Robustification of the state-space MRAC law for under-actuated systems via fuzzy-immunological computations

Omer Saleem¹, Mohsin Rizwan² and Fahim Gohar Awan³
¹Department of Electrical Engineering, National University of Computer and Emerging Sciences, Lahore, Pakistan
²Department of Mechatronics and Control Engineering, University of Engineering and Technology, Lahore, Pakistan
³Department of Electrical Engineering, University of Engineering and Technology, Lahore, Pakistan

Abstract
This paper formulates an enhanced Model-Reference-Adaptive-Controller (MRAC) that is augmented with a fuzzy-immune adaptive regulator to strengthen the disturbance-attenuation capability of closed-loop under-actuated systems. The proposed scheme employs the conventional state-space MRAC and augments it with a pre-configured fuzzy-immune mechanism that acts as a superior regulator to dynamically modulate the adaptation gains of the Lyapunov gain-adjustment law. The immunological computations increase the controller’s adaptability to flexibly manipulate the damping control effort under exogenous disturbances. The efficacy of the proposed Immune-MRAC law is comparatively analyzed under practical disturbance conditions by conducting real-time hardware experiments on the QNET rotary pendulum. The experimental outcomes validate the faster transient-recovery behavior and stronger damping effort of the proposed control law against the exogenous disturbances while preserving the system’s asymptotic stability and control energy efficiency.

Keywords
model-reference-adaptive-control, lyapunov gain-adjustment law, self-tuning, fuzzy-immune adaptation, disturbance-rejection

Corresponding author:
Omer Saleem, Department of Electrical Engineering, National University of Computer and Emerging Sciences, Lahore, Pakistan.
Email: omer.saleem@nu.edu.pk
Introduction

The Rotary-Inverted-Pendulum (RIP) is used as a standard benchmark platform to verify the robustness of any designed control strategy for nonlinear dynamical systems, due to its inherent under-actuated configuration, nonlinear characteristics, and open-loop kinematic instability. The RIP stabilization control problem is used in developing a stable gait control for legged robots, orbital stabilization of the satellites, attitude stabilization of submarines, and posture stabilization of rockets and rotorcrafts during take-off. This control problem becomes more challenging when the system encounters exogenous disturbances that pose a serious threat to the mission’s safety.

Related work

Several RIP stabilization control strategies have been proposed in the available literature. The structural simplicity of PID controllers incapacitates them to address the nonlinear disturbances. The PID gains inevitably amplify the measurement noise content which eventually corrupts the control effort. The intelligent controllers require heuristically defined logical rules which rely upon the expert’s knowledge or large sets of training data to formulate an agile control law. These control realizations inevitably involve an excessive computational burden. The sliding-mode controllers achieve the desired robustness at the cost of highly discontinuous control activity which imposes peak servo requirements upon the actuator, and thus, induces chattering in the state response. The dependance of optimal linear-quadratic-regulator on the system’s linear state-space model renders it inefficient against parametric and model variations.

The adaptive controllers use pre-configured meta-rules to dynamically adjust the critical controller parameters and redesign control law online to reject the exogenous disturbances. The nonlinear-type gain-scheduling mechanisms use well-postulated meta-rules and nonlinear scaling functions to dynamically adjust the controller parameters. However, tuning the associated hyper-parameters and deriving a sufficient stability proof is a cumbersome process. The model-predictive-controller yields time-varying controller gains by solving a finite-horizon problem. However, imprecise tuning of the hyper-parameters leads to wrong predictions which leads to frail control effort, especially under long-drifting disturbances. The derivation of accurate state-dependent-coefficient matrices used to formulate the State-Dependent-Riccati-Equation-based controller is quite difficult due to the complex geometry of the higher-order dynamical systems.

The model-reference-adaptive controller uses a Lyapunov function to minimize the tracking error between the outputs of the reference and the controlled system. This arrangement renders optimality in the control behavior while preserving its asymptotic stability. However, the fixed inner adaption-gains of the MRAC prevents it from addressing the rapid error variations.

Proposed methodology

The main contribution of this article is to methodically formulate a novel self-regulating MRAC procedure that enhances the robustness of under-actuated mechatronic systems
against bounded exogenous disturbances. The proposed scheme employs the state-space MRAC as the baseline controller that tracks an LQ-regulated reference model. This MRAC structure is robustified by retrofitting it with a pre-configured fuzzy-immune adaptation mechanism that dynamically modulates the inner adaptation gains of the Lyapunov gain-adjustment law. The adaptation law utilizes the system’s control input feedback along with well-postulated immunological rules to self-tune the state-compensator gains and achieves the desired objectives. The efficacy of the proposed Immune-MRAC (I-MRAC) law is evaluated by conducting real-time hardware experiments on the QNET rotary pendulum. The experimental outcomes validate its faster transient-recovery behavior and stronger disturbance-rejection behavior while preserving the system’s stability and control energy efficiency.

The proposed I-MRAC scheme affords several benefits that are typically unachievable via the conventional control techniques. The MRAC tracks an optimal reference model to adjust the gains while preserving the Lyapunov stability. This feature prevents the controller from imposing large control requirements and unnecessary chattering in the response. Unlike LQR, the MRAC quickly compensates for the model and exogenous disturbances. The adaptive-gain adjustment also improves the system’s immunity to measurement noise. Any set of fixed-values of MRAC’s inner adaptation-gains can only meet the control requirements for a limited range of operating conditions. This feature renders the MRAC insufficient against abrupt error variations. Hence, in this article, the fuzzy-immune mechanism acts as a superior regulator to modulate the adaptation-gains of the MRAC law and nullify this inherent problem. The fuzzy-immune mechanism mimics the adaptability of the biological immune systems to efficiently reject the exogenous disturbances. The said mechanism is formulated via well-established fuzzy-rules to strengthen the controller’s robustness. Finally, the proposed I-MRAC does not require online training process, modeling of intrinsic nonlinearities, or predictions which renders this technique computationally inexpensive and easily realizable with modern computers.

The utilization of fuzzy-immunological computations to adaptively tune the MRAC’s inner adaptation gains, to strengthen its disturbance-rejection capability, has not been attempted previously in the available literature. Hence, this idea is the novel contribution of this paper.

The rest of the paper is organized as follows: The mathematical description of the RIP system is presented in Section 2. The fixed-gain LQR and the conventional MRAC laws are constructed in Section 3. The formulation of the proposed I-MRAC law is presented in Section 4. The performance of I-MRAC is experimentally assessed in Section 5. The article is concluded in Section 6.

### System modeling

The QNET Rotary Pendulum Setup is selected to experimentally investigate the proposed control scheme. The schematic of a conventional rotary pendulum is shown in Figure 1. The system comprises a rod whose free end is coupled to a 0.1 kg mass while the pivoted end is coupled to a rotary encoder that measures the pendulum angle, $\theta$. The rod swings freely and stabilizes itself vertically about the pivoted hinge by using the energy provided
to it by the arm’s rotation. The arm is rotated via a DC-gearervo-motor. The arm’s rotation, $\alpha$, is measured via the encoder commissioned with the motor shaft. The reference value of the rod is $\pi$ radians, whereas, the reference value of the arm is its initial value, $\alpha(0)$. The RIP’s mathematical model is derived as follows.

**Mathematical model**

The dynamical model of the RIP is formulated using the Euler-Lagrange method. The derivation is initiated by computing the difference between the system’s total kinetic energy and total potential energy in terms of its state-variables ($\alpha$, $\theta$, $\dot{\alpha}$, and $\dot{\theta}$), which delivers the system’s Lagrangian. The Lagrangian is processed to deliver the following linearized equations of motion of the RIP.\(^{18}\)

$$
\ddot{\alpha} = \frac{1}{W}\left(rM_p l_p^2 g \theta - \frac{(J_p + M_p l_p^2)K_t K_m}{R_m} \dot{\alpha} + \frac{(J_p + M_p l_p^2)K_t}{R_m} V_m\right),
$$

$$
\ddot{\theta} = \frac{1}{W}\left(M_p l_p g (J_e + M_p r^2) \theta - \frac{rM_p l_p K_t K_m}{R_m} \dot{\alpha} + \frac{rM_p l_p K_t}{R_m} V_m\right)
$$

such that, $W = J_e J_p + M_p r^2 J_p s + M_p l_p^2 J_e$

These linear equations are used to derive the system’s nominal state-space model. The general state-space representation of a linear system is expressed in Eq. 2.

$$
\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t)
$$

where, $x(t)$ is the state vector, $y(t)$ is the output vector, $u(t)$ is the control-input signal, $A$ is the state matrix, $B$ is the input matrix, $C$ is the output matrix, and $D$ is the feed-forward
matrix. The state-variables and control-input variable of the RIP system are formally expressed in Eq. 3.

\[ x(t) = \begin{bmatrix} \alpha(t) & \theta(t) & \dot{\alpha}(t) & \dot{\theta}(t) \end{bmatrix}^T, \quad u(t) = V_m(t) \quad (3) \]

The nominal state-space model of the QNET RIP system is expressed below.\(^{19}\)

\[
A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_1 & a_2 & 0 \\ 0 & a_3 & a_4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ b_1 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = 0 \quad (4)
\]

where,

\[
a_1 = \frac{rM_p^2l_p^2g}{J_pJ_e + J_p^2M_p + J_pM_p^2r^2}, \quad a_2 = \frac{-K_tK_m(J_p + M_p^2l_p^2)}{(J_pJ_e + J_p^2M_p + J_pM_p^2r^2)R_m},
\]

\[
a_3 = \frac{M_p^2l_p^2g(J_e + M_p^2r^2)}{J_pJ_e + J_p^2M_p + J_pM_p^2r^2}, \quad a_4 = \frac{-rM_p^2l_pK_t}{(J_pJ_e + J_p^2M_p + J_pM_p^2r^2)R_m},
\]

\[
b_1 = \frac{K_t(J_p + M_p^2l_p^2)}{(J_pJ_e + J_p^2M_p + J_pM_p^2r^2)R_m}, \quad b_2 = \frac{rM_p^2l_pK_t}{(J_pJ_e + J_p^2M_p + J_pM_p^2r^2)R_m}
\]

The descriptions, as well as the numerical values associated with the QNET RIP’s modeling parameters, are presented in Table 1.\(^{20}\)

**Baseline model-reference-adaptive-control**

The MRAC’s design methodology is straight-forward. Initially, an optimal LQR controller is constructed. The stabilized closed loop system, given by LQR method, is declared

| Symbol | Description | Value |
|--------|-------------|-------|
| \(M_p\) | Mass of pendulum | 0.027 kg |
| \(l_p\) | Pendulum center of mass to pivot length | 0.153 m |
| \(L_p\) | Length of pendulum rod | 0.191 m |
| \(r\) | Length of horizontal arm | 0.083 m |
| \(J_m\) | Motor shaft moment of inertia | \(3 \times 10^{-5}\) kgm\(^2\) |
| \(M_{arm}\) | Mass of arm | 0.028 kg |
| \(g\) | Gravitational acceleration | 9.810 m/s\(^2\) |
| \(J_e\) | Moment of inertia about motor shaft pivot | \(1.23 \times 10^{-4}\) kg.m\(^2\) |
| \(J_b\) | Moment of inertia about pendulum pivot | \(1.1 \times 10^{-4}\) kgm\(^2\) |
| \(R_m\) | Motor armature resistance | 3.30 \(\Omega\) |
| \(L_m\) | Motor armature inductance | 47.0 mH |
| \(K_t\) | Motor torque constant | 0.028 N.m |
| \(K_m\) | Motor back-electromotive force constant | 0.028 V/(rad/s) |
as the reference model ($A_{ref}$). Next, a Lyapunov function is introduced to minimize the tracking-error between LQR reference and the actual adaptive controller. This procedure delivers the MRAC gain-adjustment law.

**LQ-regulated reference model**

The conventional LQR yields an optimal control trajectory by minimizing a quadratic-performance-index (QPI), expressed in Eq. 5, which captures the state variations and the control input associated with the linear dynamical system.21

$$J_{LQ} = \frac{1}{2} \int_0^\infty [x(t)^T Q x(t) + u(t)^T R u(t)] dt$$

(5)

where, $Q \in \mathbb{R}^{4 \times 4}$ and $R \in \mathbb{R}$ are the state and control-input weighting matrices, respectively. The QPI minimization is followed by the solution of Hamilton-Jacobi-Bellman (HJB) equation to acquire the state-feedback gains offline. The weighting matrices are selected such that $Q$ is a positive semi-definite matrix and $R$ is a positive definite matrix. For the RIP system considered in this research, the $Q$ and $R$ matrices are symbolically represented as shown in Eq. 6.

$$Q = \text{diag}(q_0, q_\phi, q_\alpha, q_\theta), \quad R = \rho$$

(6)

where, $q_i$ and $\rho$ are real-numbered coefficients of the $Q$ and $R$ matrices, respectively. The value of $\rho$ is selected as unity to maintain a reasonable control-input economy. The state-feedback gains delivered by the LQR control problem, for a specific set of $Q$ and $R$ matrices, does not always yield a good position-regulation behavior as per $J_{LQ}$. In this research, the value of $R$ is set to unity to attain an economical control activity; whereas, the $Q$ matrix is tuned by minimizing the performance criterion given in Eq. 7 to minimize the position-regulation error as well as the control-input energy.22

$$J_c = \int_0^\infty \left[ |e_\alpha(t)|^2 + |e_\theta(t)|^2 + |u(t)|^2 \right] dt$$

(7)

such that, $e_\alpha(t) = \alpha(0) - \alpha(t), \quad e_\theta(t) = \pi - \theta(t)$

where, $e_\alpha(t)$ and $e_\theta(t)$ represent the error in the angular displacement of arm and rod from their corresponding reference positions, respectively. The function $J_c$ applies equal weight to the control-minimization and error-minimization criteria. The range-space of the state-weighting-factors is restricted within [0, 100]. The tuning process is initiated with $Q = \text{diag}(1, 1, 1, 1)$. The iterative algorithm conducts an exhaustive search in the direction of descending gradient of $J_c$. In every iteration, the RIP is allowed to balance for 5.0 s, and the corresponding cost is evaluated and recorded. The iterative search is terminated when the minimum cost is achieved. The tuned coefficients of $Q$ and $R$ matrices are presented below.

$$Q = \text{diag}(32.8, 52.2, 6.1, 2.5), \quad R = 1$$

(8)
The Algebraic-Riccati-Equation (ARE) utilizes the system’s state-space model as well as the tuned $Q$ and $R$ matrices to compute the solution, $P$, as shown in Eq. 9.

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

where, $P \in \mathbb{R}^{4 \times 4}$, is a symmetric positive definite matrix. It is well-known that if the system is controllable and that $Q = Q^T \geq 0$ and $R = R^T > 0$, the solution of ARE yields an asymptotically-stable control behavior. The state-feedback gain vector, $K$, is calculated as shown in Eq. 10.

$$K = R^{-1}B^TP$$

where, $K = [k_a \ k_\theta \ k_a \ k_\theta]$. The optimal control law is expressed as follows.

$$u(t) = -Kx(t)$$

The evaluation of the gain vector yields $K = [-6.21 \ 130.56 \ -4.22 \ 17.83]$.

**State-space MRAC law design**

The conventional MRAC law updates the controller-gains online via a Lyapunov function that minimizing the tracking-error between the outputs of the LQ-regulated reference model and the actual system. Consider the linear system expressed in Eq. 2. It is desired to construct an adaptive control law that tracks and imitate the response of the LQ-regulated reference model ($A_{ref}$) as expressed via the following autonomous system.

$$\dot{x}_{ref}(t) = A_{ref}x_{ref}(t)$$

The MRAC requires a reference model with the desired stable response. Hence, the LQR-based closed-loop RIP system is used as reference model. The $A_{ref}$, given by LQR, creates a baseline for the adaptive controller to track and achieve optimality. The proposed MRAC law is given in Eq. 13.

$$u(t) = -K_a(t)x(t)$$

where, $K_a(t)$ is the self-tuning gain vector that is dynamically updated via the MRAC scheme. The closed-loop representation of the actual system is given below.

$$\dot{x}(t) = (A - BK_a)x(t) = A_c(K_a)x(t)$$

where, the matrix $A_c$ depends on the vector $K_a$.

**Compatibility condition:** Generally, it is impossible to acquire a vector $K_a$ such that the model described in Eq. 14 is equivalent to the reference model. To establish a sufficient condition for tracking the reference model, there exists a vector $\hat{K}_a$ which is described as follows.

$$A_c(\hat{K}_a) = A_{ref} = A - B\hat{K}_a$$

where, $A_{ref}$ is the reference model matrix.
This condition depicts that the columns of matrix $A - A_{\text{ref}}$ are linear combinations of the columns of the matrix $B$. In this research, the $A_{\text{ref}}$ is identified by taking $\hat{K}_a = K$. Hence, the matrix $A_{\text{ref}}$ is numerically expressed as follows.

$$A_{\text{ref}} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
55.63 & -1147.18 & 37.51 & -159.72 \\
14.27 & -263.82 & 9.62 & -40.97 \\
\end{bmatrix}$$ (16)

The tracking-error-vector, $\varepsilon(t)$, computes the difference between the state-vectors of the actual system and the reference system. It is expressed as follows.

$$\varepsilon(t) = x(t) - x_{\text{ref}}(t)$$ (17)

For the RIP’s regulatory control problem, the $\dot{x}_{\text{ref}} = [0 \ 0 \ 0 \ 0]^T$. The model-tracking error decides the convergence rate of the adaptation mechanism. The time-derivative of the tracking-error vector is given as under.

$$\dot{\varepsilon}(t) = \dot{x}(t) - \dot{x}_{\text{ref}}(t) = Ax(t) + Bu(t) - A_{\text{ref}}x_{\text{ref}}(t)$$ (18)

By simultaneously adding and subtracting the term $A_{\text{ref}}x(t)$, on the right-hand side of equation Eq. 18, the expression of tracking-error-derivative is written as follows.

$$\dot{\varepsilon}(t) = Ax(t) + Bu(t) - A_{\text{ref}}x_{\text{ref}}(t) + A_{\text{ref}}x(t) - A_{\text{ref}}x(t)$$ (19)

The variables on the left-hand side can be manipulated to yield the following expression.

$$\dot{\varepsilon}(t) = A_{\text{ref}}\varepsilon(t) - Bx(t)^T(K_a^T(t) - \hat{K}_a^T)$$ (20)

The expression in Eq. 24 is simplified as shown below.

$$\dot{\varepsilon}(t) = A_{\text{ref}}\varepsilon(t) + \delta(K_a^T(t) - \hat{K}_a^T)$$ (21)

such that, $\delta = -Bx(t)^T$

It is assumed that the conditions needed for precise model tracking have been completely fulfilled while simplifying the expression of the tracking-error-derivative. The following Lyapunov function is used to develop a stable online gain-adaptation law that modifies the values of $K_a(t)$ after every sampling interval.

$$V(\varepsilon, K_a) = \frac{1}{2} [\varphi \varepsilon(t)^T \bar{P} \varepsilon(t) + (K_a^T(t) - \hat{K}_a^T)(K_a^T(t) - \hat{K}_a^T)]$$ (22)

where, $\bar{P}$ is a positive-definite matrix and $\varphi$ is the preset positive semi-definite “adaptation-gain matrix” of the following form.

$$\varphi = \text{diag}(\varphi_\varphi \varphi_\varphi \varphi_\alpha \varphi_\theta)$$ (23)

The coefficients of the said matrix are empirically tuned by minimizing the cost function $J_c$ (and using the tuning procedure discussed in Section 3.1) to acquire accurate reference-
tracking and disturbance-compensation. The selection is done within the range $[0, 5]$. The chosen fixed adaptation-gain matrix is, $\varphi = \text{diag}(1.68 \ 0.81 \ 2.32 \ 1.55)$. The matrix $\bar{P}$ is calculated as follows.

$$A_{\text{ref}}^T \bar{P} + \bar{P} A_{\text{ref}} = -Q$$

(24)

Following the mathematical property in Eq. 28, there would always exist a pair of positive definite matrices, $\bar{P}$ and $Q$, if $A_{\text{ref}}$ is stable. The derivative of $V(.)$ is expressed as follows.

$$\dot{V}(\varepsilon, K_a) = -\frac{1}{2} \varphi \varepsilon(t)^T Q \varepsilon(t) + (\dot{K}_a^T(t) - \dot{K}_a^T(t) + \varphi \delta^T \bar{P} \varepsilon(t))$$

(25)

For $\dot{V}(\varepsilon, K_a)$ to be negative-definite, the following expression is used as the online gain adjustment law.

$$\dot{K}_a^T(t) = -\varphi \delta^T \bar{P} \varepsilon(t)$$

(26)

This expression proves that $\varepsilon(t)$ will eventually converge to zero. The simplified expression of the gain adjustment law is given in Eq. 27.

$$\dot{K}_a(t) = (\varphi x(t) B^T \bar{P} \varepsilon(t))^T$$

(27)

The gain adaptation law is implemented in the control software by programming the following solution of the first-order differential equation, expressed below.

$$K_a(t) = K_a(0) + \int_0^t \varphi(x(t)B^T \bar{P} \varepsilon(t))^T dt$$

(28)

The fixed-gain vector $K$, identified in sub-section 3.1, is utilized as $\dot{K}_a(0)$ in Eq. 32. The self-adjusting gain vector $K_a(t)$ is used in the MRAC law, expressed in Eq. 13.

**Online adaptation-gain modulation**

The matrix $\varphi$ directly influences the convergence rate and sensitivity of the Lyapunov gain-adjustment law. A fixed-valued $\varphi$ lacks the flexibility to efficiently manipulate the stiffness of the applied control input as the error conditions vary. Under perturbed conditions, a large value of adaptation-gain is selected to apply an aggressive control effort to damp the overshoots and minimize the transient recovery time. During equilibrium (or quasi-equilibrium) conditions, a small value of adaptation-gain is used to apply soft control effort that allows for gentle settlement of the response and attenuates the state-state fluctuations. Hence, in this article, the coefficients of $\varphi$ are adaptively modulated via two different fuzzy-rule-based self-tuning mechanisms. The time-varying $\varphi$ is represented as follows:

$$\varphi(t) = \text{diag}(\varphi_\varphi(t) \ \varphi_\theta(t) \ \varphi_x(t) \ \varphi_\theta(t))$$

(29)
The revised gain-adaptation law is expressed below.

$$\hat{K}_a(t) = \hat{K}_a(0) + \int_0^t (\varphi(t)\chi(t)B^T \hat{P}e(t))^T dt$$  \hspace{1cm} (30)

The fixed-gain vector $K$, identified in sub-section 3.1, is utilized as $\hat{K}_a(0)$. The self-regulating MRAC law is expressed below.

$$u(t) = -\hat{K}_a(t)x(t)$$  \hspace{1cm} (31)

**Conventional fuzzy inference adaptation**

The conventional fuzzy logic scheme (FLS) is employed as the baseline mechanism to self-regulate the MRAC law.\textsuperscript{27} It uses a pre-defined set of qualitative rules to self-tune the critical parameters.\textsuperscript{28} In this research, the FLS is formulated as an even-symmetric nonlinear function that is bounded between 0 and 1. It uses compounded state-error variable, $s(t)$, and its first-derivative, $\dot{s}(t)$, as its inputs to address the nonlinear disturbances. The variable $s(t)$ is computed as follows.

$$s(t) = F \varepsilon(t)$$  \hspace{1cm} (32)

such that, $\varepsilon(t) = x(t) - x_{ref}$ where, $F$ is the state-error-weighting vector of the order $1 \times 4$, $x_{ref}$ is the reference state-vector. It is expressed as $x_{ref} = [\alpha_{ref} \pi 0 0]^T$ in this article. The vector $F$ is empirically tuned offline by minimizing the cost-function $J_c$ (and using the tuning procedure discussed in Section 3.1). It is given as $F = [-3.16 \ 73.48 \ -3.02 \ 9.25]$. The compound error, $s(t)$, unifies all the state-error variables to inform the control system regarding the overall impact of exogenous disturbance(s). The output of the FLS is the nonlinear function $f(s, \dot{s})$. The universe of both inputs is divided into seven linguistic variables that are defined as NB – Negative Big, NM – Negative Medium, NS – Negative Small, Z – Zero, PS – Positive Small, PM – Positive Medium, and PB – Positive Big. The universe of output is defined via four linguistic variables; Z – Zero, PS – Positive Small, PM – Positive Medium, and PB – Positive Big. While defining the membership functions, the variations in both input variables are normalized within $[-1, 1]$. The variation in the output $f(s, \dot{s})$ is also restricted within $[0, 1]$. The rule-base is constructed by using the following rationale.

1. The value of $f(s, \dot{s})$ is enlarged when both $s(t)$ and $\dot{s}(t)$ have same polarities. This condition indicates that the response is diverging from the reference, and thus, requires a stiff control effort to damp the disturbances.
2. The value of $f(s, \dot{s})$ is depressed when $s(t)$ and $\dot{s}(t)$ have opposite polarities. This condition indicates that the response is either converging to reference or it is in the equilibrium state, and thus, requires a mild control effort to settle the response smoothly with minimal state-fluctuations.
These characteristics enhance the adaptability of the MRAC law. The consequent fuzzy-rule-base is shown in Table 2. The following min-max inference technique is adopted to perform the implication.

\[ \mu_{ij} = \min (h_{i1}(s), h_{j2}(\dot{s})) \]  

where, \( \mu \) is the degree of the MF, \( n \) is the number of rule, and \( h_{ij}(.) \) is the triangular input MF of the following form.

\[ h_{ij}(g) = \begin{cases} 
  1 + \frac{g - c_{ij}}{b_{ij}^-}, & -b_{ij}^- \leq g - c_{ij} \leq 0 \\
  1 - \frac{g - c_{ij}}{b_{ij}^+}, & 0 \leq g - c_{ij} \leq b_{ij}^+ \\
  0, & \text{otherwise}
\end{cases} \]  

where, \( g \) is the normalized input variable \( s \) or \( \dot{s} \), and \( b_{ij}^-\), \( c_{ij} \), and \( b_{ij}^+ \) are the left-half width, centroid, and right-half width of the \( j^{th} \) input MF of the \( i^{th} \) input, respectively. In this work, asymmetrical MFs are employed to robustify the self-regulating MRAC law against the bounded exogenous disturbance. Hence, the MFs are optimized offline by
minimizing the cost-function $J_c$. The input and output MFs are shown in Figure 2. The resulting decisions are de-fuzzified to evaluate the crisp output, $\sigma(t)$, by means of the center-of-gravity method as shown in Eq. 35.

$$f(s, \dot{s}) = \frac{\sum_{i=1}^{7} \sum_{j=1}^{7} (\mu_{ij} \times w_{ij})}{\sum_{i=1}^{7} \sum_{j=1}^{7} \mu_{ij}}$$  \hspace{1cm} (35)$$

where, $w$ is the centroid of $k^{th}$ output fuzzy MF. The value of $f(s, \dot{s})$ is restricted within 0 and 1.

$$\varphi_\alpha(t) = \varphi_{\alpha,ss} + (\varphi_{\alpha,ds} - \varphi_{\alpha,ss}) \times f(s, \dot{s})$$  \hspace{1cm} (36)$$

$$\varphi_\theta(t) = \varphi_{\theta,ss} + (\varphi_{\theta,ds} - \varphi_{\theta,ss}) \times f(s, \dot{s})$$  \hspace{1cm} (37)$$

$$\varphi_s(t) = \varphi_{s,ss} + (\varphi_{s,ds} - \varphi_{s,ss}) \times f(s, \dot{s})$$  \hspace{1cm} (38)$$

$$\varphi_{\dot{s}}(t) = \varphi_{\dot{s},ss} + (\varphi_{\dot{s},ds} - \varphi_{\dot{s},ss}) \times f(s, \dot{s})$$  \hspace{1cm} (39)$$

The parameters $\varphi_{x,ss}$ and $\varphi_{x,ds}$ represent the adaptation gains of the steady-state and the disturbed-state of RIP, respectively. These parameters are selected according to the research findings presented in. The selected values are $\varphi_{a,ds} = 2.0$, $\varphi_{\theta,ds} = 1.0$, $\varphi_{a,ds} = 3.0$, $\varphi_{\theta,ds} = 2.0$, $\varphi_{a,ss} = 1.0$, $\varphi_{\theta,ss} = 0.1$, $\varphi_{a,ss} = 0.2$, and $\varphi_{\theta,ss} = 0.1$.

![Figure 3. Block diagram of the F-MRAC.](image)
The MRAC law equipped with the aforesaid FLS is denoted as “F-MRAC”. The block diagram of F-MRAC is shown in Figure 3.

**Fuzzy-immune adaptation**

The artificial immune system is a computational intelligence technique that imitates the biological immune systems to adaptively redesign the control law online to compensate for the exogenous disturbances based on well-postulated control-input-driven meta-rules. The biological immune system is tolerant against the in-breaking of pathogens that are generally composed of antibody molecules and lymphocytes. The lymphocytes are pro-created by the B-cells, suppressor T-cells ($T_S$ cells), and the helper T-cells ($T_H$ cells). The receptors on the surface of B-cells measures the magnitude of the foreign antigen invasion. Based on their diagnosis, the B-cells trigger a suitable concentration of $T_H$ cells to assist in the generation of plasma cells. The plasma cells generate antibody molecules to counter the antigen attack. As the antigen attack weakens, the $T_S$ are activated to suppress the proliferation of antibodies. The collaborative effort of $T_S$ and $T_H$ cells balances the successive activation and inhibition of the antibody-generation process. This behavior strengthens the system’s resilience against the attacking antigens and ensures the body’s quick recovery. The total B-cell concentration generated in this process is evaluated as follows:

$$c(n) = T_H(n) - T_S(n)$$

such that, $T_H(n) = \gamma d(n)$, $T_S(n) = \gamma \lambda m(c(n), \dot{c}(n))d(n)$

where, $n$ represents the generation of antibody and antigen reproduction, $c(n)$ is B-cell concentration (or stimulus) at $n^{th}$ generation, $T_H(n)$ is the concentration of $T_H$ cells, $T_S(n)$ is the concentration of $T_S$ cells, $d(n)$ is the concentration of antigens, $m(.)$ is a pre-configured non-linear function that dynamically modifies the inhibition-rate of antibodies, and $\gamma$ and $\lambda$ are positive constants that decide the damping strength and response-speed of the immune system. The total stimulation is expressed in Eq. 41.

$$c(n) = \gamma(1 - \lambda m(c(n), \dot{c}(n)))d(n)$$

In practice, the performance of under-actuated electromechanical systems is prone to be degraded by bounded exogenous disturbances. Hence, in this research, a fuzzy-immune adaptation law is derived to efficiently reject the disturbances while avoiding highly discontinuous control activity. A logical map correlating the constituents of the typical biological immune system with the artificial-immune-adapted RIP system is presented as follows:

| Biological system | Physical system |
|------------------|----------------|
| Immune system    | RIP system     |
| The $n^{th}$ generation of antibody reproduction | The sampling interval of the RIP system |
| The antigen concentration, $d(n)$ | The classical state-error variable, $e(t)$ |
| The B-cell stimulation, $c(n)$ | The control input, $u(t)$ |
The aforementioned map yields the following artificial-immune control law.

\[ u(t) = a(t) \times e(t) \]  \hspace{1cm} (42)

The artificial-immune control law generates the control signal based on the variations in the state-error-variable. Hence, the self-adjusting fuzzy-immune gain function is expressed as follows.

\[ a(t) = \gamma (1 - \lambda_m(u, \dot{u})) \]  \hspace{1cm} (43)

The stimulation function \( m(\cdot) \) is realized by using the conventional two-input fuzzy-logic mechanism as proposed in the open literature.\(^{31}\) It employs a two-input logical rule-base that qualitatively defines the artificial-immune adaptation strategy. The control-input variables \( u \) and its derivative \( \dot{u} \) are used as the inputs to the system. The stimulus function \( m(u, \dot{u}) \) acts as its output. The universe of both inputs as well as the output is divided into seven linguistic variables that are defined as NB, NM, NS, Z, PS, PM, and PB. The variations in the input variables and the output variable are normalized within \([-1, 1]\). The rules dictated by the fuzzy-immune feedback system are used to synthesize the stimulus \( m(\cdot) \). The fuzzy-immune rule-base, used in this article, is presented in Table 3.

The fuzzy nonlinear function \( m(u, \dot{u}) \) is re-computed online, after every sampling-interval, via the same procedure presented in the previous sub-section. The said fuzzy system also uses asymmetrical MFs that are tuned offline by minimizing \( J_c \). The input and output MFs are shown in Figure 4. The coefficient of matrix \( \varphi \) are updated online via the following formulations.

\[ \varphi_{\alpha}(t) = \gamma_{\alpha}(1 - \lambda_{\alpha} m(u, \dot{u})) \]  \hspace{1cm} (44)

\[ \varphi_{\beta}(t) = \gamma_{\beta}(1 - \lambda_{\beta} m(u, \dot{u})) \]  \hspace{1cm} (45)

\[ \varphi_{\dot{\alpha}}(t) = \gamma_{\dot{\alpha}}(1 - \lambda_{\dot{\alpha}} m(u, \dot{u})) \]  \hspace{1cm} (46)

\[ \varphi_{\dot{\beta}}(t) = \gamma_{\dot{\beta}}(1 - \lambda_{\dot{\beta}} m(u, \dot{u})) \]  \hspace{1cm} (47)

| \( m(u, \dot{u}) \) | \( u \) |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|
| NB          | PB  | PB  | PM  | PM  | PS  | PS  | PS  | Z   |
| NM          | PB  | PM  | PM  | PS  | PS  | Z   | NS  | NS  |
| NS          | PM  | PM  | PS  | PS  | Z   | NS  | NS  | NS  |
| Z           | PM  | PS  | PS  | Z   | NS  | NS  | NS  | NM  |
| PS          | PS  | PS  | Z   | NS  | NS  | NM  | NM  | NM  |
| PM          | PS  | Z   | NS  | NS  | NM  | NM  | NB  | NB  |
| PB          | Z   | NS  | NS  | NM  | NM  | NB  | NB  | NB  |

Table 3. Two-input rule-base to implement the stimulus function.
The parameters $\gamma_x$ and $\lambda_x$ are tuned offline by minimizing the cost-function $J_c$ (and using the tuning procedure discussed in Section 3.1) to enhance the system’s disturbance-compensation capability. The parameters $\gamma_x$ and $\lambda_x$ are selected within the range $[0, 5]$ and $[0, 1]$, respectively.

The tuned values of these parameters are $\gamma_a = 2.24$, $\gamma_\theta = 1.35$, $\gamma_\alpha = 3.07$, $\gamma_\theta = 2.19$, $\lambda_a = 0.88$, $\lambda_\theta = 0.86$, $\lambda_\alpha = 0.72$, $\lambda_\theta = 0.68$. The MRAC directed by the immunological computations is denoted as “I-MRAC”. The block diagram of F-MRAC is shown in Figure 5.
The formulation of LQR reference model requires a well-identified linear state-space model of the dynamical system (a priori). However, this difficulty can be addressed by using modern identification tools for a given complex dynamical system. Secondly, the I-MRAC requires offline selection of a relatively larger number of hyper-parameters associated with the fuzzy-immune system. This has been addressed in this research by iteratively minimizing the cost-function $J_c$ (as discussed in Section 3.1) to optimize the said parameters. Nevertheless, the innovative traits, high scalability, and enhanced adaptability offered by the I-MRAC scheme supersedes the aforementioned issues.

**Experimental analysis**

This section details the procedure used to conduct the experiments along with the comparative analysis of the corresponding graphical results.

![Figure 6](image). The QNET rotary inverted Pendulum setup.
Experimental setup

In this work, the QNET Rotary Pendulum Board is used to conduct real-time experiments. The experimental setup is illustrated in Figure 6. The NI-ELVIS II Data-Acquisition board acquires the angular measurements at a sampling rate of 1.0 kHz, and then serially communicates the digitized data to a LabVIEW-based control application at 9600 bps. The control software is operated on a 64-bit and 1.8 GHz microprocessor with 8.0 GB RAM. The front-end of the said application is used to visualize and record the real-time state and control signal variations. The control system is implemented via the block diagram tool of the application. The adaptation laws are programed via C-language in the Math-Script tool of the LabVIEW software. This custom-built control routine processes the incoming sensor data to update the control signals that are serially fed to the hardware setup via the ELVIS board. The onboard motor driver circuit subsequently modulates and amplifies these control signals fed to drive the motor. The motor driver is durable enough to safely handle the disputed control activity.

Test and results

The aforesaid MRAC variants are experimentally examined by performing the following five test cases. The pendulum rod is erected manually and stabilized at the beginning of every trial while ensuring same initial condition. In each test-case, the responses of \( \theta(t) \), \( \alpha(t) \), \( V_m(t) \), and \( K(t) \) are analyzed. Every experimental trial is initiated with same initial condition and benchmarking criteria to ensure fairness in the acquired results. The visualization is simplified by showing the angular responses in degrees (deg.).

A. Position-regulation behavior: This test assesses the system’s reference tracking as well as the control energy consumption behavior under normal conditions in a disturbance-free environment. The test aims to vertically balance the rod while tracking the arm’s reference position with minimum fluctuations. The time-domain profile of each controller is depicted in Figure 7.

B. Impulsive-disturbance rejection: The test evaluates the controller’s ability to reject the impulse of a Newtonian force applied by the external sources. The test is conducted by applying a pulse of magnitude \(-10.0 \text{ V}\) and a duration 100.0 ms in \( V_m(t) \), every time the arm reaches its maximum position, while the rod is balancing itself vertically. The behavior of each controller is illustrated in Figure 8.

C. Step-disturbance rejection: The resilience of each MRAC variant against the deteriorating effects of step variations in the parameters, externally applied torques, or permanent load changes is tested by injecting a \(-5.0 \text{ V}\) step signal in \( V_m(t) \) at \( t \approx 7.0 \text{ s} \). The consequent response of each controller is illustrated in Figure 9.

D. Noise attenuation: The controller’s ability to mitigate the ripples and chattering effect imparted by the mechanical vibrations and sensor noise is analyzed by introducing a high-frequency and low-amplitude sinusoidal signal, \( d(t) = 1.5 \sin(20\pi t) \), in \( V_m(t) \). The lumped-disturbance compensation behavior of each controller is demonstrated in Figure 10.

E. Model-error compensation: This test case examines the controller’s robustness against the unavoidable identification-errors and model-variations by attaching a 0.10
kg mass underneath the base of the pendulum’s rotating arm via a hook, as shown in Figure 6, at $t \approx 7.0$. This modification permanently changes the system’s state-space model which leads to perturbations in the state-response. The time-domain behavior of each controller is shown in Figure 11.

**Analysis and discussions**

The performance analysis is done via the following standard-performance-indicators (SPIs): the root-mean-squared-error ($e_{x_{RMS}}$) in the responses of $\alpha$ and $\theta$, the magnitude of overshoot or undershoots ($M_{p_x}$) under disturbances, the transient-recovery time ($t_{s_x}$)
taken by the response, the offset in the arm ($\alpha_{\text{off}}$) under disturbances, the peak-to-peak amplitude of fluctuations in the arm ($\alpha_{p-p}$) under disturbance, the mean-squared-voltage ($\text{MSV}_m$) to determine the average control energy dissipation, and the peak control voltage ($V_p$) observed under disturbances. The experimental results are summarized in Table 4.

In Test-A, the standard MRAC manifests a highly deficient reference-tracking behavior while F-MRAC exhibits a mediocre improvement. The I-MRAC outperforms the aforesaid controllers by effectively attenuating the position-regulation error and minimizing the control energy dissipation. In Test-B, the MRAC shows the slowest transient recovery and weak damping control effort resulting in large overshoots and servo

Figure 8. RIP’s response under impulsive disturbances.
The F-MRAC shows a significantly improved state behavior. The I-MRAC exhibits rapid transits with strong damping against impulsive disturbances while effectively cutting-down the peak servo requirements. In Test-C, the MRAC contributes the largest value of $\alpha_{\text{off}}$ and $\alpha_p - \alpha_p$. The F-MRAC enhances the disturbance-attenuation behavior and curbs the offset. The I-MRAC effectively rejects the step-disturbance by contributing minimal $\alpha_{\text{off}}$ and $\alpha_p - \alpha_p$ while curbing the control energy demands. In Test-D, the MRAC shows a highly discontinuous control activity. The F-MRAC suppresses the chattering content while rendering a moderately discontinuous control activity. The I-MRAC robustly attenuates the noise while displaying an economical control behavior. In Test-E,

**Figure 9.** RIP’s response under step disturbances.
the proposed modification abruptly increases the system’s inertia which results in a larger (and disrupted) control energy expenditure. The MRAC contributes large fluctuations and highly expensive control activity. The F-MRAC applies a relatively stiffer damping effort. The I-MRAC exhibits strong model-error-compensation while economizing the control activity. The smooth control activity yielded by the I-MRAC also protects the motor drive system from unnecessary harm.

The performance assessment validates the enhanced position-regulation, time-optimal disturbance-rejection, and energy-efficient control activity of the I-MRAC in the aforementioned tests. Under every disturbance scenario, the I-MRAC exhibits minimal

Figure 10. RIP’s response under sinusoidal disturbances.
position-regulation error and the most economical energy consumption. The F-MRAC manifest the second-best performance. The improved robustness of I-MRAC is accredited to the dynamic adjustment of $\phi$ via immunological computations. Unlike F-MRAC that uses a fuzzy rules based on expert’s experience, the I-MRAC uses well-established biologically-inspired fuzzy-immune adaptation rules to hone the system’s robustness against environmental indeterminacies.

The fuzzy-immune adaptation addresses all the inherent shortcomings of the MRAC system by using the real-time control-input dynamics to autonomously adapt the $\phi$ matrix. This is beneficial because, unlike F-MRAC, the complete

Figure 11. RIP’s response under model variation.
knowledge of system enables the I-MRAC to perform better self-tuning of gains under disturbances. Consequently, the system automatically delivers stronger damping against disturbances. From a functional point of view, the compensator gains of I-MRAC are being self-adjusted quickly and accurately when confronted with sudden error variations. Unlike the F-MRAC, the I-MRAC efficiently responds to disturbances by inducing well-planned gain variations (as shown in the graphs) which validates its enhanced disturbance-rejection and relatively smoother control activity as compared to F-MRAC.

**Conclusion**

This paper formulates an immunologically-driven self-regulating MRAC law that strengthens the disturbance-rejection capability of under-actuated mechatronic systems. The proposed I-MRAC employs real-time control-input variations in conjunction with pre-calibrated immunological rules to modify the adaptation gains of the MRAC law. These immunological computations increase the controller’s adaptability to effectively execute self-tuning in a real-time environment. The I-MRAC flexibly manipulates the tightness of the damping control force and response speed under exogenous disturbances while economizing the control-energy expenditure. These propositions are justified by the experimental results. The proposed controller is highly-scalable and can be extended to other under-actuated systems. Furthermore, this technique does not put any recursive

| Test | SPI | MRAC | F-MRAC | I-MRAC |
|------|-----|------|--------|--------|
| A    | $e_{\theta_{\text{RMS}}}$ (degrees) | 0.62 | 0.44 | 0.29 |
|      | $e_{\alpha_{\text{RMS}}}$ (degrees) | 13.34 | 11.99 | 7.81 |
|      | $\text{MSV}_m$ ($V^2$) | 9.09 | 5.17 | 3.82 |
| B    | $e_{\theta_{\text{RMS}}}$ (degrees) | 0.71 | 0.58 | 0.35 |
|      | $|M_{\theta,0}|$ (degrees) | 2.72 | 1.95 | 1.46 |
|      | $t_{\alpha,0}$ (s) | 0.70 | 0.51 | 0.34 |
|      | $e_{\alpha_{\text{RMS}}}$ (degrees) | 11.65 | 8.44 | 5.89 |
|      | $\text{MSV}_m$ ($V^2$) | 9.65 | 5.16 | 3.36 |
|      | $V_p$ (V) | -10.62 | -11.46 | -8.23 |
| C    | $e_{\theta_{\text{RMS}}}$ (degrees) | 1.29 | 0.76 | 0.42 |
|      | $e_{\alpha_{\text{RMS}}}$ (degrees) | 34.10 | 25.16 | 16.35 |
|      | $\alpha_{\text{off}}$ (degrees) | -40.17 | -28.09 | -19.53 |
|      | $\alpha_{p-p}$ (degrees) | 30.25 | 27.69 | 18.86 |
|      | $\text{MSV}_m$ ($V^2$) | 28.71 | 22.80 | 17.78 |
|      | $V_p$ (V) | -13.03 | -13.41 | -12.85 |
| D    | $e_{\theta_{\text{RMS}}}$ (degrees) | 0.45 | 0.36 | 0.21 |
|      | $e_{\alpha_{\text{RMS}}}$ (degrees) | 10.40 | 8.79 | 6.88 |
|      | $\text{MSV}_m$ ($V^2$) | 12.88 | 10.11 | 7.76 |
| E    | $e_{\theta_{\text{RMS}}}$ (degrees) | 1.22 | 0.86 | 0.54 |
|      | $e_{\alpha_{\text{RMS}}}$ (degrees) | 18.60 | 12.15 | 8.10 |
|      | $\text{MSV}_m$ ($V^2$) | 11.81 | 9.81 | 8.52 |

Table 4. Summary of experimental results.
computational burden and can be easily realized using modern digital computers. In the future, meta-heuristic optimizers can be employed for the offline selection of the hyperparameters. Moreover, the prescribed control law can be further optimized by replacing the immune-adaptation scheme with other computational intelligence algorithms.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Omer Saleem https://orcid.org/0000-0003-2197-9302

References
1. Krafes S, Chalh Z and Saka A. A review on the control of second order underactuated mechanical systems. *Complexity* 2018; 2018: 1–17. ID 9573514.
2. Mahmoud MS (2018) *Advanced control design with application to electromechanical systems*. New York: Elsevier Science, pp.177–253.
3. Gritli H and Belghit S. Robust feedback control of the underactuated inertia wheel inverted Pendulum under parametric uncertainties and subject to external disturbances: LMI formulation. *J Franklin Inst* 2018; 355: 9150–9191.
4. Chan RPM, Stol KA and Halkyard CR. Review of modelling and control of two-wheeled robots. *Annu Rev Control* 2013; 37: 89–103.
5. Wang H, Du H, Cui Q, et al. Artificial bee colony algorithm based PID controller for steel stripe deviation control system. *Sci Prog* 2022; 105: 1–22.
6. Szuster M and Hendzel Z. *Intelligent optimal adaptive control for mechatronic systems*. Cham: Springer, 2017.
7. Wang S, Chen Y and Zhang G. Adaptive fuzzy PID cross coupled control for multi-axis motion system based on sliding mode disturbance observation. *Sci Prog* 2021; 104: 1–19.
8. Kuo TC, Huang YJ and Chang SH. Sliding mode control with self-tuning law for uncertain nonlinear systems. *ISA Trans* 2008; 47: 171–178.
9. Wen T, Xiang B and Zhang S. Optimal control for hybrid magnetically suspended flywheel rotor based on state feedback exact linearization model. *Sci Prog* 2020; 103: 1–23.
10. An-chyau H, Yung-feng C and Chen-yu K. *Adaptive control of underactuated mechanical systems*. Singapore: World Scientific, 2015.
11. Chungeng S and Ruibo Y. Adaptive robust cross-coupling position synchronization control of a hydraulic press slider-leveling. *Sci Prog* 2021; 104: 1–19.
12. Saleem O and Mahmood-ul-Hasan K. Robust stabilisation of rotary inverted pendulum using intelligently optimised nonlinear self-adaptive dual fractional order PD controllers. *Int J Syst Sci* 2019; 50: 1399–1414.
13. Chunjiang B, Jiwei F, Jian W, et al. Model predictive control of steering torque in shared driving of autonomous vehicles. *Sci Prog* 2020; 103: 1–22.
14. Batmani Y, Davoodi M and Meskin N. Nonlinear suboptimal tracking controller design using state-dependent riccati equation technique. *IEEE Trans Control Syst Technol* 2017; 25: 1833–1839.

15. Subramaniana RG and Elumalai VK. Robust MRAC augmented baseline LQR for tracking control of 2 DoF helicopter. *Rob Auton Syst* 2016; 86: 70–77.

16. Zhang D and Wei B. A review on model reference adaptive control of robotic manipulators. *Ann Rev Control* 2017; 43: 188–198.

17. Saleem O, Rizwan M, Mahmood-ul-Hasan K, et al. Performance enhancement of multivariable model-reference optimal adaptive motor speed controller using error-dependent hyperbolic gain functions. *Automatika* 2020; 61: 117–131.

18. Jian Z and Yongpeng Z (2011) Optimal linear modeling and its applications on swing-up and stabilization control for rotary inverted pendulum. In: *Proceedings of the 30th Chinese control conference, Yantai, China*. New York: IEEE, pp. 493–500. https://ieeexplore.ieee.org/xpl/conhome/5980617/proceeding

19. Balamurugan S and Venkatesh P. Fuzzy sliding-mode control with low pass filter to reduce chattering effect: an experimental validation on Quanser SRIP, *Sadhana* 2017; 42: 1693–1703.

20. Astom KJ, Apkarian J, Karam P, et al. *Student workbook: QNET rotary inverted pendulum trainer for NI ELVIS*. Ontario: Quanser Inc, 2011.

21. Lewis FL, Vrabie D and Syrmos VL. *Optimal control*. New Jersey: John Wiley and Sons, 2012.

22. Saleem O, Mahmood-ul-Hasan K and Rizwan M. An experimental comparison of different hierarchical self-tuning regulatory control procedures for under-actuated mechatronic system. *PLoS One* 2021; 16: e0256750.

23. Kavuran G, Ates A, Alagoz BB, et al. An experimental study on model reference adaptive control of TRMS by error-modified fractional order MIT rule. *Control Eng Appl Inform* 2017; 19: 101–111.

24. Chen KY. Model reference adaptive minimum-energy control for a mechatronic elevator system. *Optim Control Appl Met* 2017; 38: 3–18.

25. Astrom KJ and Wittenmark B. *Adaptive control*. 2nd ed. London: Pearson Education, 1995.

26. Hassanzadeh A, Nejadfard A and Zadi M. A multivariable adaptive control approach for stabilization of a cart-type double inverted Pendulum. *Math Probl Eng* 2011; 2011: 1–15. ID 970786.

27. Zhang H, Wang J and Lu G. Self-organizing fuzzy optimal control for under-actuated systems. *J Syst Control Eng* 2014; 228: 578–590.

28. Bhatti OS, Tariq OB, Manzar A, et al. Adaptive intelligent cascade control of a ball-riding robot for optimal balancing and station-keeping. *Adv Robot* 2018; 32: 63–76.

29. Saleem O and Mahmood-ul-Hasan K. Hierarchical adaptive control of self-stabilizing electromagnetic systems using artificial-immune self-tuning mechanism for state weighting-factor. *J Mech Sci Technol* 2021; 35: 1235–1250.

30. Lee YJ, Cho HC and Lee KS. Immune algorithm based active PID control for structure systems. *J Mech Sci Technol* 2006; 20: 1823–1833.

31. Hui L, Xingjiao L and Jing L. The research of fuzzy immune linear active disturbance rejection control strategy for three-motor synchronous system. *Control Eng Appl Inform* 2015; 17: 50–58.

**Author biographies**

**Omer Saleem** is an Assistant Professor at the Department of Electrical Engineering, National University of Computer and Emerging Sciences, Lahore, Pakistan. He received his Ph.D. in Electrical Engineering with specialization in adaptive control systems from University of
Engineering and Technology, Lahore, Pakistan. His research interests include the design of self-tuning adaptive state-feedback controllers for electro-mechanical systems.

Mohsin Rizwan is an Associate Professor at the Department of Mechatronics and Control Engineering, University of Engineering and Technology, Lahore, Pakistan. He received his Ph.D. from University of Texas at Arlington with specialization in optimal control and optimization. His research interests include nonlinear system modeling and control.

Fahim Gohar Awan is an Associate Professor at the Department of Electrical Engineering, University of Engineering and Technology, Lahore, Pakistan. He received his Ph.D. in Electrical Engineering from University of Engineering and Technology, Lahore, with specialization in electrical instrumentation. His research interests include simulation, modeling and analysis.