Separation mechanisms of reversible adhesive joining using an elastic body with variable elastic modulus

Y Sekiguchi$^1$, P Hemthavy$^1$, S Saito$^2$ and K Takahashi$^1$

$^1$ Department of International Development Engineering, Tokyo Institute of Technology, 14-11, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan
$^2$ Department of Mechanical and Aerospace Engineering, Tokyo Institute of Technology, 11-47, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan
E-mail: sekiguchi.y.aa@m.titech.ac.jp

Abstract. Reversible adhesive joining that involves altering the elastic modulus of an elastic body is proposed by considering adhesion between a semi-infinite elastic body and a rigid body that has a slightly wavy surface with defects. Separation stress of adhered surfaces is theoretically expressed as a function of the elastic modulus. However, the expression of the separation stress varies depending on the condition of the maximum half contact width of the adhesion area and a parameter that depends on the elastic modulus, the work of adhesion, and the surface roughness. Therefore, separation mechanisms are categorized into six types by considering the maximum half contact width and the above parameter, and theoretically discussed. By utilizing an elastic body which has variable elastic modulus, the separation stress can be controlled and the elastic body can be repeatedly adhered to and separated from an object.

1. Introduction
Since adhesion is thermodynamically reversible, an elastic body can be repeatedly adhered to a rigid body. Adhesion of elastic bodies has the potential to be applied to thermal reversible joining in manufacturing and fabrication processes, for example. It can contribute to the simplification and reduction of industrial processes, making them more environmentally friendly. Also, the reuse of elastic bodies leads to a reduction in the amount of adhesive material used. So creation of a reversible adhesive system is important for environmental problems. However, for practical applications of adhesive joining, the control of the separation stress is necessary.

The separation stress of adhered surfaces depends on the elastic modulus [1], and the change in the separation stress has been experimentally measured for a shape-memory gel while varying the elastic modulus of the gel by heating and cooling it [2]. Adhesion can be utilized for realizing reversible joining. The adhesion of two elastic bodies with slightly wavy surfaces has been investigated theoretically [3] by extending the solution for a contact without adhesion [4]. The separation stress is expressed as a function of the elastic modulus [1].

It has been theoretically indicated that no separation occurs at the interface after two bodies make complete contact [1]. However, in practical situations, complete contact does not occur and adhered surfaces can be separated because of defects on the surface of the bodies, such as dents or cracks. It has been suggested that a discontinuous detachment transition occurs from complete contact to partial contact due to nucleation of a crack at the bottom of a valley [5].
Also, no adhesion area has been introduced as a defect at the bottom of a valley in an adhesion contact model [2]. Within this model, several separation mechanisms exist and the separation stress for each mechanism has a different expression. The condition of the separation mechanism is important for the control of the separation stress and it has been discussed theoretically by considering the relation between the external pressure and the contact width.

2. Adhesion model

2.1. Adhesion model with a sinusoidal surface roughness

Adhesion contact between a semi-infinite elastic body and a rigid body with a sinusoidal surface roughness is considered (Figure 1). The external pressure is defined as \( p \), the half contact width as \( a \), the amplitude of the surface roughness as \( h_0 \), and the wavelength of the roughness as \( \lambda \), in which \( h_0 \ll \lambda \). Young’s moduli of the elastic body and the rigid body are given as \( E_1 \) and \( E_2 \) and Poisson’s ratios of the bodies as \( \nu_1 \) and \( \nu_2 \), respectively. The elastic modulus \( E^* \) can be written as \( 1/E^* = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2 \). Because \( E_1 \ll E_2 \), the elastic modulus can be given as \( 1/E^* = (1 - \nu_2^2)/E_1 \). The interface is assumed to be frictionless. The work of adhesion, \( \Delta \gamma \), is the work required to separate a unit area of the adhered surfaces. Most real materials show inelastic behavior, such as viscoelasticity. So during a loading and unloading cycle, the effective work of adhesion decreases during loading and increases during unloading, which is known as adhesion hysteresis [6, 7]. But in this paper, a perfect elastic body is considered and adhesion hysteresis is not expected to occur. The relation between the external pressure and the contact width has been obtained [1, 3] as

\[
\frac{\tilde{p}}{h_0} = \sin^2 \left( \frac{\pi a}{\lambda} \right) - \sqrt{\frac{2\Delta \tilde{\gamma}}{\pi h_0} \tan \left( \frac{\pi a}{\lambda} \right)}
\]

(1)

where the parameters are normalized as \( \tilde{p} = p/\pi E^* \), \( \tilde{h}_0 = h_0/\lambda \), \( \Delta \tilde{\gamma} = \Delta \gamma/(\lambda E^*) \), and \( \tilde{a} = a/\lambda \), and \( \Psi \) is given as

\[
\Psi = \frac{2\Delta \tilde{\gamma}}{(\pi \tilde{h}_0)^2} = \frac{2\lambda \Delta \gamma}{\pi^2 h_0^2 E^*}
\]

(2)

This relationship is plotted in Figure 2(a) for several values of \( \Psi \). For \( 3^{1.5}5^{2.5}/2^8 < \Psi \), there is no maximum or minimum point on the curve of Eq. (1). For \( 3^{1.5}/2^4 < \Psi < 3^{1.5}5^{2.5}/2^8 \), \( \tilde{p}/\tilde{h}_0 \) at maximum and minimum points are both tensile stresses, i.e. negative pressures. For \( 0 < \Psi < 3^{1.5}/2^4 \), \( \tilde{p}/\tilde{h}_0 \) at the minimum point is a tensile stress, whereas that at the maximum point is a pressure. The values of \( \Delta \gamma \) for rubber and metals are about 0.01 and 1.0 J/m², respectively. The elastic modulus of the rubber is about 10⁵ to 10⁷ Pa. When considering representative wavelength and amplitude of surface roughness for \( \lambda \) as 10 mm and \( h_0 \) as 0.1 to 1 mm, the value of \( \Psi \) is obtained as \( \Psi \approx 2 \times 10^{-6} \) to 2.

In Figure 2(b), the relation between the normalized external pressure and the normalized half contact width for \( \Psi = 0.2 \) is plotted. It has a minimum pressure (i.e. maximum tensile stress) at point \( A_0 \), a zero-crossing between the minimum and maximum pressures at \( A_1 \), and a maximum pressure at \( A_2 \). The pressure at \( A_3 \) is the same as that at \( A_0 \). The half contact widths at \( A_0 \), \( A_1 \), \( A_2 \), and \( A_3 \) are denoted by \( a_0 \), \( a_1 \), \( a_2 \), and \( a_3 \), respectively. When the elastic body starts contacting the rigid body at zero pressure, the contact width increases until equilibrium and it reaches \( A_1 \). When the elastic body partially contacts the rigid body (Figures 1(b) and (c)), the pressure changes following the equilibrium curve between \( A_0 \) and \( A_2 \).
Figure 1. Adhesion contact model with a sinusoidal surface roughness on the surface of a rigid body: (a) an elastic body approaches the rigid body; (b), (c) partial contact at equilibrium with contact width 2a; and (d) perfect contact.

Figure 2. Relation between the normalized half contact width and the normalized external pressure (a) for several Ψ and (b) when Ψ = 0.2.

A tensile stress is required to reduce the half contact width from a₁; it moves from A₁ to A₀ following the curve. The point A₀ is unstable. So when the half contact width reaches a₀, the elastic body separates from the rigid body. The tensile stress at A₀, i.e. the separation stress (denoted as $\sigma_{\text{Sep.}}$), can be well approximated [1] by

$$\tilde{\sigma}_{\text{Sep.}} = \frac{3}{4} \left( \frac{\tilde{E}^*}{\pi \hbar_0} \right)^{1/3},$$

where the separation stress and the elastic modulus are normalized as

$$\tilde{\sigma}_{\text{Sep.}} = \sigma_{\text{Sep.}}/(\Delta \gamma/\lambda)$$

and

$$\tilde{E}^* = E^*/(\Delta \gamma/\lambda),$$

respectively.

A pressure is required to increase the half contact width from a₁; it moves from A₁ to A₂ following the curve. The point A₂ is also unstable. When the half contact width reaches a₂, the elastic body completely contacts the rigid body; i.e., the normalized half contact width becomes $a/\lambda = 0.5$. After the surfaces make complete contact, the normalized half contact width remains $a/\lambda = 0.5$ even if the pressure is removed and the tensile stress is given.
2.2. Adhesion model with defects and surface roughness

It has been theoretically predicted that no separation occurs at the interface when there are no defects on the surface and the surfaces are in complete contact [1]. However, body surfaces have defects, such as dents or cracks, in addition to surface roughness. These defects hinder bodies from completely adhering (Figure 3(a)) so that separation is present even for complete contact. Adhesion does not occur at defects. Therefore, no adhesion area forms on the surface of the rigid body, as shown in Figure 3(b). The maximum half contact width of the adhesion area is defined as \( a^* \), as shown in Figure 3(b), i.e. the size of the defect is given as \( \lambda - 2a^* \). The elastic body approaches the rigid body and makes contact. The half contact width then increases from \( a_1 \) to \( a^* \) during the loading process. After reaching \( a^* \), a tensile stress is generated until separation occurs during unloading. As the tensile stress increases, the contact width decreases, following the curve given by Eq. (1) until separation occurs under certain conditions (Figures 4(a)–(d)). This is one of the separation mechanisms. However, for the other conditions, the contact width does not change until separation (Figures 4(e)–(h)). This is another separation mechanism.

The contact width is stable when total energy is a minimum. The total energy per unit depth \( (U_{\text{total}}) \) can be expressed as a sum of the elastic energy, the surface energy, and the mechanical potential energy, and it is normalized as \( U_{\text{total}}/h_0^2 \) [2]. The relation between the normalized total energy and the normalized maximum half contact width for \( \Psi = 0.2 \) and \( a^*/\lambda = 0.46 \) is plotted in Figure 5(a). The unloading process starts from the point \( U_1 \) and it moves to the point \( U_2 \) with decreasing pressure. Between \( U_1 \) and \( U_2 \), the gradient of the energy curve is negative. Thus, the contact width does not decrease. However, the gradient at \( U_2 \) is zero. This point is unstable, and the contact width starts decreasing after \( U_2 \). Because the curve has no minimum on the left-hand side of \( U_2 \), separation occurs. Hence, the contact width is constant until separation occurs at \( U_2 \). This is referred to as separation with constant contact width. The separation stress at this condition is the tensile stress at \( U_2 \), and it can be obtained [2] as

\[
\sigma_{\text{Sep.}} = \sqrt{2} \tan(\pi a^*) \times \dot{E}^{1/2} - \pi h_0 \sin^2(\pi a^*) \times \dot{E}^*,
\]

where \( \tilde{a}^* = a^*/\lambda \).

The relation between the normalized total energy and the normalized maximum half contact width for \( \Psi = 0.2 \) and \( a^*/\lambda = 0.42 \) is also plotted in Figure 5(b). \( a^*/\lambda \) in Figure 5(b) is smaller than that in Figure 5(a). Unloading starts from \( U_1 \) and the contact width is constant at \( a = a^* \) until reaching \( U_2 \). The gradient of the curve at \( U_2 \) is zero. So \( U_2 \) is unstable and the contact width starts decreasing. However, the curve has a minimum point \( U_3 \) on the left-hand side of \( U_2 \). The contact width reaches equilibrium at the point \( U_3 \), but separation does not occur yet. The contact width subsequently decreases following the minimum points on the energy curve, i.e. the solid curve \( A_0 U_3 \) in Figure 5(b), until separation occurs at \( A_0 \). This is referred to as separation with decreasing contact width. The separation stress for this condition is given by Eq. (3).
Figure 4. Adhesion contact model with a defect and a sinusoidal surface roughness on the surface of a rigid body: (a)–(d) separation with decreasing width and (e)–(h) separation with constant width.

Figure 5. Unloading processes with the relation between the normalized total energy and the normalized half contact width when (a) $\Psi = 0.2, a^*/\lambda = 0.46$ and (b) $\Psi = 0.2, a^*/\lambda = 0.42$.

3. Conditions of separation mechanisms
3.1. Categorization of unloading process
The separation stress is expressed as a function of the elastic modulus by Eq. (3) or Eq. (4). Therefore, it is possible to control the separation stress by changing the elastic modulus by, for example, using a shape-memory gel with a Young’s modulus in the range $2.2 \times 10^5$ Pa (above $49^\circ$C) to $1.7 \times 10^7$ Pa ($30^\circ$C) [8]. However, two different expressions for the separation stress are obtained by considering the change in the contact width during unloading. It is thus necessary to clarify the conditions for each case.

The unloading process can be categorized into several types by considering $\Psi$ and $a^*/\lambda$, as shown in Figure 6. First, the condition of $\Psi$ is considered. $\Psi$ is expressed as a function of $E^*$ (Eq. (2)). Shape-memory gel has an elastic modulus that varies with temperature. By utilizing this kind of material, $\Psi$ can be changed during processes. The stress curve changes with different $\Psi$, as shown in Figure 2(a). It can be divided into three ranges, $0 < \Psi < 3^{1.5}/2^4$, $3^{1.5}/2^4 < \Psi < 3^{1.5}5^{2.5}/2^8$, and $3^{1.5}5^{2.5}/2^8 < \Psi$, by considering the maximum and minimum points of Eq. (1).

Next, the condition of $a^*/\lambda$ is considered for each range of $\Psi$. When $0 < \Psi < 3^{1.5}/2^4$,
the maximum pressure, i.e. the pressure at A₂, is positive and the minimum pressure, i.e. the pressure at A₀, is negative. For this condition, the maximum half contact width can be divided into five range as 0 < a⁺/λ < a₀/λ, a₀/λ < a⁺/λ < a₁/λ, a₁/λ < a⁺/λ < a₂/λ, a₂/λ < a⁺/λ < a₃/λ, and a₃/λ < a⁺/λ < 0.5. These divisions are categorized as types A, B, C, D, and E, respectively, as shown in Figures 6(a)–(e). When 3₁⁻⁵/²⁴ < Ψ < 3¹⁻⁷₅²⁻⁵/²⁸, the pressure at A₂ is negative and thus there is no zero-crossing, A₁. For this condition, type B has a maximum half contact width range of a₀/λ < a⁺/λ < a₂/λ and type C does not exist. Types A, D, and E are the same in the range 0 < Ψ < 3¹⁻⁵/²¹. When 3¹⁻⁷₅²⁻⁵/²⁸ < Ψ, there is no maximum or minimum pressure in the curve given by Eq. (1). This condition is categorized as type F (Figure 6(f)). Thus, there are six conditions, as shown in Table 1.

3.2. Unloading process for each type
During loading, the contact width increases until it reaches a/λ = a⁺/λ, i.e. the maximum half contact width, at the point U₁. This is the initial point of unloading. For types A, E, and F, the pressure varies horizontally from U₁ to U₂, as shown in Figures 6(a), (e), and (f). U₂ is the intersection between a horizontal line and the curve of Eq. (1). Between U₁ and U₂, the contact width remains constant. Separation occurs when the point U₂ is reached. These types are referred to as separation with constant contact width. The separation stress for types A, E, and F is expressed by Eq. (4). Point U₂ for type B and point U₁ for type C are on the equilibrium curve of A₀A₂, as shown in Figures 6(b) and (c). The pressure varies by following the curve after these points and separation occurs at point A₀. For type D, point U₂ is unstable and the contact width starts decreasing from U₂ until equilibrium and reaches point U₃, as shown in Figure 6(d). The pressure then follows the curve A₀A₂ until separation at A₀. When the pressure decreases by following the curve of Eq. (1), the contact width also decreases. Thus, types B, C, and D are classified as separation with decreasing contact width and the separation stress for these types is expressed by Eq. (5).

3.3. Condition of Ψ and a⁺/λ for each type
The relation between Ψ and a⁺/λ for the different types of separation is plotted in Figure 7. The upper side of the solid curve shows separation with constant contact width and the lower side shows separation with decreasing contact width. In addition, the dashed curves show the conditions for six types (A–F).

The solid curve between types A and B in Figure 7 shows the condition a⁺/λ = a₀/λ for Ψ < 3¹⁻⁷₅²⁻⁵/²⁸. The dashed curves between B and D and between C and D show the condition a⁺/λ = a₂/λ for Ψ < 3¹⁻⁷₅²⁻⁵/²⁸. These two conditions are given by

\[
\Psi = \frac{2}{E^* \left( \frac{\pi h_0}{\lambda} \right)^2} = 16 \sin^3(\pi a^* / \lambda) \cos^5(\pi a^* / \lambda).
\] (5)

The dashed curve between B and C shows the condition a⁺/λ = a₁/λ for Ψ < 3¹⁻⁷₅²⁻⁴. The solid curve between D and E shows the condition a⁺/λ = a₃/λ for Ψ < 3¹⁻⁷₅²⁻⁵/²⁸. These two conditions are numerically obtained. The border line of F shows the condition Ψ = 3¹⁻⁷₅²⁻⁵/²⁸.

4. Control of separation stress
4.1. Relation between separation stress and elastic modulus
To increase the elastic modulus, the separation stress is varied, as shown in Figure 8. The circles, ●, in Figure 8 represent the points where the expression for the separation stress changes. The
right- and left-hand sides of the circles show the separation stresses given by Eqs. (3) and (4), respectively.

The separation stress is in good agreement with the Griffith theory of brittle fracture when the amplitude and the elastic modulus are sufficiently small, where the separation stress of the Griffith theory can be expressed as
Table 1. Conditions of $\Psi$ and $a^*/\lambda$ for each type.

| Type | $\Psi$ | $a^*/\lambda$ | Separation stress |
|------|--------|---------------|-------------------|
| A    | $0 < \Psi < 3^{1.5/2^5}/2^8$ | $0 < a^*/\lambda < a_0/\lambda$ | Eq. (4) |
| B    | $0 < \Psi < 3^{1.5/2^4}$ | $a_0/\lambda < a^*/\lambda < a_1/\lambda$ | \(3^{1.5/2^4} < \Psi < 3^{1.5/2^5}/2^8\) | $a_0/\lambda < a^*/\lambda < a_2/\lambda$ | Eq. (3) |
| C    | $0 < \Psi < 3^{1.5/2^4}$ | $a_1/\lambda < a^*/\lambda < a_2/\lambda$ |
| D    | $0 < \Psi < 3^{1.5^{2.5}/2^8}$ | $a_2/\lambda < a^*/\lambda < a_3/\lambda$ |
| E    | $0 < \Psi < 3^{1.5^{2.5}/2^8}$ | $a_3/\lambda < a^*/\lambda < 0.5$ | Eq. (4) |
| F    | $3^{1.5^{2.5}/2^8} < \Psi$ | $0 < a^*/\lambda < 0.5$ |

\[
\sigma_{\text{Griffith}} = \sqrt{\frac{2E^*\Delta \gamma}{\pi(0.5 - \bar{a}^*)\lambda}}
\]  
and it is plotted as dashed-dotted line in Figure 8 for the case $a^*/\lambda = 0.46$.

$a^*/\lambda$ is constant during unloading. Thus, when the elastic modulus changes during the processes, only $\Psi$ changes and the condition moves vertically in Figure 7. The expression for the separation stress changes when the condition crosses a solid curve in Figure 7. Thus, the circles in Figure 8 are related to the solid curve in Figure 7.

4.2. Reversible adhesive joining process

A small elastic modulus is preferable for creating adhesion contact when the elastic body starts contacting the rigid body. Heating is required when using the shape-memory gel as the elastic body. After the bodies have adhered, the elastic modulus is increased to give strong adhesion, i.e. to increase separation stress. Thus, cooling is required for a shape-memory gel. However, when $a^*/\lambda$ is close to 0.5, for example when $a^*/\lambda = 0.42$ or 0.46 as plotted in Figure 8, the separation stress decreases just before the transition point between Eqs. (4) and (3), i.e. the circles. Thus, the bodies can be separated by increasing the elastic modulus under these conditions, e.g. cooling the shape-memory gel. On the other hand, the separation stress does not decrease when $a^*/\lambda$ is small, such as $a^*/\lambda = 0.15$. For this condition, it is necessary to reduce the elastic modulus for separation, e.g. heating the shape-memory gel. It is important to carefully discuss the relation between the separation stress, $\Psi$, and $a^*/\lambda$, especially near the transition points between Eqs. (4) and (3) using Figures 7 and 8, in terms of controlling the separation stress.

5. Conclusions

Adhesion contact between a semi-infinite elastic body and a rigid body is considered to determine the separation stress. The surface of the rigid body has a sinusoidal surface waviness and defects. By considering the total energy of the system and the relation between the external pressure and the contact width, the separation stress is obtained as a function of the elastic modulus. It is possible to separate adhered surfaces by changing the elastic modulus of the elastic body,
Figure 7. Condition of unloading processes in terms of the relation between $\Psi$ and $a^*/\lambda$. The upper and lower sides of the solid curve respectively represent separation with constant contact width and separation with decreasing contact width.

Figure 8. Normalized separation stress as a function of the normalized elastic modulus for $a^*/\lambda = 0.42$ and 0.46 with $h_0/\lambda = 0.010$ and $a^*/\lambda = 0.15$ and 0.46 with $h_0/\lambda = 0.100$. The left and right side of the circles, ●, represent separation with constant contact width and separation with decreasing contact width, respectively. The Griffith theory of brittle fracture for $a^*/\lambda = 0.46$ is also plotted.

e.g. shape-memory gel, during the processes. However, the expression for the separation stress changes with the maximum half contact width and a parameter that depends on the elastic modulus, the work of adhesion, and surface roughness parameters. The unloading process is categorized into six types by considering the change in the contact width during unloading. The conditions for each type are obtained by considering the relation between the maximum half contact width and the parameter. Reversible adhesive joining process is proposed and it is discussed by considering the curve of stress against elastic modulus.

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