A NEW EMPIRICAL MODEL FOR THE STRUCTURAL ANALYSIS OF EARLY-TYPE GALAXIES, AND A CRITICAL REVIEW OF THE NUKER MODEL

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ABSTRACT

The Nuker law was designed to match the inner few (≈3–10) arcseconds of predominantly nearby (≤30 Mpc) early-type galaxy light profiles; it was never intended to describe an entire profile. The Sérsic model, on the other hand, was developed to fit the entire profile; however, because of the presence of partially depleted galaxy cores, the Sérsic model cannot always describe the very inner region. We have therefore developed a new empirical model consisting of an inner power law, a transition region, and an outer Sérsic model to connect the inner and outer structure of elliptical galaxies. We have additionally explored the stability of the Nuker model parameters. Surprisingly, none are found to be stable quantities; all are shown to vary systematically with a profile’s fitted radial extent, and often by more than 100%. Considering elliptical galaxies spanning a range of 7.5 mag, we reveal that the central stellar densities of the underlying host galaxies increase with galaxy luminosity until the onset of core formation, detected only in the brightest elliptical galaxies. We suggest that the so-called power-law galaxies may actually be described by the Sérsic model over their entire radial range.

Key words: galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: nuclei — galaxies: photometry — galaxies: structure

1. INTRODUCTION

Early Hubble Space Telescope (HST) observations of elliptical galaxies and the bulges of disk galaxies, hereafter collectively referred to as “bulges” (Crane et al. 1993; Ferrarese et al. 1994; Jaffe et al. 1994; Forbes, Franx, & Illingworth 1995), confirmed the ground-based conclusions of Kormendy (1985) and Lauer (1985): galaxy models with flat cores (e.g., King’s 1966 model) do not describe the majority of elliptical galaxies, or at least the resolved part of the profile. In almost all galaxies surveyed, the surface brightness profile continued to rise inward until resolution was lost. Subsequently, with the enhanced image quality afforded by HST have come new models to describe the centers of nearby bulges.

Ferrarese et al. (1994) introduced two classes of galaxies according to the behavior of the inner surface brightness profile. Those with a resolved core flattening toward the center were labeled as “type I,” and those that roughly follow a steep power law all the way into the center were designated as “type II” galaxies. These authors introduced a four-parameter double power-law model to quantify the shape of the galaxy profile within the inner ∼10″. A core radius marked the transition between the inner and outer power laws, having slopes $\beta_1$ and $\beta_2$, respectively. All of their “type I” galaxies had an inner slope shallower than −0.31 (none of them had a slope of zero); all but one of their “type II” galaxies had a slope steeper than −0.47.

Modeling a larger galaxy sample, Lauer et al. (1995; see also Kormendy et al. 1994) confirmed the above result though they interpreted it differently—and introduced a model with an additional parameter ($\alpha$), which better controlled the transition between the two power laws. This model was designated the “Nuker law” by these authors. It can be written as

$$I(r) = I_h \left( \frac{r}{r_h} \right)^{-\gamma} \left[ 1 + \left( \frac{r}{r_h} \right)^{\alpha} \right]^{-(\gamma+\beta)/\alpha}. \quad (1)$$

The intensity at the core radius, also known as the break radius $r_h$, is denoted by $I_h$. The inner power-law slope is now denoted by $\gamma$, and the outer power-law slope is denoted by $\beta$. This model reduces to the form proposed by Ferrarese et al. (1994) when $\alpha = 2(\beta_2 - \beta_1)$. Lauer et al. (1995) refer to galaxies with $\gamma < 0.3$ as “core” galaxies and galaxies with $\gamma > 0.5$ as “power law” galaxies.

Application of the Nuker model has proved extremely popular, and there are physical grounds to interpret the reduction in central profile slope and the implied core depletion. Many authors have discussed how the inner region of a galaxy may have been partially evacuated by the coalescence of merging supermassive black holes (SMBHs; e.g., Ebisuzaki, Makino, & Okumura 1991; Makino & Ebisuzaki 1996; Faber et al. 1997; Quillen, Bower, & Stritzinger 2000; Alexander & Livio 2001; Milosavljević & Merritt 2001). Conversely, the presence of “power law”...
cusps has been used to argue for adiabatic growth of central
black holes, with the growing black hole reshaping the
central region (e.g., van der Marel 1999), although
Ravindranath, Ho, & Filippenko (2002) have used the fitted
power-law slopes to argue against this scenario. To better
understand galaxy “cores,” one would like to measure
changes to a galaxy’s inner profile relative to its original
shape.

Recently, the overall “shape” of a bulge’s light profile (as
parameterized by the Sérsic [1968] $n$ shape index $n$) has
been shown to correlate strongly ($r_s = 0.92$) with the mass
of its central SMBH (Graham et al. 2001a, 2003; Erwin,
Graham, & Caon 2003). This implies a strong connection
between the formation and structure of the entire bulge and
the formation of the central black hole. The central regions
of bulges are thus directly related to the global bulge struc-
ture, and so one would like to connect these two regimes.

The Nuker model—with five free parameters—can only
describe the inner light profile of a bulge; it was never
designed to model an entire profile and is thus unable to
make a connection between the inner profile and the overall
bulge structure.7 The Sérsic model—with three free param-
ters—matches the entire radial extent of most bulge light
profiles remarkably well, with the exception of the inner few
arcseconds for some galaxies. By joining, at the break
radius, an outer Sérsic profile with an inner power law, one
might hope to be able to describe the complete light profiles
of bulges when the Sérsic model alone is inadequate.

This issue will be addressed here and in a companion
paper (Trujillo et al. 2003, hereafter Paper II). To do this,
the merits of the individual Nuker parameters will first be
explored in § 2. Section 3 then describes the Sérsic model,
which, for a number of illustrative purposes, is applied here
to the central, early-type galaxy light profiles presented by
Lauer et al. (1995). Given the shortcomings of both models
to describe the complete light profiles of all bulges, a new
empirical model is introduced in § 4 and is illustrated with
application to both a “power law” and a “core” galaxy
profile. In Paper II, we apply the new model to radially com-
plete profiles from a larger sample of early-type galaxies.
Correlations between the global and core properties will be
presented in a forthcoming paper. A recapitulation of the
main points in this paper is provided in § 5.

2. THE NUKER MODEL

In those bright galaxies where Faber et al. (1997 and
references therein) detected a “core,” the break radius $r_b$
and the intensity at this radius ($I_b$) are thought to denote the
onset of a physical transition in a galaxy’s profile. Together
with the central velocity dispersion, Faber et al. (1997; see
also Faber et al. 1987) constructed a “core fundamental
plane,” from which they concluded the following: cores are
in dynamical equilibrium; $r_b$, and $I_b$ are meaningful dynami-
cal parameters (at least in the case of “core” galaxies); velocity
anisotropy does not vary greatly among “core” galaxies; for
most galaxies, the mass of any central SMBH does not domi-
nate the core potential; and the core mass-to-light ratio varies smoothly over the fundamental plane.

Faber et al. (1997) also noted, however, that the value of
$r_b$ (and hence $I_b$) is not robust for “power law” galaxies.
The difficulty the Nuker model has in obtaining stable
parameters for such galaxies, and hence some quantity that
reflects some fixed physical structure, is a consequence of
their smooth, continuously curving profiles, which have no
obvious core.

The Sérsic model has a smooth, continuous profile that in
fact resembles the observed “power law” profiles. The Sérs-
ic model has also recently been shown to provide a good
description of both the outer and inner profiles of HST-
resolved, low-luminosity elliptical galaxies (after accounting
for the central excess flux; Stiavelli et al. 2001; Graham &
Guzmán 2003). Consequently, we explore the natural ques-
tion: Are the so-called power-law galaxies simply the bright
end of these Sérsic $r^{1/n}$ galaxies without (resolved) break
radii, depleted cores, or true central power laws? To further
this idea, we will look at a compilation of galaxies spanning
a large range in absolute magnitude.

Figure 1 shows the central surface brightnesses of the
Nuker team’s elliptical galaxies plotted against their abso-
lute magnitudes. The absolute V-band magnitudes were
obtained from Faber et al. (1997; their Table 2) and con-
verted back into B-band magnitudes using their $B-V$ color
term (their Table 1). The central surface brightnesses shown
here are those of the Nuker model at $r = 0''1$, corrected for
Galactic extinction, and converted to the B band in the same
manner as done for the magnitudes.8 Faber et al. (1997)

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7 A possible exception to this remark comes from the observation that
the outer light profiles of the more massive brightest cluster galaxies can be
well approximated by a power law (e.g., Graham et al. 1996). It should per-
haps further be noted that the Sérsic model tends toward a power law for
large values of $n$.

8 When no $B-V$ color term was given by Faber et al. (1997), a value of
0.9 has been adopted. This may lead to slightly increased scatter in Fig. 1.

![Figure 1](image-url)
wrote that “severe nuclei were ignored in fitting Nuker laws.” This can be seen in the Nuker models fitted to both the “core” (e.g., NGC 6166 and Abell 2052) and “power law” galaxies by Byun et al. (1996). It is noted that the overall placement of the Nuker team’s galaxies in Figure 1 is the same when the actual surface brightness within the inner 0.1, as given in Table 4 of Lauer et al. (1995), is used. Surface brightness values from Nuker models extrapolated to \( r = 0 \) were not used, because of this model’s power-law behavior and, hence, overestimation of the true (finite) central surface brightness.

Added to this diagram are the Virgo and Fornax elliptical galaxies, imaged by Caon, Capaccioli, & D’Onofrio (1993) and D’Onofrio, Capaccioli, & Caon (1994). The S0 galaxies have again been excluded, as we are interested in the properties of bulges (not bulges and disks combined). The observed, model-independent central surface brightness and model-independent magnitude of these elliptical galaxies are shown; there has been no recourse to the Sérsic model to determine these values, although readers are reminded that these galaxies were very well fitted with a Sérsic model by the above authors (see also D’Onofrio 2001). The dwarf elliptical galaxies of Binggeli & Jerjen (1998; their Table 1) are also included in Figure 1. These authors also avoided the nuclear point sources, and so their central surface brightness measurements are from a Sérsic model fitted to the underlying bulge (extrapolated to \( r = 0 \)), uncontaminated by possible nuclear star clusters. Finally, the dwarf elliptical galaxies imaged with HST by Stiavelli et al. (2001) are shown. The magnitudes are from their Table 1, and the central surface brightness values have been read off from the Sérsic fits in their Figure 1 (these readings are probably accurate to 0.05 mag arcsec\(^{-2}\)). The Sérsic fits were also made after excluding any excess central flux. Both the magnitude and central surface brightness measurements from Stiavelli et al.’s galaxy sample have been converted here from the \( V \) band to the \( B \) band assuming a constant \( B-V \) color of 0.9.

It should also be understood that the total central surface brightnesses of many galaxies included in Figure 1 are brighter than what is shown there; what is shown are estimates of the central surface brightness values of the underlying host galaxies. With this understanding, the central galaxy intensity is seen to increase with bulge luminosity (see also Caldwell 1983) such that

\[
M_B \propto -\left(3/2\right) \log I_{0,B} = 3 \mu_{0,B}/5 \quad \text{(Fig. 1),}
\]

but above a certain threshold \( (M_B \lesssim -20.5) \), one observes a reversal of this trend. The bright end of this trend can be seen in Phillips et al. (1996, their Fig. 6) and Faber et al. (1997, their Fig. 4c). Hot galaxies have projected central stellar densities that increase with galaxy luminosity and mass until core formation occurs and a break in the light profile is detected. Our finding would therefore appear to conflict with the interpretation by Faber et al. (1997), who wrote, “A major conclusion is that small hot galaxies are much denser than large ones.” They attributed the observed reduction in central stellar intensity as one progressed to magnitudes fainter than \( M_B \sim -20.5 \) as a resolution effect, using the rare “compact elliptical” galaxy M32 to support this view.

Past modeling of M32’s ground-based light profile excluded the excess flux observed over the inner 10’-15’ (e.g., Kent 1987; Choi, Guhathakurta, & Johnston 2002). If M32 were at the distance of the Virgo Cluster, this central excess would show up only within the inner 1’ and the galaxy would likely be considered “nucleated.” Excluding this unusually sharp core in M32 (Schweizer 1979; Tonry 1984, Graham 2002a) found the underlying bulge component has an exponential-like profile, that is, it has a relatively shallow inner slope, and derived a central surface brightness of \( 15.3 \text{ mag arcsec}^{-2} \) for the underlying bulge. This roughly translates into a central \( V \)-band surface brightness of \( \sim 15.7 \text{ mag arcsec}^{-2} \) and places it in better agreement with the other “power law” galaxies in Figure 4c of Faber et al. (1997). In any case, because there is evidence suggesting M32 contains an outer envelope or disk (Graham 2002a), it has not been included in our Figure 1.

It is noted that the “power law” galaxies in Figure 1 (considered to be intermediate-luminosity elliptical galaxies; Faber et al. 1997) form a continuous extension to the dwarf (low luminosity) elliptical galaxies, which are known to be well described by the Sérsic model (Davies et al. 1988; Young & Currie et al. 1994; Jerjen, Binggeli, & Freeman 2000). Excluding the “core” galaxies, there is no apparent dE-E dichotomy in Figure 1 (see also Jerjen & Binggeli 1997). Since the Sérsic fits to the HST dwarf elliptical profiles of Graham & Guzmán (2003) do an excellent job of describing both the outer and the inner profiles (with a point source used by Graham & Guzmán to fit those galaxies that are nucleated), and since Jerjen et al. (2000) were able to fit the highly resolved (because of their proximity) Milky Way and M31 dwarf spheroidal profiles at all radii with the Sérsic model, it appears that the “power law” centers of low-luminosity elliptical galaxies are simply the inner part of their overall Sérsic profile. Moreover, using the Sérsic-derived (finite) central surface brightness values from the Virgo and Fornax galaxies from Caon et al. (1993) and D’Onofrio et al. (1994), galaxies having \( M_B \gtrsim -20.5 \) overlap exactly with the Nuker team’s “power law” galaxies (Graham & Guzmán 2003). That is, galaxies having the same magnitude as the “power law” galaxies have the same central surface brightness when derived from the inward extrapolation of the outer profile’s best-fitting Sérsic model.

It therefore seems reasonable that the “power law” galaxies may indeed simply be Sérsic \( r^{1/n} \) galaxies without cores. This is of interest because it not only helps to provide a unifying picture of galaxy structure, but also reveals that the break radius for the so-called power-law galaxies is not something intrinsically physical to these galaxies but is simply a parameter in a model that provides a good reproduction of the observed inner light profile. This idea is pursued (and confirmed) in Paper II (see also Fig. 10 in this paper), in which we model the HST profiles of Lauer et al. (1995) and Rest et al. (2001) combined with the outer galaxy profile. That the Sérsic model can describe “power law” galaxies over their entire radial extent is also of interest because one replaces five parameters that have no clear physical meaning with three parameters that do, and which fit the entire profile. Furthermore, the notion that the inner regions of low-luminosity bulges should be treated differently than the outer regions (i.e., that the inner regions are described by a power law and the outer regions by a different function) can be replaced with a single unifying model that treats both regions simultaneously.

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\(^{9}\) It should be noted that the central surface brightnesses of these galaxies were obtained with ground-based CCD imaging under \(-1^\circ\) seeing; the true central surface brightness is therefore higher than shown here.
2.1. $\alpha$, $\beta$, $\gamma$, and $\gamma'$

The function of the parameter $\alpha$ in the Nuker model (eq. [1]) is to allow for varying degrees of curvature in the surface brightness profile, providing a smoother, less abrupt transition between the two power laws. While the transition region is apparently better matched in this way, a problem is known to arise when the transition region is apparently large and $\alpha$ becomes too small. When this occurs, the slope of the power-law components becomes less and less representative of the observed mean logarithmic slope on either side of the break radius, and more representative of the slope to the extrapolated model beyond the boundaries of the fitted galaxy profile. For smaller values of $\alpha$ (i.e., $\lesssim 1$), the presence of two power laws often fails to emerge; instead, one continuous curving arc describes the profile. As a result, the observed inner profile slope of galaxies having small values of $\alpha$ (i.e., $\lesssim 0.1$) is sometimes zero or often does not appear to reflect the observed inner slope, or both. This aspect of the Nuker model’s ability to provide an accurate quantification of the observed inner profile slope was discussed by Rest et al. (2001), who also noted the additional difficulty with the Nuker model either when the break radius is smaller than the image resolution, or when there simply is no apparent break radius. Therefore, in an effort to quantify the innermost resolved profile slope, they used an additional quantity to accompany the Nuker model. Rest et al. (2001) computed the negative logarithmic slope of the Nuker model at 0.1, which they denoted $\gamma'$.

In practice, one may find that the observed inner profile is well approximated by a real power law and the derivative at 0.1 matches the Nuker model parameter $\gamma$, in which case nothing is gained. Conversely, one may find that the inner profile is indeed curved (or its slope is poorly parameterized by the Nuker model because of a small value of $\alpha$) and the local derivative $\gamma'$ does not match $\gamma$. One thus derives a new quantity ($\gamma'$) that is dependent on the radius where it is measured—that is, when the inner profile is not a power law, $\gamma'$ is an apparent, rather than absolute, quantity. If identical galaxies are observed at different distances, then they can have different values of $\gamma'(r = 0.1)$. This was noted by Seigar et al. (2002) but not explored or quantified.

The extent of such changes is illustrated here by computing $\gamma'(r = 0.1)$ from the slope of the Nuker model (Rest et al. 2001, their eq. [8]) fitted to a range of $r^{1/n}$ profiles having $r_e = 10''$ and $n = 1, 2, 3$, and 4 (see § 3). By effectively moving each of these galaxy models 3 times farther away, that is, by simply reducing $r_e$ by a factor of 3, $\gamma'$ changes from 0.00, 0.09, 0.30, and 0.51 to 0.13, 0.36, 0.63, and 0.83 for the $n = 1, 2, 3$, and 4 models, respectively. Thus, initially three galaxies would have been classified as core galaxies, but now only one would be classified as such, even though the actual galaxy structures did not change. Half of the galaxies in the sample of Rest et al. (2001) that are classified as core galaxies using the Nuker model parameter $\gamma$ are not classified as core galaxies using the derivative at 0.1. Some of this mismatch is probably due to the distances the galaxies are at and, hence, what physical radius $\gamma'$ was measured at.

To conclude, $\gamma$ is known to be an extrapolated quantity that does not reflect the observed inner profile slope of galaxies having small values of $\alpha$. The value of $\gamma'$ is the slope of the profile at the innermost resolved point. It is however an apparent rather than an absolute quantity (unless, of course, the inner profile does follow a real power law) and as such does not reflect anything intrinsic to a galaxy and should therefore not be used as such. This statement is of course also true when using the mean logarithmic slope $\langle \gamma' \rangle$ measured over $0'.1 < r < 0'.5$ (e.g., Lauer et al. 1995; Carollo & Stiavelli 1998). Comparisons between the value of $\gamma'$ (or $\langle \gamma' \rangle$) for different galaxies should be made with caution. For example, diagrams showing these apparent quantities versus absolute galaxy magnitude are subject to the distance effects just mentioned.

The outer power-law slope ($\beta$) of the Nuker model depends on how much of the profile’s radial extent one fits; it is therefore definitely not a reliable parameter. This was recognized from the start, and Byun et al. (1996) wrote, “Even galaxies which show good agreement with the Nuker law within 0'' in general will also fail at much larger radii beyond the field covered by the present HST data, as the profiles follow a curving de Vaucouleurs law, not a power law there.” It is therefore not a parameter that need be preserved in any new model that in addition fits the outer light profiles of early-type galaxies.

2.2. Robustness of the Nuker Model Parameters

Figure 2 shows a synthetic “core galaxy” profile. It represents a typical $r^{1/4}$ profile having an inner core. The structural parameters are such that it has an outer de Vaucouleurs profile with effective radius $r_e = 25''$, a break radius of $0'.5$ at $\mu = 14$ mag arcsec$^{-2}$, and an inner power law with slope $\gamma = 0.2$. The radial extent that is fitted with the Nuker model is increased in each subsequent panel (left to right and top to bottom) in Figure 2 in order to demonstrate how the parameters of the fit change.

Not surprisingly, the value of $\beta$ is strongly dependent on the fitted radial range; this was previously known, but possibly never quantified. What will be surprising to many is the unstable nature of all the Nuker model parameters—not just $\beta$—even when fitting a (noise- and dust-free) “core galaxy.” As the fitted radial extent is increased to values typically used by Rest et al. (2001), the Nuker model break radius marches steadily outward. When the mean difference between the synthetic data and the Nuker model reaches $\sim 0.03$ mag arcsec$^{-2}$ (the average value reported by the Nuker team in their fits), the derived break radius is twice the true break radius.

This effect is illustrated again with two real “core galaxy” profiles: NGC 3348, from Rest et al. (2001), and NGC 4636, from Lauer et al. (1995). These are shown in Figures 3 and 4, respectively. Exactly the same behavior as seen in Figure 2 is observed. It turns out that, as a result of the curvature in the profile beyond the break radius, this behavior is common to many “core galaxies” fitted with the Nuker model. Indeed, simply by looking at the published “core galaxy” profiles fitted with the Nuker model (e.g., Ravindranath et al. 2001; Laine et al. 2003), one can see for oneself how the break radii have been overestimated.

Although the covariance error analysis presented for three galaxies by Byun et al. (1996; their Fig. 6) reveals that the $10\sigma$ $\chi^2$-ellipses span typically $\pm 6\%$ of the fitted Nuker model break radius, we have just witnessed that such parameter coupling is not the only source of uncertainty for the Nuker model parameters. Figure 3 reveals that reducing

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10 A factor of 3 in distance corresponds to the range of distances in the galaxy sample of Carollo & Stiavelli (1998), who computed $\langle \gamma' \rangle$ over $0'.1$--$0'.5$. 

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the fitted radius by a factor of 5—equivalent to imaging the same galaxy 5 times closer using the fixed HST Planetary Camera aperture—can change \( r_b \) (in kiloparsecs) by a factor of 3. These previously unconsidered systematic errors dominate over the random errors considered by Byun et al. (1996). In addition, because \( \alpha \) couples with \( \beta \) in the Nuker model, its value is also dependent on the radial range used. This can in turn affect the value of the inner power-law slope, \( \gamma \). As a result, the Nuker model's parameters are not always robust quantities: they are sensitive to the radial region that is fitted.

3. THE SÉRSIC MODEL

Sérsic's (1968) \( r^{1/n} \) generalization of de Vaucouleurs's (1948) \( r^{1/4} \) model has proved hugely successful in describing the light profiles of dwarf and ordinary elliptical galaxies and the bulges of spiral galaxies. Early work includes that by Davies et al. (1988), Capaccioli (1989), Caon, Capaccioli, & D’Onofrio (1993, 1994), Young & Currie (1994), James (1994), and Andredakis, Peletier, & Balcels (1995).

Recently, Graham, Trujillo, & Caon (2001b; see also Graham 2002b) showed a strong correlation \( r > 0.8 \),
significance greater than 99.99%) exists between the Sérsic shape parameter $n$ and literature velocity dispersion measurements for those early-type galaxies studied by Caon et al. (1993) and D’Onofrio et al. (1994). Central stellar velocity dispersions are, of course, completely independent from estimates of $n$ obtained from the galaxy light profile. This clearly shows that the Sérsic index $n$ is not simply an extra parameter added to improve the fits of bulge light profiles, but traces real physical differences in galaxies. A number of authors have suggested that the different profile shapes are connected to the gravitational potentials and total masses of the bulges (e.g., Caon et al. 1993; Andredakis et al. 1995; Hjorth & Madsen 1995; Seigar & James 1998; Márquez et al. 2000, 2001; Trujillo, Graham, & Caon 2001).

The radial intensity distribution of the Sérsic model is given by the expression

$$I(r) = I_e \exp \left\{ -b_n \left[ \left( \frac{r}{r_e} \right)^{1/n} - 1 \right] \right\},$$

where $I_e$ is the intensity at the half-light radius $r_e$. The quantity $b_n$ is a function of the shape parameter $n$ and is defined so that $r_e$ is the radius enclosing half of the light of the galaxy model; it can be approximated by $b_n \approx \frac{2.2}{n}$.

![Fig. 3. — Nuker model fits (solid and dotted lines), using progressively smaller radial ranges, applied to the major-axis F702W surface brightness profile of NGC 3348 (from Rest et al. 2001). The radial extent of the fitted data (filled circles) decreases from left to right and top to bottom. Although every fit looks acceptable, the actual Nuker model parameters (listed in each panel) vary systematically with the fitted radial extent (cf. Fig. 2). Rest et al. (2001) reported a break radius of $r_b = 0^\circ 99$ for this galaxy, based on their Nuker model fit.]}
1.9992n − 0.3271, for 1 ≤ n ≤ 10 (see, e.g., Caon et al. 1993; Graham 2001). Figure 5 illustrates the behavior of the $r^{1/n}$ model for values of $n$ ranging from 1 to 10; $n = 4$ reproduces the de Vaucouleurs model, while $n = 1$ reproduces an exponential profile.

It is clear from Figure 5 that profiles with low values of $n$ (which observations show us are the low-luminosity bulges; see, e.g., Fig. 11 of Graham et al. 1996) have “cores” (using the definition $\gamma < 0.3$), while profiles with large values of $n$ (the brighter bulges) would be described as “power law” galaxies. However, this is exactly the opposite of what Faber et al. (1997) found (see, e.g., their Fig. 4; see also Fig. 7 of Rest et al. 2001): cores are found in the brighter bulges, while it is the relatively fainter bulges that have power-law centers. To avoid confusion, it is important to distinguish between what might be called “apparent cores” from low-$n$ galaxies and cores that have possibly been created by supermassive black holes in high-luminosity (high-$n$) galaxies. The former should perhaps not even be referred to as “cores” at all, because they do not represent any departure from the inward extrapolation of the outer galaxy profile. Because there were very few low-luminosity galaxies (with probable Sérsic indexes ≤3) in the Lauer et al. (1995) sample, the ambiguity between “apparent” and “real” cores did not become an issue. Studies of lower
luminosity elliptical galaxies and spiral galaxy bulges are, however, more problematic.

Figure 5 also reveals that the inner profile slopes, when measured over the same radial range (in terms of fraction of the effective radius), should be equal for any sample of bulges with the same Sérsic shape. If, however, one used a fixed angular range (e.g., in arcseconds), for a sample of galaxies at a range of distances (and/or with intrinsically different scale lengths), then one will obtain a range of different inner profile slopes, even if the galaxies all have the same structural shape (as illustrated in § 2.1).

For the Sérsic model, it is simple to show that

$$\gamma'(r') = -d \log I(r') / d \log r$$

is equal to

$$\gamma' = 2(r'/r_e)^{1/n}. \tag{4}$$

This can be approximated by $2(r'/r_e)^{1/n}$. Thus, at constant $(r'/r_e)$, $\gamma'$ is a monotonically increasing function of the Sérsic index $n$. Solutions for $\gamma'$ are shown in Figure 6.

The value of $n$ is well known to increase with bulge luminosity, and $n$ is consequently a function of position along the L-shaped trend seen in Figure 1. Given the correlation between $n$ and $\gamma'$ in Figure 6, one would expect to see $\gamma'$ increase with bulge magnitude until a core starts to appear at the higher luminosity end.$^{11}$ This is indeed what is found in Graham & Guzmán (2003, their Fig. 8).

It was suggested in § 2 that the so-called power-law galaxies are actually Sérsic $r^{1/n}$ galaxies. To explore how the Nuker model can imitate a pure Sérsic profile when fitted to a restricted radial range, Figure 7 displays the results of fitting Nuker models to four $r^{1/n}$ models with values of $n$ equal to 1, 2, 3, and 4. The fitting has been done in such a way as to try to match the break radius to the radius where the change in the logarithmic profile slope is observed to be a maximum. What this means is that the fitting routine was prevented from setting the break radius to infinity$^{12}$ (although, given the results in the literature, $r_b$ never tends to infinity—possibly because of restrictions placed in the codes to keep $r_b$ bound). One can clearly see that fitting a five-parameter function (the Nuker model) to a limited radial extent of a three-parameter function (the Sérsic model) can result in what many would consider a satisfactory fit.

### 3.1. Modeling the Nuker Profiles with a Sérsic Model

Major-axis surface brightness profiles for 42 predominantly early-type galaxies imaged with HST are given in tabular form by Lauer et al. (1995). A logarithmically spaced sampling of the light profiles was used, providing greater detail in the central regions. These profiles were extracted from pre-refurbishment HST Planetary Camera images (taken with the F555W filter) and then deconvolved to account for the effects of spherical aberration (see Lauer et al. 1995 for details). Nuker models were fitted by Byun et al. (1996).$^{13}$

Figure 8 shows the results of fitting the three-parameter Sérsic model to the first eight NGC galaxies in the sample of Lauer et al. (1995). Figures for the remaining galaxies show the same behavior and are therefore not shown here. The data within the inner 0"13, the radius inside of which the profiles were deemed unreliable by Lauer et al., were not included in the fitting routine. Quite clearly, the inner $\sim 10''$ of some galaxies are very well modeled with the three-parameter Sérsic model. This is true for galaxies labeled by the Nuker team as either "power law" galaxies (e.g., NGC 1023, NGC 1172) or "core" galaxies (e.g., NGC 720, NGC 1399). Relative to the Sérsic model, NGC 1331 displays evidence for a large excess flux within the inner 0"3. On the

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$^{11}$ The Sérsic model is known to fit the bulk of a bulge’s light profile, but for large values of $n$ its rising inner profile cannot describe the presence of cores in large luminous elliptical galaxies (Kormendy 1985; Lauer 1985). From Fig. 1 and Fig. 6, it is predicted that a better sampling of galaxies with $M_B \approx -17$ mag and Sérsic indices $1.5 \leq n \leq 3$ (i.e., the brighter dwarf elliptical galaxies) should reveal more galaxies with $0.3 < \gamma' < 0.5$. However, there is the issue of galaxy distance.

$^{12}$ The Nuker model is equivalent to the $r^{1/n}$ model when $r_b \to \infty$, $\gamma = 0$, and $\alpha = 1/n$ (Byun et al. 1996).

$^{13}$ Byun et al. (1996) fitted the Nuker model to the mean profiles, rather than the major-axis profiles given by Lauer et al. (1995).
other hand, NGC 1400 appears to have a small core within the inner ∼0″2.

Despite the results of Figure 8, we are not trying to argue that one should fit the inner ∼10″ of an appreciably larger profile with a Sérsic model. Instead, we want to demonstrate the inadequacy of the inner profile for drawing reliable conclusions unless one has further information. For example, with only knowledge of the inner profile, is there really an evacuated core in NGC 720? Fitting a Sérsic model, one would say no; fitting a Nuker model, one would say yes.

In the following section, we advocate a new definition for a core, specifically, a deficit of central starlight (not due to dust) relative to the inward extrapolation of the outer light profile. This differs from the Nuker team’s definition, which is dependent on the inner profile slope.

4. A NEW EMPIRICAL LIGHT-PROFILE MODEL

We have discussed how fitting a Nuker model could lead to a false conclusion regarding the existence of a (partially evacuated) core. In addition, we have just seen in Figure 8 that if one only has the inner portion of a larger profile, then a Sérsic model is also capable of fitting the data—even when a real core may be present. One obvious way to avoid this potential confusion and at the same time enable one to connect the inner and outer galaxy structure is to increase the radial extent of the galaxy’s surface brightness profile one is investigating. Given that galaxies with evacuated cores probably do exist, we need a new model to describe the entire radial extent of a profile. Of course, the additional data points in the outer profile provide more information than contained in the inner few arcseconds and therefore enable one to determine an additional parameter beyond the capabilities of the Nuker model. Specifically, one can fit for the curvature in the outer profile, where a power law is known to be inadequate for the majority of galaxies.

Modifying the Sérsic model through the inclusion of an inner power law, or, similarly, modifying the Nuker model through the transformation of the outer power law to a Sérsic function, one obtains the expression

\[ I(r) = I_0 \left[ 1 + \left( \frac{r_b}{r} \right)^{\frac{n-1}{n}} \right] \exp \left[ -b_n \left( \frac{r^n + r_b^n}{r_e^n} \right)^{1/(an)} \right], \]  

where \( r_b \) is the break radius separating the inner power law, having logarithmic slope \( \gamma \), from the outer Sérsic function, having a shape parameter \( n \) and effective half-light radius \( r_e \). The quantity \( b_n \) is a function of \( n \) and has the usual meaning (see § 3). By leaving \( b_n \) defined this way, the value of \( r_e \) is the effective half-light radius of the outer \( r^{1/n} \) profile beyond the transition region, and not the half-light radius of the new model. The effective surface brightness of the outer Sérsic profile is obtained by setting \( r = r_e \) and \( r_b = 0 \) in equation (5), while \( r_b \) retains its value in equation (6) below. The intensity \( I_b \) at the break radius \( r_b \) can be evaluated from the expression

\[ I' = I_b 2^{-\gamma/a} \exp \left[ \ln \left( \frac{r_b}{r_e} \right)^{1/n} \right]. \]  

The final parameter, \( \alpha \), controls the sharpness of the transition between the inner (power law) and outer (Sérsic) regimes—higher values of \( \alpha \) indicate sharper transitions. It can be held fixed (e.g., \( \alpha = 100 \) is a good approximation to a perfectly sharp transition), or it can be varied if one is interested in accurately matching the transition region. In

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**Fig. 7**—Four Sérsic profiles (circles) with \( n = 1, 2, 3, \) and 4, \( r_e = 10" \), and \( \mu_0 = 20 \text{ mag arcsec}^{-2} \). The solid lines are Nuker model fits to the data points (filled circles); the dashed lines are extrapolations outside the region of the fit. The Nuker model break radii are indicated with arrows.
practice (see Paper II), we find that a sharp transition ($\alpha \gtrsim 3$) may be preferable; high values of $\alpha$ also minimize possible coupling of the parameters, which may compromise the inner profile slope (as can happen with the Nuker model).

Despite the previously discussed issues associated with the inner power-law slope of the Nuker model (§ 2.1), $\gamma$ has remained a quantity of interest in the literature. Even with a sharp transition (i.e., large values of $\alpha$), our new empirical model has curvature built into it through the parameter $n$. Consequently, the problems that the Nuker model had in trying to accommodate curvature (via small values of $\alpha$) should not plague this new model, and values of $\gamma$ derived from fitting equation (5) should more accurately reflect the observed inner profile slope.

Despite appearances, equation (5) is remarkably simple. When $r$ is less than $r_b$, equation (5) approximates a simple power law with slope $\gamma$; when $r$ is greater than $r_b$, equation (5) represents a Sérsic model. There is however a variable transition region, as mentioned above, which depends on the value of $\alpha$. Setting $r_b$ and $\gamma$ equal to zero, one recovers a pure $r^{1/n}$ model at all radii. In addition, when $n \to \infty$ it can be shown that equation (5) approximates two power laws separated at $r_b$. Figure 9 shows equation (5) for different sets of parameters for this new model.

Figure 10 presents two examples in which the new empirical model has been applied to real galaxies. One can see that the "power law" galaxy NGC 5831 from Rest et al. (2001) is well described by a Sérsic model over its entire observed radial range ($0''1$ to $\sim 3r_e$). On the other hand, NGC 3348...
from Rest et al. (2001) clearly displays a flattening of the inner profile relative to the outer Sérsic profile; this break is well matched by the new model. In Paper II, we present these and similar fits to approximately 20 bona fide elliptical galaxies. Using archival HST data matching that of the inner profiles from Lauer et al. (1995) or Rest et al. (2001), we verify that “power law” galaxies have pure Sérsic profiles, while core galaxies are well fitted by our new model.

We have explored the stability of the parameters in the new model and found that in order to obtain a robust estimate of the curvature in the outer profile, one requires a profile that extends to, typically, at least the half-light galaxy radius. However, we would advocate that one fit as much of the profile as possible, preferably out to (2–3)r_e. As our main drive has been the issue of recovering core sizes, we remark here on our fits to the “core” galaxy NGC 3348. Fitting equation (5) to our full 70″ profile (Fig. 10), we obtained a break radius of 0′′45, μ_b = 15.31 mag arcsec−2, γ = 0.18, r_e = 21″82, and n = 3.87. Truncating the profile one data point at a time, these numbers changed by at most 5%. Upon reaching a profile with a radial extent of 21.5″ (∼1r_e), application of equation (5) yielded r_e = 0′′45, μ_b = 15.31 mag arcsec−2, γ = 0.18, r_e = 20″84, and n = 3.81, with an rms scatter of 0.028 mag for the fit. Thus, use of the complete galaxy light profile (or at least enough of the profile to reliably determine the curvature beyond the break radius) can enable one to make robust estimates of the core size. Doing this, the core size we obtained for NGC 3348 is more than a factor of 2 smaller than the value published in Rest et al. (2001). The reason for this is apparent from Figures 2–4. Fitting the full 55″ profile of NGC 5831 yielded r_e = 28″28 and n = 4.91; fitting to only 27″3 (∼1r_e) gave r_e = 27.00 and n = 4.83, within 5% of each other.

Lauer et al. (1995) defined a “core” to be “the region interior to a sharp turndown or break in the steep outer brightness profile, provided that the profile interior to the break has γ < 0.3.” We suggest an alternative definition, such that a core refers to a deficit (not due to dust) of flux relative to the inward extrapolation of the outer Sérsic profile. A “core” would thus refer to something that is likely to be real, rather than an apparent feature that can appear in plots of μ versus log r (see Fig. 7).

Consideration of this new model (eq. [5]) is also suggested for studies requiring a realistic gravitational lens model for elliptical galaxies at intermediate redshifts (e.g., Evans & Wilkinson 1998; Muñoz, Kochanek, & Keeton 2001; Chae 2002; Keeton 2003). Other interesting areas of research are the feeding of SMBHs via the capture of stars and dark matter (e.g., Zhao, Haehnelt, & Rees 2002) and the evaluation of the central mass deficit possibly excavated by coalescing massive black holes (Ebisuzaki et al. 1991; Faber et al. 1997; Milosavljević & Merritt 2001; Milosavljević et al. 2002; Ravindranath et al. 2002; Komossa et al. 2003). Current research assumes that galaxies initially had the same inner profile15 [such as an n1/4 profile or a density profile in which ρ(r) ∼ r−2; e.g., Volonteri, Haardt, & Madau 2003] and compares this with how the profile actually looks. It might be of interest to replace this assumption with the inward extrapolation of the observed outer Sérsic profile. Bulges are not structurally homologous systems; their properties vary systematically with total luminosity and mass. To assume structural homology very likely introduces a systematic bias into these types of analysis. Lastly, Ravindranath et al. (2002) have used the break radii from the Nuker model to estimate the evacuated core masses in their sample of galaxies (Ravindranath et al. 2001) and those from Rest et al. (2001). This may explain why they found a weaker trend than expected, and one with considerable scatter, between the central SMBH mass and the ejected mass. This issue is explored in Paper II.

5. SUMMARY

None of the five parameters of the Nuker model are found to be robust. Hence, they cannot represent any fixed physical quantity. Recent methods that have tried to circumvent one of the Nuker parameters by measuring the inner power-law slope at some fixed radius in arcseconds (γ′), but without taking galaxy distance or size into account, are shown (quantitatively) to be subject to strong biases. Measured values of γ′ (and γ′) are not intrinsic to a galaxy and can change considerably if the same galaxies are located at different distances without any actual change in the intrinsic galaxy structure.

As observed by previous authors (e.g., Caldwell 1983; Jerjen & Binggeli 1997), dwarf elliptical galaxies form a continuous extension to the intermediate-luminosity elliptical galaxies in the central surface brightness–absolute magnitude plane (Fig. 1), such that M_B ∝ 3μ0.8/5. Faber et al. (1997) wrote, “A major conclusion is that small hot galaxies are much denser than large ones” and that “the apparent turndown in [central] surface brightness at faint magnitudes . . . is probably a resolution effect.” However, observations of the underlying host galaxy (i.e., excluding nuclear sources) reveal, over a range of 7.5 mag, that small hot

14 NGC 4636 (Fig. 4) is not shown because, given its size of 6′ × 4′, we have no outer light profile in the HST image.

15 Current assumptions that faint elliptical galaxies (the assumed building blocks of core galaxies) have isothermal cusps with ρ ∼ r−2 may not be correct. Low-luminosity elliptical galaxies have small values of n and, therefore, shallow inner cusps (Fig. 5; see also Graham & Guzmán 2003), although they may possess additional central components.
galaxies are actually less centrally dense than larger hot galaxies—this is not an artifact of resolution. We instead attribute the observed turndown in central surface brightness at bright magnitudes (M_B ≤ -20.5) to the presence of galactic cores. Lastly, it is noted that the lower luminosity elliptical galaxies are known to be well described by the Sérsic model, and it is suggested here that the “power law” galaxies may in fact simply be Sérsic r^{1/n} galaxies with no resolved core. The three-parameter Sérsic model provides a remarkably good fit to the 42 inner galaxy light profiles initially studied by the Nuker team. In some cases the quality of the fit may be due to the so-called power-law galaxies’ being Sérsic r^{1/n} galaxies all the way into the resolution limit, while in other cases it is probably a result of the limited radial extent of the profiles.

Whether a “core” represents a real physical change or just an apparent change in profile slope can be determined by looking at the entire light profile, rather than just the central profile, and we advocate a new definition for a “core” as a deficit in central flux relative to the outer Sérsic profile. The results of doing this are shown in Paper II for a sample of bona fide elliptical galaxies. In order to model the entire light profile, a new empirical model comprising an outer Sérsic function and an inner power law has been developed and presented here and will be described in greater mathematical detail in Paper II.

By combining an inner power law with an outer Sérsic profile, one should be better able to

1. Explore where and how the r^{1/n} model fails to provide a good match to the inner light profile and thereby test whether the so-called power-law galaxies are actually galaxies described by an r^{1/n} model down to the resolution limit (i.e., having no resolvable cores, and not having an inner power-law profile);

2. Quantify central excess fluxes known to exist in many galaxies;

3. Search for connections between the shape of a galaxy’s outer profile, as represented by n, and the properties of its core;

4. Quantify the slope and break radii of “cores” normalized to the galaxy’s effective half-light radius;

5. Model the gravitational lensing deflection caused by distant elliptical galaxies; and

6. Search for correlations between supermassive black hole mass (possibly derived from the log n–log M_{BH} relation; Graham et al. 2001a, 2003; Erwin et al. 2003), break radii, and the central flux/mass deficit in galaxies having partially evacuated cores.

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Fig. 10.—Two HST major-axis surface brightness profiles of elliptical galaxies. The inner r ≤ 15″ are the (deconvolved) profiles from Rest et al. (2001), while the outer points are from the full WFCPC mosaic of the same exposures (see Paper II for full details). The solid lines are fits using the new empirical model of eq. (5), with inner and outer extrapolations indicated by the dotted extensions. Extrapolations of the outer, Sérsic-like part of the model inward, past the break radius, are indicated by the dashed lines. For NGC 5831, a “power law” galaxy according to Rest et al. (2001) with r_b = 1.78, our best fit is essentially a pure Sérsic model. In NGC 3348, which Rest et al. classified as a “core” galaxy with break radius r_b = 0.99, there is a clear inner break from the Sérsic profile; we find r_b = 0.45. The rms scatter Δ mag for the fit is given in each panel.
