Improved Worst-Group Robustness via Classifier Retraining on Independent Splits

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Abstract

High-capacity deep neural networks (DNNs) trained with Empirical Risk Minimization (ERM) often suffer from poor worst-group accuracy despite good on-average performance, where worst-group accuracy measures a model’s robustness towards certain subpopulations of the input space. Spurious correlations and memorization behaviors of ERM trained DNNs are typically attributed to this degradation in performance. We develop a method, called CRIS, that address these issues by performing robust classifier retraining on independent splits of the dataset. This results in a simple method that improves upon state-of-the-art methods, such as Group DRO, on standard datasets while relying on much fewer group labels and little additional hyperparameter tuning.

1 Introduction

Modern high-capacity deep neural networks (DNNs) can achieve super-human performance in tasks where the test distribution is identical to and independent of the train distribution or i.i.d. generalization. However, it has been well-known that DNNs perform substantially worse when the test distribution differs from its train distribution. This is known as the problem of out-of-distribution (o.o.d.) generalization or domain generalization. Oftentimes, performance beyond i.i.d. generalization is desired. This is crucial in safety critical applications like self-driving cars [Filos et al., 2020] or medical imaging [Oakden-Rayner et al., 2020], where any error can be detrimental. Furthermore, fairness issues can occur in applications like loan approval [Hardt et al., 2016] or facial recognition [Buolamwini and Gebru, 2018] when models make predictions using spurious features. Hence, addressing the problem of o.o.d. generalization is crucial for real-world deployment of deep learning models.

A special case of the o.o.d. generalization problem is the group-shift problem, where the proportion of different “groups” is different at test time than at training time. In this setting, there are predefined attributes that divide the input space into different groups of interest. Here, the goal is to find a model that performs well across these predefined groups [Sagawa et al., 2020a]. Similarly to other problems in domain generalization, DNNs trained by Empirical Risk Minimization (ERM) are observed to suffer from poor worst-group performance despite good on-average performance.

The poor robust performance is often attributed to the phenomenon of “shortcut learning” [Geirhos et al., 2020] or “spurious correlation” [Sagawa et al., 2020a, Arjovsky et al., 2019] in DNNs trained with ERM. Shortcut learning poses that the ERM procedure favors those models that discriminate based on “simpler” and/or “spurious” features of the data. However, one wishes for the learning algorithm to produce a model that uses features (i.e. correlations) that performs well not only on the train distribution, but also on all potential distributions that a task may generate, like that of a worst-group distribution.

In recent years, the group-shift problem has received considerable attention. [Sagawa et al., 2020a] first investigates distributional robust optimization (DRO) [Duchi et al., 2021, Ben-Tal et al., 2013] in this setting and introduces its Group DRO (GDRO) algorithm that attempts to directly optimize for the worst-group error. Since then, the GDRO algorithm has been the standard and state-of-the-art method for producing group-robust models. However, one of the biggest weaknesses of GDRO is that it requires group annotations.
Figure 1: tSNE projection of the features of an ERM trained ResNet50 on seen (top) versus unseen (bottom) examples from Waterbird. This was obtained by using 50% of the data to train a high capacity DNN and saving the rest to generate the features for the unseen examples. The features of the minority groups (orange and yellow) from the unseen examples are better separated from the majority groups than that of the seen examples. Using unseen examples for robust classifier retraining plays a major role in improving worst-group performance as shown in this paper.

Our contribution. In this work, we propose a new approach to achieve a group-robust model that reduces the amount of group labels needed without relying on a multitude of hyperparameters and large scale tuning, which is a growing concern in the community. Our method combines insights from previous works on representations of ERM trained models and ERM’s potential theoretical promises in domain generalization with findings on memorization behaviors of overparameterized DNNs in the group shift and data imbalance setting.

In short, our method takes advantage of the supposedly good features of a high-capacity DNN trained with ERM while also overcoming the deficiency of its memorization behavior by partitioning the training set into two independent splits: one group-unlabeled split to train the feature extractor and one group-labeled split to retrain the classifier with a robust learning algorithm like GDRO. This allows for the ability to incorporate any amount of group-unlabeled data to improve the training algorithm and separate the potentially sensitive robust training phase to only a low-capacity linear layer. Our experimental results show competitive robust performance to GDRO on standard datasets (Waterbird, CelebA, MultiNLI, and CivilComments) even when using only a fraction of group annotations and not much hyperparameter tuning. For example, CRIS outperforms GDRO by 2.3% on CelebA using 30% of group labels that GDRO needs and yields competitive performance with GDRO on Waterbird and MultiNLI when using 30% of the group labels. CRIS’s performance provides further evidences of ERM trained DNNs containing good features on not only vision datasets (per Menon et al. 2021) but also on natural language tasks.

In this work, good features (w.r.t. a certain task) means that there exists a linear classifier utilizing these features that perform well on such desired task, where features refer to the inputs of the final linear layer of a DNN.
1.1 Related Works

There are three main settings that previous works in the group-shift area have considered: (1) training group labels are available, (2) a limited amount of group labels is available, and (3) no group label is available. Other related areas include domain generalization, causal learning, and long-tailed classification.

**With Training Group Labels.** Most methods here revolve around up-weighting minority groups, subsampling minority groups [Sagawa et al., 2020b], or performing GDRO [Sagawa et al., 2020a]. Follow-up works in the first setting where training group labels are available include integrating data augmentation via generative model [Goel et al., 2020] or selective augmentation [Yao et al., 2022] to a robust training pipeline.

**Limited Access to Group Labels and No Group Label.** In this setting, most works try to infer more group labels for the group-unlabeled data. These pseudo-group-labels are usually generated by training a referenced model that performs the labeling. For example, [Liu et al., 2021] utilizes a low-capacity model that create groups by labeling whether an example is correctly classified by the referenced model or not. Similarly, works like [Creager et al., 2021, Dagaev et al., 2021, Krueger et al., 2021, Nam et al., 2022] and [Nam et al., 2020] are variants of this approach of inferring pseudo-group-labels using a referenced model. These methods then proceed to use a group-robust algorithm like GDRO [Sagawa et al., 2020a] or Invariant Risk Minimization (IRM) [Arjovsky et al., 2019] to perform robust training on a new network with the newly generated pseudo-group-labels. [Pezeshki et al., 2021] modifies the dynamic of gradient descent to avoid learning spurious features. Most of the time, the use of real group labels in this setting are for validation and hyperparameter-tuning purposes.

The no-group label setting simply removes the ability to validate as well as knowledge of potential groups. This makes the problem significantly more difficult as it is unknown for which correlation to look for. While [Sohoni et al., 2020] is the only work in this space, it popularizes the pseudo-group-labels inferring and retraining approach.

**Domain Generalization and Causal Learning.** Domain Generalization and Causal Learning captures the group-shift problem. An important work by [Gulrajani and Lopez-Paz, 2021] performs a mass-scale experiment to find out that most algorithms that claim to improve o.o.d. generalization over ERM actually do not, given the same amount of hyperparameter tuning and whether model-selection is based on knowledge of the test distribution or not. Similarly, [Rosenfeld et al., 2021] proves that ERM is optimal in the linear domain generalization setting.

Related to the domain generalization problem is the fundamental problem of causal learning or causal inference [Peters et al., 2017]. This framework promises that underlying every task is a correlation that explains such task, say through a generative model or a structural equation model. As a result, causal learning frameworks like IRM [Arjovsky et al., 2019] has been proposed, which are widely used as a group-robust algorithm in the group-shift setting. However, it still seems out of grasp to provably pin down the problem of causal learning, as pointed out by [Rosenfeld et al., 2020].

**Long-tailed Classification.** The long-tailed problem concerns with certain classes having significantly less training examples than others (see for example [Haixiang et al., 2017] or [Zhang et al., 2021] for a survey). In this setting, one cares not only about the average performance but also performance across all classes, even those with fewer training examples. Hence, in the group shift setting, if one considers classifying the groups rather than classifying the labels, one might hypothesize that poor worst-group performances comes from the data imbalances between groups. Some techniques from the long-tail literature, like [Cao et al., 2019], has been applied to the group-shift setting to account for the groups imbalances, as in [Sagawa et al., 2020a]. Insights from applications of representation learning in the long-tail problem [Kang et al., 2019] gives valuable evidences for ERM trained DNNs containing good features which will be central in developing the method detailed in this paper.

2 Preliminaries

Suppose that we care about performing well on a certain classification task $T$ that is to predict labels in $\mathcal{Y}$ from inputs in $\mathcal{X}$. We are given training examples $\{(x_i, y_i)\}_{i=1}^{n}$ that are drawn i.i.d. from some train
distribution $D_{\text{train}}$. In the domain generalization setting, we want good performance on some unknown test distribution $D_{\text{test}}$ that is different but related to $D_{\text{train}}$ through the task $T$. More explicitly, we wish to find a classifier $f$ from some hypothesis space $\mathcal{F}$ using $D_{\text{train}}$ such that the classification error

$$L(f) = \mathbb{E}_{x,y \sim D_{\text{test}}} \left[ f(x) \neq y \right]$$

of $f$ w.r.t. $D_{\text{test}}$ is low. This framework encapsulates many problems like adversarial robustness, domain adaptation, long-tail classification (which includes few-shot learning), and to be discussed, group-shift.

In the group shift setting as in [Sagawa et al., 2020a], we further assume that associated with each data point $x$ is an attribute $a(x)$ (some sub-property or statistics of $x$) from a set of possible attributes $\mathcal{A}$. These attributes along with the labels form the set of possible groups $\mathcal{G} = \mathcal{A} \times \mathcal{Y}$ that each example can take. We denote an input $x$’s group label as $g(x) \in \mathcal{G}$. We then define the classification error of a predictor $f$ (w.r.t. a fixed implicit distribution) restricted to a group $g \in \mathcal{G}$ to be

$$L_g(f) := \mathbb{E}_{x,y \mid g(x) = g} \left[ f(x) \neq y \right].$$

The notion of worst-group error upper bounds the error of $f$ w.r.t. any group

$$L_{wg}(f) := \max_{g \in \mathcal{G}} L_g(f).$$

Using this notation, the group-shift problem aims to discover a classifier in

$$\arg \min_{f \in \mathcal{F}} \{ L_{wg}(f) \} = \arg \min_{f \in \mathcal{F}} \{ \max_{g \in \mathcal{G}} L_g(f) \}.$$

The GDRO algorithm solves this objective by performing a minimax optimization procedure that alternates between the model’s weight and relaxed weights on the groups. Here, we see that the group-shift problem is just a special case of the domain generalization problem when $D_{\text{test}}$ is the distribution consisting of only the points $(x, y)$ with $g(x)$ being restricted to the worst-group of $f$ in $\mathcal{G}$. 

### 2.1 Spurious Correlations

Consider the Waterbird dataset in [Sagawa et al., 2020a], where the task is to distinguish whether an image of a bird is a waterbird or a landbird. The authors in [Sagawa et al., 2020a] use the bird images from the CUB dataset [Welinder et al., 2010] and modify their background with water or land backgrounds from the PLACES dataset [Zhou et al., 2017]. These backgrounds denote the attribute $\mathcal{A}$ for each image, and along with the type of bird, form four groups: $\mathcal{G} = \mathcal{Y} \times \mathcal{A} = \{\text{waterbird, landbird}\} \times \{\text{water, land}\}$.

This dataset was constructed so that the proportion of birds on matching backgrounds (i.e. waterbird on water background and landbird on land background) is significantly more than those of the mismatched backgrounds. This is so that the backgrounds could be spuriously correlated with the labels, as predicting the background alone would achieve a high on average accuracy w.r.t. the train distribution already. As expected, for ERM trained models, the groups with the highest error are the minority groups where the background mismatches the type of the bird, giving evidences that the model is actually predicting using the background instead of the bird. Furthermore, the fact that these high-capacity models achieve zero training error leads to the conclusion that these models not only utilize spurious features like the background to make its predictions, but also must have memorized the minority groups during its training process [Sagawa et al., 2020b]. In the next section, we propose a method that attempts to circumvent these downsides.

### 3 Proposed Method: Classifier Retraining on Independent Splits

In this section, we present our main method: Classifier Retraining on Independent Splits or CRIS. Algorithm II provides an outline for CRIS. The method is simple: (1) partition the group-labeled data into
Algorithm 1 Classifier Retraining on Independent Splits

1: **Input:** Training data $D_L$ with group labels, training data without group labels $D_U$, and validation data $D_{val}$ with group labels.
2: **Additional parameters:** Split proportion $p$ and classifier retraining algorithm $R$.
3: Partition $D_L$ into two parts, $D_1$ and $D_2$, each with proportion $|D_1| = (1-p) \cdot |D_L|$ and $|D_2| = p \cdot |D_L|$.
4: Obtain the initial model $f$ by running ERM on $D_1 \cup D_U$ and then selecting the best model in terms of average accuracy on $D_{val}$.
5: Perform classifier retraining $R$ with feature extractor $f$ on $D_2$ and then select the best model in terms of worst-group accuracy on $D_{val}$ as the final output.

3.1 Why Classifier Retraining? Why Independent Splits?

Inspired by works that demonstrate the potential of simple ERM trained DNNs on a variety of o.o.d. tasks (as in [Gulrajani and Lopez-Paz, 2021] and [Rosenfeld et al., 2021]) and long-tailed tasks (as in [Kang et al., 2019]), our method focuses on developing a simple method around ERM trained models. As soon to be discussed, a strength of an ERM trained DNN is that there are good evidences that it contains **good features**. On the other hand, its **weaknesses** involve memorizing examples and using spurious features. CRIS has been designed to alleviate these weaknesses while taking advantage of the good features of an ERM trained DNN.

**Models Trained by ERM Contains Good Features.** While not being the first work to notice that ERM trained models contain good features, [Kang et al., 2019] demonstrates this hypothesis by performing extensive experiments on several long-tailed vision datasets to investigate different learning strategies for obtaining a feature extractor as well as ways to fine-tune the classifier layer. This paper finds that an ERM trained feature extractor combined with a non-parametric method of rescaling 2 the classifier layer achieves (then) state-of-the-art results on all three datasets. A similar study [Menon et al., 2021] has been done to confirm this hypothesis on vision datasets in the group-shift setting. This suggests that one key to the data-imbalance problem (for which the group-shift problem suffers) lies in correcting the classifier layer, for which forms the first phase of CRIS. Now, it is peculiar that rescaling the classifier works best whereas intuition suggests a data-dependent method like classifier retraining would be better. We hypothesize that this is related to the next issue of our discussion.

**Memorization Behavior of High Capacity DNNs** It has been well known of the remarkable ability of high-capacity DNNs to memorize training examples [Zhang et al., 2017]. In the group shift setting, this behavior has been investigated by [Sagawa et al., 2020], which provides empirical and theoretical justifications for DNNs’ memorization behavior of minority groups’ training examples. Furthermore, this memorization behavior of minority (or atypical) examples have been observed in the broader framework of data imbalances [Feldman and Zhang, 2020, Feldman, 2020]. This memorization behavior of high-capacity DNNs and ERM’s tendency to favor simpler features have been believed to be the cause of spurious correlations which results in poor robust performance. One way to circumvent memorization is to control the model’s capacity by incorporating some combinations of high $\ell_2$ regularization, early stopping, and other correctional parameters. However, this often results in costly parameter search.

**Circumventing Memorization with Independent Splits.** Examples being memorized must inevitably impact their ability to be useful in subsequent usage. As memorized examples’ (i.e. already correctly classified) loss must be low, their gradients contain little information to be of much use. Furthermore, the features of memorized examples might not be representative of their group during test time: Figure 1 presents a visualization of the features between seen versus unseen examples which suggests that the features between

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2 Rescaling each row of the linear classifier using the row’s norm to some power. See tau-norm in [Kang et al., 2019].
different groups of unseen examples are “better separated” than that of seen examples. Hence, CRIS attempts to circumvent the problem of memorization by using unseen examples ($D_2$ in Algorithm 1) to perform robust classifier retraining. The hope is that these unseen examples’ features, especially of the minority or memorized groups, are more representative of those seen at test time and hence provide sufficient information for robust classifier retraining.

4 Experiments

In this section, we detail our experimental setup along with the main results of our experiments. We then provide additional experiments and key discussions.

4.1 Experimental Setup

Datasets. We experiment on four standard datasets:

- **Waterbird** [Sagawa et al., 2020a]. Combining the bird images from the CUB dataset [Welinder et al., 2010] with water or land backgrounds from the PLACES dataset [Zhou et al., 2017], the task is to classify whether an image contains a landbird or a waterbird without confounding with the background. There are 4,795 total training examples, where the minority group (waterbird, land background) has only 56 examples.

- **CelebA** [Liu et al., 2015]. CelebA is a popular image dataset of celebrity faces. The task is to detect whether the celebrity in the image is blond or not blond, with being male or not male as the confounding attribute. There are 162,770 total training examples, and the smallest group (blond male) has 1,387 examples.

- **MultiNLI** [Williams et al., 2017]. MultiNLI is a natural language inference (NLI) dataset that attempts to determine whether a sentence’s hypothesis is entailed by, is neutral with, or contradicts its premise. The spurious attribute is thought to be the presence of negation words like no, never, or nothing. This task has 6 groups in total, with 206,175 total training examples and 1,521 examples in the minority group (is entailed and contains negation).

- **CivilComments-WILDS** [Borkan et al., 2019, Koh et al., 2021]. A natural language dataset where the task is to classify whether a sentence is toxic or non-toxic. The confounding variables are various demographics mentioned in the sentences: male, female, white, black, LGBTQ, Muslim, Christian, and other religion. This forms 16 groups that overlap, because a comment can contain multiple demographics. Following [Koh et al., 2021], we evaluate on all 16 groups but only train using the attribute black along with the label. There is a total of 269,038 training examples with (other religion, toxic) being the minority group containing 1,045 training examples. In our method, the proxy minority group, (black, toxic), contains 3,111 training examples.

Models. We use ResNet50 [He et al., 2016] with ImageNet initialization and batch-normalization for CelebA and Waterbird and use BERT [Devlin et al., 2018] for MultiNLI and CivilComments. We use the original train-val-test split in all the datasets and report the test results while using the validation set only for model selection (except in Section 4.3). Cross-entropy is used as the base for all objectives. SGD with momentum 0.9 is used for the vision datasets and the AdamW optimizer with dropout and a fixed linearly-decaying learning rate is used for BERT. We use a batch size of 16 for CivilComments and 32 for the rest of the datasets. We do not use any additional data augmentation or learning rate scheduler in our results.

Hyperparameters We use a similar setup as [Sagawa et al., 2020a]. One main difference in our experiments is that, unless explicitly specified, we fix the hyperparameters of both the ERM and the robust classifier retraining phase, using standard parameters for ERM (see Appendix A).

4.2 Main Results

We present our main results for the four datasets in Tables 1, 2, 3, and 4. We report the Test Average Accuracy (Test Avg Acc) and Test Worst-Group Accuracy (Test Wg Acc). In this section, we stress that the
validation set is used only for validation (i.e. model selection) and not for classifier retraining (in contrast to section 4.3). In these tables, the numbers in parentheses denote one standard deviation from the mean over three random seeds.

In this section, we use GDRO as the classifier retraining algorithm for CRIS. In Tables 1 and 2, GA denotes Group Adjustments \cite{Cao:19} as used in \cite{Sagawa:20a}. These numbers for GDRO and GDRO + GA are taken from Table 1 and 2 of \cite{Sagawa:20a}, respectively. We also provide an ablation study of CRIS with different values of split proportion $p$. For each proportion $p$, we also provide the expected number of minority examples, $\min n_g$, sampled from performing random split with $p$ to obtain the group-labeled split $D_2$. Furthermore, to study the effects of independent split, we experiment with performing classifier retraining with GDRO using the same data from the first phase i.e. without independent splits: this is denoted as Naive CRT in the tables.

Table 1: Results for Waterbird. GA refers to adding Group Adjustment. \textsuperscript{†}From Table 1 and 2 in \cite{Sagawa:20a}.

| $p$ | $\min n_g$ | CRIS with GDRO (Ours) | Naive CRT | GDRO + GA\textsuperscript{†} | CRIS 0.3 + GA |
|-----|-------------|------------------------|-----------|-----------------------------|-------------|
| 0.10 | 5           | 95.4 (1.10)            | 83.5 (3.24) | 93.7                        | 90.5        |
| 0.30 | 16          | 90.8 (0.35)            | 89.6 (1.15) | 90.5                        | 90.3        |
| 0.50 | 28          | 90.4                   | 89.3       | 90.5                        | 90.3 (0.62) |
| 0.70 | 39          | 90.3                   | 90.0       | 90.5                        | 90.3 (0.62) |

**Waterbird.** Table 1 presents the main results for Waterbird. We use the standard train-val-test split and report the weighted test average accuracy due to the skewed nature of the val and test set – this is standard per \cite{Sagawa:20a}. Here, CRIS with GDRO yields better worst-group performance than standard GDRO even when using only 30% of the group labels.

In \cite{Sagawa:20a}, Group Adjustment (GA) \cite{Cao:19} is observed to improve Waterbird’s worst-group performance (but not for CelebA nor MultiNLI). Hence, we also experiment GDRO with Group Adjustment as the classifier retraining objective and observe similar (but less dramatic) improvements. Finally, we remark that for $p = 0.10$, WG accuracies are a lot more varied since it heavily depends on the amount of minority group’s examples sampled for the classifier retraining phase.

**CelebA.** Table 2 presents the main results for CelebA. Here, CRIS outperforms GDRO even when only using 5% of the group-labeled examples. Overall, setting $p$ to 0.30 seems to achieve a good balance between having enough examples to obtain a good feature extractor and having enough examples to adjust the biased ERM classifier.

**MultiNLI.** Table 3 contains the main results for MultiNLI. Again, without much hyperparameter tuning and using only a fraction of the amount of group annotations, CRIS is competitive with GDRO. This gives some of the first evidence that an ERM trained BERT model contains good features for the group-shift task.

**CivilComments.** Table 4 contains the main results for CivilComments. Again, CRIS yields competitive performance to GDRO while using fewer group labels. Given the relatively severe data imbalance between the groups (the minority group consist of only 0.4% of the dataset), we see that using a larger proportion of group-labeled examples allows obtaining more minority group examples for classifier retraining, of which we observe to improve robust performance and produce stabler results. Hence, the amount of minority examples the algorithm incorporates is more important than the proportion $p$ used.
Table 2: Results for CelebA. GA refers to adding Group Adjustment. †From Table 1 and 2 in Sagawa et al., 2020a.

|                  | Test Avg Acc | Test Wg Acc |
|------------------|--------------|-------------|
| ERM              | 95.6         | 44.4        |
| GDRO†            | 91.8         | 88.3        |
| \( p \) \( \min n_g \) | CRIS with GDRO (Ours) |            |
| 0.05 69          | 91.9 (0.50)  | 88.9 (1.10) |
| 0.10 138         | 91.3 (0.36)  | 90.3 (0.82) |
| 0.30 416         | 91.3 (0.44)  | **90.6 (0.95)** |
| 0.50 693         | 92.2         | 86.1        |
| 0.70 970         | 92.2         | 87.2        |
| Naive CRT        | 93.9         | 69.2        |
| GDRO + GA†       | 93.4         | 87.8        |

Table 3: Results for MultiNLI. †From Table 1 in Sagawa et al., 2020a.

|                  | Test Avg Acc | Test Wg Acc |
|------------------|--------------|-------------|
| ERM              | 82.8         | 66.0        |
| GDRO†            | 81.4         | 77.7        |
| \( p \) \( \min n_g \) | CRIS with GDRO (Ours) |            |
| 0.05 76          | 81.8 (0.15)  | 73.8 (1.54) |
| 0.10 152         | 80.8 (0.51)  | 75.3 (2.06) |
| 0.30 456         | 80.0 (0.31)  | **77.9 (0.17)** |
| 0.50 760         | 79.9         | 74.8        |
| 0.70 1,064       | 77.8         | 72.1        |
| Naive CRT        | 82.3         | 67.9        |

4.3 Limited Availability: Group labels from only the validation set

In this setting, we only utilize the group labels from the validation set and none from the training set as have commonly been done in recent works Liu et al., 2021, Nam et al., 2022. While most methods in this setting have to infer pseudo-group-labels that add additional hyperparameters, CRIS readily adapts to this setting by training the feature extractor with all the group-unlabeled training data and saving some validation data for the classifier retraining phase. Table 4 contains our main results for all the datasets where the validation set is split in half. Note that since different datasets have different validation set sizes and distributions, the result here for some datasets might not be comparable with the previous section. This is especially true with Waterbird since the validation set is skewed towards the minority group. Hence, we compare CRIS against other methods in the same setting. The results show that CRIS outperform other methods on most datasets even without much tuning.

4.4 Further Ablation Studies

Effects of Different Classifier Retraining Methods. Table 5 contains results on using different classifier retraining methods. We see that GDRO gives the best robust performance as expected (after all, GDRO is designed to minimize such worst-group objective). Reweighing and subsampling seem to be highly effective in the vision datasets, but those methods fail to do the same for the other datasets.

Effects of Validation Accuracies on Features’ Quality. An ablation study on the effects of different
Table 4: Results for CivilComments. Since we use a proxy minority group for training, we do not include \( \min n_g \) here.

| Method             | Test Acc | Test Wg Acc |
|--------------------|----------|-------------|
| ERM                | 92.1     | 63.2        |
| GDRO               | 89.6 (0.23) | 70.5 (2.10) |
| \( p \) CRIS with GDRO (Ours) |          |             |
| 0.05               | 90.8 (0.40) | 63.3 (7.82) |
| 0.10               | 89.5 (1.81) | 68.7 (1.72) |
| 0.30               | 89.7 (0.33) | 68.6 (1.53) |
| 0.50               | 89.5 (0.70) | 71.0 (1.50) |
| 0.70               | 89.4 (0.23) | 69.0 (0.12) |

Table 5: Experimental results for CRIS when only using the validation set. Here, CRIS is performed with half of the validation set being used for robust classifier retraining and the other half for validation. Results are reported over three random seeds. Results for JTT and SSA are taken from [Liu et al., 2021] and [Nam et al., 2022], respectively.

| Method       | Waterbird | CelebA | MultiNLI | CivilComments |
|--------------|-----------|--------|----------|--------------|
|              | Avg Acc   | Wg Acc | Avg Acc  | Wg Acc       | Avg Acc  | Wg Acc | Avg Acc  | Wg Acc |
| JTT [Liu et al., 2021] | 93.9      | 86.7   | 88.0     | 88.1         | 78.6     | 72.6   | 91.1     | 69.3   |
| SSA [Nam et al., 2022]    | 92.2 (0.87) | 89.0 (0.55) | 92.8 (0.11) | 89.8 (1.28) | 79.9 (0.87) | 76.6 (0.66) | 88.2 (1.95) | 69.9 (2.02) |
| CRIS (Ours)               | 92.1 (0.20) | 90.9 (0.12) | 91.6 (0.41) | 88.5 (0.87) | 81.4 (0.96) | 77.4 (1.21) | 90.6 (0.20) | 70.3 (0.34) |

validation accuracies on the feature extractor’s quality (measured by robust performance after classifier retraining) is presented in Appendix B.2. The data show that there is a positive correlation between validation average accuracy and the model’s features’ quality. This observation serves as a proxy for CRIS’s model selection criterion. A similar positive correlation is observed when validation worst-group accuracy is used.

**Effects of Early Stopping on Features’ Quality.** We investigate whether different epochs have an effect on the classifier retraining phase in Appendix B.2. A model’s features’ quality can roughly be measured by how well classifier retraining performs. The result in Figure 2 shows that early stopping can be beneficial to avoid overfitting. However, since the highest validation average accuracy is usually realized before the overfitting point, CRIS manages to implicitly avoid this problem and hence reduce the burden of tuning for the feature extractor’s early stopping.

### 4.5 Discussions

The main results have shown that CRIS improves upon GDRO even while using much fewer group labels and requiring little additional parameter tuning. This shows that CRIS is in an effective method that is fast to train and easy to tune. Next, we highlight several key discussions.

**The Importance of Independent Splits on Classifier Retraining.** The results in Section 4.2 shows that performing robust classifier retraining on independent splits is competitive to GDRO on standard datasets. This result would no longer hold without incorporating independent splits in the classifier retraining phase, as evident in the results for *Naive CRT* across all four datasets.

**ERM Trained Models Contain Good Features for The Group-Shift Problem.** The positive
Table 6: For $p = 0.3$, we perform 3 different classifier retraining methods: reweighing, subsampling, and GDRO.

| CRIS ($p = 0.30$) | **Waterbird** | **CelebA** | **MultiNLI** | **CivilComments** |
|-------------------|--------------|------------|--------------|------------------|
| **Retraining Method** | **Avg acc** | **Wg Acc** | **Avg acc** | **Wg Acc** | **Avg acc** | **Wg Acc** | **Avg acc** | **Wg Acc** |
| **Reweighing** | 95.2 | 87.1 | 92.1 | 85.0 | 78.9 | 67.0 | 88.3 | 56.4 |
| **Subsampling** | 95.8 | 81.1 | 91.6 | 86.1 | 78.6 | 64.0 | 91.8 | 59.3 |
| **GDRO** | 91.4 | **90.2** | 91.6 | **90.4** | 80.3 | **78.0** | 89.7 | **69.1** |

result for our decoupled training procedure provides another strong evidence for ERM trained models containing good features for the group-shift problem. While this is consistent with findings in the literatures on vision datasets [Kang et al., 2019, Menon et al., 2021], our work further provides some of the first evidences of this hypothesis in non-vision tasks, where the same result would not have been possible without independent split, as evident in the result for Naive CRT.

**Tradeoffs Between Group-Labeled Data and Group Unlabeled Data.** From the results across the datasets, allocating more examples towards training the feature extractor seems to generally yield higher on-average accuracies. The final worst-group error from classifier retraining has a more complex interaction, as it depends on both the quality of the feature extractor and the amount of group-labeled examples available to perform classifier retraining – this is in contrast to the more well-known tradeoff between average accuracy and worst-group accuracy. While varying the proportion $p$ in our experiments gives a rough estimate of this tradeoff, we observe that the proportion of minority group examples available is the most important for obtaining a robust classifier (see discussion for CivilComments).

**Simplified Model Selection and Hyperparameter Tuning.** The model selection criterion of picking the best average validation accuracy model significantly simplifies hyperparameter tuning in comparison with similar two-phases methods in this group-shift setting. This decision has been chosen mainly from our ablation experiments in Section 4.4, where we observe that higher average validation accuracy generally suggests better features. A similar phenomenon in the long-tailed setting [Kang et al., 2019] has also been observed. Furthermore, running ERM on a high capacity model and limiting GDRO on a low-capacity linear layer make the training process less sensitive to different hyperparameter configurations (also see Appendix B.3).

**Incorporating Group-Unlabeled Data with The Split Proportion $p$.** The split proportion $p$ is used in our experiments primarily because all the datasets used contain full group labels with no separate group-unlabeled set $D_U$. Random sampling simulates the process of obtaining group labels on a portion of the data in hope of improving group-robustness. While we vary the split proportion $p$ in our experiments as a simple mean to investigate the tradeoff with having fewer group labels (in Section 4.2), in reality, there might only be just a few group-labeled examples available say only for $D_{val}$, and most examples would be in the group-unlabeled split $D_U$. In this setting, most works have focused on inferring (relatively costly) pseudo-group-labels. For CRIS, the main decision would on dividing the examples between validation and classifier retraining. As shown in Section 4.3, CRIS is effective in this setting when only using the validation split with a lot less parameter tuning.

**Alleviating a Weakness of GDRO** Another evidence for the ease of tuning of CRIS is that simply using the same parameters as the ERM phase for the classifier retraining phase with GDRO achieves good worst-group performance already. This is in contrast to previous methods requiring high $L_2$ regularization or early stopping to control the model’s capacity and avoid overfitting [Sagawa et al., 2020a]. The low model-capacity’s requirement for GDRO is already in-built for CRIS by restricting GDRO to only the final linear-layer. Hence, this simplifies parameter tuning and closes the generalization gap between training and testing for GDRO.
5 Conclusion and Future Works

In this paper, we propose Classifier Retraining on Independent Split (CRIS) as a simple and easy-to-tune method to reduce the amount of group annotations needed for improving worst-group performance. The experimental results not only show the effectiveness of our method but also provide evidences that ERM trained models contain good features for the group-shift problem, and the key to fit a good classifier on top of those features is to use unseen examples.

**Future Works.** We envision that our findings can further simplify and speed-up the two-phase pseudo-group-labelling methods by only requiring a small amount of pseudo-group-labels to be inferred so that those can be used to perform robust classifier retraining on features of an ERM trained model. On a broader note, while most works in representation learning focus on producing good features (either with supervised, unsupervised, or self-supervised approaches), further examinations into different ways to perform classifier retraining in different settings (as in our work) could give a fuller picture to the features of different methods. Finally, it would be of great interest to obtain better a theoretical understanding of the features that DNNs extract (say w.r.t. ERM using SGD) and such features’ relationship to the task.

Reproducibility

We make our implementation of CRIS in PyTorch [Paszke et al., 2019] publicly available. Our implementation is built on top of the code base from Liu et al. [2021]. Experimental data were collected with the help of the tool Weights & Biases [Biewald, 2020].

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Table 7: Hyperparameters used in the experiments. The slash indicates the parameters used in the first phase (feature extractor) versus the second phase (classifier retraining).

|                      | Waterbird | CelebA | MultiNLI | CivilComments |
|----------------------|-----------|--------|----------|--------------|
| **Learning Rate**    | $10^{-4}/10^{-4}$ | $10^{-4}/10^{-4}$ | $2 \times 10^{-5}/2 \times 10^{-5}$ | $10^{-5}/10^{-5}$ |
| **$\ell_2$ Regularization** | $10^{-4}/10^{-4}$ | $10^{-4}/10^{-4}$ | 0/0 | $10^{-2}/0$ |
| **Number of Epochs** | 250/250 | 20/20 | 20/20 | 6/6 |

### A Experimental Details

#### A.1 Hyperparameters

Table 7 contains the hyperparameters used in our experiments in section 4.2 and 4.3. Note that these are the standard parameters for these datasets as in previous studies [Sagawa et al., 2020a, Liu et al., 2021]. The only difference is that we train Waterbird and CelebA for slightly shorter epoch due to finding no further increase in validation accuracies after those epochs.

Except for $p$, we do not tune for any other hyperparameters. For the second phase of CivilComments, we do not use any regularization due to the implicit regularization of just training the final layer. However, adding further regularization does not seem to have much of an effect as in section B.3.

### B Further Ablation Studies

#### B.1 Impact of The Feature Extractor’s Algorithms

Here, we provide evidences that ERM trained models produce the best features for worst-group robustness. We conduct an experiment on Waterbird, where instead of using ERM to obtain a feature extractor, we perform GDRO and Reweighing instead on the first phase. The results are in table 8. While using reweighing or GDRO for the first phase defeats the purpose of reducing the amount of group labels needed (that ERM doesn’t need), it is informative to examine the features alone.

Table 8: Effects of different methods for obtaining a feature extractor on test average accuracy and test worst-group accuracy (with ResNet50 on Waterbird).

| Feature Extractor by | Test Avg Acc | Test Wg Acc |
|----------------------|--------------|-------------|
| Reweighing           | 90.1         | 88.8        |
| GDRO                 | 90.8         | 88.6        |
| ERM                  | 90.5         | **90.2**    |

Here, we see that even though ERM does not use group labels, it provides the best features for robust classifier retraining on independent split.

#### B.2 Impact of Early Stopping and Validation Accuracies on The Feature Extractor

In this section, we present an ablation study of how different early stopping epoch (figure 2), average validation accuracy (figure 3 left), and worst-group accuracy (figure 3 right) of the initial ERM trained model (feature extractor) affect the GDRO retraining phase of CRIS. The results here are from performing CRIS with GDRO and $p = 0.30$ on Waterbird across a wide variety of epochs. Table 9 presents the full data generated for this section.
Figure 2: The effect of using different epochs for the feature extractor (phase 1) on classifier retraining’s (phase 2) test accuracies.

Figure 3: The effect of different validation average accuracies (left) and validation worst-group accuracies (right) from the feature extractor (phase 1) on classifier retraining’s (phase 2) test accuracies.

B.3 Impact of Regularization on Classifier Retraining with GDRO

We briefly investigate whether additional regularization would be helpful to classifier retraining with GDRO. This also shows the sensitivity of CRIS under different hyperparameter configurations.

Table 10: Effects of weight decay on classifier retraining with GDRO on CivilComments.

| $\ell_2$ Regularization | $p = 0.5$ | $p = 0.7$ | $p = 0.9$ |
|-------------------------|-----------|-----------|-----------|
|                         | Avg Acc   | Wg Acc    | Avg Acc   | Wg Acc    | Avg Acc   | Wg Acc    |
| 0                       | 89.5 (0.70) | 71.0 (1.50) | 89.7 (0.33) | 68.6 (1.53) | 89.5 (1.81) | 68.7 (1.72) |
| $10^{-2}$               | 89.4 (0.99) | 70.6 (0.42) | 89.5 (0.29) | 68.5 (0.87) | 88.6 (1.70) | 70.2 (1.63) |
| 1                       | 88.8 (1.30) | 70.0 (1.63) | 89.5 (0.35) | 66.4 (1.99) | 89.3 (1.69) | 68.9 (2.64) |

Table 10 shows the result between running classifier retraining with GDRO using two different levels of regularization. Here, we see that adding extra regularization does not seem to be beneficial due to the already low model capacity of the linear layer.
Table 9: CRIS with GDRO \((p = 0.30)\) on Waterbird. Average \((\text{Avg Acc})\) and Worst-group \((\text{Wg Acc})\) Accuracies for various epochs of the feature extractor (“Phase 1”) and the corresponding test accuracies for classifier retraining (“Phase 2”). While training for longer epochs seem to help with average and worst-group accuracy for phase 2, the benefit is small. Hence, simply selecting the best validation average accuracy model, row Epoch 131 and denoted \(\text{BEST}\) here, yields good enough features that simplify our training procedure and model selection criteria.

| CRIS \((p = 0.3)\) | Feature Extractor (Phase 1) | Classifier Retraining (Phase 2) |
|--------------------|-----------------------------|---------------------------------|
|                    | Val Avg Acc  | Val WG Acc  | Val Avg Acc | Val WG Acc | Test Avg Acc | Test WG Acc |
| 0                  | 91.3         | 0.05         | 87.8        | 85.6       | 86.6         | 85.2        |
| 1                  | 94           | 13.5         | 88          | 87.6       | 86           | 85.5        |
| 2                  | 95.2         | 20.3         | 89.6        | 88         | 88.3         | 86.9        |
| 3                  | 95.7         | 25.6         | 90.1        | 88         | 88.5         | 88.2        |
| 4                  | 95.8         | 25.6         | 88.9        | 88.2       | 87.1         | 86.9        |
| 5                  | 96.4         | 32.3         | 89.2        | 88.7       | 87.3         | 87.1        |
| 6                  | 96.5         | 38.4         | 90.5        | 88.7       | 89.3         | 88.9        |
| 7                  | 96.3         | 29.3         | 90.6        | 88.7       | 89.3         | 88.5        |
| 8                  | 97.1         | 38.4         | 90.1        | 89.3       | 88           | 87.7        |
| 9                  | 97.1         | 42.9         | 90.4        | 89.9       | 88.2         | 87.9        |
| 10                 | 96.7         | 40.6         | 91.2        | 90.2       | 89.4         | 88          |
| 20                 | 97.3         | 54.9         | 91.4        | 91         | 90.3         | 89.5        |
| 50                 | 97.4         | 55           | 91.5        | 91         | \textbf{90.4} | \textbf{89.7} |
| 100                | 97.2         | 57.1         | 90.5        | 90.2       | 89.5         | 89.2        |
| 131 (Best)         | \textbf{97.6} | 55.6         | 90.9        | 90.2       | 89.7         | \textbf{89.6} |
| 150                | 97.2         | \textbf{60.2} | 90.7        | 90.2       | 89.6         | 89.4        |
| 200                | 97.5         | 52.6         | 91          | 90.2       | 88.8         | 88.2        |
| 250                | 97.2         | 59.4         | 91          | \textbf{90.4} | 88.3         | 87.7        |