Modeling Fatigue Damage Accumulation in Power Engine Structures

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Abstracts. The article describes the development of a new design technology for power engine structures (PES) hulls of increased reliability, based on the methods of system analysis using computer modeling of damage accumulation in structural elements, as well as the development of technical solutions for creating the PES hull design using new heat-resistant composite carbon-carbon and carbon-ceramic materials. To calculate the accumulation of damage, the chemical criteria of durable strength is used, for numerical calculation of the stress-strain state of structures taking into account creep, an iterative method for solving a three-dimensional problem of thermomechanics using the finite element method is applied. An example of the calculation of a high-pressure structural element, performed in the work, showed the possibility of practical implementation of the proposed technique and its effectiveness, in terms of reducing or replacing experimental studies to substantiate the reliability of structures.

1. Introduction

In connection with the current work on the implementation of the government program for the development of a promising space nuclear power plant, research has been actively begun to ensure the reliability of the installation being created as part of the space station. The object of this study is the reliability and durability of the hull of the power engine structures (PES). High environmental requirements for the operation of facilities of this type, high financial risks caused by the consequences of making incorrect technical decisions – all this indicates the extreme importance of the problem of ensuring the reliability of the PES. The situation is complicated by the fact that to ensure the reliability of such a system, it is not possible to apply traditional methods based on the existence of a representative statistical sample of data on the performance of a large batch of products [1-12]. In these conditions, it is necessary to develop a fundamentally new approach to creating a methodology for evaluating and predicting the reliability of PES. The system approach proposed in this paper is based on a new concept for substantiating the reliability of well-structured systems-by conducting a series of numerical supercomputer experiments to model the durability of PES elements using experimental data on the properties of materials. Substantiating reliability in the framework of this concept is through the use of complex mathematical models [12-17], describing the prolonged physical and mechanical behavior of materials under operating conditions of PES, as well as complex mathematical models to predict the structural changes limit the strength and durability of hull structural materials under long-term cyclic thermal loading.

2. Mathematical formulation of the problem

When creating a high-pressure vessel (HPV), metal materials and alloys are mainly used, as well as non-metallic materials as part of electrical insulation compounds, nodes for sealing cable routes and fixing electrical elements; fuel and absorbing compositions; winding wires with mineral and polymer insulation. Almost all of these materials are isotropic [14-19].

We will assume that the theory of small deformations is valid, for which there is an additive summation of the deformations of elasticity, plasticity and creep:

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_c + \varepsilon^0$$ (1)
Where \( \mathbf{\varepsilon}, \mathbf{\varepsilon}_e, \mathbf{\varepsilon}_p, \mathbf{\varepsilon}_c \) – the tensor of total deformation, elastic deformation, plasticity deformation and creep deformation; \( \mathbf{\varepsilon}^0 \) – the tensor of the thermal deformation.

Then, to calculate the stress-strain state (SSS) of the HPV element, we can formulate a problem of mechanics of a deformable solid body within the framework of the combined thermo-elastic-plasticity and thermal creep model, consisting of equations of mechanical equilibrium, determining relations, Cauchy relations, boundary and initial conditions. In tensor form, this problem has the following form [13, 14]:

\[
\nabla \cdot \mathbf{\sigma} + \rho \mathbf{f} = 0, \quad \mathbf{\varepsilon} = \frac{1}{2} \left( \nabla \otimes \mathbf{u} + \nabla \otimes \mathbf{u}^T \right);
\]

\[
\mathbf{\sigma} = \left( \lambda I + 2 \mu I \right) \left( \mathbf{\varepsilon} - \mathbf{\varepsilon}_c \right) - \left( 3 \lambda + 2 \mu I \right) \mathbf{I} \mathbf{\varepsilon}^0 \mathbf{E} + 2 \mu \left( \mathbf{\varepsilon} - \mathbf{\varepsilon}_p - \mathbf{\varepsilon}_c \right); \quad (2)
\]

\[
\dot{\mathbf{\varepsilon}}_p = \frac{\dot{\mathbf{h}}}{\sigma_3^2} \left( \mathbf{\sigma} - \mathbf{H} \mathbf{\varepsilon}_p - \frac{1}{3} I \left( \mathbf{\sigma} - \mathbf{H} \mathbf{\varepsilon}_p \right) \mathbf{E} \right), \quad \mathbf{\varepsilon}_c = \frac{1}{\eta} \left( \frac{\mathbf{\varepsilon}_u}{\sigma_u} \right)^7 \left( \mathbf{\sigma} - \frac{1}{3} I \left( \mathbf{\sigma} \right) \mathbf{E} \right)
\]

\[
\mathbf{\sigma} \mid_{\mathbf{\varepsilon}_c} \cdot \mathbf{n} = \mathbf{S}_e, \quad \mathbf{u} \mid_{\mathbf{\varepsilon}_c} = \mathbf{u}_e, \quad t = 0: \mathbf{\varepsilon}_c = 0, \mathbf{\varepsilon}_p = 0.
\]

Where \( \mathbf{\sigma} \) – the Cauchy stress tensor; \( I_1 (\mathbf{\sigma}) \) – the first invariant of the tensor; \( \sigma_u \) – the second invariant of the tensor [15]; \( \mathbf{Y} \) – the intensity of the strain tensor creep; \( \lambda, \mu \) – Lame constants; \( \mathbf{u}_e \) – vector of set displacements on a part of the surface \( \Sigma_u \); \( \mathbf{S}_e \) – vector of set forces on a part of the design boundary \( \Sigma_e \) (as a rule, this is the set pressure \( \mathbf{S}_e = -p \mathbf{n} \)); \( \mathbf{f} \) – the vector of density of mass forces; \( \rho \) – density of construction material; \( \nabla \) – differential nabla-Hamilton operator; \( \otimes \) – sign of the tensor product; \( \phi \) – sign of the scalar product.

### 3. Chemical criterion of durable strength

The process of fatigue crack development is well studied and is random. In a continuum of PES structures can undergo hundreds of thousands of crack formation processes in series or in parallel. For an adequate study of the totality of such random events as the appearance of defects, the using system analysis methods is required.

The accumulation of damage in the structural materials of the HPV is taken into account using the damage parameter \( z(t) \), which consists of two parts – reversible and irreversible damage [17-21]:

\[
z(t) = z_r(t) + z_d(t)
\]

Reversible damage parameter \( z_r(t) \) according to the model [2, 20] for isotropic media has the form:

\[
z_r(t) = B_0 \sigma + B_1 \sigma^2 + B_2 \sigma_c^2 + K_0 \int_0^t \frac{\sigma_r^2(\tau)d\tau}{(t-\tau)^\gamma} - T(\int_0^t \frac{\sigma_u^2(\tau)d\tau}{(t-\tau)^\beta} )^2 \quad (4)
\]

Where \( K_0, \Gamma_0, \gamma, \beta \) – damage model constants; \( \sigma = I_1 (\mathbf{\sigma}), \sigma_r, \sigma_c, \sigma_s \) – limits of static tensile, compressive and shear strength, \( B_0, B_1, B_2 \) – strength constants:

\[
B_0 = \frac{\sigma_c^2 - \sigma_r^2}{\sigma_c^2 + \sigma_r^2}, \quad B_1 = \frac{2}{\sigma_c^2 + \sigma_r^2} - \frac{1}{3 \sigma_s^2}, \quad B_2 = \frac{1}{3 \sigma_s^2}
\]

For the irreversible part of the damage \( z_d(t) \) there is following expression [19, 20]:

\[
z_d(t) = D_0 \int_0^t \sigma_u^2(\tau)d\tau - W_0 \int_0^t \sigma_u(\tau) e^{-\phi(\tau-x)} \sigma_u(x)dxd\tau
\]
Where $D_\alpha, W_0, \phi$ – constants of the model.

According to the chemical criterion of durable strength, destruction at some point $x^*$ of construction at the time moment $t^*(x^*)$ happens, if at this point the damage parameter reaches the value 1:

$$z(t^*(x^*)) = 1.$$  \hspace{1cm} (7)

For brittle materials, for which the consequence of the initial failure is almost instantaneous crack growth, leading to the complete destruction of the HPV element, the time $t^*(x^*)$ is the desired durability of the HPV.

The damage accumulation model makes it possible to predict not only the durability, but also the preservation of the tightness of structures, the violation of which is caused by the development of micro-damage in materials. The model takes into account creep deformations of structural materials.

The chemical criterion of durable strength for cyclic loading and for block loading is given. In real operating conditions, loading is usually a random process. In this case, the damage parameter is already an implementation of a random process $z(t)$. Calculate the mathematical expectation and variance of the damage parameter. Due to the fact that all values, except the amplitude of oscillations $p_o$ at $z(t)$ at this stage of the method, are assumed to be deterministic, then, using the properties of the mathematical expectation of linear random functions, we obtain:

$$M(z(t)) = A_0 \bar{p} + A_1(\bar{p}^2 + M(p_o^2)) + \sigma_o^2 \left( \frac{1}{2} M(p_o^2) \right) B_s(t) - \bar{p}^2 B_4(t),$$  \hspace{1cm} (8)

Where $A_0 = B_0 \sigma^0, A_1 = B_1 \sigma^2 + \sigma_o^2 B_2$, as $M(p_o) = 0$.

Similarly, we calculate the variance of the damage parameter:

$$D(z(t)) = M((z-M(z))^2) = M\left( A_0 p_o + A_1(p_o^2 + 2p_o \bar{p} - M(p_o^2)) + \frac{1}{2} B_s(p_o^2 - M(p_o^2))^2 \right)$$

$$= A_0 M(p_o^2) + A_1^2 M(p_o^4) + 4M(p_o^2)\bar{p}^2 - M^2(p_o^2) +$$

$$+ B_s(t)(\frac{1}{4} B_s(t) + A_1(M(p_o^4) - M^2(p_o^2)) + 4A_0 A_1 M(p_o^2))\bar{p}. \hspace{1cm} (9)$$

Assuming that the density of the distribution of the damage parameter corresponds to the normal law:

$$f(z(t)) = \frac{1}{\sqrt{2\pi D(z(t))}} \exp\left((-z(t) - M(z(t)))^2 / D(z(t))/2\right),$$  \hspace{1cm} (10)

obtain that the maximum of random values of the damage parameter is achieved when $z_{max} = M(z)$. If $z_{max}$ reaches the value 1 at some point in time $\bar{T}^*$ (i.e., when the condition is met $z_{max}(\bar{T}^*) = M(z(\bar{T}^*)) = 1$), then the structural element is destroyed at this point. This moment of time $\bar{T}^*$ we call the average durability of the PHP element at a given level of random loads, it, generally speaking, does not coincide with the mathematical expectation of random durabilities $M(t^*)$, since the dependence $z(t^*)$ is nonlinear $M(z(t^*)) \neq z(M(t^*))$. The average durability $\bar{T}^*$ is calculated using the equation:

$$M(z_{max}(\bar{T}^*)) = 1,$$  \hspace{1cm} (11)

substituting in which expression (8), we obtain the equation for the average durability $\bar{T}^*$:

$$A_0 \bar{p} + A_1(\bar{p}^2 + M(p_o^2)) + \sigma_o^2 \left( \frac{1}{2} M(p_o^2) \right) B_s(\bar{T}^*) - \bar{p}^2 B_4(\bar{T}^*) = 1.$$  \hspace{1cm} (12)

This equation sets the average fatigue curve of the PHP element under random loading – in the form of a dependence $\bar{p}$ on $\bar{T}^*$. By entering the average value of the maximum load in the oscillation cycle $\bar{p}_m = \bar{p} + \sqrt{M(p_o^2)}$ and the average coefficient of asymmetry of the oscillation cycle $\bar{k} = (\bar{p} - \sqrt{M(p_o^2)}) / \bar{p}_m$, equation (12) can be written in the form typical of fatigue theory:

$$A_0 \bar{p}_m(1 + \bar{k}) + A_1 \bar{p}_m^2(1 + \bar{k}^2) + \sigma_o^2 \bar{p}_m^2 B(\bar{T}^*, \bar{k}) = 2,$$  \hspace{1cm} (13)
Where \( B(\Bar{\tau}^*,\Bar{k}) = \frac{1}{4} (3 + 2\Bar{k} + 3\Bar{k}^2) B_1(\Bar{\tau}^*) - \frac{1}{2} (1 + \Bar{k})^2 B_4(\Bar{\tau}^*) \), coefficients \( \sigma^0_0, \sigma^0_\mu \) are actually invariants of the stress concentration tensor and characterize the degree of inhomogeneity of the stress state of the HPV element.

assuming the damage parameter in equation (10) to be equal to 1: \( z(t^*) = 1 \), we obtain the density of the durability distribution \( t^* \):

\[
    f(t^*) = \frac{1}{\sqrt{2\pi} D(z(t^*))} \exp\left(-\frac{(1 - M(z(t^*)))^2}{D(z(t^*)) / 2}\right),
\]

(14)

Where \( M(z(t^*)) \) and \( D(z(t^*)) \) expressed by the formulas (8), (9), moreover, the argument \( z(t^*) = 1 \) for these functions is different from 1, and changes with the change \( t^* \).

The distribution \( f(t^*) \) is already different from normal. Analytical expressions (8-10), (14) make it possible to significantly reduce the volume of numerical calculations of durability, abandoning the calculations of the SSS of the elements of the HPV for different moments of time

4. Modeling

Based on the results of calculating the implementations of the random variable – durability \( t^* \), at different values of the parameters of structural materials and loads \( E_0, \sigma_T, K_0, \Gamma_0, p_0 \), the distribution density \( w(t^*) \) of the random variable of failures was calculated (Figure 1). Then function of the reliability of the PES hull structure was calculated (Figure 2).

**Figure 1.** Calculated failure distribution density of the PES hull structure (durability distribution density), from the results of numerical simulation

**Figure 2.** Calculation function of the reliability of the PES hull structure, from the results of numerical simulation
Based on the results of statistical modeling, you can find the values of the reliability indicators of the hull structures: the probability of failure-free operation during continuous operation for 10 years; the average operating time to failure; gamma-percent uptime.

5. Conclusion
Reliability and durability indicators of the high pressure hull of a nuclear power engine system are obtained. The scientific results of the theory of reliability and mechanics of a deformable solid are combined by a common methodology that allows, based on the processing of information about the elastic-strength characteristics of PES structural materials, to derive their optimal values for a given model of hull elements. The proposed method can be applied in the calculation of the strength and durability of PES hulls as macrosystems consisting of a shell and internal devices. The new system methodology does not require the use of large amounts of data on failures, so it can be used for nuclear, aviation and space technology.

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