Hexadecapole interaction and the $\Delta I=4$ staggering effect in rotational bands

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17 October 1994

Abstract

A role of the multipole interaction in the description of the $\Delta I=4$ staggering phenomenon is investigated in a model consisting of a single-$j$ shell filled by identical nucleons. Exact diagonalization of the quadrupole-plus-hexadecapole Hamiltonian shows that the hexadecapole-hexadecapole interaction can produce a $\Delta I=4$ periodicity in the yrast sequence.

PACS numbers: 21.10.Re, 21.30.+y, 21.60.Cs

1 Introduction

Recently a $\Delta I=4$ staggering effect in the dynamical moment of inertia of some superdeformed bands has been observed [1, 2, 3]. This can be interpreted as a bifurcation of the yrast band into two sequences with spins varying by 4 units within each sequence. The corresponding energy splitting is very small, i.e., about 100 eV. This effect may occur due to remnants of the $C_4$ symmetry of the system and suggests that presence of hexadecapole deformations or hexadecapole-type multipole interactions may be responsible for the staggering phenomenon. Recently, the origin of the staggering has been discussed by means of a phenomenological Hamiltonian containing higher-order terms in angular momentum [4]. In this work we investigate the effect by means of a more microscopic approach involving schematic multipole-multipole interactions.
2 Theoretical Model

The model to test the $\Delta I=4$ staggering phenomenon should be able to take into account the interplay between rotation and shape dynamics of a many-body system. A degenerate single-$j$ shell occupied by identical nucleons [5, 6], interacting via quadrupole-quadrupole and hexadecapole-hexadecapole multipole forces has this property. The corresponding Hamiltonian may be written as

$$\hat{H} = -\chi_2 \hat{Q}_2 \cdot \hat{Q}_2 - \chi_4 \hat{Q}_4 \cdot \hat{Q}_4 ,$$

where $\hat{Q}_2$ and $\hat{Q}_4$ are the quadrupole and hexadecapole moment operators defined as $\hat{Q}_\lambda = \sum_{m\mu}(j m j m|\lambda \mu)\hat{a}_m^+\hat{a}_{m'}^+\hat{a}_{m'}\hat{a}_m$ where $\hat{a}_m = Ta_m T^{-1}$, $T$ is the time reversal operator, and the dot symbol stands for the scalar product $\hat{Q}_\lambda \cdot \hat{Q}_\lambda = \sum_{\mu}(-)^{\mu}\hat{Q}_{\lambda\mu}\hat{Q}_{\lambda,-\mu}$. In the harmonic oscillator approximation [7] the values of coupling constants $\chi_2$ and $\chi_4$ can be estimated as

$$\chi_2 = \frac{176}{A} \text{ MeV} , \quad \chi_4 = \frac{165}{A} \text{ MeV} .$$

The results of the diagonalization depend solely on the ratio $\chi_4/\chi_2$. Therefore, in the following, we assumed $\chi_2 = 1$ MeV (i.e., $\chi_2$ is used to fix energy scale), while the parameter $\chi_4$, describing the relative strength of hexadecapole and quadrupole interactions, was varied to investigate the conditions for an appearance of the staggering effect.

In this paper we discuss the case of $N=8$ particles in the $j=15/2$ shell, although we have performed a series of calculations for several different shells and particle numbers. A choice of $N=8$ particles corresponds to a half-filled shell, and gives a regular collective yrast band. The calculations were performed for the parameter $\chi_4$ ranging from $\chi_4=0$ to $\chi_4=10$ MeV. For larger values of $\chi_4$, one only obtains an energy scaling corresponding to the dominating hexadecapole interaction.

To analyse a $\Delta I=4$ staggering effect in collective bands, we extracted the smooth reference curve according to Ref. [8]. Quantities $\Delta E_\gamma(I) \equiv E_\gamma(I+2) - E_\gamma(I)$ and $\Delta E_\gamma^\text{ref}(I) \equiv \frac{[\Delta E_\gamma(I+2) + 2\Delta E_\gamma(I) + \Delta E_\gamma(I-2)]}{4}$ were obtained in this way. Then staggering parameter $\Delta E_\gamma(I) - \Delta E_\gamma^\text{ref}(I)$ was evaluated and plotted in each case.

3 Results and Discussion

Fig. 1 shows a complete quadrupole spectrum of Hamiltonian (1) in case of $\chi_4=0$. Solid lines connect states of spins $I$ and $I+2$ with the largest reduced matrix elements of the quadrupole moment operator. Several quite regular rotational bands were obtained. The staggering parameter was evaluated for each case, but no $\Delta I=4$ periodicity was found. In Figs. 2a and 2b the collective energies $E(I)$ and the parameters $\Delta E_\gamma(I) - \Delta E_\gamma^\text{ref}(I)$ of the yrast sequence are presented as open circles. When the hexadecapole interaction is switched on, the staggering $\Delta I=4$ appears. This is seen in Fig. 2b, where full circles denote the staggering parameters of the yrast band for $\chi_4=0.3$, 0.4, and 0.5 MeV, while the corresponding yrast spectra are plotted in Fig. 2a. The staggering effect is small and therefore difficult to be directly seen in the yrast spectra.
The amplitude of the staggering increases with $\chi_4$, but for the values of $\chi_4$ larger than 0.5 MeV, the yrast states are no longer connected by the enhanced E2 transitions; i.e., the band structure is lost. Fig. 3a shows the yrast states for $\chi_4$=4 MeV. This corresponds to the situation where the hexadecapole interaction dominates and the yrast sequence is strongly perturbed. There are no strong E2 transitions linking the yrast states, but the staggering parameter extracted from the energies of the yrast sequence exhibits the $\Delta I=4$ periodicity (Fig. 3b).

Similar effects can be obtained for other values of $j$ and $N$. We would like to stress that it was impossible to obtain similar staggering by using the quadrupole-quadrupole interaction alone. On the other hand, for $j=15/2$ and $N=8$, the staggering appears only in the yrast band.

The examples presented here show that the hexadecapole interaction may generate the $\Delta I=4$ irregularities. The proper treatment of the hexadecapole interaction seems to be crucial for the understanding of this intriguing staggering phenomenon.

Acknowledgements

This research was supported in part by the Polish State Committee for Scientific Research under Contract Nos. 20450 91 01 and 2 P 302 056 06, and by the computational grant from The Interdisciplinary Centre for Mathematical and Computational Modeling (ICM) of Warsaw University. Oak Ridge National Laboratory is managed for the U.S. Department of Energy by Martin Marietta Energy Systems, Inc. under Contract No. DE-AC05–84OR21400. The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the members and by the Department of Energy through Contract No. DE-FG05-87ER40361 with the University of Tennessee. Theoretical nuclear physics research at the University of Tennessee is supported by the U.S. Department of Energy through Contract No. DE-FG05-93ER40770.

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Figure 1: The exact spectrum for the $j=15/2$ shell filled with $N=8$ particles. The coupling constants $\chi_2=1\text{ MeV}, \chi_4=0$ were used.

Figure 2: The results for the $j=15/2$ shell filled with $N=8$ particles. The quadrupole coupling constant was $\chi_2=1\text{ MeV}$. The hexadecapole strengths $\chi_4=0$ (open circles) and $\chi_4=0.3, 0.4, 0.5\text{ MeV}$ (full circles) were used. Part (a) shows the yrast spectra, part (b) the corresponding staggering parameters $\triangle E_\gamma(I) - \triangle E_\gamma^{\text{ref}}(I)$ plotted as a function of spin.

Figure 3: The exact yrast spectrum (a) and the corresponding staggering parameter $\triangle E_\gamma(I) - \triangle E_\gamma^{\text{ref}}(I)$ (b) for the $j=15/2$ shell filled with $N=8$ particles. The quadrupole and hexadecapole coupling constants were $\chi_2=1\text{ MeV}, \chi_4=4\text{ MeV}$, respectively.
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