Which hadronic decay modes are good for $\eta_b$ searching: double $J/\psi$ or something else?

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Abstract

It has been controversial whether $\eta_b$ can be discovered in Tevatron Run 2 through the decay $\eta_b \rightarrow J/\psi J/\psi$ followed by $J/\psi \rightarrow \mu^+\mu^-$. We clear this controversy by an explicit calculation which predicts $\text{Br}[\eta_b \rightarrow J/\psi J/\psi]$ to be of order $10^{-8}$. It is concluded that observing $\eta_b$ through this decay mode in Tevatron Run 2 may be rather unrealistic. The $\eta_b$ may be observed in the forthcoming LHC experiments through the 4-lepton channel, if the background events can be significantly reduced by imposing some kinematical cuts. By some rough but plausible considerations, we find that the analogous decay processes $\eta_b \rightarrow VV, D^*\overline{D}^*$ also have very suppressed branching ratios, nevertheless it may be worth looking for $\eta_b$ at LHC and Super B factory through the decay modes $\eta_b \rightarrow K_S K^\pm \pi^\mp, D^*\overline{D}$. 


I. INTRODUCTION

Since the discovery of Υ(1S) nearly three decades ago, extensive search for its pseudoscalar partner, η_b, has been conducted in various experiments. Unfortunately, to date there is still no conclusive evidence that this elusive particle has been found [1].

The existence of η_b is a solid prediction of QCD. On the theoretical side, many work have attempted to unravel its various properties. In particular, its mass is believed to be among the simplest and most tractable observables. Numerous estimates for Υ − η_b mass splitting span the range 20–140 MeV [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Among different theoretical approaches, perturbative QCD is believed by many people to yield reliable predictions, because of decently heavy b mass and η_b being the lowest-lying b̅b state. By far the most sophisticated prediction along this direction, facilitated by the NRQCD renormalization group technique, gives \( M_{\eta_b} = 9.421 \pm 0.013 \) GeV [10]. An eventual unambiguous observation of η_b and precise measurement of its mass will decisively test the weakly-coupled picture of the b̅b ground state.

Much efforts are spent to search η_b from γγ collisions in the full data samples of LEP 2, where approximately two hundreds of η_b are expected to be produced. ALEPH has one candidate event \( \eta_b \rightarrow K_S (\rightarrow \pi^+\pi^-)K^-\pi^+\pi^-\pi^+ \) (possibly missing a \( \pi^0 \)) with the reconstructed mass of 9.30 ± 0.03 GeV, but consistent to be a background event [12]. ALEPH, L3, DELPHI have also set upper limits on the branching fractions for the η_b decays into 4, 6, 8 charged particles [12, 13, 14]. Based on the 2.4 fb\(^{-1}\) data taken at the Υ(2S) and Υ(3S) resonances, CLEO has searched distinctive single photons from hindered M1 transitions \( \Upsilon(2S), \Upsilon(3S) \rightarrow \eta_b\gamma \) and \( \Upsilon(3S) \rightarrow h_b\pi^0, h_b\pi^+\pi^- \) followed by E1 transition \( h_b \rightarrow \eta_b\gamma \), but no signals have been seen [15].

Hadron collider experiments provide an alternative environment to search for η_b. Unlike the e^+e^- machines which is limited by the low yield of η_b, hadron colliders generally possess a much larger production cross sections for η_b, which in turn allows for searching it through some relatively rare decay modes yet with clean signature. However, one should bear in mind that a noteworthy disadvantage also accompanies with hadron collision experiment, i.e., that the corresponding background events may also be copious, so the effectiveness of these decay modes might be seriously discounted (Such an example may be the decay mode \( \eta_b \rightarrow \gamma\gamma \), with branching fraction \( \sim 10^{-4} \), but the combinatorial background γ events can
be enormous).

Encouraged by the large observed width of $\eta_c \to VV$ ($V$ stands for light vector mesons), Braaten, Fleming and Leibovich (hereafter BFL) have suggested that the analogous decay process $\eta_b \to J/\psi J/\psi$, followed by both $J/\psi$ decays to muon pairs, may be used as a very clean trigger to search for $\eta_b$ at Tevatron Run 2 \cite{16}. Assuming $\text{Br}[\eta_b \to J/\psi J/\psi] \sim 1/m_b^4$, they rescale the measured branching ratio of $\eta_c \to \phi \phi$ by a factor of $(m_c/m_b)^4$ to estimate
d\begin{equation}
\text{Br}[\eta_b \to J/\psi J/\psi] = 7 \times 10^{-4 \pm 1}.
\end{equation}
Combining with the knowledge about the production rate of $\eta_b$ at Tevatron, BFL conclude that the prospect of observing $\eta_b$ through the $4\mu$ decay mode in Run 2 is promising.

Following this suggestion, CDF has searched for the $\eta_b \to J/\psi J/\psi \to 4\mu$ events in the full Run 1 data sample \cite{19}. A small cluster of seven events are seen in the search window, where 1.8 events are expected from background, with the statistical significance of 2.2 $\sigma$. A simple fit infers the cluster’s mass to be $9.445 \pm 0.006$ (stat) GeV. If this cluster is truly due to $\eta_b$, then the product of its production cross section and decay branching ratio is close to the upper end of BFL expectation.

In a recent work, Maltoni and Polosa (henceforth MP) nevertheless argue that the BFL estimate, (1), may be overly optimistic \cite{20}. MP suspect that the analogy between $\eta_c \to \phi \phi$ and $\eta_b \to J/\psi J/\psi$ is only superficial. It is known that perturbative QCD (pQCD) framework has difficulty to account for the large observed widths of the $\eta_c \to VV$ decay processes \cite{21, 22}. The consensus is that some nonperturbative mechanisms should be invoked to reconcile the discrepancy between the pQCD prediction and the actual measurement \cite{23, 24}. On the other hand, due to heavy $b$ and $c$ masses, it is rather reasonable to expect that $\eta_b \to J/\psi J/\psi$ can be safely tackled within the pQCD scheme. Therefore, the rescaling procedure used by BFL, whose validity should reside only in the domain of pQCD, may well be illegitimate when taking the measured ratio of $\eta_c \to \phi \phi$ as input, which is essentially dictated by nonperturbative dynamics.

The persuasive evidence in favor of MP’s argument comes from their explicit calculation for the inclusive decay rate of $\eta_b$ to 4-charm states,
\begin{equation}
\text{Br}[\eta_b \to \bar{c}cc] = 1.8^{+2.3}_{-0.8} \times 10^{-5}.
\end{equation}

\footnote{Note that $\text{Br}_{\text{exp}}[\eta_c \to \phi \phi]$ has shifted from $(7.1 \pm 2.8) \times 10^{-3}$ given in 2000 PDG edition \cite{17}, which was quoted by BFL, to the latest PDG value $(2.7 \pm 0.9) \times 10^{-3}$ \cite{18}.}
This ratio is even smaller than the lower limit of Br[η_b → J/ψJ/ψ] estimated by BFL! This is clearly at odds with the usual thought that the exclusive decay rate should be much smaller than the inclusive one.

One of the major concerns of this paper is to dispose of this controversy by performing an explicit calculation. Heavy b and c quark masses set hard scales so that we can confidently utilize pQCD to tackle this decay process, expecting those nonperturbative contributions plaguing the decay η_c → VV play an insignificant role here. Since each involved particle is a heavy quarkonium, we work with the color-singlet model, in line with the calculation done for double charmonium production at e^+e^- colliders [25, 26, 27, 28, 29, 30, 31]. It is found that at the lowest order in α_s, retaining the transverse momentum of c inside J/ψ is vital to obtain a non-vanishing result, and consequently, the correct asymptotic behavior of the hadronic decay branching ratio is α_s^2 v_c^0 (m_c/m_b)^8 (v_c stands for the typical velocity of c in J/ψ). Numerically, we predict

\[ \text{Br}[\eta_b \rightarrow J/\psi J/\psi] = (0.5 - 6.6) \times 10^{-8}, \]

which is much smaller than the BFL estimate, (1). Simple analysis indicates that the cluster reported by CDF [19] is extremely unlikely to be affiliated with η_b. We further argue that, the potential of discovering η_b through this decay mode is gloomy even in Run 2.

To better guide experimental search for η_b, an important issue is to know that, besides η_b → J/ψJ/ψ, which hadronic decay modes can serve as efficient triggers to detect η_b? Heavy η_b mass opens a huge phase space available for innumerable decay modes, but at a price that the branching ratio of each individual mode has been greatly diluted with respect to that from η_c decay.

At any rate, it is valuable to know which hadronic decay modes of η_b possess the largest branching ratios. We examine numerous hadronic decay channels, mostly two-body ones, e.g., η_b decays to two light mesons, and two charmed mesons. By some crude but plausible estimate, we find η_b → φφ, D^*D also have very tiny branching ratios, of the same order as that for η_b → J/ψJ/ψ. In sharp contrast, the branching ratios for η_b → D^*D might be as large as 10^{-5}, therefore they can be used as searching modes. Furthermore, stimulated by the experimental fact that η_c decays to three pseudoscalars have largest branching ratios, we urge experimentalists to look into such 3-body channels as η_b → K\overline{K}\pi, η(η')\pi\pi, D\overline{D}\pi. The corresponding ratios are estimated to be of order 10^{-4}.
FIG. 1: Lowest-order QCD diagrams which contribute to \( \eta_b \rightarrow J/\psi J/\psi \).

The remainder of the paper will proceed as follows. In Sec. II, we discuss the asymptotic behavior of \( \text{Br}[\eta_b \rightarrow J/\psi J/\psi] \) in the limit \( m_b \gg m_c \gg \Lambda_{\text{QCD}} \), and elucidate the peculiarity of this decay process. We then present the actual calculation of the ratio in Sec. III employing the color-singlet model which incorporates velocity expansion. In Sec. IV, we discuss the observation potential of \( \eta_b \) through this mode in Tevatron and LHC. In Sec. V, by some simple scaling analysis, we estimate the branching ratios for \( \eta_b \) decays to various final states, e.g., to two light mesons, to three light pseudo-scalar mesons, and to two charmed mesons. For the decay process \( \eta_b \rightarrow VV \), we also compare our estimates with those obtained from some nonperturbative mechanism. In Sec. VI we summarize and give an outlook.

II. ASYMPTOTIC BEHAVIOR AND UNNATURAL DECAY PROCESS

Before launching into the actual calculation, it is instructive to first envisage the general property of this exclusive decay process. A powerful tool in pQCD to count kinematical suppression factors for hard exclusive processes is the so-called hadron helicity selection rule, originally developed for the reactions involving light hadrons \([32]\). This rule has recently been applied to analyze the double charmonium production in \( e^+e^- \) annihilation in the limit \( \sqrt{s} \gg m_c \) \([25]\). One can work out the asymptotic behavior for \( \text{Br}[\eta_b \rightarrow J/\psi J/\psi] \) in an analogous way. The decay \( \eta_b \rightarrow J/\psi J/\psi \) can be initiated by either strong or electromagnetic interaction, with the corresponding lowest-order diagrams shown in Fig. 1 and Fig. 2 respectively. Generally speaking, the QCD contribution is dominating over the QED contribution. However, for the sake of completeness, the latter will be also included in our analysis.
For the lowest order strong decay process depicted in Fig. 1, a simple consideration may suggest that in the limit $m_b \to \infty$ with $m_c$ fixed,

$$\text{Br}_{\text{str}}[\eta_b \to J/\psi(\lambda) + J/\psi(\bar{\lambda})] \sim \alpha_s^2 v_c^6 \left(\frac{m_c^2}{m_b^2}\right)^{2+|\lambda+\bar{\lambda}|},$$

(4)

where $\lambda$ and $\bar{\lambda}$ represent the helicities of two $J/\psi$ viewed in the $\eta_b$ rest frame, and $v_c$ denotes the characteristic velocity of $c$ in $J/\psi$. Obviously the scaling behavior depends on the helicity configurations of both $J/\psi$. The factor $v_c^6$ is expected because there is a factor of wave function at the origin, $\psi_{J/\psi}(0)$, for each $\bar{c}c$ pair to emerge with small relative momentum to form S-wave bound state, and $\psi_{J/\psi}(0) \sim (m_c v_c)^{3/2}$.

The expectation (4) is compatible with the helicity selection rule that the decay configuration which conserves the hadron helicity, i.e. $\lambda + \bar{\lambda} = 0$, exhibits the slowest asymptotic decrease, $\text{Br}_{\text{str}} \sim 1/m_b^4$. This occurs because there are two hard gluons carrying large momentum transfers. The only helicity state bearing this least suppressed ratio which is also compatible with the angular momentum conservation $\lambda = \bar{\lambda}$ is thus $(\lambda, \bar{\lambda})=(0, 0)$. The helicity conservation can be violated either by the nonzero charm mass $m_c$ or by the transverse momentum of $c$ inside $J/\psi$, $q_{\perp}$. For every unit of violation of the selection rule, there is a further suppression factor of $m_c^2/m_b^2$ or $q_{\perp}^2/m_b^2$. For the other physically allowed configurations $(\lambda, \bar{\lambda})=(\pm 1, \pm 1)$, the helicity conservation is violated by two units, so one expects $\text{Br}_{\text{str}} \sim 1/m_b^8$. By writing (4) the way as it is, we have temporarily assumed that the cause of violation is entirely due to the quark mass $m_c$.

The essential assumption of BFL is the “leading twist” scaling behavior $\text{Br}_{\text{str}} \sim 1/m_b^4$, which is tacitly associated with the $\eta_b$ decay to two longitudinally polarized $J/\psi$. However, one may recall that $\eta_b \to J/\psi J/\psi$, like $\eta_c \to VV$, belongs to a class of so-called unnatural decay processes [33], for which the helicity state $(0, 0)$ is strictly forbidden due to the conflict between parity and angular momentum conservation $^3$. In operational basis, it arises because the decay amplitude for such a process involves the Levi-Civita tensor and there are no enough number of independent Lorentz vectors to contract with it for vector mesons are

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$^2$ The decay $\eta_b \to J/\psi \eta_c$ would exhibit the “leading twist” scaling behavior, $\text{Br} \sim \alpha_s^2 v_c^6 (m_c/m_b)^4$, were it not inhibited by $C$ invariance.

$^3$ An unnatural process, according to Ref. [33], is defined as a heavy quarkonium two-meson decay process that does not conserve a multiplicative quantum number called naturalness, which is defined by $\sigma = (-1)^J P$ ($J$, $P$ stand for the spin and parity of a meson).
longitudinally polarized. Further examples of the \textit{unnatural} processes for a bottomonium decays to two $S$-wave charmonia include $\Upsilon (\chi_{b2}) \rightarrow J/\psi \eta_c$ and $\chi_{b1} \rightarrow J/\psi J/\psi$, in contrast with the \textit{natural} decay processes such as $\chi_{b0,2} \rightarrow J/\psi J/\psi \eta_c$, $\eta_c \eta_c$, and $h_b \rightarrow J/\psi \eta_c$.

We may wish to examine this assertion closely for our case. Parity and Lorentz invariance constrain the decay amplitude to have the following tensor structure:

$$\mathcal{M}(\lambda, \bar{\lambda}) \propto \epsilon^{\mu\nu\alpha\beta} P_\mu \bar{P}_\nu \varepsilon_\alpha^*(\lambda) \bar{\varepsilon}_\beta^*(\bar{\lambda}),$$

(5)

where $P$, $\bar{P}$, $\varepsilon$ and $\bar{\varepsilon}$ are momenta and polarization vectors for both $J/\psi$. If both $J/\psi$ are longitudinally polarized, $\varepsilon$ and $\bar{\varepsilon}$ then can be expressed as linear combinations of $P$ and $\bar{P}$, $M(0,0)$ thus vanishes $^4$. It is worth emphasizing that this result is based solely on the basic principle of parity and angular momentum conservation, so it will not depend on dynamical details. For example, it will be true irrespective of whether this process is initiated by QCD or QED, whether higher order perturbative corrections are included or not, and whether nonperturbative QCD effects are incorporated or not.

In passing, one may notice an equivalent but more intuitive explanation for $M(0,0) = 0$ for the unnatural decay processes $^2$. It can be attributed to a peculiar property of Clebsch-Gordon coefficients that $\langle 10|10;10 \rangle = 0$. We take $\eta_b \rightarrow J/\psi J/\psi$ as an example to illustrate this. Due to parity and angular momentum conservation, two $J/\psi$ must have the relative orbital angular momentum $L = 1$ and the total spin $S = 1$. However, it is impossible for the two longitudinally polarized $J/\psi$ to couple to a $(S, S_z) = (1, 0)$ state because of this vanishing Clebsch-Gordon coefficient.

Since both $J/\psi$ in the final state must be transversely polarized, the helicity conservation is violated by two units, Eq. $^2$ then indicates $\text{Br}_{\text{str}}[\eta_b \rightarrow J/\psi \perp J/\psi \perp] \sim \alpha_s^2 v_c^6 \left(\frac{m_c}{m_b}\right)^8$. Nevertheless, as the explicit calculation will reveal, this behavior is not quite correct in counting powers of $v_c$. It turns out that the true asymptotic behavior is even more suppressed,

$$\text{Br}_{\text{str}}[\eta_b \rightarrow J/\psi(\pm1) + J/\psi(\pm1)] \sim \alpha_s^2 v_c^6 \left(\frac{m_c}{m_b}\right)^2 \left(\frac{q_\perp^2}{m_b^2}\right)^2 \sim \alpha_s^2 v_c^{10} \left(\frac{m_c}{m_b}\right)^8,$$

(6)

where $q_\perp \sim m_c v_c$ is assumed in the last term. This implies that, at the lowest order in $\alpha_s$, the violation of the rule should be ascribable to the nonzero transverse momentum of $c$ in $J/\psi$.

$^4$ Exactly with the same argument, the decay $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi(\lambda = 0) + \eta_c$ turns out to be strictly forbidden.
FIG. 2: Lowest-order QED diagrams that contributes to $\eta_b \rightarrow J/\psi J/\psi$. Only the fragmentation type diagrams are retained, whereas the other two, which can be obtained by replacing the gluons in Fig. 1 by photons, have been suppressed.

instead of the nonzero $m_c$. This is compatible with the earlier finding that the amplitude for $\eta_c \rightarrow V V$ vanishes in the collinear quark configuration, even though the quark masses are kept nonzero [21]. After $q_\perp$ is included for the light vector mesons, the decay rate then becomes nonzero, though still too tiny to account for the measured rates [22].

We can also infer the asymptotic behavior of the electromagnetic contribution in Fig. 2. The photon fragmentation produces transversely polarized $J/\psi$. Since the hard scale is set by the virtuality of fragmenting photon $\sim m_c^2$, no suppression factor $\propto 1/m_b^2$ can arise in the branching ratio. A simple counting rule suggests that the QED fragmentation contribution to the ratio exhibits the following behavior

$$\text{Br}_{em}[\eta_b \rightarrow J/\psi(\pm 1) + J/\psi(\pm 1)] \sim \left(\frac{\alpha^4}{\alpha_s^2}\right) v_c^6. \quad (7)$$

Although the electromagnetic contribution is free of suppression by inverse powers of $m_b$, in general it still cannot counteract the adversity caused by $\alpha \ll \alpha_s$.

There is also the interference term between QCD and QED contributions, with a scaling behavior intermediate between (6) and (7).

III. COLOR-SINGLET MODEL CALCULATION

In this section, we present a calculation for $\eta_b \rightarrow J/\psi J/\psi$ in perturbative QCD scheme. As stressed before, this scheme is expected to generate reliable answer for this process, since the annihilation of $\bar{b}b$ and creations of $c\bar{c}$ pairs take place in rather short distances. This decay process should be contrasted with the analogous process $\eta_c \rightarrow V V$. To account for the unnaturally large measured width of the latter, it is proposed that some nonperturbative mechanisms, e.g., the mixing among pseudo-scalar states $\eta - \eta' - \eta_c$, either due to QCD axial
anomaly \textsuperscript{23}, or due to perturbative box diagram \textsuperscript{24}, together with quark pair creation from vacuum, should play a prominent role. The impact of final state interaction on the doubly-OZI-suppressed process $\eta_c \rightarrow \omega \phi$ has also been addressed \textsuperscript{35}. Fortunately, neither of these nagging complications will bother us, because the $\eta_b$ is too heavy to bear a significant mixing with $\eta_c$, and $c$ is too heavy to easily pop out of the vacuum. The weak inter-quarkonium Van der Vaals interaction \textsuperscript{36} implies that the final state effects are also unimportant.

We first consider the QCD contribution in Fig. 1 for $\eta_b(K) \rightarrow J/\psi(P) + J/\psi(\bar{P})$, where $K$, $P$ and $\bar{P}$ signify momenta of each quarkonium. In this work, we will only consider the color-singlet Fock state of the quarkonium, completely ignoring the possible color-octet effect, which is difficult to analyze for exclusive processes in a clear-cut way \textsuperscript{5}.

In the color-singlet model calculation, it is customary to begin with the parton level matrix element $b(k) \overline{b}(\overline{k}) \rightarrow c(p) \overline{c}(\overline{p}) + c(\bar{p}) \overline{c}(\bar{\overline{p}})$, then project this matrix element onto the corresponding color-singlet quarkonium Fock states. It is worth stressing that because of impossibility to find a frame such that two back-to-back fast moving $J/\psi$ become simultaneously at rest, it is imperative to admit a manifestly Lorentz-covariant projector.

For the $c\bar{c}$ pair with total momentum $P$, we assign the momentum carried by each constituent as

$$
p = \frac{P}{2} + q,
\overline{p} = \frac{P}{2} - q, \tag{8}
$$

where $q$ is the relative momentum satisfying $P \cdot q = 0$. In the rest frame of $c\bar{c}$ pair, $P^\mu$ is thus purely time-like and $q^\mu$ purely space-like. Since the $c\bar{c}$ pair forms $J/\psi$, it is necessarily in a spin-triplet color-singlet state, and one can replace the product of the Dirac and color spinors for $c$ and $\bar{c}$ in the final state with the following Lorentz-covariant projector \textsuperscript{25, 38}:

$$
v(p) \overline{\tau}(p) \rightarrow \int \frac{dq}{4\sqrt{2}E(E + mc)} (p - m) \not\! \! \! \varphi(\lambda)(P + 2E)(p + m)
\times \left( \frac{1}{\sqrt{m_c}} \phi_{J/\psi}(-q^2) \right) \otimes \left( \frac{1}{\sqrt{N_c}} \right).
\tag{9}
$$

\textsuperscript{5} Note the color-singlet model can be viewed as a truncated version of the Nonrelativistic QCD (NRQCD) factorization approach \textsuperscript{37}, which still admits a factorization of the calculable short-distance part and universal long-distance factors in the color-singlet channel. For the exclusive processes involving heavy quakonium, the color-single model and NRQCD are often used interchangeably in literature.
where \( p^2 = m_c^2 \), \( E = \sqrt{P^2/2} = \sqrt{m_c^2 - q^2} \) is a Lorentz scalar, and \( \varepsilon^* \) is the polarization vector of \( J/\psi \) satisfying \( \varepsilon(\lambda) \cdot \varepsilon^*(\lambda') = -\delta^{\lambda\lambda'} \) and \( P \cdot \varepsilon^* = 0 \). \( N_c = 3 \) and \( 1_c \) stands for the unit color matrix. The momentum space wave function \( \phi_{J/\psi}(-q^2) \) is explicitly included and the Lorentz-invariant measure is defined as

\[
\int [dq] \equiv \int \frac{d^4q}{(2\pi)^3} 2 E \delta(P \cdot q).
\]

The introduction of the covariant projector (9) unites the non-relativistic internal motion and highly relativistic external motion of \( J/\psi \) in a coherent fashion, which is indispensable if one wishes to systematically implement relativistic corrections for highly energetic processes like this one. At the final stage of the calculation, one can always choose to perform the momentum integrals in the rest frame of each \( J/\psi \) successively, thanks to the Lorentz-invariant measure (10). We take the following integral, which arises in the zeroth order of relativistic expansion, as an example:

\[
\int [dq] \phi_{J/\psi}(-q^2) = \int \frac{d^3q}{(2\pi)^3} \phi_{J/\psi}(q^2) \bigg|_{\text{rest frame}} = \psi_{J/\psi}(0),
\]

where \( \psi_{J/\psi}(0) \) is the spatial Schrödinger wave function at the origin for \( J/\psi \).

The above formulas obviously also apply to the second \( J/\psi \), once the replacements \( P \rightarrow \tilde{P}, q \rightarrow \tilde{q}, E \rightarrow \tilde{E}, \) and \( \varepsilon(\lambda) \rightarrow \tilde{\varepsilon}(\bar{\lambda}) \) are made.

For the \( \eta_b \) in the initial state, it turns out that the \( O(v_b^2) \) correction is less relevant, so we will neglect the relative momentum and simply take \( k = \bar{k} = K/2 \). Consequently, the following simplified projector will be used:

\[
u(k) \bar{\nu}(\bar{k}) \longrightarrow \frac{1}{2\sqrt{2}} (K + 2m_b) i\gamma_5 \times \left( \frac{1}{\sqrt{m_b}} \psi_{\eta_b}(0) \right) \otimes \left( \frac{1_c}{\sqrt{N_c}} \right).
\]

For processes involving heavy quarkonium, one usually organizes the amplitude in powers of the relative momenta, to accommodate the NRQCD ansatz. Some simple algebra shows that the \( \eta_b \) part of the matrix element contains the factor

\[
\text{Tr}[\gamma_5 \bar{K}\gamma^\mu(q - \bar{q})\gamma^\nu] = 4i \epsilon^{\mu\nu\rho \sigma} K_\rho(q - \bar{q})_\sigma.
\]

In the leading order in \( v_c \) expansion, \( q = \bar{q} = 0 \), the amplitude hence vanishes. As a matter of fact, Anselmino et al have drew the same conclusion for \( \eta_c \rightarrow VV \) while using light-cone scheme in the collinear quark configuration but retaining nonzero light quark masses [21].
This color-singlet-model result can be viewed as a specific example of theirs by invoking narrow-peak approximation to the light-cone wave function for vector mesons.

In order to obtain a non-vanishing amplitude, one must go to the next-to-leading order in $v_c$. In particular, one should expand the remaining part of amplitude to linear order in $q$ and $\tilde{q}$, to pair up with the ones in Eq. (13) \(^6\).

First we expand the product of three propagators in Fig. 1:

$$\frac{1}{(q - \tilde{q})^2 - m_b^2} \frac{1}{(K/2 - q + \tilde{q})^2} \frac{1}{(K/2 + q - \tilde{q})^2} \approx -\frac{1}{m_b^6} \left[ 1 + \mathcal{O}\left( \frac{q^2}{m_b^2}, \frac{\tilde{q}^2}{m_b^2}, \frac{q \cdot \tilde{q}}{m_b^2} \right) \right].$$

Because the sub-leading terms are at least quadratic in $q$ or $\tilde{q}$, they can be dropped.

The missing $q$ or $\tilde{q}$ thus must arise from the $J/\psi$ part of the matrix element, or more precisely, from the projectors for $J/\psi$, Eq. (9). After some straightforward Dirac trace algebra, one can express the full QCD amplitude in the Lorentz-invariant form:

$$\mathcal{M}_{str} = -4\sqrt{2} C_{str} g_4 \frac{\psi_{\eta b}(0)}{m_{13/2}} A,$$

where the color factor $C_{str} = N_c^{-3/2} \text{Tr}(T^a T^b) \text{Tr}(T^a T^b) = 4^{-1} N_c^{-3/2} (N_c^2 - 1)$, and the double integral $A$ reads

$$A = \int \int [dq] [d\tilde{q}] \phi_{J/\psi}(-q^2) \phi_{J/\psi}(-\tilde{q}^2) \times \left[ \epsilon_{\mu\nu\alpha\beta} P^\mu \tilde{P}^\nu q^\alpha (\bar{\epsilon}^* \beta q \cdot \epsilon - \epsilon^* \beta q \cdot \tilde{\epsilon}) + (q \leftrightarrow \tilde{q}) \right],$$

where only the terms with the intended accuracy of $\mathcal{O}(v_c^2)$ are kept. We have substituted $E, \tilde{E}$ everywhere by $m_c$, which is legitimate in the current level of accuracy \(^7\).

One immediately observes that, to survive when contracted with the antisymmetric tensor, $q$ and $\tilde{q}$, as well as $\epsilon^*$ and $\tilde{\epsilon}^*$ in (16), must be transverse. We thus confirm the earlier assertion that transverse momentum is the agent to violate the helicity conservation.

Because the transverse components of a 4-vector are invariant under the boost along the moving direction of $J/\psi$, we can perform the integrals in the rest frame of $J/\psi$ without

\(^6\) Retaining the relative momentum in $\eta_b$, but not $q$ and $\tilde{q}$ still leads to vanishing amplitude.

\(^7\) For the same reason, we have not bothered to include explicitly the relativistic effects due to 2-body phase space and normalization of $c\bar{c}$ states as considered in Ref. [25, 38], which are powers of $E/m_c$ or $\tilde{E}/m_c$. 
concerning about the boost effect. Using the spherical symmetry of $S$-wave wave function, we have

$$
\int [dq] q'^3_q \phi_{J/\psi}(-q^2) = \frac{\delta^{ij}}{3} \left. \int \frac{d^3q}{(2\pi)^3} q^2 \phi_{J/\psi}(q^2) \right|_{\text{rest frame}} = \frac{\delta^{ij}}{3} m_c^2 \langle v^2 \rangle_{J/\psi} \psi_{J/\psi}(0). \tag{17}
$$

Here $\langle v^2 \rangle_{J/\psi}$ is a quantity governing the size of relativistic corrections. Loosely speaking, it is related to the second derivative of wave function at the origin for $J/\psi$, and characterizes the average $v^2$ of $c$ inside $J/\psi$. Inspecting Eq. (17), however, one immediately realizes at large $q$, the Coulomb wave function should dominate and this quantity turns out to be linearly ultraviolet divergent, hence its meaning becomes obscure. In fact, this factor admits a more rigorous definition as a ratio of NRQCD matrix elements $[25, 37]$:

$$
\langle v^2 \rangle_{J/\psi} = \frac{m_c^2 \langle v^2 \rangle_{J/\psi} \psi_{J/\psi}(0)}{\epsilon \cdot \langle J/\psi(\epsilon) | \psi^\dagger (\epsilon^\dagger) \sigma (-D^2) \chi | 0 \rangle}, \tag{18}
$$

where $\psi$ and $\chi$ represent Pauli spinor fields in NRQCD, $D$ is spatial covariant derivative. The matrix elements appearing in the above ratio should be understood to be the renormalized ones. Lattice QCD extraction of this quantity has been available long ago, but the precision is quite poor $[39]$. We will specify our choice for the numerical value of $\langle v^2 \rangle_{J/\psi}$ in Section IV.

Substituting (17) into (16), we obtain

$$
A = -\frac{4}{3} m_c^2 \langle v^2 \rangle_{J/\psi} \psi_{J/\psi}(0) \epsilon_{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \epsilon^*_{\alpha} \tilde{\epsilon}^*_{\beta}, \tag{19}
$$

which has the desired tensor structure as in (15). The strong decay amplitude then reads

$$
\mathcal{M}_{\text{str}} = \frac{512 \sqrt{6} \pi^2 m_c^2 m_b^{13/2} \psi_{\eta_b}(0) \psi_{J/\psi}^2(0) \langle v^2 \rangle_{J/\psi} \epsilon_{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \epsilon^*_{\alpha} \tilde{\epsilon}^*_{\beta}}{27 m_b^{3/2} m_c} \left(1 - \frac{2m_b^2}{m_c^2}\right)^{-1}. \tag{20}
$$

Next we turn to the electromagnetic contribution to $\eta_b \to J/\psi J/\psi$. Two QED diagrams which have the same topology as Fig. 1 but with gluons replaced by photons, lead to the amplitude of the same form as (20) except $\alpha_s^2$ is replaced by $\epsilon_g^2 \epsilon_c^2 \alpha^2$. Obviously, their contributions are much more suppressed than those from the fragmentation diagrams in Fig. 2 hence will not be considered. The QED fragmentation contribution to the amplitude can be easily worked out,

$$
\mathcal{M}_{\text{em}} = \frac{24 \sqrt{6} \pi^2 \epsilon_g^2 \epsilon_c^2 \alpha^2}{m_b^{5/2} m_c^2} \psi_{\eta_b}(0) \psi_{J/\psi}^2(0) \left(1 - \frac{2m_b^2}{m_c^2}\right)^{-1} \epsilon_{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \epsilon^*_{\alpha} \tilde{\epsilon}^*_{\beta}. \tag{21}
$$
Adding (20) and (21) together, squaring, summing over transverse polarizations of both $J/\psi$ and integrating over half of the phase space, we then obtain the partial width $\Gamma[\eta_b \rightarrow J/\psi J/\psi]$. Nevertheless, it is more convenient to have a direct expression for the branching ratio, where $\psi_{\eta_b}(0)$ drops out,

$$\text{Br}[\eta_b \rightarrow J/\psi J/\psi] = \frac{2^{13} \pi^2 \alpha_s^2}{34} \frac{m_c^2}{m_b^2} \psi_{J/\psi}^4(0) \left( 1 - \frac{4m_c^2}{m_b^2} \right)^{3/2}$$

$$\times \left[ \langle v^2 \rangle_{J/\psi} + \left( \frac{9 e_b e_c \alpha}{8 \alpha_s} \frac{m_c^2}{m_b^2} \right)^2 \left( 1 - \frac{2m_c^2}{m_b^2} \right)^{-1} \right]^2. \quad (22)$$

In deriving this, we have approximated the total width of $\eta_b$ by its gluonic width:

$$\Gamma_{\text{tot}}[\eta_b] \approx \Gamma[\eta_b \rightarrow gg] = \frac{8\pi\alpha_s^2}{3m_b^2} \psi_{\eta_b}^2(0), \quad (23)$$

where the LO expression in $\alpha_s$ and $v_b$ is used for simplicity.

Equation (22) constitutes the main formula of this paper. One can readily confirm the asymptotic behavior of QCD and QED contributions first given in Sec. II, Eq. (6) and (7).

**IV. OBSERVATION POTENTIAL OF $\eta_b \rightarrow J/\psi J/\psi$ AT TEVATRON AND LHC**

We now explore the phenomenological implication of (22). The input parameters are $m_b$, $m_c$, $\alpha$, $\alpha_s$, $\psi_{J/\psi}(0)$ and $\langle v^2 \rangle_{J/\psi}$, all of which can be inferred from other independent sources. The wave function at the origin for $J/\psi$ can be extracted from its dielectron width:

$$\Gamma[J/\psi \rightarrow e^+e^-] = \frac{4\pi e_c^2 \alpha_s^2}{m_c^2} \psi_{J/\psi}^2(0). \quad (24)$$

(The LO formula in $\alpha_s$ and $v_c^2$ is used for simplicity.) Using the measured dielectron width 5.55 keV [18], we obtain $\psi_{J/\psi}(0) = 0.205$ GeV$^{3/2}$ for $m_c = 1.5$ GeV.

Among various input parameters, the least precisely known is $\langle v^2 \rangle_{J/\psi}$. Since this is a subtracted quantity, it can be either positive or negative. There is a useful relation, first derived by Gremm and Kapustin using equation of motion of NRQCD [40], relating this quantity with the pole mass of charm quark and the $J/\psi$ mass $^8$. To our purpose this relation reads $^8$

$$\langle v^2 \rangle_{J/\psi} \approx \frac{M_{J/\psi}^2 - 4m_{c,pole}^2}{4m_{c,pole}^2}. \quad (25)$$

$^8$ For a reformulation of Gremm-Kapustin relation from a even lower energy effective theory of NRQCD, dubbed potential NRQCD, we refer the interested readers to Ref. [41].
In the analysis of the process $e^+e^- \rightarrow J/\psi \eta_c$ at $B$ factory \cite{25}, the charm quark pole mass is taken as the commonly quoted value, 1.4 GeV, consequently $\langle v^2 \rangle_{J/\psi} = 0.22$ and $\langle v^2 \rangle_{\eta_c} = 0.13$. However, since $m_{c\text{pole}}$ contains renormalon ambiguity, it cannot be determined better within an accuracy of order $\Lambda_{\text{QCD}}$. Therefore one should not be surprised that somewhat larger value of $m_{c\text{pole}}$ may be occasionally reported in literature. For example, a recent QCD moment sum rule analysis claims $m_{c\text{pole}} = 1.75 \pm 0.15$ GeV \cite{42}. For this specific value of charm quark pole mass, one would obtain instead $\langle v^2 \rangle_{J/\psi} = -0.22 \pm 0.15$.

The situation seems to be rather obscure since even the sign of $\langle v^2 \rangle_{J/\psi}$ cannot be unambiguously determined. To proceed, we take a practical attitude and adopt the value $\langle v^2 \rangle_{J/\psi} = 0.25 \pm 0.09$ from a recent Cornell potential model based analysis \cite{43}. This positive value seems to be most welcomed to alleviate the alarming discrepancy between the predicted and the measured cross section for $e^+e^- \rightarrow J/\psi + \eta_c$ \cite{25}, also seems compatible with a recent QCD sum rule determination of $\langle v^2 \rangle_{\eta_c}$ \cite{44}. However, we leave the possibility open that this quantity may turn out to be negative after future scrutiny. Taking $m_b = M_{\eta_b}/2 \approx 4.7$ GeV, $m_c = 1.5$ GeV, $\alpha = 1/137$ and $\alpha_s(m_b) = 0.22$, we then find

$$\text{Br}[\eta_b \rightarrow J/\psi J/\psi] = 2.4^{+4.2}_{-1.9} \times 10^{-8}. \quad (26)$$

The uncertainty is estimated by varying $m_b$ and $m_c$ in the $\pm 100$ MeV range, varying $\alpha_s(\mu)$ in the $-0.04$ range (which corresponds to slide the scale from $\mu = m_b$ to $2m_b$), as well as taking into account the errors in measured $\Gamma_{e^+e^-}$ (of $\pm 0.14$ keV) and in $\langle v^2 \rangle_{J/\psi}$. The constructive interference between electromagnetic and strong amplitudes has modest effect, i.e., neglecting the QED contribution decreases the branching ratio by few to ten per cents.

Our prediction for the branching ratio is at least three order-of-magnitudes smaller than the BFL estimate, \cite{1}. Despite the large uncertainties inherent in various input parameters, we believe our prediction captures the correct order of magnitude, $10^{-8}$. It is also worth noting that our prediction for this exclusive decay ratio is about one thousandth of the inclusive 4-charm ratio, \cite{2}, which seems fairly reasonable.

Experimentally $J/\psi$ can be tagged cleanly by its decay to muon pair. Multiplying (26) by the branching ratios of 6% for each of the decay $J/\psi \rightarrow \mu^+\mu^-$, we obtain $\text{Br}[\eta_b \rightarrow J/\psi J/\psi] \sim 10^{-8}$.

\footnote{Note the actual prediction, $\text{Br} \sim 10^{-8}$, is much larger than the expectation based on the asymptotic scaling behavior $\text{Br} \sim \alpha_s^2 v_c^{10}(m_c/m_b)^8 \sim 10^{-11}$. This can be attributed to the large prefactors in the right hand side of (22).}
$J/\psi J/\psi \rightarrow 4\mu \approx (0.2 - 2.4) \times 10^{-10}$. The total cross section for $\eta_b$ production at Tevatron energy is about 2.5 $\mu$b [20]. Therefore, the production cross section for this $4\mu$ decay mode is about 0.05 – 0.6 $\text{fb}$. For the full Tevatron Run 1 data of 100 pb$^{-1}$, we then obtain between 0.005 and 0.06 produced events. We now can safely assert that the seven $4\mu$ events reported by Ref. [19] must come from sources other than $\eta_b$ decay.

Tevatron Run 2 plans to achieve an integrated luminosity of 8.5 fb$^{-1}$ by 2009. Assuming equal $\sigma(p\overline{p} \rightarrow \eta_b + X)$ at $\sqrt{s} = 1.96$ and 1.8 TeV, we then estimate there are about 0.4-5 produced events. Since the kinematical cuts, as well as taking into account the acceptance and efficiency for detecting muon, will further cut down this number, we conclude it is not realistic to search $\eta_b$ through this decay mode in Tevatron Run 2.

To fathom the observability of $\eta_b$ through this mode at LHC, we need first know the inclusive $\eta_b$ production rate. There are rough estimates for the $\chi_{b0,2}$ cross sections at LHC, which are about 6 times larger than the corresponding cross sections at Tevatron [34]. Assuming the same scaling also holds for $\eta_b$, we then expect the cross section for $\eta_b$ at LHC to be about 15 $\mu$b, subsequently the production cross section for the $4\mu$ events to be about 0.3-3.6 $\text{fb}$. For a 300 fb$^{-1}$ data, which is expected to be accumulated in one year running at LHC design luminosity, the number of produced events may reach between 100 to 1000. The product of acceptance and efficiency for detecting $J/\psi$ decay to $\mu^+\mu^-$ is estimated to be $\epsilon \approx 0.16$, which is perhaps a conservative estimate for LHC. Multiplying the number of the produced events by $\epsilon^2$, we expect between 1 and 10 observed events per year. If we loose the constraint that $J/\psi$ must be tagged by $\mu^+\mu^-$ pair and also allow its reconstruction through $e^+e^-$ mode, we can have 4-40 observed 4-lepton events per year.

The above analysis seems to indicate that, the chance of observing $\eta_b$ at LHC through the 4-lepton mode subsists, but critically hinges on whether the signal events can be singled out from the abundant background events. The most important background events may come from the direct double $J/\psi$ production from $gg$ fusion [47, 48]. From previous analysis, we know that all of the seven $4\mu$ candidate events selected by CDF based on Tevatron Run 1 data [19], should be regarded as this kind of background events, which seem to outnumber the expected signal events by several orders of magnitude. The same situation may also apply to LHC. It might be possible for experimentalists to judiciously choose kinematical cuts to significantly suppress the background events while retaining as many signal events as possible.
V. OTHER DECAY CHANNELS OF $\eta_b$

The decay channel we have considered so far, $\eta_b \to J/\psi J/\psi$, which has very clean signature, is unfortunately very much suppressed because of its maximal violation of the helicity selection rule\(^\text{10}\). There are other two-body decay channels, e.g., $\eta_b$ decays to two light mesons and to two charmed mesons, some of which do conserve the hadron helicity, thus may have much larger branching ratios. In this section, we attempt to estimate the order of magnitude of the branching ratios for these processes. In addition, we also present some crude estimation for the $\eta_b$ decays to three pseudo-scalar states.

A. $\eta_b (\eta_c)$ decays to two and three light mesons

When contending with light mesons in hard exclusive processes, the most appropriate description of them is in terms of light-cone expansion approach. On the other hand, the constituent quark model, which treats the light mesons as non-relativistic bound states, is also frequently invoked as an alternative method for a quick order-of-magnitude estimate. In this sense, the preceding formulas derived for $\eta_b \to J/\psi J/\psi$ can be applied to describe the unnatural decay processes $\eta_b \to VV$, once we understand we are working with the constituent quark model.

We take $\eta_b \to \phi\phi$ as an representative. By regarding $\phi$ as a strangeonium, we can directly use (22), only with some trivial changes of input parameters. We take the constituent quark mass $m_s \approx M_\phi/2 = 0.5$ GeV. The wave function at the origin of $\phi$, $\psi_\phi(0)$, can be extracted analogously from its measured dielectron width of $1.27 \pm 0.04$ keV [18] through (24). We take $\langle v^2 \rangle_\phi \approx 1$ to reflect the fact that $s$ is inherently not a heavy quark and $\phi$ is not truly a heavy quarkonium. Taking $m_b = 4.7 \pm 0.1$ GeV, varying the strong coupling constant between $\alpha_s(m_b) = 0.22$ and $\alpha_s(2m_b) = 0.18$, and including the experimental uncertainty in $\Gamma_{e^+e^-}$, we obtain

$$\text{Br}[\eta_b \to \phi\phi] = (0.9 - 1.4) \times 10^{-9}. \quad (27)$$

The interference effect is more pronounced in this case due to larger ratio of $m_b$ to $m_s$.

\(^{10}\) This process is the only possible one for $\eta_b$ decays to two ground charmonium states allowed by $C$ and $P$ invariance.
Neglecting QED contribution will decrease the branching fraction by about 20%. If this estimate is reliable, such a rare decay mode perhaps will never be observed experimentally.

It is interesting also to consider the similar decay process $\eta_c \to \phi\phi$, which has been measured long ago. Parallel to the preceding procedure, varying the strong coupling constant from $\alpha_s(m_c) = 0.36$ to $\alpha_s(2m_c) = 0.26$, taking $m_c = 1.5 \pm 0.1$ GeV, we obtain

$$\text{Br}[\eta_c \to \phi\phi] = (0.3 - 1.5) \times 10^{-5}. \quad (28)$$

The interference effect is rather modest, i.e., neglecting QED contribution only decreases the branching ratio by less than one per cent. This prediction is consistent with an early constituent quark model based analysis using Bethe-Salpeter bound state formalism.\footnote{The predictions of Ref. \cite{22}, which employ three different forms of BS wave functions for $V$, are duplicated in Table 1 of Ref. \cite{49}.}

Note this estimate is indeed much smaller than the measured value $\text{Br}_{\text{exp}}[\eta_c \to \phi\phi] = (2.7 \pm 0.9) \times 10^{-3}$, which reflects a generic symptom in pQCD calculation, also present in the $\eta_c$ decays to $\rho\rho$ and $K^*\overline{K}^*$.

As stressed several times before, some nonperturbative mechanisms must be called for to rescue this discrepancy. Among different proposals, a particularly attractive and predictive scheme has been put forward by Feldmann and Kroll (henceforth FK) \cite{23}, by generalizing their influential work together with Stech on $\eta - \eta'$ mixing in quark flavor basis \cite{50}, to include $\eta_c$. FK models the amplitude of $\eta_c \to VV$ as the product of a factor governing the small admixture amplitude between $\eta_c$ and $\eta^{(')}$, which presumably can be inferred from QCD $U_A(1)$ anomaly, times a soft vertex function parameterizing the amplitude for virtual $\eta, \eta'$ transiting into $VV$. To be specific, the $\eta_c \to \phi\phi$ in their ansatz can be described as

$$\Gamma_{\text{FK}}[\eta_c \to \phi\phi] = \frac{1}{32\pi M_{\eta_c}} \left(1 - \frac{4M_{\phi}^2}{M_{\eta_c}^2}\right)^{1/2} \left|c^{\text{mix}}_{\phi\phi} g^{\text{mix}}(\eta_c)\right|^2, \quad (29)$$

where the subscript FK is used to distinguish from our earlier prediction based on the pQCD plus constituent quark model ansatz, Eq. \cite{28}. The parameter $c^{\text{mix}}$ depends on the specific flavor content of $VV$ states but not on the initial charmonium state, whereas the charm mass dependence is encoded in

$$g^{\text{mix}}(\eta_c) = \frac{1}{f_{\eta_c}} F_{\eta^{(')}VV}(s = M_{\eta_c}^2) = \frac{1}{f_{\eta_c}} F_{\eta^{(')}VV}(s = 0) \left(\frac{\Lambda^2}{M_{\eta_c}^2 - \Lambda^2}\right)^n, \quad (30)$$
where \( f_{\eta_c} \) is the decay constant of \( \eta_c \), the second factor indicates the on-shell coupling \( \eta^{(0)} VV \), and the last factor parameterizes the \( s \)-dependence of this vertex function. \( \Lambda \) is the cutoff of typical size around 1 GeV, and the variable \( n = 1 \) corresponds to the familiar monopole ansatz for form factor, and \( n = 2 \) characterizes the dipole ansatz. One noteworthy fact is that, this simple model seems able to account for both the absolute and the relative strengths of partial widths for each decay channel (\( VV = \rho \rho, K^* K^* \) and \( \phi \phi \)), to a reasonably satisfactory degree.

One curious question is to ask, whether the FK mixing mechanism can be extrapolated to \( \eta_b \rightarrow VV \), and if so, whether the corresponding prediction to the branching ratios differs drastically from our pQCD-based estimate. Let us now examine this question. After some straightforward derivation, it is easy to find

\[
\frac{\text{Br}_{FK}[\eta_b \rightarrow \phi\phi]}{\text{Br}_{FK}[\eta_c \rightarrow \phi\phi]} \approx \left(1 - \frac{4M_{\phi}^2}{M_{\eta_c}^2}\right)^{-1/2} \left(\frac{f_{\eta_c} M_{\eta_c}^n}{f_{\eta_b} M_{\eta_b}^n}\right)^4 = 0.16 \times 0.01^n, \tag{31}
\]

where we have resorted to heavy quark spin symmetry for decay constants \( f_{\eta_c} \approx f_{J/\psi} \) and \( f_{\eta_b} \approx f_T \), and consequently taken \( f_{\eta_c} = 417 \text{ MeV} \) and \( f_{\eta_b} = 715 \text{ MeV} \) through the relation \( f_{J/\psi} = \sqrt{12/M_{J/\psi} \psi_{J/\psi}(0)} \). For simplicity, we have utilized the hierarchy \( m_b, m_c \gg \Lambda \) in deriving (31). If \( \text{Br}_{FK}[\eta_c \rightarrow \phi\phi] \) is identified with the measured value \( 2.7 \times 10^{-3} \), we then obtain \( \text{Br}_{FK}[\eta_b \rightarrow \phi\phi] \) to be about \( 4 \times 10^{-6} \) for \( n = 1 \) and \( 4 \times 10^{-8} \) for \( n = 2 \). It is easy to see that the \( \text{Br}_{FK}[\eta_b \rightarrow VV] \) scales as \( m_b^{-4(1+n)} v_b^{-6} \), since \( f_{\eta_b} \sim m_b v_b^{3/2} \). This indicates this nonperturbative mechanism, also resemble the pQCD analysis in that the branching ratio is highly suppressed with respect to “leading twist” scaling \( \propto 1/m_b^4 \). Despite large uncertainty, the predicted branching ratios of \( \eta_b \rightarrow VV \) in this mixing ansatz are (much) larger than our earlier predictions based on “NRQCD” plus constitute quark model, (27). It is reasonable to question the validity of the above nonperturbative mechanism at \( \eta_b \) energy, in our opinion, perhaps the pQCD result in (27) is more trustworthy. At any rate, it will be of interest for future experiments to distinguish these different mechanisms, though the task of recording this rare decay mode in hadron machine could be extremely challenging.

One related class of decay channels may deserve some attention. The helicity-conserving decay, such as \( \eta_c \rightarrow K_{||} \bar{K}_{||} (“||” \text{ implies longitudinally-polarized}) \), has still not been observed
yet. The experimental bound for this channel is $\text{Br}_{\text{exp}}[\eta_c \to K^*\bar{K} + \text{c.c.}] < 1.28\%$.  

\begin{equation}
(32)
\end{equation}

Since the coupling $\eta^{(0)}VP$ is negligible, it is reasonable to assume that the aforementioned nonperturbative mechanism responsible for $\eta_c \to VV$, may not play a significant role for $\eta_c \to VP$. One may then make use of the scaling behaviors derived earlier to interconnect each of them. Firstly we expect that flavor $SU(3)$ is respected in pQCD calculation of $\eta_c \to VV$, accordingly our prediction is $\text{Br}[\eta_c \to K^*\bar{K}] \sim \text{Br}[\eta_c \to \phi\phi] \sim 10^{-5}$. As we have learned from Sec. II at tree level, the maximal helicity violation in $\eta_c \to J/\psi J/\psi$ is entirely due to the charm quark transverse momentum, as is evident in (6). We assume the similar pattern occurring here, i.e., the branching ratios of $\eta_c \to VV$ are also suppressed by $(q_\perp/m_c)^4$ relative to those of the corresponding helicity-conserving channels. We then have

\begin{equation}
\text{Br}[\eta_c \to K^*\bar{K}] \sim \text{Br}[\eta_c \to \phi\phi] \left( \frac{m_c}{\Lambda_{\text{QCD}}} \right)^4 \epsilon_{SU(3)}^2 \sim 10^{-3} \times \epsilon_{SU(3)}^2, 
\end{equation}

where we have taken $q_\perp \sim \Lambda_{\text{QCD}} = 500$ MeV. We have intentionally included a parameter $\epsilon_{SU(3)}$, to embody the extent of $SU(3)$ flavor violation at the amplitude level. Obviously, this parameter should be proportional to current quark mass difference $m_s - m_{d,u}$, divided by some typical hadronic scale (presumably independent of $m_c$). In contrast to the isospin violation effect, the U-spin violating process should not receive too severe suppression. If assuming somewhat optimistic value $\epsilon_{SU(3)} \approx 0.3$, we then obtain $\text{Br}[\eta_c \to K^*\bar{K}] \sim 10^{-4}$, which is safely below the experimental bound, (32). Needless to say, it will be very beneficial for the ongoing high-luminosity charmonium facility such as BES III to impose more tight constraint on this branching ratio.

Carrying over the same line of argument to $\eta_b$ decay to $VP$, we obtain

\begin{equation}
(34)
\end{equation}

If we again take $\epsilon_{SU(3)}^2 \approx 0.1$, the branching ratio is estimated to be around $10^{-6}$, about one hundred times larger than $\text{Br}[\eta_b \to J/\psi J/\psi]$. Of course, one should not take this crude estimate too seriously, and a concrete pQCD calculation based on light-cone expansion approach, which incorporates $m_s$ and $m_d$ difference, might be illuminating. If this estimate

\footnote{Although this decay channel is favored by helicity selection rule, it violates the U-spin conservation.}
is trustworthy, one then expects there are already about $O(10^2)$ produced events in the Tevatron Run 1. Since $K^*$ almost exclusively decays to $K\pi$, one needs to select those resonant $K\overline{K}\pi$ events. However, there are practical problems about usefulness of this kind of decay mode in hadron collision experiments. Because of copiously produced kaons and pions in a typical hadron collision, huge combinatorial backgrounds might make it very difficult to identify the true signal. On the other hand, the prospective Super $B$ factory, which runs at several $\Upsilon(nS)$ resonances with an unprecedented luminosity \cite{51}, may produce an enormous amount of $\eta_b$ through $M1$ transition from $\Upsilon(nS)$ states. With much suppressed backgrounds, Super $B$ factory may offer a viable environment to detect this decay mode.

Finally we exploit one experimental fact that some 3-body channels, i.e., $\eta_c$ decays to three pseudo-scalar states, have exceedingly large branching ratios. To be concrete, three $\eta_c$ decay modes with largest branching ratios are \cite{18}

\begin{align}
\text{Br}_{\exp}[\eta_c \to K\overline{K}\pi] &= (7.0 \pm 1.2)\% . \quad (35) \\
\text{Br}_{\exp}[\eta_c \to \eta\pi\pi] &= (4.9 \pm 1.7)\% . \quad (36) \\
\text{Br}_{\exp}[\eta_c \to \eta'\pi\pi] &= (4.1 \pm 1.8)\% . \quad (37)
\end{align}

Although we do not know how to calculate these processes in practice, some general pattern may still be identified. Subtracting off the phase space effects, one finds these three amplitudes roughly respect the SU(3) flavor symmetry. This may be taken as the sign that these processes could in principle be described by the pQCD scheme. These processes proceed as two steps. Firstly $\eta_c$ annihilates to two highly virtual gluons, which then transit into four energetic light quarks. This is a short-distance process. Subsequently these light quarks materialize into three pseudo-scalars via soft nonpertubative dynamics, which is a long-distance process. Factorization is expected to hold between these two stages due to the hard scale set by large $c$ mass. Because of their large decay ratios, these processes are naturally expected to possess the “leading twist” scaling behaviors, $\text{Br} \sim 1/m_c^4$. This scaling assumption can then be used to infer the ratios for $\eta_b$ decays to the same pseudo-scalar states. For example, we may expect

\begin{equation}
\text{Br}[\eta_b \to K\overline{K}\pi] \sim \text{Br}_{\exp}[\eta_c \to K\overline{K}\pi] \left(\frac{m_c}{m_b}\right)^4 \sim 10^{-4} . \quad (38)
\end{equation}

This is by far the largest branching ratio we have found in all exclusive hadronic decay channels of $\eta_b$. If this is the case, these channels will be worth pursuing.
the $\eta\pi\pi$ mode may not be very useful in hadron hadron collider experiments, since the ordinary way of reconstructing $\eta$, which goes through the $2\gamma$ decay, may suffer enormous contamination from combinatorial background. For similar reason, the $K^+K^-\pi^0$ channel may also be difficult to detect due to presence of $\pi^0$. In contrast, the decay mode $K_S K^\pm \pi^\mp$ is much more advantageous since $K_S$ can be reconstructed cleanly via its decay to $\pi^+\pi^-$. In any event, all these decay channels have promising observation potential at future Super $B$ factory.

B. $\eta_b$ decays to two charmed mesons

Heavy $\eta_b$ mass opens the gate for it to decay into charmed particles as well as light ones. In fact, Maltoni and Polosa have suggested that $\eta_b$ decays to two charmed mesons could be the most promising channels for observing $\eta_b$ in Tevatron Run 2 [20]. They first perform a perturbative calculation for $\eta_b$ decay to the inclusive $c\bar{c}g$ state,

$$\text{Br}[\eta_b \rightarrow c\bar{c}g] = 1.5^{+0.8}_{-0.4}\%,$$  \hspace{1cm} (39)

then interpret this value as an upper limit for the ratios of the exclusive decays to double charmed mesons. MP continue to argue that, the exclusive decays to $D^*\overline{D}^{(*)}$ (the charge conjugate is implicitly implied) may saturate the inclusive charmeful decay ratio, and subsequently estimate $10^{-3} < \text{Br}[\eta_b \rightarrow D^*\overline{D}^{(*)}] < 10^{-2}$.

In our opinion, this saturation assumption seems to be physically unwarranted, consequently MP’s predictions for the ratios may well be an overestimate. Firstly, there seems no reason to expect that the parton process $\eta_b \rightarrow c\bar{c}g$ will be dominated by the two-body exclusive decays, since the $g$ jet may readily hadronize in an independent direction, which will result in a multi-body decay configurations. This can be exemplified by the fact we just mentioned, that $\eta_c$ decays to 3 pseudo-scalars have larger branching ratios than any 2-body decay channels. One may notice at the lowest order in $\alpha_s$, $\eta_b$ decay to double charm mesons is also depicted by Fig. 1 with one $c\bar{c}$ pair replaced by a $q\overline{q}$ pair. The gluon which is on shell in the inclusive $\eta_b \rightarrow c\bar{c}g$ process, now has to convert to a light quark pair with large invariant mass, so is highly virtual, the corresponding ratios of double charm mesons thus must be at least suppressed by one factor of $\alpha_s$ and one factor of $1/m_b^2$ relative to (39). Taking into account the nonperturbative binding probability, which is much less than 1, will
further suppress the exclusive 2-body decay rates. Moreover, there seems also no strong reason to believe that the binding of $c$ with $ar{q}$ will necessarily be saturated by the ground state charm meson only.

Despite the lack of an explicit pQCD calculation, one may still proceed with some physical consideration. The branching ratios of $\eta_b$ decays to two charm mesons can depend on three dimensional parameters: $m_b$, $m_c$ and $\Lambda_{\text{QCD}}$. Since the decay $\eta_b \to D^{\ast+}\bar{D}^{\ast-}$ conserves the helicity, we thus expect the corresponding branching fraction scales as $1/m_b^4$. For each pair of $c$ and $\bar{q}$ to form a $D^{(*)}$ meson, there is a factor proportional to $\Lambda_{\text{QCD}}/m_c$ which characterizes the binding probability \cite{45}. This is the only place where $\Lambda_{\text{QCD}}$ dependence can enter. Therefore, we expect the following asymptotic behavior:

$$\text{Br}[\eta_b \to D^{\ast+}\bar{D}^{\ast-}] \sim \alpha_s^2 \left(\frac{\Lambda_{\text{QCD}}^2 m_c^2}{m_b^4}\right) \sim 10^{-5}, \quad (40)$$

where $m_c^2$ is inserted to make the ratio dimensionless. Notice this value is reasonably compatible with the inclusive upper bound, \cite{39} \cite{13}.

We next turn to the other charmful decay channel, $\eta_b \to D^{\ast+}\bar{D}^{\ast-}$ ($\eta_b \to D\bar{D}$ is forbidden by $P$ invariance), which violates the helicity selection rule maximally. Assuming again the helicity violation is caused by the transverse momentum of light quark in $D$ meson, the same as in $\eta_b \to J/\psi J/\psi$, we then obtain

$$\text{Br}[\eta_b \to D^{\ast+}\bar{D}^{\ast-}] \sim \text{Br}[\eta_b \to D^{\ast+}\bar{D}^{\ast-}] \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^4 \sim \alpha_s^2 \left(\frac{\Lambda_{\text{QCD}}^6 m_c^2}{m_b^8}\right) \sim 10^{-8}, \quad (41)$$

where $q_\perp \sim \Lambda_{\text{QCD}} = 500$ MeV is used. Rather small branching ratio renders this decay mode not so useful for detecting $\eta_b$. Nevertheless, it might be possible that a large prefactor may arise in the actual calculation, like what occurs in $\eta_b \to J/\psi J/\psi$ (see footnote \cite{9}), so that the actual ratio may be somewhat larger than this naive estimate. In any event, a reliable pQCD calculation of the decay rates for $\eta_b \to D^{\ast+}\bar{D}^{\ast-}$ will be helpful.

Based on the previous discussions about 3-body decay of $\eta_b$, one may wish that the branching ratio for $\eta_b \to D\bar{D}\pi$ might be as large as $10^{-4}$, similar to that for $\eta_b \to K\bar{K}\pi$. However, this decay mode may not be as competitive as $K_S K^\pm \pi^\mp$ mode, since $D$ meson does not possess a clean signature comparable with $K_S$.

\footnote{However, as will be reported in Ref. \cite{46}, the decay rate of this process vanishes if $D$ and $D^*$ are exactly degenerate. Therefore, the non-vanishing decay amplitude must be induced by heavy-quark-spin-symmetry violating effects, $\propto \Lambda_{\text{QCD}}/m_c$. This indicates the estimate \cite{40} may well be subject to further suppression.}
To summarize, the $\eta_b \to D^+D^-$ channel, with an expected branching ratio $\sim 10^{-5}$, may be regarded as a valuable searching mode. The $D^{\ast 0}D^0$ channel (charge conjugate state implicitly included) may not be so useful, since $D^{\ast 0}$ predominantly decays to $D^0\pi^0$ and $D^0\gamma$, where neither $\pi^0$ nor $\gamma$ can be cleanly tagged in hadron collision environment. In contrast, it is more advantageous to use $D^{\ast +}D^-$ mode as a trigger. $D^{\ast +}$ can be tagged through its decay to $D^0\pi^+$, subsequently $D^0$ may be reconstructed from $K^-\pi^+$, while $D^-$ can be tagged through its decay to $K^+\pi^-\pi^-$. It is worth emphasizing that due to the proximity of $D^{\ast +}$ mass to the sum of masses of $D^0$ and $\pi^+$, $D^{\ast +}$ can be cleanly identified with a rather narrow peak in $D\pi$ invariant mass spectrum. Using the measured branching ratios $\text{Br}[D^{\ast +} \to D^0\pi^+] \approx 70\%$, $\text{Br}[D^0 \to K^-\pi^+] \approx 4\%$ and $\text{Br}[D^- \to K^+\pi^-\pi^-] \approx 10\%$ [18], we estimate $\text{Br}[\eta_b \to D^{\ast +}D^- \to K^+K^-\pi^+\pi^-\pi^+\pi^-] \sim 10^{-8}$. There are roughly $O(1)$ produced events in Tevatron Run 1, about $O(10^2)$ produced events in Run 2, and about $O(10^4)$ produced events in one year run at LHC. The statistics seems to be enough in the forthcoming hadron collider program for observing $\eta_b$ through this decay mode, provided that the signal events are not swallowed by the possibly large combinatorial backgrounds.

VI. SUMMARY AND OUTLOOK

The major motif of this work is to clarify a controversy about whether double $J/\psi$ can be a useful decay mode to detect $\eta_b$ in Tevatron Run 2. We have shown this process is subject to large kinematical suppression due to the maximal violation of the helicity selection rule. By an explicit pQCD calculation based on NRQCD, we predict the corresponding branching ratio to be only of order $10^{-8}$, thus making the search for $\eta_b$ through this mode rather unrealistic in Tevatron Run 2. Nevertheless, we anticipate that at LHC, the observation potential of this decay mode may not be so pessimistic, if experimentalists can find a good way to suppress the rather copious QCD background events.

The large mass of $\eta_b$ renders any of its exclusive decay channels in general very small. To provide some useful guidance for experimental search for this elusive particle, it is valuable to identify those decay modes with relatively large branching fractions and also with clean signature. To make progress along this direction, in the following we outline some issues which we think may deserve further studies:

1. Stimulated by rather large QCD radiative correction to exclusive double charm...
production at $B$ factory $^{30,31}$, one may inquire how large the effect of the next-to-leading order QCD correction to $\eta_b \rightarrow J/\psi J/\psi$ is. It turns out that, at NLO in $\alpha_s$ for this process, the helicity selection rule can be violated by finite charm mass, consequently one obtains the non-vanishing result even at the the leading order in velocity expansion $^{32}$. At the amplitude level, the ratio of the radiative correction piece to the relativistic correction piece as considered in this work, $^{20}$, is about $\frac{\alpha_s}{\pi}: \langle v^2 \rangle_{J/\psi} \sim \mathcal{O}(1)$, which implies both effects are equally important, and the more accurate prediction will crucially depend on their relative phase. To this end, a precise determination of the quantity $\langle v^2 \rangle_{J/\psi}$, in particular its sign, would be helpful.

2. The process $\eta_b \rightarrow K^* K + c.c.$, favored by the helicity selection rule but at the same time violating U-spin symmetry, is estimated to have a branching fraction $\sim 10^{-6}$-$10^{-7}$. It might be worthwhile if an actual pQCD calculation which implements $m_s - m_d$ difference, can be carried out in the light cone expansion scheme, to compare with this rough estimate. Moreover, the eventual experimental sighting of $\eta_c \rightarrow K^* K$ in charmonium factory like BES III will definitely enrich our understanding toward this class of helicity-conserving yet flavor SU(3) violating quarkonium decay processes.

3. The individual decay modes of $\eta_c$ with largest branching ratios are $\eta_c \rightarrow K\overline{K}\pi, \eta\pi\pi$ and $\eta'\pi\pi$. Stimulated by this experimental fact, one may hope that the 3-body decay channels of $\eta_b$, such as $K_S K^\pm \pi^\mp$, with an expected branching ratio of order $10^{-4}$, might be a potentially useful searching mode for $\eta_b$ in current and forthcoming hadron collider programs, if it can survive from the copious combinatorial background events. This mode will definitely have promising potential to be observed in the prospective Super $B$ factory.

4. Another helicity-conserving decay process, $\eta_b \rightarrow D^* D + c.c.$, with an expected branching ratio of order $10^{-5}$, may also be well worth searching for experimentally. It is also profitable to carry out a concrete calculation of this exclusive double charm decay process from pQCD scheme, but one may be obliged to incorporate the heavy-quark-spin-symmetry violating effect $^{46}$.

5. In this work we haven’t discussed the possibility of observing $\eta_b$ through its decay to baryon pair, such as $\eta_b \rightarrow p\overline{p}$. If the corresponding ratio is not too small, this
is potentially a good searching mode thanks to relatively fewer baryonic background events in hadron collision experiments \(^{14}\). Experimentally \(\eta_c \to p\bar{p}\) is observed to have a branching ratio of order \(10^{-3}\) \(^{18}\). However, one should be aware that this process also violates the helicity selection rule, and the available pQCD prediction, when taking into account the nonzero light quark mass but still in the collinear quark configuration, is still far smaller than the measured value \(^{53}\), therefore some nonperturbative mechanism needs to be called for to explain this discrepancy \(^{23}\). Because of rather heavy mass of \(\eta_b\), one may hope pQCD framework should provide a reliable prediction for \(\eta_b \to pp\). It will be valuable if a more systematic calculation can be carried out.

**Note added.** While the revised version of the manuscript is to be submitted, there appears an eprint in arXiv by BABAR collaboration \(^{54}\), which claims the first unambiguous discovery of \(\eta_b\) through the hindered M1 transition process \(\Upsilon(3S) \to \eta_b \gamma\). It is interesting to note the rather precisely measured \(\eta_b\) mass, \(9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})\) MeV, seems not compatible with the predictions from most potential models as well as the weakly-coupled potential NRQCD, instead consistent with the lattice QCD prediction within error \(^{11}\).

**Acknowledgment.** I wish to thank Augustine Chen, Tao Huang, Chang-Zheng Yuan, Qiang Zhao and Bing-Song Zou for helpful discussions, Yeo-Yie Charing, Bin Gong and Jian-Xiong Wang for enjoyable collaborations on topics closely related to this work. This research was partially supported by National Natural Science Foundation of China under Grant No. 10605031.

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\(^{14}\) I thank C. Z. Yuan for suggesting this possibility.
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