Quarks in nuclear medium

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Abstract

Using the quark-meson coupling (QMC) model we study nuclear matter from the point of view of quark degrees of freedom. As the nucleon model we adopt the MIT bag model and the relativistic constituent quark model, where a square well and harmonic oscillator potentials are used to confine the quarks. We introduce the Lorentz-vector type confining potential as well as the Lorentz-scalar type one in order to examine how the vector confining potential contributes to the properties of the nucleon and nuclear matter. Next, we perform a re-definition of the scalar field in matter and transform the QMC model to a QHD-type model with a non-linear scalar potential. The result obtained from QMC is then compared with the potentials which are determined so as to fit various properties of finite nuclei and nuclear matter in relativistic mean-field models. The QMC model provides the parameters $\kappa \sim 20 - 40$ (fm$^{-1}$) and $\lambda \sim 80 - 400$ for the standard, non-linear scalar potential. We discuss a relationship between the QMC and QHD-type models in detail.

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I. INTRODUCTION

In the conventional nuclear physics we have assumed that the nucleon properties in the nuclear medium are not changed from those of the free nucleon. Thus, low- and medium-energy nuclear physics phenomenology has been successfully described in terms of (point-like) nucleons. However, after the EMC effect in the nuclear structure functions was observed [1] there has been considerable interest in the modification of hadron properties in the nuclear medium. It is one of the central questions in nuclear physics how the effect of the quark substructure of hadrons emerges in nuclear phenomena.

At present rigorous studies of quantum chromodynamics (QCD) are limited to matter system with high temperature and zero baryon density. Because of the complex non-perturbative features of QCD it is very difficult to calculate properties of nuclei at low- and medium-energy. Thus, we need to build models which help to bridge the discrepancy between nuclear phenomenology and the underlying theory. Such models may be necessarily crude because the nuclear many-body problem at the fundamental QCD level is intractable. However, it is very important and interesting to challenge this problem. Then, much effort has been devoted to the study of effective models for nuclear matter and finite nuclei based on various quark models [2].

About a decade ago, Guichon [3] proposed a relativistic quark model for nuclear matter, where it consists of non-overlapping nucleon bags bound by the self-consistent exchange of isoscalar, scalar ($\sigma$) and vector ($\omega$) mesons in mean-field approximation (MFA). This model has been further developed as the quark-meson coupling (QMC) model [3–5]. It was shown that the model can reproduce the saturation properties of nuclear matter and that it provides a fair description of finite nuclei (for recent reviews, see Ref. [6]). Since in QMC the mesons couple not to the nucleons but to the quarks directly the nucleon properties are modified self-consistently, depending on the scalar field in a medium.

On the other hand, recent theoretical studies show that various properties of finite nuclei can be described by Quantum Hadrodynamics (QHD) [7] very well. Although the original
version of QHD [7] provides the large reduction of the nucleon mass in nuclear matter, which is the source of the large spin-orbit splitting in finite nuclei, the model also gives the large nuclear compressibility around 550 MeV. In order to improve this Boguta and Bodmer [8] first introduced self-interaction terms for the $\sigma$ meson. It is now well established that such non-linear interactions are necessary to reproduce the bulk properties of finite nuclei and nuclear matter [8].

Our purpose here is to study a relation between QHD-type mean-field models and the QMC model and to examine how the internal structure of the nucleon sheds its effect on effective nuclear models. In QMC the nucleon mass in matter is given by a decreasing function of the $\sigma$ field. By applying a field re-definition to the scalar field in matter, the QMC model can be cast in a form equivalent to a QHD-type model with a non-linear scalar potential and a non-linear coupling to the gradient of the scalar field [10]. Therefore, in principle, if the effective nucleon mass is known as a function of the scalar field, it could provide a prediction of non-linear interactions of the scalar field in nuclear matter through the procedure of the field re-definition. Inversely, it may imply that if a form of the scalar self-interactions in matter could be determined by analysing various experimental data we could get significant information on the quark substructure of the in-medium nucleon.

The QMC model relies on the choice of a quark model of the nucleon. The MIT bag model has been used as the nucleon model in our previous works [3–6]. In the bag model the inside of the bag is assumed to be the perturbative vacuum and the quark moves with a current quark mass, which is nearly zero. There is an alternative idea for the nucleon model, namely the constituent quark model [11]. In this model, the quark takes a constituent quark mass of about 300 MeV in the nucleon, which is offered by spontaneous chiral symmetry breaking. Recently, Shen and Toki [12] have proposed a new version of the QMC model, where the constituent quark model with a harmonic oscillator potential was used, and studied nuclear matter and finite nuclei — they refer the model as the quark mean-field (QMF) model. Although the quark acquires a heavy mass in this picture it is still important to treat it relativistically because the quark kinetic energy is not small compared with the
constituent quark mass. Hence, in this paper we will use both the bag model and the relativistic constituent quark model in QMC and study the properties of nuclear matter and self-interactions of the scalar field.

The outline of the present paper is as follows: In Sec. II, we present relativistic quark models for the nucleon, which can be solved analytically. Using those quark models we calculate the nucleon properties in Sec. III. We also present our numerical results for nuclear matter. In Sec. IV, by performing a re-definition of the scalar field we transform the QMC model to a QHD-type model. Then, we discuss non-linear scalar potentials generated from the QMC model. In Sec. V we give some discussions and conclusion.

II. QUARK MODELS FOR THE NUCLEON

In this section we summarize relativistic quark models to describe the nucleon. First we discuss the constituent quark model with a square well (SW) potential or a harmonic oscillator (HO) potential to confine the quarks. We introduce a Lorentz-vector type potential as well as a scalar type one to examine the role of the vector type confining potential in nuclear matter. In the constituent quark model it is assumed that the quark moves in the Nambu-Goldstone vacuum and hence the quark takes a constituent quark mass (around 300 MeV). Next, we review the MIT bag model shortly. In the bag model, the quark moves in the perturbative vacuum (or the Wigner phase) and it takes a small, current quark mass.

A. Constituent quark model

We consider a light (u or d) quark (of mass $m_q$) moving under a potential $V(r)$. The Dirac equation for the quark field $\psi_q(r)$ is then given by

$$[i\gamma \cdot \partial - m_q - V(r)]\psi_q(r) = 0,$$

(1)

where we take

$$V(r) = (1 + \beta\gamma_0)U(r),$$

(2)
with a confining potential \( U(r) \) and a parameter \( \beta (\geq 0) \) to control the strength of the Lorentz-vector type potential. We assume that the shape of the Lorentz-scalar type confining potential is the same as that of the vector type one.

We first consider a square well potential and find a solution for the quark field à la Bogolioubov [13]. The potential is then given by

\[
U(r) = \begin{cases} 
0, & \text{for } r \leq R, \\
M, & \text{for } r > R,
\end{cases}
\]  

(3)

where \( R \) is the radius of the spherical well and \( M \) is the height of the potential outside the nucleon. After finishing all calculations, we take the limit \( M \to \infty \).

The solution for the quark field can be found easily [13]. Inside the well the lowest energy state is

\[
\psi_q(r) = \frac{N}{\sqrt{4\pi}} e^{-iEt} \left( f(r) \hat{\sigma} \cdot \hat{r} g(r) \right) \chi_s,
\]

(4)

with the normalization constant \( N \), spin state \( \chi_s \) and the quark energy \( E \). Here the upper \( f(r) \) and lower \( g(r) \) components of the quark wave function are respectively given by the spherical Bessel functions, \( j_0(r) \) and \( j_1(r) \). (For details, see Appendix.) Outside the nucleon volume we find a similar form of the solution with modified spherical Bessel functions. Note that since the quark wave function is proportional to \( e^{-M\sqrt{1-\beta^2}r} \) outside the well the condition \( \beta < 1 \) is necessary to confine the quark.

We then demand that \( f \) and \( g \) must be continuous at \( r = R \). This gives an eigenvalue problem, and we obtain a matching condition in the limit \( M \to \infty \):

\[
j_0(x) = \sqrt{\frac{(1-\beta)(E-m_q)}{(1+\beta)(E+m_q)}} j_1(x),
\]

(5)

with the eigenvalue of the confined quark, \( x \). The total energy is given by \( 3\alpha/R \), where \( \alpha^2 = x^2 + \lambda^2 \) and \( \lambda = Rm_q \). As in the MIT bag model there exist the spurious center of mass (c.m.) motion and gluon fluctuation corrections etc. Here we add the familiar form, \(-z/R\), with a parameter \( z \) for those corrections to the energy [13]. Thus, the nucleon mass \( M_N \) (at rest) is given by
\[ M_N = \frac{3\alpha - z}{R}. \]  

(6)

Using the solution Eq.(4) we can calculate various nucleon properties analytically (see Appendix).

This system may be described by a Lagrangian density

\[ \mathcal{L}_{SW} = \bar{\psi}_q (i \gamma \cdot \partial - m_q) \psi_q \delta (R - r) - \frac{1}{2} \bar{\psi}_q (1 + \beta \gamma \cdot a) \psi_q \delta (r - R), \]  

(7)

where \( a^\mu \) is the unit vector in time direction: \( a^\mu = (1, \vec{0}) \). This Lagrangian provides a boundary condition at \( r = R \)

\[ i \gamma \cdot n \psi_q = (1 + \beta \gamma \cdot a) \psi_q, \]  

(8)

where \( n^\mu \) is the unit normal outward from the potential surface. This is equivalent to the condition Eq.(5) and ensures that the quark is confined permanently. (Note that actually the quark current flows in the azimuthal direction [14].)

In the case where the confining potential is a harmonic oscillator potential with \( \beta = 1 \) and \( c \) the oscillator strength

\[ V(r) = \frac{1}{2} (1 + \gamma_0) cr^2, \]  

(9)

one can again find the quark wave function analytically. The solution to the Dirac equation is expressed by Eq.(4), where the upper and lower components are given in terms of gaussian functions [13]. (For details, see Appendix.) The condition to determine the quark energy \( E \) is then obtained

\[ \sqrt{E + m_q (E - m_q)} = 3 \sqrt{c}. \]  

(10)

In the HO model the c.m. energy can be estimated exactly as in the non-relativistic harmonic oscillator. It is just one third of the total energy [16]. Thus, the nucleon mass is

\[ M_N = 2E - E_s, \]  

(11)
where $E_s$ describes gluon fluctuation corrections etc. Note that an equally weighted (Lorentz) scalar-vector potential does not produce a spin-orbit splitting in the baryon spectrum, which may be consistent with the fact that the spin-orbit splitting in the observed baryon spectra is very small.

**B. MIT bag model**

In the MIT bag model the inside of the nucleon bag is considered to be the perturbative vacuum, and the quark takes a current quark mass. It requires extra energy to make a nucleon bag in which the quark moves freely, so we have to add a latent heat term to the Lagrangian density Eq.(7)

$$\mathcal{L}_B = \mathcal{L}_{SW} - B\theta(R - r),$$  \hspace{1cm} (12)

where the bag constant $B$ describes the energy gap between the inside and outside of the nucleon.

The quark wave function and the eigenvalue condition are given by the same forms as in the SW model. Thus, the nucleon mass is given as

$$M_N = \frac{3\alpha - z}{R} + \frac{4\pi}{3}BR^3,$$  \hspace{1cm} (13)

where the $z$ parameter is again introduced to take into account the sum of the c.m. and gluon fluctuation corrections etc.

**III. QMC MODEL FOR NUCLEAR MATTER**

**A. Effect of nucleon structure in a nuclear medium**

Here we consider how Eq.(1) is modified when the nucleon is bound in static, uniformly distributed nuclear matter. In the QMC model it is assumed that each quark feels scalar $V_s^q$ and vector $V_v^q$ potentials, which are generated by the surrounding nucleons, as
well as the confinement potential $V(r)$. Since the typical distance between two nucleons around normal nuclear matter density ($\rho_0 = 0.15 \text{ fm}^{-3}$) is surely larger than the typical size of the nucleon (the radius is about 0.8 fm), the interaction (except for the short-range part) between the nucleons should be colour singlet, namely a meson-exchange potential. Therefore, this assumption seems appropriate when the baryon density $\rho_B$ is not high. (For high density, see Ref. [17].) If we use the mean-field approximation for the meson fields, Eq.(11) may be rewritten as

$$[i\gamma \cdot \partial - (m_q - V^q_s) - V(r) - \gamma_0 V^q_v] \psi_q(r) = 0.$$  

(14)

The potentials generated by the medium are constants because the matter distributes uniformly. As the nucleon is static, the time-derivative operator in the Dirac equation can be replaced by the quark energy, $-iE$.

By analogy with the procedure applied to the nucleon in QHD [7], if we introduce the effective quark mass by $m^*_q = m_q - V^q_s$, the Dirac equation Eq.(14) can be rewritten in the same form as that in free space, with the mass $m^*_q$ and the energy $E - V^q_v$, instead of $m_q$ and $E$. In other words, the vector interaction has no effect on the nucleon structure except for an overall phase in the quark wave function Eq.(4), which gives a shift in the nucleon energy. This fact does not depend on how to choose the confinement potential $V(r)$. Then, the nucleon energy (at rest) $E_N$ in the medium is

$$E_N = M^*_N(V^q_s) + 3V^q_v,$$  

(15)

where the effective nucleon mass $M^*_N$ depends on only the scalar potential in the medium. This important observation was also obtained by non-local field theory model [18], where a non-local $\sigma$-$\omega$ model containing short distance vertex form factors is used to simulate an underlying QCD substructure.

Although we have discussed the QMC model using the specific model, namely the bag model, in our previous works [3-6], the qualitative features we have found may be correct in any model in which the nucleon contains relativistic quarks and the (middle- and long-range)
attractive and (short-range) repulsive N-N forces have Lorentz-scalar and vector characters, respectively. To confirm this point we will study nuclear matter using the quark models discussed in the previous section.

In Eq.(14) the scalar and vector potentials are respectively mediated by the $\sigma$ and $\omega$ mesons. Their mean-field values are introduced by $V^q_\sigma = g^q_\sigma \sigma$ and $V^q_\omega = g^q_\omega \omega$, where $g^q_\sigma$ ($g^q_\omega$) is the coupling constant of the quark-$\sigma$ ($\omega$) meson. (We here ignore the $\rho$ meson and the Coulomb field, for simplicity. For those fields, see Ref. [4].) Then, in MFA the effective Lagrangian density in the QMC model would be given

$$L_{\text{QMC}} = \overline{\psi}[i\gamma \cdot \partial - M^*_N(\sigma) - g_\omega \gamma_0 \omega] \psi - \frac{1}{2}[(\nabla \sigma)^2 + m^2_\sigma \sigma^2] + \frac{1}{2}[(\nabla \omega)^2 + m^2_\omega \omega^2],$$

(16)

where $\psi$ is the nucleon field, and $m_\sigma$ and $m_\omega$ are respectively the masses of the $\sigma$ and $\omega$ mesons — we take $m_\sigma = 550$ MeV and $m_\omega = 783$ MeV. The $\omega$-N coupling constant $g_\omega$ is related to the corresponding quark-$\omega$ coupling constant as $g_\omega = 3g^q_\omega$ [4]. In an uniform nuclear matter the derivative terms of the meson fields vanish because the fields do not depend on time and position.

Here we consider the nucleon mass in matter further. The nucleon mass is a function of the scalar field. Because the scalar field is small at low density the nucleon mass can be expanded in terms of $\sigma$

$$M^*_N = M_N + \left( \frac{\partial M^*_N}{\partial \sigma} \right)_{\sigma=0} \sigma + \frac{1}{2} \left( \frac{\partial^2 M^*_N}{\partial \sigma^2} \right)_{\sigma=0} \sigma^2 + \cdots.$$  

(17)

Since the interaction between the quark and the $\sigma$ field is described in terms of the local scalar coupling, the derivative of $M^*_N$ with respect to $\sigma$ is given by

$$\left( \frac{\partial M^*_N}{\partial \sigma} \right) = -3g^q_\sigma \int_{V_N} d\vec{r} \overline{\psi}_q \psi_q \equiv -3g^q_\sigma S_N(\sigma).$$

(18)

Here we have defined the quark-scalar charge in the nucleon $S_N(\sigma)$, which is itself a function of the scalar field, by Eq.(18). Because $S_N$ is positive, the nucleon mass decreases in matter at low density. Note that since the nucleon mass is given by Eq.(11) in the HO model the derivative turns to
\[ \left( \frac{\partial M_N^*}{\partial \sigma} \right) = -2g_\sigma^2 S_N(\sigma). \] (19)

We furthermore define the scalar-charge ratio, \( S_N(\sigma)/S_N(0) \), to be \( C_N(\sigma) \) and the \( \sigma \)-N coupling constant \( g_\sigma \):

\[ C_N(\sigma) = S_N(\sigma)/S_N(0) \quad \text{and} \quad g_\sigma = 3g_\sigma^2 S_N(0). \] (20)

Using those quantities, we find that the nucleon mass in matter is

\[ M_N^* = M_N - g_\sigma \sigma - \frac{1}{2} g_\sigma C_N'(0) \sigma^2 + \cdots. \] (21)

(In the HO model, we find \( M_N^* = M_N - \frac{2}{3} g_\sigma \sigma - \frac{1}{2} g_\sigma C_N'(0) \sigma^2 + \cdots \).) In general, \( C_N \) is a decreasing function because the quark in matter is more relativistic than in free space (because of the attractive force due to the \( \sigma \)). Thus, \( C_N'(0) \) takes a negative value. If the nucleon were structureless \( C_N \) would not depend on the scalar field, that is, \( C_N \) would be constant (\( C_N = 1 \)). Therefore, only the first two terms in the right hand side of Eq.(21) remain, which is exactly the same as the equation for the effective nucleon mass in the original QHD model.

**B. Numerical results**

We consider an isosymmetric nuclear matter with Fermi momentum \( k_F \), which is given by \( \rho_B = 2k_F^3/3\pi^2 \). From the Lagrangian density Eq.(16), the total energy per nucleon \( E_{\text{tot}} \) can be written

\[ E_{\text{tot}} = \frac{4}{\rho_B(2\pi)^3} \int^{k_F} dk \sqrt{M_N^2 + k^2} + \frac{m_{\sigma}^2}{2\rho_B} \sigma^2 + \frac{g_\omega^2}{2m_\omega^2} \rho_B, \] (22)

where the effective nucleon mass in matter is calculated by the quark model. The value of the \( \omega \) field is determined by baryon number conservation as \( \omega = g_\omega \rho_B / m_\omega^2 \).

The scalar mean-field is given by a self-consistency condition

\[ \sigma = -4 \left( \frac{2\pi}{3m_{\sigma}^2} \right)^3 \int^{k_F} dk \frac{M_N^*}{\sqrt{M_N^* + k^2}} \left( \frac{\partial M_N^*}{\partial \sigma} \right), \]

\[ = \frac{4g_\sigma C_N(\sigma)}{(2\pi)^3 m_{\sigma}^2} \int^{k_F} dk \frac{M_N^*}{\sqrt{M_N^* + k^2}}, \] (23)
where the derivative of the effective nucleon mass is given by the quark-scalar charge $S_N(\sigma)$. (See Eqs. (18) and (20). For the HO model we should replace $C_N$ by $\frac{2}{3}C_N$.)

In the previous section we have prepared the relativistic constituent quark model and the MIT bag model to describe the nucleon. We use $m_q = 300$ MeV in the constituent quark model, while the quark is assumed to be massless in the bag model. In the SW model we set the radius of the potential to be $R = 0.8$ fm and determine $z$ so as to fit the free nucleon mass $M_N (= 939$ MeV). The parameter $\beta$ is chosen to be 0 (the confining potential is purely Lorentz-scalar), 0.25 and 0.5 to examine the effect of Lorentz-vector type confining potential. The quark eigenvalue $x$ can be found by solving the boundary condition Eq. (5) at the potential surface. In the HO model, there are two adjustable parameters, $c$ and $E_s$. Those are determined so as to fit the free nucleon mass (see Eq. (11)) and the root-mean-square (rms) (charge) radius of the free nucleon: $r_q(= \langle r^2 \rangle^{1/2}) = 0.6$ fm (see Appendix). The quark energy $E$ is given by the condition Eq. (10). We found that $c = 1.591$ fm$^{-3}$, $E_s = 344.7$ MeV and $E = 641.8$ MeV for the free nucleon. In nuclear matter we keep $c$ and $E_s$ constant and the quark energy $E$ varies depending on the scalar field.

As in Ref. [4], in the bag model the bag constant $B$ and the parameter $z$ are fixed to reproduce the free nucleon mass under the condition that the nucleon mass be stationary under variation of the bag radius $R$. As in the SW model, we choose the bag radius of the free nucleon to be 0.8 fm. (Variations of the quark mass and the radius only lead to numerically small changes in the calculated results [4].) We found $B^{1/4} = 170.3$ MeV for massless quarks. In Table I we list up the parameter $z$, quark eigenvalue and rms radius of the free nucleon in the SW and bag models.

Now we are in a position to determine the coupling constants: $g_\sigma^2$ and $g_\omega^2$ are fixed to fit the binding energy ($-15.7$ MeV) at the saturation density ($\rho_0 = 0.15$ fm$^{-3}$) for symmetric nuclear matter. The coupling constants and some calculated properties for matter are listed in Table II. We also present the ratios of the quark eigenvalue, rms radius, axial vector coupling constant ($g_A$), magnetic moment of the proton ($\mu_p$) and quark-scalar charge at saturation density to those in free space. (See also Ref. [19]. For explicit expressions of
We note that the present quark models (except B(0.25) and B(0.5)) provide the similar results in the properties of the in-medium nucleon and nuclear matter. In particular, they give values for the nuclear compressibility well within the experimental range ($K \sim 200 - 300 \text{ MeV}$), which is caused by the non-linear dependence of the $\sigma$ field on the nucleon mass. In the cases of B(0.25) and B(0.5) the reduction of the nucleon mass in matter is relatively small, which is due to the strong Lorentz-vector type confining potential. Furthermore, in the HO model the increase of the rms radius at saturation density is about $10\%$, which is large compared with the observed value [21].

We briefly comment on the rms radius, axial vector coupling constant and magnetic moment of the nucleon in nuclear matter. In our calculation those quantities are calculated using only the (valence) quark wave function and other important contributions from the meson (mainly pion) cloud effect etc are not included. In the present quark picture those quantities involve integrals over the lower and upper components of the quark wave function. Because of the attractive force due to the scalar field the quark becomes more relativistic in matter than in free space, that is, the lower component is more enhanced in matter. This fact gives the decrease in $g_A$ and the increase in the rms radius and $\mu_p$ in matter.

In Fig. 1 we present the total energy per nucleon as a function of nuclear density. Around normal nuclear matter density the quark models provide the similar saturation curves, while for high density the curves obtained from the models of B(0.25) and B(0.5) are relatively lower than those of the others. This corresponds to the fact that a quark model with a larger value of $\beta$ gives a lower nuclear compressibility (see Table I). We show the scalar mean-field value and the effective nucleon mass for symmetric nuclear matter in Figs. 2 and 3, respectively. From those figures we again see that in the cases of B(0.25) and B(0.5) the scalar field is much weaker than those given by the other quark models and the decrease of the nucleon mass is fairly small even at high density, which is due to the small scalar field.

We note that in the SW and bag models with massless quarks the quark-scalar charge vanishes in the limit $\beta \to 1$, which means that the $\sigma$ meson does not couple to the nucleon.
(For details, see Appendix.) This fact implies that as $\beta$ is larger the $\sigma$-N coupling is weaker in matter. Thus, we may conclude that qualitatively a large mixture of the Lorentz-vector type confining potential leads to a weak scalar mean-field and a large effective nucleon mass in nuclear matter. It is well known that within MFA a smaller effective nucleon mass (and hence a stronger scalar field) in matter is favorable to fit various properties of finite nuclei \cite{7}. Therefore, the confining potential including a strong Lorentz-vector type one may not be suitable for describing a nuclear system.

**IV. TRANSFORMATION FROM QMC TO QHD**

The main difference between the QMC and QHD models lies in the dependence of the scalar field on the nucleon mass in matter. By performing a re-definition of the scalar field, the QMC Lagrangian density at hadronic level can be cast into a form similar to a QHD-type mean-field model, in which the nucleon mass depends on the scalar field linearly, with self-interactions of the scalar field \cite{22,10}. Here we will briefly review the field re-definition and discuss, as an example, the derivative (scalar) coupling model of QHD (i.e., the Zimanyi-Moszkowski (ZM) model) \cite{23}. Then, we will calculate non-linear scalar potentials generated from the QMC model and compare with the potentials phenomenologically determined by fitting the properties of finite nuclei and nuclear matter.

**A. Field re-definition**

In the QMC model the nucleon mass in matter is given by a function of the scalar field $\sigma$

$$M_{N,QMC}^* = M_{N,QMC}^*(\sigma), \quad (24)$$

while in QHD the mass depends on a scalar field $\phi$ linearly,

$$M_{N,QHD}^* = M_N - g_0 \phi. \quad (25)$$
This fact suggests a re-definition of the scalar field in QMC:

$$g_0 \phi(\sigma) = M_N - M_{N,QMC}(\sigma),$$

(26)

where $g_0$ is a constant chosen so as to normalize the new scalar field $\phi$ in the limit $\phi \to 0$:

$$\phi(\sigma) = \sigma + O(\sigma^2).$$

(27)

Thus, $g_0$ is given by

$$g_0 = - \left( \frac{\partial M_{N,QMC}(\sigma)}{\partial \sigma} \right)_{\sigma=0}.$$  

(28)

Comparing with Eqs. (18) and (20), we find $g_0 = g_{\sigma}$ for the SW and bag models and $g_0 = \frac{2}{3} g_{\sigma}$ for the HO model.

The contribution of the scalar field to the total energy, $E_{scl}$, is now rewritten in terms of the new field $\phi$. From Eq. (13) we find

$$E_{scl} = \frac{1}{2} \int d\vec{r} \left[ (\nabla \sigma)^2 + m_{\phi}^2 \sigma^2 \right] = \int d\vec{r} \left[ \frac{1}{2} h(\phi)^2 (\nabla \phi)^2 + U_s(\phi) \right],$$

(29)

where $U_s$ describes the self-interactions of the scalar field

$$U_s(\phi) = \frac{1}{2} m_{\phi}^2 \sigma(\phi)^2,$$

(30)

and

$$h(\phi) = \left( \frac{\partial \sigma}{\partial \phi} \right) = \frac{1}{m_{\sigma} \sqrt{2U_s(\phi)}} \left( \frac{\partial U_s(\phi)}{\partial \phi} \right).$$

(31)

Note that in uniformly distributed nuclear matter the first derivative term in $E_{scl}$ does not contribute to the total energy. (The effect of this term on the properties of finite nuclei was studied in Ref. [10].)

The QMC model is now re-formulated in terms of the new scalar field $\phi$, and it is of the same form as a QHD model with the non-linear scalar potential $U_s(\phi)$, which is generated from the effect of the internal structure of the nucleon, and with the coupling $h(\phi)$ to the gradient of the scalar field. However, we should keep in mind that this re-formulation is
performed only at hadronic level and that the non-linear potential $U_s$ is simply determined so as to simulate the effective nucleon mass given by the quark model in matter. Thus, only on the energy of nuclear matter the QHD-type mean-field model derived here is identical to the QMC model. We will again discuss this point in the last section.

The standard form of the non-linear scalar potential is [8,9]

$$U_s(\phi) = \frac{1}{2} m_\sigma^2 \phi^2 + \frac{\kappa}{6} \phi^3 + \frac{\lambda}{24} \phi^4.$$  \hfill (32)

In more sophisticated version of QHD [22], inspired by modern methods of effective field theory, many other terms of meson-meson and meson-nucleon couplings are considered. However, rather than attempting to compete with those models, in this paper we concentrate only on the parameters $\kappa$ and $\lambda$ to make our discussion simple. It is well known that the non-linear scalar potential Eq.(32) is practically indispensable to reproduce the bulk properties of finite nuclei and nuclear matter [8,9,22–25].

If the effective nucleon mass could be determined experimentally as a function of the scalar field, this procedure leading to Eq.(30) would allow to predict a part of the non-linear potential, which is originated by the quark substructure of the nucleon in nuclear matter. Inversely, if the non-linear potential obtained by fitting various experimental data could be separated into a portion due to the effect of the internal structure of the nucleon and the inherent self-interaction part of the scalar field, we could get significant information on the quark structure of the nucleon in matter by using the inverse field transformation,

$$\sigma(\phi) = \sqrt{\frac{2U_s(\phi)}{m_\sigma}}.$$  \hfill (33)

It is potentially of very interesting. However, it may be quite difficult.

As an example, we here consider the derivative coupling model of QHD, namely the Zimanyi-Moszkowski (ZM) model [23]. By re-defining the scalar field the ZM model can be exactly transformed to a QHD-type model with a non-linear potential.

In the ZM model the interaction term between the $\sigma$ and the nucleon is given by [23]

$$\mathcal{L}_{\text{int}}^{ZM} = (g_\sigma M_N) \bar{\psi} \gamma \cdot \partial \psi,$$  \hfill (34)
in addition to a similar coupling to the vector field. Then, after rescaling the nucleon field, the Lagrangian density turns to be the same form as that of the original QHD, with the effective nucleon mass

\[ M_N^* = \frac{M_N}{1 + (g_\sigma \sigma / M_N)}, \tag{35} \]

instead of the mass depending on the scalar field linearly. Thus, the model involves higher order couplings between the \( \sigma \) and the nucleon, which provides a good nuclear compressibility around 230 MeV and \( M_N^*/M_N \simeq 0.85 \).\[23]\]

Let us define a new scalar field \( \phi \) by

\[ g_0 \phi(\sigma) = M_N - M_{N,ZM}^*(\sigma), \tag{36} \]

where the mass \( M_{N,ZM}^*(\sigma) \) is given by Eq.(35). Then, we easily find

\[ \phi(\sigma) = \frac{\sigma}{1 + g' \sigma}, \tag{37} \]

with \( g' = g_\sigma / M_N \). Inversely, we also find

\[ \sigma(\phi) = \frac{\phi}{1 - g' \phi}. \tag{38} \]

The non-linear potential is thus given by Eq.(39)

\[ U_s(\phi) = \frac{1}{2} m_\sigma^2 \sigma(\phi)^2 = \frac{1}{2} m_\sigma^2 \left( \frac{\phi}{1 - g' \phi} \right)^2, \tag{39} \]

which can be expanded up to order \( O(\phi^4) \)

\[ U_s(\phi) \simeq \frac{1}{2} m_\sigma^2 \phi^2 + g_\sigma \left( \frac{m_\sigma^2}{M_N} \right) \phi^3 + \frac{3}{2} g_\sigma^2 \left( \frac{m_\sigma}{M_N} \right)^2 \phi^4. \tag{40} \]

(The shape of the potential Eq.(39) is shown in Fig.5.) Hence, the ZM model is identical to the QHD-type model with the non-linear scalar potential expressed by Eq.(39).

**B. Non-linear scalar potentials generated from QMC**

In general the nucleon mass in matter may be given by a complicated function of the scalar field. However, in QMC the mass can be parametrized by a simple expression up to \( O(g_\sigma^2) \) (see Fig.3)
\[ \frac{M_N}{M_N} \simeq 1 - ay + by^2, \]  
with a dimensionless scale \( y = g_\sigma \sigma / M_N \) and two (dimensionless) parameters \( a \) and \( b \). From the discussion around Eq. (21) we can expect that \( a \sim 1 \) for the SW and bag models while \( a \sim 2/3 \) for the HO model. This parametrization is accurate up to \( \sim 4\rho_0 \). Using Eq. (11) the calculated nucleon mass in nuclear matter can be fitted with the method of least squares. We show the parameters \( a \) and \( b \) for the various quark models in Table III. We find that the present quark models (except B(0.25) and B(0.5)) give \( b \sim 0.2 - 0.5 \).

Now we shall re-define the scalar field using Eq. (26). We find \( g_0 = a g_\sigma \) and

\[ \phi(\sigma) = \sigma - d\sigma^2, \] 

with \( d = bg_\sigma/aM_N \). Eq. (42) can be solved exactly, and we obtain

\[ \sigma(\phi) = \frac{1 - \sqrt{1 - 4d\phi}}{2d}, \] 

which satisfies the condition \( \sigma \to 0 \) in the limit \( \phi \to 0 \). The non-linear potential is then calculated

\[ U_s(\phi) = \frac{m_\sigma^2}{2} \left( \frac{\sigma(\phi) - \phi}{d} \right), \] 
and we can expand \( U_s \) as

\[ U_s(\phi) = \frac{m_\sigma^2}{2} \phi^2 + g_\sigma r \left( \frac{m_\sigma^2}{M_N} \right) \phi^3 + \frac{5}{2} g_\sigma^2 r^2 \left( \frac{m_\sigma}{M_N} \right)^2 \phi^4 + \mathcal{O}(g_\sigma^3), \] 

where \( r = b/a \).

Comparing with Eq. (32) we can find the parameters \( \kappa \) and \( \lambda \) in the non-linear scalar potential generated by QMC. They are listed in Table IV. The quark models we have adopted here lead to the non-linear potential with \( \kappa \sim 20 - 40 \) (fm\(^{-1}\)) and \( \lambda \sim 80 - 400 \), except for the models of B(0.25) and B(0.5). (The models B(0.25) and B(0.5) give quite large values of \( \kappa \) and \( \lambda \), in addition to the large effective nucleon mass and the small scalar field in matter, which seems impractical.) From the discussion in the subsection III A we
see that both $a$ and $b$ are positive in QMC because $a$ and $b$ are respectively proportional to $S_N(0)$ and $-C'_N(0)$. Hence, from Eq.(13) we can expect that the effect of the quark substructure of the nucleon in matter is cast into a non-linear potential with positive $\kappa$ and positive $\lambda$ in the QHD-type mean-field model.

In Table V, for comparison we show the parameters $\kappa$ and $\lambda$, which are phenomenologically determined in various relativistic mean-field (RMF) models. In such models $\kappa$ and $\lambda$ are sometimes taken to be negative. Furthermore, in Figs. 4 and 5 we respectively illustrate the shapes of the non-linear scalar potentials calculated from the QMC model and those in the RMF models. (In Fig. 4 we did not show the results of B(0.25) and B(0.5).) From Fig. 4 we can see that the various quark models lead to the similar non-linear potentials, despite of the big difference in the confinement mechanism. It may imply that we cannot discriminate the quark models for the nucleon only from the point of view of the energy of nuclear matter. On the other hand, the non-linear potentials in the RMF models show quite different behaviors in Fig. 5. In the region of $\phi > 0$ the potentials in the G2, NLB and ZM models are close to those produced by QMC.

We here comment on the coupling $h(\phi)$ to the gradient of the scalar field. The QMC model gives

$$h(\phi) = 1 + g_{s1} \left( \frac{\phi}{M_N} \right) + g_{s2} \left( \frac{\phi}{M_N} \right)^2 + O(g^3_{s}),$$  \hspace{1cm} (46)

where $g_{s1} = 2g_\sigma r$ and $g_{s2} = 6g^2_\sigma r^2$. We find that the present model suggests $g_{s1} \sim 4 - 8$ and $g_{s2} \sim 20 - 100$, while the ZM model gives $g_{s1} \sim 30$ and $g_{s2} \sim 580$. In the RMF model, for example, the G1 and G2 models lead to $g_{s1} \sim 9$, which is consistent with the values obtained in QMC. This coupling $h(\phi)$ affects the surface properties of finite nuclei.[10]

V. DISCUSSION AND CONCLUSION

We have first studied the MIT bag model and the relativistic constituent quark model for the nucleon, where a square well and harmonic oscillator potentials are used to confine
the quarks. We have considered not only the Lorentz-scalar type confining potential but also the Lorentz-vector type one in order to study the role of the vector potential in nuclear matter. Then, we have calculated the properties of the nucleon and nuclear matter using QMC with those quark models. As a mixture of the Lorentz-vector type confining potential is larger the effective nucleon mass in matter is larger and the strength of the scalar field is weaker. In particular, it is preferable in the bag model that the Lorentz structure of the confinement is purely scalar (or including a very weak Lorentz-vector type potential).

Next, we have performed a re-definition of the scalar field in matter and transformed the QMC model to a QHD-type model with a non-linear scalar potential. We then compared our results with the potentials which are determined so as to fit the properties of finite nuclei and nuclear matter in the RMF models. The QMC model provides the parameters $\kappa \sim 20 - 40$ (fm$^{-1}$) and $\lambda \sim 80 - 400$ for the non-linear scalar potential. The shapes of the potentials generated from QMC are thus very close to one another, despite of the different confinement mechanism in the quark models. On the other hand, the parameters phenomenologically determined in the RMF models take various values and the shapes of the potentials are quite different. In general, the phenomenological potential may consist of a part, which is caused by the quark substructure of the nucleon, and the inherent self-couplings of the scalar field. Therefore, if the part due to the internal structure of the nucleon could be known by nuclear experiments, we could get significant information on the quark structure of the in-medium nucleon.

We should emphasize that the field re-definition performed here enables us to make a QHD-type effective Lagrangian from the QMC model. However, the created Lagrangian is identical to the QMC model only on the energy of a nuclear system. In QMC the quark wave function in the nucleon is modified by the nuclear environment self-consistently at each nuclear density. We can know it explicitly. It is the big difference between the QHD-type and QMC models, and is of great advantage to the QMC model. Using the change of the quark wave function in the medium provided by QMC we can explain many intriguing nuclear phenomena [3], for example, the nuclear EMC effect [26], charge symmetry breaking...
in nuclei (the Okamoto-Nolen-Schiffer anomaly) \[27\], change of electromagnetic form factors of the bound nucleon \[28\], longitudinal response functions in electron scattering \[29\] etc.

We have one comment on the modified quark-meson coupling (MQMC) model proposed by Jin and Jennings \[30\]. The original QMC model \[3–5\] generates a (relatively) small scalar field in nuclear matter. On the other hand, it is well known that the MQMC model can generate the scalar field, which is close to that in QHD (see also Ref. \[10\]). However, unfortunately the increase of the nucleon size at \(\rho_0\) is about 40\% of the free-space value, which is apparently too large \[21,28\]. Although it must be true that the bag constant decreases as the density grows up, the reduction of the bag constant around normal nuclear matter density may not be large. For example, the Dyson-Schwinger approach at finite chemical potential \[31\] suggested that the bag constant is reduced very little at \(\rho_0\), while it decreases very rapidly near the phase transition point.

It is possible to extend the present QMC model to a model which includes quark degrees of freedom in the mesons, as well as in the nucleons — we call it QMC-II \[5,32\]. The QMC-II model can provide a lot of effective coupling terms among the meson fields because the mesons have structure. In particular, the QMC-II Lagrangian automatically offers non-linear terms with respect to the meson fields. Those interaction terms may correspond to the terms of meson-meson and meson-nucleon couplings considered in the new version of QHD \[22\] or higher order terms appearing in the chiral effective lagrangian for nuclear matter \[33\]. It will be very interesting to construct a QMC model, in which both the quark substructure of hadrons and non-linear potentials due to inherent self-couplings of the meson fields are involved (see, for example, Ref. \[12\]), and explore the connection between various coupling strengths found empirically in the phenomenological models and those predicted in the QMC model.
APPENDIX

In the relativistic quark models we adopt here the lowest energy state for the confined quark is given by Eq. (4). In the SW model the upper and lower components of the quark wave function inside the well are respectively given by the spherical bessel functions: $f(r) = j_0(\alpha x)$ and $g(r) = \sqrt{\alpha^2 - \lambda^2}/\alpha j_1(\alpha x)$, where $\alpha = Re = \sqrt{x^2 + \lambda^2}$, $\lambda = m_q R$ and $k = x/R$. The normalization constant is then given by

$$N^2 = \frac{x^2(1 - \beta)}{R^3 \sqrt{x^2 + \lambda^2} D_{SW}}, \quad (47)$$

where $D_{SW} = 2\alpha(\alpha - \sqrt{1 - \beta^2} + \beta \lambda) + \lambda \sqrt{1 - \beta^2}$.

Using those quantities the quark-scalar charge $S_N$ is calculated

$$S_N(m_q) = \int_{V_N} d\vec{r} \overline{\psi} \psi_q, \quad (48)$$

We note that $S_N$ vanishes if $m_q = 0$ MeV and $\beta \to 1$, which may be accidental in the SW and bag models. In this case there is no coupling between the $\sigma$ meson and the nucleon, which implies that the nuclear matter cannot be described by this model.

The root-mean-square (rms) (charge) radius of the nucleon is given by

$$r_q^2 \equiv \langle r^2 \rangle = \int_{V_N} d\vec{r} r^2 \overline{\psi} \psi_q = R^2 \frac{N_{rms}}{3x^2 D_{SW}}, \quad (49)$$

where

$$N_{rms} = \alpha [2x^2(\alpha - \sqrt{1 - \beta^2}) + 2\alpha(2 + \beta) - 3] - 3\lambda \left[ \lambda(1 + 2\beta) - x^2 \sqrt{1 - \beta^2} - \frac{3}{2} \right] + 2\alpha \lambda \left( \frac{x^2 - \frac{3}{4}}{4} \right) \beta - 2. \quad (50)$$

The axial vector coupling constant ($g_A$) and the magnetic moment ($\mu$) of the nucleon are also calculated analytically. In the present quark model $g_A$ is given by $g_A = (5/3) \times g_A^0$, where the factor $(5/3)$ is the expectation value of the spin-isospin operator with respect to the nucleon state. Then, $g_A^0$ is
\[ g_0^A = \frac{2\alpha^2 + 4\alpha\lambda - 3\lambda\sqrt{1 - \beta^2} + 2\beta\lambda(\alpha + 2\lambda)}{3D_{SW}}. \tag{51} \]

The magnetic moment is given by \( \vec{\mu} = \sum_i \mu_i \vec{\sigma}_i Q_i \), where \( Q_i \) is the quark charge operator and the sum \( i \) runs over all quarks in the nucleon. Then, \( \mu_i \) is

\[
\mu_i = \frac{R}{6} \cdot \frac{4\alpha + 2\lambda - 3\sqrt{1 - \beta^2} + 2\beta(\alpha + 2\lambda)}{D_{SW}}. \tag{52} \]

In the MIT bag model, those quantities are given by the same expressions as in the SW model.

In the HO model, the quark wave function is again given by Eq. (4). The upper and lower components are respectively given by gaussian functions:

\[ f(r) = e^{-r^2/2r_0^2} \text{ and } g(r) = (r/\xi r_0^2)e^{-r^2/2r_0^2}, \]

where \( r_0 = (c/\xi)^{-1/4} \) with \( \xi = E + m_q \). Then, the normalization constant is

\[
N^2 = \frac{8}{\sqrt{\pi}} \cdot \frac{\xi^2}{D_{HO}}, \tag{53} \]

where \( D_{HO} = 2(\xi r_0^2) + 3 \).

Using the quark wave function, the quark-scalar charge is calculated

\[
S_N(m_q) = \frac{2(\xi r_0^2) - 3}{D_{HO}}. \tag{54} \]

We note that \( S_N \) approaches 1/3 when \( m_q = 0 \) MeV (and \( \beta = 1 \)). Similarly, \( g_0^A \) and \( \mu_i \) are respectively given by

\[
g_0^A = \frac{2(\xi r_0^2) - 1}{D_{HO}}, \tag{55} \]

\[
\mu_i = \frac{\xi r_0^2}{D_{HO}}. \tag{56} \]

When the nucleon is embedded in a nuclear medium, those quantities can be expressed by the same forms with the effective quark mass \( m^*_q \) (see below Eq. (14)), instead of the free quark mass \( m_q \).
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TABLE I. Parameter $z$, quark eigenvalue $x$ and rms radius ($r_q$). The bag model is denoted by $B$. The value of $\beta$ is shown in the parentheses.

|       | SW(0) | SW(0.25) | SW(0.5) | B(0)  | B(0.25) | B(0.5) |
|-------|-------|----------|---------|-------|---------|--------|
| $z$   | 4.396 | 4.792    | 5.164   | 3.273 | 3.974   | 4.637  |
| $x$   | 2.449 | 2.596    | 2.732   | 2.043 | 2.276   | 2.498  |
| $r_q$ (fm) | 0.5415 | 0.5264 | 0.5137 | 0.5832 | 0.5721 | 0.5593 |

TABLE II. Coupling constants and calculated properties of the nucleon and symmetric nuclear matter at normal nuclear density. (The quantities with $\ast$ are those quantities calculated at $\rho_0$.) The nuclear compressibility $K$ is quoted in MeV. The last row is for QHD.

|       | $g_\sigma^2$ | $g_\omega^2$ | $M_N^*/M_N$ | $K$   | $x^*/x$ | $r_q^*/r_q$ | $g_A^*/g_A$ | $\mu_p^*/\mu_p$ | $S_N^*/S_N$ |
|-------|--------------|--------------|-------------|-------|---------|-------------|-------------|-----------------|-------------|
| SW(0) | 84.41        | 104.1        | 0.725       | 329   | 0.937   | 1.03        | 0.936       | 1.11            | 0.886       |
| SW(0.25) | 74.75      | 83.16        | 0.769       | 307   | 0.956   | 1.03        | 0.919       | 1.11            | 0.850       |
| SW(0.5)  | 66.61       | 65.21        | 0.807       | 287   | 0.971   | 1.03        | 0.903       | 1.10            | 0.811       |
| HO(1.0) | 146.5       | 64.51        | 0.805       | 309   | —       | 1.10        | 0.891       | 1.23            | 0.812       |
| B(0)   | 67.55       | 66.10        | 0.805       | 278   | 0.837   | 1.02        | 0.909       | 1.08            | 0.814       |
| B(0.25) | 54.65       | 35.49        | 0.869       | 235   | 0.900   | 1.03        | 0.877       | 1.06            | 0.711       |
| B(0.5) | 47.99       | 10.92        | 0.921       | 169   | 0.941   | 1.02        | 0.841       | 1.05            | 0.541       |
| QHD    | 91.64       | 136.2        | 0.556       | 540   | —       | —           | —           | —               | —           |
TABLE III. Parameters $a$ and $b$. Note that $a \sim 1$ for the SW and bag models, while $a \sim 2/3$ for the HO model.

|       | SW(0) | SW(0.25) | SW(0.5) | HO(1.0) | B(0) | B(0.25) | B(0.5) |
|-------|-------|-----------|---------|---------|------|---------|--------|
| $a$   | 1.008 | 1.006     | 1.017   | 0.6868  | 0.9982| 1.005   | 0.9933 |
| $b$   | 0.2146| 0.3229    | 0.4965  | 0.2452  | 0.4346| 0.9666  | 2.206  |

TABLE IV. Parameters $\kappa$ and $\lambda$ generated by the QMC model.

|       | SW(0) | SW(0.25) | SW(0.5) | HO(1.0) | B(0) | B(0.25) | B(0.5) |
|-------|-------|-----------|---------|---------|------|---------|--------|
| $\kappa$ (fm$^{-1}$) | 19.16 | 27.20     | 39.03   | 42.31   | 35.05| 69.65   | 150.7  |
| $\lambda$ | 78.78 | 158.7     | 326.8   | 384.1   | 263.5| 1041    | 4872   |

TABLE V. Parameters $\kappa$ and $\lambda$ for the RMF models.

|       | ZM | NL1 | TM1 | NLB | G1 | G2 |
|-------|----|-----|-----|-----|----|----|
| $\kappa$ (fm$^{-1}$) | 74.8| $-24.3$ | $-14.5$ | 4.06 | 29.4 | 46.0 |
| $\lambda$ | 720 | $-218$ | 3.71 | 10.0 | $-279$ | 19.7 |
FIG. 1. Energy per nucleon for symmetric nuclear matter. The saturation curves for the SW model and the bag model are respectively shown by solid and dotted curves. The dot-dashed curve presents the result of the HO model. The curves with squares (triangles) are for the results with $\beta = 0.25(0.5)$, while the curves without any marks are for $\beta = 0$. 
FIG. 2. Scalar mean-field value. The curves are labeled as in Fig. 1.
FIG. 3. Effective nucleon mass in symmetric nuclear matter. The curves are labeled as in Fig. [1].
FIG. 4. Non-linear scalar potentials generated from the QMC model. The solid curve shows $U_s = \frac{m^2}{2} \phi^2$. The dashed curve with open (solid) circles is for the SW model with $\beta = 0.25(0.5)$, while the dashed one without any marks is for the SW model with $\beta = 0$. The result of the B(0) model is shown by the dotted curve. The dot-dashed curve is for the HO model.
FIG. 5. Non-linear scalar potentials in the RMF models. The solid curve shows $U_s = \frac{m^2}{2} \phi^2$. The dashed curve with open (solid) circles is for the G2 (NLB) model [22], while the dashed one without any marks is for the G1 model [22]. The dashed curve with crosses is for the ZM model [23]. The potentials in the TM1 [25] and NL1 [24] models are respectively shown by the dotted and dot-dashed curves.