Investigation of local maxima of the modified modular correlation function

D M Malinichev¹, E A Melnikova¹ and Y V Prus²

¹ Russian State Social University, 4 Wilhelm Pieck str., Moscow, 129226, Russia
² Gubkin Russian State University of Oil and Gas, 65 Leninsky Prospekt, Moscow, 119991, Russia

E-mail: mmm_63@list.ru

Abstract. Recently, due to the development of communication systems for various purposes, there is a need to use indirect methods for calculating the correlation function. There are several algorithms for calculating indirect correlation functions, one of which is modular. The article examines the properties of local maxima of correlation functions of various types.

1. Introduction
Various types of noise-like signals have been widely used in radio engineering and communication systems. This is due to their high noise immunity, the possibility of code separation when working in the common frequency band and the ability to measure the motion parameters of objects with high resolution.

Currently, a large number of noise-like signals with good correlation properties are known [1, 2]. The defining property of such signals is that their autocorrelation function (ACF) has one main peak and several small side peaks. Minimization of the side peaks of the ACF is carried out for the best separation from noise of noise-like signals at the output of matched filters.

The use of such signals can increase the secrecy of the transmission and their detection against the background of interference. Various types of noise-like signals are known for which the side peaks of the multiplicative autocorrelation function are quite small [1]. These include Barker signals, pseudo-random M-sequences, Jacobi, Legendre sequences and others. In addition, multi-valued systems of phase-shifted signals are known, which include Walsh systems, Quaternary Velti E-codes, Frank codes, multiphase signal systems, and others [1].

2. The comparative analysis of the modular multiplier and the modified correlation functions
Most often, when processing discrete signals, a multiplicative correlation function is used, which is determined by the formula:

\[ B(\tau) = \sum_{i=0}^{N-1} x_i(t)y_i(t - \tau), \]

where \( x(t) \) is the reference signal, \( y(t) \) is the input signal; \( N \) is the signal length; \( \tau \) is the shift amount.
It is proposed to use a modified modular function as an alternative to the multiplicative correlation function. When passing to the discrete values of the signals \( x(t) \) and \( y(t) \), we obtain the estimate \( T(\tau) \) in the form:

\[
T(\tau) = \sum_{i=0}^{N-1} (A - |x_i(t) - y_i(t - \tau)|),
\]

where \( N \) is sample size.

The value is associated with the values of the input signal \( y(t) \) with the ratio: \( A=|Y_{\text{max}}| \). With this choice of \( A \), minimization of the side walls of the autocorrelation modified modular function is achieved. With another choice of the value \( A \), a constant component appears at the output of the device that implements algorithm: positive for \( A > Y_{\text{max}} \) and negative for \( A < Y_{\text{max}} \). This does not change the shape of the autocorrelation function, but increases the level of the side lobes.

A multi-valued M-sequence can be formed by a system of \( n \)-linear shift storage registers and \( N \) adders, as well as a feedback loop. The length of the M-sequence is

\[
N = p^n - 1,
\]

where \( n \) is the number of shift storage registers, and \( p \) is the number of characters in the alphabet (in the case of a binary sequence, 2). For more information about the method of generating sequences based on shift registers and polynomials, see the corresponding literature [3-5].

Noise-like signals obtained on the basis of multi-valued M-sequences have fairly good correlation properties. The appropriateness of the use of multi-valued M-sequences in radio engineering and communication systems is due to the fact that to ensure a given level of maximum lateral peak of the autocorrelation function, a significantly shorter sample length is required than binary M-sequences. It is impractical to use a multiplicative algorithm for processing signals obtained from multi-valued M-sequences, since in this case the maximum lateral peak of the autocorrelation function does not depend on the alphabet, but is determined only by the length of its sample.

Below are a few obtained multivalued M-sequences and their characteristics. The graph shows the actual part of the aperiodic autocorrelation function. In addition to \( p \) and \( N \), the main characteristics are also given \( T_{\text{max}} - \) the absolute value of the maximum lateral peak, \( T_{\text{maxnorms}} - \) the same value normalized with respect to the main peak.

### Table 1. The modified modular autocorrelation function for M-sequences  \((p = 2)\).

| \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) |
|---|---|---|---|---|---|---|---|
| -15 | 0 | -7 | 0.133 | 1 | 9 | - | 0.133 |
| -14 | -6 | 2 | - | 10 | - | 0.133 |
| -13 | -0.067 | 0.067 | 0.067 | -0.2 | - | 11 |
| -12 | 0.2 | -4 | -0.2 | 4 | -0.2 | 12 | 0.2 |
| -11 | -3 | -5 | 0.267 | 0.133 | - | 13 |
| -10 | 0.2 | -2 | 0.067 | 0.133 | - | 14 |
| -9 | -0.133 | -1 | 0.267 | 0.067 | - | 0.133 |
| -8 | -0.2 | 0 | 1 | 8 | -0.2 | - | 0.133 |

### Table 2. The modified modular autocorrelation function for M-sequences \((p = 4)\).

| \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) | \( \tau \) | \( T(\tau) \) |
|---|---|---|---|---|---|---|---|
| -15 | 0 | -7 | -0.033 | 1 | 0.133 | 9 | -0.033 |
| -14 | -0.067 | -6 | 0.1 | 2 | 0.133 | 10 | 0 |
| -13 | -0.067 | -5 | 0.2 | 3 | 0.167 | 11 | -0.033 |
| -12 | -0.1 | -4 | 0.1 | 4 | 0.1 | 12 | -0.1 |
Table 2. The modified modular autocorrelation function for M-sequences (p = 4).

| τ  | $T(\tau)$ | τ  | $T(\tau)$ | τ  | $T(\tau)$ | τ  | $T(\tau)$ |
|----|----------|----|----------|----|----------|----|----------|
| -11| -0.033   | -3| 0.167    | 5 | 0.2      | 13| -0.067   |
| -10| 0        | -2| 0.133    | 6 | 0.1      | 14| -0.067   |
| -9 | -0.033   | -1| 0.133    | 7 | -0.033   | 15| 0        |
| -8 | 0.1      | 0 | 1        | 8 | 0.1      |  -| -        |

Figure 1. The modified modular autocorrelation function for M-sequences p = 2, N=15, 00100110101111, p = 4, N=15, 011310221203323.

Table 3. The multiplicative autocorrelation function for M-sequences (p = 2).

| τ  | $B(\tau)$ | τ  | $B(\tau)$ | τ  | $B(\tau)$ | τ  | $B(\tau)$ |
|----|----------|----|----------|----|----------|----|----------|
| -15| 0        | -7| 0.133    | 1 | 0        |  9| -0.133   |
| -14| -0.067   | -6| 0.067    | 2 | -0.067   | 10| -0.2     |
| -13| 0        | -5| 0.133    | 3 | -0.267   | 11| 0.133    |
| -12| 0.2      | -4| -0.2     | 4 | -0.2     | 12| 0.2      |
| -11| 0.133    | -3| -0.267   | 5 | 0.133    | 13| 0        |
| -10| -0.2     | -2| -0.067   | 6 | 0.067    | 14| -0.067   |
| -9 | -0.133   | -1| 0        | 7 | 0.133    | 15| 0        |
| -8 | -0.2     | 0 | 1        | 8 | -0.2     |  -| -        |

Table 4. The multiplicative autocorrelation function for M-sequences (p = 4).

| τ  | $B(\tau)$ | τ  | $B(\tau)$ | τ  | $B(\tau)$ | τ  | $B(\tau)$ |
|----|----------|----|----------|----|----------|----|----------|
| -15| 0        | -7| -0.25    | 1 | 0        |  9| -0.08    |
| -14| -0.11    | -6| -0.03    | 2 | 0        | 10| 0        |
| -13| -0.11    | -5| 0.222    | 3 | 0.083    | 11| -0.08    |
| -12| -0.19    | -4| -0.03    | 4 | -0.03    | 12| -0.19    |
| -11| -0.08    | -3| 0.083    | 5 | 0.222    | 13| -0.11    |
| -10| 0        | -2| 0        | 6 | -0.03    | 14| -0.11    |
| -9 | -0.08    | -1| 0        | 7 | -0.25    | 15| 0        |
| -8 | 0.139    | 0 | 1        | 8 | 0.139    |  -| -        |
Figure 2. The multiplicative autocorrelation function for M-sequences $p = 2$, $N=15$, $000100110101111$, $p = 4$, $N=15$, $011310221203323$.

For periodic M-sequences, the value of the maximum side lobe of the autocorrelation function of the phase-manipulated signals is inversely proportional to their length: $R_m=1/N$ [6,7]. For non-periodic M-sequences, the value of the maximum side lobe is inversely proportional to the square root of the length of the sequence $R_m=1/\sqrt{N}$. This property of M-sequences is confirmed by our experimental studies.

Machine modeling of this dependence shows that the value of the maximum side peak level is expressed as follows:

$$T_{max} = \exp(-0.05p + 0.1) \frac{1}{\sqrt{N}}$$

Thus, with an increase in the number of levels in the signal, the value of the maximum side peak of the modified modular correlation function decreases exponentially. This is a positive feature of processing multi-valued M-sequences using a modified modular algorithm.

A feature of the modified modular correlation function when processing multi-level discrete signals obtained on the basis of multi-valued M-sequences is that it is a consequence of the difference-modular correlation function, since:

$$A = M(|x_i(t) + y_i(t)|),$$

$$T(\tau) = A(N - \tau) - \frac{2\sigma_Y}{\sqrt{\pi}} \sqrt{1 - \rho(\tau)},$$

where $\sigma_Y^2$ is the variance of the signal.

Thus, the relationship between $T(\tau)$ and $B(\tau)$ is determined by the square root function. In the case of a linear dependence of the functions $B(\tau)$ and $T(\tau)$, the level of the maximum lateral peak from remains constant. However, if the relationship between $B(\tau)$ and $T(\tau)$ is defined as the square root, then using the modified modular algorithm allows us to reduce the maximum side peak level when processing signals compared to the multiplicative algorithm.

Let us estimate the level of the maximum lateral peak of the modified modular algorithm at $N \to \infty$. The use of a modified modular algorithm for processing multiphase FM signals in comparison with the multiplicative algorithm allows, in the limit, to reduce the maximum side lobe level by no more than two times: $T_{max}=R_{max}/2$.

3. Conclusion

Thus, the same level of maximum lateral peak can be obtained either by changing the number of different values in the M-sequence, or by changing the length of its sample. Each of these methods has its own characteristics:
• an increase in the length of the sample of signals leads to an increase in the processing time of the signals, an increase in the complexity of calculations, as well as a wider spectrum.
• increasing the number of characters in the alphabet reduces the sample length of the signal. In this case, calculation algorithms are somewhat more complicated, given the fact that the ACF in this case is complex.
• calculations show that at \( p = 2,3,4,5,7,11 \) the level value of the maximum lateral peak can be \( T_{\text{max}} = 0.5 / \sqrt{N} \) for \( p > 2 \).

References
[1] Varakin L E 1985 Communication systems with noise-like signals (Moscow, USSR: Radio and communication) p 384
[2] Malinichev D M, Boykov V V and Bolnokin V E 2012 Analysis of correlation properties of multi-valued M-sequences Dynamics of complex systems 6 (2) 83-6
[3] Salmon B P, Olivier J C , Kleynhans W and Schwegmann C P 2016 Transforming the autocorrelation function of a time series to detect land cover Proc. IEEE Int. Geoscience and Remote Sensing Symposium, IGARSS 2016 (Beijing, China: IEEE) pp 5181-4
[4] Garcia O E and Theodorsen 2017 A Auto-correlation function and frequency spectrum due to a super-position of uncorrelated exponential pulses Physics of Plasmas 24 (3) 032309
[5] Nguyen C M, Pathirana P N and Trinh H 2018 Robust observer design for uncertain one-sided Lipschitz systems with disturbances Int. J. Robust Nonlinear Control 28 1366-1380
[6] Patra M and Sharyn S 2015 On cross-correlation of a hyperfunction and a real analytic function International Journal of Mathematical Analysis 9 (4) 95-100
[7] Gorbenko I D, Zamula A A, Semenko A E and Morozov V L 2017 Method for complex improvement of characteristics of orthogonal ensembles based on multiplicative combining of signals of different classes Telecommunications and Radio Engineering 76 (18) 1581-94