Lifetimes of tidally limited star clusters with different radii

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ABSTRACT

We study the escape rate of stars, \( \dot{N} \), from clusters with different radii on circular orbits in a tidal field using analytical predictions and direct \( N \)-body simulations. We find that \( \dot{N} \) depends on the ratio \( \mathcal{R} \equiv r_h/r_j \), where \( r_h \) is the half-mass radius and \( r_j \) the radius of the zero-velocity surface around the cluster. For \( \mathcal{R} \gtrsim 0.05 \), the ‘tidal regime’, there is almost no dependence of \( \dot{N} \) on \( \mathcal{R} \). To first order this is because the fraction of escapers per half-mass relaxation time, \( t_{\text{rh}} \), scales approximately as \( \mathcal{R}^{3/2} \), which cancels out the \( r_h^{-3} \) term in \( t_{\text{rh}} \). For \( \mathcal{R} \lesssim 0.05 \), the ‘isolated regime’, \( \dot{N} \) scales as \( \mathcal{R}^{-3/2} \). The dissolution time-scale, \( t_{\text{dis}} \), falls in three regimes. Clusters that start with their initial \( \mathcal{R} \), \( \mathcal{R}_i \), in the tidal regime dissolve completely in this regime and their \( t_{\text{dis}} \) is, therefore, insensitive to the initial \( r_h \). Our model predicts that \( \mathcal{R}_i \) has to be \( 10^{-20} - 10^{-10} \) for clusters to dissolve completely in the isolated regime. This means that realistic clusters that start with \( \mathcal{R}_i \lesssim 0.05 \) always expand to the tidal regime before final dissolution. Their \( t_{\text{dis}} \) has a shallower dependence on \( \mathcal{R}_i \) than what would be expected when \( t_{\text{dis}} \) is a constant times \( t_{\text{rh}} \). For realistic values of \( \mathcal{R}_i \), the lifetime varies by less than a factor of 1.5 due to changes in \( \mathcal{R}_i \). This implies that the ‘survival’ or ‘vital’ diagram for globular clusters should allow for more small clusters to survive. We note that with our result it is impossible to explain the universal peaked mass function of globular cluster systems by dynamical evolution from a power-law initial mass function, since the peak will be at lower masses in the outer parts of galaxies. Our results finally show that in the tidal regime \( t_{\text{dis}} \) scales as \( N^{0.65}/\omega \), with \( \omega \) the angular frequency of the cluster in the host galaxy.

Key words: stellar dynamics – methods: \( N \)-body simulations – globular clusters: general – galaxies: star clusters.

1 INTRODUCTION

Stars escape from clusters due to internal two- and three-body encounters in which stars get accelerated to velocities higher than the escape velocity. The resulting dissolution time-scale, \( t_{\text{dis}} \), depends on the number of stars, \( N \), and the escape rate, \( \dot{N} \), as \( t_{\text{dis}} \equiv -N/\dot{N} \). For a constant \( \dot{N} \) the instantaneous value of \( t_{\text{dis}} \) is the remaining time to total dissolution. For clusters in isolation \( t_{\text{dis}} \) scales linearly with the half-mass relaxation time, \( t_{\text{rh}} \) (Ambartsumian 1938; Spitzer 1940). The presence of a tidal field speeds up \( \dot{N} \) by roughly an order of magnitude (Hénon 1961; Spitzer & Chevalier 1973; Giersz & Heggie 1997). When treating the tidal field as a radial cut-off, as is often done in Fokker Planck and \( N \)-body simulations (Chernoff & Weinberg 1990; Gnedin & Ostriker 1997; Takahashi & Portegies Zwart 2000), \( t_{\text{dis}} \) also scales with \( t_{\text{rh}} \) (Baumgardt 2001, hereafter B01).

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Fukushige & Heggie (2000) demonstrated that for a realistic tidal field it is important to consider the finite time it takes stars to escape through one of the Lagrange points. B01 showed that then \( t_{\text{dis}} \propto t_{\text{rh}}^{3/4} \). This scaling was also found for more realistic \( N \)-body simulations that include a stellar mass function and stellar evolution (Vesperini & Heggie 1997; Baumgardt & Makino 2003, hereafter BM03). BM03 only considered the dependence of \( t_{\text{rh}} \) on cluster mass, \( M \), since in their runs the initial half-mass radius, \( r_h \), and the initial tidal radius are linked, so \( r_h \propto M^{1/3} \).

Observations of young (extragalactic) clusters show that the scaling of \( r_h \) with cluster mass, \( M \), and galactocentric distance, \( R_G \), is considerably shallower, \( r_h \propto M^{0.1} R_G^{0.1} \) (Zepf et al. 1999; Larsen 2004; Scheepmaker et al. 2007), than the Roche lobe filling relation \( (r_h \propto M^{1/3} R_G^{2/3} ) \), implying that massive clusters and clusters at large \( R_G \) are initially underfilling their Roche lobe.

The relation between \( t_{\text{dis}} \) and the median cluster radius, \( \bar{r} \), was modelled by Wielen (1971) for clusters up to \( N = 250 \). He found that \( t_{\text{dis}} \) (in Myr) scales with \( \bar{r}^{3/2} \) for \( \bar{r} \lesssim 0.01 \, r_h \), with \( r_h \) the Jacobi radius, being the radius of the zero-velocity surface around the...
cluster imposed by the tidal field. This is because for these clusters $t_{\text{dis}} \propto t_{\text{h}}$. For $F \gtrsim 0.05 r_1$ this scaling is not followed any more and $t_{\text{dis}}$ is shorter. Large clusters are even more vulnerable to disruption when the effect of passing molecular clouds is included (King 1958; Wielen 1985; Gieles et al. 2006).

Tanikawa & Fukushige (2005) modelled the evolution of clusters of larger $N$ that are initially Roche lobe underfilling using collisionless $N$-body simulations. They confirm that for Roche lobe filling clusters $t_{\text{dis}} \propto r_{\text{h0}}$, with $x = 3/4$ as was found by B01, and $x$ somewhat larger for Roche lobe underfilling clusters. They do not discuss the effect of radius on $t_{\text{dis}}$. More realistic $N$-body simulations, including various stellar initial mass function and stellar evolution, of Roche lobe underfilling clusters were considered by Engle (1999). She concludes that Roche lobe underfilling clusters survive longer. However, this is for a fixed $r_{\text{h0}}$ and different $r_1$. Since her simulations included stellar evolution, it is not possible to scale these results to the same $r_1$.

Predictions for the survival probability of globular cluster, such as the survival triangle (Fall & Rees 1977; Gnedin & Ostriker 1997), rely on the assumption that $t_{\text{dis}}$ is a constant times $t_{\text{h}}$. This is also an important assumption in a recent attempt to explain the shape and the universality of the turnover of the globular cluster mass function (GCMF; McLaughlin & Fall 2008). To be able to judge the applicability of such models it is of importance that the relation between $t_{\text{dis}}$ and $r_{\text{h}}$ is better understood for clusters with larger $N$.

The interplay between internal relaxation effects and external tidal effects is the topic of this Letter. In Section 2 we present a set of $N$-body simulations of clusters with different initial Roche lobe filling factors. We introduce a simple analytical model for $N$ that includes internal dynamics and the external tides in Section 3. In Section 4 we confront our model with the simulations, and our conclusions are presented in Section 5.

## 2 DESCRIPTION OF THE RUNS

We simulate the evolution of clusters containing between $N = 1024$ and 32 768 particles, without primordial binaries and mass loss by stellar evolution, orbiting with angular frequency $\omega$ in a steady point mass tidal field to simulate circular orbits in a galactic potential. The stellar masses are randomly drawn from a power-law mass function with index $-2.35$ with the maximum mass 30 times larger than the minimum mass.

For the density distribution of the clusters we use King (1966) models, with $W_0 = 5$. We define an initial Roche lobe filling factor as $\xi \equiv r_1/r_{\text{h1}}$, with $r_{\text{h1}}$ the initial Jacobi radius and $r_1$ the King tidal radius, that is, the radius where the stellar density of the King (1966) model drops to zero. We model four values of $\xi$, from 0.125 to 1. The $W_0$ parameter and $\xi$ set the ratio of the initial $r_{\text{h0}}$, $r_{\text{h1}}$, and $r_{\text{h2}}$, which we denote by $\nu_i (\equiv r_{\text{h0}}/r_{\text{h1}})$. Clusters with $N = [1024, 2048, 4096, 8192]$ are run [16, 8, 4, 2] times to reduce statistical variations. We also run an $N = 4096$ simulation in isolation up to $t_{1/2}$ to determine the mass-loss parameters.

All clusters are scaled to $N$-body units, such that $G = M = 1$ and $E = -0.25$ (Heggie & Mathieu 1986). Here $E$ is the total (potential and kinetic) internal energy of the cluster. In these units the virial radius, $r_{\text{v}} \equiv GM/(-4E)$, equals unity and the crossing time at $r_1$ is $2\sqrt{2}$.

The half-time, $t_{1/2}$, is when half of the initial number of stars have become unbound, where bound is defined as the number of stars within $r_1$. We multiply $t_{1/2}$ (in $N$-body times) by $\omega$ to compare the results of different $\nu_i$. The dimensionless $\omega t_{1/2}$ results are equivalent to physical times for clusters at the same $R_0$. A summary of the runs and the resulting $t_{1/2}$ and $\omega t_{1/2}$ values is given in Table 1.

All $N$-body calculations were carried out with the KIRA (McMillan & Hut 1996; Fortegies Zwart et al. 2001) integrator on the special purpose GRAPE-6 boards (Makino et al. 2003) of the European Southern Observatory.

## 3 ANALYTICAL MODEL FOR THE ESCAPE RATE OF TIDALLY LIMITED CLUSTERS

### 3.1 The ‘classical’ ansatz

Let us first assume that a cluster, consisting of $N$ stars, loses a constant fraction $\xi_c$ of its stars each $t_{\text{h}}$, so that we can write for $N$ (for example Spitzer 1987, hereafter S87),

$$N = -\xi_c N / t_{\text{h}},$$

where $t_{\text{h}}$ is conventionally expressed as (Spitzer & Hart 1971)

$$t_{\text{h}} = 0.138 \sqrt{\frac{N^{1/2}}{t_{\text{h}}}} \frac{\gamma}{\sqrt{\ln \Lambda}},$$

where $\Lambda = \gamma N$ and $\gamma = 0.11$ (Giersz & Heggie 1994a), $\gamma$ is the gravitational constant and $\bar{m}$ is the mean stellar mass. The crossing time at $r_n$, $t_{\text{cr}}$, is given by

$$t_{\text{cr}} = k \left( \frac{r_n^2}{GM} \right)^{1/2},$$

where $k$ is a constant of order unity depending on the cluster density profile.

The escape energy of stars in an isolated cluster, $E_{\text{esc}}$, is four times the mean kinetic energy of stars in the clusters, so $E_{\text{esc}} = 0.8GM/(\nu_0)^2$ and the escape velocity, $v_\text{esc}$, scales with the stellar...
root mean square velocity in the cluster, \(v_{\text{rms}}\), as \(v_{\text{rms}}^2 = 4v_{\text{rms}}^2\). From integration over a Maxwellian velocity distribution the fraction of stars with \(v^2 > 4v_{\text{rms}}^2\) can be determined. We refer to this escape fraction for isolated clusters as \(\xi_{e0}\) and S87 showed that \(\xi_{e0} = 0.0074\).

3.2 The escape fraction as a function of cluster radius

When a cluster evolves in a tidal field the critical energy for escape is

\[
E_{\text{crit}} = \frac{3GM}{2r_1}. \tag{4}
\]

For a point-mass galaxy, \(r_1\) depends on \(\omega\) and \(M\) as

\[
r_1 = \left(\frac{G}{3\omega^2}\right)^{1/3} M^{1/3}, \tag{5}
\]

where \(\omega \equiv V_0/R_0\), with \(V_0\) the circular velocity. A large \(\omega\) means a strong tidal field, which results in a small \(r_1\).

The ratio \(E_{\text{crit}}/E_{\text{esc}}\) gives the relative reduction of the escape energy due to the tidal field (following S87):

\[
\Gamma = \frac{E_{\text{esc}}/E_{\text{crit}}}{1 - 0.8GM/r_0} = \frac{15}{8} \mathcal{R}, \tag{6}
\]

where we have used \(\mathcal{R} = r_0/r_1\). Note that this \(\Gamma\) is a factor of 1.5 higher than the original definition in S87, since its result was based on \(E_{\text{esc}} = GM/r_1\), which he later refines to equation (4). We calculate \(\xi_e\) as a function of \(\mathcal{R}\) by numerically integrating Maxwellian velocity distributions for different \(\mathcal{R}\) to determine the fraction of stars with velocities \(v^2 \geq 4(1 - \Gamma) v_{\text{rms}}^2\).

In Fig. 1 we show that \(\xi_e\) increases exponentially for increasing \(\mathcal{R}\) and can be well approximated by \(\xi_{e0}\exp(10\mathcal{R})\). For \(\mathcal{R} \geq 0.05\), the expression for \(\xi_e\) scales approximately as \(\mathcal{R}^{1/2}\), which has important consequences for \(N\) (see equations 1 and 2). We approximate \(\xi_e\) by

\[
\xi_e = \xi_{e0}, \quad \mathcal{R} < \mathcal{R}_1
\]

\[
= \xi_{e0} \left(\frac{\mathcal{R}}{\mathcal{R}_1}\right)^{3/2}, \quad \mathcal{R} \geq \mathcal{R}_1, \tag{7}
\]

where \(\mathcal{R}_1 = 0.05\) is the boundary between the ‘isolated’ regime (\(\mathcal{R} \leq \mathcal{R}_1\)) and the ‘tidal’ regime (\(\mathcal{R} > \mathcal{R}_1\)).

Substituting equations (2) and (7) in equation (1) and using equation (5) we find for \(N\) in the tidal regime:

\[
N = -\left(\frac{\sqrt{3}\xi_{e0} \ln \Lambda}{0.138\mathcal{R}_1^{1/2}}\right) \omega. \tag{8}
\]

So \(N\) is independent of \(r_0\) in the regime where \(\xi_e \propto \mathcal{R}^{3/2}\). This is because for a smaller(larger) star cluster the shorter(longer) \(t_{\text{dis}}\) is balanced by the lower(higher) \(\xi_e\). Equation (8) also shows that \(N\) depends only marginally on \(N\) through \(\ln \Lambda\).

3.3 Including the escape time

Fukushige & Heggie (2000) consider the time-scale of escape for stars in a cluster evolving in a tidal field. This time-scale is non-zero because stars with energies (slightly) larger than the escape energy still need a finite time to find one of the Lagrange points, where the escape energy is lowest, to leave the cluster.

B01 derives an expression for \(N\) of stars in the potential escaper regime, that is, with energies higher than the escape energy, but still trapped in the potential, and shows that it scales as \(Nt_{\text{dis}}^{-3/4} t_{\text{crit}}^{-1/4}\), instead of \(Nt_{\text{dis}}^{-1}\) (equation 1). We include \(\xi_e\) and define \(N\) analogous to equation (1) as

\[
N = -\xi_e \frac{N}{t_{\text{dis}}^{1/4} t_{\text{crit}}^{1/4}} \omega. \tag{9}
\]

With the expressions for \(\xi_e\), \(t_{\text{dis}}\) and \(t_{\text{crit}}\) we then find for \(N\) in the tidal regime, including the escape time,

\[
N = -\left(\frac{\sqrt{3}\xi_{e0}}{0.138\mathcal{R}_1^{1/2}}\right) \left(\frac{\ln \Lambda}{\omega}\right)^{3/4} N^{1/4} \omega. \tag{10}
\]

From a comparison between equations (8) and (10) we see that \(N\) becomes \(N\) dependent when we include the escape time, but is still independent of \(r_0\). The dissolution time-scale in the tidal regime, which we define as \(t_{\text{dis}} = –N/N\), is then

\[
t_{\text{dis}} = A \left(\frac{N}{\ln \Lambda}\right)^{3/4} \frac{1}{\omega}, \tag{11}
\]

with \(A = 0.138^{3/4} \mathcal{R}_1^{1/4} k^{1/4} (\sqrt{3}\xi_{e0})\). For a \(W_0 = 5\) cluster \(k = 3.85\), which together with the values for \(\mathcal{R}_1\) and \(\xi_{e0}\) from Section 3.2 results in \(A = 0.277\). The term \((N/\ln \Lambda)^{3/4}\) can be well approximated by \(B N^\eta\), with \(\eta \approx 0.65\) (Lamers, Gieles & Portegies Zwart 2005a). The values of \(B\) and \(\eta\) depend slightly on the value of \(\gamma\) in \(\Lambda = \gamma N\). We find the best agreement with the Roche lobe filling simulations for \(\gamma = 0.2\), which results in \(B = 0.5\) and \(\eta = 0.65\), so we can write \(t_{\text{dis}} = 0.138 N^{0.65}/\omega\). This scaling of \(t_{\text{dis}}\) with \(N\) was also derived from observations (Boutloukos & Lamers 2003; Gieles et al. 2005; Lamers et al. 2005a). To compare the model to the results of the simulations we derive \(t_{1/2}\) in the tidal regime, \(t_{1/2} = t_{\text{dis}}/\xi_e\), from \(t_{\text{dis}}^{\text{obs}}\). Lamers et al. (2005b) show that when \(t_{\text{dis}}^{\text{obs}} = B N^\eta\), with \(B\) a constant, then \(t_{1/2} = (B N^\eta/\eta)[1 - (1/2)^\eta]\), so

\[
t_{1/2} = 0.077 N^{0.65}/\omega. \tag{12}
\]

1 King noticed a remarkable similarity between this result and equation (53) in his 1966 paper. There he shows that the escape rate of stars from a Roche lobe filling cluster is independent of position within the cluster. This is probably because of the same physical reason, but he derived it in a different way.
We assume that the effect of the escape time is the same for all $\mathcal{R} > \mathcal{R}_1$, so that we can apply equation (12) in this regime. With these relations we also assume that $N$ in the pre-collapse and post-collapse phase is the same.

### 3.4 Dissolution in the isolated regime

We assume that clusters with $\mathcal{R} < \mathcal{R}_1$ evolve in the same way as clusters in isolation, up to the moment that $\mathcal{R}$ becomes equal to $\mathcal{R}_1$. This assumption is justified by equation (6) from which we see that for a cluster with $\mathcal{R} = \mathcal{R}_1$ the relative contribution of the tidal field to the escape energy is less than $10\%$ per cent. In the absence of primordial binaries and mass loss by stellar evolution $\tau_0$ and $N$ remain roughly constant up to the moment of core collapse, $t_{cc}$ (Baumgardt, Hut & Heggie 2002, hereafter B02). After $t_{cc}$, isolated clusters evolve in a self-similar way ($t_{\mathrm{ref}} \propto t$), which results in two fundamental relations for the evolution of $N$ and $r_h$ (Goodman 1984; S87; B02):

$$N(t) = N_{cc} \left( \frac{t}{t_{cc}} \right)^{-v},$$  

$$r_h(t) = r_{h,cc} \left( \frac{t}{t_{cc}} \right)^{(2+v)/3},$$

where $r_{h,cc}$ and $N_{cc}$ are $r_h$ and $N$ at $t_{cc}$. More complicated relations, assuming a non-zero origin for these relations, exist (Giersz & Heggie 1994b). Core collapse happens after a multiple number of the initial $r_h$, $t_{cc}, n_{cc} = n_{h,cc} t_{cc}$.

From equation (13) we find that $t_{1/2}$ for clusters evolving completely in the isolated regime, $t_{1/2}^{iso}$, is

$$t_{1/2}^{iso} = \left( \frac{0.138 \times 2^{1/v} n_{cc}}{3} \right) \frac{N \mathcal{R}_1^{3/2}}{\ln \Lambda / \omega}.$$  

Because clusters expand after $t_{cc}$, not all clusters that start with $\mathcal{R}_1 < \mathcal{R}_1$ will reach $t_{1/2}^{iso}$ before they reach $\mathcal{R}_1$. The maximum $\mathcal{R}_1$ for which equation (15) applies is found from equations (14) and (15) and depends on $\mathcal{R}_1$ and $v$ as

$$\mathcal{R}_2 = \mathcal{R}_1 \left( \frac{1}{2} \right)^{(2+2v)/3v},$$

which for $\mathcal{R}_1 = 0.05$ (Fig. 1) and $v = 0.05\sim 0.1$ (B02) results in $\mathcal{R}_2 \approx 6 \times 10^{-3} \sim 6 \times 10^{-3}$. If we define complete dissolution as the moment where only 5 per cent of the original number of stars is still bound, then the corresponding value of $\mathcal{R}_1$ reduces to $\sim 10^{-10}$. This implies that realistic clusters never dissolve completely in the isolated regime.

### 3.5 Combining the isolated and the tidal regime

Clusters that start with $\mathcal{R}_2 < \mathcal{R}_1 < \mathcal{R}_1$ evolve partially in the isolated regime and partially in the tidal regime. Though our model allows to numerically compute $t_{1/2}$ for clusters in this regime, we simply connect $\log[t_{1/2}^{iso}(\mathcal{R}_1)]$ and $\log[t_{1/2}^{iso}(\mathcal{R}_1)]$ with a straight line. The slope of this line, representing the $\mathcal{R}_1$ dependence of $t_{1/2}$, is $\log[t_{1/2}^{iso}(\mathcal{R}_1)] / \log[t_{1/2}^{iso}(\mathcal{R}_1)]$. This slope is slightly $N$ dependent, and by using equations (12) and (15) and the parameters from the isolated run (Table 2) we find that it decreases from 0.55 to 0.35 between $N = 1024$ and 32 768. For $N = 10^6$ the slope would be 0.15.

In Fig. 2 we show the $\omega t_{1/2}$ following from our model for different $N$ and a range of two orders of magnitude around $\mathcal{R}_1$. The results of the simulations are shown as dots and are discussed in the next section.

### 4 COMPARISON TO N-BODY SIMULATIONS

The results for $\omega t_{1/2}$ of the simulations of clusters with different $N$ and $\mathcal{R}_1$ are presented in Fig. 2. The $\omega t_{1/2}$ results for $\mathcal{R}_1 = 0.5$ clusters are nearly the same as those for $\mathcal{R}_1 = 1$. For $\mathcal{R}_1 = 0.25(0.125)$ the $\omega t_{1/2}$ values are approximately 10 per cent (25 per cent) shorter compared to the Roche lobe filling results, whereas a scaling with $t_{\mathrm{ref}}$ predicts a difference of a factor of $4^{1/3}(8^{1/3}) \approx 8(23)$. From Table 1 we see that clusters that start Roche lobe underfilling have evolved for a much larger number of relaxation times than Roche lobe filling clusters by the time they reach $t_{1/2}$.

### 5 CONCLUSIONS

The dissolution time-scale of clusters evolving in a tidal field, in dimensionless units ($\omega t_{1/2}$) or in physical units, is almost independent of the initial half-mass radius, $r_h$, when $r_h$ relative to the initial Jacobi (or tidal) radius, $r_{\mathrm{J}}$, is larger than $\mathcal{R}_1(r_{\mathrm{J}}/r_h) \gtrsim 0.05$. For clusters that start with $\mathcal{R}_1 < 0.05$, $t_{\mathrm{ref}}$ scales mildly with $\mathcal{R}_1$ between $\mathcal{R}_1^{0.35}$ and $\mathcal{R}_1^{0.35}$ for the range of $N$ we consider and even flatter for larger $N$. Only clusters that start with $\mathcal{R}_1 \lesssim 10^{-4}$ can lose half their stars before they reach the influence of the tidal field. We find that in the tidal regime $t_{\mathrm{ref}}$ is mainly determined by $N$ and the angular frequency: $t_{\mathrm{ref}} \propto N^{3/2}(1/2)$, that is, what was also found by BM03 for Roche lobe filling, multimass clusters dissolving in tidal fields.

Gnedin & Ostriker (1997) construct the ‘vital’ diagram of globular clusters, which is a triangle in $M$ versus $r_h$ space, outside which clusters should have been destroyed. There are numerous globular clusters.
clusters with small $r_h$ outside this triangle, which the authors denote as lucky survivors. We show that small clusters can in fact survive.

Models that try to explain the evolution of the GCMF from an initial power law to a (universal) peaked distribution by stellar dynamical processes are in difficulties, since such models will always produce less dissolution in the outer parts of galaxies. McLaughlin & Fall (2008) have recently proposed a solution to this problem by assuming that $\frac{dM}{dt} \propto M/t_h$ and $\frac{dM}{dt} \propto \sqrt{\rho_h}$ (equations 1 and 2, with $\xi_e$ constant), that is, their $t_{\text{dis}}$ is determined by internal relaxation effects only and is independent of the strength of the tidal field. However, our results show that $\xi_e$ is not constant and, therefore, $\frac{dM}{dt}$ does not scale as $\sqrt{\rho_h}$. This makes the universality of the GCMF a problem (again), when trying to explain this by dynamical evolution alone.

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