The Tsallis Distribution for $p − p$ collisions at the LHC.

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Abstract. The Tsallis distribution has been used widely in high energy physics to describe the transverse momentum distributions of particles produced in $p − p$ collisions. In this talk we review some recent developments.

There exists a rich and wide variety of distributions covering a large range of applications [1, 2]. Those having a power law behaviour have attracted considerable attention in physics in recent years but there is a a long history in other fields such as biology and economics [3].

In high energy physics power law distributions have been applied by a very large number of scientists [4, 5, 7, 6, 8] to the description of transverse momenta of secondary particles produced in $p − p$ collisions. Indeed the available range of transverse momenta has expanded considerably with the advent of the Large Hadron Collider (LHC).

Collider energies up to 7 TeV are now available in $p − p$ collisions and transverse momenta of hundreds of GeV are now a common occurrence. In this presentation the focus will be on various forms of distributions first proposed by C. Tsallis about twenty-five years ago [9]. Applications of the Tsallis distribution to high energy $e^+ − e^−$ annihilation has been considered by Bediaga, Curado and de Miranda [10]. A recent review of power laws in elementary and heavy-ion collisions can be found in [11].

In the analysis of the new data, a Tsallis-like distribution gives excellent fits to the transverse momentum distributions as shown by the STAR [4] and PHENIX [5] collaborations at RHIC and by the ALICE [6], ATLAS [7] and CMS [8] collaborations at the LHC. In this talk we review recent work on the parameterization used by these groups and discuss a slightly different one which might lead to a more consistent interpretation and has the bonus of being thermodynamically consistent [12, 13].

For high energy physics a consistent form of Tsallis statistics (see e.g. [13] and references therein) for the particle number, energy density and pressure is given by the expressions given...
below

\[ N = gV \int \frac{d^3p}{(2\pi)^3} \left[ 1 + \left( q - 1 \right) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}}, \]  
\[ \epsilon = g \int \frac{d^3p}{(2\pi)^3} E \left[ 1 + \left( q - 1 \right) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}}, \]  
\[ P = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \left[ 1 + \left( q - 1 \right) \frac{E - \mu}{T} \right]^{-\frac{2}{q-1}}. \]  

where \( T \) and \( \mu \) are the temperature and the chemical potential, \( V \) is the volume and \( g \) is the degeneracy factor. A similar version has been presented in [14, 15, 16, 17] but with different power-law behaviour.

As is well-known the Tsallis distribution [9, 18] introduces a new parameter \( q \) which for transverse momentum spectra is always close to 1, typical values for the parameter \( q \) obtained from fits to the transverse momentum distribution are in the range 1.1 to 1.2. In this talk we always assume \( q > 1 \).

The expressions (1), (2) and (3) are thermodynamically consistent, e.g. it can be easily shown [13] that relations of the type

\[ N = V \left. \frac{\partial P}{\partial \mu} \right|_T, \]  

are satisfied. It has been verified that the proposed expressions have thermodynamic consistency [12, 13].

It follows from (1) that the momentum distribution is given by,

\[ \frac{d^3N}{d^3p} = gV \frac{E}{(2\pi)^3} \left[ 1 + \left( q - 1 \right) \frac{E - \mu}{T} \right]^{-q/(q-1)}, \]  

or, expressed in terms of transverse momentum, \( p_T \), transverse mass, \( m_T \), and rapidity \( y \)

\[ \frac{d^2N}{dp_T dy} = gV \frac{p_T m_T \cosh y}{(2\pi)^2} \left[ 1 + \left( q - 1 \right) \frac{m_T \cosh y - \mu}{T} \right]^{-q/(q-1)}. \]  

At mid-rapidity \( y = 0 \) and for zero chemical potential \( \mu = 0 \) this reduces to

\[ \left. \frac{d^2N}{dp_T dy} \right|_{y=0} = gV \frac{p_T m_T}{(2\pi)^2} \left[ 1 + \left( q - 1 \right) \frac{m_T}{T} \right]^{-q/(q-1)}. \]  

This is the expression used in [12, 13] to fit the LHC transverse momentum spectra.

It is well-known since 1988 [9] that in the limit where the parameter \( q \) goes to 1 this reduces to the standard Boltzmann distribution:

\[ \lim_{q \to 1} \frac{d^2N}{dp_T dy} = gV \frac{p_T m_T \cosh y}{(2\pi)^2} \exp \left( -\frac{m_T \cosh y - \mu}{T} \right). \]  

The parameterization given in Eq. (7) is close to the one used e.g. by the STAR [4], PHENIX [5], ALICE [6], ATLAS [7] and CMS [8] collaborations where the following form is used:

\[ \left. \frac{d^2N}{dp_T dy} \right|_{y=0} = p_T \frac{dN}{dy} \frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left[ 1 + \frac{m_T - m_0}{nC} \right]^{-n}. \]  

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where $n$, $C$ and $m_0$ are fit parameters. Indeed, after substituting

$$n \rightarrow \frac{q}{q - 1}$$

and

$$nC \rightarrow \frac{T + m_0(q - 1)}{q - 1},$$

Eq. (9) becomes

$$\left. \frac{d^2N}{dp_T dy} \right|_{y=0} = \frac{p_T}{dy} \frac{dN}{dy} \frac{(n - 1)(n - 2)}{nC(nC + m_0(n - 2))} \left[ \frac{T}{T + m_0(q - 1)} \right]^{-q/(q-1)} \left[ 1 + (q - 1) \frac{m_T}{T} \right]^{-q/(q-1)}.$$ 

Which has the same dependence on the transverse momentum as (7) apart from an additional factor $m_T$ on the right-hand side. It has to be pointed out that the inclusion of the rest mass in the substitution Eq. (11) is not in agreement with the Tsallis distribution as it breaks $m_T$ scaling which is present in the Tsallis form (6) but not in Eq. (9).

The inclusions of the factor $m_T$ leads to a more consistent interpretation of the variables $q$ and $T$ [12, 13].

The distribution (7) has been used to fit the data for identified particles, $\pi$, $K$ and $p$ for the ALICE [6] collaboration and $K^0_s$, $\Lambda$ and $\Xi$ for the CMS collaboration in $p-p$ collisions at 900 GeV [12, 13]. The resulting values for $q$ and $T$ at 900 GeV beam energy are listed in Table 1. The transverse momentum distributions for the ALICE [6] data are shown in Fig. (1). The results obtained are comparable to the ones obtained recently in [20, 25].

Interesting results were obtained in Refs. [20, 21, 22, 23] where spectra for identified particles were analyzed and the resulting values for the parameters $q$ and $T$ were considered.

The results obtained for the values of $q$ are shown in Fig. (2) for two different LHC beam energies. Note that the values for $q$ at 7 TeV are slightly higher than those obtained at 0.9 TeV.

### Table 1. Fitted values of the $T$ and $q$ parameters measured in $p-p$ collisions by the ALICE and CMS collaborations using the Tsallis form (7) for the momentum distribution.

| Particle | $q$       | $T$       |
|----------|-----------|-----------|
| $\pi^+$  | 1.166 ±0.004 | 0.0649 ±0.0010 |
| $\pi^-$  | 1.162 ±0.003 | 0.0648 ±0.0008 |
| $K^+$    | 1.145 ±0.008 | 0.0776 ±0.0044 |
| $K^-$    | 1.142 ±0.003 | 0.0769 ±0.0013 |
| $K^0_S$  | 1.135 ±0.002 | 0.0919 ±0.0015 |
| $p$      | 1.109 ±0.004 | 0.0949 ±0.0025 |
| $\bar{p}$| 1.110 ±0.018 | 0.0973 ±0.0015 |
| $\Lambda$| 1.106 ±0.004 | 0.0796 ±0.0048 |
| $\Xi^-$  | 1.081 ±0.004 | 0.1262 ±0.0037 |
Figure 1. Fit to the $\pi, K, p$ transverse momentum distributions in $p - p$ collisions as measured by the ALICE collaboration [6] using the Tsallis distribution function as given by (7).

This has been analyzed in a systematic way in [26] where the parameter $q$ was extracted from results for charged hadrons at different beam energies. A similar analysis was recently made in [27, 28], the energy dependence is compatible but the values of $q$ are systematically lower than those obtained in [26] due to a different power in the Tsallis distribution.

In conclusion we can say that the use of the Tsallis parameterization presented in (1) leads to a good description of identified particles in $p - p$ collisions at the LHC with a reasonably consistent set of parameters.

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Figure 2. Values of the Tsallis parameter $q$ for different species of hadrons. The values were extracted from results obtained by the ALICE [6] and CMS [8] collaborations.

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Figure 3. (Color online) Energy dependence of the Tsallis parameter $q$ appearing in the Tsallis distribution [26].