Second Harmonic Generation by ultra-fast Lasers in Plasma.

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Abstract

The interaction of laser with plasma gives rise to several novel relativistic nonlinear optical effects. Higher harmonic generation is an important nonlinear effect. High power ultrafast lasers uses chirped pulse amplification technique to reduce duration of pulse and hence produces pulses of extremely high power densities which in turn oscillates electrons in the medium. The velocity of oscillating electron beats with applied frequency giving rise to higher order harmonics. This paper presents the idea of generation of second harmonic waves with the ultra-fast lasers in plasma, phase matching conditions of second harmonic generated by self modulation under paraxial approximations and its application in imaging microscopy.

Key words: Ultra fast Lasers, Pulse amplification.

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1. Introduction

Plasma which is an ionic form of matter interacts with laser to give rise to different phenomenons. Introduction of Laser in plasma oscillates electrons and oscillating frequencies beats with laser frequency to give rise to different harmonics like second, third harmonics etc. The electrons recolliding with the parents ions are responsible for higher harmonic generation[1]. Permanant changes in the properties of materials have also been observed due to laser’s interactions with materials.[2] In the presence of laser field, the non linearity of the medium enables the incident field frequencies to interact with each other in medium and produces fields with different frequencies.[3] High intensity short pulse lasers leads to minimum deposition of thermal energy in materials and give more precise outputs. Pulse energy is one of the factors that decide size of a laser. Reduction in size leads to production of high densities. Frequency mixing of ultra fast pulse laser’s fundamental and second harmonic fields generates electric current whose magnitude and directions depends upon phase between two fields. The time scale of current obtained depends on the duration of laser pulse. For ultra fast laser terahertz frequencies can be generated.[4] The presence of wiggler magnetic field also enhances the amplitude of second harmonic.[5] SHG can be used to obtain highly sensitive and high resolution information of the collagen in the body. SHG microscopy on collagen fibers is done to examine arterial fibrillation.[6] The technique of amplification of short pulse involving chirping and stretching of pulses to be amplified to saturated value then restoring to original width by using optical compressor. The intensity within the amplifier has to stay below a critical level at which non linear effects become significant.[7] Femtosecond fibre CPA systems like Ytterbium doped fibre chirp CPA system delivers as short as 800fs pulses with nJ
level pulse energies at average power of more than 100W at repetition rates of 200 KHz . A stretcher compressor unit consisting of dielectric diffraction gratings produces pulse with least distortions.[8] The other technique called Optical Parametric Chirped Pulse Amplification OPCPA can generate ultra fast laser beams by employing periodically poled crystals as amplifying media which produces phase controlled optical cycle pulses.[9] Self phase modulation along with self-focusing effects arises due to change in refractive index of material with laser intensity.[10] Harmonic generation and electron acceleration by lasers in plasma have also been studied under different parameters like chirp pulse, magnetic field giving similar conclusions.[11-15] The interaction of ultra fast lasers with high intense beams in plasma leads to better self focusing of beam which in turn leads to increase in oscillating frequency of electrons. This interaction generates second harmonic waves with high intensity as well as higher harmonics but with low intensities. The dispersion and non linear effects can be reduced by using thin crystals. The reduction in dispersive effects leads to the condition \( n(2\omega) = n(\omega) \), which implies \( k(2\omega) = k(\omega) \) and \( \lambda_{\omega} = 2\lambda_{2\omega} \). If phase mismatch occurs it SH generated may go out of phase.[16]

In this paper section 2 contains derivation for amplitude of second harmonic in normalized form, section 3 contains results and discussions, section 4 explains its application in imaging microscopy and conclusion is presented in section 5.

### 2. Theoretical considerations

Consider the short pulse laser through n type semiconductor in the presence of wiggler magnetic field. The electromagnetic waves can be represented as

\[
\vec{E}_1 = \hat{x} A_1(z,t) e^{i(\omega_1 t - k_1 z)} ,
\]

\[
\vec{B}_1 = \frac{k_1 \times \vec{E}_1}{\omega_1} , \quad \quad \quad (1)
\]

\[
\vec{B}_w = \gamma B_0 e^{i k_w z} \quad \quad \quad (2)
\]

B\(_w\) is Transverse wiggler magnetic field, \( A_1 \) is the amplitude of fundamental laser pulse. Laser pulse interacts with semiconductor non linearly and produce second harmonic. Its electric vector can be expressed as

\[
\vec{E}_2 = \hat{x} A_2(z,t) e^{i(\omega_2 t - k_2 z)} , \quad \text{where} \quad \omega_2 = 2\omega_1 \quad \quad \quad (4)
\]

Linear dispersion relation followed by fundamental and 2\(^{nd}\) harmonic is

\[
k_2^2 = \frac{\omega_2^2}{c^2} (\varepsilon_L - \frac{\omega_p^2}{\omega_2^2}) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (5)
\]

where \( \varepsilon_L \) is lattice permittivity .

and \( \omega_p^2 = 4m_e e^2 / m \) is called plasma frequency.

Here \( k_2 > 2k_1 \), the difference is delivered to second harmonic by wiggler magnetic field when

\[
k_2 = 2k_1 + k_o
\]
Using eq (1) and (2)

\[ v_1 = \frac{eE_1}{m(\omega_i + i\nu)} \]  

(7)

It beats with magnetic field of eq (3) to exert force \( F_1 \) and velocity \( v_1' \) is produced

\[ v_1' = \frac{-e^2 A B_0 e^{-(\omega t - (k_1 + k_2)z)}}{2m^2 \omega_i (\omega_i + i\nu)} z \]  

(8)

Now this velocity beats with magnetic field of eq (2) to give second harmonic and provides oscillatory velocity \( v_2 \) at \( 2\omega_1, 2k_1 + k_o \)

\[ v_2 = \frac{-e^3 A^2 B^2_0 k_1 e^{-(2\omega t - (2k_1 + k_2)z)}}{16m^3 \omega_i^4 (\omega_i + i\nu)} x \]  

(9)

Now velocity in eq (9) beats with wiggler magnetic field in eq. (3) to produce a transverse pondermotive force at \( 2\omega_1, 2k_1 + k_o \)

\[ v_2^{NL} = \frac{-e^2 A^2 B^2_0 k_1 e^{-(2\omega t - (2k_1 + k_2)z)}}{128m^3 \omega_i^4 (\omega_i + i\nu)} z \]  

\[ v_2^L = \frac{-eE_2}{m i(2\omega_i + i\nu)} x \]  

(10)

Second harmonic current density

\[ J_2^L = \frac{-n_o e^2 E_2}{m i(2\omega_i + i\nu)} x \]  

\[ J_2^{NL} = \frac{-n_o e^2 A^2 B_0^2 k_2 e^{-(2\omega t - (2k_1 + k_2)z)}}{128m^4 \omega_i^4 (\omega_i + i\nu)} z \]  

(12)

Now wave equation for Second harmonic for electric field of eq. (4)

\[ \nabla^2 E_2 = \frac{4\pi J_2}{\varepsilon_i c^2 \partial t} + \frac{\partial^2 E_2}{c^2 \partial t^2} \]  

(14)
On solving eq(14) using eq. (4),(12),(13)

\[
\frac{\partial A_2}{\partial z} + \frac{2\omega_1}{k_z c^2} \frac{\partial A_2}{\partial t} = -\frac{4m_e e^5 B_2^2 k_1^2 A_1^2}{128\epsilon_L c^2 k_2 m^4 \omega_1^4 (\omega_1 + i\nu)}
\]
\[
\frac{\partial A_2}{\partial z} + \alpha \frac{\partial A_2}{\partial t} = \beta A_1^2
\]

where

\[
A_1^2 = [F(z-v_{g1}t)]^2
\]
\[
v_{g1} = c\sqrt{\epsilon_L - \omega_p^2 - \omega_1^2}
\]
\[
v_{g2} = c\sqrt{\epsilon_L - \omega_p^2 - \omega_1^2} \quad \alpha = \frac{2\omega_1}{k_z c^2}, \quad \beta = \frac{4m_e e^5 B_2^2 k_1^2}{128\epsilon_L k_2 m^4 \omega_1^4 (\omega_1 + i\nu)}
\]

Using eq(17) eq(16) can be written as

\[
\frac{\partial A_2}{\partial z} + \frac{1}{v_{g2}} \frac{\partial A_2}{\partial t} = \beta[F(z-v_{g2}t)]^2
\]

Now considering temporal profile of laser pulse to be Gaussian

\[
F(z-v_{g2}t) = A_0 e^{-\frac{(z-v_{g2}t)^2}{\tau^2 v_{g2}^2}}
\]

Laser pulse length is \(\tau\)

Normalized amplitude of second harmonics

\[
\left| \frac{A_2}{A_1} \right| = \left[ \frac{1}{4\alpha_1^2 c^2 (\epsilon_L - \omega_p^2 - \omega_1^2)} \right]^{\frac{1}{2}} \frac{4m_e e^5 B_2^2 k_1}{128\epsilon_L c^2 m^4 \omega_1^4 (\omega_1 + i\nu)} \frac{A_0}{2\beta} \sqrt{\pi} \left[ \text{erf}(z'-t') - \text{erf}((1-\beta')z'-t') \right]
\]

3. Results and discussion

In this paper semiconductor plasma has been considered. In semiconductors charge carriers electrons and holes are created in pairs. Mobility of holes contributes to the absorption process, which behaves like electrons. Here the generation of second harmonic and the variation of normalized amplitude \(A_2/A_1\) (eq(20)) with normalized distance \(z'\) under the effect of different parameters have been studied. For typical case \(n\) type Ge semiconductor with a doping level of \(n_o \approx 10^{20} \text{cm}^{-3}\) is irradiated with CO2 laser with \(\omega_p = 1.8 \times 10^{14} \text{ rad/s}\), with collisional frequency of \(\nu = 10^{14} \text{ Hz}\). Figure 1 shows the variation of normalized amplitude \(A_2/A_1\) with normalized distance \(z'\) at \(\xi_0 k_1 \sqrt{\pi} / 2\beta' = 0.5, e A_0 / m_0 c = 9, eB_0 / m_0 \omega_0 = 0.8\) for \(t' = 2, \omega_p / \omega_1 = 0.4, 0.6, 0.8\) and Figure 2 shows variation at \(t' = 4\) and it has been observed that as time increases, the increase in plasma frequency enhances the wave amplitude producing more focused and sharp second harmonic which can lead to better imaging outputs.
Figure 1 shows variation of normalized amplitude $A_2 / A_0$ with $z'$ at different values of $\omega_p / \omega_i$ at $t'=2$.

Figure 2 shows variation of normalized amplitude $A_2 / A_0$ with $z'$ at different values of $\omega_p / \omega_i$ at $t'=4$.

4. Second harmonic waves in imaging microscopy
Second harmonic imaging microscopy is a powerful tool in laser scanning systems with high resolution, high contrast and three dimensional studies of live cells and tissues. SHG has been used to study collagen fibers which showed that collagen fibrils are regularly packed as polycrystalline lattice. The study of sclera fibrils revealed their structure as inhomogenous tube like structures comprising thin hard shells[6]. The use of SHG in imaging microscopy has helped to study those structures which earlier were thought to be studied by electron microscopy. The use of ultra fast lasers can even provide more advantage in terms of clarity of image and to study the events which last for picoseconds to femtoseconds. Another advantage it provides is the reduced amount of collateral damage.

5.Conclusion

The interaction of ultra fast lasers with matter generates higher harmonics with low dispersion and better self focusing. Due to low pump power required by these lasers, their conversion efficiency is better. Increased amplitude of incident beam leads to better focused second harmonic at very low dispersion rates. The SHG by ultra fast lasers provides advantage to imaging microscopy in terms of better resolution and study of events lasting from picoseconds to femtoseconds. The future aspect of the use of SHG by ultra fast lasers may include the study of tumour formation in chronic diseases which may be proved beneficial in their treatment.

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