On the fairness of the main galaxy sample of SDSS

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Abstract  Flux-limited and volume-limited galaxy samples are constructed from SDSS data releases DR4, DR6 and DR7 for statistical analysis. The two-point correlation functions $\xi(s)$, monopole of three-point correlation functions $\zeta_0$, projected two-point correlation function $w_p$ and pairwise velocity dispersion $\sigma_{12}$ are measured to test if galaxy samples are fair for these statistics. We find that with increment of sky coverage of SDSS, $\xi(s)$ of flux-limited sample is extremely robust and insensitive to local structures at low redshift. But for volume-limited samples fainter than $L^*$ at large scales $s > \sim 10 h^{-1}\text{Mpc}$, deviation of $\xi(s)$ and $\zeta_0$ of DR7 to those of DR4 and DR6 increases with larger absolute magnitude. In the weakly nonlinear regime, there is no agreement between $\zeta_0$ of different data releases in all luminosity bins. Furthermore, $w_p$ of volume-limited samples of DR7 in luminosity bins fainter than $-M_{r,0.1} = [18.5, 19.5]$ are significantly larger, and $\sigma_{12}$ of the two faintest volume-limited samples of DR7 display very different scale dependence than results of DR4 and DR6. Our findings call for cautions in understanding clustering analysis results of SDSS faint galaxy samples, and higher order statistics of SDSS volume-limited samples in the weakly nonlinear regime. The first zero-crossing points of $\xi(s)$ of volume-limited samples are also investigated and discussed.

Key words: galaxies: distances and redshifts — galaxies: statistics — cosmology: observation — cosmology: large-scale structure

1 INTRODUCTION

Clustering analysis of galaxy samples thrives for the availability of modern massive galaxy surveys. The two mostly successful and biggest galaxy surveys to date are the two-degree field galaxy redshift survey (2dFGRS, Colless et al. 2003) and the Sloan Digital Sky Survey (SDSS, York et al. 2000). The final data release of the 2dFGRS offers 3-D mapping of roughly a quarter of million galaxies, the SDSS achieves spectra of $\sim 0.9$ million galaxies (Abazajian et al. 2009). The unprecedented number of galaxies and enormous volume surveyed by SDSS defines its unique role in the era of precision cosmology (Komatsu et al. 2010), its power spectra and the two-point correlation functions (2PCF) at large scales (e.g. Tegmark et al., 2004; Eisenstein et al., 2005; Percival et al., 2010; Reid et al., 2010).

Another highly appreciated application of clustering analysis of galaxies is to relate galaxy distribution to dark matter and halos, aiming at inferring processes galaxies experienced during their formation and evolution. Interpretation of statistics of galaxy samples provided by SDSS prevails in category of the $\Lambda$CDM+halo model and relevant extensions such as the halo occupation distribution (HOD, e.g. Berlind & Weinberg, 2002; Kravtsov et al. 2004; Zheng et al. 2005) and the conditional luminosity function (CLF, Yang et al. 2003). For example, works of Zehavi et al. (2002, 2005, 2010) systematically explored the luminosity and color dependence of galaxy 2PCFs and extensively quantified HOD
parameters of galaxies; Cooray (2006) derived the occupation of central and satellite galaxies in halos and their corresponding conditional luminosity functions from a compilation of correlation functions of SDSS, attempting to draw clues of galaxy evolution with reference to high redshift samples; Li et al. (2007) rather directly compared projected correlation functions and the pairwise velocity dispersion (PVD) of SDSS with those of mock galaxy samples populated from N-body simulations by semi-analytic models (SAM) of Kang et al. (2005) and Croton et al. (2006), they find that SAM can roughly reproduced observed clustering of SDSS galaxies but have to reduce faint satellite fraction in massive halos in the prescription of SAM by $\sim 30$ percent to resolve discrepancies in PVD.

Yet there are challenges to the fairness of SDSS galaxy samples, i.e. whether galaxy samples of SDSS are complete and have enough volume to be a fair representation of the Universe. In fact prudence in reading out physics from measured statistics especially correlation functions has been called. Nichol et al. (2006) disclosed that exclusion of the Sloan Great Wall (at $z \sim 0.08$, Gott et al., 2005) would change the 2PCF by $\sim 40\%$ and the three-point correlation function (3PCF) by as much as $\sim 70\%$ of the sample defined by the $r$-band absolute magnitudes $-22 \leq M_{r,0.1} \leq -19$. The apparent influence of super structures on estimated correlation functions at large scales is somehow against intuition since one already takes it for granted that the SDSS galaxy sample’s depth and sky coverage is sufficient to accomplish homogeneity, spatial averaging would suppress the variance induced by a particular structure in a small patch of sky. Sylos Labini et al. (2009) noticed that the zero-crossing point of 2PCF of SDSS main galaxy sample varies with luminosity and sample depth, and anti-correlation is absent in the mostly recent measured 2PCF of SDSS luminous red galaxy (LRG) sample (e.g. Martínez et al., 2009; Kazin et al., 2010). By the extreme-value statistical analysis, Antal et al. (2009) purport that either the SDSS suffers from severe sample volume dependent intrinsicsystematical effects or there is persistent density fluctuation not fading away over scales beyond standard $\Lambda$CDM model prediction.

It is therefore important for one to check the fairness of galaxy samples used in order to endorse the confidence of relevant analysis. It is understood that fairness means differently for different statistics and also samples constructed in various ways. Zehavi et al. (2010) have laboriously evaluated finite volume effects and impact of super structures, they compared 2PCFs of volume-limited galaxy sub-samples in full depth with of the same sub-sample but limited in a smaller volume overlapped with the volume-limited sub-sample defined in the luminosity bin one dex lower. Their experiment leads to the conclusion that finite volume effects are insignificant for anisotropic and projected 2PCFs in nonlinear regime for their sub-samples of luminosity higher than $M_{r,0.1} = -19$. But they then find that including faint galaxies causes weird regulation to 2PCF, which is similar but of smaller amplitude to the discovery in Zehavi et al. (2005) using an early release of SDSS. We notice that such analysis for galaxies of luminosity lower than $-18$ is missed though 2PCF of their faintest sub-sample $M_{r,0.1} \in [-18, -17]$ is adopted for estimation of biasing and HOD parameters.

These works mainly concentrate on changes to two-point statistics by altering sample depth, we rather check the fairness by sky coverage enlargement, not only of 2PCFs but also of monopole of 3PCFs in redshift space, projected 2PCFs and PVDs. There are data releases 4, 6 and 7 of SDSS main galaxy catalogue (DR4, DR6 and DR7 by Adelman-McCarthy et al. 2006, 2008; Abazajian et al., 2009, respectively), the increment of sky coverage from DR4 to DR6 is roughly the same from DR6 to DR7. An advantage to investigate effects of sample volume on correlation function by sky angular coverage other than survey depth is that the restriction of apparent magnitudes of the survey definition limits permitted range of depth adjustment, in particular for those faint galaxies which are visible only at low redshift and support very shallow sample space. And one of our purpose is to see how correlation function evolves naturally with the progress of a real survey.

Section 2 describes SDSS data and estimation methods of statistics we used, results are shown in section 3, the last section is of summary and discussion.
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2 GALAXY SAMPLES AND ESTIMATION OF CORRELATION FUNCTIONS

2.1 sample construction

The safe galaxy sample of the New York University Value-Added Galaxy Catalog (NYU-VAGC, Blanton et al., 2005) is a catalog of low redshift galaxies (mostly below \( z \sim 0.3 \)) defined by apparent magnitudes of \( 14.5 < m_r < 17.6 \). Three data releases in chronological order are selected, namely DR4, DR6 and DR7, which spectroscopic surveyed areas are about 4,783, 6,860 and 8,032 square degrees respectively. As spectroscopic coverage of SDSS is not uniform, we use only those regions of spectroscopic completeness greater than 0.9. We did not perform fibre collision correction to improve completeness, the correction only becomes significant at scales \(< 0.2 h^{-1}\text{Mpc}\) for SDSS galaxies (Zehavi et al., 2002). To ensure the correct geometry, galaxies in the three catalogues are also filtered with their own accompanied survey windows, bright star masks and completeness masks.

Flux-limited samples defined by \( r \)-band apparent magnitude range \( 14.5 < m_r < 17.6 \) and redshift \( 0.01 < z < 0.23 \) are generated. Consequently we obtain 300, 661 galaxies in DR4, 447, 407 in DR6, and 535, 845 in DR7. In order to explore influence of local galaxies on correlation functions, we also constructed confined flux-limited galaxy samples by near-end redshift cut of \( z_{\min} = 0.037, 0.046, 0.071 \). Volume-limited sub-samples are also produced in consecutive luminosity bins starting from \( M_{r,0.1} = -17 \) to \( -22.5 \) in step of 0.5 magnitude and bin width of one magnitude, the absolute magnitude in NYU-VAGC is corrected to redshift \( z = 0.1 \) and is \( K \) corrected, but e-correction is not taken into account. We noticed that there are some galaxies having different apparent magnitudes in DR7 than in early data releases, so we constructed a couple of additional volume-limited samples from DR7 but filtered with masks of DR4 for comparison, measurements indicate that such differences have little influence on statistics employed.

Details of these samples are shown in Tables 1 & 2 and Figure 1. Comoving distances of galaxies are calculated in a flat \( \Lambda \)CDM universe with \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \) and \( h = 0.7 \).

\[ \text{http://sdss.physics.nyu.edu/vagc} \]
Table 1 Numbers of galaxies in flux limited samples defined by $r$-band apparent magnitude $14.5 < m_r < 17.6$ and redshift $z_{min} \leq z \leq 0.23$.

| $z_{min}$ | 0.010 | 0.037 | 0.046 | 0.071 |
|----------|-------|-------|-------|-------|
| DR4      | 300,661 | 281,400 | 268,247 | 216,573 |
| DR6      | 447,407 | 417,426 | 397,543 | 321,915 |
| DR7      | 535,845 | 498,445 | 473,980 | 382,921 |

Table 2 Volume limited samples. Distances are in unit of $h^{-1}$Mpc.

| Label | Luminosity | Redshift | Comoving Distance | Number of Galaxies |
|-------|------------|----------|-------------------|-------------------|
|       | $M_{r,0.1} - 5 \log_{10} h$ | $z_{min}$ | $z_{max}$ | $d_{min}$ | $d_{max}$ | DR4 | DR6 | DR7 |
| VL1   | $-18.0, -17.0$ | 0.011 | 0.029 | 33.89 | 87.31 | 4,223 | 6,389 | 8,219 |
| VL1+  | $-18.5, -17.5$ | 0.014 | 0.037 | 42.53 | 108.95 | 7,292 | 11,543 | 14,343 |
| VL2   | $-19.0, -18.0$ | 0.018 | 0.046 | 53.32 | 135.61 | 31,807 | 47,565 | 57,363 |
| VL2+  | $-19.5, -18.5$ | 0.022 | 0.057 | 66.77 | 168.27 | 19,209 | 29,463 | 35,932 |
| VL3   | $-20.0, -19.0$ | 0.028 | 0.071 | 83.51 | 207.96 | 50,719 | 75,162 | 89,654 |
| VL3+  | $-20.5, -19.5$ | 0.035 | 0.087 | 104.24 | 255.70 | 60,132 | 89,602 | 107,207 |
| VL4   | $-21.0, -20.0$ | 0.044 | 0.107 | 129.83 | 312.96 | 59,215 | 87,295 | 103,924 |
| VL4+  | $-21.5, -21.0$ | 0.054 | 0.131 | 161.21 | 381.91 | 31,807 | 47,565 | 57,363 |
| VL5   | $-22.0, -21.0$ | 0.068 | 0.160 | 199.41 | 462.71 | 46,264 | 69,499 | 82,239 |
| VL5+  | $-22.5, -21.5$ | 0.083 | 0.194 | 245.46 | 555.88 | 24,002 | 36,677 | 43,631 |

2.2 Estimation of correlation functions

2.2.1 Redshift space correlation functions

Isotropic 2PCF $\xi(s)$ of separation $s$ in redshift space is measured with the estimator of Landy & Szalay (1993),

$$\xi = \frac{DD - 2DR + RR}{RR},$$

in which $DD$, $RR$ and $DR$ are respectively the normalised numbers of weighted galaxy-galaxy, random-random and galaxy-random pairs at given separation. To proceed the estimation with Eq. 1, a corresponding random sample is generated following distributions of redshift, magnitude, geometric constraints, spectroscopic completeness and survey masks of each individual galaxy sample but with 20 times of numbers of points. Each galaxy and random point is assigned a weight according to their redshift and angular position to minimize the variance in estimated $\xi$ (Efstathiou, 1988; Hamilton, 1993),

$$w_i = \frac{1}{1 + 4\pi n(z)\Phi_i J_3(s)},$$

where $\Phi_i$ is the selection function at the location of the $i$th galaxy, $n(z)$ is the mean number density, and $J_3(s) = \int_0^s \xi(s)ds$. The $J_3(s)$ is computed using a power-law $\xi(s)$ with correlation length $s_0 = 8 h^{-1}$Mpc and $\gamma_0 = 1.2$ (Zehavi et al., 2002).

Calculation of 3PCFs of all those galaxy samples lasts too long, we turn to measure the monopole of the 3PCF instead (Pan & Szapudi, 2005), which is a degenerated version of 3PCF defined as

$$\zeta_0(s_1, s_2) = 2\pi \int_{-1}^{1} \xi(s_1, s_2, \theta)d\cos\theta,$$

and estimated via

$$\zeta_0 = \frac{DDD - 3DDR + 3DRR - RRR}{RRR},$$
where combined symbols of D and R are normalized numbers of triplets counted within and between
data sets of galaxies and random points, e.g. if the number of galaxies around galaxy i in bin \((s_1^h, s_1^\ell)\)
is \(n_i(s_1)\), in bin \((s_2^h, s_2^\ell)\) is \(n_i(s_2)\), the \(DDD\) in Eq.\(\text{[4]}\) reads

\[
DDD = \left\{ \begin{array}{ll}
\sum_{N_p}^{\infty} n_i(n_1) n_i(n_2) \\ N_p(N_p - 1)(N_p - 2)
\end{array} , \quad if \ s_1 \neq s_2
\right. , \quad \left(5\right)
\]

\[
\sum_{N_p}^{\infty} n_i(n_1) (n_i(s_2) - 1) \\ N_p(N_p - 1)(N_p - 2)
\end{array} , \quad if \ s_1 = s_2
\right.
\]

2.2.2 Projected 2PCF and PVD

To minimize the effect of redshift distortion due to galaxy’s peculiar motion, the separation \(s\) (or \(r\) in real space) is divided into two components, the parallel part \(\pi\) and the perpendicular part \(\sigma\) with respect
to line-of-sight, the anisotropic 2PCF is measured on grids of \((\sigma, \pi)\). Integration of \(\xi(\sigma, \pi)\) over \(\pi\) then
yields a redshift distortion free function, the projected 2PCF,

\[
w_p(\sigma) = \int_{-\pi_{\text{max}}}^{\pi_{\text{max}}} \xi(\sigma, \pi) d\pi = \sum_i \xi(\sigma, \pi_i) \Delta \pi_i , \quad \left(6\right)
\]

which has practically an integration limit \(\pi_{\text{max}} = 50 \, h^{-1} \text{Mpc}\).

It is well known that the redshift distortion consists of two components dominated in different regimes,
coherent infall is responsible for the clustering enhancement at large scales while the smearing
of correlation strength at small scales is attributed to random motions. At large scales the boost to the
2PCF by the peculiar velocities takes a particularly simple form [Kaiser, 1987, Hamilton, 1992],

\[
\xi'(\sigma, \pi) = \xi_0(s) P_0(\mu) + \xi_2(s) P_2(\mu) + \xi_4(s) P_4(\mu) , \quad \left(7\right)
\]

where \(P_\ell(\mu)\) is Legendre polynomials, \(\mu\) is the cosine of the angle between \(r\) and \(\pi\). Assuming \(\xi = (r/r_0)^{-\gamma}\) there are relations

\[
\xi_0(s) = \xi(s) = \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right) \xi(r)
\]

\[
\xi_2(s) = \left(\frac{4\beta}{3} + \frac{4\beta^2}{7}\right) \left(\frac{\gamma}{\gamma - 3}\right) \xi(r)
\]

\[
\xi_4(s) = \frac{8\beta^2}{35} \left(\frac{\gamma(2 + \gamma)}{(3 - \gamma)(5 - \gamma)}\right) \xi(r) , \quad \left(8\right)
\]

where \(\beta \approx \Omega_0^{0.6}/b\), and \(b\) is the linear bias parameter, note that the first equation is independent on the
functional form of \(\xi(r)\).

To incorporate effects of random motion, the anisotropic 2PCF in redshift space is approximated
by a convolution of \(\xi'(\sigma, \pi)\) in Eq.\(\text{[7]}\) with the distribution function of the pairwise velocity \(f(v_{12})\) (c.f.
Peebles, 1993),

\[
\xi(\sigma, \pi) = \int_{-\infty}^{+\infty} \xi'(\sigma, \pi - v_{12}/H_0) f(v_{12}) dv_{12} , \quad \left(9\right)
\]

and in general \(f(v_{12})\) is assumed to obey an exponential distribution with PVD \(\sigma_{12}\)

\[
f(v_{12}) = \frac{1}{\sigma_{12} \sqrt{2} \pi} \exp \left(-\frac{\sqrt{2}v_{12}}{\sigma_{12}}\right) . \quad \left(10\right)
\]

The parameter \(\beta\) usually is derived from the ratio of \(\xi(s)\) to \(\xi(r)\) at large scales via the first equation
in Eq.\(\text{[8]}\) then other model parameters can be determined by combining Eq.\(\text{[7]}\)– Eq.\(\text{[10]}\) to fit the \(\xi(\sigma, \pi)\)
data grids. Note that [Jing et al., 1998] assumed a slightly different exponential distribution function of
pairwise velocity which is followed by [Li et al., 2007].
2.2.3 Covariance Matrix

Covariance matrices our results are computed with the jack-knife technique (Lupton, 1993; Zehavi et al., 2002). Each galaxy sample is divided into twenty separate slices of approximately equal sky area, then we perform the analysis twenty times, at each time leave a different slice out. Covariance matrices are generated accordingly with these twenty measurements, for instance, covariance of 2PCF measured in two bins of $i$ and $j$ is simply

$$\text{Cov}(\xi_i, \xi_j) = \frac{N - 1}{N} \sum_{\ell=1}^{N} (\xi_{i,\ell} - \overline{\xi}_i)(\xi_{j,\ell} - \overline{\xi}_j),$$

in which $N = 20$ is the number of jack-knife sub-samples we used.

3 RESULTS

3.1 Flux-limited samples

Isotropic 2PCFs of flux-limited samples in Table 1 are calculated firstly. Figure 2 manifest that the redshift space 2PCFs of flux-limited samples show little variation against data versions of SDSS. $\xi(s)$ of DR4 exhibits some deviation at large scales $\sim 100 \, h^{-1}\text{Mpc}$, but is hardly significant for the huge cosmic variance at these scales. $\xi(s)$ of flux-limited samples of the same data release are displayed in the right panel of Figure 2, there is no visible change to redshift space 2PCF of SDSS when galaxies with low redshift are excluded even when the lost of number of galaxies is as much as $\sim 25\%$ (Table 1).

Thus eliminating local volume and enlarging sky coverage from DR4 to DR7 have little influence on the clustering strength measured, unlikely there is any significant sample volume dependent effects. As we are not interested in general discussion of the SDSS main galaxy catalogue as a whole, we stop preforming further analysis with other statistical measures.

It is well known that faint galaxies have much lower linear bias than luminous ones (e.g. Tegmark et al., 2004; Zehavi et al., 2003; Li et al., 2007), when we throw away many faint galaxies by
imposing near-end redshifts limit it is expected that $\xi(s)$ should display higher amplitude when lower redshift cuts increase. It could be that the lost in number of galaxies (after proper weighting) is too small to raise any serious deviation (Table 1), or in another words $\xi(s)$ of flux-limited sample is dominated by galaxies around the redshift distribution peak.

### 3.2 Volume-limited samples

#### 3.2.1 2PCF and monopole of 3PCF in redshift space

2PCFs $\xi(s)$ and monopoles of 3PCFs $\zeta_0(\mathbf{s})$ of volume-limited samples of the three SDSS data releases are measured to probe possible differences. In this paper we only present the $\zeta_0(\mathbf{s})$ which amplitude is the strongest among configurations of $(\mathbf{s}_1, \mathbf{s}_2)$ (Pan & Szapudi, 2005). As seen in Figure 3 in nonlinear regime major discrepancies appear in the VL1 sample of the lowest luminosity, differences between results of DR4 and DR7 are around $2\sigma$ at scales as small as $\sim 3h^{-1}$Mpc, while consistency of $\xi(s)$ and $\zeta_0$ of brighter volume-limited samples of SDSS is perfect at scales $s < 10h^{-1}$Mpc.

At scales greater than $10h^{-1}$Mpc, for subsamples of VL3+ – VL5+ $\xi(s)$ of different data releases are in good agreement within errorbars, but $\zeta_0$ have variations at level of $\sim 1\sigma$ (Figure 4). For the five faint galaxy samples of VL1 – VL3, disagreement in $\xi(s)$ of DR7 to DR4 is already apparent in the regime, which is confirmed by their $\zeta_0$. We conclude that modulation to correlation functions in redshift space resulted from enlargement of sky coverage mainly occurs at scales ranging roughly from

![Figure 3](image-url)
Fig. 4 2PCFs $\xi(s)$ and monopoles of 3PCF $\zeta_0(s_1 = s_2)$ at large scales in redshift space of volume-limited samples.

$\sim 10$ to $\sim 50 \, h^{-1}\text{Mpc}$ where is usually classified as weakly nonlinear regime in structure formation theory. Those applications and conclusion appear somehow suspicious based on 3PCFs of volume-limited samples of SDSS at large scales. For three-point correlation functions in redshift space, fairness of volume-limited samples is guaranteed only at small scales, i.e. in strongly nonlinear regime.

3.2.2 The First Zero-crossing points of 2PCFs

To investigate the charge of [Sylos Labini et al. (2009)] the first zero crossing scales of $\xi(s)$ against median luminosity of volume-limited samples are plotted in Figure 5. Estimated $\xi(s)$ is effectively averaged over a scale bin $[s_{lo}, s_{hi}]$ and the quoted scale is set to be $s = \sqrt{s_{lo} s_{hi}}$. Unlikely we can right hit all zero points of $\xi(s)$ by our scale binning, so we choose to show the range of scales within which $\xi(s)$ experiences zero-crossing which is drawn as errorbars over the geometric mean of the pair of scales. From Figure 5 it is clear that in general the brighter is the characteristic luminosity of the sample, the larger the first zero crossing scale will be. The five faint volume-limited samples (VL1 – VL3) have roughly the same first zero crossing scale with mild variation between $\sim 30 - 50 \, h^{-1}\text{Mpc}$, then the crossing scale ascends abruptly to as large as more than $100 \, h^{-1}\text{Mpc}$ and even higher than the largest scale we measured ($\sim 170 \, h^{-1}\text{Mpc}$).

For faint galaxy samples, their depths are typically small and so be their effective volumes, the systematical effect of integral constraint can not be ignored ([Landy & Szalay, 1993; Bernstein, 1994]).
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Fig. 5 Luminosity dependence of the first zero crossing scales of $\xi(s)$ of volume-limited samples. Lower caps of error bars are scales where $\xi > 0$ and higher caps of error bars are the adjacent scales where $\xi$ immediately becomes negative, those points showing with only lower caps denote that the first zero crossing point is actually larger than the scale probed in this work, larger than $\sim 170 h^{-1} \text{Mpc}$.

In the weak correlation limit, the cosmic bias resulted from integral constraint can be approximated by

$$b_\xi = \frac{\hat{\xi}}{\xi} - 1 \approx -\frac{\bar{\xi}(R)}{\xi}, \quad \text{if } |\xi|, |\bar{\xi}(R)| \ll 1,$$

in which $\hat{\xi}$ is the estimated 2PCF, $R$ is the smallest size of the sample and $\bar{\xi}(R)$ is the average of the 2PCF over the sample volume, i.e. density variance at sample volume (Landy & Szalay, 1993). There is no a priori correction method to this bias unless we assume something to model the shape of the 2PCF. Since $\bar{\xi}$ is positive, naturally $\xi \approx \xi - \bar{\xi}(R)$ will have a smaller first zero-crossing scale than $\xi$. If as usual we assume that galaxy bias $b$ is linear and scale independent, $\bar{\xi} = b^2(\xi - \bar{\xi}(R))$, the correction to the first zero-crossing scale depends on sample volume only. As $\bar{\xi}$ decreases with scale slowly, it is expected that the first zero-crossing scales of faint galaxy samples will gradually become larger when sample volume increases, which is true for VL2 – VL3. However, surprisingly, it is not for VL1 and VL1+. The faintest two subsamples have the smallest sample volume, but the first-zero crossing scale of VL1 does not change when SDSS marching from DR4 to DR7, while of VL+ the scale of DR7 becomes smaller than of DR4. Furthermore, the difference between depths of VL3 and VL3+ is not very large (Table 2), but the first zero-crossing scales of their $\xi$ differ hugely. Integral constraint alone could not explain the findings.

The increment of sky coverage from DR4 to DR6 is approximately the same as the gain from DR6 to DR7, the first zero crossing scales of DR7 only differ from DR6 slightly in two luminosity bins, while DR4 does not agree with other data releases significantly, which makes it difficult to clutch at other simple geometric explanation, such as assuming fractal galaxy distribution. Ergodicity bias could not be resorted too, for low luminosity samples with low characteristic redshift, the correction $\Delta \xi$ is positive (Pan & Zhang, 2010) and would push zero point to larger scales, which obviously contradicts
observation. Neither could be redshift distortion, as on large scales redshift distortion acts on galaxy 2PCF as a multiplication.

The sudden change of the first zero-crossing scale from faint galaxies to bright galaxies probably implies that the composition of faint galaxy samples is very different compared with bright galaxy samples, which may be attributed to the shifting of leading role from satellite galaxies to central galaxies in samples brighter than $M_r, 0.1 > 20$ (Li et al., 2007). Whatever the physical mechanism is, mathematically the effect to 2PCF is fully packed into a simple function, the galaxy bias. The linear biasing model assumes that on large scales the galaxy 2PCF $\xi_g = b \xi_m$ in which $b \neq 0$ is a deterministic, scale independent bias parameter and $\xi_m$ is the 2PCF of dark matter, obviously if the model holds, the zero point of $\xi_g$ will not change no matter what $b$ could be, e.g. scale dependent. If we presume that the problem of zero crossing is in biasing, then either stochastic or nonlinear bias has to be invoked. Simple calculation indicates that if we adopt the parametrization to bias of Fry & Gaztanaga (1993) and include the second-order bias parameter in 2PCF, to the leading order the effect is again multiplicative and can not shift the first zero point of 2PCF. It appears that stochastic biasing have to be considered. Details of the calculation however is beyond scope of this paper and will be presented elsewhere.

Another interesting aspect is that the first zero-crossing scales 2PCFs of samples VL4 and VL4+ of DR4 are larger than the largest scale of our measurements, but not of DR6 and DR7. The lack of anti-correlation in the two luminosity bins of DR4 is probably an evidence of the modulation due to the Sloan Great Wall as revealed by Zehavi et al. (2005) and Nichol et al. (2006), the increased sky coverage of DR6 and DR7 just successfully weakens the influence of the super structure (Zehavi et al., 2010).

### 3.2.3 Projected 2PCF and PVD

$\xi(s)$ is a mixture of real space 2PCF and PVD. The entanglement can be sorted with the projected 2PCF $w_p$. Measurements of $w_p$ are shown in Figure 5 actually we cross checked our $w_p$ of DR7 with available results of Zehavi et al. (2010), the agreement is excellent except for the sample VL2 of which our $w_p$ differs at scales $\sigma > 4 \text{Mpc}/h$. As seen in Figure 4, $w_p$ of DR4 and DR6 are in good agreement at scale range probed in most luminosity bins, $w_p$ of DR6 are slightly larger at large scale around $\sigma \sim 10 \h^{-1} \text{Mpc}$ in several faint samples but of low significance for the size of error bars. For VL1 and VL2, their $w_p$ of DR7 are boosted by more than 70% in amplitude relative to of DR4, but the shape does not change. For subsamples in other luminosity bins, their $w_p$ are stable against data version, though for VL1+ and VL2+ there are some minor changes within errorbars.

Figure 7 demonstrates the scale dependence of PVDs $\sigma_{12}$ of different luminosity samples while Figure 8 is of the luminosity dependence of PVDs measured at scales of $\sigma = 0.27, 0.87, 2.7, 8.7 \h^{-1} \text{Mpc}$ respectively. $\sigma_{12}$ of subsamples VL1, VL2 and VL2+ of DR7 are significantly different to measurements of DR4. For VL2+, PVD of DR7 agrees with earlier data at small scales but then turns to be higher at scales $\sigma > 1 \h^{-1} \text{Mpc}$ which makes the scale dependence very weak; for VL2, $\sigma_{12}$ of DR7 roughly keeps the shape of DR4 but has a much larger amplitude. $\sigma_{12}$ of VL1 of DR7 has steeper scale dependence and stronger amplitude at small scales than results of DR4 and DR6. $\sigma_{12}$ of VL1 subsamples of DR4 and DR6 are rather flat, and do not follow the general trend that PVDs of galaxy samples with lower luminosities should rise faster at smaller scales (also see PVDs of SAMs in Fig.5 of Li et al., 2007), but now DR7 reverts VL1 to track. Comparing distributions in celestial sphere of galaxies in the lowest luminosity bin of the three SDSS data releases reveals the variation is just induced by a big structure locates in area roughly of RA $166^\circ - 188^\circ$ and DEC of $16^\circ - 26^\circ$ (Figure 1). It is another example of impact of super structure on clustering analysis of LSS in addition to the Sloan Great Wall.

Li et al. (2007) realised that $w_p$ and PVDs of faint volume-limited samples of DR4 are too low to match prediction of SAMs. Instructed by the experiment of Slosar et al. (2006), they reduced fraction of satellite galaxies in massive halos in SAMs ad hoc by around 30% and reproduced approximately the actual measurements, which then becomes a serious conflict for people to reconcile between models and observation. An eyeball check of our results with the SAMs prediction in Li et al. (2006) denotes that the amplitude boost in $w_p$ and PVDs of DR7’s faint volume-limited samples roughly compensate for the
space between DR4 and SAMs, or at least ameliorate difficulties in theoretical modelling, although we do not have SAMs data to quantify the improvement. So unlike the Sloan Great Wall, the existence of a large structure in the Universe is actually positive to our working models, which somehow casts doubts in the proclaimed practice of cutting off super structures from original data to fit into an unified picture. After all, it is still early to say which is closer to the true clustering property of those faint galaxies, we might need a deeper and wider survey than the present SDSS DR7 to reach good fairness and reduce huge uncertainties.

4 SUMMARY AND DISCUSSION

By extensive comparison of different data releases of SDSS main galaxy catalogue with 2PCFs in redshift space for flux-limited samples, 2PCFs/monopole of 3PCFs in redshift space for volume-limited samples, projected 2PCFs and PVDs for volume-limited samples, we have the following findings about galaxy clustering properties against the expansion of sky coverage of SDSS.

1. 2PCF $\xi(s)$ in redshift space of flux-limited sample is extremely robust against sample volume change, which subsequently secures relevant application; $\xi(s)$ is also insensitive to local structures at low redshift.

2. 2PCFs in redshift space $\xi(s)$ of volume-limited samples of SDSS DR7 in luminosity bins brighter than $-M_{r,0.1} = [17, 18]$ are in good agreement with earlier data releases at scales $s <$~
Fig. 7 The scale dependence of pairwise velocity dispersions in the volume-limited samples.

10 $h^{-1}$Mpc. As scale being larger, the consistency is broken for volume-limited samples fainter than $-M_{r,0.1} = [19.5, 20.5]$, and in general the deviation of DR7 to DR6 and DR4 grows with larger absolute magnitude. Zero crossing points of DR7’s $\xi(s)$ do not differ much to DR6’s, but shifts away from DR4’s apparently.

3. Volume-limited samples of SDSS display convergence in $\zeta_0$ at scales $s < 10 h^{-1}$Mpc except the one in the faintest luminosity bin; while in the weakly nonlinear regime, there is no agreement between $\zeta_0$ of different data releases in all luminosity bins.

4. Projected 2PCFs $w_p$ of volume-limited samples in luminosity bins brighter than $-M_{r,0.1} = [18.5, 19.5]$ are robust against data version, but for samples in fainter bins, $w_p$ of DR7 are significantly higher than those of earlier data. Similar phenomenon is also seen in PVDs, PVDs of the two faintest volume-limited samples also appear much steeper along scale in DR7 and then become flatter at higher luminosity, which actually turn to be closer to what SAMs predict shown in Li et al. (2007).

5. The faintest volume-limited sample of $-M_{r,0.1} = [17, 18]$ is very peculiar, it suffers of the biggest variance due to enlargement of sky coverage, agreement of $\xi(s)$ and $\zeta_0$ of DR7 in redshift space with early data is breached at scales as small as $\sim 3 h^{-1}$Mpc; $w_p$ of the sample is enhanced by around $\sim 70\%$ and PVDs distinguish much in amplitude and scale dependence from measurements of earlier data.
Fig. 8 The luminosity dependence of pairwise velocity dispersions at fixed scales.

Fairness of a galaxy sample is assessed by statistical functions, one can not claim a general fair sample hypothesis without specifying the statistical measure used. It is possible that a galaxy sample is fair for one statistical function but not for another function. With our measurements, we conclude that current SDSS is not able to provide reliable 2PCFs (both of redshift space and projected) and PVDs of samples of characteristic luminosity fainter than $L^*$, and third-order statistics in the weakly nonlinear regime for nearly all volume-limited samples.

For faint volume-limited subsamples, probably due to their very shallow depths, measurements suffers of greater finite volume effects such that enlarging sky coverage has larger influence on measurements of statistics than for bright subsamples. The inconsistency observed is manifestation of cosmic variance due to insufficient sample volume. The variances are comparable to the $1\sigma$ jack-knife error-bars which usually are regarded as good and robust approximation to the true errorbars (Zehavi et al., 2002). Now it seems that the technique underestimates the true variance, corresponding results about the habitation of faint galaxies in halos withdrawn from clustering analysis, e.g. Li et al. (2007) and Zehavi et al. (2010) are not very concrete. Conclusions about faint galaxies utilizing galaxy group catalogue constructed from SDSS DR4 (Yang et al., 2007, 2008) might also be problematic, we conjecture that a new group catalogue from DR7 may provide a very different paradigm.

In our analysis PVDs are derived under an general assumption that galaxy pairwise velocities are following closely to exponential distribution. The assumption might not be exact for satellite galaxies which pairwise velocity distribution can be better described by Gaussian (Tinker, 2007). For galaxies
of low luminosity, they are mostly likely satellite, the obtained $\sigma_{12}$ based on exponential distribution is biased and so be the relation of PVDs with galaxy luminosity presented in Figure 8. Nevertheless, our PVDs of different versions of VL1 are biased in the same way, the systematical bias will not affect our basic conclusion that PVD of VL1 of DR7 is very different to what is of DR4.

Recently there are several works applying 3PCF of SDSS (e.g. Sefusatti et al., 2006; Kulkarni et al., 2007; Marín et al., 2008; Marín, 2010; McBride et al., 2010), either to help determining cosmological parameters and galaxy biasing or to diagnose models of galaxy formation. Some results are using measurements of volume-limited subsamples of the SDSS main galaxy catalogues in the weakly nonlinear regime, our analysis however points out that one needs to be very cautious in taking relevant conclusions.

Another problem worthy of more discussion is the first zero-crossing point of 2PCF. Of course, part of the problem is induced by finite volume of samples, at least integral constraint is a serious systematic to low luminosity galaxy subsamples. But for subsamples with large volume of bright galaxies, absent of anti-correlation at large scales is still puzzling. Instead of assaulting validity of $\Lambda$CDM models it is probably better to activate stochastic bias in models of galaxy 2PCF. Halo model alone can not solve the problem since at large scales 2PCF in halo model boils down to simple multiplication of bias parameter with linear 2PCF of dark matter. In the bucket of parameters of cosmological application, galaxy 2PCF at large scales by default is fully described by linear bias parameter and 2PCF of dark matter, the single bias parameter is largely degenerated with some other parameters, such as the normalization of density fluctuation $\sigma_8$ and the matter density parameter $\Omega_m$, it is unclear if present estimation of cosmological parameters is significantly biased by the ignorance of possible exotic bias (e.g. the proposal of Coles & Erdogdu, 2007).

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