Breaking SU3\(_f\) Spontaneously Down to Isospin.

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Abstract

The mechanism where flavour symmetry of is broken spontaneously is demonstrated for a model involving nonets of pseudoscalar and vector mesons. Degenerate bare nonets of vector pseudoscalar mesons coupling sufficiently strongly to each others lead to unstable self-consistency equations, such that the SU3\(_f\) symmetric spectrum breaks down into a stable isospin symmetric mass spectrum similar to the physical spectrum.

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Conventionally one breaks flavour symmetry by adding effective non-degenerate quark masses to the QCD Lagrangian, whereby the pseudoscalars obtain (small) masses and the degeneracy of all flavour multiplets is split. Most of the chiral quark masses are assumed to come from a short distance regime, where weak interactions, and the Higgs mechanism are relevant.

Here I shall discuss an alternative way of breaking flavour symmetry, which is a generalisation of the arguments given in a recent paper to the physical case of three light flavours. In that paper I discussed a mechanism where a meson mass spectrum, obeying exact flavour symmetry, the Okubo-Zweig-Iizuka (OZI) rule and general constraints of analyticity and unitarity, is unstable with respect to quantum loops involving quark pair creation. The instability followed from the nonlinear self-consistency equations for inverse
meson propagators containing loops described diagrammatically in Fig. 1.

The insight that such a spontaneous flavour symmetry breaking mechanism, rather than the conventional explicit symmetry breaking through quark masses, might be the true mechanism in the real world, has emerged from earlier studies of meson masses and widths including meson loops [3], where most of the flavour symmetry breaking comes from the threshold positions in loops.

A very important question which arises in this connection is: Where are the Goldstone degrees of freedom and the Goldstone bosons expected whenever a symmetry is spontaneously broken? In Ref. [4] I argue within a scalar QCD model that actually the scalar or longitudinal confined gluons are the would-be Goldstone bosons, not scalar mesons carrying flavour as was expected in early work [5].

In order to make the mechanism for spontaneous flavour symmetry breaking as transparent as possible I made in [2] the simplifying assumption of \( C \)-degeneracy of multiplets. Thereby one obtains an OZI rule obeying model, which already for two flavours demonstrates why, for the exactly symmetric solution one can have an unstable situation, - a small symmetry breaking \( \Delta m \) in the masses of the loop generates an even bigger symmetry breaking in the output mass spectrum. A stable self-consistent mass spectrum is obtained only when the degeneracy of the flavour multiplet is broken down to an approximately equally spaced spectrum, which is ideally mixed and obeys the OZI rule.

The physical mass spectrum, after confinement and chiral symmetry breaking in the vacuum, is certainly not \( C \)-degenerate. Any confining mechanism should give the quark model rule \( C = (-1)^{L+S} \), and masses which depend on spin \( (S) \) and angular momenta \( (L) \). Therefore, the \( C \)-degeneracy and consequently the OZI rule is violated in the real world. In particular, the pseudoscalars \( (P) \), for which \( C = + \), are not degenerate with the vectors \( (V) \), for which \( C = - \), and consequently the OZI rule must be violated for loops like \( P \to PV \to P \) or \( V \to PP \to V \).

Here I shall present a model consistent with this fact, i.e. I relax the \( C \)-degeneracy condition and apply the arguments to the ground state nonets of \( P \) and \( V \) mesons. Thereby the model can approximate the real world for three light flavours, and if correct it should generate SU3\(_f\) breaking, singlet-octet splittings, and deviations from ideal mixing of the right sign and approximate magnitude. The same mechanism which mixes \( s\bar{s} \) with \( u\bar{u} + d\bar{d} \) also mixes \( u\bar{u} \) and \( d\bar{d} \). A priori, it is possible that one finds that precisely for the linear combinations \( u\bar{u} \pm d\bar{d} \) of pure isospin one obtains a stable solution. Then the isospin subgroup
is stable i.e., not broken spontaneously. Here I shall, in fact, show that this is what actually happens in the model presented.

In the discussion it is useful to distinguish four steps of symmetry breaking from the largest symmetry down to the physical meson spectrum (Fig. 2):

(i) The formation of an SU3\textsubscript{f} symmetric, but unstable, spectrum after confinement and spontaneous chiral symmetry breaking of the vacuum. Here one allows for explicit vector-pseudoscalar splitting. The pseudoscalar octet \( P8 \) is composed of massless Goldstone bosons, while \( P1, V1 \) and \( V8 \) must at the same time be massive and nondegenerate in order to have a spectrum self-consistent with meson loops.

(ii) One generates mass to the pseudoscalar octet, but the members remain degenerate. The spectrum is thus still SU3\textsubscript{f} symmetric although chiral symmetry is broken.

(iii) The SU3\textsubscript{f} spectrum is broken down to an isospin symmetric spectrum. Conventionally this breaking is attributed to the \( s - d \) quark mass splitting, but in this paper I shall instead argue that the situation after (ii) is unstable, and a spontaneous symmetry breaking mechanism gives a similar result in the spectrum.

(iv) Finally a small conventional explicit isospin breaking from electroweak interactions splits the isospin multiplets.

The three first steps are pictured in Fig. 2.

The physical flavourless mesons are mixtures of \( u\bar{u}, d\bar{d} \) and \( s\bar{s} \) states determined by an orthogonal 3\times3 mixing matrix \( \Omega \). If the unmixed states \( q_i\bar{q}_i \) are described by \( \text{flavour matrices} \Lambda^{ii} \) the physical states are described by \( \Lambda^\alpha = \sum_i \Omega_{\alpha i} \Lambda^{ii} \). Here \( \Omega \) mixes the the \( u\bar{u} \) and \( d\bar{d} \) states to \( u\bar{u} \pm d\bar{d} \) when isospin is exact, and furthermore mixes the \( u\bar{u} + d\bar{d} \) and \( s\bar{s} \) states by a mixing angle \( \delta \), measuring the deviation from ”ideal” mixing \( \delta = \theta^{SU3} + \arctan(\sqrt{2}) \).

This mixing matrix \( \Omega \) will be determined by the self-consistency equation, which is obtained below for the 3 \times 3 mass matrix in the inverse propagators, and which is diagonalized by \( \Omega \). Once \( \Omega \) is determined the flavour related couplings for a vertex \( A^\alpha \to B^\beta C^\gamma \) are given by the general formula

\[
g_{\alpha\beta\gamma}^{ABC,S} = g_{ABC} \text{Tr}[\Lambda_A^\alpha \Lambda_B^\beta \Lambda_C^\gamma]_S,
\]

where \( S \) is determined by the charge conjugation quantum numbers of the three multiplets, \( S = C_A C_B C_C \). Thus the symmetric \( (S = +, \text{for "D-coupling"}) \) or antisymmetric \( (S = -, \text{for "F-coupling"}) \) trace, must be taken. Therefore singlet states decouple when \( S = - \), but couple with twice the strength compared to \( 8 \to 8 + 8 \) transitions if \( S = + \). Eq.(1) allows
for OZI rule violation only through particle mixings in the meson propagator. With isospin
exact Eq.(I) implies that all $g$’s for one group of thresholds like $P \rightarrow PV$ are determined
by only one mass dependent mixing angle ($\delta$) and one overall coupling.

We can now construct the inverse propagators for the pseudoscalars including the loop
contributions from the $PV$ and $VV$ thresholds:

$$P_{\alpha\beta}(s) = m_0^2 - s + \frac{1}{4\pi} \sum_{\beta,\gamma=1}^9 \left[ g_{\alpha\beta\gamma}^{P\gamma} - g_{\alpha'\beta'\gamma}^{P\gamma} \right] F(s, m_{P\beta}^2, m_{V\gamma}^2, \Lambda) +$$

$$+ \frac{1}{4\pi} \left[ g_{\alpha\beta\gamma}^{PV\gamma} + g_{\alpha'\beta'\gamma}^{PV\gamma} \right] F(s, m_{V\beta}^2, m_{V\gamma}^2, \Lambda)$$

where the function F (See [2]) must contain the unitarity cut and the phase space of one
loop diagrams. SU6 predicts that the overall couplings are equal: $g_{PPV} = g_{PVV}$, a constraint
which shall be used here.

Summing over isospin states, and in order not to have too cumbersome notation choosing
the special case of ideally mixed ($\delta_V = 0$) vector mesons and pseudoscalars threshold masses
mixed as in the pure SU3 frame ($\beta_{P SU3} = 0$) the different thresholds contribute to the $\pi$, $K$
mass matrix with the following weights composed of Clebsch-Gordan-like coefficients in the
partial sums $\sum [g_{ABC}^{\alpha\beta\gamma} g_{ABC}^{\alpha'\beta'\gamma}]/(g_{ABC})^2$:

$$\pi : 4\rho + 2K\bar{K} + 4\rho \omega_u + 2K*\bar{K}$$

$$K : \frac{3}{2}K*\pi + \frac{3}{2}K\rho + \frac{1}{2}K\omega_u + K\phi_s + \frac{3}{2}\eta_8K* + 3K*\rho + K*\omega_u + 2K*\phi_s$$

Similarly for the $\eta - \eta'$ mass matrix one has:

$$\begin{pmatrix} \eta_{88} & \eta_{81} \\ \eta_{18} & \eta_{11} \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} K\bar{K} + \frac{2}{3} \begin{pmatrix} 1 & -2^{\sqrt{2}} \\ -2^{\sqrt{2}} & 2 \end{pmatrix} (3\rho + \omega_u \omega_u) +$$

$$\frac{4}{3} \begin{pmatrix} 0.5 \sqrt{2} \\ \sqrt{2} \end{pmatrix} K*\bar{K} + \frac{4}{3} \begin{pmatrix} 2 \sqrt{2} \\ \sqrt{2} \end{pmatrix} \phi_s \phi_s$$

There are other possible flavour symmetric, but OZI rule violating couplings, involving at least
one singlet, such as $TrA_A TrA_B TrA_C$ or $Tr[A_A \Lambda_B] TrA_C$ + permutations. For simplicity I do not
here include the extra parameters involved with these. They would shift details of the spectrum
through shifting the octets and singlets, but the spontaneous generation of "$s$-quark mass" would
still occur.
The notation in (3-5) should here be obvious [e.g. 4πρ stand for 4F(s, m^2, m^2, L)], except for the fact that K\bar{K} stands for \bar{K}K* and, more precisely e.g., K*ω should be replaced by (cos δV + √2 sin δV)^2Kω, and 2K*φs should be replaced by (sin δV − √2 cos δV)^2Kω etc. This is done in the numerical work through Ω and the formula (1), where in addition Ω and the mixing angles depend on mass, e.g. the pseudoscalar mixing angle is obtained when diagonalizing Eq. (5) at the η and η' mass, and similarly δV(mω) is not exactly equal to δV(mφ).

For the vector mesons one has similar equations as Eq. (2), involving PP, PV, and VV thresholds. There SU6 predicts that these should be added with the relative weights \frac{1}{6}, \frac{4}{6}, \frac{7}{6}, but such relations are known to agree rather poorly with experiment. For our demonstrative purpose it is sufficient to add the V → PP thresholds (for which S = −) and the V → PV thresholds (for which S = +) with equal weights such that in fact we have g_{PPV} = g_{PVV} = g_{VPP} = g_{VPV} = g. Thus for the vectors we have very similar equations as those of Eq. (2) for the pseudoscalars, but with PP replacing PV and PV replacing VV.

Now consider the SU3f limit after step (ii) assuming as above the four overall couplings to be equal. One can sum over the degenerate octets, and θ^{SU3} must vanish. For the octet and singlet pseudoscalar propagators one finds:

\[
P_{P8}^{-1}(s) = \frac{2 g^2}{34\pi} [9F(s, m_{P8}^2, m_{V8}^2, L) + 5F(s, m_{V8}^2, m_{V8}^2, L) + 4F(s, m_{V1}^2, m_{V8}^2, L)] + m_{0,P}^2 - s,
\]

\[
P_{P1}^{-1}(s) = \frac{4 g^2}{34\pi} [8F(s, m_{V1}^2, m_{V8}^2, L) + F(s, m_{V1}^2, m_{V8}^2, L)] + m_{0,P}^2 - s,
\]

and similar equations for the vectors as discussed above.

When the bare masses are equal (m_{0,P} = m_{0,V} = m_0) there exists always a self-consistent SU6 solution with all physical masses equal: \(m_{P}^2 = m_{V}^2 = m_{0}^2 + Δm^2\) where Δm^2 = \(\frac{g^2}{4\pi}12F\). All inverse propagators are then the same: \(P_{P8}^{-1} = P_{P1}^{-1} = P_{V8}^{-1} = P_{V1}^{-1} = \frac{g^2}{4\pi}12F(s, m_0^2 + Δm^2, m_0^2 + Δm^2, L) + m_0^2 - s\). In fact, in this limit one has a very similar situation as the one discussed in [2]. There I showed that this solution is unstable, and that flavour symmetry will be broken spontaneously by the loops if g is sufficiently large. The resulting spectrum of the stable solution, which must be found numerically even for the simplest possible model for F, obeys approximately the equal spacing rule.
But once the vectors are heavier than the near massless pseudoscalars, $m^V_0 > m^P_0$, the situation is more complicated. Already when SU3$_f$ remains exact the singlet masses must be different from the octet masses as can be seen from the self consistency equations (6-7) and as was anticipated in Fig.1. The singlets will be heavier since the nearest $PV$ thresholds shift only the octet down. Then for sufficiently large $g$ this unstable SU3$_f$ symmetric configuration will be spontaneously broken down in a way which must compromise between the ideally mixed solution and the OZI rule violating SU3$_f$ symmetric solution.

Also this solution can of course only be found numerically, as is done in Fig.3 for different values of the cutoff $\Lambda$, and for $g$ very large. The unstable SU3$_f$ symmetric solution for the singlet and octet states are shown as the dashed curves, while the stable flavour symmetry violating masses are the full drawn curves. To the left are the physical masses indicated. The $\rho$ and $\pi$ masses are used as input to fix the two subtraction constants $m_{0,P}$ and $m_{0,V}$, while $g$ is for simplicity assumed very big. As can be seen already this crude scheme gives a reasonable spectrum. The mixing angles $\delta$ are of right sign, although a little too big compared to experiment for the vectors (when $\Lambda = 2$ GeV $\delta(m_\omega) = 18.3^\circ$, $\delta(m_\phi) = 22.6^\circ$) and too small for the pseudoscalars ($\delta(m_\eta) = 25.1^\circ$, $\delta(m_{\eta'}) = 25.7^\circ$).

Why does the solution stabilize itself to an isospin symmetric one? Isospin is not broken spontaneously, because the linear combinations $u\bar{u} \pm d\bar{d}$ distribute the flavour probabilities $u\bar{u}$ and $d\bar{d}$ equally in the physical mesons. Therefore the instability in terms of the $r$ parameter introduced in [2] is decreased by a factor of 3, since only the loop involving an $s\bar{s}$ state contribute to the numerator of $r$, i.e.:

$$r = \frac{N_f F_{m^2}}{-N_f F_s + 4\pi/g^2} \rightarrow \frac{F_{m^2}}{-3F_s + 4\pi/g^2} \approx \frac{1}{3} < 1.$$ (8)

where $F_s = \frac{\partial F(s,m_1^2,m_2^2,1)}{\partial s}|_{s=m_1^2=m_2^2}$ and $F_{m^2} = \frac{\partial F(s,m_1^2,m_2^2,1)}{\partial m_1^2}|_{s=m_1^2=m_2^2}$. The value of $r$ decreases from slightly above one (see Fig. 3 of [2]) to about one third which is well below the instability limit $r > 1$. In detail this can be understood by looking at the $K^0 - K^+$ mass splitting and the $u\bar{u}$ and $d\bar{d}$ loop contributions to the $K^0$ and $K^+$ masses. For $K^+ \rightarrow (\bar{s}u)(\bar{u}u) \rightarrow K^+$ or $K^0 \rightarrow (\bar{s}d)(\bar{d}d) \rightarrow K^0$ the $u\bar{u}$ or $d\bar{d}$ pairs are replaced by $(u\bar{u} \pm d\bar{d})/\sqrt{2}$ after the isospin rotation in $\Omega$. The latter distribute the flavour probabilities democratically and neutralizes the instability created if $d$ is made heavier than $u$. Therefore the isospin symmetric solution is a stable solution. Of course isospin will still be broken by electroweak effects. Thus to understand the small physical isospin breaking one needs the conventional explicit symmetry breaking mechanism. Renormalization effects are of course also then present, but should
only slightly enhance the driving term from electroweak physics.

The whole of SU3 cannot remain unbroken this way, since the P-V splitting can stabilize only one degree of freedom, not two. With three flavours the neutral $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ probabilities cannot be distributed democratically for all three orthogonal flavourless states. In particular the octet states $u\bar{u} - d\bar{d}$ and $u\bar{u} + d\bar{d} - 2s\bar{s}$ contain very different amount of $s\bar{s}$.

The numerical work in this paper (Fig.3) has been done only for demonstration, with no ambitions to fit the data exactly. As input parameters there was only the $\pi$ mass, the $\rho$ mass and the cut off $\Lambda$, but still a spectrum not too far from the physical one was obtained. The essential thing which was demonstrated was that an effective ”strange quark mass” can be generated spontaneously within strong interaction loops, and that its magnitude is related to the $\rho$ and $\pi$ masses. Numerous improvement of the actual fit to the spectrum could be done by including more thresholds (such as those involving P-wave $q\bar{q}$ mesons), detailed fits for $g_{PPV}$, $g_{PVV}$ and singlet coupling parameters like $\text{Tr}\Lambda^A \text{Tr}[\Lambda^{B}\Lambda^{C}]$ mentioned in the footnote with pure gluonic contributions, more detailed function for $F$ etc.

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FIG. 1. The self-consistency equations diagrammatically. Iterating the equation gives a sum of multiloop diagrams.

FIG. 2. The sequence of steps of symmetry breaking from a fully degenerate SU6 symmetric mass (the value of which depends on how the vacuum is defined) down to the near isospin symmetric physical spectrum (see text).
FIG. 3. The predicted pseudoscalars and vector meson masses as functions of $\Lambda$. The $\pi$ and $\rho$ masses are input and fix the subtraction constants. The dashed lines shows the unstable SU3$_f$ octet and singlet masses before the spontaneous breaking (but with the same subtraction constants as for the stable solution). To the left the experimental masses are shown. Note in particular the very weak dependence on the cutoff $\Lambda$. 