On the Efficiency of Connection Charges—
Part I: A Stochastic Framework

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Abstract—This two-part paper addresses the design of retail electricity tariffs for distribution systems with distributed energy resources such as solar power and storage. In particular, the optimal design of dynamic two-part tariffs for a regulated monopolistic retailer is considered, where the retailer faces exogenous wholesale electricity prices and fixed costs on the one hand and stochastic demands with inter-temporal price dependencies on the other. Part I presents a general framework and analysis for revenue adequate retail tariffs with advanced notification, dynamic prices and uniform connection charges. It is shown that the optimal two-part tariff consists of a dynamic price that may not match the expected wholesale price and a connection charge that distributes uniformly among all customers the retailer’s fixed costs and a price-volume risk premium. A sufficient condition for the optimality of the derived two-part tariff among the class of arbitrary ex-ante tariffs is obtained. Numerical simulations quantify the substantial welfare gains that the optimal two-part tariff may bring compared to the optimal linear tariff (without connection charge). Part II focuses on the impact of two-part tariffs on the integration of distributed energy resources.

Index Terms—Retail tariff design, connection charges, dynamic pricing, distributed energy resources, optimal demand response.

I. INTRODUCTION

The electric power industry is experiencing an important transformation driven by disruptive innovation in distributed renewable generation and energy storage systems [1]. A concern of this transformation is the impact of the inclining adoption of said distributed energy resources (DERs) on the financial viability of regulated distribution grid operators [2]. In particular, under the restriction to volumetric and net-metering tariffs, the gradual decline in energy sales could compromise the ability of grid operators to recover their predominantly fixed operational and capital expenditures. This could result in the need to increase retail prices further above wholesale electricity prices, thereby amplifying the entailed economic inefficiencies, inter-customer cross-subsidies, and incentives for DER adoption in a vicious circle.

This two-part paper aims to shed lights on the effectiveness of connection charges as a means to mitigate the negative impacts of the sustained adoption of DERs. To this end, Part I develops a framework to analyze the efficiency of retail electricity tariffs set for a regulated retailer who serves a heterogeneous population of residential customers under demand and wholesale price uncertainties.

In particular, we are interested in two practical and fairly general ex-ante retail pricing models: a volumetric linear tariff and a two-part tariff consisting of a volumetric linear charge and a connection charge. Our goal in Part I is to gain insights into the structure of the optimal revenue adequate linear and two-part tariffs that allow us to analyze, in Part II, the effects of integrating customer and retailer-owned DERs.

A. Related Work

There is a vast literature on efficient retail pricing of electricity, the economic foundations of which reside in the classical theory of peak-load pricing [3]—known more recently as dynamic pricing [4]. Recent reviews of the subject along with a brief history of its adoption by regulators and electric utilities can be found in [5], [6]. In this context, connection charges have been considered as a means to raise additional revenue to recover the predominantly fixed costs of electric utilities [3]. In the U.S., mild connection charges are prevalent with exceptions such as California, where the large investor-owned utilities have default volumetric residential tariffs with virtually no connection charge [7].

In the last two decades, while the adoption of time-varying prices has been particularly slow in the U.S. [4], the advent of cheaper smart meters, small-scale renewable energy installations, battery storage technologies and home energy management tools has stimulated research in dynamic pricing (see [8], [9] and references therein) as sophisticated technologies can enable customers to react to price signals [10]. Of particular interest is real-time pricing (RTP), a form of dynamic pricing widely known to be a critical feature of efficient electricity markets [11]. An overview of dynamic pricing and a recent analysis of its limited adoption in the U.S. are available in [11] and [4], respectively.

Economic approaches to dynamic pricing often rely on functional demand models to characterize competitive equilibrium prices when smart meters become available to customers [12]–[14]. The most relevant analysis is [14] where the socially optimal linear and two-part retail tariffs subject to a retailer revenue sufficiency constraint are derived. Unlike our work, however, this analysis does not accommodate inter-temporal demand dependencies nor the integration of DERs.

Most engineering approaches, on the other hand, focus on analyzing demand response models in smart grids [8], [9]. These approaches often involve modeling customer behavior

1PG&E and SDG&E have no connection charge whereas SCE’s charges $0.99/month. While these utilities have a minimum bill of $10/month or less, it is binding on extremely few customers, and thus practically irrelevant [7].
and wholesale price uncertainty in Section III. In particular, we show that the optimal ex-ante two-part tariff consists of a time-varying retail price that not always matches the expected wholesale price and a connection charge that allocates uniformly among all customers the retailer’s fixed costs and risk-related costs caused by the ex-ante determination of the tariff. We further show that the optimal volumetric tariff, referred hereafter as linear tariff, is characterized by a time-varying price markup—relative to the optimal two-part tariff’s price—that depends on the retailer’s fixed costs and the price elasticity of demand.

We further compare the efficiency of the linear and two-part tariff in Section III-C. Specifically, we present a parametric characterization of the social welfare (SW) or total surplus and the consumer surplus (CS) as a function of the retailer’s fixed costs. We show that the two-part tariff achieves the same SW regardless of the retailer’s fixed costs. For the linear tariff, in contrast, the SW decreases as the fixed costs increase, thus characterizing a trade-off between fixed costs and efficiency. We also provide a sufficiency condition under which the two-part tariff is optimal among all ex-ante nonlinear tariffs.

We demonstrate the performance of the derived tariffs numerically using publicly available data from NYISO and the largest utility company in New York City in Section IV. Contingent on the deployment of enabling technologies and smart meters, our results estimate that the optimal day-ahead linear tariff could bring losses (4.8% of the utility’s revenue) relative to the utility’s suboptimal two-part flat tariff due to the lack of a connection charge. The optimal day-ahead two-part tariff, on the other hand, could bring significant gains (8.1% of the utility’s revenue). From a societal perspective, these losses and gains manifest themselves as reductions and increments in electricity consumption, respectively. These estimates assume a realistic own-price elasticity of demand and a stylized model for TCL. Some concluding remarks and proofs are included in Section V and the Appendix, respectively.

C. Notations

We use $\pi = \mathbb{E}[x]$ to denote the expectation of a random vector $x \in \mathbb{R}^n$ and $\Sigma_{x,y} = \text{Cov}(x,y) \in \mathbb{R}^{n \times m}$ to denote the cross-covariance matrix of two random vectors $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$. Let also $x_k$ denote the $k^{th}$ entry of a vector $x \in \mathbb{R}^n$ and $x^\top$ its transpose.

II. Model

Given our focus on the retail electricity market, we assume the state of the wholesale market is represented by an exogenous discrete-time random process $\lambda_k \in \mathbb{R}_+$, which represents the wholesale RTP of electricity at time $k$ in a single location of interest. We assume that the time periods $k = 1, \ldots, N$ partition a billing cycle, which is the time horizon relevant for our formulation. Moreover, we assume the wholesale RTP accurately reflects the social marginal cost of electricity [7].

A. A Retail Tariff Model

In this paper, we consider time-differentiated retail electricity tariffs that are set and announced in advance (i.e., ex-ante) by a regulated retailer with a fixed lag time. These
tariffs (i) are fixed before the beginning of a billing period of certain length (e.g., a month or a day) with a fixed lag time (e.g., several days or hours), (ii) specify a pricing rule that depends on the temporal consumption profile within the billing period rather than on the accumulated consumption, and (iii) are allowed to vary dynamically from one billing period to the next. In the context of retail tariffs, the tariff lag time induces a tradeoff between advanced price notification and price signal accuracy. The tariff model considered here captures both the traditional long term flat tariff that has months or years of lag time as well as more sophisticated dynamic tariffs such as those with days or hours of advanced notification, but it generally excludes ex-post tariffs such as those indexed to the wholesale RTP.

Formally, some time before the billing cycle starts, the retailer announces a tariff $T : \mathbb{R}^N \rightarrow \mathbb{R}$ that maps the metered consumption power profile $q \in \mathbb{R}_+^N$ of each customer to a scalar charge $T(q) \in \mathbb{R}$. While the $k$th entry of $q$ is a single customer's metered consumption in period $k$ of the billing period, the amount $T(q)$ (in dollars) represents the total bill. Note that this form of tariff captures the intertemporal dependencies of pricing and consumption within each billing cycle (but not between several billing cycles).

Given a tariff $T$, customers rationally choose in real-time how much electricity to purchase from the retailer during each consumption period of the current billing cycle. The retailer then pays for the aggregate demand at the wholesale RTP.

Although in practice retailers buy certain portions of the aggregate demand in forward markets (including the day-ahead market), we can neglect such purchases in our formulation without loss of generality for the following reason. In perfectly competitive and well-functioning two-settlement markets, forward transactions are essentially used to hedge against the volatility of the RTP. Here, we consider risk-neutral decision makers that deal with uncertainty by taking expectations. Thus, in our setting, forward markets would bring no significant advantages to any stakeholder. This justifies the reliance of the retailer in the RTP to purchase electricity, which is an assumption that simplifies our exposition considerably.

### B. Consumer Model

We consider $M$ customers (indexed by $i$) who obtain a monetary benefit (i.e., gross surplus) $S^i(q^i, \omega^i) \in \mathbb{R}$ from consuming a power profile $q^i \in \mathbb{R}_+^N$ throughout the billing cycle. This benefit is contingent on $\omega^i = (\omega^i_1, \ldots, \omega^i_N) \in \mathbb{R}_+^N$, where $\{\omega^i_k\}_{k=1}^N$ is an exogenous random process that represents customer $i$’s local state. We assume that $S^i$ is continuously differentiable in $(q^i, \omega^i)$.

Accordingly, customer $i$ exhibits a consumption profile $q^i = q^i(T, \omega^i)$ when facing a tariff $T$ and a sequence of local states $\{\omega^i_k\}_{k=1}^N$. Customers are rational in that sense that the sequence of consumptions $\{q^i_k(T; \omega_1, \ldots, \omega_k)\}_{k=1}^N$ solves the multistage stochastic program

$$\mathbb{E}\{S \cdot (q^i(T, \omega^i), \omega^i) - T(q^i(T, \omega^i)) \},$$

where the expectation is taken over $\omega^i$, and $\overline{S}^i(T)$ represents customer $i$’s expected surplus. Correspondingly, a tariff $T$ yields an (aggregate) expected consumer surplus

$$\overline{S}(T) = \mathbb{E}\left[\sum_{i=1}^M S^i(q^i(T, \omega^i), \omega^i) - T(q^i(T, \omega^i))\right],$$

where the expectation is taken over $\omega = (\omega^1, \ldots, \omega^M)$.

Of particular interest is the demand response to tariffs $T$ with constant gradient $\nabla T = \pi \in \mathbb{R}^N$, where $\pi \in \mathbb{R}^N$ is a time-varying per-unit price, such as the tariff with the affine form $T(q^i) = A + \pi^T q^i$. For such tariffs $T$ we use the notation

$$D^i(\pi, \omega^i) = q^i(T, \omega^i)$$

for customer $i$’s demand profile, thus implicitly assuming that it depends on $T$ only through $\pi$. Hence, $D^i$ is a standard demand function which we assume to be nonnegative and continuously differentiable in $\pi$, and its Jacobian $\nabla_{\pi} D^i \in \mathbb{R}^{N \times N}$, with $(k, t)$ entry $\partial D^i_k / \partial \pi_t$, negative definite. Under the regularity assumptions made on $S^i$ and $D^i$, one can show that $\overline{S}(T)$ is decreasing and convex in $\pi$ (see Prop. 3 in Appendix).

For example, for a linear tariff $T(q^i) = \pi^T q^i$, the consumption of a TCL may be modeled with a linear demand function $D^i(\pi, \omega^i) = \omega^i - G \pi$, with deterministic and positive definite $G_i \in \mathbb{R}^{N \times N}$. Such demand function can be derived from an additive and temporally-separable quadratic benefit function $S^i$ via stochastic dynamic programming.

### C. Retailer Model

We consider the case of a retail monopoly and refer to the single entity as the retailer, utility, or load-serving entity (LSE). In procuring an aggregate demand profile $q = \sum_{i=1}^M M^i \in \mathbb{R}^N$, we assume that the retailer incurs a variable cost $\lambda^T q$, where $\lambda = (\lambda_1, \ldots, \lambda_N) \in \mathbb{R}_+^N$ is the wholesale RTP. Hence, a tariff $T$ yields the expected retailer surplus

$$\overline{R}(T) = \mathbb{E}\left[\sum_{i=1}^M T(q^i(T, \omega^i)) - \lambda^T q^i(T, \omega^i)\right],$$

where the expectation is taken over the global state $\lambda = (\lambda_1, \ldots, \lambda_N) \in \mathbb{R}_+^N$. For notational convenience, we define the RS collected from the volumetric charge $\pi$ of an affine tariff $T(q) = A + \pi^T q$ as $\phi(\pi) = (\pi - \lambda)^\top D(\pi, \omega)$ so that $\overline{R}(T) = \phi(\pi) + MA$ and

$$\phi(\pi) = (\pi - \lambda)^\top \mathbb{E}[D(\pi, \omega)] - \text{Tr}(\text{Cov}(\lambda, D(\pi, \omega))).$$

### III. Retail Tariff Design

In our retail tariff design framework, we assume that the regulator mandates the retailer to choose a tariff $T$ that maximizes the expected consumer surplus. Moreover, in order to recover the upstream fixed costs incurred to deliver electricity, the tariff should satisfy the revenue adequacy constraint $\overline{R}(T) = F$, where $F$ is a target approved by the regulator.

Formally, the regulator’s problem can be stated as

$$\max_{T(\cdot)} \overline{S}(T) \quad \text{s.t.} \quad \overline{R}(T) = F.$$
In general, this problem falls in the category of Ramsey-Boiteux pricing and peak-load pricing in economics [28], which are main components of the theory of public utility pricing [3, Sec. 4.5]. See [6] for a recent overview of this problem in the context of electricity pricing.

In this section, we study linear and two-part tariffs—two of the most widely used tariffs in electricity industry. The restriction to two-part tariff can in fact be made without loss of generality under certain conditions (Theorem 3). We begin by making the following assumption that guarantees the existence and uniqueness of solutions to problem (6).

**Assumption 1.** \( g(\pi) = \mathbb{E}[\nabla_{\pi} D(\pi, \omega)(\pi - \lambda)] \) is such that the Jacobian matrix \( \nabla g(\pi) \) is negative definite (nd).

This assumption—made mainly for analytical convenience—is common in economics [14] and essentially imposes a limitation on the curvature of the demand function. Intuitively, the demand can be generally linear, concave, or convex in \( \pi \); however, when convex, restrictions on the “amount” of convexity are required for Assumption 1 to hold.

### A. Structure of Optimal Two-Part Tariff

By restricting the regulator’s problem to two-part tariffs of the form \( T(q) = A + \pi^T q \), problem (6) can be reformulated as a convex program under Assumption 1. We emphasize here that our analysis implicitly assumes that no customer chooses to avoid the connection charge by not consuming electricity at all. The following result characterizes the optimal solution.

**Theorem 1.** (Optimal two-part tariff) The two-part tariff \( T^* \) that solves problem (6) is characterized by

\[
\pi^* = \bar{\lambda} + \mathbb{E}[\nabla_{\pi} D(\pi^*, \omega)]^{-1} \mathbb{E}[\nabla_{\pi} D(\pi^*, \omega)(\lambda - \bar{\lambda})],
\]

(7)

\[
A^* = \frac{1}{M} \left( F - \bar{G}(\pi^*) \right).
\]

(8)

Theorem 1 implies that the optimal price \( \pi^* \) is characterized by a *period-specific* price markup relative to the expected RTP, \( \bar{\lambda} \). Examination of (7) reveals that this markup is essentially determined by the cross-covariance between the price sensitivity of demand and the RTP. To gain intuition into (7), consider a demand independent across time case in which

\[
\pi_k^* = \bar{\lambda}_k + \varepsilon_{kk}(\pi^*),
\]

(9)

for each \( k = 1, \ldots, N \), where we use

\[
\varepsilon_{kk}(\pi) = \frac{\partial D_k(\pi, \omega)/\partial \pi_k}{\mathbb{E}[D_k(\pi, \omega)/\pi_k]},
\]

(10)

4In particular, for a linear demand \( D(\pi, \omega) = b(\omega) - G(\omega, \pi) \), \( \nabla g(\pi) = \mathbb{E}[\nabla_{\pi} D(\pi, \omega)(\pi - \lambda)] \) is nd since \( \nabla_{\pi} D(\pi, \omega) \) is nd. Moreover, for a demand with additive disturbances \( D(\pi, \omega) = b(\omega) + \mathcal{D}(\pi) \), Assumption 1 holds for \( \pi \geq \bar{\lambda} \) if each \( D_k(\pi) \) is concave in \( \pi \) since \( \nabla g(\pi) = \nabla D(\pi) + \sum_{k=1}^N (\lambda_k - \bar{\lambda}_k) \nabla^2_{\pi^2} D_k(\pi) \). Concave demand functions are common in economic models since they guarantee profit and welfare maximization problems to be well defined [29]. See, for example, Prop. 4 in the Appendix.

5This assumption is widely accepted for services such as electricity and water since “it is extremely unlikely that a customer will drop out of the market, however high the tariff” [6, Sec. 4.5]. Studies suggest, however, that more cost-effective DERs might challenge this assumption in future years [2].

6In expression (4), the second expectation is a second-order expectation that can be thought as the cross-covariance between a matrix and a vector.

7That is, a demand with \( D_k(\pi, \omega) \) independent of \( \pi_t \) for all \( t \neq k \) to represent the (own or cross-time) price elasticity of demand at time \( k \) with respect to the price at time \( t \). The latter result resembles the (second-best optimal) ex-ante two-part tariff derived in [14, Sec. 3] for the single period case (\( N = 1 \)).

The expression (8) for the optimal connection charge \( A^* \) also has an intuitive interpretation. The first term corresponds to a *uniform contribution* towards the retailer’s target \( F \). And, the second term corresponds to a *uniform preallocation* of the surplus that the retailer expects to collect from the volumetric charge \( \pi^*, \bar{G}(\pi^*) \), which—as noticeable from (6)—may be positive or negative in general.

To gain additional insights into these results, we have the following corollary.

**Corollary 1.** If \( \nabla_{\pi} D(\pi, \omega) \) and \( \lambda \) are uncorrelated, then \( \pi^* = \bar{\lambda} \) and \( A^* = \frac{1}{M} \left( F + \mathbb{E}[\text{Cov}(\lambda, D(\bar{\lambda}, \omega))] \right) \).

Corollary 1 indicates that \( T^* \) has a very simple and appealing structure that resembles the result for the deterministic case where \( \pi^* = \lambda \) and \( A^* = F/M \). Note that the assumption made in Corollary 1 is valid for many situations. It is certainly true for demands that are not much affected by consumers’ local randomness, such as the charging of electric vehicles and typical household appliances. Even for loads from smart HVAC systems that are affected by random temperature fluctuations, the assumption in Corollary 1 holds because the demand function takes the form \( D(\pi, \omega) = \omega + D(\pi) \) [26].

As for the simpler structure of \( T^* \), it may not be surprising since the efficiency of marginal cost pricing (i.e., \( \pi^* = \bar{\lambda} \)) is a classical result for the deterministic case [3, Sec. 4.5] [14]. Intuitively, marginal cost pricing is efficient because it induces customers to increase consumption until the derived marginal benefit matches the marginal cost of procuring electricity.

The expression for \( A^* \) in Corollary 1 also has an intuitive interpretation. While the first term remains unchanged from (8), the second term becomes a risk premium associated to the cross-correlation that the demand and the RTP may exhibit. When such cross-correlation is positive (as in practice [50]), the retailer is likely to face additional variable costs since the expected variable cost \( \mathbb{E}[\lambda^T D(\bar{\lambda}, \omega)] \) is larger than the variable revenue \( \bar{\lambda} \mathbb{E}[D(\bar{\lambda}, \omega)] \). Intuitively, this fee represents a *uniform risk premium* that customers pay to face a deterministic price rather than the volatile RTP. Presumably, the inter-customer cross-subsidies arising from the uniform allocation of this risk premium are negligible compared to the differences. However, the integration of behind-the-meter renewables could make these cross-subsidies worth adjusting, for example, through the use of discriminatory connection charges consistent with the cost-causation principle described in [51]. A discussion on cross-subsidies is held in Part II [?].
B. Structure of the Optimal Linear Tariff

A tariff of the form \( T(q^t) = \pi^t q^t \) is a linear tariff—i.e., an ex-ante two-part tariff with no connection charge. While such purely volumetric tariff may be simpler, it has two fundamental disadvantages. First, a closed form expression of the optimal linear tariff is not available under general assumptions. Second, such restriction introduces a fundamental trade-off between the retailer surplus target and the attainable social welfare. These drawbacks are noticeable in Theorem 2 and Corollary 3 respectively.

When restricted to linear tariffs, a unique solution to problem (6) can be obtained due to Assumption 1. We characterize the optimal solution in the following result.

**Theorem 2. (Optimal linear tariff)** Consider the regime where \( F \) is large, i.e., \( F \geq \bar{\phi}(\pi^*) \). If feasible, the linear tariff \( T^1 \) that solves problem (6) is characterized by
\[
\pi^t = \pi^* - \frac{2}{\gamma} \mathbb{E}[\nabla_\pi D(\pi^t, \omega)]^{-1} \mathbb{E}[D(\pi^t, \omega)], \tag{11}
\]
or, equivalently, by
\[
\sum_{t=1}^{N} -\varepsilon_{kt}(\pi^t) \left( \frac{\pi^t_k - \pi^*_k}{\pi^*_k} \right) = \frac{\gamma - 1}{\gamma}, \quad \forall \; k = 1, \ldots, N, \tag{12}
\]
where \( \gamma \), the Lagrange multiplier of (9), satisfies \( \frac{\gamma - 1}{\gamma} \in [0, 1] \) and is such that \( \bar{\mathfrak{R}}(T^1) = F \). In this regime of \( F \), the problem is feasible if and only if \( F \leq \bar{\phi}(\pi^M) \), where \( \pi^M \) is the price that maximizes \( \mathfrak{R}(T) \) over \( \pi \), which matches \( \pi^* \) as \( \gamma \to \infty \).

In Theorem 2, expression (11) reveals that the structure of the optimal linear tariff is characterized by a period-specific price markup relative to the price of the optimal two-part tariff \( \pi^* \). The scalar \( \frac{\gamma - 1}{\gamma} \in [0, 1] \), often called the Ramsey number, adjusts markups in all periods uniformly to the point where the expected retailer surplus matches the target \( F \). A closer examination of (11), which can be rewritten as (12), shows that the own and cross price elasticities of demand determine altogether the markup for each period within the billing cycle.

To understand (12), it is informative to consider the case where the demand is independent across time, namely, \( \varepsilon_{kt}(\cdot) = 0 \) for \( t \neq k \). In this case, the product of the markup \( (\pi^t_k - \pi^*_k)/\pi^*_k \) and the own-price elasticity \( \varepsilon_{kk}(\pi^*) \) remains constant in time and equal to the Ramsey number. This means that periods with inelastic demands get high markups and periods with elastic demands get low markups. For this reason, this pricing rule is known in economics as the inverse elasticity rule [3, Sec. 3.3].

Even simpler is the single period case, also derived in [14, Sec. 3]. Notably, when \( N = 1 \), the scalar price \( \pi^t \) can be obtained directly from the constraint \( \bar{\mathfrak{R}}(T^1) = F \), and it must be set so that it pays for the average total cost of the procured electricity, i.e., \( \pi^t = (\mathbb{E}[\lambda \cdot D(\pi^t, \omega)] + F)/\mathbb{E}[D(\pi^t, \omega)] \).

A specialized application of this result was developed in [26], where a model of TCLs under a day-ahead hourly pricing scheme was considered. In this case, the demand function for each consumer is linear and the surplus function is quadratic [26]. The aggregated demand is therefore also linear. The consumers as a collective have a quadratic aggregated surplus [26]. Specifically,
\[
D^i(\pi, \omega^i) = \omega - G^i \pi, \tag{13}
\]
\[
S^i(D^i(\pi, \omega^i), \omega^i) = \delta^i(\omega^i) - \frac{1}{2} \pi^T D^i(\pi, \omega^i), \tag{14}
\]
where \( G^i \in \mathbb{R}^{24 \times 24} \) is deterministic, positive definite (and symmetric). Letting \( G = \sum_{i=1}^{N} G^i \) and \( \Omega = \sum_{i=1}^{N} \omega^i \) and applying Theorem 2 readily yields
\[
\pi^t = \bar{\lambda} + \frac{\rho}{1+\rho}(\pi^o - \bar{\lambda}), \tag{15}
\]
where \( \pi^o = G^{-1} \Omega \) induces \( \mathbb{E}[D(\pi^o, \Omega)] = 0 \) and \( \rho = \frac{\gamma - 1}{\gamma} \) is the Ramsey number, which is set so that \( \mathfrak{R}(T^1) = F \). Intuitively, \( \rho \) varies within [0, 1] inducing prices that vary between \( \bar{\lambda} \) and the profit-maximizing price \( \pi^M = \frac{1}{\gamma}(\pi^o + \bar{\lambda}) \) as \( F \) varies between \( \bar{\phi}(\bar{\lambda}) \) and the maximum profit \( \bar{\phi}(\pi^M) \).

C. Tariff Performance Comparison

We now discuss the performance of the derived tariffs in terms of social welfare (expected total surplus) leveraging the graphical representation provided in Fig. 1. Therein, a Pareto front for each tariff illustrates the expected CS and SW induced by the tariff for different RS targets \( F \in \{\bar{\phi}(\pi^*), \bar{\phi}(\pi^M)\} \). On one hand, Theorem 1 has the following implication.

**Corollary 2.** As a tariff parametrized by \( F \), the two-part tariff \( T^* \) induces an expected total surplus \( \text{SW}^* \) that is independent of \( F \) and \( \text{CS}(T^*) = \text{SW}^* - F \).

In Corollary 2, \( \text{SW}^* \) denotes the constant SW attained by the price \( \pi^* \), where \( \text{SW}^* = \text{SW}(T^*) = \text{CS}(T^*) + \text{RS}(T^*) \). Implicit in this result is that for any affine tariff one can check that
\[
\text{SW}(T) = \sum_{i=1}^{M} \mathbb{E}\left[ S^i(D^i(\pi^t, \omega^i), \omega^i) - \lambda^T D^i(\pi^t, \omega^i) \right] \tag{16}
\]
depends on \( \pi \) but not on \( A \). Corollary 2 thus implies that under the tariff \( T^* \), collecting additional revenue from customers to cover larger fixed costs embedded in \( F \) reduces consumer welfare but does not compromise social welfare. This “iso-efficient” trade-off between retailer and consumer surplus is illustrated in Fig. 1 with a linear Pareto front with negative and unitary slope in the CSRS plane. Intuitively, suboptimal two-part tariffs\(^{10}\) can achieve any point in the CSRS plane in addition to bill stability and alleged equity concerns imposed by regulators.

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\(^{10}\)As the ones currently used by most utilities in the U.S., due in part to additional bill stability and alleged equity concerns imposed by regulators.
the shaded area below the linear Pareto front in Fig. 1 but no ex-ante two-part tariff can achieve points above this front.

Theorem 2 on the other hand, has a analogous implication.

**Corollary 3.** The quantities $\mathbf{SW}(T')$ and $\mathbf{CS}(T')$ induced by $T'$ as a tariff parametrized by $F$ are decreasing and concave in $F \in [\bar{\phi}(\pi^*), \bar{\phi}(\pi^M)]$ with $\mathbf{SW}(T') = \mathbf{SW}^*$ and $\mathbf{CS}(T') = \mathbf{SW}^* - F = \bar{\phi}(\pi^*)$. 

Corollary 3 reveals that, unlike the tariff $T^*$, the optimal linear tariff $T'$ compromises not only consumer welfare but also social welfare when collecting additional revenue from customers is required to cover larger fixed costs embedded in $F$. This trade-off is depicted in Fig. 1 with a decreasing and concave Pareto front in the $\mathbf{CS}$-$\mathbf{CS}$ plane that bends away from the efficiency level $\mathbf{SW}^*$ attained by the tariff $T^*$ as $F$ increases from $\bar{\phi}(\pi^*)$ until it reaches $\bar{\phi}(\pi^M)$. As before, suboptimal linear tariffs can achieve any point in the shaded area below the curved Pareto front in Fig. 1 but no ex-ante linear tariff can achieve points above this front.

From the previous analysis, it is clear that two-part tariffs dominate linear tariffs in terms of expected consumer surplus in the regime of practical relevance where $F \geq \bar{\phi}(\pi^*)$. A natural question to ask is whether two-part tariffs can be dominated by more complex nonlinear ex-ante tariffs. We now argue that, given sufficient condition, the two-part tariff $T^*$ is indeed optimal for the regulator’s problem (6) among all ex-ante arbitrary tariffs. To establish such result it suffices to show that $T^*$ induces the same expected consumer surplus that would be achieved by a social planner who makes consumption decisions on behalf of customers with the unconstrained objective of maximizing the expected total surplus. This is because the social planner’s problem provides a trivial upper bound to the regulator’s problem.

Because we are interested in comparing ex-ante tariffs only, the social planner’s problem should incorporate such implicit restriction. The restriction to ex-ante tariffs translates into a restriction for the social planner to use only the information observable by each customer $i$ when choosing their consumption, namely their local state $\omega^i$. Hence, the social planner’s problem can be stated as

$$
\max_{\{q^i(\omega^i)\}_{i=1}^{M}} \mathbf{SW} = \mathbb{E} \left[ \sum_{i=1}^{M} S^i(q^i(\omega^i), \omega^i) - \lambda^i q^i(\omega^i) \right],
$$

where the expectation is taken with respect to $\xi = (\lambda, \omega)$, and $q^i(\omega^i)$ is causally contingent on (i.e., adapted to) the local state $\omega^i$. Finally, under the assumption that each $\omega^i$ and $\lambda$ are independent, we show that the optimal solution to (17) is $q^i(\omega^i) = D^i(\pi^*, \omega^i)$, which matches the demand induced by the optimal two-part tariff. This result and the implied optimality of $T^*$ are summarized in the following Theorem.

**Theorem 3.** If (A2) the wholesale RTP $\lambda$ and the local state $\omega^i$ of each customer $i = 1, \ldots, M$ are statistically independent, then the two-part tariff $T^*$ is an optimal solution of (6) among all arbitrary tariffs with the same lag time.

Theorem 3 indicates that the restriction to two-part tariffs may imply no loss of generality. This applies—less generally than Corollary 1—for demands that are not affected by consumers’ local randomness\cite{26}, such as washers and dryers, computers, and the charging of electric vehicles. For all the other cases, where the sufficient condition (A2) does not hold, Theorem 3 sheds lights on the performance of the optimal ex-ante two-part tariff $T^*$. While it is clear that the ex-ante restriction entails some efficiency loss when (A2) is not satisfied\cite{13} it is not clear whether the restriction to two-part tariffs does entail efficiency losses. In other words, is there a necessary condition for $T^*$ to be an optimal solution of (6)?

**IV. NUMERICAL EXAMPLE**

We now estimate the performance of the optimal linear and two-part day-ahead tariffs in a practical setting. Using publicly available data from ConEdison (New York City’s largest utility) and NYISO for the 2015 Summer season, we estimate the average daily gains in consumer surplus that both tariffs would have brought relative to the utility’s default two-part tariff with flat rate. Here we assume a linear demand model\cite{23,24} and day-ahead linear and two-part tariffs.

The utility’s monthly residential energy sales\cite{11} and an estimated residential hourly load profile\cite{19} were used to obtain 2,208 (92 days $\times$ 24 hr) aggregate hourly consumption data points. We used these points as iid realizations $\{x_j\}_{j=1}^{N}$ of the random vector $D(1\pi^{CE}, \omega) = \omega - G1 \pi^{CE} \in \mathbb{R}^{24}$, where $\pi^{CE}$ is ConEdison’s flat rate\cite{30} and then to estimate $\pi$. Due to the its low-dimensional structure, fitting $\pi$ can be reduced to determining a scaling parameter after assuming certain own-price elasticity of demand at $\pi^{CE}$\cite{26}.

$$
\pi^{CE} = \frac{\partial \mathbb{E}[D(1\pi^{CE}, \omega)^{1}] / \partial \pi^{CE}}{\mathbb{E}[D(1\pi^{CE}, \omega)^{1}] / \pi^{CE}} = \frac{1}{\mathbb{E}[x]} \frac{G1}{1/\pi^{CE}},
$$

where $\mathbb{E}[x]$ denotes the sample mean of $\{x_j\}$. While here we assume a value of $\pi^{CE} = -0.3$, which is a reasonable estimate of the short-term own-price elasticity of electricity demand\cite{33}, a sensitivity analysis over $\pi^{CE}$ is presented in [34]. We further assumed a total of $M = 2.2$ million of residential customers and used ConEdison’s default residential connection charge, which amounts to $A^{CE} = 0.52$ $\$/day, to roughly estimate the utility’s average daily revenue from residential customers as $\mathbb{E}[T^{CE}] = \mathbb{E}[D(1\pi^{CE}, \omega)^{1}] \pi^{CE} + MA^{CE}$ or $\$7.19$ million USD. As for the prices $\lambda$, we used the day-ahead prices for NYC as iid realizations to estimate $\bar{\lambda}$ and $\Sigma_{\lambda, \omega}$ with sample mean and covariance estimators.

\footnote{In other words, demands that, given a retail price vector $\pi$, are independent from random exogenous factors affecting the wholesale prices $\lambda$.}

\footnote{This is because relaxing the ex-ante restriction enables the use of the ex-post two-part tariff $T(q') = F/M + \lambda' q'$ which trivially achieves the maximum social welfare a social planner could achieve (first-best).}

\footnote{For June through August 2015, which can be found in the EIA-826 database at http://www.eia.gov/electricity/data/eia826.html.}

\footnote{For a residential building in NYC, available in the NREL OpenEI building load database http://en.openei.org/datasets/files/961/pub/}. The location/model with ID 725033-TMY3-BASE was used.

\footnote{NYC’s residential default flat rate during Jun-Aug 2015 was $\pi^{CE} = 17.2$ cents/kWh (47.7% of which correspond to supply charges and 53.3% to delivery charges). Available at http://www.coned.com/rates/supply_charges.asp and http://www.coned.com/documents/elecPSC10/SCs.pdf.}

\footnote{This is facilitated by assuming a homogeneous thermal parameter $\alpha^i$ across customers, which implies some loss of customer heterogeneity. See \cite{26} for details of the model, where $\alpha^i = 0.2$ is used in a case study.}
We plot in Fig. 2a the Pareto fronts
\[
\{ (\Delta \text{CS}(F), \Delta \text{RS}(F)) \mid F \in [\phi, \phi(M)] \}
\] (18)
induced by the optimal linear and two-part tariffs and three other relevant tariffs that satisfy the revenue sufficiency constraint: an optimized linear flat tariff with rate \( \pi^*(F) \), an optimized two-part tariff with fixed connection charge \( A^F \) and rate \( \pi^F(F - A^F M) \), and an optimized two-part flat tariff with fixed connection charge \( A^F \) and rate \( \pi^{CE} + \Delta(F) \). The latter can be thought as ConEdison’s adjusted tariff. In (18), \( \Delta \text{CS}(F) \) and \( \Delta \text{RS}(F) \) denote the surplus gains (losses if negative) relative to the corresponding surplus achieved by \( T^{CE} \). For instance, for the optimal two-part tariff \( \Delta \text{CS}(F) = \text{CS}(T^*) - \text{CS}(T^{CE}) \).

Fig. 2a compares the tariffs’ performances in consumer surplus gains for different retail surplus targets. At ConEdison’s estimated net revenue level \( F = \text{RS}(T^{CE}) \), which corresponds to \( \Delta \text{RS}(F) = 0 \), significant performance differences can be observed among the computed tariffs. These differences are more evident in Fig. 2b which magnifies Fig. 2a around the origin. Due to its nonzero connection charge, Con Edison’s tariff clearly outperforms the tariffs without connection charges, but is outperformed by the other tariffs with connection charges, which are further optimized. It is particularly interesting that switching to the optimal linear tariff would bring losses in CS (−4.8% or −$345k USD/day). Namely, by virtue of a connection charge, even a simple flat tariff can outperform a fairly sophisticated day-ahead hourly volumetric tariff. Moreover, fully optimizing ConEdison’s rate (but not its connection charge) brings rather limited CS gains (1% or $72k USD/day). However, switching to the optimal two-part tariff would bring significant gains in CS (8.1% or $582k USD/day). This corroborates how effective connection charges can be at increasing the retailer surplus without sacrificing economic efficiency. This optimal two-part tariff, which features a connection charge of \( A^* = 2.65 \) $/day or nearly 80 $/month, induces bill reductions for customers 6.62% larger than the average customer and bill increments for all other customers. Clearly, such charges may be politically unacceptable for low-income customers and may require cross-subsidized reduced tariffs, which have been an industry standard [3 Sec. 7.4].

V. CONCLUSIONS

In this first part, we derive consumer-welfare-maximizing, revenue adequate, and ex-ante linear and two-part dynamic tariffs from the perspective of a regulated retailer. This initial analysis is for the case without renewables or storage in the distribution system. Our results generalize previous works by deriving said tariffs for a stochastic and multi-period demand model with intertemporal dependencies and a predetermined lag time between the announcement of the tariff and the beginning of the billing period. We established that if the wholesale prices and each customer’s consumption are statistically independent, then the optimal two-part tariff is optimal among the class of arbitrary tariffs with the same lag time.

While the optimal two-part tariff mitigates inefficiencies induced by the optimal linear tariff, inequity concerns inconsistent with cost causation arise from the structure of the connection charge. These concerns may become significant with the sparse adoption of behind-the-meter renewables. While tariff design criteria beyond efficiency and revenue adequacy are out of the scope of our work, it is worth mentioning that allowing discriminatory connection charges can give flexibility to the regulator to achieve different objectives (such as intercustomer cost- causation equity) and provide effective long- term signals (e.g., location within the distribution network and investment in on-site generation) [31].

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**APPENDIX**

**Proof of Theorem 1**

Solving $\mathbf{F}_i(T) = F$ in (6) for $A$ and substituting in the objective $\mathbf{E}_i(S(T))$ yields the expression

$$\mathbf{E}_i(S(T)) = \sum_{i=1}^{M} \mathbf{E}_i \left[ S_i(D_i(T, \pi, \omega_i), \omega_i) - \lambda_i D_i(T, \pi, \omega_i) \right].$$

Differencing the objective $\mathbf{E}_i(S(T))$ over $\pi$ yields

$$\nabla_{\pi} \mathbf{E}_i(S(T)) = \sum_{i=1}^{M} \mathbf{E}_i \left[ \nabla_{\pi} D_i(T, \pi, \omega_i) \left( \nabla_{\omega_i} S_i(D_i(T, \pi, \omega_i), \omega_i) - \lambda \right) \right] = \mathbf{E}_i \left[ \nabla_{\omega_i} D_i(T, \pi, \omega_i) (\pi - \lambda) \right]$$

(19)

where the second equality follows from Prop. 2 below. Now, the assumption that each $\nabla_{\omega_i} D_i(T, \pi, \omega_i)$ is negative definite implies that $\mathbf{E}_i[\nabla_{\omega_i} D_i(T, \pi, \omega_i)]$ is invertible. Hence, one can rewrite the necessary first-order condition (FOC), which is obtained from equating $\nabla_{\omega_i} \mathbf{E}_i(S(T)) = 0$, as (7). Given the expression (7) for $\pi^*$, $\mathbf{E}_i$ can be obtained by solving the equality constraint $\mathbf{E}_i(F(T^*)) = F^*$ for $A^*$. Lastly, these FOC are sufficient for the optimality of $(A^*, \pi^*)$ since $\mathbf{E}_i(S(T) = \mathbf{E}_i(S(T^*) + \mathbf{E}_i(F(T^*))$ is strictly concave in $\pi$ according to Prop. 4.

**Proposition 1.** For each $k = 1, \ldots, N$, $D_{i}(\cdot, \cdot)$ satisfies

$$\mathbf{E}_i \left[ \partial S_i(D_i(T, \pi, \omega_i), \omega_i) / \partial q_i^k \right] \omega_i^1, \ldots, \omega_i^k \} = \pi_k$$

(20)

where the conditional expectation is taken over $\omega_i^1, \ldots, \omega_i^k$ conditioned on $\omega_i^k$.

**Proof:** (20) are FOCs of customer $i$’s multi-stage decision problem (11) of sequentially choosing $q_i^1, \ldots, q_i^k, q_i^N$ contingent on the so-far observed local states $\omega_i^1, \ldots, \omega_i^k$ to maximize his expected net utility or surplus, i.e.,

$$\max_{q_i^k} \mathbf{E}_i \left[ S_i(q_i^k(\omega_i^k), \omega_i^k) - T(q_i^k(\omega_i^k)) \right]$$

(21)

given a tariff $T(q_i^k) = A + \pi q_i^k$ known in advance (ex-ante). Hence, the optimal solution $q_i^* = D_i(\cdot, \cdot)$ of (21) must satisfy these necessary optimality conditions.

Each of these stationarity (KKT) FOCs is obtained by differentiating the objective in (21) with respect to $q_i^k$ and equating the result to zero. To see that, note that for each time $k = 1, \ldots, N$ one can use the law of total expectation to rewrite the objective in terms of an expectation conditioned on the information observed up to $k$, i.e.,

$$\mathbf{E}_i \left[ S_i(q_i^k(\omega_i^k), \omega_i^k) - T(q_i^k(\omega_i^k)) \right] \omega_i^1, \ldots, \omega_i^k \right] = \mathbf{E}_i \left[ \nabla_{q_i^k} S_i(q_i^k(\omega_i^k), \omega_i^k) \right] \omega_i^1, \ldots, \omega_i^k \right].$$

The FOC in (20) follows since $\partial T(q_i^k(\omega_i)) / \partial q_i^k = \pi_k$. [Q.E.D.]

**Proposition 2.** $D_{i}(\cdot, \cdot)$ satisfies the equation

$$\partial E_i[S_i(D_i(\pi, \omega_i), \omega_i)] / \partial \pi = \mathbf{E}_i \left[ \nabla_{\omega_i} D_i(T, \omega_i) \right]$$

(22)
where the expectations are taken over $\omega^t$.

Proof: We establish this vectorial identity proving each component separately. Assuming the differentiation and expectation operators can be exchanged, we apply the chain rule to the $k$-th component of the left-hand-side of (22) yielding the following sequence of equalities

$$
\frac{\partial \mathbb{E}[S^i(D(\pi,\omega^t),\omega^t)]}{\partial \pi_k} = \mathbb{E} \left[ \sum_{t=1}^T \frac{\partial S^i}{\partial q^t_i} \cdot \frac{\partial D^i_1}{\partial \pi_k} \right] = \sum_{t=1}^T \mathbb{E} \left[ \frac{\partial S^i}{\partial q^t_i} \cdot \omega^t_1, \ldots, \omega^t_t \right] \frac{\partial D^i_1}{\partial \pi_k} = \pi^T \mathbb{E} \left[ \frac{\partial D^i_1}{\partial \pi_k} \right],
$$

where the second equality follows from the law of total expectation, the third equality is due to the causality of $D^i(\pi,\omega^t)$, and the last equality is due to Prop. 1. The identity (22) readily follows.

Proposition 3. For any affine tariff $T(q) = A + \pi^T q$, $\mathbb{S}(T)$ is strictly convex and (componentwise) decreasing in $\pi$.

Proof: Differentiating $\mathbb{S}(T)$ with respect to $\pi$ (using the chain rule) and leveraging on the expression (22) from Prop. 2, one readily obtains $\nabla_\pi \mathbb{S}(T) = -\mathbb{E}[D(\pi,\omega)]$. Because $D^i(\pi,\omega^t)$ is nonnegative, the expression above implies that $\mathbb{S}(T)$ is componentwise decreasing in $\pi$. Moreover, we have that

$$
\nabla_\pi^2 \mathbb{S}(T) = -\mathbb{E}[\nabla_\pi D(\pi,\omega)].
$$

Recall that a function is strictly convex (over a convex domain) if its Hessian is positive definite (over said domain). Hence, the strict convexity of $\mathbb{S}(T)$ in $\pi$ readily follows from the assumed negative definiteness of each matrix $\nabla_\pi D^i(\pi,\omega)$. ■

Proposition 4. Consider an affine tariff $T(q) = A + \pi^T q$. If Assumption 7 holds then $\mathbb{R}(T)$ and the weighted surplus $\mathbb{S}(T) + \gamma \mathbb{R}(T)$ are strictly concave in $\pi$ for any $\gamma \geq 1$.

Proof: First, we differentiate twice $\mathbb{R}(T)$ with respect to $\pi$. Using Prop. 2 and some algebra one obtains the Hessian

$$
\nabla_\pi^2 \mathbb{R}(T) = \mathbb{E}[\nabla_\pi D(\pi,\omega)] + \nabla g(\pi).
$$

Recall that a function is strictly concave (over a convex domain) if its Hessian is negative definite (over said domain). Hence, the assumed negative definiteness of $\nabla_\pi D^i(\pi,\omega)$ and of $\nabla g(\pi)$ (Assumption 7) readily imply the negative definiteness of $\nabla_\pi^2 \mathbb{R}(T)$ and thus the strict concavity of $\mathbb{R}(T)$ in $\pi$.

It remains to show that $\nabla_\pi^2 (\mathbb{S}(T) + \gamma \mathbb{R}(T))$ is negative definite for $\gamma \geq 1$. From (23) and (24) we have that

$$
\nabla_\pi^2 (\mathbb{S}(T) + \gamma \mathbb{R}(T)) = (\gamma - 1) \mathbb{E}[\nabla_\pi D(\pi,\omega)] + \gamma \nabla g(\pi).
$$

Similar arguments yield the desired result since both terms on the right hand side are negative definite for $\gamma \geq 1$. ■

Proof of Corollary 7 If $\nabla_\pi D(\pi,\omega)$ and $\lambda$ are uncorrelated, then clearly $\mathbb{E}[\nabla_\pi D(\pi,\omega)\lambda] = \mathbb{E}[\nabla_\pi D(\pi,\omega)] \lambda$. It follows that the expression for $\pi^*$ in (7) reduces to $\pi^* = \lambda$. In turn, the expression for $\nabla g(\pi^*)$ in (8) reduces to $\nabla g(\lambda) = -\mathbb{E}[\nabla g(\pi)] = \nabla g(\lambda)$. Thus, readily simplifying (9) to

$$
A^* = \frac{1}{\mathbb{F}} (F + \mathbb{F}(\lambda, D(\lambda,\omega)))
$$

1) Proof of Corollary 7 Leveraging on (2) and (4), the expected total surplus induced by the tariff $T^*$ characterized in Thm. 1 can be written as

$$
\mathbb{S}(T^*) = \mathbb{S}(T^*) - \mathbb{R}(T^*) = \mathbb{S}(T^*) - F.
$$

Clearly, $\mathbb{S}(T)$ is a function of $\pi$ but not of $F$ or $A$. It remains to show that $\pi^*$ does not depend on $F$. But the FOC characterizing $\pi^*$ is obtained by differentiating $\mathbb{S}(T)$ with respect to $\pi$ and equating it to zero. It follows that $\pi^*$ does not depend on $F$. Moreover, since $\mathbb{S}(T^*) = F$ must hold at optimality, it readily follows that

$$
\mathbb{S}(T^*) = \mathbb{S}(T^*) - \mathbb{R}(T^*) = \mathbb{S}(T^*) - F.
$$

2) Proof of Theorem 2 Consider the Lagrangian of problem 6

$$
\mathcal{L}(\pi,\gamma) = \mathbb{S}(\pi) + \gamma(\mathbb{R}(\pi) - F),
$$

where $\gamma \in \mathbb{R}$ is the multiplier of the equality constraint. Differentiating $\mathcal{L}(\pi,\gamma)$ over $\pi$ yields

$$
\nabla_\pi \mathcal{L} = \nabla_\pi \mathbb{S}(\pi) + \gamma \nabla_\pi \mathbb{R}(\pi)
$$

$$
= \mathbb{E}[\nabla_\pi D(\pi,\omega)(\pi - \lambda) + (\gamma - 1)D(\pi,\omega)],
$$

where we use Prop. 2 to obtain the second equality. The necessary FOC in (11) follows from equating (26) to zero after some algebra. These operations use the negative definiteness and thus invertibility of the matrix $\mathbb{E}[\nabla_\pi D(\pi,\omega)]$ and the expression for $\pi^*$ in (7) (Thm. 1).

The expressions in (12) characterizing $\pi^1$ componentwise can be obtained from (11) after some algebraic manipulations using the definition of price elasticity in (19). In particular, subtracting $\pi^*$ from both sides of (11) and left-multiplying by $\mathbb{E}[\nabla_\pi D(\pi,\omega)]$ one obtains

$$
\mathbb{E}[\nabla_\pi D(\pi,\omega)](\pi^1 - \pi^*) = -\frac{1}{\gamma} \mathbb{E}[D(\pi,\omega)].
$$

Componentwise, for each $k = 1, \ldots, N$, we have

$$
\mathbb{E}[\partial D_k(\pi^1,\omega)/\partial q^t_k](\pi^1_k - \pi^*_k) = -\frac{1}{\gamma} \mathbb{E}[D_k(\pi^1,\omega)]
$$

$$
\sum_{t=1}^T \mathbb{E}[\partial D_k(\pi^1,\omega)/\partial q^t_k](\pi^1_k - \pi^*_k) = \frac{1}{\gamma} - 1
$$

$$
\sum_{t=1}^N \mathbb{E}[\partial D_k(\pi^1,\omega)] = \frac{1}{\gamma}.
$$

As for the last statement of the theorem, consider that maximizing $\mathbb{R}(T)$ (or equivalently $\nabla g(\pi)$) over $\pi$ yields the related stationarity FOC

$$
\pi^M = \pi^* - \mathbb{E}[\nabla_\pi D(\pi^M,\omega)] \mathbb{E}[D(\pi^M,\omega)].
$$
This condition, which is similar to (11), characterizes the unregulated monopoly price $\pi^M$. Indeed, (27) can be obtained from (11) by replacing $\frac{2\gamma - 1}{\gamma}$ by 1, or equivalently, by letting $\gamma \to \infty$ on both sides of (11).

Now, because $\pi^M$ maximizes $\bar{\theta}(\pi)$ and $\bar{\pi}(T)$ over $\pi \geq 0$, there does not exist $\pi \geq 0$ such that $\bar{\pi}(T^*) > \bar{\theta}(\pi^M)$. Hence, when restricted to linear tariffs, problem (6) is infeasible for all $T > \bar{\pi}(\pi^M)$ for all other values of $T$ within the considered regime, i.e., $F \in \{\bar{\theta}(\pi^M), \bar{\theta}(\pi^M)\}$, the fact that $\pi^*$ and $\pi^M$ achieve $\bar{\theta}(\pi^*)$ and $\bar{\theta}(\pi^M)$, respectively, and the concavity of $\bar{\pi}(T)$ (Prop. 4) imply the feasibility of problem (6) over said interval of $F$. Concluding, in the regime $F \geq \bar{\theta}(\pi^*)$, problem (6) over linear tariffs is feasible if $F \leq \bar{\theta}(\pi^M)$ and unfeasible otherwise.

3) Proof of Corollary 3. At optimality, we have that $\bar{\pi}(T^*) = F$ and thus $d\bar{\pi}(T^*)/dF = 1$. The envelope theorem further implies that the derivative of the value function $\bar{\pi}(T)$ of problem (6) with respect to the parameter $F$ can be computed as $d\bar{\pi}(T^*)/dF = \partial L(\pi^*, \gamma)/\partial F = -\gamma$. The total derivative of the expected total surplus with respect to $F$ is then $d\bar{SW}(T^*)/dF = 1 - \gamma$. Hence, to show that $\bar{\pi}(T^*)$ and $\bar{SW}(T^*)$ are decreasing concave functions of $F$ over $F \in [\bar{\theta}(\pi^*), \bar{\theta}(\pi^M)]$, it suffices to show that $\gamma$, as a function of $F$, satisfies $\gamma \geq 1$ and $d\gamma/dF \geq 0$ over $F \in [\bar{\theta}(\pi^*), \bar{\theta}(\pi^M)]$.

Moreover, it is clear from the Lagrangean $L(\pi, \gamma)$ in (23) that $\bar{\pi}(T^*)$ must be a decreasing function of the parameter $\gamma$. Conversely, the constraint $\bar{\pi}(T^*) = F$ implies that $\bar{\pi}(T^*)$ is a strictly increasing function of $F$. Hence, we have that $\gamma$ increases as $F$ increases, and thus $d\gamma/dF \geq 0$.

Finally, note that $\gamma = 1$ and $\pi^* = \pi^M$ are optimal for $F = \bar{\theta}(\pi^*)$ since they satisfy the FOC (11) and the constraint $\bar{\pi}(T^*) = F$. Recall also from Thm. 2 that the problem at hand remains feasible if $F \leq \bar{\theta}(\pi^M)$. Because $d\gamma/dF \geq 0$, we can conclude that $\gamma \geq 1$ for $F \in [\bar{\theta}(\pi^*), \bar{\theta}(\pi^M)]$, thus completing the proof.

4) Proof of Theorem 3. We prove this result by showing that the optimal two-part tariff $T^*$ characterized by Theorem 1 attains an upper bound for the performance of all ex-ante tariffs. This upper bound is the performance achieved by a social planner who directly makes all decisions on behalf of consumers. The social planner is unlike the regulator who is limited to coordinate such decisions indirectly through a tariff. To obtain a tight upper bound for ex-ante tariffs only (rather than a looser bound for all possibly ex-post tariffs), the social planner makes customers’ decisions relying only on the information observable by each of them (i.e., $\omega^i$) as opposed to based on global information (e.g., $\xi = (\lambda, \omega^1, \ldots, \omega^M)$).

Consider the social planner’s problem in (17), which corresponds to the regulator’s problem (6) in the absence of any DERs. Therein, the notation $q^i(\omega^i)$ indicates the restrictions of the social planner to make (causal) decisions contingent only on the local state of each customer $\omega^i$. Recall from (11) that the expected consumer surplus for a given ex-ante tariff is given by $\bar{CS}(T) = \sum_{i=1}^M CS^i(T)$, where

$$CS^i(T) = \max_{q^i(\omega^i)} E \left[ S^i(q^i(\omega^i), \omega^i) - T(q^i(\omega^i)) \right] = E \left[ S^i(q^i(T, \omega^i), \omega^i) - T(q^i(T, \omega^i)) \right],$$

and the corresponding expected retailer surplus is given by

$$\bar{RS}(T) = E \left[ \sum_{i=1}^M T(q^i(T, \omega^i)) - \lambda^T q^i(T, \omega^i) \right],$$

and the expected total surplus by

$$\bar{SW}(T) = \bar{CS}(T) + \bar{RS}(T) = E \left[ \sum_{i=1}^M S^i(q^i(T, \omega^i), \omega^i) - \lambda^T q^i(T, \omega^i) \right].$$

The following sequence of equalities/inequalities shows that problem (17) provides an upper bound to problem (6).

$$\max_{T^*} \{ \bar{CS}(T) \mid \bar{RS}(T) = F \} + F$$
$$= \max_{T^*} \{ \bar{CS}(T) + F \mid \bar{RS}(T) = F \}$$
$$= \max_{T^*} \{ \bar{CS}(T) + \bar{RS}(T) \mid \bar{RS}(T) = F \}$$
$$= \max_{T^*} \{ \bar{SW}(T) \mid \bar{RS}(T) = F \}$$
$$\leq \max_{\{q^i(\omega^i)\}_{i=1}^M} \bar{SW}.$$ (28)

In particular, the inequality in (29) holds because $\bar{SW}(T)$ depends on $T$ only through $q^i(T, \omega^i)$. This implies that maximizing $\bar{SW}(T)$ directly over $\{q^i(\omega^i)\}_{i=1}^M$ rather than indirectly over $T^*$ is a relaxation of the optimization in (28). Clearly, the problem in (29) corresponds to the social planner’s problem in (30) and (17).

It suffices to show now that $T^*$ attains the upper bound in (30). To that end, we use the independence sufficient condition $\omega \perp \lambda$. We show that, under said condition, the expected total surplus $\bar{SW}(T^*)$ matches the upper bound. First note that the condition $\omega \perp \lambda$ allows to rewrite the upper bound in (29) and (30) as follows.

$$\max_{\{q^i(\omega^i)\}_{i=1}^M} \bar{SW}$$

$$= \sum_{i=1}^M \max_{q^i(\omega^i)} E_{\omega^i} \left[ S^i(q^i(\omega^i), \omega^i) - E \left[ \lambda^T |\omega^i| q^i(\omega^i) \right] \right]$$

$$= \sum_{i=1}^M \max_{q^i(\omega^i)} E_{\omega^i} \left[ S^i(q^i(\omega^i), \omega^i) - \lambda^T q^i(\omega^i) \right]$$

$$= \sum_{i=1}^M E_{\omega^i} \left[ S^i(D^i(\lambda, \omega^i), \omega^i) - \lambda^T D^i(\lambda, \omega^i) \right].$$

The last equality follows from the definition of the demand function $D^i(\pi, \omega^i)$ in (5) as the optimal response of customers to deterministic prices.

The result follows since the tariff $T^*$ induces the same
expected total surplus if $\omega \perp \lambda$, i.e.,

$$
\mathbb{E} \left[ \sum_{i=1}^{M} S^i(D^i(\pi^*, \omega_i), \omega_i) - \lambda^\top D^i(\pi^*, \omega_i) \right] = 
\sum_{i=1}^{M} \mathbb{E}_{\omega^i} \left[ S^i(D^i(\bar{\lambda}, \omega_i^i), \omega_i^i) - \bar{\lambda}^\top D^i(\bar{\lambda}, \omega_i^i) \right].
$$