ON THE SIMULATION OF ADAPTIVE MEASUREMENTS VIA POSTSELECTION

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ABSTRACT. In this note we address the question of whether any any quantum computational model that allows adaptive measurements can be simulated by a model that allows postselected measurements. We argue in the favor of this question and prove that adaptive measurements can be simulated by postselection. We also discuss some potentially stunning consequences of this result such as the ability to solve \#P problems.

1. Introduction

In [3] Aaronson introduced a complexity class \( \text{PostBQP} \), which is a complexity class consisting of all of the computational problems solvable in polynomial time on a quantum Turing machine with postselection and bounded error. It was also shown equivalent to \( \text{PP} \) which is the class of decision problems solvable by a probabilistic Turing machine in polynomial time, with an error probability of less than 1/2 for all instances. Aaronson then raised an interesting question which asks whether adaptive measurements made by a quantum computational model be simulated with postselected measurements. In this note we address this question by asserting that it is possible to simulate adaptive measurements by postselection on the quantum circuit model of computation. We also explore the consequences of being able to at least theoretically perform this simulation, it is known that \( \text{P}^{\text{PP}} = \text{P}^{\# \text{P}} \) which implies that the complexity of \( \text{PP} \) is equivalent to that of \( \text{P}^{\# \text{P}} \) which is an \( \text{NP} \). So if an adaptive (non-projective) measurement such as a weak measurement can be simulated, following the work of Lloyd et al. logical gates can be constructed that allow us to solve \( \text{P}^{\# \text{P}} \) problems.

2. Proof

Before we show how the simulation would work, we want to establish some definitions to make an easier transition to the proof itself.

Definition 1. An adaptive measurement is an incomplete measurement is made on the system, and its result used to choose the nature of the second measurement made on the system, and so on (until the measurement is complete). A complete measurement is one which leaves the system in a state independent of its initial state, and hence containing no further information of use.

Definition 2. Postselection is the power of discarding all runs of a computation in which a given event does not occur.

We will be using the quantum circuit model which is the standard model in quantum computation theory and most other computational models have been shown to be equivalent to it. The equivalence also allows us to simulate those models on the circuit model.

Lemma 3. Measurement based quantum computation (MBQC) employs adaptive local measurements on a resource state.

Proof. See [4] for this.

Lemma 4. Measurement based models can be simulated on the quantum circuit model.
Proof. Any one-way computation can be made into a quantum circuit by using quantum gates to prepare the resource state [5].

Axiom 5. From Lemma 3 and Lemma 4 we can deduce that the quantum circuit model can simulate measurement based computation which is a model that allows for adaptive measurements.

The above mentioned axiom completes the first part of the correspondence, we now have to show that the same model that can simulate postselected measurements to complete the correspondence. Postselected measurements fall under the complexity class PostBQP and we will also use the equivalence of PostBQP and PP shown by Aaronson in [3]. This switch between complexity classes makes this proof simplistic.

Axiom 6. BQP ⊂ PP

Lemma 7. PP ∩ BQP ≠ ∅

Proof. Let us assume that no problem exist at the intersection of PP and BQP However, we know the aforementioned axiom to be true so there must atleast be one problem that exist at the intersection of the complexity classes. That particular problem, by the virtue of being at the intersection will be both PP and BQP which is self-evident. We will represent the problems present at the intersection of the two complexity classes by the set \( \tau \).

Lemma 8. From Lemma 7 we can deduce that elements of \( \tau \) can be simulated on a quantum computer

Proof. The elements of \( \tau \) fall in the class BQP which can be simulated on a quantum computer therefore the elements of that set can also be simulated by a quantum computation model.

Lemma 9. Elements of \( \tau \) can exhibit postselected measurements

Proof. The members of \( \tau \) are both PP and BQP where BQP can be simulated on a quantum computer and since PP = PostBQP, elements of \( \tau \) can be simulated through the use of postselected measurements.

Axiom 10. From the preceding proof and Axiom 5, we see the correspondence is complete. We can indeed take a quantum computational model (in our case, it is the standard quantum circuit model) that allows for adaptive measurements (Axiom 5) and simulate it with a model that allows for postselected measurements (again the quantum circuit model)

The preceding axiom presents the completed proof, in the following section we will discuss some speculative consequences of this result.

3. Consequences

Although machines capable of postselected measurements are implausible, the ability to simulate a particular type of adaptive measurement called weak measurement has some very interesting consequences. Lloyd et.al [1] showed that when weak measurements are made on a set of identical quantum systems, the single-system density matrix can be determined to a high degree of accuracy while affecting each system only slightly. If this information can then be fed back into the system by coherent operations, the single-system density matrix can be made to undergo arbitrary nonlinear dynamics such as dynamics governed by a nonlinear Schrödinger equation. Nonlinear corrections to quantum mechanical evolution can then be used to construct nonlinear quantum gates which can solve \#P and NP-Complete problems as shown by Lloyd and Abrams [2].

References

[1] Lloyd, Seth and Slotine, Jean-Jacques E. Quantum feedback with weak measurements. Phys. Rev. A (2000).
[2] Abrams, Daniel S. and Lloyd, Seth. Nonlinear Quantum Mechanics Implies Polynomial-Time Solution for NP-Complete and \#P Problems. Phys. Rev. Lett. (1998).
[3] Taylor, P.L., Heinonen, O.: A Quantum Approach to Condensed
[4] R. Raussendorf, D. E. Browne, and H. J. Briegel. Measurement based Quantum Computation on Cluster States. Phys. Rev. A (2003).

[5] H. M. Wiseman, D. W. Berry, S. D. Bartlett, B. L. Higgins, and G. J. Pryde. Adaptive Measurements in the Optical Quantum Information Laboratory. IEEE Journal of Selected Topics in Quantum Electronics (2009).

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