Research on the Phase Aberration Correction with a Deformable Mirror Controlled by a Genetic Algorithm

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Abstract. In order to improve laser beam quality, a real number encoding genetic algorithm based on adaptive optics technology was presented. This algorithm was applied to control a 19-channel deformable mirror to correct phase aberration in laser beam. It is known that when traditional adaptive optics system is used to correct laser beam wave-front phase aberration, a precondition is to measure the phase aberration information in the laser beam. However, using genetic algorithms, there is no necessary to know the phase aberration information in the laser beam beforehand. The only parameter need to know is the Light intensity behind the pinhole on the focal plane. This parameter was used as the fitness function for the genetic algorithm. Simulation results show that the optimal shape of the 19-channel deformable mirror applied to correct the phase aberration can be ascertained. The peak light intensity was improved by a factor of 21, and the encircled energy strehl ratio was increased to 0.34 from 0.02 as the phase aberration was corrected with this technique.

1. Introduction

Adaptive optics (AO) technology can measure and then correct the phase aberration in the laser wave-front caused by atmosphere turbulence in real time, thus it has been widely used in many fields, such as, laser communication, astronomical observation and so on [1, 2]. Nowadays, AO technology is also applied to compensate the phase aberration in some kinds of laser resonators and then improve the laser beam quality accordingly. It is known that a typical adaptive optical system often consists of wave-front detecting, wave-front reconstruction and wave-front correction three parts [3]. Moreover, the wave-font detecting techniques include direct and indirect two ways. Direct wave-front detecting technique which measures the wave-front slope or the curvature straightly is often used in those high speed adaptive optical systems. However, it is not necessary to obtain the phase aberration information such as the amplitude or the sort of aberration in a laser beam in advance using indirect wave-front detecting technique. With this technique, we detect the image quality affected by phase aberration in laser wave-front instead of measuring the phase aberration itself, and then the image quality is taken as a sharpness function. When wave-front phase aberration is corrected completely, the sharpness function reaches its maximum value.

Genetic algorithm (GA) is a stochastic parallel algorithm based on natural selection and biological genetics. It is often used to search the global optimum value of some multi-object problems [4]. In past a few years, the GA community has turned much of its attention to the optimization problems of
industrial engineering. Literature [5] firstly reported that a kind of GA was successfully used in an adaptive optical system to control a deformable mirror (DM) to optimize the laser intensity distribution. That paper shown that the optimum laser intensity distribution could be searched by using GA as long as the amplitude of the phase aberration is in the stroke range of the DM. However, the GA introduced in literature [5] is the simple GA based on binary encoding. There are some disadvantages in this binary encoding GA. Because the binary string is not a natural encoding, thus when the binary string is a little short, GA may not satisfy the precision request of the given problem, but when the binary string is long enough, the search space will become too large for GA to search rapidly. These drawbacks of the binary encoding indicate that in many GA application fields, especially for those problems from industrial engineering world, the simple GA based on binary encoding was difficult to apply directly. So as to overcome the shortage of the GA based on binary encoding, a real number encoding genetic algorithm based on adaptive optics technology was presented. This algorithm was applied to control a 19-channel deformable mirror to correct phase aberration in laser beam. Some simulations have been done and the results prove that this kind of GA can work well in correcting phase aberration in laser wave-front.

2. Overview of a real number encoding GA

2.1. Basic principle and realization of GA based on a real number encoding

The Genetic Algorithm (GA), as a powerful and broadly applicable stochastic search and optimization technique, is perhaps one of the most widely known Evolutionary Computation methods today. It can search the optimum solution adaptively by simulating the selection, crossover and mutation ways of biological population and by taking the fitness function value of each individual in a population as the criterion. The larger the fitness function is, the closer the individual approach the optimum object. Figure 1 is the Flowchart of automatic correction method based on genetic algorithm. The whole course of realization of the GA based on a real number encoding is as follows:

![Figure 1. The flowchart of GA.](image)
2.1.1. Encoding strategy. How to encode a solution of the problem into a chromosome is a key issue for genetic algorithms. Real number encoding is the natural description of the solution to a certain problem, for example, consider a problem to be optimized has three variable $x_1, x_2, x_3$, then $X=[x_1, x_2, x_3]$ is a chromosome which is also an encoding form for the variables of the this problem. $x_1, x_2$ or $x_3$ is not only a gene of the chromosome respectively, but also stands for a real value in their own range. In comparison with binary encoding, real number encoding can improve calculation complexity, increase the solution precision and speed the operation rate without encoding and decoding error. In this paper, we just use the real number encoding GA.

2.1.2. Fitness evaluation and selection operator. Before selection operation, the fitness of every individual in a population must be evaluated firstly. A roulette wheel approach is adopted as the selection procedure, which is one of the fitness-proportional selections and can select a new population with respect to the probability distribution based on fitness values. The roulette wheel can be constructed with as following steps: firstly, suppose that there is a population which includes $M$ individuals (chromosome), the fitness of the $k$th individual is $F_k$, then the selection probability of the $k$th individual $P_k$ can be calculated as follow:

$$P_k = \frac{F_k}{\sum_{j=1}^{M} F_j} \quad (K=1,2,\ldots,M) \quad (1)$$

The second step is to calculate cumulative probability $q_k$ for each chromosome $V_k$:

$$q_k = \sum_{j=1}^{k} P_j \quad (K=1,2,\ldots,M) \quad (2)$$

The third step is to generate a random number $r$ from the range $[0,1]$; the last step is to estimate: If $r < q_1$, then select the first individual $V_1$; or select the $k$th individual $V_k$ in terms of $q_{k-1} \leq r < q_k \ (2\leq k \leq M)$.

2.1.3. Crossover operator. Crossover operator is the main manner to generate the new individuals from the old individual, and the global search ability of GA is also determined by crossover operator, there are several crossover methods such as Simple crossover, Direction-based crossover, Arithmetical crossover and so on. In our paper, we adopted the non-uniform Arithmetical crossover operator. The
operation processes are as follow: suppose that \( Y_A^t \), \( Y_B^t \) are two individuals which are selected from an old population by a crossover probability \( P_c \), then the two new individuals \( Y_A^{t+1}, Y_B^{t+1} \) are generated as follows:

\[
Y_A^{t+1} = m \times Y_B^t + (1-m) \times Y_A^t
\]

\[
Y_B^{t+1} = m \times Y_A^t + (1-m) \times Y_B^t
\]

The variable \( m \) which is decided by the evolution generation can be described as following formula:

\[
m = r^{a(t+1)/T}
\]

Where \( r \) is a random number from the range \([0,1]\); \( T \) is the total evolution generation; \( t \) stands for the current generation which is at the range of \([1,T]\); \( a \) is a weight factor, in our paper, \( a \) is set to 2.

2.1.4. Mutation operator. The local search ability of GA is determined by mutation operation, crossover operation and mutation operation are two indispensable part of GA. There are also kinds of mutation operation ways, the mutation used here is uniform mutation operator and the mutation operation procedure is as follows: choose the gene of a chromosome randomly, then alters this gene with a probability equal to the mutation rate \( P_m \). Assume that the kth gene of a chromosome \( X=[x_1, x_2, x_3, \ldots, x_k, \ldots, x_n] \) is selected for mutation operation, the new chromosome after mutation is \( X'=[x_1, x_2, x_3, \ldots, x'_k, \ldots, x_n] \), then the kth gene \( x'_k \) can be obtained from following formula:

\[
x'_k = L_{\min} + R \times (L_{\max} - L_{\min})
\]

The values of \( x_k \) (\( k=1,2,\ldots,n \)) are in the range of \([L_{\min}, L_{\max}]\).

2.2. GA used in AO

Figure 2 is the schematic diagram of wave-front correction AO system with a GA. A wave-front with phase aberration is incident on the DM, if the phase aberration will not removed from wave-front, it is difficult to focus the wave-front effectively. However, the phase aberration can be compensated completely or mostly through changing the shape of the DM controlled by a GA. The DM used here is a 19-channel piezoelectricity DM, the voltage that each channel can apply to is in the range of \([-300v, 300v]\). GA based on a real number encoding first generates several mirror shapes which correspond to the same number of chromosomes. Each chromosome has 19 genes which correspond to 19 voltages. We take the light intensity that behind the pinhole of the focus plane of a focus lens as the fitness function, then let the GA works according to the process that section 2.1 described.

3. Simulation

3.1. Phase aberration analysis

To test the performance of the correction of laser wave-front aberration using this GA, some simulation work has been done, a wave-front with phase aberration is got in a real diode pumped Nd:YAG laser beam instead of generated randomly. It is known that any kinds of wave-front phase aberration can be described by a serial of orthodox Zernike polynomial in unit circle area [6,7]. In general, any wave-front phase aberration can be described as follows:

\[
\varphi(x,y) = \sum_{k=1}^{l} A_k Z_k(x,y)
\]

Where \( Z_k(x,y) \) is the kth Zernike polynomial and \( A_k \) is the coefficient of it accordingly. The first 35 polynomials are used to describe the phase aberration in our paper. Figure 3 is the polynomial coefficients distribution of wave-front phase aberration in a Nd:YAG laser before correction. Transverse coordinate stands for the Zernike sequence number (from 1 to 35) and vertical coordinate is the peak to valley (PV)values (\( \lambda = 632.8nm \)) which is used to represent the amplitude of every sort of phase aberrations. From figure 3 we can known that defocus is the main phase aberration in the Nd:YAG laser (excluding tip-tilt). Astigmatism, spherical and lower order coma also occupy great percentage in the phase aberration in the Nd:YAG laser [8]. So as to obtain good beam quality, these aberrations should be corrected mostly.
3.2. Parameters selection and simulation outcomes

The Nd:YAG laser wave-front with phase aberration is focalized on the focal plane through Fourier transform. The size of a pinhole which is set in the center of the focal plane can be altered by software way. The total light intensity that passes through the pinhole is used as the fitness function. It is obvious that, the more phase aberrations corrected the larger the light intensity can be obtained and the better the beam quality is. According to the principle of GA, the research space will become very slow for GA to find the optimum mirror shape when the number of initial population is large, whereas it is more likely to find the global optimum solution in such case. Considering both advantage and disadvantage, the number is set to 30. The crossover probability $P_c$ and the mutation rate $P_m$ of the real number encoding GA are set at 0.8 and 0.05 respectively. To eliminate the occasionally error, the iterative generation of program is set at 100, 200, 400 and 800 respectively, and program runs three times for each case. The results shown that the largest fitness function value was obtained at the 615th generation, and then the 19 voltages on the channels of the DM according to the 615th generation were saved. The mirror shape of the DM under this set of voltages is the optimum shape to correct the laser phase aberrations.

The four pictures (A B C D) in figure 4 (I) are the comparisons of the near-field wave-front distribution before and after correction. A, B are the 2 dimension and 3 dimension figures of the near-field wave-front distribution before correction while C, D are the figures according to the correction case. The PV value and root mean square (RMS) value of the phase aberration in near-field wave-front are brought down from $4.01\lambda$ ($\lambda=632.8\text{nm}$) and $0.27\lambda$ to $1.80\lambda$ and $0.08\lambda$ respectively. The four pictures (a b c d) in figure 4 (II) are the comparisons of the far-field light intensity distribution before and after correction. Figure a and figure b are the 2 dimension and 3 dimension far-field wave-front distribution before correction while c, d are the figures according to the correction case. The transverse coordinates of picture a, picture b, picture c, picture d are the size of simulation focal plane, and the vertical coordinates of picture b, picture d represent the normalized light intensity. It can be seen from picture c and picture d that the far-field light intensity centered on the center of the light spot, and the normalized peak light intensity is improved from 0.017 to 0.35 after correction.
Figure 4. (I) The comparisons of the near-field distribution before and after correction; (II) The comparisons of the far-field light intensity distribution before and after correction.

Figure 5. The far-field encircled energy strehl ratio curves.
Figure 5 are the far-field Encircled energy strehl ratio curves in the case of ideal plane wave-front, Nd:YAG laser wave-front with phase aberration and Nd:YAG laser wave-front with phase aberration corrected. Encircled energy strehl ratio is the ratio for the energy of an actual far-field light on focus to the energy of an ideal wave-front far-field light on focus in the range of Airy disc ($r_0$). In Figure 5, the size of encircled window is 20 times the size of $r_0$, and the Encircled energy strehl ratio is raised from 0.02 to 0.34 after correction. Figure 6 is the distribution of 35 orders Zernike polynomial coefficients before and after phase aberration is corrected. Where the black solid bars stand for the coefficients distribution before correction and the blank bars represent the coefficients distribution after correction. Figure 6 shows that the main aberrations: defocus(A3), astigmatism(A5), spherical (A10) and coma (A6,A7,A8) are reduced greatly after correction.

![Figure 6. The comparison of Zernike polynomial coefficients distribution.](image)

4. Conclusion
This paper tried to use GA in the AO system on the base of introducing the basic theory of GA. A real number encoding GA which was used to control a 19-channel DM to correct the phase aberration in a laser wave-front was simulated. The results shown that the optimum mirror shape of the DM for correcting phase aberration in a laser wave-front can be found with this GA without measuring the phase aberration specially; the far-field peak light intensity and the focusing ability of laser wave-front was enhanced a lot after correction with GA, that also means the light beam quality is improved greatly.

References
[1] Jiang Wenhan, Tang Guomao and Li Mingquan 1995 Adaptive optical image compensation on stellar Objects Opt.Engng. 34 15-20
[2] Jiang Wenhan and Li Huagui 1990 Hartmann-Shack wave-front sensing and wave-front control algorithm Proc. SPIE. 1237 64-67
[3] Rao Changhui, Zhang Xuejun and Jiang Wenhan 2002 Image Deconvolution from Hartmann-ShackWave-front Sensing Indoors Experimental Results Acta Optica Sinica 22 789-793
[4] Goldberg D E 1989 Genetic algorithms in search, optimization and machine learning[M]. Reading M A. (USA: Addison-Wesley Publishing Company. Inc)
[5] Oshichi Nemoto, Takuya Nayuki and Takashi Fujii 1997 Optimum control of the laser beam
intensity profile with a deformable mirror *Appl. Opt.* **36** 7689-95

[6] Li Xinyang and JiangWenhan 2002 Zernike Modal Wave-front Rescontruction Error of Hartmann-Shack Wave-front Sensor *Acta Optica Sinica.* **22** 1236-40

[7] Wang J W and Silva D E 1980 Wave-front interpretation with Zernike polynomials *Appl. Opt.* **19** 1510-18

[8] Ping Yang, Shijie Hu and Xiaodong Yang 2005 Test and analysis of the time and space characteristics of phase aberration in a diode-side-pumped Nd:YAG laser *Proc. SPIE.* **6108** 182-191