Abstract We present a superselection rule (SSR) which restricts the validity of the superposition principle in the rigged Hilbert space of states of a particle taking part in a one-dimensional completed scattering (OCS). This rule is induced by a dichotomous physical context which determines the quantum dynamics of a particle in the nonoverlapped spatial regions located at the asymptotically large distances from the barrier, on the different sides from it. The role of a superselection operator is played by the Pauli matrix $\sigma_3$ that divides the Hilbert space into two coherent sectors associated with this context. The unitary Schrödinger evolution crosses these sectors, in the case of the OCS, and hence this process must be treated as the result of superimposing two coherent inseparable subprocesses – transmission and reflection, and all physical observables can be introduced only for them. We show how to restore in-states for both subprocesses, according to the corresponding out-states.

Keywords superselection rules · superposition principle · tunneling · pure state · mixed state

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1 Introduction

The contemporary model of scattering a quantum particle on a one-dimensional (1D) rectangular potential barrier is included in many textbooks on quantum mechanics as a model which allows one to adequately and exhaustively present all physical properties inherent to this quantum process. But is it so? Simple analysis shows that the real efficiency of this model is extremely low. Yes, in the case of one-dimensional completed scattering (OCS) it shows in detail how the wave packet that describes the ensemble of scattering particles splits, in the course of scattering, into two parts – transmitted and reflected. It also provides a clear rule how to calculate the transmission and reflection coefficients. However, it gives no prescription how to unambiguously interpret this wave-packet splitting, as well as how to interpret the temporal dependence of the expectation values of the particle’s position and momentum. And, lastly, this model gives no rule, even for the rectangular barrier, how to unambiguously define the tunneling time – the problem which is moot up to now (see, e.g., [1]).

In our approach to the tunneling time problem (see, e.g., [2,3]) we argue that to explain the OCS is as complex as to explain the two-slit experiment and to solve the cat paradox. This is so because, in the final analysis, all these problems are the consequence of the existing, fundamentally erroneous practice of a quantum-mechanical description of micro-cat states – coherent superpositions of macroscopically distinct pure states associated with a micro-object. In the case of the OCS they are the final states of transmitted and reflected particles; in the two-slit experiment they are ‘one-slit’ states which separately
describe the passage of a particle through the first or second slit in the screen; in the cat paradox, they are the alternative states of a radioactive atom – "undecayed atom" and "decayed atom".

The fundamental error in this practice is that it implies treating micro-cat states as pure states. In particular, in line with this practice, a radioactive atom after the end of its half-value period, in the laboratory frame of reference, exists as decayed and undecayed simultaneously; a particle in the experiment with two slits in the screen passes through both the slits simultaneously; a particle at the final stage of the OCS moves along the OX-axis to the plus and minus infinity simultaneously.

It is usually said that the properties of micro-cat states look as non-physical because we attempt to explain them in terms of classical conceptions, what is erroneous because the micro-world is allegedly "unspeakable" in principle. The cat paradox is said to arise only at the macro-level, when we attempt to extend these properties onto macro-objects. Hence this paradox must be treated as a measurement or macro-objectification problem and the only way of solving this problem is to appeal to an external agent (environment, gravitation field, etc.). Its influence on a micro-system is said to be unavoidable (there are no closed micro-systems) and it leads to transforming the initial pure cat-state of this system into a mixed state of the composite macro-system – 'microsystem+external agent'; as is assumed, this influence nullifies at the macro-level the role of the main ‘culprit’ of the cat paradox – the quantum-mechanical superposition principle.

What however is unsatisfactory in this approach to the cat paradox is that it transforms quantum mechanics into a weak theory: from the viewpoint of this approach quantum mechanics, as a theory of the macro-world, is insolvent and, as a theory of the micro-world, it has no physical content. At the same time there is every reason to believe that the cat paradox must be resolved precisely at the level of micro-objects. The point is that the above practice of description of micro-cat states has been based on the assumption that the superposition principle is valid for such states without any restrictions. But this assumption is questionable. As will be shown below by the example of the OCS, the cat-states of closed micro-systems must be under the jurisdiction of (missed) superselection rules (SSRs), by which such states should be treated as mixed states.

Note that the idea of solving the cat paradox on the basis of SSRs is already used in the physical literature (see [15] as well as [6,7]). However, this idea is used here as a way of solving this paradox as a measurement problem. It was applied to the macro-cat states of composite systems, when a micro-system under study interacts with an environment or with a macro-system (with an infinite number of degrees of freedom) which plays the role of a measuring device. That is, in fact, the studies [4,5] concern macro-cat states. As regards micro-cat states, the known SSRs (the general theory of SSRs is presented in [11,15]) forbid coherent superpositions of particles with different spins, electrical charges, masses, etc. (see, e.g., [8,10] and references therein). For one-particle micro-cat states, where each of these parameters takes the only value, the issue of SSRs was never considered in the physical literature.

As it was stressed in [7] (see p.379) with regards to SSRs in the non-relativistic quantum mechanics, "Skepticism about particular superselection rules, as well as superselection rules in general, was expressed in the physics literature as late as 1970. On the whole, however, the physics community seems to have quickly accepted superselection rules as facts of quantum life. But for the most part superselection rules were treated as curious inconveniences – certainly worth noting, but once noted to be shoved aside to let quantum life proceed per usual."

The present study confirms that the role of SSRs in the non-relativistic quantum mechanics is underestimated. This results in the erroneous formulations of the superposition principle in the case of micro-cat states, and namely due to these formulations quantum mechanics is incompatible with classical physics. The contemporary theory of micro-cat states must be revised with taking into account of (yet unknown) SSRs which govern such a kind of states. To show this by the example of the OCS is the main purpose of this paper.

The key idea of our approach is that in the case of the OCS the (rigged) Hilbert space of particle’s states consists of two superselection sectors, associated with two macroscopically distinct physical contexts which determine the asymptotical behavior of the ensembles of transmitted and reflected particles at the plus and minus infinity on the OX-axis. The latter is important because "Two collectives of particles moving under two macroscopically distinct contexts form two different statistical ensembles" [17].

Besides, as is known, "probabilistic data generated by a few collectives...cannot be described by a single Kolmogorov space" (ibid); see also [18]. In this connection we are also aimed to show that at the
final stage of the OCS the non-Kolmogorovian (quantum) probability space is reduced, owing to the SSR to be presented, to the sum of two Kolmogorovian (classical) probability subspaces.

2 Backgrounds

Let us consider a spinless non-relativistic particle scattering on a semitransparent potential barrier $V(x)$ which is nonzero only in the bounded interval $[-a, a]$; $d = 2a$ is the barrier width. As is known, solving this scattering problem is reduced to solving the stationary Schrödinger equation for a particle with a given energy $E = (hk)^2/2m$. In the general case the stationary wave function $\Psi(x; k)$, outside the interval $[-a, a]$, can be written in the form

$$\Psi(x; k) = \begin{cases} A_{L,\text{in}}(k) e^{ikx} + A_{L,\text{out}}(k) e^{-ikx}, & x \leq -a; \\ A_{R,\text{out}}(k) e^{ikx} + A_{R,\text{in}}(k) e^{-ikx}, & x \geq +a. \end{cases}$$

(1)

According to the transfer-matrix approach [19] the amplitudes of the wave function in regions $x \leq -a$ and $x \geq a$, irrespective of the shape of $V(x)$, are connected by the transfer matrix $Y(k)$ as follows:

$$\begin{pmatrix} A_{L,\text{in}} \\ A_{L,\text{out}} \end{pmatrix} = Y \begin{pmatrix} A_{R,\text{out}} \\ A_{R,\text{in}} \end{pmatrix}; \quad Y = \begin{pmatrix} q & p \\ p & q^* \end{pmatrix};$$

(2)

$$q = \frac{1}{\sqrt{T(k)}} e^{i(kd-J(k))}, \quad p = i \sqrt{\frac{R(k)}{T(k)}} e^{iF(k)}$$

where $R = 1 - T$. Note that (real) transmission $T$ and reflection $R$ coefficients, as well as two phases $J$ and $F$ can be calculated (analytically or numerically) for any shape of the potential barrier in the interval $[-a, a]$. For this purpose one can use (see [19]) the explicit analytical expressions (if $V(x)$ is the rectangular barrier or $\delta$-potential) or recurrence relations (if this potential represents a piecewise continuous function); the phase $F$ for any symmetric barrier takes only two values, 0 or $\pi$.

Thus we may further assume that the transfer matrix $Y$ of the potential barrier $V(x)$ and the corresponding scattering matrix $S$ are known. The latter links incoming and outgoing waves:

$$\begin{pmatrix} A_{R,\text{out}} \\ A_{L,\text{out}} \end{pmatrix} = S \begin{pmatrix} A_{L,\text{in}} \\ A_{R,\text{in}} \end{pmatrix}; \quad S = \frac{1}{q} \begin{pmatrix} 1 & -p \\ p^* & 1 \end{pmatrix}; \quad S^{-1} = S^\dagger = \frac{1}{q^*} \begin{pmatrix} 1 & p \\ -p^* & 1 \end{pmatrix}.\quad (3)$$

Note that $A_{R,\text{in}}(k) \equiv 0$ ($A_{L,\text{in}}(k) \equiv 0$) when a particle is emitted by the left (right) source of particles. In the general case, i.e., for a bilateral incidence of a particle (when it is emitted in each experiment either by the left or by the right source of particles), both these amplitudes are nonzero; $|A_{L,\text{in}}(k)|^2 + |A_{R,\text{in}}(k)|^2 = 1$.

Thus, in the general case the stationary in- and out-states of a particle, in the $x$-representation, can be written as the two-component wave functions

$$\Psi_{\text{in}}(x; k) = \begin{pmatrix} A_{L,\text{in}}(k) e^{ikx} \\ A_{R,\text{in}}(k) e^{-ikx} \end{pmatrix}, \quad \Psi_{\text{out}}(x; k) = \begin{pmatrix} A_{R,\text{out}}(k) e^{ikx} \\ A_{L,\text{out}}(k) e^{-ikx} \end{pmatrix},$$

which are connected with each other as follows

$$\Psi_{\text{out}}(x; k) = S(x; k) \Psi_{\text{in}}(x; k); \quad S = D S D^{-1}, \quad D = \begin{pmatrix} e^{ikx} & 0 \\ 0 & e^{-ikx} \end{pmatrix}; \quad S^{-1} = S^\dagger. \quad (4)$$

The term 'incoming' implies that the amplitudes $A_{L,\text{in}}(k)$ and $A_{R,\text{in}}(k)$ depend only on the positive values of $k$. However, in the time-dependent scattering problem, the particle’s in-asymptote corresponding to the limit $t \to -\infty$ must represent a 'physical' state. That is, it must be well localized in the $x$-space in order to ensure the existence of the expectation values of all powers of the position $\hat{X}$ and momentum $\hat{P}$ operators. Thus, the in-asymptote in the $x$-representation must belong (see, e.g., [20]) to the Schwartz space. And, since its Fourier-transform (belonging too to the Schwartz space) is determined on the whole real $k$-space, we must assume further that $k \in (-\infty, \infty)$. 

Due to the temporal reversibility of the OCS all the four amplitudes possess the properties

\[
A_{L,\text{in}}(-k) = A_{L,\text{in}}^*(k) \equiv A_{L,\text{out}}'(k), \quad A_{R,\text{in}}(-k) = A_{R,\text{in}}'(k) \equiv A_{R,\text{out}}(k),
\]

where \( A_{L,\text{in}}'(k) \) and \( A_{R,\text{in}}'(k) \) as well as \( A_{L,\text{out}}'(k) \) and \( A_{R,\text{out}}(k) \) represent amplitudes of new possible incoming and outgoing waves linked by the \( \tilde{S} \)-matrix:

\[
\begin{pmatrix}
A_{R,\text{out}} e^{ikx} \\
A_{L,\text{out}} e^{-ikx}
\end{pmatrix} = \tilde{S} \begin{pmatrix}
A_{L,\text{in}}' e^{ikx} \\
A_{R,\text{in}}' e^{-ikx}
\end{pmatrix}.
\]

Thus, due to this symmetry, the spaces of stationary in- and out-states constitute, in fact, the same space, and what is important is that this space is complete (see, e.g., p. 257 in [21]).

We have to recall (see, e.g., [20,22]) that the states of a particle taking part in the OCS belong, in the \( x \)-representation, to the rigged (equipped) Hilbert space \( \mathcal{H}^\text{rig} \) – a Gelfand triplet \( \Phi \subset \mathcal{H} \subset \Phi^\times \), where \( \mathcal{H} \) is a Hilbert space; \( \Phi \) is its dense Schwartz-like subspace; \( \Phi^\times \) is the space of antilinear functionals over \( \Phi \), which include ket-eigenvectors of the one-particle operators \( \hat{X} \) and \( \hat{P} \) (bra-eigenvectors belong to the corresponding space \( \Phi^\times \) of linear functionals over \( \Phi \)).

Thus, each component of the stationary in- and out-states \( \Psi_\text{in}(x;k) \) and \( \Psi_\text{out}(x;k) \) belongs to the space \( \Phi^\times \). As regards wave packets (in- and out-asymptotes) built of these stationary states in the limits \( t \to \pm \infty \) (i.e., long before and after the scattering event), they belong in fact to the Schwartz space, rather than to the Schwartz-like space defined in [20,22]. This is so, because these packets are located far from the potential barrier and, thus, far from its discontinuities.

Now the term "incoming" implies simply that

\[
\int_0^\infty |A_{L,\text{in}}(k)|^2 dk < 1, \quad \int_0^\infty |A_{R,\text{in}}(k)|^2 dk < 1.
\]

In this case, one can be sure that the in- and out-asymptotes will be well localized (see [20]) in the appropriate spatial regions and, besides, the back-flow effect (see, e.g., [23]) will be negligible for them.

Under these conditions we can be sure that, in the \( x \)-representation, the left and right parts of the two-component in-asymptote (out-asymptote) represent wave packets that move towards (away from) the barrier in the macroscopically distinct spatial regions located, respectively, on the left and right sides of the barrier, asymptotically far from it. That is, in this case, we can be sure that the scalar product of the left and right in-asymptotes (as well as the scalar product of the corresponding pair of out-asymptotes) is zero.

Let \( \phi_k^{(1)}(x) \equiv x|\phi_k^{(1)}\rangle = \begin{pmatrix} e^{ikx} \\ 0 \end{pmatrix} \) and \( \phi_k^{(2)}(x) \equiv x|\phi_k^{(2)}\rangle = \begin{pmatrix} 0 \\ e^{-ikx} \end{pmatrix} \) be two-component wave functions which determine the basis vectors \( |\phi_k^{(1)}\rangle \) and \( |\phi_k^{(2)}\rangle \); here \( |x\rangle \) is the eigenvector of \( \hat{X} \).

Note that \( \langle x|\phi_k^{(1)}\rangle \equiv (e^{-ikx}, 0) \) and \( \langle x|\phi_k^{(2)}\rangle \equiv (0, e^{ikx}) \). Hence their norms and scalar product are

\[
\langle \phi_k^{(1)}|\phi_k^{(1)}\rangle = \langle \phi_k^{(2)}|\phi_k^{(2)}\rangle = 1, \quad \langle \phi_k^{(1)}|\phi_k^{(2)}\rangle = \langle \phi_k^{(2)}|\phi_k^{(1)}\rangle = 0.
\]

The last equality reflects the fact that any two wave packets, built in the space \( \Phi \) from the stationary states \( \phi_k^{(1)}(x) \) and \( \phi_k^{(2)}(x) \), are orthogonal because they are localized in the non-overlapping spatial regions.

3 The Pauli matrix \( \sigma_3 \) as a superselection operator for the OCS

Note that \( |\phi_k^{(1)}\rangle \) and \( |\phi_k^{(2)}\rangle \) are eigenvectors of the Pauli matrix \( \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \) and the operator \( \hat{P} \):

\[
\sigma_3|\phi_k^{(1)}\rangle = |\phi_k^{(1)}\rangle, \quad \sigma_3|\phi_k^{(2)}\rangle = -|\phi_k^{(2)}\rangle; \quad \hat{P}|\phi_k^{(1)}\rangle = \hbar k|\phi_k^{(1)}\rangle, \quad \hat{P}|\phi_k^{(2)}\rangle = -\hbar k|\phi_k^{(2)}\rangle.
\]

The set of eigenvectors of \( \sigma_3 \) is evident to coincide with the space \( \Phi^\times \) associated with the OCS. And this set is much wider than that of the operator momentum \( \hat{P} \). In particular, it contains also the eigenvectors \( \phi_x^{(1)}(x) = \begin{pmatrix} \delta(x - x') \\ 0 \end{pmatrix}^T \) and \( \phi_x^{(2)}(x) = \begin{pmatrix} 0 \\ \delta(x - x') \end{pmatrix}^T \) of the operator \( \hat{X} \).
Thus, though the sets of eigenvectors of the operators $\hat{X}$ and $\hat{P}$ are different, both belong to the set of eigenvectors of $\sigma_3$. In other words, though $\hat{X}$ and $\hat{P}$ do not commute with each other, each of them commutes with the operator $\sigma_3$. And this concerns all self-adjoint operators, associated with a spinless particle taking part in the OCS, whose eigenvectors belong to the set of eigenvectors of the operator $\sigma_3$. Of importance (see [15]) is also to emphasize that the operator $\sigma_3$ can be expressed via the projections operators $P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. Namely, $\sigma_3 = P_+ - P_-$. Since the space $\Phi^\times$ associated with the OCS is complete the self-adjoint operator $\sigma_3$ can be treated (see [15] as well as [16]) as a superselection operator that divides the space $\Phi^\times$ into two superselection sectors: $\Phi^\times = \Phi^\times_1 \oplus \Phi^\times_2$; we will imply that $\phi^{(1)}_k \in \Phi^\times_1$ and $\phi^{(2)}_k \in \Phi^\times_2$. Thus, $\mathcal{H}^{\text{rig}} = \mathcal{H}^{\text{rig}_1} \oplus \mathcal{H}^{\text{rig}_2}$. By the superselection theory [11] [16] the superposition of two pure states from the same sector gives another pure state in this sector, but superposing two pure states from different sectors gives a mixed state.

Let $\hat{B}$ be a self-adjoint operator, $\psi_1$ and $\psi_2$ be normalized by 1 states from the sectors $\Phi_1$ and $\Phi_2$, respectively. Let also $\psi_\lambda = \frac{1}{\sqrt{2}}(\psi_1 + e^{i\lambda}\psi_2)$ and $\psi_\nu = \frac{1}{\sqrt{2}}(\psi_1 + e^{i\nu}\psi_2)$; $\lambda$ and $\nu$ are real phases. Then

$$\langle \psi_\lambda | \hat{B} | \psi_\nu \rangle = \langle \psi_\nu | \hat{B} | \psi_\lambda \rangle = \frac{1}{2} \left( \langle \psi_1 | \hat{B} | \psi_1 \rangle + \langle \psi_2 | \hat{B} | \psi_2 \rangle \right).$$

Thus, in fact, the phases $\lambda$ and $\nu$ are unobservable. This means that the states $\psi_\lambda$ and $\psi_\nu$, being the superpositions of pure states, possess the properties of mixed states (see also [7]).

4 The OCS as a mixture of two alternative subprocesses – transmission and reflection

Note that Hamiltonian $\hat{H}$ associated with the OCS does not commute with the superselection operator $\sigma_3$ because its eigenvectors cannot be associated with a single superselection sector. Hence $\hat{H}$, though being a self-adjoint operator, does not represent an observable in this scattering problem. The corresponding Schrödinger evolution crosses the superselection sectors. In particular, it evolves the pure states $\phi^{(1)}_k(x)$ and $\phi^{(2)}_k(x)$ into the mixed states, respectively, $\psi^{(1)}_{k,\text{out}}(x) = \mathcal{S}\phi^{(1)}_k(x) = [\phi^{(1)}_k(x) + p^*\phi^{(2)}_k(x)]/q$ and $\psi^{(2)}_{k,\text{out}}(x) = \mathcal{S}\phi^{(2)}_k(x) = [-pe^{i\lambda}(x) + \phi^{(2)}_k(x)]/q$ (see also Exps. (3) and (4)).

The absence of a physical observable which might be associated with the OCS’s Hamiltonian is manifested in the fact that out-asymptotes, in both the cases, represent mixed states. Thus, contrary to the conventional description of the OCS, this process must be treated as a result of superimposing the two alternative subprocesses – transmission and reflection; and only their individual dynamics has physical meaning. That is, all physical observables and characteristic times are allowed only for them, rather than for the whole OCS. This means, in its turn, that the adequate quantum model of the OCS must provide the possibility to trace not only their out-asymptotes, but also in-asymptotes.

In order to show the main points of this question, let us consider $|\psi^{(1)}_{k,\text{out}}\rangle$ – the mixed out-state of a particle impinging on the barrier from the left. In this mixture, the pure out-state $|\psi^{\text{tr}}_{k,\text{out}}\rangle = \frac{1}{\sqrt{2}}|\phi^{(1)}_k\rangle$ describes transmitted particles that move to the plus infinity on the $OX$-axis, and the pure out-state $|\psi^{\text{ref}}_{k,\text{out}}\rangle = \frac{\nu}{\sqrt{2}}|\phi^{(2)}_k\rangle$ does reflected particles that move to the minus infinity. As is evident, none of these pure out-states, taken alone, can be matched with the pure in-state $|\phi^{(1)}_k\rangle$ by an unitary transformation.

Thus, in-states of both subprocesses are unknown in the current model of the OCS. Hence we face the problem of reconstructing them according to the corresponding out-states. In particular, we have to find such two in-states $|\psi^{\text{tr}}_{k,\text{in}}\rangle$ and $|\psi^{\text{ref}}_{k,\text{in}}\rangle$ which are related with $|\psi^{\text{tr}}_{k,\text{out}}\rangle$ and $|\psi^{\text{ref}}_{k,\text{out}}\rangle$, respectively, and obey the following two obvious requirements:

(a) $|\psi^{\text{tr}}_{k,\text{in}}\rangle + |\psi^{\text{ref}}_{k,\text{in}}\rangle = |\phi^{(1)}_k\rangle$;
(b) the probability current densities associated with $|\psi^{\text{tr}}_{k,\text{in}}\rangle$ and $|\psi^{\text{ref}}_{k,\text{in}}\rangle$ must be equal; for $|\psi^{\text{tr}}_{k,\text{in}}\rangle$ and $|\psi^{\text{ref}}_{k,\text{in}}\rangle$ they must be equal by absolute value.

Since all three in-states – $|\psi^{\text{tr}}_{k,\text{in}}\rangle$, $|\psi^{\text{ref}}_{k,\text{in}}\rangle$ and $|\phi^{(1)}_k\rangle$ – must correspond to the same ray, let us search for the unknown in-states in the form $|\psi^{\text{tr}}_{k,\text{in}}\rangle = A^{\text{tr}}_{\text{in}}(k)\cdot |\phi^{(1)}_k\rangle$ and $|\psi^{\text{ref}}_{k,\text{in}}\rangle = A^{\text{ref}}_{\text{in}}(k)\cdot |\phi^{(1)}_k\rangle$. 

From (a) and (b) it follows that
\[ A_{in}^{tr} + A_{in}^{ref} = 1; \quad |A_{in}^{tr}|^2 = |1/q|^2 \equiv T, \quad |A_{in}^{ref}|^2 = |p^*/q|^2 \equiv R. \]
A simple analysis shows that the last two equalities, related to the requirement (b), are fulfilled if
\[ A_{in}^{tr} = \sqrt{T}(\sqrt{T} + i\sqrt{R}); \quad A_{in}^{ref} = \sqrt{R}(\sqrt{R} \pm i\sqrt{T}). \] (5)

Thus, not only \( A_{in}^{tr} + A_{in}^{ref} = 1 \), but also \( |A_{in}^{tr}|^2 + |A_{in}^{ref}|^2 = 1 \). That is, whilst the requirement (a) implies that the pure in-states \( |\psi_{k,in}^{tr}> \) and \( |\psi_{k,in}^{ref}> \) are in the same superselection sector \( \Phi_1^\chi \), and hence their superposition gives a new pure state \( (|\phi_k^{(1)}> \) in this sector; the requirement (b) implies that these two pure in-states (as being associated with the out-states \( |\psi_{k,out}^{tr}> \) and \( |\psi_{k,out}^{ref}> \) from the sectors \( \Phi_1^\chi \) and \( \Phi_2^\chi \), respectively) possess some properties of mixed states. Indeed, on the one hand, as states from the same superselection sector they interfere with each other: \( <\psi_{k,in}^{tr} | \psi_{k,in}^{ref}> \equiv \pm i\sqrt{TR} \neq 0 \); on the other hand, \( <\psi_{k,in}^{tr} | \psi_{k,in}^{ref}> + <\psi_{k,in}^{ref} | \psi_{k,in}^{ref}> = 1 \) what is inherent to mixed states.

5 Conclusion

So, in the rigged Hilbert space of in- and out-states of a particle taking part in the OCS, there is a SSR induced by a dichotomous physical context which determines the quantum dynamics of a particle in the macroscopically distinct spatial regions located at the asymptotically large distances from the barrier, on different sides from it. The role of a superselection operator is played here by the Pauli matrix \( \sigma_y \) whose eigenvectors divide the Hilbert space into two superselection sectors. Since the unitary Schrödinger evolution of the OCS crosses these sectors, neither physical observable nor characteristic time can be unambiguously defined for the OCS. This one-particle quantum process must be treated as consisting of two coherent inseparable subprocesses, transmission and reflection, and all physical quantities can be defined only for these subprocesses. Of course, due to the interference between them an experimentalist can realize only an indirect study of their temporal evolution.

Thus, reacting to Earman’s phrase ”the classicality of the world we observe is not built into the structure of observables” (see p.403 in [7]), we can now discard the word ‘not’ in this phrase. Within a quantum-mechanical description of the OCS, based on the SSR, quantum probabilities are compatible with classical ones at the asymptotically large distances from the barrier. At these distances, the quantum (non-Kolmogorovian) probability space, associated with the OCS, is reduced to the sum of two classical (Kolmogorovian) probability spaces. Of course, at the very stage of scattering, when the ensemble of particles interacts with the potential barrier, the quantum probability space is irreducible to the sum of classical ones.

All these peculiarities of the OCS’s subprocesses have been revealed in our alternative approach \[2,3\] to the OCS and the tunnelling time problem, which is based on the idea to represent the OCS as a complex process consisting of two subprocesses and hence is fully compatible with the found SSR. This means, in particular, that the present study should be considered as a substantiation of our previous one \[2,3\].

The main lesson to follow from this study is that the role of the superposition principle in the existing quantum-mechanical practice of description of micro-cat states must be revised. This practice is based on the implicit assumption that the superposition principle is valid for such states without any restrictions. However, as was shown by the example of the OCS, this is not the case. We believe that there should be SSRs which restrict the validity of this principle for all quantum processes where micro-cat states arise, and searching for SSRs for the two-slit experiment as well as for the process of decaying a radioactive atom is the next important step for realizing this program.

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