Neutrino mass limits from SDSS, 2dFGRS and WMAP

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We investigate whether cosmological data suggest the need for massive neutrinos. We employ galaxy power spectrum measurements from the Sloan Digital Sky Survey (SDSS) and the Two Degree Field Galaxy Redshift Survey (2dFGRS), along with cosmic microwave background (CMB) data from the Wilkinson Microwave Anisotropy Probe (WMAP) and 27 other CMB experiments. We also use the measurement of the Hubble parameter from the Hubble Space Telescope (HST) Key Project. We find the sum of the neutrino masses to be smaller than 0.75 eV at 2\(\sigma\) (1.1 eV at 3\(\sigma\)).

Neutrino oscillation experiments provide substantial evidence that the three known neutrinos have a combined mass \((\Sigma \equiv \sum m_\nu)\) of at least about \(\sqrt{\delta m^2} \sim 0.045\) eV\(^1\), which can be improved to \(\Sigma \approx 1\) eV by the KATRIN experiment [4]. Large-scale structure (LSS) data from SDSS [5] combined with CMB data from WMAP [6] alone, yield \(\Sigma \leq 1.7\) eV at the 95\% C. L. [7], with no strong priors or assumptions. With 2dFGRS data [8], CMB data and significantly stronger priors, a 95\% C. L. upper limit of 1 eV was found in Ref. [9]. An even stronger bound, \(\Sigma \leq 0.63\) eV was obtained by the WMAP collaboration from a combination of 2dFGRS and CMB data and the 2dFGRS measurement of the galaxy bias parameter \(b \equiv \sqrt{P_g(k)/P_m(k)}\), where \(P_g\) and \(P_m\) are the galaxy and matter power spectra, respectively. The 2dFGRS collaboration measured a scale-independent bias over scales \(k \sim 0.1 - 0.5\) h/Mpc [10]; the WMAP collaboration adopted this bias for \(k < 0.2\) h/Mpc. A preference for massive neutrinos found in Ref. [11] is controversial because of the input of a linear clustering amplitude \(\sigma_8\) (defined as the rms mass fluctuations in spheres of radius 8 h\(^{-1}\) Mpc) [12]; at present there is no experimental consensus on the determination of \(\sigma_8\) (e.g., see Table 5 of Ref. [7]). All the above cosmological constraints were placed under the assumption of a flat Universe in accord with the predictions of inflation.

We analyze a large set of LSS and CMB data at scales where the matter power spectrum is linear, i.e., \(k \lesssim 0.15\) h/Mpc. We include the power spectrum determinations from 205,443 and 147,024 galaxy redshifts measured by SDSS and 2dFGRS, respectively. The CMB data comprise all WMAP data and a combination of the 151 band power measurements from 27 other CMB experiments [13] including CBI [14] and ACBAR [15], with multipoles \(l\) up to 1700 (or \(k \sim 0.15\) h/Mpc) [16]. Throughout, we impose a top-hat prior on the Hubble constant \(h (H_0 = 100h \text{ km/s/Mpc})\), from the HST [17]. We do not include Ly-\(\alpha\) forest data [18] in our analysis because an inversion from the flux power spectrum to the linear power spectrum is nonlinear and model-dependent [19].

Effects of neutrino mass on the power spectrum:

Neutrinos of eV masses are relativistic when they decouple, and so their final number density is independent of their mass, \(n_\nu = 3/11n_s\). Since \((E_\nu) = 2.7T_\gamma\), and \(\Omega_\gamma h^2\) is essentially the energy density of the CMB with \(T_\gamma = 2.725\) K, \(n_\nu\) is known, and

\[
\omega_\nu \equiv \Omega_\nu h^2 = n_\nu \Sigma \simeq \frac{\Sigma}{94.1\text{ eV}}. \tag{1}
\]

Neutrinos freestream on scales smaller than their Jeans length scale, which is known as the freestreaming scale. While neutrinos freestream, their density perturbations are damped, and simultaneously the perturbations of cold dark matter and baryons grow more slowly because of the missing gravitational contribution from neutrinos. The freestreaming scale of relativistic neutrinos grows with the horizon. When the neutrinos become nonrelativistic, their freestreaming scale shrinks, they fall back into the potential wells, and the neutrino density perturbation resumes to trace those of the other species. Freestreaming suppresses the power spectrum on scales smaller than the horizon when the neutrinos become nonrelativistic. (For eV neutrinos, this is the horizon at

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\(^1\) \(\delta m^2\) is the mass-squared difference of atmospheric neutrino oscillations. The mass-squared difference of solar neutrino oscillations is significantly smaller with \(\sqrt{\delta m^2} \sim 0.008\) eV.
matter-radiation equality). Lighter neutrinos freestream out of larger scales and cause the power spectrum suppression to begin at smaller wavenumbers [20],

$$k_{nr} \simeq 0.026 \left( \frac{m_\nu \omega_M}{1 \text{ eV}} \right)^{1/2} \text{Mpc}^{-1},$$

assuming almost degenerate neutrinos. Here, $\omega_M \equiv \Omega_M h^2$ is the total matter density (which is comprised of baryons, cold dark matter and massive neutrinos). On the other hand, heavier neutrinos constitute a larger fraction of the matter budget and suppress power on smaller scales more strongly than lighter neutrinos [21]:

$$\frac{\Delta P_m}{P_m} \approx -8 f_\nu \simeq -0.8 \left( \frac{\Sigma}{1 \text{ eV}} \right) \left( \frac{0.1}{\omega_M} \right),$$

where $f_\nu \equiv \Omega_\nu / \Omega_M$ is the fractional contribution of neutrinos to the total matter density.

Analyses of CMB data are not sensitive to neutrino masses due to the fact that at the epoch of last scattering, eV mass neutrinos behave essentially like cold dark matter. (WMAP data alone allow the dark matter to be entirely constituted by massive neutrinos [7]). However, an important role of CMB data is to constrain other parameters that are degenerate with $\Sigma$. Also, since there is a range of scales common to the CMB and LSS experiments, CMB data provides an important constraint on the bias parameters. Sensitivity to neutrino masses results from the complementarity of galaxy surveys and CMB experiments.

Figure 1 shows that the suppression of power caused by massive neutrinos is much greater for the galaxy power spectrum than for the CMB TT spectrum. We do not show the effect of neutrino masses on the CMB TE spectrum because it is tiny. Note that we have normalized the spectra to emphasize the power suppression at small scales.

**Analysis:**

We compute the CMB TT and TE power spectra $\delta T^2_l = l(l + 1)C_l/2\pi$, and the matter power spectrum $P_m(k)$, all in the linear approximation, using the Code for Anisotropies in the Microwave Background or CAMB [22] (which is a parallelized version of CMBFAST [23]). We assume the Universe to be flat, $\Omega_{tot} = 1$, that the dark energy is in the form of a cosmological constant $\Lambda$, and that there are three neutrino species. We calculate the angular power spectra on a grid defined by $\omega_M$, $f_\nu$, the baryon density $\omega_B \equiv \Omega_B h^2$, the Hubble constant $h$, the reionization optical depth $\tau$, and the spectral index $n_s$ of the primordial power spectrum.

We employ the following grid:

- $0.05 \leq \omega_M \leq 0.27$ in steps of size 0.02, and $\omega_M = 0.14, 0.16$.
- $0 \leq f_\nu \leq 0.15$ in steps of size 0.01.

**FIG. 1:** Upper panel: CMB TT power spectra. Lower panel: Galaxy power spectra. The curves are the spectra for $\Sigma = 0.28 \text{ eV}$ (solid, best-fit parameters; the galaxy power spectrum is shown for the SDSS best-fit normalization), $\Sigma = 1.5 \text{ eV}$ (dotted) and $\Sigma = 3 \text{ eV}$ (dashed). The latter two spectra have all other parameters (except the normalization, bias parameters and $\Omega_{cdm}$) fixed at the best-fit values. All curves are for a flat universe. The CMB TT spectra are normalized to have identical powers at the first peak. The galaxy power spectra are normalized to have identical powers at $k = 0.017 h/\text{Mpc}$. In the upper panel, the data points marked by circles (squares) represent the binned TT spectrum from WMAP (pre-WMAP experiments). In the lower panel, the data points marked by circles (squares) represent the galaxy power spectra from the 17 SDSS (32 2dFGRS) bands used in our analysis.

- $0.018 \leq \omega_B \leq 0.028$ in steps of size 0.001.
- $0.64 \leq h \leq 0.80$ in steps of size 0.02.
- $0 \leq \tau \leq 0.3$ in steps of size 0.025.
- $0.8 \leq n_s \leq 1.2$ in steps of size 0.02.
The normalization of the primordial power spectrum $A_s$, is a continuous parameter.

The bias parameters $b_{SDSS}$ and $b_{2dF}$ are scale-independent\(^2\) and continuous.

The suppression of small scale power depends directly on $f_\nu$ and indirectly on $\Sigma$. From Eq. (3), $\omega_M$ is strongly degenerate with $\Sigma$, requiring independent knowledge of $\omega_M$ to break the degeneracy. The SDSS collaboration only used WMAP data to provide this information and found the somewhat weak, but conservative, 95\% C. L. bound $\Sigma \leq 1.7$ eV \([7]\). Their analysis also gave a 1\% constraint $h = 0.645_{-0.040}^{+0.048}$, which lies at the lower end of the HST measurement $h = 0.72 \pm 0.08$ \([17]\). It is known that less stringent constraints on $\Sigma$ are obtained for lower values of $h$ \([25]\) because CMB data then allow larger $\Omega_M$ \([26]\). Aside from the fact that we are using a significantly larger dataset than the SDSS collaboration, we expect our analysis to yield a stronger constraint on $\Sigma$ simply because we constrain $h$ by the HST measurement.

Note that our grid-range for $\omega_M$ is almost identical to a combination of the 3\% range of $\Omega_M$ allowed by SN Ia redshift data \([28]\), and the HST prior on $h$.

In our analysis, we conservatively include only the first 17 SDSS band powers, for which $0.016 \leq k \leq 0.154$ /Mpc, and the power spectrum is in the linear regime. We use the window functions and likelihood code provided by the SDSS collaboration \([5]\), and leave the bias parameter $b_{SDSS}$ free.

For 2dFGRS, only 32 band powers with $0.022 \leq k \leq 0.147$ /Mpc are included. The window functions and covariance matrix have been made publicly available by the 2dFGRS collaboration \([8]\). The bias parameter $b_{2dF}$ is left free.

The WMAP data are in the form of 899 measurements of the TT power spectrum from $l = 2$ to $l = 900$ \([29]\) and 449 data points of the TE power spectrum \([30]\). We compute the likelihood of each model of our grid using Version 1.1 of the code provided by the collaboration \([31]\). The WMAP code computes the full covariance matrix under the assumption that the off-diagonal terms are sub-dominant. This approximation breaks down for unrealistically small amplitudes. When the height of the first peak is below 5000 $\mu$K$^2$ (which is many standard deviations away from the data), only the diagonal terms of the covariance matrix are used to compute the likelihood.

We include the combined CMB data from pre-WMAP experiments, by using the 28 pre-WMAP band powers, the window functions and the correlation matrix compiled in Ref. \([13]\).

We obtain the $\pm 1\%$ range of $\Sigma$, by selecting those parameter sets with $\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}} \leq 1$, after minimizing over all other parameters.

**Results and conclusions:**

Our analysis yields the best-fit parameters $f_\nu = 0.02$, $\omega_M = 0.15$, $\omega_B = 0.023$, $h = 0.66$, $\tau = 0.075$, $n_s = 0.96$, $A_s = 21.38 \times 10^{-10}$ with $\chi^2 = 1499.83$ for 1425 – 9 = 1416 degrees of freedom. From the normalizations of the power spectra required to fit the SDSS and 2dFGRS data, we obtain $b_{SDSS} = 1.13_{-0.13}$ and $b_{2dF} = 1.20_{-0.13}^{+0.08}$ (1\% ranges).

Figure 2 shows $\Delta\chi^2$ versus $\Sigma$. The neutrino mass bound is $\Sigma \leq 0.75$ eV at 2\% ($\Sigma \leq 1.1$ eV at 3\%). Thus, cosmological data do not require a significant neutrino dark matter component, and are increasingly rejecting a quasi-degenerate neutrino mass spectrum.

Eventually, lensing measurements of galaxies and the CMB by large scale structure are expected to probe a hierarchical neutrino mass spectrum with $\Sigma \approx 0.04$ eV \([32]\).

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\(^2\) On large scales, galaxy bias is expected to be a scale-independent constant \([24]\). This has been confirmed by the 2dFGRS collaboration \([10]\).

\(^3\) As a proof of principle, it was shown in Ref. \([25]\) that a flat non-$\Lambda$CDM model with $h = 0.45$, $\Omega_M = 1$ and $\Sigma = 3.8$ eV, provides as good a representation of the 2dFGRS and pre-WMAP CMB data as a flat $\Lambda$CDM model with $h = 0.7$, $\Omega_M = 0.3$ and massless neutrinos. This is true even including the WMAP data provided different power-laws are used to describe the spectrum for l above and below the first peak \([27]\).
[1] For a recent review see, V. Barger, D. Marfatia and K. Whisnant Int. J. Mod. Phys. E 12, 569 (2003) [arXiv:hep-ph/0308123].

[2] V. Barger, S. L. Glashow, D. Marfatia and K. Whisnant, Phys. Lett. B 532, 15 (2002) [arXiv:hep-ph/0201262].

[3] C. Weinheimer, arXiv:hep-ex/0210050; V. M. Lobashev et al., Nucl. Phys. Proc. Suppl. 91, 280 (2001).

[4] A. Osipowicz et al. [KATRIN Collaboration], arXiv:hep-ex/0109033.

[5] M. Tegmark et al. [SDSS Collaboration], arXiv:astro-ph/0301207.

[6] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0301207].

[7] M. Tegmark et al. [SDSS Collaboration], arXiv:astro-ph/0301207.

[8] W. J. Percival et al., arXiv:astro-ph/0105252; M. Colless et al., arXiv:astro-ph/0105252.

[9] S. Hannestad, JCAP 0305, 004 (2003) [arXiv:astro-ph/0303076].

[10] L. Verde et al., Mon. Not. Roy. Astron. Soc. 335, 432 (2002) [arXiv:astro-ph/0112161].

[11] S. W. Allen, R. W. Schmidt and S. L. Bridle, arXiv:astro-ph/0306386.

[12] S. W. Allen, A. C. Fabian, R. W. Schmidt and H. Ebeling, Mon. Not. Roy. Astron. Soc. 342, 287 (2003) [arXiv:astro-ph/0208394].

[13] X. Wang, M. Tegmark, B. Jain and M. Zaldarriaga, arXiv:astro-ph/0212417.

[14] T. J. Pearson et al., arXiv:astro-ph/0205388.

[15] C. I. Kuo et al. [ACBAR collaboration], arXiv:astro-ph/0212289.

[16] M. Tegmark and M. Zaldarriaga, Phys. Rev. D 66, 103508 (2002) [arXiv:astro-ph/0207047]; D. Scott, J. Silk and M. J. White, Science 268, 829 (1995) [arXiv:astro-ph/9505015].

[17] W. L. Freedman et al., Astrophys. J. 553, 47 (2001) [arXiv:astro-ph/0012376].

[18] R. A. Croft et al., Astrophys. J. 581, 20 (2002) [arXiv:astro-ph/0012324]; P. McDonald, J. Miralda-Escude, M. Rauch, W. L. W. Sargent, T. A. Barlow, R. Cen and J. P. Ostriker, arXiv:astro-ph/9911196.

[19] U. Seljak, P. McDonald and A. Makarov, Mon. Not. Roy. Astron. Soc. 342, L79 (2003) [arXiv:astro-ph/0302571].

[20] W. Hu and D. J. Eisenstein, Astrophys. J. 498, 497 (1998) [arXiv:astro-ph/9710216]; D. J. Eisenstein and W. Hu, Astrophys. J. 511, 5 (1997) [arXiv:astro-ph/9710252].

[21] W. Hu, D. J. Eisenstein and M. Tegmark, Phys. Rev. Lett. 80, 5255 (1998) [arXiv:astro-ph/9712057].

[22] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473 (2000) [arXiv:astro-ph/9911177]; http://camb.info/

[23] U. Seljak and M. Zaldarriaga, Astrophys. J. 469, 437 (1996) [arXiv:astro-ph/9603033].

[24] A. Taruya, H. Magara, Yi. P. Jing and Y. Suto, PASJ 53, 155 (2001).

[25] O. Elgaroy and O. Lahav, JCAP 0304, 004 (2003) [arXiv:astro-ph/0303089].

[26] J. A. Rubino-Martin et al., Mon. Not. Roy. Astron. Soc. 341, 1084 (2003) [arXiv:astro-ph/0205367].

[27] A. Blanchard, M. Douspis, M. Rowan-Robinson and S. Sarkar, arXiv:astro-ph/0304237.

[28] J. L. Tonry et al., Astrophys. J. 594, 1 (2003) [arXiv:astro-ph/0305008].

[29] G. Hinshaw et al., Astrophys. J. Suppl. 148, 135 (2003) [arXiv:astro-ph/0302217].

[30] A. Kogut et al., Astrophys. J. Suppl. 148, 161 (2003) [arXiv:astro-ph/0302213].

[31] L. Verde et al., Astrophys. J. Suppl. 148, 195 (2003) [arXiv:astro-ph/0302218].

[32] W. Hu and M. Tegmark, Astrophys. J. Lett. 514, 65 (1999) [arXiv:astro-ph/9811168]; K. N. Abazajian and S. Dodelson, Phys. Rev. Lett. 91, 041301 (2003) [arXiv:astro-ph/0212216]; M. Kaplinghat, L. Knox and Y. S. Song, arXiv:astro-ph/0303344.