From Notes to Chords in QCD

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After a very brief overview recollecting the ‘classic’ parts of QCD, that is its application to describe hard processes and static properties of hadrons, I survey recent work – some very recent – on QCD at non-zero temperature and density. At finite temperature and zero density there is a compelling theoretical framework allowing us to predict highly specific, non-trivial dependence of the phase structure on the number of flavors and colors. Several aspects have been rigorously, and successfully, tested against massive numerical realizations of the microscopic theory. The theoretical description of high density is nowhere near as mature, but some intriguing possibilities have been put forward. The color/flavor locked state recently proposed for three flavors has many remarkable features connected to its basic symmetry structure, notably including chiral symmetry re-breaking and the existence (unlike for two flavors) of a gauge invariant order parameter. I survey potential applications to heavy ion collisions, astrophysics, and cosmology. A noteworthy possibility is that stellar explosions are powered by release of QCD latent heat.

One main goal of physics is to determine the fundamental laws, where “fundamental” is taken in the strict reductionist sense, that is, laws incapable of being derived from other, more universal principles. I believe it is profoundly wrong, however, to portray this as the only goal, or even necessarily as the most important goal. It is as if after playing each note of the piano once each, you were to say “and all the rest is just combinations”. True enough – but somehow misleading. In QCD we first played the basic notes twenty-five years ago, and certainly by fifteen years ago it was clear to most reasonable people that there were no more notes to be found. But it would be quite foolish to see this as the end of the subject. Indeed, several interesting, attractive chords have been discovered already, and large parts of the keyboard have barely been touched.

1. Pure Tones at High Frequency: Jets, Running of Couplings, Proton Gluonization

The fundamental structures of QCD, both as regards its degrees of freedom and their dynamics, are laid bare in the phenomena of jets. Asymptotic freedom says that radiation events involving large changes in energy and momentum are rare. Thus when highly
energetic quarks and antiquarks are produced, say following $e^+e^-$ annihilation at high energy, including the important special case of $Z$ production, they imprint their pattern of energy and momentum flow. Accordingly one expects that soft radiation events can rearrange the particle types, and of course ensure that the quarks, antiquarks, and gluons materialize as hadrons, but that the underlying energy-momentum structure remains visible, now as jets of hadrons instead of individual quarks. Hard radiation events, although relatively rare, do happen, and in that case one has three jets, or, still more rarely, four or more. All aspects of these events – the relative cross-sections for different numbers of jets, how this varies with energies, and the angle and energy distributions – can be calculated directly from the microscopic theory. In this way, the precise structure of the fundamental quark-gluon and gluon-gluon couplings has been rigorously, and successfully, tested in detail.

Jets provide perhaps the most striking and direct demonstrations of the Yang-Mills structure of QCD and of asymptotic freedom, but these were by no means the first test of the theory, nor are they the most accurate. A very wide variety of experiments at various energy scales has been used to test the theory [1]. In particular, the predicted running of the coupling has been convincingly observed (Figure 1).

![Figure 1](image-url)

Figure 1. Comparison of theory and experiment in QCD, illustrating the running of couplings. Several of the points on this curve represent hundreds of independent measurements, any one of which might have falsified the theory. Figure from M. Schmelling, hep-ex/9701002.

Before leaving this subject I would like to especially mention one of the most dramatic,
and to me most gratifying, experimental verifications of QCD. This is the rise in the nucleon structure functions at small x, now clearly observed at HERA [2]. These large-factor 2 or more—violations of naive scaling were among the very first consequences derived from asymptotic freedom [3]. Regrettably, some of the early analysis of this data got enmeshed with murky speculations about hard pomerons and presented as a duel between inappropriate acronyms. Further work has made it clear that the original ideas and calculations were right on the money.

2. Pure Tones at Low Frequency

Since we will be considering the effects of having different numbers of flavors, there is one additional basic concept worth bearing in mind. It is quite far removed from realistic phenomenology, but might provide us with a nice theoretical laboratory for special questions, as I will discuss further below. For large enough numbers of quarks the coupling never gets strong, and one can calculate the behavior both at small and at large distances very simply. In addition to the usual ultraviolet stable zero of the $\beta$ function at zero coupling, there is an infrared stable zero at non-vanishing coupling. When the number of massless quarks is close to 16 this second zero occurs at a small value of the coupling, and one can use perturbation theory at all scales [4]. The theory becomes scale invariant and free at short distances, and scale invariant again with non-trivial exponents at long distances. There is no chiral symmetry breaking and no mass gap. The same qualitative pattern might persist to smaller numbers of quarks, as in supersymmetric versions of QCD [5], but the infrared fixed point will move to strong coupling, making accurate quantitative calculation much more difficult.

3. Hadrons

For more realistic quark spectra, and in the real world, the coupling becomes large, and a mass gap appears, at some characteristic scale usually denoted $\Lambda_{\text{QCD}}$. Of course, there can be a few Nambu-Goldstone modes appearing below this scale.

As you can see from Figure 1, the effective coupling has a focusing property: for a wide range of $\Lambda_{\text{QCD}}$, one obtains very nearly the same value of the effective coupling at $M_Z$. This realizes, in the context of QCD, Pauli’s dream of calculating the fine structure constant. Concretely, it means that one has formulated a wealth of quantitative predictions for physical phenomena, in which no material parameters whatsoever appear.

A clearer view of this remarkable aspect of QCD appears if we consider the idealized limiting theory containing just two massless quarks (the idealized $u$ and $d$). On the one hand, this idealization provides a reasonably accurate representation of the “everyday world” of nuclear physics and of the physics of non-strange resonances, probably at the 10% level, apart from those few phenomena that depend sensitively on the pion masses.

On the other hand, it is a theory whose equations, when expressed in units with $\hbar = c = 1$, contain no free parameters at all, apart from the pure numbers 3 (colors) and 2 (flavors). Classically there is a gauge coupling $g$ that appears to be an arbitrary number, but the running of the coupling implies that the physical coupling changes with mass or distance scale, and we can work with any value we like, by choosing the scale appropriately. This is the phenomenon of dimensional transmutation. Thus everything is fixed and calculable...
in terms of the units $\hbar$, $c$, and the whole numbers 3 and 2, except for the overall scale of mass, which in any case has no intrinsic meaning for QCD as such. QCD is a theory Pythagoras ("All things are numbers") would have loved.

I feel it is vitally important for a scientific theory to have algorithms in hand to deliver some concrete returns on its abstract promises. Without such algorithms, the promises are hollow. Fortunately, the asymptotic freedom of QCD allows us to discretize this theory in a controlled manner. By putting the fields on a lattice one is throwing out the high-frequency modes, but these modes are weakly coupled, so one can assess their residual effects accurately and approach the limit of vanishing lattice spacing with confidence.

After years of hard work, much of it pushing the frontiers of computing technology, heroic practitioners of numerical QCD have not only made it clear that the microscopic theory generates the qualitative phenomenology of confinement and chiral symmetry breaking, but also produced a convincing approximation to the low-lying spectrum, at the 10% level.

Much work remains to be done in lattice gauge theory, to improve the accuracy of results for static quantities and to calculate matrix elements of interest for analyzing weak hadronic processes. In addition, there are challenges of a more conceptual sort: Can one incorporate manifest chiral symmetry in a usable non-perturbative algorithm? Can one isolate appropriate degrees of freedom to obtain a semi-microscopic, more easily tractable version of QCD? There are some promising ideas for both classes of problems – notably domain wall fermions, and the instanton or monopole-instanton liquid – that need to be pushed harder. Finally and perhaps most important, there are questions that touch fundamental phenomenological issues: What is the mass of the H particle? Are there stable or metastable strangelets? Is it possible that the mass of the $u$ quark vanishes, solving the strong P, T problem? In principle QCD is adequate to address these issues, but they remain open due to our weakness in calculation.

4. Finite Temperature

As a heuristic principle, asymptotic freedom leads us to expect that at high temperature one should approach the behavior of the free theory. In particular, confinement should no longer apply and chiral symmetry should no longer be broken. These qualitative changes entail, in favorable cases, strict phase transitions.

It is instructive first to consider the pure glue theory, both for $SU(2)$ and $SU(3)$. In the pure glue theory there is a strict criterion for confinement, namely the vanishing of the Polyakov loop. This loop represents the effect of inserting a source of non-zero duality (for $SU(2)$) or triality (for $SU(3)$). Unbroken symmetry under the global transformations associated with the center of the gauge group, $Z_2$ or $Z_3$ respectively, requires the expectation value of the Polyakov loop to vanish. This can be interpreted, alternatively, as a sign that the flux associated with the source cannot be screened. Its influence extends to infinity, and it produces a disturbance costing a finite energy per unit volume over an infinite volume, hence an infinitely massive state. At high temperature there will be screening, the Polyakov loop will not vanish, and the global center symmetry will be violated. Since there is a change of symmetry, there must be a sharp phase transition somewhere between zero and infinite temperature.
The next standard question to ask is the order of the phase transition. A powerful approach to this problem is to consider whether it is possible for the transition to be second order. The singularities at second order transitions are supposed to be governed by scale invariant theories at the critical point, and to have a universal character. Thus one can analyze much simpler models than QCD, that have the appropriate symmetries, while obtaining results that apply quite rigorously to QCD. The long-wavelength modes, that alone are important for the singular critical behavior, should not depend on the microscopic details.

For $SU(2)$, with its center $Z_2$, an appropriate model is the three dimensional Ising model \cite{12}. It has a second order transition, with critical exponents that have been accurately calculated. The critical behavior of pure glue $SU(2)$ has been calculated quite accurately. The critical exponents agree with those of the 3 dimensional Ising model, within 1-2%. This is a remarkable example of universality \cite{13}.

For $SU(3)$, with its center $Z_3$, an appropriate model is the 3-state Potts model. The cubic coupling allowed in this universality class inevitably leads to a first-order transition. Thus pure glue $SU(3)$ is predicted to \cite{12}, and does \cite{13}, have a first-order deconfining transition.

For the more realistic case of gauge theory with dynamical quarks, there is no strict symmetry associated with confinement; at any non-zero temperature, the quarks will screen triality, so that the Polyakov loop will never strictly vanish and the global central gauge symmetry is always broken. However if we have more than one massless quark species there are non-trivial chiral symmetries, and we can apply the same style of analysis.

For two flavors the chiral symmetry is $SU(2)_L \times SU(2)_R$, which is broken at zero temperature to the diagonal $SU(2)_{L+R}$. In isomorphic but more suggestive notation the breaking is $SO(4) \rightarrow SO(3)$, and we recognize the universality class of a four-component magnet. This universality class has been closely studied both analytically and numerically. It has a second order transition, with accurately determined exponents and critical equation of state. Using this information, one can test the hypothesis that two massless flavor QCD is in the universality class $SO(4)$. In very beautiful work, Iwasaki, Kanaya, Kaya, and Yoshie \cite{15} have, to my mind quite convincingly, demonstrated that it is. They were able, for example, clearly to distinguish a fit to their data using the $SO(4)$ exponents ($\chi^2/df = .72$) from one using mean field exponents ($\chi^2/df \geq 3.3$).

For three flavors the chiral symmetry is $SU(3)_L \times SU(3)_R$, which is broken at zero temperature to the diagonal $SU(3)_{L+R}$. The simplest model in this universality requires a 3x3 matrix of scalar fields, and has two independent quartic couplings. The infrared fixed point present in the mean field theory is unstable when fluctuations are considered. Thus there is no candidate model for a second order transition with the appropriate symmetry, and one predicts a fluctuation-driven first order transition \cite{14}. Numerical experiments with three flavors of massless quarks confirm this expectation \cite{16}.

For more flavors one also expects a first order restoration of chiral symmetry, if it was broken at zero temperature, for the same reason. This is also consistent with the numerical experiments.

Since two and three massless flavors give such different pictures, and it is not clear \textit{a priori} which is a better approximation to reality, it becomes important to consider how the two pictures might morph from one to the other. Concretely, one can imagine starting
with vanishing strange quark mass, and cranking it up. At the beginning of this process we have a first order transition, and at the end a second order transition. The simplest possibility is that the transitions are continuously connected. The switch happens at a tricritical point, which has universal properties that can be rigorously characterized \[17\]. Available numerical data, although sparse, is consistent with this picture. Another question is the nature of the transition, that is first or second order, for the physical value of the strange quark mass. Different groups studying this question have reached opposite conclusions \[18,16\], which may indicate that reality is close to tricritical!

Of course in reality the $u$ and $d$ quarks are not strictly massless. This will round the second order transition into a crossover, and change the tricritical point into an Ising-like transition. However insofar as the light quark masses are small it will be fruitful to regard them as perturbations within the more symmetric framework, joining onto a small region very near the critical point where they dominate.

There are many other important issues that can be addressed in finite temperature QCD, besides the universal features of the phase transitions. Let me just mention the simplest one, the transition temperature. First of all, one finds that the ratio of string tension to transition temperature is markedly different for the pure glue and dynamical quark theories, presumably reflecting the fundamentally different character of the light degrees of freedom, and of the transition, in these two cases, despite the fact that the gluons overwhelmingly dominate the short-distance dynamics. The actual value of the critical temperature is of course very important for phenomenology. For the dynamical quark theory, it appears to lie within the range $T_c \sim 150$-200 Mev \[19\]. It is striking that this value is so small. Just below the transition one has essentially only pions – three degrees of freedom; while above, if one imagines a free phase, there are fifty-two (eight gluons with two helicities each, plus three flavors of quarks each with three colors and two helicities, and their antiparticles)!

5. Finite Density

We can form reasonable expectations for the behavior of QCD at very high densities. At high density – assuming weak coupling – the quarks will form a large Fermi surface. We want to see if this trial state, which is embedded in a nearby continuum, is stable. We are doing nearly degenerate perturbation theory, involving small numbers of particle-hole excitations near the Fermi surface. Since other possibilities are blocked by the exclusion principle, the leading interactions are elastic scatterings among these particles and holes. Because the momenta are large, such scatterings, over the bulk of possible angles, involve large momentum transfers. By asymptotic freedom, they are characterized by weak coupling, so our starting point is self consistent. Singular infrared behavior is eliminated by screening, so I suspect that the neglected “small angle” contributions are truly negligible asymptotically, and the rough argument sketched here can be made fully rigorous, but this has not been attempted. In any case for physical purposes we are not going to be satisfied with asymptotics, and we shall have to dirty our hands.

So what is it reasonable to expect? Ordinary chiral symmetry breaking, as seen at zero density, is a quark-antiquark pairing. It will become ever more difficult to maintain at high density, because the ‘quark’ parts of the pairs for low momentum are definitely
occupied in the Fermi sea, and their number cannot vary, so there is no possibility for correlated pairs. And at high momentum, the quark-antiquark pairs have large energy. On the other hand, there are attractive quark-quark channels. Indeed, a quark pair in a color antitriplet has reduced its flux, and should be lower in energy than the two quarks separately. The energy denominators in this channel vanish near the Fermi surface. Since a BCS-type instability can be triggered by an arbitrarily weak interaction at the Fermi surface, we expect pairing condensations of some sort in the quark-quark channel [20].

A fundamental question for QCD is the nature of the transition that restores chiral symmetry. It is very attractive, on general physical grounds, to believe that it is first order. To see one reason why, imagine setting up a medium of uniform small density with chiral symmetry broken. If the chiral symmetry restoration is first order, this medium will be mechanically unstable. It will break up into regions of non-zero density wherein chiral symmetry is restored, and regions of zero density where chiral symmetry is broken. This seems very reasonable, and indeed this picture could be used to motivate, and provide a rigorous approach to, something like the MIT bag model. On the other hand if the uniform medium were stable – what could it be? I don’t know of any candidates in the physical world. Numerical work based on effective instanton interactions, in agreement with some but not all other approaches, supports the idea of a first-order transition [21].

Now let us consider what’s on the far side of the transition, asymptotically at very large density, keeping in mind that there could be one or more intervening transitions (e.g. K condensation) near nuclear density. Once again, it is important (and instructive) to consider different numbers of flavors.

For two flavors, triplet pairing of the form

\[ \langle q^\alpha_a(p) C\gamma_5 q^{\beta}(-p) \rangle = \kappa(p^2) \epsilon^{\alpha\beta3} \epsilon_{ab} \]  

is possible [21] [22]. It puts the affected quarks (colors 1 and 2) in a color antitriplet, space-time scalar s-wave condensate. We might expect it to be especially favorable because it maintains so much symmetry, so that the interaction of a given pair can receive coherent contributions from other pairs involving different particle combinations all over the various Fermi surfaces. Indeed, it breaks the original color \( SU(3) \) down to \( SU(2) \) while leaving chiral \( SU(2)_L \times SU(2)_R \) intact. It might seem that the \( U(1) \) of baryon number is violated, but actually there is a combination of the original baryon number and color hypercharge which leaves the condensate invariant, so a modified global \( U(1) \) persists. Estimates of the energetics using effective instanton interactions make it plausible that substantial gaps, of order 100 Mev or more, can be generated at moderately high chemical potentials, corresponding to densities a few times nuclear. The corresponding critical temperatures range from several tens to perhaps 100 Mev [23].

This pairing leaves quarks of the third color untouched, and one can ask whether the residual ungapped Fermi surface is stable. In our crude effective instanton model [21] we found that there could be an additional condensation of the form

\[ \langle q^3_a(p) C\sigma_{0i} q^3_b(-p) \rangle = \eta(p^2) \hat{p}_i \delta_{ab} . \]  

This violates rotation symmetry. Condensation in this channel is not very robust and gave us gaps of order several keV at best.
For three flavors, there is what I think is a much more beautiful and compelling possibility for condensation \[24\], of the form

\[
\langle q_\alpha^a(p) C \gamma_5 q_\beta^b(-p) \rangle = \kappa_1(p^2) \delta_\alpha^a \delta_\beta^b + \kappa_2(p^2) \delta_\alpha^b \delta_\beta^a.
\]

This can be written so it appears more obviously as a generalization of the 2 flavor condensation, by noting that if \( \kappa_1 = -\kappa_2 \) then the color-flavor structure is proportional to \( e^{\alpha \beta I} e_{abI} \), summed over \( I \). It is reminiscent of the B phase of liquid helium 3, where now color and flavor rather than nuclear and orbital spin become locked together. We have found, in an effective model abstracted from one-gluon exchange, that this condensation leads to very substantial gaps, easily of order 100 Mev or more.

A state with this condensation has many remarkable features:

The symmetry \( SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_{L+R} \) of color times chiral \( SU(3) \) times baryon number breaks down to the diagonal \( SU(3) \).

In particular, chiral symmetry is broken by a mechanism that is qualitatively new. That is, the left-handed quarks lock their flavor quantum numbers to color and so, essentially independently, do the right-handed quarks. But because the kinetic terms in the Hamiltonian are only invariant under vectorial color transformations, left and right chiral symmetries become locked to each other, indirectly.

The \( U(1) \) of baryon number is genuinely broken. This leads to superfluidity, as in liquid helium. Since baryon number is – for present purposes – an exact symmetry of the Hamiltonian, its spontaneous violation is associated with a true Nambu-Goldstone mode, with a linear dispersion relation at small momenta (but velocity equal to the speed of zero sound, close to \( c/\sqrt{3} \)).

There are gaps in all single-particle channels, \( i.e. \) everywhere around all the Fermi surfaces.

There are approximate Nambu-Goldstone modes associated with the spontaneous chiral symmetry breaking. When quark masses are included, these modes acquire small gaps too. Then all the charged modes, both for weak and electromagnetic interactions – but not the neutral one mentioned above – have gaps.

All the gluons acquire a mass, essentially by the Higgs mechanism.

Although ordinary electromagnetic gauge invariance is broken, there is a combination of this transformation and color hypercharge that leaves the condensate invariant. Under this transformation, all the elementary excitations – quarks, gluons, and collective modes – carry integer charge (yes, in multiples of the electron charge)! Furthermore since there is a gap to all charged excitations, this ultra-dense material is transparent, like diamond.

It is a very interesting enterprise, to derive the Landau-Ginzburg effective description of the low energy excitations around this state. By the way it also supports Skyrme-type textures, which carry baryon number (measured by the effective coupling of the Nambu-Goldstone mode!) equal to 2.

For very large numbers of flavors, I am not sure what happens – perhaps multiple copies of the color-flavor locked state for groups of three quarks, with some fudge for any residuals. In any case when the number gets close to 16 it is a truly weak coupling problem at any finite density, and one should be able to make progress analytically.

In this discussion I have been rather cavalier in dealing with gauge-dependent quantities. The same difficulty arose, and was faced down, in BCS theory \[25\]. In the pairing wave
function, taken at face value, charge is not conserved, whereas of course in reality it had better be. This formal problem can be ameliorated by projecting on a state of definite number, by doing a Fourier transform on the conjugate phase variable $\theta$. This operation will have little practical effect, in a large system, because states with significantly different values of $\theta$ have very little overlap, nor do local operators constructed from reasonable numbers of field operators connect them. The important physical phenomenon captured in the pairing wave function is the correlation among however many pairs do exist, and this is not affected by global constraints on the (large) number of pairs. The same remarks apply, *mutatis mutandis*, for color superconductivity.

As a consequence of the projection on color singlets, however, naive attempts to characterize ordered states using formally defined order parameters that are not gauge invariant will fail. The primary stage of our two flavor diquark pairing apparently does not support any gauge invariant order parameter. In this respect it resembles the electroweak sector of the Standard Model. As in that case, the logical possibility of a crossover rather than a sharp transition to nearby states with the same symmetry but qualitatively different physical behavior thereby arises.

On the other hand, there are gauge invariant order parameters both for the complete (two-stage) two flavor condensation and, importantly, for the three flavor color-flavor locking condensation. In the latter case, an appropriate gauge-invariant order parameter is the expectation value of a six-quark operator of the type $(qqq)^2$, which captures the spontaneous $U(1)$ violation. States supporting these condensations must therefore be separated from the surrounding states, having different symmetry, by sharp phase transitions.

With so many interesting possibilities being suggested for various versions of QCD at finite density, it would be very desirable to have the kind of definitive checks that only numerical realization of the theories can provide. Unfortunately, there are very great difficulties with numerical work on QCD at finite density, which have for many years now prevented the subject from getting off the ground. The fundamental problem is that the Fermion determinants appearing in the functional integral measure for the grand canonical ensemble are not positive definite configuration by configuration. Thus importance sampling is stymied. One can use the measure associated with zero chemical potential, and evaluate the expectation value including chemical potentials, but this has been found to converge very (exponentially) slowly due to cancellations of many unwanted zero density contributions, which are much larger in absolute size than the residual terms of interest. The deeply nasty character of this problem has only been appreciated recently.

One can avoid the problem in various ways. One class of ideas is to work with models that don’t suffer from it, but still do share some features with QCD. For example with two colors the problem doesn’t arise. Using two colors is far from innocuous in the context of finite density QCD, because it makes the baryons into bosons, but one might hope to learn something. Or one can have a chemical potential which is positive for some quarks and negative for an equal number of others. This will favor production of mesons with appropriate quantum numbers, as well as baryons (and antibaryons), so it too is far from innocuous. Or one can look at models which have qualitative features in common with QCD but do not suffer from the sign problem – the 2+1 dimensional Gross-Neveu model is a good candidate. Or one can use an imaginary chemical
potential. This I think is a particularly interesting possibility, and I’d just like to say a few words about when it is likely to be useful, since the considerations apply more generally. An imaginary chemical potential does not systematically bias the ensemble to large density, so that to pick out the effect of states of non-zero baryon number density one must rely on fluctuations (which, when they occur, are appropriately weighted by the action). These fluctuations will occur most readily when the temperature is high, and the gap in the baryon number channel is small. And even then one can only realistically hope, on the small lattices likely to be practical, to fluctuate to a few baryons. So a reasonable procedure would seem to be to start with a high temperature and work down, looking for qualitative changes as a function of temperature. In this way, one could realistically hope to study how properties of baryons are affected by the presence of a thermal meson medium, and in particular, the transition from a loose association of almost massless quarks to something resembling a normal baryon.

Another possibility is to work with large numbers of quark species, close to 16. Then there is no mass gap to baryons, so fluctuations are cheap, and also the contribution of interest, due to the quarks, is not swamped by gluons. So the cancellations are probably not so bad even at real chemical potential, and the imaginary chemical potential approach should also work better (since the fluctuations will be cheap).

Still, these tricks are not going to help us directly on realistic problems. For those, new ideas of a more profound character seem to be required. Perhaps the condensed matter theorists, who have been wrestling with related sign problems increasingly successfully recently, have something to teach us here.

6. Phenomenology

Finally I’d like to discuss some potential phenomenology associated with QCD in extreme conditions.

There is a very large literature concerned with various ways of establishing, phenomenologically, that a shift from hadronic to quark-gluon degrees of freedom as the appropriate description of strong interaction physics takes place under conditions of high effective temperature. One good idea in this field, which has already received significant experimental support, is that the liberation of gluons from hadrons hardens their distribution function, permitting them to dissociate \(J/\psi\) particles more readily. Closely related to this is the suggested phenomenon of hard jet suppression. Also impressive, though less clear-cut, are suggestions of strangeness and entropy enhancement in high energy nuclear collisions, already visible in existing data. I think there is little doubt that by correlating various signals of these and other sorts one will be able to construct a convincing, richly detailed case that at high temperatures the description of strongly interacting matter as semi-free quarks and gluons is appropriate and fruitful.

Within the domain of heavy ion physics, I’d like to advertise three specific phenomena that could provide signatures and guidance for elucidating more subtle, collective aspects of the behavior.

**DCC and anomaly kick:** The idea that as the chiral condensate first melts and then relaxes it puts excess power into long-wavelength modes has been developed over the past several years, and there are excellent reviews available. Recently a promising new idea
was introduced by Asakawa, Minakata, and Muller [35], which apparently makes some associated phenomenology much more robust and accessible than it previously appeared. This is the idea of the anomaly kick. The Adler-Bell-Jackiw anomaly gives a coupling of the $\pi^o$ to the product of electric and magnetic fields, and in a heavy ion collision large coherent electric and magnetic fields are present, simply due to the charge and motion of the nuclei. By including this term in the effective field theory, one seeds the long-wavelength modes in the $\pi^o$ direction. If these are then amplified in the predicted manner, a substantial, systematic enhancement of low-momentum neutral pions is expected. It can, of course, and should be measured relative to the charged pion background.

**Tricritical point**: In recent, beautiful work Stephanov, Rajagopal, and Shuryak [36] pointed out that a continuation of tricritical point we discussed above – or, in view of non-vanishing light quark masses, simply critical point – could be experimentally accessible. We accessed it notionally by dialing the strange quark mass; these authors point out that one might hope to access it experimentally, by using instead baryon density (in addition to temperature) as a control parameter. We are speaking here of a true critical point, involving long-range correlations. SRS suggested a number of specific, striking observable phenomena that could allow a convincing identification of the predicted critical point.

**Flavor flow**: Although it has not yet been properly quantified, several of us have for some time been discussing the possibility that tendencies for formation of condensates with non-trivial flavor structure could induce observable flavor flow. Probably the most hopeful, and certainly the most fundamentally interesting, case arises for the color-flavor locked state. In a heavy ion collision one starts with essentially no strange (or antistrange) quarks. If condensation of the color-flavor locking type is energetically favorable, at high density – preferably, at low temperature – one could find it favorable to rearrange strange-antistrange pairs, as they are produced, so the the strange quarks are retained in the condensate, while the antistrange are allowed to drift away. In the extreme case, it could even become favorable to produce pairs spontaneously classically, if the rest mass of the strange pair is made up by condensation energy. Qualitatively, the signature would be a concentration of low-momentum strange quarks – evolving into strange baryons – in the central collision region, and a compensating broad enhancement of antistrangeness flowing out of it.

Turning to astrophysics, I’d like to mention two possibilities, one relatively conservative, the other potentially of dramatic consequence for the theory of stellar explosions. The (relatively) conservative possibility is that quark matter deep inside neutron stars is in the color-flavor locked state. The major consequence of this is that there is a gap to all charged excitations, so that it is difficult to lose heat by normal electromagnetic or weak processes, while there is no gap to the superfluid modes associated with spontaneous baryon number violation, so that there is a repository for heat. The qualitative effect of this would be to keep neutron stars warm for a longer time than was otherwise possible with quark matter.

A more dramatic possibility is opened up by possibility, arising perhaps because of color superconductivity and almost certainly from color-flavor locking, that QCD makes a transition to a *highly ordered* state at high density, and one having radically different properties with respect to the weak interaction. This state could be reached directly from compressed nuclear matter, or (perhaps more likely) through an intermediate phase
with chiral symmetry restoration. Transition to the ordered state will be accompanied by release of latent heat. The available energy, being naturally measured in terms of the basic interactions responsible for the bulk of nucleon masses, could be very substantial on the scale of nuclear astrophysics. It could provide the key to the whole problem of getting collapsing stars to explode – a problem that has had a long, checkered history, and continues to lack a clear, robust solution. This inspiring possibility deserves much further attention.

Let me emphasize that these possibilities do not necessarily conflict with one another, nor need they be connected with the more radical \[11\], possibly problematic \[37\], idea that an exotic form of strongly interacting matter is the ground state at zero pressure.

Turning from astrophysics to cosmology, the main message from QCD studies appears to be the satisfactory though perhaps disappointing one, that no dramatic relics of the QCD phase transition seem likely to survive processing through the big bang. I say this is satisfactory, because there is no hint of any such relic, e.g., inhomogeneous nucleosynthesis or exotic baryonic dark matter. It might have been otherwise, if the phase transition at high density and nearly zero chemical potential were strongly first order. Schmid, Schwarz, and Widerin \[38\] have pointed out that the modification of the equation of state during a QCD phase transition or crossover, specifically the pressure deficit, could enhance the growth of fluctuations in kinetically decoupled dark matter (e.g., axions) on small scales. If this is a significant quantitative effect, it could have a drastic, negative effect on prospects for axion detection in ongoing and planned experiments. To decide this question for sure, we need better control of the equation of state through the phase transition or crossover.

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