Modal analysis of the cantilever type piezo-electric generator characteristics with active based on numerical simulation

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Abstract. Finite element simulation of a piezo-electric generator (PEG) of a cantilever type with an active termination is considered. Finite element modeling was performed in the Ansys complex. The generator has a bimorphic arrangement of piezoelectric elements. A special feature of PEG is that the generator has two types of piezoelements: (1) elements located on the substrate in the form of a bimorph and (2) cylindrical piezoelements, which fix the cantilever plate. PEG can be used as a device for generating energy. A generator design oscillations modal analysis - was performed. For a given scheme of electrical connection of the PEG elements and various resistive loads, the results of the output voltage and power are given.

Introduction
The direction of development of systems for the generation of electric energy is quite relevant at present and requires research in this area. One of such systems are elements in the form of piezoelectric generators (PEG). For studies of such generators, it is necessary to carry out calculations and develop measurement systems for evaluating their real output properties. So, in the works [1-5] actual studies of the characteristics of PEG with the use of measuring equipment and the corresponding experimental techniques are given.

Among the various problems arising in the development of PEG, we can highlight the most significant of them, including the choice of energy-efficient piezoceramic materials (PCM), the development of electrical circuits for the accumulation of electrical energy with minimal charge leakage, the search for geometric configurations and the assembly technology of the PEG sensitive element, which cause maximum output power. Solving these problems, we use different approaches, which differ depending on the type and use of PEG. The reviews [5-13] contain materials on PEG systems.

In addition to analytical methods for describing PEG models, the finite element method (FEM) using the Ansys package is widely used. Examples of modeling can be presented in [14,15].

Finite element modelling of PEG.

Continuous models of composite elastic, electro-elastic and electroacoustic medium
Piezoelectric energy harvesting device is a composite elastic and electro-elastic body, which makes small relative oscillations in the moving coordinate system. Rectilinear vertical motion of the system is given by the law \( y(t) \) or, in case of the external force excitation, by \( F(t) \) (or pressure \( p(t) \)), according to which the device’s base is moving. In these conditions, the initial boundary value problem of linear electrodynamics theory is quite adequate mathematical model, which describes the functioning of such device [16].

In the present paper, we use the linear theory of elasticity and electrodynamics, based on the dissipation of energy, which is realized in the ANSYS software [17], as well as the equations of motion of liquids and gases in the acoustic approximation [18].

For piezoelectric medium, we have:

\[
\rho \ddot{u}_i + \alpha \rho \dot{u}_i - \sigma_{ij,i} = f_i; \quad D_{ij} = 0, \tag{1}
\]

\[
\sigma_{ij} = c_{ijkl} \varepsilon_{kl} + \beta \varepsilon_{kl} E_k; \quad D_i = \varepsilon_{ijkl} \varepsilon_{ij} + \alpha_{ij} E_k, \tag{2}
\]

\[
\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2; \quad E_k = -\varphi_{,k}, \tag{3}
\]

where \( \rho \) is the density of the material; \( u_i \) are the components of the vector-function of displacement; \( \sigma_{ij} \) are the components of the stress tensor; \( f_i \) are the components of the vector of the density of mass forces; \( D_i \) are the components of the electric induction; \( \varepsilon_{ijkl} \) are the components of the fourth rank tensor of the elastic moduli; \( \varepsilon_{ijkl} \) are the components of the third rank tensor of piezoelectric coefficients; \( \varepsilon_{ij} \) are the components of strain tensor; \( E_k \) are the components of the electric field; \( \varphi \) is the electric potential; \( \beta \) are the components of the second rank tensor of the dielectric constants; \( \alpha, \beta, \varepsilon_{ij} \) are non-negative damping coefficients (in ANSYS \( \varepsilon_{ij} = 0 \)).

For elastic medium:

\[
\rho \ddot{u}_i + \alpha \rho \dot{u}_i - \sigma_{ij,i} = f_i; \tag{4}
\]

\[
\sigma_{ij} = c_{ijkl} \varepsilon_{kl}; \tag{5}
\]

\[
\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2 \tag{6}
\]

**Modeling**

The full-scale FE model of PEG has a console structure in the form of a bimorph. Thin symmetrical piezoelectric elements are polarized in thickness. Sizing piezo-elements to the base is not taken into account. The geometrical dimensions of the PEG are shown in Figure 1(a): the substrate has dimensions \( l\times b \times h = 160 \times 13.5 \times 1.5 \text{ mm}^3 \), the piezoelectric elements consist of two identical piezoelectric plates, polarized in thickness with dimensions \( l_p \times b_p \times h_p = 54 \times 6 \times 0.5 \text{ mm}^3 \). The center of the attached mass is fixed at a distance of \( l_m \) from the cantilever clamp. The range of location of the cargo \( l_m \) can vary from 65 to 145 mm. Dimensions adopted in the model: \( a_m = 7 \text{ mm}, h_m = 6 \text{ mm}, b_m = 29.8 \text{ mm} \). The electrical connection diagram of the PEG with active load is shown in Fig. 1(b).

The size of the added mass can vary from 3 to 50 grams, depending on the density of the material. Dimensions of piezo-cylinders: \( D_xh = 10 \times 10 \text{ mm} \). The material of the piezoceramic elements is CTS-19. The main properties of the PEG design are also given in Table 1, 2.

**Table 1. Physical parameters of CTS-19**

|       | \( C^{E}_{11} \) | \( C^{E}_{12} \) | \( C^{E}_{13} \) | \( C^{E}_{33} \) | \( C^{E}_{44} \) | \( e_{31} \) | \( e_{33} \) | \( e_{15} \) | \( \varepsilon^{33}_{11}/\varepsilon_0 \) | \( \varepsilon^{33}_{33}/\varepsilon_0 \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|------------|------------|------------|-------------------------------|-------------------------------|
| **CTS-19** | 10.9            | 6.1             | 5.4             | 9.3             | 2.4             | -4.9       | 14.9       | 10.6       | 820                           | 840                           |

In the headings of the tables are decoded as: elastic moduli: \( C^{E}_{pq} \times 10^{10} \text{ Pa} \); Piezoelectric coefficients: \( e_{ij} \) (C/m); Relative permittivity (at room temperature): \( \varepsilon^{33}_{ij}/\varepsilon_0 \);

**Table 2. Mechanical properties of the structural materials**
| №  | Material             | ρ, [kg/m³] | E×10¹⁰ [Pa] | ν   |
|----|----------------------|------------|-------------|-----|
| 1  | Substrate fiberglass | 1600       | 0.6         | 0.25|
| 2  | Attached Mass        | 2645       | 0.3         | 0.33|
| 3  | Piezo CTS-19         | 7280       | -           | 0.33|
| 4  | Base                 | 7700       | 21          | 0.33|

**Figure 1.** (a) Electric scheme of compound PEG under active load and (b) structure scheme of PEG with proof mass: 1 – piezoelectric element; 2 – substrate; 3 - proof mass; 4 - place of PEG fixing (B - movable base); 5- piezoelectric cylinders element

**Figure 2.** Three-dimensional finite element model of PEG: 1,5 - piezoelectric elements; 2 - substrate; 3 - proof mass; 4 - place of generator fixing

The results of the modal calculation of the first 12 eigenmodes of PEG oscillations are given at the position of the attached mass of \( L_m = 80 \) (located near the piezoelectric plates). An analysis of the self-oscillation modes shows that 1,2,5,7,9 forms of oscillations are transverse modes in the direction of the Ox axis (\( TrX \)). Modes of oscillations 3,6 are transverse with respect to the axis OZ (\( TrZ \)). The modes of oscillations 4,10 are torsional with respect to the axis OX (\( TorX \)). Modes of oscillations 11,12 begin to oscillate the base of the structure (\( OscB \)) and mode 12 has a mixed mode \( OscB+ TrZ \).

Figure 4 shows the distribution of natural frequencies depending on the location of the attached mass. The analysis shows that the first natural frequency varies from 50 Hz to 22 Hz, while the load is
located, respectively, from 65 to 145 mm. Analysis of the output power and voltage shows that it is
different at different positions of the added mass and the active load (Fig. 5). The highest generator
power can be obtained in the vicinity of an active load of 100 kΩ.

![Figure 3. Own modes of oscillation piezo-generator](image)

![Figure 4. Dependence of the first 4 natural frequencies on the location of the added mass Lm](image)

![Figure 5. Dependencies of output power and output voltage on active load for two locations of proof mass](image)
Summary
The numerical simulation of a cantilever PEG with added mass and an active base of piezo-cylinders is considered. The attached mass was based in the region of the free end of the generator cantilever at $l_m = 65...145$ mm. Analysis of the AC characteristics showed that the first resonant frequency of the PEG is $50...22$ Hz. The maximum power removed from piezo-plates with an active load of $100$ kΩ with an output voltage of $8.3$ V was $670$ mWK. A more detailed analysis of the output power of PEG requires calculation with different variations of the added mass and taking into account other properties of the substrate material and dimensions of piezoelectric elements.

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