Satellite-Based Continuous-Variable Quantum Communications:
State-of-the-Art and a Predictive Outlook

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Abstract—The recent launch of the Micius quantum-enabled satellite heralds a major step forward for long-range quantum communication. Using single-photon discrete-variable quantum states, this exciting new development proves beyond any doubt that all of the quantum protocols previously deployed over limited ranges in terrestrial experiments can in fact be translated to global distances via the use of low-orbit satellites. In this work we survey the imminent extension of space-based quantum communication to the continuous-variable regime - the quantum regime perhaps most closely related to classical wireless communications. The CV regime offers the potential for increased communication performance, and represents the next major step forward for quantum communications and the development of the global quantum internet.

I. MOTIVATION AND INTRODUCTION

Moore’s Law has remained valid for half-a-century! As a result, contemporary semi-conductor technology is approaching nano-scale integration. Hence nano-technology is about to enter the realms of quantum physics, where many of the physical phenomena are rather different from those of classical physics. Hence this treatise contributes towards completing the 'quantum jig-saw puzzle' by paving the way from classical wireless systems to their perfectly secure quantum-communications counterparts, as heralded in [1], [2].

• The Inspiration: In order to circumvent the specific limitations of the classical wireless systems detailed in [1], we set out to bridge the separate classical and quantum worlds into a joint universe, with the objective of contributing to perfectly secure quantum-aided communications for anyone, anywhere, anytime across the globe, as indicated by the stylized vision of the near-future quantum communications scenario seen in Fig. 1.

• The Reality: However, quantum processing is far from being flawless - it has substantial challenges, as detailed in this contribution. Nonetheless, at the time of writing long-range quantum communications via satellites has become a reality.

Amongst its numerous intriguing attributes, quantum communication has the potential to achieve secure communications at confidence levels simply unattainable in classical communications settings. This is due to the fact that quantum physics introduces a range of phenomena which have no counterpart in the classical domain, such as quantum entanglement and the superposition of quantum states [1]. The exploitation of such effects, both before and after the transmission of information in the quantum domain, can in effect lead to communications possessing ‘unconditional’ security.

Quantum communication entails the transfer of quantum states from one place to another via a quantum channel. In a generic form, quantum communication consists of three steps: (i) the preparation of quantum states - where the original classical information is encoded into quantum states; (ii) the transmission of the prepared quantum states over a quantum channel such as optical fiber or a free-space optical (FSO) channel - where the states are transmitted from a transmitter, held by Alice, to a receiver, named Bob; and (iii) detection -

1The superposition of a logical one and zero may be viewed as a coin spinning in a box, where we cannot claim to show its state being ‘head’ or ‘tail’. When we stop spinning the coin, and lift the lid of the box, the superposition-based quantum state collapses back into the classical domain as a consequence of us observing it.
where the received states are decoded using quantum measurement resulting in some output classical information. A schematic including these three steps is shown in Fig. 2.2

A key motivation for quantum communication is that the quantum information, mapped for example to the polarization of a photon, can be shared more securely than classical information. The well-known example of this is quantum key distribution (QKD) [3], whose unconditional security has been theoretically proved (classical cryptography schemes are not proved to be secure). We also note the close connection between quantum communication and quantum entanglement. A pair of quantum states are said to be entangled if, for example, changing the polarization of a photon results in an instantaneous polarization change for its entangled pair. Einstein referred to this as a ‘spooky action at a distance.’ Important quantum communication protocols utilizing entangled states include QKD, quantum teleportation [4], [6], and entanglement swapping (teleportation of entanglement) [7].

In terms of representing the quantum states in quantum communications, discrete-variable (DV) and continuous-variable (CV) descriptions have been used [3], [9]. In the former, information is mapped to discrete features such as the polarization of single photons [3]. The detection of such features would then be realized by single-photon detectors. In DV technology information is mapped to two (or to a finite number) of basis states. The standard unit of DV quantum information is mapped to two (or to a finite number) of basis states. The standard unit of DV quantum information is mapped to two (or to a finite number) of basis states.

In DV technology, information is usually encoded onto the quadrature variables of the optical field [10]–[15], which constitute an infinite-dimensional Hilbert space. Detection of these variables is normally realized by high-efficiency homodyne (or heterodyne) detectors, which can be capable of operating at a faster transmission rate than single-photon detectors [16]–[18].

The field’s quadrature components (representing the quantum state) can be considered as related to the amplitude and phase of the laser light. In quantum mechanics, the quadrature components can also be considered as corresponding to the position and momentum of a harmonic oscillator.

There are generally three quantum communication scenarios, namely, the use of optical fibers, the use of terrestrial FSO channels, or the use of FSO channels to satellites. These scenarios are complementary and all may be expected to play a role in the emerging global quantum communication infrastructure. Fiber technology has the key advantage that once in place, an unperturbed channel from A to B exists. In fact, in fiber links the photon transfer is hardly affected by external conditions such as background light, the weather or other environmental obstructions. However, fiber suffers both from optical attenuation and polarization-preservation problems, which therefore limit its attainable distance to a few hundred kilometers [19]–[30]. These distance limitations may be overcome by the development of suitable quantum repeaters [31]. Losses in fiber are due to inherent random scattering processes, which increase exponentially with the fiber length. Explicitly, the transmissivity determining the fraction of energy received at the output of a fiber link of length L is given by $\tau = 10^{-\alpha L/10}$, where the value of $\alpha$ is highly dependent on the wavelength. Losses are minimised at the wavelength of 1550 nm, where for silicon fiber $\alpha \simeq 0.2$ dB/km.

Replacing the fiber channel with a FSO channel has the immediate advantage of lower losses [32]–[35], largely because the atmosphere provides for low absorption. The atmosphere also provides for almost unperturbed propagation of the polarization states. Additionally, FSO channels offer convenient flexibility in terms of infrastructure establishment, with links to moving objects also feasible [36]–[38]. However, terrestrial FSO quantum communications remain ultimately distance-limited, due to (amongst other issues) the curvature of the Earth, potential ground-dwelling line-of-sight (LoS) blockages, as well as atmospheric attenuation and turbulence.

FSO quantum communication via satellites [39]–[63], [65]–[70] has the additional advantage that communications can still take place, even when there is no direct free-space LoS from A to B. That is, assuming that LoS paths from a satellite to two ground stations exist, satellite-based FSO communication

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2Note we have utilised the standard quantum mechanical notation for a vector in a vector space, i.e. $|\psi\rangle$, where $\psi$ is a label for the vector (any label is valid). The entire object $|\psi\rangle$ is sometimes called a ‘ket’. Note also that $\langle\psi|$ is called a ‘bra’ which is the Hermitian conjugate or adjoint of the ket $|\psi\rangle$. In quantum mechanics, bra-ket notation is a standard notation for describing quantum states.
can still proceed. The range of satellite-based communication is also potentially much larger than that allowed for by direct terrestrial FSO connections, since the former circumvents the terrestrial horizon limit and there are lower photonic losses at high altitudes. In satellite-based FSO communications, only a small fraction of the propagation path (less than 10 km) is through the atmosphere - meaning most of the propagation path experiences no absorption and no turbulence-induced losses. The utilisation of satellites also allows for fundamental studies on the impact of relativity on quantum communications. The key disadvantage of satellite-based quantum communications is, however, atmospheric turbulence-induced loss.

QKD constitutes the most studied quantum communication protocol, and has been deployed over both fiber and FSO channels. Indeed, the implementation of QKD over optical fibers has already been commercialised. Terrestrial FSO quantum communications have been successfully deployed over very long distances. In 2007 entanglement-based QKD and decoy-state QKD was realized over a 144 km FSO link between the Canary Islands of La Palma and Tenerife. In addition to QKD, long-distance terrestrial FSO experiments have also been carried out to implement both entanglement distribution and quantum teleportation. The above long-distance FSO quantum communication experiments have been implemented at night. However, in a recent experiment (by choosing an appropriate wavelength, spectrum filtering and spatial filtering) FSO terrestrial QKD over 53 km has also been demonstrated during the day. Nonetheless, in both fiber and FSO QKD implementations, the increasing levels of channel attenuation and noise tend to limit the maximum distance of successful key distribution to a few hundred kilometers.

A promising way of extending the deployment range of QKD is through the use of satellites. Indeed, it is widely anticipated that the reliance on satellites will assist in the expansion of quantum communication to global scales. Full-scale verifications of satellite-based QKD have been reported by demonstration of QKD between an aeroplane and a ground station, in by demonstration of QKD using a moving platform on a turntable, and in by demonstration of QKD from a stationary transmitter to a moving receiver platform traveling at an angular speed equivalent to a 600 km altitude satellite). Furthermore, several satellite-based quantum communication projects have been reported. In a satellite-to-ground single-photon downlink was simulated by reflecting weak laser (coherent) pulses (emitted by the ground-based station) off a low-Earth-orbit (LEO) satellite. In addition to experimental demonstrations, quantum communications with orbiting satellites have also been investigated by a growing number of feasibility studies. Recently, the in-orbit operation of a photon-pair source aboard a nano-satellite has been reported, which demonstrates photon-pair generation and polarization correlation under space conditions.

Quantum communication via satellites has very recently been given an enormous boost with the launch of the world's first quantum satellite, Micius, by China. Building on the previously mentioned experiments, this new LEO satellite creates entangled photon pairs, sending them down to Earth for subsequent processing in a diverse range of communication scenarios. For example, using Micius, satellite-based distribution of entangled photon pairs in the downlink to two terrestrial locations separated by 1203 km has been demonstrated. Quantum teleportation of single-photon qubits from a ground station to Micius through an uplink channel has also been demonstrated. Extensions of this technology to significantly smaller satellites has just been reported for a Japanese micro-satellite and an optical ground station.

All of the previous FSO quantum communication systems referred to above have been focussed on DV technologies. They are based on single-photon technology and use single-photon detectors. Such detectors are impaired by background light, and involve spatial, spectral and/or temporal filtering in order to reduce this noise. By contrast, in CV quantum communication, homodyne detection (in which the signal field is mixed with a strong coherent laser pulse, called the “local oscillator”) is used for determining the field quadratures of light. Homodyne detectors offer better immunity to stray light since the local oscillator is also capable of assisting in both spatial and spectral filtering. Also, such homodyne detectors are more efficient than single-photon detectors, since the PIN photodiodes used in them offer higher quantum efficiencies than the avalanche photodiodes of single-photon detectors. Hence, CV-QKD can generally be considered to be more robust against background noise than DV-QKD.

In the feasibility of a point-to-point CV-QKD (with coherent polarization states of light) has been demonstrated over a 100 m FSO link. The nonclassical properties of CV quantum states propagating through the turbulent atmosphere have been analysed. Gaussian entanglement distribution through a single point-to-point atmospheric channel and its applicability to CV-QKD have been studied. The entanglement properties of quantum states in the turbulent atmosphere have also been studied. Satellite-based CV quantum communication in the context of Gaussian and non-Gaussian entanglement distribution, and its application to CV-QKD, have been investigated in detail. The results presented apply for both a single point-to-point atmospheric channel, and in combined satellite-based atmospheric channels where the satellite acts as a relay. Recently, a point-to-point CV quantum communication experiment relying on the coherent polarization states of light has been established over a 1.6 km FSO link in an urban environment. The distribution of polarization squeezed

3Gaussian quantum states are CV states with field quadratures exhibiting a Gaussian probability distribution.
TABLE I
COMPARISON OF THIS STUDY WITH AVAILABLE SURVEYS

| Approach                        | Satellite-based quantum communication | Atmospheric fading quantum channels | CV quantum systems | Quantum communication protocols | QKD | Gaussian CV quantum communication | Non-Gaussian CV quantum communication | CV quantum teleportation | CV entanglement swapping |
|---------------------------------|---------------------------------------|----------------------------------|-------------------|---------------------------------|-----|-----------------------------------|---------------------------------------|--------------------------|-------------------------|
| Braunstein and van Loock [9]    | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Pirandola and Mancini [97, 98]  | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Adesso and Illuminati [99]      | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Gisin and Thew [100]            | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Scarani et al [101]             | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Andersen et al [102]            | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Wang et al [103]                | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Weedbrook et al [104]           | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Lo et al [105] and Diamanti et al [106] | ✓                                | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Diamanti and Leverrier [107]    | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Marshall and Weedbrook [108]    | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Bedington et al [63]            | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Li et al [109]                  | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| Shenoy-Hejamadi et al [110]     | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |
| This survey                     | ✓                                     | ✓                                | ✓                 | ✓                               | ✓   | ✓                                | ✓                                     | ✓                        | ✓                       |

In quantum optics, there is an uncertainty relationship for the quadrature components of the light field, stating that the product of the uncertainties in both quadrature components is at least some quantity times Planck’s constant. Hence, the uncertainty relationship dictates some lowest possible noise (i.e., uncertainty) amplitudes for the quadrature components of the light. In squeezed light, a further reduction in the noise amplitude of one quadrature component is carried out by squeezing the uncertainty region of that quadrature component, which is at the expense of an increased noise level in the other quadrature component. 

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states[4] of light through an urban 1.6 km FSO link has also been demonstrated [94]. Recently, an experiment has been carried out relying on homodyne detection at a ground station of optical signals transmitted from a geostationary satellite [95]. This experiment is important in that it clearly demonstrates the feasibility and potential of satellite-based CV-QKD implementations.

The current work aims to survey and characterise the capabilities of CV quantum technology in satellite-based quantum communications. Since CV entanglement has been widely known as a basic resource for CV-QKD [96], our survey is focussed on satellite-based CV quantum communication in the context of CV entanglement distribution and its application to CV-QKD. A brief comparison of this survey to the other published surveys on topics related to CV quantum communication is presented in Table I.

In the context of satellite-based quantum communication we are faced with two different channels, namely, the uplink (ground-to-satellite) channels and the downlink (satellite-to-ground) channels. In the uplink, the ground station transmits signals to the satellite receiver, and in the downlink, the satellite transmits signals to the ground station receiver. Correspondingly, there are several possible architectures for implementing satellite-based quantum communication depending on the types of links utilized. Some of these configurations are illustrated in Fig. 3 and will be studied in this treatise in terms
of entanglement distribution and CV-QKD implementation.

In Fig. 3, the schemes (a) and (b) illustrate the uplink and downlink channels, respectively (both links have been demonstrated in the DV domain [66], [67], [69]). In scheme (c) of Fig. 3 the deployment of quantum technology at the satellite is minimized, since the satellite is utilized only in a reflector mode (i.e. a simple relay). As a proof of concept for the reflecting paradigm, we note the recent experimental tests of [46]–[48], where single photons (weak laser coherent pulses) emitted by the ground station were reflected (and subsequently detected on the ground) by a LEO satellite via the satellite’s cube retro-reflectors. In scheme (c) the complex quantum engineering components are limited to the ground stations, since the source of quantum states is located in one of the ground stations and the receiver of quantum states is located in the other ground station. Although satellite reflection towards another station constitutes a sophisticated engineering task in its own right, it does not require onboard generation of quantum communication information. There are many practical advantages in deploying quantum communication technology at the ground stations, such as lower-cost maintenance, and the ability to rapidly upgrade as new quantum technology matures.

The other schemes, (d) and (e), in Fig. 3 can be considered as space-based high-complexity schemes, since they involve the deployment of quantum technology at the satellite. In scheme (d) (again already demonstrated for DV states [68]) the source of quantum states is located on board the satellite, with both ground stations acting as receivers. In scheme (e) the two ground stations transmit quantum states to the satellite. In the satellite, quantum measurements are performed on the received states and the classical measurement results are communicated back to the ground stations. Scheme (e) can be utilized in support of entanglement swapping and measurement-device-independent protocols so as to implement QKD between the two ground stations.

For the readers’ convenience, the outline of this paper is listed below.

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II. Free-space channels to and from satellites

A. Sources of loss in FSO channels

The main sources of loss in FSO communication are diffraction, absorption, scattering and atmospheric turbulence [111]–[114].

Diffraction: Diffraction is a ubiquitous form of natural wave propagation phenomenon experienced by light beams, and leads to beam-spreading (beam-broadening).

Absorption and scattering: Absorption and scattering are imposed by the constituent gases and particles of the atmosphere. Both effects are strongly wavelength-dependent, and both impose attenuation on an optical wave. However, in this treatise we will assume that both scattering and absorption can be neglected, since they can be largely mitigated by an appropriate choice of the communication wavelength. Explicitly, there is a negligible absorption at the visible wavelengths spanning from 0.4 to 0.7 mm. For these reasons, scattering and absorption was also neglected in [18], [53], [83]–[85], [93], [113]–[117].

Atmospheric turbulence: Atmospheric turbulence arises due to random fluctuations in the refractive index caused by stochastic variations of temperature. The atmosphere contains turbulent random inhomogeneities of various scales - also referred to as turbulent eddies [113]. They range from a large-scale (the outer scale of turbulence) to a small-scale (the inner scale of turbulence). These eddies affect optical wave-propagation through the atmosphere in different ways, depending on their size. In general, large scales produce refractive effects and hence predominate to distort the phase of the propagating wave, while small scales are mostly diffractive in nature and therefore distort the amplitude of the wave [112], [113]. The most important effects resulting from the atmospheric eddies are beam-wandering, beam-spreading and beam-scintillation [111]–[114]. We describe each of these three effects in more detail: (i) Random deviation of the beam from its original path is referred to as beam-wandering, which is caused by large-scale turbulent eddies, whose size is large compared to the beam-width. Beam-wandering causes time-varying power fades [53], [111], [113], [114]. (ii) Atmospheric turbulence results in a randomly fluctuating beam-width in the receiver aperture plane. The broadening of the beam-width (when averaged over time) beyond that due to diffraction is termed as turbulence-induced beam-spreading [53], [56], [84], [111], [113], [118]. (iii) We define scintillation by fluctuations in the received irradiance (intensity) within the beam cross section. Scintillation includes the temporal variation in the received irradiance and spatial variation within the receiver aperture. Scintillation is mainly caused by small-scale turbulent eddies [111], [114].

B. Sources of loss in FSO channels to and from satellites

In satellite-based quantum communications, the uplink and downlink channels are very different, since the atmospheric turbulence layer only occurs near the transmitter on an uplink, and only near the terrestrial receiver on a downlink. In the following, we briefly highlight how these two channels are affected by the above-mentioned turbulence-induced effects.

Uplink channels: For typical dimensions of the aperture size embedded in the ground station, the uplink optical beam first propagates through the turbulent atmosphere and its beam-width is much narrower than the large-scale turbulent eddies [53], [111], [113], [114]. This makes beam-wandering the dominant effect in the uplink [53], [111], [113], [114]. Turbulence-induced beam-spreading also occurs to some extent in the uplink [53], [113]. As a result, the beam received by the satellite (when averaged over time) is wider than that associated with diffraction [53], [113]. Fig. 4 illustrates these two atmospheric effects, namely beam-wandering and beam-spreading in the uplink. Scintillation is not dominant in the uplink [111], [113].

Downlink channels: In contrast to the uplink case, the downlink optical beam propagates through the turbulent atmosphere only in the final part of its path. Considering the typical aperture size of the optical system embedded in the satellite, the beam-width at its entry into the atmosphere is likely to be larger than the scale of the turbulent eddies. As such, beam-wandering in the downlink tends to be less important relative to uplink channels [53], [111], [113], [114]. The photonic losses in the downlink are likely to be dominated by diffraction effects [53], [56]. Scintillation can occur to some extent in the downlink [111], [113]. However, as a consequence of aperture averaging, the downlink scintillation effects imposed on the detector can be assumed negligible when the receiver includes a large-diameter (> 0.5 m) telescope [111], [113].

C. Atmospheric fading channels

In atmospheric channels the transmissivity, $\eta_t$, fluctuates due to turbulence-induced effects. These fading channels can be characterized by the probability distribution of the transmission coefficients, $\eta$ (where $\eta = \sqrt{\eta_t}$), which is denoted
by $p(\eta)$. For a fading channel associated with the probability distribution $p(\eta)$ the mean fading loss in dB is given by $-10\log_{10}\left(\int_0^{\eta_0} \eta^2 p(\eta) d\eta\right)$, where $\eta_0$ is the maximum value of $\eta$.

As discussed in Sec. [11] beam-wandering is the dominant turbulence-induced effect in the uplink. As an aside, we note beam-wandering is expected to dominate the fading contributions in many terrestrial atmospheric communication scenarios [83], [85], [93], [94], [116].

D. Beam-wandering model

Here, we describe the probability distribution of the channel coefficients when the channel effects are dominated by beam-wandering. In the first instance we will assume that the beam-width at the receiver aperture is fixed. That is, initially we will ignore any fluctuations in the beam-width caused by atmospheric turbulence.

In practice, beam-wandering causes the beam-center to be randomly displaced (along the x and y coordinates) from the center of the receiver aperture plane. More explicitly, the beam-center position, $(x_0, y_0)$, fluctuates randomly around a fixed point, $(x_d, y_d)$, in the receiver aperture plane according to a two-dimensional Gaussian distribution [83]

$$p(x_0, y_0) = \frac{1}{2\pi \sigma_0^2} \exp \left( -\frac{(x_0 - x_d)^2 + (y_0 - y_d)^2}{2\sigma_0^2} \right), \quad (1)$$

where $\sigma_0$ is the beam-wandering standard deviation. Thus, the beam-deflection distance, $l = \sqrt{x_0^2 + y_0^2}$, i.e. the distance between the beam-center and the aperture-center at $(0, 0)$ fluctuates according to the Ricean distribution [83]

$$p(l) = \frac{l}{\sigma_0^2} f_0 \left[ \frac{ld}{\sigma_0^2} \right] \exp \left( -\frac{l^2}{2\sigma_0^2} \right), \quad (2)$$

where $d = \sqrt{x_0^2 + y_0^2}$ is the distance between the aperture-center and the fluctuation-center $(x_d, y_d)$, and $f_0 [\cdot]$ is the modified Bessel function. Note that $d = 0$ means that the beam-center fluctuates around the aperture-center. In beam-wandering the channel transmission coefficient, $\eta$, is a function of the beam-deflection distance, $l$, and is given by [83]

$$\eta^2 = \eta_0^2 \exp \left( -(\frac{l^2}{\gamma^2}) \right), \quad (3)$$

where $\gamma$ is the shape parameter, $S$ is the scale parameter and $\eta_0$ is the maximum value of $\eta$. The latter three parameters are given by

$$\gamma = 8h \exp (-4h) I_1 (4h) \left[ \ln \left( \frac{2\eta_0^2}{1-\exp (-4h)} \right) \right]^{-1},$$

$$S = \beta \left[ \ln \left( \frac{2\eta_0^2}{1-\exp (-4h)} \right) \right]^{-1} \left( \frac{1}{\gamma} \right),$$

$$\eta_0^2 = 1 - \exp (-2h),$$

where $I_1 [\cdot]$ is the modified Bessel function, and where $h = (\beta/W)^2$, with $\beta$ being the receiver aperture radius and $W$ the beam-spot radius at the receiver aperture. Note, $\beta$ and $W$ have the same units (meter). A schematic illustration of beam-wandering is shown in Fig. 5. According to Eqs. (2) and (3), the probability distribution $p(\eta)$ can be described by the log-negative Weibull distribution [83]

$$p(\eta) = \frac{2\eta^2}{\sigma_0^2} \left[ 2 \ln \left( \frac{2\eta_0}{\eta} \right) \right]^{(2-1)} \left[ \frac{2\eta_0}{\sigma_0} \right] \left( \frac{2 \ln \left( \frac{2\eta_0}{\eta} \right)}{\sigma_0} \right)^{\frac{1}{\gamma}} \exp \left( -\frac{l^2}{2\sigma_0^2} \right), \quad (5)$$

for $\eta \in [0, \eta_0]$, with $p(\eta) = 0$, otherwise. In some of the earlier literature, e.g. [119], the log-normal distribution was used. However, we now know the log-negative Weibull distribution more accurately describes the operationally-important distribution tail [83]. In Fig. 6 the log-negative Weibull distribution is shown for fixed values of the beam-wandering standard deviation $\sigma_b$ and the receiver aperture radius $\beta$, and for different values of the beam-spot radius at the receiver aperture $W$ (the mean fading loss increases with increasing $W$). In Fig. 7 the log-negative Weibull distribution is shown for the fixed values of $W$ and $\beta$, with different values of $\sigma_b$ (the mean fading loss increases with increasing $\sigma_b$).

Now we analyse the influence of beam-width fluctuations (caused by atmospheric turbulence) to the beam-wandering model just given. We refer to this effect as turbulence-induced beam-spreading. In doing this analysis, we will assume beam-deformation does not occur - meaning the beam shape remains circular as it traverses the atmospheric channel (beam-deformation has been analysed in [84]). In turbulence-induced beam-spreading, the beam-spot radius $W$ randomly changes in the receiver aperture plane [83] with the probability distribution $p(W)$. Including this effect in our beam wandering model, the transmission coefficient of the channel, $\eta$, is now a function of the two random variables $l$ and $W$ according to Eqs. (1) and (2). We define a new variable $\Theta$ by setting $\Theta = 2 \ln \left( \frac{W}{w_0} \right)$, where $w_0$ is the initial beam-spot radius at
the radiation source. This is useful since θ randomly changes according to a normal distribution with the mean value \( \langle θ \rangle \) and standard deviation \( σ_θ \) \([84]\). That is

\[
p(θ) = \frac{1}{\sqrt{2πσ_θ^2}} \exp\left(-\frac{(θ - \langle θ \rangle)^2}{2σ_θ^2}\right).
\]

With the inclusion of beam-width fluctuations in beam wandering, the calculation of a closed-form solution for \( p(η) \) is not straightforward. However, knowing the probability distribution of \( p(l) \) of Eq. (2) and \( p(θ) \) of Eq. (6), we can calculate certain important quantities after averaging over all values of the channel’s transmission coefficient. For instance, the mean fading loss in dB of a fading channel with the inclusion of beam-width fluctuations is now given by

\[
-10\log_{10}\left(\int \eta^2(l, θ)p(l, θ)dldθ\right).
\]

Assuming that atmospheric turbulence is isotropic \([84]\) and \( d = 0 \), the mean fading loss in dB of a fading channel (after the inclusion of beam-width fluctuations in the beam-wandering model) is given by

\[
-10\log_{10}\left(\int \eta^2(l, θ)p(l)p(θ)dldθ\right).
\]

Note, with the inclusion of beam-width fluctuations, the maximum value of the channel’s transmission coefficient \( η_0 \) is no longer fixed but rather randomly changes.

Optical losses in the downlink are usually orders of magnitude lower relative to uplinks \([63]\), \([67]\)–\([69]\). This means that if the “price” is paid in terms of placing the critical quantum technology on board the satellite (rather than the easier case of maintaining the quantum technology in ground stations), then much better quantum communication channels can be obtained. As alluded to earlier, the principal reason for this improvement is that in the downlink, diffraction of the beam is the main contributor to photon losses - not beam-wandering as in the uplink (see Fig. 8). The important fact is that by the time the downward-link beam hits the main turbulence-inducing layers of the atmosphere (this layer commences at about 20 km from ground level) the beam is much closer to its target and therefore any induced beam-wandering is less effective. Clearly, as opposed to most communication channels, there will be no directional reciprocity in channel throughput for quantum communications with satellites. The recent experimental deployments of quantum communication in space have mostly exploited the more favourable downlink channel conditions \([67]\), \([68]\). The losses in the downlink can then be modelled quite simply (to first order) through diffraction-only effects with the beam divergence following a \( λ/D \) scaling, where \( D \) is the diameter of the satellite telescope and \( λ \) is the transmission wavelength \([63]\).

E. Estimation of a FSO channel

Note that the rate of atmospheric fluctuations we consider are the order of a few kHz, which is at least a thousand times slower than the typical transmission rates \([113]\). This means that the channel’s transmission coefficient can be measured at the cost of additional (classical) transmission and receiver complexity \([17]\), \([115]\), \([116]\), \([120]\). These channel measurements may be carried out using several schemes, e.g., by transmitting coherent (classical) light pulses that are intertwined with the quantum information \([115]\), \([116]\) or by transmitting a local oscillator (i.e., a strong coherent laser pulse which is mixed with the signal field in the homodyne detection and
serves as a phase reference) \[17\]. In \[17\] measurement of the atmospheric channel’s transmission coefficients was carried out in real time at the receiver by passing a local oscillator through the channel in a mode orthogonally polarized to the signal. The technique of measuring the atmospheric channel’s transmission coefficient by an auxiliary classical laser beam was introduced in 2012 \[118\], and its practical employment was demonstrated for a one-way communication link in 2015 \[119\]. The same technique based on the intensity of the signal itself was introduced and realized in \[120\].

### III. Introduction to CV Quantum Systems

One form of a CV quantum system is that represented by \( N \) bosonic modes, such as those corresponding to \( N \) quantized radiation modes of the electromagnetic field \[9\], \[99\], \[102\]–\[104\], \[121\], \[122\]. A single photon has four degrees of freedom, helicity (polarization) and the three components of the momentum vector. In principle, quantum information can be encoded into any one of these degrees of freedom. A single ‘mode’ of an electromagnetic field refers to a specific combination of these photonic degrees of freedom. In many circumstances different modes can be simply represented by different frequencies (since frequency is related to momentum). For a beam of photons, the number of photons in the beam constitutes another means to encode quantum information. Quantum information encoded into the quadratures of the electromagnetic field (formally defined below) are related to an encoding in this additional degree of freedom. Since the quadrature operators have continuous spectra, we can describe the values of such operators as CV variables.

A single mode of a CV system can be described as a single quantum harmonic oscillator of a specific frequency, where the electric and magnetic fields play the ‘roles’ of the position and momentum \[123\]. It will be useful to further illustrate this concept. Consider the case of a single-frequency radiation field confined to a one-dimensional cavity with walls that are perfectly conducting. Assume the \( z \)-axis is parallel to the length of the cavity and the cavity walls are located at \( z = 0 \) and \( z = L \). The electric field within the cavity will form a standing wave. Without loss of generality, we can take the electric field to be polarized perpendicular to the \( z \)-axis, and in the positive \( x \)-direction (we take the \( x \) and \( z \) coordinates to be in the same plane and the \( y \) plane perpendicular to the \( x \) plane). In terms of the distance vector \( r \) and time \( t \), the electric field can then be written as \( E(r,t) = e_x E(x,z,t) \), where \( e_x \) is a unit-length polarization vector. Given our boundary conditions, and assuming a radiation source-free cavity, the electric field satisfying Maxwell’s equations can be written as \[123\]

\[
E_x(z,t) = \sqrt{\frac{2\omega^2}{\epsilon_0 c^2}} q(t) \sin(kz), \tag{7}
\]

where \( k = \omega/c \) is the wave number (\( \omega \) is the frequency of the mode and \( c \) is the speed of light in vacuum), \( \epsilon_0 \) is the vacuum permittivity, \( q(t) \) is a time-dependent factor having the dimension of length (meters), and \( V_o \) is the effective volume of the cavity\(^5\). For the present purposes we will assume the frequency is one of those allowed by the boundary conditions, namely, \( \omega_n = c(n\pi/L) \), where \( n = 1, 2, ... \).

Similarly, the magnetic field can be written \( B(r,t) = e_y B_y(z,t) \), where \( e_y \) is a unit-length polarization vector, and where \[123\]

\[
B_y(z,t) = \frac{\mu_0 \epsilon_0}{k^2} \sqrt{\frac{2\omega^2}{\epsilon_0 \mu_0}} p(t) \cos(kz). \tag{8}
\]

Here \( p(t) = \dot{q}(t) \), where the dot denotes the time derivative, and \( \mu_0 \) is the vacuum permeability. Based on these equations it is then straightforward to show that the Hamiltonian, \( H_o \), of the electromagnetic field can be written \[123\]

\[
H_o = \frac{1}{2} \int dV_o \left( \epsilon_0 E_x^2(z,t) + \frac{1}{\mu_0} B_y^2(z,t) \right). \tag{9}
\]

Substituting \( E_x(z,t) \) and \( B_y(z,t) \) in \( H_o \) from Eq. (7) and Eq. (8) respectively and exploiting that \( \sin^2(\pm z) + \cos^2(\pm z) = 1 \) the Hamiltonian of the single-mode electromagnetic field can be written as

\[
H_o = \frac{1}{2} (p^2 + (\omega q)^2). \tag{10}
\]

This equation can be compared with the Hamiltonian of the classical harmonic oscillator for a particle of mass \( m \) viz., \( H_o = \frac{1}{2}(p^2/m + m\omega^2q^2) \), where we have taken the generalised coordinate \( q = x \) and set \( p = m\dot{x} \), \( x \) being the position. Comparing these two Hamiltonians, it can be seen that a single-mode electromagnetic field is formally equivalent to a harmonic oscillator of unity mass, where the electric and magnetic fields play roles similar to that of the position and momentum of a particle\(^6\).

In quantum systems we replace variables, such as \( q, p \) and \( H \) of the classical system, by their corresponding operators\(^7\), e.g. \( \hat{q}, \hat{p} \) and \( \hat{H} \). Then the Hamiltonian of the single-mode electromagnetic field becomes \( \hat{H}_o = \frac{h}{2} (\hat{p}^2 + (\omega \hat{q})^2) \). As such, we can now see how a single mode of a CV system can indeed be described as a single quantum harmonic oscillator. Furthermore, note that the operators \( \hat{q} \) and \( \hat{p} \) are Hermitian (or self-adjoint). In quantum physics Hermitian operators correspond to observable quantities, where an observable is an operator that corresponds to a physical quantity, such as position or momentum, that can be measured.

However, it will be useful to introduce non-Hermitian operators \( \hat{a} \) (the annihilation operator) and \( \hat{a}^\dagger \) (the creation operator). These can be written as

\[
\hat{a} = (2\omega \hbar)^{(-1/2)} (\omega \hat{q} + i\hat{p}), \tag{11}
\]

\[
\hat{a}^\dagger = (2\omega \hbar)^{(-1/2)} (\omega \hat{q} - i\hat{p}), \tag{12}
\]

where \( \hbar = h/2\pi \), with \( h \) being Planck’s constant. These bosonic field operators satisfy the commutation relation

\(^5\)To apply this formalism to the free field we calculate the physical observables we are interested in and then simply take the limit \( V_0 \to \infty \).

\(^6\)We emphasize that the terms ‘position’ and ‘momentum’ here simply refer to the similar roles played by the field quadratures and position and momentum of a particle - e.g. the ‘position quadrature’ does not in any manner refer to the position of a photon.

\(^7\)Note that operators can be regarded as matrices. In fact, the operator and matrix viewpoints turn out to be completely equivalent.\[8\]
bosonic field operators over the Fock states is given by \[9\],

photons (quanta) of frequency \(\omega_n\) the state of the electromagnetic field containing exactly \(|n\rangle\) the Fock state spanned by the Fock, or number-state basis, \(\delta\) the commutation relationships \(k\) In terms of the quadrature operators we can then re-write \(E_x(z, t)\) as

Removing the time dependence in the creation and annihilation operators by re-setting \(\hat{a} = \hat{a}(0)\) and \(\hat{a}^\dagger = \hat{a}^\dagger(0)\), we can in turn define the quadrature operators (see later discussion on the freedom to choose the specific form of these)

\[
\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger),
\]

\[
\hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger).
\]

In terms of the quadrature operators we can then re-write \(E_x(z, t)\) as

\[
E_x(z, t) = 2\sqrt{\left(\frac{\hbar}{V_0 \epsilon_0}\right)} \sin(kz)\left[\hat{X}_1 \cos(\omega t) + \hat{X}_2 \sin(\omega t)\right].
\]

As such, we can see that the quadratures \(\hat{X}_1\) and \(\hat{X}_2\) can be considered as the amplitudes of the electric field’s time-dependent cos and sin components, respectively. Clearly, these components are 90\(^\circ\) out of phase with each other - hence the name, quadratures. The quadratures satisfy the commutation relation \([\hat{X}_1, \hat{X}_2] = i/2\epsilon_0\).

A CV system of \(N\) modes follows a similar description to that we have just given for a single mode, except of course the Hilbert space containing the multimode system is larger. The \(N\)-mode system may be described by a Hilbert space given by the tensor product \(\mathcal{H} = \otimes_{k=1}^N \mathcal{H}_k\), where \(\mathcal{H}_k\) is a single-mode Hilbert space associated with the \(k\)-th mode. The creation and annihilation operators for each mode then satisfy the commutation relationships

\[
[\hat{a}_k, \hat{a}_{k'}^\dagger] = [\hat{a}_k^\dagger, \hat{a}_{k'}] = 0, \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'},
\]

where \(\delta_{kk'}\) is the Kronecker delta function.

Consider again the single-mode Hilbert space \(\mathcal{H}_k\). This is spanned by the Fock, or number-state basis, \(|n\rangle\) for \(n = 0, 1, 2, \ldots\) where the Fock state \(|n\rangle\) is the eigenstate of the number operator \(\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k\), i.e., \(\hat{n}_k |n\rangle = n |n\rangle\). Put simply, \(|n\rangle\) represents the state of the electromagnetic field containing exactly \(n\) photons (quanta) of frequency \(\omega_k\). Note that for each mode \(k\) there exists a vacuum state which contains no quanta of the field, namely, \(|0\rangle\), satisfying \(\hat{a}_k |0\rangle = 0\). The action of the bosonic field operators over the Fock states is given by \[9\],

\[
\hat{a}_k |n\rangle = \sqrt{n} |n-1\rangle_k, \quad \hat{a}_k^\dagger |n\rangle_k = \sqrt{n+1} |n+1\rangle_k.
\]

This is due to the constraint imposed by quantum mechanics that \([\hat{a}, \hat{a}^\dagger] = 1\), where the commutator between two operators \(\hat{x}\) and \(\hat{y}\) is defined to be \([\hat{x}, \hat{y}] = \hat{x}\hat{y} - \hat{y}\hat{x}\). Note that since the annihilation and creation operators are non-Hermitian, they correspond to non-observable quantities.

It can be easily shown that our new non-Hermitian operators have a time dependence, under free evolution, which can be expressed as \(\hat{a} = \hat{a}(0) \exp(-i\omega t)\) and \(\hat{a}^\dagger = \hat{a}^\dagger(0) \exp(i\omega t)\). As such, the electric field operator can then be re-written as

\[
\hat{E}_x(z, t) = \frac{1}{2} \left(\frac{\hbar}{V_0 \epsilon_0}\right) \sin(kz) \left(\hat{X}_1 \cos(\omega t) + \hat{X}_2 \sin(\omega t)\right).
\]

Having now formally defined the vacuum state, it is probably useful to note for the unwary that some apparent inconsistency lies lurking in the literature (including the many references of this work). This applies to both the constant value applied to \(\hbar\), as well as the nomenclature itself. We note that our quadrature operators, as defined thus far, can be used to form \(\hat{q} = \sqrt{2\hbar/\omega} \hat{X}_1\) and \(\hat{p} = \sqrt{2\hbar/\omega} \hat{X}_2\); from which we can easily show consistency with \([\hat{q}, \hat{p}] = i\hbar\). In many works we will find that \(\hat{q}\) and \(\hat{p}\) written in this form (and also in ‘dimensionless’ form with, say, \(\hbar = \omega = 1\)) are also referred to as the ‘quadratures.’ Also, in many works the cofactor of \(1/2\) in front of our definitions of \(\hat{X}_1\) and \(\hat{X}_2\) is replaced by some other constant, e.g., \(1/\sqrt{2}\) or 1 - allowable re-definitions of course. It is straightforward to determine the vacuum expectation value for any well-defined defined operator (or function of that operator), e.g. \(\langle 0 | \hat{X}_1^2 | 0 \rangle = 1/4\) and \(\langle 0 | \hat{q}^2 | 0 \rangle = \hbar^2/(2\omega)\). It is common to set \(\hbar\) to some numerical constant, usually \(1/2\), 1 or 2. However, no consistency exists in the literature on this either. Setting \(\hbar = 2\) has the convenience of setting the vacuum-state variance of the \(\hat{q}\) and \(\hat{p}\) operators to 1 (when \(\omega\) set to unity).

Bearing in mind the above discussion of inconsistency in nomenclature, we adopt henceforth that \(\hbar = 2\) and \(\omega = 1\) (unless stipulated otherwise). We also redefine the ‘quadrature’ operators to be \(\hat{q}_k\) and \(\hat{p}_k\), now given by the simpler form \(\hat{q}_k = \hat{a}_k + \hat{a}_k^\dagger\) and \(\hat{p}_k = i(\hat{a}_k^\dagger - \hat{a}_k)\). This will make the notation to follow less cluttered.

Defining the vector of quadrature operators for \(N\) modes as \(\hat{R} = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_N, \hat{p}_N)\), the commutation relationship between the quadrature operators can be written as \([\hat{R}_i, \hat{R}_j] = 2i\hbar \Omega_{ij}\), where \(\Omega_{ij}\) is the \(i\)-th (\(j\)-th) element of the vector \(\hat{R}\), and \(\Omega_{ij}\) is the element of the matrix

\[
\Omega = \begin{pmatrix}
\frac{1}{2} & \cdots & \frac{1}{2} \\
\vdots & \ddots & \vdots \\
\frac{1}{2} & \cdots & \frac{1}{2}
\end{pmatrix},
\]

Since a Hermitian operator has an orthogonal set of eigenvectors with real-valued eigenvalues, the quadrature operator \(\hat{q} (\hat{p})\) (which is Hermitian) is an observable with continuous eigenspectra, i.e., \(\hat{q} | q \rangle = q | q \rangle\) \(\hat{p} | p \rangle = p | p \rangle\) with orthogonal eigenvectors or eigenstates \(|q\rangle\) \(|p\rangle\) having continuous eigenvalues \(q \in \mathbb{R}\) \(p \in \mathbb{R}\). Note that the two sets of eigenstates \(|q\rangle\) and \(|p\rangle\) identify two different bases (i.e., two different sets of orthogonal and complete eigenstates), and each set constitutes a common basis for CV quantum information. A CV quantum state can be defined as a continuous-valued superposition of the field’s eigenstates.

All the physical information about a CV system is contained in its quantum state, represented by a density operator \(\hat{\rho}\), which is a trace-one positive operator. A quantum state \(\hat{\rho}\) is said to be a pure state, when we have \(\hat{\rho}^2 = \hat{\rho}\). A pure state can be described as \(\hat{\rho} = |\psi\rangle \langle \psi|\), where \(|\psi\rangle\) is the vector representation of the pure quantum state. A mixed quantum state is defined as a statistical ensemble of pure states, which cannot be described by a single vector. Instead, it is described by its associated density operator. The density operator describing a
mixed state is in the form of \( \hat{\rho} = \sum_i p_i |\psi_i\rangle \langle \psi_i| \), where \( p_i \) is the specific fraction of the ensemble found in each pure state \( |\psi_i\rangle \).

A quantum state \( \hat{\rho} \) of a N-mode CV system can also be described in terms of a characteristic function \( \chi_{\rho}(\xi) = \text{Tr}(\hat{\rho} \hat{D}(\xi)) \), where \( \text{Tr} \) denotes trace, \( \hat{D}(\xi) = \exp(iR \xi \Omega) \) is the Weyl operator \([9], [104]\), and \( \xi \in \mathbb{R}^{2N} \). The quantum state \( \hat{\rho} \) can also be described in terms of a Wigner function (quasi-probability distribution), which is given by the Fourier transform of the characteristic function \( \chi_{\rho} \) as \([9], [104]\).

\[
W(R) = \int_{\mathbb{R}^{2N}} e^{2iN\xi} \frac{(2\pi)^{-N}}{2} \exp(-iR\xi) \chi_{\rho}(\xi),
\]

where \( R = (q_1, p_1, \ldots, q_N, p_N) \) is the vector of quadrature variables, with the real-valued variables \( q \) and \( p \) being the eigenvalues of the quadrature operators. Note that for a single-mode quantum state the probability distribution of the quadrature measurement (marginal distribution) is obtained from the Wigner function of the quantum state by integration over the conjugate quadrature.

The CV quantum states can be visualized using their Wigner function in a phase-space representation, where the axes are defined by a pair of conjugate quadrature variables \( q \) and \( p \). In such a phase space, a classical optical field is represented by a single point corresponding to its complex-valued field amplitude. However, the quantum states of light cannot be represented by a single point, since conjugate quadrature variables cannot be measured simultaneously with arbitrary precision due to the Heisenberg uncertainty relationship.

Hence the Wigner function is utilized to represent the quantum states in the phase space \([9], [102]-[104]\).

In a N-mode CV system the Heisenberg uncertainty relationship is defined for the quadrature operators of each mode \( k \), and is given by \( V(\hat{q}_k)V(\hat{p}_k) \geq 1 \), where \( V \) is the variance of the quadrature operator, and is given by \( V(\hat{R}_k) = \langle \hat{R}_k^2 \rangle - \langle \hat{R}_k \rangle^2 \), where \( \langle \cdot \rangle \) denotes the mean value. Note again, that the quadrature variance of the vacuum state of a single mode is one, i.e., we have \( V(\hat{q}) = V(\hat{p}) = 1 \), which is the lowest possible variance reachable symmetrically by the \( \hat{q} \) and \( \hat{p} \) quadratures according to the uncertainty relationship.

### A. Gaussian quantum states

Gaussian quantum states (for a detailed review, see \([103]\), \([104]\), \([122]\)) are completely characterized by the first moment (or the mean value) of the quadrature operators \( \langle \hat{R} \rangle \) and a covariance matrix \( \mathbf{M} \), i.e. a matrix of the second moments of the quadrature operators defined as

\[
M_{ij} = \frac{1}{2} \langle \hat{R}_i \hat{R}_j + \hat{R}_j \hat{R}_i \rangle - \langle \hat{R}_i \rangle \langle \hat{R}_j \rangle.
\]

(21)

The CM of a N-mode quantum state is a \( (2N \times 2N) \) real symmetric matrix, which must satisfy the uncertainty principle, viz., \( \mathbf{M} + i \mathbf{\Omega} \geq 0 \). By definition, a Gaussian state having \( N \) modes is a CV state whose Wigner function is a Gaussian distribution of the quadrature variables. That is,

\[
W(R) = \exp \left( -\frac{1}{2} \left( R - \langle R \rangle \right)^T \mathbf{M}^{-1} \left( R - \langle R \rangle \right) \right) \frac{(2\pi)^N}{\sqrt{\text{det}(M)}}. \tag{22}
\]

Some important examples of Gaussian states are vacuum states \([9], [103], [104], [122]\), coherent states \([9], [103], [104], [123]\), thermal states \([9], [103], [104], [123]\) and squeezed states \([9], [103], [104], [123]\). We discuss some of these Gaussian states further.

**Vacuum state:** The Wigner function of the vacuum state with respect to the conjugate quadrature variables \( q \) and \( p \) is shown in Fig.9(a), in which the Wigner function is centered at \((0,0)\), which means that the vacuum state has a zero mean. The covariance matrix of the vacuum state is the identity matrix, which means that a vacuum state has a symmetric distribution of the quadrature components (see Fig.9(a)) with both the quadrature components having noise variance of one. This noise is usually termed the vacuum noise or quantum shot noise.

**Coherent state:** A coherent state is generated by applying the displacement operator \( \hat{D} \) to the vacuum state formulated as \( |\alpha\rangle = \hat{D}(\alpha) |0\rangle \), where \( \hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) \) is the displacement operator and \( \alpha = (q + ip)/2 \) is the complex amplitude. Since the displacement operator does not change the variance of the quadratures, coherent states - similarly to vacuum states - exhibit the lowest possible variance reachable symmetrically by the \( \hat{q} \) and \( \hat{p} \) quadratures. The coherent state is the eigenstate of the annihilation operator, which is formulated as \( \hat{a} |\alpha\rangle = \alpha |\alpha\rangle \). To elaborate a little further, this state has a mean value of \( \langle \hat{R} \rangle = (q,p) \), and the covariance matrix is equal to the identity matrix, which means that a coherent state has a symmetric distribution of the quadrature components with both the quadrature components having noise variance equal to one. This symmetric distribution can be seen in Fig.9(b), where the Wigner function of the coherent state with a mean value of \( (3,5) \) (which is the centre of the Wigner function) is shown with respect to the conjugate quadrature variables \( q \) and \( p \). Note that coherent states are much easier to generate in the laboratory than any other Gaussian state. For example, the laser field is in a coherent state. As an important application in the context of quantum communication, coherent states are used to distribute secret keys in Gaussian CV-QKD protocols \([13], [14], [124], [125]\).

**Thermal state:** Thermal states can be described as a mixture of coherent states. The thermal state has a zero mean and a covariance matrix \( \mathbf{M}_{\text{th}} = n I \) associated with \( n = 2\bar{n} + 1 \), where \( n \) is the noise variance of each quadrature component, \( \bar{n} > 0 \) is the average number of photons and \( I \) is the \( (2 \times 2) \)-element identity matrix. This form of the covariance matrix means that a thermal state has a symmetric distribution of the quadrature components, which can be seen in Fig.9(c) where the Wigner function of the thermal state with \( n = 5 \) is shown with respect to the conjugate quadrature variables \( q \) and \( p \). Note that in the generic form of quantum communication the quantum noise of the channel is in a thermal state, called
thermal noise.

**Single-mode squeezed vacuum state**: According to the Heisenberg uncertainty relationship, the lowest possible variance reachable symmetrically by the \( \hat{q} \) and \( \hat{p} \) quadratures is one i.e., the noise variance of the vacuum state. A reduction in the variance of the \( \hat{q} \) (or \( \hat{p} \)) quadrature below the vacuum noise is possible by squeezing. In squeezing, the variance of one continuous variable is in fact decreased below the vacuum noise, while the variance of the conjugate variable is increased. For instance, in a \( \hat{q} \)-squeezed light, the variance of the \( \hat{q} \) quadrature is reduced below the vacuum noise, while the variance of the \( \hat{p} \) quadrature is increased above the vacuum noise. A single-mode squeezed vacuum state is generated by applying the single-mode squeezing operator of \( \hat{S}_r(r_s) = \exp[r_s(\hat{a}^2 - \hat{a}^{\dagger 2})/2] \) \([9], [103], [104], [123]\) to the vacuum state, where \( r_s \in [0, \infty) \) represents the single-mode squeezing parameter.\(^{11}\) Such a squeezed state has zero mean and a covariance matrix of \( M = \text{diag}[\exp(-2r_s), \exp(2r_s)] \) when the quantum fluctuations of the \( \hat{q} \) quadrature have been squeezed. In this case for the single-mode squeezing represented by \( r_s > 0 \) we have \( V(\hat{q}) < 1 \) and \( V(\hat{p}) > 1 \). This means that a single-mode squeezed state does not have a symmetric distribution of the quadrature components, since the variance of one of the quadratures is reduced by squeezing at the expense of an increase in the variance of the conjugate quadrature by the counterpart operation of anti-squeezing. Note, the state still obeys the Heisenberg uncertainty relationship. Such an asymmetric distribution of quadrature components can be seen in Fig.\( [9] d) \), where the Wigner function of the single-mode squeezed vacuum state with \( r_s = 0.5 \) is shown. Here, the \( \hat{q} \) quadrature is squeezed. In terms of applications in quantum communications, single-mode squeezed vacuum states are also utilized to distribute secret keys in Gaussian CV-QKD protocols \([12], [126]\). Note that for \( r_s = 0 \), the single-mode squeezed state corresponds to the vacuum state.

**Two-mode squeezed vacuum state**: A two-mode squeezed vacuum (TMSV) state is generated by applying the two-mode squeezing operator of \( \hat{S}_r(r) = \exp[r(\hat{a}\hat{b} - \hat{a}^{\dagger}\hat{b}^{\dagger})/2] \) \([9], [103], [104], [123]\) to a pair of vacuum states \( |0\rangle\langle 0| \), where \( r \in \mathbb{R} \) is the two-mode squeezing parameter, and the indices 1 and 2 represent the two modes. A TMSV state is described in the Fock basis as \([9], [103], [104], [123]\)

\[
|\text{TMSV}\rangle = \sum_{n=0}^{\infty} q_n |n\rangle_1 |n\rangle_2, \tag{23}
\]

where \( q_n = \sqrt{1 - \lambda^2}\lambda^n \), and \( \lambda = \tanh r \). The two-mode squeezing in dB is given by \(-10\log_{10} [\exp(-2r)]\). Such a squeezed state has a zero mean, and a covariance matrix in the following form \([9], [103], [104], [123]\)

\[
M = \begin{pmatrix}
\frac{v}{\sqrt{v^2 - 1}} & Z \\
\sqrt{v^2 - 1} & \frac{v}{\sqrt{v^2 - 1}} Z
\end{pmatrix}, \tag{24}
\]

where \( v = \cosh(2r) \) is the quadrature variance of each mode, and \( Z = \text{diag}(1, -1) \). Note that the two-mode squeezing operator \( \hat{S}_r \) cannot be factorised into the product of the two single-mode squeezing operators \( \hat{S}_r \). Hence, the TMSV state is not a product of the two single-mode squeezed vacuum states. In fact, the squeezing (anti-squeezing) operation applied to the quantum fluctuations does not squeeze (anti-squeeze) the variance of the individual modes, but rather that of the superposition of the two modes, so that we have \( V(\hat{q}_-) = V(\hat{p}_+) = \exp(-2r) \) and \( V(\hat{q}_+) = V(\hat{p}_-) = \exp(2r) \), where \( \hat{q}_- = (\hat{q}_1 - \hat{q}_2)/\sqrt{2} \), \( \hat{p}_+ = (\hat{p}_1 + \hat{p}_2)/\sqrt{2} \), \( \hat{q}_+ = (\hat{q}_1 + \hat{q}_2)/\sqrt{2} \), and \( \hat{p}_- = (\hat{p}_1 - \hat{p}_2)/\sqrt{2} \). For a two-mode squeezing operation with \( r > 0 \), we have \( V(\hat{q}_-) = V(\hat{p}_+) < 1 \) and \( V(\hat{q}_+) = V(\hat{p}_-) > 1 \). The correlations between the quadratures of the two modes are known as Einstein-Podolski-Rosen (EPR) correlations, which indicate the presence of bipartite entanglement. Hence, for the two-mode squeezing operation with \( r > 0 \) the two modes are entangled, where the entanglement increases upon increasing \( r \). The TMSV state associated with \( r > 0 \) is the most commonly used Gaussian entangled state \([9], [99], [103], [104], [121], [122]\). In the limit of \( r \to \infty \) we have a maximally entangled state having perfect correlations, yielding \( \hat{q}_1 = \hat{q}_2 \) and \( \hat{p}_1 = -\hat{p}_2 \). Note that for \( r = 0 \) the TMSV state corresponds to two (non-entangled) vacuum states.

The Gaussian entangled squeezed states can be generated by parametric down conversion in a non-degenerate optical parametric amplifier \([127]–[131]\), where a crystal having an optical nonlinearity is pumped by a bright laser beam. A photon of the incoming pumping beam spontaneously transfigures in the non-linear crystal into a lower-energy pair of photons, termed as the signal and the idler \([127]–[131]\). In Type-II parametric down conversion, which is known as a source of entangled states in the CV domain, the signal and idler are in orthogonal polarizations, forming a Gaussian entangled squeezed state \([127]–[131]\). In this process, the pump photons of frequency \( 2\omega_p \) are converted into pairs of entangled photons having a pair of different-frequency modes, namely modes 1 and 2 of frequency \( \omega_1 \) and \( \omega_2 \), where \( 2\omega_p = \omega_1 + \omega_2 \). An alternative way of generating the Gaussian entangled squeezed state is by mixing two orthogonally single-mode squeezed vacuum states, where one of the states is squeezed in the \( \hat{q} \) quadrature and the other one is squeezed in the \( \hat{p} \) quadrature. This mixing can be achieved by a balanced (or 50:50) beam splitter. Note that the single-mode squeezed vacuum state can be generated by Type-I parametric down conversion in a degenerate optical parametric amplifier, where the pump photons of frequency \( 2\omega_p \) are split into pairs of photons having the same frequency and polarization \([131]\).

Finally, note that by invoking local unitary operators the first moment of every two-mode Gaussian state can be set to zero and the CM can be transformed into the following standard form \([103], [104], [122]\)

\[
M_n = \begin{pmatrix}
A & C \\
C^T & B
\end{pmatrix}, \tag{25}
\]

where we have \( A = aI, B = bI, C = \text{diag}(c_+, c_-) \), \( a, b, c_+, c_- \in \mathbb{R} \).\(^{11}\)Note, in general, squeezing parameters are complex numbers. For simplicity (and to be consistent with most of the literature) we limit them here to real numbers.
signal mode.
of quantum mechanics, and is widely recognized as a basic resource for quantum information processing and quantum communications (for review, see [59], [104], [121], [122]). We now attempt to quantify the entanglement property of CV states more carefully. We focus our attention on bipartite CV entanglement, which relies on the entanglement between two CV quantum systems. Let us consider the pair of CV quantum systems $A$ and $B$ having Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$, respectively. The Hilbert space of the composite system is given by the tensor product $\mathcal{H}_A \otimes \mathcal{H}_B$. By definition, a bipartite quantum state $\hat{\rho}_{AB}$ relying on the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ is said to be separable, if it can be formulated as a probability distribution over a pair of uncorrelated states expressed as $\hat{\rho}_{AB} = \sum_i \rho_i^A \otimes \rho_i^B$, where the quantum state $\rho_i^A (\rho_i^B)$ acts on the Hilbert space $\mathcal{H}_A (\mathcal{H}_B)$, $\rho_i \geq 0$, and $\sum_i \rho_i = 1$. If a quantum state $\hat{\rho}_{AB}$ is separable, then its partial transpose $\hat{\rho}_{AB}^{PT}$ with respect to either subsystem is positive [132]. The partial transposition of $\hat{\rho}_{AB}$ represents the transposition with respect to only one of the two subsystems, for example to system $B$. By definition, a state is stated to be entangled, when it is not separable in the above-mentioned sense.

The grade (or quantifiable measure) of entanglement in a pure bipartite quantum state $|\psi\rangle$ (with density operator $\hat{\rho}_{AB} = |\psi\rangle \langle \psi|$) can be quantified by the entropy of entanglement $E_{\nu}(\hat{\rho}_{AB})$. The entropy of entanglement stipulates the number of entangled qubits (measured in ebits) that can be extracted from the state. Also it can be considered as the amount of entanglement required to generate the state. The entropy of entanglement is given by the von Neumann entropy of the reduced density operators $\hat{\rho}_A$ or $\hat{\rho}_B$, where $\hat{\rho}_A = \text{Tr}_B (\hat{\rho}_{AB})$ and $\hat{\rho}_B = \text{Tr}_A (\hat{\rho}_{AB})$, with $\text{Tr}_A$ and $\text{Tr}_B$ denoting the partial trace [99], [104], [121], [122].

For a Gaussian state $\hat{\rho}$, the von Neumann entropy $S(\hat{\rho})$ is given by $S(\hat{\rho}) = \sum_k g(n_k)$, where we have $g(x) = [(x + 1)/2] \log_2 [(x + 1)/2] - [(x - 1)/2] \log_2 [(x - 1)/2]$, and $\nu_k$ are the symplectic eigenvalues of the covariance matrix of the state. For a pure two-mode entangled state in the form of $|\psi\rangle = \sum_{n=0}^{\infty} |n\rangle_1 |n\rangle_2$, the entropy of entanglement is given by $E_{\nu}(\hat{\rho}_{AB}) = -\sum_{n=0}^{\infty} \nu^n \log_2 \nu^n$.

Among the different quantifiable measures used as a grade of entanglement for a mixed bipartite quantum state $\hat{\rho}_{AB} = \sum_i \rho_i |\psi_i\rangle \langle \psi_i|$, the most well-known is perhaps the entanglement of formation [133], [134], $E_f$. This is defined as $E_f (\hat{\rho}_{AB}) = \min_{\{\{i\} \in I_2\}} \sum_i \rho_i E_f (\psi_i)$, where the minimum is taken over all the possible pure-state decompositions of the mixed state $\hat{\rho}_{AB}$. The entanglement of formation gives the minimal amount of entanglement of any ensemble of pure states realizing the given state $\hat{\rho}_{AB}$ - meaning it quantifies the minimum amount of entanglement needed to prepare the quantum state $\hat{\rho}_{AB}$ from a mix of pure entangled states. In fact, given an entangled state $\hat{\rho}_{AB}$, the entanglement of formation expresses the number of maximally entangled states we need to create $\hat{\rho}_{AB}$. In general, this measure of entanglement is difficult to calculate.

The distillable entanglement is another measure for entanglement, and is the amount of entanglement that can be distilled from a given mixed state [121]. This quantity is also hard to calculate in general, since it would require optimization over all possible distillation protocols. However, there is an entanglement measure which is easy to compute, and gives an upper bound on the amount of distillable entanglement. This measure is the so-called logarithmic negativity [135], [136].

The logarithmic negativity exhibits the following properties. (i) $E_{LN}$ is a non-negative function, $E_{LN}(\hat{\rho}_{AB}) \geq 0$. (ii) If $\hat{\rho}_{AB}$ is separable, $E_{LN}(\hat{\rho}_{AB}) = 0$. (iii) $E_{LN}(\hat{\rho}_{AB})$ does not increase on average under local (quantum) operations and classical communications. The logarithmic negativity of a bipartite state $\hat{\rho}_{AB}$ is defined as [135]

$$E_{LN}(\hat{\rho}_{AB}) = \log_2 [1 + 2N(\hat{\rho}_{AB})],$$

where $N(\hat{\rho}_{AB})$ is the negativity defined as the absolute value of the sum of the negative eigenvalues of $\rho_{AB}^{PT}$. The logarithmic negativity quantifies as to what degree the quantum state fails to satisfy the positivity of the partial transpose condition.

In the special case of two-mode Gaussian states, we are able to determine the logarithmic negativity through the use of the covariance matrix [99], [104], [122]. Given a two-mode Gaussian state associated with a covariance matrix $M = \{A, C; C^T, B\}$ where $A = A^T$, $B = B^T$, and $C$ are $2 \times 2$ real matrices, the logarithmic negativity is given by [99], [104], [122]

$$E_{LN}(M) = \max \{0, -\log_2 (\tilde{\nu}_-)\},$$

where $\tilde{\nu}_-$ is the smallest symplectic eigenvalue of the partially transposed $M$. This eigenvalue is given by [99], [104], [122]

$$\tilde{\nu}_- = \left(\Delta - \sqrt{\Delta^2 - 4 \det(M)}\right) / 2,$$

where $\Delta = \det(A) + \det(B) - 2 \det(C)$.

D. Gaussian lossy quantum channel

Consider a fixed-attenuation channel described by a transmissivity of $0 \leq \tau \leq 1$ and thermal noise variance of $V_n \geq 1$. Note that in the optical frequency domain the average number of photons is very low even at room temperature (300K), hence the thermal noise has a negligible impact on the signal. In fact, in the optical frequency domain the noise variance is effectively unity, simply representing the vacuum noise. However, in the millimeter-wave domain the thermal noise exhibits a variance, $V_n$, which is much higher than unity. More specifically, we have $V_n = 2n + 1$ with $n$ being the average number of photons [137]–[140]. In order

$\Delta = \det(A) + \det(B) - 2 \det(C)$
Transmitter $\hat{\beta}$ 

Beam splitter ($\tau$) 

Mode 1 

Mode 2 

Receiver $\hat{\rho'}$ 

Fixed-attenuation channel ($\tau, V_n$) 

Fig. 11. The beam splitter representation of a fixed-attenuation channel with transmissivity $\tau$ and thermal noise variance $V_n$.

to suppress the thermal noise, the system has to be operated at very low temperatures, e.g. $< 100 \text{mK}$. The average number of photons for a single mode is given by $n = \exp(hf/k_B T_b) - 1^{-1}$, where $f$ is the frequency of the mode, $k_B$ is the Boltzmann’s constant, and $T_b$ is the temperature.

A fixed-attenuation channel is a Gaussian channel, which transforms the Gaussian input states into Gaussian states. For example, if a single-mode Gaussian quantum state is transmitted through a fixed-attenuation channel, it will remain Gaussian at the output of the channel even though it has experienced channel loss. We can model the impact of a fixed-attenuation channel of transmissivity $\tau$ and thermal noise variance $V_n$ on the single-mode input Gaussian state $\hat{\rho}$ by a beam splitter transformation, with the transmissivity of the beam splitter being $\tau$ and reflectivity $1 - \tau$.

In this channel representation shown in Fig. 11, the Gaussian input state is combined with the thermal noise in the beam splitter, such that one input mode of the beam splitter is the Gaussian input state $\hat{\rho}$ having the corresponding quadratures of $\hat{q}_1, \hat{p}_1$ and the second input mode is the thermal noise with corresponding quadratures of $\hat{q}_2, \hat{p}_2$. As a result of the beam splitter transformation we have the output modes $\hat{1}'$ (corresponding to the received quantum state $\hat{\rho}'$ at the output of the channel) and $\hat{2}'$ with corresponding quadratures of $\hat{q}'_1, \hat{p}'_1$ and $\hat{q}'_2, \hat{p}'_2$ respectively. These output quadratures can be described by

$$\hat{R}_{out} = \begin{pmatrix} \sqrt{\tau} I & \sqrt{1 - \tau} I \\ -\sqrt{1 - \tau} I & \sqrt{\tau} I \end{pmatrix} \hat{R}_{in}, \quad (30)$$

where $\hat{R}_{in} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2)$, and $\hat{R}_{out} = (\hat{q}'_1, \hat{p}'_1, \hat{q}'_2, \hat{p}'_2)$. As a result, the quadrature variance of the received quantum state at the output of the channel is given by $V(\hat{q}'_1) = \tau V(q_1) + (1 - \tau) V_n$, and $V(\hat{p}'_1) = \tau V(p_1) + (1 - \tau) V_n$.

Let us now use such a channel representation to analyse the evolution of a two-mode Gaussian quantum state over a fixed-attenuation channel (the general multimode case can be significantly more complex, e.g. [141]). We consider a TMSV state with zero mean and covariance matrix in the following form $[104], [142]$

$$M_{sm} = \begin{pmatrix} \frac{v I}{\sqrt{\tau} \sqrt{v^2 - 1} Z} & \frac{\sqrt{\tau} \sqrt{v^2 - 1} Z}{\sqrt{\tau} \sqrt{v^2 - 1} Z} \\ \sqrt{\tau} \sqrt{v^2 - 1} Z & \frac{(v + 1 - \tau) V_n}{Z} \end{pmatrix}, \quad (31)$$

where $v = \cosh(2\tau)$ is the quadrature variance of each mode in the input TMSV state ($\tau$ being the two-mode squeezing parameter).

**IV. CV-QKD**

At the time of writing most of the classical cryptography schemes are based on the Rivest-Shamir-Adleman (RSA) protocol [143] in which the encryption key is public. These cryptography schemes are based on the concept of one-way functions, i.e. on functions which are easy to compute but extremely difficult to invert. Hence, the security of these schemes cannot be proved in principle. In fact, the security of these schemes is not unconditional, since they are based on certain computational power assumptions. Thus, if quantum computers were available today with a substantial amount of computational power, RSA cryptography schemes could be broken. However, unconditional security is indeed possible using the one-time pad scheme of [144], where a symmetric, random secret key is shared between the transmitter and receiver. In the one-time pad scheme, the transmitter (Alice) encodes the message by applying a modular addition between the plaintexts and an equal amount of random bits of the shared secret key. At the receiver, Bob decodes the received message by applying the same modular addition between the received ciphertext and the shared secret key. If Alice and Bob do not reuse their key, the one-time pad scheme of [144] cannot be broken, in principle. However, the main problem of this scheme is the generation of the secret key - a key which is as long as the message itself and must be used only once. This problem becomes severe, when a large amount of information has to be securely transmitted. Partially because of this limitation, public-key cryptography is more widely used than the one-time pad scheme.
QKD is the most well-developed and most widely known protocol of quantum communications. QKD, which is based on the laws of quantum physics, allows Alice and Bob to generate secret keys that can later be used to communicate with information-theoretic (unconditional) security, regardless of any future advances in computational power. A QKD protocol can be divided into two main stages. Firstly, a quantum communication part where a pair of distant and trusted parties, Alice and Bob, generate two sets of correlated data through the transmission of a significant number of quantum states over an insecure quantum channel. Secondly, by the use of a classical post-processing protocol \[145, 146\] operated over a public but authenticated (meaning that the transferred data is known to be unaltered) classical channel, Alice and Bob extract from their correlated data a secret key that is unknown to a potential eavesdropper, Eve. The final key, which is unconditionally secure can then be used to transmit secret messages \[101, 147\]. Note that in QKD the quantum channel is open to any possible manipulation from Eve, which means that Eve has full access to the quantum channel without any computational (classical or quantum) limitation other than those imposed by the laws of quantum physics. However, Eve can only monitor the public classical channel, without modifying the messages (since the channel is authenticated). A schematic of a QKD system is shown in Fig. 12.

The security of QKD is based on some of the fundamental principles of quantum physics. From an attack perspective we could consider that Eve’s ultimate goal is to have a perfect copy of the quantum state sent by Alice to Bob. However, this outcome is impossible owing to the no-cloning theorem of quantum physics, which states that it is impossible to create an identical copy of an arbitrary unknown quantum state while keeping the original state intact \[148, 149\]. This simple, but crucial, observation can be traced back to the fact that quantum mechanics is a linear theory.

There are two main techniques of implementing QKD, DV-QKD where the key information is mapped to a single photon’s phase or polarization \[8, 74, 75\], and CV-QKD where the key information is mapped to the quadrature variables of the optical field \[10–15\]. In the DV-QKD technology detection is realized by single-photon detectors, while in the CV-QKD technology detection is realized by homodyne (or heterodyne) detectors. In this review we will focus our attention on the CV technology to implement QKD.

CV-QKD is mostly implemented experimentally in a prepare-and-measure (PM) scheme \[12–14, 20, 21, 23–25, 126, 150–153\], where Alice prepares CV quantum states and encodes the key information onto the quantum states, which are then transmitted over an insecure quantum channel to Bob. At the output of the channel Bob receives the quantum states and measures them using homodyne or heterodyne detectors. As a result, correlated data is created between Alice and Bob. Each PM scheme of CV-QKD can be represented by an equivalent entanglement-based (EB) scheme \[15, 96, 104, 126, 154, 155\], where Alice generates a two-mode entangled state, with one mode being held by Alice and the other mode being transmitted through an insecure quantum channel to Bob. Alice and Bob then proceed by measuring their own modes using homodyne or heterodyne detectors in order to create correlated data. Following the generation of the correlated data, Alice and Bob proceed with classical post-processing over a public, but authenticate, classical channel (in both the PM scheme and EB scheme), so as to generate a secret key even in the presence of Eve.

CV-QKD protocols using Gaussian quantum states have been richly analysed in theory \[12, 13, 15, 104, 126, 154, 155\], and they have also been implemented experimentally \[14, 20, 21, 23–25, 96, 150–153\]. In the Gaussian PM scheme which is shown in Fig. 13, the CV quantum states prepared by Alice are Gaussian states (squeezed states or coherent states) which are modulated by Gaussian distributions \[12–14, 20, 21, 24, 25, 126, 150, 151, 153, 154\]. In fact, Alice encodes a classical random variable drawn from a Gaussian distribution onto a Gaussian quantum state, which is transmitted to Bob, and then measured by him, thus extracting a classical random variable which is correlated to Alice’s. In the Gaussian PM scheme, the measurements of the received quantum states are made by Gaussian measurements, namely by homodyne or heterodyne detection.

CV-QKD protocols using squeezed states \[12\] can be described as follows. Alice generates a real random Gaussian-distributed variable \(a\) with zero mean and variance \(\sigma_m\). She also generates a random bit \(u\), and then prepares a single-mode squeezed vacuum state having the covariance matrix \(M = diag(1/v, v)\), where \(v = \exp(2r_s)\), and where \(r_s\) is the single-mode squeezing. The squeezed state prepared is then modulated (displaced) by an amount \(a\), where the modulation variance satisfies \(\sigma_m = v = 1/v\). In fact, depending on the value of the random bit \(u\), Alice sends a \(q\)-squeezed state having a first moment of \((\sigma_p, 0)\), \(\sigma_p = a\) or a \(p\)-squeezed state associated with the first moment \((0, a_p)\), \(a_p = a\). Hence, Alice randomly chooses to squeeze and displace either the \(\hat{q}\) or the \(\hat{p}\) quadrature. The prepared and modulated squeezed states are then transmitted over an insecure quantum channel to Bob. For each incoming state, depending on his own random bit \(u'\), Bob measures either the \(\hat{q}\) or the \(\hat{p}\) quadrature using homodyne
distribution from a Gaussian variable (a) classically random of \( u \) by applying sifting, where Bob reveals for each pulse the value \( u \). When the quantum communication is finished, classical post-processing over a public channel is commenced and all the incoming states have been measured by Bob, respectively. When the quantum communication is finished, classical post-processing over a public channel commences via a sifting operation. In this operation, Alice and Bob reveal to each other which of the two quadratures they used for preparing (Alice) and measuring (Bob) the information, discarding any incompatible data (i.e., \( a \neq b \)). In fact, Alice reveals for each pulse the value of \( u \) (i.e., whether she displaced the \( \hat{q} \) or the \( \hat{p} \) quadrature), and Bob only retains the cases, where he measured the relevant quadrature (i.e., \( u = u' \)).

Another squeezed-state protocol was developed in [125], in which Bob uses homodyne detection rather than homodyne detection and measures both the \( \hat{q} \) and \( \hat{p} \) quadratures for obtaining \((b_q, b_p)\). In the sifting step of this protocol, Bob then disregards one of his quadrature measurements, depending on Alice’s specific choice of quadrature preparation. This protocol can be seen as a noisy version of the protocol with squeezed states and homodyne detection, since the homodyne detection introduces a vacuum noise into the measurement. When Bob’s data are the reference of error correction (see below) in the classical post-processing, the homodyne detection protocol has a better robustness against the channel noise than the protocol associated with homodyne detection [126].

In contrast to the above CV-QKD protocols using squeezed states, CV-QKD protocols using coherent states [13], [14], [124] can be described as follows. Alice generates a pair of random real numbers, \( a_q \) and \( a_p \), chosen from two independent Gaussian distributions of variance \( \nu_m \). Alice then prepares a coherent state, which is then modulated (displaced) by the amounts of \( a_q \) and \( a_p \), where \((a_q, a_p)\) represents the mean value of the coherent state. The prepared and modulated coherent states are then transmitted over an insecure quantum channel to Bob. For each incoming state, depending on his own random bit \( u' \), Bob measures either the \( \hat{q} \) or the \( \hat{p} \) quadrature using homodyne detection, obtaining a real variable \( b_q \) or \( b_p \), respectively. When the quantum communication is finished and all the incoming states have been measured by Bob, classical post-processing over a public channel is commenced by applying sifting, where Bob reveals for each pulse the value of \( u' \) (i.e., whether he measured the \( \hat{q} \) or the \( \hat{p} \) quadrature), and Alice keeps \( a_q \) or \( a_p \) depending on the value of \( u' \). Note that in this protocol only one of the two real random variables generated by Alice is used for the key after the sifting stage.

Another coherent-state protocol was developed in [125], where Bob uses heterodyne detection rather than homodyne detection and measures both the \( \hat{q} \) and \( \hat{p} \) quadratures for obtaining \((b_q, b_p)\) at the cost of introducing a vacuum noise into the measurement. In this protocol, sifting is no longer needed, since both of the real random variables generated by Alice are used for the generation of the key, hence potentially resulting in higher secret key rates.

For all QKD protocols parameter estimation is performed (in the classical post-processing stage, following the sifting step), where the two parties reveal a randomly chosen subset of their data. This allows them to estimate parameters of the channel, such as the channel’s transmissivity and the channel noise. This allows them to limit the maximum amount of information Eve can have about their values. This step is followed by a reconciliation procedure - which encompasses error correction. As discussed more later, this procedure normally proceeds via the use of low density parity check (LDPC) codes [20]. QKD can be operated in two reconciliation scenarios, direct reconciliation [156] and reverse reconciliation [13], [14].

In the direct reconciliation protocol Alice’s data constitute the reference and she sends classical correction information to Bob which may be overheard by Eve. Then Bob corrects his key elements to arrive at the same values as Alice. By contrast, in the reverse reconciliation protocol Bob’s data constitute the reference and must be estimated by Alice (also by Eve) [13], [14]. Since the upper bound on Eve’s information is estimated during the parameter estimation stage, Alice and Bob apply a privacy amplification protocol (for discarding the information that may be known to Eve) to produce a shared binary secret key.

Note that there are eight protocol choices for characterising Gaussian CV-QKD in a PM scheme. This is because we must consider the type of quantum state (squeezed states or coherent states) which Alice prepares, and also the type of detection (homodyne or heterodyne detection) which Bob applies to the received states, as well as the specific type of reconciliation (direct reconciliation or reverse reconciliation). However, recalling all PM schemes have an equivalent EB scheme, we note all the PM protocols can be described in an unified way.

Fig. 13. Schematic of Gaussian CV-QKD protocols in a PM scheme.
Fig. 14. The EB representation of Gaussian CV-QKD protocols.

using the EB scheme [104], [154] shown in Fig. 14. Here Alice
generates a TMSV state, which we refer to as $\hat{\rho}_{AB}$. She keeps
mode $A$, and sends mode $B$ to Bob. At some time later, Alice
and Bob use an unbalanced beam splitter of transmissivity ($T_A$

A. CV-QKD security analysis

The most powerful, and most general, attack that Eve can
implement against QKD is known as a coherent attack [104],
[154]. In this attack, Eve prepares her ancillary system in a

global quantum state, which means she prepares an arbitrary
joint (entangled) state of the ancillae. After the interaction of
the global ancillary system with the signals sent by Alice, the
output ancillary system is stored in a quantum memory.
once the classical post-processing relying on the public channel
is finished, Eve applies an optimal joint measurement over the
ancillary system stored in the quantum memory to maximize
her knowledge on the quantum information of the trusted
parties. The security analysis of CV-QKD in the face of
cohert attacks is very complex. However, under some trivial
constraint imposed on the classical post-processing protocol,
collective attacks are just as detrimental as coherent attacks
[157]. In a collective attack against QKD Eve prepares her
ancillary system in a product state of identically prepared
ancillae. After interaction of each ancilla with a single signal
sent by Alice, the output ancilla is stored in a quantum
memory. Once the classical post-processing is completed, Eve
applies an optimal joint measurement over the ensemble of
ancillae in the quantum memory.

For a realistic reconciliation algorithm, the asymptotic CV-
QKD key rate (bits per pulse) against collective attacks is given
by [104], [154] $K = \xi I_{AB} - I_E$, where $I_{AB}$ is the mutual

variable, $a$, as well as Bob’s variable, $b$, and $0 < \xi < 1$
is the reconciliation efficiency. This efficiency reflects that in
a realistic reconciliation algorithm, Alice and Bob acquire not
all of the maximum attainable mutual information. Note that
for a perfect reconciliation algorithm we will have $\xi = 1$.
Furthermore, $I_E$ is the Holevo bound defined in [104], [154]
as an upper bound on the quantum information stolen by
Eve. In the reconciliation step, if we assume that Alice’s data
represents the reference, then $I_E = I_{AE}$ is the Holevo bound
on the mutual information between Eve’s quantum memory
and Alice’s variable. By contrast, if we assume that Bob’s
data is the reference, then $I_E = I_{BE}$ is the Holevo bound
on the mutual information between Eve’s quantum memory
and Bob’s variable. Note that $I_{AB}$ remains the same, regardless
of whose data represents the reference of reconciliation. It
was also shown [158] that in the family of collective attacks,
Gaussian attacks based on Gaussian operations [14] are the
optimal attacks Eve can implement so as to minimize the secret
key rate $K$.

Let us consider a Gaussian CV-QKD protocol in the EB
scheme, where Alice generates a TMSV state $\hat{\rho}_{AB}$, and keeps
mode $A$ while sending mode $B$ to Bob over an insecure
quantum channel. In the optimal collective Gaussian attack
(which is also referred to as the entangling-cloner attack [14])
shown in Fig. 15 Eve models the quantum channel (with
transmissivity of $0 \leq \tau \leq 1$ and thermal noise variance
of $\omega \geq 1$) by a TMSV state $\hat{\rho}_{\omega}^{B,E}$ having a quadrature
variance of $\omega$ and a beam splitter of transmissivity $\tau$.
In fact, the quadrature variance of $\hat{\rho}_{\omega}^{B,E}$ and the transmissivity
of the beam splitter in Fig. 15 are tuned in order to inject
the same noise and to impose the same attenuation as in
the original channel, respectively. In this beam splitter Eve
combines the signal mode gleaned from Alice (mode $B$) with
one mode (mode $E_1$) of the TMSV state. The first output of
the beam splitter (mode $B'$) which is the quantum signal
received by Bob is given by $\hat{q}_{B'} = \sqrt{\tau} \hat{q}_B + \sqrt{1-\tau} \hat{q}_{E_1}$,
and $\hat{\rho}_{B'} = \sqrt{\tau} \hat{\rho}_B + \sqrt{1-\tau} \hat{\rho}_{E_1}$. The second output of the
beam splitter (mode $E'_2$) and mode $E_2$ of the TMSV state
\(\hat{\rho}_{E_1,E_2}\) are stored by Eve in a quantum memory. Once the
classical post-processing over the public channel is completed,
this quantum memory is detected by means of an optimal
joint measurement which estimates Alice’s data (in direct
reconciliation) or Bob’s data (in reverse reconciliation). Note
that in a Gaussian CV-QKD protocol, the asymptotic key rate
against optimal collective Gaussian attacks can be calculated
through the equivalent EB scheme based on the covariance
matrix of the two-mode entangled state shared between Alice
and Bob

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14Gaussian operations are linear operations with respect to the quadrature
amplitudes. Such operations maintain the Gaussian character of Gaussian
states.

15Gaussian collective attacks are as strong as coherent attacks in the limit
of an infinite number of quantum states exchanged, however, it is not known
this is the case for a realistic finite-length key protocols.
V. ENTANGLEMENT DISTRIBUTION AND CV-QKD IMPLEMENTATION VIA SATELLITE

A. Entanglement distribution and standard QKD protocols

Let us reconsider the quantum communication architectures of Fig. 3 for CV entanglement distribution and for CV-QKD implementation. We assume that the source of quantum communication in the transmitter(s) is a two-mode entangled state associated with modes 1 and 2. In the scheme (a) (the scheme (b)) of Fig. 3 a two-mode entangled state is generated by Alice at the ground station (satellite) with one mode, mode 1, kept by Alice, while the other mode, mode 2, is transmitted to Bob located at the satellite (ground station) over the uplink (downlink). In the scheme (c) of Fig. 3 a two-mode entangled state is generated by Alice at the ground station transmitter with one mode, mode 1, held at the ground station transmitter and the other mode, mode 2, transmitted over the uplink to the relay satellite. The received mode is then reflected in the satellite and transmitted through the downlink to Bob at the ground station receiver. In the scheme (d) of Fig. 3 a two-mode entangled state is generated on board of the satellite with both modes then sent over the separate downlinks to Alice and Bob located at the separate ground stations. In the scheme (e) of Fig. 3 Alice and Bob are located in the separate ground stations, both initially possessing a two-mode entangled state. One mode of each entangled state is kept by a ground station transmitter and the second mode of each state is transmitted over the uplink to the relay satellite, in which on-board entanglement swapping is performed on the arriving modes. To elaborate a little further, entanglement swapping is a standard quantum protocol conceived for establishing entanglement between distant quantum systems that have never interacted [159–162]. It is the central mechanism of quantum repeaters [31], enabling the distribution of entanglement over large distances. In the scheme (e) of Fig. 3 the received modes are swapped at the satellite via a CV Bell measurement [163], where the two modes are mixed through a balanced beam splitter. Explicitly, the \( q \) quadrature of one of the output modes of the beam splitter and the \( \tilde{r} \) quadrature of the output mode are separately measured by two homodyne detectors. This process is sometimes described by saying that the two output modes of the beam splitter are conjugately homodyned [163]. The classical outcome of the Bell measurement is then communicated to Alice and Bob so that they can optimally displace their modes, according to the measurement outcome, in order to maximize the resultant entanglement shared between the two ground stations. This entanglement swapping scheme between two ground stations via satellite is shown more explicitly in Fig. 16.

As a result of the entanglement distribution in each quantum communication scheme of Fig. 3 there would exist an entangled state shared between Alice and Bob. Once the entangled states have been shared between the stations, for each scheme of Fig. 3 Alice and Bob are able to invoke CV-QKD protocols in the EB scheme by applying homodyne or heterodyne detection of their own modes. The level of entanglement produced by the quantum communication schemes considered here as well as the quantum key rates of the EB CV-QKD protocols in these schemes have recently been analyzed in [88–92].

In the schemes (a), (b), and (c) of Fig. 3 the entangled source originates from one of the trusted parties (Alice). However, in the scheme (d) of Fig. 3 the entangled source originates from the satellite, which in some circumstances may be controlled by the eavesdropper, Eve. In [155], it has been shown that in the context of the EB CV-QKD protocols Alice and Bob can still generate a secure key, even when Eve controls the entanglement source.

B. Measurement-device-independent QKD protocols

In the scheme (e) of Fig. 3 the entangled source originates from both trusted parties (Alice and Bob), however, the Bell measurement at the satellite may be controlled by Eve. In [164], it has been demonstrated that in CV-QKD protocols the secret key to be shared between the two trusted parties can be generated by the measurement of an untrusted intermediate relay. In measurement-device-independent (MDI) protocols
of QKD [164]–[166], Alice and Bob are not connected by direct links, and an intermediate relay is used for completing
the communication link. In MDI protocols the measurement
device is the intermediate relay, whose operation may be
controlled by an adversary. Fig. 16 is in fact one example of
a scenario over which a MDI protocol may be implemented.

The security of CV-MDI protocols is usually analysed using
EB schemes that invokes CV entanglement swapping at the
relay similar to that shown in Fig. 16. Although CV-MDI
protocols are practically implemented in a PM scheme (see
below).

In the EB equivalent of the Gaussian MDI-QKD protocols, a
pair of TMSV states associated with the quadrature variance
$v = \cosh (2r)$ (where $r$ is the two-mode squeezing), is initially
owned by Alice and Bob. One mode of each entangled state
is held by Alice and Bob, while the second mode of each state
is transmitted to the intermediate relay over the insecure channel.
The received modes are swapped via a CV Bell measurement
at the intermediate relay. The swapping process continues by
the relay communicating the Bell measurement result through
a classical public channel to Alice and Bob. After receiving the
Bell measurement outcome, Bob displaces his mode, while Al-
ice keeps her mode unchanged. Then Alice and Bob measure
their modes by homodyne (or heterodyne) detectors to create
correlated data. After the establishment of a sufficiently large
amount of correlated data, Alice and Bob proceed with the
classical post-processing over an authenticated public channel
to create a secret key.

In the EB scheme of the Gaussian MDI-QKD protocols, if
Alice and Bob apply a homodyne detection of their modes, the
scheme becomes equivalent to the PM scheme, in which Alice
and Bob prepare squeezed states, and if Alice and Bob apply
a heterodyne detection of their modes, the scheme becomes
equivalent to the PM scheme in which Alice and Bob prepare
coherent states. We discuss these PM schemes next.

The MDI implementation of Gaussian CV-QKD protocols
in the PM scheme depends on whether the Gaussian resource
is a squeezed or a coherent state. If a squeezed state, Alice
prepares here mode in a squeezed state with the quadrature
variance $v = \exp (2r_s)$, where $r_s$ is the single-mode squeezing.
Which one of the two quadratures is to be squeezed is based
on a randomly generated bit. The chosen quadrature is then
modulated by a random Gaussian-distributed variable with
zero mean and variance $v_m$ conditioned on $v_m = v - 1/v$.
The same procedure is applied independently at Bob’s side.
If the Gaussian resource is a coherent state, Alice prepares
her coherent-state mode with each quadrature independently
modulated by a random Gaussian-distributed variable having
zero mean and variance of $v_m$. Likewise Bob.

Following transmission to the satellite of the modes belong
to Alice and Bob, and irrespective of the Gaussian resource
used, the satellite makes a CV Bell measurement on each mode
pair, announcing the results. Alice and Bob undertake some
modification of their data based on these results and undergo
some classical post-processing to end up with a shared key.
More details of this process can be found in [91].

Note the modulation variance $v_m$ (in the protocol using
coherent states) can reach very high values, e.g., $v_m = 60$
[164]. With the use of squeezed states, however, achieving high
values of squeezing remains experimentally challenging. As
such, quadrature variance $v$ and of the modulation variance $v_m$
are limited in the range of values attained. Note that $v = 5.05$
is equivalent to the two-mode squeezing of 10 dB [167]. Note
also that vacuum squeezing at 15 dB is currently the highest
obtainable in any experiment [168].

Previous contributions on MDI-QKD protocols have mainly
been focussed on fixed-attenuation channels [30], [164],
[169]–[178]. In [91], a MDI implementation has been in-
vestigated in order to establish Gaussian CV-QKD protocols
between two ground stations, where the communication occurs
between the ground stations via a LEO satellite over a pair of
independent atmospheric channels. In this CV-MDI protocol
the measurement device is the satellite itself, which can be
controlled by an adversary. In [91], it has been demonstrated
that while the CV-MDI protocol is only feasible for low-loss
fixed-attenuation channels, the protocol is capable of achieving
a beneficial secure key rate even for transmission over high-
loss atmospheric channels. Note that in MDI-QKD the devices
of Alice and Bob have to be trusted [30], [164], [169]–
[178]. Nonetheless, it has recently been shown that QKD
is possible even when the device of one of the parties is
untrusted [179]–[181]. The security of this one-sided device-
independent protocol using CV quantum states has recently
been investigated both theoretically and experimentally [182],
[183].

We note that MDI protocols represent a step closer to full
device-independent protocols. These latter protocols are based
on Bell violation measurements at the receivers, and represent
the most robust form of QKD (the form that requires the
least number of assumptions). Although some work has been
carried out in relation to CV states in device independent
QKD (e.g. [184]), practical progress is limited due to the very
low key rates expected. CV MDI-QKD protocols, with their
reduced assumptions on how the measurement device must
operate, currently represent the most robust form of QKD that
still lead to reasonable key rates. The MDI protocols remain
unconditionally secure in their generation of keys - the best
an adversary in charge of the measurement device can do
is drive the key rate to zero (e.g. by broadcasting false Bell
measurement results).

C. Entanglement determination and quantum key rate compu-
tation

The evolution of quantum states as they prorogate through
atmospheric fading channels can be considered in two different
scenarios. In the first scenario, the transmission coefficient
$\eta$ of the atmospheric fading channel is unknown, while in
the second scenario it is known. In this latter scenario, it is
assumed that the transmission coefficient can be measured in
real time at the receiver.

1) Scenario 1. The transmission coefficient of the fading
channel is unknown: Here, we consider the distribution of a
two-mode entangled state over satellite-based atmospheric
fading channels. In fact, we assume that the transmitter initially
possesses a two-mode (mode 1 and mode 2) entangled state $\rho$,
with one (or more) of the modes transmitted to the receiving station(s) through atmospheric fading channels. This leads to two operational settings.

**Single-mode transfer:** In this setting we assume that mode 1 of \( \hat{\rho} \) remains at the ground station (satellite), while mode 2 of \( \hat{\rho} \) is transmitted to the satellite (ground station) over the fading uplink (downlink) characterized by the probability distribution \( p(\eta) \) and the maximum transmission coefficient of \( \eta_0 \). The density operator of the two-mode state at the ground station and satellite for each realization of the transmission coefficient \( \eta \) is given by \( \hat{\rho}(\eta) \). Since \( \eta \) is a random variable, the elements of the total density operator of the resultant mixed state \( \hat{\rho}'_t \) are calculated by averaging the elements of the density operator \( \hat{\rho}(\eta) \) over all possible transmission coefficients of the fading channel, giving the ensemble-averaged state of \( \hat{\rho}'_t = \int_0^{\eta_0} p(\eta) \hat{\rho}(\eta) \, d\eta. \) (33)

Now, let us consider the initial two-mode entangled state \( \hat{\rho} \) at the transmitter being a Gaussian state \([85], [86], [88], [89], [186] \). In this case the resultant ensemble-averaged state \( \hat{\rho}'_t \) is a non-Gaussian mixture of the Gaussian states \( \rho'(\eta) \) obtained for each realization of \( \eta \). Since the resultant ensemble-averaged state shared by the ground station and the satellite is a non-Gaussian state, it cannot be completely described by its first and second moments. Therefore, the final entanglement computed based on the covariance matrix of the resultant ensemble-averaged state will represent only the Gaussian entanglement between the ground station and the satellite, but not the total distributed entanglement \([85], [86], [88], [186] \). In order to calculate the total shared entanglement between the stations, the entanglement has to be computed based on the density operator of the resultant ensemble-averaged state \( \hat{\rho}'_t \).

Note that if we use the shared entanglement created for subsequent use in QKD, i.e. a EB CV-QKD protocols operating over atmospheric fading channels\(^{16} \), then the same concept (use of ensemble averaged states) is invoked when the quantum key rate is calculated. Note that when the quantum key rate is in fact calculated based on the covariance matrix of the resultant ensemble-averaged state \( \hat{\rho}'_t \), the key rate computed is only related to the Gaussian component of \( \hat{\rho}'_t \).

**Two-mode transfer:** In this setting we assume that the satellite initially possesses a two-mode entangled state \( \hat{\rho} \), with mode 1 transmitted to ground station 1 over a fading downlink obeying the probability distribution of \( p_1(\eta_1) \) and having the maximum transmission coefficient of \( \eta_{01} \), while mode 2 is transmitted to ground station 2 over a different fading downlink characterized by the probability distribution \( p_2(\eta_2) \) and having the maximum transmission coefficient of \( \eta_{02} \). Here, the two fading downlinks are assumed to be independent. The density operator of the two-mode state at the ground stations for each realization of the transmission coefficients \( \eta_1 \) and \( \eta_2 \) is given by \( \hat{\rho}'(\eta_1, \eta_2) \). The elements of the total density operator of the resultant mixed state \( \hat{\rho}'_t \) are calculated by averaging the elements of the density operator \( \hat{\rho}'(\eta_1, \eta_2) \) over all possible transmission coefficients of the two separate fading channels, giving the ensemble-averaged state of \( \hat{\rho}'_t = \int_0^{\eta_{01}} \int_0^{\eta_{02}} p_1(\eta_1) p_2(\eta_2) \hat{\rho}'(\eta_1, \eta_2) \, d\eta_1 \, d\eta_2. \) (34)

2) **Scenario 2. The transmission coefficient of the fading channel can be measured:** Let us now assume a modified scenario, in which the variable transmission coefficient of the atmospheric fading channel is measured with the aid of a separate coherent signal. For example, when a local oscillator in a polarized mode orthogonal to the signal is sent through the channel. Although this increases the complexity of the system, the grade of entanglement (and hence the quantum key rate of the EB CV-QKD protocols implemented based on this entanglement) generated between the stations will be increased.

When considering this scenario in the single-mode transfer setting where the transmission coefficient \( \eta \) is measured at the receiving station, the final entanglement can be calculated as \[ E = \int_0^{\eta_0} p(\eta) E[\rho'(\eta)] \, d\eta, \] (35)

where \( E[\rho'(\eta)] \) is the grade of entanglement of a state received through the channel of transmission coefficient \( \eta \).

In this scenario, when the initial two-mode entangled state \( \hat{\rho} \) at the transmitter is a Gaussian state, the mixed states \( \rho'(\eta) \) collected at the receiver during each transmission coefficient window remain Gaussian, because within each (small) fading bin we can assume that the transmission coefficient is constant and therefore the states during that particular bin remain Gaussian. In this case, the grade of entanglement of the mixed Gaussian state \( \rho'(\eta) \) i.e., \( E[\rho'(\eta)] \) can be calculated based on the covariance matrix of \( \rho'(\eta) \), which results in \( E \) of Eq. (35) representing the total entanglement shared between the stations \([90] \).

Considering this scenario in the EB CV-QKD protocols communicating over atmospheric fading channels, which are implemented based on the shared entangled states between the stations, the same concept is true when the quantum key rate is calculated. In fact, due to the relatively long coherence time of the atmospheric channel, it may be possible to devise a scheme, in which quantum key rates are derived for each realization of the fading (each fading bin realized), and summed \([90]–[92], [187] \). Indeed, the quantum key rate \( K[\rho'(\eta)] \) resulting from the mixed Gaussian state \( \rho'(\eta) \) can be calculated based on the covariance matrix of \( \rho'(\eta) \), and then the total key rate shared between the stations is calculated by \( K = \int_0^{\eta_0} p(\eta) K[\rho'(\eta)] \, d\eta \) \([90]–[92] \).

Similarly, considering this scenario in the two-mode transfer setting, where the transmission coefficients \( \eta_1 \) and \( \eta_2 \) are measured at the two receiving stations, the final grade of entanglement can be calculated as \[ E = \int_0^{\eta_{01}} \int_0^{\eta_{02}} p_1(\eta_1) p_2(\eta_2) E[\rho'(\eta_1, \eta_2)] \, d\eta_1 \, d\eta_2, \] (36)

\(^{16}\)Note that in \([185] \) a fast-fading channel has been considered where the users are only able to estimate the probability distribution of the channel’s transmission coefficient but not its instantaneous values, while the eavesdropper has full control of the fast-fading channel, so that she chooses the instantaneous transmission coefficient of the channel.
where $E[\rho'(\eta_1, \eta_2)]$ is the entanglement of a state that has traversed two channels having the transmission coefficients of $\eta_1$ and $\eta_2$ [90–92].

### D. Enhancement of quantum communication performance

Satellite-based communication channels tend to suffer from high uplink losses on the order of 25-30 dB (and beyond) for a LEO satellite receiver [51], [63], [113], while single downlink channels are anticipated to have losses of 5-10 dB for a LEO satellite transmitter [51], [63], [113]. Under such high losses, entanglement distribution and QKD via satellite will remain a fruitless endeavor without the beneficial intervention of the post-selection strategy [85] and entanglement distillation techniques [186] detailed below.

1) **Post-selection:** Although atmospheric fading degrades both the entanglement and the quantum key rate, its effects may be mitigated. Post-selection of high transmission-coefficient windows, as introduced in [85] for the case of a single point-to-point fading channel, is capable of improving both the entanglement and the quantum key rate. To elaborate a little further, in the post-selection strategy, a subset of the channel transmittance distribution, namely that associated with the high transmission coefficient, is selected to contribute to the resultant post-selected state and to the post-selected key rate.

To elaborate on the post-selection strategy, in addition to the quantum states, coherent (classical) light pulses are transmitted through the channel in order to estimate the channel’s transmission coefficient $\eta$ at the receiver. The received quantum state is either retained or discarded, conditioned on the channel’s transmission coefficient being higher or lower than the post-selection threshold $\eta_{th}$. Although this post-selection strategy can be invoked for enhancing the grade of entanglement and the quantum key rate between the transmitter and receiver, estimation of the channel’s transmission coefficient will impose additional complexity on both the transmitter and receiver. The operation of this form of post-selection in the scheme (c) of Fig. 3 has been invoked in [88] for enhancing the Gaussian entanglement between the ground stations (which consequently leads to an improvement in the quantum key rates of the EB CV-QKD protocols).

Note that when entangled states are conveyed over a fading channel, both the above-mentioned post-selection and entanglement distillation strategies act as “Gaussification” methods in the sense that the resultant conditioned states approach a Gaussian form due to the enhanced concentration of low-loss states in the final ensemble-averaged state. Note also that using the above-mentioned post-selection and entanglement distillation strategies, the entanglement established between the transmitter and receiver is only probabilistically increased.

Another entanglement distillation technique for is based on applying an initial non-Gaussian operation to the Gaussian entangled states (that again increases the entanglement probabilistically), which is followed by a Gaussification step that iteratively drives the output non-Gaussian state towards a Gaussian state. Non-deterministic noiseless linear amplification has been identified as a method of distilling Gaussian entanglement [196], [204]. It has been shown that the non-deterministic noiseless linear amplification is capable of distilling improved CV entanglement [197], [200], [201] and enhancing CV-QKD performance [203], [204] when applied after the lossy channel to the quantum states received. The non-Gaussian operations which result in the generation of non-Gaussian entangled states will be discussed in detail in the next section.

### VI. **Non-Gaussian CV quantum communication over atmospheric channels**

In the CV domain, previous efforts invested in entanglement distribution and QKD over atmospheric channels have been predominately focussed on Gaussian states [16], [81], [85], [86], [88], [89], [91], [93], [94]. Although Gaussian quantum states are well understood both from a theoretical and from an experimental perspective [103], [104], [122], the employment of CV non-Gaussian quantum states for quantum communication has also garnered interest [205], [225]. Non-Gaussian quantum states are valuable resource for a range

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17Note that only pure states having a positive Wigner function are Gaussian states. However, the Wigner function of non-Gaussian pure states takes on negative values.
of protocols, including teleportation [205], [209], [213], [215], cloning [223], [224] and CV-QKD protocols [220], [222], [225]. For two important reasons, entangled non-Gaussian states are particularly interesting in the context of quantum communication via satellite. The first of these reasons is that the distillation of Gaussian entanglement is impossible using only Gaussian operations [193]–[195]. However, mixed non-Gaussian states can undergo entanglement distillation without any additional requirements. The second reason is that, relative to Gaussian entanglement, non-Gaussian entanglement can be shown in some circumstances to be more robust against decoherence [213], [218], [219].

### A. Non-Gaussian entangled states

CV non-Gaussian states are mostly generated by applying non-Gaussian operations, such as photon subtraction [205], [206], [208]–[211], [214], [215], photon addition [207], [208], [210], [212], [215] and photon replacement [213], [215] to incoming Gaussian states. We discuss here non-Gaussian entangled states which are created probabilistically by applying non-Gaussian operations to (i.e, at the receiver) Gaussian TMSV states. Note that a non-Gaussian operation can be applied to either a single mode, or to both modes, of the incoming Gaussian entangled state. Also note the non-Gaussian operation can be applied to the incoming mode at the sender (i.e., incoming from the local TMSV production site), or at the receiver side (after propagation through the atmosphere).

Unless otherwise stated, we will consider the former process in the following.

For the generation of an entangled photon-subtracted squeezed (PSS) state [205], [206], [208]–[211], [214], [215], each mode of an incoming TMSV state interacts with a vacuum mode in a beam splitter. One of the outputs of each beam splitter feeds a photon number resolving detector. When both detectors simultaneously register k photons, which are considered to be non-Gaussian measurements, a pure non-Gaussian state is heralded with a probability of $0 < P_{ab} < 1$. This photon-subtraction operation is shown in Fig. [17](a) for $k = 1$. A PSS state can also be generated by applying the photon subtraction technique described above to a single mode of the TMSV state [215]. The generation of non-Gaussian states via photon subtraction as described above has been experimentally demonstrated in [226]–[228]. Note that in the photon-subtraction operation, other types of photon detectors such as on/off photon detectors (which only distinguish the presence and absence of photons, and are considered a non-Gaussian measurement) can also be used for generating a PSS state from a TMSV state [206], [209]. In this case the non-Gaussian output state is a mixed state.

An entangled photon-added squeezed (PAS) state [207], [208], [210], [212], [215] is generated by adding a single photon to each mode of a TMSV state. This single-photon addition is performed at a beam splitter, as shown in Fig. [17](b), with one of the outputs of each beam splitter being detected by an on/off photon detector. A pure non-Gaussian state is then generated (with a probability of $0 < P_{ab} < 1$) when a vacuum state is registered in both detectors simultaneously.

Note that the final creation probability of a PAS state is obtained by multiplying $P_{ab}$ by the probability of creating the two additional photons required. A PAS state can also be generated by applying the photon addition technique described above to a single mode of the TMSV state [215]. Note that the addition of single photons to coherent states and to thermal states of light has been experimentally realized in [229], [230].

By contrast, an entangled photon-replaced squeezed (PRS) state [213], [215] is generated according to Fig. [17](c), where each mode of a TMSV state interacts with a single photon in a beam splitter, with one of the outputs of each beam splitter being detected by a photon number resolving detector. When both detectors register a single photon simultaneously, a pure non-Gaussian state is heralded with a probability of $0 < P_{rb} < 1$. The final creation probability of a PRS state is obtained by multiplying $P_{rb}$ by the probability of creating the two additional photons required. A PRS state can also be generated by applying the photon replacement process described above to a single mode of the TMSV state [215].

### B. Evolution of non-Gaussian entangled states over a lossy channel

Unlike Gaussian states, the evolution of non-Gaussian states cannot be analysed solely through the covariance matrix. Previous contributions have analysed the evolution of non-Gaussian states for transmission over fixed-attenuation channels relying on the so-called Master equation approach of [216], the characteristic function approach of [213] or the Kraus operator approach of [218]. Here we discuss the general approach of Kraus representation [231] of the channel in order to directly analyze the evolution of the entangled states (Gaussian or non-Gaussian) through the channel. Considering a quantum state associated with the density operator $\hat{\rho}_{in}$ as the input of a trace-preserving completely positive channel, the output density operator of the channel can be described in an operator-sum representation of the form $\hat{\rho}_{out} = \sum_{\ell=0}^{\infty} G_{\ell} \hat{\rho}_{in} G_{\ell}^\dagger$, where the Kraus operators $G_{\ell}$ satisfy $\sum_{\ell=0}^{\infty} G_{\ell}^\dagger G_{\ell} = I$, with $I$ being the identity operator. In [231], the Kraus operators of a wide range of channels including a fixed-attenuation channel subject to vacuum noise (i.e., $V_n = 1$ in Fig. [11]) are given. In [218], the Kraus operators of a fixed-attenuation channel subject to vacuum noise but with additional Gaussian noise is given. The results of [231] have been generalized to a fixed-attenuation channel subject to thermal noise (i.e., $V_n > 1$ in Fig. [11]) in [140].

### C. Entanglement determination and quantum key rate computation

Following the evolution of pure non-Gaussian states over the lossy channel(s), the quantum state of the channel output is a non-Gaussian mixed state. In general it is not possible to analytically determine the total grade of entanglement of the mixed non-Gaussian states after transmission over a lossy channel. Since the grade of entanglement is determined by
1. **Implementation of non-Gaussian operations on the Gaussian TMSV state:**

Fig. 17 shows the implementation of non-Gaussian operations on the Gaussian TMSV state; (a) photon subtraction, (b) photon addition, and (c) photon replacement.

- **(a)** Photon subtraction: The TMSV state is transformed into a state where one photon is removed.
- **(b)** Photon addition: A photon is added to the TMSV state, creating a higher photon number state.
- **(c)** Photon replacement: A photon is removed, and another one is added, preserving the state's photon number distribution.

2. **Output Density Operator:**

The output density operator $\hat{\rho}_{\text{out}}$, which possesses an infinite number of elements, is approximated by its truncated-dimensional version, as discussed in [90], [92], [140], [206], ensuring that the trace of the truncated matrix is close to 1.

3. **Energy Grade of Entanglement:**

Given the non-deterministic nature of the non-Gaussian operations, in the context of non-Gaussian entanglement distribution, there are two key performance indicators: the grade of entanglement $E$ between two stations following the transmission of a pulse through the lossy channel(s), and the entanglement-generation rate $R_E$, where we have $R_E = P_c E$, with $P_c$ being the creation probability of the initial non-Gaussian state. The evolution of a wide range of non-Gaussian entangled states in both single-mode and two-mode transfer over atmospheric fading channels has been investigated both when the transmission coefficient of the atmospheric fading channel is unknown and when it is estimated in real time [90].

4. **CV Entanglement Generation:**

The work of [90] considered operational scenarios where the non-Gaussian entangled states transmitted through the atmospheric channel are created “just-in-time” via non-Gaussian operations applied to the Gaussian entangled input states that would otherwise be transmitted directly over the communication channel. In this scenario, transmitting the incoming Gaussian state directly over the atmospheric channel would be the best option in terms of maximizing the entanglement-generation rate. However, if the transmission rates of all the states through the channel could be equalized for example with the aid of quantum memory (see [90] for more details), some non-Gaussian states lead to enhanced entanglement transfer relative to that obtained by Gaussian state transfer.

5. **CV-QKD Protocols Analysis:**

The performance of CV-QKD protocols has been analyzed in [92] for transmission over atmospheric fading channels, where the source is constituted by PSS states in the context of EB CV-QKD protocols. In [92], one mode of the PSS state remains at the ground station (satellite), while the other photon-subtracted mode is transmitted to the satellite (ground station) over the fading uplink (downlink) channel characterized by the probability distribution $p(\eta)$ and maximum transmission coefficient of $\eta_0$. When the transmission coefficient of the atmospheric channel can be measured in real time, after acquiring each realization of $\eta$, the key rate $K(\eta)$ is calculated based on the covariance matrix of the mixed non-Gaussian state at the output of the channel. The final key rate is then computed as $K = P_c \int_{\eta_0}^{\eta} K(\eta)p(\eta) d\eta$ in units of bits per pulse, with $P_c$ being the creation probability of the initial non-Gaussian entangled state. The resultant key rate represents a lower bound on the actual key rate of the CV-QKD protocol. However, to determine the actual resultant key rates (not just its lower bounds), $K(\eta)$ must be computed based on the density operator of the mixed non-Gaussian output state.

6. **Comparison with Discrete-Variable Technologies:**

In [90], [92] the non-Gaussian operations are first applied to the initial Gaussian states, with the resultant non-Gaussian states being transmitted through the atmospheric fading channel. An alternative approach would be to transmit the initial Gaussian states through the atmospheric channel, and then apply the non-Gaussian operations after the atmospheric channel to the quantum states received. In [213], the distillation of CV entanglement using a coherent superposition-based non-Gaussian operation has been studied, where the non-Gaussian operation is the superposition of the photon subtraction and of the photon addition operations, and where the non-Gaussian operation is applied either before or after a fixed-attenuation channel.

VII. **Comparison with Discrete-Variable Technologies**

The family of DV systems invoked for satellite-based quantum communications constitutes an alternative technology, which has been deployed in Micius [67]–[69]. In space-based
deployment, a range of pragmatic issues comes into play when considering the pros and cons of DV vs. CV implementations. Perhaps the strongest argument in favour of DV systems in the space-based context is that photon losses have a less grave impact on quantum information processing in DV systems. In CV systems the photon losses in the channel introduce vacuum noise, leading to a reduction in the correlation between Alice and Bob’s data. By contrast, in DV systems, photon losses reduce the communication efficiency, but they do not trigger a false single-photon detection event. A photon is either lost in the channel, in which case Bob does not register anything, or it is simply detected at Bob’s detector. In high-loss scenarios, this effect can lead to advantages for DV systems. However, this benefit may be outweighed by other considerations, as discussed briefly below. More details on satellite-based DV quantum communication can be found elsewhere, for example in [63].

The performance of DV-QKD [64] is limited both by the difficulty of single-photon generation, as well as by the expense of single-photon detectors. It is a challenge to construct a true single-photon source owing to implementation challenges. Alternatively, single-photon sources can be approximated using an attenuated laser (weak coherent state pulses) [232], [233]. By contrast, CV-QKD systems rely on low-cost implementations and are potentially capable of supporting higher key rates than DV-QKD systems. Recall that CV-QKD can be implemented by modulating both the amplitude and phase quadratures of a coherent laser and can be subsequently measured in the receivers using homodyne detectors, which operate faster and more efficiently than the single-photon detectors. Moreover, CV-QKD systems are more compatible with standard telecommunication encoding, transmission and detection techniques. All these advantages potentially allow CV-QKD protocols to achieve higher secret key rates than DV-QKD systems.

Furthermore, the single-photon detectors of DV systems are very sensitive to background light sources. By contrast, the homodyne detectors used for CV systems offer beneficial robustness to background light. Indeed, an explicit advantage of using a local oscillator is that it has an ‘automatic’ spectral-domain filtering effect. Consequently, homodyne detectors remain to a large extent unimpaired in daylight conditions without the extra filtering that are needed by the single-photon detectors [16]. Furthermore, in CV systems, a tapped component of the local oscillator can be simply obtained and measured, thereby allowing for direct monitoring of atmospheric fluctuations effects, such as beam wandering (which can then be compensated for using adaptive optics [16], [81], [93]).

Nonetheless, the issue of whether DV or CV systems should be deployed as the quantum information carrier in space-based quantum communications remains very much an open issue at the time of writing. Ultimately, it could well be that hybrid DV+CV architectures, accommodating time-variant atmospheric conditions, turn out to be the most beneficial in many circumstances. The employment of such hybrid architectures has been extensively studied for example in [234].

VIII. Future directions

Quantum communication via satellite is in its infancy. Building on the early work and verification studies (both experimental and theoretical) of many researchers e.g. [16], [52]–[63], [65]–[70], [74]–[95], [235], [236], the pioneering experimental result of the Micius [67]–[69] collaboration has now provided us with the first glimpse of what is truly achievable via space-based platforms. However, there remains much to do before quantum communications via satellites can be considered mainstream. This is especially so in the CV quantum domain, where no space-based deployments have yet been achieved, despite the numerous theoretical studies e.g. [16], [81]–[94]. We briefly mention here some of the research topics within space-based CV quantum communications that we consider of particular interest to any multi-disciplinary engineering community.

A. Channel transmissivity measurements

The Micius [67]–[69] data provides us with our first real insight into the channel conditions experienced by quantum states, as they traverse through the turbulent atmosphere, to and from Earth. The measured photonic losses in the downlink [67], [68] and in the uplink [69] are now available (the losses in the latter case were a minimum of 41 dB). Leveraging this data for better understanding the channel conditions experienced by CV states as they travel to and from Earth would be an insightful, but costly endeavour. As discussed earlier in Sec. VII the loss of photons in the CV context fundamentally affects any subsequent information processing, as opposed to the DV case, where photons not received can be simply ignored. Ultimately, the study of how the CV states are affected by the atmosphere reduces to a determination of the statistical distribution of the channel transmissivity. Detailed knowledge of this distribution has wide ranging implications for studies pertaining to non-classical signatures of CV states traversing through atmospheric channels [87], as well as for a host of CV-based applications. The latter outcome is due to the fact that many applications are very sensitive to the channel’s transmissivity [88]–[92]. As discussed previously, beyond the dominant effects of beam wandering and beam broadening, other more subtle effects induced by the atmosphere can play a non-negligible role. These effects include beam deformation, attenuation, absorption and scattering. Sophisticated theoretical studies of these effects are now becoming available, and in general these models are found to be consistent with terrestrial experiments carried out over a wide range of turbulence conditions [84], [237], [238]. Experimental confirmations of existing turbulence models in the realm of Earth-to-satellite (and vice versa) channels would be very important. Of particular importance would be a robust validation of the beam-wandering models used for the transmissivity statistics in the Earth-to satellite channels [83], [85], and the validation of the beam-broadening models expected to dominate the satellite-to-Earth channels [56].
B. LDPC codes

The reconciliation phase of any QKD protocol is perhaps the area of quantum communications most closely associated with classical communications. In the DV scenario, long LDPC codes can be used to correct transmission errors. For scenarios, where DV quantum measurements are mapped directly to binary outcomes, the transmission of bits via a classical binary symmetric channel can be adopted as the underlying model. A range of high-performance LDPC codes which approach reconciliation factors close to 1 in the large key length limit are known for such channels [239]–[241]. However, in the CV setting the extraction of binary information is substantially more involved. Currently, there are two main techniques that are widely adopted in this regard, namely, slice reconciliation [20], [242], and multi-dimensional reconciliation [243], [244].

For the low signal to noise ratios (SNRs) routinely anticipated for satellite communications, the multi-dimensional reconciliation technique is likely to be more appropriate. In this context, multi-dimensional reconciliation via multi-edge LDPC codes is considered by many as the most appropriate path due to the high performance of such codes at low SNRs [244].

Nonetheless, numerous open research issues remain. Perhaps the most important of these is constituted by the finite key effects. Much of the work in formally determining the security of a key within QKD systems assumes having an infinite key length. However, in reality, this assumption is never satisfied and the consideration of the finite-length key effects must be analysed. This is an issue that affects both the DV [245] and CV security analyses [182], [246]–[249]. This problem is of particular concern for space-based QKD due to the short transit times of LEO satellites. Hence, the finite-length key processing invoked in the context of CV-QKD conceived for satellites has to be considered. Naturally, this analysis will be strongly dependent on the specific CV-QKD protocol adopted. Finite-length key based analyses of standard coherent state protocols [250], of MDI protocols [251], [252] and of full device-independent protocols [253] follow quite distinct paths.

Beyond the finite-length effects within the reconciliation decoding phase, the construction of near-capacity adaptive-rate LDPC codes for CV space-based implementations would be useful. Again, these issues are particularly relevant to satellite-based communication due to the time-variant properties of the channel. For LEO satellites we can expect the SNR to exhibit quite rapid variations versus time, as the satellite appears above the horizon and disappears again. Furthermore, for a given set of orbital parameters, we could anticipate the SNR’s evolution versus time to be reasonably predictable. Adaptive-rate LDPC codes well suited for counteracting the SNR vs time evolution should be constructed. The employment of puncturing techniques [254] used for multi-edge LDPC codes appears to be an appropriate pathway to achieving this [255]. These studies are only in their early phases of development, hence further research into the design of adaptive-rate codes as a path to low-complexity CV-QKD via satellites is expected to be fruitful. An important focus of such future studies should be the maintenance of high reconciliation efficiencies over the anticipated range of SNRs [256].

Finally, we note that in principle other codes beyond LDPC codes could be used in the CV-QKD reconciliation phase. Currently, however, limited work has been reported in this area. Nonetheless, we do note some work on turbo codes [257] applied to the CV domain, as reported in [258] (for use of such codes in the DV domain see [259], [260]). Furthermore, polar codes [261] have recently been invoked for CV-QKD in [262]. These contributions suggest that further performance comparisons using various error correction codes for the CV-QKD reconciliation phase may become fruitful.

C. CV quantum error correction codes

Of special importance for CV quantum communications are the non-Gaussian operations that form the basis of quantum error correction. Such operations are required due to the no-go theorem, stipulating that Gaussian errors cannot be corrected by purely Gaussian operations [263]. It is possible to build a pathway from standard classical LDPC codes to qubit error correction codes, and then to CV error correction codes. Following on from the original CV error correction protocols of [264]–[266], there are several examples of CV quantum error correction codes appearing in the recent literature [198], [267]–[273]. However, in the context of space-based implementations there is evidence to suggest that direct non-Gaussian measurement at the receiver is likely to be the most fruitful pathway to CV error correction - at least in the short term.

In Sec. VI-A we have discussed a host of non-Gaussian operations in the form of photon subtraction and addition techniques that were used to form our non-Gaussian states, as seen in Fig. 17. Such operations can also be used for producing CV entanglement distillation - a form of quantum error correction for CV variables. Photon subtraction and addition techniques are becoming mainstream in laboratories throughout the world and the imminent integration of such techniques directly into future satellite communications is expected. In QKD implementations though, a balance must be struck between the relatively low probabilities of success for the subtraction/addition operations required and the resultant degradation of the key rates. More detailed studies of these design options for space-based communications are warranted.

D. The interface with classical terrestrial networks

Although fundamentally a breakthrough, the birth of space-based quantum communications can be seen from a more pragmatic perspective - it will allow for the creation of the global “Quantum Internet”. This new Internet will interconnect a vast range of devices, from mobile devices all the way through to the much anticipated quantum computers. These devices will be able to transfer quantum information and communicate with each other in an unconditionally secure manner. Importantly, this new network will consist of not only quantum communication channels but also of classical communication channels. As such, consideration of how best to accommodate integration of the quantum information received via satellites into a wider integrated network will be required. Currently,
very little detailed thought has been given to this ambitious enterprise, and therefore there is much opportunity for high-impact future research in the context of the integrated system-oriented vision of Fig. 1.

In the CV setting, perhaps the integration of CV quantum information into the microwave setting is the most important example. The implementation of quantum communication protocols in the optical frequency domain is usually preferred, which is an explicit benefit of the negligible background thermal radiation at optical frequencies, hence all of our discussions have been in this domain. However, the advent of super-conducting microwave quantum circuits have led to an increasing interest in the implementation of quantum communication protocols in the microwave regime \(^{137–139}\), \(^{274–280}\). These interests are further fuelled by advances in macro electro-optomechanical resonators that are capable of coupling quantum information with the microwave-optical interface \(^{277},^{279},^{280}\). With the advent of this technology, quantum information created via super-conducting circuits may be readily upconverted to the optical regime for direct transfer to an overhead satellite. The satellite could then communicate that information optically to a second terrestrial receiver with subsequent conversion back to the microwave regime for storage, error correction or further information processing. Such a scenario could well represent how future quantum computers will share information globally through the quantum Internet. We also note that it is even possible to directly transmit quantum information via microwave carriers to nearby wireless receivers \(^{140}\). The development of such integration techniques for the quantum Internet is still in its infancy.

IX. CONCLUSIONS

We have discussed the recent research advances that are most relevant to CV quantum communication via low-Earth-orbit satellites. Recent experimental results gleaned from the Micius satellite on a range of DV-based quantum communication protocols indicate that CV quantum communication via large distances over the ether has become entirely plausible. We have outlined many of the technical advances in the field of CV quantum communication encompasses and highlighted a range of technical challenges it faces. However, the many advantages of this intriguing technology warrant its experimental deployment to make the vision of the perfectly secure future quantum-communications scenario portrayed in Fig. 1 a reality.

Our hope is valued Colleague that you would join this community-effort...

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