Quark mass dependence of the QCD temperature transition in magnetic fields

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December 17, 2013

Abstract

Vacuum energy of quarks, \( \varepsilon_{\text{vac}}^{(q)} = \sum q m_q |\langle \bar{q}q \rangle| \) participates in the pressure balance at the temperature transition \( T_c \) and defines the dependence of \( T_c \) on \( m_q \). We first check this dependence in absence of magnetic fields \( eB \) vs known lattice data, and then take into account the known strong dependence of the quark condensate on \( eB \). The resulting function \( T_c(eB,m_q) \) is valid for all \( eB, m_q < \sqrt{\sigma} \) and explains the corresponding lattice data.

1 Introduction

The QCD matter is believed to undergo a temperature transition from the low temperature confining state to a deconfined state of quarks and gluons. On the theoretical side the most detailed information on this transition was obtained in numerical lattice studies, see \[ \[ \text{[1, 2, 3]} \] \] for reviews. On the experimental side only indirect data on this transition in heavy ion collisions are available \[ \[ \text{[4, 5]} \] \], however the full theory is highly needed both for the experiment and for astrophysical applications, e.g. in the study of neutron stars, see e.g. \[ \[ \text{[6]} \] \]. Analytic studies of the QCD temperature transition still predominantly based on models, which can describe partial features of the transition \[ \[ \text{[7, 8, 9]} \] \], but the full analytic theory is still lacking.
In 1992 one of the authors has suggested a simple but internally consistent mechanism of the QCD temperature transition \[10\], based on a general non-perturbative approach in QCD, called the Field Correlator Method (FCM) which was formulated for the temperature theory in \[11, 12, 13\].

In FCM both perturbative and nonperturbative (np) dynamics are given by field correlators, which can be computed within the method itself \[14\], or taken from lattice data \[15\]. Moreover, the total thermodynamic potential (free energy) of a state includes the vacuum energy, which can be different in the confining and nonconfining states.

It was exploited in \[10\], that the vacuum energy of the confined state contains gluon vacuum energy \(\varepsilon_{\text{vac}}^g\equiv\varepsilon_{\text{vac}}^g(\text{mag}) + \varepsilon_{\text{vac}}^g(\text{el})\), with the color-magnetic \(\varepsilon_{\text{vac}}^g(\text{mag})\) and colorelectric \(\varepsilon_{\text{vac}}^g(\text{el})\) parts, whereas in the deconfined state the (confining) colorelectric vacuum fields \(\varepsilon_{\text{vac}}^g(\text{el})\) are absent. In this way the transition temperature \(T_c\) was calculated from the equality of the confined state pressure \(P_I\) and the deconfined one \(P_{II}\), \(P_I = P_{II}\), at \(T = T_c\), where

\[
P_I = |\varepsilon_{\text{vac}}^g(\text{el}) + \varepsilon_{\text{vac}}^g(\text{mag})| + P_{\text{hadr}}(T),
\]

\[
P_{II} = |\varepsilon_{\text{vac}}^g(\text{mag})| + P_q(T) + P_g(T).
\]

Taking for \(|\varepsilon_{\text{vac}}^g|\) the standard values of the gluon condensate \[16, 17\] and assuming, that \(|\varepsilon_{\text{vac}}^g(\text{mag})|\) is equal to one half of the total, as it is for zero temperature, and neglecting in the first approximation \(P_{\text{hadr}}(T)\), one obtains reasonable values of the transition temperature \(T_c\) for different values of the number of flavors \(n_f\) as it is shown in Table I, the upper part, in comparison with available lattice data. Note, that in this calculation the input parameters are \(|\varepsilon_{\text{vac}}^g| = 0.006 \text{ GeV}^4\) which is a basic parameter in QCD and the nonperturbative interaction \(V_1(r,T)\), generating Polyakov line average, \(L_f = \exp\left(-\frac{V_1(\infty,T)}{2g^2}\right)\), which is calculated from the field correlator \[11, 12, 13\]. The actual parameter, entering Polyakov loops is the same, as in \[13\], \(V_1(\infty,T_c) \approx 0.5 \text{ GeV}\).

These calculations, however, have been done neglecting the quark vacuum energy,

\[
|\varepsilon_{\text{vac}}^g| = \sum_{i=1}^{n_f} m_i |\langle q_i \bar{q}_i \rangle|,
\]

which is possible for zero quark masses, and can be a good approximation for \(n_f = 2\), but not for the \((2+1)\) case, where \(m_s(2 \text{ GeV}) \approx 0.1 \text{ GeV}\).
Table 1: Transition temperature $T_c$ for massless quarks, $n_f = 0, 2, 3$ (the upper part), and for different nonzero $m_q$ and $n_f$ (the lower part) in comparison with lattice data.

| $n_f$ | $m_u$, MeV | $m_d$, MeV | $m_s$, MeV | $T_c$, MeV | $T_c$ (lat), MeV |
|-------|------------|------------|------------|------------|-----------------|
| 0     | -          | -          | -          | 268        | 276 [18]        |
| 2     | 0          | 0          | -          | 188        |                 |
| 3     | 0          | 0          | 0          | 174        |                 |
| 2     | 3          | 5          | -          | 189        | 195-213 [19]    |
| 2+1   | 3          | 5          | 100        | 182        | 175 [20, 21]    |
| 3     | 100        | 100        | 100        | 195        | 205 [26]        |

Thus for a realistic case the quark vacuum energy (3) should be included in $P_I, P_{II}$, with the resulting difference $\Delta \varepsilon_{\text{vac}}^{(q)} = \varepsilon_{\text{vac}}^{(q)(I)} - \varepsilon_{\text{vac}}^{(q)(II)} \equiv \sum_q Z_q m_q \langle \bar{q}q \rangle$.

Here $Z_q$ is the factor, which takes into account the nonvanishing of $|\langle \bar{q}q \rangle|$ in the deconfined region as a result of a slower decrease of $|\langle \bar{q}q \rangle|$ with temperature due to the finite mass $m_q$ and a finite limiting value due to the finite mass $m_q$. Indeed, as it was found in the lattice study of the $n_f = 2 + 1$ QCD [20, 21] the transition temperature, obtained from strange quark susceptibility is 10-15 MeV higher than from the light quarks.

In what follows we shall exploit the effective quark condensate $Z_q \langle \bar{q}q \rangle \equiv \langle \bar{q}q \rangle_{\text{eff}}$ to distinguish from the standard quark condensate $\langle \bar{q}q \rangle_{\text{st}} = (0.27 \text{ GeV})^3$. We shall omit below the subscript $\text{eff}$, keeping the notation $\langle \bar{q}q \rangle_{\text{st}}$ for the standard condensate of $(0.27 \text{ GeV})^3$.

In principle one can check, whether this procedure of the $\varepsilon_{\text{vac}}^{(q)}$ inclusion is correct, calculating the quark mass dependence of $T_c(m_q)$ for zero m.f. and comparing it with the lattice data [20, 21, 22]. As will be shown, indeed in both cases $T_c(m_q)$ grows with $m_q$, and the concrete agreement is obtained for reasonable values of gluon and quark condensates, $G_2 = 0.006 \text{GeV}^4$, $|\langle \bar{q}q \rangle| = (0.13 \text{ GeV})^3$.

One important property of the vacuum quark energy is that the quark condensate $\langle \bar{q}q \rangle$ grows with the increasing magnetic field (m.f.) when a constant magnetic field $B$ is imposed in the system, see [23] for lattice data and [24] for analytic studies. It was shown recently in the accurate lattice studies [23, 25, 26], that $T_c(B)$ is decreasing with $B$, and the same result was ob-
tained in [27, 28] and in our latest work [29], where $\epsilon^{(q)}_{\text{vac}}$ was not taken into account, and $T_c(B)$ was slowly tending to zero at large $B$.

In the present paper, after checking the $T_c(m_q)$ dependence for zero m.f., but with $\epsilon^{(q)}_{\text{vac}}$ taken into account, we calculate $T_c(B)$ for the $(2 + 1)$ case with physical quark masses and find that $T_c(B)$ is again decreasing with growing $B$, but is tending to a constant limit at large $B$ when a reasonable (observed in $N_c = 3$) dependence of $|\langle \bar{q}q \rangle(B)|$ is accounted for. We also observe a quite different behavior of $T_c(B)$ in another case, when $|\langle \bar{q}q \rangle(B)|$ is growing faster, than linearly, as it happens in $SU(2)$, $n_f = 4$ lattice data [30].

The paper is organized as follows. In the next section we assemble together all formalism necessary to calculate transition temperature for nonzero quark masses and m.f. $B$. In section 3 we calculate $T_c(m_q)$ and compare with available lattice data, fixing in this way the starting ($B = 0$) quark vacuum energy.

In section 4 the full derivation of $T_c(m_q, B)$ is given for arbitrary m.f. and results of calculations are compared with lattice data. Discussion of results and prospectives are given in the concluding section.

2 General formalism

We are basing the contents of this section on the results of [10, 11, 12, 13], adding to that the contribution of the quark vacuum energy $\epsilon^{(q)}_{\text{vac}}$. Thus in the confined state one has

$$P_f = |\epsilon^{(q)}_{\text{vac}}| + |\epsilon^{(g)}_{\text{vac}}| + P_{\text{hadr}}(T)$$

with

$$\epsilon^{(q)}_{\text{vac}} = \frac{\beta(\alpha_s)}{8\pi} \langle (F_{\mu\nu}^a)^2 \rangle \cong - \left( \frac{11}{3} N_c - \frac{2}{3} n_f \right) \frac{\alpha_s}{32\pi} \langle (F_{\mu\nu}^a)^2 \rangle \equiv$$

$$\equiv - \left( \frac{11}{3} N_c - \frac{2}{3} n_f \right) \frac{G_2}{32} \equiv \epsilon^{(q)}_{\text{vac}}(el) + \epsilon^{(q)}_{\text{vac}}(mag),$$

where $G_2 = \frac{2\alpha_s}{\pi} \langle (E_i^a)^2 + (H_i^a)^2 \rangle$, and $\epsilon^{(q)}_{\text{vac}}$ given in [13], while $P_{\text{hadr}}$ in absence of m.f. can be approximated by the free hadron gas expression,

$$P_{\text{hadr}}(T) = \mp \sum_i \frac{T d_i}{2\pi^2} \int_0^{\infty} dk k^2 \ln(1 \mp e^{(\mu - \epsilon_i)/T}),$$

4
with degeneracy factors $d_i$, and hadron energies $\varepsilon_i = \sqrt{k_i^2 + m_i^2}$. The minus and plus signs refer to mesons and baryons respectively.

Note, that Eq. (6) does not take into account high density and interaction corrections, as well as the decay width and off-shell effects.

In the deconfined state $\langle (E^a_i)^2 \rangle = 0$, while the vector color electric interaction $V_1$ produces fundamental Polyakov loop \[31\ 32\], $L_f = \exp \left(-\frac{V_1(\infty,T)}{2T}\right)$, hence the pressure is as in (2) with \[11\ 12\]

$$\frac{1}{V_3 T^4} P_q(T) = \frac{4N_c n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} \phi_q^{(n)} L_f^n \cosh \frac{\mu n}{T} \equiv p_q, \quad (7)$$

where

$$\phi_q^{(n)} = \frac{n^2 m_q^2}{2T^2} K_2 \left( \frac{m_q n}{T} \right). \quad (8)$$

For gluons the pressure contains adjoint Polyakov loops $L_{adj}^n, L_{adj} = L_f^{3/4} = \exp \left(-\frac{g V_1(\infty)}{8T}\right)$

$$\frac{1}{V_3 T^4} P_g(T) = \frac{2(N_c^2 - 1)}{\pi^2} \sum_{n=1}^{\infty} \frac{L_{adj}^n}{n^4} \equiv p_{gl}. \quad (9)$$

As a result the transition temperature can be obtained from the equality $P_I = P_{II}$ in the form

$$T_c(\mu) = \left( \frac{\Delta |\varepsilon_{vac}| + \Delta \varepsilon_{hadr}(T_c)}{p_q + p_g} \right)^{1/4} \quad (10)$$

and

$$\Delta |\varepsilon_{vac}| = \Delta |\varepsilon^{(q)}_{vac}| + |\varepsilon^{(g)}_{vac}(el)|. \quad (11)$$

We now come to the exact form of the $g\bar{q}$ interaction $V_1(r)$ produced by $np$ nonconfining correlator $D_1^F(x), \ [11\ 12\ 31]$, which gives a one-quark contribution $V_1(\infty, T)$,

$$V_1(r, T) = \int_0^{1/T} d\nu (1 - \nu T) \int_0^r \xi d\xi D^F(\sqrt{\xi^2 + \nu^2}). \quad (12)$$

It was argued in \[12\ 31\], that $V_1(\infty, T)$ at $T$ around $T_c$ can be approximated by the formula

$$V_1(\infty, T) = \frac{0.175 \text{ GeV}}{1.35 \left( \frac{T}{T_c} \right) - 1}, \quad V_1(\infty, T_c) \approx 0.5 \text{ GeV.} \quad (13)$$
The form (13) agrees approximately with the known lattice data \[36, 37\]. In what follows we shall follow the reasoning of our previous studies \[12, 31\] and use $V_1(\infty) \equiv V_1(\infty, T_c) \approx 0.5$ GeV in the fundamental Polyakov line $L_f = \exp \left( -\frac{V_1(\infty)}{2T} \right)$ and the adjoint line $L_{adj} = \exp \left( -\frac{9V_1(\infty)}{8T} \right)$.

At this point one must take into account, that the interaction $V_1(r, T)$ is able to bind the $q\bar{q}$ pairs into bound states, see \[3\] for a review and \[31, 32\] for analytic studies. Moreover, as shown on the lattice in \[33, 34, 35\], the $Q\bar{Q}$ interaction in the unquenched case changes smoothly with temperature around $T_c$, where the confining interaction $V_{conf}(R, T)$ is replaced by the nonconfining $V_1(R, T)$. Therefore one can introduce the pressure of the bound pair and triple terms $P_{\text{bound}}$ and assume that the difference $\Delta P_{\text{hadr}} \equiv P_{\text{hadr}}(I) - P_{\text{bound}}(II)$ is small quantity near $T_c$, which can be neglected in the first approximation. Note also, that in the quenched case this transition from $V_{conf}$ to $V_1$ has a different structure, see e.g. \[36\]. As it is, we are not yet able to explain why this smooth transition of $V_{conf}$ to $V_1$ happens in the unquenched case and how it leads to the resulting QCD temperature transition, (work in this direction is going on).

As was discussed in \[31, 32\] and known from the lattice data, see \[3\] for a review, the binding properties of $V_1(R, T)$ are concentrated in a narrow region of temperatures around $T_c$, while $V_1(\infty, T)$ gives a piece of selfenergy for each quark and antiquark, decreasing at large $T$. In this way the confined pairs and triples of quarks and antiquarks near $T \approx T_c$ go over into pairs and triples connected by the interaction $V_1(r, T)$ and isolated quarks and antiquarks with energies augmented by a constant piece $V_1(\infty, T)$. In some sense this transition is similar to the process of ionization of neutral gas at increasing temperature, which finally produces the ion-electron plasma in a smooth continuous way.

In what follows we shall consider $\Delta P_{\text{hadr}}(T_c)$ as a small term as compared with $|\Delta \varepsilon_{\text{vac}}|$ and shall disregard it in the first approximation. We now can proceed with calculation of $T_c$ and we start with the case $\mu = 0, \quad B = 0$, when one can retain the first terms with $n = 1$ in the sums in \[7\] and \[9\].
3 The quark mass dependence of the transition temperature without magnetic fields

To check the influence of $\varepsilon^{(q)}_{\text{vac}} \equiv \sum_q \langle \bar{q}q \rangle m_q$ we first take it into account in the case of zero m.f., comparing $T_c(m_q, eB = 0)$ and $T_c(0, eB = 0)$. Using the solution of (10) for $T_c$ in two cases, when the $|\Delta \varepsilon_{\text{vac}}|$ is $\frac{1}{2} |\varepsilon^{(g)}_{\text{vac}}|$ and $\Delta \varepsilon_{\text{vac}} = \frac{1}{2} |\varepsilon^{(q)}_{\text{vac}}| + |\Delta \varepsilon^{(q)}_{\text{vac}}|$, the corresponding values of $T_c$ are denoted as $T_c^{(0)}$ and $T_c^{(q)}$, one obtains (using Eq. (14) from [12])

$$T_c^{(q)} = \frac{1}{2} \tau^{(q)} \left( 1 + \sqrt{1 + \frac{\kappa}{\tau^{(q)}}} \right) \left( 1 + \frac{m_q^2}{16(T_0^{(q)})^2} \right),$$

(14)

$$T_c^{(0)} = \frac{1}{2} \tau^{(0)} \left( 1 + \sqrt{1 + \frac{\kappa}{\tau^{(0)}}} \right),$$

(15)

where $\kappa = \frac{1}{2} V_1(\infty)$, $\tau^{(0)} = \left( \frac{(11-\frac{4}{3} n_f) \pi^2 G_2}{64 \cdot 12 n_f} \right)^{1/4}$ and

$$\tau^{(q)} = \left[ \frac{\pi^2}{12 n_f} \left( \frac{(11 - \frac{2}{3} n_f) G_2}{64} + \sum_q m_q |\langle \bar{q}q \rangle| \right) \right]^{1/4}. \quad (16)$$

For $n_f = 3$ and $G_2 = 0.006$ GeV$^4$ one obtains $\tau^{(0)} = 123$ MeV and $T_c^{(0)} = 168$ MeV.

Now, taking $m_q = 0.1$ GeV for the $s$ quark and the contribution of $u, d$ quarks with masses $\approx 3$ and $5$ MeV respectively, one obtains $\tau^{(q)} = 135$ MeV and $T_c^{(q)} = 181$ MeV for $|\langle \bar{s}s \rangle| \approx (0.16 \text{ GeV})^3$ and $T_c^{(q)} = 192$ MeV for $|\langle \bar{s}s \rangle| = (0.2 \text{ GeV})^3$. This $10 \div 15\%$ increase of $T_c^{(q)}$ as compared with $T_c^{(0)}$ is in line with lattice calculations of the same quantities in [22, 20, 21].

To check quantitatively the value of the effective condensate $|\langle \bar{q}q \rangle|$, or equivalently of the factor $Z_q = \frac{|\langle \bar{q}q \rangle|}{(0.27 \text{ GeV})^3}$, one can use the lattice numerical data of [22] for $T_c(m_q)$ in the $n_f = 3$ case, presented in Table 2 and compare with our predictions for $|\langle \bar{q}q \rangle| = (0.13 \text{ GeV})^3$. One can see a good agreement, which enables as choose this value of $|\langle \bar{q}q \rangle|$ for our calculations for nonzero m.f. in the next section. Note also, that in the region $m_q \geq \sqrt{\sigma} \sim 0.4$ GeV the quark condensate may have a nontrivial dependence on $m_q$, which influences the resulting values of $T_c(m_q)$, as can be seen in Table 2.
Table 2: Quark mass dependence of transition temperature from Eq. \((\ref{14})\) with \(|\langle \bar{q}q \rangle| = (0.13 \text{ GeV})^2\) in comparison with the lattice data from \([22]\).

| \(m_q, \text{ MeV} \) | 25  | 50  | 100 | 200 | 400 | 600 | 1000 |
|-------------------|-----|-----|-----|-----|-----|-----|------|
| \(T_{c \text{ (lat)}, \text{ MeV}} \) | 180 | 192 | 199 | 213 | 243 | 252 | 270  |
| \(T_{c, \text{ MeV}} \)           | 179 | 185 | 195 | 213 | 245 | 273 | 320  |

In a similar way one can include \(\varepsilon^{(q)}_{\text{vac}}\) in the transition equation for nonzero chemical potential \(\mu\), neglecting the difference \(\Delta P_{\text{hadr}}(T_c)\) as before, one has an equality \((\ref{10})\), which can be rewritten as

\[
T_c(\mu) = \left( \frac{1}{p_q(\mu) + p_g} \left| \varepsilon^{(g)}_{\text{vac}} \right| + \left| \varepsilon^{(q)}_{\text{vac}} \right| \right)^{1/4}, \tag{17}
\]

where \(p_q(\mu)\) according to \([12]\) is

\[
p_q(\mu) = \frac{nf}{\pi^2} \left[ \Phi_\nu \left( \frac{\mu - V_i(\infty)}{2T} \right) + \Phi_\nu \left( \frac{-\mu + V_i(\infty)}{2T} \right) \right], \tag{18}
\]

where \(\nu = m_q/T\) and

\[
\Phi_\nu(a) = \int_0^\infty \frac{z^4dz}{\sqrt{z^2 + \nu^2}} \frac{1}{e^{\sqrt{z^2 + \nu^2} - a} + 1}, \tag{19}
\]

and \(p_{gl}\) to lowest order is independent of \(\mu\) and is given by

\[
p_{gl} = \frac{2(N_c^2 - 1)}{\pi^2} \sum_{n=1}^\infty \frac{L_{adj}}{n^4}. \tag{20}
\]

We are now in a position to turn on the external magnetic field.

4 Transition temperature with the quark vacuum energy in external magnetic field

In principle the magnetic field influences both phases of matter: 1) the quark vacuum energy \(\varepsilon^{(q)}_{\text{vac}}\) via the quark condensate \(\langle \bar{q}q \rangle(B)\), 2) the gluon condensate via internal quark pair creation, 3) hadron gas pressure, 4) quark gas pressure.
We shall disregard as before the hadron gas contribution $\Delta P_{\text{hadr}}$ and start with the quark condensate. In this section we shall exploit the same mechanism of the temperature transition with the full (quark plus gluon) vacuum energy, as in the previous section, but now in the external constant magnetic field. To this end one can write the basic equilibrium equation

$$\left| \frac{1}{2} \varepsilon^{(g)}_{\text{vac}} + \varepsilon^{(q)}_{\text{vac}}(B) \right| = P_g^{(0)} + \sum_q P_q(B), \quad (21)$$

where $P_g^{(0)} \equiv p_g T^4$, and use the previously found $P_q(B)$ from [29], valid for all values of $B$ and $m_q$, $e_q \equiv |e_q|$.

$$P_q(B) = \frac{N_c e_q B T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} L_f^n \left\{ m_q K_1 \left( \frac{nm_q T}{T} \right) + \right.$$  

$$\left. + \frac{2T}{n} \left( \frac{e_q B + m_q^2}{e_q B} \right) K_2 \left( \frac{n T}{T} \sqrt{e_q B + m_q^2} \right) - \frac{ne_q B}{12T} K_0 \left( \frac{n T}{T} \sqrt{m_q^2 + e_q B} \right) \right\}. \quad (22)$$

For large $e_q B$ one can write $\bar{P}_q = \sum_q P_q(B)$ in the form

$$\bar{P}_q(B) \approx \frac{N_c B T L_f}{\pi^2} \sum_{q=1}^{n_f} e_q m_q K_1 \left( \frac{m_q T}{T} \right). \quad (23)$$

We take into account, that $|\langle \bar{s}s \rangle(B)|$ grows with $eB$ in the same way as $|\langle \bar{d}d \rangle(B)|$, while for $|\langle \bar{u}u \rangle|$ the charge $e_u$ is twice as big, so that according to [21], one can write

$$|\langle \bar{q}q \rangle(B)| \equiv |\langle \bar{q}q \rangle(0)| \sqrt{1 + \left( \frac{e_q B}{M_q^2} \right)^2}, M_q^2 \approx 0.27 \text{ GeV}^2. \quad (24)$$

As a result the transition temperature can be found from the relation

$$\frac{|\langle \bar{q}q \rangle(0)|}{M_q^2} \sum_q e_q m_q = \frac{N_c T L_f}{\pi^2} \sum_q e_q m_q K_1 \left( \frac{m_q}{T} \right), \quad (25)$$

which yields $T_c(\infty) \approx 100 \text{ MeV}$. In this way the asymptotic behavior of $T_c(B)$ becomes more moderate and its negative slope decreases at large $eB$, again...
in agreement with lattice data of [25]. The resulting behavior of $T_c(B)$ with the account of quark vacuum energy is shown in Fig. 1 for different values of $|\langle \bar{q}q \rangle|$. Another illustration of quark mass influence is given in Fig. 2, where a comparison is presented of our results for the solution of Eq. (21) and the corresponding lattice data from [23, 25] for two cases: $n_f = 2 + 1$ with physical strange and light quark masses, and $n_f = 3$, where all quarks have the masses of the strange quark (lattice data from [26] are given by two points at $eB = 0$ and 0.8 GeV$^2$). One can see a good ($\sim 10\%$) agreement between two sets of results. One can see from Fig. 2, that the increasing role of $\varepsilon_{\text{vac}}(g)$ leads to the flattening of the resulting dependence of $T_c(eB)$.

Note however, that we have neglected the possible dependence of $|\varepsilon_{\text{vac}}(g)|$ on m.f., which should appear in higher orders of $\alpha_s$. One can expect, that $|\varepsilon_{\text{vac}}(B)|$ should decrease for growing $eB$ due to the fact, that $\alpha_s(eB)$ decreases, since the quark loop contribution in the denominator of $\alpha_s(eB)$ in m.f. is growing with $eB$, as shown in [38].

A decreasing behavior of $\varepsilon_{\text{vac}}(g)$ in m.f. is obtained within the framework of chiral perturbation theory in [39]. Therefore one can expect, that the net effect of m.f. on the $|\Delta\varepsilon_{\text{vac}}(B)|$ would be a more mild linear growth, which

Figure 1: Transition temperature $T_c(m_q, eB, |\langle \bar{q}q \rangle|_{\text{eff}})$ in GeV as a function of $eB$ for different values of the effective quark condensate, defined at zero m.f. Lattice data of [24] are shown by dotted lines.
Figure 2: Transition temperature $T_c(eB)$ in GeV in two cases: $n_f = 2 + 1$ (solid line with points) and $n_f = 3$ with $m_q = 0.1$ GeV and $|\langle\bar{q}q\rangle| = (0.17$ GeV)$^3$ (horizontal solid line). Lattice data for $n_f = 2 + 1$ from [25] – the area between dotted lines, and for $n_f = 3$ – two points at $eB = 0$ and 0.8 GeV² from [26].

implies the partial cancellation of the effect of the quark condensate.

5 Discussion of results and prospectives

We have taken into account in the present paper the effect of the quark vacuum energy on transition temperature both with or without m.f. The case of no m.f. helps us to fix the starting value of the strange quark condensate. Its later growth with m.f. is predicted by the theory, based on the chiral Lagrangian, augmented by the quark degrees of freedom, as well as recent lattice data. This theory was developed before in [24] and the resulting behavior of the quark condensate was in good agreement with recent accurate lattice data in [23].

Comparing our present calculations with the previous ones in [29], one can see an important role of the strange quark vacuum energy (s.q.v.e.) $m_s|\langle s\bar{s}\rangle|$ at zero m.f. and even larger role for growing m.f. One can see in Fig. 1, that not only the asymptotics of $T_c(B)$ is changed by s.q.v.e. but also the slope in
the region $eB \leq 1$ GeV$^2$ becomes more flat, in better agreement with lattice data from [25].

One more consequence of the s.q.v.e. inclusion is that the phenomenon of temperature transitions is now strongly dependent on the admixture of strangeness in the matter, which can be important for the physics of neutron stars. As a general outcome, one can conclude, that the main features of the quark mass dependence of the transition temperature are accounted for by the vacuum quark energy $\varepsilon_{\text{vac}}^{(q)}$, and this holds both with or without m.f.

The authors are grateful for useful discussions to M.A. Andreichikov and B.O. Kerbikov.

References

[1] Z.Fodor and S.D.Katz, arXiv:0908.3341 [hep-ph]; S.Borsányi, Z.Fodor, C.Hoelbling et al.; arXiv:1309.5258 [hep-lat].

[2] O.Philipsen, Prog. Part. Nucl. Phys. 70, 55 (2013) [arXiv:1207.5999 [hep-lat]].

[3] P.Petreczky, PoS Confinement X (2012) 028 [arXiv:1301.6188 [hep-lat]]; J.Phys. G39, 093002 (2012), arXiv:1203.5320 [hep-lat].

[4] P.Jacobs and X.-N.Wang, Matter in extremis: ultrarelativistic nuclear collisions at RHIC, Prog. Part. Nucl. Phys. 54, 443 (2005).

[5] K.Fukushima and T.Hatsuda, Rept. Prog. Phys. 74, 014001(2011).

[6] P.Haensel, A.Y.Potekhin and D.G.Yakovlev, Neutron stars I, Equation of State and structure, Astrophysics and Space science Library, v. 326, Springer, New York, 2007.

[7] J.Kapusta and C.Gale, Finite-temperature Field Theory: Principles and Applications, Cambridge Univ. Press, 2006.

[8] T.Hatsuda and T.Kunihiro, Phys. Rept. 247, 221 (1994).

[9] K.Rajagopal and F.Wilczek, At the Frontier of Particle Physics, v3 (World Scientific, 2001).
[10] Yu.A.Simonov, JETP Lett. 55, 605 (1992); Phys. At. Nucl. 58, 309 (1995), hep-ph/9311216

[11] Yu.A.Simonov, Ann. Phys. (NY) 323, 783 (2008);
E.V.Komarov and Yu.A.Simonov, Ann. Phys. (NY) 323, 1230 (2008).

[12] Yu.A.Simonov and M.A. Trusov, Phys. Lett. B 650, 36 (2007).

[13] A.V.Nefediev, Yu.A.Simonov and M.A. Trusov, Int. Mod. Phys. E 8, 549 (2009).

[14] Yu.A.Simonov, Phys. At. Nucl. 69, 528 (2006), hep-ph/0501182;
Yu.A.Simonov and V.I.Shevchenko, Adv. High En. Phys. 2009, 873061 (2009).

[15] A. Di Giacomo and H.Panagopoulos, Phys. Lett. B 285, 133 (1992);
M.D’Elia, A. Di Giacomo, and E. Meggiolaro, Phys. lett. B 408, 315 (1997);
A. Di Giacomo, E. Meggiolaro, and H.Panagopoulos, Nucl. Phys. B 483, 371 (1997);
G.S.Bali, N.Brambilla and A.Vairo, Phys. Lett. B421, 265 (1998);
M.D’Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Rev. D 67, 114504 (2003).

[16] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. B 147, 385, 448 (1979).

[17] B.L.Ioffe and K.Zyablyuk, Eur. Phys. J. C 27, 229 (2003).

[18] A. Ali Khan et al., Nucl.Phys.Proc.Suppl. 83, 384 (2000).

[19] B.Brandt et al., arXiv:1310.8326.

[20] Y.Aoki et al., Phys. Lett B 643, 46 (2006).

[21] S.Borsanyi et al., JETP 1009, 073 (2010).

[22] F.Karsch, E.Laermann and A.Peikert, Nucl. Phys. B 605, 579 (2001).

[23] G.S.Bali, F.Bruckmann, G.Endrödi, Z.Fodor, S.D.Katz and A.Schäfer, Phys. Rev. D 86, 071502 (2012), arXiv:1206.4205
[24] Yu.A.Simonov, arXiv:1212.3118 [hep-ph].

[25] G.S.Bali, F.Bruckmann, G.Endrödi, et al., JHEP 1202, 044 (2012).

[26] G.S.Bali, F.Bruckmann, M.Constantinou et al., PoS ConfinementX 198 (2012), arXiv:1301.5826

[27] N.O.Agasian and S.M.Fedorov, Phys. Lett. B 663, 445 (2008).

[28] E.S.Fraga and A.J.Mizher, Nucl. Phys. A 820, 034016 (2011); E.S.Fraga and L.F.Palhares, Phys. Rev. D 86, 016008 (2012).

[29] V.D.Orlovsky and Yu.A.Simonov, arXiv: 1311.1087 [hep-ph].

[30] E.-M.Ilgenfritz, M.Müller-Preussker, B.Petersson and A.Schreiber, arXiv:1310.7876.

[31] Yu.A.Simonov, Phys. Lett. B 619, 293 (2005).

[32] A.Di Giacomo, E.Meggiolaro, Yu.A.Simonov, and A.I.Veselov, Phys. Atom. Nucl. 70, 908 (2007).

[33] O.Kaczmarek and F.Zantow, Phys. Rev. D 71, 114310 (2005), hep-lat/0503017.

[34] A.Bazavov and P.Petrezky, Nucl.Phys.A 904-905, 599c-602c (2013), arXiv: 1210.6314 [hep-lat].

[35] C.Allton, G.Aarts, A.Amato et al, arXiv: 1310.5135 [hep-lat].

[36] O.Kaczmarek, F.Karsch, E.Laermann and M.Lütgemeier, Phys.Rev. D 62, 034021 (2000), arXiv: hep-lat/9908010

[37] O.Kaczmarek, F.Karsch, P.Petrezky, and F.Zantow, Phys. Lett B 543, 41 (2002), hep-lat/0207002

[38] M.A.Andreichikov, V.D.Orlovsky and Yu.A.Simonov, Phys. Rev. Lett. 110, 162002 (2013), arXiv: 1211.6568 [hep-ph].

[39] N.O.Agasian and I.Shushpanov, Phys. Lett. B472, 143 (2000).