Impact of Adopting Quick Response and Agility on Supply Chain Competition with Strategic Customer Behavior

M. Kaviyani-Charati\textsuperscript{a}, S. H. Ghodsypour\textsuperscript{a}, M. Hajiaghaei-Keshteli\textsuperscript{b,∗}
\textsuperscript{a} Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, 424 Hafez Avenue, Tehran, Iran
\textsuperscript{b} Department of Industrial Engineering, University of Science and Technology of Mazandaran, Behshahr, Iran

First author: mohammad.kaviyani.ie91@gmail.com, +989396678188
Second author: ghodsypo@aut.ac.ir, +982164545390
Third author: mostafahaji@mazust.ac.ir, +989112169896
P.O. Box: 48518/78195, Behshahr, IRAN.

Abstract
A growing trend towards computerization and competition in supply chains results in uncertainty and quick variability that make the decisions difficult for both levels of retailers and manufacturers. In this paper, two Bi-Level Stackelberg models are developed under non- and agile conditions in the presence of strategic customers. Our main novelty approach in this paper is to consider both levels competing with each other in a sequential game to determine the optimal production and order quantities and prices with and without agile abilities. In addition, both proposed models are simplified single-level using the Karush-Kuhn-Tucker (KKT) approach. Then, they are remodeled by a Robust Optimization technique due to existing uncertain parameters. To have a better assessment of the models’ efficiency and applicability, they have been implemented in a real case and finally, the results are compared and analyzed.

Keywords: Competitive Supply Chain, Quick Response, Agile Manufacturer, Bi-Level Stackelberg, Exact Solution

1. Introduction

Today’s, recent advances in technologies, internet access, and online shopping have led to lower demand and a wide variety of requirements [1]. Also, the importance of the consumers in supply chains have been recently recognized and all the members of a supply chain try to satisfy and keep their customers [2]. Thus, Quick Response (QR) is identified as a key factor in a company’s success to meet the customer’s requirements [3, 4]. In addition, agility is known as a useful tool for achieving competitive advantages under constant and unexpected circumstances when the abilities make it possible for agile manufacturers (AMs) to react quickly to unforeseen circumstances, e.g. the consumers’ demand and expectations [5].
Two main features of the agility are flexibility and quick production [6]. In this fast-changing world and unstable market, retailers would be faced with various problems in their orders [7]. These situations will result in either commodity shortage or clearance sales at the end of the sales season [3, 4]. Hence, QR in retailing is a feature used to reduce the delivery time, improve the supply flexibility, and accelerate the logistics operations with technologies such as advanced information system [8-10]. QR has also employed in a different industries such as toys, bags and shoes, fashion and other inconstant changing markets [8].

The consumers are generally divided into three groups in the literature: myopic, bargain-hunting, and strategic as they are reacted to different circumstances [11, 12]. In general, strategic consumers constitute the majority of the market so that customers in this scientific work will be considered as strategic ones. Strategic customers firstly assess the market situation and then decide whether to purchase the product at full price or not [3, 4]. International marketing, increasing competition-based performance, and rapid technological and economic changes have led to an increase in uncertainty [13]. Uncertainty and keen competition have greatly influenced the decision at different levels of the supply chain in general. Therefore, considering the competition, uncertainty, and clearance sale would make the model more realistic.

The uncertainty has been modeled and solved using various approaches in the literature [14, 15]. In that regard, robust optimization (RO) has identified as an efficient method in which the unknown values of parameters will be considered and the model would be solved and tested in various scenarios [16]. Furthermore, the market is highly variable in ordering quantity and demand types. Therefore, a considerable amount of goods will end up clearance sales, if the uncertainty is not taken into account.

According to the literature, very little attention has been paid to the bi-level models which both levels have employed QR and agile systems. In contrast to good theoretical models and evaluation, scant research has been found that investigated such a mathematical model in reality with an appropriate case study. Contrary to the previous works, in this paper, two bi-level models of the competitive supply chain with strategic consumers are firstly proposed. In the first model, the manufacturer and the retailer compete with each other in a traditional way (ordering and producing goods just before starting the sale’s season). Whereas in the second one, the model is modified by adding QR and agility abilities. The problems were developed and formulated using the Stackelberg game (SG). Then the Bi-level Stackelberg models (BLSMs) would be simplified single-level with the KKT approaches. Due to the quick change and variability, the RO method is applied to addressing the uncertainty. Next, the models are implemented in the real case to illustrate the applicability and efficiency of the proposed models. Finally, the results and analysis of different models are presented.

The rest of this paper is organized as follows. The second section is devoted to the literature review and the gap discussion. In the third section, the problem and its modeling are explained. In the fourth section, the solution approaches are presented and discussed. Also,
the results are given and analyzed in this section. Finally, conclusion and suggestions for further research are proposed in the late section.

2. Literature Review

This work contains three major parts related to competitive supply chain, QR in supply chain, and agile production discussed as follows.

While demand, replacement lead time (LT), and product’s life are widely fluctuated, long, and short, respectively, QR is highly useful [8, 10, 17]. Many papers were published in the areas of utilizing QR in supply chains [3, 4, 8, 10, 17] which most of them were related to the reduction of delivery time and improvement of performance in the uncertain situation, and a few of them are close to our work. For instance, Yang et al., [4] studied the effect of QR in pricing and making a decision on the stock in a decentralized and centralized supply chain with strategic consumers using revenue sharing. Cachon & Swinney [3] discussed separately the effects of QR in production; High changeability of capacity and product design for markets with unstable changes.

They had also developed the model in the fashion market under uncertainty in demand and strategic consumers for deciding what quantity and price were selected for maximizing the profit. The application of QR in retail business considering a single product and non-deterministic demand was conducted by Cachon & Swinney [11]. In that work, a simultaneous game (Nash game) between the retailer and the consumer was proposed for optimizing the ordering quantity and price. Similarly, Li et al. [18] examined how a manufacture can reduce LT for short lifetime products in a bi-level supply chain (BLSC) involving the retailer and manufacturer. They developed a theory for determining the ordering quantity, time, and price while the risk of members’ behavior as well as extra costs for reducing LT were considered. The proposed model was evaluated by a numerical example. Also, Choi, T. M. [19] investigated the value of QR considering the rational behavior of customers in the retail business of fashion products, accordingly, an inventory decision making was mathematically formulated.

In a similar work, Dong & Wu [12] studied how strategic customers affects the price and inventory in a QR system and the yield of some scenarios had been increased. Likewise, a simple Stackelberg model for two-period pricing with a price reduction in a BLSC involving a retailer and a manufacturer was presented by Zhang et al. [20]. In that paper, three pricing scenarios were examined including QR, no QR, and limited QR, respectively. Regarding retailing problems, a newspaper supply chain inventory management model was proposed by Darwish et al. [21]. The retailers had two opportunities for ordering goods at the beginning and during the season after updating demand information using the Bayesian approach. The demand distribution and quantities were optimally determined. Recently, Khouja et al. [22] also optimized the order quantity for retailers to enhance profits using a retail off-price in non-deterministic situations. In this study, a newspaper model was developed for achieving optimal order quantity under different conditions (before and after solving demand uncertainty).
The intensity of global competition and the growing importance of customers have made the manufacturers change their strategy from the efficient system to the responding one so give priority to their consumers [23]. In that respect, agility plays a major role in competitive marketing and supply chain and also helps manufacturers supply suitable products at the right time [24]. Quick production response, as one of the most important features of agility, aimed at reducing the LT and responded to the consumer’s requirements [1, 23]. To date, many papers are published investigating agility in the different levels of supply chains.

A conceptual framework was provided by Sindhwani, & Malhotra [23] in which the interactions between different enablers of agile manufacturing system were analyzed. The research sought to find a structural model for meeting customer’s requirements. In the operations research area, a scheduling problem for an AM in an order-based-production environment in various conditions were investigated by Lalmazloumian et al., [25]. They developed a multi-product, multi-period, multi-level mixed-integer linear programming model and solved the proposed model by CPLEX. Also, Lago et al. [26] focused on the two major variables: the commodity design duration and LT. Thus, a continuous inventory-based sales model was developed, consequently, LT, time-to-market, design time, and sales’ speed were assessed in unique circumstances. Recently, Mahmoodi [27] designed an agile supply chain network in which both production and supply quantities were optimized considering uncertain demand. The agility was taken into account to mitigate the demand uncertainty and improve network flexibility.

An increase in competition can make the supply chain more complicated because supply chains have different members and levels. In this case, game theory is a highly helpful method to analyze a wide variety of possibilities where a player’s decision can affect the others' payoff [28]. Therefore, many papers have been published in competitive situations in a BLSC. A bi-level rice supply chain was given by Cheraghalipour et al. [29] for minimizing the agricultural costs. The model was solved using two meta-heuristics and hybrid algorithms and a numerical example inspired by a real case was employed to show the model’s applicability. Also, Giri et al. [30] developed a BLSC with a manufacturer and a retailer in centralized and decentralized situations. The model was optimized in terms of pricing and advertising strategies. In a decentralized situation, a supply chain with a retailer and some manufacturers were studied to investigate pricing coordination and cooperative advertising [31]. To address the problem, a multi-follower bi-level model was developed and then the model was solved by simulated annealing algorithm. The positive effects of positive relationship between the factors are clarified.

Another work surveyed the effects of manufacturer’s social responsibility investment [32]. The proposed model contains two retailers and a manufacturer for a competitive supply chain and solved using backward induction method. The results proved that selling price is sensitive to the social cooperation effort. A bi-level mixed integer model for closed-loop supply chain to collect more used products [33]. In that research, the government and private centers were taken as a leader and followers, respectively. Based on the assumptions, a network to increase their profits is designed considering the governmental policies. To
solve the model, metaheuristic algorithms and min-max regret scenario-based RO approach were employed, consequently, an optimized network was proposed according to the results.

In other situations where the consumers are sensitive to the prices, Seifbarghy et al. [34] obtained the optimal price and quality considering revenue sharing and Ren et al., [35] presented a Stackelberg model with a manufacturer and a retailer concerning make-to-order and deterministic demand. The findings of this work revealed that personalized products can boost profits in this specific matter. In addition, the effect of uncertain competition and demand on the Nash structure and supply chain profit was examined by Mahmoodi & Eshghi [36] using a numerical example. Taken risk into consideration, the green supply chain concept for the design of a lean and agile company under uncertainty and risk is developed by Golpîra et al. [37] and proposed a bi-level model in that sense. Then, the KKT approach is applied to converting the bi-level model to the single-level one. To validate the model application, the model was solved by a numerical example using a RO approach. Similarly, Yue & You [38] presented a BLSM in the decentralized supply chain for designing an optimized supply chain. The bi-level model was simplified single level using the KKT approach and evaluated through a case study. Another study was devoted to modeling supply chain competition under uncertainty [39]. The model was firstly proposed to determine distribution facilities’ locations and number, retailers’ allocation, and selling price. Then, the model was modified by real assumptions of risk and failure probabilities. For addressing the uncertainty, they used a two stage stochastic model and the models was handled by a hybrid genetic algorithm.

Also, a BLSC of supplier/manufacturer and retailer considering fashion goods with short life cycle and uncertain demand in that the manufacturer and the retailer respectively decided on the price and quantity of the order was offered by Shen et al. [40]. Likewise, Yaghin et al. [41] proposed a model considering pricing integration, markdown pricing, and production programming in a BLSC involving a retailer and a manufacturer for several periods and products in a fuzzy situation. The model was solved by a fuzzy approach and showed its results using a numerical example. Under uncertain conditions, a new hybrid multi-criterion decision making model in which AHP and G-TOPSIS fuzzy were taken into consideration was developed by Tian et al. [42] for deriving weights of influence criteria and assessing automated components remanufacturing production patterns. To show its robustness, a sensitivity analysis was employed.

Another bi-level model is presented to reduce the demand satisfaction costs in the interdictor’s attacking condition. Regarding that, a new heuristic is designed and the model is optimized using the algorithm and CPLEX software [43]. In addition, a bi-level model for competitive supply chain for obtaining optimal inventory and pricing was presented by Mahmoodi [44]. A modified threshold-accepting algorithm had been applied to solving the problem in which the modified algorithm efficiency against a differential evolution approach was proved in this matter. In another research area, Fatollahi-fard et al. [45] proposed a two-stage stochastic programming approach and a Lagrangian relaxation-based algorithm was applied to solve the model. Also, both upper and lower bounds of the problem were considered for improving a better performance. Kaviyani-Charati et al. [46] employed a RO
method to cope with uncertainty and the outputs were investigated by a Mont-Carlo simulation. The efficiency of the RO approach was confirmed in that respect.

Despite the information provided above, Table 1 shows a summary of the literature review of other previous studies.

2.1. Research gap and discussion

Most of the papers published in the field of QR and agility focus only on the retailer level. In other words, various features of QR in retailing were conceptually investigated with little attention to competitive and integrated situations. Thus, the optimal ordering quantity and price will be obtained under competitive circumstances. It is worth mentioning that a retailer with QR needs a manufacturer who can respond to the retailer at the right time. Besides, the effects of QR and agility on profits and clearance sales are investigated by comparison with the traditional supply chain using a real case study. The main aim of this study is to investigate the differences between conventional and modified models. In addition to the discussions, there is little attention to the features of responsiveness and agility in mathematical models in those papers worked on a BLSC despite emphasizing the importance of the consumers' needs and personalized products in a conceptual way. With regard to the features of the fashion market and its uncertainty, a search of the literature revealed studies that employed the RO method are still limited and need to focus more on. Thus, novel bi-level mathematical models for both supply chains in which ordering and production quantities and prices are optimized.

Apart from that, customers play a crucial role in today’s competitive world so that suppliers and retailers have striven to attract customer’s attention by providing better service and product quality, and reasonable prices. Although the consumers' demand and behavior have been considered separately in the previous papers, we consider the mentioned items simultaneously in this work. In addition to a determination of the manufacturer’s and retailer’s strategy, QR and agility would be employed to achieve the strategic customer’s satisfaction and elevate the supply chain members’ profit. These characteristics are applied in this study using some assumptions to increase the satisfaction and profit of the supply chain members.

Also, Customers are usually seeking products followed profit and satisfaction. For instance, in the smartphone industry, two competitors namely Samsung and Apple, are struggling to gain a higher market share by presenting more advanced models. Taking two Samsung smartphone models 7 and 8 as an example, the customers reasonably decide whether to buy the new model after the introduction immediately or wait until buying it with a price reduction. In the meanwhile, the producer tries to introduce a new product to the market due to a decline in the demand and meeting customers’ requirements and expectations. This can lead to attracting more customers and satisfy the customer’s needs, consequently, maximize the producer’s profit in this situation. The former example is an
actual one for an explanation of this article’s application in which tried to obtain the optimal price, production, and ordering quantity in two levels of the supply chain. Thus, a decoration industry will be employed in this research work as a case study to prove its applicability and efficiency in reality.

The main contributions of this work are listed as follow.
1. Two BLSMs for two echelon supply chain with strategic customers are developed.
2. Two different conditions with and without agile abilities are considered for developing the mathematical models.
3. To cope with the uncertain circumstance, a scenario-based RO approach is applied.
4. A real case study of chandelier’s industry is employed to show the models applicability and efficiency.

In summary, two bi-level models will be developed. The first model involves a manufacturer and a retailer without any agility; however, the second one comprises a manufacturer with the agility and a retailer with a QR feature which can be able to produce and satisfy customers’ demand in a timely manner. Additionally, the consumers’ behavior is taken into account in both models and their requirements are considered in the second one. To tackle the uncertainty, the RO approach is employed to enhance the efficiency of the results under real conditions. Also, the manufacturer and the retailer compete with each other in a sequential SG.

3. The problem

Formerly, the producers had efficient and expense-based approaches in which wholesale with lowered manufacturing costs had been preferred. In the ordering of traditional production systems, the retailer can only order once before the sale season due to the long LT [3, 4, 11]. Customization in the fashion market is high [52] and uncertainty in such markets is of great importance. Thus, it could result in losing the mathematical model efficiency in reality if the uncertainty was ignored. In such markets, quick changes in the customer’s behavior and requirements are also observed. The detailed problems are discussed in the following two sub-sections.

3.1. The first problem

A bi-level problem is suggested containing a manufacturer and a retailer who offer the strategic consumers only a product in a selling season with a full and a discount price. Strategic consumers are consumers who behave quite rationally in regards to purchasing commodities and select the time of their purchase after assessing the situation [3, 4, 11].

Given its nature and considering the existing limitations, the manufacturer initially decides on the price and production quantity as a leader and then the retailer offers his best strategies after observing the decisions of the leader. Some hypotheses are considered in this paper and listed as follows.

- The retailer has a limited capacity and his ordering quantity cannot exceed a certain limit.
• The manufacturer and the retailer are chosen as the leader and follower in the SG, respectively.
• The behavior of the strategic consumers influences the decisions of the players.
• The full and discount prices are considered for the retailer.
• Throughout the selling season, the price reduction happens only once and the rest of the products are sold at the end of the selling season at a reduced price.
• The retailer also has budget limitations.
• The consumers are homogeneous and shortage is not allowed.

The notations used to describe the problem are:

**Index**

$s$ Uncertainty scenarios for all $s \in S$

**Parameters**

$C$ The production costs of each product for the manufacturer
$S$ The reduced price for each product for the manufacturer
$Cap$ The manufacturer's production capacity
$\alpha$ The selling commission for the retailer
$v$ The commodity value for the consumer
$\delta$ The discount factor of the price of each product for the retailer
$D_s$ The consumer’s demand for the retailer under scenario $s$
$\gamma$ The percentage of the reduction of commodity value for the consumer during the time
$B$ budget limitation for the retailer
$w_s$ Ordering capacity for the retailer under scenario $s$

**Variables**

$P_I$ The manufacturer’s price
$Q_{ls}$ The production quantity for the manufacturer under scenario $s$
$P_f$ The retailer’s price
$Q_{fs}$ The ordering quantity for the retailer under scenario $s$

After giving some explanation and notations, the traditional model is proposed in the following.

**3.1.1. Mathematical model**

$$\text{Max } \pi_{ls} (Q_{ls}, P_I) = \sum_s \left( (P_I - C).Q_{ls} + (S - C). (Q_{ls} - Q_{fs})^+ \right)$$

S.t:

1. $Q_{ls} \leq Cap$ \hspace{1cm} $\forall s \in S$
2. $Q_{ls} \geq Q_{fs}$ \hspace{1cm} $\forall s \in S$
\[ \text{Max } \pi_f(Q_{fs}, P_f) = \sum_i \left( (P_l - P_i)D_i + (\delta P_f - P_i)(Q_{fs} - D_i) \right) \]

S.t:

\[ (1+\alpha)P_i \leq P_f \]
\[ P_f \leq \frac{\nu (1-\gamma)}{1-\delta} \]
\[ Q_{fs} \leq w_s \quad \forall s \in S \]
\[ P_i, Q_{fs} \leq \beta \quad \forall s \in S \]
\[ Q_{ls}, P_f, Q_{fs}, P_f \geq 0 \quad \forall s \in S \]

Constraint (1) shows the objective function of the leader. Constraint (2) guarantees that the production quantity would not exceed the manufacturer capacity. Constraint (3) guarantees that the manufacturer satisfies the consumer’s order. Constraint (4) is the objective function of the follower. The price announced by the leader cannot exceed the follower's price shown in the constraint (5). Constraint (6) guarantees that the follower's price cannot exceed a certain limit. The retailer's capacity for sales is given in constraint (7). Constraint (8) ensures which the total costs paid for the quantity of ordering the commodity would not exceed a certain limit. Finally, the constraint (9) shows the positivity of the variables.

As explained in the literature and considered the real assumptions and issues, the first problem is modified by adding the QR and agile ability to the retailer and the manufacturer, respectively. The second problem is fully described in the following sub-section.

### 3.2. The second problem

Instant changes in customer’s demands and needs as well as the remaining commodities at the end of the selling season make decision-making really difficult for the retailers and the manufacturers in this highly competitive market. QR, as a strategy, has employed new systems for reducing the inventory level and quick reaction to the consumer’s requirements and demand. The concept of QR, with regard to [3, 4, 8, 11], tries to reduce the inventory level and correctly anticipates the consumer’s demand by reducing the LT. To order the goods required by the customers, the retailer needs a manufacturer who can respond properly to the ordering quantity in a reasonable time according to the consumer’s requirements. The AM can quickly meet the retailers’ demand and needs in an unstable market using useful information sharing [1, 24]. In this problem, a bi-level model is developed in which the retailer and the manufacturer try to determine the price, ordering quantity, and production quantity considering strategic consumers to optimize the objective functions with agile abilities. Regarding the previous works [3, 4, 8, 11], the retailer can order twice throughout the selling season, once before the starting of the season and once after updating the demand during the season. The retailer and manufacturer should pay extra costs in order to use QR and agility per product unit.
In addition to the assumptions of the first problem, the following assumptions are added to this problem:

- There are two times throughout the selling season in that the retailer and the manufacturer can be supplied and the demand type cannot be the same (in terms of feature and quantity).
- Throughout the season, the requirements of the market and the customers will change once.
- The innovations and changes in the product are applied in accordance with the consumers’ demand and requirements under uncertainty.
- The cost added to a product production depends on the rate of innovation and change in the product.
- The value added to the product, offered based on the changes in the market and the customers’ requirements, depends on the rate of innovation and changes in the product.

Regarding real assumptions and agile abilities, this model is developed as follows.

### 3.2.1. Mathematical Modeling

The general model for \( I \) \((i = 1,\ldots,I)\) products is shown as follows:

\[
\text{Max } \pi_i (Q_{hi}, P_{hi}) = \sum_i \left( \left( P_{hi} - (C + n_i) \right) Q_{fi} + \left( S_i - (C + n_i) \right) (Q_{hi} - Q_{fi})^+ \right)
\]

S.t:
\[
Q_{hi} \leq \text{cap}_{pi} \quad \forall i \in I \tag{11}
\]
\[
Q_{hi} \leq Q_{fi} \quad \forall i \in I \tag{12}
\]
\[
\text{Max } \pi_f (Q_{fi}, P_{fi}) = \sum_i \left( (P_{fi} - P_{hi}) D_i + (\delta_i P_{fi} - P_{hi}) (Q_{fi} - D_i) - C_Q Q_{fi} \right) \tag{13}
\]
S.t:
\[
(1 + \alpha_i) P_{hi} \leq P_{fi} \quad \forall i \in I \tag{14}
\]
\[
P_{fi} \leq \frac{(v + m_i)(1 - \gamma_i)}{1 - \delta_i} \quad \forall i \in I \tag{15}
\]
\[
Q_{fi} \leq \omega_i \quad \forall i \in I \tag{16}
\]
\[
P_{hi} Q_{fi} \leq \beta_i \quad \forall i \in I \tag{17}
\]
\[
Q_{hi}, P_{hi}, Q_{fi}, P_{fi} \geq 0 \quad \forall i \in I \tag{18}
\]

Assumed that the cost \( C \) for the first product and also \( m_i, n_i \) and \( C_Q \) would be zero in this sense. Considering two types products, \( i = 1, 2 \) the description and mathematical model are shown as follows.
Parameters

\( C \)   The cost of producing per product unit
\( S_1, S_2 \)   The clearance price for product types 1 and 2 for the manufacturer
\( Cap_1, Cap_2 \)   Production capacity for products type 1 and 2
\( \alpha \)   The selling commission for the retailer
\( \nu \)   The value of the product for the customers
\( \delta_1, \delta_2 \)   The reduced rates of products' prices, types 1 and 2 for the retailer throughout time
\( D_{1s}, D_{2s} \)   Demand for products, types 1 and 2 under scenario \( s \)
\( \gamma_1, \gamma_2 \)   The reduced rates of products' values, types 1 and 2 for the retailer throughout time
\( C_\theta \)   The cost of QR adoption for the retailer
\( m_s \)   The additional value added to the product for the customers under scenario \( s \)
\( n_s \)   The extra cost added to the purchasing cost for the consumers under scenario \( s \)
\( w_{1s}, w_{2s} \)   The ordering capacity of the product types 1 and 2 for the retailer under scenario \( s \)
\( b_1, b_2 \)   The budget of the retailer for the product types 1 and 2

Variables

\( Q_{1s}, Q_{2s} \)   The production quantity of the product types 1 and 2 for the manufacturer under scenario \( s \)
\( P_{1l} \)   The first product price for the manufacturer
\( P_{2l} \)   The second product price for the manufacturer under scenario \( s \)
\( Q_{f1s}, Q_{f2s} \)   The ordering product quantity types 1 and 2 for the retailer under scenario \( s \)
\( P_{f1} \)   The first product price for the retailer
\( P_{f2s} \)   The second product price for the retailer under scenario \( s \)

3.2.2. Mathematical model

Max \( \pi_\theta (Q_{1s}, P_{1l}, Q_{f2s}, P_{f2s}) = \)
\[
\sum_s \left( (P_{1l} - C) \cdot Q_{f1s} + (P_{2l} - (C + n_s)) \cdot Q_{f2s} + (S_1 - C) \cdot (Q_{1s} - Q_{f1s})^+ \right) \]
\[
+ (S_2 - (C + n_s)) \cdot (Q_{f2s} - Q_{f2s})^+ \right) \]
S.t:
\( Q_{1s} \leq \text{cap}_1 \)  \( \forall s \in S \)
\( Q_{2s} \leq \text{cap}_2 \)  \( \forall s \in S \)
\( Q_{1s} \geq Q_{f1s} \)  \( \forall s \in S \)
\( Q_{2s} \geq Q_{f2s} \)  \( \forall s \in S \)
\[
\begin{align*}
\text{Max } & \pi_f(Q_{f_{1s}}, P_{f_1}, Q_{f_{2s}}, P_{f_{2s}}) = \sum_x \left( (P_{f_{1s}} - P_{f_{1s}})D_{x1} + (P_{f_{2s}} - P_{f_{2s}})D_{x2} + (\delta_1 P_{f_{1s}} - P_{f_{1s}})(Q_{f_{1s}} - D_{x1}) 
+ (\delta_2 P_{f_{2s}} - P_{f_{2s}})(Q_{f_{2s}} - D_{x2}) - C_Q Q_{f_{2s}} \right) \\
\text{s.t.:} & \quad (1+\alpha)P_{f_{1s}} \leq P_{f_{1s}} \quad \forall s \in S \quad 25 \\
& \quad (1+\alpha)P_{f_{2s}} \leq P_{f_{2s}} \quad \forall s \in S \quad 26 \\
& \quad P_{f_{1s}} \leq v \cdot (1-\gamma_1)/(1-\delta_1) \quad \forall s \in S \quad 27 \\
& \quad P_{f_{2s}} \leq (v + m_s) \cdot (1-\gamma_2)/(1-\delta_2) \quad \forall s \in S \quad 28 \\
& \quad Q_{f_{1s}} \leq \omega_{s_1} \quad \forall s \in S \quad 29 \\
& \quad P_{f_{1s}}Q_{f_{1s}} \leq \beta_1 \quad \forall s \in S \quad 30 \\
& \quad Q_{f_{2s}} \leq \omega_{s_2} \quad \forall s \in S \quad 31 \\
& \quad P_{f_{2s}}Q_{f_{2s}} \leq \beta_2 \quad \forall s \in S \quad 32 \\
& \quad Q_{f_{1s}1}, P_{f_{1s}}, Q_{f_{2s}}, P_{f_{2s}}, Q_{f_{1s}}, P_{f_{1s}}, Q_{f_{2s}}, P_{f_{2s}} \geq 0 \quad \forall s \in S \quad 33 
\end{align*}
\]

Constraint (19) shows the leader's objective function including costs, clearance sales, and sales. The production quantity of the products types 1 and 2 would not exceed its capacity ensured by constraints (20 and 21). Constraints (22 and 23) guarantee the manufacturer satisfies the ordering quantity of the retailer. The follower's objective function is shown in constraint (24). Constraints (25 and 26) demonstrate that the leader cannot announce a price higher than that of the follower. Constraints (27 and 28) ensure that the retail price cannot exceed a certain limit since customers do not be disposed to incur extra expenses. The ordering capacity and available budget for the follower are limited by constraints (29-32). And finally, the constraint (33) shows the positive variables.

4. Solution procedure

Several theories have been offered for solving linear bi-level programming problems [37, 38, 43, 44, 53, 54, 55]. Thus, this paper applies the KKT approach to converting the proposed bi-level models into single-level ones. In the following sub-section, the KKT method is briefly discussed and then, the models are linearized as the KKT's precondition. After that, they are simplified single levels and finally, the models would be solved by scenario-based RO and analyzed its results considering some various conditions.

4.1. The KKT Approach

The KKT approach is initially written for the lower level, then added to the upper one, and the bi-level model will be simplified a single-level one. It is fully addressed in the literature of the KKT [37, 38, 54-57]. However, if the model contains any non-linear formula,
they shall firstly be converted into linear one. Then the linear mathematical model can be turned to a single-level by the KKT approach. Therefore, the proposed models need to be firstly linearized.

4.2. Linearization

The $P_f Q_{fs}$ and $P_r Q_{fs}$ formulas in the first model and $Q_{f1}, P_{f1}, Q_{f2s}, P_{f2s}, Q_{f1s}, P_{f1s}$, $Q_{f2s}, P_{f2s}$ formulas in the second one are non-linear. So, they should be changed to linear by the method suggested in [58].

Taking the non-linear formula, $P_f Q_{fs}$, in the first model. On the one hand, the price per product for the manufacturer cannot be lower than its production costs. On the other hand, the maximum price that the manufacturer can offer would not be more than a certain limit owing to the nature of the problem. Thus, the high and low levels of the unit price in the first model are

$$\frac{v \cdot (1 - \gamma)}{(1 + \alpha)(1 - \delta)}$$

and $C$ respectively. The limitation related to the two non-linear variables is shown as follows:

$$C \leq P_f \leq \frac{v \cdot (1 - \gamma)}{(1 + \alpha)(1 - \delta)}$$  

Similarly, for the second variable in this formula, we would have:

$$0 \leq Q_{fs} \leq Cap ~ \forall S \in S$$

Now the non-linear formula is replaced with a new variable called $y_{1s}$ and its associated constraints are:

$$y_{1s} = P_f Q_{fs}$$

$$C \leq P_f \leq \frac{v \cdot (1 - \gamma)}{(1 + \alpha)(1 - \delta)}$$  

$$C Q_{fs} \leq P_f Q_{fs} \leq \frac{v \cdot (1 - \gamma)}{(1 + \alpha)(1 - \delta)} Q_{fs}$$

$$C Q_{fs} \leq y_{1s} \leq \frac{v \cdot (1 - \gamma)}{(1 + \alpha)(1 - \delta)} Q_{fs}$$

And the rest of non-linear formulas are similarly changed to linear, the details of them are shown in appendix A. Regarding the previous explanations, the linearized models are simplified single-level ones using the KKT approach illustrated in section 4.3 [53-57]:

4.3. Single-level models

The bi-level models are transformed into single ones using KKT approach and presented in the following models.

4.3.1. First model

$$\max \pi_{1s}(Q_{fs}, P_f) = \sum_s \left( y_{1s} - C Q_{fs} + (S - C)(Q_{ls} - Q_{fs})^+ \right)$$

s.t:
\[ Q_s \leq \text{cap} \quad \forall s \in S \]  38

\[ Q_{1s} \geq Q_{fs} \quad \forall s \in S \]  39

\[ C \cdot Q_{fs} \leq y_{1s} \leq \frac{v \cdot (1 - \gamma)}{(1 + \alpha) \cdot (1 - \delta)} Q_{fs} \quad \forall s \in S \]  40

\[ (1 + \alpha) P_t \leq P_f \]  41

\[ P_f \leq \frac{v \cdot (1 - \gamma)}{1 - \delta} \]  42

\[ Q_{fs} \leq w_s \quad \forall s \in S \]  43

\[ P_t \cdot Q_{fs} \leq \beta \quad \forall s \in S \]  44

\[ C \cdot (1 + \alpha) Q_{fs} \leq y_{2s} \]  45

\[ y_{2s} \leq \frac{v \cdot (1 - \gamma)}{(1 - \delta)} Q_{fs} \quad \forall s \in S \]  46

\[ \delta + U_4 - U_5 + U_6 = 0 \]  47

\[ (1 - \delta) D_s + U_1 - U_2 + U_7 = 0 \quad \forall s \in S \]  48

\[ -U_3 - C \cdot (1 + \alpha) U_4 + \frac{v \cdot (1 - \gamma)}{(1 - \delta)} U_5 + U_6 = 0 \]  49

\[ U_4 \cdot (C \cdot (1 + \alpha) Q_{fs} - y_{2s}) + U_5 \cdot \left( y_{2s} - \frac{v \cdot (1 - \gamma)}{(1 - \delta)} Q_{fs} \right) + U_8 \cdot y_{2s} = 0 \quad \forall s \in S \]  50

\[ U_1 \cdot ((1 + \alpha) P_t - P_f) + U_2 \cdot \left( P_f - \frac{v \cdot (1 - \gamma)}{1 - \delta} \right) + U_7 \cdot P_f = 0 \]  51

\[ U_3 \cdot (Q_{fs} - w_s) + U_4 \cdot (C \cdot (1 + \alpha) Q_{fs} - y_{2s}) + U_5 \cdot \left( y_{2s} - \frac{v \cdot (1 - \gamma)}{(1 - \delta)} Q_{fs} \right) \quad \forall s \in S \]  52

\[ +U_6 \cdot Q_{fs} = 0 \]

\[ y_{1s}, y_{2s}, U_1, \ldots, U_8, Q_{fs}, P_t, Q_{fs}, P_f \geq 0 \quad \forall s \in S \]  53

The second model can also be reformulated as follows.

### 4.3.2. Second model

\[ \text{Max } \pi_s (Q_{1s}, P_{1s}, Q_{2s}, P_{2s}) = \]

\[ \sum_s \left( y_{1s} - C \cdot Q_{f1s} + y_{2s} - (C + n_s) \cdot Q_{f2s} + (S_1 - C) \cdot (Q_{1s} - Q_{f1s}) + \right. \]

\[ \left. + (S_2 - (C + n_s)) \cdot (Q_{2s} - Q_{f2s}) \right) \]

\[ \text{S.t.} \]

\[ Q_{1s} \leq \text{cap}_1 \quad \forall s \in S \]  55

\[ Q_{1s} \leq \text{cap}_2 \quad \forall s \in S \]  56
\[ Q_{1s} \geq Q_{f,1s} \quad \forall s \in S \]
\[ Q_{12s} \geq Q_{f,2s} \quad \forall s \in S \]
\[ C \cdot Q_{f,1s} \leq y_{1s} \leq \frac{v \cdot (1 - \gamma_1)}{(1 + \alpha) \cdot (1 - \delta_1)} \cdot Q_{f,1s} \quad \forall s \in S \]
\[ (C + n_s) \cdot Q_{f,2s} \leq y_{2s} \leq \frac{(v + m_s) \cdot (1 - \gamma_2)}{(1 + \alpha) \cdot (1 - \delta_2)} \cdot Q_{f,2s} \quad \forall s \in S \]
\[ (1 + \alpha) \cdot P_{1s} \leq P_{f,1} \quad \forall s \in S \]
\[ (1 + \alpha) \cdot P_{12s} \leq P_{f,2s} \quad \forall s \in S \]
\[ P_{f,1} \leq \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} \quad \forall s \in S \]
\[ P_{f,2s} \leq \frac{(v + m_s) \cdot (1 - \gamma_2)}{1 - \delta_2} \quad \forall s \in S \]
\[ Q_{f,1s} \leq \omega_{1s} \quad \forall s \in S \]
\[ P_{1s} \cdot Q_{f,1s} \leq \beta_1 \quad \forall s \in S \]
\[ Q_{f,2s} \leq \omega_{2s} \quad \forall s \in S \]
\[ P_{12s} \cdot Q_{f,2s} \leq \beta_2 \quad \forall s \in S \]
\[ (1 + \alpha) \cdot C \cdot Q_{f,1s} \leq y_{3s} \quad \forall s \in S \]
\[ y_{3s} \leq \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} \cdot Q_{f,1s} \quad \forall s \in S \]
\[ (C + n_s) \cdot (1 + \alpha) \cdot Q_{f,2s} \leq y_{4s} \quad \forall s \in S \]
\[ y_{4s} \leq \frac{(v + m_s) \cdot (1 - \gamma_2)}{1 - \delta_2} \cdot Q_{f,2s} \quad \forall s \in S \]
\[ (1 - \delta_1) \cdot D_{1s} + U_1 - U_3 + U_{11} = 0 \quad \forall s \in S \]
\[ (1 - \delta_2) \cdot D_{2s} + U_2 - U_4 + U_{12} = 0 \quad \forall s \in S \]
\[ \delta_1 + U_7 - U_8 + U_{13} = 0 \]
\[ \delta_2 + U_9 - U_{10} + U_{14} = 0 \]
\[ -U_5 - (1 + \alpha) \cdot C \cdot U_7 + \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} \cdot U_8 + U_{15} = 0 \quad \forall s \in S \]
\[ -C \cdot Q_{Q} - U_6 - (1 + \alpha) \cdot (C + n_s) \cdot U_9 + \frac{(v + m_s) \cdot (1 - \gamma_2)}{1 - \delta_2} \cdot U_{10} + U_{16} = 0 \quad \forall s \in S \]
\[ U_1 \cdot [(1 + \alpha) \cdot P_{f,1} - P_{f,1}] + U_3 \left( P_{f,1} - \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} \right) + U_{11} \cdot P_{f,1} = 0 \quad \forall s \in S \]
\[ U_2 \cdot [(1 + \alpha) \cdot P_{f,2s} - P_{f,2s}] + U_4 \left( P_{f,2s} - \frac{(v + m_s) \cdot (1 - \gamma_2)}{1 - \delta_2} \right) + U_{12} \cdot P_{f,2s} = 0 \quad \forall s \in S \]


\[ U_7(1 + \alpha) C \cdot Q_{f,1s} - y_{3s} + U_8\left(y_{3s} - \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} Q_{f,1s}\right) + U_{13} y_{3s} = 0 \quad \forall s \in S \quad 81 \]

\[ U_9\left((C + n_s)(1 + \alpha) Q_{f,2s} - y_{4s}\right) + U_{10}\left(y_{4s} - \frac{v + m_s}{1 - \delta_2} (1 - \gamma_2) Q_{f,2s}\right) \quad \forall s \in S \quad 82 \]

\[ + U_{14} y_{4s} = 0 \]

\[ U_5(Q_{f,1s} - w_{1s}) + U_7\left((1 + \alpha) C \cdot Q_{f,1s} - y_{3s}\right) + U_8\left(y_{3s} - \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1} Q_{f,1s}\right) \quad \forall s \in S \quad 83 \]

\[ + U_{15} Q_{f,1s} = 0 \]

\[ U_6(Q_{f,2s} - w_{2s}) + U_9\left((C + n_s)(1 + \alpha) Q_{f,2s} - y_{4s}\right) + \]

\[ U_{10}\left(y_{4s} - \frac{v + m_s}{1 - \delta_2} (1 - \gamma_2) Q_{f,2s}\right) + U_{16} Q_{f,2s} = 0 \quad \forall s \in S \quad 84 \]

\[ y_{1s}, y_{2s}, y_{3s}, y_{4s}, U_1, \ldots, U_{16}, Q_{f,1s}, P_{11}, Q_{f,2s}, P_{12}, Q_{f,3s}, P_{13}, Q_{f,4s}, P_{14} \geq 0 \quad \forall s \in S \quad 85 \]

### 4.4. Reduction the Single-level Model

In this paper, the single-level models can be equalized the simplest form due to the nature of the problems and the KKT approach [56]. Based on the problem, the value of variables would not be zero. For this purpose, proved that the limitations of stationarity and complementary slackness are always correct. Therefore, they can be removed from the single-level models, and the simplified models can be analyzed. For instance, the \( P_{f,1} \) variable does not equal zero so that the dual variable associated with it would be zero. Similarly, \( U_{11}, \ldots, U_{16} \) should be zero for the second model. Considering constraints (69 and 70) related to the equation, \( y_{3s} = P_{f,1} Q_{f,1s} \). In these constraints, if \( P_{f,1} \) is equal to its upper limit, the upper limit of the constraint (70) will be satisfied as well. With regard to the explanation provided before, the problems are mathematically modeled to maximize the profit. Thus, the manufacturer and the retailer seek the maximum price in this competition. In that regard, the price would equal the upper limit, and the constraint (69 and 70) would be as follows: \((1 + \alpha) C \cdot Q_{f,1s} - y_{3s} \neq 0 \) and \( y_{3s} - v \cdot \frac{(1 - \gamma_1)}{1 - \delta_1} Q_{f,1s} = 0 \). Hence, the value of \( U_7 \) should be zero, but, by contrast, the value of \( U_8 \) could not be zero. In the following, given the constraint, \( \delta_1 + U_7 + U_8 + U_{13} = 0 \), and the value of \( U_{13} \) being zero, the value of \( U_8 \) would be \( \delta_1 \). Then, all constraints of stationarity and complementary slackness will be calculated and found in the same way (Appendix B).

After going through some mathematical processes, a brief description of the RO method is provided for solving the models in the next sub-section.

### 4.5. Robust Optimization
Despite developing various RO methods, the scenario-base RO will be utilized in this paper. The scenario-based RO fits the problem considered in this research. Mulvey believes that if a model in reality is infeasible under some scenarios, the model will lose its efficiency. Consequently, Mulvey et al. [59] introduced a new model for the RO in which uncertainties are regarded as various scenarios with certain probabilities. This model shows the answers which would be the results of a tradeoff between the robustness of the model and answer. An answer can be robust when it remains near to the optimum under all scenarios. Similarly, a model could be robust when it remains feasible for all considered scenarios with high probability. Find more information in [59, 60].

The robust models for the first and second mathematical models are presented, respectively as follows.

The first model:

\[
\begin{align*}
\text{Maxw} &= \sum_{s=1}^{S} \rho_s \cdot \zeta_{ls} - \lambda \left[ \sum_{s=1}^{S} \rho_s \cdot \left( \sum_{j=1}^{S} \rho_j \cdot \zeta_{lj} - \sum_{j=1}^{S} \rho_j \cdot \zeta_{lj}^+ \right) + 2 \theta_s \right] - \\
\sum_{s=1}^{S} \rho_s \cdot \left( W_1 Z_{1s} + W_2 Z_{2s} + W_3 Z_{3s} + W_4 Z_{4s} + W_5 (Z_{5s}^+ + Z_{5s}^-) + W_6 Z_{6s} \right) & \quad \text{86} \\
\text{S.t:} & \quad \zeta_{ls} = y_{ls} - C Q_{fs} + (S - C) (Q_{fs} - Q_{fs})^+ & \quad \forall s \in S \quad \text{87} \\
\zeta_{ls} - \sum_{j=1}^{S} \rho_j \cdot \zeta_{lj}^+ + \theta_s & \geq 0 & \quad \forall s \in S \quad \text{88} \\
Q_{ls} - Z_{ls} & \leq \text{cap} & \quad \forall s \in S \quad \text{89} \\
Q_{ls} & \geq Q_{fs} - Z_{2s} & \quad \forall s \in S \quad \text{90} \\
C Q_{fs} - Z_{3s} & \leq y_{1s} & \quad \forall s \in S \quad \text{91} \\
y_{ls} & \leq \frac{\gamma \cdot (1 - \gamma)}{(1 + \alpha) (1 - \delta)} Q_{fs} + Z_{4s} & \quad \forall s \in S \quad \text{92} \\
Q_{fs} & = W_s + Z_{5s}^+ - Z_{5s}^- & \quad \forall s \in S \quad \text{93} \\
y_{ls} & \leq \beta + Z_{6s} & \quad \forall s \in S \quad \text{94} \\
C (1 + \alpha) Q_{fs} - Z_{7s} & \leq y_{2s} & \quad \forall s \in S \quad \text{95} \\
y_{2s} & = \frac{\gamma \cdot (1 - \gamma)}{(1 - \delta)} Q_{fs} + Z_{8s}^+ - Z_{8s}^- & \quad \forall s \in S \quad \text{96} \\
(1 + \alpha) P_f = P_f & \quad \text{97} \\
P_f & = \frac{\gamma \cdot (1 - \gamma)}{1 - \delta} & \quad \text{98} \\
Z_{1s}, Z_{2s}, Z_{3s}, Z_{4s}, Z_{5s}^+, Z_{5s}^-, Z_{6s}, Z_{7s}, Z_{8s}^+, Z_{8s}^- & \quad \forall s \in S \quad \text{99}
\end{align*}
\]
The second model:

\[
\text{Max } \pi_b \left( Q_{11s}, P_{11}, Q_{12s}, P_{12} \right) = \sum_{s=1}^{S} \rho_s \zeta_{ls} - \lambda \left[ \sum_{s=1}^{S} \rho_s \left[ \left( \zeta_{ls} - \sum_{s'} \rho_{s'} \zeta_{ls'} \right) + 2\theta_s \right] \right] - \\
\sum_{s=1}^{S} \rho_s \left[ W_1 Z_{1s} + W_2 Z_{2s} + W_3 Z_{3s} + W_4 Z_{4s} + W_5 Z_{5s} + W_6 Z_{6s} + W_7 Z_{7s} + \\
W_8 Z_{8s} + W_9 \left( Z_{9s}^+ + Z_{9s}^- \right) + W_{10} \left( Z_{10s}^+ + Z_{10s}^- \right) + W_{11} \left( Z_{11s}^+ + Z_{11s}^- \right) + \\
W_{12} Z_{12s} + W_{13} Z_{13s} + W_{14} Z_{14s} + W_{15} \left( Z_{15s}^+ + Z_{15s}^- \right) + W_{16} Z_{16s} + \\
W_{17} \left( Z_{17s}^+ + Z_{17s}^- \right) + W_{18} \left( Z_{18s}^+ + Z_{18s}^- \right) \right]
\]

S.t:

\[ y_{1s} + y_{2s} - C Q_{f1s} - (C + n_s) Q_{f2s} + (S_1 - C) (Q_{11s} - Q_{f1s})^+ \]
\[ + (S_2 - C - n_s) (Q_{12s} - Q_{f2s})^+ = \zeta_{ls} \]
\[ \zeta_{ls} - \sum_{s'=1}^{S} \rho_{s'} \zeta_{ls'} + \theta_s \geq 0 \]
\[ Q_{11s} - Z_{1s} \leq \text{cap}_1 \]
\[ Q_{12s} - Z_{2s} \leq \text{cap}_2 \]
\[ Q_{11s} + Z_{3s} \geq Q_{f1s} \]
\[ Q_{12s} + Z_{4s} \geq Q_{f2s} \]
\[ C Q_{f1s} - Z_{5s} \leq y_{1s} \]
\[ y_{1s} \leq \frac{v \cdot (1 - \gamma_1)}{(1 + \alpha) \cdot (1 - \delta_1)} Q_{f1s} + Z_{6s} \]
\[ (C + n_s) Q_{f1s} - Z_{7s} \leq y_{2s} \]
\[ y_{2s} \leq \frac{v \cdot (1 - \gamma_2)}{(1 + \alpha) \cdot (1 - \delta_2)} Q_{f2s} + Z_{8s} \]
\[ (1 + \alpha) P_{f2s} = P_{f2s} + Z_{18s}^+ - Z_{18s}^- \]
\[ P_{f2s} = \frac{v \cdot (1 - \gamma_2)}{1 - \delta_2} + Z_{9s}^+ - Z_{9s}^- \]
\[ Q_{f1s} = \omega_1 + Z_{10s}^+ - Z_{10s}^- \]
\[ Q_{f2s} = \omega_2 + Z_{11s}^+ - Z_{11s}^- \]
\[ y_{1s} \leq \beta_1 + Z_{12s} \]
\[ y_{2s} \leq \beta_2 + Z_{13s} \]
As seen in the two models, the first part of the objective function in both models consists of the solution robustness part which seeks to optimize the mean profit in case each scenario occurs. The second part is to minimize the variance of the objective function related to the solution robustness and the last part is also associated with the penalty function and model robustness. Constraints 88 and 102 are for linearization of the robust objective function.

A real case of a chandelier’s industry is investigated for evaluating the QR and agility performance in the realty. In this case, the applicability and efficiency of the models are proved and also drawing a comparison between models would be provided.

4.6. Case Study

The decoration and chandelier manufacturing company of Shomal was started in 1991 in Northern Iran. The second level of the supply chain is the decoration- and chandelier-shop Milad which has served as the pioneering retailer in this field for many years, selling luxury chandeliers and decorations in the North of Iran. In this case, the traditional model is primarily applied and then the second model is implemented. The market for these types of products is highly unstable, consequently, quick changes are observed in this market. Notice that the manufacturer only produces conventional and almost predictable products with low variations in demand. In addition, the retailer does not have much interest in considering the variations of customers’ demands. The retailer’s and manufacturer’s strategies are in a way that lets the retailer order only once during a sale season. In this model, the optimal ordering and production quantities and prices for two levels of the supply chain will be obtained. For solving the model, the interview method and analyzing historical data are employed and then the certain values of parameters are shown in Table 2. Furthermore, the marketing and sales managers do not have accurate information about some parameters because of uncertain
circumstances. Therefore, the values of non-deterministic parameters, different scenarios, and an occurrence probability of each one are demonstrated in Table 3.

***** Please insert Table 2*****

Seven scenarios are considered for uncertain parameters whose values are reported together with the occurrence probability of each scenario. Concerning the information received by the experts in this area, different scenarios have pertained to each other.

***** Please insert Table 3*****

After solving the model in the GAMS 24.0.1, the optimal production and order quantity are given in Table 4, the price and objective function regarding various scenarios for each level are illustrated in the following.

***** Please insert Table 4*****

The leader has chosen the best possible strategy with regard to the model constraints and the follower should accept that as its constraints are not violated in the leader's decision. Thus, the manufacturer should offer a strategy in that all follower constraints are fully satisfied. Regarding the problem nature and the supply chain, Table 4 presents the values of clearance sales for the manufacturer, the leader of the SG. Moreover, important to note that the leader could not propose any price desired due to the limitations and also the follower would not accept the proposed price. If this is the case, the retailer will not order anything so the manufacturer shall make a decision rationally.

***** Please insert Table 5*****

Moreover, optimal prices are provided in Table 5. For the model nature, the manufacturer aims at a price that obtains the maximum profit. The leader of the SG cannot offer his favorite price since otherwise, the follower may not order at all. Thus, this strategy is not preferred by the manufacturer. Similarly, the retailer's price cannot exceed a certain limit either because the customers have a certain value in their mind for the goods. So if the price is higher than their expectation, they will not have any full-price purchase. This situation is also not favorable for the retailer. The mean values of the objective function for the retailer and the manufacturer when all scenarios are taken into consideration are presented as follows.

*****Please insert Figure 1*****

The values of the objective function for the manufacturer and retailer while two members of the supply chain take traditional approaches are shown in Fig. 1. The objective function for the manufacturer is almost $2,000,000 more than that of the retailer since the manufacturer as the leader of the game has no clearance sale in the system. However, the retailer mostly encounters clearance sales which fall his profits due to the nature of the market.
After implementing the first model in the real case, now the second one is solved and then analyze its results. For applying the model, the retailer and manufacturer by accepting extra costs can respond quickly and properly to their market, thus they can meet the customers’ expectations and demands at the right time. The retailer can issue an order less than before considering the market demand using QR ability. He can also identify the market requirements and demands and update his information after starting the sale season and give the manufacturer his new order depending on the customers’ demands and requirements [3, 4, 8, 11]. The AM can also recognize the unpredictable changes in the market and satisfy them based on both information sharing among retailer and manufacturer and cutting-edge technology [1, 24]. These matters have been taken into account by the extra cost in this paper.

Having such capabilities necessitates paying some costs which can make them able to refill or produce an order based on consumer preferences and requirements. To gather the preferred data, the interview with the retailer and the manufacturer was conducted and then the values of the parameters are illustrated in Table 6.

***** Please insert Table 6*****

Subsequently, parameters depending on the scenarios with its occurrence probabilities are presented in Table 7.

***** Please insert Table 7*****

Given the values in Tables 6 and 7, the output of the second model is reported in the following tables.

***** Please insert Table 8*****

Table 8 also shows production and order quantities for the retailer and manufacturer under different scenarios. The retailer can order twice during the sale season by incurring extra costs in which he could satisfy customer’s demands and needs. Furthermore, he can boost profits by minimizing the clearance sales at the end of the sale season. Besides, the manufacturer has the ability to identify variation in the market by accepting extra costs and look for an appropriate response to the market requirements in a short time. In Table 8, the manufacturer can respond to the conventional and unconventional demand at the delivery time based on the market demands under each scenario.

***** Please insert Table 9*****

Moreover, Table 9 shows the price of the first product which would receive before starting the season. In other words, the retailer orders a certain quantity of the product and provide it with a price independent of the scenario.

***** Please insert Table 10*****
The selling price of the second type will pertain to the scenarios if the parameters associated with the demand for the product are considered uncertain. As can be seen in Table 10, the prices of the second type of product under different scenarios are not the same. This price is calculated using the product value for the customers and other important parameters in the pricing.

The average of the objective function for the second model concerning different scenarios is presented in Fig. 2.

*****Please insert Figure 2*****

The average profit of the retailer is about $5.14e+6 more than that of the manufacturer demonstrated in Fig. 2. Although the costs for the retailer and manufacturer are increased by using agility, they would have better planning for the unstable market and offer products regarding the demands and instantaneous changes of the market that would lead to the improvement in customer satisfaction and also the product value is enhanced. The increase in the value for the customers has persuaded them to pay more money for their desired product and this has led to boosting the profit of supply chain members. Thus, they have also reduced the quantity of the product subjected to clearance sales. Now, the first and second models and the real situation of the market in a similar situation are compared as follows.

*****Please insert Figure 3*****
*****Please insert Figure 4*****

With regard to Fig. 3 and 4, the objective functions for the second model has been improved by the suggested tools. The retailer should look for two important actions in the world of fashion and luxury products: 1) to prevent the clearance sales of goods at the end of the sales season. 2) To respond quickly and appropriately to the instant changes at the market level. Furthermore, the AM with the features such as quick production response and flexibility has the capacity to timely consider the newest demands at the market level in the production and respond to the needs. Thus, the manufacturer could acquire such capability by covering the cost so that could increase his customers’ satisfaction, finally, his profit.

By QR, the retailer could reduce the extra chandeliers and seeing the low product quantity, so, in that regard, the strategic customers are more willing to buy chandeliers at full price and pay higher prices for the ones that have been satisfied their requirements. The results of this model show that the retailer and manufacturer can enhance their profits by using useful information, and up-to-date methods for production, ordering, and pricing systems. These modern methods have consequences as well, including 1) by employing agility, the production level of the supply chain can react to the instantaneous variation in the needs and demands of the market in a certain time. Consequently, the value of the goods has risen for the customers and the manufacturer’s profit would improve. 2) By taking the customers’ requirements and behavior into consideration, the retailer can attract more potential customers and raise their willingness to buy at full price. 3) Taking advantage of QR to bring down both the waste of resources and the off-sales can result in satisfaction for both
customers and retailers because customers’ needs would be satisfied as perfectly as possible and the retailer, on the other hand, could turn a better profit in this sense. 4) These features in the manufacturing and retailing allow them to do strategic planning to act better in response to uncertainty and sudden changes in the market.

Achievements of the second model have culminated in reducing delivery time, shortening LT, lowering prices, and customer satisfaction. To enhance customer satisfaction, the manufacturer attempts to introduce a new product or model based on their needs and demand and meet the quality standards. Introducing new products considering customers’ needs has driven the supply chain members into providing concentrated and compatible programming and requiring sufficient processes with an integrated information system.

The supply chain members incurring additional costs to provide an integrated system for sharing information and having close relationships with customers could cut the total costs (such as the cost of obsolete products), reduce LT, enhance customers’ satisfaction, identify immediate variations and finally introduce products depending on the market needs.

Improving flexibility and managerial expertise, employing skilled workers, and providing an integrated information system in the supply chain would help the competitors to get the highest market share. Furthermore, the acceleration of responding and identifying market demands and variations using exact information from customers could satisfy their demands and requirements. Finally, utilizing advanced technology make them able to react appropriately and quickly to various circumstances; both conventional and non-conventional demands. However, these capabilities are acquired by devoting more effort, time, and also costs.

4.6.1. Sensitivity analysis

In the following, the sensitivity of the model parameters is analyzed by changes in their values; consequently, variations in the average profit in the various situations are measured and presented.
In Fig. 7, the average objective function for the retailer is analyzed in a situation that the retailer has the ability of QR and chooses the non-AM. In this way, the average profit has increments rather than the first model in a situation for the retailer that the cost of QR is equal to or less than $1,500. However, the mean objective function for the retailer is declined by increasing the QR costs, which in such a situation, utilizing QR is not economically reasonable.

Now, the mean objective function in which the retailer selects the AM is analyzed. The retailer’s profits with different costs are presented in Fig. 8. Regarding that, the less the QR costs are, the more the profit is generated. By increasing the costs up to $50,000 for the retailer, a profit is earned more than it was in the first model. In this situation, if the QR costs are risen from $5,000 to $50,000, utilizing this capability will be economically reasonable.

In Fig. 9 (a), the average profits are analyzed while the costs of agility in the second scenario are changed. As can be observed, by increasing its costs up to $83,000, the manufacturer makes more profit than the first model. On the contrary, the profit decreases after that point. Fig. 9 (b) shows the changes of mean profit with variations in the agility costs for all scenarios. The average profit is increased more than that was in the first model as the costs of agility is up to $40,000 in each scenario. Thus, these limited costs would be economically reasonable for the manufacturer if the other parameters were constant.

Fig. 10 shows the variations of mean profit as the second type of product is differently valued for the customers. Assuming other parameters being constant, if this parameter has up to $50,000 reduction compared with that of the case study, the manufacturer’s average profit will be more than that of the first model. However, the manufacturer can gain a better profit than before by using agile capabilities.

The average profit changes when there is a variation in the demand are depicted in Fig. 11. This figure is aimed at showing economic justification in low demand situations. As illustrated, the retailer’s average profit moves further than that in the first model by the falling demand for the second type of product (around 390) in each scenario.

The average profit of the second product with the initial cost factor (up to 0.717) is more than that in the first model is demonstrated in Fig. 12.
While the cost factor for the customers is reduced during the time, changes in the retailer’s profit are illustrated in Fig. 13. As observed, a decline in the cost up to 0.75 results in more profit for the retailer compared to the first model and seems to be more economically reasonable.

****Please insert Figure 13****

In Fig. 14, the retailer’s average profit changes are illustrated as the QR costs are increasing in value under a situation in which there have been demand changes in each scenario. In this analysis, five different types of demand with distinctive values are taken into account analyzed by various costs of the QR. For instance, the demands given in the first scenario are 573, 608, 445, 479, 686, 580 and 505, respectively. In that situation, the earnings economically justify the retailer to accept QR costs up to $10,000. The rest of the results are shown in the Fig. 14.

****Please insert Figure 14****

In Fig. 15, the retailer’s average profit against variations in the QR costs is investigated in a situation in that demands, a reduction in the product value and in price of the second product are differently changed. Maximum objective function for the retailer is obtained through the initial price factor of 0.78 and QR costs of zero. What is taken from this figure is that a reduction in the demand and the price of the second product shows greater effects on a decline in the retailer’s profit.

****Please insert Figure 15****

This study confirms that agile abilities are associated with customer satisfaction and profits. The previous discussions and results compared to the real situations confirm the models’ applicability and efficiency. In contrast to the case, more products would be sold to customers in comparison with the real situation. Besides, the customers are convinced to pay more for the products in which their requirements would be met. Although the first model has not enjoyed any agile ability, the profit of the model was more than that of reality. Because the proposed model considered competition and also the possibilities of all scenarios in that the model could provide decision-makers with better strategies and results. Regarding the second model, the clearance sales are noticeably reduced compared with the first model and reality, consequently, the observed decline in that amount could be attributed to the manufacturer’s and retailer’s profits. Differences between the scientific findings and reality can also have influenced the waste resources. In other words, natural resources such as wood, oil, and water are wasted as the products have become obsolete. Thus, these findings will help practitioners to understand how to develop their work optimally and reduce their clearance sales. Last but not least, the outputs and discussions have important implications for developing an agile system.

The manufacturers require cutting-edge software (e.g. SAP and modular assembly), skilled and flexible workforce, and quick production system to develop products based on market variability. These actions can reduce natural resources waste so that they can recoup
an amount of their investment in this regard. Also, the wholesale price will have risen as they could optimize their production quantity based on the market, thus, their profits will be boosted. In the retail business, retailers should record background information about customers’ requirements, sales, and complaints. Besides, the integrated information system makes it possible for retailers to optimally adjust orders depending on the market demand. Thus, the clearance sales would be noticeably decreased and the retailers could enhance their earnings although they need to employ a computerized database and system. The system could provide them with useful information on customers’ demand, complaints, and sales figures; consequently, the available data analysis will be valuable to improve service level and quality based on customer requirements and expectations. As such, the customers will be satisfied and accordingly, a more market share could be derived.

It would be better if the supply chain members was trying to develop a more centralized system in the whole supply chain. As known, the competitive supply chain can fall the members’ profits due to the specific situations of competition. For more explanation, the members have to reach a decision because of some limitations under competitive circumstances. Therefore, good decisions will be made if the supply chain has become more centralized using different contracts such as revenue sharing. Additionally, the retailers and manufacturers should facilitate good communication with an integrated system (e.g. SAP). In this regard, the manufacturers can receive the required information on demand and expectations and then, will adopt a new approach in their production line to meet their inevitable demand.

5. Conclusion

Due to the high variability and inconsistency of the fashion market, it is shown that a successful retailer and manufacturer are the ones who can adopt the right decisions and policies according to the market needs and variations. Considering the customers’ behavior, competitive situations, and the goods value, the method of ordering, pricing, and production shall be improved.

Therefore, in this study, two BLSMs including the manufacturer and retailer were developed under uncertain circumstances to determine the optimal production and order quantities and price. Then the proposed models were implemented in the real case. In the first model, the traditional manufacturer, regardless of the market needs and variations, produces a certain volume of goods and the retailer also issues an order for once due to the long LT. In this situation, the results were a little enhanced compared to the reality in terms of prices and clearance sales. However, in the second model, an AM was introducing the products based on the instant variations in the market needs and demands considering the uncertainty supplied depending on the changes of the customers’ needs.

The results of the proposed models show that the manufacturer and retailer can identify the market variations within a short time using their agile abilities. They can also respond to these changes at the right time, accordingly, the customers will be more satisfied so that they will be persuaded to spend more money on the market. The additional costs of adopting the
abilities would be incurred by each retailer and manufacturer considered in the model. But the second product value is higher than the first one. The main reason is that the value of goods produced concerning the market variations will increase the customers’ motivation toward paying more for buying a product.

There are lots of possible directions for future research. The presented model in this study could be developed by considering the simultaneous and sequential competition between the retailers and manufacturers, in which just some of them may be agile. For more explanation, firstly, the retailers and the manufacturers compete with each other at their levels (Simultaneous Game) and then two levels (the dominant retailer and manufacturer) will sequentially compete for more profit (SG). Moreover, the proposed models can be solved by heuristic and metaheuristic algorithms to compare the obtained results with the ones found in this study. Therefore, a further study with more focus on the solution approach is suggested and the efficiency of the method is more examined.

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Appendices:

Appendix A

\[
C_r(1+\alpha) \leq P_j \leq \frac{v_r(1-\gamma)}{(1-\delta)}
\]

A-1

\[0 \leq Q_{fs} \leq cap\quad \forall s \in S\]

A-2

\[y_{2s} = P_j Q_{fs}\quad \forall s \in S\]

A-3

\[C_r(1+\alpha) \leq P_j \leq \frac{v_r(1-\gamma)}{(1-\delta)} \implies C_r(1+\alpha)Q_{fs} \leq P_j Q_{fs} \leq \frac{v_r(1-\gamma)}{(1-\delta)}Q_{fs}\]

A-4

\[y_{3s} = P_{11}Q_{f1s}\quad \forall s \in S\]

A-5

\[(1+\alpha)C \leq P_{11} \leq \frac{v_r(1-\gamma_1)}{1-\delta_1}\quad \forall s \in S\]

A-6

\[C \leq P_{11} \leq \frac{v_r(1-\gamma_1)}{(1+\alpha). (1-\delta_1)}\quad \forall s \in S\]

A-7

\[0 \leq Q_{f1s} \leq Cap\quad \forall s \in S\]

A-8

\[C \leq P_{11} \leq \frac{v_r(1-\gamma_1)}{(1+\alpha). (1-\delta_1)} \implies C Q_{f1s} \leq P_{11} Q_{f1s} \leq \frac{v_r(1-\gamma_1)}{(1+\alpha). (1-\delta_1)} Q_{f1s}\]

A-9

\[C + n_s \leq P_{12s} \leq \frac{(v + m_s)(1-\gamma_2)}{(1+\alpha). (1-\delta_2)} \implies (C + n_s)Q_{f2s} \leq P_{12s} Q_{f2s} \leq \frac{(v + m_s)(1-\gamma_2)}{(1+\alpha). (1-\delta_2)} \]

A-10
\[(C + n_s)(1 + \alpha) \leq P_{f,2s} \leq \frac{(v + m_s)(1 - \gamma_s)}{(1 - \delta_2)} \Rightarrow (C + m_s)(1 + \alpha)Q_{f,2s} \leq P_{f,2s}Q_{f,2s} \leq \frac{(v + m_s)(1 - \gamma_s)}{(1 - \delta_2)}Q_{f,2s} \Rightarrow (C + n_s)(1 + \alpha)Q_{f,2s} \leq y_{4s} \leq \forall s \in S \quad \text{A-11}\]

**Appendix B**

**First model**

\[U_5(C.(1 + \alpha)Q_{f,5} - y_{2s}) = 0 \quad \forall s \in S \quad \text{B-1}\]

\[U_6\left(y_{2s} - \frac{v.(1 - \gamma)}{(1 - \delta)}Q_{f,5}\right) = 0 \quad \forall s \in S \quad \text{B-2}\]

\[U_9y_{2s} = 0 \quad \forall s \in S \quad \text{B-3}\]

\[U_{11}(1 + \alpha).P_i - P_f = 0 \quad \forall s \in S \quad \text{B-4}\]

\[U_{12}\left(P_f - \frac{v.(1 - \gamma)}{1 - \delta}\right) = 0 \quad \forall s \in S \quad \text{B-5}\]

\[U_{8}P_f = 0 \quad \forall s \in S \quad \text{B-6}\]

\[U_{3}(Q_{f,5} - w_i) = 0 \quad \forall s \in S \quad \text{B-7}\]

\[U_5(C.(1 + \alpha)Q_{f,5} - y_{2s}) = 0 \quad \forall s \in S \quad \text{B-8}\]

\[U_6\left(y_{2s} - \frac{v.(1 - \gamma)}{(1 - \delta)}Q_{f,5}\right) = 0 \quad \forall s \in S \quad \text{B-9}\]

\[U_{7}Q_{f,5} = 0 \quad \forall s \in S \quad \text{B-10}\]

**Second model**

\[U_{11}(1 + \alpha).P_{i,1} - P_{f,1} = 0 \quad \text{B-11}\]

\[U_{12}\left(P_{f,1} - \frac{v.(1 - \gamma)}{1 - \delta}\right) = 0 \quad \text{B-12}\]

\[U_{11}P_{f,1} = 0 \quad \forall s \in S \quad \text{B-13}\]

\[U_{12}(1 + \alpha).P_{f,2s} - P_{f,2s} = 0 \quad \forall s \in S \quad \text{B-14}\]

\[U_{12}\left(P_{f,2s} - \frac{(v + m_s)(1 - \gamma_s)}{1 - \delta_2}\right) = 0 \quad \forall s \in S \quad \text{B-15}\]

\[U_{12}P_{f,2s} = 0 \quad \forall s \in S \quad \text{B-16}\]

\[U_{7}(1 + \alpha).CQ_{f,1s} - y_{3s} = 0 \quad \forall s \in S \quad \text{B-17}\]
\[ U_8^s \left( y_{3s} - \frac{v \cdot (1 - \gamma_1)}{1 - \delta_1}, Q_{1fs} \right) = 0 \quad \forall s \in S \quad \text{B-18} \]

\[ U_{13} \cdot y_{3s} = 0 \quad \forall s \in S \quad \text{B-19} \]

\[ U_9 \cdot \left( (C + n_f) \cdot (1 + \alpha) Q_{2s} - y_{4s} \right) = 0 \quad \forall s \in S \quad \text{B-20} \]

\[ U_{10} \cdot \left( y_{4s} - \frac{(v + m_s) \cdot (1 - \gamma_2)}{(1 - \delta_2)} Q_{2s} \right) = 0 \quad \forall s \in S \quad \text{B-21} \]

Mohammad Kaviyani-Charati is a data analyst in Kalleh Dairy Company and graduated in Industrial Engineering from Amirkabir University of Technology, Tehran, Iran. His research interests are sustainable supply chain management, operations research, and data analytics. He has also published his papers in international journals and conference including Journal of Manufacturing Systems and International Journal of Engineering.

Seyyed Hassan Ghodsypour is a professor of Industrial Engineering in the Department of Industrial Engineering and Management Science at Amirkabir University of Technology, Tehran, Iran. He received his PhD in Operations Research from Nottingham University in 1996. He has published various papers in different peer-reviewed journals and conferences including International Journal of Production Economics, European Journal of Operations Research, Applied Soft Computing, Computers & Industrial Engineering, Advances in Engineering Software, Applied Mathematics and Computation, Fuzzy Sets and Systems, Automation in Construction etc. His research interests are Supply Chain Management, Project Management, Portfolio Management, and Science and Technology Policy.

Mostafa Hajiaghaei-Keshteli earned his B.Sc. from Iran University of Science & Technology, Tehran, Iran (2004); M.Sc. from University of Science & Culture, Tehran, Iran (2006); and Ph.D. from Amirkabir University of Technology (Tehran-Polytechnic), Tehran, Iran (2012); all in Industrial Engineering. He has over 10 years of experience in Business Development, System Analysis, Inventory and Project Management. Mostafa also has worked for many corporations in Iran and has held the positions of consulter, Planning and project manager and VP. The main focus of his research has been Supply Chain Network, Transportation, Meta-heuristics, and Inventory Control. He has
published more than 70 articles in leading scientific and engineering conferences and journals such as CAIE, IEEE, ESWA, KNOSYS, IJAMT, NCAA, and ASOC in his research area. Currently, he is an assistant professor in University of Science and Technology of Mazandaran.

Fig. 1. Objective function of the retailer and manufacturer for the first model.

Fig. 2. Average profit of the objective function for the second model.

Fig. 3. The retailer’s profit function in the first model, real situation of the market and the second model.
**Fig. 4.** The manufacturer’s profit function in the first model, real situation of the market and the second model.

**Fig. 5.** Trade-off between model and solution robustness in the first model.

**Fig. 6.** Trade-off between model and solution robustness in the second model.
Fig. 7. Average profit function of the retailer versus cost of quick response regardless of the manufacturer agility.

Fig. 8. Analysis of the retailer’s profit versus the changes in quick response costs.

Fig. 9. The manufacturer’s average profit versus the variations of the second scenario (a) and all of the scenarios (b) for the agility costs.

(a)
Fig. 10. Variations of the manufacturer's average profit versus variations of the product value for the customers.

Fig. 11. Variations of the average profit versus variations of the demand.
**Fig. 12.** Variations of the retailer’s average profit versus the percentage of price reduction of the second type product.

**Fig. 13.** Variations of the retailer’s average profit versus cost reduction for the customers.
Fig. 14. Variations of the average profit versus variations of the demand and costs of quick response.

Fig. 15. Comparison of the average profit with the demand variations, reductive factor of the product value and price factor of the second type product versus variations of the quick response costs.
| Authors and Years | Number of level | Competition | Type of game | Agility for manufacturer | Uncertainty | Decision variable | Maximization | QR for retailer | Customer behavior | Type of uncertainty | Journal name |
|------------------|----------------|-------------|--------------|--------------------------|-------------|--------------------|--------------|----------------|------------------|-------------------|--------------|
| [3]              | 1              | ✔  | Simultaneously | ✔     | ✔  | Price and quantity | ✔  | ✔  | Strategic | Stochastic | MS            |
| [4]              | 2              | ✔  | Sequential    | ✔  ✔  | ✔  | Price and quantity | ✔  | ✔  | Strategic | Stochastic | Omega         |
| [11]             | 1              | ✔  | Simultaneously | ✔  | ✔  | Price and quantity | ✔  | ✔  | Strategic, Myopic and Bargain-hunting | Stochastic | MS            |
| [12]             | 2              | ✔  | Simultaneous  | ✔   | ✔  | Inventory and pricing | ✔  | ✔  | Strategic | Stochastic | IJPE          |
| [17]             | 2              | ×  ×  ×  ×  | Risk analysis and sensitivity | ✔  | ✔  | - | Stochastic | DS            |
| [18]             | 2              | ×  ×  ×  | Time and quantity | ✔  | ✔  | - | Stochastic | CIE          |
| [19]             | 2              | ×  ×  ✔  | Price and quantity | ✔  | ×  | - | Normal probability distribution | MOQ          | ITIOR |
| [21]             | 2              | ×  ×  ×  ✔  | Service level and inventory management | ✔  | ×  | - | - | IJPR          |
| [25]             | 4              | ✔  ×  ×  ×  | Quantity and inventory | ✔  | ×  | - | RO | AOR          |
| [34]             | 2              | ✔  | Sequential    | ×  | ×  | Price and quality | ✔  | ×  | - | - | IJPE         |
| [35]             | 2              | ✔  | Sequential    | ✔  | ×  | Price | ✔  | ×  | - | - | CIE          |
| [47]             | 2              | ✔  | Sequential    | ×  | ✔  | Price, quantity and advertising | ✔  | ✔  | - | - | EJOR         |
| [48]             | 2              | ✔  | ✔  | ✔  Quantity | ✔  | ✔  | - | Stochastic | IEEE         |
| [49]             | 2              | ✔  | Sequential    | ×  | ×  | Production, advertising and investment | ✔  | ×  | - | - | COR          |
| [50]             | 2              | ✔  | Sequential    | ×  | ×  | Production, quantity and price | ✔  | ×  | - | - | IJAMT        |
| [51]             | 2              | ✔  | Sequential    | ×  | ×  | Price, percent of reduce price and quantity | ✔  | ×  | - | - | IJCM         |
| This paper       | 2              | ✔  | ✔  | ✔  Quantity and price | ✔  | ✔  | Strategic | RO            |

Table 1

Literature review on the subject areas.
### Table 2

Value of deterministic variables.

| Parameter (P) | Value (V) |
|---------------|-----------|
| C             | 170,000   |
| \( \delta \)  | 70%       |
| \( \gamma \)  | 70%       |
| S             | 150,000   |
| Cap           | 2,400     |
| V             | 300,000   |
| \( \alpha \)  | 0.213     |
| B             | 800,000,000 |

### Table 3

The values of scenario (S)-dependent parameters.

| s/p | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-----|-----|-----|-----|-----|-----|-----|-----|
| \( D_s \) | 1670 | 1780 | 1930 | 1520 | 2010 | 1745 | 1890 |
| \( w_s \) | 1900 | 1980 | 2040 | 1600 | 2100 | 1800 | 1950 |
| \( \rho_s \) | 0.09 | 0.15 | 0.23 | 0.08 | 0.22 | 0.10 | 0.13 |

### Table 4

Optimal solution (OS) of production and ordering quantity.

| S/(OS) | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|--------|-----|-----|-----|-----|-----|-----|-----|
| \( Q_{ls} \) | 1900 | 1980 | 2040 | 1600 | 2100 | 1800 | 1950 |
| \( Q_{fs} \) | 1900 | 1980 | 2040 | 1600 | 2100 | 1800 | 1950 |

### Table 5

Optimal price (OP) in the first model.

| OP     | V               |
|--------|-----------------|
| Retailer | 300,000         |
| Manufacturer | 247,320.692   |

### Table 6

The values of certain parameters of the model.

| Parameter (P) | Value (V) |
|---------------|-----------|
| C             | 170,000   |
| \( \delta_1 \) | 70%       |
| \( \delta_2 \) | 75%       |
| \( \gamma_1 \) | 70%       |
| \( \gamma_2 \) | 72%       |
| \( s_1 \)     | 60,000    |
| \( s_2 \)     | 170,000   |
| V             | 300,000   |
| Parameter | Value |
|-----------|-------|
| $\alpha$  | 0.213 |
| $\beta_1$ | 400,000,000 |
| $\beta_2$ | 400,000,000 |
| $\text{Cap}_1$ | 1.600 |
| $\text{Cap}_2$ | 1,300 |
| $C_Q$ | 5,000 |

**Table 7**
Values of uncertain parameters depended to the scenario.

| S/P | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----|----|----|----|----|----|----|----|
| $D_{1s}$ | 1080 | 1180 | 1240 | 1040 | 950 | 1050 | 1180 |
| $D_{2s}$ | 895 | 970 | 1076 | 869 | 835 | 998 | 963 |
| $n_s$ | 30000 | 26000 | 20000 | 33000 | 38000 | 23000 | 27000 |
| $m_s$ | 61000 | 51000 | 39000 | 65000 | 72000 | 48000 | 53000 |
| $w_{1s}$ | 1200 | 1260 | 1310 | 1100 | 980 | 1120 | 1240 |
| $w_{2s}$ | 950 | 1005 | 1100 | 900 | 870 | 1040 | 980 |
| $\rho_s$ | 0.14 | 0.15 | 0.09 | 0.13 | 0.23 | 0.15 | 0.11 |

**Table 8**
The optimum quantity of production and order under different scenarios.

| S/V | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----|----|----|----|----|----|----|----|
| $Q_{f1}$ | 1200 | 1260 | 1310 | 1100 | 980 | 1120 | 1240 |
| $Q_{f2}$ | 950 | 1005 | 1100 | 900 | 870 | 1040 | 980 |
| $Q_{l1}$ | 1200 | 1260 | 1310 | 1100 | 980 | 1120 | 1240 |
| $Q_{l2}$ | 950 | 1005 | 1100 | 900 | 870 | 1040 | 980 |

**Table 9**
The price of the first product.

|             | Retailer | 247,320.692 |
|-------------|----------|-------------|
| Manufacturer |          | 300,000     |

**Table 10**
The price ($P_{r}$) of the second type product ($\times 10^2$)

| S/Pr | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|------|----|----|----|----|----|----|----|
| $P_{l2}$ | 3333 | 3241 | 3130 | 3370 | 3435 | 3213 | 3259 |
| $P_{f2}$ | 4043 | 3931 | 3797 | 4088 | 4166 | 3898 | 3954 |