A Mode Switching Extended Kalman Filter for Real-Time Traffic State and Parameter Estimation

Yue Zhou\(^1\), Kaan Ozbay\(^1\), Michael Cholette\(^2\), and Pushkin Kachroo\(^3\)

Abstract—Traffic state estimators (TSEs) based on the cell transmission model (CTM) are vulnerable to biased initial estimates of traffic flow parameters, in particular the critical density. For example, an overestimated (underestimated) initial estimate of critical density can cause a delayed (premature) switching of a TSE from free-flow working mode to congestion working mode, hence distorting estimates of traffic states. Only augmenting the traffic flow parameters into the state vector cannot resolve the issue of biased initial estimate of the critical density, because the critical density is unobservable under free-flow mode. To overcome this issue, this paper proposes an innovative supervisory observer to inform the TSE of correct instants for mode switching. In particular, the proposed supervisory observer requires no \textit{a priori} knowledge of any traffic flow parameter. The idea is to make decisions for mode switching through capturing anomalies in the residuals of the extended Kalman filter (EKF) of the TSE. This paper also proposes, for the first time in relevant literature, to augmenting the traffic flow parameters into the state vector so that its value can be calibrated online. Simulation experiments show that the proposed method can correctly capture mode switching times and generate satisfactory estimates of traffic states and track time-varying traffic flow parameters.

I. INTRODUCTION
A. Overview of CTM Based Traffic State Estimation

Real-time traffic state estimators (TSE) play an extremely important role in Intelligent Transportation Systems. Because of its simplicity, Cell Transmission Model (CTM) [1] has been widely used to serve as the underlying model of traffic flow dynamics based on which a TSE is built. A prominent feature of CTM is that for any given cell, the model of its dynamics switches between two modes – the free-flow mode and the congestion mode. For each of these modes, the traffic flow dynamics are linear in the state variable, i.e. the traffic density. A large amount of previous TSE studies have employed CTM, with the assumption that traffic flow parameters, i.e. the critical density and free-flow speed, are known \textit{a priori} and their values do not change with time, e.g. [2]–[6]. Because of this assumption, the resulting state-space model switches between linear modes. Thus Kalman filtering (KF) techniques can be applied to each mode to estimate traffic densities. For brevity, we call such an approach as the CTM-KF approach. A major shortcoming of the CTM-KF approach is that it can be vulnerable to poor knowledge of traffic flow parameters. Since traffic flow parameters are fixed values in this approach, if their values are significantly biased from the truth, estimation of traffic densities will be seriously undermined. In real world practice this can occur if offline calibration of these parameters is poorly made, or external conditions have altered after the calibration, e.g. change of weather or lighting conditions. Note that there are also TSE studies that have employed second-order macroscopic traffic flow models, e.g. [7]–[9], which are beyond the scope of this paper.

To mitigate the above shortcoming of the CTM-KF approach, there are efforts, although very few, e.g. [10], that considered the free-flow speed and critical density as unknown and time-varying, and augmented them into the state vector to be estimated jointly with traffic densities. Because of the conversion of the traffic flow parameters into state variables, the resulting traffic flow dynamics are no longer switching between linear modes, but between nonlinear modes. Consequently, the extended Kalman filter (EKF) (or other nonlinear filters) has to be used instead of KF. We call such an approach as the CTM-EKF approach, following [10]. The CTM-EKF approach is less vulnerable to poor knowledge of the free-flow speed than the CTM-KF approach, because the free-flow speed is observable under both the free-flow and congestion modes, hence can always be corrected by measurements. However, the CTM-EKF approach can still be vulnerable to poor initial estimate of the critical density because the critical density is unobservable under the free-flow mode. As pointed out in [11], an underestimated (overestimated) initial critical density can cause the CTM-EKF estimator a premature (delayed) switch from the free-flow mode to the congestion mode, while in reality the road section has not yet been congested (has already been congested). Traffic state estimation results could be significantly distorted due to such mismodeling.

As an attempt to mitigate the above shortcoming of conventional CTM-EKF approach, [11] proposed to couple a supervisory observer with the CTM-EKF estimator to command the latter to switch mode. The mechanism used by the supervisory observer in [11] to make switching decisions was straightforward: It takes advantage of the fact that a drop in the discharge flow rate from an active bottleneck (i.e. the capacity drop phenomenon) is always associated with a congestion within the bottleneck. However, as found in [11], such a method for making mode switching decisions requires an accurate \textit{a priori} knowledge of the value of the
capacity drop proportion. If the value has altered after the offline calibration, due to changes in external conditions, then the supervisory observer can make wrong mode switching decisions. Moreover, if the magnitude of capacity drop is slight, then the supervisory observer can have a difficulty in capturing it from noisy measurements.

B. Contributions of the Paper

In light of the above discussion, this paper attempts to further improve the CTM-EKF approach as an extension of [11]. This paper has two contributions. The first feature is a supervisory observer that uses a fundamentally different mechanism from the one used in [11] to make switching decisions. The supervisory observer in this paper no longer makes mode switching decisions based on whether a capacity drop has occurred or cleared. Instead, switching decisions are made solely on information generated by the CTM-EKF estimator itself. The idea is that the supervisory observer monitors (in real time) the EKF residuals of a measurement variable which is influenced by the onset and clearance of congestion; if an anomaly in the residuals is detected, it marks the presence of a mismatch between the current working mode of the CTM-EKF estimator and the traffic mode in reality (known as “mismodeling” [12] in control theory literature), and hence the CTM-EKF estimator should switch the mode. The idea is similar to the so-called multi-model adaptive filtering [13] in control theory. Therefore, the proposed supervisory observer does not require any a priori knowledge of the traffic flow parameters. That is, it does not need to know in advance the value of the capacity drop proportion, nor the values of the critical density and free-flow speed. Hence it is robust to biased initial knowledge of these values due to either poor offline calibration or changes in external conditions after the calibration. This is a fundamental difference from all existing relevant studies.

The second contribution of this paper is that the value of capacity drop proportion is to be estimated in real time (during congestion period). That is, it is no longer treated as a known, fixed-value parameter, but is rather augmented into the state vector to be estimated together with the traffic state and the other traffic flow parameters.

The remainder of this paper is organized as follows. Section II sketches the CTM framework for a highway section with a lane-drop bottleneck. But note that the proposed method applies in general to other types of bottlenecks as well. Section III introduces, in a general manner, how to construct a real time TSE based on the CTM, and explains in detail why the TSE is vulnerable to biased initial estimates of the traffic flow parameters. Section IV is the core of this paper, proposing an EKF residual based supervisory observer for detecting mode switching. Section V and VI present results from experiments using CTM synthesized measurements and using micro-simulation synthesized measurements, respectively. Section VII summarizes this study and points out future research directions.

II. CTM FOR A HIGHWAY SECTION WITH A LANE-DROP BOTTLENECK

In this paper we study the same highway section with a lane-drop bottleneck as in [11]. However, the method can be extended to other types of bottlenecks such as an on-ramp merge. The interested highway section is illustrated by Fig. 1. Because of the lane drop, a large amount of lane changing takes place within cell \(N - 1\) when the flow rate approaches the capacity of cell \(N\), and a congestion will initiate from within cell \(N - 1\). It is assumed that there is no more restrictive bottleneck downstream; and if there is a more restrictive bottleneck downstream, the tail of the congestion initiated from that bottleneck will never reach this one.

![Fig. 1: A highway section with a lane-drop bottleneck](image)

The CTM of the above highway section is composed of three major components:

1. Conservation law

\[
\rho_k = \rho_{k-1} + \frac{\Delta t}{\lambda_i} (q_{k-1,i} - q_{k,i+1})
\]

2. Interface flow: demand-supply interaction

\[
q_{k-1,i} = \min\{D_{k-1,i}, S_{k-1,i}\}
\]

\[
q_{k,i+1} = \min\{D_{k,i}, S_{k,i+1}\}
\]

3. Demand \((D)\) and supply \((S)\) functions: triangular fundamental diagram

For cell \(i = 1,2,...,N-1\),

\[
D_{k-1,i} = \lambda_i v^f \min\{\rho_{k-1,i}, \rho^c\}
\]

\[
S_{k-1,i} = \lambda_i v^f \rho^c \min\{1, \frac{\rho_{k-1,i} - \rho^c}{\rho_{k-1,i} - \rho^c}\}
\]

For the last cell \((i = N)\),

\[
D_{k-1,N} = \lambda_N v^f \rho^c
\]

\[
S_{k-1,N} = \lambda_N v^f \rho^c (1 - \theta), \text{ otherwise}
\]

In the above, \(\rho^c\) is the density of cell \(i\) at time step \(k\); \(\rho^c\) is the critical density; \(v^f\) the free flow speed; \(\theta\) the capacity drop proportion; \(\lambda_i\) the number of lanes for cell \(i\).

Note that, under the CTM framework, the critical density \(\rho^c\) will not come into the play until the condition \(\rho_{k-1,N}^{-1} \geq \frac{\lambda_N}{\lambda_N - \rho^c}\) is satisfied. To see this: When \(\rho_{k-1,N}^{-1} < \frac{\lambda_N}{\lambda_N - \rho^c}\), it is obvious that all the cells are in free-flow mode and all the interface flows should be determined by the supply functions of the corresponding upstream cells, which do not involve \(\rho^c\). As soon as \(\rho_{k-1,N}^{-1} \geq \frac{\lambda_N}{\lambda_N - \rho^c}\) is satisfied, the
As a result, the process model is:

\[
\begin{align*}
    v^r_k &= \rho^r_{k-1} + \frac{\Delta t}{\lambda} \left[ q_{k-1}^{i-1,i} - q_{k-1}^{i,i+1} \right] + \xi_{k-1}^v \\
    f^r_k &= \xi_{k-1}^{fr} + \xi_{k-1}^{cr}
\end{align*}
\]

where in (13) \( i = 1, 2, \ldots, N \). In other words, the state vector is \( \mathbf{x}_k = [\rho^r_1, \rho^r_2, \ldots, \rho^r_k, v^r_k, \xi^v_{k-1} \xi^{fr}_{k-1} \xi^{cr}_{k-1}] \), and that \( f^r_k \) is nonlinear in \( \mathbf{x}_k \). Therefore, EKF needs to be applied in place of KF.

The above treatment can resolve the issue of biased initial estimate of \( v^r \), as \( v^r \) is always observable regardless of free-flow or congestion modes. However, the issue of biased initial estimate of \( \rho^r \) remains. To see this, suppose that the initial critical density is somehow underestimated. As we explained earlier, the critical density estimate will not come into the play, and hence will not be able to be updated, until the condition \( \rho^r_{N-1} \geq \frac{\lambda}{\lambda + \mu} \rho^* \) is satisfied. However, the underestimated initial value of \( \rho^* \) will lead to a premature satisfaction of this condition, hence erroneously rendering a switching of \( S^N \) from its free-flow form to its congestion form, at some instant when in real-world it is still under the free-flow regime.

The above issue is a paradox: The traffic state estimator cannot correct the biased initial estimate of \( \rho^* \) until a certain condition is satisfied; however, the decision of whether this condition has been satisfied is dependent on the biased initial estimate of \( \rho^* \) itself.

IV. AN EKF BASED SUPERVISORY OBSERVER FOR DETECTING MODE SWITCHING

Per the analysis in the previous section, it is desirable to have a supervisory observer to tell the traffic state estimator the correct instants to switch between the free-flow mode and the congestion mode. Zhou et al. [11] is the first to propose such a supervisory observer. However, as introduced in Section I.A., the supervisory observer in [11] is dependent on a priori knowledge of the capacity drop proportion, and thus can be vulnerable if the knowledge is biased. Ideally, the supervisory observer should not require any a priori knowledge of traffic flow parameters, including the capacity drop proportion. Such a supervisory observer is proposed in this section.

The idea is actually simple, and is described as follows. As we know, at each time step, a Kalman filter updates the a priori estimate of the system state by incorporating the discrepancy between the predicted system output (i.e. computed based on the a priori estimate) and the actually measured system output (i.e. the measurements). That is:

\[
\hat{x}_{k+} = \hat{x}_{k-} + K_k (z - h(\hat{x}_{k-}))
\]
be stationary, otherwise there should arise anomalies in the pattern of the residuals. The concept of EKF residual is the same.

In our application, rather than monitor the residuals of all the measurement variables, we choose to monitor in real time the residual of the interface flow rate between cell \( N - 1 \) and cell \( N \), i.e.

\[
r_k^{N-1,N} = zq_{k}^{N-1,N} - q_{k}^{N-1,N}
\]  
(18)

The reason why the interface flow rate between cell \( N - 1 \) and cell \( N \) is chosen over other measurement variables is because cell \( N - 1 \) will always be the first cell to be influenced by a congestion and the last cell in which the congestion clears. If the working mode of the EKF correctly matches the traffic mode in reality, then the time series of \( r_k^{N-1,N} \) should be stationary. An abrupt change in the pattern of the time series implies that the current working mode of the EKF no longer matches the actual traffic mode, therefore the EKF needs to switch its working mode. The above idea is illustrated by Fig. 2.

![Fig. 2: A schematic representation of the proposed methodology](image)

It remains to design the supervisory observer to detect anomalies in the pattern of \( r_k^{N-1,N} \) time series. We apply the so-called cumulative sum (CUSUM) control chart [17]. CUSUM is a simple statistical process-monitoring technique that has been widely applied in many engineering and science disciplines. In this paper, we employ a specific CUSUM method called standardized two-sided CUSUM [17], which was first proposed by [18]. The principle of the standardized two-sided CUSUM is straightforward and is represented mathematically as ([17], [19]):

\[
C_k^+ = \max \{0, z_k - \delta + C_{k-1}^+\} \\
C_k^- = \min \{0, z_k + \delta + C_{k-1}^-\}
\]  
(19) \hspace{1cm} (20)

where \( z_k \) is the standardized deviation of the monitored process value at the current sampling instant, i.e.

\[
z_k = \frac{x_k - \mu}{\sigma}
\]  
(21)

In (19) and (20), \( \delta \) is the slack constant which is a predetermined parameter. In (21), \( \mu \) and \( \sigma \) are predetermined process mean and standard deviation, respectively. If \( C_k^+ \) or \( C_k^- \) has surpassed the predefined thresholds \( \pm h \), then it is deemed that an anomaly in the pattern of the monitored process has occurred. We have found that in our application, the lower-side CUSUM is more efficient in detecting anomalies in the EKF residuals of the interface flow between cell \( N - 1 \) and cell \( N \). Therefore we only utilize the lower-side CUSUM in the proposed supervisory observer. The proposed supervisory observer is described by Algorithm 1.

The mechanism of Algorithm 1 and the meaning of the parameters are explained as follows. At the initialization step, the “current mode” is set as free flow. This is in consistent with the aforementioned assumption that the TSE task starts from free flow condition. \( t_{SLS} \) denotes elapsed time since last mode switching. This parameter is given an initial value of zero. Every time a mode switching occurs, it will be reset to zero. \( T_w \) denotes warm-up period. It refers to a certain length of time duration immediately after a mode switching. During the warm-up period, the supervisory observer will do nothing, because the EKF residuals generated within this period may not be stationary. \( T_p \) denotes preparation period. Preparation period refers to a certain length of time duration immediately after the warm-up time. During the preparation time, the supervisory observer will store the concerned EKF residuals. At the end of the preparation period, the supervisory observer will compute the mean and standard deviation of the residuals sampled over the preparation period. The obtained mean and standard deviation will be used to compute the standardized deviations of residuals that come later. If the standardized deviation of the residual at some time step has exceeded a predefined threshold value \( h_1 \), then it is deemed that the pattern of the residuals has changed and thus the supervisory observer will command the current mode to switch from free flow to congestion; and reset \( t_{SLS} \) to be zero. Now that the current mode is congestion, and if at some other time step the standardized deviation of the residual has surpassed another predefined threshold value \( h_2 \), then it is deemed that the pattern of the residuals has changed again. This time, the supervisory observer will command the current mode to switch from congestion to free flow; and reset \( t_{SLS} \) to be zero. Determination of proper values for \( h_1 \) and \( h_2 \) are based on trial-and-error.

V. APPLICATION TO CTM SYNTHESIZED MEASUREMENTS

As verification of the proposed CTM-EKF approach, it is applied to measurements synthesized by CTM. We are interested in whether the proposed approach can reasonably recover the truth signals from measurements that are corrupted by significant, artificial random noises.

We assume that the concerned highway section is 3500 m long, and we divide it into 5 cells, each cell being 700 m long. We assume that the first four cells consist of 3 lanes, and the last cell consists of 2 lanes. We use the CTM framework introduced in Section II.A. for simulation that generates true traffic densities and true interface flow rates and speeds. The true values of the traffic flow parameters used for the simulation are as follows: The free-flow speed is 100 veh/hr, critical density 20 veh/hr/ln, and capacity drop
Algorithm 1: Mode Observer for EKF Traffic State Estimator

Data: EKF residuals of the interface flow rate from cell $N-1$ to cell $N$ for the entire estimation horizon, i.e. $r_{k}^{QN-1,N}$, for $k = 1, 2, ..., K$

Result: traffic mode of current time step, $k$

initialize current mode $\leftarrow$ free flow
initialize $t_{SLS} \leftarrow 0$

for $k = 1, 2, ..., K$ do
    if $T_{w} < t_{SLS} < T_{w} + T_{p}$ then
        Store $r_{k}^{QN-1,N}$
    else if $t_{SLS} = T_{w} + T_{p}$ then
        Calculate $\mu$ and $\sigma$ based on stored $r_{k}^{QN-1,N}$
    else if $t_{SLS} \geq T_{w} + T_{p}$ then
        Calculate $C_{k}$
        if current mode = free flow then
            if $|C_{k}| > h_{1}$ then
                current mode = congestion
                $t_{SLS} = 0$
            end
        else
            if $|C_{k}| > h_{2}$ then
                current mode = free flow
                $t_{SLS} = 0$
            end
        end
end

proportion 10%. We then corrupt the obtained true interface flow rates and speeds with significant random noises. We want to recover true traffic densities as well as true traffic flow parameters from the noise corrupted interface measurements, by applying the improved CTM-EKF approach.

Fig. 3 presents the interface flow rates between cell 4 and cell 5. Fig. 3 indicates that, the congestion initiates at around 1500 seconds, and clears at around 3250 seconds. Fig. 4 (a) presents the EKF residuals of the interface flow rate between cell 4 and cell 5, i.e. $r_{k}^{QN-1,N}$. Fig. 4 (a) indicates that, at the instants when the congestion initiates and clears, there exist anomalies in the pattern of $r_{k}^{QN-1,N}$ time series. Fig. 4 (b) presents the CUSUM chart for the standardized deviation of $r_{k}^{QN-1,N}$. We see that the CUSUM is able to make the anomalies to stand out.

Fig. 5 presents the estimated traffic flow parameters against the true values. We see that, the proposed approach is able to quickly correct biased initial estimate of the free-flow speed as soon as the estimation starts, thanks to the fact that the free-flow speed is always observable; the proposed approach is able to quickly correct biased initial estimates of the critical density and capacity drop proportion as soon as a congestion initiates.

Fig. 6 presents traffic density estimates against the true density values for cell 2, 3, and 4. We see that the estimates match the true values well. Due to space limit we do not present results for cell 1 and 2, but the results are equally well.

VI. APPLICATION TO MICRO-SIMULATION SYNTHESIZED MEASUREMENTS

To evaluate the proposed approach’s potential in real world applications, the proposed approach is applied to micro-simulation synthesized measurements. We use a commercial micro-simulation package, Aimsun, to simulate traffic over a highway section with a lane drop bottleneck section that is similar to the one used for the CTM case in the previous section. The highway section of interest is divided into 5 400m long cells. The first 4 cells have 3 lanes, and the last cell has 2 lanes. That is, cell 4 is still the bottleneck cell, as in the CTM case in the previous section. The true free-flow speed is specified as follows: 100 km/hr from 0 sec to 2400 sec; 90 km/hr from 2400 sec to 5400 sec. It is not possible
Fig. 5: Estimated traffic flow parameters: (a) free-flow speed; (b) critical density; (c) capacity drop proportion

Fig. 6: Estimated traffic densities: (a) cell 3; (b) cell 4; (c) cell 5

Fig. 7: Estimated traffic flow parameters: (a) free-flow speed; (b) critical density; (c) capacity drop proportion to specify true critical density and capacity drop proportion in Aimsun.

The measured interface flow rate between cell 4 and cell 5, as shown in Fig. 7, indicates that a congestion initiates at around 1600 sec and clears at around 3800 sec. Fig. 8 presents the residuals of the interface flow rate between cell 4 and cell 5, and the CUSUM plot of the standardized deviation of the residuals. It can be seen from Fig. 8 that, as expected, residuals of Aimsun data are much more noisy than the CTM data. Fortunately, taking advantage of the CUSUM conversion, the proposed supervisory observer is still able to correctly identify the instants of mode switching, i.e. around 1600 sec and 3800 sec.

Fig. 9 presents the estimated traffic flow parameters, namely, the free-flow speed, critical density, and capacity drop proportion. Fig. 9(a) indicates that the proposed approach is able to quickly correct biased initial estimate of the free-flow speed and to quickly capture the abrupt change of the true value in the middle of the estimation process.

Fig. 9(b) plots the estimated critical density against “true” critical density. Note that in Aimsun it is not possible to directly specify true critical density. However, note that cell 5 is the lane-dropped cell and no more restrictive bottleneck is downstream of it, so cell 5’s density should be around the critical density reduced by the capacity drop proportion, when a congestion is active upstream. Therefore, in the figure the true critical densities are approximated by the value of true cell 5 densities divided by $1 - \hat{\theta}$. The figure indicates that the estimated critical densities are a bit underestimated. This is because, as can be seen by comparing Fig. 9(c) and Fig. 7, the capacity drop proportions are a bit overestimated. As a result, the approximate true critical densities, which are calculated as $\rho_5/(1 - \hat{\theta})$, are accordingly overestimated. This is due to the fact that for the CTM, the interface flow between cell 4 (3 lanes) and cell 5 (2 lanes), as well as the discharge flow from cell 5, are all equal to the free-flow speed, even during congestion. This arises from a fundamental assumption of CTM – traffic flow speed can change instantly, i.e. vehicles have infinite capabilities of
accelerations and decelerations, which is inconsistent with Aimsun-simulated vehicle behaviors. In the next step, we will attempt to remedy this shortcoming of the CTM and incorporate the remedy into the CTM-EKF estimator.

Fig. 10 presents traffic density estimates against the true density values for cell 3, 4, and 5. We see that the estimates match the true values reasonably. Due to space limit we omit results for cell 1 and 2, but the quality of the estimates are comparable.

VII. CONCLUSIONS

This paper proposed a supervisory observer that monitors in real time the residuals of the CTM-EKF estimator, so that mismatches between the current working mode of the estimator and the traffic mode in reality, if occur, can be captured, from which the estimator knows about instants for mode switching. It was also proposed, for the first time in relevant literature, to augment capacity drop proportion into the state vector, so that its value can be updated in real time when the bottleneck is active. Simulation experiments using CTM synthesized measurements and micro-simulation synthesized measurements both showed that the proposed supervisory observer is able to correctly detect the instants for mode switching. Consequently, the CTM-EKF estimator is able to generate satisfactory estimates for traffic densities and the traffic flow parameters, including the free-flow speed, the critical density, and the capacity drop proportion. It was also noted that the estimator tends to overestimate the capacity drop proportion from the Aimsun synthesized measurements, although it is able to accurately recover the true capacity drop proportion from the CTM synthesized measurements. A remedy has been discussed and will be investigated in the next step research. In the next step, we will also couple the proposed CTM-EKF estimator with ramp metering control and mainline VSL control, both of which
need good knowledge of traffic states and the traffic flow parameters to achieve good performances.

REFERENCES

[1] C. F. Daganzo, “The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory,” Transportation Research Part B: Methodological, vol. 28, no. 4, pp. 269–287, 1994.

[2] L. Miuño, X. Sun, R. Horowitz, and L. Alvarez, “Traffic density estimation with the cell transmission model,” in Proceedings of the 2003 American Control Conference, IEEE, 2003, pp. 3750–3755.

[3] D. B. Work, O.-P. Tossevainen, S. Blandin, A. M. Bayen, T. Iwuchukwu, and K. Tracton, “An ensemble kalman filtering approach to highway traffic estimation using gps enabled mobile devices,” in 2008 47th IEEE Conference on Decision and Control, IEEE, 2008, pp. 5062–5068.

[4] L. Mihaylova, R. Boel, and A. Hegyi, “Freeway traffic estimation within particle filtering framework,” Automatica, vol. 43, no. 2, pp. 290–300, 2007.

[5] J. Thai, B. Prodhonne, and A. M. Bayen, “State estimation for a discretized lwr pde using explicit polyhedral representations of the godunov scheme,” in 2013 American Control Conference. IEEE, 2013, pp. 2428–2435.

[6] I.-C. Morărescu and C. Canudas-de Wit, “Highway traffic model-based density estimation,” in Proceedings of the 2011 American Control Conference. IEEE, 2011, pp. 2012–2017.

[7] C. Nanthawichit, T. Nakatsuji, and H. Suzuki, “Application of probe-vehicle data for real-time traffic-state estimation and short-term travel-time prediction on a freeway,” Transportation research record, vol. 1855, no. 1, pp. 49–59, 2003.

[8] Y. Wang and M. Papageorgiou, “Real-time freeway traffic state estimation based on extended kalman filter: a general approach,” Transportation Research Part B: Methodological, vol. 39, no. 2, pp. 141–167, 2005.

[9] T. Seo and A. M. Bayen, “Traffic state estimation method with efficient data fusion based on the aw-rascle-zhang model,” in 2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC). IEEE, 2017, pp. 1–6.

[10] C. M. Tampire and L. Immers, “An extended kalman filter application for traffic state estimation using ctmm with implicit mode switching and dynamic parameters,” in 2007 IEEE Intelligent Transportation Systems Conference. IEEE, 2007, pp. 209–216.

[11] Y. Zhou, E. Chung, M. E. Cholette, and A. Bhaskar, “Real-time joint estimation of traffic states and parameters using cell transmission model and considering capacity drop,” in 2018 21st International Conference on Intelligent Transportation Systems (ITSC). IEEE, 2018, pp. 2797–2804.

[12] P. D. Hanlon and P. S. Maybeck, “Characterization of kalman filter residuals in the presence of mismodeling,” IEEE Transactions on Aerospace and Electronic systems, vol. 36, no. 1, pp. 114–131, 2000.

[13] R. F. Stengel, Optimal control and estimation. Courier Corporation, 1994.

[14] M. Hadziuzzaman and T. Z. Qiu, “Cell transmission model based variable speed limit control for freeways,” Canadian Journal of Civil Engineering, vol. 40, no. 1, pp. 46–56, 2013.

[15] H.-Y. Jin and W.-L. Jin, “Control of a lane-drop bottleneck through variable speed limits,” Transportation Research Part C: Emerging Technologies, vol. 58, pp. 568–584, 2015.

[16] D. Simon, Optimal state estimation: Kalman, H infinity, and nonlinear approaches. John Wiley & Sons, 2006.

[17] D. C. Montgomery, Introduction to statistical quality control. John Wiley & Sons, 2007.

[18] J. M. Lucas and R. B. Crosse, “Fast initial response for cusum quality-control schemes: give your cusum a head start,” Technometrics, vol. 24, no. 3, pp. 199–205, 1982.

[19] B. Barratt, R. Atkinson, H. R. Anderson, S. Beevers, F. Kelly, I. Midway, and P. Wilkinson, “Investigation into the use of the cusum technique in identifying changes in mean air pollution levels following introduction of a traffic management scheme,” Atmospheric Environment, vol. 41, no. 8, pp. 1784–1791, 2007.