Quantum fluctuations and glassy behavior of electrons near metal-insulator transitions

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ABSTRACT

Glassy behavior is a generic feature of electrons close to disorder-driven metal-insulator transitions. Deep in the insulating phase, electrons are tightly bound to impurities, and thus classical models for electron glasses have long been used. As the metallic phase is approached, quantum fluctuations become more important, as they control the electronic mobility. In this paper we review recent work that used extended dynamical mean-field approaches to discuss the influence of such quantum fluctuations on the glassy behavior of electrons, and examine how the stability of the glassy phase is affected by the Anderson and the Mott mechanisms of localization.

Keywords: Electron glass, quantum fluctuations, localization

1. GLASSY BEHAVIOR AS A PRECURSOR TO THE METAL-INSULATOR TRANSITION

Understanding the metal-insulator transition (MIT) poses one of the most basic questions of condensed matter physics. It has been a topic of much controversy and debate starting from early ideas of Mott, Anderson, but the problem remains far from being resolved. Quite generally, when a system is neither a good metal nor a good insulator, both the localized and the itinerant aspects of the problem are important. In this intermediate regime, several competing processes can be simultaneously present. As a result, the system cannot "decide" whether to be a metal or an insulator until a very low temperature $T^*$ is reached, below which a more conventional description applies. This situation is typical of systems close to a quantum critical point, which describes a zero temperature second order phase transition between two distinct states of matter. Understanding the nature of low energy excitations in the intermediate regime between a metal and an insulator is of crucial importance for the progress in material science.

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Figure 1. Three basic routes to localization
The primary reason for theoretical difficulties is related to the fact that both the Mott and the Anderson transition find themselves in regimes where traditional, perturbative approaches cannot be straightforwardly applied. To make the problem even more difficult, simple estimates are sufficient to appreciate that in many situations the effects of interactions and disorder are of comparable magnitude and thus both should be simultaneously considered. So far, very few approaches have attempted to simultaneously incorporate these two basic routes to localization.

Another aspect of disordered interacting electrons poses a fundamental problem. Very generally, Coulomb repulsion favors a uniform electronic density, while disorder favors local density fluctuations. When these two effects are comparable in magnitude, one can expect many different low energy electronic configurations, i.e. the emergence of many metastable states. Similarly as in other “frustrated” systems with disorder, such as spin glasses, these processes can be expected to lead to glassy behavior of the electrons, and the associated anomalously slow relaxational dynamics. Indeed, both theoretical and experimental work has found evidence of such behavior deep on the insulating side of the transition. However, at present very little is known as to the precise role of such processes in the critical region. Nevertheless, it is plausible that the glassy freezing of the electrons must be important, since the associated slow relaxation clearly will reduce the mobility of the electrons. From this point of view, the glassy freezing of electrons may be considered, in addition to the Anderson and the Mott mechanism, as a third fundamental process associated with electron localization. Interest in understanding the glassy aspects of electron dynamics has experienced a genuine renaissance in the last few years, primarily due to experimental advances. Emergence of many metastable states, slow relaxation and incoherent transport have been observed in a number of strongly correlated electronic systems. These included transition metal oxides such as high Tc materials, manganites, and ruthenates. Similar features have recently been reported in two-dimensional electron gases, and even three dimensional doped semiconductors such as Si:P.

2. EXTENDED DMFT APPROACHES FOR DISORDERED ELECTRONS

A number of experimental and theoretical investigations have suggested that the conventional picture of disordered interacting electrons may be incomplete. Most remarkably, the characteristic “critical” behavior seen in many experiments covers a surprisingly broad range of temperatures and densities. This is more likely to reflect an underlying “mean-field” behavior of disordered interacting electrons than the asymptotic critical behavior described by an effective long-wavelength theory. Thus a simple mean-field description is needed to provide the equivalent of a Van der Waals equation of state, for disordered interacting electrons. Such a theory has long been elusive, primarily due to a lack of a simple order-parameter formulation for this problem. Very recently, an alternative approach to the problem of disordered interacting electrons has been formulated, based on dynamical mean-field theory (DMFT) methods. This formulation is largely complementary to the scaling approach, and has already resulting in several striking predictions.

Figure 2. In dynamical mean-field theory, the environment of a given site is represented by an effective medium, represented by its “cavity spectral function” $\Delta_i(\omega)$. In a disordered system, $\Delta_i(\omega)$ for different sites can be very different, reflecting Anderson localization effects.
The DMFT approach focuses on a single lattice site, but replaces its environment by a self-consistently determined “effective medium”, as shown in Fig. 2. For itinerant electrons, the environment cannot be represented by a static external field, but instead must contain the information about the dynamics of an electron moving in or out of the given site. Such a description can be made precise by formally integrating out all the degrees of freedom on other lattice sites. In presence of electron-electron interactions, the resulting local effective action has an arbitrarily complicated form. Within DMFT, the situation simplifies, and all the information about the environment is contained in the local single particle spectral function $\Delta_i(\omega)$. The calculation then reduces to solving an appropriate quantum impurity problem supplemented by an additional self-consistency condition that determines this “cavity function” $\Delta_i(\omega)$.

The precise form of the DMFT equations depends on the particular model of interacting electrons and/or the form of disorder, but most applications to date have focused on Hubbard and Anderson lattice models. The approach has been very successful in examining the vicinity of the Mott transition in clean systems in which it has met spectacular successes in elucidating various properties of several transition metal oxides, heavy fermion systems, and Kondo insulators.

When appropriately generalized to disordered systems, these methods are able to incorporate all the three basic mechanisms of electron localization. In particular, the DMFT approach is able to present a consistent picture for the glassy behavior of electrons, and discuss its emergence in the vicinity of metal-insulator transitions. In this paper we review recent results obtained in this framework, and discuss their relevance to several experimental systems.

3. SIMPLE MODEL OF AN ELECTRON GLASS

The interplay of the electron-electron interactions and disorder is particularly evident deep on the insulating side of the metal-insulator transition (MIT). Here, both experimental and theoretical studies have demonstrated that they can lead to the formation of a soft “Coulomb gap”, a phenomenon that is believed to be related to the glassy behavior of the electrons. Such glassy freezing has long been suspected to be of importance, but very recent work has suggested that it may even dominate the MIT behavior in certain low carrier density systems. The classic work of Efros and Shklovskii has clarified some basic aspects of this behavior, but a number of key questions have remained unanswered.

As a simplest example displaying glassy behavior of electrons, we focus on a simple lattice model of spinless electrons with nearest neighbor repulsion $V$ in presence of random site energies $\epsilon_i$ and inter-site hopping $t$, as given by the Hamiltonian

$$H = \sum_{<ij>} (-t + \epsilon_i \delta_{ij}) c_i^\dagger c_j + V \sum_{<ij>} c_i^\dagger c_i c_j^\dagger c_j.$$  

This model can be solved in a properly defined limit of large coordination number $z$, where an extended dynamical mean-field (DMF) formulation becomes exact. We concentrate on the situation where the disorder (or more generally frustration) is large enough to suppress any uniform ordering. We then rescale both the hopping elements and the interaction amplitudes as $t_{ij} \to t_{ij}/\sqrt{z}; \quad V_{ij} \to V_{ij}/\sqrt{z}$. As we will see shortly, the required fluctuations then survive even in the $z \to \infty$ limit, allowing for the existence of the glassy phase. Within this model:

- The universal form of the Coulomb gap proves to be a direct consequence of glassy freezing.
- The glass phase is identified through the emergence of an extensive number of metastable states, which in our formulation is manifested as a replica symmetry breaking instability.
- As a consequence of this ergodicity breaking, the zero-field cooled compressibility is found to vanish at $T=0$, suggesting the absence of screening in disordered insulators.
- The quantum fluctuations can melt this glass even at $T=0$, but the relevant energy scale is set by the electronic mobility, and is therefore a nontrivial function of disorder.
We should stress that although this model allows to examine the interplay of glassy ordering and quantum fluctuations due to itinerant electrons, it is too simple to describe the effects of Anderson localization. These effects require extensions to lattices with finite coordination, and and will be discussed in the next section.

For simplicity, we focus on a Bethe lattice at half filling, and examine the \( z \to \infty \) limit. This strategy automatically introduces the correct order parameters, and after standard manipulations\(^{23} \) the problem reduces to a self-consistently defined single site problem, as defined by an the effective action of the form

\[
S_{eff}(i) = \sum_a \int_0^\beta \int_0^\beta d\tau d\tau' \left[ \epsilon_i^{ta}(\tau)(\delta(\tau-\tau')\partial_\tau + \epsilon_i + t^2G(\tau,\tau'))\right]c_i^a(\tau') + \frac{1}{2}V^2\delta n_i^a(\tau)\chi(\tau,\tau')\delta n_i^a(\tau') \right] + \frac{1}{2}V^2 \sum_{a\neq b} \int_0^\beta \int_0^\beta d\tau d\tau' \delta n_i^a(\tau) q_{ab} \delta n_i^b(\tau'). \tag{2}
\]

Here, we have used functional integration over replicated Grassmann fields\(^{23} \) \( c_i^a(\tau) \) that represent electrons on site \( i \) and replica index \( a \), and the random site energies \( \epsilon_i \) are distributed according to a given probability distribution \( P(\epsilon_i) \). The operators \( \delta n_i^a(\tau) = (\epsilon_i^{ta}(\tau)c_i^a(\tau) - 1/2) \) represent the density fluctuations from half filling. The order parameters \( G(\tau-\tau') \), \( \chi(\tau-\tau') \) and \( q_{ab} \) satisfy the following set of self-consistency conditions

\[
G(\tau-\tau') = \int d\epsilon_i P(\epsilon_i) < \epsilon_i^{ta}(\tau)c_i^a(\tau') >_{eff}, \tag{3}
\]

\[
\chi(\tau-\tau') = \int d\epsilon_i P(\epsilon_i) < \delta n_i^{ta}(\tau)\delta n_i^b(\tau') >_{eff}, \tag{4}
\]

\[
q_{ab} = \int d\epsilon_i P(\epsilon_i) < \delta n_i^{ta}(\tau)\delta n_i^b(\tau') >_{eff}. \tag{5}
\]

### 3.1. Order parameters

In these equations, the averages are taken with respect to the effective action of Eq. (2). Physically, the “hybridization function” \( t^2G(\tau-\tau') \) represents the single-particle electronic spectrum of the environment, as seen by an electron on site \( i \). In particular, its imaginary part at zero frequency can be interpreted\(^{24} \) as the inverse lifetime of the local electron, and as such remains finite as long as the system is metallic. We recall\(^{23} \) that for \( V = 0 \) these equations reduce to the familiar CPA description of disordered electrons, which is exact for \( z = \infty \). The second quantity \( \chi(\tau-\tau') \) represents an (interaction-induced) mode-coupling term that reflects the retarded response of the density fluctuations of the environment. Note that similar objects appear in the well-known mode-coupling theories of the glass transition in dense liquids.\(^{25} \) Finally the quantity \( q_{ab} (a \neq b) \) is nothing but the familiar Edwards-Anderson order parameter \( q_{EA} \). Its nonzero value indicates that the time averaged electronic density is spatially non-uniform.

### 3.2. Equivalent Infinite Range model

From a technical point of view, a RSB analysis is typically carried out by focusing on a free energy expressed as a functional of the order parameters. In our Bethe lattice approach, one directly obtains the self-consistency conditions form appropriate recursion relations,\(^{23} \) without invoking a free energy functional. However, we have found it useful to map our \( z = \infty \) model to another infinite range model, which has exactly the same set of order parameters and self-consistency conditions, but for which an appropriate free energy functional can easily be determined. The relevant model is still given Eq. (1), but this time with random hopping elements \( t_{ij} \) and random nearest-neighbor interaction \( V_{ij} \), having zero mean and variance \( t^2 \), and \( V^2 \), respectively. For this model, standard manipulations\(^{23} \) result in the following free energy functional...
\[
F[G, \chi, q_{ab}] = - \frac{1}{2} \sum_a \int_0^\beta \int_0^\beta d\tau d\tau' \left[ t^2 G^2(\tau, \tau') + V^2 \chi^2(\tau, \tau') \right] - \frac{1}{2} \sum_{a \neq b} (\beta V)^2 q_{ab}^2 \\
- \ln \left[ \int d\xi_i P(\xi_i) \int Dc_i^{\dagger} Dc_i \exp \left\{ -S_{eff}(i) \right\} \right],
\]

with \( S_{eff}(i) \) given by Eq. (2). The self-consistency conditions, Eqs. (4-6) then follow from

\[
0 = \delta F / \delta G(\tau, \tau'); \quad 0 = \delta F / \delta \chi(\tau, \tau'); \quad 0 = \delta F / \delta q_{ab}.
\]

We stress that Eqs. (3-5) have been derived for the model with uniform hopping elements \( t_{ij} \) and interaction amplitudes \( V_{ij} \), in the \( n \rightarrow \infty \) limit, but the same equations hold for an infinite range model where these parameters are random variables.

### 3.3. The glass transition

In our electronic model, the random site energies \( \varepsilon_i \) play a role of static random fields. As a result, in presence of disorder, the Edwards-Anderson parameter \( q_{EA} \) remains nonzero for any temperature, and thus cannot serve as an order parameter. To identify the glass transition, we search for a replica symmetry breaking (RSB) instability, following standard methods. We define \( \delta q_{ab} = q_{ab} - q \), and expand the free energy functional of Eq. (6) around the RS solution. The resulting quadratic form (Hessian matrix) has the matrix elements given by

\[
\frac{\partial^2 F}{\partial q_{ab} \partial q_{cd}} = (\beta V)^2 \delta_{ac} \delta_{bd} - V^4 \int_0^\beta \int_0^\beta \int_0^\beta d\tau_1 d\tau_2 d\tau_3 d\tau_4 \left[ < \delta n_a(\tau_1) \delta n_b(\tau_2) \delta n_c(\tau_3) \delta n_d(\tau_4) >_{RS} \\
- < \delta n_a(\tau_1) \delta n_b(\tau_2) >_{RS} < \delta n_c(\tau_3) \delta n_d(\tau_4) >_{RS} \right],
\]

where the expectation values are calculated in the RS solution. Using standard manipulations, and after lengthy algebra, we finally arrive at the desired RSB stability criterion that takes the form

\[
1 = V^2 \left[ \left( \chi_{loc}(\varepsilon_i) \right)^2 \right]_{dis}.
\]

Here, \([...]\)_{dis} indicates the average over disorder, and \( \chi_{loc}(\varepsilon_i) \) is the local compressibility, that can be expressed as

\[
\chi_{loc}(\varepsilon_i) = \frac{\partial}{\partial \varepsilon_i} \frac{1}{\beta} \int_0^\beta d\tau < \delta n_i(\tau) >,
\]

and which is evaluated by carrying out quantum averages for a fixed realization of disorder. The relevant expectation values have to be carried with respect to the full local effective action \( S_{eff}(i) \) of Eq. (2), evaluated in the RS theory. In general, the required computations cannot be carried out in close form, primarily due to the unknown “memory kernel” \( \chi(\tau - \tau') \). However, as we will see, the algebra simplifies in several limits, where explicit expressions can be obtained.

### 4. CLASSICAL ELECTRON GLASS

In the classical \( (t = 0) \) limit, the problem can easily be solved in close form. We first focus on the replica symmetric (RS) solution, and set \( q_{ab} = q \) for all replica pairs. The corresponding equation reads

\[
q = \frac{1}{4} \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2/2} \tanh^2 \left[ \frac{1}{2} x \left( (\beta V)^2 q + (\beta W^2)^2 \right)^{1/2} \right],
\]

where we have considered a Gaussian distribution of random site energies of variance \( W^2 \). Note that the interactions introduce an effective, enhanced disorder strength

\[
W_{eff} = \sqrt{W^2 + V^2 q},
\]
since the frozen-in density fluctuations introduce an added component to the random potential seen by the
electron. As expected, \( q \neq 0 \) for any temperature when \( W \neq 0 \). If the interaction strength is appreciable as
compared to disorder, we thus expect the resistivity to display an appreciable increase at low temperatures. We
emphasize that this mechanism is different from Anderson localization, which is going to be discussed in the next
section, but which also gives rise to a resistivity increase at low temperatures.

Next, we examine the instability to glassy ordering. In the classical \( (t = 0) \) limit Eq. (9) reduces to

\[
1 = \frac{1}{16} (\beta V)^2 \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2/2} \cosh^{-4} \left[ \frac{1}{2} x \beta W_{\text{eff}}(q) \right],
\]

with \( W_{\text{eff}}(q) \) given by Eq.(12). The resulting RSB instability line separates a low temperature glassy phase
from a high temperature “bad metal” phase. At large disorder, these expressions simplify, and we find

\[
T_G \approx \frac{1}{6 \sqrt{2 \pi} V^2}, \quad W \to \infty.
\]

We conclude that \( T_G \) decreases at large disorder. This is to be expected, since in this limit the electrons drop
in the lowest potential minima of the random potential. This defines a unique ground state, suppressing the
frustration associated with the glassy ordering, and thus reducing the glassy phase. It is important to note
that for the well known de Almeida-Thouless (AT) line \( T_{\text{RSB}} \) decreases exponentially in the strong field limit.
In contrast, we find that in our case, \( T_G \sim 1/W \) decreases only slowly in the strong disorder limit. This is
important, since the glassy phase is expected to be most relevant for disorder strengths sufficient to suppress
uniform ordering. At the same time, glassy behavior will only be observable if the associated glass transition
temperature remains appreciable.

### 4.1. The glassy phase

To understand this behavior, we investigate the structure of the low-temperature glass phase. Consider the
single-particle density of states at \( T=0 \), which in the classical limit can be expressed as

\[
\rho(\varepsilon, t = 0) = \frac{1}{N} \sum_i \delta(\varepsilon - \varepsilon_i^R),
\]

where \( \varepsilon_i^R \equiv \varepsilon_i + \sum_j V_{ij} n_j \) are the renormalized site energies. In the thermodynamic limit, this quantity is nothing
but the probability distribution \( P_R(\varepsilon_i^R) \). It is analogous to the “local field distribution” in the spin-glass models,
and can be easily shown to reduce to a simple Gaussian distribution in the RS theory, establishing the absence
of any gap for \( T > T_G \). Obtaining explicit results from a replica calculation in the glass phase is more difficult,
but useful insight can be achieved by using standard simulation methods\(^{28, 29}\) on our equivalent infinite-range
model; some typical results are shown in Fig. 1. We find that as a result of glassy freezing, a pseudo-gap emerges
in the single-particle density of states, reminiscent of the Coulomb gap of Efros and Shklovskii (ES).\(^{16}\) The low
energy form of this gap appears universal,

\[
\rho(\varepsilon) \approx C \varepsilon^\alpha / V^2; \quad C = \alpha = 1,
\]

independent of the disorder strength \( W \), again in striking analogy with the predictions of ES. To establish
this result, we have used stability arguments very similar to those developed for spin-glass (SG) models,\(^{29}\)
demonstrating that the form of Eq. (16) represents an exact upper bound for \( \rho(\varepsilon) \). For infinite-ranged SG
models, as in our case, this bound appears to be saturated, leading to universal behavior. Such universality is
often associated with a critical, self-organized state of the system. Recent work\(^{29}\) finds strong numerical
evidence of such criticality for SG models; we believe that the universal gap form in our case has the same
origin. Furthermore, assuming that the universal form of Eq. (11) is obeyed immediately allows for an estimate
of \( T_G(W) \). Using Eq. (16) to estimate the gap size for large disorder gives \( T_G \sim E_g \sim V^2 / W \), in agreement with
Eq. (14).
Figure 3. Single particle density of states in the classical \((t = 0)\) limit at \(T = 0\), as a function of disorder strength. Results are shown from a simulation on \(N = 200\) site system, for \(W/V = 0.5\) (thin line) and \(W/V = 1.0\) (full line). Note that the low energy form of the gap takes a universal form, independent of the disorder strength \(W\). The dashed line follows Eq. (16).

The ergodicity breaking associated with the glassy freezing has important consequences for our model. Again, using the close similarity of our classical infinite range model to standard SG models,\(^{22}\) it is not difficult to see that the zero-field cooled (ZFC) compressibility vanishes at \(T = 0\), in contrast to the field-cooled one, which remains finite. Essentially, if the chemical potential is modified after the system is cooled to \(T = 0\), the system immediately falls out of equilibrium and displays hysteretic behavior\(^{29}\) with vanishing typical compressibility. If this behavior persists in finite dimensions and for more realistic Coulomb interactions, it could explain the absence of screening in disordered insulators.

4.2. Arbitrary lattices and finite coordination: mean-field glassy phase of the random-field Ising model.

Simplest theories of glassy freezing\(^{22}\) are obtained by examining models with random inter-site interactions. In the case of disordered electronic systems, the interactions are not random, but glassiness still emerges due to frustration introduced by the competition of the interactions and disorder. As we have seen for the Bethe lattice,\(^{21}\) random interactions are generated by renormalization effects, so that standard DMFT approaches can still be used. However, one would like to develop systematic approaches for arbitrary lattices and in finite coordination. These issues already appear on the classical level, where our model reduces to the random-field Ising model (RFIM).\(^{30}\) To investigate the glassy behavior of the RFIM, we developed\(^{31}\) a systematic approach that can incorporate short-range fluctuation corrections to the standard Bragg-Williams theory, following the method of Plefka\(^{32}\) and Georges et al.\(^{33}\) This work has shown that:

- Corrections to even the lowest nontrivial order immediately result in the appearance of a glassy phase for sufficiently strong randomness.
- This low-order treatment is sufficient in the joined limit of large coordination and strong disorder.
- The structure of the resulting glassy phase is characterized by universal hysteresis and avalanche behavior emerging from the self-organized criticality of the ordered state.
5. QUANTUM MELTING OF THE ELECTRON GLASS

Next, we investigate how the glass transition temperature can be depressed by quantum fluctuations introduced by inter-site electron tunneling. As in other quantum glass problems, quantum fluctuations introduce dynamics in the problem, and the relevant self-consistency equations cannot be solve in closed form for general values of the parameters. In the following, we will see that in the limit of large randomness, an exact solution is possible.

5.1. Quantum phase diagram

The main source of difficulty in general quantum glass problems relates to the existence of a self-consistently determined “memory kernel” $\chi(\tau - \tau')$ in the local effective action. By the same reasoning as in the classical case, one can also ignore this term since this quantity is also bounded.

![Phase diagram](image)

Figure 4. Phase diagram as a function of quantum hopping $t$, temperature $T$ and disorder strength $W$. Glass transition temperature $T_G$ decreases only slowly (as $1/W$) in the strong disorder limit. In contrast, the critical value of the hopping element $t_G$ remains finite as $W \to \infty$.

The remaining action is that of noninteracting electrons in presence of a strong random potential. The resulting local compressibility then takes the form

$$\chi_{loc}(\epsilon) = \frac{\beta}{4} \int_{-\infty}^{+\infty} d\omega \rho_{\epsilon}(\omega) \cosh^{-2}\left(\frac{1}{2} \beta \omega\right).$$

(17)

Here, $\rho_{\epsilon}(\omega)$ is the local density of states, which in the considered large $z$ limit is determined by the solution of the CPA equation

$$\rho_{\epsilon}(\omega) = -\frac{1}{\pi} \text{Im} G(\omega); \quad G(\omega) = \int \frac{d\epsilon P(\epsilon)}{\omega + i\eta - \epsilon - t^2 G(\omega)}.$$ (18)

In the limit $W/t >> 1$, it reduces to a narrow resonance of width $\Delta = \pi t^2 P(0) \sim t^2/W$

$$\rho_{\epsilon}(\omega) \approx \frac{1}{\pi} \frac{\Delta}{(\omega - \epsilon)^2 + \Delta^2}.$$ (19)
The resulting expression for the quantum critical line in the large disorder limit takes the form

\[ t_G(T = 0, W \to \infty) = V/\sqrt{\pi}. \] (20)

At first glance, this result is surprising, since it means that a finite value of the Fermi energy is required to melt the electron glass at \( T = 0 \), even in the \( W \to \infty \) limit! This is to be contrasted with the behavior of \( T_G \) in the classical limit, which according to Eq. (17) was found to decrease as \( 1/W \) for strong disorder. At first puzzling, the above result in fact has a simple physical meaning. Namely, the small resonance width (or “hybridization energy”) \( \Delta \sim \hbar^2/W \) can be interpreted as the characteristic energy scale for the electronic motion. As first pointed by Anderson, according to Fermi’s golden rule, the transition rate to a neighboring site is proportional to \( \Delta \) and not \( t \), and thus becomes extremely small at large disorder. Thus the “size” of quantum fluctuations, that replace the thermal fluctuations at \( T = 0 \), is proportional to \( \delta \sim 1/W \), and thus becomes very small in the large \( W \) limit. We can now easily understand the qualitative behavior shown in Eq. (24) by replacing \( T \to \Delta \sim t^2/W \) in Eq. (17). The leading \( W \) dependence cancels out, and we find a finite value for \( t_G \) in the \( W \to \infty \) limit.

More generally, we can write an expression for the glass transition critical line in the large disorder limit, as a function of \( \beta = 1/T \) and \( t \) in the scaling form

\[ 1 = (V/t)^2 \phi(\beta t^2/W), \] (21)

with

\[ \phi(z) = \frac{1}{4} z^2 \int_{-\infty}^{+\infty} dx \left[ \int_{-\infty}^{+\infty} dy \frac{1}{\pi} \frac{1}{1 + (x - y)^2} \cosh^2 \left( \frac{1}{2} \sqrt{z} y \right) \right]^2. \] (22)

At finite disorder an exact solution is not possible, but we can make analytical progress motivated by our discussion of the large \( W \) limit. Namely, one can imagine evaluating the required local compressibilities in Eq. (13) by a “weak coupling” expansion in powers of the interaction \( V \). To leading order, this means evaluating the compressibilities at \( V = 0 \), an approximation which becomes exact for \( W \) large. Such an approximation can be tested for other spin glass problems. We have carried out the corresponding computations for the infinite range Ising spin glass model in a transverse field, where the exact critical transverse field is known from numerical studies. We can expect the leading approximation to underestimate the size of the glassy region, i.e. the critical field, since the omitted “memory kernel” introduces long range correlations in time, which make the system more “classical”. Indeed, we find that the leading approximation underestimates the critical field by only about 30%, whereas the next order correction makes an error of less than 5%. Encouraged by these arguments, we use this “weak-coupling” approximation for arbitrary disorder strength \( W \). Again, the computation of the compressibility reduces to that of noninteracting electrons in a CPA formulation; the resulting phase diagram is shown in Fig. 4.

5.2. Quantum critical behavior of the electron glass

So far, we have seen how our extended DMFT equations can be simplified for large disorder, allowing an exact computation of the phase boundary in this limit. In our case, this quantum critical line separates a (non-glassy) Fermi liquid phase, and a metallic glass phase which, as we will see, features non-Fermi liquid behavior. If one is interested in details of dynamics of the electrons near the quantum critical line, the above simplifications do not apply, and one is forced to self-consistently calculate the form of the “memory kernel” (local dynamic compressibility) \( \chi(\tau - \tau') \). Fortunately, this task can be carried out using methods very similar to those developed for DMFT models for metallic spin glasses. Formulating such a theory is technically possible because the exact quantum critical behavior is captured when the relevant field theory is examined at the Gaussian level in the considered limit of large dimensions.

Because of technical complexity of this calculation, we only report the main results, while the details can be found in Ref. [36]. In this paper, the full replica-symmetry broken (RSB) solution was found both around the quantum critical line and in the glassy phase. In the Fermi liquid phase, the memory kernel was found to take the form

\[ \chi(\omega_n) = D(\omega_n) + \beta g_{\text{EA}} \delta_{\omega_n,0}, \]

where \( D(\omega_n) \) is the single-particle density of states, \( g_{\text{EA}} \) is the electron-atom interaction constant, and \( \beta \) is the inverse temperature.

\[ V^2 \chi(\omega_n) = D(\omega_n) + \beta g_{\text{EA}} \delta_{\omega_n,0}, \]
with \[ D(\omega_n) = -yq_E A / V^4 - \sqrt{|\omega_n|} + \Delta. \]

Here, \( \Delta \) is a characteristic energy scale that vanishes on the critical line, which also determines a crossover temperature scale separating the Fermi liquid from the quantum-critical regime. In contrast to conventional quantum critical phenomena, but similarly as in metallic spin glasses, the "gap" scale \( \Delta = 0 \) not only on the critical line, but remains zero throughout the entire glassy phase. As a result, the excitations in this region assume a non-Fermi liquid form

\[ D(\omega_n) = -yq_E A / V^4 - \sqrt{|\omega_n|}. \]

This behavior reflects the emergence of soft "replicon" modes\(^{22}\) describing in our case represent low energy charge rearrangements inside the glassy phase. At finite temperatures, electrons undergo inelastic scattering from such collective excitations, leading to the temperature dependence of the resistivity that takes the following non-Fermi liquid form

\[ \rho(T) = \rho(0) + AT^{3/2}. \]

Interestingly, very recent experiments\(^{37}\) on two dimensional electron gases in silicon have revealed precisely such temperature dependence of the resistivity. This behavior has been observed in what appears to be an intermediate metallic glass phase separating a conventional (Fermi liquid) metal at high carrier density, from an insulator at the lowest densities.

Another interesting feature of the predicted quantum critical behavior relates to disorder dependence of the crossover exponent \( \phi \) describing how the gap scale \( \Delta \sim \delta r^\phi \) vanishes as a function of the distance \( \delta r \) from the critical line. Calculations\(^{38}\) show that \( \phi = 2 \) in presence of site energy disorder, which for our model plays a role of a random symmetry breaking field, and \( \phi = 1 \) in its absence. This indicates that site disorder, which is common in disordered electronic systems, produces a particularly large quantum critical region, which could be the origin of large dephasing observed in many materials near the metal-insulator transition.

5.3. Effects of Anderson localization

As we have seen, the stability of the glassy phase is crucially determined by the electronic mobility at \( T = 0 \). More precisely, we have shown that the relevant energy scale that determines the size of quantum fluctuations introduced by the electrons is given by the local “resonance width” \( \Delta \). It is important to recall that precisely this quantity may be considered\(^2\) as an order parameter for Anderson localization of noninteracting electrons. Very recent work\(^{13, 24}\) demonstrated that the typical value of this quantity plays the same role even at a Mott-Anderson transition. We thus expect \( \Delta \) to generally vanish in the insulating state. As a result, we expect the stability of the glassy phase to be strongly affected by Anderson localization effects, as we will explicitly demonstrate in the next section.

6. GLASSY BEHAVIOR NEAR THE MOTT-ANDERSON TRANSITION

On physical grounds, one expects the quantum fluctuations\(^{39}\) associated with mobile electrons to suppress glassy ordering, but their precise effects remain to be elucidated. Note that even the amplitude of such quantum fluctuations must be a singular function of the distance to the MIT, since they are dynamically determined by processes that control the electronic mobility.

To clarify the situation, the following basic questions need to be addressed: (1) Does the MIT coincide with the onset of glassy behavior? (2) How do different physical processes that can localize electrons affect the stability of the glass phase? In the following, we provide simple and physically transparent answers to both questions. We find that: (a) Glassy behavior generally emerges before the electrons localize; (b) Anderson localization\(^2\) enhances the stability of the glassy phase, while Mott localization\(^1\) tends to suppress it.

In order to be able to examine both the effects of Anderson and Mott localization, we concentrate on extended Hubbard models given by the Hamiltonian

\[ H = \sum_{ij\sigma} (-t_{ij} + \varepsilon_i \delta_{ij}) c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{ij} V_{ij} \delta n_i \delta n_j. \]
Here, \( \delta n_i = n_i - \langle n_i \rangle \) represent local density fluctuations (\( \langle n_i \rangle \) is the site-averaged electron density), \( U \) is the on-site interaction, and \( \varepsilon_i \) are Gaussian distributed random site energies of variance \( W^2 \). In order to allow for glassy freezing of electrons in the charge sector, we introduce weak inter-site density-density interactions \( V_{ij} \), which we also also choose to be Gaussian distributed random variables of variance \( V^2 / z \) (\( z \) is the coordination number). We emphasize that, in contrast to previous work,\(^{21}\) we shall now keep the coordination number \( z \) finite, in order to allow for the possibility of Anderson localization. To investigate the emergence of glassy ordering, we formally average over disorder by using standard replica methods,\(^{40}\) and introduce collective \( \bar{Q} \)-fields to decouple the inter-site \( V \)-term.\(^{40}\) A mean-field is then obtained by evaluating the \( \bar{Q} \)-fields at the saddle-point level. The resulting stability criterion takes the form similar as before

\[
1 - V^2 \sum_j [\chi_{ij}^2]_{\text{dis}} = 0. 
\]

(23)

Here, the non-local static compressibilities are defined (for a fixed realization of disorder) as

\[
\chi_{ij} = -\partial n_i / \partial \varepsilon_j, 
\]

(24)

where \( n_i \) is the local quantum expectation value of the electron density, and \( [\cdot \cdot \cdot]_{\text{dis}} \) represents the average over disorder. Obviously, the stability of the glass phase is determined by the behavior of the four-order correlation function \( \chi^{(2)} = \sum_j [\chi_{ij}^2]_{\text{dis}} \) in the vicinity of the metal-insulator transition. We emphasize that this quantity is to be calculated in a disordered Hubbard model with finite range hopping, i.e. in the vicinity of the Mott-Anderson transition. The critical behavior of \( \chi^{(2)} \) is very difficult to calculate in general, but we will see that simple results can be obtained in the limits of weak and strong disorder, as follows.

6.1. Large disorder

As the disorder grows, the system approaches the Anderson transition at \( t = t_c(W) \sim W \). The first hint of singular behavior of \( \chi^{(2)} \) in an Anderson insulator is seen by examining the deeply insulating, i.e. atomic limit \( W \gg t \), where to leading order we set \( t = 0 \) and obtain \( \chi_{ij} = \delta(\varepsilon_i - \mu) \delta_{ij} \), i.e. \( \chi^{(2)} = [\delta^2(\varepsilon_i - \mu)]_{\text{dis}} = +\infty \) diverges! Since we expect all quantities to behave in qualitatively the same fashion throughout the insulating phase, we anticipate \( \chi^{(2)} \) to diverge already at the Anderson transition. Note that, since the instability of the glassy phase occurs already at \( \chi^{(2)} = V^{-2} \), the glass transition must precede the localization transition. Thus, for any finite inter-site interaction \( V \), we predict the emergence of an intermediate metallic glass phase separating the Fermi liquid from the Anderson insulator. Assuming that near the transition

\[
\chi^{(2)} \simeq \frac{A}{W^2} ((t/W) - B)^{-\alpha} 
\]

(25)

\((A \text{ and } B = t_c/W \text{ are constants of order unity})\), from Eq. (23) we can estimate the form of the glass transition line, and we get

\[
\delta t(W) = t_G(W) - t_c(W) \sim V^{2/\alpha} W^{1-2/\alpha}; W \to \infty. 
\]

(26)

The glass transition and the Anderson transition lines are predicted to converge at large disorder for \( \alpha < 2 \), and diverge for \( \alpha > 2 \). Since all the known exponents characterizing the localization transition seem to grow with dimensionality, we may expect a particularly large metallic glass phase in large dimensions.

6.1.1. Anderson localization on Bethe lattice

In order to confirm this scenario by explicit calculations, we compute the behavior of \( \chi^{(2)} \) at the Anderson transition of a half-filled Bethe lattice of coordination \( z = 3 \). We use an essentially exact numerical approach\(^{24}\) based on the recursive structure of the Bethe lattice.\(^{31}\) In this approach, local and non-local Green’s functions on a Bethe lattice can be sampled from a large ensemble, and the compressibilities \( \chi_{ij} \) can be then calculated by examining how a local charge density \( n_i \) is modified by an infinitesimal variation of the local site energy \( \varepsilon_j \) on another site. To do this, we have taken special care in evaluating the local charge densities \( n_i \) by numerically computing the required frequency summations over the Matsubara axis, where the numerical difficulties are
minimized. Using this method, we have calculated $\chi(2)$ as a function of $W/t$ (for this lattice at half-filling $E_F = 2\sqrt{2}t$), and find that it decreases exponentially as the Anderson transition is approached. We emphasize that only a finite enhancement of $\chi(2)$ is required to trigger the instability to glassy ordering, which therefore occurs well before the Anderson transition is reached. The resulting $T = 0$ phase diagram, valid in the limit of large disorder, is presented in Fig. 1. Note that the glass transition line in this case has the form $t_G(W) \sim W$, in agreement with the fact that exponential critical behavior of $\chi(2)$ corresponds to $\alpha \to \infty$ in the above general scenario. These results are strikingly different from those obtained in a theory which ignores localization, where $t_G(W)$ was found to be weakly dependent on disorder, and remain finite as $W \to \infty$. Anderson localization effects thus strongly enhance the stability of the glass phase at sufficiently large disorder. Nevertheless, since the Fermi liquid to metallic glass (FMG) transition occurs at a finite distance before the localization transition, we do not expect the leading quantum critical behavior at the FMG transition to be qualitatively modified by the localization effects.

6.1.2. Typical medium treatment of Anderson localization

As an alternative approach to the Bethe lattice calculation, in this section we introduce Anderson localization to the problem by using the formalism of "Typical Medium Theory" (TMT). We calculate the cavity field $\Delta_{TYT}(\omega)$ by solving the relevant self-consistency condition, which in turn allows us to find local compressibilities:

$$\chi_{ii} = -\frac{\partial n}{\partial \varepsilon_i} = \frac{1}{\pi} \frac{\partial}{\partial \varepsilon_i} \int_{-\infty}^{0} d\omega Im G(\varepsilon_i, \omega, W)$$

$$G(\varepsilon_i, \omega, W) = \frac{1}{\omega - \varepsilon_i - \Delta_{TYT}(\omega)}$$

needed to determine the critical line of the glass transition. These calculations were performed using a model of semicircular bare DOS $\rho_0(\omega)$ and box distribution of disorder $P(\varepsilon_i)$. The resulting phase diagram is shown in Fig. 6. The intermediate metallic glassy phase still exists, but shrinks as $W \to \infty$, reflecting the small value of the critical exponent $\alpha = 1$, which can be shown analytically within TMT. A more realistic values of this exponent, corresponding to $d = 3$ require more detailed numerical calculations, which remains a challenge for future work.
6.2. Low disorder - Mott transition

In the limit of weak disorder $W \ll U, V$, and interactions drive the metal-insulator transition. Concentrating on the model at half-filling, the system will undergo a Mott transition as the hopping $t$ is sufficiently reduced. Since for the Mott transition $t_{\text{Mott}}(U) \sim U$, near the transition $W \ll t$, and to leading order we can ignore the localization effects. In addition, we assume that $V \ll U$, and to leading order the compressibilities have to be calculated with respect to the action $S_{\text{el}}$ of a disordered Hubbard model. The simplest formulation that can describe the effects of weak disorder on such a Mott transition is obtained from the dynamical mean-field theory (DMFT). This formulation, which ignores localization effects, is obtained by rescaling the hopping elements $t \rightarrow t/\sqrt{z}$ and then formally taking the limit of large coordination $z \rightarrow \infty$. To obtain qualitatively correct analytical results describing the vicinity of the disordered Mott transition at $T = 0$, we have solved the DMFT equations using a 4-boson method. At weak disorder, these equations can be easily solved in close form, and we simply report the relevant results. The critical value of hopping for the Mott transition is found to decrease with disorder, as

$$t_c(W) \approx t_c^0 (1 - 4(W/U)^2 + \cdots),$$

where for a simple semi-circular density of states $t_c^0 = 3\pi U/64$ (in this model, the bandwidth $B = 4t$). Physically, the disorder tends to suppress the Mott insulating state, since it broadens the Hubbard bands and narrows the Mott-Hubbard gap. At sufficiently strong disorder $W \geq U$, the Mott insulator is suppressed even in the atomic limit $t \rightarrow 0$. The behavior of the compressibilities can also be calculated near the Mott transition, and to leading order we find

$$\chi^{(2)} = \left[ \frac{8}{3\pi t_c^0} (1 - \frac{t_c(W)}{t} \right)^2 (1 + 28(W/U)^2).$$

Therefore, as any compressibility, $\chi^{(2)}$ is found to be very small in the vicinity of the Mott transition, even in presence of finite disorder. As a result, the tendency to glassy ordering is strongly suppressed at weak disorder, where one approaches the Mott insulating state.

Finally, having analyzed the limits of weak and strong disorder, we briefly comment on what may be expected in the intermediate region $W \sim U$. On general grounds, we expect a global phase diagram as shown in Fig.7. The Mott gap cannot exist for $W > U$, so in this region and for sufficiently small $t$ (i.e. kinetic energy), one enters an gapless (compressible) Mott-Anderson insulator. For $W \sim U$, the computation of $\chi^{(2)}$ requires

![Figure 6. Phase diagram from Typical Medium Theory of Anderson localization, giving $\alpha = 1$. The intermediate metallic glassy phase shrinks as disorder $W$ grows, as expected. Compare this to the Bethe lattice case Fig. 5, where $\alpha = \infty$.](image-url)
Figure 7. Schematic phase diagram for an extended Hubbard model with disorder, as a function of the hopping element $t$ and the disordered strength $W$, both expressed in units of the on-site interaction $U$. The size of the metallic glass phase is determined by the strength of the inter-site interaction $V$.

the full solution of the Mott-Anderson problem. The required calculations can and should be performed using the formulation of Ref. 13, 24, but that difficult task is a challenge for the future. However, based on general arguments presented above, we expect $\chi^{(2)}$ to vanish as one approaches the Mott insulator ($W < U$), but to diverge as one approaches the Mott-Anderson insulator ($W > U$). Near the tetracritical point M (see Fig. 2), we may expect $\chi^{(2)} \sim \delta W^{-\alpha}\delta t^\beta$, where $\delta W = W - W_{Mott}(t)$ is the distance to the Mott transition line, and $\delta t = t - t_c(W)$ is the distance to the Mott-Anderson line. Using this ansatz and Eq. (23), we find the glass transition line to take the form

$$\delta t = t_G(W) - t_c(W) \sim \delta W^{\beta/\alpha}; \ W \sim W_M. \ (31)$$

We thus expect the intermediate metallic glass phase to be suppressed as the disorder is reduced, and one approaches the Mott insulating state. Physically, glassy behavior of electrons corresponds to many low-lying rearrangements of the charge density; such rearrangements are energetically unfavorable close to the (incompressible) Mott insulator, since the on-site repulsion $U$ opposes charge fluctuations. Interestingly, very recent experiments on low density electrons in silicon MOSFETs have revealed the existence of exactly such an intermediate metallic glass phase in low mobility (highly disordered) samples. In contrast, in high mobility (low disorder) samples, no intermediate metallic glass phase is seen, and glassy behavior emerges only as one enters the insulator, consistent with our theory. Similar conclusions have also been reported in studies of highly disordered InO$_2$ films, where the glassy slowing down of the electron dynamics seems to be suppressed as the disorder is reduced and one crosses over from an Anderson-like to a Mott-like insulator. In addition, these experiments provide striking evidence of scale-invariant dynamical correlations inside the glass phase, consistent with the hierarchical picture of glassy dynamics, as generally emerging from mean-field approaches such as the one used in this work.
7. CONCLUSIONS

Recent years have witnessed enormous renewed interest in the metal-insulator transition. Scores of new and fascinating materials are being fabricated, with properties that could not be anticipated. A common theme in many of these systems is the presence of both the strong electron-electron interactions and disorder, a situation which proved difficult to analyze using conventional theoretical methods. In this paper, we have described a novel approach to this difficult problem, and shown that it can capture most relevant processes. This formulation can easily be adapted to many realistic situations and will open new avenues for the development of materials science research.

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