Modeling of Powder Deposition into Shallow Dies for Three Filling Methods

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Abstract

Fundamental understanding of the deposition of powders into dies is essential to optimizing processes such as compaction. Toward this end, models were developed and verified to simulate the filling characteristics for three deposition methods into shallow dies. Understanding of the filling process for shallow dies represents the key first step towards gaining fundamental knowledge of filling deep dies. The three different filling methods modeled were: the feed shoe, rotational rainy, and point feed, which represent the commonly used methods to fill dies and containers. A free flowing, spray dried battery powder mixture (d50=600 μm) was filled into a circular shallow die (35 mm diameter x 6.5 mm deep) at 20 mm/s feed shoe speed equivalent to 26 g/min filling rate. A physics based explanation of the filling process, i.e., pressure vs. time for a specific location at the bottom of the die, is included that provides a rational basis for the use of Chapman-Richards model. In order to evaluate the goodness of the model, the average root mean square error (RMSE) and the mean value of average relative difference (ARD) of the models were calculated. The results showed that 1) the RMSE for feed shoe, rotational rainy, and point feed were 0.16 dm (dm=decimeter, fill head equivalent of measured pressure), 0.44 dm, and 0.32 dm, respectively, whereas, the ARD for feed shoe, rotational rainy, and point feed were 7%, 16%, and 11%, respectively; 2) the deposition profile for feed shoe and rainy fill were sigmoidal in shape, while for the point feed it was linear.

Keywords: die filling, mathematical model, deposition method, feed shoe deposition, rainy fill, point feed

1. Introduction

Any industry that deals with particulate materials must have a clear understanding of particle behavior during the handling and processing operations. Generally, materials in particulate form are easier and more economical to handle due to the flexibility in producing a variety of finished products. Given the widespread use of powders, it would be impossible to improve the handling and processing techniques without a precise knowledge of how particulate materials behave under various conditions (inputs) (Kessler and Greenkorn, 1999). Some engineering mathematical models are built based on purely experimental observations, which treat variables in a physically most intuitive manner. The objective is to determine simple correlations of the behavior of the dependent and independent variables. At a more fundamental level, physical, chemical, and/or biological laws, are used to form the basis for mathematical model. In this context, the parameters that are embedded in the model have well defined physical meaning. In this research, the latter approach was attempted to formulate a mathematical model for the powder deposition process in dies.

No continuum models and methods have been published to simulate the deposition characteristics in shallow dies. Xie (2006) developed an overall rate equation for feed shoe filling process for cylindrical dies using a battery powder mixture (BPM). The overall rate equation was \( \frac{dP_p}{dt} = \alpha P_p F(\tau) + \beta \), where \( P_p \) is prorated pressure, \( \tau \) is normalized time, \( \alpha \) and \( \beta \) are coefficients, and \( F(\tau) \) is a stage-specific depositi-
tion force function. Although, the model successfully described the feed shoe deposition method, no clear and complete physical interpretation of \( \alpha \) and \( \beta \), and origin of \( F(\tau) \) were presented (Xie and Puri, 2008). Therefore, the objective of this research was to develop and validate time-dependent models for feed shoe, rainy fill, and point feed filling methods based on process physics.

2. Mathematical Formulation

The two cornerstones of the deposition model are the overall force balance and mass balance equations. In this study, the static force balance was used as the basis for developing the prevailing dynamic conditions. Furthermore, it was assumed that the developed equations are point-specific located at the die bottom. In order to better describe and analyze the die filling process, it was divided into two stages: 1) powder deposition process into a die that is partially to nearly completely filled, and 2) powder deposition process during surcharge build-up. The analysis and mathematical formulations are described in the following subsections.

2.1. Analysis of powder within die

**Force Balance in powder mass:** The most widely used equation for predicting vertical and lateral pressure in dies, bins, and containers is the Janssen’s equation (Manbeck and Puri, 1995). Janssen’s solution assumes the bulk density of material \( (n_b) \), angle of internal friction \( (\phi) \), and coefficient of friction between the die wall and powders \( (\mu) \) are constant throughout the particle bed. In order to gain a better insight into the effect of loading conditions and, in particular, to simplify the mathematical simulation, the system was assumed to be axisymmetric, thus pressure distributions vary with depth but not with location around the circumference. A schematic of the free body diagram of the cylindrical die and force balance on powder mass at depth is shown in Fig. 1.

Summing forces acting on the differential elements in the vertical direction (Fig. 1b) yields Equation (1) (Roudsari, 2007):

\[
(P_v + \Delta P_v)A + P_v C dy - P_v A - \rho g A dy = 0
\]

(1)

where: \( A= \) cross-section; \( C= \) die circumference; \( P_v= \) vertical friction on the wall; \( P_v= \) vertical pressure in the mass; \( P_h= \) lateral pressure; \( g= \) gravitational constant; \( \mu= \) coefficient of friction; \( \rho_b= \) bulk density; and \( y= \) actual filled height of powder in the die.

Since the number of symbols used is large, for convenience these are listed also in the Nomenclature section.

Upon integration, Equation (1) yielded:

\[
P_v = \frac{\rho_b g R_k \mu}{k \mu} \left(1 - e^{-k \mu y R}ight)
\]

(2)

where, \( k \) is the ratio of lateral to vertical pressure within the powder mass \( (k = \frac{P_v}{P_h}) \) and \( R = A/C \).

**Mass balance in powder mass:** The physical principle of mass balance is given in Equation (3).

\[
\dot{m}_2 - \dot{m}_1 + \frac{d m}{d t} = 0
\]

(3)

where \( \dot{m}_2 = \) rate of outflow, \( \dot{m}_1 = \) rate of inflow, \( \frac{d m}{d t} = \) accumulation rate. In the case of filling a container with no outflow, Equation (3) can be rewritten as:

\[
\frac{d m}{d t} = \dot{m}_1 , \text{ but } m_1 = \rho v = Q \text{ and } \dot{m}_1 = \dot{Q}
\]

(4)

Also \( \rho \frac{d v}{d t} = \rho \dot{Q} \), \( v=A . y \), where, \( v=\)volume, then:

![Fig. 1 Free body diagram of (a) cylindrical die during rainy filling process, and (b) force balance on powder mass inside the die (Manbeck and Puri, 1995).](image-url)
Next, Equation (2) can be rewritten in terms of general parameters $\gamma_1$, $\gamma_2$, and $y$, this reduces to:

$$P_v = \frac{1}{\gamma_1} \left( 1 - e^{-\gamma_2 y} \right)$$  \hspace{1cm} (6)

where, $\gamma_1 = \rho_b g R/\mu m$ and $\gamma_2 = k \mu / R$

Rearranging Equation (6) in terms of $y$ and replacing in Equation (5) and integration leads to (Roudsari, 2007):

$$P_v = \frac{1}{\gamma_1} \left( 1 - e^{-\gamma_2 y} \right)$$

Substituting $K = \frac{1}{\gamma_1}$ and $b = \frac{\gamma_2}{A \gamma_1^2} \dot{q}$, yields:

$$P_v = K \left( 1 - e^{-bt} \right)$$  \hspace{1cm} (7)

Rewriting pressure $P_v$ as equivalent height $h^* P_v / (\rho_p \times g)$ or $h^* = K (1 - e^{-at})$  \hspace{1cm} (8)

where $K$ and $b$ are experimentally determined parameters and $\rho_p$ is the particle density.

2.2. Analysis of powder within surcharge mass

Equation (8) is a typical exponential response for pressure distribution in dies and containers; however, the most typical observed die filling pressure response was sigmoidal. For all three fill methods, there was always a powder surcharge that is a likely cause of the slowdown in pressure build-up. For the feed shoe, the pressure distribution is more complex and is believed to be caused by the cumulative effect of the feed shoe tube walls and powder head in the feed shoe tube. Since the surcharge of powders in a rainy fill provides a clearer explanation of the process physics, it is followed here. In addition, the point feed filling method’s analysis is included.

As stated previously, the die was overfilled during the filling process, which is a normal practice in industry. Following overfill, the excess powder is scraped off and the free surface of the powder is leveled. A simplified free body diagram of a heap (mound) of the surcharge powder above the die is shown in Fig. 2.

Following the force balance analysis carried out for filled powder below the die height ($y < h$), the vertical pressure was obtained as:

$$P_v = A (1 - e^{-bt(H_0 - h)})$$  \hspace{1cm} (9a)

Or, in terms of equivalent height ($h^*$) and time ($t$) (Roudsari, 2007):

$$(H_0 - h) = \gamma_3 (1 - e^{-a(t - t_f)})$$  \hspace{1cm} (9b)

where, $t_f = t$ to fill the die corresponding to $y = h$, and time at the end of filling process including surcharge $t > t_f$, and $\gamma_3$ and $\gamma_2$ are coefficients

Based on visual observations during experiments with the rainy fill method, the surcharge powder profile can be approximated by a spherical cap (ABCDEFA) as shown in Fig. 2a. The vertical pressure as equivalent height from the surcharge powder is (Roudsari, 2007):

$$h^* = \frac{\rho_b \pi r^2 \theta}{2 \gamma_3} \left( 1 - e^{-\gamma_4(t - t_f)} \right) + \frac{\rho_b \pi}{6 \gamma_3} \left( 1 - e^{-\gamma_4(t - t_f)} \right)^3$$  \hspace{1cm} (10)

where, $\rho_b$ bulk density, $g$ = gravitational constant, $\theta$ = (segment AC, Fig. 2)/$r$, and $r$ = radius of sphere.

The form of Equation (10) is sigmoidal, which is comparable to Chapman-Richards equation (Seber and Wild, 1989) shown below:

$$h^* = \alpha (1 - e^{-bt})^c$$  \hspace{1cm} (11)
where, $a$, $b$, and $c$ are parameters. The interpretation of these coefficients is: (1) $α$ represents the asymptotic equivalent height, i.e., the maximum $h^*$ when time approaches $∞$ for instance, when filling a deep die, (2) $b$ represents the reciprocal of the characteristics time ($t^*$) of the deposition process exponent, i.e., the rapidity with which $h^*$ is approached, and (3) $c$ represents the build-up characteristics of pressure at bottom of the die due to the continuously increasing surcharge powder.

For the point feed method, the observed shape of surcharge powder was conical. For this fill method, the surcharge pressure (based on volume of the cone) can be written as (Roudsari, 2007):

$$h^* = \frac{\pi \rho_b \tan \delta (\gamma_3)^3 \left(1 - e^{-\gamma_4(t-t_f)}\right)^3}{3 \rho_p} \quad (12a)$$

where, $δ$ is the cone half angle.

In the point feed fill method; the deposited powder’s base expands until it reaches the walls of the die. During this time period, most of the die is unfilled, i.e., the walls of the shallow die do not directly influence the filling pressures. In this case, retaining the first non-zero term of the Taylor series for the Equation (12a) yields (Roudsari, 2007):

$$h^* = \beta(t-t_f)^c \quad (12b)$$

where, $β$ is a coefficient determined from experimental data.

Equation (12b) is the generalized pressure build-up profile for the point feed fill method. As will be shown later $c \approx 1$ for BPM at slow filling rates. Given the generality of Equation (11), it was used for modeling the three filling methods.

3. Experimental Design

In this study, a battery powder mixture (BPM) was used to fill a shallow circular die of 35 mm in diameter that was 6.5 mm deep (i.e., $h=6.5$ mm). The bulk density and particle density of BPM were 1.65 g/cm³ and 4.7 g/cm³, respectively (Xie, 2006). It is stable under ambient conditions. The granule size range of BPM powder was from less than 80 to 1000 $μm$ with median size ($d_{50}$) of 600 $μm$. BPM powder is very friable; some granules easily fragment into finer granules during handling and transportation. Powder characteristics of relevance and importance during deposition and compaction have been reported in the literature (such as, Doelker, 1993; Hjortsberg and Bergquist 2002; Niesz, 1996, and Wu et al., 2003)

The second generation pressure deposition tester (PDT-II) (Fig. 3a) was used to measure the powder pressure distribution characteristics (Xie and Puri, 2007). An innovative rotational rainy filling device (Fig. 3b) was designed and fabricated. This versatile device can be used to measure filling characteristics at different rotational speeds (Roudsari, 2007). The point feed (Fig. 3c) method with a funnel of 30 mm inlet diameter and 4.2 mm outlet diameter opening was used to fill the circular shallow dies. The pressure sensor strip, P-1500, was placed at the bottom of the die (Fig. 3d), which was connected to a data acquisition systems and data analysis software. There are 16 pressure sensors in one pressure sensor strip, the cross-section of each sensor is 2 mm × 5 mm with center-to-center spacing of 2 mm. The pressure sensor strip was positioned at eight different angles, 22.5 degrees apart, which covered the entire die; i.e., 0°–180°, 22.5°–202.5°, 45°–225°, 67.5°–247.5°, 90°–270°, 112.5°–292.5°, 135°–315°, and 157.5°–337.5°. All sixteen sensors were exposed to record the pressure increase profile. The sensitivity of pressure sensor (P-1500) was 0.1% of the full scale range, or 5 Pa (4.8 mg on an area of 2 mm × 5 mm). The highest pressure that the P-1500 can measure was 551.6 kPa (0.529 kg on an area of 2 mm × 5 mm). The sensor calibration was repeated every 50 tests (Roudsari, 2007). Throughout this study, the data capture rate of 30 Hz per channel was used. Herein, results of BPM powder only are presented. The impact of particle size distribution and particle shape for a powder finer than BPM on the filling process is presented and discussed at length in Roudsari (2007).

4. Results and Discussion

4.1. Feed shoe filling process

A typical pressure profile obtained by the pressure sensor element located at the center during filling of the cylindrical die with the BPM at feed shoe speed of 20 mm/s is shown in Fig. 4. The plot represents the average of six runs to ensure that the experimental values had coefficient of variation generally less than 15% (Mittal et al., 2001; Wu et al., 2003; and Xie and Puri, 2006 and 2007). In order to convert the pressure profile of BPM, each pressure value was divided by the particle density (4.7 g/cc) multiplied by gravity ($\frac{L}{\rho g}$). As mentioned earlier, this can be interpreted as equivalent height ($h^*$). The equivalent height scale $h^*$ is in dm, dm is decimeter, i.e., 1 dm=0.1 m. Due to the complexity, the entire pressure profile could not be simulated by one value each for coefficients $α$, $b$, and $c$ of Equation 11. Therefore, the...
The entire filling process was divided into various phases and Phase I was divided into two stages. Every phase was modeled using a simple rate equation. The positions of circular feed shoe tube during the filling process with reference to center of the die that were used to calculate the start and end times of various phases are given in Fig. 4 and 5.

4.1.1. Feed shoe filling phases

The feed shoe filling profile shown in Fig. 4 was divided into three distinct phases. Phase I was from time 0 to time T1 which corresponded to forward stroke while the actual filling process occurred. Phase I was divided further into two stages which corresponded to feed shoe tube movement during the forward stroke. The two stages were, T1a and T1b where the first subscript denotes the phase and second represents the stage. During Phase II for time duration T2, the feed shoe started to leave the die. The pressure applied by the powder inside the feed shoe tube started to be released. Phase III for time duration T3 was post-filling process during which time the powder gradually approached an equilibrium value.
4.1.1 Phase I

In Phase I, actual filling of the die occurred during the forward stroke of feed shoe tube. The relative positions of feed shoe tube at two stages marked the start and end times are shown in Fig. 5. These particular positions of feed shoe tube with circular cross-section were used to identify and discriminate filling stages in Phase I. Corresponding to these positions, the specific times for the feed shoe tube to reach these positions are listed below:

- T1a: when feed shoe tube nose reached middle of the die, i.e., sensor location (0.49 s).
- T1b: when the back wall of feed shoe tube reached the leeward direction of the die (1.31 s).

Stage I (T1a): The majority of filling was accomplished in this stage. The filling occurs rapidly showing an exponential deposition profile.

Stage II (T1b): In this stage, the die was overfilled and surcharge powder covered the top of the powder mass.

At time 0, the feed shoe tube front wall reached the leeward direction of the die (Fig. 5), while only a small fraction of the powder in the front of feed shoe was available to enter the die. The filling rate is very low at this time. With the feed shoe movement and increase of time, more powder in the front wall of feed shoe tube filled the die. With the increase in time, the die continued to fill and the distance between the top surface of the powder in the die and the bottom of the powder in the feed shoe gradually decreased; at this time, the die was filled, i.e., could not accommodate more powder.

Two stages of the Phase I could be mathematically represented by the following rate Equation (13):

\[ h^* = 32.63(1 - e^{-5.69t})^3 \] (13)

The R-Square value of this regression was 0.97.

Phase I is responsible for most of the powder filling during the deposition process, i.e., about 90% (Xie, 2006). During Phase I, the most pressure measured by the pressure sensor strip was exerted directly from the powder deposited into the circular die. Minimal-to-no pressure was exerted or influenced by the feed shoe wall.

4.1.1.2 Phase II

Based on experimental results, the rate Equation (11) of this phase can be simplified to:

\[ \frac{dh^*}{dt} = m_2 \] (14)

where \( m_2\) is slope of the straight line. The slope \( m_2 = -6.79 \). The R-square value of this regression was 0.99.

4.1.1.3 Phase III

In this phase, the pressure reached an equilibrium state, and this state could best represent the pressure value of the end of filling process. The pressure profile decreased slowly in the beginning of this phase, followed by a close-to-horizontal profile. The reason for slow decline might be that some of the pressures at the end of forward stroke was gradually released. The rate of equivalent height Equation (11) in Phase III is simply \( \frac{dh^*}{dt} = 0 \)

4.1.1.4 Overall equation for all the three phases

To effectively account for Phases II and III, the overall rate Equation (11) for shallow die can be represented alternately by Equation (15).

\[ \frac{dh^*}{dt} = \alpha h^* F(t) + \beta \] (15)

where \( \alpha \) and \( \beta \) are coefficient, and \( F(t) \) is a function of time. The specific form of function \( F(t) \) and the corresponding applicable range of times are shown in Table 1. The Phases II and III did not have the function \( F(t) \) (their \( \alpha \) value was 0), since the changing rates of their pressure values were constant.

Root mean square error [RMSE, Equation (16)] and average relative difference [ARD, Equation (17)] values were used to determine the quality of the model predictions.

\[ RMSE = \sqrt{\frac{\sum (h^*_\text{mod} - h^*_\text{measured})^2}{n - 1}} \] (16)

\[ ARD = 100 \frac{\sum |h^*_\text{mod} - h^*_\text{measured}|}{h^*_\text{measured}} \] (17)

where \( h^*_\text{mod} \) = modeled equivalent height,

\( h^*_\text{measured} \) = measured equivalent height,

\( n \) = number of total data points.

RMSE and ARD for data were 0.16 and 7%, respectively.

Table 1 Model parameters for all three phases at the center (r=0 mm) of the cylindrical die filled with the BPM powder at 20 mm/s feed shoe speeds

| Phase # | \( h^*F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|---------|---------------|-----------|----------|---------------|
| I       | 32.63         | 0         | -6.79    | 0 \( \leq t \leq 1.8 \) |
| II      | 0             | 0         | -6.79    | 1.8 \( \leq t \leq 3.4 \) |
| III     | 0             | 0         | 0        | 3.4 \( \leq t \leq 7.8 \) |
4.1.2 Model validation

Tables 2 and 3 show the coefficients for \( r = 4 \text{ mm} \) obtained from data points, and \( r = 2 \text{ mm} \) which was obtained from interpolation of \( r = 0 \text{ mm} \) and \( r = 4 \text{ mm} \), respectively. The RMSE and ARD were 0.12 dm and 5%, respectively, which showed that the calculated values are in good agreement with measured values. The pressure profiles for measured and modeled data points at \( r = 2 \text{ mm} \) are shown in Fig. 6.

Table 2 Model parameters for all three phases of at \( r = 4 \text{ mm} \) of the cylindrical die filled with the BPM powder at 20 mm/s feed shoe speed.

| Phase # | \( h^*_F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|---------|-----------------|-------------|-------------|---------------|
| I       | \( h^*_F(e^{5.56t - 1}) \) | 29.87       | 0           | 0 \( \leq t \leq 1.8 \) |
| II      | 0               | 0           | -6.46       | 1.8 \( \leq t \leq 3.4 \) |
| III     | 0               | 0           | 0           | 3.4 \( \leq t \leq 7.8 \) |

Table 3 Model parameters for all three phases at \( r = 2 \text{ mm} \), which were obtained from interpolation of \( r = 0 \text{ mm} \) and \( r = 4 \text{ mm} \) of the cylindrical die filled with the BPM powder at 20 mm/s feed shoe speed.

| Phase # | \( h^*_F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|---------|-----------------|-------------|-------------|---------------|
| I       | \( h^*_F(e^{5.43t - 1}) \) | 31.25       | 0           | 0 \( \leq t \leq 1.8 \) |
| II      | 0               | 0           | -6.13       | 1.8 \( \leq t \leq 3.4 \) |
| III     | 0               | 0           | 0           | 3.4 \( \leq t \leq 7.8 \) |

4.2 Rainy fill

Fig. 7 shows a typical pressure profile for a cylindrical shallow die filled with BPM at 26 g/min filling rate. The plot represents the average of six runs with pressure sensor strip located at 0°-180° orientation. The entire filling profile was divided into four phases as follows:

- Phase I (\( T_1 \)) = filling process (the rotational rainy filled the die completely at this phase (43 s));
- Phase II (\( T_2 \)) = leveling process (removing surcharge powder manually) (46 s);
- Phase III (\( T_3 \)) = end of leveling process (49 s);
- Phase IV (\( T_4 \)) = equilibrium period (end of filling process) (68 s).

4.2.1 Phase I

Phase I was the filling process during which the die was overfilled. In rotational rainy fill, the average time for this process was 43 seconds. The powder filling rate (the slope of mass ratio curve) increased rapidly with time and gradually decreased after 20 seconds. The die was over-filled after 20 seconds and surcharge powder started to accumulate after this time as discussed previously. Phase I had a sigmoidal shape curve which was represented by Equation (11); the specific form was

\[
h^* = 19.32(1 - e^{-0.44t})^3\]

with R-square of 0.94.

4.2.2 Phase II

During this phase, the surcharge powder was removed from the die. The leveling process generated high pressure inside the die due to increase in the shear stress among particles. The leveling process was accomplished manually by using a fiberglass tube (Roudsari, 2007). Based on experimental results, the rate Equation (11) reduces to:

\[
\frac{dh^*}{dt} = m_2
\]

where \( m_2 \) = slope of the straight line (\( m_2 \) = 0.16). The R-square value of this regression was 0.94.

4.2.3 Phase III

During this phase, the rate Equation (11) takes on the following form:

\[
\frac{dh^*}{dt} = m_3
\]

where \( m_3 \) = slope of the straight line (\( m_3 \) = 5.08). The R-square value of this regression was 0.89. The main reason for Phase III not being a straight line might be due to the pressure release inside the die and
rearrangement that occurred after completion of the leveling process.

### 4.2.4 Phase IV

Phase IV was the last stage of rainy filling process, where the pressure became stable. The rate Equation (11) for Phase IV reduces to \( \frac{dh}{dt} = 0 \).

For the entire deposition process, the root mean square error (RMSE) and average relative difference (ARD) for rainy fill were 0.44 dm and 16%, respectively. The differences between measured data and model calculated values were relatively higher vs. feed shoe filling. The coefficient and specific form of the function \( F(t) \) [Equation (15)], and the range of times are shown in Table 4.

### 4.2.5 Model validation

Tables 5 and 6 show the coefficients for \( r=4 \) mm obtained from data and \( r=2 \) mm obtained from interpolation of \( r=0 \) mm and \( r=4 \) mm, respectively. The RMSE and ARD for \( r=2 \) mm from interpolation values were 0.18 dm and 15%, respectively (Fig. 8), which are comparable with magnitudes to the \( r=0 \) and \( r=4 \) data sets; therefore, the model validation is acceptable.

#### 4.3 Point Feed

Fig. 9 shows the entire filling profile for the point feed filling method. The complete filling process can be divided into four phases. Corresponding to each phase, a specific time with each phase of the filling process is listed below:

- Phase I (T₁): filling process (the point feed filled the die completely at this phase) (102 s);
- Phase II (T₂): leveling process (removing surcharge powder manually) (105 s);
- Phase III (T₃): end of leveling process (108 s);
- Phase IV (T₄): end of filling process (120 s).

### 4.3.1 Phase I

Phase I had almost constant deposition rate as explained previously. The powder filling increased with time. The die was filled with constant deposition rate at 26 g/min. For this phase, based on experimental results, the rate Equation (11) takes on the following form:

\[
\frac{dh}{dt} = m_1 \times (20)
\]

where \( m_1 \) = slope of the straight line (\( m_1 = 0.031 \)) and R-square was 0.99.

The filling profile for point feed method is consid-

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### Table 4 Model parameters for all four phases at the center (\( r=0 \) mm) of the cylindrical die filled with the BPM powder at 26 g/min

| Phase | \( h^{*}F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|-------|----------------|-----------|---------|---------------|
| I     | \( h^{*}(e^{0.49t} - 1) \) | 19.32 | 0 | 0 \( \leq t \leq 43 \) |
| II    | 0 | 0 | 0 | 43 \( \leq t \leq 46 \) |
| III   | 0 | 0 | 0 | 46 \( \leq t \leq 49 \) |
| IV    | 0 | 0 | 0 | 49 \( \leq t \leq 68 \) |

### Table 5 Model parameters for all four phases at \( r=4 \) mm of the cylindrical die filled with the BPM powder at 26 g/min

| Phase | \( h^{*}F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|-------|----------------|-----------|---------|---------------|
| I     | \( h^{*}(e^{0.55t} - 1) \) | 17.18 | 0 | 0 \( \leq t \leq 43 \) |
| II    | 0 | 0 | 0 | 43 \( \leq t \leq 46 \) |
| III   | 0 | 0 | 0 | 46 \( \leq t \leq 49 \) |
| IV    | 0 | 0 | 0 | 49 \( \leq t \leq 68 \) |

### Table 6 Model parameters for all four phases for \( r=2 \) mm, which were obtained from interpolation of \( r=0 \) mm and \( r=4 \) mm of the cylindrical die filled with the BPM powder at 26 g/min

| Phase | \( h^{*}F(t) \) | \( \alpha \) | \( \beta \) | Time (second) |
|-------|----------------|-----------|---------|---------------|
| I     | \( h^{*}(e^{0.49t} - 1) \) | 18.25 | 0 | 0 \( \leq t \leq 43 \) |
| II    | 0 | 0 | 0 | 43 \( \leq t \leq 46 \) |
| III   | 0 | 0 | 0 | 46 \( \leq t \leq 49 \) |
| IV    | 0 | 0 | 0 | 49 \( \leq t \leq 68 \) |

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Fig. 8 Comparison of measured and modeled equivalent height for cylindrical die using rotational rainy fill at 26 g/min.

Fig. 9 Average filling pressure profile using point feed method at the center of circular die with BPM powder at 26 g/min filling rate.
erably different from feed shoe and rainy fill. The powder deposited in the die with the point feed method is not directly influenced by the die walls; hence, a nearly linear profile. Furthermore, the particles’ accumulation profile continuously failed along angle of internal friction during the filling process which generated a constant head increase rate.

### 4.3.2 Phase II

During this phase, the leveling process occurred, i.e., the surcharge powder was removed manually from the die. The pressure rate rapidly increased due to increase in shear stress during the leveling process. The rate Equation 11 for this phase simplifies to:

\[
\frac{dh^*}{dt} = m_2 \left[\frac{1}{10^2} \right],
\]

where \(m_2 = 8.25\) (slope of the line) and R-square was 0.99.

### 4.3.3 Phase III

Phase III was end of the leveling process. The pressure quickly decreased after the surcharge powder was completely removed from the die. Due to high shear stress during the manual leveling process, the pressure was slowly released that resulted in a non-linear declining pressure profile; this was approximated by a straight line. Therefore, the rate Equation (11) for Phase III was:

\[
\frac{dh^*}{dt} = m_3 \left[\frac{1}{10^8} \right] (21)
\]

where \(m_3 = -19.52\) (slope of the line) and R-square was 0.94.

### 4.3.4 Phase IV

Phase IV was modeled as a horizontal line. The modeled equivalent height was 3.11 dm. This was obtained by taking the average of all the data points within this phase. The rate Equation (11) for this phase was \(dh^*/dt = 0\).

For the entire deposition process, the root mean square error (RMSE) and average relative difference (ARD) for point feed filling method using a battery powder mixture at 26 g/min were 0.32 dm and 11%, respectively, which represents good validation for this study (Fig. 10).

The entire pressure increase profile for cylindrical die filled with battery powder mixture can be divided into different stages. All the stages could be simulated by a rate equation, based on the data collected and the physics of the filling process. The overall rate equation \(dh^*/dt = \alpha h F(t) + \beta\) can serve as a quantitative tool for further investigation of different powders and die shapes.

Although the scope of this study was limited to one powder and one die shape, the parameters of simulation are general in nature and applicable to various powder and die shapes.

### Table 7

| Phase | \(m\) | Time (second) |
|-------|-------|---------------|
| I     | 0.0311| \(0 \leq t < 10^2\) |
| II    | 8.25  | \(10^2 \leq t < 10^5\) |
| III   | -19.52| \(10^5 \leq t < 10^8\) |
| IV    | 0     | \(10^8 \leq t < 120\) |

### Table 8

| Phase | \(m\) | Time (second) |
|-------|-------|---------------|
| I     | 0.055 | \(0 \leq t < 10^2\) |
| II    | 8.87  | \(10^2 \leq t < 10^5\) |
| III   | -18.65| \(10^5 \leq t < 10^8\) |
| IV    | 0     | \(10^8 \leq t < 120\) |

### Table 9

| Phase | \(m\) | Time (second) |
|-------|-------|---------------|
| I     | 0.043 | \(0 \leq t < 10^2\) |
| II    | 8.56  | \(10^2 \leq t < 10^5\) |
| III   | -19.08| \(10^5 \leq t < 10^8\) |
| IV    | 0     | \(10^8 \leq t < 120\) |

![Fig. 10 Comparison of measured and modeled equivalent height for cylindrical die using point feed filling method 26 g/min.](image-url)
powders and die shapes. In addition, the mathematical model permits consideration of simultaneous filling of multiple shallow dies. The mathematical models, which defines some or all of the deposition physics-based parameters, can be a very time efficient, cost effective, and reliable tool for identifying and addressing complex flow and filling issues.

Conclusions

Experimental data showed that the deposition profiles for feed shoe and rainy fill were sigmoidal in shape vs. a linear profile for point feed. For the complex filling profiles, a physics based model was used to explain the sigmoidal pressure build up profile for the feed shoe and rainy filling methods and the linear profile for the point feed. The following specific conclusions were arrived at from this study:

1) The overall rate equation for all three filling methods was: $\frac{dh^*}{dt} = \alpha h F(t) + \beta$.

2) The root mean square error (RMSE) for feed shoe, rainy, and point feed were 0.16, 0.44, and 0.32, respectively; whereas, the average relative differences (ARD) were 7%, 16%, and 11%, respectively.

3) For model validation, the RMSE values for feed shoe, rainy, and point feed were 0.15, 0.18, and 0.14, respectively; whereas, the ARD values were 5%, 15%, and 8%, respectively.

The validated powder deposition models can be used to optimize the slow filling of shallow dies so as to minimize defects associated with the filling process during compaction. Following the successful modeling of die filling at slow speeds, development and validation of models at higher deposition rates are ongoing. Incorporation of the particle characteristics through a multi-scale formulation of the deposition model would represent a leap-forward in the field of high pressure compaction.

Nomenclature

$A$ Cross-section
$b$ Parameter
$c$ Coefficient
$C$ Die circumference
$F(\tau)$ Stage-specific deposition rate function
$F(t)$ Deposition rate function
$g$ Gravitational constants
$h$ Die height
$h^*$ Equivalent height
$H$ Total height of powder in die

$H_o$ Surcharge height in die
$k$ Ratio of lateral to vertical pressure
$K$ Coefficient
$m$ Rate of outflow
$n_{i1}$ Mass
$n_{i2}$ Rate of inflow
$P_v$ Vertical friction on the wall
$P_r$ Vertical pressure in the mass
$P_l$ Lateral pressure
$P$ Pressure
$\dot{Q}$ Flow rate
$r$ Radius of sphere
$R$ hydraulic radius
$t_f$ Time of filling
$t$ Time
$v$ Volume
$y$ Actual filled height of die

Greek Letters

$\alpha$ Coefficient
$\beta$ Coefficient
$\delta$ Cone half-angle
$\varphi$ Angle of internal friction
$\rho_p$ Particle density
$\rho_b$ Bulk density
$\mu$ Coefficient of friction
$\pi$ Normalized time
$\theta$ Chord radius/r
$\gamma_1, \gamma_2, \gamma_3, \gamma_4$ Coefficients

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Author’s short biography

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Saed Sayyar Roudsari was a graduate student in the Department of Agricultural and Biological Engineering at Pennsylvania State University from August 2004 to September 2007. He received his BS in Soil and Water Engineering from Iran and MS in Civil Engineering from North Carolina A&T State University. He is currently working as a professional engineer in County of Los Angeles Department of Public Works. He is an active member of ASABE, ASCE, and ISPE. This paper is a part of his Ph.D. work at Pennsylvania State University.

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Virendra M. Puri, University Distinguished Professor, has been involved in research in the field of powder science, engineering, and technology for nearly three decades. He has served as the Acting Director of the NSF/IUCRC (Industry University Cooperative Research Center) – the Particulate Materials Center. Professor Puri has co-authored over 500 publications and co-inventor and holder of patents for four test devices in the area of powder flow, deposition and compaction. In addition, he has a Copyright for multi-purpose computational software dealing with powder processing applications. Professor Puri has been invited to serve on Editorial Boards, International Advisory Boards, and Chairpersons of several bulk solids-related publications and professional activities. He is Editor-in-Chief of Particulate Science and Technology, An International Journal. Professor Puri regularly offers seminars, courses, and workshops in powder mechanics to industry and academia. He has received several teaching and research awards.