A probabilistic key agreement scheme for sensor networks without key predistribution

V. Liagkou\textsuperscript{1,3}, E. Makri\textsuperscript{4}, P. Spirakis\textsuperscript{1-3}, and Y.C. Stamatiou\textsuperscript{2,3}

\textsuperscript{1} University of Patras, Dept. of computer Engineering, 26500, Rio, Patras, Greece
\textsuperscript{2} Mathematics Department, 451 10, Ioannina, Greece. e-mail: istamat@uoi.gr
\textsuperscript{3} Research and Academic Computer Technology Institute, N. Kazantzaki, University of Patras, 26500, Rio, Patras, Greece
\textsuperscript{4} University of the Aegean, Department of Mathematics, 83000, Karlovassi, Samos, Greece.

\textbf{Abstract.} The dynamic establishment of shared information (e.g. secret key) between two entities is particularly important in networks with no pre-determined structure such as wireless sensor networks (and in general wireless mobile ad-hoc networks). In such networks, nodes establish and terminate communication sessions dynamically with other nodes which may have never been encountered before, in order to somehow exchange information which will enable them to subsequently communicate in a secure manner. In this paper we give and theoretically analyze a series of protocols that enables two entities that have never encountered each other before to establish a shared piece of information for use as a key in setting up a secure communication session with the aid of a shared key encryption algorithm. These protocols do not require previous pre-distribution of candidate keys or some other piece of information of specialized form except a small seed value, from which the two entities can produce arbitrarily long strings with many similarities.

1 Introduction

Wireless Sensor Networks (WSNs) have some constraints, with regard to battery life, processing, memory and communication ([3]) capacity, and as such are deemed unsuitable for public crypto-based systems. Thus, symmetric key cryptosystems are more appropriate for these types of networks, but lead to problems with key distribution. These problems are mitigated with key \textit{pre-distribution schemes}, in which candidate keys are distributed to members of the network before the start communication.

Many innovative and intuitive key pre-distribution schemes for WSNs have been proposed for solving the problem of key distribution in sensor networks. On the two ends of the spectrum are key pre-distribution schemes that use a single master key as the encryption key distributed amongst all the nodes, and all pairwise keys, where a unique key exists for every pair of sensors. The former provides the most efficient usage of memory and scales well, but an attack on one node compromises the

\* Partially supported by the IST Programme of the European Union under contact number IST-2005-15964 (AEOLUS) and by the ICT Programme of the European Union under contract number ICT-2008-215270 (FRONTS).
whole network, whereas the latter provides excellent resilience but does not scale well. In addition, schemes exist which are in essence probabilistic, relying on the fact that any two neighbouring nodes have some probability $p$ of successfully completing key establishment. Some such schemes are presented in the sequel, but the list is by no means exhaustive. The authors of [10] propose a key distribution scheme which consists of three phases. In the first phase, namely the key pre-distribution phase, $k$ random keys are drawn from a generated pool of $P$ keys, and are preloaded onto the sensor nodes before their deployment. These $k$ keys constitute the key ring of each sensor node. The key identifiers of the key rings are loaded onto controller nodes, along with the identifiers of the corresponding nodes and the keys shared between the controller nodes and the sensor nodes. Once the nodes are deployed, the second phase takes place, the shared-key discovery phase, which allows nodes to discover their neighbours within communication range and with which they share keys. This phase establishes the topology of the sensor network, as seen by the routing layer. The third, and final phase, is the path-key establishment phase which assigns a path-key to pairs of sensor nodes in wireless communication range that do not share a common key. The authors of [4] present an alternative to this scheme, the $q$-composite random key predistribution scheme, whereby instead of one single common key to be shared between two nodes, $q$ keys are required, resulting in strengthened security under small scale attacks, trading off increased vulnerability on large scale attacks. The authors also propose the establishment of the key-path along multiple paths, and a random-pairwise keys scheme which provides, amongst others, node-to-node authentication. The authors of [5] also take the scheme of [10] and modify it by taking into consideration that different locations of nodes require different security needs, and as such propose a subgrouping approach in order to isolate any node captures that might take place in specific subgroups. The scheme also provides scalability for key pre-distribution, by taking into account the probability of node compromise for each of the subgroups. Chan and Perrig ([6]) address the lack of scalability of existing symmetric-key key distribution protocols, and increase the security against node compromise. In their proposed PIKE protocol, sensor nodes are used as trusted intermediaries to establish shared keys between nodes in the wireless sensor network. The authors of [8] propose a pair-wise key pre-distribution scheme, which is based on the scheme proposed by [1]. Their work involves the the pre-loading of crypto shares from multiple key spaces onto each sensor node, and after deployment, two nodes may establish a shared key between them only if they contain crypto shares from the same key space, leading to an improvement in scalability. Finally, the same authors ([9]) propose a scheme which utilises deployment knowledge, and generates pairwise keys between nodes and their neighbours, guaranteeing that each node can establish a secure communications link with its
neighbours post deployment, with a high probability. The previous solutions all pre-suppose that the sensor nodes have been loaded with some pre-existing information (i.e. the key, or sets of keys) prior to network deployment, except for Liu and Cheng ([11]). They propose a self-configured scheme whereby no prior knowledge is loaded onto the sensor nodes, but shared keys are computed amongst the neighbours. The authors propose \( SBK \) which is also a topology adaptive scheme, achieves high connectivity with a small storage overhead, and \( iSBK \), and improved version of \( SBK \).

In this paper, we propose a key agreement scheme whereby network nodes are not pre-loaded with candidate keys, but generate pairs of symmetric keys from two, initially, random bits strings. The initial research conducted ([12]) proposed a protocol that involved the examination of random positions of subsets of size \( k \), and the elimination of a random position if the two bit strings were found to disagree on more than half the examined positions. In that paper, however, the nodes cannot secretly compute the number of differing positions, a problem that is resolved in the present paper using secret circuit computations. In addition, the present protocols do not eliminate differing bits but flips them, depending on the number of bit difference in the examined subset of \( k \) bits. This leads to a different stochastic process that called for a different theoretical analysis. The scheme we propose has the following four properties: (i) scalability, since new nodes may enter the network whenever they desire without the need to equip them with some suitable key set (ii) connectivity, since any two nodes of the network can reach a large degree of similarities on the strings the possess (iii) no storage overhead, since the string sizes are independent on the network size and (iv) resilience against node captures, since capturing any number of nodes of the network does not affect the strings other nodes possess. On the negative side, the protocol we give have some increased communication overhead. However, once two nodes reach a position where their two strings have sufficiently much similarity, they can create from them a large number of candidate keys (possibly using error correction techniques to create even more similarities) for their current and future communication needs.

2 The bit-similarity problem

Two entities, say 0 and 1, initially possess an \( N \)-bit string, \( X_N^0 \) and \( X_N^1 \), respectively. The entities’ goal is to cooperatively transform their strings so as to increase the percentage of positions at which their strings contain the same bits, which we denote by \( X(i) \), with \( i \) being the time step of the protocol they execute. Then \( X(0) \) is the initial percentage of the positions at which the two strings are the same.

Below we provide a randomized protocol in which the two entities examine randomly chosen subsets of their strings in order to see whether they differ in at least half of the places. If they do, one of the entities (in turn) randomly flips a subset of these positions. This process continues
up to a certain, predetermined number of steps. The intuition behind
this protocol is that when two random substrings of two strings differ in
at least half of their positions, then flipping some bits at random in one
of the substrings is more likely to increase the percentage of similarities
between the strings than to decrease it. In the description of the protocol
$X_N^c[S]$ denotes a substring of string $X_N^c$ defined by the position set $S$.

Protocol for user $U_c$, $c = 0, 1$
Protocol parameters known to both communicating parties: (i) $k, l$, the subset sizes,
(ii) $T$, the number of protocol execution steps, (iii) the index (bit position) set $N$, (iv)
The circuit $C$ with which the two entities jointly compute whether there are at least
$[k/2]$ similarities between randomly chosen subsets of their strings.

1. $i ← 1$ /* The step counter. */
2. while $i ≤ T$ /* $T$ is a predetermined time step limit (discussed in Section 5). */
3. begin /* while */
4. $S ← \text{JOINT}_{\text{RAND}}(k, \{1, \ldots, N\})$ /* Shared random set of $k$ positions. See
text. */
5. same\text{pos} ← C(X_N^c[S], X_N^{(c+1 \ mod 2)}[S]) /* A secret computation of number of
positions with same contents (see Section 3). */
6. if (same\text{pos} ≥ $\lceil \frac{k}{2} \rceil$ \text{ and } odd$(i + c)$) then /* Users 0 and 1 alternate. */
7. begin
8. $SF ← \text{RAND}(l, S)$ /* Random set of $l$ positions from within $S$ to be flipped
by the user whose turn it is to flip. */
9. flip the bits of $X_N^c[SF]$
10. end
11. SYNCHRONIZE /* Users 0 and 1 wait to reach this point simultaneously
(barrier synchronization). */
12. $i ← i + 1$
13. end /* while */

An important requirement of the protocol is the existence of a random
number generator at each party which produces the same values,
started from a (small) seed value shared by both parties. This is in order to
allow the two parties to avoid sending over the communication channel
the chosen substring positions, as they are generated in synchrony by the
two parties (see Line 4 of the protocol).

With regard to the protocol, we are interested in the evolution of the
random variable $X(i)$, i.e. the percentage of the positions of $X^0_N$ and $X^1_N$
at which they are the same, after the $i$th step of the protocol.

3 Secret two-party function computation

During the execution of the protocol, it is necessary for the two commun-
icating parties to see whether they agree on at least half of the positions
they have chosen to compare (line 13-14 of the protocol). Thus, the two
parties need to perform a computation: compute the number of positions
on which the corresponding bits in the two chosen subsets of $k$ bits are
the same. This is an instance of an important, general problem in cryp-
tography: Secure Computation.

Secure computation techniques allow two (or more) communicating
parties to compute a function on their inputs so that nothing is revealed to
each party except what can be inferred from own input and the computed value. Moreover, no information is revealed about the bit values to a party not involved in the computation (e.g. eavesdropper).

More formally, let $A$ and $B$ be two parties with inputs of $n_A$ and $n_B$ bits respectively. The objective is to jointly compute a function $f : \{0,1\}^{n_A} \times \{0,1\}^{n_B} \rightarrow \{0,1\}$ on their inputs. The issue, here, is that $A$ and $B$ cannot, simply, exchange their inputs and compute the function since they will learn each other’s inputs, something that is not desirable in a secure computation setting. More importantly, even if $A$ and $B$ are willing to share their inputs, they would not allow an eavesdropper to acquire these inputs too. This leads to the problem of secure function computation.

In our context, we consider the following two Boolean functions:

$$f_r : \{0,1\}^k \times \{0,1\}^k \rightarrow \{0,1\} \text{ with } w_A, w_B \in \{0,1\}^k \text{ and } 0 \leq r \leq k :$$

$$f(w_A, w_B) = \begin{cases} 1, & \text{if } X(w_A, w_B) \geq r \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

We are interested in $r = \lceil \frac{k}{2} \rceil$.

$$f_X : \{0,1\}^k \times \{0,1\}^k \rightarrow \{0,1\}^{\lceil \log_2(k) \rceil} \text{ with } w_A, w_B \in \{0,1\}^k :$$

$$f_X(w_A, w_B) = x, \text{ with } x = X(w_A, w_B) \text{ written in binary.} \quad (2)$$

The function $f_X$ is, strictly, an ordered tuple $(f_X^0, f_X^1, \ldots, f_X^{\lceil \log_2(k) \rceil - 1})$ of $\lceil \log_2(k) \rceil$ 1-bit Boolean functions, where the function $f_X^i$ computes the $i$th most significant bit of $x = X(w_A, w_B)$ (with $i = 0$ we take the most significant bit and with $i = \lceil \log_2(k) \rceil - 1$ we take the least significant bit). Using techniques from oblivious function computation (see [7] for a survey on these techniques), we can prove that the computation of $f_r$ and $f_X$ can be done with randomized protocols using $O(|C_{f_r}|)$ and $O(|C_{f_X}|)$ communication steps respectively, with $C_{f_r}$ and $C_{f_X}$ being the Boolean circuits that are employed for the computation of $f$ and $f_X$ respectively. Since both $f_r$ and $f_X$ are easily seen to be polynomial time computable Boolean functions, we can construct for their computations circuits of size polynomial in their input sizes, i.e. circuits $C_{f_r}$ and $C_{f_X}$ such that $|C_{f_r}| = O(k^{c_1})$ and $|C_{f_X}| = O(k^{c_2})$, with constants $c_1, c_2 \geq 0$. Since $k$ is considered a fixed constant, we conclude that we can compute $f_r$ and $f_X$ in a constant number of rounds. The number of random bits needed by each step of the randomized protocol is in both cases $O(k)$ and, thus, constant.

To sum up, the functions $f_r$ and $f_X$ can, both, be evaluated on two $k$-bit inputs $w_A, w_B$ held by two parties $A, B$ using a constant number of rounds and a constant number of uniformly random bits. In what follows, we will assume that the communicating parties use the function $f_{\lceil \frac{k}{2} \rceil}$.

With regard to the required randomness, we assume that each of the two parties has a true randomness source, i.e. a source of uniformly random bits. Such a randomness source can be easily built into modern
devices. This randomness source is necessary in order to implement the randomized oblivious computation protocols for the computation of the function $f_{\frac{k}{2}}$. In addition, it will be used in order to produce the randomly chosen positions, are required by Step 6 of the protocol. Since each position can range from 1 up to $N$ (the string size), to form a position index we need to draw $\lceil \log_2(N) \rceil$ random bits. Alternatively, if we allow the two parties to share a small (in relation to $N$) seed, they can produce the random positions in synchronization and, thus, avoid sending them over the communication channel.

4 Theoretical analysis of the protocol

In order to track the density of positions where two strings agree, we will make use of Wormald’s theorem (see [13]) to model the probabilistic evolution of the protocol described in Section 2 using a deterministic function which stays provably close to the real evolution of the algorithm. The statement of the theorem is in the Appendix for completeness. What the theorem essentially states is that if we are confronted with a number of (possibly) interrelated random variables (associated with some random process) such that they satisfy a Lipschitz condition and their expected fluctuation at each time step is known, then the value of these variables can be approximated using the solution of a system of differential equations. Furthermore, the system of differential equations results directly from the expressions for the expected fluctuation of the random variables describing the random process.

We will first prove a general lemma that gives the probability of increasing the similarity between two strings through flipping, at random, the contents of a certain number of positions.

Lemma 1. Let $w_1, w_2$ be two strings of 0s and 1s of length $k$. Let also $j$, $0 \leq j \leq k$, be the number of places in which the two strings differ. Then if $l$ positions of one string are randomly flipped, the probability that $s$ of them are differing positions is the following:

$$P_{k,j,l,s} = \frac{\binom{j}{s} \binom{k-j}{l-s}}{\binom{l}{s}}.$$  \hspace{1cm} (3)

Proof In Equation (3) the denominator is the number of all subsets of positions of cardinality $l$ of the $k$ string positions while the numerator is equal to the number of partitions of the $l$ chosen positions such that $s$ of them fall into the $j$ differing positions and the remaining $l-s$ fall into the remaining $k-j$ non-differing positions of the two strings. Thus their ratio gives the desired probability. \hfill \Box

The following lemma, which is easy to prove based on general properties of the binomial coefficients, provides a closed form expression for a sum that will appear later in some probability computations.
Lemma 2. The following identity holds:

\[
\sum_{s=0}^{l}(2s - l)\binom{j}{l-s}\binom{k-j}{l} = \left(\frac{2j}{k} - 1\right)l.
\]  

(4)

We will now derive the deterministic differential equation that governs the evolution of the random variable \(X(i)\) manipulated by the protocol in Section 2 using Wormald’s theorem.

Theorem 1. The differential equation that results from the application of Wormald’s theorem on the quantity \(X(i)\) (places of agreement at protocol step \(i\)) as it evolves in the agreement protocol is the following:

\[
\mathbb{E}[X(i+1) - X(i)] = \sum_{j=\lceil \frac{k}{2} \rceil}^{k} \sum_{s=0}^{l} (s - (l - s))P_{k,j,l,s}P_{n,n-X(i),k,j}.
\]

(5)

Proof We will determine the possible values of the difference \(X(i+1) - X(i)\) along with the probability of occurrence for each of them.

The protocol described in Section 2 flips \(l\) positions within the \(k\) examined positions, whenever these \(k\) positions contain \(j \geq \lceil \frac{k}{2} \rceil\) differing positions in the two strings. From the flipped \(l\) positions, \(s\) of them (\(0 \leq s \leq l\)) are disagreement positions, then the two strings will have gained \(s\) agreement positions, losing \(l - s\). The net total is \(s - (l - s)\). The probability that this total occurs, for a specific value of \(s\) and a specific value of \(j\) is equal to \(P_{k,j,l,s}P_{n,n-X(i),k,j}\). Summing up over all possible values of \(s, j\) we obtain (5). □

Corollary 1. The following holds:

\[
\mathbb{E}[X(i+1) - X(i)] = \sum_{j=\lceil \frac{k}{2} \rceil}^{k} l \left(\frac{2j}{k} - 1\right) \frac{(n-X(i))}{n} \frac{X(i)}{k-j}.
\]

(6)

Proof Equation (6) follows from (5) using Equation (3) of Lemma 1 with \(k = n, l = k, s = j, n - j = X(i)\), in conjunction with Lemma 2.

Corollary 2. Using Wormald’s Theorem (Theorem 3), the evolution of the random variable \(X(i)\) whose mean fluctuation is given in (6) can be
approximated by the following differential equation:
\[
\frac{dx(t)}{dt} = \sum_{j=\left\lfloor \frac{k}{2} \right\rfloor}^{k} l \left( \frac{2j}{k} - 1 \right) \binom{k}{j} [1 - x(t)]^j x(t)^{k-j}.
\] (7)

**Proof** By applying the approximation
\[
\binom{N}{k} = \frac{N^k}{k!} \left( 1 + O \left( \frac{1}{N} \right) \right)
\]
of the binomial coefficients which is valid for \( k = O(1) \) on the three binomials which appear on the the right-hand side of (6), we obtain the following:
\[
\frac{(n-X(i)^j)(n)}{\binom{n}{k}} \sim \binom{k}{j} \left( 1 - \frac{X(i)}{n} \right)^j \left( \frac{X(i)}{n} \right)^{k-j}.
\]
Using Wormald’s theorem, we make the correspondence \( x(t) = \frac{X(i)}{n} \) and \( \frac{dx(t)}{dt} = E[X(i+1) - X(i)] \), which results in the required differential equation (7). □

5 Efficiency of the protocol

From Equation (7) we see that the percentage of similar positions, represented by the function \( x(t) \), is a monotone increasing function since its first derivative is always positive. In what follows, we will estimate how fast this percentage increases depending on its initial value \( x(0) \) as well as the parameters \( l \) and \( k \).

**Lemma 3.** The solution \( x(t) \) to the differential equation given in (7) is monotone increasing.

**Proof** From the differential equation, we see that the first derivative of the function \( x(t) \), which is equal to the right-hand side of the differential equation, is strictly positive, since \( 0 < x(t) < 1 \). Thus, the function \( x(t) \) is monotone increasing. □

**Lemma 4.** Let \( x(t_1) \) be the value of the function \( x(t) \) at time instance \( t_1 \) and \( x(t_2) \) be the value at time instance \( t_2 \), \( t_1 < t_2 \). Let, also, \( c(t_1) \) be the absolute value of the point at which the tangent line to the point \( (t_1, x(t_1)) \) of \( x(t) \) cuts the \( t \)-axis and \( c(t_2) \) the corresponding value for \( t_2 \). Let, also,
\[
p(x) = \sum_{j=\left\lfloor \frac{k}{2} \right\rfloor}^{k} l \left( \frac{2j}{k} - 1 \right) \binom{k}{j} (1 - x)^j x^{k-j}.
\] (8)
Then

\[ p(x(t_1)) = \frac{x(t_1)}{c(t_1) + t_1}, \quad p(x(t_2)) = \frac{x(t_2)}{c(t_2) + t_2}. \]  \hspace{1cm} (9)

**Proof** Let \( \epsilon_1 \) and \( \epsilon_2 \) be the two tangent lines to the function \( x(t) \) at the points \( (t_1, x(t_1)) \) and \( (t_2, x(t_2)) \) respectively, as shown in Figure 1. Due to the monotonicity of \( x(t) \), the points at which the two lines intersect with the \( t \)-axis are negative. Let \( c(t_1) \) and \( c(t_2) \) be the absolute values of these two points for lines \( \epsilon_1 \) and \( \epsilon_2 \) respectively. Then from the two right angle triangles that are formed we have \( \tan(\phi_1) = \frac{x(t_1)}{c(t_1) + t_1} \) and \( \tan(\phi_2) = \frac{x(t_2)}{c(t_2) + t_2} \). From the definition of the derivative, \( \tan(\phi_1) = \frac{dx(t)}{dt} \bigg|_{t_1} \) and \( \tan(\phi_2) = \frac{dx(t)}{dt} \bigg|_{t_2} \). From (7) and (8), we have \( \frac{dx(t)}{dt} = p(x(t)) \) and, thus, the statement of the lemma follows. \( \square \)

**Theorem 2.** Let \( t' \) be the time instance at which \( x(t') = hx(0) \), with \( 1 \leq h \leq \frac{1}{x(0)} \). Then

\[ t_2 \leq \frac{x(0)}{p(hx(0))} \cdot (h - 1). \]  \hspace{1cm} (10)

**Proof** We set \( t_1 = 0, \, t_2 = t' \) and \( x(t') = hx(0) \) in Lemma 4 and we obtain, from Equations (9) the following:

\[ p(x(0)) = \frac{x(0)}{c(0)}, \quad p(hx(0)) = \frac{hx(0)}{c(t') + t'}. \]

From these equations we obtain the following:

\[ \frac{p(x(0))}{p(hx(0))} = \frac{c(t') + t'}{hc(0)}. \]
Solving for $t'$ we obtain the following:

\[ t' = \frac{hp(x(0))c(0)}{p(hx(0))} - c(t'). \]  

(11)

Since $p(x(0))$ is the inclination of the tangent line to $x(t)$ at the point $(0, x(0))$, it holds that $p(x(0)) = \frac{x(0)}{c(0)}$. Thus, (11) becomes

\[ t' = \frac{hx(0)}{p(hx(0))} - c(t'). \]  

(12)

Since $x(t)$ is monotone increasing, the point at which the tangent to this function cuts the $x(t)$-axis at any point is greater than or equal to $x(0)$ (see Figure 1). Let $x_c$ be this point. Then

\[ p(x_c) = x_c, \]

since $x_c \geq x(0)$, and

\[ c(t') \geq x(0), \]

which is the required. \[ \square \]

Lemma 5. The following lower bounds hold for the polynomial in (8):

If $x < 1/2$ then

\[ p(x) \geq l[x^k + 1 - 2x]. \]  

(13)

If $x \geq 1/2$ then

\[ p(x) \geq l \left( 1 - x \right)^k \left( \frac{k}{k} \right) \left( \frac{k}{2} \right). \]  

(14)

Proof In (8), allowing the sum index to cover all the range from 1 to $k$ reduces the value of the sum since it adds negative terms. Thus

\[ p(x) \geq l \sum_{j=1}^{k} \left( \frac{2j}{k} - 1 \right) \binom{k}{j} (1 - x)^j x^{k-j}. \]

Since the sum evaluates to $l[x^k + 1 - 2x]$ the first statement of the lemma follows.

If, on the other hand, $x \geq 1/2$, then lower bound given for the first statement of the lemma is not good since it may even become negative. In this case we observe that the term $(1 - x)^j x^{k-j}$ is minimized for $j = k$. Setting $j = k$ in (8) we obtain the second statement of the lemma. \[ \square \]

Corollary 3. The following bounds hold for the time instance $t'$:

If $hx(0) < 1/2$ then

\[ t' \leq \frac{x(0)(h-1)}{[hx(0)]^{k+1} + 1 - 2hx(0)}. \]  

(15)
If $hx(0) \geq 1/2$ then

$$t' \leq \frac{kx(0)(h - 1)}{l[1 - hx(0)]^k [\left(\frac{k}{l}\right) \left\lceil \frac{k}{2} \right\rceil]}$$

(16)

From (15) we see that the percentage of similarities grows fast, if we start from $x(0)$ aiming at $hx(0)$, with $h \geq 1$ and $hx(0) < 1/2$ (and (15) is only a very coarse upper bound). If the target is, however, at $hx(0)$ with $x \geq 1/2$ the upper bound is not good as the denominator tends to 0 fast. However, since this denominator is simply the first derivative of $x(t)$ at $hx(0)$, this derivative fast tends to 0 if $hx(0) \geq 1/2$ and, thus, the tangent at this point tends to become parallel to the $t$-axis. Thus, we have again fast convergence.

6 Conclusions

In this paper we described a series of protocols that can be used in order to increase the percentage of similarities between two strings held by two communicating parties without revealing their values. The protocols involve the examination of random position subsets of size $k$ and the flipping of a randomly chosen subset of these positions if the two strings are found to disagree in more than half the positions. In this way, the random process governing the protocol is directed towards flipping disagreement positions more than the flipping of agreement positions. The propose protocols are, in fact, general and may be used in any situation involving either wireless or conventional networks in which there is no trusted third party or key management authority among the network nodes.

References

1. R. Blom, “An optimal class of symmetric key generation systems,” Proceedings of EUROCRYPT 84 (Thomas Beth, Norbert Cot and Ingemar Ingemarsson, eds.), Lecture Notes in Computer Science, Springer-Verlag, 209:335-338. 1985.
2. C. Blundo, C. Galdi, and G. Persiano, “Low-randomness constant-round private XOR computations,” Int. J. Secur., 6:15–26, 2007.
3. D.W Carmen, P.S. Kruss, B.J. Matt, “Constraints and approaches for distributed sensor network security,” NAI Labs Technical Report #00-010, September 2000.
4. H. Chan, A. Perrig, D. Song, “Random Key Predistribution Schemes for Sensor Networks,” Proceedings of the IEE Symposium of Privacy and Security, May 2003. pp. 197-213.
5. S-P. Chan, R. Poovendran, M-T Sun, “A key management scheme in distributed sensor networks using attack probabilities,” IEEE Global Telecommunications Conference (GLOBECOM ’05), December 2005.
6. H. Chan, A. Perrig, “PIKE: Peer intermediaries for key establishment in sensor networks,” Proceedings of the IEEE Infocom, March 2005.
7. R. Cramer, “Introduction to secure computation,” in Proc. Lectures on Data Security, pp. 16–42. LNCS, Springer-Verlag, 1999.
8. W. Du, J. Deng, Y. S. Han, P. Varshney, “A pairwise key pre-distribution scheme for wireless sensor networks,” *CCS’03*, pp. 42–51, ACM Press, New York, NY, 2003.

9. W. Du, J. Deng, Y. S. Han, S. Chen, P. Varshney, “A key management scheme for wireless sensor networks using deployment knowledge,” *Proceedings of the IEEE Infocom*, March 2004, pp. 586-597.

10. L. Eschenauer, V.D. Gligor, “A key-management scheme for distributed sensor networks”, *Proceedings of the 9th ACM Conf. Computing and Comm. Security (CCS ’02)*, Nov. 2002.

11. F. Liu, X. Cheng, “A Self-Configured Key Establishment Scheme for Large-Scale Sensor Networks,” *Proceedings of the 3rd IEEE International Conference on Mobile Ad-hoc and Sensor Systems (MASS 2006)*, October 2006, pp. 447-456.

12. E. Makri, Y. Stamatiou, “Distributively increasing the percentage of similarities between strings with application to key agreement”, *5th International Conference on AD-HOC Networks and Wireless*, August 2006.

13. N.C. Wormald, “The differential equation method for random graph processes and greedy algorithms,” *Ann. Appl. Probab.* 5, 1217–1235, 1995.
APPENDIX

Approximating stochastic processes with deterministic functions: Wormald’s Theorem [13]

**Definition 1.** A function $f$ satisfies a Lipschitz condition on $D \subset \mathbb{R}^j$ if there exists some constant $L > 0$ such that

$$|f(u_1, \ldots, u_j) - f(v_1, \ldots, v_j)| \leq L \sum_{i=1}^{j} |u_i - v_i|$$

for all $(u_1, \ldots, u_j)$ and $(v_1, \ldots, v_j)$ in $D$.

**Definition 2.** Given a random variable $X$ depending on $n$, denoted by $X^{(n)}$, we say that $X^{(n)} = o(f(n))$ always if

$$\max \{x : \Pr[X^{(n)} = x] \neq 0\} = o(f(n)).$$

**Theorem 3.** Let $Y_i^{(n)}(t)$, $n \geq 1$, be a sequence of real-valued random variables, $1 \leq i \leq k$ for some fixed $k$, such that for all $i$, all $t$ and all $n$, $|Y_i^{(n)}(t)| \leq Bn$ ($n > 0$) for some constant $B$. Let $H(t)$ be the history of the sequence, i.e. the matrix $(\overline{Y}(0), \ldots, \overline{Y}(t))$, where $\overline{Y}(t) = (Y_1^{(n)}(t), \ldots, Y_k^{(n)}(t))$.

Let $I = \{(y_1, \ldots, y_k) : \Pr[\overline{Y}(0) = (y_1n, \ldots, y_kn)] \neq 0 \text{ for some } n\}$. Let $D$ be some bounded connected open set containing the intersection of $\{(s, y_1, \ldots, y_k) : s \geq 0\}$ with a neighborhood of $\{(t/n, y_1, \ldots, y_k) : (y_1, \ldots, y_k) \in I\}$. (That is, after taking a ball around the set $I$, $D$ is required to contain the part of the ball in the half-space corresponding to $s = t/n$, $s \geq 0$.)

Let $f_i : \mathbb{R}^{k+1} \to \mathbb{R}, 1 \leq i \leq k$, and suppose that for some $m = m(n), (i)$ for all $i$ and uniformly over all $t < m$, always

$$\mathbf{E}[Y_i^{(n)}(t+1) - Y_i^{(n)}(t) | H(t)] = f_i(t/n, Y_1^{(n)}(t)/n, \ldots, Y_k^{(n)}(t)/n) + o(1),$$

(ii) for all $i$ and uniformly over all $t < m$,

$$\Pr[|Y_i^{(n)}(t+1) - Y_i^{(n)}(t)| > n^{1/5}] = o(n^{-3}), \text{ always,}$$

(iii) for each $i$, the function $f_i$ is continuous and satisfies a Lipschitz condition on $D$.

Then

(a) for $(0, z^{(0)}, \ldots, z^{(k)}) \in D$ the system of differential equations

$$\frac{dz_i}{ds} = f_i(s, z_0, \ldots, z_k), 1 \leq i \leq k$$

has a unique solution in $D$ for $z_i : \mathbb{R} \to \mathbb{R}$ passing through $z_i(0) = z^{(i)}, 1 \leq i \leq k$, and which extends to points arbitrarily close to the boundary of $D$;
(b) almost surely $Y_i^{(n)}(t) = z_i(t/n) \cdot n + o(n)$, uniformly for $0 \leq t \leq \min\{\sigma, m\}$ and for each $i$, where $z_i(s)$ is the solution in (a) with $\hat{z}^{(i)} = Y_i^{(n)}(0)/n$, and $\sigma = \sigma(n)$ is the supremum of those $s$ to which the solution can be extended.