Analysis and assessment on the catastrophe progression evaluation method of slope safety

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Abstract: Comprehensive assessment of slope safety based on catastrophe theory has been studied and improved in recent years, because of its advantage of simplicity and objective. However, as the evaluation index of slope safety, the result of catastrophe progression has problems on the accuracy and the threshold, and there is no solution available. So the ROC curve (receiver operating characteristic curve), which is commonly used in medical, has been used in the slope engineering for the first time. The ROC curve can intuitively reflect the accuracy of the evaluation method and calculate the threshold of catastrophe progression. Thus, the concept of degree of safety comes forward, which can quantify the safety of slope. The study results of engineering cases show that this method of results evaluation using ROC curve is simple and intuitive, and it plays a good role on the examination of evaluation method. Besides, it can be used as a comparative analysis of different evaluation methods, which provides a new idea for the assessment of evaluation methods of slope safety.

1. Introduction
In recent years, the slope safety has become a key issue in slope engineering and even the entire geotechnical engineering [1, 2]. The safety factor method can obtain the quantitative evaluation results of slope safety [3], but the calculation is excessively fussy. Li and Liu used the fuzzy comprehensive evaluation to analyze the safety and stability of slope. But the method needs to determine the weight of each control index artificially, which has a strong subjectivity, resulting in the evaluation result is not objective and accurate. In this situation, the comprehensive assessment of slope safety based on catastrophe theory came into being. Catastrophe theory has its advantages in Multi-objective Decision, but there is no test method to evaluate the accuracy of the final evaluation results of catastrophe theory, also no method to determine the safety grade or threshold of slope corresponding to catastrophe progression.

In this paper, we introduce the concept of receiver operating characteristic curve (ROC), which is commonly used in medical, has been used in the slope engineering for the first time. The ROC curve is used to analyze the accuracy of the catastrophe progression obtained by the catastrophe progression evaluation method for slope safety, and the threshold of the catastrophe progression corresponding to the slope safety is calculated, so as to further put forward the concept of safety degree and quantify the slope safety.
2. Catastrophe progression evaluation method of slope safety

2.2 The fundamental principles of catastrophe theory
Catastrophe theory classifies the critical points of a system according to its potential function, and induces the elementary catastrophe model according to the characteristics of discontinuous variation near each critical point. Then, by solving the first and second derivatives of the potential function $f(x)$ of the catastrophe model, the two equations are solved simultaneously, and the bifurcation point set equation of the catastrophe system is obtained. The equation indicates that when the control variable satisfies the equation, the system will mutate. Rene Thom summarized seven mutation models, the most common of which are cusp mutation model, swallow tail mutation model and butterfly mutation model. According to the potential function of each specific model, the bifurcation set equation can be obtained, and the normalization formula can be obtained. Finally, the normalization formula can be used for comprehensive evaluation[4].

2.2 Comprehensive evaluation of slope safety based on Catastrophe Theory
Because the specific engineering geological conditions of slope project are different and the monitoring methods used are different, it is difficult to find a uniform and complete comprehensive evaluation system for slope safety. Therefore, Liang et.al[5] took into account most factors affecting the slope safety and adopted six control indexes in three categories. Among them, strength index: cohesion $c$, internal friction angle $\phi$; geometric parameter index: slope angle $\phi$, slope height $H$; external load index: severity $\gamma$, pore water pressure ratio $\gamma$. Based on this, an evaluation system was established, through which 25 slope samples are comprehensively evaluated for slope safety, and finally the catastrophe progression of each slope is obtained. However, according to the result of the final abrupt progression, the so-called “excellent” degree of slope safety can only be judged based on the principle of “bigger is better”, but the slopes safety cannot be accurately judged. Therefore it is necessary to find a method which can test the accuracy of the catastrophe progression evaluation method and obtain a threshold of slope safety.

3. ROC Curve Evaluation Method of Evaluation Results
3.1 Introduction of ROC curve analysis
ROC curve which was originally used to evaluate radar performance, and late has been applied in the field of meteorology, material inspection, psychophysics and so on. Lusted first applied ROC analysis to medical diagnosis, and then with a lot of research and development, ROC analysis is now widely used in medical diagnosis. The ROC curve is a curve drawn based on a series of different two classification methods, with the true positive rate (sensitivity) as the ordinate and the false positive rate (1-specificity) as the abscissa. Among them, the closer the curve is to the upper left corner of the coordinate, the better diagnostic is, that is, the area under the curve (AUC) represents the accuracy of the judgment. Generally speaking, its classification is shown in Table 1.

| Scope of AUC | Effect   |
|--------------|----------|
| 0.5~0.6      | poor     |
| 0.6~0.8      | common   |
| 0.8~1.0      | good     |

In this paper, it is considered that the evaluation of slope safety is also a problem to be "diagnosed", and its evaluation index is equivalent to the diagnosis index of disease.

However, the evaluation result based on the catastrophe theory -- the catastrophe progression as an optimized "diagnostic" index to judge whether the slope is safe or not has not been solved yet, and the determination of the threshold, which is a problem that can be solved by ROC curve. Therefore, the ROC curve is introduced, and the slope samples and their mutation Series in [5] are quoted to analyze and evaluate the evaluation results.
3.2 ROC analysis of slope safety evaluation results

Based on catastrophe theory, comprehensive evaluation of slope safety has the advantages of quick and easy. This paper uses ROC curve to evaluate and analyze the final result. Here, the slope safety calculation samples in[5] are still used for analysis, and the specific situation is shown in Table 2.

| No. | Slope | Safety factor | actual safety | catastrophe progression | No. | Slope | Safety factor | actual safety | catastrophe progression |
|-----|-------|--------------|---------------|-------------------------|-----|-------|--------------|---------------|-------------------------|
| 1   | 0.86  | 0            | 0.672         | 14                      | 1.24| 1     | 0.663        |               |                         |
| 2   | 1.16  | 0            | 0.692         | 15                      | 1.35| 0     | 0.772        |               |                         |
| 3   | 1.3   | 1            | 0.812         | 16                      | 1.03| 0     | 0.702        |               |                         |
| 4   | 1.249 | 1            | 0.748         | 17                      | 1.28| 0     | 0.801        |               |                         |
| 5   | 1.01  | 0            | 0.702         | 18                      | 1.63| 1     | 0.8          |               |                         |
| 6   | 0.625 | 0            | 0.512         | 19                      | 1.71| 1     | 0.837        |               |                         |
| 7   | 1.12  | 0            | 0.711         | 20                      | 1.49| 1     | 0.802        |               |                         |
| 8   | 1.2   | 0            | 0.732         | 21                      | 1.45| 1     | 0.798        |               |                         |
| 9   | 1.37  | 1            | 0.822         | 22                      | 1.62| 1     | 0.814        |               |                         |
| 10  | 2.05  | 1            | 0.841         | 23                      | 1.2 | 0     | 0.678        |               |                         |
| 11  | 1.15  | 1            | 0.703         | 24                      | 1.09| 0     | 0.675        |               |                         |
| 12  | 1.44  | 1            | 0.832         | 25                      | 1.11| 0     | 0.777        |               |                         |
| 13  | 1.27  | 1            | 0.654         |                         |     |       |              |               |                         |

It should be noted that the actual safety of the slope here "0" represents an unstable slope, and "1" represents a stable slope. Using the SPSS software, we can get the ROC curve shown in Figure 1 and Table 3.

![Fig.1 ROC curve of catastrophe progression and safety factor](image)

Table 3 Related dates of test variable

| Variable          | AUC   | STD   | P Value | 95% confidence interval | Lower limit | upper limit |
|-------------------|-------|-------|---------|-------------------------|-------------|-------------|
| catastrophe progressions | 0.795 | 0.097 | 0.012   | 0.604                   | 0.986       |             |
| Safety factor     | 0.923 | 0.052 | 0.000   | 0.822                   | 1.025       |             |

From Figure 1 combined with Table 3, it can be seen that the AUC of the catastrophe progression is
From Table 1, it can be proved that the catastrophe progression method, as a method of evaluating slope safety, has a better judgment effect. For comparison, the safety factor is added here as a reliable indicator for evaluating the slope safety. The ROC curve is obviously closer to the upper left corner of the coordinate than the catastrophe progression, and the AUC also reaches 0.923, which also proves that when the safety factor can be calculated more accurately, it has a more accurate discrimination effect than the catastrophe progression. Considering that the catastrophe progression method is more convenient and fast in the evaluation of slope safety, it has greater advantages from the perspective of economy and engineering. Therefore, it can achieve such a discriminant effect, which shows that the catastrophe progression method can be used to evaluate the slope safety.

Another important function of the ROC curve is to ascertain the threshold value. From the ROC curve, we can get the variable coordinate points as shown in Table 4 (only some coordinate points are intercepted in the table).

| Variable                | Threshold(P) | Sensitivity | 1-Specificity(R) |
|------------------------|--------------|-------------|------------------|
| catastrophe progression| 0.76         | 0.692       | 0.25             |
|                        | 0.7745       | 0.692       | 0.167            |
|                        | 0.7875       | 0.692       | 0.083            |
|                        | 0.799        | 0.615       | 0.083            |
|                        | 0.8005       | 0.538       | 0.083            |
|                        | 0.8015       | 0.538       | 0            |

The false positive is $R$, the corresponding threshold value is $P$, which means that when the accuracy is $(1 - R) \times 100\%$, the threshold value corresponding to the catastrophe progression can be calculated according to the line interpolation method:

$$\frac{R_1 - R_2}{R_1 - R_3} = \frac{P_1 - P_2}{P_1 - P_3}$$  \hspace{1cm} (1)

Specifically, $R_1$ is the set false positive, $P_1$ is the corresponding threshold value, $R_2$ is the false positive rate just less than $R_1$, $R_3$ is the false positive rate just greater than $R_1$, $P_1$ and $P_2$ the corresponding threshold values of $R_1$ and $R_2$ respectively. In this paper $R_1 = 0.1$, that is, when the discrimination accuracy is required to reach 90%, the relevant data in Table 4 can be used to calculate the corresponding catastrophe progression threshold value according to Eq.(1): $P_1 = 0.7849$. That is to say when the catastrophe progression is $\geq 0.7849$, the slope is judged as stable, and the accuracy is 90%. In other words, the safety of the slope reaches 90%. At the same time, it can also be found that when the accuracy of discrimination reaches 100%, the threshold value of catastrophe progression indicates that the slope with catastrophe progression $\geq 0.8015$ can be determined as stable slope, and its safety reaches 100%, which can also be verified from the results in Table 2. The safety shown here is 90% or 100%, which can be understood as the corresponding safety degree of the slope in a certain catastrophe series. Therefore, the concept of safety degree is proposed in this paper: the corresponding safety degree of the slope catastrophe series. It is not difficult to understand that safety is actually another form of "safety factor".

The ROC curve analysis shows that the catastrophe progression evaluation method has more accurate discriminant ability for slope safety, and can calculate the threshold value of catastrophe progression corresponding to a given degree of safety (i.e., safety), on the other hand, it can also calculate the corresponding degree of safety of a catastrophe progression, which is a further step for the quantitative analysis of slope safety.
4. Conclusion

(1) This paper attempts to apply ROC curve analysis to slope engineering for the first time. The analysis shows that ROC analysis is suitable for the evaluation of slope safety. It has a strong intuition for the inspection and analysis of evaluation methods, and can be used for the comparative analysis of different evaluation methods. Therefore, we can look forward to applying this method to other slope projects and even the entire geotechnical engineering, which is worthy of further study.

(2) As a new method of slope safety evaluation, catastrophe progression method is simpler than the complicated calculation of safety factor, and more objective than fuzzy comprehensive evaluation method. Finally, the catastrophe progression is used as the judgment result, which is verified by ROC analysis in this paper. It shows that it has high accuracy, and can calculate the corresponding threshold value of slope safety degree and the corresponding safety degree of each catastrophe progression. It is proved that the catastrophe progression method is worthy of further study and promotion.

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