Observational constraints on sign-changeable interaction models and alleviation of the $H_0$ tension

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We investigate various scenarios which include interaction forms between dark matter and dark energy that exhibit sign reverse, namely where the transfer of energy between the dark fluids changes sign during evolution. We study the large-scale inhomogeneities in such interacting scenarios and we confront them with the latest astronomical data. Our analysis shows that the sign-changeable interaction models are able to produce stable perturbations. Additionally, the data seem to slightly favor a non-zero interaction, however, within 1σ confidence level (CL) the scenarios cannot be distinguished from non-interacting cosmologies. We find that the best-fit value of the dark-energy equation-of-state parameter lies in the phantom regime, while the quintessence region is also allowed nevertheless at more than 2σ CL. Examining the effect of the interaction on the CMB TT and matter power spectra we show that while from the simple spectra it is hard to distinguish the interacting case from ΛCDM scenario, in the residual graphs the interaction is indeed traceable. Moreover, we find that sign-changeable interaction models can reconcile the $H_0$ tension, however the $σ_8$ tension is still persisting. Finally, we examine the validity of the laws of thermodynamics and we show that the generalized second law is always satisfied, while the second derivative of the total entropy becomes negative at late times which implies that the universe tends towards thermodynamic equilibrium.

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I. INTRODUCTION

Recent observations of various origin suggest that around 69% of the universe consists of the dark energy sector, while around 26% constitutes of the cold dark matter one [1]. One usually assumes that these two sectors do not mutually interact, and the resulting scenario is capable of describing very efficiently the various sets of independent observations. Nevertheless, since the underlying microscopic theory of both these sectors is unknown, there is not any field-theoretical argument against the consideration of a possible mutual interaction. Furthermore, such an interaction could alleviate the known coincidence problem [2, 3, 4], namely why are the energy densities of dark matter and dark energy currently of the same order although they follow completely different scaling laws during evolution. Hence, in the literature one can find many such interacting scenarios (see [5, 6] for review and references therein), independently of the specific dark-energy nature, namely whether it arises from fields [2] or through a gravitational modification [9, 10, 11]. The interacting scenarios can be very efficient in describing late-time universe, and moreover they seem to be slightly favored comparing to non-interacting ones [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. Additionally, they seem to be efficient in addressing the $H_0$ tension [30, 31, 32, 33, 34] as well as the $σ_8$ tension [35, 36].

In the above scenarios the interaction term, that determines the interaction rate and thus the flow of energy between the dark matter and dark energy sectors, is introduced phenomenologically. Although there is a large variety in such choices, they are usually assumed to have the same sign, namely the energy flow is from one component to the other during the whole universe evolution. However, an interesting question arises, namely what would happen if we allow for a sign change of the interaction term during the cosmological evolution. Such a consideration might be further useful to investigate the dark sectors’ physics. Hence, in the present work we desire to investigate this possibility and in particular to confront the obtained scenarios with different observational data coming from probes like the cosmic microwave background radiation (CMB), supernovae type Ia (SNIa), baryon acoustic oscillations (BAO), and Hubble parameter measurements.

We organize the present work in the following way. In Section I we provide the cosmological equations at background and perturbative levels, in presence of arbitrary coupling in the dark sector. In Section II we present the models that we wish to study in this work. In Section III we describe the observational datasets and the fitting

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methodology. In Section V we provide the constraints on the models, and we perform a Bayesian analysis in comparison with $Λ$CDM cosmology. Finally, we close the present work in Section VII with a brief summary of the results.

II. INTERACTING COSMOLOGY

In this Section we briefly review interacting cosmology. We consider a homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) line element of the form

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j,$$

where $a(t)$ is the expansion scale factor of the universe. Furthermore, we consider the universe to be filled with baryons, cold dark matter, radiation, and the dark energy sector (which may be of effective origin or not), all of which considered as barotropic perfect fluids. Thus, the Friedmann equations that determine the universe evolution are written as

$$H^2 = \frac{8\pi G}{3} \rho_t,$$

$$2\dot{H} + 3H^2 = -8\pi G \rho_t,$$

with $G$ the Newton’s constant and $H = \dot{a}/a$ the Hubble function (dots denote derivatives with respect to $t$). In the above equations we have introduced the total energy density and pressure respectively as $\rho_t = \rho_r + \rho_b + \rho_c + \rho_x$ and $p_t = p_r + p_b + p_c + p_x$, with the subscripts $r$, $b$, $c$, $x$ denoting radiation, baryon, cold dark matter and dark energy.

Although the Bianchi identities lead to the conservation of the total energy momentum tensor, they do not imply anything for the separate sectors, and thus one can assume that some of them mutually interact [2 3 4 57]. In this work we allow the dark matter and dark energy sectors to interact, while radiations and baryonic matter are considered to be conserved. In particular, we assume that

$$\dot{\rho}_b + 3H \rho_b = 0,$$

$$\dot{\rho}_r + 4H \rho_r = 0,$$

$$\dot{\rho}_c + 3H \rho_c = -Q,$$

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = Q,$$

where $w_x$ is the dark-energy equation-of-state parameter (for baryonic and dark matter we consider the dust case $w_b = w_c = 0$, while for radiation as usual $w_r = 1/3$). The introduced quantity $Q$ is a phenomenological descriptor of the interaction, and its form is considered arbitrarily. If $Q > 0$ then the energy transfer is from cold dark matter (pressureless dark matter) to dark energy, while if $Q < 0$ then it is from dark energy to dark matter. Moreover, as usual the conservation equations for baryonic matter and radiation give $\dot{\rho}_b = \rho_{0b} a^{-3}$ and $\dot{\rho}_r = \rho_{0r} a^{-4}$ respectively, with $\rho_{0i}$ ($i = r, b$) the value of $\rho_i$ at present time. In summary, if the interaction function is given then the Friedmann equation (2) alongside the conservation equations (6) and (7) can determine the evolution of the universe.

One can see that the conservation equations (6) and (7) can be written in an alternative way as

$$\dot{\rho}_c + 3H(1 + w^\text{eff}_c) \rho_c = 0,$$

$$\dot{\rho}_x + 3H(1 + w^\text{eff}_x) \rho_x = 0,$$

where $w^\text{eff}_c$, and $w^\text{eff}_x$ are the effective equation-of-state parameters for cold dark matter and dark energy, given as

$$w^\text{eff}_c = \frac{Q}{3H\rho_c},$$

$$w^\text{eff}_x = w_x - \frac{Q}{3H\rho_x}.$$

Hence, as we observe, the interaction affects the equation of state of these components. In particular, dark matter may depart from dust while dark energy may be quintessence or phantom like even if the initial $w_x$ is fixed to one regime.

We proceed to the investigation of the above scenarios at the level of perturbations. We consider scalar perturbations around an FRW metric given by [38 39 40]

$$ds^2 = a^2(r) \left\{ -(1 + 2\phi)dt^2 + 2\partial_i Bdr dx^i + [(1 - 2\psi)\delta_{ij} + 2\partial_j B] dx^i dx^j \right\} ,$$

where $r$ represents the conformal time and the quantities $\phi$, $B$, $\psi$, $E$, denote the gauge-dependent scalar perturbations. Thus, in the case of interacting cosmology with $w_x \neq -1$ the perturbation equations in the synchronous gauge ($\phi = B = 0$, $\psi = \eta$, and $k^2 E = -h/2 - 3\eta$), with $k$ the Fourier mode and $h, \eta$, being the metric scalar perturbations [69], are written as [11 22 43]:

$$\delta_x' = -\left(1 + w_x\right) \left( \frac{\theta_x + h'}{2} \right) - 3H w_x \frac{\theta_x}{k^2}$$

$$-3H (c^2_{sx} - w_x) \left[ \frac{\delta_x + 3H(1 + w_x)}{k^2} \theta_x \right]$$

$$+ \frac{aQ}{\rho_x} \left[ -\delta_x + \frac{\delta Q}{Q} + 3H (c^2_{sx} - w_x) \frac{\theta_x}{k^2} \right] ,$$

$$\theta_x' = -H (1 - 3c^2_{sx}) \theta_x + \frac{c^2_{sx}}{1 + w_x} k^2 \delta_x$$

$$+ \frac{aQ}{\rho_x} \left[ \frac{\theta_x - (1 + c^2_{sx}) \theta_x}{1 + w_x} \right] ,$$

$$\delta_c' = - \left( \frac{\delta_c + h'}{2} \right) + \frac{aQ}{\rho_c} \left( \delta_x - \frac{\delta Q}{Q} \right) ,$$

$$\theta_c' = -H \theta_c .$$

In the above expressions we have introduced the overdensities $\delta_i = \delta \rho_i/\rho_i$, as well as the velocity perturbations $\theta_i$, with primes denoting derivatives with respect to the
conformal time $\tau$, with $\mathcal{H} = \frac{a'}{a}$ the conformal Hubble function. Moreover, $c_{sx}^2$ is the adiabatic sound speed, which in the following will be set to $1$ \cite{11, 12, 13} (the adiabatic sound speed for the dark matter in the dust case is $c_{sd}^2 = 0$).

Now, for the case where $w_x = -1$, namely in the case of interacting vacuum scenario, the perturbation equations are slightly different. In the synchronous gauge the momentum conservation equation for dark matter reduces to $\dot{\theta}_x = 0$ \cite{44}, while the density perturbations can be recast into \cite{44}

$$\delta_c = -\frac{\dot{h}}{2} + \frac{Q}{\rho_c} \delta_c , \quad (17)$$

However, in this gauge the vacuum energy is spatially homogeneous, i.e. $\delta\rho_x = 0$.

Finally, it is important to remark that although in the present work, as well as in many works in the literature, the spatial curvature of the universe is not considered, a series of works shows that the observational data do not exclude the non-zero curvature \cite{45, 46, 17, 33, 49, 50, 51, 52, 53}, and hence the general universe picture should include the spatial curvature as a free parameter. We will address such a complete investigation in a separate work.

III. SIGN-CHANGEABLE INTERACTION MODELS

Since the appearance of interacting cosmology, a variety of phenomenological coupling functions $Q$ have been introduced and investigated in the literature. In general, most of the coupling functions indicate a particular direction of energy transfer, namely either the transfer of energy from dark matter to dark energy or vice versa. This includes functions of the form $Q \propto H\rho_c$, $Q \propto H\rho_x$, $Q \propto H(\rho_c + \rho_x)$, $Q \propto H(\rho_c \rho_x)/(\rho_c + \rho_x)$, $Q \propto H\rho_c^2/(\rho_c + \rho_x)$, nonlinear forms, etc \cite{25, 32, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85}.

However, in principle one could also have the case in which the interaction function, i.e. $Q$, changes sign during the evolution, namely the energy transfer between the interacting sectors changes direction \cite{86, 87, 88, 89, 90}. In the following subsections we examine two of such models separately.

A. Interaction function $Q = 3H\xi(\rho_c - \rho_x)$

The first interacting model in which the interacting function changes sign during evolution is

$$Q(t) = 3H\xi(\rho_c - \rho_x), \quad (18)$$

where $\xi$ is the coupling parameter. Note the interesting feature that when $\rho_c = \rho_x$ then the interaction becomes zero even if $\xi \neq 0$. This kind of feature was observed in a different interaction model \cite{106}. Here, we consider two choices for the dark energy equation-of-state parameter $w_x$, namely the cosmological constant case $w_x = -1$ (which corresponds to the interacting vacuum scenario, from now on model IVS1), and the case $w_x \neq -1$ (from now on model IDE1). The latter case is interesting since it allows to extract constraints on the dark energy equation-of-state parameter and examine whether it lies in the quintessence or in the phantom regime.

We proceed by numerically elaborating the background cosmological equations, using as independent variable the redshift, $z = a_0/a - 1$, setting the present scale factor $a_0$ to 1. Moreover, we introduce the density parameters of the various components through $\Omega_i = 8\pi G \rho_i/(3H^2)$ and we set their current values as $\Omega_{\odot} \approx 0.69$, $\Omega_{\odot} \approx 0.25$, $\Omega_{\odot} \approx 0.05$ and $\Omega_{\odot} \approx 10^{-4}$ in agreement with observations \cite{11}.

In order to obtain a picture for the above interaction form, in Fig. 1 we first depict the evolution of $Q$ normalized by $Q_0 = H_0\rho_{00}$ (with $H_0$ and $\rho_{00}$ the present values of the Hubble parameter and the total energy density $\rho_t$ respectively), for various values of the coupling parameter $\xi$ in units where $8\pi G = 1$, for $w_x = -0.98$ (upper graph) and for $w_x = -1.01$ (lower graph).

![Graph](image_url)
\( \rho = \rho r = q \) of IDE1 for various values of the coupling parameter \( \xi \) in units where \( 8\pi G = 1 \). We have set \( \Omega_{\Lambda0} \approx 0.69, \Omega_{c0} \approx 0.25, \Omega_{b0} \approx 0.05 \) and \( \Omega_{r0} \approx 10^{-4} \) in agreement with observations.

In Fig. 2 we depict the evolution of the deceleration parameter for model IDE1, for \( w_x > -1 \) and \( w_x < -1 \), and choosing various values of the coupling parameter \( \xi \). As we observe, the value of \( \xi \) may make faster or delay the transition from deceleration to acceleration, which is a significant advantage in fitting observations precisely. Additionally, note that the qualitative features do not change for either \( w_x > -1 \) or \( w_x < -1 \). A similar graph can be obtained in the case of model IVS1, namely when \( w_x = -1 \).

We mention that in this scenario one can extract the solution for the ratio \( \frac{\rho_c}{\rho_x} \) analytically. In particular, inserting (18) into (6) and (7) gives

\[
\frac{\rho_x}{\rho_c} = -\frac{w_x}{2\xi} + \sqrt{1 + \frac{w_x^2}{4\xi^2} \left( \frac{1 - 6w_x \sqrt{4\xi^2 + w_x^2}}{1 + 6w_x \sqrt{4\xi^2 + w_x^2}} \right)},
\]

where

\[
b = -\frac{2 \xi \Omega_{\Lambda0}}{\Omega_{c0}} + w_x + \sqrt{4\xi^2 + w_x^2} = -\frac{2 \xi \Omega_{\Lambda0}}{\Omega_{c0}} + w_x - \sqrt{4\xi^2 + w_x^2}.
\]

We proceed by investigating the coincidence parameter \( r = \rho_c/\rho_x \) for this interaction model. As it is known, alleviation of the coincidence problem requires a non-zero value for \( r \) at late times. The evolution of the coincidence parameter for the model IDE1 is presented in Fig. 3 for different dark energy equation of states and setting a typical value of \( \xi = 0.01 \) (for different values of \( \xi \) the qualitative nature of the curves remains unaffected).

Moreover, for comparison we also depict the results for some well known interaction scenarios of the literature, namely \( Q = 3H^2 \rho_c, Q = 3H^2 \rho_x \) and \( Q = 3H^2 (\rho_c + \rho_x) \).

As one can see, as \( z \to 0 \) all curves tend to a non-zero value, which implies that the coincidence problem is alleviated. Additionally, although at low redshifts all models present the same behavior, at high redshifts the sign-changeable interaction models considered in this work are distinguishable from the known interacting models of the literature.

Finally, concerning the perturbations in this scenario, they are determined by (13)-(16), with

\[
\frac{\delta Q}{Q} = \frac{\delta \rho_c - \delta \rho_x}{\rho_c - \rho_x} + \frac{2\theta + h'}{6H},
\]

with \( \theta = \delta \rho / \rho \) the volume expansion of the total fluid.
The evolution of the deceleration parameter as a function of the redshift, for the interaction model IDE2 of (22), for various values of the coupling parameters \( \alpha \) and \( \beta \) in units where \( 8\pi G = 1 \), for \( w_x = -0.98 \) (upper graphs) and for \( w_x = -1.01 \) (lower graphs).

The evolution of the normalized interaction function \( Q/Q_0 \), where \( Q_0 = H_0 \rho_0 \) (with \( H_0 \) and \( \rho_0 \) the present values of the Hubble parameter and the total energy density \( \rho \), respectively), for the interaction model IDE2 of (22), for various values of the coupling parameters \( \alpha \) and \( \beta \) in units where \( 8\pi G = 1 \), for \( w_x = -0.98 \) (upper graphs) and for \( w_x = -1.01 \) (lower graphs).

We have set \( \Omega_{c0} \approx 0.74, \Omega_{b0} \approx 0.025, \Omega_{\Phi0} \approx 0.05 \) and \( \Omega_{c0} \approx 10^{-4} \) in agreement with observations.
FIG. 6: The evolution of the coincidence parameter $r = \rho_c / \rho_x$, for the interaction model IDE2 of (22), for various values of the coupling parameters $\alpha$ and $\beta$ in units where $8\pi G = 1$, for $w_x = -0.98$ (upper graphs) and for $w_x = -1.01$ (lower graphs). For comparison we additionally depict the results for some well known interaction scenarios, namely $Q = 3H\xi\rho_x$, $Q = 3H\xi\rho_c$, and $Q = 3H\xi(\rho_c + \rho_x)$, taking a typical value $\xi = 0.01$ in units where $8\pi G = 1$.

B. Interaction function $Q = 3H(\alpha\rho_c - \beta\rho_x)$

We proceed with the investigation of the interacting model

$$Q = 3H(\alpha\rho_c - \beta\rho_x),$$

(22)

where $\alpha$, $\beta$ are the coupling parameters considered to have the same sign. In the case where $\alpha = \beta = \xi$ the above model coincides with the one of the previous subsection, namely (18). We consider two choices for $w_x$, namely $w_x = -1$ (from now on model IVS2), and the case $w_x \neq -1$ (from now on model IDE2), which will be further divided into $w_x > -1$ and $w_x < -1$.

In Fig. 4 we depict the evolution of $Q$ normalized by $Q_0 = H_0^2\rho_0$, for various values of the coupling parameters $\alpha$ and $\beta$. Additionally, in Fig. 5 we present the evolution of the deceleration parameter for model IDE2, for $w_x > -1$ and $w_x < -1$ and choosing various values of $\alpha$ and $\beta$. As we observe, the values of the coupling parameters significantly determine the exact transition redshift from deceleration to acceleration, while the exact value of $w_x$ does not have a significant effect. Similar graphs can be obtained in the case of IVS2 model, namely when $w_x = -1$.

Inserting (22) into (6) and (7) provides as the analytical solution for the ratio $\frac{\rho_x}{\rho_c}$, namely

$$\frac{\rho_x}{\rho_c} = \frac{\alpha - \beta - w_x}{2\beta} - \frac{\sqrt{(\alpha - \beta - w_x)^2 + 4\alpha\beta}}{2\beta} \cdot \left(1 - \frac{ba^3\sqrt{(\alpha - \beta - w_x)^2 + 4\alpha\beta}}{1 + ba^3\sqrt{(\alpha - \beta - w_x)^2 + 4\alpha\beta}}\right),$$

(23)

with

$$b = \frac{2\beta\Omega_m0 - (\alpha - \beta - w_x) + \sqrt{(\alpha - \beta - w_x)^2 + 4\alpha\beta}}{2\beta\Omega_m0 - (\alpha - \beta - w_x) - \sqrt{(\alpha - \beta - w_x)^2 + 4\alpha\beta}}.$$

(24)

We now investigate the evolution of the coincidence parameter $r = \rho_c / \rho_x$ for this interaction model. In Fig. 6 we depict the evolution of $r$ for various values of the coupling parameters $\alpha$ and $\beta$ and for different dark energy equation of states. Furthermore, for comparison we also depict the results for some well known interaction scenarios, namely $Q = 3H\xi\rho_x$, $Q = 3H\xi\rho_c$, and $Q = 3H\xi(\rho_c + \rho_x)$. Similarly to model IDE1 above, for the present model IDE2 we can see that for $z \to 0$ all curves tend to a non-zero value, which implies that the coincidence problem is alleviated. Additionally, although at low redshifts all models present similar behavior, at high redshifts the sign-changeable interaction models considered here are distinguishable from the known interacting models of the literature.
Lastly, concerning the perturbations, in this scenario they are determined by (13)-(16), with
\[
\frac{\delta Q}{Q} = \frac{\alpha \delta \rho_c - \beta \delta \rho_x}{\alpha \rho_c - \beta \rho_x} + \frac{2 \theta + h'}{6 H}.
\] (25)

IV. THE DATA AND METHODOLOGY

In this section we briefly describe the observational datasets and the methodology we follow in order to constrain the aforementioned interaction models.

1. Cosmic microwave background (CMB) radiation: We consider the CMB data from Planck 2015 measurements [91,92], and in particular we use the high-\(\ell\) and low-\(\ell\) temperature and polarization data from [91,92].

2. Baryon acoustic oscillation (BAO): We use data from BAO distance measurements from the following sources. Data from 6dF Galaxy Survey (6dFGS) (redshift measurement at \(z_{\text{eff}} = 0.106\) [93], data from Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) \(z_{\text{eff}} = 0.15\) [95], and data from CMASS and LOWZ samples of the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) \(z_{\text{eff}} = 0.57\) [96] and \(z_{\text{eff}} = 0.32\) [96].

3. Supernovae Type Ia (SNIa): We use the most latest compilation of SNIa, consisting of 1048 data points spanned over the redshift interval \(z \in [0.01, 2.3]\) known as the Pantheon sample [93].

4. Cosmic Chronometers (CC): We use the Hubble parameter measurements from the cosmic chronometers. The total number of data is 30 and the measurements are spanned over the redshift interval \(0 < z < 2\) [97]. For technical details we further refer to [97].

In order to perform the analysis and extract the observational constraints, we use the Markov chain Monte Carlo package CosmoMC [98,99] where a convergence diagnostic by Gelman-Rubin is included [100], which in addition supports the Planck 2015 likelihood code [92]\(^1\). In the case of IDE1 model we have the eight-dimensional parameter space:
\[
P_1 = \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, w_x, n_s, \log[10^{10} A_S], \xi \},
\]
while for IVS1 we have one parameter less since \(w_x = -1\). For IDE2 we have the nine-dimensional parameter space
\[
P_2 = \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, w_x, n_s, \log[10^{10} A_S], \alpha, \beta \},
\]
and similarly for IVS2 we have one parameter less. In the above expressions \(\Omega_b h^2\) is the physical baryons density, \(\Omega_c h^2\) is the cold dark matter density, \(100 \theta_{MC}\) is the ratio of sound horizon to the angular diameter distance, \(\tau\) is the optical depth, \(w_x\) is the equation-of-state parameter for dark energy, \(n_s\) is the scalar spectral index, and \(A_S\) is the amplitude of the initial power spectrum. The remaining parameters in \(P_1\) and \(P_2\), namely \(\xi, \alpha\) and \(\beta\) are the coupling parameters for the interaction models. Finally, in Table I we present the flat priors imposed on the free parameters of the prescribed interacting scenarios.

| Parameter | Prior |
|-----------|-------|
| \(\Omega_b h^2\) | [0.005, 0.1] |
| \(\Omega_c h^2\) | [0.01, 0.99] |
| \(\tau\) | [0.01, 0.8] |
| \(n_s\) | [0.5, 1.5] |
| \(\log[10^{10} A_S]\) | [2.4, 4] |
| \(100 \theta_{MC}\) | [0.5, 10] |
| \(w_x\) | \([-2, 0]\) |
| \(\xi\) | \([-1, 0]\) |
| \(\alpha\) | \([-1, 0]\) |
| \(\beta\) | \([-1, 0]\) |

TABLE I: The flat priors on the cosmological parameters used in the present analyses.

V. OBSERVATIONAL CONSTRAINTS

In this section we provide the observational constraints on the sign-changeable scenarios and we discuss their consequences. In order to acquire a complete picture we consider three different combinations of the observational datasets described above, namely CMB+BAO, CMB+BAO+Pantheon and CMB+BAO+Pantheon+CC.

A. Interaction function \(Q = 3H(\rho_c - \rho_x)\)

The observational summary for IDE1 model is presented in Table III. Additionally, in Fig. 7 we provide the corresponding 2D contour plots of various parameters at 1\(\sigma\) and 2\(\sigma\) confidence level (CL), alongside the 1D marginalized posterior distribution. From Table III we can notice that the addition of Pantheon or Pantheon+CC to the observational combination

\(^1\) See the publicly available code at http://cosmologist.info/
CMB+BAO, slightly improves the constraints by reducing their error bars, nevertheless the improvement is not significant. Additionally, from Fig. 7 we can see that the parameters \((H_0, w_x)\) and \((H_0, \Omega_{m0})\) are negatively correlated to each other.

Concerning the dark energy equation-of-state parameter \(w_x\), from Fig. 7 we infer that a phantom nature of \(w_x\) is clearly preferred at more than 2\(\sigma\) CL. Additionally, concerning the coupling \(\xi\) and the dark energy equation-of-state parameter \(w_x\), from Table II it is clearly observed that the evidence for an interaction is small (\(\xi = -0.00014^{+0.00013}_{-0.00021}\) at 1\(\sigma\) for CMB+BAO, \(\xi = -0.00013^{+0.00013}_{-0.00020}\) at 1\(\sigma\) for CMB+BAO+Pantheon, \(\xi = -0.00013^{+0.00013}_{-0.00020}\) at 1\(\sigma\) for CMB+BAO+Pantheon+CC).

### Table II: Summary of the 68% (1\(\sigma\)) and 95% (2\(\sigma\)) confidence-level (CL) constraints on the interaction model IDE1 of (18), using various combinations of the observational data sets. Here, \(\Omega_{m0}\) denotes the present value of \(\Omega_m = \Omega_b + \Omega_c\) and \(H_0\) is in the units of km/s/Mpc.

| Parameters | CMB+BAO | CMB+BAO+Pantheon | CMB+BAO+Pantheon+CC |
|------------|---------|------------------|---------------------|
| \(\Omega_b h^2\) | 0.1198\(+0.0013^{+0.0024}_{-0.0026}\) | 0.1196\(+0.0012^{+0.0024}_{-0.0023}\) | 0.1190\(+0.0011^{+0.0024}_{-0.0023}\) |
| \(\Omega_c h^2\) | 0.0223\(+0.0016^{+0.0032}_{-0.0029}\) | 0.0223\(+0.0015^{+0.0029}_{-0.0031}\) | 0.0223\(+0.0015^{+0.0030}_{-0.0029}\) |
| 1000\(\theta MC\) | 1.04051\(+0.0004^{+0.00066}_{-0.00066}\) | 1.04054\(+0.0003^{+0.00064}_{-0.00064}\) | 1.04053\(+0.0003^{+0.00057}_{-0.00058}\) |
| \(\tau\) | 0.082\(+0.017^{+0.032}_{-0.032}\) | 0.084\(+0.017^{+0.030}_{-0.031}\) | 0.082\(+0.016^{+0.032}_{-0.034}\) |
| \(n_s\) | 0.9741\(+0.0041^{+0.0078}_{-0.0080}\) | 0.9741\(+0.0040^{+0.0080}_{-0.0080}\) | 0.9742\(+0.0037^{+0.0073}_{-0.0074}\) |
| \(\ln(10^{10} A_s)\) | 3.104\(+0.034^{+0.066}_{-0.063}\) | 3.104\(+0.033^{+0.065}_{-0.061}\) | 3.100\(+0.032^{+0.066}_{-0.066}\) |
| \(w_x\) | -1.0818\(+0.0614^{+0.0867}_{-0.0916}\) | -1.0529\(+0.0572^{+0.0483}_{-0.0555}\) | -1.0545\(+0.0524^{+0.0482}_{-0.0545}\) |
| \(\xi\) | -0.00014\(+0.00014^{+0.00019}_{-0.00019}\) | -0.00013\(+0.00013^{+0.00013}_{-0.00020}\) | -0.00013\(+0.00013^{+0.00013}_{-0.00020}\) |
| \(\Omega_{m0}\) | 0.299\(+0.011^{+0.020}_{-0.019}\) | 0.305\(+0.007^{+0.014}_{-0.013}\) | 0.305\(+0.007^{+0.014}_{-0.014}\) |
| \(\sigma_8\) | 0.845\(+0.017^{+0.036}_{-0.034}\) | 0.840\(+0.015^{+0.027}_{-0.027}\) | 0.839\(+0.014^{+0.030}_{-0.028}\) |
| \(H_0\) | 69.19\(+0.93^{+2.0}_{-1.90}\) | 68.39\(+0.68^{+1.39}_{-0.69}\) | 68.43\(+0.67^{+1.44}_{-0.70}\) |

### Figures

**FIG. 7:** The 1\(\sigma\) and 2\(\sigma\) CL contour plots for several combinations of various quantities and using various combinations of the observational data sets, for the interaction model IDE1 of (18), and the corresponding 1-dimensional (1D) marginalized posterior distributions. Here, \(\Omega_{m0}\) denotes the present value of \(\Omega_m = \Omega_b + \Omega_c\) and \(H_0\) is in the units of km/s/Mpc.
FIG. 8: The temperature anisotropy in the CMB TT spectra for the interaction model IDE1 of [18], considering three different combinations of the observational datasets, namely CB (= CMB+BAO), CBP (= CMB+BAO+Pantheon) and CBPC (= CMB+BAO+Pantheon+CC), as well as the curve for ΛCDM paradigm (upper graph), and the corresponding residual plot with reference to ΛCDM scenario (lower graph).

Hence, within 1σ the non-interacting cosmology is allowed. In summary, our observational confrontation shows that this interaction model at the background level essentially mimicks a $w_x$CDM-type cosmology with $w_x < -1$ at more than 2σ CL.

We now proceed by examining the tensions on the two main parameters, namely $H_0$ and $\sigma_8$. As we observe from both Table III and Fig. 8 (specifically from the posterior distribution of $H_0$ which is the extreme right plot of the bottom panel), $H_0$ acquires slightly higher values...
FIG. 10: The 1σ and 2σ CL contour plots for several combinations of various quantities and using various combinations of the observational data sets, for the interaction model IVS1, and the corresponding 1D marginalized posterior distributions. Here, $\Omega_{m0}$ denotes the present value of $\Omega_m = \Omega_b + \Omega_c$ and $H_0$ is in the units of km/s/Mpc.

FIG. 10: The 1σ and 2σ CL contour plots for several combinations of various quantities and using various combinations of the observational data sets, for the interaction model IVS1, and the corresponding 1D marginalized posterior distributions. Here, $\Omega_{m0}$ denotes the present value of $\Omega_m = \Omega_b + \Omega_c$ and $H_0$ is in the units of km/s/Mpc.

compared to Planck [1], with slightly higher error bars. Although the local estimation of the Hubble constant obtained by Riess et al. [107], i.e. $H_0 = 73.24 \pm 1.74$, is certainly greater than the estimated mean values of $H_0$ for this interaction model, due to the increased error bars the tension is reduced to the level of 2σ CL. Thus, the interaction between the dark components provides a way to reduce the tension from $> 3\sigma$ CL. Nevertheless, concerning $\sigma_8$ tension we deduce that the present interaction model is not able to alleviate it.

Let us now examine the effect of the interaction on the large scale observables, and mainly on the CMB TT and matter power spectra. In the upper graph of Fig. 8 we show how the interaction affects the CMB TT spectra, considering the constraints on the parameters extracted from all observational datasets, namely CMB+BAO, CMB+BAO+Pantheon and CMB+BAO+Pantheon+CC, in which for completeness we add the non-interacting case of ΛCDM cosmology. From this graph it is hard to distinguish the interacting case from ΛCDM scenario. However, in the lower graph of Fig. 8 we depict the corresponding residual plot (with reference to ΛCDM model), and one can indeed trace a distinction between interacting and non-interacting cosmologies, mainly in the lower multipoles. Similarly, we investigate the effects of the interaction through the matter power spectra presented in Fig. 9. Although in the upper graph the distinction between interacting and non-interacting cosmologies cannot be observed, in the lower graph the deviation from the non-interacting ΛCDM cosmology is clear. This is one of the main results of the present work.

We close the analysis of this model by focusing on the case where $w_x = -1$, thus $w_x$ is not a free parameter and is fixed to the cosmological constant value, namely we examine the interacting vacuum scenario. In this case the case of no-interaction seems to be favored, while $H_0$ acquires smaller values, and therefore the $H_0$ tension is not alleviated. Concerning $\sigma_8$ tension we deduce that it cannot be released either. Finally, at large scales this specific interacting scenario does not return different results in the CMB TT and matter power spectra comparing to ΛCDM cosmology, and therefore we do not explicitly present the corresponding plots.
TABLE IV: Summary of the 1σ and 2σ CL constraints on the interaction model IDE2 of (22), using various combinations of the observational data sets. Here, Ω_{m0} denotes the present value of Ω_{m0} = Ω_{b0} + Ω_{c0} and H₀ is in the units of km/s/Mpc.

| Parameters | CMB+BAO | CMB+BAO+Pantheon | CMB+BAO+Pantheon+CC |
|------------|---------|------------------|---------------------|
| H₀ | 0.80 | 0.88 | 0.96 | 1.04 | 1.12 |
| Ω_m0 | 0.96 | 0.279±0.032±0.122 | 0.787±0.040±0.061 | 0.879±0.043±0.063 | 0.879±0.043±0.063 |
| σ⁸ | 0.80 | 0.08 | 0.04 | 0.04 | 0.04 |
| x_c | 0.275 | 0.325 | 0.325 | 0.325 | 0.325 |
| Ω_b h² | 0.000015±0.000015 | 0.00015±0.00015 | 0.00015±0.00015 | 0.00015±0.00015 | 0.00015±0.00015 |
| Ω_c h² | 0.000015±0.000015 | 0.00015±0.00015 | 0.00015±0.00015 | 0.00015±0.00015 | 0.00015±0.00015 |
| 100θMC | 0.973±0.0037±0.0075 | 0.973±0.0037±0.0075 | 0.973±0.0037±0.0075 | 0.973±0.0037±0.0075 | 0.973±0.0037±0.0075 |
| ln(10^{10} A_s) | 3.101±0.032±0.062 | 3.101±0.032±0.062 | 3.101±0.032±0.062 | 3.101±0.032±0.062 | 3.101±0.032±0.062 |
| w_x | -0.933±0.0529±0.0777 | -0.933±0.0529±0.0777 | -0.933±0.0529±0.0777 | -0.933±0.0529±0.0777 | -0.933±0.0529±0.0777 |
| α | -0.000014±0.000004±0.000019 | -0.000014±0.000004±0.000019 | -0.000014±0.000004±0.000019 | -0.000014±0.000004±0.000019 | -0.000014±0.000004±0.000019 |
| β | -0.02444±0.02444±0.02444 | -0.02444±0.02444±0.02444 | -0.02444±0.02444±0.02444 | -0.02444±0.02444±0.02444 | -0.02444±0.02444±0.02444 |
| Ω_m0 | 0.279±0.023±0.034 | 0.292±0.023±0.023 | 0.292±0.023±0.023 | 0.292±0.023±0.023 | 0.292±0.023±0.023 |
| σ₈ | -0.72±0.72 | -0.72±0.72 | -0.72±0.72 | -0.72±0.72 | -0.72±0.72 |

B. Interaction function \(Q = 3H(\alpha \rho_c - \beta \rho_x)\)

The observational summary of model IDE2 is shown in Table IV while the corresponding 2D contour plots are presented in Fig. 11. Similarly to model IDE1, one can notice that the addition of Pantheon or Pantheon+CC to the combined analysis CMB+BAO, improves the parameters space only slightly. Moreover, from Fig. 11 we can see that the parameters \((H₀, w_x)\) and \((H₀, \Omega_{m0})\) are negatively correlated to each other.

Concerning the coupling parameters α and β, our analysis shows that the zero values are allowed within 1σ. Additionally, the dark-energy equation-of-state parameter prefers the phantom regime for all datasets, namely...
we see that $w_x < -1$ at more than 2σ. Furthermore, similarly to IDE1 model, in the present IDE2 scenario we also find that the estimations of $H_0$ are slightly higher compared to the ΛCDM-based Planck estimation [1], and due to the higher error bars on $H_0$ the relevant tension can be slightly reconciled due to the interaction. However, concerning $\sigma_8$ we see that the tension is not released.

We proceed by investigating the effect of the interaction on the CMB TT and matter power spectra. In the upper graph of Fig. 12 we depict the CMB TT spectra considering the constraints on the parameters extracted from all observational datasets, namely CMB+BAO, CMB+BAO+Pantheon, and CMB+BAO+Pantheon+CC, in which for completeness we add the non-interacting case of ΛCDM cosmology. Moreover, in the lower graph of Fig. 12 we present the corresponding residual plot (with reference to ΛCDM model). As we observe, this interaction model is distinguished from the non-interacting ΛCDM cosmology at both lower and higher multipoles. We further investigate the effects of the interaction on the matter power spectra, depicted in Fig. 13. Although in the upper graph the distinction between the interacting and non-interacting cosmologies cannot be observed, in the lower graph the deviation from the non-interacting ΛCDM cosmology is clear even for the small values of the coupling parameters $\alpha$, $\beta$ that were obtained from the three different observational datasets. This is one of the main results of the present work.

We proceed by analyzing the case $w_x = -1$, namely the interacting vacuum scenario. The results of the analyses are presented in Table V while in Fig. 14 we depict the corresponding contour plots. From these we deduce that the coupling parameter $\alpha$ acquires the zero value within $1\sigma$ CL irrespectively of the datasets, while $\beta$ has a tendency towards non-zero values nevertheless the value zero is allowed within $1\sigma$ except for the final combination CMB+BAO+Pantheon+CC. Additionally, concerning $H_0$, for CMB+BAO+Pantheon and CMB+BAO+Pantheon+CC datasets we find that its estimations are relatively high compared to ΛCDM-based Planck estimation [1], while the error bars on $H_0$...
The observational data sets, for the interaction model IVS2, and the corresponding 1D marginalized posterior distributions. Here, $\Omega_m$ cannot be alleviated. Hence, this enables $H_0$ to acquire values close to its local estimation $H_0^{\text{loc}}$, and thus the tension on $H_0$ is weakly resolved. However, the $\sigma_8$ tension cannot be alleviated.

### C. Bayesian analysis

We close the observational confrontation by presenting observational viabilities of the models using the Bayesian evidence. The Bayesian analysis is an important part of the cosmological model selection that quantifies the fitting results compared to a reference scenario. The com-

| Parameters | CMB+BAO | CMB+BAO+Pantheon | CMB+BAO+Pantheon+CC |
|------------|---------|------------------|----------------------|
| $\Omega_c h^2$ | 0.114$^{+0.0048+0.0069}_{-0.0020-0.0085}$ | 0.108$^{+0.0106+0.0126}_{-0.0044-0.0159}$ | 0.105$^{+0.0111+0.0149}_{-0.0053-0.0181}$ |
| $\Omega_b h^2$ | 0.0233$^{+0.00013+0.00026}_{-0.00013-0.00028}$ | 0.0223$^{+0.00016+0.00030}_{-0.00015-0.00030}$ | 0.0223$^{+0.00014+0.00030}_{-0.00015-0.00029}$ |
| $100\theta_{MC}$ | 1.0408$^{+0.00035+0.00086}_{-0.00037-0.00077}$ | 1.0411$^{+0.00044+0.00107}_{-0.00058-0.00102}$ | 1.0413$^{+0.00045+0.00123}_{-0.00060-0.00109}$ |
| $\tau$ | 0.087$^{+0.018+0.033}_{-0.016-0.035}$ | 0.088$^{+0.017+0.032}_{-0.017-0.032}$ | 0.088$^{+0.017+0.034}_{-0.017-0.034}$ |
| $\eta_s$ | 0.9766$^{+0.0036+0.0079}_{-0.0037-0.0080}$ | 0.9758$^{+0.0040+0.0079}_{-0.0039-0.0076}$ | 0.9753$^{+0.0038+0.0076}_{-0.0036-0.0076}$ |
| $\ln(10^{10} A_s)$ | 3.114$^{+0.036+0.062}_{-0.031-0.069}$ | 3.111$^{+0.032+0.064}_{-0.032-0.062}$ | 3.109$^{+0.034+0.064}_{-0.033-0.066}$ |
| $\alpha$ | $-0.000092^{+0.000092+0.000092}_{-0.000019-0.0000153}$ | $-0.000099^{+0.000099+0.000099}_{-0.000023-0.0000157}$ | $-0.000094^{+0.000094+0.000094}_{-0.000022-0.0000147}$ |
| $\beta$ | $-0.01474^{+0.01474+0.01474}_{-0.00223-0.002488}$ | $-0.03330^{+0.03330+0.03330}_{-0.00844-0.00914}$ | $-0.04238^{+0.04238+0.04238}_{-0.01469-0.05246}$ |
| $\Omega_{m0}$ | 0.299$^{+0.014+0.025}_{-0.009-0.027}$ | 0.284$^{+0.026+0.036}_{-0.015-0.042}$ | 0.276$^{+0.029+0.042}_{-0.016-0.048}$ |
| $\sigma_8$ | 0.857$^{+0.019+0.068}_{-0.037-0.058}$ | 0.902$^{+0.032+0.143}_{-0.084-0.106}$ | 0.927$^{+0.039+0.163}_{-0.093-0.127}$ |
| $H_0$ | 67.71$^{+0.61+1.11}_{-0.57-1.15}$ | 68.09$^{+0.67+1.24}_{-0.64-1.29}$ | 68.24$^{+0.64+1.50}_{-0.80-1.30}$ |

**TABLE V:** Summary of the 1σ and 2σ CL constraints on the interaction model IVS2, using various combinations of the observational data sets. Here, $\Omega_{m0}$ denotes the present value of $\Omega_m = \Omega_b + \Omega_c$ and $H_0$ is in the units of km/s/Mpc.

**FIG. 14:** The 1σ and 2σ CL contour plots for several combinations of various quantities and using various combinations of the observational data sets, for the interaction model IVS2, and the corresponding 1D marginalized posterior distributions. Here, $\Omega_{m0}$ denotes the present value of $\Omega_m = \Omega_b + \Omega_c$ and $H_0$ is in the units of km/s/Mpc.
Computation of Bayesian evidence is performed with the code MCEvidence [104, 105]², which directly computes the evidences of the model with respect to the reference ΛCDM scenario.

In Bayesian analysis one needs to calculate the posterior probability of the model parameters θ, subject to a particular observational dataset x and any prior information for the underlying model M. Recalling the Bayes theorem one can write that

\[ p(\theta|x, M) = \frac{p(x|\theta, M) \pi(\theta|M)}{p(x|M)}, \] (26)

in which \( p(x|\theta, M) \) is the likelihood function (depending on the model parameters \( \theta \) with the given data set), and where \( \pi(\theta|M) \) refers to the prior information. The quantity \( p(x|M) \) is the Bayesian evidence. Given two models, namely \( M_i \) and \( M_j \), where \( M_i \) is the model under investigation and \( M_j \) is the reference model (here the ΛCDM scenario), the posterior probability is given by

\[ \frac{p(M_i|x)}{p(M_j|x)} = \frac{\pi(M_i)}{\pi(M_j)} \frac{p(x|M_i)}{p(x|M_j)} B_{ij}, \] (27)

The quantity \( B_{ij} = \frac{p(x|M_i)}{p(x|M_j)} \) is the Bayes factor of the considered model \( M_i \) relative to the reference model \( M_j \), and quantifies how the observational data support the model \( M_i \) over \( M_j \). In Table VI we show the corresponding classification following [103].

| \( \ln B_{ij} \) | Strength of evidence for model \( M_i \) |
|----------------|---------------------------------|
| 0 ≤ \( \ln B_{ij} < 1 \) | Weak |
| 1 ≤ \( \ln B_{ij} < 3 \) | Definite/Positive |
| 3 ≤ \( \ln B_{ij} < 5 \) | Strong |
| \( \ln B_{ij} \geq 5 \) | Very strong |

TABLE VI: The Revised Jeffreys scale from [103] that quantifies the fitting efficiency of the investigated model \( M_i \) comparing to the reference model \( M_j \).

In Table VII we present the computed values of \( \ln B_{ij} \) for all sign-changeable interacting scenarios considering all observational datasets. As we can see, the values of \( |\ln B_{ij}| \) for IDE2 are greater than the values of \( |\ln B_{ij}| \) for IDE1 and this is true for all datasets. This was expected since IDE2 scenario has one extra free parameter compared to IDE1. Similarly, the values of \( |\ln B_{ij}| \) for model IVS2 are greater than those IVS1. Nevertheless, overall, ΛCDM cosmology is still favored over the present interacting models.

² See the freely available code in [github.com/yabebalFantaye/MCEvidence]

| Dataset | Model | \( \ln B_{ij} \) |
|---------|-------|----------------|
| CB      | IDE1  | -4.8           |
| CB      | IDE2  | -2.4           |
| CB      | IDE1  | -2.9           |
| CB      | IVS1  | -3.9           |
| CB      | IDE2  | -3.1           |
| CB      | IDE1  | -4.2           |
| CB      | IVS2  | -5.1           |
| CB      | IVS2  | -3.6           |
| CB      | IVS2  | -4.0           |

TABLE VII: The values of \( \ln B_{ij} \) for all interaction scenarios comparing to the reference paradigm of ΛCDM, for all observational datasets. Here, CB = CMB+BAO, CBP = CMB+BAO+Pantheon and CBPC = CMB+BAO+Pantheon+CC. The negative values of \( \ln B_{ij} \) imply that the ΛCDM paradigm is preferred over the interaction scenarios.

VI. LAWS OF THERMODYNAMICS IN SIGN-CHANGEABLE INTERACTION MODELS

In this section we shall investigate the thermodynamical laws in a universe governed by sign-changeable interacting dark energy. In order to investigate the thermodynamical properties of a specific cosmological model one assumes that the universe is a thermodynamical system bounded by a cosmological horizon, and then he applies arguments from black hole thermodynamics [108, 109, 110]. In particular, one considers that the universe is bounded by the apparent horizon with radius \( r_h = \left( \frac{H^2 + k/a^2}{2} \right)^{-1/2} \) [111] which, for a spatially flat universe becomes the Hubble horizon \( r_h = 1/H \). Hence, one can show that the first law of thermodynamics can lead to the first Friedmann equation [112].

We proceed by examining the validity of the generalized second law of thermodynamics, which states that the total entropy of the universe, namely the entropy of the various cosmological fluids plus the entropy of the horizon, should be a non-decreasing function of time [113]. Concerning the entropy of the various fluids that constitute the universe one has \( S_r + S_b + S_c + S_x \), where \( S_i \) (\( i = r, b, c, x \)) denotes the entropy of the \( i \)-th fluid. Thus, the first law of thermodynamics for each individual fluid becomes [109, 110, 111, 112, 113, 114, 115, 116, 117, 118].

\[
TdS_r = dE_r + pdV, \quad (28)
\]
\[
TdS_b = dE_b + pdV = dE_b, \quad (29)
\]
\[
TdS_c = dE_c + pdV = dE_c, \quad (30)
\]
\[
TdS_x = dE_x + pdV, \quad (31)
\]

where \( V = 4\pi r_h^3/3 \) is the volume of the universe, \( E_i \)
stands for the internal energy of the $i$-th fluid given by $E_i = \frac{1}{2} \pi r_h^6 \rho_i$, and $p_i$ is the corresponding pressure. Note that the various fluids are considered to have the same temperature. However, we mention that over the entire cosmic evolution the temperatures of different cosmic fluids are not the same [119], as they evolve differently. In particular, the temperatures of radiation and dark energy remain different from the horizon temperature for a long period of time, while for the non-relativistic matter its temperature becomes and remains equal with the horizon one. Nevertheless, if at some epoch in the universe evolution the horizon temperature comes close or become equal to the temperature of the dark energy sector, then they will remain the same for most of the expansion of the universe. Therefore, the assumption, that the temperatures of different cosmic fluids are the same, is not unjustified [119].

The entropy of the horizon is taken to be that of a black hole, namely $S_h = k_B A / (4 l_{pl}^2)$, with $k_B$ the Boltzmann’s constant, $h$ the Planck’s constant, and $l_{pl} = (\sqrt{\hbar G/c^3})$ the Planck’s length, however taking the area to be that corresponding to the horizon, i.e. $A = 4\pi r_h^2$ [108] [109] [110]. Therefore, in units where $\hbar = k_B = c = 8\pi G = 1$, the horizon entropy [32] reduces to

$$S_h = 8\pi^2 r_h^2.$$  

Moreover, concerning the temperature of the apparent horizon one can use the black hole result but with the apparent horizon instead of the black-hole one, namely $T_h = 1/(2\pi r_h)$ [109] [110] [111] [112] [113] [114] [115] [116] [117] [118].

Using all the above relations together with the assumption that the temperature of the fluids $T$ should be equal to $T_h$ one can find that

$$\dot{S} = \dot{S}_r + \dot{S}_b + \dot{S}_c + \dot{S}_x + \dot{S}_h$$

$$= 4\pi^2 H r_h^6 \left[ \rho_r (1 + w_r) + \rho_b + \rho_c + (1 + w_x) \rho_x \right]^2,$$  

where we have replaced the involved $\dot{r}_h = -H/H^2$ using the Friedmann equations [2] [3]. As we can see, the total entropy is always a non-decreasing function, and thus the generalized second law of thermodynamics is satisfied. Hence, although the sign change of the interaction between dark matter and dark energy could lead to a local entropy decrease at the microscopic level, the total entropy of the universe plus the one of the horizon is always non-decreasing. The above relation for the first law of thermodynamics in interacting cosmology has been established earlier in [52].

Now, let us examine whether the universe with sign-changeable interaction in the dark sectors will result to thermodynamic equilibrium. As it is discussed extensively in the literature (see for instance [120] [121]), in order for this to be achieved one needs to have a total entropy whose first derivative is positive, while at late times its second derivative should be negative [122], and thus the entropy asymptotically tends towards a constant value. Using $x = \ln a$ as the independent variable the above require that $S' > 0$ and $S'' < 0$, where $S = S_r + S_b + S_c + S_x + S_h$, with primes denoting differentiation with respect to $x$.

Equation (33) can be rewritten as

$$S' = \frac{16\pi^2}{H^4} (H')^2,$$  

and thus it leads to

$$S'' = 2 S' \left( \frac{H''}{H} - \frac{2H'}{H} \right) = 2 S' \left( \frac{h''}{h} - \frac{2h'}{h} \right) = 2 S' \Delta,$$  

where $h = H/H_0$ and $\Delta = \left( \frac{h''}{h'} - \frac{2h''}{h} \right)$. In Fig. 15 we depict the evolution of $S''$ for both IDE1 and IDE2 models. As we observe, in both models at late times, namely at present epoch ($\ln a = 0$) and in the future, the total entropy is convex, i.e. $S'' < 0$, and hence the universe tends towards thermodynamic equilibrium. This can also be seen by the fact that asymptotically $S''$ goes to zero. These features hold also for the two models IVS1 and IVS2. Moreover, note that the individual curves are practically indistinguishable.

Hence, in summary, we conclude that for cosmological scenarios with sign changeable interaction functions the thermodynamical laws remain valid.

VII. CONCLUDING REMARKS

Interacting cosmology has attracted the interest of the literature, since on one hand it cannot be excluded from the field theoretical point of view, and on the other hand it may offer a solution to the coincidence problem. However, almost all phenomenologically introduced interacting functions have constant sign, namely the energy flow maintains its direction throughout the whole universe evolution.

In the present work we investigated sign-changeable interacting scenarios, in which the interaction function, and thus the energy flow, changes sign during the evolution of the universe, since there is not any theoretical reason of not considering such forms. We considered various models and we extracted the involved equations at both the background and perturbation levels. Then we used various data combinations from cosmic microwave background (CMB), baryon acoustic oscillations (BAO), Supernovae Type Ia (SNIa) and cosmic chronometers (CC) in order to constrain the model parameters. Finally, we performed a Bayesian analysis in order to compare the fitting efficiency of the examined models with the reference $\Lambda$CDM paradigm.

For both examined sign-changed interacting models, namely $Q = 3H(\rho_c - \rho_x)$ and $Q = 3H(\alpha \rho_c - \beta \rho_x)$, we found that the dark-energy equation-of-state parameter
FIG. 15: The evolution of the second derivative of the total entropy $S''$ according to (35), for IDE1 (upper graph) and IDE2 (lower graph), for the combined analyses CMB+BAO, CMB+BAO+Pantheon and CMB+BAO+Pantheon+CC. In the subgraph we focus on late times, namely at present epoch ($\ln a = 0$) and in the future, where we can see that $S'' < 0$ and thus the entropy is convex as required for thermodynamic equilibrium.

$w_x$ prefers the phantom regime for all datasets, at more than 2$\sigma$. Concerning the coupling parameters we saw that although the best-fit values might be non-zero, the zero value, namely no interaction, is included within 1$\sigma$. Moreover, we showed that this results is maintained if we impose $w_x$ to take the cosmological constant value $-1$, namely considering the interacting vacuum model.

We proceeded by examining the effect of the interaction on the CMB TT and matter power spectra. As we showed, while from the simple CMB TT spectra it is hard to distinguish the interacting case from $\Lambda$CDM scenario, from the residual plot (with reference to $\Lambda$CDM model) one can indeed trace a distinction between the interacting and non-interacting cosmologies, mainly in the lower multipoles. Similarly, the simple matter power spectra cannot be used to examine the interaction, however using the corresponding residual graphs we showed that the deviation from the non-interacting $\Lambda$CDM cosmology is clear. The fact that the residual spectra plots can be used to distinguish the models from $\Lambda$CDM paradigm, even if at the background level the latter is allowed withing 1$\sigma$ CL, is one of the main results of the present work.

Concerning $H_0$, we saw that in all cases its obtained values are slightly higher compared to the $\Lambda$CDM-based Planck estimation, and thus the $H_0$ tension seems to be alleviated as a result of the interaction, although only partially. Hence, we deduce that although the interaction may be small, it is adequate to alleviate the $H_0$ tension. Nevertheless, concerning $\sigma_8$ tension we found that the present sign changeable interaction models are not able to release it. These features show that sign-changeable interacting scenarios might be worthy for further investigations.

Finally, we examined the validity of the laws of thermodynamics in a universe with sign-changeable interactions in the dark sector. As we saw, the generalized second law is always satisfied, namely the total entropy, constituting from all fluids as well as from the horizon entropy, is always increasing. Additionally, its second derivative becomes negative at late times, which implies that the universe tends towards thermodynamic equilibrium.

Last but not least, it is interesting to study the sign changeable models in extended parameter spaces, allowing the spatial curvature to be non-zero. The inclusion of massive neutrinos could also be appealing. These projects are left for future investigation.

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