On Performance of Quantized Transceiver in Multiuser Massive MIMO Downlinks

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Abstract—Low-resolution digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) are considered to reduce cost and power consumption in multiuser massive multiple-input multiple-output (MIMO). Using the Bussgang theorem, we derive the asymptotic downlink achievable rate w.r.t the resolutions of both DACs and ADCs, i.e., $b_{DA}$ and $b_{AD}$, under the assumption of large antenna number, $N$, and fixed user load ratio, $\beta$. We characterize the rate loss caused by finite-bit-resolution converters and reveal that the quantization distortion is ignorable at low signal-to-noise ratio (SNR) even with low-resolution converters at both sides. While for maintaining the same rate loss at high SNR, it is discovered that one-more-bit DAC resolution is needed when more users are scheduled with $\beta$ increased by four times. More specifically for one-bit rate loss requirement, $b_{DA}$ can be set by $\left\lceil b_{AD} + \frac{1}{2} \log \beta \right\rceil$ given $b_{AD}$. Similar observations on ADCs are also obtained with numerical verifications.

Index Terms—Massive MIMO, DAC, ADC, quantization.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) has become a candidate technique for the fifth generation (5G) wireless communication system [1]. Despite its several advantages [2], nevertheless, hundreds of antennas significantly increase cost and power consumption partly because each antenna requires a digital-to-analog converter (DAC) unit at transmitter or an analog-to-digital converter (ADC) at receiver. Currently, there exist two kinds of approaches for potential solutions. One is to exploit hybrid precoding with a very limited number of radio-frequency (RF) chains serving all antennas, like in [3]. Alternatively, each antenna element is as usual connected with a dedicated RF chain which is, however, served by low-resolution, or even 1-bit, ADC/DACs, e.g., [4].

In particular, [5] showed that 1-bit ADCs achieve satisfactory performance in uplink massive MIMO. Mixed ADCs were considered in [6], which revealed that the quantization distortion caused by low-resolution ADCs can be compensated logarithmically by more antennas. Recently, some researchers studied quantized precoding with low-resolution DACs for downlinks. [7] considered a special case of 1-bit DACs and that showed the performance depends on user load ratio. Most existing studies focused only on either low resolution DACs or ADCs at BS side for the sake of tractability. However, finite-bit converters are generally employed at user side. In a single-antenna system, it is directly known by the Shannon theory that the achievable rate depends only on the lower resolution between transmitting DAC and receiving ADC. For multi-antenna setups, however, the joint design of finite-bit DACs and ADCs, as well as its effect on performance, remains unknown and should intuitively rely on the antenna number, especially for the multiuser massive MIMO case.

This letter considers both finite-bit DACs at BS and finite-bit ADCs at user side for downlink massive MIMO. We derive an asymptotic expression for the downlink achievable rate. The rate loss due to transceiver quantization is accordingly characterized. At high signal-to-noise ratio (SNR), the DAC resolution, $b_{DA}$, can be approximately set as $\left\lceil b_{AD} - \frac{1}{2} \log (2^r - 1) + \frac{1}{2} \log \beta \right\rceil$ given maximum allowed rate loss, $r_1$, and fixed ADC resolution, $b_{AD}$. It is revealed that if $\beta$ increases by four times, $b_{DA}$ should increase about one bit for maintaining the same rate loss. While for low SNR, the rate loss is proven ignorable.

The rest of this paper is organized as follows. System model is described in Section II. In Section III, we derive an asymptotic expression for the achievable rate and analyze the rate loss due to quantization. Section IV presents simulation results. Conclusions are drawn in Section V.

Notations: $A^T$, $A^*$ and $A^H$ represent the transpose, conjugate and conjugate transpose of $A$, respectively. $A_{i,j}$ represents the $(i,j)$th element of $A$. $\operatorname{tr}\{A\}$ denotes the trace of $A$ and $\operatorname{diag}\{A\}$ keeps only the diagonal entries of $A$. $\mathbb{E}\{\cdot\}$ is the expectation operator. $\rightarrow$ denotes almost sure convergence. $\lfloor \cdot \rfloor$ is the ceiling function.

II. SYSTEM MODEL

A. System Description

Consider a downlink multiuser massive MIMO system illustrated in Fig. 1. The BS is equipped with $N$ antennas and serves $M$ single-antenna users simultaneously with user load ratio, $\beta \triangleq \frac{N}{M} < 1$ in general.

At the transmitter, data vector $s \in \mathbb{C}^{M \times 1}$ with $\mathbb{E}\{s s^H\} = I_M$ is processed by $P \in \mathbb{C}^{N \times M}$. The precoded signal, i.e., $x \triangleq P s$, is then passed through finite-bit DACs before transmission. The signal vector after DACs, $x_q \in \mathbb{C}^{N \times 1}$, is expressed as

$$x_q = Q_{DA}(x) = Q_{DA}(Ps),$$

(1)
where \( Q_{DA}(\cdot) \) denotes the quantization operation of DACs.

At the receiver, the received signal vector \( \mathbf{y} \in \mathbb{C}^{M \times 1} \) equals
\[
\mathbf{y} = \mathbf{Hx} + \mathbf{n},
\]
where \( \mathbf{n} \sim \mathcal{CN}(0, \mathbf{\sigma}_n^2) \) is the additive white Gaussian noise (AWGN) and \( \mathbf{H} \in \mathbb{C}^{M \times N} \) is a Rayleigh fading channel whose entries are i.i.d. \( \mathcal{CN}(0, 1) \). Let \( Q_{AD}(\cdot) \) denote the ADC quantization. The quantized received signal vector is
\[
\mathbf{y}_q = Q_{AD}(\mathbf{y}) = Q_{AD}(\mathbf{Hx} + \mathbf{n}).
\]

**B. Quantization Models**

Since the quantization of both finite-bit DACs and ADCs are nonlinear, it is difficult to conduct exact characterizations on the quantization operations. Fortunately, an approximate linear representation of quantization is available by the Bussgang theorem [6]. For a Gaussian source data \( \mathbf{s} \), the quantization input signals can be considered as Gaussian as well [9]. Thus for the Gaussian input, we can apply the Bussgang theorem to decompose the quantized signal into two uncorrelated parts.

Taking ADC for example, we have
\[
\mathbf{y}_q = Q_{AD}(\mathbf{y}) = \mathbf{Fy} + \mathbf{n}_{AD},
\]
where \( \mathbf{F} \in \mathbb{C}^{M \times M} \) is a diagonal matrix consisting of the quantization gains and \( \mathbf{n}_{AD} \) denotes the quantization noise uncorrelated with \( \mathbf{y} \). Note that the diagonal entries of \( \mathbf{F} \) are considered equal here under the assumption of equal average power of each received signal which asymptotically holds because the rows of \( \mathbf{H} \) tend quasi-orthogonal with large \( N \) in the massive MIMO setup. According to [9, Eqs. (5) and (25)], \( \mathbf{y}_q \) can be reduced to
\[
\mathbf{y}_q = Q_{AD}(\mathbf{y}) = (1 - \rho_{AD})\mathbf{y} + \mathbf{n}_{AD},
\]
where \( \rho_{AD} = \frac{\mathbb{E}[\|\mathbf{y} - \mathbf{y}_q\|^2]}{\mathbb{E}[\|\mathbf{y}\|^2]} \) denotes the distortion factor and [9, Eqs. (6), (25) and (28)]
\[
\mathbb{E}\{\mathbf{n}_{AD}\mathbf{n}_{AD}^H\} = \rho_{AD}(1 - \rho_{AD})\mathbb{E}\{\text{diag}(\mathbf{y}\mathbf{y}^H)\}.
\]

Let \( b_{AD} \) be the quantization bits of ADC. As in most existing works, e.g., [6, 9] and [10], we consider the optimal non-uniform ADCs since it provides a tractable and effective way of well characterizing the quantization performance. For uniform quantizers used in practice, this can serve as a performance bound and the gap between the optimal and uniform ones is in general marginal, especially for popular quantization levels [11]. The value of \( \rho_{AD} \) is exemplified as [0.3634, 0.1175, 0.03454, 0.009497, 0.002499, 0.0006642, 0.0001660, 0.00004151] with \( b_{DA} = \{1, 2, 3, 4, 5, 6, 7, 8\} \) [11]. In the condition of moderate to high-resolution quantizations, e.g., \( b_{AD} \geq 3 \), we have [9]
\[
\rho_{AD} \approx \frac{\pi \sqrt{3}}{2} 2^{-b_{AD}}.
\]

While for DAC, the Bussgang Theorem can also be applied for making the operation linearly approximated as exemplified in [2] and [12]. Similarly, we have the following representation:
\[
\mathbf{x}_q = Q_{DA}(\mathbf{x}) = \sqrt{1 - \rho_{DA}}\mathbf{x} + \mathbf{n}_{DA},
\]
where \( \rho_{DA} \) is determined by \( b_{DA} \) and
\[
\mathbb{E}\{\mathbf{n}_{DA}\mathbf{n}_{DA}^H\} = \rho_{DA}\mathbb{E}\{\text{diag}(\mathbf{x}\mathbf{x}^H)\}.
\]

Compared to ADCs, a scalar factor, \( \sqrt{\frac{1}{\rho_{DA}}} \), is multiplied for DACs in order to guarantee that \( \mathbb{E}\{\|\mathbf{x}_q\|^2\} = \mathbb{E}\{\|\mathbf{x}\|^2\} \).

**III. Asymptotic Performance Analysis**

This section derives the asymptotic downlink achievable rate and characterizes the rate loss due to finite-bit DAC/ADCs under the assumption of large \( N \) but fixed user load ratio \( \beta \).

**A. Asymptotic User Rate**

Substituting (2) and (8) into (5), the quantized signal received by users becomes
\[
y_q = (1 - \rho_{AD})\sqrt{1 - \rho_{DA}}\mathbf{H}\mathbf{x} + (1 - \rho_{AD})\mathbf{n}_{DA} + (1 - \rho_{AD})\mathbf{n},
\]
where \( n_{AD,k} \) and \( n_k \) are, respectively, the \( k \)-th element of \( \mathbf{n}_{AD} \) and \( \mathbf{n} \). Then, the signal-to-interference, quantization and noise ratio (SIQNR) of the \( k \)-th user, \( \gamma_k \), can be expressed as
\[
\gamma_k = \frac{S_k}{I_k + Q_{1k} + Q_{2k} + N_k},
\]
where \( S_k, I_k, Q_{1k}, Q_{2k} \), and \( N_k \) respectively denote the power of received signal, multiuser interference, DAC quantization noise, ADC quantization noise, and Gaussian noise. Obviously from (11), we have \( N_k = (1 - \rho_{AD})^2\sigma_n^2 \). Considering \( \mathbf{P} \) as a typical zero-forcing (ZF) precoder, asymptotic expressions of \( S_k, I_k, Q_{1k} \) and \( Q_{2k} \) under the assumption of large \( N \) but fixed \( \beta \) are, respectively, given as
\[
S_k \rightarrow (1 - \rho_{AD})^2(1 - \rho_{DA})P \left( \frac{1}{\beta} - 1 \right),
\]
\[
I_k \rightarrow 0,
\]
\[
Q_{1k} \rightarrow (1 - \rho_{AD})^2\rho_{DA}P,
\]
\[
Q_{2k} \rightarrow \rho_{DA}(1 - \rho_{DA})(1 - \rho_{DA})P \left( \frac{1}{\beta} - 1 \right) + \rho_{AD}\rho_{DA} \times (1 - \rho_{DA})P + \rho_{DA}(1 - \rho_{DA})\sigma_n^2,
\]
where the derivations are detailed in Appendix. By substituting (13)-(16) into (12), the asymptotic SIQNR is given by
\[
\bar{\gamma}(b_{DA}, b_{AD}) = \frac{(1 - \rho_{AD})(1 - \rho_{DA})(\frac{1}{\beta} - 1)\gamma_0}{\rho_{DA}\gamma_0 + \rho_{AD}(1 - \rho_{DA})\left(\frac{1}{\beta} - 1\right)\gamma_0 + 1},
\]
where \( \gamma_0 = \frac{\rho_{DA}}{\rho_{DA}\gamma_0 + \rho_{AD}(1 - \rho_{DA})\left(\frac{1}{\beta} - 1\right)\gamma_0 + 1} \) represents the average system SNR. Supposed that the quantization and interference noise suffers the worst case, i.e., Gaussian distribution, the asymptotic achievable rate can be bounded by
\[
R(b_{DA}, b_{AD}) = \log(1 + \bar{\gamma}(b_{DA}, b_{AD}))
\]
\[
= \log \left( 1 + \frac{(1 - \rho_{AD})(1 - \rho_{DA})(\frac{1}{\beta} - 1)\gamma_0}{\rho_{DA}\gamma_0 + \rho_{AD}(1 - \rho_{DA})(\frac{1}{\beta} - 1)\gamma_0 + 1} \right).
\]
B. Quantization Rate Loss Analysis

On the basis of (18), we are ready to analyze the rate loss caused by both finite-bit DACs and ADCs. Considering three special cases, i.e., using ideal (infinite-bit) DACs or ideal ADCs, or both, as benchmarks, we have

\[ R(\infty, b_{AD}) = \log(1 + \alpha_{AD} \gamma_{ideal}), \]  
\[ R(b_{DA}, \infty) = \log(1 + \alpha_{DA} \gamma_{ideal}), \]  
\[ R(\infty, \infty) = \log(1 + \gamma_{ideal}), \]

where \( \gamma_{ideal} \equiv (\frac{1}{2} - 1) \gamma_0 \) is regarded as the nominal SNR with ideal converters at both sides. Here, \( \alpha_{AD} \equiv \frac{(1 - \rho_{DA})}{\rho_{DA}(\frac{1}{2} - 1) \gamma_0 + 1} \) and \( \alpha_{DA} \equiv \frac{(1 - \rho_{DA})}{\rho_{DA}(\frac{1}{2} - 1) \gamma_0 + 1} \) are multipliers indicating the equivalent SNR degradation factors due to the use of finite-bit ADCs and DACs, respectively.

Remark 1. Assuming the same resolution for ADCs and DACs, i.e., \( b_{AD} = b_{DA} \), we observe that \( \alpha_{AD} > \alpha_{DA} \) for typical user load values, \( \beta < \frac{1}{2} \) in massive MIMO, which implies that ADC quantization always causes more pronounced SINR degradation than DAC in this case.

Now by subtracting (18) from (19), the rate loss caused by finite-bit DACs can be characterized as

\[ \Delta R_{DA}(b_{DA}) = \log \left( 1 + \frac{(1 - \rho_{AD})(\frac{1}{2} - 1) \gamma_0}{\rho_{AD}(\frac{1}{2} - 1) \gamma_0 + 1} \right) - \log \left( 1 + \frac{(1 - \rho_{AD})(1 - \rho_{DA})(\frac{1}{2} - 1) \gamma_0}{\rho_{DA} + \rho_{DA}(1 - \rho_{DA})(\frac{1}{2} - 1) \gamma_0 + 1} \right), \]  

where \( \rho_{DA} \) is a variable while \( \rho_{AD} \) is a constant.

For low SNRs with \( \gamma_0 \to 0 \), we have

\[ \Delta R_{DA}^{low}(b_{DA}) = \log \left( \frac{1 + (1 - \rho_{AD})(\frac{1}{2} - 1) \gamma_0}{1 + (1 - \rho_{AD})(1 - \rho_{DA})(\frac{1}{2} - 1) \gamma_0} \right) \to 0, \]

which implies that the rate loss due to DAC quantization can be ignorable at low SNRs even when low-resolution converters are employed at both sides. Furthermore, we normalize \( \Delta R_{DA}^{low}(b_{DA}) \) by \( \gamma_0 \) as

\[ \frac{\Delta R_{DA}^{low}(b_{DA})}{\gamma_0} \to \frac{1}{\ln 2} \rho_{DA}(1 - \rho_{AD}) \left( \frac{1}{\beta} - 1 \right), \]

where we use \((1 + x)^{rac{1}{x}} \to e \) when \( x \to 0 \). It implies that the DAC quantization loss per energy is directly proportional to \( \rho_{DA} \) and decreases with \( \beta \) increasing. Similar observations can be made with fixed \( b_{DA} \) but varying \( b_{AD} \).

For high SNRs with \( \gamma_0 \to 1 \), we have

\[ \Delta R_{DA}^{high}(b_{DA}) = \log \left( 1 + \frac{1}{\rho_{DA} - 1}(\frac{1}{\beta} - 1) + 1 \right). \]

It is obvious that the rate loss is in general not ignorable. Given fixed \( b_{AD} \), we therefore aim to determine the DAC resolution, \( b_{DA} \), in order to guarantee the rate loss no larger than \( r_1 \) w.r.t the case using ideal DACs. Accordingly, characterizing \( \Delta R_{DA}^{high} \leq r_1 \) with (25) yields:

\[ b_{DA} = \begin{cases} \log \left[ \frac{2}{\sqrt{3\pi}(2^{r_1} - 1)\left( \frac{1}{\beta} - 1 \right)} \right]^{-\frac{1}{2}} \approx b_{AD} - \frac{1}{2} \log (2^{r_1} - 1) - \frac{1}{2} \log \left( \frac{1}{\beta} - 1 \right) \quad \text{(a)} \; \beta \geq 3, \\
\approx b_{DA} - \frac{1}{2} \log (2^{r_1} - 1) + \frac{1}{2} \log \beta, \quad \text{(b)} \; \beta < 3, \end{cases} \]

where (a) utilizes (7) with \( \rho_{AD} \ll 1 \) for \( b_{AD} \geq 3 \) and (b) is under the fact that \( \beta \ll 1 \) usually holds in massive MIMO.

Remark 2. It can be observed that \( b_{DA} \) should increase about one bit if \( r_1 \) becomes two bit smaller. This is intuitively reasonable because the real and imaginary components are quantized separately. Besides, \( b_{DA} \) should increase with increasing \( \beta \), i.e., serving more users or equipping less transmitting antennas. Higher DAC resolution retains more transmitting symbol information and consequently can serve more users. In particular from (26), \( b_{DA} \) should increase about one bit if \( \beta \) grows by four times. For a special choice of \( r_1 = 1 \), \( b_{DA} \approx \left[ b_{AD} + \frac{1}{2} \log \beta \right] \),

On the other hand, given fixed-bit DACs, the resolution of ADCs can be analogously derived as

\[ b_{AD} \approx \left[ b_{DA} - \frac{1}{2} \log (2^{r_2} - 1) - \frac{1}{2} \log \beta \right], \]

where \( r_2 \) is the maximum allowed rate loss due to finite-bit ADCs.

Remark 3. Comparing (27) with (26), the difference of characterizing \( b_{AD} \) and \( b_{DA} \) only lies on the sign of last item, \( \frac{1}{2} \log \beta \). When more transmitting antennas are employed at BS, higher ADC resolution is needed to detect more information with \( \beta \) decreasing.

IV. Simulation Results

In this section, we test the rate under the assumption of large \( N \) but fixed \( \beta \). Fig. 2 compares the achievable rate with infinite and finite-bit DAC/ADCs. Markers correspond to simulation results while solid lines correspond to the derived expressions. We observe that finite-bit ADCs cause more rate loss than...
DACs with the same resolution, which verifies Remark 1. Rate loss at low SNRs is rather marginal because thermal noise is dominating in this case.

Fig. 3 displays the contour of asymptotic rate. Obviously, $R(b_{DA}, b_{AD})$ increases when both $b_{DA}$ and $b_{AD}$ grow. We find that 99% $R(\infty, \infty)$ can be achieved by using 5-bit DACs and ADCs because AWGN is rather more pronounced than quantization noise when $\gamma_0 = -10$ dB. In other words, low resolution converters are suitable to low SNR conditions as expected. Fig. 4 shows the achievable rate with varying $b_{DA}$ and fixed $b_{AD}$. Given $\beta = \frac{1}{8}$, $b_{AD} = 6$ and $t_1 = 6, 4, 2, 0.5$ we have $b_{DA} = 2, 3, 4, 6$ according to (26). We observe that the rate loss caused by finite-bit DACs at high SNR decreases about two bits with $b_{DA}$ increasing one bit when $b_{DA}$ is small.

V. CONCLUSION

We derive the asymptotic downlink rate affected by both DAC and ADC quantizations in multiuser massive MIMO and reveal that the resolution of DACs should increase with the user load ratio in order to maintain a desired rate loss.

APPENDIX

From (11), the evaluation of $S_k$ and $I_k$ are, respectively,

$$S_k = (1 - \rho_{AD})^2 (1 - \rho_{DA}) |h_k^T p_j|^2,$$

$$I_k = (1 - \rho_{AD})^2 (1 - \rho_{DA}) \sum_{j \neq k} |h_k^T p_j|^2.$$  

Then using (11) and substituting (9), we have

$$Q_{1k} = (1 - \rho_{AD})^2 h_k^T E\{n_{DA} n_{DA}^H\} h_k^*$$

$$= (1 - \rho_{AD})^2 \rho_{DA} h_k^T \text{diag}(PP^H) h_k^*,$$

where we use $x = Ps$ and $E\{ss^H\} = I_M$. Similarly,

$$Q_{2k} = E\{n_{AD,k}^2\} = \rho_{AD} (1 - \rho_{AD}) E\{\text{diag}(yy^H)\}.$$  

Then using (11) and substituting (9), we have

$$Q_{1k} = (1 - \rho_{AD})^2 h_k^T E\{n_{DA} n_{DA}^H\} h_k^*$$

$$= (1 - \rho_{AD})^2 \rho_{DA} h_k^T \text{diag}(PP^H) h_k^*.$$  

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