Runout transition and clustering instability observed in binary-mixture avalanche deposits

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Abstract
Binary mixtures of dry grains avalanching down a slope are experimentally studied to determine the interaction among coarse and fine grains and their effect on the morphology of the deposit. The distance traveled by the massive front of the avalanche on the horizontal plane of deposition area is measured as a function of mass content of fine particles in the mixture, grain-size ratio, and flume tilt. A sudden transition of the runout is detected at a critical content of fine particles, with a dependence on the grain-size ratio and flume tilt. This transition is explained in terms of the depth-averaged segregation models that describe how large particles are transported preferentially towards the avalanche front and accumulate there. Segregation by sizes during the avalanching and deposition stages produces distinct morphologies of the final deposit as the coarse-particle content is increased until full segregation and a split-off of the deposit into two well-defined separated deposits occur for certain size ratios. The formation of a separated distal deposit, in turn, depends on a critical number of coarse particles. A large number of dispersed coarse particles allows the condensation of the pure-coarse deposit around a small, initial seed cluster, which grows rapidly by braking and capturing subsequent colliding coarse particles. For different grain-size ratios, keeping the total mass constant, the change in the amount of fine particles needed for the transition to occur is found to be always less than 7%. For avalanches with a total mass of 4 kg we find that, most of the time, the runout of a binary avalanche is larger than the runout of monodisperse avalanches of corresponding constituent particles, due to lubrication on the coarse-dominated side or to drag by large particles on the fine-dominated side.

Keywords Binary avalanches · Granular flows · Granular clustering · Segregation process

1 Introduction
Rock avalanches and geophysical granular flows have been broadly studied to understand the potential risks they represent and the modifications they make on the landscape. They mobilize millions of cubic meters of rocks, with sizes ranging from microns up to tens of meters, running at velocities that, in some cases, are faster than 100 km/h [1–5]. Since their behavior depends on a myriad of parameters, such as particle-size distribution, hardness, textural characteristics, friction and restitution coefficients, kind of triggering event, and the environment in which they develop, travel, and deposit, they are very complex phenomena, which are difficult to predict and/or to be modeled. Any interstitial
Fluid, like hot gas (such as in pyroclastic density currents) or water (like in mudflows and lahars), also plays a fundamental role in the flow behavior, due to lubricating effect, which, in turn, increase the associated risk. For this reason, experiments with model particles and simple geometries are performed at laboratory scale to understand the main features of the avalanching process. In this simplifying spirit, monodisperse avalanches and the role that basal friction has on the runout have been systematically studied, by [6]. They found a minimum basal friction, which depends on particle size, and developed a simple model to determine how the geometric parameters of the rough base modify the runout of the avalanche. Several groups have studied the origin of the scaling laws that describe the length and height of the deposit as a function of the initial conditions [7, 8] and coefficients of restitution of materials [9]. Other observations relate the distance traveled by avalanches with the topography of the terrain or with the presence of model forests at the slope change [10]. Several other mechanisms may affect the mobility of avalanches, for example, dynamic fluidization [11], fluid pore-pressure increment, or ball-bearing effects induced by the presence of fine particles within large inter-particle voids [12, 13]. Several hypotheses regarding the mechanisms that lead to long runouts and the mobility of rock avalanches are summarized in [14] and references therein. Increased mobility of avalanches containing large and fine grains is also observed in numerical simulations [15]. In summary, particle–particle and particle–base interactions have been the object of thorough research to shed some light on the problem of how the multiple factors of a real avalanche are related with the final developed deposit and its runout, height and length, to better predict the risks, they could represent. During their development, avalanches present an irregular flow characterized by an intermittent motion and density heterogeneities due to position-dependent flume grain interactions and heterogeneous dilation. Van Gassen and Cruden [16] developed a model in which they describe the increments in runout as due to intermittent deposition of a dry granular flow. Later, stick-slip instability in the motion of the flow down a slope was reported by [17] by means of a medium-size, densely instrumented experimental flume [18] and found evidence of this intermittent behavior in polydisperse, dry granular flows and an increment of the affected area through the ejection of ballistic projectiles. Several other kinds of instabilities such as fingering [19] and Kelvin Helmholtz-like patterns [20] were investigated in small flume experiments showing striking similarities with natural deposits. Stick-slip motion will very likely produce reminiscent features in the final deposit. Seeking for these reminiscences, [21] performed analogous model experiments aimed to reproduce and understand the origin of hummock formation and the dynamics of spreading and aggregation. The intermittent motion causes concomitant faulting and sliding zones that will finally form the characteristic elevations and mounds scattered on top of the deposition area. In field observations, intriguing hummocks, constituted mainly by coarse grained rocks and situated beyond the avalanche front, have been described and interpreted by [2]. On increasing complexity, mixtures of two particle sizes in different proportions have also been investigated. In these cases, ubiquitous segregation occurs in different forms: levees, vertical segregation by kinetic sieving, inertial dispersion of ballistics, etc. Several authors report a non-monotonic behavior of the runout (mobility) of avalanching masses as a function of fine-particle content [10, 22–24]. Likewise, [25] used rough, irregularly shaped materials mixed with smooth, rounded grains to describe how both materials interact with each other and with the base while they flow down the slope. They observed a levee formation that guides the avalanche by channeling or confining the flow of fine grains by the so-called “outline effect”. Similarly, [25] observed the growth of the runout up to a maximum occurring at a critical proportion of fine particles. In binary-mixture avalanches, kinetic sieving due to dilation and percolation of small particles towards the bottom of the granular flowing bed occurs as it slides down the flume. Large particles are thus segregated towards the top of the moving granular bed and are further transported to the avalanche front due to faster movement of upper layers with respect to lower ones. A recirculation of this coarse-grained avalanche front produces avalanche profiles of fine particles sandwiched and preceded by coarse ones observed [26] and elegantly described theoretically by Gray and coworkers [27] by means of the derivation of a segregation-remixing equation. This model allowed a comparison against carefully conducted experiments showing a strikingly good agreement [28]. Fingering instability in binary mixture avalanches has been addressed by Baker and coworkers [29], who observed that mixtures of rough small particles with large smooth ones does not produce fingering instability, but instead, large particles just shear off the top of fines, which eventually deposit on top of the chute. In those observations, the segregation of fine and large particles leads to differentiated deposits. However, up to our knowledge, these separated deposits have never been systematically investigated. On the other hand, [30] performed two-dimensional simulations of bi-disperse material propagating down an inclined plane showing the development of an unstable flow front rich in large particles, which subsequently breaks into a series of finger-like structures, each bounded by coarse-grained lateral levees. This fingering instability arises from incorporating viscous terms in the momentum balance equations, which were derived by the Groupement de Recherche Milieux Divisés (GDR-MiDi) [31], the so called µ(I)-rheology for dense granular flows, showing good agreement with experiments [7, 31–33]. Furthermore, by coupling a depth-averaged description of the
preferential transport of large particles to a shallow-layer model of a granular avalanche and incorporating segregation-mobility feedback effects on the avalanche dynamics into the avalanche model by allowing the bulk friction coefficient of the avalanche to vary as a function of the local concentration of small particles, [30] were able to numerically reproduce fingering formation in bi-disperse avalanches.

In summary, a critical fine-particle content is responsible for the maximum mobility of the avalanche by lubricating interactions among large particles and between large particles and the base acting as bearing balls, and it has been well established for binary avalanches. All these reported data suggest that the runout is a continuous function of the fine content. However, in this work we report that for constant volume avalanches each size component has its own very well differentiated runout. During the flow, the avalanche spreads out and segregates by size, continuously changing the effective composition of the mixture in different locations. The avalanching mass is thus a non-homogeneous mixture of two dynamically changing proportion of species along the channel until it completely stops. The different runouts of each species would lead, in the case of a complete separation during the flow, to a twofolded runout value or a discontinuous transition of the runout as a function of fine-particle content, never observed or reported before. The main objective of this work is to systematically address the runout value in binary avalanches with enough contrast in grain-size species, finely exploring the behavior of the avalanche runout around the critical value where the runout reaches its maximum, modifying the fine-particle content in steps as small as 2.0%. Changes in runout account for a different rheological behavior and could help in developing risk-management strategies and in providing a more accurate interpretation of past geological flows from their deposits. The purpose of our experiments is to understand the interactions between dry, irregular, natural particles running down a flume. Particle–particle interactions (collisions and friction) may be changing dynamically, so the microscopic (grain-level) behavior, must influence the macroscopic (flow-level) behavior and, hence, the velocity of the flow and its energy dissipation per unit time. The final runout and the morphology of the deposit must be a direct consequence of the interactions of all the grains.

2 Experimental setup

We built a laboratory medium-scale (2.5-m long, 15-cm wide), variable-tilt (0–45 flume) see Fig. 1a, followed by a 1.8-m-long and 1.2-m-wide, horizontal deposition area. Both the flume and the deposition area were built using 15-mm thick, smooth, medium-density fiberboard (MDF) sheets. The flume base was covered by a hard, smooth Formica sheet to reduce the basal friction, while the deposition area was brush painted with a vinyl paint to obtain some roughness. The walls in both sections consisted of a 15-cm-height, 6-mm-thick, transparent, tempered glass. An open-frame, metallic structure (50 cm by 50 cm by 2.5 m) was employed to hold the flume at the desired tilt. A manual gate of the same 15 cm height was placed 2 m up-stream (from the break in slope) to accumulate the granular material with its free surface horizontal and start the avalanche with zero velocity. This gate was connected to a micro-switch that allowed the automatic trigger of a high resolution (2208 × 1244 px), high-frame-rate (120 fps) video camera (Sony HDR-HR150). We also painted a square grid, 5 cm in side, over the base of the deposition area to allow a better scaling and measurements of the runout and the morphology of the avalanche deposit. For all the experiments, we used irregular, rough, natural particles collected in the same area to ensure uniformity of density (2.6 kg/dm³), morphology,
and textural features (Fig. 1b). This material (andesite clasts from a deposit of a block and ash flow at Nevado de Toluca volcano in Mexico) was carefully sieved in order to obtain grain-sizes classes, based on the Wentworth scale [34], from $-4\phi$ (16 mm) to $4\phi$ (0.0625 mm) mean grain diameter. We also sieved the granular material to obtain intermediate grain-size classes: $-1.5\phi$ (3 mm), $0.5\phi$ (1.5 mm) and $1.5\phi$ (0.375 mm). The total mass obtained for each granular class was 4 kg. We used standard sieves, so a monodisperse set of grains means that it contains all the grains that pass through the mesh, moreover, to ensure a better grain-size selection, the loads of grains were sieved twice.

3 Methods

Different granulometric classes were characterized by determining their repose angle and static friction coefficients. For doing so, collapsing-granular-column experiments and tilted table tests were performed ten times for each particle size. For repose angle determination, an 8-cm-radius tube was filled with 469 g of granular material, and the tube was slowly and vertically removed in such a way that the granular column collapses into a conical mound. The true shape of the mound is shown in Fig. 2, were three different angles are defined. The repose angle is represented by $\sigma$, whereas $\beta$ and $\varepsilon$ represent respectively the smooth slope of the material that collapses after the pile formation and the full-pile angle related to $\sigma$ and $\beta$.

Static friction coefficients were determined by gluing a layer of granular material on the base of a wood slab, which was weighted and checked for the critical angle at which it starts sliding down over a smooth metallic plate.

Static-friction angles and repose angles are plotted in Fig. 3 as a function of the granulometric class, $\phi$, showing an increasing trend as the particle size diminishes. Static friction coefficients and repose angles for some particle sizes were measured again at the end of experiments, corroborating that no changes were induced by fragmentation on the material due to frictional and collisional interactions.

We performed experiments aimed to understand the behavior of dry avalanches containing only particles of two different sizes mixed together. We combined different grain classes maintaining the total mass of the granular material constant (4 kg) and then changing the relative proportion of each class. The first run was always performed with the coarsest grained sample, while the second was the finest grained one, both cases having 4 kg of particles, as stated before. This methodology rules out the possibility of contamination with fine particles of larger grains masses. Before each experiment, we slowly poured all the grains (previously sieved) in a bucket, slowly turning with a ladle, trying to get the best mixture for a given proportion by weight of both particle sizes. We performed experiments with the flume tilted at 32° and at 37°, maintaining, in all cases, the deposition area horizontally. The set of experiments is summarized in Table 1.

The video camera was placed in front of the flume and perpendicular to the deposition area, to have almost all the flume and part of the deposition area in the field of view. The camera was triggered automatically by a micro-switch connected to the gate, which was used to start the avalanche. Several markers on the flume and on its walls and the grid painted on the deposition surface helped to measure the velocity and the horizontal displacement of the granular flow body and that of scattered high-velocity

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**Fig. 2** Experimental setup and definition of the angles measured after the formation of the pile

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After each experiment, we sieved the granular material, weighted the amount to be used in the next run, and mixed all the grains again. To ensure that the experimental conditions were almost the same for all the experiments, we carefully cleaned the flume and the deposition area, because fine and very fine grains, in small quantities, increase the mobility of the avalanche, as we will show in the next section. The flume was cleaned each time, first with a brush and, if very small particles were found on the base or walls, with a wet cloth. If this was the case, the flume was dried with a hairdryer. The granular material used in each experiment was always from the same initial set of 4 kg, to maintain the same textural and morphological characteristics. All the materials selected for the flume (Formica and tempered glass) and the deposition area are almost static-free. We performed several tests in order to find out which grain size was the most affected by electrostatic attraction, sprinkling the flume at several tilt angles. It turned out that only 62-µm (4φ) and finer particles were significantly adhered to the base and the walls of the flume; consequently, we chose to not perform experiments with those grain sizes.

### 4 Results and discussion

In the following sections, we will discuss the results of the experiments and compare them to show that they are consistent. We will show that segregation of particles depends on particle size and their content ratio. Finally, we propose that granular clustering is partially responsible of the peculiar deposit morphologies observed experimentally. We also propose that granular clustering can be an alternative or an associated process involved in the hummock-formation mechanism for which there are several explanations in the literature [2, 4, 5, 21, 35, 36].

#### 4.1 General description of the deposit

In Fig. 4a, a typical deposit of a binary avalanche is shown. In this case, the coarsest particles measure 8 mm (55% by mass), while fine grains are 0.375 mm (45% by mass). Clearly, the deposit is segregated, presenting a region of only fine particles (pink), a region with a mixture of coarse and fine particles (the finest at the bottom and the coarsest on top) and a region in which just coarse particles are deposited (dark gray). In some cases, a gap opens up between the mixed and the coarse-particle regions. If the mobility of the avalanche is not large enough to overpass the break in slope and to deposit on the horizontal deposition area, the fine-particle deposit and part of the mixed-particle deposit remain on the flume, as can be seen in Fig. 4b. In this deposit, the largest particles have a mean diameter of 4 mm (62.5% by mass), and the finest ones measure 0.25 mm (37.5% by mass). The characteristic lengths of the different parts of the deposit (see Fig. 4) are defined as the “distance from break point” (DBP), which is measured from the break point to the tail of the fine-particle deposit. The fine particle deposit length (FPD) refers to the distance from DBP to the point in which a mixture of particles can be seen. At this point starts the mixed
deposit, which consists of two perfectly reverse-graded layers (RGLD), and its length is measured to the point in which particles are no longer in mutual contact. Between the mixed particle deposit and the coarse-particle deposit (CPD) there are scattered particles forming a gap (GAP), consisting of a low-density cloud of coarse particles. This low-density cloud of scattered particles ends where the density of particles suddenly increases, forming a cluster of coarse particle deposit (CPD). Outside this last region, particles are scattered and form a large V-shaped cloud. The distance from the break in slope to the tip of the coarse particle deposit will be called hereafter “run from the break point” (RBP). All these lengths are measured on the horizontal deposition area along the symmetry axis of the flow and will be very important at describing the complex phenomenology of the granular, binary-mixture avalanches.

Registering the dynamics of the flow is complex due to the high speed acquired by particles and the formation of dust clouds during the flow development which obscures particle tracking. However, it is worth noting that segregation of large particles on top of the moving body of the avalanche occurs at very early stages of the event during the first half of the total length of the flume. In this early stage, and for mixtures with a content of large particles larger than 20%, the avalanche front shows plenty of large particles in a “saltational” regime followed by the main body composed of a mixture of both sizes forming a tail of only small particles. This structure of the flow is reproduced in the deposit once the flow has completely halted. It should be remarked that the avalanche body spreads out during the development of the flow giving rise to longer and thinner flows for smaller particles being always far from a steady state.

4.2 Detailed deposition sequence

In Fig. 5, a sequence of pictures of the final deposits for ten different fine contents is shown. The avalanche contained a mixture of 8-mm particles (large, black) and 0.375-mm fine particles (pink). These experiments were performed with a flume tilt of 37°. The fifth pictured deposit, corresponding to 55% of large particles and 45% of fine clasts, is the one shown previously in Fig. 4a. It is interesting to observe that, when the mixture contains 55% of large particles and 45% of fine particles (see the fifth and sixth pictures in Fig. 5), the deposit shows a bi-modal structure; in some cases, there is a large cluster of coarse particles at the distal part of the deposit and, in other cases, a very small one. This double behavior can be obtained by changing the content of fines by a factor of 3% or less. The sequence of deposits shown in Fig. 6 was produced by an avalanche of 4-mm particles (dark gray) mixed with 0.25-mm ones (light gray). The content of fine particles in Fig. 5 and Fig. 6 increases from left to right, growing from 0 to 100% by mass. The sudden change in the deposit morphology and, hence, the retreat of the runout, is obtained when the fine-particle content in the mixture changes from 43.75 to 50%, when the large cluster at the distal part of the deposit suddenly retreats (see the fifth and sixth pictures in Fig. 6).

From the analysis of Figs. 5 and 6 and all the experiments listed in Table 1, we get the different plots shown in Fig. 7, in which the run from break point (RBP) is depicted as a function of fine-particle content. It can be seen that, for a null fine content, the coarse particles form a single, large cluster
surrounded by scattered particles towards the front. When a few percent of fines are added to the mixture, the deposit separates into three clearly defined zones: an only-fine-particle deposit, a mixed deposit and an only-coarse-particle deposit. It is worth noting that the mixed deposit always consists of two reverse-graded layers, where fine grains are at the bottom and coarse particles are on top of them.

An increment in the run from break point (RBP) is observed, and its value depends on the grain-size ratio and the flume tilt. This trend continues until the RBP reaches an almost constant value. As more fine particles are added to the mixture, the pure-fine deposit is more clearly defined and the coarse-particle deposit separates from the mixed deposit, leaving a gap full of scattered particles in between. Further addition of fine particles induces the dilution of the distal, large cluster containing only coarse particles into one or more small clouds of particles (small clusters). A small (around 5–7%) increment in the fine-particle content in the mixture produces a sudden reduction in the RBP, and both the gap and the clusters of coarse particles disappear. If the fine-particle fraction is still increased, the few coarse particles in the flow are not able to overcome the fine deposit, thus getting stuck over what would be, otherwise, the fine deposit, forming a mixed deposit. However, some coarse particles abandon this last deposit to conform the scattered cloud of particles far from the massive deposit. The trend observed in the three cases for which the flume was tilted 32° is similar to that when the flume tilt was set to 37°, in other words, there is not an apparent change in the critical content of fines nor in the GAP length for different angles, nevertheless the RBP increases for larger angles due to a larger residual kinetic energy at the brake in slope.
the structure of the flow with large particles segregated at the very front, a mixture of small and large particles at the avalanche body, and a tail of only small particles is acquired at very early stages of the flow development, the reported observations on the deposit morphology are independent on the avalanche mass, and on inclination and length of the flume. For this reason, we decided to show only one result for each flume tilt and discuss them in general terms. Clearly a small percentage of fine particles increases the runout and, hence, the horizontal travel (RBP) of the avalanche before it settles down and deposits. We attribute this behavior to a bearing-ball-like “lubrication” produced by fine particles among large particles and between the avalanche body and the flume surface. On the other hand, a large number of fine particles acts as a “sand-trap” for coarse ones. Regardless of the grain size, they always get stuck inside the massive avalanche and the deposit of fine particles. A large particle colliding against thousands of smaller particles loses

Fig. 6 Sequence of deposits created by the binary avalanche containing 4-mm (large, gray) and 0.25-mm (small, gray) particles. The fine-particle content in the mixture increases from left to right. A dramatic change in the deposit morphology is clear when the fine-particle content is increased from 43.75 to 50%. The grid on the horizontal deposition area is 5 cm by 5 cm.
suddenly almost all its kinetic energy. As the sizes ratio tends to unity, the contrast in the flow regimes is reduced, the separation of the deposits become more difficult, and the whole behavior will be the one of a monodisperse avalanche. We should highlight that we have selected mixtures with coarse particles larger than 1 mm and fine particles

| FLUME TILT degree | PARTICLE diameter ratio | Δ RBP m | Δ fine content % |
|-------------------|-------------------------|---------|-----------------|
| 32                | 32                      | 0.41    | 7               |
| 32                | 8                       | 0.43    | 7               |
| 37                | 21.3                    | 0.42    | 3               |
| 37                | 8                       | 0.31    | 7               |

Fig. 7 Total distance travelled by the avalanche on the horizontal deposition area “run from break point” (RBP) plotted as a function of the fine-grain content (%). Three results a–c with the flume tilted 32° and two other d, e for the flume tilted 37° are shown. In all cases, a sudden change in the RBP length is observed. The number of fines needed to produce this transition depends on the grain-size ratio in the granular flow mixture. Five different zones can be recognized in all plots. For clarity, they are highlighted in plot D: Zone 1, RBP increment; Zone 2, RBP almost constant; Zone 3, RBP transition; Zone 4: RBP almost constant; Zone 5, RBP decrement or almost constant. The black filled square (on each plot) is the run from break point of the large-particle monodisperse avalanche (left side) and that of the small particles (on the right side)
with less than 1 mm, mean diameter. This is because the behavior of monodisperse avalanches containing particles larger than 1 mm is very different to that corresponding to granular flows containing particles smaller than this size. We have experimentally determined (see Fig. 8) that the coarse-grained (> 1 mm), monodisperse-avalanche travels in a gravity-dominated regime due to large inertia of particles, and its RBP (and the corresponding runout) grows proportionally to grain size, showing an increment five times larger, or more, for a 16-fold increment in particle diameter. It is worth noting in Fig. 3, that the repose angle as a function of grain size (a static measure), shows consistency with the dynamic response shown by RBP in the sense that friction is more important for small grains than it is for large ones. However, the dynamic response measured through RBP involves the acceleration stage, as well as the deceleration produced at the break in slope, which is a much more complex phenomenon.

For particles larger than 1 mm, the rate at which kinetic energy is acquired at the expense of the potential-energy loss is much higher than the rate at which energy is lost by friction and collisions. On the other hand, monodisperse avalanches containing grains smaller than 1 mm behave like a friction-dominated granular flow, maintaining a much slower increase (they show a less-than-twofold increment for a 16-fold increment in particle diameter) in the runout and RBP values, regardless of the grain size. In this case, the energy lost by dissipative processes (friction and collisions) almost equals the kinetic-energy increment due to their down-slope movement in a given time interval. We hypothesize that this is because, as grain size is reduced for the same mass, the number of particles increases as the cube of their radius, so the number of mutual collisions and contacts between particles under shear also increases at the same proportion.

To validate the proposed model of increasing dissipation for smaller particle size, we performed time-driven molecular dynamic calculations of avalanching dumbbell-like particles made by overlapping discs in two dimensions (2D), described in detail by [37]. Dumbbell-like particles were used to simulate the non-sphericity of real particles which prevents them from acquiring rotation when they lay on the flume surface or on top of a layer of other particles. Due to limitation on computational power available, we cannot obtain results for particles smaller than the 1 mm size (for comparison with those reported in this paper) since the simulations take an exponentially diverging time as the particle size is reduced.

In our simulated dumbbell-like particles, both discs have the same radii \( \sigma \) and a center to center separation equal to their radii. Tangential friction forces were calculated by a Cundall-Strack model [38], and normal dissipative interactions were obtained by spring-dashpot model [39]. Coefficients of restitutions for grain-grain collisions and for grain-flume collisions were 0.4 and 0.25, respectively, and static friction coefficients for grain-grain interactions and for grain-flume interactions were 0.4 and 0.92 respectively, in agreement with granite-granite and granite-wood coefficients.

Each simulation was run keeping the total mass 4 kg and the flume inclination at 39°. Ten simulations were performed for each value of \( \sigma \).

All the simulated grains were placed in a square lattice within a triangular container at a given height on top of the flume and, afterwards, a vertical wall was removed allowing the grains to travel down the flume developing the avalanche flow until they completely stop their movement and were deposited on the horizontal deposition area.

During the avalanche process, the instantaneous total energy of the grains, i.e., the sum of potential energies and rotational and translational kinetic energies of all the grains, was calculated every 10 ms. From this, the average dissipated power was calculated as the difference between two successive values of the total energy divided by the 10-ms time interval.

Our hypothesis that a granular medium under shear containing a larger number of particles per unit mass will travel a smaller distance after the break in slope is supported thus by our numerical simulations and is related to a larger dissipation per unit time of the gravitational energy acquired by the avalanching mass for more finely divided particles.

![Fig. 8](image-url) Run from break point (RBP) plotted as a function of grain size for monodisperse avalanches. Each point is the mean value after running five experiments. The mass of grains in each avalanche is 4 kg. A kink can be seen in both trends at a grain size of 1 mm. The RBP for particles larger than 1 mm increases proportionally to grain size, while for particles smaller than 1 mm it remains with much smaller rate of change with smaller radii. This plot shows a dramatic change in the behavior of the avalanche from a gravity dominated regime (particles larger than 1 mm) to a friction-dominated regime (particles smaller than 1 mm), as discussed in text.
Our simulation results are shown in Fig. 9. In this figure, the rate of dissipated energy is plotted as a function of time for different number of particles, \( N \), maintaining the total mass constant. Time axis spans from the triggering of the avalanche to the moment at which the avalanche front reaches the deposition zone. As it can be seen, the dissipated energy per unit time is always larger for smaller grain sizes, in agreement with the aforementioned hypothesis. Having a larger number of particles implies more collisional events and, hence, more dissipated energy by finer grains. This is consistent with the experimental results shown in Fig. 8, in which runout decreases monotonically with decreasing grain size.

Pouliquen and coworkers [7, 31] have developed a method for determining an empirical effective dynamic friction coefficient which accounts for the roughness of the flume, the volume fraction of the flow, the density of particles, and the mean velocity of steady flows, where an effective friction seems to compensate the shear exerted by gravity on a layer of granular material due to the inclination angle, the velocity of the flow and the thickness of the avalanche, allowing a steady motion of the flow. “Using dimensional analysis they [the GDR-MiDi group 31] identified two independent non-dimensional parameters in these systems; the effective friction coefficient \( \mu_{\text{eff}} \), which is the ratio of the shear stress to the confining pressure, and an inertial number \( I \), which is the ratio of a typical time-scale for particle rearrangement to a typical time scale for deformation \( \gamma d / \sqrt{P / \rho} \), where \( \gamma \) is the shear rate, \( d \) is the particle diameter, \( P \) is normal stress, and \( \rho \) the particle density. This suggested a simple local rheology, \( \mu_{\text{eff}} = \mu(I) \), in which the effective friction was a function, \( \mu \), of the inertial number, \( I \)” [40]. The method of determining the \( \mu_{\text{eff}} \) consists in establishing a steady flow of height \( h \) on the inclined flume and slowly decreasing the angle until flow stops giving \( \theta_{\text{stop}} \). Further increasing of the angle until the flow starts again gives \( \theta_{\text{start}} \) and this flow will eventually stop once the thickness of the granular layer has reached a critical height \( h_{\text{stop}} \). Proceeding this way for different thickness of the steady flow the function \( h_{\text{stop}}(\theta) \) can be determined, or conversely the \( \theta_{\text{stop}}(h) \) function.

It is very important to remark that the relations of effective friction coefficients of the MiDi’s or Pouliquen’s \( \mu(I) \) model cannot be applied to flows for which \( h \) is of the order of \( h_{\text{stop}} \), since the flow becomes diluted enough to be considered as a granular gas. In other words, the \( \mu(I) \) model works only for dense steady flows, flowing on rough surfaces. In this respect, the Pouliquen’s dynamic effective friction coefficient can hardly be applied to our experiments for the set of parameters explored, since we used smooth surfaces of the flume, relatively large angles (32° and 37°), and small quantities of material (4 kg), which develop into thin layers (h/d close to one) during the avalanching process, for at least one of the species.

In trying to measure Pouliquen’s \( \mu(I) \) coefficients for our experimental conditions, the smoothness of our flume precludes the determination of this dynamic effective friction coefficient as reported previously (see for example inset of Fig. 5c in “On Dense Granular Flows” [31], in which the role of \( h_{\text{stop}}/d \) as a function of the roughness of the flume shows how \( h_{\text{stop}} \) tends to zero as the flume becomes smoother).

Moreover, measuring the Pouliquen’s dynamic effective friction coefficient spanning two orders of magnitude in particle diameters is an enormous task beyond the scope of this article. However, we have started experimenting in that direction in order to determine the validity of the dependence of the inertial number hypothesis on particle size, put forward by Pouliquen, by means of measuring such coefficients as a function of particle diameter for those particle sizes reported here, sliding down a rough inclined flume instead of on a smooth one as in the present study.

As a final comment, the Pouliquen’s effective friction coefficient satisfies the empirical relation \( \mu_{\text{eff}} = \tan(\theta_1) + (\tan \theta_2 - \tan \theta_1) \exp\left(-\beta h / \sqrt{gh / L d u}\right) \), where \( L \) is a typical dimensionless thickness over which \( h_{\text{stop}} \) varies, \( u \) the front velocity of the avalanche, and \( \theta_1 \) and \( \theta_2 \), the angles at which \( h_{\text{stop}} \) diverges or vanishes respectively. At first glance, this empirical relation seems to give a growing value of \( \mu_{\text{eff}} \) with particle size, but this is only for their experimental conditions of steady flows on rough surfaces, in which h/d can assume a wide range of values. However a closer analysis shows that for constant volume avalanches spreading on the flume, the ratio h/d tends to one (or h \( \to \) d) as the avalanche spreads out on top of the flume, and it does more slowly for smaller particles than for larger ones during the avalanche flow development and deposition (h/d is always larger for small particles than for larger ones during the flow), producing a diminishing effective friction coefficient with growing
particle size (when the flow spreads enough to form a monolayer, in the exponential function argument \( h^{3/2}/d \) tends to \( d^{1/2} \)), explaining, in part the paradox of the \( \mu(I) \) rheology model which predicts larger values of friction coefficients for larger particle sizes in apparent contradiction to our runout measurements for which smaller particles present a more frictional dynamic behavior.

This phenomenon further enhances the segregation of large and small particles during the flow as they are subjected to very different shear rates, due to the inverse-graded particle size profile developed at very early stages during the avalanche development. Large particles are subjected to smaller normal stress, friction coefficients (they conform a thin layer on top of the granular flowing bed), and the shear rate they are subjected to is larger due to larger velocities they acquire, making their effective viscosity much smaller than the one of small particles [32].

We should emphasize that the \( \mu(I) \) rheology model, being empirically determined for monodisperse steady flows flowing on rough inclined surfaces cannot be applied to bi-disperse flows on sliding down on smooth surfaces, case for which an extension of the model should be implemented based on laboratory experiments and simulations. However, it represents a starting point from which apparent contradictions or counterintuitive behaviors of dense granular flows can be pointed out (increasing friction with particle size), in the sake of better understanding and describing them with an extended model. In this sense, more theoretical effort should be done to understand increasing mobility or runout (but larger effective friction coefficient) of monodisperse avalanches with increasing particle size, and the underlying mechanism behind the transition in runout vs particle size at a critical size around 1 mm.

Finally, it should be pointed out that since the current model for the effective dynamic friction coefficient \( \mu_{eff} \) is designed for a continuum rheology description, it certainly takes into account the way in which the shear stress propagates from the flume to higher layers on the granulate and for energy dissipation through the velocity of the flow front, but due to its continuum character does not makes explicit the role of microscopic interactions (frictional grain to grain sliding or colliding events), nor the number of movement degrees of freedom available for those dissipative interactions (trough the number of particles).

### 4.3 Comparison of experimental results

In this section, we show the results obtained after running the avalanches with a change of the fine-particle content of 2.5% in each step (in almost all cases). This finely tuned exploration of the run from the break point as a function of the fraction of fine particles in the mixture was made because a sudden transition was detected during the experiments. As can be seen in Figs. 5, 6, and 7, as the proportion of fine grains increases from zero to one, several behaviors can be noted. In all plots in Fig. 7, the run from the break point (RBP) of mixed avalanches is plotted. Five different regions can be observed in all plots depicted in Fig. 7 but, for clarity, we have highlighted them only in Fig. 7d. In some cases, differences among regions are too subtle to be discerned. However, the main transition reported in this paper is always present and is notoriously sharp. First, at very small fine percentages, around 5–15%, after the first coarse-grained, monodisperse-avalanche RBP value (the first black square), the increase of the RBP as a function of fine-particle content is recorded (depicted as zone 1), followed by a much slower growth rate (zone 2), until a sudden step-function-like fall of the RBP occurs (zone 3). After this sudden transition, a slow decreasing of the RBP follows (zone 4) and, finally, for all cases, except the one shown in Fig. 7d. This tendency continues until the value of the RBP of the fine-grained, monodisperse avalanche is reached, which is always less than or very similar to the last RBP of a mixture. This tendency of the RBP occurs when the proportion of fine particles is larger than 90% by mass (zone 5). The observed transition in zone 3 produces larger changes in the RBP for larger particle-size ratios and occurs at smaller fine-particle contents for larger size ratios, as it can be seen in Table 2.

The fourth column in Table 2 shows that the critical content of fine grains shifts inversely with the grain-size ratio for a constant flume tilt. It is worth noting that the transition width is, in all cases, close to a 7%, or less. This variation in fine-particle content of the mixture was observed for experiments performed at all flume tilts. The separation between

| Flume Tilt | Particle diameters ratio | \( \Delta \) RBP (%) | Content of fine-grains at the transition (%) | Tolerance of fine-grains content at the transition (%) |
|------------|--------------------------|---------------------|---------------------------------------------|--------------------------------------------------|
| 32\(^a\) | 32                       | 0.41                | 43                                          | 7                                                |
| 32\(^a\) | 16                       | 0.43                | 48                                          | 7                                                |
| 32\(^o\) | 8                        | 0.25                | 68                                          | 5                                                |
| 37\(^o\) | 16.3                     | 0.42                | 45                                          | 3                                                |
| 37\(^o\) | 8                        | 0.31                | 68                                          | 7                                                |
mixed and coarse-grain deposits increases linearly with the fine-particle content until it falls abruptly again to zero (the two deposits can no longer be distinguished). The run from the break point is increased due to the separation of the avalanche into two fronts: a gravity dominated one (coarse particles) and a more frictional dominated one (mixed particles). The sudden transition occurs at the point in which the content of fine particles is so large (there is a small number of coarse particles) that coarse particles get trapped between the fine ones, and the few ones that can overcome the fine-particle deposit cannot form a well-defined deposit. The beginning or the end of a deposit is always measured from the point in which the density of scattered particles grows, forming large (> 1/3 of the flume width) clusters, allowing the stability of superimposed particles in two or more layers.

The way in which the tip and tail of the deposits are defined is critical to understand the discontinuous character of the observed transition. The addition of few coarse grains to a mixture composed mostly of fine particles only traps these coarse grains within the massive fine-particle deposit. The avalanche behavior is now dominated by the friction dominated nature of the flow of fine particles. Further addition of coarse particles provokes that some of these much larger grains can overcome the deposit of fine and mixed particles, which can be found randomly scattered beyond the main deposit, forming a disperse cloud. As this process of increasing the coarse fraction is continued, more and more particles are added to the coarse-particle cloud, until they start clustering together by inelastic interactions. The density of the scattered cloud of coarse particles increases until a critical density is reached and large clusters form. The first cluster stops the particles coming from the avalanche and the just-formed coarse deposit can grow as more coarse particles are added to the mixture. This is a coarse particle deposit, and the subsequent behavior of the run from the break point will be dominated by large (inertial) particles. In this way, one may say that the discontinuous transition of the RBP at a critical content of fine grain is actually the separation of the deposit into two main deposits: the one of mixed particles and the cluster of scattered, coarse particles that condense to form the coarse-grain deposit at the very front. Before the onset of clustering there is not a well-defined coarse deposit, and just a diffuse cloud of randomly distributed, coarse particles can be identified. Thus, the massive front of the deposited material is composed of coarse particles trapped by the fine-grain deposit. At this point, let us mention, for the sake of clarity, a few words on clustering and inelastic-collision driven transitions in granular matter.

“Inelastic collapse” [41] is a phenomenon in which simulations of granular gases cooling down by inelastic collisions seems to stop. The apparent halt of the simulation occurs because when two particles approach, sandwiching a third particle, they cause this third particle to perform a diverging number of collisions per unit time. The collision routine is thus invoked a diverging number of times but the relative positions of the particles involved in the formation of this cluster evolve very slowly as if the simulation were halted. On the other hand, “granular clustering”, closely related to inelastic collapse, is the phenomenon in which the restitution coefficient depends on the relative collision velocity (as in the case of real grains) and has been reported in quasi-two-dimensional granular gases as responsible for segregation in compartmentalized granular gases [42–47]. In the clustering instability, the energy dissipated in the collisions grows as the inter-particle separation decreases, because the number of collisions diverges when they come closer. In this sense, as more particles cluster together, it is easier for them to trap particles from the cloud within the cluster. Thus, clustering is a self-enhanced process. The formation of two different deposits separated by a gap, shown in Figs. 5 and 6, could be explained as a consequence of a clustering process. This process produces the second well-defined deposit transversal to the flow direction of the avalanche, since local stresses are of compressive nature. In other words, no extensional stresses are possible due to discontinuous granular interactions among grains in this horizontal deposition area. This clustering-driven formation of a separated deposit and small, dispersed clusters, in our lab-scale experiments, recalls the hummocks observed in natural rock avalanches. It should be remarked that avalanche-runout increments due to fine particles have been reported previously [22, 24, 25]. However, in our case, lubrication or bearing-ball effects, levee formation, etc., do account for just a small RBP increment until the deposit separates into two clearly defined deposits. In the case of a binary mixture, there are in fact two avalanches segregated from each other during the sliding down of the avalanching material. In our experiments, particles of granulometric classes corresponding to gravity-dominated (inertial) and friction-dominated regimes were chosen to configure each mixture, in order to discern the effect, the flow of fine particles has on the flow of coarse particles and vice versa. This competition leads to the prevalence of one behavior over the other, depending on the relative content of each species, conferring the RBP a dual behavior. On the one hand, if the coarse-grained avalanche dominates, the RBP shows a very large increment, and the deposit can split into two parts: a distal, massive cluster of only coarse particles and a proximal, massive deposit consisting of only fine particles at the closest distance to the break in slope, followed by a layered, reverse-graded, mixed part of the deposit. However, if a gap does not form, the structure of the deposit remains the same with three very well differentiated zones: a pure-fine, proximal one; a reverse-graded, mixed, intermediate one; and purely coarse-particle, distal zone forming a single deposit. On the other hand, if the fine-grain, friction-dominated avalanche dominates, the trend...
of RBP is towards diminishing. There is only one massive deposit with just two well defined zones: the proximal one of only fine particles attached to the reverse-graded mixed zone; the pure coarse-grained deposit is absent. Our findings show that the contrast in grain size of the mixture makes an always abrupt transition (within a 7% or less variation of fines content). Additionally, for fixed-total mass experiments (4 kg), we have found that the RBP of a binary avalanche is always greater than (or very similar to) the RBP of each corresponding monodisperse avalanches, depicted as a filled black square in Fig. 7.

On the coarse-dominated side (gravity-dominated or inertial regime) before the transition, fine particles within interstices, formed by large particles, lubricate (bearing ball effect) the interaction among large particles or between large particles and flume surfaces. On the other side, where fine particles are dominant (the whole behavior is friction-dominated), the RBP is larger than the one of only fine particles, due to the drag exerted by large inertial particles on the fine, “friction-dominated” particles. The formation of the distal deposit depends on a critical quantity of coarse particles, which allows the condensation of the deposit around a small initial cluster (a “seed”). This seed grows rapidly by braking and capturing subsequent coarse colliding particles. Note that now we are discussing in terms of coarse-particle content, instead of fine-particle content, because the distal cluster will form or not depending on the density of scattered particles in the disperse cloud at the very front of the deposit. From the data presented one can see that the critical content of fine particles diminishes as the particle-diameter ratio grows. This is because a small quantity of large particles is not enough to constitute a well-defined, coarse-grain deposit, because the majority of them are scattered ahead of the massive deposit. This implies that the formation of the coarse-grain cluster is not possible until the coarse-particle density in the disperse cloud reaches a critical value that allows the formation of large clusters.

On the other hand, as the particle diameter ratio tends to one, one could expect that the transition will never occur since there are not enough kinetic energy contrast among different particle to get segregated and overtake the small particle deposit. In this sense, a critical value of particle diameter ratio should be expected for which the fine-particle content diverges and no transition is possible below this particle diameter ratio.

In Fig. 10a, we show several clusters of large particles (inscribed by red circles) that are typical morphological features obtained in our small scale granular avalanche experiments. They resemble large hummocks observed in geophysical rock avalanches.

Remarkably, for mixtures in which two deposits form, (a coarse-grained, distal one, and an inversely graded proximal one) the morphology of the whole deposit strikingly resembles the hummocky deposits of the Parinacota debris avalanche in northern Chile, first described by [2] as those of the Mount Shasta flank collapse [48] and the large debris avalanche (Fig. 10b) from Tata Sabaya volcano in Bolivia.

Fig. 10 a Hummock-like clusters of coarse particles formed ahead of an experimental deposit of a binary mixture avalanche consisting of a 0.5 mm fine particles mixed with an 8 mm coarse particles. b Google Earth image of the Tata Sabaya (Bolivia) rock avalanche deposit showing large hummocks. c Lunar granular avalanche showing several clusters of large boulders at the distal part of the avalanche deposit. The resolution of the image is 0.51 m/px. Image number M112074670RE taken by the NASA Lunar Reconnaissance Orbiter Camera on November 5, 2011. (https://wms.lroc.asu.edu/lroc/view_lroc/LRO-L-LROC-2-EDR-V1.0/M112074670RE)
As described by these authors, distal hummocks are placed ahead the avalanche front and are constituted mainly by coarse-grain rocks that show collision marks that we attribute to the clustering process we have just described. As can be seen in Fig. 10c, granular clustering (hummocks) is a common phenomenon that occurs on solid planetary surfaces.

4.4 Mobility

Mobility of rock avalanches is often measured using the ratio $H/L$. The height of fall, $H$, is measured vertically from the crest of the initial avalanche mass to the lowest point of reach, whereas the length of fall, $L$, is measured horizontally from the crest of the avalanche to the most distant point of reach (denoted as $H_{\text{max}}$ and $L_{\text{max}}$ in Fig. 1). The Fahrböschung angle, $[50, 51]$, is defined as $\theta = \tan^{-1}(H_{\text{max}}/L_{\text{max}})$. We decided to characterize the mobility of our avalanches in this way because the travel angle $\alpha_G$, defined as the angle between the center of gravity of the initial mass and that of the deposit, is much less used, since it is very difficult to be determined in natural deposits $[52]$. In our case, since the deposit sometimes splits into two well differentiated sequential deposits, for the Fahrböschung angle calculation, we always measure the distance from the rear of the initial granular pile to the tip of the most distant condensed or clustered front, either when the deposit is splitted or it is a continuous one (see Figs. 4, 5, 6 as an example of splitted or a massive deposit). The excess of runout or excess of travel distance $L_e = L_{\text{max}} - L_f$ is the difference between the distance traveled by the granular flow ($L_{\text{max}}$) and the one that may be expected ($L_f$) for a mass sliding down an inclined plane, in the case when a “normal” coefficient of friction “$\tan(32^\circ) = 0.62$” is considered:

$$L_f = H_{\text{max}}/\tan(32^\circ)$$

$$L_e = L_{\text{max}} - L_f$$

This friction coefficient (0.62) is expected in a purely frictional model when a block slides down an inclined plane, defined by $[51, 54]$. The percentage of the excess of runout plotted in Fig. 11 is then:

$$L_e\% = 100(L_{\text{max}}/L_f) - 100$$

It can be seen that the excess runout is larger for mixtures in which the mass of large particles dominates over that of small particles. To compare the RBP of bi-disperse avalanches (Fig. 7) with that of mono dispersive avalanche (Fig. 8), it is important to note that both sets of experiments were performed using exactly the same material and the same experimental flume. As it can be seen, the fine-particle content (less than a critical quantity) modifies the mobility (or the RBP), with a tendency of increasing it. This can be due to the bearing-ball-like lubrication produced by interstitial fine particles, when coarse particles are the largest proportion by mass, or to coarse particles dragging fine particles, when coarse particles are a minority compared to the fine ones by mass. Instead, with a larger than- critical mass of fine grains, collisions and friction tend to slow down the avalanche trapping the coarse particles between the fine ones, reducing the mobility and the runout of the flow, thus acting like a “sand-trap”. As stated previously, further experimental and theoretical effort is needed in order to interpret at the light of currently available models (or extend them), in order to better understand the larger mobility of coarse particles, in spite of the larger effective friction coefficients predicted by such models, and the transition from short to long runout of binary avalanches reported in this study.

5 Conclusions

We have reported the run from break point (RBP) of a binary-mixture avalanche as a function of its content of fine particles. Two competing behaviors, a friction-dominated one, associated to fine particles, and a gravity-dominated one, associated to coarse particles, regulate the global behavior of the mixed avalanche, depending on the proportion of each granulometric class within the mixture. We have found a tendency of increment of the RBP, which is produced when a small percent of fine particles are added to a coarse grain avalanche, due to the bearing-ball-like “lubrication” among coarse grains and between coarse grains and the flume surface, produced by the presence of interstitial fine particles. This RBP increment saturates as more fine particles are added, until a sudden transition makes the RBP diminish abruptly to small values closer to those corresponding to the RBP of a mono-disperse avalanche of fine particles. Here, the avalanche is dominated by a friction-dominated behavior, characteristic of these fine particles flowing down the inclined plane. The sudden transition is caused by a clustering instability and occurs when the coarse-particle population, which overcomes the deposit of fine or mixed particles after segregation, reaches a critical amount, at which a new deposit of coarse particles condenses from the dispersed cloud. By looking at those clusters, formed from dispersed coarse particles at the very front, we suggest that clustering could be at the origin of some hummock-like structures formed at the front of long-runout, natural rock avalanches. The variation in the content of fine particles in the avalanching mixture required for the transition to occur is found experimentally to be lower than 7% by mass. The formation of well-defined regions of only fine and only coarse
particles and a mixed, reverse-graded region in the deposit shows that a complete segregation can be achieved for a large enough size contrast and/or flume length. The critical content of fines at the transition is related to the particle-size ratio as a power law for all the studied cases.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Robinson, T.R., Davies, T.R.H., Reznichenko, N.V., De Pascale, G.P.: The extremely long-runout Komansu rock avalanche in the Trans Alai range, Pamir Mountains, southern Kyrgyzstan. Landslides 12, 523–535 (2015)
2. Clavero, J.E., Sparks, R.S.J., Huppert, H.E., Dade, W.B.: Geological constraints on the emplacement mechanism of the Parinacota debris avalanche, northern Chile. Bull. Volcanol. 64, 40–54 (2002)
3. Fauche, L., Strecker, M.R.: Large rock avalanche deposits (Sturzströme, sturzstroms) at Sierra Aconcagua, northern Sierras Pampeanas Argentina. Eclogae Geol Helv 81, 572–599 (1988)
4. Siebert, L.: Large volcanic debris avalanches: characteristics of source areas, deposits, and associated eruptions. J. Volcanol. Geotherm. Res. 22, 163–197 (1984)
5. Ui, T.: Volcanic debris avalanche deposits--Identification and comparison with nonvolcanic debris stream deposits. J. Volcanol. Geotherm. Res. 18, 135–150 (1983)
6. Goujon, C., Thomas, N., Dalloz-Dubrujeaud, B.: Bidisperse dry granular flows on inclined planes: role of roughness. Eur. Phys. J. E 11, 147–157 (2003)
7. Pouliquen, O.: Scaling laws in granular flows down rough inclined planes. Phys. Fluids 11, 542 (1999)
8. Andreotti, B., Daerr, A., Douady, S.: Scaling laws in granular flows down a rough plane. Phys. Fluids 14, 415–418 (2002)
9. Campbell, C.: Granular material flows—an overview. Powder Tech. 162, 208–229 (2006)
10. Yang, Q., Cai, F., Ugai, K., Yamada, M., Su, Z., Ahmed, A., Huang, R., Xu, Q.: Some factors affecting mass-front velocity of rapid dry granular flows in a large flume. Eng. Geol. 122, 249–260 (2011)
11. Huerta, D.A., Sosa, V., Vargas, M.C., Ruiz-Suárez, J.C.: Archimed’s principle in fluidized granular systems. Phys. Rev. E 72, 031307 (2005)
12. Pacheco-Vázquez, F., Ruiz-Suárez, J.C.: Sliding through a super-light granular medium. Phys. Rev. E 80, 060301(R) (2009)
13. Hungr, O., Evans, S.G.: Entrainment of debris in rock avalanches: an analysis of a long run-out mechanism. Geol. Soc. Am. Bull. 116, 1240–1252 (2004)
14. Charrière, M., Humair, F., Froese, C., Jaboyedo, M., Pedrazzini, A., Longchamp, C.: From the source area to the deposit: collapse, fragmentation, and propagation of the Frank Slide. Geol. Soc. Am. Bull. 128, 332–351 (2015)
15. Linares-Guerrero, E., Goujon, C., Zenit, R.: Increased mobility of bi disperse granular avalanches. J. Fluid Mech. 593, 475–504 (2007)
16. Van Gassen, W., Cruden, D.M.: Momentum transfer and friction in the debris of rock avalanches. Can. Geotech. J. 27, 698–699 (1990)
17. Bartali, R., Sarocchi, D., Nahmad-Molinari, Y.: Stick-slip motion and high speed ejecta in granular avalanches detected through a multi-sensors flume. Eng. Geol. 195, 248–257 (2015)
18. Bartali, R., Sarocchi, D., Nahmad-Molinari, Y., Rodríguez-Sedano, L.A.: Estudio de flujos granulares de tipo geológico por medio del simulador multimedia GRANFLOW-SIM. Boletín de la Sociedad Geológica Mexicana 64, 265–275 (2012)
19. Valderrama, P., Roche, O., Samaniego, P., Van Wyk Des Vries, B., Araujo, G.: Granular fingering as a mechanism for ridge formation in debris avalanche deposits: laboratory experiments and implications for Tutupaca volcano, Peru. J. Volcanol. Geotherm. Res. 349, 409–418 (2018).
20. Rowley, P.J., Kokelaar, B.P., Menzies, M., Waltham, D.: Shear-derived mixing in dense granular flows. J. Sediment. Res. 81, 874–884 (2011)
21. Paguican, E.M.R., Van Wyk de Vries, B., Lagmay, A.: Hummocks: how they form and how they evolve in rockslide-debris avalanches. Landslides 11, 67–80 (2014).
22. Phillips, J.C., Hogg, A.J., Kerswell, R.R., Thomas, N.H.: Enhanced mobility of granular mixtures of fine and coarse particles. Earth Planet. Sci. Lett. 246, 466–480 (2006)
23. Moro, F., Faug, T., Bellot, H., Ousset, F.: Large mobility of dry snow avalanches: insights from small-scale laboratory tests on granular avalanches of bidisperse materials. Cold Regions Sci. Technol. 62, 55–66 (2010). https://doi.org/10.1016/j.coldregions.2010.02.011
24. Goujon, C., Dalloz-Dubrujeaud, B., Thomas, N.: Bidisperse granular avalanches on inclined planes: a rich variety of behaviors. Eur. Phys. J. E 23, 199–215 (2010).
25. Kokelaar, B.P., Graham, R.L., Gray, J.M.N.T., Vallance, J.W.: Fine-grained linings of levede channels facilitate runout of granular flows. Earth Planet. Sci. Lett. 385, 172–180 (2014)
26. Gray, J.M.N.T., Ancy, C.: Segregation, recirculation and deposition of coarse particles near two-dimensional avalanche fronts. J. Fluid Mech. 629, 387–423 (2009)
27. Gray, J.M.N.T., Kokelaar, B.P.: Large particle segregation, transport and accumulation in granular free-surface flows. J. Fluid Mech. 652, 105–137 (2010)
28. Wiederseiner, S., Andreini, N., Épely-Chauvin, G., Moser, G., Monnereau, M., Gray, J.M.N.T., Ancy, C.: Experimental investigation into segregating granular flows down chutes. Phys. Fluids 23, 013301 (2011)
29. Baker, J.L., Johnson, C.G., Gray, J.M.N.T.: Segregation-induced finger formation in granular free-surface flows. J. Fluid Mech. 609, 168–212 (2006)
30. Woodhouse, M.J., Thornton, A.R., Johnson, C.G., Kokelaar, B.P., Gray, J.M.N.T.: Segregation-induced fingering instabilities in granular free-surface flows. J. Fluid Mech. 709, 543–580 (2012)
31. On dense granular flows: GDR-MiDi. Eur. Phys. J. E 14, 341–365 (2004)
32. Jop, P., Forterre, Y., Pouliquen, O.: A constitutive relation for dense granular flows. Nature 44, 727–730 (2006)
33. Pouliquen, O., Forterre, Y.: Friction law for dense granular flows: application to the motion of a mass down a rough inclined plane. J. Fluid Mech. 453, 133–151 (2002)
34. Wentworth, C.K.: A scale of grade and class terms for clastic sediments. J. Geol. 30, 377–392 (1922)
35. McColl, S.T., Davies, T.R.: Evidence for a rock-avalanche origin for ‘The Hillocks’ ‘moraine’ Otago. N. Z. Geomorphol. 127, 216–224 (2011)
36. Glicken, H.: Rockslide-Debris Avalanche of May 18, 1980, Mount St. Helens Volcano, Washington. United States Geological Survey Open File Report 96-677 (1996)
37. Pérez, G.: Numerical simulations in granular matter: the discharge of 2D silo. Pramana J. Phys. 70, 989–1007. https://doi.org/10.1007/s12043-008-0104-2 (2008).
38. Cundall, P.A., Strack, O.D.L.: A discrete numerical model for granular assemblies. Géotechnique 29(1), 47–65 (1979). https://doi.org/10.1680/geot.1979.29.1.47
39. Schäfer, J., Dippel, S., Wolf, D.: Force schemes in simulations of granular materials. J. Phys. I, EDP Sci. 6(1), 5–20 (1996). https://doi.org/10.1051/jp1:1996129
40. Gray, J.M.N.T., Edwards, A.N.: A depth-averaged μ(I)-rheology for shallow granular free-surface flows. J. Fluid Mech. 755, 503–534 (2014). https://doi.org/10.1017/jfm.2014.450
41. Goldhirsh, I., Zanetti, G.: Clustering instability in dissipative gases. Phys. Rev. Lett. 70, 1619 (1993)
42. Olafsen, J.S., Urbach, J.S.: Clustering, order, and collapse in a driven granular monolayer. Phys. Rev. Lett. 81, 4369 (1998)
43. Sapozhnikov, M.V., Aranson, I.S., Olafsen, J.S.: Coarsening of granular clusters: Two types of scaling behaviors. Phys. Rev. E 67, 010302(R) (2003)
44. Bordallo-Favela, R.A., Ramírez-Saito, A., Pacheco-Molina, C.A., Perera-Burgos, J.A., Nahmad-Molinarri, Y., Pérez, G.: Effective potentials of dissipative hard spheres in granular matter. Eur. Phys. J. E 28, 395–400 (2009)
45. Perera-Burgos, J.A., Pérez-Ángel, G., Nahmad-Molinari, Y.: Diffusivity and weak clustering in a quasi-two-dimensional granular gas. Phys. Rev. E 82, 051305 (2010)
46. Mikkelsen, R., Van der Meer, D., Van der Weele, K., Lohse, D.: Competitive clustering in a bidisperse granular gas. Phys. Rev. Lett. 89, 214301 (2002)
47. Mikkelsen, R., Van der Weele, K., Van der Meer, D., Van Hecke, M., Lohse, D.: Small-number statistics near the clustering transition in a compartmentalized granular gas. Phys. Rev. E 71, 041302 (2005)
48. Crandell, D.R., Miller, C.D., Glicken, H.X., Christiansen, R.L., Newhall, C.G.: Catastrophic debris avalanche from ancestral Mount Shasta volcano. Calif Geol 12(3), 143–146 (1984)
49. Godoy, B., Clavero, J., Rojas, C., Godoy, E.: Facies volcánicas del depósito de avalancha de detritos del volcán Tata Sabaya Andes Centrales. Andean Geol. 39(3), 394–406 (2012)
50. Shreve, R.L.: The Blackhawk Landslide. GSA Special Papers 1081–48 (1968)
51. Hsü, K.J.: Albert Heim, observations on landslides and relevance to modern interpretations. In: Voight B (ed) Rockslides and Avalanches: Developments in Geotechnical Engineering, vol. 14A, Amsterdam, Elsevier, pp. 71–92 (1978)
52. Bowman, E.T., Take, W.A., Rait, K.L., Hann, C.: Physical models of rock avalanche spreading behavior with dynamic fragmentation. Can. Geotech. J. 49, 460–476 (2012)
53. Heim, A.: Der Bergsturz von Elm. Zeitschrift der Deutschen Geologischen Gesellschaft 34, 74–115 (1882)
54. Hsü, K.J.: Catastrophic debris streams generated by rock falls. Geol. Soc. of Am. Bull. 86, 129–140 (1975)
55. Corominas, J.: The angle of reach as a mobility index for small and large landslides. Can. Geotech. J. 33, 260–271 (1996)
56. Legros, F.: The mobility of long-runout landslides. Eng. Geol. 63, 301–331 (2002)