Measuring the coefficient of unit surface conductance of steel balls for frying without cooking oil by using dimensional analysis

Siswantoro, A Margiwiyatno and H Dwiyanti
Department of Agricultural Technology, Faculty of Agriculture, Jenderal Soedirman University, Purwokerto, Indonesia

E-mail: siswantoro253@gmail.com

Abstract. Frying process is basically exchange of thermal energy through direct contact between a transferring heat medium and the food in high temperature. Cooking oil is the most commonly used as frying medium, but there is an alternative frying process without cooking oil by using sand and steel balls. Even though frying process is a very popular, theory of the frying process was long neglected and poorly understood. In frying process, coefficient of unit surface conductance is an important thermal property for analyzing heat transfer process. A common notation used for the coefficient is \( h \) and the unit is J/s.m\(^2\).C. Objective of this research was to measure the coefficient of unit surface conductance (\( h \)) by using dimensional analysis for constructing mathematical model. Laboratory experiments were conducted to find the \( h \) of steel balls with various diameters. An aluminum plate was used to simulate food being fried. A cylindrical frying machine was employed for frying process. The \( h \) value was determined based on dimensionless temperature ratio. The \( h \) values were then used to develop an empirical mathematical model using dimensional analysis. The developed mathematical model showed a good agreement with measurement results with average of errors was less than 10%.

1. Introduction
Hot sand frying method (frying without cooking oil) has been long applied in Indonesia, China, and India. In this method, sand is used as heating medium. Product and bulk of sand are filled into a frying chamber. Frying is conducted by heating the frying chamber and then the heat is transferred for heating the mixture of product and bulk of sand. During the frying process, the mixture is flowing in the chamber as it is continuously stirred. In this process, the flowing bulk of sand will have property like fluid. Unlike in frying using cooking oil whereas heat transfer phenomenon occurred in the process is only convection, two types of heat transfer phenomenon are occurred in the hot sand frying process, in conduction and convection.

Mohsenin [1] used a term, so called unit surface conductance, for describing the conduction and convection phenomenon. This term refers to thermal conductivity of a relative stagnant layer of fluid that is assumed to adhere to the surface of the solid during heating. It defines the rate of heat transfer per degree of temperature difference across solid-fluid interface per unit of solid’s surface area.

Heat transfer in frying process has been got a great concern by researchers [2-5]. However, the heat transfer phenomenon in the frying without oil method has not yet scientifically investigated. Understanding the phenomenon will give considerable contribution to develop and promote non-
cooking oil frying method as an alternative healthy cooking method because there is indicative evidence that cooking food with oil could risk human health, such as high blood pressure and cancer [6, 7]. Other comparative advantages of the frying without oil (using sand or steel balls) are: (1) no rancidity problem of product, (2) lower cost on frying process, and (3) abundance availability of medium materials.

Considering the benefits of the non oil cooking method, Siswantoro has conducted experimental investigation on thermal properties of sand and applied the experimental results for frying several kinds of chip [8]. It was found that unit surface conductance (h) value was influenced by diameter of the sand. The h value of sand with diameter 0.4 mm and 3.0 mm were 95.0 J/s.m2.C and 39.9 J/s.m2.C respectively. This indicates that the greater the diameter of sand, the smaller the value of h. However, it was also found that the sand diameter should not less than 0.75 mm as the sand will stick on the chip surface. This encouraging results should be further elaborated to find other possible materials for avoiding dirtyness of the product. For this purpose, use of steel balls is main concern in this paper. Thus, the objective of this research was to measure unit surface conductance coefficient (h) of steel balls by using dimensional analysis and to construct a mathematical model.

2. Materials and methods

2.1. Materials and apparatus

For laboratory experiments, steel balls with different diameters (3.2 mm and 4.7 mm) were used as heating medium, a piece of aluminum plate was used to simulate a product being fried, and a rotating cylinder frying machine was employed to simulate frying process. Other apparatus used in the experiments were tachometer, thermocouple, and data acquisition unit.

2.2. Methods

2.2.1. Temperature of steel balls and aluminum plate measurements. During frying simulation, temperature of steel balls was measured in every 5 second. The same measurement was also made to aluminum plate. Figure 1 shows schematic diagram of experimental setup.

Treatments applied in the experiment were steel balls outer diameters (3.2 mm and 4.7 mm), rotation (rpm) of fryer cylinder, and volume of medium. The frying process was assumed to follow “Newtonian Law of Heating” whereas thermal conductivity (k) is high in comparison to the unit surface conductance (h), and so Biot number (Bi) is approximately less than 0.2 [1].

![Figure 1. Schematic view of the experiment for measuring coefficient of unit surface conductance](image-url)
Energy balance on the frying process can be written as follows:
\[ q = h.A.(T_{ps} - T_\theta) = V.\rho_a.\ C_{pa}.(dT/d\theta) \] .................. (1)

The following equation was used to calculate the \( h \) value based on value of \( T_\theta \) (aluminum plate temperature at time \( \theta \)), \( T_{ps} \) (temperature of steel balls during frying process), \( A \) (aluminum plate area), \( V \) (volume of Aluminum plate), \( \rho_a \) (mass density of aluminum plate), and \( C_{pa} \) (specific heat of aluminum plate):

\[
\frac{dT}{(T_{ps} - T_\theta)} = \frac{h.A.d\theta}{V.\rho_a.C_{pa}}
\]

\[
\int_{T_\theta}^{T_{ps}} \frac{dT}{(T_{ps} - T_\theta)} = \int_{0}^{\theta} \frac{h.A.d\theta}{V.\rho_a.C_{pa}}
\]

\[
-\ln\left(\frac{T_\theta - T_{ps}}{T_i - T_{ps}}\right) = \frac{h.A(\theta - 0)}{V.\rho_a.C_{pa}}
\]

\[
h = -\ln\left(\frac{T_\theta - T_{ps}}{T_i - T_{ps}}\right) \left(\frac{A.\theta}{V.\rho_a.C_{pa}}\right) \] ......................... (2)

2.2.2 Development of mathematical model. Value of unit surface conductance coefficient (\( h \)) in frying process is depended on the following parameters: conductivity (\( k \)) of steel balls, bulk density (\( \rho \)) of steel balls, specific heat (\( C_p \)) of steel balls, diameter (\( d \)) of steel balls, rotation (\( n \)) of fryer cylinder, diameter (\( D \)) of fryer cylinder, gravitation force (\( g \)), volume (\( V_p \)) of steel balls stored in frying chamber, and volume of frying chamber (\( V_s \)). Therefore, mathematical equation for calculating the \( h \) was written in the following form:

\[
h = f(k, \rho, C_p, d, n, D, g, V_p, V_s) \] ................. (3)

The equation (3) can be solved by dimensional analysis with basic dimension of mass (\( M \)), length (\( L \)), time (\( \theta \)), and temperature (\( T \)). Variables involved in the analysis are presented in Table 1.

| Name of variable                        | Symbol | Unit       | Dimension |
|-----------------------------------------|--------|------------|-----------|
| Coefficient of unit surface conductance | \( h \) | J/s.m\(^2\).\(^\circ\)C | \( M \ \theta^{-3} \ T^{-1} \) |
| Thermal conductivity of steel balls     | \( k \) | J/s.m.\(^\circ\)C | \( M \ L^{-1} \ T^{-1} \) |
| Bulk density of steel balls              | \( \rho \) | kg/m\(^3\) | \( M \ L^{-3} \) |
| Specific heat of steel balls            | \( C_p \) | J/kg.\(^\circ\)C | \( M \ L^{-2} \ T^{-1} \) |
| Diameter of steel balls                 | \( D \) | m          | \( L \) |
| rpm of fryer cylinder                   | \( n \) | \( s^{-1} \) | \( \theta^{-1} \) |
| Cylinder diameter                       | \( D \) | m          | \( L \) |
| Gravitation of earth                    | \( g \) | m/s\(^2\)   | \( L \ \theta^{-2} \) |
| Bulk volume of steel balls              | \( V_p \) | m\(^3\)    | \( L^3 \) |
| Volume of fryer cylinder                | \( V_s \) | m\(^3\)    | \( L^3 \) |

Solution of dimensional analysis was made using “Buckingham \( \pi \) Theorem” [8, 9]. Relation form of dimensionless number (\( \pi \)) function was written in the following form:
\[ \Sigma \text{ Number of } \pi = \Sigma \text{ Total number of variable} - \Sigma \text{ Number of basic variables} \]

Hence: \[ \pi = 10 - 4 = 6 \]

\[ \pi_1 = \frac{hd}{k}; \quad \pi_2 = d^3 \rho^2 \text{Cp}^2 \frac{g}{k^2}; \quad \pi_3 = n^3 D^3 \rho \text{Cp}/gk \]

\[ \pi_4 = d / D; \quad \pi_5 = D^3 / Vp; \quad \pi_6 = Vp / Vs \]

Hence the basic function can be written in the following form:

\[ \pi_1 = F(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \] ................................. (4)

The dimensional equation corresponding to equation (4) is

\[ (hd/k) = F\left(d^3 \rho^2 \text{Cp}^2 \frac{g}{k^2}, n^3 D^3 \rho \text{Cp}/gk, d / D, D^3 / Vp, Vp / Vs\right) \] ........................ (5)

The dimensionless number (\(\pi\)) was calculated based on the laboratory experiment results. Afterwards, the value of dimensionless number (\(\pi\)) was then used in matrix solution ([8]; [10]) as follow:

\[ b_1 = \pi_1 - (b_2)\pi_2 - (b_3)\pi_3 - (b_4)\pi_4 - (b_5)\pi_5 \]

\[ \pi_1 = b_1 + (b_2)\pi_2 + (b_3)\pi_3 + (b_4)\pi_4 + (b_5)\pi_5 \]

\[ \bar{\pi} = \text{average value of } \pi \]

A logarithmic equation that corresponds to equation (4) was solved using matrix solution, as follow:

\[ \log \pi_1 = F(\log \pi_2, \log \pi_3, \log \pi_4, \log \pi_5, \log \pi_6) \] ............................... (6)

\[ \log \pi_1 = b_1 + (b_2)\log \pi_2 + (b_3)\log \pi_3 + (b_4)\log \pi_4 + (b_5)\log \pi_5 + (b_6)\log \pi_6 \] ........... (7)

\[ \pi_1 = 10^{b_1 \pi_2 + b_2 \pi_3 + b_3 \pi_4 + b_4 \pi_5 + b_5 \pi_6} \] ................................. (8)

Result of matrix solution was a mathematical model that can be used as a general prediction equation of unit surface conductance (h). The equation is written in the following form:

\[ \pi_1 = 10^{-3.001 \pi_2 + 0.330 \pi_3 + 0.076 \pi_4 - 0.189 \pi_5 - 2.477 \pi_6 - 2.461} \] ............................... (9)

Validation of the model was made by comparing prediction results and measurement results.

3. Results and discussions

Value of unit surface conductance was calculated using equation (2). An example of temperature measurement which used to calculate coefficient unit surface conductance is shown in Figure 2
Figure 2. Temperature aluminum plate and steel balls during frying process

From the result of temperature measurement on figure (2) can be plotted a graph about the relation of dimensionless temperature ratio:

\[- \ln \left( \frac{T_A - T_{\mu}}{T_i - T_{\mu}} \right) \text{ as a Y axis, and } \left( \frac{A \theta}{V \cdot \rho \cdot C_p} \right) \text{ as an X axis, hence be obtained Figure 3.}\]

From Figure 3 and corresponding to equation (2), value of coefficient unit surface conductance (h) obtained as a gradient or slope of equation line. Hence the value of h is 191.07 J/s.m².

Figure 3. An example graph for deciding value of unit surface conductance (h) (steel balls on diameter 4.7 mm at rpm 12)

A mathematical model of unit surface conductance obtained in the research is an empirical model. The model is a power function, and constructed from 6 number of dimensionless variables of phi (π). Equation of a mathematical model can be written in the following form:
The mathematical model of equation (10) can be used for predicting value of unit surface conductance coefficient \( h \) on steel balls in a good result, due to the average error of prediction 2.8 percent. Result of measurement (observation) value of \( h \) and predicting value using the mathematical model can be seen in the Figure 4.

\[
\left( \frac{hd}{k} \right) = 10^{-3.001} \left( \frac{d^3 \rho Cp^2 g}{k^2} \right)^{0.330} \left( \frac{n^3 D^3 \rho Cp}{gk} \right)^{0.076} \left( \frac{d}{D} \right)^{-0.189} \left( \frac{D}{Vp} \right)^{-2.477} \left( \frac{Vp}{Vs} \right)^{-2.461} \quad .... (10)
\]

**Figure 4.** Prediction and observation value of unit surface conductance coefficient \( h \)

Value of unit surface conductance was predicted from the mathematical model in the range 161.4 J/s.m\(^2\).\(^0\)C to 329.2 J/s.m\(^2\).\(^0\)C whereas the average value was 242.3 J/s.m\(^2\).\(^0\)C. These figures are higher than the \( h \) value of river and ferric sand. The average value of \( h \) ferric sand and river sand are 119,9 J/s.m\(^2\).\(^0\)C and 56,4 J/s.m\(^2\).\(^0\)C respectively [8]. The differences are caused by thermal characteristic of the medium, i.e. thermal conductivity, bulk density, and thermal diffusivity.

Increasing value of unit surface conductance on frying process not only caused by increasing rotation number (rpm) of cylinder fryer, but also caused by decreasing size of medium diameter. It was predicted that increasing rotation number and decreasing size of diameter will cause intensity and contact surface area to increase. Hence quantity of heat transfer in a certain time will be greater [8, 11-14].

4. Conclusion
The developed mathematical model could be used for predicting value of unit surface conductance of steel balls with average error about 2.8%. Value of unit surface conductance coefficient of steel balls was 242.3 J/s.m\(^2\).\(^0\)C. Increase of frying chamber rotation number (rpm) caused increase of unit surface conductance value. While the value was decreased with increase of the steel balls diameter. Thermal conductivity, diffusivity, bulk density, and specific heat of steel balls were 43.25 J/s.m.\(^0\)C, 2.05735E-05 m\(^2\)/s, 4500 kg/m\(^3\), and 473 J/kg.\(^0\)C respectively.
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