A dual-hop equivalent structure of a generalised multi-hop free-space optics network

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Abstract
The performance of free-space optics links strongly depends on scintillation effects caused by the atmospheric turbulence appearance. A widely adopted way to counterbalance the expected deterioration due to scintillation is to employ cooperative and multi-hop arrangements. However, the use of multiple relays increases significantly the complexity of the mathematical models used to describe the overall performance of the system. Taking the latter into account, we approximate a generic multiple relay-assisted free-space optical configuration with a dual-hop link in order to simplify the performance analysis. To this end, we provide new mathematical derivations for the evaluation of the outage probability and the average bit error rate by assuming typical turbulence models.

1 | INTRODUCTION

Free-space-optics (FSO) has attracted increasing research and commercial interest mostly due to the benefits against other wireless technologies, including radiofrequency or millimetre wave systems, in cases where point-to-point connections are required [1–4]. The most important of them is the high bandwidth access, the operation in the unlicensed spectrum, the low installation and operation cost, the enhanced security provision and the immunity to the electromagnetic interference [5–7]. However, the operation of FSO links strongly depends on several impairments, including scintillation caused by atmospheric turbulence effect [4].

In order to counterbalance the expected deterioration, cooperative and multi-hop deployments have been proposed in the open technical literature. However, the employment of multiple relays significantly raises the complexity of the mathematical expressions used to describe the overall performance [8]. To handle this issue, an approximation methodology for multi-hop configurations with amplify and forward (AF) relay nodes was proposed in [9].

In this work, a valid approximation is used, which can be applied in any multi-hop FSO configuration with decode and forward (DF) relays, to produce the equivalent dual-hop link. DF relays, which re-modulate and retransmit the received signals, are often used in FSO transmission since they are more reliable, although they require more processing compared to AF relaying. To this end, we consider intensity modulation/direct detection with on-off keying (OOK) modulation technique, which is the most commonly adopted and straightforward modulation scheme that has been used for experimental or commercial FSO systems [3]. The approximation process leads to a significant simplification of the complicated and widely adopted expressions for the outage probability (OP) and the average bit error rate (ABER) used for the performance evaluation of multi-hop networks.

The remainder of the work is organised as follows. In the section that follows, the channel model is introduced while the approximation methodology is presented and analysed in Section 3. Next, in Section 4, we proceed to the overall performance estimation of the DF-relayed FSO system in both OP and ABER metrics, while the corresponding numerical results
In what follows, we emphasise on the DF-relayed FSO link of Figure 1, which is transformed into the dual-hop scheme of Figure 2. In such a configuration, the source, \( S \) (node 0), emits optical signals towards several relay nodes, and then every relay node towards the rest relays or the destination \( D \) (node \( L+1 \)), respectively.

Hence, the received signal at each hop is given as [10]

\[
\mathcal{Y}_{x,y} = \eta I_{x,y} \gamma_{x,y} + n_{x,y},
\]

where \( x,y \) represent the specific hop, that is, from node \( x \) towards node \( y \), \( \eta \) is the effective photocurrent conversion ratio, \( I_{x,y} \) stands for the normalised received irradiance due to turbulence effects at the corresponding receiver, \( \gamma_{x,y} \) is the modulated signal, and \( n_{x,y} \) stands for the additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \).

Based on the above, the instantaneous signal-to-noise ratio (SNR) at each hop is expressed as [11]

\[
\gamma_{x,y} = \left( \frac{\eta I_{x,y}}{\sigma_n} \right)^2,
\]

and the average SNR as [11]

\[
\bar{\gamma}_{x,y} = \left( \frac{\eta E[I_{x,y}]}{\sigma_n} \right)^2,
\]

where \( E[I_{x,y}] \) stands for the expected normalised irradiance value.

Turbulence is assumed to follow either the gamma, which holds for weak turbulence conditions or the gamma-gamma model, which is appropriate for weak to strong turbulence conditions. The probability density function (PDF) of the gamma model is expressed as [12]

\[
f_{\mathcal{Y}_{x,y}}(\mathcal{Y}_{x,y}) = \frac{\zeta_{x,y}^{\mathcal{Y}_{x,y} - 1}}{\Gamma(\zeta_{x,y})} \exp(-\zeta_{x,y}\mathcal{Y}_{x,y}),
\]

where \( \Gamma(\cdot) \) is the standard gamma function [13, eq. (8.310.1)], and \( \zeta_{x,y} \) stands for the parameter of the gamma distribution determined as

\[
\zeta_{x,y} = \left( a_{x,y}^{-1} + b_{x,y}^{-1} + a_{x,y}^{-1} b_{x,y}^{-1} \right)^{-1},
\]

while the parameters \( a_{x,y} \) and \( b_{x,y} \) are calculated as

\[
a_{x,y} = \exp\left( \frac{0.49\delta_{x,y}^2}{1 + 0.18d_{x,y}^2 + 0.566\delta_{x,y}^{12/5}/7/6} \right) - 1 \]

\[
b_{x,y} = \exp\left( \frac{0.51\delta_{x,y}^2}{1 + 0.69\delta_{x,y}^{12/5}/5/6} \frac{d_{x,y}^2}{1 + 0.9d_{x,y}^2 + 0.62d_{x,y}^2\delta_{x,y}^{12/5}/5/6} \right) - 1
\]

where, \( \delta_{x,y}^2 \) represents the Rytov variance, which is given as

\[
\delta_{x,y}^2 = 0.5C_n^2 k_{x,y}^{-7/6} L_{x,y}^{-11/6},
\]

with \( k = 2\pi/\lambda \) being the wave number, where \( \lambda \) represents the operational wavelength, \( d_{x,y} = 0.5D_{x,y}\sqrt{2\pi}\lambda^{-1}L_{x,y}^{-1} \), \( D_{x,y} \) is the receiver’s aperture diameter, \( L_{x,y} \) is the link length, and \( C_n^2 \) is a parameter proportional to the atmospheric turbulence strength [12].

After a simple power transformation of Equation (4), the PDF for the SNR is deduced as

\[
f_{\mathcal{Y}_{x,y}}(\mathcal{Y}_{x,y}) = \frac{\zeta_{x,y}^{\mathcal{Y}_{x,y} - 1/2} \Gamma(\zeta_{x,y})^{1/2}}{2\Gamma(\zeta_{x,y})^{1/2}} \exp\left( -\zeta_{x,y}^{1/2} \sqrt{\mathcal{Y}_{x,y}/\zeta_{x,y}} \right).
\]

Moreover, the PDF for the SNR of the gamma-gamma model is given according to [14] as

\[
f_{\mathcal{Y}_{x,y}}(\mathcal{Y}_{x,y}) = \frac{\zeta_{x,y}^{\mathcal{Y}_{x,y} - 1/2}}{\Gamma(\zeta_{x,y})} \exp\left( -\zeta_{x,y}^{1/2} \sqrt{\mathcal{Y}_{x,y}/\zeta_{x,y}} \right)
\times K_{\gamma_{x,y}} \left( \sqrt{4a_{x,y}b_{x,y}\mathcal{Y}_{x,y}^{1/2} - \bar{\gamma}_{x,y}^{1/2}} \right),
\]

where \( K_{\gamma}(\cdot) \) stands for the \( \gamma \)-th order modified Bessel function of the second kind [13, eq. (8.432.2)].

In what follows, we omit the indices \( x,y \) considering the same parameter values in all links.
3 | APPROXIMATION METHOD

By following the methodology of [9], the dual-hop approximation is depicted in Figure 2. More specifically, the approximated normalised PDF for the gamma model described in Equation (8) is derived as

$$f_t, y = \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right),$$

(10)

while for the gamma-gamma distribution, Equation (9), we get

$$f_t, y = \sum_{p=1}^{r} \frac{\pi p, \rho (a b) y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(11)

where

$$\frac{1}{\Gamma p, \rho} = \frac{1}{\Gamma a(\xi)} + \frac{1}{\Gamma b(\xi)}, \quad \rho r, p, \rho r, p | p=1 = \frac{\Gamma a(\xi) \Gamma b(\xi)}{\Gamma a(\xi)},$$

$$\pi r, p = \sum_{p=1}^{r} \frac{\pi p, \rho (a b) y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(13)

Next, by using [13, eq. (3.81.8)], we obtain

$$F_t, y = \frac{\zeta}{2 \Gamma(\xi)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(14)

where $\psi(\cdot, \cdot)$ stands for the lower incomplete gamma function [13, eq. (8.310.1)]. For the gamma-gamma model, the CDF is expressed by substituting Equations (11) into (12) as

$$F_t, y = \frac{(a b) y^{(s-2)/2}}{4 \pi \Gamma(\xi) \Gamma(\rho)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(15)

Then, by using $K_t(\sqrt{3}) = 0.5 C_{1/2}^0 (\xi/4) \sqrt{2/\xi}$ from [15], along with the corresponding integral transformation from [16], the CDF expression is provided in terms of the Meijer-G function [13, eq. (9.301)] as

$$F_t, y = \frac{(a b) y^{(s-2)/2}}{4 \pi \Gamma(\xi) \Gamma(\rho)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(16)

4 | PERFORMANCE ESTIMATION

In this section, the overall performance of the multi-hop network in terms of the OP and ABER, with OOK modulation, is assumed. OP is a significant quantity in order to evaluate the reliability of a communication system and represents the probability that the instantaneous SNR falls below a critical threshold $\gamma_d$, while the ABER is a crucial metric for the reliability and performance estimation of such communication systems [3].

4.1 | Outage probability

The OP for a given threshold, $\gamma_d$, can be estimated as [14, 17]

$$P_{out,t} = F_t(\gamma_d).$$

(17)

Hence, the OP of the first hop, that is, from $s$ towards $R_1$, for gamma and gamma-gamma models, is obtained from Equations (14), (16), and (17) as

$$P_{out,t} = \frac{1}{\Gamma(\xi)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(18)

and

$$P_{out,t} = \frac{(a b) y^{(s-2)/2}}{4 \pi \Gamma(\xi) \Gamma(\rho)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(19)

respectively. To derive the OP of the second link, that is, from $R_1$ towards $D$, we substitute Equations (8) into (12), and then using Equation (17), we get for the gamma distribution

$$P_{out,D} = \frac{1}{\Gamma(\xi)} \sum_{p=1}^{r} \frac{\pi p, \rho y^{(s-2)/2}}{2 \Gamma a(\xi) \Gamma b(\xi)} \exp \left(-\frac{y}{\Gamma p, \rho} \right) \times K_r, b \left(\sqrt{4 a b y / \Gamma a(\xi) b(\xi)}\right),$$

(20)
while, for the gamma-gamma, is expressed as [14]

\[
P_{\text{est},D} = \frac{(ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)} \left( \frac{\gamma}{\bar{\gamma}} \right)^{\frac{a+b}{2}}
\times G_{1,3}^{2,1} \left( ab \sqrt{\frac{\gamma}{\bar{\gamma}}} \left| \begin{array}{c}
\frac{2-a-b}{2}, \frac{2-a-b}{2}, -a-b
\end{array} \right. \right).
\] (21)

Then, the OP for the entire network can be evaluated as [10, 18, 19]

\[
P_{\text{est}} = 1 - (1 - P_{\text{est},r}) (1 - P_{\text{est},D}).
\] (22)

## 4.2 Average bit error rate

The BER of the first approximated link can be calculated from the following expression as [14]

\[
P_{b,\gamma}(\gamma) = \frac{1}{2} \mathrm{erfc}\left( \frac{1}{2}\sqrt{\gamma} \right).
\] (23)

By substituting Equations (23) and (10) or (11) into the following integral

\[
P_{b,\gamma;\tilde{r}} = \int_{0}^{\infty} P_{b,\gamma}(\gamma)f_{\tilde{r}}(\gamma)d\gamma,
\] (24)

we can estimate the ABER expression of the approximated link, that is, from \( \tilde{r} \) towards \( R_{L} \), for the gamma and the gamma-gamma turbulence, respectively. More specifically, for the gamma distribution, the ABER takes the following form

\[
P_{b,\gamma;\tilde{r}} = \frac{\xi^{\frac{1}{2}}}{4\pi(\bar{\gamma})} \sum_{p=1}^{r} \pi_{\gamma,p} \frac{\xi}{\Gamma_{r,p}^{1/2}}
\times \int_{0}^{\infty} \gamma^{\frac{2}{2} - \frac{1}{2}} \mathrm{erfc}\left( \frac{\sqrt{\gamma}}{4} \right) \exp\left( -\xi \sqrt{\frac{\gamma}{\Gamma_{r,p}}} \right) d\gamma.
\] (25)

Then, by using \( \mathrm{erfc}(\sqrt{\xi}) = (\sqrt{\pi})^{-1} G_{1,2}^{2,0}(\xi | 0, 1/2) \) and \( \sqrt{\pi} \) from [15], along with the corresponding integral transformation from [16], the ABER expression is derived in terms of the Meijer-G function [13, eq. (9.301)] as

\[
P_{b,\gamma;\tilde{r}} = \frac{\xi}{2\pi^{2\gamma} \Gamma(\bar{\gamma})} \sum_{p=1}^{r} \pi_{\gamma,p} \frac{\xi}{\Gamma_{r,p}^{1/2}}
\times G_{2,3}^{1,3} \left( \frac{\xi^{2}}{\Gamma_{r,p}} \left| \begin{array}{c}
\frac{2-\xi}{2}, \frac{1-\xi}{2}, \frac{\xi}{2}
\end{array} \right. \right).
\] (26)

while the gamma-gamma case is obtained as

\[
P_{b,\gamma;\tilde{r}} = \frac{(ab)^{\frac{a+b}{2}}}{\pi \sqrt{\pi \Gamma(a)\Gamma(b)}} \sum_{p=1}^{r} \pi_{\gamma,p} \frac{a+b}{\Gamma_{r,p}^{1/2}}
\times \int_{0}^{\infty} \gamma^{\frac{a+b-1}{2}} \mathrm{erfc}\left( \frac{\sqrt{\gamma}}{4} \right) K_{\gamma-1} \left( \sqrt{4ab \frac{\gamma}{\Gamma_{r,p}}} \right) d\gamma.
\] (27)

Finally, by using the same transformations as above, the ABER expression is estimated as

\[
P_{b,\gamma;\tilde{r}} = \frac{4^{\frac{a+b}{2}}}{\pi \Gamma(a)\Gamma(b)} \sum_{p=1}^{r} \pi_{\gamma,p} \frac{a+b}{\Gamma_{r,p}^{1/2}}
\times G_{2,3}^{1,3} \left( \frac{(ab)^{2}}{4\Gamma_{r,p}} \left| \begin{array}{c}
\frac{4-a-b}{4}, \frac{2-a-b}{4}, \frac{4-a-b}{4}, \frac{4-a-b}{4}, \frac{4-a-b}{4}
\end{array} \right. \right).
\] (28)

The ABER of the second link for gamma turbulence is calculated by properly using Equation (24) along with Equation (8). After some algebra, it can be obtained as

\[
P_{b,\gamma;\tilde{r}} = \frac{\xi^{\frac{1}{2}}}{2^{2\gamma} \pi \Gamma(\bar{\gamma}) \Gamma_{L,D}^{1/2}} G_{2,3}^{1,3} \left( \frac{\xi^{2}}{\tilde{r}_{L,D}} \left| \begin{array}{c}
\frac{2-\xi}{2}, \frac{1-\xi}{2}
\end{array} \right. \right),
\] (29)

while the corresponding expression for the gamma-gamma distribution is expressed as [14]

\[
P_{b,\gamma;\tilde{r}} = \frac{4^{\frac{a+b}{2}}}{\pi \Gamma(a)\Gamma(b)} \tilde{r}_{L,D}^{1/2} \sum_{p=1}^{r} \pi_{\gamma,p} \frac{a+b}{\Gamma_{r,p}^{1/2}}
\times G_{2,3}^{1,3} \left( \frac{(ab)^{2}}{4\tilde{r}_{L,D}} \left| \begin{array}{c}
\frac{4-a-b}{4}, \frac{2-a-b}{4}, \frac{4-a-b}{4}, \frac{4-a-b}{4}, \frac{4-a-b}{4}
\end{array} \right. \right).
\] (30)

Finally, the total ABER of the approximated system is calculated by the following equation [3, 20, 21]:

\[
P_{b,\gamma;\tilde{r}} = P_{b,\gamma;\tilde{r}} \left[ 1 - 2P_{\gamma;\tilde{r}} \right] + P_{b,\gamma;\tilde{r}}.
\] (31)

## 5 NUMERICAL RESULTS

Here, we provide some typical numerical results for the FSO configuration as illustrated in Figure 3 operating over weak or moderate turbulent channels. By properly using Equations (5) to (7), we estimate the values for the parameters of both distributions. More precisely, for the gamma case, we consider weak turbulence conditions, \( C_{0}^{\gamma} = 7 \times 10^{-15} \text{m}^{-2/3} \), with \( L = 7 \text{ km} \),
\( \lambda = 1.55 \ \text{\textmu}m \) and \( D = 0.1 \ \text{m} \), leading to \( \zeta = 1.69 \), while for the gamma-gamma case we assume moderate turbulence conditions \( C_n^2 = 2 \times 10^{-14} \text{m}^{-2/3} \), with \( L = 5 \ \text{km} \), \( \lambda = 1.55 \ \text{\textmu}m \) and \( D = 0.02 \ \text{m} \), which leads to \( a = 2.08 \) and \( b = 1.82 \). Any other set of valid parameter values leading in different \( \alpha \), \( b \) and \( \zeta \) can be used in both distribution models depending on the turbulence strength. Also, it is assumed that \( \bar{\gamma}_{0,1} = \bar{\gamma}_{1,2} = \bar{\gamma}_{2,3} = \bar{\gamma}_{3,4} = \bar{\gamma}_{L,D} = \bar{\gamma} \) and \( \bar{\gamma}_{0,2} = \bar{\gamma}_{0,3} = \bar{\gamma} \). Thus, by properly using [10, eqs. (16) and (17)], the original OP for the entire network is determined as \( P_{\text{tot}} = 1 - [(1 - P_{\text{out,D}})(1 - P_{\text{out,D}} P_1)] \), with \( P_1 = 1 - (1 - P_{\text{out,D}})(1 - P_2) \) and \( P_2 = [1 - (1 - P_{\text{out,D}})^2]P_{\text{out,D}} \). To further test the accuracy of the method, we also provide results for the OP of the system, obtained without using our approximation methodology.

Figure 4 demonstrates the OP versus the normalised outage threshold \( \bar{\gamma} / \gamma_{\text{th}} \), while Figure 5 presents the ABER versus electrical SNR. We can readily observe that the results for the OP are quite similar either with or without approximation. This is quite promising since the adoption of the described approximation assures the derivation of simplified mathematical expressions, which are useful to researchers who wish to evaluate the performance of complex relayed FSO networks. Additionally, the gamma distribution outperforms in both metrics, the gamma-gamma one as expected, due to the different \( C_n^2 \) values.

6 | CONCLUSION

A dual-hop approximation of multi-hop FSO configurations with DF relays over gamma or gamma-gamma turbulence models was adopted and further analysed in this work. The derived OP and ABER approximated expressions were significantly simplified and the corresponding numerical results revealed the accuracy of the approximation. The work can readily be extended to consider the investigation of AF relays, the adoption of other turbulence models and the evaluation of various performance metrics.

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REFERENCES

1. In: Uysal, M., et al.: Optical Wireless Communications: An Emerging Technology. Springer, Cham (2016)
2. Ghassemlooy, Z., Popoola, W.O., Rajbandari, S.: Optical Wireless Communications: System and Channel Modelling with MATLAB®, 2nd edition. CRC Press, Milton (2019)
3. Androutsos, N.A., et al.: Serial DF relayed FSO links over mixture gamma turbulence channels and nonzero boresight spatial jitter. Computer 7(3), 34 (2019)
4. Andrews, L.C., Philips, R.L.: Laser Beam Propagation Through Random Media, 2nd edition. SPIE, Bellingham, WA (2005)
5. Ma, J., et al.: Exact error rate analysis of free-space optical communications with spatial diversity over Gamma Gamma atmospheric turbulence. J. Mod. Opt. 63(3), 252–260 (2016)
6. Chaman-Motlagh, A., Ahmad, V., Ghassemlooy, Z.: A modified model of the atmospheric effects on the performance of FSO links employing single and multiple receivers. J. Mod. Opt. 57(1), 37–42 (2010)
7. Popoola, W.O., Ghassemlooy, Z.: BPSK subcarrier intensity modulated free-space optical communications in atmospheric turbulence. J. Lightwave Technol. 27(8), 967–973 (2009)
8. Khalighi, M.A., Uysal, M.: Survey on free space optical communication: A communication theory perspective. IEEE Commun. Surv. Tutorials. 16(4), 2231–2258 (2014)
9. Lim, S., Ko, K.: Approximation of multi-hop relay to dual-hop relay and its error performance analysis. IEEE Commun. Lett. 21(2), 342–345 (2017)
10. Varotsos, G.K., et al.: Mixed topology of DF relayed terrestrial optical wireless links with generalised pointing errors over turbulence channels. Technology 6(4), 121 (2018)
11. Sandalidis, H.G., et al.: Performance of free-space optical communications over a mixture composite irradiance channel. Electron. Lett. 53(4), 260–262 (2017)
12. Varotsos, G.K., et al.: FSO links with diversity pointing errors and temporal broadening of the pulses over weak to strong atmospheric turbulence channels. Optik 127(6), 3402–3409 (2016)
13. Gradshteyn, I.S., Ryzhik, I.M.: Table of Integrals, Series, and Products, 8th ed. Academic Press, Waltham, MA (2015)
14. Stassinakis, A.N., Nistazakis, H.E., Tombras, G.S.: Comparative performance study of one or multiple receivers schemes for FSO links over gamma-gamma turbulence channels. J. Mod. Opt. 59(11), 1023–1031 (2012)
15. The mathematical functions site. http://functions.wolfram.com (2020). Accessed 07 July 2020
16. Adamchik, V.S., Marichev, O.I.: The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system. In: Proceedings of the International Symposium on Symbolic and Algebraic Computation, Tokyo, Japan, pp. 212–224 (1990)
17. Prabu, K., Seiram Kumar, D.: Outage analysis of relay-assisted BPSK-SIM based FSO systems over strong atmospheric turbulence with pointing errors. Int. J. Comput. Commun. Eng. 3(5), 317–320 (2014)
18. Feng, M., et al.: Outage performance for parallel relay-assisted free-space optical communications in strong turbulence with pointing errors. In: 2011 International Conference on Wireless Communications and Signal Processing (WCSP), Nanjing, China (2011)
19. Safari, M., Uysal, M.: Relay-assisted free-space optical communication. IEEE Trans. Wireless Commun. 7(12), 5441–5449 (2008)
20. Morgado, E., et al.: End-to-end average BER in multihop wireless networks over fading channels. IEEE Trans. Wireless Commun. 9(8), 2478–2487 (2010)
21. Sheng, M., et al.: End-to-end average BER analysis for multi-hop free-space optical communications with pointing errors. J. Opt. 15(5), 055408 (2013)

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