

At time $T = \frac{R}{v}$, when the charge holders are at the meeting point, the total electric field measured at the position of the observer is equal to zero ($E_{tot}\left(\frac{R}{v}\right) = 0$). If the observer is at any other position not equidistant from the initial positions of the charge holders, Equation (2) still gives $E_{tot} = 0$ at the instant of interpenetration $T = \frac{R}{v}$ (since it is still $r_+\left(\frac{R}{v}\right) = r_-\left(\frac{R}{v}\right)$ and $\sin^2 \theta_+\left(\frac{R}{v}\right) = \sin^2 \theta_-\left(\frac{R}{v}\right)$).

However, at the same instant of time, the charge holders are seen at positions that are still $\frac{d}{c} \approx \frac{v}{c} d$ (for $v \ll c$) away from the meeting point.

This is because, at time $T = \frac{R}{v}$, the observer is still receiving the light (the image of the charge holders) emitted nearly $\frac{d}{c}$ time before (the exact value is $\frac{d}{c}/\sqrt{1 - \frac{v^2}{c^2}}$).

That means that the observer will instantaneously measure the cancellation of the total field, and thus the apparent zeroing of the total charge, no matter where or how distant this observer may be from the charges. And, it is so despite him still seeing (i.e., receiving the image of) the charges as separated in their retarded positions.
3 Moving electric fields and faster-than-light signaling

It is possible to make a simple addition to the above shrinking electric dipole setup with which we could transmit information faster than light.

Imagine the same setup as in Fig. 2, this time with a human operator close to the meeting point. In what follows, we shall also consider what happens after the instant of interpenetration: since there is ideally no friction, and since the smaller and the larger sphere are still forced to move at a constant velocity, after the interpenetration, they recede from one another at a constant velocity. Moreover, the operator, at his own will, can allow or not allow the electrical contact between the two metallic charge holders during the interpenetration (see Fig. 3). Notice that the operator makes the electrical contact (and thus the charge neutralization) happen only when the smaller sphere is entirely inside the larger one (with overlapping centers), and thus the contact happens when, from the outside, the perceived total charge is already equal to zero (superposition of equal and opposite electric fields). If the operator decides to allow the electrical contact, the total charge cancels out, and the total field is equal to zero from that moment on (at least in the proximity of the operator). On the other hand, if the operator decides not to allow the electrical contact, the field is equal to zero only during the instant of time in which the smaller sphere is entirely inside the larger one (and their centers overlap), and goes back to being different from zero (and inverted) when the smaller and the larger sphere recede still at a (forced) constant velocity.

Now, consider what the observer at distance \(d\) from the meeting point measures. It should be clear that the distant observer is informed \emph{instantaneously} on the operator’s decision: if the observer measures a definitive cancellation of the field, the operator has decided to allow the electrical contact. If instead, the cancellation of the field is not definitive (it goes to zero and then increases again in the opposite direction), the operator has decided not to allow the electrical contact. And, according to the analysis of the shrinking electric dipole made in Section 2, the observer becomes aware of both these scenarios at the same instant of time in which they happen close to the operator and thus without any information lag.

There is an objection to this result, though, that can be readily advanced. It can be objected that even in the case in which the operator chooses to allow...
Figure 3: Pictorial representation of the shrinking dipole variant proposed in the text as a tool for the faster-than-light transmission of information. Whether or not the operator decides to neutralize the overall charge at the interpenetration point (through electrical contact), owing to the superposition of equal and opposite electric fields, the total charge and field are always perceived to be equal to zero at the exact instant of complete interpenetration (when the two spheres’ centers overlap).
the electrical contact, the distant observer will see at first an instantaneous
cancellation of the field (owing to the field property), but soon he will measure
the rising of an opposite field as if the two charge holders, still moving after
the interpenetration, had not been set in contact and retained their original
charge. The information of the electrical contact (like, for instance, the
image of the operator setting the charge holders in contact) needs a time
nearly equal to \(d/c\) to reach the distant observer. Only after that time, the
observer would measure the definitive cancellation of the field.

This objection follows the common interpretation of the results coming
from the application of the Liénard-Wiechert potentials to derive the field
property described in Section 2 \[5, 6, 10\]. Since the potentials depend only
on what the charges are doing at the retarded time, when the charge holders
start to recede from the meeting point, the potentials (and thus the fields)
at the distant location of the observer will be the same whether the charge
holders recede with the same charge or whether they recede charge-less due
to the electrical contact after the interpenetration. And, this situation will last
until the information front of the electrical contact (traveling at the speed of
light) reaches the observer.

However, we shall show that this objection is incompatible with Gauss’s
law. Gauss’s law holds with every closed surface, at any instant of time,
and also when charges are moving (see, for instance, Sections 5.3 and 5.4 of
reference \[5\]).

Suppose that the operator decides to allow the electrical contact (overall
charge neutralization). Let us consider a closed spherical surface \(S\) tangent
to the interpenetration point, with the center lying on the trajectory of the
charge holders and with a diameter at least as big as the distance \(d\) of the
distant observer (see Fig. 4). If a time \(\Delta t\) has passed after the interpenetra-
tion (and the overall charge neutralization), the information that the field
is now definitely equal to zero has allegedly reached the distance \(c\Delta t\) from
the interpenetration point. Suppose that \(c\Delta t \leq d\). That means that if we
consider the intersection of the spherical surface \(S\) and the surface of the in-
formation front with radius \(c\Delta t\) and centered at the interpenetration point,
the portion \(S'\) of \(S\) that is inside the sphere of radius \(c\Delta t\) has no electric field
on it (\(E = 0\)). The remaining part of \(S\), which we call \(S - S'\), is instead still
crossed by the field that allegedly should be there since the information of
the definitive cancellation is not yet arrived.

The flux of the electric field \(E\) across the closed surface \(S\) is
Figure 4: At the interpenetration point (instant $t = 0$ s), the charge holders become neutral. After a time $\Delta t$, the information front of the neutralization has reached a distance $c\Delta t$. To apply Gauss’s law, we choose a closed spherical surface $S$ such that $S$ is tangent to the interpenetration point, and its center lies on the trajectory of the charge holders (with a diameter at least as big as the distance $d$ of the distant observer). According to the received interpretation, although the charge holders are now electrically neutral, the electric field $\mathbf{E}$ should still be different from zero beyond the information front, and corresponding to the dipole field generated by the charge holders as if they retained their original charge.
where $\text{dA}$ is a vector representing an infinitesimal element of area of the surface $S$, and $\cdot$ represents the dot product of two vectors. After the time $\Delta t$, the electric field $\textbf{E}$ is allegedly non-zero only across the portion $S - S'$ of the total surface $S$ ($S - S'$ is an open surface).

Moreover, and most importantly, the dipole field $\textbf{E}$ is always only outgoing or incoming across $S$, depending upon the sign of the former charge on the charge holder inside it. Since, by construction, $S$ encloses only the portion of space where there is allegedly only the positive or the negative charge of the dipole, the sign of the dot product $\textbf{E} \cdot \text{dA}$ is always positive or negative, respectively, for every $\text{dA}$ on the surface $S$.

Now, according to Gauss’s law, the flux in Eq. (3) must be equal to zero since there is no charge inside $S$,

$$\oint_S \textbf{E} \cdot \text{dA} = \oint_{S - S'} \textbf{E} \cdot \text{dA} = \frac{Q_{\text{inside } S}}{\epsilon_0} = 0. \tag{4}$$

Notice that, at the considered instant of time, the part of the first integral in Eq. (4) calculated on $S'$ is equal to zero.

Because the sign of the dot product $\textbf{E} \cdot \text{dA}$ is always the same for every $\text{dA}$ on the surface $S$, the only solution to Eq. (4) is that the electric field $\textbf{E}$ is identically zero at every point on $S$.

Since both $S$ and $\Delta t$ have been chosen arbitrarily, in the case of the operator allowing the electrical contact, the dipole field $\textbf{E}$ becomes equal to zero at the moment of interpenetration and remains equal to zero from then on for every observer in space, however distant. No information lag is thus possible.

This result has a non-trivial, but not entirely unexpected either, impact on the relativity of simultaneity.

According to special relativity, the simultaneity of two events occurring at two distinct places depends upon the observer’s reference frame. If one event occurs at point $x_1$ at time $t_0$ and the other event at $x_2$ and $t_0$ (the same time and the same reference frame), we find that the two corresponding times $t_1'$ and $t_2'$ in the reference frame of a moving observer differ by an amount

$$t_2' - t_1' = \frac{v(x_1 - x_2)/c^2}{\sqrt{1 - v^2/c^2}}, \tag{5}$$
where \( v \) is the velocity of the observer relative to the reference frame where points \( x_1 \) and \( x_2 \) are at rest. That derives from a straightforward application of the Lorentz transformation of the time coordinate.

The conclusion reached with Gauss’s law proves, however, that the cancellation of the dipole field is an event simultaneous at every point in space and for every observer, regardless of the inertial reference frame in which the observer is at rest.

Consider two points, A and B, separated in space and at rest relative to the observer and the center of mass of the dipole (Fig. 2). At the instant in which the dipole ‘shrinks to zero’, and there is the electrical contact, the definitive cancellation of the field is instantaneous at every point in space, and thus the cancellation of the field in A and B is simultaneous for the observer.

If now the observer moves at a constant velocity relative to A and B and the center of mass of the dipole, the application of Gauss’s law is still possible in the reference frame of the observer. In that frame, the charge holders move no longer head-on and with equal and opposite velocities, but, as mentioned before, Gauss’s law also holds when charges move arbitrarily. As happens in Fig. 4, Gauss’s law guarantees that, for the observer, the dipole field becomes definitively equal to zero just after the interpenetration and the electrical contact between the charge holders, and it happens instantaneously at every point in space. Thus, even for the moving observer, the cancellation of the field is simultaneous at points A and B.

Notice that Gauss’s law can be equivalently applied to show that if two equal and opposite charges are generated (e.g., through triboelectric charging) and separated, then the electric field of each charge should come into existence instantaneously at every point in space. Even in this case, no information lag is consistent with Gauss’s law.

Consider the situation depicted in Fig 5. A smaller and a larger sphere, both neutral, meet at the interpenetration point. They move at a constant velocity. After the interpenetration, they recede from each other at the same (controlled) constant velocity and with an equal and opposite electric charge (again, the larger sphere with \(+Q\) and the smaller one with \(-Q\)) acquired through triboelectric charging.

Does the dipole field they produce come into existence instantaneously at every point in space (no lag)? The answer is yes, and it comes from the application of Gauss’s law.

Consider a closed spherical surface \( S \) tangent to the interpenetration
Figure 5: At the interpenetration point (instant $t = 0$ s), the charge holders acquire a charge $Q$ through triboelectric charging. After a time $\Delta t$, the information that the electric field is now different from zero has reached a distance $c\Delta t$. To apply Gauss’s law, we choose a closed spherical surface $S$ such that $S$ is tangent to the interpenetration point, and its center lies on the trajectory of the charge holders (with a diameter at least as big as the distance $d$ of the distant observer). Furthermore, an open surface $S''$ is considered that is entirely inside the information front and has as a boundary the (closed) intersection line between surface $S$ and the information sphere of radius $c\Delta t$. Surface $S''$, together with $S'$, makes a closed surface that includes the charged sphere.
point, with the center lying on the trajectory of the positively charged sphere and with a diameter at least as big as the distance $d$ of the distant observer. If a time $\Delta t$ has passed after the interpenetration (and the charging of the sphere), the information that the field is now different from zero has allegedly reached a distance $c\Delta t$ from the interpenetration point. Suppose that $c\Delta t \leq d$. That means that if we consider the intersection of the spherical surface $S$ and the surface of the information front with radius $c\Delta t$ and centered at the interpenetration point, only the portion $S'$ of surface $S$ inside the sphere of radius $c\Delta t$ has an electric field different from zero on it ($\mathbf{E} \neq 0$). The remaining part of $S$, which we call $S - S'$, is not yet crossed by the field. The field cannot be there since the information of its generation is not arrived (see Fig. 5).

Now, according to Gauss’s law, the flux of the dipole field $\mathbf{E}$ must be equal to the charge inside $S$ divided by the vacuum permittivity $\epsilon_0$

$$\oiint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside } S}}{\epsilon_0}. \quad (6)$$

Notice that, at the considered instant of time, the integral in Eq. (6) is non-zero only on the portion $S'$ of the whole surface $S$,

$$\oiint_S \mathbf{E} \cdot d\mathbf{A} = \oiint_{S'} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside } S}}{\epsilon_0}. \quad (7)$$

Consider a second open surface $S''$ that is entirely inside the information front and has as a boundary the (closed) intersection line between surface $S$ and the information sphere of radius $c\Delta t$. Then, $S''$ together with $S'$ make a closed surface that includes the charged sphere (see Fig. 5). We have then

$$\oiint_{S' + S''} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside } S}}{\epsilon_0}. \quad (8)$$

Focusing on the first members of Eqs. (6) and (8), we can write

$$\oiint_S \mathbf{E} \cdot d\mathbf{A} = \oiint_{S' + (S - S')} \mathbf{E} \cdot d\mathbf{A} = \oiint_{S'} \mathbf{E} \cdot d\mathbf{A} + \oiint_{(S - S')} \mathbf{E} \cdot d\mathbf{A} = \oiint_{S' + S''} \mathbf{E} \cdot d\mathbf{A} = \oiint_{S'} \mathbf{E} \cdot d\mathbf{A} + \oiint_{S''} \mathbf{E} \cdot d\mathbf{A}, \quad (9)$$

and thus
\[ \iint_{(S-S')} \mathbf{E} \cdot d\mathbf{A} = \iint_{S'} \mathbf{E} \cdot d\mathbf{A}. \]  

(10)

Since the flux of the dipole field through the surface \( S'' \) cannot be equal to zero (\( S'' \) is inside the information front, and the field is non-zero there), the same must be for the flux of the electric field through the surface \( S - S' \). Since the choice of \( S \) is arbitrary, the only possibility for \( \iint_{(S-S')} \mathbf{E} \cdot d\mathbf{A} \) to be different from zero is that the electric field \( \mathbf{E} \) is already different from zero (and equal to that generated by the dipole) on the whole surface \( S \) before the arrival of the information front.

With reference to Fig. 5 and to Eqs. (1) and (2) (and as far as Eq. (1) holds), the electric field \( \mathbf{E}_{\text{tot}}(t) \) instantaneously generated and measured at point \( \mathbf{d} \) in space is given by

\[
\begin{align*}
\mathbf{E}_{\text{tot}}(0) &= 0 & \text{since } Q = 0 \text{ at } t = 0, \\
\mathbf{E}_{\text{tot}}(t) &= -\frac{Q}{4\pi \epsilon_0 |\mathbf{d} - t \cdot \mathbf{v}_-|^2} \left( \frac{1 - \frac{v_-^2}{c^2}}{1 - \frac{v_-^2 \sin^2 \theta_-(t)}{c^2}} \right)^{3/2} \frac{\mathbf{d} - t \cdot \mathbf{v}_-}{|\mathbf{d} - t \cdot \mathbf{v}_-|} + \\
&\quad + \frac{Q}{4\pi \epsilon_0 |\mathbf{d} - t \cdot \mathbf{v}_+|^2} \left( \frac{1 - \frac{v_+^2}{c^2}}{1 - \frac{v_+^2 \sin^2 \theta_+(t)}{c^2}} \right)^{3/2} \frac{\mathbf{d} - t \cdot \mathbf{v}_+}{|\mathbf{d} - t \cdot \mathbf{v}_+|} \quad \text{for } t > 0,
\end{align*}
\]

where \( \mathbf{d} \) is the distance from the center of the splitting dipole to the point where the field is measured, \( \mathbf{v}_- \) and \( \mathbf{v}_+ \) are respectively the velocities of the negative and positive charge measured in the reference frame of the observer, and 

\[ \sin^2 \theta_\pm(t) = \left( \frac{[\mathbf{v}_\pm \times (\mathbf{d} - t \cdot \mathbf{v}_\pm)]}{|\mathbf{v}_\pm| |\mathbf{d} - t \cdot \mathbf{v}_\pm|} \right)^2. \]

When the center of mass of the splitting dipole is stationary with the reference frame of the observer, we have that \( \mathbf{v}_+ = -\mathbf{v}_- \). If instead, the observer moves with velocity \( \mathbf{u} \) relative to the center of mass of the splitting dipole, then \( \mathbf{v}_+ \neq -\mathbf{v}_- \) since velocity \( \mathbf{u} \) must be subtracted from the equal and opposite velocities of the charges measured in the center of mass of the dipole.

The above analysis suggests that field lines exist in space or do not exist. They cannot be existent in a finite portion of space and non-existent in the remaining (infinite) portion of space. For instance, for emission and absorption of charged particles, Dirac explicitly writes \[\text{[11]}\]: “Whenever an electron is emitted, the Coulomb field around it is simultaneously emitted,
forming a kind of dressing for the electron. Similarly, when an electron is absorbed, the Coulomb field around it is simultaneously absorbed”.

4 Discussion and conclusions

In the previous Sections, we have seen that the electric field of a uniformly moving charge is measured by a distant observer as always directed away from, or pointing towards, the instantaneous present position of the charge and not towards or away from its retarded position. We have shown how this fact has important consequences for the relativity of simultaneity and the theoretical feasibility of a faster-than-light communication of information.

If we consider an electric dipole with inter-charge distance shrinking to zero at uniform velocity, then the cancellation of the total field seems to be instantaneous everywhere in space and in every reference frame. Moreover, a simple variant of the shrinking electric dipole thought experiment appears to allow faster-than-light (actually, instantaneous) communication of information.

Notice that faster-than-light communication of information does not violate causality per se. It does so only if simultaneity is relative. The relativity of simultaneity implies that time passes at different rates in reference frames in relative motion. Then, instantaneous communication of information may lead to the paradox of receiving a message from an observer in motion relative to you before you ask him to send the message to you (this situation is particularly evident when represented in a Minkowski diagram). However, a consequence of what we have found in Section 3 is that simultaneity appears to be not relative, at least with the cancellation or the generation of the field of two interpenetrating or separating equal and opposite charges. In general, if simultaneity is not relative, then time flows at the same pace in every reference frame. In that case, faster-than-light or even instantaneous signaling cannot let the effect precede the cause in any reference frame.

The theory behind the present results is well-known and well-established, and the derivation is simple and direct. All this works in favor of our findings in a twofold way. First, unless the physics theory behind it is fundamentally flawed, our results should be corresponding to physical facts. Second, owing to the underlying simplicity of our derivation, its conclusions can be unambiguously tested in the laboratory, at least in principle.

The amplitude of a standard e.m. wave depends on the distance from
the source as $1/r$, and thus the intensity of the wave scales as the square of the amplitude ($1/r^2$). However, in all the examples previously made, we deal with a quasi-static dipole field. The amplitude of the quasi-static dipole fields depends on the distance as $1/r^3$. The considerable decrease in the amplitude with distance might prevent this field from being easily detected if not purposely searched for.

Concerning already performed experimental tests, our findings, and above all what has been derived with the application of Gauss’ law in Section 3, seem to provide simple theoretical support to the conclusions of the experiments conducted by the Frascati Group on the propagation speed of Coulomb fields [12, 13, 14] and by Kholmetskii et al. on the propagation speed of magnetic fields [15, 16]. Moreover, as already suggested in [15, 16] for those results, it would be interesting to explore any possible connection between our findings and quantum non-locality. In particular, if the present results are sound and experimentally confirmed, it would be of some interest to know whether they may have a role in the apparent action-at-distance behavior of many quantum experiment results (i.e., instantaneous correlations between the properties of remote systems).

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