1. INTRODUCTION

The Galactic cosmic rays (CRs), with proton energies below a few $10^{15}$ eV, are generally believed to be accelerated in shell-type supernova remnants (SNRs). To establish that the Galactic SNRs are indeed the main sources of the Galactic CRs, one needs at least a handful of SNRs with clearly determined astronomical parameters, such as the type of supernova explosion, the SNR age, the distance, and the properties of the circumstellar medium, together with the nonthermal emission characteristics from which the energetic particle spectra can be deduced with a theoretical model. The most suitable objects are Type Ia supernovae which have rather old and low-mass progenitors, not associated with star-forming regions, and thus supposedly explode into a uniform circumstellar medium—at least into one which is not modified by mass loss in the form of a stellar wind. The type of explosion also restricts the total explosion energy rather tightly and fixes the ejecta mass to a value close to the Chandrasekhar mass.

One such object is the historical remnant SN 1006: the distance was determined using optical measurements with relatively high precision (Winkler et al. 2003) and all other distance was determined using optical measurements with the Chandrasekhar mass. The type of explosion also restricts the total explosion energy—of the order of 5% of the total hydrodynamic explosion energy, and is predicted to rise with time by a factor of ~2. The relevance of CR escape from the SNR for the spectrum of the $\gamma$-ray emission is demonstrated. The sum of the results suggests that SN 1006 is a CR source with a high efficiency of nuclear CR production, as required for the Galactic CR sources, both in flux as well as in cutoff energy.

**Key words:** acceleration of particles – cosmic rays – gamma rays: general – radiation mechanisms: non-thermal – shock waves – supernovae: individual (SN 1006)

**Online-only material:** color figures

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**ABSTRACT**

The properties of the Galactic supernova remnant (SNR) SN 1006 are theoretically re-analyzed in light of the recent H.E.S.S. results. Nonlinear kinetic theory is used to determine the momentum spectrum of cosmic rays (CRs) in space and time in the supernova remnant SN 1006. The physical parameters of the model—proton injection rate, electron-to-proton ratio, and downstream magnetic field strength—are determined through a fit of the result to the observed spatially integrated synchrotron emission properties. The only remaining unknown astronomical parameter, the circumstellar gas number density, is determined by a normalization of the amplitude of the $\gamma$-ray flux to the observed amplitude. The bipolar morphology of both nonthermal X-ray and $\gamma$-ray emissions is explained by the preferential injection of suprathermal nuclei and subsequent magnetic field amplification in the quasi-parallel regions of the outer supernova shock. The above parameters provide an improved fit to all existing nonthermal emission data, including the TeV emission spectrum recently detected by H.E.S.S., with the circumstellar hydrogen gas number density $N_H \approx 0.06$ cm$^{-3}$ close to values derived from observations of thermal X-rays. The hadronic and leptonic $\gamma$-ray emissions are of comparable strength. The overall energy of accelerated CRs at the present epoch is of the order of 5% of the total hydrodynamic explosion energy, and is predicted to rise with time by a factor of ~2. The relevance of CR escape from the SNR for the spectrum of the $\gamma$-ray emission is demonstrated. The sum of the results suggests that SN 1006 is a CR source with a high efficiency of nuclear CR production, as required for the Galactic CR sources, both in flux as well as in cutoff energy.

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**NONTHERMAL EMISSION OF SUPERNOVA REMNANT SN 1006 REVISITED: THEORETICAL MODEL AND THE H.E.S.S. RESULTS**

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Received 2012 July 12; accepted 2012 August 30; published 2012 October 11
2. SUMMARY OF ASSUMPTIONS AND EARLIER RESULTS

As a Type Ia supernova, SN 1006 presumably ejects roughly a Chandrasekhar mass \( M_{\odot} = 1.4 \, M_{\odot} \). Since the gas density is observed to vary only mildly across the SNR (Acero et al. 2007), it appears reasonable to assume the circumstellar gas density and magnetic field to be roughly uniform. The ISM mass density, \( \rho_0 = m_p N_{\text{ISM}} \approx 1.4 \, m_p \, N_{\text{H}} \), is characterized by the hydrogen number density \( N_{\text{H}} \). The value \( d = 2.2 \, \text{kpc} \) is adopted as the most reliable distance estimate (Winkler et al. 2003).

As in BKV09, a nonlinear kinetic theory of CR acceleration in SNRs (Berezhko et al. 1996; Berezhko & Völk 1997) is applied. With one exception, this theory includes equations for the most important physical processes that influence CR acceleration and SNR dynamics: shock modification by CR back-reaction, MHD wave damping and thus gas heating within the shock transition, and synchrotron losses of CR electrons under the assumption of a strongly amplified magnetic field within the remnant \( B(t) \).

The values of three scalar parameters in the governing equations (proton injection rate \( \eta \), electron-to-proton ratio \( K_{\text{ep}} \), and the upstream magnetic field strength \( B_0 \)) can be determined from a fit of the solutions that contain these parameters to the observed spatially integrated synchrotron emission data at the present epoch. Both, \( \eta \) and \( K_{\text{ep}} \), are assumed to be independent of time. The parameter values for SN 1006, evaluated in this way, agree very well with the Chandra measurements of the X-ray synchrotron filaments and were obtained in BKV09 by the analysis of the radio data compiled by Allen et al. (2004) (see also Allen et al. 2001, 2008) and of the most accurate X-ray data of Chandra (Allen et al. 2004) and Suzaku (Bamba et al. 2008).

3. RESULTS

In order to explain the detailed \( \gamma \)-ray spectrum, values of the hydrodynamic supernova explosion energy \( E_{\text{sn}} = 2.4 \times 10^{51} \, \text{erg} \) and \( E_{\text{sn}} = 1.9 \times 10^{51} \, \text{erg} \) are taken to fit the observed shock size \( R_s = 9.5 \pm 0.35 \, \text{pc} \) and shock speed \( V_s = 4500 \pm 1300 \, \text{km} \, \text{s}^{-1} \) (Moffett et al. 1993; Katsuda et al. 2009) at the current epoch \( t_{\text{SN}} \approx 10^3 \, \text{yr} \) for the ISM hydrogen number densities \( N_{\text{H}} \approx 0.08 \, \text{cm}^{-3} \) and \( N_{\text{H}} \approx 0.05 \, \text{cm}^{-3} \), respectively. These densities are consistent with the observed level of the VHE emission, as shown below. The best-fit value of the upstream magnetic field strength \( B_0 = 30 \, \mu \text{G} \) is quite insensitive to \( N_{\text{H}} \). The resulting current total shock compression ratios \( \sigma \) for \( N_{\text{H}} = 0.08 \, \text{cm}^{-3} \) and \( N_{\text{H}} = 0.05 \, \text{cm}^{-3} \) become now \( \sigma = 5.1 \) and 4.9, respectively, whereas the subshock compression ratios \( \sigma_s \) both remain close to \( \sigma_s = 3.7 \). Note that the explosion energy value \( E_{\text{sn}} = 2.4 \times 10^{51} \, \text{erg} \) is somewhat lower compared with the value \( E_{\text{sn}} = 3 \times 10^{51} \, \text{erg} \) that one would expect by extrapolating the value \( E_{\text{sn}} = 3.8 \times 10^{51} \, \text{erg} \) obtained earlier (Ksenofontov et al. 2005) for \( N_{\text{H}} = 0.1 \, \text{cm}^{-3} \) according to the expected dependence \( E_{\text{sn}} \propto N_{\text{H}} \). However, this difference is well within the range determined by the uncertainties of the observed values \( R_s \) and \( V_s \) when taking into account the relation \( R_s \propto E_{\text{sn}}^{1/5} \).

Since the properties of the accelerated CR nuclear and electron spectra and their dependence on the physical parameters, as well as the dynamical properties of the system, were described in detail in BKV09, they will not be discussed here again.

![Figure 1](image1.png)

**Figure 1.** Overall energy \( E_c \) of accelerated CRs, normalized to the total hydrodynamic energy release \( E_{\text{sn}} \), as a function of time, calculated for \( N_{\text{H}} = 0.08 \, \text{cm}^{-3} \) (solid line) and for \( N_{\text{H}} = 0.05 \, \text{cm}^{-3} \) (dashed line). Here and in all following figures, the quantity \( B_0 = B_0/\sigma \) denotes the amplified upstream magnetic field strength, where \( \sigma \) is the overall shock compression ratio and \( B_0 \) is the downstream magnetic field strength. The vertical dotted line shows the current evolutionary stage \( t_{\text{SN}} \).

### 3.1. Acceleration Efficiency

In Figure 1, the time dependence of the fractional energy \( E_c/E_{\text{sn}} \) contained in accelerated CRs during the SNR evolution is presented. Note that the value of \( E_c \) is reduced by a factor of \( f_{\text{re}} = 0.2 \) (see below) compared with the value calculated within the spherically symmetric model. According to Figure 1, \( E_c/E_{\text{sn}} \approx 0.05 \) and \( 0.065 \) for \( N_{\text{H}} = 0.05 \, \text{cm}^{-3} \) and \( N_{\text{H}} = 0.08 \, \text{cm}^{-3} \), respectively. This is lower than the value \( \approx 0.1 \) which is required on average for each SNR for their population to be the main source of CRs in the Galaxy. The reason is that SN 1006 is a quite young object in an evolutionary sense: according to Figure 1, \( E_c(t)/E_{\text{sn}} \) is expected to approach this canonical value in the subsequent evolution. On the other hand, the requirement for an average efficiency of \( \approx 0.1 \) is based on an assumed Galactic average value \( E_{\text{sn}} = 10^{51} \, \text{erg} \), which is about one-half of our value \( E_{\text{sn}} = 2 \times 10^{51} \, \text{erg} \). Therefore, the calculated efficiency \( E_c/E_{\text{sn}} \approx 0.05 \) fulfills the average requirement even for the present epoch.

### 3.2. Overall Nonthermal Spectra

Figure 2 illustrates the consistency of the synchrotron and \( \gamma \)-ray spectra, calculated with the best set of parameters \( \eta = 2 \times 10^{-4} \) as well as \( K_{\text{ep}} = 4.5 \times 10^{-4} \) for \( N_{\text{H}} = 0.05 \, \text{cm}^{-3} \), and \( \eta = 2 \times 10^{-4} \) as well as \( K_{\text{ep}} = 3.2 \times 10^{-4} \) for \( N_{\text{H}} = 0.08 \, \text{cm}^{-3} \), with the observed spatially integrated spectra. The H.E.S.S. data (Acero et al. 2010) for the northeastern (NE) and southwestern (SW) limbs, respectively, have been multiplied by a factor of two in order facilitate comparison with the full deduced \( \gamma \)-ray flux. As can be seen from Figure 2, the calculated synchrotron spectrum fits the observations both in the radio and the X-ray ranges (Allen et al. 2004; Bamba et al. 2008) very well. Note that the Chandra flux (Allen et al. 2004) is from a small region of the bright NE limb with minimal contributions from thermal X-rays; this X-ray flux was normalized to the overall Suzaku
ments (Acero et al. 2010), is consistent with the prediction of
in hard X-rays by Rothenflug et al. (2004) and Cassam-Chène (2004).
Such a geometry has also been found for the H.E.S.S. instrument by Naumann-Godo et al. (2008).

The reason is that in the analysis of BKV09, the total calculated γ-ray flux from the source is the sum of the fluxes from these
regions correspond to quasi-parallel shock regions are given by
Reynoso et al. (2011). Their analysis of the polarization of radio
emission led them to the conclusion that the magnetic field in
SN 1006 is radial at the NE and SW lobes but tangential in the
southeast (SE) of the radio shell. In addition, they established the
maximum fractional polarization in the SE which implies that
the magnetic field is highly ordered there. On the other hand, the
low fractional polarization in the lobes suggests a considerable
randomization of the magnetic field. The fact that a turbulent
magnetic field coexists with the brightest synchrotron emission
in the SW strongly supports the efficient acceleration of the CR
nuclear component, followed by magnetic field amplification
which provides its own directional randomization.

The question, whether these bright NE and SW regions of
SN 1006 represent quasi-parallel portions of the SN shock
or not, is still debated (Petruk et al. 2009; Schneider et al.
2010; Morlino et al. 2010). On the other hand, Bocchino et al.
(2011) have recently argued from the radio morphology that
the radio limits—morphologically similar to the X-ray and
γ-ray limits—are polar caps and that electrons are accelerated
with quasi-parallel injection efficiency. Like the aforementioned
theoretical arguments and X-ray analyses, this argues against a
scenario where the magnetic field is perpendicular to the shock
normal (Fulbright & Reynolds 1990). Bocchino et al. (2011) also concluded that the remaining asymmetries (converging
limbs and different surface brightness), also clearly visible in
the H.E.S.S. γ-ray data (Acero et al. 2010), could be explained
by a gradient of the ambient ISM magnetic field strength.

Additional arguments which confirm that the NE and SW
regions correspond to quasi-parallel shock regions are given by
Reynoso et al. (2011). Their analysis of the polarization of radio
emission led them to the conclusion that the magnetic field in
SN 1006 is radial at the NE and SW lobes but tangential in the
southeast (SE) of the radio shell. In addition, they established the
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magnetic field coexists with the brightest synchrotron emission
in the SW strongly supports the efficient acceleration of the CR
nuclear component, followed by magnetic field amplification
which provides its own directional randomization.

3.4. Relative Contributions to the γ-Ray Flux

Figure 3 shows the π^0-decay, the inverse-Compton (IC),
and the total (π^0-decay plus IC) γ-ray energy spectra of the
remnant, calculated for N_{HI} = 0.05 cm^{-3} and N_{HI} = 0.08 cm^{-3},
respectively. It can be seen that the γ-ray spectrum produced by
the nuclear CRs is rather close to the IC emission spectrum
produced by CR electrons alone, especially in the case of
N_{HI} = 0.05 cm^{-3}. Even for the case N_{HI} = 0.035 cm^{-3}, which is

Figure 2. Spatially integrated spectral energy distribution of SN 1006, calculated for
γ
0.08 cm^{-3} (solid line) and γ
0.05 cm^{-3} (dashed line), and compared with observational data. The observed radio spectrum is from a compilation of Allen et al. (2004). The form of the Chandra X-ray spectrum was measured for a small region of the bright northeastern (NE) rim of SN 1006 (Allen et al. 2004), whereas the Suzaku spectrum (blue color; Bamba et al. 2008) is for the entire remnant. The corresponding Chandra X-ray flux has been multiplied by a factor of 60 (red color) such as to be consistent with the Suzaku flux for energies ϵ > 2 keV. The H.E.S.S. data (Acero et al. 2010) are for the NE region (green color) and the SW region (magenta color). Both these H.E.S.S. data sets have been multiplied by a factor of two, in order to facilitate a comparison with the global theoretical spectra.

(A color version of this figure is available in the online journal.)

flux (Bamba et al. 2008) at energies ϵ > 2 keV (for details, see
BKV09).

The only important parameter which cannot be determined
from the analysis of the synchrotron emission data is the external
gas number density N_{HI}. Figure 2 shows that the spectrum of
synchrotron emission is almost non-sensitive to the ambient gas
density. Consequently, numerical solutions have been calculated
for the pair of values N_{HI} = 0.08 cm^{-3} and N_{HI} = 0.05 cm^{-3} that
appear to bracket the density range consistent with the H.E.S.S.
γ-ray measurements. It should be noted that this estimate for
N_{HI} is larger by a factor of ~1.5 than the estimate of BKV09.

The reason is that in the analysis of BKV09, the total calculated
γ-ray energy flux was compared with the energy flux from the
NE rim. However, the latter flux corresponded only to about
50% of the total observed energy flux, as it had been reported
for the H.E.S.S. instrument by Naumann-Godo et al. (2008).
A detailed comparison of the γ-ray spectrum with the calculated
spectrum will be made in the context of Figure 4.

3.3. γ-Ray Morphology

The γ-ray morphology, as found in the H.E.S.S. measure-
ments (Acero et al. 2010), is consistent with the prediction of
a polar cap geometry on account of the strongly preferred injection of
nuclear particles in the quasi-parallel regions of the shock (Völk et al. 2003). Such a geometry has also been found experimentally from an analysis of the synchrotron morphology in hard X-rays by Rothenflug et al. (2004) and Cassam-Chène et al. (2008). This means that the γ-ray emission calculated in the spherically symmetric model must be renormalized (reduced) by a factor f_{re} ≈ 0.2, as in Ksenofontov et al. (2005) and BKV09. This renormalization factor is applied here. The total γ-ray flux from the source is the sum of the fluxes from these two regions.

This morphology is also a key argument for the existence of an energetically dominant nuclear CR component in SN 1006, because only such a component can amplify the magnetic field to the observed degree. The accelerated electrons are unable to amplify the magnetic field to the required level (BKV09).
close to the minimum density \(N_H = 0.03 \text{ cm}^{-3}\) for the remnant, as argued by Acero et al. (2007), the total predicted \(\gamma\)-ray flux is still quite similar in spectral shape to the total flux for \(N_H = 0.05 \text{ cm}^{-3}\) (and essentially also for \(N_H = 0.08 \text{ cm}^{-3}\)), as shown in Figure 6 of BKV09. Unless the actual gas density would be small compared to \(N_H = 0.03 \text{ cm}^{-3}\), it would therefore be very difficult to observationally distinguish a hypothetical dominant IC scenario from the mixed \(\gamma\)-ray spectrum discussed here. Therefore, in the VHE range—except perhaps in the last decade of energy, near the cutoff, see Section 3.6—the observed \(\gamma\)-ray spectrum alone is not able to discriminate between the hadronic \(\pi^0\)-decay and the leptonic IC \(\gamma\)-ray components. However, it was already shown by Ksenofontov et al. (2005) that such a low total VHE emission flux, with a highly depressed IC \(\gamma\)-ray flux due to the synchrotron losses of high-energy electrons, is only possible if the nuclear CR component is efficiently produced with accompanying strong magnetic field amplification.

### 3.5. Radial Profile of \(\gamma\)-Ray Brightness

A possibility for an experimental discrimination between \(\pi^0\)-decay and IC \(\gamma\)-rays is given by the measurement of the radial profile of the \(\gamma\)-ray brightness. It was shown early on (Berezhko et al. 2002) that the radial profile of the TeV \(\pi^0\)-decay \(\gamma\)-ray emission has a peak near the remnant rim with a width of about 20\% of the remnant radius. On the other hand, the peak of the IC \(\gamma\)-ray emission was predicted to be much narrower, actually narrower by an order of magnitude. This latter characteristic is physically identical with the filamentary structure detected subsequently with Chandra in keV X-rays (Bamba et al. 2003; Long et al. 2003; and for Cas A by Vink & Laming 2003), because the keV synchrotron emission and the TeV \(\gamma\)-ray IC emission are produced by the same electrons, with energies approaching 100 TeV.

In Figure 4, radial profiles of the \(\gamma\)-ray brightness \(J_\gamma(\rho)\) are shown as functions of radial projection distance \(\rho\) corresponding to energies \(\epsilon_\gamma > 0.5 \text{ TeV}\), calculated for \(N_H = 0.05 \text{ cm}^{-3}\). It is seen that the profile of the IC component is indeed dramatically thinner than the profile of the \(\pi^0\)-decay component. The profile of the total \(\gamma\)-ray emission, which is the sum of these two components, has a width at half-maximum of about 0.15 \(R_s\), which is smaller than that observed by H.E.S.S. The total radial emission profile, smoothed to the H.E.S.S. point-spread function, agrees with the H.E.S.S. observations for \(\rho > 0.5 R_s\), but exceeds the H.E.S.S. values considerably at low radial distances \(\rho < 0.5 R_s\). The reason for this discrepancy is simple. As was said in Section 3.3, only those nuclear CRs that occupy two cones with opening angle 20°, with symmetry axes going from the remnant center toward the NE and SW directions, respectively, should be taken into account to model the hadronic emission of the actual remnant due to the strong angular dependence of the nuclear injection rate. The radial profiles of the \(\gamma\)-ray emission lobes, corresponding to the NE–SW directions produced by these CRs, are well consistent with the H.E.S.S. data (Figure 4). In this case, the line of sight at \(\rho < 0.7 R_s\) does not intersect the outer spherical region of the remnant which contains most of the accelerated CRs that provide the brightness depression at \(\rho < 0.7 R_s\).

A discrimination between these \(\pi^0\)-decay and IC \(\gamma\)-ray components would require a \(\gamma\)-ray instrument with an angular resolution that is an order of magnitude higher than that of H.E.S.S., or of any other existing \(\gamma\)-ray instrument. It is, however, to be noted that the measured radial H.E.S.S. profile, which gives a width of the measured peaks of 20\% of the remnant radius (Acero et al. 2010), is clear evidence that the
nuclear CR component is accelerated efficiently. Indeed, if the energy content in CR nuclei were small due to inefficient proton acceleration, then one would have to conclude that the magnetic field in the remnant is not amplified to any substantial degree. In such an inefficient scenario, electrons with energies of tens of TeV, which produce the TeV γ-ray IC emission, would be extremely smoothly distributed across the remnant—almost uniformly in the spherically symmetric case (Berezhko et al. 2002, 2003). Therefore, the expected radial width of the γ-ray emission region would be considerably larger than that expected from efficiently accelerated protons.

3.6. External Density

In order to exhibit the most detailed comparison of the calculated γ-ray spectra with the observations, Figure 5 the theoretical spectra for two ambient densities are presented together with the observed H.E.S.S. spectra for each of the polar caps on an expanded energy scale. For ease of comparison, the theoretical spectra are divided by a factor of two, which would correspond to the emission from a single polar cap region for an ideal dipolar morphology.

The H.E.S.S. spectra from the two regions are compatible with decreasing power-law distributions \( dF_\gamma/d\epsilon_\gamma \propto \epsilon_\gamma^{-\Gamma} \), with \( \Gamma \approx 2.3 \) and somewhat different fluxes \( \Phi(>1 \text{ TeV}) \), in the detected energy range \( 11.4 \leq \log \epsilon_\gamma \leq 13.2 \) (Acero et al. 2010). As Figures 2, 3, and 5 show, the H.E.S.S. data are also compatible with the more complex total theoretical γ-ray spectral energy flux density over this range that has a maximum at \( \log \epsilon_\gamma \approx 12.6 \). Above 10 GeV, the theoretical spectral energy flux can be approximated analytically by a power law with an exponential cutoff:

\[
dF_\gamma/d\epsilon_\gamma \propto \epsilon_\gamma^{-\Gamma} \exp(-\epsilon_\gamma/\epsilon_c),
\]

with cutoff energies \( \epsilon_c = 30 \text{ TeV} \) and 37 TeV for \( N_H = 0.05 \text{ cm}^{-3} \) and \( N_H = 0.08 \text{ cm}^{-3} \), respectively. These two analytical curves represent the best fit to the H.E.S.S. data. At the same time, they coincide within 4%, respectively, with the theoretical curves presented in Figure 5.

The quality of fit to the H.E.S.S. data is characterized by the values \( \chi^2/\text{dof} = 1.05 \) and 2.2 for the NE and SW parts of the remnant, respectively. It is comparable with the quality of fit by the pure power-law distributions \( dF_\gamma/d\epsilon_\gamma \propto \epsilon_\gamma^{-\Gamma} \), with \( \Gamma \approx 2.3 \) and different fluxes (Acero et al. 2010), which have, according to our calculation, a \( \chi^2/\text{dof} = 1.3 \) and 2.5 for the two regions, respectively.

In order to discriminate between those different spectral forms, the observational energy range should be extended both below the present H.E.S.S. threshold energy and above the present maximum energy detected by H.E.S.S.

According to Figure 5, the H.E.S.S. data are consistent with an ISM number density in the interval 0.05 \( \leq N_H \leq 0.08 \text{ cm}^{-3} \). There is also some indication that the actual number density is closer to the lower end of this interval, say \( N_H \approx 0.06 \text{ cm}^{-3} \). However, this is a weak conclusion, given the approximations made in the theoretical model in terms of the reduction factor \( f_{\text{red}} \) (see above). It should be noted nevertheless that the lower end of this interval is also preferred from the point of view of supernova explosion theory: the corresponding hydrodynamic explosion energy \( E_{\text{sn}} \approx 1.9 \times 10^{51} \text{ erg} \) is close to the upper end of the typical range of Type Ia SN explosion energies that vary by a factor of about two (Gamezo et al. 2005; Blinnikov et al. 2006). In addition, also from their X-ray measurements,
would expect in the simple case of energy-independent overlap of CR protons with the gas distribution.

This spread of CRs into the upstream region $r > R_s$, which for the highest energies $\epsilon \sim \epsilon_{\text{max}}(t)$ is faster than the shock expansion $R_s(t)$, represents the diffusive escape of CRs from the expanding SNR and was predicted by Berezhko & Krymsky (1988). Such a CR escape is expected in particular in the Sedov phase when the maximum energy $\epsilon_{\text{max}}(t) \propto B_0 R_s V_s$ of CRs, accelerated in the given evolutionary phase, decreases with time, because the value of $\epsilon_{\text{max}}$ is shock-size limited (see Berezhko 1996, for details) rather than time limited. At any given phase $t > t_b$ (where $t_b$ is the sweep-up time), efficient CR acceleration at the SN shock takes place only for energies $\epsilon \lesssim \epsilon_{\text{max}}(t)$, whereas for CRs with energies $\epsilon_{\text{max}}(t) < \epsilon < \epsilon_{\text{max}}(t)$, produced during earlier times, the acceleration process becomes inefficient and these CRs expand into the upstream region. Within the SNR model discussed in this paper, CR escape is relatively slow because the model assumes Bohm diffusion in the amplified magnetic field $B_0$ everywhere upstream for any CR energy. In reality, this is expected to be true only for CRs with energies $\epsilon < \epsilon_{\text{max}}(t)$ in the vicinity of the shock, where these CRs produce significant magnetic field amplification. At large distances upstream of the shock, $r - R_s \gg 0.1 R_s$, or/and for higher CR energies $\epsilon > \epsilon_{\text{max}}(t)$, CR diffusion is much faster than Bohm diffusion. Therefore, in actual SNRs, CR escape is expected to be faster and more intense than the present model predicts (see Ptuskin & Zirakashvili 2003; Drury 2011, for more detailed considerations). Since SN 1006 is only at the very beginning of the Sedov phase, this underestimate of the magnitude of escape is however not very critical.

### 4. SUMMARY

Based on a nonlinear kinetic model for CR acceleration in SNRs, the physical properties of the remnant SN 1006 were examined in a detail which corresponds to that of the available observations. Since the relevant astronomical parameters as well as the synchrotron spectrum of SN 1006 are measured with high accuracy, the values of the relevant physical parameters of the model can be estimated with similar accuracy for this SNR: proton injection rate $\eta \approx 2 \times 10^{-4}$, electron-to-proton ratio $K_{ep} \approx 3.8 \times 10^{-3}$, and downstream magnetic field strength $B_0 \approx 150 \mu G$. As a result, the flux of TeV emission detected by H.E.S.S. is predicted to agree with an ISM hydrogen number density $N_H \approx 0.06 \text{ cm}^{-3}$, consistent with the expectation from X-ray measurements. The corresponding hydrodynamic SN explosion energy $E_{\text{SN}} \approx 1.9 \times 10^{51}$ erg is somewhat above but fairly close to the upper end $E_{\text{SN}} = 1.6 \times 10^{51}$ erg of the typical range of Type Ia SN explosion energies. The efficiency of CR production is predicted to be between 5% and 6% up to the present epoch and expected to approach of the order of 10% during the further evolution. Normalized to a standard value of $E_{\text{SN}} = 10^{51}$ erg, the present efficiency is already about 10%.

The magnetic field amplification properties of this SNR can be understood as the result of azimuthal variations of the nuclear ion injection rate over the projected SNR circumference in terms of injection at quasi-parallel shocks only and the corresponding acceleration. This predicts the dipolar $\gamma$-ray emission morphology and is compatible with a polar cap-type X-ray synchrotron morphology. The $\gamma$-ray morphology is the key argument for the existence of an energetically dominant nuclear CR component in SN 1006. The magnetic field amplification allows the proton spectrum to extend to energies $\approx 10^{15}$ eV.
The difference between the $\gamma$-ray emission from the NE polar region and that from the SW polar region can be attributed to a small density difference between those regions.

It is shown that the high-energy tail of the $\gamma$-ray spectrum is affected by CR escape from the SNR interior, consistent with the comparatively low $\gamma$-ray cutoff near $10^{14}$ eV.

It is also concluded that the radial profile of the TeV $\gamma$-ray emission measured by H.E.S.S. is evidence that the nuclear CR component is indeed efficiently produced. In the opposite case of inefficient nuclear CR production, the magnetic field would not be expected to be amplified and the radial profile of the IC-dominated $\gamma$-ray emission would be expected to be significantly smoother than observed.

The sum of all these results suggests the conclusion that SN 1006 is a CR source with a high efficiency of nuclear CR production, required for the Galactic CR sources, both in flux as well as in cutoff energy.

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