Economic Stimulation Signal on Power Supply Optimization From Distribution Cost Allocation Based on Shapley Value in Integrated Energy Park

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Abstract. This paper discussed the economic incentives of the distribution cost distribution based on the Shapley value of cooperative game in the integrated energy service park for the optimization of the park's power supply mode. The application of cooperative game in the distribution of power distribution costs gives an economic signal reflecting the difference in power supply to users in the park. In the concept of multiple solutions in cooperative game theory, the Shapley value determines the power flow contribution of each user to the power grid with its uniqueness, determines the user’s power distribution cost based on the power flow, and its economic background also clarifies the user’s responsibilities, and the user chooses differently. The power distribution costs borne by the power supply are also different, thereby promoting power supply optimization. The 5-node calculation example verifies the effectiveness of the distribution cost allocation based on the Shapley value for the economic incentive of power supply optimization.

1. Introduction
In the integrated energy systems park, the economical way to supply power for users can save power distribution costs. And with the economical way, integrated energy service providers also have the motivation to optimize the power supply. There are many ways to allocate power grid costs, each with its own strengths. A class of methods based on cooperative games has a good economic and mathematical background [1, 2]. The Shapley value is convenient and unique in the concept of various solutions of cooperative games. Based on the Shapley value method of apportioning the fixed costs of the transmission grid, this paper clarifies the differences in the distribution costs of different ways of supplying power to users, incentivizing users to select reasonable locations and power sources to generate power, and guide the optimization of power supply through economic signals to save costs.

2. Shapley value of cooperative game
An important problem solved by cooperative games is to find one or a group of allocations, so that every player in the game gets their payment according to this group of allocations, and everyone has no opinion. The solution of the cooperative game must satisfy individual rationality and overall rationality. The individual rationality conditions indicate that each player's income is at least not less than his income when he does it alone. The group rationality condition shows that the sum of the profit distributed by
each person is just the total maximum profit of various alliance forms. Therefore, the solution of the cooperative game can always give users a cost allocation with the greatest personal profit.

Define a model \( I(N, v) \) of cooperative game with \( n \) players, \( v \) is a feature function, \( N=\{1,2,...,n\} \) is a set of \( n \) players, \( S \subseteq N \) called an alliance. For each subset \( S \) of the game, the function value \( v(S) \) is the maximum profit that alliance \( S \) can obtain. When the players in \( S \) become an alliance, regardless of the strategy adopted by the players outside \( S \), the players in \( S \) can always obtain this maximum profit [3]. Let \( x(I)=\{x \in \mathbb{R}^n \mid x_i \geq v(\{i\}), i=1,2,...,n; x(N)=v(N)\} \), \( x \in x(I) \) is the column vector, \( x \) is called an allocation, and the allocation of this game is denoted as \( E(v) \).

The Shapley value of cooperative games involves two related concepts which are dummy elements and pillars. For the cooperative game \( I=(N, v) \) with \( n \) players, if the player \( i \) satisfies \( v(S \cup \{i\}) = v(S) \), \( \forall S \subseteq N \), then \( i \) is a dummy element. For \( T \subseteq N \), if \( v(S) = v(S \cap T) \), \( \forall S \subseteq N \), then \( T \) is said to be the pillar of the game \( I=(N, v) \). The Shapley value of the cooperative game \( (N, v) \) with \( n \) players refers to a vector that satisfies the following three axioms.

\[
\varphi(v) = (\varphi_1(v), \varphi_2(v), \ldots, \varphi_n(v))
\]

One is validity axiom. For any pillar \( T \) of \( v \), there is \( \sum_{i \in T} \varphi(v) = v(T) \). This axiom means that dummies need not be taken into account when allocating payments.

Two is symmetry axiom. If there is a certain arrangement \( A \) of \( N \) with \( v(AS) = v(S) \), \( \forall S \subseteq N \), then \( \varphi_A(v) = \varphi(v) \), \( \forall i \in N \). This axiom says that if there is a permutation \( A \) such that any alliance \( S \) and \( AS \) have the same payment, then each player \( i \in S \) in the game and player \( AI \) will be allocated the same share.

Three is additivity axiom. Let \( v \) and \( u \) be a cooperative \( n \) players game, then \( \varphi(v+u) = \varphi(v) + \varphi(u) \), \( \forall i \in N \). This axiom says that the share allocated by the player in the sum game is equal to the sum of his allocated shares in the two games.

The Shapley value is intuitively the average value of the marginal contribution made by a player in the various alliances that he can participate in. This marginal contribution is due to the increase in the total profit of the players joining the various alliances. Its calculation is as follows.

\[
\varphi_i(v) = \sum_{s \in \mathcal{F} \setminus \{S\}} \frac{s(n-s-1)!}{n!} (v(S \cup i) - v(S))
\]

In the formula, \( s=|S| \) represents the number of players in the league \( S \), and \( S \cup i \) represents the new alliance formed after player \( i \) joins the league \( S \).

3. Shapley model of distribution cost allocation

The Shapley model of distribution cost allocation is based on line active power flow. In this model, the grid user (excluding the power generation user) is the player. The Shapley value gives the contribution of each user to the active power flow of the distribution network, that is, the sum of the active power flow on all lines caused by the distribution of this user in the distribution network.

In the distribution cost allocation, user loads are all measured by active power. Assuming that the distribution cost is converted to a certain trading period as \( K \), the maximum total active power flow of all lines of the distribution network under the maximum load of the system during this period, then the unit active power flow cost \( \lambda \) of this distribution network is [4]

\[
\lambda = \frac{K}{F_{\text{total}}}
\]
Suppose there are no other users in the distribution network except user $i$. At this time, the distribution network only supplies power to user $i$. The maximum value of the sum of active power flow of all lines in this period is $F_i$. Correspondingly, $F_S$ is the maximum value of the sum of active power flow of all lines when the distribution network is only supplying power to the members of the alliance $S$. The actual situation of the operation of the distribution network in the integrated energy park is that all grid users must transmit power, which means that all users participate in the same alliance. This kind of alliance in which all members of the bureau participates is a major alliance. In the case of a major alliance, the sum of the active power flows of all lines in the distribution network is equal to $F_{\text{total}}$.

For alliance $S$, $n_S$ is the number of people participating in alliance $S$. The sum of the line active power flow $F_i$ caused by power supply to each member user is $\sum_{i=1}^{n_S} F_i$, and the line active power flow caused by power transmission to all members of the alliance is $F_S$. For the distribution network, the maximum value of the system active power flow is decentralized with $\sum_{i=1}^{n_S} F_i \geq F_S$. The difference between $\sum_{i=1}^{n_S} F_i$ and $F_S$ will win the trend of each user participating in the alliance, that is, the characteristic function is $v(S)$.

$$v(S) = \sum_{i=1}^{n_S} F_i - F_S$$

(4)

A cooperative game model $\Gamma(N, \nu)$ with $n$ players is used to represent the cooperative game relationship of users in a comprehensive energy park with $n$ users. The trend win caused by alliance formation is the characteristic function. The Shapley value can be used to calculate the participation profit $x_i$ of each user.

$$x_i = \phi_i$$

(5)

If $F_i'$ is contribution from user $i$ to the active power flow of the power grid, then

$$\begin{cases} 
F_i' = F_i - x_i \\
\sum F_i' = F_{\text{total}}
\end{cases}$$

(6)

Then the power distribution cost $C_i'$ of user $i$ is as follows.

$$C_i' = \lambda F_i'$$

(7)

4. Economic incentive signal

As the non-storability of electric energy, in the integrated energy park, when the power user increases the load, the service provider must increase the corresponding power. In addition to the restriction of grid security, users can choose power sources at any node, which will inevitably produce different power flow results in the distribution network, and the distribution fees allocated to users will also be different. Users choose power sources under the economic incentive of distribution fees. And users get electricity service in the most cost-saving way.

The distribution cost allocation based on the Shapley value is an economic incentive for the user to increase the unit load for the user. When the power source of a certain node is selected, the corresponding increase in the distribution cost of Shapley is so allocated to the user. The greater the value of the distribution cost allocated to the user, the less economical it is for the user to choose the power supply at this node, and the lower selectivity of the power supply at this node.
When the grid is operating normally, according to the Shapley value, the distribution cost allocated to user at node \( j \) is \( C'_{jk} \). For the increased load \( \Delta P_j \), if the distribution cost allocated to the user is \( C'_{jk} \) when the power supply of node \( k \) is selected. And the increase of the distribution cost allocated to this user is \( \Delta C'_{jk} \), then \( \Delta C'_{jk} = C'_{jk} - C'_{j} \). The economic incentive to the user based on the distribution cost allocated by the Shapley value is \( \gamma_{jk} \) and \( \gamma_{jk} = \Delta C'_{jk} / \Delta P_j \). The physical meaning of this value is the increase in the distribution cost of the load per unit increment, which reflects the economic profits of the user’s choice of source on node \( k \). The smaller its value, the more economical the user chooses the power supply here.

5. Quantitation of economic incentive based on DC power flow and shapley
The economic incentives of power distribution costs to users are two situations. In the first case, the node where the user is located also has power supply, that is, \( j=k \). When the user load increases, the power supply of the same node is capable of meeting the load demand. So it is obviously the most economical for the user to choose the same node power supply. The increased load at this point with the internal digestion of the nodes will not cause an increase in the power flow in the grid. And the burden of the power distribution of the grid will not increase. So the user does not need to bear the additional distribution cost for the increased load and \( \gamma_{jk} \) is zero.

The second situation is that the user node has no power supply or the power supply of the same node has been saturated. When the user load increases, other node power sources must be selected to provide power, which will inevitably increase the load on the power distribution network. In the power flow calculation, the user loads are all active power according to the DC power flow model.

\[
P = B \theta
\] (8)

Power grid nodes are \( j \) and \( k \). The node injected power vector is \( P \), where the elements \( P_j = P_{oj} - P_{dj} \) (\( j=1,2,...,n-1 \), \( n \) is a balanced node). The power supply and load of node \( j' \) are \( P_{oj} \) and \( P_{dj} \) respectively. The imaginary part of the node admittance matrix is \( B \), where the elements are \( B_{jk} = -\frac{1}{x_{jk}} \) and \( B_{\theta} = \sum_{j'k} \frac{1}{x_{jk}} \). The node voltage phase angle vector is \( \theta \), where the voltage phase angles of nodes \( j \) and \( k \) are \( \theta_j \) and \( \theta_k \) respectively which can be regarded as the node voltage in the DC power flow equation. The active power of branch \( jk \) is as follows.

\[
P_{jk} = -B_{jk} \theta_{jk} = \frac{\theta_j - \theta_k}{x_{jk}}
\] (9)

The power user load increment of node \( j \) is \( \Delta P_j \). The power supply of node \( k \) is selected to provide increased power. The increment of the \( j \)th element of the node injecting power vector \( P_j \) is \( -\Delta P_j \). The increment of the \( k \)th element \( P_k \) is \( \Delta P_j \). Then the voltage phase angle increments at nodes \( j' \) and \( k' \) are as follows.

\[
\Delta \theta_{j'} = \Delta P_j (B^{-1})_{j'k} - (B^{-1})_{j'j}
\] (10)

\[
\Delta \theta_{k} = \Delta P_j (B^{-1})_{k'k} - (B^{-1})_{k'j}
\] (11)

The active power increment of branch \( j'k' \) is as follows.
The distribution of the active power flow of the entire distribution network changes. $L$ represents the branch set of the distribution network. And the total active power flow increment is as follows.

$$\Delta P_{jk'} = -B_{jk'} \Delta \theta_{jk'} = \Delta \theta_j - \Delta \theta_{k'} = \Delta P_j B_{jk'} ([B^{-1}]_{kj} - [B^{-1}]_{k'j} - [B^{-1}]_{jk} + [B^{-1}]_{jk'})$$ \hspace{1cm} (12)$$

The expression of the active power increment of branch $j'k'$ shows that this value is only related to the branch parameters including nodes $j$ and $k$ and the load increment of node $j$. Therefore, except for user $j$, the contribution $F_i$ of each user to the active power flow of the line does not change when each user does not participate in any alliance. Except for the alliance that user $j$ participates in, the total active power flow $F_S$ of the park distribution network caused by alliance $S$ is also unchanged. Then the increase in the contribution of this user or the alliance that this user participates to the power flow of the park distribution network caused by the increase of the load of user $j$ is $\sum_{j \in k} \Delta P_{jk'}$.

According to equation (4), if the alliance $S$ contains user $j$, $\sum_{i \in S} F_i$ has the same power flow contribution increment as $F_j$ and the value of the characteristic function does not change. According to the Shapley value, the profit $x_i$ of each user who participates in the big alliance is also unchanged. Therefore, economic incentives can be solved by simple calculation based on branch parameters and distribution network active power flow.

6. An example analysis

Take a simple 5-node system as shown in Figure 1 as an example to show the increase in user load in the integrated energy park and the change in the distribution cost based on the Shapley value. In the figure, node 1 is a balanced node, and the remaining nodes are PQ nodes. The data of each node is shown in Table 1. Symbols ①-⑦ indicate branch numbers, and branch parameters have been marked. Suppose that every MW of electricity transaction in a certain period of time should share 50 yuan.

![Five-bus system](image_url)

**Figure 1.** Five-bus system.
Table 1. Data of buses.

| bus | Bus voltage Magnitude(p.u.) | Phase | Generator power Active(MW) | Reactive(Mvar) | Load power Active(MW) | Reactive(Mvar) |
|-----|-----------------------------|-------|---------------------------|---------------|----------------------|---------------|
| 1   | 1.06                        | 0.00  | 130.5                     | 26.5          | 0.0                  | 0.0           |
| 2   | 1.04                        | -2.64 | 20.0                      | 20.0          | 0.0                  | 0.0           |
| 3   | 1.01                        | -4.81 | 0.0                       | 0.0           | 45.0                 | 15.0          |
| 4   | 1.01                        | -5.13 | 0.0                       | 0.0           | 40.0                 | 5.0           |
| 5   | 1.00                        | -5.98 | 0.0                       | 0.0           | 60.0                 | 10.0          |

The distribution cost allocation of the original load structure of the distribution network based on the Shapley value calculates as follows. The users on nodes 3, 4, and 5 bear the power distribution costs of 3551, 4004, and 3609.5 yuan respectively. The user of node 3 increases the load by 50 MW, so every MW of electricity transaction in this period should share 50 yuan. If the power source of node 1 is selected, users on nodes 3, 4, and 5 respectively allocate the power distribution costs of 5223.7, 3124.3, and 2816.5 yuan, and $\gamma_{1i} = 104.5$ yuan/MWh. If selecting the power source of node 2, and the users on nodes 3, 4, and 5 respectively share the power distribution costs of 5735.2, 2855.3, and 2574 yuan and $\gamma_{2i} = 114.7$ yuan/MWh.

According to equations (12) and (13), it is similar to the value by the distribution costs and the definition of economic incentives. It is proved that when the Shapley value is used to allocate the distribution costs of users based on the DC power flow model, economic incentives can be directly quantified using equations (12) and (13). Comparing the value of economic incentives, it is more economical for users of node 3 to choose node 2 for power source.

7. Conclusion

In the distribution network of the integrated energy service park, although there are many ways to incentive users to use power more effectively, economic ways are indispensable. The distribution cost allocated by the Shapley value of the cooperative game encourages service providers to actively optimize power supply from the financial perspective. Users actively cooperate with service providers to share economic benefits. So both service providers and users achieve a win-win effect. However, this economic incentive signal is calculated under the ideal state that the power supply capacity can always meet the user’s increase. This signal will guide the user to select the power source only from an economic view, which may cause the problem of power imbalance in some nodes.

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