Overview of Image Smoothing Algorithms

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Abstract. In the process of image generation and transmission, it is easy to be polluted and affected by various kinds of noise, which reduces the quality of the image. In order to avoid the adverse effects of image processing and image visual effect, we can use image smoothing technology to improve the adverse effects, so as to make the image clearer. There are a variety of image smoothing techniques, which had great smoothing effect, but there are also different defects. Based on the analysis of different noise models, this paper uses mean smoothing, median smoothing, Gaussian smoothing, bilateral filtering and other techniques to study and analyze the image processing algorithm, the implementation method and efficiency, which provides efficient theoretical value for image processing and image visual effect in various fields involved in image processing.

1. Introduction

In today's real society of science and technology, images can be touched anytime and anywhere. Images are the basis of human vision, and people can intuitively understand the information that images convey to us. Moreover, this way of communication is more intuitive and rich than any other form. In our real life, the images we get always contain different noises. In order to avoid the adverse effects of image processing and image visual effect, we can use image smoothing technology to improve the adverse effects, suppress noise and improve image quality, so as to make the image clearer. At the same time, image processing technology is the use of computer technology to improve the image quality of a technology.

At present, for image processing technology, has been very deep and universal development, and the requirements of image processing in different fields are gradually increasing. But technology also plays a wide role in the following two aspects: on the one hand, it makes the image processing technology more and more close to our life, on the other hand, it also makes the image processing technology more and more deeply into modern technology. Digital image processing involves many aspects of knowledge and specific methods are also diverse. At the same time, with the development of the new image processing technology, these technologies are constantly updated and developed, and more and more can better meet the needs of human information processing. In recent years, image processing methods are widely used in many areas of application, and there are a large number of new processing methods, which have received unprecedented attention and become an important topic.
2. Basic methods of image smoothing

There are many kinds of image smoothing algorithms, so a targeted smoothing algorithm can be designed according to the relevant mathematical model established by the specific noise function (as shown in Equations 1 and 2) and the characteristics of the image.

\[
P(z) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)
\]

The gray value of the image pixel is expressed by the parameter \( Z \), and \( \sigma \) is its expectation and standard deviation respectively.

Salt and pepper noise model function:

\[
P(z) = \begin{cases} 
  P_a, & z = a \\
  P_b, & z = b \\
  0, & \text{other}
\end{cases}
\]

Where, when the condition \( P_a \) is equal to zero, it is shown as "salt" noise; When the condition \( P_b \) is equal to zero, then it is "pepper" noise [1, 2].

We smoothed the image from two aspects: spatial domain and frequency domain. In the frequency domain, the smoothing processing method in the frequency domain is to treat the image as a two-dimensional signal and enhance the signal. This enhancement method can remove the noise contained in the image through the two-dimensional Fourier transform and the appropriate low-pass filtering method. Low-pass filtering in the frequency domain will take the grainy noise of the image, the edge of the image and the relative jump part of the image as the corresponding high-frequency components representing the image signal, and the low-pass component of the image signal is the part with large background area, which will be the effect when analyzing the frequency characteristics of the image signal. The noise of the high frequency part of the image can be removed by Gaussian filtering, so as to achieve the effect of smoothing the image and improving the image quality. In terms of spatial domain, linear, nonlinear and adaptive smoothing algorithms are used in the smoothing processing of spatial domain. Linear smoothing is commonly used for mean filtering and Gaussian filtering. The operation of linear smoothing mean filtering is to replace the pixel gray value of the image with its field value to the corresponding gray value of each pixel that matches with its own. The field size is set as: \( N \times N \), where \( N \) is odd.

There are some defects in the linear smoothing processing of images. Their processing will make the edges and details of the image blurred. The main reason is that the operation is like a two-dimensional low-pass filter. Compared with linear smoothing, nonlinear smoothing has more advantages in smoothing image, even can be said to improve the linear smoothing. The next problem we need to solve is the adaptive median filter proposed according to these algorithms, which is the image adaptive smoothing algorithm first proposed by J. S. Lee in 1980 [3], which is a smoothing method improved on the median filter and mean filter.

3. Image smoothing algorithm analysis

3.1. Mean filtering method

Mean filtering is a simple spatial domain processing method, or a linear algorithm. Its operation is to provide a specific template for the specified pixels in the image, and the adjacent pixels belong to the template. For example, for the 3x3 templates, eight adjacent pixels around the target pixel are used to form a filter template, and then the original pixels are replaced by the average value of all the corresponding pixels in the template.

Neighborhood average method in mean filtering is an algorithm of spatial domain processing. Let an image be \( f(x, y) \), which is \( N \times N \) array, and the processed image be \( g(x, y) \). The gray level of each pixel is determined by the average gray level of several pixels contained in the predetermined neighborhood of \( (x, y) \) [4], which can be expressed by the following equation 3.

\[
g(x, y) = \frac{\sum f(i, j)}{m}, (i, j) \in s
\]
Where $x, y = 0, 1, 2, \ldots, M$ is the number of pixels in the neighborhood, and $S$ is the coordinates of each pixel in the domain. In the process of averaging, each pixel is replaced by the average value of its neighborhood, resulting in the change of the gray value of no noise pixels, which makes the image blurred. Moreover, when the template size becomes larger, the degree of blur also deepens, especially the edge and detail parts. Therefore, it must be improved, using the weighted domain average method, using the weighted average as the output value of the center pixel. The commonly used smoothing operators are shown in formula 4:

$$
\begin{align*}
H_1 &= \frac{1}{9} \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix} \\
H_2 &= \frac{1}{10} \begin{bmatrix}
1 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 1
\end{bmatrix} \\
H_3 &= \frac{1}{16} \begin{bmatrix}
1 & 2 & 1 \\
2 & 4 & 2 \\
1 & 2 & 1
\end{bmatrix} \\
H_4 &= \frac{1}{8} \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\end{align*}
$$

(4)

### 3.2. Median filtering method

The median filter algorithm in nonlinear smoothing can filter out the noise in the image. Its operation process is to replace the gray value of the image pixel with the median value of its domain gray value. The two-dimensional median filter can be expressed by the following formula 5:

$$Y_{ij} = \text{Med}\{f_{ij}\}
$$

(5)

Where, $\{f_{ij}\}$ represents a two-dimensional data sequence. The smoothing effect of the median filter is mainly reflected in that it can not only remove the noise in the image but also protect the edge of the image, so it can improve the image quality and achieve a certain restoration effect. Nevertheless, there are still problems that cannot be solved. For example, for some points, lines, top details of a lot of images, median filtering smoothing method cannot be very good processing, so in this case, we need to adopt weighted median filtering algorithm. The function of this method is to display the middle points and those pixels that are very close to the middle. The basic operation of this method is to modify the number of window variables so that the variable of a pixel is equal to the value of the point, and then calculate the median value of the increased number set. The weighted median filter can recover the edge of the image which is disturbed by the noise well and protect other details of the image well.

### 3.3. Adaptive median filtering method

The adaptive median filter algorithm is a further modification of the median filter algorithm. Compared with the image smoothing of the median filter, the smoothing operation of the adaptive median filter is able to change the size of the window in the filter no statically under the conditions that have been designed in advance. At the same time, the adaptive median filter can deal with the large proportion of impulse noise very well in addition, the smoothing is not impulsive noise is also able to maintain more image details. Its realization is as follows:

The smoothing method of adaptive median filtering is the following two processes, A and B:

1) A layer of algorithm
   - $A_1 = Z_{\text{med}} - Z_{\min}$, $A_2 = Z_{\text{med}} - Z_{\max}$
   - If $A_1 > 0$ and $A_2 < 0$, then jump to B.
   - Otherwise, increase the size of the window.
   - If the increased window size is $\leq S_{\text{max}}$, then repeat A.
   - Otherwise, $Z_{\text{med}}$ is directly output.

2) B layer algorithm
   - $B_1 = Z_{xy} - Z_{\min}$, $B_2 = Z_{xy} - Z_{\max}$
   - Output $Z_{xy}$ if $B_1 > 0$ and $B_2 < 0$.
   - Otherwise, $Z_{\text{med}}$ is output.

Where: $S_{xy}$ represents the mask window of the center point $(x,y)$ at a given time, namely, the action area of the filter, and the area covered by the filter window. The center point of this area is a pixel point in the $y$ row and $x$ column of the image. $Z_{\min}$ represents the smallest gray level value in $S_{xy}$; $Z_{\max}$ represents the maximum grayscale value in $S_{xy}$; $Z_{\text{med}}$ represents the median of all gray
values in SXY; Zxy represents the gray value of the pixels in row y and column x of the image; Smax represents the maximum size allowed by Sxy.

Whether Zmed is a pulse can be determined by A layer, and whether Zxy is a pulse can be determined by B layer. When the results judged by ZMED and Zxy are not one pulse, the algorithm outputs a constant pixel value, Zxy, to replace the median value in the neighborhood to avoid unnecessary detail loss [5].

3.4. Gaussian filtering method
The weight of the Gaussian filter is chosen by the appearance of the Gaussian function, and the filter is linearly smooth. This filter is very useful for filtering noise following normal distribution [6, 7]. The Gaussian function of one-dimensional zero mean is as follows:

$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$  (6)

Where, $\sigma$ is the standard deviation. For this function, this parameter can represent the width of Gaussian filter. For two-dimensional zero mean discrete Gaussian function, it is:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$  (7)

Where $\sigma$ is the same as above, for this two-dimensional Gaussian function, it can also be called Gaussian radius, and its corresponding value range is [0.1 ~ 250]. $\sigma^2$ is the variance. From the perspective of probability theory, this paper explains that the formula represents the probability near $\mu$, $f(x)$ represents the probability, and $\mu$ is the mean value, that is, the expectation. The closer to $\mu$, the smaller the $\sigma$, the greater the probability; the farther away from $\mu$, the larger the $\sigma$, the smaller the probability.

In the preprocessing stage of computer vision algorithm, Gaussian smoothing can enhance images with different scales. The process of Gaussian smoothing of image is the combination of image and normal distribution. Generally speaking, Gaussian image sliding technology is the low frequency part, and the high frequency part is filtered. The function of Gaussian smoothing is to suppress the noise and achieve the effect of smoothing, but there is a phenomenon that the edge of the image and the details of the image are blurred.

3.5. Bilateral filtering method
Bilateral filter is an anisotropic filter invented by Tomasi and Manduchi in 1998. It also belongs to a kind of nonlinear filtering, the basic principle of the filter is to take into account both the spatial domain and the value domain, so that it can keep the edge of the image and filter the noise at the same time. The output image pixel value of the bilateral filter comes from the input image and is locally weighted average. The output image pixel value is as follows:

$$\hat{f}(x, y) = \frac{\sum_{(i,j) \in D_{xy}} g(x,y) w(x,y,i,j)}{\sum_{(i,j) \in D_{xy}} w(x,y,i,j)}$$  (8)

Where $D_{xy}$ represents the domain pixel of $2N+1 \times 2N+1$ of the center point $(x, y)$ and the value of $\hat{f}(x, y)$ depends on the weighted average of the domain pixel value $g(x, y)$. The weight coefficient $w(x, y, i, j)$ depends on the product of domain kernel and range kernel.

$$w_d(x, y, i, j) = \exp\left(-\frac{(i-x)^2+(j-y)^2}{2\sigma^2}\right)$$  (9)

$$w_s(x, y, i, j) = \exp\left(-\frac{|g(x,y)-g(i,j)|^2}{2\sigma^2}\right)$$  (10)

So,

$$w(x, y, i, j) = w_d(x, y, i, j) * w_s(x, y, i, j)$$  (11)

Where: where, $w_d$ is the spatial proximity factor, $w_s$ is the luminance similarity factor. From equation (11), we can see that the weighting coefficient of the bilateral filter is the nonlinear combination of $w_d$ and $w_s$.

When the Euclidean distance between the pixel and the center point gradually increasing, $w_d$ When the difference between the brightness of pixels becomes larger and larger, $w$ decreases $w_s$ decreased. When the image is in a relatively flat region, the difference between the brightness values in the domain
is small. At this time, the filter is like a Gaussian filter. When the image is in a relatively flat region, the difference between the brightness values in the domain is large. At this time, the filter replaces the brightness value of the original image with the brightness value adjacent to the image edge. So the image after bilateral filtering is smoothed and the edge is kept.

4. Analysis of algorithm experimental results

4.1. The result after adding noise:
In order to make the experimental effect more obvious, we first add noise to the image, as shown in Figure 1. In the figure below, from left to right and from top to bottom are: image converted to gray level in Figure 1; image added with 50000 salt and pepper noise in Figure 2; image added with 10% Gaussian noise in Figure 3; image added with 20% Gaussian noise in Figure 4:

![Figure 1. Add different types of noise.](image)

4.2. Figure of filtering processing results:
Figure 2 shows the original image and the image with 50000 salt and pepper noises. The structure of Figure 3 is the image after Gaussian convolution filtering, median filtering, mean filtering and adaptive median filtering. From the perspective of image display, median filter is better than Gaussian filter and mean filter in dealing with salt and pepper noise, and the image after adaptive median filter is basically similar to the original image.

![Figure 2. Gaussian noise processing](image)
Figure 3. Images after filtering

Figure 4 shows the original image and the image after adding 40% Gaussian noise, and figure 5 shows the image after Gaussian convolution filter, median filter, mean filter and adaptive median filter. According to the image display, the filtering effect of Gaussian filter and mean filter on Gaussian noise is obtained, which makes the image blurred. Adaptive median filter can remove Gaussian noise, but there are still details and edge blur.
4.3. Results of bilateral filtering

Figure 6. Original picture.

a. Qualitative comparison of different sigma shows that $s = c = 25$ (bilateral filtering) is better.

Figure 7. Sigma qualitative comparison

b. When $\text{sigma}_\text{space} < 10$, $\text{sigma}_\text{color} / \text{sigma}_\text{noise} = 2 \sim 3$, the effect is the best and the original image is the best. In this validation, it is found that the effect is good.

Figure 8. $\text{sigma}_\text{color} / \text{sigma}_\text{noise} = 2 \sim 3$

5. Conclusions
Mean filter is simple and fast in calculation, it has a good filtering effect on Gaussian noise, but the effect of eliminating the salt and pepper noise of the image is not as good as that of eliminating Gaussian noise. In addition, it can not protect the image details after filtering the image, and also make the processed image become more fuzzy, which can not achieve the goal of removing the image noise points very well effect.

Median filtering algorithm is relatively simple and easy to implement, can filter out the salt and pepper noise of the image, and then can achieve the effect of filtering out noise, protecting details and image clarity. However, it is easy to cause discontinuous lines in the image, which will result the noise filtering and the protection of image details, that is, there will be conflicts between the two.
Gaussian filter is a kind of linear smoothing, it can filter Gaussian noise well, can make the image blurred, eliminate noise, so as to achieve the effect of smoothing. Gaussian filtering can reduce the image noise when the signal can be preserved, but at the same time, this method will flatten the edge because it is invalid to approach the edge. The filter can suppress the noise from positive distribution. Bilateral filtering can keep the edge information of the image well. It is a nonlinear filtering method. It takes into account not only the relationship between space and pixel distance, but also the similarity between pixels, so that the original image can be retained in a large block, and then the edges can be retained. It can also achieve the effect of noise elimination.

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