Goldstone-Mode Relaxation in a Quantized Hall Ferromagnet in the Presence of Smooth Random Potential

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We discuss the spin relaxation of a strongly correlated two-dimensional (2D) electron gas (2DEG) in the quantized Hall regime when the filling factor is close to an odd-integer. As the initial state we consider a coherent deviation of the spin system from the $B$ direction and investigate a break-down of this Goldstone-mode state due to the spin-orbit (SO) coupling and smooth disorder. The spin relaxation (SR) process is considered in terms of annihilation transitions in the system of spin excitons (magnons).

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Great bulk of recent SR measurements and theoretical studies deal with 2D electrons confined in quantum wells or dots. However, during the last two decades only a few of works have been devoted to the SR in the quantized Hall regime proper. Specifically, we imply conditions under which a 2DEG is in a strong perpendicular magnetic field ($B \geq 10$ T) and in the absence of holes and magnetic impurities. We exclude from the consideration the effects of electrons in the edge states (cf. Ref. 6), and restrict our study to an odd-integer filling.

In the studied problem the relevant SR time is actually not a spin dephasing time but a time of Zeeman energy relaxation conditioned by a spin-flip process. Indeed, any spin-flip means actually a reduction of the Zeeman energy $|g\mu_B B\Delta S_z|$ ($\vec{B} \parallel \hat{z}$, $\Delta S_z = S_z - S_0$ is the $S_z$ component deviation from the equilibrium value $S_0$, $g \approx -0.44$). The mechanism, which makes the relaxation irreversible, has thereby to provide the energy dissipation. Another necessary condition is a spin-flip mechanism non-conserving $S_z$.

In the present work the SO coupling is considered as the cause mixing spin states and the disorder [to be more precise, the smooth random potential (SRP)] as a dissipation. A crystal lattice is still implicitly assumed to be present as a “cooler” for 2DEG excitations. We will suppose that all electron-phonon relaxation processes which do not change the 2DEG spin
state occur much faster than the SR. [Thermodynamic relaxation times are estimated to be \(\lesssim 1\) ns, i.e. well shorter than the SR time (see the corresponding estimate given at the end of the paper).]

We thus study the case where the 2DEG is a quantum Hall “ferromagnet” (QHF); i.e. the filling factor is \(\nu = N / N_\phi \approx 2\kappa + 1\), where \(N\) and \(N_\phi = L^2 / 2\pi l_B^2\) are the numbers of electrons and magnetic flux quanta (\(L^2\) is the 2DEG area, \(l_B\) is the magnetic length). In the high magnetic field limit, which really represents the solution to the first order in the ratio \(r_c = (e^2 / \varepsilon l_B) / \hbar \omega_c\) considered to be small (\(\omega_c\) is the cyclotron frequency, \(\varepsilon\) is the dielectric constant), we get the ground state with zeroth, first, second,... and \((\kappa - 1)\)-th Landau levels fully occupied and with \(\kappa\)th level filled only by spin-up electrons aligned along \(B\). Only a bare handful of experimental results on the SR in such a QHF were obtained, first indirectly (the line-widths of the electron spin resonance (ESR) were measured in Ref. 1) and then directly (in communication of Ref. 2 the photoluminescence dynamics of spin-up and spin-down states was studied). The measured times are 5 – 10 ns and exceed by 1-2 orders the SR times observed in quantum wells where the SR process is governed by the spin interaction with band holes.\(^7\)

As the spin-system perturbation in the QHF we study a coherent deviation, when the total \(S\) number is not changed. Namely, the initial state is a Goldstone mode which represents a quantum precession of the vector \(S\) around the \(B\) direction: \(|i\rangle = (\hat{S}_-)^N |0\rangle\). Here \(|0\rangle\) stands for the QHF ground state and \(\hat{S}_- = \sum_j \hat{\sigma}_-(j)\) is the lowering spin operator [\(j\) labels electrons; \(\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y) / 2\), where \(\hat{\sigma}_{x,y,z}\) are the Pauli matrices]. The number \(N\) is assumed to be macroscopically large: \(0 \ll N \ll N_\phi\). The spin numbers of the \(|i\rangle\) state are \(S = S_0 = N_\phi / 2\) and \(S_z = N_\phi / 2 - N\) (i.e. \(\Delta S_z|_{t=0} = -N\)). The total Hamiltonian has the form \(H_{\text{tot}} = \sum_j H_1^{(j)} + H_{\text{int}}\). Here \(H_{\text{int}}\) is the many-electron (Coulomb interaction) part of the Hamiltonian which has the usual form (see, e.g., Ref. 11), and \(H_1\) is the single-electron part:

\[
H_1 = \hbar^2 \hat{q}^2 / 2m^*_e - \varepsilon_Z \hat{\sigma}_z / 2 + H_{SO} + \varphi(r),
\]

where \(\hat{q} = -i \nabla + eA / c\hbar\) is a 2D operator, \(\varepsilon_Z = |g| \mu_B B\) is the Zeeman energy of one spin-flipped electron, and \(\varphi(r)\) is the SRP field [\(r\) has components \((x, y)\)]. The SO Hamiltonian is specified for the (001) GaAs plane,

\[
H_{SO} = \alpha (\hat{q} \times \hat{\sigma})_z + \beta (\hat{q}_y \hat{\sigma}_y - \hat{q}_x \hat{\sigma}_x) ,
\]

2
and presents a combination of the Rashba term \(^{12}\) (with the coefficient \(\alpha\)) and the crystalline anisotropy term \(^{13}\) (see also Refs. 3–5, 8). The parameters \(\alpha\) and \(\beta\) are small: \(\alpha, \beta \ll l_B \hbar \omega_c\) (moreover, really \(\alpha < \beta \sim 10^{-7} \text{K} \cdot \text{cm} < l_B \varepsilon_Z\)). This enable us to account \(H_{SO}\) perturbatively.

If the SRP is assumed to be Gaussian, then it is determined by the correlator \(K(\mathbf{r}) = \langle \varphi(\mathbf{r}) \varphi(0) \rangle\), where \(\varphi(\mathbf{r})\) is the SRP field. We choose also \(\langle \varphi(\mathbf{r}) \rangle = 0\), i.e. the SRP energy is measured from the center of the Landau level. In terms of the correlation length \(\Lambda\) and Landau level width \(\Delta\), the correlator is \(K(\mathbf{r}) = \Delta^2 \exp \left( -\frac{r^2}{\Lambda^2} \right)\). In the realistic case \(\Delta \approx 5 - 10 \text{K}, \Lambda \sim 30 - 50 \text{nm}\). We will study the case

\[
T \ll T^* \ll \varepsilon_Z < \Delta \ll \varepsilon^2/l_B \hbar \omega_c, \quad \text{and} \quad \Lambda > l_B
\]  

\((T\) is the temperature which is actually assumed to be zero in the calculations; the value \(T^*\) will be defined subsequently). All results which follow are obtained in the leading approximation corresponding to these inequalities.

The QHF is also remarkable for the following reason: when neglecting the last two terms in Eq. (1) the spin excitons (SEs) are actually exact (to the first order in \(r_c\)) lowest-energy eigen states. The most adequate description of the SE states is realized by the SE creation operators.\(^{3–5,14}\) However, before writing them out we choose the bare single-electron representation. As previously\(^{4,5}\) we use the spinor basis which diagonalizes the first three terms in the Hamiltonian (1) to the first order in \(u = \beta \sqrt{2/l_B \hbar \omega_c}\) and \(v = \alpha \sqrt{2/l_B \hbar \omega_c}\):

\[
\Psi_{\kappa p} = \left( \begin{array}{c} \psi_{\kappa p} \\ u \sqrt{\kappa+1} \psi_{\kappa+1 p} + i v \sqrt{\kappa} \psi_{\kappa-1 p} \end{array} \right), \quad \Psi_{\kappa p} = \left( \begin{array}{c} -v \sqrt{\kappa} \psi_{\kappa-1 p} + i u \sqrt{\kappa+1} \psi_{\kappa+1 p} \\ \psi_{\kappa p} \end{array} \right),
\]

(4)

where \(\psi_{\kappa p} = L^{-1/2} e^{i p y} \phi_\kappa(p l_B^2 + x)\) is the wave function of an electron in the Landau gauge \((\phi_\kappa\) is the harmonic oscillatory function). We note that here and in the following we present only the perturbation expansion to within the framework of the leading order in \(u\) and \(v\).

The exciton creation operator is

\[
Q_{ab}^\dagger_{\kappa q} = \frac{1}{\sqrt{N_\phi}} \sum_p e^{-i q y p l_B^2} b_{p+q y}^\dagger a_{p-q y}^\dagger,
\]

(5)

where \(a_p\) and \(b_p\) are the Fermi annihilation operators corresponding to the states (4). The annihilation excitonic operator is \(Q_{ab} = \langle Q_{ab}^\dagger \rangle = Q_{ba}^\dagger_{-q}\) and we employ also the “shift” operators \(A_q = N_\phi^{-1/2} Q_{aa}^\dagger_{-q}\) and \(B_q = N_\phi^{-1/2} Q_{bb}^\dagger_{-q}\) \((A_q = A_q^\dagger, B_q = B_q^\dagger)\). In Eq. (5) the orbital index \(\kappa\) is dropped since for our purposes the approximation of projection onto a
single Landau level is quite sufficient. In the following we drop also the “spin-orbit” index ab at the operator (5). Such single-level exciton operators constitute a Lie sub-algebra which is a part of an Excitonic Representation (ER) algebra (e.g., see Ref. 15 and references therein). In our case the relevant commutation rules are as follows:

\[
[Q_{q_1}, Q_{q_2}^+] = e^{i\theta_{12}} A_{q_1-q_2} - e^{-i\theta_{12}} B_{q_1-q_2}, \quad \text{and} \quad e^{i\theta_{12}} [A_{q_1}, Q_{q_2}] = e^{-i\theta_{12}} [B_{q_1}, Q_{q_2}] = N_\phi^{-1} Q_{q_1+q_2},
\]

where \( \theta_{12} = l_B^2 (q_1 \times q_2) \partial /2 \). Besides evidently \([Q_{q_1}, Q_{q_2}] = [A_{q_1}, B_{q_2}] = 0 \). The ground state \(|0\rangle\) is completely determined by the equations \( A_{q_1} |0\rangle = \delta_{q_0} |0\rangle \) and \( B_{q_1} |0\rangle = 0 \). Single-exciton states are normalized: \(<0| Q_{q_1}^\dagger Q_{q_2}^\dagger |0\rangle = \delta_{q_1,q_2} \).

In the limit \( \Delta \to 0 \), \( H_{SO} \to 0 \) and at \( r_c \ll 1 \) the state

\[
|N; 1; q\rangle = Q_{q}^\dagger (Q_{0}^\dagger)^N |0\rangle
\]

is the eigen state of the system studied. It has the spin numbers \( S = N_\phi/2 - 1 \) and \( S_z = N_\phi/2 - 1 - N \) [see below the expressions (8) which should be used to calculate \( S \) and \( S_z \)]. The corresponding energy is \((N+1)\varepsilon_Z + \mathcal{E}_q\), where \( \mathcal{E}_q \) is the exchange part of the SE energy. The small momentum approximation \( ql_B \ll 1 \) is quite sufficient for our problem, and therefore \( \mathcal{E}_q = (ql_B)^2/2M_{x,\kappa} \) (general expressions for the 2D magneto-excitons can be found in Ref. 11). Here \( M_{x,\kappa} \) is the SE mass at \( \nu = 2\kappa+1 \), namely: \( 1/M_{x,0} = (e^2/\varepsilon l_B) \sqrt{\pi}/8, 1/M_{x,1} = 7/4M_{x,0}, \ldots \)

Spin operators in terms of the ER are invariant with respect to \( H_{SO} \):

\[
\hat{S}_z = N_\phi (A_0 - B_0) /2, \quad \hat{S}_- = N_\phi^{1/2} Q_0^\dagger, \quad \hat{S}^2 = N_\phi Q_0^\dagger Q_0 + \hat{S}_z^2 + \hat{S}_z, \quad (8)
\]

At the same time in the basis (4) the operators \( \varphi(r) \) and \( H_{int} \) acquire corrections proportional to \( u \) and \( v \). Specifically, calculating \( \int \Psi^\dagger \varphi(r) \Psi d^2r \), where \( \Psi = \sum_p (a_p \Psi_{\kappa pa} + b_p \Psi_{\kappa pb}) \), we get the terms responsible for a spin-flip process:

\[
\varphi = N_\phi^{1/2} l_B \sum_q \varphi(q) (iuq_+ - vq_-) Q_q + \text{H.c.} \quad (\text{at} \quad ql_B \ll 1). \quad (9)
\]

Here \( \varphi(q) \) is the Fourier component [i.e. \( \varphi = \sum_q \varphi(q) e^{iqr} \)], and \( q_\pm = \mp i(q_x \pm iq_y)/\sqrt{2} \).

We stress one essential feature of the states (7). In spite of the existence of a formal operator equivalence \( Q_0^\dagger \equiv \lim_{q \to 0} Q_q^\dagger \) we find that \(|N; 1; 0\rangle\) and \( \lim_{q \to 0} |N; 1; q\rangle \) present different states. Indeed, in these states the system has different total spin numbers \( S = N_\phi/2 \) and \( S = N_\phi/2 - 1 \) respectively. \( [S_z = N_\phi/2 - 1 - N \text{ is the same for both.}] \) So, the excitation of a
“zero” exciton (with zero 2D momentum) corresponds to the transition \( S_z \rightarrow S_z - 1 \) without any change of the total \( S \) number, but each “nonzero” SE changes the spin numbers by 1: \( S \rightarrow S - 1, \ S_z \rightarrow S_z - 1 \). Let us introduce the notation \( |N\rangle = \left( Q_0^\dagger \right)^N |0\rangle \). The initial state \(|i\rangle\) in the ER is actually a “Goldstone condensate” (GC) containing \( N \) zero excitons: \(|i\rangle = |N\rangle\).

Our goal is to study the process of the GC break-down. The state \(|N\rangle\) is certainly degenerate and we will solve the problem in terms of the quantum-system transitions within a continuous spectrum. The transition probability is determined by the Fermi Golden Rule: \( w_{fi} = \frac{(2\pi/\hbar)|\mathcal{M}_{fi}|^2}{\delta(E_f - E_i)} \). In our case evidently the final state \(|f\rangle\) is obviously the state where a part of the Zeeman energy has been converted into an exchange energy. Since it is exactly the single-electron terms that constitute the perturbation responsible for the \( \mathcal{M}_{fi} \) matrix element, we could find that such a transition is the \( 2X_0 \rightarrow X_{q^*} \) process in the lowmost order of the perturbative approach (we denote the zero exciton by \( X_0 \) and the nonzero one by \( X_q \)). In other words the final state for this transition is \(|f\rangle = |N-2; 1; q^*\rangle\). The value \( q^* \) is determined by the energy conservation equation \( E_f = E_i \) which reads \( 2\epsilon_Z = \epsilon_Z + \mathcal{E}(q^*) \), i.e. \( q^* = \sqrt{2M_{x,y}^e\epsilon_Z}/l_B \).

We point out that the SO interaction (2) alone does not provide a quantum fluctuation from the GC to any state with a different electron density (in particular, to the state \(|N-2; 1; q^*\rangle\)). This feature may be verified by means of direct analysis of the SO influence on the 2DEG spectrum (c.f. Ref. 4). Besides, this can be recognized from general considerations. Indeed, each of the components \( \hat{q}_i \) \((i=x,y)\) commutes with any component of the operator \( \hat{P} = \sum_j [\hbar\dot{\hat{q}}_j - (e/c)B \times \hat{r}_j] \). The latter commutes with the Hamiltonian \( \sum_j \hbar^2 \hat{q}_j^2/2m^*_e + H_{\text{int}} \) and one can also check that \( \left[ \hat{P}, Q_q^\dagger \right] = q Q_q^\dagger \) [if \( Q_q^\dagger \) is defined by Eq. (5) to within the zero order in \( H_{SO} \)]. The operator \( \hat{P} \) hence plays the role of 2D momentum in the magnetic field.\(^15\) Its permutability with the total 2DEG Hamiltonian reflects the spatial homogeneity of the system under consideration in the “clean limit”. Since \( [H_{SO}, \hat{P}] \equiv 0 \), the SO coupling by itself cannot destroy the homogeneity in any order of the perturbation approach.

The transition is thereby determined by the operator (9). The corresponding matrix element \( \mathcal{M}_{if} = \langle (q^*; 1; N-2|\mathcal{P}|N)\rangle [R(N)R(N-2; 1; q^*)]^{-1/2} \) was actually calculated in Ref. 4. (The SRP plays the same role as the phonon field studied there.) Here and in the following the notation \( R(\ldots) \) stands for the norm of the state \(|\ldots\rangle\).\(^14\) The result is \( |\mathcal{M}_{if}|^2 = \frac{N(N-1)}{2N_o} (u^2 + v^2) |q^*l_B\mathcal{P}(q^*)|^2 \), and we obtain the rate of the \( i \rightarrow f \) transition.
\((2\pi/\hbar) \sum_\mathbf{q} |\mathcal{M}_{ij}|^2 \delta(q^2l_B^2/2M_{x,\kappa} - \epsilon_Z) = N(N-1)/\tau N_\phi \quad (N \geq 1), \) where

\[ 1/\tau = 8\pi^2(\alpha^2 + \beta^2)M_{x,\kappa}\epsilon_Z \overline{K}(q^*)/\hbar^3 \omega^2l_B^4. \] (10)

Here \( \overline{K} \) stands for the Fourier component of the correlator: \( \overline{K}(q) = L^2 |\overline{\varphi}(q)|^2/4\pi^2. \)

The quantum transition to the state \(|f\rangle = |N-2; 1, \mathbf{q}^* \rangle \) is certainly a first step in the SR process. This state is thermodynamically unstable. In a time which is much shorter than \( \tau \) it turns to a state \(|N-2; 1, \mathbf{q}_0 \rangle \), where \( q_0 \) takes the lowest possible nonzero value. In fact, relevant values of \( q_0 \) are determined by the SRP field. The SE interaction with the SRP incorporates the energy \( U_{x\text{-SRP}} \sim q^2l_B^2\Delta/\Lambda \) (the “nonzero” SE possesses the dipole momentum \( el_B^2[\mathbf{q} \times \mathbf{\hat{z}}] \), see Ref. 11). The latter determines the inhomogeneous uncertainty of the SE momentum \( \delta q \sim M_{x,\kappa}\Delta/\Lambda \) (this follows from the equation \( \delta q \partial \mathcal{E}_\mathbf{q} / \partial q = U_{x\text{-SRP}} \)). Therefore “quasi-zero” wave numbers in the range defined by inequalities

\[ 0 < q_0 \lesssim M_{x,\kappa}\Delta/\Lambda, \] (11)

present the lowest physical limit for momenta of the nonzero SEs. Under the conditions (3) we find that \( q_0 \ll q^* \), and the SE exchange energy \( (q_0l_B^2)^2/2M_{x,\kappa} \) is smaller than the value \( T^*=M_{x,\kappa}(\Delta l_B/\Lambda)^2 \). This in turn is negligible in comparison with \( \epsilon_Z \).

To solve the problem in a complete form we have obviously to study the general state of the type:

\[ |N; M_1, M_2, \ldots, M_K \rangle = (\mathcal{Q}^\dagger_{\mathbf{q}_01})^{M_1}(\mathcal{Q}^\dagger_{\mathbf{q}_02})^{M_2} \ldots (\mathcal{Q}^\dagger_{\mathbf{q}_0K})^{M_K} |N\rangle. \] (12)

All the wave-vectors \( \mathbf{q}_0k \) are assumed to satisfy the condition (11). We will also use for this state a shorthand notation \(|N; M\rangle \), where \( M = \sum^K_k M_k \) is the total number of the nonzero SEs. If \( M \gg 1 \), we assume that \( 1 \ll K \ll N_\phi \). In the framework of our approach the state (12) is an approximate eigen state of a QHF having energy \((N+M)\epsilon_Z\) and spin numbers \( S_z = N_\phi/2 - N - M \) and \( S = N_\phi/2 - M \). This value of \( S_z \) is the exact one. It can be calculated employing the representation of Eqs. (8) and the commutation rules (6). The same algebra allows us to find that \( \hat{S}|N; M\rangle = S(S+1) (|N; M\rangle + |\bar{\varepsilon}\rangle) \), where the norm of the state \(|\bar{\varepsilon}\rangle \) is small compared with the norm of \(|N; M\rangle \), namely: \( R(\bar{\varepsilon})/R(N; M) = \mathcal{O}(m^3 n/K) \). The notations \( n = N/N_\phi \) and \( m = M/N_\phi \) are used for “reduced” quantum numbers. In the special case when \( N = 0 \), the state \(|0, M\rangle \) can be treated as a “thermodynamic condensate” (TDC) which arises if \( M \) is larger than the critical number of nonzero SEs. The latter is estimated
at \( N_\phi B \int \frac{q^2}{2\pi} \{ \exp \left[ \left( E_q + |U_{\alpha\beta}| \right) / T \right] - 1 \} \) (e.g., c.f. Ref. 4), and in our case (3) it is at least smaller than \( N_\phi M_{x,y} T \). At the same time, \( M \) is determined by the spin \( S \) of the system, therefore at a given \( M=N_\phi / 2 - S > N_\phi M_{x,y} T \) we find that below some threshold temperature the nonzero SEs necessarily form a TDC. For macroscopically large \( N \) and \( M \) the state (12) hence features a coexistence of GC and TDC. It should also be noted that specific values \( q_{0k} \) as well as specific distribution given by \( M_k \) numbers have no physical meaning. The final results should not depend on them but only on \( M \) and \( N \).

We can now write the kinetic equations corresponding to the relevant spin transitions. The rate \( dN/dt \) is determined by the \( 2X_0 \rightarrow X_q^* (\rightarrow X_q) \) process (which presents a GC depletion with a simultaneous “flow” to TDC) and by the \( X_0 + X_q \rightarrow X_q^* (\rightarrow X_q) \) one. The rate \( dM/dt \) is also formed by the \( 2X_0 \rightarrow X_q^* (\rightarrow X_q) \) transition (which provides a TDC evolution) and by the \( X_q + X_0 \rightarrow X_q^* (\rightarrow X_q) \) one (determining a TDC depletion). [Values of \( q_0, q'_0 \) and \( q_0'' \) belong to the region (11).] The corresponding equations are derived again with help of the Fermi Golden Rule and Eqs. (6) (with vanishing \( \theta_{12} \) in the latter):

\[
\frac{dn}{dt} = -(2\mu_{nn} + \mu_{nm})/\tau, \quad \text{and} \quad \frac{dm}{dt} = (\mu_{nn} - \mu_{mm})/\tau, \tag{13}
\]

where

\[
\mu_{nn} = \frac{|\langle M; N-2|Q_q^* Q_{-q^*}|N; M \rangle|^2}{R(N; M)R(N-2; M+1; q^*)} = \frac{N^4 R(N-2; M+1; q^*)}{N^2 R(N; M)} \left[ 1 + O \left( \frac{m}{nN_\phi} \right) \right],
\]

\[
\mu_{nm} = \sum_k \frac{|\langle M_1; ..., M_{k-1}, ..., M_{M}; N-1|Q_{q^* q_{0k} q_{0}} Q_{-q^*}|N; M \rangle|^2}{R(N; M)R(N-1; M; q^*)} = \frac{4M^2 N^2 R(N-1; M; q^*)}{N^2 R(N; M)} [1 + O (K/N_\phi)],
\]

\[
\mu_{mm} = \sum_{k<i} \frac{|\langle M_1; ..., M_{k-1}, ..., M_{i-1}, ..., M_{M}; N|Q_q Q_{-q^* q_{0k} q_{0}} Q_{-q^*}|N; M \rangle|^2}{R(N; M)R(N; M-1; q^*)} = \frac{2M^4 R(N; M-1; q^*)}{N^2 R(N; M)} [1 + O (K/N_\phi)]
\]

\([R(N; M+1; q^*)] \) is the norm of the \( Q_{q^*} |N; M \rangle \) state. In this way we find that the norms appearing in these equations satisfy the conditions \( R(N; M+1; q^*)/R(N; M) = r \), \( R(N+1; M)/R(N; M) = N r_n \) and \( R(N; M_1; ..., M_{k+1}; ..., M_{M})/R(N; M) = M_k r_m \), so that

\[
\mu_{nn} = n^2 r/r_n^2, \quad \mu_{nm} = 4mn r/r_n r_m, \quad \mu_{mm} = 2m^2 r/r_m^2, \tag{14}
\]

where the factors \( r, r_n, r_m \) are determined by the equations

\[
1 = (1 - n - 2m)/r_n + O(m^3/nK), \quad 1 = (1 - 2n - 2m)/r_m + n^2/r_n^2 + O(m^2), \tag{15}
\]

\[
1 = (1 - 2n - 2m)/r + 4mn r/r_n r_m + n^2/r_n^2 + 2m^2/r_m^2 + O(1/N_\phi).
\]
(Positivity of $r_n \approx 1 - n - 2m$ provides the physically obvious requirement $|S_z| < S$.) The last terms in Eqs. (15) would just depend on the specific set of the $M_k$ numbers. We can therefore calculate $r_m$ and obtain the final result only in the $m \ll 1$ case. Meanwhile, the values of $m(t)$ are determined by the initial value $n(0)$. According to Eqs. (13)-(15) $\max(m) \approx n(t^*) [1 - n(t^*)] / \sqrt{2}$ ($t^*$ is the time at which $m$ peaks), i.e. at least $m^2 < 1/32$. [In particular, at $n(0) = 0.5$, $\max(m) \approx 0.1$.] The problem (13)-(15) is thus solved to the leading order in $m$. We should put $r = r_n = r_m = 1$ in $\mu_{mn}$ and $\mu_{mm}$, but $r_n = 1 - n$ and $r = (1 - n)^2$ in $\mu_{nn}$. This yields the analytical result $n(t) = 1/[2n(0)(t/\tau)^2 + 2(t/\tau) + 1/n(0)]$ and $m = n(t)n(0)t/\tau$.

The dependences are shown in Fig. 1. The vector $\mathbf{S}(t)$ at moments $t = 0, \tau, 2\tau, ...$ is depicted in the inset. The time (10) hence governs the breakdown of the GC. (Under the realistic conditions above, $\tau \sim 10^{-8} - 10^{-7}$ s.) However the SR occurs certainly non-exponentially and the actual time is increased by a factor of $N_\phi/\Delta S_z(0)$.

Experimentally, the GC could probably be created by microwave pumping at the electron frequency $\epsilon Z/\hbar$. This should cause to “rotate” the QHF spin without changing of the $\mathbf{S}$ modulus. As to observing of the SR variation with time, one can think that the optical technique$^2$ is relevant in the case.

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\[16\] The operator $\hat{P}$ appeared as the momentum operator of a magneto-exciton in the work of L.P. Gor’kov and I.E. Dzyaloshinskii in Sov. Phys. JETP, \textbf{26}, 449 (1968). To calculate the commutator with operator $Q_q^\dagger$ we choose the Landau gauge [$A = (0, Bx, 0)$] and present $\hat{P}$ in terms of the secondary quantization: $\hat{P}_x = (2\pi i \hbar /L) \sum_{\kappa, p, q, \sigma} \delta'(q) a_{\kappa, p - q / 2, \sigma}^\dagger a_{\kappa, p + q / 2, \sigma}$ and $\hat{P}_y = (\hbar /l_B) \sum_{\kappa, p, \sigma} p a_{\kappa, p, \sigma}^\dagger a_{\kappa, p, \sigma}$. ($\delta'$ is the derivative of the $\delta$-function, the operator $a_{\kappa, p, \sigma}$ corresponds to the Landau-gauge state $|\kappa, p, \sigma\rangle$).
FIG. 1. Time dependences of $|\Delta S_z|/N_\phi=n(t)+m(t)$ and of $|\Delta S|/N_\phi=m(t)$ are shown in the main picture for $n(0)=|\Delta S_z(0)|/N_\phi=0.455$. The vectors $S(t)$ at equidistant moments of time are plotted in the inset with step $\tau$. The dotted line is the arc with radius $S_0=N_\phi/2$. The gap between the dashed and dotted lines reflects the deviation of the spin modulus from the value of $S_0$. 