ROBUST OPTIMUM DESIGN OF TUNED MASS DAMPERS FOR HIGH-RISE BUILDINGS SUBJECT TO WIND-INDUCED VIBRATION

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Abstract. Control devices are commonly applied to suppress structural displacement due to dynamic loads. In this work, a study of the optimum tuned mass dampers (TMDs) design was carried out, installed in a high-rise building subject to wind-induced vibration. Tuned mass dampers are the most known passive energy device and their design are an important area of study. A mathematical model to consider the wind force in the time domain was introduced. A procedure to obtain the robust design of tuned mass dampers was proposed through the optimization under uncertainties, which considers the uncertainties present in the structural properties of the building and also in the dynamic excitation. This method led to the robust design of TMDs, whereas the device performance became insensitive to the randomness of the input variables of the optimization problem. The robust design was compared with a design obtained by a deterministic optimization and the advantages of using an optimization under uncertainties are shown. In addition, the proposed methodology was compared with traditional TMD design methods, showing again the superiority of the proposed methodology.

1. Introduction. The tendency to construct tall and slender buildings becomes these structures susceptible to vibrations caused by wind action. Often, dynamic wind loading can cause unexpected displacements, ranging from discomfort to users to structural problems. The use of passive dissipation devices helps to control the dynamic response in this type of situation.

The most well-known structural vibration control device is the tuned mass damper (TMD), which consists of a mass, with its own damping and stiffness, connected to the main structure and, usually, tuned in the building’s fundamental frequency.

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The TMD has existed for many years and has been used satisfactorily in structures to suppress vibrations from both wind loading and earthquakes. According to [24], tuned mass dampers are designed to increase the overall effective damping of a structure, in order to control the dynamic response of this structure.

Unlike the dimensioning of other structural elements, the tuned mass dampers cannot be oversized in favor of safety, because the effectiveness of these devices depends on the tuning with one (or more) vibration modes of each building where they are installed. In view of this, the use of optimization techniques aiming to reach the parameters, the location and the ideal quantity of devices is an important area of research.

The optimization of tuned mass dampers under dynamics excitations has been discussed in the literature for many years (e.g., [24]–[6]). Tanaka and Mak [24] analyzed the efficiency in the use of tuned mass dampers to energy dissipation in a building subjected to wind loading. They verified the effectiveness of the use of TMD and obtained a reduction in the dynamic response of the building in the range of 30 to 60%. According to the authors, the smaller the width of the excitation frequency range, the more effective the control system will have, the tuning of the frequency of the damper and the frequency of the structure being relevant.

Kawaguchi, Teramura and Omote [12] presented a computational method to predict the response of the building with TMD in the frequency domain. As a result, the damper reduced the first mode vibration of the structure up to 60%. Ierimonti et al. [8] proposed the Life-Cycle Cost Wind Design to analyze the performance of a tall building, equipped with TMD, against synoptic wind hazards. Liu et al. [16] discussed the major advantages of tuned mass dampers used as control devices on tall buildings. They listed some existing structures with are installed TMDs. The authors proposed the optimization of the parameters of TMDs under wind excitation and verified the optimum design in a thirty stories building. Kareem, Kijewski and Tamura [11] presented an overview of control devices to use in wind-excited buildings aiming to reduce the motion of these structures. Besides the tuned mass dampers, other devices were discussed by the authors, as tuned liquid dampers, hybrid mass damper and active mass damper. Liu et al. [14] carried out experiments in wind tunnels to study the vibration caused by the wind force in buildings with tuned liquid column dampers. The authors compared the results with a developed computational procedure, showing that the dampers are more useful in decreasing the dynamic wind response to higher wind speed values.

However, it is known that engineering problems involve uncertainties in some variables. Thus, more recently, some authors also studied the robust optimization applied to the tuned mass dampers [29]–[25], especially for buildings under seismic excitation. The robust optimization results in a design that is not very sensitive to changes in the parameters and, at the same time, it provides an effective TMD performance.

In this context, the present study proposes a complete procedure for the optimal design of tuned mass dampers to be used in high-rise buildings subject to wind action. As the studied problem is complex, involving mixed design variables (discrete for the quantity of TMDs and continuous for the parameters of these TMDs), this problem needs to be solved through optimization algorithms capable of dealing with this type of problem. Thus, the Search Group Algorithm (SGA) was used. Additionally, unlike most works that perform only a deterministic optimization, the present work proposes a robust optimization methodology, in which uncertainties
in the parameters of the building, of the wind excitation and also in the TMDs parameters are taken into account, thus making the design less sensitive to parameter variations. The Monte Carlo Simulation was employed to carry out the robust optimum design and the Latin Hypercube Sample (LHS) was used to reduce the computational cost.

The proposed methodology allows optimizing not only the TMDs parameters, but also the quantity and best position of these dampers. In other words, it is a complete methodology that allows the simultaneous optimization (in a single stage) of the quantity, positions and parameters of a single and multiple TMDs at the same time, taking the uncertainties into account. In this way, as all the optimization is done simultaneously in a single stage, it is faster and more effective than other methods that initially only optimize the positions and later, in a second stage, perform the optimization of parameters, for example. Furthermore, the chosen optimization algorithm (SGA) is a modern algorithm, faster than traditional optimization algorithms like Genetic Algorithms (GA).

2. Mathematical Models. This section describes the mathematical procedures to model structures using a shear building model with TMDs coupled. A mathematical model to simulate the wind force in the time domain is proposed. At the end of this section, the procedure used to obtain the robust optimization of the tuned mass dampers is explained.

2.1. Structural model. The motion equation governing a N degrees of freedom system with $N_{TMD}$ tuned mass dampers coupled and subject to wind excitation is given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t)$$

(1)

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ correspond to the $(N + N_{TMD}) (N + N_{TMD})$ mass, damping and stiffness matrices, respectively. $N$ defines the number of degrees of freedom of the structure and $N_{TMD}$ gives the number of tuned mass dampers. Acceleration, velocity and displacement vectors are given by $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ e $\mathbf{x}(t)$, respectively. The external wind-excitation force vector is obtained by $\mathbf{F}(t)$.

The mass matrix for a shear building model is achieved by a lumped matrix whose principal diagonal is occupied by the mass of each story, following by the mass of each TMD considered. The stiffness matrix can be expressed as follows:

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & \ldots & 0 & 0 & \ldots & 0 \\ -k_2 & k_2 + k_3 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & 0 & \vdots & 0 \\ 0 & 0 & \ldots & k_N + \sum k_{N_{TMD}} & -k_{1_{TMD}} & \ldots & -k_{N_{TMD}} \\ 0 & 0 & \ldots & -k_{1_{TMD}} & k_{1_{TMD}} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & -k_{N_{TMD}} & 0 & \ldots & k_{N_{TMD}} \end{bmatrix}$$

(2)

For the damping matrix $\mathbf{C}$, a Rayleigh damping is supposed as: $\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K}$.

The resolution of the dynamic equation (Eq.(1)) is given by the Newmark direct integration method.
2.2. Wind-action simulation. The wind is considered a stochastic process, decompose into a parcel of fluctuating wind velocity and the parcel of the average wind velocity. Only the along-wind direction was considered in this research because it is the only direction where the average wind has expressive values.

The fluctuating component was calculated by the spectral representation method. Kaimals along-wind load spectrum $S_W$ is applied [10], which is obtained by the following equations:

$$\frac{f S_W}{u_*^2} = \frac{200 n}{(1 + 50 n)^{5/3}}$$

In which:

$$n = \frac{f_z}{V(Z)}$$

$$u_* = 0.4 \bar{V} (Z_{ref}) \ln \left( \frac{Z_{ref}}{Z_0} \right)$$

where $Z_{ref}$ is the reference height to the ground, considered equal to 10 meters; $n$ is the dimensionless frequency, related to the Taylor hypothesis; $Z_0$ the is the roughness coefficient, which depends on the surface roughness of the ground and $u_*$ is the friction velocity. $\bar{V}(Z)$ is the average velocity in function of the height, assumed to follow the power-law given by:

$$\bar{V}(Z) = b \bar{V}_p \left( \frac{Z}{10} \right)^p$$

The parameters $b$ and $p$ are defined by Table 21 of ABNT NBR 6123 [1], for a time period of 600s. $\bar{V}_p$ is the design velocity, which corresponds to the average velocity over 10 minutes of time interval at 10 meters of ground height, presented as follows:

$$\bar{V}_p = 0.69 V_0 S_1 S_3$$

where $V_0$ is the reference mean wind velocity 10m above the ground. $V_0$ must be adequate to the location of the construction, and can be obtained by the isopleths of the basic wind velocity given in [1].

The method of Shinozuka and Jan [23] allows determining the wind fluctuating component by the superposition of harmonic waves obtained by the power spectral density, via the following equation:

$$\Delta \bar{V}(t) = \sum_{j=1}^{N_f} \sqrt{2 S_W (f_j)} \Delta f \cos(2\pi f_j t + \phi_j)$$

where $\phi_j$ is the phase angle, which is a random variable, with a uniform probability distribution function, ranging from 0 to $2\pi$. $f_j$ is the frequency corresponding to a whirl; and $\Delta f$ is obtained by the division interval of the frequency band of interest.

The correlated wind field is obtained employing the procedure described by Miguel et al. [19]. For that purpose, the vertical correlation length is estimated according to the expression given by [19]. Thus, the studied building is inserted into the correlation plane and the velocity for each node of the structure is obtained through linear interpolation.

Finally, the total correlated velocity field can be obtained as the sum of the velocity components (average wind velocity plus fluctuating wind velocity), as follows:

$$\bar{V}(Z, t) = \bar{V}(Z) + \Delta \bar{V}(Z, t)$$
After obtaining the total correlated field of wind velocities, the wind forces, which should be imposed on the structures nodes, are obtained following the procedure given in technical standards, as [1].

2.3. Robust optimization procedure. The scope of this work is to reach the robust design of tuned mass dampers. In this work, the robust parameters sets are given by the values that guarantee a limited dispersion of the distribution of maximum building displacement, when the system is subject to uncertainties from the structural properties and the excitation force.

The beginning of the robust optimization procedure is the definition of the probability distribution of the stochastic variables. The uncertain parameters were modeled as uncorrelated random variables with Gaussian distribution, with known mean and coefficient of variation.

The Monte Carlo method was applied for the simulation of a large number of samples needed. Aiming to reduce the large computational effort required by the Monte Carlo method, the Latin Hypercube Sampling (LHS) technique was adopted for the generation of the stochastic variables. The LHS aims to divide the input distribution into intervals with equal probability and select a sample from each interval. Consequently, LHS produces more uniform samples within the distribution. Thus, a smaller number of samples is required and the computational cost is reduced.

The next step of the robust optimization procedure is to solve the optimization problem. The objective function is not convex and strives to minimize the mean of the maximum displacement of the building $E(u_{\text{max}})$. The design variables are the quantity of TMDs ($N_{\text{tmd}}$) and their parameters, i.e., spring and damping constants $E(k_{\text{tmd}})$ and $E(c_{\text{tmd}})$ of each TMD, which are also considered as a random variable with Gaussian distribution with known mean values and coefficients of variation. The restraints are the lower ($k_{\text{tmin}}$) and upper ($k_{\text{tmax}}$) bounds of each TMD stiffness constant, the lower ($c_{\text{tmin}}$) and upper ($c_{\text{tmax}}$) bounds of each TMD damping constant, the TMD total mass ($m_{\text{tmd}}$), assumed as a percentage of the structure total mass and the maximum number of TMDs ($N_{\text{tmdmax}}$). Thus, the robust optimization problem can be written as:

Find:

$$E(k_{\text{tmd},i}) \text{ and } E(c_{\text{tmd},i}) \text{ with } i=1,2,\ldots,N_{\text{tmd}}$$

Minimize:

$$E(u_{\text{max}})$$

Subject to:

$$m_{\text{tmd}} = \text{total structure mass } \%$$

$$k_{\text{tmin}} \leq k_{\text{tmd},i} \leq k_{\text{tmax}}$$

$$c_{\text{tmin}} \leq c_{\text{tmd},i} \leq c_{\text{tmax}}$$

$$N_{\text{tmd}} \leq N_{\text{tmdmax}}$$

This complex robust optimization problem can be solved by employing the Search Group Algorithm summarized in the following section.
3. **Search Group Algorithm (SGA).** In order to solve the optimization problem, the Search Group Algorithm (SGA) was employed. The Search Group Algorithm is a meta-heuristic algorithm, capable to solve complex optimization problems. The SGA was developed by Goncalves, Lopez and Miguel [7].

The first step of SGA is the creation of the initial population by random values in the problem domain. The objective function of each population individual was evaluated, and different sizes were given according to the individual fitness. The bigger the individual is, the better is the result of the objective function of this individual. After that, the search group was formed by a standard tournament selection and continues by the mutation of a search group individual. The mutation processes increase the exploration capability. The next SGAs phase is the family generation, on each search group an individual generated a family with new individuals. In the first iteration processes, the individual can be located in any place inside the search domain. As iterations go by, the families generated are located next to the generator member. Another important fact is that bigger individuals generate a bigger family. The new search group is formed by the best member of each family in the exploration phase. In the exploitation phase, the new search group is formed by the best individuals among all families, refining the results.

One of the remarkable advantages of this method is the possibility to balance, as necessary by the problem, the exploration and exploitation phases. More information about the SGA can be obtained in [7].

4. **Numerical Example.** To illustrate the proposed methodology for the robust optimal design of TMDs, this section presents the example of a 40-story building subjected to wind excitation. All simulations are carried out in Matlab software, employing subroutines developed by the authors.

4.1. **Description of the structure and the equivalent dynamic model.** The building chosen as the case study was developed by [15] for time-domain analyses of wind-induced vibration with a tuned mass damper and multiple tuned mass dampers (MTMD). The 40-story building has 160 m high and each side has 40 m long. The structure was modeled as a shear building. The TMD or MTMD can be installed on the top of the building.

As explained previously, the robust optimization procedure considers the uncertainties present in the parameters of the structure and of the excitation force, as well as in the stiffness and damping constants of the TMDs. The structure parameters were modeled as uncorrelated random variables, by a Gaussian distribution, with known mean and coefficient of variation (CV) (see Table 1). Note that the story stiffness decreases linearly as the \( Z_i \) (story) increases. The TMD spring and damping constants were the design variables, while the TMD mass was assumed to be equal to 2% of the total mass of the building and this value was divided equally by the number of resulting TMDs.

Table 1 presents the mean values and the coefficients of variation of the random variables related to the structure and TMDs parameters.

4.2. **Wind-force modeling.** In this study, it is assumed that the building is located in Balneario Cambori city, Brazil. This choice is because this city has the tallest buildings in Brazil, and an elevated reference mean wind velocity of 43m/s. This parameter was also modeled as a random variable, given by a Gaussian Distribution, with a known coefficient of variation (Table 2).
### Table 1. Input random variables of the building and of the TMDs.

| Random Variable                        | Mean Value       | CV  |
|----------------------------------------|------------------|-----|
| Story mass                             | $m_i$            | 9.8×10^5 kg | 5% |
| Story stiffness                         | $k_1$            | 2.13×10^9 N/m | 10% |
| Story stiffness                         | $k_{40}$         | 9.98×10^8 N/m | 10% |
| Damping ratio                          | $\zeta$          | 0.016 | 10% |
| Spring constant of each TMD            | $k_{tmd}$        | Design variable | 5% |
| Damping constant of each TMD           | $c_{tmd}$        | Design variable | 5% |

### Table 2. Input random variable of the wind excitation.

| Random Variable | Mean Value | CV |
|-----------------|------------|----|
| Mean wind velocity | $V_0$ | 43 m/s | 15% |

The frequency range for the Kaimals spectrum varies between 0 and 4Hz. The duration of the wind-action was 600s. A record from one of the runs by Monte Carlo can be seen in Figure 1, which illustrates the final velocity at the building top, given by the sum of fluctuating wind velocity and average wind velocity.

![Figure 1. Total wind velocity for the Z = 160m](image)

4.3. **Robust optimization of MTMD.** In this section the robust design optimization of the tuned mass dampers was carried out, aiming to reduce the mean maximum displacement of the building. As shown before, the Monte Carlo Simulation was employed. A study of convergence of the mean and the standard derivation of $u_{max}$ without the control system was performed to establish the number of samples of the Monte Carlo Simulation Figure 2.

Observing Fig.2 it can be noted that a minimum of around 400 samples is required to stabilize the convergence curves, wherefore, 400 samples were employed in the robust optimization procedure.

The objective function is the minimization of the mean maximum displacement caused in the building by the wind load. The design variables are the number of
Figure 2. Convergence by the number of samples. (a) Convergence of the mean of $u_{max}$, (b) Convergence of the standard derivation of $u_{max}$.

Table 3. Resulting TMD robust design (scenario with 1 TMD on the top).

| Run | $N_{TMDs}$ | $E(k_{tmd})$ [kN/m] | $E(c_{tmd})$ [kNs/m] | $E(u_{max})$ [m] |
|-----|-------------|---------------------|---------------------|-----------------|
| -   | Without dampers |                     |                     | 0.628           |
| 1   | 1           | 1907.808            | 269.153             | 0.411           |
| 2   | 1           | 1964.466            | 250.027             | 0.416           |
| 3   | 1           | 1916.927            | 281.217             | 0.414           |

TMDs ($N_{tmd}$) and the parameters of each TMD. The constraints are the TMD total mass equal to 2% of the total mass of the building, the quantity of TMDs varying between 0 and 10 devices, the stiffnesses of the TMDs varying between 0 and $10^7 N/m$, and the damping constants of the TMDs varying between 0 and $10^6 Ns/m$. In this study, the following SGA parameters were used: $N_{POP}$ (size of the population) equal to 100, $it_{max}$ (maximum number of iterations) equal to 100 and $it_{maxglobal}$ (iteration percentage dedicate to the exploration phase) equal to 60%.

Table 3 shows that the dampers number and the mean maximum displacement founded in three independent runs are very similar, indicating the robustness of the proposed method. The resulting TMD spring and damping constants for the runs are also corresponding. The robust design indicates that only one TMD, located on the tallest floor (on the top) of the building, is necessary to obtain the best result in minimizing the building displacement when the uncertainties are taken into account. It is also possible to observe that a reduction of about 35% in the mean maximum displacement is obtained when the optimized TMD is installed.

It is also interesting to note that the probability function curve for the $u_{max}$ with control was slenderer, which indicates a greater precision of the displacement response of the structure in face of the wind load Figure 3. That can also be noted when the variance and standard derivation for the robust design for the situation with and without dampers were compared (Table 4). Variances have been reduced by more than 60%, and the standard derivation by more than 37%. As the variance
Table 4. Resulting TMD robust design.

| Parameter | Without TMD | With TMD | Reduction |
|-----------|-------------|----------|-----------|
| $E(u_{max})[m]$ | 0.6283 | 0.411 | 34.6% |
| $Var(u_{max})[m^2]$ | 0.0458 | 0.0180 | 60.7% |
| $Std(u_{max})[m]$ | 0.214 | 0.1344 | 37.2% |
| $E(a_{max})[m/s^2]$ | 1.417 | 0.8669 | 38.8% |

and standard deviation were smaller, the probability of the displacement to be near the mean value is greater.

Additionally, as can be observed in Table 4, the mean of maximum acceleration $E(a_{max})$ was also significantly reduced with the inclusion of the optimized TMD in the building. In other words, notwithstanding the aim of the objective function is to reduce the displacement, the acceleration also showed an important decrease for the building, in more than 38% when compared to the no controlled case.

Figure 3. Probability density function of maximum displacement.

The robust design provided a decrease in the displacement along the building height, as can be seen in Figure 4. It can be noted that the maximum reduction occurs for the maximum displacement, on the top of the building.

Another scenario of optimization under uncertainties was proposed. The difference was in the $N_{tmd}$, which was fixed in 5 for this new scenario. Consequently, the resulting design should obligatorily be composed of MTMD, with five devices, all of them located on the tallest floor (on the top) of the building. Again, three independent runs were carried out, resulting in similar values. Thus, the results for one run are shown in Table 5.

Another important fact to note is that the resulting displacement was very close to the robust design obtained by only one TMD. The sum of the spring constant of each TMD is equal to 2019.33 kN/m, that is, only about 5.5% greater than the spring value obtained for the only one TMD resulting. In other words, the design
solution resulting in one TMD or in MTMD can be considered similar, without change significantly the performance in control the vibrations.

It is interesting to point out that, as described in some papers (e.g., Chen and Wu [24], and Patil and Jangid [21]), using MTMD has some advantages over using only one TMD, mainly due to: (1) when MTMD are installed, if one or more TMDs stop working, the entire effectiveness of the system will not necessarily be compromised, different from the case when a single TMD is installed and it ceases to function; (2) MTMD can better handle structural stiffness uncertainties compared to a single TMD; (3) the individual masses of MTMD are smaller than the mass of a single TMD, therefore, MTMD are easier to be transported and installed in old or retrofitted structures; and (4) MTMD can be tuned to different frequencies, controlling several modes. Thus, even in the case of this numerical example, in which the mode to be controlled is the first, the designer can still choose to install more than one TMD due to the advantages presented above.

Additionally, it should be noted that the example presented here takes into account the uncertainties present in the structure and excitation parameters, and that
Table 6. Comparing the robust optimization with a deterministic optimization.

| Parameter          | Robust Optimization | Deterministic Optimization |
|--------------------|---------------------|----------------------------|
| Number of TMDs     | 1                   | 1                          |
| $k_{tmd}[kN/m]$    | 1907.78             | 1727.77                    |
| $c_{tmd}[kN/m]$    | 269.11              | 147.47                     |
| $E(u_{max})[m]$    | 0.411               | 0.435                      |
| $Var(u_{max})[m^2]$| 0.018               | 0.0224                     |

according to Igusa and Xu [9], for instance, MTMD can be much more stable (robust), keeping the same level of effectiveness or even more effective than a single TMD with the same total mass. So, MTMD are typically installed to reach broader effectiveness to compensate for variations in modal parameters with occupation.

For comparison purposes, the same analysis performed by the robust optimization procedure was performed for deterministic optimization. The deterministic optimization method considers that the input values are fixed and not given by a Gaussian random distribution, i.e., just one of the 400 random samples is employed. Therefore, the deterministic optimization does not need to use the Monte Carlo method to calculate the average of the responses. In this way, the entire optimization process has a lower computational cost. However, this computational gain does not bring benefits in the long term since the parameters obtained for the TMD’s by deterministic optimization do not consider the uncertainties that occur in the structure and in the wind, thus it may be less effective in decreasing the building’s vibrations. The deterministic optimization should only be used in specific cases in which all input parameters can be accurately predicted.

It is also important to mention that, in the deterministic optimization alternative, the objective function, the design variables and the constraints are the same employed in the first robust optimization, and the SGA parameters, such as the population size, the number of iterations and the percentage dedicated to the exploration phase are also the same. The difference between the two processes lies in the input values, which in the deterministic processes, instead of evaluating all the 400 random samples, only 1 random sample is considered, assuming the mean values of the input variables given in Tables 1 and 2. That is, 1 random structure is optimized for 1 random excitation and not for the 400 random excitations. It is also important to note that although the mean wind velocity is considered constant (43 m/s), the phase angle given in Eq. (8) remains a random number.

Thus, the results for the TMD given by the deterministic optimization were applied in the structure with the same uncertainties of the robust optimization to compare the two methods, considering all the 400 random samples. As can be seen, the optimum design given by the deterministic method resulted in a worse displacement performance Table 6.

The variance increased more than 24% in the deterministic optimization results, showing that this alternative is more sensitive to the uncertainties presents in the structure and in the wind force. Thus, can be concluded that the robust optimization has more stability in the results under the uncertainties. From that comparisons, it was noted the importance to find the optimum solution considering the uncertainties presents in the building properties and in the excitation force, whereas this represents a real situation on that type of problem.
4.4. **Comparison with other approaches.** To show the superiority of the proposed methodology in additional ways, this section presents a comparison of the results obtained through the proposed methodology with the results obtained through other usual approaches for TMD design. Initially, the proposed methodology is compared with the classic approaches of Den Hartog [5] and Warburton [27]. As the expressions proposed by Den Hartog [5] and Warburton [27] are for SDOF systems, the procedure proposed by Rana and Soong [22] is implemented, allowing the use of the equations proposed by [5, 27] for MDOF systems.

Thus, to use the procedure of [22], the approaches of [5, 27], and illustrate the results Figure 5, the mean values of the random variables are adopted and it is assumed that the coefficients of variation of all random variables are zero. So, the following parameters are obtained for the TMD: $k_{tmd_{DH}} = 1933.6\, \text{kN/m}$ and $c_{tmd_{DH}} = 311.4\, \text{kN}\cdot\text{s/m}$ (Den Hartog’s approach) and $k_{tmd_{W}} = 1890.6\, \text{kN/m}$ and $c_{tmd_{W}} = 252.8\, \text{kN}\cdot\text{s/m}$ (Warburton’s approach). For the proposed optimization methodology (employing this random sample), the parameters are: $k_{tmd} = 1622.9\, \text{kN/m}$ and $c_{tmd} = 221.4\, \text{kN}\cdot\text{s/m}$. Thus, Figure 5 shows the dynamic response.

As can be seen in Figure 5, as much the proposed method as the Den Hartog [5] and Warburton [27] methods are able to reduce the maximum displacement at the top by 38.65%, 27.81% and 29.27%, respectively. However, the maximum displacements obtained through the Den Hartog’s method ($u_{max} = 0.4350\, \text{m}$) and through the Warburton’s method ($u_{max} = 0.4262\, \text{m}$) are 17.66% and 15.28%, respectively, greater than the maximum displacement obtained through the proposed optimization methodology ($u_{max} = 0.3697\, \text{m}$), highlighting the superior performance of the proposed method.

![Figure 5](image.png)

**Figure 5.** Displacement at the top of the building for the cases: without TMD (yellow curve), with 1 TMD designed by Den Hartog’s method (blue curve), with 1 TMD designed by Warburton’s method (red curve), and with 1 TMD optimized by the proposed optimization method (green curve), for coefficients of variation equal zero for all random variables.

Now, to attest again the effectiveness of the proposed methodology, the results obtained via the proposed method are compared with the results obtained using a classical Genetic Algorithm (GA).
Table 7. Comparing the robust optimization with SGA and GA.

| Parameter | Robust Optimization with SGA | Robust Optimization with GA |
|-----------|-----------------------------|-----------------------------|
| Number of TMDs | 1                           | 1                           |
| $k_{tmd}[kN/m]$ | 1907.78                    | 2057.38                    |
| $c_{tmd}[kN/m]$ | 269.11                     | 761.65                     |
| $E(u_{max})[m]$ | 0.411                      | 0.449                      |

To perform a fair comparison, the parameters used for the GA, as the number of iterations and population, were the same as those of the SGA. Thus, the results of the robust optimization obtained with the GA are shown in Table 7.

As can be seen in Table 7, the results obtained with the GA are similar to those obtained with the SGA. However, it can be observed that the SGA presented a superior performance, as much in terms of the maximum displacement as in terms of computational time, confirming once again the effectiveness of the proposed method. As can be seen, the mean maximum displacement $E(u_{max})$ obtained via GA is approximately 9% higher than that obtained via the proposed method. Furthermore, the proposed method resulted in lower values of $k_{tmd}$ and $c_{tmd}$. It is also important to remark that the computational time required by the GA is about 21% higher than the computational time required by the SGA. These results show that the use of a modern and efficient optimization algorithm brings important gains to the vibration reduction design.

5. Conclusions. In this work, a robust optimization procedure to obtain the design of tuned mass dampers as a vibration control device for tall buildings subject to the dynamic action of the wind was proposed. Unlike most works in the literature that perform only a deterministic optimization, the procedure proposed in this work took into account the uncertainties present in the building properties, in the wind action and also in the TMDs parameters. A mathematical model to simulate the wind force in the time domain was introduced. An illustrative example of a building located in Brazil was studied.

The first results found that a single TMD on the top of the building was the best design and showed a considerable reduction in the mean maximum displacement of the building, and in the statistical values of the response in displacement. According to the three independent runs, the design can be considered robust. Another scenario for optimization under uncertainties was also carried out, considering mandatory to result in a MTMD scenario, with five TMDs on the top of the building. A comparison between this robust optimization design with 5 TMDs with the robust optimization design with a single TMD indicated a similarity in the responses, and the opportunity to choose the best design, single TMD or MTMD, according to the building fitness in each case. In other words, even in the case of the presented example, in which the wind force basically excites the first mode shape, and then only 1 TMD on the top would solve the problem, the use of MTMD has advantages and can be chosen as the solution of the problem. It can be easier to install small devices than installing a single TMD with a large mass. Moreover, if one or more TMDs stop working, the entire effectiveness of the system will not necessarily be compromised, different from the case when a single TMD is installed.
and it ceases to function, and, in addition, MTMD can better handle structural stiffness uncertainties compared to a single TMD.

The robust design was compared with a deterministic design, also obtained by an optimization procedure, but without considering the uncertainties to obtain the optimum parameters and quantity of devices. The deterministic design was similar to the robust design, resulting in a single TMD on the top as well. However, the comparison of the robust design with the deterministic design showed that the T-MD of the robust optimization improved the performance in minimizing the mean maximum displacement of the structure in relation to the case of the deterministic optimization, in addition to reducing the statistical variables of displacement. Additionally, it is important to highlight that the procedure of optimization under uncertainties became the response of the building to wind-action less sensitive to the variation of parameters. Therefore, it is advisable that in real design situations, in which there are uncertainties, the optimal design should be obtained through robust optimization.

To demonstrate the success of the proposed method in additional ways, the optimal solution for 1 TMD on the top of the building was compared with solutions obtained by usual TMD design methods. The results showed that the maximum displacements obtained via the Den Hartog and Warburton methods were about 18% and 15%, respectively, greater than the maximum displacement obtained through the proposed optimization methodology, emphasizing the superior performance of the proposed method.

The comparison of the results obtained through the proposed method with the results obtained through a GA showed that the mean maximum displacement obtained via the GA is about 9% higher than that obtained via the SGA. Moreover, the computational time required by the GA is around 21% higher than the computational time required by the SGA, once again confirming the best performance of the proposed methodology.

Thus, the robust optimization methodology proposed in this work can be an excellent tool for TMD and MTMD designs.

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