Optimization of the Vehicle Movement Modes

E Kudryavev¹, D Mollinedo¹
¹Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

E-mail: sdm@mgsu.ru

Abstract. Article describe process of optimization of vehicle movement modes. This task belongs to the class of combinatorial tasks. When in our task we take only six sector of the road and five transfers in each sector then number of the possible vehicle movement modes will be 18750. For decision of the task we use equation recurrence of Bellman, according to which the optimal choice has the property of independently from the initial state and solutions at the initial moment the subsequent solutions must be optimal. This allows to replace search of extremum of a function with many variables by search of extremum of a function with one variable. For optimization we use Mathcad system. The proposed procedures of optimization of vehicle movement modes allow to reduce time and costs of such calculations at least in several hundred times and effectively to carry out researches in any usage mode and exploitation of vehicles, providing good visual presentation of received results.

1. Introduction

Article describe process of optimization of vehicle movement modes. This task belongs to the class of combinatorial tasks. When in our task we take only six sector of the road and five transfers in each sector then number of the possible vehicle movement modes will be 18750. For decision of the task we use equation recurrence of Bellman, according to which the optimal choice has the property of independently from the initial state and solutions at the initial moment the subsequent solutions must be optimal. This allows to replace the extremum search of a function with many variables by extremum search of a function with one variable. For optimization we use Mathcad system.

2. Statement of the problem

It is known, when build roads, it is necessary to execute large amount of work. At the same time, on each road sector has different loading on vehicle: slopes of road, resistance to movement and so on. We know expense of fuel $s(i,j,k)$ on each road sector $i$ at transition from $j$ th transfer to $k$ th. The initial information for vehicle movement optimization is presented in table 1, the data of which will be presented later as a expenses matrix. For simplification we will to use transmission with four transfers.
### Table 1.

| Rout sector $i$ | Vehicle movement on $j$-th transfer on $(i-1)$-th road sector | |
|----------------|-------------------------------------------------|---|
|                | $j = 1$                                          | $j = 2$ | $j = 3$ | $j = 4$ |
| $i$            |                                                 |         |         |         |
| $k = 1$        | $k = 2$                                          | $k = 3$ | $k = 4$ | $k = 1$ |
| $j = 1$        | $j = 2$                                          | $j = 3$ | $j = 4$ | $j = 1$ |
| $k = 2$        | $k = 3$                                          | $k = 4$ | $k = 1$ | $k = 2$ |
| $j = 2$        | $j = 3$                                          | $j = 4$ | $k = 1$ | $k = 2$ |
| $k = 3$        | $k = 4$                                          | $k = 1$ | $k = 2$ | $k = 3$ |

| Vehicle movement on $k$-th transfer on $i$-th road sector | |
|-----------------------------------------------------------|---|
| $k = 1$                                                    | $k = 2$ | $k = 3$ | $k = 4$ | $k = 1$ |
| $i$                                                        |         |         |         |         |
| $k = 2$                                                    | $k = 3$ | $k = 4$ | $k = 1$ | $k = 2$ |
| $j = 2$                                                    | $j = 3$ | $j = 4$ | $j = 1$ | $j = 2$ |
| $k = 3$                                                    | $k = 4$ | $k = 1$ | $k = 2$ | $k = 3$ |
| $j = 3$                                                    | $j = 4$ | $k = 1$ | $k = 2$ | $k = 3$ |

| Expense of fuel $s(i, j, k)$ on each rout sector $i$ at transition from $j$ th transfer to $k$ th |
|--------------------------------------------------------------------------------------------------|
| $1$                                                                                           | $14$ | $12$ | $13$ | $11$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $2$                                                                                           | $15$ | $13$ | $12$ | $11$ | $16$ | $15$ | $14$ | $13$ | $15$ | $14$ | $13$ | $16$ | $14$ | $13$ | $12$ |
| $3$                                                                                           | $18$ | $16$ | $13$ | $12$ | $17$ | $15$ | $14$ | $13$ | $20$ | $21$ | $16$ | $14$ | $22$ | $20$ | $12$ |
| $4$                                                                                           | $17$ | $15$ | $14$ | $13$ | $11$ | $13$ | $15$ | $14$ | $15$ | $14$ | $13$ | $16$ | $15$ | $14$ | $13$ |
| $5$                                                                                           | $13$ | $-1$ | $-1$ | $-1$ | $14$ | $-1$ | $-1$ | $15$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

3. **Method selection**

All possible vehicle movement modes represent in form of a marked graph of transfers (fig. 1). Each transfer on each road sector represent in form of a circles (fig. 1). Vehicle movement represent in form of arrows. Above each arrow give expense of fuel $s(i, j, k)$ on each road sector $i$ at transition from $j$ to $k$ (fig. 1).

**Figure 1.** Marked graph of transfers for all possible vehicle movement modes.

The circle from which the arrow get out indicates the moment when the vehicle transfer changes to the transfer of movement on the current road sector. The circle in which arrow get in defines the moment
when the vehicle transfer changes on the next road sector. In our task, for vehicle movement modes optimization we will take 5 sectors of road.

Representation of all possible vehicle movement modes in the form of a marked graph of transfers provides clearness and ease of forming an acceptable set. This task belongs to the class of combinatorial tasks, in which the possible vehicle movement modes determine by the calculation.

\[ N = m_1 \times m_2 \times m_3 \times \cdots \times m_i = 4 \times 4 \times 2 = 96, \]

where \( m_i \) is the possible number of transfers of the vehicle on a on each road sector \( i \).

So, if in our task we take only five sectors and in each sector five transfers, then the possible vehicle movement modes will be 3125.

\[ N = 5 \times 5 \times 5 \times 5 = 3125. \]

We will use equation recurrence of Bellman, according to which the optimal choice has the property of independently from the initial state and solutions at the initial moment, the subsequent solutions must be optimal. This allows to replace the search of extremum of a function with many variables by search of extremum of a function with one variable.

\[ y_{\text{min}}(i, j) = \min_{k=1,2, \ldots, m_i} [c(i, j, k) + y_{\text{min}}(i + 1, k)]. \]

where: \( y_{\text{min}}(i, j) \) — minimum fuel expenses for the partial vehicle movement modes with the \( i \)-th sector and \( j \)-th transfer;

\( y_{\text{min}}(i + 1, k) \)-the same, after the \((i+1)\)-th sector and with the \( k \)-th transfer of the vehicle.

The algorithm of the dynamic programming method includes two stages.

At the first stage, sequential optimization of vehicle movement modes is performed for partial route starting from the last sector, using equation recurrence of Bellman.

At the second stage, using the results of calculating the optimal vehicle movement modes, the optimal one is found, which provides the minimum fuel consumption.

The first stage include definition of the minimum total expenses: on a current sector of road \( i \) and the minimum expenses for the subsequent sectors of road \((i - 1)\) with allocation of expenses on the added sector of the road, provided the minimum expenses to all partial route of movement of a vehicle:

1. Determine the minimum expense for sectors of the route, after the 5th. In our example, the minimum fuel consumption after the 5th sector, as there are no existing sector, is 0.

\[ y_{\text{min}}(6, 1) = 0. \]

2. Determine the minimum expense for partial vehicle movement mode starting with the 5th sector:

\[ y_{\text{min}}(5, 1) = \min_{k=1} \left\{ s(5, 1, 1) + y_{\text{min}}(6, 1) \right\} = \min \{13+0\} = 13; \]

\[ y_{\text{min}}(5, 2) = \min_{k=1} \left\{ s(5, 2, 1) + y_{\text{min}}(6, 1) \right\} = \min \{14+0\} = 14; \]

\[ y_{\text{min}}(5, 3) = \min_{k=1} \left\{ s(5, 3, 1) + y_{\text{min}}(6, 1) \right\} = \min \{15+0\} = 15. \]

Mark the arrows on the 5th sector: (1-1), (2-1), and (3-1).

3. Determine the minimum expense for partial vehicle movement mode starting with the 4th sector:

\[ y_{\text{min}}(4, 1) = \min_{k=1,2,3} \left\{ s(4, 1, 1) + y(5, 1) \right\} = \min_{k=1,2,3} \left\{ 17+13 \right\} = 28. \]

\[ y_{\text{min}}(4, 2) = \min_{k=1,2,3} \left\{ s(4, 1, 2) + y(5, 2) \right\} = \min_{k=1,2,3} \left\{ 15+14 \right\} = 13+15. \]

\[ y_{\text{min}}(4, 3) = \min_{k=1,2,3} \left\{ s(4, 1, 3) + y(5, 3) \right\} = \min_{k=1,2,3} \left\{ 17+13 \right\} = 28. \]
4. Determine the minimum expense for partial vehicle movement mode starting with the 3rd sector:

\[ y_{\min}(3, 1) = \min_{k=1, 2, 3} \left\{ \frac{s(3, 1, 1) + y(4, 1)}{s(3, 1, 2) + y(4, 2)} \right\} = \min_{k=1, 2, 3} \left\{ \frac{18 + 28}{16 + 24} \right\} = 40; \]

\[ y_{\min}(3, 2) = \min_{k=1, 2, 3} \left\{ \frac{s(3, 2, 1) + y(4, 1)}{s(3, 2, 2) + y(4, 2)} \right\} = \min_{k=1, 2, 3} \left\{ \frac{17 + 28}{15 + 24} \right\} = 39; \]

\[ y_{\min}(3, 3) = \min_{k=1, 2, 3} \left\{ \frac{s(3, 3, 1) + y(4, 1)}{s(3, 3, 2) + y(4, 2)} \right\} = \min_{k=1, 2, 3} \left\{ \frac{20 + 28}{21 + 24} \right\} = 45; \]

\[ y_{\min}(3, 4) = \min_{k=1, 2, 3} \left\{ \frac{s(3, 4, 1) + y(4, 1)}{s(3, 4, 2) + y(4, 2)} \right\} = \min_{k=1, 2, 3} \left\{ \frac{22 + 28}{20 + 24} \right\} = 44. \]

Mark the mark on the 3rd sector of the arrow: (1-2), (2-3).

5. Determining the minimum expense for partial vehicle movement mode starting with the 2nd sector:

\[ y_{\min}(2, 1) = \min_{k=1, 2, 3, 4} \left\{ \frac{s(2, 1, 1) + y(3, 1)}{s(2, 1, 2) + y(3, 2)} \right\} = \min_{k=1, 2, 3, 4} \left\{ \frac{15 + 40}{13 + 39} \right\} = 52; \]

\[ y_{\min}(2, 2) = \min_{k=1, 2, 3, 4} \left\{ \frac{s(2, 2, 1) + y(3, 1)}{s(2, 2, 2) + y(3, 2)} \right\} = \min_{k=1, 2, 3, 4} \left\{ \frac{16 + 40}{15 + 39} \right\} = 54; \]

\[ y_{\min}(2, 3) = \min_{k=1, 2, 3, 4} \left\{ \frac{s(2, 3, 1) + y(3, 1)}{s(2, 3, 2) + y(3, 2)} \right\} = \min_{k=1, 2, 3, 4} \left\{ \frac{15 + 40}{13 + 39} \right\} = 52; \]

\[ y_{\min}(2, 4) = \min_{k=1, 2, 3, 4} \left\{ \frac{s(2, 4, 1) + y(3, 1)}{s(2, 4, 2) + y(3, 2)} \right\} = \min_{k=1, 2, 3, 4} \left\{ \frac{16 + 40}{14 + 39} \right\} = 53. \]
Mark the mark on the 2nd sector of the arrow: \((1-2)\) \((2-2)\), \((3-2)\) \((4-2)\).

6. Determine the minimum expense for partial vehicle movement mode starting with the 1st sector:

\[
y_{\min}(1,1) = \min_{i=1,2,4} \left\{ \begin{array}{l}
s(1,1,1) + y(2,1) \\
s(1,1,2) + y(2,2) \\
s(1,1,3) + y(2,3) \\
s(1,1,4) + y(2,4) \\
\end{array} \right. \\
= \min_{i=1,2,4} \left\{ \begin{array}{l}
14 + 52 \\
12 + 54 \\
13 + 52 \\
11 + 53 \\
\end{array} \right. = 64.
\]

The first stage of calculation is finished when the minimum expenses of fuel are found for all vehicle movement modes. Minimum expenses of fuel for all route beginning with 1st sector, equal 64 conditional units \((y_{\min}(0,1) = 64)\). Results of calculation are given on fig. 2

\[\text{Figure 2.} \quad \text{Graph of possible vehicle movement modes on different sectors of the road with calculation results.}\]

The second stage includes determination of the minimum expense for all route starting with the 1st sector with allocation of expenses on each sector if the rout

- on the first sector allot the arrow \((1-4)\);
- on the second sector allot the arrow \((4-2)\);
- on the third sector allot the arrow \((2-2)\);
- on the fourth sector allot the arrow \((2-1)\);
- on the fifth sector of allot arrow \((1-4)\).

All these arrows are highlighted in bold on the graph of possible vehicle movement modes Fig. 2.

Below is an algorithm and an example of optimization of the vehicle movement modes by the Bellman method in Mathcad system.
4. Algorithm of optimization

Optimization of the vehicle movement modes in Mathcad system

Initial data
- initial value of an index
- number of sites on road
- number of used transfers on road
- number of used transfers on each site of road
- matrix of expenses in the size \((n \times m)\).

\[ t \left( \begin{array}{cccc}
14 & 12 & 13 & 11 \\
15 & 13 & 12 & 13 \\
18 & 16 & 1000 & 1000 \\
17 & 15 & 13 & 1000 \\
13 & 1000 & 1000 & 1000 \\
\end{array} \right) \]

\[ \begin{array}{cccc}
1000 & 1000 & 1000 & 1000 \\
16 & 14 & 13 & 12 \\
22 & 20 & 1000 & 1000 \\
1000 & 1000 & 1000 & 1000 \\
\end{array} \]

\[ \begin{array}{cccc}
1000 & 1000 & 1000 & 1000 \\
15 & 13 & 14 & 13 \\
20 & 21 & 1000 & 1000 \\
1000 & 1000 & 1000 & 1000 \\
\end{array} \]

\[ \begin{array}{cccc}
1000 & 1000 & 1000 & 1000 \\
14 & 1000 & 1000 & 1000 \\
15 & 1000 & 1000 & 1000 \\
\end{array} \]

Algorithm of calculation:
1. Number of possible vehicle movement modes:

\[ N = \prod_{i=1}^{n} t_i \quad k := 1 \ldots m \quad y_{n+1,k} := 1000 \quad y_{n+1,1} := 0 \quad N = 96 \]

2. The program of definition of the minimum expenses of fuel for optimum partial high-speed modes of movement of a vehicle, using equation recurrence of Bellman

\[ y := \begin{array}{l}
\text{for } i \in n \ldots 1 \\
\text{for } j \in 1 \ldots m \\
y_{i,j} := \begin{cases}
\text{for } k \in 1 \ldots t_i \\
\min(z) \end{cases}
\end{array} \]

\( (S_j)_{i,k} \) expenses on i-th site of road on transfer k after j-th transfer on (i-1)-th site of road.

2. Calculation result of the minimum expenses of the fuel for optimum partial high-speed modes of movement of a vehicle, using equation recurrence of Bellman

\[ y = \begin{pmatrix}
64 & 1.052 \times 10^3 & 1.052 \times 10^3 & 1.052 \times 10^3 \\
52 & 54 & 52 & 53 \\
40 & 39 & 45 & 44 \\
38 & 34 & 1.013 \times 10^3 & 1.013 \times 10^3 \\
13 & 14 & 15 & 1 \times 10^3 \\
0 & 1 \times 10^3 & 1 \times 10^3 & 1 \times 10^3 \\
\end{pmatrix} \]

\[ y_{min} = y_{1,1} = 64 \]
5. Conclusion

The proposed procedures of optimization of vehicle movement modes allow to reduce time and costs of such calculations at least in several hundred times and effectively to carry out researches in any usage mode and exploitation of vehicles, providing good visual presentation of received results.

6. References

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