M2-branes on a resolved $C_4/Z_4$

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**Abstract**

We write down M2-branes on the resolved $C_4/Z_4$ orbifold space. The resolved spatial geometry is such that it interpolates between $R^2 \times CP_3$ near the branes and $C_4/Z_4$ asymptotically. The near horizon geometry of these branes is a deformation of $AdS_4 \times S^7/Z_4$. An interesting aspect is that for $k = 4$ Chern-Simons theory coupling becomes vanishing near the IR cutoff leading to spontaneous compactification to type IIA.
1 Introduction

The study of spacetime solutions which represent extended p-brane objects has led to many new ideas in string theory as well as in field theory [1, 2]. Interestingly, very recently there has been a fast activity in this field, see [3-4], where certain type of Chern-Simons field theories in 2 + 1 dimensions have been proposed to be dual to M-theory on $AdS_4 \times S^7/Z_k$ spacetime. Namely, the Aharony-Bergman-Jafferis-Maldacena (ABJM) Chern-Simons theory [21], which has $\mathcal{N} = 6 SU(N) \times SU(N)$ superconformal symmetry, is conjectured to be dual to M-theory on $AdS_4 \times S^7/Z_k$ with level $k > 2$. While the originally proposed Bagger-Lambert (BL) membrane theory based on compact tri-algebras has maximal $\mathcal{N} = 8$ superconformal symmetry but is known only for $SO(4) \times SO(4)$ R-symmetry [3, 4]. Although by allowing noncompact tri-algebras, BL theory can be extended to admit full $SU(N) \times SU(N)$ symmetry [12]. But these theories have a ghost field in the spectrum which when gauge-fixed to a constant value gives rise to $SU(N)$ superconformal Yang-mills theory [19]. These developments are necessary to understand the M-theory origin of superconformal $SU(N)$ Yang-Mills gauge theory which lives on the D2-branes over $AdS_4 \times S_6$, and vice-versa.

For the current purpose the paper is organised as follows. In the section-2, we review basic properties of the resolved $C_4/Z_k$ orbifold geometry and write down the M2-brane solution on the resolved space. In section-3 we discuss spontaneous compactification to type IIA background. We discuss the nature of the singularity at the IR cut off scale. Near the IR cutoff the string (Chern-Simons) coupling vanishes but curvature also become large. The results are summarised in the last section.

2 M2 on resolved $C_4/Z_k$

The flat metric on $C_4/Z_k$ eight-dimensional space can be written as

$$ds^2_{C_4/Z_k} = dr^2 + \frac{r^2}{k^2}(dz + kA)^2 + r^2 ds^2_{CP_3}$$

where $r^2 = (y^m)^2$. The $y^m$'s are eight Cartesian coordinates which define the base space of the geometry, that is $\mathbb{R}^8$ or $C_4$. The round Fubini-Study metric on unit size $CP_3$ space can be read from [13, 21] and it is

$$ds^2_{CP_3} = d\xi^2 + \cos^2 \xi \sin^2 \xi (\tilde{\psi})^2 + \frac{\cos^2 \xi}{4}(d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{\sin^2 \xi}{4}(d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2)$$

where the coordinate ranges are $0 \leq \xi < \frac{\pi}{2}$, $0 \leq z < 2\pi$, $0 \leq \theta_i < \pi$, $0 \leq \phi_i < 2\pi$. The $\tilde{\psi}$ and the 1-form along the Hopf fibre $z$ are given as

$$\tilde{\psi} \equiv d\psi + \frac{\cos \theta_1}{2}d\phi_1 - \frac{\cos \theta_2}{2}d\phi_2$$
\[ A = \frac{1}{2} \left( (\cos^2 \xi - \sin^2 \xi) d\psi + \cos^2 \xi \cos \theta_1 d\phi_1 + \sin^2 \xi \cos \theta_2 d\phi_2 \right) \]  

(3)

The space is asymptotically locally Euclidean (ALE) but has (orbifold) conical singularity at \( r = 0 \) for all \( k \geq 2 \).

The M2-brane solution on this transverse space is given by

\[
\begin{align*}
    ds_{11}^2 &= h^{-\frac{1}{2}}(-dx_0^2 + dx_1^2 + dx_2^2) + h^{\frac{1}{3}}ds_{C_4/Z_k}^2 \\
    F_{(4)} &= d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2
\end{align*}
\]

(4)

where the harmonic function is

\[ h(r) = 1 + \frac{2^5\pi^2 Nk l_p^6}{r^6}. \]  

(5)

The background preserves \( 3/8 \) supersymmetries. The near horizon limit (\( r \to l_p^2 U, \ l_p \to 0 \)) of (4) gives us M2-branes on \( AdS_4 \times S_7/Z_k \) spacetime

\[
\begin{align*}
    ds_{11}^2 &\sim R^2(U^4(-dx_0^2 + dx_1^2 + dx_2^2) + \frac{dU^2}{U^2}) + R^2ds_{S_7/Z_k}^2 \\
    F_{(4)} &\sim 6R^3 \text{vol}(AdS_4)
\end{align*}
\]

(6)

where \( (R/l_p)^2 = (2^5\pi^2 Nk)^{1/3} \). Here \( l_p \) is the eleven-dimensional Planck length.

The number of M2-branes in (4) is taken as \( N \cdot k \), so that the flux through \( S^7/Z_k \) remains integral of \( N \). The doubling of supersymmetries in the near horizon region suggests that the \( AdS_4 \) geometry will preserves 24 supersymmetries. The holographic dual boundary Chern-Simons theory in large \( N \) (\( k > 2 \)) limit is conjectured to be the \( N = 6 \) \( SU(N)_k \times SU(N)_{-k} \) superconformal Chern-Simons field theory living on the worldvolume of \( Nk \) M2-branes [21]. While in the large \( N \) 't Hooft limit, but with fixed \( N/k \) ratio, the theory reduces to weakly coupled superconformal Chern-Simons theory of corresponding \( N \) D2-branes on \( AdS_4 \times CP_3 \) [21].

### 2.1 Special case of \( C_4/Z_4 \)

Our interest is in the special case of orbifold space \( C_4/Z_4 \) where we are able to resolve the conical singularity at the origin. The modified metric on \( C_4/Z_k \) is taken as Eguchi-Hanson type [44],

\[
    ds_{C_4/Z_4}^2 = \frac{dr^2}{f(r)} + \frac{r^2}{k^2}f(r)(dz + kA)^2 + r^2ds_{CP_3}^2
\]

(7)

which can be solved exactly for Ricci flatness. We determine that there is a unique solution for any \( k \) value

\[ f(r) = (1 - r_0^8/r^8) \]  

(8)

where \( r_0 \) is an integration constant. The coordinate ranges are fixed as

\[ r_0 \leq r \leq \infty, \quad 0 \leq z \leq 2\pi. \]  

(9)
To know if the metric is regular near \( r = r_0 \) region, we can define a local coordinate patch

\[
r^2(1 - r_0^8/r^8) = (k\rho)^2
\]

with \( \rho \) being infinitesimal radial coordinate. The \( r = r_0 \) neighborhood geometry then becomes

\[
ds^2 \simeq \frac{k^2}{16}d\rho^2 + \rho^2(dz + kA)^2 + r_0^2 ds_{CP^3}^2
\]

(10)

So the metric will be resolved only when \( k = 4 \). For \( k = 4 \) only, eq.(10) will have a smooth \( R^2 \times CP^3 \) geometry in the neighborhood of \( r = r_0 \). This makes the fibered circle \( z \) to be well behaved everywhere. One can think of the geometry in (10) as if every point in the \( CP^3 \) has a small \( R^2 \) patch attached, while the \( CP^3 \) space has a large but constant radius given by \( r_0 \). This resolution is similar in manner as to the resolved Eguchi-Hanson 4D instantons \([44]\) and the resolution of Calabi-Yau cones in higher dimensions, see \([45, 46, 47, 48, 49]\). For a detailed study of deformations and resolutions of various Calabi-Yau spaces in higher dimensions with fluxes turned on one can see \([50]\).

**M2-brane solution**

Correspondingly the 4\( N \) branes background on resolved \( C_4/Z_4 \) space can be obtained by solving

\[
\partial_r r^7 f \partial_r h = 0
\]

The complete solution is as in equation (4) but with a new harmonic function

\[
h(r) = 1 + \frac{Q}{4r_0^6} \left( \arctan \left( \frac{r^2}{r_0^2} \right) - \frac{1}{2} \log \left( \frac{r^2 - r_0^2}{r^2 + r_0^2} \right) \right)
\]

(11)

Near \( r = r_0 \) tip, this harmonic function behaves as

\[
h \simeq \frac{Q}{8r_0^6} \log \left( \frac{r_0^2}{2\rho^2} \right).
\]

So the solution diverges but logarithmically only. While for \( r \gg r_0 \) it becomes

\[
h \sim 1 + \frac{Q}{6r_0^6} + \frac{Qr_0^8}{14r_0^{14}} + \cdots
\]

Comparing it with \([15]\) we fix \( Q/6 \equiv 2^5 \pi^2 4Nl_p^6 \).

The near horizon decoupled geometry in this case is

\[
ds_{11}^2 \simeq l_p^2 h^4 U^2 \left( \frac{-(dx_1^2 + dx_2^2 + dx_3^2)}{U^2 h} \right) + \frac{dU^2}{U^2 f} + \frac{f}{16} (dz + 4A)^2 + ds_{CP^3}^2
\]

\[
F(4) = l_p^3 d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2
\]

(12)

1The resolved manifold is topologically a complex line bundle over \( CP^3 \) \([50]\).
2We comment that one can anyway study M2-branes on the transverse space in (7) for any \( k \) so long as we do not bother about the orbifold singularity at \( r = 0 \). Actually the space patch inside \( r = r_0 \) no longer exists, since \( r \geq r_0 \). But in the \( \rho \) coordinate which is appropriate coordinate in the \( r = r_0 \) region the singularity will exist which can be seen from (10).
with
\[ h(U) \simeq \frac{Q_0}{4U_0^6} \left( \frac{1}{2} \log \frac{U^2 + U_0^2}{U^2 - U_0^2} + \arctan \frac{U^2}{U_0^2} \right) \] (13)
and \( f = 1 - \frac{U_8}{U^8} \). Here we identify \( \frac{Q_0}{6} \equiv 2^5 \pi^2 (4N) \gg 1 \) so that the overall curvature of the spacetime is small in the Planck units. It is obviously a deformation of the \( AdS_4 \times S^7/Z_4 \) discussed above as it can be seen that in the far UV regime \( (U \gg U_0) \) the geometry in (12) becomes exactly
\[ ds^2_{11} \sim l_p^2 (Q_0/6)^{\frac{1}{2}} \left( \frac{U^4 (-dx_0^2 + dx_1^2 + dx_2^2)}{(Q_0/6)} + \frac{dU^2}{U^2} + \frac{1}{16} (dz + 4A)^2 + ds^2_{CP^3} \right) \]
\[ F_{(4)} \sim l_p^3 \frac{36}{Q_0} U^5 dU \wedge dx^0 \wedge dx^1 \wedge dx^2 \equiv l_p^3 \sqrt{6Q_0} vol(AdS_4) \] (14)
which is the near horizon geometry \( AdS_4 \times S^7/Z_4 \) corresponding to \( 4N \) M2-branes on unresolved \( C_4/Z_4 \).

3 D2-branes

In order to study corresponding type IIA string picture we need to compactify along the fibre \( z \) in \( S^7/Z_4 \). As the \( U \) decreases, at some value the effective radius of the \( z \) circle will become smaller than the eleven-dimensional Planck length and we have to think in terms of type IIA strings. We can then compactify along \( z \) and the corresponding ten-dimensional metric and dilaton are obtained from (12)\(^3\)
\[ ds^2_{str} = e^{2\phi} h^{\frac{1}{3}} U^2 \left( \frac{(-dx_0^2 + dx_1^2 + dx_2^2)}{hU^2} + \frac{dU^2}{U^2 f} + ds^2_{CP^3} \right) \]
\[ e^{\frac{4\phi}{3}} = h^{\frac{1}{3}} f U^2 \] (15)
where \( h \) is as in (13) and \( f = 1 - \frac{U_8}{U^8} \) and string length is set to one. This should be thought of as a background due to \( N \) D2-branes on deformed \( AdS_4 \times CP_3 \) in the IR region where string coupling is weak. Only in the far UV limit \( (U \gg U_0) \) or when \( U_0 = 0 \) we will get \( AdS_4 \times CP_3 \). Specially, near \( U = U_0 \) (cut-off) IR region strings become essentially non-interacting as \( f \) vanishes there, however the radius of curvature given by \( e^{\frac{4\phi}{3}} h^{\frac{1}{3}} U \) also becomes small at the same time. So there is curvature singularity which will require higher order \( \alpha' \) corrections to the string geometry. In order to get actual IR behavior we define \( (1 - \frac{U_8}{U^8}) = (4u)^2 \) in the neighborhood of \( U = U_0 \) which is a chosen lowest energy scale in our theory. In this neighborhood the radius of curvature, \( R_{(10)} \), and the string coupling behave as
\[ (R_{(10)})^2 \sim x \sqrt{\log \frac{1}{2x^2} \sqrt{4N}} \]
\(^3\) Our convention is \((l_p/R_{(11)})^2 = 1/g_s^2\).
\[ e^{2\phi} \sim x^3 \sqrt{\log \frac{1}{2x^2} \sqrt{4N}} \]  
(16)

where \( x \equiv \frac{u}{u_0} \). As \( x \to 0 \) it can be seen that string coupling vanishes faster than the curvature radius of the string metric. It means that the strings become non-interacting. However higher order world-sheet corrections have to be included in order to know the dynamics at the IR cutoff. We have plotted this behavior of quantities in eqs. (16) in the graph below.

\[ \text{Figure 1: This represents the plot of } (R_{(10)}/l_s)^2 \text{ (upper) and } e^{2\phi} \text{ (lower) Vs } x. \text{ The string coupling vanishes faster.} \]

4 World-volume theory

We shall now comment on the \( N = 6 \) world-volume theory of the M2-branes on the resolved \( AdS_4 \times S^7/Z_4 \) for finite level \( k = 4 \). As we have noted above the \( N = 6 \) Chern-Simons theory always flows to the weak coupling in IR for \( k = 4 \) being finite. We recover the D2-brane theory on \( AdS_4 \times CP_3 \) in the IR.

In the BL theory there is an additional scalar field which is considered to be a free field. Its vev controls the strength of the 3D gauge coupling. In the paper [16], it was shown that there is such a field which represents the center of mass scalar field of M2-branes corresponding to the location of the branes in the flat transverse space. We would like to see whether this survives the interpretation once the transverse \( C_4/Z_4 \) space is resolved or is no longer flat Euclidean.

Here we just write down the Born-Infeld action. The world-volume metric on M2-brane in flat Euclidean transverse space can be written as

\[ G_{\mu\nu} \equiv \eta_{\mu\nu} + \sum_{M=1}^8 G_{MN}(Y) \partial_\mu Y^M \partial_\nu Y^N \]  
(17)
where \((\mu, \nu = 0, 1, 2)\) and \((M, N = 3, 4, \cdots, 10)\). So it has an explicit \(SO(8)\) invariance. Now identifying the \(Y^{10} \equiv z\) and defining \(G_{1010} = e^{4\phi/3}\) and taking all \(G_{MN}\) independent of \(z\) field, we can write

\[
- \int d^3x \sqrt{\text{Det}(G)} = - \int d^3x e^{-\phi} \sqrt{|g_{\mu\nu} + g_{mn}\partial_\mu Y^m \partial_\nu Y^n + e^{2\phi} \partial_\mu z \partial_\nu z|}
\]

where the scalars \(Y^m, z\) constitute the all 8 scalars and \(g\) is the string metric. Doing this spontaneously breaks the overall \(SO(8)\) symmetry to \(SO(7) \times U(1)\). We can also see that in the limit when string coupling vanishes, the \(z\) kinetic terms become subleading and could be dropped, leaving behind DBI action for D2-branes. Writing down the full action for the background with a resolved transverse space metric

\[
d_{5\tilde{C}_4/Z_4}^2 = \frac{dr^2}{f(r)} + \frac{r^2}{k^2} f(r)(dz + kA)^2 + r^2 ds_{\mathbb{C}P_3}^2
\]

we get an effective BI action (for \(K = 4\))

\[
- \int d^3x \sqrt{|\eta_{\mu\nu} + \frac{1}{f(R)} \partial_\mu R \partial_\nu R + \frac{R^2}{16} f(R)(\partial_\mu z + 4A_\mu)(\partial_\nu z + 4A_\nu) + \cdots|}
\]

where \(f(R) = 1 - r_0^8/R^8\). Thus we can see that there can be an effective potential for scalar field \(R\) once \(z\) is Higgsed by gauge field \(A\), however it can be a free modulus only if \(dz + 4A\) is vanishing. While \(z\) as it appears only through derivatives will be a free field. The gauge fields in the above do not have kinetic terms (as those are not dynamical) but they can have Chern-Simons like terms. This indicates that the complete Chern-Simons theory would be a gauged version of \(N = 6\) ABJM theory (for \(k = 4\)) with appropriate superpotential for scalar fields. It will be interesting to know such a theory for \(k = 4\).

## 5 Summary

In this short note we have constructed M2-brane solutions on resolved \(C_4/Z_4\) Euclidean 8-manifolds. These solutions possess new properties which to our knowledge have not been explored earlier in the literature. The near horizon geometry is a deformation of the \(AdS_4 \times S^7/Z_4\) spacetime of the M2-branes. Although there is essential singularity in the IR region where curvatures become large but the string (Yang-Mills) coupling vanishes. This is juxtaposite of the superconformal Yang-Mills theories for which have strong coupling fixed point in the IR. We conclude that the holographic dual \(SU(N) \times SU(N)\) Chern-Simons theory is free near the IR cutoff. Although in IR the curvature of spacetime becomes very small but the string coupling vanishes there. In other words the core of the resolved M2-branes dissolves into noninteracting D2-branes and the string coupling vanishes. Since the IR region has an inbuilt cutoff \(U = U_0\) near which curvature becomes high and therefore higher order worldsheet corrections should be taken into account. We have also commented that the complete Chern-Simons theory would be a gauged version of \(N = 6\) ABJM theory (for \(k = 4\)) with appropriate superpotential for scalar fields.
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