Naturally light right-handed neutrinos in a 3-3-1 Model

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In this work we show that light right-handed neutrinos, with mass in the sub-eV scale, is a natural outcome in a 3-3-1 model. By considering effective dimension five operators, the model predicts three light right-handed neutrinos, weakly mixed with the left-handed ones. We show also that the model is able to explain the LSND experiment and still be in agreement with solar and atmospheric data for neutrino oscillation.

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I. INTRODUCTION

With the exception of nonzero neutrino mass and mixing, all the other collected experimental data in particle physics are consistent with the predictions of the standard model of the electroweak and strong interactions [1]. Concerning neutrinos, the understanding of the smallness of their masses and the largeness of their mixing, dictated by the neutrino oscillations experiments [2, 3], is a real puzzle in particle physics at the present.

On the theoretical side, if only left-handed neutrinos exist, then the most economical way they can acquire small masses is through the effective dimension-five operator [4]

\[
\mathcal{L}_{MP} = \frac{f_{ab}}{\Lambda} (\Phi^C_L \epsilon_{am}) (\Phi^C_n L_{bp}) \epsilon_{lm} \epsilon_{np} + \text{H.c.}
\]

where \(\Phi = (\phi^+ , \phi^0)^T\) and \(L_a = (\nu_{aL} , \epsilon_{aL})^T\). Thus when \(\phi^0\) develops a nonzero vacuum expectation value (VEV), \(v\), left-handed neutrinos automatically develop Majorana mass

\[
(M_L)_{ab} = \sqrt{2} f_{ab} v^2 / \Lambda,
\]

On supposing that such effective operator is realized in some high energy GUT scale, then we naturally have light masses for the left-handed neutrinos.

It could be that right-handed neutrinos also exist and they could be light too, but this feature is not naturally obtained in the standard model. For instance, to generate light right-handed neutrinos in the standard model, by effective dimension-five operators or by any other mechanism, an intricate combination of symmetries is required, usually accompanied by a considerable increasing in the particle content [5].

On the other hand, in the standard model, light right-handed neutrinos are interesting only if they can explain the LSND experiment [6]. This requires that the light right-handed neutrinos get weakly mixed with the left-handed ones. People usually refer to these light and weakly mixed right-handed neutrinos as sterile neutrinos.

In this letter we examine the problem of generating light right-handed neutrino masses in the framework of the 3-3-1 model where those neutrinos are already part of the spectrum. Interesting enough, we show that light right-handed neutrino masses are a natural outcome in this model. Our approach to the subject is through effective dimension-five operators. Basically, we construct all the effective operators allowed by the symmetries and particle content of the model and show that they yield light Majorana and Dirac mass terms for the neutrinos. Consequently, we will have three light active neutrinos and three light sterile ones [7], which makes this 3-3-1 model capable of easily explaining the LSND experiment.

II. THE MODEL

The model we consider is the 3-3-1 model with right-handed neutrinos [8, 9]. It is one of the possible models allowed by the \(SU(3)_C \otimes SU(3)_L \otimes U(1)\) gauge symmetry where the fermions are distributed in the following representation content. Leptons come in triplets and singlets

\[
L_a = \begin{pmatrix}
\nu_{aL} \\
\epsilon_{aL}
\end{pmatrix} \sim (1, 3, -1/3), \quad \epsilon_{aR} \sim (1, 1, -1),
\]

where \(a = 1, 2, 3\) refers to the three generations. After the spontaneous breaking of the 3-3-1 symmetry to the standard symmetry, the triplet above splits into the standard lepton doublet \(L_a = (\nu_{aL} , \epsilon_{aL})^T\) plus the singlet \((\nu_{aR})^C\). Thus this model recovers the standard model with right-handed neutrinos.

It is not a trivial task to generate light masses to the right-handed neutrinos in any simple extension of the standard model. However, in the 3-3-1 model in question, right-handed neutrinos can naturally obtain small masses through effective dimension-five operators. This is due, in part, to the fact that, in the model, the right-handed neutrinos compose, with the left-handed neutrinos, the same triplet \(L\). As we will see in the next section, it is this remarkable feature that turns feasible the raise of light right-handed neutrinos.
In the quark sector, one generation comes in the triplet and the other two compose an anti-triplet with the following content,

\[
Q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ d'_i \end{pmatrix}_L, \quad \sim (3, \bar{3}, 0), \quad u_{iR} \sim (3, 1, 2/3),
\]
\[
d_{iR} \sim (3, 1, -1/3), \quad d'_{iR} \sim (3, 1, -1/3),
\]
\[
Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ u'_3 \end{pmatrix}_L, \quad \sim (3, 3, 1/3), \quad u_{3R} \sim (3, 2/3),
\]
\[
d_{3R} \sim (3, 1, -1/3), \quad u'_{3R} \sim (3, 1, 2/3)
\] (4)

where \(i = 1, 2\). The primed quarks are the exotic ones but with the usual electric charges.

In the gauge sector, the model recovers the standard gauge bosons and disposes of five other gauge bosons called \(V^\pm, U^0, U^{01}\) and \(Z'\). Also, the model possesses three scalar triplets, two of them transforming as, \(\eta \sim (1, 3, -1/3)\) and \(\chi \sim (1, 3, -1/3)\) and the other as, \(\rho \sim (1, 3, 2/3)\), with the following vacuum structure \(\mathbb{R}\)

\[
\langle \eta \rangle_0 = \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle \rho \rangle_0 = \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle \chi \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}.
\] (5)

These scalars are sufficient to engender spontaneous symmetry breaking and generate the correct masses for all massive particles.

In order to have the minimal model, we assume the following discrete symmetry transformation over the full Lagrangian

\[
(\chi, \eta, \rho, e_{aR}, u_{aR}, u'_{3R}, d_{aR}, d'_{iR}) \rightarrow - (\chi, \eta, \rho, e_{aR}, u_{aR}, u'_{3R}, d_{aR}, d'_{iR}),
\] (6)

where \(a = 1, 2, 3\) and \(i = 1, 2\). This discrete symmetry helps in avoiding unwanted Dirac mass term for the neutrinos \(\mathbb{R}\) and implies a realistic minimal potential \(\mathbb{R}\).

With this at hand, the model ends up with the following Yukawa interactions,

\[
\mathcal{L}^Y = \lambda_4^1 \bar{Q}_L \chi^* d_{iR} + \lambda_4^2 \bar{Q}_3L \chi u'_{3R} + \lambda_4^3 \bar{Q}_L \eta^* d_{aR} + \lambda_4^4 \bar{Q}_3L \eta^* u_{aR} + \lambda_4^5 \bar{Q}_L \rho^* u_{aR} + \lambda_4^6 \bar{Q}_3L \rho d_{aR} + G_{ab} f_{ab} \bar{L}_R \rho e_{bL} + \text{H.c.}
\] (7)

which generate masses for all fermions, with the exception of neutrinos.

### III. NEUTRINO MASSES

In this section we construct all possible effective dimension-five operators in the 3-3-1 model with right-handed neutrinos that lead to neutrino masses. The first one involves the triplets \(L\) and \(\eta\). With these triplets we can form the following effective dimension-five operator

\[
\mathcal{L}_{ML} = \frac{f_{ab}}{\Lambda} \left( \overline{L}_a \eta^* \right) \left( \eta^T L_b \right) + \text{H.c.}
\] (8)

According to this operator, when \(\eta^0\) develops a VEV, \(v_\eta\), the left-handed neutrinos develop Majorana mass terms with the same form as in Eq. but now with

\[
(M_{L})_{ab} = \frac{f_{ab} v_\eta^2}{\Lambda}.
\] (9)

Due to the fact that right-handed neutrinos are not singlets in the model in question, a second effective dimension-five operator generating neutrino masses is possible. It is constructed with the scalar triplet \(\chi\) and the lepton triplet \(L\),

\[
\mathcal{L}_{MR} = \frac{h_{ab}}{\Lambda} \left( \overline{L}_a \chi^* \right) \left( \chi L_b \right) + \text{H.c.}
\] (10)

When \(\chi^0\) develops a VEV, \(v_\chi\), this effective operator provides Majorana masses for the right-handed neutrinos,

\[
(M_{R})_{ab} = \frac{g_{ab} v_\chi v_\eta}{\Lambda}.
\] (11)

Remarkably, a third effective dimension-five operator generating neutrino mass is possible, but now involving the scalar triplets \(\eta\) and \(\chi\),

\[
\mathcal{L}_{MD} = \frac{g_{ab}}{\Lambda} \left( \overline{L}_a \chi^* \right) \left( \eta^T L_b \right) + \text{H.c.}
\] (12)

which, remarkably, leads to the following Dirac mass term for the neutrinos,

\[
(M_{D})_{ab} = \frac{g_{ab} v_\chi v_\eta}{\Lambda}.
\] (13)

Thus, we have Majorana and Dirac mass terms for the neutrinos both having the same origin, i.e., effective dimension-five operators. As in the standard case, on supposing that the three effective dimension-five operators above are realized in some high energy GUT scale, we have thus light Dirac and Majorana mass terms.

In view of these neutrino mass terms, the usual manner of proceeding here is to arrange \(M_L, M_R\) and \(M_D\) in the following \(6 \times 6\) matrix,

\[
\begin{pmatrix} \nu^C_L & \bar{\nu} R \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu^C_L \\ \nu^C_R \end{pmatrix},
\] (14)

in the basis

\[
(\nu^C_L, \nu^C_R) = (\nu_e L, \nu_\mu L, \nu_\tau L, \nu'_e R, \nu'_\mu R, \nu'_\tau R),
\] (15)

with

\[
\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}.
\] (16)
At this point, two comments are in order. First, as the VEV $v_\nu$ is responsible for the breaking of the 3-3-1 symmetry to the standard symmetry, and that $v_\eta$ contributes to the spontaneous breaking of the standard symmetry, thus it is natural to expect that $v_\nu > v_\eta$, which implies $M_R > M_D > M_L$. This hierarchy among $M_R$, $M_D$ and $M_L$ leads to a feeble mixing among the left and right-handed neutrinos, characterizing the last as sterile neutrinos required to explain LSND experiment. Second, the model leads inevitably to three sterile neutrinos.

In order to check this, let us consider the case of one generation. In the basis $(\nu_{eL}, \nu_L^c)$ we have the mass matrix

$$
\frac{1}{\Lambda} \begin{pmatrix} f v_\nu^2 & g v_\eta v_{3L} \\ g v_\eta v_{3L} & h v_{\nu L}^2 \end{pmatrix},
$$

(17)

By diagonalizing this matrix for $v_{3L} > v_\eta$, we obtain the eigenvalues

$$
\frac{f h - g^2 v_\nu^2}{h} \Lambda, \quad \frac{h v_{\nu L}^2}{\Lambda},
$$

(18)

and the correspondent eigenvectors

$$
N_1 = \nu_{eL} + \frac{f h - g^2 v_\nu}{gh} \nu_{L}^c (\nu R)^C, \\
N_2 = (\nu_R)^C - \frac{f h - g^2 v_\nu}{gh} \nu_{eL}.
$$

(19)

We see that the magnitude of the mixing is basically established by the VEVs $v_\eta$ and $v_{3L}$ through the ratio $v_{3L}/v_\eta$. The typical values of such VEVs are $v_\eta = 10^2$ GeV and $v_{3L} = 10^3$ GeV. This leads to an active-sterile mixing of order of $10^{-1}$ which falls in the expected range of values required to explain LSND as discussed below.

In order to explain LSND experiment, we need at least one sterile neutrino. In the 3-3-1 model with right-handed neutrinos we have necessarily three sterile neutrinos. The masses and mixing of the neutrinos is dictated by the matrix $M$ in Eq. (17). As in the case of quarks and charged leptons, the masses and mixing angles of the active and sterile neutrinos is a question of an appropriate tuning of the couplings $f_{ab}$, $g_{ab}$ and $h_{ab}$.

Presently we have three kinds of experimental evidence for neutrino oscillation. One involves neutrino oscillation from the sun whose data are $2$. 90%CL $1.5 \times 10^{-8}$eV$^2 \leq \Delta m_{23}^2 \leq 3.4 \times 10^{-3}$eV$^2$, $\sin^2 2\theta_{23} > 0.92$. (20)

while the other evidence involves solar neutrino oscillation. The data in this case are $9$, 90%CL $7.4 \times 10^{-3}$eV$^2 \leq \Delta m_{12}^2 \leq 8.5 \times 10^{-5}$eV$^2$, $0.33 \leq \tan^2 \theta_{12} \leq 0.50$. (21)

The third evidence refers to the appearance of $\bar{\nu}_e$ in a beam of $\bar{\nu}_\mu$ observed by the LSND experiment $10$. This experiment does not have the status of the solar and atmospheric ones, since it needs to be confirmed. The Mini Boone experiment is in charge of this $11$.

The analysis of the data from LSND depends on the number of sterile neutrinos we suppose. For the case of only one sterile neutrino we have, the so called 3+1 scenario, where $12$,

$$
\Delta m_{41}^2 = 0.92eV^2, \ U_{e4} = 0.136 \text{ and } U_{\mu 4} = 0.205. \ (22)
$$

According to Ref. $13$, for the case of two sterile neutrinos, called 3+2 scenario $14$, we can have two possible schemes. In one case, the best fit leads to

$$
\Delta m_{41}^2 = 0.92eV^2, \ U_{e4} = 0.121 \text{ and } U_{\mu 4} = 0.204, \\
\Delta m_{51}^2 = 22eV^2, \ U_{e5} = 0.036 \text{ and } U_{\mu 5} = 0.224. \ (23)
$$

in the other case, we have

$$
\Delta m_{41}^2 = 0.46eV^2, \ U_{e4} = 0.090 \text{ and } U_{\mu 4} = 0.226, \\
\Delta m_{51}^2 = 0.89eV^2, \ U_{e5} = 0.125 \text{ and } U_{\mu 5} = 0.16. \ (24)
$$

We would like to provide a texture for $M$ that solves neutrino oscillation, i.e., that recovers as close as possible the neutrino data showed above. But as the 3-3-1 model provides three sterile neutrinos and we dispose of analysis considering at most two sterile neutrinos, we have to make some assumptions. We will neglect CP violation; assume that the third sterile neutrino decouples from the others; take $U_{e3} = 0$ and consider that the atmospheric angle is exactly maximal. By an appropriate choice of the free parameters $f_{ab}$, $g_{ab}$ and $h_{ab}$, and taking $v_\eta = 10^2$ GeV, $v_{3L} = 10^3$ GeV and $\Lambda = 10^{14}$ GeV, a possible texture for $M$ that incorporates such assumptions is,

$$
M = \begin{pmatrix}
0.0465 & 0.0208 & -0.0208 & 0.121 & 0.136 & 0.0 \\
0.0208 & 0.064 & -0.0166 & -0.0495 & 0.167 & 0.0 \\
-0.0208 & -0.0166 & 0.064 & 0.0495 & -0.167 & 0.0 \\
0.121 & -0.0495 & 0.0495 & 0.66 & 0.0 & 0.0 \\
0.136 & 0.167 & -0.167 & 0.0 & 0.851 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.7
\end{pmatrix} \text{ (eV)}.
$$

(25)

This mass matrix is diagonalized by the following mixing matrix,

$$
U^{(6)} \approx \begin{pmatrix}
0.847 & 0.476 & 0.0 & 0.179 & 0.154 & 0.0 \\
-0.344 & 0.581 & 0.71 & -0.0733 & 0.189 & 0.0 \\
0.344 & -0.581 & 0.71 & 0.0733 & -0.189 & 0.0 \\
-0.207 & 0.0 & 0.0 & 0.978 & 0.0 & 0.0 \\
0.0 & -0.309 & 0.0 & 0.0 & 0.951 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0
\end{pmatrix}.
$$

(26)
which leads to the following neutrino masses,

\[ m_1 \approx -5.5 \times 10^{-5} \text{eV}, \quad m_2 \approx 9.3 \times 10^{-3} \text{eV}, \]
\[ m_3 \approx 4.8 \times 10^{-2} \text{eV}, \quad m_4 \approx 6.9 \times 10^{-1} \text{eV}, \]
\[ m_5 \approx 9.4 \times 10^{-1} \text{eV}, \quad m_6 = 1.7 \text{eV}. \] (27)

The values for the neutrino masses in Eq. (27) and the pattern of \(U^{(6)}\) above, Eq. (26), accommodate the solar and atmospheric oscillation data along with the LSND experiment altogether. For the sterile neutrinos, we did not recover exactly the best fit, which we believe is due only to the set of assumptions made above.

We also would like to call the attention to the fact that our sterile neutrinos present non-standard interactions. For example, they interact directly to the charged leptons through a new charged gauge boson according to

\[ \frac{g}{\sqrt{2}} (\bar{\nu}_L) C \gamma^\mu \nu_R V^\mu_L + \text{H.c.}, \] (28)

and also couple to the active neutrinos through a new non-hermitian neutral gauge boson according to,

\[ \frac{g}{\sqrt{2}} (\bar{\nu}_L) C \gamma^\mu \nu_R U^\mu_L + \text{H.c.}. \] (29)

This turns the phenomenology of our sterile neutrinos much richer than usual. For example, in the standard case there is a conflict among cosmology and LSND result [13]. We are not sure if such conflict will persist here in view of these non-standard interactions. This means that our sterile neutrinos scenario demands that their cosmological aspects as, for instance, their abundance and temperature decoupling [13], their contribution to the dark matter [10] and, mainly, their implications for the big bang nucleosynthesis [13], must be revisited in the light of the non-standard interactions just mentioned [13].

Finally, we would like to say that although our sterile neutrinos couple directly to the active ones, see Eq. (29), they are still stable. The interactions in Eq. (28) allows the decay of the heavier sterile neutrinos in lighter neutrinos. For example we can have the following channel \(\nu_{\tau R} \rightarrow \nu_{\tau L} \nu_e L \nu_e R\). In this case we have the following expression for the decay width

\[ \Gamma = G_F^2 \frac{m_{\nu_{\tau R}} m_{\nu_e} m_W}{192 \pi^3 m_U}. \] (30)

For \(m_U = 250 \text{ GeV} \) and \(m_{\nu_{\tau R}} = 1.5 \text{ eV}\), we obtain a life-time of order of \(3.4 \times 10^{34} \text{s}\), leading, thus, to stable sterile neutrinos.

IV. CONCLUSIONS

The main achievement of this work is to show that the right-handed neutrinos that appear in a version of the 3-3-1 model can be naturally light when dimension five effective operators are included. Such neutrinos can be identified as sterile ones, offering us the possibility of explaining the LSND data. We have checked that, although the model leads to a 3+3 scenario, the results from a 3+2 scenario can be easily recovered by making an appropriate choice of the Yukawa couplings in the mass matrix \(M\). Remarkably, we have shown that besides LSND, this 3-3-1 model has all the necessary features to also explain solar and atmospheric neutrino oscillation data without adding any extra fields or intricate symmetries. This is an automatic property as far as the allowed dimension five operators here included can be embedded in some larger underlying theory, maybe a GUT or something else at higher energies than TeV scale.

Moreover, it is possible that the new non-standard interactions involving neutrinos can reveal a very different perspective concerning the conflict between LSND results and neutrino cosmology. That is something to be further analyzed but it is out of the scope of this work. However, we should stress that all this characteristics of this 3-3-1 model are fairly appealing, considering the tiny amount of assumptions we had to rely on.

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Although we refer to those neutrinos as sterile neutrinos, we call the attention to the fact that in the framework of the 3-3-1 model those neutrinos are not sterile because they interact with the active neutrinos and the charged leptons as showed in Eqs. [28] and [29].