A proposal for the implementation of quantum gates with photonic-crystal coupled cavity waveguides

Dimitris G. Angelakis1, Marcelo F. Santos2, Vassilis Yannopapas3,4, and Artur Ekert1,5

1Centre for Quantum Computation, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, CB3 0WA, UK
2Dep. de Física, Universidade Federal de Minas Gerais, Belo Horizonte, 30161-970, MG, Brazil
3Condensed Matter Theory Group, Blackett Laboratory, Imperial College, London, SW7 2BW, UK
4Department of Materials Science, University of Patras, Patras 265 04, Greece and
5Department of Physics, National University of Singapore, Singapore 117 542, Singapore

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Quantum computers require technologies that offer both sufficient control over coherent quantum phenomena and minimal spurious interactions with the environment. We argue, that photons confined to photonic crystals, and in particular to highly efficient waveguides formed from linear chains of defects doped with atoms or (quantum dots) can generate strong non-linear interactions which allow to implement both single and two qubit quantum gates. The simplicity of the gate switching mechanism, the experimental feasibility of fabricating two dimensional photonic crystal structures and integrability of this device with optoelectronics offers new interesting possibilities for optical quantum information processing networks.

In order to perform a quantum computation one should be able to identify basic units of quantum information i.e qubits, initialize them at the input, perform an adequate set of unitary operations and then read the output. Here we show that these tasks can be performed efficiently using photons propagating in the lines of defects in photonic crystals. These structures are known as coupled resonators optical waveguides (CROWS) or coupled cavity waveguides (CCWs) and support efficient low loss guiding, bending and coupling of light pulses at group velocities of the order of $10^{-3}$ the speed of light. Qubits can be represented by the “dual rail” CROW, i.e. by placing a photon in a superposition of two preselected lines of defects such that each line represents the logical basis state, 0 or 1. Quantum logic gates are then implemented by varying the length and the distance between the CROWS and by tuning the refractive index in some of the defects using external electric fields and cavity QED type enhanced non-linear interactions between the propagating photons. We start with a sketch of the underlying technology followed by a more detailed description of quantum logic gates and conclude with the estimation of the relevant experimental parameters.

Photonic crystals (PCs) are made of ordered inhomogeneous dielectric media with a dielectric constant spatially periodic on the same scale as the wavelength of the light propagating in them. They can exhibit band gaps and also defects like their electronic counterparts, the semiconductors. Point and line defects can also be introduced in them with the latter constituting very efficient waveguides. A point defect introduces a bound state of the electromagnetic field within the photonic band gap which can act as a high-Q cavity. Many point defects can be brought together to form the above mentioned CROWS. A light pulse which enters a CROW, propagates through a tunneling/hopping mechanism between neighboring defects allowing for low dispersion, small group velocities and efficient waveguiding.

Photons confined to photonic crystals, and in particular to highly efficient waveguides formed from linear chains of defects doped with atoms or (quantum dots) can generate strong non-linear interactions which allow to implement both single and two qubit quantum gates. The simplicity of the gate switching mechanism, the experimental feasibility of fabricating two dimensional photonic crystal structures and integrability of this device with optoelectronics offers new interesting possibilities for optical quantum information processing networks.
i.e. a sequence: the Hadamard gate, a phase gate, the Hadamard gate. It can be implemented by a device shown in the lower part of Fig. 2, which is a Mach-Zehnder interferometer embedded in a photonic crystal. The two Hadamard gates correspond to the two areas in which the CROWs are brought closer to each other. Relative phase \( \phi \) can be introduced by varying the length of one of the CROWs in the area between the two Hadamard gates. If a pulse of light is injected into one of the input ports it will emerge at the one of the two output ports with the probabilities \( \sin^2(\phi/2) \) and \( \cos^2(\phi/2) \), where \( \phi \) is the accumulated phase difference between the two arms. This has been demonstrated experimentally for 2D CROW structures in the microwave regime and has been extensively studied for the optical case [5, 8, 9].

The existing experimental realizations of a photonic crystal MZI had the phase shift \( \phi \) fixed by the architecture, however, one can also introduce an active phase control. It can be achieved by placing a medium with tunable refractive index into one of the arms of the interferometer in between the Hadamard gates. Defects in one of the arms can be doped with atoms or quantum dots of resonance frequency \( \omega_{ge} \). These two-level systems can be then tuned to be on and off-resonance with the propagating light of frequency \( \omega \) by applying an external electric field, i.e. by using the Stark effect. Initially the dopants are far off resonance with the light pulse, which allows the pulse to enter the CROWs without any reflections. As soon as the pulse reaches the area in between the Hadamard gates the electric field is applied bringing the dopants closer to resonance and inducing a near-resonant dispersive interaction. When the detuning \( \delta = \omega_{ge} - \omega \) is smaller than both \( \omega \) and \( \omega_{ge} \) and, at the same time, much larger than the coupling constant between the dopant and the field light \( \Omega \), i.e. when \( \omega_{ge}, \omega \gg \delta >> \Omega \), then the combined dopant-light system acquire a phase proportional to \( (\Omega^2/\delta)T \), where \( T \) is the interaction time. Both \( \delta \) and \( T \) can be controlled and we can therefore introduce any desired phase shift between the two arms of the interferometer.

Let us now show how the device shown in Fig. 2 can be used to implement a two-qubit conditional phase gate. The two qubits are represented by four CROWs labelled as \( |0\rangle_1, |1\rangle_1 \) and \( |0\rangle_2, |1\rangle_2 \) respectively for the first and the second qubit. Only two of the four CROWs enter the device. They have labels \( |1\rangle_1 \) and \( |1\rangle_2 \) and represent the binary 1 of the first and the second qubit. Thus the device operates either on vacuum (input \( |0\rangle_1 |0\rangle_2 \)), or on a single photon (inputs \( |0\rangle_1 |1\rangle_2 \) and \( |1\rangle_1 |0\rangle_2 \)) or on two photons (input \( |1\rangle_1 |1\rangle_2 \)). The desired action of the device, i.e. the conditional phase shift gate, is: \( |0\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2 \), \( |0\rangle_1 |1\rangle_2 \rightarrow |0\rangle_1 |1\rangle_2 \), \( |1\rangle_1 |0\rangle_2 \rightarrow |1\rangle_1 |0\rangle_2 \), \( |1\rangle_1 |1\rangle_2 \rightarrow |1\rangle_1 |1\rangle_2 \). The device should let the vacuum and one photon states pass through undisturbed and react only to a two photon state. We can achieve this by an interplay of dispersive interaction for single photons and resonant interactions for two photons.

Let us focus only on the CROWs modes that actually enter the device, i.e. \( |1\rangle_1 \) and \( |1\rangle_2 \), and consider their photon occupation numbers. From now on \( |nm\rangle \) means \( n \) photons in mode \( |1\rangle_1 \) and \( m \) photons in mode \( |1\rangle_2 \). If no phase shift is induced the device affects the transformation:

\[
|00\rangle \rightarrow |00\rangle \rightarrow |00\rangle, |01\rangle \rightarrow (|01\rangle - |10\rangle)/\sqrt{2} \rightarrow |01\rangle, |10\rangle \rightarrow (|01\rangle + |10\rangle)/\sqrt{2} \rightarrow |10\rangle, |11\rangle \rightarrow (|20\rangle - |02\rangle)/\sqrt{2} \rightarrow |11\rangle,
\]

where the first and the second arrow correspond to the action of the first and the second Hadamard gate, respectively. All we need is a nonlinear medium in between the Hadamard gates such that the states \( |00\rangle, |01\rangle \) and \( |10\rangle \) do not change, while the states \( |20\rangle \) and \( |02\rangle \) both acquire the same phase \( \pi \).

Following our scheme for the tunable single qubit phase gate let us now consider dopants with three-level configuration, with electronic levels \( g, h, e \) forming a cascade with transition frequencies \( \omega_{gh} \) and \( \omega_{he} \). The two transitions couple linearly to the hopping photons through electro-dipole interactions, as shown in Fig. 3. We place the dopants in both arms of the interferometer. A photon of frequency \( \omega \) is symmetrically detuned from \( \omega_{gh} \) and \( \omega_{he} \), so that \( \delta = |\omega_{gh} - \omega| = |\omega_{he} - \omega| >> g_1, g_2 \), where \( g_1, g_2 \) are the corresponding coupling constants for the two transitions. Thus a single photon can only undergo a dispersive interaction with the dopants. However, a pulse with two photons is resonant with the energy separation between the levels \( g \) and \( e \), i.e. \( 2\omega = \omega_{gh} + \omega_{he} \), and undergoes the resonant interaction. This can be quantified by the effective Hamiltonian, extensively studied in the theory of micromasers [11],

\[
H_{\text{eff}} = \frac{g_1^2}{\delta} \sigma_{gg}(a^\dagger a) + \frac{g_2^2}{\delta} (\sigma_{ee} a^\dagger a^\dagger a) + \frac{g_1 g_2}{\delta} (\sigma_{ge} a^\dagger)^2 + \sigma_{eg} a^2,
\]

where \( a^\dagger, a \) are the photon creation and annihilation operators and \( \sigma_{ij} = |i\rangle \langle j| \) with \( i, j = g, h, e \) are the corresponding atomic operators. The first two terms describe the dispersive interaction and the third term the two-photon resonant interaction.

If the dopant is initially in level \( |g\rangle \) then the joint dopant-field state evolves, after time \( t \), to [11]

\[
|g\rangle |00\rangle \rightarrow |g\rangle |00\rangle,
\]

\[
|g\rangle |01\rangle \rightarrow e^{-i\varphi} |g\rangle |01\rangle,
\]

\[
|g\rangle |10\rangle \rightarrow e^{-i\varphi} |g\rangle |10\rangle,
\]

\[
|g\rangle |20\rangle \rightarrow e^{i\varphi} [\cos \kappa t |g\rangle |20\rangle + \sin \kappa t |e\rangle |00\rangle],
\]

\[
|g\rangle |02\rangle \rightarrow e^{i\varphi} [\cos \kappa t |g\rangle |02\rangle + \sin \kappa t |e\rangle |00\rangle],
\]

where \( \kappa = \frac{(g_1 g_2 \sqrt{2})}{\delta} \), and \( \varphi = \frac{(g_1)^2}{\delta} \). For \( \kappa t = \pi \), the two-photon interaction completes a full Rabi oscillation, acquiring a total phase \( \phi = \pi + 2\varphi \), where \( \varphi = \frac{g_1 \varphi}{g_2 \sqrt{2}} \). The ratio \( g_1/g_2 = 2\sqrt{2} \) gives \( \varphi = \pi \) which means that the two-photon state acquires a minus sign while the remaining states are brought back to their originals. Under these conditions, the time-evolution showed above reproduces an instance of a two qubit conditional phase shift gate.
FIG. 2: The upper part shows a schematic of four coupled cavity waveguides (CROW) which represent two qubits. The two central waveguides, belonging to two different qubits, are brought together in a nonlinear interferometric device which is shown below the schematic. The device is integrated into a 2D, micrometer size photonic crystal. The lines of defects, shown in blue, transfer photons from left to right. The two waveguides are brought closer to each other right after the entrance and before the exit of the device, allowing photons to tunnel between them. The defects in between these two regions are doped with atoms or quantum dots which can be tuned to be on and off-resonance with the propagating light by applying an external electric field. An interplay between the resonant two photon and the dispersive one photon transitions leads to phase shifts required both for single qubit phase gates and two qubit controlled-phase gates. The green and red boxes mark the area with the electric field on and off, respectively. The field is switched on to induce the nonlinear phase shift. However, at the end of the quantum gate operation the field is selectively turned off to the right of the defect where the phase shift was induced. This is represented by the half green and half red area, as shown in the picture, and allows photons to be released back to the propagating modes.

FIG. 3: The relevant energy levels of the dopants. A photon of frequency $\omega$ is equally detuned from $\omega_{gh}$ and $\omega_{he} (\pm \delta)$ and undergoes a dispersive interaction with the dopants. However, a two photon pulse is resonant with the energy separation between the levels $g$ and $e$, i.e. $2\omega = \omega_{gh} + \omega_{he}$, and undergoes the resonant interaction.

For the photonic quantum computation, as described above, to be experimentally feasible we need doped 2D crystal structures of high quality, strong dopant-photon coupling, and reliable single photon sources together with efficient photo-detectors. We expect these requirements to be available with current near future technology. More specifically:

2D crystals with leaking losses from the CROW as small as a few percent and defect quality factors of the order of $10^5 - 10^6$ (in the near optical frequency regime) should be fabricated soon as indicated by both the experimental progress in 1D and detailed theoretical simulation in 2D [4, 8, 9]. Doping active elements in defects in the form of quantum dots and strong coupling regime has also been implemented for single defects. Although technologically challenging, the extension to many coupled defects should be feasible soon [10, 12]. In our scheme for a quality factor of $10^6$ we get a typical time-scale for undisturbed coherent quantum operations to be of the order of $T_1 = 1\text{ns}$. Both the phase shift operation and the two photon nonlinear phase shift can be performed within a time period which is shorter at least by one order of magnitude. The coupling constant $g$ for the individual atom-photon coupling, for example for the D2 atomic transition (852 nm) of a doped atom of $^{133}\text{Cs}$, is of the order $3 \times 10^9$ Hz [8, 12]. The maximum induced phase is $\sqrt{Ng^2T_1/\Delta}$ where $N$ is the number of dopants in the defects. If $\Delta \approx 3 \times 10^{10}$ Hz and $N \approx 100$ dopants, then the time required to induce any phase between 0 and $\pi$, is roughly 0.1 ns. Similarly for the two photon nonlinear phase shift; the two photon Rabi frequency is proportional to $\sqrt{Ng^2g_2/\Delta}$ and $g_1$ is very close to $g_2$, and both are of the order of $3 \times 10^9\text{Hz}$. With the same typical value of $\Delta$ we get the gate operation time to be of the order of 0.1 ns. We note that these figures can be
improved by adding more dopants to the defects making the coupling stronger or by fabricating higher quality factor defect cavities. Note that for the case of quantum dots, dipole moments are larger and they will thus couple stronger to the field. However tuning between dots in different defects might be a problem in that case. Lastly, the switching time of the external gates depends on the photon crossing time, which for a typical CROW group velocity of the order of $10^{-5}c$ is of the order of nanoseconds. Thus the required switching of the external electric fields should be performed on a timescale from nanoseconds to tens of picoseconds which is within current technology. We would also like to add here that the shifting could also occur by applying a slightly detuned laser field (AC stark shift), coupled to some other atomic level far from the hopping photon resonance and the atomic states into consideration. The size of the accessible device area will then be reduced to the focus area of the pulse which could be of the order of the wavelength, i.e. a few microns.

Decoherence due to coupling of the atoms to the vibrations of the medium is expected to negligible for the case of a suspended atom (or cold atomic cloud) inside or close to the surface of the defect. This could be achieved through the lowering of a trap for example on top of the defect. Another source of errors could be from the presence of disorder of a CROW in the optical/infrared regime. Current results in 1D and 2D show that is possible to implement CROWS in the near optical/infrared with very low disorder, in good agreement with theory\cite{4, 5, 8, 9, 10}. Intense efforts to implement coupled CROWS in 2D from numerous groups are expected to lead to a positive result very soon.

We should also note that the losses from coupling to the waveguide from outside will reduce the number of successful phase shifts per input number of photons. The losses once inside the waveguide are much smaller and are of the order of only a few percent\cite{4, 5, 8, 9, 10}, i.e. most of the photons that transverse the device will be phased shifted. Both loss mechanisms will nevertheless result to a reduction of the rate of input uncorrelated photons/output of phase shifted photons. This can be counterbalanced by increasing the rate of incident photons whenever possible or by integrating a single photon source in the waveguide. In the case of a complete network with many gates, some tuning of the individual emitters might be needed. This is definitely an interesting route to pursue in the next stage of this project. Lastly an implementation of our scheme requires good synchronization of photon pulses, single photon sources and very efficient single-photon detectors. These requirements are very similar to those for quantum computation with linear optical elements \cite{13}. However, our scheme is much less demanding in terms of resource overheads per a reliable quantum gate. Recent progress in the development of single photon sources indicate that the photonic quantum computation should be a realistic experimental proposition \cite{14}. A more detailed study of all possible error mechanisms for this scheme is under way and will appear elsewhere.

In conclusion, we have discussed how photons propagating in CROWS could generate strong non-linear interactions which could enable the implementation of both single and two qubit quantum gates. The simplicity of the gate switching mechanism using global external fields, the feasibility of fabricating two dimensional photonic crystal structures and CROWS with current or near future technology and the integrability of this device with optoelectronics should offer new interesting possibilities for optical quantum information processing networks \cite{15}.

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[15] Shortly after the submission of this work, we came aware of very recent work where similar mechanisms for nonlinearities in PBGs are proposed, using polariton and EIT techniques: I. Friedler et al. quant-ph/0410019