Some Cosmological Consequences of Non-Trivial PPN Parameters $\beta$ and $\gamma$

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ABSTRACT. We study homogeneous isotropic universe in a graviton-dilaton theory obtained, in a previous paper, by a simple requirement that the theory be able to predict non trivial values for $\beta$ and/or $\gamma$ for a charge neutral point star, without any naked singularities. We find that in this universe the physical time can be continued indefinitely into the past or future, and that all the physical curvature invariants are always finite, showing the absence of big bang singularity. Adding a dilaton potential, we find again the same features. As a surprising bonus, there emerges naturally a Brans-Dicke function, which has precisely the kind of behaviour needed to make $\omega_{bd}(\text{today}) > 500$ in hyperextended inflation.
For various reasons, as given in [1], it is worthwhile to consider alternative theories of gravity other than Einstein’s. Two of the popular ones are Brans-Dicke (BD) theory [2] and low energy string theory, which contain an extra scalar field $\phi$, called dilaton. Such alternative theories can be distinguished by the parametrised post Newtonian (PPN) parameters. Two of them, $\beta$ and $\gamma$ (both equal to one in Einstein’s theory), can be calculated from static spherically symmetric solutions.

By studying such solutions, it is shown in [3] that neither BD nor low energy string theory can predict non trivial values of the PPN parameters $\beta$ and/or $\gamma$ for a charge neutral star, without any naked singularities. In [4] we looked for, and obtained, a graviton-dilaton theory with an arbitrary function $\psi(\phi)$, requiring it only to be able to predict non trivial values for $\beta$ and/or $\gamma$ for a charge neutral point star, without any naked singularities. These requirements imposed certain constraints on $\psi$, described below. For the static spherically symmetric case, these constraints led to the novel features of the gravitational force becoming repulsive at distances of the order of, but greater than, the Schwarzschild radius $r_0$ and the absence of a horizon for $r > r_0$. These results suggest that black holes are unlikely to form in a stellar collapse in this theory. See [4] for further details.

In this letter, we study solutions describing a homogeneous isotropic universe in the graviton-dilaton theory, obtained as above. In the case of BD or low energy string theory, the homogeneous isotropic universe has a big bang singularity. By this, we mean here that there is a curvature singularity at a finite time in the past, beyond which the physical time cannot be continued. However, in the case of the graviton-dilaton theory considered here, the constraints on $\psi$ beautifully ensure that the physical time $\hat{t}$ can be continued indefinitely into the past or future, and that all the physical curvature invariants are finite for all $\hat{t}$. This shows that the big bang singularity is absent here.

We next add to the action a dilaton potential $V(\phi)$, made finite for all $\phi$ by restricting the range of a parameter. For the homogeneous isotropic universe in this theory, with $V$ present, we again find that the physical time $\hat{t}$ can be continued indefinitely into the past or future, and that all the physical curvature invariants are finite for all $\hat{t}$. This shows that the big bang singularity is absent here too.

The theory considered here has another surprising feature which emerges naturally. As described below, the graviton-dilaton action can be rewritten,
containing now a BD function $\omega_{bd}$. As a consequence of the constraints on $\psi$, the equivalent function $\omega_{bd}$ has precisely the kind of behaviour needed to make $\omega_{bd}(\text{today}) > 500$ in hyperextended inflation [5, 6].

All of these features are generic and model independent, and arise as consequences of the constraints on $\psi$. A highly non trivial aspect is that these constraints were originally derived in [4] by the simple requirements that the graviton-dilaton theory be able to predict non trivial PPN parameters $\beta$ and $\gamma$ for a charge neutral point star, without any naked singularities.

It is interesting to note that the big bang singularity is also avoided in the models of [7, 8] by totally different methods. In [7], a Higgs potential for a complex BD type scalar results in a repulsive gravity which facilitates the absence of singularity. In [8], a limiting curvature hypothesis is invoked to impose an upper bound (of Planckian magnitude) on the curvature which leads to the absence of singularity.

This letter is organised as follows. Starting from the graviton-dilaton action, with an arbitrary function $\psi(\phi)$ describing the theory mentioned above, we present solutions [9, 10, 11] of a homogeneous isotropic universe, and the expressions for various physical quantities. We then discuss the constraints on $\psi$, and their consequences on the physical quantities. Next, we add a dilatonic potential and repeat the above analysis. We then point out a surprising feature, which is relevant to the late stages of hyperextended inflation, and conclude with a few remarks.

2. Following Dicke’s approach [2], and as explained in detail in [4], the most general action for graviton and dilaton including the world line action for test particles, at least some of which have non zero rest mass, can be written as

$$S = \int d^4x \sqrt{-g} \left( R + \frac{1}{2} (\nabla \phi)^2 \right) + \sum_i m_i \int \left( -e^{-\psi} g_{\mu\nu} dx_i^\mu dx_i^\nu \right)^{\frac{1}{2}}, \quad (1)$$

where $m_i$ are constants at least some of which are non zero, the sum is over different types of test particles and, with the dilaton potential set to zero, the arbitrary function $\psi(\phi)$ characterises our graviton-dilaton theory. $\psi$ cannot be gotten rid of, except when all test particles have zero rest mass which, by assumption, is not the case here. In our notation, the signature of the metric is $(-+++)$ and $R_{\mu\nu\lambda\tau} = \frac{1}{2} \frac{\partial^2 g_{\mu\nu}}{\partial x^\lambda \partial x^\tau} + \cdots$. Note that in (4) the curvature scalar
appears canonically. For this reason, $g_{\mu\nu}$ is often referred to as ‘Einstein metric’. However, the test particles couple to dilaton now and feel both the gravitational and the dilatonic forces. Hence, they do not fall freely along the geodesics of $g_{\mu\nu}$. See [2].

The action in (1) can be written equivalently in terms of the metric

$$\hat{g}_{\mu\nu} = e^{-\psi}g_{\mu\nu}.$$  \hspace{1cm} (2)

It then becomes

$$S = \int d^4x \sqrt{-\hat{g}} e^\psi \left( \hat{R} - \omega(\hat{\nabla}\phi)^2 \right) + \sum_i m_i \int (-\hat{g}_{\mu\nu}dx^\mu dx^\nu)^{\frac{1}{2}},$$ \hspace{1cm} (3)

where $\omega = \frac{1}{2}(3\psi_{(1)}^2 - 1)$. Here $\psi_{(n)} \equiv \frac{d^n\psi}{d\phi^n}$, the $n$th derivative of $\psi$ with respect to $\phi$. In (3) and in the following, hats denote quantities involving $\hat{g}_{\mu\nu}$. Note that in (3) the curvature scalar $\hat{R}$ does not appear canonically. However, the test particles now couple to the metric only canonically and, hence, fall freely along the geodesics of $\hat{g}_{\mu\nu}$. For this reason, we refer to $\hat{g}_{\mu\nu}$ as physical metric: since the test particles follow its geodesics, the quantities related to $\hat{g}_{\mu\nu}$ are the physically relevant ones. This is the original approach of Dicke [2].

Thus, our theory is specified by the action given in (1) or, equivalently, in (3) for graviton, dilaton, and for the test particles. It is characterised by one arbitrary function $\psi(\phi)$. Note that setting $\psi = \phi$ $(3 + 2\omega_{bd})^{-\frac{1}{2}}$ in (3) one gets the Brans-Dicke theory; and, setting $\psi = \phi$ in (3) one gets the graviton-dilaton part of the low energy string theory\footnote{modulo the choice of the test particle coupling. In the string theory literature, the action in \( \text{(1)} \), but with $\psi = 0$, and that in \( \text{(3)} \), but with $\psi = \phi$, have both been used often.}

We now consider solutions describing a homogeneous isotropic universe. We first solve for $\phi$ and $g_{\mu\nu}$ using (1), and then obtain the physical metric $\hat{g}_{\mu\nu}$ using (2). The physical curvature scalar $\hat{R}$ is given by

$$\hat{R} = e^\psi \left( R + \frac{3}{2}(\nabla \psi)^2 - 3\nabla^2 \psi \right).$$ \hspace{1cm} (4)

The equations of motion obtained from (1) are

$$2R_{\mu\nu} + \nabla_\mu \phi \nabla_\nu \phi = \nabla^2 \phi = 0.$$
With $\phi = \phi(t)$, the equations of motion in the gauge
\[
ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right),
\]
where $k = 0, \pm 1$ and $d\Omega^2$ is the line element on an unit sphere, become
\[
\frac{6\ddot{a}}{a} + \dot{\phi}^2 = \frac{2\ddot{a}}{a} + \frac{4(\dot{a}^2 + k)}{a^2} = \ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = 0,
\]
where upper dots denote $t$-derivatives. Following the original work of [9] on string theoretic cosmology, a lot of work has been carried out in this arena in which equations (5), and their higher dimensional and other generalisations, have been solved [10, 11]. The solutions to (5) are of the form
\[
a = At^n, \quad e^{\phi - \phi_0} = t^m,
\]
where $A$ and $\phi_0$ are constants, $\epsilon = \pm 1$ and, by definition, $m$ is positive. We will set $\phi_0 = 0$ without any loss of generality. Equation (5) gives
\[
(n, m) = \left(\frac{1}{3}, \frac{2}{\sqrt{3}}\right), \quad k = 0,
\]
where the square root, above and in the following, is to be taken with a positive sign. The physical curvature scalar $\hat{R}$ is given by
\[
\hat{R} = (6\psi_2 - 3\psi_1) + 1)e^\psi R; \quad R = \frac{6n(1 - 2n)}{t^2} - \frac{6k}{a^2}.
\]
The parameter $t$ will vary only from 0 to $\infty$, since the dilaton $\phi$ becomes complex for $t < 0$ whose interpretation is not clear.

The physical metric $\hat{g}_{\mu\nu}$, using (2), becomes
\[
ds^2 = -d\hat{t}^2 + \hat{a}^2\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right),
\]
where $\hat{t}$ is the physical time and $\hat{a}$ is the physical scale factor, given by
\[
\frac{d\hat{t}}{dt} = e^{-\frac{\psi}{2}}, \quad \hat{a} = e^{-\frac{\psi}{2}}a.
\]
In low energy string theory, $\psi = \phi$. Using the above solutions, we get
\[
\hat{t} = \frac{\sqrt{3}}{\sqrt{3} - \epsilon} t^{1 - \frac{\epsilon}{\sqrt{3}}} + \text{constant}, \quad \hat{R} = -\frac{4}{3} t^{-2(1 - \frac{\epsilon}{\sqrt{3}})}.
\]
It follows that as $t \to 0$, the physical time $\hat{t} \to \text{constant}$, and the physical curvature scalar $\hat{R} \to \infty$. Thus there is a physical curvature singularity at a finite time in the past, beyond which the physical time $\hat{t}$ cannot be continued. Similarly, it can be seen that this singularity, referred to as big bang singularity in the following, is also present in BD theory.

3. There is another aspect of BD and low energy string theories, shown in [3]: they cannot predict non trivial values of the PPN parameters $\beta$ and/or $\gamma$ for a charge neutral point star, without any naked singularities. In [4] we looked for, and obtained, a graviton-dilaton theory of the form given in (3), requiring it only to be able to predict non trivial values for $\beta$ and/or $\gamma$ for a charge neutral point star, without any naked singularities. These simple requirements imposed certain constraints on $\psi$, to be described presently. For the static spherically symmetric case, these constraints led to the novel features of the gravitational force becoming repulsive at distances of the order of, but greater than, the Schwarzschild radius $r_0$ and the absence of a horizon for $r > r_0$, suggesting that black holes are unlikely to form in a stellar collapse in this theory.

We will now consider this theory, which is defined by any function $\psi(\phi)$ satisfying the following constraints (see [4] for details):

\begin{align}
(i) & \quad \bar{a} - \bar{b}\psi(1)(\bar{\phi}) > 0 \ , \ \bar{b}^2\psi(2)(\bar{\phi}) = \delta \ , \ \bar{b}\psi(1)(\bar{\phi}) = \epsilon \ , & (11) \\
(ii) & \quad \psi(n)(\phi) \equiv \frac{d^n\psi}{d\phi^n} = (\text{finite}) \quad \forall \ n \geq 1 \ , \ -\infty \leq \phi \leq \infty \ , & (12) \\
(iii) & \quad \lim_{\phi \to \pm\infty} \psi = -\lambda|\phi| \ , \ \lambda \geq \sqrt{3} . & (13)
\end{align}

In equation (11), $\bar{\phi}$ is the asymptotic value of $\phi$ which can be set to zero, $\delta$ and $\epsilon$ are $< 10^{-3}$ with at least one of them nonzero, $0 < \bar{a} < 1$, and $\bar{b} = \pm \sqrt{1 - \bar{a}^2} \neq 0$. Note that equations (12) and (13) imply a finite upper bound on $\psi$, i.e. $\psi \leq \psi_{\text{max}} < \infty$.

There are many functions $\psi$ satisfying these requirements, e.g. $\psi = -\lambda\sqrt{(\phi - \phi_1)^2 + c^2}$ where $\phi_1$ and $c^2$ are constants. However, an explicit form of $\psi$ is not needed, since we are concerned here only with generic, model independent features, true for any $\psi$ satisfying only the constraints (i)-(iii). We will now consider these features.
From equations (6) and (13), it follows that
\[ e^\psi \rightarrow t^{-\lambda m} \text{ as } t \rightarrow \infty , \text{ and } e^\psi \rightarrow t^{\lambda m} \text{ as } t \rightarrow 0 . \] (14)

Note that, by definition, \( m \) is positive. Also, \( \psi \leq \psi_{\text{max}} < \infty \). Therefore, \( e^{-\frac{\psi}{2}} \geq e^{-\frac{\psi_{\text{max}}}{2}} > 0 \). It then follows from (13) that the physical time \( \hat{t} \) is a strictly increasing function of \( t \), given by
\[ \hat{t} - \hat{t}_0 = \int_{t_0}^t dt \, e^{-\frac{\psi}{2}} , \] (15)

where \( t_0 \) and \( \hat{t}_0 \) are some finite constants, which are not relevant for our purposes. Using equations (8) and (10) we obtain the following result, where \( \hat{t} \) is defined up to a finite additive constant. In the limit \( t \rightarrow \infty \),
\[ \hat{t} = \frac{2}{2 + \lambda m} t^{1 + \frac{\lambda m}{2}} , \quad \dot{a} = A t^{n + \frac{\lambda m}{2}} , \quad \dot{R} = \frac{2}{3} \left( 1 - 3\lambda^2 \right) t^{-\left(2 + \lambda m\right)} . \] (16)

And, in the limit \( t \rightarrow 0 \),
\[ \hat{t} = \frac{2}{2 - \lambda m} t^{1 - \frac{\lambda m}{2}} , \quad \dot{a} = A t^{n - \frac{\lambda m}{2}} , \quad \dot{R} = \frac{2}{3} \left( 1 - 3\lambda^2 \right) t^{-\left(2 - \lambda m\right)} . \] (17)

In equation (17), \( \hat{t} = \ln t \) if \( \lambda m = 2 \).

The behaviour given in equations (14), (16), and (17) is generic and model independent, true for any function \( \psi \) satisfying the constraints (i)-(iii). It follows from (16) that as \( t \rightarrow \infty \),
\[ \hat{t} \rightarrow \infty , \quad \dot{a} \rightarrow \infty , \quad \dot{R} \rightarrow 0 , \] (18)

independently of the values of \( n \), \( m \), and \( \lambda \), which are all positive.

However, the behaviour of these physical quantities as \( t \rightarrow 0 \), where there is a potential big bang singularity, will depend on the values of \( n \), \( m \), and \( \lambda \). Using the solution (7) and the constraint \( \lambda \geq \sqrt{3} \), we have \( \lambda m \geq 2 \). As a beautiful consequence of this inequality, it follows from (17) that, as \( t \rightarrow 0 \),
\[ \hat{t} \rightarrow -\infty , \quad \dot{a} \rightarrow \infty , \quad \dot{R} \rightarrow 0 \text{ (or, constant)} , \] (19)

Hence, as \( t \rightarrow 0 \), the physical time \( \hat{t} \) can be continued indefinitely into the past, the physical scale factor of the universe becomes infinite, and the
physical curvature scalar becomes zero (or, approaches a constant). Also, since \( \psi \leq \psi_{\text{max}} < \infty \), it is clear from equation (10) that the physical scale factor \( \hat{a} \) never vanishes at any time \( \hat{t} \) and, from equation (8), that the physical curvature scalar remains finite for all \( \hat{t} \), suggesting the absence of big bang singularity.

However, for the singularity to be absent, all other curvature invariants must also be finite. As shown in the appendix, any curvature invariant is of the form given in (29). Since equation (12) is satisfied, and since \( (e^{\psi}t^2) \) is finite for all \( \hat{t} \) as can be seen from the above discussions, it follows that all the \( n^{th} \) order curvature invariants are also finite, for all \( \hat{t} \). This shows that the big bang singularity, shown earlier to be present in BD and low energy string theories, is indeed absent here.

Notice that these features are generic and model independent, true for any function \( \psi \) satisfying only the constraints (i)-(iii). Also note the crucial role of the constraint \( \lambda \geq \sqrt{3} \), which beautifully ensures that the physical time \( \hat{t} \) can be continued indefinitely into the past and that all the curvature invariants are finite for all \( \hat{t} \). This is a highly non trivial aspect of the graviton-dilaton theory considered here since the constraints (i)-(iii), which ensure the absence of big bang singularity here, originate from a totally different requirement - that the graviton-dilaton theory be able to predict non trivial PPN parameters \( \beta \) and \( \gamma \), obtained from static spherically symmetric solutions, for a charge neutral point star without any naked singularities.

4. As another illustration of the generic cosmological features of the graviton-dilaton theory presented here, consider the action (8) with a dilaton potential, \( V(\phi) = \Lambda e^{\psi + \alpha \phi} \), coupled as follows:

\[
S = \int d^4x \sqrt{-\hat{g}} e^\psi \left( \hat{R} - \omega(\hat{\nabla} \phi)^2 + \Lambda e^{\psi + \alpha \phi} \right). \tag{20}
\]

where \( \Lambda \) is a positive constant, and \( 0 < \alpha^2 \leq 3 \). If \( \alpha = 0 \) then in the solutions, \( \phi \) and, hence, \( \psi \) are constants and the non trivial consequences of our theory cannot be seen. Hence, we take \( \alpha \neq 0 \). The upper bound on \( \alpha^2 \) is set so that the potential \( V \equiv \Lambda e^{\psi + \alpha \phi} \) is finite for all \( \phi \), which is ensured by the constraint (iii).

Proceeding as before to the Einstein frame using (2), the equations of
motion for a homogeneous isotropic universe are

\[
\frac{6\ddot{a}}{a} + \phi^2 = \frac{2\ddot{a}}{a} + \frac{4(\dot{a}^2 + k)}{a^2} = -\frac{1}{\alpha} \left( \frac{\ddot{\phi}}{a} + \frac{3\dot{a}\dot{\phi}}{a} \right) = \Lambda e^{\alpha\phi} .
\]  

(21)

Such equations arising from dilaton potential and their solutions have also been considered in [9, 11] for specific values of \( \alpha \). The solutions to (21) give \( a \) and \( \phi \) of the same form as in (6), with

\[
(n, \epsilon m) = \left( \frac{1}{\alpha^2}, -\frac{2}{\alpha} \right) , \quad \Lambda e^{\alpha\phi_0} = \frac{2(3 - \alpha^2)}{\alpha^4} , \quad k = 0 ,
\]  

(22)

or

\[
(n, \epsilon m) = \left( 1, -\frac{2}{\alpha} \right) , \quad \Lambda e^{\alpha\phi_0} = \frac{4}{\alpha^4} , \quad A^2 = \frac{k\alpha^2}{1 - \alpha^2} ,
\]  

(23)

where \( \epsilon = -\text{sign}(\alpha) \) so that \( m \equiv \frac{2}{|\alpha|} \) is positive and, in equation (22), \( k = \text{sign}(1 - \alpha^2) \). If \( \alpha^2 = 1 \) then \( k = 0 \) and the constant \( A \) is arbitrary, and equations (22) apply. The physical curvature scalar \( \hat{R} \) is given by equation (8). Analysing, as before, the physical quantities in these solutions we find, for a generic choice of \( \alpha \) with \( 0 < \alpha^2 \leq 3 \), that there are curvature singularities in BD and low energy string theories, where the physical curvature scalar \( \hat{R} \rightarrow \infty \) at some physical time \( \hat{t} \).

Now consider the theory given by action (20), with the function \( \psi \) obeying the constraints (i)-(iii). The physical quantities \( \hat{t}, \hat{a}, \) and \( \hat{R} \) can be analysed as before. Equations (14)-(17) apply, leading, as \( t \rightarrow \infty \), to the behaviour given in (18). To find the behaviour as \( t \rightarrow 0 \), the values of \( (n, m) \) given in equations (22) and (23) are to be used. Since \( m = \frac{2}{|\alpha|} \), \( \alpha^2 \leq 3 \) and \( \lambda \geq \sqrt{3} \), we have again \( \lambda m \geq 2 \). Therefore, as \( t \rightarrow 0 \),

\[
\hat{t} \rightarrow -\infty , \quad \hat{R} \rightarrow 0 \ (\text{or, constant}) , \quad \hat{a} \rightarrow \infty .
\]  

(24)

and the physical scale factor \( \hat{a} \) will diverge if \( n < 3 \), or equivalently if \( \alpha^2 > \frac{1}{3} \), and vanish if \( n > 3 \), or equivalently if \( \alpha^2 < \frac{1}{3} \). Very importantly, however, the physical time \( \hat{t} \) can be continued indefinitely into the past or future, and the physical curvature scalar \( \hat{R} \) and, hence, following the same analysis as before, all other physical curvature invariants also are finite for all \( \hat{t} \). This again shows that there is no big bang singularity in the theory given by the action (20), which has a non trivial dilaton potential.
5. Another novel feature of the graviton-dilaton action follows by rewriting it as

\[ S = \int d^4x \sqrt{-\hat{g}} e^{\psi} \left( \hat{R} + \omega_{bd}(\hat{\nabla} \psi)^2 \right), \]

(25)

where \( \omega_{bd} = \frac{1-3\psi_1^2}{2\psi_1^2} \) is the BD function which, if constant, is the standard BD parameter. Also, the standard BD scalar field is related to \( \psi \) by \( \Phi_{bd} \equiv e^\psi \).

As noted before, the constraints (i)-(iii) on \( \psi \) imply a finite upper bound \( \psi_{max} \) on \( \psi \). Hence, \( \psi_{(1)} = 0 \) for at least one finite value of \( \phi \equiv \phi_c \), where \( \psi(\phi_c) = \psi_{max} \). Therefore, as \( \phi \to \phi_c, \psi_{(1)} \to 0, \) and \( \omega_{bd} \to \frac{1}{2\psi_1^2} \), which is precisely the kind of behaviour needed to make \( \omega_{bd}(\text{today}) > 500 \) in hyperextended inflation \([4, 5]\). The example of \( \psi = -\lambda \sqrt{(\phi - \phi_1)^2 + c^2} \) corresponds, as \( \phi \to \phi_c \) (\( = \phi_1 \)), to the model of \([6]\) with \( \alpha = 1 \); the examples of \( \psi \) corresponding, as \( \phi \to \phi_c \), to other values of \( \alpha \) in \([6]\) can be easily found.

This novel feature, that \( \omega_{bd} \) here has precisely the kind of behaviour needed to make \( \omega_{bd}(\text{today}) > 500 \) in hyperextended inflation, is a generic and model independent consequence of the constraints (i)-(iii). We find it quite surprising that this feature emerges naturally in the graviton-dilaton theory considered here. The question of whether other aspects of hyperextended inflation also follow in this theory is presently under study \([12]\).

6. We comment now on some aspects of the present theory which can be studied further. Note that we have not included here matter and radiation (except as test particles which do not affect the fields), which is necessary for cosmology. What is the effect of their inclusion? Our preliminary studies on this question indicate that, even with matter and/or radiation present, the big bang singularity may still be absent. Work on this is in progress.

Does the present theory stem from a fundamental theory, e.g. string theory? We do not know the answer. The only connection, if at all, to a fundamental theory that we can see is the following. 4 + 1-dimensional Kaluza-Klein gravity compactified on a circle to 3 + 1 dimensions gives a graviton-dilaton theory with \( \psi = \sqrt{3} \phi \) (see \([13]\) for example). Equation \((13)\) is then barely satisfied when \( \phi \to -\infty \), but not when \( \phi \to \infty \). Now if there is some connection between four and five dimensional theories in some limit

\[ \psi_1^2 = \frac{(\psi_1^2 - \lambda^2 \phi_1^2)}{\phi_1^2 \lambda^2}. \]
(say, as $\phi \to -\infty$), analogous to the one recently discovered between ten and eleven dimensional string theories \cite{14}, then there is just a possibility, admittedly far fetched, that one may be able to derive the present theory from a fundamental one.

We will end this letter with one irresistible speculation. The short distance repulsion that seems to be present in this theory may perhaps soften, and even eliminate, the ultraviolet divergences in quantum gravity. However, even if this were the case, the repulsive effects come into play only at short distances where the fields are often strong and where the known perturbative techniques may break down. Hence, a study of this important issue is likely to be difficult.

Thus, keeping in view various novel features of the present theory seen in \cite{4} and in this letter, we believe its further study to be fruitful.

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**Appendix**

All curvature invariants can be constructed using metric tensor, Riemann tensor, and covariant derivatives which contain ordinary derivatives and Christoffel symbols $\hat{\Gamma}_{ab}^c$. When the metric is diagonal, every term in any curvature invariant can be grouped into factors, each of which is of one of the following forms (no summation over repeated indices): (A) $\sqrt{\hat{g}^{aa} \hat{g}^{bb} \hat{g}^{cc} \hat{g}^{dd}} \hat{R}_{abcd}$, (B) $\sqrt{\hat{g}^{aa} \hat{g}^{bb} \hat{g}^{cc}} \hat{\Gamma}_{ab}^c$, or (C) $\sqrt{\hat{g}^{aa}} \partial_a$.

Taking $\hat{g}_{\mu\nu}$ given in \cite{9}, and the solutions for $a$, and $\phi$ given in \cite{1}, the above forms can be calculated explicitly. The calculation is straightforward, and the result is that (A), (B), and (C) can be written, symbolically, as

\begin{align*}
(A) & \simeq U e^{\psi t - 2}, \quad (B) \simeq V (e^{\psi t - 2})^\frac{1}{2}, \\
(C)^a \cdot (A)^p (B)^q & \simeq W_{n+2} (e^{\psi t - 2})^{p+\frac{1}{2}(q+n)},
\end{align*}
where $U$ and $V$ are functions of $t$, $\psi(1)$, and $\psi(2)$ only, and $W_k$ are functions of $t$ and $\psi(l)$, $1 \leq l \leq k$. It turns out, as a result of the way various factors are grouped, that the explicit $t$-dependent parts in $U$, $V$, and $W_k$'s are finite at $t = 0$ (in fact, they have divergence at $t = \infty$ only which is mild and such that it is suppressed by the accompanying $(e^{\psi t^{-2}})$ factors).

Hence, any curvature invariant constructed from $m$ Riemann tensors, $n$ covariant derivatives, and the requisite number of metric tensors will be of the form

$$\tilde{W}(t; \psi(1), \psi(2), \ldots, \psi(n+2)) \left( e^{\psi t^{-2}} \right)^{m+n^2}$$

where the explicit $t$-dependent parts of $\tilde{W}$ are finite at $t = 0$ (in fact, they have divergence at $t = \infty$ only which is mild and such that it is suppressed by the accompanying $(e^{\psi t^{-2}})$ factors). Note, as an example, that the curvature scalar given in (8) belongs to type (A), and has the above form with $m = 1$ and $n = 0$.

Note Added in Proof:

There is an issue which we are unable to resolve as yet: whether or not a test photon, or a massless test particle, sees a singularity in the graviton-dilaton background described here.

On the one hand, the physical metric $\hat{g}_{\mu \nu}$ can be conformally transformed to $g_{\mu \nu}$, which is singular - suggesting that a test photon sees a singularity. On the other hand, however, (i) the conformal factor is singular, so the validity of this transformation is not automatic; (ii) our calculations, the details of which will be given in [12], show that the big bang singularity is absent and the space time described by $\hat{g}_{\mu \nu}$ is regular everywhere even upon including macroscopic amount of matter and/or radiation, that modify the space time. This would suggest that a test photon should see no singularity. (iii) Moreover, in $3+1$ dimensional space time and within the context of Dicke type ansatz, the natural photon-dilaton coupling is ambiguous, and appears to call for another arbitrary function of dilaton. If so, this function may be quite relevant to the present issue.

For these reasons, we are unable to resolve the above issue definitively.

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