We propose to leverage denoising autoencoder networks as priors to address image restoration problems. We build on the key observation that the output of an optimal denoising autoencoder is a local mean of the true data density, and the autoencoder error (the difference between the output and input of the trained autoencoder) is a mean shift vector. We use the magnitude of this mean shift vector, that is, the distance to the local mean, as the negative log likelihood of our natural image prior. For image restoration, we maximize the likelihood using gradient descent by backpropagating the autoencoder error. A key advantage of our approach is that we do not need to train separate networks for different image restoration tasks, such as non-blind deconvolution with different kernels, or super-resolution at different magnification factors. We demonstrate state-of-the-art results for non-blind deconvolution and super-resolution using the same autoencoding prior.

1. Introduction

Deep learning has been successful recently at advancing the state of the art in various low-level image restoration problems including image super-resolution, deblurring, and denoising. The common approach to solve these problems is to train a network end-to-end for a specific task, that is, different networks need to be trained for each noise level in denoising, or each magnification factor in super-resolution. This makes it hard to apply these techniques to related problems such as non-blind deconvolution, where training a network for each blur kernel would be impractical.

A standard strategy to approach image restoration problems is to design suitable priors that can successfully constrain these underdetermined problems. Classical techniques include priors based on edge statistics, total variation, sparse representations, or patch-based priors. In contrast, our key idea is to leverage denoising autoencoder (DAE) networks as natural image priors. We build on the key observation by Alain et al. that for each input, the output of an optimal denoising autoencoder is a local mean of the true natural image density. The weight function that defines the local mean is equivalent to the noise distribution used to train the DAE. In other words, the autoencoder error, which is the difference between the output and input of the trained autoencoder, is a mean shift vector, and the noise distribution represents a mean shift kernel.

Hence, we leverage neural DAEs in an elegant manner to define powerful image priors: Given the trained autoencoder, our natural image prior is based on the magnitude of the mean shift vector. For each image, the mean shift is proportional to the gradient of the true data distribution smoothed by the mean shift kernel, and its magnitude is the distance to the local mean in the distribution of natural images. With an optimal DAE, the energy of our prior vanishes exactly at the stationary points of the true data distribution smoothed by the mean shift kernel. This makes our prior attractive for maximum a posteriori (MAP) estimation.

For image restoration, we include a data term based on the known image degradation model. For each degraded input image, we maximize the likelihood of our solution using gradient descent by backpropagating the autoencoder error.
and computing the gradient of the data term. Intuitively, this means that our approach iteratively moves our solution closer to its local mean in the natural image density, while satisfying the data term. This is illustrated in Figure 1.

A key advantage of our approach is that we do not need to train separate networks for different image restoration tasks, such as non-blind deconvolution with different kernels, or super-resolution at different magnification factors. Even though our autoencoding prior is trained on a denoising problem, it is highly effective at removing these different degradations. We demonstrate state of the art results for non-blind deconvolution and super-resolution using the same autoencoding prior.

2. Related Work

Image restoration, including deblurring, denoising, and super-resolution, is an underdetermined problem that needs to be constrained by effective priors to obtain acceptable solutions. Without attempting to give a complete list of all relevant contributions, the most common successful techniques include priors based on edge statistics [12, 33], total variation [28], sparse representations [1, 19], and patch-based priors [43, 24, 29]. While some of these techniques are tailored for specific restoration problems, recent patch-based priors lead to state of the art results for multiple applications, such as deblurring and denoising [29].

Solving image restoration problems using neural networks seems attractive because they allow for straightforward end-to-end learning. This has led to remarkable success for example for single image super-resolution [9, 15, 10, 25, 18] and denoising [5, 26]. A disadvantage of the end-to-end learning is that, in principle, it requires training a different network for each restoration task (e.g., each different noise level or magnification factor). While a single network can be effective for denoising different noise levels [20], and similarly a single network can perform well for different super-resolution factors [18], it seems unlikely that in non-blind deblurring, the same network would work well for arbitrary blur kernels. Additionally, experiments by Zhang et al. [41] show that training a network for multiple tasks reduces performance compared to training each task on a separate network. Previous research addressing non-blind deconvolution using deep networks includes the work by Schuler et al. [51] and more recently Xu et al. [58], but they require end-to-end training for each blur kernel.

A key idea of our work is to train a neural autoencoder that we use as a prior for image restoration. Autoencoders are typically used for unsupervised representation learning [36]. The focus of these techniques lies on the descriptive strength of the learned representation, which can be used to address classification problems for example. In addition, generative models such as generative adversarial networks [14] or variational autoencoders [20] also facilitate sampling the representation to generate new data. Their network architectures usually consist of an encoder followed by a decoder, with a bottleneck that is interpreted as the data representation in the middle. The ability of autoencoders and generative models to create images from abstract representations makes them attractive for restoration problems. Notably, the encoder-decoder architecture in Mao et al.’s image restoration work [26] is highly reminiscent of autoencoder architectures, although they train their network in a supervised manner.

A denoising autoencoder [35] is an autoencoder trained to reconstruct data that was corrupted with noise. Previously, Alain and Bengio [2] and Nguyen et al. [27] used DAEs to construct generative models. We are inspired by the insight of Alain and Bengio that the output of an optimal DAE is a local mean of the true data density. Hence, the autoencoder error (the difference between its output and input) is a mean shift vector [7]. This motivates using the magnitude of the autoencoder error as our prior.

Our work has an interesting connection to the plug-and-play priors introduced by Venkatakrishnan et al. [34]. They solve regularized inverse (image restoration) problems using ADMM (alternating directions method of multipliers), and they make the key observation that the optimization step involving the prior is a denoising problem, that can be solved with any standard denoiser. Brifman et al. [4] leverage this framework to perform super-resolution, and they use the NCSR denoiser [11] based on sparse representations. While their use of a denoiser is a consequence of ADMM, our DAE prior is motivated by its relation to the underlying data density (the distribution of natural images). Our approach leads to a different, simpler gradient descent optimization that does not rely on ADMM.

In summary, the main contribution of our work is that we show how to leverage DAEs to define a prior for image restoration problems by making the connection to mean shift. Crucially, for each image our prior is the squared distance to its local mean in the natural image distribution. We train a DAE and demonstrate that the resulting prior is effective for different restoration problems, including deblurring with arbitrary kernels and super-resolution with different magnification factors.

3. Problem Formulation

We formulate image restoration in a standard fashion as a maximum a posteriori (MAP) problem [17]. We model degradation including blur, noise, and downsampling as

\[ B = D(I \otimes K) + \xi, \]

where \( B \) is the degraded image, \( D \) is a down-sampling operator using point sampling, \( I \) is the unknown image to be recovered, \( K \) is a known, shift-invariant blur kernel, and \( \xi \sim \mathcal{N}(0, \sigma^2_d) \) is the per-pixel i.i.d. degradation
noise. The posterior probability of the unknown image is
\( p(I|B) = p(B | I)p(I)/p(B) \), and we maximize it by mini-
mizing the corresponding negative log likelihoods \( L \),

\[
\arg\max_I p(I|B) = \arg\min_I [L(B|I) + L(I)].
\] (2)

Under the Gaussian noise model, the negative data log like-
lihood is

\[
L(B|I) = ||B - D(I \otimes K)||^2 / \sigma_d^2.
\] (3)

Note that this implies that the blur kernel \( K \) is given at the
higher resolution, before down-sampling by point sampling
with \( D \). Our contribution now lies in a novel image prior
\( L(I) \), which we introduce next.

4. Denoising Autoencoder as Natural Image
Prior

Our key idea is to leverage a neural autoencoder to de-
fine a natural image prior. In particular, we are building on
denoising autoencoders (DAE) \([35]\) that are trained using
Gaussian noise and an expected quadratic loss. We are in-
spired by the results by Alain et al. \([2]\) who show how the
output of such autoencoders relates to the underlying data
density, and we will exploit this relation to define our prior.

4.1. Denoising Autoencoders

We visualize the intuition behind DAEs in Figure 2. Let
us denote a DAE as \( A_{\eta} \). Given an input image \( I \), its output
is an image \( A_{\eta}(I) \). A DAE \( A_{\eta} \) is trained to minimize \([35]\)

\[
L_{\text{DAE}} = \mathbb{E}_{\eta\sim I} \left[ \| I - A_{\eta}(I + \eta) \|^2 \right],
\] (4)

where the expectation is over all images \( I \) and Gaussian
noise \( \eta \) with variance \( \sigma^2_\eta \) and \( A_{\eta} \) indicates that the DAE
was trained with noise variance \( \sigma^2_\eta \). It is important to note
that the noise variance \( \sigma^2_\eta \) here is not related to the degrada-
tion noise and its variance \( \sigma^2_d \), and it is not a parameter to be
learned. Instead, it is a user specified parameter whose role
becomes clear with the following result by Alain et al. \([2]\):
Let us denote the true data density of natural images as \( p(I) \).
Alain et al. show that the output \( A_{\eta}(I) \) of the optimal DAE
(assuming unlimited capacity) is related to the true data den-
sity \( p(I) \) as

\[
A_{\eta}(I) = \frac{\mathbb{E}_{\eta} [p(I - \eta)(I - \eta)]}{\mathbb{E}_{\eta} [p(I - \eta)]} = \frac{\int g_{\sigma_\eta^2}(\eta)p(I - \eta)(I - \eta)d\eta}{\int g_{\sigma_\eta^2}(\eta)p(I - \eta)d\eta}. \] (5)

This means that the autoencoder output can be interpreted as
a local mean or a weighted average of images in the neigh-
borhood of \( I \). The weights are given by the true density
\( p(I) \) multiplied by the noise distribution that was used dur-
ing training, which is a local Gaussian kernel \( g_{\sigma_\eta^2}(\eta) \)
centered at \( I \) with variance \( \sigma^2_\eta \). That is, the parameter \( \sigma^2_\eta \)
of the autoencoder determines the size of the region around \( I \)
that contributes to the local mean. This reveals an interesting
connection to the mean shift algorithm \([7]\): The autoen-
coder error, that is the difference between the output and the
input of the autoencoder \( A_{\eta}(I) - I \) is a mean shift vector.
When the noise has a Gaussian distribution, it is straightfor-
ward to show that this autoencoder error is proportional to
the gradient of the log likelihood of the smoothed density,

\[
A_{\eta}(I) - I = \sigma^2_\eta \nabla \log \mathbb{E}_{\eta} [p(I - \eta)] = \sigma^2_\eta \nabla \log \left[ g_{\sigma_\eta^2} * p \right](I),
\] (6)

where \(* \) means convolution (see supplemental material for
derivation). The autoencoder error vanishes at station-
ary points, including local extrema, of the true density
smoothed by the Gaussian kernel.

![Figure 2. Visualization of a denoising autoencoder using a 2D spi-
ral density. Given input samples of a true density (a), the autoen-
coder is trained to pull each sample corrupted by noise back to its
original location. Adding noise to the input samples smooths the
density represented by the samples (b). Assuming an infinite num-
ber of input samples and an autoencoder with unlimited capacity,
for each input, the output of the optimal trained autoencoder is the
local mean of the true density. The local weighting function corre-
sponds to the noise distribution that was used during training, and
it represents a mean shift kernel \([7]\). The difference between the
output and the input of the autoencoder is a mean shift vector (c),
which vanishes at local extrema of the true density smoothed by
the mean shift kernel. Due to practical limitations (Section 4.2),
we approximate the mean shift vectors (d, red) using Equation 8.
The difference between the true mean shift vectors (d, black) and
our approximate vectors (d, red) vanishes as we get closer to the
manifold.](image-url)
4.2. Autoencoding Prior

The above observations inspire us to use the squared magnitude of the mean shift vector as the energy (the negative log likelihood) of our prior, \( L(I) = \|A_{\sigma_{\eta}}(I) - I\|^2 \). This energy is very powerful because it tells us how close an image \( I \) is to its local mean \( A_{\sigma_{\eta}}(I) \) in the true data density, and it vanishes at local extrema of the true density smoothed by the mean shift kernel. Figure 2(c), illustrates how small values of \( L(I) = \|A_{\sigma_{\eta}}(I) - I\|^2 \) occur close to the data manifold, as desired. Figure 3 visualizes a local minimum of our prior on natural images, which we find by iteratively minimizing the prior via gradient descent starting from a noisy input, without any help from a data term.

Including the data term, we recover latent images as

\[
\argmin_{I} \|B - D(I \otimes K)\|^2 / \sigma_{\eta}^2 + \gamma \|A_{\sigma_{\eta}}(I) - I\|^2.
\]

(7)

Our energy has two parameters that we will adjust based on the restoration problem. First, this is the mean shift kernel size \( \sigma_{\eta} \), and second we introduce a parameter \( \gamma \) to weight the relative influence of the data term and the prior.

Optimization. Given a trained autoencoder, we minimize our loss function in Equation 7 by applying gradient descent and computing the gradient of the prior using backpropagation through the autoencoder. Algorithm 1 shows the steps to minimize Equation 7. In the first step of each iteration, we compute the gradient of the data term with respect to image \( I \). The second step is to find the gradients for our prior. The gradient of the mean shift vector \( \|A_{\sigma_{\eta}}(I) - I\|^2 \) requires the gradient of the autoencoder \( A_{\sigma_{\eta}}(I) \), which we compute by backpropagation through the network. Finally, the image \( I \) is updated using the weighted sum of the two gradient terms.

Overcoming Training Limitations. The theory above assumes unlimited data and time to train an unlimited capacity autoencoder. In particular, to learn the true mean shift mapping, for each natural image the training data needs to include noise patterns that lead to other natural images. In practice, however, such patterns virtually never occur because of the high dimensionality. Since the DAE never observed natural images during training (produced by adding noise to other images), it overfits to noisy images. This is problematic during the gradient descent optimization, when the input to the DAE does not have noise.

As a workaround, we obtained better results by adding noise to the image before feeding it to the trained DAE during optimization. We further justify this by showing that with this workaround, we can still approximate a DAE that was trained with a desired noise variance \( \sigma_{\eta}^2 \). That is,

\[
A_{\sigma_{\eta}}(I) - I \approx 2 \left( \mathbb{E}_{\epsilon} [A_{\sigma_{\eta}}(I - \epsilon) - I] \right),
\]

(8)

where \( \epsilon \sim \mathcal{N}(0, \sigma_{\epsilon}^2) \), and \( A_{\sigma_{\epsilon}} \) is a DAE trained with \( \sigma_{\epsilon}^2 = \sigma_{\eta}^2 / 2 \). The key point here is that the consecutive convolution with two Gaussians is equivalent to a single Gaussian convolution with the sum of the variances (refer to supplementary material for the derivation). This is visualized in Figure 2(d). The red vectors indicate the approximated mean shift vectors using Equation 8 and the black vectors indicate the exact mean shift vectors. The approximation error decreases as we approach the true manifold.

During optimization, we approximate the expected value in Equation 8 by stochastically sampling over \( \epsilon \). We use momentum of 0.9 and step size 0.1 in all experiments and we found that using one noise sample per iteration performs well enough to compute meaningful gradients. This approach resulted in a PSNR gain of around 1.7dB for the super-resolution task (Section 5.1), compared to evaluating the left hand side of Equation 8 directly.

Bad Local Minima and Convergence. The mean shift vector field learned by the DAE could vanish in low density regions [2], which corresponds to undesired local minima for our prior. In practice, however, we have not observed such degenerate solutions because our data term pulls the solution towards natural images. In all our experiments the optimization converges smoothly (Figure 1, intermediate steps), although we cannot give a theoretical guarantee.

Algorithm 1 Proposed gradient descent. We express convolution as a matrix-vector product.

```
loop #iterations
    • Compute data term gradients \( \nabla_L L(I|B) \):
        \[
        K^T D^T (DKI - B) / \sigma^2_B
        \]
    • Compute prior gradients \( \nabla_L L(I) \):
        \[
        \nabla_L \nabla L(I)^T (A_{\sigma_{\eta}}(I) - I) + I - A_{\sigma_{\eta}}(I)
        \]
    • Update \( I \) by descending
        \[
        \nabla_L L(I|B) + \gamma \nabla L(I)
        \]
end loop
```
and Zhang et al. [41] (DnCNN-3), and TNRD by Chen and
Kim et al. [18] (SRCNN), Dong et al. [10] (VDSR),
weight in each iteration, leading to solutions that satisfy
the inverse square root of the iteration number. This pol-

\sigma \begin{align*}
\text{standard deviation}
\end{align*}

ImageNet dataset [8] by adding Gaussian noise with stan-
many pixels. Our network is trained on color images of the
us to compute the gradients with respect to the image more
application. We use a fully-convolutional network that allows
latent space implemented as a bottleneck is not required in
coders, our network does not have a bottleneck. An explicit
number of channels are 3 (RGB) for input and output and
64 for the rest of the layers. Unlike typical neural autoen-
coders, our network does not have a bottleneck. An explicit
latent space implemented as a bottleneck is not required in
principle for DAE training, and we do not need it for our appli-
cation. We use a fully-convolutional network that allows
us to compute the gradients with respect to the image more
efficiently since the neuron activations are shared between
many pixels. Our network is trained on color images of the
ImageNet dataset [8] by adding Gaussian noise with stan-
dard deviation \( \sigma = 25 \) (around 10%). We perform resid-
ual learning by minimizing the \( L_2 \) distance of the output
layer to the ground truth noise. We used the Caffe pack-
\begin{align*}
\text{age}\end{align*}

\sigma \begin{align*}
\text{standard deviation}
\end{align*}

\begin{align*}
\text{loss between the out-
\end{align*}

\sigma \begin{align*}
\text{standard deviation}
\end{align*}

\begin{align*}
\text{psnr of the super-resolved images from 'set5' and 'set14'
\end{align*}

\begin{align*}
\text{dataset for different noise levels.}
\end{align*}

\begin{align*}
\text{pock}\end{align*}

\begin{align*}
\text{srcnn, vdsr and dnccnn-3 train an end-to-
\end{align*}

\begin{align*}
\text{compare our results}
\end{align*}

\begin{align*}
\text{our method performs sig-
\end{align*}

\begin{align*}
\text{for scale factors \times 2, 3, 4, and 5. We com-
\end{align*}

\begin{align*}
\text{the crop size in pixels corresponds to the scale factor for all
\end{align*}

\begin{align*}
\text{for srcnn, however, we used a boundary of 13
\end{align*}

\begin{align*}
\text{provide full support for their network. While SR-
\end{align*}

\begin{align*}
\text{solve jointly on \times 2, 3 and 4 (DnCNN-3 training included also
denoising and JPEG artifact removal tasks). For \times 5 super-
\end{align*}

\begin{align*}
\text{compute psnr values over cropped rgb images (where the
crop size in pixels corresponds to the
\end{align*}

\begin{align*}
\text{methods. For SRCNN, however, we used a boundary of 13
\end{align*}

\begin{align*}
\text{provide full support for their network. While SR-
\end{align*}

\begin{align*}
\text{solve directly for mmse, our
\end{align*}

\begin{align*}
\text{solve for the map solution, which is not guaran-
teed to have better psnr. still, we achieve better results
in average. For scale factor \times 5 our method performs sig-
nificantly better since our prior does not need to be trained
for a specific scale. Figure 4 shows visual comparisons to
the super-resolution results from srcnn [10], TNRD [6],
and DnCNN-3 [41] on three example images. We exclude
results of VDSR due to limited space and visual similarity
with DnCNN-3. Our natural image prior provides clean and
sharp edges over all magnification factors.}

\begin{align*}
\text{non-blind deconvolution on
\end{align*}

\begin{align*}
\text{compare our approach, Denoising Autoencoder Prior
( DAEP ), to state of the art methods in super-resolution and
non-blind deconvolution problems. For all our experiments,
we trained the autoencoder with \( \sigma_e = 25 \) \( \sigma_n = 25\sqrt{2} \),
and the parameter of our energy (Equation 7) were set to
\( \gamma = 6.875/\sigma_e^2 \). We always perform 300 gradient descent
iteration steps during image restoration, which takes about
30 seconds for a \( 256 \times 256 \) image.}

\begin{align*}
\text{Super-Resolution}
\end{align*}

\begin{align*}
\text{The super-resolution problem is usually defined in ab-
\end{align*}

\begin{align*}
\text{absence of noise (\( \sigma_d = 0 \)), therefore we weight the prior by
the inverse square root of the iteration number. This pol-
\end{align*}

\begin{align*}
\text{pity starts with a rough regularization and reduces the prior
weight in each iteration, leading to solutions that satisfy
\( \sigma_d = 0 \). We compare our method to recent techniques by
Kim et al. [18] ( SRCNN ), Dong et al. [10] ( VDSR ),
and Zhang et al. [41] ( DnCNN-3 ), and TNRD by Chen and

\begin{align*}
\text{Table 1. Average PSNR (dB) for super-resolution on 'set5'
[3].}
\end{align*}

\begin{align*}
\text{Table 2. Average PSNR (dB) for super-resolution on 'set14'
[40].}
\end{align*}

\begin{align*}
\text{Table 3. Average PSNR (dB) for non-blind deconvolution on
Levin et al.'s [23] dataset for different noise levels.}
\end{align*}

\begin{align*}
\text{5. Experiments and Results}
\end{align*}

\begin{align*}
\text{We compare our approach, Denoising Autoencoder Prior
( DAEP ), to state of the art methods in super-resolution and
non-blind deconvolution problems. For all our experiments,
we trained the autoencoder with \( \sigma_e = 25 \) \( \sigma_n = 25\sqrt{2} \),
and the parameter of our energy (Equation 7) were set to
\( \gamma = 6.875/\sigma_e^2 \). We always perform 300 gradient descent
iteration steps during image restoration, which takes about
30 seconds for a \( 256 \times 256 \) image.}

\begin{align*}
\text{5.1. Super-Resolution}
\end{align*}

\begin{align*}
\text{The super-resolution problem is usually defined in ab-
\end{align*}

\begin{align*}
\text{absence of noise (\( \sigma_d = 0 \)), therefore we weight the prior by
the inverse square root of the iteration number. This pol-
\end{align*}

\begin{align*}
\text{pity starts with a rough regularization and reduces the prior
weight in each iteration, leading to solutions that satisfy
\( \sigma_d = 0 \). We compare our method to recent techniques by
Kim et al. [18] ( SRCNN ), Dong et al. [10] ( VDSR ),
and Zhang et al. [41] ( DnCNN-3 ), and TNRD by Chen and

\begin{align*}
\text{Method} & \times 2 & \times 3 & \times 4 & \times 5 \\
\text{Bicubic} & 31.80 & 28.67 & 26.73 & 25.32 \\
\text{SRCNN [10]} & 34.50 & 30.84 & 28.60 & 26.12 \\
\text{TNRD [6]} & 34.62 & 31.08 & 28.83 & 26.88 \\
\text{VDSR [18]} & 34.50 & 31.39 & 29.19 & 25.91 \\
\text{DnCNN-3 [41]} & 35.20 & \textbf{31.58} & \textbf{29.30} & 26.30 \\
\text{DAEP (Ours)} & \textbf{35.23} & 31.44 & 29.01 & \textbf{27.19} \\
\end{align*}

\begin{align*}
\text{Method} & \times 2 & \times 3 & \times 4 & \times 5 \\
\text{Bicubic} & 28.53 & 25.92 & 24.44 & 23.46 \\
\text{SRCNN [10]} & 30.52 & 27.48 & 25.76 & 24.05 \\
\text{TNRD [6]} & 30.53 & 27.60 & 25.92 & 24.61 \\
\text{VDSR [18]} & 30.72 & 27.81 & 26.16 & 24.01 \\
\text{DnCNN-3 [41]} & 30.99 & 27.93 & \textbf{26.25} & 24.26 \\
\text{DAEP (Ours)} & \textbf{31.07} & 27.93 & 26.13 & \textbf{24.88} \\
\end{align*}

\begin{align*}
\text{\( \sigma \) & Levin [23] & EPLL [43] & RTF-6 [30] & DAEP (Ours) \\
2.55 & 31.09 & 32.51 & 32.51 & \textbf{32.69} \\
7.65 & 27.40 & 28.42 & 21.44 & \textbf{28.95} \\
12.75 & 25.36 & 26.13 & 16.03 & \textbf{26.87} \\
\end{align*}
Figure 4. Comparison of super-resolution for scale factor 2 (top row), scale factor 3 (middle row), and scale factor 4 (bottom row) with the corresponding PSNR (dB) scores.

al. [30] in Table 3, where we show the average PSNR of the deconvolution for three levels of additive noise ($\sigma \in \{2.55, 7.65, 12.75\}$). Note that RTF-6 [30] is only trained for noise level $\sigma = 2.55$, therefore it does not perform well for other noise levels. Figure 5 provides visual comparisons for two deconvolution result images. Our natural image prior achieves higher PSNR and produces sharper edges and less visual artifacts compared to Levin et al. [23], Zoran and Weiss [43], and Schmidt et al. [30].

We performed an additional comparison on color images similar to Fortunato and Oliveira [13] using 24 color images from the Kodak Lossless True Color Image Suite from PhotoCD PCD0992 [21]. The images are blurred with a $19 \times 19$ blur kernel from Krishnan and Fergus [22] and 1% noise is added. Figure 6 shows visual comparisons and average PSNRs over the whole dataset. Our method produces much sharper results and achieves a higher PSNR in average over this dataset.
Figure 5. Comparison of non-blind deconvolution with $\sigma = 2.55$ additive noise (top row) and $\sigma = 7.65$ additive noise (bottom row) with the corresponding PSNR (dB) scores. The kernel is visualized in the bottom right of the blurred image.

5.3. Discussion

A disadvantage of our approach is that it requires the solution of an optimization problem to restore each image. In contrast, end-to-end trained networks perform image restoration in a single feed-forward pass. For the increase in runtime computation, however, we gain much flexibility. With a single autoencoding prior, we obtain not only state of the art results for non-blind deblurring with arbitrary blur kernels and super-resolution with different magnification factors, but also successfully restore images corrupted by noise or holes as shown in Figure 7.

Our approach requires some user defined parameters (mean shift kernel size $\sigma_\eta$ for DAE training and restoration, weight of the prior $\gamma$). While we use the same parameters for all experiments reported here, other applications may require to adjust these parameters. For example, we have experimented with image denoising (Figure 7), but so far we have not achieved state of the art results. We believe that this may require an adaptive kernel width for the DAE, and further fine-tuning of our parameters.

6. Conclusions

We introduced a natural image prior based on denoising autoencoders (DAEs). Our key observation is that optimally trained DAEs provide mean shift vectors on the true data density. Our prior minimizes the distances of restored images to their local means (the length of their mean shift vectors). This is powerful since mean shift vectors vanish at local extrema of the true density smoothed by the mean shift kernel. Our results demonstrate that a single DAE prior achieves state of the art results for non-blind image deblurring with arbitrary blur kernels and image super-resolution at different magnification factors. In the future, we plan to apply our autoencoding priors to further image restoration problems including denoising, colorization, or non-uniform and blind deblurring. While we used Gaussian noise to train our autoencoder, it is possible to use other types of data degradation for DAE training. Hence, we will investigate other DAE degradations to learn different data representations or use a mixture of DAEs for the prior.
Figure 6. Comparison of non-blind deconvolution methods on the 21st image from the Kodak image set [21]. For each method, we report the PSNR (dB) of the visualized image (left) and the average PSNR on the whole set (right). The results of other methods were reproduced from Fortunato and Oliveira [13] for ease of comparison.

Masked 70% of Pixels

| Method           | PSNR (dB) Visualized | PSNR (dB) Whole Set |
|------------------|----------------------|---------------------|
| Lucy Richardson  | 24.38/24.47          |                     |
| Zhou et al.      | 27.38/27.68          |                     |
| Levin et al.     | 27.04/27.37          |                     |
| Wang et al.      | 27.68/28.23          |                     |
| Wang et al.      | 28.63/29.25          |                     |
| Levin et al.     | 28.96/30.15          |                     |
| Shan et al.      | 28.97/30.01          |                     |
| Krishnan, Fergus | 29.15/30.18          |                     |
| Fortunato, Oliveira | 29.25/30.34       |                     |
| DAEP (Ours)      | 29.92/31.07          |                     |

Figure 7. Restoration of images corrupted by noise and holes using the same autoencoding prior as in our other experiments.

| Corrupted Condition | PSNR (dB) Visualized | PSNR (dB) Whole Set |
|---------------------|----------------------|---------------------|
| Masked 70% of Pixels| 6.13 dB              |                     |
| Our Reconstruction  | 30.08 dB             |                     |
| Input with 10% Noise| 20.47 dB             |                     |
| Our Reconstruction  | 31.05 dB             |                     |
References

[1] M. Aharon, M. Elad, and A. Bruckstein. k-svd: An algorithm for designing overcomplete dictionaries for sparse representation. *IEEE Transactions on Signal Processing*, 54(11):4311–4322, Nov 2006.

[2] G. Alain and Y. Bengio. What regularized auto-encoders learn from the data-generating distribution. *Journal of Machine Learning Research*, 15:3743–3773, 2014.

[3] M. Bevilacqua, A. Roumy, C. Guillemot, and M. Alberi-Morel. Low-complexity single-image super-resolution based on nonnegative neighbor embedding. In *British Machine Vision Conference, BMVC 2012*, Surrey, UK, September 3–7, 2012.

[4] A. Brifman, Y. Romano, and M. Elad. Turning a denoiser into a super-resolver using plug and play priors. In *IEEE ICIP*, pages 1404–1408. IEEE, 2016.

[5] H. C. Burger, C. J. Schuler, and S. Harmeling. Image denoising: Can plain neural networks compete with bm3d? In *Computer Vision and Pattern Recognition (CVPR)*, 2012 IEEE Conference on, pages 2392–2399, June 2012.

[6] Y. Chen and T. Pock. Trainable nonlinear reaction diffusion: A flexible framework for fast and effective image restoration. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2016.

[7] D. Comaniciu and P. Meer. Mean shift: a robust approach toward feature space analysis. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 24(5):603–619, May 2002.

[8] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. Imagenet: A large-scale hierarchical image database. In *Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on*, pages 248–255. IEEE, 2009.

[9] C. Dong, C. C. Loy, K. He, and X. Tang. Learning a Deep Convolutional Network for Image Super-Resolution, pages 184–199. Springer International Publishing, Cham, 2014.

[10] C. Dong, C. C. Loy, K. He, and X. Tang. Image super-resolution using deep convolutional networks. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 38(2):295–307, Feb 2016.

[11] W. Dong, L. Zhang, G. Shi, and X. Li. Nonlocally centralized sparse representation for image restoration. *IEEE Transactions on Image Processing*, 22(4):1620–1630, 2013.

[12] R. Fattal. Image upsampling via impose edge statistics. *ACM Trans. Graph.*, 26(3), July 2007.

[13] H. E. Fortunato and M. M. Oliveira. Fast high-quality non-blind deconvolution using sparse adaptive priors. *The Visual Computer*, 30(6-8):661–671, 2014.

[14] I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial nets. In Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger, editors, *Advances in Neural Information Processing Systems 27*, pages 2672–2680. Curran Associates, Inc., 2014.

[15] S. Gu, W. Zuo, Q. Xie, D. Meng, X. Feng, and L. Zhang. Convolutional sparse coding for image super-resolution. In *2015 IEEE International Conference on Computer Vision (ICCV)*, pages 1823–1831, Dec 2015.

[16] Y. Jia, E. Shelhamer, J. Donahue, S. Karayev, J. Long, R. Girshick, S. Guadarrama, and T. Darrell. Caffe: Convolutional architecture for fast feature embedding. In *Proceedings of the 22nd ACM international conference on Multimedia*, pages 675–678. ACM, 2014.

[17] Y. Jia, E. Shelhamer, J. Donahue, S. Karayev, J. Long, R. Girshick, S. Guadarrama, and T. Darrell. Caffe: Convolutional architecture for fast feature embedding. In *Proceedings of the 22nd ACM international conference on Multimedia*, pages 675–678. ACM, 2014.

[18] D. Kingma and J. Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

[19] D. Kingma and M. Welling. Auto-encoding variational bayes. In *ICLR 2014*, 2014.

[20] Kodak. Kodak lossless true color image suite. [http://r0k.us/graphics/kodak/](http://r0k.us/graphics/kodak/) Accessed: 2013-01-27.

[21] D. Krishnan and R. Fergus. Fast image deconvolution using hyper-laplacian priors. *In Advances in Neural Information Processing Systems*, pages 1033–1041, 2009.

[22] A. Levin, R. Fergus, F. Durand, and W. T. Freeman. Image and depth from a conventional camera with a coded aperture. *ACM transactions on graphics (TOG)*, 26(3):70, 2007.

[23] A. Levin, B. Nadler, F. Durand, and W. T. Freeman. *Patch Complexity, Finite Pixel Correlations and Optimal Denoising*, pages 73–86. Springer Berlin Heidelberg, Berlin, Heidelberg, 2012.

[24] D. Liu, Z. Wang, B. Wen, J. Yang, W. Han, and T. S. Huang. Robust single image super-resolution via deep networks with sparse prior. *IEEE Transactions on Image Processing*, 25(7):3194–3207, July 2016.

[25] X.-J. Mao, C. Shen, and Y.-B. Yang. Image restoration using very deep convolutional encoder-decoder networks with symmetric skip connections. In *Proc. Neural Information Processing Systems*, 2016.

[26] A. Nguyen, J. Yosinski, Y. Bengio, A. Dosovitskiy, and J. Clune. Plug & play generative networks: Conditional iterative generation of images in latent space. *arXiv preprint arXiv:1612.00005*, 2016.

[27] D. Perrone and P. Favaro. Total variation blind deconvolution: The devil is in the details. In *2014 IEEE Conference on Computer Vision and Pattern Recognition*, pages 2909–2916, June 2014.

[28] U. Schmidt, J. Jancsary, S. Nowozin, S. Roth, and C. Rother. Cascades of regression tree fields for image restoration. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 38(4):677–689, April 2016.

[29] U. Schmidt, J. Jancsary, S. Nowozin, S. Roth, and C. Rother. Cascades of regression tree fields for image restoration. *IEEE transactions on pattern analysis and machine intelligence*, 38(4):677–689, 2016.

[30] C. J. Schuler, H. C. Burger, S. Harmeling, and B. Schlkopf. A machine learning approach for non-blind image deconvolution. In *Computer Vision and Pattern Recognition (CVPR)*, 2013 IEEE Conference on, pages 1067–1074, June 2013.
[32] Q. Shan, J. Jia, and A. Agarwala. High-quality motion deblurring from a single image. In ACM Transactions on Graphics (TOG), volume 27, page 73. ACM, 2008.

[33] M. F. Tappen, B. C. Russell, and W. T. Freeman. Exploiting the sparse derivative prior for super-resolution and image demosaicing. In In IEEE Workshop on Statistical and Computational Theories of Vision, 2003.

[34] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg. Plug-and-play priors for model based reconstruction. In GlobalSIP, pages 945–948. IEEE, 2013.

[35] P. Vincent, H. Larochelle, Y. Bengio, and P.-A. Manzagol. Extracting and composing robust features with denoising autoencoders. In Proceedings of the 25th International Conference on Machine Learning, ICML ’08, pages 1096–1103, New York, NY, USA, 2008. ACM.

[36] P. Vincent, H. Larochelle, I. Lajoie, Y. Bengio, and P.-A. Manzagol. Stacked denoising autoencoders: Learning useful representations in a deep network with a local denoising criterion. J. Mach. Learn. Res., 11:3371–3408, Dec. 2010.

[37] Y. Wang, J. Yang, W. Yin, and Y. Zhang. A new alternating minimization algorithm for total variation image reconstruction. SIAM Journal on Imaging Sciences, 1(3):248–272, 2008.

[38] L. Xu, J. S. Ren, C. Liu, and J. Jia. Deep convolutional neural network for image deconvolution. In Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger, editors, Advances in Neural Information Processing Systems 27, pages 1790–1798. Curran Associates, Inc., 2014.

[39] J. Yang, J. Wright, T. S. Huang, and Y. Ma. Image super-resolution via sparse representation. IEEE Transactions on Image Processing, 19(11):2861–2873, Nov 2010.

[40] R. Zeyde, M. Elad, and M. Protter. On single image scale-up using sparse-representations. In Proceedings of the 7th International Conference on Curves and Surfaces, pages 711–730, Berlin, Heidelberg, 2012. Springer-Verlag.

[41] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang. Beyond a gaussian denoiser: Residual learning of deep cnns for image denoising. arXiv preprint arXiv:1608.03981, 2016.

[42] C. Zhou and S. Nayar. What are good apertures for defocus deblurring? In Computational Photography (ICCP), 2009 IEEE International Conference on, pages 1–8. IEEE, 2009.

[43] D. Zoran and Y. Weiss. From learning models of natural image patches to whole image restoration. In 2011 International Conference on Computer Vision, pages 479–486, Nov 2011.