Testing Loop Quantum Gravity and Electromagnetic Dark Energy in Superconductors

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Abstract

In 1989 Cabrera and Tate reported an anomalous excess of mass of the Cooper pairs in rotating thin Niobium rings. So far, this experimental result never received a proper theoretical explanation in the context of superconductor’s physics. In the present work we argue that what Cabrera and Tate interpreted as an anomalous excess of mass can also be associated with a deviation from the classical gravitomagnetic Larmor theorem due to the presence of dark energy in the superconductor, as well as with the discrete structure of the area of the superconducting Niobium ring as predicted by Loop Quantum Gravity. From Cabrera and Tate measurements we deduce that the quantization of spacetime in superconducting circular rings occurs at the Planck-Einstein scale \( l_{PE} = (\hbar G/c^3 \Lambda)^{1/4} \sim 3.77 \times 10^{-5} m \), instead of the Planck scale \( l_P = (\hbar G/c^3)^{1/2} = 1.61 \times 10^{-35} m \), with an Immirzi parameter which depends on the specific critical temperature of the superconducting material and on the area of the ring. The stephan-Boltzmann law for quantized areas delimited by superconducting rings is predicted, and an experimental concept based on the electromagnetic black-body radiation emitted by this surfaces, is proposed to test loop quantum gravity and electromagnetic dark energy in superconductors.

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1 Introduction

The physics of SuperConductors (SC) could be relevant for Loop Quantum Gravity (LQG) with respect to investigating:

1. The scale at which spacetime acquires a discrete structure.

2. The possibility of a spontaneous breaking of the Principle of General Covariance (PGC) in SCs.

3. The thermal time hypothesis and the physical nature of time in SCs.

2 Anomalous Cooper pair mass excess

In 1989 Cabrera and Tate [14, 15], through the measurement of the magnetic trapped flux originated by the London moment, reported an anomalous Cooper pair mass excess in thin rotating Niobium superconductive rings:

$$\Delta m = m^* - m = 94.147240(21)\text{eV}$$

Here $m^* = 1.000084(21) \times 2m_e = 1.023426(21)\text{MeV}$ is the experimentally measured Cooper pair mass (with an accuracy of 21 ppm), see Figure 1 displaying a typical data set, and $m = 0.999992 \times 2m_e = 1.002331\text{MeV}$ is the theoretically expected Cooper pair mass including relativistic corrections.

We can also express the relative excess of mass as:

$$\frac{\Delta m}{m} = 9.2 \times 10^{-5}$$

The above Cooper pair mass excess (or, equivalently, the slightly larger than expected measured magnetic field) has not been explained until now in the context of superconductor’s physics.

The principle of Cabrera and Tate experiment, is to measure the magnetic flux originated from the London moment which is trapped in a thin Niobium ring.

Integrating the current density of Cooper pairs around a closed path including the effect of a rotating frame:

$$\frac{m^*}{e^2n_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l} = n_s \frac{\hbar}{2e} - \int_{S_{\Gamma}} \vec{B} \cdot d\vec{S} - \frac{2m^*}{e} \vec{\omega} S_{\Gamma}$$

Where $n_s$ is the Cooper pair number density, $S_{\Gamma}$ is the area bounded by the closed curve, $\Gamma$, circulating inside the superconductor, $\omega$ is the SC’s angular velocity, $B = -\frac{m_e}{2} \omega$ is the London moment.
Figure 1: Bar graph showing the distribution of all data points of the Cooper pair relative mass excess, $m^*/2m_e$, in Cabrera and Tate experiment, for Nb ring’s rotation frequency intervals of 2 Hz. We see in the present case that the average value of the relative mass excess is $m^*/2m_e = 1.000075(25)$.

There exist an angular velocity $\omega_n$ for each $n$ such that the flux $\int \vec{B} \cdot d\vec{S}$ and the electric current line integral $\oint_{\Gamma} \vec{j} \cdot d\vec{l}$ are zero together, see Figure 2. This allows to define the flux null spacing by

$$\Delta \omega \equiv \omega_n - \omega_{n-1}$$

Subtracting eq.(3) for $n$ and $n-1$ we obtain:

$$\frac{h}{m^*} = S_{\Gamma} \Delta \omega$$

Which is the key formula used by Cabrera and Tate to calculate the Cooper pair’s mass.

3 Possible interpretations of Cabrera and Tate experiment

What are the physical parameters which should vary to offer alternatives to the interpretation of Cabrera and Tate’s experiment in terms of an anomalous excess of mass? From eq.(5) we see that we have two possibilities in addition to the Cooper pair mass parameter:

1. The area of the Niobium ring is different when the ring is in the normal state and when it is in the superconducting state.

2. We must add a gravitomagnetic term to $\Delta \omega$, eq.(4), i.e., we must add a GM term in the Cooper pair canonical momentum.
3.1 Fundamental quantum of area in superconductors and the Immirzi parameter

Let us first investigate possibility 1). Either we observe a real excess of mass of Cooper pairs, $m^*$, and the area of the ring in the superconducting state is the same as in the normal state:

$$m^* = \frac{h}{S_T \Delta \omega} \quad (6)$$

Where $S_T$ will be called the Euclidian area of the SCing ring’s hole. Or the Cooper pair’s mass remains unchanged and the area of the SCing ring is different from its area in the normal state:

$$m = \frac{h}{S'_T \Delta \omega} \quad (7)$$
Where $S'_\Gamma$ will be called the non-Euclidean area of the SCing ring’s hole. Subtracting eq.(7) from eq.(6) we find:

$$\frac{\Delta m}{m} = \frac{S'_\Gamma - S\Gamma}{S\Gamma} = \frac{\Delta S\Gamma}{S\Gamma}$$  \hspace{1cm} (8)

Inserting numerical values from Cabrera and Tate experiment in eq.(8) we find the fundamental quantum of area:

$$\Delta S\Gamma = 1.86 \times 10^{-7}[m^2]$$  \hspace{1cm} (9)

In LQG the principal series of eigenvalues of the area, is labeled by multiplets of half integers $j_i$, $i = 1, ..., n$ and is given by:

$$A = 8\pi\gamma \left(\frac{\hbar G}{c^3}\right) \sum_i \sqrt{j_i(j_i + 1)}$$  \hspace{1cm} (10)

where $\gamma$, called the immirzi parameter, is a free dimensionless constant of the theory [1]. In eq.(10) we are assuming that the fundamental quanta of area for a given fundamental value of $j_i$ is proportional to the Planck area, $l_P^2 = \left(\frac{\hbar G}{c^3}\right)$. Therefore we are assuming that the fundamental scale for quantum gravity is the Planck scale. However Beck, Mackey and CDM have argued that the fundamental scale for quantum gravity in superconductors should be the Planck-Einstein scale corresponding to the geometric mean between the Planck scale, $l_P = \left(\frac{\hbar G}{c^3}\right)^{1/2}$ which determines the highest possible energy in the universe, and the Einstein scale, $l_E = \Lambda^{-1/2}$, which is determined by the non-zero value of the Cosmological Constant (CC) $\Lambda$ and fixes the lowest possible energy in the universe.

$$l_{PE} = \sqrt{l_P l_E} = \left(\frac{\hbar G}{c^3\Lambda}\right)^{1/4}$$  \hspace{1cm} (11)

This would lead to a fundamental quantum of area in SCs that would be proportional to the Planck-Einstein area:

$$A_{PE} = l_{PE}^2 = \sqrt{A_P A_E} = \left(\frac{\hbar G}{c^3\Lambda}\right)^{1/2}$$  \hspace{1cm} (12)

substituting eq.(12) in the LQG expression for the area, eq.(10) we get:

$$A = 8\pi\gamma \left(\frac{\hbar G}{c^3\Lambda}\right)^{1/2} \sum_i \sqrt{j_i(j_i + 1)}$$  \hspace{1cm} (13)
for the fundamental value $j = 1/2$ we have the fundamental quantum of area:

$$A_{1/2} = 4\sqrt{3}\pi\gamma \left(\frac{\hbar G}{c^3\Lambda}\right)^{1/2}$$  \hspace{1cm} (14)

Assuming $A_{1/2} = \Delta S_T$ we can substitute eq.\(14\) in eq.\(8\) and use the measurement of Cabrera and Tate for the Cooper pairs excess of mass, as well as the Euclidian dimensions of their SCing Niobium ring $S_T = 2.02 \times 10^{-3} m^2$, to find the value of the Immirzi parameter in the SC:

$$\gamma \simeq 6$$  \hspace{1cm} (15)

### 3.2 Electromagnetic dark energy in superconductors

Let us now investigate possibility 2) above. In this case the Cooper pair canonical momentum, $\vec{\pi}$, should include a GravitoMagnetic (GM) term.

$$\vec{\pi} = m\vec{v} + e\vec{A} + m\vec{A}_g$$  \hspace{1cm} (16)

Where $\vec{A}$ is the magnetic vector potential, $\vec{v}$ is the Cooper pair velocity, and $\vec{A}_g$ is the GM vector potential, whose rotational gives the GM field:

$$\vec{B}_g = \nabla \times \vec{A}_g$$  \hspace{1cm} (17)

The Ginzburg-Landau eq.\(3\) should now read as:

$$\frac{m}{e^2n_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l} = \frac{n}{2e} \oint_{S_T} \vec{B} \cdot d\vec{S} - \frac{m}{e} \int_{S_T} \vec{B}_g \cdot d\vec{S} - \frac{2m}{e} \omega \vec{S}_T$$  \hspace{1cm} (18)

Subtracting eq\(3\) from eq.\(18\) we get:

$$\vec{B}_g = 2\vec{\omega} \left(\frac{m^* - m}{m}\right) + \left(\frac{m^* - m}{m}\right) \frac{1}{S_T\text{en}_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l}$$  \hspace{1cm} (19)

In a SC that is thick compared with the London penetration depth, the circular path $\Gamma$ can be chosen in the SC’s bulk where there is no current flowing, thus leading to a null current integral in eq.\(19\), and:

$$\frac{\Delta m}{m} = \frac{B_g}{2\omega}$$  \hspace{1cm} (20)

Eq.\(20\) is interpreted as indicating a deviation with respect to the classical GM Larmor theorem \[2\] \[3\].

Beck and CDM \[8\] \[6\] \[7\] have discussed the possibility of that the source of the gravitational and gravitomagnetic fields in SCs is the density of electromagnetic zero point energy, $\rho^*$, contained in the SC. Although this requires
a spontaneous breaking of the Principle of Generale possibility Covariance (PGC) when the material crosses its critical temperature, \( T_c \). Meaning that below \( T_c \) the density of electromagnetic zero point energy has the same physical nature as the vacuum energy, \( \rho_{CC} \): associated with the CC:

\[
\rho_{CC} = \frac{c^4 \Lambda}{8\pi G}
\]  

(21)

and above \( T_c \), \( \rho^* \) does not contribute anymore the cosmological vacuum energy, \( \rho_{CC} \). Since the currently measured value of the cosmological constant, \( \Lambda = 1, 29 \times 10^{-59} [m^{-2}] \), accounts reasonably for the density of dark energy observed in the universe, we can say that our model of gravitationally active electromagnetic zero point energy in SCs is an electromagnetic model of dark energy in superconducting matter. Each SCs would host a different density of electromagnetic dark energy proportional to the fourth power of its critical temperature [8]-[13]:

\[
\rho^* = \frac{\pi \ln^4(3)}{2} \frac{k^4}{(ch)^3} T_c^4
\]  

(22)

CDM [6] has shown that the Cooper pairs excess of mass measured by Cabrera and Tate is proportional to the ratio of electromagnetic dark energy contained in the superconductor and the cosmological density of dark energy:

\[
\frac{\Delta m}{m} = \frac{B_q}{2\omega} = \frac{3}{2} \frac{\rho^*}{\rho_{CC}}
\]  

(23)

In the case of Niobium eq.(23) is in excellent agreement with the measured value eq.(2).

In the following we will assume that:

1. Cooper pairs mass excess,

2. discrete areas at the Planck-Einstein scale,

3. non-classical inertia in superconductors,

are different equivalent phenomenological interpretations of the spontaneous breaking of the PGC in SCs, i.e, we assume:

\[
\frac{\Delta m}{m} = \frac{\Delta S_T}{S_T} = \frac{B_q}{2\omega} = \frac{3}{2} \frac{\rho^*}{\rho_{CC}} = \chi
\]  

(24)
4 Discrete spacetime and electromagnetic dark energy

To consolidate further the physical concept of a discrete structure of spacetime at the Planck-Einstein scale in SCs [8], let us mention briefly how one can deduce SC’s inertial properties as resulting from quantum fluctuations of the SC’s four volume.

4.1 Uncertainty relations and discrete spacetime

The successful resolution of the inverse CC problem [8] encourages us to start from the assumption that the spacetime volume of a superconductor is made of Planck-Einstein cells, \( l_{PE}^4 \), which will fluctuate:

\[
\Delta V \sim \sqrt{V} l_{PE}^2
\]

(25)

Since the density of vacuum energy associated with the CC, \( \rho_{CC} \) is canonically conjugated with the universe four-volume \( V \),

\[
\Delta \rho_{CC} \Delta V \sim \hbar c
\]

(26)

we use the electromagnetic dark energy density, eq.(22) and the SC’s four volume eq.(25) instead of respectively \( \rho_{CC} \) and \( \Delta V \) in eq.(26). In this way we obtain that the inertia in superconducting matter changes with respect to its classical laws due to the fluctuations of the SC’s discrete spacetime volume:

\[
\Delta \chi \sqrt{V} \sim \frac{2\pi^2}{3} l_{PE}^2
\]

(27)

Substituting eq.(24) in eq.(27) we find

\[
\Delta S_{\Gamma} \sqrt{V} \sim n \frac{2\pi^2}{3} l_{PE}^4
\]

(28)

Where \( n = S_{\Gamma}/l_{PE}^2 \) is the number of Planck-Einstein quanta of area making the Euclidean value of \( S_{\Gamma} \). This could indicate the the fundamental value of the quantum of area of surface delimited by SCs could fluctuate as a consequence of a four-volume fluctuations of the SC.

4.2 Stephan-Boltzmann law for discrete spacetime surfaces in superconducting rings

Let us now use the electromagnetic dark energy model in SCs to find a thermodynamical law of the Immirzi parameter.
From the equality of $A_{1/2}$ and $S_{Γ}$, $A_{1/2} = ΔS_{Γ}$, and substituting eq. (14) in eq. (8), and using eqs. (21)(22) and (21) we obtain:

$$\gamma = \frac{3\ln^4(3)}{32\sqrt{3}\pi^2} \left( \frac{T_c}{T_{PE}} \right)^4 \frac{S_{Γ}}{l_{PE}^3} = \frac{n}{32\sqrt{3}\pi^2} \left( \frac{T_c}{T_{PE}} \right)^4$$

(29)

Where as above $n = S_{Γ}/l_{PE}^2$ is the number of Planck-Einstein quanta of area making the Euclidean value of $S_{Γ}$, $T_{PE} = \frac{1}{\hbar} \left( \frac{c^2 k^3}{G} \right) \approx 60.71 K$ is the Planck-Einstein temperature. Using the Euclidean value of the area of the Niobium SCing ring used by Cabrera and Tate, $S_{Γ} = 2.02 \times 10^{-3} m^2$, in eq. (29) we find indeed the same result than in eq. (15), i.e, $\gamma \sim 6$. Taking the same surface for the different superconducting materials listed in table 1 below we can calculate the respective Immirzi coefficients.

| Superconductive material | $T_c[K]$ | $\gamma$ |
|--------------------------|--------|--------|
| Al                       | 1.18   | 0.0016 |
| Sn                       | 3.72   | 0.16   |
| Pb                       | 7.2    | 2.24   |
| Nb                       | 9.25   | 6.11   |
| Nb$_3$G$_2$              | 23.2   | 242.14 |
| YBCO                     | 91.0   | 57316.81 |

Table 1: Immirzi parameter $\gamma$ predicted by the model of electromagnetic dark energy in superconductors.

How do these values stand with respect to the usual discussion of the Immirzi parameter in the context of Black-hole thermodynamics [1]? Bekenstein suggested that the horizon of a Black-hole of mass, $M$, irradiates like a Black-Body at the temperature

$$T = \frac{\hbar c^3}{a32\pi kG M}$$

(30)

Where $a$ is a constant to be ultimately determined experimentally, although LQG calculations predict a value of $a$ in function of the Immirzi parameter, $\gamma$:

$$a \sim \frac{0.2375}{4\gamma}$$

(31)

For $\gamma = 0.2375$ we obtain $a = 1/4$, which leads to the Hawking prediction for the Black hole temperature:

$$T = \frac{\hbar c^3}{8\pi kGM}$$

(32)
This is the current procedure to fix the value of $\gamma$ in LQG.

Substituting eq.(31) in eq.(30) we get:

$$M = \frac{\hbar c^3 \gamma}{1.9\pi kGT} \quad (33)$$

Using the values for $\gamma$ and $T = T_c$ listed in Table 1) we can estimate what would be the mass of the black-hole which would be needed to generate the quantum gravitational effects, at the Planck scale, that we predict in superconducting rings, with area $S_T = 2.02 \times 10^{-3} m^2$, at the Planck-Einstein scale. The result is listed in table 2

| Superconductive material | $T_c [K]$ | $\gamma$ | $M_{\text{Black-Hole}} [Kg]$ |
|-------------------------|-----------|---------|---------------------------|
| Al                      | 1.18      | 0.0016  | $7.1 \times 10^{20}$      |
| Sn                      | 3.72      | 0.16    | $2.21 \times 10^{22}$     |
| Pb                      | 7.2       | 2.24    | $1.61 \times 10^{23}$     |
| Nb                      | 9.25      | 6.11    | $3.42 \times 10^{23}$     |
| Nb$_3$G$_2$             | 23.2      | 242.14  | $5.4 \times 10^{24}$      |
| YBCO                    | 91.0      | 57316.81| $3.26 \times 10^{26}$     |

Table 2: Equivalent black hole mass (Classical physical system at the Planck scale) corresponding to the LQG Immirzi parameter $\gamma$ predicted by the model of electromagnetic dark energy in superconductors at the Planck-Einstein scale.

We see that the masses of the corresponding black-holes ranges from $M_{\text{Moon}}/100$ until $4436M_{\text{Moon}}$. This "boost" of quantum gravitational effects in superconductors in the Earth laboratory, which have masses much smaller than the Moon’s mass, would be due to the important ratio between the Planck-Einstein scale and the Planck scale. The cause of this scale transformation could be a spontaneous breaking of the PGC in superconductors.

Let us now investigate the possible physical phenomenology associated with eq.(29). Defining the constant $\kappa [m^{-2}K^{-4}]$:

$$\kappa = \left(\frac{3 \ln^4(3)}{32\sqrt{3\pi^2}} T_{PE}^4 L_{PE}^2\right)^{-1} \quad (34)$$

we can re-write eq.(29) in a form analog to the Stephan-Boltzmann law.

$$\gamma = \kappa S_T T_c^4 \quad (35)$$

The total electromagnetic power $\varphi$ irradiated by a black body is given by the Stephan-Boltzmann law:

$$\varphi = \sigma S T^4 \quad (36)$$
Where $\sigma = 5.67 \times 10^{-8}[J/sm^2 K^4]$ is the Stephan-Boltzmann constant, $S$ is the irradiating area of the black body, and $T$ is its temperature.

Making equal eq.(35) and eq.(36), $S_T T^4 = S T^4$, we deduce.

$$\varphi = \frac{\sigma}{\kappa}$$

(37)

substituting eq.(35) in eq.(37) we find:

$$\varphi = \sigma S T^4$$

(38)

Which is the Stephan-Boltzmann law for the area delimited by the SCing ring, i.e. the SCing ring’s hole. At temperatures of the order of $T_c \sim 2.73$ it would be difficult to distinguish the thermal radiation coming from the ring’s hole from the one coming from the Cosmic Microwave Background (CMB). In the following section an experimental concept to detect the Black-Body thermal radiation coming from areas delimited by superconductors, is proposed.

5 Experimental concept to test LQG and electromagnetic dark energy in superconductors

The experimental concept to test LQG and dark energy according to the previous discussion, is to measure the black-body radiation of a thin SCing ring located near a parabolic reflector, and concentrate the electromagnetic radiation at the reflector’s foci, where it would be detected. The goal would be to exhibit an anomalous excess of thermal energy originated from the SCing ring (empty) hole according to eq.(38). In order to achieve this measurement one needs, at least, to subtract from the total amount of thermal energy detected:

1. The Cosmic microwave background,

2. The black body radiation coming from the reflector.

The reflector should not be in a superconducting state to avoid creating additional black-body radiating geometrical surfaces.

By measuring the spectral composition of the Planckian radiation emitted by the SCing ring’s hole, one could test the predictions of LQG about the non-existence of the Bekenstein-Mukhanov effect on Hawking’s thermal radiation emitted by black holes. This spectral analysis could also contribute to a better understanding of the spin-network underlying the surface.
6 Conclusions

A bi-dimensional Geometrical surface, empty from any matter, bounded by a closed superconducting wire, would irradiate in the same manner as a material black-body with similar radiating area and shape. When the superconductor that is drawing the frontier of the geometrical surface in question, becomes normal, above $T_c$, this radiation would disappear. This would be a direct consequence of the quantization of geometric areas delimited by SCing materials at the Planck-Einstein scale, and of the electromagnetic dark energy content of the superconductor. Ultimately this would represent different equivalent phenomenological manifestations of the spontaneous violation of the principle of general covariance in superconductors which, as is well known, would also lead to a violation of energy conservation, in the covariant sense.

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