Collective Evolution of Hot QCD Matter from the QGP to Freeze-Out

Adrian Dumitru* and Steffen A. Bass†

*Physics Department, Columbia University, 538W 120th Street, New York, NY 10027
†Nat. Supercond. Cycl. Lab., Michigan State University, East Lansing, MI 48824-1321

Abstract. We present results on the evolution of $\langle p_t \rangle$ with energy, for hadrons emerging from hadronization of a quark-gluon plasma possibly produced in high-energy heavy-ion collisions. We find that BNL-RHIC energy corresponds to the previously predicted plateau of $\langle p_t \rangle$, reminiscent of the assumed first-order QCD phase transition. Heavy hadrons are the best messengers of the transverse flow stall.

Theoretical arguments [1] and lattice QCD studies [2] suggest that at high temperatures, roughly $T \sim 200$ MeV, the thermodynamically stable state of QCD is different from that at $T = 0$, i.e. the theory of strong interactions exhibits a phase transition to a new state called “Quark Gluon Plasma” (QGP). It is the only phase transition of a fundamental theory accessible to experiments under defined laboratory conditions. In the QGP the effective number of relativistic degrees of freedom is expected to be substantially larger than below the phase transition temperature $T_c$. The search for signatures of that phase transition is one of the primary goals of high-energy heavy-ion collision experiments.

In particular, if the transition is first order with non-vanishing latent heat, it is supposed to leave fingerprints on the hydrodynamical expansion pattern [3,4]. Isentropic expansion would proceed through phase coexistence, where the pressure $p$ is independent of the energy density $\epsilon$, such that the isentropic speed of sound $c_s^2 = dp/d\epsilon = 0$. Thus, during that phase coexistence stage energy density gradients in the system do not reflect in pressure gradients, and the average transverse flow velocity does not increase substantially. As the $p_t$ spectra of the emitted hadrons are determined by the transverse boost velocity and the temperature at freeze-out, one expects a plateau of $\langle p_t \rangle$ of charged hadrons for some range of beam energies where the central region “sweeps” through the mixed phase [3].

However, the early studies of that effect suffered from some simplifications which render comparisons to existing experimental data, and extrapolation to higher energies, unrealistic. First, the latent heat of the transition (and therefore the space-time volume of the mixed phase) was overestimated because only pions were

---

1) Speaker at CIPANP2000, 7th Conference on the Intersections of Particle and Nuclear Physics, May 22-28, 2000, Quebec City, Canada.
assumed to contribute to the entropy of the hadronic phase. Second, final-state
decays of hadronic resonances (like $\rho \rightarrow \pi\pi$ or $\Delta \rightarrow \pi N$) which contribute more
than 50% to the measured pion multiplicity were not taken into account. Finally,
the most serious problem arises from the arbitrariness of the decoupling hypersurface, which can not be determined within hydrodynamics but affects the results considerably.

To reduce the uncertainties and the number of free parameters one therefore
has to improve on the treatment of the post-hadronization stage by considering
microscopic transport [5,6], e.g. within the relativistic Boltzmann equation with
collision kernel as determined by the known hadronic cross sections. The expansion
and subsequent hadronization of the high-temperature QGP state can be modelled
within relativistic hydrodynamics, assuming that the system evolves locally through
a phase coexistence. Thus, in the space-time region inbetween the initial-time
(purely space-like) hypersurface and the hadronization hypersurface (the boundary
between mixed phase and hadronic phase, which has both space-like and time-like
parts) we solve the continuity equation for the energy-momentum tensor,

$$\partial_\mu T^{\mu\nu} = 0, \quad \text{with} \quad T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}. \quad (1)$$

In the QGP we assume a gas of quasi-free $u$, $d$, $s$ quarks and gluons; for sim-
plicity, the only contribution from interactions to the QGP pressure is due to the
temperature independent vacuum pressure (bag pressure), see e.g. [1].

On the hadronization hypersurface, we switch to the Boltzmann equation,

$$p \cdot \partial f_i(x^\mu, p^\nu) = C_i, \quad (2)$$

where $f_i(x^\mu, p^\nu)$ denotes the phase-space distribution function of species $i$. The
binary collision approximation is used to construct the collision kernel $C_i$ [7], which
treats almost all hadrons from [8] explicitly. All those hadrons are also taken into
account in the equation of state of the mixed phase entering eq. (1). The switch
from hydrodynamics to microscopic transport is performed by matching the energy
momentum tensors and conserved currents on the hadronization hypersurface. The
evolution towards freeze-out is thus determined by the relevant hadronic cross-
sections and decay rates, competing with the local expansion rate $\partial_\mu u^\mu$. The initial
condition for central collisions of Pb/Au nuclei and CERN-SPS energy is that
hydrodynamic flow sets in on the $\tau_i = 1$ fm proper time hypersurface, and the
entropy per net baryon is $s/n_B = 45$, which reproduces a variety of measured final-
state hadron multiplicities as well as the $p_t$ spectra of $\pi$, $K$, $p$, $\Lambda$, $\Xi$, and $\Omega$, see [5,6].
For the higher BNL-RHIC energy, $\sqrt{s} = 200 A$ GeV, the parameters were assumed
to be $\tau_i = 0.6$ fm and $s/n_B = 205$, which yields $dN_{ch}/dy \approx 800$ at $y = 0$. Fig. 1
depicts $\langle p_t \rangle$ for a variety of hadrons, for CERN-SPS, BNL-RHIC, and CERN-LHC
energies. One observes that at each energy $\langle p_t \rangle$ increases with $m$. Reminiscent
of the collective expansion before decoupling, all hadrons flow approximately with
the same transverse velocity, and therefore their momentum essentially increases in
proportion to their mass. This effect is seen in data obtained for Pb+Pb collisions.
at the SPS [9]. The spectra of heavy hadrons are much less affected by the random thermal motion at decoupling, \( v_\text{th} \propto \sqrt{T/m} \), and are therefore better suited to measure the transverse flow build up during the evolution. One also observes that despite the five times higher entropy per baryon assumed to be achieved at RHIC energy, the average transverse momenta are predicted by this model to be similar to the lower SPS energy. This is mainly due to the first-order phase transition featuring a region of energy densities where \( c_s^2 \) is small (it is not exactly zero in this calculation due to the finite conserved baryon charge); and to some extent also due to earlier decoupling of the hadrons emerging from the hadronization of the plasma at RHIC energy [5].

If the initial energy density in the central region continues to grow towards higher energy as predicted by models assuming saturation of the transverse density of gluons [10] (see however [11]), the hydrodynamical expansion will lead to very large \( \langle p_t \rangle \) at LHC energy, see Fig. 1. Thus, RHIC-energy is possibly right in the plateau of \( \langle p_t \rangle \), if the concept of a phase mixture is applicable to high-energy collisions.

Fig. 2 shows the freeze-out volume of the pions at central rapidity, calculated as the average transverse area \( \pi \langle r_t \rangle^2 \) times the average length of the central rapidity slice, which is equal to proper time \( \tau \). The space-time volume is essentially increasing linearly with the multiplicity. Thus, the pion density at freeze-out is not affected much by the presence of the phase transition, unlike the average transverse momentum. When extrapolating each line to \( dN_\pi/dy = 0 \) we obtain a “hollow” volume \( V_0 \), which increases with \( p_t \). This is caused by the radial flow: high \( p_t \) pions can not be emitted from the center, where the collective velocity field vanishes for symmetry reasons, and where the pions have only random thermal momenta.

**Acknowledgements:** A.D. thanks the organizers for the invitation to
CIPANP2000 and acknowledges support from a DOE Research Grant, Contract No. De-FG-02-93ER-40764. S.A.B. is supported by NSF grant PHY-00-70818.

REFERENCES

1. e.g., Shuryak E., Phys. Rept. 61, 71 (1980); McLerran L., Rev. Mod. Phys. 58, 1021 (1986).
2. e.g., Oevers M., Karsch F., Laermann E., and Schmidt P., Nucl. Phys. Proc. Suppl. 73, 465 (1999).
3. Kataja M., Ruuskanen P., McLerran L., von Gersdorff H., Phys. Rev. D 34, 794 and 2755 (1986).
4. Blaizot J., and Ollitrault J., Phys. Rev. D 36, 916 (1987); Hung C., and Shuryak E., Phys. Rev. Lett. 75, 4003 (1995); Rischke D., and Gyulassy M., Nucl. Phys. A608, 479 (1996); Brachmann J., et al., nucl-th/9908010; nucl-th/9912014.
5. Dumitru A., Bass S., Bleicher M., Stöcker H., and Greiner W., Phys. Lett. B460, 411 (1999).
6. Bass S., and Dumitru A., Phys. Rev. C 61, 064909 (2000).
7. Bass S., et al., Prog. Part. Nucl. Phys. 41, 225 (1998).
8. Caso C., et al., Eur. Phys. J. C3, 1 (1998).
9. Bearden I., et al., [NA44 Collaboration], Phys. Rev. Lett. 78, 2080 (1997).
10. Krasnitz A., and Venugopalan R., Phys. Rev. Lett. 84, 4309 (2000); Eskola K., Kajantie K., Ruuskanen P., and Tuominen K., Nucl. Phys. B570, 379 (2000).
11. Dumitru A., and Gyulassy M., hep-ph/0006257.

\[ V^* = \pi \left( T, f_{\pi \pi} \right)^2 \left \langle < f_{\pi \pi} > \right \rangle \]

FIGURE 2. Freeze-out volume of the pions as a function of the pion rapidity density at central rapidity, for various \( p_T \) cuts. Increasingly high \( p_T \) pions are only emitted from a “shell”, the radius of the hollow core increasing with \( p_T \), as shown in the inset.