Competing interactions of spin and lattice in the Kondo lattice model

M. Gulacsi\textsuperscript{1}, A. Bussmann-Holder\textsuperscript{2} and A. R. Bishop\textsuperscript{3}

\textsuperscript{1} Department of Theoretical Physics, Institute of Advanced Studies
The Australian National University, Canberra, ACT 0200, Australia

\textsuperscript{2} Max-Planck-Institut für Festkörperforschung
Heisenbergstr. 1, 70569 Stuttgart, Germany

\textsuperscript{3} Theoretical Division,
Los Alamos National Laboratory
Los Alamos, NM 87545, U.S.A.

(Dated: 20 June 2003)

Abstract

The magnetic properties of a system of coexisting localized spins and conduction electrons are investigated within an extended version of the one dimensional Kondo lattice model in which effects stemming from the electron-lattice and on-site Coulomb interactions are explicitly included. After bosonizing the conduction electrons, it is observed that intrinsic inhomogeneities with the statistical scaling properties of a Griffiths phase appear, and determine the spin structure of the localized impurities. The appearance of the inhomogeneities is enhanced by appropriate phonons and acts destructively on the spin ordering. The inhomogeneities appear on well defined length scales, can be compared to the formation of intrinsic mesoscopic metastable patterns which are found in two-fluid systems.

PACS numbers: PACS No. 71.27.+a, 71.28.+d, 75.20.Hr
The interplay of spin, charge and lattice degrees of freedom has been investigated intensively in many transition metal oxides and especially in perovskite manganites, which have recently attracted new interest due to the discovery of colossal magnetoresistance (CMR). The initial understanding of the properties of manganites was based on the double-exchange (DE) mechanism within the Kondo lattice. However, the new experimental findings have revealed that this approach is incomplete and has to be extended to account for effects stemming from the lattice in order to understand the doping dependent phase diagram and the richness of phases that appear. In the following we model these complex systems within the Kondo lattice model (KLM), admitting for ferro- and antiferromagnetic couplings, and including explicitly the interaction with the lattice degrees of freedom.

The KLM considers the coupling between half-filled narrow band (localized $d$ or $f$) and conduction electrons. Even though studied intensively for the last two decades, the understanding of the KLM remains incomplete. Only in one dimension have numerical simulations and bosonization techniques been carried through which admit predictions about the phase diagram of the KLM. No investigations exist for the case where the KLM is extended to account for contributions stemming from the phonons, which is of special relevance to CMR materials. In particular, the small doping regime of these systems, which are ferromagnetic at low temperatures, seems to be appropriate to be modeled within the KLM extended by interactions with the lattice. In the following we present bosonized solutions of the KLM where on-site Coulomb and phonon contributions are explicitly included. This “extended” KLM model allows spin-and magnetoelastic-polaron formation, which we believe are of major importance in understanding these complex materials.

The Hamiltonian of the KLM in the presence of on-site Coulomb interaction reads:

\[ H_{\text{KLM}} = -t \sum_{j,\sigma} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{h.c.}) + J \sum_j S_{d,j} \cdot S_{c,j} + U \sum_j n_{j,\uparrow} n_{j,\downarrow}, \]

where $t > 0$ is the conduction electron hopping integral, $S_{d,j} = \frac{1}{2} \sum_{\sigma,\sigma'} c_{d,j,\sigma}^\dagger \sigma_{\sigma,\sigma'} c_{d,j,\sigma'}$, $S_{c,j} = \frac{1}{2} \sum_{\sigma,\sigma'} c_{j,\sigma}^\dagger \sigma_{\sigma,\sigma'} c_{j,\sigma'}$ and $\sigma$ are the Pauli spin matrices. Fermi operators $c_{j,\sigma}, c_{j,\sigma}^\dagger$ with subscript $d$ refer to localized $d$-spins, while those not indexed refer to the conduction electrons. The on-site Coulomb repulsion is given by the Hubbard term proportional to $U$. In the CMR materials the localized states are represented by the threefold degenerate Mn $t_{2g}$ $d$-electrons with total spin $3/2$. However, for reasons of transparency, the localized
spin is approximated here by spin 1/2, since the properties of the model are qualitatively independent of the magnitude of the localized spins. In the following the the Kondo coupling $J$ is measured in units of the hopping $t$ and both cases, antiferromagnetic ($J > 0$) and ferromagnetic ($J < 0$) couplings, will be considered. The conduction band filling is given by $n = N_c/N < 1$, where $N$ is the number of lattice sites and $N_c$ is the number of conduction electrons. To be able to understand the wide range of properties of the CMR materials, we also allow for the number of impurity spins, $N_d$, to vary, in such a way that $N_d/N < 1$.

In principle, the electron-phonon coupling could be of either inter-site (Su-Schrieffer-Heeger (SSH) or on-site (Holstein) character. Since we found the SSH-coupling to be irrelevant to forward scattering processes, its influence will not be discussed in the following; only terms arising from the on-site couplings, i.e., $\sum_j \alpha q_j n_j$, with coupling constant $\alpha$ and displacement $q_j$ will be included. The bare lattice Hamiltonian is: $H_{\text{latt.}} = \sum_j (p_j^2/2M + Kq_j^2/2)$, where $p_j$ is $q_j$’s conjugate momenta, $K$ the harmonic coupling and $M$ the ionic mass.

The underlying bosonization scheme follows standard procedures by first decomposing the on-site operators into Dirac fields, $c_{\tau,x,\sigma} \approx \sum_\tau e^{ik_{\text{F}}x} \Psi_{\tau,\sigma}(x)$, where $k_{\text{F}} = \pi n/2$, with spinor components $\tau = \pm$ (+/- being the right/left movers) and $k_{\text{F}} = \pi n/2$. Next we bosonize the Dirac fields with $\Psi_{\tau,\sigma} = \exp(i\Phi_{\tau,\sigma})/\sqrt{2\pi\lambda}$, where $1/\lambda$ is the ultraviolet cutoff. For the scalar Bose fields, $\Phi_{\tau,\sigma}(x)$, and their conjugate momenta, $\Pi_{\tau,\sigma}(x)$, $\Phi_{\tau,\sigma}(x) = \int_{-\infty}^{x} dx' \Pi_{\tau,\sigma}(x')$, are used in standard Mandelstam representation by means of which a momentum cutoff via the Fourier transform is introduced $\Lambda(k) = \exp(-\lambda|k|/2)$. If the distance between the impurity spins is larger than $\lambda$, the electrons will behave as collective density fluctuations. Thus, the Fermi fields can be represented in terms of density operators which satisfy Bose commutation relations: $c_{\tau,x,\sigma} \approx \exp(i\tau k_{\text{F}}x) \exp i\{\theta_{\tau}(x) + \tau \phi_{\rho}(x) + \sigma[\theta_{\sigma}(x) + \tau \phi_{\sigma}(x)]\}/2$, where the Bose fields for $\nu = \rho, \sigma$ are defined by $\psi_\nu(x) = i(\pi/N) \sum_{k \neq 0} e^{ikx} \phi_{\nu}(k) \pm \nu_{-}(k) \Lambda(k)/k$, with $+$ corresponding to the number fields $\psi_\nu = \phi_{\nu}$ and $-$ to the current fields $\psi_\nu = \theta_{\nu}$. The charge (holon) and spin (spinon) number fluctuations are defined as $\rho_{\tau}(k) = \sum_\sigma \rho_{\tau,\sigma}(k)$, and $\sigma_{\tau}(k) = \sum_\sigma \sigma_{\tau,\sigma}(k)$. All rapidly oscillating terms originating from e.g. backscattering and umklapp processes are neglected, since they contribute only at exactly half filling.

The localized $d$ electrons can neither be bosonized nor Jordan-Wigner transformed since no direct interaction exists between them. Using the continuum approximation for the phonon contribution, two components are relevant: a small momentum part $\Phi_0(p)$ and a
rapidly oscillating term at $2k_F$, $\Phi_\pi(x)$, resulting from the splitting of the conduction band electrons into right and left movers. While the former contribution causes forward scattering and is best represented in momentum space, the latter one gives rise to backscattering and requires representation in real space. The transformed Hamiltonian thus becomes:

\[
H = H^{el} + H^{ph} + H^{el-ph} + \frac{J}{2\pi} \sum_j [\partial_x \phi_\sigma(j)] S_{d,j}^z + \frac{J}{4\pi\lambda} \sum_j \{\cos[\phi_\sigma(j)] + \cos[2k_Fj + \phi_\rho(j)]\} \left(e^{-i\theta_\sigma(j)} S_{d,j}^+ + \text{h.c.}\right) - \frac{J}{4\pi\lambda} \sum_j \sin[\phi_\sigma(j)] \sin[2k_Fj + \phi_\rho(j)] S_{d,j}^z.
\]

(2)

If holes are present in the array of $d$-spins, all terms proportional to $S$ are zero. The notations used in Eq. (2) are: the forward scattering Holstein electron-phonon coupling term $H^{el-ph} = (\alpha/\sqrt{M})(\sqrt{2}/N) \sum_p [\rho_+(-p) + \rho_-(p)] \Phi_0(p)$; the bare lattice contribution $H^{ph} = (1/2N) \sum_p [\Pi^2_0(p) + \omega_0^2 \Phi_0^2(p)] + \frac{1}{2} \int dx [\Pi^2_\pi(x) + \omega_\pi^2 \Phi_\pi^2(x)]$ with $\omega_0 = \omega_\pi = \sqrt{K/M}$; and the standard spinon-holon term $H^{el} = (v_\rho/4\pi) \sum_{j,\nu} \{\Pi^2_\nu(j) + [\partial_x \phi_\nu(j)]^2\}$ with velocities $v_{\rho/\sigma} = v_F[1 \pm U/\pi v_F \mp \alpha^2/\pi K v_F]^{1/2}$.

It is important to note that a renormalization of the spinon-holon velocities appears here due to the Hubbard and phonon terms which act oppositely on the corresponding velocities. While the Hubbard term leads to a localization of the spinons and an increased hopping of the holons, thus supporting a magnetic ground state, the phonons delocalize the spins, but localize the charges and act destructively on the magnetic properties. It is worth mentioning that the Hubbard term alone already suffices to establish two time scales for the holon-spinon dynamics, but an important renormalization of the critical properties of the system is achieved through the variable phonon coupling, which as will be shown below establishes the existence of a Griffiths phase. The competition between the Hubbard and the phonon term obviously vanishes for $U = \alpha^2/K$.

In the following effects arising from the localized spin $d$ impurities, double exchange (DE), the phonons and Hubbard interactions will be discussed in more detail. The localized spin $d$ impurities act via double exchange (DE) on the hopping electrons so as to preserve their spin when moving through the lattice in order to screen the localized spins which are in excess of the conduction electrons, i.e. $N > N_c$. This, in turn, leads to a tendency to align the localized spins and results in an additional screening energy for the conduction electrons.

In order to gain a more transparent understanding of this complicated interplay, the
model is investigated first for the case of two sites and one conduction electron next in a simple continuum approximation, and finally the full bosonized solution will be presented.

In the case of ferromagnetic coupling ($J < 0$) the ground state energy is $E_{0, J<0} = -|J|/4 - t$ with wave function $|ψ_{0, J<0}\rangle ≡ |ψ_{DE, J<0, z}\rangle = |↑_z↑_z, ↑_z 0\rangle + |↓_z 0, ↑_z↑_z\rangle$, where $↑_z$ and $↓_z$ refers to the $z$ component of the impurity and conduction electron spins, respectively. Ferromagnetism arises here via an Ising type coupling, which allows for description of the ground state within a simple semiclassical approximation. For $J > 0$ the situation is completely changed due to the singlet formation of local and conduction electron spins. This causes a mixing of the total spin and an enhancement of the Hilbert space, where now 16 elements have to be considered. The ground state energy is given by $E_{0, J>0} = -J/4 - \sqrt{J^2 + 2Jt + 4t^2}/2$ with wave functions $|ψ_{0, J>0}\rangle ∝ |ψ_{KS, J>0}\rangle + [1/(J/4 - E_{0, J>0})] \{ |↑_z↓_z, ↑_z 0\rangle + |↑_z 0, ↑_z↑_z\rangle - |↑_z↑_z, ↓_z 0\rangle - |↓_z 0, ↑_z↑_z\rangle \}$, where the Kondo singlet $|ψ_{KS, J>0}\rangle$ states are $|↑_z↓_z, ↑_z 0\rangle - |↓_z↑_z, ↑_z 0\rangle + |↑_z 0, ↑_z↑_z\rangle - |↓_z 0, ↓_z↑_z\rangle$. $|ψ_{0, J>0}\rangle$ involves six basis elements (the degeneracy is partially lifted by conduction electron hopping) and hence falls outside the four dimensional space needed to establish DE for $J < 0$. In order to invoke DE as well, all three spin directions, $x$, $y$, and $z$, have to be considered: $|ψ_{0, J>0}\rangle ∝ [1 - 1/(J/4 - E_{0, J>0})] |ψ_{KS, J>0}\rangle + [1/(J/4 - E_{0, J>0})] \{ |ψ_{DE, J>0, x}\rangle + |ψ_{DE, J>0, y}\rangle + |ψ_{DE, J>0, z}\rangle \}$, where $|ψ_{DE, J>0, α=x, y, z}\rangle = \{ |↑_α↓_α, ↑_α 0\rangle + |↑_α 0, ↑_α↑_α\rangle + |↓_α↑_α, ↓_α 0\rangle + |↓_α 0, ↓_α↑_α\rangle \}$, and $|ψ_{DE, J>0, α=x, y, z}\rangle = |↑_z↑_z, ↑_z 0\rangle + |↑_z 0, ↑_z↑_z\rangle$. In spite of this extra complication, it is apparent from the above that in both cases, $J < 0$ and $J > 0$, spin polarons are formed.

Going beyond the two site approximation, spin polaron formation can be derived directly from the KLM Hamiltonian, which can be written as: $H ≈ -t \sum_i (c_i^{\dagger}c_{i+1, \sigma} + h.c.) + J/2 \sum_i (n_{i, \uparrow} - n_{i, \downarrow}) S_{i, \uparrow}^z$. Here spin-flip interactions are neglected, since these require a much higher energy and are consequently unlikely to be of importance to our results. This simplified Hamiltonian, as compared to Hamiltonian Eq. (1), can be solved when the electronic wave functions are treated within the continuum approximation and in the limit $N_c/N ≪ 1$, a case which is relevant to small doping concentrations in CMR materials. The electronic wave functions, $ψ_σ(x)$, satisfy a standard nonlinear Schrödinger equation: $\partial_x^2 ψ_σ(x) + (Jm_{el}/2)|ψ_σ(x)|^2ψ_σ(x) = 2m_{el}Eψ_σ(x)$ ($m_{el}$ being the bare electron mass) with soliton solutions $ψ_σ(x) \propto e^{ix}sech(x\sqrt{Jm_{el}/4})$.

These soliton solutions correspond to spin domain walls of finite size (kink-antikink pairs) and lead to a gain in electronic energy of $-σ$ for antiferromagnetic coupling, and of $+σ$ for
the ferromagnetic case. Physically the solutions resemble the dressing of the electron by a finite range of parallel (antiparallel) local spins and consequently represent polaronic type objects. From the previous considerations it can also be concluded that, when including the interactions with the phonons, the tendency towards charge localization is enhanced and increases this polaronic effect. Since the lattice also experiences a renormalization due to the coupling to the electronic degrees of freedom, substantial ionic displacement patterns will develop and the formation of magnetoelastic polarons takes place. Similar results are obtained by decoupling electronic and phononic degrees of freedom through a homogeneous Lang-Firsov transformation, where the localization stems from band narrowing. The localization width (polaron radius) is characterized by a length scale proportional to $1/\sqrt{J}$. This new length scale differs from the free conduction electrons mean free path and gives rise to competing time scales: slow motion of the polaronic carriers and fast motion of the free electrons thus inferring dynamics of two types of particles and a close analogy to a two fluid scenario. Since the polarons are in general randomly distributed within the local spin array, these states can be viewed as intrinsic inhomogeneities involving spin fluctuations and short-range spin correlations. In addition these new slow dynamics will exhibit a peak in the spin structure factor at $2k_F - \pi$ instead of the simple $2k_F$ RKKY signal. A similar observation has also been made using numerical approaches.

In order to investigate rigorously the ordering of the local spins due to the formation of polarons, we first apply, an infinite (to avoid truncation errors) unitary transformation, $\hat{S}$, to the bosonized Hamiltonian, Eq. (2). The most effective form of $\hat{S}$ is given by: $\hat{S} = i(J/2\pi)\sqrt{v_F/v_\sigma^3} \sum_j \theta_\sigma(j) S^{\prime}_d j$, which couples the conduction electron spins directly to the localized spins. Secondly, we explicitly take into account the Luttinger liquid character of the Bose fields, i.e., use their non-interacting expectation values such that the effective Hamiltonian for the local spins is derived as:

$$H_{\text{eff}} = -\frac{J^2v_F^2}{4\pi^2} \sum_{j,j'} \int_0^\infty dk \cos[k(j - j')] \Lambda^2(k) S^x_{d,j} S^x_{d,j'} + \frac{J}{2\pi\lambda} \sum_j \{\cos[K(j)] + \cos[2k_F j]\} S^x_{d,j} - \frac{J}{2\pi\lambda} \sum_j \sin[K(j)] \sin[2k_F j] S^z_{d,j}.$$  

(3)

Here $K(j)$ stems from the unitary transformation and counts all the $S^{\prime}_d j$'s to the right of the site $j$ and subtracts all those to the left of $j$: $K(j) = (J/2v_F) \sum_{t=1}^\infty (S^z_{d,j-t} - S^z_{d,j-t})$. This
term gives the crucial difference between the Kondo lattice and dilute Kondo lattice, as will be explained in the following. The most important term in Eq. (3) is the first one, which shows that a ferromagnetic coupling emerges even in the dilute Kondo lattice model. This coupling is non-negligible for $N_d > N_c$ and $j - j' \leq \lambda$ and its strength will decrease with the distance between impurity spins. Thus, $\lambda$ represents the effective delocalization length related to the spatial extent of the polarons, i.e., the effective range of DE. Thus DE will vanish if the distance between the impurity spins is larger than $\lambda$. In general $\lambda$ will depend on $J, N_c$ or even $N_d$, but we will use its low density value: $\lambda \approx \sqrt{2}/\sigma$. Consequently, we approximate it by its nearest neighbor form: $I = (J^2v^2\sigma/2\pi^2v_F)\int_0^\infty dk \cos k\Lambda^2(k)$.

For the Kondo lattice model, $K(j)$ is vanishingly small as the number of $d$-spins to the left and the right of a given site $j$ is the same. The effective Hamiltonian can thus be replaced by a random transverse field Ising model: $H_{\text{eff}} = -I \sum_j S^z_{d,j} S^z_{d,j+1} - \sum_j h_j S^x_{d,j}$, where the ferromagnetic coupling strictly vanishes if $\ell > \lambda$. The random fields, $h_j$, are generated by $(1 + \cos[2k_Fj])$ at large distances, where $\cos[2k_Fj]$ oscillates unsystematically with respect to the lattice. The large values $\cos[2k_Fj] \approx 1$ which are responsible for spin flips, are then widely separated and are driven by a cosine distribution similarly to spin-glasses. If we have a small concentration of holes in the array of localized spins, then - opposite to the previous case - $K(j)$ is non-vanishing since the hole spins are no longer equally distributed to the left and the right of a given site. This yields $K(j) \approx (-1)^j(J/2v_F)$, which gives rise to a staggered field and antiferromagnetic ordering.

Since our main interest here is to explore the occurrence of ferromagnetism in the presence of the Hubbard and phonon terms, we focus on the transition between the paramagnetic and the ferromagnetic phase. This is controlled by a critical coupling $J_{\text{crit.}} = (\pi/4)\sin(\pi n/2) \{1 - U/[2\pi\sin(\pi n/2)] + \alpha^2/[2\pi K\sin(\pi n/2)]\}^{1/2}$. For values $J < J_{\text{crit.}}$ a paramagnetic state exists which is dominated by polaronic fluctuations. For $J > J_{\text{crit.}}$ ferromagnetism appears. The transition between these phases is of order-disorder type with variable critical exponent $\delta = J_{\text{crit.}}/J$. It can be seen that, in accordance with our previous observation, the Hubbard term stabilizes the ferromagnetic phase, while the phonons counter this and tend to increase the polaronic regime. This paramagnetic polaron state can be viewed as a Griffiths phase, since the critical exponent is variable and the spin-spin correlation function is given by: $(\xi/x)^{5/6}e^{-(3/2)(2\sigma^2x/\xi)^{1/3}}e^{-x/\xi}$, where $\xi \approx 1/\delta^2$ is the correlation length. At finite temperature the susceptibility in this phase is proportional to
$T^{2\delta-1}(\ln T)^2$, while the specific heat follows a $T^{2|\delta|}$ dependence. This regime can be viewed as a paramagnet with locally ordered ferromagnetic regions, again manifesting the analogy to a two fluid picture.

In summary, we have derived an effective Hamiltonian from a one-dimensional Kondo lattice model extended to include effects stemming from the lattice and in the presence of an onsite Hubbard term, which accounts for the conduction electron Coulomb repulsion. The results are: i) A ferromagnetic phase appears at intermediate $|J|$ due to forward scattering by delocalized conduction electrons. ii) Ferromagnetism is favoured by the Hubbard term, while it is suppressed by the electron-phonon coupling. iii) The paramagnetic phase is characterized by the coexistence of polaronic regimes with intrinsic ferromagnetic order and ordinary conduction electrons. iv) In the paramagnetic phase, two time scales compete with each other - reminiscent of a two-fluid model - and the variability of the critical exponents suggests the existence of a Griffiths phase. The results are related to the small-doping regime of CMR materials which are ferromagnets at low temperatures, since here the coupling to the phonons has been shown to dominate the paramagnetic-ferromagnetic phase transition.

It is interesting to note the discrepancy between infinite dimensional calculations and the present one dimensional result. Many calculations to model CMR have been made in dynamical mean-field theory, which is an infinite dimensional approximation and therefore incapable of capturing spatial inhomogeneities. In the present work we approach the CMR materials via a one dimensional approximation, but with techniques able to describe fluctuations of short-range order. Our results show that strong intrinsic spatial inhomogeneities of Griffiths type dominate the behaviour of the Kondo lattice. Consequently the inhomogeneities exhibit clear statistical scaling properties as a function of the proximity to a quantum (order-disorder) critical point. The phonons enhance the inhomogeneities, which in a good approximation behave as a supercritical (metastable) phase of a two fluid model.

Even though various bosonization schemes have been used for the one-dimensional KLM, non of the previous approaches took into account phonons and the possibility of diluting the array of impurity spins. The inclusion of phonon degrees of freedom has been shown to be relevant in creating local magnetic inhomogeneities. It is important to mention that the properties of the system are driven by intrinsic inhomogeneities. This means that, in a renormalization group approach, the dimensionality should not matter. Thus, similar behaviour is expected in realistic two- and three-dimensions, which clearly merit further
detailed study.

1 C. W. Searle and S. T. Wang, Can. J. Phys. 48, 2023 (1970); K. Kubo and N. Ohata, J. Phys. Soc. Jpn 63, 3214 (1972).

2 See, for example A. J. Millis, P. B. Littlewood and B. I. Shraiman, Phys. Rev. Lett. 74, 5144 (1995); H. Röder, J. Zang and A. R. Bishop, Phys. Rev. Lett. 76, 1356 (1996); and the references cited therein.

3 M. Troyer and D. Würtz, Phys. Rev. B47, 2886 (1993); H. Tsunetsugu, M. Sigrist, and K. Ueda, Phys. Rev. B47, 8345 (1993); S. Moukouri and L. G. Caron, Phys. Rev. B52, R15723 (1995); N. Shibata and K. Ueda, J. Phys. Cond. Matter. 11, R1 (1999); I. P. McCulloch, A. Jouzapavicius, A. Rosengren, and M. Gulacsi, Phil. Mag. Lett. 81, 869 (2001), and Phys. Rev. B65, 052410 (2002); A. Jouzapavicius, I. P. McCulloch, M. Gulacsi and A. Rosengren, Phil. Mag. B82, 1211 (2002).

4 O. Zachar, S. A. Kivelson, and V. J. Emery, Phys. Rev. Lett. 77, 1342 (1996).

5 G. Honner and M. Gulacsi, Phys. Rev. Lett. 78, 2180 (1997); Phys. Rev. B58, 2662 (1998).

6 W. P. Su, J. R. Schrieffer and A. J. Heeger, Phys. Rev. Lett. 42, 1698 (1979); Phys. Rev. B22, 2099 (1980).

7 T. Holstein, Ann. Phys. (N.Y.) 8, 325; 343 (1959).

8 F. D. M. Haldane, J. Phys. C14, 2585 (1981). For a review, see J. Voit, Rep. Prog. Phys. 57, 977 (1994); M. Gulacsi, Phil. Mag. B76, 731 (1997); J. von Delft and H. Schöller, Ann. Phys. 4, 225 (1998).

9 P. W. Anderson and H. Hasegawa, Phys. Rev. 100, 675 (1955); P. -G. de Gennes, Phys. Rev. 118, 141 (1960).

10 D. S. Fisher, Phys. Rev. Lett. 69, 534 (1992); Phys. Rev. B 51, 6411 (1995).

11 See, for example, A. A. Abrikosov and S. I. Moukhin, J. Low Temp. Phys. 33, 207 (1978).

12 R. B. Griffiths, Phys. Rev. Lett. 23, 17 (1969).