Simulation Analysis of 6-DOF Manipulator

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Abstract. The basic structure of the manipulator is simplified with the 6-DOF manipulator as the research object, and the coordinate system is established based on the D-H method. At the same time, Matlab is used for simulation, and the quintic polynomial interpolation is used for trajectory planning and simulation to verify that the manipulator can move according to the established trajectory.

Keywords: 6-DOF; D-H method; Trajectory planning.

1. Introduction
Manipulator is an intelligent mechatronics system, which integrates mechanical, electronic, control, communication, and other technologies. It plays an important role in artificial intelligence and robotics and has a great impact on social progress and development. Trajectory planning of the manipulator is to achieve the desired trajectory through joint motion according to the requirements of the task [1].

2. Mathematical model of the 6-DOF manipulator
2.1. Establishment of D-H coordinate system
According to the physical structure of the 6-DOF manipulator, the coordinate system is established by the D-H method, where x0-z0 represents the base. Then the coordinate system from joint 2 to joint 6 is established successively, and x6-z6 represents the end gripper of the 6-DOF manipulator[2]. The coordinate is shown in Figure 1.

![Figure 1. Coordinate of 6-DOF Manipulator](image_url)
2.2. D-H parameters

After the D-H coordinate system is established, the D-H parameters are determined according to the position relationship between adjacent links. The parameters of each joint in the 6-DOF manipulator are shown in Table 1.

| connecting rod | $a_{i-1}$ | $\alpha_{i-1}$ | $d_i$ | $\theta_i$ |
|----------------|-----------|----------------|-------|------------|
| 1              | 0         | $90^\circ$     | $L_1$ | $\theta_1$ |
| 2              | $L_2$     | 0              | 0     | $\theta_2$ |
| 3              | $L_3$     | 0              | 0     | $\theta_3$ |
| 4              | $L_4$     | $-90^\circ$    | 0     | $\theta_4$ |
| 5              | 0         | $90^\circ$     | 0     | $\theta_5$ |
| 6              | 0         | 0              | $L_5$ | $\theta_6$ |

3. Trajectory planning in joint coordinate space

Trajectory planning is a time function of the spatial pose of a 6-DOF manipulator, which is related to the time of the manipulator motion. Generally, the interpolation of trajectory planning in joint coordinate space does not involve the pose information of the 6-DOF manipulator, and it is easy to calculate the interpolation precisely. [3] Therefore, the trajectory planning of the 6-DOF manipulator is carried out in the joint coordinate space. The most commonly used time function of quintic polynomial interpolation is used in the trajectory planning of joint coordinate space.

3.1. Quintic polynomial interpolation

The trajectory planning means that the end gripper of the 6-DOF manipulator moves from the starting point to the target point, in which the position and pose are known.

Suppose the initial time is $t = t_0 = 0$. The joint angle is $\theta = \theta_0$. When moving to the target point $t = t_f$, the joint angle at this time is $\theta = \theta_f$. Get it

\[
\begin{aligned}
\theta (0) &= \theta_0 \\
\theta (t_f) &= \theta_f
\end{aligned}
\]

(1)

When the 6-DOF manipulator is at the initial point and the target point, its speed is zero. Get it

\[
\begin{aligned}
\dot{\theta}(0) &= 0 \\
\dot{\theta}(t_f) &= 0
\end{aligned}
\]

(2)

A cubic polynomial can be obtained from the boundary conditions (1) and (2)

\[
\theta (t) = a_0 + a_1t + a_2t^2 + a_3t^3
\]

(3)

Two additional conditions can be added to the boundary condition of the cubic polynomial to make the acceleration of the 6-DOF manipulator zero at $t_0$ and $t_f$

\[
\begin{aligned}
\ddot{\theta}(0) &= 0 \\
\ddot{\theta}(t_f) &= 0
\end{aligned}
\]

(4)
From the above boundary conditions (1), (2), and (4), a quintic polynomial can be obtained:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \quad (5)$$

By deriving equation (5), the joint velocity function $\dot{\theta}(t)$ of the quintic polynomial can be obtained. Then, the acceleration function $\ddot{\theta}(t)$ of the joint can be obtained by further derivation:

$$\begin{cases}
\dot{\theta}(t) &= a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \\
\ddot{\theta}(t) &= 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 
\end{cases} \quad (6)$$

By substituting the boundary conditions into (5) and (6), we can get

$$\begin{align*}
a_0 &= \theta_0 \\
a_1 &= 0 \\
a_2 &= 0 \\
a_3 &= \frac{10(\theta_f - \theta_0)}{t_f^3} \\
a_4 &= \frac{15(\theta_0 - \theta_f)}{t_f^4} \\
a_5 &= \frac{6(\theta_f - \theta_0)}{t_f^5}
\end{align*}$$

Therefore, we can get the following formula

$$\begin{align*}
\theta(t) &= \theta_0 + \frac{10(\theta_f - \theta_0)}{t_f^3} t^3 + \frac{15(\theta_0 - \theta_f)}{t_f^4} t^4 + \frac{6(\theta_f - \theta_0)}{t_f^5} t^5 \\
\dot{\theta}(t) &= \frac{30(\theta_f - \theta_0)}{t_f^3} t^2 + \frac{60(\theta_0 - \theta_f)}{t_f^4} t^3 + \frac{30(\theta_f - \theta_0)}{t_f^5} t^4 \\
\ddot{\theta}(t) &= \frac{60(\theta_f - \theta_0)}{t_f^3} t + \frac{180(\theta_0 - \theta_f)}{t_f^4} t^2 + \frac{120(\theta_f - \theta_0)}{t_f^5} t^3
\end{align*}$$

3.2. Trajectory simulation of the 6-DOF manipulator

Quintic polynomial interpolation can be used in the trajectory planning of the 6-DOF manipulator from the joint angle, joint angular velocity, and joint angular acceleration. Due to the limitation of physical equipment, joint angular velocity and joint angular acceleration cannot be verified in the physical verification. Therefore, this design abandons the simulation of joint angular velocity and acceleration, and only simulates the joint angle [4].

The 6-DOF manipulator starts from point A, passes through A1 and A2 to point B. The time from A to A1 is 1s, 2s from A1 to A2, and 2s from A2 to B. The flow chart is shown in Figure 2.

![Figure 2. Point Flow Chart of Six Degrees of Freedom Manipulator during Trajectory Planning](image)
At the same time, the pose matrix of point A and point B are selected:

\[
\begin{bmatrix}
-0.8 & 0 & 0.6 & 100 \\
0 & 1 & 0 & 150 \\
-0.6 & 0 & -0.8 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
= \begin{bmatrix}
0.866 & -0.500 & 0 & 460 \\
0.500 & 0.866 & 0 & 145 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

From this, the angles of A, A1, A2, and B in each joint of the manipulator can be obtained. The angle data is shown in Table 2.

| Path point | Joint one (rad) | Joint two (rad) | Joint three (rad) | Joint four (rad) | Joint five (rad) | Joint six (rad) |
|------------|----------------|----------------|------------------|-----------------|----------------|---------------|
| A          | 1.1669         | 0              | 2.2865           | -2.2539         | 2.3502         | 1.7926        |
| A1         | 1.1210         | 0              | 2.1045           | -2.3503         | 2.3465         | 1.6992        |
| A2         | 0.6257         | 0              | 0.1422           | -2.8598         | 2.3067         | 0.6918        |
| B          | 0.3740         | 0              | -0.8551          | -pi             | 2.2865         | 0.1799        |

The coordinates of the end gripper of the 6-DOF manipulator can be obtained from the angle data as shown in Table 3.

| Path point | X-axis | Y-axis | Z-axis |
|------------|--------|--------|--------|
| A          | 2.312  | 10.954 | 8.836  |
| A1         | 3.234  | 11.692 | 8.140  |
| A2         | 9.248  | 9.250  | 1.801  |
| B          | 10.245 | 6.160  | 1.515  |

The joint angles of quintic polynomial trajectory are simulated by MATLAB:

Figure 3. Quintic Polynomial Trajectory of Each Joint Angle

It can be seen from Figure 3 that the quintic polynomial trajectory is a smooth curve, and the final manipulator can reach the expected position. However, the angle change of quintic polynomial cannot be determined. Therefore, it is necessary to transform the angle change of quintic polynomial from joint coordinate space to Cartesian coordinate space. The trajectory simulation is shown in Figure 4 [7].
Figure 4. Quintic Polynomial Simulation

It can be seen from Figure 4 that the simulation trajectory of the six degrees of freedom manipulator with quintic polynomial shows that the quintic polynomial can complete the given motion from point A to point B. Because the pose matrices of point A and point B are known, the angles of each joint of the 6-DOF manipulator at the initial point A and the final point B can be obtained by solving the forward kinematics and inverse kinematics of the 6-DOF manipulator, and the rotation angle of each joint can be obtained.

4. Conclusion
As the most important executive mechanism of robot, the research of manipulator is the focus of artificial intelligence and robotics. In this paper, by establishing the D-H coordinate system, the 6-DOF manipulator is modeled, and the trajectory planning of the 6-DOF manipulator is carried out by using the method of quintic polynomial interpolation. Through the MATLAB simulation, it shows that the 6-DOF manipulator can reach the expected position, which provides a new idea for the future experimental simulation.

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