RADIO SUPERNOVA SN 1998bw AND ITS RELATION TO GRB 980425

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ABSTRACT

SN 1998bw is an unusual Type Ic supernova that may be associated with the γ-ray burst GRB 980425. We use a synchrotron self-absorption model for its radio emission to deduce that the synchrotron-emitting gas is expanding into a circumstellar medium of approximately \( r^{-2} \) density profile at a speed comparable to the speed of light. We assume that the efficiencies of production of relativistic electrons and magnetic field are constant through the evolution. The circumstellar density is consistent with that expected around the massive star core thought to be the progenitor of SN 1998bw. The explosion energy in material moving with velocity greater than 0.5c is \( \sim 10^{49} - (3 \times 10^{50}) \) ergs, with some preference for the high values. The rise in the radio light curves observed at days 20–40 is inferred to be the result of a rise in the energy of the blast wave by a factor of \( \sim 2.5 \). Interaction with a jump in the ambient density is not consistent with the observed evolution. We infer that the boost in energy is from a shell of matter from the explosion that catches up with the decelerating shock front. Both the high explosion energy and the nature of the energy input to the blast wave are difficult to reconcile with energy input from the shock-accelerated high-velocity ejecta from a supernova. The implication is that there is irregular energy input from a central engine, which is the type of model invoked for normal γ-ray bursts. The link between SN 1998bw and GRB 980425 is thus strengthened.

Subject headings: gamma rays: bursts — radio continuum: stars — supernovae: general — supernovae: individual (SN 1998bw)

1. INTRODUCTION

Supernova (SN) 1998bw was discovered in an optical search of the error circle of the gamma-ray burst (GRB) source GRB 980425 (Galama et al. 1998a). The light curve of the supernova indicates an explosion time consistent with the time of GRB 980425, and Galama et al. (1998a) place the probability of a chance coincidence of the supernova and GRB at \( 10^{-4} \). The supernova spectrum implied a Type Ic event, but one with an unusually high luminosity and high material velocities. Modeling of the light curve of SN 1998bw suggested an explosion energy of \( (2-3) \times 10^{52} \) ergs (Iwamoto et al. 1998; Woosley, Eastman, & Schmidt 1999), more than an order of magnitude higher than a typical supernova energy input from the shock-accelerated high-velocity ejecta from a supernova. Kulkarni et al. (1998) discovered a variable radio source coincident with the optical supernova. Although a number of Type Ib/c supernovae have been found to be radio sources (Weiler et al. 1999), SN 1998bw was more luminous by \( \sim 10^4 \) at its peak. The unusual, energetic nature of SN 1998bw would seem to further strengthen its association with GRB 980425. Observations with BeppoSAX showed two variable X-ray sources in the error box of GRB 980425, one of which is coincident with SN 1998bw (Pian et al. 1999; Galama et al. 1998a). The other source might have been the X-ray afterglow of the GRB 980425, but the evidence for this is not compelling.

The aim of this paper is to develop a model for RSN (radio supernova) 1998bw and to relate its properties to those of SN 1998bw and GRB 980425. Here, we use the term RSN 1998bw primarily as an abbreviation for “the radio emission from SN 1998bw.” Previously, Kulkarni et al. (1998) argued that the shock velocity in SN 1998bw is relativistic based on the high brightness temperature, on a synchrotron self-absorption interpretation of the early evolution, and on scintillation results. They suggested that an early phase of this shock wave produced GRB 980425. Waxman & Loeb (1999) modeled the spectrum of RSN 1998bw and proposed that the emission is from a thermal distribution of electrons with a shock velocity of 0.3c. They use this result to infer that the supernova may be unrelated to GRB 980425. Our model uses a more complete data set and includes a relativistic dynamical model. Our conclusions are closer to those of Kulkarni et al. (1998) than to those of Waxman & Loeb (1999).

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The plan of the paper is as follows. The properties of RSN 1998bw are compared with those of other radio supernovae in § 2. Section 3 presents a nonrelativistic model for RSN 1998bw. We assume that a power-law electron spectrum is produced in the shock front. We treat dynamical and relativistic effects in § 4 and thus restrict the range of possible models. Approximate scaling laws for the range of models are discussed in § 5. Our model for RSN 1998bw is placed in the context of the properties of SN 1998bw and GRB 980425 in § 6. We present a final discussion and conclusions in § 7.

2. COMPARISON TO RADIO SUPERNOVAE

There is a growing data set on radio supernovae of Types Ib/c and II (Weiler et al. 1998, 1999, and references therein). The overall radio evolution can be described as an early phase when optical depth effects are important followed by a transition to a phase with an optically thin power-law spectrum (flux \( \propto \nu^{-(p-1)/2} \)). The power-law index for the electron spectrum, \( p \), has not been observed to vary in individual supernovae, although it does vary in the range of 2.1–3.2 for the group of radio supernovae (Van Dyk et al. 1994a). The time evolution in the optically thin regime can be described by a power-law decrease (flux \( \propto t^{-w} \)), with \( w \) ranging from 0.66 (SN 1980K; Weiler et al. 1992) to 1.6 (SN 1983N).

The evolution of the fluxes from RSN 1998bw (Kulkarni et al. 1998; Wieringa, Kulkarni, & Friail 1999; Fig. 1) and the evolution of the spectral indices (Fig. 2) show that the
radio supernovae. SN 1998bw belongs to the class of Type Ic supernovae (Galama et al. 1998a). Because Type Ib and Type Ic supernovae have similar properties and differ from each other only in the presence or absence of optical He lines near maximum light, we compare SN 1998bw to both of these types. The parameters for the late evolution of radio supernovae of these types are SN 1983N (Ib): \( p = 3.0, \ w = 1.6; \) SN 1984L (Ib): \( p = 3.0, \ w = 1.5; \) SN 1990B (Ic): \( p = 3.2, \ w = 1.3 \) (Van Dyk et al. 1994a). Radio emission has also been observed from the Type Ic SN 1994L, but the results have not been fully published. Observations at 15 and 88 GHz indicate an optically thin spectral index of \( p = 2.8 \pm 0.2 \) (Wink 1994). The radio spectrum of RSN 1998bw is flatter than those of other Type Ib/c supernovae, and its decline rate is at the high end of those observed.

The previously observed Type Ib/c supernovae have similar radio luminosities, within a factor of 2–3. Figure 3 (based on Fig. 4 of Chevalier 1998) shows the times and peak luminosities at 6 cm of radio supernovae; the Type Ib/c supernovae occupy a small area on this plot. SN 1998bw is extraordinary in that its radio luminosity is \( \sim 10^2 \) higher than that of the other Type Ib/c supernovae. It remains \( \sim 10^2 \) times stronger than SN 1983N over the full observed period of 250 days. Kulkarni et al. (1998) used the high brightness temperature of the source to argue for relativistic expansion. The straight lines in Figure 3 are from the theory described in Chevalier (1998), which assumes that the optically thick phase of the supernova is caused by synchrotron self-absorption. The electron spectrum is assumed to be a power law \( (p = 2.5 \text{ in Fig. 3}) \) that extends down to transrelativistic energies. An expression for the radius of the emitting region can then be obtained which depends on the factor \( (\text{magnetic energy density/relativistic electron energy density})^{1/2p+1/3} \). In Figure 3, the factor is
taken to be unity; the weak dependence on the energy-density ratio provides justification for the assumption. The total energy in fields and particles goes up if the system is far from equilibrium. The position of SN 1998bw in Figure 3 is suggestive of relativistic motion, which means that the non-relativistic theory used for Figure 3 breaks down.

In addition to its high luminosity, RSN 1998bw is also unusual in that its evolution leads to the late power-law decline. Figure 1 shows that the power-law decline began at high frequencies (4.80 and 8.64 GHz) on day 12 and continued through day 22. Between days 22 and 32, there was a rapid increase at these frequencies that raised the level of the power-law decay by a factor of \( \sim 2 \). This behavior has not been observed previously in the light curves of radio supernovae, although SN 1987A has shown a strong rise in its radio flux (Ball et al. 1995) that can be attributed to interaction with the dense wind from a previous evolutionary phase. The observed light curves generally do show fluctuations about smooth curves (Weiler et al. 1999) similar to that seen in the late power-law evolution of RSN 1998bw. The radio data on many supernovae are fragmentary, but there are excellent data at five frequencies on SN 1993J (Van Dyk et al. 1994b; Weiler et al. 1999). The radio evolution is smooth and can be modeled by shock-accelerated supernova ejecta running into a \( \rho \propto r^{-2} \) circumstellar medium (Fransson & Björnsson 1998). Another anomaly in the early evolution of RSN 1998bw is the rise at 4.80 and 8.64 GHz before day 10. The rise occurs with little change in spectral index, which implies it is not because of a reduction in optical depth, as is the case in other radio supernovae.

### 3. A MODEL FOR RSN 1998bw

The properties of RSN 1998bw sufficiently resemble those of other radio supernovae to consider initially a model similar to those applied to radio supernovae. The standard model for these objects involves synchrotron emission from relativistic electrons in a spherical shocked region between the supernova and a surrounding circumstellar wind (Chevalier 1982). The early optically thick phase can be dominated by either free-free absorption or synchrotron self-absorption, but the high velocities and low circumstellar densities found in Type Ib/c supernovae imply that synchrotron self-absorption is the dominant mechanism in these objects (Slysh 1990; Chevalier 1998). The same arguments apply to the Type Ic RSN 1998bw (see also Kulkarni et al. 1998). A significant new feature for RSN 1998bw is that the high shock velocity inferred for this radio supernova implies that all the postshock electrons are accelerated to relativistic energies. As in studies of GRB afterglows (Waxman 1997; Wijers, Rees, & Mészáros 1997), we assume that the distribution of relativistic electrons with the Lorentz factor \( \gamma \) takes a power-law form with the number density given by

\[
n_e(\gamma)d\gamma = C\gamma^{-p}d\gamma ,
\]

above a lower limit \( \gamma_{\text{m}} \), which is determined by the shock velocity. Support for the assumption in the GRB case can be found in the afterglow emission from GRB 970508 (Galama et al. 1998b). This distribution function distinguishes our model for RSN 1998bw from that of Waxman & Loeb (1999), who assumed a thermal (essentially monoenergetic) electron distribution.

Relativistic electrons moving with a Lorentz factor of \( \gamma \) emit synchrotron radiation of a characteristic frequency

\[
\nu_s(\gamma) = \frac{3\gamma^2 qB_\perp}{4\pi mc} ,
\]

where \( q \) and \( m \) are the charge and the mass of the electron, and \( B_\perp \) is the strength of the component of magnetic field perpendicular to the electron velocity. In the simplest case, where the frequency of interest is \( \nu \gg \nu_m = \nu_s(\gamma_m) \), the synchrotron spectrum has a \( \nu^{5/2} \) spectral dependence in the optically thick region and a \( \nu^{(p-1)/2} \) dependence in the optically thin region (e.g., Rybicki & Lightman 1979). This simple-model spectrum fails to reproduce two properties of the observed radio spectrum: (1) at the earliest times when the radio emission is most probably optically thick at the two highest frequencies (3 cm and 6 cm), the spectral index between them is closer to 2 than to 5/2, as noted by Kulkarni et al. (1998) and Waxman & Loeb (1999); and (2) the observed spectrum is significantly broader than the above simple spectrum. These two discrepancies lead us to believe that the characteristic frequency \( \nu_m \) is important in shaping the observed spectrum. We explore this possibility and try to infer the physical properties of the radio-emitting plasma from \( \nu_m \) and other parameters that determine the spectrum. A self-absorbed spectral index of 2 (rather than 5/2) for radiation produced by electrons accelerated by a relativistic shock was predicted by Katz (1994).

In this section, we restrict our discussion to emission from a nonrelativistically moving medium. It is well known that, in a synchrotron self-absorption model, the flux at a given frequency \( \nu \) is proportional to

\[
f_c \propto \nu^{5/2}(1 - e^{-\nu_m/\nu}) \int_{0}^{\infty} F(x)x^{(p-3)/2}dx ,
\]

where \( \nu_m = \nu/\nu_m \), \( F(x) \) is a known function of \( x \) (shown, e.g., on p. 179 of Rybicki & Lightman 1979), and \( \tau \) is the optical depth of synchrotron self-absorption. From the standard theory, we have

\[
\tau \propto \nu^{-(2 + p/2)} \int_{0}^{\infty} F(x)x^{(p-2)/2}dx .
\]

To simplify expressions, we define two auxiliary functions

\[
F_1(\nu, \nu_m, p) = \int_{0}^{\nu_m} F(x)x^{(p-3)/2}dx ;
\]

\[
F_2(\nu, \nu_m, p) = \int_{0}^{\nu_m} F(x)x^{(p-2)/2}dx ,
\]

and denote the flux and the optical depth at a reference frequency \( \nu_0 = 1 \) GHz by \( f_0 \) and \( \tau_0 \), respectively. The flux at other frequencies can then be obtained from

\[
\frac{f_c}{f_0} = \frac{F_1(\nu)}{F_1(\nu_0)} \left( \frac{\nu}{\nu_0} \right)^{5/2} \times \left\{ 1 - \exp \left[ -\tau_0 \left( \frac{\nu_0}{\nu} \right)^{2 + p/2} \frac{F_2(\nu)}{F_2(\nu_0)} \right] \right\} ,
\]

where the dependence of \( F_1 \) and \( F_2 \) on \( \nu_m \) and \( p \) is suppressed for clarity. Note that the relative flux \( f_c/f_0 \) depends on three parameters: \( \tau_0 \) (the optical depth at \( \nu_0 = 1 \) GHz), \( \nu_m \) (the characteristic frequency corresponding to the lowest electron energy), and \( p \) (the electron energy-distribution
index). By fitting the shape of radio spectrum to each day, we hope to determine these three parameters as a function of time.

The shape of the observed spectrum is specified by three spectral indices between 20 cm/13 cm, 13 cm/6 cm, and 6 cm/3 cm (Fig. 2). In principle, one can invert equation (6) to determine the parameters \( \tau_0 \), \( v_m \), and \( p \) uniquely from the three spectral indices. In practice, we adopt simple power-law distributions for these parameters as a function of time and compute and match the three spectral indices to the observed values as closely as possible. Through trial and error, we find that reasonable fits are obtained for

\[
\tau_0 = 25 \left( \frac{t}{10 \text{ days}} \right)^{-2} ;
\]

\[
v_m = 5.6 \left( \frac{t}{10 \text{ days}} \right)^{-1} \text{ GHz} ; \quad p = 2.5 .
\] (7)

The fits are shown in Figure 2. We should point out that the spectral indices between 6 cm and 3 cm on day 3 and day 4 are not reproduced by our fits, indicating that other than synchrotron self-absorption might be important at such early times or that the simple power-law evolution breaks down. In addition, both \( v_m \) and the self-absorption turnover frequency have moved below the observed frequency range by day 50, so these parameters are not constrained by the observations at later times. Over the time period that they are important for the observed radio emission, \( v_m \) and the self-absorption turnover frequency are close to each other.

To convert the fitted optical depth \( \tau_0 \), characteristic frequency \( v_m \), and index \( p \) into physical quantities of the synchrotron-emitting gas, we need to have a specific model for its geometry. Here we adopt the simplest model of a thin, uniform, spherical shell of radius \( r \) for its geometry. Here we adopt the simplest model of a synchrotron-emitting gas, we need to have a specific model (9), by

\[
\tau = \frac{p + 2}{2\pi m v^2} \left( \frac{4\pi n c v}{3q} \right)^{-p/2} F_2(v) CB_{l}^{(p+2)/2}(\xi r) ,
\] (8)

taking into account the effect of spherical geometry approximately (Chevalier 1998), and

\[
v_m = \frac{3\gamma^2 q B_{l}}{4\pi m c} , \quad (9)
\]

\[
f_v = \frac{\pi r^2}{d^2} \left( \frac{4\pi n c v}{3q B_{l}} \right)^{1/2} F_2(v) (1 - e^{-\gamma}) , \quad (10)
\]

where the electron distribution coefficient \( C \) is related to the electron energy density \( U_e \) by

\[
C = (p - 2) \frac{U_e}{mc^2} \gamma_m^{p-2} , \quad (11)
\]

and the minimum Lorentz factor \( \gamma_m \) is given, from equation (9), by

\[
\gamma_m = \left( \frac{4\pi n c v}{3q B_{l}} \right)^{1/2} . \quad (12)
\]

Making the usual assumption that the energy densities of electrons and magnetic fields are proportional to the total energy density \( U \), we have \( U_e = \epsilon_e U \) and \( B_{l} = (8\pi \epsilon_e U)^{1/2} \).

It turns out that only two physical quantities enter equations (8) and (10) for the optical depth and the flux. They are the radius \( r \) of the emitting region and the total energy density \( U \) or, alternatively, the total energy \( E = 4\pi \xi r^3 U \).

Straightforward but tedious algebraic manipulations show that

\[
E = 5.1 \times 10^{48} \left( \frac{0.1}{\epsilon_e} \right)^{6/17} \left( \frac{0.1}{\epsilon_e} \right)^{11/17} \left( \frac{\xi}{0.1} \right)^{6/17} \times \left( \frac{d}{40 \text{ Mpc}} \right)^{40/17} \left( \frac{f_v}{20 \text{ mJy}} \right)^{20/17} \left( \frac{p + 2}{5} \right)^{9/17} \times \left( \frac{1}{p - 2} \right)^{11/17} \left( \frac{v}{1 \text{ GHz}} \right)^{(11p - 56)/34} \left( \frac{v_m}{1 \text{ GHz}} \right)^{11(2-p)/34} \times \frac{F_2(v)^{9/17}}{F_1(v)^{20/17}(1 - e^{-\gamma})^{20/17}} \text{ ergs}
\] (13)

and

\[
r = 1.9 \times 10^{17} \left( \frac{\epsilon_e}{0.1} \right)^{1/17} \left( \frac{0.1}{\epsilon_e} \right)^{1/17} \left( \frac{0.1}{\xi} \right)^{1/17} \times \left( \frac{d}{40 \text{ Mpc}} \right)^{16/17} \left( \frac{f_v}{20 \text{ mJy}} \right)^{8/17} \left( \frac{p + 2}{5} \right)^{7/17} \times \left( \frac{1}{p - 2} \right)^{1/17} \left( \frac{v}{1 \text{ GHz}} \right)^{(p - 36)/34} \left( \frac{v_m}{1 \text{ GHz}} \right)^{(2-p)/34} \times \frac{F_2(v)^{7/17}}{F_1(v)^{8/17}(1 - e^{-\gamma})^{8/17} \text{ cm} .}
\] (14)

In these expressions, the quantities \( p \), \( v_m \), and \( \tau \) [as well as \( F_1(v) \) and \( F_2(v) \), implicitly] are given by equations (7) and (4) and obtained from fitting the spectral shape of the radio emission. The flux \( f_v \) is given by the observed radio light curves. We choose the smoothest curve of 6 cm and fit it with an analytical expression, as shown in Figure 1. Other model curves on the same figure are obtained using the fitted parameters in equation (7). They match the data reasonably well. The weak dependence of \( r \) on \( \epsilon_e \) is comparable to that found for normal radio supernovae (Chevalier 1998).

From the fitted flux at 6 cm and equations (13)–(14), we obtain the time evolution of the total internal energy and the radius of synchrotron-emitting gas. For a typical set of parameters \( \epsilon_e = \epsilon_e = \xi = 0.1 \) and \( d = 40 \) Mpc, we find a total energy between about \( 0.4 \times 10^{49} \) and \( 1.2 \times 10^{49} \) ergs and a radius between about \( 0.5 \times 10^{17} \) and \( 2.5 \times 10^{17} \) cm in the time interval from day 12 to day 80, as shown in Figures 4a and 4b. For comparison, a straight line with a slope equal to the speed of light is drawn in Figure 4b, where the radius is plotted as a function of time. The average speed in this time interval is close to the speed of light, although the instantaneous value could be somewhat different.

\(^1\) Note that the magnetic energy fraction defined in this paper, \( \epsilon_e = B_{l}^2/(8\pi U) \), is two-thirds the more conventionally used fraction, \( \epsilon_p = B_{l}^2/(8\pi U) \) (where \( B_{l} \) is the total field strength), for an isotropic electron distribution.
We can also infer the number density of the emitting electrons, $n_e$. From equation (2), we have

$$n_e = \int_{r_m}^{\infty} n_e(r) dr = \frac{p - 2}{p - 1} \frac{\epsilon_e}{m_c^2 \gamma_m} U,$$  

(15)

A straightforward substitution yields

$$n_e = 1.0 \left( \frac{0.1}{\epsilon_e} \right)^{7/17} \left( \frac{\epsilon_e}{0.1} \right)^{7/17} \left( \frac{0.1}{\xi} \right)^{10/17} \left( \frac{40 \text{ Mpc}}{d} \right)^{10/17} \times \left( \frac{20 \text{ mJy}}{F_o} \right)^{5/17} \left( \frac{5}{p + 2} \right)^{15/17} \left( \frac{p - 2}{p - 1} \right)^{7/17} \times \left( \frac{v}{1 \text{ GHz}} \right)^{(10p + 65)/34} \left( \frac{v_m}{1 \text{ GHz}} \right)^{(3 - 10p)/34} \times F_4(v)^{2/17} F_8(v)^{10/17} (1 - e^{-\gamma})^{5/17} \text{ cm}^{-3}.$$  

(16)

This quantity is shown as a function of radius in Figure 4c. Clearly, the number density of the emitting electrons drops off with radius approximately as $r^{-2}$. Since the density in the emitting region is related to the density in the ambient medium (which supplies the emitting gas with matter through a strong shock) by a simple compression factor, we conclude that the ambient medium must have a $r^{-2}$ density profile as well. Such a profile arises naturally in a stellar wind of constant mass-loss rate and constant velocity from the progenitor star of the supernova. We can estimate the mass-loss rate in the wind through

$$M_w = 4\pi \rho_w V_w r^2 = 8\pi \xi n_e m_p r^2 V_w,$$  

(17)

where the subscript "w" stands for the wind. We have adopted a nucleon-to-electron-number density ratio of 2, appropriate for the predominantly helium (and perhaps carbon/oxygen) wind of a Wolf-Rayet star, which is the likely progenitor of SN 1998bw (see § 6 for discussion). We plot the inferred mass flux as a function of radius in Figure 4d. For the typical parameters chosen, we have a stellar mass-loss rate of order $2.5 \times 10^{-7} (V_w/10^3 \text{ km s}^{-1}) M_\odot \text{ yr}^{-1}$. The dependence on the parameters $\epsilon_e$ and $\epsilon_c$ can be found from equation (16).

4. DYNAMICAL AND RELATIVISTIC EFFECTS

So far, we have worked directly with the radio data and inferred from it properties of the emitting region using a simple radiation model. Now we use these results as a guide and construct a more detailed model that takes into account the dynamics of the emitting matter and
relativity—two principal effects neglected in the above approach.

We envision an instantaneous release of a large amount of energy \( E_0 \) in a \( r^{-2} \) density external medium. The energy released in the medium drives a blast wave the dynamics of which can be deduced approximately from the following considerations. In the frame at rest with respect to the origin of the explosion (and the observer), let the shocked external material be distributed uniformly in a shell of thickness \( \Delta R_s \) (much less than the outer radius of the shell, the shock radius \( R_s \)). The physical quantities inside the shell are related to those of the external medium by a set of shock-jump conditions, which are given by Blandford & McKee (1976) for an arbitrary strong shock. Let \( \gamma \) be the bulk Lorentz factor of the shocked medium in the shell and \( \Gamma \) that of the shock front. The jump conditions are

\[
U' = \frac{\eta \gamma + 1}{\eta - 1} (\gamma - 1)n_0 m_p c^2, \quad (18)
\]

\[
n' = \frac{\eta \gamma + 1}{\eta - 1} n, \quad (19)
\]

where \( U' \) and \( n' \) are the energy- and nucleon-number densities of the shell in its comoving frame, and \( \eta \) is the adiabatic index, which equals \( 4/3 \) for ultrarelativistic shocks and \( 5/3 \) for nonrelativistic shocks. A simple interpolation between these two limits,

\[
\eta = \frac{4\gamma + 1}{3\gamma}, \quad (20)
\]

should be valid approximately for transrelativistic shocks as well (Huang, Dai, & Lu 1998). The symbol \( m_p \) denotes the mass of protons while \( n \) is the nucleon number density of the external medium,

\[
n = \frac{n_0 r_0^2}{\gamma^2}, \quad (21)
\]

where \( n_0 \) and \( r_0 \) are constants. The Lorentz factor of the shock front is given by

\[
\Gamma^2 = \frac{(\gamma + 1)[\eta(\gamma - 1) + 1]^2}{\eta(2 - \eta(\gamma - 1) + 2), \quad (22)}
\]

which depends on the shell Lorentz factor \( \gamma \).

The shell Lorentz factor \( \gamma \) is determined by the conservation of mass

\[
4\pi R_s^2 \Delta R_s \hat{n} = 4\pi n_0 r_0^2 R_s \quad (23)
\]

and the conservation of energy

\[
4\pi R_s^2 \Delta R_s \hat{U} = E_0, \quad (24)
\]

where the energy density \( \hat{U} \) and the number density \( \hat{n} \) of the shell in the frame at rest with respect to the origin are related to those in the comoving frame by the transformation

\[
\hat{U} = \frac{4\gamma^2 - 1}{3} U', \quad \hat{n} = \gamma n'. \quad (25)
\]

The transformation of the energy density is not exact in the transrelativistic regime but is intended to have the correct ultrarelativistic and nonrelativistic limits. Combining equations (23)–(25), we obtain an algebraic equation for \( \gamma \)

\[
\frac{(\gamma - 1)(4\gamma^2 - 1)}{3\gamma} = \frac{E_0}{4\pi m_p c^2 n_0 r_0^2 R_s} \quad (26)
\]

in terms of the shock radius \( R_s \). The shock radius itself evolves according to

\[
\frac{dR_s}{dt} = c\sqrt{1 - \Gamma^{-2}}, \quad (27)
\]

where \( t \) is the time since the explosion in the rest frame of the origin. This completes our discussion of the blast-wave dynamics.

To calculate emission from transrelativistic blast waves properly, one needs to take into account the finite light-travel time (Rees 1967). To facilitate discussion, let us adopt a cylindrical coordinate system \((R, \phi, z)\), with the \( z \)-axis pointing from the origin of the blast wave toward the observer (see Fig. 5). Let \( T \) be the time in the observer’s frame. A light pulse emitted at a time \( t \) in the rest frame of the origin from a location at an axial distance \( z \) is received by the observer at a time

\[
T = t - z/c. \quad (28)
\]

Our goal is to calculate the specific intensity \( I_\nu \) on the surface of the shell facing the observer at a distance \( R \) from the axis at an observer’s time \( T \), \( I_\nu(T, R) \). It depends on the emission and absorption coefficients at different values of \( z \), \( j_\nu(T, R, z) \), and \( \kappa_\nu(T, R, z) \). We need to determine which part of the shell contributes to the emission and absorption at \((T, R, z)\). In what follows, we describe a mathematical procedure for such a determination.

At any given time \( t \), the inner and outer radii of the emitting shell are known from the blast-wave dynamics. It is easy to calculate the time \( T \) for radiation emitted at a distance \( R \) from the axis to arrive at the observer from equation (28). The possible arrival time fills a band of \( V \) shape on

![Fig. 5.—Coordinate system for light-travel time effects. Note that the emitting shell is marked by a ring and that \( R \) denotes the distance of the line of sight away from the axis and \( z \), the distance along the line of sight.](image-url)
the $T$-$t$ diagram for a given distance $R$, as shown in Figure 6. Its shape depends on the thickness and dynamics of the shell as well as the off-axis distance $R$. One can read off this $T$-$t$ diagram the time $t$ when the emission from the shell at an off-axis distance $R$ can reach the observer at any given time $T$. Once the time $t$ is determined, the axial distance of the emitting region is given by

$$z = c(t - T) .$$  \hfill (29)

From equations (18) and (19), we can determine the comoving energy- and electron-number densities, $U'$ and $n'_e$ (which is equal to half of the nucleon-number density, $n'$, for Wolf-Rayet winds). It is then a simple matter to calculate the emission and absorption coefficients, $j'_e(T, R, z)$ and $\kappa'_e(T, R, z)$, in the comoving frame as a function of the comoving frequency $\nu'$. To obtain radiation quantities in the rest frame of the origin, we use the following standard transformation (e.g., Rybicki & Lightman 1979):

$$\nu = \frac{\nu'}{(1 - \beta \mu)} ,$$  \hfill (30)

$$j_e(T, R, z) = \frac{j'_e(T, R, z)}{[\gamma(1 - \beta \mu)]^2} ,$$  \hfill (31)

$$\kappa_e(T, R, z) = \kappa'_e[\gamma(1 - \beta \mu)] ,$$  \hfill (32)

where $\gamma$ and $\beta = (1 - \gamma^{-2})^{1/2}$ are the Lorentz factor and the speed (in units of $c$) of the emitting shell material, and

$$\mu = \frac{z}{\sqrt{R^2 + z^2}}$$  \hfill (33)

is the cosine of the angle between the symmetry axis and the line passing through the origin and the emitting point.

With the emission and absorption coefficients $j_e$ and $\kappa_e$ determined as a function of axial distance $z$, one can write down the radiative transfer equation along lines parallel to the axis

$$\frac{dI_e(T, R, z)}{dz} = j_e(T, R, z) - \kappa_e(T, R, z)I_e(T, R, z) .$$  \hfill (34)

With the usual definition of optical depth $d\tau_e = -\kappa_e dz$, we find that the specific intensity at the surface facing the observer (where $\tau_e = 0$) is given by

$$I_e(T, R, \tau_e = 0) = \int_{0}^{\tau_{max, e}} S_e(T, R, \tau_e)e^{-\tau_e}d\tau_e ,$$  \hfill (35)

where

$$\tau_{max, e} = \int_{0}^{\infty} \kappa_e dz$$  \hfill (36)

is the total optical depth through the shell at an off-axis distance of $R$ while

$$S_e = \frac{j_e}{\kappa_e}$$  \hfill (37)

is the source function. Finally, we integrate over the surface of the shell to obtain the total flux at the time $T$ and the frequency $\nu$

$$F_e(T) = \frac{2\pi}{d^2} \int_{0}^{R_{max}} I_e(T, R, \tau_e = 0)R dR .$$  \hfill (38)

With this formalism, we explore numerically various combinations of parameters, trying to determine the best fits to the observed radio light curves.

Our procedure was to choose $\epsilon_b$ and $\epsilon_e$ and then to vary the energy and ambient density until the best fit to the data was obtained. For the same magnetic and electron energy factors $\epsilon_b = \epsilon_e = 0.1$, power index of electron-energy distribution $p = 2.5$, and source distance $d = 40$ Mpc as in § 3, we find that a total energy of $E_0 = 1.2 \times 10^{49}$ ergs and a value of $n_0 r_0^2 = 1.2 \times 10^{44}$ cm$^{-1}$ for the external $r^{-2}$ density medium fit the radio light curves before the second bumps (around day 24) reasonably well (see Fig. 7). The value for the external medium corresponds to a stellar wind mass-loss rate of $4.0 \times 10^{-7} (V_w/10^3 \text{ km s}^{-1}) M_\odot \text{ yr}^{-1}$. Therefore, both the total explosion energy and the wind mass-loss rate are not far from those inferred in § 3. In other words, the corrections caused by dynamical and relativistic effects are relatively modest: they are factors of 2 or less.

After about day 24, the observed fluxes increase dramatically at all four frequencies. These increases are incompatible with the predictions of a simple constant-energy blast-wave model, as evident in Figure 7. The predicted fluxes are too small by a factor of 2 to 3 at later times. A resolution of this discrepancy is suggested by Figure 4a, where a sudden increase of energy supply is implied. We therefore consider the simplest case where the total energy of the blast wave increases from its initial value of $E_0$ to a final value of $E_1$ instantaneously at a time $t_1$ (in the rest frame of the origin). After some experimentation, we find that the combination of $E_1 = 3.2 \times 10^{49}$ ergs and $t_1 = 100$ days fits the late-time light curves reasonably well (Fig. 7). The fitting is not perfect, nor is it expected to be, given the idealized nature of our model. In calculating the blast-wave
dynamics and the radio emission, we have assumed that the promptly injected energy is shared instantaneously by all material in the uniform shell, which leads to a jump in the velocity and other properties of the emitting shell (see Fig. 8). The transrelativistic nature of the emitting region is clear in Figure 8, with a speed $\beta = v/c$ between 0.6 and about 0.9 in the time interval of interest. It is interesting to note that the energy injection occurs about 100 days after the explosion in the rest frame of the origin. Relativistic effects make its presence felt much earlier in the light curves in the observer’s frame, around day 22. Also, the jump in energy is substantial, to a value that is $\sim 2.6$ times higher.

The inferred total energy and dynamics of the blast wave depend on the magnetic- and electron-energy factors. To illustrate the dependence, we consider the extreme case with a tiny magnetic-energy factor $e_b = 10^{-6}$ and a maximum possible electron-energy factor $e_e = 1$. These parameters are close to those used by Waxman & Loeb (1999) in their subrelativistic model for RSN 1998bw with a thermal electron distribution. For a power index of electron-energy distribution $p = 2.5$ and source distance $d = 40$ Mpc as before, we find a reasonable fit to the observed radio emission (see Fig. 9) for the following set of parameters: a total initial energy of $E_0 = 1.7 \times 10^{50}$ ergs; a stellar wind mass-loss rate of $6.2 \times 10^{-5} (V_{w}/10^3 \text{ km s}^{-1}) M_\odot \text{ yr}^{-1}$; and a jump to a total energy of $E_\text{f} = 4.5 \times 10^{50}$ ergs at a time $t_\text{f} = 50$ days in the rest frame of the origin. Note that the rising parts of all four model light curves at days 20–40 are nearly parallel to one another, in a better agreement with observations than the case with $e_b = e_e = 0.1$ considered earlier (see Fig. 7). The reason is that the shock speed, also shown in Figure 8, is significantly lower in the $e_b = 10^{-6}$ case than in the $e_b = 0.1$ case in the time interval of interest; a lower shock speed tends to steepen the rise in the radio fluxes at short, optically thin wavelengths, since a larger portion of the reenergized shell contributes to the total emission at the given time. Compared with the model of Waxman & Loeb (1999), which was primarily intended for the observations on day 12, our model has a higher shock velocity ($0.6c$ vs. $0.3c$) and a higher energy by a factor of a few. The differences may be related to the fact that our model has a decelerating shock wave, includes relativistic effects, and assumes a power-law electron distribution.

2 Strictly speaking, the choice $e_e = 1$ is not consistent with our blast-wave model, which assumes implicitly that most of the energy is carried by nucleons. It should be adequate for a rough parameter study, however.
These models show that models with a wide range of $\epsilon_b$ can approximately represent the data. The scaling between these models is discussed in § 5. However, we have found that acceptable models are even more widely distributed and can be found with $\epsilon_e = 0.01$. It appears that changes in $v_{m,t}$ and changes in the time-lag effects can cancel each other out. The energy and ambient density of these models remains in the range of those given above.

We have also investigated models in which the flux rise is caused by a jump in the ambient density instead of an energy increase. Although it is possible to obtain an increase in the extrapolated flux going to late times, it is not possible to obtain a fit during the transition period (20–40 days). In particular, the fluxes remain flat during this period instead of rising.

In the models with low $\epsilon_e$, so that radiative losses are less important, the predicted optical emission is comparable with that observed in day 1 of SN 1998bw (Galama et al. 1998a). It points to the intriguing possibility that the rapidly fading optical transient of GRB 980425 was detected on day 1. This possibility is diminished, however, in that the model does not fit the early (less complete) radio light curves, before about day 10.

5. POWER-LAW DYNAMICS

Many of the previous discussions of GRB afterglow light curves have assumed power-law evolution based on smooth properties of the ejecta and the surrounding medium (Mészáros, Rees, & Wijers 1998 and references therein). RSN 1998bw shows a more complex evolution, but our model indicates that over substantial periods of time the evolution is described by a constant energy shell in a $p \propto r^{-2}$ medium. During these times, the radio flux evolution is described by $F_r \propto t^{-1.6}$ at optically thin wavelengths. Mészáros et al. (1998) discuss the expected power-law evolution in a $p \propto r^{-2}$ medium for a relativistic flow and find that for $v > v_{m,t}$ and constant energy, $F_r \propto t^{1.5 - 3p/4}$. For $p = 2.5$, we have $F_r \propto t^{-1.62}$, in good agreement with the observations of RSN 1998bw. The observed time period of 12–24 days is especially useful because self-absorption and $v_{m,t}$ affect the evolution and provide diagnostic information. The optical depth at 1.4 GHz is moderately high and we expect $F_r \propto r^{-1} \propto t$, which is in agreement with the observations (Fig. 1).

The fact that a constant energy explosion in a $p \propto r^{-2}$ medium describes the $t = 12–24$ day evolution can be used to find scaling relations for the acceptable models. The model parameters are $E$, $A$ (where $p = Ar^{-2}$), $\epsilon_n$, and $\epsilon_e$. The parameter $p$ is simply determined by the observations. The observational constraints on these parameters can be described by $v_{m,t}$, $F_{m,t}$, and $v_A$, where $v_A$ is the frequency at which the spectrum becomes self-absorbed. The expected evolution of these quantities for a relativistic explosion in an $r^{-2}$ medium is (see Waxman 1997; Mészáros et al. 1998)

$$v_{m,t} \propto \epsilon_n^{2/3} \epsilon_b^{1/3} E^{1/3} t^{-3/2},$$  \hspace{1cm} (39)
$$F_{m,t} \propto \epsilon_n^{2/3} \epsilon_b^{1/3} A E^{1/3} t^{-1/2},$$  \hspace{1cm} (40)
$$v_A \propto \epsilon_n^{-1} \epsilon_b^{-1/3} A^{-1/3} E^{-2/3} t^{-3/5}.$$  \hspace{1cm} (41)

There are three constraints on the four model parameters, so there is no expectation of a unique model fit to the data. One way of describing the scaling of acceptable models is to fix $v_{m,t}$, $F_{m,t}$, and $v_A$ at some time $t$ and find how three of the model parameters depend on the fourth one. For example, we find

$$E \propto \epsilon_b^{-1/5}, \quad A \propto \epsilon_b^{-2/5}, \quad \epsilon_e \propto \epsilon_b^{-1/5}. \hspace{1cm} (42)$$

If one dynamical model with $\epsilon_b = 0.1$ and $\epsilon_e = 0.1$ fits the data, then another model with $\epsilon_b = 10^{-6}$, $\epsilon_e \sim 1$, $E$ up by 10, and $A$ up by 100 should fit the data equally well. This is in approximate accord with the results in § 4. In the opposite direction, increasing $\epsilon_e$ to equipartition with the postshock gas ($\epsilon_b = 0.5$) leads to moderately small decreases in the other parameters. Our detailed models, discussed in § 4, show that the range of acceptable parameters is broader. The emission is not from a single region, as is assumed in the power-law model, but is integrated over a shell with time lags. These conditions allow a greater variety of models.

For the flux evolution in the optically thin regime, equations (39) and (40) can be combined to yield

$$F_r \propto \epsilon_b^{p-1} \epsilon_b^{(p+1)/4} AE^{(p+1)/4} t^{(1 - 3p)/4}.$$  \hspace{1cm} (43)

The observed flux increase from before day 24 to after day 32 is a factor of 2.2 (see § 2). If this increase is because of an energy increase, equation (43) shows that $E \propto F^4(p+1)$, and so $E$ must increase by a factor of 2.5 for $p = 2.5$. This result agrees well with the factor of 2.6 found in the detailed models. The failure of a density increase to account for the flux rise can also be seen from the power-law dynamics. The flux at an optically thick wavelength is expected to increase as $r^2$. A boost in the shock energy accelerates the expansion rate and accelerates the flux increase. An increase in the ambient density decelerates the shock and the flux increase. The data at 1.4 GHz, an optically thick wavelength, show an accelerating rise through the time of the flux increase. The implication is that the increase is caused by an energy, not a surrounding density, increase, which is in accord with our detailed models.

An extrapolation of the constant $E$ evolution to day 10 and earlier yields fluxes at 8.6 GHz that are higher than those observed. Equation (41) shows that an attempt to model this solely as an energy jump yields a higher value of $v_A$ at early times. This would imply that the observed spectrum should be in the self-absorbed regime, or $F_r \propto v^2$. The observed spectrum is not compatible with this form, which shows that the early flux increase requires factors in addition to or other than an energy increase.

6. RSN 1998bw AND GRB 980425

Our models have implications for the nature of RSN 1998bw in the context of SN 1998bw and the possibly associated GRB 980425. Models for the light curve and some spectra of SN 1998bw indicate that the progenitor star was the heavy element core of a massive star with a final mass of 13.8 $M_\odot$ (Iwamoto et al. 1998) or 6 $M_\odot$ (Woosley et al. 1999). Such stars are observed as Wolf-Rayet stars, which have winds with velocities in the $V_w = 1000–2500$ km s$^{-1}$ range and mass-loss rates of $M = 10^{-5}$ to $10^{-4} M_\odot$ yr$^{-1}$ (Willis 1991). Our $\epsilon_b = 0.1$ model for RSN 1998bw implies $M/V_w \approx (4 \times 10^{-7} M_\odot$ yr$^{-1})/(1000$ km s$^{-1}$) and the $\epsilon_b = 10^{-6}$ model implies $M/V_w \approx (6 \times 10^{-5} M_\odot$ yr$^{-1})/(1000$ km s$^{-1}$). The density range in the models overlaps the circumstellar density expected for a Wolf-Rayet star; the high $\epsilon_b$ models yield a density that is somewhat too low, so there is a preference for the low $\epsilon_b$ models. The low $\epsilon_b$ models also appear to fit the rise in the radio light curves at days 20–40 better.
The blast-wave energies estimated during the observed period beginning on day 12 range from $1.2 \times 10^{49}$ ergs with $\beta = v/c \gtrsim 0.8$ for the $\epsilon_b = 0.1$ model and $1.7 \times 10^{49}$ ergs with $\beta \gtrsim 0.5$ for the $\epsilon_b = 10^{-6}$ model. Immediately after the impulsive energy increase, the energies are $3.2 \times 10^{49}$ ergs and $4.5 \times 10^{49}$ ergs with $\beta \approx 0.85$ and $\beta \approx 0.6$ for the high and low $\epsilon_b$ models, respectively. These energies can be compared with those expected in the shock-accelerated high-velocity ejecta of SN 1998bw. There is the possibility that the high radio luminosity is simply because of the higher explosion energy than that normally found in Type Ib/c supernovae. Woosley et al. (1999) estimate that their spherically symmetric models in describing normal GRBs is nonrelativistic, Matzner & McKee (1999) estimate the relativistic mass ejection that can be expected from an explosion like SN 1998bw. They note that a stripped massive star may not be sufficiently compact to produce any relativistic ejecta. A $3 \times 10^{52}$ ergs explosion with $6 M_\odot$ of ejecta in a sufficiently compact star should yield~$\sim(1-2) \times 10^{48}$ ergs of relativistic ejecta. The shock-acceleration process yields a mass (and energy) that declines steeply with velocity, so the results are sensitive to the details of the model. The energy inferred for RSN 1998bw appears to be too high for the shock-accelerated ejecta of SN 1998bw in spherical models, but this result is not conclusive.

Our model also places constraints on the time dependence of the energy input to the blast wave. For $t > 12$ days, we find that the blast wave evolves with approximate constant energy except for an episode during which the energy is increased by a factor of $\sim 2.5$. The change does not appear to be caused by the circumstellar medium and is probably because of inner material that catches up with the interaction region. This type of evolution is not expected for energy injection from shock-accelerated, high-velocity supernova ejecta, which presumably have a smooth distribution. This property, along with the high energy in RSN 1998bw, suggests the presence of a central engine in the explosion. That RSN 1998bw was accompanied by a Type Ic supernova with $\sim 10 M_\odot$ of material moving at moderate velocity (Galama et al. 1998a; Iwamoto et al. 1998) implies that the high-velocity outflow giving rise to RSN 1998bw is not spherically symmetric but occupies only a fraction of the total 4$\pi$ solid angle if it originated in a central engine. MacFadyen & Wooley (1999) have proposed a model for SN 1998bw in which collimated, high-energy flows are created by a disk around a central black hole. The success of spherically symmetric models in describing normal GRBs is usually attributed to relativistic beaming; only a small part of a possibly asymmetric source is being observed. This argument does not apply to our model for RSN 1998bw because the velocities are only mildly relativistic; thus, it is surprising that a spherically symmetric model is so successful. The introduction of asymmetries in the models brings in new parameters, and we have not investigated such models.

The type of evolution that is observed in RSN 1998bw may be related to that observed in the optical afterglow of GRB 970508. In that case, there is evidence for a steady or decreasing flux before the 1.5 day rise phase leading to the maximum (Pedersen et al. 1998). An extrapolation of the early evolution implies that the rise is by a factor of 4. Mészáros, Rees, & Wijers (1998) suggest that the rise is caused by a multicomponent flow or by a lower $\Gamma$ shell catching up with the main shock front. Both of these models have been calculated in more detail by Panaitescu, Mészáros, & Rees (1998). In their model with late impulsive energy input, the energy increases from $0.6 \times 10^{52}$ to $2.4 \times 10^{52}$ ergs. Although the energies are larger than those inferred in RSN 1998bw, the factor increase in energy is comparable to the one that we have advocated for RSN 1998bw. A multicomponent flow model does not appear plausible for RSN 1998bw because of the continuity of the radio spectral indices during the time of the flux increase.

Having found that RSN 1998bw has a central engine related to those found in normal GRBs strengthens the association with GRB 980425 and suggests that the relation of the radio emission to the early $\gamma$-ray emission be investigated. However, there are several reasons why an energy-conserving synchrotron model cannot be extrapolated back simply to the very early times (see Iwamoto 1999). First, we have found evidence in the radio light curves for at least one episode of impulsive energy injection. Additional episodes also could have occurred before day 3 when the radio observations were initiated. The result would be lower flux values at early times. Second, synchrotron losses become more important at early times. The frequency at which the synchrotron loss time equals the age is $v_L \propto t^{1/2}$ for constant energy evolution in an $r^{-2}$ medium. The peak frequency evolves as $v_m \propto t^{-3/2}$, so that the blast wave was radiative in the past. Taking $v_m = 5.6$ GHz on day 10 implies radiative evolution before $t \approx 0.1$ day in our high $\epsilon_b$ model. Cooling is less important with low $\epsilon_b$. Third, the unusual behavior observed between days 3 and 12 may indicate that it may not be possible to extrapolate our model to small radii. Finally, the early evolution may be affected by the character of the initial injection of energy. If the shell has an initial Lorentz factor $\Gamma_i$, its mass is $M = E_i/\Gamma_i c^2$, where $E_i$ is the initial energy. The deceleration radius, $r_{dec}$, occurs when the shell has swept up a mass $\Gamma_i M$ in the circumburst environment. Thus

$$r_{dec} = \frac{E_i}{\Gamma_i c^2 (M/V_w)} = 2 \times 10^{12} E_{ig} \Gamma_{i1}^2 \left( \frac{M/V_w}{6.5 \times 10^{12} \text{ g cm}^{-1}} \right)^{-1} \text{ cm,}$$

where $E_{ig} = E_i/(10^{48}$ ergs), $\Gamma_{i1} = \Gamma_i/10$, and the reference value of $M/V_w$ corresponds to $M/V_w = (1 \times 10^{-5} M_\odot \text{ yr}^{-1})/(1000 \text{ km s}^{-1})$. The energy $E_i = 10^{48}$ ergs is a lower limit based on the radiated energy in $\gamma$-rays. The shell radius at $t = 10$ s is $r \approx 2 \Gamma_i^2 c t = 6 \times 10^{13} \Gamma_i^2$ cm. If $\Gamma_i$ starts at the lower end of the plausible range, or if $E_i$ is high, the burst observations may be during the initial deceleration period. Other possible factors are electron-scattering opacity and $\gamma\gamma$ opacity for pair production for the GRB. If the electron-scattering opacity is 0.4 cm$^2$ g$^{-1}$, as expected for a baryon-dominated plasma, electron scattering is not important. For the relatively low-energy burst considered here, $\gamma\gamma$ opacity is not important (see Piran 1996).

7. CONCLUSIONS

We have modeled the radio emission from SN 1998bw as the synchrotron emission from a rapidly expanding shock wave. We assumed that the electron-energy and the magnetic-energy densities are constant fractions of the post-shock energy density, as is commonly done in models of both radio supernovae and GRB afterglows. The result of
our modeling is that the radio evolution is consistent with expansion into a $\rho \propto r^{-2}$ circumstellar wind. The estimated wind density is consistent with that expected around the progenitor star that gave rise to the optical SN 1998bw and with the type of environment commonly deduced to exist around radio supernovae. However, the radio light curves show a period of flux increase that appears to require an episode of impulsive energy injection. The evolution at other times is consistent with constant blast-wave energy. We speculate that the freely expanding expansion ejecta are nonuniform and that a shell caught up with and energized the blast wave.

Katz (1999) has recently proposed a different model for RSN 1998bw in which the power for the source is provided by radioactivity. The $\gamma$-rays lines from radioactivity Compton-scatter electrons that then move out in a mildly relativistic flow. Possible problems for the model are that the $^{56}\text{Ni}$ produced in the explosion must be mixed to the surface of the debris and that the origin of the energetic electrons needed for the synchrotron emission is unclear.

The association of SN 1998bw with GRB 980425 appears likely based not only on the spatial and temporal coincidence of the events (Galama et al. 1998a) but also on the unusually high energies of the optical and radio supernova phenomena. The finding that the ejecta are nonuniform gives additional evidence for a relation between the two events because the ejecta in normal radio supernovae are inferred to be approximately smoothly distributed, as expected for shock-accelerated ejecta. The optical light curve of the GRB 970508 afterglow may indicate discrete structure in the ejecta; in other GRB cases, the evolution can be described by a constant-energy model as is also inferred for substantial parts of the evolution of RSN 1998bw. Although the environment of SN 1998bw is similar to that inferred around normal radio supernovae, the explosion event shows a link to the more energetic GRBs. Woosley et al. (1999) and MacFadyen & Woosley (1999) have also linked SN 1998bw to the GRBs based on the high energy of the event. The outflow may be powered by neutrino-antineutrino annihilations soon after the formation of a central black hole or by rotational energy extracted electromagnetically from a Kerr black hole or its nearby accretion disk (MacFadyen & Woosley 1999; Paczyński 1998).

Cosmological GRBs with well-observed afterglows, including GRB 970228 (Wijers et al. 1997) and GRB 970508 (Waxman 1997; Granot, Piran, & Sari 1999), have been inferred to be expanding into a medium with a low, constant density ($n_0 \approx 1 \text{ cm}^{-3}$). Power-law declines are observed but with a time dependence $F_\gamma \propto t^{-w}$ with $w = 1.10-1.20$. The steeper decline observed in RSN 1998bw ($w = 1.6$) is an important part of the evidence for expansion in a $\rho \propto r^{-2}$ medium. The steeper decline is also expected for the optical and X-ray afterglows, which can account for the unusual lack of an X-ray afterglow if SN 1998bw is associated with GRB 980425. The expected early radiative losses are another factor (Bloom et al. 1998).

Bloom et al. (1998) have discussed the notion that the SN 1998bw/GRB 980425 event is a prototype of a new class of GRB associated with supernovae (S-GRB). One of the attractive features of the association was that the single-pulse nature of GRB 980425 could be attributed to the shock wave generated by the smooth, shock-accelerated ejecta, which are not expected to contain internal shocks. In our model, the radio emission is driven by ejecta with considerable structure so that the association with a single-pulse GRB may not be significant. However, our work has brought out another suggestive distinction between the S-GRB and normal GRBs. RSN 1998bw shows evidence for interaction with a circumstellar wind, as expected for its massive star progenitor. In the cases where there are sufficient data to make a determination, the observations of normal GRBs are consistent with interaction with a medium with a low, constant density. The situation is not expected in the immediate vicinity of a massive star and may point to a different kind of progenitor object in these cases.

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