Cascaded Text Generation with Markov Transformers

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Abstract

The two dominant approaches to neural text generation are fully autoregressive models, using serial beam search decoding, and non-autoregressive models, using parallel decoding with no output dependencies. This work proposes an autoregressive model with sub-linear parallel time generation. Noting that conditional random fields with bounded context can be decoded in parallel, we propose an efficient cascaded decoding approach for generating high-quality output. To parameterize this cascade, we introduce a Markov transformer, a variant of the popular fully autoregressive model that allows us to simultaneously decode with specific autoregressive context cutoffs. This approach requires only a small modification from standard autoregressive training, while showing competitive accuracy/speed tradeoff compared to existing methods on five machine translation datasets.

1 Introduction

Probabilistic text generation is a ubiquitous tool in natural language processing. Originally primarily studied with respect to machine translation \([1,27]\), its progress has led to applications in document summarization \([39,44]\), data-to-text \([58]\), image captioning \([59]\), etc. State-of-the-art text generation approaches rely on fully autoregressive models such as RNNs and transformers \([51]\), in which the probability of an output word depends on all previous words. At inference time, beam search is used for decoding, a left-to-right serial procedure. To speed up decoding, researchers have proposed alternative parallel generation models. One class of non-autoregressive probabilistic models assumes that each word’s output probability is independent of other words \([13,65,28]\). While it is impressive that these models perform well, this independence assumption is very strong and often results in noticeable artifacts such as repetitions \([13,49]\).

We note that non-autoregressive models, while sufficient, are not necessary for fast probabilistic parallel generation. On parallel hardware, inference in models with bounded Markov dependencies is trivial to parallelize and requires sub-linear time w.r.t. sequence length \([42,38]\). In practice, given the right parameterization, we can explore any level of autoregressive dependencies to achieve a speed/accuracy tradeoff.

In this work, we exploit this property by proposing cascaded decoding with a Markov transformer architecture. Our approach centers around a graphical model representation of the output space of text generation. Given this model, we can employ cascaded decoding \([7,8,56,40]\) for parallel text generation, using an iterative procedure that starts from a non-autoregressive model and introduces increasingly higher-order dependencies. We combine this approach with a Markov transformer, an extension to the fully autoregressive transformer architecture. This network uses barriers during training to ensure it learns fixed high-order dependencies. At test time, a single network can be used to parameterize a cascade of different graphical models. The Markov transformer only changes self-attention masks and inputs at training, and is applicable to all transformer variants.

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Experiments on five machine translation datasets compare this approach to other beam search and non-autoregressive baselines. Our inference approach is comparably fast to non-autoregressive methods while allowing for local dependencies in a principled, probabilistic way. Results validate the competitive accuracy/speed tradeoff of our approach compared to existing methods. The code for reproducing all results is available at https://github.com/harvardnlp/cascaded-generation.

2 Related Work

There has been extensive interest in non-autoregressive/parallel generation approaches, aiming at producing a sequence in parallel sub-linear time w.r.t. sequence length [13, 52, 26, 65, 53, 14, 11, 12, 48, 15, 28, 16, 49, 55, 30, 41, 64, 62]. Existing approaches can be broadly classified as latent variable based [13, 26, 65, 28, 41], refinement-based [25, 48, 14, 15, 11, 30, 12, 62] or a combination of both [41].

Latent-variable approaches factor out the dependencies among output words, such that we can generate each word independently of each other conditioned on those latent variables. The training of these approaches usually employs variational autoencoders, since the log marginal is intractable [21, 37, 31]. The introduced latent variables enable generation in a single forward pass, achieving \(O(1)\) time complexity regardless of sequence length, but many of them suffer from generation artifacts such as repetitions [13]. While not using latent variables, our approach could be extended to incorporate them. A notable difference is that the parallel time complexity of this work is not \(O(1)\) but \(O(\log L)\) w.r.t. sequence length. In practice though, the only \(O(\log L)\) part in our approach takes a negligible fraction of total time [42], and our approach reaches comparable speedup compared to existing approaches with \(O(1)\) time complexity.

Another line of research uses refinement-based methods, where the model learns to iteratively refine a partially/fully completed hypothesis. Training usually takes the form of masked language modeling [11, 12] or imitating hand-crafted refinement policies [25, 48, 15]. Refinement-based approaches can sometimes reach better performance after multiple forward passes compared to latent variable based approaches which mostly use a single forward pass [15, 11, 41]. While our method superficially resembles refinement, our approach is probabilistic, model-based, and conceptually simpler. Training is by maximum likelihood, requires no hand-designed rules, and allows for activations to be preserved between iterations. A final benefit of our approach is that multiple lengths can be considered at no extra cost, as opposed to generating candidates under different lengths and reranking [11, 49, 28].

Our approach is motivated by structured prediction cascades (SPC) [56]. SPC is a technique in graphical models for graphical model type tasks, where we can specify the length of the sequence beforehand [56]. To the best of our knowledge, we are the first to adapt it to neural text generation. We also go beyond SPC, which uses multiple models, and show how to adapt a single Markov transformer model to learn the entire cascade. While [49] shares our motivation and combines a 0th order model with a 1st order graphical model, they do not consider higher-order models or cascades, or show how to achieve parallel sublinear time.

3 Cascaded Decoding for Conditional Random Fields

Neural text decoding can be viewed as a conditional random field (CRF) [24] over a sequence of words \(x_{1:L}\), where \(x_i \in \mathcal{V}\) with \(|\mathcal{V}| = V\), and \(\mathcal{X} = \mathcal{V}^L\) is the set of all sequences. This model defines a conditional probability distribution \(P(x_{1:L} | c)\), where \(c\) is an arbitrary conditioning term, e.g., a source sentence. Define an \(m\)-th (Markov) order CRF model as,

\[
P^{(m)}(x_{1:L} | c; \theta) \propto \exp \sum_{l=1}^{L-m} f_l^{(m)}(x_{l:l+m}, c; \theta^{(m)}),
\]

where \(f_l^{(m)}(\cdot)\)'s are any parameterized log potentials looking at \(m + 1\) words, for example local log-probabilities. For simplicity, we omit \(c\) and \(\theta^{(m)}\) through the rest of this paper. We can define two important special cases of this CRF model. With \(m = L - 1\), we can recover fully autoregressive neural text generation models such as RNNs and transformers. Using \(m = 0\) gives us non-autoregressive models.
Decoding aims to find the sequence with the highest model score, \( \max_{x' \in X} P^{(m)}(x') \). Computing this exactly can be done with the Viterbi algorithm in \( O(V^{m+1}L) \); however, even for \( m = 1 \) this is intractable since \( V \) is typically on the order of \( 10^5 \). Beam search is commonly used instead to approximate this value, but it cannot be parallelized, and alternatives to beam search remain under-explored in the literature.

We propose an alternative cascaded decoding approach based on max-marginals [56], which are used as a metric to prune “unlikely” n-grams at each position based on the score of the “best” sequence with a given n-gram. To be precise, define the notation \( \mathcal{X}(x_{i:j}) \) to be the set of sequences that contain a span \( x_{i:j} \), i.e. \( \{x' \in \mathcal{X} : x'_{i:j} = x_{i:j}\} \). The max-marginal of \( x_{i:j} \) is the maximum score in this set:

\[
\operatorname{MM}_{\mathcal{X}}^{(m)}(x_{i:j}) = \begin{cases} 
\max_{x' \in \mathcal{X}(x_{i:j})} P^{(m)}(x'_{1:L}) & \mathcal{X}(x_{i:j}) \neq \emptyset \\
0 & \text{o.w.}
\end{cases}
\]

Cascaded decoding, illustrated in Figure 1 proceeds by iteratively computing max-marginals for progressively higher-order models while filtering out unlikely spans. Starting with a complete initial set \( \mathcal{X}_0 = \mathcal{X} \), for all single word spans \( x_{i:l} \), we compute \( \mathcal{X}_0^{(0)} \) and collect the top \( K \) max-marginal values at each step,

\[
\mathcal{X}_1 = \{x_{1:L} \in \mathcal{X}_0 : x_{l:l} \in K \arg \max_{x'_{l:l} \in V^l} \operatorname{MM}_{\mathcal{X}_0}^{(0)}(x'_{l:l}) \text{ for all } l \}.
\]

We then apply a 1st order model \( (m = 1) \) and collect the top \( K \) \( x_{l:l+1} \) values with the highest max marginals \( \mathcal{X}_1^{(1)}(x_{l:l+1}) \) to further prune the search space,

\[
\mathcal{X}_2 = \{x_{1:L} \in \mathcal{X}_1 : x_{l:l+1} \in K \arg \max_{x'_{l:l+1} \in V^{l+1}} \operatorname{MM}_{\mathcal{X}_1}^{(1)}(x'_{l:l+1}) \text{ for all } l \}.
\]

We repeat the above process \( M \) times, and prune the search space to \( \mathcal{X}_M \). It can be shown that based on properties of max marginals this set is always non-empty [56]. We decode by finding the sequence \( x_{1:L} \) with the highest score \( P^{(M)}(x_{1:L}) \) in \( \mathcal{X}_M \).

**Implementation** The only non-parallel component of cascaded decoding is calculation of max-marginals for \( m \geq 1 \). With \( m = 1 \), max-marginals \( x_{l:l+1} \) can be exactly computed using a variant of the forward-backward algorithm. This algorithm requires \( O(K^2L) \) time when performed serially.

We can reduce this complexity on parallel hardware by leveraging the commutative property of max [42][33], and computing an inside-outside prefix sum. First we pad the sequence to a power of 2 and construct a balanced binary tree with words as leaves. We then perform max operations bottom-up and top down. The height of the tree dictates the parallel time of this approach, \( O(K^2 \log L) \).

For higher-order models with \( m > 1 \), we can compute max-marginals for \( x_{l:l+m} \) using a reduction to an \( m = 1 \) CRF. By construction, \( \mathcal{X}_m \) has exactly \( K \) spans \( x_{l:l+m} \) such that \( \mathcal{X}(x_{l:l+m}) \neq \emptyset \).
Algorithm 1 Parallel Cascaded Decoding

Given: max length $L$, limit $K$, log potentials $f^{(m)}$ for $m$ in $\{0, \ldots, M\}$, parameters $\theta$

function CASCADE()
    for $m = 0 \rightarrow M - 1$
        Compute potentials $f^{(m)}_l(x_{l:t+m}; \theta)$ for all $x_{t:l+m} \neq \emptyset(K)$ $\triangleright O(K^2)$
        Compute first-order state relabeling $\Phi^{(m)}_l$ for all positions $l = 1 \ldots L - m$ $\triangleright O(K)$
        Compute max-marginals $MM^{(m)}_l$ using TREEMM $\triangleright O(K^2 \log L)$
        Set $x_{m+1} = \left\{ x_{1:L} \in X_m : x_{l:l+1} \in K \arg \max_{x_{l:l+1} \in V^{m+1}} MM^{(m)}_l(x_{l:l+1}) \right\}$ $\triangleright O(K^2)$
    return $\arg \max_{x' \in X_M} P^{(M)}(x')$.

function TREEMM(First-order scores $C^0_i$, of size $L \times K \times K$)
    All $C^i, S^i, P^i$ size $2\log L - i \times K \times K$, all $j \in \{1, \ldots, 2\log L - 1\}$; $P^0, S^0 \leftarrow -\infty$
    for $i = 0 \rightarrow \log L - 1$ $\triangleright$ Chart max-scores computed bottom-up
        $C^{i+1}\_j \leftarrow \max_k C^i\_j + C^i\_{(2j+1)k}$.
    for $i = \log L \rightarrow 0$ $\triangleright$ Prefix and suffix MM scores computed top-down
        $P^i\_{2j} \leftarrow P^i\_{j}; P^i\_{2j+1} \leftarrow \max_k P^i\_j + C^i\_{(2j+1)k}$.
        $S^i\_{2j} \leftarrow S^i\_j; S^i\_{2j+1} \leftarrow \max_k C^i\_{(2j+1)k} + S^i\_j$.
    return $\exp[(\max_k P^0\_j) + C^0\_j + (\max_k S^0\_j)]$ $\triangleright O(K^2 \log L)$

for all positions $l$. We relabel these spans $x_{l:t+m}$ as $1 \ldots K$ for each position, using a mapping $\Phi^{(m)}_l(\ldots)$. This mapping implies that there are at most $K^2$ transitions between $\Phi^{(m)}_l(x_{l:t+m})$ to $\Phi^{(m)}_{l+1}(x_{l+1:t+1+m+1})$, resembling an $m = 1$ model over $\Phi$. Therefore, the total parallel computation cost of this process is $O(K^2 \log L)$.

The full procedure is given in Algorithm 1. As opposed to $O(V^{M+1} \log L)$ of exact search, the cascaded approximation can be computed in parallel in $O(MK^2 \log L)$. We note that this yields a sub-linear time yet (partially) autoregressive decoding algorithm.

Handling Length A common issue in parallel generation is the need to specify the length of the generation beforehand [13,28]. It is hard to predict the exact length and constraining search with strict length limits the maximum achievable score. We can relax the length constraint by considering multiple lengths simultaneously. We introduce a special padding symbol pad to $V$ at inference time, and add log-potentials to force pad and end-of-sentence tokens EOS to transition to pad. Candidate sequences of different lengths are padded to the same length, but trailing pad’s do not affect scores. The CRF parameterization allows us to consider all these lengths simultaneously, where extending the length only introduces log additional time. More details can be found at supplementary materials.

4 Model Parameterization: Markov Transformer

The cascaded decoding approach can be applied to any cascades of CRF models that obey the properties defined above, i.e., $m$-th order log-potentials. Given a training set $(c^{j}, x^{j})_{1:j}$ we would like $M + 1$ different parameters that satisfy the following MLE objectives:

$$\theta^{(m)} = \arg \max_{\theta^{(m)}} \sum_{j} \log P^{(m)}(x^{j}_{1:L} \mid c^{j}; \theta^{(m)}) \text{ for all } m \in \{0, \ldots, M\}$$

Naive approaches for cascading would require training $M + 1$ different models that are calibrated of trained together to produce similar outputs [56]. These also cannot be standard translation models such as RNNs or transformers [18,50,51], since they have $m = L - 1$. 
5 Experiments

Datasets We evaluate our approach on five commonly used machine translation benchmark datasets: IWSLT14 De-En [6] (~160k parallel sentences), WMT14 En-De/De-En [29] (~4M parallel sentences) and WMT16 En-Ro/Ro-En [3] (~610k parallel sentences). To process the data, we use Byte Pair Encoding (BPE) [45, 23] learned on the training set with a shared vocabulary between source and target.
target. For IWSLT14 the vocabulary size is 10k; for WMT14 the vocabulary size 40k. For WMT16 we use the processed data provided by [25]. We sample all validation datasets to be at most 3k.

Model Settings Markov transformer uses the same hyperparameters as standard transformers. The base settings are from FAIRSEQ[34]: For IWSLT14 De-En, we use 6 layers, 4 attention heads, model dimension 512, hidden dimension 1024; for WMT14 En-De/De-En and WMT16 En-Ro/Ro-En we use 6 layers, 8 attention heads, model dimension 512, hidden dimension 2048. We tie the decoder output projection matrix on all datasets [35], and we share source and target embeddings on WMT14 En-De/De-En and WMT16 En-Ro/Ro-En. It differs only in the application of attention barriers, where we set $M = 4$. The optimization settings can be found at supplementary materials.

At generation time, we predict the length $L$ using linear regression based on source length. We consider hypotheses of length $L - \Delta L$ to $L + \Delta L$ where we vary $\Delta L$ from 0 to 5. Since the Markov transformer was trained with $M = 4$, we consider applying cascaded decoding for 2 to 5 iterations (2 iterations corresponds to $M = 1$ in Algorithm [1], where more iterations consider higher local dependency orders at the cost of more computations. The limit $K$ is chosen from 16, 32, 64, 128.

Baselines For the fully autoregressive baseline, we use the same model setting and use beam size 5. We also compare to other parallel generation methods. These include a latent variable approach: FlowSeq [28]; refinement-based approaches: CMLM [11], Levenshtein transformer [12] and SMART [16]; a mixed approach: Imputer [41]; reinforcement learning: Imitate-NAT [55]; and another sequence-based approach: NART-DCRF [49] which combines a non-autoregressive model with a 1st-order CRF. Several of these methods use fully autoregressive reranking [13], which generally gives further improvements but requires a separate test-time model.

Evaluation We evaluate the BLEU score of different approaches. Following prior works [28, 49, 64], we use tokenized cased BLEU for WMT14 En-De/De-En and tokenized uncased BLEU for IWSLT14 De-En and WMT16 En-Ro/Ro-En, after removing BPE. We measure the average decoding time of a single sentence [13, 25, 16, 15, 53, 49] on a 12GB Nvidia Titan X GPU.

Extension Knowledge distillation [17, 19, 63] is a commonly used technique to improve the performance of parallel generation [13, 25, 28]. In knowledge distillation, we translate the training set using a fully autoregressive transformer and use the translated sentences as the new target for training.

5.1 Results

Results are presented in Table [1]. We show the tradeoff between speedup and BLEU score by finding the configuration that gives the best BLEU score with more than $1 \times, 2 \times, \ldots, 7 \times$ validation speedup.

Using knowledge distillation, our results get close to the fully autoregressive baseline: on WMT14 En-De, the gap between our approach and transformer is 0.5 BLEU, while being 2.4x faster ($K = 92$, iters=5). Our results are also competitive to previous works, even those using a reranker. For example, on WMT14 En-De, we can get 26.52 BLEU score at a 4.68x speedup, compared to NART-DCRF that reaches 26.80 BLEU at a 4.39x speedup using 19 candidate sentences to rerank. On IWSLT14, our BLEU scores are much better than previous works: we can reach within 0.54 BLEU score compared to transformer at a 5.88x speedup ($K = 16$, iters=2), 6 BLEU points better than FlowSeq.

Our approach is also competitive against previous works without distillation: at a speedup of 2.06x, we achieved a better BLEU score than FlowSeq-large using 30 candidates to rerank, which also has many more parameters (66M vs. 258M excluding the reranker). The one model that outperforms our approach is the Levenshtein Transformer. We note though that this model requires hand-crafted rules for training, and uses global communication, while our approach is probabilistic and only requires communicating log potentials between adjacent positions.

5.2 Analysis

Candidates Searched Unlike beam search, which is limited to a fixed number ($KL$) of candidates, cascaded search can explore an exponential number of sequences [61]. Figure 3(a) shows the number of candidate sequences scored by cascaded decoding ($f^{(2)}, f^{(3)}, f^{(4)}$) and beam search ($f_{\text{AR}}^{(L-1)}$). We additionally note that max-marginal computations are in practice extremely fast relative to transformer computation and take less than 1% of the total time, so the bottleneck is computing potentials.

https://github.com/pytorch/fairseq/tree/master/examples/translation
we can reduce the ratio of repetitions, as shown in Figure 3(c), where we measure the extent of visible artifacts in generation such as n-gram repetitions. By introducing higher-order dependencies, complexity is lower than normal transformer, since it attends to at most $M$ past words.

| Approach | Latency (Speedup) | WMT14 En-De | En-De | En-Ro | Ro-En | De-En |
|----------|------------------|-------------|-------|-------|-------|-------|
| Transformer (beam 5) | | 318.85ms ($\times 1.00$) | 27.41 | 31.49 | 33.89 | 33.82 | 34.44 |

**With Distillation**

Cascaded Generation with Speedup

- $> x 7$ (K=16, iters=2) 50.28ms ($\times 6.34$) 26.34 30.69 32.70 32.66 33.90
- $> x 6/5$ (K=32, iters=2) 52.93ms ($\times 6.02$) 26.43 30.72 32.73 32.70 34.01
- $> x 4$ (K=64, iters=2) 68.09ms ($\times 4.68$) 26.52 30.73 32.77 32.76 34.02
- $> x 3$ (K=32, iters=4) 107.14ms ($\times 2.98$) 26.80 31.22 33.14 33.22 34.43
- $> x 2$ (K=64, iters=5) 132.64ms ($\times 2.40$) 26.90 31.15 33.08 33.13 34.43
- $> x 1$ (K=64, iters=5) 189.96ms ($\times 1.68$) 26.92 31.23 33.23 33.28 34.49

**Literature**

- FlowSeq-base [28] - 21.45 26.16 29.34 30.44 27.55
- FlowSeq-large [28] - 23.72 28.39 29.73 30.75 -
- Base CMLM [11] (iters=10) - 27.03 30.53 33.08 33.31 -
- Levenshtein [43] 92ms ($\times 4.01$) 27.27 - - 33.26 -
- SMART [42] (iters=10) - 27.65 31.27 - - -
- Imputer [41] (iters=1) - 25.8 28.4 - - -
- imitate-NAT [55] ($\times 18.6$) 22.44 25.67 28.61 28.90 -
- NART-DCRF [49] ($\times 10.4$) 23.44 27.22 27.44 - -

**Literature+Reranking**

- FlowSeq-large (rescoring=30) - 25.31 30.68 - - -
- Base CMLM (iters=4, rescoring 2) - ($\times 3.0-3.1$) 25.6-25.7 - - -
- imitate-NAT (rescoring=7) - ($\times 9.70$) 24.15 27.28 31.45 31.81 -
- NART-DCRF (rescoring=9) - ($\times 6.14$) 26.07 29.68 29.99 - -
- NART-DCRF (rescoring=19) - ($\times 4.39$) 26.80 30.04 30.36 - -

**Without Distillation**

Cascaded Generation with Speedup

- $> x 7$ (K=16, iters=2) 47.05ms ($\times 6.78$) 21.34 26.91 32.11 32.53 32.95
- $> x 6/5$ (K=32, iters=2) 54.36ms ($\times 5.87$) 22.55 27.56 32.62 32.44 33.14
- $> x 4$ (K=64, iters=2) 69.19ms ($\times 4.61$) 23.09 27.79 32.78 32.43 33.25
- $> x 3$ (K=32, iters=3) 78.29ms ($\times 4.07$) 23.35 28.64 33.12 33.11 33.74
- $> x 2/1$ (K=64, iters=4) 154.45ms ($\times 2.06$) 24.40 29.43 33.64 33.19 34.08

**Literature**

- FlowSeq-base [28] - 18.55 23.36 29.34 30.44 24.75
- FlowSeq-large [28] - 20.85 25.40 29.73 30.72 -
- Levenshtein [43] 126ms ($\times 2.93$) 25.20 - - 33.02 -

**Literature+Reranking**

- FlowSeq-large (rescoring=30) - 23.64 28.29 32.20 32.84 -

**Variable Length Generation** Cascaded decoding allows for relaxing the length constraint. Figure 3(b) shows the effect of varying $\Delta L$ from $\{0, 3, 5\}$, where $\Delta L = 0$ corresponds to a hard length constraint, and $\Delta L = 3$ sequences of 7 possible length values from $L - 3$ to $L + 3$. By using $\Delta L = 3$, we get more than 1 BLEU improvement at any given speedup. Therefore, we use $\Delta L = 3$ for Table 1.

**Ratio of Repetitions** The independence assumption of non-autoregressive models often leads to visible artifacts in generation such as n-gram repetitions. By introducing higher-order dependencies, we can reduce the ratio of repetitions, as shown in Figure 3(c), where we measure the extent of repetitions using the ratio of unique ngrams. Cascaded decoding with more than 1 iterations significantly reduces the number of repetitions.

**Markov Transformer Analysis** Table 2 shows different search algorithms for the Markov transformer. We can observe that 1) a 4th-order Markov transformer is very expressive by itself: using beam search with $K = 5$, the BLEU score (35.07) is close to the BLEU score of a transformer (35.63); 2) Cascaded decoding is less effective without distillation than serial beam search; 3) With length constraint, cascaded decoding is more effective than beam search; 4) Variable length generation can improve upon enforcing strict length constraints. Finally, we want to note that Markov transformer’s complexity is lower than normal transformer, since it attends to at most $M$ past words.
Figure 3: Analysis on WMT14 En-De val. (a) Box plot of the number of candidate sequences at different dependency orders with $K = 16$. Results include cascaded decoding with 3 iterations (scored with $f(2)$), 4 iterations ($f(3)$) and 5 iterations ($f(4)$), and beam baseline ($f^{L-1}_{AR}$). (b) BLEU/speedup tradeoff as we vary $\Delta L$. The plot is drawn by varying $K$ from $\{16, 32, 64, 128\}$ and varying iterations from $\{2, 3, 4, 5\}$. (c) The ratio of n-gram repetitions evaluated using the ratio of unique ngrams as a proxy ($K = 16, \Delta L = 0$).

Table 2: Markov transformer with different search strategies on IWSLT14 De-En val w/o distillation. Column $\Delta L$ shows the length constraint ($L - \Delta L$ to $L + \Delta L$), where None denotes no constraint.

| Model            | Search | Parallel | Time | $\Delta L$ | Model Score | BLEU  |
|------------------|--------|----------|------|------------|-------------|-------|
| Transformer [51] | Beam (K=5) | N | $O(KL^2)$ | None | -11.82 | 35.63 |
|                  | Beam (K=64) | N | - | 0 | -17.79 | 33.14 |
|                  | Beam (K=1024) | N | - | 0 | -16.77 | 33.33 |
|                  | Cascade (K=64, iters=5) | Y | - | 0 | -17.44 | 33.45 |
|                  | Cascade (K=64, iters=5) | Y | - | 3 | -13.87 | 35.03 |

Multi-GPU Scaling on multiple GPUs is becoming more important, given the recent trend in bigger models [46, 5]. For multi-GPU parallelization\(^4\) each GPU takes a chunk of the sequence and forwards decoder for that chunk, while each GPU maintains full encoder states. The only communications between GPUs are the log potentials of size $L \times K \times K$ at each iteration. By using 4 GPUs, our approach can reach speedup of $2.79 \times$ compared to $1.68 \times$ using only 1 GPU when $K = 64$ and iters = 5 on WMT14 En-De test set with distillation. Note that we use batch size 1, while for most other approaches due to the global communication required between different parts of the target sentence, it is hard to reach this level of parallelism.

6 Conclusion

We demonstrate that probabilistic autoregressive models can achieve sub-linear decoding time while retaining high fidelity translations by replacing beam search with a cascaded inference approach. Our approach, based on [56], iteratively prunes the search space using increasingly higher-order models. To support this inference procedure, we utilize Markov transformers, a variant of transformer that can be used to parameterize cascades of CRFs. Experiments on five commonly used machine translation benchmark datasets validate that our approach is competitive in terms of accuracy/speed tradeoff with other state-of-the-art parallel decoding methods, and practically useful with distillation.

Our work opens up a number of exciting future directions, such as applying this approach to longer-form text generation using latent variables, extending the Markov transformer to mimic any specified graphical model, or using more powerful globally normalized energy models instead of locally normalized ones.

\(^4\)We use https://pytorch.org/docs/stable/multiprocessing.html
**Broader Impact**

Our work proposes an alternative approach to beam search that enables more efficient text generation. This work primarily uses machine translation as an application, but in the long run, it might be applied to longer-form text generation such as summarizing or translating entire documents, or be deployed to edge devices due to its faster inference and lower computational costs.

On the positive side, more efficient text generation can make these technologies more accessible to the general public. For example, machine translation can help overcome language barriers [36]; document summarization makes data more interpretable [33]. However, there are potential risks. Faster text generation has provoked concerns about generating fake news and targeted propaganda [54, 9] and might pose safety concerns if it was used to generate hate speech or to harass people [47]. Another potential problem is that it might generate language that appears fluent but fabricates facts [22].

To mitigate those issues, there have been works trying to detect machine-generated text [10, 60, 2]. While these works address some concerns over the abuse of text generation, we should be cautious that fake news detection is still a mostly unsolved technical problem and requires active future research [43, 4] as well as non-technical mitigation efforts.

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Supplementary Materials for
Cascaded Text Generation with Markov Transformers

Appendix A: Cascaded Decoding Examples

We show a decoding example in Table 3 (\(K = 5\), \(A L = 1\), iers=5). We sort states by max-marginals in descending order and use - to denote invalid states (with \(- \infty\) log max-marginals). In this simple sentence, using 1 iteration \((m = 0)\), non-autoregressive model) repeats the word “woman” \((m = 0)\), first row, \(x_{1:1+m}\). Introducing higher order dependencies fixes this issue.

Table 3: Cascaded Decoding Example. When \(m = 4\), Viterbi in \(X_4\) returns “an amazing woman . eos”. The source is “was ist passiert . eos” and the target is “an amazing woman . eos”.

| \(m\) | \(x_{1:1+m}\) | \(x_{2:2+m}\) | \(x_{3:3+m}\) | \(x_{4:4+m}\) | \(x_{5:5+m}\) | \(x_{6:6+m}\) | \(x_{7:7+m}\) | \(x_8\) |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|-------|
| 0     | an            | amazing       | woman         | woman         | eos           | eos           | eos           | pad   |
|       | amazing       | woman         |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | this          |               |               |               |               |               |               |       |
|       | remarkable    |               |               |               |               |               |               |       |
| 1     | an            | amazing       | woman         | .             | eos           | eos           | pad           | pad   |
|       | an            |               |               |               |               |               |               |       |
|       | 1            |               |               |               |               |               |               |       |
|       | this          |               |               |               |               |               |               |       |
|       | remarkable    |               |               |               |               |               |               |       |
|       | an            | amazing       | woman         |               | .             | eos           | pad           | pad   |
|       | an            |               |               |               |               |               |               |       |
|       | 2            |               |               |               |               |               |               |       |
|       | this          |               |               |               |               |               |               |       |
|       | remarkable    |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | 3            |               |               |               |               |               |               |       |
|       | this          |               |               |               |               |               |               |       |
|       | an            | amazing       | woman         | .             | .             | eos           | pad           | pad   |
|       | an            |               |               |               |               |               |               |       |
|       | 4            |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | 5            |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | 6            |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | 7            |               |               |               |               |               |               |       |
|       | an            |               |               |               |               |               |               |       |
|       | 8            |               |               |               |               |               |               |       |

Table 4: Cascaded Decoding Example. When \(m = 4\), Viterbi in \(X_4\) returns “what has happened ? eos”. The source is “was ist passiert ? eos” and the target is “what happened ? eos”.

| \(m\) | \(x_{1:1+m}\) | \(x_{2:2+m}\) | \(x_{3:3+m}\) | \(x_{4:4+m}\) | \(x_{5:5+m}\) | \(x_{6:6+m}\) | \(x_{7:7+m}\) | \(x_8\) |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|-------|
| 0     | what          | happened      | ?             | happened      | ?             | eos           | eos           | eos   |
|       | has           |               | ?             | eos           | ?             | pad           | pad           |       |
|       | now           | did           | what          | happened      | happened      | ?             | ?             |       |
|       | and           | what          | happen        | happen        | happen        | happened      | .             |       |
|       | well          | ’s            | eos           | happens       | happens       | .             | happened      |       |
| 1     | what          | has           | happened      | ?             | eos           | eos           | pad           | pad   |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |
|       | what          | happened      | ?             | eos           | ?             | pad           | pad           |       |

In Tables 3-8, we show more examples from IWSLT14 De-En val.
Table 5: Cascaded Decoding Example. When \( m = 4 \), Viterbi in \( \mathcal{X}_4 \) returns “you ’re happy . eos”. The source is “du bist glücklich . eos” and the target is “you ’re happy . eos”.

| \( m \times 1:1 \) | \( m \times 2:2 \) | \( m \times 3:3 \) | \( m \times 4:4 \) | \( m \times 5:5 \) | \( m \times 6:6 \) | \( m \times 7:7 \) | \( m \times 8 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| you ’re happy . | . | . | eos | . | eos | eos | pad |
| happy | . | . | lucky | eos | . | pad | pad |
| 0 | your | eos | gla@@ | happy | happy | . | - | - |
| and | ’s | good | lucky | ? | happy | happy | - |
| i | be | fortun@@ | full | you | ? | ? | - |
| you ’re happy . | . | . | . | eos | eos | pad | pad | pad |
| you are | . | . | . | lucky | . | eos | eos | pad |
| 1 | you be | . | . | happy | happy | . | eos | - |
| you ’s | be | happy | happy | full | eos | ? | eos | - |
| and you | ’re | lucky | happy | full | lucky | you | . | happy | eos | - |
| you ’re happy . | . | . | . | eos | eos | pad | pad | pad |
| you are | . | . | . | lucky | . | eos | eos | pad |
| you be | . | . | . | happy | happy | . | eos | eos | pad |
| you ’s | be | happy | happy | full | eos | ? | eos | ? |
| and you | ’re | lucky | happy | full | lucky | you | . | happy | eos | - |

Table 6: Cascaded Decoding Example. When \( m = 4 \), Viterbi in \( \mathcal{X}_2 \) returns “let ’s move . eos”. The source is “bewe@@ g dich . eos” and the target is “move it . eos”.

| \( m \times 1:1 \) | \( m \times 2:2 \) | \( m \times 3:3 \) | \( m \times 4:4 \) | \( m \times 5:5 \) | \( m \times 6:6 \) | \( m \times 7:7 \) | \( m \times 8 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| move | move | . | . | eos | eos | eos | eos | pad |
| let | . | . | eos | . | . | pad | pad |
| 0 | so | moving | move | ? | ? | - | - |
| just | ’s | forward | forward | here | ? | ? | - |
| now | let | moving | it | forward | here | here | - |
| let ’s | ’s move | move | . | . | eos | eos | pad | pad |
| just move | ’s moving | moving | . | . | eos | eos | pad | eos | pad |
| 1 | so move | move forward | move it | forward | here | eos | - |
| move . | . forward | move forward | ? | eos | ? | eos | - |
| move ’s | . moving | move | ? | . | forward | - | - |
| let’s move | ’s move | move eos | move eos | . eos | eos | eos | eos | pad | pad |
| let’s moving | ’s move it | move it | eos | eos | eos | eos | pad | pad |
| 2 | move ’s move forward | move forward | eos | eos | eos | eos | pad | pad |
| move , moving | ’s moving . | move eos | eos | eos | eos | eos | pad | pad |
| move ’s moving | ’s move ? | move eos | eos | eos | eos | eos | pad | pad |
| let’s move . | ’s move . | move eos | eos pad | eos pad | eos | eos | eos | pad | pad |
| let’s move it | ’s move it . | move it eos | eos | eos | eos | eos | pad | pad |
| 3 | let’s moving . | move eos | eos | eos | eos | eos | pad | pad |
| let’s move forward | ’s move forward | move eos | eos | eos | eos | eos | pad | pad |
| let’s move ? | ’s move eos | move eos | eos pad | eos pad | eos pad | eos pad | - |
Table 7: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns “very, very hard. eos”. The source is “sehr sehr schwer. eos” and the target is “very very hard. eos”.

| $m \times 1 \times m$ | $m \times 2 \div m$ | $m \times 3 \div m$ | $m \times 4 \div m$ | $m \times 5 \div m$ | $m \times 6 \div m$ | $m \times 7 \div m$ | $m \times 8$ |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|-------------|
| very               | difficult           | difficult           | .                   | eos                 | eos                 | eos                 | pad         |
| it                 | hard                | hard                | eos                 | .                   | pad                 | pad                 | -           |
| 0 really            | very                | .                   | difficult           | .                   | -                   | -                   | -           |
| extremely           | tough               | very                | hard                | .                   | difficult           | difficult           | -           |
| that               | .                   | tough               | very                | .                   | very                | hard                | hard        |
| very, very          | very                | very                | difficult           | .                   | eos                 | eos pad             | pad pad      |
| very very           | very hard           | very hard           | .                   | eos pad             | eos pad             | eos pad             | pad pad      |
| it very             | very difficult      | .                   | .                   | eos                 | eos pad             | eos pad             | pad pad      |
| extremely           | .                   | very                | very                | very                | difficult           | difficult           | .           |
| extremely           | .                   | very                | very                | very                | .                   | difficult           | .           |
| very, very          | very hard           | .                   | .                   | .                   | eos pad             | eos pad             | pad pad      |
| very very           | very difficult      | .                   | .                   | .                   | eos pad             | eos pad             | pad pad      |
| it very             | very hard           | .                   | eos                 | .                   | .                   | .                   | .           |
| extremely           | .                   | very                | very                | very                | .                   | difficult           | .           |
| extremely           | .                   | very                | very                | very                | .                   | difficult           | .           |
| very, very          | very hard           | .                   | .                   | .                   | eos pad             | pad pad             | pad pad      |
| very very           | very difficult      | .                   | .                   | .                   | eos pad             | pad pad             | pad pad      |
| it very             | very hard           | .                   | .                   | .                   | .                   | .                   | .           |
| extremely           | .                   | .                   | .                   | .                   | .                   | .                   | .           |

Table 8: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns “the opposite thing happened. eos”. The source is “das gegenteil passierte. eos” and the target is “the opposite happened. eos”.

| $m \times 1 \times m$ | $m \times 2 \div m$ | $m \times 3 \div m$ | $m \times 4 \div m$ | $m \times 5 \div m$ | $m \times 6 \div m$ | $m \times 7 \div m$ | $m \times 8$ |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|-------------|
| the                | opposite            | opposite            | happened            | eos                 | eos                 | eos pad             | pad         |
| and                | contr@              | thing               | .                   | pad                 | pad                 | -                   | -           |
| 0 so               | other               | ary                 | thing               | happened            | .                   | -                   | -           |
| but                | the                 | happened            | did                 | happening           | happened            | happened            | pad pad      |
| well               | convert@            | was                 | opposite            | happen              | happen              | happen              | -           |
| the opposite       | opposite thing      | thing happened      | happened            | . eos               | eos pad             | pad pad             | -           |
| the contr@         | contr@               | ary happened        | was happening       | . eos               | . eos               | eos pad             | -           |
| 1 and the          | the                 | opposite happened   | thing happened      | happened            | . eos               | eos pad             | -           |
| the other          | other thing         | thing               | did                 | . eos pad           | . happened eos      | -                   | -           |
| so the             | opposite opposite   | was happened        | .                   | -                   | -                   | -                   | -           |
| the opposite       | opposite thing      | thing happened      | happened            | . eos pad           | eos pad             | pad pad             | -           |
| the contr@         | contr@               | ary happened        | was happening       | . eos               | . eos               | eos pad             | -           |
| 2 and the          | the opposite        | opposite happened   | thing happened      | happened            | . eos               | eos pad             | pad pad      |
| the other          | other thing         | thing               | was happened        | happened            | . eos pad           | pad pad             | -           |
| so the             | opposite opposite   | was happened        | .                   | -                   | -                   | -                   | -           |
| the opposite thing | opposite thing      | thing happened      | happened            | . eos pad           | eos pad             | pad pad             | -           |
| the contr@         | contr@               | ary happened        | . eos               | . eos               | . eos               | eos pad             | -           |
| 3 and the          | the opposite        | opposite happened   | thing happened      | happened            | . eos               | eos pad             | pad pad      |
| the other          | other thing         | thing               | was happening       | happened            | . eos pad           | pad pad             | -           |
| so the             | opposite opposite   | was happened        | .                   | -                   | -                   | -                   | -           |
Appendix B: More Visualizations

(a) $m = 0$
(b) $m = 1$
(c) $m = 2$

Figure 4: Illustration of cascaded decoding ($K = 10$, iters=4) for $X_1$, $X_2$, $X_3$.

We include more visualizations of $X_1$, $X_2$ and $X_3$ in Figure 4 and Figure 5. These examples are taken from IWSLT14 De-En val.

Appendix C: Variable Length Generation Potentials

To handle length, we introduce an additional padding symbol pad to $\mathcal{V}$, and change the log potentials to enforce the considered candidates are of length $L - \Delta L$ to $L + \Delta L$. Note that we can only enforce that for $m \geq 1$, and for $m = 0$ we manually add pad to the pruned vocabulary.

We start cascaded search using a sequence of length $L + \Delta L + 1$. The main ideas are: 1) We make $\text{eos}$ and pad to always transition to pad such that sequences of different lengths can be compared; 2) We disallow $\text{eos}$ to appear too early or too late to satisfy the length constraint; 3) We force the last token to be pad such that we don’t end up with sentences without $\text{eos}$ endings. Putting these ideas together, the modified log potentials we use are:
\[
J_l^{(m)}(x_{l:l+m}) = \begin{cases}
0, & \text{if } x_{l+m-1} = \text{eos} \land x_{l+m} = \text{pad} \\
-\infty, & \text{if } x_{l+m-1} = \text{eos} \land x_{l+m} \neq \text{pad} (\text{eos} \rightarrow \text{pad}) \\
0, & \text{if } x_{l+m-1} = \text{pad} \land x_{l+m} = \text{pad} \\
-\infty, & \text{if } x_{l+m-1} = \text{pad} \land x_{l+m} \neq \text{pad} (\text{pad} \rightarrow \text{pad}) \\
-\infty, & \text{if } x_{l+m-1} \neq \text{pad} \land x_{l+m} \neq \text{eos} \land x_{l+m} = \text{pad} (\text{nothing else} \rightarrow \text{pad}) \\
-\infty, & \text{if } l + m = L + \Delta L + 1 \land x_{l+m} = \text{pad} (\text{the last token must be pad}) \\
0, & \text{if } l + m < L - \Delta L \land x_{l+m} = \text{eos} (\text{eos cannot appear too early}) \\
-\infty, & \text{if } l + m = L + \Delta L + 1 \land x_{l+m} \neq \text{pad} (\text{the last token must be pad}) \\
J_l^{(m)}(x_{l:l+m}), & \text{o.t.}
\end{cases}
\]

Note that we only considered a single sentence above, but batching is straightforward to implement and we refer interested readers to our code\footnote{https://github.com/harvardnlp/cascaded-generation} for batch implementations.

Appendix D: Full Results

In the main experiment table we showed latency/speedup results for WMT14 En-De. In Table 9, Table 10, Table 11 and Table 12 we show the latency/speedup results for other datasets. Same as in the main experiment table, we use the validation set to choose the configuration with the best BLEU score under speedup \( > \times 1, > \times 2, \text{etc.} \)

| Table 9: Results on WMT14 De-En. |
|-----------------------------------|
| **Model** | **Settings** | **Latency (Speedup)** | **BLEU** |
| Transformer (beam 5) | | 294.64ms (\(\times 1.00\)) | 31.49 |

**With Distillation**

**Cascaded Generation with Speedup**

- \( > \times 7 \) (K=16, iters=2) 43.41ms (\(\times 6.79\)) 30.69
- \( > \times 6 \) (K=32, iters=2) 52.06ms (\(\times 5.66\)) 30.72
- \( > \times 5 \) (K=16, iters=3) 62.06ms (\(\times 4.75\)) 30.96
- \( > \times 4/3 \) (K=32, iters=3) 79.01ms (\(\times 3.73\)) 31.08
- \( > \times 2/1 \) (K=32, iters=5) 129.67ms (\(\times 2.27\)) 31.15

**Without Distillation**

**Cascaded Generation with Speedup**

- \( > \times 6/5 \) (K=32, iters=2) 53.83ms (\(\times 5.47\)) 27.56
- \( > \times 4 \) (K=32, iters=3) 81.10ms (\(\times 3.63\)) 28.64
- \( > \times 3 \) (K=32, iters=4) 106.97ms (\(\times 2.75\)) 28.73
- \( > \times 2 \) (K=64, iters=4) 154.15ms (\(\times 1.91\)) 29.43
- \( > \times 1 \) (K=128, iters=4) 269.59ms (\(\times 1.09\)) 29.66
Table 10: Results on WMT16 En-Ro.

| Model Settings | Latency (Speedup) | BLEU |
|----------------|-------------------|------|
| Transformer    | (beam 5)          | 343.28ms ($\times 1.00$) | 33.89 |

With Distillation

Cascaded Generation with Speedup

| Settings | Latency (Speedup) | BLEU |
|----------|-------------------|------|
| > $\times 7$ | (K=16, iters=2) | 49.38ms ($\times 6.95$) | 32.70 |
| > $\times 6$ | (K=32, iters=2) | 54.56ms ($\times 6.29$) | 32.73 |
| > $\times 5$ | (K=16, iters=3) | 66.33ms ($\times 5.18$) | 32.89 |
| > $\times 4$ | (K=32, iters=3) | 77.39ms ($\times 4.44$) | 33.16 |
| > $\times 3$ | (K=64, iters=3) | 108.57ms ($\times 3.16$) | 33.23 |
| > $\times 2$ | (K=64, iters=4) | 142.23ms ($\times 2.41$) | 33.30 |
| > $\times 1$ | (K=64, iters=5) | 179.07ms ($\times 1.92$) | 33.23 |

Without Distillation

Cascaded Generation with Speedup

| Settings | Latency (Speedup) | BLEU |
|----------|-------------------|------|
| > $\times 7$ | (K=16, iters=2) | 45.18ms ($\times 7.60$) | 32.11 |
| > $\times 6$ | (K=32, iters=2) | 51.38ms ($\times 6.68$) | 32.62 |
| > $\times 5$ | (K=16, iters=3) | 60.34ms ($\times 5.69$) | 32.67 |
| > $\times 4$ | (K=32, iters=3) | 73.99ms ($\times 4.64$) | 33.12 |
| > $\times 3$ | (K=64, iters=3) | 105.46ms ($\times 3.26$) | 33.48 |
| > $\times 2$ | (K=64, iters=4) | 145.18ms ($\times 2.36$) | 33.64 |
| > $\times 1$ | (K=64, iters=5) | 325.42ms ($\times 1.05$) | 33.52 |

Table 11: Results on WMT16 Ro-En.

| Model Settings | Latency (Speedup) | BLEU |
|----------------|-------------------|------|
| Transformer    | (beam 5)          | 318.57ms ($\times 1.00$) | 33.82 |

With Distillation

Cascaded Generation with Speedup

| Settings | Latency (Speedup) | BLEU |
|----------|-------------------|------|
| > $\times 6/5$ | (K=16, iters=2) | 46.84ms ($\times 6.80$) | 32.66 |
| > $\times 4$ | (K=16, iters=3) | 62.57ms ($\times 5.09$) | 33.00 |
| > $\times 3$ | (K=16, iters=5) | 99.25ms ($\times 3.21$) | 33.04 |
| > $\times 2$ | (K=64, iters=3) | 103.85ms ($\times 3.07$) | 33.17 |
| > $\times 1$ | (K=64, iters=5) | 181.18ms ($\times 1.76$) | 33.28 |

Without Distillation

Cascaded Generation with Speedup

| Settings | Latency (Speedup) | BLEU |
|----------|-------------------|------|
| > $\times 6$ | (K=16, iters=2) | 47.58ms ($\times 6.70$) | 32.53 |
| > $\times 5$ | (K=32, iters=2) | 54.05ms ($\times 5.89$) | 32.44 |
| > $\times 4$ | (K=16, iters=3) | 60.94ms ($\times 5.23$) | 33.00 |
| > $\times 3$ | (K=32, iters=4) | 100.29ms ($\times 3.18$) | 33.10 |
| > $\times 2$ | (K=64, iters=3) | 105.21ms ($\times 3.03$) | 33.22 |
| > $\times 1$ | (K=128, iters=4) | 282.76ms ($\times 1.13$) | 33.29 |
Table 12: Results on IWSLT14 De-En.

| Model                | Settings       | Latency (Speedup) | BLEU  |
|----------------------|----------------|-------------------|-------|
| Transformer          | (beam 5)       | 229.76ms (×1.00)  | 34.44 |
| **With Distillation**|                |                   |       |
| Cascaded Generation  |                |                   |       |
| With Speedup         |                |                   |       |
| > ×6/5 (K=16, iters=2) &> ×4 (K=32, iters=3) &> ×3 (K=32, iters=4) &> ×2/1 (K=64, iters=5) | 39.38ms (×5.83)  & 60.27ms (×3.81)  & 78.27ms (×2.94)  & 117.90ms (×1.95) | 33.90 & 34.33 & 34.43 & 34.49 |
| Without Distillation |                |                   |       |
| Cascaded Generation  |                |                   |       |
| With Speedup         |                |                   |       |
| > ×5 (K=64, iters=2) &> ×4 (K=32, iters=3) &> ×3 (K=64, iters=3) &> ×2 (K=64, iters=5) &> ×1 (K=128, iters=5) | 48.59ms (×4.73)  & 60.09ms (×3.82)  & 75.64ms (×3.04)  & 121.95ms (×1.88) & 189.10ms (×1.22) | 33.25 & 33.74 & 33.96 & 34.08 & 34.15 |

Appendix E: Optimization Settings

Table 13: Optimization settings. We use the same settings for knowledge distillation experiments.

| Dataset          | dropout | fp16 | GPUs | batch | accum | warmup steps | max steps | max lr | weight decay |
|------------------|---------|------|------|-------|-------|--------------|-----------|-------|--------------|
| WMT14 En-De/De-En | 0.1     | Y    | 3    | 4096  | 3     | 4k           | 240k      | 7e-4  | 0            |
| WMT16 En-Ro/Ro-En | 0.3     | Y    | 3    | 5461  | 1     | 10k          | 240k      | 7e-4  | 1e-2         |
| IWSLT14 De-En    | 0.3     | N    | 1    | 4096  | 1     | 4k           | 120k      | 5e-4  | 1e-4         |

We used Adam optimizer [20], with betas 0.9 and 0.98. We use inverse square root learning rate decay after warmup steps [34]. We train with label smoothing strength 0.1 [32]. For model selection, we used BLEU score on validation set. For Markov transformers, we use cascaded decoding with $K = 16$ and $\Delta L = 3$ to compute validation BLEU score. Other hyperparameters can be found at Table 13.