Optimal reinsurance and investment for an insurer with the jump diffusion risk model in A-C case

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ABSTRACT
In this paper, we study the optimal reinsurance and investment problem for a class of jump-diffusion model, where the diffusion term represents the additional claims (i.e. A-C case). The insurer can purchase proportional reinsurance while allowing she/he to invest in a risk-free asset and a risky asset. The price process of the risky asset is driven by geometric Lévy process with dividend payouts. Applying stochastic control theory, the corresponding Hamilton–Jacobi–Bellman equation is established and the optimal reinsurance-investment strategies to maximize the expected exponential utility of terminal wealth are also established. Finally, the optimal strategies are analysed by the numerical simulation.

1. Introduction
Insurance plays an important role in international financial. Asset allocation is a central issue for insurers, where the key of issue is how to reduce the business risks and investment risks of an insurer. An insurer is trying to adopt different strategies to increase its wealth and minimize the corresponding risks. In general, there are two risks for insurers: unexpectedly huge claims and high-risk investments from the financial market. With respect to the risk of huge claims, the most common method is to take reinsurance, which can effectively transfer the insurer’s risks. The optimal investment-reinsurance problem is attracting more and more scholars attention.

Many scholars have focused on pursuing maximize the expected exponential utility of terminal wealth for an insurer. For example, Browne (1995) was studied the optimal investment strategy, where the surplus of an insurer is described by Brownian motion with drift and the investment strategies for an insurer under a geometrical Brownian motion (GBM) model. Irgens and Paulsen (2004) considered the surplus process with jump-diffusion, where compound Poisson process represents large claims and diffusion term represents an additional claim (A-C case). Yang and Zhang (2005) obtained the optimal investment policy to maximize the expected exponential utility of terminal wealth for jump diffusion risk model, in which the diffusion term indicates surplus of an insurance company’s uncertainty (U-S case), but without reinsurance. Wang (2007) extended Yang and Zhang (2005), considered an insurer investing in multiple risk assets and gave the optimal investment strategy. Xia, Fei, and Liu (2011) studied an insurer’s solvency ratio model under jump diffusion process. Liang and Guo (2011) discussed the optimal combining quota-share and excess of loss reinsurance to maximize the expected exponential utility of terminal wealth, and derived closed form expressions of the optimal strategies and value function. Furthermore, Deng and Yin (2015) and Li, Zeng, and Yang (2018) also researched the optimal investment and excess-of-loss reinsurance strategies. Assuming that the rate of return on investment obeys Ornstein–Uhlenbeck process (O-U), Liang, Yun, and Guo (2011) studied the optimal investment and proportional reinsurance strategy; Liang, Yun, and Cheung (2012) studied the optimal investment and proportional reinsurance with jump-diffusion risk process, where the premium satisfies principle of the mean-variance; Lin and Li (2011) and Wang, Rong, and Zhao (2018) researched the optimal reinsurance and investment to maximize the expected exponential utility of terminal wealth based on the constant elasticity of variance (CEV) model. Lin and Yang (2011) discussed an insurance company whose surplus is governed by a jump diffusion risk process, in which the price of risky asset was driven by geometric Lévy process, and the analytical formulas of corresponding optimal reinsurance and investment strategy and value function were obtained.
What’s more, optimal asset allocations of an investor under asset prices with a jump environment are investigated at Fei, Cai, and Xia (2015), Huang, Yang, and Zhou (2016) and Mao, Carson, and Ostaszewski (2017).

The above literature mainly studied the optimal proportional reinsurance for U-S case. For U-S case, the optimal investment strategy and the reinsurance strategy do not affect each other. However, in A-C case, the optimal investment and reinsurance are mutual influences. Therefore, it is very practical to consider the optimal reinsurance and investment in A-C case. But, in Lin and Yang (2011), it did not consider the reinsurance and investment strategy in A-C case. This paper will discuss the optimal reinsurance-investment of the insurer in A-C case based on literature (Lin & Yang, 2011), where the risk asset price is driven by geometric Lévy process. In addition, we also take into account the effects of dividend payments on optimal reinsurance-investment strategies.

The rest of the paper is arranged as follows. The next section provides necessary notations and states the model formulation of insurance-investment for the use of this paper. In Section 3, the random control model is established. Section 4 presents optimal strategies. Section 5 gives the analysis of our results and numerical illustration. Finally, we will conclude our paper in Section 6.

2. Model formulation

In this section, we briefly introduce the Jump-diffusion risk model and some basic models for reinsurance. In this paper, we consider a continuous-time financial model with the following standard assumptions: continuous trading in time for an insurer is allowed, no extra cost or taxes for trading in the financial or insurance market. and all assets are infinitely divisible. Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)\) be a complete filtered probability space, and satisfying the usual conditions, that is, \(\{\mathcal{F}_t\}_{t \geq 0}\) is right continuous and \(P\)-complete.

2.1. Jump-diffusion risk model

We consider the jump diffusion risk model, in which the surplus \(X(t)\) of an insurer at time \(t\) is

\[
X(t) = x_0 + ct + \beta W_1(t) - \sum_{k=1}^{N_1(t)} Y_k, \quad t \geq 0,
\]

where \(x_0 \geq 0\) is the initial capital, \(c > 0\) is the premium rate per unit of time for an insurer, and the diffusion volatility parameter \(\beta > 0\) is a constant. \(\{W_1(t), t \geq 0\}\) is a standard Brownian motion, \(\beta W_1(t)\) represents the additional claims for an insurer. \(\{N_1(t), t \geq 0\}\) is a Poisson process with an intensity \(\lambda_1 > 0\), where \(N_1(t)\) represents the total number of claims up to time \(t\), \(\{Y_k, k = 1, 2, \ldots\}\) is a sequence of positive independent and identically distributed random variables with a common distribution function \(F(y)\) with \(F(0) = 0\) and mean value \(\mu_1 = EY\) and variance value \(\mu_2 = EY^2\), where \(Y_k\) denotes the amount of the \(k\)th claim. In addition, it is assumed that the \(\{N(t), t \geq 0\}, \{Y_k, k = 1, 2, \ldots\}\) and \(\{W_1(t), t \geq 0\}\) are mutually independent.

2.2. Proportional reinsurance model

We assume the value \(a(t) \in [0, 1]\) can be regarded as the value of risk exposure. That is, the insurer will pay \(100a(t)\%\) of each claim occurring at time \(t\) while the reinsurer pays \(100(1 - a(t))\%\). At the same time, suppose that a reinsurer uses the mean-variance premium principle, the insurer will pay the premium to a reinsurer at a rate of \(\kappa(a(t)) = (1 - a(t))\lambda_1\mu_1 + \theta(1 - a(t))^2(\lambda_1\mu_2 + \beta^2)\) with safety loading \(\theta > 0\). With the proportional reinsurance being incorporated, the corresponding surplus process of an insurer evolves over time as:

\[
X_1(t) = x_0 + (c - \kappa(a(t)))t + \beta W_1(t) - \sum_{k=1}^{N_1(t)} Y_k.
\]

2.3. Consideration of proportional reinsurance model under investment

In order to maximize the profits and further expand the capital of an insurer, they often invest their surplus in a financial market consisting of a risk-free asset (bond or bank account) and a risky asset (stock or mutual fund). The process price of the risk-free asset evolves over time is assumed to following the ordinary differential equations:

\[
dB(t) = r_0 B(t) dt,
\]

where \(r_0 > 0\) is the constant risk-free interest rate. In the real financial investment environment, traditional risk estimation models have some limitations. However, Lévy process not only maintains the burnished statistical characteristics of Brownian motion, but also does not strictly require the continuity of path, allowing the existence of jumping discontinuity points. Therefor, the price \(R(t)\) of the risky asset is assumed to follow the geometric Lévy process

\[
dR(t) = R(t)[(r_1 - \delta) dt + \sigma dW_2(t) + \int_{\mathbb{R}} zN(dt, dz)],
\]

where \(r_1 > r\) and \(\sigma > 0\) are positive constants representing the expected instantaneous rate of return of the risky asset and the volatility of the risky asset price,
respectively, and \( \delta > 0 \) represents dividend payout rate of the stock. \( \{W_2(t), t \geq 0\} \) is another standard Brownian motion, which is mutually independent with \( \{W_1(t), t \geq 0\} \). \( \int_0^t \int_R zN(dt, dz) \) is a compound Poisson process, that is,

\[
\int_0^t \int_R zN(dt, dz) = \sum_{k=1}^{N_2(t)} Z_k,
\]

where \( \{N_2(t), t \geq 0\} \) is a Poisson process with rate \( \lambda_2 \), which is mutually independent with \( \{N_1(t), t \geq 0\} \), and \( \{Z_k, k = 1, 2, 3, \ldots\} \) are independent identically distributed random variables with a common distribution \( G(z) \).

We assume that \( b(t) \) is the amount of money investing in the risky asset at the time \( t \), and let \( b(t) > 0 \) that prevent the short-selling phenomenon. Then the rest of the surplus is invested in the risk-free asset. Moreover, we denote \( \pi(t) = (a(t), b(t)) \), where \( a(t) \) and \( b(t) \) are chosen by an insurer at the time of \( t \), and the corresponding strategy set is \( \Pi \). Thus, the surplus process of the insurer with proportional reinsurance and investment evolves over time as:

\[
\frac{dX_t^\pi}{X_t^\pi} = \left( (X_t^\pi - b(t)) \frac{dB(t)}{B(t)} + b(t) \frac{dR(t)}{R(t)} + dX_1(t) \right) - \frac{\lambda_2}{2} \left( \int_0^t \frac{zdz}{dz} \right) - \frac{\lambda_1}{2} \left( \int_0^t \frac{zdz}{dz} \right).
\]

### 3. The establishment of random control model

Suppose now that the insurance company is interested in maximizing the utility function for its terminal wealth at time \( T \). The utility function is denoted by \( u(x) \), and we assume that \( u(x)^I > 0 \) and \( u(x)^{II} < 0 \). Thus, we define the utility attained by the insurer from state \( x \) at time \( t \) as

\[
J^\pi(t, x) = E[u(X_T^\pi) | X_t^\pi = x].
\]

Here, we assume that the insurer has an exponential utility function

\[
u(x) = \xi - \frac{\tau}{m} e^{-mx},
\]

where \( \xi > 0, \tau > 0 \) and \( m > 0 \) is a constant representing the absolute risk aversion coefficient. Exponential utility function plays a prominent role in insurance mathematics, since they are the only utility functions under which the principle of ‘zero utility’ gives a fair premium that is independent of the level of reserves of an insurance company. Such a premium strategy is accepted by the insurance company itself and is also easily accepted by the policyholder. Therefore, the corresponding value function is

\[
V(t, x) = \sup_{\pi \in \Pi} J^\pi(t, x).
\]

**Theorem 3.1:** Assume that \( V(t, x) \in C^{1,2}([0, T] \times R_+) \). Then \( V(t, x) \) satisfies the Hamilton–Jacobi–Bellman (HJB) equation

\[
0 = \sup_{\pi \in \Pi} \left\{ V_t + \left[ (r_1 - \delta - r_0)b + r_0 x + (c - \kappa(a) + \lambda_1E[V(t, x - aY)] - V(t, x) \right] + \lambda_2E[V(t, x + bZ) - V(t, x)] \right\}
\]

with the boundary condition

\[
V(T, x) = u(x).
\]

**Theorem 3.2:** If \( U(t, x) \in C^{1,2}([0, T] \times R_+) \) is a classical solution to the HJB equation (2) that satisfies the boundary condition (3). Then, the value function given by (1) coincides with \( U(t, x) \), that is,

\[
V(t, x) = U(t, x).
\]

Furthermore, if \( \pi = \pi^*(a^*(t), b^*(t)) \) satisfies

\[
0 = U_t + \left[ (r_1 - \delta - r_0)b + r_0 x + (c - \kappa(a^*)) \right] U_k + \frac{1}{2}(a^2b^2 + \beta^2a^2)U_{xx} + \lambda_1E[U(t, x - a^*Y) - U(t, x)] - U(t, x)] + \lambda_2E[U(t, x + b^*Z) - U(t, x)]
\]

for all \( (t, x) \in ([0, T] \times R_+) \), then the policy \( \pi = \pi^*(a^*(t), b^*(t)) \) is an optimal policy.

### 4. Optimal strategy

To solve problem (2), we conjecture that the value function has the following form

\[
V(t, x) = \xi - \frac{\tau}{m} \exp[-mx \exp(g(T-t) + g(T-t))]
\]

where \( g(\cdot) \) is a suitable function such that (4) is a solution to (2). The boundary condition \( V(T, x) = u(x) \) implies that \( g(0) = 0 \).
From (4), a direct calculation yields

\[ V_t = [V(t, x) - \xi] [m_0 x e^{\rho (T-t)} - g'(T-t)], \]
\[ V_x = [V(t, x) - \xi] [-m_0 e^{\rho (T-t)}], \]
\[ V_{xx} = [V(t, x) - \xi] [m_2^2 e^{2\rho (T-t)}], \]
\[ E[V(t, x - aY) - V(t, x)] = [V(t, x) - \xi] \]
\[ \times E[\exp (aYm e^{\rho (T-t)} - 1)], \]
\[ E[V(t, x + bZ) - V(t, x)] = [V(t, x) - \xi] E[\exp (-bZm e^{\rho (T-t)} - 1)]. \] (5)

Substituting (5) into (2), we have

\[ \sup_{\pi \in \Pi} \left\{ -g'(T-t) + [(r_1 - \delta - r_0)b + (c - \kappa(a))] \right. \]
\[ - m e^{\rho (T-t)} + \frac{1}{2} \sigma^2 b^2 + \beta^2 a^2 \left[ m^2 e^{2\rho (T-t)} \right] \]
\[ + \lambda_1 E[\exp (aYm e^{\rho (T-t)} - 1)] \]
\[ + \lambda_2 E[\exp (-bZm e^{\rho (T-t)} - 1)] \} = 0. \]

In order to get the optimal strategy \( \pi = \pi^*(a^*(t), b^*(t)), \) we define

\[ f(a, b) = -g'(T-t) + [(r_1 - \delta - r_0)b + (c - \kappa(a))] \]
\[ \times [m e^{\rho (T-t)} + \frac{1}{2} \sigma^2 b^2 + \beta^2 a^2] \]
\[ \times [m^2 e^{2\rho (T-t)} + \lambda_1 E[\exp (aYm e^{\rho (T-t)} - 1)] \]
\[ + \lambda_2 E[\exp (-bZm e^{\rho (T-t)} - 1)]. \]

Let \( \partial f(a, b)/\partial a = 0, \) we have

\[ \kappa'(a) + \alpha \beta^2 m e^{\rho (T-t)} + \lambda_1 E[Y \exp (aYm e^{\rho (T-t)})] = 0, \]

so, we get

\[ \lambda_1 \mu_1 + 2\theta (1-a)(\lambda_1 \mu_2 + \beta^2) = \alpha \beta^2 m e^{\rho (T-t)} \]
\[ + \lambda_1 E[Y \exp (aYm e^{\rho (T-t)})]. \] (6)

Let

\[ h(a) = \lambda_1 \mu_1 + 2\theta (1-a)(\lambda_1 \mu_2 + \beta^2) - \alpha \beta^2 m e^{\rho (T-t)} \]
\[ - \lambda_1 \int_0^\infty y e^{\rho ym e^{\rho (T-t)}} F(dy), \]

then, we get

\[ h'(a) = -2\theta (\lambda_1 \mu_2 + \beta^2) - \beta^2 m e^{\rho (T-t)} \]
\[ - \lambda_1 \int_0^\infty y^2 m e^{\rho (T-t)} e^{\rho ym e^{\rho (T-t)}} F(dy) < 0 \]
\[ h''(a) = -\lambda_1 \int_0^\infty y^3 m^2 e^{2\rho (T-t)} e^{\rho ym e^{\rho (T-t)}} F(dy) < 0. \]

Obviously, we have \( h(a) \) is a monotone decreasing concave function with

\[ h(0) = 2\theta (1-a)(\lambda_1 \mu_2 + \beta^2) > 0 \]
\[ h(1) = \lambda_1 \mu_1 - \beta^2 m e^{\rho (T-t)} \]
\[ - \lambda_1 \int_0^\infty y e^{\rho ym e^{\rho (T-t)}} F(dy) < 0. \]

By simple calculation, we know that \( h(a) = 0 \) has a unique positive root \( a^* \) that is the optimal reinsurance strategy with \( 0 < a^* < 1. \)

In the same way, let \( \partial f(a, b)/\partial b = 0, \) we have

\[ r_1 - \delta - r_0 + \lambda_2 E[Z \exp (-b(t)Zm e^{\rho (T-t)})] \]
\[ = b(t) \sigma^2 m e^{\rho (T-t)}. \] (7)

Let

\[ H(b) = r_1 - \delta - r_0 + \lambda_2 E[Z \exp (-bZm e^{\rho (T-t)})] \]
\[ - b(t) \sigma^2 m e^{\rho (T-t)} \]
\[ = r_1 - \delta - r_0 + \lambda_2 \int_{-\infty}^{+\infty} z e^{-bzm e^{\rho (T-t)}} G(dz) \]
\[ - b(t) \sigma^2 m e^{\rho (T-t)}. \]

Then

\[ H'(b) = -\lambda_2 m e^{\rho (T-t)} \int_{-\infty}^{+\infty} z^2 e^{-bzm e^{\rho (T-t)}} G(dz) \]
\[ - \sigma^2 m e^{\rho (T-t)} < 0; \]
\[ H''(b) = \lambda_2 m^2 e^{2\rho (T-t)} \int_{-\infty}^{+\infty} z^3 e^{-bzm e^{\rho (T-t)}} G(dz) > 0. \]

So \( H(b) \) is a monotone decreasing function. As \( \lim_{b \to -\infty} H(b) > 0 \) and \( \lim_{b \to +\infty} H(b) < 0, \) \( H(b) = 0 \) has a unique finite root \( b^* \) that is the optimal investment strategy on \( R. \)

5. Analysis of our results and numerical illustration

In this section, we are devoted to illustrating the impact of parameters on optimal reinsurance and investment strategy for an insurer by some numerical examples. First of all, assume that

\[ F(y) = 1 - e^{-y}, \quad G(z) = p \eta_1 e^{-\eta_1 z} I_{z \geq 0} + q \eta_2 e^{\eta_2 z} I_{z < 0}, \]

where the constant \( p + q = 1, \eta_1 > 0, \eta_2 > 0 \) and \( \eta_2 > \eta_1. \)

According to the above assumptions, from equations (6) and (7), we get:
Table 1. Parameters values.

| Parameters | $r_0$ | $\lambda_1$ | $\beta$ | $m$ | $\delta$ |
|-----------|------|-------------|--------|-----|--------|
| Values    | 0.05 | 1.05        | 0.15   | 3   | 1.0    |
| Parameters | $r_1$ | $\lambda_2$ | $\sigma$ | $p$ | $q$ | $\eta_1$ | $\eta_2$ |
| Values    | 0.1  | 2.0         | 0.2    | 2/3 | 1/3 | 2       | 3       |

Figure 1. The impact of $m$ and $\theta$ on $a^*(t)$.

(1) Optimal reinsurance strategy $a^*$ satisfies:

$$
\lambda_1 + 2\theta(1-a^*(t))(\lambda_1 + \beta^2) = a^*(t)\beta^2 m e^{r_0(T-t)} + \frac{\lambda_1}{[1-a^*(t)m e^{r_0(T-t)}]^2}.
$$

(2) Optimal investment strategy $b^*$ satisfies:

$$
r_1 - \delta - r_0 + \frac{\lambda_2\eta_1}{[\eta_1 + b^*(t)m e^{r_0(T-t)}]^2} - \frac{\lambda_2\eta_2}{[\eta_2 - b^*(t)m e^{r_0(T-t)}]^2} = b^*(t)\alpha^2 m e^{r_0(T-t)}.
$$

And furthermore, we set the parameters as follows (Table 1):

5.1. Optimal proportional reinsurance policy

Figure 1 shows that the optimal reinsurance strategy $a^*(t)$ is reduced as the absolute risk aversion coefficient $m$ becomes larger. Namely, the larger the value of $m$ is, the less aggressive the insurer will be, and therefore the less retention level the insurer will hold. At the same time, we also know that the $a^*(t)$ is an increasing function of $\theta$, that is, a large $\theta$ yields a high retention level of proportional reinsurance, and therefore the insurer should retain a greater share of each claim.

Figure 2 shows that the optimal reinsurance strategy $a^*(t)$ is reduced as the risk-free interest rate $r_0$ becomes larger. As the larger, the value of $r_0$ is, the greater the expected income of the risk-free asset will be, and the larger income the insurer will obtain from investment. So for the sake of reducing its risk, the less risk the insurer wishes to share in each claim.
Figures 3 and 4 show that the optimal reinsurance strategy $\alpha^*(t)$ is reduced as the jump intensity of the claims $\lambda_1$ becomes larger and increased as the diffusion volatility parameter of the uncertainty claim $\beta$ becomes larger, respectively. As the larger, the value of $\lambda_1$ is, the greater the risk of large claims faced by the insurer will be, and therefore the insurer should retain a lesser share of each claim. On the other hand, as the larger, the value of $\beta$, the uncertainty claim of the insurer will be the greater, but the insurer is more willing to increase the amount of the claims to reduce the corresponding premiums due to the uncertainty claim is mainly aimed at the general small claims.

5.2. Optimal investment policy

Figure 5 shows that the optimal investment strategy $b^*(t)$ is reduced as the dividend payout rate $\delta$ becomes larger. As the larger, the value of $\delta$ is, the higher the dividend payout the insurer makes in the risk asset will be, and hence the less the insurer will wish to invest in the risky asset. But, Other literature did not consider this analysis. By the way, we also know that the optimal investment strategy $b^*(t)$ is reduced as the risk-free interest rate $r_0$ becomes larger. As the larger, the value of $r_0$ is, the greater the expected income of the risk-free asset will be, and the larger income the insurer will obtain from investing in the risk-free asset. So for the sake of reducing its risk, the insurer will be decreasing their investment in the risky assets.

Figure 6 shows that the optimal investment strategy $b^*(t)$ is reduced as the volatility of the risky asset $\sigma$ becomes larger and increased as the expected instantaneous rate of return of the risky asset $r_1$ becomes larger, respectively. As the larger, the value of $\sigma$ is, the greater the risk of the risk asset faced by the insurer will be, and hence the less the insurer will wish to invest in the risky asset. On the other hand, As the larger, the value of $r_1$ is, the greater the expected return on risky assets will be, and hence the more the insurer will wish to invest in the risky asset.

Figure 7 shows that the optimal investment strategy $b^*(t)$ is reduced as the jump intensity of the risky asset $\lambda_2$ becomes larger and increased as the probability of an upward jump of the risky asset $p$ becomes larger, respectively. As the larger, the value of $\lambda_2$ is, the greater the risk of the risk asset by the insurer will be, and hence the less the insurer will wish to invest in the risky asset. On the other hand, As the larger, the value of $p$ is, the greater the expected income of the risky asset will be, and hence the more the insurer will wish to invest in the risky asset.

6. Conclusions

In this paper, we have investigated optimal reinsurance and investment for an insurer with the jump diffusion
risk model in A-C case. The claim process of an insurer is approximated by jump diffusion risk model, and the investment is described by geometric Lévy process. Then, our find an optimal investment and proportional reinsurance policy which maximizes the expected exponential utility of terminal wealth. And the numerical simulation and its economic analysis are given. Finally, I think that our research can be further expanded to fractional Brownian motion environment on the basis of the existing research results in this paper. At the same time, it can also consider the optimal reinsurance and investment strategy with barrier dividend, stochastic interest rate, inflation and other factors on the basis of this article.

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