de Sitter Spacetimes from Warped Compactifications of IIB String Theory

P. Berglund\textsuperscript{1}
CIT-USC Center for Theoretical Physics
Department of Physics and Astronomy
University of Southern California
Los Angeles, CA 90089-0484

T. Hübsch\textsuperscript{2,3}
Department of Physics and Astronomy
Howard University
Washington, DC 20059

D. Minic\textsuperscript{4}
Institute for Particle Physics and Astrophysics
Department of Physics
Virginia Tech
Blacksburg, VA 24061

ABSTRACT
We continue our study of codimension two solutions of warped spacetime varying compactifications of string theory. In this letter we discuss a non-supersymmetric solution of the classical type IIB string theory with de Sitter gravity on a codimension two uncompactified part of spacetime. A non-zero positive value of the cosmological constant is induced by the presence of non-trivial stringy moduli, such as the axion-dilaton system for the type IIB string theory. Furthermore, the naked singularity of the codimension two solution is resolved by the presence of a small but non-zero cosmological constant.

\textsuperscript{1}e-mail: berglund@citusc.usc.edu
\textsuperscript{2}e-mail: thubsch@howard.edu
\textsuperscript{3}On leave from the “Rudjer Bošković” Institute, Zagreb, Croatia.
\textsuperscript{4}e-mail: dminic@vt.edu
The issue whether de Sitter space can be obtained from string theory has recently attracted renewed attention [1]. In this letter we address this question by examining non-singular non-static spacetimes which fall into the class of codimension two non-supersymmetric solutions of string theory studied in Refs. [2, 3, 4]. We construct classical non-supersymmetric string theory solutions in $D$ dimensions in which the metric in the uncompactified $(D - 2)$ directions is de Sitter rather than flat Minkowski space as was done in Refs. [2, 4]. The latter theories have a naked singularity in analogy with the global cosmic string [5]. The main point of this letter is that, in non-supersymmetric codimension two solutions of string theory (either type IIB or any space-time variable string vacua), a positive cosmological constant, $\Lambda_b$, in the $D - 2$ dimensional cosmic brane spacetime naturally resolves the naked singularities in the transverse 2-plane. We show how the naked singularity of the global cosmic brane configuration is deformed to a cosmological horizon while keeping the overall features of the solution away from the horizon. In fact, it is the stringy moduli, with non-trivial $SL(2, \mathbb{Z})$ transformation properties, that lead to $\Lambda_b > 0$. Thus, string theory gives rise to a smooth background with a positive cosmological constant in the effective $D - 2$ dimensional theory.

The analysis concerning the existence of a normalizable zero mode, and hence gravity, in the compactified $D - 2$ dimensional theory follows our previous work [4]. The higher KK-modes and the corrections to the Newton potential are suppressed, due to the presence of nontrivial moduli. Of course, this is intuitively expected from the viewpoint of generic Kaluza-Klein compactification. Our analysis is closely related to the situation without matter fields in which case one obtains a Rindler-space type solution [9] with non-trivial boundary conditions. In the latter case if one assumes a realistic value of the cosmological constant, the KK modes give a very large correction to the Newton potential and the effect of the lower dimensional graviton is washed out [9].

Let us contrast the above discussion with the Karch and Randall solutions [10] in which the bulk cosmological constant and the brane cosmological constant are both negative. These solutions describe localized gravity on anti-de Sitter (AdS) spaces which are natural from the point of view of supersymmetry. Note that Gregory has shown that the solutions with the negative brane cosmological constant appear as repulsive fixed points in the space of warped factors describing the codimension two solutions of the global vortex type [8]. We indeed find that the de Sitter (or accelerating universe) like solutions appear to be more generic and natural in the space of all non-supersymmetric solutions of the IIB string theory.

Let us briefly review the general framework of global cosmic branes from a string theory
perspective. We consider a higher-dimensional string theory compactified on a Calabi-Yau (complex) \(n\)-fold, some moduli of which are allowed to vary over (the ‘transversal’) part of the non-compact space \([11, 12]\). Specifically, we will consider type II B string theory (possibly compactified on a \(\text{fixed}\) manifold) in which we let the axion-dilaton system, \((\alpha, \phi)\), described by the complex modulus field \(\tau = \alpha + i \exp(-\phi)\), vary over the uncompactified spacetime. Following Ref. \([2]\), the effective action describing the coupling of the moduli to gravity of the observable spacetime can be derived by dimensionally reducing the higher dimensional Einstein-Hilbert action \([11, 12]\). Thus, the relevant part of the low-energy effective \(D\)-dimensional action of the modulus, \(\tau\), of the Calabi-Yau \(n\)-fold coupled to gravity reads

\[
S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^Dx \sqrt{-g} (R - \mathcal{G}_{\tau\tau} \partial_\mu \tau \partial_\nu \bar{\tau} + \ldots ) .
\]  

(1)

Here \(\mu, \nu = 0, \ldots, D-1, 2\kappa^2 = 16\pi G^D_N\), where \(G^D_N\) is the \(D\)-dimensional Newton constant, and \(\mathcal{G}_{\tau\tau} = -(\tau - \bar{\tau})^{-2}\) is the metric on \(\mathcal{M}\), the complex structure moduli space of a torus. We neglect higher derivative terms in this effective action and set the other fields in the theory to zero as in Ref. \([11]\). We also restrict the modulus to depend on \(x_i, i=D-2, D-1\), so that \(\partial_\alpha \tau = 0, \alpha = 0, \ldots, D-3\).

With the above setting of the \(D=10\) type IIB string theory, we find that the explicit solutions for \(\tau\) are aperiodic, but do exhibit a non-trivial \(SL(2, \mathbb{Z})\) monodromy \([2]\), which ensures this to be a stringy (although classical and non-supersymmetric) rather than merely a supergravity vacuum. Being a modulus, \(\tau\) interacts with other fields only through gravity. Therefore, although supersymmetry will turn out to be broken (see also Ref. \([2]\)), the induced potential for \(\tau\) can be safely neglected.

The absence of a potential for \(\tau\) permits the following simplification of the Einstein equation:

\[
R_{\mu\nu} = \mathcal{G}_{\tau\tau} \partial_\mu \tau \partial_\nu \bar{\tau} \overset{\text{def}}{=} \bar{T}_{\mu\nu} .
\]  

(2)

Eq. (2) affords the ‘separation of variables’ where the metric is axially symmetric, while \(\tau\) is independent of the radial distance from the cosmic brane, so \(\bar{T}_{\mu\nu} = \text{diag}[0, \ldots, 0, \frac{1}{4}\omega^2 l^{-2}]\). (From Ref. \([2]\), \(\tau = \alpha_0 + i g^{-1}_s \exp(\omega \theta)\) and in particular the dilaton of the type IIB superstring theory varies with the polar angle, not the radial distance.) Eq. (2) then defines the general class of our spacetimes as \textit{almost} Ricci-flat: \(R_{\mu\nu} = \text{diag}[0, \ldots, 0, \frac{1}{4}\omega^2 l^{-2}]\), where \(\omega^2 > 0\) is indeed related to supersymmetry breaking \([2]\) and \(l\) is the is length scale set by the global cosmic brane.

With a phenomenologically interesting \(K3\) compactification of the \(D=10\) solution in mind (upon which the metric receives \(\alpha'\) corrections), we continue with the general \(D\)-dimensional setting. The Ansatz for the metric, with \(z = \log(r/l)\), is given by:

\[
ds^2 = A^2(z) \bar{g}_{ab} dx^a dx^b + l^2 B^2(z) (dz^2 + d\theta^2) ,
\]  

(3)

\[
\bar{g}_{ab} dx^a dx^b = -dx_0^2 + e^{2\sqrt{N} x_0} (dx_1^2 + \ldots + dx_{D-3}^2) ,
\]  

(4)
Incidentally, it is easy to prove that if \( \bar{g}_{ab} \) is chosen to be any Ricci-flat metric (e.g., the Schwarzschild geometry), the solutions of Refs. [2, 4] remain unchanged. This can be done in complete analogy with Refs. [13, 14]. Hereafter, however, we focus on the de Sitter metric (4).

The \( R_{ab} = 0 \) part of Eq. (2) reduces to a single equation, giving:

\[
B^2 = 1^{-2} \Lambda_b^{-1} \left( A'' + \frac{1}{(D-3)} AA'' \right) = 1^{-2} \Lambda_b^{-1} \frac{h''h - \frac{D-4}{D-2}}{(D-2)},
\]

which determines \( B(z) \) in terms of \( A(z) \) or \( h(z) \). With this substitution, the remaining components of Eq. (2) produce the following single equation:

\[
\frac{1}{2(D-2)} \frac{h''^2}{h^2} - \frac{h''}{2h} + \frac{h'h'''}{2hh''} = -\frac{1}{8} \omega^2.
\]

This implies that \( \Lambda_b > 0 \), and that the Ansatz (3)–(4) does not permit a double Wick rotation into an AdS spacetime. To see this, note that Eq. (5) determines \( h(z) \), and hence \( A(z) \), to be independent of \( \Lambda_b \). But then, \( \Lambda_b \to -\Lambda_b \) in Eq. (5) would imply \( B(z)^2 < 0 \), making the entire plane transverse to the cosmic brane also time-like.

Furthermore, with \( h(z) = (1 - z/\rho_0)^{D-2} \), and so with

\[
A_0(z) = Z(z) \equiv (1 - z/\rho_0), \quad \text{and} \quad B_0(z) \equiv \frac{1}{l\rho_0 \sqrt{\Lambda_b}},
\]

the metric (3) satisfies the Einstein equations (2) for \( \omega^2 = 0 \), i.e., when \( \tau = \text{const} \). This solution describes the familiar Rindler space [9].

For \( \omega \neq 0 \) (\( \tau \neq \text{const} \)), Eq. (6) has a perturbative solution:

\[
A(z) = Z(z) \left( 1 - \frac{\omega^2 \rho_0^2(D-3)}{24(D-1)(D-2)} Z(z)^2 + O(\omega^4) \right),
\]
\[
B(z) = \frac{1}{l\rho_0 \sqrt{\Lambda_b}} \left( 1 - \frac{\omega^2 \rho_0^2}{8(D-1)} Z(z)^2 + O(\omega^4) \right).
\]

Notice that, as the metric depends on \( A(z)^2 \) and \( B(z)^2 \), it is well-defined for all values of \( z \). In particular, \( z \sim \rho_0 \) is the location of a putative horizon [4, 5]. It is easy to check for our solution (8) that both the Ricci scalar and tensor vanish at \( z = \rho_0 \), as does the whole Riemann tensor. In fact, these tensors as well as the \( R_{\mu\nu}R^{\mu\nu} \) and \( R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \) curvature scalars all remain bounded away from \( z = \rho_0 \). Thus, close to the horizon spacetime is asymptotically flat in agreement with the behavior of Rindler space, see Eqs. (3)–(8) [4].

\[\text{It is straightforward to show that } R_{zz} \text{ and } R_{\theta\theta} \text{ can be written as certain linear combinations of the above differential equation and its derivatives.}\]

\[\text{This solution is of the same form as that discussed by Gregory [4, 5] for the } U(1) \text{ vortex solution.}\]
In contrast, when $\Lambda_b = 0$ the solution is very different\footnote{This requires an initial guess for the value of $\omega^2 \rho_0^2$ and that the higher order corrections in the expansion of $A(z)$ in terms of $Z(z)$ fall off fast enough. Indeed, we have computed the expansion of $A(z)$ to $O(Z^{12}(z))$, and determined $\omega^2 \rho_0^2$ and the corresponding numerical values of the coefficients $h_i$ recursively.},
\begin{equation}
\tilde{A}(z) = \tilde{Z}(z)^{1/(D-2)}, \quad \tilde{B}(z) = \tilde{Z}(z)^{(D-2)\frac{1}{2(D-2)}} e^{\frac{1}{2(D-2)}(1-\tilde{Z}(z)^2)}, \tag{9}
\end{equation}
where now $\tilde{Z} = (1 - a_0 z)$, and we restrict to $a_0 > 0$. This solution exhibits a naked singularity, at $z = a_0^{-1}$ ($\tilde{Z} = 0$), for the global cosmic brane and the region $z > a_0^{-1}$ ($\tilde{Z} < 0$) is unphysical: the metric becomes complex. In the solution (8), the singularity is effectively removed by introducing the non-zero cosmological constant along the brane.

While the naked singularity has been removed by the non-zero $\Lambda_b$ we will now show that away from the horizon, the global cosmic brane solution (9) is still a good approximation to Eq. (8). We first obtain a power series solution of Eqs. (8), expanding around $z = 0$. From this we determine the lowest order terms in $h(z) = \sum_{n=0} h_n z^n$ \footnote{Recall that with $\tau = \alpha_0 + ig_s^{-1} \exp(\omega \theta)$, the $SL(2,\mathbb{Z})$ symmetry requires $g_s^D \sim O(1)$ in $D$ dimensions. However, in the $D-2$-dimensional brane-world, $g_s^{D-2} = g_s^D \sqrt{\alpha'/V_\perp}$, and since $V_\perp$, the volume of the transverse space, is large \footnote{It is large due to the single massive dilaton mode, $\phi = \omega \theta$,}, $g_s^{D-2} \ll 1$.}. Finally, we expand $A(z)$ and $B(z)$, expressed as functions of $h(z)$ and $h''(z)$ to lowest order in $z$,
\begin{equation}
A(z) = (1 + z \frac{h_1}{(D-2)}) , \quad B(z) = \sqrt{\frac{l^2 \Lambda_b^{-1} 2h_2}{D-2} \left(1 + z \frac{3h_3}{2h_2} - \frac{h_1(D-4)}{2(D-2)}\right)} . \tag{10}
\end{equation}
Here, the coefficients $h_i$ for $i > 2$ are determined in terms of $h_0, h_1, h_2$ by Eq. (8), the overall rescaling of $A(z)$ and $B(z)$ is absorbed in a rescaling of $x^a$ and $l$, respectively, and the numerical values of $h_1, h_2$ are determined by comparison with the expansions (8). Comparing now Eq. (10) with Eq. (9), expanded to first order in $z$, leads to
\begin{equation}
a_0 \approx 0.9(D-2)\rho_0^{-1} , \quad \xi \approx \frac{1}{0.9} \omega^2 \rho_0^2 \frac{\rho_0^{-1}}{8(D-2)} , \quad \text{and} \quad l^{-2} \Lambda_b^{-1} \frac{\omega^2}{2(D-2)} = 1 . \tag{11}
\end{equation}

The last of the identifications (11) has the following important consequence:
\begin{equation}
\Lambda_b = \frac{\omega^2}{2(D-2)l^2} . \tag{12}
\end{equation}
Thus, the cosmological constant is directly related to the amount of matter, or rather the non-trivial variation of the matter as a function of $\theta$! This gives a very non-trivial relation between the stringy moduli, and hence string theory itself, and a positive $\Lambda_b$. Since the dilaton is $\phi = -\omega \theta$ it also follows from Eq. (12) that we have a strongly coupled theory \footnote{Our scenario supersymmetry is explicitly broken.}.

Note also that $\Lambda_b \sim \omega^2/l^2$ is consistent with the notion that supersymmetry breaking and a non-zero cosmological constant are related. In our scenario supersymmetry is explicitly broken.

\textbf{References:}
\begin{itemize}
\item \cite{1}
\end{itemize}
broken by $\omega^2 \neq 0$. But since $\Lambda_b \sim \omega^2/l^2$, supersymmetry breaking by $\omega^2 \neq 0$ also induces a positive cosmological constant, which then can vanish only in the decompactifying limit, $l \to \infty$. In the limit $\omega^2 = 0$ we recover supersymmetry and thus have a possible F-theory background.

From the previous analysis we know the metric in two different regions: (far) away from the horizon (9), and near the horizon (7). Thus we can determine Kaluza-Klein corrections to the Newton potential. We start by analyzing the small gravitational fluctuations $\delta \eta_{ab} = h_{ab}$ (far) away from the horizon. This analysis follows to a large degree the discussion in Refs. [2, 4]. From the Einstein equations, $h_{ab}$ satisfies a wave equation of the form [16]:

$$\Box h_{ab} = \frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g} g^{\mu\nu} \partial_{\nu} h_{ab}) = 0.$$  \hspace{1cm} (13)

Following Ref. [3] we change coordinates, with $\tilde{A}, \tilde{B}$ given in Eq. (9),

$$dv = l \frac{\tilde{B}}{\tilde{A}} dz \hspace{1cm} (14)
$$

$$ds^2 = \tilde{A}^2(v) \eta_{ab} dx^a dx^b + \tilde{A}^2(v) dv^2 + \tilde{B}^2(v) l^2 d\theta^2.$$  \hspace{1cm} (15)

The isometries of the metric dictate the following Ansatz

$$h_{ab} = \epsilon_{ab} e^{ip \cdot x} e^{in \theta} \tilde{\psi}_m, \text{ where } \tilde{\psi}_0 \overset{\text{def}}{=} \sqrt{\tilde{A}^{-2} \sqrt{-g}} = \sqrt{\tilde{A}^{D-3} \tilde{B}}.$$  \hspace{1cm} (16)

With these variables [2, 3], Eq. (13) becomes a Schrödinger-like equation:

$$-\tilde{\psi}''_m + \left( \frac{\tilde{\psi}''_0}{\tilde{\psi}_0} + \frac{\tilde{A}^2}{\tilde{B}^2} n^2 \right) \tilde{\psi}_m = m^2 \tilde{\psi}_m.$$  \hspace{1cm} (17)

For simplicity, we choose $n = 0$. Integrating Eq. (14) gives

$$v = v_* \left[ 1 - \gamma_0^{-1} \gamma \left( \frac{D-3}{2(D-2)}; \frac{\xi^2}{2\alpha_0} \right) \right], \hspace{0.5cm} v_* = \frac{l}{2\alpha_0} e^{\frac{\xi}{\alpha_0} \left( \frac{2\alpha_0}{\xi} \right)^{D-3 \over 4(D-2)}}, \hspace{0.5cm} \gamma_0 = \gamma \left( \frac{D-3}{4(D-2)}, \frac{\xi}{2\alpha_0} \right).$$  \hspace{1cm} (18)

where $\gamma$ is the incomplete $\Gamma$-function. Before turning on the brane cosmological constant the naked singularity is situated at $z = 1/a_0$ or equivalently at $v = v_*$. Since the naked singularity is effectively removed when $\Lambda_b > 0$ the analysis below is only valid for $v \in [0, v_* - \delta v_*]$, where $v_* - \delta v_*$ is the location at which the true metric starts to substantially deviate from the global cosmic brane metric. One can show that $\delta v_*/v_* \approx 0.5$. When $v > v_* - \delta v_*$ we have to use the form of the metric given by the de Sitter perturbed solution (12).

\[11\text{We regard the global cosmic brane metric (9) to break down when } h_0 + h_1 z = \sum_{n=2}^{\infty} h_n z^n, \text{ i.e. when the sub-leading order corrections start to dominate.}\]
With the above change of variables the zero-mode becomes \( \tilde{\psi}_0(v) = \sqrt{l(1 - v/v_\star)} \) in the \( \xi = 0 \) limit. With this \( \tilde{\psi}_0 \) the general solution of Eq. (17) is

\[
\tilde{\phi}_m = \tilde{a}_m \sqrt{v_\star - v} J_0(m(v_\star - v)) + \tilde{b}_m \sqrt{v_\star - v} Y_0(m(v_\star - v)).
\]

When \( \Lambda_b = 0 \) it is possible to choose boundary conditions such that \( \tilde{b}_m = 0 \). However, one can show that as we approach \( v_\star - \delta v_\star \), a non-zero \( \tilde{b}_m \) is not be consistent with the appropriate boundary conditions at the location of the horizon.

We now look at small gravitational fluctuations \( \delta \eta_{ab} = h_{ab} \) of the longitudinal part of the metric close to the cosmological horizon. From the Einstein equations, \( h_{ab} \) satisfies a wave equation of the form given in Eq. (13). The isometries of the metric dictate the following Ansatz

\[
h_{ab} = \epsilon_{ab} \varphi_p(x) e^{i m \phi_m / \psi_0},
\]

where \( \varphi_p(x) \) is a mode with momentum \( p \) along the de Sitter space, with coordinates \( x \), i.e.

\[
(\square_x + 2\Lambda_b) \varphi_p(x) = m^2 \varphi_p(x),
\]

and \( \psi_0 \) is defined as in Eq. (11) with \( A, B \) given in Eqs. (8). The resulting differential equation is of the same form as in Eq. (17).

As in the analysis above, we focus on the s-wave, \( n = 0 \). We start by considering the case in which we can ignore the higher order corrections to the metric (8), or equivalently, ignore the matter contribution, i.e. \( \omega^2 = 0 \). It is then possible to explicitly invert the relation (14), so \( e^{-kv} = (1 - \rho_0^{-1} z) \) where \( k = \frac{1}{\rho m} \). Thus, the zero-mode is given by Eq. (11)

\[
\psi_0(v) = (l \frac{\delta v_\star}{v_\star})^{1/2} e^{\frac{(D-3)k}{2} (v_\star - \delta v_\star - v)}.
\]

The solutions to Eq. (17) are then given as

\[
\phi_m(v) = a_m e^{-\sqrt{\frac{(D-3)^2 k^2}{4} - m^2}} + b_m e^{\sqrt{\frac{(D-3)^2 k^2}{4} - m^2}}.
\]

When \( m^2 < (D - 3)^2 k^2/4 \) the normalizability of the modes implies that \( b_m = 0 \). When \( m^2 > (D - 3)^2 k^2/4 \) the argument of the exponential is \( \pm i \sqrt{\frac{(D-3)^2 k^2}{4} + m^2} \), i.e.

\[
\phi_m(v) = a_m \frac{m^{1/2}}{m^{1/2}} \cos \left( \sqrt{\frac{(D - 3)^2 k^2}{4} + m^2} \right).
\]

having again matched the wave functions at \( v = v_\star - \delta v_\star \); hereafter we drop the tildes.

\[\text{Since } \psi_0 \text{ is only valid for } v \in [v_\star - \delta v_\star, \infty), \text{ we have to match with } \tilde{\psi}_0 \text{ at } v \sim v_\star - \delta v_\star.\]
We are left to determine $a_m$ such that $\langle \phi_m | \phi_m \rangle = 1$. Because $J_0^2(mv_*) \sim \frac{\cos^2(mv_* - \pi/4)}{mv_*}$ when $mv_* \gg 1$, we can rewrite the normalization integral as

$$1 = \langle \phi_m | \phi_m \rangle \sim \frac{a_m^2}{m} \int_0^\infty dv \cos^2(mv) \sim \frac{a_m v_c}{m},$$

(25)

where we have regularized the integral by putting the system in a box of size $v_c$. Therefore, $a_m \sim m^{1/2}(v_c)^{-1/2}$.

The last step is to include the contribution of the matter, $\xi > 0$. This will only effect the wavefunction (far) away from the horizon, since close to the horizon the metric becomes independent of $\xi$. However, since $a_0 \sim \xi$ the ratio $\xi/a_0$ can be neglected in the definition of $v_*$, see Eq. (18). Thus, the effect of $\xi > 0$ is negligible.

Similarly, for the zero-mode normalization, or equivalently the volume of the transverse space $V_\perp$, we get

$$\langle \psi_0 | \psi_0 \rangle \sim \pi v_* \left[ 1 - \left( \frac{\delta v_*}{v_*} \right)^2 \right] + 2 \pi v_* l \frac{2}{D - 3} \frac{\delta v_*}{v_*} \frac{1}{k v_*} \approx \pi l v_* \left[ 1 - \left( \frac{\delta v_*}{v_*} \right)^2 + 2 \cdot 0.9 \left( \frac{\delta v_*}{v_*} \right) \right].$$

(26)

where we have used that $v_* \approx 2/(0.9(D - 3))$ which follows from the definitions of $k$ and $v_*$. Since $\delta v_*/v_* \sim 0.5$ we see that contributions from the two different regions are of the same order. Finally, we can rewrite the volume integral in terms of $\Lambda_b$ using Eq. (13),

$$\langle \psi_0 | \psi_0 \rangle \sim \frac{\pi}{D - 3} \frac{l}{\sqrt{\Lambda_b}}.$$

(27)

With these facts let us restrict our attention to $D = 6$, by compactifying the type IIB theory on a $K3$ or $T^4$ manifold. The Newton potential is found to have the following form:

$$U(r) \sim M_6^{-4} \frac{M_1 M_2}{r} \langle \psi_0^2 \rangle + \sum_m M_6^{-4} \frac{M_1 M_2}{r} e^{-m r} \langle \phi_m^2 \rangle,$$

(28)

where $\langle \psi_0^2 \rangle$ and $\langle \phi_m^2 \rangle$ are the wavefunctions averaged over the transverse space, i.e. $\langle \psi_0^2 \rangle = \langle \psi_0 | \psi_0 \rangle^{-1}$ and $\langle \phi_m^2 \rangle = \langle \psi_0 | \psi_0 \rangle^{-1}$. The Newton constant is determined from the first term: $G_N \sim M_6^{-4} \langle \psi_0^2 \rangle$, and the correction to Newton’s potential is

$$\Delta U(r) \sim G_N \frac{M_1 M_2}{r} \sum_m e^{-m r},$$

(29)

and can be neglected when $r \gg r_0 = \left( \langle \psi_0^2 \rangle \right)^{-1/2} \sim \Lambda_b^{-1/4} l^{1/2}$, where the mass gap is $\delta m = \left( \langle \psi_0^2 \rangle \right)^{1/2} \sim \Lambda_b^{1/4} l^{-1/2}$. With a realistic value for $\Lambda_b \sim G_N 10^{-44}$ (GeV)$^4$ and taking $l = l_{pl}$, we find that $M_6 \sim 10^3$ GeV. Furthermore, we get $r_0 \sim 10 \mu m$ and $\delta m \sim 10^{-2}$ eV, which is close to the lower bound at which gravity has been tested. Thus, the KK modes are suppressed, due to the peculiar dependence on $\Lambda_b$, in contrast with the situation for Rindler space [4].
In conclusion, in this letter we have shown that the space of non-supersymmetric codimension two solutions of string theory (either type IIB or any space-time variable string vacua) naturally have a positive cosmological constant, $\Lambda_b$ in $D - 2$ dimensions, if one requires the solution to be non-singular. The fact that our solutions were obtained in classical string theory also brings some new features: The non-zero positive value of the cosmological constant is essentially implied by the presence of non-trivial stringy moduli, such as the axion-dilaton system for the Type-IIB supergravity theory. This also implies that the string theory is strongly coupled. The fact that the value of the cosmological constant is directly related to the amount of matter invokes comparisons with various attempts to incorporate Mach’s principle in string theory [17] as well with the idea that supersymmetry breaking might have cosmological origin [18]. Note that our codimension two solutions can be intuitively understand as a strongly coupled classical non-supersymmetric solution of F-theory [15]. In the $\omega = 0$ limit we obtain a solution that has similar properties to a collection of 12 $D7$-branes [2]. Thus from the metric point of view, de Sitter like (accelerating universe) solutions appear to be more generic and natural in the space of non-supersymmetric classical vacua. The stability of this solutions, due to the presence of the bulk horizon can be presumably analyzed from a purely thermodynamic point of view.

Unlike the Randall-Sundrum set-up [19, 20], which can be in principle understood from the viewpoint of flux compactifications [21] in string theory, our codimension two solutions still lack such an approach. As shown in this letter (and as first pointed out by Gregory [7, 8] in a related context) the codimension two solutions are in a very precise sense attracted to a more general class of solutions with a positive cosmological constant. The Randall-Sundrum scenario seems to mesh more naturally with a negative brane cosmological constant [10]. In this context a codimension one AdS type brane world immersed in another AdS type bulk world can be realized, at least in the situation with no gravitational back reaction, as a background seen by a spectator $D5$ brane in the presence of a large number of $D3$ branes. Now, it has been argued that a certain (unstable) configurations of type $IIB*$ theory - $E4$ brane - realizes a de Sitter geometry in the near horizon limit [22]. It is tempting to try to understand de Sitter like brane worlds from the point of view of $E - D$ brane systems. (For example a de Sitter like brane world embedded in a bulk AdS space could be ”seen” by a spectator $E$ brane in the background of many $D$ branes.)

Finally, there are codimension one solutions of $USP(32)$ string theory [23] which share similar metric properties with our codimension two solutions - in particular the naked singularities exist in both cases at finite proper distance from the core of the solution. In particular, it is possible that an analysis analogous to ours would reveal that naked singularities appearing in Ref. [23] are also resolved by turning on a small positive brane cosmological constant. We hope to investigate some of these issues in the future.
Acknowledgments: We thank V. Balasubramanian, P. Horava, N. Kaloper and R. Myers for useful discussions. P. B. would like to thank LBL, the Berkeley Center for Theoretical Physics and in particular the CIG, Berkeley for their hospitality. The work of P. B. was supported in part by the US Department of Energy under grant number DE-FG03-84ER40168. T. H. wishes to thank the Caltech-USC Center for Theoretical Physics for its hospitality, and the US Department of Energy for their generous support under grant number DE-FG02-94ER-40854. The work of D. M. was supported, in the initial stages of this project, by the US Department of Energy under grant number DE-FG03-84ER40168.

References

[1] For recent reviews and references consult, for example: V. Balasubramanian, P. Horava and D. Minic, JHEP 0105(2001)043; E. Witten, hep-th/0106109; A. Strominger, JHEP 0110(2001)034; M. Spradlin, A. Strominger and A. Volovich, hep-th/0110007; C. M. Hull, JHEP 0111(2001)012; V. Balasubramanian, J. de Boer and D. Minic, hep-th/0110108; R. Bousso, A. Maloney and A. Strominger, hep-th/0112218.

[2] P. Berglund, T. H"ubsch and D. Minic: JHEP 09(2000)015; JHEP 02(2001)010.

[3] P. Berglund, T. H"ubsch and D. Minic: JHEP 01(2001)041.

[4] P. Berglund, T. H"ubsch and D. Minic: Phys. Lett. B512(2001)155.

[5] A.G. Cohen and D.B. Kaplan: Phys. Lett. B470(1999)52; See also, A.G. Cohen and D.B. Kaplan: Phys. Lett. B215(1988)663.

[6] S. Kachru, M. Schulz and E. Silverstein, Phys. Rev. D62(2000)045021; N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B480(2000)193.

[7] R. Gregory: Phys. Rev. Lett. 84(2000)2564.

[8] R. Gregory: Phys. Rev. D54(1996)4995.

[9] N. Kaloper, Phys. Rev. D60(1999)12350.

[10] A. Karch and L. Randall, JHEP 0105(2001)008.

[11] B.R. Greene, A. Shapere, C. Vafa and S.-T. Yau: Nucl. Phys. B337(1990)1.

[12] P.S. Green and T. H"ubsch: Int. J. Mod. Phys. A9(1994)3203–3227.

[13] A. Chamblin, S. W. Hawking and H. S. Reall, Phys. Rev. D61(2000)065007.
[14] R. Emparan, G. T. Horowitz and R. C. Myers, JHEP 0001(2000)007; R. Emparan, G. T. Horowitz and R. C. Myers, JHEP 0001(2000)021.

[15] C. Vafa: *Nucl. Phys. B*469*(1996)403.

[16] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman: *Nucl. Phys. B*581*(2000)309.

[17] P. Horava, *Phys. Rev. D*59*(1999)046004; P. K. Townsend, hep-th/9903043; P. Horava and D. Minic, *Phys. Rev. Lett.* 85*(2000)1610.

[18] T. Banks, *Int. J. Mod. Phys. A*16*(2001)910; hep-th/0007146. B. B. Deo and S. J. Gates, *Phys. Lett. B*151*(1985)195.

[19] L. Randall and R. Sundrum: *Phys. Rev. Lett.* 83*(1999)3370.

[20] L. Randall and R. Sundrum: *Phys. Rev. Lett.* 83*(1999)4690.

[21] H. Verlinde: *Nucl. Phys. B*580; C.S. Chan, P.L. Paul and H. Verlinde: *Nucl. Phys. B*581*(2000)156; S. B. Giddings, S. Kachru and J. Polchinski, hep-th/0105097.

[22] C. M. Hull, JHEP 9807*(1998)021.

[23] S. Sugimoto, *Prog. Theor. Phys. 102*(1999)685; E. Dudas and J. Mourad, *Phys. Lett. B*486*(2000)172; R. Blumenhagen and A. Font, *Nucl. Phys. B*599*(2001)241.