ANALYSIS OF THE METHOD OF PREDICTIVE CONTROL APPLICABLE TO ACTIVE MAGNETIC SUSPENSION SYSTEMS OF AIRCRAFT ENGINES

Conventional controllers are usually synthesized on the basis of already known parameters associated with the model developed for the object to be controlled. However, sometimes it proves extremely difficult or even infeasible to find out these parameters, in particular when they subject to changes during the exploitation lifetime. If so, much more sophisticated control methods have to be applied, e.g. the method of predictive control. Thus, the paper deals with application of the predictive control approach to follow-up tracking of an active magnetic suspension where the mathematical and simulation models for such a control system are disclosed with preliminary results from simulation investigations of the control system in question.

Keywords: active magnetic suspension, predictive control method, follow-up adaptive control method.

1. Introduction

Active magnetic suspension (AZM) systems benefit from the phenomenon of magnetic levitation when the force $F_m$ of magnetic attraction between an electromagnet (solenoid) and a ferromagnetic core compensates the force of gravity $F_g$ (Fig. 1). That balance serves as the operation principle for active magnetic bearings, i.e. devices that benefit from attraction and repelling forces to enable stable levitation of the rotor. Use of such devices eliminates friction between mating kinematic pairs and enables continuous monitoring and diagnostics of technical condition demonstrated by such systems through measurements of vibrations and
forces. Aircraft engines use the magnetic bearing as components of support systems for engine shafts.

But anyway, magnetic suspension systems are inherently unstable due to their structure, thus their reliable operation needs engineering of an appropriate control system to enable its stability and to achieve the required parameters of control duality.

Conventional controllers for such systems are usually synthesized on the basis of already known parameters associated with the model developed for the object to be controlled. The better accuracy is achievable for these parameters the more dependable control can be provided for the object. However, sometimes it proves extremely difficult or even infeasible to find out these parameters, in particular with regard to real objects since their parameters subject to changes during the equipment lifetime.

Fig. 1. Arrangement of the system designed to control position of a rotor within an active magnetic suspension system [3]

Fast development of digital technologies enabled implementation of really advanced methods of control, such as adaptive (follow-up), predictive or sliding [8] solutions or robust control. These algorithms grow out of time analysis and synthesis of control units with use of state variables, which makes it possible to achieve optimized control units. These algorithms take explicit account of meas-
urement uncertainties and absolve from the need to accurately know all properties of the controlled object. Adaptive control allows to changing parameters of the control unit according to embedded rules and laws of control algorithms and continuous identification of controlled object parameters. On the other hand, robust control method enables designing of control units with consideration to models of uncertainties [8].

2. A model of active magnetic suspensions

Operation of active magnetic suspension assumes that a rotor is disposed within an air gap at the working point, i.e. within the equal distance from the both poles. That distance is one of the most important parameters of the suspension and determines other parameters of the system, such as current stiffness $k_i$ and displacement stiffness $k_s$. These parameters define the attracting force $F_m$ of an electromagnet (solenoid).

Displacement of a rotor supported on an active homopolar electromagnetic bearing within an air gap is defined by the following differential equation [1, 2]:

$$m \frac{d^2 x}{dt^2} = F_m \pm F_z = k_s x + k_i i \pm F_z$$

(1)

where:

$m$ – weight of the rotor

$i$ – control current;

$x$ – displacement of the rotor from the working position;

$F_z$ – external force of disturbance.

Application of the Laplace transform to the equation (1) leads to formulation of the transition function defined by the relationship (2). It is the equation that makes it possible to derive operator functions for transfer function to define behaviour of the system for such input signals as values of control currents (3) or external forces.

$$X(s) = \frac{k_i}{ms^2 - k_s} I(s) \pm \frac{1}{ms^2 - k_s} F_z(s)$$

(2)

$$G_{x_i}(s) = \frac{X(s)}{I(s)} = \frac{k_i}{ms^2 - k_s}$$

(3)

$$G_{x,F_z}(s) = \frac{X(s)}{F_z(s)} = \pm \frac{1}{ms^2 - k_s}$$

(4)
In case of the control system subjected to analysis in this paper the input signal is understood as variations of the control current $I(s)$ whilst the rotor displacements $X(s)$ in the air gap serve as the output of the control unit. That unit was then reproduced in the Matlab-Simulink software environment according to the model disclosed in Fig. 2. It is the control loop that shall be supervised by a predictive control unit synthesized for that purpose.

Fig. 2. A model for active magnetic suspension (AZM) developed in the Matlab-Simulink software environment.

3. Analysis of predictive control solutions

Methods of predictive control can be applied to objects with structures that are inherently unstable, non-linear and non-stationary. This method is intended to find out a sequence of future values for the control signal pursuant to a reference model. The developed algorithm enables stable and undisturbed operation of even non-linear and unstable objects with no necessity to consider that property of controlled objects in the synthesis of a control system [5, 7].

There are plenty of control methods based on predictive algorithms. The most popular that found the broadest application are such algorithms as Extended Horizon Adaptive Control (EHAC), Extended Prediction Self-Adaptive Control (EPSAC) and Generalized Predictive Control (GPC). On the other hand, the second group of algorithms comprise such solutions as Model Algorithmic Control (MAC) that assumes simple adaptive control with a model of step-response, Model Predictive Control (MPC), which is a model for differential control with a model of impulse response as well as Dynamic Matrix Control (DMC) which is an algorithm for predictive control with a model of step response.

The methodology for application of predictive algorithms is illustrated in Fig. 3. The algorithms are used to predict future values of output signals $\hat{y}(i + j), j = 1, \ldots, H - 1$ for the horizon of prediction ($H$) as well as values of input control signals $u(i + j), j = 1, \ldots, L - 1$ for the horizon of control ($L$) at every mo-
Analysis of the method of predictive control applicable to active magnetic

moment of time $i$ so that to fulfil the control objective (e.g. minimization of the control deviation). Fig. 3 exhibits that increments of control signals are assumed to be zeros from the moment of $i = i + L$, i.e. $u(i + L) = u(i + L)$. For correct operation of a predictive control unit the assumption $L \geq H$ must be fulfilled.

![Fig. 3. Predictive control with displaceable horizon](image)

The class of algorithms that are suitable for predictive control and use models of objects in the form of step response and impulse response comprise the Model Predictive Control (MPC) algorithm with the model of impulse response. The algorithm assumes the model of a control system in the form of equation (5). Coefficients for the polynomial of the impulse response $V$ that occur in the relationship (5) are computed by multiplication of the discrete transfer function of the object and the $Z$ transform for a Dirac impulse [4, 7].

$$y(i) = Vu(i-1)$$

(5)

where:

- $V = v_0 + v_1 z^{-1} + \cdots + v_n z^{-nV}$ – polynomial of the impulse response;
- $v_0, v_1, \ldots, v_n$ – subsequent values of the impulse response for the B/A component.
- $A = 1 + a_1 z^{-1} + \cdots + a_n A z^{-nA}, B = b_0 + b_1 z^{-1} + \cdots + b_n B z^{-nB}$ – polynomials that describe the controlled object.

The goal objective for that algorithm is to minimize the discrepancies between the anticipated waveform of the output signal and the reference waveform according to the criterion (6) with consideration to the weight factor imposed to the deviation between the control output and the $u(i-1)$ value.

$$J = \sum_{j=1}^{H} \{[\hat{y}(i + j) - w(i + j)]^2 + p\delta u^2(i + j - 1)\}$$

(6)
where:

$$\delta u(i + j) = u(i + j) - u(i - 1)$$

The predictive algorithm that employs the model of an impulse response can be expressed in a general notation by means of equations for the RST controllers [7]:

$$R_u(i) = T_w(i) - S_Y(i)$$

where:

\[ R = \nabla (1 + z^{-1} \sum_{j=1}^{H} q_j \sum_{s=1}^{j} V^2_s) - \text{control function} \quad (8) \]

\[ T = K_m \sum_{j=1}^{H} q_j - \text{complementary sensitivity function} \quad (9) \]

\[ S = \sum_{j=1}^{H} q_j - \text{sensitivity function} \quad (10) \]

The \( K_m \) denotation that occurs in the equation for the \( T \) polynomial stands for a reference trajectory that is defined by the equation (11) for a first rank loop. The \( A_m \) and \( B_m \) parameters of that equation represent polynomials that define the reference model, whilst the \( \rho \) coefficient is the weighting parameter of the control signal and expresses how fast the output signal can reach the determined threshold.

$$\frac{B_m}{A_m} = \frac{(1-\rho)z^{-1}}{1-\rho z^{-1}}$$

(11)

Parameters \( q_j \) that stand in equations (8÷10) are components of the \( q \) vector that is defined by the (12) relationship, whilst \( V^2 \) are coefficients of the polynomial that represents an impulse response and is then subjected in accordance with the equation (13). Terms of the \( Q \) matrix denoted as \( h_i \) stand for parameters of the step response of the object.

$$q^T = [q_1, q_2, ..., q_H] = i^T [Q^T Q + \rho I]^{-1} Q^T$$

where:

$$Q = \begin{bmatrix}
0 & 0 & 0 \\
V_0 & V_0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
V_{L-1} & V_{L-2} & \cdots & h_0 \\
\vdots & \vdots & \ddots & \vdots \\
V_{H-1} & V_{H-2} & \cdots & h_{H-L}
\end{bmatrix}$$

$$i^T = [1, 0, 0, ..., 0]$$

$$V = V_j^1 + z^{-j}V_j^2$$

(13)
where:

\[
V_j^1 = v_0 + v_1 z^{-1} + \cdots + v_{j-1} z^{-j+1} \\
V_j^2 = v_j + v_{j+1} z^{-1} + \cdots + v_n z^{-n}
\]

The synthesis process for a predictive control unit is based on a model for the controlled object (4) and assumes determination of a polynomial for both a impulse response and step response of that object. Then parameters of the \( V \) polynomial are used to find out the \( Q \) matrix where the last column of that matrix is made up of parameters \( h_i \) for the step response. The MPC controller can be synthesized on the basis of coefficients representing the \( V \) polynomial as well as terms of the \( q \) vector derived from the \( Q \) matrix.

4. A simulation model developed in the Matlab-Simulink software environment for the predictive control algorithm

The Matlab-Simulink software environment with the Model Predictive Control Toolbox [9] package was used to carry out simulation studies on the predictive control algorithm for the active magnetic suspension system. Fig. 4 shows a simulation model for such a control loop. The model for magnetic suspension was developed on grounds of the equation for the rotor displacements in an air gap (2) of a homopolar magnetic bearing with permanent magnets. The simulation process made it possible to evaluate how input parameters of the intended predictive controller affect step characteristics of the system. These parameters included a control horizon and a prediction horizon parameters.

Fig. 4. The simulation model for the active magnetic suspension (AZM) system supervised by a predictive controller (Matlab-Simulink).
Fig. 5 presents characteristic curves obtained for a step-function input signal applied to the closed-loop control system. The input signal applied to the system was a discrete displacement of the suspension rotor within the air gap by a distance of $10^{-4}$ m. The controller was synthesized for selected values of control horizon parameter ($L = 1, L = 2, L = 5$) and for fixed range of the prediction horizon parameter ($H = 10$). The highest value of control overshoot was revealed for the system with the largest range of the control horizon parameter ($L = 5$). Both for such a system as well as for the system with the control horizon value equalling 2 the system response is of oscillating nature with overshoots ranging respectively to 6% and 1%. Under the foregoing conditions for the closed-loop control systems ($L = 1, L = 2, L = 5$) the settling time of the system equals respectively to 0.045 s, 0.012 s and 0.015 s. For each closed-loop control system subjected to investigations the value of control deviation under steady condition equalled to zero [6].

![Fig. 5. Characteristics for a closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller, plotted as a response to a step-function input signal for various ranges of control horizon $L$.](image)

Fig. 6 depicts waveforms of control signals produced by the closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller. The highest value of the control signal corresponds to the system with a control loop with a longest range of control horizon parameter ($L = 5$).
Fig. 6. Graphs of control signals produced by a closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller, plotted for various ranges of control horizons.

Fig. 7 shows time waveforms for a closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller. The input signal for the system was produced as a discrete displacement (step-function) of the rotor inside an air gap by the distance of $10^{-4}$ m. The controller was synthesized for selected ranges of prediction horizon parameters ($H = 5$, $H = 10$, $H = 15$) and for fixed range of the control horizon parameter ($L = 1$). The shortest settling time equal to 0.005 s combined with the highest overshoot range of 5% was found for the control loop with the shortest prediction horizon parameter ($H = 5$). Its response to the step-function is of the oscillation nature. Responses to step-functions recorded for two other systems demonstrate an inertial behaviour with zero overshoots and the settling time of 0.04 s and 0.18 s for control horizon ranges equalling respectively $H = 10$ and $H = 15$. Deviations of control under steady conditions was zero for each close-loop control system subjected to investigations.

Fig. 8 depicts waveforms of control signals produced by the closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller. These waveforms are plotted for various ranges of prediction horizon $H$. The highest value of the control signal corresponds to the system with a control loop with a shortest range of prediction horizon parameter ($H = 5$).
Fig. 7. Characteristic curves for a closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller, plotted as a response to a step-function input signal for various ranges of prediction horizon ($H$).

Fig. 8. Characteristics of control signals produced by a closed-loop control system for the active magnetic suspension (AZM) system with a predictive controller, plotted for various ranges of prediction horizon parameters.
5. Recapitulations and conclusions

Algorithms of predictive control are rated among the most sophisticated control methods that benefit from identification of a parameterized or not parameterized models for the controlled object. The first group of predictive algorithms comprised methods that use the transfer function of the object, such as the algorithm of Extended Horizon Adaptive Control (EHAC), Extended Prediction Self-Adaptive Control (EPSAC) and Generalized Predictive Control (GPC). The second group of algorithms is made up of the methods that employ the characteristic of the object response to a step-function input or to an input series of Dirac impulses, such as the Model Predictive Control (MPC), Model Algorithmic Control (MAC) that assumes simple predictive control with a model of a step-function response, as well as Dynamic Matrix Control (DMC) which is an algorithm for predictive control with a model of step response.

Dynamic properties of each predictive controller depend on already predefined control horizon and prediction horizon parameters, weighting functions for all signals and constraints imposed to both the control signals and output signals.

Simulation investigations of own-developed algorithms for predictive control made it possible to find out how the ranges of control horizon and prediction horizon affect parameters that define quality of control systems. Extension of the control horizon parameter for the active magnetic suspension (AZM) system reduces the settling time of the system but leads to increase of the overshoots parameter. On the other hand, extension of the prediction horizon entails prolongation of the settling time but mitigates the oscillation departures. All control algorithms that were implemented for closed-loop control system with a predictive controller and then subjected to investigations enabled observation that the output deviations under steady conditions were zero.

The foregoing results of completed analyzes and simulations make up the preliminary step to experimental verifications. Awareness of physical phenomena that occur in active magnetic suspension (AZM) system as well as familiarity with synthesis methods for parameters of a predictive controller for unstable structures enable determination of typical working parameters for the system in question. The subsequent phase of studies assumes that monitoring of variations demonstrated by control signals within the established control horizon and prediction horizon parameters shall enable to make judgment on technical condition of an active magnetic suspension (AZM) system. Achieved results shall be adaptable to more sophisticated technical units, such as magnetic bearing systems for shafts of turbojet engines.
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