An Efficient Approach for the Model of Thrombin Receptor Activation Mechanism with Mittag-Leffler Function

P. Veeresha\(^1\), D. G. Prakasha\(^2\), and Zakia Hammouch\(^3\)

\(^1\) Department of Mathematics, Karnatak University, 580003 Dharwad, India  
\text{viru0913@gmail.com}

\(^2\) Department of Mathematics Faculty of Science Shivagangothri, Davangere University, Davangere 577007, India  
\text{prakashadg@gmail.com}

\(^3\) Department of Mathematics, Faculty of Sciences and Techniques Errachidia, BP 509, 52000 Errachidia, Morocco  
\text{z.hammouch@fste.umi.ac.ma}

Abstract. In the present work, we hared an efficient technique called, \textit{q-homotopy analysis transform method} (\textit{q-HATM}) in order to find the solution for the model of thrombin receptor activation mechanism (TRAM) and examine the nature of \textit{q-HATM} solution with distinct fractional order. The considered model elucidates the TRA mechanism in calcium signalling, and this mechanism plays a vital role in the human body. We defined fractional derivative defined with Atangana-Baleanu (AB) operator and the projected scheme is an amalgamation of Laplace transform with \textit{q-homotopy analysis scheme}. For the achieved results, to present the existence and uniqueness we hired the fixed point hypothesis. To validate and illustrate the effectiveness of the considered scheme, we examined the projected model with arbitrary order. The behaviour of the achieved results is captured in terms of plots and also showed the importance of the parameters offered by the considered solution procedure. The attained results illuminate, the projected scheme is easy to employ and more effective in order to analyse the behaviour of fractional order differential systems exemplifying real word problems associated with science and technology.

Keywords: \textit{q-Homotopy analysis method} · \textit{Laplace transform} · Thrombin receptor activation mechanism · Fixed point theorem · Atangana-Baleanu derivative

1 Introduction

The human body is mainly the composition of six elements of about 99\%, and those are namely, phosphorus, calcium, nitrogen, hydrogen, carbon and oxygen. In our body, the most generous mineral is calcium (\textit{Ca}) and it is about 1.5\%. \textit{Ca} is the most play a vital rule in muscle contraction and protein regulation and also it’s very essential in the processes of contractions bones and their protection. Most of the phenomena including cell death and fertilization are achieved with the help of calcium oscillations. With the
exploit of the inositol phospholipid cascade by raising the cytosolic calcium levels produced, most of the pathways of signal transduction are arbitrated [1, 2]. Ca acts as emissary in information processing. For the analysis of enzyme phospholipase C (PLC), G protein is playing an important role.

In the present scenario, the study of most of the phenomena related to the human body like diseases and their behaviour, the essential components of our body and their functions; magnetize the attention of mathematicians and researchers associated to mathematics in order to model and analyse as well as predict its essential behaviours. In connection with this, authors in [3] nurtured the mathematical model in order to illustrate the mediated activation of human platelets, researchers in [4] analyse the cytosolic calcium dynamics by the aid of mathematical model and later by the help of fractional calculus (FC), authors in [5] present their viewpoint in order to understand the importance of FC while analysing the mathematical model stimulating the above-cited phenomenon.

The seed of fractional calculus (FC) is planted before 324 years, however lately become an essential tool for the distinct discipline of science and engineering, and hence fascinated the attention of authors. It was shortly discovered that fractional calculus is more appropriate for modelling the phenomena describing nature in a systematic manner as associated with integer order calculus. The calculus of arbitrary order turned out one of the most essential tools to describe biological phenomena. The human diseases which are modelled through derivative having fractional-order help us to incorporate the information about its present and past states. Diverse pioneering notions and fundamentals are prescribed by many senior researchers [6–11]. Recently, due to diverse applications and favourable properties, the concept of FC is widely hired to investigate real world problems [12–20]. Particularly, authors in [21] analysed the fractional order system exemplifying the fish farm model within the frame of new fractional operator and also the captured some simulating consequences associated to the model using efficient scheme, authors in [22, 23] investigated the numerical solution for the fractional order coupled special cases of KdV equations and presented some interesting results with respect to different fractional order. The epidemic model of childhood disease is analysed by the authors in [24] within the frame of fractional calculus and they presented the nature of the corresponding results for distinct arbitrary order. Authors in [19] analyse the evolution of 2019-nCoV and its dynamic structures with help of nonlocal operator and presented some numerical surface using efficient scheme.

The activated form of phospholipase C (PLC) hydrolyzes the diacylglycerol (DAG), 5-trisphosphate [\Ins(I, 4, 5)P_3], 5-bisphosphate [\PtdIns(4, 5)P_2] to inositol and phosphatidylinositol 4. From the endoplasmic reticulum, the 5-trisphosphate is helps to stimulate the let out of endogenous calcium. The number of activated cell surface receptor proportional to the rate of generates of the 5-trisphosphate. The thrombin is a multiprocessing serine protease aids from the endothelial cell to take calcium transient and it acts as a ligand for the present model. Here, we consider the system of the equation which described the TRS mechanism. In endothelial cells, this model provides incite of calcium arbitrated signal transduction. The release of calcium is determined by the 5-trisphosphate cytosolic level in the calcium homeostasis and the number of active surface receptors (S) aid to generate the 5-trisphosphate. The receptor-ligand complex (C) formed due to ligand binding with surface receptors and on cleavage outcomes in
activated receptors \((A)\). The above-cited phenomenon is illustrated with the aid of the system of three differential equations and concentration of thrombin \((c)\) as follows \([4, 5]\)

\[
\begin{align*}
\frac{dS(t)}{dt} &= -\delta cS(t) + \beta C(t) \\
\frac{dC(t)}{dt} &= \delta cS(t) - (\beta + \lambda)c(t), \\
\frac{dA(t)}{dt} &= \lambda C(t)
\end{align*}
\] (1)

where \(\delta\) and \(\beta\) respectively symbolize the on and off rate constant of thrombin binding.

Many nonlinear and important models are effectively and methodically examined with the assist of FC. Many senior pioneers proposed distinct definitions including, Riemann, Liouville, Caputo and Fabrizio. Soon after the invention of each notion, many researchers identify some limitations while examining specific problems. Including physical meaning of the initial conditions, kernel associated to singularity, non-locality and others associated with complex phenomena. With the assist of Mittag–Leffler function, Atangana and Baleanu \([25]\) proposed a new fractional-order operator and overcome all the above-cited consequences which play a vital role while investigating properties of the models.

Authors in \([5]\) presented the simulation for the fractional system with Caputo–Fabrizio derivative using perturbation iterative scheme, which poses interesting consequences. In the present framework, we consider with AB derivative and which as follows

\[
\begin{align*}
^AABC D_t^\alpha S(t) &= -\delta cS(t) + \beta C(t) \\
^AABC D_t^\alpha C(t) &= \delta cS(t) - (\beta + \lambda)c(t), \\
^AABC D_t^\alpha A(t) &= \lambda C(t)
\end{align*}
\] (2)

where \(\alpha\) is fractional order of the system.

As much as impartment of modelling real-world problems, finding the solution for the corresponding system is also vital and difficult. Most of the complex and nonlinear problems don’t have an analytical solution. In this connection, researchers preferred for semi-analytical or numerical schemes. One of the efficient and most widely hired methods to solve nonlinear problems is the homotopy analysis method (HAM) and which natured by Liao Shijun \([26, 27]\). This solution procedure overcomes most of the limitation arise while solving nonlinear problems with dissertation and perturbation. However, a few limitations have been pointed out by researchers in order to reduce computational work and time. The presented method is the mixture of LT with \(q\)-HAM and nurtured by Singh et al. \([28]\). Clearly, \(q\)-HATM is an enhanced algorithm of HAM; it does not require linearization, perturbation or discretization. Recently, many researchers hired the considered method due to its efficacy and reliability to understand physical behaviour numerous classes of nonlinear problems \([29–34]\). The considered scheme gives more freedom to choose problems associated with distinct initial conditions and it proposed with axillary and homotopy corresponding phenomena \([35, 36]\).
2 Preliminaries

Here, the basic notions and definitions of FC and LT are presented [25, 37–41].

**Definition 1.** In Caputo and Riemann-Liouville sense, for a function $f \in H^1(a, b)$ the fractional Atangana-Baleanu-derivative are presented respectively as follows [25]:

$$
\begin{align*}
ABC_a D^\alpha_t (f(t)) &= \frac{B[\alpha]}{1-\alpha} \int_a^t f'(\vartheta) \left( \frac{(t-\vartheta)^{\alpha-1}}{\vartheta - 1} \right) d\vartheta, \\
ABR_a D^\alpha_t (f(t)) &= \frac{B[\alpha]}{1-\alpha} \frac{d}{dt} \int_a^t f(\vartheta) \left( \frac{(t-\vartheta)^{\alpha-1}}{\vartheta - 1} \right) d\vartheta,
\end{align*}
$$

where $B[\alpha]$ is a normalization function such that $B(0) = B(1) = 1$.

**Definition 2.** The AB integral with fractional order is presented [25] as

$$
\begin{align*}
AB_a I^\alpha_t (f(t)) &= \frac{1-\alpha}{B[\alpha]} f(t) + \frac{\alpha}{B[\alpha] \Gamma(\alpha)} \int_a^t f(\vartheta) (t-\vartheta)^{\alpha-1} d\vartheta.
\end{align*}
$$

**Definition 3.** The Laplace transform (LT) Associated to AB operator is defined as

$$
L[ABR_a D^\alpha_t (f(t))] = \frac{B[\alpha]}{1-\alpha} \frac{s^\alpha L[f(t)] - s^{\alpha-1} f(0)}{s^\alpha + (\alpha/ (1-\alpha))}.
$$

**Theorem 1.** The following Lipschitz conditions satisfy respectively for the Riemann-Liouville and AB derivatives [25]

$$
\|ABC_a D^\alpha_t f_1(t) - ABC_a D^\alpha_t f_2(t)\| < K_1 \|f_1(x) - f_2(x)\|,
$$

and

$$
\|ABR_a D^\alpha_t f_1(t) - ABR_a D^\alpha_t f_2(t)\| < K_2 \|f_1(x) - f_2(x)\|.
$$

**Theorem 2.** The fractional differential equation $\frac{ABC}{a} D^\mu_t f(t) = s(t)$ has a unique solution is given by [25]

$$
\begin{align*}
f(t) &= \frac{1-\alpha}{B[\alpha]} s(t) + \frac{\alpha}{B[\alpha] \Gamma(\alpha)} \int_0^t s(\zeta)(t-\zeta)^{\alpha-1} d\zeta.
\end{align*}
$$

3 Basic idea of $q$-HATM

Here, we consider the differential equation of fractional order with respectively linear $\mathbb{R}$ and nonlinear $\mathbb{N}$ differential operator form.
\[ \begin{align*}
\frac{ABC}{a} D_t^\alpha v(x, t) + \mathcal{R} v(x, t) + \mathcal{N} v(x, t) &= f(x, t), \quad n - 1 < \alpha \leq n, \\
\text{with the initial condition} \\
v(x, 0) &= g(x),
\end{align*} \tag{10} \]

where \( \frac{ABC}{a} D_t^\alpha v(x, t) \) symbolise the AB derivative of \( v(x, t) \), \( f(x, t) \) denotes the source term. Using LT, Eq. (10) gives

\[\mathcal{L}[v(x, t)] - \frac{\varphi(x)}{s} + \frac{1}{2[n]} \left( 1 - \alpha + \frac{\alpha}{s^\alpha} \right) \{ \mathcal{L}[\mathcal{R} v(x, t)] + \mathcal{L}[\mathcal{N} v(x, t)] - \mathcal{L}[f(x, t)] \} = 0. \tag{12}\]

By the assist of HAM, \( \mathcal{N} \) is projected as

\[\mathcal{N} \varphi(x, t; q) = \mathcal{L} [\varphi(x, t; q)] - \frac{\varphi(x)}{s} \]

\[+ \frac{1}{2[n]} \left( 1 - \alpha + \frac{\alpha}{s^\alpha} \right) \{ \mathcal{L}[\mathcal{R} \varphi(x, t; q)] + \mathcal{L}[\mathcal{N} \varphi(x, t; q)] - \mathcal{L}[f(x, t)] \}. \tag{13}\]

Here, \( \varphi(x, t; q) \) is the real-valued function. Now, we have

\[(1 - nq)\mathcal{L} [\varphi(x, t; q) - v_0(x, t)] = hq\mathcal{N} [\varphi(x, t; q)], \tag{14}\]

where \( \mathcal{L} \) is signifying LT, \( q \in [0, \frac{1}{n}] (n \geq 1) \) is the embedding parameter and \( h \neq 0 \) is an auxiliary parameter. For \( q = 0 \) and \( q = \frac{1}{n} \), we have

\[\varphi(x, t; 0) = v_0(x, t), \quad \varphi \left( x, t; \frac{1}{n} \right) = v(x, t). \tag{15}\]

Thus, by intensifying \( q \) from 0 to \( \frac{1}{n} \), then \( \varphi(x, t; q) \) changes from \( v_0(x, t) \) to \( v(x, t) \).

Using Taylor theorem near to \( q \), we defining \( \varphi(x, t; q) \) in series form and then we get

\[\varphi(x, t; q) = v_0(x, t) + \sum_{m=1}^{\infty} v_m(x, t) q^m, \tag{16}\]

where

\[v_m(x, t) = \frac{1}{m!} \frac{\partial^m \varphi(x, t; q)}{\partial q^m} \bigg|_{q=0}. \tag{17}\]

For the proper chaise of \( v_0(x, t), n \) and \( h \), the series (14) converges at \( q = \frac{1}{n} \). By simplifying Eq. (14), we achieved
\[ \mathcal{L}[v_m(x,t) - k_m v_{m-1}(x,t)] = \hbar \mathcal{R}_m(\tilde{v}_{m-1}), \]  

(18) 

where the vectors are defined as
\[ \tilde{v}_m = \{v_0(x,t), v_1(x,t), \ldots, v_m(x,t)\}. \]

(19)

On employing inverse \( LT \) on Eq. (18), we get
\[ v_m(x,t) = k_m v_{m-1}(x,t) + \hbar \mathcal{L}^{-1}[\mathcal{R}_m(\tilde{v}_{m-1})], \]

(20) 

where 
\[ \mathcal{R}_m(\tilde{v}_{m-1}) = L[v_{m-1}(x,t)] - \left(1 - \frac{k_m}{n}\right) \left(\frac{\varrho(x)}{s} + \frac{1}{\mathcal{B}[\alpha]} \left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[f(x,t)]\right) \]
\[ + \frac{1}{\mathcal{B}[\alpha]} \left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[Rv_{m-1} + \mathcal{H}_m-1], \]

(21) 

and
\[ k_m = \begin{cases} 0, m \leq 1, \\ n, m > 1. \end{cases} \]

(22) 

In Eq. (21), \( \mathcal{H}_m \) signifies homotopy polynomial and which is defined as
\[ \mathcal{H}_m = \frac{1}{m!} \left[ \frac{\partial^m \varphi(x,t;q)}{\partial q^m} \right]_{q=0} \quad \text{and} \quad \varphi(x,t;q) = \varphi_0 + q\varphi_1 + q^2\varphi_2 + \ldots. \]

(23) 

By the aid of Eqs. (20) and (21), one can get
\[ v_m(x,t) = (k_m + \hbar)v_{m-1}(x,t) - \left(1 - \frac{k_m}{n}\right) \mathcal{L}^{-1} \left(\frac{\varrho(x)}{s} + \frac{1}{\mathcal{B}[\alpha]} \left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[f(x,t)]\right) \]
\[ + \hbar \mathcal{L}^{-1} \left\{ \frac{1}{\mathcal{B}[\alpha]} \left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[Rv_{m-1} + \mathcal{H}_m-1] \right\}. \]

(24) 

The \( q \)-HATM solution is presented as
\[ v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t) \left(\frac{1}{n}\right)^m. \]

(25)
4 Solution for Proposed Model

To demonstrate the efficiency and solution procedure of the projected method, in here we consider system describing considered a model with arbitrary order. By the assist of Eq. (2), one can get

\[
\begin{align*}
\frac{d^2}{dt^2} D_a^z S(t) + \delta e S(t) - \beta C(t) &= 0, \\
\frac{d^2}{dt^2} D_a^z C(t) - \delta e S(t) + (\beta + \lambda) C(t) &= 0, \\
\frac{d^2}{dt^2} D_a^z A(t) - \lambda C(t) &= 0
\end{align*}
\]

with initial conditions

\[
S(0) = S_0(t), C(0) = C_0(t), A(0) = A_0(t).
\]

Taking LT on Eq. (26) and then using the Eq. (27), we get

\[
\begin{align*}
L[S(t)] &= \frac{1}{s} (S_0(t)) + \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\delta e S(t) - \beta C(t)\}, \\
L[C(t)] &= \frac{1}{s} (C_0(t)) - \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\delta e S(t) - (\beta + \lambda) C(t)\}, \\
L[A(t)] &= \frac{1}{s} (A_0(t)) - \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\lambda C(t)\}.
\end{align*}
\]

Now, we define \( N \) as below

\[
\begin{align*}
&N^1[\varphi_1(t; q), \varphi_2(t; q), \varphi_3(t; q)] = L[\varphi_1(t; q)] - \frac{1}{s} (S_0(t)) \\
&\quad + \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\delta e \varphi_1(t; q) + \beta \varphi_2(t; q)\}, \\
&N^2[\varphi_1(t; q), \varphi_2(t; q), \varphi_3(t; q)] = L[\varphi_2(t; q)] - \frac{1}{s} (C_0(t)) \\
&\quad - \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\delta e \varphi_1(t; q) + (\beta + \lambda) \varphi_2(t; q)\}, \\
&N^3[\varphi_1(t; q), \varphi_2(t; q), \varphi_3(t; q)] = L[\varphi_3(t; q)] - \frac{1}{s} (A_0(t)) \\
&\quad - \frac{1}{B[z]} \left( 1 - \frac{\alpha}{z^2} \right) L\{\lambda \varphi_2(t; q)\}.
\end{align*}
\]
The deformation equation of $m$-th order at $\mathcal{H}(x, t) = 1$ is defined as

$$L[S_m(t) - k_mS_{m-1}(t)] = hR_{1,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right],$$

$$L[C_m(t) - k_mC_{m-1}(t)] = hR_{2,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right],$$

$$L[A_m(t) - k_mA_{m-1}(t)] = hR_{3,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right],$$

where

$$R_{1,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right] = L[S_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\left\{\frac{1}{s} (S_0(t))\right\}$$

$$+ \frac{1}{B[z]} \left(1 - \alpha + \frac{\alpha}{s^2}\right) L\{\delta e S_{m-1}(t) - \beta C_{m-1}(t)\},$$

$$R_{2,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right] = L[C_{m-1}(t)] + \left(1 - \frac{k_m}{n}\right)\left\{\frac{1}{s} (C_0(t))\right\}$$

$$+ \frac{1}{B[z]} \left(1 - \alpha + \frac{\alpha}{s^2}\right) L\{\delta e S_{m-1}(t) - (\beta + \lambda)C_{m-1}(t)\},$$

$$R_{3,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right] = L[A_{m-1}(t)] + \left(1 - \frac{k_m}{n}\right)\left\{\frac{1}{s} (A_0(t))\right\}$$

$$- \frac{1}{B[z]} \left(1 - \alpha + \frac{\alpha}{s^2}\right) L\{\lambda C_{m-1}(t)\}. $$

(31)

Eq. (31) reduces after employing inverse $LT$, as follows

$$S_m(t) = k_mS_{m-1}(t) + hL^{-1}\left\{R_{1,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right]\right\},$$

$$C_m(t) = k_mC_{m-1}(t) + hL^{-1}\left\{R_{2,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right]\right\},$$

$$A_m(t) = k_mA_{m-1}(t) + hL^{-1}\left\{R_{3,m}\left[\tilde{S}_{m-1}, \tilde{C}_{m-1}, \tilde{A}_{m-1}\right]\right\}. $$

(32)

Using $S_0(t) = R_T$, $C_0 = 0$ and $A_0(t) = 0$ we can obtain the terms of the series solution with the help of the above system

$$S(t) = S_0(t) + \sum_{m=1}^{\infty} S_m(t) \left(\frac{1}{n}\right)^m,$$

$$C(t) = C_0(t) + \sum_{m=1}^{\infty} C_m(t) \left(\frac{1}{n}\right)^m,$$

$$A(t) = A_0(t) + \sum_{m=1}^{\infty} A_m(t) \left(\frac{1}{n}\right)^m,$$

(33)
5 Existence of Solutions for the Proposed Problem

Here, to present the existence of the solution, we considered the fixed-point theorem. Now, the system (27) is considered as

\[
\begin{align*}
&\frac{ABC D^2}{0} [S(t)] = G_1(t, S), \\
&\frac{ABC D^2}{0} [C(t)] = G_2(t, C), \\
&\frac{ABC D^2}{0} [A(t)] = G_3(t, A).
\end{align*}
\]

Using the Theorem 2, Eq. (35) is transformed to the Volterra integral equation and defined as

\[
\begin{align*}
S(t) - S(0) &= \frac{(1-z)}{B(z)} \int_0^t G_1(\xi, S(t - \xi)^{2-1} d\xi, \\
C(t) - C(0) &= \frac{(1-z)}{B(z)} \int_0^t G_2(\xi, C(t - \xi)^{2-1} d\xi, \\
A(t) - A(0) &= \frac{(1-z)}{B(z)} \int_0^t G_3(\xi, A(t - \xi)^{2-1} d\xi.
\end{align*}
\]

**Theorem 3.** The kernel $G_1$ satisfies the Lipschitz condition and contraction if $0 \leq (\delta \varepsilon - \beta \lambda_2) < 1$ holds.

**Proof.** We consider the two functions $u$ and $u_1$ to prove the required result, as follows

\[
\begin{align*}
\|G_1(t, S) - G_1(t, S_1)\| &= \|\delta \varepsilon [S(t) - S(t_1)] - \beta C(t)\| \\
&\leq \|\delta \varepsilon\| \|S(t) - S(t_1)\| \leq (\delta \varepsilon - \beta \lambda_2) \|S(t) - S(t_1)\|,
\end{align*}
\]

where $\|C(t)\| \leq \lambda_2$ be the bounded function. Putting $\eta_1 = \delta \varepsilon - \beta \lambda_2$ in Eq. (37), we have

\[
\|G_1(t, S) - G_1(t, S_1)\| \leq \eta_1 \|S(t) - S(t_1)\|.
\]

Equation (38) signifies Lipschitz condition for $G_1$. If $0 \leq (\delta \varepsilon - \beta \lambda_2) < 1$, then it gives the contraction. Similarly, we have

\[
\begin{align*}
\|G_2(t, C) - G_2(t, C_1)\| &\leq \eta_2 \|C(t) - C(t_1)\|, \\
\|G_3(t, A) - G_3(t, A_1)\| &\leq \eta_3 \|A(t) - A(t_1)\|.
\end{align*}
\]

Now, we define the recursive form of Eq. (36) as with initial conditions

\[
\begin{align*}
S_n(t) &= \frac{(1-z)}{B(z)} \int_0^t G_1(\xi, S_{n-1}(t - \xi)^{2-1} d\xi, \\
C_n(t) &= \frac{(1-z)}{B(z)} \int_0^t G_2(\xi, C_{n-1}(t - \xi)^{2-1} d\xi, \\
A_n(t) &= \frac{(1-z)}{B(z)} \int_0^t G_3(\xi, A_{n-1}(t - \xi)^{2-1} d\xi.
\end{align*}
\]
and

\[ S(0) = S_0(t), \quad C(0) = C_0(t) \quad \text{and} \quad A(0) = A_0(t). \tag{40} \]

The successive difference between the terms presented as

\[
\begin{align*}
\phi_{1n}(t) &= S_n(t) - S_{n-1}(t) \\
&= \frac{(1-\alpha)}{B(\alpha)} (G_1(t, S_{n-1}) - G_1(t, S_{n-2})) + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t G_1(\zeta, S_{n-1})(t - \zeta)^{\alpha-1} d\zeta, \\
\phi_{2n}(t) &= C_n(t) - C_{n-1}(t) \\
&= \frac{(1-\alpha)}{B(\alpha)} (G_2(t, C_{n-1}) - G_2(t, C_{n-2})) + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t G_2(\zeta, C_{n-1})(t - \zeta)^{\alpha-1} d\zeta, \\
\phi_{3n}(t) &= A_n(t) - A_{n-1}(t) \\
&= \frac{(1-\alpha)}{B(\alpha)} (G_3(t, A_{n-1}) - G_3(t, A_{n-2})) + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t G_3(\zeta, A_{n-1})(t - \zeta)^{\alpha-1} d\zeta, \tag{41}
\end{align*}
\]

Notice that

\[
\begin{align*}
S_n(t) &= \sum_{i=1}^n \phi_{1i}(t), \\
C_n(t) &= \sum_{i=1}^n \phi_{2i}(t), \\
A_n(t) &= \sum_{i=1}^n \phi_{3i}(t). \tag{42}
\end{align*}
\]

By using Eq. (39) and applying the norm on the first term of Eq. (42), we have

\[
\| \phi_{1n}(t) \| \leq \frac{(1-\alpha)}{B(\alpha)} \eta_1 \| \phi_{1(n-1)}(t) \| + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_1 \int_0^t \| \phi_{1(n-1)}(\zeta) \| d\zeta. \tag{43}
\]

Similarly, we have

\[
\begin{align*}
\| \phi_{2n}(t) \| &\leq \frac{(1-\alpha)}{B(\alpha)} \eta_2 \| \phi_{2(n-1)}(t) \| + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_2 \int_0^t \| \phi_{2(n-1)}(\zeta) \| d\zeta, \\
\| \phi_{3n}(t) \| &\leq \frac{(1-\alpha)}{B(\alpha)} \eta_3 \| \phi_{3(n-1)}(t) \| + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_3 \int_0^t \| \phi_{3(n-1)}(\zeta) \| d\zeta. \tag{44}
\end{align*}
\]

Next using the above result, we have following results.

**Theorem 4.** The solution for Eq. (27) will exist and unique if we have \( t_0 \) then

\[
\frac{(1-\alpha)}{B(\alpha)} \eta_i + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_i < 1,
\]

for \( i = 1, 2 \) and 3.

**Proof.** Let \( S(t), C(t) \) and \( A(t) \) be the bounded functions admitting the Lipschitz condition. Now, we have by Eqs. (43) and (45)
\[ \|\phi_1(t)\| \leq \|S_n(0)\| \left[ \frac{(1 - \alpha)}{B(x)} \eta_1 + \frac{\alpha}{B(x) \Gamma(x)} \eta_1 \right]^n, \]
\[ \|\phi_2(t)\| \leq \|C_n(0)\| \left[ \frac{(1 - \alpha)}{B(x)} \eta_2 + \frac{\alpha}{B(x) \Gamma(x)} \eta_2 \right]^n, \]  
(45)
\[ \|\phi_3(t)\| \leq \|A_n(0)\| \left[ \frac{(1 - \alpha)}{B(x)} \eta_3 + \frac{\alpha}{B(x) \Gamma(x)} \eta_3 \right]^n. \]

This proves the continuity as well as existence. Now, we consider showing Eq. (46) is a solution for the system (27)

\[
\begin{align*}
S(t) - S(0) &= S_n(t) - \mathcal{K}_1n(t), \\
C(t) - C(0) &= C_n(t) - \mathcal{K}_2n(t), \\
A(t) - A(0) &= A_n(t) - \mathcal{K}_3n(t).
\end{align*}
\]

(46)

To achieve the required result, we consider

\[
\|\mathcal{K}_1n(t)\| = \left\| \frac{(1 - \alpha)}{B(x)} (G_1(t,S) - G_1(t,S_{n-1})) \\
+ \frac{\alpha}{B(x) \Gamma(x)} \int_0^t (t - \zeta)^{n-1} (G_1(\zeta,S) - G_1(\zeta,S_{n-1}))d\zeta \right\| \\
\leq \frac{(1 - \alpha)}{B(x)} \| (G_1(t,S) - G_1(t,S_{n-1})) \| \\
+ \frac{\alpha}{B(x) \Gamma(x)} \| (G_1(\zeta,S) - G_1(\zeta,S_{n-1})) \|d\zeta \\
\leq \frac{(1 - \alpha)}{B(x)} \eta_1 \|S - S_{n-1}\| + \frac{\alpha}{B(x) \Gamma(x)} \eta_1 \|S - S_{n-1}\| t.
\]

(47)

In the same way at \( t_0 \), we can obtain

\[
\|\mathcal{K}_1n(t)\| \leq \left( \frac{(1 - \alpha)}{B(x)} + \frac{\alpha t_0}{B(x) \Gamma(x)} \right)^{n+1} \eta_1^{n+1} M.
\]

(48)

We can see that form Eq. (49), when \( n \) approaches to \( \infty \), \( \|\mathcal{K}_1n(t)\| \) tends to 0. We can verify similarly for \( \|\mathcal{K}_2n(t)\| \) and \( \|\mathcal{K}_3n(t)\| \).

Now, we present the uniqueness. Suppose \( S^*(t), C^*(t) \) and \( A^*(t) \) be the set of other solutions, then we have

\[
\begin{align*}
S(t) - S^*(t) &= \frac{(1 - \alpha)}{B(x)} (G_1(t,S) - G_1(t,S^*)) \\
+ \frac{\alpha}{B(x) \Gamma(x)} \int_0^t (G_1(\zeta,S) - G_1(\zeta,S^*))d\zeta.
\end{align*}
\]

(49)
By employing norm on Eq. (51), we get

\[ \|S(t) - S^*(t)\| = \left\| \frac{1 - \alpha}{B(\alpha)} (G_1(t, S) - G_1(t, S^*)) + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t (G_1(\zeta, S) - G_1(\zeta, S^*)) d\zeta \right\| \]

\[ \leq \left( \frac{1 - \alpha}{B(\alpha)} \right) \eta_1 \|S(t) - S^*(t)\| + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_1 t \|S(t) - S^*(t)\|. \]

(50)

On simplification

\[ \|S(t) - S^*(t)\| \left( 1 - \frac{(1 - \alpha)}{B(\alpha)} \eta_1 - \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_1 t \right) \leq 0. \]

(51)

From the above condition, it is clear that \( S(t) = S^*(t) \), if

\[ \left( 1 - \frac{(1 - \alpha)}{B(\alpha)} \eta_1 - \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \eta_1 t \right) \geq 0. \]

(52)

Therefore Eq. (52) proves our result.

### 6 Numerical Results and Discussion

Here, we illustrated the nature of \( q \)-HATM solution for different \( \alpha \). The initial conditions for the proposed model is defined as

\[ S(0) = S_0(t) = N, C(0) = C_0(t) = 0, A(0) = A_0(t) = 0. \]

where \( N \) is the total number of receptors and which is \( 4.4 \times 10^4 \text{No.}/\text{cell} \). In order to capture the behaviour, the value of the parameters cited in Eq. (2) are considered as follows

\[ \delta = 0.0005 M^{-1}s^{-1}, \beta = 142.8 s^{-1}, \varepsilon = 1 \text{unit/mL}, \lambda = 0.12 s^{-1}. \]

The nature of results obtained by \( q \)-HATM for a considered model with different \( \alpha \) is dissipated in Fig. 1 with different fractional order. To analyse the behaviour of archived results associated with \( h \), the \( h \)-curves are plotted for distinct \( \alpha \) is captured in Fig. 2. These help us to adjust and control the convergence region of the obtained results. For a suitable \( h \), the obtained results rapidly tend to an analytical solution. Moreover, in the plots the convergence region is denoted by the horizontal line. The captured figures show the degree of freedom and more simulating consequences about the hired model with different arbitrary order and also it signifies the novelty of the fractional operator employed. Further, from all plots one can observer that the projected solution procedure is and very effective and more accurate to examine the considered nonlinear problem.
Fig. 1. Behaviour of the obtained results for (a) $S(t)$, (b) $C(t)$ and (c) $A(t)$ with different $\alpha$ at $n = 1$ and $h = -1$. 
Fig. 2. $h$-curves for (a) $S(t)$, (b) $C(t)$ and (c) $A(t)$ with distinct $z$ at $t = 0.01$ and $n = 1$. 
7 Conclusion

The $q$-HATM is employed efficiently in the present framework to find the solution for the system of equation with arbitrary order and illustrating the model of TRA mechanism in calcium signalling. Since, generalized Mittag-Leffler function is hired to define fractional-order AB integrals and derivatives, these operators help us to capture more simulating consequences and also it incorporate most essential behaviours of the models, and hence the current study exemplifies the effeteness of the projected derivative. Further, for the obtained results we presented the existence and uniqueness within the frame of fixed point hypothesis. As associated to consequences available in the literature, the results obtained by the help of projected method are more stimulating. The graphical representations show the dependence of the considered nonlinear model on parameters offered by the considered scheme and fractional order, and also it exemplifies the degree of freedom when we incorporate the fractional operator in the systems. We can be observed by the present study, the projected model is remarkably associated with the time instant and time history-based consequences, and which can be efficiently examined by the help of fractional calculus. Lastly, we can conclude that the present study can aid the researchers to analyse the nature system corresponded to very useful and interesting and consequences.

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