Holographic Principle and Topology Change in String Theory

Soo-Jong Rey

Physics Department, Seoul National University, Seoul 151-742 KOREA

abstract

D-instantons of Type IIB string theory are Ramond-Ramond counterpart of Giddings-Strominger wormholes connecting two asymptotic regions of spacetime. Such wormholes, according to Coleman, might lead to spacetime topology change, third-quantized baby universes and probabilistic determination of fundamental coupling parameters. Utilizing holographic correspondence between $AdS_5 \times M_5$ Type IIB supergravity and $d = 4$ super Yang-Mills theory, we point out that topology change and sum over topologies not only take place in string theory but also are required for consistency with holography. Nevertheless, we argue that the effects of D-instanton wormholes remain completely deterministic, in sharp contrast to Coleman’s proposal.

Keywords: Holography, Topology Change, String Theory

---

1 Work supported in part by KOSEF Interdisciplinary Research Grant 98-07-02-07-01-5, Ministry of Education Grant 98-015-D00054, and The Korea Foundation for Advanced Studies Faculty Fellowship.
One of the most vexing problems in quantum gravity has been the issue of spacetime topology change: whether topology of spacetime can fluctuate and, if so, whether spacetime of different topologies should be summed over. Any positive answer to this question would bear profound implications to the fundamental interactions of Nature. For example, in Coleman’s scenario [1] of baby universes [2], fluctuation of spacetime topology induce third quantization of the Universe and effective lon-local interactions thereof [1]. As a result, the cosmological constant is no longer a calculable, deterministic parameter but is turned into a quantity of probabilistic distribution sharply peaked around zero. Likewise, all other physical parameters are renormalized into quantities of probabilistic distribution [1]. While the original Coleman’s scenario was based on a Universe with positive cosmological constant, the topological fluctuation ought to persist to Universes with zero or negative cosmological constant. For instance, within saddle point approximation of Euclidean quantum gravity, Carlip [7] has argued that topology of the spacetime with negative cosmological constant is dominated by those with extremely complicated fundamental groups only yet having exponentially growing density of topologies.

String theory is the only known consistent theory of quantum gravity. As such, it would be desirable to address the issue of spacetime topology change within string theory and draw some definitive answers. The questions one would like to understand are: does spacetime topology change? what is the rule of summing over topologies? do topological fluctuation lead to third quantization of the Universe? are fundamental physical parameters distributed probabilistically rather than deterministic quantities?

In this paper, utilizing holographic principle [8, 9] that underlies string theory and consequent correspondence between anti-de Sitter string vacua and boundary conformal field theories [10, 11], we argue that some answers (albeit being only qualitative and technically limited) to the above questions can be obtained. In doing so, for the sake of concrete setup, we will restrict the entire foregoing discussions to semi-classical limit of Type IIB string theory on $AdS_5 \times S_5$. We will find that the holographic correspondence of the $AdS_5$ Type IIB string vacuum to large $N$ limit of $\mathcal{N} = 4$ super Yang-Mills theory [10] implies that (1) topology change does occur, (2) different topologies should be summed over, yet (3) only ‘tree-level’ processes of third quantization take place, and (4) no probabilistic distribution of physical coupling parameters is induced.

The argument is extremely simple. Among the quantum solitons in Type IIB string theory is the D-instanton [12] carrying Ramond-Ramond axion charge. As observed in [12] already, the D-instantons are nothing but Ramond-Ramond counterpart of wormholes (Euclidean gravitational

---

2 Prior to Coleman’s proposal, topology fluctuation and possible loss of quantum coherence have been raised by Hawking [3], Lavrelashvili, Rubakov and Tinyakov [4], and Giddings and Strominger [5].

3 Similar conclusions have been drawn by Giddings and Strominger [6].
instantons with axion charge) considered by Giddings and Strominger [13] previously. In string frame, these D-instantons are Einstein-Rosen bridges connecting two asymptotic Euclidean spacetime, through which dilaton and RR axion field vary monotonically.

Consider a D-instanton in $AdS_5$ Type IIB string background. The configuration has been studied recently [14, 15] and we repeat some aspects needed for foregoing discussions. After Wick rotation to Euclidean spacetime, covariant Type IIB equations of motion read

\[
\begin{align*}
R_{MN} - \frac{1}{3!}(F_5^2)_{MN} &= \frac{1}{2} \left( \partial_M \phi \partial_N \phi - e^{2\phi} \partial_M a \partial_N a \right) \\
\nabla^M \left(e^{2\phi} \nabla_M a \right) &= 0 \\
\n\nabla^2 \phi + e^{2\phi} \left( \partial a \right)^2 &= 0.
\end{align*}
\]

Here, $F_5$ denote self-dual 5-form field strength and $\phi, a$ are Type IIB dilaton and (Wick rotated) Ramond-Ramond axion fields. The complex field $\tau = (a/2\pi) + ie^{-\phi}$ transforms as a doublet under Type IIB $SL(2, \mathbb{Z})$ duality. The $AdS_5 \times S_5$ corresponds to the Type IIB background with flux of 5-form field strength

\[
\int S_5 F_5 = N
\]

while keeping the dilaton and the Ramond-Ramond axion fields constant:

\[
ds_{AdS_5 \times S^5}^2 = \alpha' \left[ \frac{U^2}{R^2} d\mathbf{x}^2 + \frac{R^2}{U^2} dU^2 + R^2 d\Omega_5^2 \right], \quad (R^2 = \sqrt{4\pi g_{st} N \alpha'})
\]

\[
F_5 = d \left( \frac{R^4 U^4}{4} \right) \wedge dy^0 \wedge \cdots \wedge dy^3.
\]

Clearly, contribution of the D-instanton to the energy-momentum tensor vanishes identically and hence the spacetime geometry in Einstein frame remains $AdS_5 \times S_5$. The D-instanton configuration is then specified uniquely by a harmonic function $H_{-1}$, which solves the Laplace equation on $AdS_5 \times S^5$. For homogeneous configuration on $S^5$, the solution is given by [14]:

\[
H_{-1}(U, \mathbf{x}; U_0, \mathbf{x}_0) = \left( e^\phi - e^{\phi_0} \right) = \frac{3N_{-1}}{\pi^4} \alpha'^4 \left( U_0^4 U_0^4 \right) \frac{U_0^4}{\left( (U^{-1} - U_0^{-1})^2 + (\mathbf{x} - \mathbf{x}_0)^2 \right)^4}.
\]

To be more precise, there is a slight distinction between the Gidding-Strominger wormholes and D-instantons. The wormholes considered by Giddings and Strominger are necessarily non-extremal NS-instantons whereas the D-instantons we will consider throughout this paper are extremal.
The instanton is centered at \((U_0, x_0)\) in \(AdS_5\). Consequently, the dilaton and the axion fields are no longer constant but varies over the spacetime. Hence, it is more natural to describe the spacetime geometry in the string frame:

\[
ds_{\text{string}}^2 = \alpha^\prime \sqrt{H_{-1}} \left[ \frac{U^2}{R^2} dx^2 + \frac{R^2}{U^2} dU^2 + R^2 d\Omega_5^2 \right].
\]

(5)

Because of the harmonic function factor in front, it is straightforward to see that the spacetime in string frame takes precisely the (Euclidean form of) Einstein-Rosen bridge connecting two asymptotic \(AdS_5 \times S_5\) regions at \(U \sim 0, \infty\). The D-instanton carries RR-axion charge \(N_{-1}\), which is quantized according to Dirac-Schwinger condition in units of inverse charges of Type IIB D7-branes.

We have seen that, in many respects, the D-instantons in \(AdS_5 \times S_5\) background behaves as Giddings-Strominger type Euclidean wormholes. Being so, one might consider semi-classical limit of Type IIB string theory in \(AdS_5\) background and ask whether Coleman’s scenario of baby universes can be realized. Suppose we place dilute gas of D-instantons on the \(AdS_5\) background (which will be referred as ‘base universe’). The effects of D-instanton wormholes are then computed in the standard way. The Type IIB supergravity fields are written as background fields, slowly varying on the string scale, and the D-instanton wormholes are placed in them. The integration over all possible D-instantons would then produce an effective interaction of the background Type IIB supergravity fields on the base \(AdS_5\) universe.

There are several possible combinatorics of D-instanton wormhole configurations. One possible configuration is to branch off a baby \(AdS_5\) universe from the base \(AdS_5\) one. Another possibility is that, if at all allowed, the two ends of D-instanton wormhole are attached both to the base \(AdS_5\) universe. From the third quantization point of view, the first possibility corresponds to a tree-level process, while the second one is a loop-process.

Taking into account of effective interactions produced by both combinatorics of D-instanton configurations, the resulting low-energy Type IIB supergravity action on the base \(AdS_5\) universe takes the form:

\[
S_{\text{eff}} = S_0 + \sum_I \int_{AdS_5 \times S_5} d^5x C_I \mathcal{O}_I(x) + \sum_{I,J} \int \int_{AdS_5 \times S_5} d^5x d^5y \ C_{IJ} \mathcal{O}_I(x) \mathcal{O}_J(y) + \ldots .
\]

(6)

Here, \(\mathcal{O}_I\) denote the supergravity local operators and \(C_I, C_{IJ}\) are numerical coefficients that encode the D-instanton wormhole combinatorics: the \(C_I\)’s are due to tree-level processes and the \(C_{IJ}\)’s are due to loop-level processes respectively.

As in the Coleman’s scenario of baby universes, an important point is that the coefficients \(C_{IJ}\)’s are independent of locations of operator insertion on \(AdS_5\). Being induced by the second

\footnote{D-instanton configuration centered at generic point in \(AdS_5\) is found by \[19\] (See also \[14\]). It is also important to keep the asymptotic values of the dilaton and the Ramond-Ramond axion fields to those of gauge coupling and \(\theta\)-parameter of the super Yang-Mills theory at the boundary.}
possibility mentioned above (i.e. loop-level processes), this class of effective interactions would then give rise to non-local effects to the low-energy Type IIB supergravity. Should the non-local, loop-level processes are permitted, then physics of semi-classical Type IIB string theory is described by a probabilistic distribution of physical coupling parameters. Introducing coherent state parameter $\alpha_I$'s, one can re-express

$$\exp \left( -S_{\text{eff}} \right) = \langle \exp (-S_{\text{IIB}}[\alpha]) \rangle_{\alpha},$$

where

$$\langle \hat{A} \rangle_{\alpha} \equiv \int \prod_I d\alpha_I \hat{A} \exp \left( -\frac{1}{2}K_{IJ}\alpha_I\alpha_J \right), \quad (K_{IJ}C_{JK} = \delta_{IK}),$$

and

$$S_{\text{IIB}}[\alpha] = S_0 + \sum_I (C_I + \alpha_I) \int_{\text{AdS}_5 \times S_5} d^5x \mathcal{O}(x).$$

In fact, from the third quantization point of view, integration over the coherent state $\alpha$-parameters corresponds to taking into account of loop processes. In the resulting low-energy supergravity action Eq.(9), the second term clearly indicates that physical coupling parameters are not deterministic but take probabilistic distributions with respect to the third-quantization $\alpha_I$-parameters. While string theory itself does not have any adjustable parameters, third quantization of the universe ought to lead effectively to infinitely many parameters that would only be determined by probability distribution. To many, this is quite an alarming allegation (See, for example, [16]).

We are thus led to ask back the question: are such non-local, loop-processes indeed possible in string theory? We now would like to argue that holographic principle does not permit them at all.

Recently, Banks and Green [17] and Witten [18] have identified that the D-instantons located in the $\text{AdS}_5$ space as, once holographically projected to the boundary, BPS instantons of the $d = 4, \mathcal{N} = 4$ super Yang-Mills theory. Being conformally invariant, the super Yang-Mills theory does not pose any strong infrared fluctuations. As such, at all energy scales, the Yang-Mills instantons are well defined BPS configurations with quantized topological charges. The coordinates of D-instantons on $\text{AdS}_5$ are holographically mapped to collective coordinates to the position and the size of the Yang-Mills instanton at the boundary. Integration over the position $d^5x$ of a D-instanton in the bulk of $\text{AdS}_5$ matches precisely to the conformally invariant collective coordinate measure $U_0^5 dU_0^{-1} d^4x_0$ of the Yang-Mills instanton (See Eq.(4)). Furthermore, as has been shown explicitly [17, 19], the tree-level branching-off processes of dilute

\footnote{which we have tacitly assumed to be so in deriving Eq.(8).}

\footnote{Collective coordinates associated with the global gauge degrees of freedom are expected to get lifted at strong 't Hooft coupling, as $SU_R(4)$ R-symmetry is non-anomalous and index theorem does not protect these zero modes from being lifted. See the discussion of [17].}
D-instantons yielding $\mathcal{R}^4$ interactions (and others related by supersymmetry, for example, sixteen fermion interactions) is precisely mapped to the Yang-Mills instanton induced correlators at the boundary.

Using the above holographic relation between D-instantons on $AdS_5$ and Yang-Mills instantons at the boundary and our knowledge of instanton physics in conformally invariant super Yang-Mills theory, we will now be able to argue that non-local, third quantization loop processes are impossible. Suppose, for the moment, that such processes are present. According to the holographic principle [8, 9], we should identify the effective Type IIB supergravity on the bulk of $AdS_5$ with the generating functional of boundary conformal field theory. Generic supergravity fields $\psi(U,x)$ are completely fixed by their boundary values $\psi_\infty(x)$. The nature of this Dirichlet problem should equally work even if the wormhole-induced effective interactions Eq.(8) are considered. Hence, the relation between bulk and boundary correlators would now be generalized to:

$$\exp^{-S_{\text{eff}}} = \langle \exp^{-S_{\text{in}}[\psi;\alpha]} \rangle_\alpha = \langle \exp^{-\int d^4y \psi_\infty \hat{O}_{\text{CFT}}} \rangle_{\text{instanton}}.$$

On the right-hand side, the correlators in $\mathcal{N} = 4$ super Yang-Mills theory are calculated in the background of Yang-Mills instantons.

The correspondence Eq.(10) poses severe consistency problem. Consider correlators of boundary conformal field theory, viz. $\mathcal{N} = 4$ super Yang-Mills theory. Calculated from the right-hand side of Eq.(10), super Yang-Mills correlators in the instanton background can be evaluated unambiguously from the standard instanton physics [17, 19] and hence are completely deterministic. On the other hand, the correlators calculated from the supergravity side do depend explicitly on the third quantization $\alpha$-parameters and thus take probabilistic distribution. As there simply is no room for probabilistic $\alpha$-parameters in the $\mathcal{N} = 4$ super Yang-Mills theory, should the holographic principle remain valid, the $\alpha$-parameter dependence somehow out to be suppressed in the effective Type IIB supergravity on $AdS_5$. Tracing back to the origin of the $\alpha$-parameter dependence in Eqs.(7-9), this is possible only if the integration over the $\alpha$-parameters are not present. This then follows if the D-instanton wormholes are not allowed to attach their both necks to the base $AdS_5$ universe and consequently induce non-local, loop processes.

Wonderfully, $AdS_5$ background achieves this in a rather transparent way. Recall that the $AdS_5 \times S_5$ space is non-dilatonic, viz. the dilaton and the Ramond-Ramond axion fields are frozen to constant values throughout the entire $AdS_5$ space. The D-instantons, once inserted to the $AdS_5$ space, would only induce local deformations of these fields around the instanton neck. On the other hand, in Eq.(3), we have noted that the dilaton and the Ramond-Ramond axion fields grows indefinetly at the center, at which it opens up another asymptotic $AdS_5$ space (baby $AdS_5$ universe). Therefore, once one of the D-instanton neck is attached to the non-dilatonic
base $AdS_5$ background, there is no way that the other end of the D-instanton wormhole can join back to the same base $AdS_5$ background while maintaining smooth local deformations of dilaton and Ramond-Ramond axion fields. Even if one tries to invoke $SL(2, \mathbb{Z})$ quotient of the dilaton and Ramond-Ramond axion fields, it is easy to see that smooth interpolation of these fields would not be possible in general.

What about the tree-level process of D-instanton wormholes, for which only one side of the wormhole is attached to the base $AdS_5$ space and the other end branches off a baby $AdS_5$ universe? As the Yang-Mills instantons in the boundary conformal field theory is realized in the AdS/CFT correspondence by D-instantons on $AdS_5$ [17, 18], the D-instanton wormholes that are attached to the base $AdS_5$ universe on their one ends (but not both) should be allowed and, in fact, are the only possible combinatorics. Once holographically projected to the boundary, each branch-off D-instanton wormhole represents a single Yang-Mills instanton. From these observations, viz. existence of well-defined Yang-Mills instantons and unambiguous calculability of their effects within super Yang-Mills theory, we can draw some answers to the question we have alluded at the beginning provided we combine them with the holographic principle. First, since instanton-induced processes are present at the boundary conformal field theory, topology change induced by D-instantons should be permitted in $AdS_5$ vacua of Type IIB string theory. In other words, there are changes of spacetime topology in string theory.

Second, the cluster decomposition property of the Yang-Mills instantons at the boundary conformal field theory requires that arbitrary numbers of D-instanton wormholes on $AdS_5$ should be considered to be consistent with holographic principle. This then implies that universes of different topology (viz. different numbers of D-instanton wormholes on $AdS_5$) should be summed over. The rule for summing over the topologies on $AdS_5$ string vacua should not be arbitrary but rather be fixed precisely such that, once holographically projected to the boundary, the instanton-induced processes in super Yang-Mills theory are reproduced.

Third, since dilaton and axion fields are fixed at the boundary and since Yang-Mills instantons of different sizes are completely independent, only one side of the D-instanton wormholes can be attached to the base $AdS_5$ universe. The other ends open up baby universes with $AdS_5$ geometries which asymptotically approaches a flat spacetime in string frame. On these ‘baby’ $AdS_5$ universes, the dilaton and axion fields are divergent. As was argued above, the baby $AdS_5$ universes bear no effect to the base $AdS_5$ space. This then implies that only tree-level processes of the third quantization take place and hence third quantization itself is unnecessary.

Fourth, the Yang-Mills instantons induce completely deterministic correlators at the boundary. Holography principle then implies that the corresponding branching-off processes of the

---

8Incidentally, since the argument relies solely on the fact that dilaton and Ramond-Ramond axion fields are constant throughout the anti-de Sitter space, the same conclusion can be drawn for other cases of AdS/CFT correspondence such as $AdS_3 \times S_3$ associated with near-horizon geometry of D1-D5 branes.
D-instantons in the $AdS_5$ bulk are also deterministic. From the third quantization point of view, since only tree-level processes are permitted, integration over $\alpha$-parameters is unnecessary. Henceforth, in Type IIB string theory, the topology change of spacetime is completely specified by the calculable coupling parameters ($C_i$'s in Eq. (6)) and local operators ($O_I(x)$ in Eq. (6)) in the effective supergravity theory.

The resulting topology change processes in string theory would then look like schematically as in Figure 1.

![Figure 1: Topology change induced by D-instanton wormholes on $AdS_5$ string vacua. Upon holographic projection, they reproduce instanton induced effects in $\mathcal{N} = 4$ super Yang-Mills theory. Note that the D-instantons should be interpreted as configurations in the Higgs branch.](image)

To summarize, we have shown that holographic principle draws an interesting picture to the issue of topology change in string theory. Not only topology change takes place but also should they be summed up. The way topology is summed up, however, is not arbitrary but should be deterministic so that, upon holographic projection to the boundary, the standard instanton effects in $\mathcal{N} = 4$ super Yang-Mills theory are to be reproduced. This forbids any extra collective coordinates such as Coleman’s $\alpha$-parameter and hence no probabilistic interpretation either. As the argument does not rely on the energy scale of the super Yang-Mills theory, the conclusion should remain valid in the flat space limit of the anti-de Sitter space. Certainly, most immediate question is whether similar argument can be applied to situations where the cosmological constant vanishes exactly as, for example, in M(atrix) theory or take a positive value (cosmological vacua).
I would like to thank T. Banks and G. Gibbons for correspondences, and to I. Klebanov, N. Seiberg, L. Susskind and E. Witten for discussions.

References

[1] S. Coleman, *Why There is Nothing Rather Than Something: A Theory of The Cosmological Constant*, Nucl. Phys. B310 (1988) 643.

[2] For excellent reviews on baby universe, see, for example, A. Strominger, *Baby Universes*, in the proceedings of TASI '88, pp. 315 - 392 (World Scientific Co. 1988, Singapore); S.W. Hawking, *Baby Universes*, in the proceedings of Friedmann Centenary Volume, pp. 81 - 92 (World Scientific Co. 1988, Singapore); S. Giddings, *Wormholes, The Conformal Factor and The Cosmological Constant*, in the proceedings of International Colloquium on Modern Quantum Field Theory, pp. 518 - 544 (World Scientific Co. 1990, Singapore); T. Banks, *Report on Progress in Wormholes*, Physicalia 12 (1990) 19.

[3] S. Hawking, *Quantum Coherence Down the Wormhole*, Phys. Lett. 195B (1987) 337.

[4] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, *Particle Creation and Destruction of Quantum Coherence by Topological Change*, JETP Lett. 46 (1987) 167.

[5] S.B. Giddings and A. Strominger, *Axion Induced Topology Change in Quantum Gravity and String Theory*, Nucl. Phys. B306 (1988) 890.

[6] S.B. Giddings and A. Strominger, *Loss of Incoherence and Determination of Coupling Constants in Quantum Gravity*, Nucl. Phys. B307 (1988) 854.

[7] S. Carlip, Phys. Rev. Lett. 79 (1997) 4071.

[8] G. ’t Hooft, *Dimensional Reduction in Quantum Gravity*, in ‘Salam Fest’ (World Scientific Co. Singapore, 1993); L. Susskind, *The World as a Hologram*, J. Math. Phys. 36 (1995) 6377.

[9] L. Susskind and E. Witten, *The Holographic Bound in Anti-de Sitter Space*, hep-th/9805114.

[10] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.

[11] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428 (1998) 105, hep-th/9802109; E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.
[12] G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and Seven-Branes in Type IIB Superstring Theory*, Phys. Lett. **370B** (1996) 37.

[13] S.B. Giddings and A. Strominger, *String Wormholes*, Phys. Lett. **B230** (1989) 46.

[14] C.-S. Chu, P.-M. Ho and Y.-Y. Wu, Nucl. Phys. **B541** (1999) 179, hep-th/9806103.

[15] I.I. Kogan and G. Luzón, Nucl. Phys. **B539** (1999) 121, hep-th/9805112.

[16] J.H. Schwarz, *Can String Theory Overcome Deep Problems in Quantum Gravity?*, Phys. Lett. **272B** (1991) 239;
L. Susskind, private communication (June, 1998).

[17] T. Banks and M.B. Green, J. High-Energy Phys. **9805** (1998) 002, hep-th/9804170.

[18] E. Witten, J. High-Energy Phys. **9807** (1998) 006, hep-th/9805112.

[19] M. Bianchi, M.B. Green, S. Kovacs and G. Rossi, J. High-Energy Phys. **9808** (1998) 013, hep-th/9807033.